Dynamical Cooling of Trapped Gases I: One Atom Problem

Luis Santos† and Maciej Lewenstein‡

†Departamento de Física Aplicada, Universidad de Salamanca, Plaza de la Merced s/n, 37008 Salamanca, Spain
‡Institut für Theoretische Physik, Universität Hannover, Appelstr. 2, D-30167 Hannover, Germany

We study the laser cooling of one atom in an harmonic trap beyond the Lamb-Dicke regime. By using sequences of laser pulses of different detunings we show that the atom can be confined into just one state of the trap, either the ground state or an excited state of the harmonic potential. The last can be achieved because under certain conditions an excited state becomes a dark state. We study the problem in one and two dimensions. For the latter case a new cooling mechanism is possible, based on the destructive interference between the effects of laser fields in different directions, which allows the creation of variety of dark states. For both, one and two dimensional cases, Monte Carlo simulations of the cooling dynamics are presented.

I. INTRODUCTION

Atomic laser cooling has become recently one of the most active research fields in atomic physics. Several techniques have been developed to cool free atoms up to fractions of the recoil temperature (i.e. the temperature associated with the recoil of a single photon). The sub-recoil laser-cooling methods are based on the crucial concept of dark states. These states cannot absorb the laser light, and therefore cannot be excited, but however can receive population via incoherent pumping, i.e. via spontaneous emission, and therefore these states behave as trapping states. The dark-state is a general concept, and can be obtained by several mechanisms, for example the destructive interference between the absorptions from the two ground states of a three level Λ-system, as in Velocity Selective Coherent Population Trapping (VSCPT). On the other hand, laser cooling of trapped atoms to the ground state of a trap has been achieved for single trapped ions using sideband cooling in the Lamb-Dicke limit (LDL). This limit corresponds to the case in which the size of the ground state of the trap is very small compared with the wavelength of the applied laser λ. More rigorously, LDL occurs when η ≪ 1, where η is the so-called Lamb-Dicke parameter defined as η = 2πa₀/λ. Sideband Cooling method only works properly in this limit, and also requires the levels of the trap to be well resolved, which is achieved in the so-called strong-confinement limit, i.e. when γ < ω, where ω is the trap frequency and γ is the spontaneous emission rate (the level width). This last condition can be easily fulfilled by using a metastable excited state and quenching its population using a third fast-decaying level. However, the problem appears in the regime beyond the LDL, in which the recoil of the spontaneously emitted photons provides a heating mechanism, and therefore new cooling strategies must be proposed. The most effective techniques have been those originally designed to cool free atoms up to subrecoil temperatures, i.e. dark-state cooling techniques. A recent paper proposes a new cooling scheme for trapped atoms beyond the LDL which combines the ideas of sideband and Raman cooling and dark-state cooling, to confine a single trapped atom in the ground state of the trap.

In this series of papers, we study laser cooling techniques similar to that of which allow the confining of an atomic sample into just one state of the trapping potential, either the ground state or an excited state. This first paper is devoted to the one-atom problem while the second one analyses the new physics appearing for the many-atom case. The one-atom problem is interesting as such in the context of quantum computation with ions, and also in the production of non-classical states of motion of single atoms or ions. Also the one-atom problem is a good starting point to understand clearly the cooling processes in the many-atom case, which we discuss in detail in; therefore this paper is a good first step to analyse the possibilities to achieve the Bose-Einstein Condensation with all-optical means.

The paper is organized as follows. In Sec. the cooling scheme is described, as well as the difficulties to cool beyond the LDL. In Sec. we study the one-dimensional case, analysing the cooling into the ground state of the trap (basically summarizing in Sec. IIIA). The one-dimensional cooling into an excited of the trap is studied in Sec. III B, where the dark-state conditions are presented. In section Sec. IV A and Sec. IV B the new physics which appears in the two-dimensional case is considered. In Sec. IV C a new interference mechanism between the lasers in different directions is presented. In Sec. IV D the cooling into the ground state is analysed, whereas in Sec. IV E we employ the dark-state conditions of Sec. III B and also the interferential dark-states of Sec. IV C, to confine the atom in an excited two-dimensional state. We conclude in Sec. V.

II. MODEL

We consider laser cooling in a three-level Λ-system as that presented in Fig. 1. The level |g⟩ is the ground state, |e⟩ is a metastable state, and |r⟩ is a third fast-decaying auxiliary state. A laser, or strictly speaking
two lasers excite coherently the resonant Raman transition \( |g⟩ \rightarrow |e⟩ \) (with some associated effective Rabi frequency \( \Omega \)), while a repumping laser in or off-resonance with the transition \( |g⟩ \rightarrow |r⟩ \) pumps optically the atom into \( |g⟩ \). With this three level scheme, one obtains an effective two-level system with an effective spontaneous emission rate \( \gamma \), which can be controlled by varying the intensity or the detuning of the repumping laser \( \bar{\Omega} \). The atom is assumed to be confined in an harmonic (isotropic) potential of frequency \( \omega \). We denote the states of the harmonic oscillator as \( |n⟩ \), or \( |n_x, n_y⟩ \) when treating the two-dimensional case. We assume \( \gamma < \omega \), i.e. we suppose the so-called strong-sonfinition limit, which is also called Festina-Lente limit, and turns to be a crucial point in the many atom case in order to avoid the reabsorption problem. We also assume \( \Omega < \gamma \) and \( \Omega^2/\gamma \ll \omega \), which allows to adiabatically eliminate the excited state \( |e⟩ \) in the expressions below.

In the Lamb-Dicke limit (LDL), i.e. when the dimensions of the trap are very small compared with the laser wavelength \( \lambda \), it is possible to use standard Sideband Cooling techniques to pump the atom into the ground state of the harmonic trap. In this cooling mechanism the laser is assumed red-detuned, with detuning \( \delta = -\bar{\eta}^2 \), where \( \bar{\eta}^2 \) is the closest integer to \( \eta^2 \). The absorption of a laser photon produces a transition \( |g, n⟩ \rightarrow |e, n - \mathcal{O}(\eta^2)⟩ \), and the spontaneous emission \( |e, n - \mathcal{O}(\eta^2)⟩ \rightarrow |g, n - \mathcal{O}(\eta^2)± n'⟩ \), where \( n' \) ranges from \( 0 \) to \( \mathcal{O}(\eta^2) \). As a result, in average some energy is lost in each cycle. Obviously, the states with \( n < \eta^2 \) remain dark, and therefore the atoms are finally confined in these states. Due to this, we call these pulses confinement pulses.

As the result of the cooling process, the atom can be confined in the ground state of the trap, as in the usual Sideband Cooling. Since the present cooling mechanism is based on sequences of different pulses, we call this cooling method Dynamical Cooling.

### 2. Rate Equations

The dynamics of the system can be well described, after adiabatic elimination of the \( |e⟩ \) states, by the following rate equation:

\[
\dot{N}_m = \sum_{n=0}^{\infty} \Gamma_{m \rightarrow n} N_n - \sum_{n=0}^{\infty} \Gamma_{n \rightarrow m} N_m,
\]

where \( N_n \) is the probability of the atom to be in the level \( |n⟩ \) of the trap. The transition rates are:

\[
\Gamma_{n \rightarrow m} = \frac{\Omega^2}{2\gamma} \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \mathcal{W}(\phi, \theta) \times \left| \sum_{l=0}^{\infty} \frac{\gamma(n)e^{ik\sin \theta \cos \phi} |l⟩⟨l|e^{ikx}|m⟩}{|\delta - \omega(l - m)|} + i\gamma \right|^2,
\]

where \( k \) is the laser wavevector and \( \mathcal{W} \) is the angular dipole pattern of the spontaneously emitted photons. Using closure relations, assuming a laser detuning \( \delta = s \omega \) with \( s \) an integer value, and neglecting the non-resonant terms, it is possible to write the rates to empty the level \( |m⟩ \) (called hereafter empty rates) in a very simple form in terms of the Franck-Condon factors:

### III. ONE DIMENSION

#### A. Cooling into the Ground State of the Trap

##### 1. Cooling Mechanism

The previous paper [3] studied the problem of laser cooling of a single trapped atom beyond the LDL into the ground state of a one-dimensional trap. In this section we briefly review that paper, and discuss a cooling mechanism which avoids the heating effects of the usual Sideband Cooling by employing the two step process:

- In the first step the atoms are confined in the levels \( n \leq \eta^2 \). This is achieved by using pulses with detunings \( \delta = -\bar{\eta}^2 \omega \), where \( \bar{\eta}^2 \) is the closest integer to \( \eta^2 \). The absorption of a laser photon produces a transition \( |g, n⟩ \rightarrow |e, n - \mathcal{O}(\eta^2)⟩ \), and the spontaneous emission \( |e, n - \mathcal{O}(\eta^2)⟩ \rightarrow |g, n - \mathcal{O}(\eta^2)± n'⟩ \), where \( n' \) ranges from \( 0 \) to \( \mathcal{O}(\eta^2) \). As a result, in average some energy is lost in each cycle. Obviously, the states with \( n < \eta^2 \) remain dark, and therefore the atoms are finally confined in these states. Due to this, we call these pulses confinement pulses.

- Using pulses of selected frequencies one can empty the trap levels \( \{g, 1⟩, |g, 2⟩, \ldots \} \) without affecting the population of \( |g, 0⟩ \). Then, in each cooling cycle there is some probability to pump into \( |g, 0⟩ \). There is also some probability of heating, and therefore we must repeat the confinement step, and so on. We call the pulses with selected frequency fine-cooling pulses.

As the result of the cooling process, the atom can be confined in the ground state of the trap, as in the usual Sideband Cooling. Since the present cooling mechanism is based on sequences of different pulses, we call this cooling method Dynamical Cooling.

#### 2. Rate Equations

The dynamics of the system can be well described, after adiabatic elimination of the \( |e⟩ \) states, by the following rate equation:

\[
\dot{N}_m = \sum_{n=0}^{\infty} \Gamma_{m \rightarrow n} N_n - \sum_{n=0}^{\infty} \Gamma_{n \rightarrow m} N_m,
\]

where \( N_n \) is the probability of the atom to be in the level \( |n⟩ \) of the trap. The transition rates are:

\[
\Gamma_{n \rightarrow m} = \frac{\Omega^2}{2\gamma} \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \mathcal{W}(\phi, \theta) \times \left| \sum_{l=0}^{\infty} \frac{\gamma(n)e^{ik\sin \theta \cos \phi} |l⟩⟨l|e^{ikx}|m⟩}{|\delta - \omega(l - m)|} + i\gamma \right|^2,
\]
\[
\Gamma_m = \sum_{n \neq m} \Gamma_{n\rightarrow m} \\
\simeq \frac{\Omega^2}{2\gamma} \begin{cases} 
\langle |m+s| e^{ikx}|m\rangle|^2, & \text{if } m+s > 0; \\
0, & \text{otherwise.}
\end{cases} \quad (3)
\]

3. Numerical results

Monte Carlo simulations of Eq. (1) yield similar results to those of [8]. In Fig. 2b the cooling dynamics for the case of \( \eta = 3.0, \gamma = 0.01\omega \) is shown, where the initial distribution was taken to be thermal with the mean \( \langle n \rangle = 6 \). We use a sequence of four pulses of detunings \( \delta_{1,2,3,4} = -9\omega, 0, -10\omega, -\omega \) and durations \( T_{1,2,3,4} = 2\gamma/\Omega^2 \) to confine the atom with almost total probability in the ground state of the trap. Pulses 1 and 3 are confining pulses, and 2 and 4 are fine-cooling pulses.

The reason of applying two different confinement pulses slightly detuned one respect to the other, is the oscillatory character of the Franck-Condon factors, which produces, as observed in Fig. 2a, that the empty rates have quasi-zero minima for each detuning. By applying two slightly detuned lasers this problem is avoided because the states that one laser does not empty the other laser does, and vice versa. The same reasoning explains the use of two fine-cooling pulses.

B. Cooling into an Excited State of the Trap

1. Condition to obtain a Dark Excited State of the Trap

In the present section we present a new mechanism that allows the confining of the atom into an excited state of the trap. This cooling mechanism is based on the properties of the Franck-Condon factors appearing in Eq. (1), which are of the form:

\[
\langle m+s|e^{ikx}|m\rangle = i^{2m+s} e^{-\eta^2/2} \sqrt{m!(m+s)!},
\]

\[
\times \sum_{l=0}^{\min(m,m+s)} \frac{(-1)^l \eta^{2m+s-2l}}{l!(m+s)!(m+s-l)!}. \quad (4)
\]

For a particular excited state \( |m\rangle \) of the trap, one can prove that for certain values of the detuning and of the Lamb-Dicke parameter, the sum in the last line of Eq. (4) cancels out. If this occurs, and following Eq. (3), one can observe that this level \( |m\rangle \) is not emptied, and therefore becomes a dark state. The dark-state conditions for the states \( |1\rangle \) and \( |2\rangle \) of the trap, are respectively:

\[
\eta^2 = s + 1 \quad \text{(5a)}
\]

\[
\eta^2 = (s + 2)(1 \pm (s + 2)^{-1/2}) \quad \text{(5b)}
\]

Certainly, these conditions have some limitations. First of all, as the detuning must be integer (i.e. \( s \) must be an integer number), only particular values of \( \eta \) are valid in order to satisfy the dark-state conditions. This is not a major problem because the trap frequency can be externally controlled. However, one must note that for the case of one-atom problem these conditions are certainly rather restrictive. Dynamical Cooling of a single atom is critically sensitive to the deviations from the dark-state conditions. We show in [8] that fortunately this is not the case in the many atom problem with bosonic atoms. The limitations are overcome due to quantum statistical effects (bosonic enhancement effect).

2. Scheme to cool into an excited state of the trap

Using the dark-state condition it is possible to design a cooling mechanism that allows the confinement of the atom into an excited state of the trap. The cooling mechanism consists as in the ground-state cooling in a sequence of several pulses:

- 1. Confinement pulse: exactly the same as in the ground-state cooling. The use of two slightly detuned confinement pulses is also recommended.
- 2. Dark-State cooling pulse: which fullfills the dark-state condition for a selected excited state.
- 3. Auxiliary pulses that avoid the oscillatory character of the empty rates of the dark-state cooling pulse, but do not affect the excited state in which the confinement is to be produced.

In Figs. 3 the case of \( \eta = 3.0, \gamma = 0.01\omega \) is analyzed. We have used cycles of four pulses of detunings \( \delta_{1,2,3,4} = -9\omega, 0, -10\omega, -\omega \) and durations \( T_{1,2,3,4} = 2\gamma/\Omega^2 \) to confine the atom with almost total probability in the first excited state of the trap. Pulses 1 and 3 are confinement pulses, the pulse 2 fulfills Eq. (3) and is therefore the dark-state pulse for the level \( |1\rangle \), and the pulse 4 is an auxiliary pulse. In Fig. 4 the empty rates corresponding to pulses 2 and 4 are depicted. Note that the both pulses do not empty the level \( |1\rangle \), and that it is the only level not emptied by them. Thus, the level \( |1\rangle \) becomes a trapping state. Fig. 3b shows the dynamics of confinement in the state \( |1\rangle \) for the considered sequence of pulses, where the initial distribution was taken to be thermal with the mean \( \langle n \rangle = 6 \).

The atom can also be confined in higher levels of the trap. Fig. 3 shows the dynamics of the confinement into level \( |2\rangle \), for the case of \( \eta = 3.065, \gamma = 0.01\omega \), the initial thermal distribution with the mean \( \langle n \rangle = 6 \), and cycles of four pulses of detunings \( \delta_{1,2,3,4} = -9\omega, 11\omega, -10\omega, -5\omega \) and durations \( T_{1,2,3,4} = 2\gamma/\Omega^2 \). As previously, the pulses 1 and 3 are confinement pulses, the pulse 2 fulfills Eq. (3) and is therefore a dark-state pulse for the level \( |2\rangle \), and the pulse 4 is an auxiliary pulse.
IV. TWO DIMENSIONS

A. Main new features in dimensions higher than one

In this section we consider the single atom cooling in two dimensions. The two dimensional problem contains essentially all of the main new physical features that appear when one considers dimensions higher than one, namely:

- The number of levels involved is larger (squared for the two-dimensional case). Therefore the atom can perform a random walk in a larger phase space, which means that the cooling is necessarily slower than in the one-dimensional case.
- Cooling in one direction means heating in the other directions. This fact makes necessary the use of lasers in different directions. Therefore, we will consider two lasers propagating in directions $x$ and $y$, providing an electric field

$$E = E_0 \left( e^{i k_x x} + A e^{i k_y y} \right) e^{-i \omega t} + c.c \quad (6)$$

where the parameter $A$ accounts for a possible difference of intensities or dephasing between both lasers. As we indicate below this parameter turns to be important under certain conditions.
- Confinement pulses are no longer pulses of detuning $\delta = -\tilde{\eta}^2 \omega$. This fact can be easily understood if we consider the case in which the atom absorbs a photon in direction $x$. Assuming a detuning $\delta = -\tilde{\eta}^2 \omega$, after an absorption the atom makes a transition $|g, n_x, n_y \rangle \rightarrow |e, n_x - \mathcal{O}(\tilde{\eta}^2), n_y \rangle$, and after the spontaneous emission $|e, n_x - \mathcal{O}(\tilde{\eta}^2), n_y \rangle \rightarrow |g, n_x - \mathcal{O}(\tilde{\eta}^2) \pm n'_x, n_y \pm n'_y \rangle$, where both $n'_x$ and $n'_y$ range from 0 to $\mathcal{O}(\tilde{\eta}^2)$. Therefore the two-dimensional quantum number $n = n_x + n_y$ changes into $n + n'$ where $n'$ ranges between $\mathcal{O}(-3\tilde{\eta}^2)$ and $\mathcal{O}(\tilde{\eta}^2)$, and therefore an average heating is produced. For the two-dimensional problem, the true confinement pulses are those with detuning $\delta = -2\tilde{\eta}^2 \omega$ (in general for $D$ dimensions $\delta = -D\tilde{\eta}^2 \omega$), as can be easily proved by using the same reasoning as above.
- New cooling mechanisms can be designed, as we show below.

B. Rate Equations

The cooling dynamics is regulated by a set of rate equations as that of Eq. (3). The transition rates in the two-dimensional laser arrangement considered are of the form:

$$\Gamma_{\bar{n} \rightarrow \bar{n}} = \frac{\Omega^2}{2\gamma} \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta d\theta W(\phi, \theta)$$

$$\times \left| \sum_{l_x, l_y=0}^{\infty} \gamma |\langle n_x | e^{i k x} | l_x \rangle \langle l_x | e^{i k y} | m_x \rangle | \right|^2$$

$$\times |\langle n_y | e^{i k y} | m_y \rangle \rangle^2$$

$$\times \langle n_x | e^{i k x} \mathcal{O} \cos \phi x | m_x \rangle \delta_{l_y, m_y} \right|^2 \quad (7)$$

where $\bar{n} = (n_x, n_y)$. Proceeding as in the one-dimensional case, it is possible to obtain a simple form for the empty rates:

$$\Gamma_{\bar{n}} \approx \frac{\Omega^2}{2\gamma} \left| \langle m_x + s | e^{i k x} | m_x \rangle \right|^2$$

$$+ |A|^2 \left| \langle m_y + s | e^{i k y} | m_y \rangle \right|^2$$

$$(A + A^*) \langle m_x + s | e^{i k x} | m_x \rangle \langle m_y + s | e^{i k y} | m_y \rangle \delta_{s, 0} \quad (8)$$

C. Destructive interference of the absorption amplitudes: New dark-states in two dimensions

Let us consider the case in which $\delta = 0$, i.e. $s = 0$. Let us also assume that both lasers in directions $x$ and $y$ have the same intensity but are $\pi$-dephased, i.e. $A = -1$. Then, Eq. (8) becomes:

$$\Gamma_{\bar{n}} \approx \frac{\Omega^2}{2\gamma} \left| \langle m_x + s | e^{i k x} | m_x \rangle - \langle m_y + s | e^{i k y} | m_y \rangle \right|^2$$

(9)

Eq. (9) shows that in the case of an isotropic trap, all the states in which $m_x = m_y$, satisfy $\Gamma_{\bar{n}} = 0$, i.e. are not emptied. Id est, the destructive interference of the absorption of the two lasers produces that the diagonal levels become dark. As we show in the following section, this fact can be employed to cool the atom into the ground state of the two-dimensional trap with more efficiency than the simple extension of the one-dimensional method to two dimensions. Moreover, if we set

$$A = -\frac{\langle m_x^0 | e^{i k x} | m_x^0 \rangle}{\langle m_y^0 | e^{i k y} | m_y^0 \rangle}$$

(10)

where $\bar{m}^0 = (m_x^0, m_y^0)$ is a particular state of the two-dimensional trap, then from Eq. (9) it is easy to observe that this particular level becomes a dark state. Therefore, we can specifically design the factor $A$ to regulate the interference between the two lasers in such a way that a selected state $\bar{m}^0$ becomes dark. As always, this fact can be employed to design a cooling mechanism to pump the atom into the level $\bar{m}^0$. An example of this selective cooling is discussed in section IV.E. Also one can use...
Eq. (10) to generalize the dark-state condition appearing from Eq. (3) for the more general case of an anisotropic trap.

Additionally, the dark-state condition for the excited states presented in Sec. IV A remains valid for the two-dimensional case, allowing the cooling into excited states of the trap, as we show in Sec. IV B.

D. Cooling into the Ground State of the Trap

In this section we demonstrate how the previously presented destructive interference between the lasers can be employed to improve the cooling into the ground state of the trap. As in one dimension, the cooling mechanism consists of several pulses:

- 1. Confinement pulses: with the same function as in the one-dimensional case but, instead of pulses with detuning \( \delta = -\eta^2 \omega \), we use, following the reasoning of Sec. IV A, \( \delta = -2\eta^2 \omega \). The use of two slightly detuned confinement pulses is also needed.

- 2. Dark-State cooling pulses: with detuning \( \delta = 0 \), and with ratio between the laser amplitudes \( A = 1 \). Following the discussion of the previous section the diagonal states \( m_x = m_y = 0 \) are dark for these pulses.

- 3. Sideband cooling pulses: of course the dark-state pulses induce trapping in the diagonal states, and not only in \( (0,0) \). In order to avoid that, a usual Sideband cooling pulse is neccesary, i.e. a pulse with detuning \( \delta = -\omega \).

- 4. Pseudo-confining pulses: as pointed out in Sec IV A, the atom performs its random walk in a larger phase space, and therefore with the only use of pulses 1, 2 and 3 the cooling is neccesary very slow. In order to improve the cooling time we use pulses with \( \delta = -\eta^2 \) and pulses with \( \delta = -\eta^2/2 \) (closest integer to this value). These pulses pseudo-confine the atom below \( n = \eta^2 \) and \( n = \eta^2/2 \), respectively. Due to this fact we call these pulses Pseudo-confinement pulses.

Fig. 3 shows the cooling dynamics for the case of \( \eta = 3.0, \gamma = 0.1 \omega, \) and an initial thermal distribution with mean \( \langle n \rangle = 6 \). We have used cycles of eight pulses of detunings \( \delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 0, -19\omega, -10\omega, -5\omega, -\omega \), all of them with duration \( T = 2\gamma/\Omega^2 \). In solid-line is represented the case of \( A = -1 \) for all the pulses, whereas in dashed-line the case \( A = 1 \) is depicted. Note that due to the form of Eq. (5), only for \( \delta = 0 \) it is relevant if \( A = 1 \) or \( A = -1 \). Pulses 1 and 5 are confinement pulses. Pulse 4 is the dark-state pulse (for the case \( A = -1 \)). Pulse 8 is the Sideband Cooling pulse. Pulses 2, 3, 6 and 7 are pseudo-confinement pulses. One can observe that for the case of \( A = -1 \) the cooling is very much effective than for the case of \( A = 1 \). Therefore, the interference mechanism which appears in two dimensions (and in general in dimensions higher than one) between the absorption amplitudes of lasers in different directions can be employed to improve the cooling technique.

E. Cooling into an Excited State of the Trap

In this section, we employ the two different kinds of dark-states, i.e. those produced by the special form of the Franck-Condon factors (Sec. IV B), and those produced by the interference of lasers in different directions (Sec. IV C), to confine the atom into an excited state of the harmonic trap. As in the previous section the use of confinement and pseudo-confinement pulses is in this case also neccesary.

Let us analyse in first place the cooling using the Franck-Condon properties. Fig. 3 shows in solid line the dynamics of the probability to occupy the level \( (1,1) \) of the trap, for the case of \( \eta = 3.0, \gamma = 0.1 \omega, \) the initial thermal distribution with the mean \( \langle n \rangle = 6 \), and a sequence of pulses of detunings: \( \delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 8\omega, -19\omega, -10\omega, -5\omega, -\omega \), all of them with duration \( T = 2\gamma/\Omega^2 \). As previously, pulses 1 and 5 are confinement pulses, and pulses 2, 3, 6 and 7 are pseudo-confinement pulses. Pulse 4 fullfills Eq. (5), and from Eq. (5) it is trivial to see that for an isotropic trap the level \( (1,1) \) of the trap becomes a dark-state for this pulse. Finally pulse 8 avoid the formation of undesired unemptied states. One can observe that an almost complete confinement of the atom into the level \( (1,1) \) can be obtained.

Fig. 3 shows in dashed line the dynamics of the probability to occupy the level \( (2,2) \) of the trap, for the case of \( \eta = 3.065, \gamma = 0.1 \omega \) and a sequence of pulses of detunings: \( \delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 11\omega, -19\omega, -10\omega, -5\omega, -\omega \), all of them with duration \( T = 2\gamma/\Omega^2 \). The pulses play the same role as in the previous case. One can observe that the atom can be confined with large probability into the level \( (2,2) \).

Finally, let us consider the cooling mechanism using the dark-states produced by the interference between the two lasers. Fig. 3 shows the cooling dynamics for the case of \( \eta = 3.0, \gamma = 0.1 \omega, \) the initial thermal distribution with the mean \( \langle n \rangle = 6 \), and a sequence of pulses of detunings: \( \delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 0, -19\omega, -10\omega, -5\omega, -2\omega \), all of them with duration \( 2\gamma/\Omega^2 \), except pulse 4 which has a duration \( 8\gamma/\Omega^2 \). For all the pulses we consider the case of \( A = -1 \), except pulse 4 for which we use \( A = 1/8 \), satisfying Eq. (10) for the level \( (0,1) \). As previously, pulses 1 and 5 are confinement pulses, and pulses 2, 3, 6 and 7 are pseudo-confinement pulses. Pulse 4 is a dark-state pulse for the level \( (0,1) \), and pulse 8 avoids the formation
of undesired unemptied states. One can observe that the atom can be confined with large probability into the level (0, 1).

**F. Limitations of the dark-state produced by the interference of absorption amplitudes**

We have shown that the destructive interference between the lasers in both directions allows the improvement of the cooling in the ground state of the harmonic trap, and also provides a new mechanism to cool the atoms in a selected excited state of the trap. Unfortunately this interference mechanism is not valid for large values of the Lamb-Dicke parameter \( \eta \). The reason is the following. The interferential mechanism is based on pulses with detuning \( \delta = 0 \), i.e. pulses whose resonant absorption produces a jump between one state of the trap and itself. For sufficiently large \( \eta \) the Debye-Waller factor \( e^{-\eta^2/2} \), which appears in the Franck-Condon factors (Eq. (4)), becomes very small, and therefore the Franck-Condon factors connecting one state with itself are very small. This fact implies that the empty rates practically vanish for all the states, i.e. all the states of the trap become dark, whatever the value of \( A \), and therefore the interference mechanism is no more valid. The interference mechanism can be employed, roughly speaking, for \( \eta \) not larger than 4.

**V. CONCLUSIONS**

The cooling dynamics of a single atom trapped in one-dimensional and two-dimensional harmonic potentials has been analysed. Using a particular sequence of confinement plus fine-cooling pulses the atom can be cooled into the ground state of the one-dimensional trap. Moreover, by using the special properties of the Franck-Condon factors, a selected excited state of the trap can be made dark, and then a cooling mechanism can be designed to cool into it. We have determined the dark-state condition, and proved via Monte Carlo simulations that the atom can be confined with almost total probability in the selected excited state. However, the formation of the dark-state is very sensitive to the deviations from the dark-state condition. This condition is by far very much less restrictive for the case of many atoms in the trap as we show in [8].

The difficulties to cool in higher dimensions have been analysed on studying the two-dimensional case. In particular one needs a new laser arrangement with two orthogonal lasers and confinement pulses with twice the detuning of the one-dimensional confinement pulses. The difference between the amplitudes of the orthogonal lasers acts as a new degree of freedom that can be used for Lamb-Dicke parameters \( \eta \), to produce a destructive interference between the absorption amplitudes of both lasers. This destructive interference can be used to create dark-states which, as we have shown numerically, allow the improvement of the effectivity of the cooling into the ground-state. Also this interference mechanism can be used to cool into selected excited states of the trap.

For larger Lamb-Dicke parameters \( \eta > 4 \) the interference method is no longer valid. In that regime, the only possibility seems to be the extension of the one-dimensional results of [8] (basically based on the use of blue-detuned lasers to empty the states different than the ground-state) to higher dimensions, by using a two-laser arrangement, and confinement and pseudo-confinement pulses. Fortunately, the dark-state condition based on the properties of the Franck-Condon factors remains effective for large Lamb-Dicke parameters, allowing the excited-state cooling also in this regime.

**Acknowledgements** Partial support from the Spanish Dirección General de Investigación Científica y Técnica (Grant No. PB95-0955) and from the Junta de Castilla y León (Grant No SA 81/96) is acknowledged. L.S. wants to thank M. L. for hospitality during his stay in the Institut für Theoretische Physik of Hannover. This work has been supported by the Deutsche Forschungsgemeinschaft under SFB 407.

[1] A. Aspect, R. Kaiser, N. Vanteenkiste and C. I. Westbrook, Physica Scripta **T58**, 69 (1995).
[2] A. Aspect, E. Arimondo, R. Kaiser, N. Vanteenkiste and C. Cohen-Tannoudji Phys. Rev. Lett., **61**, 826 (1988).
[3] M. Kasevich and S. Chu, Phys. Rev. Lett. **69**, 1741 (1992).
[4] F. Dietrich, J. C. Bergquist, W. M. Itano and D.J. Wineland, Phys. Rev. Lett. **62**, 403 (1989); C. Monroe, D. M. Meekhof, B. E. King, S. R. Jefferts, W. M. Itano, D. J. Wineland and P. Gould, Phys. Rev. Lett. **75**, 4011 (1995).
[5] S. Stenholm, Rev. Mod. Phys. **59**, 699 (1986).
[6] I. Marzoli, J. I. Cirac, R. Blatt and P. Zoller, Phys. Rev. A **49**, 2771 (1994).
[7] R. Dum, P. Marte, T. Pellizari and P. Zoller, Phys. Rev. Lett. **73**, 2829 (1994).
[8] G. Morigi, J. I. Cirac, M. Lewenstein and P. Zoller, Europhys. Lett. **39**, 13 (1997).
[9] L. Santos and M. Lewenstein, (in preparation).
[10] J. I. Cirac and P. Zoller, Phys. Rev. Lett. **74**, 4091 (1995).
[11] J. I. Cirac, A. S. Parkins, R. Blatt and P. Zoller, Adv. At. Mol. Phys. 37, 237 (1996).
[12] M. H. Anderson, J. R. Ensher, M. R. Matthews, C. E. Wieman and E. A. Cornell, Science **269**, 198 (1995); K.B. Davis, M. O. Mewes, M. R. Andrews, N. J. van Druten, D. S. Durfee, D. M. Kurn and W. Ketterle, Phys. Rev. Lett. **75**, 3969 (1995).
[13] Y. Castin, J. I. Cirac and M. Lewenstein, Phys. Rev. Lett.
80, 5305 (1998); see also J. I. Cirac, M. Lewenstein, and P. Zoller, Europhys. Lett. 35, 647 (1996).

FIG. 1. Internal atomic configuration considered throughout the paper.

\[ \begin{align*}
|g\rangle & \\
|e\rangle & \\
|r\rangle & \\
\end{align*} \]

FIG. 2. (a) Empty rates as a function of the trap level, for the case of \( \eta = 3.0, \gamma = 0.01\omega \), and for the case of a pulse with \( \delta = s\omega \), with \( s = 8, -3 \). (b) Dynamics of the occupation probability of the level \( n = 0 \) as a function of the applied cycles, starting from the thermal state with the mean number 6. Each cycle consists on a sequence of four pulses of detunings \( \delta_1,2,3,4 = -9\omega, 8\omega, -10\omega, -3\omega \) and durations \( T_1,2,3,4 = 2\gamma/\Omega^2 \).

FIG. 3. (a) Empty rates as a function of the trap level, for the case of \( \eta = 3.0, \gamma = 0.01\omega \), and a pulse with \( \delta = s\omega \), with \( s = 8, -3 \). (b) Dynamics of the occupation probability of the level \( n = 1 \) as a function of the applied cycles, starting from the thermal state with the mean number 6. Each cycle consists on a sequence of four pulses of detunings \( \delta_1,2,3,4 = -9\omega, 8\omega, -10\omega, -3\omega \) and durations \( T_1,2,3,4 = 2\gamma/\Omega^2 \).

FIG. 4. Dynamics of the occupation probability of the level \( n = 2 \) as a function of the applied cycles, starting from the thermal state with the mean number 6. We consider the case of \( \eta = 3.0, \gamma = 0.01\omega \), and cycles of four pulses of detunings \( \delta_1,2,3,4 = -9\omega, 11\omega, -10\omega, -5\omega \) and durations \( T_1,2,3,4 = 2\gamma/\Omega^2 \).
FIG. 5. Dynamics of the occupation probability of the level $(0,0)$ as a function of the applied cycles, starting from the thermal state with the mean number 6. We consider the case of $\eta = 3.0, \gamma = 0.01\omega$ and cycles of eight pulses of detunings $\delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 0, -19\omega, -10\omega, -5\omega, -\omega$, all of them with duration $T = 2\gamma/\Omega^2$. In solid-line is represented the case of $A = -1$ for all the pulses, whereas in dashed-line the case $A = 1$ is depicted.

FIG. 6. In solid line, the dynamics of the occupation probability of the level $(1,1)$ is depicted as a function of the applied cycles, starting from the thermal state with the mean number 6. We have considered the case of $\eta = 3.0, \gamma = 0.01\omega$, and cycles of eight pulses of detunings $\delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 0, -19\omega, -10\omega, -5\omega, -\omega$, all of them with duration $T = 2\gamma/\Omega^2$. In dashed-line, the dynamics of the occupation probability of the level $(2,2)$ is shown for the case of seven pulses of detunings $\delta_{1,2,3,4,5,6,7} = -18\omega, -9\omega, -4\omega, 11, -19\omega, -10\omega, -5\omega$, all of them with duration $T = 2\gamma/\Omega^2$.

FIG. 7. Dynamics of the occupation probability of the level $(0,1)$ as a function of the applied cycles, starting from a thermal state with a mean number 6. We consider the case of $\eta = 3.0, \gamma = 0.01\omega$, and cycles of eight pulses of detunings $\delta_{1,2,3,4,5,6,7,8} = -18\omega, -9\omega, -4\omega, 0, -19\omega, -10\omega, -5\omega, -\omega$, all of them with duration $T = 2\gamma/\Omega^2$, except pulse 4 which has a duration $T = 8\gamma/\Omega^2$. All of them have $A = -1$ except pulse 4 which has $A = 1/8$. 