SIMULATION OF IMPACT INTERACTION OF RAIL TRANSPORT CARRIAGE IN A BUTT ROUGHNESS ZONE

Summary. Mechanical and mathematical models of mechanical multidimensional discrete-continuous systems “carriage – track” in terms of static and impact interaction in a butt roughness zone are proposed. Their interaction is investigated with the example of a four-axle car and a track for four motion phases in the place of isolated butt roughness. Parameters of static and impact interaction of the carriage with a track in a place of butt joint which takes into account operational and constructive factors are defined.

Keywords: four-axle carriage, tram, track, butt roughness, carriage motion phase, shock interaction

1. INTRODUCTION

Experience around the world indicates in multidimensional discrete-continuum mechanical complex „carriage – track” reliability and durability indicators to depend on the rolling stock and the truck common work peculiarities, rolling stock type, rails and sleepers...
type, considered mechanical system operating conditions. As well as on the ability to resist the destructive action of shock and vibration loads emergent, that are cyclically repetitive.

It is also known, that the highest level of the ballast deposition arises under the first sleeper of a receiving rail. This is related to the fact [17,28] that in these places, the first sleeper under the receiving rail usually experiences the greatest power interaction between the carriage and the top structure of the way, due to their impact force [2,16,18,20,30,31]. Thus, the deflection of the receiving rail under the first personal sleeper tend to be an essential indicator, which corresponds to the peculiarities of the processes of static, shock and dynamic interaction of the receiving rail with the upper structure of the path in places with isolated path butt joints [5,23].

2. ANALYSIS OF LITERATURE DATA AND TASK STATEMENT

Analysis of the data, that is dedicated to the four-axle carriage and the rail trackway interaction indicates [28] that the weakest link of the tram carriage - rail track system is the isolated zone of the track joints. Particular studies consider the phases of the carriage movement due to the butt roughness [17], their results allow a more thorough analysis of this interaction. The collaboration of the rolling stock and the rail track is simulated as well, this determines the features of their mechanical interaction [6,7] while carriage passing of the butt, including shock interaction. Current studies indicate that the interaction between rolling stock and railway track components define the parameters of durability in operation, the strength and rigidity of the track [14,15,21,29]. One of the essential components of such interaction is shock interaction. In aggregate with other characteristics of dynamic interaction, it affects the technical resource and service duration.

Practice shows [1,6,28] that in the mechanical complex “carriage - rail track” reliability and durability indicators significantly depend on the processes peculiar properties of interaction between the track and the rolling stock, especially the operating conditions of the system. In addition, this interaction affects the ability of the system to withstand the destructive action of the resulting shock loads [24,29]. To simulate the interaction between the rolling stock and the track, there is need a to solve several related problems particularly static and shock ones. A lot of attention is focused on these issues and there is a sufficient number of new investigations in this sector.

2.1. Recent research analysis

Modern investigations were considered for this kind of interaction. Most of the papers are currently limited to the consideration of individual parameters of operation. A more generalised approach, which considers the totality and interaction of different factors, is necessary to ensure an adequate description. Currently, significant investments are made in the transport infrastructure in many countries, particularly the rail track [3]. Improving the quality and capacity of existing services and developing new infrastructure are necessary to meet the growing demand for qualitative and reliable logistics of goods and people. Here the efficiency and reliability of the track design are crucial for a successful operation. A lot of modern rail track studies focus on individual aspects of design and operation, such as fatigue [21,24], ballast failure [9], driving comfort [31], noise or vibration [8,12,13,30].

At this rate, some papers [20] consider processes of mechanical interaction in the system “carriage - rail track” only taking into account the vehicle motion on the jointless sectors of
the route, that is, without considering the existing dangerous zones with butt irregularities. While other studies [2] do not take into account the limiting conditions for the receiving rail, depending on the elastic characteristics of the giving rail and butt overlays. Practically, this does not correspond to the conditions of the real mechanical load of the tram carriage, the sections of the rear and receiving rails, which are located in the areas of the joints of the track. Therefore, the values of the constructive velocity of the tram carriage, determined in the investigations, appears virtual, and can not be accepted as reliable.

Thus, one can state that developing an adequate and convenient model of shock interaction between the rolling stock and the track is required, and an appropriate method of analysing their interaction, which considers the rail carriage in the form of a multi-dimensional discrete system, and the upper structure of the track – as a continuum system. In this formulation, the essential characteristics of the mechanical interaction are the shock impulse that occurs in the zone of butt roughness, as well as the aftershock velocity of the receiving rails.

2.2. Research objectives

The purpose of the work is to study the static and shock interaction of the carriage and the upper structure of the path to improve the parameters of discrete-continuum system “carriage – rail track” by rational selection and optimisation of the parameters of its components. This will provide an additional positive impact on the reliability and durability of the system in the area with isolated butt roughness.

Including the above-mentioned, the following research tasks were formulated: to create a mechanical model of static and shock interaction of four-axle carriage and rail track, taking into account the variation of load, velocity and reduced to the one wheel mass of the car, as well as the motion phase of the carriage through the rail joint; methods of numerical analysis based on the created mathematical models to determine and analyse the interaction of the components of the transport discrete-continuum mechanical complex; to establish new regularities of mechanical interaction of the four-axle carriage and the track in a zone of butt roughness.

3. MODEL AND INVESTIGATION METHODS OF THE CARRIAGE AND RAIL TRACK INTERACTION

The discrete-continuum model of static and shock interaction of a four-axle carriage with the upper structure of the track is used. It takes into account the design parameters of the path, carriage, load and speed of the vehicle. The carriage passage of the butt roughness of the way on all four phases of movement is considered. Thus, all wheels of the carriage wheel pairs settle down on the rear rail in the first phase, in the second phase – it is three of them to remain, in the third one – two and on the fourth only one wheel pair.

3.1. Mechanical model

The mechanical scheme is given in this paper in the example of the fourth phase of motion in Fig. 1. Here: 1 – vehicle carriage, 2-5 – the corresponding wheel of the wheel pair; 6-7 – the carriage central suspension; 8 – the receiving rail; 9 – the rear rail; 10 – the elastic elements of the ballast layer under the sleepers; 11 – the springing element that models the
rear rails rigidity at the end. This corresponds to the design scheme of a multispan beam on elastic supports.

![Diagram of rail deflection](image)

**Fig. 1. The scheme of passage of butt roughness**

### 3.2. Static interaction

For the static calculation of the rail deflections, a model of a multispan beam on 24 elastic supports is used (23 ties and support, which simulates a connection to the adjacent rail running through the working overlay). To calculate the rail rigidity value \( c_p \) its deflection at the end \( \delta_p \) under the action of a single force is determined, then: \( c_p = 1/\delta_p \). Taking into account the connection of the working overlay with the rigidity \( c_n \) of the giving and receiving rails, we obtain the rigidity of the rail at the end:

\[
c_{p,\kappa} = \frac{c_p \cdot c_n}{c_p + c_n}.
\]

The main force factors determining the static deflection of the receiving rail under the first elastic support are constant in magnitude external forces \( P_0 \). They correspond to the current number of wheel pairs on the rails attached to the giving rail (Fig. 2) and have the coordinates \( X_{Bj} \), where \( j = 1, 2, 3, 4 \) – the number of the carriage wheel pair.

On Fig. 2: \( l_i (i = 1–23) \); \( l_{24}=l_{p,k} = 12.5 \, m \) – geometric coordinates of elastic supports; \( P_0 \) – loads on the side of the carriage per wheel; \( F_{p,4f} = h_{B4f} \cdot c_{p,\kappa} \) – spring force applied to the end of the receiving rail by the giving rail at the end; \( h_{B4f} \) – giving rail deflection at the end of the fourth motion phase of the tram carriage; \( X_{B4} = 12.5 \, m \); \( X_{II1} = 1.9 \, m \), \( X_{II2} = 6.4 \, m \), \( X_{II3} = 8.3 \, m \), \( c \) – the ballast layer rigidity under the sleeper of the upper structure of the path.

The equation of the curved axis of the rails at the fourth motion phase will be written using the method of initial parameters, including the conditions of fastening. At the origin of the deflection and the angle of rotation of the sections of both rails are zero, so we get: \( y_0 = y_0 = 0 \). For the other three phases', there will be a difference only in the number of wheels on the giving and receiving rails.

For the rear rail is obtained
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\[ y(x) = \frac{1}{EJ} \times \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - P_0 \frac{(x-X_{B4})^3}{6} + \sum_{i=1}^{3} c_{y_i} \frac{(x-l_i)^3}{6} + c_{p.k} h_B \frac{(x-L)^3}{6} \right]. \quad (2) \]

For the receiving rail

\[ y(x) = \frac{1}{EJ} \times \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - P_0 \frac{F_{w4\Phi}}{8} \frac{(x-L)^3}{6} - \sum_{i=1}^{3} P_0 \frac{(x-X_{III})^3}{6} + \sum_{i=1}^{3} c_{y_i} \frac{(x-l_i)^3}{6} + c_{p.k} h_{II} \frac{(x-L)^3}{6} \right]; \quad (3) \]

Here \( J \) – is inertia moment of the rail cross-section relative to the neutral axis; \( E \) – elastic modulus of rail material; \( L = l_{p.k} \) – rail length; \( Q_0, M_0 \) are the transverse force and the bending moment in the coordinate origin. \( c_{y_i}, c_{p.k} h_B, c_{p.k} h_{II}, \) – reaction force of elastic supports. The main force factors that determine static deflection of the receiving rail under the first elastic support (2-3) are the external forces \( P_0 = P/8 \) (the load at the side of the car that falls on one wheel; \( P \) is the weight of the car with respect to its load), constant by their magnitude.

![Mechanical pattern of calculation of the junction height in the fourth motion phase](image)

Fig. 2. The mechanical pattern of calculation of the junction height in the fourth motion phase: a) rear rail; b) receiving rail

The current value of the butt roughness height is calculated from the defined elastic lines of the rear and receiving rails

\[ h_{4\Phi} = h_{B4\Phi} - h_{II 4\Phi}. \]
where \( h_{\text{B4p}} = y_B(l_{p,K}) \), \( h_{\text{H4p}} = y_H(l_{p,K}) \) – deflections of receiving and rear rails at the ends. Taking into account that the expressions (2) and (3) in the right part contain summands, which in turn depend on deflections, the solutions of these equations are performed numerically.

### 3.3. Shock interaction

Putting into consideration the parameters dependence of the shock impulse as well as the impact speed of the trolley wheel, passing butt roughness path from the following factors: the joint height of the type ‘gap – step up’; carriage loading; design and operational parameters of the vehicle. Hence, the carriage is presented in the form of a sprung summary mass on the truck, and it is assumed that when the wheel hits the edge of the receiving rail, its separation does not occur, as well as its sliding relative to the rail, which does not contradict the results of the papers [6,17,28]. At this stage, using the system motion angular momentum theorem [28], the parameters of the shock impulse are determined, the receiving rail will test in its shock interaction with the truck wheel. Presented according to the scheme in Fig. 3 is the shock interaction of the wheel from first wheel pair of the carriage and the receiving track rail with consideration for the motion angular momentum theorem of the mechanical system.

![Shock interaction scheme](image-url)

Fig. 3. Shock interaction scheme

Here: 1 – wheel of the corresponding wheel pair of the tram carriage; 2 – structural element that models the reduced mass of the; 3 – elastic suspension of the carriage; 4 – truck frame; 5 – wheel from the first wheel pair of the truck; 6, 7 – the rear and receiving rail of the path; \( h \) – joint height; \( h_1, h_2 \) – geometrical coordinates of the mass centres of the wheels 1, 5 and reduced mass 2 of the carriage; \( V_{1k}, V_{2k} \) – to-the-shock and after-the-shock velocity of the wheel 5; \( S, S_1 \) – shock impulses, that the receiving rail do experience while its shock interaction with wheel 5 from the first wheel pair of the truck; \( \alpha \) – angular coordinate of the wheel centre of mass 5; \( m_1, m_2 \) – reduced masses of the wheel and the carriage; \( r \) – wheel radius. Here, the shock impulse \( S^* \) equals by value to the impulse \( S \). In this setting, it corresponds to the law of conservation of momentum

\[
K_2 - K_1 = M_p(S^*),
\]

where \( K_1, K_2 \) – to-the-shock and after-the-shock system motion angular momentum relative to the axes, passing along the edge \( D \) of the receiving rail 6; \( S^* \) – external shock impulse, applied to the wheel 5 from the receiving rail; \( M_p(S^*) \) – shock impulse-momentum relative to the edge \( D \). In equation (4) the moments of the amount of motion, taking into account the scheme in Fig. 3, which is
\begin{align*}
K_1 &= mV_{1h}h_1 + \frac{m_r^2}{2} V_{1r} + m_rV_{1h}h_2 + \frac{3m_r^2}{2} V_{1r}; \quad K_2 = mV_{2h}h + \frac{m_r^2}{2} V_{2r} + m_rV_{2h}h_2 + \frac{3m_r^2}{2} V_{1r}.
\end{align*}

Taking into the account, that shock impulse \( S^* \) crosses wheel centre of masses 5, while velocities of the wheel 1 and reduced mass 2 of the carriage equals to \( V_{1k} \) and do not change own direction during the strike, equation 4 allows to indicate the value after-the-shock speed:
\[
V_2 = \frac{V_{1k}(2h_1/r + 1)}{3}.
\]

One defines a shock impulse of the mechanical scheme interaction in Fig. 3 with the flat end of the receiving rail in projections on the vertical axis \( y \), in accordance with the system motion angular momentum theorem, as (considering \( V_{1ky} = 0 \))
\[
m_1V_{2y} - m_1V_{1ky} = S^*_y = m_1V_{1k} \frac{2h_1 + r}{3r^2} \sqrt{r^2 - h_1^2}, \quad (5)
\]

where \( h_1 = r - h \). Equation 5 establishes the dependence of the parameters of the vertical component of the shock pulse on the mass and radius of the wheel, the speed of its centre of mass, as well as the height of the butt roughness at this phase of the movement. In this paper, the dependence (5) is further used to determine after-the-shock vertical cross-section velocity of the receiving rails at the end. The following approach is used. Experiencing shock impulse \( S_y \), the flat end of the rail 6 (Fig. 3) will receive at \( x = L \) (Fig. 2), given that on impact, the receiving rail bends along the same curve as under the action of a static concentrated load, under elastic deformation conditions according to (3), vertical displacement
\[
y(x) = y_a(L) y_{r}(L) \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - F \frac{x^3}{6} + \sum_{i=1}^{23} c_{y_i} \frac{(x - l_i)^3}{6} + c_{p.a.} h_{i \phi} \frac{(x - L)^3}{6} \right],
\]

where \( y_a(L) \) - peak value of deflection; \( y_{r}(L) \) - deflection of the rail flat end under the action of a single force \( F = 1 \) H when \( x = L \); \( h_{i \phi} \) - joint height on the \( i \)-th carriage motion phase. It is taken into account that the joint height of the track is a function of the motion phase. The velocity of the rail cross-section with the coordinate \( x \) will be at time \( t \), in accordance with the Fourier method of variables separation:
\[
V(t,x) = \frac{dy_a(t,L)}{dt} \cdot \frac{1}{y_{r}(L)EJ} \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - P_0 \frac{x^3}{6} + \sum_{i=1}^{23} c_{y_i} \frac{(x - l_i)^3}{6} + c_{p.a.} h_{i \phi} \frac{(x - L)^3}{6} \right], \quad (6)
\]

where \( \frac{dy_a(t,L)}{dt} \) - the change rate of the amplitude with time \( y_a(t,L) \), that is, the vertical cross-section velocity of the receiving rail. The obtained distribution of speed along the length of the rail (6), allows writing the expression for its momentum (5). This results in an expression for the vertical component, for the subsequent calculation of the after-the-shock velocity. Momentum \( Q_{py} \) of the receiving rail 6 in the projection on the axis after-the-shock interaction with the wheel 5 we define as
\[
Q_{py} = \frac{\rho F V_1}{y_{r}(L)EJ} \int_0^L \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - P_0 \frac{x^3}{6} + \sum_{i=1}^{23} c_{y_i} \frac{(x - l_i)^3}{6} + c_{p.a.} h_{i \phi} \frac{(x - L)^3}{6} \right] dx,
\]

where \( V_1 = \frac{dy_a(t,L)}{dt} \). Thus, including (5) and (6) with \( Q_{py} = S^*_y \) after-the-shock vertical velocity of the rail is determined as follows:
\[ V_i = \frac{S_i' y_F (L) E J}{\rho F} \left[ Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - P_0 \frac{x^3}{6} + \sum_{i=1}^{n} \frac{C_{ij} (x - L_i)^3}{6} + c_p h_{i,i} \frac{(x - L)^3}{6} \right] dx \] 

4. INVESTIGATION RESULTS

According to the model proposed, the numerical analysis of the parameters between the shock interaction of a four-axle vehicle with a rail track in the place of an isolated butt roughness of the “gap” type is performed on the example of a tram carriage. The calculations were conducted based on the variation of carriage loading, velocity and reduced to one of the wheels mass of the carriage according to the defined phases of motion. Fig. 4-7 show the dependences after the shock velocity \( V_i \) of the receiving rail. Table 1 shows the data of mechanical interaction at the maximum load of the carriage and the speed of movement tram (15 m/s) at all phases.

The analysis was carried out according to the calculation scheme shown in Fig. 2–3. The following design characteristics of the rail P-65 and the tram T-3 [18]: \( E = 2.6 \cdot 10^{11} \text{N/m}^2; \) \( J = 3573 \text{cm}^4; \) \( \rho = 7.8 \text{ kg/m}^3; \) \( F = 82.65 \text{ cm}^2; \) \( c = 4.225 \cdot 10^8 \text{ N/m} \) were used. The reduced to one wheel mass of the empty carriage is \( m = m_1 + m_2 = 2125 \text{ kg}, \) while maximum mass (with 193 passengers) of the loaded one \( m = 3814 \text{ kg}; m_1 = 1100 \text{ kg}. \) This corresponds to real operating conditions and design characteristics of the tram vehicle and rail track.

Tab. 1

| Characteristic                          | Phase 1 | Phase 2 | Phase 3 | Phase 4 |
|----------------------------------------|---------|---------|---------|---------|
| Height of joints [mm]                  | 3.89    | 1.01    | 1.95    | 0.67    |
| After-the-shock velocity [m/s]         | 10.9    | 4.70    | 8.23    | 2.62    |

Fig. 4. Dependence after-the-shock velocity on the operational factors at the first phase

Fig. 5. Dependence after-the-shock velocity on the operational factors at the second phase
5. SUMMARY

The laws analysis shows, for example, that the variation of reduced to the one wheel mass of the tram carriage in the range \( m = [2125 \pm 3814] \text{ kg} \) with design velocity of \( V_{ik} = 15 \text{ m/s} \) leads to a change after the shock velocity depending on the phases of the tram carriage motion, respectively, in the ranges \( V_1 = [4.635 \pm 10.93] \text{ m/s} \), \( V_2 = [2.355 \pm 4.708] \text{ m/s} \), \( V_3 = [3.729 \pm 8.234] \text{ m/s} \), \( V_4 = [1.368 \pm 2.623] \text{ m/s} \), that is, to the growth 2.36; 1.99; 2.2; 1.92 times.

One presented the results of numerical calculations of static and shock interaction parameters on the example of a tram carriage with a rail track at the junction, which takes into account operational, mechanical and geometric factors using the proposed models. It defines new regularities of interaction of the four-axle carriage with a rail track with vehicle passing through the butt roughness and allows to make improvement of operational parameters and design characteristics of the carriage and the top structure of a truck by rational choice and optimisation of parameters.

The results obtained are of significant theoretical importance in establishing the laws of influence of operational factors, and practically, they are used in the development of technical solutions to improve the junction of the track, in determining the modes of operation of tram carriages, including other four-axis, taking into account the limit values of shock impulses of interaction. Also, when creating an experimental-theoretical complex for research, calculations and improvement of the parameters of the carriage and the upper structure of the track.

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