An Alternative Scenario for Non-Abelian Quantum Hair

Amitabha Lahiri†

Theoretical Division T-8
Los Alamos National Laboratory
Los Alamos, NM 87545

Abstract

Topologically charged black holes in a theory with a 2-form coupled to a non-abelian gauge field are investigated. It is found that the classification of the ground states is similar to that in the theory of non-abelian discrete quantum hair.
1. Introduction

Until recently it was thought that the powerful classical ‘no-hair’ theorems [1] always forced an isolated black hole to radiate away almost all of its internal information, retaining only a handful of states characterized by its mass $M$, angular momentum $J$ and charges associated with massless gauge fields. However, since a black hole may be expected to form by absorbing matter originally in a pure quantum state and to decay by predominantly thermal radiation, it would seem that information is irretrievably lost in the process of formation and subsequent decay of a black hole. Unless, that is, the black hole can support a large number of ‘hair’ or internal degrees of freedom that correlate the radiated states with the information that went into its formation. This contradiction has been a source of consternation to all who believe that quantum coherence is preserved in all processes. In addition, a stable black hole may be shown to have a very large entropy, proportional to the area of its event horizon. This also suggests that a black hole should be able to access a very large number of internal degrees of freedom. In recent years, new hopes of resolving this conflict have been aroused by the emergence of the idea of ‘quantum hair’ [2]. That a black hole can carry topological charge and therefore global degrees of freedom not fully described by massless gauge fields was first postulated in [3]. Even though most of the analysis was classical, the only apparent way of detecting the charge was via an Aharonov-Bohm experiment that required a closed fundamental string whose world sheet fully enclosed the horizon of the black hole. The topological charge, corresponding to the integral of a 2-form field $B$ over a closed 2-surface around the black hole, introduced a phase factor in the wave-function of the string and its effect was therefore quantum in nature. Another type of hair was discovered soon afterwards [4], where the quantum nature of the hair, arising from the spontaneous breaking of a local continuous symmetry to a local discrete symmetry, was more evident. The ‘hair’ in this case is the charge associated with the local discrete group – the part of the continuous symmetry charge that is not screened by spontaneous symmetry breaking. An understanding of a possible relation between the two types of hair was non-existent until it was shown [5] that if the $B$ field is coupled to a $U(1)$ gauge field $A$ via an $mB \wedge F$ term (where $F = \text{d}A$), the resulting theory is a ‘dual’ description of the Goldstone mechanism. Since the ‘screening’ of charge through symmetry breaking is a crucial ingredient in the theory of discrete quantum hair, it seems plausible [2] that the two theories are actually different descriptions of the same physical phenomenon. However, a rigorous proof of a one-to-one correspondence between the states of the two theories has not yet been found.
In view of this, one is tempted to look for a non-abelian generalization of the $B$ field where the corresponding topological charge may relate to the discrete charge one obtains by breaking an $SU(N)$ group to a local discrete symmetry group. Such a generalization will be considered in what follows, and we will find that the topological charges on black holes in such a theory are indeed very interesting.

2. The Non-Abelian 2-Form

We begin by briefly summarizing some features of the abelian $B$ field. The $B$ field is a 2-form potential, i.e., an antisymmetric tensor field of rank 2 with ‘field strength’ $H$ defined by $H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]}$. (We assume that the space-time connection is torsion-free, so $\nabla_{\mu}$ can be replaced by $\partial_{\mu}$ in antisymmetric derivatives, as in the definition of $H$.) One insists that an action involving $B$ be invariant under a Kalb-Ramond symmetry transformation, $B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_{[\mu} \Lambda_{\nu]}$. When coupled to a gauge field $A$ that has an associated abelian gauge symmetry under $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$, the action that is formally invariant under both symmetry transformations is given by

$$ S = \int \left( \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m}{2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} B_{\rho\lambda} \right) \right). \quad (2.1) $$

The equations of motion following from this action are

$$ \nabla_{\nu} F^{\nu\mu} + m \epsilon^{\mu\nu\rho\lambda} H_{\nu\rho\lambda} = 0, $$
$$ \nabla_{\rho} H^{\mu\nu\rho} + m \epsilon^{\mu\nu\rho\lambda} F_{\rho\lambda} = 0. \quad (2.2) $$

These are the well known London equations of superconductivity, and it can be shown that for appropriate gauge choices these equations would lead to massive equations for either $A$ or $B$ [5]-[7]. In the context of black holes one can show that these equations force both $F$ and $H$ to vanish outside the horizon of a static, spherically symmetric black hole. It then follows that the vacuum Einstein equations hold outside the horizon, and black hole uniqueness theorems can be used to prove that the space-time metric is Schwarzschild, while the black hole may carry a topological charge,

$$ B = q \omega \equiv \frac{q}{4\pi} \sin \theta d\theta \wedge d\phi. \quad (2.3) $$

1 Here and later on, the presence of the Einstein-Hilbert term and the Einstein equations are understood.

2 The usual convention is to write the last term in (2.1) as $mB \wedge F \equiv \frac{m}{4} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} B_{\rho\lambda}$. Here we try to keep the equations neat and absorb the factor of 2 in $m$. 

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In components $B_{\theta \phi} = \frac{q}{4 \pi r^2}$, and one should note that $B$ can be thought of as a long range field even when it gains a mass via the $mB \wedge F$ term. It would be interesting to see if similar results hold in the presence of a non-abelian gauge field to which a 'non-abelian' $B$ couples, and if we are led to a non-abelian topological charge.

Our starting point is a naive non-abelianization of the action (2.1),

$$S = \int \text{Tr} \left( \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{m}{2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} B_{\rho\lambda} \right) \right),$$  \hspace{1cm} (2.4)

where we assume that $B$ belongs to the adjoint representation of SU(N) with $B_{\mu\nu} \rightarrow UB_{\mu\nu} U^{-1}$ and $A_{\mu} \rightarrow UA_{\mu} U^{-1} - \partial_{\mu} U U^{-1}$ under a gauge transformation with $U \in \text{SU}(N)$. The relevant field strengths are defined as $H_{\mu\nu\rho} = D_{[\mu} B_{\nu\rho]} \equiv \partial_{[\mu} B_{\nu\rho]} + [A_{[\mu}, B_{\nu\rho]}$, and $F_{\mu\nu} = [D_{\mu}, D_{\nu}]$. (It should be noted that the covariant transformation law of $B$ does not come from 'first principles', but is assumed in order to leave this particular action invariant. There are known theories containing an antisymmetric tensor that does not transform covariantly, or even has a well-defined local gauge transformation law \cite{8}.) Obviously, one cannot naively non-abelianize the Kalb-Ramond symmetry with these definitions, because under $B_{\mu\nu} \rightarrow B_{\mu\nu} + D_{[\mu} \Lambda_{\nu]}$ the action is not invariant. The Kalb-Ramond symmetry is responsible in the abelian case for the equivalence of $B$ with a Goldstone boson, and losing this symmetry evidently has serious consequences. (For an action that retains a non-abelian Kalb-Ramond symmetry by introducing auxiliary fields, see \cite{9}.) However, not all is lost. We will examine the question of symmetries later; first we investigate the results that follow from the action (2.4).

The equations of motion following from (2.4) are

$$D_{\nu} F^{\nu\mu} - [B_{\nu\rho}, H^{\mu\nu\rho}] + m \epsilon^{\mu\nu\rho\lambda} H_{\nu\rho\lambda} = 0,$$  \hspace{1cm} (2.5)

and

$$D_{\rho} H^{\mu\nu\rho} + m \epsilon^{\mu\nu\rho\lambda} F_{\rho\lambda} = 0.$$  \hspace{1cm} (2.6)

Operating with $D_{\mu}$ on (2.3) gives

$$-[B_{\nu\rho}, D_{\mu} H^{\mu\nu\rho}] + m \epsilon^{\mu\nu\rho\lambda} [F_{\mu\nu}, B_{\rho\lambda}] = 0,$$  \hspace{1cm} (2.7)

which contains no new information (it is the commutator of (2.6) with $B$), while operating on (2.6) first with $D_{\nu}$ and then with $D_{\mu}$ gives us

$$[F_{\nu\rho}, H^{\mu\nu\rho}] = 0,$$  \hspace{1cm} (2.8)

and

$$[F_{\nu\rho}, D_{\mu} H^{\mu\nu\rho}] = 0.$$
It follows from these equations that the non-abelian \(A-B\) system is highly constrained. In fact, an analysis of the constraints in the system shows that there are a total of two dynamical degrees of freedom left in the theory. A simple physical argument can be used to count the degrees of freedom as follows. One notes that when \(m = 0\), these equations demand that \(H = 0\) for a generic \(F\), and this sector of solutions is the same as standard gauge theory. If the space-time has trivial second cohomology (as is thought to be the case in all known experiments of particle physics) the topological modes of \(B\) can be gauged away\(^3\). As \(m\) is turned on, the term \(mB \wedge F\) induces a mixing between the modes of \(A\) and \(B\). But since this is a topological term in the action (it can be defined without the help of a background metric), it has vanishing contribution to the energy momentum tensor. Therefore it does not contribute to the energy-momentum carried by propagating modes, and also cannot generate an extra mode. This is very different from the abelian case, where setting \(m = 0\) completely decouples the two fields, and the resulting theory has three degrees of freedom to start with. In the non-abelian case the equations (2.8) show that there is no non-zero \(H\) that is independent of \(F\), even when \(m = 0\).

At this point however, we are interested in the physics generated by (2.4) in the context of black hole space-times. Our analysis will follow closely that of [5], which was done for abelian gauge fields. Consider a static space-time, \(i.e.,\) one with a hypersurface-orthogonal timelike Killing vector field \(\xi^\mu\), \(\mathcal{L}_\xi g_{\mu\nu} = 0\), \(\xi_\mu \xi^\mu = -\lambda^2\). Let \(\Sigma\) denote a hypersurface to which \(\xi\) is orthogonal, and \(\Pi^\mu_\mu = \delta^\mu_\mu + \lambda^{-2} \xi_\mu \xi^\mu\) the projection operator that projects into \(\Sigma\). Also let \(\Omega\) be a \(p\)-form on the space-time manifold \(M\) and \(\omega\) its projection on \(\Sigma\) with \(\mathcal{L}_\xi \Omega = 0\). Denoting the induced connection on \(\Sigma\) by \(\tilde{\nabla}\), it can be shown that

\[
\tilde{\nabla}_\alpha (\lambda \omega^{\alpha \mu \cdots \nu}) = \lambda \Pi^\mu_\mu \cdots \Pi^\nu_\nu \nabla_\alpha \Omega^{\alpha \mu \cdots \nu}\cdot \quad (2.9)
\]

Physically this may be understood as saying that the 3-divergence of a form is equal to its 4-divergence when all fields (including the metric) are time-independent. Let \(\tilde{D}_\mu\) denote the \(\Sigma\)-projection of the gauge-covariant derivative operator \(D_\mu\). It follows from (2.9) that

\[
\tilde{D}_\alpha (\lambda \omega^{\alpha \mu \cdots \nu}) = \lambda \Pi^\mu_\mu \cdots \Pi^\nu_\nu D_\alpha \Omega^{\alpha \mu \cdots \nu} + \lambda^{-1} \xi^\alpha \xi_\alpha [A_\alpha, \Omega^{\alpha \mu \cdots \nu}]\cdot \quad (2.10)
\]

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\(^3\) See section 3.

\(^\dagger\) Note that in a gauge \(\xi^\alpha A_\alpha = 0\) the last term vanishes. Such a gauge choice is always possible because \(\xi_\alpha \Pi^{(A)}_\alpha \approx 0\) is a constraint of the theory. However, it is not necessary to choose a gauge at this point.
Let us denote the $\Sigma$-projections of $H$, $F$, $^*H$ and $^*F$ by $h$, $f$, $d$ and $e$, respectively. Then by (2.10) and by the equations of motion, we have

$$\begin{align*}
\tilde{D}_\rho(\lambda h^{\mu\nu\rho}) &= -\lambda me^{\mu\nu} + \lambda^{-1}\xi^\rho \xi_\rho [A_{\rho'}, H^{\mu\nu\rho}], \\
\tilde{D}_\nu(\lambda f^{\nu\mu}) &= \lambda md^\mu + \lambda[B_{\nu\rho}, H^{\mu\nu\rho}]\Pi^\mu_{\mu'} + \lambda^{-1}\xi^\nu \xi_{\nu'} [A_{\nu'}, F^{\nu'\mu}].
\end{align*}$$

(2.11)

We multiply the first of these equations by $e_{\mu\nu}$, take the trace and integrate over the region $V$ between the horizon and the sphere at infinity (we have assumed a spherically symmetric space-time, $M \simeq S^2 \times \mathbb{R}^2$). This gives us

$$\int_V \text{Tr}[e_{\mu\nu}(\tilde{D}_\rho(\lambda h^{\mu\nu\rho}) + \lambda me^{\mu\nu} - \lambda^{-1}\xi^\rho \xi_\rho [A_{\rho'}, H^{\mu\nu\rho}])] = 0. \quad (2.12)$$

Integrating by parts and using assumptions of regularity of the horizon (field strengths are finite when $\lambda = 0$) and asymptotic flatness (field strengths $\to 0$ as $r \to \infty$) and using the $\Sigma$-projections of the equations of motion (2.5) and (2.6) we obtain

$$\int_V \text{Tr}\left(\lambda mh_{\mu\nu\rho}h^{\mu\nu\rho} + \lambda me_{\mu\nu}e^{\mu\nu} + \lambda[B_{\mu\nu}, (^*H)_\rho]h^{\mu\nu\rho} - \lambda^{-1}\xi^\rho \xi_\rho [A_{\rho'}, H^{\mu\nu\rho}]e_{\mu\nu}\right) = 0. \quad (2.13)$$

Similarly, by multiplying the second equation of (2.11) by $d_{\mu}$, taking the trace and going through a similar calculation one finds that

$$\int_V \text{Tr}\left(\lambda mf_{\mu\nu}f^{\mu\nu} + \lambda md_{\mu}d^{\mu} + \lambda[B_{\nu\rho}, H^{\mu\nu\rho}]d_{\mu} + \lambda^{-1}\xi^\nu \xi_{\nu'} d_{\mu}[A_{\nu'}, F^{\nu'\mu}]\right) = 0. \quad (2.14)$$

A little algebra shows that when the two equations (2.13) and (2.14) are added, the last terms cancel each other, and one is left with

$$\int_V \lambda \text{Tr}(h_{\mu\nu\rho}h^{\mu\nu\rho} + e_{\mu\nu}e^{\mu\nu} + f_{\mu\nu}f^{\mu\nu} + d_{\mu}d^{\mu}) = 0. \quad (2.15)$$

Since the metric is positive definite on the hypersurface $\Sigma$, this equation implies that $d$, $e$, $f$, $h$ all vanish. It follows that in fact $F$ and $H$ must vanish outside the horizon of a static, spherically symmetric, asymptotically flat space-time. We will see that this result has very interesting topological consequences for black hole vacua, which we define as a space-time with a black hole with vanishing field strengths outside the horizon.

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4 Here we have chosen a gauge $\xi^\mu B_{\mu\nu} = 0$, which we are always allowed to do as $\xi^\mu \Pi^\mu_{\mu'} \approx 0$ is a constraint of the theory. Note that the solutions in sec. 3 are consistent with this gauge choice.
3. Solutions

One possible solution to the equations $F = 0, H = 0$ is $A = 0, B = \vec{q}\varpi$, where $\varpi$ is again the harmonic form on the sphere, $\varpi = \sin \theta d\theta \wedge d\phi$, while $\vec{q}$ is now any vector in the Lie algebra of SU(N). This can be said to have topological charge $\vec{q} = \int_{S^2} B$. This can be thought of as embedding an abelian group in SU(N), corresponding to choosing a direction. However, the situation is not as trivial as it seems, because this is not the only solution. The vanishing of $F$ implies that at best $A$ is pure gauge, $A_\mu = -\partial_\mu U U^{-1}$. Since $H$ is covariant under SU(N) gauge transformations, it follows that $A_\mu = -\partial_\mu U U^{-1}, B = \vec{q}(x)\varpi$ with $\vec{q}(x) = U\vec{q}U^{-1}$ is also a solution to $F = 0, H = 0$. This solution evidently has a different topological ‘charge’, but can be thought of as an SU(N) gauge transform of our first solution. However, something does remain invariant under the gauge transformation – it is the ‘magnitude’ of the charge, $|\vec{q}(x)|^2 \equiv \text{Tr}(\vec{q}(x)\vec{q}(x))$. As $\vec{q}$ is an $(N^2 - 1)$-dimensional vector when $B$ belongs to the adjoint representation of SU(N), the group of transformations that leave $|\vec{q}(x)|^2$ invariant is O($N^2 - 1$). Suppose we classify the black hole vacua by the gauge-invariant ‘mean-square-charge’ $Q_{\text{rms}}^2 = \int_{S^2} |\vec{q}(x)|^2$. Then for $N > 2$ the symmetry group of the black hole vacua is O($N^2 - 1$), which contains SU(N) as a subgroup. It follows then, that different embeddings of SU(N) in O($N^2 - 1$) will provide gauge-inequivalent (in the sense of SU(N), but connected by O($N^2 - 1$) transformations) black hole vacua, which may be classified by the coset space O($N^2 - 1$)/SU(N). For $N = 2$ this is not the correct description of black hole vacua. In this situation the gauge group SU(2) is locally isomorphic to the symmetry group O(3) of the black hole vacua, so there is no ‘manifold of vacua’. However, one may say that the black hole vacua are now characterized by a winding number corresponding to embeddings of SO(3)$\times$Z$_2$ in SU(2). The vacua with different winding numbers are equivalent under SU(2) gauge transformations.

At this point we make a small digression and note that $A_\mu = 0, B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}$, where $\Lambda$ is a gauge-covariant object (i.e., $\Lambda \to U\Lambda U^{-1}$ under a gauge transformation), is a trivial solution of the $F = 0 = H$ system, but the gauge transformed solution $A_\mu = -\partial_\mu U U^{-1}, B_{\mu\nu}(U) = U\partial_{[\mu} \Lambda_{\nu]}U^{-1}$ is non-trivial in the sense that $\int_{S^2} B(U)$ does not vanish identically. However, if we write $d\Lambda = \vec{f}(\theta, \phi) \sin \theta d\theta \wedge d\phi$ on the sphere for some appropriate set of functions $f^a(\theta, \phi)$ ($a = 1, \cdots, N^2 - 1$) such that

$$\int_{S^2} f^a(\theta, \phi) \sin \theta d\theta \wedge d\phi = 0, \quad (3.1)$$

...
it can be seen that the charge $\vec{q}$ should be modified to $\vec{q} + \vec{f}(\theta, \phi)$, and then the modified mean-square-charge $\tilde{Q}_{\text{rms}}^2 = \int_{S^2} |\vec{q} + \vec{f}(\theta, \phi)|^2$ is invariant under SU(N). The group of invariance of $\tilde{Q}_{\text{rms}}^2$ is again $O(N^2 - 1)$, and our previous analysis of the classification of black hole vacua remains unaffected.

Coming back to the black hole vacuum states, one can compare the results obtained here with similar results obtained for the theory of discrete quantum hair. In the latter, one obtains discrete magnetic hair via the spontaneous breakdown of $SO(N^2 - 1)$ to $SU(N)/Z_N$. One can then show [2] that the black hole vacua in the theory are connected by $SO(N^2 - 1)$ transformations, but inequivalent under $SU(N)/Z_N$ transformations. The coset space $SO(N^2 - 1)/(SU(N)/Z_N)$ then classifies the black hole vacuum states. (We note that for $N=2$ this space gives the winding number of $SO(3)$ embeddings in $SO(3)$, which is in essential agreement with the findings above.)

4. Conclusions

It would seem that we have done no more than ‘almost’ reproducing discrete quantum hair via a rather unconventional route. We have reached a classification scheme for black hole vacua that matches a similar classification for a special class of discrete hair – the discrete magnetic hair\(^5\). So what have we gained by this analysis?

One important thing to note is that there is no scalar field involved in this theory, $B$ is not dual to a scalar, unlike in the abelian case. As a result, one need not have a non-vanishing Higgs field in space-time in order to have non-abelian topological charge on a black hole. However, it is easy to see that if the unbroken local symmetry group is $SU(N)/Z_N$, our analysis reaches the same conclusions about the manifold of black hole vacua as one does in the case of discrete magnetic hair. This seems to indicate that the $B$ field and discrete quantum hair may be describing very similar, but not identical physical situations.

Another clue to the similarity or difference between the two theories may be obtained by considering the experimental method of observing either phenomenon. Discrete hair may be observed by ‘lassoing’ a black hole by a cosmic string loop that carries non-abelian electric (or magnetic) flux along its core. A similar ‘lassoing’, but only with a loop carrying

\(^5\) In general, one hopes that $SU(N)$ gauge theories confine the ‘electric’ charge and screen the ‘magnetic’ charge, so this special class may be the only one relevant to physics.
electric flux, would also detect the non-abelian $B$ field. In both processes, the loop picks up a phase proportional to the product of the discrete or topological charge and the flux inside the cosmic string core. This leads to an Aharonov-Bohm type effect. In reality, both discrete magnetic hair and the $B$ charge couple to a closed tube of electric flux. Such objects are unstable for SU(3) in the presence of quarks. Since SU(3) is the group closest to experimental physics for which one can obtain a non-trivial topological (or discrete) charge, one should look for alternative particle physics manifestations of topological charge. The question of particle physics phenomena caused by a $B$ field brings us to a question we have avoided answering so far – Is there any symmetry (other than SU(N)) associated with the action (2.4)?

The full symmetry of the action (2.4) is at best poorly understood, as there are a lot of technical difficulties associated with the quantization of a non-abelian 2-form (for a discussion and a comprehensive list of references, see [10]). However, the following may provide a pointer towards relating the (2.4) action to particle physics. As has been mentioned before, a non-abelianization of the Kalb-Ramond gauge transformation, $B_{\mu\nu} \rightarrow B_{\mu\nu} + D_{[\mu\nu]}$, does not leave $H$ invariant. However, there is a ‘shift’, or a field redefinition associated with $B$ that leaves $H$ invariant, $B_{\mu\nu} \rightarrow B_{\mu\nu} - \alpha F_{\mu\nu}$, where $\alpha$ is a dimensionful parameter of mass dimension $-1$. In general, such a shift (similar to the Goldstone mechanism for gauge fields, where the gauge field is ‘shifted’ by the derivative of a fundamental scalar already in the theory) leaves the path-integral measure, and therefore amplitudes, invariant. Without going into the subtleties of quantization, let us assume that the amplitudes remain invariant in this theory as well. Let us now introduce an $F^*F$ term in the action,

$$S = \int \text{Tr} \left( \sqrt{-g} \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \right) + \frac{m}{2} \epsilon_{\mu\nu\rho\lambda} F_{\mu\nu} B_{\rho\lambda} + \Theta \epsilon_{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} \right). \quad (4.1)$$

Evidently, by using the ‘shift’ symmetry with $\alpha = 2\Theta/m$, one can set the $\Theta$ angle to zero and be left with (2.4). Loosely speaking, one may then expect the CP-violating effects of the gauge theory to reside entirely in topological modes of $B$ (for space-times with trivial second cohomology and in a state with finite action, one may write $B = U d \Lambda U^{-1}$ at large $x$, as we found in the previous section) and not appear in local dynamics as they do via a $\Theta F^*F$ term. The parameter $\alpha$ may be thought of as a constant or ‘invisible’ axion$^6$. We

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$^6$ Through an unfortunate nomenclature, the $B$ field has often been confused with an axion. However, the only connection between the two (at least when $B$ is non-abelian) seems to come from the above speculation.
also note that the invariance of the action under this shift symmetry may be used as a justification for the absence of other terms involving $B$, if we maintain that bare couplings (other than $\Theta$) should not be modified under a local symmetry of the action. More cannot be said without a deeper understanding of the action (2.4), the shift symmetry, and its relevance to particle physics. Work on this is in progress, and results will be reported elsewhere.

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