Illumination of Quantum Hall States

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We show that illumination by continuous white radiation saturates the population of levels so that some of the transitions are not observed. This reduces the number of observable transitions so that the resolution of neighboring transitions is improved. In the case of \( \nu = 11/2 \) the improved resolution leads to clear observation of zero in the \( \rho_{xx} \) which appears as a finite resistivity minimum in the absence of saturating radiation.

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1. Introduction

In ordinary spectroscopy, in some cases, the absorption lines are so wide that lines overlap resulting into a poorly resolved spectrum. If the system is illuminated by a pulse of radiation of a resonant frequency, the population of the upper level can be saturated so that the line corresponding to this frequency does not absorb and in fact an emission occurs from the saturated level so that there appears a hole in the absorption spectrum. This type of hole burning in the absorption spectrum is well known and a systematic study of this effect was performed by Khutsishvili et al [1-5]. If we illuminate the system with a white light, absorption can occur at many levels and the population can be disturbed. The line shape clearly shows the frequencies at which saturation has occurred. At this time, the detailed theories of “fractional charge” are believed to be “not applicable” to the actual experimental data.

In this letter, we propose an interpretation of quantum Hall resistance in terms of energy levels. It then follows that a plateau should occur at $\nu H$ with $\nu = 11/2$. The magnetic field $H$ fixed by the value $\nu H$ also describes a zero value in the $\rho_{xx}$. This means that the resistivity is highly anisotropic in the $xy$ plane. By disturbing the population of neighboring levels, we can unmask the zero value of $\rho_{xx}$ at $\nu = 11/2$, i.e., the zero value of $\rho_{xx}$ is not found at $\nu = 11/2$ when the sample is dark but only a finite value occurs. When illumination by a red light is turned on, there is absorption at the neighboring levels and the zero value at $\nu = 11/2$ becomes visible.

2. Theory

In our theory [6] the effective charge arises from the effective value of the Bohr magneton, $\mu_B = e\hbar/2mc$. The values of the orbital angular momenta $l$ and the spin $s$ are combined in such a way that an effective charge can be defined,

$$e_{\text{eff}} = \frac{1}{2}g e = \nu_\pm e$$

(1)

where

$$\nu_+ = \frac{l + 1}{2l + 1}$$

(2)
and
\[ \nu_- = \frac{l}{2l+1}. \]  
(3)

For \( l = 0 \), we obtain, \( \nu_+ = 1 \) and \( \nu_- = 0 \) and for \( l = 1 \), we obtain \( \nu_+ = 2/3 \) and \( \nu_- = 1/3 \) which describe the effective charge. The values of the effective charges which we tabulate are the same as those experimentally found. The subscript \(-\) in \( \nu_- \) indicates that spin is \(-\frac{1}{2}\) and that in \( \nu_+ \) shows that spin is \(+\frac{1}{2}\). Usually \( \hbar \omega = g \mu_B H \) gives the resonance. However, in the present case \( \hbar \omega = \nu \pm \mu_B H \). We consider that this frequency is a cyclotron frequency and transition with \( n \hbar \omega_c \) are observable. Therefore, we can multiply the values of \( \nu_\pm \) by an integer. Of course, this number, \( n \), can be identified as the Landau level quantum number. Therefore, transitions at \( 2\omega_c, 3\omega_c, \) etc. occur. With \( l = \infty \), both of the above series give \( \nu_+(l \to \infty) = \frac{1}{2} \) and \( \nu_-(l \to \infty) = \frac{1}{2} \). Since we can multiply these numbers by an integer, we predict the observable frequencies as \( n/2 \). For \( n = 11 \), we obtain \((11/2)\). As the value of \( n \) increases, the intensity of the line reduces and hence for \( n = 11 \) the transition becomes considerably weak. Now, we have to convert the absorption response to the resistivity. This conversion is straightforward [7]. So far we have obtained the correct effective charges, particle-hole symmetry [8] and doubly degenerate state at \( n/2 \). What is found as energy levels in the absorption becomes plateaus in the transverse resistivity, \( \rho_{xy} \) as a function of magnetic field. At a field slightly higher than that at \( 11/2 = 5.5 \), occurs \( 16 \times (1/3) = 16/3 = 5.33 \). If \( 16/3 \) is saturated by illumination of light, the transition at \( 16/3 \) disappears and the resolution near \( 11/2 \) improves. The improvement can be so good that while the zero of \( 11/2 \) is not clearly visible it becomes visible when the sample is illuminated by a continuous wave light.

3. **Experimental data**

Cooper et al [9] have performed the measurements of longitudinal resistance, \( \rho_{xx} \), along \([1, \bar{1}, 0]\) and along \([110]\) directions, as a function of magnetic field. At low fields, such as \( 2T \) very weak structure in the resistivity has been detected and the minima in \( \rho_{xx} \) corresponding to plateaus in \( \rho_{xy} \) have been detected at \( \nu = 9/2 \) and \( 11/2 \). The minimum
at $\nu = 11/2$ is quite clearly seen. The sample is then illuminated by a red light emitting diode. The level saturation effects are observed, the same way as predicted by us using the energy level representation for the quantum Hall effect. Due to the saturation at $16/3$, the zero value at $11/2$ becomes quite clear. Therefore, the experimental data is in agreement with what is theoretically predicted on the basis of energy levels.

4. **Interactions**

We see that $1/3$ comes from $s = -1/2$ whereas $2/3$ comes from $+\frac{1}{2}$. Therefore, there is a need of an interaction which can flip the spin when magnetic field is varied. The interaction can be of the form $1.s = l_x s_x + l_y s_y + l_z s_z$ so that both $l$ as well as $s$ have to flip. However, for going from $1/3$ to $2/3$, $l$ need not change. Therefore, the interaction is of the form $l_z s_x$ which can happen in the case of a very anisotropic interaction. Accordingly, we can write the interaction as

$$\mathcal{H'} = \sum_i \lambda_i < l^i_z > (s^i_+ + s^i_-) \quad (4)$$

where the summation is over all of the electrons. Next we consider as to how we can go from one value of $l$ to another value of $l$. This can be done by two site exchange interaction $l^i_+ l^-_j$ with pairwise summation,

$$\mathcal{H'} = \sum_{i>j} j_{ij} (l^i_+ l^-_j + l^i_- l^j_+) \quad (5)$$

It seems that the above interaction, (4) is quite sufficient to produce experimentally observed phenomenon and we need an $L \pm$ operator which should come from some where but not necessarily from (5) above, which causes an exchange interaction. The values given by ref. [6] are the same as those found by Störmer [10] and hence if $J$ is treated properly with both signs in $J = L \pm S$ no interactions are needed to locate the plateaus except the shift operator. However the width of the plateaus obviously requires the many-body electron-phonon type interactions.

5. **Flux Tubes and Composite Fermions**

The composite fermion theory requires that even number of fluxes are attached to the electron. The data of Cooper et al does not show any evidence of flux quantization
with even number of fluxes. Therefore, it is clear that composite fermion theory is not in conformity with the experimental data. In 1989, Jain has suggested [11] that even number of fluxes are attached to the electron so that the magnetic field becomes, $B^* = B - 2\rho \phi_o$ where $\rho$ is the density of electrons. The factor of 2 is used so that only even number of flux quanta, $2\phi_o$ in this case, are used. The experimental data does not agree with this “even number” quantization of magnetic field shift. There is no evidence of even number multiplied to any quantity in the experimental data. We have found [12] that the Jain’s theory of composite fermions is internally inconsistent. Therefore, it is concluded that flux tubes are not attached to the electrons.

6. Lande’s $g$-value formula

The Lande’s formula for the $g$-value of the electron in atomic physics gives only one value given in terms of $L$ and $S$ of the atom as,

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}.$$

For $J = L + S$, the values of $g/2$ for various values of $L$ and $S = \frac{1}{2}$, are given by

| $L$ | 0 | 1 | 2 | 3 |
|-----|---|---|---|---|
| $g_-/2$ | 1 | 2/3 | 3/5 | 4/7 |

For $J = L - S$, the above values become,

| $L$ | 0 | 1 | 2 | 3 |
|-----|---|---|---|---|
| $g_+/2$ | 0 | 1/3 | 2/5 | 3/7 |

Since the Bohr magneton has the charge of the electron the above $g_\pm$ give the effective charge. In the case of $L = 0$, $g_{(-)}/2$ has a zero charge solution which gives the soft mode or symmetry breaking mode. The values tabulated above agree with those found in the experimental data [10]. Usually, there is only one value of $g$ for a given value of $L$ which is measured in the electron spin resonance. In the present case of quantum Hall effect, $L$ is not a constant and changes as the magnetic field is varied. The plateaus in the quantum Hall effect therefore occur at the energy levels determined from the single electron expression of $g$ values and the relaxation times which become widths of
the plateaus are determined from the electron-phonon interaction which is a many-body interaction.

Our theory also agrees with data at large values of $L$ [13]. We have compared a lot of experimental data with our theory and found good agreement in all cases [14,15]. We find that the magnetic moment of the electron is slightly modified at large magnetic fields [16]. The polarization of the half-filled level is also predicted correctly by using Knight shifts [17]. At the half-filled Landau level, there is a symmetry breaking resulting into the appearance of a Goldstone boson in bilayers of semiconductors which are predicted correctly [18].

Modern and elegant theories in which fractional charge can be understood on the basis of appropriate generalizations of the Laughlin-type treatment are obviously irrelevant to quantum Hall effect experiments. The area of the flux quantization as well as the spin has not been treated correctly by Laughlin and in view of proper calculations of angular momenta, the need for the fractionally charged wave function completely disappears. Schoutens [19-22] has made an effort to consider the spin but the physics of the problem in his theory is not relevant to the experimental work on quantum Hall effect.

7. Conclusions.

We conclude that the plateau at $11/2$ arises from $L \to \infty$ limit in (2) and (3) multiplied by $n = 11$ because of $n\hbar\omega$ transitions. When $n$ becomes large, the intensity reduces. This predicted reduction in intensity for large values of $n$ is in agreement with the experimental data of Cooper et al [9].
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