Investigation of Frequencies Characteristics of Modified Waveguide Aperture by Wire Media

Dmytro Vovchuk* and Mykola Khobzei

Abstract—The paper is devoted to the investigation of radiation frequencies characteristics of a modified waveguide aperture by wire media (WM). Such construction allows radiating weak electromagnetic (EM) waves — their frequencies are non-corresponding to the resonant ones of the modified radiator. It is possible due to the unusual properties of metamaterials, namely the negative value of permittivity of WM. The study of the simulation shows that the changing of value of wires radius and at the same time the value of filling factor impacts on the radiation frequency. Therefore, the increase of the filling factor leads to the increase of the resonance frequency. The radiation is narrowband with $S_{11}$-parameter less than −20 dB. The experimental investigation shows that the decrease of the value of lattice period allows increase of the width of radiation frequency range from 30–40 MHz up to approximately 80 MHz at the level of 0.3 (≈ −10 dB). At the same time, the increase of wires’ radius values leads to the increase of the value of resonant frequency. Finally, the experimental study demonstrates that the value of overlap between waveguide port (source of EM waves) and WM sample negligibly impacts on the resonance frequency values and operational range for $D/L = 0...0.3$.

1. INTRODUCTION

The WM is a kind of metamaterials characterized by the negative value of permittivity, so-called $\varepsilon$-negative materials [1–3]. The simple example of WM is shown in Fig. 1 and consists of parallel metallic wires included into a dielectric matrix [4]. The length of wires $L$ and their diameter $d$ (or it can be often used as $2r$) as well as lattice period $a$ [3, 5] can be related to the important structural parameters of a WM. To simplify the model, the vacuum is utilized as a dielectric; the lengths of wires are the same; the lattice periods in two directions are defined and equal.

WM is widely studied for different applications including imaging, sensing, spectroscopy, thermophotovoltaic [1, 2, 6, 7], etc. Most of these investigations show the possibility of narrowband power transfer and radiation. Recent works [8, 9] achieve the broadband effect of EM energy transfer. It is possible in the case when the source of EM waves and their receiver are overlapped with the WM sample at input and output sides or at least are allocated at the same plane with WM interfaces [8]. However, such overlapping must be supported from both sides. In another case, the construction operates at the Fabri-Perot resonance frequency [10] that depends on the WM wires length. This effect is used in antennas technology that gives the possibility to radiate weak EM oscillations at the frequencies different from the antenna’s resonances or to modify the existent antennas such as horn, monopole, and others to improve their gain, directivity, bandwidth, etc. [11–13]. The wire metastructure for broadband radiation of EM waves which differs from conventional WM was suggested in [10]. Of course, WM cannot support the radiation in a wide frequency range as the WM brush in [10], but the question of the influence of the main parameters of WM (wires’ radius and lattice period) on the radiation frequencies is still topical.

* Corresponding author: Dmytro Vovchuk (dimavovchuk@gmail.com).
The authors are with the Department of Radio Engineering and Information Security, Yuriy Fedkovych Chernivtsi National University, Chernivtsi, Ukraine.
Figure 1. WM metamaterial that consists of metallic wires array with length and diameter of wires \( L \) and \( 2r \) respectively and lattice period \( a \).

Therefore, the impact of wires' radius and lattice period of a WM sample on frequencies characteristics is studied in the paper by simulations and experimental investigations. In the paper we would like to demonstrate how to control by the frequency value of Fabry-Perot resonance and frequency range around Fabry-Perot resonance without changing of wires length \( L \).

For this aim, the approach used in [14] where the power transfer through the WM sample between two discrete ports analyzed through the filling factor conception was considered and applied in the paper. The filling factor is described as \( f_r = \pi(r/a)^2 \) and defines the part of metallization from the general area of structure interface. By changing the value of a filling factor, the equivalent circuit of WM also changes, because an array of parallel metallic wires can be considered as a transmission line [15–17] which consists of distributed capacitance and inductance. It means that the variation of wires' radius and lattice period leads to the changing of total value of WM capacitance and inductance.

2. SIMULATION STUDYING OF FREQUENCIES CHARACTERISTICS

For the simulation investigation, the model that consists of a waveguide port and WM structure was designed in CST Studio Suite as shown in Fig. 2(a). The waveguide port was picked up as a source of EM waves. Its aperture’s dimensions correspond to the experimental ones and are equal to \( a_w = 164 \) and...
With such dimensions the waveguide port provides transverse TE\(_{10}\)-mode as a dominant mode for the frequency range from 0.9 up to 1.9 GHz which can be calculated from the formula of the critical value of wavenumber: 
\[ k_c = \sqrt{(m \pi/a_w)^2 + (n \pi/b_w)^2}, \]
where \( m \) and \( n \) are indices of TE\(_{mn}\), as well as \( k_c = 2\pi f_c \), where \( f_c \) is the critical frequency of the used waveguide port. In usual case, the aperture of waveguide port radiates well, but we are concentrated on the investigation of radiation at frequencies of Fabry-Perot resonances. Therefore, in order to avoid the waveguide port radiation, the aperture was loaded by a capacitive diaphragm to imitate the evanescent EM waves. The diaphragm consists of two parallel metallic sheets that are allocated along the longer walls of the waveguide port and inserted at 20 mm depth from the aperture inside the waveguide port (Fig. 2(b)). An air slot by the width of 10 mm is between the metallic sheets of the diaphragm. The WM slab consists of a 17 by 7 metallic wires array. The lattice period \( a \) is 10 mm, and the length of the sample \( L \) is 100 mm. WM was embedded into the waveguide port at depth \( D = 10 \) mm without any electric contacts with the used waveguide port and diaphragm.

As introduced above the filling factor of WM can be varied changing the wires radius \( r \) or the lattice period \( a \). The first case was used in our studied simulation. The parameters of the investigated model were as mentioned above, and the values of wires’ radius were changed from 0.01 up to 1 mm. They correspond to the values of \( f_c \) from \( 3.14 \times 10^{-6} \) up to \( 3.14 \times 10^{-2} \). Further increase of the radius is not relevant because it can be much smaller than the lattice period \( a (2r \approx a/10) \). Therefore, we will demonstrate below the possibility of controlling by Fabry-Perot resonances of WM with the defined and unchangeable length via tuning of filling factor of metal at the plane of the WM interface in the paper below.

The simulation shows that the structure radiates at the resonant frequency that is Fabry-Perot resonance, and this result is predictable. However, Fig. 3 depicts the strong dependence of the resonance frequency from the value of wires radius for the same wires’ length. The frequency dependences of \( S_{11} \)-parameters in Fig. 3 depict the radiation for radius values that are narrowband. The linear increase of radius value from 0.01 to 1 mm leads to the linear increase of the value of resonance frequency from 1.138 up to 1.178 GHz (40 MHz shift).

![Figure 3.](image)

The simulation results of \( S_{11} \)-parameters dispersions obtained for different values of the wires’ radius of WM with lattice period \( a = 10 \) mm.

The increase of the resonance frequency value in the case of \( r \) increasing for \( L = \text{const} \) can be explained by the decrease of self-inductance of each wire of WM and decreasing the total value of inductance of WM in general. It is described by the formula of self-inductance of a straight thin \((a \gg 2r)\) cylindrical wire [18, 19] as follows:
\[ L_w = 2L[\log(2L/r) - 0.75], \]
where \( L \) and \( r \) are the length and radius of wires as presented in Fig. 1.
3. EXPERIMENTAL INVESTIGATION OF FREQUENCIES CHARACTERISTICS

For the experimental study, two WM samples were manufactured that consist of parallel copper wires with the length $L = 100$ mm. The wires’ diameters and lattice periods were $2r = 1.5$ mm and $a = 6$ mm respectively for the first sample (experiment 1) as well as $2r = 1$ mm and $a = 10$ mm for the second one (experiment 2). The values of the filling factor are 0.049 and 0.00785, respectively. The parameters of the used waveguide port correspond to the parameters of simulated one in previous chapter. The aperture of the waveguide port was also loaded by the capacitive diaphragm. It allows providing the evanescent EM waves near the aperture of the waveguide port because of the experimentally measured $S_{11}$-parameters equal to $0.6...0.7 (\sim -4...-3$ dB).

The experimental setup is shown in Fig. 4(a). The WM slab was embedded into the port, and in

![Image](image_url)

**Figure 4.** (a) Experimental setup and the results of distributions of dispersion reflection coefficient ($S_{11}$-parameters) obtained for different values of the overlap of WM with the waveguide port for the wires’ radius $r$ and lattice periods $a$ were (b) 0.5 mm and 10 mm as well as (c) 0.75 mm and 6 mm, respectively.
the investigation process its position was changed from 0 up to 30 mm depth of overlap $D$. The distance between the diaphragm and WM slab was fixed and equal to 10 mm. Thus, the colormap of distribution of reflection coefficient ($S_{11}$-parameters) dispersions depending on the depth of overlap was measured and plotted for the experiment 1 (Fig. 4(b)). The range of values $D$ was 0...30 mm, and in such a case, the ratio $D/L = 0...0.3$. One can see from Fig. 4(b) the minimum value of $S_{11}$-parameters (maximum of radiation) approximately corresponds $0.1 \ (−20 \text{ dB})$ at $1.02 \text{ GHz}$, and at the level $0.3 \ (−10 \text{ dB})$ the operational frequency range is $30–40 \text{ MHz}$. Such $z$ value of the range keeps almost without changing for $D$ up to 20 mm that corresponds to $λ/10$.

For the experiment 2 another WM sample was picked up, and the same measurements were carried out for that. The obtained results show that the increase of the value of lattice period leads to the expansion of resonance frequency band up to $80 \text{ MHz}$ at the level $0.3$ for $D ≈ 20 \text{ mm} = λ/10$ (Fig. 4(c)). The minimum value of $S_{11}$-parameters is shifted to $1.062 \text{ GHz}$ due to the increase of the wires’ radius as found in the simulation studying. The decrease of lattice period $a$ leads to the increase of the total capacitance of WM. It can be explained by consideration of the capacitance of two parallel wires as given in [15, 20]: $C_w = πε_0L/\text{arcosh}(\frac{2r}{2a}) = πε_0L/\log(\frac{2r}{2a} + \sqrt{(\frac{2r}{2a})^2 - 1})$.

However, in order to describe the increase of the operational frequency band (to compare Figs. 4(b) and (c)), it is necessary to take into account the shunt inductance included in parallel to $C_w$ [16]. The value of shunt inductance increases with the decrease of lattice period $a$ especially in the case of the experiment 2, where $a$ is not much larger than $2r$. Thus, the equivalent circuit of WM represents a band-pass circuit scheme where by tuning $r$ and $d$, one can set the values of capacitance and both inductances.

4. CONCLUSION

In the paper, the frequencies characteristics via reflection coefficients dispersion, so-called $S_{11}$-parameters, of modified waveguide port were investigated. The modification was performed by the WM sample which was embedded into the aperture of waveguide port. As the results of simulation, the strong dependence between the radius of wires and the value of resonance frequency was found. The increase of filling factor leads to the increase of the value of resonance frequency, and for the radius changing from 0.01 up to 1 mm the resonance frequencies change from 1.138 up to 1.178 GHz due to the decrease of the WM inductance. Although the radiation at all of frequencies is narrowband, the values of all $S_{11}$-parameters at the resonance frequencies are less than $−20 \text{ dB}$. These results are presented for the defined value of lattice period that is 10 mm.

For the experimental investigation, two samples of WM were manufactured with wires radii 0.5 and 0.75 mm, lattice periods 10 and 6 mm as well as wires length 10 mm, respectively. These gave the possibility to investigate two WM samples with different values of filling factor (0.049 and 0.00785, respectively), radii of wires, and lattice periods. The obtained experimental results confirmed the simulation ones and showed that the decrease of lattice period leads to the increase of the operational frequency range. It can be explained through the occurrence of the shunt inductance between adjacent wires in parallel to the capacitance of those wires, and as a result, WM can be presented as a band-pass circuit. Finally, the experimental investigation shows that the change of the depth of overlap of the WM slab and EM wave source is possible up to values $D ≈ 20 \text{ mm} = λ/10$ without changing the operational frequency band.

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