Running of the Top Yukawa with and without Light Gluinos

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Abstract

We investigate correlations among various parameters in the solution space of minimal supersymmetric grand unification. In particular the extent to which the top quark Yukawa coupling exhibits fixed point behavior is discussed and we compare various analytic approximations to its value at the top mass with its exact value in numerical solutions.

One of the successes of the idea of supersymmetric grand unification is the prediction of the $b$ quark to $\tau$ lepton mass ratio and the accompanying prediction of a top quark mass significantly above that of the $Z$. These predictions are based on the solution of a set of coupled renormalization group differential equations involving the gauge and Yukawa couplings. At least in the case of small $\tan(\beta), (< 5)$, the solution space of minimal SUSY unification is a ten dimensional space defined by the values of the following ten parameters.

1) A unification scale $M_X$
2) A unified gauge coupling $\alpha_0(M_X)$
3) A top Yukawa at $M_X$, $\alpha_t(M_X)$

4) A Susy scale $M_S$
5) A ratio $\tan(\beta)$ of the Higgs vacuum expectation values
6) The weak angle $\sin^2(\theta_W)$

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7) The fine structure constant $\alpha(M_Z)$
8) The strong coupling constant $\alpha_3(M_Z)$
9) The value of the top quark mass $M_t$
10) The value of the $b/\tau$ mass ratio at the $b$ quark scale

A "solution" is defined as a set of values for these ten parameters which is consistent with the renormalization group running and with the experimental constraints:

$$\alpha(M_Z) = 127.9 \pm 0.2$$  \hspace{1cm} (1)

$$\sin^2(\theta_W(M_Z)) = 0.2328 \pm 0.0007$$  \hspace{1cm} (2)

$$m_b/m_\tau = 2.39 \pm 0.10$$  \hspace{1cm} (3)

The latter corresponds to a physical $b$ quark mass of $4.95 \pm 0.15 GeV$. There are some who feel that the uncertainty in this quantity is much smaller than taken here but out of respect for the complications of confinement we content ourselves with this $4.5\%$ uncertainty. The uncertainties on the other two quantities are below $1\%$ and once the top quark mass is known the current data will specify them to about $0.15\%$ due to the correlation

$$\sin^2(\theta_W) = 0.2324 - 0.002(M_t^2/(138 GeV)^2 - 1) \pm 0.0003$$  \hspace{1cm} (4)

The hope is that the increasing precision with which these numbers are known will shed light on the physics at the scales $M_S$ and $M_X$ and will enable predictions to be made for $m_t$ and $\alpha_3(M_Z)$.

In addition to the experimental constraints eqs. (1)(3)(4) we assume that $\alpha_t(Q) < 1$ for all $Q$ ("perturbativity") and that

$$100 GeV < M_S < M_{S,max}$$  \hspace{1cm} (5)

If the Susy scale were below $100 GeV$, with the expected degeneracy splittings among the Susy particles, we would have expected unacceptably large contributions to the $Z$ width due to Susy decay modes. If $M_S$ is too large the theoretical benefits of Susy in explaining the stability of a low scale for electroweak symmetry breaking is lost. $M_{S,max}$ is variously taken to be $1 TeV$ or $10 TeV$. Ideally this argument would prefer $M_S$ below one $TeV$ since the electroweak scale is in the hundred $GeV$ range. In addition, a Susy solution of the dark matter problem would require the mass of the LSP to be no higher than $200 GeV$ again suggesting an average Susy mass below one $TeV$. Nevertheless we will, for the sake of conservatism, take $M_{S,max} = 10 TeV$. 

In the light gluino scenario, $\tan(\beta)$ is restricted to be between 1 and 2.3 and, for simplicity, we limit our investigation in the heavy gluino case, to the range $1 < \tan(\beta) < 5$ so that we can neglect the effect of the $b$ Yukawa on the running of the couplings. This range is also preferred by proton decay. Radiative electroweak breaking would predict a value of $\tan(\beta)$ very close to 1.8 in the light gluino case.

In the simplest version of SUSY unification one assumes that all the GUT scale particles are degenerate at $M_S$ and all the SUSY partners of the standard model particles are degenerate at $M_S$. In the light gluino variant one assumes that the partners of the squarks and sleptons are at $M_S$ together with the heavy Higgs, while the photino and gluino are in the low energy region below $M_Z$ and the other neutralinos and charginos are at the scale of $M_Z$ as suggested by the $M_{1/2} = 0$ model. This is consistent with the current experimental gluino searches which leave open (at least) the three windows shown in fig.1. Fig.1 updates the chart published by the UA1 group in 1987 to include the LEP results which are probably the most model independent constraints together with the results of the HELIOS collaboration which searched for weakly interacting neutral particles.

In higher level variations, the GUT scale spectrum is assumed to be non-degenerate and possibly richer than in the minimal model and/or the Susy scale $M_S$ is split into different masses for the various particles. In the latter schemes for each non-degenerate spectrum of Susy particles there is an effective degenerate scale $M_S$ which leads to the same unification solution apart from small two loop effects. Each solution in the degenerate case corresponds to a family of solutions with different splittings among squarks and sleptons and, as long as $M_S$ is above $M_Z$, no solutions for the ten parameters above are lost by assuming degeneracy.

The two-loop differential equations to be solved are summarized in the papers of refs. We follow a "top-down" approach where one begins by choosing random values for the first 5 quantities in the list above, extrapolating to low energies and discarding the choices which are inconsistent with the constraints of eqs. The surviving solutions are stored in a data set that can be queried for correlations among the various parameters. The solution space forms a small connected region in the ten dimensional space. Careful checking is required around the borders of this region to insure that these are in fact the borders and that no solutions exist outside this region. In fig.2a for example we show the solution space of the minimal Susy model projected onto the $M_S - M_X$ plane in a random non-overlapping sample from 813 solutions while fig.2b shows the same projection for a non-overlapping subset of 1200 solutions in the light gluino scenario. The shape coding labels the $\alpha_3(M_Z)$ value of each solution. In the heavy gluino case solutions are found only for $0.111 < \alpha_3(M_Z) < .134$ while in the light gluino case the range is $0.122 < \alpha_3(M_Z) < .133$. These ranges are divided into quadrants indicated in the solutions of fig.2 by rectangles, triangles, ovals, and diamonds for $\alpha_3(M_Z)$ in the lowest to the highest quadrant respectively. One sees that with the light gluino option, The GUT scale is restricted to values comfortably above $10^{16} GeV$ while this is not the case in the standard Susy picture. In Susy unification, proton decay via lepto-quark gauge bosons of mass as low
as $10^{15} GeV$ is not in contradiction with current limits. However, proton decay via the super-heavy scalars requires these particles to have masses in excess of $10^{16} GeV$. Since the GUT scale, above which the theory is grand-unified, is the maximum mass of the GUT scale particles, solutions with $M_X$ below $10^{16} GeV$ are probably not acceptable. One could therefore add a sixth experimental constraint

$$M_X > 10^{16} GeV$$

which would cut the solution spaces of figs. 2a, b at the corresponding limit. The solution sets of fig. 2a, b were generated in the course of work reported in ref.[7]. One sees in figs. 2a, b the effect reported there that the $\alpha_3$ values in minimal Susy unification are significantly more constrained in the light gluino scenario than in the usual picture. This result, however, is critically dependent on the super-gravity inspired prediction that the charginos and neutralinos are relatively light (below the Z) when the gluino is light. In these data sets, the constraint of eq. 2 was not enforced, although that of eq. 4 was, so that a Susy prediction for $\sin^2(\theta_W)$ could be made. This prediction can be read from figs. 3a, b which show the solutions in the $\sin^2(\theta_W) - \alpha_3(M_Z)$ plane. One sees that in both heavy and light gluino cases, the predicted values of $\sin^2(\theta_W)$ lie between 0.230 and 0.2325. Thus the minimal model predicts the weak angle with a 1% accuracy and agrees with experiment. The shape coding indicates the quadrant values of $M_S$ in the range from 100 GeV to 10 TeV (rectangle, triangle, oval, diamond from lowest to highest quadrant). In figures 4a, b we show the heavy and light gluino solutions respectively in the $\tan(\beta) - M_t$ plane with the values of $M_S$ indicated by the shape coding. One sees that in both cases the top quark mass is bounded below by about 143 GeV and that in the light gluino case the band of solutions is appreciably broader. It is interesting to note that the preliminary evidence from Fermilab for $M_t \simeq 174$ GeV suggests a $\tan(\beta) \simeq 1.8$ as required in the light gluino scenario with radiative electroweak breaking.[8] In the current work, however, and that of [7], radiative breaking is not assumed.

Much has been written [6, 9, 10] about the quasi-fixed point behavior of the top Yukawa. This can be roughly defined by the statement that the value of the top Yukawa at the top mass given by

$$\alpha_t(M_t) = M_t^2 \cdot (173 GeV \sin(\beta) / (3\pi) + 11(\alpha_3(M_Z)/\pi)^2)^{-2}$$

is dependent only on the gauge couplings at the top scale and is independent of the GUT scale parameters at least for some range of those parameters that is consistent with the Susy unification solutions. Our present purpose is to clarify and quantify this statement.

Neglecting the effect of the $b$ and $\tau$ Yukawas on the running of $\alpha_t$ and two loop contributions, the top Yukawa in the Susy region satisfies

$$2\pi d\alpha_t/dt = \alpha_t(6\alpha_t - c_{t,i}\alpha_i)$$

(8)
where the $\alpha_i$ are the three gauge couplings, $t = \ln(Q)$, and

$$c_{t,i} = (13/15, 3, 16/3)$$ (9)

for $i = 1, 2, 3$ in that order. If $\alpha_t$ is initially (at the GUT scale) higher than $c_{t,i} \alpha_i / 6$ the naive prediction is that it will fall with decreasing $Q$ until the right hand side of eq. 8 vanishes. If $\alpha_t$ is below $c_{t,i} \alpha_i / 6$ initially, it will rise toward that value as $Q$ decreases. At the most naive level, one might expect from eq. 8 that $\alpha_t(Q)$ will approach the fixed point expression

$$\alpha_{t,f}^{(1)}(Q) = c_{t,i} \alpha_i / 6$$ (10)

From our unification solution set it is a simple matter to calculate for each solution the top Yukawa at $M_t$ from eq. 7. The gauge couplings at $M_t$ can be related to the $\alpha_i(M_Z)$ from the renormalization group expressions. We can therefore check how well $\alpha_t$ approaches eq. 10 at $Q = M_t$. In figures 5a, b, for the heavy and light gluino scenarios respectively, we show the correlation between $\alpha_t(M_t)$ and $\alpha_{t,f}^{(1)}(M_t)$. The correlation is far from the close equality one might have expected. The shape coding, rectangles, triangles, ovals, diamonds, indicates an $M_S$ value in the lowest to the highest quadrant respectively with the total range being $100 GeV < M_S < 10 TeV$. We will return later to discuss the tail of solutions out to high $\alpha_{t,f}^{(1)}(M_t)$ evident in figures 5a, b.

Ibanez and Lopez [9] have shown that eq. 8 is satisfied by a top Yukawa given by

$$\alpha_t(Q) = \frac{-\alpha_t(M_X) \dot{F}(Q)}{1 + 6 \alpha_t(M_X) F(Q)/4\pi}$$ (11)

where

$$F(Q) = -\int_{M_X}^{Q} \frac{dQ'}{Q'} \exp(-\int_{M_X}^{Q'} \frac{dQ''}{4\pi Q''} c_{t,i} \alpha_i(Q''))$$ (12)

and

$$\dot{F}(Q) \equiv Q \frac{d}{dQ} F(Q)$$ (13)

Eq. 11 is of course not an analytic solution of eq. 8 since the $Q'$ integral in eq. 12 is not analytically soluble. However it does illustrate the quasi-fixed-point behavior in that $\alpha_t(Q)$ becomes independent of $\alpha_t(M_X)$ if $F(Q)$ is sufficiently large as would happen, for example, for large enough $M_X$. From eq. 11 one can write a second expression for the fixed point by dropping the $1$ in the denominator of eq. 11.

$$\alpha_{t,f}^{(2)} = -\frac{2\pi}{3} Q \frac{d}{dQ} \ln F(Q)$$ (14)

Because we don’t have here an analytic solution for $F(Q)$, it is difficult to predict the extent of the independence of $\alpha_t(M_t)$ on $M_X$ and $\alpha_t(M_X)$. Eq. 14 does indicate dependence of $\alpha_t(M_t)$ on $M_X$ and indirectly
on $\alpha_t(M_X)$ through the two loop effects on the running of the gauge couplings. The empirical dependence of $\alpha_t(M_t)$ on $M_X$ is substantial as can be shown by projecting the solution space onto the $\alpha_t(M_t) - M_X$ plane. The Ibanez-Lopez expression for the quasi-fixed-point does not yield analytically the dependence of $\alpha_t(M_t)$ on the ten parameters of the unification solution. We seek preferably a quasi-fixed-point expression analogous to that of eq.\[10\].

One of the problems with both of the above treatments is that the running of the top Yukawa changes dramatically below the Susy scale. Then instead of eqs.\[8\] and \[9\] we have

$$2\pi \frac{d\alpha^s_{t \, m}}{dt} = \alpha^s_{t \, m} \left( \frac{9}{2} \alpha^s_{t \, m} - c^s_{t \, i \, \alpha_i} \right)$$

(15)

where below $M_S$ one defines the effective top Yukawa

$$\alpha^s_{t \, m} = \alpha_t \sin^2 \beta$$

(16)

and

$$c^s_{t \, i \, \alpha_i} = \left( \frac{17}{20}, \frac{9}{4}, 8 \right)$$

(17)

If $M_S$ is above $M_t$ one extrapolates from $M_S$ to $M_t$ using eqs.\[13\] and \[17\] and then redefines $\alpha_t(M_t)$ through eq.\[14\] before substituting in eq.\[8\]. This rescaling has only a higher order effect on the final answer so we use a rescaling by the fixed value $\sin \beta = 0.7641$ in both the numerical running and the analytic expressions below.

Thus, if the top Yukawa is following a quasi-fixed-point given approximately by eq.\[10\] down to $M_S$, below the Susy scale we would expect it to be drawn toward a naive fixed point corresponding to

$$\alpha^{(1)\, sm}_t(Q) = \frac{2}{9} c^s_{t \, i \, \alpha_i} \alpha_i(Q)$$

(18)

The corresponding $\alpha_{t; f}$ is more than twice the fixed point of the Susy regime suggested by eq.\[10\]. This effect could partially explain the large deviations of $\alpha_t(M_t)$ from $\alpha^{(1)\, sm}_t(M_t)$ evident in figures 5a, b. However, it is clear that even with $M_S$ as large as $10 TeV$, the top Yukawa does not have time to reach a standard model quasi-fixed-point. From figures 5a, b one sees a tendency for the top Yukawa to rise with increasing $M_S$ as would be expected from this effect but it never achieves the doubling expected naively from eq.\[18\]. Thus, if $M_S$ is in the 1 to 10$ TeV$ region, the top quark Yukawa is unlikely to have reached a limiting behavior. The tail of events at large $\alpha^{(1)}_{t; f}$ in figure 5 represents the events in which $M_S < M_t$ so the Susy quasi-fixed-point should be most accurately attained. Paradoxically, it is here that the discrepancy between $\alpha_t(M_t)$ and $\alpha^{(1)}_{t; f}$ is largest. Clearly a more accurate representation of the top Yukawa behavior is required.

In the SUSY regime the gauge couplings change according to the law

$$2\pi \frac{d\alpha_i}{dt} = \alpha_i^2 b_i$$

(19)
with

\[ b_i = (33/5, 1, -3) \]  

(20)

We can combine eqs. 8 and 19 with arbitrary parameters \( \lambda_i \) to write

\[
2\pi \frac{d(\alpha_t - \lambda_i \alpha_i)}{dt} = 6\alpha_t^2 - c_{t,i} \alpha_i \alpha_t - \lambda_i \alpha_t^2 b_i
\]  

(21)

where summation over repeated indices is intended and, for the purpose of an analytic approximation, we have again neglected two loop contributions to the running. Ideally, one would seek \( \lambda_i \) such that when \( \alpha_t \) reaches

\[ \alpha_{t;f} = \lambda_i \alpha_i \]  

(22)

the right hand side of eq. 21 would vanish. Thus we would seek solutions to the equation

\[ 6(\lambda_i \alpha_i)^2 - c_{t,i} \alpha_i \lambda_j \alpha_j - \lambda_i \alpha_t^2 b_i = 0 \]  

(23)

However, no set of \( \lambda_i \) exists that will satisfy eq. 23 for arbitrary values of the \( \alpha_i\). We propose therefore to write eq. 21 in the form

\[
\pi \frac{d(\alpha_t - \lambda_i \alpha_i)}{3 dt} = (\alpha_t - c_{t,i} \alpha_i/12)^2 - \delta^2
\]  

(24)

where

\[ \delta^2 = (c_{t,i} \alpha_i/12)^2 + \lambda_i \alpha_t^2/6 \]  

(25)

We will now choose

\[ \lambda_i = c_{t,i}/12 \]  

(26)

and make the approximation of ignoring the \( Q \) dependence of \( \delta \). Then eq. 24 can be integrated to write

\[
\alpha_t(Q) = \lambda_i \alpha_i(Q) + \delta \frac{y_0 + \delta + (y_0 - \delta)e^{6\delta \ln(Q/M_X)/\pi}}{y_0 + \delta - (y_0 - \delta)e^{6\delta \ln(Q/M_X)/\pi}}
\]  

(27)

where \( y_0 = \alpha_i(M_X) - \lambda_i \alpha_i(M_X) \) and \( \delta \) is defined at \( Q \). This is an identity at \( Q = M_X \) and gives an excellent approximation to \( \alpha_t(Q) \) for all \( Q > M_S \). In the limit \( M_X/Q \to \infty \), the exponentials in eq. 27 become negligible and \( \alpha_t(Q) \) is determined solely by the \( \alpha_t(Q) \) becoming independent of the GUT scale values. In this case one could talk about a quasi-fixed-point behavior. However in practice \( M_X \) is never large enough to justify dropping the exponentials. Furthermore, when \( M_t < M_S \) we must face the complication of integrating \( \alpha_t \) in the standard model region. In this case we use eq. 27 for \( \alpha_t(M_S) \) and write the approximate form using eq. 13

\[
\alpha_t(M_t) = \alpha_t(M_S) \left( 1 + \frac{\ln M_t/M_S}{2\pi} \left( 0.7641 \cdot \frac{9}{2} \alpha_t(M_S) - c_{t,i}^{\alpha_t} \alpha_t(M_t) \right) \right)
\]  

(28)
In fig.6a,b we show the correlation between the right hand side of this equation which we may call $\alpha_{t/f}^{(3)}$ and the exact numerically calculated left hand side. The prediction is seen to hold within a few percent over the entire solution space. Given $M_t$ and $M_S$, the gauge couplings at those scales can be found to sufficient accuracy by the first order extrapolation from their values at $M_Z$. If the exponentials in eq.27 were negligible, eqs.7 and 28, together with knowledge of $M_t$, $M_S$ and the gauge couplings at the $Z$ would yield an accurate measure of $\tan(\beta)$ independent of the GUT scale parameters. However, only to a crude approximation $\simeq 20\%$ can the exponentials in eq.27 be neglected.

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FIGURE CAPTIONS

Fig. 1. Low mass windows for the gluino mass as a function of the squark masses. The hatched areas are disfavored by the indicated experiments. The dot-dashed curves represent the loci of expected gluino lifetimes $10^{-6}s, 10^{-8}s, 10^{-10}s,$ and $10^{-12}s$ respectively from the highest to the lowest curve.

Fig. 2. (a) The correlation between the SUSY scale, $M_S$, and the GUT scale, $M_X$, in the heavy gluino case, $m_{\tilde{g}} = M_S$. Each solution corresponds to an allowed point in the ten dimensional space discussed in the introduction. For given $M_S$, no solutions exist outside of the broad band shown. The width of the band corresponds to summing over all other eight parameters. The $\alpha_3(M_Z)$ value for each solution is indicated by the shape coding (see text).
(b) The same correlation in the case of the light gluino ($m_{\tilde{g}} < M_S$).

Fig. 3. (a) The correlation between $\alpha_t(M_t)$ and $\sin^2(\theta_W)$ in the heavy gluino case. Solutions in the 1st, 2nd, 3rd, and 4th quadrant of the $M_S$ range, $100GeV < M_S < 10TeV$, are printed as rectangles, triangles, ovals, and diamonds respectively.
(b) The same as (a) but for the light gluino scenario.

Fig. 4. (a) The correlation between $\tan(\beta)$ and $M_t$ in the heavy gluino case. The $M_S$ quadrants are indicated as in Fig. 3. (b) The same as (a) but for light gluinos.

Fig. 5. The correlation between the ”naïve” top Yukawa fixed point $\alpha^{(1)}_{t;f}(M_t)$ and the actual top Yukawa $\alpha_t(M_t)$ found in the numerical solutions in the heavy gluino case (a) and in the light gluino case (b). 10% to 20% departures are observed. The quadrant values of $M_S$ are indicated by the shape coding as in fig. 3.

Fig. 6. The correlation between the approximate analytic value, $\alpha^{(3)}_{t;f}(M_t)$, of the top Yukawa and the exact numerical value of $\alpha_t(M_t)$ in the heavy gluino solution space (a) and the light gluino solution space (b). Agreement within 2% is found. The quadrant values of $M_S$ are indicated by the shape coding as in fig. 3.
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