Radiation Forces Constrain the FRB Mechanism

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ABSTRACT

We provide constraints on Fast Radio Burst (FRB) models by careful considerations of radiation forces associated with these powerful transients. We find that the induced-Compton scatterings of the coherent radiation by electrons/positrons accelerate particles to very large Lorentz factors (LF) in and around the source of this radiation. This severely restricts those models for FRBs that invoke relativistic shocks and maser type instabilities at distances less than about 10\textsuperscript{13} cm of the neutron star. Radiation traveling upstream, in these models, forces particles to move away from the shock with a LF larger than the LF of the shock front. This suspends the photon generation process after it has been operating for less than \(\sim 0.1\) ms (observer frame duration). We show that masers operating in shocks at distances larger than 10\textsuperscript{13} cm cannot simultaneously account for the burst duration of 1 ms or more and the observed GHz frequencies of FRBs without requiring an excessive energy budget (> \(10^{46}\) erg); the energy is not calculated by imposing any efficiency consideration, or other details, for the maser mechanism, but is entirely the result of ensuring that particle acceleration by induced-Compton forces upstream of the shock front does not choke off the maser process. For the source to operate more or less continuously for a few ms, it should be embedded in a strong magnetic field – cyclotron frequency \(\gg\) wave frequency – so that radiation forces don’t disperse the plasma and shut-off the engine.

Key words: Radiation mechanisms: non-thermal - methods: analytical - stars: magnetars - radio continuum: transients - masers

1 INTRODUCTION

Fast radio bursts (FRBs) are milli-second-duration bright (flux density \(\sim\) Jansky) transient events observed between 400 MHz and 7 GHz frequencies (Amiri et al. 2019a, 2019b; Ravi 2019a, 2019b; Ravi et al. 2019; Oslowski et al. 2019; Kocz et al. 2019; Bannister et al. 2019; Gaijar et al. 2018; Michilli et al. 2018; Farah et al. 2018; Shannon et al. 2018; Bannister et al. 2017; Law et al. 2017; Chatterjee et al. 2017; Marcote et al. 2017; Tendulkar et al. 2017; Spitler et al. 2016; Petroff et al. 2016; Spitler et al. 2014; Thornton et al. 2013; Lorimer et al. 2007). Numerous mechanisms have been suggested for the generation of the coherent radio emission of FRBs, eg. Kumar et al. (2017), Metzger et al. (2017), Yang & Zhang (2018), Lu & Kumar (2018), Metzger et al. (2019); for recent reviews see Katz (2018), Petroff et al. (2019), Codes & Chatterjee (2019). Most of these mechanisms are proposed to operate at a distance from neutron star surface of 10\textsuperscript{13} cm or less. We explore in this paper constraints on the FRB radiation mechanisms provided by rapid acceleration of charge particles by the strong electric field associated with the radiation and induced Compton scatterings of photons; induced Compton scattering refers to the scattering of a photons by electrons when the occupation number of photon quantum states is much larger than unity (for FRBs the occupation number is of order 10\textsuperscript{35}). The electron-photon scattering optical depth is increased due to the induced-Compton effect, which has been calculated by many authors, e.g. Melrose (1971), Blandford (1973), Blandford & Scharlemann (1975), Wilson & Rees (1978). We include the constraint on FRB models provided by induced-Compton (IC) optical depth, but that turns out to be much weaker than the IC acceleration that is estimated here. Particle acceleration due to the electric field of FRB radiations and its implications are considered in §2. Acceleration due to induced-Compton scatterings is described in §3, and the constraints these processes and some other general considerations impose on the FRB source and radiation mechanisms is discussed in §4.

2 PARTICLE MOTION DUE TO LARGE AMPLITUDE ELECTRIC FIELD

The RMS electric field strength associated with FRB radiation at a distance \(R\) from the source is

\[
\frac{E_0}{\sqrt{2}} = \left(\frac{L}{cR^2}\right)^{1/2} = \left(1.8\times10^9\text{ cm/s}\right)\left(L_{44}\right)^{1/2} R_{13}^{-1},
\]

where \(L\) is isotropic equivalent luminosity of the FRB. The non-linearity parameter, \(a\), associated with this field – which is a rough
measure of the energy gained by an electron traveling a distance of one wavelength in the wave-electric-field divided by its rest mass-energy – is
\[ a \equiv \frac{qE_0}{mc^2} = 4.5 \cdot \frac{1}{43} \cdot \frac{1}{R_{13}} \cdot \frac{1}{\omega_{ci}^{-1}}, \tag{2} \]
where \( q \) and \( m \) are electron charge and mass respectively, and \( \omega \) is the wave frequency (radian s\(^{-1}\)).

Let us consider the motion of a particle exposed to a linearly polarized EM wave and a uniform magnetic field that is perpendicular to the wave-vector of the EM wave. The wave EM fields and vector potentials for the EM wave and the static magnetic field are as follows:

\[ E_w = E_0 \hat{x} \sin(kz - \omega t), \quad B_w = E_0 \hat{y} \sin(kz - \omega t), \tag{3} \]
\[ A_w = -\frac{cE_0}{\omega} \hat{x} \cos(kz - \omega t), \quad A_B = (B_0 \hat{x} - B_0 \hat{y}) z. \tag{4} \]

The Lagrangian for particle motion is
\[ L = -\frac{mc^2}{\gamma} + \frac{q}{c} A \cdot \mathbf{v}, \tag{5} \]
where \( \gamma \) is the LF of the particle, \( \mathbf{v} \) is its 3-velocity, and \( A = A_w + A_B \) is the vector potential for the EM wave plus the static magnetic field. The \( x \) and \( y \) components of the particle’s canonical momentum are conserved since the Lagrangian is independent of \( x \) and \( y \) coordinates—
\[ p_x = m\gamma v_x + \frac{qA_x}{c} = m\gamma v_x - \frac{qE_0}{\omega} \cos \psi + \frac{qB_0 z}{c} = \text{constant}, \tag{6} \]
\[ p_y = m\gamma v_y - \frac{qB_0 z}{c} = \text{constant}, \tag{7} \]
where
\[ \psi = k z - \omega t. \tag{8} \]

The \( z \)-component of the momentum equation
\[ \frac{d\gamma v_z}{dt} = \frac{q}{mc} \left[ B_w v_x + v_x B_y - v_y B_x \right], \tag{9} \]
can be rewritten as
\[ \frac{d}{dt} \left[ \gamma v_z - \gamma c - \omega_B \left( x \sin \theta_B - y \cos \theta_B \right) \right] = 0, \tag{10} \]
where
\[ \sin \theta_B = \frac{B_y}{B}, \quad \text{and} \quad \omega_B = \frac{qB}{mc} \tag{11} \]
is the cyclotron frequency.

We assume that the particle is initially at rest, i.e. \( \mathbf{v} = 0 \), before it is hit by the EM wave, and that its initial position is \( \mathbf{r} = 0 \). Thereafter its velocity is obtained from equations (6), (7) and (10) by applying the initial conditions,
\[ \gamma v_x = ac \left[ \cos(kz - \omega t) - 1 \right] - \omega_B z \sin \theta_B, \tag{12} \]
\[ \gamma v_y = \omega_B z \cos \theta_B, \tag{13} \]
\[ \gamma v_z = c(\gamma - 1) + \omega_B (x \sin \theta_B - y \cos \theta_B), \tag{14} \]
where \( a \) is given by equation (2).

The particle LF is easily obtained from these equations and is
\[ \gamma = \frac{4a^2 \sin^4(\psi/2)}{2[1 - \xi_\rho \sin(\theta_B - \phi)]} + \frac{4a^2 \sin^2(\psi/2) \xi_\rho \sin(\theta_B - \phi) + \xi_\rho^2 + 1}{\xi_\rho \sin(\theta_B - \phi) + 1}/2 \tag{15} \]

where
\[ \sin \phi = y/\rho, \quad \rho^2 = x^2 + y^2, \quad \xi_\rho = \rho \omega_B/c, \quad \xi_\rho = z \omega_B/c. \tag{16} \]

The particular case of \( \omega_B = 0 \), i.e. vanishing static magnetic field, is illuminating:
\[ \gamma v_x = -2ac \sin^2(\psi/2), \quad \gamma v_y = c(\gamma - 1), \quad \gamma = 1 + 2a^2 \sin^4(\psi/2). \tag{17} \]

For \( a > 1 \), the particle LF is of order \( 2a^2 \) and its velocity vector lies within an angle \( \sim a^{-1} \) of the wave propagation direction. For \( a \ll 1 \), the velocity component along the wave vector oscillates at frequency \( 2\omega \) and perpendicular to it at frequency \( \omega \). However, for \( a > 1 \), the phase function along the particle worldline \( |\psi| = \omega t - kz(t) \sim \omega t/2(\gamma^2) \), and so the particle LF oscillates at frequency \( \sim \omega/(\gamma^2) \sim \omega/[2(1 + a^2)] \); where \( \gamma^2 \) is the time averaged LF-squared of the particle. Figure 1 shows numerical results for particle trajectories for a few different values of \( a \).
linearity parameter for those FRBs which emit a good fraction of their energy at $\sim 500$ MHz frequency is: $\alpha \sim 10 R_{13}^{-1} L_{43}^{1/2}$ (see eq. 2).

Considering that charge particles are forced to move at close to the speed of light by the electric field of the radiation out to a distance $\sim 10^{13}$ cm from the source, it might be hard for the plasma lens model (Cordes et al. 2017; Main et al. 2018) to operate within this distance of the source – a few ms duration radiation pulse cannot pass through the plasma in the “lens” if the plasma is located at a distance $\lesssim 10^{15}$ cm from the FRB source since the plasma would be forced to move at speed $\sim c$ by the wave electric field – unless the magnification factor of the plasma lens is much larger than $\sim 10^2$.

If the FRB radiation is produced in shock heated plasma (e.g. Metzger et al. 2019) at radius $R$, then particles upstream of the shock front would be accelerated to $\gamma \approx 2a^2 \sim 50 L_{43} R_{13}^{-2} c^{-2}$ (eq. 2) by the electric field of radiation moving upstream as long as the combination of the medium is $e^\pm$ and the cyclotron frequency is much less than 1 GHz. The LF of upstream particles wrt to the shock in this case is $\sim \frac{\gamma_{sf}}{2}; \frac{\gamma_{sf}}{2}$ is the LF of the shock front wrt to the undisturbed, upstream, medium. The upstream medium is also compressed by a factor $\sim 4a^2$ due to this acceleration, and the component of upstream particle four-velocity tangential to the shock-surface is $\sim a$ which varies on a time scale of the wave period. These effects would modify the growth rate of synchrotron maser instability operating near the shock front, and reduce the efficiency of converting blast wave energy to coherent GHz radiation.

Furthermore, particle acceleration upstream of the shock front would shut off radiation production for a while in the observed band, if $R \lesssim 10^{13}$ cm, until upstream particles slow down by plowing into the plasma further out. So the radiation by shocked plasma is not going to be produced continuously if $R \lesssim 10^{13}$ cm.

Another effect of particle acceleration by the intense FRB radiation is that it depletes the energy from the the outward moving radiation front – charge particles undergoing acceleration radiate – at a rate much larger than one might expect from Thomson scatterings. This is estimated in the following sub-section, where we also discuss the constraints imposed by this process on the circum-burst medium.

2.1 Radiative loss of Wave-accelerated particles

We calculate in this sub-section the energy loss suffered by FRB pulse as it travels through the CSM of the magnetar due to power radiated by particles that are accelerated by the EM field of the FRB radiation (particle acceleration was calculated in §2). The power emitted by a relativistic particle undergoing acceleration is given by (the covariant form of) the Larmor formula

$$P = \frac{2q^2}{3c} \left[ \left( \frac{du^0}{d\tau} \right)^2 + \sum_{i=x,y,z} \left( \frac{du^i}{d\tau} \right)^2 \right], \quad (18)$$

where $(u^\mu) = (c, v_x, v_y, v_z)$ is the four-velocity and $d\tau = dt/\gamma$ is the differential proper time. The total emitted energy within the interaction time $t_{int}$ is then given by $E_{rad} = \int_0^{t_{int}} P dt$, which is the amount of energy that is spontaneously scattered by the particle according to classical electrodynamics. In this picture, the scattered photons occupy very different regions of the phase space (for both frequency and direction) from the incoming photons, so we ignore stimulated emission (which will be discussed in the next section). We have checked that the radiative back-reaction force is negligible compared to the Lorentz force for the entire parameter space considered in this paper. However, as we show below, the cumulative energy loss could be significant such that a large fraction of the FRB wave energy is scattered away. This in turn provides a constraint on the gas density of the FRB environment.

Consider the FRB waves propagating through a strongly magnetized relativistic wind with luminosity $L_w$ and LF $\gamma_w$. An order unity fraction of the wind power is carried by Poynting flux and a fraction $\mu c^2 = 1$ is carried by electron (and positron) kinetic energy flux. At radius $R$ (much greater than the light cylinder of a neutron star), the magnetic field strength in the lab frame is $B \approx \sqrt{L_w/(\pi^2 c^5)}$. For a pulse of duration $t_{FRB}$, the interaction time between a particle and the FRB wave is

$$t_{int} \approx \min(R/c, \gamma_w^2 t_{FRB}). \quad (19)$$

We calculate the cumulative scattered energy for each electron in the wind comoving frame $E_{rad}^\prime$ and then Lorentz transform that to the lab frame $E_{rad} = \gamma_w E_{rad}^\prime$. The total number of electrons participating in the interaction near characteristic radius $R$ is

$$N_e \approx \frac{L_w}{\mu c \gamma_w \gamma_{w,mc}^2} \max \left( \frac{R}{\gamma_{w,mc}^2}, t_{FRB} \right). \quad (20)$$

If the total scattered energy exceeds the total wind energy $\mu c^2 N_e \gamma_w \gamma_{w,mc}^2$, then the wind should be significantly accelerated by the photon momentum which decreases $E_{rad}$. Thus, the scattered energy should generally be written as

$$E_{sca} = N_e \min(E_{rad}, \mu c^2 \gamma_w \gamma_{w,mc}^2). \quad (21)$$

Therefore, we obtain the ratio between scattered energy and the FRB wave energy

$$\frac{E_{sca}}{E_{FRB}} \approx \frac{L_w,47}{L_{43}} \min \left( \frac{E_{rad}}{\mu c^2 \gamma_w \gamma_{w,mc}^2}, \frac{1}{10^6} \right) \max \left( \frac{R_{13}}{37^2 \gamma_{2}^2 \gamma_{w,mc}^2}, 10^{-6} \right). \quad (22)$$

Fig. 2 shows log($E_{sca}/E_{FRB}$) as a function of $L_w$ and $\gamma_w$ for a typical weak burst from FRB 121102. If the FRB source is within the magnetosphere of a neutron star (below the light cylinder), we ruled out the presence of a mildly relativistic ($\gamma_w \lesssim 10^4$) wind from the progenitor star with $L_w \gtrsim 10^{39}$ erg s$^{-1}$, regardless of its composition (pair or electron-proton), since the FRB pulse would lose a large fraction of its energy. We note that the persistent radio source associated with FRB 121102 (luminosity $\sim 10^{39}$ erg s$^{-1}$, at projected distance of $< 40$ pc, Chatterjee et al. 2017, Marcote et al. 2017) may still be powered by an ultra-relativistic $\gamma_w \gg 10$ wind like that in the Crab Nebula.

3 PARTICLE ACCELERATION DUE TO INDUCED COMPTON SCATTERINGS

In this section, we calculate particle acceleration by scattering photons when the occupation numbers of both the initial and the final photon states are very large, i.e. due to induced Compton (IC) scatterings. The calculations described here are valid when the wave nonlinearity parameter “$a$” (eq. 2) is less than 1; IC scatterings for highly nonlinear waves is a complicated problem and will be taken up in a future paper, however, nonlinear effects when $a > 1$ are included in numerical calculations presented in Fig. 3.

It is best to view the process from the rest frame of an electron. A photon of wave-vector $k'$ is scattered to $k''$ in electron rest frame. The momentum kick given to the electron in this scattering is $k'' - k'$. The inverse of the process where $(k'_1 k'_2) \rightarrow k''$ gives an almost exact opposite kick to the electron; $k'' \equiv k'_1 / |k'_1|$ is a unit vector. The non-zero difference between the two is due to the electron recoil in this scattering process so that $k'_1 \neq k''$. © 0000 RAS, MNRAS 000, 000–000
The rate of momentum transfer to an electron by a narrow beam of photons with wave-vector within $d^3k'$ and its inverse process is

$$\delta \mathbf{p}' = \frac{e \hbar}{2} \frac{d\sigma}{d\Omega_k'} \left[ (1 + n_{k_1} n_{k_2}) (k' - k_1) \frac{d\Omega_k'}{4\pi^4} + (1 + n_{k_2}) n_{k_2} (k_2 - k') \frac{d\Omega_k'}{4\pi^4} \right],$$

(23)

where

$$\cos \chi' = \hat{k}' \cdot \hat{k}_1', k_1' = k' - \Delta k', k_2' = k' + \Delta k',$$

$$dk_2' = dk' (1 + 2\Delta k'/k'),$$

$$\hbar \Delta k' = e^2 k_2' mc (1 - \cos \chi')$$

(24)

is the momentum recoil suffered by the electron due to scattering one photon of momentum $\hbar k'$ by an angle $\chi'$. The factor $4\pi^4$ in the denominator of equation (23) is because the number of distinct photon quantum states in volume $d^3k'$ is $d^3k'/4\pi^3$.

We are considering the case where $n_k \gg 1$ is a function of $|k|$ within a cone, in the k-space, of opening angle $\theta_s \ll 1$. Expressing $k_1'$ and $k_2'$ in terms of $k'$ (eq. 24), and Taylor expansion of $n_{k_1'}$ and $n_{k_2'}$ in terms of $n_{k'}$ transforms equation (23) to the following expression

$$\delta \mathbf{p}' = \frac{d\sigma}{d\Omega_k'} \frac{\hbar \delta^3 k'}{4\pi^4} \left[ (k' - \hat{k}_1') d\Omega_k' - (k' - \hat{k}_1') d\Omega_k' \right]$$

$$+ \Delta k' (\hat{k}_1' d\Omega_k' + \hat{k}_1' d\Omega_k')$$

$$+ \frac{2(k_1' - k') \Delta k'}{k_1' n_{k_1'} k_1' d\Omega_k'} \left[ n_{k_1'} n_{k_1'} \right]$$

(25)

Integrating the above expression over incident and scattered photon directions ($\Omega_k'$ and $\Omega_k$) and frequency yields the total rate of momentum deposition to the electron by the induced-Compton scatterings. The $d\Omega_k'$ and $d\Omega_k$ integrals, nested inside the square bracket in eq. (25), are carried out subject to the condition that the scattering angle $\chi'$ is held fixed. Thus, the first two terms in the square bracket cancel exactly when the angular integral over photon propagation direction is performed. The integral of the last term over $\Omega_k$ is smaller than the third term by at least a factor $\theta_s^{-2}$ ($\theta_s$ is half angular size of the radiation beam in the electron rest frame). This is because both the incident and scattered photons lie within the angle $\theta_s$ for the induced-Compton scattering to be relevant and therefore, $|k_1' - k'| \leq k' \theta_s^2/2$. Thus, the last term in equation (25) can be ignored as well. This simplifies the expression for the radiation force on the electron considerably

$$\frac{d\mathbf{p}'}{dt} \approx \frac{\hbar \delta^3 k'}{4\pi^4} \left[ \int d\Omega_k' \frac{d\sigma}{d\Omega_k'} \int d\Omega_k' \hat{k}_1' \Delta k' n_{k_1'} n_{k_1'} \right].$$

(26)

Since $k' - k_1' = \Delta k' \ll k'$ (see equation 24), we can take $n_{k_1'} \approx n_{k'}$, and the integral over $\Omega_k'$ gives $(\pi \theta_s^2 \Delta k' n_{k'} n_{k'})/z$; the axis of the photon beam cone is along $z$, and the photon occupation number ($n_{k'}$) is taken to be angle-independent inside the radiation cone, i.e. for $\theta' < \theta_s$. Thus, equation (26) reduces to

$$\frac{d\mathbf{p}'}{dt} \approx \frac{3\hbar^2 \sigma_T \theta_s^2}{4\pi^4 m} \int d\chi' \sin \chi' \sin^2 \Theta' \times \sin^2 (\chi'/2) z,$$

(27)

where we made use of equation (24) for $\Delta k'$ and

$$\frac{d\sigma}{d\cos \chi'} = \frac{3\sigma_T}{4} \sin^2 \Theta',$$

(28)

which is the differential scattering cross-section of an electron for linearly polarized radiation; where $\Theta'$ is the angle between the electric vector of incident radiation and the momentum vector of scattered photons in the electron rest frame ($\Theta' \approx \pi/2$ for small angle scatterings that are relevant for IC acceleration).

The final two integrals are straightforward and are carried out assuming that $\theta_s \ll 1$ and $n_{k'}$ is a smooth function of $k'$:

$$\frac{d\mathbf{p}'}{dt} \approx \frac{3\hbar^2 \sigma_T \theta_s^2}{64\pi^2 m} \left[ \frac{n_{k'}}{n_{k_1}} \right].$$

(29)

We can transform this equation to lab frame by noting that $d\mathbf{p}'/dt = d\mathbf{p}/dt$.

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1 Scattering a photon outside the radiation beam ($\chi' \geq \theta_s$) is a weaker process than the induced Compton within the beam for the FRB parameters being considered here.
which leads to
\[ \sin \theta_s = \frac{\sin \theta_s}{\gamma (1 - \beta \cos \theta_s)} \approx 2 \gamma \theta_s, \quad (\theta_s, \gamma) \ll 1, \]
and
\[ k' = k (1 - \beta \cos \theta_s) \approx k / 2 \gamma \quad (\theta_s, \gamma) \ll 1, \quad (30) \]
which leads to
\[ \frac{dp_x}{dt} \approx \frac{3 h^2 \sigma_x \theta_s^2 \gamma_n^2}{32 \pi^2 m}. \quad (31) \]
The photon occupation number, \( n_k \), is a Lorentz invariant quantity and it can be easily shown to be
\[ n_k = \frac{c^3 L_{\nu}}{8 \pi^2 \theta^3 [R^2 h^3 \nu]} \sim \frac{c^3 L}{8 \pi^2 \theta^3 [R^2 h^3 \nu^4]} \quad (32) \]
where \( L_{\nu} \) is the specific luminosity (isotropic equivalent), \( R \) is distance from the FRB source where particle acceleration is being considered. Substituting this into equation (31) we finally obtain
\[ \frac{dp_x}{dt} \approx 3 \sigma_x \theta_s^2 L_{\nu}^2 \gamma \quad (33) \]
If the inertia of the medium is dominated by electrons and positrons then \( p_x = m e \beta \gamma \), and the equation for LF of the particle is
\[ \frac{d\beta \gamma}{dt} \approx \frac{3 \sigma_x \theta_s^2 L_{\nu}^2}{256 \pi^2 R^4 h^3 m^2 c^2}. \quad (34) \]
This result is easy to understand as follows. Since for induced-Compton scatterings, the scattered photon lies within the photon beam of opening angle \( \theta_s \), the scattering cross-section is \( \sigma_x \theta_s^2 n_k \). In each scattering, the electron is recoiled and the momentum impulse it receives – when we subtract the contribution from its inverse process – is \( \hbar^2 k^2 / (2 m c^2) \). The number of photons streaming outward per unit area and time is, \( L' / (4 \pi R^2 h \nu) \). Combining all of these pieces we find the radiative force on an electron to be, \( \sigma_x L_{\nu} h \theta_s^2 / (8 \pi R^2 m c^3) \); where we made use of Lorentz transformations, i.e. \( \theta_s' \sim \gamma \theta_s, \nu' \sim \nu / \gamma, L' \sim L / \gamma^2 \). Substituting for \( n_k \) from equation (32) we arrive at an expression for the radiative force on the electron that is within a factor 2 of that given in equation (33). It should be noted that particle acceleration is more severe when the medium through which the coherent radiation propagates is moving away from the source at relativistic speeds.

4 CONSTRAINTS ON FRB RADIATION MECHANISMS

We consider two cases in separate sub-sections. The first one is where the magnetic field in the source is weak, \( B \ll 10^4 \) G, so that the cyclotron frequency is less than 1 GHz and the scattering of FRB radiation is not affected by the magnetic field. The other case is that of a strong magnetic field which is analyzed in §4.2.

4.1 The medium through which the FRB radiation is passing has weak magnetic field (cyclotron frequency \( \lesssim \) GHz)

The FRB scenario considered in this section is where the coherent radiation is produced when a relativistic jet from a compact object interacts with the circum-stellar medium (CSM) and a fraction of the jet energy is converted to GHz photons. We provide general constraints on the viability of this model.

The ability of the shock model to reproduce the observed duration of FRBs, and their frequency can be robustly constrained by combining a few physical considerations. For much of the following discussions we will ignore factors of order unity.

If the FRB radiation were to be produced at radius \( r < 10^{13} \) cm, the nonlinear parameter associated with the radiation at this radius is \( \alpha > 5 \) (eq. 2) and electrons upstream of the shock front are accelerated to LF 2a^2 > 50 (see §2). This high LF of particles upstream completely changes the shock dynamics as well as the cyclotron/synchrotron maser instability growth rate that has been suggested for FRB radiation (Plotnikov & Sironi 2019). Furthermore, induced Compton scatterings upstream of the shock front provide important constraints on the shock model for FRB radiation.

The induced-Compton (IC) scattering optical depth ahead of the shock front is given by (e.g. Lyubarsky 2008, Lu & Kumar 2018)
\[ \tau_{ic} \propto \frac{\sigma_T L_{\nu}(c \gamma_{FRB})}{8 \pi^2 R^2 m c^3} \sim \frac{4 \pi R^3 n_w^2 c t_{FRB}}{R^2 c \nu_{FRB}} \quad (35) \]

This equation is valid for both IC scatterings within the photon beam and outside the photon beam as long as \( R = 2 c t_{FRB} / \theta_s' \); where \( \theta_s' \) is the beam size of the coherent radiation at \( \nu = \nu' \). Substituting for \( \gamma_{FRB} \) and the IC optical depth given in equation (35) is in the CSM rest frame. We need to modify this equation if the upstream CSM is a wind with LF \( \gamma_w \). The IC optical depth in this frame is \( \nu' \gamma_w \), and the burst duration in the wind frame is \( t_{\nu \gamma} \). Thus, the IC optical depth can be rewritten as
\[ \tau_{ic} \propto \frac{\sigma_T L_{\nu}(c \gamma_{FRB})}{8 \pi^2 R^2 m c^3} \sim \frac{4 \pi R^3 n_w^2 c t_{FRB}}{32 \pi^3 m^2 c^3 \mu_w R^3 v^3}, \quad (36) \]

where \( L_{\nu} \) and \( \mu_w \) are wind luminosity and magnetization parameter, and we used the relation \( L_{\nu} = 4 \pi R^2 n_w^2 m c^3 \mu_w \gamma_w^2 \) to get rid of \( n_w \). The requirement that \( \tau_{ic} < 1 \) provides an upper bound on the CSM density
\[ n_w(R) = n_w^c \gamma_w \lesssim (30 \text{ cm}^{-3}) \left( \frac{R_{\text{FRB}}^3}{L_{\text{FRB}}} \right) \gamma_w^3 \quad (37) \]

For density \( n_w \ll 10 \text{ cm}^{-3} \), the relativistic jet deceleration radius and the place where coherent radiation is produced is \( \gg 10^{13} \) cm (see §4.1.1). This alleviates the problem associated with the acceleration of upstream \( c^2 \) away from the shock front to high LF due to the electric field of the coherent FRB radiation. Moreover, at the larger radius, particle acceleration due to induced Compton scatterings also poses less of a problem as we describe next. However, the frequency of maser photons produced in the shock is below the observing band for even CHIME at this low CSM density; this point is discussed further in §4.1.1.

The LF of plasma upstream of the shock front – accelerated by induced-Compton scatterings of FRB radiation – can be calculated using equation (34), which is rewritten below for the relativistic case in a more convenient form
\[ \frac{d\gamma}{dt} = \frac{\gamma}{t_{\text{acc}}}, \quad \text{where} \quad t_{\text{acc}} \sim \frac{256 \pi^2 R^4 h^3 m^2 c^2}{3 \sigma_x \theta_s^2 L_{\nu}^2} \approx (3 \times 10^{-13}) \frac{R_{\text{FRB}}^3}{L_{\text{FRB}}} \theta_s^2. \quad (38) \]

This shows that the LF of particles increases exponentially on a very short time scale, and they attain a terminal LF of \( \sim 3 \theta_s^{-1} \approx 3 \gamma_{FW} \) when the radiation field in the particle comoving frame becomes nearly isotropic and little momentum is imparted to electrons in further scatterings. If the cold upstream medium moves away from the source with LF \( \gamma_{FW} \) then \( t_{\text{acc}} \) is smaller by a factor \( \gamma_{FW} \) in the lab frame.

The leading front of the radiation that is produced by the
This assumes that the wave frequency, ω, is at least a factor few times higher than the plasma frequency in the upstream medium. This is the case when many FRB radiation production scenarios including the cyclotron/synchrotron maser instability mechanism that operates in the shock transition layer and downstream of the shock front. If this condition were not satisfied then the FRB radiation would not be able to travel away from the shock front to be received by the observer.

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3 The Lorentz factor of the shock front is larger than the SLF of the shocked plasma (γw) by a factor ∼ (ω/μw)2 (e.g. Kenel & Coroniti, 1984).

4 The total energy of the relativistic jet (isotropic equivalent) for a typical FRB is of order 1022 erg if the radiation production efficiency is a few percent. Considering the mass duration of FRBs, the jet energy has to come from the magnetic field and is far smaller than what we see for FRBs (see §4.1). For a magnetar with surface magnetic field of 1012 G, the total magnetic energy above the radius R is 2πR3μw/3 erg. Therefore, the jet has to be launched at a radius no larger than 102 cm, i.e. Rjet ≲ 102 cm and tjet ≲ Rjet/γw ≲ 3 ms.
within \( R_{da} \): \( \langle n_w \rangle = 3n_w(R_{da}) \) for a steady wind CSM. This expression is independent of the LF of the relativistic jet, and the magnetization of the wind. The second part of equation (45) is obtained by taking the average particle mass in the wind to be the electron mass; \( R_{da} \) is smaller for an ionic wind by a factor 7. Considering the weak dependence of \( R_{da} \) on burst energy and CSM density, it is hard for the FRB source, according to the shock model scenario, to be at a radius very different from 10^{13} \, cm. However, upstream particle acceleration by the emergent radiation renders the shock model not viable at \( R \lesssim 10^{13} \, cm \).

As argued before, the magnetization parameter of the wind CSM cannot be larger than order unity otherwise the FRB duration, according to the shock model, would be much smaller than the observed duration. Thus, we take \( \mu_w \sim 1 \). Moreover, the asymptotic LF of a magnetized wind satisfies the relation \( \gamma_w \sim \mu_w^{1/2} \) (Goldreich & Julian, 1969; Granot et al. 2011). So we take \( \gamma_w \) to be of order unity as well. Thus, the expected CSM particle density at \( R_{da} \sim 10^{13} \, cm \), for an electron-positron wind, is

\[
\langle n_w \rangle \sim \frac{L_w}{4\pi R_{da}^2 c^3} \sim (3\times 10^5 \, cm^{-3})L_{w,37}R_{13}^{-2}.
\]  

where our choice of wind luminosity of \( 10^{37} \, erg \, s^{-1} \) is for a young magnetar (see eq. 51). The high particle density upstream of the shock front makes the CSM highly opaque to induced-Compton scattering (eq. 35), and that is another problem with the shock model for FRB radiation.

FRB radiation could be generated in shocks at \( R \gtrsim 10^{14} \, cm \), where upstream particles are not forced by the radiation to move at relativistic speed away from the shock front. However, if we demand that \( R \sim R_{da} \), so that the FRB radiation is produced efficiently, then that requires the energy of the relativistic jet to be \( \gtrsim 10^{46} \, erg \) or \( n \lesssim 1 \, cm^{-3} \) (see eq. 45). The large energy requirement seems problematic especially for the repeater FRB 121102, the total energy of outbursts in the last 10 years for this object exceeds the available energy in magnetic fields of even an extreme magnetar if each outburst has \( \gtrsim 10^{46} \, erg \) energy. On the other hand, if we take the CSM density to be sufficiently small so that \( R_{da} \gtrsim 10^{14} \, cm \) then the frequency of coherent radio emission produced in shocks is much smaller than the observed frequencies for FRBs (this point is amplified below).

We next explore the possibility that the FRB radiation is generated at a radius \( R_c \gtrsim 10^{14} \, cm \), and relax the efficiency considerations so that \( R_c \) can be much larger than the deceleration radius \( (R_{da}) \) of the relativistic jet.

The characteristic photon frequency for many maser instabilities in plasmas is of order the plasma or cyclotron frequency depending on the nature of the instability (the latter is \( \mu_w^{1/2} \) times the former; \( \mu_w = B^2/(8\pi n_w mc^2) \) is plasma magnetization parameter). The cyclotron frequency in the shocked wind frame is

\[
\nu'_B \approx \frac{qB'_w}{2\pi mc}, \quad (47)
\]

and in the observer frame it is

\[
\nu_B \approx \nu'_B\gamma_w \gamma_w \sim \frac{qL_{w,37}R_{14}^{1/2}}{2\pi mcR_{da} c^{3/2}}. \quad (48)
\]

We can eliminate \( \gamma_w \) using the FRB duration for the shock model, viz.

\[
t_{FRB} \approx \frac{R_c}{2(\gamma_w \gamma_w \mu_w^{1/2})}. \quad (49)
\]

where the factor \( \mu_w \) in the denominator is because the shock front LF w.r.t. the unshocked wind is \( \gamma_w \mu_w^{1/2} \) (the LF of shocked plasma w.r.t. the unshocked wind is \( \gamma_w \)). Thus, we arrive at the following expression for the cyclotron frequency of the shocked wind in the observer frame (which should be of order the FRB frequency for any maser mechanism apart for a possible factor \( \sim \mu_w^{1/2} \):

\[
\nu_B \approx \frac{qL_{w,37}R_{14}^{1/2}}{2\pi mcR_{da} c^{3/2}} \approx (7\times 10^9 \, Hz)\frac{L_{w,37}R_{14}^{1/2}}{R_{da} c^{3/2}}. \quad (50)
\]

This frequency is at the lower end of the radio band at which FRBs are observed. We see from the above equation that there is little room for \( \gamma_w \mu_w^{1/2} \) to be much larger than order unity unless \( L_w \gtrsim 10^{37} \, erg \, s^{-1} \).

The dipole wind luminosity for a 30 year old magnetar with surface field strength of \( 10^{13} \, G \) (e.g. Goldreich & Julian, 1969)

\[
L_w \sim 10^{37} \, erg \, s^{-1} B_{N5,15}^{-2} t_{9}^{-2}, \quad (51)
\]

as long as \( t \) is larger than the spin-down time \( t_{sd} \), which is given by

\[
t_{sd} \approx 500 \, s B_{N5,15}^{-2} t_{9}^{-3}, \quad (52)
\]

where \( P_{-3} \) is the pulsar rotation period in unit of \( 10^{-3} \, s \). The luminosity is roughly constant for \( t \gtrsim t_{sd} \).

The maximum dipole wind luminosity at time \( t \) after the birth of a NS is

\[
L_{w,37}^\text{max} \sim 10^{39} \, erg \, s^{-1} P_{-1}^{-2} t_{9}^{-1}, \quad (53)
\]

which corresponds to \( t_{sd} \sim t \), and the surface magnetic field of

\[
B \sim (7\times 10^{13} \, G) P_{-1}^{-1} t_{9}^{-1/2}. \quad (54)
\]

where \( P_{-1} \) is the pulsar rotation period in unit of 0.1 s at time \( t \).

It is entirely possible that the pulsar magnetic field, especially for a young system, is highly non-dipolar. Moreover, the magnetar wind might not be rotationally powered, but instead launched by magnetic field dissipation. For these cases, the wind luminosity provided by equations 51–53 does not apply, and for that reason we consider \( L_w \) as high as \( 10^{40} \, erg \, s^{-1} \) in all of our numerical calculations presented in Fig. 3.

The electron density associated with the magnetar \( e^- \) wind at \( R_s \) is \( n_w \sim \frac{L_w}{(4\pi R_s^2 mc^3 \gamma_w \mu_w)} \sim 3\times 10^{10} \, cm^{-3}L_{w,37}R_{14}^{1/2} (\gamma_w \mu_w)^{-1} \). The density marginally exceeds the upper limit given in equation (37) – to avoid the medium upstream of the shock-front to become opaque to induced Compton scatterings – \( \gamma_w \mu_w \gtrsim 1 \).

The LF of the shock at \( R_s \gtrsim 10^{13} \, cm \) should be \( \gtrsim 10^{-7} \, \gamma_w^{-1} \mu_w^{1/2} \) in order to produce a ms duration burst (eq 49). This requires the luminosity of the relativistic jet, obtained from equation (41), to be

\[
L_j \sim L_{w,37} \gamma_w^{1/2} \mu_w^{1/2} R_{da} \gamma_w^{4} \text{erg s}^{-1} \quad \text{in shock-wind interactions.}
\]

The allowed parameter space for the maser-in-shocks model of FRB is shown in Figure 3. The parameters are for a FRB with observed luminosity of \( 2 \times 10^{41} \, erg \, s^{-1} \) (top three panels), which corresponds to the low end of the luminosity of non-repeaters (e.g. Luo et al. 2018; Ravi 2019). And the lower three panels of Figure 3 show results for radio luminosity of \( 3 \times 10^{42} \, erg \, s^{-1} \) which is the medium luminosity for bursts of the repeater FRB 121102 (e.g. Michilli et al. 2018; Hessels et al. 2019). The observed luminosity is used for the calculation of particle acceleration upstream of the shock front, and for no other aspect of the maser mechanism. As the left top and bottom panels of the figure show, the energy requirement for maser-in-shock model grows very rapidly with increasing \( t_{FRB} \). For bursts of duration longer than \( \sim 2 \, ms \), \( L_j \gtrsim 10^{40} \, erg \, s^{-1} \). The energy requirement is reduced if the FRB frequency is much larger than the cyclotron frequency \( \nu_B \) considered in these calculations.

There are no solutions when the relativistic jet luminosity
by CHIME – Amiri et al. 2019a, 2019b). The parameter search is cut off at duration $t$, information is used only for the calculation of upstream particle acceleration and nothing else. We show in this figure the parameter space which yields the burst repeater FRB 121102 (e.g. Michilli et al. 2018; Hessels et al. 2019). We do not impose a prior on this luminosity as it is dependent on the details of the maser process

$\sim$ FRBs” is the IC process, we have taken the observed, isotropic, FRB luminosity to be either $2 \times 10^{43}$ erg s$^{-1}$ (e.g. Luo et al. 2018; Ravi 2019) – or $3 \times 10^{42}$ erg s$^{-1}$ (bottom three panels), which is roughly the median luminosity for the bursts of the repeater FRB 121102 (e.g. Michilli et al. 2018; Hessels et al. 2019). We do not impose a prior on this luminosity as it is dependent on the details of the maser process whereas the focus of this paper is to provide general constraints that should apply to all maser mechanisms operating in shocks. We emphasize that the luminosity information is used only for the calculation of upstream particle acceleration and nothing else. We show in this figure the parameter space which yields the burst duration $t_{FRB} > 0.4$ ms, and the peak frequency $\nu_{\text{p}}$ (given by eq. 50) greater than 0.4 GHz (which is the lowest frequency at which these bursts have been detected by CHIME – Amiri et al. 2019a, 2019b). The parameter search is cut off at $L_J > 10^{50}$ erg s$^{-1}$ and $L_w > 5 \times 10^{35}$ erg s$^{-1}$. The luminosity for the biggest magnetar flare we have ever observed (SGR 180620) was about $2 \times 10^{47}$ erg s$^{-1}$ in $\gamma$-rays (Hurley et al. 2005; Palmer et al. 2005), and the energy was smaller in relativistic outflows by a factor $\sim 10^2$ (Gelfand et al. 2005, Granot et al. 2006). The upper limit on $L_J$ of $10^{50}$ erg s$^{-1}$ we have set in our numerical calculations is a factor $\sim 10^3$ larger than the luminosity of mildly-relativistic outflows for SGR 180620. The observed FRB frequency is taken to be the cyclotron frequency $\nu_B$. What we find is that the maser-in-shock model for FRB radiation requires the isotropic luminosity of the relativistic jet responsible for the FRB radiation to be $\gtrsim 10^{50}$ erg s$^{-1}$ (top and bottom left and middle panels), and that means that the total outburst energy of the repeater FRB 121102 in just one year exceeds the magnetic field energy of a magnetar with surface field strength of $\sim 10^{15}$ G; we should point out that the high value of $L_J$ is dictated entirely by the requirement that the emission is produced at a sufficiently large radius $R_s$ and shock LF $\gamma_w$ (eq. 41), so as to avoid excessive particle acceleration upstream of the shock due to IC, and at the same time produce burst duration $\gtrsim 0.4$ ms; no constraint is placed as to how the maser mechanism operates and its efficiency. The top and bottom left panels show that the jet energy increases very rapidly with increasing burst duration. The top and bottom right panels show that the observed radiation is produced at a radius that is a few times larger than the deceleration radius of the jet, and the wind LF is between \~\,1 and 4 for reasons that are explained in §4.1.1. The upstream induced-Compton optical depth is included in these calculations, but it turns out to be a substantially weaker constraint than IC acceleration. The last point to note is that the maser-in-shocks model has no solutions for FRB luminosity $\gtrsim 10^{44}$ erg s$^{-1}$; there are solutions if we allow $L_J$ to be larger than $\sim 10^{51}$ erg s$^{-1}$, but that poses problems for the total energetics.

$L_J \lesssim 10^{49}$ erg s$^{-1}$ (top left and middle panels of Fig. 3). And that is a serious problem for the maser-in-shock model for FRBs. The well studied repeater FRB 121102 has been observed for about 10 years, and $\gtrsim 10^2$ outbursts have been detected during the small observing time invested to following this object. The object has had numerous outbursts with radio luminosity is GHz band of $\gtrsim 10^{43}$ erg s$^{-1}$. Each of these bursts require, according to the maser model, $L_J \gtrsim 10^{49}$ erg s$^{-1}$, and therefore, the total energy needed for outbursts in a year is at least $10^{48}$ erg if the efficiency for converting the magnetic energy to relativistic outflows is 100%. We know empirically that giant mag-

Figure 3. We show results of numerical calculations for FRB emission according to maser mechanisms operating in shocks that result from a relativistic jet of luminosity $L_J$ colliding with a cold wind of luminosity $L_w$, both of which are produced by the same compact object. Each panel shows a pair of parameters that survive the constraint imposed by particle acceleration upstream of the shock front that shuts off the generation of coherent radiation. Constraints are placed on the particle acceleration time due to induced-Compton (IC) scatterings $\tau_{\text{acc}}(R_s) > t_{\text{acc}}/10$; where $t_{\text{acc}}$, is burst duration in observer frame which is calculated using eq. 49, and $t_{\text{acc}}(R_s)$, given by eq. 39, is particle acceleration time due to IC at the shock radius $R_s$ (we note that our numerical calculation of $t_{\text{acc}}$ includes nonlinear effects when the wave nonlinear parameter $a > 1$ whereas eq. 39 is valid only for $a \ll 1$). For the purpose of calculating upstream particle accelerations by the IC process, we have taken the observed, isotropic, FRB luminosity to be either $2 \times 10^{43}$ erg s$^{-1}$ (the top three panels) – the median luminosity of “non-repeating FRBs” is $\sim 10^{44}$ erg s$^{-1}$ (e.g. Luo et al. 2018; Ravi 2019) – or $3 \times 10^{42}$ erg s$^{-1}$ (bottom three panels), which is roughly the median luminosity for the bursts of the repeater FRB 121102 (e.g. Michilli et al. 2018; Hessels et al. 2019). We do not impose a prior on this luminosity as it is dependent on the details of the maser process whereas the focus of this paper is to provide general constraints that should apply to all maser mechanisms operating in shocks. We emphasize that the luminosity information is used only for the calculation of upstream particle acceleration and nothing else. We show in this figure the parameter space which yields the burst duration $t_{\text{acc}} > 0.4$ ms, and the peak frequency $\nu_{\text{p}}$ (given by eq. 50) greater than 0.4 GHz (which is the lowest frequency at which these bursts have been detected by CHIME – Amiri et al. 2019a, 2019b). The parameter search is cut off at $L_J > 10^{50}$ erg s$^{-1}$ and $L_w > 5 \times 10^{35}$ erg s$^{-1}$. The luminosity for the biggest magnetar flare we have ever observed (SGR 180620) was about $2 \times 10^{47}$ erg s$^{-1}$ in $\gamma$-rays (Hurley et al. 2005; Palmer et al. 2005), and the energy was smaller in relativistic outflows by a factor $\sim 10^2$ (Gelfand et al. 2005, Granot et al. 2006). The upper limit on $L_J$ of $10^{50}$ erg s$^{-1}$ we have set in our numerical calculations is a factor $\sim 10^3$ larger than the luminosity of mildly-relativistic outflows for SGR 180620. The observed FRB frequency is taken to be the cyclotron frequency $\nu_B$. What we find is that the maser-in-shock model for FRB radiation requires the isotropic luminosity of the relativistic jet responsible for the FRB radiation to be $\gtrsim 10^{50}$ erg s$^{-1}$ (top and bottom left and middle panels), and that means that the total outburst energy of the repeater FRB 121102 in just one year exceeds the magnetic field energy of a magnetar with surface field strength of $\sim 10^{15}$ G; we should point out that the high value of $L_J$ is dictated entirely by the requirement that the emission is produced at a sufficiently large radius $R_s$ and shock LF $\gamma_w$ (eq. 41), so as to avoid excessive particle acceleration upstream of the shock front due to IC, and at the same time produce burst duration $\gtrsim 0.4$ ms; no constraint is placed as to how the maser mechanism operates and its efficiency. The top and bottom left panels show that the jet energy increases very rapidly with increasing burst duration. The top and bottom right panels show that the observed radiation is produced at a radius that is a few times larger than the deceleration radius of the jet, and the wind LF is between \~\,1 and 4 for reasons that are explained in §4.1.1. The upstream induced-Compton optical depth is included in these calculations, but it turns out to be a substantially weaker constraint than IC acceleration. The last point to note is that the maser-in-shocks model has no solutions for FRB luminosity $\gtrsim 10^{44}$ erg s$^{-1}$; there are solutions if we allow $L_J$ to be larger than $\sim 10^{51}$ erg s$^{-1}$, but that poses problems for the total energetics.

$L_J \lesssim 10^{49}$ erg s$^{-1}$ (top left and middle panels of Fig. 3). And that is a serious problem for the maser-in-shock model for FRBs. The well studied repeater FRB 121102 has been observed for about 10 years, and $\gtrsim 10^2$ outbursts have been detected during the small observing time invested to following this object. The object has had numerous outbursts with radio luminosity is GHz band of $\gtrsim 10^{43}$ erg s$^{-1}$. Each of these bursts require, according to the maser model, $L_J \gtrsim 10^{49}$ erg s$^{-1}$, and therefore, the total energy needed for outbursts in a year is at least $10^{48}$ erg if the efficiency for converting the magnetic energy to relativistic outflows is 100%. We know empirically that giant mag-
near outbursts convert less than a few percent of magnetic energy to mildly-relativistic outflows\textsuperscript{5}, and perhaps a much smaller fraction to ultra-relativistic jet that is invoked by maser-in-shocks models for FRB radio emission. This greatly exacerbates the energy problem – the energy requirement for maser-in-shocks model to support the activities of the FRB repeater for one year is \(\gtrsim 10^{49}\text{erg}\) and that is hard for magnetic fields, even those as large as \(10^{16}\text{G}\), to provide. We note that the value of \(L_j\) in our calculations is dictated solely by the shock LF \(\gamma_{\text{sh}}\) and \(R_j\) such that the observed duration of bursts comes out to be of order a few ms, and not by any efficiency considerations.

The maser model yields no solution for FRB luminosity \(\gtrsim 10^{46}\text{erg s}^{-1}\) when the acceleration of upstream particles by IC scatterings shuts off the maser mechanism in less than 1 ms unless we consider the luminosity of the relativistic jet \(L_j \sim 10^{47}\text{erg s}^{-1}\); \(L_j\) scales linearly with the FRB radio luminosity. We note that the analysis of ASKAP sample of bursts shows that there are FRBs with luminosity as high as \(\sim 10^{46}\text{erg s}^{-1}\) (Lu & Piro, 2019).

The main result of this sub-section is that it is highly unlikely that the FRB radiation is produced at a distance much larger than a few hundred neutron star radii. Particles in the region \(10^6 \lesssim R_s \lesssim 10^{15}\text{cm}\) suffer very strong radiative acceleration which disrupts the photon generation process. The energy requirement for maser-in-shocks models operating at \(R_s \gtrsim 10^{15}\text{cm}\) is very challenging \((\gtrsim 10^{48}\text{erg} \text{ in ultra-relativistic outflows in one year for repeaters such as FRB 121102})\). For \(R \lesssim 10^5\text{cm}\), the strong magnetic field of a magnetar suppresses the induced-Compton scatterings and acceleration of particles by the electric field of FRB coherent radiation (discussed in the next subsection). Therefore, the generation of coherent photons can proceed unimpeded close to the neutron star.

Although, almost all the discussions in this section have explicitly considered the scenario where FRB radiation is generated in shocks, the same physical considerations — acceleration of particles to high LF by the emergent radiation in the vicinity of the source region — apply to any other model for FRBs such as plasma maser type mechanism that operate far away from the neutron star surface and outside the light cylinder.

### 4.2 FRB radiation generation and propagation in a region of strong magnetic field

Photon-electron scattering cross-section is significantly modified in the presence of strong magnetic fields when the cyclotron frequency \(\nu_c \gtrsim \nu\). The cross-section for an X-mode photon\textsuperscript{7} is (e.g. Canuto et al. 1971)

\[
\sigma_x = \frac{\sigma_T}{2} \left( \frac{\nu^2}{(\nu + \nu_B)^2} + \frac{\nu^2}{(\nu - \nu_B)^2} \right),
\]

\[\text{and the cross-section when the wave electric field is not perpendicular to the static magnetic field is:}\]

\[
\sigma_x = \sigma_T \left[ \sin^2 \theta_{kB} + \frac{\cos^2 \theta_{kB}}{2} \left( \frac{\nu^2}{(\nu + \nu_B)^2} + \frac{\nu^2}{(\nu - \nu_B)^2} \right) \right],
\]

\[\text{where the cyclotron frequency}\]

\[
\nu_B = \frac{qB}{2\pi mc},
\]

\[\text{and}\]

\[
\sin \theta_{kB} \approx \theta_{kB}\]

\[\text{is the dot product of unit vectors along the static magnetic field and wave electric field as measured in the electron rest frame, and} \nu \text{ is EM wave frequency also in the electron rest frame. These formulae for the cross-section apply only when the EM wave nonlinearity parameter} a_j \equiv qE_j/(mc\omega) \text{ (see eq. 2) is much less than 1;} E_j \text{ is the component of wave electric vector amplitude along the static magnetic field. For the X-mode we are considering here, the angle between the wave electric field and the magnetic field is very close to} \pi/2 \text{ (eq. 60), and} a_j \ll 1; \text{ this point is further addressed below (60).}\]

We see from equation (39) that for \(R \lesssim 10^{13}\text{cm}\) the timescale for particle acceleration due to induced-Compton scatterings are extremely short, provided that the cross-section for scattering a photon by an electron into an unoccupied state is not drastically smaller than \(\sigma_T\). However, as we see from equation (55), \(\sigma_x\) is smaller than \(\sigma_T\) by a factor \((\nu_B/\nu)^2\) in the region of high magnetic field. Attenuation of O-modes\textsuperscript{8} by interactions with electrons in the medium in the vicinity of the source is also suppressed by a factor \(\sim \theta_{kB}/2\); modes with \(\theta_{kB} \ll 1\) are unlikely to be able to escape the immediate vicinity of the source region intact.

Consider an X-mode generated at radius \(R_s\) (measured from the center of the neutron star with strong magnetic field) such that \(R_s \lesssim 10^5\text{cm}\). It was shown by Lu et al. (2019) that the electric-vector of an X-mode traveling through a medium with non-uniform magnetic field rotates in such a way as to keep the wave-electric field nearly perpendicular to the local magnetic field and the wave-vector as long as the plasma density is sufficiently large; the index of refraction of the medium for this mode is very close to unity, so the wave-vector does not rotate. The RMS angle \(\langle E_w \times B \rangle \) as the wave travels away from the source region is given by (Lu et al., 2019)

\[
\theta_{kB} \approx \frac{2\pi\omega \nu}{RB_\omega^2},
\]

\[\text{where} \omega^2 = 4\pi q^2 n/m \text{ is the plasma frequency,} \nu \text{ is the electron density and} R_B \text{ is the radius of curvature of magnetic field lines at the current location of the wave at} R; R_B \sim R/\Omega_\text{pol} \text{ at polar coordinate} (R, \theta) \text{ wrt the magnetic axis. Let us take the} \varepsilon^2 \text{ density at} R = R_0 \text{ times the Goldreich-Julian density (Goldreich & Julian, 1968)}\]

\[
\frac{n}{\Omega_\text{ps}} = \frac{M B \cdot \Omega_\text{ps}}{2\pi q c} \approx \frac{MB_n \Omega_\text{ps}}{2\pi q c} \left( \frac{R_n}{R} \right)^3
\]

\[
\approx 10^{13} \text{cm}^{-3} M B_{n,15} \Omega_\text{ps} \left( \frac{R_n}{R} \right)^3,
\]

\[\text{where} \Omega_{ps} \text{ is the angular velocity of the NS.}\]

Substituting this into (58) we find

\[
\theta_{kB} \sim 6 \times 10^{-11} M^{-1} \nu_B \Omega_{ps} \left( \frac{R_n}{R} \right)^3,
\]

\[\text{inside the freeze-out radius given by equation (61).} \text{ The wave non-linearity parameter along the magnetic field} a_j \equiv qE_j/(mc\omega) =\]

\[\text{An O-mode is a linearly polarized EM wave with the direction of the wave electric field vector in the plane of static magnetic field and the wave-vector.}\]
The rotation of wave electric field ceases at a radius, called the freeze-out radius, where the plasma density becomes too small to be able to provide the current needed to rotate the wave electric vector. This occurs at a radius where (Lu et al., 2019),

\[ \frac{\omega_B^2}{\omega^2} \approx \frac{a_0}{R_{\text{B}}} \implies \frac{R_{\text{fo}}}{R_{\text{n.s.}}} \approx 2 \left( \frac{MB_{n.s.,15}c}{\Omega_{n.s.,6}} \right)^{1/2} \left( \frac{R_{\text{f.o.}}}{R_{\text{n.s.}}} \right)^{1/2}. \]

\( R_{\text{B}} \) in this equation is calculated at \( R_{\text{fo}} \). The FRB radiation might be produced along open magnetic field lines in the polar cap region which has an angular size of \( \theta_{pc} = \left[ \frac{\Omega_{n.s.}R_{n.s.}}{c} \right]^{1/2} \approx 5.8 \times 10^{-3} \left( R_{n.s.,10} \Omega_{n.s.} \right)^{1/2} \) rad. Taking \( \theta \approx \theta_{pc} \) and substituting \( R_{\text{B}} \approx R_{\text{fo}}/\theta \) in equation (61) leads to

\[ \frac{R_{\text{fo}}}{R_{\text{n.s.}}} \approx 400 \frac{MB_{n.s.,15}c}{\Omega_{n.s.,6}} \left( \frac{R_{\text{f.o.}}}{R_{\text{n.s.}}} \right)^{1/2}. \]

The angle \( \theta_{B} \) at the freeze-out radius is:

\[ \theta_{B}(R_{\text{fo}}) \approx 10^{-5} MB_{n.s.,15} R_{43}^{1/2} \Omega_{n.s.,6}^{1/2} R_{3.0}^{-3/2} \nu_0. \]

Making use of equation (60) we find that the photon-electron scattering cross-section, in the strong magnetic field regime (\( \omega_B \gg \omega \)), when the wave electric field is not exactly perpendicular to the large scale magnetic field is

\[ \sigma_\parallel \sim \sigma_\perp \frac{q^2}{\omega^2} \approx 2 \times 10^{-45} \text{cm}^2 \frac{\nu_0}{M^2 B_{n.s.,15}^2 R_{10}^{1/2} R_{n.s.,6}^{-3/2}}, \]

as long as \( \theta_{B} > \nu/\nu_0 \), which is the only case we are considering here.

The induced-Compton scattering optical depth in this case is

\[ \tau_{\text{ic}} \approx \frac{3\sigma_\perp \nu L_0^2}{4\pi^2 R_3^3\nu_0}, \]

where \( \theta_s = \min \{ \ell_s/R, \gamma^{-1} \} \) is the angular size of the photon beam at \( R \), \( \ell_s \) is the transverse size of the FRB source from which photons are received at \( R \), and \( \gamma \) is the LF of the FRB source. Making use of equations (59) & (64) we find the optical depth to IC for \( R \approx R_{\text{fo}} \)

\[ \tau_{\text{ic}} \approx 10^{-7} \frac{L_{\perp,4}^2}{\nu_0 B_{n.s.,15}^2 M_3 R_{n.s.,10} \Omega_{n.s.,1}}. \]

The IC optical depth is less than unity at all radii in the NS magnetosphere at least out to the light cylinder.

Next, we look into particle acceleration and energy loss in the source region of the FRB pulse. In a strong magnetic field region, particle acceleration time for \( e^\pm \) plasma due to IC scattering is a slightly modified form of equation (39)

\[ t_{\text{acc}} \sim \frac{256\pi^3 R^4 v^3 m^2 c^2}{3\sigma_\parallel \theta_0^2 L^2}. \]

Using equation (64) leads to the following expression for the acceleration time

\[ t_{\text{acc}} \sim \left( 10^{-5} \right)^4 \frac{\nu_0 (B_{n.s.,15} \Omega_{n.s.,6} M_3 R_{n.s.,6})^2}{L_4^{1/4} R_{10}^2} \text{ for } R \lesssim R_{\text{fo}}. \]

According to the coherent curvature model of FRBs (Kumar et al. 2017; Lu & Kumar 2018), \( M \gtrsim 10^3 \) and \( \ell_s \approx 10^4 \text{cm} \), and so the IC acceleration time within the source region of FRBs is \( \sim 10^{-5} \) s. This is longer than \( \ell_s/c \), the residency time of \( e^\pm \) in the source region, and hence the induced-Compton cannot adversely affect the generation of FRB coherent radiation. In fact, our estimate of \( t_{\text{acc}} \) inside the source region is probably too small. This is because the electric field of the radiation is exactly perpendicular to the static magnetic field (X-mode polarization) within the source and \( e^\pm \)s are stuck in the lowest Landau level and have weaker than classically expected interaction with X-mode photons. It is also the case that the coherent curvature radiation requires a strong electric field along the static magnetic field. Particle acceleration time due to this electric field is of order \( 10^{-15} \) s (Kumar et al. 2017), and the electric force on \( e^\pm \) far exceeds any non-zero IC scattering force.

Particles outside the source region, at larger radii, are accelerated to LF \( \sim \theta^{-1} \approx R/\ell_s \approx 10^3 R_{10} / \ell_{10}^4 \) when \( t_{\text{acc}} \ll t_{\text{FRB}} \); the FRB radiation becomes roughly isotropic in the particle rest frame when the LF approaches this value, and IC scattering force drops to zero. The total number of \( e^\pm \) in the magnetosphere of a magnetar of spin period 1 s is \( \sim 4\pi M R_{53}^3 \approx 3 \times 10^{30} M_3 R_{n.s.,15} \). Therefore, the energy lost by the FRB radiation as it travels through the NS magnetosphere is \( \leq 2 \times 10^{33} M_3 \) erg (isotopic equivalent). This energy loss is a tiny fraction of the total energy of the FRB coherent radiation.

The nonlinearity parameter for the FRB coherent radiation (described in §2) is \( \lesssim 10^5 \) at \( R = 10^4 \text{cm} \). In the absence of the magnetic field of the magnetar, \( e^\pm \) exposed to this radiation would be accelerated to LF \( \sim 10^{10} \). However, the strong magnetic field of a magnetar suppresses particle acceleration drastically, and the LF \( e^\pm \)s attain at \( R = 10^4 \text{cm} \) is close to unity. Therefore, particle acceleration by the electric field of FRB radiation does not change the conclusion we arrived at regarding the loss of FRB energy as photons travel through the NS magnetosphere to arrive at Earth, i.e., the loss of energy is negligible.

The main result of this subsection is that a FRB source located within a few 10s of neutron star radii of a magnetar can withstand the enormous radiation forces. Moreover, little energy is lost as the radiation travels though the NS magnetosphere. The reason for this is entirely due to the strong magnetic field of a magnetar which suppresses scattering cross-section and the efficiency of particle acceleration, i.e., in the absence of a strong magnetic field a large fraction of FRB radiation energy would be imparted to particles along its path as the radiation travels away from the source.

5 CONCLUSIONS

We have investigated the effects of an intense FRB coherent radiation on plasma around the region where the radiation is produced. The pur-

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8 One might worry that electrons and positrons accelerated to high LF could trigger a pair production avalanche, which can sap the energy from FRB radiation. However, \( e^\pm \) accelerated by FRB radiation are moving away from the neutron star, and thus see the NS surface emission highly red-shifted and not capable of pair production. Moreover, the photon density associated with pulsar nebula emission is \( \lesssim 10 \text{ cm}^{-3} \), which is too small for launching pair cascade.

9 The angle between the electric field of radiation and the local magnetic field direction minus \( \pi/2 \), defined to be \( \theta_{B,B} \), is given by eq. 60. Thus, the wave nonlinearity parameter along the magnetic field \( a_{\perp} \equiv qE_{\beta}/(mc^2) \ll 1 \) for \( R \lesssim 10^6 \text{cm} \). And, therefore, \( e^\pm \) are accelerated along the magnetic field by the electric field of the FRB radiation to speeds much smaller than \( c \) throughout most of the NS magnetosphere. The acceleration of particles perpendicular to the static magnetic field when the cyclotron frequency \( (\omega_B) \) is much larger than wave frequency \( \omega \) is determined by a modified non-linearity parameter \( a_{\perp} \equiv qE_{\beta}/(mc^2) \), which is less than 1 for \( R \lesssim 10^6 \text{cm} \). Therefore, the particle speed perpendicular to the magnetic field is also sub-relativistic even quite far from the NS surface; at \( R = 10^6 \text{cm} \), \( a_{\perp} \approx 10^{-2} \) and \( c/\omega \sim 10^2 \).

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pose is to constrain source properties and the radiation mechanism for FRBs by using some fairly general physics considerations, and determine conditions that a successful model should satisfy. In particular, we calculate particle speed due to electric field of the radiation and the highly enhanced scattering (the induced-Compton scattering) by electrons and positrons of the coherent FRB radiation where each quantum will be heated to the temperature of $\sim 10^{16}$K. We find that electrons and protons are accelerated to very high LF due to these forces. This severely restricts some, otherwise, promising models for FRBs. One such class of models invokes relativistic shocks and maser type instability operating in the shock transition zone or downstream of the shock front.

We have shown that the coherent radiation traveling upstream of the shock front stirs up the plasma violently – $e^\pm$ have LF greater than 30 due to the strong electric field of the radiation – even when the shock front is at a distance of $10^{13}$cm from the FRB compact progenitor star (see §2). Furthermore, induced-Compton scatterings push the upstream plasma away with a large LF as long as the shock front radius is less than $\sim 10^{13}$cm, thereby preventing particles from approaching and crossing the shock front to keep the generation of GHz radiation going.

At larger distances ($\gtrsim 10^{13}$cm) these forces are fairly tame. However, maser-in-shock models require an excessively large amount of energy ($\gtrsim 10^{46}$ erg) in ultra relativistic outflows to produce a burst at a few GHz frequency with luminosity $\gtrsim 10^{45}$ erg s$^{-1}$ at these large distances (see §4.1.1); as a point of reference, the total energy in relativistic outflows in the biggest magnetar flare ever observed (SGR 180620) was estimated to be $\sim 10^{44}$ erg from late time radio observations (Gelfand et al. 2005; Granot et al. 2006). The total energy in relativistic jets for bursts produced by the repeater FRB 121102 in one year, for maser models, is required to be substantially larger than $10^{46}$ erg. This exceeds the energy in magnetic fields of a NS with strength $10^{15}$G, and that is assuming an efficiency of 100% for converting magnetic energy to relativistic jets; the actual efficiency is perhaps no larger than 1% (§4.1.1).

If FRBs were to be associated with SGRs then that has consequences for the maser-in-shock models. The $\gamma$-ray photons from the SGR will be scattered by electrons upstream of the shock front heating them up. Whether the maser instability can survive this interaction is unclear. Let us consider that the SGR associated with a FRB released $10^{47}$ erg in $\gamma$-rays, and the $\gamma$-ray luminosity was $10^{46}$erg s$^{-1}$; the duration of the $\gamma$-ray pulse for SGR 180620 was about 0.1 s. The $\gamma$-ray pulse is ahead of the shock front, when it is at radius $R$, by the distance $\delta R \approx R/(2\gamma^2 w^2) \sim c_{\text{FRB}}$. Upstream electrons will be heated to the temperature of $\gamma$-rays on a time scale of $t_{\text{Larmor}} \sim (1 \text{ms})R^2_{13}/L_{\gamma,48}$. Considering this short time scale, and small Larmor radius ($\lesssim 5\times 10^3$cm), the upstream particles would have anisotropic velocity distribution as they enter the shock front, and that could affect the instability.

Stimulated Raman and Brillouin scatterings can also attenuate the FRB radiation. The stimulated Raman process has been calculated by many people e.g. Gangadhara & Krishan (1992), Thompson et al. (1994), Levinson & Blandford (1995). Lyubarsky & Ostrovskva (2016) considered Raman scatterings in the context of stellar coronae model for FRBs and concluded that it does not provide more severe constraint on the propagation of FRB radiation than the induced-Compton scatterings. However, implications of stimulated Raman and Brillouin scatterings for the more recent models for FRBs needs to be investigated.

We have shown in §4.2 that the FRB radiation source operating is a region of very strong magnetic field (where the cyclotron frequency is much larger than the FRB radio wave frequency) alleviates these problems; the radiation forces are weaker in the presence of strong magnetic field, and the plasma in the FRB source region is not dispersed quickly. One of the reasons for this is that the interaction cross-section between $e^\pm$ and X-mode photons is highly reduced in the presence of a strong magnetic field. The coherent curvature (“antenna”) model for FRBs requires very strong magnetic field for its successful operation (Kumar et al. 2017; Lu & Kumar 2018). It is shown in §4.2 that the source survives the radiative forces, and radiation traveling through the NS magnetosphere suffers negligible loss of energy.

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