A Consistent Way of Analysis of Hadrons Measured in a Finite Rapidity Interval in Relativistic Heavy-ion Collisions

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A new iteration method is proposed for analyzing both the multiplicities and the transverse momentum spectra measured only within a small rapidity interval and the low momentum limit without any inconsistency or ad hoc assumptions and applied to the hadron data by the ALICE collaboration in Pb+Pb collisions at 2.76 TeV/A. In order to correctly consider the resonance contribution only to the small rapidity interval, ratios involving only those hadrons whose transverse momentum spectrum is available are considered. In spite of the small number of ratios considered, the fitting of both the ratios and the transverse momentum spectra are excellent. Also the calculated ratios involving strange baryons with the parameters obtained agree with data surprisingly well.

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I. INTRODUCTION

Statistical models[1–6] are known to fit various hadron multiplicities measured in relativistic heavy-ion collisions very well with a few parameters such as the temperature, the baryon and strangeness chemical potentials, and interpreted the success of the fitting as the evidence of the chemical equilibration of the quark-gluon plasma formed during the collision. Also the so-called blast-wave model[7–10] is very successful in fitting the transverse momentum spectra of various hadrons. The resulting parameters, especially the temperature from the chemical analysis, $T_{ch}$, is different from that obtained in the thermal analysis of the momentum spectra, $T_{th}$.

Recently numerical viscous hydrodynamic calculations combined with processes called as hadronic after-burner using UrQMD[11, 12] or hadronic cascade[13] are developed and they are very successful in reproducing the hadron multiplicities, transverse momentum spectra, the rapidity distribution, and even the elliptic coefficients of various hadrons. At a certain temperature called as the switching temperature, $T_{ch}$ hadrons are generated by Monte Carlo simulation from each fluid cells. The hadrons thus generated undergo secondary collisions and decays via UrQMD or hadronic cascade. The hadronic after-burner, in some sense, takes care of the difference in formation momenta from that obtained in the thermal analysis of the momentum spectra , $T_{th}$.

It is assumed that during heavy-ion collisions a hot and dense quark-gluon plasma is formed and expands hydrodynamically. As the temperature decreases below the critical temperature hadronization occurs and the hadronic matter further expands. At $T_{ch}$, chemical freeze-out occurs and the chemical composition of hadrons does not change below $T_{ch}$. The system further expands radially and longitudinally and cools down maintaining thermal equilibrium until the thermal freeze-out at $T_{th}$. In between $T_{ch}$ and $T_{th}$, it is assumed that all the numbers of hadrons are kept fixed, except for those involved in a process with resonances with very large cross-section such as $\omega$ and $K^*$ mesons or $\Delta$ baryons. At thermal freeze-out at $T_{th}$ the system expands longitudinally with surface rapidity $\eta_{max}$ and transversally with surface rapidity $\rho_0$, whose values determine the transverse and rapidity spectra of various hadrons.

Below a blast-wave model recently developed[16] in order to implement the two different freeze-outs is summarized because the equations in it will be used. Basically it is same as the model by Dobler et al.[10] except the treatment of chemical potentials. Cylindrical geometry of the firewall is assumed in this model and the equation for transverse mass spectra can be written as

$$\frac{dN^{th}}{m_T dm_T} = \frac{d_4 V_{eff}}{(2\pi)^2} \int_{y_m}^{y_m} dy \int_{\eta_{max}}^{\eta_{max}} d\eta \int_0^R rdrm_T \cosh(y - \eta) \exp \left( -\frac{m_T \cosh(y - \eta) \cosh \rho - \mu_i}{T} \right) I_0 \left( \frac{p_T \sinh \rho}{T} \right)$$

Assumptions are that the longitudinal rapidity scales linearly with the distance from the center and also the linear rapidity profile is assumed for the transverse rapidity: $\rho(r) = \rho_0(r/R)$ at the radial distance $r$ from the center where $R$
is the radius to the surface. By changing the integration variable from \( r \) to \( r/R \), the radius \( R \) becomes a multiplication factor and can be absorbed into \( V_{\text{eff}} \).

Together with the thermal hadrons, those from the decay of resonances with higher mass should be added. The program to calculate \( dN_{\text{ch}}^{\text{res}}/m_T dm_T dy \) including 2 and 3 body decays has been written by Sollfrank[9] and used in our calculation. To get the number of hadrons from the resonance contribution, integration over \( y \) is needed. It should be emphasized that integrating only over the measured range of \( y \) takes care of the measured small rapidity window.

Adding the contribution from resonance decay, one gets an equation for the total number of hadrons.

\[
N_i = \int_{y_m}^{y_{\text{m}}} dy \int m_T dm_T \frac{d^2N_i^{\text{th}} + d^2N_i^{\text{res}}}{m_TM_T dm_T dy}
\]

The number of thermal hadrons is obtained from Eq. spec by integrating over \( m_T dm_T \). Since the number of hadrons is Lorentz invariant, the result should be the same as using the thermal distribution function without any Lorentz boost is used, which is essentially a statistical model.

When the system reaches chemical freeze-out earlier at \( T_{ch} \), numbers of hadrons are controlled by the two parameters, namely the baryon and the strangeness chemical potentials, \( \mu_B \) and \( \mu_S \) so that chemical potential for \( i \)-th particle \( \mu_i \) is the algebraic sum of the two,

\[
\mu_i = (n_q - n_{\bar{q}})\mu_B/3 + (n_s - n_{\bar{s}})\mu_S.
\]

Below \( T_{ch} \), numbers of each hadron species are kept fixed if small change of hadron numbers involved in reactions with large cross sections such as \( \rho \rightleftharpoons \pi + \pi \) is neglected, which is called as complete chemical freeze-out(CF) compared to the case when it is included as in the partial chemical equilibrium(PCE)[19, 20]. Here CF is assumed for convenience of calculation. When the system reaches thermal freeze-out at \( T_{th} \), chemical potential for hadron species, \( i \), should be determined such that it gives the right number of the hadrons, \( N_i \), which is already known at \( T_{ch} \). Thus

\[
\mu_i = T \ln[N_i] \int (d^2N_i^{\text{ch}}/m_T dm_T dy)]
\]

where the ′ denotes that \( \exp (\mu_i/T) \) is absent in this equation compared to Eq. (1).

Given the above equations, now one can describe the procedures of new method of both the chemical and thermal analysis and the method has been applied to the hadron data by ALICE collaboration[17, 18] in Pb+Pb collisions at 2.76 TeV/N in the rapidity windows \(-0.5 < y < 0.5\).

### III. A NEW METHOD OF CHEMICAL AND THERMAL ANALYSIS AND RESULTS

The iteration procedures of both the chemical and thermal analysis are as follows:

1. Assume any reasonable values for chemical freeze-out parameters such as \( T_{ch}, \mu_B, \mu_S \) and the overall constant and calculate all the particle numbers including that of resonances and store them.
2. Using the stored number for each hadron species, calculated the chemical potential from Eq. (4) and fit the transverse momentum spectrum with Eq. (1) to find \( T_{th}, \rho_0 \) and the overall constant. Transverse momentum spectra of \( \pi^+, \pi^-, K^+, K^-, p \) and \( \bar{p} \) measured by ALICE collaboration[17] in Pb+Pb collisions at 2.76 TeV/N in the rapidity windows \(-0.5 < y < 0.5\) has been analysed using the blast-wave model described in the previous section. The transverse momentum spectra are insensitive to the longitudinal rapidity at the surface, \( \eta_{\text{max}} \), while it is very sensitive to the width of the \( y \) distribution. Thus from the fitted values for the thermal analysis, the rapidity distribution of the total charged hadrons by ALICE collaboration[18] is used to give a value for \( \eta_{\text{max}} \).
3. From the fitted parameters at thermal freeze-out, namely \( T_{th}, \rho_0, \eta_{\text{max}} \) and an overall constant togther with \( \mu_i \)'s, numbers of thermal hadrons and decayed ones from resonances in the rapidity window \(-0.5 < y < 0.5\) can be calculated separately by integrating over \( m_T \).
4. Only after the calculation of the number of the secondary hadrons from resonance decays in the appropriate rapidity window, chemical analysis is ready to be done. Now the number of thermal hadrons in the small rapidity interval is obtained by subtracting the resonance decayed ones from the measured value. Chemical analysis can be done only for the thermal hadrons, namely for pions, kaons, protons and anti-protons. By integrating Eq. (1) over \( m_{T,0} < m_T \), one gets equation for thermal hadrons to fit and it is important to use Eq. (3) for the chemical analysis. In this way one gets parameters at chemical freeze-out, \( T_{ch}, \mu_B \) and \( \mu_S \) and overall constant, if needed. If one wants to fit ratios, the corresponding equation can be obtained easily from Eq. (1).
5. Iterating the steps from (1) to (4) until the parameters converge, one fits the ratios, transverse momentum spectra, and the rapidity distribution of charged hadrons consistently in a single blast-wave model.
TABLE I. Chemical(C.F.) and Thermal freeze-out(T.F.) parameters fitted to data by ALICE Collaboration. T and $\mu$’s are in MeV.

|       | T     | $\mu_B$ | $\mu_S$ | $\rho_0$ | $\eta_{max}$ | $\chi^2/N$ |
|-------|-------|---------|---------|----------|--------------|-------------|
| C.F.  | 150.7 | 0.37    | 0.15    | 0.9      |              | 0.9         |
| T.F.  | 112.9 |         |         |          |              |             |

TABLE II. The lower and upper limits of the transverse momentum for each hadron species measured by ALICE Collaboration.

|       |       |       |       |       |       |       |
|-------|-------|-------|-------|-------|-------|-------|
| $\pi$ | 0.1/3.0 |       |       |       |       |       |
| $K$   | 0.2/3.0 |       |       |       |       |       |
| $p$   | 0.35/4.5 | 0.6/8.0 | 1.2/7.0 |       |       |       |
| $\Xi$ |       |       |       |       |       |       |
| $\Omega$ |       |       |       |       |       |       |

Using the Lorentz boosted thermal distribution in step (4) for the chemical analysis makes it possible to consider any rapidity windows.

The hadron data by ALICE collaboration in Pb+Pb at 2.76 TeV/n have been analyzed following the steps described above and the resulting parameters are tabulated in Table I. The fit of ratios in shown in Fig. 1. Only the 4 ratios from the left are fitted and the ratios of strange baryons to pions are just calculations using the parameters obtained, but not fitted ones. However, in order to show the agreement, $\chi^2/N$ is calculated for all the ratios in the figure, which is 0.9. Here there is no discrepancy for protons and no need for the strangeness fugacity. The fitted transverse mass spectra of measured hadrons are drawn together with ALICE data up to $m_T = 3$ GeV. Except for small discrepancy in large $m_T$ region of protons and anti-protons, the agreement is very good. Whether the discrepancy at large $m_T$ region of protons and anti-protons is physical or not needs more work in fine tuning the parameters.

IV. SUMMARY

A new method of analysing both the ratios and the transverse momentum spectra of measured hadrons in a narrow rapidity window and $p_T, 0 < p_T$. The method presented is at least consistent in dealing with the finite rapidity interval and also only those hadrons decayed from resonances into the right rapidity window have been taken into account in the chemical analysis. Thus even in the ratio fitting, only those hadrons whose transverse momentum spectra are available can be fitted, which is only a few. The result of our fitting is extremely good, and other ratios thus calculated but not fitted agree with data surprisingly well. The thermal fit agrees with data up to $m_T = 3$ GeV. Thus the picture of the two freeze-outs is in agreement with data. A little more work is needed to find whether the discrepancy of proton and anti-proton transverse mass spectra at $m_T > 3$ GeV can be removed by fine tuning of parameters or it is physical effect.

![Graph](image-url)  

FIG. 1. Ratios among hadrons by ALICE collaboration[17] in Pb+Pb collisions at 2.76 TeV/N and result of fitting. It should be emphasized that only ratios involving pions, kaons, protons and anti-protons are fitted. Other ratios are calculation from the resulting parameters.
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