In general, a high-quality (Q) microresonator can accommodate abundant whispering gallery modes (WGMs) with the mode number increasing with the dimensional sizes of the microresonator. Removing the unnecessary modes while reorganizing the remaining modes is of vital importance, which, however, has been proved challenging and usually results in a tradeoff with the Q of the microresonator. Here, an effective and controllable mode trimming and clustering mechanism is revealed underlying the generation of polygon and star modes in weakly perturbed tapered fiber-coupled lithium niobate whispering gallery microresonators. Experimentally, various polygon and star modes are observed in sequence within a single microresonator by tuning the excitation wavelength or varying the coupling position between a tapered fiber and the circular microresonator, which can be well reproduced with the theoretical model. The finding offers a ubiquitous solution for a broad range of applications requiring elaborate selection and organization of the high-Q WGMs.
behind the significant efforts spent on the deformed WGM microdisks. [18–21] Unfortunately, the microresonator deformation usually leads to the reduction of the mode Q factor and in turn the light-matter interaction strength. Recently, the generation of high-quality (Q) polygon modes in tapered fiber-coupled circular microdisk resonators with weak perturbation has been reported, [22, 23] whereas the essential physics behind the phenomena is yet to be explored and clarified.

Here, we discuss the generation mechanism of polygon and star modes in a weakly perturbed tapered fiber-coupled lithium niobate WGM microresonator. Meanwhile, we reveal an effective and controllable method to precisely conduct the mode trimming and clustering. Experimentally, the complete polygon and star modes sequences are observed within the single microresonator, which is well reproduced with our theoretical model.

2. Fabrication of the Lithium Niobate Microdisks

To facilitate observation of mode patterns, an Er³⁺-doped z-cut lithium niobate circular microdisk resonator allowing visualization of the modes is utilized taking advantage of two-photon fluorescence excitation of the Er³⁺ ions. The circular microdisk resonator was fabricated using photolithography-assisted chemical-mechanical etching, and further details on the fabrication procedures can be found in Ref. [24–26]. The inset of Figure 1a shows the scanning-electron-microscope (SEM) image of the fabricated microdisk resonator with a thickness of 700 nm and a diameter of 83.44 µm.

3. Mode Trimming and Clustering of the Polygon and Star Modes

3.1. Experimental Setup

The experimental setup for investigating the polygon and star modes formation is illustrated in Figure 1a, in which a narrowlinewidth tunable diode laser with a wavelength range from 960 to 980 nm (Model: TLB-6719-P-D, New Focus, Inc.) is used as the pump light source. The pump light is coupled to the microdisk via the tapered fiber with a diameter of 2 µm, [27] and the
polarization state can be controlled to be transverse-magnetically polarized by an in-line polarization controller (PC). The input pump power can be adjusted and monitored by a variable optical attenuator (VOA) and a power meter combined with a 10/90 beam splitter. The relative position between the tapered fiber and microdisk resonator can be monitored and adjusted by an optical microscope system and a high-resolution positioning stage of 5 nm resolution. A top-view optical microscope with a 20x objective lens with a numerical aperture of 0.42, a visible charge-coupled device (CCD), and a short-pass filter (Model: FES800, Thorlabs, Inc.) are used to capture the mode patterns distributed on the plane of the microdisk. Remarkably, the captured up-conversion fluorescence patterns directly reflect the spatial distribution characteristics of the pump mode.[23]

Experimentally, to excite the polygon and star modes with high stability, the tapered fiber must be placed carefully on the top surface being in contact with the circular microdisk resonator. When the distance between the tapered fiber and the microdisk center is tuned to 38.5 μm, the triangle mode (ground state), the triangle mode with the first excited state (dual-localization triangle mode)[28] and the star mode can be excited at varied wavelengths of 972.47, 972.56, and 972.83 nm, as demonstrated in Figure 1b–d, respectively. When further increases the relative distance between the tapered fiber and the microdisk center to 39.8 μm, the square mode, pentagon mode, hexagonal mode, heptagon mode, and octagon mode can be successively excited at increasing wavelengths of 972.98, 974.48, 975.65, 978.82, and 979.86 nm, as shown in Figure 1e–i, respectively. Thus, various polygon and star modes are sequentially observed in the single microdisk by synergetically manipulating the pump wavelength and the tapered fiber position.

3.2. Polygon and Star ModeTrimming and Clustering Mechanism

To shed light on the mode trimming and clustering mechanism, we first reveal the mode distribution characteristics of the whispering gallery microresonator. For the thin circular microdisk with a diameter of R, the general form of the eigenmodes in cylindrical coordinates (r,θ) can be expressed as follows:[29]

$$\psi(r, \theta) = \begin{cases} \psi_m(nkr) e^{im\theta} & r \leq R \\ \psi_m^{(1)}(kr) e^{im\theta} & r \geq R \end{cases}$$

(1)

where m (m = 0, 1, . . .) is an integer that specifies the azimuthal quantum number related to orbital momentum, J_m(x) (H_m^{(1)}(x)) refers to the mth order Bessel function (the mth Hankel function of the first kind), n and k denote the effective index of the 2D thin film and vacuum wave vector, respectively. By imposing the boundary condition of the electromagnetic field, the quantization condition can be expressed in the following form,

$$S_m(nkr) = \frac{n}{v} J_m(nkr) - \frac{H_m^{(1)}(kR)}{H_m^{(1)}(k)} = 0$$

(2)

here J_m(x) and H_m^{(1)}(x) represent the first order derivatives of the mth order Bessel function and the mth order Hankel function of the first kind, respectively, and v is 1 (nv) for transverse magnetic (transverse electric) polarization. The quantization condition can determine the quantum numbers of the microwave modes, and the eigenvalues obtained in the microresonator can be defined as,[30]

$$nk_{p,m} R = x_{p,m} + \delta x_{p,m}$$

(3)

with $\delta x_{p,m} \ll x_{p,m}$, the correction term $\delta x_{p,m}$ introduced by the boundary conditions depends merely on the geometry characteristics of mode chord angle,[28] p (p = 1, 2, . . .) is an integer that describes the radial quantum number, and $x_{p,m}$ is the pth root of nth order Bessel function. Physically speaking, the wave vectors $k_{p,m}$ of quasistationary states depend on radial quantum number p and azimuthal quantum number m. The eigenvalue interval of arbitrary WGMs can be described as,

$$x_{p+1,0} - x_{p,0} = -\frac{P}{\sin \theta} \left( \frac{\Theta - \frac{Q\pi}{P}}{P} \right) + O\left(\chi^{-2}\right)$$

(4)

where P and Q denote the changes of the azimuthal and radial quantum numbers, respectively, $\Theta$ describes the mode chord angle with a relation of $\cos \Theta = m/x$, and $\Theta$ is the angle between $\Theta_{p,m}$ and $\Theta_{p+1,0}$. The cosine of chord angle is dominated by the azimuthal quantum number m when the eigenvalue is restricted to a specific range. According to Equation (4), those modes with eigenvalues of $x_{p,m}$ and $x_{p+1,0}$ are degenerate when the chord angle $\Theta$ of the mode is close to $\frac{Q\pi}{P}$. A more detailed discussion of the eigenvalue characteristics can be found in Supporting Information S1. For the quasi-degenerate mode sequence $x_{p,m}$, the quantum numbers of the microcavity modes, and the eigenvalues of the microcavity modes can be defined as,[30]

$$x_M - x_0 = -f_{Q,M}(x_0) M (M + \Delta_m) + O\left(\chi^{-2}\right)$$

(5)

with $x_M = x_{p,Q,M}^{(P,Q,M)}$, and M is the sequence number of quasi-degenerate modes, $f_{Q,M}(x_0)$ is a parameter that determines the quasi-degeneracy of the mode sequence. The $\Delta_m$ describes the mismatch angle between $\Theta_m$ and $\Theta_{0.m}$ for the classical trajectory and varies periodically with angular quantum number, resulting in accidental degeneracy.

The quasi-degenerate modes with distinct geometric features widely exist in the microresonator, as demonstrated by the black dots in Figure 2. The eigenvalues are distributed at different parabolic curves fitted by Equation (5) centered with $\cos \Theta$ equals to 0.5, 0.623, 0.707, 0.809, 0.866, 0.901, and 0.924, where $\frac{\Theta}{P}$ equals to 3, 7/2, 4, 5, 6, 7 and 8, respectively. The interval between the center points of the curve is $\Theta$/sin $\Theta$ including an excellent consistence with the classical polygon modes. In contrast, the interval between the parabolic curves is reduced by half at the positions of $\cos \Theta_{2\pi/m}$ = 0.623, which is consistent with the characteristics of the seven-star mode. At large $\cos \Theta$, we can see two sets of intertwined parabolas consisting of heptagon and octagon modes, indicating one mode may belong to two different degenerate sequences. As indicated by the different mode distribution features, the phase space of modes can be divided into two categories, namely, the regions of non-uniform distribution labeled by dark color near the center of high degeneracy and that of uniform distribution marked by light color at the boundary of the non-uniform regions. In the former regions of non-uniform
distribution, the neighboring WGMs with the fixed P-difference of the angular quantum number allow the mode recombination, forming the polygon and star modes with P-fold symmetry. In the latter regions of the two featuring uniform distribution, the polygon and star modes cannot be easily formed due to the low probability of mode degeneracy.

The mode-forming process under perturbation can be clearly analyzed according to the mode clustering mechanism. Take the tapered fiber coupled microlens as an example, the influence caused by the tapered fiber can be regarded as a refractive index modulation \( \Delta \) generated at the position \( r = R(1 - \Delta f(\theta)) \), where \( \Delta f(\theta) \) reflects the geometric characteristics of the modulation, and \( \Delta \) represents the effective refractive index difference of the lithium niobate with and without tapered fiber coupling. The modulation \( \Delta \) depends on the material refractive index and diameter of the tapered fiber. Theoretically, the modulation \( \Delta \) increases with the increase in the refractive index and diameter of the tapered fiber. The typical \( \Delta \) induced by the tapered fiber with a diameter of 2 \( \mu \)m is 0.02 when the tapered fiber is placed carefully on the top surface being in contact with the circular microdisk resonator. Imposing the new boundary caused by the tapered fiber of the electromagnetic field, the quantization condition can be expressed in the following form,

\[
\sum_{m} a_{m} S_{m}(x) \cos(m\theta) = \lambda \Delta f(\theta) \sum_{m} a_{m} \frac{\partial}{\partial x} S^{(m)}_{m}(x) \cos(m\theta)
\]

where \( S^{(m)}_{m}(x) \) and \( S_{m}(x) \) denote the quantization condition with and without perturbation induced by the tapered fiber, respectively. The rates of changes in quantization conditions \( \frac{\partial}{\partial x} S^{(m)}_{m}(x) \), which are proportional to the difference in refractive index inside and outside the boundary, reflect mode constraint capacity (see, Equations S1–S15, Supporting Information). Obviously, the rotational symmetry of the microcavity is broken due to the coupling of the tapered fiber, and a more detailed discussion can be found in Supporting Information S3.

The zero-order equation of the quantization condition in degenerate subspace can be obtained by multiplying \( \{\cos m\theta|m' \in P_{0}\} \) and by summing both sides over all \( m' \), we get,

\[
S_{m0}(x_{0} + \lambda x_{0}) a_{m0} = -\frac{2\lambda \Delta }{n^{2} - 1} \frac{\partial}{\partial x} S_{m0}(x) \sum_{m' \in P_{0}} A_{m',m} a_{m'}
\]

here \( x_{0} \) is the mode eigenvalue of microresonator without perturbation, \( P_{0} = \sum_{m} a_{m}^{(0)} \) is the degenerate subspace around the eigenvalue, \( \frac{\partial}{\partial x} S^{(0)}_{m}(x) \) is the ratio of mode constraint capacity with and without the tapered fiber, and the matrix \( A_{m',m} \) is acquired. Thus, degenerate modes with different families can be discussed independently, and combined with Equation (5), each family described by the quasi-degenerate sequence \( \{iP_{0}Q,M\} \) will be recombined,

\[
\sum_{M} \left\{(-i\lambda x_{0} + f_{Q}(x_{0}) M (M + \Delta_{m})) a_{M,M'} - \frac{2\lambda e^{-\gamma} \cos(m' - m) \theta}{n^{2} - 1} A_{m',m} \right\} a_{M'} = 0
\]

where \( a_{M'} = (p_{0} - M'Q, m_{0} + M'P)_{\text{sub}} \) represents the proportion of WGMs in new recombination states, \( \Delta_{m} \) is the eigenvalue correction, \( A_{M,M'} = A_{M + M',M''} \) depicts the coupling matrix element produced by the tapered fiber, and \( \gamma \) describes the coupling efficiency between recombination states and tapered fiber, is determined by the overlap integral between the microcavity mode and the tapered fiber mode. In particular, mode recombination affects the geometric characteristics of the intrinsic modes, resulting in different coupling efficiencies for various recombination states. We can observe the polygon and star modes instead of high-order WGM modes under appropriate coupling conditions. In addition, the perturbation strength can be affected by the position of the tapered fiber. When the angle \( \phi_{p} \) between the tapered fiber and the center of the circular microdisk resonator varies from 25° to 50°, the perturbation strength is a linear function of \( \phi_{p} \) (see Figure S2, Supporting Information).

Figure 2 demonstrates the mode field intensity distribution plots calculated based on Equation (8). The eigenvalue values \( x_{0} \) ranging from 571 to 578 are calculated, according to the experimental conditions with the thin film thickness of 0.7 \( \mu \)m, microresonator diameter of 83.44 \( \mu \)m, and wavelength range from 970 to 980 nm. The triangle mode, triangle mode with first-order state, and seven-star state at wavelengths of 972.22, 972.41, and 973.09 nm are reproduced in Figure 3a–c, respectively, when the distance between the tapered fiber and the microdisk center is tuned to 38.5 \( \mu \)m. When further increases the distance to 39.8 \( \mu \)m, the square mode, pentagon mode, hexagonal mode, heptagon mode and octagon mode are produced at wavelengths of 972.99, 974.55, 975.50, 978.98, and 979.95 nm, as depicted in Figure 3d–h, respectively, indicating an excellent consistency with the experimental results.

Figure 3. Cosine of the chord angle of the eigenmodes at different wavelengths. The black dots and colored dotted line represent the quasi-steady-state solution and the approximate result given by Equation (5), respectively.
the semiclassical approximation. The Equation (10) can be seen considered, we can obtain mode sequences with similar geometric characteristics are con-
formed through perturbation between adjacent quasi-degenerate modes in the short-wavelength limit. When the quasi-degenerate mode sequences, with similar geometric characteristics, are considered, we can obtain \( \Psi_{P,Q,M}^0 (R) \) by applying the semiclassical approximation. The Equation (10) can be seen as a discrete Fourier transform with respect to \( M \), we get,

\[
\Psi_{P,Q,M}^N (R, \theta) \propto e^{i \omega_0 \theta} H_N \left( \frac{\theta + \frac{Q+2K}{P} r}{\sqrt{2} \sigma_M} \right)
\]

\[
\exp \left( - \frac{\left( \theta + \frac{Q+2K}{P} r \right)^2}{4 \sigma_M^2} \right)
\]

The energy level interval increases as the increase of \( M \). For higher order perturbation strengths, the energy level interval increases and tends to be uniform. Figure 4c–f gives typical phase space plots calculated with perturbation strengths of 0, 0.02, 0.1, and 0.3, respectively. The mode sequence \( \{ x_{p,M}^{\Delta,289} \} \) under the perturbation of the tapered fiber, here we take the triangle mode family \( \{ x_{p,M}^{\Delta,289} \} \) as an example. Figure 4a gives the expectation values of the cosine of the chord angle \( \Theta \) at different perturbation strengths. In the case of no perturbation, the expected cosines of chord angle are equally spaced around the classical orbit, and the cosines of chord angle are expected to be further away from the classical orbit as the number \( M \) increases in absolute value. With the increase of perturbation strength, the expected cosines of chord angle are close to the cosines of the classical periodic orbits. The eigenvalue difference between mode with number \( M \) and the central mode \( x_{p,M}^{\Delta,1,10} \) is illustrated in Figure 4b. In the case of no perturbation, the eigenvalues are inhomogeneously distributed and the energy level interval increases as the increase of \( M \). When further increases the perturbation strengths, the energy level interval of the modes increases gradually and tends to be uniform. Figure 4c–f gives typical phase space plots calculated with perturbation strengths of 0, 0.02, 0.1, and 0.3, respectively. The mode sequence \( \{ x_{p,M}^{\Delta,289} \} \) originally distributed fully on the parabolic curve as shown in Figure 4c, gradually evolves into the linear distribution superimposed on the parabolic curve with the increasing perturbation strength as shown in Figure 4d–f. Interestingly, the information lost in phase space of the mode is transferred to the spatial distribution. As shown in Figure 4h–j, the orbits of the ground state (0th) and first excited state (1st) of the triangle become shaper with the increasing perturbation strength, forming the polygon mode observed in the experiment. According to the experimental conditions for controlling mode formation, the refractive index modulation \( \Delta \) and the angle \( \phi_m \) are 0.02 and 45 degrees, respectively. The calculated perturbation strength is 0.25, corresponding to 0.106 for \( \sigma_M \) when the eigenvalue is 577. The included angle between the two orbits of the first excited state of the triangle can be determined by the extreme points of the angle from Equation (11). For the first excited state mode, the included angle is \( 2 \sqrt{2} \sigma_M \), corresponding to 17 degrees angle between the two orbits of the first excited state of the triangle. The theoretical result of the included angle is consistent with the experimental value of 17 ± 2 degrees, reflecting the accuracy of the theoretical approximations.

For quasi-degenerate mode sequences, modes with high degeneracy at the center of the sequence evolve into polygon and star modes, while modes at the edge of the sequence still maintain the characteristics of WGM modes due to the low degeneracy. The eigenvalue intervals of the polygon and star modes and

Figure 3. Mode field intensity distribution plots calculated by experimental conditions. The mode field intensity distributions of a) triangle mode, b) triangle mode with the first excited state, c) seven-star mode, d) square mode, e) pentagon mode, f) hexagonal mode, g) heptagon mode, and h) octagon mode.

3.3. Evolution Characteristics of the Degenerate Polygon and Star Modes

To clearly describe the mode-forming process, an approximate solution of Equation (8) is performed. Assume that the perturbation occurs only in adjacent quasi-degenerate modes, we get,

\[
\sum_M \left\{ -2x_l + f_{P,Q}(x_l) M (M + \Delta_m) \right\} \delta_{M,M - \Delta_m} - g_{P,Q}(\phi_m), \delta_{M,M - 1} \right\} a_M = 0
\]

where \( g_{P,Q}(\phi_m) \) represents the perturbation strength, it is typically required that \( g_{P,Q}(\phi_m) < 1 \) in perturbation theory. Similar to the harmonic oscillator model, the mode recombination degree depends on the ratio between the degeneracy \( f_{P,Q}(x_l) \) and the perturbation strength \( g_{P,Q}(\phi_m) \). An approximate solution of the recombination modes can be obtained,

\[
\Psi_{P,Q,M}^N (R, \theta) \propto e^{i \omega_0 \theta} \sum_M H_N \left( \sqrt{\frac{2}{\sigma_M^2}} \right) \exp \left( - \frac{\left( \frac{\theta + \frac{Q+2K}{P} r}{\sqrt{2} \sigma_M^2} \right)^2}{4 \sigma_M^2} \right)
\]

The energy level interval increases as the increase of \( M \). For higher order perturbation strengths, the energy level interval increases and tends to be uniform. Figure 4c–f gives typical phase space plots calculated with perturbation strengths of 0, 0.02, 0.1, and 0.3, respectively. The mode sequence \( \{ x_{p,M}^{\Delta,289} \} \) originally distributed fully on the parabolic curve as shown in Figure 4c, gradually evolves into the linear distribution superimposed on the parabolic curve with the increasing perturbation strength as shown in Figure 4d–f. Interestingly, the information lost in phase space of the mode is transferred to the spatial distribution. As shown in Figure 4h–j, the orbits of the ground state (0th) and first excited state (1st) of the triangle become shaper with the increasing perturbation strength, forming the polygon mode observed in the experiment. According to the experimental conditions for controlling mode formation, the refractive index modulation \( \Delta \) and the angle \( \phi_m \) are 0.02 and 45 degrees, respectively. The calculated perturbation strength is 0.25, corresponding to 0.106 for \( \sigma_M \) when the eigenvalue is 577. The included angle between the two orbits of the first excited state of the triangle can be determined by the extreme points of the angle from Equation (11). For the first excited state mode, the included angle is \( 2 \sqrt{2} \sigma_M \), corresponding to 17 degrees angle between the two orbits of the first excited state of the triangle. The theoretical result of the included angle is consistent with the experimental value of 17 ± 2 degrees, reflecting the accuracy of the theoretical approximations.

For quasi-degenerate mode sequences, modes with high degeneracy at the center of the sequence evolve into polygon and star modes, while modes at the edge of the sequence still maintain the characteristics of WGM modes due to the low degeneracy. The eigenvalue intervals of the polygon and star modes and
the spatial distribution difference between the polygon or star mode and WGM mode increase with the increase of the perturbation strength. Thus, the polygon and star modes can be derived without interference from other high-order WGM modes under appropriate coupling conditions.

4. Conclusion

To conclude, we have discussed an effective and controllable trimming and clustering method of high-Q WGMs. In addition, we have also described the generation mechanism of polygon and star modes in microdisk resonators. Experimentally, complete polygon and star modes sequences are observed within the single microresonator, showing excellent consistency with the theoretical results. Besides the use of the MNF as the weak perturbation, it is also possible to use periodically arranged nanoparticles\(^{[31,32]}\) for the construction of the polygon and star mode. The weakly perturbed microcavity system demonstrated here may open new opportunities for the majority of research and application ranging from the WGM-based nonlinear photonics\(^{[22,23]}\) to the active selection of lasing modes\(^{[33]}\).

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest
The authors declare no conflict of interest.

Data Availability Statement
The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords
lithium niobate, mode trimming and clustering, weak perturbation, whispering gallery mode microcavity

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