A Hoare Logic with Regular Behavioral Specifications

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Abstract. We present a Hoare logic that extends program specifications with regular expressions that capture behaviors in terms of sequences of events that arise during the execution. The idea is similar to session types or process-like behavioral contracts, two currently popular research directions. The approach presented here strikes a particular balance between expressiveness and proof automation, notably, it can capture interesting sequential behavior across multiple iterations of loops. The approach is modular and integrates well with autoactive deductive verification tools. We describe and demonstrate our prototype implementation in SecC using two case studies: A matcher for E-Mail addresses and a specification of the game steps in the VerifyThis Casino challenge.

1 Introduction

Context of this work is the aim to specify the state machine logic of programs that have a control state and in which the sequence of events matters. A typical example are stateful protocols (e.g. the TCP handshake) and device drivers. One motivation for this paper in particular is the Casino case study from the ongoing VerifyThis discussion series since 2021.⁴ Goal of these discussions is to bring different communities together and bridge specifications using contract-based mechanisms typically found in deductive verification tools and the automata-based approaches of model-checking. Of particular interest was the integration of automata-like specifications and models with deductive program verification tools. Existing approaches include an embedding of a highly expressive process-calculus language into the Separation Logic assertions [20, 21, 25]. At the end of the spectrum are approaches where the C program itself is abstracted such that it can be checked directly using a software model checker. Such techniques can cope with temporal properties [8, 27]. Of course, the research area of attributing and verifying temporal behavior of systems has a long history [2, 17, 23].

In this paper we discuss a particular point in the design space: We embed declarative behavioral specification of event traces, written as regular expressions, into the pre- and postconditions of a Hoare-like logic. The approach

⁴ https://verifythis.github.io/casino/
combines strong reasoning about the functional correctness of programs with a light-weight and fully automatic integration of behavioral aspects into contracts.

As shown in Sec. 2, it turns out that the straight-forward approach to specifying loops just by using the repetition operator is not expressive enough, as it fails to capture sequential behavior across loop iterations, which necessitates to include logical conditions into the annotation language. In Sec. 3, we develop a Hoare-like logic with judgements \( \{ P \mid U \} c \{ Q \mid V \} \), where in addition to the ordinary pre-/postcondition pair \( P, Q \) for command \( c \), we have two regular trace specifications \( U \) and \( V \) that capture which events may be emitted during the execution of the program. The logic is proved sound in the Isabelle/HOL proof assistant and we suggest a corresponding completeness result. The clear advantage of regular expressions over more complex languages is that the additional verification obligations can be decided automatically with little additional effort in the verification. We have implemented the approach in the deductive verification tool SecC [10], as discussed with details on this automation in Sec. 4. We illustrate and discuss in depth the approach with two case studies: A matcher for E-Mail addresses (Sec. 5) and the Casino challenge (Sec. 6).

The contribution of this paper is therefore to propose this approach as natural point in an active research area (Sec. 7) and to showcase its merits and limitations on practical examples, backed by a tool implementation.

## 2 Motivation

As an example consider a loop that generates a sequence of alternating events even and odd, where the loop test nondeterministically terminates the loop but only in a state when \( b \) is false, expressed as pseudocode:

\[
\begin{aligned}
b &:= \text{false} \\
\text{while } \neg b \text{ do} \\
&\quad \text{if } b \text{ then emit odd} \\
&\quad \quad \text{else emit even} \\
&\quad b := \neg b
\end{aligned}
\]

Fig. 1: Event alternation

The specification command emit a demarks the occurrence of an event during program execution, and we are interested in specifying the behavior of programs in terms of the possible event traces that can occur at runtime. Such events are sometimes attributed to certain steps of the program’s semantics, but here we just keep them abstract and assume that the right events have been placed at appropriate locations in the source code in terms of emit statements. The behavior of the above program is captured nicely by

\[
\text{program specification} \quad (\text{even-odd})^*
\]

where \( \cdot \) denotes concatenation and \( * \) denotes repetition.

The program is indeed correct with respect to this specification, informally because the loop body is executed for the first time with \( \neg b \), thus emitting an even event first, the loop alternates between the two events, and terminates only after no event or after an odd event has been emitted. To make this argument formally precise, we aim at a verification approach that 1) is complete with respect to
regular languages and 2) integrates well into existing modular approaches that can be presented in the style of Hoare logic.

A simple but straightforward idea to specify the traces of a while loop is to repeat whatever the body produces, using the star operator. In the example above, the behavior of loop body can be abstracted by the regular expression \((\text{even} \mid \text{odd})\) from which we can conclude that the loops adheres to \((\text{even} \mid \text{odd})^*\). However, this naive approach cannot capture the alternation of even and odd events as produced by the loop. This coincides with the fact that the finite automaton that accepts the same language as (1) has two states, thus, we need two regular expressions to describe traces with respect to the current value of \(b\).

Just as with the functional component of loop specifications, we can choose whether we want to describe the events observed so far (analogously to an invariant), or alternatively whether we want to specify the traces that are still permitted to occur (analogously to a loop postcondition or summary [11, 12, 26]). For this example, the alternatives are as follows

\[
\begin{align*}
\text{loop invariant} & \quad (\text{even-odd})^* \text{ if } \neg b \quad (\text{even-odd})^* \cdot \text{even if } b \\
\text{loop summary} & \quad (\text{even-odd})^* \text{ if } \neg b \quad \text{odd} \cdot (\text{even-odd})^* \text{ if } b
\end{align*}
\]

The regular expressions are conditioned upon a formula which depends on the value of \(b\) in some arbitrary intermediate state encountered at the loop head. If \(b\) is currently true, then the invariant expresses that this current state has been reached by a trace prefix that ends in an even event, such that the subsequent iteration concatenating an odd to the end of this regular expression will again give (1). Conversely, the loop summary in this case makes it explicit that the next event expected is an odd event. In that regard, the two approaches are dual to one each other and perfectly symmetric for regular expressions because the repetition operator can be expressed as both a left-fold and as a right-fold recursively (which is not the case for context-free grammars, in general). For now we will focus on the invariant approach as it is conceptually more similar to how functional correctness is typically established.

## 3 Approach

In this section, we show an extension of Hoare logic that integrates trace specifications in terms of regular expressions with deductive proofs, where program correctness is expressed as judgements of the form \(\{ P \mid U \} c \{ Q \mid V \}\) with respect to pre-/postconditions \(P, Q\) and a command \(c\). In addition, there are two conditional regular expressions \(U\) and \(V\), subsequently called regular behavioral specifications, denoting the trace prefix and resulting trace, respectively.

**Preliminaries.** Conditions \(P, Q\) and \(\phi\) as well as binary relations \(R\) are represented syntactically, where the latter are formulas involving also primed variables, e.g., to denote successor states and nondeterministic transitions. For example \(x' > x\) encodes that the value of program variable \(x\) is strictly increased by a nondeterministic value. Ascribing a prime symbol to a predicate as in \(P'\)
is understood to prime all of its free variables, similarly \( U' \) is \( U \) with all free variables in all conditions replaced by their primed counterpart. Semantically, formulas can be evaluated over states \( s \in S \), written \( P_s \) for instance, to result in a semantic truth value. Usually, we treat primed variables just as other variables, however, occasionally we denote by \( R_s, s' \) the evaluation of relation \( R \) taking the unprimed variables from state \( s \) and the primed variables from \( s' \).

3.1 Regular Behavioral Specifications

Plain regular expressions \( u, v, w \) consist of the empty language, the empty word, symbols of an alphabet \( a \in A \), sequential composition, choice, and repetition:

\[
 u ::= \emptyset | \epsilon | a | (u \cdot v) | (u | v) | u^*
\]

The language \( \mathcal{L}(u) \) of an expression \( u \) is defined as usual as a set of words which are finite sequences of symbols, representing behavioral traces \( \tau = \langle a_1, \ldots, a_n \rangle \) here. A regular expression \( u \) is called nullable if its language contains the empty word, i.e., \( \langle \rangle \in \mathcal{L}(u) \); and \( u \) is called empty if its language is the empty set, i.e., \( \mathcal{L}(u) = \emptyset \). Language inclusion and equivalence are defined as follows:

\[
 u \subseteq v \iff \mathcal{L}(u) \subseteq \mathcal{L}(v) \quad u \equiv v \iff \mathcal{L}(u) = \mathcal{L}(v)
\]

such that \( u \) is nullable if and only if \( \epsilon \subseteq u \), and \( u \) is empty if \( u \equiv \emptyset \). Nullability and emptiness can be checked efficiently by a simple recursion over the syntax.

**Definition 1 (Regular Behavioral Specification).** A regular behavioral specification \( U, V, W \) is a state-dependent choice between plain regular expressions \( u_i \):

\[
 U ::= u_1 \text{ if } \phi_1 | \cdots | u_n \text{ if } \phi_n
\]

We tacitly interpret a plain regular expression \( u \text{ if } \text{true} \) with a trivial guard as such a specification. Evaluation in a particular state simply collects the options with a valid test, and discards all others:

\[
 (u_1 \text{ if } \phi_1 | \cdots | u_n \text{ if } \phi_n)_s = (u_1 \text{ if } \phi_1)_s | \cdots | (u_n \text{ if } \phi_n)_s
\]

where

\[
 (u \text{ if } \phi)_s = u \text{ if } \phi_s \quad \text{and} \quad (u \text{ if } \phi)_s = \emptyset \quad \text{otherwise}
\]

Later, for rule **Frame** in Sec. 3.3, we refer to sequential composition of such specifications which semantically obeys \( (U \cdot V)_s = U_s \cdot V_s \) but we refrain here from giving a general syntactic account for brevity, and because this construct may be somewhat misleading as \( U \) and \( V \) are evaluated in the same state, even though sequential composition suggests that \( V \) might occur after \( U \), and maybe after a state change in the program. However, we can freely move state-independent regular expression fragments in and out of this composition as in \( (u \cdot V)_s = u \cdot (V_s) \), which will be the use case in the implementation (Sec. 4).

In order to reason about these conditionals, we reflect language inclusion and equivalence as predicates with with the following semantics:

\[
 (U \subseteq V)_s \iff U_s \subseteq V_s \quad (U \equiv V)_s \iff U_s \equiv V_s
\]

such that, e.g., \( P \implies U \subseteq V \) can be regarded as a logical formula. This will be useful in particular to express a strong consequence rule in Sec. 3.3.

4
3.2 Programs and Behavioral Correctness

Imperative commands $c$ are formed by the grammar shown below, comprising specification statements [18] and the usual composition constructs:

$$c ::= \vec{x} : [G \rightsucceq R | U] | c_1 ; c_2 | \text{if } t \text{ then } c_1 \text{ else } c_2 | \text{while } t \text{ do } c | \cdots$$

Atomic commands are subsumed by specification statements $\vec{x} : [G \rightsucceq R | U]$, extended by a regular behavior. These can abstract over an arbitrary program fragment: Provided that guard $G$ holds in a given state, this command takes a nondeterministic transition by modifying the variables $\vec{x}$ according to a transition relation $R$, and by emitting a trace of the language of $U$. The specification statement can encode some more conventional constructs like skip and assignments, but also the emission of a single event as shown earlier, $\text{emit } a \equiv - : [\text{true } \rightsucceq \text{true } | a]$, with no modified variables and no constraints on the transition.

Commands $c$ execute according to a natural big-step semantics $k \xrightarrow{\tau}^* k'$ from an initial configuration $(\text{run } s : c)$ with state $s$ to a final configuration that is either $(\text{stop } s')$ (regular termination in final state $s'$) or $\text{abort}$ (subsuming runtime errors). The sequence of events that have happened during this particular execution is annotated as a trace $\tau$. The definition of the rules governing $k \xrightarrow{\tau}^* k'$ are entirely standard, except perhaps for the specification statement:

$$(\text{run } s : \vec{x} : [G \rightsucceq R | U]) \xrightarrow{\tau} \text{abort} \quad \text{if not } G_s \text{ and } \tau \text{ arbitrary}^5$$

$$(\text{run } s : \vec{x} : [G \rightsucceq R | U]) \xrightarrow{\tau} (\text{stop } s') \quad \text{if } G_s \land R_{s,s'} \land \tau \in U_s \text{ and } s' = s[\vec{x} \mapsto \vec{v}]$$

where $s[\vec{x} \mapsto \vec{v}]$ denotes the modified state in which the variables $\vec{x}$ have been updated to some arbitrary new values $\vec{v}$, leaving all other variables unchanged.

As a consequence, the derived statement $\text{emit } a$ behaves as expected

$$(\text{run } s : \text{emit } a) \xrightarrow{(a)} (\text{stop } s)$$

In this paper we do not address the issue of (non-)termination—diverging runs are simply not generated by this semantics, and the resulting logic will express functional correctness as well as behavioral correctness with respect to the trace for terminating executions only.

Judgements $\{ P | U \} c \{ Q | V \}$ comprise the usual constituents, precondition $P$, command $c$, and postcondition $Q$, as well as the (state-dependent) regular expressions specification $U$ over the alphabet $A$ collecting possible trace prefixes seen so far, and $V$ constraining how these may be extended by executing $c$.

**Definition 2 (Valid Hoare Triples).** A Hoare triple $\{ P | U \} c \{ Q | V \}$ is valid, if for all $s \in S$ with traces $\tau$, and for all configurations $k'$

$$P_s \text{ and (run } s : c) \xrightarrow{\tau}^* k' \text{ implies there is } s' \text{ with } k' = (\text{stop } s') \text{ and } Q_{s'} \text{ and } L(U_s \cdot \tau) \subseteq L(V_{s'})$$

Note that $P$ and $U$ are evaluated in the pre-state $s$, whereas $Q$ and $V$ are evaluated in the post-state $s'$, and that $k' \neq \text{abort}$.

$^5$ The arbitrary trace $\tau$ reflects that a diverging program could have any effect.
3.3 Hoare Logic Proof Rules

The proof rules are standard with respect to the functional correctness aspects captured by the pre-/postcondition. Regarding the trace specifications, there are some aspects that are worth discussing.

Typically, traces that occur during execution are simply added to the regular expression specification in the precondition. For example, the derived rule for the emit specification statement is just

\[
\{ P \mid U \} \text{emit} \ a \ \{ P \mid U \cdot a \} \quad \text{EMIT}
\]

It follows from the more general rule specification statement, which may be thought of as mirroring a procedure call that is verified modularly. The statement can be executed whenever guard \( G \) follows from the current precondition \( P \). Given then that binary relation \( R \) between unprimed and primed variables holds, we need to establish \( Q \) in that successor state, and we also need to ensure that the extension of the pre-traces \( U \) by any trace of \( W \) is covered by \( V \).

\[
P \implies G \quad P \land R \implies Q' \land (U \cdot W \subseteq V')
\]

\[
\{ P \mid U \} \land [G \leadsto R \mid W] \land \{ Q \mid V \}
\]

Recall that \( Q' \) and \( V' \) reference the successor state, denoted in the logical formula in terms of the primed variables constrained by \( R \), whereas \( U \) and \( W \) reference the state before executing the program in accordance with the semantics.

**Example 1.** We can represent the loop body of our motivating example in Fig. 1 by a single specification statement for demonstration purposes here

\[
b: [\text{true} \leadsto b' = \neg b \mid a] \quad \text{where} \quad a = \begin{cases} \text{even}, & \text{if } \neg b \\ \text{odd}, & \text{otherwise} \end{cases}
\]

Recall the invariant-like characterization of traces from Sec. 2, formalized as a regular expression specification \( U(b) \) over program variable \( b \):

\[
U(b) \equiv (\text{even-odd})^* \text{if } \neg b \mid (\text{even-odd})^* \cdot \text{even } \text{if } b
\]

The part of the instantiated precondition of rule \text{SPEC} that relates the pre-traces to the post-traces takes \( V = U \) to re-establish the invariant after executing the body, which leads to the following proof obligation

\[
b' = \neg b \implies U(b) \cdot a \subseteq U(b')
\]

This can be reduced by case analysis where \( U(b) \) and \( U(b') \) pick the opposite regular expression, respectively, where the most recently emitted event \( a \) is underlined:

\[
\begin{array}{ccc}
(\text{even-odd})^* \cdot \text{even} \subseteq (\text{even-odd})^* \cdot \text{even} & \text{if } \neg b \\
(\text{even-odd})^* \cdot \text{even} & \subseteq (\text{even-odd})^* & \text{otherwise}
\end{array}
\]

Clearly, both conditions are satisfied.
The consequence rule exhibits the usual co-/contravariance duality: We may wish to conduct the proof with a weaker precondition \( P_2 \) and larger set of traces seen so far \( U_2 \) to establish a stronger postcondition \( Q_2 \) than necessary together with a more precise set of extended traces \( V_2 \).

\[
P_1 \implies P_2 \land (U_1 \sqsubseteq U_2) \quad \{ P_2 \mid U_2 \} \ c \ \{ Q_2 \mid V_2 \} \quad Q_2 \implies Q_1 \land (V_2 \sqsubseteq V_1)
\]

This rule feeds contextual information from predicate logic formulas into the inclusion conditions of regular language specification, just as we have seen for the Spec rule in Ex. 1. Note that this rule makes up for not having defined sequential concatenation of trace specifications: We can manually re-arrange regular expressions with respect to their outer conditionals. The consequence rule can also feed information about the outcome of the test into the premises of the If rule (not shown).

The rule for sequential composition just chains the intermediate conditions and thereby avoids to refer to sequential composition of trace specifications:

\[
\{ P \mid U \} \ c_1 \{ Q \mid V \} \quad \{ Q \mid V \} \ c_2 \{ R \mid W \} \quad \{ P \mid U \} \ c_1; c_2 \{ R \mid W \} \quad \text{SEQ}
\]

As we have motivated by the examples above, the rule for while loops employs in addition to an ordinary invariant \( I \) a regular expression specification \( U \) that characterizes the traces observed when executing the loop.

\[
\{ t \land I \mid U \} \ B \{ I \mid U \} \quad \{ I \mid U \} \text{ while } t \text{ do } B \{ \neg t \land I \mid U \} \quad \text{WHILE}
\]

The same \( U \) occurs for all traces in the rule in four places. This reflects the fact that the loop specification must already be given in a closed form that is generalized with respect to an arbitrary number of iterations. The evaluation of \( U \) is still relative to the context for that occurrence, specifically, with the postcondition \( \neg t \land I \) in the conclusion we can narrow down \( U \) to those cases that actually match the exit state.

**Example 2.** In Sec. 2, we had specified that the loop must not exit when \( b \) is true, such that \( U(b) \) simplifies to the desired overall behavior of the program with properly paired even and odd events (even·odd)*.

The rules seen so far do not allow one to ever “forget” any prefix of the trace seen so far. This means that the trace specification of loops is not really modular—\( U \) must maintain any events that have occurred in the past. The following framing rule therefore allows one to put a subpart of the execution into the context of a prefix \( W \) that captures the past up to that point. This rule encodes that the program execution itself cannot depend on past events, because traces have no manifestation at runtime.

\[
P \land Q' \implies W \equiv W' \quad \{ P \mid U \} \ c \ \{ R \mid V \} \quad \{ P \mid W \cdot U \} \ c \ \{ Q \mid W \cdot V \} \quad \text{FRAME}
\]
Just as with the frame rule in Separation Logic [24] there is a side-condition that the frame \( W \) is independent of the program execution, which we can express by an additional premise as shown. This premise is trivially satisfied for plain regular expressions that do not contain any state-dependent conditions and our implementation uses this heavily as explained in Sec. 4.

With the help of this rule, we can justify the naive approach to loops that just repeats any observation that can be made about the executions of the loop body with out taking any sequencing constraints into account, given here by a plain regular expression \( v \) to simplify the presentation.

\[
\begin{align*}
\{ t \wedge I \mid \epsilon \} & B \{ I \mid v \} \\
\{ I \mid \epsilon \} & \text{while } t \text{ do } B \{ -t \wedge I \mid v^* \} \quad \text{While*}
\end{align*}
\]

It follows from rule While for invariant \( U = v^* \), and the frame rule for \( W = v^* \) to justify the needed intermediate condition \( \{ t \wedge I \mid v^* \cdot \epsilon \} \ B \{ I \mid v^* \cdot v \} \) from the given premise of rule While*. The rest of the justification is stitched together by rule Conseq and the algebraic laws of the repetition operator.

As an example, taking \( v = (\text{even} \mid \text{odd}) \) we can derive the weaker characterization of the loop’s behavior that was mentioned in Sec. 2.

**Theorem 1 (Soundness).** The rules presented in this paper are sound: if the premises are valid according to Def. 2 then the respective conclusion is also valid.

**Proof.** Mechanized in Isabelle/HOL, available for review\(^6\) [https://gist.github.com/gernst/6a156facbe402f6d7f8dbb8b6520c7d70](https://gist.github.com/gernst/6a156facbe402f6d7f8dbb8b6520c7d70) \(\square\)

We remark that the soundness proof does not in any way depend on the fact that the trace specifications are regular, or whether they can be presented as a finite enumeration.

**Claim (Completeness of Trace Annotations).** If there is an ordinary regular expression \( v \) that describes the traces of a given program \( c \), i.e., \( \{ P \mid \epsilon \} \ c \{ Q \mid v \} \) is valid, we can find regular expression specifications to compose this fact using the proof rules, including trace invariants for loops.

**Proof (Main Idea).** We can partition the actual state space of the program into equivalence classes with respect to their trace prefixes at each program location. Since the automaton corresponding to \( v \) has finitely many states, we can describe them by a finite number of regular expressions that are conditional upon a predicate logic formula characterizing the respective equivalence class. \(\square\)

In this paper, we do not make this claim formally precise, but we think that the intuition is clear. For example, the equivalence classes for the odd/even program from Sec. 2 would be given by \( \neg b \) and \( b \) as expected. The merit of this completeness property is that the proof rules themselves already contain sufficient information to capture any regular behavior relative to an adequate background theory in which the formulas in the conditions can be expressed.

\(^6\) The mechanization will be made available permanently at a later date, e.g., as an entry to the Archive of Formal Proofs at [https://www.isa-afp.org/](https://www.isa-afp.org/).
4 Tool Support in SecC

We have implemented support for trace specifications in SecC, which is an autoactive deductive program verifier for low-level concurrent C code with expressive security specifications. It is based on the logic SecCSL [10], which for the purposes to this paper is analogous to standard concurrent separation logic. The tool and case studies are publicly available at https://bitbucket.org/covern/secc.

The tool follows the classical design of most deductive tools, in which all C functions are specified modularly in terms of user-supplied contracts, such that they can be verified in isolation to avoid the combinatorial path explosion of interprocedural verification. To that end, SecC supports program annotations for function contracts, loop invariants, auxiliary statements such as intermediate assertions and other proof hints. It is possible, too, to specify logical functions and separation logic predicates, as well as to encode mathematical proofs (e.g. by induction) as lemmas and lemma functions, see e.g. [16].

4.1 Specification Syntax

For this work, we extended the specification language by trace annotations. We use the example from Sec. 2 as shown in Fig. 2 to explain the specification syntax in general as well as for the traces.

```c
int nondet();
void even_odd()
  _(trace {even odd}*)
  {
    int b = 0;
    while(nondet() || b)
      _(trace {even odd}*) if !b
      _(trace {even odd}*) even if b
      {
        if(!b) { _(emit even) }
        else { _(emit odd) }
        b = !b;
      }
  }

Fig. 2: Even/odd example in SecC.
```

In SecC, auxiliary code is wrapped inside ‹(...)›, an idiom taken from VCC. This is used here for different purposes, foremost, in the contract of function even_odd() to specify its behavior. An annotation ‹(trace U)› for a function with body c corresponds to a Hoare triple \{ P | \epsilon \} c \{ Q | U \} (pre-/postconditions P and Q are absent in the example). Inside the loop, the specification statement ‹(emit a)› generates the respective events. It is the proof engineer’s task to place these at the program locations of their interest—in the future we may support events that are implicitly generated, e.g., by function calls as in [9].

There is also a trace annotation for the loop, which showcases the concrete syntax for conditionals, analogous to the presentation earlier in this paper albeit in SecC each choice is listed separately. By convention, absence of a trace annotation, even for just a particular case, enforces that no events may be emitted at all.\(^7\)

\(^7\) We have omitted some annotations related to SecC’s enforcement of absence of timing side-channels, which requires an invariant that b never contains any secret
4.2 Verification Engine

In contrast to the Hoare-style rules of Sec. 3, which mirror those of the logic SecCSL behind the tool, SecC operationalizes the proof rules by a forward symbolic execution algorithm, first described here [3], similarly to VeriFast [15] or the Silicon backend of Viper [19]. The engine traverses the program using execution states, consisting of a store of symbolic variables, a path constraint, a symbolic representation of the heap, and for this work a trace prefix. The design of the logic necessitates being careful about branching not just for case distinctions in the program but also in the logic itself. To address this, SecC always takes apart relevant case distinctions and follows the branches individually, pruning those with an unsatisfiable path constraint. The same principle is applied to the regular trace specifications, which are broken apart into plain regular expressions eagerly, such that SecC can make use of the rule Frame, e.g., to concatenate the possible trace extensions after a while-loop is dispatched modularly. However, one has to be careful not to discard any execution branch vacuously just because there is no trace specified for it. Therefore, SecC always completes behavioral annotations with an empty trace that is the negation of the disjunction of all other cases (which might of course be unsatisfiable).

At the end of each execution path, SecC needs to check that the trace produced up to this point is covered by the behavioral specification annotated to the surrounding context (a C function or a loop). The inclusion check is implemented for plain regular expressions by an approach similar to the one shown in [1]. It is based on the derivative $\delta_a u$ of $u$ with respect to symbol $a$ that captures the language of suffixes of $u$ after a leading occurrence of $a$, i.e., $L(\delta_a u) = \{ \tau \mid a \cdot \tau \in L(u) \}$. From this definition we can derive an algorithmic check, $\Gamma \vdash u \sqsubseteq v$, which maintains a set $\Gamma$ of expressions seen already and proceeds recursively as shown below. Effectively, this algorithm computes in $\Gamma$ a simulation relation between $u$ and $v$ from the transitions that are possible over $u$ via the derivative.

$$\begin{align*}
\Gamma \vdash u \sqsubseteq v &\iff \\
\begin{cases}
true, & \text{if } (u, v) \in \Gamma \\
u \text{ empty,} & \text{if } v = \emptyset \\
\Gamma \cup \{u, v\} \vdash \delta_a u \sqsubseteq \delta_a v, \forall a \in \text{first}(u), & \text{otherwise}
\end{cases}
\end{align*}$$

Here, $\text{first}(u) \subseteq A$ is an overapproximation of the set of first symbols of words in $L(u)$, which can be computed recursively from the syntax. Termination is ensured by unfolding each pair of expressions once only (first line).\(^8\) The second case catches when $\delta_a v$ has become empty, which corresponds to an event $a$ of $u$ that is not covered by the specification. Conversely, in the last case when $u$ is nullable but $v$ is not, some required events were missed.

\(^8\) Note that it is crucial that the check whether a given pair of expressions is contained in $\Gamma$ already considers equivalence modulo reordering and duplicates of the choice operator, otherwise, one may keep accumulating larger and larger expressions.
5 Case Study: Regular Expression Matching

A canonical case study for the specification approach of this paper is to verify the state machine of a matcher that checks whether some input adheres to a given concrete regular expression. For instance, we aim at matching E-Mail addresses, here with much simplified format \[a-z]+[@][a-z]+[.][a-z]+\] which expects a name part and domain part with a single dot, where both parts are separated by an @ sign.

```c
void lex()
  // (trace letter+ at letter+ dot letter+ eof)
{
  int state = 0;

  while(0 <= state && state <= 5)
    // (invariant 0 <= state && state <= 6)
    // (trace ...) see running text
    {
      char c = next();
      check(c);

      if(state == 0) {
        if('a' <= c && c <= 'z') state = 1;
        else abort();
      } else if(state == 1) {
        if('a' <= c && c <= 'z') state = 1;
        else if(c == '@') state = 2;
        else abort();
      } // states 2 to 4 omitted
      ...
    } else if(state == 5) {
      if('a' <= c && c <= 'z') state = 5;
      else if(c == -1) state = 6;
      else abort();
    }
}

Fig. 3: E-Mail address matcher in SecC.
```

Often, such matchers are implemented as a library in programming languages, which at runtime translates textual representations of regular expressions into some internal automaton. For high-performance code, however, the preferred alternative is to compile this automaton upfront into source code, e.g., using the popular tool re2c [5]. Possible translation schemes make use of language features like loops/gotos and switch/conditionals, representing the automaton either implicitly via control flow, or with an outer loop and an explicit state variable, as shown in the example in Fig. 3. We will show how to verify this code with respect to the declarative specification of the expected matches in terms of the regular expression trace annotation at the top.\(^9\)

The input is read by repeated calls to some external function `next()` which returns the next input character. The switching logic of the state machine is implemented via variable `state` which is modified according to transitions encoded in the nested chain of conditionals. For example, state 0 expects a first letter symbol as the repetition is non-empty, whereas state 1 may subsequently transition over an occurrence of the symbol @. The loop terminates in state 6 after the input becomes empty, encoded by `c == -1`. To simplify matters further, the code simply calls `abort()` to denote an unexpected input character is encountered, which exposes nontermination via a

\(^9\) Available in the SecC bitbucket repository at examples/case-studies/matcher.c
postcondition \( \texttt{false} \), i.e., a call to this function will vacuously verify that branch of the computation.

The function \( \texttt{check()} \), shown in Fig. 4, denotes the events that are emitted in relation to the respective input characters, and also includes an error event for all other characters. This additional event does not occur in the top-level specification of \( \texttt{lex()} \), such that clearly the only way to satisfy the trace constraints when such an error is encountered is to reject the match via a call \( \texttt{abort()} \).

Function \( \texttt{check()} \) is a specification artifact that separates out the interpretation of inputs in terms of the four abstract events letter, at, dot, and eof. While it may appear somewhat cumbersome to draw this connection explicitly, we emphasize that this just reflects the prototypical nature of the design that does not ascribe any meaning to events upfront. Of course one may provide support for certain kinds of events out-of-the-box, perhaps complemented by character class definitions as abbreviations for finite enumerations.

```c
void check(char c);
_ (trace letter if 'a' <= c && c <= 'z')
_ (trace at if c == '@')
_ (trace dot if c == '.')
_ (trace eof if c == -1)
_ (trace error otherwise)
```

Fig. 4: Specifying the correspondence between inputs and abstract events.

They reflect the part of the input that has been matched already, starting with the empty trace, written as () in SecC in state 0, up to the final trace in state 6. Given the negated loop test at exit together with the invariant, the last line is the only one with a satisfiable guard, which is precisely the guarantee we need to satisfy the contract of \( \texttt{lex()} \). Preservation over a single iteration over the loop body is briefly discussed with respect to state 1. Its corresponding trace letter+ is initially established from a single letter event in the incoming transition from state 0. Reading the next character in the range \([a-z]\) in state 1 extends the trace to letter+ letter by another letter event from \( \texttt{check()} \), which is subsumed by letter+ in state 1 again as required. On the other hand, reading an @ symbol produces the trace specified for state 2, which is the one transitioned to at the end of this iteration. All other transitions work analogously.

### 6 Case-Study: VerifyThis Casino Challenge

In this section we discuss the Casino case study that fueled a series of online discussions in the context of the VerifyThis competition. The case study deals with
Fig. 5: Automaton specification (adapted from a model by Matthias Ulbrich, https://verifythis.github.io/casino/spec/)

As mentioned in the introduction, a particular goal of the efforts around this challenge was to bridge between different verification approaches, such as finite state models, contract-based models, and hybrid solutions. An aspect in focus is the pot of money associated with the game, which is supplied by the operator and which needs to cover the prize of the player, causing an invariant that encodes this requirement. Fig. 5 shows a high-level description that avoids draining the pot by simply restricting the operator to reduce the amount in the pot as long as an unresolved bet has been placed.

SecC models this game as follows:¹⁰ Data on the blockchain is treated as memory that has public visibility, a feature that is supported by the logic. All data passed in and out of the operations that encode the moves in the game are likewise specified to be public. Ownership, e.g., of a wallet or a payment that has been sent but not received yet, naturally maps to resources in Separation Logic, encoded into abstract predicates that cannot be duplicated.

The game itself is specified as a main function with a top-level loop that nondeterministically chooses among the next possible moves and then calls the function that implements the respective transition. Each of these functions emits one of the events shown in Fig. 5. The corresponding behavior of the game is captured by the regular expression shown below. It declaratively specifies all traces through the automaton in Fig. 5. We point out the inclusion of some

¹⁰ Available in the SecC bitbucket repository at examples/case-studies/casino.c
trailing changes to the pot. Moreover, it is noteworthy that after a place_bet event, no further remove_from_pot can happen until the game is decided.

\[ \text{trace init} \]
\[
(\text{add_to_pot} \mid \text{remove_from_pot})^* \text{create_game} \\
(\text{add_to_pot} \mid \text{remove_from_pot})^* \text{place_bet} \\
\text{add_to_pot}^* \quad \text{decide_bet}^* \\
(\text{add_to_pot} \mid \text{remove_from_pot})^*
\]

Since the game can be in one of three states (idle, game available, bet placed), the trace invariant for the loop has three parts, that analogously to those of the matcher in Sec. 5 reflect the different stages of the game. Like with the even/odd example, the trace invariant duplicates as a prefix the entire specification in order to be inductive, which results in a somewhat large, but structurally straightforward annotation, for example that right before deciding the bet we have seen everything apart from the event decide_bet.

\[ \text{trace \ldots} \]
\[
(\text{add_to_pot} \mid \text{remove_from_pot})^* \text{create_game} \\
(\text{add_to_pot} \mid \text{remove_from_pot})^* \text{place_bet} \\
\text{add_to_pot}^* \quad \text{if state == BET\_PLACED}
\]

where \(\ldots\) here omits the entire game specification as shown above.

We have experimented further with simplified variants that just include the functionality directly related to the trace behaviors.\(^{11}\) The approach taken for the full case study is complemented by the naive approach with rule WHILE*. Furthermore, we formulate the game as a set of several mutually tail-recursive procedures, each corresponding to one state. This affects the annotations in two ways: First, each function on its own can specify its contribution to the observable behaviors, and second, we are effectively encoding the summary-based approach from Sec. 2, which in comparison to an invariant reasons about the trace suffixes that are yet to be fulfilled, which in comparison to the corresponding part of the loop annotation, specifies possible additions to the pot and includes a final decide_bet before the game repeats recursively (where again \(\ldots\) omits the game but without init).

\begin{verbatim}
void game_placed()
  \text{trace (add_to_pot+ decide_bet) // \textit{behavior of just game_placed}}
  \text{(\ldots)}
\end{verbatim}

\[ 6.1 \quad \text{Discussion} \]

The two case studies demonstrate how behavioral specifications in terms of regular expressions can capture declaratively properties of C functions as part of their top-level modular contracts. However, loop annotations become quite involved, and in the approach taken the user has to come up with the equivalence classes of regular expressions corresponding to the control states of the loop.

\(^{11}\) Available in the repository at examples\_case\_studies/casino\_statemachine.c
Moreover, these expressions are typically larger than the original annotation, and are repeated in significant parts for the many different cases. We think that this can be addressed for example by allowing the user to abbreviate expressions as it is possible for example in typical scanner generators like flex. Another idea is to infer such specifications automatically, e.g., by having the user annotate the conditions that partition the state space, but not the regular expressions themselves. We think that this is feasible but we leave this idea for future work.

SecC verifies both case studies in less than 5s on a Thinkpad T470p. We do not think that these numbers are particularly meaningful, as they were done with a cold-cache JVM and they include many calls to an external SMT solver. The overhead introduced by the regular expression inclusion check contributes noticeably to the time to verify functions which have many paths, like the main loops of the case studies. Nevertheless, this was never a limiting factor here, on the contrary, thanks to the decidability of the inclusion check once the conditions are settled, there is no additional manual effort involved to help out the verifier with additional proof hints, as it is often typical with expressive functional contracts. Perhaps, when scaling up to regular expressions as they may occur in practice, an efficient automata-based implementation may be preferable.

Finally, as hinted at already in Sec. 5, built-in support for certain kinds of domain-specific events can help streamline the verification process. For example, related work has considered opening and closing of file handles [6], as well as function calls and returns [9]. These applications, however, would strongly benefit from a context-free specification language, instead of a regular one, to pair each open with a close for instance. Similarly, attaching data drawn from finite sets or even symbolic data like file handles to events is useful. However, such extensions reflect different trade-offs wrt. automation as the inclusion check may become undecidable. This opens up a research space in between the work presented here and the highly expressive approaches, in which one can experiment with practical heuristics supported by domain-specific proof hints when needed.

7 Related Work

Deductive verification tools like SecC [10] come with strong support for logical specifications, which would in principle enable to encode regular traces into lists explicitly. Nevertheless, there are several recent approaches that aim for a more first-class support of specifying behaviors, which enables one to provide certain guarantees (soundness of compositions, automation):

A common strategy is to encode behaviors into abstract permission-based predicates, which integrates event histories with expressive logical data types. For example, VeriFast supports such I/O-specifications and has support not just for safety properties but also for liveness [14, 21]. VerCors has a highly elaborate mechanism to capture behaviors as process models [20], which can then be given to a model checker to verify system-wide properties, based on the earlier work [4] that tracks histories of events similarly. Another work that cuts into this direction is Igloo [25], in which modularity and composition is a key concern.
Interestingly, all of these approaches reason in the opposite direction as we do: The precondition specifies, which traces are still allowed to occur, in other words, a forward simulation between the code and the process or recursively defined model of the externally visible behavior is maintained as part of the auxiliary state. Our initial design of the logic was in fact based on this idea, but we did not (yet) succeed to prove end-to-end soundness wrt. Def. 2. Likely this can be remedied with an additional intermediate backward simulation, as hinted at in [20], but for this work we settled for the design as presented which we think is very clean. Nevertheless, looking further into this issue is certainly of interest, in parts because [25] relies on a proof system with precisely this kind of guarantee.

Session types [13] are another very active research area: Here, the interactions between software components are captured as part of the type system. Similarly, interface automata [7] capture possible interactions at the system level. Temporal contracts have been proposed for a functional language in [9], but considering runtime checking only, and giving completeness to ease the specification burden. Work that looks into temporal properties of smart contracts is [22].

8 Conclusion

We have presented a Hoare logic that integrates external behaviors as regular expressions into deductive verification systems. This can be used to declaratively specify sequences of events occurring during the execution. The approach supports loops with control states via conditions in the specification, such that interesting sequential behavior across multiple iterations can be captured. There is much room for experimentation with different trade-offs between automation, ease of specification, and expressiveness, to be explored in the future.

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