Optimization of Sand-Bentonite Mixture for the Stable Engineered Barriers using Desirability Optimization Methodology: A Macro-Micro-Evaluation

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1. Introduction

Due to the expanding population and resource consumption rate around the world over the past few years, significant growth has been recorded in waste generation from domestic to radioactive wastes. The most cost-effective and common way of dealing with these wastes is disposal at landfill sites. Well-designed engineered barriers (EB) and subsurface repositories (SR) are required at landfill sites for controlling the leaching and movement of hazardous substances from these wastes (Frändegård et al., 2013). Sand-bentonite mixtures (SBMs), are usually used in landfills to control the leachates. The SBM is used in the compacted form as compaction packs the soil particles closely, and correspondingly decreases the soil hydraulic conductivity, which is required for the EB or SR dealing with hazardous waste (Yong et al., 1986).

Hydraulic conductivity (k) is the main design criteria to be considered in different standards for the EBs using SBM; for instance, k requirement for the landfill liner is between $10^{-9}$ – $10^{-10}$ m/s, and for the radioactive repository is between $10^{-9}$ – $10^{-11}$ m/s as per criteria of Swedish EPA-2000 and Japan Nuclear Cycle...
Development Institute, respectively (JNCDI, 2000; Swedish EPA, 2000). In literature, basic characterization of hydraulic behavior of SBM is conducted by various researchers; for instance, Srikanth and Mishra (2016) discussed the effect of sand particle size on compaction and hydraulic conductivity behavior of compacted SBMs by using different types of sand and bentonite. Abichou et al. (2002) investigated the hydraulic conductivity of SBM at different dry densities for different ratios of sand to bentonite. Several other researchers i.e., Wang et al. (2013a, 2013b), Sharma and Deka (2019) and Agus et al. (2010) investigated the permeability characteristics of clayey soil, silty clays and SBM as insulation barriers.

Apart from hydraulic conductivity, volumetric change and strength behavior could be the other essential characteristics to be considered for the design of the EB (D’Appolonia, 1980; Yong et al., 1986; Lloret et al., 2003; Li et al., 2018). For instance, the SRs are constructed below the ground level surrounded by the compacted SBM blocks to store the high-level radioactive waste (HLW materials) (D’Appolonia, 1980). Since SBM blocks contain bentonite, these blocks absorb water from the surrounding rocks and exhibit significant volumetric change upon the hydration (Wang et al., 2013b). On the other hand, the volumetric increase due to hydration of the compacted SBM material applies the stress (swelling pressure) on the adjacent restraints. Moreover, the surrounding rocks also exert confining stress against the volumetric increase (Wang et al., 2013a). Hence, it is essential to study the volumetric-change behavior i.e., swelling and compressibility behavior, and strength characteristics of a compacted SBM against the stress.

Compression, swelling and suction characteristics of different types of densely compacted SBM for safe disposal of radioactive wastes have been investigated by several researchers (Filz, 1996; Lloret and Villar, 2007; Horpibulsuk et al., 2011; Saba et al., 2014; Kaufhold et al., 2015; Chen et al., 2017; Dafalla, 2017; Ma et al., 2018). In the literature, some correlational models for the compression characteristics for SBMs are also developed (Horpibulsuk et al., 2011; Ma et al., 2018). Villar and Lloret (2008) determined the influence of water content and dry density on the swelling and hydraulic conductivity behaviors of a compacted bentonite barrier during and after 18 years of construction. Wang et al. (2013a) investigated that the swelling behavior of the compacted SBM is similar to the pure bentonite and also proposed a correlational model to predict axial stress with bentonite void ratio. In addition to this, some researchers (Filz, 1996; Villar and Lloret, 2008; Saba et al., 2014; Chen et al., 2017; Dafalla, 2017; Ma et al., 2018) have also studied the strength characteristics of the SBMs. Thus, vast literature is available on the characterization of different geotechnical properties of the SBM (Horpibulsuk et al., 2011; Tripathi and Viswanadham, 2012; Saba et al., 2014; Kaufhold et al., 2015; Proia et al., 2016; Chen et al., 2017; Dafalla, 2017; Sharma et al., 2017; Sobti and Singh, 2017; Ma et al., 2018). From the literature, it can be inferred that EBs require having low hydraulic conductivity, swelling potential, swelling pressure and compression, and high compressive and shear strength for stability, which are antagonist responses correlatively for SBM considering its mix design (Yong et al., 1986; Saba, 2013; Akgün and Mahir, 2015; Srikanth and Mishra, 2016; Khalid and Rehman, 2018; Rehman et al., 2018). Generally, in past studies, the optimal SBM is determined based on a direct interpretation of the antagonist geotechnical responses, which results in an overgeneralized mix design; thus, an accurate optimization technique is required to be applied that could astutely integrate multiple antagonist geotechnical responses. In addition, different existing design specifications e.g., Swedish EPA (2000) and Japan Nuclear Cycle Development Institute advocate determining the optimal SBM for EBs mainly based on the permeability of SBM and ignore other important geotechnical facets which are extremely important for the stability of EBs especially if they are to be built underground (JNCDI, 2000; Swedish EPA, 2000).

To the best of the authors’ knowledge based on an extensive literature review, no past study has implied any authentic optimization technique and considered all the desired geotechnical characteristics of the SBM for EBs to determine an optimal SBM, hitherto. The current study addresses this gap and implements an optimal mixture design technique called Desirability Optimization Methodology (DOM) and incorporates hydraulic conductivity ($k$), unconfined compressive strength ($q_c$), angle of internal friction ($\phi$), cohesion ($c$), compression index ($C_s$) and swell pressure ($SP$) altogether as the geotechnical responses to determine optimal SBM. DOM is predominantly effective for the assessment of the optimal conditions in industrial or natural processes containing various factors leading to the antagonist responses (Amdoun et al., 2018). Albeit DOM is frequently used in industrial engineering, biochemistry, microbiology, drugs and chemical analysis, this method has never been applied to determine the optimal mixture design of SBM for the possible best geotechnical performance of EB (Khor and Ramakrishnan, 2016; Amdoun et al., 2018; Giordano et al., 2020; Bonaccorso et al., 2021; Rehman and Khalid, 2022). In addition, in past studies, mostly SBMs are compacted with medium compaction effort; however, it is envisaged that compacting SBM with high compaction energy may decrease the quantity of bentonite required for an optimal solution.

Considering these research dimensions, the authors conducted various geotechnical experiments on compacted SBM using a modified compaction effort of around 2,700 kN-m/m$^3$ (Khalid and Rehman, 2018). In addition, scanning electron microscope (SEM) analysis is also conducted to study the microstructural changes along various mix ratios of SBM to determine the mix range having the suitable microstructure. Afterward, the statistical models for the aforementioned geotechnical responses of SBM are established and subsequently integrated into a mathematical algorithm of desirability function ($D$). Depending on the required performance of EB, the antagonist geotechnical responses are optimized by this algorithm considering criteria such as targeted minimization of $k$, minimization of $C_s$ and $SP$, and maximization of $q_c$, $\phi$ and $c$, and optimal SBM is determined with a high $D$-value.
2. Materials and Test Methods

To meet the requirement of insulating buffer material, Ravi sand and Ca-bentonite are selected for the current study as they are abundantly available in Punjab, Pakistan. Different mix proportions are adopted in this research by varying bentonite from 15% to 40% at a rate of 5% to determine an optimum quantity of bentonite and sand to be used as an impermeable barrier for the safe disposal of municipal and nuclear waste. The bentonite content (BC) in the SBM is calculated using Eq. (1):

\[
BC = \frac{m_{\text{ben}}}{m_{\text{ben}} + m_{\text{sand}}} \times 100;
\]

\[
SC = 100 − BC
\]

where \(m_{\text{ben}}\) and \(m_{\text{sand}}\) are the dry mass of the bentonite and sand in the mixture respectively, and SC is the sand content.

Some physical properties of sand and bentonite are given in Table 1. The coefficient of uniformity \((C_u)\) and coefficient of curvature \((C_c)\) of selected sand are 2.44 and 1.14, respectively. The mean grain size of the sand is around 0.2 mm and the maximum particle size of bentonite was lesser than 0.075 mm. The liquid limit \((L)\) and plasticity index \((P)\) of the bentonite are 209% and 157%, respectively. The PI of bentonite indicates that bentonite is a very high plastic cohesive material. According to the unified soil classification system (USCS), the selected sand and bentonite are classified as poorly graded sand (SP) and high plastic clay (CH), respectively. With the addition of bentonite and bentonite are classified as poorly graded sand (SP) and high plastic clay (CH), respectively. Moreover, the optimum water content \((w_{\text{opt}})\) and the maximum dry unit weight \((\gamma_{\text{dmax}})\) using modified compaction effort are determined to be 17% and 16.1 kN/m³ for bentonite and 11% and 17.6 kN/m³ for sand, respectively.

X-ray diffraction is one of the widely used methods for the identification of clay minerals and studying their crystal structure within the soils. X-ray diffraction analysis is carried out on bentonite (Fig. 1) shows that the predominant clay mineral is the montmorillonite group. In addition, quartz, kaolinite, illite, and traces of calcite are observed. Results of the chemical analysis show that bentonite is composed mostly of SiO₂ (45.80%), with 14.62% of Al₂O₃, 8.02% of CaO, 7.29% Fe₂O₃ and traces of MgO.

Geotechnical tests are conducted to investigate different engineering characteristics of these SBMs such as compaction characteristics, shear strength, swell potential, consolidation and permeability. To ensure proper compaction of samples, a series of modified compaction tests (as per ASTM D1557) are conducted on all SBMs (15 – 40% BC), for which compaction effort was 2,700 kN-m/m³ and corresponding \(w_{\text{opt}}\) and \(\gamma_{\text{dmax}}\) are determined. Afterward, for sample preparation, the corresponding amount of \(w_{\text{opt}}\) is added to each SBM and compacted in the compaction mold by employing the same compaction effort and procedure as adopted in the modified compaction test. Theoretically, for a certain soil, only a single value of density can be achieved for a particular set of moisture content and compaction effort. The real dry density of SBM compacted in a compaction mold through the aforementioned procedure was also determined and it was found to be almost similar to \(\gamma_{\text{dmax}}\). After achieving \(\gamma_{\text{dmax}}\) of an SBM, the samples for different tests i.e., swelling tests, oedometer tests, unconfined compressive strength tests and unconsolidated-undrained (UU) triaxial compression tests were extruded carefully as per the required specifications of the tests designated by ASTM. Moreover, it is important to note that in this study, heavy compaction of SBM is achieved due to the application of modified compaction effort (compaction energy = 2,700 kN-m/m³).

Swelling tests are carried out using a classical oedometer apparatus (Mujtaba et al., 2020). The test is realized according to the free swelling method (ASTM D4546). The dimensions of samples extruded from compacted SBM mass at \(\gamma_{\text{dmax}}\) are 38 mm in diameter and 19 mm in height. The sample is inundated under a surcharge pressure of 1 psi and allowed to swell freely to its total swell potential in a saturated condition. The total free swell potential is computed using the following equation:

\[
\text{Swell Potential} = \left( \frac{H_f - H_0}{H_0} \times 100 \right)
\]

where \(H_0\) is the initial height (before swelling), and \(H_f\) is the final height (after swelling) of the specimen. After achieving the full swelling potential, the sample is loaded to reach its initial

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Table 1. Properties of Sand and Bentonite

| Properties | \(D_{50}\) (mm) | \(D_{10}\) (mm) | \(D_{60}\) (mm) | \(C_u\) | \(C_c\) | Specific Gravity | Max. Dry Density (kN/m³) | Optimum Moisture Content (%) | Liquid Limit (%) | Plastic Limit (%) | Plasticity index (%) | Classification as per USCS |
|------------|----------------|----------------|----------------|--------|--------|----------------|------------------------|--------------------------|-------------------|-------------------|----------------------|---------------------------|
| Sand       | 0.09           | 0.15           | 0.2            | 2.4    | 1.1    | 2.69          | 17.6                   | 11                       | -                 | -                 | -                    | SP                        |
| Bentonite  | -              | -              | -              | -      | -      | 2.76          | 16.5                   | 17                       | 209               | 52                | 157                  | CH                        |

Fig. 1. X-Ray Diffraction of Bentonite
volume; the required pressure corresponding to achieving the initial volume after free swell is regarded as the swell pressure (SP), which is a design parameter of the SBM for EB.

Moreover, Oedometer tests are performed to determine the compressibility characteristics i.e., $C_v$, coefficient of consolidation, $C_s$, coefficient of volume compressibility ($m_v$), and yield stress ($\sigma_y$) of SBMs. For this test, a specimen with 38 mm in diameter and 19 mm in height is placed in a metal ring and saturated for 24 h. The initial moisture content and porosity of the sample were in a range of 12.56 – 13.25% and 0.29 – 0.3, respectively. In this study, the oedometer test of 24 hours incremental loading is carried out. The $k$-value is also indirectly obtained by using $C_v$ evaluated by Taylor’s approach and $m_v$ as per the following equation.

$$k = \gamma_w m_v C_v$$  \hspace{1cm} (3)

where $\gamma_w$ is the unit weight of water. To compute the mechanical characteristics of SBMs under compression and shear forces, unconfined compressive strength tests and unconsolidated-undrained (UU) triaxial compression tests are performed by following ASTM D2166 and ASTM D2850, respectively. The initial inherent properties of samples are consistent with the aforementioned tests. The $q_u$, deformation modulus ($E_{50}$), $c$ and $\phi$ are determined using the results of these tests (Ur Rehman and Khalid, 2021). Besides, measurements for all tests are taken on three individual samples for repetition, and the findings are approved only if the deviation from the average is within the 5% error allowance. In addition, in this study, to obtain an optimized mixture design different mathematical models are also developed for major geotechnical design parameters of SBM using SC and BC as the predictors. Afterward, a full-scale optimization study is carried out using these models. In addition, SEM is also conducted as per Ijaz et al. (2020a, 2020b) to analyze the microstructure of different SBMs.

### 3. Macro-Geotechnical Evaluation and Statistical Modeling

#### 3.1 Mechanical Characteristics of the Sand-Bentonite Mixture

#### 3.1.1 Unconfined Compressive Behavior

Figure 2 presents the unconfined compressive behavior of different SBMs. It is observed that as the BC is increased in the
SBM, the $q_u$ and $E_50$ increase substantially. The increase in $q_u$ is observed to be from 246 kPa to 946 kPa as the BC increases from 15% to 40%. These results are in good agreement with the results of Srikanth and Mishra (2016). The $q_u$ determines the stability of the SBM against compressive loading thus, it can be regarded as the design parameter for the SBM design. In this regard, the following relationship is drawn between $q_u$ and SC (%) and BC (%):

$$q_u = 18.9BC - 2.6SC + 0.14SC \cdot BC$$  \hspace{1cm} (4)

Statistical evaluation of this model is also done by determining, $R^2$, Adj $R^2$, and Pred $R^2$ values, all these parameters are observed to be around 0.99 and are in agreement with each other (the difference between these parameters is less than 0.2) (Table 2). This shows the high reliability of the model (Rehman et al., 2021). In addition, adequate precision measures the signal-to-noise ratio of a model, a ratio greater than 4 is desirable. For this model, the ratio of 322.6 is observed, which indicates an adequate signal-to-noise ratio. The analysis of variance (ANOVA) results also show significant health of this model as large $F$-value (21059.9) and insignificant probability, $p$-value, (<0.0001) is observed for this model (Mujtaba et al., 2021). Thus, this model could reliably be used to predict the $q_u$ for any SBM design ratio (Table 2).

### 3.1.2 Shear Response

**Figure 3** presents the $c$ and $\varphi$ values of the various SBMs obtained from UU triaxial shear tests. The plots of $c$ and $\varphi$ against BC indicate that as the BC increases, $c$ and $\varphi$ increase and decrease, respectively. These contrasting trends are attributed to the increase in the cohesive matter in the SBM with an increase in BC (Saba, 2013). The maximum rate of increase (i.e., slope) for the $c$ occurs between a bentonite content of 30% and 32.5% and is equal to about 7.7 kPa. The maximum rate of decrease for the $\varphi$ occurs between a bentonite content of 20% and 22.5% and is equal to about 0.716 degrees. The $c$ and $\varphi$ are important parameters to determine the resistance of SBM against shear failure, which is an important geotechnical consideration for the EB. To predict $c$ and $\varphi$ using SC (%) and BC (%) in an SBM, the following models are developed:

$$c = 3.66BC - 0.68SC + 0.014SC \cdot BC$$  \hspace{1cm} (5)

$$\varphi = 0.195SC + 0.006BC$$  \hspace{1cm} (6)

These models are observed to have significant and identical values of $R^2$, Adj $R^2$ and Pred $R^2$ (>0.99). In addition, adequate precision is also observed to be high for these models (64.6 and 88.1 for Eqs. (5) and (6), respectively) (Table 2). Moreover, ANOVA results show a high $F$-value (843.1 and 1965.1 for Eqs. (5) and (6), respectively) and insignificant $p$-values (<0.0001 for Eqs. (5) and (6), respectively). These statistical analyses indicate the high health of both correlations (Table 2).

### 3.2 Volumetric-Change Response of the Sand-Bentonite Mixture

#### 3.2.1 Swelling Behavior

**Figure 4** presents the swelling behavior of the subjected SBMs. It
is observed that swell potential and SP are linearly increased with an increase in the BC (Figs. 4(b) and 4(c)). The swelling behavior of the SBMs appears to be a compromised geotechnical characteristic as the BC is increased. Thus, to achieve a high-performance SBM for engineered barriers an astute mix design of SC and BC is needed to be adopted. SP can be regarded as the design parameter for the engineered barrier which is observed to have the following best fit relationship with SC (%) and BC (%) in an SBM:

\[ SP = 2.7SC - 62.8BC + 1.7SC \cdot BC + 0.14SC \cdot BC( SC - BC ) \]  

(7)

This model is also statistically validated; \( R^2 \), Adj \( R^2 \) and Pred \( R^2 \) values are observed to be as high as 0.99 and the difference between these three coefficients is found to be insignificant (Table 2). An adequate precision factor of 153.9 is observed for this model. In addition, \( F \)-value and \( p \)-value are observed to be around 4370.2 and <0.0001, respectively. Thus, these results indicate that the model is statically reliable (Table 2).

3.2.2 Compressibility Response

Figure 5 presents the compressibility characteristics of the SBMs. As can be seen from the compression curves of the SBM in Fig. 5(a), the initial void ratio (\( e_0 \)) marginally increases (0.4227 to 0.4332) with an increase in the BC (15% to 40%) (Fig. 5(b)). On the other hand, \( \sigma_y \) (300 kPa to 420 kPa) increases as the BC increases (15% to 40%) in the SBM (Fig. 5(c)) this trend is found to be in line with (Srikanth, and Mishra 2016). The \( m (0.00224 \text{ MPa}^{-1} \text{ to } 0.00372 \text{ MPa}^{-1}) \) and \( C_c (2.79 \times 10^{-8} \text{ m}^2/\text{sec} \text{ to } 0.49 \times 10^{-8} \text{ m}^2/\text{sec}) \) are observed to increase and decrease respectively with an increase in the BC (15% to 40%) (Figs. 5(d) and 5(e)). Meanwhile, \( C_c \) which can be taken as the main design parameter to compute settlement, increases as the BC increases (Fig. 5(f)). However, due to initial compaction at \( \gamma_{dmax} \) using modified effort, the value of \( C_c \) is observed to be less than 0.05 for all the subjected samples (0.007 - 0.0153), thus these SBMs could be regarded a as very insignificantly compressible (Fig. 5(f)). Since \( C_c \) is regarded as the potential geotechnical design parameter for the EB, a mathematical model is proposed to predict \( C_c \) using SC and BC in the SBM as follows:

\[ C_c = 3.2 \times 10^{-3} BC - 6.1 \times 10^{-8} SC - 5.2 \times 10^{-8} SC \cdot BC + 3.4 \times 10^{-8} SC \cdot BC(SC - BC) \]  

(8)

The values of \( R^2 \), Adj \( R^2 \) and Pred \( R^2 \) are higher than 0.98 and are found to be insignificantly different from each other (Table 2). This model has an adequate precision factor of 46.46. \( F \)-value and \( p \)-value are found to be approximately 363.62 and <0.0001, respectively. These results show that the model (Eq. (8)) is statically reliable (Table 2).
3.2.2.1 Normalized Compression Curve for Compacted SBM

Normalizing the compression curve is also very important for the geotechnical design of an SBM to assess its compression behavior. For the compression curves of reconstituted clay, Burland (1990) introduced the concept of the void index \( I_v \). According to Burland (1990), at the initial water content of 1 to 1.5 times the \( w_{L} \), the compression curves of recomposed clay can be well normalized with \( I_v \). The proposed \( I_v \) equation is given below:

\[
I_v = \frac{e - e_{100}}{e_{100} - e_{1000}}
\]

where the \( e_{100} \) and \( e_{1000} \) are the void ratios at effective vertical stresses \( (\sigma'_v) \) of 100, and 1,000 kPa, respectively. Burland (1990),
Hong et al. (2010), and Khalid and Rehman (2018) proposed the unique relationships between $I_v$ and $\sigma'$ to compute intrinsic compression line for reconstituted clay (ICL), extended intrinsic compression line for reconstituted clay (EICL), and normalized compacted compression line for compacted clays (NCCL), respectively, to normalize the compression curve for these soils, as given below.

\begin{align}
I_{\text{ICL}} &= 2.45 - 1.285(\log \sigma'') + 0.015(\log \sigma'')^2 \\
I_{\text{EICL}} &= 3 - 1.87(\log \sigma'') + 0.179(\log \sigma'')^2 \\
I_{\text{NCCL}} &= 0.215 - 0.0027(\log \sigma'') + 2.1 \times 10^{-6}(\log \sigma'')^2 - 6 \times 10^{-10}(\log \sigma'')^3
\end{align}

Since these equations were proposed for clays, hence, they are found to be non-significant for the SBM (Fig. 6). A new relationship is drawn to predict NCCL for SBM with an $R^2$-value of 0.99 as follows:

\begin{align}
I_{\text{NCCL}} &= 0.0571 - 0.0007(\log \sigma'') + 1 \times 10^{-6}(\log \sigma'')^2
\end{align}

The $I_v$ is defined in terms of $e_{100}$ and $e_{2000}$ which are void ratios effective at $\sigma'$ of 100, and 2,000 kPa, respectively, as presented in Eq. (14) in this study; whereas the $e_{100}$ and $e_{100} - e_{2000}$ could be determined with $e_0$ using the following equations (Fig. 7) (Eqs. (15) and (16)). In Eq. (14), the $e_{2000}$ is used to estimate the void index of SBMs compacted at modified compaction energy instead of $e_{100}$ because the difference between $e_{100}$ and $e_{2000}$ is very small to predict the compression curves properly.

\begin{align}
I_v &= \frac{e - e_{100}}{e_{100} - e_{2000}} \\
e_{100} &= 1.03(e_0 - 0.014) \\
e_{2000} &= 0.421(e_0 - 0.34)
\end{align}

A reliable prediction of compression curves for all SBM can be done using the equation proposed in the current study, as shown in Fig. 7 which could be vital for assessing the compression behavior of compacted SBMs in the absence of consolidation test results in designing EB.

### 3.3 Hydraulic Conductivity Analysis of Sand Bentonite Mixture

Figure 8 presents the $k$-value of SBMs against varying BC. It can be observed that $k$ decreases ($4.03 \times 10^{-11}$ m/sec to $0.84 \times 10^{-11}$ m/sec) as the BC increases (15% to 40%). The $k$ is regarded as the main design parameter for the EB, and its requirement for landfill liner is between $10^{-9}$ m/s $-$ $10^{-10}$ m/s, and for the repository is between $10^{-9}$ m/s $-$ $10^{-11}$ m/s as per criteria of Swedish EPA-2000 and Japan Nuclear Cycle Development Institute, respectively (Swedish EPA, 2000; JNCDI, 2000). It can be observed that a sufficient $k$-value is observed even at low BC since the sample is compacted using modified compaction energy. An SC and BC mix design-based model is also developed to predict the $k$-value of the SBM with almost identical $R^2$, Adj $R^2$ and Pred $R^2$ values around 0.99 as follows:

\begin{align}
\sigma' &= 0.0571 - 0.0007(\log \sigma'') + 1 \times 10^{-6}(\log \sigma'')^2
\end{align}
The adequate precision value, F-value and p-value are observed to be around 40.4, 313.7 and <0.0001, respectively (Table 2). Thus, the model is statistically validated and could reliably be used to predict the k-value of the SBM based on BC and SC mix design (Table 2).

### 4. Microstructural Evaluation

The microstructural analysis is also conducted to determine the soil structure changes in the SBM with an increase in the BC and decrease in the SC by performing SEM analyses. Micro-images are presented at 20 mm in Fig. 9 for different SBMs. Fig. 9(a) shows the microstructure of clean sand when BC is zero, it can be observed that large pores (about the size of 20 mm) are evident among sand grains which are responsible for large permeability. As the 20% BC is added to the sand (Fig. 9(b)), the large pores are converted to small and medium pores (less than 20 mm) and the cohesive matter is increased which is evident in the clay clothed sand. The decrease in the pores is desirable for EB; however, an increase in the cohesive matter may deteriorate particle interlocking. This change in structure is the reason for the increase in values of \( q_u \), \( c \), and \( C_c \) of SBM and the decrease in the \( \phi \) and \( k \)-value with increasing BC. A major shift in the microstructure occurs when BC is increased from 20% to 30% as the medium pores are converted to small and micro-pores and clay-clothed sand become more evident (Fig. 9(c)). A further increase in the BC majorly increases the cohesive matter in the form of clay-clothed sand and virgin clay, however, further decrease in the pore size remains marginal. The replacement of sand grains with virgin clay is the reason for the large increase in

![Fig. 8. Permeability of Sand-Bentonite Mixtures](image)

\[
k = 1.98 \times 10^{-8} SC - 1.8 \times 10^{-11} BC + 3.2 \times 10^{-13} SC \cdot BC - 2.8 \times 10^{-15} SC \cdot BC (SC - BC)
\]

(17)

The adequate precision value, F-value and p-value are observed to be around 40.4, 313.7 and <0.0001, respectively (Table 2). Thus, the model is statistically validated and could reliably be used to predict the k-value of the SBM based on BC and SC mix design (Table 2).

![Fig. 9. SEM Micro Images of Sand-Bentonite Mixtures: (a) Sand, (b) B20S80, (c) B30S70, (d) B40S60](image)
the swelling potential and compressibility of SBM at this high BC. Thus, optimum structure to achieve reasonable engineering properties of SBM may occur between 20% – 30% of BC.

5. Mix Design Optimization using DOM

For optimization of mix design of the SBM, the SC and BC are regarded as the independent variables and different geotechnical characteristics i.e., \( q_u, c, \phi, SP, C_s \) and \( k \)-value are regarded as the responses. For these responses, mathematical models are already developed during the geotechnical evaluation of the SBM using SC and BC as predictors. An optimization study is carried out in this paper using the approach for computing the desirability functions \((d)\) outlined for each subjected geotechnical response to determine the optimum mix design of SBM (in terms of SC:BC) to achieve possible high performance in terms of hydraulic conductivity, strength and volumetric-change behavior. In this study, the following computation model is used to compute \( d \) for all the responses individually, which are quantified by the quadratic models developed in a previous section of this paper having input parameters SC and BC (Eqs. (4) – (8), (17)):

\[
d_i(\hat{y}_i) = \begin{cases} 
0 & \text{if } \hat{y}_i(x) < L_i \\
\left(\frac{\hat{y}_i(x) - L_i}{L_i - T_i}\right)^2 & \text{if } L_i \leq \hat{y}_i(x) < T_i \\
\left(\frac{\hat{y}_i(x) - U_i}{U_i - T_i}\right)^2 & \text{if } T_i \leq \hat{y}_i(x) \leq U_i \\
0 & \text{if } \hat{y}_i(x) > U_i 
\end{cases}
\]

where \( L_i \) is the minimization value, \( U_i \) is the maximization value, and \( T_i \) is the target value. Factors \( s \) and \( t \) determine the strictness of the \( d_i \); \( \hat{y}_i(x) \) is the response, which is defined by Eqs. (4) – (8) and 17 in the current study. The overall desirability function \((D)\) is computed by taking the arithmetic mean of series of computed \( d_i \); the computation model for \( D \) in the current study is as follows:

\[
D = \left( d_1, d_2, d_3, \ldots, d_m \right)^{\frac{1}{m}}
\]

where \( m \) is the number of responses.; whereas \( d_i \) and \( D \) are measured as the index numbers. The important step in optimization is the setting of goals for each subjected response and input parameters such as maximization, minimization, the target value (also range), or combination. Input parameters SC and BC are set to be in the tested range of this study as per Eq. (1). Maximization goal is set for the strength characteristics i.e., \( q_u, c, \phi \), SP, \( C_s \) and \( k \)-value of SBM; the maximum value obtained in this study and 0.015 (as this could yield insignificant settlement) are given the weightage of 1 for SP and \( C_s \), respectively, whereas 0.01 is set to be a lower limit for \( C_s \). On the other hand, for \( k \)-value weightage of 1 is set at \( 10^{-11} \) m/s as the upper limit and the lower limit is set at \( 10^{-11} \) m/s as per the criteria of the Japan Nuclear Cycle Development Institute (JNCDI, 2000). Using these goals, \( D \) is calculated for the full range of SBMs tested in the current study (Fig. 10). It is observed that an optimized mix design with the highest \( D \) close to 1 is obtained at 75.63% SC and 24.37% BC. The value of responses for this mix design are predicted to be \( q_u = 529.0 \) kPa, \( c = 63.8 \), \( \phi = 14.9^\circ \), SP = 540.2, \( C_s = 0.0096 \) and \( k \)-value = \( 1.4 \times 10^{-11} \) m/s. For this mix design minimum \( d_i \) is obtained for SP and maximum for \( C_s, q_u, \) and \( k \)-value. A reasonably high \( D \)-value could be obtained from a mixture having SC: BC in a range of 74:26 to 78:22 (with the highest at 75:63: 24.37) (Fig. 10); this is also in line with microstructural evaluation (Fig. 9). Thus any mix design can be selected from this range depending on the availability of material. In addition, it is important to note that these mix design ratios are determined when the SBM is compacted using compaction energy around 2,700 kN-m/m³, with the change in compaction energy, the required mix-design may change (Saba et al., 2014). Moreover, it is also worth mentioning that adoption of the \( k \)-value as a lone design parameter and minimization of it as a goal in the tested range of input parameters in this study yields \( D \)-value close to 1 (0.99) for SC:BC of 60:40, however, for this mix design, strength characteristics are seriously compromised (Figs. 2 – 3). Thus, the optimized design of SBM based on all essential geotechnical parameters could yield high possible engineering performance.

6. Conclusions

In this paper, geotechnical and microstructural evaluation of the SBM is conducted to achieve an optimized SBM design by employing DOM. Different SBMs are tested to determine mechanical properties, volumetric-change behavior, and hydraulic conductivity. Mathematical models are developed and statistically validated for different design parameters using SC and BC as the predictors to integrate with DOM-based algorithm. The optimum mix design of SBM was determined by setting maximization of strength and minimization of swell pressure, compressibility, and hydraulic conductivity of compacted SBM as the goals. Following are the main findings of this study.

1. BC imparts variant impacts on different geotechnical design
responses of SBM e.g., it ameliorates \( q_u \) and \( c \) and deteriorates \( \varphi \), \( SP \), \( C \), and \( k \)-value. Different models are developed for these responses using SB and BC as the predictors for which \( R^2 \), \( Adj\ R^2 \) and \( Pred\ R^2 \) values are observed to be higher than 0.98. In addition, models are also developed to predict the compression curve of the compacted SBM, which could be useful in the design of the EB and SR.

2. Optimization of SBM is carried out by integrating developed models for the geotechnical design responses in a DOM-based algorithm, which is subsequently simulated by setting maximization of strength and minimization of swell pressure, compressibility and hydraulic conductivity of compacted SBM as the goals. A reasonably high \( D \)-value is obtained for a mixture having SC:BC in a range of 74:26 to 78:22 with the highest at 75.63:24.37.

3. Microstructural analysis shows that with an increase in the BC, the large pores in the sand microstructure are filled and converted to small and micro-pores and cohesive matter in the form of clay-clothed sand and virgin clay increases which is responsible for the increase in the \( c \), \( q_u \), and decrease in the \( \varphi \) and \( \varphi \). A major shift in the microstructure of medium pores to micro-pores occurs for the BC between 20\% – 30\%, which substantiates the optimized range obtained based on DOM.

4. It is also observed that the optimum sand bentonite combination obtained through DOM not only has lower \( k \)-value and volumetric change vulnerabilities but also has better mechanical properties than the one obtained by optimizing using singular geotechnical characteristics, such as \( k \)-value. Thus, in terms of antagonist hydraulic, volumetric change, and mechanical responses, this study manifests an effective and pragmatic strategy for designing the SBM for stable EBs.

Acknowledgments

Laboratory of Geotechnical Engineering, Civil Engineering Department, UET Lahore, is acknowledged for providing technical support during this study.

Nomenclature

| Term           | Definition                                                                 |
|----------------|----------------------------------------------------------------------------|
| ANOVA          | Analysis of variance                                                      |
| BC             | Bentonite content                                                         |
| \( e \)         | Cohesion                                                                  |
| \( C_i \)      | Compression index                                                         |
| \( C_m \)      | Coefficient of consolidation                                              |
| \( D \)        | Desirability                                                              |
| DOM            | Desirability optimization methodology                                      |
| \( v \)        | Void ratio                                                                |
| EB             | Engineered barriers                                                       |
| \( e_0 \)      | Initial void ratio                                                        |
| \( e_{100} \)  | Void ratio at 100 kPa pressure                                             |
| \( e_{1000} \) | Void ratio at 1,000 kPa pressure                                           |
| \( e_{2000} \) | Void ratio at 2,000 kPa pressure                                           |
| \( EICL \)     | Extended intrinsic compression line                                        |
| \( HLW \)      | High-level radioactive waste                                               |
| \( ICL \)      | Intrinsic compression line                                                |
| \( I_s \)      | Plasticity index                                                          |
| \( I_{vc} \)   | Void index proposed for compacted clay-bentonite mixture                  |
| \( m_{sand} \)| Mass of sand                                                               |
| \( m_v \)      | Coefficient of volume compressibility                                      |
| \( MSW \)      | Municipal solid waste                                                     |
| \( NCCL \)     | Normalized compacted compression line                                      |
| \( q_u \)      | Unconfined compressive strength                                            |
| \( SB \)       | Sand bentonite                                                            |
| \( SBM \)      | Sand bentonite mixture                                                    |
| \( SC \)       | Sand content                                                              |
| \( SR \)       | Subsurface repositories                                                   |
| \( USCS \)     | Unified soil classification system                                         |
| \( w_L \)      | Liquid limit                                                              |
| \( w_{opt} \)  | Optimum moisture content                                                  |
| \( w_p \)      | Plastic limit                                                             |
| \( \sigma_c \)| Effective consolidation pressure                                           |
| \( \sigma_y \)| Yield stress                                                               |
| \( \varphi \)  | Angle of internal friction                                                |
| \( \gamma_{max} \)| Maximum dry unit weight                                                  |

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