Testing the RRPP vertex of effective Regge action

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Abstract

In frames of effective Regge action the vertices describing conversion of two reggeized gluons to one two and three ordinary gluons was constructed. The self-consistency: Bose symmetry and gauge invariance properties checks was shown to be fulfilled. The simplest one with creation of a single gluon was intensively verified in programs of experimental and theoretical treatment since it determine the kernel of of the known BFKL equation. Here we discuss the possibility to check the vertex with creation of two real gluons, which can reveal itself in process of scalar mesons production in high energy peripheral nucleons collisions. We show that the mechanisms which include emission of two gluons in the same effective vertex contribution dominate compared with one with the creation of two separate gluons. Numerical estimations of cross section of pair of charged pions production for LHC facility give the value or order 10mb. As well we estimate the excess of production of positively charged muons (as a decay of pions) created by cosmic ray proton collisions with the atmosphere gas nuclei to be in a reasonable agreement with modern data.

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I. INTRODUCTION

The problem of unitarization of the BFKL Pomeron as a composite state of two interacting reggeized gluons \cite{1} is actual problem of QCD \cite{2}. In the circle of questions are the construction of three Pomeron vertices, and more complicated ones including as well the ordinary gluons can be solved using in terms of vertices of effective Regge action formulated in form of Feynman rules for effective Regge action build in \cite{3}. In this paper the effective vertices containing the ordinary and the reggeized gluons were build.

A lot of attention to describe the creation in the peripheral kinematics of the bound states of heavy and light quarks was paid recent time \cite{4}. Creation of gluon bound states (gluonium) in the peripheral kinematics was poorly considered in literature. This is a motivation of this paper.

To satisfy the requirement for the scattered nucleons to be colorless states (non excited barions), we must consider the Feynman amplitudes corresponding to two reggeized gluons exchange. More over in each nucleon both reggeized gluons must interact with the same quark.

Process of creation of two gluons with the subsequent conversion of them to the bound state with quantum number of $\sigma$ meson in the pionization region of kinematics of nucleons collisions:

$$ N_A(P_A) + N_B(P_B) \rightarrow N_A(P'_A) + N_B(P'_B) + g_1(p_1) + g_2(p_2); \quad g_1(p_1) + g_2(p_2) \rightarrow \sigma(p), \quad (1) $$

can be realized by two different mechanisms. One of them consist in creation of two gluons each of them is created in "collision" of two reggeized gluon RRP - type vertex. Another contains the effective vertex with two reggeized and two ordinary gluons RRPP - vertex. The second reggeized gluon in the scattering channel do not create real gluons. the relevant $RRP$ vertex have a form:

$$ V_\mu(r_1(q_1, a) + r_2(q_2, b) \rightarrow g(\mu, c)) = \Gamma^\mu_{cad}(q_1, k, q_2) = g f_{cad} C_\mu(q_1, k, q_2), \quad (2) $$

with the coupling constant $g$, $g^2 = 4\pi\alpha_s$ and $k = q_1 + q_2$,

$$ C_\mu(q_1, k, q_2) = 2[(n^-)^\mu \left( q_1^+ + \frac{q_1^2}{q_2} \right) - (n^+)\mu \left( q_2^- + \frac{q_2^2}{q_1^+} \right) + (q_2 - q_1)\mu]. \quad (3) $$

The 4-vector $C_\mu$ obey the gauge condition $k^\mu C_\mu = 0$. 

The effective RRPP vertex with the conservation law

\[ r_1(q_1, c) + r_2(q_2; d) \rightarrow g_1(p_1, \nu_1, a_1) + g_2(p_2, \nu_2, a_2) \]

have a form [3]:

\[
\frac{1}{ig^2} \Gamma_{ca_1a_2d}^{\nu_1\nu_2}(q_1, p_1, p_2; q_2) = \frac{T_1}{p_{12}^2} \xi(q_1, q_2) \gamma_{\nu_1\nu_2\eta}(-p_1, -p_2, k) + \frac{T_3}{(p_2 - q_2)^2} \Gamma_{\nu_1}(q_1, p_1 - q_1) \Gamma_{\nu_2}(p_2 - q_2) - \frac{T_2}{(p_1 - q_2)^2} \Gamma_{\nu_2}(q_1, p_2 - q_1) \Gamma_{\nu_1}(p_1 - q_2) - T_1[(n^-)_{\nu_1}(n^+)_{\nu_2} - (n^-)_{\nu_2}(n^+)_{\nu_1}] - T_2[2g_{\nu_1\nu_2} - (n^-)_{\nu_1}(n^+)_{\nu_2} - T_3[(n^-)_{\nu_2}(n^+)_{\nu_1} - 2g_{\nu_1\nu_2}] - 2q_2^2(n^+)_{\nu_1}(n_)_{\nu_2}[\frac{T_3}{p_2^2 q_1^2} - \frac{T_2}{p_2^2 q_2^2}] - 2q_1^2(n^-)_{\nu_1}(n_)_{\nu_2}[\frac{T_3}{p_1 q_2} - \frac{T_2}{p_2 q_2}],
\]

with the light-like 4-vectors

\[ n^+ = p_B/E, n^- = p_A/E, n^+ n^- = 2, (n^\pm)^2 = 0, \]

\[ \sqrt{s} = 2E \] is the total center of mass energy,

\[ \gamma_{\mu\nu\lambda}(p_1, p_2, p_3) = (p_1 - p_2)\lambda g_{\mu
u} + (p_2 - p_3)\mu g_{\nu\lambda} + (p_3 - p_1)\nu g_{\lambda\mu}, p_1 + p_2 + p_3 = 0, \]

is the ordinary three gluon Yang-Mills vertex, and the induced vertices:

\[
\Gamma^{\mu\nu'}(q_1, q_2) = 2q_1^+ g^{\mu\nu'} - (n^+)_{\nu'}(q_1 - q_2)_{\nu'} - (n^+)^{\nu'}(q_1 + 2q_2)^{\nu'} - \frac{q_2^2}{q_1^2}(n^+)^{\nu}(n^+)^{\nu'};
\]

\[
\Gamma^{\mu\nu'}(q_1, q_2) = 2q_2^- g^{\mu\nu'} + (n^-)^{\nu'}(q_1 - q_2)_{\nu'} + (n^-)_{\nu'}(-q_2 - 2q_1)^{\nu'} - \frac{q_1^2}{q_2^2}(n^-)^{\nu}(n^-)^{\nu'}.
\]

We use here the notation \( k^\pm = (n^\pm)_\mu k^\mu \) and light cone decomposition implies:

\[
q_1 = q_1^+ + \frac{q_1^+}{2} n^-, q_2 = q_2^+ + \frac{q_2^+}{2} n^+; q_1^- = q_2^+ = 0,
\]

\[
p_i = \frac{p_i^+}{2} n^- + \frac{p_i^-}{2} n^+ + p_i\perp, p_i n^\pm = 0.
\]

The color structures are

\[
T_1 = f_{a_1a_2} f_{c\perp d}, T_2 = f_{a_2r} f_{a_1d}; T_3 = f_{c\perp r} f_{a_2d},
\]

with \( f_{abc} \) is the structure constant of the color group; Jacoby identity provides the relation \( T_1 + T_2 + T_3 = 0 \). The conditions of Bose-symmetry and gauge invariance:

\[
\Gamma_{ca_1a_2d}^{\nu_1\nu_2}(q_1, p_1, p_2; q_2) p_{1\nu_1} = 0, \Gamma_{ca_1a_2d}^{\nu_1\nu_2}(q_1, p_1, p_2; q_2) = \Gamma_{ca_2a_1d}^{\nu_2\nu_1}(q_1, p_1, p_2; q_2),
\]

are satisfied.
II. POMERON MECHANISMS OF $\sigma$ MESON PRODUCTION

We consider the case when the hadrons after collision remain to be colorless. For the case of nucleon collisions it results that both exchanged reggeized gluons must interact with the same quark. The color coefficient associated with $RRPP$ vertex results to be

$$Tr t^{\mu t} \times Tr t^{\nu t} = \frac{1}{4} N \delta_{\alpha_1 \alpha_2} \Pi^{\mu \nu}_{02},$$

(11)

$N = 3$ is the rank of the color group. For the mechanism of creation of two separate gluons we have

$$Tr t^{\mu t} \times Tr t^{\nu t} \times f_{nma_1} \times f_{kla_2} \times \Pi^{\mu \nu}_{11} = \frac{1}{4} N \delta_{\alpha_1 \alpha_2} \Pi^{\mu \nu}_{11}.$$

(12)

Projecting the two gluon state to the colorless and spin-less state we use the operator:

$$\mathcal{P} = \frac{\delta_{a_1 a_2}}{\sqrt{N^2 - 1}} g^{\mu \nu}.$$

(13)

The resulting expressions are

$$\frac{1}{16} N \sqrt{N^2 - 1} \left[ \Pi_{02}, \Pi_{11} \right]$$

with

$$\Pi_{02} = -12 - \left[ \frac{1}{(p_2 - q_2)^2} \Gamma_{\eta \nu -} (q_1, p_1 - q_1) \Gamma_{\eta \nu +} (p_2 - q_2, q_2) + (p_1 \leftrightarrow p_2) \right];$$

(14)

$q_1 = l_1, q_2 = p_1 + p_2 - l_1$ and

$$\Pi_{11} = C_\mu (l - l_1, l_1 - l + p_1) C_\mu (l_1, p_1 - l_1),$$

(15)

Here $l_1$ is the 4-momentum of the gluonic loop, $l = P_A - P_{A'}$ is the transferred momentum.

In a realistic model describing interaction two reggeized gluons with the transversal momenta $\vec{l}_1, \vec{l}_2$, which form a Pomeron with quark $[5]$ we have for the corresponding vertex

$$\Phi_P (\vec{l}_1, \vec{l}_2) = -\frac{12\pi^2}{N} F_P (\vec{l}_1, \vec{l}_2),$$

(16)

with

$$F_P (\vec{l}_1, \vec{l}_2) = \frac{-3\vec{l}_1 \vec{l}_2 C^2}{(C^2 + (\vec{l}_1 + \vec{l}_2)^2)(C^2 + \vec{l}_1^2 + \vec{l}_2^2 - \vec{l}_1 \vec{l}_2)},$$

(17)

and $C = m_\rho/2 \approx 400 MeV$. We note that this form of Pomeron-quark coupling obey gauge condition: it turns to zero at zero transverse momenta of gluons. The factor 3 corresponds to three possible choice of quark into proton.
Matrix elements of peripheral processes are proportional to $s$. To see it we can use Gribov’s substitution to gluon Green functions nominators $g_{\mu\nu} = (2/s)P_{\mu A}P_{B\nu}$ with Lorentz index $\mu(\nu)$ is associated with $B(A)$ parts of Feynman amplitude. Performing the loop momenta $l_1$ integration it is convenient to use such a form of phase volume

$$d^4 l_1 = \frac{1}{2s} ds_1 ds_2 d^2 \vec{l}_1, s_1 = 2P_A l_1, s_2 = 2P_B l_1.$$  

(18)

Simplifying the nucleon nominators as

$$\bar{u}(P_A') \hat{P}_B \hat{P}_A \hat{P}_B u(P_A) = s^2 N_A, N_A = \frac{1}{s} \bar{u}(P_A') \hat{P}_B u(P_A), \sum |N_A|^2 = 2,$$  

(19)

(with the similar expression for part $B$), we find all they be equal. The integration on variables $s_{1,2}$ can be performed for the sum of all four Feynman amplitudes as

$$\int_{-\infty}^{\infty} \frac{ds_{1,2}}{2\pi i} \left[ \frac{1}{s_{1,2} + a_{1,2} + i0} + \frac{1}{-s_{1,2} + b_{1,2} + i0} \right] = 1.$$  

(20)

Combining all the factors the matrix element corresponding to the vertex RRPP can be written as

$$- iM = 2^5 3^2 s(\pi \alpha_s)^3 N_A N \sqrt{N^2 - 1} \frac{1}{N} F(\vec{\Delta}) f(\vec{l}, \vec{p}),$$  

(21)

with $\vec{p} = \vec{p}_1 + \vec{p}_2$ and the relative momenta of real gluons $\vec{\Delta} = (\vec{p}_2 - \vec{p}_1)/2$,

$$f(\vec{l}, \vec{p}) = \int \frac{d^2 l_1 C^4}{2^5 \pi^2 (l - \vec{l}_1)^2 (\vec{p} - \vec{l}_1)^2} F_P(\vec{l}_1, \vec{l} - \vec{l}_1) F_P(\vec{l}_1 - \vec{l}, \vec{p} - \vec{l}_1) \Pi_{02}(\vec{l}_1, \vec{p} - \vec{l}_1),$$  

(22)

and the similar expression for another mechanism. Here we had introduced the factor $F'(\vec{\Delta}) = [a^2 \vec{\Delta}^2 + 1]^{-2}$ which describe the conversion of two gluon state to bound state of the size $a \sim 1 fm$ which is gluonium component of scalar meson.

Let perform the phase volume of the final state as

$$d \Gamma = \frac{d^3 P_B' d^3 P_A' d^3 p_1 d^3 p_2}{2 E_B' 2 E_A' 2 E_1 2 E_2} (2\pi)^{-8} \delta^4(P_A + P_B - P_{A'} - P_{B'} - p_1 - p_2)$$

$$= d \Gamma_{AB} d \Gamma_{12} (2\pi)^{-8},$$

$$d \Gamma_{AB} = \frac{d^4 l d^4 P_B d^4 P_A \delta^4(P_A - P_{A'} - l) \times \delta^4(P_B + l - P_{B'} - p) \delta(P_{B'} - M_{B'}^2) \delta(P_{A'} - M_{A'}^2),}{2 E_1 2 E_2}$$  

(23)
Using the relation
\[ d^4 l = \frac{1}{2s} d(2lP_B) d(2lP_A) d^2 \vec{l}, \]  
(24)
we perform integration on the momenta of the scattered nucleons with the result:
\[ d\Gamma_{AB} = \frac{d^2 \vec{l}}{2s}. \]
(25)
Keeping in mind the almost collinearity of 3-momenta of real gluon we can transform the gluon part of the phase volume as:
\[ d\Gamma_{12} = d^4 p \delta(p^2 - M^2) \frac{2}{M} d^3 \Delta. \]
(26)
Integration on the \( \Delta \) can be performed in the explicit form:
\[ \int d^3 \Delta F^2(\Delta) = \frac{\pi^2}{4a^3} = \frac{2\pi^2 M_p^3}{10^3 a (fm)^3}, \]
(27)
where we had used the conversion constant \( M_p \times fm = 5 \), \( M_p \) is the nucleon mass. The last part of the phase volume of gluonic system can be arranged using light cone form of 4-momentum \( p \) (see (8)):
\[ d^4 p \delta(p^2 - M^2) = \frac{dp^+}{2p^+} d^2 \vec{p} = \frac{1}{2} L d^2 \vec{p}. \]
(28)
with the so called "boost logarithm" \( L \approx \ln(\frac{2E}{M}) \), \( E, M \) – Energy and mass of proton in laboratory reference frame. For LHC facility as well as for cosmic protons (in the knee region of spectra) we use below \( L = 15 \).

For the contribution to the total cross section we obtain
\[ \sigma_1^P = A \frac{6 L M_p^3}{M_\sigma M_\rho^4} J, \ A = \frac{6^4 N^2 - 1}{5^3} \frac{\pi^2}{N^2 a (fm)^3}, \]
(29)
with
\[ J = \int \frac{d^2 l d^2 p}{(2\pi)^2 C^4} f^2(l, \vec{p}). \]
(30)
Numerical integration give \( J = 7.4 \times 10^3 \).

Corresponding contribution to the total cross section of the single \( \sigma \) meson production is of order of 10mb.

The contribution arising from the other mechanism of production (including the interference of amplitudes) turns out be at least order of magnitude less. It determines the accuracy of the result obtained on the level of 10%.
III. SCREENING EFFECTS. SEVERAL $\sigma$- PRODUCTION

Let us now generalize the result to include the screening effects as well as the possibility to produce several $\sigma$ mesons.

At large impact parameters limit proton interact with the whole gluon field of the nucleon (or nuclei) moving in the opposite direction coherently. So the main contribution arises from the many Pomeron exchanges mechanism (compare with the "chain' mechanism essential in BFKL equation) [1]. So we must consider Pomeron s-channel iterations. Let consider three kinds of the iteration blocks. One is the pure Pomeron exchange, the second is Pomeron with the sigma meson emission from the central region. The third one is the "screening block":two blocks of the second type with the common virtual $\sigma$-meson. Contribution to the amplitude of production $n$ $\sigma$ mesons of the blocks of the third kind is associated with the "large logarithm" which arises from boost freedom of these blocks in completely analogy with QED [6].

In the similar way the closed expression (omitting the terms of order $1/N^2$ compared with the ones of order of 1) for the summed on number of s-channel iteration ladders of the first and the third type can be obtained using the relation

$$
\int \Pi_1^n \frac{d^2k_i}{(2\pi)^2} = \int \Pi_1^{n+1} \frac{d^2k_i}{(2\pi)^2} \int d^2\rho \exp\left(i\sum_{i=1}^{n+1}(\vec{k}_i - \vec{q})\right) = \\
\int d^2\rho \exp\left(-i\vec{q}\vec{\rho}\right)\Pi_1^{n+1} \frac{d^2k_i}{(2\pi)^2} e^{i\vec{k}_i\vec{\rho}}. 
$$

(31)

Accepting the assumption about color-less structure of the Pomeron as a bound state of two reggeized interacting gluons and applying the same sequence of transformations as was done in [6] we obtain for the cross section of $n$ $\sigma$ mesons production at the peripheral high energy protons collisions:

$$
\sigma_n = \int \frac{d^2p}{(2\pi)^2} \left(\frac{L\sigma_0 Z(\rho)}{n!}\right)^n \exp(-L\sigma_0 Z(\rho)), 
$$

(32)

with $\sigma_0 = 2.25 \times 10^{-3}a_s^6M_\rho^3/(M_\sigma M_\rho^4a(fm)^3)$ and

$$
Z(\rho) = \int \frac{d^2p}{(2\pi)^2}|B(\rho, p)|^2, \\
B(\rho, p) = \int \frac{d^2l}{(2\pi)^2} f(l, p) \exp(i\vec{l}\vec{\rho}).
$$

(33)

Numerical estimations of the cross section of one sigma meson production for the give $\sigma_1 = 10mb$. 

7
IV. CONCLUSION

The results given above can be applied to explain the excess of the positive muons compared with the negative ones produced by cosmic ray interaction with the Earth surface. Really one can neglect QED mechanisms of production of the charged pions in favor to strong interactions one. It turns out that the main mechanism is the peripheral production of the pions pairs (with the subsequent decay to muons) in the high energy cosmic ray proton collisions with the nuclei of nitrogen or oxygen in the Earth atmosphere.

Keeping in mind the atmosphere gas density one can be convinced that at least one direct collision of cosmic ray proton with the nuclei take place. The QCD mechanism contribution for impact parameters $\rho$ exceeding size of nuclei ($\rho \gg A^{1/3} fm$) is also small. Travelling through the nuclei the cosmic proton have a direct collision with the protons and the neutrons of the nuclei $\rho \leq 1 fm$ (such kind of collisions produce positive charged pions due to the decay of the excited resonances) or have the peripheral collisions, when the pairs of pions are produced ($1 fm < \rho < A^{1/3} fm$).

The number of positive charged pions produced in direct collisions $N_d$, is proportional to $A^{1/3}$ with the characteristic atomic number $A = 14$. The number of pion pairs produced in the peripheral collisions can be estimated as

$$N_p = M_p^2 \sigma_1 \approx 7.8.$$  \hspace{1cm} (34)

For the ratio of the positive charged muons to the negative charged ones $R = \frac{N_{\mu^+}}{N_{\mu^-}}$ we have

$$R_{th} = 1 + \frac{N_d}{N_p} = 1.32.$$  \hspace{1cm} (35)

This quantity can be compared with the recent experimental value $R$ (here only hard muons are taken into account).

$$R_{exp} = \frac{N_{\mu^+}}{N_{\mu^-}} = 1.4 \pm 0.003.$$  \hspace{1cm} (36)

These values are in reasonable agreement.

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[1] E. Kuraev, L. Lipatov and V. Fadin, Sov. Phys. JETP, 44, 443 (1976), ibid 45 (1977), 199;
I. Balitski and L. Lipatov, Sov. J. Nucl. Phys. 28, (1978), 822.
[2] V. Gribov and L. Lipatov, Sov. J. Nucl. Phys. 18 (1972), 438, 675;
L. Lipatov, Sov. J. Nucl. Phys. 20, (1975), 93; G. Altarelli and G. Parisi, Nucl. Phys. B 126, 298;
Yu. Dokshitzer, Sov. Phys. JETP, 46 (1977), 641.
[3] E. Antonov, L. Lipatov, E. Kuraev and I. Cherednikov, Nucl. Phys.
[4] V. Kiselev, F. Likhoded, O. Pakhomova and V. Saleev, Phys. Rev. D66:034030, 2002.
[5] J. Gunion and D. Soper, Phys. Rev. D 15 (1977), 2617;
M. Fukugita and J. Kwiecinski, Phys. Lett. B 83 (1979), 119.
[6] E. Bartos, S. Gevorkyan, E. Kuraev and N. Nikolaev, Phys. Lett. B538 (2002), p 45.
[7] P. Adamson et al. MINOS Collaboration, hep-ex-0701045, 2007.