Updated Running Quark Mass Values

Hideo Fusaoka
Department of Physics, Aichi Medical University
Nagakute, Aichi 480-11, Japan

and

Yoshio Koide
Department of Physics, University of Shizuoka
52-1 Yada, Shizuoka 422, Japan

Abstract

The running quark masses $m_q(\mu)$ at various energy scales $\mu$ ($\mu = 1\text{GeV}$, $\mu = m_q$, $\mu = m_Z$ and so on) are evaluated by using the mass renormalization equations systematically. Also, those at energy scales $\mu$ higher than $\mu = m_Z$ (from $\mu = 10^3 \text{GeV}$ to $\mu = 10^{16} \text{GeV}$) are evaluated by using the evolution equations of the Yukawa coupling constants for the standard model with a single Higgs boson.

* Talk presented by H. Fusaoka at the 1997 Shizuoka Workshop on Masses and Mixings of Quarks and Leptons, Shizuoka, Japan, March 1997. To appear in the Proceedings.
† E-mail: fusaoka@amugw.aichi-med-u.ac.jp
‡ E-mail address: koide@u-shizuoka-ken.ac.jp
1. Introduction

It is very important to know reliable values of quark masses $m_q$ not only for hadron physicists but also for quark-lepton physicists. For such a purpose, a review article by Gasser and Leutwyler [1] has offered useful information on the running quark masses $m_q(\mu)$. However, since Gasser and Leutwyler’s review article [1] in 1982, there have been some developments, for example, higher order calculation of perturbative QCD [2, 3], matching condition at quark threshold [4], and discovery of the top quark [5].

In this talk we report the running quark masses $m_q(\mu)$ at various energy scales $\mu$ ($\mu = 1 \text{GeV}, \mu = m_q, \mu = m_Z$ and so on) which are evaluated by using the mass renormalization equations systematically. The calculation was done by taking the matching condition at the quark flavor threshold into account. Also, those at energy scales $\mu$ higher than $\mu = m_Z$ (from $\mu = 10^3 \text{ GeV}$ to $\mu = 10^{16} \text{ GeV}$) are evaluated by using the evolution equations of the Yukawa coupling constants for the standard model with a single Higgs boson.

In the next section, we review values of light quark masses $m_u(\mu), m_d(\mu)$ and $m_s(\mu)$ at $\mu = 1 \text{ GeV}$. In Sec.3, we review pole mass values of heavy quark masses $M_c^{pole}, M_b^{pole}$ and $M_t^{pole}$. In order to estimate $m_q(\mu)$ at any $\mu$, we must know the values of the QCD parameters $\Lambda_{\text{MS}}^{(n)} (n = 3, 4, 5, 6)$. In Sec.4, the values of $\alpha_s(\mu)$ and $\Lambda_{\text{MS}}^{(n)}$ are reviewed. In Sec.5, running quark masses $m_q(\mu)$ are evaluated for various energy scales $\mu$, e.g., $\mu = 1 \text{ GeV}, \mu = m_q, \mu = M_q^{pole}, \mu = m_Z, \mu = \Lambda_W$, and so on, where $M_q^{pole}$ is a “pole” mass of the quark, and $\Lambda_W$ is a symmetry breaking energy scale of the electroweak gauge symmetry $SU(2)_L \times U(1)_Y$, i.e. $\Lambda_W \equiv \langle \phi^0 \rangle = (\sqrt{2}G_F)^{-\frac{1}{2}}/\sqrt{2} = 174.1 \text{ GeV}$. In Sec.6, the reliability of the perturbative calculation below $\mu \sim 1 \text{ GeV}$ is discussed. In Sec.7, evolution of the Yukawa coupling constants of the standard model with a single Higgs boson is estimated for energy scales higher than $\mu = \Lambda_W$. Finally, Sec.8 is devoted to summary and discussion.

2. Review: light quark masses at $\mu = 1 \text{ GeV}$

Since Gasser and Leutwyler [1] have obtained the light quark masses $m_u(\mu), m_d(\mu)$ and $m_s(\mu)$ at $\mu = 1 \text{ GeV}$, various values of light quark masses are reported. We summarize these values in Table 1.

As shown in Table 1, there is not so a large discrepancy among these estimates as far as $m_u$ and $m_d$ are concerned. But, for the strange quark mass $m_s$, two different values, $m_s \approx 175 \text{ MeV}$ and $m_s \approx 200 \text{ MeV}$ have been reported. We use weighted averages as input values in our calculations.
Table 1. Light quark mass values at 1 GeV (in unit of MeV)

|                    | $m_u$   | $m_d$   | $m_s$   |
|--------------------|---------|---------|---------|
| Gasser and Leutwyler (1982)[1] | $5.1 \pm 1.5$ | $8.9 \pm 2.6$ | $175 \pm 55$ |
| Dominguez and Rafael (1987)[6] | $5.6 \pm 1.1$ | $9.9 \pm 1.1$ | $199 \pm 33$ |
| Narison (1995)[7]            | $4 \pm 1$    | $10 \pm 1$   | $197 \pm 29$  |
| Leutwyler (1996)[8]          | $5.1 \pm 0.9$ | $9.3 \pm 1.4$ | $175 \pm 25$  |
| Weighted averages            | $4.90 \pm 0.53$ | $9.76 \pm 0.63$ | $187 \pm 16$  |

3. Review: pole masses of heavy quarks

Charm and bottom quark masses

Gasser and Leutwyler (1982) [1] have estimated charm and bottom quark masses $m_c$ and $m_b$ and Tirard and Yudurán (1994) [9] have re-estimated $m_c$ and $m_b$ precisely and rigorously. On the other hand, from $\psi$- and $\Upsilon$-sum rules, Narison (1994) [10] has estimated the running quark masses corresponding to the short-distance perturbative pole masses to two-loops and three loops. In Table 2, we summarize their values. We use weighted averages in Table 2 as input values in our calculations.

Table 2. Pole masses of charm and bottom quark

|                    | $M_c^{pole}$   | $M_b^{pole}$   |
|--------------------|----------------|----------------|
| Tirard and Yudurán (1994)[9] | $1.570 \pm 0.019 \mp 0.007$ | $4.906 \pm^{0.009}_{0.051} \mp^{0.004}_{0.011}$ |
| Narison (1994)[10] | $1.64 \pm^{0.10}_{0.07} \pm 0.03$ | $4.87 \pm 0.05 \pm 0.02$ |
| Weighted averages | $1.59 \pm 0.02$ | $4.89 \pm 0.05$ |

Top quark mass

The top quark mass values obtained by the CDF collaboration (1994) [5, 11] and the D0 collaboration [12] are summarized in Table 3. We use the values quoted by the particle data group (PDG96) [13] as the pole mass of the top quark.

Table 3. Pole mass of top quark

|          | $M_t^{pole}$   |
|----------|----------------|
| CDF (1994)[5]      | $174 \pm 10$   |
| CDF (1995)[11]     | $176 \pm 8$    |
| D0 (1995)[12]      | $199 \pm^{10}_{21}$ |
| PDG (1996)[13]     | $180 \pm 12$   |
4. Estimation of $\alpha_s(\mu)$ and $\Lambda_{\text{MS}}^{(n)}$

Prior to estimates of the running quark masses $m_q(\mu)$, we must estimate the values of $\alpha_s(\mu)$ and $\Lambda_{\text{MS}}^{(n)}$. The effective QCD coupling $\alpha_s = g_s^2/4\pi$ is given by [14]

$$\alpha_s(\mu) = \frac{4\pi}{\beta_0 L} \left\{ 1 - \frac{2\beta_1}{\beta_0^2} \ln L + \frac{4\beta_1^2}{\beta_0^3 L^2} \left[ \left( \ln L - \frac{1}{2} \right)^2 + \frac{5}{8} \beta_1 \beta_0 - \frac{5}{4} \right] \right\} + O\left( \frac{\ln^2 L}{L^3} \right),$$

(4.1)

where

$$\beta(\alpha_s) = -\frac{\beta_0}{2\pi} \alpha_s^2 - \frac{\beta_1}{4\pi^2} \alpha_s^3 - \frac{\beta_2}{64\pi^3} \alpha_s^4 + O(\alpha_s^5),$$

(4.2)

$$\beta_0 = 11 - \frac{2}{3} n_q, \quad \beta_1 = 51 - \frac{19}{3} n_q, \quad \beta_2 = 2857 - \frac{5033}{9} n_q + \frac{325}{27} n_q^2,$$

(4.3)

$$L = \ln(\mu^2/\Lambda^2).$$

(4.4)

The value of $\alpha_s(\mu)$ is not continuous at $n$th quark threshold $\mu_n$ at which the $n$th quark flavor channel is opened, because the coefficients $\beta_0$, $\beta_1$ and $\beta_2$ depend on the effective quark flavor number $n_q$.

Therefore, we use the expression $\alpha_s^{(n)}(\mu)$ with a different $\Lambda_{\text{MS}}^{(n)}$ for each energy scale range $\mu_n \leq \mu < \mu_{n+1}$. The values of $\Lambda_{\text{MS}}^{(n)}$ are evaluated by matching condition [4]. In Table 4, the values of $\Lambda_{\text{MS}}^{(n)}$ are summarized and the underlined values denote input values $\Lambda_{\text{MS}}^{(5)}$.

We show the threshold behavior of $\alpha_s^{(n)}(\mu)$ in Fig. 1. We can see that $\alpha_s^{(n-1)}(\mu)$ in $\mu_{n-1} \leq \mu < \mu_n$ connects with $\alpha_s^{(n)}(\mu)$ in $\mu_n \leq \mu < \mu_{n+1}$ continuously.

![Fig. 1. The threshold behavior of $\alpha_s^{(n)}(\mu)$](image)
Table 4. The values of $\Lambda_{\text{MS}}^{(n)}$ in unit of GeV.

| $n$ | 3   | 4   | 5   | 6   |
|-----|-----|-----|-----|-----|
| $\Lambda_{\text{MS}}^{(n)}$ | 0.333$^{+0.047}_{-0.042}$ | 0.291$^{+0.048}_{-0.041}$ | 0.209$^{+0.039}_{-0.033}$ | 0.0882$^{+0.0185}_{-0.0153}$ |

5. Estimation of running quark masses $m_q(\mu)$

From the pole mass values $M_q^{\text{pole}}$, we estimate the running mass values $m_q(\mu)$ at $\mu = M_q^{\text{pole}}$ for $q = u, d, s$ by using the relation [15]

$$m_q(M_q^{\text{pole}}) = M_q^{\text{pole}} \left[ 1 + \frac{4 \alpha_s(M_q^{\text{pole}})}{3 \pi} + K \left( \frac{\alpha_s(M_q^{\text{pole}})}{\pi} \right)^2 + O(\alpha_s^3) \right], \quad (5.1)$$

The values of $K$, $M_q^{\text{pole}}$ and $m_q(\mu)$ at $M_q^{\text{pole}}$ for $q = c, b, t$ are summarized in Table 5.

Table 5. Parameter K, Pole masses $M_q^{\text{pole}}$ and Running mass $m_q(\mu)$ at $M_q^{\text{pole}}$

|   | $K$  | $M_q^{\text{pole}}$ (GeV) | $m_q(M_q^{\text{pole}})$ (GeV) |
|---|------|---------------------------|-------------------------------|
| c | 14.47| 1.59                      | 1.213                         |
| b | 12.94| 4.89                      | 4.248                         |
| t | 10.98| 180                       | 170.1                         |

The scale dependence of a running quark mass $m_q(\mu)$ is governed by the equation [2]

$$\mu \frac{d}{d\mu} m_q(\mu) = -\gamma(\alpha_s)m_q(\mu), \quad (5.2)$$

where

$$\gamma(\alpha_s) = \gamma_0 \frac{\alpha_s}{\pi} + \gamma_1 \left( \frac{\alpha_s}{\pi} \right)^2 + \gamma_2 \left( \frac{\alpha_s}{\pi} \right)^3 + O(\alpha_s^4), \quad (5.3)$$

$$\gamma_0 = 2, \quad \gamma_1 = \frac{101}{12} - \frac{5}{18} n_q, \quad \gamma_2 = \frac{1}{32} \left[ 1249 - \left( \frac{2216}{27} + \frac{160}{3} \zeta(3) \right) n_q - \frac{140}{81} n_q^2 \right]. \quad (5.4)$$

The running quark mass $m_q(\mu)$ is given by

$$m_q(\mu) = R(\alpha_s) \tilde{m}_q, \quad (5.5)$$

$$R(\alpha_s) = \left( \frac{\beta_0 \alpha_s}{2 \pi} \right)^{2 \gamma_0/\beta_0} \left\{ 1 + \left( \frac{2 \gamma_1}{\beta_0} - \frac{\beta_1 \gamma_0}{\beta_0^2} \right) \frac{\alpha_s}{\pi} \right\}$$
\[ + \frac{1}{2} \left[ \left( \frac{2\gamma_1}{\beta_0} - \frac{\beta_1\gamma_0}{\beta_0^2} \right)^2 + 2 \frac{\gamma_2}{\beta_0} - \frac{\beta_1\gamma_1}{16\beta_0^2} + \frac{\beta_2\gamma_0}{2\beta_0^3} \right] \left( \frac{\alpha_s}{\pi} \right)^2 + O(\alpha_s^3) \right\}, \quad (5.6) \]

where \( \hat{m}_q \) is the renormalization group invariant mass which is independent of \( \ln(\mu^2/\Lambda^2) \), \( \alpha_s \) is given by (4.1) and \( \beta_i \) (\( i = 0, 1, 2 \)) are defined by (4.3).

By using \( \Lambda_{\text{MS}}^{(n)} \), we can evaluate \( R^{(n)}(\mu) \) for \( \mu < \mu_{n+1} \), where \( \mu_n \) is the \( n \)th quark flavor threshold and we take \( \mu_n = m_{q_n}(m_{q_n}) \). We show the threshold behavior of \( R^{(n)}(\mu) \) in Fig. 2. As shown in Fig. 2, the behavior of \( R(\mu) \) is discontinuous at \( \mu = \mu_n \equiv m_{q_n}(m_{q_n}) \).

![Graph showing the threshold behavior of R(n)(\mu)](image)

**Fig. 2.** The threshold behaviour of \( R^{(n)}(\mu) \)

We can evaluate the values of \( m_q(m_q) \) (\( q = c, b, t \)) by using the values of \( M_{q_n}^{\text{pole}} \) and the relation

\[ m_{q_n}(\mu) = \left[ R^{(n)}(\mu)/R^{(n)}(M_{q_n}^{\text{pole}}) \right] m_{q_n}(M_{q_n}^{\text{pole}}) \quad (\mu < \mu_{n+1}) \right. \quad \text{.} \quad (5.7) \]

Similarly, we evaluate the light quark masses \( m_q(m_q) \) by using the values \( m_q(1\text{GeV}) \) and the relation

\[ m_q(\mu) = \left[ R^{(3)}(\mu)/R^{(3)}(1\text{GeV}) \right] m_q(1\text{GeV}) \quad (\mu < \mu_4) \right. \quad \text{.} \quad (5.8) \]

Running quark mass values \( m_{q_n}(\mu) \) at \( \mu \geq \mu_{n+1} \) cannot be evaluated by using \( R(\mu)^{(n)} \) straightforwardly, because of the discontinuity of \( R(\mu) \) at quark threshold \( \mu = \mu_n \equiv m_{q_n}(m_{q_n}) \).

The behavior of the \( n \)th quark mass \( m_{q_n}^{(N)} \) (\( n < N \)) at \( \mu_N \leq \mu < \mu_{N+1} \) is
given by the matching condition [16]

\[ m_{qn}^{(N)}(\mu) = m_{qn}^{(N-1)}(\mu) \left[ 1 + \frac{1}{12} \left( x_N^2 + \frac{5}{3} x_N + \frac{89}{36} \left( \frac{\alpha_s^{(N)}(\mu)}{\pi} \right)^2 \right) \right] , \quad (5.9) \]

where

\[ x_N = \ln \left( \frac{\left( m_{qN}^{(N)}(\mu) \right)^2}{\mu^2} \right) \quad . \quad (5.10) \]

In Fig. 3, we illustrate the \( \mu \)-dependency of the light quark masses \( m_q(\mu) \) (\( q = u,d,s \)) which take the matching condition (5.9) into account. We also illustrate the behavior of the heavy quark masses \( m_q(\mu) \) (\( q = c,b,t \)) in Fig. 4.

Fig. 3. running masses of light quarks

Fig. 4. running masses of heavy quarks

The numerical results are summarized in Table 5, where input values \( m_q(1 \text{ GeV}) \) for \( q = u,d,s \) and \( m_q(M_{q}^{\text{pole}}) \) for \( q = c,b,t \) are used. The first and second errors come from \( \pm \Delta m_q \) (or \( \pm \Delta M_{q}^{\text{pole}} \)) and \( \pm \Delta \Lambda_{\text{MS}}^{(5)} \), respectively. The values with asterisk should not be taken rigidly, because these values have been calculated in the region with a large \( \alpha_s(\mu) \).

6. Reliability of the perturbative calculation below \( \mu \sim 1 \text{ GeV} \)

As we noted already, the values of the light quark masses \( m_q(m_q) \) (\( q = u,d,s \)) should not be taken rigidly, because the perturbative calculation below \( \mu \sim 1 \text{ GeV} \)
Table 5. Running quark mass values $m_q(\mu)$ at $\mu = m_q$

| $q = \Lambda$ | $u$ | $d$ | $s$ | $c$ | $b$ | $t$ |
|--------------|-----|-----|-----|-----|-----|-----|
| $M_q^{pole}$ | 0.501 ± 0.002 | 0.517 ± 0.002 | 0.681 ± 0.011 | 1.59 ± 0.02 | 4.89 ± 0.05 | 180 ± 12 |
| $\pm 0.058$ | +0.0016 | +0.0017 | +0.015 | ± 0.018 | ± 0.046 | ± 11.4 |
| $\pm 0.0002$ | +0.001 | ± 0.0017 | ± 0.016 | ± 0.018 | ± 0.046 | ± 11.4 |
| $m_q(M_q^{pole})$ | 0.0436 ± 0.0001 | 0.448 ± 0.001 | 0.549 ± 0.007 | 1.302 ± 0.028 | 4.339 ± 0.034 | 339 ± 12 |
| $m_q(m_c)$ | 0.00421 ± 0.000045 | 0.00838 ± 0.00054 | 0.160 ± 0.014 | 1.302 ± 0.018 | 5.782 ± 0.047 | 318 ± 22 |
| $m_c = 1.302$ | ± 0.00011 | ± 0.000015 | ± 0.0003 | ± 0.020 | +0.145 | ± 10 |
| $m_q(m_b)$ | 0.00312 ± 0.0002 | 0.00621 ± 0.00040 | 0.119 ± 0.010 | 0.950 ± 0.016 | 4.339 ± 0.046 | 253 ± 18 |
| $m_b = 4.339$ | ± 0.00004 | ± 0.00031 | ± 0.0006 | ± 0.052 | ± 0.029 | ± 4 |
| $m_q(m_W)$ | 0.00224 ± 0.00002 | 0.00446 ± 0.00029 | 0.0855 ± 0.0073 | 0.668 ± 0.013 | 3.029 ± 0.038 | 182 ± 13 |
| $m_W = 80.33$ | ± 0.00017 | ± 0.00035 | ± 0.0066 | ± 0.047 | ± 0.074 | ± 0.04 |
| $m_q(m_Z)$ | 0.00222 ± 0.00004 | 0.00442 ± 0.00029 | 0.0847 ± 0.0072 | 0.661 ± 0.012 | 2.996 ± 0.038 | 180 ± 13 |
| $m_Z = 91.187$ | ± 0.00017 | ± 0.00034 | ± 0.0066 | ± 0.047 | ± 0.074 | ± 0.02 |
| $m_q(m_t)$ | 0.00212 ± 0.00002 | 0.00422 ± 0.00027 | 0.0809 ± 0.0069 | 0.630 ± 0.009 | 2.847 ± 0.021 | 170.8 ± 11.5 |
| $m_t = 170.8$ | ± 0.00017 | ± 0.00033 | ± 0.0063 | ± 0.045 | ± 0.074 | ± 0.2 |
| $m_q(\Lambda_W)$ | 0.00212 ± 0.00002 | 0.00422 ± 0.00027 | 0.0808 ± 0.0069 | 0.629 ± 0.012 | 2.843 ± 0.036 | 170.5 ± 12.3 |
| $\Lambda_W = 174.1$ | ± 0.00017 | ± 0.00033 | ± 0.0063 | ± 0.045 | ± 0.075 | ± 0.3 |
| $\pm 0.00014$ | +0.00028 | +0.00053 | +0.041 | +0.069 | +0.070 | +0.3 |
seems to be not very reliable. Let us look at this more explicitly.

In order to see the reliability of the calculation of $\alpha_s(\mu)$ by using \( (4.1) \), in Fig. 5, we illustrate the values of the second and third terms in \{ \} of \( (4.1) \) separately. The values of the second and third terms exceed one at $\mu \simeq 0.5$ GeV and $\mu \simeq 0.6$ GeV, respectively. Also, in Fig. 6, we illustrate the values of the second and third terms in \{ \} of \( (5.6) \) separately. The values of the second and third terms exceed one at $\mu \simeq 0.6$ GeV and $\mu \simeq 0.7$ GeV, respectively. These mean that the perturbative calculation is not reliable below $\mu \simeq 0.7$ GeV. Therefore, the values with asterisk in Tables 5 should not be taken strictly.

7. Evolution of Yukawa coupling constants

By using the renormalization group equation, we estimate the Yukawa coupling constants in the standard model with a single Higgs boson.

The quark mass matrices $M_u$ and $M_d$ at $\mu = \Lambda_W$ are given by

$$M_a(\mu) = \frac{1}{\sqrt{2}Y_a(\mu)}v_a,$$

(7.1)

where $Y_a$ denotes a matrix of the Yukawa coupling constants $y_{ij}^a$ ($a = u, d; i = 1, 2, 3$), $(Y_a)_{ij} = y_{ij}^a$ and $v$ is the vacuum expectation value of the Higgs boson.
The renormalization scale dependence of a matrix \( H_a = Y_a Y_a^\dagger \) is given by \([3]\)

\[
\frac{d}{dt} H_a = \left[ \frac{1}{16\pi^2} \beta_a^{(1)} + \frac{1}{(16\pi^2)^2} \beta_a^{(2)} \right] H_a + H_a \left[ \frac{1}{16\pi^2} \beta_a^{(1)\dagger} + \frac{1}{(16\pi^2)^2} \beta_a^{(2)\dagger} \right].
\]

(7.2)

where \( t \) is given by \( t = \ln(\mu/m_Z) \) and the one-loop and two-loop contributions \( \beta_a^{(1)} \) and \( \beta_a^{(2)} \) are written as \( \beta_a^{(1)} = c_a^{(1)} 1 + \sum_b a_a^b H_b \) and \( \beta_a^{(2)} = c_a^{(2)} 1 + \sum_b b_a^b H_b + \sum_{b,c} b_a^{bc} H_b H_c \), respectively. The expressions of coefficients \( c_a^{(i)}, a_a^b, \) etc. have been given in Ref. \([3]\). For the input parameters, we use the quark masses \( m_q(m_Z) \) in Table 5 and the following parameters in the CKM matrix \( V \) \([13]\):

\[
|V_{us}| = 0.2205 \pm 0.0018, \quad |V_{cb}| = 0.041 \pm 0.003, \quad |V_{ub}/V_{cb}| = 0.08 \pm 0.02. \quad (7.3)
\]

For the gauge coupling constants, we use \([17]\)

\[
\alpha(m_Z) = (128.89 \pm 0.09)^{-1}, \quad \sin^2 \theta_W = 0.23165 \pm 0.000024,
\]

\[
\alpha_3(m_Z) = 0.118 \pm 0.003. \quad (7.4)
\]

The input value of Higgs boson \( m_H \) is \( \sqrt{2}\Lambda_W = 246.2 \) GeV.

If the input value \( m_H \) is less than \( 2.2 \times 10^2 \) GeV (2.3 \( \times \) \( 10^2 \) GeV) for two (one) loop evaluations, then the quartic coupling constant \( \lambda \), of the Higgs boson self interaction, becomes negative at high energy. On the other hand, if the input value \( m_H \) is more than \( 2.6 \times 10^2 \) GeV for both two and one loop calculations, the burst of \( \lambda \) occurs at high energy. In Fig. 7, we illustrate the \( \mu \)-dependency of the Yukawa coupling constants \( y_q(\mu) \) \((q = u, d, s, c, b, t)\) which take the renormalization equation into account.

Fig. 7. Evolution of the Yukawa coupling constants
8. Summary

We have evaluated the running quark mass values $m_q(\mu)$ at various energy scales below $\mu = \Lambda_W$ and the Yukawa coupling constants of the standard model with a single Higgs boson at energy scales above $\mu = \Lambda_W$. Although we have used the renormalization equation, the perturbative calculation below $\mu \sim 0.7$ GeV is not adequately reliable because the values of the second and third terms in the \{ of perturbative series (4.1) and (5.6) exceed one less than $\mu \simeq 0.7$ GeV.

We discuss the grade of parameters fitted in mass matrix models. At present, the “confidence” grade of the “observed” values of the running quark masses $m_q(\mu)$ and CKM matrix parameters are not equal at the same levels because these values are highly dependent on models or other experimental values (input values). Therefore, it is important for the model-building of quark mass matrix that we know the confidence levels of these values. Our opinion based on the present work is summarized in the following Table:

| grade           | CKM matrix element | quark mass ratio |
|-----------------|-------------------|-----------------|
| (i) (Reliable)  | $|V_{us}|$         |                 |
| (ii) (Almost reliable) | $|V_{cb}|$ | $m_d/m_s \ m_c/m_b \ m_b/m_t$ |
| (iii) (Somewhat variable) | $|V_{ub}|$ | $m_c/m_t \ m_u/m_c \ m_s/m_c$ |
| (iv) (Variable) |       | $m_u/m_d \ m_d/m_b \ m_s/m_b$ |
| (v) (Unreliable) | $|V_{td}|$         |                 |

We have classified the CKM matrix elements on the basis of the experimental and theoretical errors. In grading the quark mass ratios, we have considered ratios are reliable in the cases where (1) both two quarks are heavy quarks or light quarks and (2) the mass difference between two quarks is small.

Finally, we would like to point out that we should use the running mass values of $\mu = m_Z$ rather than $\mu = 1$ GeV for quark mass matrix phenomenology, together with the CKM matrix parameters at $\mu = m_Z$.

Acknowledgments

The authors would like to express their sincere thanks to Prof. M. Tanimoto for his helpful discussions and Prof. Z. Hioki for informing the new values of elec-
troweak parameters. This work was supported by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan (No.08640386).

References

[1] J. Gasser and H. Leutwyler, Phys. Rep. 87, 77 (1982).

[2] O. V. Tarasov, Dubna preprint JINR P2-82-900, 1982 (unpublished).

[3] T. P. Cheng, E. Eichten and L. F. Li, Phys. Rev. D9, 2259 (1974); M. Machacek and M. Voughn, Nucl. Phys. B236, 221 (1984).

[4] W. Bernreuther, Ann. Phys. 151, 127 (1983); I. Hinchiliffe, p.77 in Particle data group, R. M. Barnet et al., Phys. Rev. D54, 1 (1996).

[5] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 73, 225 (1994).

[6] C. A. Dominguez and E. de Rafael, Ann. Phys. 174, 372 (1987).

[7] S. Narison, Phys. Lett. B358, 113 (1995).

[8] H. Leutwyler, Phys. Lett. B378, 313 (1996).

[9] S. Titard and F. J. Yudurán, Phys. Rev. D49, 6007 (1994).

[10] S. Narison, Phys. Lett. B341, 73 (1994).

[11] CDF collaboration, F. Abe et al., Phys. Rev. Lett. 74, 2626 (1995).

[12] D0 collaboration, S. Abachi et al., Phys. Lett. 74, 2632 (1995).

[13] Particle data group, R. M. Barnet et al., Phys. Rev. D54, 1 (1996).

[14] O. V. Tarasov, A. A. Vladimirov and A. Yu. Zharkov, Phys. Lett. B93, 429 (1980).

[15] N. Gray, D. J. Broadhurst, W. Grafe and K. Schilcher, Z. Phys. C48, 673 (1990).

[16] W. Bernreuther and W. Wetzel, Nucl. Phys. B197, 228 (1982); W. Bernreuther, Ann. Phys. 151, 127 (1983); S. A. Larin, T. van Ritbergen and J. A. M. Vermaseren, Nucl. Phys. B438, 278 (1995).
W. Hollik, Invited talk at 11th Topical Workshop on Proton-Antiproton Collider Physics, Padua, Italy, May 26 – June 1, 1996, Univ. Karlsruhe preprint KA-TP-19-1996 (1996). See, also, Z. Hioki, Act. Phys. Polonica B27, 1569 (1996).