Crossover of the relative heat transport contributions of plume ejecting and impacting zones in turbulent Rayleigh–Bénard convection

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Abstract – Turbulent thermal convection is characterized by the formation of large-scale structures and strong spatial inhomogeneity. This work addresses the relative heat transport contributions of the large-scale plume ejecting versus plume impacting zones in turbulent Rayleigh–Bénard convection. Based on direct numerical simulations of the two dimensional (2-D) problem, we show the existence of a crossover in the wall heat transport from initially impacting dominated to ultimately ejecting dominated at a Rayleigh number of $Ra \approx 3 \times 10^{11}$. This is consistent with the trends observed in 3-D convection at lower $Ra$, and we therefore expect a similar crossover to also occur there. We identify the development of a turbulent mixing zone, connected to thermal plume emission, as the primary mechanism for the crossover. The mixing zone gradually extends vertically and horizontally, therefore becoming more and more dominant for the overall heat transfer.

Introduction. – Thermally driven turbulence is omnipresent in nature and technology and its deep fundamental understanding is of utmost relevance for answering various environmental or technological questions. As a model system for thermally driven convection, Rayleigh–Bénard convection (RBC) – the flow in a box heated from below and cooled from above – has always been the most paradigmatic and popular one \cite{1,3}. It also reflects the intrinsic difficulty of thermally driven flows, namely its spatially inhomogeneous, including in the lateral direction, due to the formation of large-scale structures. Different regions in the flow show different flow features and contribute differently to the overall heat transfer, which is the key global response of the system to some given control parameters. In the presence of sidewalls, the spatial inhomogeneity in horizontal direction is obvious. However, due to the formation of large-scale structures, it even holds in the absence of sidewalls, for periodic boundary conditions \cite{4,8}, or for very large aspect ratios $\Gamma$ defined as cell width over cell height \cite{9}.

The spatial inhomogeneity of the flow must also be reflected in any theoretical approach to understand the heat transfer in RB flow. The simplest example may be the theory of Malkus \cite{10}, which assumes a thermal shortcut in the bulk and laminar type heat transport in the thermal boundary layers (BLs), which leads to the scaling law $Nu \sim Ra^{1/3}$ for the Nusselt number (the dimensionless heat transfer) as function of the Rayleigh number (the dimensionless temperature difference between top and bottom plate). This also holds for the mixing length theory of Castaing et al. \cite{11}, the boundary layer theory of Shraiman and Siggia \cite{12}, and the unifying theory of Grossmann and Lohse \cite{13,15}, which splits the kinetic energy and thermal dissipation rates into bulk and boundary-layer/plume contributions, with different scaling behavior. Since the kinetic energy and thermal dissipation rates are additive with respect to these contributions and the total dissipation rates can exactly be connected with the overall Nusselt and Rayleigh numbers, this implies that the system response parameters, i.e., the Nusselt and Reynolds num-
bers, do not show pure scaling behavior, but a smooth crossover from the dominance of one region to another.

Indeed, different scaling behavior in different regions of the flow were measured in various experiments and direct numerical simulations (DNS). For example, for the local heat fluxes, 

Shang et al. \[16\] measured the scaling close to \( N_{\text{loc}} \sim Ra^{1/4} \) near the sidewalls and close to \( \sim Ra^{1/2} \) in the bulk. This suggests that for increasing \( Ra \), the latter may take over. This is the so-called asymptotic ultimate regime \( \nu < Ra^{1/2} \), first suggested by Kraichnan \[19\] and Spiegel \[20\] and indeed found in so-called homogeneous RB flow \[21\] \[22\], where the flow-driving hot and cold temperature boundary conditions at the plates have been replaced by a bulk driving with an overall temperature gradient, as well as in radiatively driven convection \[23\].

Different scaling behavior of the local heat flux in different regions of the flow in the lateral direction, reflecting the spatial lateral inhomogeneity of the flow, was also observed in numerical simulations with periodic boundary conditions, despite the periodicity. Van der Poel et al. \[4\] distinguishes plume ejecting and plume impacting regions, which are separated by regions dominated by wind-shielding. These regions are set by the large-scale convection rolls. In general, these large-scale rolls wiggle laterally due to the turbulent nature of the flow. Therefore, to obtain the local heat flux in the different regions, spatially moving averages have to be performed. In this way, the local dynamics can be disentangled from the slow large-scale movement of the rolls.

Assuming that the large-scale rolls are of similar size and always located at the same places, for very large \( Ra \geq 10^{13} \) and in two-dimensional (2-D) direct numerical simulations, Zhu et al. \[24\] obtain effective scaling laws: \( N_{\text{loc}} \sim Ra^{0.28} \) in the plume impacting region and \( N_{\text{loc}} \sim Ra^{0.38} \) in the plume ejecting region. This implies that for very large \( Ra \), the latter scaling wins for the overall (global) Nusselt number, \( \nu \sim Ra^{0.38} \). This is the so-called ultimate regime, which corresponds to \( \nu \sim Ra^{1/2} \) with logarithmic corrections, as predicted by Kraichnan \[19\] and by Grossmann and Lohse \[25\] \[26\]. In this regime, the kinetic boundary layer has become turbulent, which is reflected in logarithmic velocity and temperature profiles that allow for an enhanced heat transfer. Note that, without accounting for the movement of the structures in the decomposition, Zhu et al. \[24\] \[27\] find that the plume ejecting regions contribute more to the overall heat flux for all \( Ra \) explored in their study (\( 10^{11} \leq Ra \leq 4.64 \times 10^{14} \)).

In contrast to the findings by Zhu et al. \[24\], for \( Ra \) up to \( 10^{9} \) and in three-dimensional (3-D) DNS, Blass et al. \[8\] find that the plume impacting regions contribute more to the overall heat flux. These findings in large periodic boxes (\( \Gamma = 32 \)) are in line with earlier findings in more confined geometries of Reeuwijk et al. \[28\] (\( \Gamma = 4 \)) and Wagner & Shishkina \[29\] (\( \Gamma = 1 \)). Indeed, such a heat flux distribution would be expected from the fact that the boundary layer thickness grows as the fluid is advected along the plate with correspondingly reduced heat fluxes downstream (i.e. towards the ejecting region). However, Blass et al. \[8\] also find that the dominance of the plume impacting regions diminishes with increasing \( Ra \). Extrapolating their data to higher \( Ra \), they estimate a crossover from impacting to ejecting dominated at \( Ra \approx 10^{12} \), which appears consistent with the findings of Zhu et al. \[24\] in 2-D, Blass et al. \[8\] use a conditional averaging technique which is superior to a spatially moving average because it allows to extract precise statistics despite movement of the structures or changes in their number and orientation.

In this paper, we want to reconcile the results of Zhu et al. \[24\] – the dominance in the heat flux of the plume ejecting regions in 2-D DNS beyond \( Ra = 10^{11} \) and of Blass et al. \[8\] – dominance in the heat flux of the plume impacting regions in 3-D DNS up to \( Ra = 10^{9} \), obtained with the conditional averaging technique. We do this by applying the superior conditional averaging technique to the numerical data obtained by Zhu et al. \[24\] and to new 2-D direct numerical simulations (DNS) for \( Ra \) down to \( 10^{7} \). Our main result is that the observations by Blass et al. \[8\] and Zhu et al. \[24\] are consistent and robust, and that we can identify the crossover Rayleigh number at which the heat flux from the plume ejecting regions overtakes that from the plume impacting regions. This analysis is relevant in the context of the findings of Zhu et al. \[24\] \[27\], who showed that beyond \( Ra \geq 10^{13} \), both, the local heat flux in the plume ejecting regions (which grows with increasing \( Ra \)) and the overall heat flux, scale steeper than the classical Malkus scaling \( \nu \sim Ra^{1/3} \). This reflects the onset of the ultimate regime around that Rayleigh number, consistent with theoretical predictions \[13\] \[30\] and experimental measurements \[31\] \[32\].

Numerical Simulations. – In this letter, we restrict us to 2-D DNS of RB flow, with periodic sidewalls and no-slip boundary conditions (BCs). The governing dimensionless Navier–Stokes equations in the Oberbeck–Bousinesq approximations read

\[
\partial u / \partial t + u \cdot \nabla u = -\nabla p + \sqrt{PrRa} \nabla^2 u + \alpha g \beta \nabla \theta,
\]

\[
\partial \theta / \partial t + u \cdot \nabla \theta = 1/\sqrt{PrRa} \nabla^2 \theta, \quad \nabla \cdot u = 0, \tag{1}
\]

where \( u \) is the velocity, \( \theta \) the temperature, \( p \) the pressure, \( t \) the time and \( e_z \) denotes the unit vector in the vertical direction. The equations have been non-dimensionalised using the free-fall velocity \( u_{ff} \equiv (\alpha g \Delta H)^{1/2} \), the free-fall time \( H/u_{ff} \), the temperature difference \( \Delta \) between the hot and the cold plate, and the cell height \( H \). The parameters \( Ra, Pr \), and the aspect ratio \( \Gamma \) are

\[
Ra \equiv \alpha g \Delta H^3 / (\kappa \nu), \quad Pr \equiv \nu / \kappa = 1, \quad \Gamma \equiv L/H = 2,
\]

where \( L \) is the lateral extension of the periodic domain, \( \alpha \) the fluid thermal expansion coefficient, \( \nu \) the viscosity, \( \kappa \) the thermal diffusivity and \( g \) acceleration due to gravity.

The set of equations (1) is solved numerically using the finite volume code GOLDFISH for \( Ra \) from \( 10^{7} \) to \( 3 \times 10^{12} \).
Complementary, we have reanalysed the flow snapshots from the previously published data series [24] for \( Ra \) from \( 10^{11} \) to \( 10^{14} \), which was generated with the finite-difference solver AFID [32,33]. Taken together, the present study covers the parameter range \( 10^7 \leq Ra \leq 10^{14} \). These two computational codes for turbulent RBC were validated against each other, with excellent agreement [35]. Besides, we demand the same grid resolution criteria [36] in simulations with both codes. Thus, for the overlapping \( Ra \)-range, we use grids with the same number of nodes for both codes and with a similar grid nodes clustering near and in the boundary layers attached to the isothermal plates. For further details regarding the computational grids we refer to the supplementary material in Zhu et al. [24].

**Conditional averaging.** – Fig. 1(a) gives an impression of the complexity of the flow and its large-scale organization into two counter-rotating circulation rolls, driven by single zones of respectively rising hot plumes and sinking cold plumes. We decompose the flow into plume impacting and ejecting zones (the way to do this is explained later), extract statistical information and analyse their individual heat transport contributions. Evidently, this procedure depends on the robustness of the conditional averaging algorithm, which should be applicable in a broad range of \( Ra \) and which, first and foremost, should be able to reliably identify the location of the large-scale rolls.

In a 3-D domain, the first choice for the large-scale roll identification would be the technique of Krug et al. [7,8,37], which was developed to identify 3-D superstructures in turbulent RBC. The method is based on the observation that there exists a pronounced scale separation between the turbulent thermal superstructures and small-scale turbulent fluctuations so that, after applying a low-pass filter at an intermediate wave-number \( (k \approx 2/H) \), what remains is a visually convincing representation of the large-scale structures. However, the turbulence cascades (and hence the spectral energy distributions) are different in 2-D and 3-D flows. As a result, the 2-D flows studied here lack the needed distinct scale separation, which impedes the applicability of the technique [7,8] to 2-D RBC.

On the other hand, one of the advantages of the 2-D confined (\( \Gamma = 2 \)) cell is that the configuration of the large-scale circulation (LSC) is stable, which means that one can safely assume the number of the convection rolls to be fixed for all times. This is beneficial in so far, that the problem reduces to finding the horizontal positions of the hot and cold large-scale plumes as functions of time, i.e. \( x_H^*(t) \) and \( x_C^*(t) \). These positions are indicated by the dashed lines in Fig. 1(a). The functions \( x_H^*(t) \) and \( x_C^*(t) \) are generally independent so that, in particular, the distance between the plumes changes with time.

To find \( x_H^*(t) \) and \( x_C^*(t) \), we employ a pattern (or, equivalently, template) matching algorithm. The idea is simple. We check the flow fields for the presence of some pattern by moving a template (horizontally) over the flow field and measure their similarity via convolution. In doing so, the templates \( P_H \) and \( P_C \) are chosen such that they resemble the structure of the region of interest, i.e. the hot (\( H \)) upwelling and cold (\( C \)) downwelling plumes. The (circular) convolution can be formally defined as

\[
(P_H(x') \otimes \theta)(x) = \int_0^L \int_0^H P_H(x') \theta(x',z)dx' dz,
\]

and the position of the maximal correlation gives the lo-
Figure 2: Schematic overview of the iterative pattern matching algorithm that is used to identify the hot and cold plumes and the conditional average of the fields, based on the horizontal positions of the plumes.

Figure 3: Probability density distribution of the minimum plume separation $x_d^* = \min |x_H^* - x_C^*|$. The different curves represent different $Ra$. The inset figure shows the mean and one standard deviation of the minimal distance between the hot and cold plumes, for different $Ra$. The colour scale ranges from blue (smaller $Ra$) to red (larger $Ra$), according to the inset figure.

The statistical symmetry of the hot and cold plumes. In this way, all snapshots are processed and successively added to the conditional time averaged mean field, as it is shown in Fig. 1(b). Finally, from this mean field we can generate new templates $P_{H(C)}$, by reapplying the inherent symmetries and then restart the algorithm. In practice, the algorithm converges after one iteration and delivers convincing and robust positions of $x_H^*$ and $x_C^*$. We confirmed that stretching and squeezing of the flow fields while mapping does not distort the global response characteristics like $Nu$, which remained practically unaffected ($<1\%$ deviation) by these manipulations.

As noted, the templates are first initialised with vertically independent cosine functions. Therefore, the first step of the algorithm is equivalent to a cosine fit method, which is often used to identify the LSC in cylindrical cells [39, 40]. This method, however, inherently constrains the size of the LSC and fixes the relative size of the large-scale hot and cold plumes and the distance between them. To get an impression about the limitations of the cosine fit method, we evaluate the probability density function (PDF) and the mean and standard deviation of the (minimal) relative distance between $x_H^*$ and $x_C^*$, or, in simple terms, how close do the hot and cold plumes approach each other (see Fig. 3). We find that for all $Ra$ this distance varies quite substantially within the range between 0.8 to 1, which shows that the relative motion of the plumes is significant. As a consequence, the cosine fit method leads to rather “blurry” looking mean fields, while the mean fields from our conditional averaging algorithm appear more “in-focus”.

Results. – In the following analysis, we will make use of two Nusselt number definitions. The first one is based on the global heat transport and defined as

$$Nu = \sqrt{Ra\overline{Pr}(\overline{u_2}\overline{\theta})_V - \langle \partial_z \overline{\theta} \rangle_V},$$

where the overline represents the conditional time average and $\langle \cdot \rangle_V$ denotes a volume average. The second one, given by

$$\overline{Nu}(d) = -\partial_z \overline{\theta}|_{z=0} \text{ (or } z=H),$$

characterizes the local wall heat transport as a function of the conditioned horizontal coordinate $d$. As depicted...
in Fig. 1(b), $d$ is defined such that the ranges $-0.5 \leq d < 0$ and $0 < d \leq 0.5$ correspond to plume ejecting and impacting regions, respectively.

We start off by analysing the horizontal distribution of $Nu$ normalized by $Nu$. Generally, the thermal BL grows as the fluid travels downstream along the plate, i.e. when proceeding from the large scale impacting to the emitting region. Correspondingly, the local heat transfer in the laminar and weakly chaotic regime is expected to decrease along this direction $[11]$. This is seen in $[8, 28, 29]$ and consistently also here (Fig. 4) at small $Ra$, for which the heat transport reaches its maximum in the impacting zone and then gradually decreases with increasing $d$ in the ejecting zone. However, as $Ra$ increases, we observe two distinct departures from this picture, which both lead to heat transport enhancements in the ejecting zone.

The first enhancement occurs in the center of the ejecting zone, at $d > 0.35$, where a significant peak in $Nu/Nu$ emerges initially as $Ra$ is increased beyond $10^7$. However, this peak reaches a maximum at $Ra = 10^8$ and eventually subsides again with even stronger thermal driving. Based on an analysis of instantaneous flow fields, we identified the formation of small recirculation regions, which lead to secondary circulation cells, as the source of this behaviour. These recirculations disappear for $Ra \gtrapprox 10^{10}$ and therefore do not play a role beyond. Besides that, this phenomenon is likely (and certainly in it’s strength) peculiar to 2-D RBC, since it is not observed in comparable 3-D studies $[8, 28, 29]$. Of more general relevance is the peak that first emerges at $Ra \approx 10^8$ in the region $0.2 < d < 0.35$ (see Fig. 1) and which ends up dominating the overall wall heat transport for larger $Ra$. From Fig. 1(b), we can verify that this part of the domain, the "leg" of the large-scale plume, is the predominant origin of small-scale turbulent plumes that emit from the BLs. These turbulent plumes are able to effectively mix their surroundings, thus inducing an increase of the vertical and horizontal heat transport. Hence, this part of the domain can be seen as the turbulent mixing zone, as suggested by Castaing et al. $[11]$. We further note that a similar increase of the heat transfer in streamwise direction is also observed in connection with the shear-driven laminar-turbulent transition of the BL $[42]$. The peak first occurs at $d \approx 0.35$ at $Ra = 10^8$ but its location gradually shifts towards lower values of $d$ as $Ra$ is increased. This lends some support to the hypothesis of van der Poel et al. $[4]$, who surmised that the transition to ultimate scaling is driven by a spreading of the plume-ejection dominated region. However, especially at the highest $Ra$ studied here, the simultaneous growth in peak magnitude with increasing $Ra$ appears to be an even more relevant factor in shifting the balance in the heat transfer direction towards the ejecting side.

To shed more light on the local heat transport mechanisms, we proceed and decompose the convective heat transport $\overline{u_z \theta}$ into its mean field $\overline{u_z \theta}$ and turbulent contributions $\langle u'_z \theta' \rangle$ according to

$$\langle u_z \theta \rangle_d = \langle u_z \theta \rangle_d + \langle u'_z \theta' \rangle_d,$$

where $\langle \cdot \rangle_d$ denotes a horizontal average taken either across the impacting ($d < 0$) or the ejecting ($d > 0$) region. The mean and turbulent convective heat transport profiles in vertical direction, normalized with the mean thickness of the thermal BL $\lambda_0 = 1/(2Nu)$, are shown in Fig. 5.

Figures 5(a, b) show that for small $Ra$, the mean field transport is dominant in the convective heat transfer for both, impacting and ejecting zones. However, its relative contribution weakens with increasing thermal driving such that the large-scale circulation ultimately plays no significant role in the convective heat transport at high $Ra$. This behaviour is especially apparent in the impacting zone Fig. 5(a), where the mean convective transport...
vanishes almost completely, whereas in the ejecting zone (Fig. 5b), the mean convective transport remains of significant importance. Moreover, the contribution in the ejecting zone first rises before it starts to decay. This is another manifestation of the recirculation regions mentioned earlier, and therefore it is no coincidence that also here the strongest effect is observed at $Ra = 10^9$.

We turn the focus now to the turbulent convective heat transport shown in Figures 5 (c, d). As expected, turbulent mixing becomes the predominant heat transport mechanism at large $Ra$, but again, the impacting and ejecting zones behave characteristically differently. In the impacting zone (Fig. 5c), the turbulent transport contribution initially increases with increasing $Ra$ before it saturates. Above $Ra \approx 10^{10}$, the curves almost collapse onto a single curve, which was observed similarly in Blass et al. [8]. This indicates that above a certain $Ra$, the relative contribution of the turbulent heat transport in the impacting region does not increase significantly anymore. In the ejecting region ($0 \leq d \leq 0.5$), however, the relative importance of turbulent transport increases gradually with increasing $Ra$, confirming again the trends observed earlier in 3-D convection [8]. Evidently, the aforementioned ejecting plumes create an efficient turbulent mixing zone which becomes more and more important and ultimately dominates the heat transport mechanisms. Moreover, the mixing zone increases in size with increasing $Ra$, compared to the thermal BL thickness, and reaches its maximal effectiveness at $z \approx 5\lambda$. This complements the observations by Schumacher [43], who showed that the extension of this mixing zone can be significantly larger than the thermal BL. Surprisingly, the location of this maximum is relatively robust with respect to changes in $Ra$. It is further noteworthy that turbulent transport is dominant even deep inside the BL ($z < \lambda$) once $Ra \gtrsim 10^{10}$.

As mentioned in the introduction, for low and intermediate $Ra$ in 3-D RBC, the impacting zones dominate the wall heat transport [8], whereas for large $Ra$ in 2-D RBC, the ejecting zones were found to contribute the majority of the heat transport [24]. Yet a direct link between these observations has up to now been missing. With this in mind, we compare contributions of the emitting and ejecting regions to the total wall heat transport in Fig. 6. Thanks to the wide range of $Ra$ available here, we now observe a clear crossover of the contributions from the ejecting and impacting regions at $Ra \approx 3 \times 10^{11}$. At this critical Rayleigh number (highlighted as dashed lines in Fig. 4 and 5), the dominance of the contribution from the impacting zone changes to the dominance of the contribution from the ejecting zone. The data by Zhu et al. [24] also show such a crossover, if we apply the conditional averaging as proposed in the present study. Therefore, the dynamic tracking of the LSC is the key to a successful individual statistical description of the different zones. Additionally, from Fig. 5a), we find that the 3-D and 2-D cases show increasingly similar behaviour as $Ra$ increases. This gives confidence that the observed trends in the $Nu$ distribution are indeed driven by the increase in $Ra$ and not predominantly related to differences between 2-D and 3-D flows. We would expect a similar crossover to occur in 3-D as well at sufficiently high $Ra$.

**Conclusion.** – By means of direct numerical simulations and using a conditional averaging technique we explored the properties of the plume impacting and plume ejecting zones in horizontally periodic 2-D RBC. This study covers the range $10^7 \leq Ra \leq 10^{14}$, thus bridging the $Ra$ gap between the corresponding studies of Blass et al. [8] (3-D) and Zhu et al. [24] (2-D). We provide an unifying picture of the relative heat transport importance of ejecting and impacting zones across $Ra$ and show the existence of a crossover from an impacting dominated to an ejecting dominated local wall heat transfer at $Ra \approx 3 \times 10^{11}$. This trend is connected to an increase in the convective heat transport at the leg of the large-scale plume. Specifically, we identify the turbulent convective transfer to become
the dominant transport mechanism, which is reflected in a gradual growth with $Ra$ of the turbulent convective heat flux $\theta'$. The turbulent mixing zone reaches its peak efficiency at a vertical distance of about five thermal boundary layer thicknesses from the plate and it gradually expands in size with increasing $Ra$, thus occupying an even larger fraction of the domain. It remains to be verified whether such a crossover towards the dominance in the heat transport of the thermal plume ejecting regions also exists in 3-D turbulent convection, but our results strongly suggest so.

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