An electron of helium atom under a high-intensity laser field

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Abstract
We scrutinize the behavior of eigenvalues of an electron in a helium (He) atom as it interacts with electric field directed along the \(z\)-axis and is exposed to linearly polarized intense laser field radiation. To achieve this, we freeze one electron of the He atom at its ionic ground state and the motion of the second electron in the ion core is treated via a more general case of screened Coulomb potential model. Using the Kramers–Henneberger (KH) unitary transformation, which is the semiclassical counterpart of the Block–Nordsieck transformation in the quantized field formalism, the squared vector potential that appears in the equation of motion is eliminated and the resultant equation is expressed in the KH frame. Within this frame, the resulting potential and the corresponding wave function are expanded in Fourier series and using Ehlotzky’s approximation, we obtain a laser-dressed potential to simulate intense laser field. By fitting the more general case of screened Coulomb potential model into the laser-dressed potential, and then expanding it in Taylor series up to \(O(r^4, \alpha_0^0)\), we obtain the solution (eigenvalues and wave function) of an electron in a He atom under the influence of external electric field and high-intensity laser field, within the framework of perturbation theory formalism. We found that the variation in frequency of laser radiation has no effect on the eigenvalues of a He electron for a particular electric field intensity directed along \(z\)-axis. Also, for a very strong external electric field and an infinitesimal screening parameter, the system is strongly bound. This work has potential application in the areas of atomic and molecular processes in external fields including interactions with strong fields and short pulses.

Keywords: perturbation technique, hydrogen atom, laser field radiation, helium atom

(Some figures may appear in colour only in the online journal)

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1. Introduction

Lasers have emerged as one of the world’s indispensable technologies, employed in telecommunications, law enforcement, military equipment, etc. Laser pulses control various atomic or molecular processes. For instance, atoms undergo about three ionizations when probed by a laser of controlled intensity. Recent advances in laser technology have aroused the interest of many researchers to investigate new sources of laser in order to probe and control molecular structure, function and dynamics on the natural timescale of atomic motion—the femtosecond—and electron motion—attosecond—timescale [1]. To obtain an intense laser field, it is required to concentrate large amounts of energy within a short period of time, and then focus the laser light onto a small area. In an intense laser system, a train of pulses of short duration are created by the oscillator. The energy of the pulses is then increased by the amplifier, and finally the pulses are focused.

Studying atoms in intense laser fields has been a subject of active research for more than three decades due to its salient application in the invention of high-power short-pulse laser technologies. These atoms exhibit new properties that have been discovered via the study of multiphoton processes [2]. When a high-power laser is directed into a gas of atoms, the magnitude of electromagnetic field is found to be consistent with the Coulomb field, which binds a 1s electron in a hydrogen (H) atom [3]. Within this context, many outstanding results focusing on H atoms have been reported so far (see [4–8] and references therein). It was shown in [9] that in the presence of an oscillating magnetic field, the ionization rate of a hydrogen atom interacting with intense laser dwindles, and the electron density becomes ionized at a slower rate when the magnetic field strength is kept constant and the intensity of the laser increased.

On the other hand, the study of He atoms under intense laser field has also received great attention from both theorists and experimentalists [10–13]. Chattaraj [14] studied dynamic response of a He atom in an intense laser field within a framework of quantum fluid density. Chen et al [15] numerically simulated the double-to-single ionization ratio for the He atom under intense laser field. In fact electron–helium scattering in the presence of a laser field was recently reported in [16]. So, many outstanding contributions have been made on this subject; however, it is worth mentioning that—to our best knowledge—there has been no account equivalent to the current study either experimentally or theoretically. In the present work, our main focus is to scrutinize the behavior of eigenvalues of an electron in a He atom under the influence of external electric field and exposed to linearly polarized intense laser field radiation. This study will be of great interest in the areas of atomic and molecular processes in external fields including interactions with strong fields and short pulses.

2. Formulation of the problem

In this section, we derive the equation of motion for one spherically confined electron in a He atom under the influence of external electric field directed along the z-axis and exposed to linearly polarized intense laser field radiation. Achieving our goal in this section requires that we start with the following time-dependent Schrödinger wave equation

\[
\begin{align*}
\frac{i\hbar}{\partial t} \Psi(r, t) = & \left[ -\frac{\hbar^2}{2\mu} \nabla^2 - i\hbar \frac{e}{2\mu} \left( \mathbf{A}(r, t) \cdot \nabla + \nabla \cdot \mathbf{A}(r, t) \right) + \frac{e^2}{2\mu} \mathbf{A}(r, t)^2 - \phi + V(r) - Fr \right] \Psi(r, t),
\end{align*}
\]

with the scalar potential \(\phi(r, t)\) and the vector potential \(\mathbf{A}(r, t)\) which is invariant under the gauge transformation. \(\mu\) is the effective mass of the electron. Furthermore, \(Fr\) describes a radial electric field. We consider Coulomb gauge, such that \(\nabla \cdot \mathbf{A}(r, t) = 0\) with \(\phi = 0\) in empty space and then simplify the interaction term in equation (1) by performing gauge transformations within the framework of dipole approximation. In this approximation, for an atom whose nucleus is located at the position \(r_0\), the vector potential is spatially homogeneous \(\mathbf{A}(r, t) \approx \mathbf{A}(t)\). Moreover, term \(\mathbf{A}(r, t)^2\) appearing in equation (1) is considered for extremely high field strength. It is usually small and can be eliminated by extracting a time-dependent phase factor from the wave function via [17]

\[
\Psi(r, t) = \exp \left[ \frac{i e^2}{2\mu \hbar} \int_{-\infty}^{t} \mathbf{A}(t')^2 dt' \right] \Psi(r, t),
\]

(2)

to obtain velocity gauge\(^7\)

\[
\frac{i\hbar}{\partial t} \Psi(r, t) = \left[ -\frac{\hbar^2}{2\mu} \nabla^2 - i\hbar \frac{e}{\mu} \mathbf{A}(t) \cdot \nabla + V(r) - Fr \right] \Psi(r, t).
\]

(3)

A prerequisite to studying an electron in a He atom under intense high-frequency laser field is transforming equation (3) to the Kramers–Henneberger (K–H) accelerated frame. Now, with the introduction of the following unitary K–H transformation

\[
\Psi^H(r, t) = U^\dagger \Psi^v(r, t) \quad \text{with} \quad U = \exp \left[ -i \frac{\alpha(t)}{\hbar} \mathbf{p} \right],
\]

\[
\alpha(t) = \frac{e}{\mu} \int_{-\infty}^{t} \mathbf{A}(t') dt',
\]

(4)

which is the semiclassical counterpart of the Block–Nordsieck transformation in the quantized field formalism, the coupling term \(\alpha(t) \cdot \mathbf{p}\) in the velocity gauge (i.e. equation (3)) is eliminated. More explicitly, this can be done via

\[
\begin{align*}
\frac{i\hbar}{\partial t} U^\dagger \mathbf{p} \Psi^H(r, t) = & \mathbf{p} \left[ -\frac{\hbar^2}{2\mu} \nabla^2 - i\hbar \frac{e}{\mu} \mathbf{A}(t) \cdot \nabla + V(r) - Fr \right] U \Psi^H(r, t).
\end{align*}
\]

(5)

Evaluation of terms in equation (5) is straightforward and easy. However, let us try to be more explicit in evaluating the term \(U^\dagger V(r) U\). This can be done via the Campbell–Baker–Hausdorff identity: \(e^{\hat{A} \hat{B}} e^{-\hat{A}} = \hat{B} + [\hat{A}, \hat{B}] + [\hat{A}, [\hat{A}, \hat{B}]]/2! + \ldots\). Thus, we have

\[
\mathbf{p} = -i\hbar \nabla.
\]
\[ U^\dagger V(r)U = \exp \left[ \frac{i}{\hbar} \alpha(t) \cdot p \right] V(r) \exp \left[ \frac{i}{\hbar} \alpha(t) \cdot p \right] \]

\[ = V(r) + [\alpha(t), \nabla] V(r) + \frac{1}{2!} [\alpha(t), \nabla]^2 V(r) + \ldots \]

\[ = V[r + \alpha(t)], \quad (6) \]

where \( \alpha(t) \) denotes the displacement of a free electron in the incident laser field. Hence, equation (5) becomes

\[ i\hbar \frac{\partial}{\partial t} \Psi^\dagger(r, t) = \left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V[r + \alpha(t)] - Fr \right] \Psi^\dagger(r, t). \quad (7) \]

Equation (7) represents a space-translated version of the time-dependent Schrödinger wave equation with incorporation of \( \alpha(t) \) into the potential in order to simulate the interaction of atomic system with the laser field. It is worth mentioning that K–H transformation leaves the term \( Fr \) invariant. Now, for a steady field condition, the vector potential takes the form \( A(t) = (E_\parallel t) \cos(\omega t) \) with \( \alpha(t) = \alpha_0 \sin(\omega t) \), where \( \alpha_0 = eE_\parallel /\mu_0 \omega^2 \) is the amplitude of oscillation of a free electron in the field (called as the laser-dressing parameter), \( E_\parallel \) denotes the amplitude of electromagnetic field strength and \( \omega \) is the angular frequency. Now, considering a pulse where the electric field amplitude is steady, the wave function in the frame of Kramers–Henneberger takes the following Floquet form [17]:

\[ \Psi^\dagger(r, t) = e^{-i\frac{E_{KH}}{\hbar}} \sum_n \psi_n^{KH}(r)e^{-in\omega t}, \quad (8) \]

where Floquet quasi-energy has been denoted by \( E_{KH} \). The potential in the K–H frame can be expanded in Fourier series as [18]

\[ V[r + \alpha(t)] = \sum_{m = -\infty}^{\infty} V_m(\alpha_0; r)e^{-im\omega t} \quad \text{with} \]

\[ V_m(\alpha_0; r) = \frac{i}{\pi} \int_{-1}^{1} V(r + \alpha_0 \theta) \frac{T_m(\theta)}{\sqrt{1 - \theta^2}} d\theta, \quad (9) \]

where we have taken the period as \( 2\pi/\omega \) and introduced a new transformation of the form \( \theta = \sin(\omega t) \). Furthermore, \( T_m(\theta) \) are Chebyshev polynomials. Substituting equations (8) and (9) into equation (7) yields a set of coupled differential equations:

\[ \left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_m(\alpha_0; r) - Fr - (E_{KH} + n\hbar\omega) \right] \psi_n^{KH}(r) \]

\[ = -\sum_{m = -\infty}^{\infty} V_{n \rightarrow m}\psi_m^{KH}(r). \quad (10) \]

Considering \( n = 0 \) (which gives the lowest order approximation) and high frequency limit (which made \( V_m \) with \( m \neq 0 \) vanish), equation (10) becomes

\[ \left[ -\frac{\hbar^2}{2\mu} \nabla^2 + V_0(\alpha_0; r) - Fr - E_{KH} \right] \psi_0^{KH}(r) = 0, \quad (11) \]

and the coefficient of the Fourier series for the potential becomes

\[ V_0(\alpha_0; r) = \frac{1}{\pi} \int_{-1}^{1} V[r + \alpha_0 \theta] \frac{d\theta}{\sqrt{1 - \theta^2}} \]

\[ = \frac{1}{\pi} \int_{0}^{1} \left[ V[r + \alpha_0 \theta] + V[r - \alpha_0 \theta] \right] \frac{d\theta}{\sqrt{1 - \theta^2}}. \quad (12) \]

Using Ehlotzky’s approximation [19], one has

\[ V_0(r + \alpha_0 \theta) + V[r - \alpha_0 \theta]) \approx V[r + \alpha_0] + V[r - \alpha_0]. \]

Hence, by evaluating the integral, we obtain

\[ V_0(\alpha_0; r) = \frac{1}{2} \left[ V[r + \alpha_0] + V[r - \alpha_0] \right]. \quad (13) \]

Equation (13) is the approximate expression to model the laser field. Now, let us incorporate the potential to simulate the behavior of one electron in a He atom into the model. In order to model this potential, we freeze one electron of the helium at its ground state and then consider the motion of the second electron in the ion core. We would like to advise the reader to check literature [11, 20] for more details about this. Moreover, an appropriate model potential for this has been presented and studied in [20]:

\[ V(r) = -(a/r)[1 + (1 + br)a]e^{-2br}. \]

Hence, equation (11) becomes

\[ \left[ -\frac{\hbar^2}{2\mu} \nabla^2 - \frac{a}{r^2} [1 + (1 + br)a]e^{-2br} \right] \psi_0^{KH}(r) = 0, \quad (14) \]

where \( r_a \) denotes the strength coupling constant and \( b \) represents the screening parameter. The above equation (14) is the equation of motion for one spherically confined electron in a helium atom exposed to linearly polarized intense laser field radiation. Now, in order to achieve the objective of this study, in the next section, we solve equation (14) within the framework of perturbation formalism.

### 3. Eigenspectra calculation

Equation (14) is not solvable analytically. One can either use numerical procedure or perturbation formalism. Using the perturbation approach, we decompose the equation into two parts, where the first part is exactly solvable and the other part is perturbation. Consequently, the eigenvalue solutions are represented in power series with the leading term corresponding to the solution of the exactly solvable part and the other part is a correction to the energy term, which corresponds to the perturbation term. This approach has been used in numerous research reports (see [21, 22] and references therein). Now, we re-write equation (14) as

\[ \frac{\hbar^2}{2\mu} \left( \nabla^2 \chi(r) + \nabla^2 \chi_0(r) \right) + \frac{2}{\lambda} \nabla \chi(r) \nabla \chi_0(r) \]

\[ = \chi(r) - E_{KH}, \quad (15) \]

where \( \psi_0^{KH}(r) = \chi(r) \chi(r) \) with \( \chi(r) \) as the wave function of the exactly solvable part and \( \chi_0(r) \) as the moderating wave function. The effective potential \( V_{ei}(r) \) represents the Taylor series expansion of the potential terms in equation (14). This can be written as:
The first term is the main part, which corresponds to a shape invariant potential for which the superpotential is known analytically and the remaining part is taken as a perturbation, \( \Delta V_{\text{eff}}(r) \). At this juncture, one may be intrigued about why the superpotential is known analytically and the remaining part is taken as a perturbation, \( \Delta V_{\text{eff}}(r) \). It is therefore required to expand the related functions and unperturbed wave functions as 

\[
V_{\text{eff}}(r) = -\frac{4a}{r} + \left( 2ab - \frac{4}{405} ab^5 a_0^8 - \frac{8}{63} ab^7 a_0^6 - \frac{4}{5} ab^9 a_0^4 - \frac{4}{3} ab^9 a_0^2 \right) + \left( \frac{32}{1575} ab^9 a_0^8 + \frac{4}{15} ab^9 a_0^6 + \frac{16}{9} ab^9 a_0^4 + 4ab^9 a_0^2 - F \right) r + \left( -\frac{8}{385} ab^9 a_0^8 - \frac{112}{405} ab^9 a_0^6 - \frac{40}{21} ab^9 a_0^4 - \frac{24}{5} ab^9 a_0^2 - \frac{4}{3} ab^3 \right) r^2 + \left( \frac{8}{567} ab^9 a_0^8 + \frac{128}{675} ab^9 a_0^6 + \frac{4}{3} ab^9 a_0^4 + \frac{32}{9} ab^9 a_0^2 + \frac{4}{3} ab^3 \right) r^3 + \left( -\frac{88}{12285} ab^9 a_0^8 - \frac{16}{165} ab^9 a_0^6 - \frac{56}{81} ab^9 a_0^4 - \frac{40}{21} ab^9 a_0^2 - \frac{4ab^9 a_0^5}{5} \right) r^4 + O(r^5, a_0^9). \tag{16}
\]

The superpotential is known analytically and the remaining part is taken as a perturbation. We substitute these expressions into equation (17b) and then equate terms with the same power of \( \eta \) on both sides to give the following expressions

\[
2W_0(r)W_0^{(1)}(r) - \frac{\hbar}{\sqrt{2}\mu} \frac{dW_0^{(1)}(r)}{dr} = V_{\text{eff}}^{(1)}(r) - E_{\text{KH}}^{(1)}, \tag{19a}
\]

\[
[W_0^{(1)}(r)]^2 + 2W_0(r)W_0^{(2)}(r) - \frac{\hbar}{\sqrt{2}\mu} \frac{dW_0^{(2)}(r)}{dr} = V_{\text{eff}}^{(2)}(r) - E_{\text{KH}}^{(2)} \tag{19b}
\]

\[
2[W_0(r)W_0^{(3)}(r) + W_0^{(1)}(r)W_0^{(2)}(r)] - \frac{\hbar}{\sqrt{2}\mu} \frac{dW_0^{(3)}(r)}{dr} = V_{\text{eff}}^{(3)}(r) - E_{\text{KH}}^{(3)} \tag{19c}
\]

\[
2[W_0(r)W_0^{(4)}(r) + W_0^{(1)}(r)W_0^{(3)}(r)] + W_0^{(2)}(r)W_0^{(2)}(r) - \frac{\hbar}{\sqrt{2}\mu} \frac{dW_0^{(4)}(r)}{dr} = V_{\text{eff}}^{(4)}(r) - E_{\text{KH}}^{(4)} \tag{19d}
\]

Taking the superpotentials into account and then multiplying each term in equations (19a)–(19d) by \( X_0^2(r) \), we obtain first, second and third-order corrections to the energy and their superpotentials as follows:

\[
E_{\text{KH}}^{(i)} = \int_0^\infty X_0^2(r) \left( \frac{32}{1575} ab^9 a_0^8 + \frac{4}{15} ab^9 a_0^6 + \frac{16}{9} ab^9 a_0^4 + 4ab^9 a_0^2 - F \right) dr = \frac{\hbar^2 \zeta^2}{2\mu} \left( \frac{8}{525} ab^9 a_0^8 + \frac{8}{3} ab^9 a_0^6 + \frac{4}{3} ab^9 a_0^4 + 3b^9 a_0^2 - \frac{3F}{4ab^3} \right) \tag{20a}
\]

\[
W_0^{(i)}(r) = \frac{\sqrt{2}\mu}{\hbar} \int_0^\infty X_0^2(r) \left[ F_{\text{KH}}^{(i)} - \left( \frac{32}{1575} ab^9 a_0^8 + \frac{4}{15} ab^9 a_0^6 + \frac{16}{9} ab^9 a_0^4 + 4ab^9 a_0^2 - F \right) \right] d\phi = \frac{\hbar^2 \zeta}{\sqrt{2}\mu} \left( \frac{8}{1575} ab^9 a_0^8 + \frac{1}{15} ab^9 a_0^6 + \frac{4}{9} ab^9 a_0^4 + 2b^9 a_0^2 - \frac{F}{4ab^3} \right) \tag{20b}
\]
\[ E_{K}^{(2)}(r) = \int_{0}^{\infty} \mathcal{X}^{(2)}(r) \left[ -\frac{\hbar^3}{\mu} \frac{1}{2} \int^{r} \mathcal{X}^{(2)}(\varrho) \left( E_{K}^{(2)}(\varrho) + \mathcal{W}_{0}^{(1)}(\varrho) \right)^{2} \right] \varrho \, d\varrho \]

\[ = \frac{2\mu}{\hbar^{3}} \left( \frac{1}{2} \int^{r} \mathcal{X}^{(2)}(\varrho) \left( E_{K}^{(2)}(\varrho) + \mathcal{W}_{0}^{(1)}(\varrho) \right)^{2} \right) \varrho \, d\varrho \]

With equations (20a)–(20e), we obtain the approximate energy eigenvalues and the wave function for an electron in a He atom under intense laser field encircled by an electric field as:

\[ E_{K} = E_{K}^{(0)} + \left( 2ab - \frac{4}{405} ab^{6} - \frac{8}{63} ab^{7} + \frac{4}{5} ab^{6} + \frac{4}{3} ab^{5} \right) + E_{K}^{(1)} + E_{K}^{(2)} + E_{K}^{(3)} + \ldots \] \tag{21}

and

\[ \psi_{K}^{(2)}(r) \approx 2\zeta^{3/2} \varphi(\varphi) \exp \left( -\frac{2\mu}{\hbar^{2}} \int^{r} \mathcal{W}_{0}^{(1)}(\varrho) + \mathcal{W}_{0}^{(2)}(\varrho) \varrho \, d\varrho \right) \] \tag{22}

respectively. The behavior of energy eigenvalues of an electron in a He atom interacting with electric field and exposed to linearly polarized intense laser field radiation as a function of various model parameters has been shown in table 1 and figure 2.

4. Numerical result

We delineate the effective potential and its approximate expression in figure 1. Specifically, figures 1(a) and (b) show that intensifying the electric field strength will make the repulsiveness of the effective potential burgeon and, hence, the system become less attractive. It can also be seen that for a strong coupling strength and weak external electric field, the effective potential tends to continuum states. Furthermore, in figure 1(c), by considering a weak external electric field, directed along the z-axis, we found that the approximate expression to the model potential is only valid for low screening parameter values.

We anticipate that the approximate expression may be influenced by variations in external electric field and coupling strength. In order to elucidate this, we have plotted figure 1(d). It can be seen that the approximate expression diverges more from the model potential when \( (a, F) = (5, 5) \) compares to \( (a, F) = (1, 0.01) \). This connotes that in order to obtain a better approximation to the potential model, we have to consider interaction with low electric field directed along the z-axis with \( a = 1 \). However, it is worth mentioning that, supposing the electric field were directed along \( \theta = \pi \), a disparate conclusion might arise. Moreover, studying the variations in figures 1(c) and (d), one can infer that the approximation is only valid for \( br \ll 1 \).

In table 1, we scrutinize the behavior of energy levels of an electron of helium atom as it interacts with electric field directed along the z-axis and is exposed to linearly polarized...
intense laser field radiation. It can be observed from this table that as the electric field becomes more intense, the eigenvalues become more negative and the system becomes more repulsive. Moreover, by considering a weak electric field and then varying the screening parameter, we found that the system becomes weakly bound. This corroborates the results of figures 1(c) and (d), where we determine the validity condition for the approximate expression. In fact, one can predict that as the screening parameter continues to increase, there will be a critical point where a transition from bound to continuum states takes place.

Furthermore, figure 2(a) shows that the eigenvalue is inversely proportional to applied external electric field notwithstanding the choice of screening parameters (this can be seen from the slope of the graph). This figure reveals that for a very strong external electric field and an infinitesimal screening parameter $b = 0.0005$, the eigenvalue is approximately $-8.0033$, which is much smaller than the value of $-8.0176$ for $b = 0.0010$. This suggests that the system becomes more strongly bound as the screening parameter increases.
parameter, the system is strongly bound. Figure 2(b) expounds this further. It can be seen that the eigenvalues increase monotonically with an increase of screening parameter for various intensities of external electric field. As the screening parameter burgeons, the system becomes weakly bound and highly repulsive. In fact this figure authenticates our projection in table 1. At $\alpha_0$, the system tends to continuum states.

In figure 2(c), we examine the behavior of the eigenvalues as function of laser dressing parameter for a very large screening parameter and various electric field intensities. It can be observed that as $\alpha_0$ increases, $E_{KH}$ dwindles monotonically and becomes less repulsive. However, the scrutiny takes a different shape when considering a very small screening parameter. Variation in the eigenvalues is indiscernible since the energy shift $\Delta E_{KH} = 0$. In fact figures 2(c) and (d) demonstrate the susceptibility of eigenvalues of an electron in a He atom to changes in the screening parameter. One can deduce that the eigenvalues will respond to variation in the frequency of the laser only if we consider a very large value of the screening parameter. But, this will invalidate our approximation, then we can only conclude that the variation in frequency of laser radiation has no effect on the eigenvalues of an electron in He for a particular electric field intensity directed along the $z$-axis.

5. Concluding remarks

We scrutinize the behavior eigenvalues of an electron in a He atom as it interacts with electric field directed along $\theta = 0$ exposed to linearly polarized intense laser field radiation. In order to achieve this, one electron of the helium atom is frozen at its ionic ground state and the motion of the second electron in the ion core is treated via a more general case of screened Coulomb potential model. Using the Kramers–Henneberger ($KH$) unitary transformation, which is a semiclassical counterpart of the Block–Nordsieck transformation in the quantized field formalism, the squared vector potential that appears in the equation of motion is eliminated and the resultant equation is expressed in the KH frame. Within this frame, the resulting potential and the corresponding wave function have been expanded in Fourier series and, using Ehlotzky’s approximation, we obtain a laser-dressed potential to simulate intense laser field. By fitting the more general case of the screened Coulomb potential model into the laser-dressed potential, and then expanding it in Taylor series up to $O(r^4, \alpha_0)$, we obtain the solution (eigenvalues and wave function) of an electron in a He atom under the influence of external electric field and high-intensity laser field, within the framework of perturbation theory formalism. It has been shown that the variation in
frequency of laser radiation has no effect on the eigenvalues of an electron of helium for a particular electric field intensity but that for a very strong external electric field and an infinitesimal screening parameter, the system is strongly bound. This work has potential application in the areas of atomic and molecular processes in external fields including interactions with strong fields and short pulses. This work represents the continuation of our project ‘atoms and molecules interacting with external fields in Laser-Plasma’ which commenced in [21, 24, 25]. We hope that the current study will inspire future progress in exploring molecular systems under intense laser field.

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