General trends of the late period of evolution in the quasichemical model of nucleation

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Abstract

The periods after the end of the "primary" nucleation are considered. The approximate analytical description is given. The process is split into several periods which form the loop of evolution.

1 Preliminary remarks

The power of metastability in the system is described by the value of super-saturation $\zeta$ defined as the ratio of surplus number density of molecules to the saturated number density of molecules. After the end of nucleation one can show that the size droplet spectrum is rather well localized in the space of sizes. If we choose the free molecular regime of growth (in order to escape from the problems of the density profiles around droplets) it is natural to come to the natural size coordinate

$$\rho = \nu^{1/3}$$

where $\nu$ is the number of molecules inside the droplet. The law of regular growth is rather simple

$$\frac{d\rho}{dt} = \frac{\zeta}{\tau}(1 - \frac{\rho_c}{\rho})$$

with the characteristic time $\tau$ as a parameter and the critical size $\rho_c$ which is determined as

$$\rho_c = \frac{2a}{3\ln(\zeta + 1)}$$
where $a$ is the renormalized surface tension. For the small supersaturations

$$\rho_c \approx 2a/3\zeta$$

On the base of the critical size the value of supersaturation can be expressed as

$$\zeta = 2a/3\rho_c$$

The asymptotic rate of growth for the supercritical droplets is very simple

$$(\frac{d\rho}{dt})_{as} = \frac{\zeta}{\tau}$$

From the theory of nucleation it is known that the distribution function is well localized in the scale of sizes. Namely, the half-width $\Delta_1x$ in $\rho$-axis satisfies the following estimate

$$f_*(\Delta_1x)^4 \simeq \Gamma^{-1}\Phi_*$$

Here $f_*$ is the amplitude of spectrum, $\Phi_*$ is the ”quantity” of surplus vapor calculated in terms of supersaturation, parameter $\Gamma$ can be estimated as

$$\Gamma \simeq \nu_c$$

and the value $\nu_c$ is the number of molecules in the critical embryo.

The number of droplets can be estimated as

$$N = f_*\Delta_1x$$

At the final of the surplus vapor exhaustion one can write

$$N\Delta_1x^3 = \Phi_*$$

where $z_{fin}$ is an imaginary final coordinate in $\rho$-scale (actually there is no real final coordinate it is some quasistationary approximation).

We consider the situation of decay here but the situation of dynamic conditions can be easily considered by quasistationary approximations and by the first step of corresponding iteration procedures.

Then we come to

$$\frac{z_{fin}}{\Delta_1x} = \Gamma^{1/3} \sim \nu_c^{1/3}$$

(1)

Since $\nu_c \gg 1$ we see the certain hierarchy.
2 Regular relaxation of the size spectrum

When the asymptotic law of the regular growth is used the evolution of the system after the end of nucleation \[1\] can be described by means of the first order differential equation

\[ \tau \frac{dz}{dt} = \Phi_* - \sum_i \mu_i^\infty z^{3-i} C_i^3 \]

with "coefficients" \( \mu_i^\infty \) as the full momentums of the distribution function\(^1\)

\[ \mu_i^\infty = \int_{-\infty}^{\infty} x^i f(x) dx \]

Here \( f(x) \) is the distribution density determined by the nucleation theory \[1\]. The variable \( x \) is the shift at \( \rho \)-scale of the size of droplet from the "marked" size \( z \) of a droplet born at some moment.

If we choose as \( z \) the size of the droplet formed at the initial moment of time then

\[ \mu_i^\infty = \int_{0}^{\infty} x^i f(x) dx \]

The initial condition for the differential equation is the following: At \( t = t_* + \Delta_1 t \) the position of the size spectrum is \( z = \Delta_1 x \). Here \( t_* \) is the "characteristic" moment, \( \Delta_1 t \) is the duration of the "back" side spectrum formation. Approximately \( \Delta_1 t = \Delta_1 x \tau / \Phi_* \). So, \( t_* + \Delta_1 t \) is the moment of the "end" of the nucleation period. Also here \( \Delta_1 x \) is the length of the back side of the size spectrum. All these characteristics can be given by the nucleation theory.

This differential equation can be easily integrated

\[ \tau \int \frac{dz}{\Phi_* - \sum_i \mu_i^\infty z^{3-i} C_i^3} = t \]

Condition of the end of the regular relaxation is

\[ (2 \div 3) \rho_c = z - \Delta_1 x \quad (2) \]

or

\[ (2 \div 3) \rho_c = z \]

\(^1\)With appropriate renormalization.
Since
\[ \rho_c = \frac{2a}{3 \ln(\zeta + 1)} \approx \frac{2a}{3 \zeta} \]
and
\[ \zeta = \Phi_* - \sum_i \mu_i \infty z^3_i C_3^i \]
it is an ordinary algebraic equation which can be effectively solved on the base of the strong inequality
\[ \zeta \ll \Phi_* \]

At the moment when the equation (2) is satisfied one has to take into account the diffusion correction to the law of droplets growth.

The final values can be made more accurate. We can use the balance equation
\[ \Phi = \frac{2a}{3 \rho_0} + N \rho_0 \]
to get \( \rho_0 \).

3 Diffusion erosion of spectrum

The process of diffusion leads to destruction of the previous form of spectrum. Since the spectrum is rather narrow and (1) takes place one can use initial condition as \( \delta \)-function type. But it is not absolutely necessary.

There are two approaches which can be used here. At first one can assume that the spectrum is narrow according to (1) and use the Green function in \( \nu \)-axis
\[ G = \frac{1}{\sqrt{4 \pi D}} \exp(-\frac{(\nu - \nu_0)^2}{4Dt}) \]
Here \( \nu_0 = \rho_0^3 \),
\[ D = (W^+ + W^-)/2 \]

\( W^+ \) is absorbtion coefficient and \( W^- \) is desorbtion coefficient. These coefficients are taken at \( \nu = \nu_0 \).

Another possibility is to take \( \nu_0 \) as the mean coordinate corresponding to \( z^3 \) in the previous solution of the relaxation problem.

The effect of dependence of \( D \) on \( \nu \) leads to the following equation
\[ \frac{\partial f}{\partial t} = -D_0 \nu^2/3 \frac{\partial^2 f}{\partial \nu^2} \]
where

\[ D_0 = D/\nu_0^{2/3} \]

does not essentially depend on \( \nu \). The last equation can be approximately written as

\[ \frac{\partial f}{\partial t} = -D_0 \frac{\partial^2 f}{\partial S^2} \]

for

\[ S = \nu^{2/3} \]

Then the solution is the Green function and it is known

\[ G = \frac{N}{\sqrt{4\pi D_0}} \exp\left(-\frac{(S - S_0)^2}{4D_0t}\right) \] (3)

Here

\[ S_0 = \nu_0^{2/3} \]

Until the half-width \( 4Dt \) is many times less than \( \nu_0 \) there is no difference what variable (\( \nu \) or \( S \)) is used.

One can remark that the initial form of the size spectrum will disappear rather soon. Really, the characteristic width of spectrum is

\[ \Delta_2S = (z + \Delta_1x)^2 - z^2 \approx 2\Delta_1xz \]

and

\[ \frac{\Delta_2S}{S} = 2\frac{\Delta_1x}{z} \]

After the time \( t_2 \) the diffusion erosion of the size spectrum will have the scale of the initial width of spectrum and the initial form of the size spectrum completely disappears. This time allows the estimate

\[ t_2 = \frac{(\Delta_2S)^2}{4D} \]

Since \( \Delta_1x/z_{fin} \ll 1 \) one can use both \( x \) or \( S \) representation of solution.

One has to note that in the process of relaxation \( \zeta \) varies and \( D = D(\zeta) \) varies. The law \( \zeta(t) \) is already known from solution of the regular evolution equation. Then one can suggest the following approximate account of variation of \( D \): instead of \( Dt \) in denominator one can approximately use \( \int D(\zeta(t))dt \).
One has also to refine the regular evolution equation, which allows to prolong it. Really, having written the law of growth

\[ \frac{dz}{dt} = \frac{\zeta}{\tau}(1 - \frac{2a}{3z}) \]

with

\[ \zeta = \Phi - \sum_i \mu_i^\infty z^{3-i}C_i^3 \]

This differential equation will be more accurate and complex. The problem is that \( \mu_i^\infty \) begin to depend on \( t \) and \( z \).

But here fortunately appears the monodisperse approximation. Since \( \Delta_1 x \ll z_{fin} \) then

\[ z^3 \mu_0^\infty \approx N z^3 \gg z^2 \mu_1^\infty \approx N \Delta_1 x z^2 \gg z \mu_2^\infty \approx N \Delta_1^2 x z \gg \mu_3^\infty \approx N \Delta_3^2 x \]

So, one can use the monodisperse approximation. The monodisperse approximation is rather accurate. In this approximation

\[ \zeta = \Phi - N z^3 \]

Then

\[ \frac{dz}{dt} = \frac{\Phi - N z^3}{\tau}(1 - \frac{2a}{3\Phi z - 3N z^4}) \]

The main momentum is \( \mu_0^\infty \). The main advantage is that this momentum doesn’t change in the real regular law of growth instead of the asymptotic one. The last equation can be easily integrated

\[ \int \frac{dz}{(\Phi - N z^3)(1 - \frac{2a}{3\Phi z - 3N z^4})} = \frac{t}{\tau} \]

Since \( 0 < dz/dt < \zeta/\tau \) for \( z > z_c \) one can see the ”narrowing” of the spectrum.

Now we have to extract the limits of a ”pure diffusion”. One has to take into account that in reality the pure diffusion equation takes place only in the ”extended near-critical zone”. While the boundary of the ”near critical zone” is determined by condition

\[ F_c - F \approx 1 \]
the "extended near-critical zone" (ENZ) is determined by two conditions

\[ \frac{dF}{d\nu} \ll 1 \]
\[ \nu < (2 \div 3)\nu_c \equiv \beta^3\nu_c \]

The second condition is necessary for the case \( \zeta \ll 1 \) to exclude the infinite zone of big \( \nu \). If we require that the diffusion length \( \Delta S \sim \sqrt{4Dt} \) has to be compared with the regular length \( \Delta S \sim \zeta^2t^2/\tau^2 \), then we come to \( \rho^3 \sim 4/\zeta \), which is less than \( \rho_c = 2a/3\zeta \).

Certainly, since now \( dz/dt \) depends on \( z \) the values of \( \mu_i \) will change. So, here there are only two possibilities: 1). to consider \( \mu_i \) slowly varying; 2). to solve everything in the monodisperse approximation.

One can see that both "near-critical zone" and "extended near-critical zone" are not symmetrical in respect to \( \nu_c \). This requires to use more sophisticated methods instead of the leading term of the steepens descent to calculate the Zeldovitch' factor. But this can be considered as technical details.

The lower boundary where the first condition fails even for small \( \zeta \) depends on the concrete value of surface tension. We denote the boundary as

\[ \nu_l = \nu_c\alpha \quad \rho_l = \rho_c\alpha^{1/3} \]

The value of \( \alpha \) can be estimated when we require that the derivative of the free energy equals by the absolute value to the derivative of the free energy for big supercritical embryos. This leads to \( \alpha = 1/8 \).

The spread of the gaussian occurs up to the moment

\[ t_3 = (1 - \alpha^{2/3})^2S_c^2/(4D_0) \]

where \( S_c \) corresponds to the final coordinate.

When the droplet attains \( \nu_l \) then rather soon (in comparison with the time of attaining \( \alpha\nu_c \)) it will be dissolved. The probability of growth back is small.

When the essential part of spectrum is in the ENZ one can use the presented solution. One can observe and prove the relaxation to the quasiequilibrium. In quasiequilibrium the behavior of \( \rho_c \) is regulated by the balance condition

\[ \int f(\rho, t)3\rho^2 \frac{d\rho}{dt} d\rho = 0 \]
$(f(\rho, t)$ is the known distribution) or by

$$\int f(\rho, t)3\rho^2 \frac{\zeta}{\tau}(1 - \frac{\rho_c}{\rho})d\rho = 0$$

It is important that the vapor consumption by droplets with a known spectrum of sizes regulates the behavior of $\rho_c$ contrary to the method of Lifshitz-Slezov [2], where the behavior of $\rho_c$ regulated the form of spectrum.

Actually nothing happened when $f(\rho, t)$ is narrow. Let it be like $\delta$-function $\delta(\rho_a)$. Then $\rho_a$ relaxes to $\rho_c$ and the process stops until the diffusion makes spectrum wider.

4 Further evolution of spectrum in a near critical region

On the base of solution (3) one can see the transformation of spectrum. The spectrum is like a Gaussian and according to the balance condition $\rho_c$ is near the head of the Gaussian. Then it is clear that at first the lower boundary of ENR will be attained by the essential part of spectrum. The upper boundary can not be attained first.

We see that the essential part of spectrum is localized in the region where $|\nu - \nu_c| \ll 1$. Then at first there is no difference whether to use the diffusion equation in $\nu$-scale or in $S$-scale.

The time when the essential part of spectrum attains $\rho_l$ can be easily calculated

$$4Dt = (\nu_c - \nu_l)^2$$

Here we calculate time from the moment of the end of the previous period.

After this moment we have to take into account that practically all droplets at $\rho_l$ are going to dissolve. Then

$$f(\rho)|_{\rho = \rho_l} = 0$$

Solution of diffusion equation (it has to be solved now in $S$-scale) with such a boundary condition is

$$f(\rho) = 0$$

for

$$\rho < \rho_l$$
and

\[ f(S) = G_+ - G_- \]  \hspace{1cm} (4)

for

\[ S^{1/2} > \rho_t \]

Here

\[ G_+ = \frac{N}{\sqrt{4\pi Dt}} \exp\left(-\frac{(S - S_0)^2}{4Dt}\right) \]

\[ G_- = \frac{N}{\sqrt{4\pi Dt}} \exp\left(-\frac{(S - S_1)^2}{4Dt}\right) \]

and \( S_0 \) and \( S_1 \) are defined by conditions that \( S_0 = z_{\text{fin}}^2 \) and \( S_0 - S_l = S_l - S_1 \) (where \( S_l = \nu_t^{2/3} \)).

A special question appears what is here \( D \). But at first we shall analyze the properties of solution. One can see that contrary to the unbounded solution with a half-width

\[ \Delta S \sim t^{1/2} \]

and the maximum (and mean) value is staying

\[ S_{\text{max}} = \text{const} \]

the new solution has the moving maximum value

\[ S_{\text{max}} \sim t^{1/2} \]

the moving mean value

\[ S_{\text{mean}} \sim t^{1/2} \]

and the growing half-width

\[ \Delta S \leq t^{1/2} \]

which ensures the relative localization of spectrum.

So, we have the following estimate

\[ \Delta S \leq S_{\text{mean}} \]

The value of supersaturation has to be determined from

\[ \int_0^\infty d\rho \rho^2 f(\rho) \frac{d\rho}{dt} = 0 \]  \hspace{1cm} (5)
Then we get $\rho_c$ and the supersaturation.

Then in this solution we have to take into account that $D \approx D(\rho_c(\zeta))$. Again we can approximately substitute $Dt$ by $\int D(\zeta(t))dt$.

Actually, this is the end of solution. Everything presented later refers to rather small tails which can be seen only in enormously big systems.

5 Formation of a tail

The previous equation to get the supersaturation is not a unique way to do it. Rigorously speaking the supersaturation has to be derived from the substance balance equation

$$\frac{2a}{3\rho_c} = \Phi_* - \int_0^\infty \rho^3 f(\rho, t)d\rho$$  \hspace{1cm} (6)

Distribution has at first to be taken here from (4).

One can see that the part of the size spectrum with big $\rho$ is important in the substance balance due to the factor $\rho^3$. But the law of growth of the droplets of big sizes is regular, it is not the diffusion walking on the flat potential surface as it was supposed in derivation of solution (4).

We suppose that at $\rho_r = \beta \rho_c$ the law of growth will be changed to the regular asymptotic law of growth

$$\frac{d\rho}{dt} = \frac{\zeta}{\tau}$$

Here $\beta$ is a parameter, the normal value is $\beta = 1.5$

The rate of appearance of droplets at the beginning of the region of regular growth can be calculated as following. At first we can calculate $f(\rho)$ by

$$f(\rho) = f(S) \frac{dS}{d\rho} = f(S)2S^{1/2}$$

on the base of solution (4). Then

$$J = f(\beta \rho_c) \frac{\zeta}{\tau}$$
or
\[ J = f(\beta \rho_c) \frac{\zeta}{\tau}(1 - \beta) \]
for the intensity of formation of the tail. Then we get the tail formed according to
\[ f_{\text{tail}}(\rho, t) = f(\beta \rho_c(t')) \frac{\zeta(t')}{\zeta(t)} \]
where \( t' \) is defined as
\[ \rho = \beta \rho_c + \int_{t'}^t \frac{\zeta(t'')}{\tau} dt'' \]

Let \( z \) be a coordinate \( \rho \) of some droplet\(^2\). Then \( x = z - \rho \) for the given droplet. It remains constant during the regular growth. Then
\[ f_{\text{tail}}(\rho, t) = f_{\text{tail}}(x) \]
and it is a known function.

One can see the following approximation. Certainly, the position of \( S_c \) coincides in initial moments of time with \( S_0 \) and do not vary essentially. It a quasi-integral of evolution. Then denoting
\[ \zeta_0 = \frac{2a}{3\rho_0} \]

one can put \( \zeta_0 \) instead of \( \zeta(t'') \) in the previous formula. The movement of the boundary \( S_r = (\beta \rho_c)^2 \) can be neglected and one can act in a quasi-stationary approximation. As the result one gets the form of the tail.

More accurate is to take \( S_c \) from the solution (4) ignore the formation of the tail. Then
\[ \zeta_0(t'') = \frac{2a}{3S_c^{1/2}(t'')} \]

Now we can present the distribution as
\[ f = f_{\pm} + f_{\text{tail}} \]
and in the quasiequilibrium we have
\[ \int_0^\infty d\rho \rho^2 (f_{\pm} + f_{\text{tail}}) \frac{d\rho}{dt} = 0 \quad (7) \]
This gives \( \rho_c \) in quasiequilibrium and a new value of supersaturation. This closes the iteration loop. Actually only one loop is necessary.

\(^2\)More convenient is to choose \( z \) as the coordinate of the front side of the tail.
6 Dissolution of the head of spectrum

At first the tail is not important in the substance balance. But the quantity of substance $G_{\text{tail}}$ in the tail grows faster than something proportional to $t^3$. So the droplets in the tail become the essential consumers of vapor.

Now the distribution density is the following

\[ f(\rho) = 0 \]

for

\[ \rho < \rho_l \]

and

\[ f(S) = G_+ - G_- \] (8)

for

\[ \rho_r > S^{3/2} > \rho_l \]

\[ f = f_{\text{tail}} \]

for

\[ \rho > \rho_r \]

Then one can get the supersaturation from the balance equation

\[ \zeta = \Phi_* - \int_{S_l}^{S_r} f(S)S^{3/2}dS + \int_{\rho_c}^{\infty} f_{\text{tail}}(\rho)\rho^3d\rho \]

At first the vapor consumption of the droplets in the tail i.e. the evolution of

\[ G_{\text{tail}}(t) = \int_{S_r}^{\infty} f_{\text{tail}}(\rho, t)d\rho \]

determines the value of supersaturation

\[ \frac{d}{dt}\zeta = -\frac{d}{dt}G_{\text{tail}} \]

The head of the spectrum begins to dissolve since the droplets in the head of spectrum become mainly subcritical. This diminishes the quantity

\[ G_{\text{head}} = \int_{S_l}^{S_r} f(\rho, t)dt \]

and partially stabilizes the supersaturation.
The result will be the dissolution of the head of the size spectrum. The final of this period will be when

\[ N_{\text{tail}} \sim N_{\text{total}} - N_{\text{tail}} \]

One can propose the approximate (not necessary) model: at first the intensity of vapor consumption by the tail can be calculated according to

\[ \frac{d}{dt} G_{\text{tail}} = 3 \int_{\rho_r}^{\infty} f_{\text{tail}}(\rho) \rho^2 \frac{d\rho}{dt} d\rho \]

Then the number \( N_{\text{head}} \) in the head satisfies the following equation

\[ [\frac{d}{dt} N_{\text{head}}] z^3 + \frac{d}{dt} G_{\text{tail}} = 0 \]

The growth of \( G_{\text{tail}} \) occurs in the avalanche manner. So, the dissolution of the head occurs in the avalanche manner also. So, this solution can prolonged up to

\[ N_{\text{tail}} \ll N - N_{\text{tail}} \]

7 Dissolution of the essential tail of spectrum

One can spread the solution to the beginning of the dissolution of the tail of spectrum.

But when the complete dissolution of the head is finished one can propose the more effective procedure.

From

\[ \frac{2a}{3(z - x_c)} = \Phi - \sum_i C_i^3 z^{3-i} \mu_i(z) \]

where the changing momentums are

\[ \mu_i = \int_{-\infty}^{\tilde{x}} x^i f_{\text{tail}}(x) dx \]

(here \( \tilde{x} \) corresponds to \( \rho = \beta \rho_c \)). are known functions of \( x_c \) we take \( x_c \) as a function of \( z \). The region \( \rho < \beta \rho_c \) is negligible in definition of \( \mu_i \) when \( z >> \rho_c \). Now the law of growth

\[ \tau \frac{dz}{dt} = \frac{2a}{3(z - x_c)} \]
becomes the closed differential equation of the first order. Since the r.h.s. of the last equation is known function of \( z \) the last equation can be easily integrated

\[
t = \tau \int \frac{dz}{2a} \quad \frac{3(z-x_c(z))}{2a}
\]

It can be rewritten as

\[
\tau \frac{dz}{dt} = \Phi - \sum_i C_i z^{(3-i)} \mu_i(z)
\]

Since \( \mu_i(\tilde{x}(z)) \) can be well approximated as polynomials on \( z \), the integration can not cause difficulties.

Here one can see the important property which allows to solve this equation very simply: the vapor is consumed by the droplets with regular growth.

One can show the simplicity of solution by the following example: suppose that the height of tail \( f_* \) is constant. Then

\[
G_{\text{tail}} = f_*(z^4 - z_c^4\beta^4)
\]

and

\[
\tau \frac{dz}{dt} = \Phi - f_*(z^4 - z_c^4(z)\beta^4)
\]

One can integrate this equation. Rewrite this equation for evolution of \( z_c(t) \)

\[
\frac{2a}{3z_c} = \Phi + f_*z_c^4\beta^4 - f_*z^4
\]

Then \( z \) can be expressed through \( z_c \)

\[
z = f_*^{-1/4}(\Phi + f_*z_c^4\beta^4 - \frac{2a}{3z_c})^{1/4}
\]

Having differentiated the last relation and substituted it into the initial balance equation

\[
\Phi - f_*(z^4 - f_*z_c^4\beta^4) = \tau \frac{1}{3z^3} \frac{dz^4}{dt}
\]

we get

\[
\frac{2a}{3\tau z_c} = \frac{1}{3} f_*^{-1/4}(\Phi + f_*z_c^4\beta^4 - \frac{2a}{3z_c})^{-3/4}(f_*A z_c^3\beta^4 + \frac{2a}{3z_c^2}) \frac{dz_c}{dt}
\]
which can be integrated
\[ t = \tau \int \frac{3z_c}{2a} f_s^{-1/4} (f_s 4z_c^3 \beta^4 + \frac{2a}{3z_c^2} \frac{1}{3} (\Phi + f_s z_c^4 \beta^4 - \frac{2a}{3z_c})^{-3/4} dz_c \]

The end of this period takes place when the number of droplets in ENR, namely \( N_{ENR} \) becomes comparable with the rest droplets
\[ N_{ENR} \sim N_{total} - N_{ENR} \]

8 Closure of the loop of evolution

Later one has to take into account the diffusion of the rest of the tail. Again we have the spectrum of some form in ENR. Since the form of the tail is rather flat we take the flat spectrum with a gaussian front side. At first we see the dissolution of the flat part of the size spectrum. It is easy to describe because we see the one dimensional heat conductivity problem with a boundary condition of a following type: at \( S = S_l \) the distribution \( P \) is zero; at big \( S \) the distribution \( P \) vanishes, then at \( S = z^2 \) one can see the free diffusion. The initial condition is
\[ P(x, t = 0) = \Theta(S - S_l)A\Theta(S_k - S) \]

where \( A \) is the amplitude of tail (constant or slowly varying function) and \( S_k \) is the position of the front side of tail (in shifted coordinates), \( \Theta(x) \) is the Heavisaid’s function.

To keep condition \( P = 0 \) at \( S = S_l \) we can prolong the previous initial condition oddly.

The solution of diffusion problem can be found anywhere.

Having moved in \( S \)-scale the origin to the former \( S_l \) we see the solution
\[ P = \int F(x, 0) \exp \left( \frac{(S - x)^2}{4Dt} \right) dx \]

Here \( F(x, 0) \) is the distribution given by
\[ F(x, t = 0) = \Theta(S)\Theta(S_k - S_l - S)A - \Theta(-S)\Theta(S + S_k - S_l)A \]

It is quite clear that when \( A \) is constant or can be approximated by polynomials then the solution can be written via a linear combination of error functions.
It is also clear that when
\[ t \gg \frac{S_k^2}{(4D)} \]
then the solution begin to resemble the gaussian.

When the flat region is dissolved the size spectrum resembles the gaussian and we come to the already studied problem. So the procedure of all sections starting from the diffusion erosion of the size spectrum in the ENR has to be repeated. This closes the loop and these loops will be inevitably continued in future. One has to stress that the number of droplets radically decreases in every loop and to see these loops one has to observe an extremely giant system. To give description on the base of continuous approximation even in the beginning of the tail of the size spectrum there have to be many droplets. Earlier or later diffusion and stochastic growth of droplets lead to the change of the regime of evolution presented here. Also the change of regime of vapor consumption, the heat release effects, coagulation etc. destroy the presented solution.

References

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