Neutrino mass due to the neutrino-gaugino mixing

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Abstract
We study a possibility to explain neutrino masses and mixings based on supersymmetry. If we introduce a flavor diagonal but generation dependent extra U(1) gauge interaction at a TeV region, we can obtain masses and mixings of neutrinos required for the explanation of both solar and atmospheric neutrinos. In this model, differently from the usual bilinear $R$-parity violating scenario, the neutrino mass degeneracy can be resolved at a tree level by the neutrino-gaugino mixing caused by an $R$-parity violation. The model is straightforwardly extended to include the quark sector by introducing an anomalous U(1) which can be used for the Froggatt-Nielsen mechanism.

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1 Introduction

Recent observations of the solar neutrinos [1] and the atmospheric neutrinos [2] at Super Kamiokande suggest that neutrinos have small masses and there are large flavor mixings in the lepton sector. These features are quite different from the ones of the quark sector and it may be a clue to the development of a unification picture of quarks and leptons for us to ask “what is the origin of these differences?”.

The mixing in the lepton sector is represented by the so-called MNS matrix which is defined by $V_{\text{MNS}} = U^\dagger_{\ell} U_{\nu}$, where $U_{\ell}$ and $U_{\nu}$ are the mixing matrices for charged leptons and neutrinos, respectively. Thus the origin of large mixings in the lepton sector exists in either $U_{\ell}$ or $U_{\nu}$. A famous example of possible mass matrices to realize large mixings is a democratic form, which takes the form such as

$$m = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (1)$$

However, this matrix has a serious problem to apply it to the neutrino sector. It has only a nonzero eigenvalue and then both the solar and atmospheric neutrino problems cannot be explained. It needs a suitable deviation from the exact democratic form to overcome this fault. It may be worthy to study how to generate an appropriately deviated form from it based on a certain physical principle. We study this problem on the basis of supersymmetry and an extra $U(1)$ symmetry in a TeV region. The former one is related to the gauge hierarchy problem and the latter one often appears in the effective theory of superstring. So this kind of study is considered to be sufficiently motivated.

In the supersymmetric theory there generally exists a discrete symmetry called $R$-parity, which is defined by $R_p = (-1)^{3B+L+2S}$ where $B$ and $L$ are a baryon number and a lepton number, respectively and $S$ stands for a spin. Under this symmetry particles in the standard model (SM) have $R_p = +1$ and their superpartners have $R_p = -1$. Thus the SM particles cannot mix with their superpartners without an $R$-parity violation. In the minimal supersymmetric SM (MSSM) there are neutral fermions with $R_p = -1$ in addition to the neutrinos. They are called neutralinos and contain $(\lambda_W, \lambda_Y, \tilde{H}_0^1, \tilde{H}_2^0)$. $\lambda_W$ and $\lambda_Y$ are gauginos for $\text{SU}(2)_L \times U(1)_Y$ gauge symmetry and $\tilde{H}_1^0$ and $\tilde{H}_2^0$ stand for Higgsinos. If $R_p$ is not broken, neutrinos and neutralinos cannot mix. However, if there are $R_p$ violations such as an explicit breaking due to the bilinear $R$-parity violating terms $\epsilon_a L_a H_2$ in superpotential or a spontaneous breaking due to the nonzero vacuum expectation values (VEVs) of sneutrinos, mass mixings $M$ among neutrinos and neutralinos appear in the form such as

$$M(\nu, \nu, \nu) = \begin{pmatrix} \sqrt{2}g_2 \langle \tilde{\nu}_e \rangle & \sqrt{2}g_1 \langle \tilde{\nu}_e \rangle & \epsilon_e \\ \sqrt{2}g_2 \langle \tilde{\nu}_\mu \rangle & \sqrt{2}g_1 \langle \tilde{\nu}_\mu \rangle & \epsilon_\mu \\ \sqrt{2}g_2 \langle \tilde{\nu}_\tau \rangle & \sqrt{2}g_1 \langle \tilde{\nu}_\tau \rangle & \epsilon_\tau \end{pmatrix} \begin{pmatrix} -i\lambda_W \\ -i\lambda_Y \\ \tilde{H}_2^0 \end{pmatrix}. \quad (2)$$

A study of the scalar potential indicates that $\langle \tilde{\nu}_a \rangle$ is proportional to $\epsilon_a$. As a result of this feature, all column vectors of $M$ are proportional to each other. Under the assumption that gaugino masses $M_1, M_2$ and a supersymmetric Higgs mass $\mu$ are much larger than
and $\epsilon_\alpha$, we find that the neutrino mass matrix satisfies $M^\nu_{ij} \propto \langle \tilde{\nu}_i \rangle \langle \tilde{\nu}_j \rangle$ as a result of the seesaw mechanism. Unfortunately, the light neutrino mass matrix $M^\nu_{ij}$ obtained in this way has only one nonzero mass eigenvalue. For the explanation of the atmospheric and solar neutrinos, we need a mass perturbation to resolve this mass degeneracy. A well-known example of such possibilities is an inclusion of one-loop effects and several works in this direction have been done by now.

In this talk we would like to propose another possibility. We consider a tree level solution to this problem and study whether we can obtain large flavor mixings and also appropriate mass eigenvalues in the lepton sector in such a scenario.

2 A simple Model

We introduce an extra U(1) gauge symmetry to the MSSM, which is assumed to remain unbroken at a TeV region. This kind of extra U(1) has several interesting aspects to take it seriously. In fact, such a kind of symmetry often appears in the effective theory of superstring and it can also present a natural solution to the $\mu$-problem. So there is a certain physical motivation to consider such a symmetry in the model building. It should be noted that there are experimental constraints on the extra U(1) from the precision measurements at LEP. Its mass should be generally larger than 600 GeV and its mixing angle $\xi$ with the ordinary $Z$ gauge boson has to be less than $10^{-3}$.

In the present consideration we assume that the extra U(1) has flavor diagonal but generation dependent interactions such as

$$L = \bar{\nu}_a i \gamma^\mu (\partial_\mu - i g X q_a A_\mu) \nu_\alpha + i \sqrt{2} g_X (\tilde{\nu}_a^* \lambda_X q_a \nu_\alpha - \tilde{\nu}_a \bar{\lambda}_X q_a \nu_\alpha) + \cdots. \quad (3)$$

If sneutrinos get nonzero vacuum expectation values $\langle \tilde{\nu}_\alpha \rangle \neq 0$ in eq. (3), neutrino-gaugino mixing appears through $\sqrt{2} g_X q_a \langle \tilde{\nu}_\alpha \rangle$, which can break the previously mentioned proportional relation among column vectors of the extended neutrino-neutralino mixing matrix corresponding to (2). As a result, we can have two nonzero mass eigenvalues of neutrinos at the tree level. In the following discussion we study neutrino oscillation phenomena and other phenomenological features of this model.

We take charge assignments of this extra U(1) gauge symmetry for each generation of the leptons as follows,

$$\ell_{L\alpha} : (\ell_I, q_I, q_{\tilde{W}}), \quad \tilde{\ell}_{R\alpha} : (-\ell_I, -q_I, -q_{\tilde{W}}), \quad (4)$$

where $\ell_{L\alpha}$ and $\tilde{\ell}_{R\alpha}$ stand for SU(2)$_L$ doublet fields and its singlet charged ones, respectively. We assume Higgs chiral superfields have no charge of the extra U(1). At this stage we do not identify the lepton flavor. If we assume that sneutrinos get VEVs such as $\langle \tilde{\nu}_e \rangle = \langle \tilde{\nu}_\mu \rangle = \langle \tilde{\nu}_\tau \rangle \equiv u \neq 0$, we have a neutrino-gaugino mass matrix

$$M = \begin{pmatrix} 0 & M^T \\ m & M \end{pmatrix}, \quad (5)$$

$$m = \begin{pmatrix} a_2 & a_1 & b \\ a_2 & a_1 & b \\ a_2 & a_1 & c \end{pmatrix}, \quad M = \begin{pmatrix} M_2 & 0 & 0 \\ 0 & M_1 & 0 \\ 0 & 0 & M_X \end{pmatrix}. \quad (6)$$

*In this discussion we can safely ignore the contributions from the mixings with Higgsinos.
Table 1 Contributions to $\nu_\alpha \rightarrow \nu_\beta$.

$$
\begin{array}{ccc}
(\alpha, \beta) & (i, j) & -4U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}(\equiv A) \\
(\text{I, II}) & (1, 2) & \cos^2 \theta \\
 & (1, 3) & \sin^2 \theta \\
 & (2, 3) & -\sin^2 \theta \cos^2 \theta \\
(\text{I, III}) & (2, 3) & 2\sin^2 \theta \cos^2 \theta \\
(\text{II, III}) & (2, 3) & 2\sin^2 \theta \cos^2 \theta \\
\end{array}
$$

where $a_\ell = \frac{1}{\sqrt{2}}g_\ell u$, $b = \sqrt{2}g_\chi q_1 u$ and $c = \sqrt{2}g_\chi q_3 u$. If elements of $m$ are much smaller than the ones of $M$, we can use the seesaw formula and then the neutrino mass matrix is found to be written as

$$
M^\nu = m^T M^{-1} m = \begin{pmatrix}
    m_0 + \epsilon^2 & m_0 + \epsilon^2 & m_0 + \epsilon \delta \\
    m_0 + \epsilon^2 & m_0 + \epsilon^2 & m_0 + \epsilon \delta \\
    m_0 + \epsilon \delta & m_0 + \epsilon \delta & m_0 + \delta^2
\end{pmatrix},
$$

(7)

where $m_0 = \frac{g^2 u^2}{2M^2}$, $\epsilon = \frac{g_\chi q_1 u}{\sqrt{\nu}}$ and $\delta = \frac{g_\chi q_3 u}{\mu}$. A diagonalization matrix $U$ of $M^\nu$ which is defined by $U^\dagger M^\nu U = M_{\text{diag}}$ is easily found to be

$$
U = \begin{pmatrix}
    \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta \\
    -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta \\
    0 & -\sin \theta & \cos \theta
\end{pmatrix}.
$$

(8)

The mass eigenvalues and the mixing angle $\theta$ are written as

$$
m_1 = 0, \quad m_{2,3} = \frac{1}{2} \left[ 3m_0 + 2\epsilon^2 + \delta^2 \mp \sqrt{(m_0 + 2\epsilon^2 - \delta^2)^2 + 8(m_0 + \epsilon \delta)^2} \right],
$$

(9)

$$
\sin^2 2\theta = \frac{8(m_0 + \epsilon \delta)^2}{(m_0 + 2\epsilon^2 - \delta^2)^2 + 8(m_0 + \epsilon \delta)^2}.
$$

(10)

If we assume that the charged lepton mass matrix is diagonal, a transition probability for the neutrino oscillation $\nu_\alpha \rightarrow \nu_\beta$ can be written by using the elements of the mixing matrix $U$ as

$$
\mathcal{P}_{\nu_\alpha \rightarrow \nu_\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j} \sin^2 \left( \frac{\Delta m^2_{ij}}{4E} L \right),
$$

(11)

where $\Delta m^2_{ij} = |m_i^2 - m_j^2|$. We summarize the contributions of the possible modes to the neutrino oscillation $\nu_\alpha \rightarrow \nu_\beta$ in Table 1. Here we assume that an inverse hierarchy and also an approximate degeneracy between $\nu_2$ and $\nu_3$: $m_1 \ll m_2 \sim m_3$. In that case we find that the atmospheric neutrino requires

$$
2 \times 10^{-3} \text{eV}^2 \lesssim \Delta m^2_{12} \simeq \Delta m^2_{13} \lesssim 6 \times 10^{-3} \text{eV}^2.
$$

\(^1\) We can consider the same type texture of neutrino mass matrix in the ordinary seesaw framework with right-handed neutrinos by introducing suitable symmetries \[^{[10]}\].
The solar neutrino is also found to require that $\Delta m_{23}^2$ should take a value in a solution dependent way in the region such as

$$10^{-10} \text{eV}^2 \lesssim \Delta m_{23}^2 \lesssim 1.5 \times 10^{-4} \text{eV}^2.$$  

Taking account of this, we find that we should identify the flavor $(I, II, III)$ with $(\tau, \mu, e)$. Then we can rewrite eq. (8) into the usual MNS mixing matrix as

$$U_{\text{MNS}} = \begin{pmatrix} 0 & -\sin \theta & \cos \theta \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} \sin \theta \end{pmatrix}. \quad (12)$$

The atmospheric neutrino is explained by $\nu_\mu \to \nu_\tau$ which comes from the modes (A) and (B) in Table 1. On the other hand, the solar neutrino is explained by $\nu_e \to \nu_\mu$ (E) and $\nu_e \to \nu_\tau$ (D) whose total amplitude is $A = \sin^2 2\theta$. Thus if we assume $\sin^2 2\theta \sim 10^{-2}$, the small mixing MSW solution (SMA) is realized. The large mixing MSW solution (LMA) and others (LOW and VO) are possible for the case of $\sin^2 2\theta \sim 1$.

We order several aspects of other neutrino phenomenology in this model here.

(1) CHOOZ constraint [11] : It is related to the neutrino oscillations $\nu_e \to \nu_{\mu, \tau}$ with $\Delta m_{12}^2$ and $\Delta m_{13}^2$ which are relevant to the atmospheric neutrinos. Their amplitudes are proportional to $U_{\text{e1}}^\text{MNS}$ in this model. However, it is zero and then the CHOOZ constraint is trivially satisfied.

(2) Neutrinoless double $\beta$-decay [12] : The effective mass parameter relevant to the neutrinoless double $\beta$ decay can be estimated as

$$|m_{ee}| = \left| \sum_j |U_{ej}|^2 e^{i\phi_j} m_j \right| = m_2 \sin^2 \theta + m_3 \cos^2 \theta \sim m_3, \quad (13)$$
where we assume $\phi_j = 0$ and use the fact $m_2 \sim m_3$. Then we have $|m_{ee}| \sim 0.04 - 0.08$ eV independently of the value of $\sin \theta$. This value seems to be a promising one for near future experiments.

(3) Effects of the mode (C): This corresponds to the oscillation $\nu_\mu \rightarrow \nu_\tau$, which is irrelevant to the atmospheric neutrino and also the short baseline experiments because of the smallness of $\Delta m_{23}^2$. However, if $\sin^2 2\theta \simeq 1$ and $\Delta m_{23}^2 \sim 10^{-4}$ eV$^2$ which correspond to the LMA solution are satisfied, it can effectively contribute to the deviation from the simple two flavor oscillation scenario at $L \gtrsim 2000$ km.

3 Various issues of the model

In this section we will briefly comment on the issues which are crucial for the model to be a phenomenologically viable model.

(1) Realization of oscillation parameters
We have $M_A, g_A$ ($A = 2, 1, X$), $g_\alpha$ and $u$ as the parameters in our model. If we assume a gauge coupling unification and a universal gaugino mass $M_0$ at $M_{GUT}$, gaugino masses at a low energy region are determined by renormalization group equations as

$$M_2(\mu) = \frac{M_0}{g_U^2}g_2^2(\mu), \quad M_{1,X}(\mu) = \frac{M_0}{g_U^2}g_{1,X}^2(\mu).$$

(14)

Under this assumption, the number of parameters can be effectively reduced and we can take $q_I, q_{III}$ and $\frac{g_{1}}{M_0} u^2$ as the remaining parameters. Every oscillation parameter can be written down by using them. Taking account of this fact, we find that the atmospheric neutrinos require $0.017$ eV $\lesssim \frac{g_{1}}{M_0} u^2 \lesssim 0.023$ eV. This means that $u$ should be $60 - 70$ keV if we take $M_0 \sim 100$ TeV and $g_U \sim 0.72$, for example. The solar neutrinos impose $(q_I, q_{III})$ to be in a certain region, which is shown in Fig.1. From this figure we find that the LMA solution is allowed in the wider region than other solutions. Here it is a crucial problem whether the gauge symmetry including $U(1)_X$ can be anomaly free in a consistent way with the above required region of $(q_I, q_{III})$. On this point, we find that the model can be consistent, although we need to introduce additional chiral superfields such as $2_0, 1_{\pm 1}$ and $4(1_0)$ of $SU(2)_L \times U(1)_Y$, for example. If we impose the anomaly free conditions and also the massive conditions for the chiral superfields $2_0, 1_{\pm 1}$ at the TeV region which seems to be required from experiments, $q_I$ and $q_{III}$ are found to satisfy a certain relation as shown in Fig.1. From this analysis we find that this kind of model can be easily constructed in a consistent way.

(2) $Z'$ nonuniversal coupling and FCNC
We assume that $U(1)_X$ has the flavor diagonal but generation dependent interactions. In general, such kind of interactions can induce dangerous FCNC processes, for example, a coherent $\mu-e$ conversion, $\tau \rightarrow 3\nu, 3\mu, \mu \rightarrow e\gamma$ and so on. The additional new $Z'$ interactions can be written in mass eigenstates as

$$\mathcal{L}_{Z'} = -g_1 \left( \frac{g_X}{g_1} \cos \xi J^{\mu}_{(X)} - \sin \xi J^{\mu}_{(I)} \right) Z'_\mu, \quad (15)$$

$$J^{\mu}_{(X)} = \sum_{i,j} \left( \tilde{\nu}_{Li} B^{\mu}_{ij} \nu_{Lj} + \tilde{\ell}_{Li} B^{\mu}_{ij} \ell_{Lj} + \tilde{\ell}_{Ri} B^{\mu}_{ij} \ell_{Rj} \right), \quad (16)$$

6
\[ B_{ij}^\psi = V^{\psi \dagger} \text{diag}(q_{\text{III}}, q_{\ell}, q_{\ell}) V^{\psi}, \]

where \( \ell_L \) and \( \ell_R \) stand for the charged leptons. \( V^\psi \) is a diagonalization matrix of the charged lepton mass matrix and it is defined as

\[ V^{\psi \dagger} M_\psi V^\psi = \text{diagonal}, \quad M_\psi = \left\{ \begin{array}{ll} m_D^\dagger m_D & (\psi = \ell_R) \\ m_D m_D^\dagger & (\psi = \ell_L) \end{array} \right. \]

where \( m_D \) is a Dirac mass matrix defined by \( \bar{\ell}_L m_D \ell_R \). Because of the U(1)_X constraint, we find that \( B_{ij}^{\ell_L, \ell_R} \propto \delta_{ij} \). So nonuniversal coupling of U(1)_X induces no serious problem in this model. If the usual conditions \( m_{\sigma'} \gtrsim 600 \text{ GeV} \) and \( \xi \lesssim 10^{-3} \) are satisfied, no contradiction to the present experimental data appears.

(3) Sneutrino VEVs
Small but nonzero VEVs of sneutrinos \( \langle \tilde{\nu}_\alpha \rangle \) are crucial in this model. If bilinear \( R \)-parity violating terms \( \epsilon L_\alpha H_2 \) exist in the superpotential, the minimization of the scalar potential results in

\[ u \sim \epsilon \frac{\mu \langle H_1 \rangle + B_\epsilon \langle H_2 \rangle}{\bar{m}^2}, \]

where \( B_\epsilon \) is a soft breaking parameter for \( \epsilon L_\alpha H_2 \) and \( \bar{m}^2 \) is an averaged value of the squared soft scalar masses of Higgs scalars. This implies that \( u \) can be very small for the small \( \epsilon \) satisfying \( \epsilon \ll \mu, m, B_\epsilon \). The problem is how we can explain the smallness of \( \epsilon \). Several possible solutions can be considered as in the case of \( \mu \)-term \[14\]. (i) Giudice-Masiero mechanism : If Kähler potential includes a term \( ZL_\alpha H_2 \) and the supersymmetry is broken by an \( F \)-term of the chiral superfield \( Z \), for example, we have a small one such as \( \epsilon \sim \frac{F_{\text{in}}}{M_{\text{pl}}} \).

(ii) Higher dimensional term in the superpotential : We consider that the superpotential includes a nonrenormalizable term such as \( \frac{\tilde{N} \tilde{N}}{M_{\text{pl}}^2} \bar{S}L_\alpha H_2 \). If scalar components of \( \tilde{N} \) and \( N \) get the VEVs of the intermediate scale through a \( D \)-flat direction and a scalar component of \( S \) gets a TeV scale VEV, for example, we can have \( \epsilon \sim \frac{\langle N \rangle^2}{M_{\text{pl}}^2} \langle S \rangle \) which can be sufficiently small. (iii) Coupling to the superconformal sector : If the MSSM fields couple to the superconformal sector, they get large anomalous dimensions \[13\]. This can be applied to the explanation of the hierarchy among Yukawa couplings and the universality of soft scalar masses \[13\], \[19\]. This idea is also applicable to the explanation of the smallness of \( \epsilon \). For example, we consider superpotentials \( W_{\text{sc}} = y_1 \tilde{H}_1 \bar{T} S + y_2 \tilde{H}_2 \bar{T} \tilde{S} \) and \( W_{\mu} = \epsilon_\alpha L_\alpha \tilde{H}_2 + \mu_1 \tilde{H}_1 H_2 + \mu_2 H_1 \tilde{H}_2 \), where \( T, \tilde{T} \) and \( S, \tilde{S} \) are chiral superfields in the superconformal sector and \( \tilde{H}_{1,2} \) are the extended Higgs chiral superfields. As a result of the couplings with the superconformal sector, \( \tilde{H}_{1,2} \) can have a large anomalous dimension \( \gamma \) and the bilinear \( R \)-parity violating parameters \( \epsilon_\alpha \) behave as \( \epsilon_\alpha (\Lambda) = \epsilon_\alpha (\Lambda_0) \left( \frac{\Lambda}{\Lambda_0} \right)^\gamma \)[17]. We find that there is a large suppression in the case of \( \Lambda \ll \Lambda_0 \) even if \( \epsilon(\Lambda_0) \) is a weak scale. Anyway we have many candidates for the mechanism to realize the small bilinear \( R \)-parity violating terms and we can expect that they are applicable to our model.

4 An extension of the model
We consider here an extension of our simple model to include the quark sector. For this purpose we introduce two Abelian symmetries \( U(1)_F \times U(1)_X \)[18]. Although \( U(1)_X \)
This seems to have qualitatively nice features if we take \( \lambda \) the charged lepton mass matrix has the same form as \( F \) as the extra colored triplets to the SM fields, and also the existence of Higgs mixings such as gauge anomaly cancellation, the proton stability by prohibiting dangerous couplings to the following results. Here we impose several phenomenological conditions, that is, the couplings unification additionally to the above list of field contents. They have no affect to these requirements bring the constraints on the \( U(1) \) charges introduced above. We must study neutrino masses under these constraints. If \( \kappa_5 \langle S_5 \rangle / \kappa_4 \langle S_4 \rangle \) is equal to \(-\kappa_1 \langle S_1 \rangle / \kappa_2 \langle S_2 \rangle \) in eq. (22), we find that \( (H_2^1, H_2^2) \) \( \approx \sin \zeta H_1^1 + \cos \zeta H_2^2 \) can play a role of the usual Higgs fields, where the mixing angle \( \zeta \) is defined as \( \tan \zeta = \kappa_1 \langle S_1 \rangle / \kappa_2 \langle S_2 \rangle \).

Now we discuss the masses and the flavor mixings in the quark and lepton sectors. We introduce a pair of Higgs doublet chiral superfields only to guarantee the SM gauge couplings unification additionally to the above list of field contents. They have no affect to the following results. Here we impose several phenomenological conditions, that is, the gauge anomaly cancellation, the proton stability by prohibiting dangerous couplings of the extra colored triplets to the SM fields, and also the existence of Higgs mixings such as

\[
(H_2^1, H_2^2) \left( \begin{array}{cc} \kappa_1 \langle S_1 \rangle & \kappa_2 \langle S_2 \rangle \\ \kappa_4 \langle S_4 \rangle & \kappa_5 \langle S_5 \rangle \end{array} \right) \left( \begin{array}{c} H_1^1 \\ H_1^2 \end{array} \right). \]

(21)

The last condition comes from the requirement to make only one pair of Higgs doublets massless. These requirements bring the constraints on the \( U(1)_X \) charges introduced above. We must study neutrino masses under these constraints. If \( \kappa_5 \langle S_5 \rangle / \kappa_4 \langle S_4 \rangle \) is equal to \(-\kappa_1 \langle S_1 \rangle / \kappa_2 \langle S_2 \rangle \) in eq. (22), we find that \( (H_2^1, H_2^2) \approx \sin \zeta H_1^1 + \cos \zeta H_2^2 \) can play a role of the usual Higgs fields, where the mixing angle \( \zeta \) is defined as \( \tan \zeta = \kappa_1 \langle S_1 \rangle / \kappa_2 \langle S_2 \rangle \).

Now we discuss the masses and the flavor mixings in the quark and lepton sectors. We assume that \( U(1)_F \) is broken by the VEV of the scalar component of \( S_0 \) which takes a value nearly equal to \( M_{pl} \). In the quark sector \( U(1)_F \) controls the flavor mixing by regulating the number of \( S_0 \) contained in each nonrenormalizable terms through the so-called Froggatt-Nielsen mechanism \( \equiv \frac{S_0}{M_{pl}} \). If we introduce a parameter \( \lambda \equiv \langle S_0 \rangle / M_{pl} \), the quark mass matrix can be written as

\[
M_u \sim \left( \begin{array}{ccc} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{array} \right) (H_2^1), \quad M_d \sim \left( \begin{array}{ccc} \lambda^4 \sin \zeta & \lambda^3 \sin \zeta & \lambda \sin \zeta \\ \lambda^3 \sin \zeta & \lambda^2 \sin \zeta & \sin \zeta \\ \lambda^2 \cos \zeta & \lambda^2 \cos \zeta & \cos \zeta \end{array} \right) (H_2^2). \]

(22)

From these mass matrices we can obtain the mass eigenvalues and the CKM mixing as follows,

\[
m_u : m_c : m_t = \lambda^6 : \lambda^4 : 1, \\
m_d : m_s : m_b = \lambda^4 \sin \zeta : \lambda^2 \cos \zeta : \cos \zeta, \\
V_{us} \sim \lambda, \quad V_{ub} \sim \lambda^3, \quad V_{cb} \sim \lambda^2. \]

(23)

This seems to have qualitatively nice features if we take \( \lambda \sim 0.22 \). In the lepton sector the charged lepton mass matrix has the same form as \( M_d^T \) because of the \( SU(5) \) relation.
and then we obtain
\[ m_e : m_\mu : m_\tau = \lambda^4 \sin\zeta : \lambda^2 \cos\zeta : \cos\zeta. \] (24)

A diagonalization matrix \( \tilde{U}_\ell \) of the charged lepton mass matrix is found to be
\[
\tilde{U}_\ell = \begin{pmatrix}
1 & 0 & \lambda \sin\zeta \\
-\lambda \sin^2\zeta & \cos\zeta & \sin\zeta \\
-\lambda \sin\zeta \cos\zeta & -\sin\zeta & \cos\zeta
\end{pmatrix}.
\] (25)

For the neutrino sector we can directly use the result in the previous simple model. Then, for example, if we take \( \lambda \sim 0.22 \), \( \cos\theta \sim 1 \) and \( \sin\zeta \sim \cos\zeta \sim \frac{1}{\sqrt{2}} \), we find that the solar and atmospheric neutrinos can be explained simultaneously and the LMA solution can be applied to the solar neutrino. The neutrino mass spectrum has a normal hierarchy in this case and the MNS-matrix can be written as
\[
U_{\text{MNS}} = \tilde{U}_\ell^\dagger U_\nu \sim \begin{pmatrix}
\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{\lambda}{\sqrt{2}} \\
\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
-\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{\lambda}{2}
\end{pmatrix}.
\] (26)

Although the CHOOZ constraint is satisfied \( [20] \), the maximal mixing is not realized for the solar neutrino and we have a rather large value of \( U_{e3}^{\text{MNS}} \) such as \( -\frac{1}{2} \sim -0.11 \). This is an interesting prediction of this model. Neutrinoless double \( \beta \)-decay seems difficult to be found in the near future experiments since \( |M_{ee}| \sim \frac{1}{2} \sqrt{\Delta m^2_{\text{solar}}} + \frac{\lambda^2}{4} \sqrt{\Delta m^2_{\text{atm}}} \) is so small. On the model parameters, if we take \( M_0 \sim 100 \text{ TeV} \) and \( g_U \sim 0.72 \), \( u \) should be \( 0.76 - 1.6 \text{ MeV} \). The \( U(1)_X \) charges \( (q_1, q_2) \) should be in the region shown in Fig.2, in which we also draw the lines which represent a relation between \( q_1 \) and \( q_2 \) required by the
previously mentioned various constraints. Because of the flavor dependent interactions of $U(1)_X$, FCNC seems to give severe constraints on the model. However, if $m_{Z'} \gtrsim 100$ TeV and $\xi \lesssim 10^{-6}$ are satisfied, the present experimental bounds for the FCNC can be satisfied.

5 Summary

In this talk we have discussed that the supersymmetry and the neutrino masses can be closely related by considering the $R$-parity violation. If we introduce a generation dependent extra $U(1)$ gauge symmetry at a TeV region, the masses and mixings of neutrinos can be explained at a tree level due to the mixing among neutrinos and gauginos. Since the supersymmetry is a promising ingredient to solve the gauge hierarchy problem in the SM, it seems to be a very interesting possibility to explain the neutrino masses and mixings in relation to the supersymmetry.

Our model can be extended so as to include the quark sector straightforwardly. If we consider two Abelian flavor symmetries $U(1)_F \times U(1)_X$, the mass and mixing in both quark and lepton sectors seem to be successfully explained by both the Froggatt-Nielsen mechanism and the neutrino-gaugino mixing.

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