Feature Investigation of a Segmented Pole Quasi-\textit{Halbach} Tubular-Linear Synchronous Machine

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ABSTRACT The paper is aimed at an investigation of the features of a tubular-linear synchronous quasi-Halbach machine (T-LSM) where the radially-magnetized PMs are substituted by four equal segments with parallel magnetization. This substitution is done in an attempt to improve the machine cost-effectiveness which makes it a viable candidate to equip free-piston engine-based series hybrid propulsion systems. The study is initiated by an analytical formulation of the air gap flux density considering (i) the case of the conventional quasi-Halbach T-LSM and (ii) the case of the segmented pole one. The comparison between the two T-LSMs is extended to a 3D finite element analysis (FEA)-based investigation of their no- and on-load features. Following the prototyping of the quasi-Halbach T-LSM with segmented poles, selected features predicted by 3D FEA are experimentally-validated. It is shown that while the PM segmentation slightly affects the machine back-EMF and cogging force, it has a remarkable fallout on its force production capability which makes it necessary the assessment of the cost-performance trade-off.

INDEX TERMS Quasi-\textit{Halbach} PM tubular-linear synchronous machine, segmented poles, analytical model, 3D finite element analysis, no- and on-load features, cost-performance trade-off.

I. INTRODUCTION

Linear synchronous machines with \textit{Halbach} magnetized PMs in the mover exhibit attractive features, especially a sinusoidal air gap flux density which is desirable in order to achieve reduced stator iron loss, sinusoidal back-EMF waveform, and low cogging force [1, 2]. Several \textit{Halbach} magnetized PM linear topologies have been investigated, among which one can distinguish: (i) the single ring \textit{Halbach} array and (ii) the segmented \textit{Halbach} array [3]. From a manufacturing point of view, the ring \textit{Halbach} array requires a specific magnetized process, while the segmented one could be simply obtained by installing parallel-magnetized blocks with different magnetization directions, in a suitable arrangement.

Further manufacturing process simplification, aimed at the improvement of the cost-effectiveness, could be gained with the reduction of the number of the magnetic blocks [4]. Of particular interest is the so-called “quasi-\textit{Halbach}” concept where a pole is achieved by three magnetic blocks such that: a radially-magnetized PM sandwiched between two spoke-type ones [5, 6]. The concept simplicity makes it attractive in so far as it fulfils the cost/performance tradeoff.

Several works dealing with both rotating and linear quasi-\textit{Halbach} concepts have been reported in the literature. In [7], Sadeghi and Parsa developed a genetic algorithm allowing the optimization of the features of a rotating quasi-\textit{Halbach} magnetized PM machine. To do so, an analytical model depending on the materials and geometrical data, has been derived. First, an optimization of the efficiency, the output power, and the acceleration, has been carried out separately. Then, a multi-objective optimization considering these features simultaneously has led to the \textit{Pareto} optimal solution.

Beyond the genetic algorithms, several innovative multi-objective optimization procedures have been recently introduced and their effectiveness proved, such as the system-level sequential Taguchi and multiobjective system level optimization methods. The latter have been applied by Zhu et al. to the design of switched reluctance motor drives [8, 9].
In [10], Shen and Zhu established an analytical model aimed at the prediction of the features of slotted/slotless PM machines with different number of segments Halbach array, with various PM remanences, magnetisation angles and arcs. A special attention has been paid to the study of the magnetization which represents a key parameter in the modeling of PM machines with segmented Halbach array.

In [11], Zhang et al. proposed a Halbach array incorporating the third harmonic dedicated to five phase PM machines. The magnetizing angle of the proposed Halbach array and the stator teeth width have been optimized using the nonlinear programming method. A FEA based comparison of the features of the optimized machine and those of the machine equipped with a conventional Halbach array has revealed that the torque density and efficiency of the proposed concept are higher than that of the conventional one.

In [12], Chen et al. dealt with different curve-edged Halbach arrays implemented in different T-LSM actuators. A special attention has been given to the investigation of the PM with open boundary shapes. For this purpose, a differential quadrature finite element model has been developed, leading to an optimal selection of the PM edge shape. The study has been ended by a dual validation using FEA and experiments.

It has been reported in [13] that quasi-Halbach T-LSMs are currently considered as viable candidates to equip free piston engines. Embedded in series hybrid propulsion systems, free piston engines built around T-LSMs, have the merit to generate electricity with a quite acceptable efficiency reaching up to 42%; the one of internal combustion engines does not exceed 30% [14], [15].

Within the same application, Zouaghi et al. investigated in [16] the no-load operation of a quasi-Halbach T-LSM. To start with, a formulation of the flux density spatial repartition has enabled the prediction of the no-load features. The study has been extended to a sizing procedure aimed at the force ripple reduction. Then, a special attention has been paid to cancelation of the end effect. In [17], the same authors extended the investigation to the force production capability. This has been carried out using a dedicated analytical model. After a recall of the expressions of the radial and axial air gap flux density due to the mover PMs, the analytical model has been completed by the formulation of the armature magnetic reaction. This made it possible the formulation of the developed force by distinguishing its synchronizing and reluctant components.

In spite of the remarkable increase of the energy efficiency, the large penetration of T-LSM-based free piston engines in series-hybrid powertrains is compromised by the PM cost. This latter could be reduced through the minimization of the PM volume. This could be achieved considering consequent pole topologies [18]–[21].

Beyond the material, the complexity of the magnetization process affects the PM cost-effectiveness particularly the radial magnetization. Within this statement, a special attention is currently paid to the substitution of the radially-magnetized PMs by parallel-magnetized ones.

In [22], Meessen et al. developed an analytical model of a quasi-Halbach T-LSM where the radially-magnetized PMs are approximated by parallel-magnetized PM segments. The air gap flux densities of different topologies with segmented PMs have been predicted and compared to the one given by the conventional quasi-Halbach T-LSM.

It should be underlined that, beyond the manufacturing aspect, the PM segmentation has been considered in order to improve the machine performance by shaping the rotor poles using different PM segments in the case of IPM machines. Within this topic, Yang et al. carried out a comparative study between three IPM machines equipped with: (i) delta-shaped PMs, (ii) single V-shaped PMs, and (iii) double V-shaped PMs in their rotors, sharing the same volume of PMs and the same stator core and winding configuration [23]. In [24], An et al. proposed an approach to predict the armature magnetic reaction (AMR) of an IPM machine with segmented skewed poles. In a first step, the formulation of the AMR has been carried out using the vector superposition method, with the saturation and slotting effects neglected. Then, the virtual magnetic field of the rotor is introduced in order to investigate the influence of the local saturation due to AMR. The latter has been derived combining the subdomain and equivalent magnetic circuit methods. The stator slotting effect has been incorporated using the complex relative permeance.

Moreover, the PM segmentation has been proposed for the reduction of the PM eddy-current losses of fractional-slot PM machines due to the AMR dense harmonic content. This problem has been tackled in [25] where a 3D analytical model has been established for the prediction of the PM eddy-current losses. Following the verification of its validity in the case of unsegmented pole machines, it has been applied to investigate the effects of the PM radial and axial segmentations on the reduction of the PM eddy-current losses.

The present paper deals with an investigation of the effect of the pole segmentation on the features of a quasi-Halbach T-LSM, considering both no- and on-load operations. The segmentation has been limited to the radially-magnetized PMs which have been substituted by four equal segments with parallel magnetization. Fig. 1 shows a radial cross section of a mover north pole in the cases of:

(a) one radially-magnetized PM (conventional concept),
(b) four PM segments with parallel magnetization.

The predicted features are compared to those of the conventional concept. The comparative study is carried out using a dedicated analytical model as well as a 3D FEA one.
II. 3D FORMULATION OF THE FLUX DENSITY SPATIAL REPARTITION

A. TOPOLOGICAL DESCRIPTION

Fig. 2(a) illustrates a 3D view of the topology of the quasi-Halbach T-LSM under investigation. They share the layout shown in Fig. 2(b) where the geometrical parameters involved in the machine modeling are identified. The main design and geometrical parameters shared by both conventional and segmented pole quasi-Halbach T-LSMs are given in tables 1 and 2, respectively.

Prior to initiate the investigation of the PM segmentation effect on the air gap flux density, this latter is expressed in the case of the conventional concept. The formulation developed hereunder is inspired from the model established in [16].

B. CASE OF THE CONVENTIONAL QUASI-HALBACH T-LSM

Assuming a slotsless stator, the magnetic vector potential $\mathbf{A}$ could be expressed in terms of the cylindrical coordinates as:

\[
\begin{align*}
\frac{\partial}{\partial z} & \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_{1\theta}) \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_{1\theta}) \right) = 0 \\
\frac{\partial}{\partial z} & \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_{1\theta}) \right) + \frac{\partial}{\partial r} \left( \frac{1}{r} \frac{\partial}{\partial r} (r A_{1\theta}) \right) = -\mu_0 \nabla \mathbf{M}
\end{align*}
\]

where:
- $r$, $\theta$, and $z$ are the radial, circumferential, and axial coordinates, respectively,
- $A_{1\theta}$ and $A_{1\theta}$ are the magnetic potential vectors in the air gap and PM regions, respectively,
- $\mathbf{M}$ is the magnetization vector which is expressed as:

\[
\mathbf{M}(r, z) = M_r(r, z) \mathbf{e}_r + M_z(r, z) \mathbf{e}_z
\]

Accounting for equation (2), the model resolution could be reduced to a 2D problem. The resulting radial component of the flux density spatial repartition is expressed as follows:

\[
B_r(r, z) = \sum_{n=1}^{\infty} \Pi_n(r) \sin(m_n z) \hat{\lambda}(z)
\]

where:

\[
\Pi_n(r) = a_{1n} B_I \rho(m_n r) + b_{1n} B_K \rho(m_n r)
\]

where $B_I$ and $B_K$ are first and second kinds of modified Bessel functions of order $\rho$, respectively, $\hat{\lambda}(z)$ is the relative permeance function, $a_{1n}$, $b_{1n}$, and $m_n$ are constants.

The derivation of the relative permeance function $\hat{\lambda}(z)$ is initiated by the definition of a corrected air gap $g_c$ as follows:

\[
g_c = K_c g
\]

where $K_c$ is the Carter factor whose analytical prediction has been inspired from the formulation derived in [26], as:

\[
K_c = \frac{z_s}{z_s - \gamma g}
\]

where:

\[
\begin{align*}
\gamma &= 4 \pi \left( \frac{s_o}{2g} \arctan \frac{s_o}{2g} - \ln \left[ \sqrt{1 + \left( \frac{s_o}{2g} \right)^2} \right] \right) \\
z_s &= \frac{\tau_p}{N_s}
\end{align*}
\]

The slotting effect has been taken into account considering the permeance function $\lambda(r, z)$, as [26]:

\[
\lambda(r, z) = \begin{cases} 
\Lambda_0 \left[ 1 - \beta(r) \left( 1 + \cos \frac{z}{0.82z_0} \pi \right) \right] & \text{for } 0 \leq z \leq 0.8z_0 \\
\Lambda_0 & \text{for } 0.8z_0 \leq z \leq z_s/2
\end{cases}
\]
where \( \Lambda_0 = \frac{\mu_0}{2c}, z_0 = s_0, \) and where \( \beta(r) \) is defined at the axis of a stator slot as:

\[
\beta(r) = \frac{1}{2} \left( 1 - \frac{1}{1 - \left( \frac{s_0}{2g_c} \right)^2 \left( 1 + \nu^2 \right)} \right) \tag{9}
\]

where \( \nu \) is the solution of equation:

\[
\ln \left( \frac{1 + b}{1 - b} \right) + \frac{4g_c}{s_0} \arctan \left( \frac{2g_c}{s_0} \right) - \frac{2\pi}{s_0} (r + g_c - R_s) = 0 \tag{10}
\]

where:

\[
\begin{align*}
    a &= \sqrt{1 - \left( \frac{s_0}{2g_c} \right)^2} \\
    b &= \frac{\nu}{\sqrt{a^2 + \nu^2}}
\end{align*} \tag{11}
\]

For a given radial position, the spatial repartition of the air gap flux density is independent of \( r \). The permeance function \( \lambda \) is taken in the middle of the air gap for which \( r = R_s - \frac{g}{2} \).

Finally, the relative permeance function \( \hat{\lambda}(z) \) is derived as:

\[
\hat{\lambda}(z) = \frac{\lambda(z)}{\Lambda_0} \tag{12}
\]

Accounting for its trivial axial component \([16]\), the air gap flux density is assumed to be equal to its radial component, as:

\[
B(r, z) = B_z(r, z) \tag{13}
\]

C. CASE OF THE SEGMENTED POLE QUASI-HALBACH T-LSM

Let us consider a mover pole made up of \( \eta \) parallel-magnetized PM segments. Accounting for the cylindrical coordinate system \((r, \theta, z)\) assigned to each outer position located in the PM segment outer surface (see Fig. 3), the magnetization vector \( \vec{M} \) of the PM segment is expressed as follows:

\[
\begin{align*}
    M_r(r, \theta, z) &= \sum_{n=1,2,...}^{\infty} M_{rn} \sin m_n z \cos(\theta) \\
    M_z(r, \theta, z) &= \sum_{n=1,2,...}^{\infty} M_{zn} \cos m_n z
\end{align*} \tag{14}
\]

where:

\[
-\frac{\pi}{\eta} \leq \theta \leq \frac{\pi}{\eta} \tag{15}
\]

The air gap flux density expressed in the local relative coordinate system is expressed as follows:

\[
B_r(r, z, \theta) = \sum_{n=1}^{\infty} \Pi_n(r) \sin(m_n z) \cos(\theta) \hat{\lambda}(z) \tag{16}
\]

D. CASE STUDY

Let us consider the case where the mover pole is made up of 4 parallel-magnetized PM segments. The expression of \( \hat{\lambda} \) considering the global cylindrical coordinate system yields:

\[
\begin{align*}
    M_r(r, \theta, z) &= \sum_{n=1,2,...}^{\infty} M_{rn} \sin m_n z f(\theta) \\
    M_z(r, \theta, z) &= \sum_{n=1,2,...}^{\infty} M_{zn} \cos m_n z
\end{align*} \tag{17}
\]

where the function \( f(\theta) \) is defined as follows:

\[
f(\theta) = \begin{cases} 
    \cos \theta & \text{for } -\pi/4 \leq \theta \leq \pi/4 \\
    \sin \theta & \text{for } \pi/4 \leq \theta \leq 3\pi/4 \\
    -\cos \theta & \text{for } 3\pi/4 \leq \theta \leq 5\pi/4 \\
    -\sin \theta & \text{for } 5\pi/4 \leq \theta \leq 7\pi/4 
\end{cases} \tag{18}
\]

Considering the global cylindrical coordinate system, the model resolution has led to the following solution:

\[
B_r(r, z, \theta) = \sum_{n=1}^{\infty} \Pi_n(r) \sin(m_n z) f(\theta) \hat{\lambda}(z) \tag{19}
\]

The developed model enabled the assessment of the waveforms of the air gap flux density along the axial and radial directions. These have been combined in 3D plots as shown in Figs. 4 and 5 which correspond to the results yielded by the conventional and the segmented pole quasi-Halbach T-LSMs, respectively. It is to be noted that the air gap flux density is axially-modulated by the slotting effect in both machines, and is radially-modulated by the pole segmentation in the case of the proposed quasi-Halbach T-LSM.

The air gap flux density has been predicted in two axial positions corresponding to its maximum and minimum values. The obtained results are drawn by continuous lines in Fig. 6. The case of the conventional machine has been also treated, leading to results drawn by interrupted lines in Fig. 6. It clearly appears that the air gap flux density is affected by the pole segmentation especially within the PM interfaces.

To sum up, one can conclude that, in the manner of the permeance function that affects the air gap flux density in the axial direction of both quasi-Halbach T-LSMs, the pole segmentation leads to a radial degradation of the air gap flux density that turns to be significant at the PM segment interfaces. The developed model could be simply adapted to assess the effect of the pole segmentation under variable number of PM segments \( \eta \). An accurate investigation of such an effect has to be carried out. The following section develops this idea considering a 3D FEA.
III. 3D FEA-BASED FEATURE INVESTIGATION

This section is devoted to a 3D FEA-based investigation of the no- and on-load features of both quasi-Halbach T-LSMs, using a MagNet software package. The use of 3D FEA is motivated by the fact the axis-symmetry adopted in 2D FEA is not applicable in the case of quasi-Halbach T-LSMs with segmented poles. The 3D FEA has been applied to a study domain including the total machine, as shown in Fig. 7.

A. NO-LOAD FEATURES

1) Flux Density Mapping and Vectors

Figs. 8(a) and 8(b) show zoom views of the flux density mapping and vectors in radial cross sections of the segmented pole and conventional quasi-Halbach T-LSMs, respectively.

Compared to the conventional machine, the segmented pole one exhibits:

- a nonuniform repartition of the flux density in the stator teeth facing the mover poles. Indeed, it takes higher values in the tooth sections, radially-located after the shoes, facing the middles of the PM segments,
- an increase of the flux density in the PM segment lateral regions with remarkable singularities in their interfaces,
- a similarity in the behavior of the flux density in the stator teeth facing the mover poles and in the air gap, except in the regions facing the PM segment interfaces. These are characterized by high relative values of the flux density reaching 0.7T in the air gap near the mover and low values not exceeding 0.2T in the stator shoe.

For the sake of a better understanding of the phenomenon described in the last item, the flux density has been computed by 3D FEA in the middle of the air gap and for an axial position for which a mover north pole faces a stator tooth. The case of the conventional machine has been also treated considering the same position of the mover. The obtained results are shown in Fig. 9.

The comparison of the air gap flux density loci described in the polar coordinate frame has revealed the following remarks:
The PM segmentation has led to a remarkable increase of the flux density in the air gap regions facing the middle of the PM segments,

A local degradation followed by a reversal of the air gap flux density within the PM segment interfaces,

Focusing these interfaces, the negative flux density in the air gap and the positive one in the PM segments leads to the circulation of a local leakage flux. This phenomenon is confirmed by the mapping of Fig. 8(b). This latter also shows a similar leakage flux taking place between the PM interfaces and the aluminium-made shaft which is expected accounting for the similar magnetic behaviors of air and aluminium.

Further 3D FEA-based investigation of the flux density mapping has been carried out with emphasis on its normal component computed on the mover outer surface. The obtained results are illustrated in Fig. 10, referring to which it is to be noted that the PM segmentation affects the normal component of the flux density along the mover pole outer surface in the circumferential and axial directions. Moreover, Fig. 10(a) confirms the existence of a leakage flux along the outer limits of the PM segment interfaces. It also confirms that this flux circulates locally between the air gap and the PM segment interfaces and is slightly affected by the stator.
Indeed, one can notice the presence of such a leakage flux in all PM segment interfaces either the corresponding poles belong to the active or the passive parts of the mover.

2) Back-EMFs

A 3D FEA-based investigation of the line-to-line back-EMFs has been carried out and has led to the results shown in Fig. 11. This latter shows in continuous line the waveforms of the line-to-line back-EMFs in the case of the conventional T-LSM, and in interrupted line those in the case of the segmented pole T-LSM, for a mover velocity of 1.64 m/s.

One can remark that the back-EMFs have a slightly higher amplitude in the case of the conventional quasi-Halbach T-LSM. It appears that, in spite of the degradation of the air gap flux density within the interfaces, the back-EMF is slightly affected by the pole segmentation. This is due to the fact that the flux per pole is almost the same in both machines. This statement is confirmed by Fig. 9 from which one can notice that the average value of the flux density in the air gap region facing a mover pole is equal to 0.657T in the segmented pole machine and to 0.666T in the conventional one.

Against expectations, the back-EMF waveforms of both T-LSMs waveforms are far from being sinusoidal. The harmonic content of the back-EMFs is due to the machine short stroke on one hand and to the end effect phenomenon on the other hand [30]. This latter also affects the symmetry of the back-EMFs induced in the three phases, as shown in Fig. 11.

3) Cogging Force

The cogging force has been computed in the case of the conventional quasi-Halbach T-LSM. The obtained results are plotted in black in Fig. 12. These enabled the validation of the cogging force waveform analytically-predicted in [16]. Fig. 12 also shows in red, the cogging force waveform exhibited by the segmented pole machine. One can notice that its peak-to-peak value is slightly higher than the one of the conventional machine, while the periodicity of both waveforms is almost the same.

With regard to the high cogging force peak-to-peak value, whether the machine poles are segmented or not, it should be underlined that this drawback is due to the end effect that affects most if not all linear machines [16].

Basically, the cogging force \( F_c \) is the sum of the cogging forces applied to the \( N_s + 1 \) stator teeth under a null armature current, as:

\[
F_c = \sum_{i=1}^{N_s+1} F_c^i(i)
\]

where \( F_c^i(1) \) and \( F_c^i(N + 1) \) are the cogging forces applied to the stator end teeth. These forces are by far higher than the ones applied to the remaining stator teeth, resulting in a number of periods equal to 2 instead of \( N_s + 1 \) [16].

Referring to the literature, it has been reported that the cogging force could be significantly-reduced considering different design approaches. In the manner of rotating machines, the substitution of the distributed armature windings by a concentrated ones, with selected slot-pole combinations yielding a fractional slot per pole and per phase lower than unity, enables the reduction of the cogging force [27], [28]. This approach is penalized by excessive PM eddy current loss due to the dense harmonic content of the armature magnetic reaction; a drawback that could be tackled by rearranging the armature according to multi-layer distributions [29]. However, the effectiveness of this approach is compromised by the end effect in the case of linear machines. This makes it necessary the association of an end effect minimization approach, as proposed in [30]–[32].

B. ON-LOAD FEATURES

Fig. 13 shows the load characteristics of both quasi-Halbach T-LSMs, in the case of a variable resistor in the armature and for a mover velocity of 1.64 m/s. One can remark that, for a given loading level, the armature voltage is higher in the case of the conventional quasi-Halbach T-LSM than in the case of the PM segmented one. Accounting for the fact that the back-EMFs are slightly affected by the pole segmentation, the gap between the two characteristics of Fig. 13 is due to an increase of the machine inductance. Indeed, the ratio of the phase flux linkage transformed into leakage turns to be higher following the poles segmentation. This leads to a decrease of the total reluctance through which the phase flux linkage flows, and consequently an increase of the phase inductance.

Fig. 14 shows the characteristics giving the mean values of the forces developed by both quasi-Halbach T-LSMs versus...
the loading level in the case of a variable resistor in the armature and for a mover velocity of 1.64 m/s. One can notice that the pole segmentation leads to a reduction of the force production capability of 18% in the considered loading range. In spite of this limitation, it should be underlined that such a reduction would not be so significant in the case of series-hybrid propulsion systems equipped with free piston engines. Integrated in the latter, the segmented pole quasi-Halbach T-LSM operates as a generator that charges the battery pack; fundamentally considered as a R-C type load.

IV. EXPERIMENTAL VALIDATION
For the sake of validation of the FEA-based predicted features, a prototype of the segmented PM quasi-Halbach T-LSM has been built. Fig. 15 shows the face (a) and side (b) views of the prototyped machine. The used PMs are of sintered samarium-cobalt type (Sm<sub>2</sub>Co<sub>17</sub>) with a remanence <br>

The prime mover is achieved by a separately-excited DC motor mechanically-coupled to a crank-handle system that drives the prototype. Fig. 16 is a photo of the scope integrated in the developed test bench, showing the back-EMFs induced in two phases of the prototyped machine.

Fig. 17 shows the measured line-to-line back-EMFs for a linear velocity of 1.64 m/s. For the sake of comparison, the FEA results shown in Fig. 11, are recalled. One can notice a quite good agreement between FEA and experimental results.

Fig. 18 shows the waveform of the force developed by the prototyped T-LSM, for a linear velocity of 1.64 m/s and an armature current rms value of 0.165 A. It also shows the developed force computed by 3D FEA considering the same loading condition and mover velocity as in the experimental test. The difference between the FEA and experimental results is due to the assumption adopted in the experimental data-based calculation of the developed force, such that:

\[
F_{mes} = \frac{u_a i_a + u_b i_b + u_c i_c}{v}
\] (21)

instead of:

\[
F = \frac{e_a i_a + e_b i_b + e_c i_c}{v}
\] (22)

where \([e, u, i]_{(a,b,c)}\) are the back-EMFs, voltages, and currents, respectively.
Fig. 19 illustrates a linear shape of the load characteristic with a variable resistor in the armature, for a mover velocity of 1.64 m/s. This is due to the linear behavior of the magnetic circuit. Indeed, referring to Fig. 8, the no-load flux density does not exceed 1.35T. Furthermore, the loading range is relatively narrow, yielding a limited armature magnetic reaction.

Fig. 20 illustrates the variation of the mean value of the force developed by the prototyped segmented pole quasi-Halbach T-LSM with respect to the loading level and for a mover velocity of 1.64 m/s. In the manner of Fig. 19, the variation of the loading level has been achieved by a variable resistor in the armature. For the sake of comparison, the developed force, computed by 3D FEA and shown in Fig. 14, is recalled in Fig. 20. One can remark a quite acceptable agreement between the experimental and FEA results.

From the analysis of Figs. 19 and 20, one can notice a minor shift between the 3D FEA results and the experimental ones. This is due to the following causes:

- the experimental tests have been carried out under a speed of 1.64 m/s which represents actually a mean value. Indeed, made up a separately-excited DC motor mechanically-coupled to a crank-handle system, the prime mover does not provide a constant speed along the mover short stroke. This approximation concerns both Figs. 19 and 20,
- the experimental data-based calculation of the developed force is achieved used equation (21). This assumption concerns only Fig. 20.

This is why the shift between the 3D FEA results and the experimental ones is more remarkable in Fig. 20.

V. COST-PERFORMANCE TRADE-OFF

The selection of SmCo PMs was motivated by the fact that they represent the alloys of choice when high-temperature stability is needed. Indeed, accounting for the internal combustion taking place in free piston engines, the PMs capability to operate in a high temperature environment without being demagnetized is specifically required. Beyond their high-temperature stability, SmCo magnets exhibit a strong resistance to corrosion and oxidation. SmCo alloys were discovered and developed in the late 1960s by Strnat. Currently, there are two series of SmCo magnets, namely: SmCo$_5$ and Sm$_2$Co$_{17}$. The latter is reputed by a higher maximum working temperature reaching up to 350°C compared to the former with a maximum working temperature not exceeding 250°C. Beyond these two potential candidates, the SmCo$_7$ alloy with a TbCu$_7$-type is presently under investigation [33].

The cost of SmCo magnets is by far dominated by the cobalt one. About 90% of the world’s cobalt supply came from Zaire, a country that was racked by civil unrest. By 1979, cobalt prices had risen by a factor of six in two years to reach 72 USD/Kg (250 USD/Kg in current terms). The cobalt price is currently 70 USD/Kg, but remains volatile and has fluctuated between 20 and 110 USD/Kg over the past ten years, with spikes in 2008 and 2018 [34].
Dealing with PM manufacturing, the magnetization is a key step which is systematically-achieved at the end of the process. Anisotropic PMs have an intrinsic preferred magnetization direction. They exhibit interesting magnetic features providing that their magnetization is carried out in a direction parallel to their preferred one. Furthermore, their magnetization process is currently considered as a mature technology that remains simpler and cheaper in comparison with the one of their isotropic counterparts. The latter do not have a preferred magnetization direction which offers many options for designers to select different PM magnetization orientations. However, this degree of freedom is compromised by a high cost process required to build specific and complex magnetization fixtures especially in the case of rare earth PMs [35]. Among the most complex magnetization fixtures, one can distinguish those required to achieve:

- Halbach arrays in the case of rotating machines,
- multipole magnetizations of rotating machines,
- radially-magnetized PMs (namely the poles) of quasi-Halbach array in the case of linear machines.

SmCo magnets belong to the anisotropic class of PMs. They are manufactured by sintering (most common) or by injection molding and compression bonding. In the two latter forms, the oriented magnetic grains are embedded in a polymer matrix, resulting in a 50% loss of the energy product compared to one of a fully dense oriented, sintered magnet [34], [36]. In light of that, it clearly appears that achieving the radially-magnetized PMs of quasi-Halbach T-LSMs by SmCo alloys (in any form) is by far impractical. These could be achieved by isotropic magnets made from randomly oriented grains (sintered or bonded).

When it comes to find out potential substitutes of SmCo magnets, one has to account for the challenge in achieving (i) comparable magnetic and thermal performance along with (ii) a reasonable cost. Dealing with the first criterion, the only possible candidates are neodymium magnets which are made from alloys of neodymium, iron and boron (NdFeB) [37]. Beyond the conventional oriented sintered magnetization of neodymium magnets, some manufacturers, such as Nitto Denko Corporation (Japan), developed dedicated technologies to fabricate sintered neodymium magnets with controlled non-uniform orientation of the magnetic field. Specific tooling varying from shape to shape and from size to size makes these technologies far from fulfilling the reasonable cost criterion.

Within the same framework, let us discuss the candidacy of bonded neodymium magnets. These are made by melt spinning a thin ribbon incorporating randomly oriented NdFeB grains. This ribbon is then pulverized into particles, mixed with a polymer, and either compression- or injection-molded into bonded magnets [38]. Nevertheless, randomly oriented, bonded magnets are penalized by a poor energy production which is eight times less than the oriented sintered PMs [34]. This makes the bonded NdFeB PMs out of scope regarding their possible integration in quasi-Halbach T-LSMs.

To sum up, it clearly appears that, under the most optimistic scenario, the fabrication of the radially-magnetized PMs using a NdFeB alloy could only be done under a random sintered magnetization. This would be carried out at the expense of (i) a loss in high-temperature stability, (ii) a loss in the resistance to corrosion and oxidation, and (iii) a loss in the energy production, when compared to SmCo sintered PMs. Moreover, it is more likely admitted that SmCo sintered PMs remain cheaper than random sintered NdFeB ones given the specificity and the complexity of the required tooling. However, SmCo magnets with their magnetization aligned along their intrinsic one, could not fit into the radially-magnetized poles of quasi-Halbach T-LSMs. Accounting for such a technological barrier on one hand and for the high-temperature stability and the strong resistance to corrosion and oxidation of SmCo sintered PMs; priceless benefits for free piston engines, on the other hand, the present work considered building the poles of quasi-Halbach T-LSMs by four segments of SmCo sintered PMs. Nevertheless, it has been found that this approach is allied to a loss in the on-load features. This said, taking advantage of the developed analytical model, the study could be extended to a search of an optimal number of PM segments that minimizes the segmentation effect on the on-load features.

### VI. POLE SEGMENTATION: PROS AND CONS

As a summary of this work, the present paragraph enumerates the advantages and disadvantages of the pole segmentation of quasi-Halbach T-LSMs, with respect to the conventional topology. These are classified in table 3. Beyond the features investigated in the previous sections, table 3 also includes the effect of the pole segmentation on the iron loss. Regarding the PM eddy current loss, and referring to [25], they are reduced by the pole segmentation. However, and referring to Fig. 8, the pole segmentation causes a local saturation in the middle of the PM segments which leads to an increase of the iron loss. With this said, and for high speed oscillatory applications such as free piston engines, a special attention should be paid to the iron loss reduction. The increase of the number of the pole segments represents a crucial benefit for the reduction of the PM eddy current loss. Concerning the stator, and accounting for the difficulties to achieve a tubular magnetic circuit with lamination, the slotless topology could be adopted in order to reduce the iron loss. In this case, the stator coreback would be made up of isolated bars [39].
VI. CONCLUSION

This paper was devoted to an investigation of the no- and on-load features of a segmented PM quasi-Halbach T-LSM where the radially-magnetized PMs have been substituted by four equal PM pieces of parallel-magnetization. The study has been initiated by an analytical formulation of the no-load flux density spatial repartition considering (i) the case of the conventional quasi-Halbach T-LSM and (ii) the case of the segmented pole one. The established models enabled a 3D investigation of the spatial repartitions of the flux density with emphasis on the effect of the mover pole segmentation.

The comparison between both T-LSMs has been extended to a 3D FEA-based investigation of the no- and on-load features. It has been found that the PM segmentation causes a local degradation of the air gap flux density within the interfaces between the PM segments. In spite of such a degradation, it has been noticed that the back-EMF production capability is slightly affected by the PM segmentation. However, it has been shown that the PM segmentation leads to a reduction of the on-load features. Such a decrease is mainly due to an increase of the machine inductance which is allied to an augmentation of the leakage fluxes taking place in the interfaces between the PM segments. The study has been achieved by an experimental validation of the no- and on-load features predicted by 3D FEA.

In light of the obtained results, there is some way to go before the proposed concept could be regarded as a mature technology. Many continuations could be treated within the outlooks. These could be initiated by an optimization of the number of the PM segments in an attempt to minimize the segmentation effects, using the developed analytical model.

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