1 Introduction

With the miniaturisation and multifunction of the combat platform, the integration of radar and communication has become a hot topic in current research. At present, the researches of integrated waveform can be roughly divided into two types [1, 2]. The first type is based on the integration of hardware, of which communication and radar adopt different waveforms, and then merges and separates signal through data processing. The other type is to design an integrated waveform for radar and communication, of which radar waveform is modulated with communication data to achieve integration.

Obviously, the second approach, signal sharing is more integrated. Researchers have put forward many methods and produced a massive relevant achievements. An integrated radar and communication system based on pseudo-random (PN) sequence spread spectrum has been proposed in [3]. However, for bipolar PN sequence, it is difficult to obtain ideal autocorrelation property and cross-correlation property. Poor Doppler tolerance is one of the main drawbacks of bipolar PN sequence, polyphase sequence set [4, 5] was developed to optimise the problem. However, P sequence did not readily scale to communication as only a single sequence is provided. Due to the problem of Doppler sensitivity of binary phase code and the Welch limit [6] of polyphase single code, an integrated radar communication system based on complete complementary codes (CCC) is proposed in [7]. As the autocorrelation sum of each sequence set is zero (except for zero shift) and the cross-correlation sum is zero for all shift, CCC is a good choice as a spread spectrum sequence. What’s more, the large advances in modern integrated circuit technologies facilitate an efficient implementation of polyphase CCC designs. In this paper, we derive the ambiguity function based on CCC and establish the integration system based on CCC spread spectrum sequence. The influence of Doppler shift on autocorrelation is also evaluated. As a result, we prove the superiority of the CCC in the integrated waveform design.

The remainder of this paper is organised as follows. Section 2 introduces the principle of CCC and derives its ambiguity function in detail. In Section 3, an integrated model is established. The simulation and analysis of ambiguity function and the integrated performance are given in Section 4. Finally, Section 5 concludes the paper.

2 Complete complementary codes

2.1 Principle of CCC

Researches on spreading sequences have proved that the sequence with ideal correlation properties does not exist regardless of binary multi-variable or polyphase single-code sequence. The maximum autocorrelation edge peak and the zero cross-correlation value of the single-code sequence set cannot achieve at the same time; they are subject to theoretical restrictions (such as Welch limit). Based on above analysis, researchers put more emphasis on the complex sequence and then came up with the concept of CCC.

Suppose two pairs of complementary sequences \( \{ A_0, B_0 \} \) and \( \{ A_1, B_1 \} \), where \( A_i = \{ a_0^i, a_1^i, \ldots, a_N^i \} \), \( B_i = \{ b_0^i, b_1^i, \ldots, b_N^i \} \), \( i = \{ 1, 2 \} \). \( N \) is the length of the two pairs of sequences. If their correlation function satisfies the following conditions:

\[
\begin{align*}
R_{A_0B_0}(\tau) + R_{B_0A_0}(\tau) & = \begin{cases} 2N & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \\
R_{A_1A_1}(\tau) + R_{B_1B_1}(\tau) & = \begin{cases} 2N & \tau = 0 \\ 0 & \tau \neq 0 \end{cases} \\
R_{A_0B_1}(\tau) + R_{B_0A_1}(\tau) & = 0 \quad \forall \tau
\end{align*}
\]

Here, \( \tau \) denotes the number of time shifts, \( R_{A_0B_0}(\tau), R_{B_0A_0}(\tau), R_{A_1A_1}(\tau), \) and \( R_{B_1B_1}(\tau) \) denote the auto-correlation function of sequence \( A_0, B_0, A_1, \) and \( B_1, \) respectively. \( R_{A_0B_1}(\tau) \) and \( R_{B_0A_1}(\tau) \) denote the cross-correlation function between two pairs of complementary codes. Then, the two pairs constitute a group of CCC.

2.2 Derivation of ambiguity function

According to the definition of the phase-coded ambiguity function, the ambiguity function of CCC is derived. The two pairs of sequences are still used as an example. For the CCC, because of the existence of two-coded signals, it is necessary to transmit at different times at different antennas with different frequency. Assuming that the time \( t \) launches the \( A_i \) code and the time \( t + T_p \) launches the \( B_i \) code, every sequence is composed of \( N \) sub-pulses of length \( T_p \), the expression of \( u_{A_i}(t) \) and \( u_{B_i}(t) \) are as follows:

\[
u_{A_i}(t) = \begin{cases} \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} a_k^i e^{j \pi (t-kT_p)} & -NT_p < t < 0 \\ 0 & \text{else} \end{cases}
\]
\[
    u_b(t) = \begin{cases} 
    \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} b_k^* e^{j(t - kT_p)} & 0 < t < NT_p \\
    0 & \text{else} \end{cases} 
\]  

Then, \( u_{(A,B)}(t) \) can be expressed as 
\[
    u_{(A,B)}(t) = \begin{cases} 
    u_A(t) + u_B(t) & -NT_p < t < NT_p \\
    0 & \text{else} \end{cases} 
\]

According to the definition of ambiguity function, the ambiguity functions of code \( A \) and code \( B \) are given by
\[
    \chi_{A}(\tau, f_d) = \int_{-\infty}^{\infty} u_A(t) * u_A^*(t + \tau) * e^{j2\pi ft_d} dt 
\]
\[
    \chi_{B}(\tau, f_d) = \int_{-\infty}^{\infty} u_B(t) * u_B^*(t + \tau) * e^{j2\pi ft_d} dt 
\]

where \((\cdot)^*\) denotes the complex conjugate of the argument and \( f_d \) denotes Doppler shift, respectively. The ambiguity functions of \( \{A, B\} \) can be written as (see (8)). Similarly,
\[
    \chi_{(A,B)}(\tau, f_d) = \chi_{A}(\tau, f_d) + \chi_{B}(\tau, f_d) + \chi_{A,B}(\tau, f_d) 
\]

Then, the ambiguity functions of \( \{A_n, B_n\} \) and \( \{A_i, B_i\} \) are as follows:
\[
    \chi(\tau, f_d) = \chi_{A_n}(\tau, f_d) + \chi_{B_n}(\tau, f_d) + \chi_{A_nB_n}(\tau, f_d) + \chi_{A_nA_iB_i}(\tau, f_d) 
\]

Based on the characteristic of complementary sequences, that is, the sum of sequence cross-correlation function is zero, the following equation can be obtained: (see (11)). By substituting (11) into (10), the ambiguity function can be written as
\[
    \chi(\tau, f_d) = \chi_{A_n}(\tau, f_d) + \chi_{B_n}(\tau, f_d) + \chi_{A_nA_iB_i}(\tau, f_d) 
\]

Using the property of phase-coded signal, phase-encoded signal can be convolved by two sub-functions as
\[
    u(t) = v(t) \otimes \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} c_k e^{j(t - kT_p)} 
\]

where \( \{c_k\} \) is the encoded sequence, and the expression of rectangular pulse function is defined as
\[
    v(t) = \begin{cases} 
    \frac{1}{\sqrt{T_p}} & 0 < t < T_p \\
    0 & \text{else} \end{cases} 
\]

Then, \( \chi_A(t, f_d) \) and \( \chi_B(t, f_d) \) can be decomposed into the convolution of the following two ambiguity functions:
\[
    \chi_{A}(t, f_d) = \chi(t, f_d) \otimes \chi(t, f_d) 
\]
\[
    \chi_{B}(t, f_d) = \chi(t, f_d) \otimes \chi(t, f_d) 
\]

where \( \chi(t, f_d) \) denotes the ambiguity function of a rectangular pulse, \( \chi(t, f_d) \) and \( \chi(t, f_d) \) represent another ambiguity function containing the encoded information's sub-functions. From the definition of CCC and the property of convolution, the sum of (15a) and (15b) can be described as
\[
    \chi(\tau, f_d) + \chi_{B}(\tau, f_d) = \sum_{m=1}^{\infty} \chi(t - mT_p, f_d) \times \chi(t, f_d) + \chi(t, f_d) 
\]

According to (7) and (14), we obtain (see (17) and (18)) (see (18)). According to (1) and (2), we can find that only when \( m = 0 \), the above formula has non-zero value. If \( m \neq 0 \), the value of (18) is always zero. Then,
\[
    \chi(\tau, f_d) + \chi_{B}(\tau, f_d) = 2e^{j\pi f_d(t - |t|)} \sin(\pi f_d T_p) \sin(\pi f_d T_p) \quad |t| < T_p 
\]

Similarly,
\[
    \chi(\tau, f_d) + \chi_{B}(\tau, f_d) = 2e^{j\pi f_d(t - |t|)} \sin(\pi f_d T_p) \sin(\pi f_d T_p) \quad |t| < T_p 
\]
\[ x_{\Delta f}(\tau - mT_p, f_d) = \begin{cases} \frac{\sin[\pi f_d(T_p - |\tau - mT_p|)]}{\pi f_d} & |\tau - mT_p| < T_p \\ 0 & \text{else} \end{cases} \]

\[ x_{\Delta f}(mT_p, f_d) = \begin{cases} \frac{1}{N} \sum_{m=0}^{N-1} (a_m^T a_{m+K} + b_m^T b_{m+K}) e^{j2\pi f_d K T_p} & 0 \leq m \leq N - 1 \\ \frac{1}{N} \sum_{m=0}^{N-1} (a_m^T a_{m+K} + b_m^T b_{m+K}) e^{j2\pi f_d K T_p} & -(N - 1) \leq m \leq 0 \end{cases} \]

\[ x_{\Delta f}(\tau, f_d) = x_{A_B}(\tau, f_d) + x_{B_A}(\tau, f_d) = 4e^{j\pi f_d(T_p - |\tau|)} \sin(\pi f_d T_p) \sin(\pi f_d T_p) |\tau| < T_p \]

Based on (19) and (20), we can get the analytical expression of the ambiguity function of CCC, which is described as (see (21)).

### 3 Integrated system modelling

Fig. 1 presents the integrated signal generation, transmission, and reception block diagram of CCC. As we can see from Fig. 1, the spreading processing of CCC is performed on the information symbols, and then two channel carrier signals with different frequencies at \( F_0 + f_1 \) and \( F_0 + f_2 \) are, respectively, used for modulation. The frequency different between two adjacent carriers is inverse number of the sub-pulse width. After receiving signals are demodulated and processed through a low-pass filter, the correlating results of codes \( A_0, A_1 \) and codes \( B_0, B_1 \) are, respectively, at Out₁ and Out₂. Then, radar echo signal can be obtained. Total output \( \text{Out}_3 \) is the sum of \( \text{Out}_1 \) and \( \text{Out}_2 \), which reflect the communication data information.

### 4 Simulation and analysis

#### 4.1 Ambiguity function

According to [8], we consider a set of four-phase complete complementary sequence of code length \( N = 40 \). Fig. 2 shows the ambiguity function, ambiguity contour plot, autocorrelation magnitude, and normalised amplitude, respectively. In Fig. 2a, we can see that the ambiguity function of CCC is ‘drawing pin’. The narrow central peak means high distance resolution and Doppler resolution. Fig. 2c shows that the autocorrelation function of CCC is only the main lobe and no side lobe at the zero Doppler cut, which meets requirements of many radar applications with focus being on good delay-Doppler tolerance. Furthermore, the normalised amplitude shows a distinct peak at the zero delay cut, which is good for the separation between different sequences and thus supports multiple-access in communication field.

#### 4.2 Doppler shift

The influence of the Doppler shift is evaluated here. The peak side lobe ratio (PSLR) of the autocorrelation function is an important index to measure the performance of pulse compression coding. For the integrated CCC spread spectrum sequence, this section provides an example to testify the property of autocorrelation function with Doppler shift. The results of normalised Doppler shift at \( f_{\text{ind}} = 0.01 \) and \( f_{\text{ind}} = 0.04 \) are shown in Figs. 3a and 3b, respectively. The P4 code with code length \( N = 40 \) is also shown here in order to compare with CCC. The normalised Doppler frequency \( f_{\text{ind}} \) is normalised by \( 1/T_P \). Obviously, in the case of the same SNR and Doppler shift, the peak amplitude of CCC is four times that of the P4 code, which is the same as the result we deduce in the Section 2. Meanwhile, for CCC, the main lobe is still stable as Doppler shift increases. Although the side lobe of CCC is larger than the P4 code when the Doppler shift is \( f_{\text{ind}} = 0.04 \), its’ PSLR is still higher than that of P4 code.

In addition, Fig. 4 shows the PSLR of the sequence with the normalised Doppler shift increasing from 0.01 to 0.05. As we can see, with the increase of Doppler shift, the PSLR of CCC decreases. The PSLR of CCC decreases dramatically, but its PSLR is still higher than P4 code. The higher PSLR of CCC implies that CCC performs better in pulse compression performance and communication.

### 5 Conclusion

In this paper, we introduce the complete complementary codes into the design of integrated radar and communication system. On the one hand, we deduce the analytical expression of the ambiguity function of CCC, which presents good autocorrelation and cross-correlation property. On the other hand, to make use of such property of CCC, an integrated system based on CCC is established. The simulation example verifies that the ambiguity function of CCC has an excellent characteristic of correlation property comparing with P4 code, even the Doppler shift increases.
Fig. 2 Ambiguity function of complete complementary codes
(a) Ambiguity function, (b) Ambiguity contour, (c) Zero Doppler section, (d) Zero delay section

Fig. 3 Autocorrelation function with Doppler shift
(a) Normalised Doppler shift \( f_{\text{sd}} = 0.01 \), (b) Normalised Doppler shift \( f_{\text{sd}} = 0.04 \)

Fig. 4 PLSR with normalised Doppler shift \( f_{\text{sd}} \)

6 References

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