Antenna subtraction at NNLO with identified hadrons

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arXiv:2205.XXXXX

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Loops and Legs 2022, Ettal (Germany) 25-30 April 2022
Introduction

The identification of hadrons require the introduction of a **fragmentation function** (FF) to describe the fragmentation of the high-energy quark or gluon into the actually detected hadron:

\[
\frac{d\sigma}{d\eta} = \sum_p \int d\eta D_{H\leftarrow p}(\eta, \mu_a^2) \, d\hat{\sigma}_p(\eta, \mu_a^2).
\]

Whenever we identify a QCD particle, we spoil the cancellation of collinear divergences!

![Diagram](from P. Nason's lectures)

Such divergences are absorbed in the bare \( D_{H\leftarrow p}^{(b)}(\eta) \) to result in the physical \( D_{H\leftarrow p}(\eta, \mu_a^2) \).
Motivation: why identified hadrons?

Fit of light fragmentation functions e.g. BDSS21 [De Florian et al. 2015], JAF20 [Moffat et al. 2021] and NNFF1.0 [Bertone, Carrazza, et al. 2017; Bertone, Hartland, et al. 2018]
Motivation: why identified hadrons?

Heavy quarks produced in association with vector bosons e.g. $W + c$, with the $c$-quark fragmenting into the $D$-meson detected $\rightarrow$ A. Kardos’ talk

**[ATLAS 1402.6263]**

**[CMS 1811.10021]**
This talk

• **Extend the antenna subtraction formalism to include identified hadrons.**
• First progress in this direction: photon fragmentation $\rightarrow$ R. Schürmann’s talk
• We focus on an identified hadron $H$ in fully exclusive $e^+e^-$ annihilation:

$$e^+ + e^- \rightarrow H( +\text{jets} ) + X$$

Possible presence of additional jets, which may contain the identified hadron.
• A step towards hadron fragmentation in $ep$ or $pp$ collisions.

Inclusion of FFs in the sector-improved residue subtraction scheme $\rightarrow$ M. Czakon’s talk
First application: $B$-hadron production in $t\bar{t}$ events [Czakon et al. 2021]
Antenna subtraction

Perturbative expansion of the short-distance cross section $d\hat{\sigma}$:

$$d\hat{\sigma} = d\hat{\sigma}^{\text{LO}} + \left(\frac{\alpha_s}{2\pi}\right) d\hat{\sigma}^{\text{NLO}} + \left(\frac{\alpha_s}{2\pi}\right)^2 d\hat{\sigma}^{\text{NNLO}}.$$  

Required subtraction scheme to deal with infrared divergences in the intermediate steps of the calculation. In the antenna subtraction scheme [Gehrmann-De Ridder, Gehrmann, and Glover 2005]:

$$d\hat{\sigma}^{\text{LO}} = \int_n [d\hat{\sigma}^B]$$

$$d\hat{\sigma}^{\text{NLO}} = \int_{n+1} [d\hat{\sigma}^R - d\hat{\sigma}^S] + \int_n [d\hat{\sigma}^V - d\hat{\sigma}^T - d\hat{\sigma}^{\text{MF}}]$$

$$d\hat{\sigma}^{\text{NNLO}} = \int_{n+2} [d\hat{\sigma}^{RR} - d\hat{\sigma}^S] + \int_{n+1} [d\hat{\sigma}^{RV} - d\hat{\sigma}^T - d\hat{\sigma}^{\text{MF,RV}}] + \int_n [d\hat{\sigma}^{VV} - d\hat{\sigma}^U - d\hat{\sigma}^{\text{MF,VV}}]$$
Outline

Introduction

Subtraction at NLO

Subtraction at NNLO

Integration of NNLO antenna functions

Analytical checks

Conclusions and outlook
Outline

Introduction

Subtraction at NLO

Subtraction at NNLO

Integration of NNLO antenna functions

Analytical checks

Conclusions and outlook
Real subtraction term $d\hat{s}^S$

\[ M_{n+1}(\text{original momenta}) \rightarrow M_n(\text{mapped momenta}) \times X(\text{original momenta}) \]
Real subtraction term $d\hat{\sigma}^S$

New ingredient: unresolved parton between the identified parton $k_p$ and another hard parton.

$$M(k_1, \ldots, k_p, k_j, k_k, \ldots, k_{1+m}) \rightarrow M(k_1, \ldots, K_p, K, \ldots, k_{1+m}) \times X^{0, \text{id.p}}(k_p, k_j, k_k)$$

with the mapping defined as

$$z = \frac{s_{pj} + s_{pk}}{s_{pj} + s_{pk} + s_{jk}}, \quad K_p = k_p/z, \quad K = k_j + k_k - (1 - z)\frac{k_p}{z}$$

Phase space factorizes as

$$d\Phi_{m+1}(k_1, \ldots, k_p, k_j, k_k, \ldots, k_{m+1}; Q) = d\Phi_m(k_1, \ldots, K_p, K, \ldots, k_{m+1}; Q) \times \frac{q^2}{2\pi}d\Phi_2(k_j, k_k; q-k_h) z^{1-2\varepsilon} \, dz$$

with $q = k_j + k_k + k_h$. Factor $z^{1-2\varepsilon}$ from identified particle phase space.
Virtual subtraction term $d\hat{\sigma}^T$ and $d\hat{\sigma}^{MF}$

Integration of the fragmentation $X_{30}$ antenna

$$\chi^{id.p}_{30}(z) = \frac{1}{C(\epsilon)} \int d\Phi_2 \frac{q^2}{2\pi} z^{1-2\epsilon} X^{id.p}_{30}, \quad C(\epsilon) = (4\pi e^{-\gamma_E})^\epsilon / (8\pi^2)$$

over the two-particle phase space with kinematics

$$q(q^2) + (-k_p) \rightarrow k_j + k_k$$

with $s_{jk} = (q - k_p)^2 = q^2(1 - z)$. After integration, it can be directly combined with the one-loop mass factorisation kernels

$$\chi^{id.p}_{30}(z) - \mu^{-2\epsilon} \Gamma^{(1)}(z)$$

which subtracts all the explicit poles of the virtual matrix element.
Outline

Introduction

Subtraction at NLO

Subtraction at NNLO

Integration of NNLO antenna functions

Analytical checks

Conclusions and outlook
Real-real subtraction term $d\hat{\sigma}^S$

- **Single unresolved limit:**
  as in the NLO case

- **Double unresolved limit:**
  New ingredient: tree-level four-parton fragmentation antenna function $X_{pjkl}^{0,\text{id.p}}$

\[
M(k_1, \ldots, k_p, k_j, k_k, k_l, \ldots, k_{1+m}) \rightarrow M(k_1, \ldots, K_p, K, \ldots, k_{1+m})
\times \left[ X_{pjkl}^{0,\text{id.p}} - X_{pjk}^{0,\text{id.p}} X_{PKl}^{0,\text{id.P}} - X_{jkl}^{0} X_{pJL}^{0,\text{id.p}} \right]
\]

with the NNLO mapping defined as the generalized version of the NLO mapping

\[
z = \frac{s_{pj} + s_{pk} + s_{pl}}{s_{pj} + s_{pk} + s_{jk} + s_{pl} + s_{jl} + s_{kl}}, \quad K_p = k_p / z, \quad K = k_j + k_k + k_l - (1 - z) \frac{k_p}{z}
\]

It turns into a NLO phase space mapping in its single unresolved limits, as required in order to cancel the single unresolved limits of the $X_{40,\text{id.p}}^{40,\text{id.p}}$ antenna function.
Real-virtual subtraction term $d\hat{\sigma}^T$ and $d\hat{\sigma}^{MF,RV}$

- **Explicit infrared poles:** subtracted by product of $\lambda_{30}^{id,p}(z)$ with tree-level $(m+1)$-parton matrix element.

- **Single unresolved limit:**

  \[
  M_{m+1}^1(k_1, \ldots, k_p, k_j, k_k, \ldots, k_{1+m}) \\
  \rightarrow \left[X^{0, id,p}(k_p, k_j, k_k) M_{m}^1(k_1, \ldots, K_p, K, \ldots, k_{m+1})
  \\
  + X^{1, id,p}(k_p, k_j, k_k) M_{m}^0(k_1, \ldots, K_p, K, \ldots, k_{m+1}) \right]
  \]

  New ingredient: one-loop three-parton fragmentation antenna function $X^{1, id,p}$.

- ... to be combined with mass factorisation contribution containing one-loop $\Gamma^{(1)}(z)$ kernels.
Virtual-virtual subtraction term $d\hat{\sigma}^U$ and $d\hat{\sigma}^{MF,VV}$

The integrated antenna functions

$$\chi_{40}^{id,p}(z) = \frac{1}{[C(\epsilon)]^2} \int d\Phi_3 \frac{q^2}{2\pi} z^{1-2\epsilon} X_{40}^{id,p}$$

$$\chi_{31}^{id,p}(z) = \frac{1}{C(\epsilon)} \int d\Phi_2 \frac{q^2}{2\pi} z^{1-2\epsilon} X_{31}^{id,p}$$

combine with the NNLO mass factorization terms (containing two-loop $\Gamma^{(2)}(z)$ kernels or products of $\Gamma^{(1)}(z)$ kernel), in order to cancel the explicit poles of the double-virtual matrix elements.
Outline

Introduction

Subtraction at NLO

Subtraction at NNLO

Integration of NNLO antenna functions

Analytical checks

Conclusions and outlook
Integration of real-real antenna function $X_{40}^{\text{id.p}}$

- Three-particle phase space with $2 \rightarrow 3$ kinematics

$$q(q^2) + (-k_p) \rightarrow k_1 + k_2 + k_3$$

and invariants $q^2$ and $s = (q - k_p)^2 = q^2(1 - z)$.

- Well-known technique:
  
  - write phase space in terms of cut propagators
  - run reduction with REDUZE2 [Manteuffel and Studerus 2012]
  - evaluate master integral with differential equation method [Gehrmann and Remiddi 2000] in the canonical form [Henn 2015; Henn 2013].
  - solution in terms of harmonic polylogarithms (HPLs) [Remiddi and Vermaseren 2000]
  - boundary condition by internal consistency or explicit evaluation at $z = 1$

- After the master integrals have been inserted, expansion of the $(1 - z)^{-\epsilon}$ and $(1 - z)^{-2\epsilon}$ factors in terms of distributions.
9 master integrals
Deep relationship between space-like and time-like kinematics.

\[ q(q^2) + (-k_p) \to k_1 + k_2 + k_3, \quad q^2 > 0, \quad z = \frac{2 k_p \cdot q}{q^2} \]

\[ q(-Q^2) + k_p \to k_1 + k_2 + k_3, \quad Q^2 > 0, \quad x = \frac{Q^2}{2 k_p \cdot q} \]

- **Investigated in the literature e.g.** [Stratmann and Vogelsang 1997; Almasy, Moch, and Vogt 2012]
- **Same set of master integrals** appearing in the integration of the antenna functions with initial-final kinematics.
- After analytic continuation of the HPLs \( z \to 1/x \), with the package HPL [Maitre 2006; Maitre 2012], we recover the initial-final master integrals.
- Simple “recipe” to relate double-real antenna functions with space-like and time-like kinematics:
  \[ x \to 1/z, \quad Q^2 \to -q^2 \]
Integration of real-real antenna function $X_{31}^{\text{id},p}$

- Integrating one-loop matrix elements over two-particle phase space with $2 \rightarrow 2$ kinematics.
- Same chain as before, with one loop momentum plus two cut propagators.
- We find 6 master integrals. Explicit evaluation of four of them, the remaining two found with differential equation with boundary condition evaluated at $z = 1$. 

![Diagrams](image)
Again, same master integrals appearing in the integration of initial-final antenna functions. Does the recipe

\[ x \to 1/z, \quad Q^2 \to -q^2 \]

work at the real-virtual level?

- Integrals related to one-loop bubble, by “keeping track of \((-1)^{-\epsilon}\) factors”
- Integrals related to one-loop box: analytic continuation required at the integrand level → it prevents a simple relationship between master integrals which can be propagated to the antenna level
Outline

Introduction

Subtraction at NLO

Subtraction at NNLO

Integration of NNLO antenna functions

Analytical checks

Conclusions and outlook
One-particle inclusive spectrum in $e^+e^-$ annihilation

Integrated antenna functions can be related to known analytical results for one-particle inclusive cross sections in $e^+e^-$ annihilation:

$$\frac{d\sigma^H}{dx} = \sum_{i=q,\bar{q},g} \int_x^1 \frac{dz}{z} D_i^H \left( \frac{x}{z} \right) \frac{d\sigma_i^H}{dz} = \sum_j \sigma_j^{(0)} \int_x^1 \frac{dz}{z} D_j^H \left( \frac{x}{z} \right) C_j(z)$$

with $x = 2E_h/\sqrt{s}$ the energy fraction. The coefficient functions $C_j$ are the mass-renormalised and UV-renormalised version of the parton fragmentation functions $\hat{F}_j$:

$$\hat{F}_j = \hat{F}_j^{(0)} + \left( \frac{\hat{\alpha}_s}{4\pi} \right) S_\epsilon \left( \frac{\mu^2}{Q^2} \right)^\epsilon \hat{F}_j^{(1)} + \left( \frac{\hat{\alpha}_s}{4\pi} \right)^2 S_\epsilon^2 \left( \frac{\mu^2}{Q^2} \right)^{2\epsilon} \hat{F}_j^{(2)} + O(\hat{\alpha}_s^3).$$

The leading order $\hat{F}_j^{(0)}$ are just zeros or $\delta$-functions. $\hat{F}_j^{(1)}$ and $\hat{F}_j^{(2)}$ can be directly compared to a linear combination of integrated antenna functions and virtual form factors.
First derived in [Rijken and Van Neerven 1996; Rijken and
Van Neerven 1997] and independently in [Mitov, Moch, and Vogt
2006]. Quark function usually written in terms of non-singlet \( \hat{F}^{NS} \) and pure-singlet \( \hat{F}^{PS} \) components, related to:

\[
\hat{F}^{NS}|_N = 2A_{40}^{id.q} + 8A_{31}^{id.q} + \delta(1 - z)V_q^{(2)}|_N \\
\hat{F}^{NS}|_{NF} = 2B_{40}^{id.q} + \delta(1 - z)V_q^{(2)}|_{NF} \\
\hat{F}^{NS}|_{1/N} = -\tilde{A}_{40}^{id.q} - 8\tilde{A}_{31}^{id.q} - 4\tilde{C}_{40}^{id.\bar{q}} \\
- 2\tilde{C}_{40}^{id.q_1} - 2\tilde{C}_{40}^{id.q_3} + \delta(1 - z)V_q^{(2)}|_{1/N} \\
\hat{F}^{PS} = B_{40}^{id.q'} \\
\hat{F}_g|_N = 4A_{40}^{id.g} + 8A_{31}^{id.g} \\
\hat{F}_g|_{1/N} = -2\tilde{A}_{40}^{id.g} - 8\tilde{A}_{31}^{id.g}
\]
Higgs decay in the heavy-top limit

First obtained in [Almasy, Moch, and Vogt 2012]. Quark $\hat{F}_q$ and gluon function $\hat{F}_g$ related to:

\[
\hat{F}_g|_{N^2} = F_{40}^{id,g} + 4F_{31}^{id,g} + 4\delta(1 - z)V_g^{(2)}|_{N^2}
\]

\[
\hat{F}_g|_{NNF} = 2G_{40}^{id,g} + 4G_{31}^{id,g} + 4\delta(1 - z)V_g^{(2)}|_{NNF}
\]

\[
\hat{F}_g|_{NF/N} = -\tilde{G}_{40}^{id,g} + 4\tilde{G}_{31}^{id,g} + 4\delta(1 - z)V_g^{(2)}|_{NF/N}
\]

\[
\hat{F}_g|_{N^2_F} = 4\tilde{G}_{31}^{id,g} + 4\delta(1 - z)V_g^{(2)}|_{N^2_F}
\]

\[
\hat{F}_q|_{N} = -2G_{40}^{id,q} - 8G_{31}^{id,q}
\]

\[
\hat{F}_q|_{1/N} = \tilde{G}_{40}^{id,q} + 8\tilde{G}_{31}^{id,q} - 4\tilde{F}_{40}^{id,q}
\]

\[
\hat{F}_q|_{NF} = -2\tilde{H}_{40}^{id,q} - 8\tilde{G}_{31}^{id,q}
\]
Outline

Introduction

Subtraction at NLO

Subtraction at NNLO

Integration of NNLO antenna functions

Analytical checks

Conclusions and outlook
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- We have extended the antenna subtraction method to include hadron fragmentation processes up to NNLO in QCD in $e^+e^+$ collisions.
- Local subtraction scheme allows more exclusive calculations e.g. hadrons inside jets, any kind of pre-processing before hadron detection.
- We have derived integrated fragmentation antenna functions in the final-final kinematics, and compared them to known inclusive calculations.
- Final-final fragmentation antenna functions as a step towards fragmentation in $pp$ collisions. Several ingredients in the initial-final kinematics from photon fragmentation.
- Stay tuned for some first phenomenological application!