Only certain galaxies are included in surveys: those bright and large enough to be detectable as extended sources. Because gravitational lensing can make galaxies appear both brighter and larger, the presence of foreground inhomogeneities can scatter galaxies across not only magnitude cuts but also size cuts, changing the statistical properties of the resulting catalog. Here we explore this size bias, and how it combines with magnification bias to affect galaxy statistics. We demonstrate that photometric galaxy samples from current and upcoming surveys can be even more affected by size bias than by magnification bias.

I. INTRODUCTION

Any survey using either counts of galaxies or shapes of galaxies must make choices: which galaxies are in and which are out. For example, a magnitude limited survey contains all galaxies brighter than a fixed flux threshold. Because gravitational lensing can make galaxies appear brighter, a background galaxy whose line of sight passes through an overdense region might be catapulted into the sample when it otherwise would not have been included in the catalog. As was pointed out many years ago [1–6], this leads to non-zero correlations between background and foreground objects, an effect which goes by the moniker of magnification bias [7–9].

A less studied phenomenon is that the increase in size of a background galaxy by a foreground matter overdensity [10] will have a similar effect if the catalog under consideration includes a size cut. We refer to this additional source of bias as size bias, and to the net effect of gravitational lensing on the galaxy sample as lensing bias.

The purpose of this paper is to introduce size bias as an additional effect that needs to be considered in studies of galaxy statistics. Any study which correlates galaxies in a flux- and size-limited sample with some other observable is potentially sensitive to size as well as magnification bias. In addition, size and magnification bias affect shear observables as well. This effect has so far not been studied in detail and is fleshed out in a companion paper [11].

II. LENSING BIAS

Qualitative discussion. Figure 1 illustrates the basic physics behind lensing bias. The figure shows size vs. $i$ band apparent magnitude for a small random subsample of stars and galaxies in the GOODS field [12]. Our size variable is $r$, the Full Width at Half Maximum (FWHM) of the galaxies in the $i$ band. The FWHM was measured using adaptive moments [13, 14]. All images have been degraded by adding noise and 0.9" seeing typical of the upcoming Dark Energy Survey [DES, 15]. Also shown in the figure are horizontal and vertical blue lines corresponding to a size and magnitude cut of $r > 1.2"$ and $i_{AB} < 24$, respectively. The hatched regions contain galaxies included in the survey due to lensing by a foreground mass: the vertical band corresponds to the traditional magnification bias effect, while the horizontal region contains galaxies affected by the size bias introduced in this work. “Stars” at lower left with abnormally small sizes are shot noise.

FIG. 1: The distribution of galaxies (red triangles) and stars (blue) in the GOODS field as a function of size and magnitude, after downgrading images to an assumed 0.9” seeing, typical of near-future large imaging surveys such as DES and Pan-STARRS. The thick vertical and horizontal lines denote our fiducial magnitude and size cuts ($i_{AB} < 24$ and $r > 1.2$ arcsec, respectively). The hatched regions contain galaxies included in the survey due to lensing by a foreground mass: the vertical band corresponds to the traditional magnification bias effect, while the horizontal region contains galaxies affected by the size bias introduced in this work. “Stars” at lower left with abnormally small sizes are shot noise.
necessary in order to be able to robustly separate galaxies from stars, and to obtain good shape measurements of the galaxies\(^1\).

The diagonal dotted lines in Figure 1 are contours of constant surface brightness. Since lensing preserves surface brightness, a foreground matter overdensity will move all sources along these diagonal lines upwards and to the right, increasing the number of objects in the galaxy sample. Conversely, an underdense region along the line of sight can act in the opposite way. In the weak lensing limit, the flux magnification factor \(A\) is given by \(A = 1 + 2\kappa\), where \(\kappa\) is the convergence (see below). Then, the magnitude and size of a source will be (de-)magnified according to:

\[
i_{AB} \rightarrow i_{AB} - \frac{5\kappa}{\ln 10}; \quad \log r \rightarrow \log r + \frac{\kappa}{\ln 10}.
\]  

(1)

The narrow horizontal and vertical hatched bands shown in Figure 1 contain galaxies which would pass our galaxy selection cuts if they were to be lensed by matter along the line of sight. For illustration purposes, we chose a large value of the lensing convergence, \(\kappa = 0.1\). The well known magnification bias effect corresponds to the vertical hatched region. In addition, however, we see that there is a comparable population of galaxies in the horizontal hatched region which is brought into our galaxy sample not because the galaxies are made brighter, but because the galaxies become more extended.

**Quantitative treatment.** We wish to determine how the galaxy density field \(n_{\text{obs}}(\tilde{\theta})\) observed in an experiment relates to the intrinsic (i.e. un-lensed) galaxy density field \(n(\tilde{\theta})\). Our discussion follows closely the presentation found in the appendix of Hui et al. [16]. Let \(f\), \(r\), and \(\tilde{\theta}\) denote the observed flux, apparent size, and position of a galaxy, respectively, and \(f_g\), \(r_g\), and \(\tilde{\theta}_g\) denote the corresponding intrinsic quantities. Further, let \(\Phi(f, r, \tilde{\theta})\) and \(\Phi_g(f_g, r_g, \tilde{\theta}_g)\) denote the observed and intrinsic distribution of galaxies in flux, size, and position. Because the total number of galaxies is preserved, one has that

\[
\Phi(f, r, \tilde{\theta}) \, df \, dr \, d^2\tilde{\theta} = \Phi_g(f_g, r_g, \tilde{\theta}_g) \, df_g \, dr_g \, d^2\tilde{\theta}_g.
\]

(2)

In the linear regime, the intrinsic and observed galaxy properties are related via\(^2\)

\[
\tilde{\theta} = \tilde{\theta}_g + \delta \tilde{\theta}, \quad f = f_g, \quad r = \sqrt{A} r_g, \quad d^2\tilde{\theta} = A \, d^2\tilde{\theta}_g
\]

(3)

where \(\delta \tilde{\theta}\) is the gravitational lensing deflection angle, and \(A = |\partial \tilde{\theta} / \partial \tilde{\theta}_g|\) is the corresponding amplification. We work in the weak lensing regime throughout, so that \(A \approx 1 + 2\kappa\). Note that the surface brightness of a source \(S \propto f/r^2\) is unaffected by \(A\).

Let now \(\varepsilon(f, r)\) be the galaxy selection function of a survey, which we assume depends only on a source’s magnitude and size. Given this selection function, the intrinsic and observed galaxy number densities are:

\[
n_{\text{obs}}(\tilde{\theta}) = \int df \int dr \, \varepsilon(f, r) \Phi(f, r, \tilde{\theta}),
\]

\[
n_0(\tilde{\theta}_g) = \int df_g \int dr_g \, \varepsilon(f_g, r_g) \Phi_g(f_g, r_g, \tilde{\theta}_g).
\]

(4)

(5)

Using equation (3), we can write \(f\), \(r\), and \(\tilde{\theta}\) in the above expression in terms of the corresponding intrinsic quantities. Linearizing, and using equation (2) to replace \(\Phi\) by \(\Phi_g\), we find that the relation between the observed and intrinsic galaxy densities is given by

\[
n_{\text{obs}}(\tilde{\theta}) = n_0(\tilde{\theta})[1 + (2\beta_f + \beta_r - 2)\kappa],
\]

(6)

where the parameters \(\beta_f\) and \(\beta_r\) are defined via:

\[
\beta_f \equiv \frac{1}{n_{\text{obs}}} \int df \int dr \frac{\partial \varepsilon(f, r)}{\partial \ln f} \Phi(f, r),
\]

\[
\beta_r \equiv \frac{1}{n_{\text{obs}}} \int df \int dr \frac{\partial \varepsilon(f, r)}{\partial \ln r} \Phi(f, r).
\]

(7)

(8)

While strictly speaking \(\beta_f\), \(\beta_r\) are defined in terms of the intrinsic galaxy distribution \(\Phi_g\), we have replaced this with the observed galaxy distribution \(\Phi\), which is the only one accessible to observations. Any differences between the two are of order \(\kappa\) averaged over the survey area, and hence correspond to a higher order correction in our perturbative approach.

If we further assume that the selection function is given by a simple magnitude and size cut, the function \(\varepsilon(f, r)\) takes the form \(\varepsilon(f, r) = \Theta(f - f_{\text{min}})\Theta(r - r_{\text{min}})\) where \(\Theta(x)\) is a step function. In this case, the expressions for \(\beta_f\) and \(\beta_r\) simplify to

\[
\beta_f = -\frac{\partial \ln n_{\text{obs}}}{\partial \ln f} \bigg|_{r = r_{\text{min}}} \quad ; \quad \beta_r = -\frac{\partial \ln n_{\text{obs}}}{\partial \ln r} \bigg|_{f = f_{\text{min}}}.
\]

(9)

Note that in this case, one has \(\beta_f = (5/2)\partial \ln n / \partial m \equiv 5s/2\). If \(\beta_r = 0\), one obtains \(n_{\text{obs}} = n_0[1 + (5s - 2)\kappa]\), which is the standard expression for magnification bias. For \(\beta_r \neq 0\), size bias acts in a way that is analogous to magnification bias, except that it is sensitive to the slope of the galaxy size distribution rather than that of the magnitude distribution. The parameter which captures the full extent of lensing bias therefore is

\[
q = 2\beta_f + \beta_r - 2.
\]

(10)

To first order in \(\kappa\), lensing bias affects the observed number density via

\[
n_{\text{obs}} = n_0(1 + q\kappa).
\]

(11)
The next section presents estimates of $q$ for upcoming galaxy surveys.

III. FORECAST FOR UPCOMING SURVEYS

Figure 2 shows the galaxy density of the GOODS field as a function of both magnitude and size. The GOODS data is degraded to 0.9" seeing and typical noise, and the galaxies are error weighted using adaptive moments [13, 14]. In order for galaxy ellipticities to be reliably measured, one requires both $i_{AB} \lesssim 24$ and $r \gtrsim 1.2''$, which we adopt as our fiducial magnitude and size cuts.

In order to measure the $q$ parameter for this data set, we determine $\beta_f$, $\beta_r$ from the downgraded GOODS galaxy sample according to Eq. (9), for a range of cut values. Note that, consistent with our perturbative approach, we have neglected the intersection between the two hatched regions in Fig. 1 in this calculation, which would further increase the value of $q$ but is formally of order $\kappa^2$.

While we take into account realistic weights assigned to each galaxy from the measurement errors, the results presented here are mainly for illustrative purposes: sharp size and flux cuts are rarely made in practice. Nevertheless, in many cases some form of apparent size cut is applied to galaxy catalogs. For instance, the main observable of weak lensing studies is galaxy ellipticity, which can only be reliably measured if the galaxies are significantly more extended than the PSF of the instrument.

Consequently, weak lensing studies usually introduce a size and magnitude dependent error weighting function $\varepsilon(m, r)$ which operates as a selection function. In general, this selection will be a smooth function of $m$, $r$, rather than a sharp cut. In that case, the resulting $q$ will be a suitable average over the sharp-cut $q$ values presented here.

Figure 3 shows the parameters $\beta_f$ and $\beta_r$ as a function of both the magnitude and size cuts, while holding the other cut fixed. The corresponding value of $q$ is shown (solid line) as well as the $q$ obtained when neglecting size bias (long-dashed line). It is clear from the figure that size bias can have a significant impact on $q$ for size cuts larger than the instrumental PSF, eventually dominating the lensing effect for large cuts. The fact that size cuts smaller than the PSF have no impact on the total lensing bias is not surprising: since all galaxies have an apparent size at least as large as the PSF, there is effectively no size cut if $r_{\text{cut}}$ is smaller than the PSF. We conclude that the sample of source galaxies in the DES weak lensing survey will be affected by size bias at a level that is comparable to or larger than the corresponding magnification bias. Interestingly, we find that for typical cut values, the $q$ parameter including size bias is positive, while it is negative when no size cut is applied. It might be possible to confirm this prediction with existing data from galaxy surveys. As shown in the companion paper.
We have also investigated whether size bias could enhance the lensing signature in the Luminous Red Galaxy (LRG) sample in the Sloan Digital Sky Survey, and in particular in the correlation function of these galaxies. The sample does include an effective size cut in order to separate stars from galaxies. However, since these galaxies are quite bright and large, most of them lie far from the thresholds. Therefore, size bias plays an insignificant role in the interpretation of the LRG correlation function. We estimated that this will also hold for higher-redshift LRG samples derived from the SDSS. For a different reason, size bias is expected to be unimportant for quasars (QSO): since these are point-like objects, lensing does not affect their size noticeably, so that QSO are again only affected by magnification bias.

IV. EFFECT OF LENSGING BIAS ON SHEAR MEASUREMENTS

As an application of our discussion of lensing bias, we briefly present the effect on shear observables which is the subject of [11]. Since the cosmic shear field $\gamma$ can be measured only where there are background galaxies, $\gamma$ is preferentially sampled in regions with a high density of background galaxies. However, a part of the fluctuations in the galaxy density come from lensing bias which is due to the same lensing field as the shear $\gamma$ itself. Because of this, the actual measured shear field is schematically given by (see [11] for details):

$$\gamma_{\text{obs}}(\vec{\theta}) = [1 + q \kappa(\vec{\theta})] \gamma(\vec{\theta}).$$

(12)

The leading lensing bias correction to the shear power spectrum is formally identical to the reduced shear correction [17, 18]. However, the amplitude of the correction is multiplied by a factor $1 + q \approx 2 - 3$, from our estimates in Sec. III. Thus, the correction to the shear power spectrum from lensing bias is at the 5% level for $\ell \sim 1000$, and growing for higher $\ell$ [11].

V. SUMMARY AND CONCLUSIONS

Gravitational lensing magnifies galaxies, making them not only brighter, but also larger in appearance. If galaxy selection criteria include any form of size measure, it follows that gravitational lensing can scatter galaxies in and out of the sample across the applied size cut. This effect, which we have dubbed size bias, is completely analogous to magnification bias, and is important whenever a size cut is applied that removes some non-negligible fraction of galaxies from the sample. Size bias will often be just as important as magnification bias, so many previous examples subject to the latter might profitably be re-examined for traces of size bias. Our calculations in §III suggest that upcoming large galaxy surveys will be similarly afflicted by both types of lensing bias. In a companion paper [11], we evaluate the impact of lensing bias on shear observables, and demonstrate that this effect is significant at the 5% level for $\ell \sim 1000$ in case of the shear power spectrum. We estimate that for DES, neglecting lensing and size bias can lead to biases in the Dark Energy parameters estimated from cosmic shear at the $2-3\sigma$ level. Similar conclusions are expected to apply to other upcoming surveys such as Pan-STARRS [19].

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