Geometric constructions for Ramsey-Turán theory

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Combining two classical notions in extremal combinatorics, the study of Ramsey-Turán theory seeks to determine, for integers $m \leq n$ and $p \leq q$, the number $RT_p(n, K_q, m)$, which is the maximum size of an $n$-vertex $K_q$-free graph in which every set of at least $m$ vertices contains a $K_p$.

Two major open problems in this area from the 80s ask: (1) whether the asymptotic extremal structure for the general case exhibits certain periodic behaviour, resembling that of the special case when $p = 2$; (2) constructing analogues of Bollobás-Erdős graphs with densities other than $1/2$.

We refute the first conjecture by witnessing asymptotic extremal structures that are drastically different from the $p = 2$ case, and address the second problem by constructing Bollobás-Erdős-type graphs using high dimensional complex spheres with all rational densities. Some matching upper bounds are also provided.