Invited Paper

Curved holographic optical elements and applications for curved see-through displays

Kiseung Bang, Changwon Jang and Byoungho Lee

School of Electrical and Computer Engineering, Seoul National University, Seoul, South Korea

Abstract

Holographic optical elements (HOEs) have been used as important tools for implementing augmented reality (AR) and see-through displays because they are transparent and thin. Moreover, as HOEs usually come in the shape of a thin film, they can be bent, used for coating, or attached to curved surfaces. While they can be used to implement curved AR displays, however, the applications of the curved HOE have not been sufficiently studied. In this paper, an analysis method for curved/bent HOEs using the coupled-wave theory and the numerical ray tracing method is introduced. Using this method, the influence of the surface curvature on the optical characteristics of HOEs, including the aberration and diffraction efficiency, was analyzed. Also presented herein is a method of designing the optimal curvature that can reduce the aberration. Curved HOEs can be applied to see-through displays such as head-mounted displays (HMDs), head-up displays (HUDs), or transparent screens. They can be used to expand the field of view (FOV) and to reduce the form factor. The proposed analysis method provides a useful guideline for designing practical curved see-through displays.

1. Introduction

Holographic optical elements (HOEs) have been widely studied of late for use as novel optical components of display devices. HOE is a volume hologram that is capable of diffracting light in the desired manner [1]. Due to the nature of the volume hologram, HOE has many unique advantages. As it can record arbitrary wavefronts, it can be used as various optical elements, such as lenses, diffusers, and lens arrays. In addition, high efficiency and transparency can be achieved due to the selectivity of the volume grating that diffracts only light with a specific wavelength and incident angle.

Numerous studies have been conducted of late on augmented reality (AR) displays using HOEs [2–10]. A lens array HOE has been proposed to implement a see-through 3D screen for AR [2]. An off-axis lens HOE was implemented to be used for AR near-eye displays [3–5]. HOEs are also used as incouplers and outcouplers for the exit-pupil expansion of waveguide displays [6–8]. Transparent stacked diffusive screens can be used for 3D displays and head-up displays (HUDs) [9,10].

On the other hand, another important advantage of HOE is that it can be made curved or bendable. In the flat panel display market, efforts are also being made to develop curved or bendable displays. Curved displays can give users a more immersive experience and can provide an esthetic product design [11,12]. Some display companies have launched curved TVs in the market [13]. Although they have not been a big success, efforts to implement curved or bendable displays continue for the future display market. For example, in the head-mounted display (HMD), the field of view (FOV) can be enlarged using a curved half mirror, a curved wedge HOE waveguide, or a curved diffuser HOE [14–17]. In HUD, the curved surface of the windshield can operate as an image combiner without any other additional image combiner [18,19]. A curved lenslet array can enlarge the viewing angle of 3D integral imaging displays [20]. For the projection-type panoramic display, a 240° cylindrical diffuser screen can be utilized [21], and a 360° spherical diffuser screen has been used to implement the projection-type sphere display [22].

The concept of the curved or bendable displays using HOE was proposed some years ago in academia [1,23]. It is expected that HOE can be used as a light-modulating component, such as a lens or a screen, instead of emitting...
light from the device itself. Generally, photopolymer, dichromated gelatin, and silver halide are widely used materials for HOEs [1,23–27]. In particular, photopolymer can be attached to a curved surface because it is easy to manufacture as a thin-film type [26,27]. Its optical characteristics or actual applications, however, have not been studied sufficiently. Due to the nonlinear characteristics of the volume hologram, it is not easy to analyze HOE when fabricated in a curved form.

Thus, a more practical study on the implementation of AR displays using curved HOEs was conducted. In particular, the optical characteristics (e.g. optical aberration and diffraction efficiency) of curved HOEs were simulated. Also, some applications of curved HOEs for AR are shown in this paper, and an application for HMDs is demonstrated. This paper thus provides a basis for the use of curved HOEs as an AR display application.

2. Related works

2.1. Curved display systems

For various purposes, display systems include curved surfaces. In HUD, the curved windshield can be used as an optic to avoid placing additional image combiners in the driver’s sight. In industry, Continental Corp. implemented HUD using the windshield as a curved half mirror [18]. The light from the projection part is reflected by the windshield on the driver’s eye. The aberration caused by the curved windshield is compensated for by the aspherical mirrors inside the projection component. As the system uses the windshield as a simple half mirror, the optical design has a lower degree of freedom. For example, the position of the projection component is fixed by the law of reflection, and it is difficult to design a small projection part because the windshield usually has very low optical power and low magnification. On the other hand, Wayray Corp. has demonstrated an HUD system using a curved HOE on the windshield [19]. Compared to the windshield as a half mirror, the deflection angle and optical power of HOE can be controlled according to the recording method. Thus, it has greater design freedom, and the size of the projection component can be reduced.

In HMD, curved eyepieces can be used to increase the FOV. Meta Corp. has released Meta 2, a product that uses a curved half mirror. The curved half mirror images the display source at a far distance, with a wide 90° FOV [14]. As light with a wide diverging angle is virtually imaged, the eyebox is also very wide. As the curved half mirror, however, must be positioned at a 45° angle to the eye, Meta 2 has a large form factor. Guillaumée et al. demonstrated an HMD system with a wide viewing angle and eyebox and a small form factor using a curved HOE and a polarization-selective contact lens [17]. The HOE is recorded to have the wavefront of the microlens array, and the virtual image is projected onto the HOE. The contact lens operates as a small aperture for the polarization of virtual information, and it does not block the polarized light for the world scene. To achieve a larger FOV, the HOE is curved around the user’s pupil. By using the curvature of the HOE, the system can achieve a large 55° FOV.

Curved HOEs are also used in waveguide-type HMDs [15,16]. Travis et al. used a curved HOE as an outcoupler of a curved wedge waveguide. As the curved HOE surrounds the user’s head, a very large 115° FOV was achieved.

In addition, for 3D displays, Kim et al. have demonstrated an integral imaging display with a curved lenslet array [20]. The viewing angle of the 3D integral imaging display could be enlarged by the curvature of the lenslet array. Using a curved diffuser screen, a panoramic display with a 240° FOV and a spherical display with a 360° viewing angle were also proposed [21,22] (Table 1).

2.2. Diffraction theory for HOEs

Diffraction by HOE (i.e. volume grating) had been theoretically derived by several researchers. Kogelnik’s coupled-wave theory has formally described the wavelength and angular selectivities of the volume hologram [28]. The \( k \)-vector closure method (KVCM) used in Kogelnik’s theory, however, goes beyond energy conservation under off-resonance conditions. For this reason, the theory has a large error from the experiment results as it deviates from the resonance condition [29].

The discrepancy was solved through Uchida’s beta-value method (BVM) [30]. As HOE is usually a thin film, the volume grating is spatially limited along the thickness direction. Owing to the duality between the spatial and \( k \)-vector domains, the volume grating vector in the \( k \)-vector domain has a large degree of freedom only along the thickness direction, which is called ‘\( k \)-cloud.’ In BVM, a solution is obtained that can achieve not only momentum conservation but also energy conservation. The validity of BVM was verified through experiments [29].

In addition, the diffraction of the volume grating can be more accurately calculated through rigorous coupled-wave analysis (RCWA) or the dynamic diffraction theory (DDT) [31–33]. When these were compared with BVM, their results were found to be almost identical to those of BVM [29]. As the calculation of BVM is much more intuitive and faster than that of RCWA or DDT, this paper analyzes HOE based on BVM.


### Table 1. Summary of display systems using curved-surface optical elements.

| Author [year]    | Application | Principle       | Imaging elements                                                                 | Advantages                                                | Notes                                      |
|------------------|-------------|-----------------|-----------------------------------------------------------------------------------|-----------------------------------------------------------|--------------------------------------------|
| Continental Corp. [2017] | HUD        | Virtual imaging | Windshield as a curved half mirror HOE on the curved windshield                   | No additional image combiner                             | Large form factor of the projection part   |
| Wayray Corp. [2017]    | HUD        | Virtual imaging | Curved microlens array HOE                                                         | No additional image combiner                             | Moderate form factor of the projection part |
| Guillaumée et al. [2013] | AR HMD    | Dual-aperture   | Curved half mirror                                                                | Small form factor, large FOV, large eyebox               | Need to wear dual-aperture contact lenses  |
| Meta Corp. [2016]       | AR HMD     | Virtual imaging | HOE                                                                               | Large FOV, large eyebox                                  | Large form factor                         |
| Travis et al. [2018]    | AR HMD     | Wedge           | Curved HOE outcoupler                                                             | Very small form factor, very large FOV, switchable exit pupil | Small intrinsic eyebox, need for pupil tracking |
| Kim et al. [2004]       | 3D display  | Integral imaging | Curved lens array                                                                 | Enlarged viewing angle                                   | Not applied to AR                         |
| Simon and Göbel [2002]  | Panoramic display | Projection   | Curved diffuser screen                                                            | Very large FOV                                            | Not applied to AR                         |
| Benko et al. [2008]     | Spherical display | Projection     | Curved diffuser screen                                                            | 360° viewing angle                                        | Not applied to AR                         |

### 2.3. Previous research on curved HOEs

The first concept of the curved HOE was introduced as soon as the concept of HOE was suggested. Close presented a simple example of a mirror-like curved HOE without the astigmatism aberration [1]. Holographic materials such as photopolymer, dichromated gelatin, and silver halide can be used to coat an arbitrary surface. Especially, using the dip coating method, even a holographic material with a curved surface can be coated with uniform thickness [34,35]. Therefore, the fabrication of curved HOEs is not very difficult compared with the fabrication of planar HOEs.

Detailed research on curved HOEs, however, started much later. In the 1960s, for planar HOEs, Meier [36] and Champagne [37] mathematically derived the aberrations up to the third order, and they had been extended to the general expression for all the orders of aberration [38–40]. In 1986, Peng and Frankena studied the aberrations for HOEs with spherical surfaces [41]. Verboven and Lagasse introduced a method of calculating the aberration of HOE with an arbitrary curved surface expressed as a polynomial [42]. Using the pertinent theories, many applications of the curved HOE have been studied. Boker and Davidson derived a method of compensating for the aberration of an on-axis lens HOE by designing the curvature of the HOE [43]. A spherical-surface HOE was designed to operate as a Fourier lens without aberration. As an early application, curved HOEs were used to design HUDs for aircraft pilots [44].

In most of the previous relevant studies, the description of the aberration was presented through the analytic method. Although such method can accurately describe the relevant cases, the formulas are so complex that it is not easy to interpret the results intuitively. Moreover, as each theory assumes specific cases, it has limitations when applied to general and practical design. In this paper, a numerical ray tracing analysis method for curved HOEs is presented. Using such method, some typical cases of curved HOEs are analyzed.

### 3. Method

A numerical method is used to analyze curved HOEs based on the sequential ray tracing method. Assume a curved HOE whose curved surface is defined by a 2D function \( h(x, y) \) as

\[
z = h(x, y).
\]

Here \( x, y, z \) denotes the Cartesian coordinates in space. The probe wave incident on the curved HOE can be expressed as a bundle of rays. Each ray propagates to intersect the curved surface of the HOE. At each intersection point, the diffracted rays are calculated based on the diffraction theory.

In the recording process, the volume grating is formed in the shape of the interference pattern between the reference and signal waves [28,30]. When the period of the volume grating is \( \Lambda \), the local volume grating vector \( \mathbf{R} \) is defined as a vector perpendicular to the fringe planes and with a size of \( 2\pi/\Lambda \). Then, as shown by the dotted vectors in Figure 2(a), volume grating vector \( \mathbf{R} \) satisfies the following equation:

\[
\mathbf{K} = \mathbf{R} - \mathbf{S}.
\]

\( \mathbf{R} \) and \( \mathbf{S} \) refer to the \( k \)-vectors of the reference and signal waves at the local position, respectively.

For each local area, the diffracted ray is calculated as follows. In the BVM theory, the \( k \)-cloud is assumed to be formed in the shape of an extremely long cloud in the thickness direction [29,30]. In this case, the unit vector in the thickness direction is the same as the normal vector of the curved surface. As can be seen in Figure 1, the normal unit vector of the curved surface changes according to local position \( x, y \). Normal vector \( \mathbf{n} \) is expressed
The diffracted ray can be determined through BVM [30]. In this case, as the \( \mathbf{k} \) is no modification of grating vector \( \mathbf{K} \). The normal vector is different. Therefore, when the reference wave is incident, the normal vector changes. The \( k \)-space diagram in this case is shown in Figure 2(a). Therefore, when the reference wave is incident, it is diffracted in the same direction as the signal wave, which points to the Bragg matching condition. Compared with the case of the planar HOE, however, the direction of the \( k \)-cloud is different because the direction of the normal vector is different. Therefore, the direction of the diffracted light is different in the Bragg mismatching condition. In such condition, when a probe beam with \( \mathbf{p} \)-vector \( \mathbf{P} \) is incident, \( k \)-vector \( \mathbf{D} \) of the diffracted ray can be determined through BVM [30]. In this case, as the \( k \)-cloud is elongated along normal vector \( \mathbf{n} \), BVM should be modified as follows:

\[
\mathbf{D}_{\text{transverse}} = \mathbf{P} - (\mathbf{P} \cdot \mathbf{n}) \mathbf{n} - [\mathbf{K} - (\mathbf{K} \cdot \mathbf{n}) \mathbf{n}],
\]

(4)

\[
D_n = \text{sign} (\mathbf{n} \cdot (\mathbf{P} - \mathbf{K})) \| k_p^2 - | \mathbf{D}_{\text{transverse}} |^2 / \mathbf{n} ,
\]

(5)

and

\[
| \mathbf{D} | = | \mathbf{D}_{\text{transverse}} | + D_n | \mathbf{n} | .
\]

(6)

Figure 2. (a) \( k \)-space diagram of a curved-surface HOE (type 1) and (b) a bendable-surface HOE (type 2).

Type 2 is a bendable-surface case. After an HOE is recorded on a flat flexible surface, it can be bent during its use. In this case, it is possible to implement bendable devices that can change the curvature of the display surfaces. Grating vector \( \mathbf{K} \) formed in the recording process is deformed by bending the HOE. When the radius of curvature is much larger than the thickness of the HOE, the stretch effect inside the HOE is negligibly small. Grating vector \( \mathbf{K} \) will rotate by only as much as the normal vector is rotated. The \( k \)-space diagram in this case is shown in Figure 2(b). In this case, Equations (4) and (5) should be modified as follows:

\[
\mathbf{D}_{\text{transverse}} = \mathbf{P} - (\mathbf{P} \cdot \mathbf{n}) \mathbf{n} - [\mathbf{K} - (\mathbf{K} \cdot \mathbf{n}) \mathbf{n} ] ,
\]

(7)

\[
D_n = \text{sign} (\mathbf{n} \cdot (\mathbf{P} - \mathbf{K})) \| k_p^2 - | \mathbf{D}_{\text{transverse}} |^2 / | \mathbf{n} | ,
\]

(8)

where \( \mathbf{K}' \) is the rotated grating vector and \( k_p \) is the length of \( \mathbf{P} \). As the rotated grating vector \( \mathbf{K}' \) differs from the original grating vector \( \mathbf{K} \), the Bragg matching condition is not satisfied even if the reference wave is incident. Also, the direction of the diffracted wave changes from the signal wave. It can be predicted that the larger the rotation angle of the normal vector is, the greater the deviation from the Bragg matching condition will be, and the diffraction efficiency will tend to decrease. If the rotation of the local surface exceeds the bandwidth of angular selectivity, the incident light cannot be diffracted in some areas of the HOE. Furthermore, if the direction of the probe wave is not spatially homogeneous (i.e. the probe wave is not a plane wave), the probe ray may be incident on the local positions in the HOE other than the local position where the reference ray is incident.

The \( k \)-vectors of the diffracted rays obtained through the aforementioned simple calculation can be converted to wavefront \( U \) of the diffracted wave. When \( U \) denotes the wavefront on the plane at the HOE position, the wavefront \( U \) and \( k \)-vectors on the plane satisfy the following
4. Analysis

4.1. Curved-surface HOE

In this section, how the curvature of the HOE affects the optical characteristics of the HOE was analyzed for two specific cases. Figure 3 shows type 1, the curved-surface case. In this model, the HOE is recorded as having been attached to the curved surface or as having been used to coat the curved surface. The reference wave is an off-axis plane wave, and the signal wave is an on-axis spherical wave. Then the HOE operates as an off-axis lens, which is widely used to make transparent eyepieces for the AR display [3–5].

Angle $\theta_r$ of the reference wave was set at 45°, and focal length $f$ of the signal wave was set at 75 mm. It was assumed that the thickness of the holographic material was 24 $\mu$m, and that refractive index modulation $n_1$ was 0.03. These are the same as the specifications of the photopolymer that was used in the experiment. The curved surface was assumed to have had a constant radius of curvature $R$ in the $x$ and $y$ directions, which is a part of a sphere with radius $R$. To determine the effect of the curvature, radius of curvature $R$ was set as a variable. The width of the HOE was 5 $\times$ 5 cm, and all the waves had a 532 nm wavelength.

\[
\frac{dU}{dx} = k_x(x, y)/2\pi \tag{9}
\]

\[
\text{and } \frac{dU}{dy} = k_y(x, y)/2\pi. \tag{10}
\]

4.1.1. Bragg matching condition

Figure 4 shows the simulation results for the planar and curved HOEs when the reference wave is incident. As described in section 2.2, in both the planar and curved HOE cases, the Bragg matching condition is satisfied. It has the highest diffraction efficiency, and the diffracted wave has the same wavefront as the signal wave. Comparing the ray tracing results shown in Figure 4(a,c), the signal wave is reconstructed in the curved-surface HOE as well as the planar HOE, and all the rays are focused at the desired focal point. Figure 4(b,d) are graphs comparing the diffraction efficiencies of the planar and curved-surface HOEs. Although the diffraction efficiency differs slightly depending on the position of the HOE, both the planar and curved HOE have a theoretically high diffraction efficiency of 97% or higher. That is, in the Bragg matching condition, the effect of the curvature on the characteristics of the diffracted wave is very small.

4.1.2. Aberration in the Bragg mismatching condition

When the probe wave is incident at a different angle from the reference wave (i.e., in the Bragg mismatching condition), the focal point of the diffracted rays tends to shift in the horizontal direction, as shown in Figure 5. As the position of the focal point determines the position of the exit pupil in the retinal-projection display, this phenomenon can be utilized to realize a pupil-shifting retinal-projection AR display [5]. Furthermore, the off-axis lens HOE can be used as a Fourier lens, which collects the incident light into the different focal positions according to the incident angle. HOE, however, is known to have a severe aberration in the Bragg mismatching condition [36–40]. The aberration becomes more severe as the deviation gets farther from the Bragg matching angle. The aberration limits the available exit pupil shifting range of retinal-projection displays because the vignetting effect occurs when the spot size exceeds the users’ pupil size [5]. When HOE is used as a Fourier lens, the aberration limits the FOV.

Figure 6 plots each Zernike coefficient of the aberration according to probe wave angle $\theta_p$ along the $x$-axis for both the planar and curved HOEs. The aberration is defined as the difference between the aberrated wavefront and the wavefront in the absence of aberration. Fringe/Zemax indices are used for the Zernike coefficients [45]. Note that every Zernike coefficient is zero at the 45° Bragg matching angle, and the aberration increases as the incident angle deviates from the Bragg matching angle. Plotted as a solid line, the dominant aberrations of the planar HOE are vertical astigmatism, horizontal coma, and defocus aberration. Compared to the planar HOE, in the curved HOE, it will be noticed that
Figure 4. (a) Ray tracing results and (b) diffraction efficiency distribution for the planar HOE. (c) and (d) are the results of the same analysis for the curved-surface HOE with a 25 cm radius of curvature.

The vertical astigmatism and horizontal coma aberrations are lessened while the defocus aberration becomes worse.

The defocus error, however, only changes the focal distance and does not affect the spot size or the quality of the focal point. Figure 8(a,b) show the ray tracing results of the curved HOE. As shown in the figure, the focal plane becomes tilted in the curved HOE. The tendency of the tilt of the focal plane is to have a normal vector parallel to the reference rays. As the vertical astigmatism and horizontal coma aberrations, however, are less than in the planar HOE, the spot size of the focal points tends to get smaller. To utilize the curved off-axis lens HOE in the Bragg mismatching condition, the angle of the focal plane and the spot size should be considered together in designing the curvature.

4.1.3. Design of the curved HOE with reduced aberration

For some applications, the angle of the focal plane does not have to be a specific angle. For example, in HUD, the off-axis lens HOE can be used as an image combiner [19]. In this case, the image source is located at the tilted focal plane, and the light emitted from the image source is reversely diffracted towards the direction of the reference wave. The aberration, however, degrades the resolution of the virtual image. As the aberration is worse at the periphery, it is one of the important factors that limit the FOV of HUD. Therefore, the aberration needs to be eliminated regardless of the shape of the focal plane.

Regarding the aberration tendency, it can be predicted that the optimal curvature for the least aberration can be found. To quantify the total aberration of the curved HOE, the sum of the square of Zernike coefficients is calculated. Here, as the defocus aberration is not the authors’ concern, it is excluded. The Zernike coefficients for 25 incident angles of $5 \times 5$ arrays with $4^\circ$ intervals are all used in the summation. Figure 7 shows the aberration value according to the radius of curvature of the curved HOE model in Figure 3.

The graph shows that the curved HOE has the least aberration when the radius of curvature is 7.92 cm. Note that the optimal radius of curvature is found near the focal length of the 7.5 cm signal wave. This coincides with the result of the previous research on the curved on-axis lens HOE using the Abbe sine condition [43].

Figure 8(a,b) show the ray tracing results of the optimized focal points. The ray tracing results show that the focal plane is tilted at $45^\circ$, which is the angle of the reference wave. As plotted in Figure 8(c), the Zernike coefficient for the horizontal coma aberration is significantly reduced compared to that of the planar HOE in Figure 6. Figure 9(b) shows the positions of the traced rays on the
Figure 5. Ray tracing results in the Bragg mismatching condition for (a) the planar HOE and (b) the curved HOE. Five planar waves are incident at the 37°, 41°, 45°, 49°, and 53° angles. (c) and (d) are the magnified diagrams of the boxed area in (a) and (c), respectively. The + and × marks represent the focal points in the x and y directions, respectively. The focal plane is depicted as a dotted line.

Figure 6. Zernike coefficients according to the x-axis angle change for the planar HOE (solid line) and the curved HOE with a 25 cm radius of curvature (dotted line).

Figure 7. Total aberration according to the radius of curvature. focal plane of the optimized curved HOE. For comparison, the result of the planar HOE is plotted in Figure 9(a). Due to the decrease of the coma aberration, the diffracted rays of the optimized curved HOE are better focused than those of the planar HOE. This optimization result is an example using only spherical curvature. If an arbitrary curvature is considered other than the spherical curvature, the aberration can be further reduced.
Figure 8. (a) Ray tracing result of the optimized curved HOE, and (b) magnified diagram of the box area. The angles of the probe waves are $37^\circ$, $41^\circ$, $45^\circ$, $49^\circ$, and $53^\circ$. (c) Graph of the Zernike coefficients of the diffracted wave according to the incident angle of the probe wave.

Figure 9. Rays projected onto the focal plane for (a) the planar HOE and (b) the optimized curved HOE. Each color represents the ray bundle of each incident angle. The interval of the incident angle is $4^\circ$ in the $x$ and $y$ directions, and the center incident angle is the same as that of the reference wave ($45^\circ$). Focal points with a negative $x$ coordinate are formed with an incident angle larger than $45^\circ$, and vice versa.

Figure 10. Diffraction efficiency distribution in Bragg mismatching conditions (a),(b) for the planar HOE and (c),(d) for the curved HOE with a 25 cm radius of curvature. The angle of the probe wave was set to (a),(c) $41^\circ$ and (b),(d) $49^\circ$.

As a result, the available FOV can be improved in imaging systems like the HUD system. In an actual application to HUD systems, the curvature of the windshield substrate is pre-determined to achieve low air resistance, and it is usually not a spherical surface. As the optical distance from the HOE to the display can be controlled through optical methods, the distance can be treated as a variable to optimize instead of the radius of curvature of the fixed windshield.

4.1.4. Diffraction efficiency in the Bragg mismatching condition

In the Bragg mismatching condition, not only does the aberration increase; the diffraction efficiency also decreases. Figure 10 shows the diffraction efficiency distribution on the planar and curved HOEs at the $41^\circ$ and $49^\circ$ incident angles, respectively. The efficiency decreases as incident angle $\theta_p$ moves away from the Bragg matching condition ($45^\circ$). Moreover, when incident angle $\theta_p$ deviates more than the limit, a region with zero diffraction efficiency appears. If there exists an area with zero efficiency, it is difficult to compensate for it in other ways, which can be a big problem when using the HOE as a display component. In the case of the curved HOE, however, zero efficiency appears at a further deviation compared to the planar HOE. In Figure 10, it will be noticed that zero efficiency does not appear in the curved HOE while the planar HOE has zero efficiency at the same incident angle.

Figure 11 plots the minimum diffraction efficiency within the HOE area according to the probe angle $\theta_p$. If the minimum diffraction efficiency is close to zero, it means that there is an area with zero efficiency. On the contrary, if the minimum diffraction efficiency is close to 100%, there is sufficient efficiency for the whole area. The graph in Figure 11 shows that the more curved the HOE is, the wider the incident angle that can be used with sufficient efficiency.

In a reflective HOE, it is known that the angular selectivity curve becomes wider when the angle between the
Figure 11. Graph showing the minimum diffraction efficiency according to the incident angle of the probe wave. Each curve corresponds to each radius of curvature.

The reference and signal waves becomes smaller [28]. Therefore, as can be seen in Figure 10, zero efficiency appears at the right side of the HOE, whose angular selectivity curve is narrower. Besides, even if the reference and signal waves are fixed, the angular selectivity can vary as the normal vector changes. Figure 3(a) shows the k-space diagram of the curved HOE. Through BVM [30], the degree of mismatching is determined by the distance between $\mathbf{R} - \mathbf{K}$ and the circle along the normal vector $\mathbf{n}$ direction. When the direction of the normal vector $\mathbf{n}$ is parallel to the radius direction around $\mathbf{R} - \mathbf{K}$, the distance becomes the shortest. When the deviation of the probe angle is small, vector $\mathbf{R} - \mathbf{K}$ is located around the k-vector of signal wave $\mathbf{S}$. Therefore, when normal vector $\mathbf{n}$ has a similar direction as the k-vector of signal wave $\mathbf{S}$, the HOE can have high efficiency at a wider incident angle. Thus, the curved HOE has sufficient efficiency in a wider angle range.

4.2. Bendable-surface HOE

The effect of curvature on the bendable-surface HOE (type 2) was also analyzed. Figure 12 shows the model that was analyzed. All the recording conditions, such as the angle of the reference wave and the focal length of the signal wave, are the same as those for the type 1 model. It was assumed, however, that the HOE was recorded on the planar surface and then bent along only the x-axis to have a radius of curvature $R$.

4.2.1. Aberrations of the bendable HOE

Figure 13 shows how the bent HOE diffracts light with a 25 cm radius of curvature. Here, the incident angle of the probe wave is 45°, which is the same as that of the reference wave. In the absence of curvature, all the diffracted rays converge at one point because it is in the Bragg matching condition. When the HOE is bent, however, it will be noticed that the diffracted ray does not focus on one focal point. In this case, the focal points in the x and y directions appear at different distances. In Figure 13(a,b), the red circle represents the x-direction focal point and the blue circle represents the y-direction focal point. This phenomenon occurs when a diffracted wave has a vertical astigmatism aberration.

Figure 14(a) plots the x- and y-direction focal distances, changing radius of curvature $R$. As the radius of curvature becomes smaller, the focal length in the y direction is not changed much whereas the focal length in the x direction (the direction of the curvature) decreases. Figure 14(b) shows this in diopter units. In the case where the curvature is not very large, the inverse of the radius of curvature ($R$) and the reciprocal of the x-direction focal length ($f_x$) have a linear relation.

$$\frac{1}{f_x} = C \cdot \frac{1}{R} + \frac{1}{f_0},$$

where $f_0$ is the focal length of the 75 mm signal wave and C is a unitless constant. The linear approximation results in $C = 1.889$ in this case. This is similar to the focal length formula by a curved mirror, where $C = 2$ in paraxial approximation [46]. Through this, it will be noticed that the bendable HOE has a similar tendency as a curved mirror with the same curvature. The reason that C has a value slightly smaller than 2 is that the angle change of the diffracted wave in the HOE k-space is slightly smaller than the angle change of the reflected wave at the mirror with the same angle of rotation.

Figure 15 shows the aberration of the bent HOE and its Zernike coefficients. As expected, defocus and vertical astigmatism, which affect the x- and y-direction focal lengths, are the major aberrations. Figure 15(c) shows the Zernike coefficient according to the radius of
Figure 13. Ray tracing result of a bent HOE with a 25 cm radius of curvature viewed from the (a) $x$–$z$ and (b) $y$–$z$ planes, respectively. The $+$ and $\times$ marks represent the focal points in the $x$ and $y$ directions, respectively.

The aberrations other than vertical astigmatism and defocus are relatively very weak.

4.2.2. Experiments on the astigmatism of bendable HOEs

The change of the focal distance in the $x$ and $y$ directions can be proven through an experiment. To implement the bendable-surface HOE (type 2), the planar HOE was first recorded on a planar substrate. A 24-$\mu$m-thick Bayfol HX TP star (Covestro) photopolymer was used. Instead of bending the substrate, the HOE was separated from the substrate and was pasted on a curved substrate. The substrate was curved along the $x$ direction, and two samples with 25 and 12 cm radii of curvature, respectively, were fabricated. Using the setup shown in Figure 16, the point spread function (PSF) of the bent HOE was measured at various distances. The intensity distribution of the diffracted wave was observed using a lensless CCD, and the distance was adjusted using a linear stage. Figure 17 shows the PSF of the planar and bent HOEs with 25 and 12 cm radii of curvature, respectively. In the case of the planar HOE, it can be seen that the diffracted wave is focused at the desired distance. The spherical signal wave is well restored in the Bragg matching condition. On the other hand, in the case of the bent HOE, the $x$- and $y$-direction focal distances have different locations. This is consistent with the simulation result. The asterisks in Figure 14 indicate the $x$-direction focal distance and the $y$-direction focal length measured in the experiment.

4.2.3. Diffraction efficiency of the bendable HOE

When the HOE is bent, it assumes the Bragg mismatching condition; as such, the diffraction efficiency is reduced.

Figure 14. (a) Graph of the focal lengths in the $x$ and $y$ directions with the radius of curvature. (b) is drawn in diopter units. The asterisk symbol indicates the values obtained from the experiment.
Figure 15. (a) Phase aberration of a diffracted wave and (b) Zernike coefficients for the curved HOE with a 25 cm radius of curvature. (c) Graph of Zernike coefficients according to the radius of curvature of the curved surface.

Figure 16. Experiment setup for the measurement of the PSF of the bent HOE (type 2).

Figure 17. PSFs measured at focal distances for (a) a planar HOE and (b) a bent HOE with 25 and (c) 12 cm radii of curvature, respectively.

40 cm, a certain area of the HOE will have very low diffraction efficiency.

Qualitatively, zero efficiency occurs when the rotation angle of the HOE’s local normal vector is greater than that permitted by the angular selectivity. Therefore, it can be expected that as the width of the used bendable HOE becomes narrower and the HOE comes to have wider angular selectivity characteristics, the boundary of the available radius of curvature will become smaller.

5. Applications

Presented in this section are some examples of practical applications of curved HOEs. In many applications, using a curved HOE can reduce the form factor of the system or improve the system’s characteristics, such as the FOV or the aberration. The advantages of the curved AR display systems are introduced in the following sections.

5.1. Retinal-projection HMD with a curved HOE

Curved HOEs can be used to implement a retinal-projection HMD [3,5]. Figure 19(a) shows the concept of the retinal-projection display. The HOE is placed in front of the user’s eye, and the wavefront of the off-axis reflective lens is recorded in the HOE. This is identical to the model analyzed in Chapter 4.1. The pencil beams from a laser scanning projector are incident onto the HOE in the
Figure 18. (a) Diffraction efficiency distribution on the bent HOE with a 25 cm radius of curvature and on that with a (b) 12 cm radius of curvature. (c) Minimum diffraction efficiency on the bent HOE according to the radius of curvature.

Figure 19. Concept of the applications of curved HOEs. (a) Retinal-projection AR display. (b) Waveguide-type AR display. (c) HUD for automobile drivers. (d) Panoramic AR display.

same direction as the reference wave, and the HOE collects the beams in the pupil. The beams are projected onto the retina, passing through the eye lens, and the user sees the image generated by the projector. At the same time, the light coming from outside is not affected by the HOE due to the angular selectivity.

In the retinal-projection display, each pencil beam represents each pixel. As the pencil beam is hardly
affected by the aberration, the retinal-projection display can provide an all-in-focus image and can have a wide FOV, without any concern about the aberration. Previous studies have demonstrated a large 55° FOV in the horizontal direction [5]. The FOV of the retinal-projection display is determined by the angular width of the HOE seen from the position of the pupil. To achieve a larger FOV, a wider HOE and shorter eye relief are needed. It is not easy, however, to achieve short eye relief and a wider FOV that achieves user comfort and provides the necessary space for incident light.

If a curved HOE is used instead of a planar HOE, the eyepiece can be designed to surround the user’s face, as shown in Figure 19(a). Using this geometry, a retinal-projection display with a larger FOV can be implemented using the same HOE width. Figure 20 shows a retinal-projection display using a curved HOE. The HOE has a 12 cm radius of curvature only in the horizontal direction. A plane wave was used as the reference wave, and the eye relief was set to 50 mm. Figure 20(b) shows the display result. A large 65° horizontal FOV can be achieved using a curved HOE with a 55 mm lateral width.

On the other hand, the curved HOEs can also improve the exit pupil shifting method for retinal-projection displays. The retinal-projection display forms a very small exit pupil, which causes a small eyebox. To solve this problem, the exit pupil shifting method has been proposed [5]. When the angle of the probe wave deviates slightly from the Bragg matching condition, the position of the exit pupil shifts. A pupil tracking device can be used to shift the exit pupil according to the position of the pupil.

In the Bragg mismatching condition, however, the light is no longer collected at the small focal point (i.e., it is aberrated). If the focal point becomes larger than the pupil, a part of the virtual image cannot be delivered to the user’s retina, which limits the available range of exit pupil shifting. By using the analysis results in section 4.1, the curved HOEs can be used to reduce the aberrations in the Bragg mismatching condition, which allows the further utilization of the exit pupil shift. To summarize, the curved HOEs are advantageous for widening the FOV and increasing the available range of the exit pupil shift.

5.2. Waveguide-type HMD with a curved HOE

Curved HOEs can be applied to the waveguide-type HMDs, which are among the most promising optical systems for AR HMDs [47,48]. Instead of propagation through the free space, the light is transmitted using multiple total internal reflection (TIR) within a thin waveguide. As the medium used as a waveguide is usually 1–2 millimeters thick, the display system can have a very small form factor.

Recently, Travis et al. [15,16] suggested a design for the curved waveguide-type display. The concept of the system is shown in Figure 19(b). The beams from the image source are deflected to satisfy the TIR condition in the waveguide through the HOE in coupler. The beams undergo the same number of TIRs in the waveguide and reach the HOE outcoupler. The outcoupler collects the beams in the user’s pupil. In this display type, the exit pupil expansion method is not used, but a large FOV can be achieved.

For a very large FOV, the width of the outcoupler can be a limiting factor, as in the retinal-projection display. For the efficient design for a large FOV, the outcoupler should be curved and should surround the user’s face. The curved HOE can be selected as the curved outcoupler. As an empty space for the incident beam is no longer necessary, the eye relief can be shorter than that of the retinal-projection display. Therefore, a very large FOV can be achieved with the curved HOE, and the design of Travis et al. showed a large (115°) FOV [15,16].

5.3. HUD using a curved HOE

Using the curved HOE, HUD can be achieved without an additional window [19]. Figure 19(c) shows the concept of the system. The thin HOE film can be put inside the windshield and can operate as an imaging optic. The display unit is imaged by the first imaging optic, such as a curved mirror, and is imaged again by the curved HOE.
to a far distance from the user. At the same time, the light from the real world is not diffracted by the HOE due to the angular selectivity.

As a part of an imaging system, the curved HOE needs to have little aberration. As mentioned in section 4.1, the curved HOE can be optimized to have reduced aberration. For most practical automobiles, the curvature of the windshield is determined by aerodynamics or esthetic design. To maintain the shape of the windshield and to eliminate aberration, the position and angle of the imaged display unit should be chosen carefully. The result shown in section 4.1 indicates that the image of the display unit should be placed on the perpendicular axis of the HOE. The distance between the image of the display unit and the HOE should be the same as the radius of curvature of the windshield. The orientation of the image of the display unit should be perpendicular to the signal wave (i.e. the front sight). The residual aberration needs to be compensated for by the pre-distortion of the first imaging optic.

5.4. AR screen using a curved HOE

In the previous research, a panoramic virtual reality (VR) display was demonstrated [21]. The user is to be at the center of a cylindrical screen, which is projected by several projectors. The user can see the virtual images with a large (240°) FOV. The concept is shown in Figure 19(d). For VR, the screen only has to show the virtual images. If the curved HOE, however, is used as a screen, the system will provide the AR experience. As it needs to operate as a screen, the signal wave in the recording process must be the light scattered by a diffuser film. Due to the angular selectivity, the HOE is transparent for the light from outside, but the light from the projector is scattered on the HOE surface. Only with the scattered images on the screen, however, the user may notice the difference in depth between the screen and the background through motion parallax or accommodation. To address the depth difference, the projection-type multi-view display can be applied [49]. By adding a parallax barrier on the screen, a 3D image at a far distance can be generated instead of a 2D image on the screen. Using a polarization-selective parallax barrier and a quarterwave retarder, the frontal projection method is also applied for the compact system [49].

6. Conclusion

Holographic optical elements (HOEs) have been used as important elements of augmented reality (AR) and see-through displays because they are transparent and thin. Moreover, as HOEs usually come in the shape of a thin film, they can be bent, attached to, or used for coating a curved surface. The curved/bent HOEs were analyzed using the coupled-wave theory and the numerical ray tracing method. Using this method, the aberration and efficiency of the curved/bent HOEs were calculated according to the degree of curvature. Also, how to design the optimal curvature so that the aberration in the Bragg mismatching condition could be reduced was shown. The recent and possible applications of the curved HOEs, such as the head-mounted display (HMD), head-up display (HUD), and transparent screen, were also introduced. Applying the curved HOEs can help expand the field of view (FOV) or reduce the form factor. The proposed analysis method can be utilized in designing curved see-through displays.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Kiseung Bang received his B.S. Electrical Engineering and Computer Science degree from Seoul National University, Seoul, Republic of Korea in 2016. He is currently a Ph.D. student at Seoul National University. His research interests are the holographic, 3D, and near-eye displays.

Changwon Jang received his B.S. and Ph.D. Electrical Engineering and Computer Science degrees from Seoul National University in 2013 and 2018, respectively. He is currently a postdoctoral researcher at Seoul National University. His research interests are the holographic display and near-eye display design.

Byoungho Lee received his Ph.D. from University of California at Berkeley in 1993. Since September 1994, he has been part of the faculty of the School of Electrical Engineering, Seoul National University, where he is currently the head of the department. He received the Jinbojang National Badge of Korea in 2016 and is currently a fellow of SPIE, OSA, and IEEE as well as the president of the Optical Society of Korea.

References

[1] D.H. Close, Opt. Eng. 14 (5), 145408 (1975).
[2] K. Hong, J. Yeom, C. Jang, J. Hong, and B. Lee, Opt. Lett. 39 (1), 127–130 (2014).
[3] A. Maimone, A. Georgiou, and J.S. Kollin, ACM Trans. Graph 36 (4), 85 (2017).
[4] G. Li, D. Lee, Y. Jeong, J. Cho, and B. Lee, Opt. Lett. 41 (11), 2486–2489 (2016).
[5] C. Jang, K. Bang, S. Moon, J. Kim, S. Lee, and B. Lee, ACM Trans. Graph 36 (6), 190 (2017).
[6] H. Mukawa, K. Akutsu, I. Matsumura, S. Nakano, T. Yoshida, M. Kuwahara, and K. Aiki, J. Soc, Inf. Disp. 17 (3), 185–193 (2009).
[7] J.A. Piao, G. Li, M.L. Piao, and N. Kim, J. Opt. Soc. Korea 41 (11), 2486–2489 (2016).
[8] C.Jang, K.Bang, S.Moon, J.Kim, S.Lee, and B.Lee, ACM Trans. Graph 36 (6), 190 (2017).
[9] H. Mukawa, K. Akutsu, I. Matsumura, S. Nakano, T. Yoshida, M. Kuwahara, and K. Aiki, J. Soc, Inf. Disp. 17 (3), 185–193 (2009).
[10] J.A. Piao, G. Li, M.L. Piao, and N. Kim, J. Opt. Soc. Korea 41 (11), 2486–2489 (2016).
[11] H. Mukawa, K. Akutsu, I. Matsumura, S. Nakano, T. Yoshida, M. Kuwahara, and K. Aiki, J. Soc, Inf. Disp. 17 (3), 185–193 (2009).
[12] J.A. Piao, G. Li, M.L. Piao, and N. Kim, J. Opt. Soc. Korea 41 (11), 2486–2489 (2016).
[13] H. Mukawa, K. Akutsu, I. Matsumura, S. Nakano, T. Yoshida, M. Kuwahara, and K. Aiki, J. Soc, Inf. Disp. 17 (3), 185–193 (2009).