Renormalization of the chiral pion–nucleon
Lagrangian beyond next–to–leading order

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Abstract

The complete renormalization of the generating functional for Green functions of quark currents between one–nucleon states in two flavor heavy baryon chiral perturbation theory is performed to order $q^4$. We show how the heat kernel method has to be extended for operators orthogonal to the heavy fermion four–velocity. A method is developed to treat the multi–coincidence limit arising from insertions of dimension two (and higher) operators on internal baryon propagators in self–energy graphs. As examples, we study the divergences in the isoscalar magnetic moment and the scalar form factor of the nucleon.
1 Introduction

Chiral perturbation theory (CHPT) allows to systematically investigate the consequences of the spontaneous and explicit chiral symmetry breaking QCD is believed to undergo. In the presence of nucleons, such studies can be extended and related to a large variety of precisely measured processes. The basic degrees of freedom are the three (almost) massless pseudoscalar Goldstone bosons, i.e. the pions, and the spin–1/2 fields, the nucleons, treated as very heavy, static sources. The corresponding effective field theory is subject to an expansion in small momenta and quark (meson) masses or equivalently an expansion in the number of pion loops. To one loop order, divergences appear. Some of these were treated e.g. in \[1\]. A systematic treatment of the leading divergences of the generating functional for Green functions of quark currents between one–nucleon states was given in ref.\[2\]. This allows for a chiral invariant renormalization of all two–nucleon Green functions of the pion–nucleon system to order \(q^3\) in the low–energy expansion, where \(q\) denotes the various expansion parameters, in our case the pion energy \(E_\pi\) and mass \(M_\pi\) with respect to the scale of chiral symmetry breaking and with respect to the nucleon mass \(m\). Since these two scales are of comparable size, one effectively has a double expansion in powers of \(E_\pi/m\) and \(M_\pi/m\). The study of ref.\[2\] was extended to the three flavor case to the same order in the chiral expansion in \[4\]. However, a series of precise calculations of single nucleon processes like Compton scattering, neutral and charged pion photoproduction off nucleons and deuterium or the chiral corrections to Weinberg’s predictions for the S–wave pion–nucleon scattering lengths have shown that it is mandatory to go order \(q^4\), for a review see \[5\]. At that order, one has to construct one loop graphs with exactly one insertion from the dimension two pion–nucleon Lagrangian and local counterterms with a priori unknown coupling constants. These allow to absorb the divergences appearing in the loop diagrams. In this paper, we extend the calculation of \[2\] to order \(q^4\), i.e. we investigate the divergences of the generating functional for Green functions of quark currents between one–nucleon states using heat–kernel techniques. We do not treat nucleon–anti-nucleon S–matrix elements, which formally start to appear at this order. We perform a chiral invariant renormalization of all two–nucleon Green functions of the pion–nucleon system to \(\mathcal{O}(q^4)\) in the low–energy (chiral) expansion. Another argument why one has to perform the renormalization at fourth order is the observation that the effective pion–nucleon Lagrangian consists of terms with odd and even chiral dimension starting at orders one and two, respectively. Therefore, the first corrections to the dimension two tree graphs (which are sometimes large) appear at one loop and order four \[6\]. This investigation is only a first step in a systematic evaluation of isospin–violating effects at low energies and to eventually gain a deeper insight into the mechanism of this isospin violation. In a next step, virtual photons have to be included in the generating functional. This will then allow us to separate the hadronic (QCD) isospin violating effects \(\sim m_d - m_u\), i.e. the ones due to the light quark mass difference, from the purely electromagnetic ones, which are of the same size and are e.g. the main contribution to the charged to neutral pion mass difference. The novel data on pion photoproduction from MAMI and SAL as well as the level shift measurements for pionic hydrogen and deuterium at PSI have now reached such a precision that a clean separation of electromagnetic from the purely hadronic contributions

\#6 An alternative method to work out the divergence structure at third order has recently been developed in ref.\[4\].
based on a consistent machinery is called for.

The manuscript is organized as follows. In section 2 we review the path integral formalism of heavy nucleon CHPT, following closely ref.[1]. This section is mostly relevant to define our notations. In particular, we write down the dimension two pion–nucleon Lagrangian in the form that is particularly useful for our purpose. In section 3 we work out the generating functional to order $q^4$. Here and in what follows, our work closely parallels the one of ref.[2]. Since we work to one order higher, our emphasis is on discussing the novel contributions appearing beyond $O(q^3)$. To be specific, instead of the two irreducible graphs at $O(q^3)$, we have to deal with four (tadpole and three types of self–energy graphs). In section 4 we work out the renormalization of the irreducible self–energy graphs based on standard heat kernel techniques. The much more involved renormalization of the irreducible self–energy graphs is spelled out in section 5. We split this into three subsections. First, we consider the vertex–corrected self–energy diagrams which can be evaluated straightforwardly by the method developed in ref.[2] for such type of operators. We then consider the self-energy diagram with a dimension two insertion on the intermediate nucleon line (the so–called “eye graph”), which formally involves a triple coincidence limit in the proper time. Its contributions are worked out making use of an n–fold coincidence technique which we develop in one part of the section. In that context, we have to deal with operators which are no longer projected in the direction of the nucleons’ four–velocity. We show how to extend the heat–kernel methods used in [2, 3] to handle such type of operators. We proceed and work out the pertinent singularity structure. In section 6 we write down the full counterterm Lagrangian at order $q^4$ and tabulate the pertinent operators and their $\beta$–functions. This table constitutes the main result of this investigation. Section 7 contains a few sample calculations. We consider the isoscalar magnetic moment of the nucleon and the scalar form factor of the nucleon to order $q^4$. We extract the pertinent divergences by straightforward Feynman diagram evaluation and we show how to use table 1. A summary and a discussion of the various checks on our calculation is given in section 8. The appendices contain sufficiently detailed technicalities to check the calculation at various intermediate steps. In particular, in app. A we list all Seeley–deWitt coefficients for a certain class of elliptic differential operators up to dimension four. Also given are all products of singular operators with one meson and one baryon propagator as well as for one meson and two baryon propagators, see app. B and app. C, respectively. Furthermore, the contributions from the most complicated diagram, the eye graph, are separately listed in app. D. In appendix E, we discuss an alternative way of treating parts of the eye graph which allows for a good check on parts of the rather involved calculations. We also consider it useful to give the divergent operators and their $\beta$–functions from the tadpole, self–energy and eye graphs in separate tables. This is spelled out in app. F.

2 Brief exposé of the heavy nucleon effective field theory and its path integral formulation

To keep the manuscript self–contained, we briefly develop the path–integral formulation of the chiral effective pion–nucleon system. This follows largely the original work of [1], which was reviewed in [3]. The reader familiar with these methods can skip this section. Most important is
the definition of the dimension two \( \pi N \) Lagrangian given at the end of this section because it will be used extensively later on.

The interactions of the pions with the nucleons are severely constrained by chiral symmetry. The generating functional for Green functions of quark currents between single nucleon states, \( Z[j, \eta, \bar{\eta}] \), is defined via

\[
\exp \left\{ i Z[j, \eta, \bar{\eta}] \right\} = N \int [du][dN][d\bar{N}] \exp \left\{ i \left( S_{\pi\pi} + S_{\pi N} + \int d^4x \left( \bar{\eta}N + \bar{N}\eta \right) \right) \right\}, \tag{2.1}
\]

with \( S_{\pi\pi} \) and \( S_{\pi N} \) denoting the pionic and the pion–nucleon effective action, respectively, to be discussed below. \( \eta \) and \( \bar{\eta} \) are fermionic sources coupled to the baryons and \( j \) collectively denotes the external fields of vector \((v_\mu)\), axial–vector \((a_\mu)\), scalar \((s)\) and pseudoscalar \((p)\) type. These are coupled in the standard chiral invariant manner. In particular, the scalar source contains the quark mass matrix \( \mathcal{M} \), \( s(x) = \mathcal{M} + \ldots \). The underlying effective Lagrangian can be decomposed into a purely mesonic \((\pi\pi)\) and a pion–nucleon \((\pi N)\) part as follows (we only consider processes with exactly one nucleon in the initial and one in the final state)

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{\pi N} \tag{2.2}
\]

subject to the following low–energy expansions

\[
\mathcal{L}_{\pi\pi} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi\pi}^{(4)} + \ldots , \quad \mathcal{L}_{\pi N} = \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(3)} + \mathcal{L}_{\pi N}^{(4)} + \ldots \tag{2.3}
\]

where the superscript denotes the chiral dimension. The pseudoscalar Goldstone fields, i.e. the pions, are collected in the \( 2 \times 2 \) unimodular, unitary matrix \( U(x) \), \( U(\phi) = u^2(\phi) = \exp\{i\phi/F\} \) with \( F \) the pion decay constant (in the chiral limit). The external fields appear in the following chiral invariant combinations:

\[
r_\mu = v_\mu + a_\mu , \quad l_\mu = v_\mu - a_\mu , \quad \text{and} \quad \chi = 2B_0(s + ip). \]

Here, \( B_0 \) is related to the quark condensate in the chiral limit, \( B_0 = |\langle 0|\bar{q}q|0 \rangle|/F^2 \). We adhere to the standard chiral counting, i.e. \( s \) and \( p \) are counted as \( \mathcal{O}(q^2) \), with \( q \) denoting a small momentum or meson mass. The effective meson–baryon Lagrangian starts with terms of dimension one,

\[
\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi} \left( i\nabla - m + \frac{g_A}{2} \gamma_5 \right) \Psi , \tag{2.4}
\]

with \( m \) the nucleon mass in the chiral limit and \( u_\mu = i[u^\dagger(\partial_\mu - i r_\mu)u - u(\partial_\mu - il_\mu)u^\dagger] \). The nucleons, i.e. the proton and the neutron, are collected in the is–doublet \( \Psi \),

\[
\Psi = \begin{pmatrix} p \\ n \end{pmatrix} . \tag{2.5}
\]

Under \( SU(2)_L \times SU(2)_R \), \( \Psi \) transforms as any matter field. \( \nabla_\mu \) denotes the covariant derivative, \( \nabla_\mu \Psi = \partial_\mu \Psi + \Gamma_\mu \Psi \) and \( \Gamma_\mu \) is the chiral connection, \( \Gamma_\mu = \frac{1}{2} [u^\dagger(\partial_\mu - i r_\mu)u + u(\partial_\mu - il_\mu)u^\dagger] \). Note that the first term in Eq.(2.4) is of dimension one since \( (i\nabla - m) \Psi = \mathcal{O}(q^0) \). The lowest order pion–nucleon Lagrangian contains two parameters, the nucleon mass \( m \) and the axial–vector coupling constant \( g_A \), both taken at their values in the chiral limit.\(^\#7\)

\(^\#7\) We omit the often used superscript ‘\( ^\circ \)’ to keep our notation compact.
spin–1/2 fields, the chiral power counting is no more systematic due to the large mass scale \( m \), \( \partial_0 \Psi \sim m \Psi \sim \Lambda_\chi \Psi \), with \( \Lambda_\chi \sim 1 \text{GeV} \) the scale of chiral symmetry breaking. This problem can be overcome in the heavy mass formalism proposed in [8]. We follow here the path integral approach developed in [1]. Defining velocity–dependent spin–1/2 fields by a particular choice of Lorentz frame and decomposing the fields into their velocity eigenstates (sometimes called 'light' and 'heavy' components),

\[
H_v(x) = \exp\{imv \cdot x\} P_v^+ N(x), \quad h_v(x) = \exp\{imv \cdot x\} P_v^- N(x),
\]

the mass dependence is shuffled from the fermion propagator into a string of \( 1/m \) suppressed interaction vertices. The projection operators appearing in Eq.(2.6) are given by

\[
P_{\pm} = \frac{1 \pm v / \sqrt{v^2 - 1}}{2},
\]

with \( v^\mu \) the four–velocity subject to the constraint \( v^2 = 1 \). In this basis, the effective pion–nucleon action takes the form

\[
S_{\pi N} = \int d^4x \left\{ \bar{H}_v A H_v - \bar{h}_v C h_v + \bar{h}_v B H_v + \bar{H}_v \gamma_0 B^\dagger \gamma_0 h_v \right\}.
\]

The matrices \( A, B \) and \( C \) admit low energy expansions, e.g.

\[
A = A_{(1)} + A_{(2)} + A_{(3)} + A_{(4)} + \ldots.
\]

Explicit expressions for the various contributions can be found in [5]. Similarly, we split the baryon source fields \( \eta(x) \) into velocity eigenstates,

\[
R_v(x) = \exp\{imv \cdot x\} P_v^+ \eta(x), \quad \rho_v(x) = \exp\{imv \cdot x\} P_v^- \eta(x),
\]

and shift variables, \( h_v = h_v - C^{-1} (B H_v + \rho_v) \), so that the generating functional takes the form

\[
\exp[iZ] = N \Delta_h \int [dU][dH_v][dH_v] \exp\{iS_{\pi\pi} + iS'_{\pi N} \}
\]

in terms of the new pion–nucleon action \( S'_{\pi N} \),

\[
S'_{\pi N} = \int d^4x \left( \bar{H}_v (A + \gamma_0 B^\dagger \gamma_0 C^{-1} B) H_v + \bar{H}_v (R_v + \gamma_0 B^\dagger \gamma_0 C^{-1} \rho_v) + (\bar{R}_v + \bar{\rho}_v C^{-1} B) H_v \right).
\]

The determinant \( \Delta_h \) related to the 'heavy' components is identical to one, i.e. the positive and negative velocity sectors are completely separated. The generating functional is thus entirely expressed in terms of the Goldstone bosons and the 'light' components of the spin–1/2 fields. The action is, however, highly non–local due to the appearance of the inverse of the matrix \( C \). To render it local, one now expands \( C^{-1} \) in powers of \( 1/m \), i.e. in terms of increasing chiral dimension. To any finite power in \( 1/m \), one can now perform the integration of the 'light' baryon field components \( N_v \) by again completing the square,

\[
H'_v = H_v + T^{-1} (R_v + \gamma_0 B^\dagger \gamma_0 C^{-1} \rho_v), \quad T = A + \gamma_0 B^\dagger \gamma_0 C^{-1} B.
\]

*Notice that for these matrices the chiral dimensions are given as subscripts.*
Notice that the second term in the expression for $T$ only starts to contribute at chiral dimension two. To be more precise, we give the chiral expansion of $T$ up to and including all terms of order $q^4$,

$$T = A_{(1)} + A_{(2)} + A_{(3)} + A_{(4)} + \frac{1}{2m} \gamma_0 B_{(1)}^\dagger \gamma_0 B_{(1)} + \frac{1}{(2m)^2} \gamma_0 B_{(1)}^\dagger \gamma_0 (C_{(1)} - 2m) \ B_{(1)} + \frac{1}{(2m)^3} \gamma_0 B_{(1)}^\dagger \gamma_0 (C_{(1)} - 2m) (C_{(1)} - 2m) B_{(1)} + O(q^5). \quad (2.13)$$

We thus arrive at

$$\exp[iZ] = N' \int [dU] \exp\{iS_{\pi\pi} + iZ_{\pi N}\}, \quad (2.14)$$

with $N'$ an irrelevant normalization constant. The generating functional has thus been reduced to the purely mesonic functional. $Z_{\pi N}$ is given by

$$Z_{\pi N} = -\int d^4x \left\{ \bar{\rho}_v (C^{-1} B T^{-1} \gamma_0 B^\dagger \gamma_0 C^{-1} - C^{-1}) \rho_v + \bar{\rho}_v (C^{-1} B T^{-1}) R_v + \bar{R}_v (T^{-1} \gamma_0 B^\dagger \gamma_0 C^{-1}) \rho_v + \bar{R}_v T^{-1} R_v \right\}. \quad (2.15)$$

At this point, some remarks are in order. First, physical matrix elements are always obtained by differentiating the generating functional with respect to the sources $\eta$ and $\bar{\eta}$. The separation into the velocity eigenstates is given by the projection operators as defined above. As shown in ref.[10], the chiral dimension of the 'heavy' source $\rho_v \sim P_v^- \eta$ is larger by one order than the chiral dimension of the 'light' source, $R_v \sim P_v^+ \eta$. Based on that observation, we can determine the chiral dimension with which the various terms in Eq.(2.13) start to contribute. Consider first the term in the last line. It is proportional to the propagator of the 'light' fields and thus starts at order $q^{-1}$. Interactions and loops related to this term start at $O(q)$ and $O(q^3)$, respectively. Consequently, to order $q^3$, only this last term in Eq.(2.13) generates the Green functions related to the 'light' fields. The second line in Eq.(2.15) starts at two orders higher compared to the term just discussed. It thus affects tree graphs at order $q^3$ and $q^4$ and loops only at $O(q^5)$, which is beyond the accuracy of our calculation. The terms in the first line in Eq.(2.13) contribute only at three chiral orders higher than the leading term and thus lead to wave–function renormalization at $O(q^4)$. We stress again that the operator $C^{-1}$ is related to the opposite–velocity–nucleon propagator. While the propagator of the nucleon moving in the direction of $v$ does not contain the mass any more, the anti–velocity nucleon propagator picks up exactly the factor $2m$ which is nothing but the gap between the two sectors.
It is straightforward to construct the dimension two effective chiral effective Lagrangian from this action. We use the definitions of [5] but introduce some more compact notation for the later use. The dimension one and two Lagrangians thus take the form:

\[
\mathcal{L}_{\pi N}^{(1)} = \bar{u} \left\{ iu \cdot \nabla + g_A S \cdot u \right\} H_v ,
\]

\[
\mathcal{L}_{\pi N}^{(2)} = \bar{u} \left\{ \Theta_{\mu}^\mu v + \dot{\Theta}_0 v^\mu S^\nu \left\{ \nabla_\nu , u_\mu \right\} + \Theta_{1,5} \langle \chi_+ \rangle + \Theta_5 \chi_+ + \left( \Theta_{2,3}^\mu + \Theta_4 \left[ S^\mu , S^\nu \right] \right) u_\mu u_\nu + \Theta_6 \left[ S^\mu , S^\nu \right] F^+_{\mu \nu} + \Theta_7 \left[ S^\mu , S^\nu \right] \langle F^+_{\mu \nu} \rangle \right\} H_v ,
\]

(2.16)

with

\[
\Theta_{\mu}^\mu = \frac{1}{2m} (v^\mu v^\nu - v^2 g^\mu\nu) , \quad \dot{\Theta}_0 = -\frac{i g_A}{2m} , \quad \Theta_{1,5} = \left( c_4 - \frac{c_5}{2} \right) , \quad \Theta_4 = \left( c_4 + \frac{1}{4m} \right) , \quad \Theta_5 = c_5 , \quad \Theta_6 = -\frac{1}{4m} (1 + c_6) , \quad \Theta_7 = -\frac{1}{4m} c_7 .
\]

(2.17)

and \( S^\mu \) is the covariant spin–operator à la Pauli–Lubanski, \( S^\mu = \frac{i}{2} \gamma_5 \sigma^\mu\nu v_\nu \) subject to the constraint \( S \cdot v = 0 \). Traces in flavor space are denoted by \( \langle ... \rangle \). Notice that the spin–matrices appearing in the operators have all to be taken in the appropriate order. We also have \( F^\pm_{\mu \nu} = uF^L_{\mu \nu} u^\dagger \pm u^\dagger F^R_{\mu \nu} u \), with \( F^L_{\mu \nu} , F^R_{\mu \nu} \) the field strength tensors related to \( l_\mu \) and \( r_\mu \), respectively. The explicit symmetry breaking is encoded in the matrices \( \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi u^\dagger \).

3 Generating functional to one loop

In this section, we turn to the calculation of \( S_{\pi N}^{(1)} + Z_{\pi N}^{(1)} \) to one loop beyond leading order, i.e. to order \( q^4 \) in the small momentum expansion. The method has been exposed in some detail by Ecker [2] for SU(2) to \( O(q^3) \) and for SU(3) in more detail to the same order in [3]. Here, we just outline the pertinent steps following essentially the method used in ref. [2] and discuss the additional terms appearing to the order we are working. To be specific, one has to expand

\[
\mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{\pi N}^{(4)} - R_v \left[ T_{(1)} + T_{(2)} \right]^{-1} R_v
\]

(3.1)

in the functional integral Eq.(2.14) around the classical solution, \( u_{cl} = u_{cl}^{(1)} \). This leads to a set of reducible and irreducible one–loop diagrams to be discussed below. We chose the fluctuation variables \( \xi \) in a symmetric form, \( \xi_R = u_{cl} \exp \{ i \xi / 2 \} , \xi_L = u_{cl}^\dagger \exp \{ -i \xi / 2 \} \), with \( \xi_{cl} \) traceless \( 2 \times 2 \) matrices. Consequently, we have also \( U = u_{cl} \exp \{ i \xi / 2 \} u_{cl} \). To second order in \( \xi \), the covariant derivative \( \nabla_\mu \), the chiral connection \( \Gamma_\mu \) and the axial–vector \( u_\mu \) take the form

\[
\Gamma_\mu = \Gamma_{\mu}^{cl} + \frac{1}{4} \left[ u_{cl}^{cl} , \xi \right] + \frac{1}{8} \xi \nabla_{\mu}^{cl} \xi + \mathcal{O}(\xi^3)
\]

\[
\nabla_{\mu}^{cl} \xi = \partial_{\mu} \xi + \Gamma_{\mu}^{cl} \xi, \quad \xi \nabla_{\mu}^{cl} \xi = \xi \left[ \nabla_{\mu}^{cl} , \xi \right] - \left[ \nabla_{\mu}^{cl} , \xi \right] \xi
\]

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\[ u_\mu = v^\text{cl}_\mu - [\nabla^\text{cl}_\mu, \xi] + \frac{1}{8} [\xi, [u^\text{cl}_\mu, \xi]] + \mathcal{O}(\xi^3), \]
\[ \chi^\pm = \chi^\text{cl}_\pm - \frac{i}{2} \{\chi^\text{cl}_\pm, \xi\} - \frac{1}{8} \{\xi, \{\chi^\text{cl}_\pm, \xi\}\} + \mathcal{O}(\xi^3), \]
\[ F^\pm_{\mu\nu} = F^\pm_{\mu\nu} - \frac{i}{2} [F^\pm_{\mu\nu}, \xi] + \frac{1}{8} [\xi, [F^\pm_{\mu\nu}, \xi]] + \mathcal{O}(\xi^3). \quad (3.2) \]

Notice that while \( \nabla^\text{cl}_\mu \) defined here acts on the fluctuation variables (fields) \( \xi(x) \), the covariant derivative \( \nabla_\mu \) defined in Eq. (2.4) acts on the baryon fields. We are now in the position to expand the fermion propagator \( S \) to quadratic order in the fluctuations making use of the relation

\[ [A^{(1)} + A^{(2)}] \cdot S = 1. \quad (3.3) \]

To the order we are working, the expansion of the propagator around the classical solution \( S^\text{cl} \) takes the form (there is at most one insertion from the dimension two \( \pi N \) Lagrangian)

\[ S^\text{cl} = S^\text{cl}_{(1)} - S^\text{cl}_{(1)} A^{(2)} S^\text{cl}_{(1)}, \quad (3.4) \]

with \( S^\text{cl}_{(1)} \) the full lowest order classical fermion propagator, i.e. with all possible tree structures of the external sources attached, and we expand the matrix valued operators \( A^{(1)} \) and \( A^{(2)} \) around their respective classical solutions,

\[ A^{(i)} = A^\text{cl}_{(i)} + A^1_{(i)} + A^2_{(i)}, \quad i = 1, 2, \quad (3.5) \]

where the terms are of order \( \xi^0,1,2 \), in order. We have tentatively assumed the existence of the inverse of the free fermion propagator. Consequently, the fermion propagator to order \( \xi^2 \) reads

\[ A^{-1} = \left( A^{-1}\right)^{\text{cl}}_{(1)} - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^\text{cl}_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} + \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} + \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} \]

\[ - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} \]

\[ - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} + \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} \]

\[ - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} + \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} \]

\[ - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} + \left( A^{-1}\right)^{\text{cl}}_{(1)} A^1_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} \]

\[ - \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(1)} \left( A^{-1}\right)^{\text{cl}}_{(1)} A^2_{(2)} \left( A^{-1}\right)^{\text{cl}}_{(1)} + \mathcal{O}(\xi^3, \xi^4). \quad (3.6) \]

The expanded fermion propagator in Eq. (3.6) consists of irreducible and reducible parts as depicted in Fig. 1, which we now discuss briefly. The first line of Eq. (3.6) corresponds to the terms of order \( \xi^0 \). The second and third line, corresponding to the second row in Fig. 1, comprise the terms of order \( \xi \), which vanish by use of the equations of motion. For this to happen, one must chose a
consistent parametrization of the fields in the pion and the pion–nucleon sector and account for the pertinent tadpole graphs with exactly one insertion from $\mathcal{L}_{\pi\pi}^{(4)}$. The fourth line (third row in Fig. 1) corresponds to the irreducible self–energy and tadpole diagrams first worked out by Ecker. In the next three lines, corresponding to the fourth row in Fig. 1, the reducible graphs at $O(q^{4})$ are shown. As stated before, these graphs together with similar ones from the meson sector can be made to vanish for a consistent set of field definitions. In the lowest row of the figure, the novel irreducible self–energy and tadpole diagrams are shown. The eight line in Eq.(3.6) is the novel self–energy diagram with three propagators, i.e. the insertion from $\mathcal{L}_{\pi N}^{(2)}$ on the nucleon line while the pion is in the air. The next two graphs are vertex corrections to the order $q^{3}$ self–energy graph and the last diagram, corresponding to the last line in Eq.(3.6). All the diagrams starting with the third row in Fig. 1 are of $O(\xi^{2})$. The corresponding generating functional reads

$$Z_{\text{int}}[j, R_{\pi}] = \int d^{4}x' d^{4}y' d^{4}y \, \bar{\psi}_{\pi}(x) \frac{\mathcal{S}_{(1)}^{cl}(y, y') \delta(y - y')}{2 F^{2}}$$

in terms of the self–energy functionals $\Sigma_{1,2,3}$. $\Sigma_{1}$ refers to the self-energy graphs at order $q^{3}$ and the same diagram with one dimension two insertion on the nucleon line. $\Sigma_{2}$ collects the tadpoles at orders $q^{3}$ and $q^{4}$ and $\Sigma_{3}$ refers to the dimension two vertex corrected self–energy diagrams. Consider first $\Sigma_{1}^{(1)}$ and $\Sigma_{3}^{(1)}$. These take the form (we only display the one for $\Sigma_{1}^{(1)}$)

$$\Sigma_{1}^{(1)} = \frac{-2}{F^{2}} V_{i} G_{ij} \frac{1}{[A_{(1)}^{cl}]}^{-1} V_{j} = \frac{-2}{F^{2}} V_{i} G_{ij} S_{(1)}^{cl} V_{j} ,$$

with vertex functions of dimension one

$$V_{i(1)} = V_{i(1)}^{(1)} + V_{i(1)}^{(2)} ,$$

$$V_{i(1)}^{(1)} = \frac{i}{4\sqrt{2}} \left[ v \cdot u^{cl}, \tau_{i} \right] , \quad V_{i(1)}^{(2)} = -\frac{g_{A}}{\sqrt{2}} \tau_{j} S \cdot d_{ji}^{cl} ,$$

with $i, j, k = 1, 2, 3$ and the $\tau^{i}$ denote the conventional Pauli (isospin) matrices. Similarly, the vertex–corrected self–energy contribution $\Sigma_{3}^{(2)}$ is given by the same form as in Eq.(3.8) with exactly one of the following dimension two vertices

$$V_{i(2)} = \sum_{k=1}^{11} V_{i(2)}^{(k)} ,$$

$$V_{i(2)}^{(1)} = \frac{\Theta_{0}^{\mu\nu}}{2\sqrt{2}} \left[ [\nabla_{\mu}^{cl}, u_{\nu}^{cl}], \tau_{i} \right] , \quad V_{i(2)}^{(2)} = \frac{\Theta_{0}^{\mu\nu}}{2\sqrt{2}} \left[ u_{\mu}^{cl}, \tau_{j} \right] d_{ji}^{cl} ,$$

$$V_{i(2)}^{(3)} = \frac{\Theta_{0}^{\mu\nu}}{2\sqrt{2}} \left\{ [u_{\mu}^{cl}, \tau_{i}] \nabla_{\mu}^{cl} + [u_{\mu}^{cl}, \tau_{j}] \nabla_{\nu}^{cl} \right\} ,$$

$$V_{i(2)}^{(4)} = \frac{\Theta_{0}}{2\sqrt{2}} \left\{ u_{\mu}^{cl}, [u_{\mu}^{cl}, \tau_{i}] \right\} v_{\nu}^{cl} S^{\nu} , \quad V_{i(2)}^{(5)} = \frac{-2\Theta_{0}}{\sqrt{2}} \tau^{j} v \cdot d_{ji}^{cl} S \cdot \nabla^{cl} ,$$

Note that for the $V_{i}$ the chiral dimension is again given as subscript ‘(i)’.
\[
V^{(6)}_{i(2)} = -\frac{\tilde{\Theta}_0}{\sqrt{2}} \tau^k S \cdot d_{kj}^c v \cdot d_{ji}^c \quad V^{(7)}_{i(2)} = -\frac{\Theta_{i,3}^{\mu \nu}}{\sqrt{2}} \{u_{\mu}^c, \tau^j\} d_{ji}^\nu,
\]
\[
V^{(8)}_{i(2)} = -\frac{\Theta_4}{\sqrt{2}} [i_{\mu}^c, \tau^j d_{ji}^c] [S^\mu, S^\nu], \quad V^{(9)}_{i(2)} = -\frac{i\Theta_{1,5}}{2\sqrt{2}} \{\chi_-, \tau^j\},
\]
\[
V^{(10)}_{i(2)} = -\frac{i\Theta_5}{2\sqrt{2}} \{\chi_-, \tau^j\}, \quad V^{(11)}_{i(2)} = -\frac{i\Theta_6}{2\sqrt{2}} [F_{\mu\nu}^-, \tau^j] [S^\mu, S^\nu].
\]

(3.10)

Notice that there is no contribution \(\sim \Theta_7\) because \(\langle F_{\mu\nu}^-, \tau^j\rangle = 0\). The new contribution at order \(q^4\) with exactly one dimension two insertion on the intermediate nucleon line, \(\Sigma^{(2)}_1\), has the form
\[
\Sigma^{(2)}_1 = \frac{2}{F^2} V_i G_{ij} \{ [A_{(1)}^{-1}] [A_{(2)}^{-1}] [A_{(1)}^{-1}] \} V_j
\]

(3.11)

with the fermion propagator \(A^{-1}\) properly expanded around its classical solution.

The tadpole contributions \(\Sigma^{(1),(2)}_2\) read (from here on, we drop the superscript 'cl')

\[
\Sigma^{(1)}_2 = \frac{1}{8F^2} \left\{ g_A \left( \tau_i, [S \cdot u, \tau_j] \right) G_{ij} + i\tau_i [G_{ij} v \cdot \bar{d}_{jk} - v \cdot d_{ij} G_{jk}] \tau_k \right\},
\]
\[
\Sigma^{(2)}_2 = \frac{1}{F^2} \sum_{k=1}^{15} \Sigma^{(2,k)}_2,
\]
\[
\Sigma^{(2,1)}_2 = -\frac{\Theta^\mu_0}{8} \left\{ \tau_i [d_{ij}^c G_{jk} - G_{ij} \tilde{d}_{jk}^\mu] \tau_k \right\} \nabla \nu + (\nu \leftrightarrow \nu), \quad \Sigma^{(2,2)}_2 = \frac{\Theta^\mu_0}{16} \{ u_{\mu, \tau_i} [u_{\nu, \tau_j}] G_{ij} \},
\]
\[
\Sigma^{(2,3)}_2 = -\frac{\Theta^\mu_0}{8} \left\{ \tau_i [d_{ij}^\mu d_{jl} G_{ik} - G_{ij} \tilde{d}_{jl}^\mu d_{ik} + d_{ij}^\nu G_{jl} \tilde{d}_{ik}^\mu - d_{ij}^\nu G_{jl} \tilde{d}_{ik}^\nu] \tau_k \right\},
\]
\[
\Sigma^{(2,4)}_2 = -\frac{\tilde{\Theta}_0}{8} \left\{ v \cdot u, \tau_i [d_{ij}^\nu G_{jk} - G_{ij} \tilde{d}_{jk}^\nu] \tau_k \right\} S^\nu,
\]
\[
\Sigma^{(2,5)}_2 = -\frac{\tilde{\Theta}_0}{4} \left\{ \tau_i d_{ij}^\mu G_{jk}[u_{\nu, \tau_k}] + [u_{\nu, \tau_i}] G_{ij} \tilde{d}_{jk}^\mu \tau_k \right\} S^\nu v^\mu,
\]
\[
\Sigma^{(2,6)}_2 = \frac{\tilde{\Theta}_0}{4} S^\nu \left\{ \tau_i, [v \cdot u, \tau_j] \right\} G_{ij} \nabla \nu,
\]
\[
\Sigma^{(2,7)}_2 = \frac{\tilde{\Theta}_0}{8} S^\nu \left\{ G_{ij} \tilde{d}_{jk}^\nu \left[ \tau_i, [v \cdot u, \tau_j] \right] + [\tau_i, [v \cdot u, \tau_j]] d_{jk}^\nu G_{ki} \right\},
\]
\[
\Sigma^{(2,8)}_2 = \frac{\tilde{\Theta}_0}{8} S^\nu \left\{ \tau_i, [[\nabla \nu, v \cdot u], \tau_j] \right\} G_{ij},
\]
\[
\Sigma^{(2,9)}_2 = \frac{\Theta^\mu_0}{8} \left\{ u_{\mu, \tau_i} [u_{\nu, \tau_j}] \right\} G_{ij}, \quad \Sigma^{(2,10)}_2 = \frac{\Theta^\mu_0}{2} \left\{ \tau_i d_{ij}^\mu G_{jl} \tilde{d}_{ik}^\mu \tau_k \right\},
\]
\[
\Sigma^{(2,11)}_2 = \frac{\tilde{\Theta}_1}{8} [S^\mu, S^\nu][u_{\mu, \tau_i} [u_{\nu, \tau_j}]] G_{ij}, \quad \Sigma^{(2,12)}_2 = \frac{\tilde{\Theta}_1}{2} [S^\mu, S^\nu] \left\{ \tau_i d_{ij}^\mu G_{jl} \tilde{d}_{ik}^\mu \tau_k - (\mu \leftrightarrow \nu) \right\},
\]
\[
\Sigma^{(2,13)}_2 = -\frac{\tilde{\Theta}_1}{8} \left\{ \{\tau_i, \{\chi_+, \tau_j\}\} \right\} G_{ij}, \quad \Sigma^{(2,14)}_2 = -\frac{\tilde{\Theta}_1}{8} \left\{ \tau_i, \{\chi_+, \tau_j\} \right\} G_{ij},
\]
\[
\Sigma^{(2,15)}_2 = \frac{\tilde{\Theta}_1}{8} [S^\mu, S^\nu][\tau_i, [F_{\mu\nu}^+, \tau_j]] G_{ij}.
\]

(3.13)
In these equations, \( G_{ij} \) is the full meson propagator \[12\]

\[ G_{ij} = (d_{ij} d^{ij} + \sigma^{ij})^{-1} \]  

(3.14)

with

\[
\nabla_{\text{cl}}^\mu \xi = \frac{1}{\sqrt{2}} \gamma_{ij} d_{jk}^\mu \xi_k, \quad \xi = \frac{1}{\sqrt{2}} \Gamma_{\text{cl}}^\mu \tau_i \xi_i, \quad d_{ij}^\mu = \delta_{ij} \partial^\mu + \gamma_{ij}^\mu, \quad \gamma_{ij}^\mu = \frac{1}{2} \left( \Gamma_{\text{cl}}^\mu \Gamma^\iota \right) + \sigma^{ij}, \quad \sigma^{ij} = \frac{1}{8} \left( \left[ a_{\mu}^i, \tau_i \right] \left[ a_{\mu}^j, \tau_j \right] + \chi + \left\{ \tau_i, \tau_j \right\} \right). 
\]

(3.15)

Note that the differential operator \( d_{ij} \) is related to the covariant derivative \( \nabla_{\text{cl}}^\mu \) and it acts on the meson propagator \( G_{ij} \). The connection \( \gamma_{\mu} \) defines a field strength tensor,

\[
\gamma_{\mu\nu} = \partial_\nu \gamma_\mu - \partial_\mu \gamma_\nu + [\gamma_\mu, \gamma_\nu], \quad [d_\lambda, \gamma_{\mu\nu}] = \partial_\lambda \gamma_{\mu\nu} + [\gamma_\lambda, \gamma_{\mu\nu}],
\]

(3.16)

where we have omitted the flavor indices.

### 4 Renormalization of the tadpole graph

In this section, we consider the renormalization of the tadpole contribution \( \Sigma^{(1)}_{2} + (y, y') \). This is done in Euclidean space letting \( x^0 \rightarrow -i x^0, v^0 \rightarrow -i v^0, v \cdot \partial \rightarrow v \cdot \partial, S^0 \rightarrow i S^0 \) and \( S \cdot u \rightarrow -S \cdot u \). We remark that the sense of the Wick rotation is not uniquely defined, one only has to perform it consistently. In the coincidence limit \( y \rightarrow y' \), the functionals \( \Sigma^{(1)}_{2} + (y, y') \) are divergent. The divergences can be extracted by using standard heat kernel techniques since they appear as simple poles in \( d = 4 - \varepsilon \). The corresponding residua are local polynomials in the fields of \( \mathcal{O}(q^2) \) and \( \mathcal{O}(q^4) \) for \( \Sigma^{(1)}_{2} \) and \( \Sigma^{(2)}_{2} \), respectively. They can easily be transformed back to Minkowski space. The tadpole graphs are proportional to the meson propagator \( G_{ij} \), which is an elliptic second–order differential operator of the type \( A(x) = -d_{\mu} d_{\mu} + \sigma(x) \), and derivatives thereof. The methods to treat such type of operators are spelled out in detail in \[3\]. Here, we only add some additional remarks. The singular function related to the meson propagator in the coincidence limit takes the form \[14\] \[3\]

\[ G(x, x) = \frac{2}{\varepsilon} a_1(x, x) + \text{finite}, \]  

(4.1)

i.e. it singularities are given by the Seeley–deWitt coefficient \( a_1 \). For operators with one derivative acting on the meson propagator, one gets

\[ d_{\mu} G(x, y)|_{x \rightarrow y} = \frac{2}{\varepsilon} a_1(x, y)|_{x \rightarrow y} + \text{finite}, \]  

(4.2)

whereas in the case of two derivatives, an extra \( \delta \)–function appears,

\[ d_{\mu} d_{\nu} G(x, y)|_{x \rightarrow y} = \frac{2}{\varepsilon} \delta_{\mu\nu} (-2) a_2(x, y)|_{x \rightarrow y} + \frac{2}{\varepsilon} d_{\mu} a_1(x, y)|_{x \rightarrow y} + \text{finite}. \]  

(4.3)

These are the basic structures we have to deal with. Notice that we need the pertinent Seeley–deWitt coefficients and their derivatives (in the coincidence limit) to higher order than given in \[3\].
We remark that the coefficient $a_n(x,x)$ has chiral dimension $2n$. In appendix [A], the necessary formulas are collected. After some algebra, the divergent part of the tadpoles can be cast in the form (rotated back to Minkowski space)

$$\Sigma_{2}^{(1)+(2),\text{div}}(y,y) = \frac{1}{(4\pi F)^2} \frac{2}{\xi} \left[ \hat{\Sigma}_2^{(1)}(y,y) + \sum_{k=1}^{15} \hat{\Sigma}_2^{(2,k)}(y,y) \right], \quad (4.4)$$

with $\hat{\Sigma}_2^{(1),(2)}(y,y)$ finite monomials in the fields of chiral dimension three and four, in order. The explicit form of $\hat{\Sigma}_2^{(1)}(y,y)$ is given in [2]. For the new contributions at order $q^4$, we get

$$\hat{\Sigma}_2^{(2,1)} = \frac{\Theta_0^{\mu\nu}}{6} \left[ [\nabla^\tau, \Gamma_{\mu\tau}] \nabla_{\nu} + (\mu \leftrightarrow \nu) \right], \quad \hat{\Sigma}_2^{(2,2)} = -\frac{\Theta_0^{\mu\nu}}{32} \langle u_\mu J(u_\nu) \rangle \quad \hat{\Sigma}_2^{(2,3)} = \frac{\Theta_0^{\mu\nu}}{6} [\nabla^\gamma, [\nabla_{\mu}, \Gamma_{\nu\gamma}]]$$

$$\hat{\Sigma}_2^{(2,4)} = \frac{\Theta_0}{6} \{ v \cdot u, [\nabla^\tau, S^\nu \Gamma_{\nu\tau}] \}, \quad \hat{\Sigma}_2^{(2,5)} = -\frac{\Theta_0 v^\mu}{4} S^\nu \left[ \frac{2}{3} \langle u_\nu [\nabla_{\tau}, \Gamma_{\mu\tau}] \rangle + \frac{1}{2} [\eta_{\mu}(1), u_\nu] \right],$$

$$\hat{\Sigma}_2^{(2,6)} = -\frac{\Theta_0}{8} J(v \cdot u) S \cdot \nabla, \quad \hat{\Sigma}_2^{(2,8)} = -\frac{\Theta_0}{8} J([S \cdot \nabla, v \cdot u]), \quad \hat{\Sigma}_2^{(2,9)} = -\frac{\Theta_0}{2} \{ u_\mu, J(u_\nu) \},$$

$$\hat{\Sigma}_2^{(2,7)} = -\frac{\Theta_0}{8} \left[ \langle v \cdot u [S \cdot \nabla, (u^2 + \chi_+)] \rangle - v \cdot u \langle [S \cdot \nabla, (u^2 + \chi_+)] \rangle \right] - \langle v \cdot u [S \cdot \nabla, (u^2 + \chi_+)] \rangle$$

$$- 2[S \cdot \nabla, u_\tau] \langle v \cdot uu^\tau \rangle - 2u_\tau \langle v \cdot uu^\tau \rangle \bigg]$$

$$\hat{\Sigma}_2^{(2,10)} = \frac{\Theta_0^{\mu\nu}}{2} \left[ -\frac{1}{6} \eta_{\mu\nu}(1) + \frac{2}{3} \langle \Gamma_{\mu\tau}, \Gamma^\nu_\tau \rangle \right],$$

$$\hat{\Sigma}_2^{(2,11)} = -\frac{\Theta_0}{8} [S^\mu, S^\nu] \{ u_\mu, J(u_\nu) \},$$

$$\hat{\Sigma}_2^{(2,12)} = -\Theta_0 [S^\mu, S^\nu] \left[ -\frac{1}{2} \eta(\Gamma_{\mu\nu}) + \frac{1}{4} \langle \eta(1), \Gamma_{\mu\nu} \rangle + \frac{1}{3} [\nabla^\gamma, [\nabla_{\mu}, \Gamma_{\nu\gamma}]] - \frac{2}{3} [\Gamma_{\mu\gamma}, \Gamma_{\nu\tau}] \right],$$

$$\hat{\Sigma}_2^{(2,13)} = \frac{\Theta_0}{8} \langle \mathcal{C}(\chi_+) \rangle, \quad \hat{\Sigma}_2^{(2,14)} = \frac{\Theta_0}{8} \mathcal{C}(\chi_+), \quad \hat{\Sigma}_2^{(2,15)} = -\frac{\Theta_0}{8} [S^\mu, S^\nu] J(F^\mu_{\nu}),$$

where we used the abbreviations

$$\eta(X) = \frac{1}{2} \langle X + u^2 \rangle \langle X \rangle + \frac{1}{2} \langle X (X + u^2) \rangle + \frac{1}{4} \langle X (X + 1) \rangle - \frac{1}{2} \langle X, X \rangle - u_\tau \langle u_\tau X \rangle,$$

$$\eta^\mu(X) = \frac{1}{2} \langle \nabla^\mu, (u^2 + \chi_+) \rangle \langle X \rangle + \frac{1}{2} \langle X [\nabla^\mu, (u^2 + \chi_+)] \rangle + \frac{1}{4} \langle X [\nabla^\mu, \chi_+] \rangle$$

$$- \frac{1}{2} \langle X, [\nabla^\mu, \chi_+] \rangle - [\nabla^\mu, u_\tau] \langle u_\tau X \rangle - u_\tau \langle X [\nabla^\mu, u^\tau] \rangle,$$

so that

$$\eta(1) = \frac{1}{2} \langle u^2 \rangle + u^2 + \frac{3}{4} \langle X_+ \rangle,$$

$$\eta^\mu(1) = [\nabla^\mu, u^2] + \frac{1}{2} \langle [\nabla^\mu, u^2] \rangle + \frac{3}{4} \langle [\nabla^\mu, \chi_+] \rangle,$$

$$\eta^{\mu\nu}(1) = \frac{1}{2} \langle [\nabla^\mu, [\nabla^\nu, u^2]] \rangle + [\nabla^\mu, [\nabla^\nu, u^2]] + \frac{3}{4} \langle [\nabla^\mu, [\nabla^\nu, \chi_+]] \rangle,$$

(4.7)

and

$$\mathcal{J}(X) = -\langle (u^2 + \chi_+)X \rangle + \langle (u^2 + \chi_+)X \rangle - X \langle u^2 + \chi_+ \rangle - 2u_\mu \langle u^\mu X \rangle + \langle u^2 + \chi_+ \rangle \langle X \rangle.$$
for any $2 \times 2$ matrix $X$, and
\begin{equation}
\mathcal{C}(\chi_+) = u^2 \langle \chi_+ \rangle + \langle (u^2 + \chi_+) \chi_+ \rangle + 3\chi_+ \langle u^2 \rangle + \{\chi_+, u^2\} - 2\chi_+ \chi_+ - 2u_\mu \langle w^\mu \chi_+ \rangle.
\end{equation}
Furthermore, we set $u^2 = u_\mu u^\mu$.

5 Renormalization of the self–energy graphs

This section is split into three paragraphs. In the first one, we renormalize the self–energy graph at order $q^3$ together with the two vertex–corrected self–energy diagrams at order $q^4$. This can be done straightforwardly by the method proposed by Ecker [2]. In the next paragraph, we extend the heat kernel methods to deal with operators which are orthogonal to the direction given by the four–velocity of the heavy baryon. Such type of operators start to appear at fourth order, specifically in the self–energy graph with a dimension two insertion on the internal fermion line. We then consider this particular diagram, which we call the “eye graph” from here on. Formally, such a graph involves a triple coincidence limit. The eye graph has thus to be worked out making use of the novel technique to evaluate $n$–fold coincidences which is presented and discussed in the third paragraph of this section. In addition, we show in appendix [3] that as long as one works to $O(q^4)$, one can still use the method developed by Ecker to extract the singularities based on the heat kernel expansion for part of the eye graph contribution. Therefore, part of the divergent structure of the eye graph can be evaluated by two different methods, which serves as an excellent check on the rather involved algebra.

5.1 $O(q^3)$ and vertex corrected self–energy graphs at $O(q^4)$

In this paragraph, we consider the renormalization of the self–energy contributions $\Sigma^{(1)}_1(y, y)$ and $\Sigma^{(2)}_3(y, y)$. The pertinent divergences are due to the singular behavior of the product of the meson and the baryon propagators $G_{ij}(x, y) [A^{(1)}]^{-1}(x, y)$ in the coincidence limit $x \rightarrow y$. This expression is directly proportional to the full classical fermion propagator $S_{cl}^{(1)}$. However, this differential operator is not elliptic and thus not directly amenable to the standard heat–kernel expansion. Consider therefore the object [3]
\begin{align}
S^{(1)} = i D_{(1)}^\dagger \left[ D_{(1)} D_{(1)}^\dagger \right]^{-1}, \quad D_{(1)} & \equiv i A_{(1)}. \tag{5.1}
\end{align}

In fact, the operator in the square brackets in Eq.(5.1) is positive definite and hermitian. Furthermore, it is a one–dimensional operator in the direction of $v$ and we can use the heat kernel methods for such type of operators invented by Ecker [2] and discussed in detail in [3]. For completeness, we give the basic definitions. Consider an differential operator of the type [7–10]
\begin{align}
\Delta = -(v \cdot d)^2 + a(x) + \mu^2, \quad d_\mu = \partial_\mu + \gamma_\mu \tag{5.2}
\end{align}

Note that in this section and in what follows, the symbol “$d$” is sometimes used for the derivative acting on the nucleon fields instead of “$\nabla$”.

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\[ a(x) = -g_A^2(S \cdot u)^2 + g_A [iv \cdot \nabla, S \cdot u]. \]  
(5.3)

The heat kernel \( J(t) = \exp\{-\Delta t\} = J_0 K \) can be split again into its free part,
\[ J_0(t) = g(x, y) \frac{1}{\sqrt{4\pi t}} \exp\left\{-\mu^2 t - \frac{[v \cdot (x - y)]^2}{4t}\right\}, \]
(5.4)

and the interaction part \( K \), which satisfies the equation,
\[ \left[ \frac{\partial}{\partial t} - (v \cdot d)^2 + a(x) + \frac{1}{t} v \cdot (x - y) v \cdot d \right] K(x, y, t) = 0 , \]
(5.5)

using \( v^4 = 1 \). The function \( g(x, y) \) is introduced so that one can fulfill the boundary condition in the coordinate–space representation,
\[ g(x) = \int \frac{d^d p}{(2\pi)^d} \delta(k \cdot v) e^{-ipx}, \]
(5.6)

where the explicit form of the function \( g(x) \) is not needed to derive the recurrence relations for the heat kernel, but we demand that \( \partial g(x, y) \) is orthogonal to the direction of \( v \),
\[ v \cdot \partial g(x, y) = 0 . \]
(5.7)

Later when products of singular operators are constructed (see appendix B) we make use of this special form in Eq.(5.6). It is important to stress the difference to the standard case. Because \( v \cdot d \) is a scalar, one has essentially reduced the problem to a one–dimensional one, i.e. \( v \cdot d \) is a one–dimensional differential operator in the direction of \( v \). Using the heat kernel expansion for \( K \),
\[ K(x, y, t) = \sum_{n=0}^{\infty} k_n(x, y) t^n , \]
(5.8)

where \( t \) is the proper time, we derive the Seeley–DeWitt coefficients and their derivatives in the coincidence limit (denoted by the “\( \mid \)”)
\[
k_0 \mid = 1 , \quad (v \cdot d)^m k_0 \mid = 0 , \quad (v \cdot d)^m k_0 (v \cdot d)^n \mid = 0 , \quad k_1 \mid = -a , \\
(v \cdot d) k_1 \mid = -\frac{1}{2} [v \cdot d, a] , \quad (v \cdot d)^2 k_1 \mid = -\frac{1}{3} [v \cdot d, [v \cdot d, a]] , \\
k_2 \mid = \frac{1}{2} a^2 - \frac{1}{6} [v \cdot d, [v \cdot d, a]] , \quad k_1 v \cdot d \mid = v \cdot d k_1 \mid. \]
(5.9)

The propagator is given as the integral in terms of the pertinent Seeley–DeWitt coefficients,
\[ J(x, y) = \Delta^{-1}(x, y) = \int_0^\infty dt \ J(x, y, t) \]
\[ J_n(x, y) = \sum_{n=0}^{\infty} J_n(x, y) k_n(x, y) \]
\[ J_n(x, y) = g(x, y) \int_0^\infty dt \ \frac{\exp\left\{-\mu^2 t - \frac{[v \cdot (x - y)]^2}{4t}\right\}}{\sqrt{4\pi t} \sqrt{4t}} \ t^n , \]
\[ = \frac{\Gamma(n+1)}{v^2} \int \frac{dk^d}{(2\pi)^d} \left[ \frac{1}{(v \cdot k)^2 + \mu^2} \right]^{n+1} e^{-ik \cdot (x - y)}, \]
(5.10)
where in the last line we have made use of the rest–frame representation of \( g(x, y) \),
\[ g(x, y) = \delta^{d-1}(x - y) \] for \( v = (1, 0, \ldots, 0) \). The physical interpretation of this result is rather simple. It is the Fourier transform of \( n + 1 \) massive propagators since we have \( n \) insertions and the \( \Gamma \)–function simply takes care about the combinatorics. Since the particle propagator \( A_{(1)} \) is massless, one must keep \( \mu^2 \neq 0 \) in intermediate steps to get a well defined heat kernel representation without infrared singularities. In case of the vertex–corrected self–energy \( \Sigma_{3}^{(2)} \), one has exactly one of the dimension two vertices collected in Eq.\((3.10)\). For more details, in particular the recurrence relations between the Seeley–DeWitt coefficients and so on, we refer the reader to refs.\([2,3]\). We add the important remark that the coefficients \( k_m \) have chiral dimension \( 2m \). After the same type of algebra spelled out in section 6 of \( [3] \), one finds (notice that in intermediate steps covariance is destroyed by this method but a particular recombination of the terms allows to restore it)

\[
\Sigma_{1}^{(1), \text{div}}(y, y) = \frac{1}{(4\pi F)^2} \frac{2}{\epsilon} \delta^4(x - y) \sum_{i=1}^{16} \hat{\Sigma}_{1, i}^{(1)}(y) , \tag{5.11}
\]

\[
\Sigma_{3}^{(2), \text{div}}(y, y) = \frac{1}{(4\pi F)^2} \frac{2}{\epsilon} \delta^4(x - y) \sum_{i=1}^{157} \hat{\Sigma}_{3, i}^{(2)}(y) \tag{5.12}
\]

where the 16 terms corresponding to the \( \mathcal{O}(q^3) \) self–energy can be extracted from appendix C of ref.\([3]\) if one transforms the SU(3) operators given there into their SU(2) counterparts\([3]\) and the 157 new \( \mathcal{O}(q^4) \) operators and their corresponding contributions to the self–energy are collected in \([13]\).

### 5.2 Extension of the heat kernel methods

The extraction of divergences in the meson–baryon sector is based essentially on the method described in the previous section. The complete information about the interaction is hidden in the Seeley–DeWitt coefficients. Note that up to now in the meson–baryon sector we only have considered the projection of a differential operator in direction of \( v \), e.g. we have calculated operators like \( v \cdot d \). This procedure ensures the renormalization to third order as pointed out in refs.\([2,3]\). We will see that the renormalization of a part of the self–energy diagrams can be done by applying the same methods. However, to fourth order we also have to renormalize insertions on the intermediate baryon line of the form

\[
(v \cdot d)^2 - d^2 \tag{5.13}
\]

which is orthogonal to \( v \). For the complete renormalization we thus have to calculate the Seeley–DeWitt coefficients not only in the direction of \( v \). It is clear that we can not calculate

\[
d^2_{\mu}k_m(x, y) \tag{5.14}
\]

from the recurrence relation in Eq.\((5.5)\) in the coincidence limit, because this relation always projects on the direction of \( v \). A direct calculation of \( (d_{\mu}k_m(x, y))_i \) does not seem to be possible

\#11 These terms have first been worked out in \([2]\). However, since we are organizing the various contributions in a different way than Ecker, the number of terms at this intermediate stage is different.
but fortunately is not needed. The renormalization can be done by considering the sum of left and right covariant derivatives,

\[
(d^\mu k)^\mu_m(x, y) + k_m(x, y) \left|_{x=y} \right.
\]

\[
= (\partial^\mu k_m(x, y) + \Gamma^\mu_m(x) k_m(x, y) + k_m(x, y) \partial^\mu_m - k_m(x, y) \Gamma^\mu_m(y)) \left|_{x=y} \right.
\]

\[
= (d^\mu, (k_m(x, y)|_{x=y})]
\]

\[
≡ [d^\mu, k_m] .
\]

While in the first line we differentiate before taking the coincidence limit, in the last line we first perform the coincidence limit. Since we know the coincidence limit of \(k_m\), this expression is well defined as pointed out above. Using this trick, we find for example

\[
(d^\mu k_0(x, y) + k_0(x, y) \left|_{x=y} \right. = [d^\mu, k_0] = 0
\]

\[
(d^\mu k_1(x, y) + k_1(x, y) \left|_{x=y} \right. = [d^\mu, k_1] = -[d^\mu, a]
\]

\[
(d^\mu (v \cdot d k)(x, y)) + (v \cdot d k)(x, y) \left|_{x=y} \right. = [d^\mu, v \cdot d k] = -\frac{1}{2}[d^\mu, [v \cdot d, a]]
\]

This method allow us to calculate the complete eye graph as detailed in the next paragraph.

### 5.3 Eye graph at \(O(q^4)\) and treatment of \(n\)–fold coincidences

In this section, we consider the eye graph at \(O(q^4)\). It is not obvious that the method described in paragraph 5.1 can be applied to such type of diagram because if one dissects it into its lowest order (chiral dimension one) pieces, it formally involves a triple coincidence limit. We thus have develop a method to treat such terms. So far, we only dealt with the product of one meson and one baryon propagator, i.e. double coincidences to extract the pertinent short–distance singularities. Formally, one Fourier–transforms

\[
\int d^4 x \ G_m(x) J_n(x) e^{ik \cdot x} ,
\]

and obtains

\[
\frac{\Gamma(n+1)\Gamma(m+1)}{v^2} \int d^4 \ell \left[ \frac{1}{\ell^2} \right]^{m+1} \left[ \frac{1}{[(k - \ell) \cdot v] + \mu^2} \right]^{n+1} .
\]

The physical interpretation of this last formula is again very simple. For \(n\) insertions of external fields on the pion line and \(m\) insertions on the nucleon one, we get the Fourier–transform of \(n+1(m+1)\) pion (nucleon) propagators with the appropriate combinatorial factors given by the \(\Gamma–functions. For the eye graph, we have to deal with triple coincidences of the type

\[
\int d^4 x d^4 y G_1(x + y) J_m(x) J_n(y) e^{ik \cdot x} e^{iq \cdot y} .
\]
Again, in momentum–space this takes a simple form,

\[
\frac{\Gamma(l+1)\Gamma(m+1)\Gamma(n+1)}{(v^2)^2} \int \frac{d\ell^d}{(2\pi)^d} \left[ \frac{1}{|\ell|^2} \right]^{l+1} \left[ \frac{1}{[(k-\ell)\cdot v]^2 + \mu^2} \right]^{m+1} \left[ \frac{1}{[(q-\ell)\cdot v]^2 + \mu^2} \right]^{n+1}.
\]

(5.23)

The interpretation of this formula is obvious. Furthermore, it can easily be extended to n–fold coincidences which makes it useful for many other applications. It is important to note that Eq.(5.23) can be reduced to already known integrals. To be specific, introduce the abbreviations \(\omega = v \cdot k\) and \(\omega' = v \cdot q\). We can simplify the integral, which has \((m+1)\) and \((n+1)\) insertions on the two intermediate nucleon lines, with the standard trick,

\[
\left[ \frac{1}{\omega - v \cdot \ell} \right]^{2m+2} = \frac{(-)^{2m+1}}{(2m+1)!} \left( \frac{\partial}{\partial \omega} \right)^{2m+1} \frac{1}{\omega - v \cdot \ell},
\]

(5.24)

which allows us to write Eq.(5.23) in the form

\[
\frac{\Gamma(m+1)\Gamma(n+1)}{v^2} \frac{(-)^{2m+1}}{(2m+1)!} \frac{(-)^{2n+1}}{(2n+1)!} \times \frac{(\partial)^{2l+1}}{(\partial \omega)^{2m+1}} \frac{\Gamma(l+1)}{v^2} \int \frac{d\ell^d}{(2\pi)^d} \left[ \frac{1}{|\ell|^2} \right]^{l+1} \frac{1}{\omega - v \cdot \ell} \frac{1}{\omega' - v \cdot \ell}.
\]

(5.25)

The remaining loop integral, which we denote as \(I(\omega, \omega')\), has only insertions on the pion line and none any more on the internal nucleon one. It can be further simplified by partial fractions,

\[
I(\omega, \omega') = \int d^d x d^d y G_l(x+y) v \cdot \partial J_0(x) v \cdot \partial J_0(y) e^{ik \cdot x} e^{iq \cdot y}
\]

\[
= \frac{\Gamma(l+1)}{v^2} \int \frac{d\ell^d}{(2\pi)^d} \left[ \frac{1}{|\ell|^2} \right]^{l+1} \left[ \frac{1}{\omega - v \cdot \ell} - \frac{1}{\omega' - v \cdot \ell} \right] \left( \frac{1}{\omega - \omega'} \right)
\]

\[
= -\frac{I_0(\omega) - I_0(\omega')}{\omega - \omega'}
\]

(5.26)

\[
I_0(\omega) = \frac{\Gamma(l+1)}{v^2} \int \frac{d\ell^d}{(2\pi)^d} \left[ \frac{1}{|\ell|^2} \right]^{l+1} \frac{1}{\omega - v \cdot \ell} = \int d^d x G_l(x) v \cdot \partial J_0(x) e^{ik \cdot x}.
\]

The integral \(I_0(\omega)\) is, however, already known from the calculation of the leading order self–energy graph. Since \(I_0(\omega)\) is a polynom in \(\omega\), the numerator of last equation is a polynom in \(\omega - \omega'\). Consequently, the same factor appearing in the denominator can always be canceled so that \(I(\omega, \omega')\) is a rational polynom in \(\omega\) and \(\omega'\). This is the reason why the method of differentiation allows to treat such type of integrals, even in the case of arbitrary many coincidences. Let us briefly discuss this general case, i.e. a one–loop diagram with \(n\) insertions from \(L^{(k)}_{\pi N}\) \((k \geq 2)\) on the nucleon line. Denoting by \(x\) and \(x'\) the meson–nucleon interaction points and by \(x_i\) \((i = 1, \ldots, n)\) the points of the insertions, we have to deal with a structure of the type

\[
G_k(x - x') J_{m_1}(x - x_1) J_{m_2}(x_1 - x_2) \cdots J_{m_n}(x_{n-1} - x_n) J_{m_{n+1}}(x_n - x').
\]

(5.27)
Redefining \( u_1 = x - x_1, u_2 = x_1 - x_2, \ldots, u_n = x_{n-1} - x_n, u_{n+1} = x_n - x' \), this takes the form

\[
G_k(u_1 + u_2 + \ldots + u_n + u_{n+1}) J_{m_1}(u_1) J_{m_2}(u_2) \ldots J_{m_n}(u_n) J_{m_{n+1}}(u_{n+1}) ,
\]

(5.28)

and can be evaluated in momentum–space as described before, i.e. one forms

\[
\int \prod_{i=1}^{n+1} d^d u_i \ G_k \left( \sum_{i=1}^{n+1} u_i \right) \prod_{i=1}^{n+1} \left[ J_{m_i}(u_i) e^{i q_i u_i} \right] ,
\]

(5.29)

which is a rational polynom of the momenta \( q_i, P(q_1, \ldots, q_{n+1}) \). This can now be treated exactly along the same lines as shown above for the special case of a triple coincidence, i.e. with exactly one insertion from \( \mathcal{L}_{2N}^{(2)} \). The results for the pertinent products of three singular operators are collected in app. [C]. The whole eye graph can be evaluated using the method described here.

Putting pieces together, the singularities related to the eye graph can be extracted,

\[
\Sigma^{(2),\text{div}}_1(y, y) = \frac{1}{(4\pi F)^2} \frac{2}{\epsilon} \delta^4(x - y) \sum_{i=1}^{181} \hat{\Sigma}^{(2)}_{1,i}(y) ,
\]

(5.30)

with the new 181 \( \mathcal{O}(q^4) \) operators and their corresponding contributions to the self–energy are collected in appendix [D]. More precisely, these terms are already partially summed in that appendix and given are the contributions with zero, one and two covariant derivatives acting on the nucleon fields.

### 6 The counterterm Lagrangian

We are now in the position to enumerate the full counterterm Lagrangian at order \( q^4 \). The \( q^3 \) terms can be found in [2]. To bring it in a more compact form, we use the same relations as discussed in ref. [3]. To separate the finite parts in dimensional regularization, we follow the conventions of [12] to decompose the irreducible one–loop functional into a finite and a divergent part. Both depend on the scale \( \mu \):

\[
\delta^4(x - y) \left( \Sigma^{(1)}_1(x, y) + \Sigma^{(1)}_2(x, y) \right) + \delta^4(x - y) \left( \Sigma^{(2)}_1(x, y) + \Sigma^{(2)}_2(x, y) + \Sigma^{(2)}_3(x, y) \right)
\]

\[
= \delta^4(x - y) \left( \Sigma^{(1)+(2),\text{fin}}_1(x, y, \mu) + \Sigma^{(1)+(2),\text{fin}}_2(x, y, \mu) + \Sigma^{(2),\text{fin}}_3(x, y, \mu) \right)
- \frac{2L(\mu)}{F^2} \delta^4(x - y) \left[ \hat{\Sigma}^{(1)+(2)}_2(y) + \hat{\Sigma}^{(1)+(2)}_1(y) + \hat{\Sigma}^{(2)}_3(y) \right] ,
\]

(6.1)

with

\[
L(\mu) = \frac{\mu^{d-4}}{(4\pi)^2} \left( \frac{1}{d - 4} - \frac{1}{2} \left[ \log(4\pi) + 1 - \gamma \right] \right) .
\]

(6.2)

The first two terms on the left–hand–side of Eq. (5.1) are of dimension three whereas the remaining three are of dimension four. The generating functional can then be renormalized by introducing the counterterm Lagrangian

\[
\mathcal{L}^{\text{ct}}_{\pi N} = \mathcal{L}^{(3)\text{ct}}_{\pi N}(x) + \mathcal{L}^{(4)\text{ct}}_{\pi N}(x)
\]

\[
= \frac{1}{(4\pi F)^2} \sum_i b_i \tilde{N}_v(x) \tilde{O}^{(3)}_i(x) N_v(x) + \frac{1}{(4\pi F)^2} \sum_i d_i \tilde{N}_v(x) \tilde{O}^{(4)}_i(x) N_v(x)
\]

(6.3)
where the \( b_i \) and \( d_i \) are dimensionless coupling constants and the field monomials \( \tilde{O}_i^{(3)}(x) \) and \( \tilde{O}_i^{(4)}(x) \) are of order \( q^3 \) and \( q^4 \), respectively. The low-energy constants \( b_i \) and \( d_i \) are decomposed in analogy to Eq.(6.1),

\[
\begin{align*}
  b_i &= b_i^r(\mu) + (4\pi)^2 \beta_i L(\mu) , \\
  d_i &= d_i^r(\mu) + (4\pi)^2 \delta_i L(\mu) .
\end{align*}
\]

The \( \beta_i \) are dimensionless functions of \( g_A \) constructed such that they cancel the divergences of the one-loop functional to order \( q^3 \). They have been listed by Ecker [2]. The \( \delta_i \) not only depend on \( g_A \) but also on the finite dimension two LECs \( c_1,\ldots,7 \). Their numerical values have recently been determined in ref. [15] and independently in ref. [16]. There is by now yet another determination available, see ref. [17]. The \( \delta_i \) are listed in table 1 together with the corresponding operators \( \tilde{O}_i^{(4)}(x) \). For easier comparison, we also give the separate tables for the tadpole, self–energy and eye graph counterterms and respective \( \beta \)–functions in app. [F]. Note that one can make the explicit mass–dependence disappear from the \( \beta \)–functions if one makes the \( c_i \) dimensionless via

\[
c_i' = 2m c_i \quad (i = 1,\ldots,7) ,
\]

so that the \( \delta_i \) only depend on \( g_A \) and the \( c_i' \). The operators listed in table 1 constitute a complete set for the renormalization of the irreducible tadpole and self–energy functional for off-shell baryons. These are the terms where the covariant derivative acts on the nucleon fields. There are 44 terms of this type. As long as one is only interested in Green functions with on-shell nucleons, the number of terms can be reduced considerably by invoking the baryon equation of motions. In particular, all equation of motion terms of the form

\[
iv \cdot \nabla N = -g_A S \cdot u N
\]

can be eliminated by appropriate field redefinitions in complete analogy to the order \( q^3 \) calculation [11]. Also, many of the terms given in the table refer to processes with at least three Goldstone bosons. These are only relevant in multiple pion production by photons or pions off nucleons [18]. As an illuminating example we mention the elastic pion–nucleon scattering amplitude. At second and third order, one has four and five counterterms, respectively, if one also counts the deviation from the Goldberger–Treiman relation (GTR), i.e. the difference between the physical value of the pion–nucleon coupling constant and its value as given by the GTR. Although the fourth order amplitude has not yet been worked out in full detail, one can enumerate the possible new counterterms from the structure of the amplitude. This leads to only five new operators (for details, see [19]). Certainly, there are much more low energy data points for \( \pi N \) scattering. This pattern is in marked contrast to the total number of terms, which are seven [1], 31 [17] and 199 for the second, third and fourth order, respectively. Similar remarks apply to single pion production off nucleons. An analogous situation happens in the mesonic sector. While there are about 130 counterterms at sixth (two loop) order, only two new operators contribute to the elastic \( \pi \pi \) scattering amplitude (and four others amount to quark mass renormalizations of dimension four LECs).
The renormalized LECs $b_i^r(\mu)$ and $d_i^r(\mu)$ are measurable quantities. They satisfy the renormalization group equations

\[
\begin{align*}
\mu \frac{d}{d\mu} b_i^r(\mu) &= -\beta_i , \\
\mu \frac{d}{d\mu} d_i^r(\mu) &= -\delta_i .
\end{align*}
\] (6.7)

Therefore, the choice of another scale $\mu_0$ leads to modified values of the renormalized LECs,

\[
\begin{align*}
b_i^r(\mu_0) &= b_i^r(\mu) + \beta_i \log \frac{\mu}{\mu_0} , \\
d_i^r(\mu_0) &= d_i^r(\mu) + \delta_i \log \frac{\mu}{\mu_0} .
\end{align*}
\] (6.8)

We remark that the scale–dependence in the counterterm Lagrangian is, of course, balanced by the scale–dependence of the renormalized finite one–loop functional for observable quantities. In addition to the terms listed in table 1, there are also a few more finite counterterms, i.e. terms with $\delta_i = 0$. These are not needed for the renormalization discussed here. We are presently working on setting up the minimal fourth order Lagrangian containing also these finite terms.
Table 1: Counterterms to fourth order and their $\beta$-functions. Here, $\bar{A} = A - \langle A \rangle / 2$.

| i  | $\tilde{O}_i^{(4)}$ | $\delta_i$ |
|----|---------------------|------------|
| 1  | $\langle u_\mu u_{\nu} \rangle \langle u^\mu u^\nu \rangle$ | $-(c_2 - g_A^2 / 8m) / 6 + c_3 (g_A^2 + g_A^4 / 6)$ $+ (c_4 + 1 / 4m) (-49g_A^4 / 48)$ $+ (-1 / 8 + 5g_A^2 / 24 + 17g_A^4 / 12 + 9g_A^6 / 32) / 2m$ |
| 2  | $\langle u^2 \rangle \langle u^2 \rangle$ | $-(c_2 - g_A^2 / 8m) / 12 + c_3 (-7g_A^4 / 4 - 5g_A^6 / 16)$ $+ (c_4 + 1 / 4m) (g_A^4 + 19g_A^6 / 16)$ $+ (-1 / 8 - 55g_A^2 / 48 - 21g_A^4 / 16 - 27g_A^6 / 64) / 2m$ |
| 3  | $\langle (v \cdot u)^2 \rangle \langle u \cdot u \rangle$ | $+ c_3 (1 / 2 + 2g_A^2 / 8m) + 17g_A^4 / 16 + (c_4 + 1 / 4m) (-3g_A^2 / 2 - 19g_A^4 / 8)$ $+ (11g_A^2 / 3 + 37g_A^4 / 12 + 9g_A^6 / 64) / 2m$ $+ (-1 / 8 - 17g_A^2 / 12 - 97g_A^4 / 48 - 9g_A^6 / 16) / 2m$ |
| 4  | $\langle u^\mu v \cdot u \rangle \langle u_{\mu} v \cdot u \rangle$ | $(c_2 - g_A^2 / 8m) (2 / 3 - g_A^2 / 2) + c_3 (-2g_A^4 - g_A^4 / 3)$ $+ (c_4 + 1 / 4m) (g_A^4 / 2 + 4g_A^4 / 24)$ $+ (1 - 17g_A^2 / 12 - 97g_A^4 / 48 - 9g_A^6 / 16) / 2m$ |
| 5  | $\langle (v \cdot u)^2 \rangle \langle (v \cdot u)^2 \rangle$ | $+ (c_2 - g_A^2 / 8m) (1 / 2 + 3g_A^4 / 16) - c_3 g_A^4 / 12 + (c_4 + 1 / 4m) g_A^4 / 6$ $+ (3g_A^2 / 8 - 11g_A^4 / 12 - 9g_A^6 / 64) / 2m$ $+ (1 / 4 + g_A^4 / 4 + g_A^4 / 16) / 2m$ |
| 6  | $i \langle [u^\mu, u^\nu] \bar{F}^+_{\mu
u} \rangle$ | $(c_2 - g_A^2 / 8m) / 24 + c_3 (-1 / 6 - g_A^2 / 2)$ $+ (c_4 + 1 / 4m) g_A^4 / 2 + (1 + c_6) (g_A^4 / 8 - 11g_A^4 / 32) / 2m$ $+ (-53g_A^2 / 48 - 5g_A^4 / 8) / 2m$ |
| 7  | $i \langle u^\mu, u^\nu \rangle \langle F^+_{\mu\nu} \rangle$ | $(1 + c_6 + 2c_7) (-g_A^2 / 8 - g_A^4 / 32) / 2m$ |
| 8  | $i \langle [u_{\mu}, v \cdot u] \bar{F}^+_{\mu\nu} v^\nu \rangle$ | $(c_2 - g_A^2 / 8m) (-1 / 3 - g_A^2 / 2) + c_3 (-3g_A^2 / 2)$ $- (c_4 + 1 / 4m) g_A^4 + (1 + c_6) (-g_A^2 / 4 + 11g_A^4 / 16) / 2m$ $+ (1 / 2 - 113g_A^2 / 24 + 27g_A^4 / 16) / 2m$ |
| 9  | $i \langle [u_{\mu}, v \cdot u] \langle F^+_{\mu\nu} v^\nu \rangle$ | $+ (1 + c_6 + 2c_7) (g_A^2 / 4 + g_A^4 / 16) / 2m$ |
| 10 | $\langle u \cdot u \rangle \langle \chi_+ \rangle$ | $c_1 (1 - 3g_A^2 / 2 - 9g_A^4 / 8) + c_3 (-1 / 2 - 13g_A^4 / 16) + (c_4 + 1 / 4m) g_A^4 / 4$ $+ (c_2 - g_A^2 / 8m) / 4 + (1 / 2 - 3g_A^2 / 12 - 27g_A^4 / 64) / 2m$ |
| 11 | $\langle u \cdot u \rangle \bar{\chi}_+$ | $c_5 (1 / 2 + g_A^4 / 8)$ |
| 12 | $u^\mu (u_{\mu} \bar{\chi}+) v^\nu$ | $c_5 (-1 / 2 + 3g_A^2 / 2 + g_A^4 / 4)$ |
| 13 | $\langle (v \cdot u)^2 \rangle \langle \chi_+ \rangle$ | $c_1 (1 + 9g_A^2 / 8) + (c_2 - g_A^2 / 8m) (1 / 2 - 9g_A^4 / 16) + (c_4 + 1 / 4m) g_A^4 / 2$ $+ c_3 g_A^2 / 4 + (1 / 2 + 7g_A^2 / 8 + 27g_A^4 / 64) / 2m$ $- c_5 g_A^4 / 8$ |
| 14 | $\langle (v \cdot u)^2 \rangle \bar{\chi}_+$ | $-c_5 (1 / 2 + g_A^4 / 4)$ |
| 15 | $v \cdot u (v \cdot u \bar{\chi}+) \rangle$ | $-c_5 (1 / 2 + g_A^4 / 4)$ |
| 16 | $\langle \bar{F}^+_{\mu\nu} F^+_{\mu\nu} \rangle$ | $(c_2 - g_A^2 / 8m) / 12 + (1 + c_6) (g_A^2 / 4) / 2m - (47g_A^4 / 48) / 2m$ |
| 17 | $\bar{F}^+_{\mu\nu} \langle F^+_{\mu\nu} \rangle$ | $- (1 + c_6 + 2c_7) (g_A^2 / 4) / 2m$ |
| 18 | $\langle v^\mu \bar{F}^+_{\mu\nu} v^\nu F^+_{\mu\nu} \rangle$ | $- (1 + c_6) (g_A^2 / 2) / 2m + (19g_A^4 / 6) / 2m - (c_2 - g_A^2 / 8m) / 3$ |
| 19 | $v^\mu \bar{F}^+_{\mu\nu} \langle v^\nu F^+_{\mu\nu} \rangle$ | $+ (1 + c_6 + 2c_7) (g_A^2 / 2) / 2m - (3g_A^4 / 4) / 2m$ |
| 20 | $\langle v^\mu \bar{F}^+_{\mu\nu} \rangle \langle v^\nu F^+_{\mu\nu} \rangle$ | $- (9g_A^2 / 8) / 2m$ |
| 21 | $\langle \chi_+ \rangle \langle \chi_+ \rangle$ | $c_1 (3 / 4 - 9g_A^2 / 8) - 3(c_2 - g_A^2 / 8m) / 32$ $- 3c_5 / 8 - (27g_A^2 / 128) / 2m$ $c_5 (1 / 4 + 3g_A^2 / 8)$ |
Table 1: continued

| i  | $\hat{O}_0^{(4)}$ | $\delta_i$ |
|----|------------------|------------|
| 23 | $iv_\rho \epsilon_{\mu\nu} [u_{\mu}, u_{\nu}] \langle u_\sigma S \cdot u \rangle$ | $+ (c_4 + 1/4m)(-g_A^2/2 + g_A^4/24)$ |
| 24 | $iv_\rho \epsilon_{\mu\nu} \langle u_{\mu}, u_{\nu} S \cdot u \rangle \sigma$ | $(c_4 + 1/4m)(-g_A^2 - 3g_A^4/8) + (-3g_A^6/48)/2m$ |
| 25 | $iv_\rho \epsilon_{\mu\nu} \langle u_{\mu}, u_{\nu} \rangle S \cdot u$ | $c_3g_A^4/6 - 7(c_4 + 1/4m)g_A^4/24 + (-g_A^4/12 + 3g_A^6/48)/2m$ |
| 26 | $v_\rho \epsilon_{\mu\nu} F_{\mu\nu}^+ \langle u_\sigma S \cdot u \rangle$ | $-(c_4 + 1/4m)g_A^2 + (1 + c_6)(g_A^4/24)/2m$ |
| 27 | $v_\rho \epsilon_{\mu\nu} \langle F_{\mu\nu}^+ \rangle \sigma S \cdot u$ | $+ (1 + c_6 + 2c_7)(g_A^4/16)/2m$ |
| 28 | $v_\rho \epsilon_{\mu\nu} \langle F_{\mu\nu}^+ S \cdot u \rangle u_\sigma$ | $-(c_4 + 1/4m)g_A^2 - (1 + c_6)(g_A^4/8)/2m$ |
| 29 | $v_\rho \epsilon_{\mu\nu} S_{\sigma}[u_{\mu}, u_{\nu}] \langle S \cdot u \rangle$ | $(1 + c_6)(g_A^4/24)/2m$ |
| 30 | $v_\rho \epsilon_{\mu\nu} \langle S^T \tilde{F}_{\tau\sigma} \mu_{\mu} \rangle \nu_\nu$ | $-(c_4 + 1/4m)g_A^2$ |
| 31 | $iv_\rho \epsilon_{\mu\nu} S_{\sigma}[u_{\mu}, u_{\nu}] \langle u \cdot u \rangle$ | $+ c_3(g_A^4/2 + 5g_A^4/8) + (c_4 + 1/4m)(1/4 - g_A^4/16)$ |
| 32 | $iv_\rho \epsilon_{\mu\nu} S_{\sigma} \nu^\nu \langle [u_{\mu}, u_{\nu}] \nu_\kappa \rangle$ | $+ (5g_A^2/8 + 3g_A^4/4 + g_A^6/16)/2m$ |
| 33 | $iv_\rho \epsilon_{\mu\nu} S_{\sigma}[u_{\mu}, u_{\nu}] \langle u_\nu \nu_\kappa \rangle$ | $2c_3g_A^2 + (c_4 + 1/4m)(-5/12 - g_A^2/4 + g_A^4/24)$ |
| 34 | $iv_\rho \epsilon_{\mu\nu} S_{\sigma}[u_{\mu}, u_{\nu}] \langle (v \cdot u)^2 \rangle$ | $+ (25g_A^2/24 + 3g_A^4/8)/2m$ |
| 35 | $iv_\rho \epsilon_{\mu\nu} S_{\sigma} \nu \cdot u \langle [u_{\mu}, u_{\nu}] \nu_\nu \rangle$ | $c_3(-2g_A^2 + g_A^4/3) + (c_4 + 1/4m)(1/2 + g_A^2/2)$ |
| 36 | $iv_\rho \epsilon_{\mu\nu} S_{\sigma}[v \cdot u, u_{\mu}] \langle u_\nu v_\nu \rangle$ | $+(c_2 - g_A^2/8m)(g_A^2/2 + g_A^4/8)$ |
| 37 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \tilde{F}_{\mu\nu}^+ \langle u \cdot u \rangle$ | $- c_3g_A^4/2 + (c_4 + 1/4m)g_A^4/16$ |
| 38 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \tilde{F}_{\mu\nu}^+ \langle u \cdot u \rangle$ | $+ (-c_2 - g_A^2/8m)g_A^2 - 2c_3g_A^2$ |
| 39 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \tilde{F}_{\mu\nu}^+ \langle u \cdot u \rangle$ | $+ (c_4 + 1/4m)(1/2 - g_A^4/24) + (-5g_A^2/4) + g_A^4/2)/2m$ |
| 40 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \mu_{\mu} \langle F_{\mu\nu}^+ \rangle \nu_\kappa$ | $(c_2 - g_A^2/8m)g_A^2 + c_3(2g_A^2 - g_A^2/3)$ |
| 41 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \nu^\nu \langle F_{\mu\nu}^+ \nu_\kappa \rangle$ | $-(c_4 + 1/4m)g_A^2/2 + (-5g_A^2/2 + g_A^4/8)/2m$ |
| 42 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \tilde{F}_{\mu\nu}^+ \langle u_\nu \nu_\kappa \rangle$ | $+ c_3g_A^2 - (c_4 + 1/4m)/2 + (1 + c_6)(1/4 - g_A^4/16)/2m$ |
| 43 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \tilde{F}_{\mu\nu}^+ \langle (v \cdot u)^2 \rangle$ | $+ (5g_A^2/4 + 4g_A^4/4)/2m$ |
| 44 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \langle F_{\mu\nu}^+ \rangle \langle (v \cdot u)^2 \rangle$ | $(1 + c_6 + 2c_7)(g_A^2/8 + g_A^4/32)/2m$ |
| 45 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \langle F_{\mu\nu}^+ \rangle \langle (v \cdot u)^2 \rangle$ | $+ (c_4 + 1/4m)/2 + (1 + c_6)(1/4 - g_A^4/4 + g_A^4/24)/2m$ |
| 46 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \langle F_{\mu\nu}^+ v_\kappa \rangle \langle u_\nu v_\nu \rangle$ | $+ (-9g_A^2/4)/2m$ |
| 47 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \langle F_{\mu\nu}^+ v_\kappa \rangle \langle u_\nu v_\nu \rangle$ | $(c_4 + 1/4m)(2/3 + g_A^2) - (5g_A^2/3)/2m$ |
| 48 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \langle F_{\mu\nu}^+ \rangle \langle (v \cdot u)^2 \rangle$ | $2g_A^2c_3 - (c_4 + 1/4m)/3 + (7g_A^2/6)/2m$ |
| 49 | $v_\rho \epsilon_{\mu\nu} S_{\sigma} \langle F_{\mu\nu}^+ v_\kappa \rangle \langle u_\nu v_\nu \rangle$ | $2g_A^2c_3 - (c_4 + 1/4m)g_A^2 + (g_A^4/2)/2m$ |

22
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
\textbf{i} & \textbf{$\hat{O}_i^{(4)}$} & \textbf{$\delta_i$} \\
\hline
50 & $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle [u_\mu, u_\nu] \partial_+ \rangle$ & $c_5(-g^2_A/2 - g^2_A/8)$ \\
51 & $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle [u_\mu, u_\nu] \partial_+ \rangle$ & $c_1(g^2_A + g^4_A/4) + (c_4 + 1/4m)(1/4 - g^2_A/16)$ \\
52 & $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} - \tilde{F}_{\nu\mu} \rangle$ & $+c_3 g^2_A/2 + (5g^2_A/8 + 3g^4_A/32)/2m$ \\
53 & $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} - \tilde{F}_{\nu\mu} \rangle$ & $-(c_4 + 1/4m)/3 + (5g^2_A/6)/2m$ \\
54 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(g^2_A)/2m$ \\
55 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $+2c_1 g^2_A - (c_4 + 1/4m)/2$ \\
56 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $+(1 + c_6)(1/4 - g^2_A/16)/2m + (5g^2_A/4)/2m$ \\
57 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $+(1 + c_6 + 2c_7)(3g^2_A/32)/2m$ \\
58 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-c_5 g^2_A$ \\
59 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-g_A(1 + c_6)/16m$ \\
60 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-g_A(1 + c_6)/16m$ \\
61 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-g_A(1 + c_6)/16m$ \\
62 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $g_A(1 + c_6)/4m$ \\
63 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-g_A(1 + c_6)/2m$ \\
64 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $g_A(1 + c_6)/2m$ \\
65 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-g_A(1 + c_6)/2m$ \\
66 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $4c_1 g_A - 4(c_4 + 1/4m)g_A/3$ \\
67 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $+(2g_A/3 + 53g^3_A/24 + 3g^5_A/8)/2m$ \\
68 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(g^2_A)/2m$ \\
69 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(-7g^2_A/8 - 9g^5_A/32)/2m$ \\
70 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(c_4 + 1/4m)(g_A/2 + 17g_A^3/16)$ \\
71 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $+ (15g^3_A/8 - 15g^5_A/16)/2m$ \\
72 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(-g^5_A/8)/2m$ \\
73 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(3g^5_A/8)/2m$ \\
74 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $(-g^3_A/3 - 3g^5_A/8)/2m$ \\
75 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-4c_3 g^3_A/3 + (c_4 + 1/4m)(-g_A - 7g_A^3/6)$ \\
76 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $+(g_A^3/6 + 7g^5_A/4)/2m$ \\
77 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-(c_4 + 1/4m)g_A^2/2 + (1 + c_6)(g_A^3/16)/2m$ \\
78 & $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle \tilde{F}_{\mu\nu} \partial_+ \rangle$ & $-(1 + c_6)(g_A^3/16)/2m$ \\
\hline
\end{tabular}
\caption{continued}
\end{table}

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| i   | O\(^{(4)}\) | \(\delta_i\)                           |
|-----|------------|----------------------------------------|
| 79  | \(\langle u_{\mu}, S \cdot u \rangle [i\nabla^\mu, v \cdot u]\) | \(5(c_4 + 1/4m)g_A/3 + (7g_A/12 - g_A^3/24)/2m\) |
| 80  | \(\langle u_{\mu}, v \cdot [iS \cdot \nabla, u^\mu]\rangle\) | \(-(c_4 + 1/4m)g_A/3 + (g_A/6 - g_A^3/6 + g_A^3/8)/2m\) |
| 81  | \(\langle [v \cdot u, u_{\mu}] [i\nabla^\mu, S \cdot u]\rangle\) | \((c_4 + 1/4m)g_A/3 + (5g_A/12 - 5g_A^3/3 - g_A^3/8)/2m\) |
| 82  | \(\langle [v \cdot u, S \cdot u] [iv \cdot \nabla, v \cdot u]\rangle\) | \(-4g_A(c_3 - g_A^3/8m) + c_3(-4g_A - 2g_A^3/3)\) |
| 83  | \(\langle v \cdot u, v \cdot uS \cdot \nabla + h.c.\rangle\) | \(+ (c_4 + 1/4m)(-4g_A/3 + g_A^3/2)\) |
| 84  | \(\langle v \cdot u u_{\mu}\rangle u^\mu iS \cdot \nabla + h.c.\) | \(+ (73g_A/6 - 17g_A^3/12 - 9g_A^3/8)/2m\) |
| 85  | \(\langle u \cdot u \rangle S \cdot u \cdot \nabla + h.c.\) | \(+ (g_A^3/8)/2m\) |
| 86  | \(\langle S \cdot uu_{\mu}\rangle u^\mu iv \cdot \nabla + h.c.\) | \((-g_A^3/2 + g_A^3/2 - g_A^3/12)/2m\) |
| 87  | \(\langle v \cdot u u_{\mu}\rangle S \cdot u i\nabla^\mu + h.c.\) | \(c_3(4g_A + 5g_A^3/2) + (c_4 + 1/4m)(-3g_A - 5g_A^3/2)\) |
| 88  | \(\langle v \cdot u S \cdot u u_{\mu} \cdot \nabla^\mu + h.c.\rangle\) | \(+ (3g_A^3/2 - g_A^3/2 - 3g_A^3/4)/2m\) |
| 89  | \(\langle u_{\mu} u \cdot u \rangle v \cdot u i\nabla^\mu + h.c.\) | \(c_3(-4g_A - 2g_A^3) + (c_4 + 1/4m)(-g_A + g_A^3/2)\) |
| 90  | \(\langle (v \cdot u)^2 \rangle S \cdot u iv \cdot \nabla + h.c.\) | \(+ (5g_A^3/2 - g_A^3/2)/2m\) |
| 91  | \(\langle v \cdot u S \cdot u \rangle v \cdot u iv \cdot \nabla + h.c.\) | \(+ (2g_A + g_A^3/4)/2m\) |
| 92  | \(\langle (v \cdot u)^2 \rangle v \cdot u iS \cdot \nabla + h.c.\) | \((-g_A^3/12 - g_A^3/12)/2m\) |
| 93  | \(\langle [v \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle u^\mu\) | \((c_2 - g_A^3/8m)(-2g_A + g_A^3/2) + c_3(-4g_A - 2g_A^3)\) |
| 94  | \(\langle [v \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle v \cdot u\) | \(+ (c_4 + 1/4m)(g_A + 5g_A^3/2) + (7g_A + g_A^3/2 - g_A^3/5)/2m\) |
| 95  | \(\langle [\nabla^\mu, F_{\mu \nu}^S v^\nu]\rangle S \cdot u\) | \((c_2 - g_A^3/8m)(2g_A) + c_3(4g_A + 2g_A^3)\) |
| 96  | \(\langle F_{\mu \nu}^{\mu^\nu} S^\nu\rangle \nabla^\mu, S \cdot u\) | \((c_4 + 1/4m)(-g_A - g_A^3/2)/(2g_A)\) |
| 97  | \(\langle F_{\mu \nu}^{\mu^\nu} S^\nu\rangle [v \cdot \nabla, u^\mu]\) | \((-2g_A + 2g_A^3 - g_A^3)/2m\) |
| 98  | \(\langle v^\mu F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, v \cdot u]\) | \((g_A - g_A^3/24)/2m\) |
| 99  | \(\langle [v \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle v \cdot u\) | \(- (1 + c_6 + 2c_7)(g_A^3/12)/2m\) |
| 100 | \(\langle v^\mu F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, v \cdot u]\) | \((1 + c_6 + 2c_7)(g_A^3/12)/2m + (2g_A + 2g_A^3/6)/2m\) |
| 101 | \(\langle F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, v \cdot u]\) | \(- (g_A^3/4)/2m\) |
| 102 | \(\langle F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, u^\mu]\) | \(- (g_A^3/3)/2m\) |
| 103 | \(\langle [\nabla^\mu, F_{\mu \nu}^S v^\nu]\rangle v \cdot u\) | \((-1 + c_6 + 2c_7)(g_A^3/12)/2m + (2g_A + g_A^3/3)/2m\) |
| 104 | \(\langle [\nabla^\mu, F_{\mu \nu}^S v^\nu]\rangle S \cdot u\) | \((c_4 + 1/4m)(2g_A/3) + (1 + c_6)(-g_A^3/4)/2m\) |
| 105 | \(\langle [S \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle u^\mu\) | \(+ (40g_A/3 + 5g_A^3/6)/2m\) |
| 106 | \(\langle F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, v \cdot u]\) | \((1 + c_6)(g_A^3/4)/2m\) |
| 107 | \(\langle F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, u^\mu]\) | \((c_4 + 1/4m)(-2g_A) + (1 + c_6)(g_A^3/4)/2m\) |
| 108 | \(\langle F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, u^\mu]\) | \(+ (7g_A - g_A^3)/2m\) |
| 109 | \(\langle [\nabla^\mu, F_{\mu \nu}^S v^\nu]\rangle v \cdot u\) | \((c_4 + 1/4m)(2g_A) + (1 + c_6)(-g_A^3/4)/2m\) |
| 110 | \(\langle [S \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle S \cdot u\) | \(2g_A/2m\) |
| 111 | \(\langle [S \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle u^\mu\) | \((-11g_A/6 + g_A^3/3)/2m\) |
| 112 | \(\langle [\nabla^\mu, F_{\mu \nu}^S v^\nu]\rangle v \cdot u\) | \((c_4 + 1/4m)(-8g_A/3) + (-g_A/2 + 7g_A^3/4)/2m\) |
| 113 | \(\langle [S \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle v \cdot u\) | \((c_4 + 1/4m)(2g_A/3) + (-g_A/3 - 4g_A^3/3)/2m\) |
| 114 | \(\langle [S \cdot \nabla, F_{\mu \nu}^S v^\nu]\rangle u^\mu\) | \(-c_3(2g_A) + (c_4 + 1/4m)g_A\) |
| 115 | \(\langle F_{\mu \nu}^S v^\nu\rangle [v \cdot \nabla, u^\mu]\) | \(+ (1 + c_6)(g_A^3/4)/2m + (g_A/2)/2m\) |
Table 1: continued

| \( i \) | \( O_i^{(4)} \) | \( \delta_i \) |
|---|---|---|
| 108 | \( [v^\mu F_{\mu}^+, S' v, v \cdot u] v \cdot \nabla + \text{h.c.} \) | \( 2(c_2 - g_A^2/8m)g_A + c_3(4g_A) - (c_4 + 1/4m)g_A - (1 + c_6)(g_A^3/4)/2m - (6g_A)/2m \) |
| 109 | \( [\tilde{F}^+, S' v, v \cdot u] \nabla^\mu + \text{h.c.} \) | \( (g_A)/2m \) |
| 110 | \( [\tilde{F}^+ \nu v', S v, u] \nabla^\mu + \text{h.c.} \) | \( + (g_A^3/12)/2m \) |
| 111 | \( [v^\mu \tilde{F}_{\mu}^+, S' v, u_{\kappa}] \nabla^\kappa + \text{h.c.} \) | \( (g_A)/2m \) |
| 112 | \( \langle [\nabla^\kappa, v^\mu \tilde{F}_{\mu}^+] S' v, u_{\kappa} \rangle \) | \( -2g_A(c_4 + 1/4m)/3 + (-5g_A/3 + 2g_A^3/3)/2m \) |
| 113 | \( \langle \tilde{F}_{\mu \nu}^{\dagger} \nabla^\mu, S u \rangle \) | \( (g_A^3)/2m \) |
| 114 | \( \langle \tilde{F}_{\mu \nu}^{\dagger} \nu v' [S \cdot \nabla, S u] \rangle \) | \( -c_5 g_A^3/6 \) |
| 115 | \( [\tilde{\chi}_+, iv \cdot \nabla, S u] \) | \( + c_1 g_A^3 + c_3 g_A - 2(c_4 + 1/4m)g_A + (g_A/2 + 3g_A^3/8)/2m \) |
| 116 | \( [iv \cdot \nabla, \tilde{\chi}_+, S u] \) | \( (g_A)/2m \) |
| 117 | \( \langle \chi_+ u, S u \rangle iv \cdot v + \text{h.c.} \) | \( -3c_5 g_A^3/2 \) |
| 118 | \( \langle \chi_+ u, S u \rangle iv \cdot v + \text{h.c.} \) | \( -1 - g_A^2/3 - 3g_A^4/2)/2m \) |
| 119 | \( \langle \tilde{\chi}_+ S' u \rangle iv \cdot v + \text{h.c.} \) | \( -(1 + g_A^2)/2m \) |
| 120 | \( \langle v \cdot u [v, \nabla, [\nabla^\mu, u_{\mu}] \rangle \) | \( -(1 + c_6)/2m \) |
| 121 | \( \langle iv \cdot \nabla, v \cdot u [\nabla^\mu, \mu_{\mu}] \rangle \) | \( (1 + c_6)/4m \) |
| 122 | \( \langle [[\nabla^\mu, \mu_{\mu}, v \cdot u] v \cdot \nabla + \text{h.c.} \) | \( -(1 + c_6)/4m \) |
| 123 | \( \langle \nabla^\mu, \mu_{\mu}, u_{\mu} \rangle \nabla^\mu + \text{h.c.} \) | \( -(9g_A^4)/8)/2m \) |
| 124 | \( \langle v_{\nu} \rho \rho_{\mu \sigma} S_{\sigma} \langle v \cdot u \tilde{F}_{\mu \nu}^+ \rangle v \cdot \nabla + \text{h.c.} \) | \( (1 + c_6)/4m \) |
| 125 | \( \langle v_{\nu} \rho \rho_{\mu \sigma} S_{\sigma} \langle F_{\mu \nu}^- \rangle v \cdot \nabla + \text{h.c.} \) | \( 3g_A^2(c_2 - g_A^2/8m) + c_2 g_A^2 - 2(c_4 + 1/4m)g_A^2 \) |
| 126 | \( \langle v_{\nu} \rho \rho_{\mu \sigma} S_{\sigma} \langle F_{\mu \nu}^- \rangle v \cdot \nabla + \text{h.c.} \) | \( (1 + c_6)/4m \) |
| 127 | \( \langle v_{\nu} \rho \rho_{\mu \sigma} S_{\sigma} \langle u \cdot v \tilde{F}_{\mu \nu}^+ \rangle v \cdot \nabla + \text{h.c.} \) | \( -3g_A^2(c_2 - g_A^2/8m) + 5c_3 g_A^2/3 - 10(c_4 + 1/4m)g_A^2/3 \) |
| 128 | \( \langle \nabla^\mu, \mu_{\mu}, u_{\mu} \rangle \nabla^\mu + \text{h.c.} \) | \( -9g_A^4)/8)/2m \) |
| 129 | \( \langle v \cdot \nabla, v \cdot u [v \cdot \nabla, v \cdot u] \rangle \) | \( -2(c_2 - g_A^2/8m)/3 - 4c_3 g_A^2 + 2(c_4 + 1/4m)g_A^2 \) |
| 130 | \( \langle v \cdot u [v, \nabla, [\nabla^\mu, v \cdot u]] \rangle \) | \( + (19g_A^2/6 - 131g_A^4/24)/2m \) |
| 131 | \( \langle v \cdot u [v, \nabla, [\nabla^\mu, v \cdot u]] \rangle \) | \( -2(c_2 - g_A^2/8m)/3 - 14c_3 g_A^2/3 + 10(c_4 + 1/4m)g_A^2/3 \) |
| 132 | \( \langle [v \cdot u, \nabla, u_{\mu} [v \cdot \nabla, \mu_{\mu}]] \rangle \) | \( + (3g_A^2/4)/2m \) |
| 133 | \( \langle [v \cdot u, \nabla, u_{\mu} [v \cdot \nabla, \mu_{\mu}]] \rangle \) | \( -4(c_2 - g_A^2/8m) + c_3 (4 + g_A^2) \) |
| 134 | \( \langle u_{\mu} [v \cdot \nabla, [v \cdot \nabla, u_{\mu}]] \rangle \) | \( + (-7/2 + 3g_A^2/2)/2m \) |
Table 1: continued

| i | $\hat{O}^{(4)}$ | $\delta_i$ |
|---|---|---|
| 139 | $\left[ v \cdot \nabla, v \cdot u \right], u_\mu \nabla^\mu + \text{h.c.}$ | $(1/2 + 3g_A^2/16 - g_A^4/8)/2m$ |
| 140 | $\left[ [v \cdot \nabla, u_\mu], v \cdot u \right] \nabla^\mu + \text{h.c.}$ | $(1/2 - 3g_A^2/16 + g_A^4/8)/2m$ |
| 141 | $\left[ \nabla^\mu, v \cdot u \right], v \cdot u \nabla^\mu + \text{h.c.}$ | $(+1/2 - g_A^4/8)/2m$ |
| 142 | $\left[ [v \cdot \nabla, u_\mu], u^\mu \right], v \cdot \nabla + \text{h.c.}$ | $-c_3g_A^2 + (-7g_A^4/8)/2m$ |
| 143 | $\left[ [\nabla^\mu, u_\nu], u_\nu \right] \nabla^\mu + \text{h.c.}$ | $(g_A^4/8)/2m$ |
| 144 | $\left[ u_\mu, \left[ \nabla^\mu, v \cdot u \right] \right], v \cdot \nabla + \text{h.c.}$ | $(-7/12 - 89g_A^2/48)/2m$ |
| 145 | $\left[ u_\mu, \left[ \nabla^\mu, u_\nu \right] \right] \nabla^\nu + \text{h.c.}$ | $(1/12 + 5g_A^4/12)/2m$ |
| 146 | $v \cdot \nabla \left( (v \cdot u)^2 \right), v \cdot \nabla$ | $9(c_2 - g_A^2/8m)/2 - 2c_3g_A^2 + 4(c_4 + 1/4m)g_A^2$ |
| 147 | $\nabla^\mu \left( (v \cdot u)^2 \right), \nabla^\mu$ | $+(-8 - 7g_A^2 - 99g_A^4/8)/2m$ |
| 148 | $v \cdot \nabla \left( v \cdot u, u_\mu \right) \nabla^\mu + \text{h.c.}$ | $(1 + 9g_A^4/8)/2m$ |
| 149 | $v \cdot \nabla \left( u \cdot u \right), v \cdot \nabla$ | $(2 - 4g_A^2 + 9g_A^4/4)/2m$ |
| 150 | $\nabla^\mu \left( u \cdot u, \nabla^\mu \right)$ | $13c_3g_A^2/2 - 4(c_4 + 1/4m)g_A^2 + (7g_A^2 + 63g_A^4/8)/2m$ |
| 151 | $\left( \nabla^\mu, F_\mu^+ v^\nu \right), \nabla^\mu + \text{h.c.}$ | $+(-3g_A^2/2 - 9g_A^4/8)/2m$ |
| 152 | $\left( [v \cdot \nabla, F_\mu^+ v^\nu], \nabla^\mu \right)$ | $-1/6 - 9g_A^4/6)/2m$ |
| 153 | $\left( v \cdot \nabla, F_\mu^+ v^\nu \right)$ | $1/6 + 89g_A^2/24)/2m$ |
| 154 | $\left( \nabla^\mu, F_\mu^+ v^\nu \right)$ | $2c_5g_A^2$ |
| 155 | $\nabla^\mu \left( v \cdot \nabla, \nabla^\mu, \nabla^\mu \right)$ | $-3c_1g_A^2 - (c_2 - g_A^2/8m)/4 - (g_A^4/8)/2m$ |
| 156 | $\nabla^\mu \left( v \cdot \nabla, v \cdot \chi^+ \right)$ | $-3c_1g_A^2 - (c_2 - g_A^2/8m)/4 - (g_A^4/8)/2m$ |
| 157 | $\nabla^\mu \left( [v \cdot \nabla, v \cdot \chi^+] \right)$ | $-3c_1g_A^2 - (c_2 - g_A^2/8m)/4 - (g_A^4/8)/2m$ |
| 158 | $\nabla^\mu \left( [v \cdot \nabla, v \cdot \chi^+] \right)$ | $-3c_1g_A^2 - (c_2 - g_A^2/8m)/4 - (g_A^4/8)/2m$ |
| 159 | $v \cdot \nabla \left( \chi^+, \chi^+ \right)$ | $9c_1g_A^2 + (9g_A^2/2)/2m$ |
| 160 | $\nabla^\mu \left( v \cdot \nabla, \chi^+, \chi^+ \right)$ | $-(9g_A^2/8)/2m$ |
| 161 | $v \cdot \nabla \left( \chi^+, \chi^+ \right)$ | $-3c_5g_A^2$ |
| 162 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [v \cdot \nabla, v \cdot u], [\nabla^\mu, u_\nu] \right]$ | $(2g_A^2 + g_A^4/6)/2m$ |
| 163 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [\nabla^\mu, v \cdot u], [v \cdot \nabla, u_\nu] \right]$ | $(2g_A^2 + g_A^4/4)/2m$ |
| 164 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [v \cdot \nabla, \nabla^\mu, u_\nu] \right], v \cdot u$ | $(2g_A^2 + g_A^4/4)/2m$ |
| 165 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [v \cdot \nabla, \nabla^\mu, u_\nu], v \cdot u \right]$ | $(2g_A^2 - 4g_A^4/6)/2m$ |
| 166 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [\nabla^\mu, v \cdot \nabla, u_\nu], v \cdot u \right]$ | $(-g_A^4/6)/2m$ |
| 167 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [\nabla^\mu, v \cdot \nabla, u_\nu], v \cdot u \right]$ | $(7g_A^2 + 11g_A^4/12)/2m$ |
| 168 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [\nabla^\mu, v \cdot \nabla, u_\nu] \right]$ | $-(c_4 + 1/4m)g_A^2 + (3 + 2c_3g_A^2)$ |
| 169 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [\nabla^\mu, v \cdot \nabla, u_\nu] \right]$ | $(7g_A^2 + 5g_A^4/4)/2m$ |
| 170 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left[ [\nabla^\mu, v \cdot \nabla, u_\nu] \right]$ | $+10c_3g_A^2/3 - (c_4 + 1/4m)g_A^2/3$ |
| 171 | $i v_\rho \varepsilon^{\mu\nu\rho\sigma} S_\sigma \left( [v \cdot \nabla, v \cdot u] u_\nu \right) \nabla^\mu + \text{h.c.}$ | $-(c_4 + 1/4m)/3 + (-g_A^2/6 - g_A^4/6)/2m$ |

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Table 1: continued

| i | $\hat{O}_i^{(4)}$ | $\delta_i$ |
|---|------------------|---------|
| 172 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma (v \cdot \nabla, u_\mu) v \cdot u \nabla_\mu + h.c.$ | $(-g^4_A/4)/2m$ |
| 173 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma (\nabla_\mu, u_\nu) v \cdot u \nabla + h.c.$ | $(-2g^2_A - 3g^4_A/4)/2m$ |
| 174 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma (v \cdot \nabla, u_\mu, u_\nu) v \cdot \nabla + h.c.$ | $2c_3g_A^3 + (g^3_A + 7g^4_A)/4)/2m$ |
| 175 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma (\nabla_\kappa, u_\mu) u_\nu \nabla_\kappa + h.c.$ | $(-g^4_A)/2m$ |
| 176 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma v \cdot \nabla (v \cdot u, u_\mu) \nabla_\nu + h.c.$ | $(6g^2_A + g^4_A)/2m$ |
| 177 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma v \cdot \nabla [u_\mu, u_\nu] v \cdot \nabla$ | $(c_4 + 1/4m)g^3_A/2 - 4c_3g^2_A$ |
| 178 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma \nabla_\kappa [u_\mu, u_\nu] \nabla_\kappa$ | $+(6g^2_A - 7g^4_A)/4)/2m$ |
| 179 | $i\nu_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma [\nabla_\mu, [\nabla_\kappa, [u_\nu, u_\sigma]]$ | $(g^2_A + g^4_A)/2m$ |
| 180 | $v_\mu \epsilon^{\rho\mu\nu\sigma} S_\sigma [v \cdot \nabla, [v \cdot \nabla, F_{\mu\nu}^+]$ | $(-g^4_A)/2m$ |
| 181 | $v_\mu \epsilon^{\rho\mu\nu\sigma} S_\sigma [\nabla_\kappa, [\nabla_\kappa, F_{\mu\nu}^+]$ | $-(1 + c_6)(g^2_A/6)/2m + (7g^2_A)/2m$ |
| 182 | $v_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma [v \cdot \nabla, [v \cdot \nabla, F_{\mu\nu}^+]]$ | $-(c_4 + 1/4m)g^3_A/3 + (g^2_A)/2m$ |
| 183 | $v_\mu \epsilon^{\rho\mu\nu\sigma} S_\sigma v \cdot \nabla F_{\mu\nu}^+ v \cdot \nabla$ | $(1 + c_6 + 2c_7)(g^2_A/4)/2m$ |
| 184 | $v_\mu \epsilon^{\rho\mu\nu\sigma} S_\sigma \nabla_\kappa F_{\mu\nu}^+ \nabla_\kappa$ | $(2g^2_A)/2m$ |
| 185 | $v_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma v \cdot \nabla F_{\mu\nu}^+ v \cdot \nabla$ | $(-4g^2_A)/2m$ |
| 186 | $v_\mu \epsilon^{\rho\mu\nu\sigma} S_\sigma v \cdot \nabla F_{\mu\nu}^- v \cdot \nabla$ | $-(1 + c_6 + 2c_7)(3g^2_A/4)/2m + (4g^2_A)/2m$ |
| 187 | $v_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma [\nabla_\mu, [\nabla_\kappa, F_{\mu\nu}^+]]$ | $(-2g^2_A/3)/2m$ |
| 188 | $v_\mu \epsilon^{\rho\mu\nu\sigma} [v \cdot \nabla, [u_\mu, u_\kappa], [\nabla_\nu, u_\sigma]]$ | $-g_A(c_4 + 1/4m)/4$ |
| 189 | $v \cdot \nabla, [v \cdot \nabla, v \cdot u] iS \cdot \nabla + h.c.$ | $(g^2_A/3)/2m$ |
| 190 | $v \cdot \nabla, [v \cdot \nabla, S \cdot u] iv \cdot \nabla + h.c.$ | $(-g^3_A)/2m$ |
| 191 | $v \cdot \nabla, [\nabla_\kappa, S \cdot u] i\nabla_\mu + h.c.$ | $-(g^4_A/3)/2m$ |
| 192 | $\nabla_\mu, [\nabla_\mu, S \cdot u] iv \cdot \nabla + h.c.$ | $-(g^4_A/2)/2m$ |
| 193 | $S \cdot u iv (v \cdot \nabla)^3 + h.c.$ | $8g_A c_3/3 - 16(c_4 + 1/4m)g_A/3(-4g_A/3 + 2g^3_A)/2m$ |
| 194 | $v \cdot \nabla \{iS \cdot \nabla, v \cdot u\} v \cdot \nabla$ | $(-g^3_A)/2m$ |
| 195 | $v \cdot \nabla \{iv \cdot \nabla, S \cdot u\} v \cdot \nabla$ | $(-g^3_A)/2m$ |
| 196 | $\nabla_\mu \{iv \cdot \nabla, S \cdot u\} \nabla_\mu$ | $g^3_A/2m$ |
| 197 | $(v \cdot \nabla)^2 \{v \cdot \nabla, $ | $(-18g^3_A)/2m$ |
| 198 | $\nabla_\mu v \cdot \nabla \nabla_\mu \nabla_\mu v \cdot \nabla$ | $(9g^2_A)/2m$ |
| 199 | $v \cdot \nabla \nabla_\mu \nabla_\mu v \cdot \nabla$ | $(9g^2_A)/2m$ |
7 Sample calculations and checks

In this section, we perform a few sample calculations. On one hand, this shows how to use table 1 (or tables 2-4) for extracting the divergences for a process under consideration, and on the other hand, since already a large body of $q^4$ calculations exists, it serves as a good check on the rather involved manipulations leading to the final results in table 1 (or tables 2-4).

7.1 Example I: The isoscalar nucleon magnetic moment

As a first example, we consider the isoscalar nucleon magnetic moment. To order $q^3$, it is finite and divergences only appear at $O(q^4)$. Here, we are only interested in these divergent pieces. The complete expression including the finite pieces is given in ref. [20], where the first corrections to the P–wave low–energy theorems [21] in neutral pion photoproduction are worked out. To be specific, consider the one–loop graph in Fig.2a. Using the Feynman rules given in [5], its contribution to the isoscalar magnetic form factor, which we denote by $I_{sM}^s(\omega)$, is

$$I_{sM}^s(\omega) = 3 \left(1 + c_6 + 2c_7\right) \frac{g_A^2}{F_\pi^2} \frac{1}{2m} S_\mu \left[S \cdot \epsilon, S \cdot k\right] S_\nu \frac{1}{i} \int \frac{d^d l}{(2\pi)^d} \frac{l_\mu l_\nu}{(v \cdot l - \omega) v \cdot l (l^2 - M^2_\pi)},$$

(7.1)

with $\omega = v \cdot k$ and $\epsilon$ the polarization vector of the photon. The spin–matrices can be combined to give $[S \cdot \epsilon, S \cdot k]/4$ and the d–dimensional integral for $\omega = 0$, i.e. for the magnetic moment, gives a contribution of the form $(2M^2_\pi L + \text{finite}) g_{\mu\nu}$. Putting pieces together, we arrive at (we do not give the finite piece here)

$$I_{sM}^{s,\text{div}}(0) = 3 \frac{e}{4m} \left(1 + c_6 + 2c_7\right) \frac{g_A^2}{F_\pi^2} M^2_\pi \left[S \cdot \epsilon, S \cdot k\right] L,$$

(7.2)

with $L$ defined in Eq.(6.2). We see that the divergent structure is proportional to $M^2_\pi$ and the diagram is, of course, linear in the electromagnetic field strength tensor. The pertinent fourth order counter term, depicted in Fig.2b, is the one numbered 55 in table 1 (or, since we are dealing with an example of an eye graph, the operator $L_{22}$ in table 4),

$$L^{(4)}_{\pi N} = \bar{N} \{ \alpha \bar{v}_\rho \epsilon^{\mu\nu\sigma} S_\sigma \left(F^{\mu\nu}_\pi + \chi_+\right) \} N,$$

(7.3)

where the strength $\alpha$ will be specified shortly. The pertinent fourth order Feynman insertion from this part of the Lagrangian reads

$$\gamma NN – \text{vertex : } i \frac{16}{3} \alpha \frac{M^2_\pi}{4} \left[S \cdot \epsilon, S \cdot k\right].$$

(7.4)

Taking now $\alpha = \delta_{55} = (1 + c_6 + 2c_7)(3g_A^2/64m)$ from table 1, the graph $I_{M}^{\text{ct}}(0)$ contributes as

$$I_{M}^{\text{ct}}(0) = \text{finite} - 16 \frac{M^2_\pi}{4m} \left(1 + c_6 + 2c_7\right) \frac{g_A^2}{F_\pi^2} \frac{e}{64m} \left[S \cdot \epsilon, S \cdot k\right] L$$

$$= \text{finite} - 3 \frac{e}{4m} \left(1 + c_6 + 2c_7\right) \frac{g_A^2}{F_\pi^2} M^2_\pi \left[S \cdot \epsilon, S \cdot k\right] L,$$

(7.5)

so that by adding Eq.(7.2) and Eq.(7.5), we see that we are left with a finite piece. This is a particular simple example to show how to use table 1 (or the tables in app. [3]) and it checks exactly one operator.
7.2 Example II: Scalar form factor of the nucleon

The elastic pion–nucleon scattering amplitude has only been worked out to order $q^3$ \[16, 17\] in HBCHPT. It already contains divergences at that order. However, the so–called remainder at the Cheng–Dashen point, which involves the scalar form factor of the nucleon, $\sigma_{\pi N}(2M_\pi^2)$, and the isospin–even $\pi N$ scattering amplitude $\bar{D}(\nu, t)|_{\nu=0, t=2M_\pi^2}$ (the ‘bar’ means that the nucleon pole graphs with pseudovector coupling are subtracted) have been calculated at $\mathcal{O}(q^4)$ \[22\]. Here, we consider the scalar form factor of the nucleon for arbitrary values of the squared momentum transfer $t$ at order $q^4$. For the particular kinematics at the Cheng–Dashen point, we recover of course the results of ref.\[22\].

The various types of graphs are shown in fig.3. These are most conveniently evaluated in the Breit frame. However, one has to generalize the Breit frame kinematics appropriately to order $q^4$. Consequently, there are also contributions at this order from diagrams which formally start at order $q^3$. To be precise, the product $v \cdot p$ picks up pieces that are of order $q^2$ beyond the leading one $\sim q$ (which actually vanishes in the Breit frame). With this in mind, it is straightforward to work out the contributions of graphs 3b,c:

\[
I(3b) = 6c_1 \Delta_\pi \frac{M_\pi^2}{F_\pi^2},
\]

\[
I(3c) = -6c_1 \frac{M_\pi^4}{F_\pi^2} I_0(t) + 3\left(c_2 - \frac{g_2^2}{8m}\right) I_2(t) + 3c_3 \frac{M_\pi^2}{F_\pi^2} \left(M_\pi^2 I_0(t) - \Delta_\pi + tI_1(t)\right),
\]

where the loop functions $\Delta_\pi, I_{0,1,2}(t)$ are given by

\[
\frac{1}{i} \int \frac{d^d l}{(2\pi)^d} \frac{\{1, l^2, l \cdot k, (v \cdot l)^2\}}{[M_\pi^2 - l^2][M_\pi^2 - (l - k)^2]} = \{I_0(t), -\Delta_\pi + M_\pi^2 I_0(t), tI_1(t), I_2(t)\},
\]

with $t = k^2$. Putting the divergent pieces together, one gets

\[
I^{(\text{div})}_{\text{tadpole}} = 32 \frac{M_\pi^4}{F_\pi^2} \left(\frac{3}{4} c_1 - \frac{3}{32} (c_2 - \frac{g_2^2}{8m}) - \frac{3}{8} c_3\right) L - 4 \frac{M_\pi^2}{F_\pi^2} t \left(-\frac{1}{8} (c_2 - \frac{g_2^2}{8m}) - \frac{3}{4} c_3\right) L.
\]

The pertinent tadpole counterterm structures are E1 and C3 (two derivatives) from table 2 which we list here together with the pertinent Feynman insertions for a scalar–isoscalar source:

\[
\langle \chi_+ \rangle \langle \chi_+ \rangle : \quad -32i M_\pi^4, \quad \langle [\nabla_\mu, [\nabla_\nu, \chi_+]] \rangle : \quad 4i M_\pi^2 t.
\]

Injecting these in the genuine counterterm diagram Fig.3a, one finds

\[
I^{(\text{div})}_{\text{tadpole}} = -I^{(\text{ct})}_{\text{tadpole}},
\]

which is the desired result. Similarly, the divergent pieces of the self–energy graphs (cf. fig 3d,e) read

\[
I^{(\text{div})}_{\text{self}} = \left(\frac{9 g_3^2 M_\pi^4}{4 m F_\pi^2} + \frac{1}{8} m F_\pi^2 t\right) L.
\]
Again, with the operators $E_1$ and $C_3$ (two derivatives) from table 3, we find these divergences to be cancelled,
\[ I_{\text{self}}^{(\text{div})} = -I_{\text{self}}^{(\text{ct})}. \] (7.13)

Finally, consider the eye graphs (fig. 3f,g,h,i). Their divergent parts read
\[ I^{(3f)} = \left[ -18 \frac{g_\Lambda^2 M^4}{F^2} c_1 - \frac{9}{16} \frac{g_\Lambda^2 M^2}{F^2} t \right] L, \]
\[ I^{(3g)} = \left[ -18 \frac{g_\Lambda^2 M^4}{F^2} c_1 \right] L, \] (7.14)
\[ I^{(3h+i)} = \left[ -45 \frac{g_\Lambda^2 M^4}{8 F^2} + \frac{23}{16} \frac{g_\Lambda^2 M^2}{F^2} t \right] L, \]
with the first contribution stems from the dimension three operator $O_{20}$ of Ecker’s list [2]. At order $q^4$, it has the Feynman insertion $-8 M^2_\pi b_{20}^\nu v \cdot p$. The operator $v \cdot p$ vanishes to leading order but has a finite piece at second order so that this term can contribute here\(^{12}\). This remark also applies to diagram 3i, which already contributes at $O(q^3)$ but has an additional order $q^4$ piece due to the kinematics and thus is of relevance here. In addition to the already known operators $E_1$ and $C_3$ (two derivatives) from table 4, we need the structure $C_5$ from that table together with its pertinent Feynman insertion,
\[ \nabla_\mu (\chi_+) \nabla^\mu : \quad -4iM^2_\pi p_1 \cdot p_2, \] (7.15)
where $p_1 (p_2)$ is the incoming (outgoing) nucleon momentum and $p_1 \cdot p_2 = -t/4$. Adding up these three terms amounts to
\[ I_{\text{eye}}^{(\text{div})} = \left[ (-i) (-32iM^4_\pi) \left( -\frac{9}{8} \frac{g_\Lambda^2}{F^2} c_1 - \frac{45}{256} \frac{g_\Lambda^2}{m F^2} t \right) \right] L \]
\[ + \left[ (-i) (4iM^2_\pi t) \left( -\frac{5}{64} \frac{g_\Lambda^2}{m F^2} t \right) \right] L + \left[ (-i) (iM^2_\pi t) \left( -\frac{9}{16} \frac{g_\Lambda^2}{m F^2} t \right) \right] L \]
\[ = \left[ 36 \frac{g_\Lambda^2 M^4_\pi}{F^2} c_1 + 45 \frac{g_\Lambda^2 M^4_\pi}{8 m F^2} t - \frac{14}{16} \frac{g_\Lambda^2 M^2}{m F^2} t \right] L, \] (7.16)
which exactly cancels the terms in Eq. (7.14). Of course, one could have also added all the divergences from the various types of graphs and then use the operators 159, 161 and 181 from the master table 1. This leads, of course, to the same result. With that remark we conclude our discussion of the scalar form factor at order $q^4$. We have also checked the divergence structure of the complete fourth order pion–nucleon scattering amplitude, which gives many additional checks beyond the ones discussed here\(^{13}\).

\(^{12}\)In principle, the whole operator $O_{20}$ can be shifted to dimension four (and higher) by using the nucleon equations of motion. We have checked that adjusting the $Z$-factor and the other terms accordingly, one also finds a cancellation of the divergences.

\(^{13}\)We are grateful to Nadia Fettes for providing us with the Feynman graph results.

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8 Summary and conclusions

In this paper, we have performed the chiral–invariant renormalization of the effective two–flavor pion–nucleon field theory and constructed the complete counterterm Lagrangian at next–to–leading one–loop order $q^4$. To incorporate the massive spin–1/2 degrees of freedom (the nucleons), we have used heavy baryon chiral perturbation theory in the path integral formulation \cite{1}. This extends previous work by Ecker \cite{2}, who worked out the leading one–loop divergences at order $q^3$.

The pertinent results of our study can be summarized as follows:

1. At order $q^4$ in the chiral counting, one has to deal with four irreducible diagrams involving pion and nucleon propagators in the presence of external fields as shown in fig. 1. From these, the so–called eye graph involves a triple coincidence limit. In section 5.3 we have developed a method to treat such multiple coincidence limits. All other contributions can be worked out using the standard double coincidence limit \cite{12, 2, 3}. To deal with operators which are orthogonal to the direction defined by the nucleons’ four–velocity (these only start to show up at order $q^4$), we have extended the pertinent heat kernel methods as detailed in section 5.2.

2. The method used destroys covariance in some intermediate steps. Of course, the final results are covariant. This is achieved by forming appropriate combinations of the operators. Furthermore, hermiticity is restored by again combining appropriate terms. For completeness, some intermediate results for the eye–graph are given in app. D. Combining all pieces results in the complete counterterm Lagrangian given in section 6 in terms of the operators $\tilde{O}_i^{(4)}$, compare table 1. This table constitutes the central result of this work. To facilitate comparison and checks, we have also listed in app. F the resulting operators and $\beta$–functions for the tadpole, self–energy and eye graphs, respectively. The pertinent $\beta$–functions depend on the finite LECs $g_A$, $c_1$, ..., $c_6$ and the inverse of the nucleon mass.

3. We have performed a variety of checks on the rather involved manipulations leading to table 1. First, to third order we recover Ecker’s result \cite{2}. Second, since a variety of processes have been already calculated at order $q^4$ using Feynman diagram techniques including $\pi N \rightarrow \pi N$ and $\gamma N \rightarrow N$, we have checked our results against these calculations as detailed in section 6. In particular the complete fourth order amplitude for elastic $\pi N$ scattering allows to check a large number of terms.

To really address the question of isospin breaking alluded to in the introduction, it is mandatory to construct also all finite terms and to include virtual photons in the pion–nucleon system along similar lines.\footnote{For the case of the purely pionic Lagrangian, see ref. \cite{23}, and for the pion–nucleon system to order $q^3$, see ref. \cite{24}.} We hope to be able to report the results in the near future.
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A Seeley–DeWitt coefficients

In this appendix, we evaluate all possible Seeley–DeWitt coefficients up to chiral dimension four (not all of them are used in the actual calculation). The list presented below is the most general which one gets for an elliptic differential operator of the form

\[- d^x_\mu d^x_\mu + a^x \]  \hspace{1cm} (A.1)

in Euclidean space, where we use the following definitions

\[ d^x_\mu = \partial_\mu + \gamma^x_\mu, \quad \gamma^x_\mu = \partial^x_\mu - \gamma^x_\mu, \] \hspace{1cm} (A.2)

\[ \gamma_{\mu\nu} = \partial_\mu \gamma_\nu - \partial_\nu \gamma_\mu + [\gamma_\mu, \gamma_\nu] = [d_\mu, d_\nu]. \] \hspace{1cm} (A.3)

The evaluation of the coefficients in the coincidence limit is described in [3] and is rather tedious. We introduce for a string of derivatives acting on the Seeley–DeWitt coefficients in the coincident limit the notation

\[ d_\alpha...d_\omega h_n | = (d^x_\alpha...d^x_\omega h_n(x, y)) |_{x \to y} \] \hspace{1cm} (A.4)

The result of such a calculation is always given by a string of commutators.

\begin{align*}
\text{Seeley–DeWitt} & \quad \text{coefficients of dimension zero and one} \quad (A.5) \\
| h_0 | = 1 & \quad d_\mu h_0 | = 0 & \quad h_0 \gamma^x_\mu | = 0 \quad (A.6) \\
\text{Seeley–DeWitt} & \quad \text{coefficients of dimension two} \quad (A.7) \\
| d_\mu d_\nu h_0 | & = d_\mu h_0 \gamma^x_\mu | = h_0 \gamma^x_\mu \gamma^x_\nu | = \frac{1}{2} \gamma_{\mu\nu} \\
| h_1 | & = -a \\
\text{Seeley–DeWitt} & \quad \text{coefficients of dimension three} \quad (A.8) \\
| d_\lambda d_\mu d_\nu h_0 | & = d_\mu d_\lambda d_\nu h_0 | = \frac{1}{3} \left( [d_\lambda, \gamma_{\mu\nu}] + [d_\mu, \gamma_{\lambda\nu}] \right) , \\
| d_\lambda d_\mu h_0 \gamma_\nu | & = d_\mu d_\lambda h_0 \gamma_\nu | = \frac{1}{2} d_\lambda d_\mu d_\nu h_0 | = \frac{1}{6} \left( [d_\lambda, \gamma_{\mu\nu}] + [d_\mu, \gamma_{\lambda\nu}] \right) , \\
| d_\nu h_0 \gamma_\mu | & = d_\nu h_0 \gamma_\mu \gamma_\lambda | = \frac{1}{6} \left( [d_\lambda, \gamma_{\mu\nu}] + [d_\mu, \gamma_{\lambda\nu}] \right) , \\
| h_0 \gamma_\mu \gamma_\nu \gamma_\lambda | & = h_0 \gamma_\mu \gamma_\nu \gamma_\lambda | = \frac{1}{3} \left( [d_\lambda, \gamma_{\mu\nu}] + [d_\mu, \gamma_{\lambda\nu}] \right) .
\end{align*}
\[ \begin{align*}
\frac{d_{\mu} h_{1}}{} &= -\frac{1}{2} \left[ d_{\mu}, a \right] + \frac{1}{6} \left[ d_{\nu}, \gamma_{\mu\nu} \right], \\
\frac{h_{1}}{} &= d_{\mu} h_{1} - \frac{1}{3} \left[ d_{\nu}, \gamma_{\mu\nu} \right] = -\frac{1}{2} \left[ d_{\mu}, a \right] - \frac{1}{6} \left[ d_{\nu}, \gamma_{\mu\nu} \right]
\end{align*} \]

**Seeley–DeWitt coefficients of dimension four**

\[\begin{align*}
d_{\alpha} d_{\beta} d_{\gamma} d_{\delta} h_{0} &= -\frac{1}{8} \left( \{ \gamma_{\delta\alpha}, \gamma_{\beta\gamma} \} + \{ \gamma_{\gamma\alpha}, \gamma_{\beta\delta} \} + \{ \gamma_{\delta\gamma}, \gamma_{\alpha\beta} \} \right) \\
&\quad - \frac{1}{4} \left( [d_{\beta}, [d_{\gamma}, \gamma_{\delta\alpha}]] + [d_{\alpha}, [d_{\beta}, \gamma_{\delta\gamma}]] + [d_{\alpha}, [d_{\gamma}, \gamma_{\delta\beta}]] \right) \\
&= d_{\alpha} d_{\gamma} d_{\beta} d_{\delta} h_{0} = d_{\alpha} d_{\gamma} d_{\beta} d_{\delta} h_{0} = d_{\delta} d_{\gamma} d_{\beta} d_{\alpha} h_{0}
\end{align*}\]

\[\begin{align*}
d_{\alpha} d_{\beta} d_{\gamma} d_{\gamma} h_{0} &= -\frac{1}{4} \left( \{ \gamma_{\gamma\alpha}, \gamma_{\beta\gamma} \} + [d_{\beta}, [d_{\gamma}, \gamma_{\gamma\alpha}]] + [d_{\alpha}, [d_{\gamma}, \gamma_{\beta\gamma}]] \right)
\end{align*}\]

\[\begin{align*}
d_{\alpha} d_{\alpha} d_{\beta} d_{\gamma} h_{0} &= \frac{1}{2} \gamma_{\alpha\gamma} \gamma_{\alpha\gamma}
\end{align*}\]

\[\begin{align*}
d_{\alpha} d_{\beta} d_{\gamma} h_{0} &= \frac{1}{3} d_{\alpha} d_{\beta} d_{\gamma} h_{0} - \frac{1}{6} \left( \gamma_{\beta\gamma} \gamma_{\alpha\delta} + \gamma_{\gamma\alpha} \gamma_{\beta\delta} + \gamma_{\alpha\beta} \gamma_{\gamma\delta} \right)
\end{align*}\]

\[\begin{align*}
d_{\delta} d_{\gamma} h_{0} &= \frac{1}{6} \left( d_{\delta} h_{0} + d_{\gamma} h_{0} - d_{\gamma} h_{0} \right) - \frac{1}{4} \gamma_{\delta\gamma} \gamma_{\beta\alpha}
\end{align*}\]

\[\begin{align*}
h_{0} &= -\frac{1}{8} \left( \{ \gamma_{\delta\alpha}, \gamma_{\beta\gamma} \} + \{ \gamma_{\gamma\alpha}, \gamma_{\beta\delta} \} + \{ \gamma_{\delta\gamma}, \gamma_{\alpha\beta} \} \right) \\
&\quad + \frac{1}{4} \left( [d_{\beta}, [d_{\gamma}, \gamma_{\delta\alpha}]] + [d_{\alpha}, [d_{\beta}, \gamma_{\delta\gamma}]] + [d_{\alpha}, [d_{\gamma}, \gamma_{\delta\beta}]] \right)
\end{align*}\]

\[\begin{align*}
h_{0} &= \frac{1}{6} \left( \{ \gamma_{\gamma\alpha}, \gamma_{\beta\gamma} \} - [d_{\beta}, [d_{\gamma}, \gamma_{\gamma\alpha}]] - [d_{\alpha}, [d_{\gamma}, \gamma_{\beta\gamma}]] \right)
\end{align*}\]

\[\begin{align*}
h_{0} &= \frac{1}{2} \gamma_{\alpha\gamma} \gamma_{\gamma\alpha}
\end{align*}\]

\[\begin{align*}
d_{\mu} d_{\nu} h_{1} &= -\frac{1}{3} \left( [d_{\mu}, [d_{\nu}, a]] + \gamma_{\mu\nu} a + \frac{1}{2} a \gamma_{\mu\nu} \right) \\
&\quad + \frac{1}{12} \left( [d_{\gamma}, [d_{\mu}, \gamma_{\nu\gamma}]] + [d_{\nu}, \gamma_{\mu\gamma}] + \{ \gamma_{\mu\gamma}, \gamma_{\nu\gamma} \} \right)
\end{align*}\]

\[\begin{align*}
d_{\mu} d_{\mu} h_{1} &= -\frac{1}{3} \left[ d_{\mu}, [d_{\mu}, a] \right] + \frac{1}{6} \gamma_{\mu\alpha} \gamma_{\mu\alpha}
\end{align*}\]

\[\begin{align*}
d_{\mu} h_{1} d_{\nu} &= -\frac{1}{6} \left( [d_{\mu}, [d_{\nu}, a]] + \gamma_{\mu\nu} a + 2 a \gamma_{\mu\nu} \right) \\
&\quad + \frac{1}{12} \left( [d_{\gamma}, [d_{\mu}, \gamma_{\nu\gamma}]] - \{ \gamma_{\mu\gamma}, \gamma_{\nu\gamma} \} + 2 \{ \gamma_{\mu\gamma}, \gamma_{\nu\gamma} \} \right)
\end{align*}\]

\[\begin{align*}
d_{\mu} h_{1} d_{\mu} &= -\frac{1}{6} \left[ d_{\mu}, [d_{\mu}, a] \right] - \frac{1}{6} \gamma_{\mu\gamma} \gamma_{\mu\gamma}
\end{align*}\]

\[\begin{align*}
h_{1} d_{\nu} d_{\mu} &= -\frac{1}{3} \left( [d_{\mu}, [d_{\nu}, a]] + \frac{1}{2} \gamma_{\nu\mu} a + a \gamma_{\nu\mu} \right)
\end{align*}\]
In this appendix we consider the singularities arising from the product of the baryon and meson propagators in the evaluation of the self–energy diagram. The corresponding singularities can best be extracted in Euclidean $d$-dimensional Fourier space [14]. For some general remarks on the treatment of such singular products, see appendix B of [3] and references therein. Let $G_n(x, y)$ and $J_n(x, y)$ be the corresponding expansion coefficients for the meson and baryon propagators at proper time $t^n$, respectively. We thus have to deal with integrals of the type

$$\int d^d x \ G_n(x) \ J_m(x) e^{ikx} .$$

To evaluate it, we need a specific representation of the function $g(x)$, which appears in the heat kernel of the one–dimensional operator in the direction of the four–velocity $v$ [2] introduced to handle the self–energy graphs,

$$g(x) = \int \frac{d^d p}{(2\pi)^{d-1}} \delta(k \cdot v) e^{-ipx} .$$

Straightforward algebra leads to the following list (which extends the one given in ref.[2]) that contains all singular products appearing in $\Sigma_1$ (with $\varepsilon = 4 - d$). Note that derivatives only act on the functions directly to their right:

1. $G_0(x - y) J_0(x - y) \sim -\frac{4}{(4\pi)^2 v^2} \frac{1}{\varepsilon} \delta^d(x - y)$
2. $G_0(x - y) v \cdot \partial J_0(x - y) \sim \frac{4}{(4\pi)^2 v^2} v \cdot \partial \delta^d(x - y)$
3. $\partial_\mu G_0(x - y) J_0(x - y) \sim -\frac{8}{(4\pi)^2} v_\mu v \cdot \partial \delta^d(x - y)$
4. $\partial_\mu G_0(x - y) v \cdot \partial J_0(x - y) \sim -\frac{4}{(4\pi)^2} v_\mu (v \cdot \partial)^2 \delta^d(x - y)$
5. $\partial_\mu G_0(x - y) v \cdot \partial J_1(x - y) \sim -\frac{4}{(4\pi)^2} v_\mu \delta^d(x - y)$
6. $\partial_\mu G_1(x - y) v \cdot \partial J_0(x - y) \sim -\frac{1}{(4\pi)^2} v_\mu \delta^d(x - y)$
7. $\partial_\mu \partial_\nu G_0(x - y) J_0(x - y) \sim -\frac{4}{(4\pi)^2} \delta_{\mu\nu} (v \cdot \partial)^2 \delta^d(x - y)$
\[
\begin{align*}
\partial_\mu \partial_\nu G_0(x-y)v \cdot \partial J_0(x-y) & \sim -\frac{16}{(4\pi)^2 v^2 \varepsilon} \frac{1}{v_\mu v_\nu (v \cdot \partial)^2} \delta^d(x-y) \\
\partial_\mu \partial_\nu G_0(x-y)J_1(x-y) & \sim -\frac{4}{3(4\pi)^2 \varepsilon} \delta_{\mu\nu} (v \cdot \partial)^3 \delta^d(x-y) \\
+ \frac{16}{3(4\pi)^2 v^2 \varepsilon} v_\mu v_\nu (v \cdot \partial)^3 \delta^d(x-y) \\
\partial_\mu \partial_\nu G_0(x-y)v \cdot \partial J_1(x-y) & \sim -\frac{4}{(4\pi)^2 \varepsilon} \delta_{\mu\nu} v \cdot \partial \delta^d(x-y) \\
- \frac{16}{(4\pi)^2 v^2 \varepsilon} v_\mu v_\nu (v \cdot \partial) \delta^d(x-y) \\
\partial_\mu \partial_\nu G_1(x-y)J_0(x-y) & \sim -\frac{1}{(4\pi)^2 \varepsilon} \delta_{\mu\nu} \delta^d(x-y) \\
- \frac{4}{(4\pi)^2 \varepsilon} v_\mu v_\nu \delta^d(x-y) \\
\partial_\mu \partial_\tau G_1(x-y)v \cdot \partial J_0(x-y) & \sim -\frac{2}{(4\pi)^2 \varepsilon} \frac{1}{v_\mu v_\nu (v \cdot \partial)^2} \delta^d(x-y) \\
+ \frac{4}{(4\pi)^2 \varepsilon} v_\mu v_\nu (v \cdot \partial) \delta^d(x-y) \\
\partial_\mu \partial_\nu \partial_\tau G_0(x-y)J_0(x-y) & \sim -\frac{32}{(4\pi)^2 \varepsilon} \frac{1}{v_\mu v_\nu v_\tau (v \cdot \partial)^3} \delta^d(x-y) \\
- \frac{32}{(4\pi)^2 \varepsilon} v_\mu v_\nu v_\tau (v \cdot \partial)^3 \delta^d(x-y) \\
\partial_\mu \partial_\nu \partial_\tau G_0(x-y)v \cdot \partial J_0(x-y) & \sim -\frac{4}{3(4\pi)^2 v^2 \varepsilon} \delta_{\mu\nu} v_\tau (v \cdot \partial)^4 \delta^d(x-y) \\
+ \frac{8}{(4\pi)^2 \varepsilon} v_\mu v_\nu v_\tau (v \cdot \partial)^4 \delta^d(x-y) \\
\partial_\mu \partial_\nu \partial_\tau G_0(x-y)J_1(x-y) & \sim -\frac{16}{3(4\pi)^2 \varepsilon} \delta_{\mu\nu} v_\tau (v \cdot \partial) \delta^d(x-y) \\
+ \frac{32}{(4\pi)^2 \varepsilon} v_\mu v_\nu v_\tau (v \cdot \partial) \delta^d(x-y) \\
\partial_\mu \partial_\nu \partial_\tau G_0(x-y)v \cdot \partial J_1(x-y) & \sim \frac{8}{(4\pi)^2 \varepsilon} \delta_{\mu\nu} v_\tau (v \cdot \partial)^2 \delta^d(x-y) \\
- \frac{48}{(4\pi)^2 \varepsilon} v_\mu v_\nu v_\tau (v \cdot \partial)^2 \delta^d(x-y) \\
\partial_\mu \partial_\nu \partial_\tau G_0(x-y)v \cdot \partial J_2(x-y) & \sim -\frac{8}{3(4\pi)^2 v^2 \varepsilon} \delta_{\mu\nu} v_\tau \delta^d(x-y)
\end{align*}
\]
\[ \partial_\mu \partial_\nu \partial_\tau G_1(x - y) J_0(x - y) \sim + \frac{4}{(4\pi)^2} \delta_{\mu\nu} v_\tau (v \cdot \partial) \delta^4(x - y) \]

\[ \partial_\mu \partial_\nu \partial_\tau G_1(x - y) v \cdot \partial J_0(x - y) \sim - \frac{2}{(4\pi)^2} \delta_{\mu\nu} v_\tau (v \cdot \partial)^2 \delta^4(x - y) \]

\[ \partial_\mu \partial_\nu \partial_\tau G_1(x - y) v \cdot \partial J_1(x - y) \sim \frac{2}{(4\pi)^2} \delta_{\mu\nu} v_\tau \delta^4(x - y) \]

\[ \partial_\mu \partial_\nu \partial_\tau G_2(x - y) v \cdot \partial J_0(x - y) \sim - \frac{1}{(4\pi)^2} \frac{1}{v^2} \delta_{\mu\nu} v_\tau \delta^4(x - y) \]

\[ + \frac{2}{(4\pi)^2} \frac{1}{v^2} v_\mu v_\nu \delta^d(x - y). \]

\section*{C Triple products of singular operators}

The integrals of the type Eq.(5.22) can be worked out in a variety of ways. One possibility is to reduce the triple coincidence integral to a one-dimensional one,

\[ I_3 = \int d^d x \int d^d y \ G_t(x + y) J_m(x) J_n(y) e^{i k \cdot x} e^{i q \cdot y} \]

\[ = \frac{\Gamma(\frac{3-d}{2} + l)}{\sqrt{\pi} (4\pi)^{d/2} (v^2)^{n+m+2}} \sum_{\nu=0}^{\infty} \int_0^1 dt \left( \frac{d - 3 - 2l}{2\nu} \right) t^m \left[ tk + (1 - t)q \right]^{2\nu} (1 - t)^n \times \]

\[ [t(1 - t)(k - q)^2]^{-(3+m+n+l+\nu-d/2)} \Gamma\left( \frac{d - 2}{2} - l - \nu \right) \Gamma\left( \frac{6 - d}{2} + m + n + l + \nu \right). \]

It is then straightforward to extract the more complicated forms when derivative operators are acting on one, two or three of the propagator functions. In fact, only when a sufficient number of derivatives acts on the various propagator functions, one encounters singularities. For the eye graph under consideration, one has up to six derivatives inside the integral. Still, all such terms can be reduced to the form given above and it is thus appropriate to give it here.

Before listing the pertinent results, we give a short example to illustrate the calculational procedure. Consider the product

\[ \int d^d y \int d^d x \ S \cdot \partial G_t(x + y) J_m(x) J_n(y) e^{i k \cdot x} e^{i q \cdot y} \]

\[ = -\frac{1}{2} S_\mu S_\nu \delta^{\mu\nu} \int d^d y \int d^d x \ G_{t-1}(x + y) J_m(x) J_n(y) e^{i k \cdot x} e^{i q \cdot y}, \]
where we have used that $S \cdot v = 0$ to arrive at the second line. This integral is only divergent for $l = m = n = 0$ so that use of Eq. (C.1) leads to

$$
\int d^4 y \int d^4 x \, S \cdot \partial S \cdot \partial G_0(x+y) \, J_0(x) \, J_0(y) \, e^{i k x} \, e^{i q y} \sim \frac{\Gamma(1-d/2)}{\sqrt{\pi}} \frac{1}{(4\pi)^{d/2} v^4} \, \Gamma(d/2) \, \Gamma(\varepsilon/2) \, . \quad (C.3)
$$

Using $v^4 = 1$ and working out the product of the various $\Gamma$–functions and reinstating all prefactors, one arrives at the result given in Eq. (C.4) below.

Straightforward algebra allows then to extract in a similar fashion the pertinent triple products (in our case with one meson and two nucleon propagators):

$$
S \cdot \partial S \cdot \partial G_0(x+y) \, J_0(x) \, J_0(y) \sim - \frac{4 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \, \delta^d(x) \, \delta^d(y) \quad (C.4)
$$

$$
G_0(x+y) \, v \cdot \partial J_0(x) \, v \cdot \partial J_0(y) \sim \frac{4}{v^4} \, \frac{1}{(4\pi)^{d/2}} \frac{1}{\epsilon} \, \delta^d(x) \, \delta^d(y) \quad (C.5)
$$

$$
S \cdot \partial S \cdot \partial G_0(x+y) \, v \cdot \partial J_0(x) \, J_0(y) \sim \frac{4 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \left[ v \cdot \partial \delta^d(x) \, \delta^d(y) \right. \\
\left. + 2 \delta^d(x) \, v \cdot \partial \delta^d(y) \right] \quad (C.6)
$$

$$
S \cdot \partial S \cdot \partial G_0(x+y) \, v \cdot \partial J_0(x) \, v \cdot \partial J_0(y) \sim - \frac{4 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \left[ (v \cdot \partial)^2 \delta^d(x) \, \delta^d(y) \right. \\
\left. + \delta^d(x) \, (v \cdot \partial)^2 \delta^d(y) \right] \quad (C.7)
$$

$$
S \cdot \partial S \cdot \partial G_1(x+y) \, v \cdot \partial J_0(x) \, v \cdot \partial J_0(y) \sim - \frac{2 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \, \delta^d(x) \, \delta^d(y) \quad (C.8)
$$

$$
S \cdot \partial S \cdot \partial G_0(x+y) \, v \cdot \partial J_1(x) \, v \cdot \partial J_0(y) \sim \frac{4 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \, \delta^d(x) \, \delta^d(y) \quad (C.9)
$$

$$
\partial^2 \, S \cdot \partial S \cdot \partial G_0(x+y) \, J_0(x) \, J_0(y) \sim \frac{4 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \left[ 3(v \cdot \partial)^2 \delta^d(x) \, \delta^d(y) \right. \\
\left. + 4 \delta^d(x) \, (v \cdot \partial)^2 \delta^d(y) \right] \quad (C.10)
$$

$$
\partial^2 \, S \cdot \partial S \cdot \partial G_1(x+y) \, J_0(x) \, J_0(y) \sim \frac{10 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \, \delta^d(x) \, \delta^d(y) \quad (C.11)
$$

$$
\partial^2 \, S \cdot \partial S \cdot \partial G_0(x+y) \, J_1(x) \, J_0(y) \sim - \frac{4 \, S_{\mu} S_{\nu} \, \delta_{\mu\nu}}{3 \, v^2 \, (4\pi)^{d/2}} \frac{1}{\epsilon} \, \delta^d(x) \, \delta^d(y) \quad (C.12)
$$
where the perpendicular derivative is defined as follows:

\[
\partial_{\perp}^2 := \left( \delta_{\mu\nu} - \frac{v_\mu v_\nu}{v^2} \right) \partial_\mu \partial_\nu \tag{C.24}
\]
D  Eye graph contributions

In this appendix, we list the resulting contribution of the eye graphs to the divergent part of the generating functional. First, we consider the part of the second order insertion $T_{(2)}$ which does not contain any derivative acting on the nucleon propagator. These are the monomials proportional to $c_1, ..., c_7$. Furthermore we introduce the notation $T_{(2)} \equiv T_{(2)} S$ which shows explicitly the spin–dependence, with $S \in \{1, S^\mu, [S^\mu, S^\nu]\}$. Thus, the so redefined $T_{(2)}$ does not contain any spin–matrices. We find after some algebra the final result:

$$
\tilde{\Sigma}_{1,1}^{(2)} = -v \cdot u \langle v \cdot u T_{(2)} \rangle + \frac{1}{2} \langle (v \cdot u)^2 \rangle \langle T_{(2)} \rangle \\
+ g_A^2 \left[ -4 S^\mu \left( \langle \Gamma_{\mu\nu} T_{(2)} \rangle - \Gamma_{\mu\nu} \langle T_{(2)} \rangle \right) S^\nu + 2 S^\nu \eta(T_{(2)}) S^\nu \right] \\
+ \frac{4}{3} S^\nu \left( -6v \cdot \nabla \langle T_{(2)} \rangle v \cdot \nabla + 3v \cdot \nabla \langle T_{(2)} \rangle v \cdot \nabla \right) \\
+ 2\langle [v \cdot \nabla, [v \cdot \nabla, T_{(2)}]] \rangle - [v \cdot \nabla, [v \cdot \nabla, T_{(2)}]] \rangle S^\nu \\
- \frac{2}{3} ig_A^3 S^\kappa \{S, S^\nu\} S^\kappa \left( 3\langle \{T_{(2)}, u_\rho\} \rangle v \cdot \nabla + \text{ h.c.} \right) \\
+ \frac{3}{2} \{T_{(2)}, u_\rho\} v \cdot \nabla + \text{ h.c.} - \frac{1}{2} [u_\rho, [v \cdot \nabla, T_{(2)}]] + \frac{1}{2} [[v \cdot \nabla, u_\rho], T_{(2)}] \\
+ S^\kappa \{S, S^\rho\} S^\kappa \left( -\frac{3}{2} [T_{(2)}, u_\rho] v \cdot \nabla + \text{ h.c.} - \langle \{u_\rho, [v \cdot \nabla, T_{(2)}]\} \rangle \\
+ \frac{1}{2} \{u_\rho, [v \cdot \nabla, T_{(2)}]\} + \langle \{T_{(2)}, [v \cdot \nabla, u_\rho]\} - \frac{1}{2} \{T_{(2)}, [v \cdot \nabla, u_\rho]\} \rangle \right) \\
+ g_A^4 \left[ \frac{4}{3} S^\nu \left( -2\langle T_{(2)} (S \cdot u)^2 + (S \cdot u)^2 T_{(2)} \rangle + T_{(2)} (S \cdot u)^2 + (S \cdot u)^2 T_{(2)} \right) S^\nu \right] \\
- \frac{4}{3} S^\nu \left( 2\langle S \cdot u T_{(2)} S \cdot u \rangle - S \cdot u T_{(2)} S \cdot u \rangle S^\nu \right). \\
(D.1)
$$

We are considering now the part which contains exactly one derivative. The operator

$$
\frac{-ig_A}{2m} \{S \cdot \nabla, v \cdot u\}
$$

leads to the following contributions:

$$
\tilde{\Sigma}_{1,2}^{(2)} = \frac{ig_A}{2m} \left[ \langle (v \cdot u)^2 \rangle v \cdot u S \cdot \nabla + \text{ h.c.} \right] \\
+ \frac{g_A^2}{2m} \left[ \frac{3}{4} \langle (v \cdot u)^2 \rangle \eta(1) - \frac{3}{4} \langle v \cdot u \eta(v \cdot u) \rangle - 6v \cdot \nabla \langle (v \cdot u)^2 \rangle v \cdot \nabla \right] \\
+ 5\langle v \cdot u [v \cdot \nabla, [v \cdot \nabla, v \cdot u]] \rangle + 3 \langle [v \cdot \nabla, v \cdot u] [v \cdot \nabla, v \cdot u] \rangle
$$
\[
+ \frac{g_2^2}{2m} [S^\mu, S^\nu] \left[ -2\langle (v \cdot u)^2 \rangle \Gamma_{\mu\nu} - 2\langle \Gamma_{\mu\nu}v \cdot u \rangle v \cdot u \right]
+ i\frac{g_3^3}{2m} \left[ 2\langle v \cdot u, S \cdot u \rangle v \cdot u v \cdot \nabla + \text{h.c.} + \frac{1}{3} [v \cdot \nabla, [v \cdot \nabla, v \cdot u]] S \cdot \nabla + \text{h.c.} \right]
\]

\[
- v \cdot v \nabla \{ S \cdot \nabla, v \cdot u \} v \cdot \nabla - \frac{2}{3} [v \cdot \nabla, [v \cdot \nabla, v \cdot u]] S \cdot \nabla + \text{h.c.}
\]

\[
- \frac{1}{3} v \cdot u \langle [v \cdot \nabla, S^\mu \Gamma_{\mu\nu} v^\nu] \rangle - 3\langle v \cdot u [v \cdot \nabla, S^\mu \Gamma_{\mu\nu} v^\nu] \rangle
\]

\[
- \frac{1}{2} \langle \gamma(v \cdot u)S \cdot \nabla + \text{h.c.} + \frac{2}{3} ([\nabla^\mu, \Gamma_{\mu\nu} S^\nu] v \cdot u) \rangle
\]

\[
+ \frac{g_3^3}{2m} \left[ - \frac{1}{2} v_\mu \epsilon^{\mu\nu\sigma} \langle \Gamma_{\mu\nu} v \cdot u \rangle \nabla_\sigma + \text{h.c.} \right]
\]

\[
+ \frac{g_4^4}{2m} \left[ - \frac{1}{6} \langle v \cdot u u_\nu \rangle^2 - \frac{5}{12} \langle (v \cdot u)^2 \rangle + \frac{7}{12} \langle (v \cdot u)^2 \rangle \rangle u \cdot u \right]
\]

\[
+ \frac{\{ S^\mu, S^\nu \}}{4} \left[ -18 v \cdot v \nabla \langle v \cdot u u_\nu \rangle \nabla_\mu - 18 \tilde{\nabla}_\mu \langle v \cdot u u_\nu \rangle v \cdot \nabla \\
+ [\{ v \cdot \nabla, v \cdot u, u_\nu \}] \nabla_\mu + \text{h.c.} - \langle v \cdot u, [v \cdot \nabla, u_\nu \rangle \nabla_\mu + \text{h.c.} \rangle
\]

\[
- 3\langle v \cdot u, [\nabla_\mu, u_\nu \rangle v \cdot \nabla + \text{h.c.} - \langle v \cdot u, u_\nu \rangle \langle \Gamma_{\mu\tau} v^\tau \rangle - 7\langle \{ v \cdot u, u_\nu \} \Gamma_{\mu\tau} v^\tau \rangle
\]

\[
+ 6\langle [v \cdot \nabla, v \cdot u] [\nabla_\mu, u_\nu \rangle] + 9\langle \{ \nabla_\mu, v \cdot u \} [v \cdot \nabla, u_\nu \rangle + 12\langle v \cdot u[v \cdot \nabla, [\nabla_\mu, u_\nu \rangle)]
\]

\[
+ 9\langle [v \cdot \nabla, [\nabla_\mu, v \cdot u \rangle] u_\nu \rangle \rangle u \cdot u \rangle
\]

\[
+ \frac{g_4^4}{2m} [S^\mu, S^\nu] \left[ \frac{1}{2} \langle [u_\mu, u_\nu] v \cdot u \rangle v \cdot u + \frac{1}{2} \Gamma_{\mu\tau} v^\tau \langle u_\nu v \cdot u \rangle - \frac{1}{4} \langle \Gamma_{\mu\tau} v^\tau \rangle \rangle u \cdot v \cdot u \rangle
\]

\[
- \frac{1}{2} v \cdot \nabla \langle v \cdot u u_\nu \rangle \nabla_\mu - \frac{1}{2} \tilde{\nabla}_\mu \langle v \cdot u, u_\nu \rangle v \cdot \nabla
\]

\[
+ \frac{1}{4} \langle [v \cdot \nabla, [v \cdot u \rangle u_\nu \rangle \nabla_\mu + \text{h.c.} - \frac{1}{4} \langle v \cdot u [v \cdot \nabla, u_\nu \rangle \nabla_\mu + \text{h.c.} \rangle
\]

\[
- \frac{3}{4} \langle v \cdot u [\nabla_\mu, u_\nu \rangle v \cdot \nabla + \text{h.c.} + \frac{1}{6} [[v \cdot \nabla, v \cdot u], [\nabla_\mu, u_\nu \rangle] + \frac{1}{4} [[[\nabla_\mu, v \cdot u], [v \cdot \nabla, u_\nu \rangle]]
\]

\[
+ \frac{1}{4} [[v \cdot \nabla, [\nabla_\mu, v \cdot u \rangle, u_\nu \rangle] - \frac{1}{6} [[v \cdot \nabla, [\nabla_\mu, u_\nu \rangle], v \cdot u] - \frac{1}{6} [[[\nabla_\mu, [v \cdot \nabla, u_\nu \rangle], v \cdot u]]
\]

\[
+ i\frac{g_5^5}{2m} \left[ - \frac{1}{24} \langle (v \cdot u)^2 \rangle v \cdot u S \cdot \nabla + \text{h.c.} + \frac{1}{8} \langle u \cdot u \rangle v \cdot u S \cdot \nabla + \text{h.c.} \right]
\]

\[
\frac{1}{4} \langle v \cdot u u_\mu \rangle S \cdot u \nabla^\mu + \text{h.c.} - \frac{1}{4} \langle (v \cdot u)^2 \rangle S \cdot u v \cdot \nabla + \text{h.c.}
\]

\[
- \frac{1}{12} \langle v \cdot u S \cdot u \rangle u_\mu \nabla^\mu + \text{h.c.} + \frac{1}{6} \langle v \cdot u S \cdot u \rangle v \cdot u v \cdot \nabla + \text{h.c.}
\]

\[
- \frac{1}{12} \langle u_\mu S \cdot u \rangle v \cdot u \nabla^\mu + \text{h.c.} - \frac{1}{12} \langle v \cdot uu_\mu \rangle u^\mu S \cdot \nabla + \text{h.c.}
\]
leads to the following contributions:

Consider next the part which contains exactly two derivatives. The operator

$$\frac{1}{2m} \left[ (v \cdot \nabla)^2 - \nabla^2 \right]$$

leads to the following contributions:

\[ \Sigma^{(2)}_{1,3} = \frac{1}{2m} \left[ \nabla_\mu \langle (v \cdot u)^2 \rangle \nabla^\mu - 4v \cdot \nabla \langle (v \cdot u)^2 \rangle v \cdot \nabla ight. 
\]

\[ - \frac{1}{2} [v \cdot u, [\nabla_\mu, v \cdot u]] \nabla^\mu + \text{h.c.} + 2[v \cdot u, [v \cdot \nabla, v \cdot u]] v \cdot \nabla + \text{h.c.} 
\]

\[ - \langle v \cdot u [\nabla_\mu, [v \cdot \nabla, v \cdot u]] \rangle + 4 \langle v \cdot u [v \cdot \nabla, [v \cdot \nabla, v \cdot u]] \rangle 
\]

\[ + \frac{3}{8} \langle (v \cdot u)^2 \rangle \eta(1) - \frac{3}{8} \langle v \cdot u \eta(v \cdot u) \rangle 
\]

\[ + \frac{ig_A}{2m} \left[ 3 \langle v \cdot u S \cdot u \rangle v \cdot u v \cdot \nabla + \text{h.c.} - \frac{3}{2} \langle [v \cdot u, S \cdot u] [v \cdot \nabla, v \cdot u] \rangle + \eta(v \cdot u) S \cdot \nabla + \text{h.c.} 
\]

\[ - v \cdot u \eta(1) S \cdot \nabla + \text{h.c.} - 8 \langle v \cdot u, S^\mu \Gamma_\mu v^\nu \rangle v \cdot \nabla + \text{h.c.} + 2 \langle v \cdot u, S^\mu \Gamma_\mu \nabla^\nu \rangle + \text{h.c.} 
\]

\[ - \frac{14}{3} \langle v \cdot u, [\nabla^\mu, S^\nu \Gamma_{\mu\nu}] \rangle - \frac{64}{3} \langle v \cdot u [v \cdot \nabla, S^\mu \Gamma_{\mu \nu} v^\nu] \rangle + 20 \langle S^\mu \Gamma_\mu v^\nu [v \cdot \nabla, v \cdot u] \rangle 
\]

\[ - 4 \langle S^\mu \Gamma_\mu [\nabla^\nu, v \cdot u] \rangle + 4 \langle v \cdot \nabla, v \cdot u \rangle \langle S^\mu \Gamma_\mu v^\nu \rangle + 8 v \cdot u \langle [v \cdot \nabla, S^\mu \Gamma_{\mu \nu} v^\nu] \rangle 
\]

\[ + 8 v \cdot \nabla \{ S \cdot \nabla, v \cdot u \} v \cdot \nabla - 4 \langle S \cdot \nabla, [v \cdot \nabla, v \cdot u] \rangle v \cdot \nabla + \text{h.c.} 
\]

\[-4 \langle v \cdot \nabla, [v \cdot \nabla, v \cdot u] \rangle S \cdot \nabla + \text{h.c.} \]

\[ + \frac{g_A^2}{2m} \left[ -24 \langle v \cdot \nabla \rangle^2 (v \cdot \nabla)^2 + \frac{9}{2} v \cdot \nabla \langle \nabla^\mu v \cdot \nabla \rangle v \cdot \nabla + \frac{9}{2} \langle \nabla_\mu v \cdot \nabla \rangle v \cdot \nabla \nabla^\mu v 
\]

\[ + \frac{9}{8} \langle [\nabla_\mu, \Gamma_{\mu \nu \nu}] \rangle v \cdot \nabla + \text{h.c.} - \frac{3}{8} \langle [v \cdot \nabla, \Gamma_{\mu \nu \nu}] \rangle \nabla^\mu + \text{h.c.} 
\]

\[-\frac{7}{6} \langle [v \cdot u, w^\nu] v^\nu \Gamma_{\nu \nu} \rangle - \frac{3}{8} \langle (v \cdot u)^2 \rangle^2 + \frac{3}{8} \langle (v \cdot u)^2 \rangle \langle u \cdot u \rangle + \frac{9}{2} \langle \Gamma_{\mu \nu} v^\nu \rangle^2 
\]

\[ + 5 \Gamma_{\mu \nu} \langle \Gamma_{\nu \nu} v^\nu \rangle - \frac{15}{4} \left( \frac{3}{32} \langle \chi^+ \rangle^2 + \frac{1}{8} \langle \chi^+ \rangle \langle u \cdot u \rangle + \frac{1}{8} \langle u \cdot u \rangle + \frac{1}{8} \langle u \cdot u \rangle \right) 
\]

\[ - \frac{5}{24} \langle 1 \rangle_{(1)_{\mu \nu}} - \frac{35}{12} v^\mu \eta(1)_{(1)_{\mu \nu}} + \frac{49}{12} \langle \Gamma_{\mu \nu} \Gamma_{\mu \nu} \rangle - \frac{46}{3} \langle v^\mu \Gamma_{\mu \nu} v^\nu \rangle 
\]

\[ - \frac{21}{12} \langle 1 \rangle_{(1)_{\mu \nu}} - \frac{35}{12} v^\mu \eta(1)_{(1)_{\mu \nu}} + \frac{49}{12} \langle \Gamma_{\mu \nu} \Gamma_{\mu \nu} \rangle - \frac{46}{3} \langle v^\mu \Gamma_{\mu \nu} v^\nu \rangle 
\]
\[-\frac{31}{4}[\nabla^\mu, \Gamma_{\mu\nu}v^\nu] v \cdot \nabla + \text{h.c.} + \frac{7}{3}[\nabla^\mu, \Gamma_{\mu\nu}] \nabla^\nu + \text{h.c.} + \frac{5}{4}[v \cdot \nabla, \Gamma_{\mu\nu}v^\nu] \nabla^\mu + \text{h.c.}\]

\[-4v \cdot \tilde{\nabla} \langle (v \cdot u)^2 \rangle v \cdot \nabla - 2v \cdot \tilde{\nabla} \langle v \cdot u u_\mu \rangle \nabla^\mu - 2 \tilde{\nabla}_{\mu} \langle u^\mu v \cdot u \rangle v \cdot \nabla \]

\[+ \frac{10}{3} v \cdot u [v \cdot \nabla, [v \cdot \nabla, v \cdot u]] + 2 \langle [v \cdot \nabla, v \cdot u] [v \cdot \nabla, v \cdot u] \rangle - \frac{4}{3} \langle v \cdot u [v \cdot \nabla, [\nabla^\mu, u_\mu]] \rangle + \frac{7}{3} \eta(1) \Gamma_{\mu\nu} - \frac{7}{2} \eta(1) \Gamma_{\mu\nu} + \eta(1) \mu \nabla^\nu + \text{h.c.} - \frac{14}{3} [\Gamma_{\mu\nu}, \Gamma^\nu\kappa]
\]

\[+ [v^\kappa \Gamma_{\kappa\mu}, v^\nu \Gamma_{\nu\mu}] + \frac{1}{3} \langle [\nabla^\kappa, [\nabla^\kappa, \Gamma_{\mu\nu}]] + 16 \langle v \cdot \nabla, [v \cdot \nabla, \Gamma_{\mu\nu}] \rangle \rangle \]

\[+ i g_3^3 \left[ \frac{1}{6} \left( [u_\mu, S \cdot u] [\nabla^\mu, v \cdot u] - \frac{1}{6} \langle [v \cdot u, S \cdot u] [v \cdot \nabla, v \cdot u] + \frac{1}{3} \langle [v \cdot u, u_\mu] [S \cdot \nabla, u^\nu] \rangle \right) + \langle (v \cdot u)^2 \rangle v \cdot u S \cdot \nabla + \text{h.c.} - \langle u_\mu \rangle v \cdot u S \cdot \nabla + \text{h.c.} \right]
\]

\[-v \cdot \tilde{\nabla} \{v \cdot \nabla, S \cdot u\} v \cdot \nabla + \tilde{\nabla}_{\mu} \{v \cdot \nabla, S \cdot u\} \nabla^\mu \]

\[-\frac{1}{6} [S \cdot u, \Gamma_{\mu\nu}v^\nu] \nabla^\mu + \text{h.c.} + \frac{2}{3} \langle v \cdot \nabla, [v \cdot \nabla, S \cdot u] \rangle v \cdot \nabla + \text{h.c.} \]

\[-\frac{1}{6} [v \cdot \nabla, [\nabla^\mu, S \cdot u] \nabla^\mu + \text{h.c.} - \frac{1}{2} \langle [\nabla^\mu, [\nabla^\mu, S \cdot u] \rangle v \cdot \nabla + \text{h.c.} \rangle \]

\[-\frac{1}{2} S \cdot u \langle [\nabla^\mu, \Gamma_{\mu\nu}v^\nu] \rangle - \frac{2}{3} \nabla^\mu, S \cdot u \rangle \langle \Gamma_{\mu\nu}v^\nu \rangle + \frac{1}{6} \langle S \cdot [\nabla, \Gamma_{\mu\nu}v^\nu] \rangle \]

\[-\frac{1}{2} \langle u^\mu \langle S \cdot \nabla, \Gamma_{\mu\nu}v^\nu \rangle \rangle + \frac{7}{3} \langle S \cdot u [\nabla^\mu, \Gamma_{\mu\nu}v^\nu] \rangle \]

\[-\frac{5}{2} \eta(S \cdot u) v \cdot \nabla + \text{h.c.} + \frac{10}{3} S \cdot u (v \cdot \nabla)^3 + \text{h.c.} - \frac{5}{3} [v \cdot \nabla, [v \cdot \nabla, S \cdot u] v \cdot \nabla + \text{h.c.} \]

\[+ \frac{g_3^3}{2m} v_\rho \epsilon^{\rho\mu\nu\sigma} \left[ -\frac{1}{4} \langle [u_\mu, u_\nu] v \cdot u \rangle \nabla_\sigma + \text{h.c.} - \frac{1}{3} v \cdot u \langle u_\mu [\nabla^\nu, u_\sigma] \rangle \right]
\]

\[+ \frac{7}{2} \langle u_\mu \Gamma_{\nu\sigma} \rangle v \cdot \nabla + \text{h.c.} - 2 \langle u_\mu \Gamma_{\nu\sigma} v^\sigma \rangle \nabla_\sigma + \text{h.c.} \]

\[+ \frac{g_3^4}{2m} \left[ \frac{5}{16} \langle (v \cdot u)^2 \rangle \eta(1) - \frac{5}{16} \langle u \cdot u \rangle \eta(1) + \frac{7}{4} \langle \Gamma_{\mu\nu} [u^\mu, u^\nu] \rangle - \frac{7}{2} \langle \Gamma_{\mu\nu} u^\nu [u^\mu, v \cdot u] \rangle \right]
\]

\[+ \langle S^\mu, S^\nu \rangle - \frac{99}{4} v \cdot \tilde{\nabla} \langle u_\mu u_\nu \rangle v \cdot \nabla + \frac{9}{4} \tilde{\nabla}^\kappa \langle u_\mu u_\nu \rangle \nabla_\kappa \]

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also be used here for some parts of the diagram. For that, define
the insertion on the intermediate nucleon line. However, we now proof that Ecker’s method can
has just one nucleon line, where as the latter allows for two propaga-
tors, as it should be due to
in analogy to Eq.(5.1), thus

\[ D \]

One can write
appear, only terms
products of the meson and nucleon propagators. This reveals that terms of the type
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In this appendix, we discuss one possible check on the calculation of certain contributions from

E Check on some eye graph contributions

In this appendix, we discuss one possible check on the calculation of certain contributions from
the eye graph. To be precise, we note that part of the eye graph can be worked out along the
methods used for the tadpole and vertex–corrected selfenergy graphs, and only some remaining
pieces then involve a triple coincidence limit. To be precise, let us take a closer look at the singular
products of the meson and nucleon propagators. This reveals that terms of the type \( v \cdot \partial J_m \) never
appear, only terms \( \sim J_m \) (compare appendix [3]). Terms of the former type can only appear if one
has just one nucleon line, where as the latter allows for two propagators, as it should be due to
the insertion on the intermediate nucleon line. However, we now proof that Ecker’s method can
also be used here for some parts of the diagram. For that, define\(^{13}\)

\[ D_{(2)} \equiv iA_{(2)} \]  

(E.1)
in analogy to Eq.(5.1), thus \( D = i(A_{(1)} + A_{(2)}) \). We seek the inverse of the operator \([A_{(1)} + A_{(2)}]\).
One can write

\[ [A_{(1)} + A_{(2)}]^{-1} = iD^\dagger [DD^\dagger]^{-1} \equiv iD^\dagger \Delta^{-1} \]  

(E.2)

#15Again, the subscripts (i) \((i = 1, 2)\) denote the chiral dimension.
with $\Delta^{-1}$ being a hermitian and positive definite operator. The chiral expansion of this operator takes the form

$$\Delta^{-1} = \Delta_{(-2)}^{-1} - \Delta_{(-2)}^{-1}[D_{(1)}D_{(2)}^\dagger + D_{(2)}D_{(1)}^\dagger] \Delta_{(-2)}^{-1} - \Delta_{(-2)}^{-1}[D_{(2)}D_{(2)}^\dagger] \Delta_{(-2)}^{-1},$$

$$\equiv \Delta_{(-2)}^{-1} + \Delta_{(-1)}^{-1} + O(1). \quad (E.3)$$

The first term on the right hand side of Eq. (E.3) is nothing but the inverse of the elliptic operator considered in the previous paragraph. The two operators in the square brackets defining $\Delta_{(-1)}^{-1}$ are positive definite and hermitean. These have to be handled by the multi–coincidence method described below. Consequently, the inverse of $[A_{(1)} + A_{(2)}]$ can be written as

$$[A_{(1)} + A_{(2)}]^{-1} = iA_{(1)}^\dagger \Delta_{(-2)}^{-1} + iA_{(2)}^\dagger \Delta_{(-2)}^{-1} + iA_{(1)}^\dagger A_{(2)} \Delta_{(-1)}^{-1} + O(1), \quad (E.4)$$

the first term giving the already calculated counterterms at order $q^3$. The terms of order $q^4$ are thus generated by the terms $iA_{(2)}^\dagger \Delta_{(-2)}^{-1} + iA_{(1)}^\dagger A_{(2)} \Delta_{(-1)}^{-1}$. Consider only the first term of this sum. It can be interpreted as follows: By constructing the inverse of $[A_{(1)} + A_{(2)}]$, we have shifted the dependence on the intermediate point $z$, i.e. on the coordinates of the dimension two insertion, into one of the two vertices which connects the meson loop to the nucleon line. Stated differently, part of the eye graph has been transformed into a vertex–corrected type of self–energy diagram and thus can be treated along the lines outlined before. As a check, one can verify that it reproduces the correct result for the free operator $A_{(2)}$, i.e. it leads to the corrected nucleon propagator $S_N^{(2)}$

$$S_N^{(2)} = \frac{i}{2m} \left[1 - \frac{\ell^2}{(v \cdot \ell + i\eta)^2}\right], \quad (E.5)$$

with $\ell$ the small nucleon off–shell momentum and $\eta \rightarrow 0^+$. 

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### F  Tables for the tadpole, self–energy and eye graphs

In this appendix, we list separately the fourth order monomials and their $\beta$–functions arising from the tadpole, the self–energy and the eye graphs, in order. To make the comparison easier, we give the operators in a certain order, like the $A_i$ ($i = 1, 2, \ldots$) only contain combinations of $v_\mu$’s and $u_\mu$’s, all operators with exactly one $\chi_+$ are grouped together (the $C_i$) and so on. Furthermore, all operators are also ordered according to the number of nucleon covariant derivatives $\nabla_\mu$, which is zero, one, two . . . . In the tables, we use the definition $\bar{A} = A - \langle A \rangle / 2$ for $A = \chi_+, F_{\mu\nu}^+$.

#### Table 2: Tadpole counterterms

| $i$ | $\tilde{O}_i^{(4)}$ | $\delta_i$ |
|-----|------------------|-----------|
| A1  | $\langle u_\mu u_\nu \rangle \langle u_\mu^* u_\nu^* \rangle$ | $-1/16m + 2c_3/3 - (c_2 - g_A^2/8m)/6 - 2c_3/3$ |
| A2  | $\langle u_\mu^2 \rangle \langle u_\mu \rangle$ | $-1/16m + c_3/3 - (c_2 - g_A^2/8m)/12 - c_3/3$ |
| A3  | $\langle (v \cdot u)^2 \rangle \langle u_\mu u_\nu \rangle$ | $1/16m + (c_2 - g_A^2/8m)/3$ |
| A4  | $\langle u_\mu^* v \cdot u_\nu \rangle \langle u_\mu v \cdot u_\nu \rangle$ | $1/16m + 2(c_2 - g_A^2/8m)/3$ |
| B1  | $i \langle [w_\mu, w_\nu] \tilde{F}_{\mu\nu}^+ \rangle$ | $-c_3/3 + (c_2 - g_A^2/8m)/24 + c_3/6$ |
| B3  | $i \langle [w_\mu, v \cdot u] \tilde{F}_{\mu\nu}^+ v^\nu \rangle$ | $- (c_2 - g_A^2/8m)/3$ |
| C1  | $\langle u_\mu \cdot u_\nu \rangle \langle \chi_+ \rangle$ | $c_1 - 1/16m + c_3/2 - (c_2 - g_A^2/8m)/4 - c_3$ |
| C2  | $\langle u_\mu \cdot u_\nu \rangle \tilde{\chi}_+$ | $c_5/2$ |
| C3  | $u_\mu^* (u_\mu \tilde{\chi}_+)$ | $- c_5/2$ |
| C4  | $\langle (v \cdot u)^2 \rangle \langle \chi_+ \rangle$ | $1/16m + (c_2 - g_A^2/8m)/2$ |
| D1  | $\langle F_{\mu\nu}^+ \tilde{F}_{\mu\nu}^+ \rangle$ | $-c_3/3 + (c_2 - g_A^2/8m)/12 + c_3/3$ |
| D3  | $\langle v_\mu \tilde{F}_{\mu\nu}^+ v^\nu \tilde{F}_{\nu\mu}^+ \rangle$ | $- (c_2 - g_A^2/8m)/3$ |
| E1  | $\langle \chi_+ \rangle \langle \chi_+ \rangle$ | $3c_1/4 - 3(c_2 - g_A^2/8m)/32 - 3c_3/8$ |
| E2  | $\tilde{\chi}_+ \langle \chi_+ \rangle$ | $c_5/4$ |
| H1  | $iv_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma u_\mu, u_\nu \langle u \cdot u \rangle$ | $(c_1 + 1/4m)/4$ |
| H2  | $iv_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma u_\mu u_\nu \langle u \cdot u \rangle$ | $-5(c_4 + 1/4m)/12$ |
| H3  | $iv_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma [u_\mu, u_\nu] \langle u_\mu u_\nu \rangle$ | $(c_1 + 1/4m)/2$ |
| I1  | $u_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma \tilde{F}_{\mu\nu}^+ \langle u \cdot u \rangle$ | $(1 + c_6)/8m - (c_4 + 1/4m)/2$ |
| I3  | $u_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma (\tilde{F}_{\mu\nu}^+ u_\sigma)$ | $(1 + c_6)/8m - (c_4 + 1/4m)/2$ |
| I4  | $u_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma u_\mu \langle \tilde{F}_{\mu\nu}^+ u_\sigma \rangle$ | $2(c_4 + 1/4m)/3$ |
| I5  | $u_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma (\tilde{F}_{\mu\nu}^+ u_\sigma)$ | $-2(c_4 + 1/4m)/3$ |
| J2  | $iv_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma [u_\mu, u_\nu] \langle \chi_+ \rangle$ | $(c_1 + 1/4m)/4$ |
| K1  | $iv_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma [F_{\mu\nu}^+, F_{\nu\mu}^+]$ | $-(c_4 + 1/4m)/3$ |
| L1  | $u_\rho \epsilon^{\rho\mu\nu\sigma} S_\sigma F_{\mu\nu}^+ \langle \chi_+ \rangle$ | $(1 + c_6)/8m - (c_4 + 1/4m)/2$ |
| N1  | $\langle [S \cdot u, v \cdot u] I^{\mu\nu} \rangle \langle u_\mu, u_\nu \rangle$ | $g_A/12m$ |
Table 2: continued, one derivative

| i  | $\tilde{O}^{(4)}$                                                                 | $\delta_i$                  |
|----|----------------------------------------------------------------------------------|----------------------------|
| C2 | $\langle [u_\mu, S \cdot u] i [\nabla^\mu, v \cdot u] \rangle$               | $g_A/24m$                  |
| C4 | $\langle [v \cdot u, u_\mu] i [\nabla^\mu, S \cdot u] \rangle$               | $g_A/24m$                  |
| C7 | $\langle v \cdot u u_\mu \rangle u^\mu i S \cdot \nabla + h.c.$               | $-g_A/4m$                  |
| C8 | $\langle u \cdot u \rangle v \cdot u i S \cdot \nabla + h.c.$                 | $-g_A/4m$                  |
| D12| $\langle [\nabla^\mu, F_{\mu \nu}^+ S^\nu \cdot S] \cdot v \cdot u \rangle$  | $g_A/12m$                  |
| D13| $\langle [\nabla^\mu, F_{\mu \nu}^+ v^\nu \cdot S^\nu \cdot S] \cdot v \cdot u \rangle$ | $-g_A/12m$                |
| E4 | $\langle \chi^+ \rangle v \cdot u i S \cdot \nabla + h.c.$                   | $-g_A/4m$                  |
| F3 | $[\bar{\chi}^-, v \cdot u \rangle i v \cdot \nabla + h.c.$                  | $-1/48m$                   |
| F4 | $[\bar{\chi}^-, u_\nu \rangle i \nabla^\nu + h.c.$                         | $1/48m$                    |

| two derivatives |
|-----------------|
| A6 | $\langle [v \cdot \nabla, u_\mu] [v \cdot \nabla, u^\mu] \rangle$            | $-2(c_2 - g_A^3/8m)/3$   |
| A7 | $\langle [v \cdot \nabla, [v \cdot \nabla, u_\mu]] u^\mu \rangle$           | $-2(c_2 - g_A^3/8m)/3$   |
| A9 | $\langle [\nabla^\nu, u_\mu] [\nabla^\nu, u^\mu] \rangle$                   | $-2c_3/3 - (c_2 - g_A^3/8m)/3 - 4c_3/3$ |
| A10| $\langle [\nabla^\nu, [\nabla^\nu, u_\mu]] u^\mu \rangle$                  | $-2c_3/3 - (c_2 - g_A^3/8m)/3 - 4c_3/3$ |
| A17| $[u_\mu, [\nabla^\mu, v \cdot u]] v \cdot \nabla + h.c.$                      | $-1/24m$                  |
| A18| $[u_\mu, [\nabla^\mu, u_\nu]] [\nabla^\nu + h.c.$                          | $1/24m$                   |
| B4 | $[\nabla^\mu, F_{\mu \nu}^+ v^\nu \cdot \nabla + h.c.$                      | $1/12m$                   |
| B5 | $[\nabla^\mu, F_{\mu \nu}^+] i \nabla^\nu + h.c.$                           | $-1/12m$                  |
| C2 | $\langle [v \cdot \nabla, [v \cdot \nabla, \chi^+]] \rangle$                | $-(c_2 - g_A^3/8m)/4$    |
| C3 | $\langle [\nabla^\mu, [\nabla^\mu, \chi^+]] \rangle$                        | $-c_3/4 - (c_2 - g_A^3/8m)/8 - c_3/2$ |
| D8 | $i v_\mu \epsilon_{\mu \nu \rho \sigma} S_\sigma[[\nabla_\kappa, u_\mu], [\nabla^\kappa, u_\nu]]$ | $-(c_4 + 1/4m)/3$       |
| D9 | $i v_\mu \epsilon_{\mu \nu \rho \sigma} S_\sigma[[\nabla^\kappa, \nabla_\kappa, u_\mu], u_\nu]]$ | $-(c_4 + 1/4m)/3$       |
| E2 | $v_\mu \epsilon_{\mu \nu \rho \sigma} S_\sigma[[\nabla^\kappa, [\nabla^\kappa, F_{\mu \nu}^+]]$ | $-(c_4 + 1/4m)/3$       |
### Table 3: Self-energy counterterms

| i  | $\hat{O}_i^{(4)}$                                                                 | $\delta_i$                                                                 |
|----|----------------------------------------------------------------------------------|----------------------------------------------------------------------------|
| A1 | $\langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$                      | $g_4^2 c_3/2 - g_4^2 (c_4 + 1/4m)/4 + c_3 g_4^2/2 + c_3 g_4^2/6$            |
|    |                                                                                  | $(c_4 + 1/4m) g_4^2/2 - (c_4 + 1/4m) g_4^2/3$                                |
|    |                                                                                  | $(g_4^2/12)/2m + (g_4^2/6 - g_4^2/4 - 3g_4^2/16)/2m$                       |
| A2 | $\langle u^2 \rangle \langle u^2 \rangle$                                       | $-g_4^2 c_3/2 + g_4^2 (c_4 + 1/4m)/4 - c_3 g_4^2/2 + c_3 g_4^2/4$           |
|    |                                                                                  | $(c_4 + 1/4m) g_4^2/2 + (c_4 + 1/4m) g_4^2/2$                               |
|    |                                                                                  | $(-3g_4^2/8 - g_4^2/8)/2m + (5g_4^2/24 + 5g_4^2/8 + 9g_4^2/32)/2m$          |
| A3 | $\langle (v \cdot u)^2 \rangle \langle u \cdot u \rangle$                      | $g_4^2 (c_3 - c_2 + g_4^2/8m)/2 - g_4^2 (c_4 + 1/4m)/2$                     |
|    |                                                                                  | $+g_4^2 (c_2 - g_4^2/8m + c_3) + c_3 g_4^2/2 + c_3 g_4^2/2$                 |
|    |                                                                                  | $-(c_4 + 1/4m) g_4^2/2 - (c_4 + 1/4m) g_4^2$                                |
|    |                                                                                  | $+(-1/2 + g_4^2/2 + g_4^2/4)/2m + (3g_4^2/4)/2m$                           |
|    |                                                                                  | $+(-29g_4^2/24 - 7g_4^2/8 - 9g_4^2/6)/2m$                                  |
| A4 | $\langle u^\mu v \cdot u \rangle \langle u_\mu v \cdot u \rangle$             | $-g_4^2 (c_3 - c_2 + g_4^2/8m)/2 + g_4^2 (c_4 + 1/4m)/2$                    |
|    |                                                                                  | $-g_4^2 (c_2 - g_4^2/8m + c_3) - c_3 g_4^2/2 - c_3 g_4^2/3$                 |
|    |                                                                                  | $-(c_4 + 1/4m) g_4^2/2 + 2(c_4 + 1/4m) g_4^2$                               |
|    |                                                                                  | $+(1/2 - g_4^2/8 - g_4^2/6)/2m + (-3g_4^2/4)/2m$                           |
|    |                                                                                  | $+(29g_4^2/24 + g_4^2/2 + 3g_4^2/8)/2m$                                    |
| A5 | $\langle (v \cdot u)^2 \rangle \langle (v \cdot u)^2 \rangle$                 | $-c_3 g_4^2/12 + (c_4 + 1/4m) g_4^2/6 + (-g_4^2/24)/2m + (3g_4^2/32)/2m$    |
| B1 | $i [\langle \mu^\nu, \nu^\mu \rangle F_{\mu\nu}^{++} \rangle$                | $-g_4^2 c_3/2 + g_4^2 (c_4 + 1/4m)/4$                                      |
|    |                                                                                  | $+(-g_4^2/8)/2m + (g_4^2/24 + g_4^2/4)/2m$                                 |
| B3 | $i [\langle \mu_\nu, v \cdot u \rangle \tilde{F}_{\mu\nu}^{++} \nu^\nu \rangle$ | $-g_4^2 (c_3 - c_2 + g_4^2/8m)/2 - g_4^2 (c_4 + 1/4m)/2$                  |
|    |                                                                                  | $-g_4^2 (c_2 - g_4^2/8m + c_3)$                                            |
|    |                                                                                  | $+(1/2 + g_4^2/4)/2m + (g_4^2)/2m + (-3g_4^2/24 - g_4^2/4)/2m$              |
|    |                                                                                  | $(1/4)/2m + (g_4^2/4)/2m$                                                   |
| B4 | $[i \langle \mu_\nu, v \cdot u \rangle \langle F_{\mu\nu}^{++} \nu^\nu \rangle]$ | $-c_3 g_4^2/4 + (c_4 + 1/4m) g_4^2/2$                                    |
|    |                                                                                  | $+(-g_4^2/8)/2m + (3g_4^2/16 + 9g_4^2/32)/2m$                              |
|    |                                                                                  | $c_3 g_4^2/4 - (c_4 + 1/4m) g_4^2/2$                                       |
|    |                                                                                  | $+(g_4^2/8)/2m + (-9g_4^2/32)/2m$                                         |
| C1 | $\langle u \cdot u \rangle \langle \chi_+ \rangle$                             | $-c_3 g_4^2/4 + (c_4 + 1/4m) g_4^2/2$                                    |
|    |                                                                                  | $+(-g_4^2/8)/2m + (3g_4^2/16 + 9g_4^2/32)/2m$                              |
| C4 | $\langle (v \cdot u)^2 \rangle \langle \chi_+ \rangle$                         | $c_3 g_4^2/4 - (c_4 + 1/4m) g_4^2/2$                                       |
|    |                                                                                  | $+(g_4^2/8)/2m + (-9g_4^2/32)/2m$                                         |
| D1 | $\langle F_{\mu\nu}^{++} \rangle$                                              | $(g_4^2/24)/2m$                                                           |
| D2 | $\langle \nu^\mu \tilde{F}_{\mu\nu}^{++} \nu^\nu \rangle$                    | $(g_4^2/2)/2m - (7g_4^2/6)/2m$                                             |
| D3 | $\langle \nu^\mu \tilde{F}_{\mu\nu}^{++} \nu^\nu \rangle$                    | $(g_4^2/2)/2m$                                                           |
| E1 | $\langle \chi_+ \rangle \langle \chi_+ \rangle$                                | $+(9g_4^2/64)/2m$                                                       |

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| i  | $\hat{O}^{(4)}_i$                                                                 | $\delta_i$                                                                                   |
|----|-----------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------|
| F1 | $iv_\rho \epsilon^{\mu\nu\sigma} [u_\mu, u_\nu] \langle u_\sigma S \cdot u \rangle$ | $-g_A^2(c_4 + 1/4m)/2$                                                                  |
| F2 | $iv_\rho \epsilon^{\mu\nu\sigma} \langle u_\nu, u_\sigma S \cdot u \rangle u_\mu$    | $-g_A^2(c_4 + 1/4m) + (g_A^4/24)/2m$                                                      |
| F3 | $iv_\rho \epsilon^{\mu\nu\sigma} \langle [u_\mu, u_\nu] u_\sigma \rangle S \cdot u$ | $c_3g_A^4/6 + (-g_A^4/12)/2m + (-g_A^6/24)/2m$                                          |
|    |                                                                                   | $-(c_4 + 1/4m)g_A^3/3$                                                                    |
| G1 | $v_\rho \epsilon^{\mu\nu} F_{\mu\nu}^+(u_\sigma S \cdot u)$                     | $-g_A^3(c_4 + 1/4m)$                                                                      |
| G2 | $v_\rho \epsilon^{\mu\nu} (F_{\mu\nu}^+ S \cdot u) u_\sigma$                     | $-g_A^3(c_4 + 1/4m)$                                                                      |
| G3 | $v_\rho \epsilon^{\mu\nu} (S' \bar{F}_{\tau\sigma}^+ u_\mu) u_\nu$                | $-g_A^3(c_4 + 1/4m)$                                                                      |
| H1 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma [u_\mu, u_\nu] \langle u \cdot u \rangle$ | $c_3g_A^4/2 + (-g_A^4/4 - g_A^4/8 - g_A^6/24)/2m$                                       |
| H2 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma u_\kappa \langle [u_\mu, u_\nu] u_\kappa \rangle$ | $g_A^2c_3 + g_A^2c_3 + (-5g_A^4/12 - g_A^4/4)/2m$                                         |
| H3 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma [u_\nu, u_\sigma] \langle u_\mu u_\tau \rangle$ | $-g_A^2c_3 + g_A^2(c_4 + 1/4m)/2 - g_A^3c_3 + c_3g_A^4/3$                               |
| H4 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma [u_\mu, u_\nu] \langle (v \cdot u)^2 \rangle$ | $-c_3g_A^4/2 + (g_A^4/8 + g_A^6/24)/2m$                                                   |
| H5 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma v \cdot u \langle [u_\mu, u_\nu] v \cdot u \rangle$ | $g_A^2(c_2 - 2g_A^2/8m) - 2g_A^2(c_2 - 2g_A^2/8m + c_3 + (c_4 + 1/4m))$                 |
|    |                                                                                   | $(g_A^2/4)/2m + (-g_A^2/2m + (-9g_A^4/4)/2m$                                               |
|    |                                                                                   | $-g_A^3(c_2 - g_A^2/8m) - g_A^3(c_4 + 1/4m)/2$                                           |
|    |                                                                                   | $+2g_A^2(c_2 - g_A^2/8m + c_3) - c_3g_A^4/3$                                              |
|    |                                                                                   | $+(-g_A^2/4)/2m + (-g_A^2)/2m + (g_A^2/4)/2m$                                            |
| I1 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma F_{\mu\nu}^+ \langle u \cdot u \rangle$   | $-g_A^2/2 - g_A^2/2m$                                                                     |
| I2 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma u_\kappa \langle F_{\mu\nu}^+ u_\kappa \rangle$ | $-g_A^2/2m$                                                                               |
| I3 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma u_\mu \langle F_{\mu\nu}^+ u_\kappa \rangle$ | $c_3g_A^4/2 + (g_A^2/4 - g_A^4/8 - g_A^6/24)/2m$                                       |
| I4 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma u_\kappa \langle F_{\mu\nu}^+ u_\kappa \rangle$ | $g_A^2(c_4 + 1/4m) + (2g_A^2/3)/2m$                                                       |
| I5 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma u_\kappa \langle u_\mu F_{\mu\nu}^+ \rangle$   | $2g_A^2c_3 + (-g_A^2/2)/2m + (-2g_A^2/3)/2m$                                              |
| I6 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma F_{\mu\nu}^+ \langle u_\nu u_\tau \rangle$    | $+2g_A^2c_3 - g_A^2(c_4 + 1/4m) + (g_A^2/2)/2m$                                           |
| I7 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma F_{\mu\nu}^+ \langle (v \cdot u)^2 \rangle$   | $(g_A^2/2)/2m$                                                                           |
| I8 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma F_{\mu\nu}^+ \langle v \cdot u \rangle$       | $2g_A^2(c_2 - g_A^2/8m) + g_A^2(c_4 + 1/4m)$                                             |
|    |                                                                                   | $-4g_A^2(c_2 - g_A^2/8m + c_3)$                                                           |
|    |                                                                                   | $+(-g_A^2/2)/2m + (2g_A^2)/2m + (3g_A^2)/2m$                                             |
| I9 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle F_{\mu\nu}^+ v \cdot v \rangle \rangle u_\nu v \cdot u \rangle$ | $(-g_A^2)/2m$                                                                            |
| I10| $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle F_{\mu\nu}^+ v \cdot v \rangle \rangle u_\nu v \cdot u \rangle$ | $2g_A^2(c_2 - g_A^2/8m) - 4g_A^2(c_2 - g_A^2/8m + c_3)$                                  |
|    |                                                                                   | $-2(c_4 + 1/4m) + (g_A^2/2)/2m$                                                           |
|    |                                                                                   | $+(2g_A^2)/2m + (3g_A^2)/2m$                                                             |
| I11| $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle F_{\mu\nu}^+ v \cdot v \rangle \rangle u_\nu v \cdot u \rangle$ | $-g_A^2(c_4 + 1/4m) - 2(c_4 + 1/4m)$                                                      |
|    |                                                                                   | $+(2g_A^2)/2m + (-3g_A^2)/2m$                                                             |
| I12| $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle u_\mu F_{\mu\nu}^+ \rangle \rangle u_\nu v \cdot u \rangle$ | $c_3g_A^4/2 + (-g_A^2/4)/2m + (-g_A^2/16)/2m$                                             |
| J1 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle u_\mu, u_\nu \rangle (\chi_+)$     | $c_3g_A^4/2 + (-g_A^2/4)/2m + (-g_A^2/16)/2m$                                             |
| K1 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle F_{\mu
u}^+, F_{\mu
u}^+ \rangle$  | $-g_A^2/3)/2m$                                                                            |
| K2 | $iv_\rho \epsilon^{\mu\nu\sigma} S_\sigma \langle F_{\mu
u}^+, F_{\mu
u}^+ \rangle$  | $(g_A^2)/2m$                                                                              |
| L1 | $v_\rho \epsilon^{\mu\nu\sigma} S_\sigma F_{\mu
u}^+ (\chi_+)$                    | $(-g_A^2/2)/2m$                                                                           |
Table 3: continued, one derivative

|   | M1 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle \bar{F}_{\mu\nu}^- v \cdot u \rangle u_\sigma \) | \(-g_A(1 + c_6)/16m\) |
|---|---|---|---|
|   | M2 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle u_\sigma \bar{F}_{\mu\nu}^- \rangle v \cdot u \) | \(-g_A(1 + c_6)/8m + g_A(1 + c_6)/16m\) |
|   | M3 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle u_\sigma v \cdot u \rangle \bar{F}_{\mu\nu}^- \) | \(-g_A(1 + c_6)/8m\) |
|   | M4 | \( iv_\rho \epsilon^{\rho\mu\nu\sigma} \langle \bar{F}_{\mu\nu}^- \rangle v \cdot u \cdot S \) | \(-g_A(1 + c_6)/16m\) |
|   | M5 | \( i\langle S^\mu \bar{F}_{\mu\nu}^- \rangle [u^\nu, v \cdot u] \) | \(g_A(1 + c_6)/4m\) |
|   | M6 | \( i\langle S^\mu \bar{F}_{\mu\nu}^- \rangle [u^\nu, v \cdot u] \) | \(-g_A(1 + c_6)/2m\) |
|   | M7 | \( \langle S^\mu \bar{F}_{\mu\nu}^- \rangle F_{\tau\rho}^+ v^\rho \) | \(g_A(1 + c_6)/2m\) |
|   | M8 | \( \langle S^\mu \bar{F}_{\mu\nu}^- \rangle \tilde{\bar{F}}_{\tau\rho}^+ v^\rho \) | \(-g_A(1 + c_6)/2m\) |
|   | N1 | \( \langle [v \cdot u, S \cdot u] [i\nabla^\mu, u_\mu] \rangle \) | \(4g_Ac_1 - 2g_A(c_4 + 1/4m)/3 - 2(c_4 + 1/4m)g_A/3 + (-g_A/6)/2m + (2g_A^3)/2m + (-g_A/6 - g_A^3/4)/2m\) |
|   | N2 | \( \langle v^\mu \bar{F}_{\mu\nu}^- S^\nu [\nabla^\tau, u_\tau] \rangle \) | \((g_A^3)/2m\) |
|   | A2 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle [u_\mu, u_\nu] u_\sigma \rangle v \cdot \nabla + \text{h.c.} \) | \((c_4 + 1/4m)g_A/2 + (c_4 + 1/4m)g_A^3/2 + (-g_A^3/8)/2m + (g_A^3/2)/2m + (g_A^3/4 + 3g_A^3/8)/2m - c_3g_A^3/3 - g_A(c_4 + 1/4m) - 2(c_4 + 1/4m)g_A^3/3 - g_A^3c_3 + (g_A^3)/6m + (g_A^5)/2m\) |
|   | A6 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle u_\mu [v \cdot \nabla, u_\nu] \rangle u_\sigma \) | \(-g_A(c_4 + 1/4m)/2\) |
| B1 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle \bar{F}_{\mu\nu}^- v \cdot u \rangle [iv \cdot \nabla, u_\sigma] \rangle \) | \(-g_A(c_4 + 1/4m)/2\) |
| B2 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle u_\mu \bar{F}_{\mu\nu}^+ \rangle [iv \cdot \nabla + \text{h.c.} \) | \(-g_A(c_4 + 1/4m)/2\) |
| B3 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle u_\mu \bar{F}_{\mu\nu}^+ \rangle [iv \cdot \nabla + \text{h.c.} \) | \(-g_A(c_4 + 1/4m)/2\) |
| B4 | \( v_\rho \epsilon^{\rho\mu\nu\sigma} \langle u_\mu \bar{F}_{\mu\nu}^+ \rangle [iv \cdot \nabla + \text{h.c.} \) | \(-g_A(c_4 + 1/4m)/2\) |
|   | C1 | \( \langle [u_\mu, S \cdot u] [iv \cdot \nabla, u_\mu] \rangle \) | \(g_A(c_4 + 1/4m) + 2g_A^3c_3/3 + (g_A^3)/3/2m + (-g_A^3)/2m\) |
|   | C2 | \( \langle [u_\mu, S \cdot u] [i\nabla^\mu, v \cdot u] \rangle \) | \(2(c_4 + 1/4m)g_A/3 + g_A(c_4 + 1/4m) + (3g_A)/2m + (-2g_A)/2m + (5g_A^3)/12/2m\) |
|   | C3 | \( \langle [u_\mu, v \cdot u] [iS \cdot v \cdot u] \rangle \) | \(-g_A(c_4 + 1/4m)/3 + (g_A)/6/2m + (-g_A^3)/6/2m\) |
|   | C4 | \( \langle [v \cdot u, u_\mu] [i\nabla^\mu, S \cdot u] \rangle \) | \(g_A(c_4 + 1/4m)/3 + (3g_A)/2m) + (-2g_A)/2m + (g_A + 6g_A^3)/6/2m\) |
|   | C5 | \( \langle [v \cdot u, S \cdot u] [iv \cdot \nabla, v \cdot u] \rangle \) | \(-4g_A(c_4 + 1/4m)/3 - 4g_A(c_2 - g_A^3/8m) + c_3) - 2g_A(c_3)/3 + (-g_A)/2m + (-4g_A/3) - g_A^3/3/2m + (2g_A - g_A^3/6 + g_A^5)/2m\) |
|   | C6 | \( \langle u \cdot u \rangle v \cdot u iS \cdot \nabla + \text{h.c.} \) | \((+g_A)/2m + (g_A^3)/2m\) |
|   | C7 | \( \langle v \cdot u u_\mu \rangle u^\mu iS \cdot \nabla + \text{h.c.} \) | \(+(g_A)/2m\) |
|   | C8 | \( \langle u \cdot u \rangle S \cdot u iv \cdot \nabla + \text{h.c.} \) | \(4c_3g_A + 2c_3g_A^3 - (c_4 + 1/4m)g_A\) |
|   |   |   | \(-2(c_4 + 1/4m)g_A - 2(c_4 + 1/4m)g_A^3 + (-g_A - g_A^3)/2m + (-g_A^3)/2m\) |
Table 3: continued, one derivative

|  | Expression | Result |
|---|------------|--------|
| C9 | $\langle S \cdot uu_{\mu} \rangle u^\mu iv \cdot \nabla + \text{h.c.}$ | $-4c_3g_A - 2c_3g_A^3 + (c_4 + 1/4m)g_A$
|  |  | $-2(c_4 + 1/4m)g_A$
|  |  | $+(g_A^3/3)/2m$
| C10 | $\langle v \cdot u u_{\mu} \rangle S \cdot u i\nabla^\mu + \text{h.c.}$ | $(g_A^3/2m)$
| C11 | $\langle v \cdot u S \cdot u \rangle u_{\mu} i\nabla^\mu + \text{h.c.}$ | $0$ |
| C12 | $\langle u_{\mu} S \cdot u \rangle v \cdot u i\nabla^\mu + \text{h.c.}$ | $+(c_4 + 1/4m)g_A + 2g_A(c_2 - g_A^3/8m) - 2g_A^3c_3$
| C13 | $\langle (v \cdot u)^2 \rangle S \cdot u iv \cdot \nabla + \text{h.c.}$ | $+2(c_4 + 1/4m)g_A^3 - 4g_A(c_2 - g_A^3/8m + c_3) + (2g_A)/2m$
|  |  | $+(3g_A/2 + g_A^3/2m + (-g_A/2 + g_A^3/2)/2m)$
| C14 | $\langle v \cdot u S \cdot u \rangle v \cdot u iv \cdot \nabla + \text{h.c.}$ | $-(c_4 + 1/4m)g_A - 2g_A(c_2 - g_A^3/8m) + 2g_A^3c_3$
|  |  | $+(-2g_A)/2m + 4g_A(c_2 - g_A^3/8m + c_3)$
|  |  | $+(g_A/2)/2m + (g_A/2 - g_A^5/3)/2m$
| C15 | $\langle (v \cdot u)^2 \rangle v \cdot u iS \cdot \nabla + \text{h.c.}$ | $(-g_A^3)/2m$ |
| D2 | $\langle [v \cdot \nabla, u^\mu F_{\mu \nu}^+] S^\nu \rangle v \cdot u$ | $(2g_A)/2m$ |
| D7 | $\langle [v \cdot \nabla, mu F_{\mu \nu}^+] S^\nu \rangle u \cdot u$ | $2g_A(c_4 + 1/4m)/3 + (2g_A)/2m + (5g_A)/3/2m$
|  |  | $+(g_A^3)/2m + (-g_A + g_A^3/3)/2m$
| D9 | $\langle u^\mu F_{\mu \nu}^+ S^\nu [v \cdot \nabla, v \cdot u] \rangle$ | $-2g_A(c_4 + 1/4m) + (-g_A^3)/2m + (3g_A)/2m$
| D10 | $\langle F_{\mu \nu}^+ S^\nu [v \cdot \nabla, u^\mu] \rangle$ | $2g_A(c_4 + 1/4m)$
| D12 | $\langle [\nabla^\mu, F_{\mu \nu}^+] v^\nu \rangle S \cdot u$ | $(g_A/3)/2m$ |
| D13 | $\langle [\nabla^\mu, F_{\mu \nu}^+] v^\nu \rangle S \cdot u$ | $-4g_A(c_4 + 1/4m)/3 + (-g_A)/3)/2m$
| D14 | $\langle [S \cdot \nabla, F_{\mu \nu}^+] v^\nu \rangle u^\mu \rangle$ | $-4g_A(c_4 + 1/4m)/3 + (g_A^3)/2m + (-g_A^3/2)/2m$
| D15 | $\langle \nabla^\nu, F_{\mu \nu}^+ S^\nu, u^\mu \rangle v \cdot \nabla + \text{h.c.}$ | $2g_A(c_4 + 1/4m)/3 + (-g_A)/3)/2m$
| D16 | $\langle \nabla^\nu, F_{\mu \nu}^+ S^\nu, v \cdot u \rangle v \cdot \nabla + \text{h.c.}$ | $+(-g_A)/2m + (g_A^3)/3)/2m$
| D19 | $\langle u^\nu \nabla_{\mu}^+, S^\nu, u_{\nu} \rangle \nabla^\kappa + \text{h.c.}$ | $g_A(c_4 + 1/4m) - 2cg_A + (g_A/2)/2m$
| D20 | $\langle [\nabla^\nu, \nabla^\kappa, F_{\mu \nu}^+] S^\nu, u_{\nu} \rangle$ | $-g_A(c_4 + 1/4m) - 2g_A(c_2 - g_A^3/8m)$
|  |  | $+4g_A(c_2 - g_A^3/8m + c_3)$
| D21 | $\langle F_{\mu \nu}^+ v^\nu \rangle [\nabla^\mu, S \cdot u] \rangle$ | $+(-2g_A)/2m + (-3g_A)/2m + (3g_A)/2m$ |
| D22 | $\langle F_{\mu \nu}^+ v^\nu \rangle [S \cdot \nabla, u^\mu \rangle$ | $(g_A)/2m$ |
| E3 | $\langle \chi^+ \rangle S \cdot u iv \cdot \nabla + \text{h.c.}$ | $-2(c_4 + 1/4m)g_A + 3c_3g_A + (g_A/2)/2m + (-g_A^3/4)/2m$
| E4 | $\langle \chi^+ \rangle v \cdot u iS \cdot \nabla + \text{h.c.}$ | $(g_A)/2m$ |
Table 3: continued, two derivatives

|   |   |   |   |   |
|---|---|---|---|---|
| F1 | $\langle v \cdot u [v \cdot \nabla, \left[ \nabla^\mu, u_\mu \right] \rangle$ | (-1)/2m + $(g_A^2/6)/2m$ |
| F2 | $\langle [v \cdot \nabla, v \cdot u] \left[ \nabla^\mu, u_\mu \right] \rangle$ | (-1)/2m + $(2g_A^2)/2m$ |
| F3 | $\left[ \nabla^\mu, u_\mu \right] v \cdot \nabla + h.c.$ | $-2c_1 - c_5 + (g_A^2/3)/2m + (5g_A^2)/12)/2m$ |
| F4 | $\left[ \nabla^\mu, u_\mu \right], u_\mu \right] \nabla^\nu + h.c.$ | $(-g_A^2/6)/2m$ |
| G1 | $v_\mu \epsilon^{\rho \mu \nu \sigma} S_\rho \langle v \cdot u F^-_{\mu \nu} \rangle v \cdot \nabla + h.c.$ | $(1 + c_0)/2m$ |
| G2 | $v_\mu \epsilon^{\rho \mu \nu \sigma} S_\rho \langle F^-_{\mu \nu} v \cdot u \nabla + h.c. \rangle$ | $-(1 + c_0)/2m$ |
| G3 | $v_\mu \epsilon^{\rho \mu \nu \sigma} S_\rho \langle F^-_{\mu \nu} [v \cdot \nabla, v \cdot u] \rangle$ | $(1 + c_0)/4m$ |
| G4 | $v_\mu \epsilon^{\rho \mu \nu \sigma} S_\rho \langle v \cdot u, [v \cdot \nabla, F^-_{\mu \nu}] \rangle$ | $(1 + c_0)/4m$ |
| A1 | $\langle [v \cdot \nabla, v \cdot u] \left[ v \cdot \nabla, v \cdot u \right] \rangle$ | $c_3 g_A^2 + (1 + g_A^2/2)/2m$ |
| A2 | $\langle v \cdot u \left[ v \cdot \nabla, \left[ v \cdot \nabla, v \cdot u \right] \right] \rangle$ | $(-3g_A^2)/2m + (10g_A^2)/12)/2m$ |
| A3 | $\langle u_\nu \left[ v \cdot \nabla, \left[ \nabla_\mu, v \cdot u \right] \right] \rangle$ | $-2c_1 + 1/4m \rangle g_A^2$ |
| A4 | $\langle v \cdot \nabla, u_\mu \right] \nabla^\mu, v \cdot u \rangle$ | $(g_A^2)/2m$ |
| A5 | $\langle v \cdot \nabla, u_\mu \right] \nabla^\mu, v \cdot u \rangle$ | $-c_3 g_A^2 + (g_A^2)/2m$ |
| A6 | $\langle v \cdot \nabla, u_\mu \right] \nabla^\mu, v \cdot u \rangle$ | $+2(c_1 + 1/4m) g_A^2$ |
| A7 | $\langle u_\mu \left[ v \cdot \nabla, \left[ v \cdot \nabla, u_\mu \right] \right] \rangle$ | $+ (g_A^2)/2m + (9g_A^2)/6 + 11g_A^2)/12)/2m$ |
| A8 | $\langle \left[ v \cdot \nabla, v \cdot u \right] \left[ v \cdot \nabla, u_\mu \right] \rangle$ | $g_A^2 c_3 - 2g_A^2 c_3 + (g_A^2)/6)/2m$ |
| A9 | $\langle \left[ v \cdot \nabla, v \cdot u \right] \left[ v \cdot \nabla, u_\mu \right] \rangle$ | $+10(c_1 + 1/4m) g_A^2/3$ |
| A10 | $\langle v \cdot \nabla, u_\mu \right] \nabla^\mu, u_\mu \rangle \rangle$ | $(g_A^2)/2m + (10g_A^2)/6 + 5g_A^2)/12)/2m$ |
| A11 | $\langle u_\mu \left[ v \cdot \nabla, \left[ v \cdot \nabla, u_\mu \right] \right] \rangle$ | $-(g_A^2)/2m + (5g_A^2)/6)/2m$ |
| A12 | $\langle v \cdot \nabla, u_\mu \right] \nabla^\mu, u_\mu \rangle \rangle$ | $(g_A^2)/2m + (9g_A^2)/2)/2m$ |
| A13 | $\langle v \cdot \nabla, u_\mu \right] \left[ v \cdot \nabla, u_\mu \right] \rangle$ | $4(c_2 - g_A^2)/8m + c_3 + g_A^2$ |
| A14 | $\langle v \cdot \nabla, u_\mu \right] \nabla^\mu, u_\mu \rangle \rangle$ | $+(-3/2)/2m + (g_A^2)/2)/2m$ |
| A15 | $\langle v \cdot \nabla, u_\mu \right] \left[ v \cdot \nabla, u_\mu \right] \rangle$ | $(1/2)/2m + (g_A^2)/2)/2m$ |
| A16 | $\langle [v \cdot \nabla, v \cdot u] \left[ v \cdot \nabla, u_\mu \right] \rangle v \cdot \nabla \rangle$ | $(-1/2)/2m + (-g_A^2/3)/2m + (5g_A^2)/12)/2m$ |
| A17 | $\langle u_\mu \left[ v \cdot \nabla, \left[ v \cdot \nabla, u_\mu \right] \right] \rangle v \cdot \nabla \rangle$ | $(-g_A^2)/6)/2m$ |
Table 3: continued, two derivatives

| i  | $O_i^{(4)}$                                                                 | $\delta_i$                                                                 |
|----|------------------------------------------------------------------------------|----------------------------------------------------------------------------|
| A19| $v \cdot \nabla \langle (v \cdot u)^2 \rangle v \cdot \nabla$               | $-2g_A^2c_3 + 4(c_4 + 1/4m)g_A^2 + (4g_A^2/2m + (4g_A^2)/2m + (9g_A^4/2)/2m - 2g_A^2c_3 - (2c_4 + 1/4m)g_A^2 + (g_A^2)/2m + (-3g_A)^2 - (9g_A^4/2)/2m$ |
| A21| $v \cdot \nabla \langle v \cdot u u_\mu \rangle \nabla^\mu + \text{h.c.}$   |                                                                            |
| A22| $v \cdot \nabla \langle u \cdot u \rangle v \cdot \nabla$                 |                                                                            |
| B3  | $[v \cdot \nabla, F_{\mu \nu}^+ v^\nu] i \nabla^\mu + \text{h.c.}$       | $(g_A^2)/2m$                                                              |
| B4  | $[\nabla^\mu, F_{\mu \nu}^+ v^\nu] iv \cdot \nabla + \text{h.c.}$        | $(2g_A^2 / 3m + (-5g_A^2 / 6m))/2m$                                       |
| B5  | $[\nabla^\mu, F_{\mu \nu}^+] i \nabla^\nu + \text{h.c.}$                 | $(g_A^2 / 3m)/2m$                                                         |
| C2  | $\langle [v \cdot \nabla, [v \cdot \nabla, \chi_+]] \rangle$              | $(3g_A^2 / 8m + (13g_A^2 / 6m))/2m$                                       |
| C3  | $\langle [\nabla^\mu, [\nabla_\mu, \chi_+] \rangle \rangle$              | $(-3g_A^2 / 8m + (9g_A^2 / 16m))/2m$                                      |
| C4  | $v \cdot \nabla \langle \chi_+ v \cdot \nabla \rangle$                    | $(-9g_A^4 / 4m)/2m$                                                      |
| D1  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [v \cdot \nabla, v \cdot u], \nabla_\mu, u_\nu]$ |                                                                            |
| D2  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla, u_\nu$ |                                                                            |
| D3  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [v \cdot \nabla, [\nabla_\mu, v \cdot u]], u_\nu$ | $(-2g_A^2 / 2m)$                                                         |
| D4  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [v \cdot \nabla, \nabla_\mu, u_\nu], v \cdot \nabla$ | $(-2g_A^2 / 2m)$                                                         |
| D5  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [v \cdot \nabla, u_\mu], v \cdot \nabla, u_\nu$ |                                                                            |
| D6  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], \nabla_\nu, u_\lambda]$ | $2c_3g_A^2 + (-g_A^2 - (1/2)/g_A^2)/2m$                                   |
| D7  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], \nabla_\nu, u_\lambda]$ | $(-g_A^2 - (2g_A^4 / 3m)/2m + 4c_3g_A^2 - 2c_3g_A^2 / 3m)$ |
| D8  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, \nabla_\nu, u_\lambda]$ | $(-2g_A^2 / 3m)/2m$                                                      |
| D9  | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [u_\mu, \nabla_\nu, u_\lambda]$ | $(-2g_A^2 / 3m)/2m$                                                      |
| D10 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [v \cdot \nabla, v \cdot u], u_\nu, \nabla_\mu$ | $-2g_A^2 / 2m$                                                           |
| D11 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [v \cdot \nabla, v \cdot u], u_\nu, \nabla_\mu$ |                                                                            |
| D12 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $2c_3g_A^2 + (g_A^2 / 2m + (-g_A^2) / 2m)$                               |
| D13 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D14 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D15 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D16 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D17 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D18 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D19 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, v \cdot u], v \cdot \nabla + \text{h.c.}$ | $(-2g_A^2 / 2m)$                                                         |
| D20 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, [\nabla_\nu, [u_\mu, u_\nu]]$ | $(-2g_A^2 / 2m)$                                                         |
| D21 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, [\nabla_\nu, [u_\mu, u_\nu]]$ |                                                                            |
| D22 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, [\nabla_\nu, [u_\mu, u_\nu]]$ |                                                                            |
| D23 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, [\nabla_\nu, [u_\mu, u_\nu]]$ |                                                                            |
| D24 | $iv_\rho \epsilon_{\mu \nu \lambda} S_\sigma [\nabla_\mu, [\nabla_\nu, [u_\mu, u_\nu]]$ |                                                                            |
Table 3: continued, three and more derivatives

| i | $\tilde{O}_i^{(4)}$ | $\delta_i$ |
|---|---|---|
| E1 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} [v \cdot \nabla, [v \cdot \nabla, F_{\mu\nu}^+]]$ | $(-g_A^2)/2m$ |
| E2 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} [\nabla_{\kappa}^\nu, [\nabla_{\kappa}, \tilde{F}_{\mu\nu}^+]]$ | $(-g_A^3/3)/2m$ |
| E4 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} v \cdot \nabla \tilde{F}_{\mu\nu}^+ v \cdot \nabla$ | $(4g_A^2)/2m$ |
| E6 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} v \cdot \nabla \tilde{F}_{\mu\nu}^+ v \cdot \nabla$ | $(4g_A^2)/2m$ |
| E7 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} [\nabla_{\mu}, [v \cdot \nabla, F_{\mu\nu}^+ v^\nu]]$ | $(4g_A^2)/2m$ |
| E8 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} [\nabla_{\mu}, [\nabla_{\kappa}, \tilde{F}_{\mu\kappa}^+]]$ | $(-2g_A^3/3)/2m$ |
| F1 | $\nu \epsilon^{\rho\mu\nu\sigma} S_{\sigma} ([\nabla_{\mu}, \chi^+]) \nabla_{\nu} + \text{h.c.}$ | $(-3g_A^2/4)/2m$ |
| G1 | $\nu \epsilon^{\rho\mu\nu\sigma} [[v \cdot \nabla, u_{\mu}], [\nabla_{\nu}, u_{\sigma}]]$ | $-g_A(c_4 + 1/4m)/4$ |
| A1 | $[v \cdot \nabla, [v \cdot \nabla, v \cdot u]] iS \cdot \nabla + \text{h.c.}$ | $(4g_A)/2m$ |
| A2 | $[S \cdot \nabla, [v \cdot \nabla, v \cdot u]] iv \cdot \nabla + \text{h.c.}$ | $(4g_A)/2m$ |
| A3 | $[v \cdot \nabla, [v \cdot \nabla, S \cdot u]] iv \cdot \nabla + \text{h.c.}$ | $(2g_A^3/3)/2m$ |
| A6 | $S \cdot u i(v \cdot \nabla)^3 + \text{h.c.}$ | $8g_A c_3/3 - 16(c_4 + 1/4m)g_A/3 + (-4g_A/3)/2m + (-4g_A^3/3)/2m$ |
| A7 | $v \cdot \nabla \{iS \cdot \nabla, v \cdot u\} v \cdot \nabla$ | $(-8g_A)/2m$ |
| B1 | $(v \cdot \nabla)^2 (v \cdot \nabla)^2$ | $(6g_A^2)/2m$ |
Table 4: Eye graph counterterms

| i  | $\hat{O}^{(4)}_{i}$ | $\delta_{i}$                                                                 |
|----|-------------------|-------------------------------------------------------------------------------|
| A1 | $\langle u_\mu u_\nu \rangle \langle u_\mu u_\nu \rangle$ | $-(c_4 + 1/4m)g_A^2/4 - 11(c_4 + 1/4m)g_A^4/16$ $+$ $(g_A^2/24 + 7g_A^4/8 + 15g_A^6/32)/2m$ |
| A2 | $\langle u^2 \rangle \langle u^2 \rangle$            | $+(c_4 + 1/4m)g_A^2/4 - 3c_3g_A^4/16 - 3c_3g_A^6/4$ $-$ $3c_3g_A^4/8 + 11(c_4 + 1/4m)g_A^4/16$ $+$ $(-47g_A^2/48 - 29g_A^4/16 + 45g_A^6/64)/2m$ |
| A3 | $\langle (v \cdot u)^2 \rangle \langle u \cdot u \rangle$ | $+c_3/2 - (c_4 + 1/4m)g_A^2/2 - 11(c_4 + 1/4m)g_A^4/8$ $-$ $3g_A^4(c_2 - g_A^2/8m)/8$ $-$ $3(c_2 - g_A^2/8m)g_A^4/16 + 3c_3g_A^4/16 + 3c_3g_A^6/8$ $-$ $3(c_2 - g_A^2/8m)g_A^2/4$ $+$ $(3/8 + 87g_A^2/24 + 89g_A^4/24 + 45g_A^6/64)/2m$ |
| A4 | $\langle u_\mu v \cdot u \rangle \langle u_\mu v \cdot u \rangle$ | $+(c_4 + 1/4m)g_A^2/2 + 11(c_4 + 1/4m)g_A^4/8$ $+$ $(3/8 - 7g_A^2/4 - 113g_A^4/48 - 15g_A^6/16)/2m$ |
| A5 | $\langle (v \cdot u)^2 \rangle \langle (v \cdot u)^2 \rangle$ | $+(c_2 - g_A^2/8m)/2 - 3(c_2 - g_A^2/8m)g_A^4/16$ $+$ $3(c_2 - g_A^2/8m)g_A^4/8$ $+$ $(-3g_A^2/8 - 5g_A^4/12 - 15g_A^6/64)/2m$ |
| B1 | $i \langle [u^\mu, u^\nu] F_{\mu\nu}^+ \rangle$     | $+(c_4 + 1/4m)g_A^2/4 + (1 + c_6)g_A^4/16m$ $-$ $11(1 + c_6)g_A^4/64m$ $+$ $(-49g_A^2/48 - 7g_A^4/8)/2m$ |
| B2 | $i [u^\mu, u^\nu] \langle F_{\mu\nu}^+ \rangle$    | $-(1 + c_6 + 2c_7)g_A^2/16m - (1 + c_6 + 2c_7)g_A^4/64m$ |
| B3 | $i \langle [u_\mu, v \cdot u] F_{\mu\nu}^+ v^\nu \rangle$ | $-(c_4 + 1/4m)g_A^2/2 - (1 + c_6)g_A^4/8m$ $+$ $(-53g_A^2/12 + 35g_A^4/16)/2m$ $+$ $11(1 + c_6)g_A^4/32m$ |
| B4 | $i [u_\mu, v \cdot u] \langle F_{\mu\nu}^+ v^\nu \rangle$ | $+(1 + c_6 + 2c_7)g_A^2/8m + (1 + c_6 + 2c_7)g_A^4/32m$ $+$ $(g_A^4/16)/2m$ |
| C1 | $\langle u \cdot u \rangle \langle \chi_+ \rangle$ | $-3c_1g_A^4/16 - 3c_1g_A^4/4 - 3c_1g_A^4/2 - 9c_3g_A^2/16$ $+$ $(-15g_A^2/32 - 45g_A^6/64)/2m$ |
| C2 | $\langle u \cdot u \rangle \bar{\chi}_+$          | $-c_5g_A^4/8 + c_5g_A^4/4$                                                   |
| C3 | $u^\mu \langle u_\mu \bar{\chi}_+ \rangle$        | $+3c_5g_A^2/2 + c_5g_A^4/4$                                                 |
| C4 | $\langle (v \cdot u)^2 \rangle \langle \chi_+ \rangle$ | $+c_1 + 3c_1g_A^4/16 - 3c_1g_A^4/4 - 9(c_2 - g_A^2/8m)g_A^4/16$ $+$ $(3/8 + 3g_A^2/4 + 45g_A^4/64)/2m$ |
| C5 | $\langle (v \cdot u)^2 \rangle \bar{\chi}_+$       | $+c_5g_A^4/8 - c_5g_A^4/4$                                                  |
| C6 | $v \cdot u \langle v \cdot u \bar{\chi}_+ \rangle$ | $-c_5 - c_5g_A^4/4$                                                        |
Table 4: continued

| i  | $O_{i}^{[4]}$                                                                 | $\delta_{i}$                                                                 |
|----|-------------------------------------------------------------------------------|------------------------------------------------------------------------------|
| D1 | $\langle F_{\mu \nu}^+ F_{\mu \nu}^+ \rangle$                                | $+(1 + c_{6})g_{A}^{2}/8m - (49g_{A}^{2}/48)/2m$                             |
| D2 | $F_{\mu \nu}^+ \langle F_{\mu \nu}^+ \rangle$                                | $-(1 + c_{6} + 2c_{7})g_{A}^{2}/8m$                                          |
| D3 | $\langle \rho \mu \nu \sigma \rangle \rho_{\mu \nu \sigma} \rho_{\mu \nu \sigma}$ | $-(1 + c_{6})g_{A}^{2}/4m + (23g_{A}^{2}/6)/2m$                              |
| D4 | $\rho \mu \nu \sigma \rho_{\mu \nu \sigma} \rho_{\mu \nu \sigma}$            | $+(1 + c_{6} + 2c_{7})g_{A}^{2}/4m - (5g_{A}^{2}/4)/2m$                      |
| D5 | $\langle \rho \mu \nu \sigma \rangle \rho_{\mu \nu \sigma} \rho_{\mu \nu \sigma}$ | $-(9g_{A}^{2}/8)/2m$                                                         |
| E1 | $\langle \chi_{+}(\chi_{+}) \rangle$                                          | $-9c_{1}g_{A}^{2}/8 - (45g_{A}^{2}/128)/2m$                                 |
| E2 | $\tilde{\chi}_{+}(\chi_{+})$                                                  | $+3c_{5}g_{A}^{2}/8$                                                         |
| F1 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} u_{\mu} u_{\nu} \langle u_{\sigma} S \cdot u \rangle$ | $(c_{4} + 1/4m)g_{A}^{4}/24$                                                 |
| F2 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} \langle [u_{\mu} u_{\nu}] S \cdot u \rangle u_{\sigma}$ | $-3(c_{4} + 1/4m)g_{A}^{4}/8 + (-5g_{A}^{4}/48)/2m$                         |
| F3 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} \langle [u_{\mu} u_{\nu}] S \cdot u \rangle u_{\sigma}$ | $(c_{4} + 1/4m)g_{A}^{4}/24 + (5g_{A}^{4}/48)/2m$                           |
| G1 | $\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} F_{\mu \nu}^+ \langle u_{\sigma} S \cdot u \rangle$ | $(1 + c_{6})g_{A}^{4}/48m$                                                   |
| G2 | $\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} \langle F_{\mu \nu}^+ \rangle u_{\sigma} S \cdot u$ | $+(1 + c_{6} + 2c_{7})g_{A}^{4}/32m$                                         |
| G3 | $\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} \langle F_{\rho \mu \nu}^+ \rangle u_{\rho} S \cdot u$ | $-(1 + c_{6})g_{A}^{4}/16m$                                                  |
| G4 | $\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} \langle F_{\mu \nu}^+ u_{\sigma} \rangle S \cdot u$ | $(1 + c_{6})g_{A}^{4}/48m$                                                   |
| H1 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} S_{\sigma}[u_{\mu} u_{\nu}] \langle u \cdot u \rangle$ | $+c_{3}g_{A}^{2}/2 + c_{3}g_{A}^{2}/8 - (c_{4} + 1/4m)g_{A}^{4}/16$          |
| H2 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} S_{\rho \mu \nu \sigma} u_{\rho} \langle [u_{\mu} u_{\nu} u_{\rho}] \rangle$ | $+(7g_{A}^{2}/8 + 7g_{A}^{2}/8 + 5g_{A}^{4}/48)/2m$                         |
| H3 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} S_{\sigma}[u_{\mu} u_{\nu}] \langle (v \cdot u)^{2} \rangle$ | $-(c_{4} + 1/4m)g_{A}^{4}/4 + (c_{4} + 1/4m)g_{A}^{4}/24$                   |
| H4 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} S_{\rho \mu \nu \sigma} u_{\rho} \langle [u_{\mu} u_{\nu} u_{\rho}] \rangle$ | $+(35g_{A}^{2}/24 + 5g_{A}^{4}/8)/2m$                                       |
| H5 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} S_{\sigma} v \cdot u \langle [u_{\mu} u_{\nu} v \cdot u] \rangle$ | $+(c_{2} - g_{A}^{2}/8m)g_{A}^{2}/2 + (c_{2} - g_{A}^{2}/8m)g_{A}^{4}/8$   |
| H6 | $i\nu_{\rho} \epsilon_{\rho \mu \nu \sigma} S_{\rho \mu \nu \sigma} u_{\rho} \langle [u_{\mu} v \cdot u] \rangle$ | $+(c_{4} + 1/4m)g_{A}^{4}/16$                                                |
|     |                                                                              | $+(c_{4} + 1/4m)g_{A}^{4}/16$                                                |
|     |                                                                              | $+(7g_{A}^{2}/4 + g_{A}^{4}/2)/2m$                                           |
|     |                                                                              | $+(3g_{A}^{2}/2 + 2g_{A}^{4}/8)/2m$                                          |
Table 4: continued

|   | \(O_i^{[4]}\)                                                                 | \(\delta_i\)                                                                 |
|---|-----------------------------------------------------------------------------|------------------------------------------------------------------------------|
|I1 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma F^+_{\mu \nu} \langle u \cdot u \rangle\) | \(+c_3g_A^2 - (1 + c_6)g_A^4/32m + (7g_A^2/4 + 7g_A^4/4)/2m\) |
|I2 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle F^+_{\mu \nu} \rangle \langle u \cdot u \rangle\) | \((1 + c_6 + 2c_7)g_A^4/64m + (1 + c_6 + 2c_7)g_A^3/16m\) |
|I3 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma u^\mu \langle F^+_{\mu \nu} u^\nu \rangle\) | \(-(1 + c_6)g_A^2/8m + (1 + c_6)g_A^4/48m + (-7g_A^3/4)/2m\) |
|I4 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma u_\mu \langle F^+_{\nu \mu} u^\nu \rangle\) | \(- (7g_A^2/3)/2m\) |
|I5 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \bar{F}^+_{\mu \nu} v^\sigma (u \cdot u)^2\) | \((7g_A^3/3)/2m\) |
|I6 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle F^+_{\mu \nu} \rangle \langle (v \cdot u)^2 \rangle\) | \(+ (c_2 - g_A^2/8m)g_A^2 + (1 + c_6)g_A^4/32m + (-g_A^3 - 7g_A^4)/2m\) |
|I7 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle F^+_{\mu \nu} \rangle \langle (v \cdot u)^2 \rangle\) | \(+ (1 + c_6 + 2c_7)/8m - (1 + c_6 + 2c_7)g_A^4/64m\) |
|I8 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma v \cdot u \langle \bar{F}^+_{\mu \nu} v \cdot u \rangle\) | \(- (1 + c_6)/4m - (1 + c_6)g_A^4/48m - (g_A^2)/2m\) |
|I9 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma F^+_{\mu \nu} v^\sigma \langle u_\nu v \cdot u \rangle\) | \(+ (-3g_A^3 + g_A^3/4)/2m\) |
|I10| \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle F^+_{\mu \nu} \rangle \langle u_\nu v \cdot u \rangle\) | \(-g_A^4/8)/2m\) |
|I11| \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma v \cdot u \langle \bar{F}^+_{\nu \mu} u^\nu \rangle\) | \(+ (3g_A^3)/2m\) |
|I12| \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle u_\mu, u_\nu \rangle \langle \chi_+ \rangle\) | \(-c_5g_A^2/2 + c_5g_A^4/8 - c_5g_A^3/4\) |
|J1 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma [u_\mu, u_\nu] \langle \chi_+ \rangle\) | \(+ c_1g_A^2 + c_1g_A^4/12 + c_1g_A^4/6 - (c_4 + 1/4m)g_A^6/16 + (7g_A^3 + 5g_A^4/32)/2m\) |
|K1 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma [F^+_{\mu \nu}, F^+_{\nu \mu}]\) | \((7g_A^3/6)/2m\) |
|L1 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle F^+_{\mu \nu} \rangle \langle \chi_+ \rangle\) | \(+ 2c_1g_A^2 - (1 + c_6)g_A^2/32m + (7g_A^4/4)/2m\) |
|L2 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle F^+_{\mu \nu} \rangle \langle \chi_+ \rangle\) | \(+ (c_1 + c_6 + 2c_7)g_A^4/64m\) |
|L3 | \(v_\rho \epsilon^{\mu \nu \sigma} S_\sigma \langle \bar{F}^+_{\mu \nu} \rangle \langle \chi_+ \rangle\) | \(-c_5g_A^2\) |
|N1 | \(\langle [v \cdot u, S \cdot u] [i \nabla^\mu, u_{\mu}] \rangle\) | \((7g_A/6 + 11g_A^3/24 + 3g_A^5/8)/2m\) |
Table 4: continued, one derivative

| i  | \(\hat{O}_i^{(4)}\)                                                                 | \(\delta_i\)                                                                 |
|----|---------------------------------------------------------------------------------|--------------------------------------------------------------------------------|
| A1 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle [u_\mu, u_\nu] v \cdot u \rangle \nabla_\sigma + \text{h.c.} \) | \((-7g_A^3/8 - 9g_A^5/32)/2m\)                                               |
| A2 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle [u_\mu, u_\nu] u_\sigma \rangle v \cdot \nabla + \text{h.c.} \) | \(+9(c_4 + 1/4m)g_A^3/16 + (7g_A^3/8 - 15g_A^5/16)/2m\)                      |
| A3 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle \nabla_\mu, u_\nu \rangle \langle u_\sigma v \cdot u \rangle \) | \((-g_A^5/8)/2m\)                                                             |
| A4 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle \nabla_\mu, u_\nu \rangle v \cdot u \rangle u_\sigma \) | \((3g_A^5/8)/2m\)                                                             |
| A5 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle [\nabla_\mu, u_\sigma] v \cdot u \rangle \) | \((-g_A^5/8 + 1/4m)g_A^3 + 1/4m)g_A^3/2m\)                                   |
| A6 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle u_\mu [v \cdot \nabla, u_\nu] \rangle u_\sigma \) | \((5g_A^5/4)/2m - (c_4 + 1/4m)g_A^3/2m\)                                     |
| B1 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle F_\mu_\nu \cdot [iv \cdot \nabla, u_\sigma] \rangle \) | \((1 + c_6)g_A^3/32m\)                                                       |
| B2 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle [iv \cdot \nabla, F_\mu_\nu] \rangle u_\sigma \rangle \) | \(- (1 + c_6)g_A^3/32m\)                                                     |
| B3 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle F_\mu_\nu u_\sigma \rangle iv \cdot \nabla + \text{h.c.} \) | \(-9(1 + c_6)g_A^3/32m - (7g_A^5/4)/2m\)                                     |
| B4 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle F_\mu_\nu v \cdot u \rangle i\nabla_\sigma + \text{h.c.} \) | \((g_A^3/4)/2m\)                                                             |
| B5 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle u_\mu \nabla_\nu u^\kappa \rangle i\nabla_\sigma + \text{h.c.} \) | \(g_A^3/2m\)                                                                 |
| B6 | \(v_\mu \varepsilon_{\rho \mu \nu \sigma} \langle F_\mu_\nu^* \rangle u_\sigma iv \cdot \nabla + \text{h.c.} \) | \(+3(1 + c_6 + 2c)g_A^3/32m\)                                                |
| C1 | \(\langle [u_\mu, S \cdot u \rangle [iv \cdot \nabla, u^\mu] \rangle \) | \(- (c_4 + 1/4m)g_A^3/2 + (5g_A^5/4)/2m\)                                    |
| C2 | \(\langle [u_\mu, S \cdot u \rangle [iv \cdot \nabla^\mu, v \cdot u] \rangle \) | \(+ (g_A - 11g_A^3/24)/2m\)                                                  |
| C3 | \(\langle [u_\mu, v \cdot u] [iv \cdot \nabla^\mu, S \cdot u] \rangle \) | \((g_A^5/8)/2m\)                                                             |
| C4 | \(\langle [v \cdot u, u_\mu] [iv \cdot \nabla^\mu, S \cdot u] \rangle \) | \((-7g_A^3/6 + g_A^3/6 - g_A^5/8)/2m - (c_4 + 1/4m)g_A^3/2m\)               |
| C5 | \(\langle [v \cdot u, S \cdot u \rangle [iv \cdot \nabla^\mu, v \cdot u] \rangle \) | \(+ (7g_A^3/6 - 11g_A^3/12 - 13g_A^5/8)/2m\)                                  |
| C6 | \(\langle u \cdot u \rangle v \cdot u S \cdot \nabla + \text{h.c.} \) | \(+ (g_A - g_A^3 + g_A^5/8)/2m\)                                              |
| C7 | \(\langle v \cdot u u_\mu \rangle u^\mu \cdot S \cdot \nabla + \text{h.c.} \) | \(+ (g_A + g_A^3/2 - g_A^5/12)/2m\)                                           |
| C8 | \(\langle u \cdot u \rangle S \cdot u i v \cdot \nabla + \text{h.c.} \) | \(+ c_3g_A^5/2 - (c_4 + 1/4m)g_A^3/2 + (5g_A^5/4)/2m\)                      |
| C9 | \(\langle S \cdot u u_\mu \rangle u^\mu \cdot S \cdot \nabla + \text{h.c.} \) | \((c_4 + 1/4m)g_A^3/2 + (5g_A^5/2 - 5g_A^5/6)/2m\)                           |
| C10 | \(\langle v \cdot u u_\mu \rangle S \cdot u i \nabla^\mu + \text{h.c.} \) | \(+ (g_A + g_A^5/4)/2m\)                                                      |
| C11 | \(\langle v \cdot u S \cdot u \rangle u_\mu i \nabla^\mu + \text{h.c.} \) | \((g_A + g_A^3/12 - g_A^5/12)/2m\)                                            |
| C12 | \(\langle u_\mu S \cdot u \rangle v \cdot u i \nabla^\mu + \text{h.c.} \) | \((- g_A^3/12 - g_A^5/12)/2m\)                                                |
| C13 | \(\langle (v \cdot u)^2 \rangle S \cdot u i v \cdot \nabla + \text{h.c.} \) | \((c_2 - g_A^3/8m)g_A^3/2 + (c_4 + 1/4m)g_A^3/2 + (4g_A - 3g_A^5/2)/2m\)     |
| C14 | \(\langle v \cdot u S \cdot u \rangle v \cdot u i v \cdot \nabla + \text{h.c.} \) | \(- (c_4 + 1/4m)g_A^3/2\)                                                     |
| C15 | \(\langle (v \cdot u)^2 \rangle v \cdot u S \cdot \nabla + \text{h.c.} \) | \((g_A + g_A^3 - g_A^5/24)/2m\)                                               |
Table 4: continued, one derivative

| i  | $\tilde{O}^{(i)}$                                                                 | $\delta_i$                                                                 |
|----|----------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| D1 | $\langle [v \cdot \nabla, F_{\mu\nu}^+ S^\nu] \rangle u^\mu$                    | $-(1 + c_6 + 2c_7) g_A^3 / 24m$                                         |
| D2 | $\langle [v \cdot \nabla, v^\mu F_{\mu\nu}^+ S^\nu] \rangle v \cdot u$          | $(1 + c_6 + 2c_7) g_A^3 / 24m + (-4g_A + g_A^3 / 6) / 2m$                |
| D3 | $\langle [\nabla^{\mu}, F_{\mu\nu}^+ S^\nu] \rangle S \cdot u$                 | $-(g_A^3 / 4) / 2m$                                                     |
| D4 | $\langle F_{\mu\nu}^+ v^{\nu} \rangle [\nabla^\mu, S \cdot u]$                 | $-(g_A^3 / 3) / 2m$                                                     |
| D5 | $\langle F_{\mu\nu}^+ S^\nu \rangle [v \cdot \nabla, u^\mu]$                   | $(1 + c_6 + 2c_7) g_A^3 / 24m$                                         |
| D6 | $\langle v^\mu F_{\mu\nu}^+ S^\nu \rangle [v \cdot \nabla, v \cdot u]$         | $-(1 + c_6 + 2c_7) g_A^3 / 24m + (-2g_A + g_A^3 / 3) / 2m$                |
| D7 | $\langle [v \cdot \nabla, v^\mu F_{\mu\nu}^+ S^\nu] \rangle v \cdot u$         | $-(1 + c_6) g_A^3 / 8m + (32g_A / 3 + 3g_A^3 / 2) / 2m$                   |
| D8 | $\langle [v \cdot \nabla, F_{\mu\nu}^+ S^\nu] \rangle u^\mu \rangle$            | $(1 + c_6) g_A^3 / 8m$                                                   |
| D9 | $\langle v^\mu F_{\mu\nu}^+ S^\nu \rangle [v \cdot \nabla, v \cdot u]$         | $(1 + c_6) g_A^3 / 8m - (10g_A) / 2m$                                    |
| D10| $\langle \tilde{F}_{\mu\nu}^+ S^\nu \rangle [v \cdot \nabla, u^\mu]$          | $-(1 + c_6) g_A^3 / 8m$                                                  |
| D11| $\langle \tilde{F}_{\mu\nu}^+ S^\nu \rangle [\nabla^\mu, v \cdot u]$          | $(2g_A) / 2m$                                                          |
| D12| $\langle [v \cdot \nabla, \tilde{F}_{\mu\nu}^+ S^\nu] r \cdot u\rangle$       | $(-7g_A / 3 + g_A^3 / 3) / 2m$                                          |
| D13| $\langle [\nabla^{\mu}, \tilde{F}_{\mu\nu}^+ S^\nu] S \cdot u \rangle$        | $(5g_A^3 / 4) / 2m$                                                     |
| D14| $\langle [S \cdot \nabla, \tilde{F}_{\mu\nu}^+ v^{\nu}] u^\mu \rangle$        | $-2(2g_A) / 3) / 2m$                                                    |
| D15| $\langle \tilde{F}_{\mu\nu}^+ S^\nu \rangle [v \cdot \nabla] + h.c.$            | $(1 + c_6) g_A^3 / 8m$                                                   |
| D16| $\langle v^\mu \tilde{F}_{\mu\nu}^+ S^\nu \rangle [v \cdot \nabla] + h.c.$     | $-(1 + c_6) g_A^3 / 8m - (4g_A) / 2m$                                   |
| D17| $\langle \tilde{F}_{\mu\nu}^+ S^\nu \rangle [v \cdot u \rangle \nabla^\mu + h.c.$ | $g_A / 2m$                                                              |
| D18| $\langle \tilde{F}_{\mu\nu}^+ v^{\nu} \rangle S \cdot u \rangle \nabla^\mu + h.c.$ | $(g_A^3 / 12) / 2m$                                                     |
| E1 | $\langle \tilde{X}_+ [iv \cdot \nabla, S \cdot u] \rangle$                      | $c_5 g_A^3 / 6$                                                         |
| E2 | $\langle [iv \cdot \nabla, \tilde{X}^+_a] \rangle S \cdot u \rangle$            | $-c_5 g_A^3 / 6$                                                        |
| E3 | $\langle [\tilde{X}_+] S \cdot u iv \cdot \nabla + h.c. \rangle$                | $c_1 g_A^3 + (5g_A^3 / 8) / 2m$                                         |
| E4 | $\langle \tilde{X}_+ v \cdot u is \cdot \nabla + h.c. \rangle$                  | $(-g_A + g_A^3 / 8) / 2m$                                                |
| E5 | $\langle \tilde{X}_+ S \cdot u \rangle iv \cdot \nabla + h.c. \rangle$          | $-3c_5 g_A^3 / 2$                                                       |
| F1 | $\langle v \cdot u [v \cdot \nabla, \nabla^{\mu}, u^\mu \rangle \rangle$        | $(-4g_A^3 / 3 - 3g_A^3 / 2) / 2m$                                       |
| F2 | $\langle [v \cdot \nabla, v \cdot u \rangle \nabla^{\mu}, u^\mu \rangle \rangle$ | $-(g_A^3 - 3g_A^3 / 4) / 2m$                                            |
| F3 | $\langle [\nabla^{\mu}, u^\mu] v \cdot u \rangle v \cdot \nabla + h.c. \rangle$  | $(-31g_A^3 / 16 - 3g_A^3 / 8) / 2m$                                     |
| F4 | $\langle [\nabla^{\mu}, u^\mu], u^\nu \rangle \nabla^\nu + h.c. \rangle$        | $(7g_A^3 / 12) / 2m$                                                     |
Table 4: continued, two derivatives

| i   | $O_i^{(4)}$                                                                 | $\delta_i$                                               |
|-----|------------------------------------------------------------------------------|----------------------------------------------------------|
| A1  | $\langle [v \cdot \nabla, v \cdot u] [v \cdot \nabla, v \cdot u] \rangle$   | $(5g_A^2 + 33g_A^4/4)/2m - 3g_A^2(c_2 - g_A^2/8m)$       |
| A2  | $\langle v \cdot u [v \cdot \nabla, [v \cdot \nabla, v \cdot u]] \rangle$   | $(4 + 25g_A^4/3 + 45g_A^4/4)/2m - 3g_A^2(c_2 - g_A^2/8m)$ |
| A3  | $\langle u^{\mu} [v \cdot \nabla, [\nabla_{\mu}, v \cdot u]] \rangle$     | $(-2g_A^2 - 9g_A^4/8)/2m$                                |
| A4  | $\langle v \cdot u [\nabla^{\mu}, [\nabla_{\mu}, v \cdot u]] \rangle$     | $(-1 - 9g_A^4/8)/2m$                                    |
| A5  | $\langle [v \cdot \nabla, u^{\mu}] [\nabla^{\mu}, v \cdot u] \rangle$     | $(-2g_A^2 - g_A^4/8)/2m$                                 |
| A6  | $\langle [v \cdot \nabla, u^{\mu}] [v \cdot \nabla, u^{\mu}] \rangle$     | $(-35g_A^2/6 - 51g_A^4/8)/2m - 3c_3g_A^2$                |
| A7  | $\langle u^{\mu} [v \cdot \nabla, [v \cdot \nabla, u^{\mu}]] \rangle$     | $(-35g_A^2/6 - 69g_A^4/8)/2m - 3c_3g_A^2$                |
| A8  | $\langle [\nabla^{\nu}, v \cdot u] [\nabla^{\nu}, v \cdot u] \rangle$    | $(3g_A^4/3)/2m$                                          |
| A9  | $\langle [\nabla^{\nu}, u^{\mu}] [\nabla^{\nu}, u^{\mu}] \rangle$        | $(5g_A^2/12 + 3g_A^4/4)/2m$                              |
| A10 | $\langle u^{\mu} [\nabla^{\nu}, [\nabla^{\nu}, u^{\mu}]] \rangle$         | $(5g_A^2/12 + 9g_A^4/8)/2m$                              |
| A11 | $[[v \cdot \nabla, v \cdot u], v \cdot u] v \cdot \nabla + \text{h.c.}$    | $(-2 + 2g_A^4)/2m$                                      |
| A12 | $[[v \cdot \nabla, v \cdot u], u^{\mu}][\nabla^{\mu} + \text{h.c.}}$    | $(5g_A^4/16 - g_A^4/8)/2m$                               |
| A13 | $[[v \cdot \nabla, u^{\mu}], v \cdot u][\nabla^{\mu} + \text{h.c.}}$    | $(5g_A^4/16 - g_A^4/8)/2m$                               |
| A14 | $[[\nabla^{\mu}, v \cdot u], v \cdot u][\nabla_{\mu} + \text{h.c.}}$    | $(-1/2 - g_A^4/8)/2m$                                   |
| A15 | $[[v \cdot \nabla, u^{\mu}], u^{\mu}][v \cdot \nabla + \text{h.c.}}$    | $(-11g_A^4/8)/2m$                                       |
| A16 | $[[\nabla^{\mu}, u^{\nu}], u^{\nu}][\nabla_{\mu} + \text{h.c.}}$        | $(g_A^4/8)/2m$                                          |
| A17 | $[u^{\mu}, [\nabla^{\nu}, v \cdot u]] v \cdot \nabla + \text{h.c.}}$    | $(-31g_A^4/16)/2m$                                      |
| A18 | $[u^{\mu}, [\nabla^{\nu}, u^{\nu}]] \nabla^{\nu} + \text{h.c.}}$        | $(7g_A^4/12)/2m$                                        |
| A19 | $v \cdot \nabla \langle (v \cdot u)^2 \rangle v \cdot \nabla$            | $+9(c_2 - g_A^2/8m)g_A^2/2$                              |
|     | $\nabla_{\mu} \langle (v \cdot u)^2 \rangle \nabla^{\mu}$               | $+(-4 - 10g_A^2 - 135g_A^4/8)/2m$                       |
|     | $v \cdot \nabla \langle v \cdot u u^{\mu} \rangle \nabla^{\mu} + \text{h.c.}}$ | $(1 + 9g_A^4/8)/2m$                                      |
| A22 | $v \cdot \nabla \langle u^{\mu} u^{\nu} \rangle v \cdot \nabla$          | $(-2g_A^2 + g_A^4/4)/2m$                                 |
| A23 | $\nabla_{\mu} \langle u^{\mu} u^{\nu} \rangle \nabla^{\mu}$             | $+9c_3g_A^2/2 + (9g_A^2 + 99g_A^4/8)/2m$                  |
|     | $\nabla_{\mu} \langle u^{\mu} \rangle \nabla^{\mu}$                     | $+(-3g_A^2/2 - 9g_A^4/8)/2m$                            |
| B1  | $\langle [\nabla^{\mu}, F_{\mu}^{\nu} v^{\nu}] \rangle iv \cdot \nabla + \text{h.c.}}$ | $-(9g_A^4/16)/2m$                                      |
| B2  | $\langle [v \cdot \nabla, F_{\mu}^{\nu} v^{\nu}] \rangle i \nabla^{\mu} + \text{h.c.}}$ | $(3g_A^4/16)/2m$                                      |
| B3  | $[v \cdot \nabla, F_{\mu}^{\nu} v^{\nu}] i \nabla^{\mu} + \text{h.c.}}$ | $-(5g_A^2/8)/2m$                                      |
| B4  | $[\nabla^{\mu}, F_{\mu}^{\nu} v^{\nu}] iv \cdot \nabla + \text{h.c.}}$ | $(31g_A^4/8)/2m$                                      |
| B5  | $[\nabla^{\mu}, F_{\mu}^{\nu} v^{\nu}] i \nabla^{\nu} + \text{h.c.}}$ | $-(7g_A^4/6)/2m$                                      |
Table 4: continued, two derivatives

| i   | $\hat{O}_i^{(4)}$                                                                 | $\delta_i$                                                                 |
|-----|----------------------------------------------------------------------------------|--------------------------------------------------------------------------|
| C1  | $[v \cdot \nabla, [v \cdot \nabla, \bar{\chi}^+]]$                              | $c_5 g_A^2$                                                              |
| C2  | $\langle [v \cdot \nabla, [v \cdot \nabla, \chi^+]] \rangle$                    | $-(35 g_A^2/16)/2m - 3c_1 g_A^2$                                        |
| C3  | $\langle [\nabla^\mu, [\nabla^\mu, \chi^+]] \rangle$                           | $-(5 g_A^2/32)/2m$                                                      |
| C4  | $v \cdot \nabla \langle \chi^+ \rangle v \cdot \nabla$                         | $9c_1 g_A^2 + (27 g_A^2/4)/2m$                                           |
| C5  | $v \cdot \nabla \langle \chi^+ \rangle \nabla^\mu$                             | $-(9 g_A^2/8)/2m$                                                       |
| C6  | $v \cdot \nabla \bar{\chi}^+ v \cdot \nabla$                                  | $-3c_5 g_A^2$                                                            |
| D1  | $i v_\rho \rho \mu \nu \sigma \sigma_0 [\nabla^\mu, v \cdot u], [\nabla_\nu, \rho \sigma]$ | $(g_A^4/6)/2m$                                                          |
| D2  | $i v_\rho \rho \mu \nu \sigma [\nabla^\mu, v \cdot u], [v \cdot \nabla, \rho \sigma]$ | $(g_A^4/4)/2m$                                                          |
| D3  | $i v_\rho \rho \mu \nu \sigma [\nabla^\mu, v \cdot u], \rho \sigma_0$          | $(g_A^4/4)/2m$                                                          |
| D4  | $i v_\rho \rho \mu \nu \sigma [\nabla^\mu, v \cdot u], v \cdot \rho \sigma$    | $-(g_A^4/6)/2m$                                                         |
| D5  | $i v_\rho \rho \mu \nu \sigma [\nabla^\mu, v \cdot u], v \cdot \rho \sigma_0$ | $-(g_A^4/6)/2m$                                                         |
| D6  | $i v_\rho \rho \mu \nu \sigma [\nabla^\mu, \rho \sigma_0], [v \cdot \nabla, u \nu]$ | $(8g_A^3 + 17g_A^3/12)/2m - (c_4 + 1/4m) g_A^2/3$                        |
| D7  | $i v_\rho \rho \mu \nu \sigma [u \mu_\nu], [v \cdot \nabla, \rho \sigma_0]$    | $(g_A^4/3)/2m$                                                          |
| D8  | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \mu_\nu], [\nabla^{\kappa}, \rho \sigma]$ | $(8g_A^3 + 23g_A^3/12)/2m - (c_4 + 1/4m) g_A^2/3$                        |
| D9  | $i v_\rho \rho \mu \nu \sigma [u \mu_\nu], [\nabla^{\kappa}, u \nu]$          | $(g_A^2/6 - g_A^4/6)/2m$                                                |
| D10 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], \nabla_\mu + h.c.$       | $(g_A^2/6 - g_A^4/4)/2m$                                                |
| D11 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\mu, u \nu], \nabla_\mu + h.c.$         | $(2g_A^2 + g_A^4)/2m$                                                   |
| D12 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\mu, u \nu], v \cdot \nabla + h.c.$    | $-g_A^4/4)/2m$                                                          |
| D13 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], v \cdot \nabla + h.c.$ | $-(3g_A^2)/2m$                                                          |
| D14 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\mu, u \nu], v \cdot \nabla + h.c.$    | $$(11g_A^2)/4)/2m$$                                                      |
| D15 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], v \cdot \nabla + h.c.$ | $(-g_A^4)/2m$                                                           |
| D16 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\mu, u \nu], v \cdot \nabla + h.c.$    | $$(2g_A^2)/2m$$                                                          |
| D17 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], v \cdot \nabla + h.c.$ | $$(2g_A^2)/2m$$                                                          |
| D18 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], v \cdot \nabla + h.c.$ | $$(+4g_A^2 + g_A^4)/2m$$                                                |
| D19 | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], v \cdot \nabla + h.c.$ | $(c_4 + 1/4m) g_A^2/2 + (-8g_A^2 - 11g_A^4)/2m$                          |
|     | $i v_\rho \rho \mu \nu \sigma_0 [\nabla^\kappa, u \nu], v \cdot \nabla + h.c.$ | $$(g_A^2 + g_A^4)/2m$$                                                  |
Table 4: continued, two derivatives

| i  | $\hat{O}_i^{(4)}$                                                   | $\delta_i$                                      |
|----|-------------------------------------------------------------------|-------------------------------------------------|
| E1 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma [v \cdot \nabla, [v \cdot \nabla, F_{\mu\nu}^+]$ | $-(1 + c_6) g_A^2/12m + (8 g_A^2)/2m$          |
| E2 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma [\nabla_\kappa, [\nabla_\kappa, F_{\mu\nu}^+]]$ | $(g_A^2/6)/2m$                                 |
| E3 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma \langle [v \cdot \nabla, [v \cdot \nabla, F_{\mu\nu}^+]$ | $(1 + c_6 + 2c_7) g_A^2/8m$                    |
| E4 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma v \cdot \nabla F_{\mu\nu}^+ v \cdot \nabla$      | $(1 + c_6) g_A^2/4m - (16 g_A^2)/2m$            |
| E5 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma \nabla_\kappa F_{\mu\nu}^+ \nabla_\kappa$        | $(2 g_A^2)/2m$                                 |
| E6 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma v \cdot \nabla F_{\mu\nu}^+ v \cdot \nabla + h.c.$ | $-(-8 g_A^2)/2m$                               |
| E7 | $\nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma v \cdot \nabla (F_{\mu\nu}^+) v \cdot \nabla$    | $-3(1 + c_6 + 2c_7) g_A^2/8m$                  |
| F1 | $i \nu_\mu \epsilon_{\mu\nu\sigma} S_\sigma \langle [\nabla_\mu, \chi_+] \rangle v \cdot \nabla + h.c.$ | $(3 g_A^2)/4)/2m$                              |

Table 4: continued, three and more derivatives

| i  | $\hat{O}_i^{(4)}$                                                   | $\delta_i$                                      |
|----|-------------------------------------------------------------------|-------------------------------------------------|
| A1 | $[v \cdot \nabla, [v \cdot \nabla, v \cdot u]] i S \cdot \nabla + h.c.$ | $(-4 g_A + g_A^3/3)/2m$                         |
| A2 | $[S \cdot \nabla, [v \cdot \nabla, v \cdot u]] i v \cdot \nabla + h.c.$ | $-4 g_A/2m$                                    |
| A3 | $[v \cdot \nabla, [v \cdot \nabla, S \cdot u]] i v \cdot \nabla + h.c.$ | $-g_A^3/2m$                                    |
| A4 | $[v \cdot \nabla, [\nabla_{\mu}, S \cdot u]] i \nabla_\mu + h.c.$ | $-(g_A^3/6)/2m$                                |
| A5 | $[\nabla_{\mu}, [\nabla_{\mu}, S \cdot u]] i v \cdot \nabla + h.c.$ | $-(g_A^3/2)/2m$                                |
| A6 | $S \cdot u i (v \cdot \nabla)^3 + h.c.$                         | $(10 g_A^3)/3)/2m$                              |
| A7 | $v \cdot \nabla \{i S \cdot \nabla, v \cdot u\} v \cdot \nabla$ | $(8 g_A - g_A^3)/2m$                           |
| A8 | $\nabla_{\mu} \{i v \cdot \nabla, S \cdot u\} \nabla_\mu$ | $g_A^3/2m$                                      |
| A9 | $v \cdot \nabla \{i v \cdot \nabla, S \cdot u\} v \cdot \nabla$ | $-(g_A^3)/2m$                                   |
| B1 | $(v \cdot \nabla)^2 (v \cdot \nabla)^2$                        | $(-24 g_A^3)/2m$                                |
| B2 | $\nabla_{\mu} v \cdot \nabla v \cdot \nabla \nabla_\mu$ | $(9 g_A^3)/2)/2m$                              |
| B3 | $v \cdot \nabla \nabla_{\mu} \nabla_\mu v \cdot \nabla$ | $(9 g_A^3)/2)/2m$                              |
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Figure 1: Contributions to the one-loop generating functional to order $q^4$. The solid (dashed) double lines denote the baryon (meson) propagator in the presence of external fields, respectively. The circle–cross denotes an insertion from $\mathcal{L}_{\pi N}^{(2)}$. First line: Terms of order $\xi^0$. Second line: Terms of order $\xi$. Third line: Irreducible graphs of order $\xi^2$ and $q^3$ in the chiral expansion. Fourth line: Reducible graphs of order $\xi^2$ and $q^4$ in the chiral expansion. Fifth line: Irreducible graphs of order $\xi^2$ and $q^4$ in the chiral expansion.
Figure 2: (a) One-loop graph contributing to the isoscalar magnetic moment. The dot and the circle-cross denote an insertion from the dimension one and two Lagrangian, in order. Nucleons, pions and photons are depicted by solid, dashed and wiggly lines, respectively. (b) Fourth order counterterm with strength $\alpha$ that renormalizes the divergence of the loop graph (a).
Figure 3: Divergent graphs contributing to the scalar form factor of the nucleon. (a) is a genuine counterterm diagram at order $q^4$. (b) and (c) are the tadpole, (d) and (e) the self-energy and (g)–(i) the eye graphs. (f) is a counterterm from the dimension three Lagrangian, as indicated by the cross, which also contributes to this order as explained in the text. Note also that graph (i) starts at order $q^3$ but picks up a fourth order piece due to the kinematics. The dot and the circle–cross denote an insertion from the dimension one and two Lagrangian, in order. Nucleons and pions are depicted by solid and dashed lines, respectively. The double–line denotes the scalar–isoscalar source.