Research Article

The Pareto-Optimal Stop-Loss Reinsurance

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Reinsurance plays a role of a stabilizer of the insurance industry and can be an effective tool to reduce the risk for the insurer. This paper aims to provide the optimal reinsurance design associated with the stop-loss reinsurance under the criterion of value-at-risk (VaR) risk measure. In this paper, the probability levels in the VaRs used by the both reinsurance parties are assumed to be different and the optimality results of reinsurance are derived by minimizing linear combination of the VaRs of the cedent and the reinsurer. The optimal parameter values of the stop-loss reinsurance policy are formally derived under the expectation premium principle.

1. Introduction

Reinsurance is an effective risk management tool that enables an insurer to reduce the underwriting risk. An insurer must conduct the classical tradeoff between the risk retained and the premium paid to the reinsurer. Generally speaking, the more risks you have to transfer, the more you will pay. In order to balance the relationship between the risk retained and the reinsurance premium, the academics started the research of the optimal reinsurance problem. Arrow [1] first studied the reinsurance problem and showed that the stop-loss reinsurance was optimal by using the criterion of maximizing the expected utility of the terminal wealth. Heerwaarden et al. and Gollier and Schlesinger [2, 3] give the same conclusion under the second degree stochastic dominance and showed that the stop-loss reinsurance was optimal. Young [4] generalized Arrow’s result by considering Wang’s premium principle. Recently, the optimal reinsurance problem has been revisited under different risk measures. Cai and Tan [5] derived explicitly the optimal retained level of a stop-loss reinsurance minimizing the value-at-risk (VaR) and conditional tail expectation (CTE) of the insurer’s total loss under the expected premium principle. Cai et al. [6] derived the optimal ceded loss functions among the class of increasing convex loss functions. Tan et al. [7] give 17 kinds of reinsurance premium principles and studied the optimal quota-share reinsurance and the optimal stop-loss reinsurance under the criterions of VaR and CTE. Cheung [8] provided a geometric approach to re-examine the optimal reinsurance problems studied in [6] and generalized the results by studying the VaR minimization problem with Wang’s premium principle. Chi and Tan [9] analyzed the VaR-based and conditional-value-at-risk (CVaR)-based optimal reinsurance models over different classes of ceded loss functions with increasing generality. Chi [10] showed that the layer reinsurance is always optimal under both the VaR and CVaR criteria when the reinsurance premium is calculated by a variance related principle. Lu et al. [11] studied the optimal reinsurance under VaR and CTE criteria when the ceded loss functions are increasing concave functions.

As we all know, there are two parties in a reinsurance contract, an insurer and a reinsurer. They have conflicting interests. Borch [12] studied the optimal quota-share reinsurance and the optimal stop-loss reinsurance from the both sides of the reinsurance under the optimization criterion of maximizing the product of the expected utility functions of the two parties’ terminal wealth. Borch [13] showed reinsurance policy which is very attractive to the insurer may not be optimal for the reinsurer and it might
be unacceptable for the reinsurer. Since then, the study of the optimal reinsurance opened a new direction. Cai et al. and Fang and Qu [14, 15] obtained the sufficient conditions for the optimal reinsurance contract by studying the joint survival probability and the joint profitable probability of the two parties. Cai et al. [16] used the convex combination of the VaRs of the cedent and the reinsurer as the object function to research the optimal reinsurance policies. Based on the criterion of VaR under the different confidence levels, Jiang et al. [17] studied pareto-optimal reinsurance strategies and gave the optimal forms. Lo [18] discussed the generalized problems of [16] by using the Neyman–Pearson approach. Cai et al. [19] studied pareto-optimality of reinsurance arrangements under general model settings and obtained the explicit forms of the pareto-optimal reinsurance contracts under tail-value-at-risk (TVaR) measure and the expected value premium. By geometric approach, Fang et al. [20] studied pareto-optimal reinsurance policies under general premium principles and gave the explicit parameters of the optimal ceded loss functions under Dutch premium principle and Wang’s premium principle. Lo and Tang [21] characterized the set of pareto-optimal reinsurance policies analytically and visualized the insurer-reinsurer trade-off structure geometrically.

It is interesting to notice that most optimal forms of reinsurance in these cited papers are stop-loss reinsurance contracts. Inspired by these results, we mainly study the stop-loss reinsurance in this paper. We study the optimal form of reinsurance policy by minimizing the convex combination of the VaRs of the cedent and the reinsurer under the expected principle. The rest of the paper is organized as follows. In Section 2, we mainly introduce some preliminary knowledge. In Section 3, we assume that the cedent and the reinsurer have different confidence levels and then discuss the optimal stop-loss reinsurance under the expected principle by the optimization problem. In Section 4, we give numerical examples. In Section 5, we conclude the paper.

2. VaR-Based Optimal Reinsurance Model

In this section, we establish the framework of the optimal stop-loss reinsurance model-based VaR risk measure. Let the total loss for an insurer over a period of time be \( X \), where \( X \) is a nonnegative random variable and defined in the probability space \((\Omega, \mathcal{F}, \mathbb{P})\), with distribution function \( F_X(x) = \mathbb{P}(X \leq x) \), survival function \( S_X(x) = \mathbb{P}(X > x) \), mean \( \mathbb{E}[X] = \mu (0 < \mu < \infty) \), and variance \( \mathbb{D}[X] = \sigma^2 > 0 \). In a reinsurance contract, a reinsurer agrees to pay the part of the loss \( X \), denoted by \( f(X) \), to the insurer at the end of the contract term, while the insurer will pay a reinsurance premium, denoted by \( \pi(f(X)) \), to the reinsurer when the contract is signed, where the function \( f(x) (0 \leq f(x) \leq x) \) is called ceded loss function. Then, the retained loss for the insurer is \( I(X) = X - f(X) \), where the function \( I(x) \) is called retained loss function.

As we all know, stop-loss reinsurance is optimal in the sense that it gives the lowest variance for the retained risk when the mean is given. In many literature studies, stop-loss reinsurance has been shown to be optimal under certain conditions, such as [1–3]. Therefore, many articles take stop-loss reinsurance as an example to study the reinsurance, for example [5, 7, 14]. In this paper, we study the stop-loss reinsurance, that is to say \( f(X) = (X - d)^+ \), where the parameter \( d \geq 0 \) is the retention. Under stop-loss agreement, the total loss of the insurer is

\[
T_I = X - f(X) + \pi(f(X)),
\]

and the total loss of the reinsurer is

\[
T_R = f(X) - \pi(f(X)).
\]

In fact, the reinsurance aims to control the risks of the two sides of reinsurance, and this will involve their maximum aggregate loss. The risk measure most often used in practice is simply the Value-at-Risk at a certain level \( \alpha \) with \( 0 < \alpha < 1 \), which is the amount that will maximally be lost with probability \( \alpha \). The VaR of a random variable is defined as follows.

**Definition 1.** The VaR of a nonnegative random variable \( X \) at a confidence level \( \alpha \), \( 0 < \alpha < 1 \), is defined as

\[
\text{VaR}_\alpha(X) = \inf\{x: F_X(x) \geq \alpha\}. \tag{3}
\]

The VaR defined by (3) is the maximum loss which is not exceeded at a given probability \( \alpha \). We list several properties of the VaR.

**Proposition 1.** For any nonnegative random variable \( X \) with the survival function \( S_X(x) \), we have the following properties for any \( \alpha \in (0, 1) \):

1. Translation invariance: \( \text{VaR}_\alpha(X + c) = \text{VaR}_\alpha(X) + c \), \( (c \in \mathbb{R}) \)
2. Homogeneity: \( \text{VaR}_\alpha(cX) = c \text{VaR}_\alpha(X) \), \( (c \in \mathbb{R}) \)
3. If \( h(x) \) is an increasing and left-continuous function, then \( \text{VaR}_\alpha(h(X)) = h(\text{VaR}_\alpha(X)) \)

Obviously, the insurer and the reinsurer are mutually restricted and even opposed in the interests. This means, a reinsurance policy which is very attractive to the insurer may not be optimal for the reinsurer and it might be unacceptable for the reinsurer. To be fair, we consider the two parties of the reinsurance. Inspired by [17], we study the pareto-optimal reinsurance under the criterion of VaR because it can be expressed by the linear combination of the VaRs of the cedent and the reinsurer. Then, in this paper, we study optimal reinsurance policy by solving the following optimization problem:

\[
\min_{L(f)} L(f) = \min\{\beta \text{VaR}_{\alpha_c}(T_I) + (1 - \beta)\text{VaR}_{\alpha_r}(T_R)\}, \tag{4}
\]

where \( 0 \leq \beta \leq 1 \), and the probability levels in the VaRs used by the cedent and the reinsurer are possibly different, say \( \alpha_c \) and \( \alpha_r \), respectively.
3. Stop-Loss Reinsurance Optimization

Let \( f(x) = (x - d)_+ \) denote the ceded function, where \( d \geq 0 \) is the stop-loss retention; then, the objective function is

\[
L(f) = \beta \text{VaR}_\alpha(T_f) + (1 - \beta) \text{VaR}_\alpha(T_R)
\]

\[= \beta \text{VaR}_\alpha(X - f(X) + \pi(f(X))) + (1 - \beta) \text{VaR}_\alpha(f(X) - \pi(f(X))).\]  

Let \( a_c = \text{VaR}_\alpha(X) \) and \( a_r = \text{VaR}_\alpha(X); \) then, the objective function is

\[L(d) = \beta[a_c - (a_c - d)] + (1 - \beta)(a_r - d) + (2\beta - 1)\pi[(X - d)_+],\]  

and the optimization problem is

\[
\min_{d \in (0, \infty)} L(d).
\]

One of the commonly used principles is the expectation premium principle, that is to say \( \pi[(X - d)_+] = (1 + \theta)[E[(X - d)_+] = (1 + \theta) \int_0^\infty S_X(x)dx, \) where \( \theta > 0 \) is the relative safety loading. In order to get the optimal retention \( d^* \), we give the following results.

**Theorem 1.** If \( \beta = (1/2) \), we have the following conclusions:

1. When \( a_r < a_c \), the optimal stop-loss coefficient \( d^* \) is arbitrarily in \([0, a_c]\).
2. When \( a_r > a_c \), the optimal stop-loss coefficient \( d^* \) is arbitrarily in \([a_c, \infty)\).

**Proof.** Specifically, when \( \beta = (1/2) \), then the objective function is degraded to \( L(d) = (1/2)[a_c - (a_c - d)_+ + (a_r - d)_+] \).

1. When \( a_r < a_c \), we can obtain \( a_r \leq d \leq a_c \) and

\[L(d) = \begin{cases} 
\frac{1}{2}a_r, & d < a_r, \\
\frac{1}{2}d, & a_r \leq d \leq a_c, \\
\frac{1}{2}a_c, & d > a_c.
\end{cases}\]  

In conclusion, we have

\[
\min_{d \in (0, \infty)} L(d) = \frac{1}{2}a_r,
\]

so \( d^* \) is any number in \([0, a_r]\).

(2) When \( a_r > a_c \), we have \( a_r \geq a_c \) and

\[
L(d) = \begin{cases} 
\frac{1}{2}a_r, & d < a_c, \\
\frac{1}{2}(a_c + a_r - d), & a_c \leq d \leq a_r, \\
\frac{1}{2}a_c, & d > a_r.
\end{cases}
\]

So,

\[
\min_{d \in (0, \infty)} L(d) = \frac{1}{2}a_r,
\]

and \( d^* \) is any number in \([a_r, \infty)\).

We study the situation of \( \beta \neq (1/2) \) and accomplish our task by subdividing our considerations into four cases: (1) \( \beta > (1/2) \) and \( a_r < a_c \); (2) \( \beta > (1/2) \) and \( a_r > a_c \); (3) \( \beta < (1/2) \) and \( a_r < a_c \); (4) \( \beta < (1/2) \) and \( a_r > a_c \).

For convenience, we use the notations:

\[
\theta_1 = \frac{1}{1 + \theta}, \quad \theta_2 = \frac{\beta}{(2\beta - 1)(1 + \theta)}, \quad \theta_3 = \frac{\beta - 1}{(2\beta - 1)(1 + \theta)}, \quad \theta_4 = \frac{\beta a_c - (1 - \beta)a_r}{2\beta - 1}.
\]

**Theorem 2.** When \( \beta > (1/2) \) and \( a_r < a_c \), the optimal stop-loss reinsurance parameters are as follows:

1. When \( S_X(0) \leq \theta_1 \), the optimal stop-loss reinsurance coefficient is given by

\[
d^* = \begin{cases} 
0, & \mu < \theta_1 Q(\beta, a_r, a_c), \\
0 \text{ or } \infty, & \mu = \theta_1 Q(\beta, a_r, a_c), \\
\infty, & \mu > \theta_1 Q(\beta, a_r, a_c).
\end{cases}
\]

2. When \( S_X(a_r) \leq \theta_1 < S_X(0) \), the optimal stop-loss reinsurance coefficient is given by

\[
d^* = \begin{cases} 
S_X^{-1}(\theta_1), & \int_{S_X^{-1}(\theta_1)}^\infty S_X(t)dt < \theta_1 [Q(\beta, a_r, a_c) - S_X^{-1}(\theta_1)], \\
0, & \int_{S_X^{-1}(\theta_1)}^\infty S_X(t)dt = \theta_1 [Q(\beta, a_r, a_c) - S_X^{-1}(\theta_1)], \\
\infty, & \int_{S_X^{-1}(\theta_1)}^\infty S_X(t)dt > \theta_1 [Q(\beta, a_r, a_c) - S_X^{-1}(\theta_1)].
\end{cases}
\]
When \( S_X(0) \leq \theta_1 \), the optimal stop-loss reinsurance coefficient is given by
\[
d' = \begin{cases} 
0, & \int_{S_2^1(\theta_1)}^\infty S_X(t)dt < \theta_2(a_c - a_r), \\
s_2^1(\theta_2), & \int_{S_2^1(\theta_2)}^\infty S_X(t)dt = \theta_2(a_c - a_r), \\
\infty, & \int_{S_2^1(\theta_2)}^\infty S_X(t)dt > \theta_2(a_c - a_r).
\end{cases}
\]

When \( d \leq a_c \), it follows from (17) that
\[
L_r(d) = \begin{cases} 
(2\beta - 1)(1 + \theta)[\theta_1 - S_X(d)], & d < a_r, \\
(2\beta - 1)(1 + \theta)[\theta_2 - S_X(d)], & a_r < d \leq a_c.
\end{cases}
\]

Because \( \beta > (1/2) \), so \( L_r(d) \) is an increasing function. Meanwhile, \( \theta_1 \leq \theta_2 \) and \( S_2^1(\theta_2) \leq S_2^1(\theta_1) \). Now, let us discuss the following situations in turn:

1. If \( S_X(0) \leq \theta_1 \), then \( L_r(0) \geq 0 \) and \( L_r(d) \geq 0 \) for any \( d \in [0,a_c] \). So, \( d^*_1 = 0 \) and the optimal parameter \( d^* \) depends on the size of \( L(0) \) and \( L(\infty) \).

2. If \( S_X(a_r) < \theta_1 < S_X(0) \), this is equivalent to \( 0 < S_2^1(\theta_1) \leq a_r \). When \( d < S_2^1(\theta_1) \), we can obtained \( L_r(d) < 0 \), otherwise, \( L_r(d) > 0 \). This means \( d^*_1 = S_2^1(\theta_1) \), and finally \( L(d) \) depends on the relative magnitude between \( L(S_2^1(\theta_1)) \) and \( L(\infty) \).

3. If \( \theta_1 < S_X(a_r) < \theta_2 \), this means \( S_2^1(\theta_1) < a_r < S_2^1(\theta_2) \). When \( d < a_r \), we can obtained \( L_r(d) < 0 \); on the contrary, \( L_r(d) > 0 \). It shows that \( d^*_1 = a_r \). So, the optimal parameter \( d^* \) is determined by the size of \( L(a_r) \) and \( L(\infty) \).

4. If \( S_X(a_r) < \theta_2 \leq S_X(a_c) \), this is equivalent to \( a_r \leq S_2^1(\theta_2) < a_c \). When \( d < S_2^1(\theta_2) \), we can obtained \( L_r(d) < 0 \); otherwise, \( L_r(d) > 0 \). So, we proved that \( d^*_1 = S_2^1(\theta_2) \) and \( d^* \) depends on the size of \( L(d) \) and \( L(\infty) \).

5. If \( \theta_2 \leq S_X(a_c) \), obviously, \( L_r(a_c) \leq 0 \) and \( L_r(d) \leq 0 \) for any \( d \in [0,a_r] \). This implies that \( d^* = \infty \).

Theorem 3. When \( \beta > (1/2) \) and \( a_r > a_c \), the optimal stop-loss reinsurance parameters are as follows:

1. If \( S_X(0) \leq \theta_1 \), the optimal stop-loss reinsurance coefficient is given by
\[
d^* = \begin{cases} 
0, & \mu \leq \theta_1 \varphi(b_r, a_r), \\
\text{or } \infty, & \mu > \theta_1 \varphi(b_r, a_r).
\end{cases}
\]

2. If \( S_X(a_r) < \theta_1 < S_X(0) \), the optimal stop-loss reinsurance coefficient is given by
\[
d^* = \begin{cases} 
S_2^1(\theta_1), & \int_{S_2^1(\theta_1)}^\infty S_X(t)dt < \theta_1 \varphi(b_r, a_r), \\
S_2^1(\theta_2), & \int_{S_2^1(\theta_2)}^\infty S_X(t)dt = \theta_2 \varphi(b_r, a_r), \\
\infty, & \int_{S_2^1(\theta_2)}^\infty S_X(t)dt > \theta_2 \varphi(b_r, a_r).
\end{cases}
\]
(3) When \( \theta_1 \leq S_X (a_i) \), the optimal stop-loss reinsurance coefficient is given by \( d^* = \infty \).

**Theorem 4.** When \( \beta < (1/2) \) and \( \alpha_i < \alpha_c \), the optimal stop-loss reinsurance parameters are as follows:

1. When \( S_X (0) \leq \theta_2 \) and \( S_X (0) \leq \theta_1 \), the optimal stop-loss reinsurance coefficient is given by \( d^* = a_c \).
2. When \( S_X (a_i) < \theta_2 \leq S_X (0) \) and \( S_X (a_i) \notin (\theta_2, \theta_1) \), the optimal stop-loss reinsurance coefficient is given by

\[
d^* = \begin{cases} 
0, & \int_0^{a_i} S_X (t) dt > \theta_1 Q (\beta, a_i, a_c), \\
0 \text{ or } a_c, & \int_0^{a_i} S_X (t) dt = \theta_1 Q (\beta, a_i, a_c), \\
0 \text{ or } a_c, & \int_0^{a_i} S_X (t) dt = \theta_1 Q (\beta, a_i, a_c), \quad (22) \\
a_c, & \int_0^{a_i} S_X (t) dt < \theta_1 Q (\beta, a_i, a_c). 
\end{cases}
\]

3. When \( \theta_1 < S_X (a_i) < S_X (0) \leq \theta_1 \), the optimal stop-loss reinsurance coefficient is given by \( d^* = a_c \).
4. When \( \theta_2 < S_X (a_i) < \theta_1 \leq S_X (0) \), the optimal stop-loss reinsurance coefficient is given by

\[
d^* = \begin{cases} 
0, & \int_0^{a_i} S_X (t) dt > \theta_1 a_c, \\
0 \text{ or } a_c, & \int_0^{a_i} S_X (t) dt = \theta_1 a_c, \quad (23) \\
a_c, & \int_0^{a_i} S_X (t) dt < \theta_1 a_c. 
\end{cases}
\]

5. When \( \theta_1 \leq S_X (a_i) \) and \( \theta_2 \leq S_X (a_i) \), the optimal stop-loss reinsurance coefficient is given by \( d^* = 0 \).

**Theorem 5.** When \( \beta < (1/2) \) and \( \alpha_i > \alpha_c \), the optimal stop-loss reinsurance parameters are as follows:

1. When \( S_X (0) \leq \theta_1 \), the optimal stop-loss reinsurance coefficient is given by \( d^* = a_c \).
2. When \( \theta_1 < S_X (0) \) and \( S_X (a_i) \leq \theta_3 \), the optimal stop-loss reinsurance coefficient is given by

\[
d^* = \begin{cases} 
0, & \int_0^{a_i} S_X (t) dt > \theta_1 Q (\beta, a_i, a_c), \\
0 \text{ or } a_c, & \int_0^{a_i} S_X (t) dt = \theta_1 Q (\beta, a_i, a_c), \quad (24) \\
a_c, & \int_0^{a_i} S_X (t) dt < \theta_1 Q (\beta, a_i, a_c). 
\end{cases}
\]

3. When \( \theta_2 < S_X (a_i) \), the optimal stop-loss reinsurance coefficient is given by \( d^* = 0 \).

### 4. Numerical Examples and Comparison

In this section, we construct two numerical examples to illustrate the reinsurance policy that we derived in the previous sections. Specifically, we assume that the loss variable \( X \) follows the exponential distribution with the survival function \( S_X (x) = e^{-0.001x} \) for \( x > 0 \) and the mean \( \mu = 1000 \). Let the safety loading parameter \( \theta = 0.2 \). We discuss two examples specified below.

**Example 1.** Assume \( \alpha_i = 0.95 \) and \( \alpha_c = 0.99 \). In this case, \( a_i = 2995.7 \) and \( a_c = 4605.2 \). The optimal ceded loss function \( f (x) \) are shown in Table 1.

**Example 2.** Assume \( \alpha_i = 0.95 \) and \( \alpha_c = 0.99 \). In this case, \( a_i = 2995.7 \) and \( a_c = 4605.2 \). The optimal ceded loss function \( f (x) \) are shown in Table 2.

**Remark 1.** Following the abovementioned examples, we know that the optimal parameter of the stop-loss reinsurance policy depends on the combining parameter \( \beta \) when the probability levels in the VaRs are used by the both reinsurers differently.

### 5. Conclusions

Some scholars have shown that the stop-loss reinsurance is the optimal reinsurance policy under the convex combination of the both reinsurance parties. In this paper, we mainly study the pareto-optimal stop-loss reinsurance policy with the expectation premium principle. We analyze the topic from the following aspects: (1) the optimality results of reinsurance are derived by minimizing linear combination of the VaRs of the cedent and the reinsurer; (2) assuming that the probability levels in the VaRs used by the both reinsurance parties are different. Fortunately, through analysis, we finally derived the optimal parameters for the stop-loss reinsurance.

**Data Availability**

All data, models, or code generated or used during the study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.
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