Magnetic dipolar ordering and relaxation in the high-spin molecular cluster compound Mn$_6$

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Few examples of magnetic systems displaying a transition to pure dipolar magnetic order are known to date. As was recently shown, within the newly discovered class of single molecule magnets (SMMs), quite attractive examples of dipolar magnetism may be found. The molecular cluster spins and thus their dipolar interaction energy can be quite high, leading to reasonably accessible ordering temperatures even for sizable intercluster distances. In favorable cases bonding between clusters in the molecular crystal is by Van der Waals forces only, and no exchange paths of importance can be distinguished. An important restriction, however, is the requirement of sufficiently low crystal field anisotropy for the cluster spin, in order to prevent the occurrence of superparamagnetic blocking at temperatures above the dipolar ordering transition. This condition can be met for molecular clusters of sufficiently high symmetry, as for the Mn$_6$ molecular cluster compound studied here. The uniaxial anisotropy of the cluster spin $S = 12$ is as small as $D/k_B = 0.013$ K, giving a total zero-field splitting of the $S = 12$ multiplet of 1.9 K. As a result, the electron-spin lattice relaxation time remains fast ($\sim 10^{-4}$ s) down to $T_c$ and no blocking occurs. Magnetic specific heat and susceptibility experiments show a transition to ferromagnetic dipolar order at $T_c = 0.16$ K. Classical Monte Carlo calculations, performed for Ising $S = 12$ dipoles on a lattice do predict ferromagnetic ordering and account for the value of $T_c$ as well as the shape of the observed specific heat anomaly. By applying magnetic fields up to 6 T the hyperfine contributions to the specific heat arising from the $^{55}$Mn nuclei could be detected. From the time-dependence of the measured $C_M$ the nuclear-spin lattice relaxation time $T_1n$ could be determined for the same field range in the temperature region $0.2 < T < 0.6$ K. The nuclear magnetic relaxation was further studied by high field $^{55}$Mn pulse NMR measurements of both the nuclear $T_{1n}$ and $T_{2n}$ at $T = 0.9$ K (up to 7 T). The data are in good mutual agreement and can be well described by the theory for magnetic relaxation in highly polarized paramagnetic crystals and for dynamic nuclear polarization, which we extensively review. The experiments provide an interesting comparison with the recently investigated nuclear spin dynamics in the anisotropic single molecule magnet Mn$_{12}$-ac.

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I. INTRODUCTION

Experimental and theoretical interest in magnetic molecular clusters carrying a net high spin has rapidly evolved in recent years (for reviews see, e.g., Refs. 1-16). Since the cores of these molecules can be viewed as nano sized pieces of magnetic insulators, they offer attractive possibilities to study magnetic objects of dimensions in between atom and bulk. Of great importance is the fact that these (macro)molecules form stoichiometric chemical compounds, which may crystallize as molecular crystals, implying that for a given compound identical magnetic molecules are arranged on the sites of a regular three-dimensional lattice. More often than not, there is only a single molecular site per unit cell, so that the symmetry axes of all molecules in the crystal are perfectly aligned. Provided that the intermolecular magnetic interactions are sufficiently weak, macroscopic solid state techniques can then be exploited to study the properties of individual cluster spins, taking into account their couplings to the “environment” (phonons, nuclear moments) as perturbations. Accordingly, such experiments have already provided highly interesting information about the quantum tunneling properties of the cluster spins 6,7,8,9,10,11,12,13,14,15,16.

On the other hand, the long-range magnetic ordering (LRMO) phenomena expected to occur at sufficiently low temperatures as a consequence of the intercluster magnetic dipolar coupling present an interesting object of study in itself. In many of these compounds, the clusters are only or mainly coupled by van der Waals forces in the molecular crystal. Short-ranged superexchange interactions may then be neglected, leaving only the dipolar coupling between cluster spins as a source for producing LRMO. Since in the literature of magnetic phase transitions few examples of LRMO produced purely by dipol-
lar forces are yet available\textsuperscript{19,20,21} the chance to exploit molecular magnets to this end is quite attractive and could represent an important contribution to this field.

However, for most high-spin molecules studied so far, such as Mn\textsubscript{12}\textsuperscript{22} Fe\textsubscript{8}\textsuperscript{23} and the Mn\textsubscript{4} family of compounds\textsuperscript{24,25,26} the cluster spins have strong Ising-type anisotropy, associated with the zero-field splitting (ZFS) of the magnetic energy levels by the action of the crystal field. As a consequence, the cluster spins become frozen below a blocking temperature, \( T_B \), of typically a few K, with the spin direction randomly distributed between the two possible orientations along the easy axis. Evidently, this superparamagnetic blocking process is in competition with intercluster magnetic interactions that tend to establish LRMO at (usually much) lower temperatures. Although it has been shown theoretically\textsuperscript{27} that the occurrence of quantum tunneling between opposite spin directions at temperatures below \( T_B \) may in principle produce sufficient fluctuations to overcome blocking, in most of the investigated anisotropic molecules the times involved for the actual observation of the ensuing LRMO are still much too long. The recently discovered example of LRMO found at \( T_c = 0.21 \) K in the molecular magnet Mn\textsubscript{4}Me in an octahedron of Mn\textsuperscript{3+} ions has a highly symmetric cluster core, comprising six ferromagnetically coupled Mn\textsuperscript{3+} ions, yielding a total spin \( S = 12 \) for this molecular superparamagnetic particle. The gray and black spheres indicate the Br\textsuperscript{−} and O\textsuperscript{2−} ions respectively. The white arrows illustrate the ferromagnetic alignment of the Mn\textsuperscript{3+} spins inside the cluster.

The obvious route to find dipolar-induced LRMO in molecular magnetic cluster compounds is, therefore, to search for high-spin molecules with as low anisotropy as possible and with negligible superexchange interactions. In a preliminary report\textsuperscript{29} on the compound Mn\textsubscript{6}O\textsubscript{4}Br\textsubscript{4} (Et\textsubscript{2}dbm)\textsubscript{6} (hereafter called Mn\textsubscript{6}) we could show that it provides an excellent example. The Mn\textsubscript{6} molecule has a highly symmetric cluster core, comprising an octahedron of Mn\textsuperscript{3+} ions the faces of which are capped by O\textsuperscript{2−} or Br\textsuperscript{−} ions. The structure of the molecule\textsuperscript{29,30} and a sketch of its octahedral core are shown in Fig. 1. From previous magnetic studies\textsuperscript{30} above 2 K it was found that the superexchange paths formed between the Mn\textsuperscript{3+} ions (each having atomic spin \( s = 2 \)) through the intervening O\textsuperscript{2−} and Br\textsuperscript{−} ligands result in a relatively strong ferromagnetic interaction, of value \( J_f/k_B \sim +13 \) K on basis of the pair Hamiltonian \( \mathcal{H} = -2J_{Sy}S_iS_j \). As a consequence, the ground state is a \( S = 12 \) multiplet and the energy of the nearest excited state is approximately 150 K higher. The unit cell is monoclinic, with space group \( Pc \), and contains four molecules that have such a high (nearly \( T_d \)) symmetry that the net anisotropy for the cluster spin is quite small. No superexchange paths connecting neighboring clusters can be discerned indeed in the crystal structure, so that we can safely assume that the crystal binding arises solely from Van der Waals bonds. We note that, although intercluster magnetic ordering has also been reported for the molecular magnets Fe\textsubscript{19}\textsuperscript{31,32} and Mn\textsubscript{4}Br\textsubscript{24} in those cases superexchange between clusters apparently plays an important role, as evidenced, e.g., by the much higher \( T_c \) values found (1.2 − 1.3 K). However, for a Cr\textsubscript{4}S cluster\textsuperscript{33} for which the intrachannel exchange between the Cr\textsuperscript{3+} ions happens to be likewise ferromagnetic (net spin \( S = 6 \)), the low value of \( T_c = 0.17 \) K that was observed could be compatible with dipolar-induced magnetic ordering, in this case of antiferromagnetic type. More data would be needed, however, to substantiate this.

In a preliminary report\textsuperscript{29} we could already show that the magnetic anisotropy in Mn\textsubscript{6} is sufficiently low to enable measurements of its magnetic susceptibility and specific heat under thermal equilibrium conditions down to the lowest temperatures reached (\( T \approx 15 \) mK) by our experimental setups. The data did evidence a transition to LRMO, as hoped for, at a temperature of \( T_c = 0.16 \) K, corresponding apparently to a ferromagnetic arrangement of the cluster moments. Comparison with Monte Carlo simulations strongly supported the expected dipolar origin of the intercluster coupling. When applying magnetic fields of up to 6 T, the study of the temperature-dependent magnetic specific heat revealed a transition from fast relaxation to non equilibrium conditions within the experimental time window of 1 − 100 s.

In the present work we extend these experimental and theoretical studies and discuss in much more detail the results and conclusions. In addition, we have performed Mn NMR studies in varying field, enabling to draw more definite conclusions about the magnetic relaxation of both electronic and nuclear spin systems in this material. The nuclear spin-lattice relaxation time is governed by fluctuations of the cluster electronic spins, and is indeed quite fast in zero field. By applying a magnetic field these fluctuations become progressively suppressed as a consequence of the Zeeman splitting of the electronic energy levels, thereby bringing the nuclear spin system out of thermal equilibrium. This provides an interesting comparison with recent zero-field Mn NMR studies\textsuperscript{34,35,36,37} of the anisotropic molecular magnet Mn\textsubscript{12}-ac, for which the suppression of the magnetic relaxation in the thermally activated regime can be fully ascribed to the strong splitting of the cluster spin levels by the crystal field. For the present compound, crystal-field splittings play a very minor role in the relaxation
process, except for providing the necessary channel for energy transfer between spins and lattice phonons. We present a full analysis of the longitudinal and transverse nuclear relaxation in terms of previously developed theories for relaxation by paramagnetic impurities and for dynamic polarization, taking into account electron spin fluctuations by both spin-lattice relaxation and spin-spin interactions. Our NMR data in high fields prove to be in excellent qualitative as well as quantitative agreement with such theoretical predictions. The values for the longitudinal nuclear spin-lattice relaxation rate and for the effective hyperfine interaction constant deduced from our high-field time-dependent specific heat data are likewise in good accord with the NMR results.

The outline of this paper is as follows. After giving a few experimental details in the next section, the measured susceptibility and zero-field specific heat data are presented and discussed in section III A and B, followed by a Monte Carlo simulation study of the zero-field specific heat in section III C. In section III D the field-dependent specific heat measurements are discussed, followed by the nuclear resonance and nuclear relaxation data in section III E. Section IV contains an analysis of these data in terms of existing theoretical models for relaxation in highly polarized magnetic systems. Concluding remarks are given in section V, while in Appendix A we describe the calculation of the demagnetizing factor for powder samples. Systeme International. units will be used throughout the paper.

II. EXPERIMENTAL DETAILS

Polycrystalline samples of Mn$_6$ were prepared as reported in Ref. 39. Low-temperature specific heat measurements were performed in a home made calorimeter that makes use of the thermal relaxation method.3,15,38 For the measurements, a few milligrams of sample were mixed with Apiezon N grease and placed on the sapphire plate of the calorimeter. Details of this measurement technique are given in Ref. 33. An important advantage of this method is that the characteristic time $\tau_e$ of the experiment (typically, $\tau_e \approx 0.1 - 1000$ seconds at low $T$) can be varied (within limits) by changing the dimensions (and therefore the thermal resistance) of the Au wire that acts as a thermal link between the calorimeter and the mixing chamber of the dilution refrigerator. Magnetic fields up to 16 T can be applied with a superconducting magnet and the lowest temperatures reached are of the order of 50 mK.

The ac-susceptibility measurements were performed between 15 mK and 4 K in a home made susceptometer, placed inside the plastic mixing chamber of a dilution refrigerator and thermalized by the $^3$He flow.35 The susceptometer, based on the mutual inductance technique, consists of a primary coil with 250 turns of $\phi$100 $\mu$m NbTi wire, and two oppositely wound secondary coils, each with 660 turns of $\phi$40 $\mu$m Cu wire. By placing the sample inside one of the two secondary coils and feeding the primary with an ac current (typically $\sim$ 50 $\mu$A), the induced voltage across the secondary is proportional to the susceptibility of the sample. By phase-sensitive detection we can also discriminate the real and imaginary parts of the susceptibility. The excitation frequency $\nu$ was varied between 230 and 7700 Hz. Additional measurements above 1.8 K were performed using the ac option of a commercial Superconducting Quantum Interference Device (SQUID) magnetometer.

As for the NMR experiments, we introduced the sample and the four-turns NMR coil inside the plastic $^3$He pot of a pumped $^3$He cryostat, where variable magnetic fields up to 8 T could be applied by a superconducting magnet. The resonance of the $^{55}$Mn nuclei of Mn$_6$ was observed by means of the spin-echo technique, with a typical duration of the $\pi/2$ pulse of $t_{\pi/2} \approx 2\mu$s.

Estimates of the magnitude of the magnetic anisotropy have been obtained as follows. Magnetic data measured above 1.8 K (Ref. 30) can be fitted by expressions valid for fully isotropic $S = 12$ spins, i.e. they do not evidence any detectable ZFS for the total spin. The data are excellently fitted by the Brillouin curve calculated for an isotropic paramagnet with $S = 12$ and $g = 1.98$ [as derived from electron paramagnetic resonance (EPR)]. If we write a single-spin Hamiltonian for the molecule as:

$$
\mathcal{H} = -DS_z^2 - g\mu_B\mathbf{B}_a \cdot \mathbf{S},
$$

(1)

these experiments provide an upper limit of $|D|/k_B \lesssim 0.01$ K. In order to obtain an independent estimation of $D$, high-frequency EPR experiments were carried out at the NHMFL in Tallahassee by J. Krzystek, using several frequencies in the range 95 - 380 GHz. Simulations of the spectra performed using Eq. (1) agree well with the experimental results for 0.01 K $< |D|/k_B < 0.05$ K. One of $|D|/k_B \approx 0.01$ K seems therefore appropriate to describe the ZFS in Mn$_6$. When discussing the ac-susceptibility and specific heat data in sections III A and III E we shall adopt the value $D/k_B = 0.013$ K, which yields the best agreement between theory and experiment for both techniques.

The isotropic character of the molecular spin might seem paradoxical at first, considering that the individual Mn$^{3+}$ ions, being Jahn-Teller ions, experience strong anisotropy: typical values of $|D|/k_B$ for the ion are a few tenths of Kelvin. However, the net $D$ value entering in the spin Hamiltonian for the cluster can be seen in first approximation to result from the vectorial addition of the local anisotropy tensors of the individual ions, which then can give rise to a low net anisotropy for highly symmetric molecules such as Mn$_6$ (cf. Fig. 4), even for large ZFS of the constituting atoms.39 In fact, the possibility to tune the net anisotropy of the cluster spin by means of molecular synthesis is one of the attractive properties of these nanosized molecular superparamagnets.
III. EXPERIMENTAL RESULTS AND ANALYSES

A. Magnetic susceptibility

Strong evidence for the long-range ordering of the magnetic moments is provided by the magnetic susceptibility data, shown in Fig. 2. The real part $\chi'$ of the complex ac-susceptibility is plotted in Fig. 2(a), and is seen to show a sharp maximum at $T_c = 0.161(2)$ K. We first demonstrate that the value of $\chi'$ at $T_c$ is of the order of the estimated limit $1/N_{\text{eff}}$ for a ferromagnetic powdered sample, where $N_{\text{eff}}$ is an effective demagnetization factor appropriate for the (cylindrically shaped) container filled with the grains. In the Appendix we argue that $N_{\text{eff}}$ can be approximately given in terms of the demagnetizing factors $N_{\text{grain}}$ and $N_{\text{cont}}$ of, respectively, the individual grains and the container as [Eq. (A3)]:

$$N_{\text{eff}} = N_{\text{grain}} + f(N_{\text{cont}} - 1/3),$$

where $f$ denotes the volume filling fraction of the container.

Assuming the shape of the grains to be approximately spherical, we put $N_{\text{grain}} = 1/3$, while from the shape of the container, we estimate $N_{\text{cont}} \approx 0.2$. The density of the material is estimated to be $\rho_{\text{grain}} \approx 1.45$ g/cm$^3$ from the value for the similar compound Mn$_6$O$_3$Br$_2$(Me$_2$dmh)$_6$, and the filling fraction is estimated as $f \sim 1/3$. All this then leads to $N_{\text{eff}} \approx 0.29(5)$ and therefore $\chi'(T_c) \approx 1/N_{\text{eff}} = 3.5(6)$, the large error arising obviously from all the uncertainties in the above line of argument and in the estimates of the parameters involved. Next we should realize that this value would be valid for the $\chi'$ measured along the easy axis, whereas even a relatively small anisotropy will lower appreciably the $\chi'$ along the other directions.\[3\] Therefore, the powder $\chi'$ could easily be lower by a factor of 2 [see Eqs. (A3) and (A6) of the Appendix].

Given all the uncertainties, the above derived value can obviously serve as an order of magnitude estimate only, but we note that the experimental value of $\chi'(T_c) \approx 3$ is indeed rather close. As a second argument for the ferromagnetic nature of the transition we include therefore in Fig. 2(a) the powder susceptibility expected for the paramagnetic (non interacting) case, as calculated from the spin hamiltonian (Eq. 1) and applying in addition the corrections for demagnetizing effects as described in the Appendix. Besides the curve for $D/k_B = 0.013$ K appropriate for Mn$_6$, we also include for comparison the fully isotropic ($D = 0$) and infinite anisotropy limits. From this plot it is evident that, when approaching $T_c$, the susceptibility of Mn$_6$ increases appreciably above the paramagnetic limit, confirming the ferromagnetic nature of the correlations.

Below $T_c$, the powder $\chi'$ decreases rapidly, as expected for an anisotropic ferromagnet in which the domain-wall motions become progressively pinned. The associated domain-wall losses should then lead to a frequency dependent maximum around $T_c$ in the imaginary part, $\chi''$, as the experimental data of Fig. 2(b) indeed show. In fact, although the Mn$_6$ spins can be considered as nearly isotropic at high temperatures, the anisotropy energy is still large compared with the dipolar interaction energy $\mu_0 \mu^2 / 4\pi k_B r^3 \approx 0.1$ K between nearest neighbor molecules. Thus the ordering should be that of an Ising dipolar ferromagnet.

As shown in detail in the inset of Fig. 2(a), the temperature $T_{\text{peak}}$ at which the maximum value of $\chi'$ is found depends only weakly on $\nu$, which we attribute to the anisotropy. The total activation energy for the reversal of each Mn$_6$ molecular spin amounts to $DS^2 \approx 1.9$ K, i.e., about 35 times smaller than for Mn$_{12}$. Although this is quite small, one could still expect the superparamagnetic blocking of the Mn$_6$ spins to occur when $T \approx T_B(\text{Mn}_{12})/35$, that is below $\approx 0.15$ K. Since this value is very close to the actual $T_c$, one may expect that for $T \rightarrow T_c$ the approach to equilibrium begins to be hindered by the anisotropy of the individual molecular spins. We stress, however, that the frequency dependence of $\chi'$
observed here is quite different from that of the well-known anisotropic superparamagnetic clusters. A way to quantify the frequency dependence of the peak in $\chi'$ is by means of the parameter $\Delta T_{\text{peak}}/\left[T_{\text{peak}} \Delta (\log_{10} \nu)\right]$, which gives the variation of $T_{\text{peak}}$ per decade of frequency. We find here $\Delta T_{\text{peak}}/\left[T_{\text{peak}} \Delta (\log_{10} \nu)\right] \simeq 0.03 - 0.05$, to be compared with the typical values of $\sim 0.20$ for superparamagnetic blocking. In fact it is closer to the value $\sim 0.06$ found for certain types of spin glasses but the peak observed here is much higher and sharper. Also, since the cluster spins are situated on a regular crystalline lattice, a comparison with random magnetic systems would not be appropriate. The frequency dependence we observe is indeed very much weaker than is found in the LiHo$_{0.045}$Y$_{0.955}$F$_4$. Also this finding appears to exclude an interpretation in terms of a freezing transition in Mn$_6$.

We finally turn to the susceptibility as measured above $T_c$ in the paramagnetic region, which was plotted as $1/\chi'$ vs $T$ in the inset of Fig. 2 in Ref. [23]. The high-temperature susceptibility data $\chi'_0$ corrected for the demagnetizing field $\chi'_0 = \chi'/([1-N_{\text{eff}}] \chi')$ follow the Curie-Weiss law $\chi'_0 = C/(T - \theta)$ quite well down to approximately 0.3 K, with $C = 0.62(2)$ K and $\theta = 0.14(3)$ K. The constant $C$ equals, within the experimental errors, the theoretical value for isotropic spins $N^2 g^2 \mu_B^2 S(S + 1)/3k_B V_m = 0.595$ K, where $S = 12$, $g = 2$, and $V_m = 1647$ cm$^3$/mole is the molar volume. The positive $\theta$ confirms the ferromagnetic nature of the ordered phase. The fact that mean field theory is so well obeyed down to very close to $T_c$ as expected for a dipolar ferromagnet. We remark that the behavior of the powder susceptibility we observe for Mn$_6$ in Fig. 2 closely resembles previous powder data for Cs$_2$NaGdCl$_6$ (cf. Fig. 3 of [14]), and LiHo$_{0.045}$Y$_{0.955}$F$_4$ ($x = 0.46$) (cf. Fig. 1 in [49] and Fig. 4 in [42]). Both materials are considered to be examples of anisotropic dipolar ferromagnets. Although weak ferromagnetism (i.e., canted antiferromagnetism) would also lead to $\chi(T = T_c) = 1/N$, this would be accompanied by a negative Curie-Weiss $\theta$, whereas we observe a positive value. Moreover, for canted antiferromagnetism an antisymmetric interaction term of the form $d \cdot (S_i \times S_j)$ is needed (see e.g. Ref. [54]) and this is not expected for a system of equivalent magnetic moments interacting by dipolar interactions, as is the case here. For metamagnetic systems, having ferromagnetic nearest neighbor interactions and weaker further neighbor couplings, also a positive value for $\theta$ can be found but the magnetic ordering below $T_c$ due to the further neighbor interaction is basically antiferromagnetic. Consequently, although the value of $\chi(T = T_c)$ can be much higher than for nearest neighbor antiferromagnets, it will fall still far below the ferromagnetic limit of $1/N$. We therefore conclude that the evidence for ferromagnetic dipolar order in Mn$_6$ presented here is quite strong. Additional studies, for instance of the spontaneous magnetization below $T_c$, would of course be welcome to provide further proof.

### B. Zero-field specific heat

Specific heat data $c$ taken in zero applied field are shown in Fig. [20] as a function of temperature on a double logarithmic scale. Above 2 K, the specific heat is dominated by the contribution from the lattice phonons that can be reasonably fitted by the well-known low-$T$ Debye approximation: $c_{\text{latt}} \propto (T/\Theta_D)^3$, with a Debye temperature $\Theta_D \simeq 29$ K. Such low values are commonly observed in the molecular cluster compounds reflecting the weak bonding between the cluster molecules in such molecular solids. In the lowest temperature range below 0.1 K, the specific heat is seen to remain rather high valued, which can be mainly ascribed to the expected contribution from the $^{55}$Mn nuclear spins, the energy levels of which are split by the hyperfine interaction with the Mn$^{3+}$ electronic spins $s = 2$ (see section III D below). This contribution will be more clearly and directly visible in the

![Diagram](image-url)
field-dependent studies, where it will be shown to be describable by a term $c_{\text{muc}}T^2/R \approx 5.2 \times 10^{-3}$ K$^2$, which is plotted as the dashed line in Fig. 3. Anticipating the discussion below, we subtract this hyperfine contribution, as well as the phonon $\propto T^3$ term from the raw data in Fig. 3 in order to obtain the magnetic specific heat $c_{\text{el}}$ associated with the cluster electron spin $S = 12$ only, as plotted in Fig. 4. The resulting curve shows two characteristic features. At a temperature $T_c \approx 0.15(2)$ K a peak is observed, that can be associated with the transition to long-range magnetic order, in good agreement with the value for $T_c$ deduced from the peak in the ac-susceptibility extrapolated to zero frequency. Above $T_c$, one observes a widely extended “high-temperature tail”, that reflects the weak zero-field splittings (ZFS) of the $S = 12$ multiplets by the crystal field interactions. We recall that even for a $D$ value as small as 0.013 K the total splitting of an $S = 12$ multiplet will still be an appreciable $DS^2 = 1.9$ K. In the absence of magnetic intercluster interactions the ZFS of such a multiplet would lead to a multilevel Schottky curve, shown as the dashed-dotted curve in both Fig. 3 and 4, where the fit to the experiment leads to the estimate $|D|/k_B \approx 0.013$ K, in reasonably good agreement with the values quoted above. In contrast with the highly anisotropic molecular clusters Mn$_{12}$, Mn$_4$, and Fe$_8$, where the multilevel ZFS Schottky is found above 1 K and is the most pronounced feature of the experimentally observed magnetic specific heat.

The density, $\rho = 1.45$ g/cm$^3$, was estimated from the Mn$_6$O$_4$Br$_4$(Me$_2$dbm)$_6$ compound. The resulting molar volume is a few percent larger than what would be obtained from the lattice parameters used in the Monte Carlo simulation, but this discrepancy is due entirely to the approximation of orthorhombic structure. The Hamiltonian includes the dipolar interaction term as well as the anisotropy term $-DS^2_z$ given in Eq. (1). For $T < 0.5$ K the intermolecular dipolar interactions become important and remove the degeneracy of the $|m\rangle$ spin doublets. The MC simulations show that the ground state is indeed ferromagnetically ordered, as observed, and predict a shape for $c_{\text{el}}$ that is in good agreement with the experiment. In the upper panel of Fig. 4 we show $c_{\text{el}}$ as calculated assuming all molecular easy (z) axes to point along $a_z$, i.e., one of the two nearly equivalent short axes of the actual lattice. Similar results were obtained for other orientations chosen for the anisotropy (z) axis. We note that our simulations give $T_c = 0.22$ K, which is slightly lower than the experimental $T_c = 0.161(2)$ K. This difference may be due to the Ising approximation

![FIG. 4: (Color online) (a) $T$ dependence of the electronic specific heat at zero applied field (circles). The dashed-dotted line is the Schottky anomaly calculated with $D/k_B = 0.013$ K. The solid line is the Monte Carlo (MC) calculation for an orthorhombic lattice of 1024 Ising spins with periodic boundary conditions. For each point, $2 \times 10^4$ MC steps per spin were performed. (b) $T$ dependence of the entropy obtained by integration of the electronic specific heat curve.](image-url)
be the anisotropy axis $z$ along the "uniaxial" with strong preference for the spins to lie this lattice is always found ferromagnetic, it can be ei-
the anisotropy parameter. Although the ground state for
the combination of the long-range dipolar interaction and
actual ground state and the value of $c$ illustrate the complicated way in which the nature of the
agram shown in Fig. 5, which we include here since it
were varied. These calculations resulted in the phase di-
points along this direction for $M$ points in the direction (in the
The dipolar energy is minimized when the magnetization
$D > 0$ is a competing interaction only. For instance, in those models the vari-
ation of $D$ with anisotropy is restricted to about 20%
of the $T_c$ value, which moreover is highest for the Ising ("uniaxial") case. It is also interesting to compare these
Monte Carlo calculations with the predictions of simple
field theory. The latter gives $T_c = 2S^2J_{\text{eff}}/k_B$ and
$T_c = 2S(S + 1)J_{\text{eff}}/3k_B$, respectively for $D \to +\infty$ and
$D = 0$, where $J_{\text{eff}}$ is an effective interaction constant.
The ratio between these two limits, which is $3S/(S + 1)$,
is about 2.5 times larger than what is obtained by Monte
Carlo calculations.

To conclude this section we may stress that our deta-
ealed calculations have evidenced that for a dipolar mag-
et the value of $T_c$ may vary strongly with anisotropy and lattice symmetry. This illustrates the danger in drawing
conclusions about the nature of the magnetic interac-
tions, i.e., whether they are of dipolar origin or not, just
by comparing the value of $k_B T_c$ with the dipolar interac-
tion energy of a pair of nearest neighboring spins. Our
detailed calculations specific for this compound show a
good quantitative agreement assuming just dipolar cou-
ping, both as regards the value of $T_c$ and the shape of
the specific heat anomaly. Furthermore, the prediction
that the ordering is ferromagnetic, as observed, is quite
robust since it is independent of the details of the simu-
lations. All this confirms that LRMO in Mn$_6$ is mainly
driven by dipolar interactions.

D. Field-dependent specific heat: Nuclear spin
contribution

We next discuss the time-dependent magnetic specific
heat $c_m = c - c_{\text{latt}}$ measured under varying applied mag-
etic fields $B_{\text{ext}}$, as plotted in Fig. 6. Even for the low-
est $B_{\text{ext}}$ value, the ordering anomaly is already fully sup-
pressed, as expected for a ferromagnet. Accordingly, we
may account for these data with the Hamiltonian (1) ne-
eglecting dipolar interactions. The Zeeman term splits
the otherwise degenerate $|\pm m\rangle$ doublets, and already
for $B_{\text{ext}} \sim 0.5$ T the level splittings become predominantly
determined by $B_{\text{ext}}$, so that the anisotropy term can then
also be neglected. As seen in Fig. 6, the calculations
performed with $D = 0$ (dotted curves) reproduce quite
satisfactorily the data at higher temperatures.
expected to be small compared to their nuclear Zeeman hyperfine coupling constant. By contrast, we may neglect estimated by the sum of two contributions: the high-temperature limit of the nuclear specific heat can therefore be approximated electronic contribution, whereas the dashed line is the expected nuclear specific heat at zero field calculated with Eq. 3. The solid lines represent the total time-dependent \( c_m \) (electronic + nuclear) calculated accounting for the nuclear spin-lattice relaxation (NSLR) time \( T_{1n} \) for long (\( \sim 100 \) s) and short (\( \sim 1 \) s) experimental times. The dashed lines are the calculated contributions arising from the \( ^{55}\text{Mn} \) and \( ^1\text{H} \) nuclear spins.

However, when the maxima of the multilevel Schottky anomalies are shifted to higher \( T \) by increasing \( B_a \), an additional contribution is revealed at low \( T \). It is most clearly visible in the curves for \( 1 \, \text{T} < B < 2.5 \, \text{T} \), and varies with temperature as \( c_m T^2 / R \approx 4 \times 10^{-3} \). We attribute this contribution to the already-mentioned high-temperature tail of the equilibrium nuclear specific heat \( c_{\text{nucl}}^{(\text{eq})} \). As discussed in section IV, this specific heat should be dominated by the contributions of the six \( ^{55}\text{Mn} \) nuclear spins \( (I = \frac{5}{2}) \) and of the 114 protons present in each molecule. The energy levels of the former are split, even at zero field, by the strong on-site hyperfine interactions with the \( ^{55}\text{Mn}^{3+} \) electronic spins \( s = 2 \). For the specific heat analysis this interaction can be approximated by \( H_{\text{hf}} = A \mathbf{I} \cdot \mathbf{s} \), where \( A \) is an effective isotropic hyperfine coupling constant. By contrast, we may neglect the hyperfine splitting of the protons because it can be expected to be small compared to their nuclear Zeeman splitting for \( B_a > 1 \, \text{T} \). This is indeed confirmed by the NMR experiments shown below. The high-temperature limit of the nuclear specific heat can therefore be approximated by the sum of two contributions:

\[
\frac{c_{\text{nucl}}^{(\text{eq})} T^2}{R} \approx 6 \times \frac{1}{3} A^2 s^2 I (I + 1) + 114 \times \left( \frac{h \gamma_H B_a}{2 k_B} \right)^2
\]

where \( \gamma_H = 2.675 \times 10^8 \, \text{rad} \, \text{T}^{-1} \, \text{s}^{-1} \) is the protons gyromagnetic ratio. Taking \( A / k_B = 8.6 \) mK as estimated from the NMR spectra measured for the same sample (see section III E), we obtain the dashed line shown in Fig. 4 at zero field. This is the same contribution that was subtracted from the zero-field data shown in Fig. 4.

The difference between the calculated and experimental \( c_{\text{nucl}}^{(\text{eq})} \), which becomes especially evident for the \( B_a = 2 \, \text{T} \) curve, can be due to the shift of the \( ^{55}\text{Mn} \) nuclear energy splittings by the applied field, which is neglected in Eq. 3.

A remarkable feature of the experimental data that is not reproduced by these equilibrium calculations is that, at the lowest \( T \), the nuclear specific heat drops abruptly to a baseline of about \( 3 \times 10^{-3} R \). This remaining specific heat is probably a background feature arising from incomplete correction for the field-dependent addenda contributions. The crossover temperature \( T^* \) where the drop of \( c_m \) occurs depends on \( B_a \) but also on the characteristic time constant \( \tau_{e} \) of our (time-dependent) specific heat experiment: as is shown in the inset of Fig. 4 the deviation from the (calculated) equilibrium specific heat is found at a lower \( T \) when the system is given more time to relax. Interestingly, the specific heat becomes even smaller than the expected contribution of the protons. The drop therefore shows that, below \( T^* \), the nuclear spins of both the \( ^1\text{H} \) and \( ^{55}\text{Mn} \) atoms cannot attain thermal equilibrium with the lattice phonons within the experimental time \( \tau_e \) because the longitudinal nuclear spin-lattice relaxation time becomes too short. This relaxation effect can be described as follows:

\[
c_{\text{nucl}}(\tau_e) = c_{\text{nucl}}^{(\text{eq})} [1 - \exp(-\tau_e / T_{1n})],
\]

where, to simplify the discussion, we have used the same nuclear spin-lattice relaxation (NSLR) time \( T_{1n} \) for both protons and \( ^{55}\text{Mn} \) although they can obviously differ from each other. According to Eq. 4 the specific heat decreases fast when \( T_{1n} \) becomes of the order of \( \tau_e \). This crossover to a non equilibrium regime (as measured by time-dependent specific heat) provides therefore direct information on the temperature and field dependence of the nuclear \( T_{1n} \).

As is well established \( ^{55}\text{Mn} T_{1n}(T, B_a) \) can be related to the time-dependent part of the transverse hyperfine field as produced by the fluctuations of the electron spin. For the case of MnO both the electron spin fluctuations due to spin-lattice coupling and to dipolar spin-spin interactions will have to be considered. A more extensive theoretical treatment is given below in section 

\[
\frac{1}{T_{1n}} \approx \frac{\kappa_0 B_a^2}{k_B T} \exp \left( \frac{-g_B B_a}{k_B T} \right),
\]

where \( \kappa_0 \) is a constant that depends on the electronic spin-lattice relaxation rate and on the details of the relaxation mechanism. This shows that an exponential temperature dependence of the NSLR rate is expected.
at high fields. The effect of the field is to polarize the electron spins, which reduces the fluctuations of the hyperfine field, thus effectively disconnecting the nuclear spins from the lattice.

As seen from Eq. 3, the nuclear spins can be taken out of equilibrium either by decreasing $T$ down to $T^*$ at constant field (as in Fig. 6), or by increasing $B_a$ up to a given value $B^*$ at constant $T$. The latter effect is indeed also observed experimentally, as shown in Fig. 4 for $T = 0.3$ K and 0.6 K. In this figure the transition to non-equilibrium is obvious from the fact that the data measured in high field fall far below the expected nuclear contributions (dashed lines). The inset of Fig. 4 shows $B^*(T^*)$ obtained either from $T$ sweeps at constant $B$ (as in Fig. 6) or from $B$ sweeps at constant $T$ (as in Fig. 7). The two methods prove to be fully consistent with each other. The fit of $B^*(T^*)$ using Eq. 5 gives an average value of $\kappa_0 \approx 40 \text{s}^{-1}\text{T}^{-3}$. Using this value of $\kappa_0$ we have calculated the time-dependent $c_{\text{nucl}}$ from Eq. 4. Adding this to the calculated electronic specific heat yields the solid lines in Figs. 6 and 7, which can be seen to be in reasonably good agreement with the experimental data over the whole range of field and temperature.

E. $^{55}\text{Mn} - \text{NMR and nuclear relaxation}$

The nuclear spin-lattice relaxation in Mn$_6$ has been further investigated by $^{55}\text{Mn}$ nuclear magnetic resonance. Apart from the fundamental question as to by which mechanisms the magnetic relaxation proceeds in this isotropic molecular magnetic crystal, it is of interest to compare with the typical behavior observed recently for the highly anisotropic single-molecule magnets like Mn$_{12}$-acetate.$^{34,35,36}$

As is well known, it is difficult to observe NMR for nuclei of paramagnetic ions due to the very large and strongly fluctuating magnetic fields produced at the nuclei by the electron spin through the (on-site) hyperfine interactions. As a consequence, nuclear resonance lines become very broad and spin-lattice relaxation rates too fast to be measured. To enable the observation of the NMR signals, one should therefore take recourse to the low-temperature and high-field regime, in which electron spin fluctuations can be expected to be sufficiently suppressed. Accordingly, we performed our experiments at $T = 0.9$ K, using a $^3\text{He}$ cryostat, and fields in the range 3 to 7 T.

The $^{55}\text{Mn}$ NSLR was studied by measuring the recovery of the nuclear magnetization after an inversion pulse. By integrating the echo intensity we obtained recovery curves as those shown in Fig. 8a. For the ease of comparison between different curves, we renormalize the vertical scale such that $M(0)/M(\infty) = -1$ and $M(t > T_{1n})/M(\infty) = 1$, even though usually $|M(0)| < |M(\infty)|$: this is just an artifact that occurs when the NMR line is much broader than the spectrum of the inversion pulse, and does not mean that the length of the pulse is incorrect. Since the $^{55}\text{Mn}$ nuclei have spin $I = 5/2$, the recovery of the nuclear relaxation for the central line in the quadrupolar split manifold is described by:

$$
\frac{M(t)}{M(\infty)} = 1 - \left[ \frac{16}{63} e^{-(15t/T_{1n})^\alpha} + \frac{2}{35} e^{-(6t/T_{1n})^\alpha} \right],
$$

(6)

where $1/T_{1n}$ is the nuclear spin-lattice relaxation rate, and $\alpha$ is a stretching exponent, which needs to be introduced to account for the large inhomogeneity of the NMR line, which causes the inversion recovery to consist of a combination of recoveries with different rates. We typically found an optimal value of $\alpha \simeq 0.5$, although the choice of the stretching exponent does not strongly
influence the value of $1/T_{1n}$ extracted from the fit.

The transverse spin-spin relaxation (TSSR) rate $1/T_{2n}$ was studied by measuring the decay of echo intensity upon increasing the waiting time $\tau$ between the $\pi/2$- and the $\pi$-pulses. The decay of transverse magnetization $M_\perp(\tau)$ can be fitted to a single exponential:

$$\frac{M_\perp(2\tau)}{M_\perp(0)} = e^{-2\tau/T_{2n}},$$

as shown in Fig. 8(b).

The field-sweep NMR spectra in Fig. 9 clearly show that it is impossible to determine whether there are inequivalent sites in the molecule, as regards the hyperfine coupling [compare with the case of Mn$^{12}$-ac (Ref. 53)]. This may be due to the large quadrupolar splitting expected in Mn$^{3+}$ sites, plus the fact that our sample is an unoriented powder. As expected from the internal ferromagnetic structure of the cluster electron spins, the $^{55}$Mn spectrum shifts to higher fields when lowering the frequency. The spectra can be fitted by a Gaussian shape with total width $2\sigma_B \approx 2.2$ T. If this width were due to quadrupolar splitting $\Delta\nu_Q$, only, one would deduce $\Delta\nu_Q \approx 7$ MHz: this estimate can be obtained from a comparison to the Mn$^{12}$ line in Mn$_{12}$-ac, where $\Delta\nu_Q = 0.72$ MHz yields $2\sigma_B = 2.4$ MHz i.e. $2\sigma_B = 2\sigma/\gamma Mn/(2\pi) \approx 0.23$ T, where $\gamma Mn = 6.64 \times 10^7$ rad T$^{-1}$ s$^{-1}$ is the $^{55}$Mn gyromagnetic ratio. Such an estimate is thus even larger than the highest $\Delta\nu_Q \approx 4.3$ MHz found in the Mn$^{3+}$ sites of the less symmetric Mn$_{12}$-ac cluster. We expect therefore that the random orientation of the crystallites and, eventually, the presence of inequivalent Mn sites as regards the hyperfine coupling, are also contributing to the observed broadening. Indeed, when decreasing the frequency by 5 MHz the maximum of the spectrum shifts only by 0.24 T, instead of the 0.47 T that would be expected when all the local hyperfine fields are antiparallel to $B_a$. We conclude that the observed spectrum, as well as the NSLR and TSSR data, should be considered as obtained from a mixture of nuclear signals arising from randomly oriented crystallites with largely overlapping and quadrupolar-split NMR lines from all the Mn sites in the cluster. Extrapolating to $B_a = 0$ the field dependence of the peak of the spectrum, one obtains $\nu(0) \approx 360$ MHz $\Rightarrow B_{hyp} \approx 34$ T, very similar to the highest value found in the Mn$^{3+}$ site of Mn$_{12}$-ac. Finally, in connection with the discussion of the nuclear specific heat of the previous section, it is interesting to mention that the $^1$H resonance is found at a value of field given simply by $B_{hyp} \approx \omega/\gamma_H$ (the excluded region in the spectra shown in Fig. 9). This confirms that the local hyperfine fields do not appreciably shift the $^1$H resonance frequency and therefore, for sufficiently high fields ($B_a > 1$ T), the hyperfine interaction of protons can be neglected, as we did.

Figure 10 shows the field dependencies of the NSLR rate $1/T_{1n}$ and the TSSR rate $1/T_{2n}$, measured at constant frequency $\nu = 251.5$ MHz and temperature $T = 0.9$ K. From the discussion above it is clear that these data must be interpreted with a certain caution, since shifting $B_a$ at constant $\nu$ means that we are sampling each time a different portion of the NMR signal, which means different quadrupolar satellites, different orientation of the crystallites, etc. Nevertheless, the agreement with the estimate of $T_{1n}$ obtained by specific heat data (inset of Fig. 4) turns out to be satisfactory. We can directly compare the NSLR rates $1/T_{1n}(B_a)$ obtained from respectively NMR and specific heat data by plotting (dashed line in Fig. 10) $1/T_{1n}(B_a)$ as calculated from Eq. (3), with $T = 0.9$ K and fixing $\kappa_0 = 40$ s$^{-1}$T$^{-3}$ as obtained from the fit of $B_0^*(T^*)$ in the inset of Fig. 4. The agreement is seen to be reasonable. The solid line in the same figure represents a fit to the $1/T_{1n}(B_a)$ based on a model described by Eq. (5a) of the following section, with parameters given in the discussion there.

In Fig. 10 we also show the TSSR rate $1/T_{2n}(B_a)$, with ordinate axis shifted in order to compare its field depen-
treme polarization of the electron spins is induced by the 8
below, we expect 1
1
2n calculated according to Eq. (25).

highly anisotropic single-molecule magnets like Mn

6-ac provide a good basis to describe the relaxation in
dynamic nuclear polarization. These same models should
theoretical models previously developed in the field of dy-
namic nuclear polarization. These same models should
overlooked. We therefore consider worthwhile to give a
detailed overview of the different ingredients needed to
arrive at a consistent picture describing the behavior we
observe in both the NMR and the specific heat of Mn6,
being confident it will be quite useful for the other ma-
terials mentioned as well. We will briefly come back to
this point in the conclusion section, drawing a compar-
ison with our observations in Mn12-ac. We start with an
evaluation of the relevant hyperfine interactions between
the cluster spins and the various nuclear spins in the Mn6
molecular cluster. Quite generally, the hyperfine in-
teraction hamiltonian of a nuclear moment I with the
surrounding electron spins s
j can be written in the form of
the bilinear coupling

\[ \mathcal{H}_{hf} = \sum_j I \cdot \mathbf{A} \cdot s_j \] (8)

Here the hyperfine interaction \( \mathbf{A} \) is a second rank tensor
and summation is over electron spins on both the same
atom and surrounding atoms. It is often convenient to
interpret the hyperfine coupling in terms of a magnetic
hyperfine field: \( B_{hf} = -\gamma_n \hbar \mathbf{I} \cdot \mathbf{s} \), acting on the
nuclear spin in addition to the applied field \( B_a \). \( \gamma_n \) is the
nuclear gyromagnetic ratio. The total magnetic Hamilton-
ion for the nuclear spins then becomes

\[ \mathcal{H}_{hf} = -\gamma_n \hbar \mathbf{I} \cdot (\mathbf{B}_a + \mathbf{B}_{hf}) = -\gamma_n \hbar \mathbf{I} \cdot \mathbf{B}_{tot} \] (9)

In principle \( \mathcal{H}_{hf} \) will be time-dependent since both \( \mathbf{A} \) and
\( \mathbf{s} \) can depend on time due to, respectively, atomic mo-
tions (not considered here) and fluctuations of the elec-
tron spins. Provided that the frequency of these fluctu-
ations is fast compared to the nuclear Larmor frequen-
cies produced in the static case, the nuclear resonance
is still well defined be it at a frequency that is shifted
with respect to that for \( B_{hf} = 0 \), the shift being pro-
portional to the time average of of \( \langle A(t) \rangle \cdot \mathbf{s} \). In the
effective field picture the hyperfine field can be split up
into a static part, \( \langle B_{hf} \rangle \), and a time-dependent part
\( \mathbf{b}(t) = \langle B_{hf}(t) \rangle + \mathbf{b}(t) \). The time-dependent fraction,
\( \mathbf{b}(t) \), is usually much smaller and can then be treated as a
perturbation that may produce relaxation of the nu-
clear polarization. Neglecting the quadrupolar interac-
tions, the remaining magnetic hyperfine interaction may
be decomposed into contributions coming from the cou-
ping of the nuclear moment with the orbital motions of
the electrons, the dipolar interactions with the electron
spins and, in case a finite density of electrons is present
at the nuclear site, the part due to the Fermi-contact in-
teraction. Thus we may write \( \langle B_{hf} \rangle = \mathbf{B}_{hf} + \mathbf{B}_L + \mathbf{B}_F \),
in an obvious notation. As for the relative strengths of
these contributions, for nuclei (such as the present 55Mn)
residing on magnetic atoms the Fermi-contact term is
strongest by far, with \( B_F \sim 10 - 10^2 \) T, followed by

IV. DISCUSSION: MAGNETIC HYPERFINE INTERACTIONS AND NUCLEAR MAGNETIC RELAXATION

In this section we apply theoretical results for magnetic
hyperfine interactions and nuclear magnetic relaxation in
magnetic insulating solids to interpret the data of the pre-
vious sections. We recall that for such materials direct
relaxation channels such as quadrupolar interactions con-
necting nuclear spins to the lattice become ineffective at
low temperatures. Thus the electron spins present have
to serve as an intermediary between nuclei and phonons
in some way or the other and we have to consider the in-
terconnected spin dynamics of both nuclear and electron
spin systems and their coupling to the lattice. As will be
shown below, for highly polarized electron spin systems,
such as Mn6 in high fields at low temperature, the relax-
ation behavior can be adequately described in terms of
theoretical models previously developed in the field of dy-
namic nuclear polarization. These same models should
also provide a good basis to describe the relaxation in
highly anisotropic single-molecule magnets like Mn12-ac
and Fe8 below their blocking temperatures, where an ex-
treme polarization of the electron spins is induced by the
crystal field. Although several groups have described the
application to molecular magnets of some general the-
ories of nuclear relaxation45,56,57,58, it appears that the
role of the electronic dipolar coupling has so far been
overlooked. We therefore consider worthwhile to give a
detailed overview of the different ingredients needed to
arrive at a consistent picture describing the behavior we
observe in both the NMR and the specific heat of Mn6,
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In principle \( \mathcal{H}_{hf} \) will be time-dependent since both \( \mathbf{A} \) and
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tions (not considered here) and fluctuations of the elec-
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in an obvious notation. As for the relative strengths of
these contributions, for nuclei (such as the present 55Mn)
residing on magnetic atoms the Fermi-contact term is
strongest by far, with \( B_F \sim 10 - 10^2 \) T, followed by
the orbital and dipolar interactions with electrons on the same atom (on site), typically of order 10 T. For ions with closed or half-filled shells both \( B_L \) and \( B_{\text{dip}} \) vanish, whereas also for non-S-state ions, such as Mn\(^{3+}\), the orbital moment can be quenched by crystal-field splittings, so that \( B_L \) becomes negligible. Furthermore, nuclei on nonmagnetic ligand atoms directly coordinating the magnetic ions via covalent bonds may also experience substantial Fermi-contact interaction, of order \( 1 \times 10^5 \) Hz \( B_{\text{dip}} \sim 0.1 - 1 \) T.

For the \(^{55}\)Mn nuclei on the Mn\(^{3+}\) ions (with nuclear spin \( I = 5/2 \) and electron spin \( s = 2 \)) of the present compound Mn\(_6\) we deduced in section III-E a resonance frequency (extrapolated to \( B_s = 0 \)) centered around 360 MHz (corresponding to \( B_{\text{dip}} = 34 \) T), which is almost the same as the value 364 MHz Kubo et al. recently found for one of the two Mn\(^{3+}\) sites in Mn\(_{12}\)-ac. According to their analysis, the corresponding \( B_{\text{dip}} = 34.5 \) T for this site results from the combination of Fermi-contact and dipolar fields \( B_{\text{F}} = 41 \) T and \( B_{\text{dip}} = 14 \) T that are of opposite sign. As they point out, the value for \( B_{\text{F}} \) should not vary much for the same ion in comparable coordinations, quoting values for Mn\(_{12}\)-ac of, respectively, \( B_{\text{F}} = 42 \) T and 36 T with \( B_{\text{dip}} = 12 \) T and 11.5 T, again of opposite sign. We may therefore assume similar values for these fields in Mn\(_6\). As is often done, in order to estimate the resulting nuclear energy level splittings responsible for the measured hyperfine specific heat, we have in sections III B and D approximated the net time average of the diagonal part of the (slightly anisotropic) hyperfine interaction by an effective isotropic scalar interaction \( \mathcal{H}_{\text{hf,iso}} = A \mathbf{s} \cdot \mathbf{I} \), where

\[
A = -\gamma_n \hbar B_{\text{hf}} / s \text{, } s = 2 \text{ and } A / k_B = 8.7 \text{ mK (corresponding to } B_{\text{hf}} = 34 \text{ T). As for the other nuclei present in Mn\(_6\), the only sizable contribution to be expected is that arising from the long-range dipolar interactions of the proton spins with the surrounding electronic spins. Since these dipolar fields are small, their contribution may be neglected in zero applied field as compared to that of the }^{55}\text{Mn nuclei, and only becomes substantial for applied fields exceeding 1 T. Although transferred hyperfine interactions with nuclei on the oxygen and Br ions that are directly bonded covalently to the Mn atoms could be substantial we may nevertheless neglect their contributions in comparison with the other ones in view of the low abundance of the }^{17}\text{O isotope and the low number of Br atoms present.}

Considering next the nuclear spin relaxation (NSR) we remark that the longitudinal NSR rate, \( 1 / T_{1n} \), is given quite generally by the expression

\[
\frac{1}{T_{1n}} = \frac{1}{2} \sum_{m,n} W_{m,n} (E_m - E_n)^2 / \sum_mE_m^2, \tag{10}
\]

where \( W_{m,n} \) denotes the probability for a transition between nuclear energy levels \( m, n \) induced by the perturbation considered. As mentioned, we assume the main source for NSR to be the time-dependent fluctuations of the electron spins \( s(t) \) that produce fluctuating components \( b(t) \) of the hyperfine field. The theory has been developed by Moriya \cite{58,61,62} on basis of the general theory of magnetic resonance absorption of Kubo and Tomita. Two possible sources for the electron spin fluctuations have to be considered, namely electron spin-lattice relaxation, characterized by the longitudinal electron spin-lattice relaxation rate \( 1 / T_{1e} \), and spin-spin relaxation, due to magnetic interactions (dipolar or exchange) between the electron spins and characterized by the transverse electron spin relaxation rate \( 1 / T_{2e} \). Obviously, hyperfine interaction terms producing NSR should involve operator combinations as \( I^z s^z \), \( I^z s^- \) and \( I^- s^+ \) and \( I^+ s^- \). The first of these distinguishes itself in that a nuclear spin flip is not combined with an electronic spin flip. It follows that this first type involves transitions at NMR frequencies \( \omega = \omega_e \), the others at ESR frequencies \( \omega = \omega_e \pm \omega_n \approx \omega_e \). In the high-\( T \) approximation, the NSR rate can be expressed in terms of the spectral densities \( f_j^\pm(\omega) \) of the two-spin \( (i \neq j) \) and autocorrelation \( (i = j) \) functions \( \langle s_j^\alpha(0) s_j^\alpha(t) \rangle \) at these frequencies as

\[
\frac{1}{T_{1n}} = \frac{2}{3} (s + 1) \sum_j \left[ A_j f_j^+(\omega_n) + B_j f_j^-(\omega_n) \right], \tag{11}
\]

\[
f_j^\alpha(\omega) = \int_{-\infty}^{+\infty} \langle s_j^\alpha(0) s_j^\alpha(t) \rangle e^{-i\omega t} \, dt \text{ (} \alpha = \pm, z \text{).} \tag{12}
\]

The coefficients \( A_j \) and \( B_j \) are constants depending on the details of the hyperfine interactions. For the ease of discussion we approximate the Fermi-contact interaction by an isotropic scalar on-site hyperfine coupling and neglect possible orbital contributions. Adding the dipolar interaction, the hyperfine Hamiltonian becomes

\[
\mathcal{H}_{\text{hf}} = A\mathbf{i} \cdot \mathbf{s}_i + \frac{\mu_0}{4\pi} \gamma_e \gamma_n \hbar^2 \sum_j \left[ \frac{\mathbf{l}_i \cdot \mathbf{s}_j}{r_{ij}^3} - 3 \frac{(\mathbf{l}_i \cdot \mathbf{r}_{ij})(\mathbf{s}_j \cdot \mathbf{r}_{ij})}{r_{ij}^5} \right], \tag{13}
\]

where \( \gamma_e \) is the electronic gyromagnetic ratio. For the terms responsible for NSR one then obtains

\[
\mathcal{H}' = I_i^z \sum_j D_{ij}^z s_j^z(t) + I_i^+ \left[ (1/2) A s_i^- + \sum_j (D_{ij}^+ s_j^- + D_{ij}^+ s_j^0) \right] \text{ + c.c.,} \tag{14}
\]

where \( D_{ij}^z \) and \( D_{ij}^\pm \) denote the components of the dipolar coupling tensor connecting \( I_i^+ \) with \( s_j^\pm \) and with \( s_j^0 \), respectively, and c.c. stands for complex conjugates. It is important to note that for a pure scalar hyperfine interaction only the transverse spectral densities \( f_{ij}^\pm(\omega_e) \) do appear. Assuming an exponential decay of the spin-correlation functions, the scalar interaction leads to the
NSR rate

\[
\frac{1}{T_{1n}} = \frac{1}{2} A^2 \int_{-\infty}^{+\infty} \langle s^+(0) s^-(t) \rangle e^{i(\omega_e - \omega_n)t} dt = \frac{1}{3} s(s+1) A^2 \frac{T_{2e}}{1 + (\omega_e - \omega_n)^2 T_{2e}^2}.
\]

(15)

As noted above, relaxation then requires energies \(\hbar \omega_e\) of the order of the electronic level splittings. By contrast the dipolar interaction contains terms of different symmetry, so that it contributes to both transverse and longitudinal terms. In particular it contains the operator \(D_{ij}^2 \propto -(3/2) \sin \theta \cos \theta e^{-i\theta}\) that may induce a nuclear flip unaccompanied by an electron flip, thus involving the much smaller energy \(\hbar \omega_n\). The NSR rate by this process is found to be given by

\[
\frac{1}{T_{1n}} = \frac{3}{5} \left( \frac{\mu_0}{4\pi} \right)^2 (\gamma_e \gamma_n \hbar) r^{-6} \int_{-\infty}^{+\infty} \langle s^+(0) s^+(t) \rangle e^{-i\omega_n t} dt.
\]

(16)

Comparing Eqs. (15) and (18) it is clear that, unless the spin-lattice relaxation of the Electron-Zeeman reservoir.

\[
\langle s^+(0) s^-(t) \rangle = \frac{1}{3} s(s+1) \exp(-t/T_{1e}),
\]

(17)

the NSR rate by this process is found to be given by

\[
\frac{1}{T_{1n}^{(EZ)}} = \frac{2}{5} \left( \frac{\mu_0}{4\pi} \right)^2 (\gamma_e \gamma_n \hbar) r^{-6} s(s+1) \frac{T_{1e}}{1 + \omega_n^2 T_{1e}^2},
\]

(18)

an expression first derived by Bloembergen\(^a\) for the nuclear spin-lattice relaxation by paramagnetic impurities in diamagnetic crystals. The superscript (EZ) is added to indicate that this nuclear relaxation is driven by the spin-lattice relaxation of the Electron-Zeeman reservoir. Comparing Eqs. (15) and (18) it is clear that, unless the electronic line widths would be comparable to the level splittings \(\omega_e\), the process will outweigh the previous one by the large factor \((\omega_e/\omega_n)^2\). In most cases of interest one further has \(\omega_n T_{1e} \gg 1\), so that one may write approximately

\[
\frac{1}{T_{1n}^{(EZ)}} \approx \frac{2}{5} \left( \frac{\mu_0}{4\pi} \right)^2 (\gamma_e \gamma_n \hbar) r^{-6} s(s+1) \frac{T_{1e}}{\omega_n^2 T_{1e}^2} = \frac{2}{5} \left( \frac{B_{\text{dip}}}{B_{\text{tot}}} \right)^2 \frac{1}{T_{1e}}
\]

(19)

Here \(B_{\text{dip}} = (\mu_0/4\pi) \gamma_e \gamma_n \sqrt{s(s+1)} r^{-3}\) stands for the electronic dipolar field at the nuclear site, and \(B_{\text{tot}} = B_n + B_{\text{hf}} = \omega_n/\gamma_n\) is the total field responsible for the nuclear Zeeman splittings\(^a\).

In the next step we have to compare this result with the NSR rate arising from spin-spin interactions, which for Mn\(_6\) amount to the dipolar interactions between the electronic cluster spins \(S = 12\). We note that since the total spin \(S = 12\) of the Mn\(_6\) cluster results from the strong ferromagnetic intramolecular exchange between the atomic spins \(s = 2\), the fluctuations of the total spin are obviously related to those of the constituting atomic spins and vice versa. Thus, although both the hyperfine interactions and the electron spin lattice coupling basically involve the atomic spins, the atom spin fluctuations nevertheless are in a one-to-one relationship with those of the cluster spins. Due to such spin-spin interactions the spectral density will no longer be given by a Lorentzian. Instead of the exponential decay of the correlation functions, Eq. (17), one usually assumes a Gaussian approximation for the autocorrelation functions\(^b\) |

\[
\langle s^+(0) s^+(t) \rangle = \frac{1}{2} \langle s^+(0) s^-(t) \rangle = \frac{1}{3} s(s+1) \exp(-\omega_n^2 t^2).
\]

(20)

For the longitudinal NSR rate one obtains:

\[
\frac{1}{T_{1n}^{(ED)}} = \frac{\sqrt{2\pi}}{3} \gamma_n^2 B_{\text{dip}}^2 \omega_n^{-1} \exp(-\omega_n^2/2\omega_{\text{int}}^2),
\]

(21)

where \(\omega_{\text{int}}\) stands for the electronic dipolar spin-spin interaction\(^b\) which in our case can be estimated from the dipolar ordering temperature, \(\hbar \omega_{\text{int}} \approx k_B T_e\), and also corresponds to the electronic TSSR rate, \(\omega_{\text{int}} \approx 1/T_{2e}\).

At this point it is important to emphasize that the above derivations are essentially only valid at high temperatures and low applied fields, since the effects of polarization of the electronic spins by the applied field have been neglected. As noted already by Moriya\(^b\) and in later work on dynamic polarization\(^b\), the more the electron spins become polarized, the less they will be able to relax the nuclear spins. To account for this, one should replace the time dependencies of the electronic spin \(s(t)\) by its fluctuating part, \(\delta s(t) = s(t) - s_0\), where \(s_0\) denotes the thermal average of \(s\). Thus, instead of an expression as in Eq. (17) for the decay of the electronic spin, one should take\(^b\)

\[
\langle \delta s^+(0) \delta s^+(t) \rangle = \langle \delta s^+(0) \delta s^-(t) \rangle = \langle \delta s^+(0) \delta s^-(t) \rangle = \langle \delta s^0(t) \rangle \exp(-t/T_{1e}) = \langle \delta s^0(t) \rangle \exp(-t/T_{1e}) = \langle \delta s^0(t) \rangle \exp(-t/T_{1e}).
\]

(23)

using the fact that the thermal average of each Mn\(_{3+}\) spin \(s^2\) is given by \(sB_S(X),\) where \(X = g\mu_B B_n S/k_B T, s = 2,\)
and $B_S$ is the Brillouin function for the total molecular spin $S$. Accordingly, the expression for the electron spin autocorrelation function should be multiplied by the factor $3S(s+1)^{-1}\partial B_S/\partial X$. Restricting in what follows to the simplest case of spin $S = 1/2$, as appropriate for the present experiments in the high-field / low-$T$ range where only the two lowest lying electron Zeeman states are relevant, this factor reduces to $(1 - \tanh^2 X)/2$, with $X = g\mu_B B_a/2k_B T$. We thus obtain for $1/T_{1e}^{(ED)}$ instead of Eq. (19) the relation
\[
\frac{1}{T_{1e}^{(ED)}} \approx \frac{1}{5} \left( \frac{B_{\text{dip}}}{B_{\text{tot}}} \right)^2 (1 - \tanh^2 X) \frac{1}{T_{1e}} \quad (24)
\]
whereas instead of Eq. (21) one has now:
\[
\frac{1}{T_{1e}^{(ED)}} \approx \frac{1}{T_{2n}^{(ED)}} \approx \frac{\sqrt{2\pi}}{6} \frac{(\gamma_n B_{\text{dip}})^2}{\omega_{\text{int}}} (1 - \tanh^2 X) \exp(-\omega_n^2/2\omega_{\text{int}}^2). \quad (25)
\]
In both cases, since $\tanh X$ gives the degree of polarization of the electron spin, one observes that when this approachs unity the nuclear relaxation rate goes to zero, as to be expected. For the electron-dipolar relaxation channel one should notice that, although the actual electronic linewidth $1/T_{2e}$ strongly depends on the electronic polarization (the second moment of the absorption line is proportional to $1 - \tanh^2 X$, cf. Ref. [52]), $\omega_{\text{int}}$ in (25) is still given by the dipolar coupling as calculated in the high-$T$ limit.\(^65\)

Proceeding next to compare the above predictions with the high-field NMR experiment, we may already notice that Eq. (25) yields the right order of magnitude for $1/T_{2n}$. From the value of $T_\tau \approx 0.16$ K, we deduce the electronic dipolar broadening to be $\omega_{\text{int}} \approx 2 \times 10^{10}$ rad/s. With NMR frequencies of order $\omega_n \approx 1.5 \times 10^9$ rad/s the factor $\exp(-\omega_n^2/2\omega_{\text{int}}^2)$ becomes $\approx 1$. Further, we have $B_{\text{tot}} \approx 30$ T and $B_{\text{dip}} \approx (1/3)B_{\text{tot}} \approx 10$ T, yielding $\gamma_n B_{\text{dip}} \approx 6 \times 10^6$ rad/s. For applied fields $B_a > 5$ T the polarization correction factor $(1 - \tanh^2 X)$ becomes of order $10^{-3}$ to $10^{-4}$. From Eq. (25) with the numerical factors quoted above we thus find the prediction (cf. Fig. 10 dotted line): $1/T_{2n}^{(ED)} \approx 10^4$ to $10^3$ s$^{-1}$ for the transverse NSR-rate arising from electron spin-spin interactions, i.e. in the same range as the experimental transverse rate. Conversely, the data clearly show that $1/T_{1n} \ll 1/T_{2n}$, contrary to the prediction of Eq. (25). Indeed, this process basically only establishes the thermal equilibrium between the nuclear and the electron spin systems, i.e. without considering the relaxation of the latter toward the phonon bath. For the complete description of the nuclear spin-lattice relaxation process we obviously have to investigate the spin-phonon coupling mechanism as well.

In order to estimate the electronic $1/T_{1e}$, we remark that the electron spin-lattice relaxation rate arising from transitions between the two lowest Zeeman levels of the $S = 12$ multiplet due to the direct process will be given by the sum of the transition rates $w_\tau$ and $w_\gamma$ due to absorption and emission of phonons, respectively,\(^22\)
\[
\frac{1}{T_{1e}} = w_\tau + w_\gamma. \quad (26)
\]
Since the phonon modulation of the crystal field can be expected to be the main source of this coupling, we may apply to $M_{\text{NS}}$ the calculations developed by Leuenberger and Loss\(^68\) obtaining
\[
w_\tau = V_{e-ph}(g\mu_B B_a)^3 \frac{1}{\exp(2X) - 1}, \quad (27a)
\]
\[
w_\gamma = V_{e-ph}(g\mu_B B_a)^3 \frac{1}{1 - \exp(-2X)}, \quad (27b)
\]
\[
V_{e-ph} = \frac{D^2 S(2S - 1)^2}{6\pi e^2 \hbar^4}, \quad (27c)
\]
where $S = 12$, $D/k_B = 0.013$ K is the uniaxial anisotropy constant, $\rho = 1.45$ g/cm$^3$ is the density and $c_s$ the sound velocity. Within the Debye model, the latter is obtained from the experimental Debye temperature $\Theta_D = 29$ K as
\[
c_s = \frac{k_B \Theta_D}{\hbar} \left( \frac{6\pi N_A}{V_m} \right)^{-1/3} = 1.3 \times 10^3$ m/s. \quad (28)
\]
Substituting into Eq. (26) yields
\[
1/T_{1e} \approx 104 B_a^3 \coth X, \quad (29)
\] with $B_a$ in Tesla. (It should be noted that the value calculated for $V_{e-ph}$ is very sensitive to the values used for $\Theta_D$ and $D/k_B$ so that it obviously is subject to a large uncertainty margin). For instance, $B_a = 5$ T and $T = 1$ K yields $1/T_{1e} \sim 10^4$ s$^{-1}$. Because of the very small value of the anisotropy constant $D$ in Mn$_6$, $1/T_{1e}$ is thus expected to be much lower than the typical values $\sim 10^7$ s$^{-1}$ found, e.g., in Mn$_{12}$-ac. This also implies that a model for the TSSR rate based on the random changes in local hyperfine field due to electron-phonon excitations, as recently used to describe $1/T_{2n}$ in Mn$_{12}$-ac, would lead in this case to a quantitative estimate that is about three orders of magnitude lower than our experimental result.

Nuclear relaxation to the lattice can now occur in two ways, either directly via the spin-lattice relaxation fluctuations of the individual electron spins, or in a two-step process by spin-spin relaxation to the electron dipolar reservoir followed by relaxation to the lattice. The direct spin-lattice relaxation (single-ion) process is described by Eq. (25), which becomes
\[
\frac{1}{T_{1n}^{(ED)}} \approx 21 \left( \frac{B_{\text{dip}}}{B_{\text{tot}}} \right)^2 B_a^3 \coth X(1 - \tanh^2 X). \quad (30)
\]
For large $X$, $\coth X \approx 1$ and $(1 - \tanh^2 X) \approx 4 \exp(-2X)$. With $B_{\text{dip}}/B_{\text{tot}} \approx 1/3$ one obtains
\[
\frac{1}{T_{1n}^{(ED)}} \approx 9 B_a^3 \exp(-g\mu_B B_a/k_B T), \quad (31)
\]
notably in connection with the phenomenon of dynamic in diamagnetic insulators with paramagnetic impurities, has been met for nuclear relaxation in magnetic crystals or has to occur in a second step. This situation has often seen 20 that has the same field dependence as Eq. (31), but assuming a prefactor of order 25.

FIG. 11: (Color online) Block diagram of the nuclear and electronic spin systems involved in the relaxation process at high fields, and the relative rates of energy transfer.

( ...$T_1$... )

which for the case $C_{NZ} \gg C_{ED}$ would become roughly equal to $2/T_{1e}$ ($C_{ED}/C_{NZ}$). Here the symbols $C_{ED}$ and $C_{NZ}$ stand for the (field-dependent) specific heats of the ED and the NZ reservoirs. The situation is seen to be analogous to the phonon-bottleneck phenomenon, well known in paramagnetic relaxation. Applied to our present case, we calculate $C_{NZ}$ from Eq. (8) to be of order 0.006 R at $T = 0.9$ K and $B = 5$ T (cf. Figs. 6 and 7), which value depends only weakly on applied field. $C_{ED}$ in zero field can be estimated as

$$C_{ED} = 6 \left( \frac{\mu_0 g^2 \mu_B^2 S(S+1)}{4\pi} \right) \frac{1}{T^2} \sum_{j>i} \frac{1}{r_{ij}^6},$$

after averaging over the angular dependence of the dipolar coupling, as appropriate for an unoriented powder sample. Using the same lattice parameters as for the Monte Carlo simulations yields $C_{ED} \approx 0.004$ R at $T = 0.9$ K. $C_{ED}$ then depends on the electronic polarization as $C_{ED}(X) = C_{ED}(0)(1 - \tanh^2 X) \approx 4 C_{ED}(0) \exp(-g \mu_B B / k_B T)$, since for high fields only the lowest Zeeman level is available for the electron spins. All this leads to a global rate for the nuclear spin-lattice relaxation

$$\frac{1}{T_1^n} \approx 208 B_{a}^3 \coth(X) \frac{0.004(1 - \tanh^2 X)}{0.006 + 0.004(1 - \tanh^2 X)} 
\approx 550 B_{a}^3 \exp(-g \mu_B B / k_B T),$$

that has the same field dependence as Eq. (34), but with a prefactor of about 550 instead of 9, meaning that the two-step spin-spin relaxation should be the fastest process by almost two orders of magnitude in the high-field region. The solid line in Fig. 10 is obtained from Eq. (34) but assuming a prefactor of order 25.

At this point it is important to recall that the prefactors in both Eqs. (34) and (35) are affected by a large...
numerical uncertainty originating from the expression for $1/T_{1e}$, which contains the electron-phonon coupling constant $V_{e-\text{ph}}$, Eq. (27c). This constant is proportional to $D^2$ and $\Theta_D^{-5}$, and both these quantities have rather large error bars. However, the values of $D$ and $\Theta_D$ influence in the same way the relaxation rates via the Electron-Zeeman and the Electron-Dipolar channels, thus the latter is expected to dominate in any case by almost two orders of magnitude. Due to their strong influence on $1/T_{1e}$, allowing both $D$ and $\Theta_D$ to vary by only a factor 1.5 would already yield the correct quantitative prefactor in Eq. (27c).

Summarizing the results of this section, we may state that both the longitudinal and transverse nuclear relaxation that we observe at high applied fields are in excellent qualitative and even quantitative agreement with the model based on fast dipolar relaxation of the hyperfine-coupled Nuclear-Zeeman system to the Electron-Dipolar reservoir, followed by much slower relaxation of the combined systems via the electron spin-lattice channel. The direct nuclear spin-lattice relaxation process by single-ion electron spin-lattice relaxation predicts a similar field dependence but is calculated to be much slower at high fields. At low fields one will have $\omega_e \approx \omega_{\text{int}}$, so that the Electron-Dipolar and the Electron-Zeeman systems will become “on speaking terms”, and a subdivision of the two electron spin reservoirs is no longer valid. In this range, however, NSR by means of the scalar hyperfine interaction, Eq. (15), should also become important (since then no longer $\omega_n \ll \omega_e$, whereas $T_{2e} \ll T_{1e}$).

V. CONCLUDING REMARKS

In conclusion, our experiments on Mn$_6$ show that dipole-dipole interactions between molecular magnetic clusters may indeed induce long-range magnetic order at low temperatures if the anisotropy is sufficiently small. Spin-lattice relaxation is then fast enough to produce equilibrium conditions down to the low temperatures needed. We should add that similar conditions could in principle also be reached in the highly anisotropic cluster systems, for which it was shown that by applying magnetic fields perpendicular to the anisotropy axis, the spin-lattice relaxation can be tuned and made similarly fast through the process of magnetic quantum tunneling. However, it is very rare to observe magnetic ordering phenomena in those systems. One exception known to date originates from an unusually high tunneling rate already in zero field. Instead, for the majority of the anisotropic clusters it is likely that, given the magnitude of the fields needed to have a considerable increase of the relaxation rate ($B_{\perp} \gg 1$ T), any longitudinal component of the field would create a Zeeman splitting that is much larger than the energy involved in the magnetic dipolar ordering. We found indeed that in Mn$_6$ the ordering transition is removed already for relatively small fields ($\sim 0.5$ T). In a recent neutron diffraction experiment on a Mn$_{12}$-ac single crystal, however, Luis et al. achieved an extremely accurate alignment of the field (to within 0.1 degree) and obtained evidence for a ferromagnetic phase induced by the transverse field.

We have also studied the nuclear spin dynamics of Mn$_6$, both directly by NMR experiments and through the hyperfine contribution to the field-dependent specific heat. The agreement between the two techniques is very good, and also provides an interesting comparison with the nuclear spin dynamics in the anisotropic single-molecule magnet Mn$_{12}$-ac. Both qualitatively and quantitatively, the nuclear magnetic relaxation data turn out to be in good agreement with predictions obtained from theories developed earlier for relaxation in paramagnetic crystals and for dynamic polarization. In high fields, the observed nuclear relaxation is dominated by electron spin fluctuations arising from the dipolar interactions between cluster spins. In spite of the large Zeeman splittings between the cluster spin levels produced in such high fields, these fluctuations are able to relax the nuclei through the dipolar part of the hyperfine interaction. In this field range the electron dipolar and the electron Zeeman system are basically decoupled. Relaxation of the nuclear spins then proceeds in two steps, namely an initial rapid relaxation to the electron dipolar system via the electron spin-spin interaction channel, followed by a much slower relaxation of the combined nuclear-electron spin systems to the lattice through the electron spin-lattice channel. It is of interest in this regard to note that the values for the longitudinal and transverse relaxation rates as observed for Mn$_{12}$-ac in the low-temperature ($T < 0.9$ K) quantum regime, where the electron spin fluctuations can be attributed to quantum tunnelling of the cluster spins, fall only slightly below the present observations for Mn$_6$ at $T = 0.9$ K. Below the blocking temperature ($\sim 3$ K) the cluster spins of Mn$_{12}$-ac become almost fully polarized even in zero field due to the strong crystal field splittings of the electron spin levels, the distance between the first excited state from the ground state amounting to more than 10 K. The temperature independent value found for the transverse nuclear relaxation rate, $1/T_{2n} \approx 100$ s$^{-1}$, could be well explained in terms of intercluster nuclear spin diffusion, i.e., nuclear flip-flops arising from the dipolar interaction between nuclear spins in neighboring clusters. The same physical mechanism should also put a lower bound to the transverse nuclear rate in Mn$_6$, which is, however, not relevant due to the presence of the faster spin-spin relaxation process.

For the longitudinal rate $1/T_{1n}$ for Mn$_{12}$-ac a value of $\approx 0.03$ s$^{-1}$ is found below 1 K, slightly depending on temperature. This value agrees with the nuclear-spin-mediated tunneling rate estimated for the fast-relaxing molecular spins in Mn$_{12}$-ac. As argued by Morello et al. the tunneling process can at the same time provide a relaxation channel for the nuclei to the electron-dipolar system. Similar to the above-discussed case of Mn$_6$ in high field, relaxation to the lattice should then occur in a second step through the electron spin-lattice coupling.
APPENDIX A: ESTIMATION OF THE EFFECTIVE DEMAGNETIZING FACTOR

In this Appendix, we derive an approximate expression for the effective demagnetizing factor $N_{\text{eff}}$ appropriate for the cylindrically shaped container filled with the grains. This problem is notoriously complex and can only be approximately solved. In a first step the relation should be found between the applied field $H_a$ and the local field $H_{\text{loc}}$ acting on a reference grain in the container. Approximating the grains as point dipoles, the difference $(H_{\text{loc}} - H_a)$ will be due to the contributions to the field arising from all the other dipoles inside the container. Adopting the well-known Lorentz construction, the dipole summation is split into one inside a (sufficiently large) sphere around the reference grain and a contribution from the dipoles outside this sphere. For this second contribution the dipoles are usually assumed to form a homogeneous continuum so that it is just proportional to the difference in demagnetizing factors of the container ($N_{\text{cont}}$) and of the sphere (1/3). As for the first summation, it would be zero for a cubic arrangement of the dipoles. This will not be the case here since the grains are randomly packed, but since a valid estimate is not easily obtained, and we may expect it to be small, we shall just neglect it. One then obtains

$$H_{\text{loc}} = H_a - f M (N_{\text{cont}} - 1/3)$$  \hspace{1cm} (A1)

Here $f$ denotes the filling volume fraction of the grains in the container and $M$ is the magnetization of the grains.

In the next step we have to correct $H_{\text{loc}}$ for the dipolar contributions arising from the magnetic material inside the grain. In case of a ferromagnetic material, one usually only takes the shape-dependent demagnetizing correction into account. An argument for this may be found in that the magnetization process for the ferromagnet is mostly determined by the mobility of the domain walls, which will react to the macroscopically averaged internal field. For simplicity, we first consider the case of zero magnetocrystalline anisotropy for which the demagnetization factor as well as the magnetization and fields can be treated as scalars. We thus obtain for the internal field $H_i$ inside the grain

$$H_i = H_{\text{loc}} - N_{\text{grain}} M$$
$$= H_a - N_{\text{grain}} M - f M (N_{\text{cont}} - 1/3)$$  \hspace{1cm} (A2)

From the definition: $H_i = H_a - N_{\text{eff}} M$, we thus finally find

$$N_{\text{eff}} = N_{\text{grain}} + f (N_{\text{cont}} - 1/3).$$  \hspace{1cm} (A3)

If, by contrast, the grain has uniaxial magnetic anisotropy, we should distinguish between the parallel $\chi_{i,\parallel}$ and the perpendicular $\chi_{i,\perp}$ intrinsic susceptibilities in the response to the internal magnetic field. These susceptibilities depend on the magnitude of the anisotropy parameter $D$. They can be obtained, from the numerically calculated eigenstates and eigenvalues of the spin Hamiltonian, using the Van Vleck’s formalism as described in, e.g., Ref. 72. In this case Eq. (A2) becomes

$$H_i = H_{\text{loc}} - \tilde{N}_{\text{grain}} \tilde{\chi}_i H_i,$$  \hspace{1cm} (A4)

where $\tilde{N}_{\text{grain}}$ and $\tilde{\chi}_i$ are respectively the diagonal demagnetizing and intrinsic susceptibility tensors. By combining Eqs. (A1) and (A3) it is possible to find a relationship between the measured susceptibility $\chi$ and the two components of $\tilde{\chi}_i$. For the case when the anisotropy axes are randomly oriented in the sample, we find

$$\chi = \frac{\chi_{\text{eff}}}{1 + f \chi_{\text{eff}} (N_{\text{cont}} - 1/3)}$$  \hspace{1cm} (A5)

where the susceptibility $\chi_{\text{eff}}$ corrected for the demagnetizing factor of the grains equals

$$\chi_{\text{eff}} = \frac{2 \chi_{i,\perp}}{3 (1 + \chi_{i,\perp}/3)} + \frac{\chi_{i,\parallel}}{3 (1 + \chi_{i,\parallel}/3)}.$$  \hspace{1cm} (A6)

This relationship was used to calculate the theoretical powder susceptibilities shown in Fig. 2.

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