Non-Gaussianity in curvaton models with nearly quadratic potentials

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Abstract. We consider curvaton models with potentials that depart slightly from the quadratic form. We show that although such a small departure does not significantly modify the Gaussian part of the curvature perturbation, it can have a pronounced effect on the level of non-Gaussianity. We find that unlike in the quadratic case, the limit of small non-Gaussianity, $|f_{NL}| \ll 1$, is quite possible even with small curvaton energy density $r \ll 1$. Furthermore, non-Gaussianity does not imply any strict bounds on $r$, but the bounds depend on the assumptions about the higher order terms in the curvaton potential.

Keywords: cosmological perturbation theory, inflation

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1. Introduction

Possible non-Gaussian features of the cosmic microwave background (CMB) temperature anisotropy can provide important constraints on models of inflation. For instance, an observation of a significant non-Gaussianity would effectively rule out single-field inflation (for a review of non-Gaussianity, see [1]). Usually in the literature the non-Gaussianities are characterized by a non-linearity parameter $f_{NL}$, which is a measure of the non-Gaussian curvature perturbation relative to the Gaussian perturbation. Present WMAP observations yield the limit $-58 < f_{NL} < 134$ at the 95% confidence level. With polarization measurements, the Planck Surveyor Mission is expected to push the limit down to $|f_{NL}| \lesssim 2.9$ [3]. For the single-field inflation one obtains $f_{NL}$ which is of the order of the slow-roll parameters [4]; hence a detection of non-Gaussianity by Planck would indeed suffice to rule out single-field inflation.

For the curvaton models [5]–[7] it has been suggested [8] that a non-observation of non-Gaussianity would indicate that the model is ruled out, at least in the case of the quadratic potential. In curvaton models the curvature perturbation is generated after inflation by the decay of an effectively massless scalar field $\sigma$ different from the inflaton. The curvaton energy density remains subdominant until the end of inflation, so the density parameter $r \equiv 4\rho_{\sigma}/(4\rho_{r} + 3\rho_{\sigma}) \ll 1$, where $\rho_{r}$ is the energy density of radiation after inflation. Thereafter the curvaton field begins to oscillate and behaves effectively like matter. Its relative energy density grows during oscillations and when it eventually decays, the perturbation that it has received during inflation will be imprinted on the decay products, the light degrees of freedom. Because the curvaton is massless, the perturbation is predominantly Gaussian. However, there will also be a non-Gaussian contribution that arises because of the curvaton dynamics after inflation. A result well known by now is that in curvaton models with quadratic potentials the non-linearity...
parameter can be written as \[ f_{\text{NL}} = \frac{5}{3} + \frac{5}{8}r - \frac{5}{3r}, \] (1.1)
where \( r \) is evaluated at the time of curvaton decay. Thus, for low \( r \) as required for the subdominance of the curvaton during inflation, \( |f_{\text{NL}}| \) is typically much bigger than 1. However, we wish to point out that this result is considerably modified for non-quadratic potentials. Although the curvaton must be weakly self-interacting, it is highly likely that there is some departure from the quadratic potential. As we will discuss in this paper, while leaving the Gaussian perturbation essentially unchanged, a small correction to the quadratic potential can have an important effect on the non-Gaussianity parameter. Indeed, we show that if one does not insist on a strictly quadratic potential, it is quite possible to have \( |f_{\text{NL}}| \) much less than 1 in the curvaton scenario also.

2. The curvature perturbation generated in the curvaton model

We adopt here the non-linear, so-called separate universe approach presented in \([10]\), \([12]\)–\([14]\), which is valid on large scales. There one considers perturbations around the homogeneous and isotropic flat FRW universe assuming that their spatial variation outside horizon is smooth. The evolution of large scales is approximated by replacing each quantity by its spatial average inside some smoothing scale \( k_{\text{smooth}} \) and considering these smoothed, locally homogeneous and isotropic regions to evolve like separate FRW universes \([10,13,14]\). The spatial variation of perturbations outside the smoothing scale is taken into account by carrying out a first-order gradient expansion leading to different expansion rates in different smoothed regions. Here we briefly recapitulate the main features \([10,14]\) of this approach relating to non-Gaussianity in curvaton models.

In the first order in the gradient expansion, the spatial part of the metric can be written as \([14]\)
\[
\gamma_{ij} = a^2(t)e^{-2\psi(t,x)}\gamma_{ij} \tag{2.1}
\]
where \( \gamma_{ij} \) is constant, assuming the amplitude of the gravitational waves to be small. The curvature perturbation \( \psi(t,x) \) thus defined can be interpreted as a perturbation in the scale factor, \( \bar{a}(t,x) \equiv a(t)e^{-\psi(t,x)} \). As has been shown in \([10,14]\), the curvature perturbation on uniform energy density hypersurfaces stays constant outside the horizon in the absence of non-adiabatic pressure perturbation, just like its counterpart in the usual first-order \([15]\) and second-order perturbation theories \([16]\).

The amount of expansion along the worldline of a comoving observer from a spatially flat \( \psi = 0 \) slice at time \( t_1 \) to a generic slice at time \( t \) is given by \( N(t,x) = \ln(\bar{a}(t,x)/a(t_1)) \) since the expansion in a spatially flat gauge corresponds to that of the unperturbed universe. By choosing the slice at time \( t \) to have uniform energy density, the curvature perturbation on that slice can be written as \([10,14]\)
\[
\zeta(t,x) \equiv -\psi(t,x) = \ln \frac{\bar{a}(t,x)}{a(t)} = N(t,x) - N(t) \tag{2.2}
\]
where \( N(t) \) is the amount of expansion in the background universe.

Following \([10]\) we expand the curvature perturbation \( \zeta(t,x) \) up to second order in the Gaussian curvaton perturbations in order to take into account the non-Gaussian effects:
\[
\zeta(t,x) = \partial_\sigma N(t)\delta\sigma(t,x) + \frac{1}{2}\partial_\sigma^2 N(t)\delta\sigma(t,x)^2. \tag{2.3}
\]
In the limit $r \ll 1$ the curvature perturbation is almost completely generated during oscillations of the curvaton field. Assuming sudden decay at $t = t_{dec}$, the amount of expansion in the background universe during oscillations is given by [10]

$$N(\sigma_{dec}, \sigma_{osc}) = \frac{1}{3} \ln \left( \frac{1}{2} m_\sigma \sigma_{osc}^2 \right) = \frac{1}{4} \ln \left( \frac{\rho_t}{\rho_{dec} - \rho_\sigma} \right)$$

(2.4)

where $\sigma_{osc}$ is the value of the curvaton at the onset of oscillations. The total curvature perturbation is obtained by substituting equation (2.4) into (2.3):

$$\zeta(t, x) = \frac{r \sigma_{osc}^2}{2 \sigma_{osc}} \delta \sigma + \frac{1}{4} \left( \left( -\frac{3}{8} r^3 - r^2 + r \right) \left( \frac{\sigma_{osc}^2}{\sigma_{osc}^2} \right)^2 + r \frac{\sigma''_{osc}}{\sigma_{osc}^2} \right) \delta \sigma^2$$

(2.5)

where we have used the notation $\delta \sigma \equiv \delta \sigma$. The initial values for the curvaton field $\delta \sigma(t, x)$ in each smoothed region are set by inflation and the derivatives in equation (2.3) are thus taken w.r.t. the field value during inflation $\sigma$.

We point out that the non-linear part in the curvature perturbation, equation (2.5), consists of a square of the linear part $((\sigma_{osc}/\sigma_{osc}) \delta \sigma)^2$ and of an additional dynamical term. One could thus expect the non-Gaussian effects to be more dependent on the dynamics than the Gaussian part, which is indeed quite reasonable since, loosely speaking, the Gaussian part depends on the size of the perturbations while the non-Gaussian part depends on the relative size of the perturbations, compared to the background field.

From equation (2.5), the curvature perturbation can be written in the form [17]

$$\zeta = \zeta_g - \frac{5}{2} f_{NL} (\zeta_{osc}^2 - \langle \zeta_{osc}^2 \rangle)$$

where $\zeta_g$ is Gaussian and the non-linearity parameter $f_{NL}$ is independent of position. The non-linearity parameter can now be directly read off from equation (2.5):

$$f_{NL} = \frac{5}{3} + \frac{5}{8} r - \frac{5}{3} r \left( 1 + \frac{\sigma''_{osc} \sigma_{osc}}{\sigma_{osc}^2} \right).$$

(2.6)

Although this result was obtained in [10, 18], the dependence of the last term $\sigma''_{osc} \sigma_{osc}/\sigma_{osc}^2$ on the potential has not been previously examined. In the quadratic case $\sigma''_{osc} \sigma_{osc}/\sigma_{osc}^2 = 0$ but, as we will show shortly, this result may be considerably modified even if the deviation from the quadratic potential is small.

### 3. Small deviation from the quadratic potential

The curvaton equation of motion during radiation domination is given by

$$\ddot{\sigma} + \frac{3}{2t} \dot{\sigma} + V'(\sigma) = 0$$

(3.1)

where we have ignored spatial gradients which are small on large scales. We now consider the potential of the form

$$V(\sigma) = \frac{1}{2} m_\sigma^2 \sigma^2 + \lambda m_\sigma^{4-n} \sigma^n$$

(3.2)

with $\lambda \ll 1$. To describe the size of the potential correction at the end of inflation we introduce a parameter $s \equiv 2\lambda (\sigma_*/m_\sigma)^{n-2}$. The smallness of the correction in equation (3.1) requires $s \ll 2/n$. In the quadratic case the eom (3.1) is nothing but a Bessel equation with a general solution $\sigma_0(t) = A_0 J_{(1/4)}(m_\sigma t)/(m_\sigma t)^{1/4} +$
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\[ B_0J_{(-1/4)}(m_\sigma t)/(m_\sigma t)^{1/4} \equiv A_0y_1(t) + B_0y_2(t). \]

To obtain a regular solution at the end of inflation, \( m_\sigma t = 0 \), we must set \( B_0 = 0 \). Furthermore, requiring \( \sigma_0(m_\sigma t = 0) = \sigma_* \) we find

\[ \sigma_0(t) = \sigma_* \frac{\pi}{2^{5/4}\Gamma(3/4)} \frac{J_{(3/4)}(m_\sigma t)}{(m_\sigma t)^{1/4}} \]  

(3.3)

from which we see that \( \sigma'_{\text{osc}}/\sigma_{\text{osc}} = 0 \) for the quadratic case, as claimed above.

Let us now make an ansatz of the form \( \sigma = \sigma_0 + \lambda \sigma_1 \) in equation (3.1). At first order in \( \lambda \) we obtain the linearized equation of motion

\[ \ddot{\sigma}_1 + \frac{3}{2t} \dot{\sigma}_1 + m_\sigma^2 \sigma_1 + m_\sigma^{4-n} n \sigma_0^{n-1} = 0. \]  

(3.4)

The solution to the homogeneous equation is already given above; the general solution \( \sigma(t) = A(t)y_1(t) + B(t)y_2(t) \) is obtained by the method of variation of parameters. The coefficients are solved from the equations

\[ A'y_1 + B'y_2 = 0 \]  

(3.5)

\[ (A'y_1 + B'y_2)m_\sigma^2 = -m_\sigma^{4-n} n C^{n-1} y_1^{n-1} \]  

(3.6)

where \( d/d(m_\sigma t) \equiv ' \) and \( C \equiv \sigma_*(\pi/2^{5/4}\Gamma(3/4)) \). Since the correction to the quadratic potential is small at the end of inflation and even smaller at later times, we can assume that the beginning of the oscillations takes place at the same time as in the purely quadratic case, \( t_{\text{osc}} = 1/m_\sigma \). Thus the era between the end of inflation and the beginning of oscillations corresponds to \( m_\sigma t = 0 \ldots 1 \). Equations (3.5) and (3.6) can now be solved by expanding the homogeneous solutions and by a straightforward calculation we find that up to order \( O((m_\sigma t/2)^8) \)

\[ A(m_\sigma t = 1) \approx -n \sigma_*^{n-1} m_\sigma^{2-n}(1.0 - 0.10n + 6.9 \times 10^{-3} n^2 - 3.6 \times 10^{-4} n^3 + 1.4 \times 10^{-5} n^4) \]  

(3.7)

\[ B(m_\sigma t = 1) \approx n \sigma_*^{n-1} m_\sigma^{2-n}(0.82 - 0.095n + 6.8 \times 10^{-3} n^2 - 3.6 \times 10^{-4} n^3 + 1.3 \times 10^{-5} n^4). \]  

(3.8)

Thus the value of the curvaton at the onset of oscillations reads

\[ \sigma_{\text{osc}} \approx 0.81 \sigma_* + \lambda n m_\sigma^{2-n} \sigma_*^{n-1} g(n) \]  

(3.9)

with \( g(n) \equiv -0.20 + 1.2 \times 10^{-2} n - 6.1 \times 10^{-4} n^2 + 3.1 \times 10^{-5} n^3 - 2.1 \times 10^{-6} n^4 \). With the exponent values \( n \lesssim 10 \) that we are considering, \( g(n) \) is negative and roughly constant, \( g(n) \sim -0.1 \). The non-linearity parameter, valid for any potential of the type given in equation (3.2), is now obtained by substituting equation (3.9) into (2.6):

\[ f_{\text{NL}} = \frac{5}{3} + \frac{5}{8} r - \frac{5}{3r} \left( 1 + n(n - 1)(n - 2) \frac{g(n)(0.41s + 0.25s^2 ng(n))}{(0.81 + 0.50n(n - 1)sg(n))^2} \right). \]  

(3.10)
4. The behaviour of $|f_{NL}|$ in the limit $r \ll 1$

It is readily seen that the effect of the potential correction in equation (3.10) is most significant in the limit of small curvaton energy density $r \ll 1$. In the following we are working in this limit if not otherwise stated. Thus we can consider the dominating $1/r$ part of $f_{NL}$ alone and neglect the small contribution from the remaining terms $4.5 + 5r$. We now examine the effect of the potential correction by keeping $s$ fixed. Our solution to the equation of motion (equation (3.9)) is constructed in such a way that the value of the curvaton during inflation, $\sigma_*$, is fixed to be the same as in the quadratic case; this is an approximation which however should be justified as we are considering only small departures from the quadratic potential. Since we also keep the mass $m_\sigma$ unaltered, a constant $s$ means that we are considering the coupling constant as a function of the exponent $\lambda = \lambda(n)$.

As stated above, our perturbative approach to solving the equation of motion (3.1) puts limits $\lambda \ll 1$, $s \ll 2/n$. Furthermore, the dominance of the Gaussian perturbations and the masslessness of the curvaton during inflation also restrict the possible values of $\sigma_*$ and $m_\sigma$. Using equation (3.9) we find the spectrum related to the two-point correlator of the curvature perturbation (i.e. Gaussian spectrum) to be

$$P_\zeta = \left( \frac{0.81 + (sn(n-1)/2)g(n)}{0.81 + (sn/2)g(n)} \right)^2 \left( \frac{rH_*}{4\pi\sigma_*} \right)^2 \left( \frac{rH_*}{4\pi\sigma_*} \right)^2 \left( \frac{rH_*}{4\pi\sigma_*} \right)^2$$

where $(rH_*/(4\pi\sigma_*))^2$ is the purely quadratic result; $H_*$ is the Hubble parameter during inflation. In the small correction limit the prefactor in equation (4.1) is $\sim O(10^{-1})$ and hence we conclude that, although the perturbation amplitude is suppressed, the Gaussian part is not significantly affected by the correction as the suppression can be compensated by a slight increase in the scale of inflation. Thus we may use the results for the quadratic parameters [9] which typically imply the restriction $m_\sigma \ll H_* \lesssim \sigma_*$ coming from the masslessness of the curvaton field and the assumed Gaussianity of the curvaton perturbations. This means that the smallness of the potential correction, $s \ll 2/n$, requires $\lambda \ll 1$ when $n \geq 4$. Moreover, the inflaton-generated curvature perturbation is supposed to be negligible which also requires $\lambda \ll 1$ for non-renormalizable terms, $n > 4$.

In figure 1 we show the behaviour of the dominant part of the non-linearity parameter as a function of $n$ for two selected values of $s$. The value of the non-linearity parameter depends on the parameters $\lambda, \sigma_*, m_\sigma$, but nevertheless figure 1 reveals the generic qualitative behaviour of $|f_{NL}|$ as $n$ is varied. One can clearly see that when the exponent of the potential correction is increased, the amount of non-Gaussianity first begins to decrease as compared to the quadratic case. However, at large $n$ the value of $|f_{NL}|$ begins to grow rapidly.

The explanation for the behaviour of $|f_{NL}|$ and the physics involved is most transparent if we switch to the perturbative point of view. Using perturbation theory one finds [11, 19] that the $1/r$ part in equation (2.6) represents the first-order contribution while the rest is due to second-order effects. Thus the first-order theory is adequate as long as we restrict ourselves to the region where the $1/r$ term dominates. The

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4 We will show that the $1/r$ term may vanish, in which case other terms in equation (2.6) should also be taken into account. The term linear in $r$ is however negligible in the limit $r \ll 1$ and the constant part $5/3$ does not affect our qualitative conclusions.
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Figure 1. The qualitative behaviour of the non-linearity parameter with $s = 0.2$ on the left panel and $s = 0.05$ on the right panel; the values of $|f_{\text{NL}}|$ are in units $5/3r = 1$. Apart from the analytical result of equation (3.10) we also show a numerical result obtained directly from equations (2.6) and (3.1) setting $t = 1/m_\sigma$ as the beginning of the oscillations.

decrease in the amount of non-Gaussianity means that the perturbations become smaller compared to the background value. In our case this would imply that after inflation, the perturbations are damped faster than the background value. Indeed, we see that this is the case by considering the first-order equation of motion for the perturbations, $\delta \ddot{\sigma} + (3/2t)\dot{\delta} \sigma + V''(\sigma)\delta \sigma = 0$, with a potential correction of the form $\lambda m_\sigma^4 n \sigma^n$. If the potential is purely quadratic the perturbations and the background field apparently obey the same equation.

The increase of the exponent $n$ diminishes the energy density of the background field at the beginning of oscillations since we are keeping $s$ fixed. Furthermore, the energy density associated with the perturbations at the end of inflation gets bigger. With a large enough $n$ these effects become dominant over the damping of the perturbations, whence the amount of non-Gaussianity again begins to grow. From figure 1 we see that the bigger the value of $s$, the smaller the value of $n$ at which the growth begins; this is of course quite reasonable.

We should point out here that the increase in $|f_{\text{NL}}|$ with a large $n$ typically happens when the potential correction becomes significant, $s \sim 2/n$. At this point the perturbative approach breaks down and the result (equation (3.10)) can be at most in qualitative agreement with the true behaviour of $f_{\text{NL}}$. The drastic increase in the value of $|f_{\text{NL}}|$ seen in figure 1 is partly due to this effect. In figure 1 we also display the result of a numerical analysis in which we have, for simplicity, neglected the small change in the time $t_{\text{osc}}$ corresponding to the beginning of oscillations. The values of $|f_{\text{NL}}|$ thus obtained are not significantly different from the perturbative results, which is to be expected since only the region $s \lesssim 2/n$ is shown in figure 1. However, the increase in $|f_{\text{NL}}|$ is seen to be less steep, suggesting a possible levelling out in the non-perturbative region.
That this indeed is so can be seen as follows. When the correction $s$ becomes even larger, $s \gtrsim 2/n$, one can no longer ignore the effect on the beginning of oscillations. Indeed, the perturbations begin to oscillate long before the background field, and the oscillation may not initially take place in the quadratic part of potential [20]. We do not consider such large corrections in detail here but we make some general remarks on the behaviour of $f_{\text{NL}}$ justifying the use of the perturbative results in the region $s \approx 2/n$ and giving a qualitative understanding of the region $s \gtrsim 2/n$ not covered by our perturbative treatment. Using equation (3.1) we find that the condition for a local extremum in $f_{\text{NL}}(n)$ can be approximatively written as

$$\sigma_{\text{osc}}'' = \frac{1}{m_{\sigma}^2} \left( \sigma_{\text{osc}} \sigma_{\text{osc}}'' + \sigma_{\text{osc}}'' \sigma_{\text{osc}}' - 2 \frac{\sigma_{\text{osc}}''}{\sigma_{\text{osc}}} \sigma_{\text{osc}} \sigma_{\text{osc}}' \sigma_{\text{osc}}'' \sigma_{\text{osc}}' \right).$$

(4.2)

To obtain this result we have assumed the potential correction to be of the same order of magnitude as the quadratic part. In the limit of small corrections $\lambda \ll 1$ the expression in parentheses in equation (4.2) vanishes and, since $\sigma_{\text{osc}}'' < 0$ by equation (3.9), we see that in this region $f_{\text{NL}}$ is a monotonically decreasing function of $n$; this is consistent with our perturbative result, equation (3.10). However, when the correction becomes larger, the terms in equation (4.2) involving time derivatives are no longer negligible. Thus, for certain values of $n$ there exist solutions to equation (4.2) implying that the growth of the non-linearity parameter $|f_{\text{NL}}|$ in the region $s \gtrsim 2/n$ eventually ceases, whereafter $|f_{\text{NL}}|$ begins to oscillate. We do not examine the non-Gaussianity in this region more closely, but we point out that $|f_{\text{NL}}|$ might become large enough to already exclude some classes of potentials with the present WMAP limits [3].

5. Restrictions on the potential correction

So far we have considered the potential corrections only from a technical point of view. There are, however, some physical motivations for choosing non-quadratic potentials. Small corrections of the type $n = 2 + \epsilon$ typically represent the effects coming from one-loop corrections due to the light degrees of freedom that the curvaton couples to. As we have seen above, these tend to decrease the amount of non-Gaussianity. Also, it is conceivable that the curvaton is self-interacting. The $n = 4$ term, in particular, would be interesting since it implies less fine-tuning to satisfy the smallness condition of the correction $s \ll 2/n$; for $n = 4$ one would not have to require $\lambda \ll 1$ as for the higher order cases such as might arise, for example, in models [21, 22] where the curvaton field is considered to be one of the flat directions of the minimally supersymmetric Standard Model. This is also seen in figure 2 where we show $f_{\text{NL}}$ as a contour plot in $(\sigma/m_\sigma, \lambda)$ space for $n = 4$ and 6. We note that, especially in the $n = 4$ case, there is a significant region in the parameter space in which the potential correction is small, $s \lesssim 2/n$, but the value of $f_{\text{NL}}$ is highly suppressed from the quadratic case. For non-renormalizable terms ($n > 4$) the requirement of negligible inflaton-generated curvature perturbations sets an upper limit on $\lambda$ (e.g. for $n = 6\lambda \lesssim 10^{-10}$), but the region in the parameter space with small values of $|f_{\text{NL}}|$ is still considerable. In other words, it is quite possible to obtain $|f_{\text{NL}}| \ll 1$ even in the curvaton models by adding a small self-interaction term to the quadratic potential.

WMAP yields an upper limit [2] $|f_{\text{NL}}| \leq 100$, which in the case of a quadratic curvaton potential implies that $r \gtrsim 0.02$ [8] as can be seen from equation (1.1). For
Figure 2. The non-linearity parameter $|f_{NL}|$ in units $5/3r = 1$ as a contour plot with $n = 4$ in the left panel and $n = 6$ in the right panel. The scale on the $y$-axis is logarithmic. The values of $|f_{NL}|$ are evaluated only in the perturbative region $s \lesssim 2/n$ and the non-perturbative region is printed in white.

Figure 3. Allowed regions in the parameter space $(s, r)$. The labels are outside the allowed region for a given value of $n$.

In the non-quadratic case, the limit is greatly modified due to the decrease in $f_{NL}$. From equation (3.10) the part of the parameter space $(s, r)$ compatible with present observations is given by

$$r > \frac{1}{60} \left| 1 + \frac{n(n-1)(n-2)g(n)(0.41s + 0.25s^2ng(n))}{(0.81 + 0.50n(n-1)sg(n))^2} \right|. \quad (5.1)$$

The allowed region is represented in figure 3 for a choice of parameter values. It is noteworthy that the limits on $r$ are strongly dependent on the size $s$ and form $n$ of the

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potential correction. In the non-perturbative region not shown in figure 3, we expect the increase in the lower limit on $r$ to level out as a result of the behaviour of $f_{NL}$ described above.

6. Conclusions

In this paper we have examined how a small departure from the quadratic curvaton potential will affect the level of non-Gaussianity of the primordial curvature perturbation produced. For non-quadratic potentials there are two competing, opposite effects that contribute to the net non-Gaussianity. First, unlike in the quadratic case, the perturbations will be damped faster than the background value. This tends to reduce the value of the non-linearity parameter $|f_{NL}|$. Second, the increase of the exponent $n$ diminishes the energy density associated with the background field at the beginning of oscillations. With a steep enough potential, this effect becomes dominant and compensates for the damping of the perturbations. As a consequence, the value of $|f_{NL}|$ will increase as compared with that for the purely quadratic case.

The net outcome is that although a small departure from a quadratic curvaton potential does not significantly modify the Gaussian part of the perturbation, it can have a pronounced effect on the level of non-Gaussianity. In particular, the limit $|f_{NL}| \ll 1$ is allowed even with small curvaton energy densities $r \ll 1$. This is in sharp contrast with the quadratic result [8, 11] and shows that the curvaton models are not ruled out by a possible non-detection of non-Gaussianity as has been suggested in e.g. [8]. Furthermore, unlike in the quadratic case, there are no strict limits that could be placed on the curvaton energy density parameter $r$. By adding a small correction to the potential, in practice one can enable arbitrarily small $r$ to be obtained with suitably chosen parameter values. This is an interesting result in the sense that it implies that one cannot use present observations to fix the lower limit for the energy scale of an approximately quadratic curvaton potential without making further assumptions about the higher order terms.

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