A Decomposition Control of Variable Speed Control Torque Gyro

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Abstract. IPACS (energy / attitude control integrated control system) is the main problem studied in recent years. In view of this problem, on the basis of the existing flywheel control law torque analysis, the singular value decomposition of torque output in CMGs mode is carried out. Only when the CMGs mode is close to singular, the instruction torque in the singular direction will be allocated to RWs so as to give full play to the VSCMGs execution ability under the premise of ensuring the output accuracy of the VSCMGs. In order to realize the control better, the zero motion control law and robust pseudo-inverse operation rate are analyzed.

Keywords: VSCMGs, IPACS, Zero Motion Robust Control Law

Introduction

VSCMGs proposed by Ford and Hall, the flywheel speed are adjustable and has the function of avoiding singularity. SGCMG cluster singularities were first described by geometric method in Margulies 1978. In 2005, Tang Liang pointed out the singular types of SGCMG groups on the basis of existing scholars, and analyzed whether they could adopt zero motion avoidance. The torque characteristics of the flywheel with each operating law are also obtained [1].CMG singularities can be solved by reasonably designing momentum management schemes, but internal singularities are difficult to predict. Usually, zero motion gradient method, singular robust method, path planning method, frame angle and spatial restriction method are used. VSCMGs is the combination of the reaction wheel and the traditional CMG, SGCMGs the torque cannot be output in the normal direction of the torque coplanar, that is, SGCMG s singular problem. When the VSCMGs are only used as an actuator to control the attitude control system, it can change the direction of diagonal momentum as SGCMGs to output the force moment, which is called CMGs mode. The output torque is small but high precision. [4]One of the more reasonable schemes for common attitude control systems is to obtain strong moment output in CMGs/RWs mixed mode, lock each CMGs frame step by step, and obtain small moment output from single RWs mode. This paper studies this scheme in detail. If the control design is unreasonable, it is very likely that part of the rotor wheel speed is low and the torque output ability is reduced. On the contrary, part of the rotor wheel speed is too high and easy to saturate. Therefore, the wheel speed of each VSCMGs should be as balanced as possible. The proposed method makes full use of the VSCMGs execution ability under the premise of ensuring the VSCMGs output accuracy, and solves the synchronous output separately from the VSCMGs CMGs mode and the RWs
mode. The robust pseudo-inverse operation law based on singular value decomposition is used to give all the instruction torque to the CMGs mode. At the same time, the RWs mode is used to compensate the torque deviation in the singular direction when the CMG is close to singular. The pseudo-inverse operation law and robust pseudo-inverse operation law of zero motion all realize the avoidance of singular state to different degrees and are improved on the basis of pseudo-inverse operation law. The pseudo inverse operation law with zero motion can avoid the internal hidden singularities of the configuration, but it cannot solve the internal singularities problem, and the robust pseudo inverse operation law can introduce error terms while carrying out singular avoidance, which can reduce the control accuracy. Combined with the advantages of zero motion avoiding implicit singularities and robust pseudo inverse rule avoiding singularities, the control law is designed. In this paper, the VSGCMGs of pyramid configuration is introduced [2].

1. VSGCMGs System Analysis of Pyramid Configuration

1.1 Output Torque of Gyroscopes

The control torque gyroscopes consists of three parts, including the angular momentum flywheel, the frame supporting the flywheel and the servo system driving the rotation of the frame. The rotation of the frame supporting the flywheel can change the direction of the angular momentum and then output the gyroscopic torque. There are

\[ l = l_\omega dp/dt = l_\omega \omega_f \]

Where, \( l \) is the angular momentum of the flywheel, and \( \dot{l} \) is the first derivative of \( l \), \( l \) is flywheel speed inertia, \( r \) is flywheel speed, \( d \) as a differential operator, \( p \) the angular momentum of the flywheel corresponds to the unit vector, \( t \) time, \( \omega \) is the angular velocity of the frame, \( f \) is the corresponding unit vector of gyro torque. The gyro moment acts on the frame base, \( f \) has the following relationship with \( p \),

\[ f = -h \times p \]

\( h \) is the unit vector corresponding to the unit frame axis. After setting up the space rectangular coordinate system, Set up a plane quadrilateral with x axis and y axis as right angle, Place four flywheels on each side, And taking the z axis as the vertex of the pyramid configuration, If the angle between the flywheel and the vertex at the intersection point of the x axis and the envisaged plane is, \( \alpha \) is the heeling Angle of pyramid configuration, \( h_i \) \( (i = 1,...,4) \) is the unit vector corresponding to each unit frame axis, then the first unit can be expressed as \( h_1[3] \). The According to the pyramid configuration, four frame axes of SGCMG clusters can be described as:

\[ h_1 = p_1 \alpha + f_1 k, h_2 = p_2 \alpha + f_2 k, h_3 = p_3 \alpha + f_3 k, h_4 = p_4 \alpha + f_4 k \]

Among them, \( f_\alpha = \cos \alpha, p_\alpha = \sin \alpha \).

The angular momentum of SGCMG clusters can be expressed as O-xyz coordinate system:

\[ L = L_xi + L_yj + L_zk \]

Where, \( L_x, L_y \) and \( L_z \) and are the components of the angular momentum of SGCMG cluster on each axis of coordinate system O-XYZ respectively. When the pyramid configuration is configured according to the above space configuration, the angular momentum of pyramid configuration SGCMG cluster is the sum of the angular momentum generated by the four flywheel:
\[ L = l_1 + l_2 + l_3 + l_4 \]
\[ = lr \begin{bmatrix} -fa\theta_1 & f\theta_1 \\ f\theta_2 & -fa\theta_2 \\ p\theta_1 & p\theta_2 \end{bmatrix} lr \begin{bmatrix} -fa\theta_3 & f\theta_3 \\ f\theta_4 & -fa\theta_4 \\ p\theta_3 & p\theta_4 \end{bmatrix} \]
\[ = lr \begin{bmatrix} -fa\theta_1 & f\theta_1 & -fa\theta_3 & f\theta_3 \\ f\theta_2 & -fa\theta_2 & f\theta_4 & -fa\theta_4 \\ p\theta_1 & p\theta_2 & p\theta_3 & p\theta_4 \end{bmatrix} \]
\[ \text{(4)} \]

Where, \( \Theta \) is the angular position of each element frame, \( l_i \) is the angular momentum of each flywheel, \( p\theta = \sin \theta, f\theta = \cos \theta \), \( i = 1, ..., 4 \). The positive direction of the angular position of the frame is defined as rotating anticlockwise along the axis of the frame.

As the frame angular position of the SGCMG cluster is changed, the direction of the flywheel angular momentum changes and the gyroscopic torque can be output. Hence, the torque output equation of SGCMG clusters can be described as

\[ \tau = \frac{\partial L}{\partial \theta} \]
\[ \text{(5)} \]

Among them, there are \( f = Irf_i \)

\[ \begin{bmatrix} -fa\theta_1 & f\theta_1 & -fa\theta_3 & f\theta_3 \\ f\theta_2 & -fa\theta_2 & f\theta_4 & -fa\theta_4 \\ p\theta_1 & p\theta_2 & p\theta_3 & p\theta_4 \end{bmatrix} \]
\[ \text{(6)} \]

1.2 Control Law Design of CMGs Patterns

If the command torque is all assigned to CMGs mode [3], then there is

\[ M_f = -A_J\omega d[I]\omega \]
\[ \omega = \omega_r + \omega_n, \text{ In the formula, } \omega_r \text{ is the frame speed instruction that controls the torque output; } \omega_n \text{ is idle instruction, respectively satisfying} \]

\[ \begin{cases} A_J\omega d[I]\omega_r = -M_f \\ A_J\omega d[I]\omega_n = 0 \end{cases} \]
\[ \text{(8)} \]

First solve for \( \omega_r \). The pseudo-inverse optimal solution can be obtained according to the optimization method as follows:

\[ \omega_r = (J_{\omega d}[I])^{-1}A_r^T(A_JA_r)^{-1}M_f \]
\[ \text{(9)} \]

Obviously, \( (A_1A_1^T) = 0 \), the pseudo-inverse does not exist when the gyroscope group frame falls into singularity. The singular value analysis is performed for \( A_1, A_1 = VSU^T, V = [v_1 v_2 v_3], U = [u_1, ..., u_n] \) are unitary matrices, \( S = \text{diag}\{\sigma_1, \sigma_2, \sigma_3\} 0_{3x(n-3)} \), formula (9) is rewritten to

\[ \omega_r = -(J_{\omega d}[I])^{-1}A_r^T(S_3^2V_3V_3^T)^{-1}M_f \]
\[ \text{(10)} \]

When a robust pseudo-inverse operation law with minimum moment error is introduced to avoid the gyro configuration falling into singularity, the pseudo-inverse solution does not exist.

\[ \omega_r = (J_{\omega d}[I])^{-1}(S_3^2\alpha u + \frac{\sigma_3}{\sigma_3^2 + \sigma_2^2} \alpha u_3) \]
\[ \text{(11)} \]
\[ \delta = \begin{cases} 0 & \sigma_1 \geq \gamma \delta_0, \\ \delta_0 & \sigma_1 < \gamma \end{cases} \]

where \( \delta \) are constants greater than zero and \( \sigma \) are constant values. \( \alpha_1, \alpha_2, \alpha_3 \in \mathbb{R} \). The corresponding torque error is

\[ M_{\text{error}} = -(J_{\text{cmd}}[I])^{-1} \frac{\delta^2}{\sigma_j^3 + \delta} \alpha_3 u_3 \]  (12)

It can be concluded that the introduction of torque error only makes the third singular value change from 0 to \( \delta \) when the system is close to singularity, while the other two singular values remain unchanged. Only when the CMGs are close to singularity, the torque is generated in the direction with the smallest singular value. Therefore, when this control law is adopted, the torque error can be minimized while avoiding explicit and implicit singularities, which not only ensures the output accuracy of the torque, but also ensures the existence of the rotor angular acceleration solution under the RWs mode. Therefore, the design of the control law in CMGs mode is completed [4].

1.3 Control Law Design of RWs Patterns

The control law design of RWs mode can be carried out by referring to the spacecraft attitude control problem with flywheel as the system actuator. \( M_{\text{error}} \) is the command moment. Different from a single flywheel, the flywheel configuration in VSCMGs changes. Meanwhile, the flywheel configuration matrix is \( A_v \), which changes with the change of frame Angle, so there is a singular problem. Each flywheel in the flywheel set receives the command control moment allocated by the flywheel control law. Therefore, the command angular acceleration of each flywheel should be calculated, so that when the flywheel drives each flywheel to operate according to the command, the resultant torque output by the flywheel set is equal to the expected control moment. The control torque output by the flywheel set can be written as general formula

\[ M_{\text{error}} = -D \dot{r} \]  (13)

Where, \( \dot{r} \) is the angular acceleration of VSCMGs flywheel, and \( D = A_v J_{\text{cmd}} \) is the rotor installation matrix. The design of the control law is to calculate the command angular acceleration of each flywheel according to the installation matrix \( D \) of the flywheel set after obtaining the required command control moment \( M_{\text{error}} \).

The control law of minimum energy consumption is obtained

\[ \dot{r} = -D^T (DD^T)^{-1} M_{\text{error}} \]  (14)

Since the number of rotors is more freedom than the control required, the RW control law also has zero motion, thus completing the design of the control law in RW mode [5].

2. Optimal Design of Pseudo-Inverse Control Rate for Zero Motion

Aiming at the characteristics that "pseudo-inverse + zero motion" does not have the ability to avoid singular points, a perturbation robust pseudo-inverse operation law is designed. This kind of control law can be used to avoid singularities by introducing error perturbation terms [6]. The scope of application includes explicit and implicit singular points. Therefore, a new control law combining the advantages of two control laws is designed in the form of

\[ \omega = -A^T (A A^T + \phi(I_{3 \times 3} + E_{3 \times 3}))^{-1} M_f \]  
\[ I_0 + \alpha [I_6 \times 6 - A^T (A A^T)^{-1} A_1] \frac{\partial \sigma}{\partial \theta} \]  (15)

Coefficient \( \phi, \alpha \) selection rule
\[
\begin{aligned}
  \phi &= 0, \quad \alpha = \alpha_1 \quad \sigma \geq \sigma_1 \\
  \phi &= \phi_1, \quad \alpha = 0 \quad \sigma < \sigma_1
\end{aligned}
\] (16)

When the configuration is greater than the threshold value of singular metric \( \sigma_1 \), the internal hidden singular point is avoided in priority. In this case, no error term is introduced and the solution is accurate. When the configurational singularity metric is less than \( \sigma_1 \), it indicates that zero motion has little effect [7]. At this point, the perturbation term is introduced and the zero motion term is set to zero to prevent the angular velocity of the instruction frame from being too large. In this way, not only internal hidden singularities can be fully utilized, but also explicit singularities can be avoided. The selection principle of the threshold value is as follows: while enlarging the action interval of zero motion as far as possible, ensure the kinetic energy to avoid the apparent singularity in time. When the threshold \( \sigma_1 \) is selected, it should not be too large. The larger the threshold is, the smaller the interval of zero motion effect is and the larger the perturbation working interval is [8].

3. Result Verification
Taking CMG failure as an example, the robust pseudo-inverse control law and the robust pseudo-inverse control law with zero motion are respectively used for simulation. It is found that the three-axis maneuver curve obtained by using the robust pseudo-inverse control law obviously deviates from the standard trajectory at a certain time, and the deviation is above. This is caused by the interference of air jet unloading [9]. After unloading, the curve gradually converges. The maneuvering effect of the robust pseudo-inverse control law with zero motion is obviously better than the former, and the real-time tracking accuracy is higher. Based on other parameters, it can be concluded that although the internal singularity is avoided in the maneuvering process, the tracking error increases due to the extra torque output caused by perturbation. However, the robust pseudo-inverse manipulation law with zero motion makes full use of the hidden singularity point inside the angular momentum body, and the singular measure is always above 0.5, indicating that the configuration is always far away from the singular state.

4. Conclusions
In this paper, the singular causes of VSCMGs are analyzed and the mathematical expression of gyro output torque is given. The hybrid control system is designed. The advantage is that the torque error can be minimized while avoiding explicit singularity. Then, based on the previous studies, the mixed control structure and the control law with zero motion and robust control were designed, and a comprehensive control law was designed for the explicit singularities in the depth of angular momentum SGCMGs configuration. On the basis of avoiding the explicit singularities, the implicit internal singularities were fully utilized. The simulation results show that the proposed integrated control law is superior to the traditional robust pseudo-inverse control law. The control law of this design combines the advantages of robust pseudo-inverse and pseudo-inverse control law with zero motion, and has certain universality in singularity avoidance. It is also applicable to pyramid and other SGCMGs configurations, and has practical application [10]. The operating space of angular momentum is divided into two parts by the selection of threshold, but the analysis of the transition region between the two parts is lacking. Under the premise of considering the maximum frame angular velocity of SGCMG in the following research, the transition part of \( \sigma_1 \) can be added, and the pseudo-inverse control law with zero motion can be transferred to the small disturbance of the robust pseudo-inverse control law by optimizing parameter configuration.

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