A comparative study of equivalent circuit models for a Li-ion battery pack of an electric Tuk-Tuk

Natcha Rajchapanupat\textsuperscript{1} and Poowanart Poramapojana\textsuperscript{1,*}

\textsuperscript{1}Department of Mechanical Engineering, Faculty of Engineering, Kasetsart University, Bangkok 10900, Thailand

* Corresponding Author: fengpap@ku.ac.th

Abstract
This paper presents developments and validations of Lithium Nickel Manganese Cobalt Oxide (NMC) battery cell and battery pack models. A 7.4 kWh NMC battery pack is installed on an electric Tuk-Tuk equipped with a 3 kW motor. The battery pack is configured of four cells connected in parallel and 20 sets of the four cells connected in series. Based on the first-order and the second-order equivalent circuit models, battery cell and battery pack models are developed in Matlab/Simulink environment. For parameter identifications of the cell model, pulse discharge tests are performed at room temperature with 10\% State of Charge (SOC) interval. The lumped heat capacitance method is applied to the battery models. The battery cell model is validated with SOC, cell center temperatures and terminal voltage using the experimental data of constant current discharge tests. With the battery pack configuration, the battery cell model is scaled up to match the battery pack voltage and its capacity. The dynamics of the battery pack models are validated with the vehicle testing data using the no-load, wide-open throttle and real road tests. The simulation results show good agreement with the test data. Using the developed battery models, vehicle performance and its energy consumption can be improved by optimizing battery cell and pack configurations.

Keywords: Li-ion battery, Battery pack models, Electric Tuk-Tuk

1. Introduction
Tuk-Tuk is a unique three-wheeled vehicle name in Thailand. The vehicles have been widely used in many countries. Within the constraint of cost, electric Tuk-Tuks generally are installed with lead-acid batteries. With the advancement of Lithium-ion battery technology and electric powertrain, their performance and energy efficiency can be improved explicitly. The optimization of the powertrain and battery pack configuration can be achieved using accurate battery and powertrain models in the design processes [1-5].

Many battery models have been proposed to capture behaviors of battery cells. To understand electrochemical reactions in the cell, electrochemical models can achieve high accuracy. However, the models are not designed for battery management systems in electric vehicles because of their complex equations and a large number of unknown parameters [6]. For real-time and control-oriented purposes, equivalent circuit models are mostly used to manage vehicle power and battery control units. The equivalent circuit models have been developed. For example, the first-order resistance-capacitance
(RC) and second-order RC (Dual polarization model) [7-8]. Comparisons of twelve equivalent circuit models for LiNMC and Lithium Iron Phosphate (LiFePO4) cells were studied [9]. The simulation results of the study show the first-order RC model is preferred due to its accuracy and simple to estimate cell voltage and SOC.

This paper presents effectiveness comparisons of the first-order model and the second-order model from the cell model to the battery pack model. The remainder of this paper is organized as follows. In Section 2, the experimental of the battery cell testing and the on-road battery pack testing are described. Section 3 demonstrates the battery cell model developments and validations. Section 4 presents the battery pack model developments and validations. Finally, Section 5 concludes the present work.

2. Experimental

2.1 Battery Cell Testing

A diagram of a battery test system used in this present work is shown in Figure 1. The test bench consists of a battery cell, an ITECH IT8511A electronic DC load, an IT6722A DC power supply and a Graphtec GL840 data logger. The cell voltage, current, ambient temperature and cell center temperature are recorded by GL840 data logger with the sampling time of 1 second. In addition, Type K thermocouples are used in the tests. Two types of cell tests are performed at room temperature including constant current (CC) discharge and pulse current discharge tests. The CC discharge tests are conducted to determine the Open Circuit Voltage (OCV) test and to validate the cell model. The pulse current discharge tests are performed to identify battery cell parameters for the models. Moreover, the experimental data is recorded every 0.1 second for the pulse test.

For CC discharge tests, the cell is discharged from the fully-charged state to the cut-off voltage of 3.0 V. For each test, the cell is charged using a constant-current constant-voltage protocol. The cell is charged with a 1/3 C-rate constant current step until the cell voltage equals to 4.2 V. Then, the cell is charged by a constant voltage step until the cut-off current reaches 1/20 C-rate. After the cell is fully charged, one hour rest before performing the CC discharge test is used to make the cell reaches the thermal equilibrium state. For the battery parameters identification, details of the testing protocol are explained in [10].

![Figure 1. The battery cell test bench](image1)

![Figure 2. The electric Tuk-Tuk configuration](image2)

2.2 On-road Battery Pack Testing

The electric Tuk-Tuk configuration used in the present work is shown in Figure 2. The vehicle is propelled by a 3 kW motor. A 1204M Curtis motor controller is energised by a 7.4 kWh (74 VDC, 100Ah) battery pack. To record battery cell voltage in the pack, a data acquisition system is used to receive data through the Control Area Network (CAN) bus interface.

To evaluate the effectiveness of the battery pack model, three test conditions are performed to capture the dynamic behavior of the battery pack. First of all, a non-road load test is performed to simulate the
steady-state response of the system. The vehicle is lifted their rear wheels. The vehicle speed is increased with an incremental of 100 rpm and maintain at the speed for a few seconds. Secondly, a Wide-Open Throttle (WOT) test is conducted to simulate a step response of the system. The vehicle is driven with the full pedal position until the vehicle reaches the maximum speed. Finally, a real-world road test is performed on a local road for 6 minutes.

3 Battery Cell Model Validation

3.1 Equivalent Circuit Models

The resistance-capacitance network-based equivalent circuit models are shown in Figure 3. The cell terminal voltage $V(t)$ is determined using equation (1) where $V_{ocv}(SOC)$ is the open-circuit voltage (OCV) as a function of battery State of Charge (SOC). $R_0(SOC)$, $R_1(SOC)$ and $R_2(SOC)$ are the resistors. $C_1(SOC)$ and $C_2(SOC)$ are the capacitors. $V_1(t)$ is the voltage drop across $R_1(SOC)$ and $C_1(SOC)$. $V_2(t)$ is the voltage drop across $R_2(SOC)$ and $C_2(SOC)$. $I(t)$ is the current.

$$V(t) = V_{ocv}(SOC) - V_1(t) - V_2(t) - I(t)R_0(SOC)$$ (1)

The SOC can be evaluated using equation (2) where SOC₀ is the initial SOC and $Q_{bat}$ is battery cell capacity in the unit of Ah.

$$SOC = SOC_0 - \frac{1}{3600Q_{bat}} \int I \, dt$$ (2)

The rate of $V_1(t)$ and $V_2(t)$ are expressed in equation (2) and (3). By taking time integral to the equations, $V_1(t)$ and $V_2(t)$ can be obtained.

$$\frac{dV_1}{dt} = -\frac{V_1(t)}{R_1(SOC)C_1(SOC)} + \frac{I(t)}{C_1(SOC)}$$ (3)

$$\frac{dV_2}{dt} = -\frac{V_2(t)}{R_2(SOC)C_2(SOC)} + \frac{I(t)}{C_2(SOC)}$$ (4)

To estimate cell surface temperature, the energy conservation with a lumped-capacitance method [11] is shown in equation (5) where $m$ is the cell mass, $c_p$ is the specific heat capacity, $T$ is the cell surface temperature, $h$ is the heat transfer coefficient, $A_S$ is the cell surface area, $T_{amb}$ is the ambient temperature and $\dot{Q}$ is the convective heat generation rate. In this present work, $\dot{Q}$ is estimated using Equation (6). The reversible heat is neglected to simplify the problem [12-13].

$$mc_p \frac{dT}{dt} = hA_S(T - T_{amb}) + \dot{Q}$$ (5)

$$\dot{Q} = I^2(R_0 + R_1 + R_2)$$ (6)

Figure 3. The resistance-capacitance network-based equivalent circuit models

Figure 4. The pulse current discharge test for the parameters identification at 70% SOC
3.2 Parameter Identification for The Battery Cell

Based on the models shown in figure 3, the OCV, resistors and capacitors can be evaluated using the experimental data. To find the OCV equation as a function of SOC as shown in Equation (7), MATLAB curve fitting toolbox is used to obtain constants by comparing with the C/20 constant current discharge test. \(d_0\), \(d_1\), \(d_2\), \(d_3\), \(d_4\) and \(d_5\) are constants.

\[
V_{\text{OCV}} = d_0 + d_1(soc) + d_2(soc)^2 + d_3(soc)^3 + d_4e^{-d_5(soc)}
\] (7)

The resistors and the capacitors are estimated from the pulse current discharge tests. Figure 4 demonstrates the pulse current discharge test for the parameters identification at 70% SOC. At the time \(t_0\), \(R_0(soc)\) of both models is determined using Equation (8) where \(\Delta V_0\) is the voltage drop when the pulse discharge is applied. \(R_1(SOC)\) can be evaluated using equation (9). For the first-order model, \(\Delta V_1\) is the voltage drop from \(t_0\) to \(t_2\) whereas \(\Delta V_1\) for the second-order model is defined from \(t_0\) to \(t_2\). \(t_1\) is the half time of \(t_0\) and \(t_2\). \(t_2\) is the end time of the pulse discharge test. Time constants \(\tau_1(SOC)\) and \(\tau_2(SOC)\) of the first-order and the second-order can be estimated using Equation (10). \(C_1(SOC)\) can be obtained from the definition of a time constant as shown in Equation (11). Figure 5 shows the OCV, resistors and capacitors of the models.

\[
R_0(soc) = \frac{\Delta V_0}{I}
\] (8)

\[
R_1(soc) = \frac{\Delta V_1}{I}
\] (9)

\[
V_1(t) = IR_1\left(1 - e^{-\frac{t-t_0}{\tau_1}}\right)
\] (10)

\[
\tau_1(SOC) = R_1(SOC)C_1(SOC)
\] (11)

![Figure 5](image1.png)

**Figure 5.** The open-circuit voltage, resistors and capacitors of the models

![Figure 6](image2.png)

**Figure 6.** Cell model validations at room temperature

3.3 Cell Model Validations

Validation of discharge voltage at room temperature is shown in Figure 6. Noted that the cell was discharged using N3300A Agilent DC electronic load for 2C discharge [10]. The discharge current is the input of the models. The cell voltage and cell temperatures are compared with the experimental results. For validations of cell voltage, the simulation results show good agreement with the experimental data. For the first-order model, the maximum errors are 0.026 V, 0.027 V and 0.034 V for
C/3, 1C, and 2C-rates respectively. The maximum errors of the second-order model are 0.021 V, 0.024 V and 0.023 V for C/3, 1C, and 2C-rates respectively.

For cell temperatures validations, the thermal properties of the models are listed as follows. The cell mass is 0.585 kg and the specific heat capacity is 950 kJ/kg·K [10]. The convective heat transfer coefficients during the experimental validation are 3.382 W/m²K, 5 W/m²K, and 11.5 W/m²K for 1/3C, 1C, and 2C-rates respectively. For validation of the cell center temperatures, the simulation results show that the lumped-capacitance model of both models follows the experimental results. For the first-order model, the maximum errors are 4.34% (1.11°C), 4.90% (1.39°C) and 3.15% (1.03°C) for 1/3C, 1C, and 2C-rates respectively. For the second-order model, the maximum errors are 4.24% (1.08°C), 4.04% (1.14°C) and 3.03% (0.99°C) for 1/3C, 1C, and 2C-rates respectively. The errors of the temperature validation can be reduced by including the reversible heat into the heat generation equation.

4 Battery Pack Model Validation
This section presents the effectiveness of the battery pack models for the first-order and the second-order models.

4.1 Battery Pack Model
The battery pack configuration for the first-order model is shown in Figure 7. A group of four cells are connected in parallel to meet the required capacity. 20 groups of the cells are connected in series to meet the working voltage. Assuming that 80 cells in the pack have the same characteristics and capacity [14]. The resistance and the capacitance of the battery pack can be calculated using Equation (12) and (13) where $R_{0\_pack}$ is the resistance of the pack, $R_{0\_cell}$ is the resistance of the cell, $n_s$ is the number of the cell in series, $n_p$ is the number of the cell in parallel, $C_{1\_pack}$ is the capacitance of the battery pack, and $C_{1\_cell}$ is the capacitance of the battery cell. $R_{1\_pack}$ and $R_{2\_pack}$ are determined in the same manner of $R_{0\_pack}$. The battery pack model in MATLAB / Simulink environment is shown in Figure 8.

$$R_{0\_pack} = \left(\frac{n_s}{n_p}\right) R_{0\_cell}$$  \hspace{1cm} (12)

$$C_{1\_pack} = \left(\frac{n_p}{n_s}\right) C_{1\_cell}$$  \hspace{1cm} (13)

![Figure 7. Battery pack configuration for the first-order model](image7.png)

![Figure 8. Battery pack model in MATLAB / Simulink environment](image8.png)
4.2 Comparisons of Battery Pack Model Validations

Figure 9 shows current, voltage and SOC of the cells in the battery pack for the non-road load test. For the first-order model, the maximum errors are 0.74 mV, 1.46 mV and 1.66 mV for the average Root-Mean-Squared Error (RMSE), the minimum and the maximum cell voltage in the pack respectively. For the second-order model, the maximum errors are 0.74 mV, 1.29 mV and 1.81 mV for the average RMSE, the minimum and the maximum cell voltage in the pack respectively.

Figure 10 shows current, voltage and SOC of the cells in the battery pack for the Wide-Open Throttle test. For the first-order model, the maximum errors are 1.33 mV, 2.32 mV and 3.21 mV for the average RMSE, the minimum and the maximum cell voltage in the pack respectively. For the second-order model, the maximum errors are 1.18 mV, 2.38 mV and 3.23 mV for the average RMSE, the minimum and the maximum cell voltage in the pack respectively.

Figure 11.

**Figure 9.** Current, voltage and SOC of the cells in the battery pack for the non-road load test.

**Figure 10.** Current, voltage and SOC of the cells in the battery pack for the Wide-Open Throttle test.

**Figure 11.** Current, voltage and SOC of the cells in the battery pack for the real-world road test.

**Figure 12.** Model validation performance for the battery cell and the on-road battery pack tests.
Figure 11 shows current, voltage and SOC of the cells in the battery pack for the real-world road test. For the first-order model, the maximum errors are 3.00 mV, 18.66 mV and 15.50 mV for the average RMSE, the minimum and the maximum cell voltage in the pack respectively. For the second-order model, the maximum errors are 3.10 mV, 20.48 mV and 14.14 mV for the average RMSE, the minimum and the maximum cell voltage in the pack respectively.

Figure 12 shows the model validation performance for the battery cell and the on-road battery pack tests. The average RMSE, the minimum and the maximum of the cell voltage between the experimental data and the simulation results are summarized. The RMSE of the battery cell validation is increased when the discharge current is increased. For the on-road battery pack validation, the RMSE is high when the discharge current has highly fluctuated.

5 Conclusion
Based on the resistance-capacitance network, the first-order and the second-order equivalent circuit models are developed for an NCM cathode pouch cell and a battery pack. For the cell model, the parameters identification of the models is obtained from pulse current discharge tests. For cell validations of both models, the cell voltage and temperatures at three discharge rates show good agreement between the experimental data and the simulation results. The second-order model demonstrates better dynamic behavior of the cell. The single-cell model is used to estimate a group of four cells connected in parallel in the battery pack. A non-road load test, a Wide-Open Throttle (WOT) test and a real-world road test are conducted to capture steady-state response, step response and real-road drive respectively. The simulation results of the battery pack model validations show good agreement with the experimental data for both models.

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