Flow of Neon-Nitrogen-Hydrocarbon mixture through adiabatic capillary tube at cryogenic temperatures

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Abstract. Capillary tubes are widely used in J-T refrigerators owing to their simplicity, easy operation, and zero maintenance. J-T refrigerators are mainly used for domestic and commercial refrigeration and air conditioning and hence the study on flow through capillary tubes is highly focused on conventional refrigerants at relatively higher temperatures. The present work is an extension of previous work done by the authors and focuses on the flow of refrigerant mixtures consisting of neon, nitrogen, and hydrocarbons at cryogenic temperatures for the capillary tube of a J-T cryocooler. The mass flow rate through the capillary tube is studied experimentally by changing refrigerant mixture composition and inlet temperature to the capillary tube. A mathematical model is developed based on a length feedback loop to predict the mass flow rate through the capillary tube using the homogenous flow model. The model is found to predict mass flow rate through the capillary tube with an average error of 10% for almost all the experimental values.

1. Introduction
Capillary tubes are the simplest form of expansion devices as they are simple and require no maintenance. However, the capillary tubes do not offer temperature control as can be obtained with a thermostatic expansion valve. But, as the capillary tubes have no sealing requirements, it makes an ideal expansion device for low-temperature application as in a Mixed Refrigerant Joule-Thompson (MRJT) cryocooler [1]. The flow inside the capillary tube will be in two-phase throughout with vapour and liquid having a different composition. This makes the flow inside capillary tubes at cryogenic temperature difficult to analyse. However, as the mass flux inside the capillary tube is high, the flow pattern inside the capillary tube is homogeneous and thus the effect of liquid and vapour concentration difference is nullified [2].

1.1 Mixed Refrigerant J-T Cryocoolers
Mixed refrigerant J-T cryocoolers are widely used to reach cryogenic temperatures in the range of 80 – 150 K using nitrogen-hydrocarbon mixtures. However, for temperatures below 70 K, additional low boiling component like neon or helium needs to be added along with modification as the system. Narayanan and Venkatarathnam [3] have studied two approaches to reach a temperature of 70 K i.e. one which uses a phase separator and another with a pre-cooling cycle. It has been observed that the volumetric cooling capacity follows an inverted U type curve for various discharge pressure while the overall exergy efficiency of the system follows a logarithmically reducing trend. J. Lee et. al. [4] have carried out a comparison between precooled MRJT cycles and Stirling and Brayton cycles to cool HTS cables. Various cycle configurations are also analysed to obtain the best COP for a cooling temperature of 64 K to 70 K. It has been reported that the exergy efficiency of the MRJT cycle is higher than that
of the Brayton cycle but lower than that of the Stirling cycle. However, owing to the simplicity of the MRJT cycle, it has been recommended for use.

1.2 Flow of cryogenic mixture through the capillary tube

Numerous studies are available on the flow of refrigerants through capillary tubes at relatively high temperatures, but limited literature is available for the flow of refrigerants through capillary tubes at cryogenic temperatures. Walimbe [5] have carried theoretical analysis on the flow of helium/neon-nitrogen-hydrocarbon mixtures through capillary tubes at cryogenic temperatures using dimensional analysis. Inlet and outlet pressures, inlet quality and temperature, capillary tube diameter and length, and mass flow rate through capillary tubes have been used as the parameters. The correlation thus formulated have predicted the capillary tube length with an error of 8.66 %. However, such correlations are only valid for the domain of the parameters used in the study. Ardhapurkar et. al. [6] studied the pressure drop in the capillary tube for a Mixed Refrigerant Joule-Thompson cryocooler using homogeneous and various separated flow models [7-9]. A mixture of Nitrogen, Methane, Ethane, Propane, and Isobutane have been used as the fluid for the analysis. It has been reported that predictions related to pressure drop for flow through capillary tubes are close to the experimental values using the homogeneous flow model. Kruthiventi and Venkataraman [10] have shown analytically for MRJT cryocooler that at the exit of the capillary tube, the flow is close to being sonic. Thus, if an appropriate factor of safety is not considered during design, the flow might transition to a choked condition resulting in insensitivity to heat load.

The authors [11] in a previous study have studied the flow of refrigerant through the capillary tube at cryogenic temperature. However, the refrigerant mixtures used were limited to nitrogen-hydrocarbon mixtures which are used for cooling temperature down to 80 K. A mathematical model has been developed to predict the capillary tube length for given parameters. Experiments have also been conducted to obtain data in order to validate the mathematical model. The model has been found to predict the capillary tube length with an accuracy of ±20 %.

To reach lower temperatures, Neon or Helium is required to be added to the refrigerant mixture and very little data is available for the flow of such refrigerant mixtures through capillary tubes at cryogenic temperatures. In the present work, the mathematical model developed earlier [11] is used to evaluate the mass flow rate through capillary tubes at cryogenic temperatures. An algorithm to predict the mass flow rate through the capillary tube for a set of parameters is developed by modifying the ones available in the literature [12]. An experimental setup is also developed to carry out the experimental investigation of the flow of neon-nitrogen-hydrocarbon mixtures through capillary tubes at cryogenic temperatures.

2. Mathematical Model

The mathematical model developed previously [11] is used to develop a numerical model to simulate the flow of refrigerant mixture through capillary tubes. The schematic of the capillary tube is as shown in figure 1. As can be seen from the figure, the capillary tube has only the two-phase region through the capillary tube as the inlet to the capillary tube is in the two-phase state for the MRJT cryocooler. The two-phase region is divided into elemental control volumes for the numerical analysis.

Flow inside the capillary tube is assumed to be steady and one dimensional. As the mass flux inside the capillary tube is high due to the small flow area, the vapour and liquid velocities are assumed to be constant i.e. flow is homogeneous. To analyse the flow of refrigerant mass, momentum, and energy conservation equations as shown below are solved.

\[ \dot{m} = \rho_1 V_1 A = \rho_2 V_2 A \]  

(1)
\[ \frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + H_L \]  \hspace{1cm} (2)

\[ h_t + \frac{V_t^2}{2} = h_f + x(h_g - h_f) + \frac{g^2}{2} (v_f(1-x) + v_g x)^2 \]  \hspace{1cm} (3)

The detailed derivation of the mathematical model is provided by the authors in previous work [11].

For the two-phase region, the incremental length for given pressure drop \( dP \) is given as,

\[ dl = \frac{2dP}{\rho_g + \frac{dP}{\rho_i}} \]  \hspace{1cm} (4)

The two-phase density is calculated as given in equation 5 while various two-phase viscosity models are used to calculate two-phase viscosity as given in equation 6-9.

\[ \frac{1}{\rho_{tp}} = \frac{1-x}{\rho_f} + \frac{x}{\rho_g} \]  \hspace{1cm} (5)

McAdams model [13]:

\[ \mu_{tp} = \frac{x\mu_g + (1-x)\mu_f}{\frac{8}{\mu_f}} \]  \hspace{1cm} (6)

Dukler’s Model [14]:

\[ \mu_{tp} = \frac{x\mu_g + (1-x)\mu_f}{\frac{8}{\mu_f}} \]  \hspace{1cm} (7)

Beattie and Whalley's model [15]:

\[ \mu_{tp} = \alpha_{tp} \mu_g + (1 - \alpha_{tp}) \mu_f (1 + 2.5\alpha_{tp}) \]  \hspace{1cm} (8)

Lin’s Model [16]:

\[ \mu_{tp} = \mu_g + \alpha_{tp} (\mu_f - \mu_g) \]  \hspace{1cm} (9)

The two-phase friction factor is calculated using the Blasius equation as provided in equation 10

\[ f_{sp} = 0.3165 Re^{-0.25} \]  \hspace{1cm} (10)

Here the two-phase Reynolds number is evaluated using two-phase density and any of the viscosity model described above.

2.1 Solution methodology

Using the mathematical model, a numerical code is developed to determine the mass flow rate through the capillary tube for a given condition.

The algorithm for the prediction of mass flow rate is as shown in figure 2.

The algorithm is based on calculating the capillary tube length for a given mass flow rate and then comparing it with the actual capillary tube length.

The mass flow rate is increased or decreased until the desired accuracy is obtained i.e. change in mass flow rate is below a certain set value. This length feedback mechanism is observed to reduce the numerical error in the prediction of the mass flow rate through the capillary tube.

Initial guess of mass flow rate is required for the algorithm and in the present case, it is taken as 1 gm/sec as it is found to converge for all the cases. A lower value of the initial mass flow rate is recommended to assure convergence of the numerical model. It is observed that for certain numerical trials, the final solution is found to be of oscillating in nature and thus an under relaxation of the increment/decrement is recommended as it ensures convergence. However, under relaxation results in slow convergence and thus requires a
greater amount of computational resources. But, as the computational requirement for the numerical simulation is very small, the increase in computational demand is not significant.

3. Experimental investigation of flow through a capillary tube

To aid the overall study of the investigation of refrigerant flow through the capillary tube, an experimental investigation is carried out for the neon-nitrogen-hydrocarbon mixture. The schematic of such a system is as shown in figure 3. As can be seen from the figure that the system consists of a two-stage compression process with an intercooler and aftercooler. One bulk oil separator and two coalescence oil separators are used to prevent oil from entering the low temperature region. To simulate the heat exchanger of an MRJT cryocooler, Precooling coil is used to cool the refrigerant mixture before expansion. A heater is also installed after the precooler coil for better temperature control. The capillary tube is kept inside a vacuum chamber to minimise heat gain from the surroundings. Two different refrigerant mixtures are used for the study and their details are provided in table 1.

The mixture contains an equal composition of neon, nitrogen, methane, and ethane with a total charging pressure of 1 MPa. The capillary tube used for the study is of 50 cm length, 1.14 mm inner diameter, and 75 micron surface roughness.

| Mix. No. | %Ne | %N2 | %CH4 | %C2H6 |
|----------|-----|-----|------|-------|
| Mix 1    | 33.3| 33.3| 33.3 | -     |
| Mix 2    | 25  | 25  | 25   | 25    |

3.1 Methodology

Experiments are performed for the above-mentioned mixtures for different capillary inlet temperatures. Initially, the aftercooler and intercooler fans are turned on. After waiting for a few minutes both the compressors are turned on. Liquid nitrogen is poured into the precooling coil up to a fixed point and is kept constant throughout the experiments by topping up with excess liquid nitrogen as and when required. To maintain the desired temperature, the heater is turned on and the temperature sensor mounted just before the entrance of the capillary tube is used to monitor the temperature at the capillary tube inlet. Temperature and pressure data are recorded once the desired capillary inlet temperature is obtained and maintained steadily. Sample of working fluid in circulation is also taken to determine the composition of the refrigerant mixture in circulation. Flow rate at the suction of the compressor is also
noted to calculate the mass flow rate in the system during the experiment. To ensure the accuracy of the experimental results, each experiment is repeated twice or until satisfactory repeatability is established.

3.2 Instrumentation

Details of the instrumentation and uncertainties associated with the measurement are provided in table 2. For pressure and temperature measurement, piezoelectric transducers and platinum RTDs are used respectively. Flow rate is measured at the suction of the compressor using a calibrated flow meter. As there is a change in the refrigerant mixture in circulation compared to the charged one, an adsorption type gas chromatograph is used to determine the refrigerant mixture in circulation. An accurate refrigerant mixture in circulation is essential to evaluate the properties of the fluid for numerical analysis.

| Measurement quantity | Type                  | Range        | Uncertainty |
|----------------------|-----------------------|--------------|-------------|
| Flowrate             | Rotameter             | 0-20 m³/hr   | 0.5 m³/hr   |
| Temperature          | PT100 (RTD)           | -200 to 400 °C | 0.1 °C |
| Pressure             | Piezoelectric         | 0-3.5 MPa    | 1 %         |
| composition          | Gas chromatograph     | -            | 5%          |

The uncertainties in the experimentation based on uncertainties of the individual instrument can be evaluated as given in equation 10 [17].

$$\frac{\delta R}{R} = \left( \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \cdots + \frac{\delta x_m}{x_m} \right)^{0.5}$$  \hspace{1cm} (11)

where $\delta R$ is the uncertainty in the output, $X_i$ and $\delta X_i$ are the entities impacting the output (pressure, temperature, etc. in the present case) and $a$, $b$, … are the exponents of the individual entities.

4. Results and Discussion

To ensure that the experimental results are free of any error, a comparison is carried out between the temperatures obtained after expansion experimentally and theoretically. For the theoretical temperature after expansion, an isenthalpic line is drawn from the starting point of the expansion until the low pressure of the system. The comparison for Mix 2 is shown in figure 4 using a T-s chart for the capillary inlet temperature of 104 K. As can be seen from the figure that the error in the experimental temperature of the refrigerant mixture after the expansion is only 0.1 K.

Experiments are conducted using the two refrigerant mixtures as mentioned above by varying the temperature of the refrigerant mixture entering the capillary tube in steps of 25 K. Various parameters like high and low pressure across the capillary tube and mass flow rate through the capillary tube are presented. These experimental results are then used to validate the mathematical model to predict the mass flow rate through the capillary tube. The improvement in the model by using the mass feedback mechanism is evaluated in the study.

4.1 Experimental results for flow of refrigerant through capillary tubes

For the two refrigerant mixtures, experimental results are shown in table 3. Capillary inlet temperatures, inlet and outlet pressures, mass flow rate, and composition of the refrigerant mixture in circulation is provided. The first four rows show experimental results for refrigerant mix 1 while the last four rows provide results for mix 2. As can be seen from the table that as the capillary tube inlet temperature
reduces, the high pressure in the system reduces while the low pressure in the system almost remains constant.

| Sr. No. | Ti (K) | Ph (MPa) | Pl (MPa) | m (kg/hr) | Ne (%) | N2 (%) | CH4 (%) | C2H6 (%) |
|---------|--------|----------|----------|-----------|--------|--------|---------|---------|
| 1       | 104.1  | 1.48     | 0.195    | 8.2       | 32.8   | 29.8   | 21.3    | 16.1    |
| 2       | 124.0  | 1.94     | 0.22     | 8.3       | 32.7   | 26.0   | 24.4    | 16.9    |
| 3       | 148.7  | 1.85     | 0.238    | 8.3       | 27.1   | 32.5   | 23.0    | 17.3    |
| 4       | 162.0  | 2.12     | 0.258    | 7.3       | 29.2   | 25.9   | 25.2    | 19.6    |
| 5       | 98.7   | 1.42     | 0.155    | 7.8       | 37.5   | 35.7   | 26.6    | -       |
| 6       | 122.6  | 1.61     | 0.175    | 7.3       | 34.4   | 36.3   | 29.3    | -       |
| 7       | 148.1  | 1.45     | 0.155    | 7.4       | 36.5   | 35.0   | 28.5    | -       |
| 8       | 172.6  | 1.61     | 0.155    | 7.7       | 35.0   | 36.6   | 28.4    | -       |

The capillary inlet pressure varies from 1.48 to 2.12 MPa respectively for mix 1 for a capillary inlet temperature of 104.1 K to 162 K. Similarly, for mix 2 the capillary inlet pressure varies from 1.42 MPa to 1.61 MPa for a capillary inlet temperature of 98.7 K to 172.6 K. The capillary exit pressure varies slightly from 0.195 to 0.258 MPa for mix 1 while it remains almost constant at 0.15 MPa for mix 2. As a result, the pressure ratio in the system reduces as the capillary inlet temperature reduces. This is mainly due to the reduction in resistance across the capillary tube for the colder refrigerant. The mass flow rate is observed to not vary that much as the suction side pressure is remaining almost constant. For mix 1, the mass flow rate is constant at 8.2-8.3 kg/hr for capillary inlet temperature of 104.1 to 148.7 K while for a capillary inlet temperature the mass flow rate reduces slightly to 7.3 kg/hr. For mix 2, the mass flow rate is observed to be constant with slight variations from 7.3 to 7.8 kg/hr. This is because of the higher pressure ratio for the system for a higher capillary inlet temperature resulting in lower volumetric efficiency of the compressor. Due to this, a slight reduction in mass flow rate is observed at higher capillary inlet temperatures. However, as the capillary inlet temperature increases, the composition of ethane and methane increases and that of neon decreases and as a result the density of the refrigerant entering the compressor increases. This results in an increase in the mass flow rate and therefore with both these effects combined, the mass flow rate is not varying noticeably.

The composition of the mixture in circulation is not vary significantly, however, as the capillary inlet temperature reduces, the amount of low boiling component (neon and nitrogen) increases slightly and that of high boiling component (methane and ethane) reduces slightly. The change in refrigerant composition is not significant as the liquid nitrogen level in the precooling coil is kept constant and the capillary inlet temperature is regulated with the heater. Thus, the amount of liquid holdup in the precooling coil will be the same for all the experiments resulting in a slight shift in refrigerant composition compared to the charged one.

### 4.2 Numerical prediction of refrigerant mass flow rate through capillary tubes

The mass flow rate prediction for refrigerant flow through the capillary tube for mix 1 and 2 are presented in figure 5 and 6 respectively. An error bar of ±15 % is also shown for reference of the reader. Peng-Robinson equation of state [18] is used to calculate the required property data of the refrigerants using the ASPEN software package [19]. From the figures, it can be observed that the mass prediction algorithm predicts refrigerant mass flow rate through the capillary tube within an error of ±15 % for almost all the experimental values. Here, the mass flow rate prediction is not shown for mix 2 at the capillary inlet temperature of 162 K as not shown as the entrance to the capillary tube is in a superheated state and the algorithm is developed for subcooled and two-phase refrigerant flow through the capillary tube.

It can be seen that the McAdams viscosity model which is widely used for conventional refrigerant mixtures, provides a good match for mix 1 while has a larger error for predicting mass flow rate through the capillary tube for mix 2. Similarly, for Beattie and Whalley viscosity model, the mass flow rate predictions are matching very well with the actual values consistently for mix 1. However, for mix 2, the predictions are seen to have slightly higher errors. Dukler and Lin’s viscosity model are also seen
to have a reasonable match but are not consistent in predicting the mass flow rate for both refrigerant mixtures. For mix 2, at a capillary inlet temperature of 98 K, the error in prediction is high compared to others and mass flow rate prediction is lower than the actual one. Error in predicting mass flow rate for both the refrigerant mixtures is presented in table 4 for all the considered viscosity models. As can be seen from the figure, the error in predicting mass flow rate through capillary tube is higher for mix 1 compared to mix 2. It can also be observed that the average error for the mass prediction algorithm is almost the same for all the two-phase viscosity models. This is due to the inherent nature of the algorithm which searches in a broad domain of the mass flow rates to minimise the error. The average error is also less compared to the length prediction algorithm presented by authors in earlier work [11] where the error in numerical simulations was between 12 % to 19 %. This results in a reduction in numerical error of up to 45 %. Thus, a significant reduction in numerical prediction is obtained by changing the objective function of the numerical model and employing a feedback based approach.

Large error in prediction of the mass flow rate through the capillary tube for mix 1 can be attributed to the presence of very low boiling components such as neon and medium-high boiling component in the refrigerant mixture such as ethane. Interaction between such components may not be captured by the equation of state used as the binary interaction parameters are not well defined for components like neon.

5. Conclusions
The present work analyses the flow of cryogenic refrigerant mixtures through adiabatic capillary tubes at very low temperatures. Refrigerant mixtures containing neon, nitrogen, methane, and ethane are used for the analysis providing a wide range of data for flow through capillary tubes. An improvised mathematical model is developed based on the length feedback loop to predict the mass flow rate through the capillary tube. To aid the analysis, several experiments are also conducted which will not only help in the validation of the numerical model but also provide an important benchmark for further study.

From the study, it is found that the improvised algorithm provides a reduction in error by almost 50 % for the prediction of refrigerant mass flow rate through the capillary tube. Based on the study, it is found that McAdams and Bettie and Whally two-phase viscosity models are more consistent in predicting the mass flow rate compared to others.

Table 4: Error in the prediction of refrigerant mass flow rate through a capillary tube

| Viscosity Model       | Error in prediction mass flow rate through capillary tube (%) |
|-----------------------|-------------------------------------------------------------|
|                       | Mix 1            | Mix 2            |
| McAdams [10]          | 5.4              | 16.9             |
| Dukler [11]           | 9.4              | 15.9             |
| Beattie and Whalley [12] | 4.4            | 17.3             |
| Lin [13]              | 10.0             | 16.6             |

Figure 5: Mass flow rate through the capillary tube prediction (Mix 1)  
Figure 6: Mass flow rate through the capillary tube prediction (Mix 2)
6. **References**

[1] V. Brodyanskii, E. Gromov, A. Grezin, V. Yagodin, V. Nikol'skii, A. Tashchina, ‘Efficient throttling cryogenic refrigerators which operate on mixtures’, Chemical and Petroleum Engineering, vol. 7(12), pp. 1057-1061, 1971.

[2] J. Collier and J. Thome, ‘Convection Boiling and Condensation’, Oxford University Press, New York, 1994.

[3] V. Narayanan and G. Venkatarathnam, ‘Performance of two mixed refrigerant processes providing refrigeration at 70 K’, Cryogenics, vol. 78, pp. 66-73, 2016.

[4] J. Le, HG Hwang, S. Jeong, B. Park, Y. Han, ‘Design of high efficiency mixed refrigerant Joule-Thompson refrigerator for cooling HTS cable’, Cryogenics, vol. 51, pp. 408-414, 2011.

[5] N. Walimbe, ‘Investigations on Mixed Refrigerant Joule-Thompson Cryocooler for 80 K Applications’, PhD thesis, Indian Institute of Technology Bombay, 2009.

[6] P. Ardhapurkar, A. Srisharan, M. Atrey, ‘Investigation of pressure drop in capillary tube for Mixed Refrigerant Joule-Thompson cryocooler’, AIP Conference Proceedings, vol. 1573, pp. 155-162, 2014.

[7] L. Friedel, ‘Improved friction pressure drop correlations for horizontal and vertical two-phase pipe flow’, Proceeding of European Two-Phase Flow Group Meet, 1979.

[8] R. Gronnerud, ‘Investigation of liquid hold-up, flow-resistance and heat transfer in circulation type evaporators, part IV: Two-phase flow resistance in boiling refrigerants’ Bull. de l’Inst. du Froid, Annexe, 1, 1979.

[9] H. Muller-Stempnagen and K. Heck, ‘A simple friction pressure drop correlation for two-phase flow in pipes’ Chemical Engineering and Processing: Process Intensification, vol. 20, pp. 297-308, 1986.

[10] S. Kruthiventi and G. Venkatarathnam, ‘Studies on capillary tube expansion device used in J-T refrigerators operating with nitrogen-hydrocarbon mixtures’, Cryogenics, vol. 87, pp. 76–84, 2017.

[11] D. Parmar and M. Atrey, ‘Experimental and numerical investigation on the flow of mixed refrigerants through capillary tubes at cryogenic temperatures’, Applied Thermal Engineering, p.115339, 2020.

[12] D. Subodh, S. Deodhar, B. Hardik, H. Kothatdia, K. Iyer, S. Prabhu, ‘Experimental and numerical studies of choked flow through adiabatic and diabatic capillary tubes’, Applied Thermal Engineering, vol. 90, pp. 879-894, 2015.

[13] W. M. Adams, W. Wood, R. Bryan, ‘Vaporisation inside horizontal tubes Benzene-oil mixture’, ASME, vol. 64, pp. 193-1942.

[14] A. Dukler, M. Wicks, R. Cleveland, ‘Frictional pressure drop in two-phase flow’ AIChE Journal, vol. 10, pp. 38-51, 1964.

[15] D. Beattie and P. Whalley, ‘A simple two-phase frictional pressure drop calculation method’, International Journal of Multiphase Flow, vol. 8, pp. 83-87, 1981.

[16] S. Lin, C. Kwork, R. Li, Z. H. Chen, Z. Y. Chen, ‘Local frictional pressure drop during vaporization for R12 through capillary tubes’, International Journal of Multiphase Flow, vol. 1, pp. 95-102, 1991.

[17] R. Moffat, ‘Describing the uncertainties in experimental results’, Experimental thermal and fluid science, vol. 1, pp. 3-17, 1988.

[18] D. Peng and D. Robinson, ‘A New Two-Constant Equation of State’, Industrial and Engineering Chemistry: Fundamentals, vol. 15, pp. 59–64, 1976.

[19] Aspentech, ‘aspenONE’, version 11, 2019.