Magnetic moments of the SU(3) decuplet baryons
in the chiral quark-soliton model

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Abstract

Magnetic moments of baryons are studied within the chiral quark soliton model with special emphasis on the decuplet of baryons. The model is used to identify all symmetry breaking terms proportional to $m_s$. Sum rules for the magnetic moments are derived. A “model-independent” analysis of the symmetry breaking terms is performed and finally model calculations are presented, which show the importance of the rotational $1/N_c$ corrections for cranking of the soliton.

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I. INTRODUCTION

The magnetic moments of the $\Delta^{++}$ and $\Omega^{-}$ have been measured recently. The former one was obtained from the analysis of pion bremsstrahlung\[1\], $\mu_{\Delta^{++}} = (4.52 \pm 0.50) \mu_N$, while the latter one was measured by the E756 Collaboration \[2\] and found to be $\mu_{\Omega^{-}} = (-1.94 \pm 0.17 \pm 0.14) \mu_N$. These experimental data have triggered a new interest in the study of the magnetic moments of the baryon-decuplet: For example, they have been investigated in the relativistic quark model \[3,4\], in chiral perturbation theory \[5,6\], in the QCD sum rule approach \[7\], in the chiral bag model \[8\], in quenched lattice gauge theory \[9\], and in the nonrelativistic quark model with two-body exchange currents \[10\]. One should, however, bare in mind that while the world average for the $\Omega^{-}$ quoted by the Particle Data Group ’96 differs only slightly from \[2\]: $\mu_{\Omega^{-}} = (-2.02 \pm 0.05) \mu_N$, the mean value for $\Delta^{++}$ is much less constrained: $\mu_{\Delta^{++}} = (3.5 - 7.5) \mu_N$.

Recently, the octet magnetic moments have been studied in the chiral quark–soliton model (\(\chi\)QSM; also known as the semibosonized nonlinear Nambu–Jona-Lasinio model) \[11\] and it has been shown that the \(\chi\)QSM reproduces the data within about 15%. In fact, the accuracy that has been reached is more or less the upper limit that can be obtained in any model with hedgehog symmetry \[12\]. The aim of the present work is to extend our investigation to the magnetic moments of the baryon decuplet. Only after this work had been completed we were aware of Ref. \[13\] where both octet and decuplet magnetic moments have been calculated in a model identical to the one discussed here. Although the results of Ref. \[13\] are in qualitative agreement with ours, there are some quantitative differences which we discuss in section IV.D of the present paper. Moreover, in contrast to the authors of Ref. \[13\] who have contented themselves with the model calculations, we try to present ”model-independent” analysis based on the chiral quark-soliton model, which does not rely on dynamical calculations and does not suffer from the ambiguities of the SU(3) quantization.

In contrast to the mass splittings, the general analysis of the symmetry breaking for the magnetic moments is quite involved. Famous Gell-Mann–Okubo mass formulae are so simple, since matrix elements of the mass splitting operator, which is assumed to transform as an octet, can be parametrized by 2 free parameters. In the case of magnetic moments there are two parameters which parametrize magnetic moments in the chiral limit and at least 5 others which describe chiral symmetry breaking. Under these circumstances it might seem impossible to write general model-independent relations similar to those for the mass splittings. Nevertheless we will adopt the strategy in which the algebraic structure of the \(\chi\)QSM will serve us as a tool to identify the relevant symmetry breaking terms. Then the pertinent coefficients, which are of course calculable from the solitonic profile function of the model, will be treated as free parameters and fitted. Despite the large number of free parameters this procedure does have a predictive power, since the number of magnetic moments in decuplet and octet is 18; moreover there are decays governed by the same operator which can be described by the same set of parameters. This “model-independent” analysis will be at the end compared with the model predictions in line with Ref. \[11\], i.e.

\[1\] Note that the result is model-dependent.
those using the selfconsistent profile functions.

As far as the “model-independent” analysis is concerned our results can be summarized as follows: we are able to fit all but one (called \( p \)) free parameters in the octet sector; this is because the system of linear equations which we get is undetermined. Two measured magnetic moments of \( \Omega^- \) and \( \Delta^{++} \) depend very weakly on \( p \), and, as a consequence, \( p \) cannot be constrained from the data. Nevertheless, we are able to derive sum rules, both for the octet and for the decuplet and also the ones which involve particles from both multiplets, which to our knowledge are new. For example we get for the magnetic moments:

\[
\Delta^{++} - \Omega^- = \frac{3}{4} \left( 2p + n + \Sigma^+ - \Sigma^- - \Xi^0 - 2\Xi^- \right). \tag{1}
\]

Experimentally, \( \Delta^{++} - \Omega^- = 6.43 \) with a rather large error corresponding to the uncertainty of the world average for \( \mu_{\Delta^{++}} \) and the right hand side of Eq.(1)

\[
\frac{3}{4} \left( 2p + n + \Sigma^+ - \Sigma^- - \Xi^0 - 2\Xi^- \right) = 7.38.
\]

As far as model predictions are concerned we find: \( \mu_{\Delta^{++}} = 4.73 \) and \( \mu_{\Omega^-} = -2.27 \) in very good agreement with the data.

In the \( \chi \)QSM the baryon can be viewed as \( N_c \) valence quarks coupled to the polarized Dirac sea bound by a nontrivial chiral background hedgehog field in the Hartree approximation. The proper quantum numbers of baryons are obtained by the semi–classical quantization carried out by integrating over zero–mode fluctuations of the pion field around the saddle point. Hence, decuplet baryons are understood as the excited states of a collective rotation. In the process of quantizing the system, the angular velocities \( \Omega_a \) corresponding to the rigid rotation of the hedgehog are promoted to the collective quantum operators, which satisfy \( SU(3)_{\text{right}} \) (generalized spin) commutation rules. These operators do not commute with the Wigner matrices \( D_{ab}^{(\nu)} \) which naturally appear in the formalism. It has been recently shown that inclusion of the terms proportional to the commutator \([\Omega_a, D_{bc}^{(\nu)}]\) which vanish classically but are nonzero in the quantum case is of utmost importance for the phenomenological description of axial properties of the nucleon [14,21]. These new terms, first discussed in Ref. [22], are taken into account in our analysis of the magnetic moments.

One more remark should be added as far as the rotational corrections to physical quantities are concerned. Recently [23] it has been observed that the prescription used here (and in other works [11,13,14,18,20]) leads in the SU(3) case to the problem with charge quantization. Namely, because the hedgehog is not fully SU(3) symmetric (see Eq.(10)) some quantities, like tensors of inertia for example, may get spurious contributions which are totally antisymmetric. This happens only for SU(3) and not for SU(2) because the hedgehog commutes with \( \lambda_8 \) and the symmetry properties are such that terms proportional to the structure constant \( f_{abc} \) are allowed. This is reflected in the charge of the proton which due to these new contribution is shifted away from 1. So far an \( \text{ad hoc} \) prescription has been adopted in which the unwanted terms have been subtracted. It is however not clear which terms should be subtracted in case of operators different than electric charge; it is also not clear if all operators suffer from this ambiguity. Since there is no satisfactory theoretical solution to this problem for the purpose of the present work we simply take our formulae as they are with no subtractions. Needless to say that our "model-independent" analysis

\[^2\text{We thank the referee for raising this point.}\]
is free from this ambiguity. This is because it affects only the numerical values of various coefficients predicted by the model, which in the "model-independent" analysis are taken from the data.

The outline of the paper is as follows: In the next section, we sketch the basic formalism for obtaining the decuplet magnetic moments in the $\chi$QSM. In section III, we first calculate the contribution of the leading order and $1/N_c$ rotational corrections in the chiral limit, taking advantage of the former calculation of the octet magnetic moments. The results are compared with the isovector relations for the magnetic moments in the large $N_c$ limit. In section IV, we study the effects of the strange quark mass $m_s$ corrections. We also compare in section IV.D our results with those of Ref. [13]. In the last section, we summarize the present work and draw conclusions.

II. GENERAL FORMALISM

In this section we present general formulae needed to calculate decuplet magnetic moments (for details of the formalism see a recent review [16]).

The $\chi$QSM is characterized by a partition function in Euclidean space given by the functional integral over the pseudoscalar meson and quark fields:

$$Z = \int \mathcal{D}\Psi \mathcal{D}\Psi^\dagger \mathcal{D}\pi^a \exp \left( -\int d^4x \Psi^\dagger iD\Psi \right) = \int \mathcal{D}\pi^a \exp \left( -S_{\text{eff}}[\pi] \right),$$

where $S_{\text{eff}}$ is the effective action

$$S_{\text{eff}}[\pi] = -Sp \log iD.$$

$iD$ represents the intrinsic Dirac differential operator

$$iD = \beta(-i\slashed{\partial} + \hat{m} + MU^\gamma_5)$$

with the pseudoscalar chiral field

$$U^\gamma_5 = \exp (i\pi^a \lambda^a \gamma_5) = \frac{1 + \gamma_5}{2} U + \frac{1 - \gamma_5}{2} U^\dagger.$$

$\hat{m}$ is the current quark mass matrix given by

$$\hat{m} = \text{diag}(m_u, m_d, m_s) = m_0 \mathbf{1} + m_8 \lambda_8.$$ 

$\lambda^a$ represent the usual Gell-Mann matrices normalized as $\text{tr}(\lambda^a \lambda^b) = 2\delta^{ab}$. The $m_0$ and $m_8$ are respectively defined by

$$m_0 = \frac{2\overline{m} + m_s}{3}, \quad m_8 = \frac{\overline{m} - m_s}{\sqrt{3}}$$

with isospin symmetry $(m_u + m_d = 2\overline{m})$ assumed. $M$ stands for the dynamical quark mass arising from the spontaneous chiral symmetry breaking, which is in general momentum-dependent [17]. We regard $M$ as a constant and employ the proper-time regularization for convenience.
The operator $iD$ is expressed in Euclidean space in terms of the Euclidean time derivative $\partial_\tau$ and the Dirac one–particle Hamiltonian $h(U^{\gamma_5})$
\[
  iD = \partial_\tau + h(U^{\gamma_5}) + \beta \hat{m} - \beta \vec{m} \mathbf{1}
\] (8)
with
\[
h(U^{\gamma_5}) = \vec{\alpha} \cdot \nabla + \beta MU^{\gamma_5} + \beta \vec{m} \mathbf{1}.
\] (9)

Here $\beta$ and $\vec{\alpha}$ are the well–known Dirac Hermitian matrices. $U$ is assumed to have a structure corresponding to the embedding of the SU(2)-hedgehog into SU(3):
\[
  U = \begin{pmatrix} U_0 & 0 \\ 0 & 1 \end{pmatrix} \text{ with } U_0 = \exp (i \hat{r} \cdot \vec{\tau} P(r)).
\] (10)

$P(r)$ is called profile function. The partition function of Eq.(2) can be simplified by the saddle point approximation which is exact in the large $N_c$ limit. One ends up with a stationary profile function $P(r)$ which is evaluated by solving the Euler–Lagrange equation corresponding to $\delta S_{\text{eff}} / \delta P(r) = 0$. This gives the static classical field $U_0$. Note that according to Eq.(9) the profile is calculated with the tail corresponding to the massive pion. Since the masses of the up and down quarks are much smaller than that of the strange quark, we approximate the mass term in Eq.(8) as
\[
  \hat{m} - \vec{m} \mathbf{1} \approx m_s \left( \frac{1}{3} - \frac{1}{\sqrt{3}} \lambda_8 \right).
\] (11)

The magnetic moments of the baryon decuplet can be calculated from the following one-current baryon matrix element:
\[
  \langle B | \bar{\psi}(z) \gamma_\mu \hat{Q} \psi(z) | B \rangle,
\] (12)
where $\hat{Q}$ is the charge operator of quarks in SU(3) flavor space, defined by
\[
  \hat{Q} = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} = \frac{1}{2} \left( \lambda^3 + \frac{1}{\sqrt{3}} \lambda_8 \right).
\] (13)

One can relate the baryonic matrix element Eq.(12) to the correlation function:
\[
  \langle 0 | J_B(x, T) \bar{\psi}(y) \gamma_\mu \hat{Q} \psi J_B^\dagger(y, 0) | 0 \rangle
\] (14)
at large Euclidean time $T$. The baryon current $J_B$ can be constructed from $N_c$ quark fields,
\[
  J_B = \frac{1}{N_c} \epsilon^{i_1 \ldots i_{N_c}} \Gamma_{SS_3 I_{I_3 Y}}^{\alpha_1 \ldots \alpha_{N_c}} \psi_{\alpha_{i_1} i_1} \cdots \psi_{\alpha_{N_c} i_{N_c}}
\] (15)
$\alpha_1 \ldots \alpha_{N_c}$ are spin–isospin indices, $i_1 \ldots i_{N_c}$ are color indices, and the matrices $\Gamma_{SS_3 I_{I_3 Y}}^{\alpha_1 \ldots \alpha_{N_c}}$ are taken to endow the corresponding current with the quantum numbers $SS_3 I_{I_3 Y}$. $J_B(J_B^\dagger)$ annihilates (creates) the baryon state at given time $T$. The rotational corrections $1/N_c$ and
linear $m_s$ corrections being taken into account, the expression for the collective magnetic moment operator $\hat{\mu}$ can be written as follows:

$$\hat{\mu} = (w_1^1 + m_s w_1^2) D_Q^{(8)} + w_2 D_{pq3} D_Q^{(8)} \cdot \hat{S}_q + \frac{w_3}{\sqrt{3}} D_Q^{(8)} \hat{S}_3$$

$$+ m_s \left[ \frac{w_4}{\sqrt{3}} d_{pq3} D_Q^{(8)} D_{sq} + w_5 \left( D_Q^{(8)} D_{ss}^{(8)} + D_Q^{(8)} D_{ss}^{(8)} \right) + w_6 \left( D_Q^{(8)} D_{ss}^{(8)} - D_Q^{(8)} D_{ss}^{(8)} \right) \right], \quad (16)$$

where

$$w_1^1 = \frac{M_N}{3} \left( Q_0 + \frac{Q_1}{I_1} + \frac{Q_2}{I_2} \right),$$

$$w_1^2 = -2 \frac{M_N}{3} \mathcal{M}_0,$$

$$w_2 = -\frac{M_N}{3} \frac{\mathcal{I}_2}{I_2},$$

$$w_3 = -\frac{M_N}{3} \frac{\mathcal{I}_1}{I_1},$$

$$w_4 = \frac{M_N}{3} \left( 6 \mathcal{M}_2 - 2 \frac{K_2}{I_2} \mathcal{I}_2 \right),$$

$$w_5 = \frac{M_N}{3} \left( \mathcal{M}_0 + \mathcal{M}_1 - \frac{1}{3} \frac{K_1}{I_1} \mathcal{I}_1 \right),$$

$$w_6 = \frac{M_N}{3} \left( \mathcal{M}_0 - \mathcal{M}_1 + \frac{1}{3} \frac{K_1}{I_1} \mathcal{I}_1 \right). \quad (17)$$

The dynamical quantities $w_i$ are independent of the baryons involved. They are expressed in terms of the inertia parameters of the soliton which have a general structure like:

$$\sum_{m,n} \langle n | O_1 | m \rangle \langle m | O_2 | n \rangle \mathcal{R}(E_n, E_m, \Lambda), \quad (18)$$

where $O_i$ are spin-isospin operators changing the grand spin of states $|n\rangle$ by 0 or 1. The double sum runs over all the eigenstates of the intrinsic quark Hamiltonian in the soliton field and $\mathcal{R}$ is the regularization function. The numerical technique for calculating such double sums has been developed in [24–26]. Explicit forms of the inertia parameters are given in Appendix. $\hat{S}_q$ stands for an operator of the generalized spin acting on the angular variable $R(t)$ [24]. $D_{ab}^{(\nu)}(R)$ denote Wigner matrix in the representation $\nu$. Terms $Q_1/I_1 + Q_2/I_2$ arise from the time-ordering of the collective operators [13].

The operator (16) has to be sandwiched between the octet and decuplet collective wave functions. However, strictly speaking, spin 1/2 (3/2) baryon wave functions are no longer pure octet (decuplet) states. This is because the collective splitting Hamiltonian

$$\hat{H}' = m_s \left( \alpha D_{ss}^{(8)} + \beta \hat{Y} + \frac{\gamma}{\sqrt{3}} D_{8A}^{(8)} \hat{S}_A \right) \quad (19)$$

mixes the states in various SU(3) representations (here $A = 1 \ldots 3$). Constants $\alpha$, $\beta$ and $\gamma$ are given by [24].
\[ \alpha = -\sigma + \frac{K_2}{I_2}, \quad \beta = -\frac{K_2}{I_2}, \quad \gamma = 2 \left( \frac{K_1}{I_1} - \frac{K_2}{I_2} \right). \]  

Here \( K_i \) and \( I_i \) are the “moments of inertia” and \( \sigma \) is related to the nucleon sigma term \(^{27}\): \( \sigma = 1/3 \Sigma/\bar{m} \).

A spin \( S = 1/2 \) state has the following form in the first order in \( m_s \):
\[
\left| B, \frac{1}{2}, S_3 \right\rangle = \left| 8, B, \frac{1}{2}, S_3 \right\rangle + m_s c_{10}^B \left| 10, B, \frac{1}{2}, S_3 \right\rangle + m_s c_{27}^B \left| 27, B, \frac{1}{2}, S_3 \right\rangle
\]
and a spin \( S = 3/2 \) state reads:
\[
\left| B, \frac{3}{2}, S_3 \right\rangle = \left| 10, B, \frac{3}{2}, S_3 \right\rangle + m_s a_{27}^B \left| 27, B, \frac{3}{2}, S_3 \right\rangle + m_s a_{35}^B \left| 35, B, \frac{3}{2}, S_3 \right\rangle,
\]
where
\[
c_{10}^B = c_{10} \begin{bmatrix} \sqrt{5} \\ 0 \\ \sqrt{5} \end{bmatrix}, \quad c_{27}^B = c_{27} \begin{bmatrix} \sqrt{2} \\ 3 \\ \sqrt{5} \end{bmatrix}, \quad a_{27}^B = a_{27} \begin{bmatrix} \sqrt{15/2} \\ 2 \sqrt{3/2} \\ 0 \end{bmatrix}, \quad a_{35}^B = a_{35} \begin{bmatrix} 5/\sqrt{14} \\ 2 \sqrt{5/7} \\ 3 \sqrt{5/14} \end{bmatrix}
\]
in the basis \([N, \Lambda, \Sigma, \Xi]\) and \([\Delta, \Sigma^*, \Xi^*, \Omega]\) respectively, and
\[
c_{10} = \frac{I_2}{15} \left( \sigma - \frac{I_1}{K_1} \right), \quad c_{27} = \frac{I_2}{25} \left( \sigma + \frac{I_1}{3K_1} - \frac{4I_2}{3K_2} \right), \quad a_{27} = \frac{I_2}{8} \left( \sigma - \frac{5K_1}{3I_1} + \frac{2K_2}{3I_2} \right), \quad a_{35} = \frac{I_2}{24} \left( \sigma + \frac{K_1}{I_1} - \frac{2K_2}{I_2} \right).
\]

In actual calculations we shall need the explicit form of the SU(3) wave functions corresponding to the states of Eqs.\(^{21,22}\). Let us remind that the wave function of a state of flavor \( B = (Y, T, T_3) \) and spin \( S = (Y' = -1, S, S_3) \) in the representation \( \nu \) is given in terms of a tensor with two indices: \( \psi_{(\nu B), (S \Xi)} \), one running over the states \( B \) in the representation \( \nu \) and the other one over the states \( S \) in the representation \( \Xi \). Here \( \Xi \) denotes the complex conjugate of the representation \( \nu \), and the conjugate of the state \( S \) is given by: \( \Xi = (1, S, -S_3) \). Explicitly \(^{24}\):
\[
\psi_{(\nu B), (S \Xi)} = \sqrt{\dim(\nu)} (-)^{Q_S} D_{BS}^{(\nu)*},
\]
where \( Q_S \) is a charge corresponding to the SU(3) state \( S \). The explicit dependence on the SU(3) rotation matrix has been suppressed in Eq.\(^{25}\).

Now, by sandwiching \(^{10}\) between the states \(^{21,22}\) we get the following expression for the magnetic moments with \( \mathcal{O}(m_s) \) accuracy:

\(^3\)Here we use \( S \) to denote not only the spin but also the corresponding SU(3) state with hypercharge \(-1\). It is always clear from the context how \( S \) should be understood.
\[ \mu(B) = \mu_0(B) + m_s \mu_1^{(\text{op})}(B) + m_s \mu_1^{(\text{wf})}(B), \]  

(26)

where by \( \mu_0 \) we have denoted the chiral limit part of the magnetic moment, \( \mu_1^{(\text{op})} \) comes from the symmetry breaking in the magnetic moment operator (i.e. from terms proportional to \( w_4, w_5 \) and \( w_6 \)) and \( \mu_1^{(\text{wf})} \) arises from the interference between the \( \mathcal{O}(m_s) \) and \( \mathcal{O}(1) \) parts of the wave functions \([21,22]\) with \( \mu_0 \). It is quite straightforward to evaluate the SU(3) matrix elements required in order to calculate \( \mu_0 \). One has, however, to remember that the wave functions \([23]\) by construction transform under the transformations generated by the operators \( \hat{S} \) as tensors in the representation \( \hat{\nu} \) rather than \( \nu \). Then using the known formulae for integrating the products of the Wigner \( D^{(\nu)} \) functions over the SU(3) group and SU(3) coupling coefficients \([29]\), one gets for the octet \([N, \Lambda, \Sigma, \Xi]\):

\[
\mu_0(B_8) = -\frac{1}{30} \left[ \begin{array}{c} 14T_3 + 1 \\
-3 \\
5T_3 + 3 \\
-4T_3 - 4 \end{array} \right] \left( w_1^3 - \frac{1}{2} w_2 \right) S_3 + \frac{1}{60} \left[ \begin{array}{c} 2T_3 + 3 \\
1 \\
5T_3 - 1 \\
8T_3 - 2 \end{array} \right] w_3 S_3, \tag{27} \]

\[
\mu_1^{(\text{op})}(B_8) = -\frac{1}{270} \left[ \begin{array}{c} 22T_3 - 3 \\
9 \\
9T_3 - 5 \\
-4T_3 + 6 \end{array} \right] w_4 S_3 + \frac{1}{45} \left[ \begin{array}{c} -10T_3 \\
0 \\
-3T_3 + 2 \\
4T_3 - 3 \end{array} \right] w_5 S_3 - \frac{1}{15} \left[ \begin{array}{c} 2T_3 \\
0 \\
-T_3 \\
2T_3 \end{array} \right] w_6 S_3 - \frac{1}{30} \left[ \begin{array}{c} 14T_3 + 1 \\
-3 \\
5T_3 + 3 \\
-4T_3 - 4 \end{array} \right] w_1^2 S_3, \tag{28} \]

\[
\mu_1^{(\text{wf})}(B_8) = -\frac{1}{3} \left[ \begin{array}{c} 2T_3 - 1 \\
0 \\
T_3 - 1 \\
0 \end{array} \right] \left( w_1 + w_2 + \frac{1}{2} w_3 \right) S_3 + \frac{1}{45} c_{27} \left[ \begin{array}{c} 4T_3 + 6 \\
9 \\
4 \\
-4T_3 + 6 \end{array} \right] \left( w_1 + 2w_2 - \frac{3}{2} w_3 \right) S_3, \tag{29} \]

and for the decuplet \([\Lambda, \Sigma^*, \Xi^*, \Omega]\):

\[
\mu_0(B_{10}) = -\frac{1}{12} \left( w_1^3 - \frac{1}{2} w_2 - \frac{1}{2} w_3 \right) Q S_3, \tag{30} \]

\[
\mu_1^{(\text{op})}(B_{10}) = -\frac{1}{252} \left( \frac{1}{3} \left[ \begin{array}{c} 11Q - 13 \\
15Q + 2 \\
19Q + 17 \\
-9Q \end{array} \right] w_4 + 2 \left[ \begin{array}{c} 5Q - 4 \\
3Q - 1 \\
Q + 2 \\
-6Q \end{array} \right] w_5 \right) S_3 - \frac{1}{12} w_1^2 Q S_3, \tag{31} \]

\[
\mu_1^{(\text{wf})}(B_{10}) = -\frac{1}{36} a_{27} \left[ \begin{array}{c} 5Q - 10 \\
6Q - 4 \\
7Q + 2 \\
0 \end{array} \right] \left( w_1 + \frac{1}{2} w_2 + \frac{3}{2} w_3 \right) S_3 + \]
Here $Q$ is the charge and $T_3$ the third component of the isospin of the baryon $B$ while $S_3$ is its spin projection on the third axis.

A remark concerning Eqs. (30, 31) and (32) is in order. In the chiral limit one gets a simple formula in which magnetic moments in decuplet are proportional to the corresponding electric charge. This simple proportionality is broken by the $O(m_s)$ corrections, both for the “operator” and the wave function contributions. Note also that due to the symmetry properties of the Clebsch-Gordan coefficients there is no contribution proportional to $w_6$ for the decuplet. On the contrary, $w_6$ does contribute in the octet case, because of the interference between $8_a$ and $8_s$ SU(3) representations.

III. MAGNETIC MOMENTS IN THE CHIRAL LIMIT

In the case of the chiral limit ($m_s = 0$), i.e. with $U$-spin symmetry unbroken, we can relate the decuplet magnetic moments to those of the octet baryons. We introduce two parameters consisting of $w_1$, $w_2$ and $w_3$:

$$u = \frac{1}{60} \left( w_1 - \frac{1}{2} w_2 \right), \quad w = \frac{1}{120} w_3, \quad (33)$$

with $w_1 = w_1^0 + m_s w_2^0$. Using these two parameters, we can express the octet and decuplet magnetic moments as follows:

$$\mu_p = \mu_{\Sigma^+} = -8u + 4w,$$
$$\mu_n = \mu_{\Xi^0} = 6v + 2w,$$
$$\mu_A = -\mu_{\Sigma^0} = 3v + w,$$
$$\mu_{\Sigma^-} = \mu_{\Xi^-} = 2v - 6w \quad (34)$$

and for the decuplet:

$$\mu_B = -\frac{15}{2} (v - w) Q_B \quad (35)$$

Here $Q_B$ denotes the charge of the baryon $B$.

In addition to the $U$-spin symmetry, one can also see that the generalized Coleman and Glashow sum rules are satisfied in the chiral limit $[30,31]$$^4$

$^4$When we go off the chiral limit, for the purpose of the “model-independent” analysis we include the correction proportional to $w_1^2$ in the definition of $w_1$. 

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$$\frac{1}{84} a_{35} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} (Q + 2) \left( \frac{5}{2} w_2 - \frac{5}{2} w_3 \right) S_3. \quad (32)$$
\[ \mu_{\Sigma^0} = \frac{1}{2} (\mu_{\Sigma^+} + \mu_{\Sigma^-}) , \]
\[ \mu_{\Delta^-} + \mu_{\Delta^+} = \mu_{\Delta^0} + \mu_{\Delta^+} , \]
\[ \sum_{B \in \text{decuplet}} \mu_B = 0. \] (36)

In principle one can fit \( v \) and \( w \) to the experimental data within the octet and then make predictions for the decuplet. However, since the \( U \)-spin symmetry is rather strongly broken in the real world, one cannot expect this procedure to be quantitatively accurate. Indeed:

\[ 20v = 2\mu_n - \mu_p = 3\mu_{\Xi^0} + \mu_{\Xi^-} = -2\mu_{\Sigma^-} - 3\mu_{\Sigma^+} \]
\[ (-6.61) \quad (-4.40) \quad (-5.06) \] (37)

and for \( w \):

\[ 20w = 4\mu_n + 3\mu_p = \mu_{\Xi^0} - 3\mu_{\Xi^-} = -4\mu_{\Sigma^-} - \mu_{\Sigma^+} , \]
\[ (0.73) \quad (0.70) \quad (2.18) \] (38)

where numbers in parenthesis correspond to the experimental values. One can observe very strong disagreement between the theoretical formulae in the chiral limit and the experiment, especially for \( w \). This means, of course, that the sum rules of Eqs.(37,38) will be strongly violated by \( m_s \) corrections. As will be shown in the next section only the mean values of the three terms contributing to \( v \) or \( w \) in Eqs.(37,38) are free of the \( m_s \) corrections. With this in mind we get:

\[ v = -0.268 , \quad w = 0.060 . \] (39)

Had we used in Eqs.(37,38) the nucleon or the \( \Xi \) data alone to fit \( w \) the result would be a factor of 2 smaller.

Using Eq.(39) we obtain the octet magnetic moments:

\[ \mu_p \quad (2.79) = \mu_{\Sigma^+} \quad (2.46) = 2.38 , \]
\[ \mu_n \quad (-1.91) = \mu_{\Xi^0} \quad (-1.25) = -1.49 , \]
\[ \mu_{\Lambda} \quad (-0.61) = -\mu_{\Xi^0} \quad \text{(no data)} = -0.74 , \]
\[ \mu_{\Sigma^-} \quad (-1.16) = \mu_{\Xi^0} \quad (-1.25) = -0.90 \]

and for the decuplet:

\[ \mu_B = 2.46 \, Q_B \]

(in units of nuclear magneton \( \mu_N \)). In particular:

\[ \mu_{\Delta^+} \quad (4.52) = 4.92 \quad \text{and} \quad \mu_{\Omega^-} \quad (-1.94) = -2.46 , \]

where numbers in parenthesis correspond to the experimental data.

In summary let us note that with theoretical formulae derived from the \( \chi \)QSM in the chiral limit with \( \mathcal{O}(1/N_c) \) corrections we are able to describe data with accuracy of the order 25 – 30 % as far as octet baryons are concerned. The same concerns decuplet magnetic
moments where, however, the experimental situation is less clear. It is therefore evident that large corrections due to the strange quark mass are expected.

One more remark is here in order: although in the chiral limit magnetic moments depend on 3 model parameters, namely $w_1$, $w_2$ and $w_3$, only the combination $w_1 - 1/2 w_2 \sim v$ enters. As we shall see in the next section it will be possible to extract $w_1$ and $w_2$ separately only through the wave function corrections (see Eq.(26)).

Let us now turn to the results calculated within the $\chi$QSM by a selfconsistent solution of the equation of motion for the solitonic profile function. In Table I the numerical results are presented, as we vary the constituent quark mass $M = 370$, 400, 420, and 450 MeV. It is found that as the constituent quark mass increases the magnetic moments in general decrease. Once more the importance of the $O(1/N_c)$ corrections can be observed.

It is interesting to extract model predictions for $v$ and $w$ (for $M = 420$ MeV):

$$v^{NJL} = -0.269, \quad w^{NJL} = 0.029.$$  (40)

One can see from Eq.(40) that the model prediction for $w$ is 2 times smaller than the estimate of our “model-independent” analysis. As mentioned after Eq.(39) this value (40) of $w$ is compatible with the nucleon and $\Xi$ magnetic moments but contradicts the data for the $\Sigma$.

In the large $N_c$ limit, isovector magnetic moments satisfy the following relations [15]

$$\frac{\mu_{\Delta^+} - \mu_{\Delta^-}}{\mu_p - \mu_n} = \frac{9}{5} + O\left(\frac{1}{N_c^2}\right),$$

$$\frac{\mu_{\Delta^+} - \mu_{\Delta^0}}{\mu_p - \mu_n} = \frac{3}{5} + O\left(\frac{1}{N_c^2}\right),$$

$$\frac{\mu_{\Sigma^+} - \mu_{\Sigma^-}}{\mu_{\Sigma^0} - \mu_{\Sigma^-}} = \frac{3}{2} + O\left(\frac{1}{N_c^2}\right),$$

$$\frac{\mu_{\Xi^0} - \mu_{\Xi^-}}{\mu_{\Xi^0} - \mu_{\Xi^-}} = -3 + O\left(\frac{1}{N_c^2}\right).$$  (41)

It is interesting to see if the present results for $N_c = 3$ satisfy these relations. In fact, our model is exact in the large $N_c$ limit as we mentioned before. Our results in the chiral limit for $N_c = 3$ agree with Eq.(11), within $1 \sim 4\%$ except for the relation $(\mu_{\Xi^0} - \mu_{\Xi^-})/(\mu_{\Xi^0} - \mu_{\Xi^-})$, from which our result deviates by around 12%. This indicates that the extrapolation from the large $N_c$ to $N_c = 3$ for the above relation is well justified.

IV. MAGNETIC MOMENTS FOR THE FINITE STRANGE QUARK MASS

A. Mass splittings revisited

In order to proceed with the “model-independent” analysis we have to estimate mixing parameters $c_i$ and $a_i$ defined in Eqs.(23,24). For the purpose of this analysis we take the following values for the physical parameters:

$$\Sigma = 48 \text{ MeV}, \quad m_s = 180 \text{ MeV}, \quad \overline{m} = 6 \text{ MeV}.$$  (42)
With this set of parameters:

$$\sigma = 2.67. \quad (43)$$

In order to estimate the ratios $K_i/I_i$ entering Eqs.(24) we observe that:

$$M_\Sigma - M_A = m_s \left( \frac{1}{5} \sigma - \frac{3}{5} \frac{K_1}{I_1} + \frac{2}{5} \frac{K_2}{I_2} \right) = 77 \text{ MeV} \quad (44)$$

and

$$M_\Xi - M_N = m_s \left( \frac{1}{2} \sigma + \frac{1}{2} \frac{K_1}{I_1} + \frac{K_2}{I_2} \right) = 379 \text{ MeV}. \quad (45)$$

From the above equations one obtains:

$$\frac{K_1}{I_1} = 0.51, \quad \frac{K_2}{I_2} = 0.52. \quad (46)$$

For the purpose of the “model-independent” analysis it is convenient to make use of the approximate equality of the ratios:

$$\frac{K_1}{I_1} \approx \frac{K_2}{I_2}. \quad (47)$$

Then:

$$m_{s_1} = c, \quad m_{s_2} = \frac{3}{5} c, \quad m_a_2 = \frac{15}{8} c, \quad m_a_3 = \frac{15}{24} c, \quad (48)$$

where

$$c = 0.14 m_s I_2. \quad (49)$$

Unfortunately $I_2$ cannot be constrained from the mass splittings without making further assumptions about masses of the baryons belonging to higher SU(3) representations entering Eqs.(21,22). In fact $I_2$ is responsible e.g. for the octet antidecuplet splitting. There are theoretical predictions [31] that the lightest member of antidecuplet should have a mass of the order of 1530 MeV. That would mean that $I_2 \sim 0.8 \text{ fm}$; on the other hand the model predictions are somewhat smaller: $I_2 \sim 0.5 - 0.7 \text{ fm}$ depending on the constituent mass $M$. In Ref. [31], which discusses a possible physical interpretation of the antidecuplet it is suggested that $I_2 \sim 0.4 \text{ fm}$. For the purpose of further analysis we leave $c$ (or equivalently $I_2$) as a free parameter.

**B. Magnetic moments of the octet**

Let us define a new set of parameters:

$$x = \frac{1}{540} m_s w_4, \quad y = \frac{1}{90} m_s w_5, \quad z = \frac{1}{30} m_s w_6,$$

$$p = \frac{1}{6} c \left( w_1 + w_2 + \frac{1}{2} w_3 \right), \quad q = -\frac{1}{150} c \left( w_1 + 2 w_2 - \frac{3}{2} w_3 \right). \quad (49)$$
In this notation Eqs.(27,28,29) read:

\[
\begin{bmatrix}
-8 & 4 & -8 & -5 & -1 & 0 & 8 \\
6 & 2 & 14 & 5 & 1 & 2 & 4 \\
3 & 1 & -9 & 0 & 0 & 0 & 9 \\
-8 & 4 & -4 & -1 & 1 & 0 & 4 \\
2 & -6 & 14 & 5 & -1 & 2 & 4 \\
6 & 2 & -4 & -1 & -1 & 0 & 4 \\
2 & -6 & -8 & -5 & 1 & 0 & 8 \\
\end{bmatrix}
\begin{bmatrix}
v \\
w \\
x \\
y \\
z \\
p \\
q \\
\end{bmatrix}
= \begin{bmatrix}
2.79 \\
-1.91 \\
-0.61 \\
2.46 \\
-1.16 \\
-1.25 \\
-0.65 \\
\end{bmatrix}
\begin{bmatrix}
p \\
n \\
\Lambda^0 \\
\Sigma^+ \\
\Sigma^- \\
\Xi^0 \\
\Xi^- \\
\end{bmatrix}.
\]  

(50)

Note that: \(x, y, z, p, q \sim m_s\). This set of equations is not linearly independent; indeed there is one null vector corresponding to the following sum rule derived already in Ref. [8].

\[
12p + 7n - 7\Sigma^- + 22\Sigma^+12 + \Lambda + 3\Xi^+ + 23\Xi^0 = 0,
\]

where symbols denoting particles stand for their corresponding magnetic moments.

In order to solve (50) we first observe that it is possible to find linear combinations of the set of equations (50) which give

\[
\begin{align*}
v &= 0.268 = \frac{2n - p + 3\Xi^0 + \Xi^- - 2\Sigma^- - 3\Sigma^+}{60}, \\
w &= 0.060 = \frac{3p + 4n + \Xi^0 - 3\Xi^- - 4\Sigma^- - \Sigma^+}{60}, \\
z &= 0.080 = \frac{n - \Sigma^- + \Sigma^+ - \Xi^0 - p + \Xi^-}{6},
\end{align*}
\]

(51)

independently of \(x, y, p\) and \(q\). As we see from Eq.(51) \(w\) is free of \(m_s\) corrections only if we take the average of the three expressions given by Eq.(50). This value of \(w\) is larger as the one predicted by the \(\chi\)QSM and this will have consequence for the phenomenological predictions.

Since the set of equations (50) is linearly dependent we can solve it only as a function of one free parameter which we choose to be \(p\). Before we quote results for \(w\), let us note that the solution for \(q\) reads:

\[
q = 0.0118 - 0.1111p.
\]

(52)

The results for \(w\) read:

\[
\begin{align*}
w_1 &= -13.76 \frac{0.229-p}{0.265-p}, \\
w_2 &= 4.63 \frac{0.475-p}{0.265-p}, \\
w_3 &= 7.22, \\
w_4 &= -14.20 - 60.0p, \\
w_5 &= -0.40, \\
w_6 &= -2.40.
\end{align*}
\]

(53)

This set of \(w\) corresponds to

\[
c = 0.29 - 1.09p \quad \text{and} \quad I_2[\text{fm}] = 2.19 - 8.28p.
\]

(54)

\(\)From our discussion at the end of section IV.A we see that for \(I_2 \sim 0.4 - 0.8 \text{ fm} \)

\[
p = 0.17 - 0.22,
\]

(55)
however, strictly speaking, we have no handle to fix $p$ from the octet magnetic moments alone. Therefore, as already said, we leave $p$ as a free parameter for the time being.

Before proceeding to the discussion of the decuplet let us observe that with the set of parameters of Eqs.\((52,53)\):

$$\Sigma_8 = \sum_{B \in \text{octet}} \mu_B = 0.53 \quad (\text{exp.} \ 0.35).$$

It is important to observe that this sum is entirely given in terms of the wave function corrections, it is namely equal:

$$\sum_{B \in \text{octet}} \mu_B = 5p + 45q \quad (57)$$

which in view of Eq.\((52)\) is independent of $p$. In contrast to the $\chi$QSM or the Skyrme Model our “model-independent” analysis gives positive number for $\Sigma_8$. This is precisely due to the larger value for $w \sim w_3$ which is required by our fitting procedure.

C. “Model-independent” analysis of decuplet magnetic moments

Before applying the results of the previous section to the decuplet case let us first discuss sum rules which result from Eqs.\((30,31,32)\) which can be conveniently rewritten in a following form:

$$\begin{bmatrix}
-2 \\
-1 \\
0 \\
1 \\
-1 \\
0 \\
1 \\
1
\end{bmatrix}
+ \begin{bmatrix}
-9 \\
2 \\
13 \\
24 \\
-17 \\
-2 \\
13 \\
-9
\end{bmatrix}t + \begin{bmatrix}
-6 \\
-1 \\
4 \\
9 \\
-2 \\
1 \\
4 \\
-6
\end{bmatrix} \xi + \begin{bmatrix}
0 \\
5 \\
10 \\
15 \\
-2 \\
4 \\
10 \\
5
\end{bmatrix} \eta + \begin{bmatrix}
-4 \\
-3 \\
-2 \\
-1 \\
-6 \\
-4 \\
-2 \\
-3
\end{bmatrix} r + \begin{bmatrix}
\Delta^{++} \\
\Delta^+ \\
\Delta^0 \\
\Delta^- \\
\Sigma^{++} \\
\Sigma^{*0} \\
\Sigma^{*-} \\
\Xi^0 \\
\Xi^{-} \\
\Omega^-
\end{bmatrix} s = 0 \quad (58),$$

where

$$t = \frac{1}{8} \left( w_1 - \frac{1}{2} w_2 - \frac{1}{2} w_3 \right) = \frac{15}{2} (v - w),$$

$$r = \frac{5}{64} c \left( w_1 + \frac{1}{2} w_2 + \frac{3}{2} w_3 \right),$$

$$s = \frac{5}{448} c \left( w_1 + \frac{5}{2} w_2 - \frac{5}{2} w_3 \right),$$

$$\xi = \frac{15}{14} x = \frac{1}{504} m_s w_4, \quad \eta = \frac{15}{14} y = \frac{1}{84} m_s w_5. \quad (59)$$

Equation \((58)\) without wave function corrections, i.e. with $r = s = 0$ is identical to the one derived in Ref.\(8\). The authors of Ref.\(8\) diagonalize the splitting Hamiltonian \((19)\)
up to all orders in $m_s$ via the procedure introduced by Yabu and Ando [28]. Our procedure is perturbative in $m_s$ and as a result we are able to study analytically rather than only numerically the influence of the wave function corrections on the magnetic moments. Moreover, similarly to the octet case, we are able to derive sum rules for the decuplet magnetic moments. Note that in the chiral limit ($x = y = r = s = 0$) the sum of magnetic moments is proportional to the sum of charges, which is a simple consequence of the fact that the first column of Eq.(58) is simply $-Q$ (charge). For the finite $m_s$ only some of these sum rules survive:

$$
-4\Delta^{++} + 6\Delta^+ + 3\Sigma^{*+} - 6\Sigma^{*0} + \Omega^- = 0,
-2\Delta^{++} + 3\Delta^+ + 2\Sigma^{*+} - 4\Sigma^{*0} + \Xi^{*-} = 0,
-\Delta^{++} + 2\Delta^+ - 2\Sigma^{*0} + \Xi^{*0} = 0,
\Sigma^{*+} - 2\Sigma^{*0} + \Sigma^{*-} = 0,
2\Delta^{++} - 3\Delta^+ + \Delta^- = 0,
\Delta^{++} - 2\Delta^+ + \Delta^0 = 0.
$$

Moreover, we can calculate $t$ from the following differences:

$$
t = \frac{1}{3} (\Omega^- - \Delta^{++}) = \frac{1}{2} (\Xi^{*-} - \Delta^+) = \Xi^{*0} - \Sigma^{*+} = \Sigma^{*-} - \Delta^0.
$$

These relations are interesting since they are related to the octet splittings:

$$
\Delta^{++} - \Omega^- = \frac{3}{4} \left(2p + n + \Sigma^+ - \Sigma^- - \Xi^0 - 2\Xi^-\right).
$$

Experimentally: $6.46 \pm 0.59 = 7.38$, where we have neglected errors on the right hand side. As already remarked in the Introduction the error on the left hand side which corresponds to the recent measurement of Ref. [1] would dramatically increase if the world average for $\mu_{\Delta^{++}}$ would have been used.

Finally, let us present the formulae for the decuplet magnetic moments as functions of $p$. Because of very weak dependence on $p$ of the only measured magnetic moments, namely $\Delta^{++}$ and $\Omega^-$, $p$ cannot be constrained from the existing decuplet data. Therefore in the second and the third column of Table II we present two limiting cases corresponding to the limits on $p$ somewhat extended with respect to Eq.(55).

---

5 One should keep in mind that $t$ is also $m_s$ dependent through $w_1^2 m_s$; see Eq.(16).

6 For example, the mixing parameter $c_{i0}$ used in Ref. [11] corresponds to $p = 0.232$. 
D. Model predictions for the decuplet

In Table III, the numerical results of the decuplet magnetic moments, based on a self-consistent solitonic profile function, are presented. We can compare now our final results with the experimental data for $\mu_{\Delta^{++}}$ and $\mu_{\Omega}$. It is found that the present model is in a remarkable agreement with the data within about 10%. The $m_s$ corrections are by no means negligible. Their largest contribution is about 30% and occurs in the $\mu_{\Delta^-}$.

Similar conclusions have been drawn in Ref. [13], where both octet and decuplet magnetic moments have been calculated in the model identical to ours. There are however small quantitative differences about which we would like to comment. The authors of Ref. [13] present their results for the constituent quark mass of 400 MeV with a two-step regularization function as in Ref. [24], however with slightly different parameters. They also use soliton profile function with massive pion tail corresponding to the non-strange quark mass of 6 MeV. With these parameters in the chiral limit they get results which are only slightly different from ours, namely they get for $t_{\text{chiral}} = -2.28$ (see Eq.(58)), whereas we get $-2.34$. For $m_s \neq 0$ the differences are, however, larger. In our case magnetic moments of $\Delta^{++}$ and $\Omega^-$ increase with $m_s$ in absolute value, whereas in Ref. [13] they decrease. Moreover the magnetic moment of $\Xi^{*-}$ is independent of $m_s$ in [13], whereas in our case it increases in absolute value. We have explicitly checked that the results of Ref. [13] fulfill the sum rules of Eqs.(54) and (55).

Similar discrepancies have been found by authors of Ref. [13] in connection with the previous calculations of the octet magnetic moments [11] by two of the present authors. They attributed these discrepancies to the differences in the soliton profile; this is, however, not correct since in both cases the massive pion tail has been used. The differences in the decuplet case are merely a reflection of the same problem to which at the moment we do not see any straightforward explanation.

V. SUMMARY AND CONCLUSIONS

In the present paper the magnetic moments of baryons have been studied within the chiral quark soliton model with special emphasis on the decuplet of baryons. We have adopted the strategy in which the model has served as a tool to identify symmetry breaking terms proportional to $m_s$. Two sources of such terms are present in the model: (i) $m_s$ corrections to the magnetic moment operator and (ii) wave function corrections which involve higher SU(3) flavor representations. Having worked in perturbation theory in the symmetry breaker we have derived analytical formulae for magnetic moments both for the octet and decuplet.

For completeness also rotational $1/N_c$ corrections have been included. They are important for model calculations with the explicit solitonic profile function, however, they do not introduce any new algebraic structure in the general expressions for the magnetic moments.

Symmetry breaking pattern for the magnetic moments is much more involved than the one for the baryon masses, where Gell-Mann–Okubo mass formulae work very well with only 2 free parameters. Here the number of free parameters in the chiral limit is already 3 and one has 3 more if one goes off the chiral limit. Two more constants enter if one takes into account wave function corrections. It is possible to constrain all but one, called $p$, free parameters from the mass spectrum and octet magnetic moments. It is interesting that
is related to the splitting between well established octet or decuplet and exotic baryon multiplets such as spin 1/2 antidecuplet \[31\]. Unfortunately the only two known decuplet magnetic moments, namely that of \(\Delta^{++}\) and \(\Omega^-\), do not provide any handle on this splitting, since they are almost independent of \(p\). In this respect precise measurements of \(\Delta^0\), \(\Delta^-\) or \(\Sigma^{*-}\) could serve as a tool to extract \(p\) and, in consequence, the masses of the exotic states.

We have also found sum rules which the magnetic moments satisfy in this order of the perturbation theory.

Finally we have presented model results for the decuplet magnetic moments. Our results agree with the existing data reasonably well, however, as far as the value of \(w_3\) is concerned, there is for the value of \(w_3\) substantial difference between the numerics of our “model-independent” analysis (which merely uses the algebraic structure of the \(\chi\)QSM) and actual model calculations, based on selfconsistent profile functions. This is reflected in the sign difference between “model-independent” predictions and model results for \(\Delta^0\) and \(\Sigma^{*0}\). In general the two sets of predictions agree each other within 20%. At this point it is important to stress that this relatively good agreement could be achieved only by taking into account rotational \(1/N_c\) corrections. This can be clearly seen from Table III. Similarly to the axial constants \[20\], magnetic moments would be factor of 2 off experimental data if these corrections were discarded.

We have compared our results with those of Ref. \[13\]. In the chiral limit the agreement between two papers is quite satisfactory. However, we have found some numerical differences for the finite strange quark mass. Since the parameters and numerical procedures used in both cases are very similar these differences are somewhat surprising and require further study.

Let us stress at the end that parameters \(w_i\) extracted from the data in the “model-independent” analysis may be used to predict some of the constants governing semileptonic hyperon decays \[32\].

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APPENDIX

In this appendix, we present all formulae appearing in Eq.(16).

\[
I_1 = \frac{N_c}{6} \sum_n \int d^3x \int d^3y \left[ \Psi_{n}(x) \tau \Psi_{val}(x) \cdot \Psi_{val}(y) \tau \Psi_{n}(y) \frac{E_n - E_{val}}{E_n - E_{val}} \right] \\
+ \frac{1}{2} \sum_m \left[ \Psi_{m}^{\dagger}(x) \tau \Psi_{n}(x) \cdot \Psi_{m}^{\dagger}(y) \tau \Psi_{n}(y) R_x(E_n, E_m) \right],
\]

\[
I_2 = \frac{N_c}{6} \sum_{m^0} \int d^3x \int d^3y \left[ \Psi_{m^0}^{\dagger}(x) \Psi_{val}(x) \Psi_{val}(y) \Psi_{m^0}(y) \frac{E_{m^0} - E_{val}}{E_{m^0} - E_{val}} \right]
\]
\[ K_1 = - \frac{N_c}{6} \sum_n \int d^3 x \int d^3 y \left[ \frac{\Psi_n^\dagger(x) \Psi_m^0(x) \Psi_{m^0}^\dagger(y) \Psi_n(y) R_L(E_n, E_{m^0})}{E_n - E_{val}} \right], \]

\[ K_2 = - \frac{N_c}{6} \sum_{m^0} \int d^3 x \int d^3 y \left[ \frac{\Psi_{m^0}^\dagger(x) \Psi_m^0(x) \Psi_{m^0}^\dagger(y) \Psi_n(y) R_M(E_n, E_{m^0})}{E_{m^0} - E_{val}} \right], \]

\[ Q_0 = - \frac{N_c}{3} \int d^3 x \left[ \Psi_{val}(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_{val}(x) \right] \]

\[ - \frac{1}{2} \sum_n \text{sgn}(E_n) \Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_n(x) R(E_n), \]

\[ Q_1 = \frac{i N_c}{6} \sum_n \int d^3 x \int d^3 y \left[ \frac{\Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_{val}(x) \Psi_n^\dagger(y) \tau \Psi_n(y)}{E_n - E_{val}} \right], \]

\[ Q_2 = - \frac{N_c}{6} \sum_{m^0} \int d^3 x \int d^3 y \left[ \frac{\Psi_{m^0}^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_{val}(x) \Psi_{m^0}^\dagger(y) \Psi_n(y)}{E_{m^0} - E_{val}} \right], \]

\[ Q_3 = \frac{N_c}{3} \sum_n \int d^3 x \int d^3 y \left[ \frac{\Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_{val}(x) \Psi_n^\dagger(y) \tau \Psi_n(y)}{E_n - E_{val}} \right], \]

\[ Q_4 = \frac{N_c}{3} \sum_{m^0} \int d^3 x \int d^3 y \left[ \frac{\Psi_{m^0}^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_{val}(x) \Psi_{m^0}^\dagger(y) \Psi_n(y)}{E_{m^0} - E_{val}} \right], \]

\[ M_0 = \frac{N_c}{9} \sum_n \int d^3 x \int d^3 y \left[ \frac{\Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_{val}(x) \Psi_n^\dagger(y) \beta \Psi_n(y)}{E_n - E_{val}} \right], \]

\[ + \frac{1}{2} \sum_{m} \Psi_m^\dagger(x) \gamma_5 \{ r \times \sigma \} \cdot \tau \Psi_m(x) \Psi_m^\dagger(y) \beta \Psi_n(y) R_{\beta}(E_n, E_m) \]
\[ M_1 = \frac{N_c}{9} \sum_n \int d^3 x \int d^3 y \]
\[ \times \left[ \frac{\Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \Psi_{\text{val}}(x) \cdot \Psi_{\text{val}}^\dagger(y) \beta \tau \Psi_n(y)}{E_n - E_{\text{val}}} \right. \]
\[ + \frac{1}{2} \sum_m \Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \Psi_m(x) \cdot \Psi_m^\dagger(y) \beta \tau \Psi_n(y) R_\beta(E_n, E_m) \right] \]
\[ M_2 = \frac{N_c}{9} \sum_{m^0} \int d^3 x \int d^3 y \]
\[ \times \left[ \frac{\Psi_{m^0}^\dagger(x) \gamma_5 \{ r \times \sigma \} \Psi_{\text{val}}(x) \tilde{\Psi}_{m^0}(y) \beta \tau \Psi_n(y)}{E_{m^0} - E_{\text{val}}} \right. \]
\[ + \sum_n \Psi_n^\dagger(x) \gamma_5 \{ r \times \sigma \} \Psi_{m^0}(x) \tilde{\Psi}_{m^0}(y) \beta \Psi_n(y) R_\beta(E_n, E_{m^0}) \right] . \quad (63) \]

The regularization functions in Eq. (63) are as follows:

\[ R_I(E_n, E_m) = -\frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \phi(u; \Lambda_i) \left[ \frac{E_n e^{-uE_n^2} + E_m e^{-uE_m^2}}{E_n + E_m} + \frac{e^{-uE_n^2} - e^{-uE_m^2}}{u(E_n^2 - E_m^2)} \right] , \]

\[ R_M(E_n, E_m) = \frac{1}{2} \frac{\text{sgn}(E_n) - \text{sgn}(E_m)}{E_n - E_m} , \]

\[ R(E_n) = \int \frac{du}{\sqrt{\pi u}} \phi(u; \Lambda_i) |E_n| e^{-uE_n^2} , \]

\[ R_Q(E_n, E_m) = \frac{1}{2\pi} c_i \int_0^1 d\alpha \frac{\alpha(E_n + E_m) - E_m}{\sqrt{\alpha(1 - \alpha)}} \frac{\exp \left( -[\alpha E_n^2 + (1 - \alpha) E_m^2]/\Lambda_i^2 \right)}{\alpha E_n^2 + (1 - \alpha) E_m^2} , \]

\[ R_\beta(E_n, E_m) = \frac{1}{2\sqrt{\pi}} \int_0^\infty \frac{du}{\sqrt{u}} \phi(u; \Lambda_i) \left[ \frac{E_n e^{-uE_n^2} - E_m e^{-uE_m^2}}{E_n - E_m} \right] . \quad (64) \]

where the cutoff function \( \phi(u; \Lambda_i) = \sum_i c_i \theta \left( u - \frac{1}{\Lambda_i} \right) \) is fixed by reproducing the pion decay constant and other mesonic properties [16].
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TABLE I. The dependence of the magnetic moments of the SU(3) decuplet baryons on the constituent quark mass $M$ without $m_s$ corrections: $\mu(\Omega^0)$ corresponds to the leading order in the rotational frequency while $\mu(\Omega^1)$ includes the subleading order. The unit of magnetic moments are given in nuclear magneton $\mu_N$. The numbers are obtained by using a selfconsistently calculated solitonic profile function.

| Baryon | 370 MeV | 400 MeV | 420 MeV | 450 MeV | Exp     |
|--------|---------|---------|---------|---------|---------|
| $\Delta^{++}$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ | $\mu_B(\Omega^0)$ | $\mu_B(\Omega^1)$ |
| $\Delta^+$ | 2.11 | 0.74 | 2.16 | 0.77 | 2.19 | 0.80 | 2.22 | 0.83 | 2.25 | 0.86 | 2.28 | 0.89 | 2.31 | 0.92 | 2.34 | 0.95 | 2.37 | 0.98 | 2.40 |
| $\Delta^0$ | 1.05 | 0.57 | 1.15 | 0.60 | 1.24 | 0.63 | 1.34 | 0.66 | 1.44 | 0.69 | 1.54 | 0.72 | 1.64 | 0.75 | 1.74 | 0.78 | 1.84 | 0.81 | 1.94 |
| $\Delta^-$ | 0.60 | 0.29 | 0.71 | 0.34 | 0.82 | 0.38 | 0.94 | 0.42 | 1.05 | 0.46 | 1.17 | 0.50 | 1.29 | 0.54 | 1.42 | 0.58 | 1.56 | 0.62 | 1.71 |

TABLE II. “Model-independent” results to linear order in $m_s$ for decuplet magnetic moments as functions of $p$ defined in Eq. (49). The unit of magnetic moments are given in nuclear magneton $\mu_N$.

| Baryons | $p = 0.15$ | $p = 0.25$ | Exp     |
|---------|------------|------------|---------|
| $\Delta^{++}$ | $5.322 + 0.087p$ | $5.33$ | $5.34$ | $4.52 \pm 0.5$ |
| $\Delta^+$ | $2.847 - 0.715p$ | $2.74$ | $2.67$ | $--$ |
| $\Delta^0$ | $0.371 - 1.516p$ | $0.14$ | $-0.01$ | $--$ |
| $\Delta^-$ | $-2.104 - 2.318p$ | $-2.45$ | $-2.68$ | $--$ |
| $\Sigma^{*+}$ | $2.987 + 0.442p$ | $3.05$ | $3.10$ | $--$ |
| $\Sigma^{*0}$ | $0.449 - 0.537p$ | $0.37$ | $0.32$ | $--$ |
| $\Sigma^{*-}$ | $-2.089 - 1.516p$ | $-2.32$ | $-2.47$ | $--$ |
| $\Xi^{*0}$ | $0.527 + 0.442p$ | $0.59$ | $0.64$ | $--$ |
| $\Xi^{*-}$ | $-2.054 - 0.715p$ | $-2.18$ | $-2.25$ | $--$ |
| $\Omega^{-}$ | $-2.060 + 0.087p$ | $-2.05$ | $-2.04$ | $-1.94 \pm 0.31$ |
TABLE III. The magnetic moments of the SU(3) decuplet baryons with $m_s$ corrections:
$\mu(\Omega^0, m_s^0)$ corresponds to the leading order in the rotational frequency while $\mu(\Omega^1, m_s^0)$ includes the subleading order. $\mu(\Omega^1, m_s^1)$ represents our final results. The constituent quark mass $M$ is fixed to be 420 MeV. The unit of magnetic moments are given in nuclear magneton $\mu_N$. The numbers are obtained by using a selfconsistently calculated solitonic profile function.

| Baryons | $\mu_B(\Omega^0, m_s^0)$ | $\mu_B(\Omega^1, m_s^0)$ | $\mu_B(\Omega^1, m_s^1)$ | Exp.  |
|---------|--------------------------|--------------------------|--------------------------|-------|
| $\Delta^{++}$ | 1.88 | 4.47 | 4.73 | $4.52 \pm 0.50$ |
| $\Delta^+$ | 0.94 | 2.23 | 2.19 | — |
| $\Delta^0$ | 0 | 0 | −0.35 | — |
| $\Delta^-$ | −0.94 | −2.23 | −2.90 | — |
| $\Sigma^{*+}$ | 0.94 | 2.23 | 2.52 | — |
| $\Sigma^{*0}$ | 0 | 0 | −0.08 | — |
| $\Sigma^{*-}$ | −0.94 | −2.23 | −2.69 | — |
| $\Xi^{*0}$ | 0 | 0 | 0.19 | — |
| $\Xi^{*-}$ | −0.94 | −2.23 | −2.48 | — |
| $\Omega^-$ | −0.94 | −2.23 | −2.27 | $−1.94 \pm 0.31$ |