Influence of the shape of the pressureless trapezoidal channel and roughness on the pressure loss of the machine channels of the pumping stations

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Abstract. In this work, we consider the general equation of fluid motion in pressureless channels and the functional dependences of the coefficient of hydraulic friction on the Reynolds number, relative roughness, and the shape of the live section of the channel, as well as the resistance formulas for the simplest channels with respect to the cross-sectional shape (round and infinitely wide rectangular), and then for channels with a more complex cross-sectional shape. To take into account the effect (on the pressure loss) of the channel cross-sectional shape and the presence of a flow with a free surface in it, additional correction factors are introduced (using the hydraulic radius concept). In conclusion, it is concluded that the values of the mentioned correction factors can and should be refined only as a result of the relevant experiments. The work also examined the patterns of hydraulic resistance in engine channels of the correct form of a live section with uniform turbulent fluid motion. Formulas of hydraulic resistance in the machine channels of a simple and complex outline of a live section are given.

1. Introduction
To identify the patterns of hydraulic resistance in engine channels of the correct shape of a living section with uniform turbulent fluid motion, we consider the laws of hydraulic resistance in engine channels of a simple outline of a live section, and then a complex one shows the patterns of hydraulic resistance or determining the flow energy loss for round and infinitely wide rectangular pressure pipes using the logarithmic law of Karman’s velocity distribution. For round pipes, the tangential stresses τ₀ on the wall will be the same along the entire perimeter of the section, and for infinitely wide rectangular pipes, the value of τ₀ on both sides of the pipe can be neglected (figure 1).
Therefore, in both cases, we can assume that $\tau_0 = \tau_{ocr}$, where $\tau_{ocr}$ is the average tangential stress along the entire wetted perimeter. Determining the coefficient of hydraulic friction $\lambda$ from the ratio:

$$\frac{\tau_{ocr}}{\rho} = \lambda \bar{u}^2/8 \quad (1)$$

in which $\bar{u} = v$ is the average flow velocity, and bearing in mind that $y_* = \sqrt{\tau_0 c_p/\rho}$, we obtain

$$\frac{y}{y_*} = 2\sqrt{2/\lambda} \quad \text{or} \quad \frac{\theta}{\theta_*} = \sqrt{8/\lambda} \quad (2)$$

here $\bar{U}_* = \theta_*$ is the dynamic flow rate. Therefore, as a measure of hydraulic resistance, it is sufficient to consider the ratio $y/y_*$ or $\lambda$.

2. Method

Analysis of the operation of non-pressure machine channels of pumping stations in various modes, operating under different hydraulic conditions and different values of $h$ depth of flow, $R$ is a hydraulic radius and $\chi$ is the wetted perimeter of the live section of the stream, taking into account the influence of roughness of the wetted surface, the shape of the living section of the channel, and the effect of the free surface of the flow on the hydraulic resistance (on the coefficient of hydraulic friction $\lambda$) of the machine channels of pumping stations is a method for studying this work.

3. Results and Discussion

Until now, some researchers believed that the patterns expressing hydraulic resistance in pressure and pressureless flows are identical. A.P. Zegjda believed that the question of the nature of the size of the pressureless flow can be solved by replacing the pipe diameter with a hydraulic radius, and, by analogy with the pressure flow, suggested a dependence for $\lambda$ (assuming a pressureless flow)[1–4].

$$\lambda = f(ReD; \frac{k}{R}) \quad (3)$$

here $ReD = \theta D/\theta$ is the Reynolds number; $R = D/4$ is the hydraulic radius.
Figure 2. Dependence \( \lambda_h = f(Re \ h) \)

1 is the Bazin experiments, series № 2, a rectangular channel, the surface of the bottom and walls - smooth concrete, 2 is the same, series № 24, a semicircular channel, the surface of the bottom and walls - smooth concrete, 3 is the same, series № 6, rectangular channel, the surface of the bottom and walls - desks, 4 is the same, series № 26, semicircular channel, the surface of the bottom and walls - boards, 5 is the same, series № 4, rectangular channel, the surface of the bottom and walls - gravel \( d = 0.01 - 0.02 \) m, 6 is the same, series № 27, a semicircular canal, the surface of the bottom and walls - gravel \( d = 0.01 - 0.02 \) m, 7 is the experiments of the author, series №1, a rectangular channel, the surface of the bottom and walls is smoothly rubbed concrete, 8 is the author’s optics, series № 3, trapezoidal channel, the surface of the bottom and walls is smoothly grinded concrete, 9 is the same, series № 8, a rectangular channel, the surface of the bottom and walls are faces \( d = 0.5 - 0.7 \) cm.

It is pertinent to note that the indicated arrangement of the dependency curves \( \lambda_R \) of the Reynolds number will change significantly, and with it, the form of the curves themselves will change, for example, the value \( \lambda \) does not refer to the hydraulic radius \( R \), and the greatest depth \( h \) in the channel, i.e. calculate value \( \lambda_h \) and Reynolds number \( Re \ h = \vartheta / \nu \) (see figure 2).

However, the validity of this approach was not justified and requires additional analysis. Moreover, recent studies have shown [5–8], what's the attitude \( \frac{\vartheta}{\vartheta_0} \) or \( \lambda \) depends not only on \( Re_R \) and the relative roughness \( \Delta / R \) but also on the shape of the live section of the channel \( \Phi \) and has a dependence of the following form:

\[
\lambda = \lambda(Re_R; \Delta / R; \Phi) \tag{4}
\]

here \( Re_R = \vartheta R / \vartheta \) is the Reynolds number; \( \Delta / R \) is the relative roughness; \( \Phi \) is the parameter taking into account the shape of the cross-section of the channel. To justify the dependence (4), we first consider the hydraulic resistance formula (round and rectangular pipes of infinite width) and then proceed to consider machine channels of complex cross-sectional shape (for example, trapezoidal). Considering a pipe of the circular cross-section with smooth walls (figure 1, a), we turn to the expression:

\[
\frac{y}{y_*} = a_{sh} + b \ln \left( y y_* / \nu \right), \quad b = I / \kappa; \tag{5}
\]

(here \( \kappa \) is the Karman constant), which is the velocity distribution equation for a smooth surface. Multiplying both sides of the expression by \( 2PdR \), we integrate the resulting ratio in the range from \( (r_0 \)
- $\delta$) to 0, where $\delta$ is the thickness of the laminar sublayer, $\hat{r}_0$ is the radius of the pipe. The magnitude of the flow in the laminar sublayer can be neglected, and the terms with $\delta$ can be discarded. Then we get the equation for the average flow velocity in a round pipe with smooth walls:

$$\frac{\nu}{u_*} = \frac{\theta}{\theta_*} = a_{sh} - b[1.5 - \ln(\tau_0 \theta_*/\theta)]$$  \hspace{1cm} (6)

In a detailed way, one can obtain the equation for a rectangular pipe of infinite width (figure 1, b).

$$\frac{\nu}{u_*} = \frac{\theta}{\theta_*} = a_{sh} - b[1 - \ln(h \theta_*/\theta)]$$  \hspace{1cm} (7)

Here: $h$ is the half of the height of the stream. Introducing the hydraulic radii $R = \hat{r}_0/2$ and $R = h$ in the last two equations, respectively, we obtain

$$\frac{\nu}{\nu_*} = a_{sh} - b[0.81 - \ln(R \theta_*/\theta)]$$  \hspace{1cm} (8)

and

$$\frac{\nu}{\nu_*} = a_{sh} - b[1 - \ln(R \theta_*/\theta)]$$  \hspace{1cm} (9)

In equations (8) and (9), the quantities $a_{sh}$ and $b$ are determined experimentally. Dependencies (8) and (9) differ in the values of the coefficients in the second term in the first part. If we take the hydraulic radius $R$ as the characteristic size of the channel shape, then it turns out that for the considered cases the expressions of the average velocity are not the same [9–10]. The discrepancy in the mean flow velocities here will be small. When considering more complex cross-sectional shapes (for example, trapezoidal channels), complications arise due to the presence of secondary flows in the corners of the channel. Also, in the case of fluid free movement, the free surface serves as an additional source of friction.

Considering the expression for the average velocity in the machine channel of a trapezoidal shape according to Karman’s law (for cases when the bisectors of the internal angles of a given shape intersect over the living flow section), we see that neglecting the correction terms due to changes in the shear stresses on the wall (on the solid boundary), and apparent tangential stresses on the free surface can lead to an error (figure 3).

![Figure 3](image)

**Figure 3.** Hydraulic resistance in the machine channels of the correct cross-sectional shape during turbulent fluid motion: $\psi_1$ and $\psi_2$ are the internal angles of the channel cross-section; $B_0$ is the width of the base; $h$ is the depth of the stream.

We divide the live section of the channel into zones of infinitely small width $dy$ so that all their parts are at a minimum distance $y$ from the wall [11–15]. The speed at point $P'$ in one of the zones can be expressed as:
\[
\frac{y}{y_*} = a + b \ln \left( \frac{y y_*}{y} \right) - K_f \cdot \frac{y}{y_*} \quad (10)
\]

where \(y\) is the distance to the point \(P'\) normal to the wall; \(y_*\) is the dynamic speed corresponding to the local tangential stress at the base of the normal from the point \(P'\); \(K_f \cdot y / y_*\) correction taking into account the effect of the free surface; \(\bar{u}\) is the average speed in a live section; \(u_*\) is the average dynamic velocity along a solid boundary, depending on the position of the point \(P'\).

The ratio of the local dynamic velocity \(u_*\) to the average can be described as follows;

\[
\frac{u_*}{u} = I + K^* \quad (11)
\]

Further replacing \(u_*\) in dependence (10) by its value from the expression (11);

\[
\frac{u}{y_*} = a + b \ln \frac{y}{y_*} + b \ln \frac{u}{u_*} - K_f \cdot \frac{y}{y_*} \quad (12)
\]

and discarding small quantities in \(\ln u_* / u\), containing \(K_*\), we get;

\[
\frac{u}{y_*} = a + b \ln \frac{y}{y_*} - (K_f - K) \cdot \frac{y}{y_*} \quad (13)
\]

Dependence (13) more accurately describes the velocity distribution in the trapezoidal channel with smooth surfaces[16–22]. If we neglect the flow in the laminar sublayer, then the total flow rate of the fluid through the living section will be expressed;

\[
Q = \theta \cdot \omega = \int_0^b u b(y)dy = \int_0^b u d\omega \quad (14)
\]

here: \(Q\) is the channel flow rate; \(\theta = y / \omega\) is the average flow rate \(d\omega = b(y)dy\) is the live sectional area of the channel. Length \(b(y)\) any zone is expressed by the ratio;

\[
b(y) = \chi - \varphi y \quad (15)
\]

here: \(\chi\) is the wetted perimeter; \(\varphi\) is a function of angles, depending on the position of the point at which their bisectors intersect, and having the following form:

\[
\varphi = ct g \Psi_1 + ct g \Psi_2 + 2 (\cos ec \Psi_1 + \cos es \Psi_2 ) \quad (16)
\]

Live sectional area of the channel \(\omega\) defined as;

\[
\omega = \int_0^\omega d\omega = \int_0^b b(y)dy \quad (17)
\]

Substituting Value \(b(y)\) from dependence (15) into dependence (17) and integrating, we obtain;

\[
\omega = \int_0^h (\chi - \varphi y)dy = \chi h - \varphi h^2 / 2 = h (\chi - \varphi h) / 2 \quad (18)
\]

The average current velocity can be determined from the expression;

\[
\theta = \frac{1}{\omega} \int_0^\omega u \omega = \frac{1}{\omega} \int_0^h u b(y)dy \quad (19)
\]

Substituting the value of \(U\) from dependence (18) into dependence (19) we obtain:
\[ \frac{0_{\gamma_2}}{b_{\gamma_2}} = \frac{1}{\omega_0} \int_0^{\omega_0} [a + b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} - (K_1 - K_3) \frac{0_{\gamma_2}}{0_{\gamma_2}}] d\omega = \frac{1}{\omega_0} \left[ \int_0^{\omega_0} ad\omega - \int_0^{\omega_0} (K_1 - K_3) d\omega + \int_0^{\omega_0} b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} \cdot d\omega \right] \]

\[ d\omega = \frac{1}{\omega_0} \left[ \int_0^{\omega_0} ad\omega - \int_0^{\omega_0} (K_1 - K_3) d\omega + \int_0^{\omega_0} b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} \cdot d\omega + \int_0^{\omega_0} b \ln\omega \right] \quad (20) \]

Having accepted, \( \alpha = \frac{1}{\omega_0} \int_0^{\omega_0} ad\omega, \quad K = \frac{1}{\omega_0} \int_0^{\omega_0} (K_1 - K_3) d\omega \), calculating separately the terms involved in dependence (20), we have

\[ \int_0^{\omega_0} b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} d\omega = \beta \]

\[ \int_0^{\omega_0} b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} d\omega = \frac{1}{\omega_0} \int_0^{\omega_0} b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} d\omega \]

\[ \frac{1}{\omega_0} \int_0^{\omega_0} b \ln\omega d\omega = \frac{1}{\omega_0} \int_0^{\omega_0} b \ln\omega d\omega \]

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Then we get,

\[ \frac{0_{\gamma_2}}{b_{\gamma_2}} = \frac{0_{\gamma_2}}{b_{\gamma_2}} + \int_0^\gamma b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} + b \ln \theta - b - \frac{\phi h^2}{4} = a_{\gamma_2} + b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} + b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} - b - \frac{\phi h^2}{4} \quad (24) \]

If in the logarithmic term of equation (24), h is replaced by the hydraulic radius R, by substituting \( h = h \cdot R/R \), we get:

\[ b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} - \frac{R}{b} = b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} + b \ln \frac{0_{\gamma_2}}{0_{\gamma_2}} \quad (25) \]

Denoting by \( \Phi \) the difference,

\[ \ln \frac{h}{R} = \frac{\phi h^2}{4} = \phi \quad (26) \]

then the expression for the average flow velocity in the machine channel with a trapezoidal shape of a living section with a smooth surface of the bottom and walls when accepted \( \gamma = \theta = \theta \) and \( \gamma_2 = \theta_2 \) will have the form:

\[ \frac{\theta}{\theta_2} = a_{gi} - b[1 - \ln(R\theta/\theta)] - \frac{\phi h^2}{4} - \frac{\phi h^2}{4} - \frac{\phi h^2}{4} \quad (27) \]

If, the bottom and slopes of the machine channel are rough, then the \( a_{sh} \) of the dependence (27) should be replaced by \( a_{sh} \), then it takes the form for average speed:

\[ \frac{\theta}{\theta_2} = a_{sh} - b[1 - \ln(R/\Delta)] - \frac{\phi h^2}{4} - \frac{\phi h^2}{4} \quad (28) \]

If the dependence (27) and (28) is compared with the corresponding equations (8) and (9) for pipes of circular cross-section and infinite width, it can be established that they differ in the presence \( K \cdot \theta/\theta \), and \( \Phi \). These terms reflect the combined effect on the flow energy loss of the presence of a free surface and an uneven distribution of shear stresses along the wetted channel perimeter, depending on the shape of the living section. Dependencies (27,28) allow us to find the magnitude of the error in determining the pressure loss when the terms \( \Phi \) and \( K \theta/\theta \), not taken into account. Obviously, \( \Phi \) and \( K \theta/\theta \) depend on the geometry of the cross-section of the machine channel and will vary from section to section. The values \( a_{gi}, a_{sh} and b \) are determined experimentally.
4. Conclusions
For a pressure flow in a round pipe \((R = D/4)\) and infinitely wide rectangular channels (for \(b \gg h; R = h\)), as well as in machine channels, where the shear stresses are uniformly distributed \((\tau_0)\) along the entire wetted perimeter \((\tau_0 \approx \tau_{oc})\), the geometric interpretation of the hydraulic radius is justified, in other cases (here \(\tau_0 \neq \tau_{oc}\)) it makes no sense.

The non-pressure machine channel of the correct cross-section corresponds to the law of hydraulic resistance, determined by the shape of the live section \(Φ\) and \(K\) taking into account the influence of the free surface of the flow during the pressureless movement of water in the machine channels of pumping stations.

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