Control Analysis of Resonance System Based on Active Disturbance Rejection Control

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Abstract. The mass-spring-damping resonance system is modeled by the mechanism-method and Lagrange method, and the stability of the system is analyzed. The extended state observer (ESO) algorithm is used to analyze and control the mass-spring-damping system based on active disturbance rejection control (ADRC) theory. At the same time, it is compared with the traditional PID algorithm. Random noise is added to the simulation to simulate the real environment and the results show that the ADRC control algorithm is more ideal for system control.

Keywords: Extended state observer; Active disturbance rejection control; Damping system; Differential Tracker.

1. Introduction
Springs, dampers, and masses are typical components that make up a mechanical system. The spring-mass-damping system is the most common mechanical vibration system, which has quite a wide range of uses in life, and the buffer is one of them [1]. The buffer device is the main component that generates energy during the process of absorption and dissipation, and its ability to absorb and dissipate energy directly relates to the safety and stability of the system. Buffers can be seen everywhere in life, such as our car shock absorbers and buffers used to consume collision energy, the performance of their buffer system directly affects the stability of the car and the driver’s safety. The stability of the buffer system directly affects the success of the rendezvous and docking. In addition, the mass-spring-damping system is a classic teaching model of automatic control principles. Therefore, the study of spring-mass-damping system has very deep practical significance [2]. In this article, the system is modeled by using the physical method and the Lagrange method to model, and the control performance of the mass-spring-damping system is comparative analyzed by using the ADRC algorithm and traditional PID algorithm. Finally, the simulation results demonstrate the superiority of ADRC.

2. Two-level Mass-spring-damping System Modeling
The existing single-input two-stage mass-spring-damping system is shown in Figure 1. In the system, the displacement of mass \( m_1 \) is \( x \), the displacement of mass \( m_2 \) is \( y \), the elastic coefficients of the spring are \( k_1 \) and \( k_2 \), the damping coefficient of the damping system is \( c \), in which, \( m_1=m_2=1 \text{kg} \), \( k_1=k_2=0.5 \text{N/m} \). In this system, all the frictions are neglect.

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2.1. Mechanism Modeling

Taking the balance point as the coordinate origin, for $m_1$ and $m_2$

$$m_1\ddot{x}+k_1(x-y) = 0$$ (1)

$$k_1(x-y) = m_2\ddot{y} + c\dot{y} + k_2y$$ (2)

By the Laplace transform to get its transfer function as follow

$$\frac{Y(s)}{X(s)} = \frac{k_1}{m_2s^2 + cs + k_1 + k_2} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$ (3)

where $K = \frac{k_1}{k_1 + k_2}$, $\zeta = \frac{c}{2\sqrt{m_2(k_1 + k_2)}}$ and $\omega_n = \sqrt{\frac{k_1 + k_2}{m_2}}$.

2.2. Lagrange Modeling

The Lagrange equation with dissipated energy is

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} + \frac{\partial D}{\partial \dot{q}_i} = Q_i, \quad i = 1, 2, 3, \cdots, n \quad (4)$$

Where $Q_i$ is the external force of the system along the generalized coordinate $q_i$, $q_i$ is the generalized coordinates of the system, $L$ is the Lagrange of the system in generalized coordinates $(q_1, q_2, q_3, \cdots, q_n)$ and generalized speed $(\dot{q}_1, \dot{q}_2, \dot{q}_3, \cdots, \dot{q}_n)$, $T$ is the kinetic energy, $V$ is potential energy, and $D$ is the dissipated energy, where $L = T - V$. For this system, the generalized coordinates are taken as $(x, y)$, the kinetic energy, the potential energy, and the dissipated energy are as follows

$$T = \frac{1}{2} m_1x^2 + \frac{1}{2} m_2y^2, \quad V = \frac{1}{2} k_1(x-y)^2 + \frac{1}{2} k_2y^2, \quad D = \frac{1}{2} c\dot{y}^2$$ (5)

Substitute $T$, $V$, $D$ into (4), and get

$$m_1\ddot{x}+k_1(x-y) = 0$$ (6)

$$k_1(x-y) = m_2\ddot{y} + c\dot{y} + k_2y$$ (7)

By the Laplace transform, the transfer function can be obtained

$$\frac{Y(s)}{X(s)} = \frac{k_1}{m_2s^2 + cs + k_1 + k_2}$$ (8)

The result is consistent with the equation (3).
2.3. System Analysis

According to the Routh-Hurwitz stability criterion, the system stability analysis is carried out, in which the mass and spring coefficient of the mass are positive. It can be obtained that when the damping ratio \( \zeta \) of the system is positive, the system is stable. The damping ratio \( \zeta \) changes with the changing of damped coefficient \( c \), and the performance of the system will also changes as shown in Figure 2.

![Figure 2. Step response with different damping ratios](image)

3. ADRC Controller

The ADRC technology proposed by the researcher Han Jing-qing, a famous scholar in China, is a new control technology that does not depend on an accurate system model [3]. The structure of a traditional auto-disturbance controller is shown in Figure 3. First, according to the characteristics of the controlled object, Tracking Differentiator (TD) arranges a suitable transition process, quickly tracks the input signal, and obtains the generalized differential signal. Then, through Extended State Observer (ESO) real-time estimation of system state information and total disturbance information, and feedforward compensation for disturbance. Finally, the state feedback law is used to transform the nonlinear system into an integral series linear system through Nonlinear States Error Feed-back (NLSEF).

![Figure 3. Classic ADRC controller](image)

By selecting different TD, ESO, NLSEF functions, multiple types of ADRC controller [4] can be designed. In this paper, the second-order linear TD, ESO, NLSEF is used to select the proportional feedback control law. The design algorithm of the three parts is as follows.

Differential tracking and transition process arrangement:

\[
\begin{align*}
\text{fh} &= \text{fhan}(x_1(k) - v(k), x_2(k), r_0, h_0) \\
x_1(k+1) &= x_1(k) + hx_3(k) \\
x_2(k+1) &= x_2(k) + h\text{fh}
\end{align*}
\]

Expansion state observation:

\[
\begin{align*}
e(k) &= z_1(k) - y(k) \\
z_1(k+1) &= z_1(k) + h\left[z_2(k) - \beta_{01}e(k)\right] \\
z_2(k+1) &= z_2(k) + h\left[z_3(k) - \beta_{02}\text{f}_{\text{al}}\left(e, \frac{1}{2}, \delta\right) + bu\right] \\
z_3(k+1) &= z_3(k) - h\beta_{03}\text{f}_{\text{al}}\left(e, \frac{1}{4}, \delta\right)
\end{align*}
\]
Non-linear combination

\[
\begin{align*}
  e_1 &= v_1 - z_1 \\
  e_2 &= v_2 - z_2 \\
  u_0 &= \beta_1 f_a(e_1, a_1, \delta) + \beta_2 f_a(e_2, a_2, \delta)
\end{align*}
\]

Disturbance compensation forms the control amount

\[
u = \frac{u_0 - z_1}{b_0}
\]

In the algorithm, \(r_0, \beta_{01}, \beta_{02}, \beta_{03}, \beta_1, \beta_2, a_1, a_2, \delta\) and \(b_0\) are the parameters of the controller, where \(r_0\) is determined according to the needs of the speed of the transition process and the affordability of the system; The parameters \(\beta_{01}, \beta_{02}, \beta_{03}\) are determined by the sampling step size used by the system [5]. The main parameters to be adjusted in the system are \(\beta_1, \beta_2, a_1, a_2, \delta, b_0\). As shown in Figure 4, the ADRC model is built for this system.

**Figure 4.** ADRC controller for the spring-mass-damping system

3.1. Parameter Tuning of ESO

The expanded state observer is the most important part of the ADRC controller. The expanded state observer has 4 adjustable parameters. These are the nonlinear parameter \(\delta_1\) of the \(f_a\) function and the observer parameters \(\beta_1, \beta_2\) and the compensation coefficient \(b_0\), respectively. In the \(f_a\) function, \(\delta\) is the width of the linear interval of the \(f_a\) function near the origin. If \(\delta\) is too large, most of the ESO works in the linear region, which cannot reflect the advantages of non-smooth feedback and the ability to approximate nonlinear signals will also be much weaker[6]. If \(\delta\) is too small, high-frequency tremor is prone to occur near the origin, so \(\delta\) is generally about \(5h \leq \delta \leq 10h\).

Observer parameters \(\beta_1\) and \(\beta_2\) are the feedback gain of state error feedback, which affects the convergence rate of ESO. Parameters \(\beta_1\) and \(\beta_2\) are determined by the sampling step size used by the system. The same \(\beta_1, \beta_2\) can be used.

3.2. Parameter Setting of NLSEF Control Rate

In the auto disturbance rejection controller, NLSEF has 6 adjustable parameters. The non-linear parameters \(\delta_2, \delta_3\), and compensation coefficients \(b_0, b_1\) and \(\delta\) in the controller gain \(\alpha_1, \alpha_2\) and \(f_a\) functions are the same as those in ESO.

For the controller gains \(\alpha_1\) and \(\alpha_2\), the larger \(\alpha_1\) and \(\alpha_2\), the lower the rise time of the system, but excessive \(\alpha_1\) and \(\alpha_2\) will cause the system to oscillate and even cause the system output to diverge. The value of \(\alpha_1\) and \(\alpha_2\) is generally \(0 < \alpha_1 < 1 < \alpha_2\), and the compensation coefficient \(b_0, b_1\), similar
to the integral gain in PID, the parameter should be adjusted to the middle position of "basin" as much as possible.

4. Simulation Experiment Analysis

ADRC adopts a nonlinear state error feedback strategy, which can significantly improve the efficiency of feedback control. Non-linear state error feedback is based on the principle of "small error, large gain, large error, small gain", appropriate selection of parameters and linear interval for segmentation, and the use of different gain control in different intervals can obtain the effect of rapid adjustment. In order to verify the advanced nature of ADRC control, simulation experiments of PID and ADRC control were conducted respectively.

The PID control algorithm is used to control the system, and the parameters are modified for simulation. Among them, $P = 20.330$, $I = 9.648$, and $D = 10.654$.

ADRC control is a mathematical model that does not need to consider the controlled object, extracts the disturbance information according to the input and output signal information, and feeds back to the input terminal for control after the actual value is calculated. Adopt the method from small to large to adjust, through modifying the parameter debugging, get a set of ideal controller parameters. The parameter selection is $b = b_0 = 0.015$, $\beta_1 = 1500$, $\beta_2 = 80$, $a_1 = 0.6$, $a_2 = 2.3$, $\delta = 0.008$. When the disturbances are not taken into consideration, the simulation results with different damping ratio are as shown in Figure 5.

![Figure 5](image1.png)

**Figure 5.** The response of the system without disturbances for different damping ratios, where the damping ratios are taken as (a) $\zeta = 0$, (b) $\zeta = 0.3$, (c) $\zeta = 0.5$

In order to further illustrate the superiority of the designed ADRC controller, two cases of damping ratio with the same white noise are selected for comparison. The simulation results are as shown in Figure 6.

![Figure 6](image2.png)

**Figure 6.** The response of the system with disturbances for different damping ratios, where the damping ratios are taken as (a) $\zeta = 0$, (b) $\zeta = 0.3$, (c) $\zeta = 0.5$

From Figure 6, it is obvious that the system with a certain white noise cannot be stabilized without control. The simulation results demonstrate that the ADRC controller have a better performance than that of PID controller, and the effectiveness of the ADRC controller is verified.
5. Conclusion
In this paper, mechanism modeling and Lagrange modeling are used to model a typical second-order mass-spring-damping system. And then, the ADRC controller is discussed in detail. In order to verify the effectiveness of the ADRC algorithm, the PID control and ADRC control are used to stabilize the control performance of the system. Finally, the simulation results verify that the ADRC controller designed in this paper has better abilities of disturbance rejection and strong robustness.

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