MINIMAL SO(10) GRAND UNIFICATION: PREDICTIONS

FOR PROTON DECAY AND NEUTRINO MASSES AND MIXINGS

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ABSTRACT

Prospects for SO(10) as a minimal grand unification group have recently been heightened by several considerations such as the MSW resolution of the solar neutrino puzzle, baryogenesis, possibility for understanding fermion masses etc. I review the present status of the minimal SO(10) models with special emphasis on the predictions for proton lifetime and predictions for neutrino masses for the non-supersymmetric case and discuss some preliminary results for the supersymmetric case. It was generally believed that minimal SO(10) models predict wrong mass relations between the charged fermions of the first and second generations; furthermore, while the smallness of the neutrino masses in these models arises from the see-saw mechanism, it used to be thought that detailed predictions for neutrino masses and mixings require further adhoc assumptions. In this talk, I report some recent work with K.S.Babu, where we discovered that the minimal SO(10) model, both with and without SUSY, has in it a built-in mechanism that not only corrects the bad mass relations between the charged fermions but at the same time allows a complete prediction for the masses and mixings in the neutrino sector. We define our minimal model as the one that consists of the smallest set of Higgs multiplets that are needed for gauge symmetry breaking. Our result is based on the hypothesis that the complex $10$ of Higgs bosons has only a single coupling to the fermions. This hypothesis is guaranteed in supersymmetric models and in non-SUSY models that obey a softly broken Peccei-Quinn symmetry.

1. Introduction:

At the present time, the only experiments where there is some hint of new physics beyond the standard model are the ones which have detected the neutrinos emitted from the solar core[1]. The deficit of solar neutrinos reported in these experiments from Homestake, Kamiokande, SAGE, and GALLEX[2] if confirmed in the other proposed experiments such as the BOREXINO[3] and SNO[4] will confirm that the neutrinos have masses and mixings very much like the quarks. The observed deficit in the present experiments can be explained in terms of neutrino oscillations in two different ways: (i) long wave length vacuum oscillation[5], and (ii) resonant matter oscillation (the Mikheyev-Smirnov-Wolfenstein (MSW) effect[6]). Assuming a two-flavor $\nu_e - \nu_\mu$ oscillation, in the former case, the neutrino masses and mixing angle should satisfy $\Delta m^2 \sim 10^{-10}$ eV$^2$ and $\sin^2\theta_{e\mu} \simeq (0.75$ to $1)$. In case of MSW there are two allowed windows that fit all of the experimental data[7]: (a) the small mixing

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1 Invited talk presented at the workshop on "New Physics with New Experiments" held in Kaimierz, Poland, May, 1993.
angle non–adiabatic solution, which requires $\Delta m^2 \simeq (0.3$ to $1.2) \times 10^{-5} \text{eV}^2$ and $\sin^2 2\theta_{\mu \tau} \simeq (0.4$ to $1.5) \times 10^{-2}$, and (b) the large angle solution with $\Delta m^2 \simeq (0.3$ to $5) \times 10^{-5} \text{eV}^2$ and $\sin^2 2\theta_{\mu \tau} \simeq (0.5$ to $0.9)$. In all these cases, barring an unlikely scenario of near mass degeneracy among neutrinos, either $\nu_\mu$ or $\nu_\tau$ should have mass in the $(10^{-5}$ to $10^{-3}) \text{eV}$ range. Specifically for the MSW resolution, the mass has to be of order $10^{-3}\text{eV}$.

There are two other indications, though more controversial, for a non-vanishing neutrino mass; i) the apparent deficit of GeV muon neutrinos in the cosmic rays that originate from the decay of kaons and pions[8] in the atmosphere; and ii) the need for a hot component in the dark matter of the universe[9]. The first effect would be an indication of a possible oscillation between the $\nu_\mu$ and $\nu_\tau$, again indicating the existence of a non-vanishing mass for the neutrinos. The simplest candidate for the hot component of the dark matter is a neutrino with mass in the few electron volt range. The controversy surrounding the first effect has to do with the uncertainties in the atmospheric muon neutrino fluxes[10], whereas that with the second is due to possible effects of a cosmological constant and other cosmological inputs that go into the discussion of structure formation. In fact if all these three effects are to be taken together, the resulting form of the neutrino mass matrix becomes very seriously constrained[11] and one has to invoke specific kinds of family symmetries to understand them. Our goal in this article will be to see to what extent the present observations motivate the serious consideration of SO(10) as a grand unification symmetry and how much of the data can be understood in this framework.

A natural explanation for the origin of such tiny neutrino masses is the see–saw mechanism[12], which is based on a mass matrix of the following form:

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}$$

Here, the $m_D$ and $M_N$ are $3 \times 3$ matrices denoting the Dirac and the Majorana masses involving the left and the right-handed neutrinos. The diagonalization of the see-saw matrix leads to the light neutrino masses of the following form:

$$M_\nu \simeq \frac{m_D^2}{M_N}$$

The mass matrix $M_N$ corresponds to the scale of $B-L$ breaking and represents physics beyond the standard model. It therefore could be large whereas $m_D$ is characteristic of the electroweak scale and is in the GeV range, thus explaining the smallness of the neutrino masses. The solar neutrino puzzle indicates that the $B-L$ scale is in the $(10^{12} - 10^{16}) \text{GeV}$ range.

All of the observations above, viz., non–zero neutrino masses, the see–saw mechanism, and a high $B-L$ scale, fit rather naturally in grand unified models based on the gauge group $SO(10)$. In its non-supersymmetric version, experimental constraints from proton life-time and the weak mixing angle $\sin^2 \theta_W$ require that $SO(10)$ breaks not directly into the standard model, but at least in two steps. In a two-step breaking scheme, the left–right symmetric intermediate scale is around
In supersymmetric $SO(10)$ there is no need for an intermediate scale, $SO(10)$ can break directly to the standard model at around $10^{16}$ GeV.

There is also another compelling cosmological reason for the $SO(10)$ model which is absent in GUT models, which has to do with the generation of cosmological baryon asymmetry. It has been pointed out that, in $SO(10)$ models with the see-saw mechanism, the right-handed neutrino is very heavy and has a mass near the $B-L$ breaking scale of $10^{12}$ GeV or so. Therefore, its decay can generate a lepton asymmetry when the temperature of the Universe is about $10^{11}$ GeV or so. This lepton asymmetry gets subsequently converted to the baryon asymmetry due the sphaleron effects at around $T \approx 300$ GeV[14]. In investigating the survival of the lepton asymmetry from this high temperature down to the electro-weak phase transition temperature, one must make sure that the lepton violating interactions are out of equilibrium. This is guaranteed only if the $B-L$ symmetry scale is bigger than $10^{11}$ or so. This therefore again points towards an $SO(10)$ theory where such high scales are naturally generated.

To confront $SO(10)$ models with the solar neutrino data, one must make precise predictions of the neutrino masses and mixing angles. This requires, however, detailed information of the Dirac neutrino mass matrix as well as the Majorana matrix. In grand unified theories (GUTs), it is possible to relate the quark masses with the lepton masses. It used to be thought that in simple $SO(10)$ models, the charge $-1/3$ quark mass matrix is related to the charged lepton matrix and the neutrino Dirac mass matrix is related to the charge $2/3$ quark matrix at the unification scale. No simple simple way was known, in general, to relate the heavy Majorana matrix to the charged fermion observables. This prevented any prediction of light neutrino spectrum without making extra symmetry assumptions.

In a recent paper[15] it was shown Babu and this author that this situation resulted from an incomplete analysis of the Higgs sector of the model and we showed that in a class of minimal $SO(10)$ models, not only the Dirac neutrino matrix, but the Majorana matrix also gets related to observables in the charged fermion sector. This leads to a complete prediction for the neutrino masses and mixings without adding any extra Higgs multiplets or any new symmetries to the theory. We use a simple Higgs system with one (complex) 10 and one 126 that have Yukawa couplings to fermions. The 10 is needed for quark and lepton masses, the 126 is needed for the see–saw mechanism. Crucial to the predictivity of the neutrino spectrum is the observation that the standard model doublet contained in the 126 receives an induced vacuum expectation value (vev) at tree–level. In its absence, one would have the asymptotic mass relations $m_b = m_{\tau}$, $m_s = m_{\mu}$, $m_d = m_e$. While the first relation would lead to a successful prediction of $m_b$ at low energies, the last two are in disagreement with observations. The induced vev of the standard doublet of 126 corrects these bad relations and at the same time also relates the Majorana neutrino mass matrix to observables in the charged fermion sector, leading to a predictive neutrino spectrum.
2. Minimal SO(10) GUT without supersymmetry:

In this section, we shall consider non-SUSY SO(10) model to illustrate our mechanism. In the minimal scenario defined here, the SO(10) symmetry breaks to the standard model via the $SU(2)_L \times SU(2)_R \times SU(4)_C \equiv G_{224}$ chain. The breaking of SO(10) via $G_{224}$ is achieved by either a 54 or a 210 of Higgs. The 210 also breaks the discrete $D$–parity,[16] whereas the 54 preserves it. $D$–parity is a local discrete $Z_2$–subgroup of SO(10), under $D$, a fermion field $f$ transforms into its charge conjugate $f^c$. Breaking of $D$–parity at the GUT scale makes the see–saw mechanism natural[17].It further changes the evolution of the gauge coupling constants by making the Higgs spectrum left-right asymmetric. It also eliminates the cosmological domain wall problem that can arise if this $Z_2$ local symmetry survived to the intermediate scale. We will therefore work with the 210 Higgs multiplet, although our mechanism to cure the fermion mass problem and predict the neutrino masses is independent of this choice. We will denote this chain with $SU(4)_c$ as intermediate symmetry as chain A. If instead of a 210 dim. Higgs multiplet, we chose a 45 plus 54 combination, then the SO(10) symmetry would break down to $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c \equiv G_{2213}$ group.In this case, also D-parity breaks at the GUT scale, thereby making the see-saw formula natural. We will denote this case as chain B. The second stage of symmetry breaking goes via the 126 in both cases. Finally, the standard model electro–weak symmetry breaking proceeds via the 10.

2a. Mass scales and Proton decay:

Before discussing the fermion sector of the theory, let us discuss the mass scales as predicted by the low energy values of $\sin^2 \theta_W$, $\alpha_{\text{strong}}$ and $\alpha_{\text{em}}$. This analysis was first carried out for chains with D-parity breaking by Chang et. al. in ref.13 and recently been reanalyzed by Deshpande et al.[13]. The threshold corrections due to heavy particles was carried out recently by Parida and this author[18]. In the threshold correction analysis, the survival hypothesis was used to determine which Higgs submultiplets are at what mass scale and then allowing for reasonable uncertainties in the heavy Higgs boson masses, the uncertainties in the mass scales was estimated. It was found that the different mass scales and the uncertainties in their values depend on the symmetry breaking chain and we have:

Chain A:

$$M_U = 10^{15.8^{+8}_{-1.7}} \pm 2 GeV$$

$$M_I = 10^{11.5^{+2.1}_{-0.2}} \pm 0.02 GeV$$

(3)

Chain B:

$$M_U = 10^{15.8^{+4.1}_{-1.2}} \pm 25 GeV$$

$$M_I = 10^{9.5^{+3}_{-1.8}} \pm 18 GeV$$

(4)
The significance of these results is that the $M_U$ governs the life-time of the proton in $SO(10)$ models whereas the $M_I$ governs the neutrino masses via the see-saw mechanism. We will come to the neutrino masses later. The proton life-time in both symmetry breaking chains is given by [18]:

\[
\begin{align*}
\text{ChainA:} & \quad \tau_p = 1.6 \times 10^{35\pm 0.9\pm 3.2} \text{ years} \\
\text{chainB:} & \quad \tau_p = 1.6 \times 10^{35\pm 7\pm 1.0\pm 8} \text{ years}
\end{align*}
\]

In the above formulae, the last uncertainties are from the threshold corrections whereas the first and second are from the matrix element and $\alpha_{str}$ uncertainties respectively. It is clear from eq.(3) that the threshold uncertainties in $M_I$ and $M_U$ are so large that one could consider this as an almost single scale theory. This possibility has been studied in detail in ref.19 and indeed, the results turn out to be quite consistant with the present lower limits on proton lifetime.

2b. Fermion masses:

Let us now turn to the fermion sector. we denote the three families belonging to 16–dimensional spinor representation of $SO(10)$ by $\psi_a$, $a = 1 - 3$, the complex 10–plet of Higgs by $H$, and the 126–plet of Higgs by $\Delta$, the Yukawa couplings can be written down as

\[
L_Y = h_{ab} \psi_a \psi_b H + f_{ab} \psi_a \psi_b \Delta + H.C.
\]

Note that since the 10–plet is complex, one other coupling $\psi_a \psi_b \Phi$ is allowed in general. In SUSY–$SO(10)$, the requirement of supersymmetry prevents such a term. In the non–SUSY case, which we are focussing on in this section, we forbid this term by imposing a $U(1)_{PQ}$ symmetry, which may anyway be needed in order to solve the strong CP problem.

The 10 and 126 of Higgs have the following decomposition under $G_{224}$: $126 \rightarrow (1,1,6) + (1,3,10) + (3,1,\overline{10}) + (2,2,15), \quad 10 \rightarrow (1,1,6) + (2,2,1)$. Denote the (1,3,10) and (2,2,15) components of $\Delta(126)$ by $\Delta_R$ and $\Sigma$ respectively and the (2,2,1) component of $H(10)$ by $\Phi$. The vev $<\Delta_R^0> = v_R \sim 10^{12} \text{ GeV}$ breaks the intermediate symmetry down to the standard model and generates Majorana neutrino masses given by $f v_R$. $\Phi$ contains two standard model doublets which acquire vev’s denoted by $\kappa_u$ and $\kappa_d$ with $\kappa_u, \kappa_d \sim 10^2 \text{ GeV}$. $\kappa_u$ generates charge 2/3 quark as well as Dirac neutrino masses, while $\kappa_d$ gives rise to $-1/3$ quark and charged lepton masses.

Within this minimal picture, if $\kappa_u, \kappa_d$ and $v_R$ are the only vev’s contributing to fermion masses, in addition to the $SU(5)$ relations $m_b = m_c$, $m_s = m_\mu$, $m_d = m_e$, eq. (7) will also lead to the unacceptable relations $m_u : m_c : m_t = m_d : m_s : m_b$. Moreover, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix will be identity. Fortunately, within this minimal scheme, we have found new contributions to the fermion mass matrices which are of the right order of magnitude to correct these bad relations. To see this, note that the scalar potential contains, among other terms, a crucial term

\[
V_1 = \lambda \Delta \Delta \Delta H + H.C.
\]
Such a term is invariant under the $U(1)_{PQ}$ symmetry. It will be present in the SUSY $SO(10)$ as well, arising from the $210$ $F$–term or perhaps as a Planck induced term etc. This term induces vev’s for the standard doublets contained in the $\Sigma$ multiplet of $126$. The vev arises through a term $\Sigma_R \Delta_R \Sigma \Phi$ contained in $V_1$.

We can estimate the magnitudes of the induced vev’s of $\Sigma$ (denoted by $v_u$ and $v_d$ along the up and down directions) assuming the survival hypothesis to hold:

$$v_{u,d} \sim \lambda \left( \frac{v_R^2}{M_{\Sigma_{u,d}}^2} \right) \kappa_{u,d}.$$  \hspace{1cm} (9)

Suppose $M_U \sim 10^{15}$ GeV, $M_I \sim 3 \times 10^{12}$ GeV as in the chain A and $M_\Sigma \sim 10^{14}$ GeV, consistent with survival hypothesis, then $v_u$ and $v_d$ are of order 100 MeV, in the right range for correcting the bad mass relations. We emphasize that there is no need for a second fine–tuning to generate such induced vev’s. In the SUSY version with no intermediate scale, the factor $(v_R^2/M_{\Sigma}^2)$ is not a suppression, so the induced vev’s can be as large as $\kappa_{u,d}$. This is the key observation of ref.15, which leads to the predictive $SO(10)$ model in the neutrino sector, while removing the disagreement with charged fermion, masses that were thought to exist in the minimal model before.

Another way to view this is to realize that the effect of the mixing is to leave a pair of light standard model doublets are admixtures of doublets in $(2,2,1)$ and $(2,2,15)$ multiplets and these light doublets acquire vevs at low energies leading to the same mass patterns as before.

It is also worth noting that if we considered the symmetry breaking chain B, then since the intermediate scale is at most $10^{10}$GeV, the induced vevs $v_u$ and $v_d$ would be very small and would not provide any useful correction to the bad charged fermion mass relations. Thus, our scenerio would prefer the $SU(4)_C$ intermediate symmetry chain.

3. The charged fermion masses:

We are now in a position to write down the quark and lepton mass matrices of the model:

$$M_u = h\kappa_u + f v_u \hspace{1cm} M_d = h\kappa_d + f v_d$$
$$M_D^\nu = h\kappa_u - 3f v_u \hspace{1cm} M_l = h\kappa_d - 3f v_d$$

$$M_M^\nu = f v_R.$$  \hspace{1cm} (10)

Here $M_D^\nu$ is the Dirac neutrino matrix and $M_M^\nu$ is the Majorana mass matrix.

Before proceeding, we should specify the origin of CP violation in the model. We shall assume that it is spontaneous or soft, that will keep the number of parameters at a minimum. The Higgs sector described above already has enough structure to generate realistic CP violation either softly or spontaneously. The Yukawa coupling matrices $h$ and $f$ in this case are real and symmetric. Although there will be three different phases in the vev’s (one common phase for $\kappa_u$ and $\kappa_d$ and one each
for \( v_u \) and \( v_d \), only two combinations enter into the mass matrices, as the overall phase can be removed from each sector. We shall bring these two phases into \( v_u \) and \( v_d \) and hence forth denote them by \( v_u e^{i\alpha} \) and \( v_d e^{i\beta} \).

To see the predictive power of the model as regards the neutrino spectrum, note that we can choose a basis where one of the coupling matrices, say \( h \), is real and diagonal. Then there are 13 parameters in all, not counting the superheavy scale \( v_R \): 3 diagonal elements of the matrix \( h_{\kappa u} \), 6 elements of \( f_{\nu u} \), 2 ratios of vev’s \( r_1 = \kappa_d/\kappa_u \) and \( r_2 = v_d/v_u \), and the two phases \( \alpha \) and \( \beta \). These 13 parameters are related to the 13 observables in the charged fermion sector, viz., 9 fermion masses, 3 quark mixing angles and one CP violating phase. The light neutrino mass matrix will then be completely specified in terms of other physical observables and the overall scale \( v_R \). That would lead to 8 predictions in the lepton sector: 3 leptonic mixing angles, 2 neutrino mass ratios and 3 leptonic CP violating phases.

The relations of eq. (10) hold at the intermediate scale \( M_I \) where quark–lepton symmetry and left–right symmetry are intact. There are calculable renormalization corrections to these relations below \( M_I \). The quark and charged lepton masses as well as the CKM matrix elements run between \( M_I \) and low energies. The neutrino masses and mixing angles, however, do not run below \( M_I \), since the right-handed neutrinos have masses of order \( M_I \) and decouple below that scale. The predictions in the neutrino sector should then be arrived at by first extrapolating the charged fermion observables to \( M_I \).

We fix the intermediate scale at \( M_I = 10^{12} \text{ GeV} \) and use the one–loop standard model renormalization group equations to track the running of the gauge couplings between \( M_Z \) and \( M_I \).

To compute the renormalization factors, we choose as low energy inputs the gauge couplings at \( M_Z \) to be \( \alpha_1(M_Z) = 0.01688 \), \( \alpha_2(M_Z) = 0.03322 \), \( \alpha_3(M_Z) = 0.11 \). For the light quark (running) masses, we choose values listed in Ref. 20. The top–quark mass will be allowed to vary between 100 and 200 GeV. Between 1 GeV and \( M_Z \), we use two–loop QCD renormalization group equations for the running of the quark masses and the \( SU(3)_C \) gauge coupling, treating particle thresholds as step functions. From \( M_Z \) to \( M_I \), the running factors are computed semi–analytically both for the fermion masses and for the CKM angles by using the one–loop renormalization group equations for the Yukawa couplings and keeping the heavy top–quark contribution. The running factors, defined as \( \eta_i = m_i(M_I)/m_i(M_Z) \) for light quarks \((u, d, s)\) are \( \eta(u, c, t) = (0.273, 0.286, 0.506) \), \( \eta(d, s, b) = (0.279, 0.279, 0.327) \), \( \eta(e, \mu, \tau) = 0.960 \) for the case of \( m_t = 150 \text{ GeV} \). The (common) running factors for the CKM angles (we follow the parameterization advocated by the Particle Data Group) \( S_{23} \) and \( S_{13} \) is 1.081 for \( m_t = 150 \text{ GeV} \). The Cabibbo angle \( S_{12} \) and the KM phase \( \delta_{KM} \) are essentially unaltered.

Let us first analyze the mass matrices of eq. (10) in the limit of CP conservation. We shall treat spontaneous CP violation arising through the phases of the vev’s \( v_u e^{i\alpha} \) and \( v_d e^{i\beta} \) as small perturbations. This procedure will be justified a posteriori. In fact, we find that realistic fermion masses, in particular the first
family masses, require these phases to be small.

We can rewrite the mass matrices $M_l, M_D^\nu$ and $M_M^\nu$ of eq. (10) in terms of the quark mass matrices and three ratios of vev’s $r_1 = \kappa_d/\kappa_u$, $r_2 = v_d/v_u$, $R = v_u/v_R$:

$$
M_l = \frac{4r_1r_2}{r_2 - r_1} M_u - \frac{r_1 + 3r_2}{r_2 - r_1} M_d,
$$

$$
M_D^\nu = \frac{3r_1 + r_2}{r_2 - r_1} M_u - \frac{4}{r_2 - r_1} M_d,
$$

$$
M_M^\nu = \frac{1}{Rr_1 - r_2} M_u - \frac{1}{Rr_1 - r_2} M_d.
$$

(11)

It is convenient to go to a basis where $M_u$ is diagonal. In that basis, $M_d$ is given by $M_d = V M_d^{\text{diagonal}} V^T$, where $M_d^{\text{diagonal}} = \text{diag}(m_d, m_s, m_b)$ and $V$ is the CKM matrix. One sees that $M_l$ of eq. (11) contains only physical observables from the quark sector and two parameters $r_1$ and $r_2$. In the CP–conserving limit then, the three eigen–values of $M_l$ will lead to one mass prediction for the charged fermions. To see this prediction, $M_l$ needs to be diagonalized. Note first that by taking the Trace of $M_l$ of eq. (11), one obtains a relation for $r_1$ in terms of $r_2$ and the charged fermion masses. This is approximately

$$
r_1 \simeq (m_\tau + 3m_b)/4m_t
$$

(12)

(as long as $r_2$ is larger than $m_b/m_t$). Since $|m_b| \simeq |m_\tau|$ at the intermediate scale to within 30% or so, depending on the relative sign of $m_b$ and $m_\tau$, $r_1$ will be close to either $m_b/m_t$ or to $(m_b/2m_t)$. Note also that if $r_2 \gg r_1$, $M_l$ becomes independent of $r_2$, while $M_D^\nu$ retains some dependence:

$$
M_l \simeq 4r_1 M_u - 3M_d, \quad M_D^\nu \simeq M_u - \frac{4}{r_2} M_d.
$$

(13)

This means that the parameter $r_2$ will only be loosely constrained from the charged fermion sector.

We do the fitting as follows. For a fixed value of $r_2$, we determine $r_1$ from the $\text{Tr}(M_l)$ using the input values of the masses and the renormalization factors discussed above. $M_l$ is then diagonalized numerically. There will be two mass relations among charged fermions. Since the charged lepton masses are precisely known at low energies, we invert these relations to predict the $d$–quark and $s$–quark masses. The $s$–quark mass is sensitive to the muon mass, the $d$–mass is related to the electron mass. This procedure is repeated for other values of $r_2$. For each choice, the light neutrino masses and the leptonic CKM matrix elements are then computed using the see–saw formula.

4. Predictions for neutrino masses and mixings:

We find that there are essentially three different solutions. A two–fold ambiguity arises from the unknown relative sign of $m_b$ and $m_\tau$ at $M_l$. Although solutions exist for both signs, we have found that a relative minus sign tends to result in
somewhat large value of $m_s/m_d$. Our numerical fit shows that the loosely
constrained parameter $r_2$ cannot be smaller than 0.1 or so, otherwise the $d$–quark mass
comes out too small. Now, the light neutrino spectrum is sensitive to $r_2$ only when
$r_2 \sim 4m_s/m_c \sim \pm 0.4$, since the two terms in $M^D$ become comparable (for the second
family) then. Two qualitatively different solutions are obtained depending on
whether $r_2$ is near $\pm 0.4$ or not.

Numerical results for the three different cases are presented below. The input
values of the CKM mixing angles are chosen for all cases to be $S_{12} = -0.22$, $S_{23} =
0.052$, $S_{13} = 6.24 \times 10^{-3}$. Since $\delta_{KM}$ has been set to zero for now, we have allowed
the mixing angles to have either sign. Not all signs result in acceptable quark
masses though. Similarly, the fermion masses can have either sign, but these are
also restricted. The most stringent constraint comes from the $d$–quark mass, which
has a tendency to come out too small. Acceptable solutions are obtained when
$\theta_{23}$, $\theta_{13}$ are in the first quadrant and $\theta_{12}$ in the fourth quadrant.

Solution 1:

\[
\begin{align*}
\text{Input : } & m_u(1 \text{ GeV}) = 3 \text{ MeV, } m_c(m_c) = 1.22 \text{ GeV, } m_t = 150 \text{ GeV} \\
& m_b(m_b) = -4.35 \text{ GeV, } r_1 = -1/51.2, \quad r_2 = 2.0 \\
\text{Output : } & m_d(1 \text{ GeV}) = 6.5 \text{ MeV, } m_s(1 \text{ GeV}) = 146 \text{ MeV} \\
& \left( m_{\nu_e}, m_{\nu_{\mu}}, m_{\nu_\tau} \right) = R \left( 2.0 \times 10^{-2}, 9.9, -2.3 \times 10^{4} \right) \text{ GeV} \\
& V_{\text{lepton}}^{KM} = \begin{pmatrix} 0.9488 & 0.3157 & 0.0136 \\ -0.3086 & 0.9349 & -0.1755 \\ -0.0681 & 0.1623 & 0.9844 \end{pmatrix}.
\end{align*}
\]

(14)

Solution 2:

\[
\begin{align*}
\text{Input : } & m_u(1 \text{ GeV}) = 3 \text{ MeV, } m_c(m_c) = 1.22 \text{ GeV, } m_t = 150 \text{ GeV} \\
& m_b(m_b) = -4.35 \text{ GeV, } r_1 = -1/51, \quad r_2 = 0.2 \\
\text{Output : } & m_d(1 \text{ GeV}) = 5.6 \text{ MeV, } m_s(1 \text{ GeV}) = 156 \text{ MeV} \\
& \left( m_{\nu_e}, m_{\nu_{\mu}}, m_{\nu_\tau} \right) = R \left( 7.5 \times 10^{-3}, 2.0, -2.8 \times 10^{3} \right) \text{ GeV} \\
& V_{\text{lepton}}^{KM} = \begin{pmatrix} 0.9961 & 0.0572 & -0.0676 \\ -0.0665 & 0.9873 & -0.1446 \\ 0.0584 & 0.1485 & 0.9872 \end{pmatrix}.
\end{align*}
\]

(15)

Solution 3:

\[
\begin{align*}
\text{Input : } & m_u(1 \text{ GeV}) = 3 \text{ MeV, } m_c(m_c) = 1.27 \text{ GeV, } m_t = 150 \text{ GeV} \\
& m_b(m_b) = -4.35 \text{ GeV, } r_1 = -1/51.1, \quad r_2 = 0.4 \\
\text{Output : } & m_d(1 \text{ GeV}) = 6.1 \text{ MeV, } m_s(1 \text{ GeV}) = 150 \text{ MeV} \\
& \left( m_{\nu_e}, m_{\nu_{\mu}}, m_{\nu_\tau} \right) = R \left( 4.7 \times 10^{-2}, 1.4, -5.0 \times 10^{3} \right) \text{ GeV}
\end{align*}
\]
\[ V_{KM} ^{\text{lepton}} = \begin{pmatrix} 0.9966 & 0.0627 & -0.0541 \\ -0.0534 & 0.9858 & 0.1589 \\ 0.0633 & -0.1555 & 0.9858 \end{pmatrix} . \] \tag{16}

Solution 1 corresponds to choosing \( r_1 \sim m_b/m_s \). All the charged lepton masses are negative in this case. Since \( r_2 \) is large, the Dirac neutrino matrix is essentially \( M_s \), which is diagonal; so is the Majorana matrix. All the leptonic mixing angles arise from the charged lepton sector. Note that the predictions for \( m_d \) and \( m_s \) are within the range quoted in Ref. 20. The ratio \( m_s/m_d = 22 \) is within the allowed range from chiral perturbation theory estimates\[22\]. The mixing angle \( \sin \theta_{\nu_e - \nu_\mu} \) relevant for solar neutrinos is 0.30, close to the Cabibbo angle. Such a value may already be excluded by a combination of all solar neutrino data taken at the 90\% CL (but not at the 95\% CL)\[7\]. Actually, within the model, there is a more stringent constraint. Note that the \( \nu_\mu - \nu_\tau \) mixing angle is large, it is approximately \( 3|V_{cb}| \simeq 0.16 \). For that large a mixing, constraints from \( \nu_\mu - \nu_\tau \) oscillation experiments imply\[23\] that
\[ |m_{\nu_\mu}^2 - m_{\nu_\tau}^2| \lesssim 4 \text{ eV}^2 . \]
Solution 1 also has \( m_{\nu_\mu}/m_{\nu_\tau} \simeq 2.3 \times 10^{3} \), requiring that \( m_{\nu_\mu} \lesssim 0.9 \times 10^{-3} \text{ eV} \). This is a factor of 2 too small for \( \nu_e - \nu_\mu \) MSW oscillation for the solar puzzle (at the 90\% CL), but perhaps is not excluded completely, once astrophysical uncertainties are folded in. If \( \nu_\tau \) mass is around \( 2 \times 10^{-3} \text{ eV} \), \( \nu_\tau - \nu_\tau \) oscillation may be relevant, that mixing angle is \( \simeq 3|V_{ud}| \simeq 6\% \). It would require the parameter \( R = v_u/v_R \simeq 10^{-16} \) or \( v_R \sim 10^{16} \text{ GeV} \) for \( v_u \sim 1 \text{ GeV} \). Such a scenario fits very well within SUSY–SO(10).

Solution 2 differs from 1 in that \( r_2 \) is smaller, \( r_2 = 0.2 \). The ratio \( m_s/m_d = 27.8 \) is slightly above the limit in Ref. 22. The 1–2 mixing in the neutrino sector is large in this case, so it can cancel the Cabibbo like mixing arising from the charged lepton sector. As we vary \( r_2 \) from around 0.2 to 0.6, this cancellation becomes stronger, the \( \nu_e - \nu_\mu \) mixing angle becoming zero for a critical value of \( r_2 \). For larger \( r_2 \), the solution will approach Solution 1. The \( \nu_\mu - \nu_\tau \) mixing angle is still near \( 3|V_{cb}| \), so as before, \( m_{\nu_\tau} \lesssim 2 \text{ eV} \). From the \( \nu_\tau/\nu_\mu \) mass ratio, which is \( 1.4 \times 10^{3} \) in this case, we see that \( m_{\nu_\mu} \lesssim 1.5 \times 10^{-3} \text{ eV} \). This is just within the allowed range\[7\] (at 95\% CL) for small angle non–adiabatic \( \nu_e - \nu_\mu \) MSW oscillation, with a predicted count rate of about 50 SNU for the Gallium experiment. Note that there is a lower limit of about 1 eV for the \( \nu_\tau \) mass in this case. Forthcoming experiments should then be able to observe \( \nu_\mu - \nu_\tau \) oscillations. A \( \nu_\tau \) mass in the (1 to 2) eV range can also be cosmologically significant, it can be at least part of the hot dark matter. In SUSY SO(10), \( \nu_e - \nu_\tau \) oscillation (the relevant mixing is about \( 3|V_{ud}| \simeq 5\% \)), could account for the solar neutrino puzzle.

Solution 3 corresponds to choosing \( r_1 = 0.4, m_s/m_d = 24.6 \) is within the allowed range. However, the mass ratio \( \nu_\tau/\nu_\mu \) is \( \sim 3.6 \times 10^{3} \), and \( \sin \theta_{\nu_\tau} \simeq 3|V_{cb}| \) so \( \nu_e - \nu_\mu \) oscillation cannot be responsible for solar MSW. As in other cases, \( \nu_e - \nu_\tau \) MSW oscillation with a 6\% mixing is a viable possibility.

Let us now re-instating the CP–violating phases \( \alpha \) and \( \beta \) in the vev’s perturbatively. Small values of the phases are sufficient to account for realistic CP violation in the quark sector. We shall present details for the case of Solution 2 only, others
are similar. We also tried to fit all the charged fermion masses and mixing angles for large phases, but found no consistent solution.

First we make a basis transformation to go from the basis where \( M_u \) is diagonal to one where the matrix \( h_{\kappa u} \) is diagonal. It is easier to introduce phases in that basis. For \( \alpha = 3.5^0, \beta = 4.5^0 \), the CP–violating parameter \( J \) for the quark system[24] is \( J \simeq 1 \times 10^{-5} \), which is sufficient to accommodate \( \epsilon \) in the neutral \( K \) system. The leptonic CP violating phases are correspondingly small, for \( \epsilon_l \), the analog of \( J \) is \( J_l \simeq 7 \times 10^{-5} \). These small phases modify the first family masses slightly, but the effect is less than 10\%. Our predictions for the neutrino mixing angles are essentially unaltered.

Recently, three more solutions for neutrino masses have been found by Lavoura, which give good fits to the charged fermion masses[25]. We refer to ref.25 for details on them.

5. Supersymmetric minimal SO(10) and its implications:

In this section, we consider the supersymmetric version of our model. The fermions are part of the chiral superfields transforming as the \( 16 \)-dimensional spinor representation of the \( SO(10) \) group and are denoted by \( \psi \). We will choose the Higgs multiplets for the supersymmetric version the same way for the non-SUSY case, i.e., Higgs fields belonging to \( 10, \bar{126} \) and \( 210 \) representations (denoted by \( H, \bar{\Delta} \) and \( \Phi \) respectively) but we have to include a \( 126 \) dimensional multiplet (denoted by \( \Delta \)) to maintain supersymmetry down to the electro-weak scale. The most general superpotential for the model can be written as:

\[
W = W_m + \mu_1 \Phi^2 + \lambda_1 \Phi^3 + \lambda_2 \Delta \bar{\Delta} \Phi + \mu_2 \Delta \bar{\Delta} + \lambda_3 \Phi \Delta H + \lambda_4 \Phi \bar{\Delta} \bar{H} + \mu_3 H^2
\]

where

\[
W_m = h_{ab} \psi_a \psi_b H + f_{ab} \psi_a \psi_b \bar{\Delta}
\]

The gauge and supersymmetry breaking is determined by writing down the \( F \)-terms and setting them to zero at the GUT scale. The detailed analysis of this is given in ref. 26. Here, we simply quote the result of that paper which says that if we do not fine–tune any of the parameters of the superpotential in the above equation, the vanishing of \( F \)-terms requires that the \( SO(10) \) symmetry break down to the standard model in one step. This implies that \( v_R \simeq M_U \). This has several interesting implications as we will see later. Unfortunately, as in all SUSY GUT models, in the supersymmetric limit, there are several degenerate vacua corresponding to the symmetry groups i) \( SU(5) \), ii) \( SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_c \); iii) \( SU(2)_L \times U(1)_{B-L} \times SU(3)_c \) iv) \( SU(5) \times U(1) \) v) \( SO(10) \). Once supergravity corrections are included this degeneracy disappears, and by appropriate choice of parameters, we can always choose the global minimum to correspond to the standard model.

Doublet-Triplet Splitting:
The second point to notice is that in the Higgs potential derived from this superpotential, there are terms of the form $\Delta \Delta \Delta H$ proportional to $\lambda_2 \lambda_3$ and $\lambda_2 \lambda_4$ so that a vev for the $(2, 2, 15)$ sub-multiplet of $126$ is induced once $SO(10)$ and $B - L$ are broken, in analogy with the non–SUSY case. The detailed analysis of this question is tied to the question of doublet-triplet splitting of SUSY-GUT models. Let us therefore briefly discuss this question below. In the minimal $SO(10)$ model there are eight light Higgs doublets, four $H_u$ type (i.e. capable of giving masses to up quarks) and four $H_d$ type which can give mass to down type quarks. After GUT symmetry breaking, one therefore has a $4 \times 4$ matrix and for the theory to reproduce low energy physics, this matrix must have two massless eigen-modes. In general this will require some fine tuning of parameters so that all doublets do not acquire GUT scale mass. At the same time, this model has a $5 \times 5$ mass matrix involving the color triplet fields. Successful doublet-triplet splitting requires that the same fine-tuning that leaves a pair $(H_u, H_d)$ massless should not at the same time leave any triplets below the GUT scale, otherwise, not only will there be problem with proton life–time, there will also be problem with unification of couplings. The doublet–triplet splitting is called natural when the required fine–tuning is guaranteed by some symmetry.

To study this question in detail, let us isolate the various doublet and triplet fields in the model: The four up Higgs doublets are denoted by: $H_u$ from $10$, $\sigma_u$ and $\sigma^u$ from $126$ and $\overline{126}$ respectively and $\alpha_u$ from $210$ and similarly four down type doublets from the same Higgs representations. The five color triplets fields belonging to $3$ rep. and electric charge $-1/3$ are denoted by $\zeta_1$ from $10$, $\gamma_1$ from $\overline{126}$, $\gamma_2$ from $126$, $\omega$ from $(1, 3, 15)$ in $210$ and $\delta_R$ from $(1, 3, 10)$ in $\overline{126}$. There is of course a similar set belonging to $\overline{3}$ to be denoted by $\zeta_2, \gamma_2, \gamma_1, \overline{\omega}$ and $\delta_R$. The $4 \times 4$ Dirac matrix involving the doublets can be written as follows: (the basis for this matrix is $(H_u, \sigma_u, \sigma^u, \alpha_u)$ denoting the columns and corresponding fields with subscripts d denoting the rows).

$$M_D = \begin{pmatrix}
\mu_3 & \lambda_3 v_U & \lambda_4 v_U & \lambda_4 v_R \\
\lambda_3 v_U & 0 & \bar{\mu}_2 & \lambda_2 v_R \\
\lambda_4 v_U & \bar{\mu}_2 & 0 & 0 \\
\lambda_3 v_R & 0 & \lambda_2 v_R & \bar{\mu}_1 \\
\end{pmatrix} \tag{19}$$

In the above matrix, $v_R$ and $v_U$ stand for the vev’s of the $126$ and $210$ dim. Higgs multiplets respectively and the symbols $\bar{\mu}_i$ denote some combination of $\mu_i$ with the vev contributions. The color triplet matrix in the basis $(\zeta_1, \gamma_1, \overline{\omega}, \delta_R)$ for columns and corresponding $\overline{3}$ denoting rows is:
\[ MT = \begin{pmatrix} \mu_3 & \lambda_4 v_U & \lambda_3 v_U & \lambda_4 v_R & \lambda_4 v_U \\ \lambda_4 v_U & 0 & \tilde{\mu}_2 & 0 & 0 \\ \lambda_3 v_U & \tilde{\mu}_2 & 0 & \lambda_2 v_R & \lambda_2 v_U \\ \lambda_3 v_R & \lambda_2 v_R & 0 & \tilde{\mu}_1 & \lambda_2 v_R \\ \lambda_3 v_U & \lambda_2 v_U & 0 & \lambda_2 v_R & \tilde{\mu}_2 \end{pmatrix} \] (20)

We wish to point out that, the entries in the above matrices do not contain the detailed numerical coefficients nor the various vevs corresponding to the multiplets- rather just the orders of magnitudes. It is clear that, one needs fine tuning to get a light pair of Higgs doublets. For our scheme to work, we need a linear combination of \( H_u \) and \( \sigma_u \) (and similarly with subscript d) to remain massless at the GUT scale. There are several ways to achieve this. One particularly simple way is to set \( \lambda_3 = \lambda_4 = 0 \) and add to the theory a Planck scale induced term of the form \( 10^{126} \frac{126}{M_{Pl}} \) such that it balances against the \( \mu_3 \) term after GUT scale symmetry breaking. At this stage, there are two light doublets, which arise only from the \( 10 \) Higgs. But if we add a further Planck induced term of the form \( 10^{126} \frac{126}{M_{Pl}} \), then this causes an admixture of \( \sigma_u \) with \( H_u \) and similarly for \( H_d \) as required. The corresponding color triplet will however be somewhat light and considerations of proton decay then suggests that the coefficients of the Planck induced term be rather large.

The pattern of the predictions for the neutrino masses and mixings in the SUSY version is very similar to the non-SUSY version.

6. Summary and Conclusion:

In summary, we feel that if the present indications for neutrino masses in the micro-, milli-, and eV range continues to receive full confirmation, then the SUSY \( SO(10) \) may provide the correct framework for the next stage in the search for further unification of forces and matter. Our recent work described in this report would suggest that the relevant \( SO(10) \) theory could very well be the minimal one with just three Higgs multiplets needed to break the gauge symmetries appropriately. Just like the minimal \( SU(5) \) had the unique feature that it predicted a value for both proton lifetime and \( \sin^2 \theta_W \), and could therefore be tested experimentally, the minimal \( SO(10) \) grand unified model leads to definite predictions for light neutrino masses and mixing angles in terms of observables in the charged fermion sector and could therefore be tested in the next round of neutrino oscillation experiments. This in our opinion elevates minimal \( SO(10) \) theory above other competing GUT scenarios that can lead to nonvanishing neutrino masses such as \( SU(5) \times U(1) \) or \( E_6 \) theories in terms of testability. In a sense, the status of minimal \( SO(10) \) is better than the minimal \( SU(5) \) since here we fit all charged fermion masses without invoking additional Higgs multiplets. Of course if we want further predictability, say in the charged fermion sector, we will certainly need new symmetries. This line of research is currently being pursued.
We also wish to point out that our approach here has been orthogonal to some other recent attempts[27] based on grand unification; we have kept the Higgs sector as simple as possible and followed its consequences. In particular, our results are manifestly stable under radiative corrections. Moreover, we have not used any sort of family symmetry to make the model predictive. Predictivity in the neutrino sector follows from minimality. To repeat again, we have found three different types of solutions for the neutrino spectrum. In Solution 1, the $\nu_e - \nu_\mu$ mixing angle is near the Cabibbo angle, while Solutions 2 and 3 have it much smaller. In all cases, $\nu_e - \nu_\tau$ mixing angle is predicted to be near $3|V_{td}| \simeq 0.05$ and $\nu_\mu - \nu_\tau$ mixing angle is $\simeq 3|V_{cb}| \simeq 0.15$ with the mass ratio $m_{\nu_e}/m_{\nu_\mu} \geq 10^3$. None of the solutions admit the vacuum oscillation solution to the solar neutrino puzzle. If it is due to small angle non–adiabatic MSW, as in Solution 2, $\nu_\mu - \nu_\tau$ oscillation should be observable in the forthcoming experiments.

On the theoretical side, there is one important lesson for SO(10) model builders, i.e. whenever, there is a $\mathbf{10}$ contributing to charged fermion masses and $\mathbf{160}$ contributing to the right-handed neutrino masses, one must include the induced vev contribution also in the charged fermion sector. This will certainly effect the predictivity of the models in the charged fermion sector. Our model also has the potential to solve the strong CP problem: note for instance that if $\lambda_4$ is set to zero, we have a model with a softly broken PQ symmetry.

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