Single-world interpretations of quantum theory cannot be self-consistent

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Abstract
According to quantum theory, a measurement may have multiple possible outcomes. Single-world interpretations assert that, nevertheless, only one of them “really” occurs. Here we propose a gedankenexperiment where quantum theory is applied to model an experimenter who herself uses quantum theory. We find that, in such a scenario, no single-world interpretation can be logically consistent. This conclusion extends to deterministic hidden-variable theories, such as Bohmian mechanics, for they impose a single-world interpretation.

1 Introduction
Imagine an experimenter who reads notes about an experiment that she carried out in the past. According to the notes, she measured the vertical spin component of an electron that was initially polarised along the horizontal direction

$$|\rightarrow\rangle = \frac{1}{\sqrt{2}} |\downarrow\rangle + \frac{1}{\sqrt{2}} |\uparrow\rangle .$$

The measurement had two possible outcomes, \(z = -\frac{1}{2}\) and \(z = +\frac{1}{2}\), corresponding to projectors along \(|\downarrow\rangle\) and \(|\uparrow\rangle\), respectively. Unfortunately, our experimenter long forgot the details of the experiment, and the page of her lab notebook containing the measurement outcome got lost. Still, she may apply quantum theory and find that the probabilities associated to the two possible outcomes are

$$P[z = -\frac{1}{2}] = P[z = +\frac{1}{2}] = 1/2 .$$

But this statement is manifestly symmetric — it does not give any preference to either \(z = -\frac{1}{2}\) or \(z = +\frac{1}{2}\).

Suppose now that our experimenter gets told that the measurement outcome \(z = -\frac{1}{2}\) must have occurred. Given this piece of information, she may be tempted to conclude that also the following statement is true.

“Outcome \(z = -\frac{1}{2}\) occurred whereas outcome \(z = +\frac{1}{2}\) did not.” (S1)
Figure 1: A basic quantum measurement experiment. A source emits an electron polarised according to \( \psi \). The detector measures the vertical polarisation direction \( z \).

The starting point of this work is the (well known) fact that this conclusion is not the only logically possible one. Noting that the statement “outcome \( z = +\frac{1}{2} \) occurred” is not the logical negation of “outcome \( z = -\frac{1}{2} \) occurred”, the following assertion could as well be an accurate description of reality.

\[
\text{“Both outcome } z = -\frac{1}{2} \text{ and outcome } z = +\frac{1}{2} \text{ occurred.”} \quad (S2)
\]

Hence, to conclude that (S1) is the correct statement, an additional assumption is necessary. The assumption could be that there is only one “single” world, in the sense that among the different physically possible outcomes of a measurement, only one is “real”.\(^1\) It contrasts with a many-worlds assumption, according to which all possible outcomes are equally real — even though we are always only aware of one of them. The latter would imply (S2), thus retaining the symmetry of the probabilistic expression (1) obtained from quantum theory.

Physical theories that impose that quantum measurements have single outcomes are often called single-world interpretations. (The term “interpretation” alludes to the fact that textbook quantum mechanics leaves open the question of how an expression such as (1) should be translated to a statement such as (S1) or (S2). It is hence necessary to interpret (1).) The main contribution of this work is an argument which, adopting this terminology, asserts that single-world interpretations of standard quantum theory cannot be self-consistent. More specifically, we prove a theorem that corresponds to the following informal claim.

**Main result (informal version)**

There cannot exist a physical theory \( T \) that has all of the following properties:

- **(QT)** *Compliance with quantum theory:* \( T \) forbids all measurement results that are forbidden by standard quantum theory (and this condition holds even if the measured system is large enough to contain itself an experimenter).

- **(SW)** *Single-world:* \( T \) rules out the occurrence of more than one single outcome if an experimenter measures a system once.

- **(SC)** *Self-consistency:* \( T \)’s statements about measurement outcomes are logically consistent (even if they are obtained by considering the perspectives of different experimenters).

\(^1\)For our argument it will not be necessary to define the term “real”. Relevant is merely that, in a single-world view, one particular measurement outcome must be “singled out”.

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Property (QT) refers to standard quantum theory, which does not impose any constraints on the complexity of objects it is applied to [Deu85]. It is certainly legitimate to question whether a theory that accurately describes nature must respect this requirement — after all, we are still lacking experimental evidence for the validity of quantum theory on macroscopic scales. Conversely, self-consistency, as defined by (SC), is arguably an unavoidable requirement to any reasonable physical theory. We are thus left with two scenarios, depending on the outcome of future experiments.

- **Scenario 1:** Experiments reveal that quantum theory is inaccurate in certain regimes and must be replaced by a different theory. (It could be, for instance, that we discover a yet unknown physical mechanism that leads to an “objective collapse” of the wave function.)

- **Scenario 2:** Experiments confirm that quantum theory accurately describes systems as complex as experimenters who themselves apply quantum theory. In this case, the result proved here forces us to reject a single-world description of physical reality.

A nice example that illustrates the use of the main result presented here is the de Broglie-Bohm theory, also known as Bohmian mechanics or pilot-wave theory [DB27, Boh52, DT09]. According to this theory, measurement outcomes are determined by particle positions, which are well-defined at any time, so that (SW) is satisfied. Bohmian mechanics is also compatible with quantum theory (without a “collapse mechanism”), so that (QT) holds, too. We must therefore conclude that it cannot satisfy (SC). That is, if Bohmian mechanics is applied from different experimenters’ perspectives, one sometimes obtains mutually contradicting statements. In fact, the same conclusion holds more generally for any deterministic hidden variable theory.

The paper is organised as follows. In Section 2, we introduce a (rather minimalistic) framework to formally capture the relevant aspects of different interpretations of quantum theory, such as single-world vs. many worlds. The main result is stated in Section 3. For its proof, we introduce in Section 4 an extension of the Wigner’s Friend gedankenexperiment [Wig67], which is then analysed in Section 5. In Section 6 we discuss specific theories that have been proposed, such as Bohmian mechanics, in the light of the results presented here.

# 2 Framework: Physics in terms of stories

## 2.1 Physical theories as constraints on stories

The main result, as described informally in the introduction, refers to the notion of a *physical theory* $T$, and we should therefore spell out what we mean by this. Yet, all we need is a characterisation in terms of certain *necessary* criteria. The reason is that the result asserts the *non*-existence of a theory $T$ with certain properties. Hence, the looser the definition we use for characterising $T$, the stronger is the claim (cf. Section 6.4).

The approach we take is based on the paradigm that any physical theory $T$ comes with a set of laws that “forbid” certain things from happening. For example, thermodynamics forbids that heat flows from a cold to a hot body. Another example would be a “collapse theory”, which forbids that a macroscopic object can be in a superposition of being at two
Figure 2: Physical theories. $\Sigma$ is the set of all stories. Physical theories, such as quantum theory, QT, or electrodynamics, ED, rule out some of them.

Distant locations. This paradigm connects to the idea of falsifiability. If an experiment leads to an observation that is forbidden by $T$ then we have falsified $T$. Conversely, if nothing is forbidden by $T$ then $T$ cannot be experimentally falsified.

A basic ingredient to our approach is the notion of a story, i.e., an account of events that occur. Specifically, we consider stories, real or fictitious, about experiments such as the spin measurement described in the introduction (see Fig. 1). To expand on this example, suppose that the electron is emitted by a source that polarises it along a direction $\psi$ as indicated by the position of a knob. Right after, its vertical spin component is measured and the outcome $z$ is shown on the display of the measurement device. In addition, there shall be a clock that shows time $t$. A possible story may then read as follows.

$$
\begin{align*}
  s_1 &= \{ "At \ t = 0:00 \ the \ source \ is \ invoked \ with \ knob \ position \ \psi = |\rightarrow\rangle \ and \ at \\
  &\quad t = 0:01 \ the \ measurement \ device \ shows \ outcome \ z = -\frac{1}{2}."
\end{align*}
$$

Another story could be

$$
\begin{align*}
  s_2 &= \{ "At \ t = 0:00 \ the \ source \ is \ invoked \ with \ knob \ position \ \psi = |\uparrow\rangle \ and \ at \\
  &\quad t = 0:01 \ the \ measurement \ device \ shows \ outcome \ z = -\frac{1}{2}."
\end{align*}
$$

Many more stories are conceivable of course, and we will in the following denote by $\Sigma$ the set of all possible stories. For our purposes, it is not necessary to characterise this set further — it suffices to simply postulate its existence. Yet, for concreteness, one may think of $\Sigma$ as the set of all finite sequences of letters or of all sequences of English words. Most of the stories then have no well-defined meaning, but this is unproblematic as we shall see later.

Given a story $s \in \Sigma$, we may find that it contradicts certain laws of physics. In this sense, physical theories impose constraints on the set of all possible stories $\Sigma$ (Fig. 2). For example, we would say that quantum mechanics forbids story $s_2$, but not $s_1$. This motivates the following characterisation of physical theories.

\begin{boxed_text}
A physical theory, $T$, specifies a subset of $\Sigma$, whose elements are called forbidden stories. We write $s \notin T$ to indicate that $s \in \Sigma$ is forbidden by $T$.
\end{boxed_text}

The terminology is deliberately chosen in an asymmetric manner. If a story $s \in \Sigma$ is not forbidden by a physical theory $T$, then this should not be interpreted as “$s$ is allowed

\footnote{In certain contexts it is useful to impose that the set be countable, though.}
by $T'$. For example, it could be that $s$ is not precise enough for $T$ to be applicable, or it could be that $s$ simply has no meaning. (Recall that we did not impose any constraints on the set of all stories $\Sigma$.)

### 2.2 Interpreting stories in terms of their plots

A story, in the usual sense of the word, has a *plot*, corresponding to the series of *events* that happen according to it. We will use this notion to specify, in a precise manner, what a story $s$ tells us about an experiment. To illustrate this, we may once again consider the spin measurement experiment of the introduction. This experiment has the following generic structure.

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**Basic Measurement Experiment (BME)**

Given a quantum system with Hilbert space $\mathcal{H}$ and a family of measurement projectors $\{\pi_z\}_{z \in \mathbb{Z}}$ on $\mathcal{H}$, perform the following steps at the corresponding times $t$.\(^3\)

- $@ 0:00$ Prepare the system in state $\psi$.
- $@ 0:01$ Measure the system w.r.t. $\{\pi_z\}_{z \in \mathbb{Z}}$ and record the outcome $z$.
- $@ 0:02$ Halt.

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When carrying out such an experiment, which we may call $E$, we are typically interested in a well-defined set of events that may potentially occur. We denote this set by $[E]$. In the case of the spin measurement experiment, any event of interest could, for instance, be identified with a particular reading of values shown by the clock, the source (whose knob position shows the prepared state), and the measurement device. This would motivate an event space of the form

$$[E] = \{(t, \psi, z) \in T \times \mathcal{H} \times (\mathbb{Z} \cup \perp)\} ,$$

where $\perp$ indicates that no outcome has been recorded, and where

$$T = \{n : n \in \mathbb{N}_0, m \in \{0, \ldots, 59\}\}$$

is the set of possible time values $t$ shown by the clock. This latter set is discrete, capturing the idea that the clock has a finite precision.

Given the set $[E]$ of potential events that can happen in the experiment $E$, a story $s$ about $E$ now defines a *plot* $s^E$, i.e., a subset of events that actually occur according to the story. For story $s_1$ from Section 2.1, for instance, the plot could be taken to be

$$s_1^E = \{(0:00, |\psi\rangle, \perp), (0:01, 0, -\frac{1}{2})\} .$$

(The assignment $\psi = 0$ shall indicate that the measurement is interpreted as a destructive one.) Similarly, for $s_2$, a natural choice could be

$$s_2^E = \{(0:00, |\uparrow\rangle, \perp), (0:01, 0, -\frac{1}{2})\} .$$

\(^3\)Here and in the following, times indicated in an experiment should be understood as placeholders that can be substituted by any other points in time with the same order. Furthermore, we assume that the systems’ states remain unchanged unless the protocol prescribes a specific action on them. In the present example, the evolution of the system’s state between $t = 0:00$ and $t = 0:01$ shall hence be trivial.
The plots $s_1^E$ and $s_2^E$ thus correspond to interpretations of the stories $s_1$ and $s_2$, which were described in prose, declaring precisely which events “happen” according to them. This idea is captured more generally by the following definition.

An experiment, $E$, specifies a set of events, denoted by $[E]$, as well as a function that assigns to certain stories $s \in \Sigma$ a subset $s^E$ of $[E]$, called plot of $s$.

We note that, formally, there is no constraint on how a story $s$ is being interpreted in terms of a plot $s^E$ — in principle $s^E$ can be chosen arbitrarily. For our purposes, it will only be relevant that the interpretations of a story in different experiments are compatible with each other, in a sense that we are going to describe in Section 2.3 below.

In general, a story $s$ could be ambiguous about whether or not a certain event in $[E]$ occurs. It could also be that $s$ does not talk about the experiment $E$ at all. In such cases one may of course not want to assign a plot $s^E$ to $s$. The above definition accounts for this, as it does not demand that $s^E$ exists for all $s \in \Sigma$. In other words, $s \mapsto s^E$ is generally a partial function on $\Sigma$.

Importantly, the notion of a plot allows us to formally capture different interpretations of quantum theory. While, for instance, the stories $s_1$ and $s_2$ above are in agreement with a single-world interpretation, one could also consider other stories, according to which the measurement provokes a branching, with different outcomes being observed in the different branches. For example, we may define a story $\tilde{s}_1$ with plot $\tilde{s}_1^E = \{(0:00, |\psi\rangle, \perp), (0:01, 0, -\frac{1}{2}), (0:01, 0, +\frac{1}{2})\}$, i.e., both outcomes $z = -\frac{1}{2}$ and $z = +\frac{1}{2}$ occur according to this story.

2.3 Different views on the same story

Two experimenters may have different views on the same experimental setup and therefore describe it in two different ways. Consequently, their interpretation of a given story $s$ in terms of its plot will in general also be different. Nevertheless, the two plots must of course still be compatible.

To explain what we mean by this, we first consider a simple situation where an experiment $E$, as viewed by one experimenter, is a part of a larger experiment $E'$, as viewed by another experimenter. An example of such a larger experiment could be an extension of the spin measurement experiment from above where, in addition to the measurement at time $t = 0:01$, a second spin measurement with outcome $z'$ is carried out at $t = 0:02$. The event space $[E']$ of $E'$ may therefore be defined as the set of quadruples $(t, \psi, z, z')$. However, when describing $E$ we may ignore the second measurement, so that $[E]$ consists of triples $(t, \psi, z)$.

Formally, the two experiments are defined by (partial) functions $s \mapsto s^E$ and $s \mapsto s^{E'}$ from the set of stories $\Sigma$ to $[E]$ and $[E']$, respectively. The functions specify how a given story $s$ should be interpreted. While the plots $s^E$ and $s^{E'}$ may in principle be defined arbitrarily, we want to model that $E$ is a part of $E'$. This is done by imposing certain compatibility constraints. For instance, we may demand that any event $(t, \psi, z, z')$ that occurs in experiment $E'$ must correspond to an event $(t, \psi, z)$ that occurs in the part $E$. 
This could be captured by the compatibility constraint
\[(t, \psi, z) \in s^E \iff \exists z' : (t, \psi, z, z') \in s^E',\]
which should hold for any story \(s\) for which \(s^E'\) is defined. To simplify the notation, we will in the following abbreviate terms of the form \(\exists y : (x, y)\) by \((x, \ast)\). The compatibility constraint can then be rewritten as
\[(t, \psi, z) \in s^E \iff (t, \psi, z, \ast) \in s^E'.\] (4)

More generally, two experiments, \(E_1\) and \(E_2\), may just have an overlap, but not be contained in each other. Suppose for example that both of them include a measurement of time \(t\) as well as some quantity \(z\), but in addition also separate quantities, \(z_1\) and \(z_2\), so that their event spaces, \([E_1]\) and \([E_2]\), consist of triples \((t, z, z_1)\) and \((t, z, z_2)\), respectively. The compatibility constraint that models that the two experiments have the quantities \(t\) and \(z\) in common could then be formulated as the condition that
\[(t, z, \ast) \in s^{E_1} \iff (t, z, \ast) \in s^{E_2}\] (5)
holds for any story \(s\) for which both \(s^{E_1}\) and \(s^{E_2}\) are defined.

We stress that, in the logic of the framework, compatibility constraints such as (4) and (5) are not derived relations, but rather define how the different experiments are related. In other words, (4) defines what we mean when we say that experiment \(E\) is a part of experiment \(E'\). Similarly, (5) defines what we mean when we say that \(E_1\) and \(E_2\) have the quantities \(t\) and \(z\) in common.

### 2.4 Representation of physical laws

Recall that physical laws correspond to rules that forbid certain stories. To formulate these rules, it is often convenient to consider specific experiments. The rules then take the form of conditions on the plots of the stories, which can be expressed in mathematically precise terms.

To illustrate this, consider the quantum-mechanical law that the measurement of a quantum system with respect to projectors \(\{\pi_z\}_{z \in Z}\) will with certainty give outcome \(\hat{z}\) if the prepared state \(\psi\) satisfies \(\|\pi_{\hat{z}}\psi\| = 1\). To formalise this law, let \(\text{BME}_{\mathcal{H}, \{\pi_z\}}\) be the set of all experiments \(E\) that have the form of the Basic Measurement Experiment introduced in Section 2.2. Formally, their event space \([E]\) is defined by (2), and the introductory spin measurement example would correspond to the case where \(\mathcal{H} = \text{span}\{|\uparrow\rangle, |\downarrow\rangle\}\) and where \(\pi_{\uparrow}\) and \(\pi_{\downarrow}\) are projectors along \(|\uparrow\rangle\) and \(|\downarrow\rangle\), respectively. The law that \(\hat{z}\) occurs if \(\|\pi_{\hat{z}}\psi\| = 1\) can then be written as
\[(0:00, \psi, \ast) \in s^E \text{ and } \|\pi_{\hat{z}}\psi\| = 1 \implies (0:01, \ast, \hat{z}) \in s^E.\] (6)

That a theory \(T\) contains this law now means that the implication
\[(6) \text{ is violated for some } E \in \text{BME}_{\mathcal{H}, \{\pi_z\}} \implies s \notin T\] (7)
holds for any story \(s\). Applying this criterion to the above examples, we find that \(s_2\) is forbidden, whereas \(s_1\) and \(\tilde{s}_1\) are not.
The rule (7) manifestly depends on the set of experiments $\text{BME}_{\mathcal{H}, \{\pi_z\}}$. This set therefore *defines* the range of applicability of the corresponding physical law. It can in principle be chosen arbitrarily. For example, if one holds the position that quantum mechanics is only valid on microscopic scales, one may restrict $\text{BME}_{\mathcal{H}, \{\pi_z\}}$ to experiments where the measured system is small in some sense. Conversely, if one assumes that quantum theory extends to macroscopic systems then the set should also include, for instance, arbitrary quantum measurements on cats [Sch35].

In Section 3 we will use a generalisation of (7) to express formally what we mean when we say that a theory $T$ complies with quantum theory. The generalisation will include experiments where the system’s state evolves non-trivially and where the measurement may be repeated.

3 Main result

The main result, which will be stated as Theorem 1 below, refers to three properties that a physical theory may, or may not, have. In the following we describe them using the story-based approach introduced in Section 2.

3.1 Compliance with quantum theory (QT)

Property (QT) corresponds to the assumption that the laws of standard quantum theory are valid. In fact, for our purposes, it suffices to restrict to some particular rules that are implied by standard quantum theory, i.e., we do not need to provide a full characterisation of the theory. To specify these rules, we consider the following experiment.

**Repeated Measurement Experiment (RME)**

Given a quantum system with Hilbert space $\mathcal{H}$, a unitary $U$ on $\mathcal{H}$, a family of measurement projectors $\{\pi_z\}_{z \in \mathbb{Z}}$ on $\mathcal{H}$, and a value $\hat{z} \in \mathbb{Z}$, repeat the following steps for increasing $n \in \mathbb{N}_0$ until the halting criterion is satisfied.

- @ $n:00$ Prepare a quantum system in state $\psi$.
  - Let the system evolve according to $U$.
- @ $n:01$ Measure the system w.r.t. $\{\pi_z\}_{z \in \mathbb{Z}}$ and record the outcome $z$.
- @ $n:02$ Halt if $z = \hat{z}$.

To simplify the notation, we will in the following represent the measurement projectors $\{\pi_z^H\}_{z \in \mathbb{Z}}$ in the Heisenberg picture, i.e.,

$$\pi_z^H = U^\dagger \pi_z U \ .$$

While standard quantum theory generally makes probabilistic predictions, it also implies certain deterministic statements. In particular, the following must hold for any fixed $\hat{z} \in \mathbb{Z}$.

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4The halting step is not strictly needed. But since we are mostly interested in the case where $\pi_\hat{z} \psi \neq 0$, and since (QT) then implies that the experiment will end after finite time, it avoids the need for stories that talk about an infinite sequence of measurements.
(a) If the prepared state $\psi$ satisfies $||\pi_H^z \psi|| = 0$ in round $n$ then the outcome $z = \hat{z}$ does not occur in this round.

(b) If the prepared state $\psi$ satisfies $||\pi_H^z \psi|| = 1$ in round $n$ then the outcome $z = \hat{z}$ occurs in this round.

(c) If the prepared state $\psi$ satisfies $||\pi_H^z \psi|| \neq 0$ and is identical in all rounds $n$ until the halting criterion is satisfied then the outcome $z = \hat{z}$ occurs at some point.

The definition of $(QT)$ below captures the idea that any theory $T$ that complies with quantum theory must satisfy these rules.

To formalise the rules within our framework, let $\text{RME}_{H,\{\pi^\mu_{\pi}\}}$ be the set of all experiments of the type above, for any given Hilbert space $H$ and a family of measurement projectors $\{\pi^\mu_{\pi} \}$ in the Heisenberg picture. On the formal level, we demand that the event space of any $E \in \text{RME}_{H,\{\pi^\mu_{\pi}\}}$ is of the form (see also (2))

$$[E] = \{(t, \psi, z) \in T \times H \times (Z \cup \perp)\},$$

where $T$ is defined by (3). The set $\text{RME}_{H,\{\pi^\mu_{\pi}\}}$ may otherwise be chosen arbitrarily, provided that it includes the relevant parts of the Extended Wigner’s Friend Experiment defined in Section 4 below. This requirement will be captured formally by the conditions (12), (14), (16), and (18).

The properties of a story $s$ that are relevant for the formulation of $(QT)$ can be written as conditions on the plots $s^E$ for $E \in \text{RME}_{H,\{\pi^\mu_{\pi}\}}$. For example, that the experiment is repeated until the halting condition is satisfied corresponds to the implication

$$n = 0 \text{ or } ((n-1):01, *, \hat{z}) \in s^E \text{ for } z \neq \hat{z} \implies \forall t \in [n:00, n:01] : (t, *, *) \in s^E.$$  

(9)

For convenience, we also define the set

$$\Psi_n(s) = \{\psi : (n:00, \psi, *) \in s^E\},$$

telling us which states are prepared in round $n$ according to story $s$. (If $s$ talks about many worlds, for instance, this set could contain more than one element.)

$T$ satisfies $(QT)$ means that $T$ forbids all stories $s$ according to which in some experiment $E \in \text{RME}_{H,\{\pi^\mu_{\pi}\}}$, and for some $\hat{z} \in Z$ one of the following happens.

(a) In some round $n$, any $\psi \in \Psi_n(s)$ satisfies $||\pi_H^z \psi|| = 0$. Yet $(n:01, *, \hat{z}) \in s^E$ holds.

(b) In some round $n$, some $\psi \in \Psi_n(s)$ satisfies $||\pi_H^z \psi|| = 1$. Yet $(n:01, *, \hat{z}) \in s^E$ does not hold.

(c) The experiment is repeated, i.e., (9) is satisfied, with $\Psi_n(s) = \{\psi\}$ for $\psi$ fixed such that $||\psi_H^z \psi|| \neq 0$. Yet $(n:01, *, \hat{z}) \in s^E$ holds in no round $n$.

Note that, according to standard quantum theory, all these conditions hold with certainty.
3.2 Single-world (SW)

Property (SW) captures the idea that a measurement on a quantum system has only one single outcome. More specifically, we demand that this is the case for all quantum measurements relevant to one particular experimenter. (In the language of [Bru15], this could be regarded as the requirement that there are “facts” — maybe not “per se”, but at least “relative to an observer”.)

To formalise this property, let \( \mathbf{Obs} \) be a set of experiments of the form of the Repeated Measurement Experiment as defined in Section 2.2. That is, any \( E \in \mathbf{Obs} \) is also an element of \( \mathbf{RME}_{\mathcal{H}, \{\pi_H^z\}} \), where \( \mathcal{H} \) is a Hilbert space and \( \{\pi_H^z\}_{z \in \mathcal{Z}} \) a family of projectors. Crucially, the set \( \mathbf{Obs} \) may be restricted to those experiments that describe the viewpoint of one single experimenter. In the Extended Wigner’s Friend Experiment, this will be Wigner, i.e., we will demand that the experiment \( W \), defined in Section 4.2, is included in this set (cf. (18) below).

\[ T \text{ satisfies (SW) means that } T \text{ forbids all stories } s \text{ according to which for some experiment } E \in \mathbf{Obs} \text{ the set } \{z \in \mathcal{Z} : (0:01, *, z) \in s^E\} \]

has more than one single element.

We remark that (SW) is fundamentally different from the requirement that a theory \( T \) be deterministic. For example, \( T \) may prescribe that the outcome of a spin measurement is either \( z = -\frac{1}{2} \) or \( z = +\frac{1}{2} \) (not both), but still assert that this outcome is not correlated to anything that can be known before the measurement is carried out.

3.3 Self-consistency (SC)

Property (SC) is the requirement of self-consistency. Generally speaking, it demands that the laws of a theory \( T \) do not contradict each other. To express this formally, let \( \mathbf{Exp} \) be the set of all experiments that the theory should be able to talk about. For us, it is sufficient to demand that \( \mathbf{Exp} \) includes the experiments \( F_1, F_2, A, \) and \( W \), as defined in Section 4 below.

\[ T \text{ satisfies (SC) means that the condition } s^E \text{ is defined for all } E \in \mathbf{Exp} \]

does not imply that \( s \) is forbidden by \( T \).

To illustrate this, let \( E_1 \) and \( E_2 \) be the two experiments described in Section 2.3. Suppose that a theory \( T \) forbids any story \( s \) according to which \( z_1 = -\frac{1}{2} \) in experiment \( E_1 \), as well as any story according to which \( z_2 = +\frac{1}{2} \) in experiment \( E_2 \). If, in addition, \( T \) requires that \( z = z_1 \) holds in \( E_1 \) and that \( z = z_2 \) holds in \( E_2 \) then there is obviously no story \( s \) left that satisfies all rules of the theory. Hence, assuming that \( E, E' \in \mathbf{Exp} \), the
theory $T$ violates the self-consistency condition (SC), i.e., it rules out any possible story about the two experiments.

### 3.4 Theorem

We are now ready to state the main claim.

**Theorem 1.** No physical theory $T$ can satisfy $(QT)$, $(SW)$, and $(SC)$.

### 4 Extended Wigner’s Friend Experiment

The proof of Theorem 1 is based on an extension of the Wigner’s Friend gedankenexperiment [Wig67], which is similar to the Schrödinger’s cat experiment [Sch35]. Wigner considered an experimenter, the *friend*, who carries out a measurement while being enclosed in a perfectly isolated lab. The time evolution of the experimenter’s state (including the lab) then corresponds to that of a closed system. The extension is inspired by Deutsch’s idea of “undoing” this measurement and verify whether the original state can be retrieved [Deu85]. The specific setup considered here is similar to one proposed recently by Brukner, which involves multiple experimenters who carry out a Bell test [Bru15].\(^5\)

As a core ingredient we use a construction that originates in work by Hardy, where it was used to establish Bell’s theorem without inequalities [Har92, Har93].\(^6\)

We note that the purpose of the experiment is to prove Theorem 1. We therefore do not have to worry about its technological feasibility at this point. We only need to ensure that none of the steps of the experiment are forbidden by the basic laws of physics. The experiment has hence a similar status as, for instance, the Schrödinger’s cat gedankenexperiment. Nevertheless, a reader being worried about experimenters (or cats) enclosed in perfectly isolated labs may substitute them by quantum computers which simulate their actions.

In Section 4.1 we first describe the Extended Wigner’s Friend Experiment informally. We will do this in terms of an experimental protocol that prescribes the actions of the different participating experimenters from an overall perspective. Then, in Section 4.2, we provide a formal characterisation using the story-based framework introduced in Section 2. For this, we specify the views of the different experimenters individually.

#### 4.1 A bird’s eye view on the experiment

The Extended Wigner’s Friend Experiment features four experimenters (see Fig. 3). Two of them, the friends F1 and F2, shall be located in separate labs. We assume that, from an outside perspective, they can be treated as isolated quantum systems — unless there are explicit communication steps in the experimental protocol. This means that the time

\(^5\) As discussed at the end of Section 6.2, the argument presented here leads to different conclusions, though.

\(^6\) Specifically, the global state (30) between F1 and F2 corresponds to Eq. 1 of [Har93] for the case where $\alpha^2 = (3 + \sqrt{5})/6$. 

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evolution of their state is described by a unitary. We also assume that F1 is fully informed about the state of F2 (including F2’s entire lab) at the time when the experiment starts, and that this state is therefore pure. The other two experimenters, Wigner W and his assistant A, are not only informed about the initial states of F1 and F2, but can also carry out arbitrary quantum measurements on them.

### Extended Wigner’s Friend Experiment (EWFE)

Repeat the following steps for increasing \( n \in \mathbb{N}_0 \) until the halting criterion in the last step is satisfied.

- \( @ n:00 \) F1 invokes a quantum random number generator and memorises the output \( r \in \{ \text{head}, \text{tail} \} \).
- \( @ n:10 \) Depending on whether \( r = \text{head} \) or \( r = \text{tail} \), F1 sets the spin \( S \) of an electron to state \(|↓⟩_S \) or \(|→⟩_S \), respectively, and hands it over to F2.
- \( @ n:20 \) F2 measures \( S \) with respect to the basis \(|↓⟩, |↑⟩⟩_S \) and memorises the outcome \( z \in \{-\frac{1}{2}, \frac{1}{2}\} \).
- \( @ n:30 \) A measures F1 with respect to a basis \( |\text{ok}⟩_{F1}, |\text{fail}⟩_{F1}⟩ \) and records the outcome \( x \in \{ \text{ok}, \text{fail} \} \).
- \( @ n:40 \) W measures F2 with respect to a basis \( |\text{ok}⟩_{F2}, |\text{fail}⟩_{F2}⟩ \) and records the outcome \( w \in \{ \text{ok}, \text{fail} \} \).
- \( @ n:50 \) A and W compare their outcomes and halt if \( x = w = \text{ok} \).

The quantum random number generator used in the first step shall be designed such that the probabilities of the outcomes \( r = \text{head} \) and \( r = \text{tail} \) are \( \frac{1}{3} \) and \( \frac{2}{3} \), respectively. For concreteness, we may think of a mechanism that prepares a qubit \( C \), the “quantum coin”, in state

\[
\psi^0_C = \sqrt{\frac{1}{3}}|\text{head}⟩_C + \sqrt{\frac{2}{3}}|\text{tail}⟩_C
\]  

(10)

and measures it with respect to the orthonormal basis \( \{|\text{head}⟩_C, |\text{tail}⟩_C⟩ \). Furthermore, for the spin state of the electron prepared in the second step, which lives in a space \( \mathcal{H}_S \) spanned by the two orthonormal vectors \(|↓⟩_S \) and \(|↑⟩⟩_S \), we use the convention

\[
|→⟩_S = \sqrt{\frac{1}{2}}|↓⟩_S + \sqrt{\frac{1}{2}}|↑⟩⟩_S .
\]

The basis \( \{|\text{ok}⟩_{F1}, |\text{fail}⟩_{F1}⟩ \), with respect to which the measurement by A on F1 is carried out, shall be defined as

\[
|\text{ok}⟩_{F1} = \sqrt{\frac{1}{2}}|\text{head}⟩_{F1} - \sqrt{\frac{1}{2}}|\text{tail}⟩_{F1}
\]

\[
|\text{fail}⟩_{F1} = \sqrt{\frac{1}{2}}|\text{head}⟩_{F1} + \sqrt{\frac{1}{2}}|\text{tail}⟩_{F1},
\]

where \(|\text{head}⟩_{F1}⟩ \) and \(|\text{tail}⟩_{F1}⟩ \) are the states of F1 (including her lab) at time \( t = n:30 \) depending on whether she has seen \( r = \text{head} \) or \( r = \text{tail} \), respectively. Similarly, the basis \( \{|\text{ok}⟩_{F2}, |\text{fail}⟩_{F2}⟩ \) of the measurement carried out by W on F2 shall be defined as

\[
|\text{ok}⟩_{F2} = \sqrt{\frac{1}{2}}|→⟩_{F2} - \frac{1}{\sqrt{2}}|^⟩_{F2} - \frac{1}{\sqrt{2}}|+⟩_{F2}
\]

\[
|\text{fail}⟩_{F2} = \sqrt{\frac{1}{2}}|→⟩_{F2} + \frac{1}{\sqrt{2}}|^⟩_{F2} + \frac{1}{\sqrt{2}}|+⟩_{F2},
\]
Figure 3: Illustration of the Extended Wigner’s Friend Experiment. In any round $n$ of the experiment, one of Wigner’s friends, $F_1$, polarises an electron depending on a random value $r$. The other friend, $F_2$, measures its vertical polarisation $z$. Wigner’s assistant, $A$, and Wigner, $W$, measure the entire labs of $F_1$ and $F_2$, giving outcomes $x$ and $w$, respectively. The experimenters use quantum theory to derive statements about values seen by others. $F_1$, for instance, tries to infer $w$ as measured by $W$. The experiment ends when $x = w = \text{ok}$. Where $|\frac{-1}{2}\rangle_{F_2}$ and $|\frac{1}{2}\rangle_{F_2}$ are the states of $F_2$ at time $t = n:40$ in the case where $z = \frac{-1}{2}$ and $z = \frac{1}{2}$, respectively.

Following the spirit of the approach described in Section 2, we will consider stories that one can tell about this experiment. An example could be a story as follows.

$s = \begin{cases} \text{“At time } t = 0:00 \text{ the output of } F_1\text{'s random number generator is } r = \text{tail. She therefore prepares the electron spin } S \text{ in state } |\rightarrow\rangle. \text{ When } F_2 \text{ measures } S \text{ at } t = 0:20, \text{ she gets outcome } z = \frac{1}{2}. \text{ In their subsequent measurements at times } t = 0:30 \text{ and } t = 0:40, A \text{ and } W \text{ obtain outcomes } x = \text{ok} \text{ and } w = \text{ok}, \text{ respectively. They therefore halt.”} \end{cases}$

4.2 The experimenters’ views

To describe the Extended Wigner’s Friend Experiment formally, we subdivide it into four “sub-experiments”, which we label by the four experimenters, $F_1$, $F_2$, $A$, and $W$. The idea is that experiment $F_1$ is a specification of the part relevant to experimenter $F_1$, and so on.\footnote{This double use of notation is unproblematic as it is always clear from the context whether we mean the experimenter or the corresponding experiment.} The idea behind this subdivision is that each of the four experiments
corresponds, up to relabelings, to a Repeated Measurement Experiment, as introduced in Section 3.1, and can therefore be described using standard quantum theory. Note that such a description is not possible for the joint experiment, since \(z\) and \(w\), for instance, cannot simultaneously be regarded as observables according to quantum theory.

**Definition of Experiment F1**

The experimenter \(F1\) prepares the state \(\psi_S\) of \(S\) depending on the value \(r\). Furthermore, we assume that she is interested in the outcome \(w\) measured at time \(t = n:40\). The corresponding event space may therefore be defined as

\[
[F1] = \{(t, r, \psi_S, w) \in T \times \{\text{head}, \text{tail}, \perp\} \times \mathcal{H}_S \times \{\text{ok}, \text{fail}, \perp\} : \\
\text{if } t = n:10 \text{ then } r = \text{head} \iff \psi_S = |↓\rangle \text{ and } r = \text{tail} \iff \psi_S = |→\rangle \}
\]

for \(T\) as defined by (3). To formalise that \(w\) is the outcome of a quantum measurement, we relate this experiment to one of the set \(\text{RME}_{\mathcal{H}_S,(\pi^H_w)}\), where \(\mathcal{H}_S\) is the Hilbert space associated to the electron spin \(S\) and where \(\pi^H_w\) are measurement projectors in the Heisenberg picture. Specifically, taking \(U = U^n_{n:20}\) (for any \(n \in \mathbb{N}_0\)) to be an isometry from \(S\) to \(F2\) such that

\[
U|↓\rangle_S = |−\frac{1}{2}\rangle_{F2} \quad \text{and} \quad U|↑\rangle_S = |+\frac{1}{2}\rangle_{F2},
\]

we define \(\pi^H_w = U^\dagger|w\rangle\langle w|U\) for \(w \in \{\text{ok}, \text{fail}\}\). We then require that

\[
\exists E \in \text{RME}_{\mathcal{H}_S,(\pi^H_w)} : (t, *, \psi_S, w) \in s^{F1} \iff (t', \psi_S, w) \in s^E,
\]

where the mapping \(t \mapsto t'\) is such that \(n:10 \mapsto n:00\), \(n:40 \mapsto n:01\), and \(n:50 \mapsto n:02\). Note that this mapping must not be injective. While the time information contained in the plot \(s^E\) is thus more coarse-grained than that of \(s^{F1}\), it is still sufficient to apply the laws of quantum theory, as formulated by (QT).

**Definition of Experiment F2**

Experimenter \(F2\) measures the vertical spin direction \(z\). We assume that she is also interested in the state \(\psi_S\) as well as the randomness \(r\) it depends on. We therefore define the event space as

\[
[F2] = \{(t, r, \psi_S, z) \in T \times \{\text{head}, \text{tail}, \perp\} \times \mathcal{H}_S \times \{-\frac{1}{2}, +\frac{1}{2}, \perp\} : \\
\text{if } t = n:10 \text{ then } r = \text{head} \iff \psi_S = |↓\rangle \text{ and } r = \text{tail} \iff \psi_S = |→\rangle \}
\]

To specify how \(z\) arises as the outcome of a quantum measurement, we define \(\pi^H_z = |z\rangle\langle z|\) for \(z \in \{-\frac{1}{2}, +\frac{1}{2}\}\) and demand that

\[
\exists E \in \text{RME}_{\mathcal{H}_S,(\pi^H_z)} : (t, *, \psi_S, z) \in s^{F2} \iff (t', \psi_S, z) \in s^E,
\]

where \(t \mapsto t'\) is such that \(n:10 \mapsto n:00\), \(n:20 \mapsto n:01\), and \(n:40 \mapsto n:02\).
Definition of Experiment A

Experimenter A is measuring $x$ and, in addition, interested in the outcome $z$ of the measurement at $t = n:20$. Since these outcomes ultimately depend on the initial state $\psi_C \in \mathcal{H}_C$ of the quantum coin $C$, we also include it in the event space, which we choose to be

$$
\mathcal{A} = \{(t, \psi_C, z, x) \in \mathcal{T} \times \mathcal{H}_C \times \{-\frac{1}{2}, +\frac{1}{2}, \perp\} \times \{\text{ok}, \text{fail}, \perp\} : \\
\text{if } t = n:00 \text{ then } \psi_C = \psi^0_C
$$

(15)

for $\psi^0_C$ as defined by (10). To describe this as a quantum measurement, let $V = V^{n:00-n:10}$ be an isometry from $C$ to $\mathcal{F}_1 \otimes S$ such that

$$
V|\text{head}\rangle_C = |\text{head}\rangle_{F1} \otimes |\downarrow\rangle_S \text{ and } V|\text{tail}\rangle_C = |\text{tail}\rangle_{F1} \otimes |\uparrow\rangle_S
$$

and define the projectors $\pi^H_{z,x} = V^\dagger |x\rangle\langle x| \otimes |z\rangle\langle z| V$. We then require that

$$
\exists E \in \text{RME}_{\mathcal{H}_C, \{\pi^H_{z,x}\}} : \\
\text{if } t = [n:00, n:20] : (t, \psi_C, *, *) \in s^A \iff (n:00, \psi_C, *) \in s^E
$$

(16)

$$
\text{if } t = [n:20, n:30] : (t, \psi_C, z, *) \in s^A \iff (n:01, \psi_C, (z, *)) \in s^E
$$

$$
\text{if } t = [n:30, n:40] : (t, \psi_C, z, x) \in s^A \iff (n:01, \psi_C, (z, x)) \in s^E
$$

$$
\text{if } t = [n:40, n:50] : (t, \psi_C, *, x) \in s^A \iff (n:01, \psi_C, (*, x)) \in s^E,
$$

where we restrict to values $z \neq \perp$ and $x \neq \perp$.

Definition of Experiment W

Experimenter W measures $w$ and is also interested in $x$. Similarly to the above, the event space shall therefore be

$$
\mathcal{W} = \{(t, \psi_C, x, w) \in \mathcal{T} \times \mathcal{H}_C \times \{\text{ok}, \text{fail}, \perp\} \times \{\text{ok}, \text{fail}, \perp\} : \\
\text{if } t = n:00 \text{ then } \psi_C = \psi^0_C
$$

(17)

To describe this as a quantum measurement, we define the projectors

$$
\pi^H_{x,w} = V^\dagger |x\rangle\langle x| \otimes (U^\dagger |w\rangle\langle w| U) V,
$$

where $U$ and $V$ are the isometries from above. We then require that

$$
\exists E \in \text{RME}_{\mathcal{H}_C, \{\pi^H_{x,w}\}} \cap \text{Obs} : \\
\text{if } t = [n:00, n:30] : (t, \psi_C, *, *) \in s^W \iff (n:00, \psi_C, *) \in s^E
$$

(18)

$$
\text{if } t = [n:30, n:40] : (t, \psi_C, x, *) \in s^W \iff (n:01, \psi_C, (x, *)) \in s^E
$$

$$
\text{if } t = [n:40, n:50] : (t, \psi_C, x, w) \in s^W \iff (n:01, \psi_C, (x, w)) \in s^E,
$$

where we restrict to values $x \neq \perp$ and $w \neq \perp$. Note that $E \in \text{Obs}$ ensures that the single-world property (SW) holds from the viewpoint of $W$. The condition that the
experiment is repeated until the halting criterion is satisfied (which can be tested by $W$) then corresponds to (9), with $\hat{z} = (\text{ok}, \text{ok})$. This could also be restated as

$$n = 0 \text{ or } ((n - 1):40, *, x, w) \in s^W \text{ for } (x, w) \neq (\text{ok}, \text{ok}) \implies \forall t \in [n:00, n:40] : (t, *, *, *) \in s^W.$$  

(19)

By induction, it is easy to see that this condition implies that

$$\forall t \in [n:00, n:40] : (t, *, *, *) \in s^W$$

for any $n$ not larger than the minimum one satisfying $(n:40, *, x = \text{ok}, w = \text{ok}) \in s^W$.

**Compatibility conditions**

The experiments $F_1$, $F_2$, $A$, and $W$ obviously have certain overlapping elements — after all, they are all part of one big experiment. As explained in Section 2.3, the overlap between the experiments can be defined via compatibility constraints. Specifically, we demand that

$$(t, r, *, *) \in s^{F_1} \iff (t, r, *, *) \in s^W,$$  

(21)

which models that the quantity $w$ defined in the experiment $W$ is the same as the one $F_1$ refers to. Similarly, the compatibility constraints which model that $r$, $z$, and $x$ denote the same quantities in each experiment can be written as

$$(t, r, *, *) \in s^{F_1} \iff (t, r, *, *) \in s^{F_2},$$  

(22)

$$(t, *, z, *) \in s^{F_2} \iff (t, *, z, *) \in s^A,$$  

(23)

$$(t, \psi_C, *, x) \in s^A \iff (t, \psi_C, x, *) \in s^W.$$  

(24)

**5 Proof**

We split the proof of Theorem 1 in two parts. In the first, we use property (QT) to analyse the sub-experiments $F_1$, $F_2$, $A$, and $W$ separately. We recall that each of them models a part of the Extended Wigner’s Friend Experiment that is of interest to one of the four experimenters. Accordingly, the conclusions we obtain from this analysis, (25), (26), (29), and (31), are those that the experimenters would arrive at if they applied standard quantum theory. Then, in the second part of the proof, we use properties (SW) and (SC) to combine these individual conclusions, which then leads to a contradiction.

**5.1 Analysis of individual views**

For the following, let $T$ be any theory that satisfies property (QT) and let $s$ be any story that is not forbidden by $T$. 

16
Analysis of Experiment F1

By linearity, we have

\[ U|\rightarrow\rangle_S = U\left(\sqrt{\frac{1}{2}}|\downarrow\rangle_S + \sqrt{\frac{1}{2}}|\uparrow\rangle_S\right) = |\text{fail}\rangle_{F2}, \]

which immediately implies that \( \|\pi_H^{\text{fail}}|\rightarrow\rangle_S\| = 1 \). Hence, item (a) of property (QT), which applies due to (12), tells us that

\[ (n:10, *, |\rightarrow\rangle_S, *) \in s^{F1} \implies (n:40, *, * \text{ fail}) . \]

Using furthermore the constraint (11), we conclude that

\[ (n:10, r = \text{tail}, *, *) \in s^{F1} \implies (n:40, *, * \text{ fail}) \in s^{F1}. \]  

(25)

Analysis of Experiment F2

Here (QT) applies because of (14). Item (b) thus asserts that

\[ (n:20, *, *, z = +\frac{1}{2}) \in s^{F2} \implies \exists \psi : (n:10, *, \psi, *) \in s^{F2} \text{ and } \|\pi_H^{\psi}|\rightarrow\rangle_S\| \neq 0 . \]

By the constraint (13), we conclude that

\[ (n:20, *, *, z = +\frac{1}{2}) \in s^{F2} \implies (n:10, r = \text{tail}, *, *) \in s^{F2}. \]  

(26)

Analysis of Experiment A

Using the explicit form (10) of \( \psi^0_C \) we find that

\[ V\psi^0_C = \sqrt{\frac{1}{3}}|\text{head}\rangle_{F1} \otimes |\downarrow\rangle_S + \sqrt{\frac{2}{3}}|\text{tail}\rangle_{F1} \otimes |\rightarrow\rangle_S \]

\[ = \sqrt{\frac{1}{3}}|\text{head}\rangle_{F1} \otimes |\downarrow\rangle_S + \sqrt{\frac{1}{3}}|\text{tail}\rangle_{F1} \otimes |\downarrow\rangle_S + \sqrt{\frac{1}{3}}|\text{tail}\rangle_{F1} \otimes |\uparrow\rangle_S \]

\[ = \sqrt{\frac{2}{3}}|\text{fail}\rangle_{F1} \otimes |\downarrow\rangle_S + \sqrt{\frac{1}{3}}|\text{tail}\rangle_{F1} \otimes |\downarrow\rangle_S . \]  

(27)

This vector is obviously orthogonal to \( |\text{ok}\rangle_{F1} \otimes |\downarrow\rangle_S \). This means that \( \psi^0_C \) is orthogonal to \( \pi_{\frac{1}{2}}^{\text{ok}} \). Furthermore, (15) guarantees that

\[ (n:00, \psi_C, *, *) \in s^A \implies \psi_C = \psi^0_C . \]

Let \( E \) be an experiment as defined by (16). It follows from item (b) of (QT) that

\[ (n:01, *, (z = -\frac{1}{2}, x = \text{ok})) \notin s^E . \]  

(28)

By (16) we also have

\[ (n:40, *, *, x = \text{ok}) \in s^A \implies (n:01, *, (z, x = \text{ok})) \in s^E \]

\[ \implies (n:20, *, z = +\frac{1}{2}, *) \in s^A \]

for some value \( z \neq 0 \). Since, according to (28), \( z \neq -\frac{1}{2} \), we conclude that

\[ (n:40, *, *, x = \text{ok}) \in s^A \implies (n:20, *, z = +\frac{1}{2}, *) \in s^A . \]  

(29)
Analysis of Experiment W

Using (27) we find

\[ UV\psi_0^C = \sqrt{2/3} \otimes |\text{fail}\rangle_{F1} \otimes |-1/2\rangle_{F2} + \sqrt{1/3} \otimes |\text{tail}\rangle_{F1} \otimes |+1/2\rangle_{F2}. \]  

(30)

The overlap between this state and the state \(|\text{ok}\rangle_{F1} \otimes |\text{ok}\rangle_{F2}\) equals

\[ \sqrt{1/3}\langle \text{ok} | \text{tail} \rangle_{F1} \langle \text{ok} | +1/2 \rangle_{F2} = \sqrt{1/12}. \]

In other words, the state \(\psi_0^C\) has a non-zero overlap with the projector \(\pi_{\text{ok,ok}}^H\). Furthermore, (17) ensures that the system is always prepared in this state, unless no state was prepared at all. Item (c) of (QT), which applies due to (18), together with the requirement (19) that the experiment is repeated until the halting condition is satisfied, implies that there must exist a round \(n\) where \(x = w = \text{ok}\) occur, i.e.,

\[ (n:40, *, x = \text{ok}, w = \text{ok}) \in s^W \]  

holds whenever the plot \(s^W\) is defined.

5.2 Combining the views

Assume by contradiction that \(T\) satisfies the three properties (QT), (SW), and (SC). Property (SC) implies that there must exist a story \(s\) that is not forbidden by \(T\) such that all of \(s^{F1}, s^{F2}, s^A,\) and \(s^W\) are defined. In the following we consider such a story \(s\).

According to the analysis above, there must exist a round \(n\) such that (31) holds. Taking \(n\) to be the smallest such number, it follows from (20) that

\[ \forall t \in [n:00, n:40] : (t, *, *, *) \in s^W. \]  

(32)

Furthermore, by virtue of (18) we can apply property (SW), which implies

\[ |\{(x, w) \in \{\text{ok, fail}\} \times \{\text{ok, fail}\} : (n:40, *, x, w) \in s^W\}| = 1. \]  

(33)

Since we have chosen \(n\) such that (31) holds, it follows that

\[ (n:40, *, x, w) \in s^W \iff x = w = \text{ok}. \]  

(34)

Using the compatibility condition (24) we obtain

\[ (n:40, *, *, x = \text{ok}) \in s^A. \]

Constraint (29), which resulted from the quantum-mechanical analysis from A’s viewpoint, hence implies that

\[ (n:20, *, z = +1/2, *) \in s^A. \]

We now use the compatibility condition (23) to infer that

\[ (n:20, *, z = +1/2, *) \in s^{F2}. \]
Applying (26), which resulted from the quantum-mechanical analysis from F2’s viewpoint, we obtain

\[(n:10, r = \text{tail}, *, *) \in s^{F2}.\]

Using the compatibility criterion (22), this means that

\[(n:10, r = \text{tail}, *, *) \in s^{F1}.\]  
(35)

The quantum-mechanical analysis from F1’s viewpoint, (25), then gives

\[(n:40, *, *, w = \text{fail}) \in s^{F1}.\]

By the compatibility condition (21) this means that

\[(n:40, *, *, w = \text{fail}) \in s^{W}.\]

But this is in contradiction to (34), which concludes the proof of Theorem 1.

6 Possible scenarios

According to Theorem 1, if one wishes to devise a physical theory, one has to give up at least one of the assumptions (QT), (SW), or (SC). In the following three subsections, we discuss possible scenarios that arise when one of the assumptions is abandoned. They usually correspond to theories that have been proposed in the literature. The fourth subsection addresses the question whether there are other implicit assumptions built into the framework used here.

6.1 Theories that violate (QT)

Property (QT) demands that the laws of quantum theory are valid even for systems that are complex enough to contain an observer (who herself applies quantum theory to describe a system in her possession). This assumption is generally violated by theories that postulate a modification of the usual Schrödinger equation [GR95, Wei12]. Examples include spontaneous collapse models such as the GRW theory and extensions thereof [GRW86, Pea89, Tum06], as well as gravity induced collapse models [Kar66, Dió89, Pen96] (see [BLS13] for a review). The same is true for the proposal of [AG15] to supplement the basic formalism of quantum theory with the postulate that measurements must be carried out in a context (which may include the measurement device). Since, according to the postulate, this context cannot be treated itself as a quantum system, it rules out the possibility of applying quantum measurements to measurement devices, thus violating (QT).

Conversely, property (QT) is not usually violated by hidden variable theories. The reason is that (QT) only refers to statements that can be made with certainty within quantum theory. In particular, it does not impose any constraints on the statistics of measurement outcomes, and hence it does not rule out theories that provide more informative predictions than quantum theory [CR11]. To illustrate this, one may take a variant of Bohmian mechanics where the particle positions are known. This theory, although being deterministic, would never predict an outcome that is forbidden by quantum theory, and hence still satisfy (QT).
The intuition behind many-worlds. Slightly varying the Schrödinger’s cat gedankenexperiment [Sch35], a mechanism may decide, based on the outcome of a spin measurement, whether or not the cat is fed. According to a many-worlds interpretation, the two perceptions of the cat, hungry and happy, are equally real. While this may sound counter-intuitive, a much more familiar situation is obtained when the feeding mechanism is replaced by one triggered by time. Assuming the cat has no good memory, the situation remains the same for her: the two perceptions, hungry and happy, are equally real.

6.2 Theories that violate (SW)

Property (SW) demands that only one single outcome occurs when we measure a quantum system. It captures the intuition that the outcome we are aware of is the only real one. Probably the most prominent representative of a theory which abandons this intuition is the relative state formalism, which is also known as the many-worlds or the Everett interpretation [Eve57, Whe57, DeW70, Deu85, Deu97, Vai16]. It proclaims that the measurement of a quantum system results in a branching into different “worlds”, in each of which one of the possible measurement outcomes occurs. The different outcomes are therefore all equally “real”, corresponding to statement (S2) in the case of our introductory example of a spin measurement (cf. Fig. 4).

Various variants and extensions of the relative state formalism have been proposed. Among them are the many-minds interpretation [Zeh70, AL88] and the parallel lives theory [BRR13], as well as notions such as quantum Darwinism [Zur07]. Their common feature is that they do not postulate a physical mechanism that singles out one particular measurement outcome, although observers have the perception of single outcomes. They are therefore all examples of theories that are not single-world in the sense of (SW).

We recall, however, that property (SW) does not demand that one can simultaneously assign unique outcomes to any measurement made during an experiment. Rather, the requirement is that the measurement outcomes accessible to one observer (in our case
experimenter \( W \) have single values. Our analysis of the Extended Wigner’s Friend Experiment therefore leads to a conclusion that differs, for instance, from that of [Bru15], who argued that facts cannot exist “per se”, but that they may still exist “relative to observers”. Theorem 1 also excludes this latter possibility.

6.3 Theories that violate \((SC)\)

It is a consequence of Theorem 1 that deterministic hidden variable theories must violate property \((SC)\), i.e., they cannot be self-consistent. Indeed, as argued in Section 6.1, they satisfy \((QT)\), and because they are (by definition) deterministic, measurements have only one single outcome, so that \((SW)\) holds.

An example of such a hidden variable theory is Bohmian mechanics. The conclusion that Bohmian mechanics is not self-consistent may be compared to the results of [CM02, KW10]. They suggest that, in an experiment with entangled particles, the behaviour of the Bohmian particle positions is not consistent with the statistics that quantum theory would predict for position measurements. However, since Bohmian particle positions cannot in general be identified with the outcomes of position measurements [ESSW92, Vai05], this finding does not point to a fundamental inconsistency of Bohmian mechanics [Gis15]. This is in contrast to the conclusions we can draw from Theorem 1, which shows that an inconsistency arises independently of how one interprets the Bohmian particle positions.

It is certainly unsatisfactory if a theory is not self-consistent. One may therefore ask whether there is an easy fix. One possibility could be to restrict the range of applicability of the theory and add the rule that its predictions are only valid if an experimenter who makes the predictions keeps all relevant information stored. While we do not normally impose such a rule when using theories to make predictions, this would, at least in the case of Bohmian mechanics, remove the inconsistency. Indeed, since in the Extended Wigner’s Friend Experiment \( F_1 \) may not be able to store the value \( r \) until \( t = n:40 \), the theory could no longer be applied to analyse her part of the experiment. Formally, this means that \((12)\) would no longer be a valid requirement and, hence, \((25)\) could not be established.

6.4 Relaxing the notion of a physical theory

The formulation of Theorem 1 is based on a framework, as outlined in Section 2, which allows us to reason about physical theories. Hence, any structure imposed by the framework potentially limits the generality of the claim. One may therefore try to further relax it. However, compared to other frameworks used in the literature on the foundations of quantum theory, the one used here is already rather minimalistic. In particular, it is not demand that a physical theory provides predictions that can be expressed in terms of probability distributions.

To illustrate this, it is instructive to have a quick look at other approaches, such as the one taken by Bell [Bel66]. The assumptions that enter Bell’s theorem are usually formulated in terms of probability distributions of certain random variables that model the outcomes of measurements. The model therefore comes with the a priori assumption that all measurements, even if they are carried out by different experimenters, have a well-defined outcome (the value of the random variable), and that a physical theory allows us
to calculate predictions (the probability distributions assigned to these variables). The same holds true for various results based on probabilistic frameworks [Bar07, BW16] (cf. [Har11, CDP11, MM11, CR12, PBR12, OCB12, LS13] for some recent examples). In contrast to this, Theorem 1 completely avoids the use of the notion of probability, so that no such assumption is necessary.

One may now ask whether there are still other assumptions built into the framework used here. For example, the idea of considering multiple experimenters, which is central to our argument, could be problematic if one takes a more radical subjective viewpoint. A nice example is QBism, according to which probabilities represent beliefs of an agent and are therefore entirely subjective [FMS14]. Even if an agent assigns probability 1 to a particular outcome of a measurement, this should not be taken as a fact, and QBism would even allow that another agent assigns probability 0 to the same outcome. This suggests that, within QBism, it may not be sensible to carry out an analysis that is based on the combination of different experimenters’ views. Nevertheless, since it is rather common that we try to infer the behaviour of others by reasoning about their decision-making, it could still be interesting to explore how QBism should be applied to situations where one agent uses QBism to express his believes about another agent’s actions who herself applies QBism. We leave a study of the Extended Wigner’s Friend Experiment from such a Bayesian viewpoint to future research.

7 Conclusions

A natural requirement to any reasonable physical theory $T$ is that different observers who apply $T$ should not arrive at logically contradicting statements. This is captured by property (SC), which demands that there exists at least one possible story $s$ that none of the different observers would consider as forbidden. Once one accepts (SC) as an unavoidable requirement, our main result, Theorem 1 implies that we are left with the option to either deny (QT) or (SW).

It is in principle possible to decide between these two options by an appropriately designed experiment. The experiment that would need to be carried out is a test whether the predictions of standard quantum theory are valid for systems that are complex enough to count as observers. As famously noted by Bell, defining precisely what this means appears to be impossible [Bel90]. However, experiments that test quantum coherence for macroscopic systems [AH14] certainly provide evidence in favour of (QT), and hence against (SW).

While the ultimate experiment would involve human observers, one could argue that it is sufficient to consider systems that have the same level of complexity. This could be achieved once we are able to build scalable quantum computers. It therefore provides another incentive to build such machines. If they work according to the predictions of quantum theory, we know we are left with Scenario 2 described in the introduction. That is, we are forced to give up the view that there is one single reality.

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8Scalability means that arbitrarily complex computing devices can be built from elementary components by composition.
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