Creation of optimal warehouse of spare parts with minimum cost and maximum fail-safety

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Abstract. In this article we present the method of creation of spare parts warehouse, conditions for ensuring the best reliability of warehouse with lowest total cost of spare parts storage. The method is based on Poisson-law distribution of probability of consumption of each part in a given period of time. The calculation program was developed for use on a personal computer.

1. Introduction
One of the conditions for efficient functioning repair services of transport enterprises, service stations, machine-building plants, etc. is availability of spare parts required for elimination of failures and faults of machines and process facilities [1]. It is obvious that reliability of warehouse will be the higher; the more spare parts are stored in the warehouse. However excessive increase in the number of spare parts leads to increase in the costs linked with their purchasing and storage. The number of spare parts, the need for which occurs most often, should be greater than the number of rarely required parts [2,3]. At the same time it is reasonable to take into account cost of stored parts, since excessive stocks of expensive parts are less profitable than stocks of cheap parts with the same safety margin in the warehouse.

2. Materials and methods
The number of spare parts taken from a warehouse for a certain period of time is a random variable with a probability distribution according to Poisson’s law [4]:

\[ P(k) = \frac{M^k}{k!} e^{-M} , \quad (1) \]

where \( k \) – the random number taken from the warehouse of spare parts; \( M \) – mathematic expectation, i.e. average consumption of spare parts for the planned period (referring to the part of a particular name).

If there are spare parts of a certain name (i) in the \( H_i \) warehouse, the need for spare part will be satisfied when \( k \leq H_i \). Probability \( \alpha_i \) that warehouse will be fail-safe regarding \( i \)-part can be found as the sum of the probabilities expressed by the formula (1):

\[ \alpha_i = \sum_{k=0}^{H_i} \frac{M^k}{k!} e^{-M} . \quad (2) \]
When in stock $n$ types (names) of spare parts are stored, the reliability of the $\alpha_C$ warehouse is equal to the product of the fail-safe for each type of part:

$$\alpha_C = \alpha_1 \cdot \alpha_2 \cdots \alpha_n = \prod_{i=1}^{n} \alpha_i.$$  \hfill (3)

Increase in the norm of parts stored in a warehouse leads to increase in the reliability of the warehouse and also the cost of the stored parts (cost of the warehouse) [7]. Increase of efficiency $H_i$ till $H_i + 1$ when cost of the part $C_i$ may be evaluated by $\frac{\Delta \alpha}{C_i}$, where $\Delta \alpha = \alpha(H_i + 1) - \alpha(H_i)$ – increase in reliability with increase in the safety margin per one part.

For convenience of calculation we introduce the value $R_i = \ln \alpha_i$, since if reliability $\alpha_i$ is changing in the range from 0 to 1, $R_i$ changes in a wider range from $-\infty$ to 0. We change increase of fail-safety for $\Delta R_i = R(H_i + 1) - R(H_i)$, because $R_i = \ln \alpha_i$, then:

$$\Delta R_i = \ln \sum_{k=0}^{H_i+1} \frac{M^k_i}{k!} - \ln \sum_{k=0}^{H_i} \frac{M^k_i}{k!} = \ln \left\{ \frac{\sum_{k=0}^{H_i+1} M^k_i}{\sum_{k=0}^{H_i} M^k_i} \right\}.$$  \hfill (4)

Having transformed the sum by putting the common factors out of the bracket, the calculation can be made using a cyclic computer program.

Having determined the values of the sums, we find the relative value $\frac{\Delta R_i}{C_i}$ over the entire list of the parts stored in the warehouse, comparing the values obtained; choose the largest one, fixing the number (name) of the corresponding part. Increase in the storage number of selected part gives the greatest increase in warehouse reliability per unit cost of purchasing parts. Increase this rate by one part and determine the total cost of the warehouse $C = \sum_{i=1}^{n} C_i \cdot H_i$. If the cost of the warehouse is less than the total cost specified in terms of calculating, then the calculation is repeated, that is, the number of the part that gives the greatest increase in the availability of the warehouse per unit of cost is again searched. If the cost of the warehouse is compared with the specified total cost, the calculation is terminated. After that printout of the storage norms of the entire range of parts is done [5-7].

For implementation of the task of creation the optimal warehouse, an algorithm was developed that allows one to perform calculations by two ways:

**B1** - for given total cost of warehouse, the creation of a given list of spare parts of warehouse stocks (parts storage standards), ensuring maximum reliability of the warehouse;

**B2** - for given warehouse reliability, the formation of a given list of spare parts of warehouse stocks (parts storage standards), ensuring the minimum cost of the warehouse. As the initial data we use: $S_i$ – name of the item of spare part in warehouse; $M_i$ – average consumption of spare part for scheduled period; $C_i$ – part cost; $C$ – total cost of the warehouse (option B1); $\alpha$ – total fail safety warehouse (option B2).

The first calculation step is the input of initial data and choice of the calculation option: B1 or B2. The initial condition is absence of spare parts in the warehouse for all items of parts (safety margin is zero).

Next, a sequential enumeration of all parts positions is performed and increase in reliability per unit of costs (cost of the part) is determined, provided that the safety margin is increased by one part.
Option B1 or the availability of the warehouse is determined by option B2. The storage rate of this item is increased by one part and the total cost of the warehouse is calculated \( C = \sum_{i=1}^{n} C_i \cdot H_i \) when calculated by B1 option or reliability \( \alpha_0 = \prod_{i=1}^{n} \alpha_i \) by option B2. If the cost of the warehouse or the reliability is less than specified one, the calculation is repeated, that is, the number of the part that gives the greatest increase in the reliability of the warehouse per unit of cost is again sought. We continue calculations until the total value of the warehouse equals the specified value of the warehouse \( C_0 \geq C \) (for B2—with targeted total reliability: \( \alpha_0 \geq \alpha \)).

The final stage of the program is the output of the obtained data for each item of parts \( S_i, M_i, C_i, H_i, P_i \) (where \( H_i, \alpha_i \) – recommended stock availability and reliability in the warehouse for each item of parts in the warehouse as well as totals - the total cost of warehouse and the total warehouse fail-safety.

Based on the proposed algorithm, the program "Optimal warehouse v1.1." was developed.

When writing the program, language C++ was used. The total amount of memory occupied by the program is 530 kb. Computer requirements are:

a. IBMPC compatible computer with P166 processor or higher;

b. 32 MB of RAM;

c. operating system:
   - Windows NT 4.0 SP3 (or higher) with the Russian language and Russian regional settings or;
   - Windows 2000 Professional (with the Russian language and Russian regional settings);

g. as initial files*.xls (Microsoft Office Excel 2000 or higher are used).

3. Results and discussion

Identification of the role of initial parameters of creation of the optimal warehouse was made by the conditional example of the warehouse consisting of 15 items (names) of spare parts, the average consumption of which and the cost differ significantly [8,9].

In the first option of stocking, the storage rate corresponded to the average consumption of spare parts for the planned period. The probability of satisfying the need for each part, when the random variable was not more than the average consumption, was calculated according to Poisson’s law in accordance with formula (2). And the reliability of the warehouse was calculated using formula (3). The optimal storage rate was calculated using the developed program “Optimal warehouse v1.1.” from the condition of equality of the total cost of stored spare parts in the first and second versions. The results are shown in table 1.

The calculation results show that the spare parts warehouse according to the proposed program provides an increase in warehouse fail-safety almost nine times compared with the reliability of the warehouse where storage standards are equal to the average consumption of spare parts. Thus, with practically equal costs of stored parts, the number of cases when the application for the required part is not satisfied due to the lack of spare parts for the optimal warehouse will be almost nine times less [10,11].

The correlation of the cost of spare parts and their average consumption on the optimal standards for the storage of parts in a warehouse can be seen in figure 1.

4. Conclusion.

To increase the reliability of the warehouse an obvious condition is required - increase storage rates for spare parts. However, an optimal storage standard should be assigned taking into account the cost of each type of spare part and its average consumption. The average cost of spare parts is determined from the statistical data indicators of the warehouse circulation.
The proposed method of completing spare parts warehouse solves the problem of optimizing warehousing, ensuring that the most fail-safe warehouse is obtained at the lowest total cost of stored parts.

**Table 1.** Indicators of a conditional storage of 15 types of parts.

| No. | Part name | Part cost Ci | Average consumption of parts Mi | Before optimization | After optimization |
|-----|-----------|--------------|---------------------------------|--------------------|-------------------|
|     | Si        |              |                                 | Cost, ΣCi          | Fail-safety (by Poisson law) | Rate Hi | Cost, ΣCi | Fail-safety by program OCv1.1, αi |
| 1   | Part 1    | 10           | 1                               | 10                 | 0.73576            | 3       | 30        | 0.981                  |
| 2   | Part 2    | 10           | 2                               | 20                 | 0.67668            | 5       | 50        | 0.983                  |
| 3   | Part 3    | 10           | 5                               | 50                 | 0.61596            | 9       | 90        | 0.968                  |
| 4   | Part 4    | 10           | 10                              | 100                | 0.58304            | 15      | 150       | 0.951                  |
| 5   | Part 5    | 10           | 25                              | 250                | 0.55292            | 32      | 320       | 0.929                  |
| 6   | Part 6    | 50           | 1                               | 50                 | 0.73576            | 2       | 100       | 0.92                   |
| 7   | Part 7    | 50           | 2                               | 100                | 0.67668            | 3       | 150       | 0.857                  |
| 8   | Part 8    | 50           | 5                               | 250                | 0.61596            | 7       | 350       | 0.867                  |
| 9   | Part 9    | 50           | 10                              | 500                | 0.58304            | 11      | 550       | 0.697                  |
| 10  | Part 10   | 50           | 25                              | 1250               | 0.55292            | 26      | 1300      | 0.629                  |
| 11  | Part 11   | 100          | 1                               | 100                | 0.73576            | 1       | 100       | 0.736                  |
| 12  | Part 12   | 100          | 2                               | 200                | 0.67668            | 2       | 200       | 0.677                  |
| 13  | Part 13   | 100          | 5                               | 500                | 0.61596            | 5       | 500       | 0.616                  |
| 14  | Part 14   | 100          | 10                              | 1000               | 0.58304            | 9       | 900       | 0.458                  |
| 15  | Part 15   | 100          | 25                              | 2500               | 0.55292            | 21      | 2100      | 0.247                  |
|     | Total:    |              |                                 | 6880               | 0.000966           |         | 6890      | 0.008581               |

Figure 1. Change in the storage rate of spare parts as percentage of their average consumption depending on specified safety margin of the warehouse.
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