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Control of spacecraft waveguide dynamic behavior by means of support arrangement

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Abstract. The most important parameters that determine the dynamic behavior of the waveguide in orbit are the first natural vibration frequency and the critical compressive force at which the stability of the waveguide structure is lost. This paper deals with the selection of the type and location of the outermost and intermediate supports for a single waveguide between two microwave units to provide the required values of the first natural vibration frequency and critical compressive force. The most common support arrangement schemes and their corresponding coefficients for equations defining the first natural frequency and minimum critical compressive force are given. For the calculation, the analytical dependencies of beam vibration theory and stability theory are used, which allowed to obtain simple equations and to comprehensively assess the influence of various factors on the obtained results. The results can be used in the design of any extended structures to ensure their dynamic behavior by means of support arrangement.

1. Introduction

In mechanical engineering there are a large number of extended structures, which are subject to forced fluctuations and significant temperature change: pipelines, oil pipelines, steam pipelines, rails, waveguide, etc. For such structures, an important problem is the provision of their dynamic state, which is largely characterized by the first natural frequency of vibration and the minimum critical force [1-3]. Providing the first natural frequency of vibrations of the structure above the specific value serves as a condition of absence of resonance at the lower frequency, at which usually amplitude of vibrations and stresses reach maximum values. The minimum value of critical force is relevant mainly for thin-walled extended structures, in which buckling occurs even with relatively small compressive forces. In general, these conditions can be written as:

\[ f_{\text{min}} \geq \left[ f \right], \quad P_{\text{min}} \geq \left[ P_{cr} \right]. \]  \hspace{1cm} (1)

One such design is the waveguide of the spacecraft antenna-feeder system. Forced vibrations of the waveguide are experienced by spacecraft engines, so one of the most important parameters determining the dynamic behavior of the waveguide is the minimum value of the first natural frequency of vibrations. When the spacecraft is orbiting, the solar rays and attenuation of the transmitted microwave signal cause the waveguide to heat to +120°C, resulting in its thermal expansion [4]. In the event that the waveguide supports do not allow it to expand freely, especially along its longitudinal axis, this results in an equivalent compressive force \( P \) which can reach a critical value and cause loss of stability of the waveguide construction (figure 1).
Figure 1. Sun rays action on a waveguide and equivalent model.

One effective way to provide a minimum of natural frequency and critical force is to select the appropriate species and order of the edge and intermediate supports. Theoretical bases of calculation of vibrations and loss of stability are presented in the works of many scientists of the 19-20 century [5-12]. Their calculated constraints are most simple based on beam theory, which is applicable to extended thin-walled structures. On the basis of the obtained dependencies, various manuals have been developed in the future to help the ordinary calculation engineer to carry out the necessary calculations without using fundamental theoretical foundations. However, practically all existing literature is given from the point of view of solving problems of vibration theory and stability theory for different types of beam supports. In the engineer ‘s work there is usually the opposite task, to select such type of supports and to create from them such their location at which the first natural frequency of vibrations and critical compressive force do not exceed the specified permissible values. In order to solve this problem, this study has been completed. This paper addresses the issue of controlling the dynamic state of the waveguide construction, which is characterized by conditions (1), by selecting the type and order of the supports. The results obtained can be used for any extended structures experiencing forced heat vibrations and requiring the provision of minimum values of the first natural frequency of vibrations and critical compressive force.

2. Mathematical model and schemes of waveguide
Waveguide modeling be most accurate according to shell theory [13,14], but the mathematical dependencies obtained from it are complex and the solution requires the use of numerical methods. Therefore, we use the theory of beams, which is quite accurate at the length of the structure under 5-6 times more than the characteristic size of its cross-section [15-18]. We consider the waveguides only with such geometric dimensions, for which this condition is satisfied and we get the solutions of task (1) in a simple analytical form. According to the vibration theory [5-9], the dynamic state of the waveguide beam model at free vibrations is described by the differential equation:

$$EJ \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} = 0, \quad (2)$$

where: $w=(w,t)$ is the deflection of waveguide longitudinal axis; $E$ is the Young's modulus of the waveguide material; $J$ is the waveguide inertia moment; $m$ is the mass per unit length of the waveguide. According to the theory of vibrations, we use a decision function in the form:

$$w(x,t) = A \sin \left( \frac{x \pi}{l} \right) \cdot \sin (\omega t), \quad (3)$$

where: $A$ is vibration amplitude; $\omega$ is angular vibration frequency.

For solution of equation (2), it is necessary to set 4 boundary conditions which reflect conditions of waveguide supports. For example, the absence of deflections and bending moments in hinged supports can be written as:

$$w = w(x = 0, t) = w(x = l, t) = 0; \quad \frac{\partial^2 w(x = 0, t)}{\partial x^2} = \frac{\partial^2 w(x = l, t)}{\partial x^2} = 0; \quad (4)$$
After taking into account boundary conditions, we get a solution for angular frequency of oscillations converted to the value of frequency of oscillations:

\[ f = \frac{\omega}{2\pi}. \]  

(5)

For convenience, the expression of the first natural frequency of the waveguide is represented in the universal form [5-9,19,20]:

\[ f_1 = \frac{\alpha^2}{2\pi} \sqrt{\frac{E \cdot J}{m \cdot l^4}}, \]  

(6)

where \(\alpha\) is the coefficient of the type and location of the supports at vibrations; \(l\) is waveguide length. Thus, the effect of the method of fixing the waveguide on its first natural frequency only be taken into account by a dimensionless coefficient \(\alpha\). The expression for the first critical compressive force of the waveguide, according to studies [10-12], is also presented in universal form:

\[ P_1 = \frac{\pi E J}{\mu^2 \cdot l^2}, \]  

(7)

where \(\mu\) is the coefficient of the type and location of the supports at stability loss. Similarly, the effect of the method of the waveguide supports on its first critical compressive force only be taken into account by a dimensionless coefficient \(\mu\). The waveguides can be single-span and have no intermediate supports. However, in most cases, the waveguides connect two remote massive UHF-blocks, have a large length and several intermediate supports. Consider both cases of waveguide design (figures 2,3) with different types of supports.

![Figure 2. Single-span waveguide.](image)

![Figure 3. Multi-span waveguide.](image)

Using different versions of boundary conditions describing conditions of waveguide supports and substituting in them solution of equation (2) we obtain expressions for the first natural frequency of vibrations and critical force. Figures 4 and 5 show the possible types of supports and their corresponding coefficient values \(\alpha \text{ и } \mu\).

![Figure 4. Coefficients for single-span waveguide.](image)

![Figure 5. Coefficients for multi-span waveguide.](image)
Of particular interest is the dependence of the first eigenfrequency on the number of intermediate supports \( N \) for the multipath waveguide. Consider such cases in figure 6, where all supports are equidistant from each other.

![Figure 6. Multi-span waveguide with \( N \) equidistant intermediate supports.](image)

When equation (2) is solved, the values of coefficient \( \alpha \) for the first natural frequency of vibrations (figure 6) are obtained depending on the number of intermediate supports \( N \), given in table 1.

| Scheme No. | Number of intermediate support \( N \) |
|------------|-------------------------------------|
|            | 0 | 1 | 2 | 3 | 4 | 5 |
| 11         | 3.14 | 6.28 | 9.42 | 12.57 | 15.71 | 18.85 |
| 12         | 4.73 | 7.85 | 10.67 | 13.57 | 16.55 | 19.56 |

The considered ways of fixing and the found values of coefficients corresponding to them \( \alpha \) and \( \mu \) allow to carry out the assessment of the first own frequency of fluctuations and the critical squeezing force in the majority the cases of a design of supports of waveguides which are found in practice.

3. Results
For the purpose of calculation, we consider waveguide with typical cross-section size of 35x15x1.2 mm, inertia moment \( J=6.6\times10^{-9} \) m\(^4\), length \( l=0.5 \) m. The waveguide material is duralumin with properties as \( E=7.1\times10^5 \) MPa, density \( \rho=2770 \) kg/m\(^3\).

We calculate the first natural frequency and critical force for the waveguide with the specified dimensions and shown in figures 4,5 methods of supports. The results are presented in the form of graphs in figures 7 and 8, the number in the circle indicates the number of the calculation scheme.

![Figure 7. Dependence of the first natural frequency on the type of supports.](image)

![Figure 8. Dependency of the first critical force on the type of supports.](image)

Having set the numerical data depending on the multi-span waveguide (figure 6), we plot the value of the first natural frequency from the number of intermediate supports in figure 9. Also of practical
interest is the relationship showing how many times the first natural frequency of the waveguide change when \( N \) intermediate supports are added, it is shown in figure 10.

![Figure 9](image1.png) Figure 9. Dependence of the first natural frequency on the number of intermediate supports

![Figure 10](image2.png) Figure 10. Dependence of multiplier for the first natural frequency on the number of intermediate supports

The graphs in figures 7-10 are based on points showing certain ideal types of supports: hinged support without friction and absolute rigid attachment. Intermediate values between points refer to real supports having finite stiffness and friction.

4. Discussion
The solutions obtained show that control of the dynamic state of the waveguide by means of supports arrangement is an effective method. For example, for a single-span waveguide, variation of the supports allows the first natural frequency to be increased by 6 times and the first critical force to be increased by 16 times. For a multi-span beam, the introduction of each additional intermediate support initially increases the first natural frequency of vibration by about 1.5 times, but with the number of intermediate supports above 10 their influence on the solution is reduced and the first natural frequency is determined by the actual stiffness of the supports themselves.

In addition to the positioning of the supports, it is necessary to use other methods of controlling the dynamic behavior of the waveguide through the construction of the supports. For example, a radical solution to avoid buckling is to replace fixed supports with sliding ones that allow the waveguide to slip through them when heated. In this case, the compressive force be determined only by the friction forces in the support, which are usually extremely small.

5. Conclusions
This paper describes the effect of selecting the type and location of end and intermediate supports for a single waveguide between two UHF-blocks on the value of the first natural frequency of vibrations and critical compressive force. The most common support arrangement schemes are given and their corresponding coefficients are obtained for equations defining the first natural frequency and minimum critical compressive force. An example of the numerical calculation of the waveguide is given and recommendations are given when designing the supports. The results of the paper can be used in the design of any extended structures to ensure their dynamic behavior by means of support arrangement.

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References
[1] Klebanov B M, Barlam D M, Nystorm F E 2008 *Machine elements: life and design* (CRC Press: Boca Raton)
[2] Bauchau A O, Craig J I 2009 *Structural analysis. With Applications to Aerospace Structures* (Springer Netherlands: Heidelberg)
[3] Clough R W, Penzien J 2010 *Dynamics of Structures* (Computers and Structures inc.: Walnut Creek)
[4] Kudryavtsev I V, Minakov A V and Mityaev A E 2019 J. Mach. Manuf. Reliab. 48(4) 306–13
[5] Kelly S G 2011 *Mechanical vibrations: Theory and applications, SI* (Cengage Learning: Stamford)
[6] Singiresu S R 2011 *Mechanical Vibrations* (Prentice Hall: Upper Saddle River)
[7] Shabana A A 2019 *Theory of Vibration* (Springer: Heidelberg)
[8] He J, Fu Z 2001 *Modal Analysis* (Butterworth-Heinemann: Oxford)
[9] Rades M 2010 *Mechanical vibrations. Part II. Structural dynamic modeling* (București: Printech)
[10] Alfutov N A 2000 *Stability of Elastic Structures* (Springer-Verlag Berlin: Heidelberg)
[11] Bazant Z P 2010 *Stability of structures: elastic, inelastic, fracture and damage theories* (World Scientific Publishing: Singapore)
[12] Gambhir M L 2004 *Stability analysis and design of structure* (Springer International Publishing: Cambridge)
[13] Gould P L 1988 *Analysis of shells and plates* (Springer-Verlag: New York)
[14] Blauwendaal J, Hoefakker J H 2014 *Structural shell analysis* (Springer Netherlands: Heidelberg)
[15] Allen D H 2013 *Introduction to the mechanics of deformable solids: bars and beams* (Springer: Heidelberg)
[16] Librescu L 2006 *Thin-walled beams: Theory and application* (Springer: Heidelberg)
[17] Connor J J, Faraji S 2016 *Fundamentals of structural engineering* (Springer International Publishing: Cambridge)
[18] Eugster S 2015 *Geometric continuum mechanics and induced beam theories* (Springer International Publishing: Heidelberg)
[19] Blevins R D 2016 *Formulas for dynamics, acoustics and vibration* (John Wiley & Sons Ltd: Chichester)
[20] Paz M, Leigh W 2004 *Structural Dynamics-Theory and Computation* (Springer: Heidelberg)