Development of Novel Air-knife System to Prevent Check-mark Stain on Galvanized Strip Surface

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When galvanized steel strip is produced through a continuous hot-dip galvanizing process, the thickness of the adhered zinc film is controlled by normally impinging a thin plane nitrogen gas jet. In such a gas wiping process, frequently there appears stain of check-mark, hereinafter called “check mark stain”. The check mark stain is due to non-uniform zinc coating over the surface of the steel strip. Presence of such check mark stain lowers the quality, productivity and profitability of the end products. From our proceeding research, it was found that there are alternating stream-wise vortices impinging on the steel strip that move almost periodically to the right and to the left along the stagnation line due to the jet flow instability. This instability is closely related to buckling of the center sheet of the plane jet. Since higher stagnation pressure removes more molten zinc adhered on the surface, the zinc coating thickness is thinner at the high pressure point. In addition, since the strip moves upward with a constant speed, the non-uniform coating surface is formed with a variety of patterns like “W”, “V” and “X”.

In the present study, in order to avoid the appearance of the check-mark stain, a new type of the air-knife system is proposed. This system consists of the main jet and a guide jet located beneath the main jet. The main jet removes the molten zinc and the guide jet prevents the formation of alternating impinging vortices on the steel strip so that it suppresses the cause of the check-mark stain. The design concept of the proposed air-knife system is verified by investigating the 3-D turbulent flow in the impingement jet region obtained numerically by using a commercial code FLUENT. Large eddy simulation (LES) technique is used to solve the governing conservation equations of mass, momentum and heat for 3-D compressible turbulent flow field.

KEY WORDS: slot impinging jet; galvanized steel; check-mark stain; double air-knife; buckling of the jet; LES.

1. Introduction

Since the galvanized steel strip has good corrosion resistance and good characteristics for painting, welding and manufacturing, it is widely used in a variety of industrial fields such as civil engineering, automobile, shipbuilding and home appliance industries. It is produced by a continuous hot dip galvanizing process. A heat-treated steel strip is passed through a molten zinc bath maintained at high temperature and is drawn up with the molten zinc adhered on both strip surfaces as shown in Fig. 1. Usually the thickness of the adhered zinc film is about 50 times thicker than desired thickness. In order to remove the excessive zinc, the steel strip with molten zinc on its surfaces is vertically drawn up between a pair of two-dimensional, horizontal opposing plane gas jets. In this way the excessive molten zinc is wiped down by the impinging gas jets. This wiping mechanism has been well investigated theoretically and experimentally.1-4) The facility of such a nitrogen gas jet is called the air-knife system and the manufacturing method is called the gas wiping process. This method which was put to practical use in the mid-twenty century is widely used in most of the commercial plants because of good productivity and easy control of the zinc coating thickness. However, such a gas wiping process has two technical problems concerning the uniformity of the coated film thickness. One is the edge over-coating (EOC) problem in the strip edge re-
ions and the other is the check-mark stain problem in the interior region. The edge over coating problem is that the thickness of the zinc film near the strip edge regions is 1.4–1.8 times thicker than that on the surface of the interior strip region. The principal cause of the EOC was found by Kim et al. They simulated numerically the turbulent flow field around the gas wiping region formed by the two impinging, opposing jets and revealed that due to the oscillation of the opposing jets in the outside region of the strip edge, the static pressure decreases significantly toward the strip edge, which dramatically reduces the zinc wiping force. After this research, Ahn et al. suggested a feasible solution for EOC by using the Coanda flow effect to remove the oscillation of the opposing jets outside of the strip edge.

The check-mark stain is the oblique pattern that appears on the coated film surface after the gas wiping process. Depending on its seriousness, the check-mark stain is classified into four grades as shown in Fig. 2. The first grade has no stain on the surface. The surface pattern of the 2nd grade check-mark stain has surface defects such as irregular points or lines. On the other hand, the 3rd–4th grade check-mark stains have surface defects of oblique patterns such as “W”, “V” or “X”. The check-mark stain causes a number of technical problems; for example, non-uniformity of the electrical and thermal characteristics and the diffused reflection on the coated surface. When such defects are serious, the zinc-coated steel strip cannot be used in manufacturing the end products.

When a high speed plane jet collides with a plate normal to the jet axis, buckling of the plane jet axis is observed, and a non-uniform and unstable flow field is formed near the jet-plate impinging region. In our preceding numerical investigation of the 3-D turbulent flow field in the impingement region, it was found that along the span-wise direction of the plane jet there are a series of longitudinal vortices that impinge normally on the strip surface, that the strip surface pressure at the center of the vortices is low whereas it is high at the periphery of the impinging vortices, that these vortices move to the right and to the left periodically, and finally that the combination of these periodic span-wise movement of the vortices and the upward feeding of the steel strip results in such check-mark stain on the coated surface.

In the present study, in order to avoid the appearance of the check-mark stain and improve the cutting ability, a new type of the air-knife system is proposed. The new air-knife system consists of a main plane jet and a guide jet located beneath the main jet. Hereinafter the new system is referred to as a double air-knife system. The major function of the main jet is to remove the molten zinc. On the other hand, the purpose of the guide jet is to prevent the buckling of the plane jet and to suppress the periodic sideways movement of the longitudinal vortices in the main jet. In the double air-knife system, therefore, it is expected that the flow field of the jet becomes more stable and that the alternating vortices are no longer generated on the stagnation line.

In order to verify the present design concept of the double air-knife system based on the fluid dynamic point of view, the 3-D unsteady compressible flow is studied numerically by using the Large Eddy Simulation technique. Smagorinsky–Lilly SGS (sub-grid scale) model is used in the present LES study. Performance of the double air-knife system is to be compared with the existing air-knife system, based on simulation results such as static pressure, velocity and vorticity fields near the air-knife and on the steel surface.

2. Geometry and Boundary Conditions of the Flow Field Around the Air-knife System

Since the Mach number of the plane impinging jet is around 0.3–0.8 in the gas wiping process, the flow is evidently compressible, and thus the unsteady 3-D compressible turbulent flow is numerically simulated by using the commercial code, FLUENT to investigate the characteristics of the flow field. Boundary conditions and the computational domain of the gas wiping system are shown in Fig. 3.

The symbols $d_{01}$ and $L$ in Fig. 3(a) represent the lip width of the air-knife and the distance from the exit of the air-knife to the steel strip, respectively. The symbols $d_{02}$, $d_{03}$ and $d_{04}$ in Fig. 3(b) are the lip widths of the main, guide jets and distance from the main jet to the guide jet, respectively. $\theta$ is the subtended angle between the main jet and guide jet. Here, a base line of the subtended angle parallels the guide.

![Fig. 2. Photograph of check-mark with grade. (a) 1st grade, (b) 2nd grade, (c) 3rd grade, (d) 4th grade.](image-url)
jet. At this time, the same distance \( L \) is used for both the existing air-knife system and the double air-knife system.

The inside stagnation pressure \( (P_0) \) and temperature \( (T_0) \) of the air-knife are given with the respective values under which the 3rd grade check-mark stain appears by the actual existing air-knife system. Working fluid is nitrogen gas and 25 kPa (gauge pressure) and 340 K are given as the inside stagnation pressure and temperature, respectively.

The geometrical sizes of the existing and double air-knife systems are described in Table 1. Case 1 indicates the existing air-knife system and Case 3 is the double air-knife system. Meanwhile, Case 2 has the geometrically same shape as Case 1, but its lip width \( (d_{20}) \) is the sum of the main \( (d_{22}) \) and guide \( (d_{23}) \) jets in Case 3. If the flow field of the impinging jet in Case 2 is stable, double air-knife system is no longer needed. Thus, the flow characteristic of the double air-knife system is compared with Case 2. The surface of the steel strip is treated as a wall boundary moving at a speed of 2.5 m/s in \( y \)-direction at \( x/H_1 = 1005 \).

Large Eddy Simulation (LES) technique is used to produce the instantaneous flow data of the impinging jet. The smallest grid size is taken to be 0.1 mm in the jet-plate impinging region. In the \( z \)-direction, the grids are uniformly distributed, whereas in the \( y \)-direction, the first grid spacing is 0.1 mm near the impinging region and it is increased with a successive rate in the \( y \)-direction. Using this grid spacing strategy, about a total of three millions structured grids were formed with finer meshes in the impinging region as shown in Fig. 3. Time step of the unsteady solver was taken to be \( \Delta t = 5.0 \times 10^{-7} \) s, which is about 1/80 of the buckling period of the plane jet. The maximum Courant number of this grid system is about 1.0 under these calculation conditions. The PISO algorithm was used as a pressure–velocity coupling and the bounded central differencing method was employed for a discretization method, which has been appropriately proved for LES technique.

### 3. Flow Field Analysis of the Existing Air-knife System Concerning the Check-mark Stain

Simulation results of Case 1 in Table 1 will be explained in the first place. Figure 4 shows the velocity and static pressure contours of the nitrogen gas flow on the \( xy \)-plane at the middle of the steel strip. The maximum velocity of the jet is about 230 m/s obtained by the simulation. It is favorably compared with 232 m/s obtained by using the isentropic compressible gas equation, Eq. (1). \(^{15} \)

\[
\frac{P_0}{P} = \left( 1 + \frac{\gamma - 1}{2} \frac{V}{M^2} \right) \frac{T_0}{T} = \left( 1 + \frac{\gamma - 1}{2} \frac{V}{M^2} \right),
\]

Here, the atmospheric pressure \( P \) is taken to be 101 kPa and \( \gamma = 1.4 \). In general, if a gas jet collides normally with a moving strip, the gas jet is bent toward the moving direction of the strip. \(^{16,17} \) It is known that faster the jet velocity relatively to the strip movement, smaller the bending of the gas jet near the impingement. In the present study, since the gas jet velocity is significantly high in comparison with the
moving speed of the strip, bending of the gas jet is not discernible as shown in Fig. 4. However, alternating vortices are seen to be aligned above and below the gas jet center plane, and buckling of the jet center plane can be clearly seen in Fig. 4.\textsuperscript{18,19} This phenomenon is the local 3-dimensional flow characteristics obtained by using the LES technique. Such detailed flow field could not be generated by 2-D or 3-D RANS equations employing \( k-\varepsilon \) or \( k-\omega \) turbulent model.

It is known that the buckling of the jet center plane is caused by the inertial effect. There is a pair of opposing forces in the \( x \)-direction within the jet center plane itself. One is the inertial force in the stream-wise direction and the other is the reaction force due to the jet collision on the strip. Considering Bernoulli’s equation, radial equilibrium of the jet, the bending moment equation and these two forces, the buckling wavelength, \( \lambda_b \), can be derived as follows.\textsuperscript{20}

\[
\lambda_b = 2\pi\sqrt{I/A} \quad \text{...............(2)}
\]

Here, \( A \) is the jet exit area and \( I \) is the area moment of inertia. When the exit width of the 2-dimensional air-knife is \( d \), then, \( \lambda_b = (\pi^3/6)d \).

In our Case 1, since \( d = 1.5 \) mm, the buckling wavelength is about 2.72 mm. Numerical result depicted in Fig. 5 shows that the mean length of the buckling wavelength is about 2.8 mm \pm 10\%, which is reasonably consistent with the analytic solution.

Figure 5 shows the static pressure contours on the \( xz \)-plane at \( y = 1.4 \) mm. The static pressure varies almost periodically between 12 kPa and 20 kPa along the impingement stagnation line on the surface of the steel strip. Because of this sinusoidal distribution of surface pressure, the gas moves sidewise alternatively in the inner impinging region. It is evident that more molten zinc is removed at the high pressure point than at the low pressure point.\textsuperscript{21} Now, since the steel strip moves upward at a constant speed, traces of these high pressure points on the strip surface result in the check-mark stain in a pattern like “W”, “V” or “X”.

In addition, it can be observed in Fig. 5 that the static pressure is distributed in a wavelike manner at the distance a little far from the strip. When we consider the alternating vortices aligned along the span-wise direction and the buckling of the jet center plane, such wavelike distribution of the static pressure can be easily interpreted. This observation confirms the appearance of buckling of the jet center line.

4. Flow Field Analysis of the Double Air-knife System

In the present study, a double air-knife system is designed to prevent the check-mark stain on the zinc coated strip surface. The removing force of the molten zinc is one of the most important factors in the design of the air-knife system. Non-dimensional equation that describes the relationship among the impinging static pressure, wall shear stress and the zinc coating thickness can be derived from the Navier–Stokes equation as shown in Eq. (3).\textsuperscript{21} Three assumptions are introduced to formulate the above equation. These are; flow field of the molten zinc is a steady laminar flow, the surface tension of the molten zinc is negligible, and the no-slip condition of the liquid is applicable on the steel strip.

\[
(1 + \nabla\hat{P}(Y))\hat{h}^3 - 1.5\hat{T}(Y)\hat{h}^2 - 5\hat{h} + 2Q = 0 \quad \text{........(3)}
\]

Dimensionless variables are defined as follows:

\[
h_0 = \sqrt{\frac{\mu U}{\rho_l g}}, \quad \hat{h} = \frac{h}{h_0}, \quad Y = \frac{y}{L}, \quad Q = \frac{q}{q_0},
\]

\[
q_0 = \frac{2}{3} U h_0, \quad \nabla\hat{P} = \frac{\nabla P}{\rho_l g}, \quad T = \frac{\tau_w}{\tau_0}, \quad \tau_0 = \sqrt{\mu U / \rho_l g}
\]

The molten zinc removing force of the air-knife system is related to the impinging pressure gradient, \( \nabla\hat{P}(Y) \), and wall shear stress, \( T(Y) \), on the surface of the steel strip. Performance of the air-knife systems will be explained by using the numerical simulation results and Eq. (3).

Figures 6(a), 6(b) and 6(c) display distributions of the mean static pressure, static pressure gradient (\( dp/dy \)) and wall shear stress on the steel strip, respectively. In all figures, \( y = 0 \) is taken at the lower lip of the exit of the air-knife. Spreading of the jet is predicted from the distribution of the mean impinging static pressure in Fig. 6(a). The maximum static pressure in Case 1 is about 16.5 kPa. It is about 65\% of the inside stagnation pressure of the air knife, 25 kPa. In this case, mean static pressure is decreased due to the periodical distribution of the high and low pressure point along the impinging stagnation line as shown in Fig. 5. In Cases 2 and 3, the maximum static pressures are about 22.5 kPa and 24.5 kPa, respectively. In Case 1 where the lip width of the air knife is 1.5 mm, the flow field is very unstable due to the large buckling of the jet as shown in Figs. 4 and 5. Consequently, the mean static pressure of the impinging jet is decreased significantly when it collides with the steel strip. On the other hand, in Case 3 of which lip width is 2 mm, the buckling wavelength is about 3.63 mm from Eq. (2). In this case, buckling of the jet is less developed than Case 1. Note that the non-dimensional distance from the exit of the air-knife to the steel strip, \( L/d \), is decreased even with the same distance \( L \). Then, in Case 3, the flow field is more stable than Case 1 and decrease in the...
mean pressure on the impinging region is hardly observed. Figure 7 shows the velocity and static pressure contours of the nitrogen gas flow on the $x_2$-plane at the middle of the steel strip, formed by the double air-knife jet system. In contrast to the case of the existing air-knife system as shown in Fig. 4, buckling of the jet center plane is no longer presented as shown in Fig. 7. And although high and low pressure points are observed above and below the gas jet center plane in the pressure contours of right side figure in Fig. 7, its pressure differences are much smaller than in Cases 1 and 2.

This flow feature can be used to deduce the function of the guide jet. The guide jet suppresses the buckling of the main jet, therefore the flow field becomes more stable. Wall static pressure gradient along the $y$-direction and wall shear stress on the steel strip in Fig. 6 are directly related to the zinc coating thickness. Hence, the removing force is inferred by using these graphs. Wall static pressure gradient in Case 3 is about twice larger than Case 1 and one and a half times larger than Case 2 as shown in Fig. 6(b). This is also related to the jet spreading rate. When the impinging jet region is normalized by the jet lip width, Case 1 has wider impinging jet region in comparison with Case 3 as can be seen in Fig. 6(a). This feature implies that the spreading rate of the jet of the double air-knife is smaller than the existing air-knife system. Further, in Case 3, the maximum wall shear stress is highest among the three cases as shown in Fig. 6(c). At this time, it should be noted that the wall shear stress is in direct proportion to the wall jet velocity along the strip, therefore, Case 3 in which the decrease of the static pressure is smallest shows the maximum wall shear stress. On comparing Figs. 6 and 7, it is found that the flow characteristic of the jet of the double air-knife system is significantly different from that of the existing air-knife system.

The calculation results of the zinc coating thickness are shown in Fig. 8, which are obtained by substituting the values of Figs. 6(b), 6(c) into Eq. (3). $x$-Axis represents the coating thickness in $\mu$m and $y$-axis indicates the $y$ position in mm unit from the impingement stagnation line. The computed final coating thicknesses are 24.4 $\mu$m in Case 1, 19.6 $\mu$m in Case 2 and 17.9 $\mu$m in Case 3. Consequently, it is expected that the zinc removal ability can be improved by about 25% by employing the proposed double air-knife system.

Now, let’s turn our attention to the feasibility to prevent the check-mark stain by employing the double air-knife system. As it was already discussed in Introduction of this report, prevention of the check-mark stain can be assessed by scrutinizing the variation of the static pressure distribution along the impinging stagnation line. Figure 9 demonstrates the instantaneous distribution of the static pressure along the stagnation line. Here, the static pressure distribution is
taken at \( y \) position where the mean static pressure is maximum in each case. The difference between high and low static pressure that alternatively appear along the stagnation line is largest in Case 1 among the three cases, and it is smallest in Case 3. Standard deviation of the spatially fluctuating static pressure is 3,050 Pa, 2,130 Pa and 690 Pa in Case 1, Case 2 and Case 3, respectively. Detailed static pressure contours in the impinging region are shown in Fig. 10 for Cases 1 and 3. Difference between highest and lowest static pressure in Case 3 is decreased to about 20% of Case 1. Optimistically speaking, such small difference may result in indiscernible check-mark stain on the zinc coated surface.

5. Parametric Study of the Double Air-knife System

Design variables of the double air-knife system are lip width of the main jet \( (d_{02}) \), lip width of the guide jet \( (d_{03}) \), the gap distance between the main and the guide jets \( (d_{04}) \) and the subtended angle between the main and the guide jets \( (\theta) \). The distance from the exit of the air-knife to the steel strip \( (L) \) and the inside stagnation pressure of the main jet \( (P_{02}) \) are fixed at 10 mm and 25 kPa, respectively. Meanwhile, the inside stagnation pressure of the guide jet \( (P_{03}) \) is chosen as an operating variable because the inside pressure of the main and the guide jets can be set individually. The performance of the double air-knife system depends on these geometric variables and these inside stagnation pressures. The values of these design variables are listed in Table 2. Other geometrical sizes and the inside stagnation pressure of the main jet and the stagnation temperature are identical with those in Case 3.

The same numerical method was used in this parametric study. Optimum design values are determined based on the maximum static pressure on the strip surface and the rms value of the static pressure variation along the impinging stagnation line. Here the maximum static pressure is defined as the maximum value in the distribution of the mean static pressure along \( y \)-axis on the strip surface, and its rms value is calculated among the local instantaneous values of the static pressure on the strip surface along the stagnation line. High static pressure indicates good zinc removal ability and the low rms value of the pressure fluctuation implies more stable flow field in the impinging region along the stagnation line.

Figures 11(a) and 11(b) show the maximum static pressures and rms values of the pressures along the stagnation line for all sets of the design variables listed in Table 2. First, consider the effect of the lip width of the guide jet
(d_{03}) as a design variable. It is noted in the figures that the maximum static pressure is smallest in Case 3 where d_{03}=0.5 mm. The smallest rms value of the static pressure fluctuation is 475 Pa in D3-3 where d_{03}=0.6 mm, so it has the most stable flow field.

Next, consider the effect of the gap distance between the main and the guide jets (d_{04}) on the performance of the double air-knife system. From these two figures it is found that larger gap size leads to lower impinging static pressure and larger rms value of the pressure fluctuation. The reason is explained as follows: when the gap size is large, the guide jet starts to guide the main jet at relatively far distance from the exit of the jet so that the guide effect should be small. Therefore, it is concluded that smallest gap size is desirable. However, due to the difficulty of manufacturing, a gap size of about 0.2 mm is recommended.

Another design variable is the subtended angle $\theta$ between the main and the guide jets. A total of four different angles are tested: they are 0°, 1°, 1.5° and 2°. Note that in Case 3, the main jet and the guide jet are parallel to each other. When one compares the computational results in Figs. 11(a) and 11(b), it can be found that the maximum static pressures are nearly the same except the Case 3 where the maximum static pressure is lowest. But, the rms value of the pressure fluctuation is lowest of about 497 Pa in the case of $\theta=1°$.

Finally, in order to study the effect of the inside stagnation pressure of the guide jet (P_{03}), it is varied to be 5 kPa, 15 kPa, 25 kPa, 35 kPa and 45 kPa. Note that the inside stagnation pressure of the main jet was fixed at 25 kPa. The present numerical test reveals that the maximum static pressure is highest and the rms value of the pressure fluctuation is lowest under the condition that P_{03}=15 kPa. Here, it is worth noting that the maximum static pressure does not exceed 25 kPa even in the cases where the inside stagnation pressure in the guide jet is higher than that in the main jet of P_{02}=25 kPa.

In conclusion, optimum design and operating variables are found to be d_{03}=0.6 mm, d_{04}=0.2 mm, $\theta=1°$, P_{03}=15 kPa in the double air-knife system under the condition that the lip width of the guide jet (d_{02}) is 1.5 mm and the inside stagnation pressure of the main jet (P_{04}) is 25 kPa. With these optimized conditions for the double air-knife system, the maximum static pressure is 24.9 kPa and the rms value of the pressure fluctuation is 360 Pa. Thus, it is anticipated that this optimum double air-knife system has the good zinc removal ability and produces no check-mark stain on the coated steel strip.

6. Conclusion

In order to prevent the check-mark stain on the zinc coated surface in a continuous hot-dip galvanizing process, a new double air-knife system is proposed. The proposed system consists of a main jet and a guide jet installed just beneath the main jet. The 3-D flow field around the system is calculated by using the commercial code, FLUENT. LES technique was used to simulate the 3-D unsteady compressible turbulent flow field established by impinging a two-dimensional plane jet normally onto a moving steel strip in a relatively short distance. The performance of the proposed new air-knife system is compared with that by the existing air-knife system. As a result of our simulation, it was observed that the guide jet makes the flow field of the main jet more stable, and so the appearance of the check-mark stain is suppressed. Furthermore, the proposed double air-knife system has better zinc removal ability in comparison with the existing system.

In addition, in order to find out the optimum configuration of the double air-knife system, a parametric study was carried out. For this study, the maximum impinging static pressure and the rms value of the static pressure variation along the impinging stagnation line were used to search optimum values of design variables. The optimum configuration are found to be the case when d_{03}=0.6 mm, d_{04}=0.2 mm, $\theta=1°$ and P_{03}=15 kPa. It is anticipated that the optimized double air-knife system proposed herein does not produce the check-mark stain on the zinc coated steel strip.

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