TRIAXIAL BLACK HOLE NUCLEI

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ABSTRACT

We demonstrate that the nuclei of galaxies containing supermassive black holes can be triaxial in shape. Schwarzschild’s method was first used to construct self-consistent orbital superpositions representing nuclei with axis ratios of 1:0.79:0.5 and containing a central point mass representing a black hole. Two different density laws were considered: \( \rho \propto r^{-\gamma} \) and \( \gamma = \{1, 2\} \). We constructed two solutions for each \( \gamma \): one containing only regular orbits and the other containing both regular and chaotic orbits. Monte Carlo realizations of the models were then advanced in time using an N-body code to verify their stability. All four models were found to retain their triaxial shapes for many crossing times. The possibility that galactic nuclei may be triaxial complicates the interpretation of stellar kinematical data from the centers of galaxies and may alter the inferred interaction rates between stars and supermassive black holes.

Subject headings: galaxies: elliptical and lenticular, cD — galaxies: nuclei — galaxies: structure — stellar dynamics

1. INTRODUCTION

The phenomenon of triaxiality has remained of central importance to our understanding of galaxy dynamics since the demonstration by Schwarzschild (1979) of the existence of self-consistent triaxial equilibria. Schwarzschild’s models had large, constant-density cores, and the majority of the orbits were regular, respecting three integrals of the motion. But the realizations that the central densities of elliptical galaxies and bulges are very high (Crane et al. 1993) and that supermassive black holes are generic components of galaxies (Kormendy & Richstone 1995) have modified somewhat our ideas about the persistence of triaxiality. A central black hole can destroy triaxiality on large scales by rendering the center-filling box orbits stochastic (Gerhard & Binney 1985). Evolution to globally axisymmetric shapes can occur in just a few crossing times if the black hole contains on the order of \( 10^{-2} \) of the galaxy’s mass (Merritt & Quinlan 1998; Sellwood 2001).

Less is known about the possibility of maintaining triaxial shapes at the very centers of galaxies, where the gravitational force is contributed roughly equally by the stars and by the nuclear black hole. Regular orbits, both boxlike and tubelike, exist in this region (Merritt & Valluri 1999; Sambhus & Sridhar 2000; Poon & Merritt 2001, hereafter Paper I), but the boxlike orbits (the “pyramids”) have shapes that are generally contrary to the figure elongation, making them less useful than classical box orbits for maintaining a triaxial shape. Furthermore, the pyramid orbits disappear within the “zone of chaos,” which begins at a radius where the enclosed stellar mass is a few times that of the black hole (Paper I), roughly \( 10^2 \) pc in a typical bright elliptical galaxy.

Here we demonstrate that triaxial equilibria do in fact exist in the vicinity of the black hole. We first use Schwarzschild’s technique to construct self-consistent superpositions of orbits computed in a fixed triaxial potential representing the stars and a central point mass (§ 2). We then test the long-term stability of the models by using an N-body code to advance them forward in time (§ 3); we find that the N-body models maintain their triaxial shapes for many crossing times. Finally, we present some of the observable properties of the models (§ 4) and discuss the implications of triaxiality for observational and theoretical studies of galactic nuclei (§ 5).

2. SCHWARZSCHILD SOLUTIONS

We model the stellar nucleus as a triaxial spheroid with a power-law dependence of density on radius,

\[
\rho_\star = \rho_0 m^{-\gamma},
\]

\[
m = \sqrt{\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}},
\]

inside of some surface \( m = m_{\text{out}} \) discussed below. We took \( \gamma = 1 \) or 2; these values correspond roughly to the density profiles observed at the centers of bright and faint elliptical galaxies, respectively (Gebhardt et al. 1996). The triaxiality \( T \) is defined as

\[
T = \frac{a^2 - b^2}{a^2 - c^2}.
\]

We chose \( a = 1.0, b = 0.79, c = 0.5 \) for both models, corresponding to \( T = 0.5 \), “maximal” triaxiality. The central black hole is represented by a point mass with \( M_{\text{bh}} = 1 \). Expressions for the gravitational potential and forces corresponding to this mass model are given in Paper I.

Although our mass model for the stars is scale free, the presence of the black hole imposes a scale. We identify three characteristic radii associated with the black hole. At a radius of \( r_g \), the enclosed mass in stars (defined as the mass within an equidensity surface that intersects the x-axis at \( x = r_g \)) is equal to that of the black hole. In real galaxies, \( r_g \) is typically a factor of about a few greater than the “sphere of influence” \( r_p \), where

\[
r_p \equiv \frac{GM_{\text{bh}}}{\sigma_\star^2} \approx 10.8 \text{ pc} \left( \frac{M_{\text{bh}}}{10^8 M_\odot} \right) \left( \frac{\sigma_\star}{200 \ \text{km s}^{-1}} \right)^{-2}
\]

depends on the stellar velocity dispersion \( \sigma_\star \). The third fiducial radius is \( r_{\text{in}} \), the radius at which the character of the boxlike orbits undergoes a sudden transition to chaos in triaxial potentials. Table 1 gives values of \( r_g \) and \( r_{\text{in}} \) for our two mass models; values of \( r_{\text{in}} \) are approximate and are taken from Paper I. Table 1 also gives values of the dynamical times at \( r_g \) and \( r_{\text{in}} \).
defined as the period of a circular orbit of the same energy in the equivalent spherical potential, defined to have a scale length of \( a_{\text{av}} = (abc)^{1/3} \). The enclosed stellar mass within \( r_{\text{ch}} \) is about 3\( M_\odot \) for \( \gamma = 1 \) and 6\( M_\odot \) for \( \gamma = 2 \).

Any numerical realization of a stellar system must have an outer boundary. Our goal was to test self-consistency out to a radius of at least \( r_{\text{ch}} \), corresponding to \( \sim 2r_s \) for \( \gamma = 1 \) and \( \sim 6r_s \) for \( \gamma = 2 \). We therefore chose the outer surface of our mass model, \( m = m_{\text{out}} \), to be large enough that almost all of the density at \( r_{\text{ch}} \) in a real galaxy would be contributed by orbits with apocenters below this surface. In order to estimate \( m_{\text{out}} \), the isotropic distribution functions corresponding to a spherical galaxy with the density law of equation (1), \( \gamma = [1, 2] \), and a central point mass were computed and transformed to \( F(r_{\text{cp}}; r) \), the distribution of apocentric radii \( r_{\text{cp}} \) of orbits passing through \( r \). For \( \gamma = 1 \), we found that orbits with \( r_{\text{cp}} = 5r_{\text{ch}} \) contribute \( \sim 70\% \) of the density at \( r_{\text{ch}} \). For \( \gamma = 2 \), setting \( r_{\text{cp}} = 5r_{\text{ch}} \) is sufficient to give \( \sim 75\% \) of the density at \( r_{\text{ch}} \). (In the spherical models \( r_{\text{ch}} \) was defined as the radius containing the same mass as in the triaxial models.) Our corresponding choices for \( m_{\text{out}} \) are given in Table 1.

We followed standard procedures for constructing the Schwarz-

### Table 1

| Model Parameters | \( \gamma = 1 \) | \( \gamma = 2 \) |
|------------------|------------------|------------------|
| \( r_{\text{g}} \) | 0.64             | 0.20             |
| \( T_{\text{g}} (r_{\text{g}}) \) | 0.98             | 0.17             |
| \( r_{\text{ch}} \) | 1.1              | 1.3              |
| \( T_{\text{ch}} (r_{\text{ch}}) \) | 1.8              | 1.5              |
| \( m_{\text{out}} \) | 5.6              | 3.8              |
| \( T_{\text{g}} (m_{\text{out}}) \) | 5.1              | 4.6              |

#### Fig. 1.

Cumulative mass fractions \( F \) contributed by different kinds of orbits to different shells of the triaxial models. Boxlike orbits are blue, tube orbits (both \( z \)- and \( x \)-tubes) are orange, and chaotic orbits are purple. Higher energies are indicated via darker shades, defined according to the \( x \)-intercept of the equipotential that has the same energy as the orbit. Numbers below the color bar indicate radii where the equidensity shells and equipotentials intersect the \( x \)-axis. (a, c) Solutions with only regular orbits for \( \gamma = 1 \) and 2, respectively. (b, d) Solutions with both regular and chaotic orbits.

schild solutions (Schwarzchild 1993; Merritt & Fridman 1996). The mass model within \( m_{\text{out}} \) was divided into 64 equipotential shells, and each shell was divided into 48 cells per octant, for a total of 3072 cells. Shells were more closely spaced near the center (Fig. 1). Orbits were computed in two initial condition spaces: stationary start space, which yields boxlike orbits, and \( X-Z \) start space, which yields mostly tube orbits. Orbital energies were selected from a grid of 42 (52) values for \( \gamma = 1 \) (2), defined as the energies of equipotential surfaces, which were spaced similarly in radius to the equipotensity shells. The outermost energy shell intersected the \( x \)-axis at \( x = m_{\text{out}} \) for both models. Orbits were integrated for 100 dynamical times, as defined above, and their contributions to the masses in the cells were recorded. In order to distinguish regular from stochastic trajectories, the largest Liapunov exponent was computed for each orbit. For \( \gamma = 1 \), the total number of orbits integrated was 18,144, of which 9751 were found to be regular. For \( \gamma = 2 \), the numbers were 22,464 and 15,048, respectively. Orbital weights that reproduced the masses in the 3072 cells were found via quadratic programming (e.g., Dejonghe 1989).

As expected, orbital superpositions that exactly reproduced the cell masses in all of the cells, including the outermost ones, could not be found, since only a few orbits visit the outer shells. We were able to find “exact” solutions (solutions that precisely reproduced the densities in a subset of cells) by relaxing the constraints in a number of ways; for instance, by eliminating the angular constraints on the outermost shells (forcing only the integrated shell mass to be fitted) or by ignoring the outer cells entirely when imposing the constraints. For example, when only regular orbits were included, exact solutions could be found for \( \gamma = 1 \) for all cells within shell 51, corresponding to a radius of \( \sim 3.2 \). This radius is substantially greater than \( r_{\text{ch}} \) (Table 1).

When these “exact” solutions were advanced forward in time, as described below, they were found to exhibit significant evolution at large radii, due to the fact that the model’s density was not correctly reproduced in the outermost cells. We therefore constructed new solutions that were not “exact” but that were constrained to reproduce the densities in all cells within \( m_{\text{out}} \) with as high an accuracy as possible. Such solutions exhibited smaller fractional errors in the innermost cells \( (r \leq r_{\text{ch}}) \) than in the outer ones. Models constructed in this way were found to evolve much less than the exact solutions and provide the basis for the discussion below.

### Table 2

#### Orbital Content

| Region         | \( \gamma = 1 \) (regular): | \( \gamma = 1 \) (all): | \( \gamma = 2 \) (regular): | \( \gamma = 2 \) (all): |
|----------------|-----------------------------|-------------------------|-----------------------------|-------------------------|
| \( r < 0.5 \)  | 0.61 0.10 0.29 0.00         | 0.09 0.04 0.26 0.61     | 0.75 0.10 0.15 0.00        | 0.06 0.05 0.45 0.45     |
| \( r < 1.0 \)  | 0.56 0.08 0.37 0.00         | 0.10 0.03 0.25 0.62     | 0.73 0.13 0.14 0.00        | 0.05 0.05 0.44 0.47     |
| \( r < 1.5 \)  | 0.53 0.07 0.40 0.00         | 0.10 0.02 0.25 0.63     | 0.73 0.13 0.13 0.00        | 0.04 0.05 0.42 0.49     |

#### Fig. 1.

Cumulative mass fractions \( F \) contributed by different kinds of orbits to different shells of the triaxial models. Boxlike orbits are blue, tube orbits (both \( z \)- and \( x \)-tubes) are orange, and chaotic orbits are purple. Higher energies are indicated via darker shades, defined according to the \( x \)-intercept of the equipotential that has the same energy as the orbit. Numbers below the color bar indicate radii where the equidensity shells and equipotentials intersect the \( x \)-axis. (a, c) Solutions with only regular orbits for \( \gamma = 1 \) and 2, respectively. (b, d) Solutions with both regular and chaotic orbits.
γ: solutions in which only regular orbits were used and solutions in which all orbits (regular and chaotic) were provided to the modeling algorithm. The dominant family of orbits in the models containing only regular trajectories was found to be the z-tubes, orbits that circulate about the short axis of the figure. Plots of the individual orbits used in the solutions confirm that many of the z-tubes have the correct shape for reproducing the triaxial figure, being more elongated in x (the long axis) than in y (the intermediate axis). Most of the remaining contribution to the density of the regular nuclei was found to come from pyramid orbits and high-energy box orbits in the γ = 1 model, while for γ = 2, roughly equal contributions came from pyramid orbits and from the x-tubes. (See Fig. 1 of Paper I for illustrations of the various orbit families.) The most heavily occupied pyramid orbits had shapes similar to “saucer” orbits, z-tubes associated with a 2 : 1 resonance (Fig. 1c of Paper I); both models contained roughly equal numbers of saucers and pyramids. High-energy boxlike orbits are more important in the γ = 1 case than in the γ = 2 case because (1) high-energy tube orbits in the γ = 1 model are elongated opposite to the figure and (2) most of the high-energy regular box orbits in the γ = 2 model are bananas, which are not very useful for filling the outermost shells.

In the models constructed using both regular and chaotic trajectories, the number of stars on chaotic orbits is surprisingly high: ~60% for γ = 1 and ~45% for γ = 2 at all radii. Plots of the most highly populated chaotic orbits suggest that they have reached a nearly steady state in a time-averaged sense, filling the volume defined by the equipotential surface corresponding to their energy. Stationary chaotic building blocks like these have been used before when constructing triaxial models (e.g., Merritt & Fridman 1996), but no published triaxial models have contained nearly so large a fraction of stars on chaotic orbits.

3. N-BODY MODELS

We verified that our orbital superpositions correspond to true equilibria by realizing them as N-body models and integrating them forward in time in the gravitational potential computed from the N-bodies themselves and from the point mass representing the black hole. Initial conditions were prepared by reintegrating all orbits with nonzero occupation numbers. The position and velocity of each trajectory were recorded at fixed time intervals; the length of each integration was determined by the orbit’s occupation number. The sense of rotation of the tube orbits was chosen randomly such that the mean motion was everywhere zero. The number of particles was $2.42 \times 10^6$ for $\gamma = 1$ and $2.19 \times 10^5$ for $\gamma = 2$; a smaller $N$ was chosen for $\gamma = 2$ since the N-body integrations were slower for the more condensed model.

The initial conditions were then advanced in time using the N-body code GADGET (Springel, Yoshida, & White 2001), a parallel tree code with variable time steps. The interparticle softening length was chosen to be 0.005 (0.003) for $\gamma = 1$ (2). Figure 2 shows the evolution of the axis ratios of the models, computed using the iterative procedure described by Dubinski & Carlberg (1991). The axis ratios fluctuate with time, but the models retain their triaxial figures very well over many dynamical times. No significant evolution was seen in the radial distribution of matter.

4. OBSERVABLE PROPERTIES

Two observational signatures of triaxiality are isophote twists and kinematic misalignments. Isophote twists require a varia-

![Fig. 2.—Evolution of the N-body axis ratios: $a = 1$; $r$ defines the longest dimension of the ellipsoid within which the axis ratios were determined, using the algorithm described in the text. $T_d$ is the dynamical time at this radius. Black curves correspond to solutions where only regular orbits were used, and red curves to solutions where both regular and chaotic orbits were used.](image)

5. DISCUSSION

We have shown that long-lived triaxial configurations are possible for nuclei containing black holes, whether constructed purely from regular orbits or from a combination of regular and chaotic orbits. The latter solutions are remarkable given
Fig. 3.—Line-of-sight velocities of the rotating regular models described in the text, as seen along the y-axis. Contours of positive velocity are represented by solid curves and negative velocity by dashed curves.

the large fraction of the mass on chaotic orbits, on the order of 50% or more (Table 2). Such solutions violate the Jeans theorem in its standard form (e.g., Binney & Tremaine 1987) but are consistent with a generalized Jeans theorem (Merritt 1999) if we assume that the chaotic building blocks are “fully mixed,” that is, they approximate a uniform population of the accessible phase space. This appears to be the case based on visual inspection of the time-averaged chaotic trajectories. While a sudden onset of chaos can effectively destroy triaxiality in models containing a large population of regular box orbits (Merritt & Quinlan 1998; Sellwood 2001), our work shows that at least the central parts of galaxies containing black holes can remain triaxial even when dominated by chaotic orbits.

Galaxy nuclei are typically modeled as oblate axisymmetric systems when deriving black hole masses from stellar kinematical data (e.g., van der Marel et al. 1998; Joseph et al. 2001). Modeling a triaxial nucleus as axisymmetric will generally lead to biased estimates of $M_{bh}$; for instance, if an elongated bar is viewed end-on, the velocity field can mimic that produced by a black hole even in the absence of a central mass concentration (Gerhard 1988). The high-resolution data on which such mass estimates are based are usually obtained from single slits, making it difficult to rule out triaxial shapes using information like that in Figure 3.

Axisymmetry is typically also assumed when deriving rates of interaction of stars with nuclear black holes (e.g., Syer & Ulmer 1999; Magorrian & Tremaine 1999). The triaxial models that we constructed from regular orbits were dominated by $z$-tubes (Table 2), similar to the tube orbits that populate an oblate spheroid. The rate of capture or destruction of stars by the black hole in these models would be similar to the rates in an axisymmetric nucleus. However, the dominance of chaotic orbits in our second set of solutions implies a qualitatively different sort of behavior: the “loss cone” of stars captured by the black hole would be refilled on a timescale determined by the frequency of near-center passages of the chaotic orbits, rather than by two-body scattering.

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