Unification of elementary forces in gauge $\text{SL}(2N,C)$ theories

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Abstract

We argue that the gauge $\text{SL}(2N,C)$ theories may point to a possible way where the known elementary forces, including gravity, could be consistently unified. Remarkably, while all related gauge fields are presented in the same adjoint multiplet of the $\text{SL}(2N,C)$ symmetry group, the tensor field submultiplet providing gravity can be naturally suppressed in the weak-field approach developed for accompanying tetrad fields. As a result, the whole theory turns out to effectively possess the local $\text{SL}(2,C) \times \text{SU}(N)$ symmetry so as to naturally lead to the $\text{SL}(2,C)$ gauge gravity, on the one hand, and the $\text{SU}(N)$ grand unified theory, on the other. Since all states involved in the $\text{SL}(2N,C)$ theories are additionally classified according to their spin values, many possible $\text{SU}(N)$ GUTs – including the conventional one-family $\text{SU}(5)$ theory – appear not to be relevant for the standard 1/2 spin quarks and leptons. Meanwhile, the $\text{SU}(8)$ grand unification for all three families of composite quarks and leptons that stems from the $\text{SL}(16,C)$ theory seems to be of special interest that is studied in some detail.
1 Introduction

It is impossible not to notice that there is a certain similarity between gravity and the other three elementary forces when one treats gravity in a conventional gauge theory framework \([1, 2, 3]\). Indeed, the spin-connection fields gauging the local \(SL(2, C)\) symmetry group of gravity emerge much as photons and gluons appear in the Standard Model. In this connection, one may think that these spin-connection fields could be unified with the ordinary SM gauge bosons in a particular non-compact symmetry group, thus leading to the hyperunification of all known elementary gauge forces. We will refer below to such theories as the hyperunified theories (HUTs), and specifically as the \(SL(2N, C)\) HUT, when speaking about an unification of the \(SL(2, C)\) gauge gravity with the \(SU(N)\) grand unified theory, respectively.

It may not be surprising that the similar ideas have been discussed for a long time. In fact, there are many classes of models in the literature where unification of gravity and other interactions goes through the unification of the local Lorentz and internal symmetries in the framework of some non-compact covering symmetry group \([4, 5, 6, 7]\). While the spin-1 fields in the total gauge hypermultiplet of this group are proposed to mediate ordinary gauge interactions, the spin-2 fields there should provide the tiny gravity type interactions in the theory to conform to reality. The point is, however, that generally these tensor fields turn out to cause interactions being comparable with those relating to vector fields and, apart from that, lead to an existence of ghosts in the theory making it essentially unstable that can only be cured by some enormously extended Higgs sector in it.

In contrast, we show that the \(SL(2N, C)\) HUT being considered here suggests some new and crucial possibility that has not been yet actually explored. It follows from a generic option in the theory that allows to selectively weaken tensor field submultiplet as compared to the spin-1 field ones in the total \(SL(2N, C)\) gauge hypermultiplet. This is related with the fact that one can safely move on from this hypermultiplet to the new one being ”filtered out” by the accompanying tetrad fields that appears to naturally suppress just the tensor field submultiplet. As a result, these tensor fields do not propagate, their interactions essentially decouple from the other elementary forces and in the linear approximation their trace is only left in a form of Einstein-Cartan gravity in a final unified theory with the effective \(SL(2, C) \times SU(N)\) symmetry.

Note that some prototype model for the \(SL(2N, C)\) hyperunification could be the well-known \(SL(6, C)\) symmetry which has been considered quite a long ago \([8]\) as a possible relativistic version of the global \(SU(6)\) symmetry model describing the spin - unitary spin symmetry classification for mesons and baryons \([9]\). In this respect, one might expect that there would be a potential danger for our hyperunified theory due to the Coleman-Mandula theorem \([10]\) on the impossibility of combining spacetime and internal symmetries that just first appeared in connection with the \(SL(6, C)\) symmetry mentioned above. Regarding to the hyperunified theories, however, this theorem seems not to be a serious obstacle as is usually claimed on the basis of the following two heuristic arguments. The first is that it only constrains symmetries of the \(S\)-matrix and, as such, places no constraints on spontaneously broken symmetries which do not show up directly at the \(S\)-matrix level. The second is that the theorem only works if there is a mass gap in the theory that certainly
does not happen in the HUT case where in the symmetry limit all fields, as gauge bosons so the matter fields, are massless. Apart from that, however, there is an argument being very specific for the HUT considered. The point is that, while all gauge fields in it are unified in a framework of $SL(2N,C)$ symmetry group, its nondiagonal generators are solely related to the $SU(N)$ "flavored" tensor fields which appear to be naturally suppressed in the weak-field approach developed for accompanying tetrad fields. Thus the whole theory turns out to effectively possess the local $SL(2,C) \times SU(N)$ symmetry rather than the unified $SL(2N,C)$ which only provides the total hypermultiplets for gauge and matter fields in the theory. As a result, one is naturally led to the $SL(2,C)$ gauge gravity, on the one hand, and the $SU(N)$ GUT, on the other, thus actually getting rid of the constraints of the Coleman-Mandula theorem.

The paper is organized in the following way. In Section 2 we give a standard presentation of the $SL(2,C)$ gauge gravity which then it is discussed in the weak-field approach. In Section 3 the $SL(2N,C)$ HUT is presented in detail and in Section 4 some particular HUT models are considered. Our summary is given in final Section 5.

2 $SL(2,C)$ gravity

2.1 Standard presentation

We first assume that a local frame at any spacetime point possesses the global $SL(2,C)$ symmetry group. Accordingly, its transformations for the basic fermions in the theory are given by the matrix $\Omega$

$$\Psi \to \Omega \Psi, \quad \Omega = \exp \left\{ \frac{i}{4} \theta_{ab} \gamma^{ab} \right\}, \quad \gamma^{ab} = i[\gamma^{a}, \gamma^{b}]/2$$

which generally has the pseudounitary form, $\Omega^{-1} = \gamma_{0}\Omega^{+}\gamma_{0}$. Furthermore, to provide invariance of their kinetic terms, $i\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi$, one need to replace $\gamma$-matrices in them by a set of some tetrad matrices $e^{\mu}$ which transform like

$$e^{\mu} \to \Omega e^{\mu} \Omega^{-1}$$

Generally, the tetrad matrices $e^{\mu}$, as well as their conjugates $e_{\mu}$, contain the appropriate tetrad fields $e^{a}_{\mu}$ and $e^{a}_{\mu}$, respectively,

$$e_{\mu} = e^{a}_{\mu} \gamma^{a}, \quad e^{\mu} = e^{a}_{\mu} \gamma^{a}$$

They, as usual, satisfy the orthogonality relations

$$e^{a}_{\mu} e^{b}_{\nu} = \delta^{\nu}_{\mu}, \quad e^{a}_{\mu} e^{\mu}_{b} = \delta^{a}_{b}$$

and determine the metric tensors in the theory

$$g_{\mu\nu} = \frac{1}{4} Tr(e_{\mu} e_{\nu}) = e^{a}_{\mu} e^{b}_{\nu} \eta_{ab}, \quad g^{\mu\nu} = \frac{1}{4} Tr(e^{\mu} e^{\nu}) = e_{a}^{\mu} e_{b}^{\nu} \eta^{ab}$$
Going now to the case when the $SL(2, C)$ transformations become local, one have to introduce the gauge field multiplet \( I_\mu \) transforming as usual

\[
I_\mu \rightarrow \Omega I_\mu \Omega^{-1} - \frac{1}{i} (\partial_\mu \Omega) \Omega^{-1}
\]

thus providing the fermion field by covariant derivative

\[
\partial_\mu \Psi \rightarrow D_\mu \Psi = \partial_\mu \Psi + i I_\mu \Psi
\]

The \( I_\mu \) multiplet gauging the $SL(2, C)$ has by definition the form

\[
I_\mu = \frac{1}{4} T_\mu^{[ab]} \gamma^{ab}
\]

with the flat spacetime tensor field components \( T_\mu^{[ab]} \).

The tensor field \( T_\mu^{[ab]} \) may in principle propagate, while the tetrad \( e^\mu \) is not considered as a dynamical field. So, the invariant Lagrangian built from the \( T_\mu^{[ab]} \) field strength

\[
T_\mu^{[\nu\rho]} = \partial_\nu T_\rho^{[ab]} - i \eta_{cd} (T_\rho^{[ac]} T_\nu^{[bd]} - T_\rho^{[bc]} T_\nu^{[ad]})
\]

can be written in a conventional form

\[
L_G = \frac{1}{2\kappa} \epsilon_\mu^{[ab]} e_\nu^c T_\mu^{[ab]} e^\nu_e e^\nu_f, \quad \kappa \equiv \sqrt{-\det Tr(e^\mu e^\nu)/4}
\]

once the commutator for tetrads and some of standard relations for \( \gamma \)-matrices have been used. This is in fact the simplest pure gravity Lagrangian being equivalent to the Palatini formulation of the standard Einstein-Cartan gravity Lagrangian. It happens to be invariant not only under $SL(2, C)$ but under general four-coordinate transformations $GL(4, R)$ as well.

Meanwhile, the gauge invariant fermion matter coupling given by the covariant derivative \( I_\mu \) presents the spin-connection tensor field \( T_\mu^{[ab]} \) interaction with the spin-density current

\[
L^{\text{int}} = -\frac{c}{2} \epsilon^{abcd} [\bar{\Psi} e_\mu T_\mu^{[ab]} \gamma_5] \gamma_5 \Psi
\]

This is a key feature of the Einstein-Cartan gravity which eventually results in, apart from the standard GR, the tiny four-fermion interaction once the constraint equation for the non-propagating tensor field is used.

### 2.2 $SL(2, C)$ gravity in weak-field approach

Since we think that the $SL(2, C)$ gauge gravity is a part of the unified set of all elementary forces assembled in the $SL(2N, C)$ we have to be sure that its extraordinary smallness is compatible with pretty sizeable contributions of the other interactions. In this connection we propose that this smallness related to the weak nature of the gravity tensor field itself rather than its vanishingly small coupling constant.
This weakness supposedly manifests itself once spacetime described by the tetrad \( e^a_\mu \) appears to be close to the flat Minkowski one in the weak-field approximation. Note that the very process of decomposing a general spacetime described by the tetrad \( e^a_\mu \) into the flat one \( \delta^a_\mu \) plus some perturbation term \( \epsilon^a_\mu \) does not break general covariance. Actually, the perturbation tetrad \( \epsilon^a_\mu \) which, we refer further to as the "petrad", is defined in terms of the subset of a general set of diffeomorphisms on spacetime which leave \( e^a_\mu \) sufficiently small that is required by the weak-field approximation

\[
e^a_\mu = \delta^a_\mu + \epsilon^a_\mu, \quad |\epsilon^a_\mu| \ll 1 \quad (12)
\]

In a general case when no extra constraints on petrads are imposed the tetrads are no longer orthogonal but their orthogonality relations (4) include some vanishingly small deviations

\[
P^a_\nu = e^a_\nu e^a_\mu + \delta^a_\nu \delta^a_\mu + \epsilon^a_\nu \epsilon^a_\mu, \quad q^a_\nu = e^a_\nu e^a_\mu + \delta^a_\nu \delta^a_\mu + \epsilon^a_\nu \epsilon^a_\mu \quad (13)
\]

which can be directly calculated

\[
P^a_\nu = \frac{1}{4} e^a_\sigma I^\sigma_\mu, \quad p^b_\mu e^b_\nu = p^b_\mu e^b_\nu, \quad g_{\mu \nu} p^\mu_\nu = p_{\mu \nu} \quad (14)
\]

from the above tetrad parametization. In this connection, the metric tensor will also include such a deviation which we define from a similar equation

\[
e^a_\mu e^a_\nu = g_{\mu \nu} + p_{\mu \nu} \quad (15)
\]

Multiplying the basic equations (13) by the proper tetrads and also require general covariance for the shifted metric tensor (15), \( g_{\mu \nu} e^{ab} = e^{b}_\mu \), one can readily find relations between all deviations

\[
P^a_\nu e^a_\nu = q^a_\nu e^a_\mu, \quad p^a_\mu e^a_\nu = p^{a}_\nu e^a_\mu, \quad g_{\mu \nu} p^\mu_\nu = p_{\mu \nu} \quad (16)
\]

Using them the metric tensor itself can be directly calculated

\[
g_{\mu \nu} = e^a_\mu e^a_\nu \left[ \delta^a_\nu - p^a_\nu + (pp)^a_\nu \right] \quad (17)
\]

up to the second order terms in \( p \).

Now, instead of the "strong" gauge field \( I^\mu_\mu \) we will construct the new "weak" one

\[
I^\mu_\mu = \frac{1}{4} e^a_\sigma I^\sigma_\mu e^a_\alpha = \frac{1}{16} T_{\mu[ab]} q^c_\alpha (\gamma_c \gamma^{[ab]} \gamma^d) \quad (18)
\]

where due to the \( \gamma \)-matrix identity

\[
\gamma_c \gamma^{[ab]} \gamma^c = 0 \quad (19)
\]

the starting tensor field multiplet \( T_{\mu[ab]} \) is largely extinguished and only some of its very small part being proportional the tetrad nonorthogonality parameter \( q^c_\alpha \) (14) comes out. The relation between the new and old tensor multiplets in the first order in tiny petrad fields is given by

\[
I^\mu_\mu = \frac{1}{4} T_{\mu[ab]} \gamma^{ab} = \frac{1}{16} T_{\mu[ab]} e^c_\alpha (\gamma_c \gamma^{[ab]} \gamma^d) \quad (19) \quad (e^c_\alpha \equiv e^c_\nu \delta^\nu_\alpha + e^c_\alpha \delta^\nu_\nu) \quad (20)
\]
Remarkably, in weak tetrad approximation the $\gamma$-matrix identity (19) triggers an important suppression mechanism in the $SL(2,C)$ gravity theory so that the $SL(2,C)$ gauge theory can now be formulated only in terms of the weakened tensor field (20). Another important thing is that, while the new tensor multiplet $T_{\mu [ab]}$ globally transforms like the old $T_{\mu [ab]}$ one, the right gauge transformation may alternatively appear for only one of them in the theory. Indeed, as follows from (20), their gauge function matrices should satisfy the relation

$$\partial_\mu \theta^{(T)}_{ab} \gamma^{[ab]} = \partial_\mu \theta_{[ab]} \left[ c_0^{ab} (\gamma c^{ab}) \right]$$

which means that if one requires gauge invariance for the new $T_{\mu [ab]}$ field, then the old $T_{\mu [ab]}$ field (or any superposition of them) is excluded as a gauge field candidate in a theory, and vice versa.

The basic Lagrangian for the $SL(2,C)$ gravity with the new tensor field will also contain terms being linear in $T_{\mu [ab]}$ that results in the appropriate analogs of the gravity and matter field Lagrangians (10, 11), respectively. Indeed, the minimal gravity Lagrangian (10) with the proper replacements remains practically the same form

$$L_G = e^{2 \kappa T_{\mu [ab]} e_{[a}^{\mu} e_{b]}^{\nu}}$$

In a linear weak-field $T_{\mu [ab]}$ field approach taken the tensor field propagation is automatically neglected, while variation of this Lagrangian with respect to the tetrad $e_\mu^a$ (or petrad $e_\mu^a$) leads to the standard Einstein equation of motion for pure gravity.

3 Extension to $SL(2N,C)$ HUT

3.1 Some basic elements

Generally, the $SL(2N,C)$ symmetry contains, as its main subgroups, the $SL(2,C)$ which covers the orthochronous Lorentz group and the internal $U(N)$ symmetry, so that the $SU(N)$ appears as the maximal compact subgroup of the $SL(2N,C)$. Indeed, the $8N^2 - 2$ generators of $SL(2N,C)$ are formed from the tensor products of the generators of $SL(2,C)$ and generators of $U(N)$ so that the basic transformation applied to the fermions looks as

$$\Omega = \exp \left\{ \frac{i}{2} \left[ (\theta^k + i \theta_5^k \gamma_5) \lambda^k + \frac{1}{2} \theta^{K}_{[ab} \gamma^{ab} \lambda^K \right] \right\} \quad (K = 0, k)$$

where $\lambda^k$ ($k = 1, ..., N^2 - 1$) are the $SU(N)$ Gell-Mann matrices ($[\lambda^k, \lambda^l] = 2i f^{klm} \lambda^m$), while $\lambda^0$ is the unit matrix corresponding to $U(1)$ generator (all $\theta$ parameters may be constant or, in general, depend on the spacetime coordinate).

For further description of the fermion matter in the theory one needs again to introduce the generalized tetrad multiplet

$$e_\mu = (e_\mu^{aK} \gamma_a + e_\mu^{aK} \gamma_a \gamma_5) \lambda^K$$

5
which transforms, as before, according to (2) where the transformation matrix is now given by equation (23). Despite its somewhat excessive extension form which generally appears in the $SL(2N, C)$ framework, it would be natural for tetrad flat space components in (24) to have the same form as in the pure gravity case. This means that such an extension might not include the axial-vector part and $SU(N)$ symmetry components, 

$$
e_{\mu}^{aK} = \epsilon_{\mu}^{a} \delta^{K0} = (\delta_{\mu}^{a} + \epsilon_{\mu}^{a}) \delta^{K0}$$

(25)

that could be reached through some gauge invariant constraints put on tetrad.

Indeed, one can introduce for that some special non-dynamical $SL(2N, C)$ scalar multiplet in the theory

$$S = \exp\{i [(s^{k} + i p^{k} \gamma_{5}) \lambda^{k} + i T_{ab}^{K} \gamma^{ab} \lambda^{K}/2]\}$$

(26)

which transforms like as $S \rightarrow \Omega S$. With this scalar multiplet one can form a new tetrad in terms of the gauge invariant construction, $S^{-1}eS$. So, choosing appropriately the flat space components in the $S$ field one can turn the tetrad axial part to zero and establish symmetry between Greek and Latin spacetime indices [8]. In this way one may also exclude the tetrad $SU(N)$ symmetry components. This is especially apparent when the starting tetrad has a simple factorized form, $e_{\mu}^{aK} = e_{\mu}^{a} \epsilon^{K}$, where $\epsilon^{K}(x)$ is some set the $U(N)$ symmetrical functions. As a result, the $SU(N)$ members $\epsilon^{k}$ of this set can be then gauged away in $S^{-1}eS$ by the proper choice of the $s^{k}$ field components [26]. In the first order in them the corresponding conditions happen to be

$$\epsilon^{i} = f^{ijk} \epsilon^{j} s^{k} \quad (i, j, k = 1, \ldots, N^2 - 1)$$

(27)

Now, the new tetrad (26), as well as the related metric tensor, will automatically satisfy all conditions discussed above in the pure gravity case including its the weak-field approximation.

3.2 Local $SL(2N, C)$ symmetry and gauge hypermultiplet

Once the $SL(2N, C)$ transformation (23) becomes local one also need, as ever, to introduce the gauge field multiplet $I_{\mu}$ transforming as usual

$$I_{\mu} \rightarrow \Omega I_{\mu} \Omega^{-1} - \frac{1}{i} (\partial_{\mu} \Omega) \Omega^{-1}$$

(28)

thus providing the fermion multiplet by covariant derivative

$$\partial_{\mu} \Psi \rightarrow D_{\mu} \Psi = \partial_{\mu} \Psi + i I_{\mu} \Psi$$

(29)

The $I_{\mu}$ hypermultiplet includes, as follows from its decomposition to the flat spacetime component fields

$$I_{\mu} = V_{\mu} + A_{\mu} + T_{\mu} = \frac{1}{2} \left( V_{\mu}^{k} + i A_{\mu}^{k} \gamma_{5} \lambda^{k} + \frac{1}{4} T_{\mu}^{K} \gamma^{ab} \lambda^{K} \right) \quad (K = 0, k)$$

(30)

the vector and axial-vector field multiplets, and tensor field multiplet as well. Just the latter provides supposedly gravitational interaction in the framework of the $SL(2N, C)$
HUTs. In this connection, the crucial problem is how one could selectively suppress the tensor fields when they are members of the same gauge hypermultiplet as vector and axial-vector ones. Fortunately, the weak-field mechanism described above for the pure gravity case allows to naturally combine strong internal forces related to the vector fields with tiny gravity.

For that purpose, we show again that the $\text{SL}(2N,\mathbb{C})$ tetrad multiplets can project an essential part of tensor field components out of the total $\text{SL}(2N,\mathbb{C})$ gauge hypermultiplet. Thus, instead of the "strong" gauge fields contained in $I_\mu$ we will construct new "weak" ones which for the simplified tetrads (25) acquire the form

$$I_\mu = \frac{1}{4} e_\sigma e^\sigma_{\mu} e^\tau_{\mu} \gamma_\tau (\gamma_\sigma I_\mu \gamma^d)$$

Leaving again only the first order terms in the petrad components one comes to

$$I_\mu = \frac{1}{4} (\delta_\sigma^d + \epsilon_\sigma^d)(\gamma^\sigma I_\mu \gamma_d) \quad (\epsilon_\sigma^d \equiv \epsilon_\mu^d \epsilon_\sigma^\mu + \epsilon_\mu^\sigma \epsilon_\sigma^d)$$

and eventually to the new gauge field hypermultiplet in the weak-field approximation

$$I_\mu = V_\mu + A_\mu + T_\mu = \frac{1}{2} (V_\mu^k + i A_\mu^k \gamma_5) \lambda^k + \frac{1}{4} T_{\mu[ab]}^k \gamma^{ab} \chi^k$$

As one can readily confirm, the vector and axial-vector submultiplets in the starting gauge multiplet (30) practically remain in the weak tetrad approximation within the first order terms in tiny petrad fields

$$V_\mu^k + i A_\mu^k \gamma_5 = V_\mu^k - i A_\mu^k \gamma_5$$

except that the axial-vector fields change the sign that, though, seems to be unessential for what follows. Meanwhile, the new tensor field components acquire again, as in the pure gravity case above, the simple form

$$T_{\mu[ab]}^k \gamma^{ab} = \frac{1}{4} T_{\mu[ab]}^k \epsilon_\tau^d (\gamma_\tau \gamma^{ab} \gamma_d) \quad (K = 0, k)$$

and therefore are significantly weakened by the corresponding petrad fields. Thus, hyperunification of the basic elementary forces does not prevent the tensor field submultiplet of having the vanishingly small couplings with each other, as well as with a matter - as soon as the smallness of tensor fields shown in (35) is conventionally converted into the smallness of their coupling constant.

It is also worth noting that, though the new $I_\mu$ field hypermultiplet globally transforms like the old $I_\mu$ one (24), the right gauge transformation may alternatively appear for only one of them in the theory. Indeed, as follows from (33) and (35), while gauge functions for vector and axial-vector submultiplets in $I_\mu$ and $I_\mu$ can be practically the same, such a gauge function matrix for the modified tensor field multiplet should be quite different, just as we had it above in the pure gravity case (21). This means that if one chooses the $I_\mu$ as the gauge hypermultiplet in a theory then the old $I_\mu$ hypermultiplet or any superposition of $I_\mu$ and $I_\mu$ is excluded as a gauge field candidate.
3.3 Tensor fields

Let us now construct the field strength for the $I_\mu$ field multiplet and then its total Lagrangian including the matter field part. This strength is given in main terms by

$$F_{\mu\nu} = \partial_{[\mu}I_{\nu]} + i[I_{\mu}, I_{\nu}] = (V + A)_{\mu\nu} + T_{\mu\nu}$$

$$= \frac{1}{2} \partial_{[\mu} \left( V^k + i A^k \gamma_5 \right)_{\nu]} \lambda^k - \frac{1}{2} f^{ijk} (V^i + i A^i \gamma_5)_{\mu} (V^j + i A^j \gamma_5)_{\nu} \lambda^k$$

$$+ \frac{1}{4} \partial_{[\mu} T^{K}_{\nu]ab} \gamma^{ab} \lambda^K + i \frac{1}{16} T^{K}_{\mu[ab]} T^{K'}_{\nu[a'b']} \gamma^{ab} \lambda^K \gamma^{a'b'} \lambda^{K'}$$

(36)

for vector, axial-vector and tensor curls, $(V + A)_{\mu\nu}$ and $T_{\mu\nu}$, respectively.

One can see that kinetic terms of the tensor field multiplet which would provide its propagation can be neglected in the linear tensor field approximation taken. The total action will only contain terms being linear in $T^{K}_{\mu[ab]}$ field multiplet that amounts to the Palatini type Lagrangian \(10\) which for the $SL(2N,C)$ case has a form

$$\mathcal{L}_G \sim e Tr \{ F_{\mu\nu} [e^\mu, e^\nu] \}$$

(37)

Interestingly, there are no contributions from the vector and axial-vector multiplets collected in the curls $(V + A)_{\mu\nu}$ in (36). So, writing only tensor multiplet one comes - after using of standard trace relations for appearing products of $\gamma$-matrices $Tr(\gamma^a \gamma^b \gamma^c \gamma^d)$ and $Tr(\gamma^a \gamma^b \gamma^c)$ - to the general Lagrangian being practically a repetition of the pure gravity one

$$\mathcal{L}_G = e \left[ \frac{1}{2\kappa} (\partial_{[\mu} T^{[0]}_{\nu]} - i \eta_{cd}(T^{[0]ac}_{\mu} T^{0[bd]}_{\nu} + T^{k[ac]}_{\mu} T^{k[bd]}_{\nu})) \right] e^{\mu} e^{\nu}$$

(38)

The only difference are related to some extra $SU(N)$ flavored tensor self-interaction terms, while the corresponding flavor tensor kinetic terms are absent.

Let us turn now to the matter sector in the $SL(2N,C)$ hyperunified theory proposed. The gauge invariant fermion matter couplings are given in terms of the modified gauge multiplet \(33\) by

$$\mathcal{L}_M = -\frac{e}{2} \overline{\Psi} \left\{ e^\mu, \left[ \frac{1}{2} \left( \gamma^k + i A^k \gamma_5 \right) \lambda^k + \frac{1}{4} T^{K}_{\mu[ab]} \gamma^{ab} \lambda^K \right] \right\} \Psi$$

(39)

Leaving aside for the moment vector and axial-vector fields, one has after using of standard trace relations for $\gamma$- and $\lambda$-matrices for the matter Lagrangian the multiplet

$$\mathcal{L}_M^{(T)} = -\frac{e}{2} \epsilon^{abcd} \left[ \overline{\Psi} e_c \left( T^{k}_{\mu[ab]} + T^{k}_{\mu[ac]} \gamma^k \right) \gamma^5 \gamma^d \gamma^5 \Psi \right]$$

(40)

Variation of the total Lagrangian \(38\) \(40\) with respect to $e^{\mu}_a$ leads to the GR type equation of motion

$$R^{0a}_\mu + \frac{1}{2} R e^{\mu}_a = \kappa \gamma^a_\mu + \kappa \epsilon^{abcd} \overline{\Psi} T^{K}_{\mu[bc]} \lambda^K \gamma^5 \gamma^d \gamma^5 \Psi$$

(41)

containing a standard stress-energy tensor $\partial^a_\mu$ and a new spin source extension which also contains the flavored tensor fields given in \(38\). Meanwhile, variation under $T^{0}_{\mu[ab]}$ gives the constraint equation leading to the tiny 4-fermion spin density current interaction

$$\kappa \left( \overline{\Psi} \gamma^d \lambda^K \right) \left( \overline{\Psi} \gamma^5 \lambda^K \Psi \right)$$

(42)
being solely proportional to the conventional GR coupling constant $\kappa$ just as in the standard Einstein-Cartan gravity but also containing the extra $SU(N)$ symmetrical four-fermion interaction term.

### 3.4 Vector and axial-vector fields

Let us turn now to the spin-1 vector and axial-vector fields being basic carriers of the internal $SU(N)$ symmetry in this hyperunification scheme. Their gauge sector stemming from the common strength tensor (36) looks as

$$\mathcal{L}^{(VA)} = -2Tr[(V + A)_\mu(V + A)^\mu]$$

$$= -\frac{1}{4}[\partial_\mu V^k_\nu - \epsilon^{ijk}(V^i_\mu V^j_\nu + A^i_\mu A^j_\nu)]^2 - \frac{1}{4}[\partial_\mu A^k_\nu]^2 \quad (43)$$

where one can readily confirm that vector fields acquire a conventional gauge theory form, while axial-vector fields only contributed into the vector field interaction terms. Meanwhile, as follows from the matter sector of the theory (39) the vector fields interact with ordinary matter fermions

$$\mathcal{L}_M^{(VA)} = -\frac{e}{2}\bar{\Psi}[\gamma^\mu a^a \gamma^k V^k_\mu]\Psi \quad (44)$$

while axial-vector fields do not, thus being sterile to them.

We could try to adapt such fields to reality though there seems no a direct indication that they might exist. The traditional way is to make these axial-vector fields superheavy through some enormously extended Higgs sector remaining at the same time the Standard Model gauge bosons massless or enough light. This seems to be quite difficult since the axial-vector fields want to follow the same pattern of the mass formation as the vector fields do. Anyway, despite the gauge $SL(2N,C)$ invariance in the theory, the very presence of the axial-vector fields breaks the gauge $SU(N)$ invariance related to the vector fields, thus only leaving the global $SU(N)$ symmetry in the theory.

In this connection, a rather interesting way could be if these axial-vector fields were condensed, thus providing a true vacuum in the theory. Remarkably, in this vacuum, as is shown below, gauge invariance for the vector fields is completely restored, though a tiny spontaneous breaking of Lorentz symmetry at some Planck order scale $\mathcal{M}$ may appear. To get such a vacuum one can, instead of writing a conventional polynomial potential for the gauge hypermultiplet, put on it some nonlinear covariant constraint of the type

$$\frac{1}{4}Tr[(V + A)_{\mu}(V + A)^\mu] = \mathcal{M}^2 \quad (45)$$

which in the dominating field components looks as

$$(V^i_\mu)^2 + (A^i_\mu)^2 = \mathcal{M}^2 \quad (46)$$

\footnote{Such a type of constraint have been used earlier \cite{11} \cite{12} in a context of emergent gauge and gravity theories allowing to treat photons and gravitons as zero modes of the spontaneously broken Lorentz invariance and its extensions.}
The most appropriate solution to this constraint equation may be related to the special Goldstone type one

\[ A_{i\mu} = a_{i\mu} + n_{i\mu} \sqrt{M^2 - (V_{i\mu})^2} - a_2, \quad n_{i\mu}a_{i\mu} = 0 \quad (a_2 \equiv (a_{i\mu})^2) \quad (47) \]

This parametrization shows that, whereas the axial multiplet is condensed being provided by an effective Higgs mode (given by the second term), there are produced the zero mass excitations being orthogonal to the vacuum direction along the unit Lorentz vector \( n_{i\mu} \), \( n_{i\mu}n_{i\mu} = 1 \). We further use its factorized form, \( n_{i\mu} \equiv n_\mu e^i \), where \( n_\mu \) is the unit Lorentz vector (\( n_\mu^2 = \pm 1 \)), while \( e^i \) is the internal \( SU(N) \) symmetry one (\( e^i e^j = 1 \)).

Now, turning back to the spin-1 field Lagrangian (43) and substituting the \( A_{i\mu} \) expression (47) one can confirm that the first order terms in the zero mode \( a_{i\mu} \) do not show up there provided that the orthogonality relations

\[ n_\mu a_{i\mu} = 0, \quad n_\mu V_{i\mu} = 0, \quad (\partial n) a_{i\mu} = 0 \quad (48) \]

work. They can be treated as gauge conditions for zero modes \( a_{i\mu} \) and vector field \( V_{i\mu} \), respectively, while the last relation means that zero modes are supposed not to depend on the \( x \)-coordinate component along the direction where Lorentz symmetry is broken. So, neglecting all the higher zero mode terms one eventually comes to the Lagrangian

\[ L_{VA} = -\frac{1}{4}[\partial_\mu V^k_\nu - f^{ijk}V^i_\mu V^j_\nu]^2 - \frac{1}{4M^2} [n^k_\mu (V^i_\sigma \partial_\nu V^i_\sigma) - n^k_\nu (V^i_\sigma \partial_\mu V^i_\sigma)]^2 \quad (49) \]

which presents the conventional vector field gauge invariant Lagrangian plus some small non-invariant and Lorentz violating terms stemming from the square root in (47) when the lowest order terms in \( (V_{i\mu})^2/M^2 \) is taken.

We propose that the high-order terms in zero axial-vector modes are naturally very small since their vanishing promotes the global \( SU(N) \) symmetry of the Lagrangian (43) to the gauge one. Interestingly, some of these modes acquire large masses since symmetry of the constraint (46) is much higher than symmetry of the Lagrangian and part of zero modes are in fact pseudo Goldstone states. As follows from their mass term in (43)

\[ (a_{i\mu}a^{\mu'}) (n_\mu^i n^{\sigma')} f^{ijk} f^{i'j'k}M^2/2 \quad (50) \]

the modes related to the "non-diagonal" generators of the \( SU(N) \) acquire superheavy masses, while modes corresponding to the "diagonal" ones are left massless though only in the tree approximation. They will lead to various processes including those where the vector fields decay into these invisible axial-vector modes being sterile to ordinary matter. Note that the present data allows in principle such a possibility for the Standard Model vector bosons whose total width fraction into invisible modes is still quite large [13].

### 3.5 Symmetry breaking

The entire symmetry breaking scenario will crucially depends on a way the starting \( SL(2N,C) \) symmetry breaks through the proper set of scalar fields into the \( SU(N) \times SL(2,C) \) symmetry and further to the Standard Model. Actually, one does not need to cause the first stage
of symmetry breaking since all nondiagonal generators of $SL(2N,C)$ are related to the $SU(N)$ "flavored" tensor fields which in the weak-field approach appear to be significantly suppressed and can be neglected beyond the first order terms. Due to this suppression tensor fields do not propagate and the whole theory appears to practically deal with the gauge vector and axial-vector fields in a kind of the $SU(N)$ grand unification scheme. The only place where tensor fields can take part is the pure gravity sector with the Einstein-Cartan $SL(2,C)$ theory provided by the "neutral" tensor field $T^0_{\mu[ab]}$. Remarkably, in the linear gravity Lagrangian the "flavored" tensor fields $T^k_{\mu[ab]}$ contribute only to the tiny four-fermion spin current interactions.

As to the internal $SU(N)$ symmetry violation to the Standard model one actually need to have a number of scalars, its adjoint ($\Phi$) and fundamental ($H$) multiplets which under $SL(2N,C)$ transform as

$$\Phi \rightarrow \Omega \Phi \Omega^{-1}, \quad H \rightarrow \Omega H$$

respectively. For simplicity, one can again use the tetrad projection mechanism which we used above for gauge hypermultiplet to suppress the tensor field components in them. As a result, one comes to only scalar and pseudoscalar field components in the $SU(N)$ symmetry breaking multiplets, while their suppressed tensor components do not propagate and can be neglected beyond the the first order approximation. Particularly, for the "projected" adjoint and fundamental scalars one now has respectively

$$\Phi = \left( \phi^k + i\phi^k_5\gamma_5 \right) \lambda^k \quad (k = 1, ..., N^2 - 1)$$

$$H_s = h_s + ih_5s\gamma_5 \quad (s = 1, ..., N)$$

(51)

**4 Application to GUTs**

**4.1 $SU(5)$ and beyond**

Let us now consider more closely how the $SL(2N,C)$ type model can be applied to the known GUTs starting with a conventional $SU(5)$ [14] which would stem from the $SL(10,C)$ HUT. In this case some of its low-dimensional multiplets of the chiral (lefthanded for certainty) fermions can be given in terms of the $SU(5) \times SL(2,C)$ components as

$$\Psi_L^{ia}, \quad 10 = (5, 2)$$

$$\Psi_{L[ai, jb]} = \Psi_{L[ij][ab]} + \Psi_{L[ij][ab]}, \quad 45 = (10, 3) + (15, 1)$$

(52)

(53)

where we have used that a common antisymmetry on two or more joint $SL(10,C)$ indices means antisymmetry in internal indices ($i, j, k = 1, ..., 5$) and symmetry in the chiral spinor ones ($a, b, c = 1, 2$), and vice versa (dimension of representations are also indicated). One can see that while the $SU(5)$ antiquintet can easily be constructed, its decuplet is not contained in the pure antisymmetric $SL(10,C)$ representations. Moreover, the tensor corresponds in fact to the collection of vector and scalar multiplets rather than the fermion ones.

Note in this connection, that all GUTs where fermions are assigned to the pure antisymmetric representations seem to be also irrelevant since the spin values of appearing
states are not in conformity with what we have in reality. The most known example of this kind is the $SU(11)$ GUT [15] with all three quark-lepton families collected in its one-, two-, three-, and four-index antisymmetric representations. No doubt this GUT should also be excluded in the framework of the considered $SL(2N,C)$ theories. Actually, for the right $1/2$ spin value of ordinary quarks and leptons these theories should include more complicated fermion multiplets having in general the upper and lower indices rather than the pure asymmetric ones. The point is, however, that such multiplets appear enormously extended and contain in general lots of exotic states which never been detected. This could motivate to seek a possible solution in the composite nature of quarks and leptons for whose constituents - preons - the $SL(2N,C)$ unification might look much simpler.

4.2 $SU(8)$ GUT with composite quarks and leptons

Following the recent discussion [16], we introduce $N$ lefthanded and $N$ righthanded preons being the fundamental multiplets $P_{\tilde{L}a}^{\alpha}$ and $P_{\tilde{R}a}^{\alpha'}$ of the "metaflavor" $SL(2N,C)$ HUT symmetry ($i = 1, ..., N; a = 1, 2$) times some local left-right "metacolor" $SO(n)_L \times SO(n)_R$ symmetry ($\alpha = 1, ..., n; \alpha' = 1, ..., n$). Both of these symmetries are obviously anomaly-free and the numbers of metaflavors ($N$) and metacolors ($n$) are not yet determined. The metaflavor symmetry describes preons at small distances as well as their composites, which are produced due to confining forces of the above metacolor symmetry, at large ones. Some of these composites, including the observed quarks and leptons, are expected to be much lighter than their composition scale. For that, the accompanying chiral symmetry $SU(N)_L \times SU(N)_R$ of the preons should be preserved at large distances in a way that - when it is considered as the would-be local symmetry group with some spectator gauge fields and fermions - the corresponding triangle anomaly matching conditions [17] are satisfied. Namely, the $SU(N)_L^3$ and $SU(N)_R^3$ anomalies related to $N$ lefthanded and $N$ righthanded preons have to individually match those for lefthanded and righthanded composite fermions being produced by the $SO(n)^{MC}_{L}$ and $SO(n)^{MC}_{R}$ metacolor forces, respectively.

Moreover, as is turned out, just this condition, when being properly strengthened, can determine the particular metaflavor symmetry $SL(2N,C)$ in the theory. Actually, one may propose that all composites, both lefthanded and righthanded, have just the three-preon configuration ($n = 3$) and belong to a single representation of $SU(N)_L$ and $SU(N)_R$, respectively, rather than to some set of their representations. It then appears that among all third-rank representations, the corresponding anomaly matching condition is only satisfied for representations of the type $\psi_{[i,j]}^k_L$ and $\psi_{[i,j]}^k_R$ ($i, j, k = 1, 2, ..., N$) that gives individually the unique solution

$$N^2/2 - 7N/2 - 1 = 3, \quad N = 8$$

This means that between all possible chiral symmetries only the $SU(8)_L \times SU(8)_R$ symmetry could in principle provide the massless fermion composites at large distances that in turn identifies the metaflavor $SL(16,C)$ symmetry as a possible unified symmetry of the lefthanded and righthanded preons and their massless composites. Note that, in contrast to the above global chiral symmetry, in the local $SL(16,C)$ metaflavor theory, being as yet vectorlike, all metaflavor triangle anomalies are automatically cancelled out.
Turning now from the chiral symmetry multiplets $\psi_{[ij]}^{kL,R}$ to the corresponding $SL(16, C)$ composite multiplets $\Psi_{[ia, jb]}^{kcL,R}$ and presenting them in terms of the $SU(8) \times SL(2, C)$ components one has the collections

$$\Psi_{[ia, jb]}^{kc} = \Psi_{[ij]}^{kc(ab)} + \Psi_{[ij]}^{kc(ab)}; \quad 1904 = (216, 2) + (216 + 8, 4) + (280 + 8, 2) \quad (55)$$

which contains some different spin $1/2$ and $3/2$ lefthanded and righthanded composite fermion submultiplets. Meanwhile, as one can easily confirm, among all submultiplets in (55) only the $(216, 2)_{L,R}$ ones satisfy individually the anomaly matching condition for the chiral $SU(8)_L$ and $SU(8)_R$ symmetries, respectively. As a result, all the other submultiplets have then to acquire superheavy masses. This actually means that only the $SU(8) \times SL(2, C)$ subgroup of the $SL(16, C)$ HUT symmetry survives at large distances where the composite fermions emerge. Surprising as it may seem, this is consistent with what we have above in a general case though from a different point of view - the $SL(2N, C)$ symmetry with the tensor fields being weakened and the axial-vector field being condensed turns out to be reduced to the $SU(N) \times SL(2, C)$ symmetry. Now, for the particular $SL(16, C)$ symmetry case it also follows from the chiral symmetry preservation at large distances.

Remarkably, these 216-dimensional submultiplets being decomposed into the standard $SU(5)$ GUT and family symmetry $SU(3)_F$ looks as

$$(216, 2)_{L,R} = [([-5] + 10, \bar{3}) + (45, 1) + (5, 8 + 1) + (24, 3) + (1, 3) + (1, \bar{6})]_{L,R} \quad (56)$$

where the first term in the squared brackets, when taken for lefthanded states in $216_L$, describes all three quark-lepton families being the family symmetry triplets. However, there are also the righthanded states in $216_R$ in our still vectorlike $SL(16, C)$ theory. This means that, while preons are left massless being protected by their own metacolors, the composites (56) being metacolor singlets could in principle pair up and acquire the heavy Dirac masses.

To avoid this for the submultiplet of physical quarks and leptons in (56) one may propose, following the scenario developed in [16], some spontaneous breaking of the chiral symmetry just in the righthanded preon sector to induce a basic $L$-$R$ symmetry violation in the $SL(16, C)$ HUT at large distances. This violation implies that, whereas nothing really happens with the lefthanded preon composites still completing the total multiplet (56) of the $SU(8)_L$, the righthanded preon composites will form only some particular submultiplets of the $[SU(5) \times SU(3)]_R$ symmetry. Remarkably, it can be managed in a way that all submultiplets in (56), apart from the $(\bar{5} + 10, \bar{3})_L$ one, becomes heavy and decouples from the laboratory physics. So, the vectorlike metaflavor symmetry $SL(16, C)$ being first reduced to the $SU(8) \times SL(2, C)$ one then breaks, after $L$-$R$ symmetry violation, down to its chiral subgroup $[SU(5) \times SU(3)_F] \times SL(2, C)$ for composites at large distances. As a result, one eventually comes to the conventional $SU(5)$ GUT [14] together with the extra local $SU(3)_F$ family symmetry [18] describing just three standard families of composite quarks and leptons. Both types of the triangle anomalies, $SU(5)^3$ and $SU(3)_F^3$, emerging at this stage are properly cancelled out in the theory [10].

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5 Summary

We have argued that the $SL(2N,C)$ hyperunified theories may point to a possible way where all elementary gauge forces, including gravity, could be consistently unified. Such a unification only means that gravity and other interactions are provided by vector and tensor fields being the members of the $SL(2N,C)$ gauge hypermultiplet. But this does not mean, however, that all these interactions have some common universal coupling constant. Rather, such a universal constant is required for ordinary $SL(2N,C)$ gauge invariant couplings of vector and tensor fields, while gravitational interaction is related to the principally new coupling being linear in the tensor field strength (38). Remarkably, this coupling only appears due to tetrads which are the necessary ingredients of the $SL(2N,C)$ invariant theory and goes by itself with an independent coupling constant $(1/2\kappa)$ being conventionally related to the Planck mass.

At the same time, the fact that the ordinary gauge interactions of vector and tensor fields are comparable in the $SL(2N,C)$ theory might happen to be experimentally unacceptable unless the $SU(N)$ flavored tensor fields could selectively develop very heavy masses that generally is a problem. However, as was shown above, one can safely move on to the new gauge hypermultiplet being filtered out by the accompanying tetrad fields (31). As a result, while all new gauge fields are unified in the framework of $SL(2N,C)$ symmetry group, the filtered tensor field submultiplet appears to be naturally suppressed in the weak-field approach developed for tetrads. As a result, the whole theory turns out to effectively possess the local $SL(2,C) \times SU(N)$ symmetry, so as to naturally lead to the $SL(2,C)$ gauge gravity, on the one hand, and the $SU(N)$ GUT, on the other.

An essential problem related to this type of HUT models is a possible presence of ghosts. Indeed, given that the unifying gauge group is noncompact, one may expect that some of the spin-connection components have wrong sign kinetic terms [4, 5, 6]. Remarkably, in our case this generically happens to the tensor field components in the $SL(2N,C)$ gauge hypermultiplet rather than its vector and axial-vector components. But since just tensor fields appears to be naturally suppressed in the theory this problem appears to be circumvented at least in the weak-field approximation that, in turn, is protected by general covariance.

Spontaneous breakdown of the $SL(2N,C)$ HUT down to the Standard Model will lead to the variety of new processes connected as with the generalization of the gravity sector through the torsion related phenomena, so with the SM sector via new particles and new couplings which were discussed above. The quantum corrections to the largely classical approach presented here is also an important point to which one needs to pay more attention. These and related questions are planned to be considered elsewhere [19].

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