Experimental validation of fractional order internal model controller design on buck and boost converter

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Abstract
In this paper, fractional order internal model control technique is formulated for non-ideal dc–dc buck and boost converter. The fractional order internal model control approach integrates the concept of Commande Robuste d’Ordre Non Entier principle for tuning a fractional order filter with internal model control scheme. The final controller can be expressed as a series combination of proportional integral derivative controller and a fractional order low pass filter. To assess the robustness of the proposed fractional order internal model control scheme, both the servo response and regulatory response of the dc–dc converters are investigated in the presence of disturbances. The efficacy of fractional order internal model control technique is demonstrated via comparison with 2 degrees of freedom internal model control scheme. Furthermore, an experimental validation of fractional order internal model control is conducted on laboratory setup, and a dSPACE 1104 microcontroller is used for hardware implementation. The simulation results and the hardware validation are a testimony to the effectiveness of fractional order internal model control technique.

Keywords
Internal model control, controller design, buck converter

Introduction
Over the past years, switched mode power supplies have been used in a plethora of commercial and industrial applications such as dc motor drives, computers, electric vehicles, batteries and so on. The dc–dc conversion has developed into a sophisticated technology, and an enhanced control of dc–dc converters can revamp the performance of applications provided to the end user. Buck and boost dc–dc converters are among the widely used switched mode supplies and aid in the reduction and scale-up of the input voltage, respectively. In an open loop mode, the dynamic response of the converters is unsatisfactory, which necessitates the requirement of closed loop control for regulation of output voltage to the desired levels. Moreover, the boost converter exhibits non-minimum phase characteristics due to the occurrence of zero in right half plane, leading to inverse response. Therefore, the voltage regulation of dc–dc converters is a challenging problem and evinces widespread interest among control engineers.¹–³

Hitherto, numerous control techniques have been delineated in literature for the control of various industrial processes. The existing techniques can be categorized into proportional integral derivative (PID) control, adaptive control,⁴ sliding mode control,⁵ model predictive control,⁶ soft computing approaches⁷ and so on. A brief review of the PID control and the notion of automation of loop cycling approach with modern technology is discussed in Atherton.⁸ The parameters of an adaptive super-twisting sliding mode controller based on Lyapunov theory are computed via particle swarm optimization (PSO) algorithm for two-axis helicopter in Humaidi and Hasan.⁹ An extended state observer–based model predictive technique for the speed control of a permanent magnet synchronous generator is proposed in Li et al.,¹⁰ whereas a fuzzy logic controller for the control of droplet movement inside a microfluidic network is formulated in Mehmood et al.¹¹ It is deduced that although a PID controller¹² has a simple...
structure and easy tuning approaches, however, it is unable to give optimal control in the presence of environmental disturbances, whereas sliding mode control and model predictive control have a complex structure. On the other hand, application of soft computing techniques entails a large simulation time and involves numerous random parameters.

In the recent years, fractional calculus has evoked wide interests in the formulation of novel control strategies to ensure a more accurate and precise control. The notion of fractional calculus has enabled the researchers to formulate extensions of the existing control strategies such as PID, stability boundary locus and Ziegler Nichols into the fractional order (FO) domain. A fractional order proportional integral derivative (FO-PID) controller, first discovered by Podlubny, is considered to be more flexible than an integer order PID controller, since it has five tuning parameters, which can be independently tuned, in comparison to the conventional PID, wherein only three independent tuning parameters are present. Various techniques of tuning a FO-PID controller are discussed in the literature. A hybrid control scheme that incorporates the advantages of both the FOPID controller as well as the active disturbance rejection control (ADRC) technique is presented in Fang et al. for a hydroturbine speed governor system and is observed to exhibit an enhanced performance over both these control techniques. However, tuning of the parameters of a lead compensator for FO processes, approximated into an equivalent integer order model via Matsuda approximation, is achieved via minimization of integral performance indices in Dogruer and Tan. In Wang et al., a proportional integral derivative dynamic matrix control (PID-DMC) technique that inherits the advantages of both PID and DMC control approaches is formulated and tested on an industrial heating furnace process system. Internal model control (IMC) is another such control approach based on Youla parameterization, which is widely used due to its simple structure and intuitive design. In ideal conditions, it is characterized by dual stability, perfect control and zero-steady state error. To further improve the control performance, the concept of FO control can be amalgamated into IMC scheme, leading to the formulation of fractional order internal model control (FO-IMC) technique. The tuning methodology for FO-IMC entails the application of Commande Robuste d’Ordre Non Entier (CRONE) principle in controller design, wherein the FO filter is chosen to satisfy Bode ideal characteristics.

Motivation

Over the past years, PID controller has become synonymous with industrial control applications due to its simple structure, numerous tuning techniques and a successful past record. However, in real-time operating conditions, the system is exposed to variable environmental conditions and it is entirely plausible that various physical and environmental disturbances have an adverse effect on its response. Therefore, the controller design should not only be able to track the set point but also reject the disturbances effectively. Moreover, another technique, namely, IMC, ideally possesses the properties of perfect control, zero-steady state error and is interconvertible into classical feedback structure. However, the conventional PID or IMC controllers may not be able to ensure a quick rejection of disturbances. With the application of fractional calculus in PID controller, a robust FOPID controller was conceived which exhibits good disturbance rejection, efficient set point tracking and excellent robustness, since it has two more tuning parameters in comparison to conventional PID controller. FO-IMC technique inherits the advantages of both IMC as well as PID controls and thereby is a refinement over these individual control approaches.
Various FO control schemes are delineated in literature; however, their practical validation is missing from most of the publications. In this paper, FO-IMC technique is elucidated for the voltage control of buck and boost converter and practically validated on a real-hardware setup of a power converter, for the first time. It is observed that single degree of freedom FO-IMC exhibits an improved response over 2 degrees of freedom internal model control (TDF-IMC) technique and it ensures stable and an effective response for both set point tracking and disturbance rejection, respectively. Various case studies are conducted in the paper, which validate the robustness of the FO-IMC scheme.

Mathematical modelling

In this section, a non-ideal transfer function model for a buck converter and boost converter is formulated. The schematic of the buck and boost converter are shown in Figures 1 and 2, respectively. In Figures 1 and 2, elements of circuit are inductor \( L_o \), capacitor \( C_o \), load resistor \( r_{Lo} \), diode \( d_p \) and a switch \( S_o \). Furthermore, the parasitic elements are resistance of source \( r_p \), equivalent series resistance of inductor \( R_L \) and capacitor \( R_C \), diode resistance \( R_d \), switch resistance \( R_{sw} \) and forward voltage drop of diode \( V_f \). The voltages across input, output, capacitor and a switch \( V_{op} \) and \( V_{co} \), respectively, whereas the current through capacitor and inductor are designated by \( i_C \) and \( i_L \), respectively.

The ON and OFF time differential equations for a non-ideal dc-dc buck converter are derived using Kirchhoff’s current law and Kirchhoff’s voltage law as follows.31,32

**Buck converter**

**ON time**

\[
L_o \frac{di_L(t)}{dt} = \left(-r_p + R_{sw} + R_L\right)i_L(t) - \left(r_{Lo} \parallel \frac{R_C}{R_C}\right)i_{cL}(t) + \left(\frac{r_{Lo} \parallel \frac{R_C}{R_C}}{r_{Lo}} - 1\right)v_{co}(t) + v_{ip}(t)
\]

**OFF time**

\[
L_o \frac{di_L(t)}{dt} = \left(-r_p + R_{sw} + R_L\right)i_L(t) - \left(r_{Lo} \parallel \frac{R_C}{R_C}\right)i_{cL}(t) + \left(\frac{r_{Lo} \parallel \frac{R_C}{R_C}}{r_{Lo}} - 1\right)v_{co}(t) + v_{ip}(t)
\]

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Figure 1. Non-ideal model of buck converter.31.

Figure 2. Non-ideal model of boost converter.31.

\[
\frac{C_o}{d}v_{co}(t) = \left(\frac{r_{Lo} \parallel \frac{R_C}{R_C}}{r_{Lo}}\right)i_{cL}(t) - \left(\frac{r_{Lo} \parallel \frac{R_C}{R_C}}{r_{Lo}}\right)v_{co}(t)
\]

\[
v_{op}(t) = \left(r_{Lo} \parallel \frac{R_C}{R_C}\right)i_{cL}(t) + \left(\frac{r_{Lo} \parallel \frac{R_C}{R_C}}{r_{Lo}}\right)v_{co}(t)
\]

Using state space form of system representation, equations (1)–(3) can be expressed as

\[
\frac{z_1}{d} = A_mz_1(t) + B_mu_m(t)
\]

\[
y_m(t) = C_mz_1(t)
\]

where \( z_1 = [i_{cL}(t), v_{co}(t)]^T \), \( u_m = [v_{ip} V_f]^T \), \( y_m = [v_{op} i_{cL}(t)]^T \), \( r_{Lo} \parallel \frac{R_C}{R_C} \) and \( r_{Lo} \parallel \frac{R_C}{R_C} \). The state space matrices in equation (4) are given as

\[
A_m = \begin{bmatrix} -\left(-r_p + R_{sw} + R_L\right) - \left(r_{Lo} \parallel \frac{R_C}{R_C}\right) & \frac{r_{Lo} \parallel \frac{R_C}{R_C}}{r_{Lo}} \end{bmatrix} \]

\[
B_m = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad C_m = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]

Equations (6)–(8) can be expressed in state space form as

\[
\frac{z_1}{d} = A_nz_1(t) + B_nu_n(t)
\]

\[
y(t) = C_nz_1(t)
\]

The state space matrices in equation (9) can be written as
Using equations (4) and (9), the steady state averaged model is obtained via the following formulae

\[ A_{\text{avg}} = D A_m + \overline{D} A_n \]
\[ B_{\text{avg}} = D B_m + \overline{D} B_n \]
\[ C_{\text{avg}} = D C_m + \overline{D} C_n \]

where \( \overline{D} = 1 - D \), and averaged state space matrices are determined as

\[ A_{\text{avg}} = \begin{bmatrix} -D(r_L + R_L) & -D(r_0||R_0) & -\frac{r_0}{L_o} \frac{R_0}{R_{C_L}} \end{bmatrix} \]
\[ B_{\text{avg}} = \begin{bmatrix} 0 & 0 \end{bmatrix} \]
\[ C_{\text{avg}} = \begin{bmatrix} \frac{r_0}{L_o} & \frac{R_0}{R_{C_L}} \end{bmatrix} \]

Using small signal analysis, the transfer function relating output voltage to duty cycle is determined using the following formula

\[ G_{\text{buck}}(s) = C_p(sI - A_p)^{-1}B_p \]

wherein, the state space matrices are defined as

\[ A_p = \begin{bmatrix} -D(r_L + R_L) & -D(r_0||R_0) & -\frac{r_0}{L_o} \frac{R_0}{R_{C_L}} \end{bmatrix} \]
\[ B_p = \begin{bmatrix} \frac{r_0}{L_o} & \frac{R_0}{R_{C_L}} \end{bmatrix} \]
\[ C_p = \begin{bmatrix} \frac{r_0}{L_o} & \frac{R_0}{R_{C_L}} \end{bmatrix} \]

Substitution of the state space matrices from equation (14) in equation (13) yields the transfer function of non-ideal buck converter as

\[ G_{\text{buck}}(s) = \frac{v_{\text{op}}(s)}{d(s)} = \kappa \frac{p_0 + p_1 s + q_0 + q_1 s + q_2 s^2}{q_0 + q_1 s + q_2 s^2} \]

where \( v_{\text{op}}(s) \) and \( d(s) \) represent small signal variations in output voltage and duty cycle, respectively, and the numerator and denominator terms are expanded as

\[ \kappa = \frac{v_{\text{op}} (r_L + R_L)}{r_L} = \frac{v_{\text{op}} (r_0 + R_0)}{r_0} \]
\[ p_0 = 1, q_0 = 1, p_1 = C_o r_L \]

Hence, equations (15) and (16) give the final non-ideal transfer function of buck converter. In the following subsection, the non-ideal transfer function of de-de boost converter is formulated.

\[ q_1 = \frac{L_o}{D(r_L + R_L) + D(r_0||R_0) + r_{Lo} + R_{f2}} \]
\[ q_2 = \frac{L_o}{D(r_L + R_L) + D(r_0||R_0) + r_{Lo} + R_{f2}} \]
\[
(v_{op}(t)) = (r_{Lo} \parallel R_{cf})V_{Lo}(t) + \left(\frac{r_{Lo}}{R_{cf}} \right) v_{co}(t)
\]
\[
- (r_{Lo} \parallel R_{cf})V_{co}(t)
\]
Equations (22) - (24) can be represented in state space formulation as
\[
\dot{z}_2 = A_{z2}z_2(t) + B_{z2}u(t) + G_{z2}V_f
y = C_{z2}(z) + E_{z2}u(t) + H_{z2}V_f
\]
The state space matrices in equation (25) are
\[
\begin{align*}
A_{z2} &= \begin{bmatrix}
-\frac{1}{L}\left((r_{Lo} + r_p + R_{di}) + (r_{Lo} \parallel R_{cf})\right) - \left(\frac{u_{Lo}}{R_{cf}}\right) \\
0 & 1
\end{bmatrix} \\
B_{z2} &= \begin{bmatrix}
\frac{1}{L} \frac{v_{op}}{R_{cf}} \\
0
\end{bmatrix} \\
C_{z2} &= \begin{bmatrix}
r_{Lo} \parallel R_{cf} \\
1
\end{bmatrix} \\
E_{z2} &= \begin{bmatrix}
0 & -\frac{1}{L} \left(\frac{v_{op}}{R_{cf}}\right) \\
0 & 0
\end{bmatrix}, \quad G_{z2} = \begin{bmatrix}
-\frac{1}{L} \\
0
\end{bmatrix}, \quad H_{z2} = \begin{bmatrix}
0 & 0
\end{bmatrix}
\end{align*}
\]
Using equations (20) and (25), the averaged state space model of boost converter is obtained via following formulae
\[
\begin{align*}
A_{avg} &= DA_{m2} + \hat{D}A_{z2}, \quad B_{avg} = DB_{m2} + \hat{D}B_{z2} \\
C_{avg} &= DC_{m2} + \hat{D}C_{z2}, \quad E_{avg} = DE_{m2} + \hat{D}E_{z2} \\
G_{avg} &= DG_{m2} + \hat{D}G_{z2}, \quad H_{avg} = DH_{m2} + \hat{D}H_{z2}
\end{align*}
\]
where \(\hat{D} = 1 - D\). Therefore, the state space matrices of the averaged model are computed as
\[
\begin{align*}
A_{avg} &= \left[\begin{array}{cc}
\frac{(\hat{D}r_{Lo} + \hat{D}r_p + R_{di} + r_p + R_{di}) + \hat{D}r_{Lo} + \hat{D}r_p}{L} & \frac{\hat{D}r_{Lo} + \hat{D}r_p}{L} \\
0 & 1
\end{array}\right] \\
B_{avg} &= \begin{bmatrix}
\frac{\hat{D}r_{Lo}}{r_{Lo}} \\
0
\end{bmatrix}
\end{align*}
\]
Using small signal analysis, the transfer function that relates output voltage to duty cycle is determined as
\[
G_{\text{boost}}(s) = \frac{v_{op}(s)}{d(s)} = C_p(sI - A_p)^{-1}B_p + E_p
\]
where
\[
A_p = \begin{bmatrix}
-\left(\frac{r_{Lo} + R_{Lj} + DR_{sw} + (\hat{D}R_{sw})(r_{Lo} + R_{Lj}) + DR_{Lo}(\hat{D}r_{Lo} + R_{Lj})}{L}\right) & \frac{r_{Lo} + R_{Lj} + DR_{sw} + (\hat{D}R_{sw})(r_{Lo} + R_{Lj}) + DR_{Lo}(\hat{D}r_{Lo} + R_{Lj})}{L} \\
0 & 1
\end{bmatrix}
B_p = \begin{bmatrix}
\frac{\hat{D}r_{Lo}(r_{Lo} + R_{Lj})}{L} \\
0
\end{bmatrix}
E_p = \begin{bmatrix}
-\frac{I_{z0}}{r_{Lo} + R_{Lj}} \\
0
\end{bmatrix}
C_p = \begin{bmatrix}
\frac{\hat{D}r_{Lo} + \hat{D}r_p}{L} \\
0
\end{bmatrix}
\]
where \([I_{z0} V_{z0}]^T = -A_p^{-1}(B_p[V_{op} I_{z0}]^T + E_p)\) such that
\[
A_p = DA_{m2} + \hat{D}A_{z2}, \quad B_p = D\hat{B}_{m2} + \hat{D}B_{z2}, \quad E_p = DE_{m2} + \hat{D}E_{z2}
\]
and \(V_{op}\) and \(I_{z0}\) are the steady state values of input voltage and current source, respectively.

Using equation (30), equation (29) can be expressed in simplified form as
\[
G_{\text{boost}}(s) = \kappa_v \left[\left(1 + \frac{\alpha}{\kappa_w}\right)^r \left(1 + \frac{\alpha}{\kappa_w}\right)^\omega\right]
\]
where
\[
\begin{align*}
\kappa_v &= \frac{(r_{ip} + R_{Lj} + DR_{sw} + \hat{D}R_{sw})(r_{Lo} + R_{Lj}) + \hat{D}r_{Lo}(\hat{D}r_{Lo} + R_{Lj})}{L} \\
\alpha_v &= \frac{\kappa_w}{L_o(r_{Lo} + R_{Lj})(\hat{D}V_f - V_{op})} \\
\kappa_w &= (V_{ip} - \hat{D}V_f)(-r_{ip} - R_{Lj} - DR_{sw})(r_{Lo} + R_{Lj}) + V_{ip}(\hat{D}V_f + \hat{D}r_{Lo}(\hat{D}r_{Lo} + R_{Lj})) \\
\omega &= (r_{ip} + R_{Lj} + DR_{sw} + \hat{D}R_{sw})(r_{Lo} + R_{Lj}) + \hat{D}r_{Lo}(\hat{D}r_{Lo} + R_{Lj}) \\
\alpha_l &= \sqrt{\frac{(r_{ip} + R_{Lj} + DR_{sw} + \hat{D}R_{sw})(r_{Lo} + R_{Lj}) + \hat{D}r_{Lo}(\hat{D}r_{Lo} + R_{Lj})}{L_oC_o(r_{Lo} + R_{Lj})^2}} \\
\beta_l &= \sqrt{\frac{L_oC_o(r_{ip} + R_{Lj} + DR_{sw} + \hat{D}R_{sw})(r_{Lo} + R_{Lj}) + \hat{D}r_{Lo}(\hat{D}r_{Lo} + R_{Lj})}{L_o + C_o(r_{ip} + R_{Lj} + DR_{sw} + \hat{D}R_{sw})(r_{Lo} + R_{Lj}) + \hat{D}r_{Lo}(\hat{D}r_{Lo} + R_{Lj})}}
\end{align*}
\]
Proposed FO-IMC scheme

FO-IMC scheme, introduced in Maâmar and Rachid,\textsuperscript{23} extends the conventional 1 degree of freedom IMC approach via the introduction of FO filter, which is chosen in order to satisfy Bode characteristics so as to ensure plant robustness via CRONE principle.\textsuperscript{23,25,26} The principal objective of FO-IMC technique is to control an integer order process via assimilation of the desirable properties of the reference model into the system model. The fractional characteristics in the FO-IMC controller are imposed by the reference closed loop model, which exhibits robust properties such as infinite gain margin, constant phase margin and iso-damping property. IMC scheme has attracted increasing interests, which can be attributed to various desirable properties such as simple structure, internal stability and fewer tuning parameters, whereas CRONE principle introduces the criteria to ensure plant robustness. The amalgamation of both these techniques gives rise to FO-IMC approach, which can be categorized into three distinct steps, namely, reduced order modelling, IMC scheme and CRONE principle.

Consider a higher order plant, which can be expressed in the transfer function form as follows

\[ G_{\text{plant}}(s) = \sum_{i=0}^{M} \tilde{P}_i s^i \sum_{j=0}^{N} \tilde{Q}_j s^j \]

where \( N \geq M \).

To obtain a simplified controller structure, it is worthwhile to utilize the concept of reduced order modelling to obtain a simplified lower order form of the original higher order plant. Model order reduction is a tool for simplification of higher order plant to ensure retention of dominant features of the original plant in the reduced lower order plant, which simultaneously leads to reduction in computational effort and formulation of a lower order controller. To obtain the reduced order model, various techniques have been developed such as Padé approximation, Routh approximation, Mihailov criteria, Cauer approximation, Balanced truncation and so on. Reduction of plant into its second order counterpart aids in achieving PID controller structure in the final controller.\textsuperscript{33} However, in this case, both the buck and boost converter models, given in equations (15) and (31), are of second order; hence, the model order reduction step is not needed.

The reduced order model obtained in this step can be expressed as

\[ G_{\text{mod}}(s) = \frac{\sum_{i=0}^{1} \tilde{p}_i s^i}{s^2 + \sum_{j=0}^{1} \tilde{q}_j s^j} \]  

(34)

Without loss of generality, equation (34) is re-written as

\[ G_{\text{mod}}(s) = \frac{\tilde{p}_0(1 + \tilde{p}_2 s)}{s^2 + \tilde{q}_1 s + \tilde{q}_0} \]  

(35)

where \( \tilde{p}_2 = \tilde{p}_1/\tilde{p}_0 \). Since boost converter represents a non-minimum phase system; therefore, for the present analysis, \( \tilde{p}_2 < 0 \). However, the FO-IMC approach is applicable even when \( \tilde{p}_2 > 0 \) and a similar analysis can be conducted for that case as well.

The block diagram of IMC scheme is illustrated in Figure 3. The IMC controller is characterized by controller \( Q_r(s) \) and the mathematical model of the plant \( G_{\text{mod}}(s) \) that is placed in parallel to the plant. The difference between output of plant and output of process model contains information about the model–plant mismatch and disturbances, which is fed back to IMC controller \( Q_r(s) \). The IMC structure in Figure 3 can be transformed into classical feedback structure in Figure 4 via the use of Youla parameterization\textsuperscript{34} as follows

\[ C_{f0}(s) = \frac{Q_r(s)}{1 - Q_r(s)G_{\text{mod}}(s)} \]  

(36)

To begin with FO-IMC technique, the process model in equation (35) is decomposed into two distinct parts, that is, minimum phase part and the non-minimum phase part. Therefore

\[ G_{\text{mod}}(s) = G_{\text{mod}}^+(s)G_{\text{mod}}^-(s) \]  

(37)

Here, \( G_{\text{mod}}^+(s) \) and \( G_{\text{mod}}^-(s) \) are the minimum phase and non-minimum phase part of process model, respectively.
Therefore
\[ G_{\text{mod}}(s) = (1 + \frac{\tilde{p}_2 s}{s^2 + \tilde{q}_1 s + \tilde{q}_0}) \]

\[ G_{\text{ideal}}(s) = \frac{1}{\eta s^\chi} \]  
(38)

Subsequently, IMC controller is formulated as
\[ Q_{\text{fb}}(s) = (G_{\text{mod}}(s))^{-1} F(s) \]  
(39)

where \( F(s) \) denotes the transfer function of filter. The primary role of filter is to ensure robustness of the overall system to model–plant mismatch and to make the controller physically realizable. The structure of the filter in IMC controller is decided by CRONE principle. CRONE principle was formulated by Oustaloup and defines robust controller as the one which makes it possible to obtain an open loop transfer function defined by constant phase margin in a useful frequency band. In his extensive work, he recommended the ideal shape of open loop transfer function as the one earlier suggested by Bode, which is given as
\[ G_{\text{ideal}}(s) = \frac{1}{\eta s^\chi} \]  
(40)

where \( \chi \) represents the slope of Bode magnitude plot and may assume integral or non-integral values. The corresponding closed loop transfer function is computed as
\[ G_{\text{ideal}}(s) = \frac{1}{\eta s^\chi + 1} \]  
(41)

The transfer function in equation (41) exhibits various desirable properties such as infinite gain margin and constant phase margin (which depends only on \( \chi \)). Hence, it gives a closed loop system having phase margin insensitive to variations in gain of plant. Therefore, the overall closed loop system shows robustness to variations in gain and the corresponding step response displays iso-damping property. Now, if the overall closed loop transfer function of IMC structure follows the reference model in equation (41), it will ensure the inheritance of all the desirable properties of the reference model into IMC structure, since \( G_{\text{plan}}(s) = G_{\text{mod}}(s) \) implies \( Y(s) \sim F(s)R(s) \). Based on the aforementioned analysis, transfer function of filter is chosen as
\[ F(s) = \frac{1}{\frac{1}{s} + \frac{1}{\omega_0^\chi + 1}} \]  
(42)

where \( \xi \in [0, 1] \) and \( \omega > 0 \). Hence, the fractional term is introduced by the structure of overall closed loop reference model. On comparing equations (41) and (42), it can be deduced that \( \eta = \nu \) and \( \xi + 1 = \chi \).

Finally, the IMC controller is transformed into classical feedback controller via Youla parameterization given in equation (36). The final classical feedback controller \( C_{\text{fb}}(s) \) is given as
\[ C_{\text{fb}}(s) = \left( \frac{\tilde{q}_1}{\tilde{p}_0} + \left( \frac{\tilde{q}_0}{\tilde{p}_0} \right) \left( \frac{1}{s} \right) \right) \left( \frac{1}{\omega_0^\chi - \tilde{p}_2} \right) \]  
(43)

Hence, the application of FO-IMC technique gives an interesting structure, that is, series combination of the conventional PID controller and a FO low pass filter (LPF). It can be deduced that the PID controller is dependent only on system parameters. Once the plant model is ascertained, computation of PID controller is a straightforward procedure. Once PID controller is calculated, tuning of the filter parameters, namely, \( \nu \) and \( \xi \) is undertaken. Herein, we adopt the gain crossover frequency and phase margin as the yardstick for tuning of \( \nu \) and \( \xi \).

The phase margin (\( \phi \)) and gain crossover frequency (\( \omega_c \)) for the reference closed loop transfer function are calculated as
\[ \omega_c = \eta^{-1/\chi}, \quad \phi = \pi \left( 1 - \frac{\chi}{2} \right) \]  
(44)

Since, \( \eta = \nu \) and \( \xi + 1 = \chi \), the final FO-IMC controller is formulated as
\[ C_{\text{fb}}(s) = \left( \frac{\tilde{q}_1}{\tilde{p}_0} + \left( \frac{\tilde{q}_0}{\tilde{p}_0} \right) \left( \frac{1}{s} \right) \right) \left( \frac{1}{\omega_0^\chi - \tilde{p}_2} \right) \]  
(45)

where \( \xi = (\pi - \phi)/(\pi/2) - 1 \) and \( \nu = 1/\omega_0^\xi + 1 \).

**Remark.** The existing FO-IMC technique entails the random choice of \( \omega_c \) and \( \phi \) in equation (45). Alternately, the tuning parameters, that is, \( \omega_c \) and \( \phi \) for the proposed FO-IMC approach can be computed depending on user requirements or in such a manner that one obtains the desired system response with an efficient set point tracking and quick disturbance rejection capability. An iterative approach is proposed to choose appropriate values of \( \omega_c \) and \( \phi \). The iterative approach can comprise of the following steps: (a) choose initial values of \( \omega_c \) and \( \phi \), (b) using equation (44), compute \( \eta \) and \( \chi \), (c) choose the desired performance index such as overshoot, wherein the formula of damping factor (\( \zeta_d \)) is given by (\( \zeta_d = -\cos(\pi/\chi) \)), (d) if the value of \( M_p \) computed in (c) \( \neq M_p\text{desired} \), alter the value of \( \omega_c \) and go to step (b), else terminate the algorithm.

Hence, it can be concluded that FO-IMC controller is a cascade combination of PID controller and FO LPF. Furthermore, the stability and robustness analyses of FO-IMC technique are conducted as shown below.

**Stability analysis**

The overall transfer function of the FO-IMC technique with respect to the reference input is given as
\[ Y(s) = \frac{G_{\text{plan}}(s)Q_{\text{fb}}(s)}{1 + Q_{\text{fb}}(s)\left( G_{\text{plan}}(s) - G_{\text{mod}}(s) \right)} \]  
(46)
Under the assumption of plant–model matching, that is, $G_{plant}(s) = G_{mod}(s)$, the transfer function in equation (46) reduces to

$$\frac{Y(s)}{R(s)} = G_{mod}(s)Q_r(s)$$  \hspace{1cm} (47)

Substituting the expressions of $G_{mod}(s)$ and $Q_r(s)$ from equations (37) and (39) in equation (47)

$$\frac{Y(s)}{R(s)} = \frac{Y(s)}{R(s)} = \frac{G^+_{mod}(s)F(s)}{R(s)}$$  \hspace{1cm} (48)

Replacing the expression of $G^+_{mod}(s)$ and $F(s)$ from equations (38) and (42) in equation (48), the transfer function is obtained as

$$\frac{Y(s)}{R(s)} = \frac{1 + \hat{p}_s s}{1 + \nu s^{1/\zeta} + 1}$$  \hspace{1cm} (49)

The pole of the transfer function in equation (49) is given by $s = (−1/\nu)^{1/\zeta} + 1$, which can also be expressed as

$$s = [(1/\nu)e^{\pi(1/\zeta + 1)} = (1/\nu)^{1/(\zeta + 1)}e^{\pi(1/\zeta + 1)}]$$

Therefore, the argument of pole $\pi/(\zeta + 1) < \pi$, since $\zeta > 0$. Hence, the overall FO-IMC controlled system is stable.

For a step reference, the output of the system can be calculated as

$$Y(s) = \left(\frac{1}{s}\right) \frac{1 + \hat{p}_s s}{1 + \nu s^{1/\zeta} + 1}$$  \hspace{1cm} (50)

Using final value theorem of Laplace transformation, the final output response to a unit step reference is obtained as

$$y(t) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = 1$$  \hspace{1cm} (51)

Therefore, the steady state error is given by

$$e(t) = r(t) - y(t) = 1 - 1 = 0$$  \hspace{1cm} (52)

Hence, the controlled system tracks the reference input with zero-steady state error.

**Robustness analysis**

The overall transfer function of the FO-IMC technique with respect to disturbance as an input signal is given as

$$\frac{Y(s)}{D(s)} = \frac{1 - G_{mod}(s)Q_r(s)}{1 + Q_r(s)(G_{plant}(s) - G_{mod}(s))}$$  \hspace{1cm} (53)

Under the assumption of plant–model matching, that is, $G_{plant}(s) = G_{mod}(s)$, the transfer function in equation (53) reduces to

$$\frac{Y(s)}{D(s)} = \frac{1 - G_{mod}(s)Q_r(s)}{1 + Q_r(s)(G_{plant}(s) - G_{mod}(s))}$$  \hspace{1cm} (54)

Substituting the expressions of $G_{mod}(s)$ and $Q_r(s)$ from equations (37) and (39) in equation (54), the transfer function reduces to

$$\frac{Y(s)}{D(s)} = 1 - G_{mod}(s)Q_r(s)$$  \hspace{1cm} (55)

Replacing the expression of $G^+_{mod}(s)$ and $F(s)$ from equations (38) and (42) in equation (55), the transfer function is obtained as

$$\frac{Y(s)}{D(s)} = s(1 - \nu s^{1/\zeta})$$  \hspace{1cm} (56)

For a step disturbance, the output is computed as

$$Y(s) = \frac{(1 - \nu s^{1/\zeta})}{1 + \nu s^{1/\zeta} + 1}$$  \hspace{1cm} (57)

Using final value theorem of Laplace transformation, the final output response to a unit step disturbance is

$$y(t) = \lim_{t \to \infty} y(t) = \lim_{s \to 0} sY(s) = 0$$  \hspace{1cm} (58)

Hence, the FO-IMC technique effectively rejects the load disturbance and is robust as well.

**Simulation results**

In this section, the voltage regulation of buck and boost converter is analysed via application of FO-IMC control technique. A nonlinear model of converters in the presence of all parasitic elements is developed using Sim PowerSystems toolbox in Simulink environment to validate the effectiveness of FO-IMC scheme. Fractional-order modeling and control (FOMCON) toolbox in MATLAB and Simulink is used for simulation of FO controllers. The frequency range and approximation order for the fractional operator are chosen as $[0.001 \ 1000]$ rad/s and 5, respectively. The efficacy of FO-IMC control scheme is demonstrated via comparison with the PID controller tuned via single degree of freedom IMC technique (IMC-PID) as well as the TDF-IMC scheme.

The parameters of buck converter are given as $v_{ip} = 12 \text{V}, r_{ip} = 0.03 \text{Ω}, L_o = 489 \text{μH}, R_{lf} = 0.24 \text{Ω}$

$C_o = 100 \text{μF}, R_{cf} = 0.1 \text{Ω}, V_f = 0.5 \text{V}, R_{di} = 0.03 \text{Ω}$

$R_{ow} = 0.05 \text{Ω}, r_{Lo} = 10 \text{Ω}$  \hspace{1cm} (59)

The parameters of boost converter are

$v_{ip} = 5 \text{V}, r_{ip} = 0.2 \text{Ω}, L_o = 250 \text{μH}, R_{lf} = 0.24 \text{Ω}$

$C_o = 220 \text{μF}, R_{cf} = 0.12 \text{Ω}, V_f = 0.5 \text{V}, R_{di} = 0.03 \text{Ω}$

$R_{ow} = 0.05 \text{Ω}, r_{Lo} = 22 \text{Ω}$  \hspace{1cm} (60)

Using equations (15), (16) and (59), the transfer function of the buck converter is obtained as

$$G_{buck}(s) = \frac{2522.8(s + 10^3)}{s^2 + 1816s + 2.086 \times 10^4}$$  \hspace{1cm} (61)
In a similar manner, on substitution of system parameters from equation (60) in equations (31) and (32), the transfer function of boost converter is obtained as
\[
G_{\text{boost}}(s) = \frac{0.086021(s + 3.788 \times 10^4) (s + 2.362 \times 10^4)}{s^2 + 2374s + 5.403 \times 10^6}
\] (62)

To design FO-IMC controller for the buck converter, the plant model is first decomposed into minimum phase part and non-minimum phase part as follows
\[
G_{\text{mod}}(s) = \frac{2522.8(s + 10^7)}{s^2 + 1816s + 2.086 \times 10^7}, G_{\text{mod}}^+(s) = 1
\] (63)

Using equations (39) and (42), the FO-IMC controller is obtained as
\[
Q_r(s) = \frac{s^2 + 1816s + 2.086 \times 10^7}{2522.8(s + 10^7)(1 + 10^7 s^2 + 1)}
\] (64)

Finally, using Youla parameterization formula in equation (36), the controller in classical feedback form is given as
\[
C_{\beta1}(s) = \left(\frac{1816}{2522.8} + \frac{2.086 \times 10^7}{2522.8} \left(\frac{1}{s} + \frac{1}{s + 10^7}\right)\right)
\] (65)

Using equation (45), with the values of parameters as \( \omega_d = 800 \text{ rad/s} \) and \( \phi = 88^\circ \), the FO-IMC controller for the buck converter in classical feedback form is obtained as
\[
C_{\beta1}(s) = \left(\frac{668.0918 + \frac{7.6742 \times 10^6}{s} + 0.3679s}{s + 10^7}\right)
\] (66)

The damping factor of the response \( \xi_d \), obtained via application of FO-IMC technique on an ideal system, is related to the tuning parameter of the filter \( \zeta \) by \( \xi_d = - \cos(\pi/(\zeta + 1)) \). Ideally, the damping factor is desired to be close to unity to avoid higher overshoots and slow response times. Therefore, the value of \( \phi \) should take a value closer to 90\(^\circ\), so it is selected as 88\(^\circ\). Subsequently, \( \omega_{sc} \) is adjusted in an iterative fashion by investigating different values, checking the corresponding responses, and is finally chosen equal to 800 rad/s to obtain a desired response with comparatively quick response times and efficient disturbance rejection ability.

To demonstrate effectiveness of FO-IMC control scheme, an extensive comparative analysis is carried out with the TDF-IMC\(^{56} \) and IMC-PID\(^{22} \) approaches. The TDF-IMC set point tracking controller \( (Q_{\text{st}}(s)) \) and disturbance rejection controller \( (Q_d(s)) \) for buck converter are formulated using the technique proposed in Saxena and Hote\(^ {36} \) with the values of the time constant of the set point tracking filter and disturbance rejection filter as \( \lambda_s = 5 \times 10^{-2} \) and \( \lambda_d = 4 \times 10^{-3} \), respectively, and can be given as
\[
Q_{\text{st}}(s) = \frac{s^2 + 1816s + 2.086 \times 10^7}{0.1261s^2 + 1.514 \times 10^5s + 2.523 \times 10^8}
\] (67)

\[
Q_d(s) = \frac{1.495 \times 10^{-5}s^2 - 0.008638s + 1}{1.6 \times 10^{-5}s^2 + 0.008 + 1}
\]

The IMC-PID controller for the buck converter using the value of the time constant of filter \( \lambda = 6 \times 10^{-3} \) and the technique proposed in Saxena and Hote\(^ {22} \) is obtained as
\[
C_1(s) = \left(\frac{119.9725 + \frac{1.3781 \times 10^6}{s} + 0.0661s}{1} \right)
\] (68)

In a similar manner, FO-IMC controller can be formulated for the boost converter as well. The minimum phase and non-minimum phase parts of the boost converter are given as
\[
G_{\text{mod}}^-(s) = \frac{2.0318 \times 10^4(s + 3.788 \times 10^4)}{s^2 + 2374s + 5.403 \times 10^6},
\]
\[
G_{\text{mod}}^+(s) = (1 - 4.237 \times 10^{-5}s)
\] (69)

Using equations (39) and (42), the FO-IMC controller is obtained as
\[
Q_r(s) = \frac{s^2 + 2374s + 5.403 \times 10^6}{2.0318 \times 10^4(s + 3.788 \times 10^4)(1 + 10^7 s^2 + 1)}
\] (70)

Finally, using Youla parameterization formula in equation (36), the FO-IMC controller in classical feedback form is obtained as
\[
C_{\beta2}(s) = \left(\frac{2374}{2031.8} + \frac{5.403 \times 10^6}{2031.8} \left(\frac{1}{s} + \frac{1}{2031.8}\right) \right)
\]
\[
\left(\frac{1}{s + 10^7}\right) \left(\frac{1}{10^7 s + 4.237 \times 10^{-5}}\right)
\] (71)

Using equation (45), with the values of parameters as \( \omega_d = 350 \text{ rad/s} \) and \( \phi = 89^\circ \), the transfer function of FO-IMC controller for the boost converter in classical feedback form is computed as
\[
C_{\beta2}(s) = \left(\frac{1.1684 + \frac{2.6592 \times 10^5}{s} + 4.9217 \times 10^{-4}s}{s + 10^7}\right)
\]
\[
\left(\frac{1}{0.00270^{0.111} + 4.237 \times 10^{-5}}\right) \left(\frac{1}{s + 3.788 \times 10^4}\right)
\] (72)
The TDF-IMC set point tracking \( Q_r(z) \) and disturbance rejection controller \( Q_d(z) \) for boost converter are formulated using the technique proposed in Saxena and Hote\(^36\) with the values of the time constant of the set point tracking filter and disturbance rejection filter as \( \lambda_r = 3.3 \times 10^{-3} \) and \( \lambda_d = 2.5 \times 10^{-3} \), respectively, and can be given as

\[
Q_r(z) = \frac{s^2 + 2374s + 5.403 \times 10^6}{0.002213s^3 + 9.722s^2 + 5.283 \times 10^5s + 7.697 \times 10^7 -1.554 \times 10^{-6}s^2 - 0.01044s + 1 \over 6.25 \times 10^{-8}s^2 + 0.005s + 1}
\]

(73)

The IMC-PID controller for the boost converter using the value of the time constant of filter \( \lambda = 6 \times 10^{-3} \) and the technique proposed in Saxena and Hote\(^22\) is obtained as

\[
C_2(s) = \left( \frac{193.3689 + 4.409 \times 10^5}{s} + 0.0815s \right) \left( \frac{1}{s + 3.788 \times 10^4} \right)
\]

(74)

To scrutinize the effectiveness of FO-IMC scheme for buck and boost converter, various case studies are carried out that are explained below.

**Transient analysis**

**Buck converter.** The tracking response for a buck converter is investigated via the introduction of a reference signal, that is, a combination of step input and ramp input, and can be mathematically represented as

\[
r(t) = 4[u(t) - u(t - 0.03)] + \frac{4.33}{0.03} (tu(t - 0.03) - tu(t - 0.06)) + 8.33u(t - 0.06)
\]

(75)

The response of FO-IMC, IMC-PID and TDF-IMC techniques to the reference input is shown in Figure 5. It can be deduced that the controller formulated via FO-IMC technique tracks the reference input closely and swiftly, in comparison to the IMC-PID and TDF-IMC techniques, which exhibits sluggish behaviour in the transient phase.

**Boost converter.** In a similar manner, we undertake the transient response analysis for the boost converter. The response of FO-IMC, IMC-PID and TDF-IMC techniques to the reference input is shown in Figure 6. It can be ascertained that FO-IMC technique tracks the reference signal quickly with minimal overshoot, as compared to IMC-PID and TDF-IMC techniques, which track the reference signal changes slowly in the transient phase.

Hence, it can be deduced that FO-IMC exhibits an improved and an excellent tracking behaviour over the IMC-PID and TDF-IMC techniques.

**Voltage regulation**

**Buck converter.** The closed loop voltage regulation for buck converter is investigated via introduction of following disturbances:

1. Step up variation in input voltage from 12 V to 15 V.
2. Step down change in input voltage from 12 V to 9 V.

Figures 7 and 8 depict the voltage regulation performance of buck converter, while input voltage is perturbed from 12 V to 15 V and from 12 V to 9 V, respectively. It is noticed that FO-IMC scheme ensures a quicker rejection of disturbances and the output voltage returns to nominal value swiftly. However, in TDF-IMC and IMC-PID control techniques, the transient response as well as disturbance rejection of the system exhibit sluggish behaviour.
Boost converter. The closed loop voltage regulation for boost converter is investigated via introduction of following perturbations:

1. Step up variation in input voltage from 5 V to 7 V.
2. Step down decrement in input voltage from 5 V to 3 V.

Figures 9 and 10 illustrate the voltage regulation ability of the boost converter, when input voltage is perturbed from 5 V to 7 V and from 5 V to 3 V, respectively. The disturbance rejection capability of FO-IMC scheme is clearly illustrated by the smaller settling times accompanied by smaller overshoots on the introduction of perturbation in input voltage.

Figures 9 and 10 illustrate the voltage regulation ability of the boost converter, when input voltage is perturbed from 5 V to 7 V and from 5 V to 3 V, respectively. The disturbance rejection capability of FO-IMC scheme is clearly illustrated by the smaller settling times accompanied by smaller overshoots on the introduction of perturbation in input voltage.

Set point tracking

Buck converter. The closed loop set point tracking ability of FO-IMC scheme for buck converter is scrutinized for the following cases:

1. Step up increment in reference voltage from 8 V to 10 V.
2. Step down perturbation in reference voltage from 8 V to 6 V.

Figures 11 and 12 exhibit the set point tracking performance, when the reference voltage for boost converter is given a step change from 8 V to 10 V and from 8 V to 6 V, respectively. It can be clearly noticed that FO-IMC scheme tracks the changes in set point accurately and quickly in comparison to IMC-PID and TDF-IMC controllers.

Boost converter. The closed loop set point tracking ability of boost converter is examined via following case study:

1. Step up variation in reference voltage from 8.33 V to 10 V.
2. Step down variation in reference voltage from 8.33 V to 6 V.
Figures 13 and 14 reveal the set point tracking ability of FO-IMC scheme, when compared to TDF-IMC scheme, while a step change is introduced in the reference voltage from 8.33 V to 10 V and from 8.33 V to 6 V, respectively. An improved set point tracking performance is observed for the FO-IMC scheme, when compared to IMC-PID and TDF-IMC schemes.

Load regulation

Buck converter. The load regulation capability of the buck converter is examined for the following cases:

1. Step up increment in load resistance from 10 Ω to 5 Ω.
2. Step down change in load resistance from 10 Ω to 15 Ω.

Figures 15 and 16 exhibit the load disturbance rejection capability of FO-IMC technique versus existing TDF-IMC approach. Although the FO-IMC technique exhibits a slightly larger undershoot upon the occurrence of load disturbance; however, it rejects the disturbance quickly in minimum time in comparison to IMC-PID and TDF-IMC schemes. The overall effectiveness of FO-IMC technique is judged via computation of integral absolute error (IAE) and integral time absolute error (ITAE) in Table 1. It can be observed that FO-IMC technique has lower values of both IAE and ITAE in comparison to IMC-PID and TDF-IMC schemes, thereby demonstrating the efficacy of FO-IMC technique.

Boost converter. The load regulation capability of FO-IMC technique for boost converter is investigated for the following cases:

1. Change in load resistance from 22 Ω to 30 Ω.
2. Change in load resistance from 22 Ω to 20 Ω.

Figures 17 and 18 depict the load regulation ability of FO-IMC technique, in comparison to TDF-IMC
approach, when the load resistance is varied from 22Ω to 30Ω and 22Ω to 20Ω, respectively. An improved load regulation capability is seen for FO-IMC controller in comparison to TDF-IMC and IMC-PID schemes. Further investigation reveals that below the value of 20Ω, the FO-IMC controlled boost converter system does not track the reference voltage of 8.33 V, although it is still stable, which is a limitation of the proposed technique.

Furthermore, a comparative analysis of the performance indices, that is, IAE and ITAE is conducted for all the aforementioned cases in Table 1 and 2 for buck converter and boost converter, respectively. It can be deduced that FO-IMC scheme yields a less value of

| Table 1. Comparison of performance indices for buck converter. |
| --- |
| | Technique | Parameter | Variation | IAE | ITAE |
| | FO-IMC | Input voltage | 12–15 V | 0.01992 | 0.000166 |
| | Reference voltage | 12–9 V | 0.01827 | 0.0001143 |
| | | 8–10 V | 0.01981 | 0.0001616 |
| | Load resistance | 8–6 V | 0.01998 | 0.0001683 |
| | | 10–15 Ω | 0.01705 | 0.0000757 |
| | IMC-PID | Input voltage | 10–5 Ω | 0.01798 | 0.0001026 |
| | Reference voltage | 12–15 V | 0.05035 | 0.000489 |
| | Load resistance | 8–6 V | 0.05205 | 0.0005543 |
| | 10–15 Ω | 0.04332 | 0.0002494 |
| | TDF-IMC | Input voltage | 10–5 Ω | 0.04486 | 0.0003001 |
| | Reference voltage | 12–9 V | 0.05233 | 0.0005607 |
| | Load resistance | 8–6 V | 0.05205 | 0.0005543 |
| | 10–15 Ω | 0.04332 | 0.0002494 |

IAE: integral absolute error; ITAE: integral time absolute error; FO-IMC: fractional order internal model control; IMC: internal model control; PID: proportional integral derivative; TDF-IMC: 2 degrees of freedom internal model control.

| Table 2. Comparison of performance indices for boost converter. |
| --- |
| | Technique | Parameter | Variation | IAE | ITAE |
| | FO-IMC | Input voltage | 5–7 V | 0.03429 | 0.0004572 |
| | Reference voltage | 5–3 V | 0.02979 | 0.000323 |
| | Load resistance | 8.33–10 V | 0.03104 | 0.0003677 |
| | IMC-PID | Input voltage | 22–20 Ω | 0.03382 | 0.0004042 |
| | Reference voltage | 22–30 Ω | 0.02938 | 0.000325 |
| | Load resistance | 8.33–6 V | 0.03269 | 0.0003939 |
| | TDF-IMC | Input voltage | 8.33–6 V | 0.03768 | 0.0004245 |
| | Reference voltage | 8.33–10 V | 0.03741 | 0.0003939 |
| | Load resistance | 22–30 Ω | 0.04602 | 0.0009505 |

IAE: integral absolute error; ITAE: integral time absolute error; FO-IMC: fractional order internal model control; IMC: internal model control; PID: proportional integral derivative; TDF-IMC: 2 degrees of freedom internal model control.

Figure 15. Load regulation behaviour of buck converter when load resistance is changed from 10 Ω to 15 Ω.

Figure 16. Load regulation behaviour of buck converter when load resistance is changed from 10 Ω to 5 Ω.
IAE and ITAE in comparison to IMC-PID and TDF-IMC schemes.

**Parametric uncertainty**

The parameters of a power electronic converter such as resistance, inductance and capacitor vary with the change in environmental conditions. Therefore, it is vital to validate the robustness of the FO-IMC controller in the presence of parametric uncertainty. The proposed controller should exhibit desired performance, even in the presence of parametric variations, without an appreciable effect on system performance. Hence, a ±50% parametric uncertainty is considered to validate the robustness for both the buck and boost converters.

**Buck converter.** The parameters of the buck converter are varied as follows

\[
\begin{align*}
    r_{ip} &\in [0.015 \Omega, 0.045 \Omega], L_o \in [244.5 \mu H, 733.5 \mu H] \\
    R_{if} &\in [0.12 \Omega H, 0.36 \Omega H], C_o \in [50 \mu F, 150 \mu F] \\
    R_{cf} &\in [0.05 \Omega F, 0.15 \Omega F], R_{di} \in [0.015 \Omega, 0.045 \Omega] \\
    R_{sw} &\in [0.025 \Omega, 0.075 \Omega], r_{Lo} \in [5 \Omega, 15 \Omega] \\
\end{align*}
\]  

(76)

Figure 19 depicts the system response, when a ±50% uncertainty is introduced into the system parameters as given in equation (76). It can be seen from Figure 19 that the system responses in the presence of nominal parameters, lower bound uncertainty and upper bound uncertainty are almost coincident, thereby validating the robustness of the proposed technique to parametric uncertainty.

**Boost converter.** The parameters of the boost converter are varied as follows

\[
\begin{align*}
    r_{ip} &\in [0.1 \Omega, 0.3 \Omega], L_o \in [125 \mu H, 375 \mu H] \\
    R_{if} &\in [0.12 \Omega H, 0.36 \Omega H], C_o \in [110 \mu F, 330 \mu F] \\
    R_{cf} &\in [0.06 \Omega F, 0.18 \Omega F], R_{di} \in [0.015 \Omega, 0.045 \Omega] \\
    R_{sw} &\in [0.025 \Omega, 0.075 \Omega], r_{Lo} \in [11 \Omega, 33 \Omega] \\
\end{align*}
\]  

(77)

The robustness of the boost converter is demonstrated via investigation of the system response in the presence of lower bound (−50%) and upper bound (+50%) parametric uncertainty in system parameters. It can be observed from Figure 20 that even in the presence of parametric uncertainty, the proposed FO-IMC technique exhibits a stable response and tracks the set point effectively with good transient behaviour. Hence, it can be concluded from the aforementioned analysis that FO-IMC technique can be a viable approach to handle the voltage regulation, set point tracking as well as parametric uncertainty for both dc–dc buck and boost power converters.

**Experimental results**

To demonstrate, the hardware implementation of FO-IMC scheme on buck and boost converter, a practical laboratory setup\(^3\) is shown in Figure 21. Depending on availability, diode MUR1560 and MOSFET IRPF460 are chosen as switching devices. The controller is implemented via dSPACE 1104 microcontroller board. A
voltage sensor AD202JN is utilized to sense the output voltage, which is fed to ADC channel of dSPACE 1104.

The voltage regulation ability of buck converter is assessed via introduction of step up and step down perturbation in input voltage, as shown in Figures 22 and 23, respectively. The corresponding plots for boost converter are shown in Figures 24 and 25.

To evaluate the set point tracking ability of buck converter, a step change is introduced in reference voltage from 8 V to 10 V and from 10 V to 8 V. The corresponding response is illustrated in Figure 26. In a similar manner, a step change is introduced in reference voltage from 8 V to 6 V and from 6 V to 8 V, as seen in Figure 27. For the boost converter, the set point tracking capability is shown in Figures 28 and 29. Finally, the load disturbance rejection capability for the buck converter is scrutinized via change in the load resistance from 10 Ω to 22 Ω and 22 Ω to 10 Ω and the corresponding output voltage is shown in Figure 30. For the boost converter, the load regulation capability, when the load resistance is changed from 22 Ω to 11 Ω and 11 Ω to 22 Ω, is shown in Figure 31.

It can be observed from hardware results that FO-IMC technique exhibits a good input disturbance rejection capability, efficient load disturbance rejection ability and also tracks the changes in reference voltage.
**Figure 24.** Regulatory behaviour of boost converter when input voltage is changed from 5 V to 7 V and 7 V to 5 V.

**Figure 25.** Regulatory behaviour of boost converter when input voltage is changed from 5 V to 3 V and 3 V to 5 V.

**Figure 26.** Set point tracking behaviour of buck converter when reference voltage is changed from 8 V to 10 V and 10 V to 8 V.

**Figure 27.** Set point tracking behaviour of buck converter while reference voltage is changed from 8 V to 6 V and 6 V to 8 V.
**Figure 28.** Set point tracking behaviour of boost converter when reference voltage is changed from 8.33 V to 10 V and 10 V to 8.33 V.

**Figure 29.** Set point tracking behaviour of boost converter when reference voltage is changed from 8.33 V to 6 V and 6 V to 8.33 V.

**Figure 30.** Output voltage in boost converter when load resistance is changed from 10 Ω to 22 Ω and 22 Ω to 10 Ω.

**Figure 31.** Output voltage in boost converter when load resistance is changed from 22 Ω to 11 Ω and 11 Ω to 22 Ω.
effectively and quickly, thereby affirming the practical validity of FO-IMC scheme.

**Conclusion**

In this paper, a FO-IMC technique is formulated, such that the final controller is obtained as a series combination of PID controller and FO filter. The non-ideal models of both buck and boost converters are derived via the consideration of all the parasitic elements such as forward voltage of diode, equivalent resistances of inductance and capacitance and resistance of semiconductor devices, while they are working in ON mode. An extensive case study is conducted to validate the efficacy of the FO-IMC technique. The transient performance is investigated by the introduction of a reference signal, which is a combination of step and ramp reference signals. The robustness of the controller is verified via the introduction of parametric uncertainty as well as perturbations in input voltage, reference voltage and load resistance. In all the cases, it is observed that the FO-IMC technique exhibits an improved transient performance and an enhanced robustness in comparison to the existing TDF-IMC scheme. Most importantly, the experimental validation is undertaken via real-time hardware, which is a testimony to the efficacy and practical implementation of FO-IMC scheme. The future scope of this work involves the analysis of the dc–dc power converters under motor load or resistor-inductor (RL) load.

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