ERRATUM TO “GLOBAL CLASSICAL SOLUTIONS OF THE ‘ONE AND ONE-HALF’ DIMENSIONAL VLASOV-MAXWELL-FOKKER-PLANCK SYSTEM”

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Abstract. This note is an erratum to [S. Pankavich and J. Schaeffer, Comm. Math. Sci., 14(1):209–232, 2016] and corrects an \(L^2\) estimate concerning derivatives of a Green’s function for the linear Vlasov–Fokker–Planck operator. Here, the proof that relies on this estimate is corrected using alternative means.

Key words. Kinetic Theory, Vlasov, Fokker–Planck equation, global existence.

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1. Introduction

The purpose of this article is to correct an error in [1]. The notation and assumptions used in [1] will be used here, and it is assumed that the reader is familiar with [1]. In particular, the estimate

\[
\left( \int \int |\nabla w \mathcal{G}(t, x, v, \tau, y, w)|^2 \, dw \, dy \right)^{\frac{1}{2}} \leq C(t - \tau)^{-1/2},
\]

which appears just above line (4.28) of [1], is incorrect. What follows completes the proof of Theorem 1.1 of [1] without using the estimate (1.1) by proving that the solution possesses the stated regularity.

2. Correction to Proof of Theorem 1.1

Proceeding as in (4.16) of [1] we have

\[
\frac{d}{dt} \int \int v_0^a (f^{n+1} - f^{\ell+1})^2 \, dv \, dx \\
=-2 \int \int v_0^a |\nabla_v (f^{n+1} - f^{\ell+1})|^2 \, dv \, dx \\
+ \int \int (f^{n+1} - f^{\ell+1})^2 (\Delta_v v_0^a + E^n \cdot \nabla_v v_0^a) \, dv \, dx \\
+2 \int \int f^{\ell+1} (K^n - K^\ell) \cdot (v_0^a \nabla_v (f^{n+1} - f^{\ell+1}) (f^{n+1} - f^{\ell+1}) \nabla_v v_0^a) \, dv \, dx.
\]

We will use the notation

\[
\sigma^{n,\ell} \leq o^{n,\ell}
\]

to mean \(\forall \epsilon > 0 \, \exists N\) such that \(n, \ell > N \Rightarrow \sigma^{n,\ell} \leq \epsilon\). Recall that

\[
\sup_{t \in [0,T]} \|(E^n B^n) - (E^\ell, B^\ell)\|_{L^1(R)} + \sup_{t \in [0,T]} \int v_0^a (f^{n+1} - f^{\ell+1})^2 \, dv \, dx \leq o^{n,\ell}.
\]
By (4.14) of [1] we have

$$|K^n - K^\ell| \leq v_0 \left| (E^n, B^n) - (E^\ell, B^\ell) \right| + v_0^{1+\epsilon/2} |B^\ell|^{2-n\epsilon/2}$$

where we assume $\ell > n$.

Recall that $\alpha = a + 2 + \epsilon$ and note that

$$\int \int |f^{\ell+1}(K^n - K^\ell) \cdot v_0^a \nabla_v (f^{n+1} - f^{\ell+1})| dv dx$$

$$\leq o^{n,\ell} \int \int |f^{\ell+1}| v_0^{a+1+\epsilon/2} \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right| dv dx$$

$$\leq o^{n,\ell} \sqrt{\int \int v_0^a (f^{n+1} - f^{\ell+1})^2 dv dx} \sqrt{\int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx}$$

$$\leq o^{n,\ell} \sqrt{\int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx}.$$ 

Similarly,

$$\int \int |f^{\ell+1}(K^n - K^\ell) \cdot (f^{n+1} - f^{\ell+1}) \nabla_v v_0^a| dv dx$$

$$\leq o^{n,\ell} \sqrt{\int \int v_0^a (f^{n+1} - f^{\ell+1})^2 dv dx} \leq o^{n,\ell}.$$ 

So, by Equation (2.1) we find

$$\frac{d}{dt} \int \int v_0^a (f^{n+1} - f^{\ell+1})^2 dv dx$$

$$\leq - \int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx + o^{n,\ell}$$

$$+ \left[ o^{n,\ell} \sqrt{\int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx} - \int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx \right]$$

$$\leq o^{n,\ell} - \int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx.$$ 

It follows that

$$\int_0^T \int \int v_0^a \left| \nabla_v (f^{n+1} - f^{\ell+1}) \right|^2 dv dx d\tau \leq o^{n,\ell}. \quad (2.2)$$

In a similar manner we may show that

$$\int_0^T \int \int v_0^a \left| \nabla_v f^{n+1} \right|^2 dv dx d\tau \leq C. \quad (2.3)$$

Note that the exponent of $v_0$ in the inequality (2.2) is $a$, but in the inequality (2.3) it is $\alpha$. 
Next, we derive $L^p$ bounds on $G$. Considering $0 \leq \tau \leq t \leq T$, letting $p \geq 1$, $b, \theta \geq 0$, and using the change of variables
\[
u = \frac{v - w}{\sqrt{t - \tau}} , \quad z = \frac{x - y - \frac{t - \tau}{2} (v_1 + w_1)}{(t - \tau)^{3/2}}
\]
we have
\[
\iint w_0^{-b \theta} G^p dw dy = C \iint \left( \frac{1}{1 + \sqrt{t - \tau} u^2} \right)^{-b \theta} \left( t - \tau \right)^{-5/2} e^{-|u|^2/4} |e^{-3z^2}|^p (t - \tau)^{5/2} dz du
\]
\[
= C (t - \tau)^{\frac{5}{2} (1 - p)} \left( \int \left| u < \frac{1}{2} (t - \tau)^{-1/2} |v| \right| \left( \sqrt{1 + \frac{1}{|v|}} \right)^{-b \theta} e^{-p |u|^2/4} du \right)
\]
\[
+ \int \left| u > \frac{1}{2} (t - \tau)^{-1/2} |v| \right| \left( \sqrt{1 + \frac{1}{|v|}} \right)^{-b \theta} e^{-p |u|^2/4} du \right)
\]
\[
\leq C (t - \tau)^{\frac{5}{2} (1 - p)} \left[ \left( \sqrt{1 + \left( \frac{1}{2} |v| \right)^2} \right)^{-b \theta} \int e^{-p |u|^2/4} du + e^{-\frac{1}{8} (\frac{|v|}{2\tau})^2} \int e^{-|u|^2/8} du \right]
\]
\[
\leq C (t - \tau)^{\frac{5}{2} (1 - p)} \left[ \left( \sqrt{1 + |v|^2} \right)^{-b \theta} + e^{-C |v|^2} \right]
\]
\[
\leq C (t - \tau)^{\frac{5}{2} (1 - p)} v_0^{-b \theta}.
\]
So for $1 \leq p < 7/5$ we have
\[
\int_0^t \iint w_0^{-b \theta} G^p dw dy d\tau \leq C v_0^{-b \theta}.
\]
(2.4)

Here, constants may depend on $p, b,$ and $\theta$. Later, specific choices of $p, b,$ and $\theta$ are used and this dependence is removed.

Next, we bound $|\nabla_v G|$ and $|\nabla_u G|$. Note that
\[
\iint w_0^{-b \theta} \left( \frac{|v - u|}{t - \tau} \right)^p dw dy
\]
\[
= C \iint \left( \sqrt{1 + \frac{1}{|v - \sqrt{t - \tau} u|}} \right)^{-b \theta} \left[ \frac{|u|}{\sqrt{t - \tau}} (t - \tau)^{-5/2} e^{-|u|^2/4} e^{-3z^2} \right]^p (t - \tau)^{5/2} dz du
\]
\[
= C (t - \tau)^{\frac{5}{2} - 3p} \int \left( \sqrt{1 + \frac{1}{|v - \sqrt{t - \tau} u|}} \right)^{-b \theta} \left| u e^{-|u|^2/4} \right|^p du
\]
\[
\leq C (t - \tau)^{\frac{5}{2} - 3p} v_0^{-b \theta}
\]
and similarly,
\[
\iint w_0^{-b \theta} \left( \frac{|x - y - \frac{t - \tau}{2} (v_1 + w_1)|}{(t - \tau)^2} \right)^p dw dy \leq C (t - \tau)^{\frac{5}{2} - 3p} v_0^{-b \theta}.
\]
Hence,
\[
\iint w_0^{-b \theta} (|\nabla_v G|^p + |\nabla_u G|^p) dw dy \leq C (t - \tau)^{\frac{5}{2} - 3p} v_0^{-b \theta}
\]
and for $1 \leq p < 7/6$

$$
\int_0^t \int \int w_0^{-b\theta} (|\nabla_w G|^p + |\nabla_v G|^p) \, dwdyd\tau \leq Cv_0^{-b\theta}. \quad (2.5)
$$

In a very similar manner it may be shown that

$$
\int_0^T \int \int v_0^{-b\theta} G^p \, dvdxdt \leq Cv_0^{-b\theta} \quad (2.6)
$$

for $p < 7/5$ and

$$
\int_0^T \int \int v_0^{-b\theta} (|\nabla_v G|^p + |\nabla_w G|^p) \, dvdxdt \leq Cv_0^{-b\theta}
$$

for $p < 7/6$.

Next we derive two inequalities that will be used repeatedly. Let $p, q \in [1, \infty)$ and $r \in [1, \infty]$ satisfy

$$
\frac{1}{p} + \frac{1}{q} - 1 = \frac{1}{r}.
$$

We will first consider the case $r \neq \infty$, but what follows may be easily adapted to the case $r = \infty$. Let $\theta \geq 0$ and define

$$
b = \left( \frac{1}{p} - \frac{1}{r} \right)^{-1} \quad c = \left( \frac{1}{q} - \frac{1}{r} \right)^{-1}
$$

and note that

$$\frac{1}{r} + \frac{1}{b} + \frac{1}{c} = 1.
$$

By Hölder’s inequality and using the inequality (2.4), we have for $p < 7/5$ and any $h(\tau, y, w) \geq 0$

$$
\begin{align*}
\int_0^t \int \int G h \, dwdyd\tau &= \int_0^t \int \int \left[ G^{p/r} (w_0^\theta h)^{q/r} \right] \left[ G^{p/(\frac{1}{2} - \frac{1}{r})} w_0^{-\theta} \right] \left[ w_0^\theta h \right]^{q(\frac{1}{2} - \frac{1}{r})} \, dwdyd\tau \\
&\leq \left( \int_0^t \int \int G^{p} (w_0^\theta h)^{q} \, dwdyd\tau \right)^{\frac{1}{r}} \left( \int_0^t \int \int G^{p} w_0^{-b\theta} \, dwdyd\tau \right)^{\frac{1}{b}} \left( \int_0^t \int \int (w_0^\theta h)^{q} \, dwdyd\tau \right)^{\frac{1}{c}} \\
&\leq Cv_0^{-\theta} \left( \int_0^t \int \int G^{p} (w_0^\theta h)^{q} \, dwdyd\tau \right)^{1/r} \left( \int_0^t \int \int (w_0^\theta h)^{q} \, dwdyd\tau \right)^{1/c}.
\end{align*}
$$
Hence, using the inequality (2.6) we have, for \( p < \frac{7}{5} \)
\[
\left[ \int_0^T \int \left( \int_0^t \int \mathcal{G} h d\omega d\nu \tau \right)^r d\nu d\tau dt \right]^{1/r} 
\leq C \left[ \int_0^T \int \left( \int_0^t \int \mathcal{G}^p (w_0^\theta h)^q d\omega d\nu \tau \right)^{r/c} d\nu d\tau dt \right]^{1/r} 
\leq C \left( \int_0^T \int \left( w_0^\theta h \right)^q d\omega d\nu \tau \right)^{1+r/c} 
\leq C \left[ \int_0^T \int \left( w_0^\theta h \right)^q d\omega d\nu \tau \right]^{1/q}. 
\]

Similarly, using the inequality (2.5) we find for \( p < \frac{7}{6} \)
\[
\left[ \int_0^T \int \left( \int_0^t \int |\nabla w\mathcal{G}| + |\nabla_v\mathcal{G}| h d\omega d\nu \tau \right)^r d\nu d\tau dt \right]^{1/r} 
\leq C \left( \int_0^T \int \left( w_0^\theta h \right)^q d\omega d\nu \tau \right)^{1/q}. 
\]

Now we will show that \( f^n \) converges in \( L^\infty \). We have
\[
f^{n+1} - f^{\ell+1} = - \int_0^t \int \mathcal{G} (K^n \cdot \nabla_v f^{n+1} - K^\ell \cdot \nabla_v f^{\ell+1}) (\tau, y, w) d\omega d\nu \tau. 
\]

Using (4.14) of [1] and taking \( \ell > n \)
\[
|K^n \cdot \nabla_v f^{n+1} - K^\ell \cdot \nabla_v f^{\ell+1}| 
\leq |K^n - K^\ell| |\nabla_v f^{n+1}| + |K^\ell| |\nabla_v (f^{n+1} - f^{\ell+1})| 
\leq |(E^n, B^n) - (E^\ell, B^\ell)| w_0 |\nabla_v f^{n+1}| 
+ Cw_0^{1+n/2} 2^{-n\epsilon/2} |\nabla_v f^{n+1}| + Cw_0 |\nabla_v (f^{n+1} - f^{\ell+1})| 
\leq o^n w_0^{1+n/2} |\nabla_v f^{n+1}| + Cw_0 |\nabla_v (f^{n+1} - f^{\ell+1})|. 
\]

Next, we will apply the inequality (2.7) with \( p < 7/5 \) and \( q = 2 \). Note that by taking \( p \) close to 7/5, we may make \( r = \left( \frac{1}{p} + \frac{1}{q} - 1 \right)^{-1} \) close to
\[
\left[ \frac{5}{7} + \frac{1}{2} - 1 \right]^{-1} = 14/3. 
\]
Thus, applying the inequality (2.7) with $p < 7/5$, $q = 2$, $\theta = \frac{a}{2} - 1$, and $h = w_0 |\nabla_v (f^{n+1} - f^{\ell+1})|$ while using the inequality (2.2) yields

$$\left[ \int_0^T \int \int \left( v_0^{\frac{a}{2}-1} \int \int \mathcal{G} w_0 |\nabla_v (f^{n+1} - f^{\ell+1})| dwdydt \right)^r dvdxdt \right]^{1/r} \leq C \left[ \int_0^T \int \int \left( w_0^{\frac{a}{2}-1} w_0 |\nabla_v (f^{n+1} - f^{\ell+1})| \right)^2 dwdydt \right]^{1/2} \leq o^{n,\ell}$$

for $r < 14/3$. Applying the inequality (2.7) with $p < 7/5$, $q = 2$, $\theta = \frac{a}{2} - 1 - \epsilon/2$, and $h = w_0^{1+\epsilon/2} |\nabla_v f^{n+1}|$ and then using the inequality (2.3) yields

$$\left[ \int_0^T \int \int \left( v_0^{\frac{a}{2}-1-\epsilon/2} \int \int \mathcal{G} w_0^{1+\epsilon/2} |\nabla_v f^{n+1}| dwdydt \right)^r dvdxdt \right]^{1/r} \leq C \left[ \int_0^T \int \int \left( w_0^{\frac{a}{2}-1-\epsilon/2} w_0^{1+\epsilon/2} |\nabla_v f^{n+1}| \right)^2 dwdydt \right]^{1/2} \leq C$$

for $r < 14/3$. Since $\frac{a}{2} - 1 - \epsilon > \frac{a}{2} - 1$, Equation (2.9) and inequality (2.10) now yield for $r < 14/3$

$$\int_0^T \int \int \left( v_0^{\frac{a}{2}-1} |f^{n+1} - f^{\ell+1}| \right)^r dvdxdt \leq o^{n,\ell}. \quad (2.11)$$

Similarly, using the inequality (2.3) we may show that for $r < 14/3$

$$\int_0^T \int \int \left( v_0^{\frac{a}{2}-1} |f^{n+1}| \right)^r dvdxdt \leq C. \quad (2.12)$$

To use the inequality (2.11) we integrate by parts in Equation (2.9) to obtain

$$f^{n+1} - f^{\ell+1} = \int_0^T \int \int \nabla w \cdot (K^n f^{n+1} - K^\ell f^{\ell+1}) dwdydt \quad (2.13)$$

and using (4.14) of [1]

$$|K^n f^{n+1} - K^\ell f^{\ell+1}|$$

$$\leq |(E^n, B^n) - (E^\ell, B^\ell)| w_0 |f^{n+1}| + C w_0^{1+\epsilon/2} 2^{-\epsilon/2} |f^{n+1}| + C w_0 |f^{n+1} - f^{\ell+1}|$$

$$\leq o^{n,\ell} w_0^{1+\epsilon/2} |f^{n+1}| + C w_0 |f^{n+1} - f^{\ell+1}|. \quad (2.14)$$

Next, we will apply the inequality (2.8), but now with $p < 7/6$ and $q < 14/3$. Note that we may take $p$ close to $7/6$ and $q$ close to $14/3$ to make $r$ close to

$$\left( \frac{6}{7} + \frac{3}{14} - 1 \right)^{-1} = 14.$$
Thus, applying inequality (2.8) with $p < 7/6$, $q < 14/3$, $\theta = \frac{a}{2} - 2$, and
\[ h = w_0 |f^{n+1} - f^{\ell+1}| \]
and then using the inequality (2.11) yields
\[
\left[ \int_0^T \int \left( v_0^2 - 2 \int_0^t \int |\nabla w| w_0 |f^{n+1} - f^{\ell+1}| \, dw \, dy \, d\tau \right)^r \, dv \, dx \, dt \right]^{1/r} \leq C \left[ \int_0^T \int \left( w_0^{2\epsilon} |f^{n+1} - f^{\ell+1}| \right)^q \, dw \, dy \, d\tau \right]^{1/q} \leq o^n \epsilon \]
for $r < 14$. Applying inequality (2.8) again with $p < 7/6$, $q < 14/3$, $\theta = \frac{a}{2} - 2 - \frac{\epsilon}{2}$ and
\[ h = w_1^{1+\epsilon/2} |f^{n+1}| \]
yields
\[
\left[ \int_0^T \int \left( v_0^2 - 2 - \epsilon/2 \int_0^t \int |\nabla w| w_0^{1+\epsilon/2} |f^{n+1}| \, dw \, dy \, d\tau \right)^r \, dv \, dx \, dt \right]^{1/r} \leq C \left[ \int_0^T \int \left( w_0^{\frac{\epsilon}{1+\epsilon}} |f^{n+1}| \right)^q \, dw \, dy \, d\tau \right]^{1/q} \leq C
\]
for $r < 14$ by using the inequality (2.12). With these two estimates, (2.13) and (2.14) yield
\[
\int_0^T \int \left( v_0^2 - 2 |f^{n+1} - f^{\ell+1}| \right)^r \, dv \, dx \, dt \leq o^n \epsilon \]
for $r < 14$. Similarly,
\[
\int_0^T \int \left( v_0^2 |f^{n+1} - f^{\ell+1}| \right)^r \, dv \, dx \, dt \leq C.
\]
Finally, we can apply estimate (2.8) with $p = \frac{7}{6.4}$, $q = \frac{14}{12}$, and $r = \infty$. Proceeding as above we obtain
\[
\left| v_0^2 - 3 \left( f^{n+1} - f^{\ell+1} \right) \right|_{L^\infty} \leq o^n \epsilon.
\]
Defining
\[ f = \lim_{n \to \infty} f^n \]
it follows that
\[
\left| v_0^2 - 3 \left( f^n - f \right) \right|_{L^\infty} \to 0 \text{ as } n \to \infty.
\]
Next, we bound $\nabla_v f^n$ in $L^\infty$. We have
\[
f^{n+1} = H - \int_0^t \int G K^n \cdot \nabla_v f^{n+1} \, dw \, dy \, d\tau
\]
\[ |\nabla_v (f^{n+1} - H)| = \left| \int_0^t \iint \nabla_v G^n \cdot \nabla_v f^{n+1} \, dw \, dy \, d\tau \right| \leq C \int_0^t \iint |\nabla_w G| |w_0| |\nabla_v f^{n+1}| \, dw \, dy \, d\tau. \quad (2.15) \]

Applying estimate (2.8) with \( p < \frac{7}{6}, q = 2, \theta = \frac{a}{2} - 1, \) and \( h = w_0 |\nabla_v f^{n+1}|, \) and then using (2.3) yields

\[ \left[ \int_0^T \iint \left( v_0^\frac{\alpha}{2} - 1 \int_0^t \iint |\nabla_w G| w_0 |\nabla_v f^{n+1}| \, dw \, dy \, d\tau \right)^r \, dv \, dx \, dt \right]^{1/r} \leq \left[ \int_0^T \iint \left( w_0^\frac{\alpha}{2} - 1 \int_0^t \iint |\nabla_v f^{n+1}| \, dw \, dy \, d\tau \right)^2 \, dw \, dy \, d\tau \right]^{1/2} \leq C. \]

Note that taking \( p \) close to \( \frac{7}{6} \) yields \( r \) close to \( \frac{14}{5} \). Hence, using estimate (2.15) we find for \( r < \frac{14}{5} \)

\[ \int_0^T \iint \left( v_0^\frac{\alpha}{2} - 1 \right)^r \, dv \, dx \, dt \leq C. \]

We then apply estimate (2.8) three more times. In each application we take \( h = w_0 |\nabla_v f^{n+1}| \). First, using \( p < \frac{7}{6}, q < \frac{14}{3}, \) and \( \theta = \frac{a}{2} - 2 \) yields

\[ \int_0^T \iint \left( v_0^\frac{\alpha}{2} - 2 \right)^r \, dv \, dx \, dt \leq C \]

for \( r < \frac{14}{3} \). Using \( p < \frac{7}{6}, q < \frac{14}{3}, \) and \( \theta = \frac{a}{2} - 3 \) yields

\[ \int_0^T \iint \left( v_0^\frac{\alpha}{2} - 3 \right)^r \, dv \, dx \, dt \leq C \]

for \( r < 14 \). Using \( p = \frac{7}{6}, q = \frac{14}{3}, r = \infty, \) and \( \theta = \frac{a}{2} - 4 \) yields

\[ \left\| v_0^\frac{\alpha}{2} - 4 \nabla_v f^{n+1} \right\|_{L^\infty} \leq C. \quad (2.16) \]

Recall that \( a > 8 \) so \( \frac{a}{2} - 4 = \frac{a+2+\epsilon}{2} - 4 > 1 \). Now \( \forall h \in \mathbb{R}^2 \)

\[ |f^{n+1}(t, x, v + h) - f^{n+1}(t, x, v)| \leq C|h|, \]

and so

\[ |f(t, x, v + h) - f(t, x, v)| \leq C|h|. \]

Finally, we show that \( f \) is Hölder continuous in \( x \). Let \( h > 0 \) and

\[ e(t, x, v) = f^{n+1}(t, x + h, v) - f^{n+1}(t, x, v), \]
then
\[ \partial_t e + v_1 \partial_x e + K^e \cdot \nabla_v e - \Delta_v e = -(K^n(t, x + h, v) - K^n(t, x, v)) \cdot \nabla_v f^{n+1}(t, x + h, v). \]

Note that
\[
|K^n(t, x + h, v) - K^n(t, x, v)| \leq v_0 \left( |E^n(t, x + h) - E^n(t, x)| + |B^n(t, x + h) - B^n(t, x)| \right)
\leq v_0 h^{1/2} \left( \int (\partial_x E)^2 dx + \int (\partial_x B)^2 dx \right)
\leq C v_0 h^{1/2}.
\]

Thus, by the bound (2.16) we find
\[
|\partial_t e + v_1 \partial_x e + K^n \cdot \nabla_v e - \Delta_v e| \leq C v_0 h^{1/2} C v_0^{4-\alpha/2} \leq C h^{1/2}.
\]

By Lemma 2.1 of [1]
\[ |e| \leq C h^{1/2}. \]
It follows that $f$ is Hölder continuous with exponent $1/2$ in $x$.

Now, Theorem II.1 of [2] shows that $f$ possesses the regularity stated in Theorem 1.1 of [1], and this completes the argument.

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