Thorium isomer for radiative emission of neutrino pair

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ABSTRACT

It is proposed to use the isomer ionic ground state $^{229}_{4}\text{m}^{4+}{\text{Th}}$ embedded in transparent crystals for precision determination of unknown neutrino parameters. Isolation from solid environment of the proposed nuclear process, along with available experimental techniques of atomic physics, has a great potentiality for further study.

Key words
Neutrino mass, Majorana particle, Thorium isomer, Macro-coherence
Introduction  Despite of remarkable success of neutrino oscillation experiments in recent years [1] a few critically important neutrino parameters have been left untouched. To improve the situation we proposed [2, 3] to use atoms, instead of conventional nuclear target [1, 5], for the purpose of determination of undetermined important neutrino parameters such as the smallest neutrino mass, the mass type (Dirac vs Majorana), and their CP properties, not easy to access by other means (due to the energy mismatch to expected neutrino masses of a fraction of eV).

We propose in the present work yet another possibility: use of the nuclear isomer of thorium ion, $^{229m}\text{Th}^{4+}$. This nuclear isomer has an exceptionally small excitation energy as nuclear levels, the best estimated value 7.8 ± 0.5 eV at present [6], and a small decay rate of nuclear magnetic dipole transition to ground state yet to be determined. Due to protection of many surrounding electrons, nuclear isomer in thorium atom has an advantage of excellent isolation from environmental effects. It therefore has spurred great interest both in application and in fundamental physics: a possibility of the next-generation frequency standard [7], [8], [9], [10], and the precision measurement of variation of fundamental constants [11].

The process we use for the neutrino mass spectroscopy is atomic de-excitation: radiative emission of neutrino pair (RENP) from the ground ion atom of the isomer nucleus (isomer ground state) to the ground state ion of the normal (ground) nucleus, denoted by $|e\rangle \rightarrow |g\rangle + \gamma + \nu_i\nu_j$ where $\nu_i, i = 1, 2, 3$ is the neutrino mass eigen-state. Location of pair emission thresholds, $\omega_{ij} = \epsilon_{eg}/2 - (m_i + m_j)^2/(2\epsilon_{eg})$, and the photon spectrum in their vicinity gives excellent opportunities of determining all neutrino mass values. Separation into mass eigen-state thresholds is made possible by precise frequencies of trigger lasers, an essential ingredient of RENP process developing field storage along with target polarization.

Since we need a large number of target atoms, of order the Avogadro number, for RENP, a solid environment is the best target choice. Isolation of the proposed isomer ion from solid environment is a great advantage in this regard, since we need a high degree of coherence, normally difficult to achieve in solids, for developing a macro-coherent state of target and the stored field for RENP [12, 3].

$\text{Th}^{4+}$ ion in transparent solid  We consider embedding the isomer Th in transparent crystal such as CaF$_2$. The initial RENP state $|e\rangle$ is taken as the isomer ion ground state $^{229m}\text{Th}^{4+}$, while the final state $|g\rangle$ is the normal ground state $^{229}\text{Th}^{4+}$. The embedded $^{229m}\text{Th}^{4+}$ is a closed shell ion of configuration $6p^6$. CaF$_2$ solid has a large electric crystalline field at the ionic site, reaching field gradient $\sim O(5 \times 10^{18})\text{V/cm}^{-2}$ [13]. The large field gradient causes large energy shifts and large state mixing among parity even or among parity odd states. The proposed RENP process that occurs within a system of nucleus + core electrons is however insensitive to state mixing among different parity states.

Weak interaction of neutrino pair emission from electron  According to the standard electroweak theory, neutrino interaction with atomic electron is described by $H_w = \langle n|\int dx^3\mathcal{H}_{2\nu}^n(x)|n'^\prime\rangle$ with

$$H_{2\nu}^n = \frac{G_F}{\sqrt{2}} \left( \bar{\nu}_i \gamma^\alpha (1 - \gamma_5)\nu_e \bar{e} \gamma_\alpha (1 - \gamma_5)e - \frac{1}{2} \sum_i \bar{\nu}_i \gamma^\alpha (1 - \gamma_5)\nu_i \bar{e} \gamma_\alpha (1 - 4\sin^2\theta_W - \gamma_5)e \right).$$  \hspace{1cm} (1)

Since atomic electrons are described in the non-relativistic regime, we may ignore terms suppressed by the factor $1/m_e$. In terms of two component spinors,

$$H_{2\nu}^n = \frac{G_F}{\sqrt{2}} \left( \bar{e} \sum_{ij} b_{ij} \nu_{j}^\dagger \nu_i + \bar{e}^\dagger \bar{\sigma} e \cdot \sum_{ij} a_{ij} \nu_{j}^\dagger \nu_i \right) + O(\frac{1}{m_e}),$$ \hspace{1cm} (2)

$$b_{ij} = U_{ei}^* U_{ej} - \frac{\delta_{ij}}{2} (1 - 4\sin^2\theta_W), \quad a_{ij} = -U_{ei}^* U_{ej} + \frac{1}{2} \delta_{ij},$$ \hspace{1cm} (3)

where $U_{ei}$ is the mixing amplitude of $\nu_e$ with the $i$-th neutrino mass eigen-state. For neutrino pair emission from core electrons, the relevant current arises from the zero-th component of 4-vector current. This gives rise to pair emission of the same species of neutrinos, with $b_i e^\dagger e, b_i = b_{ii}$, Our process is insensitive to
Majorana CP phases which appear in off-diagonal elements of $U^*_e U_{ej}$. In a recent work [15] this type of mono-pole contribution, both from inner core electrons and from nucleus, has been considered and found to greatly enhance RENP rates.

**Coulomb assisted RENP**  We consider RENP in which the photon is emitted from the nuclear isomer and the neutrino pair from atomic core electrons. This seemingly separated process is bridged by Coulomb interaction between atomic core electron and nucleus. The strength of Coulomb interaction is estimated by Thomas-Fermi model [14], resulting in [15],

$$J_C \equiv \sum_c \langle c | Z_e | c \rangle = 1.6 \times \left(\frac{2^5/3}{(3\pi)^{2/3}}\right) 2^{7/3} \alpha^2 m_e \sim 31\text{eV} Z^{7/3}. \quad (4)$$

For $^{229}\text{Th}$ $J_C \sim 1.5 \times 10^8\text{eV}$. This matrix element is insensitive to the quantum numbers of core electrons $c$.

RENP amplitudes from the isomer ground state consist of two Feynman diagrams as depicted in Fig(1), differing in time sequence of three interaction vertexes; Coulomb interaction sandwiched between nuclear M1 emission and neutrino pair emission in the ion ground state. Two contributions have a common vertex product and differ in energy denominator in third order perturbation formula. The vertex product is two neutrino wave functions $\times J_C g e \frac{\vec{S}_N \cdot \vec{B}}{2 m_N} \sum_i b^i \nu_i \nu_i^\dagger g e \frac{\vec{S}_N \cdot \vec{B}}{2 m_N} \sum_i b^i \nu_i \nu_i^\dagger$ where $g$ is the unknown g-factor of isomer M1 transition, and $\vec{B}$ is the magnetic field of emitted photon.

![Figure 1: Th isomer RENP diagrams. Electron emitting neutrino pair is one of core electrons.](image)

Energy denominators from these contribution are given by $2/(\epsilon_m - \omega)^2$ with $\epsilon_m$ the isomer energy. Multiplying the remaining factors in amplitude gives Coulomb assisted RENP amplitude,

$$G_F \sqrt{2} \sum_i b^i \nu_i \nu_i^\dagger g e \frac{\vec{S}_N \cdot \vec{B}}{2 m_N} \sum_i b^i \nu_i \nu_i^\dagger \frac{2 J_C}{(\epsilon_m - \omega)^2}. \quad (5)$$

**RENP spectral rate**  RENP photon spectral rate is obtained from the squared amplitude, by summing over neutrino helicities and momenta and by replacing the emitted photon field strength $|E|^2$ by the extractable stored field energy density, equal to $\epsilon_m n \times$ a dynamical factor $\eta_\omega$. The quantity $\epsilon_m n$ is the energy density stored in the isomer level. The dynamical factor $\eta_\omega(t)$ is time dependent and computed numerically by solving the master equation for fields and target polarization, as given in [3]. Usually, $\eta_\omega$ is much less than unity. See below on more of the corresponding physical process.

The neutrino helicity summation has the mono-pole current squared term $\frac{1}{2}(1 + \vec{p}_1 \cdot \vec{p}_2 + \delta_M \frac{m_1 m_2}{E_1 E_2})$ multiplied by $|b_i|^2$. Here $\delta_M = 1$ for the Majorana case and zero for the Dirac case. The phase space integral of neutrino momenta, using the energy-momentum conservation of 3-body decay, reduces to

$$\int \frac{d^3 p_1 d^3 p_2}{(2\pi)^2} \delta(E_1 + E_2 + \omega - \epsilon_{eg}) \delta(\vec{p}_1 + \vec{p}_2 + \vec{k}) \frac{1}{2} \left(1 + \frac{\vec{p}_1 \cdot \vec{p}_2}{E_1 E_2} + \delta_M \frac{m_1 m_2}{E_1 E_2}\right),$$

$$\int_{E_-}^{E_+} dE_1 E_1 E_2 \frac{1}{2} \left(1 + \frac{\vec{p}_1 \cdot \vec{p}_2}{E_1 E_2} + \delta_M \frac{m_1 m_2}{E_1 E_2}\right), \quad E_2 = \epsilon_{eg} - \omega - E_1,$$
with \(E_\pm\) derived by 3-body kinematics. Amplitude squared as described above give quadratic functions of neutrino energy \(E_1\) and the energy integral over \(E_1\) can be performed explicitly. Explicitly, the spectral shape is given by \(4/(\epsilon_m - \omega)^2\) times

\[
I(\omega) = \sum_i |b_i|^2 \Delta_i(\omega) I_i(\omega) \theta(\omega_{ii} - \omega),
\]

\[
I_i(\omega) = \frac{\omega^2}{3} + \frac{2m_i^2\omega^2}{3\epsilon_{eg}(\epsilon_{eg} - 2\omega)} + m_i^2(1 + \delta_M), \quad \Delta_i(\omega) = \left(1 - \frac{4m_i^2}{\epsilon_{eg}(\epsilon_{eg} - 2\omega)}\right)^{1/2}.
\]  

(8) \hspace{1cm} (9)

In Fig(2) and Fig(3) we illustrate the spectral shape \(I(\omega)\) by taking the isomer energy of \(\epsilon_m = 7.8\,\text{eV}\). The spectral rate is decomposed into three factors; the overall rate, the squared atomic matrix element, and kinematical factor \(I(\omega)\), thus

\[
\Gamma_{2\nu\gamma} = \Gamma_n \left(\frac{2J_C}{(\epsilon_m - \omega)^2}\right)^2 I(\omega) \eta_\omega(t),
\]

\[
\Gamma_n = \frac{G_F^2}{2} \left(\frac{g_e}{2m_N}\right)^2 n^3\epsilon_{eg} \sim 1.2 \times 10^{-13}\,\text{Hz} \left(n_0 \frac{n}{10^{22}\,\text{cm}^{-3}}\right)^3 V_{\text{cm}}^3 \epsilon_{eg} \sim 1.2 \times 10^{-13}\,\text{Hz}\,\text{cm}^3 \frac{n}{10^{22}\,\text{cm}^{-3}} V_{\text{cm}}^3 \epsilon_{eg}.
\]

The actual rate for the thorium isomer, taking 7.8 eV energy, is \(4I(\omega)/(\epsilon_m - \omega)^4\) multiplied by

\[
22g^2 \text{Hz} \left(n_0 \frac{n}{10^{22}\,\text{cm}^{-3}}\right)^3 V_{\text{cm}}^3 \eta_\omega(t).
\]

(10) \hspace{1cm} (11) \hspace{1cm} (12)

The Majorana vs Dirac distinction is difficult for this large energy spacing \(\sim 7.8\,\text{eV}\). Distinction is improved for smaller energy spacings of atoms, as discussed in [16].

Macro-coherent field storage

RENP rates depend on the field storage factor \(\eta_\omega(t)\), controlled by two-photon paired super-radiance (PSR) process \(|r\rangle \rightarrow |g\rangle + \gamma\gamma\). PSR is triggered by laser irradiation of two frequencies, the one at \(\omega\) and another at \(\epsilon_m - \omega > \omega\). The dynamical factor \(\eta_\omega(t)\) is defined by a space integrated quantity of \(|E(t,x)(r_1(t,x) - ir_2(t,x))|^2\), expressed in dimensionless units, and is calculated by
The RENP spectrum: NH vs IH

Figure 3: Th isomer RENP spectral shape in threshold regions corresponding to Fig[2]. NH case in brown and IH case in green.

...numerically solving the master equation for the developed field $E(t, x)$ and target polarization $r_1 - ir_2$ of [12], [3]. It also depends on experimental conditions.

Macro-coherent two-photon PSR is controlled by diagrams consisting of two vertexes of the isomer M1 and atomic M1. Atomic M1 vertex is induced by the state mixing of $6p$ electron in the ion closed shell with electron in $7p$ orbit caused by crystalline field. Its strength is large due to large $J_C$, even if nuclear M1 amplitude $\propto 1/m_N$ is small. Thus, both RENP and PSR are assisted by Coulomb interaction with nucleus. Detailed numerical simulation is needed [17].

In summary, we proposed to use the nuclear isomer $^{229m}$Th$^{4+}$ embedded in transparent crystals for the neutrino mass spectroscopy. It has a merit of excellent isolation from solid environment. Thus, although RENP rates are not very large, a large relaxation time may compensate these moderate rates.

Work on parity violation (PV) effects is in progress. PV asymmetry along with PV rate is expected to be large for $^{229m}$Th RENP.

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[17] In [3] a result for numerical simulation of $\eta_\omega(t)$ is presented for pH$_2$ molecule target (strong source of paired super-radiance (PSR) of E1 $\times$ E1 transition, and for time dependence see Fig 14 of this reference).

Its time dependence is complicated: a fast rise in $O(2 \text{ ns})$, then a plateau region of magnitude $O(10^{-2})$ of duration of several nano-seconds, finally gradual decrease ending around $10^{-6}$ up to 12 ns (end time of calculation).

Numerical simulations based on the master equation given in [3] should be performed for specific targets considered, especially for target atoms of weaker PSR sources and large relaxation time, which is expected to give different temporal behaviors and large values.