P–odd asymmetries in polarized $\Lambda_b \rightarrow \Lambda \ell^+ \ell^−$ decay

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Abstract

We calculate various P–odd asymmetries appearing in the differential decay width for the cascade decay $\Lambda_b \rightarrow \Lambda(a+b)V^∗(\ell^+\ell^-)$ with polarized and unpolarized heavy baryons including new vector type interactions and using the helicity amplitudes. It is obtained that the study of P–odd asymmetries can serve a good test for establishing new physics beyond the SM.

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1 Introduction

Rare $B$–decays induced by the flavor–changing neutral current (FCNC) $b \to s$ or $b \to d$ transitions occur at loop level in the standard model (SM), since FCNC transitions that are forbidden in the SM at tree level provide consistency check of the SM at quantum level. Moreover, these decays are also quite sensitive to the existence of new physics beyond the SM, since new particles running at loops can give contributions to these decays. New physics appear in rare decays through the Wilson coefficients which can take values different from their SM counterpart or through the new operator structures in an effective Hamiltonian (see for example [1] and references therein).

Among the hadronic, leptonic and semileptonic decays, the last decay channels are very significant, since they are theoretically, more or less, clean, and they have relatively large branching ratio. With the help of the semileptonic decays $B \to M\ell^+\ell^−$ ($M$ being pseudoscalar or vector mesons) described by the $b \to s(d)\ell^+\ell^−$ transition, one can study many observables like forward–backward asymmetry $A_{FB}$, lepton polarization asymmetries, etc. Existence of these observables is very useful and serve as a testing ground for the standard model (SM) and in looking for new physics beyond th SM. For this reason, many processes, like $B \to \pi(p)\ell^+\ell^−$ [2], $B \to K\ell^+\ell^−$ [3] and $B \to K^*\ell^+\ell^−$ [4–11] have been studied comprehensively.

Recently, BELLE and BaBar Collaborations announced the following results for the branching ratios of the $B \to K^*\ell^+\ell^−$ and $B \to K\ell^+\ell^−$ decays:

$$B(B \to K^*\ell^+\ell^−) = \left\{\begin{array}{l}
(11.5^{+2.6}_{-2.4} \pm 0.8 \pm 0.2) \times 10^{-7} \quad [12], \\
(0.78^{+0.19}_{-0.17} \pm 0.12) \times 10^{-6} \quad [13], 
\end{array}\right.$$  

$$B(B \to K\ell^+\ell^−) = \left\{\begin{array}{l}
(4.8^{+1.0}_{-0.9} \pm 0.3 \pm 0.1) \times 10^{-7} \quad [12], \\
(0.34 \pm 0.07 \pm 0.12) \times 10^{-6} \quad [13].
\end{array}\right.$$  

Another exclusive decay which is described at inclusive level by the $b \to s\ell^+\ell^−$ transition is the baryonic $\Lambda_b \to \Lambda\ell^+\ell^−$ decay. Unlike mesonic decays, the baryonic decays could maintain the helicity structure of the effective Hamiltonian for the $b \to s$ transition [14]. Radiative and semileptonic decays of $\Lambda_b$ such as $\Lambda_b \to \Lambda\gamma$, $\Lambda_b \to \Lambda\ell\bar{\nu}_\ell$, $\Lambda_b \to \Lambda\ell^+\ell^−$ ($\ell = e, \mu, \tau$) and $\Lambda_b \to \Lambda\nu\bar{\nu}$ have been extensively studied in the literature [15–20]. More details about heavy baryons, including the experimental prospects, can be found in [21,22].

Many experimentally measurable quantities such as branching ratio [23], a polarization and single– and double–lepton polarizations, as well as forward–backward asymmetries, have already been studied in [24,25] and [26], respectively. Analysis of such quantities can be useful for more precise determination of the SM parameters and in looking for new physics beyond the SM.

In the present work we analyze the possibility of searching for new physics in the baryonic $\Lambda_b \to \Lambda\ell^+\ell^−$ decay by studying different P–odd asymmetries that characterize the angular dependence of the angular decay distributions, with the inclusion of non–standard vector type of interactions. In our analysis we use the helicity amplitude formalism and
polarization density matrix method (see the first and third references in [15]) to analyze the joint decay distributions in this decay.

The paper is organized as follows. In section 2, using the Hamiltonian that includes non-standard vector interactions, the matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ is obtained. In section 3 we calculate the different P–odd asymmetries. In the final section we study the sensitivity of various asymmetries to the non–standard interactions.

## 2 Matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay

In this section we derive the matrix element for the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay which is governed by the effective Hamiltonian describing $b \rightarrow s \ell^+ \ell^-$ transition. The effective Hamiltonian for the $b \rightarrow s \ell^+ \ell^-$ transition can be written in terms of the twelve model independent four–Fermi interactions as [5]

$$
\mathcal{M} = \frac{G_F}{\sqrt{2 \pi} V_{tb} V_{ts}^*} \left\{ C_{SL} \bar{s} \gamma_\mu b \bar{\ell} \gamma^\mu \ell + C_{BR} \bar{s} \ell \gamma_\mu \ell \bar{\ell} + C_{LL}^{\text{tot}} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + C_{RR}^{\text{tot}} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + C_{LR}^{\text{tot}} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + C_{RL}^{\text{tot}} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + C_{TR}^{\text{tot}} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + C_{TE}^{\text{tot}} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + C_{T} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell + iC_{T} \bar{s} \ell \gamma^\mu b \bar{\ell} \gamma^\mu \ell \right\},
$$

where $q = P_{\Lambda_b} - P_{\Lambda} = p_1 + p_2$ is the momentum transfer and $C_X$ are the coefficients of the four–Fermi interactions, $L = (1 - \gamma_5)/2$ and $R = (1 + \gamma_5)/2$. The terms with coefficients $C_{SL}$ and $C_{BR}$ describe the penguin contributions, which correspond to $-2m_sC_7^{\text{eff}}$ and $-2m_bC_7^{\text{eff}}$ in the SM, respectively. The next four terms in Eq. (1) with coefficients $C_{LL}^{\text{tot}}$, $C_{LR}^{\text{tot}}$, $C_{RL}$ and $C_{RR}$ describe vector type interactions, two ($C_{LL}^{\text{tot}}$ and $C_{LR}^{\text{tot}}$) of which contain SM contributions in the form $C_9^{\text{eff}} - C_{10}$ and $C_9^{\text{eff}} - C_{10}$, respectively. Thus, $C_{LL}^{\text{tot}}$ and $C_{LR}^{\text{tot}}$ can be written as

$$
C_{LL}^{\text{tot}} = C_9^{\text{eff}} - C_{10} + C_{LL} ,
C_{LR}^{\text{tot}} = C_9^{\text{eff}} + C_{10} + C_{LR} ,
$$

where $C_{LL}$ and $C_{LR}$ describe the contributions of new physics. Additionally, Eq. (1) contains four scalar type interactions ($C_{LLRR}$, $C_{RLLL}$, $C_{LLRL}$ and $C_{LRRL}$), and two tensor type interactions ($C_T$ and $C_{TE}$). Note that we will neglect the tensor type interactions throughout in this work.

The amplitude of the exclusive $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay is obtained by calculating the matrix element of $\mathcal{H}_{\text{eff}}$ for the $b \rightarrow s \ell^+ \ell^-$ transition between initial and final baryon states $\langle \Lambda | \mathcal{H}_{\text{eff}} | \Lambda_b \rangle$. It follows from Eq. (1) that the matrix elements

$$
\langle \Lambda | \bar{s} \gamma_\mu (1 + \gamma_5) b | \Lambda_b \rangle ,
\langle \Lambda | \bar{s} \sigma_{\mu\nu} (1 + \gamma_5) b | \Lambda_b \rangle ,
$$

are needed in order to calculate the $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay amplitude.
These matrix elements parametrized in terms of the form factors are as follows (see [24, 27])

\[
\langle \Lambda | \bar{s} \gamma_{\mu} b | \Lambda_b \rangle = \bar{u}_{\Lambda} \left[ f_{1 \gamma_{\mu}} + i f_{2 \sigma_{\mu \nu}} q^\nu + f_{3 q_{\mu}} \right] u_{\Lambda_b}, \tag{3}
\]

\[
\langle \Lambda | \bar{s} \gamma_{\mu} \gamma_{5} b | \Lambda_b \rangle = \bar{u}_{\Lambda} \left[ g_{1 \gamma_{\mu} \gamma_{5}} + i g_{2 \sigma_{\mu \nu} \gamma_{5} q^\nu} + g_{3 q_{\mu} \gamma_{5}} \right] u_{\Lambda_b}, \tag{4}
\]

\[
\langle \Lambda | \bar{s} \sigma_{\mu \nu} b | \Lambda_b \rangle = \bar{u}_{\Lambda} \left[ f_{T \sigma_{\mu \nu}} - i f_{T \gamma_{\mu} q^\nu - \gamma_{\nu} q^\mu} - i f_{S} \left( P_{\mu} q^\nu - P_{\nu} q^\mu \right) \right] u_{\Lambda_b}, \tag{5}
\]

\[
\langle \Lambda | \bar{s} \sigma_{\mu \nu} \gamma_{5} b | \Lambda_b \rangle = \bar{u}_{\Lambda} \left[ g_{T \sigma_{\mu \nu}} - i g_{T \gamma_{\mu} q^\nu - \gamma_{\nu} q^\mu} + i g_{S} \left( P_{\mu} q^\nu - P_{\nu} q^\mu \right) \right] \gamma_{5} u_{\Lambda_b}, \tag{6}
\]

where \( P = p_{\Lambda_b} + p_{\Lambda} \) and \( q = p_{\Lambda_b} - p_{\Lambda} \).

The form factors of the magnetic dipole operators are defined as

\[
\langle \Lambda | \bar{s} \bar{\sigma} \sigma_{\mu \nu} q^\nu b | \Lambda_b \rangle = \bar{u}_{\Lambda} \left[ f_{T} \bar{\gamma}_{\mu} + i f_{2} \bar{\sigma}_{\mu \nu} q^\nu + f_{3} q_{\mu} \right] u_{\Lambda_b}.
\]

Using the identity

\[
\sigma_{\mu \nu} \gamma_{5} = -\frac{i}{2} \epsilon_{\mu \nu \alpha \beta} \sigma^{\alpha \beta},
\]

and Eq. (5), the last expression in Eq. (7) can be written as

\[
\langle \Lambda | \bar{s} \sigma_{\mu \nu} \gamma_{5} q^\nu b | \Lambda_b \rangle = \bar{u}_{\Lambda} \left[ g_{T} \gamma_{\mu} \gamma_{5} + i g_{2} \sigma_{\mu \nu} \gamma_{5} q^\nu + g_{3} q_{\mu} \gamma_{5} \right] u_{\Lambda_b}.
\]

Multiplying (5) and (6) by \( i q^\nu \) and comparing with (7), one can easily obtain the following relations between the form factors

\[
f_{2}^{T} = f_{T} + f_{S} q^{2},
\]

\[
f_{1}^{T} = \left[ f_{1}^{V} + f_{T} \left( m_{\Lambda_b} + m_{\Lambda} \right) \right] q^{2} = -\frac{q^{2}}{m_{\Lambda_b} - m_{\Lambda}} f_{3}^{T},
\]

\[
g_{2}^{S} = g_{T} + g_{S} q^{2},
\]

\[
g_{1}^{T} = \left[ g_{1}^{V} - g_{T} \left( m_{\Lambda_b} - m_{\Lambda} \right) \right] q^{2} = \frac{q^{2}}{m_{\Lambda_b} + m_{\Lambda}} g_{3}^{T}.
\]

The matrix elements of scalar and pseudoscalar operators can be obtained by multiplying both sides of Eqs.(3) and (4) with \( q_{\mu} \) and using equation of motion, as a result of which we get,

\[
\langle \Lambda | \bar{s} b | \Lambda_b \rangle = \frac{1}{m_{b} - m_{s}} \bar{u}_{\Lambda} \left[ f_{1} \left( m_{\Lambda_b} - m_{\Lambda} \right) + f_{3} q^{2} \right] u_{\Lambda_b},
\]

\[
\langle \Lambda | \bar{s} \gamma_{5} b | \Lambda_b \rangle = \frac{1}{m_{b} + m_{s}} \bar{u}_{\Lambda} \left[ g_{1} \left( m_{\Lambda_b} + m_{\Lambda} \right) \gamma_{5} - g_{3} q^{2} \right] u_{\Lambda_b}.
\]

Using these definitions of the form factors, for the matrix element of the \( \Lambda_b \rightarrow \Lambda \ell^{+} \ell^{-} \) we get [25, 26]

\[
\mathcal{M} = \frac{G_{F}}{4\sqrt{2} \pi} V_{tb} V_{ts}^{*} \frac{1}{2} \left\{ \bar{\ell} \gamma_{\mu} (1 - \gamma_{5}) \ell \bar{u}_{\Lambda} \left[ (A_{1} - D_{1}) \gamma_{\mu} (1 + \gamma_{5}) + (B_{1} - E_{1}) \gamma_{\mu} (1 - \gamma_{5}) \right] \right\},
\]

3
\[ + i \sigma_{\mu \nu} q^\nu \left( (A_2 - D_2)(1 + \gamma_5) + (B_2 - E_2)(1 - \gamma_5) \right) u_{\Lambda_b} \]
\[ + \ell \gamma_\mu (1 + \gamma_5) \ell \bar{u}_\Lambda \left[ (A_1 + D_1) \gamma_\mu (1 + \gamma_5) + (B_1 + E_1) \gamma_\mu (1 - \gamma_5) \right] \]
\[ + i \sigma_{\mu \nu} q^\nu \left( (A_2 + D_2)(1 + \gamma_5) + (B_2 + E_2)(1 - \gamma_5) \right) \]
\[ + q_\mu \left( (A_3 + D_3)(1 + \gamma_5) + (B_3 + D_2)(1 - \gamma_5) \right) u_{\Lambda_b} \]
\[ + \frac{1}{2} \ell (1 - \gamma_5) \ell \bar{u}_\Lambda \left[ (P_1 - P_2 + R_1 - R_1)(1 - \gamma_5) + (P_1 + P_2 - R_1 - R_2)(1 + \gamma_5) \right] u_{\Lambda_b} \]
\[ + \frac{1}{2} \ell (1 + \gamma_5) \ell \bar{u}_\Lambda \left[ (P_1 - P_2 + R_2 - R_1)(1 - \gamma_5) + (P_1 + P_2 + R_1 + R_2)(1 + \gamma_5) \right] u_{\Lambda_b} \] \hspace{1cm} (9)

where

\[ A_1 = \frac{1}{q^2} \left( f_1^T - g_1^T \right) C_{SL} + \frac{1}{q^2} \left( f_1^T + g_1^T \right) C_{BR} + \frac{1}{2} (f_1 - g_1) \left( C_{LL}^{tot} + C_{LR}^{tot} \right) \]
\[ A_2 = A_1 (1 \rightarrow 2) \]
\[ A_3 = A_1 (1 \rightarrow 3) \]
\[ B_1 = A_1 \left( g_1 \rightarrow -g_1; g_1^T \rightarrow -g_1^T \right) \]
\[ B_2 = B_1 (1 \rightarrow 2) \]
\[ B_3 = B_1 (1 \rightarrow 3) \]
\[ D_1 = \frac{1}{2} (C_{RR} - C_{RL}) (f_1 + g_1) + \frac{1}{2} (C_{LR}^{tot} - C_{LL}^{tot}) (f_1 - g_1) \]
\[ D_2 = D_1 (1 \rightarrow 2) \]
\[ D_3 = D_1 (1 \rightarrow 3) \]
\[ E_1 = D_1 (g_1 \rightarrow -g_1) \]
\[ E_2 = E_1 (1 \rightarrow 2) \]
\[ E_3 = E_1 (1 \rightarrow 3) \]
\[ F_1 = \frac{1}{m_b} \left( f_1 \left( m_{\Lambda_b} - m_\Lambda \right) + f_3 q^2 \right) \left( C_{LRLR} + C_{RLLR} + C_{LRLL} + C_{RLRL} \right) \]
\[ F_2 = N_1 \left( C_{LRLR} \rightarrow -C_{LRLR}; C_{RLRL} \rightarrow -C_{RLRL} \right) \]
\[ R_1 = \frac{1}{m_b} \left( g_1 \left( m_{\Lambda_b} + m_\Lambda \right) - g_3 q^2 \right) \left( C_{LRLR} - C_{RLLR} + C_{LRLL} - C_{RLRL} \right) \]
\[ R_2 = H_1 \left( C_{LRLR} \rightarrow -C_{LRLR}; C_{RLRL} \rightarrow -C_{RLRL} \right) \]

It follows from these expressions that \( \Lambda_b \rightarrow \Lambda \ell^+ \ell^- \) decay is described in terms of many form factors. It is shown in [28] that Heavy Quark Effective Theory (HQET) reduces the number of independent form factors to two \((F_1 \text{ and } F_2) \) irrelevant of the Dirac structure of the corresponding operators, i.e.,

\[ \langle \Lambda (p_{\Lambda}) | \bar{s} \Gamma \ell | \Lambda (p_{\Lambda_b}) \rangle = \bar{u}_\Lambda \left[ F_1 (q^2) + \gamma F_2 (q^2) \right] \Gamma u_{\Lambda_b} \] \hspace{1cm} (11)
where \( \Gamma \) is an arbitrary Dirac structure and \( v^\mu = p^\mu_{\Lambda_b}/m_{\Lambda_b} \) is the four-velocity of \( \Lambda_b \). Comparing the general form of the form factors given in Eqs. (4)–(8) with (11), one can easily obtain the following relations among them (see also [24, 25, 27])

\[
\begin{align*}
g_1 &= f_1 = f_2^T = g_2^T = F_1 + \sqrt{\hat{r}_A} F_2, \\
g_2 &= f_2 = g_3 = f_3 = g_T = f_T = \frac{F_2}{m_{\Lambda_b}}, \\
g_T^S &= f_T^S = 0, \\
g_1^T &= f_1^T = \frac{F_2}{m_{\Lambda_b}} q^2, \\
g_3^T &= \frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} + m_\Lambda), \\
f_3^T &= -\frac{F_2}{m_{\Lambda_b}} (m_{\Lambda_b} - m_\Lambda),
\end{align*}
\]

where \( \hat{r}_A = m_\Lambda^2/m_{\Lambda_b}^2 \).

In order to obtain the helicity amplitudes for the \( \Lambda_b \to \Lambda \ell^+ \ell^- \) decay, it is convenient to regard this decay as a quasi two-body decay \( \Lambda_b \to \Lambda V^* \) followed by the leptonic decay \( V^* \to \ell^+ \ell^- \), where \( V^* \) is the off-shell \( \gamma \) or \( Z \) bosons. The matrix element of \( \Lambda_b \to \Lambda \ell^+ \ell^- \) decay can be written in the following form:

\[
\mathcal{M}^{\Lambda b \ell \ell} = \sum_{\lambda_{V^*}} \eta_{\lambda_{V^*}} L_{\lambda_{V^*}}^{\Lambda b} H_{\lambda_{V^*}}^{\Lambda},
\]

where

\[
\begin{align*}
L_{\lambda_{V^*}}^{\Lambda b} &= \varepsilon_{\lambda_{V^*}}^{\mu} \left( \ell^- (p_{\ell}, \lambda_{\ell}) \ell^+ (p_{\ell}, \lambda_{\ell}) | J^\mu_{\ell} | 0 \right), \\
H_{\lambda_{V^*}}^{\Lambda} &= (\varepsilon_{\lambda_{V^*}}^{\mu})^* \left( \Lambda (p_{\Lambda}, \lambda_{\Lambda}) | J^\mu_{\Lambda} | \Lambda_b (p_{\Lambda_b}) \right),
\end{align*}
\]

where \( \varepsilon_{\lambda_{V^*}}^{\mu} \) is the polarization vector of the virtual intermediate vector boson. The metric tensor can be expressed in terms of the polarization vector of the virtual vector particle \( \varepsilon_V = \varepsilon(\lambda_V) \) as

\[
-g^{\mu\nu} = \sum_{\lambda_{V^*}} \eta_{\lambda_{V^*}} \varepsilon_{\lambda_{V^*}}^{\mu} \varepsilon_{\lambda_{V^*}}^{\nu},
\]

where the summation is over the helicity of the virtual vector particle \( V, \lambda_V = \pm 1, 0, t \) with the metric \( \eta_{\pm} = \eta_0 = -\eta_t = 1 \), where \( \lambda_V = t \) is the scalar (zero) helicity component of the virtual \( V \) particle (for more details see [28–30]). The upper indices in Eqs. (13) and (14) correspond to the helicities of the leptons and the lower ones correspond to the helicity of the \( \Lambda \) baryon. Moreover, \( J^\mu_{\ell} \) and \( J^\mu_{\Lambda} \) in Eqs. (13) and (14) are the leptonic and hadronic currents, respectively.

In the calculations of the leptonic and baryonic amplitudes we will use two different frames. The leptonic amplitude \( L_{\lambda_{V^*}}^{\Lambda b} \) is calculated in the rest frame of the virtual vector boson with the \( z \)-axis chosen along the \( \Lambda \) direction and the \( x-z \) plane chosen as the virtual \( V \) decay plane. The hadronic amplitude is calculated in the rest frame of \( \Lambda_b \) baryon.
Using Eqs. (9)–(14), after lengthy calculations, we get for the helicity amplitudes:

\[
\begin{align*}
\mathcal{M}^{++}_{+1/2} &= 2m_e \sin \theta \left( H^{(1)}_{+1/2,+1} + H^{(2)}_{+1/2,+1} \right) + 2m_e \cos \theta \left( H^{(1)}_{+1/2,0} + H^{(2)}_{+1/2,0} \right) \\
&\quad + 2m_e \left( H^{(1)}_{+1/2,t} - H^{(2)}_{+1/2,t} \right) + \frac{1}{2} \sqrt{q^2} \left[ (1 + v) J^{(1)}_{+1/2,-1} - (1 - v) J^{(2)}_{+1/2,-1} \right] \\
&\quad + \frac{1}{2} \sqrt{q^2} \left[ (1 - v) J^{(1)}_{+1/2,-1} - (1 + v) J^{(2)}_{+1/2,-1} \right], \\
\mathcal{M}^{-+}_{+1/2} &= -\sqrt{q^2} \left( 1 - \cos \theta \right) \left[ (1 - v) H^{(1)}_{+1/2,+1} + (1 + v) H^{(2)}_{+1/2,+1} \right] - \sqrt{q^2} \sin \theta \left[ (1 - v) H^{(1)}_{+1/2,0} + (1 + v) H^{(2)}_{+1/2,0} \right] \\
&\quad + \frac{1}{2} \sqrt{q^2} \left[ (1 - v) J^{(1)}_{+1/2,-1} - (1 + v) J^{(2)}_{+1/2,-1} \right], \\
\mathcal{M}^{++}_{-1/2} &= -2m_e \sin \theta \left( H^{(1)}_{-1/2,+1} + H^{(2)}_{-1/2,+1} \right) + 2m_e \cos \theta \left( H^{(1)}_{-1/2,0} + H^{(2)}_{-1/2,0} \right) \\
&\quad + 2m_e \left( H^{(1)}_{-1/2,t} - H^{(2)}_{-1/2,t} \right) + \frac{1}{2} \sqrt{q^2} \left[ (1 + v) J^{(1)}_{-1/2,-1} - (1 - v) J^{(2)}_{-1/2,-1} \right] \\
&\quad + \frac{1}{2} \sqrt{q^2} \left[ (1 - v) J^{(1)}_{-1/2,-1} - (1 + v) J^{(2)}_{-1/2,-1} \right], \\
\mathcal{M}^{-+}_{-1/2} &= -\sqrt{q^2} \left( 1 + \cos \theta \right) \left[ (1 - v) H^{(1)}_{-1/2,-1} + (1 + v) H^{(2)}_{-1/2,-1} \right] - \sqrt{q^2} \sin \theta \left[ (1 + v) H^{(1)}_{-1/2,0} + (1 - v) H^{(2)}_{-1/2,0} \right] \\
&\quad + \frac{1}{2} \sqrt{q^2} \left[ (1 + v) J^{(1)}_{-1/2,-1} - (1 - v) J^{(2)}_{-1/2,-1} \right], \\
\mathcal{M}^{--}_{-1/2} &= 2m_e \sin \theta \left( H^{(1)}_{-1/2,-1} + H^{(2)}_{-1/2,-1} \right) - 2m_e \cos \theta \left( H^{(1)}_{-1/2,0} + H^{(2)}_{-1/2,0} \right) \\
&\quad + 2m_e \left( H^{(1)}_{-1/2,t} - H^{(2)}_{-1/2,t} \right) + \frac{1}{2} \sqrt{q^2} \left[ (1 - v) J^{(1)}_{-1/2,0} - (1 + v) J^{(2)}_{-1/2,0} \right] \\
&\quad + \frac{1}{2} \sqrt{q^2} \left[ (1 - v) J^{(1)}_{-1/2,0} - (1 + v) J^{(2)}_{-1/2,0} \right], \\
\end{align*}
\]

(15)

where

\[
\begin{align*}
H^{(1)}_{+1/2,\pm 1} &= H^{(1)} V_{1/2,1} \pm H^{(1)} A_{1/2,1}, \\
H^{(2)}_{+1/2,\pm 1} &= H^{(2)} V_{1/2,1} \pm H^{(2)} A_{1/2,1}, \\
H^{(1,2)}_{+1/2,0} &= H^{(1,2)} V_{1/2,0} \pm H^{(1,2)} A_{1/2,1}, \\
H^{(1,2)}_{+1/2,t} &= H^{(1,2)} V_{1/2,t} \pm H^{(1,2)} A_{1/2,t}, \\
\end{align*}
\]

(16)

where \( \theta \) is the angle of the positron in the rest frame of the intermediate boson with respect
to its helicity axes. Explicit expressions of the helicity amplitudes $H_{\lambda\lambda',\mu\nu}^{V,A}$ are

$$H_{1/2,1}^{(1)V} = -\sqrt{Q_-} \left[ F_1^V - (m_{\lambda_b} + m_\Lambda)F_2^V \right],$$

$$H_{1/2,1}^{(1)A} = -\sqrt{Q_+} \left[ F_1^A + (m_{\lambda_b} - m_\Lambda)F_2^A \right],$$

$$H_{1/2,1}^{(2)V} = H_{1/2,1}^{(1)V}(F_1^V \rightarrow F_3^V, F_2^V \rightarrow F_4^V),$$

$$H_{1/2,1}^{(2)A} = H_{1/2,1}^{(1)A}(F_1^A \rightarrow F_3^A, F_2^A \rightarrow F_4^A),$$

$$H_{1/2,0}^{(1)V} = -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_-} \left[ (m_{\lambda_b} + m_\Lambda)F_1^V - q^2 F_2^V \right] \right\},$$

$$H_{1/2,0}^{(1)A} = -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_+} \left[ (m_{\lambda_b} - m_\Lambda)F_1^A + q^2 F_2^A \right] \right\},$$

$$H_{1/2,0}^{(2)V} = H_{1/2,0}^{(1)V}(F_1^V \rightarrow F_3^V, F_2^V \rightarrow F_4^V),$$

$$H_{1/2,0}^{(2)A} = H_{1/2,0}^{(1)A}(F_1^A \rightarrow F_3^A, F_2^A \rightarrow F_4^A),$$

$$H_{1/2,t}^{(1)V} = -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_+} \left[ (m_{\lambda_b} - m_\Lambda)F_1^V + q^2 F_5^V \right] \right\},$$

$$H_{1/2,t}^{(1)A} = -\frac{1}{\sqrt{q^2}} \left\{ \sqrt{Q_-} \left[ (m_{\lambda_b} + m_\Lambda)F_1^A - q^2 F_5^A \right] \right\},$$

$$H_{1/2,t}^{(2)V} = H_{1/2,t}^{(1)V}(F_1^V \rightarrow F_3^V, F_5^V \rightarrow F_6^V),$$

$$H_{1/2,t}^{(2)A} = H_{1/2,t}^{(1)A}(F_1^A \rightarrow F_3^A, F_5^A \rightarrow F_6^A),$$

$$J_{+1/2,0}^{(1)} = J_{+1/2,t}^{(1)} = \sqrt{Q_+} (P_1 - P_2) - \sqrt{Q_-} (R_1 - R_2),$$

$$J_{+1/2,0}^{(2)} = J_{+1/2,t}^{(2)} = J_{+1/2,0}^{(1)}(P_2 \rightarrow -P_2, R_2 \rightarrow -R_2),$$

$$J_{-1/2,0}^{(1)} = J_{-1/2,t}^{(1)} = J_{+1/2,0}^{(1)}(\sqrt{Q_-} \rightarrow -\sqrt{Q_-}),$$

$$J_{-1/2,0}^{(2)} = J_{-1/2,t}^{(2)} = J_{+1/2,0}^{(2)}(\sqrt{Q_-} \rightarrow -\sqrt{Q_-}).$$

(17)

where

$$Q_+ = (m_{\lambda_b} + m_\Lambda)^2 - q^2,$$

$$Q_- = (m_{\lambda_b} - m_\Lambda)^2 - q^2,$$

and

$$F_1^V = A_1 - D_1 + B_1 - E_1,$$

$$F_1^A = A_1 - D_1 - B_1 + E_1,$$

$$F_2^V = F_1^V(1 \rightarrow 2),$$

$$F_2^A = F_1^A(1 \rightarrow 2),$$

$$F_3^V = A_1 + D_1 + B_1 + E_1,$$

$$F_3^A = A_1 + D_1 - B_1 - E_1,$$

$$F_4^V = F_3^V(1 \rightarrow 2),$$

$$F_4^A = F_3^A(1 \rightarrow 2),$$

$$F_5^V = F_3^V(1 \rightarrow 2),$$

$$F_5^A = F_3^A(1 \rightarrow 2),$$

$$F_6^V = F_3^V(1 \rightarrow 2).$$
\[ F_5^V = F_1^V(1 \rightarrow 3), \]
\[ F_5^A = F_1^A(1 \rightarrow 3), \]
\[ F_6^V = F_4^V(2 \rightarrow 3), \]
\[ F_6^A = F_4^A(2 \rightarrow 3). \]

The remaining helicity amplitudes can be obtained from the parity relations

\[ H_{\Lambda^b, \Lambda^b}^{V,(A)} = \mp H_{\Lambda^b, \Lambda^b}^{V,(A)}. \]  

The square of the matrix element for the \( \Lambda_b \rightarrow \Lambda \ell^+ \ell^- \) decay is given as

\[
|\mathcal{M}|^2 = |\mathcal{M}_{+1/2}^+|^2 + |\mathcal{M}_{+1/2}^-|^2 + |\mathcal{M}_{+1/2}^0|^2 + |\mathcal{M}_{+1/2}^-|^2 \\
+ |\mathcal{M}_{-1/2}^+|^2 + |\mathcal{M}_{-1/2}^-|^2 + |\mathcal{M}_{-1/2}^0|^2 + |\mathcal{M}_{-1/2}^-|^2.
\]

Following the standard methods used in literature (see the third reference in [15]), the normalized joint angular decay distribution for the two cascade decay

\[ \Lambda_b^{1/2+} \rightarrow \Lambda^{1/2+}(\rightarrow a(1/2^+) + b(0^-)) + V(\rightarrow \ell^+ \ell^-), \]

is given by

\[
\frac{d\Gamma}{dq^2 d\cos \theta d\cos \theta_\Lambda} = \left| \frac{G\alpha}{4\sqrt{2}\pi} V_{tb} V_{*1/2} \right|^2 \sqrt{\lambda(m_{\Lambda_b}^2, m_{\Lambda}^2, q^2)} \sqrt{\lambda(m_{\Lambda_b}^2, m_{\Lambda}^2, m_b^2)} \frac{1024\pi^3 m_{\Lambda_b}^2 m_{\Lambda}^2}{v_B(\Lambda \rightarrow a + b)} \times \frac{1}{2} \right] \left[ 1 + \alpha_{\Lambda} \cos \theta_\Lambda \right] \left[ \begin{array}{c}
(8m_\ell^2 \sin^2 \theta |A_{1/2+1/2}^+|^2 + (1 - \cos \theta)^2 q^2 |A_{1/2+1/2}^-|^2 \\
+ (1 + \cos \theta)^2 q^2 |A_{1/2+1/2}^0|^2 + 8m_\ell^2 \cos^2 \theta |A_{1/2+1/2}^0|^2 + 8m_\ell^2 |B_{1/2,+1/2}^0|^2 \\
+ \sin^2 \theta q^2 \left( 2 |A_{1/2+1/2}^0|^2 + 2v^2 |B_{1/2,-1/2}^0|^2 \right) \\
- 4m_\ell \sqrt{q^2 \left( \text{Re}(B_{1/2+1/2}^0 + B_{1/2,-1/2}^0) \right) + v^2 \cos \theta \text{Re}(C_{1/2+1/2}^0 + C_{1/2,-1/2}^0))} \right) \\
+ \frac{q^2}{2} \left( |D_{1/2+1/2}^-|^2 + 2v^2 |C_{1/2+1/2}^-|^2 \right) \\
+ (1 - \alpha_{\Lambda} \cos \theta_\Lambda) \left[ (8m_\ell^2 \sin^2 \theta |A_{-1/2-1/2}^+|^2 + (1 + \cos \theta)^2 q^2 |A_{-1/2-1/2}^-|^2 \\
+ (1 - \cos \theta)^2 q^2 |A_{-1/2-1/2}^0|^2 + 8m_\ell^2 \cos^2 \theta |A_{-1/2-1/2}^0|^2 + 8m_\ell^2 |B_{1/2,-1/2}^0|^2 \\
+ \sin^2 \theta q^2 \left( 2 |A_{1/2,-1/2}^0|^2 + 2v^2 |B_{1/2,-1/2}^0|^2 \right) \\
- 4m_\ell \sqrt{q^2 \left( \text{Re}(B_{1/2+1/2}^- + B_{1/2,-1/2}^-) \right) + v^2 \cos \theta \text{Re}(C_{1/2+1/2}^- + C_{1/2,-1/2}^-))} \right) \\
+ \frac{q^2}{2} \left( |D_{1/2-1/2}^-|^2 + 2v^2 |C_{1/2-1/2}^-|^2 \right) \right] \right). \]

In Eq. (21) we introduce the following definitions:

\[ H_{\Lambda_1, \Lambda_2}^{(1)} + H_{\Lambda_1, \Lambda_2}^{(2)} = A_{\Lambda_1, \Lambda_2}. \]
\[ H_{\lambda,\lambda W}^{(1)} - H_{\lambda,\lambda W}^{(2)} = B_{\lambda,\lambda W} \; , \\
J_{\lambda,\lambda W}^{(1)} + J_{\lambda,\lambda W}^{(2)} = C_{\lambda,\lambda W} \; , \\
J_{\lambda,\lambda W}^{(1)} - J_{\lambda,\lambda W}^{(2)} = D_{\lambda,\lambda W} \; . \]

Note that in deriving Eq. (22), we perform integration over the azimuthal angle \( \varphi \) between the planes of the two decays \( \Lambda \rightarrow a + b \) and \( V \rightarrow \ell^+ \ell^- \).

It is well known that heavy quarks \( b(c) \) resulting from \( Z \) decay are polarized. It is shown in [31, 32] that a sizeable fraction of the \( b \) quark polarization retained in fragmentation of heavy quarks to heavy baryons. Therefore, an additional set of polarization observables can be obtained if the polarization of the heavy \( \Lambda_b \) baryon is taken into account.

In order to take polarization of the \( \Lambda_b \) baryon into consideration we will use the density matrix method. The spin density matrix of \( \Lambda \) baryon is

\[
\rho = \frac{1}{2} \begin{pmatrix}
1 + \mathcal{P} \cos \theta_\Lambda^S & \mathcal{P} \cos \theta_\Lambda^S \\
\mathcal{P} \cos \theta_\Lambda^S & 1 - \mathcal{P} \cos \theta_\Lambda^S
\end{pmatrix}
\]

where \( \mathcal{P} \) is the polarization of \( \Lambda_b \), and \( \theta_\Lambda^S \) is the angle that the polarization of \( \Lambda_b \) makes with the momentum of \( \Lambda \), in the rest frame of \( \Lambda_b \).

The four–fold decay distribution can easily be obtained from Eq. (21). Obviously, there appears on the left–hand side of Eq. (21) the distribution over \( \theta_\Lambda^S \), i.e., \( d/d\cos \theta_\Lambda^S \). Hence the right–hand side of the same equation can be modified as follows:

\[
\begin{align*}
|1/2, 1|^2 & \rightarrow (1 - \mathcal{P} \cos \theta_\Lambda^S) |1/2, 1|^2 \\
|1/2, t|^2, |1/2, 0|^2, (1/2, 0)(1/2, t)^* & \rightarrow (1 + \mathcal{P} \cos \theta_\Lambda^S) \{ |1/2, 0|^2, |1/2, t|^2, \\
(1/2, 0)(1/2, t)^* \} \\
(1/2, 1)(1/2, t)^*, (1/2, 1)(1/2, 0)^* & \rightarrow \mathcal{P} \sin \theta_\Lambda^S \{ (1/2, 1)(1/2, 0)^*, (1/2, 1)(1/2, t)^* \} \\
|−1/2, −1|^2 & \rightarrow (1 + \mathcal{P} \cos \theta_\Lambda^S) |−1/2, −1|^2 \\
|−1/2, t|^2, |−1/2, 0|^2, (−1/2, 0)(−1/2, t)^* & \rightarrow (1 - \mathcal{P} \cos \theta_\Lambda^S) \{ |−1/2, 0|^2, |−1/2, t|^2, \\
(−1/2, 0)(−1/2, t)^* \} \\
(−1/2, −1)(−1/2, −t)^*, (−1/2, −1)(−1/2, 0)^* & \rightarrow \mathcal{P} \sin \theta_\Lambda^S \{ (−1/2, −1)(−1/2, t)^*, \\
(−1/2, −1)(−1/2, 0)^* \} 
\end{align*}
\]

From the expressions for the four–fold angular distribution we may define the following forward–backward asymmetries:

\[
A_{FB} = \frac{1}{\int_{−1}^{+1} d\cos \theta \int_{−1}^{+1} d\cos \theta_\Lambda \int_{−1}^{+1} d\cos \theta_\Lambda \int_{−1}^{+1} d\cos \theta_\Lambda} \frac{d\Gamma}{dq^2 d\cos \theta d\cos \theta_\Lambda d\cos \theta_\Lambda^S},
\]

(25)
Performing relevant integrations in Eqs. (25)–(27), we get:

\[
A^{FB}_{\theta} = \frac{16v\sqrt{q}}{\Delta} \bigg\{ 2\sqrt{q} \text{Re}[A_{+1/2,+1}B_{+1/2,+1} - A_{-1/2,-1}B_{-1/2,-1}] \\
- m_t \text{Re}[A_{-1/2,0}(C_{-1/2,t}^* + C_{-1/2,0}) + A_{+1/2,0}(C_{+1/2,t}^* + C_{+1/2,0})] \bigg\},
\]

\[
A^{FB}_{\theta}\Lambda = \frac{2}{3\Delta} \bigg\{ -32m_t^2 \left| A_{+1/2,+1} \right|^2 - 16q^2 \left( \left| A_{+1/2,+1} \right|^2 + v^2 \left| B_{+1/2,+1} \right|^2 \right) \\
+ 16m_t^2 \left| A_{+1/2,0} \right|^2 + 8q^2 \left( \left| A_{+1/2,0} \right|^2 + v^2 \left| B_{+1/2,0} \right|^2 \right) \\
+ 32m_t^2 \left| A_{-1/2,-1} \right|^2 + 16q^2 \left( \left| A_{-1/2,-1} \right|^2 + v^2 \left| B_{-1/2,-1} \right|^2 \right) \\
- 16m_t^2 \left| A_{-1/2,0} \right|^2 - 8q^2 \left( \left| A_{-1/2,0} \right|^2 + v^2 \left| B_{-1/2,0} \right|^2 \right) \\
+ 48m_t^2 \left| B_{+1/2,t} \right|^2 - 48m_t^2 \left| B_{-1/2,t} \right|^2 \\
- 3q^2 \left| D_{-1/2,t} \right|^2 + 24m_t\sqrt{q^2} \text{Re}[B_{-1/2,t}(D_{+1/2,0}^* + D_{-1/2,0}^*) - B_{+1/2,t}(D_{+1/2,0}^* + D_{+1/2,0}^*)] \\
+ 3q^2 \left( \left| D_{+1/2,t} \right|^2 + \left| D_{+1/2,0} \right|^2 - \left| D_{+1/2,0} \right|^2 + 2\text{Re}[D_{+1/2,0}D_{+1/2,t} - D_{-1/2,0}D_{+1/2,t}] \right) \\
+ 3q^2 v^2 \left( \left| C_{+1/2,t} \right|^2 - \left| C_{-1/2,t} \right|^2 - \left| C_{-1/2,0} \right|^2 + 2\text{Re}[C_{+1/2,t}C_{+1/2,0} - C_{-1/2,t}C_{-1/2,0}] \right) \bigg\},
\]

\[
A^{FB}_{\theta}\Lambda = \frac{2}{3\Delta} \bigg\{ 32m_t^2 \left| A_{+1/2,+1} \right|^2 + 16q^2 \left( \left| A_{+1/2,+1} \right|^2 + v^2 \left| B_{+1/2,+1} \right|^2 \right) \\
+ 16m_t^2 \left| A_{+1/2,0} \right|^2 + 8q^2 \left( \left| A_{+1/2,0} \right|^2 + v^2 \left| B_{+1/2,0} \right|^2 \right) \\
- 32m_t^2 \left| A_{-1/2,-1} \right|^2 - 16q^2 \left( \left| A_{-1/2,-1} \right|^2 + v^2 \left| B_{-1/2,-1} \right|^2 \right) \\
- 16m_t^2 \left| A_{-1/2,0} \right|^2 - 8q^2 \left( \left| A_{-1/2,0} \right|^2 + v^2 \left| B_{-1/2,0} \right|^2 \right) \\
+ 48m_t^2 \left| B_{+1/2,t} \right|^2 - 48m_t^2 \left| B_{-1/2,t} \right|^2 \\
- 3q^2 \left| D_{-1/2,t} \right|^2 + 24m_t\sqrt{q^2} \text{Re}[B_{-1/2,t}(D_{+1/2,0}^* + D_{-1/2,0}^*) - B_{+1/2,t}(D_{+1/2,0}^* + D_{+1/2,0}^*)] \\
+ 3q^2 \left( \left| D_{+1/2,t} \right|^2 + \left| D_{+1/2,0} \right|^2 - \left| D_{+1/2,0} \right|^2 + 2\text{Re}[D_{+1/2,0}D_{+1/2,t} - D_{-1/2,0}D_{+1/2,t}] \right) \\
+ 3q^2 v^2 \left( \left| C_{+1/2,t} \right|^2 - \left| C_{-1/2,t} \right|^2 - \left| C_{-1/2,0} \right|^2 + 2\text{Re}[C_{+1/2,t}C_{+1/2,0} - C_{-1/2,t}C_{-1/2,0}] \right) \bigg\},
\]
where
\[
\Delta = 4 \left\{ 32 m_t^2 \, |A_{+1/2,1}|^2 + 16 q^2 \left( |A_{+1/2,1}|^2 + v^2 |B_{+1/2,1}|^2 \right) \\
+ 16 m_t^2 \, |A_{+1/2,0}|^2 + 8 q^2 \left( |A_{+1/2,0}|^2 + v^2 |B_{+1/2,0}|^2 \right) \\
+ 32 m_t^2 \, |A_{-1/2,-1}|^2 + 16 q^2 \left( |A_{-1/2,-1}|^2 + v^2 |B_{-1/2,-1}|^2 \right) \\
+ 16 m_t^2 \, |A_{-1/2,0}|^2 + 8 q^2 \left( |A_{-1/2,0}|^2 + v^2 |B_{-1/2,0}|^2 \right) \\
+ 48 m_t^2 \, |B_{+1/2,t}|^2 + 48 m_t^2 \, |B_{-1/2,t}|^2 \\
+ 3q^2 \left( |D_{-1/2,t}|^2 + |D_{-1/2,0}|^2 + |D_{+1/2,t}|^2 + |D_{+1/2,0}|^2 + 2Re[D_{-1/2,t}D_{-1/2,0}^* + D_{+1/2,t}D_{+1/2,0}^*] \right) \\
- 24 m_t \sqrt{q^2} Re[B_{-1/2,t}(D_{-1/2,t}^* + D_{-1/2,0}) + B_{+1/2,t}(D_{+1/2,t}^* + D_{+1/2,0})] \\
+ 3q^2 v^2 \left( |C_{-1/2,t}|^2 + |C_{-1/2,0}|^2 + |C_{+1/2,t}|^2 + |C_{+1/2,0}|^2 + 2Re[C_{-1/2,t}C_{-1/2,0}^* + C_{+1/2,t}C_{+1/2,0}^*] \right) \right\}.
\]

### 3 Numerical analysis

In this section we will study the sensitivity of the P–odd asymmetries on the new Wilson coefficients. The values of the input parameters we use in our calculations are: $|V_{tb}V_{ts}^*| = 0.0385$, $m_t = 1.77$ GeV, $m_\mu = 0.106$ GeV, $m_b = 4.8$ GeV, and we neglect the mass of the strange quark. In further numerical analysis, the values of the new Wilson coefficients which describe new physics beyond the SM, are needed. In numerical calculations we will vary all new Wilson coefficients in the range $-|C_{10}^{SM}| \leq C_X \leq |C_{10}^{SM}|$. The experimental results on the branching ratio of the $B \to K^{*}\ell^+\ell^-$ decay [12, 13] and the bound on the branching ratio of $B \to \mu^+\mu^-$ [35] suggest that this is the right order of magnitude for the vector and scalar interaction coefficients. For the values of the Wilson coefficients in the SM we use: $C_7^{SM} = -0.313$, $C_9^{SM} = 4.344$ and $C_{10}^{SM} = -4.669$. The magnitude of $C_7^{eff}$ is quite well constrained from $b \to s\gamma$ decay, and hence well established. Moreover, we will fix the values of the Wilson coefficients, i.e., $C_{BR}$ and $C_{SL}$ are both related to $C_7^{eff}$ as follows: $C_{BR} = -2m_b C_7^{eff}$ and $C_{SL} = -2m_s C_7^{eff}$. As far as the Wilson coefficient $C_9^{SM}$ is considered, we take into account the short, as well as the long distance contributions coming from the production of $\bar{c}c$ pair at intermediate states. It is well known that the form factors are the main and the most important input parameters necessary in the numerical calculations. The calculation of the form factors of $\Lambda_b \to \Lambda$ transition does not exist at present. But, we can use the results from QCD sum rules in corporation with HQET [28, 33]. We noted earlier that, HQET allows us to establish relations among the form factors and reduces the number of independent form factors into two. In [28, 33], the $q^2$ dependence of these form factors are given as follows

\[
F(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2}.
\]
| F(0) | a_F | b_F |
|------|-----|-----|
| F_1  | 0.462 | -0.0182 | -0.000176 |
| F_2  | -0.077 | -0.0685 | 0.00146 |

Table 1: Form factors for $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ decay in a three parameter fit.

The values of the parameters $F(0), a_F$ and $b_F$ are given in table 1.

In order to have an idea about the sensitivity of our results to the specific parametrization of the two form factors predicted by the QCD sum rules in corporation with the HQET, we also have used another parametrization of the form factors based on the pole model and compared the results of both models. The dipole form of the form factors predicted by the pole model are given as:

$$F_{1,2}(E_\Lambda) = N_{1,2} \left( \frac{\Lambda_{QCD}}{\Lambda_{QCD} + E_\Lambda} \right)^2,$$

where

$$E_\Lambda = \frac{m_{\Lambda_b}^2 - m_{\Lambda}^2 - q^2}{2m_{\Lambda_b}},$$

and $\Lambda_{QCD} = 0.2$, $|N_1| = 52.32$ and $|N_1| \simeq -0.25\Lambda_1$ [34].

It is well known that, in addition to the short distance contributions $C_9$ receives long distance contributions coming from the production of the real $\bar{c}c$ state that can be written as

$$C_{9}^{eff}(m_b, \hat{s}) = C_{9}(m_b) \left[ 1 + \frac{\alpha_s(\mu)}{\pi} \omega(\hat{s}) \right] + Y(\hat{s}),$$

where $C_{9}(m_b) = 4.344$ and

$$Y(\hat{s}) = Y^{per}(\hat{s}) + Y_{LD}.$$

Here

$$Y^{per}(\hat{s}) = g(\hat{m}_c, \hat{s}) C^{(0)} - \frac{1}{2} g(1, \hat{s}) \left[ 4C_3 + 4C_4 + 3C_5 + C_6 \right] - \frac{1}{2} g(0, \hat{s}) \left[ C_3 + 3C_4 \right] + \frac{2}{9} \left[ 3C_3 + C_4 + 3C_5 + C_6 \right] - \lambda_u \left[ 3C_1 + C_2 \right] \left[ g(0, \hat{s}) - g(\hat{m}_c, \hat{s}) \right],$$

where

$$C^{(0)} = 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6,$$

$$\lambda_u = \frac{V_{ub}V_{ud}^*}{V_{tb}V_{td}^*}.$$
and the loop function $g(m_q, s)$ stands for the loops of quarks with mass $m_q$ at the dilepton invariant mass $s$. This function develops absorptive parts for dilepton energies $s = 4m_q^2$:

$$g(\hat{m}_q, \hat{s}) = -\frac{8}{9} \ln \hat{m}_q + \frac{8}{27} + \frac{4}{9} y_q - \frac{2}{9} (2 + y_q) \sqrt{|1 - y_q|}$$

$$\times \left[ \Theta(1 - y_q) \left( \ln \frac{1 + \sqrt{1 - y_q}}{1 - \sqrt{1 - y_q}} - i\pi \right) + \Theta(y_q - 1) 2 \arctan \frac{1}{\sqrt{y_q - 1}} \right],$$

where $\hat{m}_q = m_q/m_b$ and $y_q = 4\hat{m}_q^2/\hat{s}$ (see [36, 37]). The long distance contributions are embedded into $Y_{LD}$ whose expression is given as

$$Y_{LD}(\hat{s}) = \frac{3}{\alpha^2} \left[ - \frac{V_{ij}^* V_{cb}}{V_{ij} V_{tb}} C^{(0)} - \frac{V_{ij}^* V_{ab}}{V_{ij} V_{tb}} (3C_3 + C_4 + 3C_5 + C_6) \right]$$

$$\times \sum_{V_i = \psi(1s), \ldots, \psi(6s)} \frac{\pi \kappa_i \Gamma(V_i \rightarrow \ell^+ \ell^-) M_{V_i}}{M_{V_i}^2 - \hat{s}m_b^2 - iM_{V_i} \Gamma_{V_i}},$$

where $\kappa_i$ are the Fudge factors (see for example [38]).

From the explicit expressions of the asymmetry parameters we see that they depend on the new Wilson coefficients and $q^2$. Therefore there might appear some difficulties in studying the dependence of the physical quantities on both variables in the experiments. In order to avoid this difficulty we perform our analysis at fixed values of $C_X$.

In Fig. (1), the dependence of the P–odd asymmetry $A^{FB}_\theta$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay is presented at five different values of $C_{LL}$. From this figure we see that, outside the resonance regions, the zero–position of $A^{FB}_\theta$ is shifted compared to that of the SM result and this behavior of $A^{FB}_\theta$ is quite similar to the one determined by the coefficient $C_{LR}$. The zero–positions of $A^{FB}_\theta$ occur at the values $q^2 < 5 \text{ GeV}^2$, and therefore the zero of $A^{FB}_\theta$ is sensitive only to the short distance contributions of the new Wilson coefficients and is free of the long distance effects. When new Wilson coefficients are negative (positive), the zero–position of $A^{FB}_\theta$ in the SM shifts to right (left). Further analysis shows that the zero–position of $A^{FB}_\theta$ is practically independent of the Wilson coefficients $C_{RR}$ and $C_{RL}$ and coincides with that of the SM result. Moreover, for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay, $A^{FB}_\theta$ seems to be insensitive to the presence of any scalar type interactions in the allowed region of the new Wilson coefficients.

Depicted in Fig. (2) is the effect of the Wilson coefficient $C_{LL}$ on the dependence of $A^{FB}_\theta$ on $q^2$ for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay. We observe from this figure that, far from the resonance region, the zero–position of $A^{FB}_\theta$ for this decay channel is realized only for $C_{LL} = -4$. Similarly, another zero–position occurs at $C_{LR} = -4$. Therefore, the analysis of the zero–position, as well as determination of $A^{FB}_\theta$ between the resonance regions, can serve as a good test for establishing new physics beyond the SM. In the presence of the Wilson coefficients $C_{LL}, C_{LR}, C_{RL}$ and $C_{RR}$, the absolute value of $A^{FB}_\theta$ in the SM differs, approximately, two times compared to its absolute value in the new physics beyond the SM, between the resonance regions. Therefore, measurement of the value of $A^{FB}_\theta$ in experiments can give useful information about the new physics. It is further observed that $A^{FB}_\theta$ for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay is very sensitive to the existence of the scalar interaction with the coefficient $C_{LRRL}$, while independent of all other scalar interactions (see Fig. (3)).
Therefore measurement of $\mathcal{A}_\theta^{FB}(\Lambda_b \to \Lambda\tau^+\tau^-)$ can be quite informative for establishing the new scalar interactions.

In Figs. (4) and (5), we present the dependence of $\mathcal{A}_\theta^{FB}$ on $q^2$ at five fixed values of the Wilson coefficients $C_{RR}$ and $C_{RL}$, respectively, for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay. From these figures we see that, up to $q^2 = 18 \text{ GeV}^2$, the magnitude of $\mathcal{A}_\theta^{FB}$ decreases at all values of the Wilson coefficients. Contrary to the $\mathcal{A}_\theta^{FB}$ case, where $\mathcal{A}_\theta^{FB}$ exhibits strong dependence on $C_{LL}$ and $C_{LR}$, $\mathcal{A}_\theta^{FB}$ is practically insensitive to these coefficients.

In Fig. (6), we present the dependence of $\mathcal{A}_\theta^{FB}$ on $q^2$ for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay, at several fixed values of the Wilson coefficients $C_{RL}$. It should be noted that, $\mathcal{A}_\theta^{FB}$ shows practically similar behavior on $C_{RR}$, and for this reason we present only the result for $C_{RL}$. We observe that, the zero–position is absent for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay. However, a measurement of the magnitude of $\mathcal{A}_\theta^{FB}$ can give conclusive information about the existence of the new physics. Similar to the $\Lambda_b \to \Lambda\mu^+\mu^-$ case, $\mathcal{A}_\theta^{FB}$ is weakly dependent on the Wilson coefficients $C_{LL}$ and $C_{LR}$. It is observed that, $\mathcal{A}_\theta^{FB}$ for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay is rather sensitive to the scalar interactions with the coefficients $C_{RRL}$ and $C_{RLL}$, while it is independent of the remaining scalar interaction coefficients. Close to the end of the allowed region ($q^2 > 18 \text{ GeV}^2$), $\mathcal{A}_\theta^{FB}(\Lambda_b \to \Lambda\tau^+\tau^-)$ shows considerable departure from the SM result (see Fig. (7)).

Our analysis shows that the zero–position of $\mathcal{A}_\theta^{FB}$ for the $\Lambda_b \to \Lambda\mu^+\mu^-$ case is practically independent of the new vector interaction with coefficients $C_{LR}$, $C_{RL}$ and $C_{RR}$, and only in the presence of the coefficient $C_{LL}$, it shifts to the right (left) compared to the SM prediction, at its negative (positive) values. Moreover, the value of $\mathcal{A}_\theta^{FB}$ shows considerable departure from the SM values for the coefficients $C_{LR}$, $C_{RL}$ and $C_{RR}$ in the region $2 \text{ GeV}^2 \leq q^2 \leq 4 \text{ GeV}^2$. Here we note that the zero–position and magnitude of $\mathcal{A}_\theta^{FB}$ for the $\Lambda_b \to \Lambda\mu^+\mu^-$ decay are both insensitive to any of the scalar type interactions.

In Figs. (8), (9) and (10) we present the dependence of $\mathcal{A}_\theta^{FB}$ on $q^2$ at fixed values of $C_{LL}$, $C_{LR}$ and $C_{RL}$, respectively, for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay. Here we would like to note that the dependence of $\mathcal{A}_\theta^{FB}$ on $q^2$ at the given values of $C_{RR}$ coincides practically with that of that of its dependence on $C_{RL}$. We observe from these figures that, except the resonance region ($q^2 \approx 14.6 \text{ GeV}^2$), there are no other zero–points of $\mathcal{A}_\theta^{FB}$ for the Wilson coefficients $C_{LL}$ and $C_{RL}$. New zero–points of $\mathcal{A}_\theta^{FB}$ appear in the presence of $C_{LR}$, and they are being two zero–points at $C_{LR} = -4$ and one zero–point at $C_{LR} = -2$. Therefore, determination of the zero–position of $\mathcal{A}_\theta^{FB}$ at $q^2 \approx 17.6 \text{ GeV}^2$ and $q^2 \approx 18.8 \text{ GeV}^2$ is an unambiguous indication of the new physics, and this new physics is solely due to the presence of the Wilson coefficient $C_{LR}$.

It should be noted that $\mathcal{A}_\theta^{FB}$ asymmetry for the $\Lambda_b \to \Lambda\tau^+\tau^-$ decay is very sensitive to the presence of the new scalar type interactions $C_{LRR}$ and $C_{LRL}$ (see Fig. (11)). Since the dependence of $\mathcal{A}_\theta^{FB}(\Lambda_b \to \Lambda\tau^+\tau^-)$ on the above–mentioned scalar coefficients turned out to be very similar, we present the one for the $C_{LRL}$ case. From this figure we observe that there appears a new zero–position of $\mathcal{A}_\theta^{FB}$ which is absent in the SM, and far from the resonance regions considerable departure from the SM is predicted. For this reason study of the zero–position of $\mathcal{A}_\theta^{FB}$ can give comprehensive information about the existence of the new physics beyond the SM.
We can get additional information by measuring the magnitude of $A_{FB}^{S\Lambda}$ in the regions $14.6 \, GeV^2 \leq q^2 \leq 16 \, GeV^2$ and $17.6 \, GeV^2 \leq q^2 \leq 19.2 \, GeV^2$, which can be useful in determining the magnitude of the new Wilson coefficients.

In conclusion, we study the dependence of three P–odd forward–backward asymmetries on $q^2$ in the presence of the new vector type interactions. The results we obtain can briefly be summarized as follows:

- The zero–position of $A_{FB}^{g\Lambda}$ for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay is sensitive only to the presence of $C_{LL}$ and $C_{LR}$, and is free of the long distance effects. The location of its zero–position unambiguously allows us to determine the sign of the new Wilson coefficients.

- Determination of the value of $A_{FB}^{g\Lambda}$ for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay between the resonance regions can give invaluable information about the new physics, which is more sensitive to the presence of the vector coefficients $C_{RL}$ and $C_{RR}$, as well as scalar coefficient $C_{LRRL}$.

- It is shown that the P–odd asymmetry $A_{FB}^{g\Lambda}$ for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ and $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decays is more sensitive to the Wilson coefficients $C_{RR}$ and $C_{RL}$, while it is insensitive to the effects of $C_{LL}$ and $C_{LR}$. In the case of the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay, the same asymmetry exhibits strong dependence on the scalar coefficients $C_{RLRL}$ and $C_{RLLR}$ as well.

- Zero–position of $A_{FB}^{g\Lambda}$ for the $\Lambda_b \rightarrow \Lambda \mu^+ \mu^-$ decay is practically independent of the vector type coefficients $C_{LR}$, $C_{RL}$ and $C_{RR}$, and all type of scalar interactions; but it shows dependence only on $C_{LL}$. Its zero–point position is shifted to the right (left) compared to that of the SM result at negative (positive) values of $C_{LL}$. regions of $q^2$ are found where the value of $A_{FB}^{g\Lambda}$ depart from the SM prediction in the presence of the new vector interactions with the new Wilson coefficients $C_{LR}$, $C_{RR}$ and $C_{RL}$.

- Our analysis predicts that, except the resonance regions, there are new zero–points of $A_{FB}^{g\Lambda}$ for the negative values of $C_{RL}$, and for the scalar coefficients $C_{LRRL}$, $C_{LRLR}$ for the $\Lambda_b \rightarrow \Lambda \tau^+ \tau^-$ decay.
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References

[1] T. M. Aliev, V. Bashiry, and M. Savci, prep: hep-ph/0507324 (2005).

[2] S. R. Choudhury and N. Gaur, Phys. Rev. D 66, 094015 (2002); T. M. Aliev, M. Savci, Phys. Rev. D 60, 014005 (1999).

[3] S. R. Choudhury, A. S. Cornell, N. Gaur, and G. C. Joshi, Phys. Rev. D 69, 054018 (2004); T. M. Aliev, V. Bashiry, and M. Savci, Eur. Phys. J. C 35, 197 (2004); T. M. Aliev, M. K. Çakmak, A. Özpıneci, and M. Savci, Phys. Rev. D 64, 055007 (2001).

[4] F. Krüger and L. M. Sehgal, Phys. Lett. B 380, 199 (1996); J. L. Hewett, Phys. Rev. D 53, 4964 (1996).

[5] S. Fukae, C. S. Kim, T. Morozumi, and T. Yoshikawa, Phys. Rev. D 59, 074013 (1999).

[6] S. Rai Choudhury, A. Gupta, and N. Gaur, Phys. Rev. D 60, 115004 (1999); S. Fukae, C. S. Kim, and T. Yoshikawa, Phys. Rev. D 61, 074015 (2000); D. Guetta and E. Nardi, Phys. Rev. D 58, 012001 (1998).

[7] A. S. Cornell, N. Gaur, JHEP 0502, 005 (2005).

[8] T. M. Aliev, M. K. Çakmak, and M. Savci, Nucl. Phys. B607, 305 (2001); T. M. Aliev and M. Savci, Phys. Lett. B 481, 275 (2000).

[9] G. Burdman, Phys. Rev. D 52, 6400 (1995).

[10] T. M. Aliev, V. Bashiry, M. Savci, JHEP 0405, 037 (2004).

[11] T. M. Aliev, A. Özpıneci, M. Savci, C. Yüce, Phys. Rev. D 66, 115006 (2002); T. M. Aliev, A. Özpıneci, M. Savci, Phys. Lett. B 511, 49 (2001); T. M. Aliev, A. Özpıneci, M. Savci, Nucl. Phys. B585, 275 (2000); T. M. Aliev, C. S. Kim, Y. G. Kim, Phys. Rev. D 62, 014026 (2000); T. M. Aliev, E. O. İltan, N. K. Pak, Phys. Lett. B 451, 175 (1999); T. M. Aliev, C. S. Kim, M. Savci, Phys. Lett. B 441, 410 (1998).

[12] A. Ishikawa et. al, BELLE Collaboration, Phys. Rev. Lett. 91, 261601 (2003).

[13] B. Aubert et. al, BaBar Collaboration, prep: hep-ex/0507005 (2005).

[14] T. Mannel and S. Recksiegel, J. Phys. G 24, 979 (1998).

[15] P. Bialas, J. G. Körner, M. Krämer, and K. Zalewski, Z. Phys. C 57, 115 (1993); F. Hussain, J. G. Körner, and R. Migneron, Phys. Lett. B 248, 406 (1990); 252 723 (E) (1990); J. G. Körner and M. Krämer, Phys. Lett. B 275, 495 (1992); T. Mannel and G. A. Schulier, Phys. Rev. D 279, 194 (1992); M. Gronau and S. Wakaizumi, Phys. Rev. D 47, 1262 (1993); M. Tanaka, Phys. Rev. D 47, 4969 (1993); M. Gronau, T. Hasuike, T. Hattori, Z. Hioki, T. Hayashi, and S. Wakaizumi, J. Phys. G 19, 1987 (1993); Z. Hioki, Z. Phys. C 59, 555 (1993); B. König, J. G. Körner, and M. Krämer, Phys. Rev. D 49, 2363 (1994); M. Gremm, G. Köpp, and L. M. Sehgal, Phys. Rev. D 52, 1588 (1995); C. S. Huang and H. G. Yan, Phys. Rev. D 56, 5981 (1997); S. Balk,
J. G. Körner, and D. Pirjol, Eur. Phys. J. C 1, 221 (1998); J. G. Körner and D. Pirjol, Phys. Lett. B 334, 399 (1994); Phys. Rev. D 60, 014021 (1999).

[16] M. Gremm, F. Krüger, and L. M. Sehgal, Phys. Lett. B 355, 579 (1995); T. Mannel and S. Reckziegel, Acta Phys. Polon. B 28, 2489 (1997); C. K. Chua, X. G. He, and W. S. Hou, Phys. Rev. D 60, 014003 (1999); G. Hiller and A. Kagan, Phys. Rev. D 65, 074038 (2002).

[17] M. Krämer and H. Simma, Nucl. Phys. Proc. Suppl. 50, 125 (1996).

[18] C. H. Chen and C. Q. Geng, and J. N. Ng, Phys. Rev. D 65, 091502 (2002).

[19] T. M. Aliev, A. Özpıneci, M. Savcı, and C. Yüce, Phys. Lett. B 542, 229 (2002).

[20] C. H. Chen and C. Q. Geng, Phys. Rev. D 63, 054005 (2001).

[21] J. G. Körner, M. Krämer, and D. Pirjol, Prog. Part. Nucl. Phys. 33, 787 (1994).

[22] Z. Zhao et al., Report of Snowmass 2001 working group E2: Electron Positron Colliders from the φ to the Z, in Proc. of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001) ed. R. Davidson and C. Quigg, hep–ex/0201047 (2002).

[23] T. M. Aliev, A. Özpıneci, and M. Savcı, Phys. Rev. D 65, 115002 (2002);
T. M. Aliev, M. Savcı, J. Phys. G 26, 997 (2000).

[24] T. M. Aliev, A. Özpıneci, and M. Savcı, Nucl. Phys. B649, 1681 (2003).

[25] T. M. Aliev, A. Özpıneci, and M. Savcı, Phys. Rev. D 67, 035007 (2003).

[26] T. M. Aliev, V. Bashiry, and M. Savcı, Nucl. Phys. B709, 115 (2005).

[27] C. H. Chen and C. Q. Geng, Phys. Rev. D 64, 074001 (2001).

[28] T. Mannel, W. Roberts, and Z. Ryzak, Nucl. Phys. B355, 38 (1991).

[29] K. Hagiwara, A. D. Martin, and M. F. Wade, Nucl. Phys. B327 569 (1989).

[30] J. G. Körner and G. A. Schuler, Z. Phys. C 46, 93 (1990).

[31] G. Abbiendi et al, OPAL Collaboration, Phys. Lett. B 444, 539 (1998).

[32] D. Buskulic et al, ALEPH Collaboration, Phys. Lett. B 374, (1996) 319.

[33] C. S. Huang, H. G. Yan, Phys. Rev. D 59, 114022 (1999).

[34] C. H. Chen, C. Q. Geng, Phys. Rev. D 64, 114024 (2001).

[35] M. C. Chang et al., BELLE Collaboration, Phys. Rev. D 68, 111101 (2003).

[36] M. Misiak, Nucl. Phys. B393, 23 (1993); Phys. Rev. D B439, 461(E) (1995).

[37] A. J. Buras and M. Münz, Phys. Rev. D 52, 186 (1995).

[38] A. Ali, P. Ball, L. T. Handoko, and G. Hiller, Phys. Rev. D 61, 074024 (2000).
Figure captions

Fig. (1) The dependence of the P–odd forward–backward asymmetry $A_{FB}^{θ^B}$ on $q^2$ at five different fixed values of the vector type Wilson coefficient $C_{LL}$ for the $Λ_b \rightarrow Λμ^+μ^−$ decay.

Fig. (2) The same as in Fig. (1), but for the $Λ_b \rightarrow Λτ^+τ^−$ decay.

Fig. (3) The same as in Fig. (2), but at five different fixed values of the scalar type Wilson coefficient $C_{LRRL}$.

Fig. (4) The dependence of the P–odd forward–backward asymmetry $A_{FB}^{θ^B}$ on $q^2$ at five different fixed values of the vector type Wilson coefficient $C_{RL}$ for the $Λ_b \rightarrow Λμ^+μ^−$ decay.

Fig. (5) The same as in Fig. (4), but at five different fixed values of the vector type Wilson coefficient $C_{RR}$.

Fig. (6) The same as in Fig. (4), but for the $Λ_b \rightarrow Λτ^+τ^−$ decay.

Fig. (7) The same as in Fig. (6), but at five different fixed values of the scalar type Wilson coefficient $C_{RLRL}$.

Fig. (8) The dependence of the P–odd forward–backward asymmetry $A_{FB}^{θ^B}$ on $q^2$ at five different fixed values of the vector type Wilson coefficient $C_{LL}$ for the $Λ_b \rightarrow Λτ^+τ^−$ decay.

Fig. (9) The same as in Fig. (8), but at five different fixed values of the vector type Wilson coefficient $C_{LR}$.

Fig. (10) The same as in Fig. (8), but at five different fixed values of the vector type Wilson coefficient $C_{RL}$.

Fig. (11) The same as in Fig. (8), but at five different fixed values of the scalar type Wilson coefficient $C_{LRRL}$. 
Figure 1:

Figure 2:
\[ A_{FB}(\Lambda_b \rightarrow \Lambda r^+) \]

Figure 3:

\[ A_{FB}(\Lambda_b \rightarrow \Lambda \mu^- \mu^+) \]

Figure 4:
Figure 5:

Figure 6:
Figure 7:

Figure 8:
Figure 9:

\[ A_{q^2}(\Lambda_b \rightarrow \Lambda^+ \tau^-) \]

Figure 10:

\[ A_{q^2}(\Lambda_b \rightarrow \Lambda^+ \tau^-) \]

$q^2 (GeV^2)$
Figure 11: $A_{FB}^{b \to \Lambda \tau^- \tau^+}(C_{LRLL})$ vs. $q^2$ (GeV$^2$) for different values of $C_{LRLL}$.