A Stable Control Algorithm for Multi Robot Formation

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Abstract. This paper presents the developed trajectory tracking controller for a formation of nonholonomic robots, which combines features from the leader-follower and virtual-structure approaches. The implemented decentralized control strategy allows the robots to be relatively independent and to switch easily between the executed individual tasks and the collective tasks. Convergence is thoroughly analyzed and guarantied using the Lyapunov approach.

1. Introduction

In the recent years, the cooperation of multiple robots like unmanned aerial vehicles (UAVs), autonomous underwater vehicles (AUVs), mobile robots systems (MRSs) is gaining significant importance in the control and robotics communities. Using formation of multiple robots to accomplish a mission or objective offers has obvious advantages such as increased feasibility, scalability, accuracy, flexibility, cost and energy efficiency, and so on. Potential applications include geophysical survey, search and rescue, farm aerial spraying, forest fire suppression, pipeline and power line patrol, and also many military applications such as general marine mine sweeping, exploration, surveillance and tracking, etc.

Several approaches have been proposed to solve the formation control problem. Behavior-based approach, leader-follower approach, virtual-structure, artificial potential, and graph theory are classified as the main formation control methods.

In the behavior-based approach [1], [2] numerous desired behaviors are supported for each robot, and the effective formation control is obtained as a result from a weighted summation of individual behavioral outputs.

A powerful technique to design control laws comes from the use of artificial potential functions where the formation control and collision avoidance can be solved through a combination of attractive and repulsive vector fields [3].

Other formation schemes are based on formation graphs and the so-called consensus problem. Some tools of graph theory and linear systems are used to prove the convergence to a desired formation [4].

In the virtual structure approach [5], the entire formation is managed as one unit. The desired posture for every robot is calculated and each robot has to track its own trajectory.

In the leader-following approach some agents are designated as leaders, where-as others are designated as followers [6], [7]. Each follower robot has been controlled to track the state that is obtained from the states of one or two leaders. The leader-following method has a simple structure and if it is necessary, it can rely on local sensor data only.
In this paper an original trajectory tracking controller for multi-robot formation, which combines features from leader-follower and virtual-structure approaches is proposed and its performance is evaluated. The designed control law is obtained as a generalization of an earlier presented trajectory tracking controller for a single nonholonomic mobile robot.

2. The control problem statement
The task of the control algorithm is to drive a group of robots to follow a desired trajectory while keeping the predefined space structure of the formation. The adopted approach concerns that the leader of the formation has to track the reference trajectory, while the other robots have to keep their positions in the formation relative to the position of the leader. So this approach can be classified as a leader following one. It is possible to modify the control problem discussed above if we assume that the leader of the formation is not a physical, but a virtual robot following perfectly the reference trajectory. In this case, the approach can be classified as a virtual structure approach. In this investigation, we will assume the case where the leader is a physical robot.

The structure of the formation is defined by specifying the position and orientation of each robot as a position and orientation of a leader with added offsets by constant values. In this way, there are two possibilities: (i) the offsets of the coordinates relative to the leader’s coordinates can be considered in the inertial coordinate frame; (ii) the offsets of coordinates can be considered in the moving coordinate frame attached to the leader. We will consider the first case where the offsets are defined in the inertial coordinate frame.

In order to implement the above strategy a kinematic controller, initially proposed for trajectory tracking for a single robot \cite{8} will be applied. The usage of this controller for the multi-robot system case ensures an asymptotic convergence of the trajectories of the controlled robots to the leader’s trajectory shifted by some constant displacement.

2.1. Robot Movement Representation
For the description of the motion of each robot on a 2D plane two coordinate frames have to be used: a global inertial frame \((O, x, y)\) and a local frame fixed to the center of mass \(G_i\) of the \(i\)-th robot \((G_i, x_i, y_i)\).

The configuration,

\[
p_i = \begin{bmatrix} x_i \\ y_i \\ \theta_i \end{bmatrix}
\]

of the \(i\)-th robot represents the position and orientation of the local coordinate system in the global frame, where the orientation \(\theta_i\) is taken counterclockwise from the global X-axis. The motion of the mobile robot is controlled by its linear and angular velocities \(- v(t)\) and \(\omega(t)\) respectively. The relation between \([v(t) \ \omega(t)]^T\) and the velocities in Cartesian frame is given by equation (2), where \(J(\theta)\) denotes the Jacobian matrix.

\[
\dot{p_i} = \begin{bmatrix} \dot{x}_i \\ \dot{y}_i \\ \dot{\theta}_i \end{bmatrix} = J(q_i) \begin{bmatrix} \cos(\theta_i) & 0 \\ \sin(\theta_i) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}
\]

2.2. The Controller Design
The control task for a leader following behavior is to ensure that the trajectory of the \(i\)-th robot \(p_i(t)\) will follow the trajectory of the leading robot \(p_L(t)\) conceivably shifted to some predefined constant
displacement $c_i$. The displacement $c_i = [c_{ix} \ c_{iy} \ c_{i\theta}]^T$ represents the desired distance between the configuration of $i$-th robot and the leading robot in the global frame.

Then the tracking error in global reference frame $E_i(t)$ is defined as follows:

$$E_i(t) = \begin{bmatrix} E_{ix} \\ E_{iy} \\ E_{i\theta} \end{bmatrix} = p_x(t) - p_i(t) - c_i = \begin{bmatrix} x_i(t) - x_i(t) - c_{ix} \\ y_i(t) - y_i(t) - c_{iy} \\ \theta_i(t) - \theta_i(t) - c_{i\theta} \end{bmatrix}$$

(3)

![Figure 1. Formation consisting of a leading robot and two following robots.](image)

Let $e_i(t)$ denotes the expression of $E_i(t)$ in the $i$-th robot local reference frame:

$$e_i(t) = \begin{bmatrix} e_{ix} \\ e_{iy} \\ e_{i\theta} \end{bmatrix} = T \begin{bmatrix} E_{ix} \\ E_{iy} \\ E_{i\theta} \end{bmatrix}$$

(4)

where $T(\theta)$ is a transformation matrix. From equation (2) it is possible to obtain the following two equations:

$$u_i = \dot{x}_i \cos(\theta_i) + \dot{y}_i \sin(\theta_i)$$

(5)

$$0 = \dot{x}_i \sin(\theta_i) - \dot{y}_i \cos(\theta_i)$$

(6)

The time derivative vector $\dot{e}_i(t)$ can be calculated from (4) and using equations (5) and (6) the following expressions for its elements can be obtained:
\[
\dot{\theta}_i(t) = (\dot{x}_L - \dot{x}_i) \cos \theta_i + (\dot{y}_L - \dot{y}_i) \sin \theta_i - (x_L - x_i - c_{\omega}) \dot{\theta}_i \sin \theta_i + (y_L - y_i - c_{\nu}) \dot{\theta}_i \cos \theta_i = \\
= \dot{x}_L \cos \theta_i + \dot{y}_L \sin \theta_i - \nu_i + \epsilon_i \dot{\theta}_i = \\
= \dot{x}_L \left[ \cos \left( \theta_L - (c_{\omega} + e_{\omega}) \right) \right] + \dot{y}_L \sin \left( \theta_L - (c_{\omega} + e_{\omega}) \right) - \nu_i + \epsilon_i \dot{\theta}_i = \\
+ \left[ \sin \left( \theta_L \right) \cos \left( c_{\omega} + e_{\omega} \right) \right] + \left[ \cos \left( \theta_L \right) \sin \left( c_{\omega} + e_{\omega} \right) \right] - \nu_i + \epsilon_i \dot{\theta}_i = \\
= \left( \dot{x}_L \sin \left( \theta_L \right) + \dot{y}_L \cos \left( \theta_L \right) \right) \sin \left( c_{\omega} + e_{\omega} \right) + \\
+ \left( \dot{x}_L \cos \left( \theta_L \right) - \dot{y}_L \sin \left( \theta_L \right) \right) \cos \left( c_{\omega} + e_{\omega} \right) - \nu_i + \epsilon_i \dot{\theta}_i = \\
= \nu_i \cos \left( c_{\omega} + e_{\omega} \right) - \nu_i + \epsilon_i \dot{\theta}_i \\
\tag{7}
\]

\[
\dot{e}_2(t) = - (\dot{x}_L - \dot{x}_i) \sin \theta_i + (\dot{y}_L - \dot{y}_i) \cos \theta_i - (x_L - x_i - c_{\omega}) \dot{\theta}_i \cos \theta_i - (y_L - y_i - c_{\nu}) \dot{\theta}_i \sin \theta_i = \\
= - \dot{x}_L \sin \theta_i + \dot{y}_L \cos \theta_i + \dot{x}_L \sin \theta_L + \dot{y}_L \cos \theta_L - e_i \dot{\theta}_i = \\
= - \dot{x}_L \left[ \sin \left( \theta_L \right) \cos \left( c_{\omega} + e_{\omega} \right) \right] - \cos \left( \theta_L \right) \sin \left( c_{\omega} + e_{\omega} \right) + \\
+ \dot{y}_L \left[ \cos \left( \theta_L \right) \cos \left( c_{\omega} + e_{\omega} \right) \right] + \sin \left( \theta_L \right) \sin \left( c_{\omega} + e_{\omega} \right) - e_i \dot{\theta}_i = \\
= \left( - \dot{x}_L \sin \left( \theta_L \right) + \dot{y}_L \cos \left( \theta_L \right) \right) \cos \left( c_{\omega} + e_{\omega} \right) + \\
+ \left( \dot{x}_L \cos \left( \theta_L \right) + \dot{y}_L \sin \left( \theta_L \right) \right) \sin \left( c_{\omega} + e_{\omega} \right) - e_i \dot{\theta}_i = \\
= \nu_i \cos \left( c_{\omega} + e_{\omega} \right) - e_i \dot{\theta}_i \\
\tag{8}
\]

\[
\dot{e}_3(t) = \dot{\theta}_L - \dot{\theta}_i = \omega_L - \omega_i \\
\tag{9}
\]

Then the linear and angular velocities of the \(i\)-th robot are calculated according to the following control law:

\[
q_i(t) = \begin{bmatrix}
\nu_i \\
\omega_i
\end{bmatrix} = \begin{bmatrix}
\nu_i \cos(c_{\omega} + e_{\omega}) + k_i e_{\omega} \\
\omega_i + k_i \nu_i e_{\omega} + k_i \nu_i \sin(c_{\omega} + e_{\omega})
\end{bmatrix} \\
\tag{10}
\]

The analysis of stability can be implemented by defining the Lyapunov function candidate \(V(t)\):

\[
V_i(t) = \frac{1}{2} (e_{\omega}^2 + e_{\omega}^2) + \frac{1 - \cos(c_{\omega} + e_{\omega})}{k_i} \\
\tag{11}
\]

The time derivative \(\dot{V}_i(t)\) has to be calculated:

\[
\dot{V}_i = \dot{\epsilon}_i e_{\omega} + \dot{\epsilon}_{\omega} e_{\omega} + \frac{\sin(c_{\omega} + e_{\omega})}{k_i} \\
\tag{12}
\]

Substituting equations (7), (8), and (9) into (12) and applying after that the control law (10) for \(\nu_i(t)\) and \(\omega_i(t)\) leads to the following result:
\[ V_i = \left[ \nu_i \cos (c_{i\theta} + \epsilon_{i}) - \nu_i + \epsilon_{i,2} \omega_{i,2} \right] e_{i,1} + \left[ \nu_i \sin (c_{i\theta} + \epsilon_{i}) - \epsilon_{i,1} \omega_{i,1} \right] e_{i,2} + \left[ \omega_{i,2} - \omega_{i,1} \right] \frac{\sin (c_{i\theta} + \epsilon_{i})}{k_2} = \]

\[ = -k_i e_{i,1}^2 - k_i \nu_i \frac{\sin^2 (c_{i\theta} + \epsilon_{i})}{k_2} \leq 0 \]

If the linear velocity \( \nu_i(t) \) of the leading robot has nonnegative values and the controller parameters \( k_i, k_2 \), and \( k_1 \) are also positive, then the inequality (13) is satisfied and the system is uniformly asymptotically stable around \( e_i(t) = 0 \). This means that the trajectory of the \( i \)-th robot will converge to the trajectory of the leader shifted by some constant displacement \( c_i = \left[ c_{i\theta}, 0, 0, \epsilon_{i,3} \right]^T \).

3. Simulation experiments

In order to investigate more systematically the dynamics of the collaboration between the robots in the formation, we have conducted a simulation experiment in the Webots environment, a 3D simulator for autonomous robots (figure 2). Webots simulator computes trajectories and sensory inputs of the robots in an area corresponding to the physical set-up. The simulation is sufficiently faithful for the controllers to be transferred to the real robots without changes, and for the robot behaviors to be very similar to those of the real robots.

The functionality of the proposed trajectory tracking controller has been tested on a scenario, where three robots have to perform a cooperative mission. A formation consisting of one leading robot and two following robots has been considered. The spatial structure of the formation has been defined by the following offsets:

\[ c_1 = [c_{1x}, c_{1y}, c_{1z}, c_{1\theta}]^T = [-20 \text{ cm}, 10 \text{ cm}, 0 \text{ rad}]^T \]
\[ c_2 = [c_{2x}, c_{2y}, c_{2z}, c_{2\theta}]^T = [-20 \text{ cm}, -10 \text{ cm}, 0 \text{ rad}]^T \]

which represents an isosceles triangle.

The desired trajectory of the leading robot has been selected to be a square with a side length of 100 cm and rounded corners (figure 3). The linear velocity on the straight-line parts of the path has to be 1.246 cm/s, while on the curves the linear velocity has to be 1.09 cm/s and the angular velocity has to be 0.12 rad/s.

![Figure 2. The simulation setup using software Webots.](image-url)
In addition, we have introduced initial tracking errors for the two followers:

\[
E_i(0) = \begin{bmatrix}
E_{x1}(0) \\
E_{y1}(0) \\
E_{\theta 1}(0)
\end{bmatrix} = \begin{bmatrix}
x_1(0) - x_L(0) - c_{1x} \\
y_1(0) - y_L(0) - c_{1y} \\
\theta_1(0) - \theta_1(0) - c_{1\theta}
\end{bmatrix} = \begin{bmatrix}
0 \text{ cm} \\
-10 \text{ cm} \\
0 \text{ rad}
\end{bmatrix}
\]

\[
E_2(0) = \begin{bmatrix}
E_{x2}(0) \\
E_{y2}(0) \\
E_{\theta 2}(0)
\end{bmatrix} = \begin{bmatrix}
x_2(0) - x_L(0) - c_{1x} \\
y_2(0) - y_L(0) - c_{1y} \\
\theta_2(0) - \theta_1(0) - c_{1\theta}
\end{bmatrix} = \begin{bmatrix}
-8.4 \text{ cm} \\
15 \text{ cm} \\
0 \text{ rad}
\end{bmatrix}
\]

The trajectories of the leading robot and the two following robots are shown on figure 3. It can be seen that by using the proposed control law, the two following robots keep the desired displacement with respect to the leader and the initially introduced tracking error tends asymptotically to zero. The time varying errors of both followers are shown on figure 4 and figure 5 respectively.

**Figure 3.** Reference trajectory of the leading robot and the obtained trajectories for the following robots.
Figure 4. The errors $E_{1X}$, $E_{1Y}$, and $E_{1\theta}$ of the first following robot along the trajectory.

Figure 5. The errors $E_{2X}$, $E_{2Y}$, and $E_{2\theta}$ of the second following robot along the trajectory.
4. Conclusion
In this paper, a control algorithm for a multi robot formation is proposed. The aim of the algorithm is to provide a reliable tracking control of the mobile robots. The formation movement strategy consists of two tasks: 1) the leader of the formation has to track the desired trajectory and 2) the robots in the formation have to track the leader by keeping at the same time the predefined structure of the formation. The presented control law is an extension of the control law proposed by Kanayama et al. (1991) and allows the accomplishment of both tasks mentioned above. The asymptotic convergence of the error is proved for the case of formation trajectory tracking. The presented simulation experiment confirm the convergence and the stability of the system.

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