Why Decoherence has not Solved the Measurement Problem:  
A Response to P.W. Anderson

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ABSTRACT

We discuss why, contrary to claims recently made by P.W. Anderson, decoherence has not solved the quantum measurement problem.

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It has lately become fashionable to claim that decoherence has solved the quantum measurement problem by eliminating the necessity for Von Neumann’s wave function collapse postulate. For example, in a recent review in *Studies in History and Philosophy of Modern Physics*, Anderson (2001) states “The last chapter... deals with the quantum measurement problem....My main test, allowing me to bypass the extensive discussion, was a quick, unsuccessful search in the index for the word ‘decoherence’ which describes the process that used to be called ‘collapse of the wave function’. The concept is now experimentally verified by beautiful atomic beam techniques quantifying the whole process.” And again, in his response to the author’s response (Anderson, 2001), “Our difference about ‘decoherence’ is real. I find this, and the cluster of ideas around it much preferable to more traditional ways of treating the quantum paradoxes because the ‘classical’ apparatus is treated as a quantum system as well....; and as I remarked, recent experiments have verified this approach.” In a somewhat similar vein, Tegmark and Wheeler (2001) state in a recent *Scientific American* article discussing the “many-worlds” interpretation of quantum mechanics and decoherence, “…it is time to update the quantum textbooks: although these infallibly list explicit non-unitary collapse as a fundamental postulate in one of the early chapters, ...many physicists ... no longer take this seriously. The notion of collapse will undoubtedly retain great utility as a calculational recipe, but an added caveat clarifying that it is probably not a fundamental process violating Schrödinger’s equation could save astute students many hours of frustrated confusion.”

These striking statements to the contrary, I do not believe that either detailed theoretical calculations or recent experimental results show that decoherence has resolved the difficulties associated with quantum measurement theory. This will not be a surprise to
many workers in the field of decoherence; for example, in their seminal paper on decoherence as a source of spatial localization, Joos and Zeh (1985) state “Of course no unitary treatment of the time dependence can explain why only one of these dynamically independent components is experienced.” And in a recent review on decoherence, Joos (1999) states “Does decoherence solve the measurement problem? Clearly not. What decoherence tells us is that certain objects appear classical when observed. But what is an observation? At some stage we still have to apply the usual probability rules of quantum theory.” Going back a few years, an informative and lively debate on these issues can be found in the Letters column of the April 1993 *Physics Today* (starting on page 13 of that issue and continuing over many pages), in response to an earlier article in that journal by Zurek (1991). An enlightening discussion of the measurement problem has been given by Bell (1990), and there also are extensive discussions of both the measurement problem and the role of decoherence in the philosophy of physics literature. A careful analysis of the measurement problem has been given by Brown (1986), who reviews earlier work of Fine (1969) and others. Rebuttals to the claim that decoherence solves the measurement problem have been given in the books of Albert (1992), Barrett (1999) and Bub (1997), with Bub’s treatment closest in spirit to the formulation given below. A detailed analysis of decoherence within the consistent histories approach has been given by Kent and McElwaine (1997), and discussions of decoherence in the context of the many-worlds approach can be found in Bacciagaluppi (2001) (who gives an extensive bibliography on decoherence as it relates to the measurement problem) and in Butterfield (2001). Despite the existence of these and other prior discussions, I think it worthwhile to revisit the substantive issues, particularly in the light of recent claims that decoherence resolves the measurement problem.
Let me begin by setting up a simple quantum mechanical model for measurement, in which the effects of decoherence can be explicitly taken into account. Let us consider a two state system $X$ with initial ($t = 0$) state vector

\[ |\psi_0\rangle_X = \alpha|\psi^{(A)}\rangle_X + \beta|\psi^{(B)}\rangle_X, \]

with $|\alpha|^2 + |\beta|^2 = 1$, which could be, for example, an atomic ion (or nucleus) with up and down spin states and a high enough energy gap for internal excitations so that these can be neglected. This system is probed by an apparatus with initial state vector $|\phi_0\rangle_{APP}$, and the system and apparatus interact in turn with an environment with initial state vector $|\phi_0\rangle_{ENV}$. Thus, at the start of the measurement process, the total wave function is the direct product

\[ |\Phi_0\rangle = |\psi_0\rangle_X |\phi_0\rangle_{APP} |\phi_0\rangle_{ENV}. \]

Before going on to discuss the time development of this system, let me comment on assumptions that are implicit in Eq. (2). First of all, there is nothing subjective about the separation into system, apparatus, and environment. At the low energies characterizing atomic beam experiments, atomic nuclei are conserved species of particles, with an independent conservation law for each atomic number and for each isotope. Hence with a system consisting, say, of a silver ion or nucleus, an apparatus constructed from iron, copper, etc., and an environment consisting of the atmospheric gases together with electromagnetic radiation, there is an objective distinction between system, apparatus, and environment, even when indistinguishability of like particles is taken into account. Secondly, convergent calculations presuppose a finite dimensional Hilbert space (unless careful precautions are taken). Once high energy or short distance physics is subsumed in renormalized parameters, this assumption is obeyed for the system and apparatus, which contain a finite number of atoms.
How many particles must be considered in the environment? Letting $T$ be the time allotted for the measurement (typically of order $10^{-3}$ seconds in an atomic beam experiment), only those environmental particles that are within a radius $R = cT$ of the apparatus, with $c$ the velocity of light, can causally influence the experimental outcome. We shall use this criterion to define the number of particles in the environment; it is clearly tens of orders of magnitudes smaller than the number of particles in the visible universe. While it may be questionable whether the Schrödinger equation can be applied to the entire universe, there should be no difficulty, if quantum mechanics is an exact theory, in applying the Schrödinger equation to the state vector for all environmental particles within the causal horizon $R$ for the apparatus.

Let us now allow the initial state vector of Eq. (2) to evolve in time according to the deterministic unitary evolution $U = \exp(-iHt)$. In general, the two system states, the apparatus states, and the environment states become entangled in a generic way, so that all we can say is that the evolved wave function is $|\Phi(t)\rangle = U|\Phi_0\rangle$, which conveys no useful information. However, for a cleverly devised apparatus, such as that used in a Stern-Gerlach or similar molecular beam experiment, we can guarantee that the state that evolves from the initial state $|\Phi_0\rangle$ has the form

$$|\Phi(t)\rangle = \alpha|\psi^{(A)}\rangle_X|\phi^{(A)}(t)\rangle_{APP+ENV} + \beta|\psi^{(B)}\rangle_X|\phi^{(B)}(t)\rangle_{APP+ENV},$$

(3)

where $|\phi^{(A)}(t)\rangle_{APP+ENV}$ and $|\phi^{(B)}(t)\rangle_{APP+ENV}$ are entangled states of the apparatus and environment that by time $T$ are macroscopically distinguishable. The presence or absence of $|\phi^{(A,B)}(T)\rangle_{APP+ENV}$ is registered by a recording device, which long after $T$ can be read by an observer who travels in from outside the causal horizon of the apparatus.

Even in this generality, without invoking a specific model, we can state the effect
of decoherence. What decoherence does is to cause the rapid decay with time of the inner product

$$\langle \phi^{(A)}(t) | \phi^{(B)}(t) \rangle_{\text{APP+ENV}} ,$$  \hspace{1cm} (4)

which at time \( t = 0 \) was unity. As a consequence, interference effects between the system states \( |\psi^{(A)}\rangle_X \) and \( |\psi^{(B)}\rangle_X \), which are initially present, rapidly disappear as time evolves. A widely used model for decoherence, originated by Harris and Stodolsky (1981) in the context of optically active molecules, and applied to spatial localization of macroscopic objects by Joos and Zeh (1985), fits readily into this framework. In this model one neglects the interactions of the environmental particles with each other and with the system \( X \), and treats their scattering from the apparatus in an elastic, static, instantaneous approximation, so that there is no excitation of internal degrees of freedom of the apparatus, recoil of the apparatus is ignored, and the effect of scattering of a particle of the environment on the apparatus is described by the action of the single particle \( S \) matrix on the particle initial state. In this approximation,

$$|\phi^{(J)}(t)\rangle_{\text{APP+ENV}} = |\phi^{(J)}(t)\rangle_{\text{APP}} \prod_i S_{i(J)} |\phi_0\rangle_{\text{ENV},i} \prod_k |\phi_0\rangle_{\text{ENV},k} , \quad J = A, B ,$$  \hspace{1cm} (5a)

with \( |\phi_0\rangle_{\text{ENV},i} \) the initial state of the \( i \)th environmental particle that has scattered by time \( t \), with \( |\phi_0\rangle_{\text{ENV},k} \) the initial state of the \( k \)th environmental particle that has \textit{not} scattered by time \( t \), and with \( S_{i(J)} \) the \( S \) matrix for the \( i \)th particle to scatter from the state \( (J) \) of the apparatus. (Thus the products over \( i \) and \( k \) together extend over all particles in the environment.) In the approximation of Eq. (5a), the inner product of Eq. (4) becomes

$$\langle \text{APP+ENV} | \phi^{(A)}(t) | \phi^{(B)}(t) \rangle_{\text{APP+ENV}} \simeq \langle \text{APP} | \phi^{(A)}(t) | \phi^{(B)}(t) \rangle_{\text{APP}} \prod_i \langle \phi_0 | S_{i(A)}^\dagger S_{i(B)} |\phi_0\rangle_{\text{ENV},i} .$$  \hspace{1cm} (5b)
Since each factor \( E_{\text{ENV};i} \langle \phi_0 | S_{i(A)^\dagger} S_{i(B)} | \phi_0 \rangle_{E_{\text{ENV};i}} \) is in general of magnitude less than 1, and since the number of factors in the product over \( i \) grows approximately linearly with the elapsed time, the off diagonal matrix element of Eq. (5b) approaches zero as \( \exp(-\Lambda t) \), with \( \Lambda \) defining the decoherence rate, as \( t \) becomes large. This formalism has been applied to estimate decoherence rates in a large variety of processes of physical interest, and as we have already noted, fits seamlessly into the framework of our more general discussion.

Returning to the general formula of Eq. (3), the quantum measurement problem consists in the observation that Eq. (3) is \textit{not} what is observed as the outcome of a measurement! What is seen is not the superposition of Eq. (3), but rather \textit{either} the unit normalized state

\[
|\psi^{(A)}\rangle_X |\phi^{(A)}(t)\rangle_{\text{APP+ENV}},
\]

\( (6a) \)

\textit{or} the unit normalized state

\[
|\psi^{(B)}\rangle_X |\phi^{(B)}(t)\rangle_{\text{APP+ENV}}.
\]

\( (6b) \)

But because these states are orthogonal, they cannot both have evolved from a single initial state by a deterministic, unitary evolution, since it is a property of unitary transformations in Hilbert space that if \( |(A)\rangle = U|0\rangle \), and \( |(B)\rangle = U|0\rangle \), then \( \langle(A)|(B)\rangle = \langle0|U^\dagger U|0\rangle = 1 \). Thus, when quantum mechanics is applied uniformly at all levels, to the apparatus and its environment as well as to the system, we are faced with a contradiction. This contradiction is in no way ameliorated by decoherence, since the inner product of Eq. (5b) plays no role in the final state vector of Eq. (6a) or Eq. (6b) that describes the outcome of the measurement. Note also that to see this contradiction we do not need an infinite sequence of repetitions of the experiment, as would be needed to discuss the probabilities of the outcomes \( A \) and
(B), since only enough repetitions are needed to achieve an outcome (A) and an outcome (B) at least once.\footnote{The preceding argument is based on a simplified, “bare bones” statement of the measurement problem, which suffices to demonstrate the ineffectiveness of decoherence. More precise treatments of the measurement problem can be found in the article of Brown (1986) and in the book of Bub (1997). Additionally, the assumption of a deterministic unitary evolution can be weakened to that of a deterministic linear evolution, as discussed for example by Bassi and Ghirardi (2000).}

What are the ways out of this dilemma? One route, discussed in detail in the book of DeWitt and Graham (1973), is to insist that the superposition of Eq. (3) \textit{is} the final outcome of the measurement, not Eqs. (6a) or (6b), with the world state vector splitting into two branches, only one of which we observe. A measure is then postulated on the space of world state vectors, with respect to which outcomes obeying the Born rule are typical, while outcomes not obeying the Born rule are of measure zero. A second route, represented by Bohmian quantum mechanics [Bohm (1952) and Dürr, Goldstein and Zanghi (1992)] and, alternatively, Ax-Kochen quantum mechanics [Ax and Kochen (1999)], supplements the usual quantum mechanical formalism with an associated configuration space, which plays a role in picking individual outcomes, in such a way that Born rule probabilities emerge for the observer who cannot access the additional information contained in the auxiliary configuration space. These approaches are all \textit{interpretations} of quantum mechanics, in the specific sense that by design they reproduce all physical predictions of quantum mechanics, and so are empirically indistinguishable from the orthodox theory, while changing the mathematical foundations so as to resolve the difficulties associated with measurement theory.
If we insist on having only one world existing within the standard arena of states and operators in Hilbert space, we must instead discard one or more of the assumptions made in our analysis above, by injecting new physics. One alternative is to preserve the deterministic unitary evolution of quantum mechanics, and to drop the assumption that the environment and/or apparatus can always be prepared in a specified initial state. One might then attempt to show that the discrete choice of experimental outcome is tied to details of the initial state, giving a sense in which “decoherence”, as understood more generally to mean environmental influence, could be said resolve the measurement problem. A calculation showing how this might happen has never been given, and in fact, G. Grübl (2002) has recently pointed out that a modification of the generalized formulation of the measurement problem given by Bassi and Ghirardi (2000) shows, under very weak assumptions, that initial state environmental effects cannot explain the occurrence of definite experimental outcomes.

Another alternative, which can also be formulated within the standard state vector and operator apparatus of Hilbert space, is to abandon the assumption of a deterministic unitary evolution, and to suppose instead that the evolution is stochastic unitary, in the sense that while the wave function for an individual system evolves unitarily, this evolution has a random, or stochastic component. This approach was pioneered by Ghirardi, Rimini, and Weber (1986), Pearle (1976, 1979), Gisin (1984) and Di’osi (1988), and has been studied by many others.\(^2\) It is implemented by replacing the standard Schrödinger evolution of the state vector by a stochastic differential evolution, in which \(d|\psi\rangle\) receives a contribution

\(^2\) See Adler, Brody, Brun and Hughston (2001) and Adler (2002) for recent mathematical and phenomenological analyses, respectively. These papers give further references to the literature on the stochastic reduction approach.
proportional to $dt$ together with an additional small, nonlinear contribution proportional to $dW_t$, with $dW_t$ a stochastic differential obeying $dW_t^2 = dt$. Heuristically, the idea here is that quantum mechanics may be modified by a low level universal noise, akin to Brownian motion, possibly arising from new physics at the Planck scale, which in certain situations causes reduction of the state vector. This approach, which has been developed in great detail, reproduces the observed fact of discrete outcomes governed by Born rule probabilities, and predicts the maintenance of coherence where that is observed (including in systems with large numbers of particles, such as recent superconductive tunneling and molecular diffraction experiments), while predicting state vector reduction when the apparatus parameters are those characterizing measurements. The stochastic approach may ultimately be falsified by experiment, but it constitutes a viable phenomenological solution to the measurement problem. Decoherence, in the absence of a detailed theory showing that it leads to stochastic outcomes with the correct properties, has yet to achieve this status.
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