MAGNETAR-DRIVEN MAGNETIC TOWER AS A MODEL FOR GAMMA-RAY BURSTS AND ASYMMETRIC SUPERNOVAE

Dmitri A. Uzdensky\(^1\) and Andrew I. MacFadyen\(^2,3\)

Received 2006 September 1; accepted 2007 June 26

ABSTRACT

We consider a newly born millisecond magnetar, focusing on its interaction with the dense stellar plasma in which it is initially embedded. We argue that the confining pressure and inertia of the surrounding plasma act to collimate the magnetar’s Poynting-flux-dominated outflow into tightly beamed jets and increase its magnetic luminosity. We propose this process as an essential ingredient in the magnetar model for gamma-ray-burst and asymmetric supernova central engines. We introduce the “pulsar in a cavity” as an important model problem, representing a magnetized rotating neutron star inside a collapsing star. We describe its essential properties and derive simple estimates for the evolution of the magnetic field and the resulting spin-down power. We find that the infalling stellar mantle confines the magnetosphere, enabling a gradual buildup of the toroidal magnetic field due to continuous twisting. The growing magnetic pressure eventually becomes dominant, resulting in a magnetically driven explosion. The initial phase of the explosion is quasi-isotropic, potentially exposing a sufficient amount of material to \( ^{56}\text{Ni}\)-producing temperatures to cause a bright supernova. However, if significant expansion of the star occurs prior to the explosion, then very little \( ^{56}\text{Ni}\) is produced, and no supernova is expected. In either case, hoop stress subsequently collimates the magnetically dominated outflow, leading to the formation of a magnetic tower. After the star explodes, the decrease in bounding pressure causes the magnetic outflow to become less beamed. However, episodes of late fallback can reform the beamed outflow, which may be responsible for late X-ray flares.

Subject headings: gamma rays: bursts — magnetic fields — pulsars: general — stars: magnetic fields — stars: neutron — supernovae: general

1. INTRODUCTION

In the classical collapsar scenario for long-duration gamma-ray bursts (GRBs), the core of a rotating massive star collapses to form a black hole, whereas the overlying stellar material possesses enough angular momentum to form an accretion disk that persists for at least several seconds, long enough for its jet to break out from the star (Woosley 1993; Paczynski 1998; MacFadyen & Woosley 1999). This accretion disk–black hole system then acts as a central engine for the GRB. The power for the explosion comes both from accretion energy, released via neutrinos and perhaps via a magnetic mechanism (e.g., the magnetic tower mechanism as proposed by Uzdensky & MacFadyen 2006), and from the black hole rotational energy, released via the Blandford & Znajek (1977) mechanism.

In the present paper we investigate an alternative scenario, in which the central object formed as a result of the core collapse is not a black hole, but rather a rapidly rotating (millisecond) magnetar with a large-scale poloidal magnetic field of the order of \(10^{15}\) G. Such a strong magnetic field can be produced, for example, by a turbulent \(\alpha-\delta\) dynamo driven by convection in a proto–neutron star (PNS) subject to neutrino cooling (Duncan & Thompson 1992; Duncan & Thompson 1992; Thompson 1994; Yi & Blackman 1998; Nakamura 1998; Spruit 2002; Wheeler et al. 2000, 2002; Ruderman et al. 2000; Lyutikov & Blandford 2002; Thompson et al. 2004; Metzger et al. 2007). The present paper is also devoted to investigating the millisecond-magnetar scenario, but viewed within the overall context of a collapsing star.

At the most basic level, the main idea is that a GRB (or a supernova) explosion is powered by the magnetic extraction of rotational energy of the newly born, rapidly rotating magnetar. This magnetic luminosity operates alongside the much stronger neutrino cooling, which is the main avenue for releasing the gravitational binding energy of the young, still-contracting neutron star. However, most of the neutrinos escape to infinity without sharing their energy with the stellar-envelope gas (unless their spectrum is strongly modified by coronal processes; see Ramirez-Ruiz & Socrates 2005). Magnetic fields, on the other hand, couple to the gas tightly and this makes them a very efficient explosion agent. Energetically, magnetic GRB models are usually quite plausible. For example, assuming a typical surface magnetic field \(B_s = 10^{15}\) G, a rotation rate \(\Omega = 10^4\) s\(^{-1}\), and a radius \(R_s = 10\) km, it is easy

\(^{1}\) Department of Astrophysical Sciences, and Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas Princeton University, Peyton Hall, Princeton, NJ 08544; uzdensky@astro.princeton.edu.

\(^{2}\) Institute for Advanced Study, Princeton, NJ 08540; aim@ias.edu.

\(^{3}\) Department of Physics, New York University, New York, NY 10003.
to see that the resulting basic energetics and timescales fall just in the right ballpark to make the millisecond magnetar a plausible candidate for a GRB central engine (e.g., Thompson 1994). Indeed, the total rotational energy of a millisecond-period neutron star is $E_{\text{rot}} \approx 5 \times 10^{52}$ ergs, which is more than enough to drive a long-duration GRB. The timescale for the energy extraction can be estimated by dividing this available energy by the total magnetic luminosity (the spin-down power), $L_{\text{magn}}$. The latter can be roughly estimated by the usual pulsar luminosity formula $L_{\text{magn}} \approx B^2 R_5^3 \Omega_5^3 c^3$, which for the above parameters yields $\sim 3 \times 10^{50}$ ergs s$^{-1}$, corresponding to the characteristic timescale of order 100 s. Thus, from the point of view of the overall energetics and timescales, the millisecond-magnetar central engine is just a scaled-up version of the Ostriker & Gunn (1971) model for regular pulsar-powered supernovae (with the magnetic field scaled up by 3 orders of magnitude and the time scaled down by 6 orders of magnitude).

It has to be noted, however, that the plausible overall energetics and timescales are, by themselves, not sufficient for making a good GRB central engine model. This is because there are some extra physical requirements mandated by observations. In particular, to make a successful GRB, the central engine has to be capable of producing an energetic outflow that is (1) ultrarelativistic, (2) highly collimated, and (3) baryon-free. The pioneering works cited above have focused mostly on the energetics and timescales, but not on the mechanisms for producing an outflow that satisfies these requirements (see, however, Wheeler et al. [2000] and Bucciantini et al. [2006] for a discussion of collimation). Also, most of these previous models, with the notable exceptions of Wheeler et al. (2000) and Arons (2003) (see § 2.3), have considered a magnetar in isolation; that is, they have completely ignored the effect of any surrounding stellar gas on shaping the outflow. This may be a good approximation for the accretion-induced collapse of a white dwarf, but it is not appropriate in the collapsing-star scenario.

In contrast, in this paper we stress that the infalling stellar gas is still present during the explosion and needs to be taken into account. Thus, an important new element that distinguishes our model from those previous works is the consideration of the interaction between a newly born magnetar and the stellar plasma in which it is initially embedded. Specifically, we argue that the pressure and inertia (i.e., the ram pressure) of the surrounding stellar gas act as a natural collimator forcing the magnetized outflow into two tightly beamed jets. It also plays a crucial role in magnetic extraction of rotational energy from the magnetar.

In order to illustrate these ideas, we introduce the pulsar-in-a-cavity problem as a basic-physics paradigm for this scenario. We describe this problem in detail in § 2. We first give a general description of the problem and its various versions. Then, in § 2.1, we consider the simplest special case of a rotating force-free magnetosphere inside a fixed rigid cavity. In that section, we first demonstrate that differential rotation of the magnetic field lines is inevitably established inside the cavity, even if the pulsar itself is rotating uniformly; as a result, a strong toroidal magnetic field gradually builds up. We then study the long-term evolution of the magnetic field inside the cavity and show that the magnetic luminosity increases with time. We also show that a massive, non-force-free plasma strip unavoidably arises in the equatorial plane beyond the light cylinder. In § 2.2 we discuss the subtle issue of hoop-stress collimation and argue that external confinement and differential rotation are two important ingredients for collimating relativistic Poynting-flux-dominated outflows. We then consider, in § 2.3, the case of a magnetosphere surrounded by a cavity with a fixed external pressure (instead of a fixed radius).

In § 3 we discuss a specific example relevant to the core collapse of a massive star: a cavity formed behind the stalled bounce-shock at the center of the collapsing star. The radius of the shock stays roughly stationary on the timescale for magnetic fields in the cavity to grow. At the same time, both the ram pressure of the gas falling onto the cavity and the neutrino energy deposition inside it decrease with time. We therefore argue that at some point, a fraction of a second after bounce, the magnetic field will inevitably start to dominate the force balance, leading to a magnetically driven explosion.

In § 4 we further explore some of the astrophysically interesting aspects of our model. Thus, in § 4.1 we discuss the possibility of magnetohydrodynamic (MHD) instabilities (e.g., kink) developing in the twisted magnetosphere and their implications for our model. In § 4.2 we address an important issue of $^{56}$Ni production and argue that the two-phase nature of the explosion in our model is well suited to explain a large amount of $^{56}$Ni inferred from observations. In § 4.3 we briefly discuss the possibility of restarting the GRB engine by the fall-back of the postexplosion material. In § 4.4 we describe an extension to our model: a “magnetar in a tube,” motivated by the fact that the material along the rotation axis does not experience a centrifugal barrier and hence falls onto the PNS faster. In § 4.5 we discuss the implications of our model for pulsar kicks. Finally, in § 4.6 we suggest some directions for future numerical simulations of this problem. We draw our conclusions in § 5.

2. THE PULSAR-IN-A-CAVITY PROBLEM

In order to understand how a millisecond-magnetar central engine operates in the collapsar context, it is first necessary to consider the following basic physics problem: what happens when an axisymmetric pulsar is placed inside a conducting cavity filled with a low-density, infinitely conducting plasma (see Fig. 1)? Specifically, we are interested in a situation where the cavity radius $R_0$ is much larger than the pulsar light-cylinder radius $R_{\text{LC}}$. We call this idealized problem the pulsar-in-a-cavity problem (Uzdensky & MacFadyen 2006), and we propose it as the first essential step in building up a physical understanding of the problem. It is a modification of the famous problem of an axisymmetric rotating magnetic dipole in free space, considered by Goldreich & Julian (1969) as a model for an isolated pulsar’s magnetosphere. In some aspects, it is also similar to the system considered by Kardashev (1970), J. P. Ostriker (1970, unpublished), and Ostriker & Gunn (1971).

We would like to point out that adding a cavity makes the problem, in a sense, less fundamental, since the behavior of the system depends, in general, on the assumed physical properties of the cavity. To make the situation less arbitrary, we shall fix the electromagnetic properties of the cavity by assuming that its walls are perfectly conducting, as is the plasma that fills the cavity. We shall also assume that all the field lines close back to the pulsar inside the cavity, i.e., that there are no field lines connecting the pulsar to the cavity wall. This choice is most natural when the bulk of the magnetic flux has been produced by a dynamo operating inside the neutron star and then emerged through its surface (as is in the case of magnetars), as opposed to a situation where the pulsar field lines connect directly to the wall (i.e., to the outer stellar envelope, as considered, e.g., by Kardashev [1970] and Goldreich et al. [1971]). At the same time, we are still left with a lot of freedom regarding the mechanical properties of the cavity. Thus, we are dealing not with one unique problem, but
In order to gain a more complete understanding of the interaction between the central magnetar and the surrounding stellar material, a full MHD description that includes plasma pressure and inertial effects will eventually be required. Of particular interest would be the confinement of the expanding magnetosphere by the surrounding plasma and the dynamical response of the star to the expanding magnetosphere at its center. The full-MHD approach is especially relevant if there is a strong wind driven off the PNS by neutrinos and/or by the magnetocentrifugal mechanism, as considered by Thompson et al. (2004) and Bucciantini et al. (2006). A useful simplification may come from noting that the main difference between the dense-plasma case and the relativistic force-free case is simply the difference between the Alfvén speed and the speed of light (J. Ostriker 2006, private communication). Then, the MHD case may be treated similarly to the relativistic force-free case, but with the light cylinder replaced by a smaller Alfvén surface.

For simplicity, however, in this paper we restrict ourselves to the force-free case. That is, we assume that the plasma density inside the cavity is so low that electromagnetic forces dominate the dynamics almost everywhere inside the cavity (but outside the neutron star of course). The only exception is the part of the equatorial plane outside the light cylinder, where plasma inertia needs to be taken into account (see below). While not realistic, given the large plasma densities present in the center of a massive star, the force-free description may nonetheless reflect some essential features of the full solution. It is of relevance especially for late phases of the evolution when the magnetic field outside the neutron star has been amplified to large values.

As we have already mentioned, we also assume that the plasma inside the cavity can be accurately represented by an infinitely conducting fluid. We expect this key assumption to be well justified throughout most of the cavity, owing to the very large plasma densities and temperatures. Indeed, the high plasma density ensures that the plasma (including photons) is highly collisional and hence is well described by resistive MHD; this means that the resistivity due to particle-particle or photon-particle collisions dominates over all other nonideal terms in generalized Ohm’s law. On the other hand, because of the very high plasma temperature, the resistivity is actually quite small; i.e., the magnetic Reynolds number is very high. All this makes ideal MHD a good approximation in the environment of a collapsing star (see Uzdensky & MacFadyen [2006] for more discussion). At the same time, we do acknowledge that this assumption may break down in some special regions, in particular, inside the equatorial plasma strip (see below) and at the cavity boundary, where various fluid instabilities may lead to enhanced turbulent energy dissipation. In any case, however, we expect ideal MHD to be much better justified inside a collapsing star than the force-free assumption. For this reason, in this paper we ignore any finite-resistivity effects, leaving them for a future study. In addition, because of the very high density, the plasma inside the cavity is completely optically thick to electromagnetic radiation, so the photons are tightly coupled to the gas. What this means is that there is no radiative (apart from possible neutrino cooling that we ignore in this basic problem) cooling in our system. Therefore, all the energy that is extracted from the pulsar stays inside the cavity, as long as we do not allow the cavity to expand.

Throughout most of this discussion we ignore all numerical factors (e.g., $4\pi$). In addition, we assume that the magnetar rotation rate $\Omega$, stays approximately constant on the timescales under consideration. However, ultimately one will have to consider the effect of decreasing rotation rate as the pulsar slows down.

Finally, we would like to stress that there are important differences between the problem of an isolated pulsar magnetosphere and our pulsar-in-a-cavity problem. In particular, in the isolated pulsar case one usually seeks a steady state (although perhaps employing time-dependent simulations to achieve it). In the pulsar-in-a-cavity case, on the other hand, we do not expect...
a stationary solution; the problem is intrinsically time-dependent and it is the time evolution of the system that is of particular interest. In addition, it is believed that the wind of a normal isolated pulsar crosses the fast magnetosonic surface somewhere far beyond the light cylinder and then reaches the termination shock. This is important because it implies that the inner pulsar magnetosphere is causally disconnected from the outside; in particular, the inner magnetosphere’s structure and the pulsar spin-down power cannot be influenced by the boundary conditions at very large distances. In sharp contrast, our case of a magnetosphere enclosed within a finite-sized cavity is qualitatively different, because we now lack that huge separation of radial scales. In practical terms, this means that we draw a dividing line between the isolated pulsar magnetosphere and the bounded pulsar magnetosphere based on the presence or absence of the fast magnetosonic surface inside the cavity. In particular, in our present study we are interested in the case of a cavity formed inside the stalled supernova shock (see §3). Its radius may be about 100–200 km, i.e., only moderately larger than the light cylinder radius of a millisecond magnetar (about 30 km). Then, the plasma outflow may not have enough range to reach the fast magnetosonic surface. As a result, our bounded pulsar magnetosphere always remains in causal contact with the outer boundary. Correspondingly, the inner structure of the magnetosphere and the pulsar spin-down power are affected by the confining cavity.

2.1. Pulsar in a Fixed Spherical Cavity

We start with our problem 1, in which the walls of the cavity are fixed. For definiteness, we take the cavity to be spherical in shape. The main results obtained in this section should also be approximately valid for the case of an expanding (or contracting) cavity, as long as the expansion (contraction) speed is slow compared with the speed of light.

Let us try to think physically about how the magnetic field will evolve after the pulsar is spun-up instantaneously at $t = 0$. In the Goldreich & Julian (1969) model for an isolated pulsar, the field lines extending beyond the light cylinder bend backward and tend to become open. (Strictly speaking, the field lines actually always close, but very far away, in the so-called boundary zone [Goldreich & Julian 1969].) As long as the cavity boundary (the outer edge of the magnetosphere) lies outside the fast magnetosonic surface of the outflow, there is no feedback of this boundary on the inner magnetosphere. Then, the pulsar continuously spins down, losing its rotational energy and angular momentum to magnetic braking by these effectively open field lines. In our case, on the other hand, such an immediate field-line opening is not possible, since the entire magnetosphere is contained inside the cavity of a finite size. This is one of the most critical differences between the isolated pulsar case and our case.

2.1.1. Development of Differential Rotation

One important point that one needs to take into account is the establishment of differential rotation in the magnetosphere. This is nontrivial, since, by assumption, the magnetar rotates uniformly. However, as we now show, the field lines that extend beyond the pulsar light cylinder nevertheless necessarily undergo differential rotation. As a result, these field lines are continuously twisted and hence toroidal flux is continuously injected into the cavity.

To see how this comes about, let us consider a field line $\Psi$ (Fig. 2) and compare the angular velocities at two points on this line: point $A$, where the field line attaches to the pulsar, and point $B$, where it intersects the equator. Since this field line extends beyond the light cylinder, it cannot remain purely poloidal: a toroidal field has to develop so that the plasma particles can slide backward and out along the line, like beads on a wire. This toroidal field leads to a continuous braking of the star so that there is an outward flow of angular momentum and a Poynting flux of energy along the line. However, the toroidal magnetic field at point $B$ has to be exactly zero because of the assumed reflection symmetry with respect to the mid-plane. Therefore, the plasma can no longer slide toroidally; this means that the toroidal velocity of the field line is equal to that of the plasma at this point. The angular momentum and rotational energy of the pulsar extracted by the magnetic field are partly accumulated and stored in the magnetic form and partly transferred to the equatorial plasma. Thus, the material at point $B$ is continuously torqued by the magnetic field. Then, since the confining wall prevents the material from moving out freely in the radial direction, the toroidal velocity of the plasma becomes closer and closer to the speed of light. However, it can never exceed the speed of light; therefore, the plasma and hence field line, angular velocity at point $B$ is bounded: $\Omega_B \approx c/R_B = \Omega_* R_{LC}/R_B$. On the other hand, the angular velocity at point $A$ is of course just the rotation rate of the pulsar: $\Omega_A = \Omega_*$. This means that the field line experiences differential rotation at a rate $\Delta \Omega = \Omega_A - \Omega_B \geq \Omega_*(1 - R_{LC}/R_B)$. For field lines that cross the equator well outside the light cylinder, $R_B \gg R_{LC}$, we then have $\Delta \Omega \approx \Omega_*$. Thus, differential rotation is established over a timescale of order the light-crossing time across the cavity, $t_0 \equiv R_0/c \gg \Omega_*^{-1}$.

This differential rotation is important because it leads to a continuous toroidal stretching of the field lines and thus to a continuous injection of toroidal magnetic flux (of opposite signs) into the upper and lower hemispheres. Since all this toroidal flux has to be contained within a cavity of fixed size, the toroidal magnetic field at any given point grows, roughly speaking, linearly in...
the radial return current back to (or from) the neutron star (see below).

2.1. Magnetic Field Structure at Late Times

Now let us try to estimate the toroidal field evolution and distribution inside the cavity on long timescales ($t \equiv N_0$, where $N \gg 1$, and $t_0 \equiv R_0/c$ is the light-crossing time across the cavity) and at distances much larger than the light-cylinder radius.

As we discussed above, because of the differential rotation, the bounded magnetosphere cannot be stationary: toroidal magnetic flux is constantly being injected into a finite volume. Hence, the toroidal field strength continuously increases, whereas the poloidal magnetic field does not. The poloidal electric field, $E_{\text{pol}}$, may become much larger than $B_{\text{pol}}$ but in any case cannot exceed the value $B_{\text{pol}}\Omega R_0/c = B_{\text{pol}} R_0/R_{\text{LC}}$. Thus, after several light-crossing times the magnetosphere outside the light cylinder becomes toroidal-field-dominated: $B_0 \gg E_{\text{pol}}, B_{\text{pol}}$.

Next, although the configuration is time-dependent, after many light-crossing times the evolution slows down. Indeed, the poloidal field structure readjusts (e.g., in response to a change in the toroidal field strength) on a timescale of order the fast magnetoionic crossing time across the cavity; for a force-free plasma, this coincides with $t_0 \equiv R_0/c$. Since the toroidal flux grows linearly in time, the relative change in the toroidal field strength over $\Delta t \sim t_0$ becomes small (of order $N^{-1}$) at late times, $t = N_0$, $N \gg 1$. An approximate force-free equilibrium is then established separately in each of the two hemispheres, with poloidal current being approximately constant on poloidal flux surfaces: $I \sim I(\Psi)$. The magnetic field structure in such an equilibrium is governed by the relativistic force-free Grad-Shafranov equation (also known as the pulsar equation). In the limit where the toroidal magnetic field totally dominates the dynamics, this equation reduces to $II' = 0$ so that the poloidal current function becomes independent of $\Psi$; $I(\Psi) = I_0$ = const. This corresponds to the vacuum field produced by a singular line current $I_0$ (which grows linearly in time) flowing along the rotation axis. The toroidal magnetic field is $B_{\text{pol}}(t, R, Z) = I_0(t)/R$, i.e., $B_0 =$ const. on cylinders, and the equilibrium can be described as the balance between the toroidal field tension and pressure. In other words, the $j \times B$ force becomes relatively small inside the cavity because the poloidal current becomes spatially separated from the toroidal magnetic field: it flows out of the pulsar along the axis (in both hemispheres), then as a surface current along the cavity walls, and finally returns to the pulsar along the non-force-free equatorial current sheet. The bulk of the magnetosphere is thus almost current-free. In this regard, the electric-current structure of the cavity is similar to that of the magnetic bubble considered by Lyutikov & Blandford (2002, 2003) in their model for Poynting-flux-dominated GRB outflows (although we apply our model deep inside the collapsing star, that is, on different spatial and temporal scales compared with their model).

Let us now estimate the magnitude of the poloidal line current $I_0(t)$ and hence the characteristic strength of the toroidal field in the cavity. We shall express magnetic quantities characterizing the field in the cavity in terms of the total poloidal magnetic flux that extends beyond the light cylinder, which we shall call $\Psi_0$.

Up to a factor of order unity, this flux can be estimated from the pure dipole magnetic field; i.e.,

$$\Psi_0 \sim \Psi_{\text{dipole}}(R_{\text{LC}}) = B_0 \frac{R_0^3}{R_{\text{LC}}}, \quad (1)$$

This estimate is justified because inside the light cylinder the poloidal field remains close to dipole. Moreover, even in the extreme case of an unbounded, isolated pulsar magnetosphere, in which the field is completely open outside the light cylinder, the poloidal flux crossing the light cylinder differs from the dipole formula only by a small amount (e.g., Contopoulos et al. 1999; Komissarov 2006; McKinney 2006b; Spitkovsky 2006). Thus, this estimate should be quite good in our case as well.

Now, what is the characteristic poloidal magnetic field strength in the cavity at distances $r \sim R_0$? Here the dipole formula ($B_{\text{pol}} \sim r^{-3}$), describing a fully closed nonrotating field, and the split-monopole formula, describing the fully open magnetosphere of an isolated pulsar, differ. In our case, all the field lines are closed—i.e., they intersect the equator within $R_0$—so one might think that the dipole-field estimate should be more applicable. However, as we show below, most of the field lines crossing the light cylinder actually intersect the equator in a narrow strip near the outer wall; therefore, the characteristic poloidal field at distances of order $R_0$ from the center and off the equatorial plane should be estimated as

$$B_{\text{pol}} \sim B_0 \equiv \frac{\Psi_0}{R_0^3}. \quad (2)$$

For $\Psi_0$ given by equation (1), this estimate gives a value $B_{\text{pol}} \sim B_0 (R_0^3/R_{\text{LC}}^3)$, which is by a factor $R_0/R_{\text{LC}}$ larger than a pure dipole field at these distances.

Now let us estimate the poloidal current and the toroidal magnetic field. In general, the poloidal current flowing through a region enclosed by an axisymmetric flux surface $\Psi$ can be calculated by following the shape of a field line corresponding to $\Psi$:

$$I(\Psi, t) = \Delta \Omega t \left[ \int d_{\text{pol}} \frac{1}{\Psi B_{\text{pol}} R^2 (I_{\text{pol}})} \right]^{-1}, \quad (3)$$

where $d_{\text{pol}}$ is the path length along the poloidal field. The main contribution to the integral comes from large distances, $R \sim R_0$, and thus the integral can be estimated as being of order $R_0/\Psi_0$. Then, since $\Delta \Omega \approx \Omega_* = c/R_{\text{LC}}$, the axial poloidal current can be estimated as

$$I_0(t) \sim \Omega_* t \frac{\Psi_0}{R_0} \sim \frac{\Psi_0}{R_{\text{LC}}} \frac{t}{t_0}. \quad (4)$$

Thus, we see that for $t \gg t_0$ the poloidal current becomes much stronger than the typical poloidal current in the unbounded pulsar magnetosphere ($I \sim \Psi_0/R_{\text{LC}}$). Using the estimate (eq. [1]) for $\Psi_0$, we can express $I_0$ as

$$I_0(t) \sim B_0 \frac{R_0^3}{R_{\text{LC}}^3} \frac{t}{t_0}. \quad (5)$$

Correspondingly, the characteristic toroidal magnetic field at distances of order $R_0$ is

$$B_0(R_0) = \frac{I_0}{R_0} \sim B_0 \Omega_* t, \quad (6)$$
which is similar to the estimate presented by Kardashev (1970) for the toroidal field of a pulsar inside an expanding supernova cavity. We see that, after many light-crossing times across the cavity, $B_0(R_0)$ becomes much larger than the toroidal field of an isolated pulsar at these distances \[ B_{\psi}^{\text{isolated}} \sim \Omega_0/(R_0 R_{\text{LC}}) = B_0(R_0/R_{\text{LC}}) = B_0\Omega_0 t_0 \ll B_0\Omega_0 t. \]

Finally, we would like to remark on how to determine the structure of the poloidal field, $\Psi(r, \theta)$. Usually, when studying steady state axisymmetric magnetospheres, one uses an iterative procedure (e.g., Contopoulos et al. 1999). First, one makes a guess for the poloidal current $i(\Psi)$, plugs it into the Grad-Shafranov equation, and solves this equation for $\Psi(r, \theta)$. Then one uses equation (3) to determine the new function $i(\Psi)$ and repeats the steps until the procedure converges. In our case, however, this approach does not appear to be feasible, since to lowest order the Grad-Shafranov equation simply gives $i(\Psi) = I_0 = \text{const.}$ We therefore advocate for an inverted approach where the poloidal flux function is determined from equation (3). How to realize such an approach in practice is not clear. One thing to note, however, is that this calculation should depend on $\Psi(R, Z = 0)$ as a boundary condition, and this has to be determined from considering the redistribution of the poloidal flux across the equatorial mid-plane. This issue is discussed in § 2.1.3.

### 2.1.3. Centrifugal Force in the Equatorial Plane

As we noted above, the magnetosphere outside the pulsar light cylinder cannot be entirely force-free. Because the toroidal magnetic field reverses across the equator (due to the assumed reflection symmetry), the magnetic field tension continuously accelerates the equatorial plasma in the toroidal direction. Correspondingly, this tension force performs mechanical work on the equatorial plasma, so a certain part of the rotational energy extracted from the pulsar by the magnetic field is deposited in the equatorial plane (the rest is stored in the bulk of the cavity as the toroidal magnetic field energy). Since the plasma in the equatorial plane rotates ultrarelativistically, the added energy leads to an increase in the relativistic “mass” of the plasma, $\Delta m \sim t^2$. An important consequence is that this relativistically rotating massive equatorial sheet experiences an outward radial centrifugal force, $F_{\text{cent}}$. This force cannot be balanced by the toroidal magnetic field because the latter is zero at the equator. Consequently, the equatorial plasma moves toward the wall and compresses the poloidal magnetic field, until finally the centrifugal force is balanced by the $j \times B$ force due to the non-force-free part of the toroidal current $j_0$. Thus, the poloidal magnetic flux in the equatorial plane outside the light cylinder is pushed against the wall and is strongly concentrated in a narrow band of ever-decreasing width $d(t) \ll R_0$ near the wall (see Fig. 3). Because of this effect, we can expect nearly all the poloidal flux $\Psi_0$ that extends beyond the light cylinder to cross the equator inside this strip, i.e., at cylindrical radii $R \approx R_0$. At the same time, in the magnetosphere above and below the equatorial plane, the poloidal field lines that emanate from this band have to fan out because they have to fill the cavity volume. Thus, the characteristic poloidal magnetic field in the cavity is of the order $B_0 = \Psi_0/R_0^2$ (see eq. [2]) and is much weaker (by a factor of $d/R_0$) than in the equatorial strip.

Let us assess the centrifugal force quantitatively. The total torque exerted on the massive equatorial strip by the magnetic field is given by $\tau(t) = \int \mathbf{r}(\Psi, t) d\Psi \approx I_0 t_0 \Psi_0$. Since the toroidal velocity of this strip is close to the speed of light, the total work per unit time due to this torque (i.e., the total Poynting flux that arrives at the strip) is $P_{\text{strip}} \approx \tau c/R_0 = B_0 c \Psi_0$. This power goes into accelerating the rotation of plasma in the strip, and some part of it may in principle be dissipated into heat. Since the rotation here is already ultrarelativistic, the result of this acceleration is an increase in the rotation and/or thermal $\gamma$-factors, i.e., of the relativistic mass $m$ of the plasma in the strip: $d/\text{d}t(mc^2) = P_{\text{strip}}$. As a result, the relativistic mass grows with time as

$$m(t)c^2 \sim \Omega_0^2 t^2 \frac{R_{\text{LC}}}{R_0} \frac{\Psi_0^2}{R_0} \sim \left( \frac{t}{t_0} \right)^2 \frac{R_{\text{LC}}}{R_0} \frac{\Psi_0^2}{R_0} \sim \frac{R_{\text{LC}}}{R_0} B_0^2(t)R_0^2;$$

that is, the plasma energy in the equatorial strip always remains small compared with the energy $B_0^2(t)R_0^2$ stored in the toroidal magnetic field at these distances. The centrifugal force acting on the equatorial strip can be estimated as

$$F_{\text{cent}}(t) = \frac{m(t)c^2}{R_0} \sim B_0^2 R_0^2 \Omega_0^2 I^2 \frac{R_{\text{LC}}}{R_0} \sim B_0^2(t)R_0^2 \left( \frac{R_{\text{LC}}}{R_0} \right).$$

We see that this force grows quadratically with time, just as the toroidal field pressure, but always remains small (by a factor of $R_{\text{LC}}/R_0 \ll 1$) compared with the overall horizontal force exerted on the side wall by the toroidal field.

A detailed analysis of the internal structure of the massive equatorial plasma strip, including its vertical structure, lies beyond the scope of this paper. However, we present here a simple estimate for the Lorentz factor due to rotation, $\gamma_{\text{rot}}$, in terms of the strip width $d$ and half-thickness $h$. This estimate is derived under a certain very restrictive set of assumptions and serves for illustration only.

Let us consider the vertical force balance inside the strip in the corotating frame, and let us neglect the contribution from electric force for simplicity. Then the toroidal magnetic field pressure outside the strip has to be balanced by the plasma pressure inside: $P_{\text{pol}} = B_0^2/8\pi$. Next, let us make the assumption that the plasma in the strip is a light relativistic fluid with the adiabatic index 4/3.

---

4 The contribution from the force-free part, $j_0^2(z = 0) = \rho_0 c(z = 0)$ is exactly canceled by the radial electric force, $\rho_0 E_z$, provided that the ideal-MHD condition $\mathbf{v} \times \mathbf{B} = \mathbf{E}$ holds in the equatorial strip.
Then, the comoving energy density is $\rho_{\text{co}} c^2 = 3p_{\text{co}}$. On the other hand, the total plasma energy $mc^2$ inside an annular strip of radius $R_0$, width $d$, and thickness $2h$ can be written in the lab frame as $mc^2 = 4\pi R_0^2 dh \rho_{\text{co}} c^2 \gamma_{\text{rot}}^2$. By combining all these expressions with equation (7) for $mc^2$, we find

$$\gamma_{\text{rot}}^2 \approx \frac{2}{3} \frac{R_0 R_{\text{LC}}}{dh}. \quad (9)$$

On the other hand, it may be possible that a significant amount of plasma accumulates in the equatorial strip or that the plasma there is strongly compressed by the toroidal field pressure. Then, the baryon number density $n_b$ may become so large that the comoving energy density is dominated by the nonrelativistic component, i.e., by the baryon rest mass, $\rho_{\text{co}} c^2 \approx n_b \rho_{\text{co}} m_p c^2 \gg p_{\text{co}} = B_0^2/8\pi$. Since $n_b \approx \gamma_{\text{rot}}^{-1} n_b$, the condition that this is true can be written as

$$\gamma_{\text{rot}}^{-1} \gg \sigma \equiv \frac{B_0^2}{4\pi n_b m_p c^2}. \quad (10)$$

Provided that we are in this regime, the total plasma energy in the strip is dominated by the kinetic energy of the baryons: $mc^2 = 4\pi R_0^2 dh \gamma_{\text{rot}} n_b m_p c^2$. Then, using equation (7), we get

$$\gamma_{\text{rot}} \sim \frac{R_0 R_{\text{LC}}}{dh} \sigma_{\text{strip}}. \quad (11)$$

By substituting this expression into condition (10), we see that the comoving energy density is dominated by the rest mass of the baryons only when

$$\sigma \ll \sqrt{\frac{dh}{R_{\text{LC}} R_0}} \ll 1. \quad (12)$$

Correspondingly, we have

$$\gamma_{\text{rot}} \ll \sqrt{\frac{R_0 R_{\text{LC}}}{dh}}. \quad (13)$$

2.1.4. Magnetic Spin-Down Power of a Pulsar in a Fixed Cavity

Another extremely important point is that the rate at which the magnetic field in a bounded magnetosphere extracts rotational energy from the central rotating conductor actually grows with time. This is because the magnetic torque per unit area is proportional to the toroidal field at the conductor’s surface, and the latter grows linearly with time. Thus, the magnetic power generated by a spinning pulsar inside a cavity increases linearly with time as long as the cavity does not expand (or expands slowly) and the spin rate of the pulsar stays constant. We can estimate the spin-down power as

$$P(t) = I(t) \Omega \cdot \frac{\Psi_0^2}{R_0} \sim \frac{P_{\text{isolated}} ct}{R_0}, \quad (14)$$

where $P_{\text{isolated}} \sim B_0^2 R_0^6 \Omega^4/c^3$ is the spin-down power of an isolated, unbound pulsar. As we see, after many light-crossing times, the power of a pulsar in a cavity greatly exceeds that of a classical isolated pulsar. This is our answer to the apparent paradox raised by Lyutikov (2006).

We thus emphasize that the energy extraction from a magnetar in a cavity can be a runaway process. This is because the twisting of a magnetic field confined by an external boundary results in an increase in the field’s strength at the light cylinder and hence in a growing rate of energy extraction from the magnetar.

This effect can be attributed to a positive feedback that exists between the energy that has been already extracted from the pulsar, and the strength of the agent that extracts the energy (the toroidal magnetic field). Namely, most of the extracted energy is stored in the toroidal magnetic field, and since the volume occupied by this field is kept finite, the toroidal field strength increases with time. Because the magnetosphere remains in a quasi-equilibrium, the toroidal field constantly readjusts everywhere, including within the light cylinder. In other words, because the system is not hyperbolic but elliptic, the inner magnetosphere feels the presence of the outer confining wall. In particular, the toroidal field at the very surface of the pulsar increases linearly with time, and hence so does the spin-down torque exerted by the magnetic field on the pulsar. This picture is similar to what is happening in the combustion chamber of a rocket, for example. In that case, the gas temperature and pressure increase as the chemical energy of the fuel is released in the combustion process. At the same time, the rate at which fuel burning occurs increases with an increase in the ambient temperature. As a result, rapid and efficient burning demands high pressure and hence a strong confining chamber capable of withstanding this pressure. Similarly, in our case of a pulsar placed inside a cavity, the presence of strong cavity walls leads to an increased energy extraction rate from the pulsar.

In a realistic situation, this steady power growth might not last indefinitely. It may be limited, for example, by the development of the kink instability, which would result in the conversion of the toroidal flux to poloidal flux and to partial dissipation of magnetic energy (see § 4.1 for more discussion).

2.2. Hoop-stress Collimation: Contrast with the Isolated Pulsar

The toroidal field generated by the differential rotation exerts a constantly growing pressure on the cavity walls. If we now relax the assumption that the walls are fixed and allow them to move, this pressure will make the cavity inflate. We then want to understand how rapidly such inflation will proceed and whether it will be isotropic or, say, collimated along the axis. We discuss the collimation issue in this subsection.

Generally speaking, since the toroidal field pressure in the lateral direction is partly negated by the field’s tension (the hoop stress), which has no vertical component, one may expect the resulting expansion to be predominantly vertical. However, notice that here we are interested in a situation where the (differential) rotation is relativistic: $\Delta \Omega R_0 / \Omega R_0 \gg c$. On the other hand, Lynden-Bell’s (1996) magnetic tower model, for example, was developed for the nonrelativistic regime. It is well known that hoop-stress collimation is not a trivial issue in the relativistic case. Thus, it is not immediately obvious that the hoop-stress collimation mechanism can be applied to the pulsar-in-a-cavity scenario considered in this paper. We therefore would like to discuss this issue in some detail here.

At first, one might think that there should be no problem collimating the outflow: the magnetic field is predominantly toroidal even without differential rotation. And it is the toroidal field’s hoop stress that is usually credited for collimating astrophysical jets. However, as is well known, hoop-stress collimation does not work as well when applied to ultrarelativistic magnetically dominated outflows, as it does in the nonrelativistic case. The quintessential example of this lack of collimation is the isolated-pulsar wind inside the termination shock. The basic reason for
this difficulty is the decollimating force due to the poloidal electric field, \(E_{\text{pol}}\). Indeed, in the case of an unbounded relativistic, uniformly rotating, force-free magnetosphere (e.g., an isolated aligned pulsar magnetosphere) in a steady state, the poloidal electric and toroidal magnetic fields have to be nearly equal in strength at large distances from the central axis (Goldreich & Julian 1969). It is important to note that this balance can be realized in an uncollimated, quasi-spherical poloidal magnetic field configuration; an excellent example of this is Michel’s (1973) split-monopole solution. A rough argument explaining this lack of hoop-stress collimation in the relativistic-rotation case goes as follows. Let us consider an uncollimated field configuration; the poloidal magnetic field is open outside the light cylinder and has a split-monopole geometry, i.e., drops off with distance as \(r^{-2}\). In a steady state, the poloidal electric field is \(E_{\text{pol}} = B_{\text{pol}} R / R_{\text{LC}}\), where \(R\) is the cylindrical radius. It therefore drops off along radial rays as \(r^{-1}\). But the toroidal magnetic field also drops off as \(r^{-1}\). Moreover, at the light cylinder, \(E_{\text{pol}}\) and \(B_{\text{pol}}\) are comparable: \(E_{\text{pol}} \sim B_{\text{pol}}\). Since outside the light cylinder they both decrease as the same power of \(r\), they remain comparable to each other (both being much larger than \(B_{\text{pol}}\) at large distances. Moreover, as Goldreich & Julian (1969) showed, \(E_{\text{pol}}\) and \(B_{\text{pol}}\) even become equal asymptotically as \(r \to \infty\). The bottom line is that a quasi-spherical relativistic force-free equilibrium can be established as a balance between the collimating pinch force (the sum of the toroidal magnetic field pressure and its tension) and the opposing electric force. Hoop-stress collimation is suppressed as a result of this balance.

Now, in the case of a rotating magnetosphere enclosed inside a rigid cavity of a fixed radius \(R_0 > R_{\text{LC}}\), the situation is different and hoop-stress collimation can in fact work. Indeed, as we showed above, after many light-crossing times \((t \gg R_0/c)\), the toroidal magnetic field filling the cavity becomes stronger than both \(B_{\text{pol}}\) and \(E_{\text{pol}}\), in contrast to the isolated pulsar case. Moreover, this toroidal field is distributed nonuniformly; it is basically inversely proportional to the cylindrical radius. Correspondingly, the stress exerted by the toroidal magnetic field on the cavity walls is also nonuniform: the magnetic pressure pushing vertically against the top and bottom walls is much higher than the lateral magnetic pressure acting on the side walls. Therefore, if we now allow the cavity to expand under this pressure, we expect any subsequent expansion to be mostly vertical (see Fig. 4), at least as long as the expansion velocity is slow compared with the speed of light. Then we effectively find ourselves in a situation similar to the nonrelativistic magnetic tower proposed by Lynden-Bell (1996). We therefore envision that the eventual, long-term result of this process will be the creation of a pair of oppositely directed magnetic towers (Uzdensky & MacFadyen 2006). The interaction of the expanding towers with the surrounding stellar envelope aids in their confinement, similarly to jet collimation seen in hydrodynamical simulations of the collapsar model (MacFadyen & Woosley 1999; Aloy et al. 2000; MacFadyen et al. 2001; Zhang et al. 2003). In the scenario considered in the present paper, these towers are driven not by a differentially rotating disk, but by a rapidly rotating magnetar. This suggests that considering the magnetosphere of a pulsar inside a cylindrical, as opposed to spherical, cavity may represent yet another interesting and important problem for future study (see § 4.4).

An important element in the above discussion is the fact that the electric field is small compared with the toroidal magnetic field. This is directly related to the fact that the toroidal field is generated not as a part of an outgoing large-scale electromagnetic wave driven by the pulsar rotation, but as a result of differential rotation. This observation points to the important role played by differential rotation (as opposed to uniform relativistic rotation) in collimating relativistic force-free outflows.

2.3. Pulsar Magnetosphere Confined by a Constant External Pressure

Let us now consider the case when the pulsar magnetosphere is confined by some fixed and uniform external gas pressure, \(P_{\text{ext}}\), instead of a cavity of fixed radius \(R_0\). We are interested in this particular setup because it is closest to that considered by Lynden-Bell in his original magnetic tower paper (Lynden-Bell 1996), and we here want to compare his nonrelativistic disk model with a pulsar in a similar setting.

Like Lynden-Bell, let us assume that the external pressure is weak compared with the magnetic field pressure \(B^2 / 8\pi\) in the immediate vicinity of the rotating conductor. Moreover, because we are interested in exploring relativistic effects, we want our pulsar magnetosphere to be able to expand well beyond the light cylinder. Therefore, we shall also assume that the external pressure is small compared with the magnetic pressure of a pure dipole field at the light cylinder: \(8\pi P_{\text{ext}} \ll B_{\text{dipole}}^2 (R_{\text{LC}}) \sim B_0^2 (R_{\text{LC}}) \ll B_0^2\).

Let us imagine, as is frequently done in time-dependent pulsar magnetosphere studies (e.g., Komissarov 2006; McKinney 2006b; Spitkovsky 2006), that we start with a nonrotating star with a dipole field and then spin it up suddenly at \(t = 0\). The initial evolution of the magnetic field is then similar to that of an isolated pulsar: the field lines that extend beyond the light cylinder start to wind up and expand radially at the speed of light; i.e., \(R_0 \sim ct\). This stage of uninhibited quasi-spherical expansion proceeds until the magnetic field pressure at the outer edge of the expanding magnetosphere becomes as small as the external gas pressure. In order to estimate when this happens, we need to evaluate the toroidal magnetic field pressure at \(R = R_0(t)\). The toroidal field
changes with time because of two opposing factors: continuing injection of the toroidal magnetic flux, \( \chi(t) \sim \Psi_0 \Omega, t = \Psi_0 R_0(t)/R_{LC} \), and the increasing volume of the cavity. The net result is that the toroidal field drops off according to

\[
B_0[R_0(t), t] \sim \frac{\chi(t)}{R_0^2(t)} \sim \frac{\Psi_0}{R_{LC} R_0(t)} \sim \frac{\Psi_0}{R_{LC} c t},
\]

(15)

Another way to obtain this estimate is to note that the main result of this free expansion is the establishment of the stationary isolated-pulsar magnetosphere inside the radius \( R_0(t) \). The toroidal magnetic field in an isolated pulsar magnetosphere scales as \( B_0(r) \sim \Psi_0/R_{LC} c r \) (Goldreich & Julian 1969), which is equivalent to the above estimate.

Eventually, the pressure of the toroidal magnetic field drops to a level where it becomes equal to the external gas pressure (there is an equilibrium value, \( R_{eq} \)) and is of order \( c \). These two outward forces are opposed by the accretion ram pressure, \( I_0 \sim R_{eq} \) \( c \) \( \psi_0 \), corresponding to the cavity radius reaching an equilibrium value,

\[
R_{eq} = c t_{eq} = \frac{\psi_0}{\sqrt{8 \pi P_{ext} R_{LC}}},
\]

(16)

After this, the expansion continues, but changes its character: the lateral expansion slows down, and the expansion becomes mostly vertical. Eventually, at \( t \gg t_{eq} \), a magnetic tower forms, similar to Lynden-Bell’s (1996) tower. One important difference is that the radius of the tower is much larger than \( R_{LC} \): a proper analysis requires relativistic treatment, so Lynden-Bell’s nonrelativistic theory is not directly applicable. In particular, we expect the vertical expansion of the tower to be relativistic. This can be seen from the following argument. As the tower grows, its radius stays roughly constant, of order \( R_{eq} \), whereas its height increases linearly with time, with the velocity \( V_{top} \). The continuously injected toroidal flux goes into filling the expanding volume of the tower with toroidal magnetic field, so that, roughly speaking,

\[
\chi = \Psi_0 \Omega, t \sim \frac{B_0 V_{top} R_{eq}}{\psi_0}.
\]

Assuming \( B_0 \sim (8 \pi P_{ext})^{1/2} \), we therefore arrive at the estimate

\[
V_{top} \sim c.
\]

(18)

This result can be understood naturally by noting that the problem has no mass or density parameter, so there is no characteristic velocity scale other than the speed of light \( c \) (scales like \( \Omega R_{eq} \) are even larger than \( c \)).

The toroidal magnetic field stays roughly constant during this stage, so the poloidal current flowing through the tower is also constant and is of order

\[
I_{0,eq} \sim B_0 V_{top} R_{eq} \sim \frac{\psi_0}{R_{LC}},
\]

(19)

the same as the poloidal current in the isolated pulsar case. Correspondingly, the magnetic luminosity (i.e., the spin-down power of the pulsar) stays at a constant level of order \( P_{isolated} \). However, unlike the isolated pulsar case, this luminosity is not quasi-spherical, but is channeled predominantly in the vertical direction.

Provided that the expansion of the tower is submagnetosonic, an approximate relativistic force-free equilibrium is established inside the tower (at least away from the top lid of the tower). As in the fixed-cavity case, the work done by the toroidal field’s magnetic tension on the equatorial current sheet goes into accelerating the equatorial plasma to ultrarelativistic velocities. The relativistic mass of this plasma and hence also the radial centrifugal force grow linearly with time, as does the overall magnetic pressure force on the outer wall (because of the steadily increasing height of the tower).

3. MAGNETAR INSIDE A COLLAPSING STAR: AN OUTLINE OF THE GENERAL SCENARIO

Previous studies of core-collapse supernovae (SNe) have shown that, when the core of a massive star collapses into a PNS, a bounce shock is launched back into the star (see the reviews by, e.g., Bethe & Wilson 1985; Woosley & Weaver 1986; Bethe 1990). However, as was also shown in these studies, the shock quickly stalls at a radius of about 200 km. The explosion then enters a relatively long (~1 s) quasi-stationary phase (see Fig. 5). During this phase, accreting material constantly moves through the shock and gets heated to very high temperatures. The shock looks stationary in the Eulerian frame and the shock jump condition can be viewed as a balance between the ram pressure of the infalling material, which tends to quench the shock, and the thermal pressure of the postshocked gas, which is supported mostly by the continuous heating due to neutrino deposition in the dense plasma behind the shock. Gradually, both the neutrino luminosity and accretion rate decline with time. Eventually, one of two things has to happen as an outcome of the competition between neutrinos and accretion. If neutrinos win, the shock engulfs the entire star and one gets a successful SN explosion. If they lose, the shock dies and the PNS gains mass beyond the critical mass and collapses into a black hole, which then subsequently swallows the rest of the star, without a SN.

In our model, we add a third dynamical component: the magnetic field. The magnetic force is pushing out, helping the explosion, as is the thermal pressure of the neutrino-heated gas. These two outward forces are opposed by the accretion ram pressure. Our main idea is that, generally speaking, the two outward forces evolve differently with time, and thus the explosion may be a two-stage process. In particular, we suggest that the magnetic pressure force is unimportant during the stalled-shock phase that lasts a few hundreds of milliseconds. However, we note that during this time the magnetar makes several hundred revolutions, resulting in a great amplification of the toroidal magnetic

---

**Fig. 5.—** Stalled-shock phase of a core-collapse explosion.
conditions for a GRB. Note that this jet is driven by the magnetar-level (i.e., $10^{15}$ G) field and is therefore stronger and faster than the LeBlanc & Wilson (1970) jet that may have been launched a few seconds earlier, during the core-collapse process (Wheeler et al. 2000).

4. DISCUSSION

4.1. Effect of MHD Instabilities

The physical picture presented in this paper, with its smooth coherent magnetic structure, is of course an idealization necessary for obtaining a basic physical insight into the system’s dynamics and for getting the main ideas across in the clearest possible way. The actual magnetic field is likely to be different from such a simple system of nested axisymmetric flux surfaces. Instead, it may consist of many loops of different sizes and orientations. It may thus have a complex substructure on smaller scales, both temporal and spatial. This substructure may arise naturally from the beginning, especially if the protomagnetar’s magnetic field is produced by a turbulent dynamo. On the other hand, it may also result from a nonlinear evolution of various MHD instabilities that may develop in the system. The effect of MHD instabilities in our highly twisted magnetosphere is one of the greatest uncertainties in our model. This section is devoted to the discussion of two such instabilities: kink and Rayleigh-Taylor.

4.1.1. Kink Instability

As the confined magnetosphere is twisted up, it may become prone to a nonaxisymmetric kink-like instability. This may happen both during the pulsar-in-a-cavity phase and during a later magnetic tower phase.

The kink is probably the most dangerous instability in our scenario. In its nonlinear stage, it may lead to a significant disruption. Such a disruption, however, is not necessarily a bad thing: it is likely to be only temporary and the tower may be able to reform after being disrupted, as is seen in laboratory experiments by Lebedev et al. (2005). The resulting nonsteady evolution may then provide a plausible mechanism for rapid variability in GRBs. In addition, as a result of such disruption, a fraction of the toroidal magnetic field energy may be dissipated into thermal energy (Eichler 1993; Begelman 1998). As was shown by Drenkhahn & Spruit (2002; see also Giannios & Spruit 2005, 2006, 2007), this may contribute to the acceleration of the Poynting-flux-dominated outflow and to powering the prompt gamma-ray emission at later times. Also, such kink-driven magnetic dissipation in the magnetosphere may be seen as a manifestation of “coronal activity” that may modify (harden) the emitted neutrino spectrum (Ramirez-Ruiz & Socrates 2005).

As far as we know, the stability of the pulsar in a cavity has not yet been studied. However, several nonrelativistic three-dimensional MHD simulations (Kato et al. 2004b; Nakamura et al. 2007; Ciardi et al. 2007) have recently addressed the kink instability of magnetic towers (although not in the GRB context). They seem to indicate that during the first few rotation periods, a tower is stabilized by the surrounding high-pressure gas, but at later times a large-scale external kink does develop. As a result, the tower’s overall shape becomes helical. This, however, does not immediately lead to the total disruption of the tower; although the configuration is nonaxisymmetric, its main morphological features remain similar to those in the axisymmetric case (Nakamura et al. 2007). Similar conclusions have been reached by Nakamura & Meier (2004) in their three-dimensional MHD study of Poynting-flux-dominated jets propagating through a stratified external
medium. These authors found that the jet stability strongly depends on the background density and pressure profiles along the jet. In particular, a steep external pressure gradient forestalls the instability onset. When the instability does eventually develop, the resulting helical structures saturate and do not develop into full MHD turbulence. An important theoretical evidence supporting external pressure stabilization follows from Königl & Choudhuri’s (1985) analysis of a force-free magnetized jet confined by an external pressure. They argued that a nonaxisymmetric helical equilibrium state becomes energetically favorable (conserving the total magnetic helicity in the jet) only when the pressure drops below a certain critical value. If this happens and the external kink mode does become unstable, then this nonaxisymmetric equilibrium can be interpreted as the endpoint of the nonlinear development of the instability.

In addition to the above nonrelativistic studies, a few first steps have recently been taken toward understanding the stability of relativistic jets, in particular, in the framework of relativistic force-free electrodynamics (Gruzinov 1999; Tomimatsu et al. 2001). However, to the best of our knowledge, to date there have been no formal stability studies of relativistic magnetic towers or of confined pulsar magnetospheres. Such studies, both analytical and numerical, are clearly needed. They may involve a linear perturbation analysis or a nonaxisymmetric relativistic MHD or force-free simulation. They would have to take into account several stabilizing effects. First, as Tomimatsu et al. (2001) found in their linear stability analysis of a narrow rotating relativistic force-free jet, rapid field-line rotation inhibits kink instability. Second, regarding the stability of a rapidly growing magnetic tower, we expect that the tower expansion should quickly transition to the relativistic regime, eventually reaching a very large $\gamma$-factor. Once this happens, the relativistic time delay may effectively stabilize the outflow (see Giannios & Spruit 2006). This is because MHD instabilities grow on the local Alfvén-crossing time in the fluid frame and hence much slower in the laboratory frame. As a result, even if instabilities are excited, they do not have enough time to develop before the breakout of the flow from the star.

We would like to point out that, in this problem, we are actually interested not so much in the instability onset or its early linear development, but rather in its long-term (many rotation periods) nonlinear evolution and its overall effect on the magnetosphere. Such long-term behavior is very poorly understood and needs to be investigated in the future. Therefore, here we can only provide a hypothetical discussion. In its nonlinear stage, the kink instability may lead to conversion of some of the toroidal magnetic flux to poloidal flux (in our geometry). Some of this new poloidal flux may become detached from the star via reconnection (which, in reality, may be strongly inhibited deep inside the collapsing star; see Uzdensky & MacFadyen 2006). This could lead, in principle, to the break up of the single coherent magnetosphere into a number of smaller spheromak-like plasmoids (Fig. 7). That is, instead of further twisting up of the entire magnetosphere or, at a later stage, lengthening of the magnetic tower, one would effectively get continuous injection of new plasmoids into the system. Hoop stress still works inside each of them, so the overall dynamical effect may be the same as that of a single magnetosphere, at least qualitatively. The resulting multicomponent structure of the outflow may be responsible for the observed intermittency in GRBs. It is a very interesting scenario that should be considered in future research.

An important consideration that then needs to be taken into account is the conservation of magnetic helicity. Differential rotation leads to a continuous injection of helicity into the system (of opposite signs in the two hemispheres). The kink instability may convert toroidal flux to poloidal but it will not destroy the magnetic helicity accumulated in the cavity. Thus, whatever the resulting configuration might be, it will have to be consistent with a growing amount of helicity. One may in fact imagine a cyclic process involving twisting up the magnetosphere for several rotation periods, followed by flux conversion due to the kink instability, followed by reconnection and the production and detachment of a plasmoid carrying the magnetic helicity (and some of the magnetic energy) injected during the given cycle. One can hypothesize that if this cyclical process is robust, then over time the bulk of the cavity may become filled with spheromak-like plasmoids. The picture then would be somewhat similar to that in Figure 7, except it would not have to be axisymmetric. Each of these plasmoids would have some net magnetic helicity and magnetic energy and would be in a magnetostatic equilibrium configuration, confined laterally by the overall pressure of the neighboring plasmoids. It may also contain thermal energy produced as a result of reconnection during plasmoid creation. Since the magnetic field is closed within each plasmoid, each plasmoid is not magnetically connected to the star and the magnetic field inside of it is not subject to any additional twisting. Helicity then stays constant within each plasmoid. As the number of such plasmoids grows with time, they occupy a larger and larger fraction of the cavity volume. Correspondingly, the part of the volume that is directly connected to the rotating neutron star shrinks with time. As long as some part of the neutron star field lines extends beyond the light cylinder, differential rotation continues on these field lines, resulting in more twisting and more generation of plasmoids. At some point, however, the part of the magnetosphere that is directly connected to the neutron star—the NS magnetosphere proper—will be squeezed by the surrounding
plasmoids to such a degree that it will be confined entirely inside the light cylinder. If this happens, twisting will stop, the neutron star’s proper magnetosphere will be corotating with the neutron star, and magnetic energy extraction will cease, at least if the boundary between the proper magnetosphere and the surrounding plasmoids is axisymmetric. If this boundary is not axisymmetric, then energy and angular momentum will continue to be extracted at some level through a process akin to the magnetic propeller effect (Illarionov & Sunyaev 1975). On the other hand, it is possible that magnetic reconnection will restore the link between the magnetosphere proper and the outer plasmoids, even if temporarily. If this happens, the situation will become more complicated; the spin-down torques will again be modified. In fact, as was pointed out to us by the referee of this paper, this resulting situation may become somewhat analogous to the case of a pulsar magnetosphere compressed by a strong relativistic wind of another pulsar, such as in the case of the double pulsar PSR J0737–3039. Such magnetosphere-wind interaction, along with the resulting pulsar spin-down torque, was considered by Lyutikov (2004) and Arons et al. (2005). As they point out, the torque due to the reconnected field lines can become much larger than the usual spin-down torque of an isolated pulsar.

It is also interesting to make the following comment. An unbounded relativistic force-free outflow driven by a rotating conductor is expected to be stable. On the other hand, a closed confined magnetosphere with field lines subject to differential rotation, such as our pulsar-in-a-cavity problem or a magnetic tower, may still be unstable, due to the Kelvin-Helmholtz instability. In light of this work, we cannot rule out the possibility that our magnetic cavity and/or the subsequently formed magnetic towers may also suffer from fragmentation into several Rayleigh-Taylor fingers. However, initially small-scale fingers quickly merge with one another to form a small number of large ones in the nonlinear stage. Therefore, we do not expect strong mixing of the baryons from the stellar envelope into the magnetosphere. The exact geometry of the outflow may change and a strong time variability may develop, but, overall, we expect the outflow to survive. More research is needed in order to assess the implications of this instability for our scenario.

Finally, we would like to reiterate that a proper treatment of these problems requires a time-dependent three-dimensional relativistic force-free or full (preferably relativistic) MHD analysis and simulations (see § 4.6).

4.2. Nickel Production

A central issue for the central engine of long-duration GRBs is the required production of $^{56}$Ni. The SNe that have been observed to accompany long-duration GRBs (SN GRBs) are classified as Type Ibc (SNe Ibc; see, e.g., Soderberg 2006; Kaneko et al. 2007). Modeling of the optical light curves of SNe Ibc requires the presence of radioactive $^{56}$Ni to heat the ejecta after initial postexplosion expansion of the star. The $^{56}$Ni masses inferred from the peak optical brightness of SN GRBs have a broad range, with the brightest—e.g., SN1998bw and SN2003dh—requiring several 0.1 $M_\odot$. On average, however, SN GRBs are not required to produce more $^{56}$Ni than the local population of SNe Ic (Soderberg 2006). In fact, as with low-luminosity SNe, e.g., the “tailless” SNIa, some SNe GRBs may produce little or no $^{56}$Ni (MacFadyen 2003), as recent observations indicate for GRB 060505 and GRB 060614, two relatively nearby ($\sim 0.1$) long GRBs with no detected supernova component (Fynbo et al. 2006; Della Valle et al. 2006; Gal-Yam et al. 2006).

In models of (non-GRB) core-collapse supernovae, $^{56}$Ni is produced hydrodynamically in material heated to $T \gtrsim T_{\text{Ni}} \sim 5 \times 10^7$ K by the explosion shock launched in the core of the star. The amount of $^{56}$Ni produced depends on the mass inside of the expanding shock when its temperature declines below $T_{\text{Ni}}$. This occurs when its radius has expanded to

$$R_{\text{Ni}} \sim \left( \frac{3E}{4\pi a T_{\text{Ni}}} \right)^{1/3} \sim (3.7 \times 10^{51})E_{51} \text{ cm},$$

where $E = E_{51} \times 10^{51}$ ergs is the explosion energy and $a$ is the radiation constant. The mass inside this radius depends on the density structure of the progenitor star and on how much expansion or contraction occurs before the shock reaches a given mass element. In particular, little or no $^{56}$Ni is produced by a shock, even if extremely powerful, if it is launched into a low-density environment. This may occur if a weak initial explosion expands the stellar core so that little mass remains within a few $10^8$ cm when the strong shock arrives. Production of $\sim 0.1$ $M_\odot$ of $^{56}$Ni occurs for many presupernova stars if $T_{\text{Ni}} \sim 10^{51}$ ergs is deposited isotropically by a (quasi-)spherical shock on a timescale of $\sim 1$ s so that little preexpansion of the star occurs before the shock arrives. Some of the brightest SNe, e.g., SN1998bw, require energies of up to $\sim 10^{52}$ ergs to make the $\sim 0.5$ $M_\odot$ inferred from light-curve modeling.

The requirement of fast ($\lesssim 1$ s) isotropic deposition of energy for hydrodynamical production of $^{56}$Ni presents a serious challenge for models of the SN GRB central engine: first, because the GRB engine must typically last 10 s or more for relativistic ejecta to escape the star, and second, because GRBs are believed to be highly asymmetric explosions. The high degree of beaming and long timescale for energy deposition render collapsar jets themselves incapable of producing anywhere near the required $^{56}$Ni masses (MacFadyen & Woosley 1999), since little mass ($<0.001$ $M_\odot$) can be heated to sufficiently high temperatures. Therefore, in the original collapsar model, with a black hole accretion disk as the
central engine, the $^{56}\text{Ni}$ is produced in a nonrelativistic biconical wind blown from the disk and constituting a distinct explosion component (MacFadyen & Woosley 1999; MacFadyen 2003).

A fundamental problem for the magnetar model, if it is to produce a GRB and a SN, is the requirement that it produce both an isotropic explosion for the $^{56}\text{Ni}$ production and beamed relativistic ejecta. In our model, $^{56}\text{Ni}$ can be produced behind a roughly spherical hydrodynamical shock driven by the initial quasi-isotropic expansion of the magnetosphere. The expansion becomes collimated, and the tower formation begins only after the stress of the magnetosphere becomes sufficient to balance the postshock pressure. The collimation process of the magnetar wind thus involves a quick isotropic expansion followed by a beamed component. We feel that this modification to the magnetar scenario, i.e., the inclusion of the magnetosphere interaction with the exterior star, strengthens its viability as a model for the long GRB central engine.

4.3. Restarting the Engine

We note that the same magnetar can power explosions with the degree of collimation that depends on the magnitude of the outer bounding pressure. A quasi-spherical SN or a highly beamed jet may result from the same star at different times as the bounding pressure changes. In the collapse and explosion of a massive star, the pressure of stellar gas bounding the central magnetar may have a complex time history. The star may initially collimate the embedded magnetar power into a tightly collimated tower responsible for GRB emission. Subsequently, after the star expands and the pressure bounding the magnetar decreases, the magnetar power will no longer be strongly beamed and a normal quasi-spherical magnetar outflow will result. Later, however, if material not ejected is not accreted, the magnetar will again be surrounded by a bounding pressure and its power will be recollimated. X-ray flares observed following some GRBs (Burrows et al. 2005; Falcone et al. 2006; Romano et al. 2006) could result from this process (see also, e.g., Proga & Zhang 2006; Perna et al. 2006).

4.4. Preshaping the Cavity

In previous sections, we have shown that the toroidal field makes the expanding plasma self-collimating due to hoop stress, and propose a spherical cavity with constant wall properties (i.e., no dependence on polar angle) as the simplest model problem. However, the cavity is expected in many cases to have lower density near the polar axis at fixed radius due to various processes acting as the star collapses. Among these are rotational flattening and the asymmetric stress from an early magnetized wind. First, in order to produce a millisecond magnetar, the progenitor star must have been rapidly rotating. We therefore expect the collapsed core to be strongly modified by rotational effects. In particular, the material near the rotation axis experiences no centrifugal barrier inhibiting its accretion, resulting in a relatively low density in the polar region. A separate effect is that a weaker, nonrelativistic MHD jet may have been launched along the axis earlier, during the collapse of the stellar core (LeBlanc & Wilson 1970; Wheeler et al. 2000). In addition, an initial MHD wind from the protomagnetar may be concentrated to the poles as in Bucciantini et al. (2006). This will push out the cavity in the polar region. The subsequent relativistic wind will then expand into a cavity preshaped by the previous MHD wind. At a fixed radius, the pressure and density of the wall will be decreased at the poles relative to the equatorial values. If these effects are extreme, the cavity is significantly weakened in the polar direction, and a model problem consisting of a “magnetar in a tube” is of interest.

4.5. Pulsar Kicks

Note that in our picture, most of the magnetically extracted rotational energy of the neutron star travels vertically through the two oppositely directed channels. Correspondingly, a significant amount of linear momentum is transported up and down from the neutron star and, correspondingly, a back-reaction force is exerted on the neutron star from both the top and the bottom. The two back-reaction forces are oppositely directed and nearly cancel each other. However, even a slight imbalance in the force may have important consequences for the overall momentum imparted to the neutron star and hence for its terminal velocity. For example, taking the total initial rotational energy of the PNS to be $E_{\text{rot}} = 5 \times 10^{52}$ ergs, the momentum transported out in each direction is $P = E_{\text{rot}}/2c \sim 10^{42}$ cgs. Therefore, just a 10% imbalance would result in the terminal velocity of the neutron star of order $v_{\text{term}} \simeq 0.1P/M_{\text{NS}} \sim 300$ km s$^{-1}$.

4.6. Prospects for Numerical Simulations

In order to gain a solid physical understanding of the fundamental physical processes controlling the interaction of a magnetar with its birth environment, we suggest a sequence of numerical investigations employing a range of well-tested plasma descriptions. Of particular usefulness are limiting cases that allow for simplified analysis, making the key physics more transparent. Simulations should cover regions of parameter space where limiting cases overlap with more complete plasma descriptions. For example, force-free (degenerate) electrodynamics (FFDE) is a useful tool for studying highly magnetized plasma for which pressure and inertia are negligibly small. In this extreme case, the cavity wall would have to be represented by a rigid perfectly conducting outer boundary condition. While this case may not be of general relevance for the realistic physical environment, some basic aspects of a bounded rotating magnetosphere may be understood using this description. In addition, the FFDE description has the advantage of requiring fewer parameters to specify the initial and boundary conditions for a given model problem. The results of the time integration can then be more easily understood with a minimum of complicating factors. Time-dependent force-free codes have already been used successfully in recent years to study pulsar magnetospheres (e.g., Komissarov 2006; McKinney 2006b; Spitkovsky 2006).

It is possible that the full magnetar-in-a-star problem can be successfully investigated by a hybrid simulation that would employ a relativistic force-free code inside the cavity and a relativistic hydrodynamic simulation outside (e.g., R. D. Blandford 2005, private communication; McKinney 2006a).

The next step would be to treat the plasma in the fully relativistic MHD regime. There are several relativistic MHD codes in existence that have reached the required level of maturity (Koida et al. 1999; Gammie et al. 2003; Del Zanna et al. 2003; De Villiers et al. 2003; Fragile 2005; Komissarov 2005; Nishikawa et al. 2005). Of interest would be a set of simulations with a range of plasma $\beta$. The low-$\beta$ simulations should match the FFDE case, at least qualitatively. Once these simulations are analyzed and the basic physical processes elucidated, $\beta$ can be gradually increased, enabling an understanding of how plasma inertia and pressure affect the dynamics of the magnetosphere expansion and collimation.

The basic process of tower formation and collimation can initially be explored with two-dimensional axisymmetric simulations. However, to investigate tower stability to nonaxisymmetric disruptions, fully three-dimensional simulations are necessary.
Finally, note that the general processes we describe here are of interest for many astrophysical systems, including nonrelativistic central objects (e.g., planetary nebulae; see Blackman et al. 2001; Matt et al. 2006). For this reason, nonrelativistic MHD simulations of this problem are of interest to themselves, as well as a first step toward fully relativistic MHD. Recent nonrelativistic MHD simulations indicate that the magnetic tower mechanism can operate successfully in a variety of astrophysical environments (e.g., Romanova et al. 2004; Kato et al. 2004a, 2004b; Nakamura et al. 2006, 2007), including core-collapse supernovae (Burrows et al. 2007).

5. CONCLUSIONS

In this paper we have investigated the millisecond-magnetar scenario for the central engine of gamma-ray bursts and core-collapse supernovae. We have focused on the interaction between the rapidly rotating magnetar magnetosphere and the surrounding infalling stellar envelope. We have argued that the stellar material provides a confining (ram) pressure that has a strong effect on both the size and the shape of the magnetosphere. In particular, it can channel the highly magnetized outflow originating from the proto–neutron star into two collimated magnetic towers.

More specifically, we suggest that the stalled bounce shock—a common feature in models of core-collapse supernovae—effectively plays the role of a cavity that confines the magnetosphere. The cavity’s radius, determined by the balance between the pressure of the hot neutrino-heated gas and the ram pressure of the infalling material, stays quasi-stationary at $R_0 \approx 200$ km during the first few hundred milliseconds after the bounce. To get a feeling for what happens to the magnetar magnetosphere during this stage, we introduce a simplified fundamental-physics problem that we call the pulsar-in-a-cavity problem. A large part of our paper (§ 2) is devoted to investigating this problem. We show that since the radius of the cavity is larger than the pulsar light-cylinder radius, the magnetic field inside the cavity continuously winds up. Correspondingly, both the toroidal field strength and the magnetic spin-down luminosity of the pulsar increase roughly linearly with time. The magnetic energy in the cavity then grows quadratically with time. We then demonstrate that in the context of a millisecond magnetar inside a collapsing star, the magnetic field becomes dynamically important a fraction of a second after the bounce. This leads to a subsequent revival of the stalled shock and may result in a successful magnetically driven explosion. As long as the expansion of the cavity is nonrelativistic, the toroidal magnetic field inside it remains larger than the poloidal magnetic and electric fields. As a result, the hoop stress collimates the Poynting-flux-dominated outflow into two vertical channels that are similar to Lynden-Bell’s (1996) magnetic towers (see Uzdensky & MacFadyen 2006).

Finally, we discuss the implications of the model for several observationally motivated questions relevant to GRBs and core-collapse supernovae, such as $56\text{Ni}$ production, late-time X-ray flares, and pulsar kicks. We also outline a set of numerical studies that we feel need to be done.

We are grateful to A. Beloborodov, E. Blackman, J. Goodman, A. Königl, R. Kulsrud, M. Lyutikov, J. McKinney, J. Ostriker, C. Thompson, and to the anonymous referee for encouraging conversations and critical remarks. A. I. M. acknowledges support from the Keck Fellowship at the Institute for Advanced Study. D. A. U.’s research has been supported by the National Science Foundation under grant PHY-0215581 (PFC: Center for Magnetic Self-Organization in Laboratory and Astrophysical Plasmas).

REFERENCES

Akiyama, S., Wheeler, J. C., Meier, D. L., & Lichtenstadt, I. 2003, ApJ, 584, 954
Aloy, M. A., Müller, E., Ibáñez, J. M., Martí, J. M., & MacFadyen, A. 2000, ApJ, 531, L119
Andeljan, N. V., Bisnovatyi-Kogan, G. S., & Moiseenko, S. G. 2005, MNRAS, 359, 313
Arons, J. 2003, ApJ, 589, 871
Arons, J., Backer, D. C., Spitkovsky, A., & Kaspi, V. M. 2005, in ASP Conf. Ser. 328, Binary Radio Pulsars, ed. F. A. Rasio & I. H. Stairs (San Francisco: ASP), 95
Begelman, M. C. 1998, ApJ, 491, 291
Bethe, H. A. 1990, Rev. Mod. Phys., 62, 801
Bethe, H. A., & Wilson, J. R. 1985, ApJ, 295, 14
Blackman, E. G., Frank, A., & Welch, C. 2001, ApJ, 546, 288
Blandford, R. D., & Znajek, R. L. 1977, MNRAS, 179, 433
Bucciantini, N., Thompson, T. A., Arons, J., Quataert, E., & Del Zanna, L. 2006, MNRAS, 368, 1717
Burrows, A., Dessart, L., Livne, E., Ott, C. D., & Murphy, J. 2007, ApJ, 664, 416
Burrows, D. N., et al. 2005, Science, 309, 1833
Ciardi, A., et al. 2007, Phys. Plasmas, 14, 056501
Contopoulos, I., Kazanas, D., & Fendt, C. 1999, ApJ, 511, 351
Della Valle, M., et al. 2006, Nature, 444, 1050
Del Zanna, L., Bucciantini, N., & Londrillo, P. 2003, A&A, 400, 397
De Villiers, J.-P., Hawley, J. F., & Krolik, J. H. 2003, ApJ, 599, 1238
Drenkhahn, G., & Spruit, H. 2002, A&A, 391, 1141
Duncan, R. C., & Thompson, C. 1992, ApJ, 392, L9
Eichler, D. 1993, ApJ, 419, 111
Falcke, H. D., et al. 2006, ApJ, 641, 1010
Fragile, P. C. 2005, preprint (astro-ph/0503305)
Fynbo, J. P. U., et al. 2006, Nature, 444, 1047
Gal-Yam, A., et al. 2006, Nature, 444, 1053
Gammie, C. F., McKinney, J. C., & Tóth, G. 2003, ApJ, 589, 444
Giannios, D., & Spruit, H. 2005, A&A, 430, 1
———. 2006, A&A, 450, 887
———. 2007, A&A, 469, 1
Goldreich, P., & Julian, W. H. 1969, ApJ, 157, 869
Goldreich, P., Pacini, F., & Rees, M. J. 1971, Comments Astrophys. Space Phys., 3, 185
Gruzinov, A. 1999, preprint (astro-ph/9908101)
Illarionov, A. F., & Sunyaev, R. A. 1975, A&A, 39, 185
Koide, S., Shibata, K., & Kudoh, T. 1999, ApJ, 522, 727
Koide, S., Shibata, K., & Kudoh, T. 1999, ApJ, 522, 727
Koch, T., Shibata, K., & Kudoh, T. 2006, MNRAS, 359, 801
Koide, S., Shibata, K., & Kudoh, T. 2006, MNRAS, 367, 19
Königl, A., & Choudhuri, A. R. 1985, ApJ, 288, 173
Lebedev, S. V., et al. 2005, MNRAS, 361, 97
LeBlanc, J. M., & Wilson, J. R. 1970, ApJ, 161, 541
Lyutikov, M. 2003, ApJ, 599, 1238
Lyutikov, M. 2004, MNRAS, 353, 1095
Lyutikov, M., & Blandford, R. 2002, in Proc. Workshop on Beaming and Jets in Gamma Ray Bursts (NBSI), ed. R. Ouyed (Copenhagen), 146
———. 2003, preprint (astro-ph/0312347)
MacFadyen, A. I. 2003, in AIP Conf. Proc. 662, Gamma-Ray Burst and Afterflow Astronomy 2001 (Melville: AIP), 202
MacFadyen, A. I., & Woosley, S. E. 1999, ApJ, 524, 262
MacFadyen, A. I., Woosley, S. E., & Heger, A. 2001, ApJ, 550, 410
Matt, S., Frank, A., & Blackman, E. G. 2006, ApJ, 647, L45
Michel, F. C. 1973, ApJ, 180, L133
Nakamura, M., & Meier, D. L. 2004, ApJ, 617, 123
Nakamura, T. 1998, Prog. Theor. Phys., 100, 921
Nishikawa, K.-I., Richardson, G., Koide, S., Shibata, K., Kudoh, T., Hardee, P., & Fishman, G. J. 2005, ApJ, 625, 60
Ostriker, J. P., & Gunn, J. E. 1971, ApJ, 164, L95
Paczynski, B. 1998, ApJ, 494, L45
Perna, R., Armitage, P., & Zhang, B. 2006, ApJ, 636, L29
Proga, D., & Zhang, B. 2006, MNRAS, 370, L61
Ramirez-Ruiz, E., & Socrates, A. 2005, preprint (astro-ph/0504257)
Romano, P., et al. 2006, A&A, 450, 59
Romanova, M. M., Ustyugova, G. V., Koldoba, A. V., & Lovelace, R. V. E. 2004, ApJ, 616, L151
Ruderman, M. A., Tao, L., & Kluzniak, W. 2000, ApJ, 542, 243
Soderberg, A. 2006, in AIP Conf. Proc. 838, Gamma-Ray Bursts in the Swift Era, ed. S. S. Holt & N. Gehrels, & J. A. Nousek (Melville: AIP), 380
Spitkovsky, A. 2006, ApJ, 648, L51
Spruit, H. 1999, A&A, 341, L1
Thompson, C. 1994, MNRAS, 270, 480
Thompson, C., & Duncan, R. C. 1993, ApJ, 408, 194
Thompson, T. A., Chang, P., & Quataert, E. 2004, ApJ, 611, 380
Tomimatsu, A., Matsumoto, T., & Takahashi, M. 2001, Phys. Rev. D., 64, 123003
Usos, V. V. 1992, Nature, 357, 472
Uzdensky, D. A., & MacFadyen, A. I. 2006, ApJ, 647, 1192
Wheeler, J. C., Meiier, D. L., & Wilson, J. R. 2002, ApJ, 568, 807
Wheeler, J. C., Yi, I., Höfflich, P., & Wang, L. 2000, ApJ, 537, 810
Woosley, S. E. 1993, ApJ, 405, 273
Woosley, S. E., & Weaver, T. A. 1986, ARA&A, 24, 205
Yi, I., & Blackman, E. G. 1998, ApJ, 494, L163
Zhang, W., Woosley, S. E., & MacFadyen, A. I. 2003, ApJ, 586, 356