Parametric studies on thin plates with holes

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Abstract. Plates are most important structural members finding applications in composite construction. Plates usually provided with holes meant of structural connections, service ducts, and ventilation services. The impact of holes in thin plates induces higher stress concentration and stress distribution parameters around the holes. This paper aims to analyze the impact of holes on thin plate subjected to uniform tensile load at its ends and its stress concentration factor. The parameters involved for the studies are number and the shape of the holes. The comparative study is made with analytical and numerical investigation on the plates with different d/D ratios. The variations in stress concentration factor for different d/D ratios are determined based on finite element analysis using ANSYS.

1. Introduction

The rectangular plate with holes finds its application in various structural engineering initiatives of steel design. The steel plates are often provided with holes for service ducts, connections causes pitting corrosion which reduces the mechanical strength leading to the failure of structure under service loads. Therefore, it is essential to analyze the state of stress around the holes to ensure safety and to evaluate the load bearing capacity of these structures. The solution for stress state around the hole is as shown in figure 1 and figure 2 below.

Figure 1. Stress distribution on plate. Figure 2. Stress concentration around holes.

The study is attempted to analyze the plate with circular and square holes with variations in number of holes. The numerical and analytical investigations are made to compare the impact of shape of hole
and the stress concentration factor on a rectangular plate subjected to tensile load. The rectangular plate of size 1000mm x 500mm x 15mm provided with diameter of hole varies from 100mm to 400mm respectively.

2. Literature survey

Gokul [1] investigated the analysis of thin plates with holes. The analysis includes the shape and size of the hole on the stress distribution of plate. The diameter of hole increases with increase in stress distribution, whereas the circular holes are inducing less concentration when compare to square opening. Ai-Azzawi and sadeq Aziz Abed [2] attempted on numerical analysis of reinforced concrete hollow core slabs, the hollow holes are used to reduce the cracking load and ultimate capacity of about (13.5%) and increased deflection of (39.5%). The finite element technique to assess stress distribution within a rectangular plate with two holes subjected to in-plane loading is also made. Durgadevagi and NA jabez [3] studied the comparative analysis of slab with different shape of cut out for various end conditions using ANSYS. They aimed to know the variation of displacement, stress and strain with different end conditions of the slab comparing the slabs with different boundary conditions with opening. Based on the result, square slab with circular opening is found to be efficient by providing fixed support on all edges. Lotfi Toubal et al. [4] investigated the stress concentration in a circular hole in composite plate. In this study, stress concentration has been calculated, in which Stress near holes in experiment is lesser when compare to analytical and numerical model. Babulan K S et al., [5] investigated the isotropic rectangular plate with centrally located holes which is subjected to axial uniform tension of 50Mpa along the thickness direction. From this analysis the stress around the holes is larger than the stress at other region of the plate, which was defined that in terms of stress concentration factor. The holes in the plate includes d/D ratio of 0.2, 0.4, 0.6. By finite element analysis shows that deformation of plate depends upon d/D ratio of hole. From comparison of manual and software analysis the percentage variation of stress in manual analysis have been 4.8 %, 2.9%, and 4.7% higher than software analysis for 0.2, 0.4 &0.6 respectively. The maximum stress occurred around the holes of the plate. Saksham Dhanjal [6] described that plate without hole subjected to loads causes the uniform stress employed on all region of the hole but presence of holes, notches and keyways often leads to stress concentration around the hole. The three dimensional finite analysis was conducted on rectangular plate with hole at centre was subjected to uniform tension at two adjacent edges. The comparisons were made for different sizes on same plate. From the obtained results size of holes increases with increasing in stress concentration. Dheeraj Gunwant [7] investigated the comparison of stress and displacement on different ratio of major to minor axis of elliptical holes on rectangular plate subjected to uniform tension. The ratio from minor axis to major axis is assumed as 0.2, 0.4, 0.6, 0.8 & 1.0. From this analysis the maximum deflection of plate when the ratio of elliptical hole is 1.0. The maximum stress is situated at all four corners of elliptical hole.

3. Analytical approach

The analytical investigation is performed to study the stress concentration factor (Kt), a dimensionless factor used to locate the concentrated stress in a plate material and defined as the ratio of the maximum principal stress in the element to the normal stress.

\[ Kt = \frac{\sigma_{max}}{\sigma_{nor}} \]

3.1. Rectangular plate with single and two holes

The rectangular plates of uniform size and thickness as shown in figure 3 and figure 4 are taken for the study in which the numbers of holes are varied to predict the stress concentration parameters. The variation of d/D ratios for different diameter of holes are shown in table 1 and table 2 respectively for single and double holes.
Figure 3. Rectangular plate with single hole.  

Figure 4. Rectangular plate with two holes.  

Table 1. Rectangular Plate with single hole.  

| S. No. | Size of plate          | Diameter of hole | d/D ratio |
|-------|-----------------------|------------------|-----------|
| 1     |                       | 0                | 0         |
| 2     | 100mm                 | 0.2              |
| 3     | 500×500×15mm          | 0.4              |
| 4     | 300mm                 | 0.6              |
| 5     | 400mm                 | 0.8              |

The stress concentration factor (Kt) for rectangular plate with single hole is given by the below equation (1),

\[ K_t = 3.0 - 3.14\left(\frac{d}{D}\right) + 3.667\left(\frac{d}{D}\right)^2 - 1.527\left(\frac{d}{D}\right)^3 \]  
Eqn. (1)

Where, \(0 < \frac{d}{D} < 1\)

Table 2. Rectangular Plate with two hole.  

| S. No. | Size of plate          | Diameter of holes | d/D ratio |
|-------|-----------------------|-------------------|-----------|
| 1     |                       | 0                 | 0         |
| 2     | 100mm                 | 0.2               |
| 3     | 1000×500×15mm         | 0.4               |
| 4     | 300mm                 | 0.6               |
| 5     | 400mm                 | 0.8               |

The stress concentration factor (Kt) for rectangular plate with two holes is given by the equation (2) below,

\[ K_t = 3.0 - 0.712\left(\frac{d}{L}\right) + 0.271\left(\frac{d}{L}\right)^2 \]  
Eqn. (2)

Where, \(0 < \frac{d}{L} < 1\)

The variation of maximum stresses based on the analytical approach and the numerical approach using ANSYS for plate with single and two holes are compared in the table 3 and table 4 respectively.
Table 3. Comparison of stresses for plate with single hole.

| S. No. | d/D ratio | Kt | Stress by analytical approach $\sigma_{\text{max}}$ (Mpa) | Stress by numerical approach $\sigma_{\text{max}}$ (Mpa) |
|--------|-----------|----|---------------------------------|---------------------------------|
| 1      | 0         | 3  | 20.01                           | 20.34                           |
| 2      | 0.2       | 2.506 | 20.87                           | 21.53                           |
| 3      | 0.4       | 2.233 | 25.00                           | 24.35                           |
| 4      | 0.6       | 2.106 | 35.14                           | 30.63                           |
| 5      | 0.8       | 2.026 | 68.00                           | 47.59                           |

Table 4. Comparison of stresses for the plates with two holes.

| S. No. | d/D ratio | Kt | Stress by analytical approach $\sigma_{\text{max}}$ (Mpa) | Stress by numerical approach $\sigma_{\text{max}}$ (Mpa) |
|--------|-----------|----|---------------------------------|---------------------------------|
| 1      | 0         | 3  | 20.01                           | 20.27                           |
| 2      | 0.2       | 2.87 | 23.92                           | 21.53                           |
| 3      | 0.4       | 2.76 | 30.65                           | 23.33                           |
| 4      | 0.6       | 2.67 | 44.51                           | 27.28                           |
| 5      | 0.8       | 2.6  | 86.67                           | 48.26                           |

3.2. Rectangular plate with square opening

In case of rectangular plate with square opening, the stress concentration is too high near the sharp edges. The rectangular plate of size 500×500×15mm with square opening of size 100mm, 200mm and 300mm respectively are analysed. The variation in Stress concentration factor for plate with square hole are calculated by the equation (3) below,

$$K_t = C_1 + C_2 (a/b) + C_3 (a/b)^2 + C_4 (a/b)^3$$

The variation of $K_t$ for the plate with square opening is too high when compared to circular opening. The maximum stresses for square hole plate based on the analytical approach and the numerical approach using ANSYS are compared as shown in Table 5.

Table 5. Comparison of stresses for the plates with square hole.

| S. No. | d/D ratio | Kt | Stress by analytical approach $\sigma_{\text{max}}$ (Mpa) | Stress by numerical approach $\sigma_{\text{max}}$ (Mpa) |
|--------|-----------|----|---------------------------------|---------------------------------|
| 1      | 0.2       | 7.075 | 58.68                           | 21.56                           |
| 2      | 0.4       | 7.075 | 78.26                           | 26.219                          |
| 3      | 0.6       | 7.075 | 67.41                           | 47.526                          |

4. Results and Discussion

The variation of stress concentration factor with d/D ratio for rectangular plates with single and two holes were simulated using ANSYS workbench 19.2. The analytical results were compared with the numerical finite element simulation with the variation of 15%. The variation of stresses increases with increases in number of holes in the plate as shown in figure 5, wherein the stress concentration factor increases with reduction in cross section of plates.
It indicates that the diameter of holes increases with increase in stress concentration factor due to reduction on cross section. Thus large variation of stresses in the plate around the holes. The maximum stress concentration factor under comparative analysis had a slight variation with numerical approach. The variation gets reduced by finer mesh using finite element analysis. The variation in d/D ratio for rectangular plate with two holes increases linearly with increase in Kt values gradually as shown in figure 6.

The above figure 7 shows that the percentage variations of stress concentration factor (Kt) on analytical and numerical analysis. When the d/D ratio increases gradually with increase in percentage variation of Kt upto certain limit wherein the optimum percentage variation adopted at d/D ratio from 0.4 to 0.6 respectively for the plates with two holes.

The variations of stress distribution are compared for the plates with single holes as shown in the below figure 8 and figure 9. Higher the d/D ratio, stress variation on the plate is also higher. This variation may be minimized by increases the mesh size in the ANSYS software. The variation increases from 1.28 % to 34.02% with increase in d/D ratio of 0.8.
5. Numerical Approach

From the figure 10 to 22 shows the simulation studies using finite element analysis for variation of rectangular plate without hole, with single hole and two holes. Comparative analysis on stress concentration factor (Kt) for d/D ratio increases with increase in % variation. The stress concentration near the holes are so negligible for the plate with single hole, where in it increases with increase in diameter of the hole to the boundary, and certainly negligible at plate with two hole distance.
Figure 14. Single hole, $d/D=0.8$.

Figure 15. Two hole, $d/D=0$.

Figure 16. Two hole, $d/D=0.2$.

Figure 17. Two hole, $d/D=0.4$.

Figure 18. Two hole, $d/D=0.6$.

Figure 19. Two hole, $d/D=0.8$.

Figure 20. Single square hole, $d/D=0.2$.

Figure 21. Single square hole, $d/D=0.4$. 
6. Conclusions
Following conclusions are reported based on analytical and numerical investigation using finite element analysis,

- The maximum stress concentration occurs around the hole for all cases of plates irrespective of diameter and the shape of the holes.
- The stress concentration is much smaller near one diameter distance from the edge of the hole, and negligible at two diameters distance from the edge of the hole.
- The rectangular plates with two holes have higher stress concentration between the two holes wherein in the number of holes increases with increase in stress variation due to reduction in cross sectional area.
- ANSYS provides a very suitable way of determining stresses and the obtained results are not very accurate as compared to the analytical results but they can be used for the simulation of complex geometries. The accuracy can be achieved by means of reducing the mesh size to suitable limits.
- The percentage variation on analysis of rectangular plate with single hole results in reduction in variation of $K_t$ for 0.6 to 0.8 $d/b$ ratio.
- The percentage variation on analysis of rectangular plate with two holes results in reduction in variation of $K_t$ for 0.8 $d/b$ ratio.
- When the shape of hole varies from square to circle the stress concentration factor also varies invariably as for square hole, sharp corners leads to abrupt changes as per $d/b$ ratio.

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