Calculating processes of laminar and turbulent heat transfer around the elements of the aircraft

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Abstract. The physical-mathematical model and a numerical method for solving the nonstationary two-dimensional one-fluid (one-temperature) Reynolds equations closed with turbulence models, taking into account the diffusion and thermal conductivity processes for bodies of simple geometric forms were described and verified. The effects of the temperature field inside the streamlined body and at changing its shape under the action of temperature stresses were analysed. Temperature fields and thermal stresses in the simple structural elements have been obtained. The corrections to the value of convective heat flux due to nonuniform surface temperatures, the presence of temperature gradients and the shape change may be significant and have to be considered to increase the accuracy of aerothermodynamic modelling.

1. Introduction
A great interest of researchers is attracted to the problems of high-speed aircraft aerothermodynamics. Commercial software does not give a solution of the required accuracy for the case of hypersonic flows. This is confirmed by numerous works in which the LAURA software package is used for these complex purposes. LAURA is not a commercial US code and is not sold by the US government [1]. Therefore, this problem is new and relevant. It is necessary to know the surface temperature of high-speed aircraft for accurate calculation of its external aerothermodynamics (e.g. when there is a need to take into account chemical reactions). Many computational techniques use the approximation of a constant surface temperature and often greatly underestimate it. This can lead to unaccounted heat fluxes, which are proportional to the temperature gradients of the solid and its vicinity [2-9]. The second important effect is the change in the body shape (in particular, radius of curvature of the body) under the influence of thermal stress. Since the thermal flows depend on the form of the streamlined body, the problem has a self-adjoint character in the general case [10-16].

This paper aims to describe, validate and verify the engineering-physical-mathematical model and a numerical method for solving the nonstationary two-dimensional one-fluid (one-temperature) Reynolds equations closed with turbulence models, taking into account the diffusion and thermal conductivity processes for bodies of simple geometric forms. Another task is to study the influence of thermal effects on the accuracy of aerothermodynamic calculations. The mathematical statement of the
problem and a brief description of the numerical technique are given. For verification and validation of the developed computer codes, two test problems are solved. The second test is based on the estimates of heat fluxes in a blunt cylinder with a streamlined high-speed air flow. Heat flows that are obtained on the basis of the solution of the problem of external aerothermodynamics under the assumption of a constant surface temperature are known.

2. The description of the mathematical model
A mathematical model of the considered processes is based on multicomponent Reynolds equations. To solve these equations by the finite-difference method a coordinate transformation $r = r(\xi, \eta, \zeta)$, $z = z(\xi, \eta, \zeta)$, $\phi = \phi(\xi, \eta, \zeta)$ is introduced. These equations have the following form:

$$\frac{\partial \rho c_i}{\partial t} + \text{Div}(\rho c_i \vec{V}) = -\alpha \frac{\rho c_i u}{r} + \text{Div}(\rho D_i \nabla c_i) + \left( \frac{\partial \rho c_i}{\partial t} \right)_x, \quad \frac{\partial \rho}{\partial t} + \text{Div}(\rho \vec{V}) = -\alpha \frac{\rho u}{r}, \quad (1)$$

where $S_r, S_z$ are the forces [6, 17], arising due to the presence of viscous friction forces, $S_e$ is the volumetric (specific) energy release, $Re$ is the Reynolds number, $Pr$ is the Prandtl number, $u(r, z, t), v(r, z, t)$ is the projection of the velocity vector $\vec{V}(r, z, t)$ on the axis $R$ and $Z$, $e$ is the specific internal energy of gas, $J = \partial(r, z)/\partial(\xi, \eta)$ is the Jacobian of transition from a cylindrical coordinate system $r, z$ to a curvilinear coordinate system $\xi, \eta$, $V_\xi = \xi_r u + \xi_z v, V_\eta = \eta_r u + \eta_z v$ are the contravariant components of the velocity vector $\vec{V}(r, z, t)$ in the curvilinear coordinate system $\xi, \eta$, $\rho, P$ are the density and pressure of gas, $\sum_i q_i \xi, \sum_i q_i \eta$ are the projections of the vector of the flux density of radiant energy $\vec{q}$ on the axis of the curvilinear coordinate system $\xi$ and $\eta$, $\alpha = 0$ corresponds to the flat geometry, and $\alpha = 1$ corresponds to the one-dimensional cylindrical geometry.

The transfer of broadband radiation is considered using a multigroup diffusion approximation, the equations of which have the following form:

$$1 \frac{\partial (J q_i \xi)}{\partial \xi} + \frac{\partial (J q_i \eta)}{\partial \eta} + \chi_i c U_i = 4 \chi_i \sigma T^4, \quad \frac{c \partial U_i}{3 \partial \xi} + \chi_i q_i \xi = 0, \quad \frac{c \partial U_i}{3 \partial \eta} + \chi_i q_i \eta = 0, \quad (2)$$

where $U_i(y, z, t)$ is the bulk density of broadband radiation for $i$–th spectral group, and $\chi_i$ is the spectral absorption coefficient.

The optical parameters $\chi_i(T, \rho)$ were calculated using a computer system ASTEROID [7]. The turbulent coefficients of viscosity $\mu_\Sigma$ and thermal conductivity $\lambda_\Sigma$ were calculated with the help of the $q - \omega$ Coakley model in a curvilinear coordinate system $\xi, \eta$.

The solution to two-dimensional non-stationary equations of viscous one-temperature radiation gas dynamics, written in vector semi-divergence form in a curvilinear coordinate is constructed using the method of splitting into physical processes and directions. These equations are:

$$\frac{\partial \vec{U}}{\partial t} + \frac{\partial \vec{F}}{\partial \xi} + \frac{\partial \vec{G}}{\partial \eta} + \vec{S} = \left[ \frac{1}{\text{Re}} \frac{\partial \vec{F}_v}{\partial \xi} + \frac{\partial \vec{G}_v}{\partial \eta} + \vec{S}_v \right]. \quad (3)$$

The type of vectors included in the system of equations is specified in [6, 17]. This system of equations (the time $t$ is a variable) includes ordinary differential equations of the first order, which can
be solved using the multi-stage Runge-Kutta method (this work applies the three-step variant of this method).

In the first fractional time step $t \in [t, t + \Delta t / 3]$, we use a nonlinear quasimonotone compact-polynomial finite-difference scheme of high order of accuracy, developed in [6, 17]:

$$
\frac{\partial U_{i,j}}{\partial t} + \frac{F_{i+1/2,j} - F_{i-1/2,j}}{\Delta \xi} + \frac{G_{i,j+1/2} - G_{i,j-1/2}}{\Delta \eta} + S_{i,j} = 0. \tag{4}
$$

On the second fractional time step $t \in [t + \Delta t / 3, t + 2\Delta t / 3]$ the “parabolic” (“viscous”) part of the partial differential equation system was solved by the explicit method with adhesion boundary conditions. The first and second spatial derivatives included in this system of equations on the second time step, were found using quasimonotone compact-polynomial finite-difference schemes of high order of accuracy.

At the third fractional time step $t \in [t + 2\Delta t / 3, t + \Delta t]$ the “hard” part of the $\omega - \omega$ model system of equations is calculated using an implicit Rosenbrock method. The time step $\Delta t$ required to integrate this differential scheme is selected taking into account the criterion of Courant – Friedrichs – Levy.

A modified alternating-triangular method using a three-layer iteration scheme is applied for the solution of radiative transfer equations. In this iteration scheme the “time” step is found using the method of conjugate directions [16].

3. Verification and results of numerical simulation

The validation and verification process for the above engineering-physical-mathematical model of heat exchange near the surface of the aircraft can be divided into several stages:

- 2D testing of the developed engineering-physical-mathematical model and a numerical method for solving the Euler equations using bodies of simple geometric forms;
- 3D validation and verification by methods, using unstructured grids for the sample of a complex geometric shape.

For 2D testing, developed with the use of the engineering-physical and mathematical methods of the aircraft, the following set of bodies of simple geometric shapes (separate elements of the HA construction) is used here:

- a wedge interfaced with a plate;
- a cone conjugate to a cylinder;
- a cylinder coupled with a plate;
- a cone blunt in the sphere and conjugated to a cylinder;
- a hypersonic flow around the blunted axisymmetric bodies (in this case, we can use the calculated and experimental data [14]).

In 2D testing, the search for a solution corresponding to the high-temperature heterogeneous gas flow near the aircraft surface may be based on the solution of nonstationary two-dimensional one-fluid (one-temperature) Reynolds equations closed with $\omega - \omega$ turbulence models, taking into account the diffusion and thermal conductivity processes [18-23]. A special case of such a system of equations is the system of Euler equations. The computational domain includes the flow field in the undisturbed flow behind the front of the shock wave, in the wake of the streamlined body. The procedure for the numerical solution of the Euler equations with the help of a nonlinear quasimonotonic compact polynomial difference scheme of higher order of accuracy consists of three stages:

- integration of equations with respect to the time variable $t$ and transition to a new time layer $t + \Delta t$;
- reconstruction of the values of variables on the faces of the calculated cells from the values in the cell centers;

- 2D validation and verification by methods, using unstructured grids for the sample of a complex geometric shape.
• calculation of numerical flows through the faces of computed cells by means of an exact or approximate solution of the problem of disintegration of a discontinuity (or its analogues) between values on two (opposite) sides of each face.

For 2D testing of the numerical method for solving the Euler equations, the following parameters of the air flow running onto a body of simple geometric shape were used:

- Pressure in the flowing stream: \( P = 2060 \) Pa;
- Speed in the flowing stream: \( V = 1860 \) m/s;
- Temperature in the flowing stream: \( T = 223 \) K;
- Mach number in the flowing stream: \( M = 6 \);
- Gas of the flowing stream: Air;
- Height from the surface of the Earth: \( H = 25 \) km;

All gas-dynamic characteristics behind a flat or conical shock wave (the wedge flow past the plate is considered) are determined [2] (for a given Mach number of the unperturbed gas flow \( M_\infty \)) by the angle of the wedge or cone \( \beta \) solution, streamlined by supersonic gas flow, and angle of inclination \( \alpha \) of the front of the shock wave (figure 1).

In the planar case [2], knowing the angle \( \alpha \) (figure 1), we can determine the Mach number \( M_{\infty,n} \), which is calculated from the normal velocity component (to the front of the shock wave). This Mach number \( M_{\infty,n} \) allows finding the values of all gas-dynamical variables behind the front of the shock wave.

In the case of supersonic flow around the cone for 2D testing of the numerical method for solving the Euler equations, one can use the graphical dependence of the slope angle \( \beta \) of the shock wave on the angle of deflection of the supersonic gas flow for different values of the \( M_\infty \) number. It should be also noted here that, since the thickness of the shock wave is very small, the formulas for calculating the gasdynamic parameters behind the plane of the plane-parallel oblique shock \( (P_2/P_\infty, \rho_2/\rho_\infty, w_2/w_\infty \) and e.t.c.) are applicable to the calculation of the parameters behind an axisymmetric shock wave (the spatial effects across the shock front are small).

Figure 1. Scheme of the shockwave front location relative to the wedge or cone in the case of supersonic gas flow at zero angle of attack.
Figure 2. Temperature distribution in flow around an acute wedge.

From the analysis of the graphical dependences shown in figure 2 [17], it follows that the error in the deviation of the inclination angle $\beta$ of the shock wave is a value $\leq 1\%$, and in gas-dynamic quantities it is of the order of $3\%$.

The following problem was to find the distribution of temperature and displacements in the structural element of the aircraft - in a cylinder blunted over the sphere. In solving this problem, the following geometry of the aircraft element (spherical blunting radius of 1.27 cm, cylinder length of 10 cm) was given, and the convective heat flux incident on the surface of the element under consideration was specified.

Therefore, at high velocities and, respectively, large deceleration temperatures, while the blunting radius decreases at the critical point, the values of the convective and radiation fluxes sharply increase (if the inverse heat flux $q_w$ of the body surface temperature $T_w$ is not taken into account). These formulas also demonstrate a noticeable effect on the convective heat flux $q_w$ of the surface temperature $T_w$ of the aircraft element.

To find the temperature distribution and displacements in a spherically blunted cylinder the following geometry of the aircraft element (spherical blunting radius is 1.3 cm) and the convective heat flux distribution were taken from [4]. The velocity of the oncoming stream corresponds to the Mach number $M = 10$. We note that these aerothermophysical values correspond to the hypersonic regime of gas flow.

The calculations of this work (figures 3, 4) allow us to estimate the effect of the blunting radius, the geometric shape, the structural material and the surface temperature distribution on the convective heat flux $q_w$.

It is assumed that the loss of internal energy by an airborne element is realized only through thermal radiation. The element of the aircraft itself is made of tungsten and rigidly embedded. Figures below show the results of calculations based on the above technique for the element of the aircraft under consideration.

Figure 3. The temperature distribution $T$ [K] in a spherically blunted cylinder.
Figures 3, 4 show, that the temperature distribution of the surface \( T_w = 1600 - 2100 \) K, which is streamlined by the external flux of the aircraft element, differs from the constant and significantly exceeds the temperature of the element surface, which was accepted as the boundary condition in the calculation of the aerothermodynamics of the aircraft (this temperature is usually \( \sim 300 \) K). The blunting radius changes by 5%. At the same time, the convective heat flux at the front critical point due to the change in the radius of curvature increases by 3%. However, if the substantial growth of the surface temperature is taken into account, then the change will be at the level of 10% near the critical point and 65-70% along the surface of the cylinder.

4. Conclusion
The engineering-physical-mathematical model and a numerical method have been developed and verified by the acute wedge flow test task. The error in the deviation of the inclination angle of the shock wave was a value \( \leq 1\% \), and in gas-dynamic quantities it was of the order of 3%.

It has been shown that the temperature of the surface of a cylinder, blunted over a sphere and streamlined by a hypersonic air flow, is not constant and substantially exceeds the surface temperature of the element, usually taken as a boundary condition in the calculation of aerothermodynamics of aircraft. The convective heat flux at the front critical point of the elements under consideration may increase by 10-20% due to a significant increase in the surface temperature and a change in the radius of blunting due to thermal deformation. Corrections to the value of convective heat flux in the problems of external aerodynamics must be taken into account. The consideration of the conjugate problem will increase the accuracy of aerothermodynamic modeling.

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