Universal spin-resolved Kirchhoff’s laws of thermal radiation for nonreciprocal planar media

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A chiral absorber of light can emit spin-polarized (circularly polarized) thermal radiation based on Kirchhoff’s law which is applicable only for reciprocal media at thermal equilibrium. No such law exists for nonreciprocal media. Here we prove three spin-resolved Kirchhoff’s laws of thermal radiation applicable for reciprocal and nonreciprocal planar media. These laws are applicable to birefringent crystals (uniaxial or biaxial), Weyl semimetals, magnetized insulators, semiconductors and plasmas (gyroelectric/gyromagnetic media) as well as magneto-electric topological insulators, metamaterials and multi-ferroic media. We further propose an experiment to verify these laws using a single system of doped Indium Antimonide (InSb) thin film in an external magnetic field. Our work highlights the fundamentally intriguing role of photon spin in the context of nonreciprocal media and paves the way for novel practical applications based on nonreciprocal thermal systems.

One formulation of Kirchhoff’s law of thermal radiation states that emissivity (η) is equal to absorptivity (α) for both left circular polarization (LCP) and right circular polarization (RCP) states. Its mathematical form is:

\[ \eta_{(+,-)}(\omega, \mathbf{n}) = \alpha_{(+,-)}(\omega, \mathbf{n}) \]

where (+) denotes RCP, (-) denotes LCP, \( \omega \) is the frequency and \( \mathbf{n} \) denotes the direction. This spin-resolved Kirchhoff’s law is valid only for reciprocal media at thermal equilibrium. Based on it, many works have designed reciprocal chiral absorbers [1–3] which can emit partially spin-polarized thermal radiation and few works have demonstrated it in experiments [4, 5]. This conventional law is not applicable for nonreciprocal media (with broken time reversal symmetry) such as semiconductors in external magnetic fields. Naturally, the question then arises whether new forms of Kirchhoff’s laws exist for nonreciprocal media [6].

Thermal radiation from nonreciprocal media has been explored recently but primarily in the context of radiative heat transfer [7, 8]. Here we focus on photon-spin-related phenomena. Interestingly, that leads us to discover three modified Kirchhoff’s laws for planar media which reveal themselves only upon analyzing spin-resolved thermal emission and absorption. While the first law for the reciprocal media is intuitively expected, the other two laws are new. They demonstrate a subtle balance of spin-resolved emission and absorption of thermal radiation, which maintains thermal equilibrium of non-reciprocal planar media with the surrounding. We further propose an experiment which can validate these laws conveniently using a single material system of a doped InSb thin film in the presence of magnetic field.

We prove these laws based on thermodynamic balance of energy and angular momentum exchange as well as Rytov’s fluctuational electrodynamics. They are derived for several classes of non-reciprocal media including gyrotropic semiconductors or metals in a magnetic field, Weyl semimetals, ferromagnetic materials as well as magneto-electric topological insulators, metamaterials and multi-ferroics. While our previous work [9] provides a similar comprehensive analysis of photon spin in the thermal near field, this work reports surprising features in the far field. Fundamentally, these inquiries highlight and motivate new connections between photon spin and electromagnetic nonreciprocity. Practically, they will be useful for developing novel functionalities such as directional radiative heat transfer [7, 9], optimized energy harvesting [10], spin-polarized light sources [11], and fluctuations-induced torque [12, 13].

**Theory:** We consider an extended planar slab much larger than the thermal wavelength at thermal equilibrium with the blackbody radiation (ideal gas of photons). As shown in fig.1(a), the power emitted per unit surface area \( dA \) of the body at an angle \( \theta \) to the surface normal within the solid angle \( d\Omega \) per unit frequency interval \( d\omega \) is given by,

\[ P_{\text{rad}}(\omega, \theta, \phi, \hat{e}) = \eta(\omega, \theta, \phi, \hat{e}) I_b(\omega, T) \frac{\cos \theta d\omega d\Omega dA}{2} \]

where \( \eta \) is the dimensionless emissivity dependent on frequency \( \omega \), emission direction \( \theta, \phi \) in spherical coordinates and orthogonal polarization states \( \hat{e} \). The blackbody radiance at temperature \( T \) given by \( I_b(\omega, T) = \omega^2 \Theta(\omega, T) / (4\pi^3 c^2) \) with \( \Theta(\omega, T) = h\omega / [\exp(h\omega/k_BT) - 1] \) being the Planck’s function, is divided by two to account for two polarization states separately. Similarly, the power absorbed per unit surface area \( dA \) due to the blackbody radiance incident at an angle \( \theta \) to the surface normal within a solid angle \( d\Omega \) per unit frequency interval \( d\omega \) is,

\[ P_{\text{abs}}(\omega, \theta, \phi, \hat{e}) = \alpha(\omega, \theta, \phi, \hat{e}) I_b(\omega, T) \frac{\cos \theta d\omega d\Omega dA}{2} \]

For linear material response and regular reflection from the surface, the frequency \( \omega \) and the angle \( \theta \) are not changed upon reflection. Hence, we can
FIG. 1. (a) We provide a universal perspective of spin-resolved thermal radiation from reciprocal and nonreciprocal planar media. The figure shows the typical conventions used to describe the thermal radiation from the planar surface in a specific direction. Figure (b) illustrates the spectral radiance of thermal photons in \( (\theta, \phi) \) direction when the planar slab is at thermal equilibrium with the surrounding vacuum. These energy flux rates are described using the spin-resolved blackbody radiance \( (I_{\pm}) \), emissivities \( (\eta_{\pm}) \) and interconversion reflectances \( (R_{\pm \mp}) \) for radiation incident along the conjugate \( (\theta, \phi + \pi) \) direction.

apply the principle of total energy conservation for energy-exchange channels characterized by \( \omega, \theta \) separately and focus on the polarization or spin-dependent properties. Instead of the usual \( s, p \)-polarization basis, we consider RCP \((\hat{e}_{+})\) and LCP \((\hat{e}_{-})\) polarization states and show that this spin-resolved analysis provides useful, new insights related to the angular momentum of thermal radiation. The circular polarization basis states in the basis of s,p polarization states \((\hat{e}_{s}, \hat{e}_{p})\) are \(\hat{e}_{\pm} = (\hat{e}_{s} \pm i\hat{e}_{p})/\sqrt{2}\).

Figure 1(b) depicts the energy flux rates at frequency \( \omega \) in the direction \( (\theta, \phi) \) which contain emitted, reflected and incident radiation. We describe these fluxes at equilibrium within the radiometry paradigm [14]. As shown in the rightmost figure, the incident radiation contains both RCP \((I_{++})\) and LCP \((I_{--})\) radiation. Since the incident blackbody radiation is unpolarized, it follows that \(I_{++} = I_{--} = I_{0}/2\). The emitted radiation is described using the spin-resolved emissivities as \(\eta_{++}I_{++}(\theta, \phi, \theta, \phi + \pi)\) and \(\eta_{--}I_{--}\). The reflected radiation arises from the radiation incident along the conjugate direction \( (\theta, \phi + \pi) \) and is described using the polarization interconversion reflectances. For instance, as shown in the second figure in 1(b), the incident RCP radiation \((I_{++})\) gets reflected as \(R_{++}(\theta, \phi + \pi)I_{++}(\theta, \phi, \theta, \phi + \pi)\) (RCP) and \(R_{--}(\theta, \phi + \pi)I_{++}(\theta, \phi, \theta, \phi + \pi)\) (LCP) along \( (\theta, \phi) \) direction. These interconversion reflectances can be measured separately in an experiment or can be calculated for the planar slab using the well-known Fresnel reflection coefficients \((r_{ss}, r_{sp}, r_{ps}, r_{pp})\) with the help of following expressions (omitting the dependence on \( \theta, \phi \) for brevity):

\[
R_{++/-} = \left| (r_{ss} + r_{ps}) \pm i(r_{sp} - r_{pp}) \right|^2 / 4
\]

\[
R_{--/+} = \left| (r_{ss} - r_{ps}) \pm i(r_{sp} + r_{pp}) \right|^2 / 4
\]

The Fresnel reflection coefficient \(r_{jk}\) for \( j,k = [s,p] \) denotes the amplitude k-polarized incident wave due to unit amplitude j-polarized reflected wave due to unit amplitude k-polarized incident wave. Since the overall radiation in the far-field is isotropic and unpolarized at thermal equilibrium, the spin-resolved energy flux rates in opposite directions are equal. This is further justified by our previous work [9] which shows that the equilibrium spin angular momentum density of thermal radiation is always zero in the far-field although it can be nonzero in the near-field of certain nonreciprocal media. By equating the incoming and outgoing radiation flux rates in the far-field, we obtain the following spin-resolved emissivities in terms of reflectances:

\[
\eta_{++}(\theta, \phi) = 1 - R_{++}(\theta, \phi, \theta, \phi + \pi) - R_{--}(\theta, \phi + \pi)
\]

\[
\eta_{--}(\theta, \phi) = 1 - R_{--}(\theta, \phi, \theta, \phi + \pi) - R_{++}(\theta, \phi + \pi)
\]

Similarly, it is straightforward to obtain the spin-resolved absorptivities by considering the reflection of incident polarized radiation \(I_{\pm}\) separately. By subtracting the reflected flux from the incident radiation flux, we obtain the following spin-resolved absorptivities:

\[
\alpha_{++}(\theta, \phi) = 1 - R_{++}(\theta, \phi, \theta, \phi + \pi) - R_{--}(\theta, \phi + \pi)
\]

\[
\alpha_{--}(\theta, \phi) = 1 - R_{--}(\theta, \phi, \theta, \phi + \pi) - R_{++}(\theta, \phi + \pi)
\]

We note that the emissivities (Eq.4) can also be obtained within the scattering matrix formulation of fluctuational electrodynamics theory as described in the supplement. We further validate these expressions (Eqs. 4, 5) in the supplement by proving thermodynamic consistency condition of zero angular momentum exchange of the planar slab with the environment at thermal equilibrium. In the following, we analyze the far-field thermal emission from reciprocal and nonreciprocal planar media.

As a superset of all types of materials, we consider a bianisotropic medium whose optical properties can be described using the following constitutive relations (in the frequency domain) assuming local material response:

\[
D = \varepsilon\varepsilon_{0}E + \frac{\varepsilon_{1}}{c}H, \quad B = \frac{\varepsilon_{1}}{c}E + \mu\mu_{0}H
\]

where \(\varepsilon,\mu\) are dimensionless permittivity and permeability tensors and \(\varepsilon_{1},\mu_{1}\) are magneto-electric coupling tensors. Such a material is reciprocal if these parameters satisfy the conditions \(\varepsilon = \varepsilon^{T},\mu = \mu^{T},\varepsilon_{1} = -\varepsilon_{1}^{T}\) where \((\cdot)^{T}\) denotes the matrix transpose. It is nonreciprocal if any one of these conditions is violated. We consider a list of several material classes as shown in Table I, which includes isotropic, uni/biaxial anisotropic, gyroelectric, gyromagnetic and...
TABLE I. Reciprocal and nonreciprocal material classes

| No. | Material Class | Description | Examples | Kirchhoff’s law |
|-----|----------------|-------------|----------|-----------------|
| 1   | Reciprocal isotropic | $\mathcal{P}, \mathcal{P}$ are scalars, $\xi = \zeta = 0$ | Common dielectric and metallic materials | SKL-1 |
| 2   | Reciprocal anisotropic | Diagonal $\mathcal{P}$ with unequal entries, scalar $\mathcal{P}$, $\xi = \zeta = 0$ | Uniaxial and biaxial crystals [15] | SKL-1 |
| 3   | Nonreciprocal gyroelectric | $\mathcal{P} \neq \mathcal{P}'$, scalar $\mathcal{P}$, $\xi = \zeta = 0$. | Weyl semimetals [16], metals and semiconductors in magnetic field [17, 18] | SKL-2,3 |
| 4   | Nonreciprocal gyromagnetic | $\mathcal{P} \neq \mathcal{P}'$, scalar $\mathcal{P}$, $\xi = \zeta = 0$. | Ferromagnets, ferrites [19] | SKL-2,3 |
| 5   | Reciprocal magneto-electric | scalar $\mathcal{P}$ and $\mathcal{P}$, nonzero $\xi = -\zeta$. | Chiral metamaterials [20, 21] | SKL-1 |
| 6   | Nonreciprocal magneto-electric | scalar $\mathcal{P}$ and $\mathcal{P}$, nonzero $\xi = \zeta'$. | Topological insulators [22], Multi-ferroic media [23] | SKL-2,3 |

Spin-resolved Kirchhoff’s law for reciprocal materials: For a reciprocal bianisotropic planar slab, by writing the Green’s function on the vacuum side of the geometry, we can infer the following relations [24]:

\[
\begin{align*}
 r_{ss,pp}(\theta, \phi) &= r_{ss,pp}(\theta, \phi + \pi), \\
 r_{sp}(\theta, \phi) &= -r_{ps}(\theta, \phi + \pi)
\end{align*}
\]

Using these relations in Eqs.3,4 and 5, we find that the spin-resolved emissivity is equal to the spin-resolved absorptivity for reciprocal media:

\[
\begin{align*}
\eta_+(\omega, \theta, \phi) &= \alpha_+(\omega, \theta, \phi)
\end{align*}
\]

Spin-resolved Kirchhoff’s laws for nonreciprocal materials: We numerically find that for planar slabs of most nonreciprocal material classes, the Fresnel reflection coefficients satisfy other relations which allow us to obtain new spin-resolved Kirchhoff’s laws. In particular, for gyrotropic materials with the gyrotropy axis perpendicular to the planar surface and for nonreciprocal, (isotropic) magneto-electric materials with diagonal magneto-electric coupling, we find the following relations between the reflection coefficients:

\[
\begin{align*}
 r_{ss,sp,ps,pp}(\theta, \phi) &= r_{ss,sp,ps,pp}(\theta, \phi + \pi), \\
 r_{sp}(\theta, \phi) &= r_{ps}(\theta, \phi + \pi)
\end{align*}
\]

Using these relations in Eqs.3,4 and 5, we obtain:

\[
\begin{align*}
\eta_+(\omega, \theta, \phi) &= \alpha_{(+)}(\omega, \theta, \phi), \\
\eta_-(\omega, \theta, \phi) &= \alpha_{(-)}(\omega, \theta, \phi)
\end{align*}
\]

Based on this analysis, we provide three spin-resolved Kirchhoff’s laws (SKL) of thermal radiation:

- **SKL-1** [Eq.7]: For all reciprocal planar media, the spin-resolved emissivity in $(\theta, \phi)$ direction is equal to the spin-resolved absorptivity in the same direction for each spin (polarization) state.

- **SKL-2** [Eq.8]: For nonreciprocal planar media that preserve the rotational symmetry in the plane of interface such as gyrotropic media with gyrotropy axis perpendicular to the surface and isotropic magneto-electric media, the spin-resolved emissivity in $(\theta, \phi)$ direction is equal to the spin-resolved absorptivity in the same direction but of opposite spin state.

- **SKL-3** [Eq.9]: For gyrotropic media with gyrotropy axis parallel to the surface and anisotropic nonreciprocal magneto-electric media which lead to cross coupling between the components of $\mathbf{E}, \mathbf{H}$ fields lying parallel and perpendicular to the interface, the spin-resolved emissivity in the direction $(\theta, \phi)$ is equal to the spin-resolved absorptivity for the conjugate direction $(\theta, \phi + \pi)$ for each spin state.

We note that the above derivation is for a semi-infinite planar slab, but the laws also hold for planar slabs of non-reciprocal magneto-electric materials. We find that the Fresnel reflection coefficients which determine the polarization reflectances (Eq.3) satisfy certain conditions for most of these material classes. This leads to simplifying relations between the spin-resolved emissivities (Eq.4) and absorptivities (Eq.5) which are called as the spin-resolved Kirchhoff’s laws.

Using these relations in Eqs.3,4 and 5, we find that the spin-resolved emissivity is equal to the spin-resolved absorptivity for reciprocal media:

\[
\begin{align*}
\eta_+(\omega, \theta, \phi) &= \alpha_+(\omega, \theta, \phi) \\
\eta_-(\omega, \theta, \phi) &= \alpha_-(\omega, \theta, \phi)
\end{align*}
\]
of finite thickness on a typically considered substrate of isotropic medium (Table I), as we show in the experimental proposal. Furthermore, these laws also hold for finite planar slabs with vacuum or air on both sides as demonstrated separately in the supplement. While SKL-1 for reciprocal media is intuitively expected, the laws for nonreciprocal planar media (SKL-2 and SKL-3) are new and subtle. We note that these laws are not applicable for nonreciprocal anisotropic magnetoelectric media which cause $E_x - H_y$ or $E_y - H_x$ cross coupling and for composite planar media which combine (non-isotropic) material types following different spin-resolved Kirchhoff’s laws. Despite these limiting cases, these laws will be useful for thermal-radiation engineering for many nonreciprocal media noted above for all frequencies and emission directions.

**Experimental proposal:** We consider a doped Indium Antimonide (InSb) slab of thickness $t = 1\mu m$ and doping concentration $n = 10^{17} cm^{-3}$ (available at vendors like MTI Corp.) on top of a glass substrate of constant permittivity $\varepsilon_r = 2.25$. We use the experimentally well-characterized Drude-Lorentz oscillator model [25–27] to calculate the permittivity of InSb with or without applied magnetic field [9]. In the absence of magnetic field, InSb slab has isotropic permittivity ($\bar{\varepsilon}$) and permeability ($\bar{\mu}$), and $\bar{\mu}/\bar{\varepsilon} = 0$. Therefore, it is reciprocal in nature and follows SKL-1. In the presence of magnetic field, $\bar{\varepsilon}$ has nonzero off-diagonal entries and InSb slab acts as a gyrotropic medium with the gyrotropy axis parallel to the applied field. Consequently, when the magnetic field is perpendicular or parallel to the surface, thermal radiation from InSb slab will follow either SKL-2 or SKL-3, respectively. In an experiment, the measurement of spin-resolved emissivities (Eq.4) and absorptivities (Eq.5) can be performed using a specific combination of polarizers and quarter-wave-plate optical components [5] in suitable directions of emission. Here, we obtain them numerically as a function of frequency.

Figure 2 summarizes the three spin-resolved Kirchhoff’s laws for the InSb slab (top schematic) and figures (a,b,c) demonstrate the calculated spectra of emissivities (left figures) and absorptivities (right figures). For brevity, we focus on the direction $(\theta = \pi/4, \phi = \pi/4)$. As shown in fig. 2(a), in the absence of magnetic field, the spin-resolved emissivities and absorptivities are equal for both spin states, RCP (red) and LCP (blue). The plots lie on top of each other. Figure 2(b) demonstrates SKL-2, where a magnetic field of strength 1T is applied perpendicular to the slab. Evidently, $\eta(+) = \alpha(-)$ and $\eta(-) = \alpha(+)$, which verifies SKL-2. Since $\eta(+) \neq \eta(-)$ in the given emission direction, the thermal emission from the slab will be partially spin-polarized. In figure 2(c), we consider a magnetic field of strength 2T applied along the $x$-axis of the geometry. This example demonstrates that spin-resolved emissivities in $(\theta, \phi)$ direction are equal to spin-resolved absorptivities for the same spin state but in the conjugate $(\theta, \phi + \pi)$ direction. Thus, SKL-3 can be verified.

**Conclusion:** We proved three spin-resolved Kirchhoff’s laws of thermal radiation applicable for reciprocal and a vast majority of nonreciprocal planar media. We proposed an experiment to verify these laws conveniently using a single material. The extension of our analysis for the calculation of a nonequilibrium fluctuations-induced torque [12] (discussed in the supplement) and for 2D materials is straightforward. Our work reveals surprising connections between photon spin and nonreciprocity, motivates new analogies for a broad set of materials and
invites new fundamental explorations.

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Universal spin-resolved Kirchhoff’s laws of thermal radiation for nonreciprocal planar media: Supplementary Materials

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We show that the emissivities derived within the radiometry paradigm in the main manuscript can be obtained using Rytov’s fluctuational electrodynamics. We further validate the expressions for spin-resolved emissivities and absorbivities by showing that they satisfy the thermodynamic consistency condition that the net exchange of angular momentum between the planar slab and its environment is zero at thermal equilibrium. We also describe the calculation of a fluctuations-induced torque that can be experienced by a planar slab upon emitting partially spin-polarized thermal radiation into the external environment at low temperature. Finally, we derive the spin-resolved Kirchhoff’s laws for planar slabs of finite thickness with vacuum on both sides, where the transmittances must also be taken into account in addition to reflectances.

I. FLUCTUATIONAL ELECTRODYNAMIC DERIVATION OF EMISSIVITY

The following basis-independent trace formula for the thermal emission power from an object at temperature $T$ into the external vacuum at temperature $T_e$ in terms of its scattering operator ($S$) has been derived previously [1–3]:

$$H = \int \frac{d\omega}{2\pi} [\Theta(\omega, T) - \Theta(\omega, T_e)] \text{Tr}\{ I - SS^\dagger \}$$

Substituting the scattering matrix for the planar interface in the basis of RCP ($\hat{e}_{(+)})$ and LCP ($\hat{e}_{(-)}$) plane waves, we obtain:

$$H_s = \int \frac{d\omega}{2\pi} [\Theta(\omega, T) - \Theta(\omega, T_e)] \text{Tr}\{ \int_0^{\omega_0} dk |k||dk| \int_0^{2\pi} d\phi \left( I - \begin{bmatrix} r_{(++)} & r_{(+-)} \r_{(--)} & r_{(-+)} \end{bmatrix} \begin{bmatrix} r_{(++)}^* & r_{(+-)}^* \r_{(--)}^* & r_{(-+)}^* \end{bmatrix} \right) \}$$

Here $H_s$ is the Poynting flux or heat radiation per unit area of the slab in the normal ($\hat{e}_z$) direction and $r_{(jk)}$ denotes the amplitude of j-polarized reflected circularly polarized (CP) wave due to unit amplitude k-polarized incident CP wave. $R_{(jk)} = |r_{(jk)}|^2$ denotes the polarization interconversion reflectance used in the calculation of emissivities and absorbivities. $k_\parallel$ is the in-plane wavevector which is related to the vacuum wavevector $k_0 = \omega/c$ as $k_\parallel = k_0 \sin \theta$. Using $d\Omega = \sin \theta d\theta d\phi$ and assuming the surrounding to be at $T_e = 0K$, we obtain:

$$H_s = \int d\omega \int d\Omega \left[ 1 - R_{(+)}(\theta, \phi) - R_{(-)}(\theta, \phi) + 1 - R_{(+)}(\theta, \phi) - R_{(-)}(\theta, \phi) \right] \frac{I_b(\omega, T)}{2} \cos \theta$$

Here $I_b(\omega, T) = \omega^2 \Theta(\omega, T)/(4\pi^3c^2)$ is the blackbody radiance at temperature $T$ and $\Theta(\omega, T) = h\omega/\exp(h\omega/k_B T) - 1$ is the Planck’s function. It follows that the integrand of the above expression reproduces the expression for the emitted power per unit area ($dA$), per unit frequency $d\omega$ within the solid angle $d\Omega$ given by Eq.1 in manuscript. We note that this derivation is not new and has been presented before for isotropic planar media and in the usual basis of (s,p)-polarization states [1]. A derivation based on the second kind of fluctuation-dissipation theorem can be found for isotropic, dielectric/metals materials in Ref. [4] and for nonreciprocal gyroelectric or magneto-optic planar media in Ref. [5].

II. PHOTONIC ANGULAR MOMENTUM EXCHANGE AT THERMAL EQUILIBRIUM

A circularly polarized plane wave normalized to describe a single photon carries a net angular momentum of $\pm \hbar$ along its direction of propagation. It carries only spin angular momentum since the orbital contributions are zero. Intrinsic orbital angular momentum is zero because of the plane wavefront and extrinsic orbital angular momentum
is zero because of the cancellation over its infinite transverse extent [6, 7]. It also follows from the spin angular momentum density expression \([\sim \text{Im}\{E^*(\omega) \times E(\omega)\}]\) that there are no cross-polarization interaction terms in the net angular momentum when the radiation consists of both RCP \((\hat{e}_x(+)\)) and LCP \((\hat{e}_z(-))\) photons. Therefore, similar to the energy flux, the angular momentum flux can also be considered separately for both polarization states.

We first obtain the rate of change of angular momentum along \(z\)-direction due to emission, absorption and reflection of thermal photons. Using Eq. 1 in the manuscript, the number of RCP and LCP photons per unit time \((dn/\,dt)\) carrying angular momentum \(+h\) and \(-h\) respectively along the emission direction \((\theta, \phi)\) is:

\[
\frac{dn_\pm}{dt} = \eta_\pm(\omega, \theta, \phi) \frac{I_b}{2\hbar\omega} \cos \theta \, d\omega \, d\Omega \, dA
\]  

(4)

The rate of change of angular momentum along \((+\)\(z\))-direction per unit area \(dA\) of the planar body per unit frequency \(d\omega\) due to the emission, absorption and reflection of photons travelling in \((\theta, \phi)\) direction within the solid angle \(d\Omega\) is:

\[
\frac{dJ_z(\theta, \phi)}{dt} = \left[ -[\eta_+(\omega) - \eta_-(\omega)] - [\alpha_+(\omega) - \alpha_-(\omega)] + [2R_+(\omega) - 2R_-(\omega)] \right] \frac{I_b}{2\hbar\omega} \cos^2 \theta \, d\omega \, d\Omega \, dA
\]

(5)

Here, the terms inside the brackets depend on \((\omega, \theta, \phi)\) but we have dropped that dependence for brevity and simplicity. The additional \(\cos \theta\) term in the expression denotes the projection of the angular momentum along the \(z\)-direction. The negative sign outside the brackets of the term corresponding to emitted photons indicates the loss of angular momentum due to the emission process. The negative sign outside the term corresponding to the absorbed photons indicates that the associated angular momentum component along \(z\)-direction of the incident photons (which get absorbed) is negative. The term corresponding to the reflected photons is obtained by considering the reflection of both RCP and LCP radiation incident along \((\theta, \phi)\) direction. For instance, for incident RCP \((+)\) radiation, the fraction of photons that gets reflected is \([R_+(+) + R_-(+)]\). The initial angular momentum of these photons along \(z\)-axis is proportional to \(\cos \theta[R_+(+) + R_-(+)]\). When these photons get reflected as both RCP and LCP photons, their final angular momentum is proportional to \(\cos \theta[R_+(+) - R_-(+)]\). The angular momentum imparted to the slab upon reflection is negative of the angular momentum change of these photons. Similarly, we obtain the following expressions for the angular momentum change per unit time (torque) per unit area along \(x\)-axis and \(y\)-axis (parallel to the surface):

\[
\frac{dJ_x(\theta, \phi)}{dt} = \left[ -\eta_+(\omega) + \eta_-(\omega) - \alpha_+(\omega) + \alpha_-(\omega) + 2R_+(\omega) - 2R_-(\omega) \right] \frac{I_b}{2\hbar\omega} \cos \theta \sin \theta \cos \phi \, d\omega \, d\Omega \, dA
\]

(6)

\[
\frac{dJ_y(\theta, \phi)}{dt} = \left[ -\eta_+(\omega) + \eta_-(\omega) - \alpha_+(\omega) + \alpha_-(\omega) + 2R_+(\omega) - 2R_-(\omega) \right] \frac{I_b}{2\hbar\omega} \cos \theta \sin \theta \sin \phi \, d\omega \, d\Omega \, dA
\]

(7)

Integration of the above quantities over the solid angle \((d\Omega)\) and the frequency \((d\omega)\) will yield the total torque per unit area experienced by the planar slab along the respective axes.

Substituting the spin-resolved emissivities (Eq.4 in the manuscript) and absorptivities (Eq.5 in the manuscript), it follows that, for conjugate directions of radiative energy exchange characterized by \((\theta, \phi)\) and \((\theta, \phi + \pi)\), the total rate of change of angular momentum is zero.

\[
\frac{dJ_x(\theta, \phi)}{dt} + \frac{dJ_y(\theta, \phi + \pi)}{dt} = 0, \quad \text{for} \ k=\{x,y,z\}
\]

Consequently, the total torque per unit area obtained by integrating over all directions \((\theta, \phi)\) is always zero (for all materials) at thermal equilibrium with the surrounding.

When the planar slab is at a different temperature than the surrounding environment (out-of-equilibrium), it can experience a fluctuation-induced torque due to the net angular momentum loss via thermal emission process [8]. To compute the thermal emission power or angular momentum transfer rates under out-of-equilibrium conditions, the flux rates are scaled proportionate to the temperature-dependent blackbody radiance \(I_b(\omega, T)\). Assuming the planar slab to be held at constant temperature \(T\) and the external environment is at temperature \(T_e\), it follows that the flux rates corresponding to emitted photons contain the blackbody radiance term \(I_b(\omega, T)\) and those corresponding to absorbed and reflected photons contain the blackbody radiance term \(I_b(\omega, T_e)\). It then follows from the above expressions that the spectral torque per unit area experienced by an out-of-equilibrium planar slab due to thermal emission into the vacuum at \(T_e = 0\,\text{K}\), can be expressed as \(\mathbf{\tau}_t \frac{I_b}{\hbar \omega}\), where \(\mathbf{\tau}_t\) is dimensionless vector quantity obtained by integrating the terms inside square brackets in Eqs. 5,6,7 over all directions. For real, dispersive materials, this expression will be integrated over the frequency to obtain the total torque per unit area.
FIG. 1. This figure illustrates the spectral radiance of thermal photons in \((\theta, \phi)\) direction when the finite thickness planar slab is at thermal equilibrium with the surrounding vacuum on both sides. These energy flux rates are described using the spin-resolved blackbody radiance \((I_{\pm})\), emissivities \((\eta_{\pm})\) and interconversion reflectances and transmittances. The emissivities are obtained by the equality of spin-resolved flux rates in opposite directions in the far-field under thermal equilibrium condition.

### III. SPIN-RESOLVED KIRCHHOFF’S LAWS FOR FINITE THICKNESS SLABS IN VACUUM

We now consider a finite thickness slab surrounded by vacuum on both sides. In this case, the incident radiation, in addition to getting reflected, also gets transmitted to the other side of the slab. It is straightforward to obtain the emissivities in terms of reflectances and transmittances using similar energy balance considerations. In particular, as shown in fig.1, the radiation in the direction \((\theta, \phi)\) contains emitted, transmitted, reflected and incident photons. The transmitted radiation in \((\theta, \phi)\) direction arises from the radiation incident in the direction \((\pi - \theta, \phi + \pi)\). The transmittances are calculated from the associated Fresnel transmission coefficients \((t_{ss}, t_{sp}, t_{ps}, t_{pp})\) with the help of following expressions:

\[
T_{(+ +) / (- -)} = \frac{1}{4}[(t_{ss} + t_{ps}) \pm i(t_{sp} - t_{pp})]^2, \quad T_{(- -) / (+ +)} = \frac{1}{4}[(t_{ss} - t_{sp}) \pm i(t_{ps} + t_{pp})]^2
\]

The Fresnel transmission coefficient \(t_{jk}\) for \(j, k = [s, p]\) denotes the amplitude of \(j\)-polarized transmitted wave due to unit amplitude \(k\)-polarized incident wave. Equating outgoing and incoming radiation flux rates in the opposite directions in the far-field, we obtain the following expressions for spin-resolved emissivities:

\[
\eta_{(+)}(\theta, \phi) = 1 - R_{(+)}(\theta, \phi + \pi) - R_{(+)}(\theta, \phi + \pi) - T_{(+)}(\pi - \theta, \phi + \pi) - T_{(+)}(\pi - \theta, \phi + \pi)
\]
\[
\eta_{(-)}(\theta, \phi) = 1 - R_{(-)}(\theta, \phi + \pi) - R_{(-)}(\theta, \phi + \pi) - T_{(-)}(\pi - \theta, \phi + \pi) - T_{(-)}(\pi - \theta, \phi + \pi)
\]

Similarly, the absorptivities are obtained by subtracting the reflected and transmitted flux rates from the incident polarized flux rates.

\[
\alpha_{(+)}(\theta, \phi) = 1 - R_{(+)}(\theta, \phi) - R_{(+)}(\theta, \phi) - T_{(-)}(\pi, \phi) - T_{(+)}(\theta, \phi)
\]
\[
\alpha_{(-)}(\theta, \phi) = 1 - R_{(+)}(\theta, \phi) - R_{(-)}(\theta, \phi) - T_{(-)}(\pi, \phi) - T_{(-)}(\theta, \phi)
\]

We now obtain the relations between the spin-resolved emissivities (Eq.9) and absorptivities (Eq.10) using the relations between Fresnel reflection and transmission coefficients for various materials.

**Spin-resolved Kirchhoff’s laws for reciprocal finite thickness slabs:** For reciprocal materials, the Fresnel coefficients satisfy the following relations:

\[
r_{ss, pp}(\theta, \phi) = r_{ss, pp}(\theta, \phi + \pi) = r_{ss, pp}(\pi - \theta, \phi + \pi)
\]
\[
t_{ss, pp}(\theta, \phi) = t_{ss, pp}(\theta, \phi + \pi) = t_{ss, pp}(\pi - \theta, \phi + \pi)
\]
\[
r_{sp}(\theta, \phi) = -r_{ps}(\theta, \phi + \pi), \quad t_{sp}(\theta, \phi) = -t_{ps}(\pi - \theta, \phi + \pi)
\]
Using these relations, we obtain:

\[ R(+\pi, \phi + \pi) = R(-\pi, \phi), \quad R(+\phi, \phi + \pi) = R(-\phi, \phi) \]
\[ R(+\pi, \phi + \pi) = R(-\pi, \phi), \quad R(+\phi, \phi + \pi) = R(-\phi, \phi) \]
\[ T(+\pi, \phi + \pi) = T(-\phi, \phi), \quad T(+\phi, \phi + \pi) = T(-\phi, \phi) \]
\[ T(+\pi, \phi + \pi) = T(-\phi, \phi), \quad T(+\phi, \phi + \pi) = T(-\phi, \phi) \]

It then follows that

\[ \eta(+\phi, \phi) = \alpha(+\phi, \phi), \quad \eta(-\phi, \phi) = \alpha(-\phi, \phi) \]

This is the first spin-resolved Kirchhoff’s law applicable for all reciprocal media.

**Spin-resolved Kirchhoff’s law for nonreciprocal media which preserve the rotational symmetry:**

This class of materials includes gyrotropic media with gyrotropy axis perpendicular to the surface and isotropic magnetoelectric media with diagonal coupling \((\vec{\pi}, \vec{\zeta})\) tensors.

For gyrotropic media, we find the following relations:

\[ r_{ss,pp}(\theta, \phi) = r_{ss,pp}(\theta, \phi + \pi) = r_{ss,pp}(\pi - \theta, \phi + \pi) \]
\[ t_{ss,pp}(\theta, \phi) = t_{ss,pp}(\theta, \phi + \pi) = t_{ss,pp}(\pi - \theta, \phi + \pi) \]
\[ r_{sp}(\theta, \phi) = r_{ps}(\theta, \phi + \pi), \quad t_{sp}(\theta, \phi) = t_{ps}(\pi - \theta, \phi + \pi) \]
\[ r_{sp}(\theta, \phi) = r_{ps}(\theta, \phi), \quad t_{sp}(\theta, \phi) = -t_{ps}(\theta, \phi) \]

This allows us to relate the reflectances and transmittances in different directions as the following:

\[ R(+\pi, \phi + \pi) = R(-\phi, \phi), \quad R(+\phi, \phi + \pi) = R(-\phi, \phi) \]
\[ R(+\phi, \phi + \pi) = R(-\phi, \phi), \quad R(+\phi, \phi + \pi) = R(-\phi, \phi) \]
\[ T(+\phi, \phi + \pi) = T(-\phi, \phi), \quad T(+\phi, \phi + \pi) = T(-\phi, \phi) \]
\[ T(+\phi, \phi + \pi) = T(-\phi, \phi), \quad T(+\phi, \phi + \pi) = T(-\phi, \phi) \]

Substituting these relations in the expressions for emissivities,

\[ \eta(+\phi, \phi) = \alpha(-\phi, \phi), \quad \eta(-\phi, \phi) = \alpha(+\phi, \phi) \]

This is the second spin-resolved Kirchhoff’s law. It turns out that for an isotropic magnetoelectric slab, the same relations between the Fresnel coefficients apply except the last relation for the Fresnel transmission coefficients. For these materials, \( t_{sp}(\theta, \phi) = t_{ps}(\theta, \phi) \). This leads to slightly different relations between the transmittances. However, substituting them in the expressions for emissivities, the same spin-resolved Kirchhoff’s law is obtained.

\[ R(+\phi, \phi + \pi) = R(-\phi, \phi), \quad R(+\phi, \phi + \pi) = R(-\phi, \phi) \]
\[ R(+\phi, \phi + \pi) = R(-\phi, \phi), \quad R(+\phi, \phi + \pi) = R(-\phi, \phi) \]
\[ T(+\phi, \phi + \pi) = T(-\phi, \phi), \quad T(+\phi, \phi + \pi) = T(-\phi, \phi) \]
\[ T(+\phi, \phi + \pi) = T(-\phi, \phi), \quad T(+\phi, \phi + \pi) = T(-\phi, \phi) \]

**Spin-resolved Kirchhoff’s law for other nonreciprocal media:** Based on our analysis of thick planar slabs in the main text, we consider gyrotropic media with gyrotropy axis parallel to the surface and magneto-electric media that lead to cross coupling between \( E - H \) field components in perpendicular and parallel directions. All these materials satisfy the following relations between the Fresnel coefficients:

\[ r_{sp}(\theta, \phi) = -r_{ps}(\theta, \phi) \]
\[ t_{sp}(\pi - \theta, \phi + \pi) = -t_{ps}(\theta, \phi + \pi), \quad t_{ps}(\pi - \theta, \phi + \pi) = -t_{sp}(\theta, \phi + \pi) \]
\[ t_{ss,pp}(\pi - \theta, \phi + \pi) = t_{ss,pp}(\theta, \phi + \pi) \]
We then obtain the following simplifying relations:

\[
R_{(+)}(\theta, \phi + \pi) = R_{(-)}(\theta, \phi + \pi), \quad r_{sp} + r_{ps} = 0
\]

\[
T_{(++, -)}(\pi - \theta, \phi + \pi) = T_{(++, -)}(\theta, \phi + \pi)
\]

\[
T_{(+)}(\pi - \theta, \phi + \pi) = T_{(-)}(\theta, \phi + \pi), \quad T_{(-)}(\pi - \theta, \phi + \pi) = T_{(+)}(\theta, \phi + \pi)
\]

Substituting these relations in the expressions for emissivities, we obtain:

\[
\eta_{(+)}(\theta, \phi) = \alpha_{(+)}(\theta, \phi + \pi), \quad \eta_{(-)}(\theta, \phi) = \alpha_{(-)}(\theta, \phi + \pi)
\]  

This is the third spin-resolved Kirchhoff’s law. The Fresnel reflection and transmission coefficients are obtained by matching the boundary conditions at the interfaces. Their derivation for generic material classes is described in our previous work [9] and the tool developed to compute them can be provided upon reasonable request to the author. We note that for anisotropic, magnetoelectric media that lead to coupling between \( E \) – \( H \) field components lying parallel to the surface, no such simplifying relations can be obtained. Also, for composite media which combine non-isotropic materials satisfying different spin-resolved Kirchhoff’s laws in the same geometry, the above spin-resolved Kirchhoff’s laws are not applicable.

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