Recovery of Bathymetry from Altimeter Data

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1 Introduction

With the advent of satellite altimetry it has been proved that this new technology has had an impact not only on geodesy, but also on other scientific fields, such as oceanography and geophysics. The satellite altimetry technique provides direct measurements of sea surface heights with respect to the ellipsoid, i.e., the geometrical reference surface of the Earth, and makes it possible to determine the gravity field of the oceans on a global scale. This set of data can be used to predict the bathymetry of deep-seafloor features. As we know, one of the major achievements in the use of satellite altimetry in geodesy is the determination of the marine geoid with high accuracy and high resolution. After corrections for the sea surface topography, satellite altimetry can provide an estimation of the marine geoid with a precision level of about 5-10 cm. Such a precise geoid offers a good opportunity further for determining the marine gravity field over the oceans in the world. Several works were achieved by Sandwell and Smith (1997), Knudsen and Andersen (1997), Andersen and Knudsen (1998), and Hwang, Kao and Parsons (1998) [1-4].

Since geoid anomalies decrease slowly with the distance r from a mass anomaly (proportional to $r^{-1}$), the shape of the geoid reflects the distribution of a relatively deep-seated mass within the Earth. The gravity anomaly decreases more rapidly (proportional to $r^{-2}$) and hence is more sensitive to shallow mass anomalies. Nevertheless, a major contribution to the marine geoid is made by topographic anomalies in the very shallow rock-water interface on the ocean bottom, since this surface represents a large density contrast. Consequently, there is a strong correlation between the shape of the geoid and ocean...
bottom topography (bathymetry). Although this fact was first declared in the 19th century by Siemens (1876)\cite{5}, and he suggested that ocean depths could be inferred from sea surface gravimetry, it was more than a century later that Dixon, et al. (1983) demonstrated that this technique was feasible\cite{6}. Since then, modeling bathymetry with satellite altimetry has been carried out by many researchers\cite{7,10}. Caimant and Baudry (1996) gave a comprehensive review of the techniques and data used in bathymetric modeling\cite{11}.

At present, there exist two methods used to recover the bathymetry from altimeter data, i.e. the deterministic method and the stochastic method. The former is based on the formula of Parker (1972)\cite{12} and the three plate models of Watts (1978)\cite{13}, and the latter on the least-square collocation derived from the stochastic concept. The bathymetric model from altimeter data will not be sufficiently accurate for such purposes as maritime navigation and hazard prevention, but can be very useful in oceanography.

2 Depth estimation model

2.1 The model of the deterministic method

Seafloor is the shallowest density interface in the oceanic domain. Depth variations of the seafloor can be considered as height variations of mass elements, the density $\Delta \rho$ of which is given by the contrast between rock and sea water densities. These seafloor depth variations will disturb the local gravity field. From Caimant and Baudry (1996)\cite{11}, the disturbing potential $T(r)$ due to a given mass element of volume $V$ is:

$$T(r) = G \Delta \rho \int_{V} \frac{dv}{r - r'}$$  \hspace{1cm} (1)

where $G$ is the gravitational constant; $r$ is the coordinate vector of location at which the disturbing potential is computed; $r'$ is the coordinate vector of the center of the mass element. The well-known Brun's formula relates the geoid height $N$ to the disturbing potential $T$. The discretized form of the Brun's formula corresponding to Eq. (1) is:

$$N(r) = \frac{G \Delta \rho}{\gamma} \sum_{r'} \Delta \Omega(r') \left[ \int_{Z_b}^{Z_t} \frac{dZ}{r - r'} \right]$$  \hspace{1cm} (2)

where $\gamma$ is the normal gravity acceleration at the surface of the Earth; $\Delta \Omega(r')$ is the element of the integrated surface; $Z_b$ and $Z_t$ are the depth of the bottom and the top of the mass element centered on $r'$, respectively; the Z axis is vertical. Eq. (2) is convolution operation. Thus, it can be dealt with in the Fourier domain. The Fourier Transform of Eq. (2) is, in plane approximation\cite{12}:

$$F[N(r)] = \frac{2\pi G \Delta \rho}{\gamma} \exp(-ik|z_0|) \cdot \sum_{n=1}^{\infty} \frac{|k|^n - 2}{n!} F[Z^n(r')]$$  \hspace{1cm} (3)

where $r$ is now the vector formed by coordinates in the x-y plane; $k$ is the modulus of the wave number vector; $z_0$ is the reference depth of the density interface; $Z(r')$ stands for the depth variations of the interface relative to $z_0$. Details concerning the derivation of the Fourier expansions can be found in Reference\cite{12}. A linear form of Eq. (3) is obtained by limiting the series to the first term:

$$F[N(r)] = \frac{2\pi G \Delta \rho}{\gamma} \cdot \exp(-ik|z_0|) F[Z(r')]$$  \hspace{1cm} (4)

The above linear approximation allows an easy recovery of the topography of an uncompensated interface $b(r)$ from a given $N(r')$:

$$b(r) = F^{-1}[[Z_N(k)]^{-1}F[N(r')]]$$  \hspace{1cm} (5)

with

$$Z_N(k) = \frac{2\pi G \Delta \rho}{\gamma} |k|^{-1} \exp(-|k|z_0)$$  \hspace{1cm} (6)

Similarly, when the gravity anomalies $\Delta g (r')$ is given, $b(r)$ is recovered as

$$b(r) = F^{-1}[[Z_g(k)]^{-1}F[\Delta g(r')]]$$  \hspace{1cm} (7)

with $Z_g(k) = 2\pi G \Delta \rho \exp(-|k|z_0)$ \hspace{1cm} (8)

where $Z_N(k)$ and $Z_g(k)$ are called the Admission functions. The above two sets of equations including Eq. (5) to Eq. (8) are considered as the basic formulas of the deterministic method for modeling bathymetry from altimeter-derived geoid height and gravity anomalies, respectively. According to Eq. (7), a general form of the depth estimation model can be written as the following linear relationship in the frequency domain:

$$B(u, v) = K(u, v) \Delta G(u, v)$$  \hspace{1cm} (9)

where $B$ and $\Delta G$ are the Fourier transforms of
depth and gravity anomaly, respectively; $K$ is the transfer function; $u$, $v$ are the two spatial frequencies. In practical application, we can set $K$ to be

$$K(u, v) = \frac{1}{2\pi G\Delta \rho} e^{2\pi i u^2 + v^2}$$

where $\Delta \rho$ is the difference between the densities of seafloor material and sea water; $G$ is the gravitational constant; $d$ is the mean depth. Due to the $e^{2\pi i u^2 + v^2}$, bathymetric estimation is generally regarded as the downward continuation problem. In mathematics, the downward continuation problem is regarded as an ill-posed problem whose solution will be non-unique and unstable, and it requires some regularization treatment. Considering these problems, Smith and Sandwell (1994) simply removed components with wavelengths shorter than 15 km in their final bathymetric model, thus a detailed procedure for computing a predicted bathymetric model was provided.

### 2.2 The model of the stochastic method

According to the formula of Parker (1972) and the three plate models of Watts (1978), the bathymetric model between the residual fields of depth and gravity anomaly of the deterministic method can be constructed. In fact, however, given the complex composition of a studied seafloor, any three plate model by Watts (1978) will hardly match the reality; even if it does match, the parameters such as plate thickness and flexural rigidity are difficult to compute correctly. Due to this uncertainty, it is useful to model bathymetry using a stochastic approach. In 1994, Tscherning et al. experimented first with a least-squares collocation method in which gravity anomalies and apriori given depths were used to produce new depths in an area where the depth data may be considered erroneous. A lot of similar work has been carried out in the following years for the same purpose. Arabelos (1997) made a study on the possibility to estimate ocean bottom topography from marine gravity and satellite altimeter data by using collocation. Hwang (1999) used the following equation to construct his bathymetric model for the South China Sea (SCS):

$$h = \mu + \mathbf{C}_{h\mathbf{g}}^{-1}(\Delta g - \mu_{\Delta g})$$

(11)

where $h$ is the depth; $\Delta g$ is the gravity anomaly; $\mu$ and $\mu_{\Delta g}$ are the expected values of the depth and the gravity anomaly, respectively; $\mathbf{C}_{h\mathbf{g}}$ is the cross-covariance matrix between depth and gravity anomaly; $\mathbf{C}_{\mathbf{g}}$ is the auto-covariance matrix for gravity anomaly. In practical application, the expected values of the depth and the gravity anomaly can be represented by the regional trends. Let the residual depth and gravity anomaly be

$$\bar{h} = h - \mu$$

$$\bar{\Delta g} = \Delta g - \mu_{\Delta g}$$

(12)

(13)

then, from Eq. (11), we have

$$\bar{h} = \mathbf{C}_{h\mathbf{g}}^{-1}\bar{\Delta g}$$

(14)

which is equivalent to the formula of least-squares collocation. The covariance matrices $\mathbf{C}_{h\mathbf{g}}$ and $\mathbf{C}_{\mathbf{g}}$ can be constructed from covariance functions. However, it is rather difficult to model the cross-covariance function needed for $\mathbf{C}_{h\mathbf{g}}$ due to the fact that $h$ and $\Delta g$ come from different types of parameter fields. Because of this, Hwang (1999) proposed to use the concept of frequency domain collocation to speed up the computations, in which the covariance functions were replaced by the power spectrum density functions. Following this idea, he had computed a corrected bathymetric model for SCS, which had a better agreement with the ship-borne depths as compared with the predicted model.

According to the theory of least-squares collocation, Eq. (14) is originated from the following model:

$$\Delta g = \mathbf{C}_{h\mathbf{g}}^{-1}(\mathbf{s} + \mathbf{n})$$

(15)

where we have just decomposed the observation $\Delta g$ into a “signal part” $\mathbf{s}$ and the “noise part” $\mathbf{n}$. Obviously, Eq. (15) does not give a deterministic functional relation between the gravity anomaly $\Delta g$ and the depth $h$. In Eq. (14), the link relating $\Delta g$ to $h$ is their cross-covariance function. In regional gravity field approximation of physical geodesy, height information is usually used to improve the accuracy of predicted free-air gravity anomaly. The prediction model can be written as

$$\Delta g = a + bh + \mathbf{s} + \mathbf{n}$$

(16)

where $a$ and $b$ are the unknown parameters. A lot
of researches have shown that the above prediction model could always give better anomaly estimates than the simple model (see Eq. (15)). It inspires us to modify the prediction model of depth in a similar way. Suppose that there is a linear correlation between the gravity anomaly $\Delta g$ and the depth $h$. Similar to Eq. (16), we can rewrite the prediction model of depth as

$$h = a + b \Delta g + s + n$$  

(17)

where $a$ and $b$ are still the unknown parameters, differently, here $s$ represents the “signal part” of observation $h$, and $n$ represents the “noise part” of observation $h$. The observation equations can be expressed in matrix notation as

$$L = AX + S + V$$  

(18)

where $L$ represents the depth observation vector; $X$ is the unknown parameter vector, which comprises $a$ and $b$; $A$ is a given matrix expressing the effect of the parameters $X$ on the observations $L$, which comprises 1 and $\Delta g$; $S$ is the signal vector; $V$ is the correction vector of $L$. Eq. (18) is an ultimate model of least-squares collocation with parameters. The general minimum principle can be used to derive optimal estimates for $X$ and $S$. The least square solutions of Eq. (18) are:

$$X = (A^T\bar{C}^{-1}A)^{-1}A^T\bar{C}^{-1}L$$  

(19)

$$S = C_n\bar{C}^{-1}(L - AX)$$  

(20)

$$V = C_m\bar{C}^{-1}(L - AX)$$  

(21)

$$H = L - V$$  

(22)

where $\bar{C} = C_s + C_m$, which is called the total covariance matrix of $L$; $C_s$ and $C_m$ are the covariance matrices of the signal $s$ and the noise $n$, respectively; $H$ is the adjusted depth vector. From Eq. (19) to Eq. (22), on the basis of a given initial depth model, we can obtain an improved prediction depth model using an iterative procedure. For the first iteration, the covariance matrix of the signal $s$ should be replaced by that of the initial depth model $h$ owing to the fact that the signal $s$ is unknown in the first step. When the computed results are to be stable, the iterative procedure is finished.

As mentioned above, the difference between Eqs. (15) and (17) is that the statistical relation between the gravity anomaly $\Delta g$ and the depth $h$ in the former model has been changed to a deterministic linear relation in the latter. The advantage of this modification is that the new model has no requirement of determining the cross-covariance function needed for $C_{gh}$. The gains of new model in computation efficiency will be illustrated further in the case study of this paper.

3  A case study

In order to evaluate the new method proposed above, some practical computations and a comparison with ship-measured depth are carried out in this paper. In this case study, the computation area is defined as $(2^\circ N-25^\circ N, 105^\circ E-125^\circ E)$. The data file of altimeter-derived gravity anomaly from Huang et al. (2001) has been used as input information, which is given on a $2^\prime \times 2^\prime$ grid and derived from the altimeter data of Seasat, Geosat, ERS-1 and Topex/Posidon. The newly developed $5^\prime \times 5^\prime$ global digital topographic model JGP95E has been used as the initial depth model for iterative computation. The following formulas have been applied to compute the covariance matrix:

$$C(d) = \frac{1}{2} [C(d)_h + C(d)_v]$$  

(23)

$$C(0) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} (\Delta h_{ij})^2$$  

(24)

$$C(d)_h = \frac{1}{m(n-q)} \sum_{i,j=1}^{m} \Delta h_{ij} \Delta h_{i,j+q}$$  

(25)

$$C(d)_v = \frac{1}{n(m-q)} \sum_{i,j=1}^{n} \Delta h_{ij} \Delta h_{i,j+q}$$  

(26)

where $d$ is the distance between two data points; $C(d)_h$ and $C(d)_v$ represent the covariance in horizontal and vertical directions, respectively; $m$ and $n$ are the grid numbers in the two directions; $\Delta h$ is the difference between the depth $h$ and its regional mean value. According to Reference[16], we give an error of 400 m for the initial depth model JGP95E.

In order to evaluate the accuracy of the altimeter-derived depth, here the computed results are compared with the shipborne measurements of depth requested from our study in the region $(7^\circ N-20^\circ N, 110^\circ E-120^\circ E)$. The data came from numerous cruises during 1990 to 2000. It is believed that the accuracy of the ship depth is better than 1%. Before comparison, the depths from the altimeter-derived grid are interpolated to each ship depth location by using weighted-mean method. The comparisons are
made at all (152,979) stations. First of all, the statistics of the ship depths themselves used for comparison are shown in Table 1.

Table 2 gives the statistics of the differences between the altimeter-derived and the shipborne depths.

| Year | Max/m | Min/m | Mean/m | RMS/m | STD/m |
|------|-------|-------|--------|-------|-------|
| 1990 | 3,037 | 4,395 | 308    | 2,079 | 2,216 |
| 1991 | 2,545 | 4,448 | 493    | 2,424 | 2,571 |
| 1993 | 3,833 | 4,613 | 622    | 4,173 | 4,197 |
| 1994 | 2,532 | 4,303 | 258    | 2,794 | 2,944 |
| 1995 | 2,492 | 4,359 | 576    | 3,474 | 3,610 |
| 1996 | 2,405 | 4,441 | 1,164  | 3,302 | 3,404 |
| 1997 | 13,874| 4,463 | 273    | 2,037 | 2,272 |
| 1998 | 9,677 | 4,276 | 218    | 2,637 | 2,722 |
| 1999 | 55,314| 4,296 | 222    | 3,775 | 3,832 |
| 2000 | 57,237| 4,514 | 463    | 3,616 | 3,665 |

Table 3 Statistics of discrepancies between JGP95E and the shipborne depth

| Year | Max/m | Min/m | Mean/m | RMS/m | STD/m | Relative error/% |
|------|-------|-------|--------|-------|-------|------------------|
| 1990 | 820.4 | -825.8| 35.4   | 224.6 | 221.8 | 10.8             |
| 1991 | 890.2 | -889.0| 41.8   | 260.7 | 257.3 | 10.8             |
| 1993 | 1154.7| 1163.7| 114.9  | 272.6 | 247.2 | 6.5              |
| 1994 | 1028.3| 1020.9| 2.5    | 281.9 | 281.9 | 10.1             |
| 1995 | 736.0 | -748.0| 16.6   | 246.9 | 246.4 | 7.1              |
| 1996 | 673.7 | -686.7| 1.5    | 236.6 | 234.8 | 7.6              |
| 1997 | 651.7 | -682.6| 19.3   | 233.6 | 232.2 | 11.6             |
| 1998 | 702.6 | -703.1| 2.6    | 281.9 | 281.9 | 10.1             |
| 1999 | 647.0 | -658.8| 12.9   | 217.3 | 217.3 | 6.0              |

Table 4 Statistics of discrepancies between Reference [7] and the shipborne depth

| Year | Max/m | Min/m | Mean/m | RMS/m | STD/m | Relative error/% |
|------|-------|-------|--------|-------|-------|------------------|
| 1990 | 628.2 | -624.8| 30.9   | 188.4 | 185.8 | 9.1              |
| 1991 | 625.1 | -628.9| 49.5   | 197.3 | 191.0 | 8.1              |
| 1993 | 697.8 | -752.5| 25.0   | 204.6 | 203.1 | 4.9              |
| 1994 | 731.7 | -728.3| 52.3   | 228.5 | 219.6 | 8.6              |
| 1995 | 703.4 | -736.2| 19.2   | 242.7 | 241.9 | 7.0              |
| 1996 | 605.9 | -697.1| 33.3   | 220.0 | 217.5 | 6.7              |
| 1997 | 654.1 | -671.0| 58.6   | 202.9 | 194.2 | 10.0             |
| 1998 | 118.7 | -846.5| 25.4   | 267.6 | 266.4 | 10.1             |
| 1999 | 761.2 | -764.2| 18.5   | 208.5 | 207.7 | 5.5              |
| 2000 | 565.9 | -566.1| 9.9    | 154.9 | 154.6 | 4.3              |

4 Conclusions

This paper has concentrated on the recovery of depth by using a modified stochastic model from the altimeter-derived gravity anomalies. The comparisons with the ship-borne depths have shown that the result obtained here is satisfactory. It should be pointed out that such depth model from altimeter data will not be sufficiently accurate for such purposes as maritime navigation and hazard prevention, but can be very useful in many research fields of oceanography. Future improvements will be made by combining more accurate altimeter-derived gravity anomalies and the existing sparse shipboard depths.

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