On Some Cases of Plastic Deformations Under Complex Loading

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Abstract. The experimental results under complex loading in view of theory of processes are explained in this issue. The opportunity for reducing of external efforts at metal forming is shown here. These results and investigations can be expanded for anisotropic material and for spatial processes of material forming.

1. Introduction
At present the energy saving technologies and optimal control of deformation at metallurgy has a great importance. The use of new isotropic and polymer materials demands to investigate more widely their mechanical properties and ways for optimal reinforcing. The account of complex loading processes is vital here. It has been experimentally established that at some complex loading processes the force of plastic deformation is less than at simple loading (like tension or torque). The Il’yushin’s theory of elasto-plastic processes can explain this effect [1]. So, at complex loading the energy at metal deformation can be reduced compared with simple loading. The possibility and mathematical explanation of this problem are described in this issue. The theory of elasto-plastic processes by Il’yushin is the base for this approach. For deformation trajectories with bend points the approach for decreasing of energy at metal processing are shown. Given effect can be used at development of new technology at plastic processing of metals and application it in practice. Thereby, taking into account the complex loading and investigated effect appearing here, aside from definition of reserves of bearing capability of structures, allows to minimize the work of external forces at technological processes by optimal controlling of strain path at metal forming processing.

Mathematical problems of plasticity under complex loading [2] cannot be solved by creating the universal constitutive relations. The models [3-7] which developed for complex loading processes can be applicable only for specific processes of loading and for different materials in each case separately.

2. Main part
Many boundary plasticity problems have not reliable solution [8,9]. Particularly, reason for it the initial errors in gradient plasticity positions. The Drucker’s Postulate [10] and its gradient principle can’t always at any loading adequately reflect the processes of strain-stress state in continua. For instance are structured-unstable materials, material with falling “strengthening” diagram, etc. At investigation of complex loading for isotropic metals also appear the breaches of the Drucker’s Postulate. This is due to not only deformational anisotropy. One of them is a breach of the gradient principle. In generalized Hodge and Prager’s view [11] for increments of full deformation are:

\[ d\varepsilon_i = H_{ij} d\sigma_{ij} + 0.5\left[P(J_2,J_3)s_{ij} + Q(J_2,J_3)\frac{\partial J}{d s_y}\right] df \]
here first summand is elastic increments of strain, the second summand is plastic component of one, \( J_2 \) and \( J_3 \) are invariants of stress deviators, \( s_{ij} \) are components of stress deviators, \( H_{ijkl} \) is matrix of elastic components, \( df \) is yield surface, \( d\varepsilon_{ij} \) and \( d\sigma_{kl} \) are increments of strain tensors and stresses respectively.

As possible see when moving on yield surface (neutral loading) have to be absent the increments of plastic forming deformation. The yield surface in strain space gives the same results for neutral deformation. However, experiments and theoretical investigations [12-14] under neutral loading both in stress space and in strain space (neutral deformation) shows that this condition is not executed. Under neutral loading on yield surfaces lead to strain growing. Under loading under constant intensity deformation the situation is vice versa, fall of stress intensity. The typical process of stress falling is expressed on Figure 1 at different value of plastic deformation. These experimental data give the cause to consider influence of complex loading on value of work under plastic deformation can be greatly lower, under simple loading.

Also at process of the metal processing by pressure was noticed that work of external power under different path of deformation has smaller importance, than under simple loading. Particularly is obviously at processes of loading or deformation with bend points. This effect can be explained on base of plastic theories of processes by Il’yushin [1] and principle of delay of vector and scalar characteristic of materials.

Lemma: Minimum of the work of external force for achievement of the given form under plastic deforming is reached only at realization of the strain processes with bend-point.

Proof: On the base of generalized Clapeyron theorem the work of external force on corresponding displacement is equal to accumulated potential energy of deformed rigid:

\[
A = \int_V \sigma_{ij} d\varepsilon_{ij} dV = \int_V \frac{1}{3} \sigma_0 d\varepsilon_0 dV + \int_V s_{ij} d\varepsilon_{ij} dV
\]

here \( s \) is length of strain’s path, \( d\varepsilon_{ij} \) are components of strain deviator’s increments, \( \sigma_0 \) is a module of spherical part of stress tensor and \( d\varepsilon_0 \) is increment of the average relative elongation, \( s_{ij} \) - components of deviator of stress tensor.

Since work of forces on elastic deformation does not depend on strain path because field of the stress vector on deformation before start yielding is conservative. The work of hydrostatic pressure on
three dimensional compressions is conservative also. It is follow assumption to fact that material is plastic incompressible.

Herein:

\[ \int_0^s s_j \, ds_j = \int_0^s \sigma d\mathcal{F} = \int_0^s \sigma \cos \theta \, ds = \int_0^s \sigma ds + \int_0^s \sigma \cos \theta ds \]

here \( \theta \) is angle of deviation between stress vector \( \mathbf{\sigma} \) and vector of strain’s increment \( d\mathcal{F} \).

At active loading process the minimum of functional of the work is reached when angle of deviation is \( \pm \pi/2 \). It is possible only at normalcy of stress vector to Frenét’s ort \( \mathbf{\alpha}_1 \) (tangent to strain paths). Follow to principle of delay the angle of deflection cannot be constant on all deformation process. It is means that at loading before exhausting of trace of delay of vector characteristics \( \mathbf{\alpha} \) of material is strain path must has break point.

Conclusion: At metal forming processing by pressure, stampings, etc. important is aside from the whole the possibility of optimum management for loading process. Thereby, follows it in condition should be put the optimum quantity of section of streak line for optimal solving of problem. Thereof possible to put define on which stage to produce the break for strain path. Here must know aside from classical physical parameter of material, as well as its trace of delay \( \mathbf{\alpha} \).

Remark: At realization of the loading processes in stress space with bend-point appears the situation when scalar product of stress vector \( \mathbf{\sigma} \) and strain’s increments vector \( \mathbf{\alpha}_1 \) can have a minimum also. However, difference between loading processes in strain and stress spaces consist in change vector characteristic material in the first event because here occurs dropping in stresses and it gives minimum work at strain process of loading.

Let’s consider the multilink strain process of homogenous specimen and evaluate its work value. At plastic incompressibility of material the first invariants of stress tensor \( \mathbf{\sigma}_{ij} \) and Lagrangian strain tensor \( \mathbf{\epsilon}_{ij} \) can be considered as external parameters. Then, plastic deformation occur cause forming only. Deviator of the stresses \( \tilde{\mathbf{\sigma}}_{ij} \) and strains \( \tilde{\mathbf{\epsilon}}_{ij} \) are linear dependent. And only five of theirs are independent.

Let’s consider the processes with break-point in Ilyushin vector space \( \mathcal{E}^5 \) \([1]\) which generated by strain vector. The stress vector in this space is presented as accompanying to strain path.

The forming work can be expressed in the next view:

\[ \int_0^s \tilde{\mathbf{\sigma}} d\mathcal{F} = \int_0^s \sigma \cos \theta \, ds \]

Obviously, that under determined strain path the work at complex loading wills less than at simple one. However, impose the certain restrictions on strain path geometry are necessary. Follows previous lemma for minimization of forming work it is necessary to consider the processes with bend-point. At such choice in general case a quantity of section \( n \), lengths of section \( \Delta s^{(i)} \) and corner-angles \( \beta^{(i)} \) are unknown. In real processes to reach of given points in strain or stress space are required.

Here considered the strain trajectory Figure 2 where all section of the deformation process and strain vector are coplanar. It is correspond to plane problems of plasticity.

Without restriction of generality, according to Isotropy postulate, let’s consider processes in \( \mathcal{E}_1 \sim \mathcal{E}_2 \) plane. In this case problem of the determination directing cosine of stress vector possible on the base of the Convergence Law. Enter the following values: \( \alpha^{(i)} \) (\( l=1,n \)) are angles of inclination of strain path to axis \( O\mathcal{E}_1 \), \( \beta^{(i)} = \alpha^{(i-1)} - \alpha^{(i)} \) (\( l=2,n \)) are corner-angles of strain path, \( \gamma_i = [\tilde{\mathbf{\sigma}} \Delta s^{(i)}] \) (\( l=1,n \)) are angles of inclination of stress vector in bend-point to previous section of strain path, \( \theta^{(i+1)} = \gamma_i + \beta^{(i+1)} \) (\( l=1,n-1 \)) are angles of inclination of stress vector in bend-point to next section of path.
strain path. The positive direction angles are defined if tumbling from previous to the following section goes clockwise.

As was it earlier noted, in experiments was noticed versatility of $\vartheta$ function for each material under same break-angles in trajectory of deformation. For given reasons the empirical formulas from experiments as (1-3) for approximation of $\vartheta$ function were offered. On the base of above mentioned approximations the graph is built. Here the convergence process of stress vector and strain path is shown on Figure 3. The trace of delay accepted as: $\bar{\lambda} = 5\varepsilon_x$.

$$\vartheta = \frac{\theta}{\exp(k(1 + \frac{1}{\lambda})\Delta s)}, \quad (1)$$

here $k = \ln 16/(1 + \bar{\lambda})$, $\theta$ - angle of bend

$$\vartheta = \exp\left(\frac{l}{\Delta s - \bar{\lambda}b}\right) - H, \quad (2)$$

here $H = 1$, $b = \frac{\ln(\theta/16 + 1)}{\ln(\theta/16 + 1) - \ln(\theta + 1)}$, $\theta$ - angle of bend

$$\vartheta = \text{Carctg}\left(\frac{r}{\bar{\lambda}}\Delta s\right), \quad (3)$$

here $C = \frac{2\theta}{\pi}$, $r = \text{ctg}\left(\frac{\pi}{32}\right)$, $\theta$ - angle of bend

There is possibility for reduction of the plastic forming work value is possible see from graph. The ratio as (1) better reflects the vector characteristic of the material before exhausting the trace of delay $\bar{\lambda}$, but quicker decreases after passing of it than in experiment. The approximations as (2) and (3) on the contrary quicker decrease, than experimental curves, but after exhausting of trace of delay $\bar{\lambda}$ approaches to experimental curve.
For ideal plastic material:
\[
\int_0^s \varepsilon d\bar{s} = \sigma \mathcal{E}[n] \quad \text{at simple loading;}
\]
\[
\int_0^s \varepsilon d\bar{s} = \sigma \int_0^s \cos \alpha ds \quad \text{at complex loading.}
\]

If accept all section of multi-link trajectory equal to \( \Delta s \), all angles of deflection on each section is identical (i.e. angle of alteration are changes by one law), that:
\[
\sigma \int_0^s \cos \alpha ds = \sigma \int_0^s \cos \alpha ds.
\]

Let’s show the opportunity to find the multi-link strain path with work of stress on increments of deformations will less than at same deformation under simple loading and condition when it can be reached. For simplification, consider the processes with equal angles in bend points on strain paths in the manner of zigzag \( \beta_i = \pm x \). In this case the value of strain vector is:
\[
|\mathcal{E}| = n\Delta \sqrt{\frac{1 + \cos(x - \alpha_1)}{2}} \quad \text{at even quantity of sections.}
\]
\[
|\mathcal{E}| = \Delta \sqrt{\frac{n^2 + 1}{2} + \frac{n^2 - 1}{2} \cos(x - \alpha_1)} \quad \text{at odd quantity of sections.}
\]

Not limiting generalities are possible to accept \( \alpha_i = 0 \). Then for multi-link zigzag trajectory with angles of breaks equal to \( x \) the condition when work at complex loading at plastic deformation less than one under simple loading is for ideal plastic material:
\[
\sigma \int_0^s \cos \alpha ds \leq \sigma \Delta \sqrt{\frac{1 + \cos(y)}{2}}
\]

or \[
\sigma \int_0^s \cos \alpha ds \leq \sigma \Delta \sqrt{\frac{1 + n^2}{2} + \cos(y) \frac{n^2 - 1}{2}}
\]
Simplifying get:

\[ \int_0^\Delta \cos \theta \, ds \leq \Delta \sqrt{\frac{1 + \cos(y)}{2}} \]  

Remark: Active processes of loading are considered here, i.e. at \( \beta_t \leq \pi / 2 \).

3. Conclusion

This condition is sufficient, but not necessary one. It is possible to show as at case of strengthening material this condition is sufficient too. The condition as (4) even under "worst" variant is wholly workable. For it reason is enough to constraint of corner-angle with length of each link of strain path. Thereby, it is shown opportunity in spite of increase the strain path under complex loading compare with simple one to reduce external efforts for achieving the same form at plastic deformation.

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