SINKHOLE VULNERABILITY MAPPING: RESULTS FROM A PILOT STUDY IN NORTH CENTRAL FLORIDA

Clint Kromhout  
Florida Department of Environmental Protection, Office of the Florida Geological Survey, 3000 Commonwealth Blvd., Suite 1, Tallahassee, FL 32303, Clint.Kromhout@dep.state.fl.us

Alan Baker  
Florida Department of Environmental Protection, Office of the Florida Geological Survey, 3000 Commonwealth Blvd., Suite 1, Tallahassee, FL 32303, Alan.Baker@dep.state.fl.us

Abstract  
At the end of June in 2012, Tropical Storm Debby dropped a record amount of rainfall across Florida which triggered hundreds, if not thousands, of sinkholes to form which resulted in tremendous damage to property. The Florida Division of Emergency Management contracted with the Florida Department of Environmental Protection’s Florida Geological Survey to produce a map depicting the state’s vulnerability to sinkhole formation. The three-year project began with a pilot study in three northern Florida counties: Columbia, Hamilton and Suwannee. Utilizing the statistical modeling method Weights of Evidence, results from the pilot study yielded a 93 percent success rate of predicting areas where the geology is conducive to sinkhole formation. Lessons learned and field mapping techniques developed during the pilot study are now being applied to map the entire State’s vulnerability to sinkhole formation.

Introduction  
Florida is underlain by several thousand feet of carbonate rock (limestone and dolostone) with a variably thick mixture of sands, clays, shells, and other near surface carbonate rock units. These several thousand feet of carbonate rocks are host to one of the world’s most productive aquifers, the Floridan aquifer system. Natural erosional processes, both physical and chemical, have acted upon these carbonate rocks as water flows through them, both horizontally and vertically, dissolves and physically erodes the rock. Those erosional processes create cavities within the rock. The dissolution and cavity collapse within the rocks has created Florida’s karst topography which is characterized by sinkholes, swallets, caves (wet and dry), springs, disappearing / reappearing streams, and subterranean groundwater flow.

The Florida Geological Survey (FGS) was contracted by the Florida Division of Emergency Management (FDem) to produce a map depicting the State’s vulnerability to sinkhole formation following a mass sinkhole event triggered by record rainfall from Tropical Storm Debby in June of 2012. The three-year project began with a pilot study in three northern Florida counties: Columbia, Hamilton and Suwannee. Prior to Tropical Storm Debby’s record rainfall, the state had been experiencing a multi-year drought leading to reduced groundwater levels within the Floridan aquifer system. The leading hypothesis is that cavities which may have normally been water-filled had developed unsaturated air space. The lack of hydrostatic buoyancy meant the overburden (the sands and clays over the carbonate rocks) no longer had adequate support and collapsed when the record rainfall from TS Debby added increased hydrostatic loading and lubrication of overburden soils by rising groundwater levels over a very short time period (Figure 1).

Sinkholes are a geological hazard that place people’s property and even lives at risk. Vulnerability of an area to sinkhole formation is dependent upon both natural

Figure 1. Groundwater levels in Suwannee County (Live Oak, FL) prior to and post Tropical Storm Debby.
As Florida's population continues to increase, the potential for encountering a sinkhole hazard increases.

Current sinkhole hazard maps that are available to Florida Department of Emergency Management (FDEM) are insufficient and poorly substantiated by available geologic data. The FDEM presently relies on two sources of activity: 1) a non-scientific qualitative self-assessment of risk reported by each county, and 2) publically-available and statistically-biased subsidence incidence reports that are broadly generalized to the scale of entire counties without application of a scientifically defensible method.

The FGS maintains a database of voluntarily reported subsidence incidents, which are largely unverified reports of sinkholes; however, other subterranean events can cause holes, depressions or subsidence of the land surface that may mimic sinkhole activity. These include 1) subsurface expansive clay or organic layers which compress as water is removed, 2) collapsed or broken sewer and drain pipes or broken septic tanks, 3) improperly compacted soil after excavation work, and 4) buried trash, logs and other debris. Often a depression is not verified by a licensed professional geologist or engineer to be a true sinkhole, and the cause of subsidence is not known. As such, one of the primary goals of the pilot project was to map existing and recently formed sinkholes for usage as model training point sites.

Criteria for pilot study area site selection

Two important criteria for pilot area selection are geologic diversity and contrast. In order for a model to be tuned and validated using a pilot study in preparation for statewide application, the area chosen must contain a broad range of diversity and contrast.

Geologic diversity is defined as an area that contains multiple geomorphic terrains and districts. Terrains are small geomorphic areas which contain similar landforms formed under similar processes. A terrain is a sub-unit area to a larger area termed a district. Districts are larger generalized regional geomorphic areas which formed under similar processes. The pilot study area (Suwannee, Hamilton, and Columbia) (Figure 2) contains seven geomorphic terrains (Figure 3) and three geomorphic districts (Figure 4). In general, the greater the number of terrains and districts an area contains, the greater the underlying geologic diversity.

Geologic contrast is defined as an area which contains both variable overburden sediment thicknesses and content types and variable depths to carbonate rock. Two
Closed topographic depressions serve as targets to visit which may be sinkholes. Those identified sites were termed “points of interest” (POI). POIs were researched using GIS from which a POI GIS layer was created. GIS layers typically used during that process were: digital elevation models (DEM), LiDAR (light detection and ranging) high resolution elevation data, closed topographic depressions (CTDs) (DEM and LiDAR derived), streams, swallets, springs, geology, aerial imagery, the Florida National Hydrologic Dataset (NHD), and subsidence incident reports (SIRs) (a database of unverified sinkholes maintained by the FGS). The POIs serve two purposes. First, POIs may be used as training point sites for future modeling, provided that field investigations find them to be sinkholes. Second, the complete set of POIs serves as a planning tool that helps guide systematic and efficient navigation of the field area.

In order to assure adequate spatial coverage of the pilot study area, the study area was split into two primary grids: a 10 kilometer grid and a one kilometer grid. With in each 10 kilometer grid cell a minimum of four POIs were identified for onsite visitation by field staff. When possible, more POIs were identified within a 10 kilometer cell. The one kilometer grid cells were used as an arbitrary minimum spacing between each POI within a 10 kilometer cell. There was no limit to the number of sites documented, although effort was made to traverse at least a kilometer before documenting another site.

Field Methods
Field work within the pilot study area was conducted over thirteen days from early November, 2013 through the end of March, 2014. While navigating from one POI to the next, field staff would visually survey both their physical surroundings and the GIS data looking for clues potentially indicating the presence of a sinkhole. If a potential sinkhole was identified or sighted, that site was added as a POI within the associated GIS POI dataset. Effort was then made to investigate that POI.

When on site, efforts were first made to determine whether or not the POI being observed was truly a sinkhole within the best professional judgment of field staff. If the POI was determined not to be a sinkhole, then notes were made in the comments field of the POI GIS layer indicating such. Identification of non-sinkhole features which mimic the topographic profile of a sinkhole were equally important to documenting actual sinkholes. Non-sinkhole features identified during fieldwork included: old rock quarries, old hard-rock phosphate mine pits, borrow pits, test pits, dug drainage ponds, decomposing tree roots and root mats, and cypress domes.
When a POI was judged to be a sinkhole, data was collected and entered into a GIS shapefile. Those data included a GPS location, photos, general comments, and dimensions (which were recorded either via tape measure, laser range finder or measured in GIS using a DEM or LiDAR layer). In some instances, the size of a sinkhole was subjective because the sinkhole’s dimensional boundaries may have been 1) part of a nested cluster which had begun to coalesce, 2) was partly or completely within a stream channel, 3) was obscured by thick vegetation impeding measurement, or 4) infrastructure, such as roads, were built through it and made the sinkhole’s dimensional boundaries difficult to discern. In other instances, some measurements were not able to be made because safety was a concern due to the dangers of the sinkhole itself, livestock, or passing vehicles. When possible, in those instances diameters and/or depths were read from DEM or LiDAR datasets. At some sites multiple sinkholes were documented, and in those circumstances attempts were made to record a range of dimensions under a singular POI site. All measurements were recorded in feet. Distances measured via laser range finder registered in yards and were converted to feet.

**Documented Sinkholes**
Within the pilot study area a total 236 POI sites were visited. 207 of those sites were determined to be sinkholes. The remaining 29 were depressional features determined not to be sinkholes such as old hard-rock phosphate mine pits, borrow pits, test pits, dug drainage ponds, and cypress domes, which can all have circular to semi-circular map profiles and have closed topographic depression elevations. The median diameter of the documented sinkholes was 25.9 meters (85 feet). Diameters ranged from 0.6 meters to 179 meters (2 feet to 587 feet). The median depth was 4.6 meters (15 feet). Depths ranged from 0.3 meters to 18.3 meters (1 foot to 60 feet).

**Observations for future project mapping**
An important goal of the pilot project was to identify a set of karst feature field observations by which sinkholes could be differentiated from similar shaped features non-karst features. Those observations would then be used in the statewide mapping phase of the project. A two-year timeline to map the whole state and a limited project budget meant using investigative geophysical surveys or drilling could not be employed to further confirm if a depression was truly a sinkhole below the subsurface. Therefore, the karst feature field observations are important in order to map existing and recently formed sinkholes with reasonable confidence.

**Karst feature field observations**
Without technical on-site subsurface investigation, only professional judgment can be used to make a reasonable determination based upon known information and site observations.

**Observations**
- Depression has complete topographic closure (i.e. once a liquid or sediment crosses the topographic threshold it cannot flow out)
- Signs of surficial deformation past & present
  - Vertical to sub-vertical soil cracks concentric to depression’s perceived center point which may create a complete to partial ring around the depression
  - Sagged(ing) ground in relation to the near-vicinity ground surface topography
  - Soil creep or slumped(ing) soil
  - Arcing trunks of trees and shrubs attempting to re-straighten to vertical orientation within the depression perpendicular to depression’s perceived center point, indicating soil creep
  - Trees, shrubs, or other vertical features that are leaning or sagging into the depression
  - Exposed rock or semi-indurated sediments which otherwise woul n’ot be exposed at near-vicinity ground surface topography
  - Water marks on foliage indicating the depression is actively internally draining
  - Water flow marks on ground which orient sediments or foliage litter towards the lowest elevation(s) within the depression
  - Vegetation showing signs of stress or dying within the depression

**Model explanation, Weights of Evidence modeling technique**
Use of the Weights of Evidence (WofE) modeling technique involves the combination of diverse spatial data that are used to describe and analyze interactions and generate predictive models (for a detailed discussed of this statistical modeling technique see Bonham-Carter, 1994 and Raines et al., 2000). WofE is a data-driven process that relies on mathematical relationships between known occurrences as model training sites to create maps from weighted continuous input data layers. These input data layers, known as evidential themes, are then combined to yield an output data layer (or result of the model), known as a response theme (Raines, 1999). WofE was adapted to mineral potential mapping in a GIS and is based on the application of Bayes’ Rule of Prob-
ability, with an assumption of conditional independence, which occurs when an evidential theme does not affect the probability of another evidential theme (Raines et al., 2000). Although Bayesian theory has been applied to ground-water related issues in recent years (e.g., Soulsby et al., 2003; Meyer and Nicholson, 2003; and Feyen et al., 2004), the specific application of WofE to the potential for sinkhole formation is not known.

When applied in this project, WofE was used to generate sinkhole favorability response themes (expressed in probability maps). These response themes were generated in the Environmental Systems Research Institute (ESRI) ArcGIS version 10.2 environment. WofE was executed using the Spatial Data Modeler Tools (ArcSDM toolbox) which is public domain and available through the ESRI arcscripts pages. The fundamental approach and basic nomenclature of WofE is described in the following sections.

**Study Area**

The initial step in implementing a WofE model is the identification and delineation of a study area extent (i.e., pilot county boundaries). This is a critical step since the area identified is used in the calculation of weights and probabilities throughout the modeling process.

**Training Sites Theme and Prior Probability**

Training sites are locations of known features, also known as occurrences in the literature. In mining applications for example, existing mines are known as occurrences. In an aquifer vulnerability assessment, wells with water quality indicative of high recharge are potential known occurrences. In this study, existing or known, true karst features are considered occurrences. Training points are used in WofE to calculate the following parameters: prior probability, weights for each evidential theme, and posterior probability of the response theme.

Training points are converted to represent a unit area of the study area, such as a grid cell within a GIS application. For the sinkhole favorability model, each cell size represents one square kilometer. The prior probability is calculated by dividing the training point unit area (total number of training points multiplied by 1 km) by the total study area and represents the probability that a training point will occupy any given unit area within that study area, independent of any evidential theme data. In less complex terms, the prior probability is based on prior knowledge of the problem without the benefit of supporting evidence. In the sinkhole study example, prior probability could be described as the proportion of known sinkholes within the study area.

**Evidential Themes**

An evidential theme is defined as a set of continuous spatial data that is associated with the location and distribution of known occurrences, i.e., training points. In GIS terms, an evidential theme is analogous to a data layer or coverage. Evidential themes in the mining example might include the location of hydrothermal ore deposits or proximity to faults. In the sinkhole project, proximity to closed topographic depressions and overburden thickness are examples of evidential themes.

Weights calculated in WofE establish spatial associations between training points and the evidential themes. Depending on the data comprising an evidential theme, in order to deal with random processes, it may be necessary to re-classify the data into categories prior to analysis. This is completed by grouping large sets of data into fewer, more manageable categories that are meaningful. For example, if an evidential theme consisted of a data layer of confining unit thickness divided into one-foot thickness intervals, it might be necessary to classify the data into 3 meters (10ft.) or 6.1 meters (20ft.) intervals to generalize the dataset and make it more manageable and can maximize the spatial association between the map pattern and the pint pattern.

Weights are calculated for each evidential theme based on the presence or absence of training points with respect to the study area. A positive weight is calculated for areas that have more points than would be expected by chance; the weight is associated with occurrence of evidence. Conversely, a negative weight would be calculated for areas that have fewer points than expected; the weight is not associated with occurrence of evidence (or non-evidence). A weight of zero indicates that there is no association between training points and the evidential theme, or that the evidential theme is not a discriminating layer.

While performing the initial sinkhole pilot study several data sets were evaluated but not used because they were not discriminating and therefore added nothing to the model. This reaffirms the idea of using a data-driven model versus an expert knowledge model in that two of the layers that were deemed logical as predictors of favorable areas for sinkhole formation did not, in reality, work. These were themed layers depicting the distance to surface streams or surface water bodies since karstic areas are internally drained. Swallets and streams may appear in sinkhole prone areas but they are often dry streams and only flow during heavy rainfall events. The logic is that sinkholes are strongly associated with areas that do not have streams or surface water features. It
turns out that some of the water features are sinkholes that breach the water table and are classified as lakes. It may be more accurate to classify water filled sinks differently or look at density of water bodies based on area instead of the presence or absence of either feature. It is also worth noting that the data layers, in their current state, were insufficient as predictor maps and therefore were excluded from this analysis.

Weights can be calculated using three distinct methods: categorical, cumulative ascending, or cumulative descending. The categorical method is used to calculate weights for evidential themes where the theme’s values are not ordered (e.g., a geologic map). The cumulative ascending method is used to calculate cumulative weights in a proximity analysis. In this case areas nearest a training point have a strong association while those farther away have a weak association. In this method, areas represented by smaller values of an evidential theme have a stronger association with training points, and those represented by larger values of an evidential theme have a weaker association with training points. Area and number of points are determined cumulatively from the first class to the last class. This method is applicable for themes where the points are mainly associated with the lower values of the evidential theme (e.g., overburden thickness). The cumulative descending method is used to calculate the cumulative weights from the last class to the first class in the opposite way of cumulative ascending. This method is applicable for themes where the points are mainly associated with the higher values of the evidential theme (e.g., soil hydraulic conductivity).

Generalization of evidential themes follows calculation of weights in the WofE modeling process. Themes are generalized in an effort to establish which areas of the evidential layers share a greater association with locations of training points. During the calculation of weights for each evidential theme, a contrast value is calculated, which is the difference between the positive and negative weights (positive weight – negative weight) described above. Contrast is a measure of a theme’s significance in predicting the location of training points and helps to determine the threshold or thresholds that maximize the spatial association between the evidential theme map pattern and the training point theme pattern (Bonham-Carter, 1994).

Confidence of the evidential theme is also calculated for each class, and equals the contrast divided by its standard deviation (Studentized contrast) for a given evidential theme. Confidence provides a useful measure of significance of the contrast due to the uncertainties of the weights and areas of possible missing data (Raines, 1999). Also, a contrast value that is significant, based on its confidence, suggests that an evidential theme is a useful predictor of training points. Evidential themes that do not meet the minimum confidence level of significance are not included in the models.

Following the calculation of weights, contrast is used as a threshold to generalize or break evidential themes into categories. These breaks delineate which areas of the model for each evidential layer within the study area have more association with the training points. The simplest and most common method of categorizing an ordered evidential theme is to select the maximum contrast as a threshold to determine where to place a break in the evidential data theme thereby creating two categories: one with strong(er) association with the training point theme and one with weak(er) association with the training point theme. In a few cases, more complex statistical contrast patterns are inherent in the data and may justify the creation of multiple classes in the evidential theme data.

Response Theme
Following the generalization of evidential themes, WofE output results are generated and are known as response themes. A response theme is an output data layer showing the probability (posterior probability) that a unit area contains a training point based on the evidence (evidential theme) provided. Areas of higher posterior probability indicate that an area is more likely to contain a training point, whereas areas of lower posterior probability indicate that an area is less likely to contain a training point. As it relates to the sinkhole mapping project, a response theme can be understood as a favorability map that is displayed in classes of relative favorability based on selected karst features used as training points.

A response table is generated during calculation of each response theme and that table contains a list of evidential themes and their respective weights, contrast and confidence (of the evidential theme generalized break). In general, a positive weight (W1) for an evidential theme indicates areas where training points are likely to occur, while a negative weight (W2) for an evidential theme indicates areas where training points are not likely to occur. Contrast is the difference between the highest and lowest weights and is a measure of how well an evidential theme predicts training points. Contrast is also used to rank the evidential themes. Higher contrast values indicate those evidential themes that best predict training point locations and which are more important in the model. Conversely, a negative weight that is stronger than a positive weight indicates that an evidential theme is a better predictor of where training points are not like-
ly to occur (i.e., low favorability) as opposed to where they were likely to occur.

**Pilot study area results**

A preliminary favorability map of the Weights of Evidence Model was generated using four evidential themes that showed the strongest association with the training point theme and therefore were considered the strongest for predicting sinkhole areas. Those layers were overburden (Figure 5), proximity to closed topographic depressions (Figure 6), a layer depicting the difference between the water-table surface and the top of limestone (Figure 7) and soil hydraulic conductivity that utilizes the weighted average of the soil column thickness (Figure 8).

Each of the model evidential layers were calculated against the study area training points. A calculated weights table was used to pick the break between areas that are associated with training sites and areas not asso-

**Figure 5.** Thickness of overburden on top of limestone surface: Layer showing the thickness of overburden on top of limestone units susceptible to dissolution. This layer that showed the strongest association with the training sites. In areas where the overburden was 32.3 meters (106ft.) or less in thickness (in red) are considered more closely associated with sinkhole formation. Areas with overburden thicknesses greater than 32.3 meters (106ft.) are not associated with the training sites.
Figure 6. Proximity to closed topographic depressions: Layer showing areas that are proximal to closed topographic depressions. Closed topographic depressions were taken from the USGS 1:24,000 topographic maps and filtered by their circularity index. The layer that showed the strongest association with active karst areas has a circularity index of 0.9 (or 90 percent round) when compared to the area of a circle with the same perimeter. The resulting polygon layer was buffered and then intersected with the training sites in order to show areas that are and are not associated with sinkholes. Red areas are more associated and are generally less than 1,390 meters (4,560.4 ft.) away. Areas with values more than 1,390 meters (4,560.4 ft.) are not associated with training sites.
Figure 7. Difference between groundwater level and the top of limestone: Top of limestone data points are used to create a layer depicting the surface of limestone that is susceptible to dissolution. The layer was subtracted from a groundwater level surface and then intersected with training sites to show areas that are and are not associated with sinkholes. Red areas are more associated with training sites and have groundwater levels that are generally 0 - 1.5 meters (0 - 5ft.) from the top the limestone. Areas with values more than 0 - 1.5 meters (0-5ft.) are not associated with training sites.
Figure 8. Soil hydraulic conductivity (weighted average): Soils data is intersected with training sites to show areas that are and are not associated with the training point dataset. Red areas are more associated and have hydraulic conductivity rates of 207 millimeters per hour (8.15 inches per hour) and greater. Areas with hydraulic conductivity values between 131.1 and 206.8 millimeters per hour (5.16 and 8.14 inches per hour) are moderately associated with training sites and areas with conductivity values less than 130.8 millimeters per hour (5.15 inches per hour) are not associated with training sites.
Overburden thickness was calculated by taking the top of limestone surface and subtracting it from land surface. Values in the pilot study area ranged from 95.1 meters (312 ft.) thick in the extreme northeastern portion of the region to 0 meters (0ft.) which occurs mostly in the lower lying areas along the major area rivers. Intersecting the training sites with this evidential layer revealed that training sites occurred in areas with 32.3 meters (106ft.) or less of overburden (Figure 5).

Closed topographic depressions are taken from the United States Geological Survey (USGS) 1:24,000 topographic maps and are the hachured closed isolines on the map. The depression features were filtered based on an index of circularity or circular index (Denizman, 2003). Since sinkholes tend to be highly circular, filtering by circularity index allows for the removal of closed topographic depressions that are highly linear (e.g., a drainage ditch). The circularity index of a feature is the ratio of the area of a perfect circle with the same perimeter as the closed depression.

The circularity value is displayed as a ratio where 1.0 is a perfect circle and lower values are more elongated. For the WofE analysis, multiple circularity index values were queried and buffered. Values investigated ranged from 0.5 up to 0.9. Ultimately, closed topographic features with a circularity index of 0.9 or greater had the strongest association with the training point sites (Figure 6).

In some instances multiple layers can be combined into a single layer to select for complex interactions between layers. For example, the difference between the top of limestone layer and the top of the potentiometric surface are two layers that have been combined into a single evidential theme. The combined layer references the difference between water table surface and top of limestone. The complex layer helps reveal the areas in the pilot area where the top of soluble rock is near the potentiometric surface. Presumably, this is a zone where the hydraulic pumping of the aquifer is most pronounced, thereby actively flushing sediments from cavities within the underlying soluble limestone rock layers (Figure 7).

The rate at which water moves through the soil can be an important factor in locating areas favorable to sinkhole formation. Soil hydraulic conductivity is the ability of the soil to transmit water. Soil hydraulic conductivity values were calculated for each soil horizon based on its thickness and applied to the entire soil column. Values ranged from 7.6 millimeters per hour (0.30 inches per hour) to 887.7 millimeters per hour (34.95 inches per hour).

Based on calculated weights, the soil hydraulic conductivity theme had justification for a multiple class generalization. One class that is strongly associated with known occurrences of sinkholes, one that is moderately associated, and lastly one that is not related to sinkhole formation (Figure 8).

The four evidential themes were combined in the WofE model to build the response theme, shown in Figure 9. The model revealed a strong contrast depicting areas with favorable sinkhole formation. An independent set of data points, called the Subsidence Incident Report (SIRs) database was brought in as a way of analyzing the results of the model (Figure 10). In the pilot study area there were 261 total sites reported. Of those, 163 or 62 percent fell in the highest favorability category. Another 81 sites or 31 percent were in the highly possible areas. Conversely only 1 of the 261 sites reported fell in an area determined to be unlikely. Overall the model is a better predictor of where the geology is not favorable for sinkhole formation than where the geology is favorable.
Figure 9. Results from preliminary pilot study with training sites: Weights of evidence output map from combining the four evidential themes; overburden, proximity to closed depressions, difference between water table aquifer and top of rock and soil hydraulic conductivity.
Figure 10. Results from preliminary pilot study with Subsidence Incident Reports: The Subsidence Incident Reports data was used to analyze the results of the modeled WoFE response theme. 93 percent of the reports fell into the highest and highly probable areas where the geology is favorable to sinkhole formation.
References
Bonham-Carter GF. 1994. Geographic Information Systems for Geoscientists, Modeling with GIS: Oxford, Pergamon. 398 p.
Denizman C. 2003. Morphometric and spatial distribution parameters of karstic depressions, Lower Suwannee River Basin, Florida: Journal of Cave and Karst Studies 65 (1): 29-35.
Feyen L, Dessalegn AM, DeSmedt F, Gebremeskel S, Batelaan O. 2004. Application of a Bayesian Approach to Stochastic Delineation of Capture Zones: Ground Water 42 (4): 542-551.
Meyer PD, Nicholson TJ. 2003. Analysis of hydrogeologic conceptual model and parameter uncertainty, in Mishra, S., editor, Symposium on Groundwater quality modeling and management under uncertainty: Reston, VA, American Society of Civil Engineers, Conference Proceedings, p. 47-57.
Raines GL. 1999. Evaluation of Weights of Evidence to Predict Epithermal-Gold Deposits in the Great Basin of the Western United States: Natural Resources Research 8 (4): 257-276.
Raines GL, Bonham-Carter GF, Kemp L. 2000. Predictive Probabilistic Modeling Using ArcView GIS: ArcUser 3 (2): 45-48.
Soulsby C, Petry J, Brewer MJ, Dunn SM, Ott B, Malcolm IA. 2003. Identifying and assessing uncertainty in hydrological pathways; a novel approach to end member mixing in a Scottish agricultural catchment. Journal of Hydrology 274 (1-4): 109-128.