Study on frequency domain response characteristics of straight pipe conveying fluid

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Abstract. In this paper, a transfer matrix method (TMM) for solving the frequency domain response of the straight pipe conveying fluid is established based on 14-equation model by Laplace transformation. The reliability of the method is verified by the frequency domain response results of fluid velocity and structural axial stress in the pipe under free boundary conditions. Then, based on the transfer matrix method, the influence of boundary conditions and fluid density on the frequency domain response characteristics of pipeline vibration is analyzed. The numerical results show that the boundary conditions have a significant effect on the amplitude of the vibration response of the pipeline, but the amplitude changes little under the fixed and simply supported boundary conditions. The vibration frequency is inversely proportional to the density, while the axial vibration response amplitude increases with the increase of fluid density, and the change of density has little effect on the transverse vibration response.

Key words: fluid-solid coupling; straight pipe; transfer matrix method; fluid density; frequency domain response.

1. Introduction

With the continuous improvement of the stability and safety requirements of the pipeline system, the study of flow induced vibration has been paid more and more attention. Researchers have carried out a lot of research on the vibration problems of pipeline systems with different types, different medium effects or different boundary conditions by means of theoretical analysis, numerical simulation and experimental verification.

For the pipeline system with large length diameter ratio (i.e. large span and small diameter pipe), the influence of respiratory mode can be ignored, and its bending vibration mode is similar to that of beam structure. Therefore, the Euler Bernoulli beam and Timoshenko beam model are used to simplify the coupled vibration model for large-span and small-diameter pipeline system. Based on the Poisson coupling mechanism, Walker and Phillips established four equation models of conservative and non-conservative systems, and used the model to study the dynamic characteristics of pipeline under transient valve closing excitation. Ruoff further optimizes the 6-equation model to improve its calculation accuracy, which fully considers the influence of flow strength, damping and physical parameters. In addition, the 12-equation model has also been used. However, 4-equation model, 6-equation model, 8-equation model and 12-equation model can only meet the calculation requirements of simple straight
pipeline. For complex pipeline system or high calculation accuracy model, the above equation models are no longer practical. At present, the 14-equation model is widely used by researchers.

Because of its high efficiency, simplicity and flexibility, transfer matrix method is widely used in coupled vibration analysis of pipeline system. The core idea of transfer matrix method is to construct the transfer relationship between discrete elements and nodes by mechanical balance, and to solve the dynamic model by combining with the boundary conditions. Chaudhry introduced the application of this method in the coupled vibration analysis of pipeline. Based on the transfer matrix method, the effects of supports, structural characteristics and fluid parameters on the dynamic response and natural frequency of the pipeline are analyzed. Zhang Lixiang gives the frequency domain solutions of 4-equation model, 8-equation model and 14-equation model based on transfer matrix method. Secondly, based on the 14-equation dynamic model, Li Zhenbin obtained the vibration mode and frequency of the pipeline system with elastic support boundary by transfer matrix method. Liu Gongmin derived the frequency domain solution of N branch transfer matrix. The influence of fluid pressure on the dynamic response of pipeline system was analyzed by adding friction coupling term to 14-equation model.

In conclusion, based on the assumption of beam model, the 14-equation model of fluid structure coupling vibration is established, and the transfer matrix method is used to solve the coupling vibration characteristics of liquid filled pipe section under different fluid medium, boundary conditions and fluid density. In addition, the mechanism of fluid structure coupling vibration is explored.

2. Dynamic model of pipe section

2.1. Coupled vibration 14-equation

For micro deformation pipeline system, the effect of structure on flow field is small, so the change of flow field caused by structure deformation can be ignored, and only the influence of fluid on structure can be considered. The simplified model of the direct pipeline is shown in Figure 1. The following assumptions are made in this paper.

1) The fluid medium in the pipe is uniform, one-way and compressible, considering the influence of fluid viscosity;
2) The fluid medium in the tube is liquid, and there is no hole phenomenon, and the influence of radial deformation on the fluid is not considered;
3) The motion of fluid pressure wave in the tube is described by one-dimensional wave equation;
4) The pipe structure is isotropic elastic body, which is axisymmetric. Timoshenko beam model is used to describe the vibration of the pipe;
5) The circumferential pressure and flow rate of the fluid on the same pipe section are the same and constant.

In conclusion, considering the effects of Poisson coupling and fluid friction, the 14 equation model can be described as follows:

4-equation of axial vibration:

\[ \rho_f \frac{\partial V_f}{\partial t} + \frac{\partial p}{\partial z} + \frac{2\pi f R}{r} = 0 \]  \hspace{1cm} (1)
\[ \rho_s A_s \frac{\partial w_{zz}}{\partial t} + \frac{\partial f_{sz}}{\partial z} - \frac{A_s}{\delta} \tau_f = 0 \] (2)

\[ \frac{\partial w_{zz}}{\partial z} + \frac{1}{E A_s} \frac{\partial f_{zz}}{\partial t} + \frac{\mu}{E} \frac{R^2}{(R + \frac{z}{2})^3} \frac{\partial p}{\partial t} = 0 \] (3)

\[ K \frac{\partial p}{\partial t} + 2 \mu \frac{\partial f_{zz}}{\partial t} + \frac{\partial V}{\partial z} = 0 \] (4)

4-equation of transverse x-z plane vibration:

\[ m \frac{\partial w_{sx}}{\partial t} + \frac{\partial f_{sx}}{\partial z} = 0 \] (5)

\[ EI \frac{\partial \theta_x}{\partial z} + \frac{\partial M_x}{\partial t} = 0 \] (6)

\[ \frac{\partial w_{sx}}{\partial z} - \frac{\theta_x}{\delta} + \frac{1}{k g A_x} \frac{\partial f_{sx}}{\partial t} = 0 \] (7)

\[ \frac{\partial M_y}{\partial z} + f_{sy} + (\rho_s I_s + \rho_f I_f) \frac{\partial \theta_x}{\partial t} = 0 \] (8)

4-equation of transverse y-z plane vibration:

\[ m \frac{\partial w_{sy}}{\partial t} + \frac{\partial f_{sy}}{\partial z} = 0 \] (9)

\[ EI \frac{\partial \theta_y}{\partial z} + \frac{\partial M_y}{\partial t} = 0 \] (10)

\[ \frac{\partial w_{sy}}{\partial z} - \frac{\theta_y}{\delta} + \frac{1}{k g A_y} \frac{\partial f_{sy}}{\partial t} = 0 \] (11)

\[ \frac{\partial M_x}{\partial z} + f_{sx} + (\rho_s I_s + \rho_f I_f) \frac{\partial \theta_y}{\partial t} = 0 \] (12)

2-equation of torsional vibration

\[ \frac{\partial M_z}{\partial z} + \rho J \frac{\partial^2 \theta_z}{\partial t^2} = 0 \] (13)

\[ \frac{\partial M_z}{\partial t} + G J \frac{\partial \theta_z}{\partial z} = 0 \] (14)

Where, \( \rho_f, p, V \) are fluid density, pressure and velocity; \( \tau_f \) is friction stress between fluid and pipe wall; \( V_{Fz} \) is the axial vibration velocity; \( R, R_0 \) are the inner or outer diameter of the pipe; \( f_z, w_z, \theta, M \) are the force, velocity, angular velocity and moment of the pipeline in different directions; \( \rho_s, K, m, E, \delta, \mu, G \) are the density, bulk modulus, mass, Young's modulus, thickness, Poisson's ratio and shear modulus of the pipeline respectively; \( k \) is section shear coefficient; \( A_s = \pi (R_0^2 - R^2) \) is the cross-sectional area of the structure; \( I_s = \frac{\pi}{4} (R_0^4 - R^4), I_f = \frac{\pi}{4} R^4 \) are the moment of inertia of pipe and fluid; \( J_s = \frac{l_s}{2}, J = \rho_s J_s dz \) is the torsional inertia of the pipe.

### 2.2. Frequency domain of 14-equation

The 14 equation is transformed into frequency domain by Laplace transform, and it is written as a matrix

\[ s \Phi(z, s) + A^{-1} B \frac{\partial \Phi(z, s)}{\partial z} = A^{-1} R(z, s) \] (15)

Where, \( \Phi(z, s) = \ell(y(z, t)), A = \hat{A} + \frac{c}{s}, R(z, s) = \ell(\hat{r}(z, t)) + A \Phi(z, 0) \).

### 2.3. Boundary condition

According to the balance relationship between transverse force and displacement, the boundary conditions of pipeline can be established by linear elastic stiffness and bending stiffness, as shown in Figure 2. By simplifying the boundary conditions in Figure 2, the boundary conditions under any support can be replaced.
(1) Under the free boundary condition, the corresponding stiffness values of six degrees of freedom in three directions are all 0;
(2) With fixed support boundary conditions, the degrees of freedom in three directions tend to be infinite;
(3) Under the simply supported boundary condition, the bending stiffness and the linear elastic stiffness are infinite;
(4) When the elastic stiffness of each direction is not zero, it is the boundary condition of elastic support.

In order to meet the needs of TMM, the boundary conditions can be expressed in matrix form.
\[
\begin{align*}
H(0, s) \Phi(0, s) &= W(0, s) \\
H(L, s) \Phi(L, s) &= W(L, s)
\end{align*}
\]

Where, \(k_{x,0}, k_{y,0}, k_{z,0}, k_{x,\alpha}, k_{y,\alpha}, k_{z,\alpha}\) are the linear elastic stiffness at the starting position; \(k_{x,0\beta}, k_{y,0\beta}, k_{z,0\beta}, k_{x,\alpha\beta}, k_{y,\alpha\beta}, k_{z,\alpha\beta}\) are the bending strength of different positions.

\[
\begin{align*}
&k_{x,0}, k_{y,0}, k_{z,0}, k_{x,\alpha}, k_{y,\alpha}, k_{z,\alpha}
\end{align*}
\]

\[
\begin{align*}
k_{x,0\beta}, k_{y,0\beta}, k_{z,0\beta}, k_{x,\alpha\beta}, k_{y,\alpha\beta}, k_{z,\alpha\beta}
\end{align*}
\]

\[
\begin{align*}
&k_{x,0}, k_{y,0}, k_{z,0}, k_{x,\alpha}, k_{y,\alpha}, k_{z,\alpha}
\end{align*}
\]

\[
\begin{align*}
k_{x,0\beta}, k_{y,0\beta}, k_{z,0\beta}, k_{x,\alpha\beta}, k_{y,\alpha\beta}, k_{z,\alpha\beta}
\end{align*}
\]

Fig. 2 Schematic diagram of pipeline structure boundary conditions

### 3. Model reliability verification

#### 3.1. Model parameter

In this paper, Dundee tube model is used to verify the rationality of transfer matrix method and boundary condition simplification. The boundary condition of the model is the free boundary at both ends of the pipeline, and the test results are given by the model test designed by Vardy and fan. Dundee tube model parameters and internal fluid physical parameters are given in Table 1, and the two ends of the model are sealed with plugs. Ignoring the effect of fluid additional damping, the damping ratio of the structure is 0.002, and an exciting force of 9.4kn and lasting for 2S is applied at the initial end.

| parameters                      | value | parameters                      | value |
|--------------------------------|-------|--------------------------------|-------|
| Pipe length (m)                | 4.51  | z=0 end plug quality (kg)      | 1.321 |
| Pipe inner diameter (mm)        | 26    | z= end plug quality (kg)       | 0.3258|
| Pipe wall thickness (mm)        | 3.945 | Bulk modulus (Gpa)             | 2.14  |
| Pipe Young's modulus (Gpa)      | 168   | Fluid density (kg/m³)          | 999   |
| Pipe Poisson's ratio            | 0.3   | Pipe density (kg/m³)           | 7980  |

#### 3.2. Verification method

The frequency domain response curves of straight pipe calculated by this method and measured in reference [25] are shown in Fig. 3 (a) and (b). The comparison shows that the calculation results of this method are in good agreement with the literature results, and the errors are mainly caused by the following two reasons:

1) The influence of fluid gravity is also considered in the literature;
2) The influence of friction coupling term is not considered in the literature;

The calculation error is within the engineering allowable range, so it is proved that the method established in this paper is reliable and the calculation results are accurate.
4. Numerical results and analysis

4.1. Influence of boundary conditions on vibration response

The coupled vibration response curves at the midpoint of the straight pipe under different boundary conditions are shown in Fig. 4. Where (a) is the fluid coupling vibration velocity response and (b) is the frequency domain response of axial vibration velocity of pipeline structure. The results show that the boundary condition has a significant effect on the coupled vibration response of the pipeline, while the fixed support has a small effect on the axial vibration velocity and fluid vibration velocity of the pipeline.

4.2. Influence of fluid density on vibration response

Fig. 5 shows the influence of fluid density on the fluid structure coupling vibration response characteristics of liquid filled straight pipe under free boundary conditions (the density is 500kg / m$^3$, 800kg / m$^3$ and 1000kg / m$^3$). Among them, the flow velocity, axial vibration velocity and response amplitude decrease with the increase of fluid density, and the vibration frequency decreases with the increase of fluid density. This is mainly due to the increase of fluid density and the influence of additional mass.
In this paper, based on the 14-equation model of straight pipe, the method of solving the transfer matrix of frequency domain analysis is established by Laplace transform. The reliability of the method is verified, and the influence of boundary conditions and fluid density on the frequency domain response of pipe is explored. The results show that:

1. The calculation results are in good agreement with the experimental results in literature;
2. The results show that the boundary conditions have a significant effect on the vibration response amplitude of the pipeline, but the change of the amplitude is small under the fixed support boundary;
3. The vibration frequency is inversely proportional to the density, and the axial vibration response amplitude increases with the increase of fluid density.

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