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Electromagnetic form factors  
in the relativized Hypercentral CQM  

E. SANTOPINTO  
Università degli Studi di Genova & INFN  
Dipartimento di Fisica  
Via Dodecaneso, 33  
16146 Genova  
ITALY  

These are preliminary lecture notes, intended only for distribution to participants
Electromagnetic form factors in the relativized Hypercentral CQM

E. Santopinto

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M. Ferraris, M. Giannini, M. Pizzo, E. Santopinto, L. Tiator, Phys. Lett. B 364, 231 (1995).
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M. De Sanctis, M.M. Giannini, L. Repetto, E. Santopinto, Phys. Rev. C 62, 025208 (2000).
De Sanctis, M. Giannini, E. Santopinto, A. Vassallo, Nucl. Phys. A 755, 294 (2005)
Introduction

The hypercentral Constituent Quark Model

The elastic nucleon form factors (relativistic effects)

Conclusion

Work done in collaboration with M. Giannini, A. Vassallo, M. De Santis
New Jlab data on nucleon f.f.

Give rise to problems:

• compatibility with old (Rosenbluth plot) data

• why the ratio $G_E/G_M$ decreases?

• is there any zero in the f.f.?
VMD models (fits)

$$FF = F_{\text{intr}} \times \text{VMD propagators}$$

Mesons $F_{\text{intr}}$

- JIL $\rho \ \omega \ \phi$
  - dipole
  - monopole

- G K $\rho \ \omega \ \phi$
  - QCD-interpolation
Skyrme Model

Holzwarth 1996,2002

ff given by soliton + VMD
dip at $Q^2 3 \text{ GeV}^2$

Boosting of the soliton
dip at $Q^2 10 \text{ GeV}^2$

Skyrmion (nucleon) mass 1648 MeV
The Hypercentral Constituent Quark Model

M. Ferraris et al., Phys. Lett. B364, 231 (1995)
**SPACE WAVE FUNCTION**

### Jacobi coordinates

\[
\rho = \frac{r_1 - r_2}{\sqrt{2}}
\]

\[
\lambda = \frac{r_1 + r_2 - 2r_3}{\sqrt{6}}
\]

### Hyperspherical coordinates

\((\rho, \lambda, \Omega_\rho, \Omega_\lambda) \rightarrow (x, \xi, \Omega_\rho, \Omega_\lambda)\)

\[
x = \sqrt{\rho^2 + \lambda^2}
\]

\[
\xi = \arctan \frac{\rho}{\lambda}
\]

(size)

(shape)
Hypercentral Hypothesis

\[ V = V(x) \]

- **Factorization**

  \[ \psi(x, t, \Omega_\rho, \Omega_\lambda) = \psi_{\nu\gamma}(x) \quad Y_{[\gamma, l_\rho, l_\lambda]} \]

  ("dynamics") ("geometry")

  \( \nu \) hyperradial excitation \quad \( \gamma \) grand angular quantum number

- **3 Body Forces**
Motivations

- QCD fundamental mechanism

- Flux tube model

- Hypercentral approximation

\[ \sum_{i<j} V(r_{ij}) \approx V(x) + \ldots \]
Hypercentral Model

\[ H_{3q} = 3m + \sum_{i=1}^{3} \frac{p_i^2}{2m} + V(x) + H_{hyp} \]

M. Ferraris, M. M. Giannini, M. Pizzo, S. Santopinto, L. Tiator, Phys. Lett. B364 (1995), 231

- \( V(x) = -\frac{\tau}{x} + \alpha x; \quad H_{hyp} = A \left[ \sum_{i<j} V^S(r_i, r_j) \sigma_i \cdot \sigma_j + \text{tensor} \right] \)

- 3 parameters \( \tau, \alpha, A \) ← fixed to the spectrum, \( m = \frac{M}{3} \)

---

\[ \begin{array}{c c c}
N & \Delta \\
1900 & \quad & \quad \\
1700 & \quad & \quad \\
1500 & \quad & \quad \\
1300 & \quad & \quad \\
\end{array} \]

\( \tau = 4.59 \)
\( \alpha = 1.61 \text{ fm}^{-1} \)
\( A \leftarrow (N - \Delta) \)
3-quark lattice potential

G.S. Bali
Phys. Rep. 343, 1 (2001)
Electromagnetic properties

- Photocouplings
  M. Aiello et al., PL B387, 215 (1996)

- Helicity amplitudes (transition f.f.)
  M. Aiello et al., J. of Phys. G24, 753 (1998)

- Elastic form factors of the nucleon
  M. De Sanctis et al., EPJ A1, 187 (1998)

- Structure functions
  to be published

Fixed parameters $\rightarrow$ predictions
$A_{1/2}^P N(1535) S11$
Dynamical model (Mainz group)

--- pion cloud contribution

is included in phenomenological approach

but not in the constituent quark model

- $Q^2$ dependent fit (superglobal fit)
- hypercentral constituent quark model
please note

- the calculated proton radius is about 0.5 fm (value previously obtained by fitting the helicity amplitudes)
- not good for elastic form factors
- there is lack of strength at low $Q^2$ (outer region) in the e.m. transitions
- emerging picture: quark core (0.5 fm) plus (meson or sea-quark) cloud
ELASTIC FORM FACTORS

1. - Relativistic corrections to the elastic form factors

2. - Results with the semirelativistic CQM

3. - Introduction of Quark form factors
1.- Relativistic corrections to form factors

- Breit frame
- Lorentz boosts applied to the initial and final state
- Expansion of current matrix elements up to first order in quark momentum
- Results

\[
A_{\text{rel}}(Q^2) = F \ A_{n,\text{rel}}(Q^2_{\text{eff}}) \\
F = \text{kin factor} \\
Q^2_{\text{eff}} = Q^2 \left(\frac{M_N}{E_N}\right)^2
\]
Elastic Form Factors in the hCQM

M. De Sanctis, M.M. Giannini, L. Repetto, E. Santopinto, Phys. Rev. C 62, (2000) 025208.

\[ \mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} \]

\[ Q^2 \text{(GeV)} \]

calculated

NO QUARK FORM FACTORS

\[ T_{NR}, \text{ BOOSTS, CURRENT EXPANSION} \]
2.- Results with the semirelativistic kinetic energy

M. De Sanctis, M. Giannini, E. Santopinto, A. Vassallo,
Nucl. Phys. A 755, 294 (2005)

- Relativistic kinetic energy
- Boosts to initial and final states
- Expansion of current to any order
- Conserved current
\[ H_{3q} = \sum_{i=1}^{3} \sqrt{p_i^2 + m^2} + V(x) + H_{hyp} \]

V(x) = - \frac{\tau}{x} + \alpha x

\[ V(x) = - \frac{\tau}{x} + \alpha x \]

\[ \alpha \text{ and } \tau \text{ not much different from the NR case} \]
Calculated values!
3.- Quark form factors

M. De Sanctis, M. Giannini, E. Santopinto, A. Vassallo,
Nucl. Phys. A.755,294(2005)

• Quark form factors are added into the current

\[ ff = F_{\text{intr}} \ast \text{quark form factors} \]

\[ \text{semirelativistic hCQM input (calculated)} \]

• Monopole + dipole form for qff

• Fit of:

  – \( G_{E}^{p}/G_{M}^{p} \) ratio (polarization data)
  – \( G_{M}^{p}; G_{E}^{n}; G_{M}^{n} \)
Conclusion

• The hCQM provides a consistent framework for the description of all the 4 nucleon electromagnetic form factors

• Relativity is crucial in explaining the decrease of the ratio $G_E/G_M$

• Quark form factors are necessary in order to get a good reproduction of the $Q^2$ behaviour