COMPACT OBJECTS WITH LARGE MASSES.

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Abstract

It is shown that gravitational theory allows stable equilibrium configurations of the degenerated Fermi-gas, whose masses $M \gg M_\odot$ and sizes are less than $\alpha = 2GM/c$ ( $G$ is the gravitational constant and $c$ is the speed of light). They have no events horizon at the distance $r = \alpha$ from the center. This are objects with low luminosity and are a new candidate in Dark Matter in the Universe. Orbits of test particles near the above objects are considered.

1 Introduction.

Our knowledge about the limiting masses of equilibrium stars configurations and about the fate of collapsing objects is based on the predictions of Einstein’s general relativity. The astrophysical data do not contradict these results, however, they do not prove their validity unambiguously. When analyzing the observation data it is necessary to take into consideration the fact that there also exist other gravitation theories which satisfy (like the general relativity) the post-Newtonian tests, predicting, however, other physical results in strong fields. (See, for example, [1] and [2]). In this paper, starting from the gravitational equations proposed before [3], we show that stable equilibrium configurations of a degenerated Fermi-gas can exist with
the following properties: their masses \( M \) are many times as large as the Sun and their sizes are smaller than \( \alpha = 2GM/c \), where \( G \) is the gravitational constant and \( c \) is the speed of light.

2 Spherically-Symmetric Gravitational Field.

The above mentioned gravitation equations are

\[
B^\gamma_{\alpha\beta,\gamma} - B^\epsilon_{\alpha\delta} B^\delta_{\beta\epsilon} = 0
\]

(1)

These are vacuum bimetric equations for the tensor

\[
B^\gamma_{\alpha\beta} = \Pi^\gamma_{\alpha\beta} - \Pi^\gamma_{\alpha\beta}
\]

(2)

(Greek indices run from 0 to 3), where

\[
\Pi^\gamma_{\alpha\beta} = \Gamma^\gamma_{\alpha\beta} - (n + 1)^{-1} \left[ \delta^\gamma_\alpha \Gamma^\epsilon_{\beta\epsilon} - \delta^\gamma_\beta \Gamma^\epsilon_{\epsilon\alpha} \right],
\]

(3)

\[
\Pi^\gamma_{\alpha\beta} = \Gamma^\gamma_{\alpha\beta} - (n + 1)^{-1} \left[ \delta^\gamma_\alpha \Gamma^\epsilon_{\beta\epsilon} - \delta^\gamma_\beta \Gamma^\epsilon_{\epsilon\alpha} \right],
\]

(4)

\( \Gamma^\gamma_{\alpha\beta} \) are the Christoffel symbols of the pseudo-Euclidean space-time \( E \) whose fundamental tensor is \( \eta_{\alpha\beta} \), \( \Gamma^\gamma_{\alpha\beta} \) are the Christoffel symbols of the Riemannian space-time \( V \) of dimension \( n \), whose fundamental tensor is \( g_{\alpha\beta} \). The comma in eqs. (1) denotes the covariant differentiation in \( E \).

A peculiarity of eqs.(1) is that they are invariant under arbitrary transformations of the tensor \( g_{\alpha\beta} \) retaining invariant the equations of motion of a test particle, i.e. geodesics in \( V \). In other words, eqs.(1) are the geodesic-invariant. Thus, the tensor field \( g_{\alpha\beta} \) is determined up to geodesic mappings of space-time \( V \) (in the way analogous to that the potential \( A \) in electrodynamics is determined up to gauge transformations). Therefore, additional conditions can be imposed on the tensor \( g_{\alpha\beta} \). In particular, if the tensor \( g_{\alpha\beta} \) satisfies the conditions

\[
Q_\alpha = \Gamma^\sigma_{\alpha\sigma} - \Gamma^\sigma_{\alpha\sigma},
\]

(5)

then eqs (1) will be reduced to vacuum Einstein equations \( R_{\alpha\beta} = 0 \), where \( R_{\alpha\beta} \) is the Ricci tensor. As distinguished from the \( g_{\alpha\beta} \) or \( \Gamma^\gamma_{\alpha\beta} \) the tensor \( B^\gamma_{\alpha\beta} \) is
invariant under geodesic mappings of space-time \( V \). It is in a sense analogous to strength tensor \( F_{\alpha\beta} \) in electrodynamics which is invariant under the gauge transformations. Let us find the spherically symmetric solution of eqs.\(^{[4]}\) for a distant observer. We suppose that the observer’s frame of reference is an inertial one and space-time is pseudo-Euclidean. Choose a spherical coordinate system. Then, if a test particle lagrangian is invariant under the mapping \( t \to -t \), the fundamental metric form of space-time \( V \) can be written as

\[
ds^2 = -A(dr)^2 - B[(d\theta)^2 + \sin^2(\theta)(d\varphi)^2] + C(dx^0)^2,
\]

where \( A, B \) and \( C \) are the functions of the radial coordinate \( r \). Proceeding from the above-stated, we shall find the functions \( A, B \) and \( C \) as the solution of the equations system

\[
R_{\alpha\beta} = 0 \quad (7)
\]

and

\[
Q_\alpha = 0 \quad (8)
\]

which satisfy the conditions:

\[
\lim_{r \to \infty} A = 0 \quad \lim_{r \to \infty} B = 0 \quad \lim_{r \to \infty} B = 0 \quad (9)
\]

The equations \( R_{11} = 0 \) and \( R_{00} = 0 \) can be written as

\[
BL_2 - 2CL_1 = 0 \quad (10)
\]

and

\[
B'C' - 2BL_2 = 0 \quad (11)
\]

where

\[
L_1 = R_{1212} = B''/(4B) - (B'/4A)/(4B) - B'/4A,
\]

\[
L_2 = R_{1010} = -C''/2 + C'(AC)'/(4AC)
\]

and the differentiation of \( A, B \) and \( C \) with respect to \( r \) is denoted by primes.

Eqs.\(^{(8)}\) give

\[
B^2AC = r^4 \quad (12)
\]

First, the combination of eqs.\(^{(10)}\) and \(^{(11)}\) gives

\[
4CL_1 - B'C' = 0
\]
\[ 2B'' - (B')^2/B - B'(AC)'/(AC) = 0. \]  \hfill (13)

Also, taking the logarithm of eq. (12) we obtain

\[ (AC)'/(AC) = 4/r - 2B'/B = 0. \]  \hfill (14)

Equation (13) then becomes

\[ 2B'' + (B')^2/B - 4B'/r = 0, \]

or

\[ u'/u = 4/r, \]  \hfill (15)

where \( u = (B')^2B \). By using (9) we find

\[ B = (r^3 + \beta^3)^{2/3}, \]  \hfill (16)

where \( \beta \) is a constant.

Next, from eq. (11) we find by using eqs. (9)

\[ C = 1 - \alpha/B^{1/2}, \]  \hfill (17)

where \( \alpha \) is a constant. The non-relativistic limit should be considered in order to find this constant. The equations of motion of the particle will coincide with the Newtonian ones if \( \alpha = 2GM/c^2 \).

Finally, we can find from eq. (12) the function \( A \). The constant \( \beta \) in eq. (16) remains indeterminate. Since \( Q_\alpha \) is a vector, the gauge condition (8) is covariant. Different constants \( \beta \) cause the different solutions of eq. (1) in the used coordinate system. Setting \( \beta = \alpha \) we shall obtain the functions \( A, B \) and \( C \) which have no singularity at \( r \to 0 \):

\[ A = (f')^2(1 - \alpha/f)^{-1}, \quad B = f^2, \quad C = 1 - \alpha/f, \]  \hfill (18)

where

\[ f = (r^3 + \alpha^3)^{1/3} \]

and \( f' = df/dr \).

We consider solution (18) as the basis for the analysis below.
3 Equilibrium configurations.

The gravity force in the spherically-symmetric field is given by

$$F^i = -m(B^i_{\beta\gamma} - B^0_{\alpha\beta} \dot{x}^i) \dot{x}^\alpha \dot{x}^\beta$$  \hspace{1cm} (19)

where $\dot{x}^i = dx^i/dt$. (Latin indices run from 1 to 3). The gravitational force affecting the a test particle of the mass $m$ in rest ($\dot{x}^i = 0$) is given by

$$F = -\frac{G m M}{r^2} \left(1 - \frac{\alpha}{f}\right),$$ \hspace{1cm} (20)

where $G$ is the gravitational constant, $\alpha = 2GM/c^2$, $c$ is the speed of light, $f = (\alpha^3 + r^3)^{1/3}$. Fig. 1 shows the plot of the function $F_1 = -(1/2 \tau^2)(1 - \alpha/f)$ against the distance $\tau = r/\alpha$.

![Plot of F_1 vs \tau](image)

Fig. 1 The plot of the function $F_1$ against the $\tau = r/\alpha$

It follows from Fig.1 that the $|F|$ reaches its maximum at $r$ of the order of $\alpha$ and tends to zero at $r \to 0$. It would therefore be interesting to know what the limiting masses of the equilibrium configurations the gravitational force $F(r)$ (20) can admit. To answer this question we start from the equation

$$\frac{dp}{dr} = -\frac{G p M}{r^2 \tau^2} (1 - \alpha/f)$$ \hspace{1cm} (21)
In this equation $\rho$ is the pressure, $M = M(r)$ is the matter mass inside of a sphere of the radius $r$, $\rho = \rho(r)$ is the matter density at the distance $r$ from the center, $\alpha$ and $f$ is the function of $M(r)$.

Suppose the equation of state is $p = K\rho^{\Gamma}$, where $K$ and $\Gamma$ are constants. For numerical estimates we shall use their values [4]:

For a degenerated electron gas:
- $\Gamma = \frac{5}{3}$, $K = 1 \cdot 10^{13}$ SGS units at $\rho \ll \rho_0$, where $\rho_0 = 10^6$ gm/cm$^3$;
- $\Gamma = \frac{4}{3}$, $K = 1 \cdot 10^{15}$ SGS units at $\rho \gg \rho_0$.

For degenerated neutron gas:
- $\Gamma = \frac{5}{3}$, $K = 5 \cdot 10^9$ SGS units at $\rho \ll \rho_0$, where $\rho_0 = 5 \cdot 10^{15}$ gm/cm$^3$;
- $\Gamma = \frac{4}{3}$, $K = 1 \cdot 10^{15}$ SGS units at $\rho \gg \rho_0$.

For rough estimates we replace $dp/dr$ by $-p/r$, where $p$ is the average matter pressure and $R$ is its radius. Under the circumstances we obtain from eq.(21)

\[
\frac{p}{\rho c^2} = \frac{\alpha}{2R} (1 - \alpha/f).
\] (22)

If $R \gg \alpha$, then the term $\alpha/f$ is negligible. Setting $M(R) \approx \rho R^3$ we find the mass of equilibrium states as a function of $\rho$:

\[
M = \left( \frac{K}{G} \right)^{3/2} \rho \left( \Gamma - 4/3 \right)^{3/2}.
\] (23)

It follows from eq.(23) that there is the maximal mass [4] $M = \left( \frac{K}{G} \right)^{3/2}$ at $\rho \gg \rho_0$.

However, according to eq.(22), there are also equilibrium configurations at $R < \alpha$. In particular, at $R \ll \alpha$ we find from eq.(22) that the masses of the equilibrium configurations are given by

\[
M = c^{9/2} 10^{-1} K^{-3/4} G^{-3/2} \rho^{-\left( \Gamma - 1/3 \right)/3/4}.
\] (24)

These are the configurations with very large masses. For example, the following equilibrium configurations can be found:

- the nonrelativistic electrons: $\rho = 10^5$ gm/cm$^3$, $M = 1.3 \cdot 10^{42}$ gm, $R = 2.3 \cdot 10^{12}$ cm;
- the relativistic electrons: $\rho = 10^7$ gm/cm$^3$, $M = 2.3 \cdot 10^{40}$ gm, $R = 1.3 \cdot 10^{11}$ cm;
- the nonrelativistic neutrons: $\rho = 10^{14}$ gm/cm$^3$, $M = 3.9 \cdot 10^{35}$ gm, $R = 1.6 \cdot 10^7$ cm.
The reason of the two types of configurations existence can be seen from Fig. 2, where for $\rho = 10^{15} \text{ gm/cm}^3$ the plots of right-hand and left-hand sides of Eq.(3) against the mass $M$ are given.

![The plot of right-hand ($W_2(M)$) and left-hand ($W_1(M)$) sides of Eq.(22) against $M$.](image)

The following conclusions can be made after considering the plots of the above kind:
1. There are no equilibrium configurations whose the density is larger than a certain value $\rho_{\text{max}} \sim 10^{16} \text{gm/cm}^3$.
2. For each value of $\rho < \rho_{\text{max}}$ there are two equilibrium states (with $R > \alpha$ and $R < \alpha$).

Are the configurations with large masses stable?

The total energy of the degenerate gase is $E = E_{\text{int}} + E_{\text{gr}}$, where $E_{\text{int}}$ is the intrinsic energy and $E_{\text{gr}}$ is the gravitational energy. The gravitational energy of a sphere is

$$E_{\text{gr}} = \int_{\infty}^{R} dM(r) \chi(r) M(r),$$

(25)

where

$$\chi(r) = \int_{r}^{\infty} dr' (r')^{-2}(1 - 1/f),$$
\[ \alpha = 2GM(r)/c^2, \quad f = (\alpha(r)^3 + (r')^3)^{1/3}, \]

\[ M(r) = 4\pi \int_0^r dr' \rho (r'^2). \]

The function \( \chi(r) \) is approximately

\[ \chi(r) = (1/r)(1 - \exp(-r/\alpha)). \quad (26) \]

Therefore, at \( p = \text{const} \) up to a constant of the order one

\[ E_{gr} = -\frac{GM^2}{R}(1 - \exp(-R/\alpha)). \quad (27) \]

The intrinsic energy \( E_{\text{int}} = \int u \, dM \), where \( u \) is the energy per the mass unit. For the used equation of state \( u = K(\Gamma - 1)^{-1}\rho^{\Gamma - 1} \). Thus, up to a constants of the order of one

\[ E = KMP^{-1} - GM^{5/3}\rho^{1/3}[1 - \exp(-QM^{-2/3}\rho^{-1/3})], \quad (28) \]

where \( Q = c^2/2G \). As an example, Fig.3 and Fig.4 show the plot of the function \( E = E(\rho) \) for the nonrelativistic neutron gas of the mass \( M = 10^{36} \, \text{gm} \) and \( M = 10^{33} \, \text{gm} \) (neutron stares) correspondingly.

Fig. 3 The plot of the function \( E = E(\rho) \) for the neutron configuration of the mass \( M = 10^{36} \, \text{gm} \).
Fig. 4 The plot of the function $E = E(\rho)$ for the neutron star of the mass $M = 10^{33}$ gm.

The analysis of such plots show that the function $E = E(\rho)$ has the minimum. Thus, the above equilibrium states of large masses are stable.

4 Orbits of non-zero mass particles.

The equations of motion of a test particle of non-zero mass in the spherically symmetric field are given by

$$r^2 = (c^2C/A)[1 - (C/E)(1 + \alpha^2\overline{J}^2/B)],$$

$$\dot{\phi} = c C\overline{J}\alpha/(B\overline{E})$$

where $(r, \phi, \theta)$ are the spherical coordinates (\theta is supposed to be equal to $\pi/2$), $\dot{r} = dr/dt$, $\dot{\phi} = d\phi/dt$, $\overline{E} = E/(mc)$, $\overline{J} = J/(amc)$, $E$ is the particle energy, $J$ is the angular momentum. Let $\overline{\mu} = 1/\overline{J}$, where $\overline{J} = (1 + \overline{r}^3)^{1/3}$ and $\overline{r} = r/a$. Then the differential equation of the orbits, following from eqs. (29) and (30) can be written as

$$d\overline{\mu}/d\phi = \mathcal{G}(\overline{\mu}),$$

9
where

$$G(\pi) = \pi^3 - \pi^2 + \pi / J^2 + (E^2 - 1) / J^2.$$ 

Eq. (31) differs from the orbit equations of general relativity [3] by the function $u$ instead of the function $1 / r$. Therefore, the distinction in the orbits becomes apparent only at the distances $r$ of the order of 1 or less than that.

Setting $\dot{r} = 0$ in eq. (29) we obtain $E^2 = N(r)$, where

$$N(r) = (1 - 1/J)(1 + J^2 / E^2)$$

is the effective potential [3]. Fig. 5 shows the function $N = N(\pi)$ at the typical value of $J = 4$.

Fig. 5 The effective potential $N = N(\pi)$ at $J = 4$. The horizontals $E = E_1$ and $E = E_2$ show the types of particle orbits due to lack of the event horizon.

The function $N(\pi)$ differs from the one in general relativity in two respects: 1) it is determined at every point of the interval $(0, \infty)$ and 2) it tends to zero when $\pi \to 0$.

Possible orbit types can be shown by the horizontals $E = Const$. The orbits with energies $E_1$ and $E_2$ in Fig. 5 have the peculiarity due to the lack of the event horizon. The first orbit type, with energy $E_1$, exceeding the
maximum of the function $\nu(r)$, shows that for each value of $\mathcal{J}$ there exists such value of $\mathcal{E}$ that the gravitational field cannot keep a particle escaping from the center. Fig.6 shows the difference in the orbits of the above type and the ones in general relativity.

Fig.6 The particle orbits at $\mathcal{E} = 1$ and $\mathcal{J} = 1.99$ according to eq.(31) (curve 1) and the one in general relativity (curve 2, dots). At the distances $\tau > 1$ the curves are very close together. The coordinates $x$ and $y$ are determined in the following way: $x = \tau \cos(\varphi)$ and $y = \tau \sin(\varphi)$.

The orbits of the second type, with the energy $\mathcal{E}_2$, are the ones of particles kept by the gravitational field near the field center.

It follows from eqs. (23) and (31) that the velocity of a test particle freely falling to the point mass $M$ tends to zero when $r \to 0$. The time of the motion of the particle from some distance $r = r_0$ to $r = 0$ is infinitely large.

In the effective potential plot the stable circular orbits are shown by the minimum points of the function $\mathcal{N}(\tau)$, the unstable ones — by the maximum points. The minimums of the function $\mathcal{N}(\tau)$ exists only at $\mathcal{J} > \sqrt{3}$ which corresponds to the value of the function $f(\tau) > 3$. Therefore, stable circular orbits exist only at $\tau > \tau_{cr}$, where $\tau_{cr} = \sqrt{26} \alpha \approx 2.96 \alpha$. The orbital speed of the particle with $r = r_{cr}$ is equal to $0.4c$. At $r < r_{cr}$ unstable
circular orbits can exist. At $J \to \infty$ the location of the maximums tends to $\mathcal{f} = 3/2$. Therefore, the minimum radius of the nonstable circular orbit is $r_{\text{min}} = 1.33\alpha$. (In general relativity it is equal to $1.5\alpha$). The speed of the motion of a particle on this orbit is equal to $0.51c$. The binding energy $E = 0.0572$, just as it occurs in general relativity.

The rotation frequency $\omega = \dot{\phi}$ of the circular motion will be

$$\omega = \left[c(\mathcal{f} - 1)(\mathcal{f}^3\alpha)(\mathcal{J}/E)\right] (33)$$

In a circular motion $\mathcal{r}$ is the constant and, therefore, the function $\mathcal{N}(\mathcal{r})$ has the minimum. Consequently, from the equation $d\mathcal{N}/dr = 0$ we find

$$\mathcal{J}^2 = \mathcal{f}^2/(2\mathcal{f} - 3) \quad (34)$$

Using (29) we have at $\dot{r} = 0$ for $E^2 = \mathcal{N}$:

$$E^2 = 2(\mathcal{f} - 1)/[\mathcal{f}(2\mathcal{f} - 1)] \quad (35)$$

From the eqs. (32) — (34) we obtain

$$\omega = \alpha^{1/2}c/\left[\sqrt{2/\mathcal{f}^3}\right].$$

Hence, the circular orbits have rotation period

$$T = 2\sqrt{2}\pi c^{-1}\alpha(1 + \mathcal{r}^3)^{1/2} \quad (36)$$

(3 rd Kepler law). In comparison with general relativity the change in $T$ is 2% at $\mathcal{r} = 3$ and 20% at $\mathcal{r} = 1.33$.

Let us find the apsidal motion. For ellipsoidal orbits the function $G(\pi)$ has 3 real roots $\pi_1 < \pi_2 < \pi_3$. The apsidal motion per one period is

$$\delta \phi = 2|\Delta| - 2\pi \quad (37)$$

where

$$\Delta = \int_{\pi_1}^{\pi_2} [G(\pi)]^{1/2} d\pi \quad (38)$$

We have

$$\Delta = 2(\pi_3 - \pi_1)^{1/2}\mathcal{F}(\pi/2, q) \quad (39)$$
where
\[ q = \left[ (\bar{u}_2 - \bar{u}_3) / (\bar{u}_3 - \bar{u}_1) \right]^{1/2} \tag{40} \]
and
\[ F(\pi/2, q) = \int_0^{\pi/2} (1 - q^2 \sin^2(\beta))^{1/2} d\beta \tag{41} \]

Let us introduce (by analogy with general relativity) the following notations:
\[ \bar{u}_1 = (1 - e) / p, \quad \bar{u}_2 = (1 + e / p), \quad \bar{u}_3 = 1 - 2 / p, \tag{42} \]
where the parameter \( p \) at \( r \to \infty \) becomes the focal parameter \( p \) divided by \( \alpha \) and parameter \( e \) becomes the eccentricity \( e \) divided by \( \alpha \).

At \( r \gg 1 \) the value of \( q \approx (2e / p)^{1/2} \ll 1 \) and, therefore,
\[ F(\pi/2, q) = (\pi/2)(1 + q^2/4 + 9q^4/64 + ...) \tag{43} \]

Using eqs. \((38)\) and \((41)\) we find with accuracy up to \( 1/p^2 \)
\[ \Delta \varphi = 3\pi/p + (\pi/(8p^2))(-e^2 + 16e + 54 + ...) \tag{44} \]

For the orbits of Mercury or a binary pulsar (such as PSR 1913 + 16) the value of \( \bar{u} \) differs very little from the value of \( \alpha/r \). Consequently, the values of \( \bar{p} = 2/(\bar{u}_1 + \bar{u}_2) \) and \( \bar{e} = (\bar{u}_2 - \bar{u}_1) / (\bar{u}_2 + \bar{u}_1) \) differs very little from the values of \( p/\alpha \) and the \( e/\alpha \). Hence, their apsidal motion differs very little from the general relativity prediction. Even, for example, at \( \bar{p} = 10 \) and \( \bar{e} = 0.5 \), the difference in \( \Delta \varphi \) is about \( 6 \cdot 10^{-4} \text{rad} \).

5 Photon Orbits.

The equations of motion of a photon in the spherical symmetric field are given by
\[ \dot{r}^2 = (c^2 C/A)(1 - C b^2 / B), \quad \dot{\varphi} = c C b / B, \tag{45} \]
where \( b \) is the impact parameter.

The differential equation of orbits can be written as
\[ d\bar{u} / d\varphi = \mathcal{G}_1(\bar{u}), \tag{46} \]
where
\[ \mathcal{G}_1(\bar{u}) = \bar{u}^3 - \bar{u}^2 + \bar{b}^2 \]
and \( \bar{b} = b/\alpha \).

Setting \( \dot{r} = 0 \) in eqs. (46) we obtain
\[
\bar{b} = (BC)^{1/2} = \frac{\bar{f}}{(1 - 1/\bar{f})}.
\]
Fig 7 shows \( \bar{b} \) as a function of \( \bar{\tau} \), i.e. the location of the orbits turning points.

![Graph showing \( b = b(\tau) \)](image)

Fig. 7 The function \( b = b(\tau) \)

The function \( \bar{b}(\tau) \), in contrast to general relativity, is determined on all the axes, which is caused by the lack of events horizon. The minimal value of \( \bar{b} \), i.e. \( \bar{b}_{\text{min}} \), is equal to 2.6. It is reached at \( \bar{\tau} = 1.5 \). The motion of photons can be shown by the horizontal \( \bar{b} = \text{Const} \). In Fig 7 two types of the orbits are shown which have the peculiarities due to the lack of event horizon. The orbit with impact parameter \( b = b_1 \) show that the attracting mass cannot keep a photon escaping from the center at the impact parameter \( \bar{b} < \bar{b}_{\text{min}} \). This orbit type also shows the gravitational capture of a photon. The photon finishes at the field center, unlike general relativity, where it end on the Schwarzshild sphere.

The orbit with \( b = b_2 \) shows that the attracting mass can keep a photon escaping from the center only if the impact parameter \( \bar{b} \) is greater than \( \bar{b}_{\text{min}} \). The angle of the light deflection at the distances close to \( \bar{\tau} = 1.33 \) is given by
\[
\theta = \ln(4.021/\delta^2),
\] (47)
where
\[ \delta = 2(\mathcal{J}_{\text{min}} - 3/2) \text{ and } \mathcal{J}_{\text{min}} = (1 + r_{\text{min}}^3)^{1/3} \]

6 Conclusion Remarks.

It follows from eq. (27) that the gravitational energy released by a collapse is more than it was supposed to be hitherto. At the same time the gravitational potential on the surface of a stable massive configuration of the degenerate Fermi-gas is of the order of

\[ V = \frac{GM}{R}(1 - \exp(-R/\alpha)) \tag{48} \]

Hence, it follows from the virial theorem that the above objects with \( R \ll \alpha \) are the ones with low temperatures and, therefore, with low luminosities. They, more probably, refer to "dark" matter of the Universe. If their luminosities are caused by an accretion, then the Eddington limit of luminosity is approximately

\[ \mathcal{L} = \mathcal{L}_{\text{Edd}}^0[1 - \exp(-R/\alpha)], \tag{49} \]

where \( \mathcal{L}_{\text{Edd}}^0 \approx 1 \cdot 10^{38} M_\odot \text{erg/s} \). Hence, if \( R/\alpha \ll 1 \), its luminosity is \( \mathcal{L} \ll \mathcal{L}_{\text{Edd}}^0 \).

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