Effect of deformation induced nonlinear and anisotropic elasto-plasticity on sheet forming simulations

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Abstract. Anisotropy in initial yield and subsequent hardening in flow stress has been modeled in the finite element simulations to enhance the accuracy of formability and springback in sheet metal products. As continued efforts in the field of constitutive modeling, the evolution of anisotropy during the plastic deformation and its dependency on deformation path were also studied by many researchers. In addition to the plastic behavior, the elastic nonlinearity in pre-deformed sheet metals has been also regarded as a key factor which influences the accuracy of the sheet metal forming simulation, particularly in the springback simulation. In this study, recent reviews on the constitutive modeling for the deformation induced nonlinear and evolution of anisotropy are provided mainly using the published data [1,2]. To model the initial anisotropy the Hill 1948 yield function with non-associated flow rule and Yld2000-2d non-quadratic yield function were considered. Also, these yield surfaces evolve as a function of equivalent plastic strain. For better modeling the nonlinearity of elastic behavior during the deformation, recently developed multi-surface elasticity model was implemented in the simulation. To investigate the effect of the considered anisotropic plasticity and nonlinear elasticity on forming simulation, cylindrical circular cup drawing was experimentally conducted and its earing profile and springback after splitting of the formed cup in the circumferential direction were compared with the simulated results. The results showed that the finite element simulations could predict the deformed shapes after forming and springback with enhanced accuracy when the constitutive model could represent the complexity in the anisotropy and nonlinearity during plastic deformation.

1. Introduction

The constitutive equations in the metal forming simulations include elasticity matrix, yield functions and hardening laws. In the conventional sheet metal forming simulations under isothermal condition, the elasticity is assumed to be linear isotropic and the Hooke’s law identified by Young’s modulus and Poisson’s ratio has been usually adopted. For the yield functions characterizing the initial yield and plastic anisotropy, anisotropic yield functions under plan stress condition in either quadratic or non-quadratic forms have been used to consider anisotropic nature of rolled sheets for metal forming applications. In terms of the strain or work hardening behavior of sheet metals, conventional isotropic
hardening laws have been employed, but more complex anisotropic hardening models based on either 
kine\michael's hardening or distortional hardening also drew considerable attentions for the metal forming 
simulations, especially for the springback. Conventionally the sheet metal forming simulations based on 
the above mentioned constitutive models made positive contributions to the predictions of formability 
and springback of automotive parts with good accuracy but within the reasonable computational cost. 
However, the shape of parts and forming processes become more complicated than those in conventional 
 stamping as the emergence of advanced high strength steels (AHSS) and lightweight alloys, which led 
to non-conventional forming technologies. Under the non-conventional forming with sheet made of 
AHSS or lightweight alloys, larger springback due to the high strengths or smaller elastic constants is 
expected and the deformation paths are more complex and changeable during the forming operations. 
Recently, numerous efforts have been made to model such complex mechanical behavior by developing 
more accurate constitutive models which reproduce the anisotropic responses of sheet metals under non-

\[\text{constant strain paths for advanced sheet metals. As examples, in this study, the evolutions of yield} \]
\[\text{functions and elastic property as a function plastic deformation are presented and validated by the} \]
\[\text{comparisons of simulated results with experiments. The modeling efforts presented in this study were} \]
\[\text{mostly published, thus interested readers are suggested to refer to the authors' other publications [1,2].} \]

2. Summary of constitutive model 
The constitutive equations used in this study are briefly summarized as following. 
For the modeling of elasticity of sheet metals, three cases are considered.

- Constant elastic modulus defined by the initial Young’s modulus (E0).
- Chord modulus (Chord) defined by the slope of the two points between initiation of unloading 
  and fully unloaded states in the true stress-strain curve. The Chord modulus is defined as a 
  function of equivalent plastic strain [3,4].
- Multi-surface elasticity model (MS) to reproduce the realistic hysteric loops during the loading-
  unloading-reloading of stress-strain curve as a function of plastic deformation [2].

The Chord and MS models were investigated in this study because many previous studies observed 
the nonlinearity of apparent elastic behavior during unloading (or reloading) after prior plastic 
deformation [5-7]. The unloading behavior of the MS and Chord models becomes identical if the stress 
is fully recovered, but difference occurs for the deformation with considerable residual stress.

For the anisotropic yield functions, the following two models were applied to the finite element 
simulations.
- A quadratic yield function by Hill [8] identified by the yield stresses or r-values along three 
  different material orientations (Hill48)
- Yld2000-2d for a non-quadratic yield function [9] identified by three yield stresses and r-values 
  under uniaxial deformation and additional stress and plastic anisotropy under equi-biaxial stress 
  state (Yld2000)

Additionally, the evolutions of yield functions are also considered to represent the distortion of initially 
defined yield surfaces as the plastic deformation proceeds. This evolution of yield function is sometimes 
referred to the differential hardening [1,10].

To study the effect to hardening on the predictive capability for sheet metal forming simulations, 
two distinctive hardening laws were considered
- Classical isotropic hardening law based on Power law type (or Swift hardening law) in which 
  the yield function expands proportionally (ISO)
- Distortional hardening model [11,12] to represent the Bauschinger effect under strain path 
  changes, especially under reversed loading condition (HAH). The HAH model represents
homogeneous yield function based anisotropic hardening model which is not based on the kinematic hardening concept.

All the constitutive models were implemented in the user material subroutine of ABAQUS software with semi-implicit stress update algorithm [13].

3. Material identification procedure
The material parameters for each constitutive model were determined from proper experimental procedure. For the nonlinear elastic behavior during unloading (and reloading), the uniaxial tensile tests with repeated loading-unloading after prescribed prior plastic strains were conducted. Then, the change of apparent Chord modulus and the parameters for the MS model could be fitted. Examples of the Chord modulus fitted to the exponential function and the loading-unloading-reloading hysteric loop are shown in Fig. 1(a) and (b), respectively. Detailed identification procedure can be referred to Lee et al. [1,5].

![Figure 1. Identification of nonlinear apparent elastic modulus for (a) Chord modulus and (b) multi-surface elasticity models [2]](image)

The anisotropic yield functions, Hill 48, Yld2000-2d and other models such as BBC model [14], are identified by the uniaxial tensile tests along three different sheet orientations, rolling, transverse and diagonal directions, from which the yield stresses and r-values were determined. Either r-values or yield stresses were selectively used for Hill48. Additionally, the hydraulic bulge test was conducted for the Yld2000-2d in which the equi-biaxial stress and plastic strain rate ratio between rolling and transverse directions were determined. In this study, r-value based Hill48 was used. The eight parameters were calculated by Newton-Raphson iteration for the Yld2000-2d. Moreover, the evolutions of initial yield functions were implemented by considering the change of the yield function coefficients as a function of effective plastic strain, which is shown in Fig. 2(a) and (b) for Yld2000-2d and Hill48, respectively.
Finally, the hardening models were calibrated by either uniaxial tensile test for isotropic hardening law or tension-compression tests for anisotropic hardening model. For both cases, the tests were performed along the rolling direction. The best fitting parameters using the nonlinear least square method were chosen as the parameters of the considered hardening laws. The details on the identifications are omitted here.

4. Results and discussion
The constitutive models were implemented in the finite element models and validations were conducted using simple applications. To investigate the predictive capability of finite element model in consideration of advanced constitutive laws the standard dent test for sheet metal was first illustrated. The two sets of model were compared. The “conventional” set denotes the FE model with all conventional constitutive laws with constant elastic properties, isotropic hardening and isotropic yield function without evolution. In contrast, “advanced” set includes nonlinear elastic modulus after pre-strains (Chord or MS modulus models), the HAH model (or kinematic hardening [15,16]), and Yld2000-2d anisotropic yield function. As shown in Fig. 3 the indentation displacement-load curve could be well reproduced by the advanced set, while the conventional set cannot predict the residual indentation displacement with considerable difference from experiment. This is because the denting was made after 2% pre-stretching in this experiment, which involves prior plastic deformation, or deformation path change. Also, the deformation mode during denting signifies the role of accurate elasticity model and biaxial stress state.

Figure 2. Evolutions of yield function coefficients for (a) Yld2000-2d, and (b) Hill48 [1].

Figure 3. Denting experiment and comparison of indent displacement-load curves between simulations and experiment [2]
The effect of anisotropic yield function with its evolution is shown in Fig. 4(a). When the anisotropic yield function Yld2000-2d was considered the earing profile could be well predicted. Moreover, the height of earing could be better predicted when the evolution of yield function was additionally implemented. For the evolution of anisotropy, other rigorous approach is also available [17]. Finally, the effect of nonlinear elastic behavior after plastic deformation was studied by the split-ring springback simulation. As shown in Fig. 4(b), the profile of split ring could be better predicted for the FE model with nonlinear elasticity approach.

Figure 4. (a) Earing predictions by the Yld2000-2d with or without differential hardening [1], and (b) comparison of split ring springback with different elasticity models (more rigorous study on this is under investigation)

5. Conclusions
A short review of the effect of nonlinear and evolution of anisotropy on the mechanical behavior of sheet metals was presented. The considered constitutive models include quadratic or non-quadratic anisotropic yield functions, isotropic or anisotropic hardening laws, and nonlinear apparent elasticity models based on Chord or multi-surface law. In addition, the evolution of initial yield surface as a function of plastic stain was also implemented. The effect of anisotropic yield function and its evolution was evaluated by the earing of circular cylindrical forming, in which the non-quadratic Yld2000-2d with evolution resulted in most accurate prediction compared to other conventional models. The importance of nonlinear elasticity model was illustrated by the springback of split ring and the dent experiments. Both tests involve the deformation path changes before springback or indentation. Therefore, only finite element simulations with advanced constitutive models could predict the experimentally measured springback profile or indentation displacement-load curve.

Acknowledgement
This work has been supported by National Research Foundation of Korea (NRF 2017R1A2A2A05069619 and 2012R1A5A1051500).

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