How does the supervision stringency affect systemic risk based on the differential dynamic model?

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ABSTRACT

This paper focused on the supervision stringency and studied its impact on the financial system risk contagion mechanism. This paper adopts Susceptible-Exposed-Infected-Recovered epidemical model and sets supervision stringency as the principal parameter. The model was formed by a differential equation set and financial system are set with Susceptible group, Exposed group, Infectious A, Infectious B, and Removed group. Based on the theoretical research, this paper gave the steady-state solution and the robust conditions for equilibrium. The conclusion is that in a short time, a small portion of Susceptible group and Exposed group will become Removed group, while a big portion rapidly becomes Infectious A, B. The rate and quantity of other institutions infected is much higher than systematically important financial institutions. Meanwhile, it is known that enhancing supervision stringency is instrumental in alleviating risk spill-over effect of other institutions and risk contagion among institutions. Under system equilibrium, the number of infectious institutions gradually decreases with supervision stringency increasing. Furthermore, the appropriate enhancement of supervision stringency can avoid risk eruption, whereas risk contagion could outbreak if supervision was overstringent.

1. Background and literature review

After the financial crisis in 2008, financial systemic risk prevention has become the main objective of government regulators and international organizations. Financial systemic risk refers to the possibility that individual financial institution suffers from economic loss due to system\textsuperscript{1} volatility, crises or paralysis caused by external shocks or internal factors. Generally, systemic risk includes policy risk, economic cyclical fluctuation risk, interest rate risk, purchasing power risk, exchange rate risks etc., in which both macro-external risk and micro-internal risk are included. Among all, individual institution risk attracts much attention: under certain conditions\textsuperscript{2}, individual risk often fails to be handled timely and spreads to the system through the trading network, then it infects the whole system and triggers system risk. Risk introducers becomes risk exporters, then leads to large-scale systemic risk contagion.

To avoid this, financial supervisory authorities need to act, but consensus on supervision effect has not been reached in academia. Many scholars thought financial supervision can reduce bankruptcy risk of individual banks (Barth, Caprio, & Levine, 2004; Buch & DeLong, 2008; Laeven & Levine, 2009), and alleviate the accumulation of financial systemic risk (Hart & Zingales, 2011; Kashyap, Rajan, & Stein, 2008). Faten Ben Bouhenni (2014) pointed out restrictions on bank activities, supervisors’ power and capital adequacy decrease risk-taking and supervision enhance bank’s stability. The study of John Kandrac and Schlusche (2017) shows the institutions affected by exogenous reduction in supervisory attention take on more risk than unaffected counterparts. The empirical result of Morshedur, Ali, and Mouri (2018) shows a significant negative relation between risk-taking and capital regulation. Ailian Zhang and Liu (2019) assessed the effectiveness of implemented regulation in 2013 on interbank financing for Chinese commercial banks and proved that bankruptcy risk decreases after the regulation. Ningyu, Kezhong, Changjun, and Badar (2019) proved that under the explicit deposit insurance scheme, higher regulatory ability could reduce the risk of banking during normal times. These papers believe that more stringent supervision did have an effect on reducing risk-taking. However, some bankers raised doubt about it, they thought that stricter fund requirements and supervision organization with greater power will deteriorate...
bank performance and then destabilize the financial system. Barth et al. (2004) argued that stricter capital requirements do not necessarily improve the financial system stability; Repullo (2004) further believes that capital supervision will instead increase bank risk activities due to highly clustered market. Zhang and Jiang (2018) present empirical evidence that higher capital requirement pressure brings about a lower charter value for banks, which, in turn, increases their risk-taking ability. Hai, Jinyi, and Chen (2019) finds that requiring banks to build up too much capital buffer is more likely to result in greater risk-taking ability for high-risk banks. Gamze and Pelin (2019) provided evidence that higher activity restrictions strongly increase the bank risk for developed markets, and powerful supervisory agencies will lead to an increase in bank risk. Faizul Haque (2019) used data of 144 conventional banks from 12 Middle East and North Africa (MENA) countries and found official supervisory power has a positive association with bank risk and capital stringency increases bank risk. Based on these arguments, this paper will analyse systemic risk contagion mechanism and the impact of supervision stringency on it. The individual risk of financial institution is uncertain and has its own characteristics, but supervision stringency will greatly affect risk-taking ability: If the requirement for capital buffer is too high, it will take banks more cost filling the capital gap and cause problems including slowdown in credit and economic growth, shrink in profit margins and decrease of investment projects. Furthermore, if the supervision requirements exceed the level that individual financial institutions are willing to accept, it may further induce supervision arbitrage. Or, the financial institutions may strengthen its risk control construction and reduce risk-taking activities under supervision; if the supervision stringency is low, under market competition, individual financial institutions will spare no effort to pursue the interests of investors and shareholders, resulting in improper risk control and increasing risk behaviour.

Although adjusting supervision stringency can improve industrial risk control ability, it still varied among interconnected individual institutions in the financial system, mainly constrained by capital, human resources, technology and major business etc. For this reason, when individual financial institution breaks into risk and transmits risk, its associated financial institutions behave differently: Some institutions control risk well and they can mitigate risks, stop risk transmission and form an immune mechanism; Some get infected and become risk exporters; Some are greatly affected and go bankrupt. Therefore, the entire financial system will undergo dynamic changes. There are always some financial institutions that become risk exporters or go bankrupt then exit, and the state of financial institutions changes over time in the process.

Based on the above points, this paper holds that the financial system has dynamic characteristics in essence, especially when considering the risk contagion. There are always some enterprises facing risks, exporting risks, and even going bankrupt and exit, so it will be necessary to conduct a dynamic analysis. From a macro-perspective, factors that affect system equilibrium and activities around equilibrium point have a great reference significance, such as whether drastic change would happen in small neighbourhood and its evolution direction. This paper focuses on how supervision stringency affects the systemic risk contagion process.

More importantly, this essay considers connection between micro-risks and macro-risks. The nature of financial system determines different reflections on the macro-factors, and the statistical characteristics of many micro-risk factors are usually representations of macro-risks. The most typical example is chaos dynamics, which refers to that a small change in one condition may lead to a great change in the entire system in the complex dynamical system. Taking financial system as an example, an institution’s default may cause a wide-scale system default. We mainly studied this phenomenon in this paper, that is, how micro-actions trigger macro-risks. In reality, for regulators, in addition to the existing modes, measures about chaos risk need to be considered urgently.

Financial risk could be quantified at the initial stage, but the impact of risk transmission might be more reflected by the state of financial institutions, and the regulators also perceive risk through their state. In other words, the state of institution is already an excellent dynamic factor. This paper mainly uses a dynamic system to combine the macro-factors with enterprises’ behaviour, and makes research on the impact of supervision stringency.

This research is based on the dynamic characteristics of the financial risk, which originates from the business connection in the financial system. To a certain extent, these connections are similar to infectious diseases transmission. Therefore, aimed to analyse the financial risk contagion mechanism, economists started to apply the epidemiological Susceptible-Infected-Recovered (SIR) model to research studies. The SIR model was first proposed by Kermack and McKendrick (1927), who established the ordinary differential equation (ODE) to analyse the dynamic transmission process among sensitive state, infection state and immune state of individuals and studied methods to measure the infection rate, transmission threshold and equilibrium point. Based on the SIR model, a series of models have been derived,
including the Susceptible-Exposed-Infected-Recovered (SEIR) model with the incubation period which has been applied in many financial areas.

In international studies, Garas, Argyrakis, Rozenblat, Tomassini, and Havlin (2010) first applied SIR model to the financial crisis contagion among national financial institutions. Based on the global corporate ownership network and international trade network, the author built an intercountry network model, set all network nodes as a risk-sensitive group, individual nodes as infection resources, successful risk controllers as risk-immune groups. Through simulation, 12 central susceptible infectious sources are obtained which are highly likely to trigger a global financial crisis. Notably, six of them are small- and medium-sized countries instead of big countries like the United States. Drawing on experience from Gara, Toivanen (2013) further studied risk contagion among European interbank banks and found that the concentration degree, business links and risk exposure are more likely to cause financial systemic risk rather than high total market size. Furthermore, when risks disrupt, liquidity operations and other policies conducted by central banks can only slightly mitigate financial risk contagion, supervisory authorities should strengthen risk prevention measures. Based on the SIR model, Demiris, Kypraios, and Smith (2013) considered the change in infection rate through national trade channel and financial business channel under financial crisis and found establishing the risk control mechanism could mitigate the impact of financial crisis. Through the SEIR model, Derbali (2016) studied the 12 default risk transmissions of 18 Greek companies from 2006 to 2012. The study showed that strengthening supervision is not always accompanied by the stable financial system, instead an open and transparent information system is the key to preventing financial systemic risk. Alberto, Davide, Danilo, and Simone (2019) assumed that banks can be only susceptible to or infected by speculation, and upon recovery from speculation banks do not ever become immune. Thus, they characterize financial contagion as a susceptible-infected-susceptible (SIS) model, which represents interbank dynamics more realistically than in an SIR set-up. In Chinese studies, Cao and Zhu (2012) simulated the risk contagion process of banks under the SIR model based on the Complex Network Theory. They found that the number of ultimate risk-immunized banks is dependent on the infection rate and network structure, and the contagion threshold is vital in controlling risk contagion scale. Ma, Zhuang, and Li (2013) studied the stock market crisis transmission with the SIR model and found that crisis spreads extremely fast when the capital chain breaks. Using the SIR model, Mi, Liu, and Mi (2007) analysed enterprise’s internal infection mechanism and found that improving enterprise immunity could effectively prevent risk infection. People’s bank of China Nanning branch Research Group (2017) applied the Susceptible-Infected-Recovered-Susceptible model to simulate risk cross-infection mechanism in the financial market, and obtained the risk spread threshold. Their simulation showed it is inefficient to use ex-ante risk prevention strategy alone or ex-post risk mitigation strategy alone to prevent and control risks. Risk contagion can be effectively controlled only when both strategies are used.

Financial supervision is a huge systematic project. It needs to be put in place too much on the stringency of supervision. The excessive supervision is easy to passivate the innovation of the market and hinder the innovation and prosperity of the industry. If the supervision is too little, it will cause the market from reality to virtuality, which is not conducive to the development of the financial service real economy. Controlling financial supervision is conducive to improving the flexibility of financial supervision and promoting financial regulation to better balance the relationship between industry development and industry management.

The particularity of the financial industry determines that the supervision stringency needs to maintain a certain flexibility. If financial supervision is too loose, it will give the lawless elements the opportunity to take advantage of the opportunity to bring losses to the society and consumers, causing the market to ‘out of control.’ Taking P2P network lending as an example, due to the lack of standardized management of online lending in 2013–2015, thousands of counterfeit online lending platforms illegally raise funds and financial fraud through Internet channels, which not only brought loss of equity to investors. And also, it has increased the cost and difficulty of supervision for industry rectification and supervision in 2016–2018. The flexible financial supervision thinking needs to master the principle of combining power delegation and tightened oversight. At the same time, the flexibility of financial supervision needs to be fully adjusted in conjunction with the different development goals of the real economy in different development periods.

Although many papers had already introduced the epidemiological SIR model into the studies of systemic risk contagion, there are few similar studies in China, and research studies mainly introduce the financial risk SIR model to individual case or particular industry but dynamic contagion (from Susceptible to Infectious to Removed) simulation is still in its vacancy. More importantly, risk contagion process varied among different organizations, nevertheless, existing studies did not take this into consideration, and the results might be far from practice. This paper modified this defect and
classified organizations as systemically important financial institutions and other organizations. Specifically, we rely on an epidemiological approach to characterize how financial activities’ interconnection between organizations can determine the risk contagion of the financial system, which would finally reach system stability. Besides, we also take into account a more realistic condition that system characteristics and risk infection rates are assumed to be varying with supervision stringency. This extension allows us to provide a simple and intuitive explanation of how risk infection rate affects final dynamic system equilibrium.

Based on the SIR model, this paper will introduce supervision stringency, using a dynamic system to combine macro-factors with micro-enterprises behaviours, and analyse the impact of supervision stringency on financial systemic risk contagion according to how the state changes dynamics of enterprises. The second part is model construction. It introduces a risk contagion model which considers supervision stringency and is suitable for Chinese financial system. The third part is equilibrium point analysis. System analysing and proving will be carried out. The fourth part is simulation. The fifth part is the conclusion.

2. Preliminaries

2.1. Hypothesis

In the Chinese financial system, capital and information of financial institutions, such as banks, securities traders and fund companies are highly intensive and of strong liquidity, and financial business amongst them are close-linked. Financial risks are transmitted to relevant institutions through trading networks, financial instruments, funds and other risk carriers. Referring to the epidemiological SEIR dynamic model, in the following section, this paper will build a risk contagion model among financial institutions. The research hypotheses will be explained as follows:

Hypothesis 1: The system network constituted of financial institutions is limited but not closed. Every financial institution is a node in the network system. In the risk contagion process, financial institutions normally enter or exit the system in the same proportion.

Hypothesis 2: There are differences among financial institutions on every node. Some nodes can obtain immunity through risk prevention measures; Some institutions are infected and then become risk-immune group through various risk prevention measures; Some institutions fail to become immune and exit the system unusually.

Hypothesis 3: There are two types of risk sources, which respectively represent infected systemically-important financial institutions and other infected institutions (non-systemically important financial institutions). After being affected, the systemically-important financial institutions downsize their business and might become other institutions after then.

2.2. Variables and parameter assumptions

Consider a financial system, it consists of numerous financial enterprises that are interconnected through financial businesses and form a tight network. It is financial businesses that enable these enterprises to transmit financial risks among them: some enterprises may fall into risk because unable to cash cheques (or execute); some enterprises may become risk sources affected by market risks. Because of the interconnection among financial institutions, the system shows obvious dynamic characteristic during risk contagion. To illustrate this system, we established SEIR model, which originates from epidemiology SIR model. By setting ordinary differential equation, SIR model describes dynamic process among different status including of sensitive status (S), infected status (I), immune status (R). Based on the original model, a series of derivative models were developed, such as SEIR model containing incubation period. Moreover, considering that supervision greatly affects financial system, risk contagion and enterprises risk management methods, we take supervision stringency into consideration and set as $\omega$, which refers to the proportion of risk immunized in newly entered enterprises, while $1 - \omega$ refers to the proportion of risk susceptible institutions. It is noted that when enterprises become risk exposed (E) and become risk source (I) after infected, these risk sources can be divided into systemically-important risk sources and other risk source according to business scale.

Specifically, we consider the following types of enterprises (institutions) in the system to simulate how supervision stringency affects systemic financial risk contagion: Healthy group, Susceptible group, Exposed group, Infectious A, Infectious B, Removed group.

For convenience, this paper considered the enterprises that enter the market normally as Healthy group, which may become Removed group after influenced by supervision measures, or become Susceptible group after increasing business connections with other financial institutions.

(1) Susceptible group ($S$): mainly refers to the enterprises which have not established effective risk prevention mechanism, get infected easily and then become risk exporter. To a certain extent, the Susceptible group can be measured by the proportion of financial business in total business.
Figure 1. Risk contagion process of the financial system.

(2) Exposed group (E): refers to enterprises before they become Infectious A or Infectious B. At this time, the enterprises have already been impacted by financial risks and are trying to deal with risks.

(3) Infectious A (I₁): refers to systemically important financial institutions. They break into risk and transmit risk, and can be characterized by a wide connection, great influence, rapid action and quick reflections on the entire financial system and related enterprises. These institutions may go bankruptcy or become Removed group under various measurements or downsize business then become Infectious B.

(4) Infectious B (I₂): refers to institutions other than the Infectious A, i.e. non-systemically important financial institutions. They break into risk and transmit risk. Here these two classes are used as representatives for convenience.

(5) Removed group (R): refers to enterprises that can avoid or suffer less losses. Removed group often obtains immunity in two ways:

The first way is reducing and cutting off channels of financial risk contagion, and reducing business connections. However, in modern society, capital operation has become an important business operation method, so it is very difficult to reduce financial connection in this context.

The second way is risk hedging and risk arrangement which prevent enterprises from being affected by financial risks. For example, the market exchange rate risk can be hedged through exchange or derivatives such as options and futures. Credit risk can be hedged through asset connections and credit default swaps. Although hedging needs cost and cut down return, it can avoid certain impact of financial risks.

In addition to the above two ways, the policies and guidance of supervision authorities, the industry characteristics etc., will affect enterprises’ judgment on risks, and further affect the risk-handling method. Here consider supervision policies, the stricter the supervision policies, the higher the probability that the enterprise become immune, which means a greater proportion of newly entered enterprise might become Removed group.

According to hypothesis 1: \( T = S + E + I_1 + I_2 + R \), \( T \) is the number of financial institution nodes. According to their state dynamics in risk contagion, the process of financial system is shown in Figure 1.

Model parameters are set as follows:

\( \omega \): supervision stringency, i.e. the proportion of newly entered companies become Removed group directly while \( 1 - \omega \) represents the proportion of companies which become Susceptible group.

\( \beta_1, \beta_2 \): the risk infection rate of systemically important financial institutions and other institutions, respectively.

\( \sigma_1, \sigma_2 \): the possibility that Exposed group turns into Infectious A and Infectious B, respectively.

\( \alpha_1, \alpha_2 \): the probability that Infectious A and Infectious B become susceptible because of risks, respectively.
b: the probability that Infectious A turns into Infectious B, which means Infectious A downsizes business after being impacted and become Infectious B.

d, m1, m2: the probability that Infectious A and Infectious B converting into immune state, respectively.

which: the exit rate for all institutions.

2.3. Model deduction

The five states above together with Healthy state cover all the state categories anytime of financial institutions in the financial system. Under risk impact, the number of enterprises varies in different states. As a whole, the financial system can be described by the following differential equations:

\[
\frac{dS}{dt} = \mu(1 - \omega)T - \mu S - (\beta_1h_1 + \beta_2l_2)S
\]
\[
\frac{dE}{dt} = (\beta_1h_1 + \beta_2l_2)S - \sigma_1E - \sigma_2E - \mu E
\]
\[
\frac{dl_1}{dt} = \sigma_1E - r - m_1l_1 - (\mu + \alpha_1)l_1
\]
\[
\frac{dl_2}{dt} = \sigma_2E + r - m_2l_2 - (\mu + \alpha_2)l_2
\]
\[
\frac{dR}{dt} = m_1l_1 + m_2l_2 + \omega E - \mu R
\]

For \( T = S + E + l_1 + l_2 + R \), the equilibrium point can be obtained from Equation (1) as follows:

\[
S^* = T(1 - \omega)E^* = 0
\]
\[
E_0: \quad l_1^* = 0
\]
\[
E_1: \quad E^* = \begin{cases} 
\frac{\mu + \sigma_1 + \sigma_2}{r + m_1 + \alpha + \mu} & \text{if } \frac{\sigma_1}{m_1} + \frac{\sigma_2}{m_2} > 0 \\
\frac{\mu(1 - \omega)T - \mu S}{\mu + \sigma_1 + \sigma_2} & \text{otherwise}
\end{cases}
\]
\[
\frac{dl_1^*}{dt} = \frac{-\sigma_1}{r + m_1 + \alpha + \mu}E^*
\]
\[
\frac{dl_2^*}{dt} = \frac{-\sigma_2}{r + m_2 + \alpha + \mu}E^*
\]

Basic reproduction number can be obtained as follows:

\[
\rho = \frac{(1 - \omega)T\beta_1\sigma_1}{(\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)} + \frac{(1 - \omega)T\beta_2\sigma_2}{(\sigma_1 + \sigma_2 + \mu)(m_2 + \mu + \alpha_2)} + \frac{(1 - \omega)T\beta_2\sigma_1}{(\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)}
\]

This is the risk spread threshold of financial institutions. When \( \rho < 1 \), risks among institutions can be effectively controlled and gradually eliminated. When \( \rho > 1 \), individual risk will spread to other institutions, and gradually will form risk contagion.

3. Stability analysis of equilibrium point

Now considers equilibrium point \( E_1 \), sets \( l_1^* = k_1E^*, l_2^* = k_2E^* \), then \( S^*, E^*, l_1^*, l_2^* \) can be denoted as follows:

\[
S^* = \frac{(1 - \omega)T}{\rho}, \quad E^* = \frac{(\rho - 1)}{k_1\beta_1 + k_2\beta_2}
\]
\[
l_1^* = k_1E^*, \quad l_2^* = k_2E^*
\]

in which

\[
k_1 = \frac{\alpha_1}{r + m_1 + \mu + \alpha_1}, \quad k_2 = \frac{\alpha_2 + r\k_1}{m_2 + \mu + \alpha_2}
\]

For \( k_1 > 0, k_2 > 0, \beta_1 > 0, \beta_2 > 0, \mu > 0, (1 - \omega) > 0 \)

The positive equilibrium point exists if and only if \( \rho > 1 \)

Because \( l_1^*, l_2^* \) and \( E^* \) are in linear relationship, either all are equal to 0 or none is equal to 0.

Solving the Equation (2) in equation system (1) \([k_1(1 - \omega)T - \mu S - (\beta_1h_1 + \beta_2l_2)S = 0\]

When \( E^* = 0 \), \( S^* = \frac{\sigma_1^* + \sigma_2^* + \mu}{\beta_1^* + \beta_2l_2} \) can be obtained.

Solving Equation (1) in equation system \( (1 - \omega)T - \mu S - (\beta_1h_1 + \beta_2l_2)S = 0 \)

When \( E = 0, S = (1 - \omega)T \) can be obtained.

When \( E = 0, (k_1(1 - \omega)T - \mu S = \mu(1 - \omega)T - \mu S^* \)

and \( E = \frac{\mu(1 - \omega)T - \mu S^*}{k_1\beta_1 + k_2\beta_2} \)

At point \( P_0(S_0, 0, 0, 0), S_0 = (1 - \omega)T \)

\[
J_0 = \begin{bmatrix}
-\mu & 0 & 0 & 0 \\
0 & -\sigma_1 - \sigma_2 - \mu & 0 & 0 \\
0 & \sigma_1 & 0 & \sigma_2 \\
0 & \sigma_2 & \beta_1S_0 & -\beta_2S_0 \\
-\beta_1S_0 & -\beta_2S_0 & \beta_1S_0 & 0 \\
r_1 & 0 & 0 & -m_2 - \mu - \alpha_2
\end{bmatrix}
\]

\[
\det(sI - J_0) = \begin{bmatrix}
\mu & 0 & 0 & 0 \\
0 & s + (\sigma_1 + \sigma_2 + \mu) & 0 & 0 \\
0 & 0 & -\sigma_1 & -\sigma_2 \\
\beta_1S_0 & \beta_2S_0 & -\beta_1S_0 & -\beta_2S_0 \\
\beta_1S_0 & -\beta_2S_0 & 0 & 0 \\
\beta_1S_0 & -\beta_2S_0 & 0 & 0 \\
-s + (r + m_1) & s + (r + m_1) & 0 & 0 \\
-s + (r + m_1) & 0 & -r & s + (m_2 + \mu + \alpha_2)
\end{bmatrix}
\]

There are in linear relationship, either all are equal to 0 or none is equal to 0.
$a_1 = (\sigma_1 + \sigma_2 + \mu) + (r + m_1 + \mu + \alpha_1)$
$+ (m_2 + \mu + \omega_2)$
$a_2 = (\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)$
$+ (\sigma_1 + \sigma_2 + \mu)(m_2 + \mu + \omega_2) + (r + m_1$ $+ \mu + \alpha_1)(m_2 + \mu + \omega_2) - (\sigma_1\beta_1 + \sigma_2\beta_2)S_0$
$a_3 = (\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)(m_2 + \mu + \omega_2)$
$- \left[ \sigma_1\beta_2 + \sigma_2\beta_2(r + m_1 + \mu + \alpha_1)$ $+ \sigma_1\beta_1(m_2 + \mu + \omega_2) \right]$
$s_1 = -\mu s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3 = 0$, there are three roots with a negative real part.
If $\rho < 1$, then $a_3 > 0$ and $(\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1) > \sigma_1\beta_1 S_0$.

$(\sigma_1 + \sigma_2 + \mu)(m_2 + \mu + \omega_2) > \sigma_2\beta_2 S_0; a_2 > 0$

$a_1 a_2 - a_3 = a_1[(\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)$
$+ (\sigma_1 + \sigma_2 + \mu)(m_2 + \mu + \omega_2)$
$- (\sigma_1\beta_1 + \sigma_2\beta_2)S_0]$ $+$
$(\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)(m_2 + \mu + \omega_2)$
$- (\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)$
$x (m_2 + \mu + \omega_2)$
$+ [(\sigma_1 + \sigma_2 + \mu)(r + m_1 + \mu + \alpha_1)$
$+ (\sigma_1 + \sigma_2 + \mu)(m_2 + \mu + \omega_2)$
$- (\sigma_1\beta_1 + \sigma_2\beta_2)]$
$\alpha_1 \alpha_2 - \alpha_3 > 0$

At point $P^*(S^*, E^*, I_1^*, I_2^*)$, the characteristic polynomial is

$det(sI_1 - J^*) = \begin{vmatrix} s + \mu + \beta_1 I_1^* + \beta_2 I_2^* & 0 \\ -\beta_1 I_1^* - \beta_2 I_2^* & s + (\sigma_1 + \sigma_2 + \mu) \\ 0 & -\sigma_1 \\ 0 & -\sigma_2 \\ \beta_1 S^* & \beta_2 S^* \\ -\beta_1 S^* & -\beta_2 S^* \\ s + (r + m_1) \\ +\mu + \alpha_1 \\ -r & s + (m_2 + \mu + \omega_2) \end{vmatrix}$

$s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4 = 0$ has four roots with a negative real part.

If $b_1, b_2, b_3, b_4 > 0$, $b_1 b_2 > b_3$, $b_1 b_2 b_3 > b_3^2 + b_1^2 b_4$, then the process is similar to that of $P_0$, in theory, $\rho > 1$, the equilibrium point $P^*$ is locally asymptotically stable.

4. Simulation
4.1. Analysis of risk contagion mechanism

This paper used MATLAB software to write computation program of the SIR model, and the simulated the risk contagion process among financial institutions under different supervision stringencies. The discreteness of individual institutions was not considered. Set the total number of mechanisms $T = 1$, and set the number of institutions in each state:

$S(0) = 0.2T, E(0) = 0.05T,$
$l_1(0) = 0.1T, l_2(0) = 0.15T, R(0) = 0.5T,$

The initial value of the parameter:

$\mu = 0.01, \omega = 0.8, b_1 = 0.6, b_1 = 0.7, \sigma_1 = 0.4,$
$\sigma_2 = 0.5, r = 0.05, \alpha_1, \alpha_2 = 0.05, m_1, m_2 = 0.03$

and time $t \leq 400$.

By using the initial value for simulation, the number and its trend of Susceptible group, Exposed group, Infectious A, Infectious B and Removed group can be obtained, as shown in Figure 2.

This paper sets factors including contact rate, infection rate, infected enterprises removal rate to reflect the whole risk transmission process. When a few systemically important financial institutions and other institutions get infected and start risk transmission, some Susceptible institutions and Exposed institutions will then be infected within short time and become Infectious A and Infectious B. Specifically, systemically important institutions get infected slightly faster than other institutions, while the infection period is relatively short. For Infectious A, the original state of systemically important financial institution is closely related to other institutions and its individual risk will quickly affect other institutions through balance sheet connection, transaction network and other carriers, then evolve into system risk further. Due to its large scale, complex business and irreplaceable functions, it has a much stronger risk-taking capacity than other small institutions and they are ‘too big to fail.’

Simulation showed the trend that the number of Infectious A slightly increases in the initial short stage, while the number of Infectious B increased sharply. However, the number of Susceptible group showed an decreasing trend at first because some Susceptible group became Removed group due to their strong risk control ability. When the risk transmission stopped, financial institutions began to adjust business, strengthen risk control and establish the risk prevention mechanism, which causes sharp decline in the number of Infectious A, Infectious B, meanwhile the number of Susceptible group begins
4.2. State analysis under different supervision stringency

Considering changes in supervision stringency, we analysed the risk contagion process again. Keep other parameters unchanged, set $\omega$ as 0, 0.2, 0.5, 0.8, 1, the institutions under different supervision stringency are shown in Figures 3 and 4.

In the risk contagion model, when the supervision stringency $\omega = 0$, newly entered companies become Susceptible institutions under this condition. When $\omega = 1$, they become Removed group, and when $\omega \in (0, 1)$, they gradually become Removed group from Susceptible group with supervision stringency increasing.

When $R_0 > 1$, as the risk spreads in the financial market, the number of Susceptible groups will change in three stages. In the first stage, Susceptible group gets infected and becomes risk sources, so their number will decline drastically in a short time, then reaches the lowest point. In the second stage, the number increases at a
diminishing marginal rate. In the third stage, after reaching its peak, the number slowly declines and gradually stabilizes. It is notably that with supervision stringency increasing, peak point defers and peak value increases. The equilibrium values are roughly identical, all of which are lower than the initial number of Susceptible groups.

In practice, since the supervision stringency can never reach 100% and the number of Susceptible institutions is consistent in the long term, it is always possible that some institutions get infected and transmit risk to others, and then trigger financial systemic risk eruption. However, if supervision stringency is too high, the peak value of the number of Susceptible groups will increase. As Susceptible institutions can easily become risk sources when risks erupt, the system would be less stable, which may lead to another round of systematic risk eruption.

Notably, in the first and second stage, as the supervision stringency increases, the declining rate of Susceptible group quantity increases. After reaching bottom, its quantity increases at a diminishing marginal rate. When $\omega = 1$, the supervision stringency reaches 100%, the number of Susceptible decreases sharply and goes to 0.

In the right picture, the number of Removed groups fluctuates more obviously and stable value varied under different supervision stringency. It can be easily observed that the stronger the supervision stringency, the greater the ultimate number of Removed groups. To be specific, under different supervision stringency, the number of Removed groups always increases rapidly in the initial short stage and the growth rate is positively correlated with supervision stringency. In particular, at a low supervision stringency level, after a temporary increase, the number of Removed groups slowly decreases and gradually stabilizes. At a high supervision stringency level, the number of Removed groups increases at a diminishing marginal rate and the stable value is higher than the initial value. When the supervision stringency reaches 100%, there are only Removed group in the system. The middle graph is the change in the number of potential risk exporters. It can be seen that they play the role of the intermediate transition, which have fallen to a s’ value in the short term.

This indicates that supervision stringency cannot be too low, or the number of Removed institutions under equilibrium would decline. A state of 0.5 should be adopted to ensure that the number of Removed would not decline, and that the whole system stability will not be weakened because of overly high supervision stringency.

When the risk spreads, Infectious A, such ‘too big to fail’ financial institutions, are closely linked with other institutions, so their individual risk will quickly affect other institutions, and eventually increase systemic risk. Infectious B, non-systemically important institutions (others institutions), get infected when systemic risks erupt because of business connection. As the above graph above shows Infectious A and Infectious B change in the same pattern and can be summarized that rising rapidly

Figure 4. Infectious A (left); Infectious B (right).
initially in the short term and the peak value is little affected by supervision stringency.

In the initial stage, the number of Infectious B grows rapidly, much faster than the number of Infectious A, while the subsequent decline is significantly smaller than that of Infectious A. Under the same supervision stringency, the ultimate number of Infectious B far exceeds that of Infectious A.

Infectious B is more sensitive to supervision stringency. In the short term, the increased extent of Infectious B quantity inversely changes with supervision stringency. The subsequent decline extent is in positive correlation. When it finally reaches system equilibrium, the supervision stringency is negatively related with the number of Infectious B and Infectious A.

It can be concluded that during short-term after risk erupts, risk infection status will not be affected by supervision stringency. Once the risk erupts, some institutions will inevitably get infected due to business connection. Constrained by the capital adequacy ratio, the ex-ante risk

Figure 5. Susceptible group (left); Exposed group (medium); Removed group (right).

Figure 6. Infectious A (left) or Infectious B (right).
preventions are ineffective in the short term. These supervision measurements can work well only on the long-term basis, and the institutions can control risk and further become Removed group. Besides, the higher the supervision stringency, the more the Removed institutions.

4.3. Model optimization

In fact, connection coefficients $\beta_1$, $\beta_2$ are often affected by supervision stringency, and they further affect the dynamical system equilibrium. The following conditions are added:

$$\beta_1 = \alpha_1 \sigma, \beta_2 = \alpha_2 \sigma, \alpha_1, \alpha_2 \geq 0,$$

this formula indicates that $\beta_1$ and $\beta_2$ are positively correlated with supervision stringency.

Keep other factors unchanged, when $\alpha_1 = 0.2$, $\alpha_2 = 0.3$, $\omega = 0$, $0.2$, $0.5$, $0.8$, $1$, the institutions under different states are shown in Figures 5 and 6.

Different from independent connection coefficient, when the coefficient is proportional to the supervision stringency, the distribution of Susceptible institutions quantity is completely different, and exposed group have different equilibrium values. When the risk sources are independent, the equilibrium values of Susceptible institutions quantity are identical, and the number of exposed groups eventually is zero. As Susceptible institutions play an important role in the financial system and supervision stringency affects system significantly, thus, regulators should pay much attention to act appropriately.

In general, the above trend is in line with assumption. More specifically, different curves have intersection points, but the ultimate equilibrium value are different. This indicates the systems may reach the same status in the process: however, the supervision stringency affected more on the declining rate and system equilibrium.

5. Conclusion and suggestions

According to the state of institutions in the financial system during the risk contagion process, this paper built an epidemic dynamic model including Susceptible group, Exposed group, Infectious A (Infectious A), Infectious B (Infectious B), and Removed group. This paper use simulation to analyse the risk contagion mechanism in financial system under supervision constraints, and further studied the impact of different supervision stringency on the financial system. Conclusions and suggestions are listed as follows:

Conclusions are drawn that the number of other institutions infected peaks later and higher than systemically important financial institutions, and other institutions are more sensitive to supervision stringency. Regulators should implement differentiated measurements for different institutions, including monitoring systemically important financial institutions, establishing risk isolation mechanisms, mitigating and cutting off risk contagion with other institutions. Meanwhile, they should strengthen risk prevention construction of other institutions, and block risk contagion channels, so as to reduce the risk exposure of market resonance.

Besides, regulators should take moderate ex-ante supervision measures, which can cut down the number of risk institutions in the long run, increase the number of risk-immune institutions (Removed group) and avoid negative impact of overly high supervision stringency. The supervision department should pay more attention to ex-ante prevention than ex-post mitigation and improve supervision system, conduct market risk daily monitoring, timely capture of early warning information, strengthen abnormal transactions supervision, and adopt various risk prevention measurements.

Afterall, even if high supervision stringency does not work in the short period after risk eruption and can not suppress risk contagion, it is effective in the long term. Regulators should provide positive information guidance when risks erupt, prevent negative information, spread and maintain financial system stability. Policy-makers and regulators should have clear crisis consciousness, and continuously promote reform, improve risk supervision system, legal system and technical tools.

Notes

1. The institutional system or market system, where the financial institution engages in financial activities.
2. Certain conditions may include unfavorable macro-conditions, close-connected financial businesses, high-speed risk contagion or faulty risk control mechanisms.
3. In this paper, organizations other than systemically important financial institutions refer to non-systemically important financial institutions.
4. Here refers to that the institutions exit system normally.

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