Stability of Accelerated Expansion in Nonlinear Electrodynamics

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Abstract
This paper is devoted to study the phase space analysis of isotropic and homogenous universe model by taking a noninteracting mixture of electromagnetic and viscous radiating fluids whose viscous pressure satisfies a nonlinear version of the Israel-Stewart transport equation. We establish an autonomous system of equations by introducing normalized dimensionless variables. In order to analyze stability of the system, we find corresponding critical points for different values of the parameters. We also evaluate power-law scale factor whose behavior indicates different phases of the universe model. It is concluded that bulk viscosity as well as electromagnetic field enhances the stability of accelerated expansion of the isotropic and homogeneous universe model.

Keywords: Phase space analysis; Bulk viscosity.
PACS: 04.20.-q; 95.36.+x; 98.80.-k.

1 Introduction
Many astronomical observations (type Ia supernova, large scale structure and cosmic microwave background radiation) predict that our universe is expanding at an accelerating rate in its present stage [1]. These observations

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suggest two cosmos phases, i.e., the cosmic state before radiation (the pri-
mordial inflationary era) and ultimately the present cosmos phase after the
matter dominated era. In the last couple of decades, it is speculated that the
source for this observed cosmic acceleration with unusual anti-gravitational
force may be an anonymous energy component dubbed as dark energy (DE).
The existence of this energy with large negative pressure can be recognized
by its distinctive nature from ordinary matter which may lead to cosmic
expansion. The study of the dominant contents of matter distribution in
the universe has remained one of the most challenging issues. Recent ob-
servations show that the visible part of our universe is made up of baryonic
matter contributing only 5% of the total budget while the remaining ingre-
dients yield the total energy density composed of non-baryonic fluids (68%
DE and 27% dark matter) [2].

Several cosmological proposals have been introduced in literature to ex-
perience the ambiguous nature of DE. The cosmological constant ($\Lambda$) governed
by a negative equation of state (EoS) parameter ($\gamma = -1$) is taken to be the
simplest characterization of DE. However, this identification has two well-
known problems, i.e., fine-tuning and cosmic coincidence. In addition, there
are several cosmological models which can be considered as an alternative
to $\Lambda$ like scalar field model [3], phantom model [4], tachyon field [5] and k-
essence [6] that also suggest expanding behavior of the universe. Another
approach involves the generalization of simple barotropic EoS to more exotic
forms such as Chaplygin gas [7] and its modification [8]. It has also been
demonstrated that a fluid with bulk viscosity may cause accelerated expan-
sion of the universe model without cosmological constant or scalar field [9].
Our main concern is to find another approach which can minimize exotic
forms of matter by introducing dissipation through viscous effects of fluids.

During the last few years, cosmological models including nonlinear elec-
tromagnetic fields have attained remarkable interest. The application of this
electrodynamics to different universe models may lead to many significant
results. Nonlinear electrodynamics (NLED) is the generalization of Maxwell
theory which is considered as the most viable theory to remove the initial
singularities. Vollick [10] considered FRW universe model with NLED and
found that the respective model will show a period of late-time acceleration
for $E^2 < 3B^2$. Kruglov [11] found that the universe tends to accelerate
in magnetic background at the early era due to NLED model. Ovgun [12]
formulated analytical nonsingular extension of isotropic and homogeneous
solutions by presenting a new mathematical model in nonlinear magnetic
monopole fields.

The study of possible stable late-time attractors has attained remarkable significance for different universe models. A phase space analysis manifests dynamical behavior of a cosmological model through a global view by reducing complexity of the equations (converting the system of equations to an autonomous system) which may help to understand different stages of evolution. Copeland et al. [13] studied a phase plane analysis of standard inflationary models and found that these models cannot solve density problem. Guo et al. [14] explored phase space analysis of FRW universe model filled with barotropic fluid and phantom scalar field in which phantom dominated solution is found to be a stable late-time attractor.

Garcia-Salcedo [15] examined the dynamics of FRW universe with NLED and found that the critical points have no effects. Yang and Gao [16] discussed phase space analysis of k-essence cosmology in which critical points play an important role for the final state of the universe. Xiao and Zhu [17] analyzed stability of FRW universe model in loop quantum gravity via phase space portraits by taking barotropic fluid as well as positive field potential. Acquaviva and Beesham [18] studied phase space analysis of FRW spacetime filled with noninteracting mixture of fluids (dust and viscous radiation) and found that nonlinear viscous model shows the possibility of current cosmic expansion.

This paper is devoted to study the phase space analysis of FRW universe model with nonlinear viscous fluid. The plan of the paper is as follows. In section 2, we provide basic formalism for NLED and general equations as well as a nonlinear model for bulk viscosity. An autonomous system of equations is established to analyze stability of the system by introducing normalized dimensionless variables in section 3. Section 4 provides the formulation of power-law scale factor. Finally, we conclude our results in the last section.

2 Nonlinear Electrodynamics and General Equations

The standard cosmological model is successful in resolving many issues but still there are some issues which remain to be solved. One of them the initial singularity which leads to a troubling state of affairs because at this point, all known physical theories break down. If the early universe is governed by
Maxwell’s equations, then there will be a spacelike initial singularity in the past. However, if Maxwell’s equations become modified in the early universe, when the electromagnetic field is large, it might help avoiding the occurrence of cosmic singularities [19]. For the situations where strong electromagnetic field occurs, it makes sense to couple gravitation with NLED. The coupling of Einstein gravity with NLED is defined by the action

$$ S = \frac{1}{16\pi} \int \sqrt{-g} [R - \mathcal{L}(F, F^*)] d^4x. $$ (1)

We consider nonlinear extension of Maxwell Lagrangian density up to second order terms in the field invariants $F = F_{\mu\nu}F^{\mu\nu}$ and $F^* = F_{\mu\nu}^*F^{\mu\nu}$ given by [10]

$$ \mathcal{L} = \mathcal{L}(F, F^*) = -\frac{1}{4\mu_0} F + \alpha F^2 + \beta F'^2, $$ (2)

where $\mu_0$ denotes magnetic permeability, $\alpha, \beta > 0$ are arbitrary constants which yield linear density for $\alpha, \beta \to 0$ and $F^*$ is the dual of electromagnetic field tensor. We do not consider the term $FF^*$ involving $F^*$ in order to preserve the parity [20, 21]. The linear term of this Lagrangian dominates during radiation dominated era while the quadratic terms dominate in the early universe that corresponds to the bouncing behavior of the universe to avoid initial singularity [22]. The mechanisms behind the bounce have been demonstrated in [23, 25]. The energy-momentum tensor associated with this Lagrangian has the following form

$$ T_{\mu\nu}(EM) = -4\partial F^\mu F^\nu + (\partial F^* F^* - \mathcal{L}) g_{\mu\nu}. $$ (3)

In order to fulfill the requirement of isotropic and homogeneous universe, i.e., the electromagnetic field to act as its source, the energy density and the pressure corresponding to the electromagnetic field can be computed by averaging over volume [21, 23]. It is assumed that electric and magnetic fields have coherent lengths that are much shorter than the cosmological horizon scales. After several conditions, the energy momentum tensor of the electromagnetic field associated with $\mathcal{L}(F, F^*)$ can be written as that of a perfect fluid

$$ T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, $$ (4)

such that

$$ \rho_{EM} = -\mathcal{L} - 4E^2 \partial_F \mathcal{L}, $$ (5)

$$ p_{EM} = \mathcal{L} - \frac{4}{3}(2B^2 - E^2) \partial_F \mathcal{L}, $$ (6)
where $\partial_F$ represents partial derivative with respect to $F = F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2)$. $E$ and $B$ denote the averaged electric and magnetic fields, respectively.

We consider isotropic and homogeneous universe model given by

$$ds^2 = -dt^2 + a(t)(dr^2 + r^2d\theta^2 + r^2\sin^2\theta d\phi^2), \quad (7)$$

where $a(t)$ is the scale factor. We assume the universe model to be filled with two cosmic fluids, i.e., a noninteracting electromagnetic fluid with energy density $\rho_{EM}$ as well as pressure $p_{EM}$ and a viscous fluid having energy density $\rho_v$ as well as pressure $p = p_v(\rho_v) + \Psi$. Here $p_v$ represents the equilibrium part of viscous pressure whereas $\Psi$ is the non-equilibrium part, i.e., bulk viscous pressure satisfying an evolution equation. Bulk viscosity plays an important role to stabilize the density evolution and overcomes the rapid changes in cosmos. It also promotes negative energy field in the fluid and hence can play the role of dark energy to describe the dynamics of cosmos. It has been suggested that a fluid with bulk viscosity may cause an accelerated expansion of the universe model without cosmological constant or scalar field [23]. The main contribution of bulk viscosity to the effective pressure is its dissipative effect. We obtain Raychaudhuri and constraint equations from the field equations given by

$$\dot{\Theta} = -\frac{1}{3}\Theta^2 - \frac{1}{2}\left[\rho_{EM} + \rho_v + 3(p_{EM} + p_v + \Psi)\right], \quad (8)$$

$$\dot{0} = \rho_{EM} + \rho_v - \frac{1}{3}\Theta^2, \quad (9)$$

where dot means derivative with respect to time. The conservation of energy-momentum tensor yields the following evolution equations for viscous and electromagnetic field components

$$\dot{\rho}_v = -[\rho_v + p_v + \Psi]\Theta, \quad (10)$$

$$\dot{\rho}_{EM} = -[\rho_{EM} + p_{EM}]\Theta. \quad (11)$$

We consider a barotropic EoS for viscous fluid defined by

$$p_v = (\gamma - 1)\rho_v, \quad (12)$$

where $1 \leq \gamma \leq 2$. Using Eqs. (8) and (9), Raychaudhuri and conservation equations for viscous fluid turn out to be

$$\dot{\Theta} = -\frac{1}{2}\Theta^2 - \frac{3}{2}[p_{EM} + (\gamma - 1)\rho_v + \Psi], \quad (13)$$

$$\dot{\rho}_v = -[\gamma\rho_v + \Psi]\Theta. \quad (14)$$
We characterize the viscous pressure variable by the following evolution equation \[26\]

\[
\tau \dot{\Psi} = -\zeta \Theta - \Psi \left(1 + \frac{\tau_*}{\zeta} \right)^{-1} - \frac{1}{2} \tau \Psi \left[\Theta + \frac{\tau}{\zeta} - \frac{\dot{\zeta}}{\zeta} - \frac{\dot{T}}{T}\right],
\]

(15)

where \(\zeta, T, \tau\) and \(\tau_*\) denote bulk viscosity, local equilibrium temperature, linear relaxation time and characteristic time in nonlinear background, respectively. This equation is derived by using a nonlinear model describing a relationship between thermodynamic flux \(\Psi\) and thermodynamic force \(\chi\) in the form

\[
\Psi = -\frac{\zeta \chi}{1 + \tau_* \chi}.
\]

(16)

This is a nonlinear extension of Israel-Stewart equation which reduces to its linear form as \(\tau_* \to 0\). The nonlinear term in Eq.(15) must be positive for thermodynamic consistency and positivity of entropy production rate. The parameters involved in Eq.(15) can be defined by the relations \(\zeta = \zeta_0 \Theta (\zeta > 0), \tau = \frac{\zeta}{\gamma \nu^2 \rho_v}, \tau_* = k^2 \tau\) and \(T = T_0 \rho^{(\gamma - 1)/\gamma}\). Here \(k\) is a constant such that \(k = 0\) gives linear (Israel-Steward) case while \(T_0\) represents constant temperature. Also, \(\nu\) corresponds to the dissipative effect of the speed of sound \(V\) such that \(V^2 = c_s^2 + \nu^2\), where \(c_s^2\) is its adiabatic contribution. By causality, \(V \leq 1\) and \(c_s^2 = \gamma - 1\) which yields

\[
\nu^2 \leq 2 - \gamma, \quad 1 \leq \gamma \leq 2.
\]

(17)

The explicit form of evolution equation by using the above relations yields

\[
\dot{\Psi} = -\gamma \nu^2 \rho_v \Theta - \frac{\gamma \nu^2 \Psi \rho_v}{\zeta_0 \Theta} \left(1 + \frac{k^2 \Psi}{\gamma \nu^2 \rho_v}\right)^{-1} - \frac{1}{2} \Psi \left[\Theta + \left(\frac{2\gamma - 1}{\gamma}\right) \frac{\dot{\rho_v}}{\rho_v}\right].
\]

(18)

3 Phase Space Analysis

In this section, we discuss the phase space analysis of isotropic and homogeneous universe model for radiation case. Due to many arbitrary parameters, it seems difficult to find analytical solution of the evolution equation. In this context, we define normalized dimensionless variables \(\Omega = \frac{3 \rho}{\rho_v}\) and \(\tilde{\Psi} = \frac{3 \Psi}{\rho_v}\) such that the corresponding dynamical system can be reduced to autonomous one. We also define a new variable \(\tilde{\tau}\) for time through which the corresponding derivative is represented by prime such that \(\frac{d}{d\tilde{\tau}} = \frac{d}{\tilde{\tau}}\). Here each term is
associated with some physical explicit background since the chosen dimensionless variables $\Omega$ and $\tilde{\Psi}$ occur due to physical impact of viscous energy density and pressure, respectively. The system of Eqs. (13) and (14) in terms of these normalized variables takes the form

$$\frac{\Theta'}{\Theta} = -\frac{3}{2} \left[ 1 + p_{EM} + (\gamma - 1)\Omega + \tilde{\Psi} \right], \quad (19)$$

$$\frac{3\rho'_{v}}{\Theta^{2}} = -3[\gamma\Omega + \tilde{\Psi}]. \quad (20)$$

Differentiation of the dimensionless variable for energy density gives

$$\Omega' = \frac{3\rho'_{v}}{\Theta^{2}} - 2\Omega\frac{\Theta'}{\Theta}. \quad (21)$$

Using Eqs. (19) and (20), this equation turns out to be

$$\Omega' = 3(\Omega - 1)[\Omega(\gamma - 1) + \tilde{\Psi} + 3p_{EM}]. \quad (22)$$

Now we introduce the concept of a new evolution equation for $\tilde{\Psi}$. The first derivative of $\tilde{\Psi}$ with respect to $\tilde{\tau}$ through Eq. (19) leads to an evolution equation of the form

$$\tilde{\Psi}' = -3\gamma \nu^{2} \Omega \left[ 1 + \frac{\tilde{\Psi}}{3\zeta_{0}} \left( 1 + \frac{k^{2}\tilde{\Psi}}{\gamma \nu^{2} \Omega} \right)^{-1} \right]$$

$$\quad + 3\tilde{\Psi} \left[ 1 + 3p_{EM} \left( 1 - \frac{3}{\Omega} \frac{2\gamma - 1}{2\gamma} \right) \right] - 3(\gamma - 1)\tilde{\Psi}(1 - \Omega). \quad (23)$$

It is mentioned here that Eqs. (22) and (23) play a remarkable role to describe the respective dynamical system for phase space analysis.

In order to find the critical points $\{\Omega_{c}, \tilde{\Psi}_{c}\}$, we need to solve the dynamical system by imposing the condition $\Omega' = \tilde{\Psi}' = 0$. The stability of FRW universe model can be analyzed according to the nature of critical points. Here we restrict the phase space region to a condition which is necessary for the positivity of entropy production rate given by [18, 26]

$$\tilde{\Psi} > -\frac{\gamma \nu^{2} \Omega}{k^{2}}. \quad (24)$$

This condition tends the possible negative values of $\tilde{\Psi}$ towards zero for $k^{2} \gg \nu^{2}$. Contrarily, the bulk pressure will be less restrictive if $k^{2} \ll \nu^{2}$. It is
noted that finite values of $k$ allow only positive values of bulk pressure in the limit $\nu \to 0$. It would be more convenient to consider $k^2 \leq \nu^2$ along with $\nu^2 \leq 2 - \gamma$ and $\tau_s = k^2 \tau$ which leads to the fact that the characteristic time for nonlinear effects $\tau_s$ does not exceed the characteristic time in linear background $\tau$. We characterize the critical points by deceleration parameter $q = -1 - \frac{\Omega'}{\Omega}$ and effective EoS parameter $\gamma_{\text{eff}} = -\frac{2\Omega'}{3\Omega}$ which yield

$$q = \frac{1}{2} \left[ 1 + 9p_{EM} + 3(\gamma - 1)\Omega + 3\Psi \right], \quad \text{(25)}$$

$$\gamma_{\text{eff}} = 1 + 3p_{EM} + (\gamma - 1)\Omega + \Psi. \quad \text{(26)}$$

To examine a region of phase space undergoing accelerated expansion, we impose $q < 0$ in Eq.(25) which gives

$$\Psi < -\frac{1}{3} - 3p_{EM} - (\gamma - 1)\Omega. \quad \text{(27)}$$

The possibility of accelerated expansion in the physical phase space is determined by comparing Eqs.(24) and (25) through $q < 0$ given by

$$\frac{\nu^2}{k^2} > \frac{1 + 9p_{EM} + 3(\gamma - 1)\Omega}{3\gamma\Omega}. \quad \text{(28)}$$

Substituting $\Omega' = 0$ in Eq.(22), we find the following conditions

$$\Omega_c = 1, \quad (\gamma - 1)\Omega_c + \Psi_c + 3p_{EM} = 0. \quad \text{(29)}$$

We insert these conditions in $\Psi'$ to find the location of critical points. This analysis is carried out by characterizing the viscous fluid through the choice of its EoS parameter $\gamma$ (radiation). We consider $0 < k^2 = \nu^2 \leq 2 - \gamma$ for which the case of stiff matter ($\gamma = 2$) is excluded from the analysis as it yields $\nu^2 = 0$.

### 3.1 Radiation Case ($\gamma = \frac{4}{3}$)

We consider the radiation case for phase space analysis by taking $\gamma = \frac{4}{3}$. Imposing the condition (29) and $\Psi' = 0$ in Eq.(23), we have

$$\frac{3\nu^2}{4\zeta_0} \Psi^3 - \frac{\nu^2}{\zeta_0} \Psi^2 - 3\Psi \left( 1 - \frac{4\nu^2}{9\zeta_0} - \frac{21p_{EM}}{8} \right) + 4\nu^2 = 0. \quad \text{(31)}$$
This cubic equation yields three roots among which we retain only those roots that lie in the physical phase space. The general form of the dynamical system is given by

\[ \Omega' = X(\Omega, \bar{\Psi}), \quad \bar{\Psi}' = Y(\Omega, \bar{\Psi}). \]  

The eigenvalues of the system can be determined by the Jacobian matrix

\[ Z = \left( \frac{\partial X}{\partial \Omega} \quad \frac{\partial X}{\partial \bar{\Psi}} \right)_{\mid P^\pm}. \]

The eigenvalues for the above stability matrix corresponding to the points \( P_r^\pm \) are given by

\[ \lambda_1 = 1 + 3\bar{\Psi} + 9p_{EM}, \quad (34) \]
\[ \lambda_2 = -\frac{16\alpha^2}{3\zeta_0} \left[ \frac{1}{4 + \bar{\Psi}} - \frac{\bar{\Psi}}{(4 + \bar{\Psi})^2} \right] - \frac{63p_{EM}}{8} + 6\bar{\Psi} + 3. \]  

The fixed point is called a source (respectively, a sink) if both eigenvalues consist of positive (respectively, negative) real parts. In case of viscous radiating fluid, we can explore source and sink according to the sign of eigenvalues as well as direction of the trajectories. We investigate two critical points \( P_r^+ = \{1, \bar{\Psi}_c^+\} \) and \( P_r^- = \{1, \bar{\Psi}_c^-.\} \) corresponding to positive (\( \bar{\Psi}_c^+ \)) and negative (\( \bar{\Psi}_c^- \)) roots, respectively. By taking \( \Omega_c = 0 \) and the second condition \( (30) \) with \( \bar{\Psi}_c = -\frac{\Omega}{3} - 3p_{EM} \), we obtain \( P_r^0 = \{0, -3p_{EM}\} \).

3.1.1 Case I:
We are interested to analyze the impact of electromagnetic field on stability of the critical points in the presence of nonlinear bulk viscosity. The energy density \( \rho_{EM} \) and pressure \( p_{EM} \) are given by

\[ \rho_{EM} = \frac{1}{2\mu_0}(B^2 + E^2) - 4\alpha(B^2 - E^2)(B^2 + 3E^2), \quad (36) \]
\[ p_{EM} = \frac{1}{6\mu_0}(B^2 + E^2) - \frac{4\alpha}{3}(B^2 - E^2)(5B^2 - E^2). \]  

The dynamical behavior of critical points for different values of electric and magnetic fields as well as other parameters is shown in Figures 1-2. The
Figure 1: Plots for the phase plane evolution of viscous radiating fluid with $\gamma = 4/3$, $\nu = k = \sqrt{1/5}$, $\zeta_0 = 0.2$, $\alpha = 0.01$ and different values of $B$ and $E$.

green trajectory depicts a flow from the point $P^+_d$ towards $P^-_d$. The white region corresponds to the negative entropy production rate that diverges on its boundary whereas the green region shows accelerated expansion of the universe ($q < 0$). Here the point $P^0_d$ shows varying behavior, i.e., either it is a saddle point or a sink depending on the values of different parameters.

In these plots, we have taken $\zeta_0 = 0.2, 1$ by varying $\nu, k, B$ and $E$. For $\nu = k = \sqrt{1/5}$ and $\zeta_0 = 0.2$, it is found that the global attractor $P^-_d$ lies in green region showing accelerated expansion for the same values of $B$ and $E$. This region tends to decrease by increasing $E$ such that the point $P^-_d$ lies in the deceleration region. By increasing $\zeta_0$, we find accelerated expansion with different values of $B, E$ and larger values of the parameters $\nu$ and $k$. For $\nu = k = 1$ and $\zeta_0 = 1$, we find accelerated expansion of the universe model for all choices of electric and magnetic fields. The point $P^0_d$ behaves as a sink for $\zeta_0 = 0.2$ which becomes a saddle point for larger values of $\zeta_0$. We observe that the increasing value of bulk viscosity increases the region for accelerated expansion in the presence of NLED. In the following, we discuss two different cases for electric as well as magnetic universe.

3.1.2 Case II ($E = 0$):

It is well-known that NLED helps to diminish the initial singularity in the early universe where only the primordial plasma identifies matter [27]. Some recent results indicate that a magnetic universe is appropriate to avoid the
Figure 2: Plots for the phase plane evolution of viscous radiating fluid with \( \gamma = 4/3, \nu = k = 1, \zeta_0 = 1, \alpha = 0.01 \) and different values of \( B \) and \( E \).

initial singularity and ultimately shows late time accelerated expansion \cite{28}. Here we assume the squared electric field \( < E^2 > \) to be zero such that the magnetic field \( (F = 2B^2) \) rules over the universe known as magnetized universe. Thus the energy density \( (5) \) and pressure \( (6) \) take the form

\[
\rho_B = \frac{B^2}{2\mu_0} (1 - 8\mu_0\alpha B^2), \quad (38)
\]

\[
p_B = \frac{B^2}{6\mu_0} (1 - 40\mu_0\alpha B^2). \quad (39)
\]

The respective evolution plots are given in Figure 3. For \( \nu = k = \sqrt{2/3} \) and \( \zeta_0 = 1 \), we find that sink lies in green region showing the stability of accelerated expansion for the magnetized universe. This region tends to decrease by increasing the value of magnetic field \( B \). The point \( P_0^d \) behaves as saddle for small values of magnetic field. It is mentioned here that increasing values of bulk the viscosity and the parameters \( \nu \) as well as \( k \) with different values of \( B \) give rise to the stability of accelerated expansion of the universe for different choices of \( B \). We also find that a smaller value of bulk viscosity show decelerated expansion with increasing values of \( B \).
Figure 3: Plots for the phase plane evolution of viscous radiating fluid with $\gamma = 4/3$, $\nu = k = \sqrt{2/3}$, $\zeta_0 = 1$, $\alpha = 0.01$ and $B = 0.2, 0.8$.

3.1.3 Case III ($B = 0$):

Here, we deal with the electric universe by setting $\langle B^2 \rangle = 0$. The corresponding energy density and pressure are given by

$$\rho_E = \frac{E^2}{2\mu_0} (1 + 24\mu_0\alpha E^2), \quad (40)$$

$$p_E = \frac{E^2}{6\mu_0} (1 - 8\mu_0\alpha E^2). \quad (41)$$

The plots corresponding to different choices of electric field $E$ are shown in Figure 4. For $\nu = k = \sqrt{2/3}$ and $\zeta_0 = 1$, we analyze the sink $P_d^-$ in green region showing accelerated expansion of the universe for different values of $E$. The point $P_d^0$ behaves as a saddle for small values of magnetic field. We find that the region for accelerated expansion tend to decrease by increasing electric field $E$. It is observed that accelerated expanding region exists for increasing values of bulk viscosity and parameters $\nu$ as well as $k$ with all choices of $E$. It supports the fact that the role of bulk viscosity and electric field is to increase the stability of accelerated expansion of the universe model. The summary of our results filled with viscous radiating fluid is given in Table 1.
\[ P^+/P^- E = 0.2 \]

\[ 0.0 \quad 0.2 \quad 0.4 \quad 0.6 \quad 0.8 \quad 1.0 \]

\[ /Minus 1.0 \quad /Minus 0.5 \quad 0.0 \quad 0.5 \quad 1.0 \quad 1.5 \quad 2.0 \]

\[ \Omega /CapPsi /OverTilde \]

Figure 4: Plots for the phase plane evolution of viscous radiating fluid with \( \gamma = 4/3, \nu = k = \sqrt{2/3}, \zeta_0 = 1, \alpha = 0.01 \) and \( E = 0.2, 0.8 \).

Table 1: Stability Analysis of Critical Points for Radiation Dominated Fluid

| Critical Point | Behavior      | Stability    |
|---------------|---------------|--------------|
| \( P^+ \)     | Source        | Unstable     |
| \( P^- \)     | Sink          | Stable       |
| \( P^0 \)     | Saddle/Sink   | Unstable/Stable |

4 Power-Law Scale Factor

In this section, we discuss the power-law behavior of scale factor corresponding to the critical points. For this purpose, we integrate Eq. (19) which leads to

\[
\dot{\Theta} = -\frac{1}{2} \left[ 1 + 3\rho_{EM} + (\gamma - 1)\Omega + \tilde{\Psi} \right] \Theta^2. \quad (42)
\]

For \( \Theta \neq 0 \), we formulate power-law scale factor whenever \( 1 + 3\rho_{EM} + (\gamma - 1)\Omega + \tilde{\Psi} \neq 0 \). Solving \( \Theta = \frac{3a}{a} \) for \( a(t) \), we obtain the generic critical point as

\[
a = a_0(t - t_0)^{\frac{2}{3[1 + 3\rho_{EM} + (\gamma - 1)\Omega + \tilde{\Psi}]}}. \quad (43)
\]

The following condition must hold for exponentially expanding models (identified by the condition \( 1 + 3\rho_{EM} + (\gamma - 1)\Omega + \tilde{\Psi} = 0 \)) to be present in the
physical phase space region (bounded by Eq. (24))

\[(1 - \gamma)\Omega_c - 3p_{EM} - 1 > -\frac{\gamma\nu^2}{k^2} \Omega. \quad (44)\]

This condition is not satisfied in the physical phase space for \(\nu^2 = k^2\). If \(\nu^2 > k^2\), the above inequality must be satisfied in the following physical phase space region

\[(1 + 3p_{EM}) \left[ 1 - \gamma \left( 1 - \frac{\nu^2}{k^2} \right) \right]^{-1} < \Omega \leq 1. \quad (45)\]

It is mentioned here that sign of the term \(1 + 3p_{EM} + (\gamma - 1)\Omega + \bar{\Psi}\) is quite important to evaluate different cosmological stages. If \(1 + 3p_{EM} + (\gamma - 1)\Omega + \bar{\Psi} = 0\), it corresponds to the exponential expansion of the universe model. Also, \(1 + 3p_{EM} + (\gamma - 1)\Omega + \bar{\Psi} \geq 0\) yields accelerated expansion or contraction of the cosmological model, respectively. If \(\nu^2 < k^2\), the possibility of having accelerated expansion will narrow down. Figure 5 shows the physical phase space region (excluding the white region with negative entropy production rate) whereas yellow and dark gray regions correspond to accelerated and exponential expansion of the universe model for \(\nu^2 > k^2\), respectively. Table 2 provides the polynomial behavior of power-law scale factor for different critical points with \(1 + 3p_{EM} + (\gamma - 1)\Omega + \bar{\Psi} \neq 0\).

**Table 2: Power-law Scale Factors for Different Critical Points**

| Critical Point | Scale factor for \(\gamma = 4/3\) |
|---------------|----------------------------------|
| \(P^0\)       | \(a_0(t - t_0)^{-\frac{3p_{EM}}{2}}\) |
| \(P^+\)       | \(a_0(t - t_0)^{\frac{3(3p_{EM} + \bar{\Psi})}{2} + \frac{1}{2}}\) |
| \(P^-\)       | \(a_0(t - t_0)^{\frac{3(3p_{EM} + \bar{\Psi})}{2} + \frac{1}{2}}\) |

## 5 Outlook

In this paper, we have discussed the impact of NLED on the phase space analysis of isotropic and homogeneous universe model by taking noninteracting mixture of electromagnetic and viscous radiating fluids. This analysis
Figure 5: Plot of qualitative phase space analysis for power-law scale factor with $v^2 > k^2$. Yellow and dark gray regions indicate the accelerated and exponential expansion of the universe model, respectively.

has been proved to be a remarkable technique for the stability of dynamical system. An autonomous system of equations has been developed by defining normalized dimensionless variables. We have evaluated the corresponding critical points for different values of the parameters to discuss stability of the system. We have also calculated eigenvalues which characterize these critical points. We summarize our results as follows.

Firstly, we have discussed stability of critical points through their eigenvalues corresponding to different values of $E$ and $B$ for viscous radiation dominated universe model. It is found that the critical points $P^+_d$ and $P^-_d$ correspond to source (unstable) and sink (stable), respectively (Figures 1-2). It is mentioned here that the green region corresponds to accelerated expansion of the universe. The point $P^-_d$ is a global attractor in the physical phase space region which leads to an expanding model dominated by viscous matter for various choices of cosmological parameters. In the presence of both electric and magnetic fields, we find that bulk viscosity increases the region for accelerated expansion while the increasing values of $E$ shows deceleration region for smaller values of bulk viscosity. It is mentioned here that large values of bulk viscosity as well as other parameters correspond to accelerated expansion of the respective universe model for all choices of electric and magnetic fields.
We have also studied electric and magnetic universe cases separately. It is found that sink lies in the green region showing accelerated expansion of the magnetized universe for smaller values of bulk viscosity and other parameters while increasing value of magnetic field decreases this region. For $B = 0$, we have analyzed accelerated expansion of the universe model corresponding to large values of the parameters which tends to decrease by increasing $E$. It is worth mentioning here that the role of bulk viscosity is to increase the green region for accelerated expansion with different choices of $E$ and $B$ for both electric as well as magnetic universe. Moreover, we have also studied the behavior of power-law scale factor corresponding to the critical points. It is found that the power-law scale factor indicates various phases of evolution (accelerated or exponential expansion) of the respective universe model.

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