Metastable SUSY Breaking and Supergravity at Finite Temperature

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ABSTRACT

We study how coupling to supergravity affects the phase structure of a system exhibiting dynamical supersymmetry breaking in a metastable vacuum. More precisely, we consider the Seiberg dual of SQCD coupled to supergravity at finite temperature. We show that the gravitational interactions decrease the critical temperature for the second order phase transition in the quark direction, that is also present in the global case. Furthermore, we find that, due to supergravity, a new second order phase transition occurs in the meson direction, whenever there is a nonvanishing constant term in the superpotential. Notably, this phase transition is a necessary condition for the fields to roll, as the system cools down, towards the metastable susy breaking vacuum, because of the supergravity-induced shift of the metastable minimum away from zero meson vevs. Finally, we comment on the phase structure of the KKLT model with uplifting sector given by the Seiberg dual of SQCD.

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1 Introduction

Understanding how supersymmetry breaking occurs is a major problem on the road to connecting the underlying supersymmetric theory with the observed world\footnote{For a recent review of supersymmetry breaking see \cite{1}.}. The idea, that supersymmetry can be broken due to dynamical effects \cite{2}, has long been considered phenomenologically very promising, since it naturally leads to a large hierarchy between the Planck and the susy breaking scales. However, dynamical supersymmetry breaking has turned out to be quite difficult to implement in a supersymmetric gauge theory. The
reason is that only rather complicated examples \[3\] satisfy the strict conditions, necessary for the absence of a global supersymmetric vacuum.

It was realized recently \[4\] that the situation changes dramatically, if one abandons the prejudice that the phenomenologically relevant vacuum has to be a global, and not just local, minimum of the effective potential. In this case, one can relax the requirement that the theory lacks a global supersymmetric vacuum and search for models with meta-stable, sufficiently long-lived, susy breaking vacua. From this new point of view, models with non-zero Witten index and without a conserved $U(1)$ R-symmetry can be considered phenomenologically viable for supersymmetry breaking. The spectrum of susy breaking theories is then significantly enriched. In particular, as shown in \[4\], meta-stable dynamical supersymmetry breaking occurs even in $\mathcal{N} = 1$ SQCD with $SU(N_c)$ gauge group and $N_f$ massive fundamental flavors. This can be established by going to the Seiberg (magnetic) dual description of this theory, where supersymmetry is broken at tree-level.\footnote{For convenience, we will call this the ISS model from now on.}

During the last year, many more examples of meta-stable dynamical susy breaking were found in various phenomenologically appealing settings \[5\]. Progress was also made on understanding the embedding of those field-theoretic models in string/M-theory \[6\].

However, once we consider phenomenology in a local, instead of a global, minimum of the zero-temperature effective potential, the following question arises. How natural is it for the high-temperature system, that is the early Universe, to end up in the metastable state after cooling down? To address this question, the recent works \[7, 8\] studied the ISS model at finite temperature. They found that the metastable vacuum is thermodynamically preferable compared to the supersymmetric global ones.\footnote{Although their conclusions agree, their approaches are different and, in a sense, complimentary. In \[7\], they consider a path in field space, which extrapolates between the susy breaking vacuum and a global vacuum, and construct the effective potential along this path. Using this they show that, even if at high temperature the system starts at a susy vacuum, it will end up in the metastable one as it cools down. On the other hand, \[8\] studies in great detail the phase structure around the origin of field space, which is a local minimum of the nonzero temperature potential. They assume that at high temperature the quark and meson fields of the ISS model are localized near this point, which is reasonable since the number of light degrees of freedom at the origin is largest and hence this state maximizes the entropy. With this starting point, \[8\] investigates the phase structure of the free-energy as the}

\footnote{We will review more details on that in the next Section.}

\footnote{There are $N_c$ of them as we review in Section\[2\]}
temperature decreases and finds out that there is a critical temperature, $T^Q_c$, for a second order phase transition in the quark direction (that is, the direction towards the metastable minimum). On the other hand, in the meson direction (i.e., the direction towards a global susy vacuum) they find that only a first order phase transition can occur, at temperature smaller than $T^Q_c$, and that it is quite suppressed by a high potential barrier.

We follow the approach of [8] for the ISS model coupled to supergravity. This is certainly necessary for more realistic cosmological applications of the idea of dynamical supersymmetry breaking in a metastable vacuum. We will compute the one-loop effective potential at finite temperature by using the results of [21, 22] for chiral multiplets coupled to supergravity. The nonrenormalizability of the latter theory is not an issue as it is supposed to be viewed as an effective low-energy description only, not as a fundamental theory. For more details on one-loop (albeit at $T = 0$) calculations in supergravity coupled with various matter multiplets, see [23]. However, we should note that the considerations of [21, 22] treat the zero-temperature classical supergravity contribution $V_0$ to the $T \neq 0$ effective potential as an effective potential itself. In other words, the $T = 0$ loop corrections are viewed as already taken into account in the standard sugra potential $V_0$. So it might seem conceivable that these results may be affected (despite susy being broken) by regularization subtleties similar to those at $T = 0$. In the latter case, a regularization compatible with supersymmetry was developed in the last five references of [23] and was shown rather recently in [24] to sometimes have an impact on quantities of phenomenological interest, like the flavor-changing neutral currents. For the trivial Kähler potential, that we will need, this is not the case. Nevertheless, it is important, although going well beyond the scope of our paper, to address this issue in full generality at $T \neq 0$. Another remark is due. Every gravitational system exhibits instability under long-wave length gravitational perturbations [25]; this Jeans instability occurs also at finite temperature [26]. While it is certainly of great importance for structure formation in the early Universe, it is a subleading effect on cosmological scales on which the Universe is well-approximated by a homogeneous fluid. So we will limit ourselves to considering the leading effect, by using the formulae of [21, 22] for the effective potential, and will not address here the Jeans instability.

We will show that the supergravity corrections decrease the critical temperature for a second order phase transition in the quark direction, $T^Q_c$, for any $N_c$ and $N_f$. While this is only a small quantitative difference with the rigid case, in the meson branch a significant qualitative difference can occur. The reason is that in the relevant field-space

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7 Reference [9] discusses in detail the suppression of this transition in a class of O’Raireataigh models.
region there are no contributions to the tree-level meson masses in the rigid limit and so the supergravity corrections are the leading ones. As a result, it turns out that, whenever the superpotential contains a nonvanishing constant piece $W_0$, there is a second order phase transition in the meson direction at a temperature $T_{\phi}^c$, smaller than $T_{Q}^c$. However, this is not a phase transition towards any of the global supersymmetric minima, as it occurs at nonvanishing quark vevs. Notably though, our new phase transition is precisely what is needed for the system to roll towards the metastable vacuum since, due to the supergravity interactions, the latter is shifted away from the origin of the meson direction whenever $W_0 \neq 0$, as shown in [12].

Considering the ISS model plus supergravity is, in fact, the first step towards a full investigation of the phase structure of the KKLT scenario [10] with ISS uplifting sector at finite temperature. It was already argued in [11, 12] that metastable susy breaking provides a natural way of lifting the AdS KKLT vacuum to a de Sitter one, avoiding the problems encountered previously in the literature. Recall that the original proposal was to introduce anti-D3 branes, which break supersymmetry explicitly, whereas the later idea to use nonvanishing D-terms [13] turned out to be quite hard to realize [14]. Studying the phase structure of the KKLT-ISS system is a big part of our motivation. However, in this case the computations become much more technically challenging. We make the initial step by showing that the origin of the ISS field space is no longer a local minimum of the temperature-dependent part of the effective potential. The shift of the high temperature minimum away from the ISS origin is related to the vev of the KKLT volume modulus.

This paper is organized as follows. In Section 2 we review necessary background material about the ISS model. In Section 3 we compute the one-loop temperature-dependent contribution to the effective potential or, equivalently, the free energy of the ISS model coupled to supergravity. To achieve that, in Subsection 3.1 we derive the mass matrices for both quark and meson vevs nonzero, that are coming from the F-terms; in Subsection 3.2 we take into account the D-terms. In Section 4 we expand the general results of Section 3 in terms of the small parameter $M_P^{-1}$, where $M_P$ is the Planck mass. This allows us to read off the leading supergravity corrections to the rigid theory, considered in [8]. In Section 5 we compute the critical temperature for a second order phase transition in the quark direction to $O(M_P^{-2})$. In Section 6 we show that there is also a second order phase transition in the meson direction and estimate the critical temperature for it. In Section 7 we consider the KKLT model with ISS uplifting sector and argue that the origin of the ISS field space is no longer a local minimum of the high temperature effective potential. The shift of the minimum away from this origin is determined by the vev of the KKLT
volume modulus. In Section 8 we discuss the implications of our results for the phase structure of the ISS model coupled to supergravity and for the end point of this system’s evolution at low temperature. We also outline open problems. Finally, in Appendix A we give some useful formulae for mass matrices near the origin of field space and in Appendix B we show that no new supersymmetric minima appear in a small neighborhood of the origin in the ISS model coupled to supergravity, in the field directions of interest.

2 ISS model

It was argued in [15] that SQCD with $SU(N_c)$ gauge group and $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry has a dual (magnetic) description in terms of an $SU(N_f - N_c)$ gauge theory coupled to certain matter fields. When the condition $N_f < 3N_c/2$ is satisfied the magnetic theory is IR free. The matter content of the dual theory comprises two chiral superfields $q$ and $\tilde{q}$, that are transforming in the $(N_f,1)$ and $(1,N_f)$ representations of the flavor symmetry group respectively and are in the fundamental and antifundamental representations of $SU(N_f-N_c)$ respectively (and so are called quarks), and a gauge-singlet chiral meson superfield $\Phi$ in the $(N_f,N_f)$ flavor group representation. Hence the index structure of the magnetic quark and meson fields is the following: $q^a_i$, $\tilde{q}^\dagger_a$ and $\Phi^i_j$ with $i = 1,...,N_f$ and $a = 1,...,N$, where $N = N_f - N_c$. For easier comparison with the literature, we will also use the notation $N_m = N$ and $N_c = N_e$, implying in particular that $N_f = N_m + N_e$. In terms of this notation the above condition for IR free dual theory is $N_e > 2N_m$. In the following we will only consider this case.

The magnetic theory has the following tree-level superpotential:

$$W = h \text{Tr } q^\dagger \Phi \tilde{q} - h \mu^2 \text{Tr } \Phi.$$  \hfill (2.1)

The second term breaks the flavor group to its diagonal subgroup and corresponds to a quark mass term in the microscopic theory (i.e., the original $SU(N_c)$ gauge theory). The Kähler potential is the canonical one:

$$K = \text{Tr } q^\dagger q + \text{Tr } \tilde{q}^\dagger \tilde{q} + \text{Tr } \Phi^\dagger \Phi.$$  \hfill (2.2)

The magnetic description can be used to prove the existence of a metastable vacuum, which breaks supersymmetry at tree-level [4]. Indeed, it is immediate to see that the F-term condition

$$F_{\Phi^i_j} = h \left( q^a_i \tilde{q}^\dagger_a - \mu^2 \delta^i_j \right) = 0$$  \hfill (2.3)
cannot be satisfied as the matrix \( q_i^a \tilde{q}_d^j \) has at most rank \( N_m \) while \( \delta_i^j \) has rank \( N_f \). The moduli space of metastable vacua can be parameterized as:

\[
q = \begin{pmatrix} Q \\ 0 \end{pmatrix}, \quad \tilde{q}^T = \begin{pmatrix} \tilde{Q} \\ 0 \end{pmatrix}, \quad \Phi = \begin{pmatrix} 0 & 0 \\ 0 & \varphi \end{pmatrix},
\]

(2.4)

where \( \varphi \) is an \((N_f - N) \times (N_f - N)\) matrix while \( Q \) and \( \tilde{Q} \) are \( N \times N \) matrices satisfying the condition \( QQ = \mu I_{N \times N} \). The point of maximum global symmetry is at

\[
\langle q_1 \rangle = \langle q_1^T \rangle = \mu I_N, \quad \langle q_2 \rangle = \langle \tilde{q}_2 \rangle = 0, \quad \langle \Phi \rangle = 0,
\]

(2.5)

where we have denoted: \( q^T \equiv (q_1, q_2) \) with \( q_1 \) and \( q_2 \) being \( N \times N \) and \( N \times (N_f - N) \) matrices respectively. It will also be useful for the future to introduce the following notation for the generic components of the \( N_f \times N_f \) matrix \( \Phi \):

\[
\Phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix},
\]

(2.6)

where \( \phi_{11} \) is \( N_m \times N_m \), \( \phi_{12} \) is \( N_m \times N_e \), \( \phi_{21} \) is \( N_e \times N_m \) and finally \( \phi_{22} \) is an \( N_e \times N_e \) matrix.

The value of the scalar potential in each metastable minimum in (2.4) is:

\[
V_{\text{min}} = (N_f - N) h^2 \mu^4.
\]

(2.7)

Usually, when supersymmetry is spontaneously broken, the moduli space of classical vacua is not protected against quantum corrections. As a result, the quantum moduli space is typically smaller and one may wonder whether any of the metastable vacua survive in it. In this regard, it was shown in [4] that the classically flat directions around the maximally symmetric vacuum (2.5) acquire positive masses at one-loop through the supersymmetric Coleman-Weinberg potential [16]. This metastable minimum is therefore tachionic-free and from now on we will always mean (2.5), when we talk about a supersymmetry breaking vacuum.

In addition to the perturbative corrections that we just discussed, there are also non-perturbative ones. Namely, gaugino condensation in the magnetic gauge group \( SU(N) \) induces the Affleck-Dine-Seiberg superpotential [17]:

\[
W_{\text{ADS}} = N \left( h^{N_f} \frac{\det \Phi}{\Lambda^{N_f - 3N}} \right),
\]

(2.8)

where \( \Lambda \) is the UV cutoff of the magnetic theory, i.e. the scale above which the magnetic description becomes strongly coupled and hence not well-defined. Adding this dynamically
generated contribution to the classical ISS superpotential leads to $N_c$ supersymmetric vacua, characterized by nonvanishing meson vevs: \[ h\langle \Phi_j^i \rangle = \mu \epsilon^{\frac{3N-N_f}{N_c}} \delta_j^i, \quad \langle q \rangle = \langle \bar{q}^T \rangle = 0 , \] (2.9)

in agreement with the Witten index \[18\] of the microscopic theory. The metastable vacuum can be made long-lived by taking $\epsilon$ parametrically small as in that case the tunnelling to the supersymmetric vacuum is strongly suppressed.

Since the $ADS$ superpotential is suppressed by powers of the UV cutoff, for small meson fields it is completely negligible compared to the tree-level one, (2.1). So in the following we will drop $W_{ADS}$ from our considerations, as we will study the finite temperature effective potential only in a neighborhood of the origin of field space.

### 3 One-loop effective potential at nonzero $T$

In the present section we compute the one-loop effective potential at finite temperature for the ISS model coupled to supergravity. Its analysis in subsequent sections will enable us to deduce the phase structure of this theory near the origin.

Let us start by recalling some generalities about the path-integral derivation of the effective potential in a theory with a set of fields $\{\chi^I\}$. An essential step in that is to shift $\chi^I$ by a constant background $\hat{\chi}^I$. Equivalently, we expand the Lagrangian around a nonzero background, $\{\hat{\chi}^I\}$, for the fields. Using this expansion, one can derive with functional methods a formula for the effective potential. The original derivation of \[19\] was only for zero-temperature renormalizable field theory. The same kind of considerations apply also for finite temperature and up to one-loop give \[20\]:

\[ V_{eff}(\hat{\chi}) = V_{tree}(\hat{\chi}) + V_0^{(1)}(\hat{\chi}) + V_T^{(1)}(\hat{\chi}) , \] (3.1)

where $V_{tree}$ is the classical potential, $V_0^{(1)}$ is the zero temperature one-loop contribution, encoded in the Coleman-Weinberg formula, and finally the temperature-dependent correction is:

\[ V_T^{(1)}(\hat{\chi}) = -\frac{\pi^2 T^4}{90} \left( g_B + \frac{7}{8} g_F \right) + \frac{T^2}{24} \left( Tr M^2_v(\hat{\chi}) + 3 Tr M^2_\nu(\hat{\chi}) + Tr M^2_f(\hat{\chi}) \right) + O(T) . \] (3.2)

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8We denote by $\epsilon$ the quantity $\mu \Lambda^{-1}$.

9Recall that $N_f > 3N$ and hence for very small $\epsilon$ the meson vev, $\langle \Phi \rangle$, in (2.9) becomes very large.
For convenience, from now on we will denote this last expression simply by $V_T$. Here $g_B$ and $g_F$ are the total numbers of bosonic and fermionic degrees of freedom respectively. $\text{Tr} M_s^2$, $\text{Tr} M_v^2$ and $\text{Tr} M_f^2$ are the coefficients of the quadratic terms of scalar, vector and fermion fields computed from the shifted classical potential or, in other words, the mass matrices of those fields in the classical background $\{\hat{\chi}^I\}$. The expansion (3.2) is valid in the high temperature regime, more precisely when all masses are much smaller than the energy scale set by the temperature. The above result for the one-loop effective potential at finite temperature was shown in [21] to also hold for coupling to supergravity.

We turn now to computing $\text{Tr} M_s^2$, $\text{Tr} M_v^2$ and $\text{Tr} M_f^2$ for our case. The classical background $\hat{\chi}$ around which we will be expanding is:

$$\langle q_1 \rangle = \langle \tilde{q}_1 \rangle = Q \mathbb{I}_{N_m \times N_m}, \quad \langle \phi_{11} \rangle = \varphi_1 \mathbb{I}_{N_m \times N_m}, \quad \langle \phi_{22} \rangle = \varphi_2 \mathbb{I}_{N_e \times N_e}, \quad (3.3)$$

with zero vevs for all remaining fields and with $Q$, $\varphi_1$, $\varphi_2$ all being real.

### 3.1 F-terms

In this subsection we consider the contribution from the F-terms. The D-terms will be taken into account in the next one. For convenience, from now on we denote collectively all components of the fields $q$, $\tilde{q}$ and $\Phi$ simply by $\chi^I$.

#### 3.1.1 Preliminaries

Recall that the classical F-term supergravity potential is:

$$V = e^K \{K^{IJ} D_I W D_J \overline{W} - 3 |W|^2 \}, \quad (3.4)$$

where $I, J$ run over all scalar fields in the theory, $K^{IJ}$ is the inverse of the Kähler metric and the Kähler covariant derivative is:

$$D_I W = \partial_I W + \partial_I K W. \quad (3.5)$$

The supergravity Lagrangian is invariant under Kähler transformations:

$$K(\chi^I, \bar{\chi}^I) \rightarrow K(\chi^I, \bar{\chi}^I) + F(\chi^I) + \bar{F}(\bar{\chi}^I),$$

$$W(\chi^I) \rightarrow e^{-F(\chi^I)} W(\chi^I). \quad (3.6)$$

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10Note that this is different from the number of fields. For example, for $N_B$ chiral superfields the number of scalar degrees of freedom is $g_B = 2N_B$. In fact, below we will denote by $N_B$ the number of complex scalars.

11In (3.2) the quantity $\text{Tr} M_f^2$ is computed summing over Weyl fermions.

12In this section we set $M_P = 1$. The explicit dependance on the Planck mass will be reinserted later when needed.
One can exploit this invariance, by taking \( F(\chi^I) = \log W(\chi^I) \), in order to show \[27\] that the scalar potential depends only on the combination

\[
G = K + \ln|W|^2,
\]

but not on \( W \) and \( K \) separately. In terms of this function, we can rewrite \[3.4\] as follows:

\[
V = e^G (G^I G_I - 3)
\]

where

\[
G_I \equiv \frac{\partial G}{\partial \chi^I}, \quad G^J \equiv \frac{\partial G}{\partial \bar{\chi}^J}.
\]

This notation utilizes the fact that for us the Kähler potential is canonical i.e. \( K_{I\bar{J}} = \delta_{I\bar{J}} \), see \[2.2\], and so \( \bar{\chi}^J \equiv K_{J\bar{L}} \chi^{\bar{L}} \).

For such a Kähler potential, the expressions for the scalar and fermionic mass matrices are \[21\] \[13\]

\[
\text{Tr}M^2_s = \langle 2e^G \{(G^I J + G^I G^J)(G_{IJ} + G_I G_J) + (N_B - 1)G^I G_I - 2N_B \} \rangle \quad (3.10)
\]

and

\[
\text{Tr}M^2_f = \text{Tr}M^2_{1/2} + \text{Tr}M^2_{3/2} = \langle e^G \{(G^I J + G^I G^J)(G_{IJ} + G_I G_J) - 2 \} \rangle,
\]

respectively. Here \( N_B \) is the number of complex scalars. Using \[3.8\], it is easy to show that \[3.10\] follows from

\[
\text{Tr}M^2_s = 2 \frac{\partial^2 V}{\partial \chi^I \partial \bar{\chi}^J}.
\]

The derivation of the fermion mass-squared is a bit more involved since one has to disentangle the mixing between the gravitino and the goldstino in the supergravity Lagrangian. After having dealt with that, one finds \( \text{Tr}M^2_{3/2} = -2e^G \). The anomalous negative sign in the gravitino contribution is due to the fact that in \( \text{Tr}M^2_{1/2} \) we have summed over the physical matter fermions and the goldstino.

Before starting the actual computations, two remarks are in order. First, it may seem that it is more illuminating to perform the calculations as in \[8\], i.e. to compute separately the mass-squareds of every field and then add them. However, for us this becomes rather cumbersome, whereas the trace formulae above provide a very efficient way of handling things. And second, the formulation of the supergravity Lagrangian in terms

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\[13\] We will consider a non-canonical Kähler potential in Section \[7\] when we address the KKLT set-up with ISS uplifting sector.
of the function $G$, (3.7), appears to encounter a problem for vanishing superpotential, as $W$ enters various terms in the denominator. That will be an issue for the D-terms in Subsection 3.2 and we will use there a more modern formulation that is equally valid for $W \neq 0$ and $W = 0$. Here we simply note that for the F-terms there is no problem, since the apparent negative powers of $W$, coming from derivatives of $G$, are cancelled by the positive powers from $e^G$. (This will be made more explicit in Subsection 3.1.2.) So the F-term results never contain division by zero. This is an important point as in later sections we will be interested in the effective potential at the origin of field space, where the ISS superpotential vanishes.

3.1.2 Mass matrices

We are finally ready to find the F-term mass matrices $\text{Tr}M^2_i$ and $\text{Tr}M^2_f$ for the ISS model coupled to supergravity. To do so more efficiently, we note that, instead of finding separate expressions for the various ingredients $G^{IJ}G_{IJ}$, $G^{IJ}G_I G_J$ and $G^I G^J G_I G_J$ which enter the mass formulae, it is computationally much more convenient to calculate the whole combination $(G^{IJ} + G^I G^J)(G_{IJ} + G_I G_J)$ for each particular choice of $I$ and $J$. This avoids introducing a big number of terms that have to cancel at the end, as we now explain. From the definition of $G$, i.e. $G = K + \ln W + \ln \bar{W}$, we have that

$$G_I = K_I + \frac{W_I}{W}, \quad G_{IJ} = -\frac{W_IW_J}{W^2} + \frac{W_{IJ}}{W}. \quad (3.13)$$

Therefore, in the expressions $G^{IJ}G_{IJ}$, $G^{IJ}G_I G_J$ and $G^I G^J G_I G_J$ one seems to obtain many terms proportional to $1/W^2$. Taking into account the $|W|^2$ factor coming from $e^G$, one is left with many terms $\sim 1/|W|^2$. However, it is clear that they have to cancel at the end, since the scalar potential (3.4) does not include any negative powers of $|W|^2$ and so if we were computing the masses of each field separately and adding them (as in [8]) we could not possibly obtain terms $\sim 1/|W|^2$. This cancellation can be incorporated from the start by using (3.13) to write:

$$G_{IJ} + G_I G_J = \frac{W_{IJ}}{W} + \frac{K_IW_J + K_J W_I}{W} + K_I K_J \quad (3.14)$$

or equivalently

$$G_{IJ} + G_I G_J = \frac{W_{IJ}}{W} + G_I K_J + K_I G_J - K_I K_J. \quad (3.15)$$

It is evident now that this expression does not contain any $1/W^2$ terms. Incidentally, this also makes it obvious that, as expected, the expression $e^G(G^{IJ} + G^I G^J)(G_{IJ} + G_I G_J)$ does not contain any powers of $W$ in the denominator.
Therefore, one immediately finds:

\[ R^{\phi_{11}\phi_{22}} \equiv (G^{\phi_{11}\phi_{22}} + G^{\phi_{11}\phi_{22}})(G_{\phi_{11}\phi_{22}} + G_{\phi_{11}\phi_{22}}). \]  

(3.16)

To make use of (3.15), we note that:

\[ W_{\phi_{11}\phi_{22}} = 0, \quad \langle K_{\phi_{11}} \rangle = \varphi_1 \mathbb{1}_{N_m \times N_m}, \quad \langle K_{\phi_{22}} \rangle = \varphi_2 \mathbb{1}_{N_e \times N_e}, \]

\[ \langle G_{(\phi_{11})} \rangle = \left[ \varphi_1 + \frac{\hbar}{W_0}(Q^2 - \mu^2) \right] \delta_i^j, \quad \langle G_{(\phi_{22})} \rangle = \left[ \varphi_2 - \frac{\hbar \mu^2}{W_0} \right] \delta_k^l. \]  

(3.17)

Therefore, one immediately finds:

\[ \langle R^{\phi_{11}\phi_{22}} \rangle = \left[ \left( \varphi_1 + \frac{\hbar(Q^2 - \mu^2)}{W_0} \right) \varphi_2 + \left( \varphi_2 - \frac{\hbar \mu^2}{W_0} \right) \varphi_1 - \varphi_1 \varphi_2 \right]^2 (\delta_i^j \delta_k^l)(\delta_j^i \delta_l^k), \]  

(3.18)

where we denoted by \( W_0 \) the value of the ISS potential in the background (3.3). The last factor gives \((\delta_i^j \delta_k^l)(\delta_j^i \delta_l^k) = N_m N_e\). So we obtain

\[ \langle R^{\phi_{11}\phi_{22}} \rangle = \left[ \varphi_1 \varphi_2 + \frac{\hbar(Q^2 - \mu^2)}{W_0} \varphi_2 - \frac{\hbar \mu^2}{W_0} \varphi_1 \right]^2 N_m N_e. \]  

(3.19)

Similarly, it is very easy to compute:

\[ \langle R^{\phi_{11}\phi_{11}} \rangle = \varphi_1^2 \left( \varphi_1 + \frac{2 \hbar(Q^2 - \mu^2)}{W_0} \right)^2 N_m^2, \]

\[ \langle R^{\phi_{22}\phi_{22}} \rangle = \varphi_2^2 \left( \varphi_2 - \frac{2 \hbar \mu^2}{W_0} \right)^2 N_e^2, \]

\[ \langle R^{q_1\phi_{11}} \rangle = \langle R^{q_1\phi_{11}} \rangle = \left( \frac{\hbar Q}{W_0} \right)^2 N_m^2 + Q^2 \left[ \left( \frac{\hbar \varphi_1}{W_0} + 1 \right) \varphi_1 + \frac{\hbar(Q^2 - \mu^2)}{W_0} \right]^2 N_m, \]

\[ + \frac{2 \hbar Q}{W_0} \left[ \left( \frac{\hbar \varphi_1}{W_0} + 1 \right) \varphi_1 + \frac{\hbar(Q^2 - \mu^2)}{W_0} \right] \varphi_1, \]

\[ \langle R^{q_1\phi_{22}} \rangle = \langle R^{q_1\phi_{22}} \rangle = Q^2 \left[ \varphi_2 + \frac{\hbar}{W_0} (\varphi_1 \varphi_2 - \mu^2) \right] \varphi_2 - \frac{\hbar \mu^2}{W_0} \varphi_1 \right]^2 N_m N_e, \]

\[ \langle R^{q_2\phi_{21}} \rangle = \langle R^{q_2\phi_{12}} \rangle = \left( \frac{\hbar Q}{W_0} \right)^2 N_m N_e, \]

\[ \langle R^{q_1 q_1} \rangle = \langle R^{q_1 q_1} \rangle = Q^4 \left( 1 + \frac{2 \hbar}{W_0} \varphi_1 \right)^2 N_m^2, \]

\[ \langle R^{q_1 q_1} \rangle = \left( \frac{\hbar \varphi_1}{W_0} \right)^2 N_m^2 + Q^4 \left( 1 + \frac{2 \hbar \varphi_1}{W_0} \right)^2 N_m^2 + \frac{2 \hbar \varphi_1 Q^2}{W_0} \left( 1 + \frac{2 \hbar \varphi_1}{W_0} \right) N_m, \]

\[ \langle R^{q_2 q_2} \rangle = \left( \frac{\hbar \varphi_2}{W_0} \right)^2 N_m N_e. \]  

(3.20)
For all remaining pairs $\langle R^i \chi^j \rangle = 0$.

The last ingredient in (3.10), that we need to compute, is $\langle G^i G_I \rangle$. The only nonvanishing components are for $I = \phi_{11}, \phi_{22}, q_1, \tilde{q}_1$ and we find:

$$\langle G^i G_I \rangle = \left( \varphi_1 + \frac{h(Q^2 - \mu^2)}{W_0} \right)^2 N_m + \left( \varphi_2 - \frac{h\mu^2}{W_0} \right)^2 N_e + 2Q^2 \left( \frac{h\varphi_1}{W_0} + 1 \right)^2 N_m. \quad (3.21)$$

One can notice that all expressions above depend only on $Q^2$, not on $Q$ alone. Hence the temperature dependent part of the one-loop effective potential as a function of $Q$ is of the form $^{14}$

$$V_T = e^{Q^2}(AQ^6 + BQ^4 + CQ^2 + D) \quad (3.22)$$

for any values of the meson vevs $\varphi_1$ and $\varphi_2$. Therefore $Q = 0$ is always an extremum. In the meson directions things are not so apparent, as there are odd powers of $\varphi_1$ and $\varphi_2$. However, one can see that all of them multiply either a power of $Q^2$ or the first power of $W_0$. Since $W_0$ is linear in the meson vevs, for $Q = 0$ the dependence of $V_T$ on $\varphi_1$ and $\varphi_2$ is, in fact, at least quadratic (by that we also mean mixed terms, i.e. with $\varphi_1 \varphi_2$). We will see in Section 4.1 that the point $(Q, \varphi_1, \varphi_2) = (0, 0, 0)$ is a local minimum of $V_T$, as was also the case for vanishing supergravity interactions $^8$. Before that, however, let us first consider the D-term contributions to the mass matrices of the various fields.

### 3.2 D-term masses

The D-terms for Super Yang-Mills coupled to supergravity were derived for the first time in $^{27}$. However that formulation, entirely in terms of the function $G$ of eq. (3.7), is not convenient for our purposes since the D-terms have a singular dependance on the superpotential. For example, denoting by $g$ the gauge coupling constant and by $T_\alpha$ the generators of the gauge group, the D-term scalar potential was found to be

$$V_D = \frac{1}{2} D^\alpha D^\alpha, \quad (3.23)$$

with

$$D^\alpha = g G^I T_\alpha {}^I \chi^J = \frac{1}{2} g T_\alpha {}^I \chi^J \frac{D_I W}{W}, \quad (3.24)$$

where in the last equality we have substituted $G^I = K^I + W^I / W$. Clearly, $V_D$ is not well-defined when the superpotential $W$ vanishes. The same problem, i.e. division by $W$, $^{14}$We drop from now on the piece that is $\sim T^4$, as it does not depend on the vevs of the fields and so it does not contribute to the derivatives of $V_T$, which are the quantities that will be of interest for us.

$^{15}$This is no longer true if one includes the KKLT sector, as we argue in Section 7.
appears also in the fermionic mass terms. So if we adopt the formulation given in [27], Tr$M_f^2$ seems to diverge for $W = 0$. That is problematic since we would like to study the effective potential at the origin of field space, where the ISS superpotential vanishes.

It has been noted long ago that the above formulation is not suitable in the presence of a vanishing superpotential and the latter case has to be studied separately, without the use of the function $G = K + \ln |W|^2$. However, a careful study of the supergravity Lagrangian that is valid both for $W \neq 0$ and $W = 0$ was performed only recently (to the best of our knowledge) in [28]. What is relevant for us is that $V_D$ can be written as

$$V_D = \frac{g^2}{2} (i\xi^I_\alpha \partial_I K - 3i r_\alpha)^2, \quad (3.25)$$

where $\xi^I_\alpha$ are the gauge transformations of the scalar fields, i.e. $\delta_\alpha \chi^I = \xi^I_\alpha (\{\chi^J\})$, and $r_\alpha$ are functions determined by the gauge variations of the superpotential:

$$\delta_\alpha W = \xi^I_\alpha \partial_I W = -3r_\alpha W, \quad (3.26)$$

where the second equality is required for gauge invariance of the action. These functions also characterize the gauge-non-invariance of the Kähler potential:

$$\delta_\alpha K(\chi, \bar{\chi}) = 3(r_\alpha (\chi) + \bar{r}_\alpha (\bar{\chi})). \quad (3.27)$$

In the case of non-vanishing superpotential one can use (3.26) to express $r_\alpha$ in terms of $W$ and $\partial_I W$. Substituting the result in (3.25), one finds (3.23)-(3.24) upon using $\xi^I_\alpha = iT^I_\alpha J \chi^J$. This, indeed, shows that the formulation of [28] reduces to the one in [27] when $W \neq 0$.

Since the ISS superpotential is gauge invariant, we have $r_\alpha = 0$ and

$$V_D = \frac{g^2}{2} \sum_{\alpha=1}^{N^2_m - 1} \left( \text{Tr}(q_1^\dagger T_\alpha q_1 - \bar{q}_1 T_\alpha \bar{q}_1^\dagger) \right)^2 \quad (3.28)$$

as in [8]. We can then borrow the results, found in the global supersymmetry case, to obtain a $4g^2Q^2(N^2_m - 1)$ contribution to Tr$M_s^2$. Also, the vector boson mass is the same as in [8] and so it gives a $4g^2Q^2(N^2_m - 1)$ contribution to Tr$M_v^2$.

Let us now come to the fermionic sector. The mass matrices are [28]:

$$M^{IJ} = \mathcal{D}^I \mathcal{D}^J M \quad (3.29)$$

$$M_{I\alpha} = -i \left[ \partial_I \mathcal{P}_\alpha - \frac{1}{4} (\text{Re} f)^{-1} \beta \gamma \mathcal{P}_\beta \partial_I f_{\gamma \alpha} \right] \quad (3.30)$$

$$M_{\alpha \beta} = -\frac{1}{4} \partial_I f_{\alpha \beta} K^{IJ} M_J, \quad (3.31)$$
where \( f_{\alpha\beta} \) are the gauge kinetic functions, the action of the covariant derivative \( \mathcal{D}^I \) on \( M \equiv e^{K/2}W \) is
\[
\mathcal{D}^I = \partial^I + \frac{1}{2} \partial^I K,
\]
(3.32)
\( M^I \equiv \mathcal{D}^I M \) and finally
\[
P_\alpha = i \xi_\alpha^I \partial_I K.
\]
(3.33)
These expressions are clearly well defined and non singular even when \( W = 0 \), unlike the analogous formulae in [27]. In our case, obviously the index \( I \) runs now only over the quark fields. For a flat Kähler potential, one can easily verify from (3.29) that:
\[
M^{I\bar{J}} M_{I\bar{J}} = e^G \left( G^{I\bar{J}} + G^{I\bar{J}} \right) \left( G_{I\bar{J}} + G_{I\bar{J}} \right),
\]
so that we recover correctly the contribution from the matter fermions and the goldstino in (3.11). Since for us \( f_{\alpha\beta} = 1/g^2 = \text{const} \) for all \( \alpha \) and \( \beta \), \( M_{\alpha\beta} = 0 \) while the contribution to \( \text{Tr} M^2 \) from the mixing of gaugino and hyperini can be found from (3.30) using \( P_\alpha = q_1^\dagger T_\alpha q_1 - \tilde{q}_1 T_\alpha \tilde{q}_1^\dagger \). It reads
\[
\langle 2M_{I\alpha} M_{I\alpha}^\dagger \rangle = 8g^2 Q^2 (N_m^2 - 1).
\]
(3.35)
In the supergravity Lagrangian there is one more mixing between fermions, which could potentially add a term to \( \text{Tr} M^2 \), namely the mixing between the gravitino and gaugino. In [28] it is of the form \( \psi P_\alpha \chi^\alpha \). However, in our case \( \langle P_\alpha \rangle = 0 \) and so this mixing does not contribute. To recapitulate, the D-terms give exactly the same contribution as in the rigid case considered in [8].

4 Expansion in \( M_P^{-1} \) and rigid susy limit

In Section 3, we computed all ingredients of the finite temperature one-loop effective potential for the ISS model coupled to supergravity. Now we will make connection with the globally supersymmetric theory by inserting in the relevant formulae the explicit dependance on the Planck mass, \( M_P \), and expanding in powers of \( M_P^{-1} \). For later use, we will extract the leading supergravity corrections to the rigid results.

Let us start with the tree-level supergravity potential
\[
V = M_P^4 e^G \left( M_P^2 G^I G_I - 3 \right) + \frac{1}{2} D^a D^a
\]
(4.1)
with
\[
G = \frac{K}{M_P^2} + \log \frac{|W|^2}{M_P^2}.
\]
(4.2)
As discussed in the previous section, the D-term contribution is the same as in the rigid limit. It is easy to see that the expansion of (4.1) gives:

\[ V = W'W_I + \frac{1}{2} D^a D^a + \frac{1}{M_P^2} (KW'W_I + 2\text{Re}(K_IW'W) - 3|W|^2) + \mathcal{O}(M_P^{-4}), \]  

(4.3)

where obviously the first two terms are the standard global susy result. For future use, we note that taking \( I = \phi_{11}, \phi_{22} \) in the last equation gives, to leading order in the supergravity corrections, the following contribution to the classical F-term potential for the quarks:

\[
V_F \supset \left( h^2 \text{Tr} q_1 q_1^\dagger \tilde{q}_1 - h^2 \mu^2 (\text{Tr} q_1 q_1 + \text{Tr} q_1^\dagger q_1^\dagger) + h^2 \mu^4 (N_m + N_e) \right) \left( 1 + \frac{\text{Tr} q_1 q_1^\dagger + \text{Tr} q_1 q_1^\dagger}{M_P^2} \right). 
\]  

(4.4)

Inserting the \( M_P \) dependence in the thermal one-loop potential \( V_T \) yields:

\[
V_T = \frac{T^2}{24} M_P^2 \langle \epsilon^G \left( 3M_P^2 \sum_{I,J} R^{IJ} + 2(N_B - 1)M_P^2 G^I G_I - 2(2N_B + 1) \right) \rangle + \langle V_D \rangle, \]  

(4.5)

where we have denoted by \( R^{IJ} \) the quantity \((G^{IJ} + G^I G^J)(G_{IJ} + G_I G_J)\), as before. To obtain the explicit powers of \( M_P \) in \( R^{IJ} \), we note that due to (4.2) equation (3.15) becomes:

\[
G_{IJ} + G_I G_J = \frac{W_{IJ}}{W} + \frac{G_I K_J}{M_P^2} + \frac{G_I K_I}{M_P^2} - \frac{K_I K_J}{M_P^4}. 
\]  

(4.6)

Now, expanding (4.5) we find:

\[
V_T = \frac{T^2}{24} \left( 3W^{IJ} W_{IJ} + 3 \frac{K}{M_P^2} W^{IJ} W_{IJ} + 6 \frac{|W|^2}{M_P^2} \text{Re} \left[ \frac{W_{IJ}}{W^2} (W^I K^J + K^I W^J) \right] \right) + 2 \frac{(N_B - 1)}{M_P^2} W^I W_I \rangle + \langle V_D \rangle + \mathcal{O}(M_P^{-4}). \]  

(4.7)

Together, equations (4.3) and (4.7) give the general expression for the zeroth and first orders in the \( M_P^{-1} \) expansion of the one-loop effective potential at finite temperature.

Let us now apply the above formulae for the background (3.3), but with vanishing meson vevs. To see what (4.7) leads to, note that the only contributions to the first two terms come from \( \langle |W_{q_1 \phi_{11}}|^2 \rangle = \langle |W_{q_2 \phi_{21}}|^2 \rangle = h^2 Q^2 N_m^2 \) and \( \langle |W_{q_2 \phi_{22}}|^2 \rangle = h^2 Q^2 N_m N_e \) and that the only nonzero \( \langle W^I \rangle \) are \( \langle W^{\phi_{11}} \rangle = h(Q^2 - \mu^2) \) and \( \langle W^{\phi_{22}} \rangle = -h\mu^2 \). Hence we obtain:

\[
V_T = \frac{T^2}{2} h^2 Q^2 (N_m^2 + N_m N_e) \left( 1 + 2 \frac{Q^2 N_m}{M_P^2} \right) + T^2 Q^2 g^2 (N_m^2 - 1) + \frac{T^2}{M_P^2} \left( h^2 Q^2 (Q^2 - \mu^2) N_m + \frac{1}{12} (N_B - 1) [h^2 (Q^2 - \mu^2)^2 N_m + h^2 \mu^4 N_e] \right) + \mathcal{O}(M_P^{-4}). \]  

(4.8)

\[ \text{Recall that, as we mentioned in Footnote 14, we drop for brevity the constant } \sim T^4 \text{ contribution to effective potential in (3.3), as it does not affect our considerations.} \]
Taking $M_P \rightarrow \infty$, we find the global supersymmetry result for $V_T$\footnote{Our result is in agreement with that of \cite{8}, upon correcting a typo there. Namely, they have overcounted by a factor of two the number of Weyl fermions (in their notation) $\Psi_{\phi_{11}}$ and $(\Psi_{\tilde{q}_{1}} + \Psi_{\tilde{\tilde{q}}_{1}})/\sqrt{2}$.}

4.1 Local minimum at the origin

At high temperature, the temperature-dependent contribution $V_T$ completely dominates the effective potential and so the minima of $V_{\text{eff}}$ are given by the minima of $V_T$. Let us now address the question whether the origin of field space is a local minimum of $V_T$.

In principle, this could be a complicated problem, as we have to consider a function of three variables, $V_T(Q, \varphi_1, \varphi_2)$. However, things are enormously simplified by the fact that, as we noted at the end of Subsection 3.1.2, $Q = 0$ is an extremum for any $\varphi_1$ and $\varphi_2$. Since it is also a local minimum in the rigid limit, it will clearly remain such when taking into account the subleading supergravity corrections in our case. So we are left with investigating a function of two variables, $\varphi_1$ and $\varphi_2$. The presence of terms linear in any of them could, potentially, shift the position of the minimum away from the point $(\varphi_1, \varphi_2) = (0, 0)$\footnote{This is what happens for the KKLT-ISS model, as we will see in Section 7.}. However, such terms in the ISS model coupled to supergravity appear only multiplied by powers of $Q^2$, as we noted below eq. (3.22). So, given that $(\varphi_1, \varphi_2) = (0, 0)$ is a local minimum in the global supersymmetry case, it will remain such after coupling to supergravity, to all orders in the $1/M_P$ expansion.

It is, nevertheless, instructive to write down explicitly the expression for $V_T$ to leading order in the $1/M_P$ corrections:

$$V_T(Q, \varphi_1, \varphi_2) = \frac{T^2}{2} h^2 Q^2 (N_{m}^2 + N_e N_m) + T^2 Q^2 g^2 (N_{m}^2 - 1) + \frac{T^2}{4} h^2 (\varphi_1^2 N_{m}^2 + \varphi_2^2 N_m N_e)$$
$$+ \frac{T^2}{M_P^2} \left[ h^2 Q^2 (2\varphi_1^2 + Q^2 - \mu^2) N_m + \frac{(N_B - 1)}{12} \left( h^2 ((Q^2 - \mu^2)^2 + 2\varphi_1^2) N_m \right. \right.$$}
$$+ h^2 \mu^4 N_e \} + (2Q^2 N_m + \varphi_1^2 N_m + \varphi_2^2 N_e) \left( \frac{1}{2} h^2 Q^2 (N_{m}^2 + N_m N_e) \right)$$
$$+ \left. \frac{1}{4} h^2 (\varphi_1^2 N_{m}^2 + \varphi_2^2 N_m N_e) \right] + \mathcal{O} \left( \frac{1}{M_P^4} \right). \quad (4.9)$$

Clearly, this is consistent with (4.8). We have collected the terms that survive in the $M_P \rightarrow \infty$ limit on the first line. Obviously, the origin of field space $(Q, \varphi_1, \varphi_2) = (0, 0, 0)$ is a minimum. One can also notice that to this order $V_T$ is a function of the form $V_T(Q^2, \varphi_1^2, \varphi_2^2)$, i.e. it does not depend on odd powers of any of the vevs. One can verify
that the terms linear in $\phi$ and multiplying powers of $Q^2$, that we mentioned above, start appearing at $\mathcal{O}(1/M_6^6)$, while terms linear in $\phi_1\phi_2$ appear at $\mathcal{O}(1/M_4^4)$.

It will be useful for the next sections to extract from (4.9) a couple of special cases. One case is the second derivative in the quark direction only:

$$\frac{\partial^2 V_T}{\partial Q^2} \bigg|_{Q=0,\phi_1=0,\phi_2=0} = T^2\left[h^2(N_m^2+N_mN_e) + 2g^2(N_m^2-1)\right] - \frac{T^2}{M_5^2} \frac{1}{3} h^2 \mu^2 N_m (N_B+5).$$

(4.10)

The other interesting case is the potential in the meson directions only:

$$V_T(0,\phi_1,\phi_2) = \frac{T^2}{4} \left[h^2 \left(\phi_1^2 N_m + \phi_2^2 N_m N_e\right) + \frac{h^2}{M_5^2} \left(\phi_1^2 N_m + \phi_2^2 N_e\right)^2 N_m\right] + \frac{1}{M_5^2} \frac{1}{3} h^2 \mu^4 \left(N_B - 1\right)\left(N_m + N_e\right).$$

(4.11)

Finally, let us note that if we want to study other minima of the potential, which are away from the origin of field space $(Q,\phi_1,\phi_2) = (0,0,0)$, we would have to include in our considerations the non-perturbative superpotential $W_{ADS}$, see eq. (2.8).

5 Phase transition in quark direction

As we saw in the previous section, at high temperature the origin of field space is a local minimum of the effective potential. Lowering the temperature, one reaches a point at which this minimum becomes unstable and the fields start evolving towards new vacua. As recalled in Section 2, at zero temperature the ISS model possess supersymmetric vacua at non-zero meson vev and a metastable vacuum in the quark branch. Adding the supergravity interactions results in a slight shift of the positions of those minima. In particular, the metastable vacuum shifts to small vevs for some of the meson directions [12]. Hence, in order to end up in it, the system must undergo a second order phase transition in those same meson directions, as well as in the quark ones. In the next section we will see that this is indeed the case. In the present section, we will find the critical temperature, $T^Q_{c}$, for the onset of a second order phase transition towards nonvanishing quark vevs, while keeping all meson vevs at zero.

Before turning to the supergravity corrections to $T^Q_{c}$, let us recall how to compute the critical temperature in a generic field theory [20]. Suppose that we have a theory with a set of fields $\{\chi^I\}$. To find the effective potential, one has to shift $\chi^I$ by constant background fields $\hat{\chi}^I$, as we reviewed in Section 3. The effective potential is a function of
\( \hat{\chi}^I \) only and we consider the case when it is of the form \( V_{\text{eff}}(\hat{\chi}^2) \). The location of the minima is then determined by:

\[
\frac{\partial V_{\text{eff}}(\hat{\chi}^2)}{\partial \hat{\chi}^I} = 2 \hat{\chi}^I \frac{\partial V_{\text{eff}}(\hat{\chi}^2)}{\partial \hat{\chi}^2} = 0.
\] (5.1)

Clearly, \( \hat{\chi}^I = 0 \) satisfies this condition. Other minima, at \( \hat{\chi}^I \neq 0 \), can only occur when \( \partial V_{\text{eff}}/\partial \hat{\chi}^2 = 0 \). The critical temperature, at which rolling towards such minima begins, can be found by requiring [20]:

\[
\frac{\partial V_{\text{eff}}}{\partial \hat{\chi}^2} \bigg|_{\hat{\chi}=0} = \frac{\partial V_0}{\partial \hat{\chi}^2} \bigg|_{\hat{\chi}=0} + \frac{\partial V_T}{\partial \hat{\chi}^2} \bigg|_{\hat{\chi}=0} = 0,
\] (5.2)

where we have split the one-loop effective potential into zero-temperature and temperature-dependant contributions, \( V_0 \) and \( V_T \) respectively. This is equivalent to [20]

\[
\frac{\partial V_T}{\partial \hat{\chi}^2} \bigg|_{\hat{\chi}=0} = -\frac{m^2}{2},
\] (5.3)

where \( m^2 \) is the unshifted tree-level mass-squared of the field, whose nonzero vev characterizes the new vacuum. This last equation can only be solved if \( m^2 < 0 \). In the global supersymmetry case, among the quark fields, which have nonzero vevs at the metastable minimum, the only ones with negative tree-level mass-squared are the components of \( \text{Re}(q_1 + \tilde{q}_t^1)/\sqrt{2} \equiv \text{Re}q_+ \) [8]. Since for small vevs the gravitational corrections are sub-leading, we are guaranteed that this will be the case for us as well.

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19 This will be the case for us. However, even in principle this assumption is not a restriction but merely a simplification, as we explain in a subsequent footnote.

20 The assumption that \( V_{\text{eff}} \) depends only on \( \hat{\chi}^2 \), but not on \( \hat{\chi} \) alone, guarantees that \( \hat{\chi} = 0 \) is an extremum of the effective potential and thus simplifies the computations technically. In the general case, instead of (5.3) one has to solve, symbolically, the following system: \( V'_{\text{eff}}(\hat{\chi}_c, T_c) = 0, V''_{\text{eff}}(\hat{\chi}_c, T_c) = 0 \) in order to find the critical temperature \( T_c \), at which the phase transition occurs, and the vev \( \hat{\chi}_c \), at which the relevant extremum of \( V_{\text{eff}} \) is situated. Clearly, here we have denoted by ‘ and ” first and second derivatives w.r.t. to \( \hat{\chi} \), respectively.

21 In principle, \( m^2 \) gets also contributions from the Coleman-Weinberg potential. However, in our case the origin of field space is not a local minimum of the zero temperature potential and so perturbation theory around it does not make sense. Hence, for us, \( m^2 \) is purely classical. In fact, to be more precise, we should note that at the origin the Coleman-Weinberg potential becomes imaginary. As shown in [29], the imaginary part of the effective potential encodes the decay rate of a system with a perturbative instability. This decay process leads to the breaking up of the initial homogeneous field configuration into various domains, in each of which the fields are rolling toward a different classical solution, and should not be confused with a non-perturbatively induced tunneling between different minima of the potential. We will not dwell on that further in the present paper.
5.1 Mass matrix diagonalization

We will now compute the tree-level squared masses of the fields \( R e q_+ \), that are necessary for finding the critical temperature in the quark-direction. Before considering the supergravity corrections, it will be useful to give the derivation of these masses in the global supersymmetry limit. For more generality, we keep \( Q \neq 0 \) in this latter case, although we will take \( Q = 0 \) when we turn to the supergravity contributions to \( m^2 \) in (5.3).

From the tree-level scalar potential of the global theory, \( V = K^{IJ} \partial_I W \partial_J \bar{W} \), one finds [8]:

\[
h^2 Q^2 (|q_1|^2 + |\tilde{q}_1|^2) + h^2 Q^2 \text{Tr}(q_1 \tilde{q}_1^\dagger + \tilde{q}_1^\dagger q_1) + h^2 (Q^2 - \mu^2) (\text{Tr} \tilde{q}_1 q_1 + h.c.).
\] (5.4)

When we turn on gravitational interactions we will have a similar expression but, generically, with different coefficients. So it is of benefit to consider the general expression:

\[
A(|q_1|^2 + |\tilde{q}_1|^2) + B(\text{Tr} q_1 \tilde{q}_1^\dagger + \text{Tr} \tilde{q}_1^\dagger q_1) + C(\text{Tr} \tilde{q}_1 q_1 + h.c.)
\] (5.5)

with arbitrary \( A, B \) and \( C \). It can be diagonalized easily by introducing the combinations

\[
q_+ = \frac{q_1 + \tilde{q}_1^\dagger}{\sqrt{2}}, \quad q_- = \frac{q_1 - \tilde{q}_1^\dagger}{\sqrt{2}}.
\] (5.6)

Substituting the inverse transformation,

\[
q_1 = \frac{q_+ + q_-}{\sqrt{2}} \quad \text{and} \quad \tilde{q}_1^\dagger = \frac{q_+ - q_-}{\sqrt{2}},
\] (5.7)

in (5.5), we find:

\[
A(|q_+|^2 + |q_-|^2) + B(|q_+|^2 - |q_-|^2) + C \frac{1}{2} \text{Tr}(q_+^2 - q_-^2 + q_+^2 - q_-^2).
\] (5.8)

Finally, we decompose \( q_\pm = R e(q_\pm) + i I m(q_\pm) \) and obtain the following mass terms:

\[
(A+B+C)(R e(q_+))^2 + (A+B-C)(I m(q_+))^2 + (A-B-C)(R e(q_-))^2 + (A-B+C)(I m(q_-))^2.
\] (5.9)

Reading off the values of \( A, B \) and \( C \) from (5.4), we obtain from (5.9):

\[
m_{Req_+}^2 = h^2 (3Q^2 - \mu^2), \quad m_{Imq_+}^2 = h^2 (Q^2 + \mu^2),
\]

\[
m_{Req_-}^2 = h^2 (\mu^2 - Q^2), \quad m_{Imq_-}^2 = h^2 (Q^2 - \mu^2).
\] (5.10)

We see that, as already mentioned above, only the components \( R e q_+ \) of the field \( q_+ \) have negative mass-squareds for zero shift \( Q \). Therefore, only their masses should appear on

\[\text{Clearly, the negative mass-squared of } I m q_- \text{ is irrelevant as in the metastable supersymmetry breaking vacuum } \langle q_- \rangle = 0.\]
the right hand side of eq. (5.3). We note that the masses for the fields $Im q_+$ and $Im q_-$ differ from those given in table 5 of [8]. Fortunately their typos cancel out in the total $Tr M_s^2$.

Let us now apply the result (5.9) for the diagonalization of the expression (5.5) to the case of interest for us. Namely, we consider the ISS model coupled to supergravity and we want to compute the following derivatives:

$$\partial^2 q \, \partial^2 q^\dagger, \partial^2 \tilde{q} \, \partial^2 \tilde{q}^\dagger, \partial^2 \tilde{q} \, \partial^2 q$$ etc.,

which give the coefficients $A$, $B$ and $C$. As before, $V$ is the tree-level supergravity scalar potential. The only nonvanishing derivatives, upon setting $Q = 0$, are:

$$m_{q q}^2 = m_{\tilde{q} \tilde{q}}^2 = \frac{1}{M_P^2} h^2 \mu^4 (N_m + N_e), \quad m_{q \tilde{q}}^2 \neq m_{\tilde{q} q}^2 = -h^2 \mu^2.$$  (5.12)

Hence, applying (5.9), we find that the tree-level mass of each of the $N_m^2$ real fields in $Re q_+$ is:

$$m_{Req_+}^2 = -h^2 \mu^2 + \frac{1}{M_P^2} h^2 \mu^4 (N_m + N_e).$$  (5.13)

The first term corresponds to the global supersymmetry result, as can be seen from (5.10), while the second is a correction due to the supergravity interactions.

Note that we did not need to include the D-terms in the computation of the unshifted masses. The reason is that, as we have seen in Section 3.2, the contribution of these terms to the mass-squared of the fields is always proportional to $Q$ and so vanishes for zero shift.

### 5.2 Critical temperature

To find the critical temperature in the quark direction, recall that we shift the fields $q_{i a}$ and $\tilde{q}_{i b}$ by constant matrices $Q \delta^a_i$ and $Q \delta^b_{ij}$ respectively, see eq. (3.3). In the basis of $q_+$ and $q_-$, see equation (5.6), the only fields which get shifted are the $N_m$ diagonal components of $q_+$. From those, only the $N_m$ fields $Re q_+$ have negative tree-level mass-squared as we saw above. Therefore we can find the critical temperature from:

$$\frac{\partial^2 V_T}{\partial Q^2} \bigg|_{Q=0} = -N_m m_{Req_+}^2.$$  (5.14)

For more details see Appendix A. Note that (5.12) are exact expressions, i.e. to all orders in $1/M_P$. However, since they happen to be at most of $\mathcal{O}\left(\frac{1}{M_P^2}\right)$, they can also be read off from the part of the scalar potential (4.4), which contains only the relevant leading order supergravity corrections. Clearly, this is equivalent to (5.3) as $\partial^2 V_T/\partial Q^2 = 2 \partial V_T/\partial (Q^2)$. 
Recall that, to leading order in the supergravity corrections, we have (see (4.10)):

$$\frac{\partial^2 V_T}{\partial Q^2} |_{Q=0} = T^2 \left[ h^2(N_m^2 + N_m N_e) + 2g^2(N_m^2 - 1) \right] - \frac{1}{M_p^2} \frac{T^2 h^2 \mu^2 N_m (N_B + 5)}{3}.$$  \hspace{1cm} (5.15)

Together with (5.13), this then leads to:

$$\left( T_c^Q \right)^2 = \frac{h^2 \mu^2 N_m}{h^2(N_m^2 + N_m N_e) + 2g^2(N_m^2 - 1)} - \frac{1}{M_p^2} \frac{h^2 \mu^4 (N_m + N_e) N_m}{h^2(N_m^2 + N_m N_e) + 2g^2(N_m^2 - 1)} + \frac{1}{M_p^2} \frac{h^4 \mu^4 N_m^2 (N_B + 5)}{3 \left[ h^2(N_m^2 + N_m N_e) + 2g^2(N_m^2 - 1) \right]^2} + O \left( \frac{1}{M_p^4} \right).$$  \hspace{1cm} (5.16)

To see whether the order $1/M_p^2$ correction increases or decreases the critical temperature, recall that $N_f > 3N$ i.e. $N_e > 2N_m$. Let us write

$$N_e = 2N_m + p, \quad p \in \mathbb{Z}_+$$  \hspace{1cm} (5.17)

and substitute this in the total numerator of the $O(1/M_p^4)$ terms in (5.16):

$$\text{Numerator} = -h^4 \mu^4 \left( \frac{4N_m^2 + 10}{3} N_m p + \frac{2p^2 - 5}{3} \right) N_m^2 - 2h^2 \mu^4 g^2 (3N_m^2 + pN_m)(N_m^2 - 1).$$  \hspace{1cm} (5.18)

Clearly, for any $N_m$ and for any $p > 1$ every term in the above expression is negative definite. For $p = 1$ there is a positive contribution from the last term in the first bracket: $h^4 \mu^4 N_m^2$. However, it is outweighed by the first two terms in that bracket for any value of $N_m$. So the conclusion is that the supergravity interactions cause $T_c^Q$ to decrease compared to the rigid case for any $N_m$ and $N_e$.

## 6 Phase transition in meson direction

In the global supersymmetry case, a second order phase transition is only possible in a field space direction with nonzero squark vevs \[8\] 25 \[26\]. However, in our case things may be quite different due to the supergravity-induced contributions to the tree-level meson masses. Note that, in the approximation of neglecting $W_{\text{dyn}}$, the meson fields have zero classical mass in the global limit \[8\] and so the supergravity corrections are the leading ones\[26\]. Thus the possibility occurs that in the region of small vevs, for which neglecting the dynamically generated superpotential is well-justified, some of the meson directions

\[25\] Although a first order phase transition can still occur in the meson direction \[8\].

\[26\] Recall that we do not have to take into account contributions from zero-temperature one-loop effects in the rigid theory as the origin of field space is not a local minimum of the zero-temperature potential.
may develop negative tree-level mass-squareds due to supergravity. We will see in the following when that happens.

Let us address this issue in a slightly more general set-up as in [12], namely by adding to the ISS superpotential a constant piece. I.e., we consider \( W = W_0 + W_{ISS} \), where \( W_{ISS} \) is as in (2.1) and \( W_0 = \text{const} \). This is also a useful preparation for the KKLT-ISS model that we will discuss more in the next subsection. It will turn out that we need to compute the tree-level supergravity-induced meson masses not only at the origin of field space but also along the quark direction, i.e. for \( Q \neq 0 \) and \( \varphi_1, \varphi_2 = 0 \). Also, we will have to keep track of terms of up to \( O(1/M_\text{Pl}^2) \) to see the novel effect.

Turning to the computation, from (A.6) we find:

\[
\begin{align*}
 m^2_{(\phi_{11})_{ij}(\bar{\phi}_{11})_{k}\ell} &= 2h^2 Q^2 \delta^i_j \delta^k_\ell + \frac{1}{M_\text{Pl}^2} \left\{ \delta^i_k \delta^j_\ell h^2 \left[ (Q^2 - \mu^2)^2 N_m + \mu^4 N_e \right] + \delta^i_j \delta^k_\ell h^2 \left[ (Q^2 - \mu^2)(3Q^2 + \mu^2) + 4Q^4 N_m \right] \right\} + \frac{1}{M_\text{Pl}^2} \left\{ \delta^i_j \delta^k_\ell \left[ -2W_0^2 + 2h^2 Q^2 N_m \left[ (Q^2 - \mu^2)^2 N_m + \mu^4 N_e \right] \right] + \delta^i_k \delta^j_\ell 4h^2 Q^4 N_m \left[ 2(Q^2 - \mu^2) + Q^2 N_m \right] \right\}, \\
 m^2_{(\phi_{22})_m(\bar{\phi}_{22})_n} &= \frac{1}{M_\text{Pl}^2} \left\{ \delta^a_m \delta^b_n h^2 \left[ (Q^2 - \mu^2)^2 N_m + \mu^4 N_e \right] - h^2 \mu^4 \delta^a_m \delta^b_n \right\} + \frac{1}{M_\text{Pl}^2} \left\{ \delta^a_m \delta^b_n \left[ -2W_0^2 + 2h^2 Q^2 N_m \left[ (Q^2 - \mu^2)^2 N_m + \mu^4 N_e \right] \right] \right\}, \\
 m^2_{(\phi_{11})_{ij}(\bar{\phi}_{22})_{k}\ell} &= -h^2 \mu^2 \left\{ \frac{Q^2 + \mu^2}{M_\text{Pl}^2} + \frac{4Q^4 N_m}{M_\text{Pl}^2} \right\} \delta^i_j \delta^k_\ell, \\
 m^2_{(\phi_{22})_m(\bar{\phi}_{11})_{k}\ell} &= -h^2 \mu^2 \left\{ \frac{Q^2 + \mu^2}{M_\text{Pl}^2} + \frac{4Q^4 N_m}{M_\text{Pl}^2} \right\} \delta^a_m \delta^k_\ell, \\
 & \quad (6.1)
\end{align*}
\]

where clearly \( i, j, k, l = 1, \ldots, N_m \) and \( m, n, p, q = 1, \ldots, N_e \). Note that \( W_0 \) appears only at order \( 1/M_\text{Pl}^4 \). All terms of the form \( m^2_{\phi_{11}\phi_{11}}, m^2_{\phi_{22}\phi_{22}} \) and \( m^2_{\phi_{11}\phi_{22}} \) vanish (see Appendix A). To understand which mass matrix one has to diagonalize, let us recall that the meson fields are shifted as follows: \( (\phi_{11})_{ij} \) by \( \varphi_1 \delta^i_j \) and \( (\phi_{22})_m \) by \( \varphi_2 \delta^m_n \), see eq. (3.3). Hence there are \( N_m \) fields shifted by \( \varphi_1 \) (let us denote them by \( \phi_1 \)) and \( N_e \) fields shifted by \( \varphi_2 \) (let us denote them by \( \phi_2 \)). In the quark direction we could factor out an overall \( N_m \) (since both \( q_1 \) and \( \bar{q}_1 \) have the same number of components) and diagonalize the remaining mass matrix.\(^{28}\) Now things are somewhat more complicated as there are different numbers of \( \phi_1 \)

\(^{27}\)For reasons that will become clear below, we will be interested only in the range \( Q \in [0, \mu] \). So we are still allowed to neglect \( W_{dyn} \).

\(^{28}\)This is why we ended up with \( m^2 = N_m m^2_{\text{req}_+} \) in (6.14), where \( m^2_{\text{req}_+} \) was the result of the diagonalization.

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and $\phi_2$ fields. More precisely, from (6.1) we see that we have to diagonalize the expression:

$$M^2_\phi = \left( 2h^2Q^2 N_m + \frac{1}{M_P^2} (N_m C_1 + N_m^2 C_2) + \frac{1}{M_P^2} (N_m C_3 + N_m^2 C_4) \right) \phi_1 \bar{\phi}_1$$

$$+ \left( \frac{1}{M_P^2} h^2 (Q^2 - \mu^2)^2 N_m N_e + \frac{1}{M_P^2} N_e C_3 \right) \phi_2 \bar{\phi}_2$$

$$- h^2 \mu^2 N_m N_e \left( \frac{Q^2 + \mu^2}{M_P^2} + \frac{4Q^2 N_m}{M_P^2} \right) (\phi_1 \bar{\phi}_2 + \phi_2 \bar{\phi}_1),$$

(6.2)

where

$$C_1 = h^2 \left[ (Q^2 - \mu^2)^2 N_m + \mu^4 N_e \right],$$

$$C_2 = h^2 \left[ (Q^2 - \mu^2)(3Q^2 + \mu^2) + 4Q^4 N_m \right],$$

$$C_3 = -2W_0^2 + 2Q^2 N_m C_1,$$

$$C_4 = 4h^2 Q^4 N_m \left( 2(Q^2 - \mu^2) + Q^2 N_m \right).$$

(6.3)

Notice that $W_0$ enters the above formulae only at order $1/M_P^4$. Before proceeding further, let us make an important remark about the position of the local minimum that is our starting point at high temperature. From (3.20) and (3.21), one can see that for $W_0 \neq 0$ terms linear in $\varphi_1$ and $\varphi_2$ start appearing at order $1/M_P^4$. This implies that the position of the minimum is shifted to some point $(Q, \varphi_1, \varphi_2) = (0, \varphi_1^*, \varphi_2^*)$ with $\varphi_1^*, \varphi_2^* \neq 0$. However, one can easily verify that both $\varphi_1^*$ and $\varphi_2^*$ are of $O(1/M_P^4)$.

Therefore, in all terms of (6.1) with an explicit $1/M_P$ dependence one can take $\varphi_1^*, \varphi_2^* = 0$ since we are working to order $1/M_P^4$. Hence the only place the nonzero $\varphi_1^*, \varphi_2^*$ can make a difference in is the zeroth order terms. This translates to the zeroth order term in (A.6). However, that term is independent of the meson vevs as the superpotential is linear in those and the indices $I, J$ are along meson directions. Thus the results (6.1)-(6.3) are unchanged by the nonzero $\varphi_1^*, \varphi_2^*$. For convenience, in the following we will keep referring to the local minimum at $(0, \varphi_1^*, \varphi_2^*)$ as 'the minimum at the origin of field space'.

Let us now go back to (6.2). Since the coefficients of $\phi_1 \bar{\phi}_1$ and $\phi_2 \bar{\phi}_2$ are not the same, we cannot diagonalize this expression immediately by using the results of Subsection 5.1. However, it is still useful to change basis to the real components of the fields: $\phi_1 = Re \phi_1 + iIm \phi_1$ and $\phi_2 = Re \phi_2 + iIm \phi_2$. Then $M^2_\phi$ acquires the form:

$$M^2_\phi = \sum_{i=1}^{2} (M_{11} x_i^2 + 2M_{12} x_i y_i + M_{22} y_i^2),$$

(6.4)

More precisely, one finds $\varphi_1^* = \frac{4 \mu^2 W_0}{h N_m} \frac{1}{M_P^2} + O(\frac{1}{M_P^4})$ and $\varphi_2^* = \frac{4 \mu^2 W_0}{h N_m} \frac{1}{M_P^4} + O(\frac{1}{M_P^6})$. We have not looked at the subleading orders, so we abstain from claiming that $\varphi_1^* = \varphi_2^*$.
where \((x_1, y_1) = (Re \phi_1, Re \phi_2)\) and \((x_2, y_2) = (Im \phi_2, Im \phi_2)\). Thus the problem is reduced to the diagonalization of the \(2 \times 2\) matrix \(M_{ab}\) with \(a, b = 1, 2\) and so the eigenvalues are given by:

\[
m^2_{\pm} = \frac{\text{Tr} M \pm \sqrt{(\text{Tr} M)^2 - 4 \det M}}{2}.
\]

(6.5)

Before proceeding further with the \(Q \neq 0\) considerations, let us first take a look at what happens at the point \((0, \varphi_1^*, \varphi_2^*)\), which for us, as explained above, is the same as looking at the origin of field space.

For \(Q = 0\), the expressions (6.1) and (6.2) simplify significantly. Namely, we have:

\[
m^2_{(\phi_{11})_1^i(\phi_{11})_1^j}|_O = \frac{h^2 \mu^4}{M_P^2} \left[ \delta^k_i \delta^j_l (N_m + N_e) - \delta^j_i \delta^k_l \right] - \frac{2W_0^2}{M_P^2} \delta^k_i \delta^j_l,
\]

\[
m^2_{(\phi_{22})_{11}^n(\phi_{22})_{11}^p}|_O = \frac{h^2 \mu^4}{M_P^2} \left[ \delta^k_i \delta^j_l (N_m + N_e) - \delta^j_i \delta^k_l \right] - \frac{2W_0^2}{M_P^4} \delta^k_i \delta^j_l,
\]

\[
m^2_{(\phi_{11})_2^i(\phi_{22})_2^j}|_O = - \frac{h^2 \mu^4}{M_P^2} \delta^i_j \delta^k_l, \quad m^2_{(\phi_{22})_{11}^n(\phi_{11})_1^k}|_O = - \frac{h^2 \mu^4}{M_P^2} \delta^0_i \delta^k_l,
\]

(6.6)

where \(|_O\) denotes evaluation at the origin, and therefore:

\[
M^2_{\phi} |_O = \left[ \frac{h^2 \mu^4}{M_P^2} N_m N_e - \frac{2W_0^2}{M_P^4} N_m \right] \phi_1 \bar{\phi}_1 + \left[ \frac{h^2 \mu^4}{M_P^2} N_m N_e - \frac{2W_0^2}{M_P^4} N_e \right] \phi_2 \bar{\phi}_2
\]

\[- \frac{h^2 \mu^4}{M_P^2} N_m N_e (\phi_1 \bar{\phi}_2 + \phi_2 \bar{\phi}_1).
\]

(6.7)

Applying (6.5), we find that the two eigenvalues are:

\[
m^2_+ = \frac{2h^2 \mu^4}{M_P^2} N_m N_e - \frac{W_0^2}{M_P^4} (N_m + N_e), \quad m^2_- = - \frac{W_0^2}{M_P^4} (N_m + N_e).
\]

(6.8)

So, clearly, for any \(N_m\) and \(N_e\) there is a negative meson mass-squared direction. However, note that its value is of lower order in \(1/M_P\) than the leading term (which is of zeroth order) in the negative squark mass-squared that is driving the quark phase transition; see (5.13). Hence even without calculating the critical temperature \(T^c_q\), that would correspond to \(m^2_-\), we can be sure that it would be much smaller than \(T^c_Q\) of Subsection 5.2. Therefore, by the time the temperature starts approaching \(T^c_q\), the system would have already undergone the second order phase transition in the quark direction and would be rolling along the \(Q\) axis. So let us get back to considering (6.5) with \(Q \neq 0\).
One easily finds that, up to order $1/M_P^4$, the two eigenvalues of the matrix $M$ are:

\[
\begin{align*}
m^2_+ &= 2h^2Q^2N_m^2 + O\left(\frac{1}{M_P^2}\right), \\
m^2_- &= \frac{h^2(Q^2 - \mu^2)^2N_mN_e}{M_P^2} + \frac{1}{M_P^4} \frac{N_e}{2Q^2} \left[-4Q^2W_0^2 + 4h^2Q^4(Q^2 - \mu^2)^2N_m^2 + 4h^2\mu^4Q^4N_mN_e - h^2\mu^4(Q^2 + \mu^2)^2N_e\right].
\end{align*}
\] (6.9)

The expression for $m^2_-$ seems divergent for $Q = 0$. However, we just saw in the previous paragraph that at the point $Q = 0$ things are completely regular.\[30\] The reason for the apparent problem in (6.9) is the following. To obtain the last formulae, we had to expand the square root in (6.5) for small $1/M_P$. This is perfectly fine when $Q$ is of order $\mu$. However, when $Q \to 0$ one has to be more careful as the "leading" zeroth order contribution, $2h^2Q^2$, in the above mass formulae becomes $\ll \mu^4/M_P^2$ (keep in mind that $\mu^2/M_P$ may be small, but it is definitely finite). So to extract the correct behaviour of (6.5) in the limit $Q \to 0$, one has to first expand in small $Q$ and then expand this result in small $1/M_P$ to the desired order. Doing that, one recovers (6.8) at the zeroth order in the $Q$-expansion.

The lesson we learn from the above considerations is that (6.9) is valid in a neighborhood of the point $Q = \mu$, in which $Q$ and $\mu$ have comparable orders of magnitude. In that neighborhood both $m^2_\pm$ are positive definite except at $Q = \mu$, where one has:\[31\]

\[
\begin{align*}
m^2_+ &= 2h^2\mu^2N_m^2 + O\left(\frac{1}{M_P^2}\right), \\
m^2_- &= -\frac{2W_0^2N_e}{M_P^4} + \frac{2h^2\mu^6N_m^2(N_m - 1)}{M_P^4}.
\end{align*}
\] (6.10)

So, in principle, the sign of $m^2_-|_{Q=\mu}$ depends on the relative magnitudes of $W_0$ and $h\mu^3$, except for the case $N_m = 1$. However, if one wants the positive vacuum energy density in the metastable state to be very small, then the following relation should be satisfied:\[12, 11\]:

\[
\frac{W_0^2}{M_P^4} \approx h^2\mu^4N_e,
\] (6.11)

where $\approx$ means that the two sides are of the same order of magnitude. If one assumes this relation, then the first term in $m^2_-$ is dominant as by order of magnitude it is $\sim h^2\mu^4/M_P^2$ and so $m^2_-|_{Q=\mu} < 0$. Hence, when (6.11) holds, the new effect due to $W_0 \neq 0$ corrects the order $1/M_P^2$ results instead of those at $O(1/M_P^4)$.

\[30\]This is, in fact, the main reason we performed that computation explicitly; it was already clear from (6.1) that if there is a critical temperature $T_r^c$ at $Q = 0$, then it must be that $T_r^c \ll T_Q^c$.

\[31\]Strictly speaking, (6.10) is also valid in the small neighborhood in which $Q - \mu \to 0$, i.e. when $Q - \mu \ll \mu^2/M_P$. The argument is analogous to the one below eq. (6.9).
Let us see what conclusions one can make for the sign of $m^2$ throughout the interval $[0, \mu]$ for the cases when the $W_0$ contribution is of order $1/M_P^4$ and of order $1/M_P^2$, respectively. In either case, the sign of the eigenvalue $m^2_-$ is determined by the sign of $\det M$, see (6.5). However, in the first case, the leading order contribution is $\sim Q^2(Q^2 - \mu^2)^2$ and so is positive-definite except at $Q = 0$ and at $Q = \mu$. At these two points the subleading term (of order $1/M_P^4$) determines the sign and it is negative due to the negative $W_0$ contribution. On the other hand, in the case when (6.11) holds we should only look at the terms of up to $O(1/M_P^2)$. The leading contribution to $\det M$, which is first order in $1/M_P^2$, is now $\det M^{(1)} \approx Q^2 h^2 (Q^2 - \mu^2)^2 N_m N_e - Q^2 W_0^2 N_e/M_P^2$, which upon using (6.11) gives

$$\det M^{(1)} \approx Q^2 h^2 [(Q^2 - \mu^2)^2 N_m N_e - \mu^4 N_e^2] < 0 \quad (6.12)$$

at any point in $(0, \mu]$ as $N_m < N_e$. To recapitulate: when (6.11) is imposed, we find that $m^2_- < 0$ for any $Q \in [0, \mu]$, while without (6.11) $m^2_-$ is negative only in small neighborhoods of $Q = 0$ and $Q = \mu$. In a similar way one can show that $m^2_+ < 0$ is positive-definite without (6.11), whereas with (6.11) it is negative only in a small neighborhood of the origin.

The above conclusions imply that at some temperature $T_\phi^c$, below $T_Q^c$ of Section 5, there will be another second order phase transition, this time in the meson direction corresponding to $m^2_-$. We will explain in Section 8 that this phase transition is actually necessary in order for the system to roll towards the metastable vacuum, due to the shifting of the latter from $(\varphi_1, \varphi_2) = (0, 0)$ to $(\varphi_1, \varphi_2) \sim (\mu^2/M_P, \mu^2/M_P)$. However, computing the critical temperature is now much more complicated than in the quark case. One reason is that the phase transition occurs at some $Q \neq 0$. (With (6.11) imposed, this could apriori be any point in the interval $[0, \mu]$, whereas without this constraint it has to occur at $Q = \mu$.) Another, much more serious issue is that for temperatures below $T_Q^c$ there are masses that are greater than the temperature and so the high-temperature approximation (3.2) cannot be used. Instead, one should consider the full integral expression:

$$V_{\text{eff}}(\hat{\chi}) = V_0(\hat{\chi}) + \frac{T^4}{2\pi^2} \sum_I \pm n_I \int_0^\infty dx \ x^2 \ln \left(1 \mp \exp(-\sqrt{x^2 + m_I^2(\hat{\chi})/T^2})\right), \quad (6.13)$$

where $n_I$ are the numbers of degrees of freedom and the upper (lower) sign is for bosons (fermions).

Unfortunately, that means that we cannot obtain a simple analytic answer for the meson critical temperature $T_\phi^c$. However, we can make an estimate of its magnitude in

---

If $\det M < 0$, then the expression under the square-root is greater than $(\text{Tr} M)^2$ and so $m^2_- < 0$.

More precisely, except in very small neighborhoods of those points; see the remark in footnote 31.
the vein of \[7\]. Namely, let us consider a path in field space connecting the point along the Q-axis, at which the rolling in the meson direction starts, with the point \((Q, \varphi_1, \varphi_2) \neq (0, 0, 0)\), where the metastable minimum is, and take into account only fields whose masses change significantly along this path\[^{34}\]. Now, let us assume that all relevant masses are either much smaller or much greater than \(T\). We will see below that our results are consistent with this assumption. Then for fields with \(m \ll T\) we can still use (3.2), whereas for fields with \(m \gg T\) we can utilize the approximation (see (3.5) of [7]):

\[\pm \int_0^\infty dx x^2 \ln \left(1 + \exp\left(-\sqrt{x^2 + m^2/T^2}\right)\right) \sim T^4 \left(\frac{m}{2\pi T}\right)^{3/2} \exp\left(-\frac{m}{T}\right).\]  

(6.14)

In principle, the full temperature-dependent part of the effective potential is obtained by summing over all fields. Note however, that the exponential in (6.14) strongly suppresses the contributions of the fields with mass \(m \gg T\). In other words, as long as there is at least one field with \(m \ll T\), one can neglect the heavy fields to leading order.

As we saw above, without imposing (6.11) the phase transition occurs at \(Q = \mu\). Let us assume that this is true also with (6.11). Clearly, for the latter case this assumption will only give us a lower bound on \(T_\varphi\), but this is as good as one can get in that case without studying the full dynamical evolution of the system. At \(Q = \mu\), the heavy masses in our system are \(m_{Req_1}, m_{Imq_1}, m_{Req_2}, m_{Imq_2}\) and \(m_0^2\). All of them are \(\sim h^2 \mu^2 + \mathcal{O}(1/M_P^2)\) and to leading order remain constant along the above field space path. The light masses that determine \(T_\varphi\) in this approximation are \(m_{Req_1}, m_{Imq_1}, m_{Req_2}, m_{Imq_2}\) and \(m_0^2\)\[^{35}\].

Hence we can obtain an estimate for the critical temperature by solving

\[\partial^2_{\varphi_-} V_T^l = -m_-^2, \quad \partial_{\varphi_-} V_T^l = -\partial_{\varphi_-} V_0 \quad \text{at} \quad (Q, \varphi_1, \varphi_2) = (\mu, \varphi_1^*, \varphi_2^*).\]  

(6.15)

In this system of equations \(V_T^l\) denotes the temperature-dependent part of the effective potential, that results from (3.2) by summing only over the light fields, and \(\varphi_-\) is the linear combination of \(\varphi_1\) and \(\varphi_2\) that corresponds to the mass-squared eigenvalue \(m_-^2\). One then finds that

\[T_\varphi \sim \frac{W_0}{h M_P^2} \approx \frac{\mu^2}{M_P},\]  

(6.16)

where in the second equality we have used (6.11). Note that the heavy masses (let us denote them by \(m_h\) with \(h\) running over all heavy fields) are all of zeroth order in \(1/M_P\),

\[^{34}\text{Recall that the reason for this is that fields with (nearly) constant masses contribute only to the field-independent } T^4 \text{ term in the effective potential, which we are not interested in.}\]

\[^{35}\text{All of them vary significantly (meaning that the magnitude of their change is comparable to the magnitude of their leading order) along the path of interest and so contribute essentially to the effective potential.}\]
whereas the light masses (denote them by \( m_l \) with \( l \) running over the light fields) are all multiplied by the small constant \( h \) compared to \( 6.16 \). So our assumption, that at temperatures near \( T_c^f \) we have \( m_h > > T > > m_l \), is consistent with the estimate in \( 6.16 \).

Before concluding this section, let us comment on the first order phase transition found in [8]. Its critical temperature is of order \( (\hat{T}_\phi c)^2 \sim h\mu^2 \). When it is reached, tunnelling becomes possible between \( \langle \Phi \rangle = 0 \) and a minimum away from the origin in the meson direction \( \langle \Phi \rangle \sim \Pi_{N_f} \), which at zero temperature becomes the supersymmetric vacuum at \( \langle \Phi \rangle = h^{-1}\mu(3N-2N_f)/(N_f-N) \Pi_{N_f} \). On the other hand, our second order phase transition occurs in the field direction \( \varphi^- = \varphi_2 + \frac{N_e}{N_f} \frac{m_h}{M_p^2} \varphi_1 + O(1/M_p^4) \) and at \( \langle q_1 \rangle = \langle \bar{q}_1 \rangle \neq 0 \). Which one of \( T_c^f \) and \( \hat{T}_c^f \) is greater depends on the relative magnitudes of \( h \) and \( (\mu/M_p)^2 \). However, regardless of that, the second order phase transition is much more likely to take precedence since the first order one, as any tunnelling event, is exponentially suppressed.

To gain a better understanding of the phase structure in the meson direction and, in particular, to be able to estimate the supergravity corrections to the height of the potential barrier relevant for the first order phase transition, we would need to take into account the non-perturbative dynamically generated contribution to the superpotential. We leave that for future research.

7 Towards KKLT-ISS at finite \( T \)

The proposal of [10] is a significant progress towards finding dS vacua in string theory with all moduli stabilized. However, the uplifting of their AdS minimum to a de Sitter one has been rather difficult to implement in a controlled way. It was shown recently in [11, 12], that this can be achieved easily by using the ISS model as the uplifting sector. They considered the following coupling:

\[
W = W_{ISS} + W_{KKLT}, \quad K = K_{ISS} + K_{KKLT},
\]

(7.1)

where \( W_{ISS} \) and \( K_{ISS} \) are given by \( 2.1 \) and \( 2.2 \), whereas:

\[
W_{KKLT} = \tilde{W}_0 + ae^{-b\rho} \quad \text{and} \quad K_{KKLT} = -3 \ln(\rho + \bar{\rho}).
\]

(7.2)

In the string context, the constant \( \tilde{W}_0 \) is due to nonzero background fluxes. By tuning it suitably, one can achieve an almost vanishing cosmological constant and a light gravitino mass [11], which is an important improvement compared to the models with D-term

\[36\] This expression for \( \varphi^- \) is valid with or without \( 6.11 \) as \( W_0 \) appears for first time at order \( 1/M_p^9 \).
uplifting. Therefore, it would be quite interesting to investigate the phase structure of the KKLT-ISS model \((7.1)\) at finite temperature. In this section, we limit ourselves to a discussion of the fate of the \((Q, \varphi_1, \varphi_2) = (0, 0, 0)\) ISS minimum when the KKLT field \(\rho\) is included.

In Subsection 4.1, we noted that the presence of terms linear in the meson vevs \(\varphi_1\) and \(\varphi_2\) can shift the minimum of the effective potential away from the origin of the ISS field space. This, of course, refers to terms that are not multiplying \(Q^2\) since \(Q = 0\) is always a local minimum as long as \(\mu \ll M_P\), see eq. \((4.10)\). However, in that section such linear terms were not appearing. Now the situation is very different, as the inclusion of the KKLT sector introduces many terms linear in \(\varphi_1\) and \(\varphi_2\). One source of them comes from the constant piece \(\tilde{W}_0\) in the KKLT superpotential, as one can easily convince oneself by looking at the form of \(\langle G^I G_I \rangle\) in \((3.21)\)\(^{37}\). In addition, there are many more contributions, linear in the meson vevs, from mixed terms between the KKLT and ISS sectors in the total \(\text{Tr} M^2\). For example, the total fermionic \(\text{Tr} M_f^2\) is now

\[
\text{Tr} M_f^2 = \langle e^G [K^{AB} K^{CD} (\nabla_A G_C + G_A G_C) (\nabla_B G_D + G_B G_D) - 2]\rangle, \tag{7.3}
\]

where \(A = \{\rho, I\}\) and the index \(I\) runs over the ISS fields. Writing this out, we have

\[
\text{Tr} M_f^2 = \langle e^G [K^{\rho \rho} K^{\bar{\rho} \bar{\rho}} (\nabla_\rho G_{\rho} + G_\rho G_\rho) (\nabla_{\bar{\rho}} G_{\bar{\rho}} + G_{\bar{\rho}} G_{\bar{\rho}}) + 2K^{\rho \bar{\rho}} K^{I \bar{J}} (\nabla_\rho G_I + G_\rho G_I) (\nabla_{\bar{\rho}} G_J + G_{\bar{\rho}} G_J) + (G_{IJ} + G_I G_J) (G^{IJ} + G^I G^J) - 2]\rangle, \tag{7.4}
\]

where in the second line we have used that \(\nabla_\rho G_I = \nabla_I G_\rho\)\(^{38}\). The last line is the familiar ISS plus supergravity contribution, which for \(W = W_{ISS} + W_{KKLT}\) gives the linear term mentioned in the beginning of this paragraph. Let us now take a more careful look at the remaining terms. One can verify that the second line contains, among many others, the terms

\[
\langle \frac{3}{(\rho + \bar{\rho})^3} [WK_I W_I + \bar{W} K^I W_I]\rangle. \tag{7.5}
\]

It is easy to see that these also lead to contributions which are linear in \(\varphi_1\) and \(\varphi_2\). Further linear terms come from the first line of \((7.3)\) and also from the mixed terms in the scalar potential

\[
V = e^K (K^{AB} D_A W D_B \bar{W} - 3|W|^2) \tag{7.6}
\]

\(^{37}\)Recall that the full expression is \(\langle e^G G^I G_I \rangle\). Hence the mixed terms in \((3.21)\) give contributions proportional to \(\varphi_1 \tilde{W}_0\) and to \(\varphi_2 \tilde{W}_0\).

\(^{38}\)The relation \(\nabla_\rho G_I = \nabla_I G_\rho\) is due to \(\partial_I G_{\rho} = \partial_\rho G_I = -W_\rho W_I / W^2\) together with the fact that the only nonvanishing Christoffel symbols are \(\Gamma^{\rho}_{\rho \rho}\) and \(\Gamma^{\bar{\rho}}_{\rho \bar{\rho}}\).
which determines the bosonic $\text{Tr}M_b^2$ via [22]:

$$\text{Tr}M_b^2 = 2 \left\langle K^{CD} \frac{\partial^2 V}{\partial \chi^C \partial \chi^D} \right\rangle. \quad (7.7)$$

Therefore, in the coupled KKLT-ISS model, generically, the origin of the ISS field space $(Q, \varphi_1, \varphi_2) = (0, 0, 0)$ is not a local minimum anymore. The coefficients of the terms linear in the meson vevs are functions of the KKLT field $\rho$. Hence $\langle \rho \rangle$ is related to the magnitude of the shift of the minimum in the ISS plane. Although very interesting, we leave further analysis of the KKLT-ISS system for future work.

8 Discussion

We studied the effective potential at finite temperature for the ISS model coupled to supergravity. Assuming that at high temperature the fields are at the origin of field space, which is a local minimum, we investigated the phase structure of the system as it cools down. In the quark direction, the situation is analogous to the rigid case [8]. Namely, there is a second order phase transition at certain critical temperature, $T_{Qc}$. The effect of the supergravity corrections is to decrease $T_{Qc}$ compared to its global supersymmetry counterpart. In the meson branch however, a new feature appears whenever the superpotential contains a nonvanishing constant piece $W_0$. Recall that in the global theory all meson fields always had a local minimum at the origin of the meson direction, with no tree-level contributions to their masses-squared. Now we find that, when $W_0 \neq 0$, for some of them this ceases to be true at some temperature $T_{\varphi_c}$, below which negative tree-level supergravity corrections to their effective masses-squared are outweighing the positive one-loop temperature dependent contributions. Hence, the supergravity interactions lead to the occurrence of a new second order phase transition whenever $W_0 \neq 0$.

Since $T_{\varphi_c} < T_{Qc}$, as we saw in Section 6, the fields first start rolling away from the origin in the quark direction. When the temperature decreases enough, the same happens also in the meson direction. However, unlike in the rigid case, the second phase transition does not imply that the system is moving away from the supersymmetry breaking vacuum. The reason is that the coupling to supergravity leads to slight shifting of the position of the metastable minimum [12]. \footnote{This is not a trivial consequence of including supergravity, as in the ISS model coupled to sugra with $W_0 = 0$ there is no shifting of the metastable vacuum compared to the global case.} Whereas in the rigid theory it was given by $\langle q_1 \rangle = \langle \tilde{q}_1 \rangle = \mu \mathbb{I}_{N_m}$ and $\langle \Phi \rangle = 0$, in the locally supersymmetric case some of the meson vevs...
also acquire nonzero value: \((\langle \text{diag} \varphi_{11} \rangle, \langle \text{diag} \varphi_{22} \rangle) = (\varphi_1, \varphi_2) \sim (\mu^2/M_P, \mu^2/M_P) \ll \langle q \rangle\). Hence the latter phase transition is, in fact, a necessary condition for the system to evolve towards the metastable vacuum. Of course, to follow with more precision the evolution of this system as the temperature decreases, one has to study the full effective potential for both \(\langle q \rangle \neq 0\) and \(\langle \Phi \rangle \neq 0\) away from a small neighborhood of the origin.

In the above paragraph, we reached the conclusion that the final state of the system at \(T \sim 0\) is likely to be the metastable vacuum. However, one should be cautious since, similarly to \([8]\), our considerations assume thermal equilibrium. Hence, although suggestive, they are not completely conclusive. As was pointed out in \([8]\), for proper understanding of the evolution of the system one should address also the dynamics of the fields at finite temperature. Actually, even before worrying about dynamics, one may be concerned that in our case the situation is complicated by the existence of new supergravity-induced supersymmetric minima. Indeed, it was shown in \([11]\) that such a minimum occurs in the KKLT-ISS model. However, this new vacuum only appears due to the interaction with the KKLT sector; it is easy to see that the last condition in the solution for this minimum, eq. (19) of \([11]\), is only satisfied with an appropriate choice of (some of) the tunable KKLT parameters \(W_0, a\) and \(b\). Still, one may wonder whether there could be a solution to the supersymmetry preserving equations for the ISS plus supergravity sector alone. In Appendix B we show that this is not possible (at least in the field-space directions of interest, i.e for an ansatz for the vevs that is of the same type as the one in \([11]\)).

The new solution of \([11]\) is only one indication that the KKLT-ISS system is quite intricate to study. Another is the fact that, as we saw in Section \([7]\), the interaction with the KKLT sector leads to shifting of the high-temperature minimum of the effective potential away from the origin of the ISS field space. Understanding the phase structure of this system is of great interest. However, the technical complications involved are rather significant. Therefore, it may be beneficial to gain preliminary intuition about it by considering the recently proposed O’KKLT model \([30]\), as the latter is much simpler while having quite similar behavior. In addition, the O’KKLT model has a significance of

\(^{40}\)That is, susy minima other than those induced by the non-perturbative superpotential \(W_{\text{ADS}}\), see eq. (2.8). Recall that the Witten index gives only the number of global susy minima of globally supersymmetric theories. Hence, in the present context it is not applicable and so one cannot immediately rule out the presence of additional solutions.

\(^{41}\)It is true that this minimum is much further out in field space than the metastable one, but its very existence raises the possibility that it may be quite premature to make conclusions about the final state at low temperature, based solely on studying the immediate neighborhood of the origin.
its own, as it was argued in [30] to be of value in studies of cosmological inflation. We hope to address this in the future.

Acknowledgements

We would like to thank V. Khoze, R. Russo and G. Travaglini for useful discussions. The work of L.A. is supported by the EC Marie Curie Research Training Network MRTN-CT-2004-512194 Superstrings.

A Useful mass matrix formulae

By differentiating the F-term part of (4.1) w.r.t. to $\chi^I$ and $\bar{\chi}^J$ one finds:

$$\partial_I \partial^J V = e^G [M_p^4(G^{L^I} + G^{L}G^J)(G_{LI} + G_L G^I) - M_p^4 G_I G^J + \delta_I^J (M_p^4 G^L G_L - 2 M_p^2)].$$  \hspace{1cm} (A.1)

Substituting $e^G = e^{K/M_p^2} |W|^2/M_p^6$ and

$$G_{LI} + G_L G^I = \frac{1}{M_p^2} \left( \frac{W_{LI} + K_L W_I + K_I W_L}{W} + \frac{K_L K_I}{M_p^4} \right),$$  \hspace{1cm} (A.2)

we see that all powers of $W$ in the denominator cancel out. We want to compute the value of the resulting expression for two cases: One is for zero background vevs of all scalars. And the other is for $\varphi_1, \varphi_2 = 0$ but $Q \neq 0$.

In the first case, i.e. for $Q, \varphi_1, \varphi_2 = 0$, one has that $\langle K \rangle = 0$ and $\langle K_I \rangle = 0$ for all $I$. Let us denote $\langle W \rangle \equiv W_0$ and for the moment consider $W_0 \neq 0$ for more generality. We find that

$$\langle \partial_I \partial^J V \rangle = \langle W_{IL} \bar{W}^{L} \rangle + \frac{1}{M_p^2} \left( \delta_I^J \langle W_L \bar{W}^L \rangle - \langle W_I \bar{W}^I \rangle \right),$$  \hspace{1cm} (A.3)

where all higher orders in the $1/M_p$ expansion vanish due to $\langle K \rangle, \langle K_I \rangle = 0$ regardless of the value of $W_0$. Realizing that the vevs of all double derivatives of $W$ vanish at the origin of field space, we finally obtain:

$$\langle \partial_I \partial^J V \rangle = \frac{1}{M_p^2} \left( \delta_I^J \langle W_L \bar{W}^L \rangle - \langle W_I \bar{W}^I \rangle \right).$$  \hspace{1cm} (A.4)

Note that this result, apart from being exact to all orders, is also completely independent of $W_0$.

\footnote{In fact, this statement only applies to a variant of O’KKLT considered in Section 3 there. This same variant is also the model that is most useful as an approximation of KKLT-ISS since it is the one, whose supersymmetric global minimum is at finite field values.}
Applying (A.4) to compute the mass matrices $m_{qq}^2$, where $q$ is either $q_1$ or $\tilde{q}_1$ and $\bar{q}$ is either $\tilde{q}_1$ or $\tilde{\tilde{q}}_1$, one arrives at half of the relations in (5.12). The other half, i.e. of types $m_{qq}^2$ and $m_{q\bar{q}}^2$, can be derived in a similar way. (It is perhaps more convenient to differentiate the potential in the form (3.4).) One finds that at the origin of field space:

$$\langle \partial_K \partial_L V \rangle = \langle K^{IJ} W_{KLI} \overline{W}_J \rangle,$$

which is exactly the expression for the rigid case, since all second derivatives of the superpotential vanish for zero background fields. And again, this is exact result to all orders in $1/M_P$.

In the case of $\varphi_1, \varphi_2 = 0$ but $Q \neq 0$, we will be interested in the meson mass matrices. For $I, J$ running over the meson field components only, we still have that $\langle K_I \rangle = 0$, although now $\langle K \rangle \neq 0$. Keeping $W_0 \neq 0$ and expanding (A.1) up to order $1/M_P^4$, we find:

$$\langle \partial_I \partial^J V \rangle = \langle W_{IL} \overline{W}^{IL} \rangle$$

$$+ \frac{1}{M_P^2} \left\{ \delta^I_J \langle W_L \overline{W}^L \rangle - \langle \overline{W}_I W^J \rangle + \langle W_I K_L \overline{W}^{IL} + W_{1L} K^L \overline{W}^I + K W_{IL} \overline{W}^{IL} \rangle \right\}$$

$$+ \frac{1}{M_P^2} \left\{ \delta^I_J \langle K W_L \overline{W}^L \rangle - 2 W_0^2 \rangle + \langle K \rangle \langle W_I K_L \overline{W}^{IL} + W_{1L} K^L \overline{W}^I \rangle + \langle K^2 \rangle \right\}.$$

Obviously, for $Q = 0$ the above formula agrees with (A.4). However, recall that the $I, J$ indices in it run only over the meson fields, whereas $L$ runs also over the squarks.

Finally, one can easily convince oneself that if $K, L$ run only over the mesons, whereas the squarks are the only fields with nonzero vevs, then $\langle \partial_K \partial_L V \rangle = 0$ to all orders and for any value of $W_0$.

**B On existence of new susy solutions**

We investigate here whether there are solutions to the supersymmetry preserving equations for the ISS model coupled to supergravity, with the non-perturbative superpotential $W_{ADS}$ of eq. (2.8) still neglected.

The susy equations are:

$$D_{q_1} W = 0, \quad D_{\tilde{q}_1} W = 0, \quad D_{\Phi_{11}} W = 0, \quad D_{\Phi_{22}} W = 0.$$  

(B.1)

Similarly to [11], we consider the following ansatz for the expectation values of the quark and meson fields in the solutions we are seeking:

$$\langle q_1 \rangle = \mu_1 \mathbb{1}_{N_m}, \quad \langle \tilde{q}_1 \rangle = \mu_2 \mathbb{1}_{N_m}, \quad \langle \Phi_{11} \rangle = \nu_1 \mathbb{1}_{N_m}, \quad \langle \Phi_{22} \rangle = \nu_2 \mathbb{1}_{N_e}.$$  

(B.2)
where all vevs are real. Evaluating (B.1) in this background gives:

\[ D_{q_1} : \ h\mu_2\nu_1 + \mu_1\langle W \rangle = 0 \ , \quad D_{\tilde{q}_1} : \ h\mu_1\nu_1 + \mu_2\langle W \rangle = 0 \, , \]
\[ D_{\Phi_{11}} : \ h(\mu_1\mu_2 - \mu^2) + \nu_1\langle W \rangle = 0 \ , \quad D_{\Phi_{22}} : \ -h\mu^2 + \nu_2\langle W \rangle = 0 \ . \]  
(B.3)

The two equations on the first line of (B.3) imply that \( \mu_1^2 = \mu_2^2 \). On the other hand, the ones on the second line lead to:

\[ \nu_2 = -\frac{\mu^2}{\mu_1\mu_2 - \mu^2}\nu_1 \, . \]  
(B.4)

Now, using this last relation and the equations for \( D_{q_1} \) and \( D_{\Phi_{22}} \), we find:

\[ \nu_1^2 = \frac{\mu_1}{\mu_2} (\mu_1\mu_2 - \mu^2) \ , \quad \nu_2 = -\mu^2 \left( \frac{\mu_1}{\mu_2} \right)^{1/2} \left( \frac{\mu_1}{\mu_2 (\mu_1\mu_2 - \mu^2)} \right) \, . \]  
(B.5)

So far, we have expressed all vevs in terms of one of them, which could be either \( \mu_1 \) or \( \mu_2 \). Let us choose this to be \( \mu_1 \). To obtain an independent equation for it, we need to use the explicit vev of the superpotential:

\[ \langle W \rangle = h \left( \mu_1\mu_2\nu_1N_m - \mu^2(\nu_1N_m + \nu_2N_e) \right) \, . \]  
(B.6)

Combining this with (B.4) and the \( D_{q_1} \) equation in (B.3), we find:

\[ -\frac{\mu_2}{\mu_1} = \frac{1}{M_P^2} \left[ \mu_1\mu_2N_m - \mu^2 \left( N_m - \frac{\mu^2N_e}{\mu_1\mu_2 - \mu^2} \right) \right] \, , \]  
(B.7)

where we have reinserted the explicit dependence on \( M_P \) that comes from \( D_IW = \partial_IW + (K_I/M_P^2)W \). Let us now consider first the case \( \mu_2 = \mu_1 \). Then (B.7) becomes a quadratic equation for \( \mu_1^2 \), whose solutions are:

\[ \left( \frac{\mu_1}{M_P} \right)^2 = \left( \frac{\mu}{M_P} \right)^2 - \frac{1}{2N_m} \pm \frac{1}{2N_m} \sqrt{1 - 4 \left( \frac{\mu}{M_P} \right)^4 N_mN_e} \, . \]  
(B.8)

Since we would like both \( \mu_1 \ll M_P \) and \( \mu \ll M_P \) in order to have a reliable field theory description, only the plus sign in (B.8) is meaningful. Hence, we have

\[ \left( \frac{\mu_1}{M_P} \right)^2 = \left( \frac{\mu}{M_P} \right)^2 - \left( \frac{\mu}{M_P} \right)^4 N_e + \mathcal{O} \left( \frac{\mu^8}{M_P^8} \right) \, . \]  
(B.9)

Note that, up to now, we have not used the explicit form of \( \langle W \rangle \). So our results (B.5), together with \( \mu_1^2 = \mu_2^2 \), are valid also for the KKLT-ISS set-up considered in [11]. In fact, the solution given in their eq. (19) is only valid for \( \mu_1 = \mu_2 \), in which case one can see that it agrees with (B.5).
This implies that $\mu_1^2 - \mu_2^2 < 0$, which is inconsistent with (B.5) since the vevs $\nu_1$ and $\nu_2$ are real. If we take in turn $\mu_2 = -\mu_1$ and repeat the above steps, we end up with

$$\left( \frac{\mu_1}{M_P} \right)^2 = - \left( \frac{\mu}{M_P} \right)^2 + O \left( \frac{\mu^4}{M_P^4} \right),$$

which is again inconsistent for real vevs. So we conclude that coupling to supergravity does not increase the number of vacua of the ISS model.

**References**

[1] K. Intriligator and N. Seiberg, “Lectures on supersymmetry breaking,” arXiv:hep-ph/0702069.

[2] E. Witten, *Dynamical Breaking of Supersymmetry*, Nucl. Phys. B188 (1981) 513.

[3] I. Affleck, M. Dine and N. Seiberg, *Calculable Nonperturbative Supersymmetry Breaking*, Phys. Rev. Lett. 52 (1984), 1677; I. Affleck, M. Dine and N. Seiberg, *Dynamical Supersymmetry Breaking in Four Dimensions and Its Phenomenological Implications*, Nucl. Phys. B256 (1985) 557; K. Intriligator and S. Thomas, *Dynamical Supersymmetry Breaking on Quantum Moduli Spaces*, Nucl. Phys. B473 (1996) 121, hep-th/9603158; K. Izawa and T. Yanagida, *Dynamical Supersymmetry Breaking in Vector-like Gauge Theories*, Prog. Theor. Phys. 95 (1996) 829, hep-th/9602180; see also G. Giudice and R. Rattazzi, *Theories with Gauge-mediated Supersymmetry Breaking*, Phys. Rept. 322 (1999) 419, hep-th/9801271 and references therein.

[4] K. Intriligator, N. Seiberg and D. Shih, *Dynamical SUSY Breaking in Meta-Stable Vacua*, hep-th/0602239.

[5] H. Ooguri and Y. Ookouchi, *Landscape of Supersymmetry Breaking Vacua in Geometrically Realized Gauge Theories*, hep-th/0606061; T. Banks, *Remodeling the Pentagon After the Events of 2/23/06*, hep-th/0606313; S. Förste, *Gauging Flavour in Meta-Stable Susy Breaking Models*, hep-th/0608036; A. Amariti, L. Girardello and A. Mariotti, *Non-supersymmetric Meta-Stable vacua in SU(N) SQCD with adjoint matter*, JHEP 0612 (2006) 058, hep-th/0608063; M. Dine, J. Feng and Eva Silverstein, *Retrofitting O’Raifeartaigh Models with Dynamical Scales*, hep-th/0608159; M. Dine and J. Mason, *Gauge Mediation in Metastable Vacua*, hep-ph/0611312.

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R. Kitano, H. Ooguri and Y. Ookouchi, Direct Mediation of Meta-Stable Supersymmetry Breaking, hep-ph/0612139; H. Murayama and Y. Nomura, Gauge Mediation Simplified, hep-ph/0612186; C. Csáki, Y. Shirman and J. Terning, A Simple Model of Low-scale Direct Gauge Mediation, hep-ph/0612241; O. Aharony and N. Seiberg, Naturalized and Simplified Gauge Mediation, hep-ph/0612308; S. Abel and V. Khoze, Metastable SUSY Breaking within the Standard Model, hep-ph/0701069; A. Amariti, L. Girardello and A. Mariotti, On Meta-Stable SQCD with Adjoint Matter and Gauge Mediation, hep-th/0701121.

[6] V. Braun, E. Buchbinder and B. Ovrut, Dynamical SUSY Breaking in Heterotic M-theory, Phys. Lett. B639 (2006) 566, hep-th/0606166; V. Braun, E. Buchbinder and B. Ovrut, Towards Realizing Dynamical SUSY Breaking in Heterotic Model Building, hep-th/0606241; S. Franco, I. Garcia-Etxebarria and A. Uranga, Non-supersymmetric Meta-stable Vacua from Brane Configurations, JHEP 0701 (2007) 085, hep-th/0607218; I. Bena, E. Gorbatov, S. Hellerman, N. Seiberg and D. Shih, A Note on (Meta)stable Brane Configurations in MQCD, JHEP 0611 (2006) 088, hep-th/0608157; C. Ahn, Brane Configurations for Nonsupersymmetric Meta-Stable Vacua in SQCD with Adjacent Matter, hep-th/0608160; R. Tatar and B. Wetenhall, Metastable Vacua, Geometrical Engineering and MQCD Transitions, JHEP 0702 (2007) 020, hep-th/0611303; C. Ahn, Meta-Stable Brane Configuration with Orientifold 6 Plane, hep-th/0701145.

[7] S. Abel, C. Chu, J. Jaeckel and V. Khoze, SUSY Breaking by a Metastable Ground State: Why the Early Universe Preferred the Non-supersymmetric Vacuum, hep-th/0610334.

[8] W. Fischler, V. Kaplunovsky, C. Krishnan, L. Mannelli and M. Torres, Meta-Stable Supersymmetry Breaking in a Cooling Universe, hep-th/0611018.

[9] N.J. Craig, P.J. Fox and J.G. Wacker, Reheating Metastable O’Raifeartaigh Models, hep-th/0611006.

[10] S. Kachru, R. Kallosh, A. Linde and S. Trivedi, de Sitter Vacua in String Theory, Phys. Rev. D68 (2003) 046005, hep-th/0301240.

[11] E. Dudas, C. Papineau and S. Pokorski, Moduli Stabilization and Uplifting with Dynamically Generated F-terms, hep-th/0610297.
[12] H. Abe, T. Higaki, T. Kobayashi and Y. Omura, Moduli Stabilization, F-term Up-lifting and Soft Supersymmetry Breaking Terms, Phys. Rev. D75 (2007) 025019, hep-th/0611024.

[13] C. Burgess, R. Kallosh and F. Quevedo, de Sitter String Vacua from Supersymmetric D-terms, JHEP 0310 (2003) 056, hep-th/0309187.

[14] K. Choi, A. Falkowski, H. Nilles and M. Olechowski, Soft Supersymmetry Breaking in KKLT Flux Compactification, Nucl. Phys. B718 (2005) 113, hep-th/0503216; A. Achucarro, B. de Carlos, J. Casas and L. Doplicher, de Sitter Vacua from Uplift-D-terms in Effective Supergravities from Realistic Strings, JHEP 0606 (2006) 014, hep-th/0601190; M. Haack, D. Kreft, D. Lust, A. Van Proeyen and M. Zagermann, Gaugino Condensates and D-terms from D7-branes, JHEP 0701 (2007) 078, hep-th/0609211.

[15] N. Seiberg, Electric-Magnetic Duality in Supersymmetric Non-Abelian Gauge Theories, Nucl. Phys. B435 (1995) 129, hep-th/9411149.

[16] S. Coleman and E. Weinberg, Radiative Corrections as the Origin of Spontaneous Symmetry Breaking, Phys. Rev. D7 (1973) 1888.

[17] I. Affleck, M. Dine and N. Seiberg, Supersymmetry Breaking by Instantons, Phys. Rev. Lett. 51 (1983) 1026; Dynamical Supersymmetry Breaking in Supersymmetric QCD, Nucl. Phys. B241 (1984) 493.

[18] E. Witten, Constraints on Supersymmetry Breaking, Nucl. Phys. B202 (1982) 253.

[19] R. Jackiw, Functional Evaluation of the Effective Potential, Phys. Rev. D9 (1974) 1686.

[20] L. Dolan and R. Jackiw, Symmetry Behavior at Finite Temperature, Phys. Rev. D9 (1974) 3320.

[21] P. Binetruy and M. Gaillard, Temperature Corrections, Supersymmetric Effective Potentials and Inflation, Nucl. Phys. B254 (1985) 388.

[22] P. Binetruy and M. Gaillard, Temperature Corrections in the Case of Derivative Interactions, Phys. Rev. D32 (1985) 931.
[23] J. Burton, M. Gaillard and V. Jain, *Effective One-loop Scalar Lagrangian in No-scale Supergravity Models*, Phys. Rev. D41 (1990) 3118; Mary K. Gaillard, A. Papadopoulos and D.M. Pierce, *A String Inspired Supergravity Model at One Loop*, Phys. Rev. D45 (1992) 2057; M. Gaillard and V. Jain, *Supergravity Coupled to Chiral Matter at One Loop*, Phys. Rev. D49 (1994) 1951, [hep-th/9308090](http://arxiv.org/abs/hep-th/9308090); M. Gaillard, *Pauli-Villars Regularization of Supergravity Coupled to Chiral and Yang-Mills Matter*, Phys. Lett. B342 (1995) 125, [hep-th/9408149](http://arxiv.org/abs/hep-th/9408149); M. Gaillard, V. Jain, K. Saririan, *Supergravity Coupled to Chiral and Yang-Mills Matter at One Loop*, Phys. Lett. B387 (1996) 520, [hep-th/9606135](http://arxiv.org/abs/hep-th/9606135).

[24] M. Gaillard and B. Nelson, *On Quadratic Divergences in Supergravity, Vacuum Energy and the Supersymmetric Flavor Problem*, Nucl. Phys. B751 (2006) 75, [hep-ph/0511234](http://arxiv.org/abs/hep-ph/0511234).

[25] J. Jeans, Philos. Trans. Rev. Soc. London A199 (1902) 491.

[26] D. Gross, M. Perry and L. Yaffe, *Instability of Flat Space at Finite Temperature*, Phys. Rev. D25 (1982) 330.

[27] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Yang-Mills Theories with Local Supersymmetry: Lagrangian, Transformation Laws and SuperHiggs Effect*, Nucl. Phys. B212 (1983) 413.

[28] R. Kallosh, L. Kofman, A. Linde and A. Van Proeyen, *Superconformal Symmetry, Supergravity and Cosmology*, Class. Quant. Grav. 17 (2000) 4269, [hep-th/0006179](http://arxiv.org/abs/hep-th/0006179).

[29] E. Weinberg and A. Wu, *Understanding Complex Perturbative Effective Potentials*, Phys. Rev. D36 (1987) 2474.

[30] R. Kallosh and A. Linde, *O'KKLT*, [hep-th/0611183](http://arxiv.org/abs/hep-th/0611183).