Electro-osmotic optimized flow of Prandtl nanofluid in vertical wavy channel with nonlinear thermal radiation and slip effects

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Abstract

The simulations have been performed for the nonlinear radiative flow of Prandtl nanofluid following the peristaltic pumping in a wavy channel. The applications of entropy generation for the electrokinetic pumping phenomenon are also focused as a novelty. The complex wavy channel induced the flow of Prandtl nanofluid. Moreover, the formulated problem is solved by using the convective thermal and concentration boundary conditions. The Keller Box numerical procedure is adopted as a tool for the simulation task. The results are also verified by implementing the built-in numerical technique bvp4c. The comparison tasked against obtained numerical measurement has been done with already reported results with excellent manner. The physical characteristics based on the flow parameters for velocity, heat transfer phenomenon, concentration field, and entropy generation pattern is visualized graphically. It has been observed that the presence of thermal slip and concentration enhanced the heat transfer rate and concentration profile, respectively. The skin friction coefficient declines with electro-osmotic force and slip parameter. The increasing variation in Nusselt number is observed for electro-osmotic parameter for both linear and non-linear radiative phenomenon.

Keywords

Prandtl nanofluid, entropy generation, wavy channel, electro-osmosis flow, Keller box method

Introduction

Peristalsis is a topic of interest for researchers from past three decades because of its applications in physiological and industrial processes. The design of dialysis and heart-lungs machines involves peristaltic mechanism. Peristaltic pumps play an important role in agricultural processes like pushing of chemicals/water from tank/river into the unfertile field. Important industrial applications of peristalsis include the roller and finger pumps to transport fluids. Transporting fluids through this process is helpful in avoiding pollution in industrial expenditures/wastages/consumption. Choudhari et al.¹

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assumed the peristaltic motion of Bingham material due to the elastic tube with porous walls and distinct flow characteristics. Prasad et al. identified the role of slip features for Casson fluid flow due to the peristaltic movement in inclined channel. The peristaltic investigation for non-Newtonian magnetized material in view of electroosmosis aspect was visualized by Tanveer et al. Divya et al. observed the hemodynamics applications regarding the peristalsis flow of Jeffrey fluid and additional under the consideration of distinct flow properties. Saunier and Yagoubi presented the novel contribution on the peristaltic flow with the surface impact and compressive stresses. The Powell-Eyring nanofluid thermal observation based on the peristaltic phenomenon has been focused by Ahmed et al. Khazayinejad et al. reported a radiative analysis for the biological material subject to the peristaltic pumping and porous space. Parveen et al. observed the thermophysical classification of nanofluid with peristaltic activity and slippage features. Abo-Elkhair et al. focused on the peristaltic determination of hybrid nano-materials in view of uniform Reynolds number assumptions. Javid et al. determined the cilia transport problem for the peristaltic mechanism and electrokinetic features.

The electroosmotic flow is the flow induced by the application of an electric field across the channel in the presence of electric double layer at channel walls. The electroosmotic flow is most significant in small channels because characteristic length of the channel is much greater than electric double layer (EDL) in small channels and in all heterogeneous fluids based system electric double layers exists. Electroosmotic flow plays a vital role in microfluidic devices such as, micro-reactors, biosensors and DNA analysis, etc. Ramesh and Prakash explained the inspiration of electroosmosis phenomenon in microfluidic vessel with inspection of heat transfer. Sharma et al. discussed the diffusive convective flow regarding the electroosmosis enrollment due to nanofluid flow. Lodhi and Ramesh implemented the Darcy theory for a electroosmotic problem subject to the viscoelastic fluid. Noreen et al. observed the electroosmotic activity against the bio-fluid flow via microchannel. The research communicated by Lv et al. explored the electroosmosis significance for the viscoelastic material in a metallic cylinder. Noreen and Waheed determined the electroosmotic evaluation for the microtube flow with porous activity. Ghosh et al. indentified the insight significances of electrophoretic due to electrical layer. Zhou et al. prescribed the physiological applications for the electroosmosis pattern along with the joule heating effects. The Prandtl nanofluid determination with electroosmosis assumptions was noticed by Abbasi et al. Enhancing heat transfer in a channel is important for designing more compact heat exchangers, which are used in a variety of engineering applications including electronic device cooling, air-conditioning equipment, and ocean thermal energy conversion technologies, among others. For researchers and engineers, the advancement of these technologies is a major concern. However, in engineering and technology departments, poor thermal conductivity is the main impediment to heat transfer of these possible fluids. Choi generated the idea of nanofluid and introduced us to its high thermal conductivity compared to other fluids. Saleem et al. addressed the hybrid nanofluid thermal performances for the peristaltic flow subject to the curved tube. Narla and Tripathi inspected the optimized thermal analysis regarding the peristaltic phenomenon in curved channel. Akram et al. determined the heat transfer rate based on the utilization of hybrid nanofluid following the peristaltic pattern and electroosmotic approach. Akhtar et al. determined the heat transfer investigation for the peristaltic transport of nanofluid in elliptic duct. Song et al. investigated the ciliated motion of nanofluid with double diffusion convection with implementation of Hall features. Ge-JiLe et al. observed the nanofluid thermal properties for the creeping flow with claim of biotechnology applications. Mahanthesh numerically investigated the flow and heat transport of nanoliquid with aggregation kinematics of nanoparticles. In an other study Mahanthesh highlighted the influence of quadratic thermal radiation on the heat transfer characteristics. Mahanthesh et al. discussed the impacts of non-linear thermal radiation, Hall current and heat source on the dynamics of dusty nano fluid.

The creation of entropy is linked to the irreversibility of thermodynamic processes; as irreversibility grew, the system’s useful work dropped and its efficiency fell. As a result, researchers aim to come up with a realistic way to reduce the rate of entropy formation in order to improve a system’s performance. For the first time Bejan discovered that the rate of entropy generation can be used to calculate irreversibility. A rich research on the optimized flow of various materials has been presented by research. This research inspired the nonlinear radiative flow of Prandtl Eyring nanofluid presence of entropy generation phenomenon. The peristaltic motion with applications of electroosmotic pumping regarding the Prandtl Eyring nanofluid has been considered in the wavy channel. The mixed convection and chemical reaction consequences are also taken into consideration. Additionally, the problem is further extended by utilizing the electric field and joule heating effects. The whole analysis is performed with implementation of thermal and concentration slip constraints. The solution procedure has been followed by employing implicit finite difference method. After carefully observing the scientific research, it is claimed that such novel thermal
investigation for the electroosmotic phenomenon and entropy generation features is not focused by researchers yet. The electroosmotic phenomenon presents applications in capillary electro-chromatography, medicine, microfluidic devices, capillary electrophoresis, microchips, geomechanics, petroleum engineering, etc. Moreover, the motivations for considerations the entropy generation effects are justified due to its significances in the energy storage devices, cooling systems, extrusion processes, solar collectors, exchangers turbo apparatus, combustion, heat, etc.

Mathematical modeling for electro-osmotic pumping of Prandtl Eyring nanofluid

We consider the two-dimensional flow of Prandtl Eyring nanofluid in a symmetric vertical channel by due to peristaltic wave. The applications of electroosmotic phenomenon have been accounted. Moreover, the energy equation is modified in view of joule heating and thermal radiation. For chemical reactive fluid, the chemical reaction features are utilized in the concentration equation. The flow geometry is presented in Figure 1.

Furthermore, the left wall $-H$ and right wall $H$ may be defined as

$$-H(\xi, t) = -A - a_1 \sin\left(\frac{c_1}{\lambda} (\xi - ct)\right)$$

$$- a_2 \sin\left(\frac{c_2}{\lambda} (\xi - ct)\right) - a_3 \sin\left(\frac{c_3}{\lambda} (\xi - ct)\right),$$

$$H(\xi, t) = A + a_1 \sin\left(\frac{c_1}{\lambda} (\xi - ct)\right) + a_2 \sin\left(\frac{c_2}{\lambda} (\xi - ct)\right)$$

$$+ a_3 \sin\left(\frac{c_3}{\lambda} (\xi - ct)\right).$$

(1)

Where, $a_i (i = 1 - 3)$ are non-similar wave amplitudes, $\lambda$ is wave length, $t$ is time, $A$ is half width of the channel, $\xi$ is axial co-ordinate, $c_k (k = 1 - 3)$ are parameters related to wave amplitude, and $c$ is speed of complex peristaltic wave. For the mathematical formulation of the problem under consideration the basic conservation laws of mass, momentum, energy, and concentration along with Nernst-Planck and Poisson equations are used which expressed as.

Continuity equation

The equation describes that rate at which the fluid enters the channel is equal to the rate at which liquid leaves the channel and fluid mass in dynamic system remain conserved.

$$\nabla \cdot \mathbf{V} = 0,$$  \hspace{1cm} (3)

Momentum equations

The momentum equations actually describing the Newton’s second law of motion. The left hand side is describing the inertial forces while the terms on right hand side are describing pressure gradient forces, viscous forces, and body forces.

$$\rho \frac{d\mathbf{V}}{dt} = \text{div} \mathbf{\tau} + \mathbf{\rho B},$$  \hspace{1cm} (4)

Energy equation

By employing the first law of thermodynamics, the energy balance equation is

$$(\rho C_p) \frac{dT}{dt} = K_f \nabla^2 T + D_{nc} \nabla^2 C + \sigma e E^2$$

$$+ \mathbf{\tau} : \nabla \mathbf{V} - \mathbf{\nabla q_r},$$

(5)

Concentration equation

The concentration equation in presence of chemical reaction in differential form is

$$(\frac{dC}{dt}) = D_f \nabla^2 T + D_c \nabla^2 C - k_c (C - C_0),$$

(6)

In above equations, $D_{nc}$ is Dufour diffusivity, $D_c$ is Soret diffusivity, $\rho_f$ is density of fluid, $C_f$ is heat capacity of nanofluid, $K_f$ is thermal conductivity, $D_e$ is Dufour diffusivity, $d/dt$ is material time derivative, $T$
is temperature having specific value $T_0$ at both left and right walls, $q_r$ is radiative thermal heat flux, $\bar{C}$ is concentration of mass, and $k_c$ is chemical reaction constant. $B$ represents the body forces and $\tau = -Pf + S$ is stress tensor and for Prandtl fluid

$$\bar{S} = \frac{1}{\gamma} A_{2} \sinh^{-1} \left( \frac{\gamma}{C_1} \right) \bar{A}_1$$

(7)

where

$$\dot{\gamma} = \sqrt{2 \Pi}$$

(8)

in which $\Pi = tr(\bar{A}_1)^2$, $\bar{A}_1 = \nabla V + (\nabla V)^T$, $A$ and $C_1$ denotes the material parameters. The rheological equations of mathematical model under consideration are:

$$\frac{\partial \bar{U}}{\partial t} + \bar{U} \frac{\partial \bar{U}}{\partial \xi} + \bar{V} \frac{\partial \bar{U}}{\partial \eta} = - \frac{\partial \bar{p}}{\partial \xi} + \frac{\partial S_{\xi \xi}}{\partial \xi} + \frac{\partial S_{\xi \eta}}{\partial \eta}$$

$$+ \rho \xi \xi + \rho \eta \eta \left( \beta_T (\bar{T} - T_0) + \beta_C (\bar{C} - C_0) \right)$$

(10)

$$\rho \eta \left( \frac{\partial \bar{V}}{\partial t} + \bar{U} \frac{\partial \bar{V}}{\partial \xi} + \bar{V} \frac{\partial \bar{V}}{\partial \eta} = - \frac{\partial \bar{p}}{\partial \eta} + \frac{\partial S_{\eta \xi}}{\partial \xi} + \frac{\partial S_{\eta \eta}}{\partial \eta} \right)$$

(11)

In the presence of electro-static and mixed convection the equation of momentum for the Prandtl Eyring fluid is:

$$\left( \rho C_p \right) \left( \frac{\partial \bar{T}}{\partial t} + \bar{U} \frac{\partial \bar{T}}{\partial \xi} + \bar{V} \frac{\partial \bar{T}}{\partial \eta} = K_f \left( \frac{\partial^2 \bar{T}}{\partial \xi^2} + \frac{\partial^2 \bar{T}}{\partial \eta^2} \right) \right)$$

$$+ \sigma \xi E^2 \xi \frac{\partial \xi}{\partial \eta}$$

$$D_n \left( \frac{\partial \bar{C}}{\partial \xi} + \frac{\partial \bar{C}}{\partial \eta} \right) + \left( S_{\xi \xi} \frac{\partial \bar{U}}{\partial \xi} + S_{\xi \eta} \left( \frac{\partial \bar{U}}{\partial \eta} + \frac{\partial \bar{V}}{\partial \xi} \right) + S_{\eta \eta} \frac{\partial \bar{V}}{\partial \eta} \right)$$

(12)

The right hand side of above equation is material derivative of heat transfer, the first term on right hand side is due to thermal conductivity, second term is joule heating, third term is due to non-linear Radiative heat flux fourth term is mass convention and last term is viscous dissipation. In view of Roseland approximation heat flux can be written as $q_r = -\alpha^* \frac{\partial T}{\partial \eta}$, in which $\alpha^*$ is the Stefan-Boltzmann constant and $k^*$ mean absorption coefficient.

$$\left( \frac{\partial \bar{C}}{\partial t} + \bar{U} \frac{\partial \bar{C}}{\partial \xi} + \bar{V} \frac{\partial \bar{C}}{\partial \eta} \right) = \bar{D} \left( \frac{\partial^2 \bar{C}}{\partial \xi^2} + \frac{\partial^2 \bar{C}}{\partial \eta^2} \right)$$

$$+ \bar{D}' \left( \frac{\partial \bar{T}}{\partial \xi} + \frac{\partial \bar{T}}{\partial \eta} \right) - k_c (\bar{C} - C_0)$$

(13)

**Electric field**

The electric potential function $\phi$ can be calculated from Poisson equation

$$\nabla^2 \phi = - \frac{\rho_e}{\varepsilon_1}$$

(14)

with $\varepsilon_1$ (dielectric constant) $\rho_e$ (electrolyte electric density) which are presented as:

$$\rho_e = e \xi (\bar{n}_+ - \bar{n}_-)$$

(15)

$z, n_+$, and $n_-$ be charge balance, electric charge magnitude, negative ions and positive ions, respectively. Following Nernst-Planck expression:

$$\frac{\partial \bar{n}_+}{\partial t} + \bar{U} \frac{\partial \bar{n}_+}{\partial \xi} + \bar{V} \frac{\partial \bar{n}_+}{\partial \eta} = D \left( \frac{\partial \bar{n}_+}{\partial \xi} + \frac{\partial \bar{n}_+}{\partial \eta} \right)$$

(16)

with chemical species diffusivity ($D$), average electrolyte solution temperature $T$, and Boltzmann constant $K_B$.

**Boundary conditions**

The flow constraints for model are:

$$\bar{U} + \beta_1 S_{\xi \eta} = 0, K_h \frac{\partial \bar{T}}{\partial \eta} = - \eta_1 (\bar{T} - T_0), K_m \frac{\partial \bar{C}}{\partial \eta}$$

$$= - \eta_2 (\bar{C} - C_0) \text{ at } \bar{\eta} = \bar{H}$$

$$\bar{U} - \beta_1 S_{\xi \eta} = 0, K_h \frac{\partial \bar{T}}{\partial \eta} = \eta_1 (\bar{T} - T_1), K_m \frac{\partial \bar{C}}{\partial \eta}$$

$$= \eta_2 (\bar{C} - C_1) \text{ at } \bar{\eta} = - \bar{H}$$

(17)

with heat coefficient ($K_h$) and mass coefficients ($K_m$), thermal conductivity coefficients ($\eta_1$) and mass diffusivity coefficients ($\eta_2$).

**Linear transformations and dimensionless formulation**

Transforming equations from laboratory frame to moving frame we use following linear transformation and dimension less variables for dimension less formulation.
\[ \xi = \frac{z - c_t \cdot \eta}{A}, \quad \eta = \frac{y}{c_0}, \quad \bar{u}(\xi, \eta) = \bar{U}(\xi, \eta, t) - c, \quad \bar{v}(\xi, \eta) \]
\[ = \bar{V}(\xi, \eta, t) \]
\[ \bar{p}(\xi, \eta) = \bar{P}(\xi, \eta, t), \quad T^*(\xi, \eta) = \bar{T}(\xi, \eta, t), \quad C^*(\xi, \eta) \]
\[ = C(\xi, \eta, t) \]
\[ \begin{cases} \xi^* = \xi - c_t \cdot \eta, \quad \eta^* = \eta, \quad \bar{u}(\xi^*, \eta^*) = \bar{U}(\xi^*, \eta^*, t) - c, \quad \bar{v}(\xi^*, \eta^*) \\ = \bar{V}(\xi^*, \eta^*, t) \\ \bar{p}(\xi^*, \eta^*) = \bar{P}(\xi^*, \eta^*, t), \quad T^*(\xi^*, \eta^*) = \bar{T}(\xi^*, \eta^*, t), \quad C^*(\xi^*, \eta^*) \end{cases} \]
\[ = C(\xi^*, \eta^*, t) \]
\[ (18) \]

\[ \begin{align*}
\xi &= \frac{x}{A}, \quad \eta = \frac{y}{c_0}, \quad \bar{u} = \bar{U} \frac{c_0}{c_0}, \quad \bar{v} = \bar{V} \frac{c_0}{c_0}, \quad \theta = \frac{T - T_0}{T - T_0}, \\
\phi &= \frac{C - C_0}{C_1 - C_0}, \quad \rho = \frac{\alpha^2 \bar{p}}{C_1 - C_0}, \\
S_n &= \frac{c m}{A}, S_f, \quad \frac{\delta}{A}, \quad \bar{R} = \frac{c m}{A}, \quad \varphi = \frac{\varepsilon c_0}{K_B T_B} \phi, \\
n &= \frac{n}{n_0}, U_1 = -E_{c_0 f} K_B T_B, \\
Pr &= \frac{\mu_f (C_f)}{K_f}, S = \frac{\sigma c E_0^2 A^2}{K_f (T_1 - T_0)}, \quad \alpha = \frac{A_2}{C_1}, \\
\beta &= \frac{\alpha c^2}{64 c^2 c_1}, S_c = \frac{c m}{A}, D_c, N_v = \frac{C_0 D_c}{K_f T_0}, \\
N_c &= \frac{D_c T_0}{D_f C_0}, \quad \bar{E} = \frac{\varepsilon c}{K_f (T_1 - T_0)}, \quad \bar{R}_n = \frac{16 c^2 A^2}{3 K_f c^2}, \\
\Omega &= \frac{T_1}{T_0}, \quad \gamma = \frac{\rho_f k A^2}{\nu_f}, \\
Gr &= \frac{g \beta_f T_1 (T_1 - T_0) A^2}{c v_T}, \quad G_c = \frac{g \beta_f (C_1 - C_0) A^2}{c v_T}, h = \frac{h}{A} 
\end{align*} \]

In which \( Re \) is Reynolds number, \( U_1 \) is Helmholtz-Smoluchowski velocity, \( \alpha \) and \( \beta \) be materials constants, \( Sc \) be Schmidt number, \( N_c \) is Soret parameter, \( N_v \) is Dufour number, \( Gr \) is thermal Grashof number, \( G_c \) is the concentration Grashof number, \( Pr \) Prandtl constant, \( \gamma_1 \) is chemical reaction parameter, \( Ec \) is Eckert number, \( \beta_1 \) is velocity slip parameter, \( S \) Joule heating constant, \( B_i \) is thermal Biot number, and \( C_i \) is concentration Biot number.

After using the stream function \( \psi \) defined by \( u = \frac{\partial \psi}{\partial \eta} \) and \( v = -\frac{\partial \psi}{\partial \xi} \) continuity equation (1) satisfied while flow model retained as:
\[ \frac{\partial^2 \psi}{\partial \eta^2} + \frac{Pr}{N_v} \frac{\partial^2 \phi}{\partial \eta^2} + S + R_n \frac{\partial}{\partial \eta} \left[ (1 + (\Omega - 1) \theta)^3 \frac{\partial \theta}{\partial \eta} \right] \\
+ \frac{Br}{A} \left( \frac{\partial^2 \phi}{\partial \eta^2} \right)^2 + \beta \left( \frac{\partial^2 \phi}{\partial \eta^2} \right)^4 = 0, \]
\[ \begin{cases} \xi^* = \xi - c_t \cdot \eta, \quad \eta^* = \eta, \quad \bar{u}(\xi^*, \eta^*) = \bar{U}(\xi^*, \eta^*, t) - c, \quad \bar{v}(\xi^*, \eta^*) \\ = \bar{V}(\xi^*, \eta^*, t) \\ \bar{p}(\xi^*, \eta^*) = \bar{P}(\xi^*, \eta^*, t), \quad T^*(\xi^*, \eta^*) = \bar{T}(\xi^*, \eta^*, t), \quad C^*(\xi^*, \eta^*) \end{cases} \]
\[ = C(\xi^*, \eta^*, t) \]
\[ (18) \]

Incorporating the dimensionless expressions and following constraints of \( Re, Pe, \delta \ll 1 \), Poisson equation (14) transformed as:
\[ \frac{\partial^2 \phi}{\partial \eta^2} = -k^2 \left( \frac{n_+ - n_-}{2} \right), \]
\[ (23) \]

in which \( k = \varepsilon c_0 \sqrt{\frac{2 m}{c N_v T_0}} \) is the electro-osmotic parameter. The dimensionless form of Nernst-Planck equation is:
\[ \frac{\partial^2 n_+}{\partial \eta^2} + \left( \frac{\partial}{\partial \eta} \left( n_+ \frac{\partial \phi}{\partial \eta} \right) \right) = 0, \]
\[ (24) \]

The solution of the equation (24) subject to the conditions \( n_+ = 1 \) at \( \phi = 0 \) and \( \frac{\partial n_+}{\partial \eta} = 0 \) at \( \frac{\partial \phi}{\partial \eta} = 0 \) is obtained in the form
\[ n_+ = \text{Exp}(\pm \phi), \]
\[ (25) \]

Imposing the boundary condition \( \phi = 0 \) at \( y = h \) and \( \frac{\partial \phi}{\partial \eta} = 0 \) at \( \eta = 0 \). For analytical simulations of equation (23) in view of \( n_+ \), one get
\[ \phi = \frac{\cosh(k \eta)}{\cosh(k h)}, \]
\[ (26) \]

Boundary conditions are:
\[ \psi = \frac{F_1}{2}, \quad \frac{\partial \psi}{\partial \eta} - \beta_1 \left[ \frac{\partial^2 \psi}{\partial \eta^2} + \beta \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^3 \right] \]
\[ = -1, \quad \frac{\partial \psi}{\partial \eta} - B_i (\theta - 1) = 0, \quad \frac{\partial \psi}{\partial \eta} - B_i (\phi - 1) = 0 \]
\[ \text{at} \quad \eta = -h \]
\[ \psi = \frac{F_1}{2}, \quad \frac{\partial \psi}{\partial \eta} + \beta_1 \left[ \frac{\partial^2 \psi}{\partial \eta^2} + \beta \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^3 \right] \]
\[ = -1, \quad \frac{\partial \psi}{\partial \eta} + B_i (\theta) = 0, \quad \frac{\partial \psi}{\partial \eta} + B_i (\phi) = 0 \]
\[ \text{at} \quad \eta = h \]
\[ (27) \]

Where \( h(\eta) = 1 + a_1 \sin(\alpha_1 \xi) + a_2 \sin(\alpha_2 \xi) + a_3 \sin(\alpha_3 \xi) \).

The repossenation of skin fraction, Nusselt number and Sherwood number is:
\[ C_f = \frac{\partial h}{\partial \xi} \left[ \frac{\partial \psi}{\partial \eta^2} + \beta \left( \frac{\partial^2 \psi}{\partial \eta^2} \right)^3 \right] \]
\[ (28) \]
\[ Nu = \frac{\partial h}{\partial \xi} \left|_{\eta = -h} \right. \]
\[ (29) \]
\[ Sh = \frac{\partial h}{\partial \xi} \left|_{\eta = -h} \right. \]
\[ (30) \]
**Entropy generation phenomenon**

The relation for volumetric entropy is

\[
E_G = \frac{K_f}{T_m} \left( 1 + \frac{16\sigma^* T^3}{3K_f k^*} \right) \frac{\partial T}{\partial \xi} \frac{(\partial T)}{2} + \frac{\Phi}{T_m} + \frac{RD}{C_m} \left( \frac{(\partial \psi^*)^2}{\partial \xi} + \frac{(\partial \psi^*)^2}{\partial \eta} \right) + \frac{RD}{T_m} \left( \frac{\partial \psi^*}{\partial \xi} + \frac{\partial \psi^*}{\partial \eta} \right) + \frac{\sigma E_{1}^2}{\partial \xi} \frac{\partial \psi^*}{\partial \eta} \right) \]

Diffusion irreversibility

In which \( \Phi = S_{\xi} \frac{\partial \psi^*}{\partial \xi} + S_{\eta} \frac{\partial \psi^*}{\partial \eta} + S_{\eta} \frac{\partial \psi^*}{\partial \eta} \).

with entropy generation number

\[
E_S = \frac{E_G}{E_C} = \frac{E_G}{K_f(T_1 - T_0)},
\]

Following equation (18):

\[
E_S = \left( 1 + RN(1 + (\Omega - 1))^3 \right) \left( \frac{\partial \psi^*}{\partial \eta} \right)^2 + Br \left( \alpha \left( \frac{\partial \psi^*}{\partial \eta} \right)^2 + \beta \left( \frac{\partial \psi^*}{\partial \eta} \right)^4 \right) + \frac{L \theta^* \partial \theta^*}{\Delta \eta \eta} + L \frac{\partial \theta^*}{\Delta \eta} \frac{(\partial \theta^*)^2}{\partial \eta} + S \frac{(\partial \theta^*)^2}{\Delta}.
\]

In which \( \Delta = \frac{T_1 - T_0}{T_m}, \omega = \frac{C_m - C_i}{C_i} \) and \( L = \frac{RD(C_i - C_m)}{k} \).

The Bejan number is related as:

\[
Be = \frac{1 + RN(1 + (\Omega - 1))^3 \left( \frac{(\partial \theta^*)}{\partial \eta} \right)^2}{E_S},
\]

**Numerical method**

Due to highly complexity, the analytical solution task for problem is almost not possible. Therefore, the finite difference numerical scheme is followed for simulation process.\(^{34}\) Let us inserting new variables like \( \psi' = I, \theta = m, q = \vartheta, \phi = Z \) in governing equations:

\[
Y_j - Y_{j-1} + Pr \left( \frac{Z_j - Z_{j-1}}{h_j} + S + \frac{1}{1 + (\Omega - 1)^3} \frac{(1 + (\Omega - 1)\vartheta_{j-1})^3 Y'}{3(1 + (\Omega - 1)\vartheta_{j-1})^2 (\Omega - 1)} \right) + Br \left( \alpha \left( \vartheta_{j-1} \right)^2 + \beta \left( \vartheta_{j-1} \right)^4 \right) = 0,
\]

\[
\frac{1}{Sc} \frac{Z_j - Z_{j-1}}{h_j} + Sr \frac{Y_j - Y_{j-1}}{h_j} - \gamma_1 \vartheta_{j-1} = 0,
\]

\[
\alpha q' + 3\beta m q' + 6\beta mq^2 + Gr Y + Ge Z
\]

\[
+ k^2 U_1 \text{sech}(kh) \cosh(k \eta_{j-1}) = 0,
\]

\[
Y' + Pr \left( N_\text{f} Z' + S + \frac{Pr}{Pr + \frac{3}{2} \left( 1 + (\Omega - 1)^3 \right) \vartheta_{j-1}} \right) = 0,
\]

Subject to boundary conditions

\[
I - \left[ am + \beta(m)^3 \right] = -1, Y - Bi_\text{r}\theta = 0, Z - Bi_\text{r}\phi = 0 \quad \text{at} \quad \eta = -h
\]

\[
I + \left[ am + \beta(m)^3 \right] = -1, Y + Bi_\text{r}\theta = 0, Z + Bi_\text{r}\phi = 0 \quad \text{at} \quad \eta = h
\]

In next step the central difference approximations are used for derivative and average for rest of dependent variables, we have

\[
\frac{\vartheta_{j+1} - \vartheta_{j-1}}{h_j} = \frac{\vartheta_{j-1} - \vartheta_{j-2}}{h_j}, m_j = \frac{m_{j-1} - m_{j-2}}{h_j} = q_j, q_j = \frac{q_{j-1} - q_{j-2}}{h_j}
\]

\[
\frac{\vartheta_{j+1} - \vartheta_{j-1}}{h_j} = \frac{\vartheta_{j-1} - \vartheta_{j-2}}{h_j}, m_j = \frac{m_{j-1} - m_{j-2}}{h_j} = q_j, q_j = \frac{q_{j-1} - q_{j-2}}{h_j}
\]

\[
\frac{\vartheta_{j+1} - \vartheta_{j-1}}{h_j} = \frac{\vartheta_{j-1} - \vartheta_{j-2}}{h_j}, m_j = \frac{m_{j-1} - m_{j-2}}{h_j} = q_j, q_j = \frac{q_{j-1} - q_{j-2}}{h_j}
\]

\[
\frac{\vartheta_{j+1} - \vartheta_{j-1}}{h_j} = \frac{\vartheta_{j-1} - \vartheta_{j-2}}{h_j}, m_j = \frac{m_{j-1} - m_{j-2}}{h_j} = q_j, q_j = \frac{q_{j-1} - q_{j-2}}{h_j}
\]

\[
\frac{\vartheta_{j+1} - \vartheta_{j-1}}{h_j} = \frac{\vartheta_{j-1} - \vartheta_{j-2}}{h_j}, m_j = \frac{m_{j-1} - m_{j-2}}{h_j} = q_j, q_j = \frac{q_{j-1} - q_{j-2}}{h_j}
\]
For linearization process:

\[ \psi_{j+1} = \psi_j + \delta \psi_j \]

so the equations (39)-(42) takes the form

\[ \delta \psi_j - \delta \psi_{j-1} - \frac{h_j}{2} \left( \delta l_j + \delta l_{j-1} \right) = (r_1)_{j-\frac{1}{2}} \quad (43) \]

\[ \xi_1 \delta \psi_j + \xi_2 \delta \psi_{j-1} + \xi_3 \delta l_j + \xi_4 \delta l_{j-1} + \xi_5 \delta m_j + \xi_6 \delta m_{j-1} + \xi_7 \delta q_j + \xi_8 \delta q_{j-1} + 
\]

\[ \xi_9 \delta \theta_j + \xi_{10} \delta \theta_{j-1} + \xi_{11} \delta Y_j + \xi_{12} \delta Y_{j-1} + \xi_{13} \delta \phi_j + \xi_{14} \delta \phi_{j-1} + \xi_{15} \delta Z_j + \xi_{16} \delta Z_{j-1} = (r_2)_{j-\frac{1}{2}} \quad (44) \]

Where

\[ \lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0, \lambda_5 = Br \alpha h m_{j-\frac{1}{2}} + 2 Br \beta h \left( m_{j-\frac{1}{2}} \right)^3 \quad (45) \]

\[ \lambda_0 = \frac{Pr Re}{2} \left( Y_j - Y_{j-1} \right) \left( 3(\Omega - 1)^3 \theta_j \frac{1}{4} + 3(\Omega - 1) + 6(\Omega - 1)^2 \theta_j \frac{1}{4} \right) + 3 Pr Re \left( 1 + (\Omega - 1)^3 \theta_j \frac{1}{4} + 3(\Omega - 1) \theta_j \frac{1}{4} \right) \]

\[ \lambda_{11} = 1 + Pr Re \left( 1 + (\Omega - 1)^3 \theta_j \frac{1}{4} \right) \]

\[ \lambda_{12} = -1 - Pr Re \left( 1 + (\Omega - 1)^3 \theta_j \frac{1}{4} \right) \]

\[ \lambda_{13} = 0 = \lambda_{14}, \lambda_{15} = Pr Nt c, \lambda_{16} = -Pr Nt c. \quad (46) \]

\[ \gamma_{11} = \gamma_{22} = \gamma_3 = \gamma_4 = \gamma_5 = \gamma_6 = \gamma_7 = \gamma_8 = 0, \]

\[ \gamma_9 = 0 = \gamma_{10}, \gamma_{11} = N c t, \gamma_{12} = -N c t, \]

\[ \gamma_{13} = -\gamma_{14} = \gamma_{15} = \gamma_{16} = \frac{1}{\lambda_{14}} = \frac{1}{\lambda_{15}}. \quad (47) \]

**Entropy generation**

After using the above process the expressions for entropy generation number and Bejan number are

\[ E_s = \left( 1 + Rn(1 + (\Omega - 1))^3 \right) \left( Y \right)^2 + \left( \frac{Br}{\Delta} \left( \alpha(m)^3 + \beta(m)^3 \right) \right) + \frac{L}{\Delta} \left( \frac{\Omega \Delta}{\Delta} \right) \left( Y \right)^2 + \frac{S}{\Delta}, \quad (48) \]

\[ Be = \left( 1 + Rn(1 + (\Omega - 1))^3 \right) \left( Y \right)^2 + \left( \frac{Br}{\Delta} \left( \alpha(m)^3 + \beta(m)^3 \right) \right) + \frac{L}{\Delta} \left( \frac{\Omega \Delta}{\Delta} \right) \left( Y \right)^2 + \frac{S}{\Delta}, \quad (49) \]

**Special cases**

- Results for simple peristaltic flow can be obtained by \( a_2 = a_1 = 0.0 \).
- If \( U_1 = 0 \), the results are in absence of electric field.
- Results for no-slip flow case can be obtained by choosing \( \beta_1 = 0, B i_i \to \infty, B i_m \to \infty \).
Results for viscous fluid case can be recovered by neglecting Prandtl Eyring parameters. That is \( \alpha = \beta = 0.0 \).

Results and discussion

The physical interpretations are presented in this section.

Axial velocity

The physical attribution and mechanism of axial velocity \( u(\eta) \) has been testified for electro-osmotic constant \( k \), rheological fluid constants \( (\alpha, \beta) \), electro-osmotic velocity \( (U_1) \), slip parameter \( (\beta_1) \), thermal Grashof number \( (Gr) \), and concentration Grashof number \( (Gc) \) in Figures 2 to 7 for non-similar values of flow rate. During the variation of other parameters the fixed values of the involved parameters are \( a_1 = 0.1, a_2 = 0.2, a_3 = 0.3, a_4 = 1.0, a_5 = 2.0, a_6 = 3.0, \alpha = 1.0, \beta = 0.1, k = 0.5, U_1 = 2.0, Pr = 1.0, Sc = 0.7, N_{ct} = 0.5, N_{co} = 0.5, Br = 0.1, \gamma = 1.0, Gr = 0.5, Gc = 0.5, \beta_1 = 0.1, B1 = 3.0, Rn = 0.1, \Omega = 1.4, S = 2.0 \) and \( Bim = 3.0 \).

Figure 2 pronounced the axial velocity change for electro-osmotic constant for \( Q = -1.0 \). A reducing change in core channel surface is noted for electrosomotic constant. However, arising observations are predicted in the symmetric surface channel. In view of electric double layer, a reverse relation between electric double layer and electro-osmotic constant is noticed. A reducing electric double layer with higher electro-osmotic constant is noted near the surface. With iterative values of \( k \) for \( Q = 5 \), velocity rate in heart of surface and near the surface is opposite. The determination in axial velocity with role of slip factor is observed in Figures 3 and 4. The low velocity observations in core surface channel is observed against iterative non-Newtonian numerical values (Figure 5). The arising attribution of velocity for Prandtl fluid is noticed as compared to viscosus material. The decline in axial velocity is due to rheological behavior of the fluid which restrict the motion in the core region of flow as a result fall in velocity is noticed near the central line. The decline in velocity is noticed at the heart of the channel and the effects of slip are dominant near both walls for dissimilar values of \( Q \). It is observed that the increase in slip dominants its impact on velocity near walls and the effects of slip parameter vanishes as the fluid moves toward origin. Figure 6 observed the silent features of electro-osmotic velocity parameter \( U_1 \) for longitudinal velocity \( u(\eta) \). The retarded observations in

\[ \text{Figure 2. Axial velocity } u(\eta) \text{ against } k \text{ with } \alpha = 1.0. \]

\[ \text{Figure 3. Axial velocity } u(\eta) \text{ against } \alpha \text{ with } k = 0.5. \]

\[ \text{Figure 4. Axial velocity } u(\eta) \text{ against } \beta \text{ with } \alpha = 1.0. \]

\[ \text{Figure 5. Axial velocity } u(\eta) \text{ against } \beta_1 \text{ with } \beta = 0.1. \]
u(η) due to $U_1$ are noticed. In core regime of surface, the velocity declined with electro-osmotic constant. However, the vicinity regime surface of boundary, the increasing aspect of velocity is noted. The appearance of electro-osmotic force is attributing the role of resistive force. Moreover, a reverse velocity change with $U_1$ is resulted for $\dot{Q} = 5$. Figures 7 and 8 focused the influence of thermal Grashof constant $Gr$ and mass concentration Grashof number $Gc$. The leading change in velocity rate in left channel surface is resulted while velocity reduces in all remaining zone with Grashof number. Here, the absence of buoyancy forces is denoted for. The trend of velocity change for enhancing mass concentration Grashof number $Gc$ is shown contrary behavior as compared to $Gr$.

**Temperature distribution**

Figures 9 to 13 illustrates the interest of petameters like $Bi_t, Nt_t, Ec, Rn,$ and $Gr$ on $\theta$ temperature profile $\theta(\eta)$ in view of temperature ratio parameter $\Omega$. During the analysis the numerical values of fixed parameters are $a_1 = 0.1, a_2 = 0.2, a_3 = 0.3, \alpha_1 = 1.0, \alpha_3 = 3.0, Q = -1.0, \alpha = 1.0, \beta = 0.1, k = 0.5, U_1 = 2.0, Pr = 1.0, Sc = 0.7, N_{cl} = 0.5, N_t = 0.5, Br = 0.5, \gamma = 1.0, Gr = 0.5, Gc = 0.5, \beta_1 = 0.1, Bi_t = 3.0, Rn = 0.5, S = 2.0$ and $Bi_m = 3.0$. Figure 9 reports the nature of temperature profile for Biot number $Bi_t$. The observations are visualized for linear and non-linear radiation impact. The improving change in Biot number $Bi_t$ result decrement in $u(\eta)$. Owing to temperature jumps, the lower heat transfer is obtained. Figure 10 reports the dynamic of $\theta(\eta)$ for Dufour constant $Nt_t$. The increasing attention in temperature due to $Nt_t$ has been observed. By definition, the Dufour constants explores the ratio between temperature and concentration differences. Figure 11 claimed the change in $u(\eta)$ for Eckert number $Ec$. The declining temperature rate in left half and right half is observed, respectively. Physically, the increment claimed in Eckert number improves the internal kinetic energy due to dissipation factor and subsequently temperature raise. Figure 12 conveyed the contribution of radiation parameter $Rn$, the temperature profile decreases by strengthen the radiation process continuously and the fall in temperature occur due to the Radiative process the fall in internal kinetic energy of the fluid and as a result fall...
in temperature is noticed. The observations for \( \theta(\eta) \) and \( Gr \) are defined in Figure 13. When \( Gr \) increases, lower heat transfer rate is noted. The association of electric force in flow regime present reduced temperature change due to Grashof number. For \( O = 1.2 \) that is for non-linear radiation the fall in temperature has maximum magnitude.

Concentration profile

The graphical predictions for \( Bim, \gamma_1, Sc, \) and \( Nct \) on \( \phi(\eta) \) profile when other involved parameters are fixed with \( a_1 = 0.1, a_2 = 0.2, a_3 = 0.3, \alpha_1 = 1.0, \alpha_2 = 2.0, \alpha_3 = 3.0, Q = -1.0, \alpha = 1.0, \beta = 0.1, k = 0.5, U_1 = 2.0, Pr = 1.0, Sc = 0.7, N_{le} = 0.5, N_{tc} = 0.5, B_{fr} = 0.5, \gamma = 1.0, Gr = 0.5, \gamma_1 = 0.5, \beta_1 = 0.1, B_{im} = 3.0, Rn = 0.5, \Omega = 1.4 \) and \( B_{im} = 3.0 \). The concentration profile observations Joule heating constant is explored via Figure 14. The declining concentration change due to \( B_{im} \) is noted for \( S = \pm 2 \). The profile of concentration due chemical reaction parameter \( \gamma_1 \) is noted in Figure 15. The lower concentration rate due to \( \gamma_1 \) is resulted. The physical influence of \( Sc \) and \( N_{ct} \) on concentration field in discussed in Figures 16 and 17, respectively. With change in \( Sc \) and \( N_{cr} \), concentration profile rises.

Nusselt number and Sherwood number

In this section the Figures 18 to 22 are plotted to show the variation of Nusselt number and Sherwood number for various parameters of interest for both linear and non-linear radiation parameter. The geometrical
parameters are taken with same values as discussed in above sections. Other involved parameters are $a = 1.0$, $\beta = 0.1$, $k = 0.1$, $U_1 = -2.0$, $Pr = 1.0$, $Sc = 0.7$, $N_{cr} = 0.1$, $N_{tc} = 0.5$, $Br = 0.5$, $\gamma = 1.0$, $Gr = 0.5$, $Ge = 0.5$, $B_t = 0.1$, $Bi_1 = 3.0$, $Rn = 0.5$, $\Omega = 1.4$ and $Bi_m = 3.0$. Figure 18 reported the signified impact of $k$ and $U_1$ on Nusselt number. When electro-osmotic parameter $k$ allocate maximum numerical values, the change in Nusselt number is increased. This trend is same for linear and nonlinear radiative phenomenon. The increasing framework of Nusselt number is predicted for nonlinear and linear radiative case. The physical reason behind such fact is presence of kinetic energy. However, with increasing $U_1$, Nusselt number rises up to remarkable range. The features explored in Figure 19 illustrates the association of Dufour number $N_{tc}$ on Nusselt and Sherwood number. The boosting profile of $Nu$ and $Sh$ due to $N_{tc}$ is observed. However, no significant change is noted due to joule heating constant. Figure 20(a) predicted the influence of Nusselt against $Br$. When $Br$, the Nusselt number get peak trend. With Brinkman number $Br$, heat transfer rate grows up. In Figure 20(b) the response of Nusselt number against thermal radiation parameter for both $\Omega = 1.0$ and $\Omega = 1.4$ is reported. It is observed that increase in thermal radiation parameter decreases the $Nu$ for both linear and non-linear Radiative flows.

A larger numerical variation in Nusselt number with enhancing joule heating constant $S$ is resulted (Figure 21(a). The boosted impact of joule heating parameter for linear radiative and nonlinear radiative phenomenon is obtained. The results focused in Figure 21(b)
show the impact of $g$ on Sherwood number. The rising behavior of Sherwood number with larger $g$ is founded.

**Wall shear force**

The impact of wall shear force defined via relations (24) is visualized for electro-osmotic velocity ($U_1$) in Table 1. For assisting flow case ($U_1 = -2.0$), the wall shear force fluctuated when electro-osmotic constant gets vary. The deacceleration in wall shear force due to electro-osmotic force has been noted. Here, for $U_1 = 0$, the role of electro-osmotic constant dismiss which is due to electric double layer. The higher wall shear force magnitude for slip appearance and electro-osmotic is noted. For no-slip appearance, the wall shear force declined.

**Entropy generation**

Figure 22 is plotted to analyzed the response of axial pressure for involved values of radiation parameter $Rn$ for both linear ($\Omega = 1.0$) and non-linear ($\Omega = 1.2$) radiative flows. The flow parameters and geometrical parameters are fixed as in above sections. The involved parameters are $\beta_1 = 0.1, k = 0.1, U_1 = -2.0, Pr = 1.0, Sc = 0.7, N_u = 0.1, N_{ic} = 0.5, Br = 0.5, \gamma = 1.0, Gr = 0.5, Gc = 0.5, Bi_t = 3.0, Rn = 0.5, \Omega = 1.2, L = 1.0, \omega = 5, \Delta = 1.0$ and $Bi_{in} = 3.0$.

The rise in thermal radiation declines the entropy generation number along the cross-section with large magnitude for $\Omega = 1.2$ but opposite results are noted for $\Omega = 1.0$. In Figure 22(b) the entropy generation number rises for the increasing values of Brinkman number $Br$. For non-linear radiation case the magnitude of entropy generation number is large due to large heat transfer into the fluid. From Figure 22(c) it is clear that increase in joule heating parameter cause a remarkable increase in entropy generation number for $\Omega = 1.0$ and $\Omega = 1.4$. Furthermore, the magnitude of entropy generation number is large for $\Omega = 1.4$. It is also clear in Figure 22(d) and (e), the rise in thermal Biot number $Bi_t$ and $N_{ic}$ also cause a fall in entropy generation number.
The irreversibility in the present study is defined by Bejan number. The analysis is carried out for both linear radiation phenomenon $\Omega = 1.0$ and non-linear radiation process $\Omega = 1.4$. In Figure 23(a) the effects of $Rn$ of Bejan number is plotted. The Bejan number $Be$ declines for thermal radiation parameter $Rn$ for linear Radiative flow while rises for non-linear Radiative flow. The rise in non-linear Radiative case is due to rise in temperature ratio which cases the irreversibility. The Bejan number for enhancing viscous dissipation is plotted in Figure 23(b) for several values of $\Omega$. A decline in Bejan number is intended for $Br$. In Figure 23(c) the rise in joule heating parameter $S$ cause a large increase in $Be$. The rise in thermal Biot number and Doufor number $Ntc$ reduces irreversibility which is interpreted in Figure 23(d) and (e).

### Verification of results

The verification and confirmation of simulation for obtained data is checked first by making comparison of present results with the work of and Ramesh and Prakash. Figure 24 is plotted to show the accuracy of simulation for fluid parameters. Moreover, Figure 25 is plotted to show the velocity profile for Prandtl Eyring nanofluid. The velocity profile shows a good agreement by Keller box scheme and bvp4c solver for flow parameters.

The value of involved parameters in Figure 25 is $a_1 = 0.1, a_2 = 0.2, a_3 = 0.3, \alpha_1 = 1.0, \alpha_2 = 2.0, \alpha_3 = 3.0, Pr = 1.0, Sc = 0.7, N_a = 0.5, N_b = 0.5, Br = 0.5, \gamma = 1.0, Gr = 0.5, Gc = 0.5, S = -2.0 \text{ and } B_n = 3.0$

| $k$ | $\alpha$ | $\beta$ | $U_1 = -2.0$ | $U_1 = 0.0$ | $U_1 = 2.0$ |
|-----|---------|--------|-------------|-------------|-------------|
|     |         |        | $\beta_1 = 0.0$ | $\beta_1 = 0.1$ | $\beta_1 = 0.0$ | $\beta_1 = 0.1$ | $\beta_1 = 0.0$ | $\beta_1 = 0.1$ |
| 0.1 | 1       | 0.1    | 1.211008    | 2.946895    | 1.211018    | 2.946902    | 1.211028    | 2.946908    |
| 1.0 |         |        | 1.1415859   | 2.903274    | 1.211018    | 2.946902    | 1.28105284  | 2.990869    |
| 2.0 |         |        | 0.684856    | 2.616074    | 1.211018    | 2.946902    | 1.771263    | 3.297207    |
| 2   |         |        | 1.650318    | 3.657448    | 2.1717109   | 3.943750    | 2.7050304   | 4.238234    |
| 2.5 |         |        | 2.133142    | 4.079132    | 2.6524049   | 4.345999    | 3.1799227   | 4.618509    |
| 3   |         |        | 2.6156729   | 4.449116    | 3.133215    | 4.698605    | 3.656813    | 4.952117    |
| 1   | 0.0     |        | 0.457698    | 1.841334    | 0.822670    | 1.560170    | 1.238135    | 2.621687    |
| 0.1 |        |        | 0.513633    | 1.885405    | 0.867868    | 2.94690    | 1.295918    | 3.297207    |
| 0.2 |        |        | 0.562383    | 1.887580    | 0.905025    | 3.36480    | 1.335255    | 3.6720635   |

**Figure 21.** (a) Nusselt number $Nu$ for $S$. (b) Sherwood number $Sh$ for $\gamma$.  

**Table 1.** Wall shear force at upper channel surface when $\alpha_1 = 0.1, \alpha_2 = 0.2, \alpha_3 = 0.3, \alpha_1 = 1.0, \alpha_2 = 2.0, \alpha_3 = 3.0, Pr = 1.0, Sc = 0.7, N_a = 0.5, N_b = 0.5, Br = 0.5, \gamma = 1.0, Gr = 0.5, Gc = 0.5, S = -2.0, B_n = 3.0
$Q = -1.0, \alpha = 1.0, \beta = 0.1, k = 1.5, U_1 = 2.0, Pr = 1.0, Sc = 0.7, N_c = 0.5, N_{tc} = 0.5, Br = 0.1, \gamma = 0.5, Gr = 0.5, Gc = 0.5, \beta_1 = 0.1, Bi = 3.0, Rn = 0.1, \Omega = 1.4, S = 2.0 \text{ and } Bi_m = 3.0.$

**Concluding Remarks**

The entropy generation flow of Prandtl nanofluid with applications of electro-osmotic is numerically investigated. The considered flow is induced due to wavy channel. The simplification in the governing model has been done by following the Debye-Huckel linearization approach. The finite difference numerical analysis is reported. The major outcomes are:

- The axial velocity change in core channel surface decline with electro-osmotic constant.
- The increasing change in velocity rate in left channel surface has been obtained for Grashof number.
Figure 23. (a) Bejan number $Be$ for $Rn$. (b) Bejan number $BeE_\gamma$ for Br. (c) Bejan number $Be$ for $S$. (d) Bejan number $Be$ for $Bi_k$. (e) Bejan number $Be$ for $N_{tc}$.

Figure 24. Comparison of velocity profile.

Figure 25. Comparison of velocity profile.
• The temperature rate enhanced with Dufour constant however, the impact of Eckert number is reverse.
• When chemical reaction parameter increases, the lower concentration rate is noted.
• The entropy generation number rises for the increasing values of Brinkman number.
• The magnitude of entropy generation is larger for nonlinear radiative phenomenon as compared to linear radiative framework.
• The Bejan number enhanced with joule heating parameter.
• With enhancing Dufour number, the local Nusselt number also increases.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University, Abha 61413, Saudi Arabia for funding this work through research groups program under grant number R.G.P–1-303-42.

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The text contains references to various research papers on fluid mechanics, nanofluid flow, heat transfer, and entropy analysis. It also includes a section on notations used in the text. The references cover topics such as the flow and heat transport of nanomaterials, the impact of hall current and exponential heat source, and the effectiveness ofHall current and exponential heat source. The text also mentions the use of artificial neural network (ANN) analysis for heat and entropy generation in microchannel.

**Notations**

- $\bar{x}$: axial co-ordinate
- $Br$: Brinkman number
- $\beta_C$: concentration
- $C_f$: heat capacity of nanofluid
- $S$: Joule heating constant
- $\tilde{C}$: concentration
- $-H$: concentration Biot number
- $Na$: Nusselt number
- $a(i = 1 – 3)$: wave amplitude
- $Be$: Bejan number
- $D_{st}$: Soret diffusivity
- $Sc$: Schmidt number
- $\eta_1$: thermal conductivity
- $K_b$: heat coefficient
- $z$: charge balance
- $K_B$: Boltzmann constant
- $D_f$: Dufour diffusivity
- $Re$: Reynolds number
- $\Phi$: viscous dissipation
- $N_{ct}$: Soret parameter
- $n_+ + n_-$: positive and negative ions
- $c$: speed peristaltic wave
- $\rho_e$: current density
- $Gr$: thermal Grashof number
- $T$: temperature
- $\Omega$: temperature ratio
- $k$: electro-osmotic parameter
- $k_e$: chemical reaction
- $Gc$: concentration Grashof number
- $\rho_f$: density of fluid
- $\epsilon_1$: dielectric constant
- $D$: diffusivity of the chemical species
- $N_F$: Dufour number
- $E_{3}$: entropy generation number
- $\alpha$ & $\beta$: fluid parameters
- $g$: gravitational acceleration
- $A$: half width
- $U_1$: Helmoltz-Smoluchowski velocity
- $e$: magnitude of electric charge
| Symbol | Description                          | Symbol | Description                          |
|--------|-------------------------------------|--------|-------------------------------------|
| $K_m$  | mass coefficient                    | $S_{ij}$, $S_{k\eta}$, $S_{\eta\eta}$ | stress components               |
| $\eta_2$ | mass diffusivity                   | $Bi_T$ | thermal Biot number                |
| $Pr$   | Prandtl constant                    | $K_r$  | thermal conductivity              |
| $\bar{h}$ | pressure                           | $\beta_T$ | thermal expansion coefficient |
| $Rn$   | radiation parameter                | $\beta_1$ | velocity slip parameter         |
| $H$    | right wall                          | $c_k(k = 1 - 3)$ | wave amplitude             |
| $Sh$   | Sherwood number                     | $\lambda$ | wave length                     |
| $C_f$  | skin fraction                       |        |                                     |
| $\psi$ | stream function                     |        |                                     |