Engineered quantum tunnelling in extended periodic potentials

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Abstract. Quantum tunnelling from a tilted, but otherwise periodic potential is studied. Our theoretical and experimental results show that, by controlling the system’s parameters, we can engineer the escape rate of a Bose-Einstein condensate to an exceptional degree. Possible applications of this atom-optics realization of the open Wannier-Stark system are discussed.

1. Introduction

One of the first manifestations of quantum mechanics was radioactive decay, in which – according to Gamov’s theory – a particle can overcome a potential barrier because of an even exponentially small tail of its spatial wave function at the unbounded side of the barrier [1]. Besides this static problem of over-the-barrier tunnelling, also dynamical tunnelling mechanisms are well-know today [2]. The latter situation is found in systems where particles can escape from dynamical barriers (such as regular elliptic islands in classical phase space) on the grounds of dynamically induced coupling processes (see, e.g., [3] for realistic examples).

Quantum tunnelling has found many technological applications, such as, for instance, in scanning tunnelling microscopes [4] and in superconducting squid devices [5]. Arguably the mostly used application of tunnelling is present in tunnelling diodes and related integrated semiconductor devices which go back to the pioneering work of Leo Esaki [6]. The latter also proposed to exploit resonantly enhanced tunnelling (RET) for technical use, and since the 1970’s much progress has been made in producing artificial super-lattice structures [7], in which RET of fermionic quasiparticles could be demonstrated [8,9].

Here we report the realization of RET using Bose-Einstein condensates which are held in optically produced potentials (“optical lattices”). The counter-propagating beams creating the lattice can be easily and in a controlled manner accelerated with respect to each other such as to mimic an additional static linear potential in the moving frame of reference [10]. Tunnelling in this Wannier-Stark system occurs between the quantised energy levels (the Wannier-Stark levels) in various wells of the potential. An example of RET between next-nearest potential wells is illustrated in Figure 1: In such a situation of tunnelling between energetically degenerate levels, the escape rate can be varied by orders of magnitude by a slight change of the static tilting force [11].
The part of the condensate which has tunnelled through the barrier is in good approximation just accelerated to infinity by the static force. As a consequence, we are dealing with an open, i.e. non-hermitian decay problem \([11,12]\), for which theoretical interest has recently revived within the more general context of avoided level crossings in the complex energy plane \([11,13,14]\).

In contrast to solid-state realizations of RET (see, e.g., \([9]\)), our system with ultracold atoms offers a near-perfect and easily reproducible control over parameters (the tilting force and the potential depth), the geometry (tunnelling along an almost arbitrarily long array of potential wells and the condensate density), and over the initial conditions (energy and quasimomentum of the prepared condensate). Decay rates for non-interacting cold atoms were measured for the Wannier-Stark system in a small interval of the Stark force \([15]\), and a systematical study of RET with such cold atomic systems has been proposed in ref. \([11]\). Here we show that RET is accessible to modern experiments with ultracold atomic gases, not only for the case of independently moving atoms (described by the usual “linear” Schrödinger equation) but even in the presence of a nonlinearity (with a mean-field nonlinear term mimicking the interatomic interactions).

![Figure 1. RET: tunnelling of atoms from a tilted lattice is resonantly enhanced when the energy difference between the levels within an energetically equidistant Wannier-Stark ladder (e.g. made up of the lower levels in all the wells) matches the distance between the energy levels in the wells.](image)

2. The nonlinear Wannier-Stark system and experimental results

Our experimental setup is well-modelled by the following three-dimensional Gross-Pitaevskii equation \([16,17]\):

\[
i\hbar \frac{\partial}{\partial t} \psi(\vec{r},t) = \left[ -\frac{\hbar^2}{2M} \nabla^2 + \frac{1}{2} M \left( \omega_x x^2 + \omega_y y^2 + \omega_z z^2 \right) + V \sin^2 \left( \frac{\pi x}{d_L} \right) + F x + g N |\psi(\vec{r},t)|^2 \right] \psi(\vec{r},t) . \tag{1}
\]

\(\psi(\vec{r},t)\) represents the wave function of Bose-Einstein condensate, and the various potential terms can be used to engineer the dynamics and the tunnelling decay of the quantum gas. The three frequencies \(\vec{\omega}\) describe the longitudinal and the transversal trap frequencies which can be varied to change the density of the condensate within the wells. In such a way, we can adapt the effective nonlinearity \(gN |\psi(\vec{r},t)|^2\), where \(g = 4\pi \hbar^2 a_s / M\), with the \(s\)-wave scattering length \(a_s\) and the total number of atoms \(N\). Not only can we control the many-body evolution of the condensate by adapting the atomic density, but we are also able to guide the dynamical behaviour by varying the potential depth \(V\) and the force \(F\). As a consequence, we have all possible handles to switch between “fast” and “slow” tunnelling escape of the condensate using in particular the RET effect sketched in figure 1. The optimal extraction of the tunnelling rates from the experimentally observable momentum distributions of the condensate at different evolution times is described in full detail in a recent theoretical paper \([17]\). Here we restrict to show the main experimental results which can be found in the figures 2 and 3.
Figure 2. Upper panel: measured Wannier-Stark tunnelling rates (•) for \( g \approx 0 \) in a lattice of depth \( V_0 \approx 3.5 \). The full line shows the theoretical expectation obtained by the methods described in [11, 12], while the dashed curve represents the Landau-Zener prediction \( \Gamma_{LZ} \). Lower panel: the ratio of the theoretical (——) and the measured (•) rates and \( \Gamma_{LZ} \). Even if the modulation here is relatively small due to the chosen small lattice depth, we note the extension of the measured rates over two orders of magnitude in the upper panel. The nicely resolved central peak around \( F_0 \approx 1.35 \) corresponds to RET between second-nearest potential wells, while the left one at smaller \( F_0 \approx 0.5 \) is a consequence of RET between third-nearest well. The shoulder on the right shows signatures of the next-nearest neighbour RET peak around \( F_0 \approx 2.7 \), corresponding to the situation sketched in figure [1].

Figure 3 (a) highlights the impact of the mean-field nonlinearity on the tunnelling rates. The effective nonlinearity parameter is conveniently expressed by \( C \equiv \frac{g n_0}{8 E_{rec}} \), with the peak density of the condensate \( n_0 \), and \( C \) can be directly estimated from independent measurements [16]. In accordance with the theoretical prediction of [17], the repulsive interatomic interactions have two main effects. First, they tend to enhance the decay rate for all values of \( F \). Second, they wash out the RET peak structure due to the different scaling of the tunnelling rates with the nonlinear parameter \( C \) at the peak maximum and in the region where the Landau-Zener
formula applies [19]. In this latter regime, the scaling is found to be linear in $C$ [19], as expected from perturbation theory, which, on the other hand, does not apply at the peak maxima where energy levels are degenerate.

**Figure 3.** (a) same theoretical data as in the upper graph of figure 2 with measured tunnelling rates for a small nonlinearity $C \approx 0.025$ (■). The mean-field, repulsive nonlinearity washes out the RET peak structures of figure 2, and this can be used as an additional experimental handle to globally enhance the rates. (b) and (c) show numerical data from a Bose-Hubbard model of 7 atoms in 6 potential wells for $V_0 = 3$ (fixing the hopping constant $J \approx 0.22$), interaction constant $U = 0.2$, and $F_0 = 0.47$ (b) or $F_0 = 0.16$ (c). The broad distribution at the larger force arises from the regular Bloch oscillation dynamics of the atoms. The distribution in (c) shows that there is a small number of preferred channels to tunnel to the first excited levels originating from the quantum chaotic motion of the atoms along the wells (leading e.g. to interaction-induced decoherence of the Bloch oscillations [22]).

While the experiment can access the decay rates for short times (up to about ten Bloch periods), for longer times, the density dependent decay naturally leads to a nonexponential scaling of the survival probability of the condensate in the open Wannier-Stark system. Such a nonexponential tunnelling might be observable in future experiments which could either realize larger nonlinearities (we estimate from numerical data $C > 0.25$ [17]), or be sensitive enough to measure the long-time behaviour of the survival probability close to a RET peak [20].

We would like to emphasise that the presence of atom-atom interactions in ultracold atomic gases does not hinder possible applications of the here observed RET effect. On the contrary, the effective interaction can be controlled and used as an additional handle to enhance (for repulsive interactions) or to decrease (for attractive interactions) the quantum tunnelling in an extended potential [17]. In the here presented experiments, the interactions can be controlled by the nonlinearity parameter $C \approx 0 \ldots 0.1$ [16,19].

Other experimental setups can reduce the density of atoms to the order of one atom per lattice well [21], which makes necessary a true many-body description of the problem going
beyond the mean-field equation (1). An adequate description of the many-body dynamics is complicated but possible for small lattices and filling factors of the order one [22]. Based on the Bose-Hubbard model, we could deduce that the tunnelling between the lowest and the first excited Wannier-Stark levels depends indeed crucially on the dynamical properties of the many-body problem. The latter are controlled again by the Stark force $F$. If the many-body dynamics shows signatures of quantum chaos, we have found that the system has preferred decay channels, whilst for regular (i.e. close to integrable motion) of the atoms along the lattice, the distribution of the tunnelling rates is relatively broad [23]. The latter numerical results were obtained from a single-band Bose-Hubbard model for which the decay to the excited energy levels on each lattice site was perturbatively included [23]. The corresponding distributions are collected in the panels (b) and (c) of figure 3 as examples of regular and chaotic motion, respectively.

3. Conclusions

In summary, our combined theoretical and experimental analysis of the tunnelling decay of ultracold atoms in periodic potentials show that quantum tunnelling can be easily controlled by adapting the parameters of the system. Experiments with Bose-Einstein condensates in optically produced potentials allow us the required degree of control making them an ideal playground to study solid-state models such as the Wannier-Stark system.

In consequence, we can propose the RET effect as an experimental handle to engineer quantum tunnelling, and as a possible starting point for future applications. We may think of diode-like switches between spatially separated condensates, or continue the route to study true many-body tunnelling for which dynamical properties of the system as a whole (including strong interatomic interactions [24]) and possible external couplings (e.g. to leads or to heat baths [25]) conspire to lead to new interesting physical phenomena.

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