A Mathematical Technique for Analyzing Folds With the Computer Program “FOLDPI”

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ABSTRACT

A mathematical technique for analyzing folds was proposed instead of the tedious and slow graphical method. Procedure of this technique comprises converting the data of bedding planes to pole attitudes, calculation of the mean pole vector of fold limbs, obtaining the best fit $\pi$-circle, determining the fold geometric properties and finding fold cylindricity. This procedure was carried out by FOLDPI, a GWBASIC computer program written for the purpose of this application. Most of the geometrical properties of fold were dealt with. In addition, an example taken from the Sinjar Anticline was used for testing the validity of this technique. The results of testing the program against manually obtained solutions proved that this technique can be very helpful in getting faster and more accurate results.

INTRODUCTION

Conventionally, folds were analyzed using the common graphical technique of $\pi$-diagram, which is one of the stereographic projection applications. This graphical method has a worldwide usage and it has an advantage of graphically showing fold geometry but it is tedious and time consuming specially in plotting and counting the S-poles on the
stereonet. Recently, and for the sake of faster and easier techniques, many structural geologists have attempted to modify their related methods towards the trend of mathematics and computer programming approaches. Accordingly, the present author suggests a mathematical technique for digital execution of this $\pi$-diagram and the determination of the geometric parameters of folds using mathematics and computer program.

Previously, some authors made contributions in this trend. They performed some steps in this respect. Ramsay (1967) suggested two mathematical techniques in the scope of fold analysis. The first was applied for determination of unimodal poles distribution by vectors of directional cosines. While the second method was used for determining the best-fit $\pi$-circles of cylindrical folds. Bengston (1980) mentioned, marginally, about this idea throughout his study of tangent diagram. Ramsay and Huber (1987) described the methods of Ramsay (1967) by “the accuracy contained in such methods is only justified if it is imperative to exploit the full potential of very precise primary data”.

$\pi$-diagram of fold analysis is a method applied by using an equal area stereographic projection (stereonet) to plot the perpendiculars to the bedding planes (S-poles). Attitudes of such bedding planes are collected along traverse which must be transverse to the fold axis. Plotting the S-poles of such a fold (point diagram), counting the concentration of points in 1% of the stereonet area, and lastly contouring the counted values in the form of contour percentages (contour %) to produce the $\pi$-diagram of this fold. The advantage of this diagram is the geometric view of the fold parameters. Such parameters are fold axis, fold plunge, fold symmetry, interlimb angle and attitude of axial plane.

In $\pi$–diagram, if the fold is perfectly cylindrical, the bedding S-poles fall, perfectly, along a great circle of the stereonet ($\pi$-circle). When the fold transforms geometrically to non-cylindrical types, the scattering of the poles around $\pi$-circle becomes more pronounced. The perpendicular to the $\pi$-circle is called $\pi$-axis that is coinciding with the
fold axis. Consequently, if the fold is perfectly cylindrical the angle between each S-pole and fold axis is perfectly 90° (Fig. 1). Practically, the S-poles of any natural fold do not lie exactly on a certain great circle but fall in zone around this circle. Nevertheless, the human error in field measurements plays a role in the accuracy of these measurements; which amount to ±2 degree (Ramsay and Huber, 1987). However, the mean reason responsible for the scattering of S-poles is the absence of perfectly cylindrical folds in the field. Many of natural folds are of cylindrical, sub-cylindrical and non-cylindrical styles. In this respect Ramsay (1967) designed his method for determining the best-fit π-circles for perfectly cylindrical folds. This is because only in this type of folds the π-axis makes a right angle with each S-pole; and he built up his mathematics on this property. It must be mentioned that in Ramsay (1967) terminology, the term cylindrical fold is analogous to perfectly cylindrical one. In the more recent synthesis, Ramsay and Huber (1987) classified the folds into perfectly cylindrical, cylindrical, sub-cylindrical and non-cylindrical types (Fig. 2). Previously, folds classified as cylindrical and non-cylindrical and some time cylindroid (Fleuty, 1964). In the classification of Ramsay and Huber (1987) the perfectly cylindrical fold has its S-poles lie perfectly on the π-circle.

While if more than 90% of the S-poles fall within an angle of ±10° from the constructed π-circle the fold should be termed cylindrical. But if more than 90% of the data lie within ±20° of the π-circle the fold is then called sub-cylindrical fold. Folds with more than 10% of their S-poles falling outside the limit of ±20° are termed non-cylindrical (Fig. 2). In addition to the scope of the present work, the author extended the method of Ramsay (1967) from its application only on perfectly cylindrical to include all types of folds described by Ramsay and Huber (1987). This extension was based on the styles of poles distribution a round the best-fit π-circle.
Nabeel K. Al-Azzawi

**METHODOLOGY**

The proposed mathematical method comprises the following procedures:

1st- Converting the data of bedding plane attitudes of both fold limbs (strike direction/dip amount when strike was taken clockwise from dip direction) to pole attitude (dip direction/dip amount) and finally to their corresponding directional cosine vectors ($\alpha$, $\beta$, & $\gamma$).

2nd- Calculation of the mean vector of each fold limb and hinge area by unimodal poles distribution method (summing method).

3rd- Obtaining the best-fit \(\pi\)-circle for these two means of fold limbs, or three means (two limbs and a hinge area)

4th- Determination of fold parameters which reflect its geometry.

5th- Determination of the cylindricity of folds according to Ramsay and Huber (1987).

**1st- Determination of directional cosines:**

Mathematical equations needed for the determination of directional cosines from attitudes of bedding planes are derived in this work. So the angles $\alpha$, $\beta$, & $\gamma$ are determined from strike directions and dip amounts. The following steps explain this procedure.

1. Conversion of bedding plane attitudes (strike direction /dip amount) to pole attitude (dip direction / dip amount).

   So, $dr = 90 + ds$, $dp = 90 – dpl$, $cdr = 90 – dr$

   Where $dr$ and $dp$ are the angles of dip direction and dip amount of the pole, $ds$ and $dpl$ are of strike direction and dip amount of the bedding plane and $cdr$ is the complimentary angle of $dr$.

2. Determination of the directional cosines. These can fall into four cases. Each case represents one of the upper four quarters of the Cartesian coordinate. It must be mentioned that in the four cases the angle $\gamma$ is always equal to $(90 – dp)$ with negative sign (Fig.4).

**The first case:** if the pole of any plane falls in the first quarter (Fig.3).

   Determination of the angle $\alpha$:

   In Figure (3), suppose the line $OA$ in the triangle $OAB$ is unity.

   Then

   $AB = \sin dp$ and $OB = \cos dp$. 

![Fig. (3): Determination of the angles $\alpha$ and $\beta$ from strike direction and dip amount of the bedding plane.](image)
In the triangle OBC the angle OCB is a right angle, then
\[ \sin \, cdr = \frac{CB}{OB} \quad \text{so} \quad CB = \cos \, dp \cdot \sin \, cdr \]
Also
\[ \cos \, cdr = \frac{OC}{OB} \quad \text{so} \quad OC = \cos \, cdr \cdot \cos \, dp \]
According to Pythagorean Theorem
\[ (AC)^2 = (AB)^2 + (BC)^2 \]
Therefore,
\[ (AC)^2 = \sin^2 \, dp + \cos^2 \, dp \cdot \sin^2 \, cdr \]
And, in the triangle OAC
\[ (AC)^2 = (OC)^2 + (OA)^2 - 2 \cdot OC \cdot OA \cdot \cos \, \alpha \]
\[ \cos \, \alpha = \frac{(OC)^2 + (OA)^2 - (AC)^2}{2 \cdot OC \cdot OA} \]
Resultant, the equation which is shown below is used for the determination of \( \cos \alpha \):

\[
\cos \alpha = \frac{\cos^2 \, cdr \cdot \cos^2 \, dp + 1 - \sin^2 \, dp - \cos^2 \, dp \cdot \sin^2 \, cdr}{2 \cdot \cos \, cdr \cdot \cos \, dp}
\]

**Determination of the angle \( \beta \):**

In the triangle \( OAB \), \( OBA \) is a right angle, suppose the hypotenuse \( OA \) is unity (Fig. 3). Therefore,
\[ OB = \cos \, dp \quad \text{and} \quad AB = \sin \, dp \]
The angle \( BCO \) in the triangle \( BCO \) was drawn to be 90°. So,
\[ \sin \, dr = \frac{CB}{\cos \, dp} \quad \text{so} \quad CB = \sin \, dr \cdot \cos \, dp \]
\[ \cos \, dr = \frac{OC}{\cos \, dp} \quad \text{so} \quad OC = \cos \, dr \cdot \cos \, dp \]
\( ABC \) is a right triangle, So according to the Pythagorean theorem
\[ (AC)^2 = (CB)^2 + (AB)^2 \]
Then,
\[ (AC)^2 = \sin^2 \, dr \cdot \cos^2 \, dp + \sin^2 \, dp \]
Lastly, in the triangle \( AOC \)
\[ (AC)^2 = (OA)^2 + (OC)^2 - 2 \cdot OA \cdot OC \cdot \cos \, \beta \]
\[ \cos \, \beta = \frac{(1 + (OC)^2 - (AC)^2)}{2 \cdot OC} \quad \text{------} \quad OA \text{ is unity} \]
Resultant, the equation listed below, determine \( \cos \beta \):

\[
\cos \beta = \frac{\cos^2 \, dr \cdot \cos^2 \, dp + 1 - \sin^2 \, dp - \cos^2 \, dp \cdot \sin^2 \, dr}{2 \cdot \cos \, dr \cdot \cos \, dp}
\]

It must be noted that this equation is similar to that equation of \( \cos \alpha \), accept the angle \( cdr \) is replaced by the angle \( dr \). In this case, when the angle \( \alpha \) fall in the first quarter it has a positive sign whereas the angle \( \beta \) is negative (Fig. 4).
The second case: If the pole of any plane falls in the second quarter (Fig. 3). Therefore,\
\[ \text{dr}_{\text{new}} = \text{dr}_{\text{old}} - 90 \]
\[ \text{cdr} = 90 - \text{dr} \]
Similar to the first case, the angle \( \alpha \) and \( \beta \) can be determined by:

\[ \cos \alpha = \frac{\cos^2 \text{dr}. \cos^2 \text{dp} + 1 - \sin^2 \text{dp} - \cos^2 \text{dp}. \sin^2 \text{dr}}{2 \cos \text{dr}. \cos \text{dp}} \]

And,

\[ \cos \beta = \frac{\cos^2 \text{cdr}. \cos^2 \text{dp} + 1 - \sin^2 \text{dp} - \cos^2 \text{dp}. \sin^2 \text{cdr}}{2 \cos \text{cdr}. \cos \text{dp}} \]

In this case, both \( \alpha \) and \( \beta \) have positive signs (Fig. 4).

The third case: If the pole of any plane falls into the third quarter.
Then, \[ \text{dr}_{\text{new}} = \text{dr}_{\text{old}} - 180 \]
\[ \text{cdr} = 90 - \text{dr} \]
Similarly,

\[ \cos \alpha = \frac{\cos^2 \text{cdr}. \cos^2 \text{dp} + 1 - \sin^2 \text{dp} - \cos^2 \text{dp}. \sin^2 \text{cdr}}{2 \cos \text{cdr}. \cos \text{dp}} \]

And

\[ \cos \beta = \frac{\cos^2 \text{dr}. \cos^2 \text{dp} + 1 - \sin^2 \text{dp} - \cos^2 \text{dp}. \sin^2 \text{dr}}{2 \cos \text{dr}. \cos \text{dp}} \]

In this quarter, \( \alpha \) has negative sign. While \( \beta \) is positive (Fig. 4).

The forth case: When the pole lies in the forth quarter.
Therefore, \[ \text{dr}_{\text{new}} = \text{dr}_{\text{old}} - 270 \]
\[ \text{cdr} = 90 - \text{dr} \]
Similar to the previous cases, the angles \( \alpha \) and \( \beta \) can be determined by the following equations:

\[ \cos \alpha = \frac{\cos^2 \text{dr}. \cos^2 \text{dp} + 1 - \sin^2 \text{dp} - \cos^2 \text{dp}. \sin^2 \text{dr}}{2 \cos \text{dr}. \cos \text{dp}} \]

And
In this quarter, the angles $\alpha$ and $\beta$ have negative signs. Resultant, data of poles (directions and dip amounts) representing bedding planes are converted to directional cosines which represented by the cosines of the angles $\alpha$, $\beta$ and $\gamma$.

2nd- The unimodal pole distribution method:

The unimodal pole distribution was suggested and described by Ramsay (1967). It is also called method of summing vectors. Considering the pole as a unit vector, the method is used for determining the mean vector of poles of any geologic planes. This is done after determining the angles $\alpha$, $\beta$ and $\gamma$ of each pole (unit vector) with respect to the coordinate axes $x$, $y$ and $z$ respectively (Fig. 5A). These angles, in the present work, are an output of the previous procedure (mentioned in 1st). The $x$, $y$, and $z$ components of the side of the vector box (Fig. 5B) for each measurement are then calculated by $\cos \alpha$, $\cos \beta$ and $\cos \gamma$ respectively. These directional cosines are determined for all poles and sums of the vector components are calculated ($\Sigma \cos \alpha$, $\Sigma \cos \beta$ and $\Sigma \cos \gamma$). These sums give the dimensions of the $x$, $y$, and $z$ components of the total vector sum and the diagonal of this box gives the strength of the total vector sum ($TVS$) which is equal to:

$$TVS = \sqrt{(\Sigma \cos \alpha)^2 + (\Sigma \cos \beta)^2 + (\Sigma \cos \gamma)^2}$$  \hspace{1cm} (Ramsay, 1967)

Therefore, the mean vector direction with respect to $x$, $y$ and $z$ axes are given by:

$$\cos \alpha = \frac{\Sigma \cos \alpha}{TVS} \hspace{1cm} \text{------------------------ 1}$$
$$\cos \beta = \frac{\Sigma \cos \beta}{TVS} \hspace{1cm} \text{------------------------ 2}$$
$$\cos \gamma = \frac{\Sigma \cos \gamma}{TVS} \hspace{1cm} \text{------------------------ 3}$$

In the course of this work, data that were taken from a fold can be differentiated into two or three concentrations. If the folds are of chevronic or mostly chevronic style, two concentrations of pole distribution are found. That is because the hinge is angular and there is no hinge area then the two concentrations representing the two limbs. Whereas, in the box type folds or near this shape, three concentrations are found. Two of them for the limbs and the third represent the hinge area. It means that each fold has two or three mean vectors. Consequently, the users of this method (Unimodal poles distribution) must process each concentration of fold poles and determine the mean vector of each one. These mean vectors (with the angles $\alpha$, $\beta$ and $\gamma$) can be used for determining the best-fit
π-circle of such a fold. Mean vectors can be plotted manually by setting the angles α, β and γ on the stereonet or they can be processed mathematically to determine the fold π-circle.

3rd- Determination of best fit π-circle of folds:

Fold π-circle, as it mentioned above, is a stereographic best-fitted great circle to bedding S-poles. The method for determining the best-fit π-circles was suggested by Ramsay (1967). In this method, π-axis was determined mathematically by finding the normal to each bedding S-poles of perfectly cylindrical fold. Then π-circle can be drown, stereographically, perpendicular to this normal. The method was based on the usual technique of minimizing the squares of the deviations of the observed bedding S-poles from this surface.

Ramsay (1967; pp.18-20) designed this method for number of poles that fall perfectly on the π-circle, so this method is constrained for perfectly cylindrical folds only.

The present author made a simple modification for wider range of applications including cylindrical and sub-cylindrical fold, which are dominant in the field. Therefore, it is modified by obtaining the mean vector of large number of poles for the two fold limbs or two-fold limbs and hinge area by the unimodal pole distribution. Then, determining of the π-circle best fitted to the two or the three concentrations. By this way of drawing the π-circle, limits of ±10º and ±20º from π-circle can be plotted and type of fold cylindricity can be found (As in E). According to this modification many types of folds can be identified.

The method was described in (Ramsay, 1967), and it is summarized here as the following:

Suppose α, β and γ are angles between π-axis vector of any fold and the three coordinate axes X, Y and Z respectively. Also, suppose a, b and c are angles between any bedding S-pole vector and the same coordinate axes (Fig. 6).

**Fig. (6)** Calculation of α, β, γ, a, b and c and determination of best-fit π-circle to no. of S-poles (Ramsay, 1967)
A Mathematical Technique for Analyzing Folds…

\[ A = \frac{\left( \sum lm \sum mn - \sum l n \sum m2 \right)}{\left( \sum l2 \sum m2 - (\sum lm)^2 \right)} \]
\[ B = \frac{\left( \sum lm \sum ln - \sum mn \sum l2 \right)}{\left( \sum l2 \sum m2 - (\sum lm)^2 \right)} \]

Where, \( \cos a = l \), \( \cos b = m \), \( \cos c = n \), \( \cos a / \cos \gamma = A \) and \( \cos \beta / \cos \gamma = B \)

Ramsay (1967) derived three equations for determination of \( \alpha \), \( \beta \) and \( \gamma \). These equations are described below.

Because, \( \cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1 \)
And \( A = \cos \alpha / \cos \gamma \) and \( B = \cos \beta / \cos \gamma \)
Therefore \( A^2 \cos 2 \gamma + B^2 \cos 2 \gamma + \cos 2 \gamma = 1 \)
And \( \cos^2 \gamma = 1 / (A^2 + B^2 + 1) \)
so \( \cos \gamma = \left( 1 + A^2 + B^2 \right)^{-1/2} \)

Similarly, equations responsible for determining \( \alpha \) and \( \beta \) were derived.

\( \cos \alpha = A \left( 1 + A^2 + B^2 \right)^{-1/2} \)
\( \cos \beta = B \left( 1 + A^2 + B^2 \right)^{-1/2} \)

By the values and signs of these angles the Cartesian coordinate position of \( \pi \)-axis was found. Fold \( \pi \)-circle can be drawn considering this \( \pi \)-axis normal to it.

4th- Determination of fold geometry:

Most of the important geometric properties of folds can be determined from this technique. Such properties are fold axis, fold plunge, interlimb angle, fold symmetry, attitude of axial plane and fold cylindricity.

Fold axis:

The angles \( \alpha \), \( \beta \) and \( \gamma \) which were calculated in (3rd page 6 ) are used for determining the attitude of fold axis (dip direction / dip amount).

From figure (7),

\( dp = 90 - \gamma \) when \( dp \) is the dip amount of fold axis

Also from the same figure, the triangle \( OAC \) has the right angle \( OCA \) and the hypotenuse \( OA \) suppose to be unity.

Fig.(7). Conversion of fold axis attitude from directional cosines to dip dirs and dip amounts
Also in the triangle, $OBA$ is a right angle.

Whereas in the triangle $ABC$

$$AC^2 = AB^2 + CB^2$$

$CB = \sqrt{(\sin^2 \beta - \sin^2 \alpha)}$

In the triangle $OBC$,

$$CB^2 = OC^2 + OB^2 - 2 \cdot OC \cdot OB \cdot \cos \delta$$

$$\sin^2 \beta - \sin^2 \alpha = \cos^2 \beta \cdot \cos^2 \alpha - 2 \cdot \cos \beta \cdot \cos \alpha \cdot \cos \theta$$

$$2 \cdot \cos \beta \cdot \cos \alpha \cdot \cos \theta = \cos^2 \beta + \cos^2 \alpha - \sin^2 \beta + \sin^2 \alpha$$

When $\delta$ is the dip direction of fold axis. The signs of the angles $\alpha$ and $\beta$ serve as indicators to show in which quarter of Cartesian coordinate the fold axis was fall (Table 1).

| Sign of the angle $\alpha$ | Sign of the angle $\beta$ | Position   |
|---------------------------|---------------------------|------------|
| Positive                  | Negative                  | First quarter |
| Positive                  | Positive                  | Second quarter |
| Negative                  | Positive                  | Third quarter |
| Negative                  | Negative                  | Fourth quarter |

If the fold axis falls in the first quarter, $\delta$ remains without change. Whereas, if it falls in the second, third and forth quarter, then $90^\circ$, $180^\circ$ and $270^\circ$ are added to the angle $\delta$ for obtaining its correct dip direction.

| Attitude of fold axis = dip direction (\delta) / dip amount (\alpha) |

**Fold plunge:**

Fold plunge depends upon dip amount of fold axis ($\alpha$). When ($\alpha$) is equal to zero, the fold is nonplunging. While if ($\alpha$) is more than zero, the fold becomes plunging. And increasing of $\alpha$ angle means increasing of degree of plunging.

| Fold Plunge = $\alpha$ of fold axis |

**Interlimb angle:**

According to this procedure, interlimb angle can be calculated by adding the absolute value of $\gamma_1$ (of the first limb) to the absolute value of $\gamma_2$ (of the second one). The angles $\gamma_1$ and $\gamma_2$ are always having negative sign (Fig.4) then it must be considered their absolute values when the interlimb angle was determined. This calculation must be done along a plane perpendicular to the fold axis. Then concentrations of the two limbs must be rotated until the fold axis becomes horizontal and determination of the interlimb angle was done along the N-S vertical plane (Fig. 8).
A Mathematical Technique for Analyzing Folds…

Interlimb angle = γ\text{limb1} + γ\text{limb2} = 180° - (\text{adpl}_{1} + \text{adpl}_{2})

Where \text{adpl}_{1} and \text{adpl}_{2} are mean dips of the two limbs

Folds symmetry:
Fold symmetry can be found by comparing \text{adpl}_{1} with \text{adpl}_{2} or \text{γlimb1} with \text{γlimb2}. Resultant, if \text{dp1} is equal to \text{dp2}. Then the fold is symmetrical, otherwise it is asymmetrical with the vergency being towards the limb which has greater pole dip angle (dp).

\[
\text{Folds symmetry : } \text{adpl}_{1} \leq \text{adpl}_{2} \quad \text{OR} \quad \text{γlimb1} \leq \text{γlimb2}
\]

Attitude of axial plane:
This attitude will be possible if the attitude of axial trace is known (Ramsay and Huber, 1987). While if the fold is of parallel type, axial plane can be obtained by joining the bisector of interlimb angle with the fold axis. This is because in the parallel fold the axial plane always bisects the interlimb angle. In this work the axial plane determination was restricted to parallel fold type because other types are more complicated to be processed mathematically.

Axial plane attitude can be represented by its dip amount Apdp and dip direction Apdr:

\[
\text{Apdp} = 90° - (\text{γmax} + \text{γmin} / 2) - \text{γmin}
\]

Where \text{γmax} and \text{γmin} are dip angles of steep and gentle limbs respectively.

\text{Apdr} coincide with the dip direction of gentle limb.

\[
\text{Attitude of axial plane} = \text{Apdp} / \text{Apdr}
\]

5th- Determination of fold cylindricity:
Ramsay and Huber (1987) classified the folds into perfectly cylindrical, cylindrical, sub-cylindrical and non-cylindrical, which were described previously in the introduction (Fig. 2).
Different natural folds show different properties or different $\pi$-diagram models. And it is very complicated to put a solution for each model. So a simplification was made for putting a general procedure for all these models. This is to revolve and rotate the data (mentioned below) to make a general solution for all the cases and keeping the entire relative geometrical relationships constant with each other but not with the coordinate axes.

Revolving and rotating fold data

Many authors suggested various methods for rotation of oriented data. Saha (1987) designed a FORTRAN program for rotation of data by transformation matrix. Al-Azzawi and Al-Jumaily (2000) proposed a mathematical procedure for rotation of joint planes relative to bedding rotation by trigonometric method.

For the sake of applying this idea, two stages are used. Firstly, revolving of data, horizontally, until direction of $\pi$-axis coincides with east direction of stereonet. This is done by adding $(90 - dr)$ when $dr$ less than $90^\circ$, and subtracting $(dr - 90)$ when $dr$ more than $90^\circ$ to or from the angle $dr$.

Secondly, rotation of all data around $Y$ coordinate axis (N-S line in the stereonet) until $\pi$-axis becomes horizontal; it means rotation angle ($R$) = $dp$ of $\pi$-axis (Fig. 8). When $\pi$-axis rotated by the angle $R$, $\pi$-circle became vertical plane. This is because $\pi$-axis is always perpendicular to the plane containing bedding S-poles ($\pi$-circle). This rotation can be done by transformation matrix. Many authors such as Arfken (1970) and Saha (1987) described this method. And it is summarizes here by the followings:
1- Determination of $L, M & N$ components for each S-pole vector before rotation.

\[
L = \cos dp \cdot \sin dr, \quad M = \cos dp \cdot \cos dr \quad \text{and} \quad N = \sin dp
\]

2- Multiplication of the components $L, M & N$ by the transformational matrix that is responsible for rotation of vectors anticlockwise around $Y$-axis and through an angle $R$.

\[
\begin{array}{ccc}
L & \cos R & 0 & \sin R \\
M & 0 & 1 & 0 \\
N & -\sin R & 0 & \cos R \\
\end{array}
\]

The resultant are new $L, M & N$ components (after rotation) which are used to determine the direction $dr$ and dip angle $dp$ of a pole vector.

\[
dp = \sin^{-1} N_{new} \quad \text{and} \quad dr = \sin^{-1} (L_{new} / \cos dp) \quad \text{or} \quad dr = \cos^{-1} (M_{new} / \cos dp)
\]

Mathematically, this fold classification could not be applied without revolving and rotating data (as it mentioned above). Rotating all data until $\pi$-axis becomes horizontal standardize all natural cases in into one form. Figure (9) shows the rotated state of Figure (2) which responsible for this classification. Figure (9) can be used to plot the revolved and rotated bedding S-poles of any fold and to find the type of this fold according to its cylindricity.
Ramsay and Huber (1987) suggested limits for defining fold cylindricity (Fig. 2). These limits are ±10 and ±20 around π-circle of perfect cylindrical fold. These limits were divided the stereonet into fields; each field represents one of fold types. Figure (9) showed these limits after revolution and rotation. Empirically, the present author derived mathematical equations to deal with these limits during the present technique. And using Lagrangian interpolation method mentioned in (Al-Azzawi, 2004) does this. For more explanation, the author exhibited curves shown in figure (10) representing these limits.

S-pole directions of stereonet normally range from 1° to 360°. So that, direction of each S-pole can be tested and determined in which field it fall. Counting of these poles and obtaining the percentage of each field was done. Then the fold can be classifying as in the followings:

1- If all S-poles fall on the plane of perfectly cylindrical (π-circle) (Fig. 11), then it is called perfectly cylindrical fold.
2- If 90% of S-poles fall between plane-10 and plane+10 around the mean π-circle (Fig.9) or on and above the standard curve of cylindrical fold (Fig. 10), then the fold named cylindrical type.
3- If 90% of them fall between plane-20 and plane+20 (Fig.9) or on and above the standard curve of sub-cylindrical fold (Fig.10), the fold classified as sub-cylindrical type.

4- Otherwise or when more than 10% of the S-poles lie outside plane-20 and plane+20 (Fig.9) or fall below the standard curve of sub-cylindrical fold (Fig.10), the fold becomes non-cylindrical type.

So, mathematically or by computer programming, the number of bedding S-poles that fall between these limits can be determined.

The procedure for analyzing fold was designed in GWBASIC computer program called FOLDPI (see the Appendix)

Tested sample:

Data for checking the validity of this technique has been taken from Sinjar Anticline. Al-Azzawi (1982) studied this anticline through three traverses. They named Gaulat, Sinjar and Jeribi. The first one was used for this test. The anticline in this traverse is, stratigraphically, comprise Shiranish formation of Upper Cretaceous age (the older formation). And it is followed by Aliji Fn. (Paleocene –L. Eocene), Jaddala Fn (Middle to Upper Eocene), Avana Fn. (M. to U. Eocene), Euphrates Fn. (L. Miocene), Serigakni Fn. (L. Miocene), Jeribe Fn. (L. Miocene), Alfatha Fn. (M. Miocene) and Injana Fn. (U. Miocene), (Al-Azzawi, 1982; Numan and Al-Azzawi, 2002). Tested data was taken from Serikagni Fn. (both Sinjar and Gaulat traverse) and analyzed graphically by π-diagram.
Consequently, the geometric properties of this fold are shown in a $\pi$-diagram of Figure (11). This Figure showed that the mean pole attitudes of first and second limb are $340/74$ and $182/43$, attitude of fold axis is $087/5$, it is asymmetrical fold with N vergency, plunging fold with 7 degrees, the interlimb angle is $120^\circ$, dip amount of axial plane is $76^\circ$ toward SE and more than 90% of bedding S-poles are fall within $\pm 10$ around the mean $\pi$-circle then the fold classified as cylindrical fold. A computer program output for this analysis is exhibited in Figure (12) that shows not only encouraging result but it is more easy, fast and accurate than the graphical method specially when it done by computer scheme.

![Output of the program FOLDPI showing the geometric analysis of Sinjar Anticline.](image)

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Nabeel K. Al-Azzawi

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APPENDIX

10 REM
20 REM PROGRAM FOR ANALYZING FOLDS BY NEW MATHEMATICAL TECHNIQUE
30 REM DEPENDING ON THE PI-DIAGRAM PRINCIPALS. WRITTEN BY DR. NABEEL K.
40 REM AL-AZAWI, JONE, 2005. DEPARTMENT OF GEOLOGY/UNIVERSITY OF MOSUL.
50 CLS.
60 PRINT
70 PRINT "GEOMETRIC ANALYSIS OF FOLDS"
80 PRINT
90 PRINT
100 N1=50
110 N2=15
120 N3=0
130 K=N1+N2
140 M=N1+N2+N3
150 DIM DS(M), DPL(M), DR(M), DR1(M), DR2(M), DR3(M), DRR(M), DP(M), DP1(M), DP2(M), DPP(M),
CDR(M), ALFA(M), BETA(M), GAMMA(M), OFIE1(M), OFIE2(M), COSALF(M), COSBET(M), COSGAM(M),
X(10), FX(10), LU(10), LD(10)
160 CINDY=1
170 REM INPUT DATA OF BEDDING PLANES AS STRIKE DIRECTIONS / DIP AMOUNTS.
180 IF CINDY=1 THEN PRINT "*********ALFA, BETA & GAMMA OF FIRST LIMB"
190 IF CINDY=2 THEN PRINT "*********ALFA, BETA & GAMMA OF SECOND LIMB"
200 IF CINDY=3 THEN PRINT "*********ALFA, BETA & GAMMA OF HINGE AREA"
210 ON CINDY GOTO 220,230,240
220 N = N1 : GOTO 250
230 N = N2 : GOTO 250
240 N = N3
250 SDPL=0
260 SDDR=0
270 FOR I = 1 TO N
280 READ DS(I) , DPL(I)
290 SDPL = SDPL + DPL(I)
300 DDR = DS(I) – 90
310 SDDR = SDDR + DDR
320 NEXT I
330 DATA 234,16, 240,17, 246,15, 242,18, 240,18, 242, 17, 260,21,242,18, 252,20, 240,23,248, 23,15,
250,13, 236, 13, 250,10, 270,18, 248,20
340 DATA 236,19, 240, 18, 242,12, 256, 12, 250,10, 244,12, 250,14, 236, 16, 240, 20, 260,18, 268,20, 250,
18,264, 21, 252, 18,264, 20, 240,20
350 DATA 230, 20,240,20, 260, 20, 286,22,242, 20, 232, 22, 250, 23, 230,32,228, 30,230,18, 252, 20, 254,12,18,
248, 15,258, 20, 258,15
360 DATA 100, 25, 110, 28, 90, 60, 89, 69, 100, 60, 80, 42, 74, 43, 90, 42, 100, 37, 87,45,100,45, 88, 49, 94, 42,
98, 34, 79, 45
370 ADPL(CINDY) = INT(SDPL/N)
380 ADDR(CINDY) = INT(SDDR/N)
390 IF ADDR(CINDY) < 0 THEN ADDR(CINDY) = 360 – ADDR(CINDY)
400 SDR=0
410 FOR I = 1 TO N
420 REM DR AND DP ARE DIP DIRECTION AND DIP AMOUNT OF S-POLE,
430 REM WHERE CDR IS THE COMPLIMENTARY ANGLE OF DR.
440 DR(I) = 90+ DS(I) : DP(I) = 90- DPL(I) : IF DR(I) > 360 THEN DR(I) = DR(I) –360
450 SDR = SDR + DR(I)
460 REM THE THREE STATEMENTS BELOW ARE TO PUT ALL READINGS (OF 2 LIMBS AND HINGE AREA)
470 IF DR(I) = 1 THEN DR2(I) = DR(I) : DP2(I) = DP(I)
480 IF DR(I) = 2 THEN DR2(I) = DR(I) : DP2(I) = DP(I)
490 IF DR(I) = 3 THEN DR2(I) = DR(I) : DP2(I) = DP(I)
500 IF DR(I) <=90 THEN DR1(I) = DR(I)
510 IF DR(I) <=90 AND DR(I) <=180 THEN DR1(I) = DR(I) - 90
Nabeel K. Al-Azzawi

520 IF DR(I) > 180 AND DR(I) <= 270 THEN DR1(I) = DR(I) – 180
530 IF DR(I) > 270 AND DR(I) <= 360 THEN DR1(I) = DR(I) – 270
540 CDR(I) = 90 – DR1(I)
550 DR1(I) = DR1(I) * 3.132857 / 180
560 DP(I) = DP(I) * 3.142857 / 180 : CDR(I) = CDR(I) * 3.142857 / 180
570 NEXT I
580 ADR(CINDY) = SDR/N
590 REM DETERMINATION OF DIRECTIONAL COSINES: THERE ARE FOUR CASES.
600 REM GAMMA = 90 – DP WITH -VE SIGN.
610 FOR I = 1 TO N
620 GAMA(I) = 90 – DP(I) * 180 / 3.142857
630 GAMA(I) = GAMA(I) * 3.142857 / 180
640 IF GAMA(I) > 0 THEN GAMA(I) = 0 – GAMA(I)
650 OFIE1(I) = ((COS(CDR(I)) ^ 2 * (COS(DP(I)) ^ 2 + 1 – (SIN(DP(I))) ^ 2 – (COS(DP(I))) ^ 2)
                    * (SIN(DR1(I))) ^ 2) / ((2 * COS(DR1(I)) * COS(DP(I)))))
670 IF DR(I) > 90 THEN 750
680 COSALF(I) = OFIE1(I)
690 COSBET(I) = OFIE2(I)
700 ALFA(I) = ATN(SQR(1 – (OFIE1(I)) ^ 2) / OFIE1(I))
710 BETA(I) = ATN(SQR(1 – (OFIE2(I)) ^ 2) / OFIE2(I))
720 IF ALFA(I) < 0 THEN ALFA(I) = 0 – ALFA(I)
730 IF BETA(I) > 0 THEN BETA(I) = 0 – BETA(I)
740 GOTO 980
750 IF DR(I) > 180 THEN 830
760 COSALF(I) = OFIE2(I)
770 COSBET(I) = OFIE1(I)
780 ALFA(I) = ATN(SQR(1 – (OFIE2(I)) ^ 2) / OFIE2(I))
790 BETA(I) = ATN(SQR(1 – (OFIE1(I)) ^ 2) / OFIE1(I))
800 IF ALFA(I) < 0 THEN ALFA(I) = ALFA(I) * (-1)
810 IF BETA(I) < 0 THEN BETA(I) = BETA(I) * (-1)
820 GOTO 980
830 IF DR(I) > 270 THEN 910
840 COSALF(I) = OFIE1(I)
850 COSBET(I) = OFIE2(I)
860 ALFA(I) = ATN(SQR(1 – (OFIE1(I)) ^ 2) / OFIE1(I))
870 BETA(I) = ATN(SQR(1 – (OFIE2(I)) ^ 2) / OFIE2(I))
880 IF ALFA(I) > 0 THEN ALFA(I) = 0 – ALFA(I)
890 IF BETA(I) > 0 THEN BETA(I) = 0 – BETA(I)
900 GOTO 980
910 REM WHEN DR MORE THAN 270.
920 COSALF(I) = OFIE2(I)
930 COSBET(I) = OFIE1(I)
940 ALFA(I) = ATN(SQR(1 – (OFIE2(I)) ^ 2) / OFIE2(I))
950 BETA(I) = ATN(SQR(1 – (OFIE1(I)) ^ 2) / OFIE1(I))
960 IF ALFA(I) > 0 THEN ALFA(I) = 0 – ALFA(I)
970 IF BETA(I) > 0 THEN BETA(I) = 0 – BETA(I)
980 NEXT I
990 REM DETERMINING THE SUMMATION OF COSALF, COSBET & COSGAM
1000 SCOSALF = 0 : SCOSBET = 0 : SCOSGAM = 0
1010 FOR I = 1 TO N
1020 COSGAM(I) = COS(GAMA(I))
1030 SCOSALF = SCOSALF + COSALF(I)
1040 SCOSBET = SCOSBET + COSBET(I)
1050 SCOSGAM = SCOSGAM + COSGAM(I)
1060 NEXT I
1070 REM DETERMINATION OF UNIMODAL POLE DISTRIBUTION BY SUMMING METHOD. COSALF, COSBET & COSGAM ARE DIRECTIONAL COSINES OF THE MEAN OF POLE VECTORS.
1080 TVS = ((SCOSALF)^2 + (SCOSBET)^2 + (SCOSGAM)^2)^1/2
1090 COSA(CINDY) = SCOSALF / TVS
1100 COSB(CINDY) = SCOSBET / TVS
1110 COSC(CINDY) = SCOSGAM / TVS
A Mathematical Technique for Analyzing Folds

1120 \( A(\text{CINDY}) = \text{ATN}(\text{SQR}(1 - \text{COSA(\text{CINDY})}^2 / \text{COSA(\text{CINDY})}) \))
1130 \( B(\text{CINDY}) = \text{ATN}(\text{SQR}(1 - \text{COSB(\text{CINDY})}^2 / \text{COSB(\text{CINDY})}) \))
1140 \( C(\text{CINDY}) = \text{ATN}(\text{SQR}(1 - \text{COSC(\text{CINDY})}^2 / \text{COSC(\text{CINDY})}) \))
1150 \text{PRINT “THE DIR. OF LIMB NO. “; CINDY; “ IS “; ADR(\text{CINDY})}
1160 \text{IF ADR(\text{CINDY}) <= 90 THEN 1200}
1170 \text{IF ADR(\text{CINDY}) <= 180 THEN 1210}
1180 \text{IF ADR(\text{CINDY}) <= 270 THEN 1220}
1190 \text{IF ADR(\text{CINDY}) > 270 THEN}
1200 \( A(\text{CINDY}) = \text{ABS}(A(\text{CINDY})) \) \times (-1) : \( B(\text{CINDY}) = \text{ABS}(B(\text{CINDY})) \) \times (-1) : \( C(\text{CINDY}) = \text{ABS}(C(\text{CINDY})) \) \times (-1) : \text{GOTO 1240}
1210 \( A(\text{CINDY}) = \text{ABS}(A(\text{CINDY})) \) \times (-1) : \( B(\text{CINDY}) = \text{ABS}(B(\text{CINDY})) \times (-1) : \( C(\text{CINDY}) = \text{ABS}(C(\text{CINDY}) \times (-1) \times (-1) : \text{GOTO 1240}
1220 \( A(\text{CINDY}) = \text{ABS}(A(\text{CINDY})) \times (-1) : \( B(\text{CINDY}) = \text{ABS}(B(\text{CINDY})) \times (-1) : \( C(\text{CINDY}) = \text{ABS}(C(\text{CINDY}) \times (-1)
1230 \( A1(\text{CINDY}) = A(\text{CINDY}) \times 180 / 3.142857 : B1(\text{CINDY}) = B(\text{CINDY}) \times 180 / 3.142857 : C1(\text{CINDY}) = C(\text{CINDY}) \times 180 / 3.142857
1250 \text{PRINT A1(\text{CINDY}),B1(\text{CINDY}),C1(\text{CINDY}),CINDY}
1260 \text{IF CINDY = 1 THEN CINDY = CINDY + 1 : GOTO 170}
1270 \text{IF CINDY = 2 THEN INPUT “TO INPUT DATA OF HINGE AREA PRESS ..2 OTHERWISE PRESS ..0”; XXX: IF XXX= 2 THEN CINDY = CINDY + 1: GOTO 170}
1280 \text{REM DETERMINATION OF BEST -FIT PI-CIRCLE OF FOLD.}
1290 \text{SLM= SMN=SLN=SM2=SL2=0}
1300 \text{FOR I = 1 TO 2}
1310 \( L(I)= \text{COS}(A(I)) : M(I)= \text{COS}(B(I)) : N(I) = \text{COS}(C(I)) \)
1320 \text{IF A(I)>0 THEN L(I) = L(I) \times (-1)}
1330 \text{IFB(I) > 0 THEN M(I) = M(I) \times (-1)}
1340 \text{IF C(I) <0 THEN N(I)= N(I) \times (-1)}
1350 \text{SLM= SLM + L(I) \times M(I)}
1360 \text{SMN= SMN + M(I) \times N(I)}
1370 \text{SLN= SLN + L(I) \times N(I)}
1380 \text{SL2= SL2 + (L(I)) \times 2)}
1390 \text{SM2= SM2 + (M(I)) \times 2)}
1400 \text{NEXT I}
1410 \text{AA=} (SLM* SMN – SLN * SM2 / (SL2*SM2 – (SLM)^2)
1420 \text{BB = (SLM*SLN – SMN*SL2) / (SL2SL2*SM2 – (SLM)^2)
1430 \text{COSALF1= AA*(1+AA^2+BB^2)^(-1/2)}
1440 \text{COSBET1 = BB*(1+AA^2 +BB^2)^(-1/2)}
1450 \text{COSGAM1= 1+AA^2+BB^2^(-1/2)}
1460 \text{ALFA1= ATN(SQR(1 - (COSALF1)^2) / COSALF1)
1470 \text{BETA1= ATN(SQR(1 - (COSBET1)^2) / COSBET1)
1480 \text{GAMA1= ATN(SQR(1 - (COSGAM1)^2) / COSGAM1)
1490 \text{IF GAMA1 > 0 THEN 1530}
1500 \text{GAMA1= GAMA1 \times (-1)}
1510 \text{BETA1= BETA1 \times (-1)}
1520 \text{ALFA1= ALFA1 \times (-1)}
1530 \text{ALFA2= ALFA1 \times 180 / 3.142857}
1540 \text{BETA2= BETA1 \times 180 / 3.142857}
1550 \text{GAMA2= GAMA1 \times 180 / 3.142857}
1560 \text{PRINT “********ALFA, BETA & GAMA OF FOLD AXIS”}
1670 \text{PRINT ALFA2,BETA2,GAMA2}
1580 \text{REM THIS ALFA,BETA AND GAMMA ARE OF PI-AXIS THAT IS NORMAL TO PI-CIRCLE}
1590 \text{REM IF ALFA IS +VE & BETA –VE -----PI-AXIS FALL IN THE FIRST QUARTER.}
1600 \text{REM IF ALFA IS +VE & BETA +VE -----PI-AXIS FALL IN THE SECOND QUARTER}
1610 \text{REM IF ALFA IS -VE & BETA +VE -----PI-AXIS FALL IN THE THIRD QUARTER}
1620 \text{REM IF ALFA IS +VE & BETA --VE -----PI-AXIS FALL IN THE FIRST QUARTER}
1630 \text{REM DP (HERE) IS DIP OF PI-AXIS AND DR IS ITS DIP DIRECTION.}
1640 \text{DP= 90 – ABS(GAMA2)}
1650 \text{DP1 = DP \times 3.142857 / 180}
Nabeel K. Al-Azzawi

\[ \text{AXIS} = \left( (\cos(\text{ALFA1})^2 + (\cos(\text{DP1}))^2 - (\sin(\text{ALFA1}))^2 + (\sin(\text{DP1}))^2) / (2*\cos(\text{ALFA1}) * \cos(\text{DP1})) \right) \]

\[ \text{DR} = \arctan\left( \sqrt{\left| 1 - \text{AXIS}^2 \right|} / \text{AXIS} \right) \]

\[ \text{DR} = \text{DR} + 8.3142857 \]

\[ \text{IF ALFA1 > 0 AND BETA1 < 0 THEN QR = 1} \]

\[ \text{IF ALFA1 > 0 AND BETA1 > 0 THEN QR = 2} \]

\[ \text{IF ALFA1 < 0 AND BETA1 < 0 THEN QR = 3} \]

\[ \text{IF ALFA1 < 0 AND BETA1 < 0 THEN QR = 4} \]

\[ \text{IF QR = 1 THEN DR = 90-DR} \]

\[ \text{IF QR = 2 THEN DR = DR+90} \]

\[ \text{IF QR = 3 THEN DR = DR+180} \]

\[ \text{IF QR = 4 THEN DR = DR+270} \]

\[ \text{PRINT} \]

\[ \text{PRINT} \text{"THE FOLD AXIS IS FALL IN THE QUARTER NO.";QR} \]

\[ \text{REM DETERMINING THE ATTITUDE OF FOLD AXIS} \]

\[ \text{REM_-----------------------------------------------------------------} \]

\[ \text{PRINT} \text{"******** THE ATTITUDE OF FOLD AXIS IS (";INT(\text{DR});"/";INT(\text{DP}));" \}

\[ \text{REM DETERMINATION OF FOLD PLUNGE} \]

\[ \text{REM------------------------------------------------------} \]

\[ \text{IF DP = 0 THEN PLUNGE$ = "NONPLUNGING"} \]

\[ \text{IF DP > 0 THEN PLUNGE$ = " PLUNGING"} \]

\[ \text{PRINT} \text{"********THIS FOLD IS";PLUNGE$} \]

\[ \text{PRINT} \text{"********AMOUNT OF PLUNGING IS";INT(\text{DP});"DEGREE"} \]

\[ \text{REM DETERMINATION OF INTERLIM ANGLE} \]

\[ \text{REM_-----------------------------------------------------------------} \]

\[ \text{FOR I = 1 TO 2} \]

\[ \text{IF DR < 90 THEN REV = 90- DR} : \text{ADDR(I) = ADDR(I) + REV} \]

\[ \text{IF DR > 90 THEN REV = DR - 90} : \text{ADDR(I) = ADDR(I) - REV} \]

\[ \text{ADPL(I) = ADPL(I) * 3.142857 / 180} \]

\[ \text{ADDR(I) + ADDR(I) * 3.142857 / 180} \]

\[ \text{R = DP * 3.142857 / 180} \]

\[ \text{REM CALCULATING THE PARAMETERS L, M AND N BEFOR ROTATION} \]

\[ \text{REM FOR THE SAKE OF DETERMINING INTERLIMB ANGLE, DATA OF THE TWO LIMBS MUST} \]

\[ \text{BE ROTATED TO VERTICAL PLANE} \]

\[ \text{FOR I = 1 TO 2} \]

\[ \text{IF C(I) > C(2) THEN 2190} \]

\[ \text{IF A(2) > 0 AND B(2) > 0 THEN VERG$ = " NW"} \]

\[ \text{IF A(2) > 0 AND B(2) < 0 THEN VERG$ = " SW"} \]

\[ \text{IF A(2) < 0 AND B(2) > 0 THEN VERG$ = " NE"} \]

\[ \text{IF A(2) < 0 AND B(2) < 0 THEN VERG$ = " SE"} \]

\[ \text{GOTO 2230} \]

\[ \text{IF A(1) > 0 AND B(1) > 0 THEN VERG$ = " NW"} \]

\[ \text{IF A(1) > 0 AND B(1) < 0 THEN VERG$ = " SW"} \]

\[ \text{IF A(1) < 0 AND B(1) > 0 THEN VERG$ = " NE"} \]

\[ \text{IF A(1) < 0 AND B(1) < 0 THEN VERG$ = " SE"} \]

\[ \text{GOTO 2230} \]

\[ \text{2190 PRINT "********THE INTERLIMB ANGLE OF THIS FOLD IS = "; INT(INTERLIMB) ;"DEGREE"} \]

\[ \text{2110 REM FOLD SYMMETRY} \]

\[ \text{2120 REM} \]

\[ \text{2130 IF C(1) > C(2) THEN 2190} \]

\[ \text{2140 IF A(2) > 0 AND B(2) > 0 THEN VERG$ = " NW"} \]

\[ \text{2150 IF A(2) > 0 AND B(2) < 0 THEN VERG$ = " SW"} \]

\[ \text{2160 IF A(2) < 0 AND B(2) > 0 THEN VERG$ = " NE"} \]

\[ \text{2170 IF A(2) < 0 AND B(2) < 0 THEN VERG$ = " SE"} \]

\[ \text{2180 GOTO 2230} \]

\[ \text{2190 IF A(1) > 0 AND B(1) > 0 THEN VERG$ = " NW"} \]

\[ \text{2200 IF A(1) > 0 AND B(1) < 0 THEN VERG$ = " SW"} \]

\[ \text{2210 IF A(1) < 0 AND B(1) > 0 THEN VERG$ = " NE"} \]

\[ \text{2220 IF A(1) < 0 AND B(1) < 0 THEN VERG$ = " SE"} \]

\[ \text{2230 IF C(1) = C(2) THEN PRINT "********THE FOLD IS SYMMETRICAL" ELSE PRINT "********THE} \]

\[ \text{FOLD IS ASYMMETRICAL AND VERGENT TOWARD";VERG$} \]

\[ \text{2240 REM} \]

\[ \text{------------------------------------------------------------------------------------------------------------------------} \]
A Mathematical Technique for Analyzing Folds…

2250 C1(1) = INT(ABS( C1(1)))
2260 C1(2) = INT(ABS(C1(2)))
2270 IF C1(1)>C1(2) THEN CC=90-((C1(1)+C1(2))/2)–C1(2) ELSE CC=90-((C1(1)+C1(2))/2)– C1(1)
2280 APDP= INT(ABS(CC))
2290 IF APPL(1) > APPL(2) THEN APDR = ADDR(2) ELSE APDR=ADDR(1)
2300 IF APDR > 0 AND APDR< 90 THEN APDRS = “ NE”
2310 IF APDR > 90 AND APDR< 180 THEN APDRS = “ SE”
2320 IF APDR >180 AND APDR<270 THEN APDRS = “ SW”
2330 IF APDR >270 AND APDR<360 THEN APDRS = “ NW”
2340 IF APDR = 0 OR APDR = 360 THEN APDRS = “N”
2350 IF APDR = 90 THEN APDRS = “E”
2360 IF APDR =180 THEN APDRS = “S”
2370 IF APDR = 270 THEN APDRS = “W”
2380 PRINT“********DIP OF AXIAL PLANE IS “;APDP;”TOWARD”;APDR$;”(IF PARALLEL FOLD)”
2390 REM TO DETERMINE THE CYLINDRICITY OF FOLD, DATA MUST BE REVOLVING & ROTATING. THIS IS TO MAKE ALL DATA MOVE UNTIL PI—CIRCLE COINCIDE N-S LINE ON STEREOMET.
2400 REM 1- REVOLVING OF PI-DIAGRAM UNTIL PI-AXIS POINT COINCIDE WITH E-W LINE OF STEREOMET.
2410 REM ----------------------------------------------------------------------------------------------------------------------------------------
2420 FOR I = 1 TO M
2430 IF DR < 90 THEN REV = 90 – DR : DR2(I)=DR2(I) + REV
2440 IF DR > 90 THEN REV = DR – 90 : DR2(I) = DR2(I) – REV
2450 IF DR2(I)= 90 OR DR2(I) = 270 THEN DR2(I) = DR2(I) –1
2460 IF DR2(I) > 90 AND DR2(I) < 270 THEN DR2(I) = 360 + DR2(I)
2470 IF DR2(I) > 90 AND DR2(I) < 270 THEN DR2(I) = 90 – DR2(I)
2480 IF DR2(I) > 90 AND DR2(I) < 270 THEN DR2(I) = 270 – DR2(I)
2490 IF DR2(I) > 90 AND DR2(I) < 270 THEN DR2(I) = 360 – DR2(I)
2500 DP2(I) = DP2(I) * 3.142857 / 180
2510 DR2(I) = DR2(I) * 3.142857 / 180
2520 R = DP * 3.142857 / 180
2530 REM DETERMINATION OF THE PARAMETERS L, M & N BEFORE ROTATION.
2540 L1 = COS(DP2(I)) * SIN(DR2(I))
2550 M1 = COS(DP2(I)) * COS(DR2(I))
2560 N1 = SIN(DP2(I))
2570 REM DETERMINATION OF THE PARAMETER AFTER ROTATION
2580 L2 = L1 * COS (R ) + N1 * SIN(R )
2590 M2 = M1
2600 N2 = L1 * (1-N2) + N1 * COS(R )
2610 DP1(I) = ATN((1-N2^2) (1/2)
2620 ANG = ABS(COS(DP1(I))
2630 DR1(I) = ATN((L2 / ANG) / (1- (L2 / ANG) )^ (1/2)
2640 IF DR1(I) > 180 AND DR2(I) < 270 THEN DR1(I) = 180 – DR1(I)
2650 IF DR1(I) > 270 AND DR2(I) > 180 THEN DR1(I) = 360 + DR1(I)
2660 IF DR1(I) > 360 THEN DR1(I) = DR1(I) – 360
2670 IF DP1(I) < 0 THEN DP1(I) = ABS(DP1(I)) : DR1(I) = 180 + DR1(I)
2680 NEXT I
2690 REM *****************************************************************************************************************************************
2700 REM DETERMINATION OF FOLD CYLINDRICITY.
2710 PER=0 : CYN=0 : SUB=0 : NON=0
2720 REM READ THE STANDARD CURVE FOR PERFECT CYLINDRICAL FOLD.
2730 FOR I = 1 TO 10
2740 READ X(K), FX(K)
2750 DATA 0, 0, 1, 36, 2, 78, 3, 82, 4, 86, 5, 86, 6, 86, 7, 87, 8, 88, 9, 89
2760 NEXT K
2770 FOR I = 1 TO M
2780 IF DR1(I) <= 90 THEN DR3(I) = INT(DR1(I))
2790 IF DR1(I) > 90 AND DR1(I) <= 180 THEN DR3(I) = INT( 180 – DR1(I))
2800 IF DR1(I) > 180 AND DR1(I) <= 270 THEN DR3(I) = INT( DR1(I) – 180)
2810 IF DR1(I) > 270 AND DR1(I) <= 360 THEN DR3(I) = INT( 360 – DR1(I))
IF DR3(I) = > 10 AND DP1(I) = 90 THEN PER = PER + 1 : GOTO 2880
2830 IF DR3(I) = > 10 AND DP1(I) < 90 THEN 2880
2840 IF DR3(I) < 1 THEN DR3(I) = 0
2850 XX = DR3(I)
2860 GOSUB 3240
2870 IF INT(DP1(I)) >= TLX THEN PER=PER+1
2880 NEXT I
2890 REM STANDARD CURVE FOR CYLINDRICAL FOLD.
2900 FOR K = 1 TO 10
2910 READ X(K), FX(K)
2920 DATA 0, 0, 10, 0, 20, 60, 30, 70, 40, 75, 50, 77, 60, 79, 70, 80, 90, 90, 90, 80
2930 NEXT K
2940 FOR I = 1 TO M
2950 IF DR3(I) =< 10 THEN SYN=SYN +1 : GOTO 3020
2960 IF DR3(I) >= 70 AND DP1(I) >= 80 THEN SYN=SYN +1 : GOTO 3020
2970 IF DR3(I) >= 70 AND DP1(I) < 80 THEN 3020
2980 IF DR3(I) < 1 THEN DR3(I) = 0
2990 XX= DR3(I)
3000 GOSUB 3240
3010 IF INT(DP1(I)) >= INT(TLX) THEN SYN=SYN+1
3020 NEXT I
3030 REM STANDARD CURVE FOR SUB-CYLINDRICAL FOLD.
3040 FOR K = 1 TO 10
3050 READ X(K), FX(K)
3060 DATA 0, 0, 10, 0, 20, 0, 30, 45, 40, 58, 50, 64, 60, 66, 70, 68, 80, 70, 90, 70
3070 NEXT K
3080 FOR I = 1 TO M
3090 IF DR3(I) =< 20 THEN SUB=SUB +1 : GOTO 3160
3100 IF DR3(I) >= 80 AND DP1(I) >= 70 THEN SUB = SUB +1 :GOTO 3160
3110 IF DR3(I) >= 80 AND DP1(I) < 70 THEN 3160
3120 IF DR3(I) < 1 THEN DR3(I) = 0
3130 XX = DR3(I)
3140 GOSUB 3240
3150 IF INT(DP1(I)) >= INT(TLX) THEN SUB=SUB+1
3160 NEXT I
3170 NON = M – SUB
3180 REM THE PERCENTAGE DETERMINATION OF EACH TYPE
3190 IF PER / M >= .9 THEN PRINT "******** THIS FOLD IS PERFECT CYLINDRICAL" :GOTO 3230
3200 IF SYN/M>=.9 THEN PRINT "******** THIS FOLD IS CYLINDRICAL":GOTO 3230
3210 IF SUB/M>.9 THEN PRINT "******** THIS FOLD IS SUB-CYLINDRICAL":GOTO 3230
3220 IF NON/M>.1 THEN PRINT "******** THIS FOLD IS NON-CYLINDRICAL "
3230 END
3240 REM**************************************************************************
3250 REM A SUBROUTINE FOR LAGRANGIAN INTERPOLATING A POINT WITHIN A CURVE.
3260 TLX=O
3270 FOR K = 1 TO 10
3280 LU(K) = 1 : LD(K) = 1
3290 FOR J = 1 TO 10
3300 IF K = J THEN 3330
3310 LU(K) = LU(K) * (XX – X(J))
3320 LD(K) = LD(K) * (X(K) – X(J))
3330 NEXT J
3340 LX(K) = LU(K) / LD(K) * FX(K)
3350 TLX = TLX + LX(K)
3360 NEXT K
3370 RETURN
3380 REM**************************************************************************