Negotiation-Aware Reachability-Based Safety Verification for Autonomous Driving in Interactive Scenarios

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Abstract—Safety assurance is a critical yet challenging aspect when developing self-driving technologies. Hamilton-Jacobi backward-reachability analysis is a formal verification tool for verifying the safety of dynamic systems in the presence of disturbances. However, the standard approach is too conservative to be applied to self-driving applications due to its worst-case assumption on humans’ behaviors (i.e., guard against worst-case outcomes). In this work, we integrate a learning-based prediction algorithm and a game-theoretic human behavioral model to online update the conservativeness of backward-reachability analysis. We evaluate our approach using real driving data. The results show that, with reasonable assumptions on human behaviors, our approach can effectively reduce the conservativeness of the standard approach without sacrificing its safety verification ability.

I. INTRODUCTION

Our society is rapidly advancing towards self-driving cars that interact with human-driven cars on public roads. Safety assurance is a critical yet challenging aspect when designing such autonomous systems. Hamilton-Jacobi (HJ) reachability analysis is a formal safety verification tool for verifying the safety of dynamic systems [1], [2]. Under HJ-reachability paradigm, both forward [3]–[5] and backward-reachability analysis [6]–[8] can be used to ensure safety in human-robot interaction applications. For instance, forward-reachability analysis can be used to compute the human’s forward-reachable set (FRS) that should be avoided by the robot. Often, the FRS leads to overly conservative robot behaviors due to its open-loop spirit. Different from forward-reachability analysis, backward-reachability analysis formulates the human-robot interaction as a differential game and computes the full backward-reachable tube (BRT) that contains states that may result in the human-robot joint system being unsafe, assuming the human is adversarial and has full control bound within some time horizon. When operating online, the BRT can either be used as a safety monitor that switches the robot’s controller to the optimal safety controller when the human is about to breach the BRT [8] or infused into the robot’s low-level controller design as constraints [6], [7]. Backward-reachability analysis is less conservative since it allows the robot to react to the human’s adversarial actions. However, it is still impractical for self-driving applications due to its worst-case assumption on humans’ behaviors (adversarial human with full control bound). Therefore, mitigation strategies are needed for generating a practical yet effective BRT.

Since the conservativeness of the full BRT is induced by assuming that the human-driven car utilizes its full control bound to drive the joint system to unsafe states, a natural solution is to directly limit the human-driven car’s control bound. Such a technique has been exploited in both forward and backward-reachability analysis. For instance, [5] assumes that the human operates under a probabilistic model parameterized by some unknown parameters and casts the human state prediction problem as a forward-reachability problem in the joint state space of the human and the belief over the unknown model parameters. When computing the human’s FRS, [5] limits the human’s control bound based on his/her likely actions. Despite being robust to misspecified human models and priors, [5] separates the human from the robot and neglects the effects from interactions. In traffic settings, self-driving cars might get stuck with such an approach. The approach in [8] exploits a learning-based predictor to forecast the human-driven car’s future trajectory and classifies the prediction into one of the pre-defined driving modes (e.g., turn left). Each driving mode is associated with a less-conservative human-driven car’s control bound, and the corresponding BRT can be computed offline and queried online for safety verification. Such an approach works well if the learning-based predictor offers the human driver’s future trajectories based on the self-driving car’s future plan candidates, instead of only on map information and historical observations (environmental states). Without that, the estimated human-driven car’s control bound can still be conservative in interactive scenarios since it doesn’t account for the negotiation between the human-driven car and the self-driving car.

Our key insight is that: in interactive scenarios, the distribution of human-driven car’s actions is not only constrained by environmental states but also affected by the human driver’s internal state that governs his/her behaviors in negotiations. Estimating the human-driven car’s control bound online with both environmental states and the human driver’s internal state accounted for can help to obtain a less-conservative BRT.

Overall, we make the following contributions towards...
practical safety verification for self-driving cars:

Developing a framework for constructing a practical yet
effective BRT by leveraging learning-based prediction
and game-theoretic reasoning. We propose to hierarchi-
cally estimate the human-driven car's control bound. We
first utilize a learning-based method to estimate a control
bound with the environmental information accounted for, and
further modify the bound by reasoning about the human’s
willingness to cooperate in a game-theoretic fashion. Then
the updated control bound is used to query an appropriate
BRT online for safety verification.

Verifying the effectiveness of our approach with real
driving data. We compare our approach with the standard
full BRT using real traffic data. We show that our approach
is significantly less conservative while maintaining safety
verification ability.

II. PRELIMINARIES

A. Hamilton-Jacobi reachability analysis

In this section, we briefly introduce the HJ backward-
reachability analysis. More details can be found in [1], [2].

Backward-reachable tube. We consider the evolution of
a dynamical system governed by the differential equation:
\[ \dot{s}(t) = f(s(t), u(t), d(t)), \]
where \( s \in \mathbb{R}^{n_s} \) denotes the state of
the system, \( u \in \mathbb{R}^{n_u} \) denotes the bounded control, and
\( d \in \mathbb{R}^{n_d} \) denotes the bounded disturbance. We assume
that we have access to a target set \( \mathbb{G} \) that contains
the unsafe states of the dynamical system. Then we define
the backward-reachable tube (BRT) of the dynamical system (\( O_r \))
with respect of \( \mathbb{G} \) as a set of initial states from which the
disturbance can drive the dynamical system to \( \mathbb{G} \) within a
time horizon \( |\tau| \) despite the optimal control efforts, namely,
\[
O_r = \{ s_0 \in \mathbb{R}^{n_s} : \exists d(\cdot), \forall u(\cdot), \exists t \in [\tau, 0], s(t|f, s_0, d(\cdot), u(\cdot)) \in \mathbb{G} \},
\]
where \( \tau \) is a negative number since the system dynamics are
propagated backwards in time.

Level-set method. The BRT \( O_r \) can be characterised as a
zero sub-level set of a value function \( V(s, \tau) \):
\[
O_r = \{ s \in \mathbb{R}^{n_s} : V(s, \tau) < 0 \},
\]
where the value function \( V(s, t) \) is the viscosity solution
of the following Hamilton-Jacobi Isaacs partial differential
equation [9]:
\[
\min \left( \frac{\partial V(s, t)}{\partial t} + \min_{d \in \mathbb{D}} \max_{u \in \mathbb{U}} \nabla V(s, t)^T f(s, u, d), \right) V(s, 0) - V(s, t) = 0, t \in [\tau, 0],
\]
and \( V(s, t) \) can be computed via dynamic programming [10]
with terminal condition \( V(s, 0) = l(s) \), where \( l(s) \) is defined
as \( l(s) \leq 0 \iff s \in \mathbb{G} \).

B. Backward-reachability analysis in self-driving settings

System dynamics. We consider pairwise interactions be-
tween the self-driving car and each human-driven car in the
environment (i.e., we verify the self-driving car’s safety with
respect to each human-driven car separately). Following [8],
we consider a dynamical system that encodes the relative
dynamics between the self-driving car and the human-driven
car in a pairwise interaction. Specifically, the dynamics of the
human-driven car is represented using an extended uni-cycle
model and the dynamics of the self-driving car is modeled
using a high-fidelity bicycle model. The state of the relative
dynamic system is defined as \( s = \{ x_{rel}, y_{rel}, \psi_{rel}, v_r, v_h \} \),
where \( x_{rel} (y_{rel}) \) is the x(y)-coordinate of the human-driven
car in a coordinate frame centered at the geometric center
of the self-driving car with \( x \)-axis aligned with the heading
of the self-driving car, \( \psi_{rel} \) denotes the relative heading
between the two cars, and \( v_r (v_h) \) denotes the speed of
the self-driving car (human-driven car). We let \( u_r(t) = (a_r(t), \delta_f(t)) \)
denote the control input of the self-driving car with \( a_r \) denoting the acceleration and \( \delta_f \) denoting the front
wheel rotation, and let \( u_h(t) = (a_h(t), \omega_h(t)) \) denote the
control input of the human-driven car with \( a_h \) denoting the
acceleration and \( \omega_h \) denoting angular speed. The evolution
of the relative system is then governed by the following
differential equation
\[
\dot{x}_{rel} = \frac{v_r}{l_r} \frac{y_{rel}}{l_r} \sin(\beta_r) + v_h \cos(\psi_{rel}) - v_r \cos(\beta_r),
\dot{y}_{rel} = -v_r \frac{\dot{x}_{rel}}{l_r} \sin(\beta_r) + v_h \sin(\psi_{rel}) - v_r \sin(\beta_r),
\dot{\psi}_{rel} = \omega_h - \frac{v_r}{l_r} \sin(\beta_r),
\dot{v}_r = a_r,
\dot{v}_h = a_h,
\]
where \( l_f (l_r) \) denotes the front (rear) axle length of
the self-driving car and \( \beta_r \) is computed via \( \beta_r = \tan^{-1}\left( \frac{l_f}{l_r + l_f \tan(\delta_f)} \right) \).

BRT as a safety monitor. We treat the human-driven car’s
control as disturbance of the relative dynamical system (4),
then we can compute the BRT that contains initial relative
states that could potentially lead to collisions. Such a BRT
can be used as a safety monitor that switches the self-driving
car’s low-level controller to the optimal safety controller
when the current relative state is within the BRT [8] or can be
infused into the self-driving car’s low-level controller design
as constraints [6], [7].

Disturbance bound. It can be observed from (4) and (3)
that the conservativeness of the BRT highly depends on the
human-driven car’s control bound \( \mathbb{D} \). The BRT computed
with the full human-driven car’s control bound is overly
conservative in general.

III. NEGOTIATION-AWARE BRT

In this section, we propose a negotiation-aware BRT
computation approach that estimates the human-driven car’s
control bound based not only on environmental information but also on the potential negotiation with the self-driving car. We propose to estimate such a bound hierarchically: 1) first utilize a learning-based method to estimate a control bound with the environmental information accounted for, and further modify the bound by reasoning about the internal state of the human driver in a game-theoretic fashion.

A. Estimate disturbance bound via learning-based predictor

As in [8], one way to obtain a practical estimate on \( D \) is to leverage a learning-based prediction algorithm. Typically, a learning-based prediction algorithm will generate the human-driven car’s most-likely future trajectory and the statistical prediction error given the map information and the joint history states [11]. Hence, in the first layer, we utilize the most-likely trajectory and the prediction error to generate an initial control bound estimation. More specifically, we construct the Frenet frame taking the most-likely path as the reference path. In the Frenet frame, we can obtain an acceleration bound \( D_a = [a_{\text{lower}}, a_{\text{upper}}] \) from the most-likely trajectory and an angular speed bound \( D_\omega = [0, 0] \). The statistical prediction error \( \epsilon \) can further provide additional safety margins to improve the predicted control bound’s robustness in the presence of long-tailing problems in learning-based approaches.

B. Negotiation modeling via game-theoretic reasoning

One limitation of the first-layer’s estimation approach is that the obtained control bound does not account for negotiations between agents (e.g., a human will slow down if a self-driving car gradually merges in front of him/her [12]). Such a limitation can also lead to a conservative BRT when active maneuvers are needed in interactive scenarios. To tackle this problem, in the second layer, we exploit a game-theoretic approach to explicitly reason about negotiations and correct the predicted human-driven car’s acceleration bound. We emphasize that in the second layer, the game-theoretic approach is exploited for its ability to model intense interactions between agents, mimic humans’ decision-making behaviors, and reveal their internal states.

Assumptions and simplifications. We assume that the human reacts to the self-driving car via acceleration in the Frenet frame described in Section III-A. In other words, when modeling negotiations, we assume that the human-driven car will operate along its predicted most-likely path (reference path in the Frenet frame). Such an assumption is reasonable in light of two observations: 1) learning-based predictors predict a decent potential path of a human-driven car in structured roads, as they explicitly encode features of road environments during training; 2) a human driver tends to follow a reference path and react to other drivers mainly by adjusting his/her acceleration. Since the current motion plan of the self-driving car is available in the context of safety verification, we further assume that the self-driving car operates along its current planned path when modeling negotiations (such an assumption limits the self-driving car’s mobility and tends to make the human’s acceleration bound prediction more conservative, but it provides a computational gain by making the negotiation modeling online usable).

Negotiation as a Stackelberg game. With the above assumptions, we model the negotiation between the human-driven car and the self-driving car as a Stackelberg game [13] since it explicitly considers one player’s advantages over the other player, which can be used to model different roles in a traffic negotiation. The dynamics of the game is governed by \( s_{t+1} = g(s_t, a_t^h, a_t^l) \), where the subscript denotes a discrete time step, \( a \) denotes acceleration, and the dynamics function \( g \) governs the state evolution along agents’ respective paths.

We assume that both the human-driven car and the self-driving car have a finite set of acceleration controllers in their negotiation. A controller \( \pi \in \Pi \) defines an agent’s controls for a short planning horizon \( T \). At a time step \( t \), an agent’s controller \( \pi \) maps a time increment \( \tau (\tau \in [0, T]) \) to the acceleration command at \( \Delta t_t + \tau \), where \( \Delta t_t \) is the time integral between successive time steps.

In a negotiation, the human-driven car can either be a follower who is willing to cooperate or be a leader who aims to dominate the interaction. Specifically, a follower maximizes his/her own reward function while accommodating the self-driving car. Assuming that humans are noisy-rational, then a follower selects controller in response to the self-driving car’s current motion plan according to the following quantal response model:

\[
\mathbb{P}(\pi_h | s_t, \pi_f^g) \propto \exp(\beta Q^h(s_t, \pi_h, \pi_f^g)),
\]

\[
Q^h_t(s_t, \pi_h, \pi_f^g) = \sum_{n=0}^{T/\Delta t} R_h(s_{t+n}, \pi_h(n \Delta t), \pi_f^g(n \Delta t)),
\]

where \( \pi_f^g \) denotes the self-driving car’s current planned controls along its desired path, and \( R_h \) is human’s reward function learned via Inverse Reinforcement Learning [12], [14]. In contrast to a follower, a leader makes decisions assuming the self-driving car is a follower who optimally responds to his/her controller:

\[
\mathbb{P}(\pi_h | s_t) \propto \exp(\beta Q^f_h(s_t, \pi_h, \arg \max_{\pi_f \in \Pi} Q^f_t(s_t, \pi_h, \pi_f))),
\]

where the expression of \( Q^f_t \) is defined analogously to (6) with reward function \( R_f \).

Leading-following role inference. Since the leading-following role of the human-driven car in a negotiation is unknown, we treat it as an internal state that needs to be inferred online. Normally the self-driving car’s sensing module runs at a much higher frequency than the safety verification module. We let \( \xi_t \) denote the collection of controls of the human-driven car in the previous \( T[s] \) observed by the self-driving car. We let \( \theta \in \Theta = \{l, f\} \) denote the human-driven car’s leading-following role and define \( b_l(\theta) = \mathbb{P}(\theta^* = \theta) \) as the belief distribution over \( \Theta \) at step \( t \). Then the self-driving
car can update the human-driven car’s role via Bayesian inference:

\[ b_t(\theta) \propto Z(\xi_t | \theta) b_{t-1}(\theta), \quad (8) \]

where \( Z(\xi_t | \theta) \) is an observation model that specifies the probability of observing \( \xi_t \) if the human-driven car’s role were \( \theta \). We define a distance measure \( d \) that evaluates the accumulative difference (Euclidean distance) between the observed human-driven car’s controls \( (\xi_t) \) and controls obtained from a controller \( \pi \in \Pi \) evaluated at the times steps that the observations were made, and use \( d \) to find the controller that best matches the observed controls. Then the observation model is defined as:

\[ Z(\xi_t | \theta) = \begin{cases} \{ \arg \min_{\pi_h \in \Pi} d(\pi_h, \xi_t) | s_{t-1}, \pi_i^r \}, & \text{if } \theta = f, \\ \{ \arg \min_{\pi_h \in \Pi} d(\pi_h, \xi_t) | s_{t-1}, \pi_i^r \}, & \text{if } \theta = l, \end{cases} \quad (9) \]

where the probabilities are computed using (5) and (7).

With (3) and (9), we can “correct” the human’s predicted acceleration bound from the learning-based predictor by explicitly modeling and inferring the human’s willingness to be negotiated.

C. Online update the backward-reachable tube.

Since \( \Pi \) is finite, we have a finite set of potential acceleration bounds \( (\mathcal{D}_a) \) of the human-driven car associated with the controllers. For each acceleration bound \( \mathcal{D}_a \), we offline compute a BRT, \( \mathcal{O}_+^\tau \), using the control bound \( (\mathcal{D}_a, \mathcal{D}_\omega) \), where \( \mathcal{D}_\omega \) denotes the human-driven car’s angular speed bound obtained from the learning-based predictor with safety margin described in Section III-A. When operating online, at each step, we first update \( b_t(\theta) \) based on the current observation, then run the following algorithm to augment BRTs based on the distribution over the human-driven car’s future control bounds with respect to \( b_t(\theta) \). The notation \( \Delta \) denotes a desired confidence bound (we aim to augment BRTs such that their marginal probability is larger than \( \Delta \)). When \( \Delta = 1 \), the resulting BRT is equivalent to the one that considers the whole acceleration bound (most conservative). Note that the obtained BRT is in the Frenet frame described in Section III-A.

Algorithm 1: Update BRT

1. For each controller \( \pi_i^r \in \Pi \), compute its expected probability \( \mathbb{P}(\pi_i^r) \) with respect to \( b_t(\theta) \);
2. Sort \( \Pi \) with respect to \( \mathbb{P}(\pi_i^r) \) in descending order;
3. Initialize \( \mathcal{O}_+^\tau \leftarrow \emptyset \), \( P = 0 \);
4. for \( i = 1, 2, ..., |\Pi| \) do
5. \( P = P + \mathbb{P}(\pi_h) \);
6. \( \mathcal{O}_+^\tau \leftarrow \mathcal{O}_+^\tau \cup \mathcal{O}_+^\tau \);
7. if \( P \geq \Delta \) then
8. \( \text{break} \);
9. end if
10. end for
11. Return \( \mathcal{O}_+^\tau \);
C. Result

In the right plot of Fig. 1, we show the relative speed/distance between two cars when the human-driven car breaches the self-driving car’s BRT. It can be observed that, even though all 15 interactions are safe, the human-driven car breaches the full BRT in all the interactions in our experiment, which shows the conservativeness of the full BRT. With the prediction-based BRT, 9 interactions are identified as unsafe interactions, which indicates that leveraging end-to-end trajectory prediction can reduce some of the conservativeness of the full BRT. With our approach, only 4 interactions are identified as unsafe interactions by the BRT. Moreover, the average ground-truth minimal time-to-collision (TTC) [17] of the identified unsafe interactions is \( \text{TTC} = 2.4 \text{s} \), which shows that our approach to dynamically updating the BTR further reduces the conservativeness of the full BRT while maintaining the safety verification ability of backward-reachability analysis. In Fig. 2 we show an instance of our experiment in a strong negotiation. In (a-1)-(a-2) and (b-1)-(b-2), both the full BRT and the prediction-based BRT identify the interaction as unsafe, and the safety controller stops the self-driving car, although the human-driven car is trying to yield and stops at \( 6 \text{m} \) away from the self-driving car. In contrast, our approach identifies that the human-driven car is willing to cooperate and quickly adjusts the BRT to ensure a safe and effective interaction. The preliminary experiment result shows that, indeed, estimating the human-driven car’s control bound online with both environmental states and the human driver’s internal state accounted for can help to obtain a less-conservative while safety-preserving BRT.

V. Conclusion

Summary. Safety assurance is a critical yet challenging aspect when developing self-driving technologies. Hamilton-Jacobi backward-reachability analysis is a formal verification tool for verifying the safety of dynamic systems in the presence of disturbances. However, the standard approach is too conservative to be applied to self-driving applications due to its worst-case assumption on humans’ behaviors. In this work, we integrated a learning-based prediction algorithm and a game-theoretic negotiation model to online update the conservativeness of backward-reachability analysis. We evaluated our approach using real driving data. The results showed that, with reasonable assumptions on human behaviors, our approach can effectively reduce the conservativeness of the standard approach without sacrificing its safety verification ability.

Limitations and future works. Our work is limited in many ways. One of the limitations is the simplifications made in the negotiation modeling (i.e., assuming agents operate along pre-defined paths using pre-defined acceleration controllers). Although such simplifications may affect the accuracy of the control bound estimation, they make integrating learning-based prediction and game-theoretic reasoning more practical, and they provide computational benefits that make the negotiation modeling online usable. Besides, the BRT augmentation step in Algorithm 1 provides some robustness to the effects induced by these simplifications. An alternative approach is to solve the full Stackelberg game with discretized state-action space offline [18] and use the pre-computed leader and follower policies for online inference. We only compared our approach with the baselines in a signal traffic scenario using a small number of traffic interactions. More comprehensive experiments are needed to validate our approach in various traffic scenarios. In this work, we used a controller-switching strategy when using the BRT to ensure the safety performance of the self-driving car. Further experiments are needed to validate our approach’s advantages over the baselines in a setting where the BRT is infused into the low-level controller’s design process [6].

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3Typically, a two-car interaction with less than 2[s] of TTC can be viewed as a dangerous interaction.

Fig. 2. (a-1)-(a-2) show two sequential steps in an experiment with the full BRT. (b-1)-(b-2) and (c-1)-(c-2) show steps in similar simulations with the BRT generated by the prediction-based reachability analysis [8] and our approach, respectively. The BRTs showed in the cartesian frame above are projected from the Frenet frame.
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