Proximity and Josephson effects in superconductor - two dimensional electron gas planar junctions.

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Abstract

The DC Josephson effect is theoretically studied in a planar junction in which a two dimensional electron gas (2DEG) infinite in lateral directions is in contact with two superconducting electrodes placed on top of the 2DEG. An energy gap in the excitation spectrum is created in the 2DEG due to the proximity effect. It is shown that under certain conditions, the region of the 2DEG underneath the superconductors is analogous to a superconducting region with an order parameter \( \varepsilon_g \exp(i\phi) \), where \( \varepsilon_g \) (\( \varepsilon_g < \Delta \)) depends on the interface transmittance and the Fermi velocity mismatch between the superconductors and the 2DEG.

Key words: energy gap, Josephson effect, proximity effect, two-dimensional electron gas, weak links
I. INTRODUCTION

In recent years significant progress has been made in the preparation and study of Josephson junctions, in which the weak coupling is realized through a two dimensional electron gas (2DEG). These systems are prepared using semiconductor heterostructures with superconducting contacts. The structure Nb-InAs(2DEG)-Nb appears to be the most promising\textsuperscript{1–3}, although it may be possible to obtain the Josephson effect also in other structures based, for example, on GaAs\textsuperscript{4–6}. In Josephson junctions with a 2DEG as a weak link, one expects phenomena analogous to the conductance quantization in quantum point contacts\textsuperscript{7–9}. One of these phenomena is the predicted quantization of the critical Josephson current $I_c$\textsuperscript{10,11}. In short quantum contacts ($L < \xi$, $\xi = \hbar v_F/\pi \Delta$ is the coherence length), the critical current is predicted to increase with increasing contact width in a step like way, with a step height equal to $e\Delta/\hbar$.

The geometry of the contacts shown in Fig. 1a has been considered in previous theoretical works\textsuperscript{10,11}. In this case, electrons move in a channel or 2D region of finite length and experience Andreev reflections at the SN interfaces. The excitation spectrum of the 2DEG (or 1DEG) is changed due to the interference of waves reflecting at the opposite SN boundaries; in particular, bound states decaying into the S regions and corresponding to energies $\varepsilon < \Delta$ appear in the system\textsuperscript{12–14}. These bound states give the main contribution to the Josephson current $I_c$ at low temperatures.

It is, because of its practical significance, of interest to study the DC Josephson effect in the geometry shown in Fig. 1b. In this geometry, the 2DEG is unbounded in the $(x, y)$ plane (the $z$ axis is directed perpendicular to the 2DEG plane). We will find the condensate Green’s functions in the N layer, i.e., in the layer with the 2DEG, and show that due to the proximity effect, the properties of the N layer at $|x| > L/2$, where $L$ is the distance between both superconductors, are analogous to those of a superconductor with an effective order parameter $\varepsilon_g \exp(i\phi)$ with $\varepsilon_g$ dependent on the interface transmittance and the Fermi velocity mismatch. We note that the pair potential in the N layer $\Delta_N$ determined from the selfconsistency equation equals zero because the electron-phonon coupling constant in the N layer is supposed to be vanishing. For a general discussion of the proximity effect in terms of a tunneling model see Ref. 15. We adopt the following simplified model of the system shown in Fig. 1b. The transmittance of the SN interface $T_b(x)$ is proposed to be dependent on the coordinate $x$; it varies from a given value $T_b$ at $|x| > L/2$ to zero at $|x| < L/2$ on a characteristic length $x_0$. We assume that $x_0$ is much larger than the Fermi wave length $k_F^{-1}$ in the N layer, but is smaller than the coherence length in the 2DEG $\xi_{2D} = \hbar v_F/\pi \varepsilon_g$. Then, we can use an adiabatic approximation to calculate the energy spectrum of the 2DEG at $|x| > L/2$.

II. THE PROXIMITY EFFECT IN THE 2DEG.

Let us consider the energy-diagram for a system shown schematically in Fig. 2. The region at $z < 0$ is occupied by a superconductor, and there is a quantum well with the 2DEG in the layer $0 < z < d$. The Fermi momenta in the S and N regions ($p_F = \sqrt{p_{Fx}^2 + p_{Fz}^2}$ and $k_F = \sqrt{k_{Fx}^2 + k_{Fy}^2}$, respectively) differ greatly from each other due to a significant difference
in electron concentrations in these regions; namely, \( p_F = (3\pi^2 n_S)^{1/3} \gg k_F = (2\pi n_{2D})^{1/2} \) (the formula for \( k_F \) is written for the case when only the lowest subband in the quantum well is filled). Generally speaking, there exists a Schottky barrier at \( z = 0 \), which we model by a potential of the form \( U_b a_b \delta(z) \), where \( U_b \) and \( a_b \) are the height and width of the barrier varying smoothly at \( \mid x \mid \approx L/2 \).

In order to calculate the excitation spectrum and wave functions of the 2DEG, we write the well known Bogoliubov-de Gennes equations

\[ \hat{H} \hat{\Psi}_k = \varepsilon_k \hat{\Psi}_k. \]  

Here \( \hat{\Psi}_k \) is the two component wave function, \( \varepsilon_k \) is the excitation energy, and \( \hat{H} \) is the Hamiltonian of the system,

\[ \hat{H} = \left[ \left( -\frac{1}{2m_1} \nabla^2 + U_b a_b \delta(z) - \varepsilon_F \right) \hat{\sigma}_z + \hat{\sigma}_x \Delta \right] \theta(-z) + \left[ -\frac{1}{2m_2} \nabla^2 + U_{Sm} - \varepsilon_F \right] \hat{\sigma}_z \theta(z) \theta(d - z). \] 

Here \( m_1 \) and \( m_2 \) are the effective masses in the S and N regions respectively, and \( U_{Sm} \) is the difference between the potentials of the conduction band edges in the S and N regions (see Fig. 2).

One can show that propagating (into the S region) states with \( \varepsilon_k > \Delta \) and bound states with \( \varepsilon_k < \Delta \) exist. The latter states correspond to a branch of the spectrum with an effective energy gap \( \varepsilon_0 \), which is small if the barrier transmittance is small. For low temperatures \((T \ll \Delta)\), the main contribution to the critical current \( I_c \) originates from these bound modes. We will restrict ourselves to the case of low temperatures. A solution describing the bound states has the form

\[ \hat{\Psi}_{k_0}(r) = \exp(ik_{||}r_{||}) \left\{ \left[ B_+ \exp(\kappa z + ip_z z) \begin{pmatrix} u \\
v \end{pmatrix} - B_- \exp(\kappa z - ip_z z) \begin{pmatrix} u \\
v \end{pmatrix} \right] \theta(-z) + \left[ E_+ \exp(ik_z z) \begin{pmatrix} 1 \\
0 \end{pmatrix} - E_- \exp(-ik_z z) \begin{pmatrix} 1 \\
0 \end{pmatrix} + H_+ \exp(ik_z z) \begin{pmatrix} 0 \\
1 \end{pmatrix} \\
- H_- \exp(-ik_z z) \begin{pmatrix} 0 \\
1 \end{pmatrix} \right] \theta(z) \theta(d - z) \right\}. \]  

Consider first the wave function in the quantum well (the second term in Eq.(3)). The first two terms \((E_\pm)\) describe an electron excitation moving forward and backward and the second term \((H_\pm)\) correspond to a hole excitation. Momenta of these excitations along the \( z \) axis can be presented in the form \( k_z = k_n + \delta k_n \) and \( \bar{k}_z = k_n + \delta \bar{k}_n \), where \( k_n = (\pi/d)(n + 1) \) and \( |\delta k_n, \delta \bar{k}_n| \ll k_n \). Here we will consider the case when only the lowest subband, with an energy \( \varepsilon_0 = \pi/d^2 m_2 \), is filled \((n = 0)\) and other subbands \((n \geq 1)\) are empty. So we are interested in the states with \( n = 0 \). A relation between \( \delta k_0 \) and \( \delta \bar{k}_0 \) can be obtained from Eq.(1),

\[ \varepsilon_k = \xi_k + v_0 \delta k_0 = -(\xi_k + v_0 \delta \bar{k}_0). \] 

Here \( \xi_k = (k_{||}^2 - k_{F0}^2)/2m_2 \) is the kinetic energy of electrons in the 2DEG relative to the Fermi energy, \( \varepsilon_F = (k_{F0}^2 + k_{F0}^2)/2m_2 + U_{Sm} \), \( k_{F0} \) is the Fermi momentum in the limit of infinite barrier height, \( v_0 = k_0/m_2 \).
Consider now the wave functions in the S region, which decay over the length \( \kappa^{-1} \). The coefficients \( (u, v) \) have the usual form,

\[
u^2 = \frac{1}{2}(1 + \xi_p/\varepsilon_p), \quad v^2 = \frac{1}{2}(1 - \xi_p/\varepsilon_p),
\]

but in this case the functions \( \xi_p \) are purely imaginary

\[
\xi_p = -i\kappa p_z/m_1, \quad \varepsilon_p = (\Delta^2 + \xi_p^2)^{1/2}.
\]

The momentum \( p_z \) approximately coincides with the Fermi momentum \( p_F \) where we assume the reflection at the interface to be specular, i.e. \( p_\parallel = k_\parallel \) and, as noted above, \( k_\parallel \ll p_F \). Hence for the Fermi energy, we can write

\[
\varepsilon_F = p_F^2/2m_1 = (p_z^2 + p_\parallel^2)/2m_1 \approx p_z^2/2m_1.
\]

In order to find the excitation spectrum, i.e. the dependence of \( \varepsilon_k \) on the momenta in the \((x, y)\) plane \( k_\parallel \), we must use boundary conditions at the interface \((z = 0)\). These conditions consist in continuity of \( \hat{\Psi}_{k_\parallel}(z) \) at \( z = 0 \) and in a relationship between the derivatives \( \partial_z \hat{\Psi}_{k_\parallel}(z) \) at \( z = 0 \)

\[
\hat{\Psi} _{k_\parallel}(+0) = \hat{\Psi} _{k_\parallel} (-0), \quad \frac{1}{2m_2} \partial_z \hat{\Psi} _{k_\parallel}(z)|_{+0} - \frac{1}{2m_1} \partial_z \hat{\Psi} _{k_\parallel}(z)|_{-0} = U_b a_b \hat{\Psi} _{k_\parallel}(0).
\]

Substituting Eq.\((3)\) into Eq.\((8)\), we get a set of algebraic equations for the coefficients \( E_\pm, H_\pm \) and \( B_\pm \). The solvability condition results in a dispersion relation for the excitation spectrum at \( \varepsilon_k < \Delta \)

\[
1 + \alpha_k \bar{\alpha}_k (w^2 + s^2) + w(\alpha_k + \bar{\alpha}_k) + is(\varepsilon_k/\xi_p)(\alpha_k - \bar{\alpha}_k) = 0 \tag{9}
\]

Here \( \alpha_k = \delta k_0 d \ll 1 \) and \( \bar{\alpha}_k = \delta \bar{k}_0 d = -(\alpha_k + 2\xi_k/\varepsilon_0) \). \( w = (2U_b a_b m_2/k_0) \) is a dimensionless parameter characterizing the barrier transmittance, the factor \( s = p_F m_2/k_0 m_1 \) depends on the mismatch of the Fermi momenta and the effective masses. Eq.\((3)\) determines the spectrum of bound states with energies \( \varepsilon_k < \Delta \). Eq.\((9)\) can be rewritten in the form

\[
\varepsilon_k^2 \left[ 1 + \frac{2\varepsilon_{g0}}{(\Delta^2 - \varepsilon_k^2)^{1/2}} \right] = (\xi_k - \frac{w}{s} \varepsilon_{g0})^2 + \varepsilon_{g0}^2, \tag{10}
\]

where \( (\xi_k - \frac{w}{s} \varepsilon_{g0}) \) is the relative kinetic energy of electrons moving in the \((x, y)\) plane. The quantity \( \varepsilon_{g0} = \varepsilon_0 s/(w^2 + s^2) \) is the energy gap in the excitation spectrum of the 2D electron gas induced by the proximity effect in the case of very low barrier transmittances. Indeed, under the condition

\[
(\varepsilon_0/\Delta) \frac{s}{w^2 + s^2} \ll 1 \tag{11}
\]

one can neglect the second term in the square brackets in Eq.\((10)\) and obtain for \( \varepsilon_k \) not too close to \( \Delta \)

\[
\varepsilon_k = \pm \left[ \varepsilon_{g0}^2 + (\xi_k - \frac{w}{s} \varepsilon_{g0})^2 \right]^{1/2}. \tag{12}
\]
Therefore, the dependence of $\varepsilon_k$ on $k_\parallel$ is nearly the same as in a 2D superconductor with the energy gap

$$
\varepsilon_g \approx \varepsilon_{g0} = \varepsilon_0 - \frac{s}{w^2 + s^2},
$$

(12')

where $\varepsilon_0$ is the subband energy for $n = 0$. This dependence is shown in Fig. 3 for several values of $w$. In Fig. 3 it is shown that, with changing $w$, not only the value of $\varepsilon_g$ is changed, but also the $k_\parallel$ at which the minimum in $\varepsilon_k$ occurs. This may be understood intuitively, since increasing the SN-barrier transparency will alter the exact form of the electron-hole wave function in the 2DEG, $\Psi_{k_\parallel}(r)$. Since $k_F$ is increased as compared to $k_{F0}$ when the barrier transmittance is finite, the minimum in $\varepsilon_k$ is expected to occur at larger $k_\parallel$.

With increasing the temperature $T$ the energy gap in the S region $\Delta(T)$ is diminished, and the condition Eq.(11) is violated at $T$ sufficiently close to $T_c$. The dependence of the energy gap in the 2DEG $\varepsilon_g$ on $\Delta(T)$ is determined from Eq.(10) if we put $(\xi_k - \frac{w}{s}\varepsilon_{g0}) = 0$, which means putting $\varepsilon_k$ at a minimum. Then we obtain the equation for $\varepsilon_g$

$$
\varepsilon_g^2 \left[ 1 + \frac{2\varepsilon_{g0}}{(\Delta(T))^2 - \varepsilon_g^2} \right] = \varepsilon_{g0}^2.
$$

(13)

This dependence is shown in Fig. 4. The maximal value of $\Delta(T)/\varepsilon_{g0}$ equals $\Delta(0)/\varepsilon_{g0}$. If this value is very large, the energy gap in the excitation spectrum of the 2DEG coincides with $\varepsilon_{g0}$ in a wide range of $T$. The characteristic temperature $T^*$ determining a transition of $\varepsilon_g$ from $\varepsilon_{g0}$ to $\Delta(T)$ is given by the equation $\Delta(T^*) \approx \varepsilon_{g0}$. At low temperatures ($\Delta(T) = \Delta(0)$) we can also calculate the influence of the barrier, expressed in $w$, from Eq.(13). Fig. 5 shows the energy gap $\varepsilon_g$ in the 2DEG as function of the transmittance $T_b = 1/((w/2s)^2 + 1)$ of the S-2DEG interface, for different values of $s$.

If the excitation energy $\varepsilon_k$ exceeds $\Delta$, the wave functions in the S region do not decay, but oscillate, and the $k$ vector runs over a continuous set of values. These wave functions describe the propagation of two electrons (incident on the barrier and reflected from it) and a hole that appears as a result of Andreev reflection. The bound states obtained above are closely related to those studied earlier in the 3D case.

The wave functions corresponding to the bound states are determined by Eq.(8). By introducing new variables

$$
\tilde{\varepsilon}_k \equiv \varepsilon_k/\varepsilon_0 = \alpha_k + \xi_k/\varepsilon_0, \ t_k = \xi_k/\varepsilon_0 - t_w, \ t_w = \frac{w}{w^2 + s^2},
$$

(14)

we can write for the coefficients from Eq.(8) the relations

$$
E_- = E_+ \exp(2i\alpha), \ H_- = H_+ \exp(2i\alpha)
$$

$$
B_+ = 2\tilde{\varepsilon}_k [(E_+u_p + H_+v_p)\tilde{\varepsilon}_k - (E_+u_p - v_pH_+)(t_k + t_w)] / (\delta^2 - \tilde{\varepsilon}_k)^{1/2}
$$

$$
B_- = 2\tilde{\varepsilon}_k [(E_+v_p + H_+u_p)\tilde{\varepsilon}_k - (E_+v_p - u_pH_+)(t_k + t_w)] / (\delta^2 - \tilde{\varepsilon}_k)^{1/2}.
$$

(15)

Using these relations and Eq.(8) for the wave functions, we can find the Green’s functions $\hat{G}^{R(A)}$ for the 2DEG in the system shown in Fig.1b and calculate the critical Josephson current.
III. THE GREEN’S FUNCTIONS AND THE DC JOSEPHSON EFFECT

Consider the system shown in Fig. 1b. Electrons move in the quantum well in the \((x, y)\) plane. As shown before, the wave functions in the 2DEG change drastically due to the proximity effect. In particular, the condensate Green’s functions \(G^{R(A)}\) are induced in the quantum well, and therefore the Josephson effect is possible in this system. In order to determine the critical current, we need to know the functions \(G^{R(A)}\), that is, the nondiagonal elements of the matrix \(G^{R(A)}_{\alpha\beta} : F^{R(A)}(z, z'; q) = G^{R(A)}_{12}(z, z'; q)\). We are interested in the current \(I\) averaged over the quantum well width, which means that we must find \(F^{R(A)}(k) = \langle F^{R(A)}(z, z'; q) \rangle\), where the brackets denote averaging over \(z\) \((0 < z < d)\). As is well known, the matrix components of \(G^{R(A)}\) are expressed through the components of the wave functions \(\Psi_{\alpha}(z, q)\)

\[
G^{R(A)}_{\alpha\beta}(z, z'; k) = \sum_{i=\pm 1} \frac{\langle \Psi_{\alpha}(z, k)\Psi_{\beta}^{*}(z, k) \rangle_i}{\varepsilon \pm i0 - \varepsilon_{ki}}, \quad (16)
\]

where the sum is taken over the two branches of the spectrum determined by Eq.(11). We suppose that the transmittance of the SN barrier is small, implying \(\varepsilon_0 \ll \Delta\), and the conditions \(s/w \ll 1\) and \(s/w^2 \ll \Delta/\varepsilon_0\) are fulfilled. Then, one can show that \(E_+\) and \(H_+\) are coupled by the relation

\[
E_+(\varepsilon_k - t_k) = \varepsilon_0 H_+. \quad (17)
\]

Taking into account Eqs.(17) and (12), one can find from Eq.(16) that the components of the Green’s functions are equal to

\[
G^{R(A)}_{11}(k) = \frac{1}{d} \left\{ \frac{1}{2\varepsilon_k} \left[ \frac{\varepsilon_k + \varepsilon_0 t_k}{\varepsilon \pm i0 - \varepsilon_k} + \frac{\varepsilon_k - \varepsilon_0 t_k}{\varepsilon \pm i0 + \varepsilon_k} \right] \right\},
\]

\[
G^{R(A)}_{12}(k) = \frac{1}{d} \left\{ \frac{\varepsilon_0}{2\varepsilon_k} \left[ \frac{1}{\varepsilon \pm i0 - \varepsilon_k} + \frac{1}{\varepsilon \pm i0 + \varepsilon_k} \right] \right\}. \quad (18)
\]

Here \(\varepsilon_k = (\varepsilon^2_0 + \varepsilon^2_0 t^2_k)^{1/2}\) is the excitation energy, \(\varepsilon_0\) is the energy gap in the excitation spectrum in the 2DEG at \(|x| > L/2\) (see Eq.(13)). These functions are identical to the Green’s functions of an ordinary two dimensional superconductor with a spatially dependent energy gap because, as supposed, the parameter \(w(x)\) varies from a constant value \(w\) at \(|x| > L/2\) to \(\infty\) at \(|x| < L/2\). The characteristic length of the \(w(x)\) variation is \(x_0\), which is small as compared to the coherence length in the 2DEG, \(\xi_{2D} = \hbar v_F/\pi\varepsilon_0\) and large as compared to the Fermi wave length \(k_F^{-1}\). Therefore the system under consideration is equivalent to a 2DEG contacting at \(|x| > L/2\) the superconducting 2DEG with the effective order parameter \(\varepsilon_0 \exp(\pm i\phi/2)\), where \(\varepsilon_0 \ll \Delta\) (strictly speaking this magnitude is achieved at \(|x| > L/2\) over distances of the order of the coherence length as it takes place in ordinary SNS junctions). Hence for the critical current \(I_c\) one can use the formulae obtained in Refs. 13 14 where the width of the N layer was assumed to be large or in Ref. 10 where a one dimensional channel is analyzed. If the width, \(L_y\), of a channel in the 2DEG shown in Fig. 1b is comparable with the Fermi wave length \(k_F^{-1}\) at \(|x| < L/2\) and is much larger at \(|x| > L/2\), we can use the expression for \(I_c\) obtained in Ref. 10

\[
I_c = N(\varepsilon_0/\hbar) \sin(\phi/2) \tanh((\varepsilon_0/2T) \cos(\phi/2)), \quad (19)
\]
where $N$ is the number of subbands below the Fermi level. This means that $I_c$ will increase step wise with increasing the electron density in the quantum well, and the height of the steps equal $e\varepsilon_g/h$.

IV. CONCLUSIONS

We have found the critical Josephson current $I_c$ for a 2DEG in contact with two superconductors. In contrast to previously analyzed systems the effective order parameter in the 2DEG $\varepsilon_g \exp(i\phi)$ is reduced in comparison with $\Delta \exp(i\phi)$ in the superconductor, and its magnitude is determined by the SN barrier (Schottky barrier) transmittance and by a mismatch of the Fermi momenta and the effective masses in the S and N regions (here $\phi$ is the macroscopic phase of the superconductors). The barrier transmittance characterized by the parameter $w$ may depend on the carrier density in the 2DEG, $n_{2D}$. Therefore, the critical current $I_c$ will depend on $n_{2D}$ even when only the lowest subband is filled.

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FIGURES

FIG. 1. Schematic representation of S-2DEG-S Josephson junctions, as studied in Refs. 10,11 (1a) and in the present paper (1b). The spacing between the superconductors is $L$, the width of the 2DEG is $d$.

FIG. 2. Energy diagram of the system under consideration – a superconductor and a 2DEG are separated by a potential barrier $U_b$. Hatched areas denote states filled with electrons (we suppose that only the lowest subband in the quantum well, with energy $\varepsilon_0$, is occupied). The energy gap in the excitation spectrum of the 2DEG is induced due to the presence of the superconductor.

FIG. 3. The normalized excitation energy in the 2DEG, $\varepsilon_k/\Delta$, vs the longitudinal momentum of electrons, $k_{||}$, at different values of the parameter $\varepsilon_g/\Delta = (\varepsilon_0/\Delta)s/(s^2 + w^2)$: $\varepsilon_g/\Delta = 0.2$ (dashed line); 0.3 (solid line); 0.6 (dotted line).

FIG. 4. The normalized energy gap in the excitation spectrum of the 2DEG $\varepsilon_g/\varepsilon_g0$ vs the normalized energy gap in the S region $\Delta/\varepsilon_g0$. When the temperature is increased towards $T_c$, $\Delta(T)$ goes to 0, thus violating the condition of Eq.(11).

FIG. 5. The normalized energy gap $\varepsilon_g/\Delta$ vs the transparency $T_b$, expressed in the dimensionless parameter $w$, of the barrier between superconductor and 2DEG, for different values of the mismatch, $s = p_F m_2/k_0 m_1$: $s = 0.5$ (solid line); 1 (dashed line); 2 (dotted line).