Elastic light-by-light scattering in a non-minimal Lorentz violation scenario

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In the last years the Lorentz Violation scheme was implemented in QFT, in special in standard model of particle physics, in an attempt to explain our actual open problems. In this work we analyze non-minimal couplings between a Lorentz-violating 4-vector and the photon field and the leptonic current of QED. The 1-loop contribution of the elastic photon-photon scattering is showed and the novel characteristics which arose are pointed out.

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I. INTRODUCTION

The Standard Model (SM) has been successful over the past years in the attempt to describe the high energy phenomena process. Measurements in the very known particle accelerators have been shown with great agreement between theory and experiment. However, some issues have shown up. For instance the hierarchy, the strong CP and the cosmological constant problems \cite{1,2}, which the SM has not been able to explain satisfactorily. Therefore, some extensions have been proposed with the goal to seek some clues about these phenomena. Some researches proposed extra symmetries, like Grand unification models, or super symmetric extensions of the SM \cite{3,4}.

Another approach recently proposed was made through the search for Lorentz Symmetry Violation (LSV), the so called minimal SM extension \cite{5,6}. The main idea is that some fields could acquire non-null vacuum expectations values, as vector or tensor fields. It implies that the Lorentz symmetry could not be an exact symmetry, but an (worth) approximation. Up to now, the bounds in the LSV parameters are strong, and corroborates with the Lorentz symmetry as a good symmetry, leastwise in our present cosmological time. Going further, some non-minimal LSV couplings have been analyzed as well, and the possible effects in the SM are also examined.

In this work we analyze the contribution to some specific non-minimal coupling LSV parameters in the quantum electrodynamics (QED), specifically to the elastic scattering between two photon, also called elastic light-by-light (LbyL) scattering.

This paper is organized as follows: in the Section II we show the major details about the photon-photon scattering. In the Section III we introduce the non-minimal couplings and in the Section IV we calculate the contribution of the non-minimal couplings to elastic LbyL scattering. Finally, in Section V we discuss the results.

II. ELASTIC LIGHT-BY-LIGHT SCATTERING

Interest in nature and phenomena linked to light has always been present in physics. Even when quantum mechanics brought with it the concept of wave-particle dualism, reconciling the undulatory and corpuscular models of light that already intrigued names like René Descartes, Isaac Newton, Robert Hooke, Christiaan Huyghens, new interests continued to emerge. Maxwell’s equations of classical electrodynamics had the inclusion of light in theory as a great success, but their linearity forbids the existence of processes that quantum point of view would predict. As early as 1933 the concern with the properties of the vacuum and interaction between quanta of energy \cite{8} exemplified well the paradigm shift. Theoretical work on nonlinear electrodynamics first appeared in the 1930s with Born, Infeld, Euler and Heisenberg \cite{9,12}, followed by subsequent years \cite{13,14}. The implemented corrections resulted in the possibility of scattering between two photons by means of quantum vacuum fluctuations \cite{11,14}, including the prediction of the cross section of the process \cite{15,17}.

Non-linear phenomena such as the scattering of a photon in a Coulomb field \cite{18} or the splitting of a photon in two in the presence of an external field \cite{19} were studied and already observed experimentally \cite{20,25}. On the other hand, elastic LbyL scattering, given its small cross section, had only recently achieved experimental evidence \cite{26}. These effects are represented in the lowest order by square diagrams, but in some cases we replace real photons by a line representing an external field, as the examples in Fig. (1) show.

From the six Feynman diagrams associated with elastic LbyL scattering, obtained by the different combinations of the photonic legs, we can calculate the scattering amplitudes and the differential cross section for non-polarized photons. The loop can contain different kinds of virtual charged particles (quarks, leptons, W\textsuperscript{±} \cite{27}) according to available energy in the experiment.
Considering only vertices of the QED, in a low energy regime ($\omega \ll m$), the differential cross section of the process is given by [15, 16, 18]

$$\frac{d\sigma}{d\Omega} = \frac{139\alpha^4}{(180\pi)^2 m^2} \left(\frac{\omega}{m}\right)^6 (3 + \cos^2 \theta)^2,$$

whereas in the ultra-relativistic case [28]

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^4}{\pi^2 \omega^2} \log^2 \frac{1}{\lambda},$$

that is suitable for small scattering angles ($m/\omega \ll \theta \ll 1$). In both results we are using natural units ($\hbar = c = 1$).

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In an extended version of QED coupling $\xi^\mu$ and $F_{\mu\nu}$, our Lagrangian presents the following dimension-5 term

$$\mathcal{L}_{LV} = \xi^\mu \bar{\psi} \gamma^\nu \gamma^\rho \gamma^\sigma \psi F_{\mu\nu\rho\sigma},$$

where $\xi^\mu$ is constant and has canonical dimension of inverse mass, and $\mathcal{L}_{LV}$ is CPT-even whenever $\xi^\mu$ transforms under T-symmetry as $\xi^\mu \rightarrow (\xi_0, \xi) \rightarrow (\xi_0, \xi) = (0, \frac{1}{\xi}).$ On the other hand, $\mathcal{L}_{LV}$ would be CPT-odd if $\xi^\mu = (\xi_0, \xi) \rightarrow (\xi_0, \xi) = (0, \frac{1}{\xi}).$ It describes a sort of transition electric dipole moment, which connects the relativistic dominant component of the fermion with the weak relativistic component. Differently from minimal coupling (e.g., Carrol-Field-Jackiw [31]), in this case the fermionic current structure will be modified. In this coupling, together with the QED Lagrangian

$$\mathcal{L}_{QED} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \bar{\psi}(i\gamma \cdot D - m)\psi,$$

where $\gamma = \gamma^\mu (\partial_\mu + ieA_\mu)$, the modified Maxwell equations will be given by

$$\partial_\mu F^{\mu\nu} = e\bar{\psi} \gamma^\nu \gamma^\rho \gamma^\sigma \psi + \xi^{[\nu} \partial_\rho (\bar{\psi} \gamma^{\mu]}) \psi$$

$$= \left[(e\partial_\mu + \xi^{\nu} \partial_\mu - \xi_\mu \partial^{\nu}) (\bar{\psi} \gamma^{\nu} \psi)\right]$$

and

$$i\gamma^\mu \partial_\mu + m e \gamma^\mu A_\mu + \gamma^\mu \xi^{\nu} F_{\mu\nu} \psi = 0.$$
Here, Eq. (5) represents the new sourced Maxwell equations and Eq. (6) is the modified Dirac equation. Through the Eq. (7), we can show that an extended charge will be conserved, instead of the QED electric charge. We have \( \partial_{\mu} J^{\nu} = 0 \) where \( J^{\nu} = (e \delta_{\nu}^\mu + \xi^\nu \partial_{\mu} - \xi_{\mu} \delta_{\nu}^\mu) \psi \gamma^\mu \psi \) and

\[
Q' = \int d^3 x J^0 = Q + \partial_{t}[\int d^3 x (\xi \cdot J)], \tag{7}
\]

where \( Q = e \int d^3 x \psi^\dagger \psi \) and \( J^i = \psi^\dagger \gamma^0 \gamma^i \psi \). Though the charge is defined for the free particle in introducing \( \mathcal{L}_{LV} \) the new conserved 4-current is \( J_{\mu}' \). However, the fermionic charge is not affected by the extra contribution. In a quantum point of view, the coupling in Eq. (3) modifies the QED vertex of Feynman diagrams, which now reads

\[
\Gamma^\mu = e \gamma^\mu - i q \xi^\mu + i (\xi \cdot q) \gamma^\mu. \tag{8}
\]

As previously mentioned, we can consider other non-minimal interaction, coupling our background 4-vector with the dual of the electromagnetic field strength \( F_{\mu\nu} \). The new LSV term modifying the standard QED Lagrangian is then

\[
\mathcal{L}_{LV} = \xi^\mu \bar{\psi} \gamma^\nu \psi \tilde{F}_{\mu\nu}, \tag{9}
\]

where \( \tilde{F}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} F^{\alpha\beta} \). Differently from the previous term, this term do not could contribute for CP-violating decays but contributes to a sort of transition magnetic dipole moment (MDM) of the fermions. The modified Maxwell equations becomes

\[
\partial_{\mu} F^{\mu\nu} = e \bar{\psi} \gamma^\nu \psi + e \varepsilon_{\alpha\beta\mu\nu} \xi^\alpha \partial_{\mu} \bar{\psi} \gamma^\beta \psi = (e \delta^\mu_\alpha - \xi^\mu_\beta \varepsilon^\beta_\alpha \psi^\dagger \psi) \bar{\psi} \gamma^\alpha \psi \tag{10}
\]

and

\[
(i \gamma^\mu \partial_{\mu} + m + e \gamma^\mu A_\mu + \gamma^\mu \xi^\nu \tilde{F}_{\mu\nu}) \psi = 0 \tag{11}
\]

A important detail in this case is that the conserved charge do not changes, i.e., \( Q' = Q = e \int d^3 x \psi^\dagger \psi \). Going further, in this situation the extension of the usual QED vertex is written as

\[
\Gamma^\mu = e \gamma^\mu - \xi^\mu_\alpha \gamma^\alpha \xi^\nu \gamma^\nu q^\beta. \tag{12}
\]

By using those vertices, which are a deformation of the original QED vertex, we can find the quantum corrections for leptonic EDM and MDM, for instance [7]. Once we are interested in determining the effect of LSV to the elastic LbyL scattering, the next step in this paper will be to use Eq. (3) and Eq. (12) to calculate scattering amplitudes and the differential cross sections of this process.

**IV. ELASTIC LIGHT-BY-LIGHT SCATTERING WITH LSV**

**A. F Coupling**

The scattering which we want to analyze depends on a tensor whereas, in a 1-loop approximation, can be found by the following expression

\[
T_{\mu\nu\alpha\beta} (q_1, q_2, q_3, q_4) = \int \frac{d^4 p}{(2\pi)^4} T^\mu_\nu (p) \gamma^\alpha (p-q_4) \gamma^\beta (p-q_1) \gamma^\gamma (p-q_3) \gamma^\delta (p-q_4) \times \gamma^\alpha (p-q_4) \gamma^\beta (p-q_1) \gamma^\gamma (p-q_3) \gamma^\delta (p-q_4). \tag{13}
\]

The key to visualize the contribution of the LSV modification is rewrite the vertex as

\[
\Gamma^\mu (q) = e \gamma^\alpha (\delta^\mu_\alpha + i e^{-1} \xi \cdot q q^\mu - i e^{-1} q q^\mu) = e \gamma^\alpha M^\mu_\alpha (\xi, q). \tag{14}
\]

and, since the loop integral do not depends on the \( q_i \) \( (i = 1, ..., 4) \) momentum, i.e., do not depends on external momentum which coupling with the vertex, the tensor with the LV contribution reads

\[
T_{\mu\nu\alpha\beta}' (q_1, q_2, q_3, q_4) = T_{\mu\nu\alpha\beta} (q_1, q_2, q_3, q_4) \times M^\mu_\mu (\xi, q_1) M^\nu_\nu (\xi, q_2) M^\alpha_\alpha (\xi, q_3) M^\beta_\beta (\xi, q_4). \tag{15}
\]

A another point of view is that the result could be seen by the coupling between the tensor \( T_{\mu\nu\alpha\beta} \) and a redefinition of the polarization vectors \( \epsilon_\mu (q) \). Therefore, the LV modifies the polarization vector itself, i.e.,

\[
\epsilon_\mu (q) = M^\mu_\alpha (q) \epsilon_\alpha (q) = (1 + ie^{-1} \xi \cdot q) \epsilon_\alpha (q) - i e^{-1} \xi q \cdot \epsilon (q). \tag{16}
\]

Considering physical external photons, we have \( q \cdot \epsilon (q) = 0 \) and we reach

\[
\epsilon_\mu (q) = (1 + ie^{-1} \xi \cdot q) \epsilon_\alpha (q). \tag{17}
\]

Using the scattering matrix

\[
\mathcal{M}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}^{QED} = (e^{\lambda_1}_\mu (q_1) e^{\lambda_2}_\mu (q_2) e^{\lambda_3}_\alpha (q_3) e^{\lambda_4}_\beta (q_4) \times T_{\mu\nu\alpha\beta} (q_1, q_2, q_3, q_4)), \tag{18}
\]

with the LSV contribution we reach the following modified scattering matrix

\[
\mathcal{M}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}^{LV} = (1 + C) \mathcal{M}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}^{QED}, \tag{19}
\]

where

\[
C = i e^{-1} (q_1 + q_2 - q_3 - q_4) \cdot \xi + e^{-2} [(q_1 \cdot \xi) (q_2 \cdot \xi) + (q_1 \cdot \xi) (q_3 \cdot \xi) + (q_1 \cdot \xi) (q_4 \cdot \xi) + (q_2 \cdot \xi) (q_3 \cdot \xi) + (q_2 \cdot \xi) (q_4 \cdot \xi) - (q_3 \cdot \xi) (q_4 \cdot \xi)] + O(\xi^3). \tag{20}
\]

Since, by momentum conservation, \( q_1 + q_2 - q_3 - q_4 = 0 \), the LSV only contributes in second order of the \( \xi \) parameter. Finally, we will have \( |M_{\mu\nu}|^2 \) given by

\[
|M_{\mu\nu}|^2 = (1 + C) (1 + C) |M_{\mu\nu}^{QED}|^2 \approx (1 + 2Re(C)) |M_{\mu\nu}^{QED}|^2. \tag{21}
\]
Using a referential where, in Lorentz gauge, \( q_1 = (\omega, q), q_2 = (\omega, -q) \), \( q_3 = (\omega, k) \) and \( q_4 = (\omega, -k) \), the LSV contribution is written as

\[
C = e^{-2}[(k \cdot \xi)^2 + (q \cdot \xi)^2 + 2(\omega \xi_0)^2] + O(\xi^3).
\]

where \( \xi^\mu = (\xi_0, \xi). \) Taking the components of \( q, k \) and \( \xi \) in terms of intensity and angles \( \theta, \phi, \theta_\xi \) and \( \phi_\xi \), the result will be in general

\[
\frac{1}{4} \sum |M^\text{LV}|^2 \approx \frac{1}{4} \sum \left( 1 + \omega^2 \rho^2 \right) |M^\text{QED}|^2, \tag{22}
\]

where \( \rho^2 = \rho^2(\theta, \phi, \theta_\xi, \phi_\xi) = 2\text{Re}(C)/\omega^2. \)

This way, the final result for modified differential cross section can be written as

\[
\frac{d\sigma^{\gamma\gamma\xi}}{d\Omega} = \frac{1}{64\pi^2 (2\omega)^2} \sum |M^\text{QED}|^2 = \frac{d\sigma^{\gamma\gamma\xi}}{d\Omega} \left( 1 + \omega^2 \rho^2(\theta, \phi, \theta_\xi, \phi_\xi) \right). \tag{23}
\]

**B. \( \tilde{F} \) Coupling**

For the \( \tilde{F} \) coupling the line of thought is the same. However, the result differs due to the new structure of the Levi-Civita tensor. Rewriting the result in Eq. (12) in an analogous way to what we did in (14)

\[
\Gamma^\mu &= e\gamma^\alpha(\delta^\mu_\alpha - e^{-1}\epsilon^\mu_\alpha\beta\xi^\beta q^\alpha) \\
&= e\gamma^\alpha N^\mu_\alpha(\xi, q). \tag{24}
\]

Thus, in order to calculate the elastic LbyL scattering differential cross section with this kind of LV term, we rewrite the polarization vector. But, in this case, we use \( N^\mu_\alpha. \) Then

\[
\epsilon^\mu(q) = N^\mu_\nu(q)e^\nu(q) \\
= e^\mu(q) - ie^{-1}\epsilon^\mu_\nu\alpha\beta\xi^\alpha q^\beta \epsilon^\nu(q), \tag{25}
\]

and in the same path, we reach the following scattering matrix

\[
M^\text{LV}_{\lambda_1,\lambda_2,\lambda_3,\lambda_4} = (e_1^\lambda_1)^\mu_{(\xi_3)^\nu}(e_3^\lambda_3)^\alpha_\nu(e_4^\lambda_4)^\beta_\alpha T_{\mu\nu\alpha\beta} \\
= N^\mu_\mu N^\nu_\nu(N^\xi)^\alpha_\alpha(N^\xi)^\beta_\beta \times \\
\times (e_1^\lambda_1)^\mu_{(\xi_2)^\nu}(e_2^\lambda_2)^\nu_\nu(e_3^\lambda_3)^\alpha_\alpha(e_4^\lambda_4)^\beta_\beta T_{\mu\nu\alpha\beta}. \tag{26}
\]

So, in a short way

\[
|M^\text{LV}|^2 = (NN)^\mu_\nu(NN)^\nu_\mu(NN)^\alpha_\alpha(NN)^\beta_\beta \times \\
\times T_{\mu\nu\alpha\beta}(T^\alpha_\nu)^\alpha\nu\beta_\nu, \tag{27}
\]

where \( (NN)^\mu_\nu = N^\mu_\mu N^\nu_\nu \) and the contractions in parenthesis can be expanded as

\[
(NN)^\mu_\nu = \delta^\mu_\nu (1 - (q \cdot \xi)^2) + q^2(\delta^\mu_\nu \xi^\nu - \xi^\mu \xi^\nu) + O(\xi^3). \tag{28}
\]

Here we ignore proportional terms to \( q_\mu \), since, due to gauge invariance, they cancel when contracted with any index of \( T. \) Finally, we can find \( |M|^2 \) and the differential cross section. For external photons we have then

\[
\frac{d\sigma^{\gamma\gamma\xi}}{d\Omega} = \frac{d\sigma^{\gamma\gamma\xi}}{d\Omega} \left( 1 - \omega^2 \rho^2(\theta, \phi, \theta_\xi, \phi_\xi) \right), \tag{29}
\]

where the \( \rho \) is the same function as the previous section.

In both non-minimal couplings, the LV contributions shows up in the region \( \omega_\xi \approx e/|\rho|. \) Since the LV parameters typically came from high energy effects, an analysis in the UV limit should be more productive. Quite independently will be the relation \( \frac{d\sigma^\text{QED}}{d\Omega} \). Since the \( \rho^2 \) and the contractions in parenthesis shows up in the region \( \omega_\xi \approx e/|\rho|. \) However, this third case also present a superposition of observed effects on the first two and therefore it will not be analyzed.

**V. DIFFERENTIAL CROSS SECTION: LSV EFFECTS**

In order to shed light in the LV effects we will take some particular choices for the configuration of the background 4-vector. The first will be a timelike scenario and the second one spacelike. Such a division is arbitrary and if \( \xi \) exists, it will be in general a nontrivial mixture of temporal and spacial components. We could, for example, consider a lightlike 4-vector, i.e., \( \xi = (\xi_0, 0, 0, \zeta). \) However, this third case also present a superposition of observed effects on the first two and therefore it will not be analyzed.

**A. Timelike background 4-vector**

In the case of timelike LV, i.e., \( \xi = (\xi_0, 0), \) our result takes the simplest form. The contribution will be given by \( \rho^2(\theta, \phi, \theta_\xi, \phi_\xi) = 4e^{-2}\xi_0^2 \) and thus

\[
\left| \frac{\partial \sigma}{\partial \Omega} \right|_{\text{QED}} - 1 \approx 4e^{-2}\xi_0^2 \omega^2 \tag{30}
\]

The effects of the time-like LV will show up in frequencies near to \( \omega_\xi = e\xi_0^{-1}. \)
Figure 2: Photon-photon cross section profile for QED (Black) and for QED plus time-like Lorentz violation contribution with $\omega_{\xi} = 0.01 m$ (Gray).

Since the difference between the QED-cross section and the LV increases with the photon energy, the best regime to detect possible LV effects is in the high energy process, which QED contribution turns down and the LV effect could rise up in this background.

By introducing cutoff parameters $\Lambda_{\pm}$ in a modified differential cross section of the process $[54]$ and by comparison between the new differential cross section and experimental results, it is possible to parameterize deviations from QED. As soon as experimental data for elastic LbyL scattering are available with good precision, such analysis can be developed to find the values of $\Lambda_{\pm}$ in an adequate confidence level, and finally get upper bounds for our LSV parameters in the same spirit as it has already been done for other QED processes $[47, 48]$.

B. Spacelike background 4-vector

We consider next the case of a pure spacelike background 4-vector, i.e., $\xi^{\mu} = (0, \xi)$. With this particularization we will investigate the angular profile of differential cross section in the low and high energy regimes for elastic LbyL scattering.

Without lost of generality, a referential where the incoming photons are in the z axis, i.e., $q = \omega \hat{z}$ and $k \cdot \hat{z} = \omega \cos \theta$ can be chosen. When we dealing with $\xi$ in an arbitrary direction

$$\xi = |\xi| \{\sin \theta_{\xi} \cos \phi_{\xi} \hat{z} + \sin \theta_{\xi} \sin \phi_{\xi} \hat{y} + \cos \theta_{\xi} \hat{x}\},$$

and then

$$k \cdot \xi = |\xi| \omega \left(\sin \theta \sin \theta_{\xi} \cos (\phi - \phi_{\xi}) + \cos \theta \cos \theta_{\xi}\right),$$

$$q \cdot \xi = |\xi| \omega \cos \theta_{\xi}.$$

From this choice of reference, we can write the total differential cross section as a part due to only QED and another part due to VSL, i.e.

$$\frac{d\sigma_{\gamma \gamma}}{d\Omega} = \frac{d\sigma_{\gamma \gamma}}{d\Omega} + \frac{d\sigma_{LV}}{d\Omega}.$$

We can then verify the effects of the modification on the differential cross section graphs, both in the low power regime and in the high energy regime. If $\omega \ll m$, a general result for the space-like 4-vector case is written as

$$\frac{d\sigma_{\gamma \gamma \xi}}{d\Omega} = \frac{2 |\xi|^2}{e^2} \frac{139 \alpha^4 \omega^8}{(180\pi)^2 m^8} (3 + \cos^2 \theta)^2 \left[\left(\cos \theta_{\xi}\right)^2 + \left(\sin \theta \sin \theta_{\xi} \cos (\phi - \phi_{\xi}) + \cos \theta \cos \theta_{\xi}\right)^2\right],$$

which can have better visualization of the effects in two different paticularizations. Taking first a background vector parallel to the z axis ($\theta_{\xi} = 0$) we find

$$\frac{d\sigma_{\gamma \gamma \parallel}}{d\Omega} = \frac{2 |\xi|^2}{e^2} \frac{139 \alpha^4 \omega^8}{(180\pi)^2 m^8} \left[9 + 15 \cos^2 \theta + 7 \cos^4 \theta + \cos^6 \theta\right],$$

from where we can see that there is a change in the angular dependence in $\theta$ and there is no resulting azimuthal dependence. On another hand, if we consider the background vector in transverse plane xy ($\theta_{\xi} = \pi/2$),

$$\frac{d\sigma_{\gamma \gamma \perp}}{d\Omega} = \frac{2 |\xi|^2}{e^2} \frac{139 \alpha^4 \omega^8}{(180\pi)^2 m^8} \left[(3 + \cos^2 \theta)(\sin \theta \cos (\phi - \phi_{\xi}))\right]^2,$$

whose instantaneous angular profile is plotted in Fig. [3] for different relative orientations of the background in the transverse xy plane. This LSV piece is clearly $\phi$ dependent, feature very distinctive in comparison with the QED and which can be used as a way to look for LSV in the experimental results.

Now we turn to the high energy regime, under the conditions of validity of Eq. (2). Similarly to the previous one, taking a background vector parallel to the z axis

$$\frac{d\sigma_{\gamma \gamma \parallel}}{d\Omega} = \frac{2 |\xi|^2}{e^2} \frac{139 \alpha^4 \omega^8}{(180\pi)^2 m^8} \left[1 + \cos^2 \theta\right],$$

and again, more interesting is the second scenario with a transverse background in which the LV piece in differential cross section becomes

$$\frac{d\sigma_{\gamma \gamma \perp}}{d\Omega} = \frac{2 |\xi|^2}{e^2} \frac{139 \alpha^4 \omega^8}{(180\pi)^2 m^8} \log^4 \frac{1}{\theta} \left(\sin^2 \theta \cos^2 (\phi - \phi_{\xi})\right).$$

This LV contribution is also plotted in Fig. [4] for different choices of the azimuthal angle $\phi_{\xi}$. Once more, we have an anisotropy profile in the scattering cross-section.

When we analyze the cases of differential cross section modified by a 4-vector background in transverse plane, the angular profile in low energy regime for QED graphs shows minimum values in $\theta = \pi/2$ while the LSV surfaces have maxima in the same $\theta = \pi/2$. These maximums occur in $\phi = 0, \pi, 2\pi$ for $\xi \parallel \hat{z}$ and in $\phi = \pi/2, 3\pi/2$ for $\xi \perp \hat{z}$. Such specifications are those in which it would be easier to observe LSV effects in experiments, since $|\xi|$ is expected as a very small intensity term.

In the high energy regime, the LSV effects will be more pronounced in the same $\phi = 0, \pi, 2\pi$ for $\xi \parallel \hat{z}$ and in $\phi = \pi/2, 3\pi/2$ for $\xi \perp \hat{z}$, but the lower the value of $\theta$ the more intense it will be.
Figure 3: Instantaneous angular profile of differential cross sections for QED (top) and pure spacelike background (\(\xi \perp \hat{z}\), i.e., \(\theta_\xi = 0\)) LV scenario, in low energy regime. The vertical axes are given by 

\[
N'_\sigma = \left[\alpha^4/\omega^2\right]^{-1}d\sigma^{\gamma\gamma}/d\Omega \\
N_\sigma = \left[2\alpha^4/\omega^8/e^2m^8\right]^{-1}d\sigma^{\gamma\gamma,\perp}/d\Omega,
\]

with \(\phi_\xi = 0\) (middle) and \(\phi_\xi = \phi/2\) (bottom).

Furthermore, Eqs. (37) and (38) show that the extra LSV contribution up to \(O(\xi^3)\) is energy independent, while the differential cross section of QED fall with \(\omega^{-2}\). In the case of \(\xi \perp \hat{z}\), in particular, a plateau can be displayed with experiments performed at increasingly higher energies for small \(\theta\). It is important to remember that the limits of validity of Eq. (2) \((m/\omega \ll \theta \ll 1)\) must be respected.

If the whole previous analysis were developed for \(\tilde{F}\) coupling, the same resulting azimuthal dependence would be observed, but with a global minus sign, as indicated in Eq. (29). Similar results were reported with this \(\phi\)-dependence, considering nonminimal couplings to modify the QED Lagrangian, in processes such as Compton and Bhabha scatterings [47, 48].

An important remark might be done: the LV considerations above apply to a truly fixed, time-independent, background. These requirements are only explicit in an
inertial reference frame, which is not the case of the Earth due to its sidereal and orbital motions: in the lab the background would seem to rotate. A convenient and approximately inertial frame is the so-called Sun-centered frame (SCF) [33], broadly used in the literature [32].

In order to translate the accessible, time-dependent, background as observed on Earth, $\xi_{\text{lab}}$, in terms of combinations of the constant $\xi_{\text{Sun}}$, we place a general Lorentz transformation, i.e., $\xi^\mu_{\text{lab}} = \Lambda^\mu_\nu \xi^\nu_{\text{Sun}}$, where $\Lambda^\mu_\nu$ is given in ref. [33]. If we ignore sub-dominant boost effects of order $\beta \lesssim 10^{-4}$, we may write $\xi_{\text{lab}} = \xi^z_{\text{Sun}} \equiv 0$ and $\xi_{\text{lab}} = R^{\chi} \chi, T_{\text{lab}} \xi_{\text{Sun}}$, where the rotation matrix is explicitly time-dependent ($T_{\text{lab}}$ is the time in the SCF).

Usually experiments are conducted over long time-scales, the LV signatures observed in Earth-bound experiments would be effectively time averaged. The only non-vanishing (time-averaged) spatial components are then $\xi^x_{\text{lab}} = - \sin \chi \xi^x_{\text{Sun}}$ and $\xi^y_{\text{lab}} = \cos \chi \xi^y_{\text{Sun}}$, with $\chi$ the colatitude of the experiment. Depending on the experiment, the parallel and perpendicular components of the LV vector will be related with the above expressions.

VI. CONCLUDING REMARKS

In this work we investigated two specific nonminimal Lorentz violating couplings between the fermion and gauge fields, and its effects in the differential cross section of elastic photon-photon scattering as described by QED. When considering the coupling with $F$, a new charge conservation scheme was derived, dependent of $\xi$, but the electric charge conservation is not affected. However, if we consider only a timelike background 4-vector, this contribution vanishes. On the other hand, the second coupling with $F$, generates the same kind of contribution to the elastic LbyL scattering without modification in the charge conservation law.

Other advantage is that this couplings bring with itself a transition EDM and MDM’s, which can open the opportunity to be bounded by neutrino processes, for instance.

In our development, we did not present results dependent on the more general case for $\xi$, with temporal and spatial components mixture, but, in another way, we have focused just on purely timelike or spacelike LV background. This choice, besides being common in works on LV, give us differential cross section modifications whose characteristics can be more easily observed. From them, and having new experimental results, it will be possible to estimate upper bounds for the LV parameters of our models. Since the data yield of experiments involving ultra-peripheral Pb+Pb collisions at LHC is expected to double at the end of 2018 and increase tenfold after LHC Run 4 [26], scheduled to start in 2026, we expect to have accurate data to find these upper bounds in the next few years.

When we analyze the behavior of the differential cross section in the space-like case, we observe that new terms arose bring new angle dependences, either in $\theta$ and $\phi$. The $\phi$ contribution is the most interesting, onde the QED are independent of azimuthal variation, and in our case would be no more true. Since the high as the low energy limits there are a rise of a periodic pattern in the $\phi$ angle (Fig. 3 and Fig. 4) which would be a visible signal of Lorentz violation in a experimental point of view. This azimuthal dependence is a characteristic present in other LV+QED process, as Compton, Bhabha e Moller scatterings [47, 48] considering the couplings cited in e [9]. Besides that, since that the QED contribution to photon-photon scattering decreases in the High energy limit whereas the LV contribution increases, this regime would be the most fruitful regime to Lv searches.

One appointment which need to be done is that these couplings are in some sense inspired in the Carrol-Field-Jackiw model (CFJ) [55], but here the charged current replaces the photon field $A$. Due to this trait, we figure that this non-minimal LV effect should shows up and be observed easily that in the CFJ model. Instead of modify the propagator of the photon field, the non-minimal couplings modifies the vertex of the interactions between the photon and the electric current. Therefore this approach maintains the dispersion relations of the massless photon of QED. Other advantage of this approach is that derivative couplings naturally shows up in high energy limits, thus it could be easily observed in high energy experiments as LHC than minimal LV coupling.

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Evidence for light-by-light scattering

Photon-photon

Calculation of the polarization tensors of \( Z \rightarrow \gamma \gamma \) at the LHC

Electronic energy loss in heavy-ion collisions with the ATLAS detector

Spontaneous breaking of Lorentz symmetry in string theory

Non-linear interactions between electromagnetic fields

The scattering of light by light

The dispersion approach to photon-photon scattering

The scattering of photons by photons

Non-minimal coupling between electromagnetic fields

Nonlinear corrections to Maxwell electrodynamics using \( \gamma \gamma \) scattering

\( \mu \) light by light contribution to the \( (g-2) \) on the lattice

Hadronic light-by-light scattering contribution to the sixth-order magnetic moment of the muon

Ultra-peripheral collisions and hadronic structure

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Limits on Lorentz-and parity-violating modification of electrodynamics

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