Holographic s-wave condensation and Meissner-like effect in Gauss-Bonnet gravity with various non-linear corrections

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Abstract

In this paper we have studied the onset of holographic s-wave condensate in the (4 + 1) dimensional planar Gauss-Bonnet-AdS black hole background with several non-linear corrections to the gauge field. In the probe limit, performing explicit analytic computations, with and without magnetic field, we found that these higher order corrections indeed affect various quantities characterising the holographic superconductors. Also, performing a comparative study of the two non-linear electrodynamics it has been shown that the exponential electrodynamics has stronger effects on the formation of the scalar hair. We observe that our results agree well with those obtained numerically [Z. Zhao et. al., Nucl. Phys. B 871 [FS] (2013) 98].

1 Introduction

The AdS/CFT correspondence[1]-[2] is considered to be one of the most remarkable achievements in modern physics. Since its discovery, it is gradually becoming a powerful tool to analyse various strongly coupled phenomena in quantum field theory using results from a gravity background which is weakly coupled. According to this correspondence, the d-dimensional gauge theory at the boundary completely encodes the dynamics of the (d + 1)-dimensional gravity theory in the bulk[3]. Among several other applications, most important implementations of the AdS/CFT duality are found in fluid/gravity correspondence and in the computations of gluon scattering amplitudes[7]-[14]. It also has a large impact on the diverse areas of mathematics. For a review on the applications of AdS/CFT correspondence see ref. [15].

Apart from the applications mentioned above, the AdS/CFT correspondence has also been widely used in condensed matter physics, particularly in superconductivity. In order to have gravity dual of a superconductor there must be a charged hairy black hole in the bulk AdS space-time in which the scalar hair is minimally coupled to an Abelian gauge field[16]-[17]. The black hole admits the scalar hair below a certain critical temperature through the mechanism of spontaneous $U(1)$ gauge symmetry breaking[18]-[19]. Then the AdS/CFT duality enables us to construct gravity duals of the transition from normal to superconducting phase in the boundary CFT.

Gravity theories in higher dimensions (greater than four) have been an interesting topic of research for the past few decades. The primary reason for this consideration is that these theories provide an appreciable framework for unifying gravity with other interactions exist in nature. String theory, being a suitable candidate for such a unified theory, requires the consideration of higher dimensional space-time for its consistency. In order to describe several aspects of string theory one needs to

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For excellent reviews on this correspondence see [3]-[6].
associate gravity theories with it. Likewise, other successful theories, such as brane theory, theory of supergravity etc. need to study in higher dimensions. All these facts necessitated the study of gravity in higher dimensions [20]. For excellent reviews on this topic see refs. [21, 22]. The effect of string theory on gravity is studied by considering a low-energy effective action which describes classical gravity [23]. This effective action must contain higher curvature terms and is needed to be ghost free [24]. It is found that the only effective action that satisfies these criteria is the Lovelock action [25]. The higher curvature terms of this action may be identified as the Gauss-Bonnet term, the third order Lovelock term etc.

Motivated by the successes of various non-linear electrodynamic theories, interests have grown among physicists to study gravity theories in the presence of these non-linear theories. One of the reasons for studying gravity theories with non-linear electrodynamics lies in the fact that it emerges in the low-energy limit of the heterotic string theory [26]-[28]. Moreover, several other interesting features have been explored while studying gravity theories that incorporate non-linear effects [29]-[48]. These non-linear theories correspond to the higher derivative corrections to the gauge fields in the Abelian theory. In fact, it is found that non-linear electrodynamics has several advantages over the Maxwell theory [48]-[50].

Besides the conventional Born-Infeld non-linear electrodynamics [50], there are several other non-linear electrodynamics which are of significant importance [48]. Two of them have been considered very recently in refs. [48, 49], namely the exponential form of non-linear electrodynamics (ENE) and the logarithmic form of non-linear electrodynamics (LNE), in the context of static charged asymptotic black holes. The novel property of these non-linear theories is that, their asymptotic black hole solutions are the same as that of a Reissner-Nordström black hole [49]. Moreover, these types of non-linear theories satisfy the known properties of Born-Infeld non-linear electrodynamics such as, regularity of electromagnetic field, absence of shock waves, birefringence etc. At this stage of discussion it must be added that, unlike the logarithmic electrodynamics, the exponential electrodynamics does not completely remove the divergence of electric field at \( r = 0 \). But, the divergence is much weaker than the usual Einstein-Maxwell gravity so that it can be neglected [49].

For the past several years, enormous amount of efforts have been given in order to understand number of vexing issues associated with the holographic superconductors with both linear (Maxwell case) as well as non-linear electrodynamics [18]-[19], [51]-[84]. Apart from the study of holographic superconductors in the framework of conventional Einstein gravity, several interesting attempts have been made in order to study these objects in the presence of higher curvature corrections to the Einstein gravity in higher dimensions [51, 52, 53, 60, 67, 71, 79, 82, 84]. The motivations for incorporating these corrections may be emphasized as follows:

(i) The dimensionality of the space-time affects the boundary behaviors of the fields coupled to gravity. It is also observed that the conformal dimensions of the dual operators of the boundary theory depends strongly on the dimensionality of the theory under consideration.

(ii) Till date most of the models of holographic superconductors have been built by considering nonstringy backgrounds. This is due to the lack of knowledge of stringy backgrounds narrowing down the class of field theories that can be studied holographically. The most popular way to cure this problem is to introduce the higher curvature corrections in the effective gravity theory.

(iii) The higher curvature corrections are added in the theory with suitable coupling constants. These can be treated as \( \alpha' \) (inverse of the string tension, \( T \)) expansion in the string theory. In fact the Gauss-Bonnet term is the first \( O(\alpha') \) low energy correction to the gravitational action.

Apart from the above points there are other generic quantitative features that distinguish the higher dimensional holographic superconductors with higher curvature corrections from the usual holographic superconductors in Einstein gravity. For example, the charge density of the previous systems vary as \( T^3 \) whereas for the later this varies as \( T^2 \), \( T \) being the temperature of the boundary field theory. In addition to it, the observed constancy of the ratio of the frequency gap of the real part of the conductivity to the critical temperature of the superconductor [55] breaks down for Gauss-Bonnet
superconductors. In certain generalized cases of holographic superconductors different values of the Gauss-Bonnet correction changes the order of the phase transition. Moreover, the ratio of the shear viscosity to the entropy density \((\eta/s \geq 1/4\pi)\) in CFT dual to the Einstein-Gauss-Bonnet gravity changes significantly with the Gauss-Bonnet coupling.

Although the points mentioned in the previous paragraphs explicitly express the importance of studying Gauss-Bonnet holographic superconductors with non-linear electrodynamics, there are in fact very few works in this direction known to us. Most of these analysis are based on numerical techniques. Thus, a systematic analytic analysis is still lacking in this particular context. In this regard it must be stressed that, several aspects of holographic superconductors with exponential and logarithmic electrodynamics has been studied recently in ref. in the planar Schwarzschild-AdS black hole background. Thus, no curvature correction has been taken into account in that work. Moreover, the computations have been done using numerical technique; no analytical analysis has yet been performed for this particular models. Therefore, it would be nice to perform an analytic analysis by incorporating the higher curvature Gauss-Bonnet term along with the two aforementioned non-linear electrodynamic fields in the action.

It is noteworthy that, although, the numerical results have always been substantiated by the analytical results in the literature, the analytic approach is always more preferable over the numerical technique particularly in the vicinity of the absolute temperature, \(T \to 0\). This is due to the fact that, in the zero temperature limit, the numerical solutions to the equations of motion becomes extremely difficult and therefore the determination of the quantities characterising the condensates becomes strenuous. Thus, the only way to survive this uneasy situation is to adopt a suitable analytic approach.

Inspired by all the above mentioned facts, we plan to make an extensive analytic investigation of the holographic superconductors subject to the aforementioned higher order corrections. The primary motivation of our study may be pointed out as follows:

(i) To make a comparative study between the Maxwell theory and the non-linear electrodynamic theories regarding their effects on the formation of the scalar condensates. On top of that, we want to study the effects of the higher derivative as well as the higher curvature corrections on several quantities characterising the holographic superconductors, such as the critical temperature \((T_c)\) for condensation and the condensation width \((\langle O \rangle)\). Also, the study of holographic superconductors in the presence of an external magnetic field is one of the primary issues in order to investigate the response of the superconductors in the magnetic field.

(ii) To compare the non-linear electrodynamic theories in an attempt to see which one has stronger effects on the formation of the scalar condensates.

In order to carry out the analysis, we have considered a holographic model of s-wave superconductor in the background of Gauss-Bonnet-AdS black hole. As a result, the higher curvature correction is included in the gravity action. On the other hand, to study the effects of higher derivative corrections on the condensates, we have included exponential and logarithmic electrodynamics in the matter action.

We have immersed the holographic superconductors in an external static magnetic field and have studied the response of the condensates to the magnetic field. At this point, it must be mentioned that the effects of an external magnetic field on holographic condensates have been studied earlier. There it was observed that below the critical temperature, the external magnetic field expels the holographic condensates. This phenomena resembles closely with the Meissner effect in ordinary superconductors. Due to this close analogy we may call it “holographic Meissner effect”. Thus, there is a similarity between the AdS/CFT s-wave holographic superconductors and ordinary superconductors.

In order to study analytically the effects of these higher order corrections on our s-wave holographic Gauss-Bonnet superconductor with two types of non-linear electrodynamics (ENE and LNE), we have used a well known analytic technique known as the Matching method. It is observed that, in
presence of higher curvature as well as the higher derivative corrections, the critical temperature for condensation indeed decreases making the scalar hair harder to form. On the other hand, the normalized order parameters for condensation and the critical values of the magnetic field increases with the increase of these corrections which in turn makes the condensation harder. The mathematical form regarding the dependence of the normalized order parameters on temperature establishes the fact that the holographic condensates indeed show mean field behaviour. Moreover, we have been able to determine the critical exponents associated with the order parameters. These are found to be $1/2$ for both the cases (with ENE and LNE). This confirms that there is a second order phase transition in going from normal to superconducting phase. Also, there is a close resemblance between our holographic superconductor and ordinary type-II superconductor. At this point of discussion, it is worth mentioning that while solving the equations of motion for the gauge fields we encountered some serious mathematical difficulties. However, by considering a perturbative solution of the gauge field we have been able to evade the difficulties.

As a crucial part of our investigation, we have made a qualitative comparison, based on the graphical analysis, between these superconductors and the holographic superconductor with usual Maxwell theory by setting the value of the non-linearity parameter ($b$) to zero in the results we have obtained. This comparative study reveals the fact that these higher order corrections have stronger effects on the formation of the holographic condensates, namely, these corrections make the condensation harder to form compared to the Maxwell case. Finally, based on our results, we have made a comparative study between exponential and logarithmic electrodynamics. It has been found that exponential electrodynamics has stronger effects in the formation of scalar condensates than the logarithmic electrodynamics.

But, before presenting analytic details, let us mention briefly the plan of presentation of our paper. In section 2, the basic setup for $s$-wave holographic superconductor with two different non-linear electrodynamics (ENE and LNE) in the planar $(4 + 1)$-dimensional Gauss-Bonnet AdS black hole background is given. In section 3, we have calculated various properties of the $s$-wave holographic superconductor with exponential electrodynamics (ENE) which include the critical temperatures for condensation and the expectation values of the condensation operator in the absence of external magnetic field. In section 4, we have discussed the effects of an external static magnetic field on this holographic superconductor and calculated the critical magnetic field for condensation. We have drawn our conclusions and discussed some of the future scopes in section 5. Finally, in appendix A we have given the expressions of the aforementioned quantities for the $s$-wave holographic superconductor with logarithmic electrodynamics (LNE) and derived a necessary mathematical relation.

## 2 Basic set up

The effective action for the higher curvature Lovelock gravity in an arbitrary dimension $d$ may be written as $^{[25]}$

$$ S_{\text{grav}} = \frac{1}{16\pi G_d} \int d^d x \sqrt{-g} \sum_{i=0}^{[d/2]} \alpha_i \mathcal{L}_i $$

where, $\alpha_i$ is an arbitrary constant, $\mathcal{L}_i$ is the Euler density of a $2i$ dimensional manifold and $G_d$ is the Gravitational constant in $d$-dimensions. In our subsequent analysis we shall consider the coordinate system where $G_d = \hbar = k_B = c = 1$.

In this paper we shall be mainly concerned with the $(4 + 1)$-dimensional Einstein-Gauss-Bonnet gravity in anti-de Sitter (AdS) space. Thus, the effective action $(\Pi)$ can be written as,

$$ S_{\text{grav}} = \frac{1}{16\pi} \int d^5 x \sqrt{-g} \left( \alpha_0 \mathcal{L}_0 + \alpha_1 \mathcal{L}_1 + \alpha_2 \mathcal{L}_2 \right) $$

$$ = \frac{1}{16\pi} \int d^5 x \sqrt{-g} \left( -2\Lambda + \mathcal{R} + \alpha \mathcal{L}_2 \right) $$

(2)
where $\Lambda$ is the cosmological constant given by $-6/l^2$, $l$ being the AdS length, $\alpha_2 \equiv \alpha$ is the Gauss-Bonnet coefficient, $L_1 = \mathcal{R}$ is the usual Einstein-Hilbert Lagrangian and $L_2 = \mathcal{R}_{\mu\nu\rho\sigma} \mathcal{R}^{\mu\nu\rho\sigma} - 4 \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \mathcal{R}^2$ is the Gauss-Bonnet Lagrangian.

The Ricci flat solution for the action (2) is given by \[51, 96, 97\]
\[
d s^2 = -f(r) d t^2 + f(r)^{-1} d r^2 + r^2 (d x^2 + d y^2 + d z^2) \tag{3}\]
where the metric function is \[51, 96, 97\]
\[
f(r) = \frac{r^2}{2a} \left( 1 - \sqrt{1 - 4a \left( 1 - \frac{M}{r^4} \right)} \right). \tag{4}\]

In (4), $M$ is the mass of the black hole which may be expressed in terms of the horizon radius $(r_+)$ as, $r_+ = M^{1/4} \left[51, 60, 71\right]$; the parameter $a$ is related to the coefficient $\alpha$ as, $a = 2\alpha$. It is to be noted that, in order to avoid naked singularity we must have $a \leq 1/4 \left[51, 71\right]$. Also, in the asymptotic infinity $(r \to \infty)$ we may write the metric function (4) as,
\[
f(r) \sim \frac{r^2}{2a} \left( 1 - \sqrt{1 - 4a} \right). \tag{5}\]

Thus, the effective AdS radius can be defined as \[51, 60, 71\],
\[
L_{\text{eff}}^2 = \frac{2a}{1 - \sqrt{1 - 4a}}. \tag{6}\]

Note that, in the limit $a \to 1/4$, $L_{\text{eff}}^2 = 0.5$. This limit is known as the Chern-Simons limit \[98\].

The Hawking temperature of the black hole may be obtained by analytic continuation of the metric at the horizon $(r_+)$. Following this prescription we may determine the Hawking temperature as \[51, 60, 71\],
\[
T = \frac{1}{4\pi} \left( \frac{\partial f(r)}{\partial r} \right)_{r=r_+} = \frac{r_+}{\pi}. \tag{7}\]

In this paper we shall study the $s$-wave holographic superconductor in the framework of various non-linear electrodynamics in the $(4 + 1)$-dimensional planar Gauss-Bonnet AdS (GBAdS) black hole background. For this purpose, we shall consider a matter Lagrangian which consists of a charged $U(1)$ gauge field, $A_\mu$, and a charged massive complex scalar field, $\psi$. Thus, the matter action for the theory may be written as \[16, 19, 51, 60]\,
\[
S_{\text{matter}} = \int d^5 x \sqrt{-g} \left( L(F) - |\nabla_\mu \psi - iA_\mu \psi|^2 - m^2 \psi^2 \right) \tag{8}\]
where $m$ is the mass of the scalar field. Moreover, we shall carry out all the calculations in the probe limit \[18\]. In this limit, gravity and matter decouple and the backreaction of the matter fields (scalar field and gauge field) can be suppressed in the neutral AdS black hole background. This is achieved by rescaling the matter action (8) by the charge, $q$, of the scalar field and then considering the limit $q \to \infty$. The generosity of this approach is that we can simplify the problems without hindering the physical properties of the system. The term $L(F)$ in (8) corresponds to the Lagrangian for the non-linear electrodynamic field. In different non-linear theories the Lagrangian $L(F)$ can take the following forms \[48, 80]\:
\[
L(F) = \begin{cases} 
\frac{1}{4b} \left( e^{-bF_{\mu\nu}}F^{\mu\nu} - 1 \right), & \text{for ENE} \\
\frac{-2}{b} \ln \left( 1 + \frac{1}{8}bF_{\mu\nu}F^{\mu\nu} \right), & \text{for LNE}
\end{cases} \tag{9}\]

\[2\] Without loss of generality, we can choose $l = 1$, which follows from the scaling properties of the equation of motion.
Note that in the limit \( b \to 0 \), we recover the usual Maxwell term: \( L(F) |_{b=0} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} \).

In order to study various properties of the holographic superconductor, we need to solve the equations of motion corresponding to the gauge field, \( A_\mu \) and the scalar field, \( \psi \). The variation of the action \( \mathcal{S} \) with respect to \( \psi \) and \( A_\mu \) gives

\[
\begin{align*}
\partial_\mu \left( \sqrt{-g} \partial^\mu \psi \right) - i \sqrt{-g} A^\mu \partial_\mu \psi - i \partial_\mu \left( \sqrt{-g} A^\mu \psi \right) - \sqrt{-g} A^2 \psi - \sqrt{-g} m^2 \psi &= 0, \\
\frac{1}{\sqrt{-g}} \partial_\mu \left( \frac{\sqrt{-g} F^{\mu\nu}}{e^{bF_2}} \right) - i \left( \psi^* \partial^\nu \psi - \psi (\partial^\nu \psi)^* \right) - 2 A^\nu |\psi|^2 &= 0, \quad \text{for ENE} \tag{11a}
\end{align*}
\]

\[
\begin{align*}
\frac{1}{\sqrt{-g}} \partial_\mu \left( \frac{\sqrt{-g} F^{\mu\nu}}{1 + \frac{bF_2}{8}} \right) - i \left( \psi^* \partial^\nu \psi - \psi (\partial^\nu \psi)^* \right) - 2 A^\nu |\psi|^2 &= 0, \quad \text{for LNE} \tag{11b}
\end{align*}
\]

respectively.

In order to solve the above equations of motion we shall choose the following ansatz for the two fields concerned \([18, 19]\):

\[
\begin{align*}
A_\mu &= \left( \phi(r), 0, 0, 0 \right), \tag{12a} \\
\psi &= \psi(r). \tag{12b}
\end{align*}
\]

It is to be noted that, the above choices of the fields are justified since it is seen that under the transformations \( A_\mu \to A_\mu + \partial_\mu \theta \) and \( \psi \to \psi e^{i\theta} \) the above equations of motion remain invariant. This demands that the phase of \( \psi \) remains constant and we may take \( \psi \) to be real without any loss of generality.

In the rest of our analysis we shall change variables and set \( z = \frac{r}{r_+} \). This makes the analysis a lot simpler. Thus the horizon of the black hole \((r = r_+)\) corresponds to \( z = 1 \) and the asymptotic limit \((r \to \infty)\) corresponds to \( z = 0 \).

With the above choices and using \([10]\) the equation of motion for the scalar field \( (\psi(z)) \) may be obtained as \([51, 60]\):

\[
\psi''(z) + \left( \frac{f'(z)}{f(z)} - \frac{1}{z} \right) \psi'(z) + \frac{\phi^2(z)\psi(z) r_+^2}{z^4 f^2(z)} - \frac{m^2 \psi(z) r_+^2}{z^4 f(z)} = 0 \tag{13}
\]

while using \((11a)\) and \((11b)\) the equations of motion for the scalar potential \( (\phi(z)) \) corresponding to the two aforementioned non-linear electrodynamics (ENE and LNE) may be written as \([80]\):

\[
\left( 1 + \frac{4bz^4 \phi^2(z)}{r_+^2} \right) \phi''(z) - \frac{1}{z} \phi'(z) + \frac{8bz^3}{r_+^2} \phi^3(z) - \frac{2\psi^2(z) \phi(z) r_+^2}{f(z) z^4} e^{-2bz^4 \phi^2(z)/r_+^2} = 0 \tag{14}
\]

\[
\left( 1 + \frac{bz^4 \phi^2(z)}{4r_+^2} \right) \phi''(z) - \frac{\phi'(z)}{z} + \frac{5bz^3}{4r_+^2} \phi^3(z) - \frac{2\psi^2(z) \phi(z) r_+^2}{f(z) z^4} \left( 1 - \frac{bz^4 \phi^2(z)}{4r_+^2} \right)^2 = 0. \tag{15}
\]

Before we proceed further, let us first mention the boundary conditions which may be put as follows:

(i) At the horizon \((z = 1)\) one must have, for \( m^2 = -3 \)^\(^3\)

\[
\phi(1) = 0, \quad \psi'(1) = \frac{3}{4} \psi(1) \tag{16}
\]

\(^3\)For the rest of the analysis of our paper we choose \( m^2 = -3 \). This ensures that we are above the Breitenlohner-Freedman bound\([99, 100]\).
(ii) In the asymptotic AdS region \((z \to 0)\) the solutions for the scalar potential and the scalar field may be expressed as,

\[
\phi(z) = \mu - \frac{\rho}{r_+^2} z^2, \quad (17a)
\]
\[
\psi(z) = D_- z^{\lambda_-} + D_+ z^{\lambda_+} \quad (17b)
\]

where \(\lambda_\pm = 2 \pm \sqrt{4 - 3L_{eff}^2}\) is the conformal dimension of the condensation operator \(O_i (i = 1, 2)\) in the boundary field theory, \(\mu\) and \(\rho\) are identified as the chemical potential and the charge density of the dual field theory, respectively. It is interesting to note that, since we have considered \(m^2 < 0\) in our analysis, we are left with the two different condensation operators of different dimensionality corresponding to the choice of quantization of the scalar field \(\psi\) in the bulk. We choose \(D_- = 0\). Then, according to the AdS/CFT correspondence \(D_+ \equiv \langle O_2 \rangle\), the expectation value of the condensation operator in the dual field theory.

### 3 s-wave condensate and its critical behaviour without magnetic field

In this section we shall analytically derive the critical temperature for condensation, \(T_c\), for the holographic s-wave condensate with two types of non-linear electrodynamics mentioned in the previous section. As a next step, we shall determine the normalized condensation operator and the critical exponent associated with the condensation values in the presence of these non-linear theories in the background of \((4+1)\)-dimensional Gauss-Bonnet AdS black hole. In this way we would be able to demonstrate the effects of the Gauss-Bonnet coupling coefficient \((a)\) as well as non-linear parameter \((b)\) on these condensates.

In order to carry out our analysis we have adopted a well known analytic technique which is known as the matching method\([51]\). In this method, we first determine the leading order solutions of the equations of motion \((13), (14), (15)\) near the horizon \((1 \geq z > z_m)\) and at the asymptotic infinity \((z_m > z \geq 0)\) and then match these solutions smoothly at the intermediate point, \(z_m\). It is to be noted that the matching method helps us to determine the values of the critical temperature as well as of condensation operator only approximately, in the leading order of the non-linear parameter, \(b\). Moreover, this method provides us a much better understanding of the effects the Gauss-Bonnet coefficient \((a)\) as far as analytic computation is concerned\([51]\).

In this section we shall perform the analysis for the holographic s-wave superconductor with exponential electrodynamics only. Since, the analysis for the holographic superconductor with logarithmic electrodynamics closely resembles to that of the previous one, we shall only present the results corresponding to this model in appendix A.1.

Let us first consider the solutions of the gauge field, \(\phi(z)\), and the scalar field, \(\psi(z)\), near the horizon \((z = 1)\). We Taylor expand both \(\phi(z)\) and \(\psi(z)\) near the horizon as\([51]\),

\[
\phi(z) = \phi(1) - \phi'(1)(1 - z) + \frac{1}{2} \phi''(1)(1 - z)^2 + \cdots \quad (18)
\]
\[
\psi(z) = \psi(1) - \psi'(1)(1 - z) + \frac{1}{2} \psi''(1)(1 - z)^2 + \cdots \quad (19)
\]

It is to be noted that, in \([18]\) and \([19]\), we have considered \(\phi'(1) < 0\) and \(\psi(1) > 0\) in order to make \(\phi(z)\) and \(\psi(z)\) positive. This can be done without any loss of generality.

Near the horizon, \(z = 1\), we may write from \([13]\)

\[
\psi''(1) = \left[ \frac{1}{z} \psi'(z) \right]_{z=1} - \left[ \frac{f'(z) \psi'(z)}{f(z)} \right]_{z=1} - \left[ \frac{\phi^2(z) \psi(z) r_+^2}{z^4 f^2(z)} \right]_{z=1} - \left[ \frac{3 \psi(z) r_+^2}{z^4 f(z)} \right]_{z=1}. \quad (20)
\]
Using the L’Hôpital’s rule and the values \( f'(1) = -4r_+^2 \), \( f''(1) = 4r_+^2 + 32ar_+^2 \), we may express (20) in the following form:

\[
\psi''(1) = -\frac{5}{4}\psi'(1) + 8a\psi'(1) - \frac{\phi'^2(1)\psi(1)}{16r_+^2}.
\]  

Finally, using the boundary condition (16), the Taylor expansion (19) can be rewritten as,

\[
\psi(z) = \frac{1}{4}\psi(1) + \frac{3}{4}\psi(1)z + (1-z)^2 \left[ \frac{15}{64} + \frac{3a}{2} - \frac{\phi'^2(1)}{64r_+^2} \right] \psi(1).
\]  

Near the horizon, \( z = 1 \), from (14) we may write

\[
\phi''(1) = \frac{1}{(1 + \frac{4b}{r_+}\phi'^2(1))} \left[ \phi'(1) - \frac{8b}{r_+^2}\phi'^2(1) - \frac{\psi^2(1)\phi'(1)}{2}e^{-2b\phi'^2(z)/r_+^2} \right].
\]  

In obtaining (23) we have considered that the metric function, \( f(z) \), can also be Taylor expanded as in (18), (19).

Substituting (23) in (18) and using (16) we finally obtain,

\[
\phi(z) = -\phi'(1)(1-z) + \frac{1}{2}(1-z)^2 \left[ 1 - \frac{8b}{r_+^2}\phi'^2(1) - \frac{\psi^2(1)\phi'(1)}{2}e^{-2b\phi'^2(z)/r_+^2} \right] \left[ \phi'(1) \right] \left( 1 + \frac{4b}{r_+}\phi'^2(1) \right).
\]  

Now, using the method prescribed by the matching technique[51], we match the solutions (17a), (17b), (22) and (24) at the intermediate point \( z = z_m \). It is very much evident that the matching of the two asymptotic solutions smoothly at \( z = z_m \) requires the following four conditions:

\[
\mu - \frac{\rho z_m^2}{r_+^2} = \beta(1 - z_m) - \frac{\beta}{2}(1 - z_m)^2 \left[ \frac{1 - 8b\beta^2}{1 + 4b\beta^2} - \frac{\alpha^2}{2} e^{-2b\beta^2} \right] \tag{25}
\]

\[-2\rho z_m = -\beta + \beta(1 - z_m) \left[ \frac{1 - 8b\beta^2}{1 + 4b\beta^2} - \frac{\alpha^2}{2} e^{-2b\beta^2} \right] \tag{26}
\]

\[D_+ z_m^\lambda_+ = \frac{\alpha}{4} + \frac{3\alpha z_m}{4} + \alpha(1 - z_m)^2 \left[ -\frac{15}{64} + \frac{3a}{2} - \frac{\beta^2}{64} \right] \tag{27}
\]

\[\lambda_+ D_+ z_m^\lambda_+ = \frac{3\alpha z_m}{4} - 2\alpha z_m(1 - z_m) \left[ -\frac{15}{64} + \frac{3a}{2} - \frac{\beta^2}{64} \right] \tag{28}
\]

where we have set \( \psi(1) = \alpha \), \( -\phi'(1) = \beta \) (\( \alpha, \beta > 0 \)), \( \tilde{\beta} = \frac{\beta}{r_+} \) and \( D_- = 0 \) [cf.(17b)].

From (26), using (7), we obtain,

\[
\alpha^2 = \frac{2z_m}{(1 - z_m)} e^{2b\tilde{\beta}^2} \left( 1 + \frac{4b\tilde{\beta}^2(3 - 2z_m)}{z_m} \right) \left( \frac{T_c}{T} \right)^3 \left( 1 - \frac{T^3}{T_c^3} \right). \tag{29}
\]

Since, in our entire analysis we intend to keep terms which are only linear in the non-linear parameter, \( b \), (29) can be approximated as,

\[
\alpha^2 \approx \frac{2z_m}{(1 - z_m)} \left( 1 + \frac{6b\tilde{\beta}^2(2 - z_m)}{z_m} \right) \left( \frac{T_c}{T} \right)^3 \left( 1 - \frac{T^3}{T_c^3} \right). \tag{30}
\]
Here the quantity $T_c$ may be identified as the critical temperature for condensation and is given by,

$$
T_c = \left[ \frac{2\rho}{\beta \pi^3} \left( 1 - \frac{12b^2(1 - z_m)}{z_m} \right) \right]^{\frac{1}{4}}.
$$

(31)

Now from (27) and (28) we obtain,

$$
D_+ = \left[ \frac{(3z_m^2 + 5z_m)}{4(\lambda_+ + (2 - \lambda_+)z_m)} \left( \frac{1}{z_m} \right)^{\lambda_+} \right] \alpha,
$$

(32)

$$
\tilde{\beta} = 8 \left[ -\frac{15}{64} + \frac{3a}{2} - \frac{(3z_m^2 + 5z_m)(1 + \lambda_+)}{4(\lambda_+ + (2 - \lambda_+)z_m)(1 - 4z_m + 3z_m^2)} + \frac{(1 + 6z_m)}{4(1 - 4z_m + 3z_m^2)} \right]^{\frac{1}{4}}.
$$

(33)

Finally, using (7), (30) and (32), near the critical temperature, $T \sim T_c$, we may write the expectation value, $\langle O_2 \rangle$, of the condensation operator in the following form:

$$
\frac{\langle O_2 \rangle}{T_c^{\lambda_+}} = \left( \frac{\pi}{z_m} \right)^{\lambda_+} \left[ \frac{(3z_m^2 + 5z_m)}{4(\lambda_+ + (2 - \lambda_+)z_m)} \right]^{\lambda_+} \left[ \frac{6z_m}{(1 - z_m)} \left( 1 + \frac{6b\beta^2(2 - z_m)}{z_m} \right) \right] \left( 1 - \frac{T}{T_c} \right)^{\frac{\beta}{4z_m^2}}.
$$

(34)

In (34) we have normalized $\langle O_2 \rangle$ by the critical temperature, $T_c$, to obtain a dimensionless quantity $\mathcal{O}_2$. In the similar fashion we can also calculate the critical temperature, $T_c$, and condensation operator, $\langle O_2 \rangle$, for the holographic superconductors with logarithmic electrodynamics (LNE). The corresponding expressions for the above mentioned quantities are given in Appendix A.1.

In the next section we shall be mainly concerned with the effects of magnetic field on this $s$-wave holographic superconductor with the two different types of non-linear electrodynamics mentioned earlier. But, before that we would like to make some comments on the results obtained so far. These may be put as follows:

(i) From (31) (and (A.1)) it is evident that in order to have a meaningful notion of the critical temperature, $T_c$, there must have an upper bound to the non-linear coupling parameter, $b$. The upper bounds corresponding to two non-linear theories are given below:

$$
b \leq \begin{cases} 
\frac{z_m(\lambda_+ + 2z_m - \lambda_+ z_m)}{12(1 + 96a\lambda_+ - 6(32a - 13)(\lambda_+ - 1)z_m + 3(32a - 5)(\lambda_+ - 2)z_m^2)}, & \text{for ENE} \\
\frac{2z_m(\lambda_+ + 2z_m - \lambda_+ z_m)}{3(1 + 96a)(\lambda_+ - 18(32a - 13)(\lambda_+ - 1)z_m + 9(32a - 5)(\lambda_+ - 2)z_m^2)}, & \text{for LNE}
\end{cases}
$$

(35)

For particular values of $\tilde{\beta}$ and $z_m$, this upper bound is smaller for ENE and larger for LNE.

(ii) The critical temperature, $T_c$, decreases as we increase the values of the non-linear parameter, $b$ (see table 1). This feature is general for the two types of holographic superconductors considered in this paper. It must be remarked that, without any non-linear corrections ($b = 0$) the critical temperature is larger than the above two cases. For example, $T_c = 0.1907\rho^{1/3}$ for $a = 0.2$, $z_m = 0.5$. This suggests the onset of a harder condensation. Another nontrivial and perhaps the most interesting feature of our present analysis is that, for a particular value of the non-linear parameter, $b$, the value of the critical temperature, $T_c$, for the holographic condensate with exponential electrodynamics is smaller than that with logarithmic electrodynamics (table 1) showing stronger effects of the former on the condensation. It is also noteworthy that similar feature was obtained numerically by the authors of [80] in the planar Schwarzschild-AdS black hole background.

(iii) The condensation gap for the holographic condensate with non-linear electrodynamics is more than that with Maxwell electrodynamics (figure 1). On top of that, holographic superconductors

\footnote{Here we have used the relation $(1 - t^3) = (1 - t)(1 + t + t^2)$ for any arbitrary variable $t$.}
Table 1: Values of the coefficient of $T_c$ for different values of the parameters $a$, $b$ and $z_m$.

| $a$ | $b$ | $a$ | $b$ |
|-----|-----|-----|-----|
| $0.01$ | $0.0008$ | $0.2$ | $0.0002$ |
| $0.2$ | $0.0007$ | $z_m = 0.3$ | $b = 0.0001$ |
| ENE | $0.1713$ | LNE | $0.2261$ |
| $0.2198$ | $0.2306$ | $0.0933$ | $0.2030$ |
| $0.2018$ | $0.2102$ | $0.0002$ | $0.1882$ |
| $b = 0.0009$ | $b = 0.0003$ |
| $z_m = 0.5$ | $b = 0.0010$ | $b = 0.0006$ | $b = 0.0010$ |
| ENE | $0.1256$ | LNE | $0.1945$ |
| $0.1783$ | $0.1985$ | $0.0770$ | $0.1830$ |
| $0.1684$ | $0.1882$ | $b = 0.0004$ | $b = 0.0010$ |
| $b = 0.0004$ | $b = 0.0009$ |
| $z_m = 0.7$ | $b = 0.0012$ | $b = 0.0006$ | $b = 0.0010$ |
| ENE | $0.0645$ | LNE | $0.1661$ |
| $0.1399$ | $0.1698$ | $0.0537$ | $0.1605$ |
| $0.1424$ | $0.1648$ | $b = 0.0004$ | $b = 0.0010$ |

with ENE exhibit larger gap compared with that with LNE. This suggests that the formation of the scalar hair is difficult for the holographic condensate with ENE.

(iv) The Gauss-Bonnet coupling coefficient, $a$, also has important consequences in the formation of the holographic condensate. From table 1 it is clear that as we increase the value of $a$ the critical temperature for condensation decreases. This means that the increase of $a$ makes the formation of scalar hair difficult. This indeed shows that both $a$ and $b$ has the same kind of influences on the formation of the hair.

(v) The expectation value of the condensation operator, $\langle O_2 \rangle$, vanishes at the critical point $T = T_c$ and the condensation occurs below the critical temperature, $T_c$ (see the right panel of figure 1). Moreover, form (34) (and (A.2)) we observe that $\langle O_2 \rangle \propto (1 - T/T_c)^{1/2}$ which shows the mean field behaviour of the holographic condensates and signifies that there is indeed a second order phase transition (critical exponent 1/2). This also admires the consistency of our analysis.

4 Effects of external magnetic field with non-linear corrections

In this section we intend to study the effect of an external magnetic field on the holographic superconductors with the non-linear electrodynamics mentioned in section 3. In order to do so, we need to add an external static magnetic field in the bulk. According to the gauge/gravity duality, the asymptotic value of the magnetic field in the bulk corresponds to a magnetic field in the boundary field theory, i.e., $B(x) = F_{xy}(x, z \to 0)$[87, 89]. Considering the fact that, near the critical magnetic field, $B_c$, the value of the condensate is small, we may consider the scalar field $\psi$ as a perturbation near $B_c$. This allows us to adopt the following ansatz for the gauge field and the scalar field[74, 75, 87, 89]:

$$A_\mu = \left( \phi(z), 0, 0, Bx, 0 \right),$$

$$\psi = \psi(x, z).$$

With the help of (8), (36a) and (36b) we may write the equation of motion for the scalar field $\psi(x, z)$ as [74, 75, 84],

$$\psi''(x, z) - \frac{\psi'(x, z)}{z} + \frac{f'(z)}{f(z)} \psi'(x, z) + \frac{r^2_+ \phi^2(z) \psi(x, z)}{z^4 f^2(z)} + \frac{1}{z^2 f(z)} \left( \partial_z^2 \psi - B^2 x^2 \psi \right) + \frac{3r^2_+ \psi(x, z)}{z^4 f(z)} = 0 \quad (37)$$
Figure 1: Plots of critical temperature \((T_c - \rho)\) and normalized condensation operator \(\langle O_2 \rangle \approx T_c - T/T_c\) for \(s\)-wave holographic superconductors with different electrodynamical theories. The green, red and blue curves correspond to Maxwell, ENE and LNE cases, respectively.

In order to solve (37) we shall use the method of separation of variables\,[87, 89]. Let us consider the solution of the following form:

\[
\psi(x, z) = X(x)R(z). \tag{38}
\]

As a next step, we shall substitute (38) into (37). This yields the following equation which is separable in the two variables, \(x\) and \(z\).

\[
z^2f(z) \left[ \frac{R''(z)}{R(z)} + \frac{R'(z)}{R(z)} \left( \frac{f'(z)}{f(z)} - \frac{1}{z} \right) + \frac{r^2_0 \phi^2(z)}{z^4 f^2(z)} + \frac{3r^2_0}{z^4 f(z)} \right] - \left[ -\frac{X''(x)}{X(x)} + B^2 x^2 \right] = 0. \tag{39}
\]

It is interesting to note that, the \(x\) dependent part of (39) is localized in one dimension. Moreover, this is exactly solvable since it maps the quantum harmonic oscillator (QHO). This may be identified as the Schrödinger equation for the corresponding QHO with a frequency determined by \(B\)^\[^74, 75, 87, 89\],

\[-X''(x) + B^2 x^2 X(x) = C_n BX(x) \tag{40}\]

where \(C_n = 2n + 1 \ (n = integer)\). Since the most stable solution corresponds to \(n = 0\)^\[^74, 75, 87\], the \(z\) dependent part of (39) may be expressed as

\[
R''(z) + \left( \frac{f'(z)}{f(z)} - \frac{1}{z} \right) R'(z) + \frac{r^2_0 \phi^2(z) R(z)}{z^4 f^2(z)} + \frac{3r^2_0 R(z)}{z^4 f(z)} = \frac{BR(z)}{z^2 f(z)}. \tag{41}
\]

Now at the horizon, \(z = 1\), using (16) and (41), we may write the following equation:

\[
R'(1) = \left( \frac{3}{4} - \frac{B}{4r^2_0} \right) R(1). \tag{42}
\]
On the other hand, at the asymptotic infinity, \( z \to 0 \), the solution of (44) can be written as

\[
R(z) = D_- z^{\lambda_-} + D_+ z^{\lambda_+}.
\]  (43)

It is to be noted that in our analysis we shall choose \( D_- = 0 \) as was done in section 3.

Near the horizon, \( z = 1 \), Taylor expansion of \( R(z) \) gives

\[
R(z) = R(1) - R'(1)(1 - z) + \frac{1}{2} R''(1)(1 - z)^2 + \cdots
\]  (44)

where we have considered \( R'(1) < 0 \) without loss of generality.

Now calculating \( R''(1) \) from (44) and using (42) we may write from (44)

\[
R(z) = \frac{1}{4} R(1) + \frac{3z}{4} R(1) + (1 - z) \frac{B}{4 r_+^2} R(1) + \frac{1}{2} (1 - z)^2 \left[ 3a - \frac{15}{32} + (1 - 16a) \frac{B}{16 r_+^2} + \frac{B^2}{32 r_+^4} - \frac{\phi^2(1)}{32 r_+^4} \right] R(1)
\]  (45)

where in the intermediate step we have used the Leibniz rule [cf. (20)].

Finally, matching the solutions (43) and (45) at the intermediate point \( z = z_m \) and performing some simple algebraic steps as in section 3 we arrive at the following equation in \( B \):

\[
B^2 + 2Br_+^2 \left[ \frac{8 (\lambda_+ - (\lambda_+ - 1)z_m)}{(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)} + (1 - 16a) \right] + \left[ \frac{(1 + 3z_m)\lambda_+ - 3z_m}{2(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)} \right] + \left( 3a - \frac{15}{32} - \frac{\phi^2(1)}{32 r_+^2} \right) 32 r_+^4 = 0.
\]  (46)

Equation (46) is quadratic in \( B \) and its solution is found to be of the following form [51]:

\[
B = r_+^2 \left[ \frac{1}{(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)} + (1 - 16a) \right] - \frac{16 [(1 + 3z_m)\lambda_+ - 3z_m]}{(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)} \right] + \left( 96a - 15 - \frac{\phi^2(1)}{r_+^2} \right) \left( \frac{8 (\lambda_+ - (\lambda_+ - 1)z_m)}{(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)} + (1 - 16a) \right).
\]  (47)

We are interested in determining the critical value of the magnetic field strength, \( B_c \), above which the superconducting phase disappears. In this regard, we would like to consider the case for which \( B \sim B_c \). Interestingly, in this case the condensation becomes vanishingly small and we can neglect terms that are quadratic in \( \psi \). Thus, the equation of motion corresponding to the gauge field (equation 44), \( \phi \), may be written as

\[
\left( 1 + \frac{4b^2 \phi^2(z)}{r_+^2} \right) \phi''(z) - \frac{1}{z} \phi'(z) + \frac{8b^2 z^3}{r_+^2} \phi^3(z) = 0.
\]  (48)

In order to solve the above equation we shall consider a perturbative solution of the following form:

\[
\phi(z) = \phi_0(z) + \frac{b}{r_+^2} \phi_1(z) + \cdots
\]  (49)

where \( \phi_0(z) \) is the solution of \( \phi(z) \) for \( b = 0 \) and so on. After some algebraic calculations we may write the solution of (49) as:

\[
\phi(z) = \frac{\rho}{r_+^2} (1 - z^2) \left[ 1 + \frac{b}{r_+^2} - \frac{2b^2}{r_+^4} (1 + z^4)(1 + z^2) \right] .
\]  (50)

\(^5\text{See Appendix A for the derivation.}\)
At this point of discussion, it must be stressed that we have considered terms which are linear in the non-linear parameter $b$.

At the asymptotic boundary of the AdS space, $z = 0$, the solution \( \phi(z) \) can be approximated as

\[
\phi(z) \approx \frac{\rho}{r_+^2} \left[ 1 + \frac{b}{r_+^2} - \frac{2b \rho^2}{r_+^6} \right] - \frac{\rho}{r_+^2} \left( 1 + \frac{b}{r_+^2} \right) z^2.
\]  
(51)

Now, comparing (51) with (17a) we may identify the chemical potential, \( \mu \), as

\[
\mu = \frac{\rho}{r_+^2} \left[ 1 + \frac{b}{r_+^2} - \frac{2b \rho^2}{r_+^6} \right]
\]  
(52)

Near the horizon, $z = 1$, we may write from (48)

\[
\phi''(1) = \phi'(1) - \frac{12b}{r_+^2} \phi^3(1) + O(b^2)
\]  
(53)

Substituting (53) into (18) and using the boundary condition (16) we may write

\[
\phi(z) = -\phi'(1)(1 - z) + \frac{1}{2}(1 - z)^2 \left( \phi'(1) - \frac{12b}{r_+^2} \phi^3(1) \right)
\]  
(54)

Matching the solutions (54) and (17a) at the intermediate point $z_m$ and using (52) we can find the following relation:

\[
(\beta - 2\eta)(2\eta^3 - 6\beta^3 - \eta) = 0
\]  
(55)

where we have set $-\phi'(1) = \beta$ and $\frac{\rho}{r_+^2} = \eta$. One of the solutions of this quartic equation can be written as

\[
\beta = 2\eta
\]

which implies

\[
\phi'(1) = -\frac{2\rho}{r_+^2}.
\]  
(56)

As a final step, substituting (56) into (47) and using (17) and (31) we obtain the critical value of the magnetic field strength as

\[
\frac{B_c}{T_c^2} = \pi^2 \left( 1 + \frac{12b\beta^2(1 - z_m)}{C^2z_m} \right) \left[ \beta C - \mathcal{M} \left( \frac{T}{T_c} \right)^3 \right]
\]  
(57)

In a similar manner we can determine the critical value of magnetic strength, $B_c$, for the holographic superconductor with logarithmic electrodynamics. The expression for $B_c$ in this model is given in Appendix A.1.

In (57) the terms $C$ and $\mathcal{M}$ can be identified as

\[
C = \left( 1 - \frac{A - \mathcal{M}^2}{\beta^2 x^6} \right) ^{\frac{1}{2}}
\]

\[
\mathcal{M} = 1 - 16a + \frac{8(\lambda_+ - (\lambda_+ - 1)z_m)}{(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)}
\]

where

\[
A = \left[ 96a - 15 + 16 \left( \frac{(1 + 3z_m)\lambda_+ - 3z_m}{(1 - z_m)(\lambda_+ - \lambda_+ z_m + 2z_m)} \right) \right].
\]

Here, we must mention that we have normalized $B_c$ by the square of the critical temperature, $T_c$, such that the critical magnetic field strength, $B_c$, becomes dimensionless.
Figure 2: Plots of critical magnetic field strength ($B_c/T_c^2 - T/T_c$) for different values of $b$ (Upper panel) and $a$ (Lower panel). The green, red and blue curves correspond to Maxwell, ENE and LNE cases, respectively.

In figure 2 we have plotted (57) (and (A.3)) as a function of $T/T_c$. These plots help us to understand the effect of the Gauss-Bonnet coupling parameter ($a$) and the non-linear parameter ($b$) on the holographic condensates. From figure 2 we observe that, although, the qualitative nature of the graphs remain same for all values of $z_m$, the critical value of the magnetic field strength ($B_c$) increases with the increase of $z_m$. If we keep the value of $z_m$ and $a$ fixed and increase the value of $b$ the value of $B_c$ gradually increases. We also observe that the critical magnetic field strength ($B_c$) corresponding to the Maxwell case ($b = 0$) is lower than those for the ENE and LNE cases. This indicates that the critical magnetic field strength is higher in presence of the non-linear corrections than the usual Maxwell case. Moreover, this increment is larger for the holographic condensate with ENE than that with LNE. On the other hand, if we vary $a$ while keeping $z_m$ and $b$ constant similar effects are observed, i.e. the critical field strength increases with the increase of the higher curvature coupling parameter ($a$). From the preceding discussion we may infer that the higher order corrections make the condensation harder to form. Moreover, between the two non-linear electrodynamics, the exponential electrodynamics has stronger effects on the formation of the holographic s-wave condensate, namely, the formation of the scalar hair is more difficult for holographic superconductor with ENE. This can be explained by noting that the increase in the critical field strength tries to reduce the condensate away making the condensation difficult to set in [74, 75, 87].

5 Conclusions

In this paper, considering the probe limit, we have studied a holographic model of superconductor in the planar Gauss-Bonnet-AdS black hole background. We have also taken into account two different types of non-linear electrodynamics (exponential and logarithmic form of non-linear electrodynamics) in the matter Lagrangian which may be considered as higher derivative corrections to the gauge fields.
in the usual Abelian theory. In addition to that, we have made an analytic survey to study the effects of an external magnetic field on these superconductors.

The aim of the present article is twofold, namely, (i) to make a comparative study between the Maxwell electrodynamics and the two non-linear electrodynamics (ENE, LNE) regarding their effects on the holographic condensates as well as to study the effects of the higher order corrections (to the usual Einstein-Maxwell theory) and an external magnetic field on these condensates and (ii) to compare the aforementioned non-linear theories in order to investigate which one has stronger effect. The higher order corrections manifest themselves through the Gauss-Bonnet coupling parameter ($a$) and the non-linear parameter ($b$) respectively. We have been able to calculate the critical temperature ($T_c$) and the expectation value of the order parameter ($\langle O_2 \rangle$) for the holographic $s$-wave condensates corresponding to these two different models. In order to do so, we have used an effective analytic technique known as the Matching method[51]. From our analytic results we have demonstrated the effects of the various correction terms on the holographic condensates with ENE and LNE. We have further extended our analysis by immersing the superconductor into an external static magnetic field. In doing so, we have been able to successfully extend the matching technique in order to determine the critical value of the magnetic field ($B_c$) for condensation. In an attempt to perform the analysis we faced difficulties regarding the solutions of the gauge fields near the critical magnetic field $B_c$. However, we have successfully circumvented these difficulties by considering perturbative solutions of the gauge fields (see equation (49)). As in the previous case, we have been able to show the effects of various correction terms on the critical value of the magnetic field which is directly related to the formation of the holographic condensates.

The analytic results of the present article shows that the quantities which characterize the holographic condensates (with or without the magnetic field) are indeed affected in the presence of these correction terms. These can be put as follows:

(i) The non-linear electrodynamics has stronger effect on the condensates than the usual Maxwell case. Moreover, the critical temperature for condensation ($T_c$) decreases as we increase the values of the non-linear parameter ($b$) as well as the Gauss-Bonnet coupling parameter ($a$) (see figure 1 and table 1) whereas, the normalized order parameter ($\langle O_2 \rangle^{1/\lambda^+} / T_c$) increases with the increase of $b$ and $a$ (figure 1). This implies that, in the presence of the higher order corrections the formation of the scalar hair become difficult. On the other hand, the critical magnetic field strength ($B_c$) increases as we increase $b$ and $a$. The increasing magnetic field strength tries to reduce the condensate away completely. Thus, the formation of condensates become difficult.

(ii) From figures 1,2 and table 1 we observe that for particular parameter values $T_c$ is less in holographic superconductor with ENE than that with LNE whereas $\langle O_2 \rangle^{1/\lambda^+} / T_c$ and $B_c$ is more in the previous one. This results suggest that the exponential electrodynamics exhibit stronger effects than the logarithmic electrodynamics.

It is interesting to note that, our analytic computations reveal that the order parameters for condensation exhibit a mean field behaviour, $\langle O_2 \rangle \propto (1 - T/T_c)^{1/2}$. Note also that, the value of the associated critical exponent is 1/2 which further ensures that the holographic condensates indeed undergo a second order phase transition in going from normal to superconducting phase. This result is consistent with that obtained earlier[51].

At this point it must be added that, similar conclusions were drawn in ref. [80] where numerical computations were performed in this direction. Our analytic calculations provide further confirmations regarding this issue. Moreover, the novel feature of our present analysis is that we have been able to study the effect of the higher curvature corrections which was not performed explicitly in ref. [80].

Although, we have been able to explore several issues regarding $s$-wave holographic superconductors with different non-linear corrections, there are several other nontrivial and important aspects which we intend to explore in the future. These can be written in the following order:

(i) We have performed our entire analysis in the probe limit. It will be interesting to carry out the analysis by considering the back-reaction on the metric.
We can further extend our analysis considering higher curvature gravity theories beyond Gauss-Bonnet gravity. Also, the study of holographic $p$-wave and $d$-wave superconductors in presence of these various higher order corrections may be considered as an open and challenging issue.

6 Acknowledgements

The authors would like to thank Rabin Banerjee and Dibakar Roychowdhury for useful discussions. S. D would like to thank Department of Science and Technology (D. S. T), Government of India, and A. L would like to thank Council of Scientific and Industrial Research (C. S. I. R), Government of India, for financial support.

A Appendix

A.1 Holographic condensate with LNE:

The critical temperature for condensation:

\[ T_c = \left[ \frac{2\rho}{\beta \pi^2} \left( 1 - \frac{3b \tilde{\beta}^2(1 - z_m)}{2z_m} \right) \right]^{\frac{1}{3}}. \]  

Normalized order parameter:

\[ \langle O_2 \rangle = \left( \frac{\pi}{z_m} \right) \left[ \frac{(3z_m^2 + 5z_m)}{4(\lambda_+ + (2 - \lambda_+)z_m)} \right]^{\frac{1}{3}} \left[ \frac{6z_m}{(1 - z_m)} \left( 1 + \frac{3b \tilde{\beta}^2(2 - z_m)}{4z_m} \right) \left( 1 - \frac{T}{T_c} \right) \right]^{\frac{1}{3}}. \]  

Critical magnetic field strength:

\[ \frac{B_c}{T_c^2} = \pi^2 \left( 1 + \frac{3b \tilde{\beta}^2(1 - z_m)}{2C^2z_m} \right) \left[ \beta C - M \left( \frac{T}{T_c} \right)^3 \right]. \]  

The quantities $C$, $M$ and $A$ are identified in section 4.

A.2 Solution of gauge field ($\phi(z)$) near $B_c$:

The equation for the gauge field near $B_c$ for the holographic superconductor with ENE is given by

\[ \left( 1 + \frac{4bz^4\phi^2(z)}{r_+^2} \right) \phi''(z) - \frac{1}{z} \phi'(z) + \frac{8bz^3}{r_+^2} \phi^3(z) = 0. \]  

Let us consider the following perturbative solution of \((A.4)\):

\[ \phi(z) = \phi_0(z) + \frac{b}{r_+^2} \phi_1(z) + \cdots \]  

where $\phi_0(z)$, $\phi_1(z)$, $\cdots$ are independent solutions and the numbers in the suffices of $\phi(z)$ indicate the corresponding order of the non-linear parameter ($b$).

Substituting \((A.5)\) in \((A.4)\) we obtain

\[ \left[ \phi_0'(z) - \frac{\phi_0''(z)}{z} \right] + \frac{b}{r_+^2} \left[ \phi_1'(z) - \frac{\phi_1''(z)}{z} + 4z^4\phi_0'(z)\phi_0''(z) + \frac{8z^3}{r_+^2}\phi_0^3(z) \right] + O(b^2) = 0. \]  

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Equating the coefficients of $b^0$ and $b^1$ from the l.h.s of (A.6) to zero we may write

$$b^0 : \quad \phi''_0 (z) - \frac{\phi'_0 (z)}{z} = 0 \quad (A.7a)$$

$$b^1 : \quad \phi''_1 (z) - \frac{\phi'_1 (z)}{z} + 4 z^4 \phi''_0 (z) \phi'_0 (z) + \frac{8 z^3}{r^2} \phi''_0 (z) = 0 \quad (A.7b)$$

Now, using the boundary condition (17a) we may write the solution of (A.7a) as

$$\phi_0 (z) = \frac{\rho}{r^2} (1 - z^2) \quad (A.8)$$

Using (A.8) we may simplify (A.7b) as

$$\phi''_1 (z) - \frac{\phi'_1 (z)}{z} - 96 z^6 \left( \frac{\rho}{r^2} \right)^3 = 0 \quad (A.9)$$

As a next step, using the asymptotic boundary condition (17a), from (A.9) we obtain the solution of $\phi_1 (z)$ as

$$\phi_1 (z) = \frac{2 \rho^3}{r^6} (z^8 - 1) - \frac{\rho}{r^2} (z^2 - 1) \quad (A.10)$$

Substituting (A.8) and (A.10) in (A.5) we finally obtain the solution of the gauge field as

$$\phi (z) = \frac{\rho}{r^2} (1 - z^2) \left[ 1 + \frac{b}{r^2} - \frac{2 b \rho^2}{r^6} (1 + z^4)(1 + z^2) \right] \quad (A.11)$$

The solution of the gauge field for the holographic superconductor with LNE may be obtained by similar procedure and is given below:

$$\phi (z) = \frac{\rho}{r^2} (1 - z^2) \left[ 1 + \frac{b}{r^2} - \frac{b \rho^2}{4 r^6} (1 + z^4)(1 + z^2) \right] \quad (A.12)$$

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