Power Control and Receiver Design for Energy Efficiency in Multipath CDMA Channels with Bandlimited Waveforms

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Abstract—This paper is focused on the cross-layer design problem of joint multiuser detection and power control for energy-efficiency optimization in a wireless data network through a game-theoretic approach. Building on work of Meshkati, et al., wherein the tools of game-theory are used in order to achieve energy-efficiency in a simple synchronous code division multiple access system, system asynchronism, the use of bandlimited chip-pulses, and the multipath distortion induced by the wireless channel are explicitly incorporated into the analysis. Several non-cooperative games are proposed wherein users may vary their transmit power and their uplink receiver in order to maximize their utility, which is defined here as the ratio of data throughput to transmit power. In particular, the case in which a linear multiuser detector is adopted at the receiver is considered. The proposed games are shown to admit a unique Nash equilibrium point, while simulation results show the effectiveness of the proposed solutions, as well as that the use of a decision-feedback multiuser receiver brings remarkable performance improvements. Index Terms—Power control, non-cooperative games, energy-efficiency, CDMA, Multipath fading.

I. INTRODUCTION

Game theory [1] is a branch of mathematics that has been applied primarily to social science and economics to study the interactions among several autonomous subjects with contrasting interests. Recently, it has been discovered that it can also be used for the design and analysis of communication systems, mostly with application to resource allocation algorithms [2], and, in particular, to power control [3]. As examples, the reader is referred to [4], [5]. Here, for a multiple access wireless data network, noncooperative and cooperative games are introduced, wherein users choose their transmit powers in order to maximize their own utilities, defined as the ratio of the throughput to transmit power. While the above studies consider the issue of power control assuming that a conventional matched filter is available at the receiver, the recent paper [6] considers for the first time the problem of joint linear receiver design and power control so as to maximize the utility of each user. In particular, it is shown here that the inclusion of receiver design in the considered game brings remarkable advantages, and, also, results based on the powerful large-system analysis are presented.

All of the cited studies, while laying the foundations of the game-theoretic approach to utility maximization in wireless data networks, focus on a very simple model, i.e. a synchronous direct sequence code division multiple access (DS/CDMA) channel subject to flat-fading. In this paper, instead, we extend the game-theoretic framework to a more practical and challenging scenario, namely we explicitly take into account (a) the possible system asynchrony across users; (b) the use of bandlimited chip-pulses; and (c) the multipath distortion induced by the wireless propagation channel. Note that in such a scenario intersymbol and interchip interference arises, thus implying that the appealing mathematical relationships between the signal-to-interference plus noise ratio (SINR) and the transmit power (as revealed in [6]) do not hold any longer, and this makes system analysis much more involved than it is for the case in which no self-interference exists. A further contribution of this paper is the consideration of non-linear multiuser receivers. Indeed, while previous studies have considered the case in which either a matched filter (see, e.g., [5]) or a linear multiuser detector [6] is adopted at the uplink receiver, here we also consider the case in which a non-linear decision feedback receiver is employed at the receiver.

Notation: (·)T denotes transpose, while ∗ and × denote linear convolution and ordinary product, respectively.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider the uplink of an asynchronous DS/CDMA system with K users, employing bandlimited chip pulses and operating over a frequency-selective fading channel. The received signal at the access point (AP) may be written as

\[ r(t) = \sum_{p=0}^{B-1} \sum_{k=1}^{K} \sqrt{\bar{p}_k} b_k(p) s'_k(t - \tau_k - \bar{p}_k) \ast c_k(t) + w(t) \]  \hspace{1cm} (1)

\[ 1\text{For the sake of simplicity we assume here a real signal model; however, the extension to complex signals to account for I and Q components is trivial.} \]
In the above expression, $B$ is the transmitted frame or packet length, $T_b$ is the bit-interval duration, $p_k$ and $\tau_k \geq 0$ denote the transmit power and timing offset of the $k$-th user, $b_k(p) \in \{+1, -1\}$ is the $k$-th user’s information symbol in the $p$-th signaling interval (extension to modulations with a larger cardinality is straightforward). Moreover, $c_k(t)$ is the impulse response modeling the channel effects between the receiver and the $k$-th user’s transmitter, while $w(t)$ is the additive noise term, which is assumed to be a zero-mean, Wide-Sense Stationary (WSS) white Gaussian process with Power Spectral Density (PSD) $N_0/2$. It is also assumed that the channel coherence time exceeds the packet duration $B T_b$, so that the channel impulse responses $c_0(t), \ldots, c_{K-1}(t)$ may be assumed to be time-invariant over each transmitted frame. As to $s'_k(t)$, it is the $k$-th user’s signature waveform and is written as

$$s'_k(t) = \sum_{n=0}^{N-1} \beta_k^{(n)} h_{\text{SRRC}}(t-n T_c),$$

with $\{\beta_k^{(n)}\}_n^{N-1}$ the $k$-th user’s spreading sequence, $N$ the processing gain, $T_c = T_b/N$ the chip interval, and $h_{\text{SRRC}}(\cdot)$ a square-root raised cosine waveform with roll-off factor $\alpha \in [0, 1]$. We assume here that $h_{\text{SRRC}}(t)$ is zero outside the interval $[0, 4 T_c]$ and attains its maximum value in $t = 2 T_c$.

The receiver front-end consists of a filter with impulse response $h_{\text{SRRC}}(-t)$, followed by a sampler at rate $M/T_c$; in our simulations we will assume that $M = 2$. Denoting by $y(t)$ the signal at the output of the receiver matched filter, it can be easily shown that

$$y(t) = r(t) * h_{\text{SRRC}}(-t) = \sum_{p=0}^{B-1} K \sum_{k=1}^{K} \sqrt{p_k b_k(p)} \times s_k(t - \tau_k - p T_b) * c_k(t) + w(t) * h_{\text{SRRC}}(-t),$$

or in the simplified form

$$y(t) = \sum_{p=0}^{B-1} K \sum_{k=1}^{K} \sqrt{p_k b_k(p)} h_k(t - p T_b) + n(t).$$

Notice that the waveform $h_k(t)$ is supported in the interval $[\tau_k + T_b + T_m + 7 T_c]$, where $T_m$ denotes the maximum channel multipath delay spread over the $K$ active users. Assuming that $\tau_k + T_m < T_b$, the support of the waveform $h_k(t - p T_b)$ is contained in the interval $[p T_b, (p + 2) T_b + 7 T_c]$, thus implying that, for a system with processing gain larger than 7, in the symbol interval $I_p = [p T_b, (p + 2) T_b]$ the contribution from at most four symbols for each user (i.e. the $p$-th, the $(p-1)$-th, the $(p+2)$-th and the $(p+1)$-th ones) is observed. Accordingly, sampling the waveform $y(t)$ at rate $M/T_c$, the $2MN$-dimensional vector $y(p)$ collecting the data samples of the interval $I_p$ can be expressed as

$$y(p) = \sum_{k=1}^{K} \sqrt{p_k b_k(p-2)} h_{k,-2} + b_k(p-1) h_{k,-1} + b_k(p) h_{k,0} + b_k(p+1) h_{k,+1} + n(p).$$

In (4), the vector $h_{k,i}$ is $2MN$-dimensional, and contains the samples of the signature $h_k((t-(p+i))T_b)$ coming from $I_p$, while the vector $n(p)$ contains the noise contribution, and is a Gaussian random vector with covariance matrix $M$. We assume that the data vector $y(p)$ will be used in order to detect the information symbols $b_1(p), b_2(p), \ldots, b_K(p)$, i.e. the $p$-th epoch data symbols for all the users.

Assume now that each mobile terminal is interested both in having its data received with as small as possible error probability at the AP, and in making optimal use of the energy stored in its battery. Obviously, these are conflicting goals, since error-free reception may be achieved by increasing the received SNR, i.e. by increasing the transmit power, which of course comes at the expense of battery life.

A useful approach to quantify these conflicting goals is to define the utility of the $k$-th user as the ratio of its throughput, defined as the number of information bits that are received with no error in the unit time, to its transmit power $[4], [5]$, i.e.

$$u_k = \frac{T_k}{P_k}.$$  

Note that $u_k$ is measured in bits/Joule. Denoting by $R$ the common rate of the network and assuming that each packet of $B$ bits contains $L$ information bits and $B-L$ overhead bits, reserved, e.g., for channel estimation and/or parity checks, the throughput $T_k$ can be expressed as

$$T_k = R \frac{L}{B} P_k,$$

wherein $P_k$ denotes the probability that a packet from the $k$-th user is received error-free. In the considered DS/CDMA setting, the term $P_k$ depends formally on a number of parameters such as the spreading codes of all the users, their transmit powers and their channel impulse responses; however, a customary approach is to model the overall interference as a Gaussian random process, and assume that $P_k$ is an increasing function of the $k$-th user’s SINR $\gamma_k$, which is a good model for many practical scenarios.

For the case in which a linear receiver is used to detect the data symbol $b_k(p)$, according, i.e., to the decision rule

$$\hat{b}_k(p) = \text{sign} \left[ d_k^T y(p) \right],$$

with $\hat{b}_k(p)$ the estimate of $b_k(p)$ and $d_k$ the $2MN$-dimensional vector representing the receive filter for user $k$, it is easily seen

\[\text{Of course there are many other strategies to lower the data error probability, such as for example the use of error correcting codes, diversity exploitation, and implementation of optimal reception techniques at the receiver. Here, however, we are mainly interested in energy efficient data transmission and power usage, so we assume that only the transmit power and the receiver strategy can be varied to achieve energy efficiency.}\]
that for the case at hand the SINR $\gamma_k$ can be written as
\[
\gamma_k = \frac{p_k(d_k^T h_k,0)^2}{d_k^T M d_k + \sum_{i \neq k, j=-2}^{1} \sum_{j=0}^{1} p_i(d_k^T h_{i,j})^2 + p_k(d_k^T h_{k,j})^2}.
\]

The exact shape of $P_k(\gamma_k)$ depends not only on $\gamma_k$, but also on other factors such as the modulation and coding type. However, in all cases of relevant interest, it is an increasing function of $\gamma_k$ with a sigmoidal shape, and converges to unity as $\gamma_k \to +\infty$; as an example, for binary phase-shift-keying (BPSK) modulation coupled with no channel coding, it is easily shown that
\[
P_k(\gamma_k) = \left[1 - Q(\sqrt{2\gamma_k})\right]^B,
\]
with $Q(\cdot)$ the complementary cumulative distribution function of a zero-mean random Gaussian variate with unit variance.

It should be noted however that substituting Eq. (9) into (6), and, in turn, into (5), leads to a strong incongruence. Indeed, for $p_k \to 0$, we have $\gamma_k \to 0$, but $P_k$ converges to a small but non-zero value (i.e. $2^{-B}$), thus implying that an unboundedly large utility can be achieved by transmitting with zero power. To circumvent this problem, a customary approach [5], [6] is to replace $P_k$ with an efficiency function, say $f_k(\gamma_k)$, whose behavior should approximate as close as possible that of $P_k$, except that for $\gamma_k \to 0$ it is required that $f_k(\gamma_k) = o(\gamma_k)$. The function $f_k(\gamma_k) = \left(1 - e^{-\gamma_k/\sqrt{2}}\right)^2$ is a widely accepted substitute for the true probability of correct packet reception, and in the following we will adopt this model. This efficiency function is increasing and $S$-shaped, converges to unity as $\gamma_k$ approaches infinity, and has a continuous first order derivative.

Summing up, substituting (6) into (5) and replacing the probability $P_k$ with the above defined efficiency function, we obtain the following expression for the $k$-th user’s utility:
\[
u_k = R^L \frac{f(\gamma_k)}{B} p_k, \quad \forall k = 1, \ldots, K.
\]

### III. NON-COOPERATIVE GAMES WITH LINEAR RECEIVERS

In what follows we illustrate three different noncooperative games wherein each user aims at maximizing its own utility by varying its transmit power, and, possibly, its linear uplink receiver. Formally, the considered game $G$ can be described as the triplet $G = [K, \{S_k\}, \{u_k\}]$, wherein $K = \{1, 2, \ldots, K\}$ is the set of active users participating in the game, $u_k$ is the $k$-th user’s utility defined in eq. (10), and $S_k = [0, P_{k,\text{max}}] \times \mathcal{R}^{2NM}$, is the set of possible actions (strategies) that user $k$ can take. It is seen that $S_k$ is written as the Cartesian product of two different sets, and indeed $[0, P_{k,\text{max}}]$ is the range of available transmit powers for the $k$-th user (note that $P_{k,\text{max}}$ is the maximum allowed transmit power of user $k$), while $\mathcal{R}^{2NM}$, with $\mathcal{R}$ the real line, defines the set of all possible linear receive filters.

#### A. Power control with plain matched filter

We first consider the case in which $S_k = [0, P_{k,\text{max}}]$ and the uplink receiver is a matched filter, i.e. we assume that each user tunes its transmit power in order to maximize its own utility, but the uplink receiver is a matched filter. Consequently, the $k$-th user’s SINR is expressed as
\[
\gamma_k = \frac{p_k||h_{k,0}||^4}{h_{k,0}^T M h_{k,0} + \sum_{i \neq k} \sum_{j=0}^{1} p_i(h_{i,0}^T h_{i,j})^2 + p_k(h_{k,0}^T h_{k,j})^2},
\]
and the noncooperative game can be cast as the following maximization problem
\[
\max_{S_k} \nu_k = \max_{p_k \in [0, P_{k,\text{max}}]} u_k(p_k) = \max_{p_k \in [0, P_{k,\text{max}}]} \frac{f(\gamma_k(p_k))}{p_k}, \quad \forall k = 1, \ldots, K.
\]

**Proposition 1:** The non-cooperative game defined in (13) admits a unique Nash equilibrium point $p^*_k$, for $k = 1, \ldots, K$, wherein $p^*_k = \min\{|\bar{p}_k, P_{k,\text{max}}\}$, with $\bar{p}_k$ denoting the $k$-th user’s transmit power such that the $k$-th user’s SINR $\gamma_k$ equals $\bar{\gamma}_k$, i.e. the unique solution of the equation
\[
\frac{B}{2a_k} \gamma(a_k - b_k \gamma) = \exp(\gamma/2) - 1,
\]
with $a_k = ||h_{k,0}||^4$ and
\[
b_k = \sum_{j \neq 0} (h_{k,0}^T h_{k,j})^2.
\]

**Proof:** The proof is omitted for brevity.

In summary, Proposition 1 states that a Nash equilibrium for the noncooperative game (13) always exists, and it can be found with the following steps. First, the unique solution $\bar{\gamma}_k$ of the equation (14) is determined. Then, each user adjusts its transmit power to achieve its target SINR $\bar{\gamma}_k$. These steps are repeated until convergence is reached.

#### B. Power control and receiver design with no ISI

Let us now consider the case in which not only the transmit power, but also the linear receiver can be tuned so as to maximize utility for each user; moreover, let us also impose the condition that the receive filter be orthogonal to the subspace spanned by ISI. Denoting by $O_k$ a $2NM \times (2NM - 3)$-dimensional matrix containing in its columns a basis for the orthogonal complement of the subspace spanned by the $k$-th user’s ISI, i.e. by the vectors $h_{k,-2}$, $h_{k,-1}$, and $h_{k,1}$, we assume that the decision rule to detect the symbol $b_k(p)$ can be written as
\[
\hat{b}_k(p) = \text{sign} \left[ x_k^T O_k^T y(p) \right],
\]

**Note:** that for an oversampling factor $M > 1$ a whitening transformation would in principle be required prior to matched filtering; for the sake of simplicity, however, noise whitening is not performed here.
with \( x_k \) a \((2NM - 3)\)-dimensional vector. The \( k \)-th user’s SINR is now written as
\[
\gamma_k = \frac{p_k (x_k^T O_k^T h_{k,0})^2}{x_k^T O_k^T M O_k x_k + \sum_{j \neq k} p_j (x_k^T O_k^T h_{k,j})^2},
\]
(16)
namely the \( k \)-th user’s transmit power appears only in the numerator in the RHS of (16), thus implying that the relation \( \frac{\partial \gamma_k}{\partial p_k} = \frac{\gamma_k}{p_k} \) holds. We now consider the following maximization problem
\[
\max_{S_k, u_k} u_k = \max_{p_k} u_k(p_k, x_k), \quad \forall k = 1, \ldots, K.
\]
(17)
Given (16), the above maximization can be also written as
\[
\max_{p_k, d_k} f(\gamma_k(p_k, x_k)) = \max_{p_k} f\left(\frac{\max_{x_k} \gamma_k(p_k, x_k)}{p_k}\right),
\]
(18)
i.e. we can first take care of SINR maximization with respect to linear receivers, and then focus on maximization of the resulting utility with respect to transmit power. We now have the following:

**Proposition 2:** Let \( M_{yy} \) denote the covariance matrix of the vector \( y(p) \). The non-cooperative game defined in (17) admits a unique Nash equilibrium point \((p_k^*, x_k^*)\), for \( k = 1, \ldots, K \), wherein
- \( x_k = \sqrt{p_k} \left( O_k^T M_{yy} O_k \right)^{-1} O_k^T h_{k,0} \) is the unique (up to a positive scaling factor) \( k \)-th user’s receiver filter that maximizes the SINR \( \gamma_k \) in (17). Denote \( \gamma_k^* = \max_{x_k} \gamma_k \).\( \gamma_k^* \)
- \( p_k^* = \min\{\bar{p}_k, \bar{p}_{k,\max}\} \), with \( \bar{p}_k \) the \( k \)-th user’s transmit power such that the \( k \)-th user’s maximum SINR \( \gamma_k^* \) equals \( \gamma_k \), i.e. the unique solution of the equation \( f(\gamma) = f'(\gamma) \), with \( f(\gamma) \) denoting the derivative of \( f(\gamma) \).

**Proof:** The proof is omitted due to lack of space. Note however that, due to the constraint that the receive filter is orthogonal to the ISI contribution, the mathematical structure of the maximization in (17) is similar to that of the noncooperative game proposed in [6], and the proof can thus be adapted from there.

The above equilibrium can be reached according to the following procedure. For a given set of users’ transmit powers, the receiver filter coefficients can be set according to the relation \( x_k = \sqrt{p_k} \left( O_k^T M_{yy} O_k \right)^{-1} O_k^T h_{k,0} \). Each user can then tune its power so as to achieve the target SINR \( \gamma_k \). These steps are repeated until convergence is reached.

**C. Power control and unconstrained receiver design**

Finally, we consider the case in which no constraint is imposed on the receive filter, so that the \( k \)-th user’s SINR is written as in Eq. (3). We now consider the following maximization
\[
\max_{S_k, d_k} u_k(p_k, d_k) = \max_{p_k} f\left(\frac{\max_{x_k} \gamma_k(p_k, d_k)}{d_k}\right),
\]
(19)
\(\forall k = 1, \ldots, K\), wherein the fact that the efficiency function is non-decreasing has been exploited. Now, the maximization of \( \gamma_k \) with respect to \( d_k \) is trivial, since it is well known that the linear receiver that maximizes SINR is the minimum mean square error multiuser receiver. As a consequence, denoting by \( d_k^* \) the maximizer of \( \gamma_k \), we have
\[
d_k^* = \sqrt{p_k} M_{yy}^{-1} h_{k,0} ;
\]
(20)
let us denote by \( \gamma_k(p_k) \) the \( k \)-th user’s SINR with \( d_k = d_k^* \). Maximizing the utility with respect to the transmit power requires instead solving the equation
\[
f(\gamma_k(p_k)) = f'(\gamma_k(p_k)) \gamma_k'(p_k) p_k ;
\]
(21)
with \( f' \) denoting first-order derivative with respect to \( p_k \). Now, (21) appears to be quite complicated and unmanageable. Indeed, note that letting \( H_k = [h_{k,-2} \ldots h_{k,-1} h_{k,1}] \), we have
\[
M_{yy} = M_{yy}(p_k) = Q_k + p_k H_k H_k^T + p_k h_{k,0} h_{k,0}^T ;
\]
(22)
with \( Q_k \) the covariance matrix of the thermal noise and of the multiuser interference for the \( k \)-th user, thus implying that \( \gamma_k(p_k) \) is expressed as
\[
\gamma_k(p_k) = \frac{p_k (h_{k,0}^T M_{yy}^{-1}(p_k) h_{k,0})^2}{h_{k,0}^T M_{yy}^{-1}(p_k) h_{k,0}^2}.
\]
(23)
It is clear that substituting (23) and its first-order derivative into (21) and solving with respect to \( p_k \) is quite complicated. Accordingly, we have not been able in this case to formally prove the existence of a Nash equilibrium point. However, we have numerically evaluated the utility function and (21), and we have found in every case considered that (21) admits a unique solution and that the resulting game admits an equilibrium point. We thus state the following conjecture.

**Conjecture 1:** The non-cooperative game defined in (19) admits a unique Nash equilibrium point \((p_k^*, d_k^*)\), for \( k = 1, \ldots, K \), wherein
- \( d_k^* \) is the linear MMSE receiver (see Eq. (20)), which maximizes the SINR \( \gamma_k \) in (3). Denote \( \gamma_k^* = \max_{d_k} \gamma_k(d_k) \).
- \( p_k^* = \min\{\bar{p}_k, \bar{p}_{k,\max}\} \), with \( \bar{p}_k \) the unique solution of Eq. (21).

Also in this case, the equilibrium can be reached through an iterative procedure. For a given set of users’ transmit powers, the receiver filter coefficients can be set equal to the MMSE multiuser receiver; each user can then tune its power to \( p_k^* \), and these steps are repeated until convergence is reached.

**IV. NON-COOPERATIVE GAMES WITH DECISION-FEEDBACK RECEIVERS**

Consider now the case in which a non-linear decision feedback receiver is used at the receiver. We assume that the users are indexed according to a non-increasing sorting of their channel gains, i.e. we assume that \( \|h_{1,0}\| > \|h_{2,0}\| > \ldots, \|h_{K,0}\| \). We consider a serial interference cancellation
the ISI subspace for each user, i.e. our decision rule is

\[ \hat{b}_k(p) = \text{sign} \left[ d_k^T \left( y(p) - \sum_{j<k} \sum_{i=-2}^{0} \sqrt{p_j} b_{j,i} h_{j,i} \right) \right] . \]  

(24)

Accordingly, if past decisions are correct, users that are detected later enjoy a considerable reduction of multiple access interference, and indeed the SINR for user \( k \), under the assumption of correctness of past decisions, is written as

\[ \gamma_k = \frac{p_k (d_k^T h_{k,0})^2}{\zeta_k} . \]  

(25)

with \( \zeta_k = d_k^T M d_k + \sum_{j<k} \sum_{i=-2}^{0} p_j (d_k^T h_{j,i})^2 + \sum_{j>k} \sum_{i=-2}^{0} p_j (d_k^T h_{j,i})^2 \).

A. Power control and receiver design with no ISI

Replicating the path of the previous section, we start imposing the constraint that the receive filter be orthogonal to the ISI subspace for each user, i.e. our decision rule is

\[ \hat{b}_k(p) = \text{sign} \left[ x_k^T O_k^T \left( y(p) - \sum_{j<k} \sum_{i=-2}^{0} \sqrt{p_j} b_{j,i} h_{j,i} \right) \right] . \]  

(26)

and the \( k \)-th user SINR is

\[ \gamma_k = \frac{p_k (x_k^T O_k^T h_{k,0})^2}{\varrho_k} , \]  

(27)

with \( \varrho_k = x_k^T O_k^T M O_k x_k + \sum_{j<k} \sum_{i=-2}^{0} p_j (x_k^T O_k^T h_{j,i})^2 + \sum_{j>k} \sum_{i=-2}^{0} p_j (x_k^T O_k^T h_{j,i})^2 \). Given receiver \( \{26\} \) and the SINR expression \( \{27\} \), we consider here the problem of utility maximization with respect to the transmit power, and receiver vectors \( x_1, \ldots, x_K \):

\[ \max_{p_k, x_k} \frac{f(\gamma_k(p_k, x_k))}{p_k} , \quad \forall k = 1, \ldots, K . \]  

(28)

The following result can be shown to hold.

**Proposition 3**: Let \( J_k \) be a matrix having as columns the vectors in the set

\[ \{ \sqrt{p} h_{i,1} \}_{i=1}^{K} \cup \{ \sqrt{p} h_{i,j} \}_{i \geq k, j=-2,-1,0} \]

and define \( M_k = (J_k x_k^T + M) \): The non-cooperative game defined in \( \{29\} \) admits a unique Nash equilibrium point \( (p_k^*, x_k^*) \), for \( k = 1, \ldots, K \), wherein

- \( x_k^* = \sqrt{p_k} (O_k^T M_k O_k)^{-1} O_k^T h_{k,0} \) is the unique \( k \)-th user receive filter \( \{27\} \) that maximizes the SINR \( \gamma_k \) given in \( \{27\} \).
- \( p_k^* = \min \{ \tilde{p}_k, p_{k,\max} \} \), with \( \tilde{p}_k \) the \( k \)-th user’s transmit power such that the \( k \)-th user’s maximum SINR \( \gamma_k \) equals \( \frac{\gamma_k}{5} \), i.e. the unique solution of the equation \( f(\gamma_k) = \gamma f'(\gamma_k) \), with \( f' \) the derivative of \( f(\gamma_k) \).

**Proof**: The proof is omitted due to lack of space.

B. Power control and unconstrained receiver design

Finally, we consider the case in which no constraint is imposed on the receive filter, so that the \( k \)-th user’s SINR is written as in \( \{25\} \), and the decision rule is given by \( \{24\} \). We now consider the following maximization

\[ \max_{u_k} \frac{f(\max_{d_k} \gamma_k(p_k, d_k))}{d_k} , \quad \forall k = 1, \ldots, K . \]  

(29)

let us denote by \( \bar{\gamma}_k(p_k) \) the \( k \)-th user’s SINR as \( d_k = \bar{d}_k \). Maximizing the utility with respect to the transmit power requires instead solving the equation

\[ f(\gamma_k(p_k)) = f'(\gamma_k(p_k)) \gamma_k(p_k) p_k . \]  

(31)

Now, \( \{31\} \) is formally equivalent to \( \{21\} \) and is quite complicated to manage. Accordingly, the same considerations of Section III.C apply here as well, and, supported by extensive computer simulations, we conjecture the existence of a unique Nash equilibrium. We thus have the following

**Conjecture 2**: The non-cooperative game defined in \( \{29\} \) admits a unique Nash equilibrium point \( (p_k^*, d_k^*) \), for \( k = 1, \ldots, K \), wherein

- \( d_k^* \) is given by Eq. \( \{30\} \), which maximizes the user \( k \) SINR \( \gamma_k \) in \( \{25\} \). Denote \( \bar{\gamma}_k = \max_{d_k} \gamma_k \).
- \( p_k^* = \min \{ \bar{p}_k, p_{k,\max} \} \), with \( \bar{p}_k \) the unique solution of Eq. \( \{27\} \).

Also in this case, the equilibrium can be reached through an iterative procedure. For a given set of users’ transmit powers, the receiver filter coefficients can be set equal to the receiver in \( \{30\} \); each user can then tune its power to \( p_k^* \), and these steps are repeated until convergence is reached.

V. Numerical Results

We consider now an uplink DS/CDMA system with processing gain \( N = 7 \), and assume that the packet length is \( B = 120 \). Users may have random positions with a distance from the AP ranging from 10m to 500m. The channel impulse response \( c_k(t) \) for the generic \( k \)-th user is assumed to be equal to

\[ c_k(t) = \sum_{\ell=1}^{3} \alpha_{k,\ell} \delta(t - \tau_{k,\ell}) , \quad \forall k = 1, \ldots, K \]  

such that \( \tau_k + \tau_{k,\ell} \) is uniformly distributed in \( [0, T_b] \) and \( \alpha_{k,\ell} \) is a Rayleigh distributed random variable with mean equal to \( d_k^{-2} \delta_k \), with \( d_k \) being the distance of user \( k \) from the AP, and \( [\bar{r}_1, \bar{r}_2, \bar{r}_3] = [0.5, 0.3, 0.2] \). For the thermal noise level, we take \( N_0 = 10^{-9} \text{W/Hz} \), while the maximum allowed power \( p_{k,\max} \) is 25dB. We present here results of averaging over...
5000 independent realizations for the users locations, fading channel coefficients and set of spreading codes.

Figs. 1 - 2 report the achieved average utility (measured in bits/Joule) and the average user transmit power for the proposed non-cooperative games. As expected, the power control game with matched filter at the receiver is the one with the poorest performance, while the best performance is attained by the non-linear decision-feedback receivers. It is seen that for $K > N$ the average utility achieved by the non-linear receivers is twice the average utility achieved by the linear receivers. Moreover, constrained receivers are outperformed by unconstrained receivers, even though the gap is not that large.

Fig. 3 reports the average fraction of users that transmit at the maximum available power, i.e. the probability that a user implementing a certain game is not able to achieve its target SINR and ends up transmitting at its maximum power. As expected, it is seen that the larger fraction corresponds to the use of a matched filter at the receiver, while using non-linear decision feedback receivers permits minimizing this fraction, which, moreover, increases as the network load (i.e. number of users) increases.

VI. Conclusions

In this paper we have considered the problem of utility maximization in a wireless data network through the use of a game-theoretic approach. The cross-layer issue of multiuser receiver design and power control for utility maximization has been considered for the practical scenario of an asynchronous, bandlimited and multipath distorted CDMA system. The case in which a non-linear decision feedback detector is adopted has been considered. First we have derived the non-linear decision feedback receiver maximizing the utility for each user; then, we have shown how the use of a non-linear multiuser receiver provides significant performance gains, especially in the case in which the number of users is close to or larger than the system processing gain. Overall, it can be stated that game theory is an attractive mathematical tool that can be effectively used for the design of utility-maximizing resource allocation algorithms in wireless networks operating in practical scenarios.

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