Abstract—Inter-area oscillations in bulk power systems are typically poorly controllable by means of local decentralized control. Recent research efforts have been aimed at developing wide-area control strategies that involve communication of remote signals. In conventional wide-area control, the control structure is fixed a priori typically based on modal criteria. In contrast, here we employ the recently-introduced paradigm of sparsity-promoting optimal control to simultaneously identify the optimal control structure and optimize the closed-loop performance. To induce a sparse control architecture, we regularize the standard quadratic performance index with an $\ell_1$-penalty on the feedback matrix. The quadratic objective functions are inspired by the classic slow coherency theory and are aimed at imitating homogeneous networks without inter-area oscillations. We use the New England power grid model to demonstrate that the proposed combination of the sparsity-promoting control design with the slow coherency objectives performs almost as well as the optimal centralized control while only making use of a single wide-area communication link. In addition to this nominal performance, we also demonstrate that our control strategy yields favorable robustness margins and that it can be used to identify a sparse control architecture for control design via alternative means.

Index Terms—wide-area control, inter-area modes, sparsity-promoting control, alternating direction method of multipliers

I. INTRODUCTION

Bulk power systems typically exhibit multiple electromechanical oscillations. Local oscillations are characterized by single generators swinging relative to the rest of the network, whereas inter-area oscillations are associated with the dynamics of power transfers and involve groups of generators oscillating relative to each other. With steadily growing demand, deployment of renewables in remote areas, and deregulation of energy markets, long-distance power transfers outpace the addition of new transmission facilities. These developments lead to a maximum use of the existing network, result in smaller stability margins, and cause inter-area modes to be ever more lightly damped. In a heavily stressed grid, poorly damped inter-area modes can even become unstable. For example, the blackout of August 10, 1996, resulted from an instability of the 0.25Hz mode in the Western interconnected system [2].

Local oscillations are typically damped by generator excitation control via power system stabilizers (PSSs) [3]. However, these decentralized control actions can interact in an adverse way and destabilize the overall system [4]. Sometimes inter-area modes cannot be even stabilized by PSSs [5], unless sufficiently many and carefully tuned PSSs are deployed [6]–[8].

Regarding tuning of conventional PSSs, high-gain feedback is necessary in some networks [5] but may destabilize other networks [6], [7]. Furthermore, even when decentralized controllers provide stability they may result in poor performance.

In principle, the above problems can be solved by distributed wide-area control (WAC), where controllers make use of remote measurements and control signals. WAC is nowadays feasible thanks to recent technological advances including fast and reliable communication networks, high-bandwidth and time-stamped phasor measurement units (PMUs), and flexible AC transmission system (FACTS) devices. We refer to the surveys [9]–[11] and the articles in [12] for a detailed account of technological advances and capabilities. Several efforts have been directed towards WAC of oscillations based on robust and optimal control methods; see [12]–[18] and references therein. The chosen performance metrics include frequency domain and root-locus criteria, signal amplifications from disturbance inputs to tie line flows, inter-area angles, or machine speeds.

Typically, the controllers are designed for pre-specified sensor and actuator locations and an a priori fixed sparsity pattern that induces the necessary communication structure. In an attempt to identify optimal sensor or actuator placements and to reduce the communication complexity and the interaction between control loops, different strategies aim at identifying few but critical control channels [18]–[21]. These strategies rely on modal perspectives and aim at maximizing geometric metrics such as modal controllability and observability residues. As a result, the control channels are typically chosen through combinatorial SISO criteria and not in an optimal way.

Another body of literature relevant to our study is optimal control subject to structural constraints, for example, a desired sparsity pattern of the feedback matrix in static state feedback design [22]. In general, control design subject to structural constraints is hard, stabilizability is not guaranteed, and optimal control formulations are not convex for arbitrary structural constraints [23]. Furthermore, in the absence of pre-specified structural constraints, most optimal control formulations result in controllers without any sparsity structure according to which measurements and control channels can be selected, and they typically require a centralized implementation. In order to overcome these limitations of decentralized optimal control, alternative strategies have recently been proposed that simultaneously identify the control structure and optimize the closed-loop performance; see [24]–[28]. The proposed strategies combine classic optimal and robust control formulations with recent advances in compressed sensing.

Here we investigate a novel approach to WAC design for bulk power systems. We follow the sparsity-promoting optimal control approach developed in [27] and find a linear static state
feedback that simultaneously optimizes a standard quadratic cost criterion and enhances a sparse control structure. Our choice of performance criterion is inspired by the classic work \cite{29}-\cite{31} on slow coherency. We propose a novel performance criterion that encourages the closed-loop system to imitate a homogeneous network of identical generators with no inter-area oscillations. Besides this physical insight, an additional advantage of our performance criterion is that the optimal controller makes use of readily accessible state variables such as angles and frequencies. In the performance assessment of our sparsity-promoting optimal control synthesis, we emphasize robustness to time delays, tuning uncertainties, and variable operating conditions arising in WAC implementation.

We consider a coordinated PSS design for the IEEE 39 New England power grid model to illustrate the utility of our approach. This compelling example shows that, with only a single WAC link, it is possible to achieve nearly the same performance achieved by the optimal centralized controller. Besides this nominal performance level, we also show that the identified control structure is not sensitive to the chosen operating point, and that robustness margins gracefully deteriorate as the control structure becomes increasingly sparse. Finally, we show that our approach can also be used to identify a sparse architecture for control design via alternative means. In particular, we use nonlinear simulations to demonstrate that the closed-loop performance can be significantly improved with a proportional feedback of a single remote measurement (identified by our sparsity-promoting framework).

The remainder of the paper is organized as follows: Section \[4\] describes the sparsity-promoting optimal control approach with the slow coherency objectives. Section \[III\] shows the implementation and a link to our source code can be found in Appendix \[A\]. Finally, Section \[IV\] concludes the paper.

II. PROBLEM SETUP AND THEORETICAL FRAMEWORK

A. Modeling of generation, transmission, and control

A power network is described by the dynamics of generators and control equipment as well as the algebraic load flow, generator stator, and power electronic circuit equations. Some loads are also modeled by detailed differential-algebraic equations \[5\]. Here, we initially consider a detailed, nonlinear, and differential-algebraic power network model of the form

\[
\dot{x}(t) = f(x(t), z(t), u(t), \eta(t)),
0 = g(x(t), z(t), u(t), \eta(t)),
\]

where the dynamic and algebraic variables \(x(t) \in \mathbb{R}^n\) and \(z(t) \in \mathbb{R}^q\) constitute the state, \(u(t) \in \mathbb{R}^p\) is the control action through either power electronics (FACTS), generator excitation (PSS) or governor control, and \(\eta(t) \in \mathbb{R}^q\) is a white noise signal accounting for fluctuations in generation and load.

Next we linearize the system (1) at a stationary operating point, solve the resulting linear algebraic equations for the variable \(z(t)\), and arrive at the linear state-space model

\[
\dot{x}(t) = Ax(t) + B_1\eta(t) + B_2u(t),
\]

where \(A \in \mathbb{R}^{n \times n}, B_1 \in \mathbb{R}^{n \times q}, \) and \(B_2 \in \mathbb{R}^{n \times p}.\)

B. Review of slow coherency theory

To obtain an insightful perspective on inter-area oscillations, we briefly recall the classic slow coherency theory \cite{29}-\cite{31}. Let the state variable \(x\) of the power network model (1) (or its linearization (2)) be partitioned as \(x = [\theta^T, \dot{\theta}^T, z^T]\), where \(\theta, \dot{\theta} \in \mathbb{R}^n\) are the rotor angles and frequencies of \(n\) synchronous generators, and \(z \in \mathbb{R}^{n-2n}\) are the remaining state variables, which typically correspond to fast electrical dynamics. In the absence of higher-order generator dynamics, for purely inductive lines, and for constant-current loads, the power system dynamics (1) can be reduced to the electromechanical swing dynamics of the generators \[4\].

\[
M_i\ddot{\theta}_i + D_i\dot{\theta}_i = P_{red,i} - \sum_{j=1}^{n_g}[Y_{red,ij} E_i E_j \sin(\theta_i - \theta_j)],
\]

Here, \(M_i\) and \(D_i\) are the inertia and damping coefficients, \(E_i\) is the q-axis voltage, \(P_{red,i}\) is the generator power injection in the network-reduced model, and \(Y_{red}\) is the Kron-reduced admittance matrix \[32\]. When the swing equations (3) are linearized at an operating point \((\theta^*, \dot{\theta}^*)\), they read as

\[
M\ddot{\theta} + D\dot{\theta} + L\theta = 0,
\]

where \(M\) and \(D\) are the diagonal matrices of inertia and damping coefficients, and \(L\) is a Laplacian matrix with off-diagonals \(L_{ij} = -[Y_{red,ij}/E_i E_j \cos(\theta_*^i - \theta_*^j)]\) and diagonal elements \(L_{ii} = -\sum_{j=1,j\neq i}^{n_g}L_{ij}\). Notice that (4) is a dissipative mechanical system with kinetic energy \((1/2) \cdot \dot{\theta}^T M \dot{\theta}\) and potential energy \((1/2) \cdot \theta^T L \theta\). Since \(M\) and \(D\) are diagonal, the interactions among generators in (4) are entirely described by the weighted graph induced by the Laplacian matrix \(L\).

Inter-area oscillations may arise from non-uniform inertia and damping coefficients (resulting in slow and fast responses), clustered groups of machines (swinging coherently) and sparse interconnections among them, as well as large inter-area power transfers. Assume that the set of generators \(\mathcal{V} = \{1, \ldots, n_g\}\) is partitioned in multiple coherent (and disjoint) groups of machines (or areas), that is, \(\mathcal{V} = \mathcal{V}_{\text{loc}} \cup \mathcal{V}_{\text{rem}} \cup \ldots\). Then it can be shown \cite{29}-\cite{31} that the long-time dynamics of each area \(\alpha\) with nodal set \(\mathcal{V}_\alpha\) are captured by the aggregate variable

\[
\delta_\alpha = \left(\sum_{i \in \mathcal{V}_\alpha} M_i \theta_i\right)/\left(\sum_{i \in \mathcal{V}_\alpha} M_i\right)
\]

describing the center of mass of area \(\alpha\). The slow inter-area dynamics are obtained as

\[
\ddot{\delta}_\alpha + D\dot{\delta}_\alpha + L\delta = 0,
\]

where \(\delta = [\delta_\alpha, \delta_\beta, \ldots]^T\) and \(\ddot{\delta}, \dot{\delta}, \) and \(L\) are the aggregated inertia, dissipation, and Laplacian matrices.

C. Local and wide-area control design

As illustrated in the block diagram\[5\] in Fig. 1 we seek for linear time-invariant control laws and follow a two-level control strategy of the form \(u(t) = u_{\text{sec}}(t) + u_{\text{wac}}(t)\).

In the first step, the local control \(u_{\text{sec}}(t)\) is designed based on locally available measurements and with the objective of

1A large power transfer between two nodes \(i, j\) in distinct areas amounts to a large steady state difference angle \(|\theta_*^i - \theta_*^j|\). Equivalently, in the linearized model (4), the coupling \(|L_{ij}| = |Y_{red,ij}/E_i E_j \cos(\theta_*^i - \theta_*^j)|\) is small.

2In Fig. 1 the system dynamics (A matrix) includes the power network dynamics and the local control loops. The WAC signal is added to the PSS input. Alternatively, the WAC signal can be added to the automatic voltage regulator (AVR). Both strategies have been advocated, see \cite{6}, \cite{17}, \cite{18}.
stabilizing each isolated component. For example, \( u_{\text{loc}}(t) \) can be obtained by a conventional PSS design with the objective of suppressing local oscillations \([3]\). Next, the wide-area control \( u_{\text{wac}}(t) \) is designed to enhance the global system behavior and suppress inter-area oscillations.

For this second design step, the local control \( u_{\text{loc}}(t) \) is assumed to be absorbed into the plant \([2]\). Additionally, since \( u_{\text{wac}}(t) \) relies on the communication of remote signals, the two-level control strategy guarantees a nominal performance level in case of communication failures.

D. Sparsity-promoting linear quadratic control

As discussed in Section I, an inherent problem in WAC is the proper choice of control architecture, that specifies which quantities need to be measured and which controller needs to access which measurement. Here, we confine our attention to quantities need to be measured and which controller needs to the proper choice of control architecture of remote signals, the two-level control strategy guarantees a

An equivalent formulation of the optimal control problem \([6]\) is given via the closed-loop observability Gramian \( P \) as

\[
\begin{aligned}
J_\gamma(K) &\triangleq \text{trace} \left( B^T P B_1 \right) + \gamma \sum_{i,j} w_{ij} |K_{ij}| \\
&\quad \text{subject to} \quad (A - B_2 K)^T P + P(A - B_2 K) = -(Q + K^T R K).
\end{aligned}
\]

The latter formulation \((7)\) is amenable to an iterative solution strategy using the alternating direction method of multipliers (ADMM); see \([25], [27]\). The cost function in \((7)\) is a linear combination of the \( \mathcal{H}_2 \)-norm of the closed-loop system

\[
\begin{aligned}
\dot{x}(t) &= (A - B_2 K) x(t) + B_1 \eta(t), \\
y(t) &= \begin{bmatrix} Q^{1/2} \\ -R^{1/2} K \end{bmatrix} x(t),
\end{aligned}
\]

and the sparsity-promoting term \( \gamma \sum_{i,j} w_{ij} |K_{ij}| \). In what follows, for a fixed value of \( \gamma \geq 0 \), we denote the minimizer to \((7)\) by \( K_* \) and the optimal cost by \( J_* = J(K_*) \). For \( \gamma = 0 \) the problem \((7)\) reduces to the standard state-feedback \( \mathcal{H}_2 \)-problem \([34]\) with the optimal gain \( K_0^* \) and the optimal cost \( J_0^* \). On the other hand, for \( \gamma > 0 \) the weighted \( \ell_1 \)-norm promotes sparsity in the feedback gain \( K_* \), thereby identifying essential pairs of control inputs and measured outputs.

To solve the sparsity-promoting optimal control problem \((7)\), we rely on the approach proposed in \([27]\) using ADMM. See Appendix A for details on the algorithmic implementation.

E. Choice of optimization objectives

The design parameters \( Q, R, B_1, \gamma \) need to be chosen with the objective of damping inter-area oscillations. The resulting feedback \( u_{\text{wac}}(t) \), the control variables \( K_* x(t) \), the communication structure (the sparsity pattern of the off-diagonals of \( K_* \)), and the control effort depend solely on \( Q, R, B_1 \), and \( \gamma \).

State cost: Slow coherency theory shows that an ideal power system without inter-area oscillations is characterized by uniform inertia coefficients and homogenous power transfers among generators. Equivalently, the Laplacian and inertia matrices in the linearized swing equations \([4]\) take the form

\[
L = L_{\text{unit}} = \ell \cdot (I_{n_g} - (1/n_g) I_{n_q} I_{n_q}^T), \quad M = M_{\text{unit}} = m \cdot I_{n_q},
\]

where \( \ell, m > 0 \) are constants, \( I_{n_g} \) is the \( n_g \)-dimensional identity matrix, and \( I_{n_q} \) is the \( n_q \)-dimensional vector of ones.

Inspired by the above considerations, we choose the following performance specifications for the state cost:

\[
x^T Q x = \frac{1}{2} \theta^T L_{\text{unit}} \theta + \frac{1}{2} \theta^T M_{\text{unit}} \hat{\theta} + \varepsilon \cdot \| \theta \|_2^2,
\]

where \( \varepsilon \geq 0 \) is a small regularization parameter. For \( \varepsilon = 0 \), the state cost \( x^T Q x \) quantifies the kinetic and potential energy of a homogenous network composed of identical generators, and it penalizes frequency violations and angular differences. The cost \( x^T Q x \) does not penalize deviations in the generator voltages, the states of the excitation system, or the local control.
$u_{loc}(t)$ included in the $A$ matrix. The regularization term $\varepsilon \|\theta\|_2^2$ assures numerical stability and detectability of $(A, Q^{1/2})$.

As we will see in Section III, the state cost (8) results in an improved average closed-loop performance with all inter-area modes either damped or distorted. If the objective is to reject disturbances preceding the inter-area dynamics (5) suggests the cost function penalizes signal variations (i.e., power flows) [28], [35]. To guarantee a simple numerical treatment of the resulting Ricatti equations using standard software, an inclusion of a small positive regularization parameter $\varepsilon > 0$ is necessary.

\[x^T Q x = \ell \cdot \|\delta_\alpha - \delta_\beta\|_2^2 + m \cdot \|\dot{\delta}_\alpha - \dot{\delta}_\beta\|_2^2 + \varepsilon \cdot \|\theta\|_2^2, \quad (9)\]

where $\ell, m > 0$ are gains, $\delta_\alpha, \delta_\beta$ are the aggregate variables, and $\varepsilon \geq 0$ is a small regularization parameter. Alternative cost functions penalize selected generator frequency deviations or branch power flows $|L_{ij}|(\theta_i - \theta_j)$ to assure coherency and guarantee (soft) thermal limit constraints. Finally, linear combinations of all cost functions can also be chosen.

In summary, the state costs in (8) and (9) reflect the insights of slow coherency theory, and, as we will see later, they also promote the use of readily available control variables.

Control cost: For simplicity and in order to minimize interactions among generators the control effort is penalized as $u^T R u$, where $R$ is a positive definite and diagonal matrix.

System noise: In order to mitigate the impacts of noisy or lossy communication among spatially distributed controllers, one may choose $B_1 = B_2$. Otherwise, $B_1$ can be chosen to include the uncertainties in load and generation in $l$.

Promoting sparsity: For $\gamma = 0$, the optimal control problem (7) reduces to a standard state-feedback $H_2$-problem whose solution can be obtained from the positive definite solution to the algebraic Riccati equation. Starting from this initial value, we iteratively solve the optimal control problem (7) for increasingly larger values of $\gamma$. A logistically spaced sequence of $\gamma$-values performs well in practice. In the end, the resulting sequence of optimal controllers is analyzed, and a value of $\gamma$ is chosen to strike a balance between the closed-loop performance and sparsity of the controller.

F. Robustness, time delays, and tuning uncertainties

In comparison to local control loops, WAC involves the communication of remote signals and may suffer from additional vulnerabilities, such as uncertain communication channels as well as time delays. In particular, time delays may arise from communication delays, latencies and multiple data rates in the SCADA (supervisory control and data acquisition) network, and asynchronous measurements. As a result, local control signals and measurements may have different rates and time stamps than WAC signals obtained from remote sites.

Given the above vulnerabilities, the wide-area control loop needs to be robust to gain uncertainties $\Delta_g$ and multiplicative uncertainties $\Delta_m$, as depicted in Fig. 2. For instance, if the channel from measurement $i$ to control input $j$ features a time-delay, then the $(i,j)$th entry of $\Delta_g$ is a pure phase delay.

$\frac{1}{3}$If the power system model (1) entails a slack bus, then $\varepsilon$ can be set to zero. Otherwise, there is a marginally stable and unobservable mode corresponding to a uniform shift of all angles. The optimal control problem (7) is still feasible, and the marginally stable mode is not unaccounted for controllers using only angular differences (i.e., power flows) [38], [39]. To guarantee a simple numerical treatment of the resulting Ricatti equations using standard software, an inclusion of a small positive regularization parameter $\varepsilon > 0$ is necessary.

We refer to [34] Section 7.4 for the modeling of delays with multiplicative uncertainties $\Delta_m$. In a general robust control theory framework, the uncertainty blocks $\Delta_g$ and $\Delta_m$ are unknown, stable, and proper dynamical systems that typically satisfy certain norm bounds on their input-output behavior.

Standard frequency domain arguments show that robustness to delays is directly related to phase margins [34]. For a non-interacting (i.e., diagonal) uncertainty $\Delta_g$, the optimal centralized feedback gain $K_g^\ast$ results in guaranteed phase margins of $\pm 60^\circ$, lower gain (reduction) margins of 0.5, and upper gain (amplification) margins of $\infty$ in every channel [36]. For an unstructured multiplicative uncertainty $\Delta_m$, the optimal centralized feedback gain $K_0$ guarantees closed-loop stability provided that its $L_2$-induced input-output norm is bounded by 0.5 corresponding to a multivariable phase margin of $30^\circ$ [37].

Yet another source of uncertainty in WAC results from the tuning of the local PSSs throughout the network. The PSS gains are typically subject to rescheduling depending on the operating conditions of the overall system or the individual machines. Hence, at a given point in time the PSS gains may not be known throughout the system and may be substantially different from the gains used to construct the linearized model (2). The gain margins discussed in preceding paragraphs provide safeguards against such PSS tuning uncertainties.

For the $\gamma$-parameterized family of sparsity-promoting controllers, we explicitly verify robustness to phase and gain uncertainties in Section III. In particular, we observe that the phase and gain margins decay gracefully for $K_0^\ast$ as $\gamma$ increases. Alternatively, robustness to known time delays can be included in the control design by explicitly accounting for delays via Padé approximations (absorbed in the plant) [34].

III. COORDINATED SUPPLEMENTARY PSSs DESIGN

In this section, we use the IEEE 39 New England power grid model to validate performance of the proposed WAC strategy. This model consists of 39 buses and 10 two-axis generator models, where generator 10 is an equivalent aggregated model.

A. Local control design and inter-area dynamics

The Power System Toolbox [38] was used to obtain the nonlinear differential-algebraic model (1) and the linear state...
The open-loop system is unstable, and the generators are equipped with PSS excitation controllers designed with washout filters and lead/lag elements. For generator \( i \), the local PSS reads in the Laplace domain as

\[
u_{\text{local},i}(s) = k_i \cdot \frac{T_{w,i}s}{1 + T_{w,i}s} \cdot \frac{1 + T_{d1,i}s}{1 + T_{d1,i}s} \cdot \frac{1 + T_{d2,i}s}{1 + T_{d2,i}s} \dot{\theta}_i(s). \tag{10}
\]

The controller gains are chosen according to the tuning strategy \( [8] \) as \( T_{w,i} = 3, T_{d1,i} = T_{d2,i} = 0.1, T_{d1,i} = T_{d2,i} = 0.01 \) for \( i \in \{1, \ldots, 9\} \), \( k_i = 12 \) for \( i \in \{1, 2, 3, 5, 6, 9\} \), \( k_4 = 10 \), \( k_7 = 11.03 \), and \( k_8 = 9.51 \). As in \( [8] \), we adopt the first-order exciter model, and choose uniform time constants \( T_a = 0.015 \) s and regulator gains \( K_a = 200 \) for all generators. The static DC gains \( K_a \) \( k_i \) in the exciter and PSS control channels are slightly different compared to the highly tuned data in \( [8] \). We made this choice for the sake of illustration: the PSSs \( [10] \) provide good damping for the local modes and stabilize the otherwise unstable open-loop system \( [2] \). On the other hand, the chosen PSSs do not severely distort the shapes of the inter-area modes unless they are very carefully tuned to the linearized model (depending on the particular operating condition).

An analysis of the closed-loop modes and participation factors reveals the presence of five dominant inter-area modes. These five modes are reported in Table I and the groups of coherent machines and the frequency components of the associated eigenvectors are illustrated in Fig. 3.

### TABLE I

| mode no. | eigenvalue | damping ratio | frequency [Hz] | coherent groups |
|----------|------------|---------------|----------------|----------------|
| 1        | \(-0.6347 \pm 13.76j/2\) | 0.16614 | 0.59956 | 10 vs. all others |
| 2        | \(-0.7738 \pm 16.70j/84\) | 0.11358 | 1.0772 | 1.8 vs. 2-7,9,10 |
| 3        | \(-1.1310 \pm 15.73j/04\) | 0.19264 | 0.91202 | 1.2,3,8,9 vs. 4-7 |
| 4        | \(-1.1467 \pm 15.90j/95\) | 0.19049 | 0.94052 | 4.5,6,7,9 vs. 2,3 |
| 5        | \(-1.5219 \pm 15.80j/23\) | 0.25009 | 0.93778 | 4.5 vs. 6,7 |

B. WAC design and nominal performance

To provide additional damping for the inter-area oscillations, we design a supplementary WAC signal \( u_{\text{wac}}(t) \) which additively enters the AVRs of the controlled generators. The sparsity-promoting optimal control strategy \( [6] \) and \( \ell_1 \)-norm \( \ell_1 \)-regularized optimization \( \ell_1 \)-control design is used to design \( u_{\text{wac}}(t) \), where the state cost \( [8] \) is selected with \( \varepsilon = 0.1 \) and \( (\ell, m) = (2, 2) \). To share the control burden equally we set the control weight to be identity \( R = I \). This choice results in a WAC signal \( u_{\text{wac}}(t) \) of the same magnitude as the local control signal \( u_{\text{loc}}(t) \), that is, \( \max_{t \in \mathbb{R}_{\geq 0}} ||u_{\text{wac}}(t)||_\infty \approx \max_{t \in \mathbb{R}_{\geq 0}} ||u_{\text{loc}}(t)||_\infty \), and it avoids input saturation. Furthermore, to reject communication noise in the WAC implementation, we choose \( B_1 = B_2 \). Finally, we solve the optimal control problem \( [7] \) for 40 logarithmically spaced values of \( \gamma \) in the interval \( [10^{-4}, 10^0] \). Our results are reported in Fig. 4.

For \( \gamma = 0 \), the optimal feedback gain \( K_0^* \) is fully populated, thereby requiring centralized implementation. As \( \gamma \) increases, the off-diagonal elements of the feedback matrix \( K_0^* \) become significantly sparser whereas the relative cost \( (J_0^* - J_0)/J_0^* \) increases only slightly; see Fig. 4 and Fig. 5. Additionally, as \( \gamma \) increases, the state cost \( [8] \) promotes the use of angles and speeds in the off-diagonal elements of \( K_0^* \), and most nonzero elements of \( K_0^* \) correspond to local feedback. For \( \gamma = 1 \), the optimal controller \( K_1^* \) is within 1.5882% of the optimal centralized performance even though only a single signal needs to be communicated: the controller at generator 1 needs to access \( \theta_9(t) \). As expected, as \( \gamma \) increases most of the control burden is on generator 1, which has the largest inertia of all controlled generators. Likewise, the angle of loosely connected \( * \) generator 9 needs to be measured and communicated to generator 1. The identified WAC channel 9 \( \rightarrow 1 \) appears to be necessary to suppress the inter-area mode 2 in Table I which is mainly dominated by generators 1 and 9.

The optimal feedback gain \( K_1^* \) does not necessarily increase the damping of the eigenvalues associated with inter-area modes. Rather, the closed-loop performance is improved by distorting the associated eigenvectors. Apart from one poorly damped pair of eigenvalues located at \(-0.6044 \pm 1.5203 \) in terms of graph theory, the sum of effective resistances \( [32] \) between generator 9 and the others is very large compared to remaining network.
Fig. 5. The sparsity patterns of $K^*_\gamma$ illustrate the control architecture. A nonzero $(i,j)$ element of $K^*_\gamma$ (denoted by a colored dot) implies that controller $i$ needs to access state $j$. The diagonal blocks (framed) with blue dots correspond to local feedback, and the off-diagonal blocks with red dots correspond to remote feedback signals. As $\gamma$ increases, the information exchange becomes sparser, and angles and frequencies (the first two states of each vertical block) become the sole signals to be communicated.

Fig. 6. Subfigure (a) displays the frequency components of the least damped oscillatory mode of the closed loop with local PSSs and WAC. Subfigure (b) shows the WAC signal $u_{\text{wac}}(t)$. The initial conditions are aligned with the eigenvector of the dominant open-loop inter-area mode $1$, and $u_{\text{wac}}(t)$ is subject to additive white noise with zero mean and standard deviation 0.01.

(possibly corresponding to the inter-area mode 1 in Table I), all other complex-conjugate eigenvalue pairs are left of the asymptote $\text{Real}(s) = -12.74$. From the frequency components of its eigenvector (in Fig. 6(a)) and the frequency time series (in Fig. 7(a) and 7(b)), it can be seen that this poorly damped mode does not anymore correspond to generators oscillating against each other. As a consequence, poorly damped power flow oscillations between the areas $\mathcal{V}_a = \{1, \ldots, 9\}$ and $\mathcal{V}_b = \{10\}$ have been suppressed; see Fig. 7(c) and 7(d).

C. Implementation issues, robustness, and delays

As discussed in Subsection II-F, multivariable phase margins are well suited robustness measures to account for communication delays, latencies in the SCADA network, and asynchronous measurements of generator rotor angles. Here, we investigate how the general multivariable phase margin of the WAC closed-loop changes with the sparsity-promoting parameter $\gamma$; see Fig. 8. Similar to the performance and sparsity levels of $K^*_\gamma$ in Fig. 4, the multivariable phase margin is nearly a monotonic function of $\gamma \in [10^{-4}, 10^0]$. Over the range of examined values of $\gamma$, the phase margin experiences a modest drop of about $4^\circ$. Similar observations can be made about the multivariable gain reduction and amplification margins displayed in Fig. 8. They deteriorate gracefully, in a monotonic way, and by comparatively small amounts. Hence, each PSS gain can differ from its nominal gain (used to construct the linearized model (2)) and independently from other PSS gains by a factor of order [0.2, 5] even for $\gamma = 1$. We thus conclude that the controller resulting from the sparsity-promoting optimal control problem (6) enjoys nearly the same robustness properties as the optimal centralized controller.

The wide-area control signal can be further decomposed as

$$u_{\text{wac}}(t) = u_{\text{wac}}^{(\text{loc})}(t) + u_{\text{wac}}^{(\text{rem})}(t),$$

where $u_{\text{wac}}^{(\text{loc})}(t)$ corresponds to block-diagonal state feedback, and $u_{\text{wac}}^{(\text{rem})}(t)$ corresponds to feedback that utilizes the remote measurement signal $\theta_\text{R}(t)$ (which has to be communicated to the AVR controller at generator 1). Since the block-diagonal state feedback $u_{\text{wac}}^{(\text{loc})}(t)$ depends only on generator internal
states, it can be implemented locally. The required generator state variables can be reconstructed online using appropriate filters and readily available measurements.\(^{39}\)

For the wide-area controller \(u_{\text{wac}}(t) = -K_x^t x(t)\), if the local control \(u_{\text{loc}}(t)\) is absorbed in the plant, then we obtain a 58.303° phase margin with respect to the remote control input \(u_{\text{rem}}(t)\). The corresponding tolerable time-delay margin is 3.964 s. Additionally, we found that the information structure identified by the WAC \(u_{\text{wac}}(t) = -K_x^t x\) is not sensitive to the actual operating and linearization point in the dynamics.\(^1\)

To validate this hypothesis, we randomly altered the power demand at each load to create different loading conditions and operating points. Additionally, we perturbed the nominal PSS gains in (10). As a result, even if the PSS gains are randomly altered by factors within \([0.5, 4]\), and the load power demand is randomly altered within \(\pm 25\%\) of the nominal demand (leading to different linearization matrices in (2)), the sparsity pattern of \(K_x^t\) is identical to the one shown in Fig. 5.

We conclude that the feedback resulting from the sparsity-promoting optimal control problem (3) is not only characterized by low communication requirements and good closed-loop performance but also by favorable robustness margins.

**D. Sparsity identification and alternative control schemes**

We note that while the local state feedback \(u_{\text{loc}}(t)\) essentially requires a retuning of the local generator excitation control, the remote control signal \(u_{\text{rem}}(t)\) needs to be communicated. Recall that one of our motivations for the sparsity-promoting optimal control formulation (3) – besides finding a stabilizing, robust, and optimal feedback – was the identification of an appropriate control architecture for \(u_{\text{rem}}(t)\). For the considered model, our sparsity-promoting strategy identified a single crucial WAC channel: the rotor angle \(\theta_0(t)\) serves as measurement to the AVR control at generator 1. After having identified this WAC channel, alternative control schemes can be developed. For instance, in absence of the local “retuning” control \(u_{\text{loc}}(t)\), \(u_{\text{wac}}(t)\) reduces to the proportional control

\[
\begin{equation}
\tilde{u}_{\text{wac}}(t) = \left[ k (\theta_0(t) - \theta_1(t)) \right]^T, \quad (12)
\end{equation}
\]

Here, \(k > 0\) is the \((1, 9)\) element of \(K_x^t\) and we use the local rotor angle \(\theta_1(t)\) as a reference for \(\theta_0(t)\). Hence, no absolute angle measurements are required\(^3\) and the WAC control (12) can be implemented, for example, by integrating the frequency difference \(\dot{\theta}_0(t) - \dot{\theta}_1(t)\). The simple yet crucial proportional remote control signal (12) together with an appropriate retuning of the local PSSs (10), then yields the WAC signal \(u_{\text{wac}}(t)\).

To illustrate the utility of this sparsity identification scheme, we apply the proportional WAC signal (12) to the full nonlinear differential-algebraic power network model (1) equipped with the PSSs (10). To demonstrate the importance of the WAC channel \(9 \rightarrow 1\), we do not retune the local PSSs to emulate \(u_{\text{loc}}(t)\). Our simulation scenario is based on a three phase fault at line \([3, 4]\) at 0.1 s, which is cleared at 0.2 s. The WAC input (12) is subject to a delay of 0.1 s. We compare the system responses in presence and absence of the WAC signal (12); see Fig. 9. As in the linearized case, the decay rates of the frequencies are comparable but the inter-area modes are distorted in WAC closed loop; see Fig. 9(a) and 9(b). Due to the lack of tuned local feedback \(u_{\text{loc}}(t)\) in (12), generator 10 still slightly swings against the remaining generators in WAC closed loop. On the other hand, the implementation of the WAC signal (12) by generator 1 results in a reduced effort in providing stabilizing control signals; see Fig. 9(c) and 9(d).

We conclude that the simple proportional feedback (12) identified by the sparsity-promoting framework significantly improves the closed-loop performance. Of course, now that the control channel \(9 \rightarrow 1\) is identified more sophisticated control strategies can be developed via alternative means.

**IV. CONCLUSIONS**

We proposed a novel approach to wide-area control of inter-area oscillations. We followed a recently introduced paradigm to sparsity-promoting optimal control, and our performance objectives were inspired by the well-known slow coherency theory. We validated the proposed control strategy on the IEEE 39 New England power grid model, which demonstrated nearly optimal performance, low communication requirements, and robustness of the sparsity-promoting controller.

Our initial results are very promising, and we are currently working on further extensions. Of course, further scrutiny of the proposed sparsity-promoting optimal WAC approach on other types of power networks is needed. The authors also envision an application of the proposed control strategy to enhance the performance of purely decentralized primary and secondary control schemes\(^{40}\) with supplementary WAC. Since sparsity is a function of the state space coordinates, the identified control scheme may not be sparse in other coordinates. Hence, an interesting question is to identify state space representations of power system models, which are amenable to a sparsity-promoting control design. For example, it is of interest to extend the proposed design methodology to sparse, differential-algebraic, and structure-preserving models.

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A. Algorithmic methods for sparsity-promoting optimization

We briefly summarize the algorithmic approach to the optimization problem (7) and refer to [27] for further details:

(i) Warm-start and homotopy: The optimal control problem (7) is solved by tracing a homotopy path that starts at the optimal centralized controller with $\gamma = 0$ and continuously increases $\gamma$ until the desired value $\gamma_{\text{des}}$.

(ii) ADMM: For each value of $\gamma \in [0, \gamma_{\text{des}}]$, the optimization problem (7) is solved iteratively using ADMM:

(iii) Updates of weights: In each step of ADMM, the weights are updated as $w_{ij} = 1/|k_{ij}| + \varepsilon$ with $\varepsilon > 0$. We have conducted 5 update steps with $\varepsilon = 10^{-3}$.

(iv) Polishing: Once the desired sparsity pattern $K$ is identified, a structured optimal control problem is solved:

\[
\text{minimize } J_{\gamma}(K) = \text{trace}(B^T P B),
\]

subject to $K \in K$,

\[
(A - B_2 K) P + P(A - B_2 K) = -(Q + B_1^T R K).
\]

This iterative approach is convergent under a local convexity assumption, and stability of $A - B_2 K$ is guaranteed. The algorithms developed in [27] have been implemented in MATLAB.

The associated software as well as the numerical power network data used in the present paper can be downloaded at www.ece.umn.edu/users/mlhallo/software/lqrsp/.

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