Quantum fluctuations and vortex-antivortex unbinding in the 2D BCS-BEC crossover

L. Salasnich and G. Bighin

Abstract: Very recently quasi two-dimensional (2D) systems made of attractive fermionic alkali-metal atoms with a widely tunable interaction due to Fano-Feshbach resonances have been realized. In this way it has been achieved the 2D crossover from the Bardeen-Cooper-Schrieffer regime of weakly-interacting Cooper pairs to the Bose-Einstein condensate regime strongly bound dimers. These experiments pave the way to the investigation of 2D strongly-interacting attractive fermions during the Berezinskii-Kosterlitz-Thouless (BKT) transition from a low-temperature superfluid phase characterized by quasi-condensation and algebraic long-range order to a high-temperature normal phase, where vortex proliferation driven by quantum and thermal fluctuations completely destroys superfluidity. In this paper we discuss our preliminary theoretical results on the behavior of the BKT critical temperature across the crossover. Our microscopic calculations are based on functional integration taking into account renormalized Gaussian fluctuations and the crucial 2D effect of vortex-antivortex unbinding.

Keywords: BCS-BEC crossover · Ultracold atoms · Dimensional regularization

1 Introduction

The study of superfluid properties of Fermi systems is one of the most interesting areas of research in the field of ultracold atoms because of direct implications for superconductivity in solid-state materials as well as for nuclear matter and other many-body quantum systems [12]. An extremely important achievement in this field has been the realization of the crossover from the Bardeen-Cooper-Schrieffer (BCS) superfluid phase of loosely bound pairs of fermions to the Bose-Einstein condensate (BEC) of tightly bound composite bosons [3]. The actual experimental realization of the crossover was made possible by the use of Fano-Feshbach resonances, a tool from atomic physics, with no counterpart in solid-state experiments, which allows to change at will the strength and sign of the interparticle interactions [3].

Several exciting research lines are now focused on Fermi superfluids in low spatial dimensions, where quantum fluctuations are strongly enhanced and beyond-mean-field theories are needed to obtain a reliable description of the observed phenomena. While for one-dimensional (1D) bosonic and fermionic systems with contact interaction there are exact analytical solutions [4-6,7,8,9,10,11], for two-dimensional (2D) quantum systems there are no exact solutions. In 2D systems with a continuous symmetry the Mermin-Wagner-Hohenberg theorem precludes spontaneous symmetry breaking and condensation at finite temperature [12]. The Berezinskii-Kosterlitz-Thouless critical temperature marks the transition from a superfluid phase characterized by quasi-condensation and algebraic long-range order to a normal phase, where vortex proliferation completely destroys superfluidity. As opposed to conventional off-diagonal long-range order typical of 3D superfluid systems, algebraic long-range order is driven by strongly enhanced quantum and thermal fluctuations [11,12]. For this reason of particular relevance are studies aimed at understanding the pairing of...
fermions in strongly interacting 2D Fermi gases, both in the homogeneous case and in the presence of an optical lattice. This subject is of great importance for condensed matter physics also in view of the not yet fully understood character of the corresponding mechanisms in high-temperature superconductors [12]. Strongly interacting Fermi gases made of alkali-metal atoms in quasi-2D configurations have been realized only very recently [13][14][15][16] and some evidences of superfluid behavior, such as the condensation of pairs [14] and the algebraic decay of the first-order correlation function [15] have been observed. However, genuine signatures of the superfluid state such as the second-sound mode are still missing.

2 Results at zero temperature

The first theoretical analysis of 2D Fermi superfluid in the full BCS-BEC crossover based on Gaussian corrections to the mean-field equations was carried out very recently by our group both at zero [17][18] (see also [19]) and finite temperature [20]. These 2D theoretical results [19][20] are obtained performing the regularization of zero-point energy which appears in the Gaussian fluctuations (for a comprehensive review in any dimension see [21]). The zero-point energy is due to both fermionic single-particle excitations and bosonic collective excitations, and its regularization (dimensional regularization or convergence-factor regularization) gives remarkable results in the full BCS-BEC crossover and reliable analytical prediction in the BEC regime of composite bosons, which are in extremely good agreement with $^6$Li experimental data of the zero-temperature equation of state in the crossover [13].

In 2014 Makhalov et al. [13] have realized a quasi-2D Fermi system with widely tunable s-wave interactions nearly in a ground state, investigating an ultracold gas of $^6$Li atoms by measuring the pressure $P$ as a function of the scattering length. The experiment [13] covers physically different regimes corresponding to weakly or strongly attractive Fermi gases or a Bose gas of tightly bound pairs of fermions. In Fig. 1 we plot the pressure $P$ in units of the ideal pressure $P_{\text{ideal}}$ as a function of the adimensional gas parameter $a_B n_B^{1/2}$, where $a_B$ is the scattering length of composite bosons (made of two fermions) and $n_B$ is the bosonic density. In the full crossover the figure shows a very good agreement between the experimental data (filled squares with error bars) and our theoretical results (solid curve) based on zero-temperature beyond-mean-field theory with renormalized Gaussian fluctuations [18][19][20].

![Fig. 1](image-url) Scaled pressure $P/(2P_{\text{ideal}})$ of the 2D gas of composite bosons as a function of the bosonic gas parameter $a_B n_B^{1/2}$, where $P_{\text{ideal}} = 2\pi \hbar^2 n_B^2/m_B$ is the pressure of an ideal 2D gas with $m_B = 2m$ the mass of each bosonic particle (made of two fermions with mass $m$), $a_B$ is the s-wave scattering length of bosons, and $n_B = n/2$ is the bosonic 2D density (with $n$ the fermionic density). On the left there is the BEC regime of deeply bound Cooper pairs (forming bosonic molecules) while on the right there is the BCS regime of weakly bound Cooper pairs. The filled squares with error bars are the experimental data of Makhalov et al. [13]. The solid curves are obtained from the zero-temperature beyond-mean-field theory with renormalized Gaussian fluctuations [18][19][20]. Notice that the mean-field theory (dashed line) predicts a completely wrong result: a constant pressure, independent of the scattering length.

3 Preliminary results at finite temperature

As previously stressed, the study of the 2D BCS-BEC crossover is very interesting also for high-$T_c$ superconductivity: the phase diagram of cuprate superconductors can be interpreted in terms of a BCS-BEC crossover as doping is varied and the critical temperature $T_c$ has a wide fluctuation region with pseudo-gap effects not yet fully understood [12][22]. Moreover, it has been recently suggested that iron-based superconductors have composite superconductivity, consisting of strong-coupling BEC in the electron band and weak-coupling BCS-like superconductivity in the hole band [23].

The superfluid transition in two dimensions is of the Berezinskii-Kosterlitz-Thouless (BKT) type, featuring no true long-range coherence and making therefore the observation of the ordered state more subtle. A reliable microscopic analysis of the BKT phase transition must include the quantization of circulation. This quantization is a peculiar consequence of the existence of an underlying compact real field, whose spatial gradient determines the local superfluid velocity of the system. This compact real field, the so-called Nambu-Goldstone field, is the phase angle of the
complex bosonic field which describes, in the case of attractive fermions, strongly-correlated Cooper pairs of fermions with opposite spins \[2,17,18,19,20,21\]. The compactness of the Nambu-Goldstone field implies the presence of quantized vortices and antivortices which regularize the superfluid density of the system as the temperature increases (enhancement of thermal fluctuations) \[2\].

By means of functional integration with renormalized Gaussian fluctuations we have calculated the finite-temperature equation of state in the 2D BCS-BEC crossover \[17,18,19,20,21\]. We have also derived the bare superfluid density \[20\] and then we have renormalized it by using the Kosterlitz’s renormalization group equations \[24\]: the renormalized superfluid density jumps discontinuously from a finite value to zero as the temperature reaches the BKT critical temperature. Above \(T_{BKT}\) there is the unbinding of vortex-antivortex pairs and the proliferation of free vortices \[2,24\].

Our theoretical determination of the critical temperature \(T_{BKT}\) across the whole crossover in reported in Fig. 2 (solid line). The rapid decrease of \(T_{BKT}\) approaching both the BCS and the BEC limit is a consequence of the fermionic single-particle excitations and bosonic collective excitations dominating the superfluid density, respectively, rapidly decreasing the normal density as either limit is approached. A consequence of this interplay is that the critical temperature is higher in the intermediate regime (\(\epsilon_B \sim \epsilon_F\)), where the superfluid density is neither fermion-dominated nor boson-dominated.

The current approach, involving the inclusion of Gaussian fluctuations in the equation of state \[20\] along with a renormalization group analysis \[24\], is able to reproduce the downward trend as the interaction get stronger (i.e. by increasing the binding energy \(\epsilon_B\) of Cooper pairs). The renormalization group analysis on top of a mean-field theory is not sufficient to reproduce the correct trend, as shown by the dashed line in Fig. 2. In other words, as also observed elsewhere \[17,18,19,20,21\], Gaussian fluctuations are required in order to correctly describe the physics of an interacting Fermi gas in the strongly-coupled limit.

Notice that that experimental data of Fig. 2 may be affected by errors larger than the bars. The algebraic decay of the first-order correlation function, presented in Ref. \[15\] as the signature of the superfluid state, could be interpreted in terms of the strong-coupling properties of a normal-state, as suggested in Ref. \[25\]. Moreover, in the experiment \[15\] the fermionic superfluid is not trapped in a strictly 2D configuration.

4 Conclusions

A complete understanding of the effects of quantum and thermal fluctuations on the critical temperature is of paramount importance for ultracold atoms physics but it is also of direct interest for engineering novel high-temperature superconductors. The results discussed here are a step towards a clear description of the behavior of the critical temperature of 2D Fermi superfluids in the BCS-BEC crossover. As a matter of fact, the behavior of the critical temperature of 2D Fermi superfluids both in the intermediate and strong-coupling regimes is still unknown and it is of primary importance to overcome this lack of knowledge, not only for ultracold atomic gases but also for high-T\(_c\) superconductors \[12,22,26\]. More generally, the results and techniques developed for superfluid alkali-metal atoms are of direct interest for strictly related phenomena in quite different physical systems, for instance high-temperature superfluidity in double-bilayer graphene \[27\], BCS-BEC in atomic nuclei \[28\], and color superconductivity in quark matter \[29\].

Acknowledgements This work was partially supported by MIUR through the PRIN Project ”Collective Quantum Phenomena: from Strongly-Correlated Systems to Quantum Simulators”.

References

1. J. Annett, Superconductivity, Superfluids and Condensates (Oxford Univ. Press., Oxford, 2004); A.J. Leggett,
4

Quantum Liquids (Oxford Univ. Press, Oxford, 2006).
2. B. Svistunov, E. Babaev and N. Prokof’ev, Superfluid States of Matter (CRC Press, 2015).
3. W. Zwerger, The BCS-BEC Crossover and the Unitary Fermi Gas, Lecture Notes in Physics 836 (Springer, Berlin, 2012).
4. M. Girardeau, J. Math. Phys. 1, 516 (1960).
5. E.H. Lieb and W. Liniger, Phys. Rev. 130, 1605 (1963).
6. E.H. Lieb, Phys. Rev. 130, 1616 (1963).
7. J.B. McGuire, J. Math. Phys. 5, 622 (1964).
8. M. Gaudin, Phys. Lett. A 24A, 55 (1967).
9. C.N. Yang, Phys. Rev. Lett. 19, 1312 (1967).
10. A. Luther and V. J. Emery, Phys. Rev. Lett. 33, 589 (1974).
11. J. N. Fuchs, A. Recati, and W. Zwerger, Phys. Rev. Lett. 93, 090408 (2004).
12. A. Larkin and A. Varlamov, Theory of Fluctuations in Superconductors (Oxford Univ. Press, Oxford, 2005).
13. V. Makhalov, K. Martiyanov, and A. Turlapov, Phys. Rev. Lett. 112, 045301 (2014).
14. M.G. Ries, et al., Phys. Rev. Lett. 114, 230401 (2015).
15. P.A. Murthy, et al., Phys. Rev. Lett. 115, 010401 (2015).
16. I. Boettcher, et al., Phys. Rev. Lett. 116, 045303 (2016).
17. L. Salasnich, P.A. Marchetti, and F. Toigo, Phys. Rev. A 88, 053612 (2013).
18. L. Salasnich and F. Toigo, Phys. Rev. A 91, 011604(R) (2015).
19. L. He, H. Lu, G. Cao, H. Hu, and X.-J. Liu, Phys. Rev. A 92, 023620 (2015).
20. G. Bighin and L. Salasnich, Phys. Rev. B 93, 014519 (2016).
21. L. Salasnich and F. Toigo, Phys. Rep. 640, 1 (2016).
22. Q. Chen, J. Stajic, S. Tan, and K. Levin, Phys. Rep. 412, 1 (2005).
23. K. Okazaji et al., Sci. Rep. 4, 4109 (2014).
24. D.R. Nelson and J.M. Kosterlitz, Phys. Rev. Lett. 39, 1201 (1977).
25. M. Matsumoto, D. Inotani, and Y. Ohashi, Phys. Rev. A 93, 013619 (2016).
26. D.J. Scalapino, Rev. Mod. Phys. 84, 1383 (2012).
27. A. Perali, D. Neilson, A.R. Hamilton, Phys. Rev. Lett. 110, 146803 (2013).
28. K. Hagino, H. Sagawa, J. Carbonell, and P. Schuck, Phys. Rev. Lett. 99, 022506 (2007).
29. R. Anglani, et al., Rev. Mod. Phys. 86, 509 (2014).