Probing thermoelectric transport with cold atoms

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We propose experimental protocols to reveal thermoelectric and thermal effects in the transport properties of ultracold fermionic atoms, using the two-terminal setup recently realized at ETH. We show in particular that, for two reservoirs having equal particle numbers but different temperatures initially, the observation of a transient particle number imbalance during equilibration is a direct evidence of thermoelectric (off-diagonal) transport coefficients. This is a time-dependent analogue of the Seebeck effect, and a corresponding analogue of the Peltier effect can be proposed. We reveal that in addition to the thermoelectric coupling of the constriction a thermoelectric coupling also arises due to the finite dilatation coefficient of the reservoirs. We present a theoretical analysis of the protocols, and assess their feasibility by estimating the corresponding temperature and particle number imbalances in realistic current experimental conditions.

Keywords: Quantum transport, ultracold Fermi gases, thermoelectricity

Thermoelectricity has been a recurrent theme in condensed matter physics. The increasing demand for sustainable energy sources, as well as progress in materials science, have triggered a marked increase of interest and research in the subject over the past two decades [37–39, 40]. A better understanding of fundamental processes controlling thermal and thermoelectric transport is likely to bring progress in the field at large.

Ultra-cold atomic gases offer a remarkably clean and controllable set-up to investigate interacting quantum systems. Phenomena involving the transport of atoms have been the focus of several experiments in this context, for example probing the reaction to external forces [5, 6] as observing Bloch oscillations in optical lattices [7, 8], or the expansion dynamics of atoms in disordered [9–11] and lattice potentials [12]. The transport of impurities [13–15] and spin diffusion [16] have also been investigated. Recently, increasing interest has been devoted to closer analogues of mesoscopic transport devices, e.g. modelling ‘quantum pumps’ and ‘batteries’ [17–19]. In a recent experiment transport of fermionic atoms between two reservoirs connected through a tunable constriction was realized [20].

In this letter, we go beyond atom transport and design a proposal to observe both thermal and thermoelectric transport in the context of cold atoms, in the two terminal geometry of Ref. [20] (Fig. 1). We suggest protocols specifically geared at revealing the off-diagonal transport coefficients controlling thermoelectric effects (inset of Fig. 2). These protocols can be viewed as time-dependent analogues of the Seebeck and Peltier effects. We estimate the magnitude of the expected effects in the simplest case of non-interacting fermions, for a ballistic or diffusive constriction with different geometries. We conclude that the proposed effects should be experimentally observable in the currently available set-up.

The experimental realization of thermal and thermoelectric transport in cold atomic gases would open many possibilities. A particularly appealing opportunity offered by cold atomic gases is the study of these effects i) in the regime of high temperature where \( T \) is a sizeable fraction of the Fermi temperature \( T_F \) and ii) in the absence of any phonon excitations. Purely electronic contributions to high-temperature transport and thermopower (as captured e.g. by Heike’s formula) have been often discussed in the context of materials with strong electronic correlations [21, 22] and are directly relevant to thermoelectric properties of oxide materials for example. In the solid state context however, separating the different contributions of electrons and phonons is highly involved. A setup in which phonon contributions can be suppressed is therefore invaluable to reach a deeper fundamental understanding of thermoelectric transport of quantum interacting particles, which may well in turn provide guidance for better solid-state materials design.

Figure 1. Two-terminal transport setup. The left (L) and right (R) reservoirs are characterized by their temperatures \( T_L, T_R \), chemical potentials \( \mu_L, \mu_R \) and their thermodynamic coefficients \( \kappa, C, \alpha \), cf. Eq. (1). The constriction (region between dashed lines) is characterized by its matrix of linear response coefficients \( \mathcal{L} \). Due to the different temperatures and chemical potentials of the reservoirs, a particle \( (I_N) \) and entropy \( (I_S) \) current flow in the constriction (in our conventions, \( I > 0 \) corresponds to a flow from right to left).

General framework. The transport setup under consid-
eration is depicted in Fig. 1. Two reservoirs of fermionic atoms are connected by a constriction. The reservoirs are characterized by their temperature $T_{L,R}$ and their chemical potential $\mu_{L,R}$ which determine the particle number $N(T, \mu)$ and entropy $S(T, \mu)$ of each reservoir through the grand-canonical equation of state. Important thermodynamic properties of the reservoirs are the compressibility $\kappa$, the dilatation coefficient $\alpha$ and the heat capacity $C_\mu$ (at constant $\mu$ or $N$), defined by:

$$\kappa = \frac{\partial N}{\partial \mu} \bigg|_T, \quad \alpha = \frac{\partial N}{\partial T} \bigg|_\mu, \quad C_\mu,N = \frac{\partial S}{\partial T} \bigg|_{\mu,N}$$  \tag{1}$$

The particle and entropy currents flowing through the constriction $I_N = \frac{d}{dt}(N_L - N_R)$, $I_S = \frac{d}{dt}(S_L - S_R)$ are related to the (small) chemical potential and temperature differences $\Delta \mu = \mu_L - \mu_R$ and $\Delta T = T_L - T_R$ by:

$$\begin{pmatrix} I_N \\ I_S \end{pmatrix} = \mathcal{L} \begin{pmatrix} \Delta \mu \\ \Delta T \end{pmatrix}, \quad \text{with} \quad \mathcal{L} = \begin{pmatrix} L_{11} & L_{12} \\ L_{12} & L_{22} \end{pmatrix} \tag{2}$$

$L$ is the matrix of transport coefficients associated with the constriction. Its symmetry is insured by Onsager’s relations [24–26].

We consider a time-dependent process in which the reservoirs are prepared with given initial particle numbers and temperatures, and equilibrate through exchange of particles and entropy through the constriction.

Using (2) and the properties of the reservoirs given by (1), we derive equations ruling the time evolution of the particle and temperature imbalance:

$$\tau_0 \frac{d}{dt} \begin{pmatrix} \Delta N/\kappa \\ \Delta T \end{pmatrix} = -\mathcal{A} \begin{pmatrix} \Delta N/\kappa \\ \Delta T \end{pmatrix}, \quad \mathcal{A} = \begin{pmatrix} 1 & -S/\ell \\ -S/\ell & L+S^2/\ell \end{pmatrix} \tag{3}$$

These equations resemble the discharge of a capacitor (the reservoirs) in a resistor (the constriction), taking into account thermal transport as well. The characteristic time-scale $\tau_0 \equiv \kappa / L_{11}$ corresponds to the transport time-scale $RC$ in a capacitor, with $R = 1/L_{11}$ the resistance of the constriction and $\mathcal{L} (\sim \kappa)$ the capacitance [20]. The Lorenz number [27] of the constriction $L \equiv L_{22}/L_{11} - (L_{12}/L_{11})^2$ is the ratio $R/(TR_T)$ of the resistance to thermal resistance. The thermodynamic coefficient $\ell \equiv C_\mu/\kappa T - (\alpha/\kappa)^2 = C_N/\kappa T$ characterizes the reservoir and has a form similar to $L$. Finally, $S \equiv \alpha/\kappa - L_{12}/L_{11}$ is the effective thermoelectric (Seebeck) coefficient. Note that $S$ has the dimension of the Boltzmann constant $k_B$, while $L$ and $\ell$ have dimension $k_B^2$ and $\kappa$ the dimension of the inverse of an energy.

Two remarks are in order. (i) In the absence of any thermoelectric effect ($S = 0$), the time constants for particle and thermal relaxation are $\tau_0$ and $\tau_0 L/\ell$, respectively. At low temperature, the usual form of the Wiedemann-Franz law, when applicable, dictates that $L \rightarrow \pi^2/3$ and the thermodynamics of a free Fermi gas yields the same limit for $\ell \rightarrow \pi^2/3$. Hence, in this case, the Wiedemann-Franz law expresses the fact that the timescales for particle and heat relaxation are identical when $S = 0$, an interpretation which to our knowledge has not been formulated before.

(ii) The coupling between heat and particle transport (off-diagonal elements of $\mathcal{A}$) is determined by the effective Seebeck coefficient $S_r$. In the present context, this coefficient has two competing contributions: one from the constriction $S_c = -L_{12}/L_{11}$ and one from the reservoirs $S_r = \alpha/\kappa$. In particular, the presence of $S_r$ induces a coupling between thermal and electric transport even when the off-diagonal transport coefficient $L_{12}$ of the constriction can be neglected.

On Fig 3, we display $S_c$, $S_r$, $L$ and $\ell$ as a function of $T/T_F$, calculated within a Landauer-Büttiker [28–30] formalism for a diffusive constriction (see below for details). The plot illustrates the competition between the contribution of the reservoir $S_r$ and that of the constriction $S_c$. In the case of Fig. 3, this results in a positive (negative) total Seebeck coefficient in the absence (presence) of transverse harmonic confinement in the constriction. Note also on Fig. 3 the deviations from the Wiedemann-Franz law at finite temperature.

The general solution of Eq. (3) reads, given an initial particle and temperature difference $\Delta N_0$ and $\Delta T_0$:

$$\Delta N(t) = \left\{ \frac{1}{2} \left[ e^{-t/\tau_c} + e^{-t/\tau_+} \right] - \left[ 1 - L+S^2/\ell \right] \frac{e^{-t/\tau_c} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta N_0 + \frac{S_k}{\lambda_+ - \lambda_-} \left[ e^{-t/\tau_c} - e^{-t/\tau_+} \right] \Delta T_0 \tag{4}$$

$$\Delta T(t) = \left\{ \frac{1}{2} \left[ e^{-t/\tau_c} + e^{-t/\tau_+} \right] - \left[ L+S^2/\ell - 1 \right] \frac{e^{-t/\tau_c} - e^{-t/\tau_+}}{2(\lambda_+ - \lambda_-)} \right\} \Delta T_0 + \frac{S}{\kappa(\lambda_+ - \lambda_-)} \left[ e^{-t/\tau_c} - e^{-t/\tau_+} \right] \Delta N_0 \tag{5}$$

The inverse time-scales $\tau_{\pm}^{-1} = \tau_0^{-1} \lambda_{\pm}$ are given by the eigenvalues of the transport matrix $\mathcal{A}$

$$\lambda_{\pm} = \frac{1}{2} \left( 1 + \frac{L+S^2}{\ell} \right) \pm \sqrt{\frac{S^2}{\ell} + \left( \frac{1}{2} \frac{L+S^2}{\ell} \right)^2}. \tag{6}$$

In principle, eqs. (4) and (5) enable one to extract the thermodynamic and transport coefficients from experimental measurements. In particular, the Wiedemann-Franz law and its possible violations could be tested. However, thermoelectric effects are more difficult to extract. This is due to the fact that in the presence of both a particle and temperature imbalance at $t = 0$, the
time-evolution is typically dominated by the exponential decay involving the symmetric combination of exponentials, and terms responsible for off-diagonal transport are masked.

Hence, we propose two specific protocols in order to reveal thermo-electric effects, estimate the expected magnitude of the signal and confirm experimental feasibility.

Experimental proposal for off-diagonal (thermoelectric) transport. The first protocol focuses on the particle current induced by a temperature difference, and is a transient analogue of the Seebeck effect. The system is prepared with equal number of particles in the two reservoirs $\Delta N_0 = 0$, but with a temperature difference (see inset of Fig. 2). The temperature difference can for example be prepared by heating one of the reservoirs (e.g. by using laser light) with a closed constriction, which prevents particle transfer. After reopening the constriction, the off-diagonal coupling between particle and heat transport will lead to an evolution of the particle number difference given by:

$$\frac{\Delta N(t)}{N_0} = \frac{S T_F \kappa}{N_0} \frac{e^{-t/\tau_+} - e^{-t/\tau_-}}{\lambda_+ - \lambda_-} \frac{\Delta T_0}{T_F}, \quad (7)$$

where $N_0, T_F$ refer to reservoirs at equilibrium. Hence, a transient particle imbalance during equilibration is a ‘smoking gun’ observation revealing the existence of a non-zero Seebeck coefficient. Since typically $\tau_+ < \tau_-$, the particle imbalance first reaches an extremum at a time $t_{max} = \tau_0 (\ln \lambda_+ - \ln \lambda_-)/\lambda_+ - \lambda_-$ and then at long times relaxes exponentially to zero with a characteristic time $\tau_-$. This behaviour is exemplified in Fig. 2. The sign of the particle imbalance is given by the sign of the effective Seebeck coefficient and thus depends on the considered situation. If the reservoir effect is the dominant mechanism ($S > 0$), then particles tend at first to flow from the cold to the hot reservoir (see Fig. 2), due to its lower chemical potential. This behaviour is in contrast to the classical intuition of particles flowing from the high pressure (warmer) to the low pressure (colder) side. On the contrary, if the thermo-electric properties of the constriction dominate ($S < 0$), then particles first flow from the hot to the cold reservoir. In both cases the temperature imbalance equilibrates monotonically. As expected, the entropy current ($I_S = SI_N - \Delta T/R_G T$) is flowing from the hot to the cold reservoir, because the contribution of thermal diffusion to entropy flow (second term) always dominates over the thermo-electric contribution from particle transport (first term). A second complementary protocol can be proposed in which the system is prepared initially with a particle number imbalance but equal temperatures in the two reservoirs. This is the analogue of a Peltier experiment, in which temperature and particle number imbalances have reversed roles.

In order to estimate the magnitude of the expected effects and assess whether they can be observed experimentally, we have considered the simplest case of non-interacting fermions and computed the transport coefficients of the constriction using a Landauer-Büttiker formalism, in the spirit of mesoscopic physics [28–30]. In this framework, the transport coefficients $\mathcal{L}_{ij}$ of the constriction can be expressed in terms of the moments $M_n$ of an energy-dependent (dimensionless) transport function $\Phi\left(\epsilon\right)$ as: $\mathcal{L}_{11} = 2 M_0 / h$, $\mathcal{L}_{22} = 2 k_B^2 / h \cdot M_2 / (k_B T)^2$ and $\mathcal{L}_{12} = 2 k_B / h \cdot M_1 / (k_B T)$, with:

$$M_n = \int_{-\infty}^{\infty} d\epsilon \Phi\left(\epsilon\right) \left(-\frac{\partial f}{\partial \epsilon}\right) (\epsilon - \mu)^n \quad (8)$$

where $f$ is the equilibrium Fermi function. The precise form of the transport function $\Phi\left(\epsilon\right)$ depends on the dispersion relation and on the scattering mechanism in the constriction. We used a relaxation time approximation with an energy-independent scattering time, and considered either a ballistic or a diffusive constriction (both cases can be realized experimentally [20] using speckle noise of adjustable strength). The constriction is two-dimensional, with or without harmonic confinement in the transverse direction, and the reservoirs are taken as three dimensionally harmonically trapped non-interacting Fermi gases.

In order to observe the particle imbalance which is induced by the temperature imbalance, its amplitude should be sizable. For the typical parameters used in Fig. 2, the amplitude of the maximum $|\Delta N_0(t_{max})|/N_0$ is seen to exceed 10%, which makes it within reach of

![Figure 2](image-url)
current experimental setups [20]. To further quantify the effects, we define a parameter \( \eta \equiv (\Delta N(t_{\text{max}})/N_0) \cdot (T_F/\Delta T_0) \) which can be understood as a thermoelectric efficiency of the system. It compares the maximum relative particle imbalance to the initial temperature imbalance in units of the Fermi temperature. A similar quantity can be defined for the Peltier-like protocol, leading to an efficiency \( \eta_P = \eta/l \) (hence expected to be smaller than \( \eta \) since \( l > 1 \) - see Fig. 3). Figure 4 displays \( \eta \) as a function of \( T/T_F \), comparing four cases - ballistic and diffusive and with or without transverse confinement in the constriction. In particular, one sees that \( \eta \approx 50\% \) at a commonly reached experimental temperature \( T/T_F = 0.4 \) [20], meaning that an initial relative temperature imbalance \( \Delta T_0/T_F = 20\% \) with reservoirs containing \( N_0 = 100000 \) particles would lead to a maximal particle imbalance of the order of 10000 particles, which is experimentally sizeable. We note (Fig 4) that a very peculiar situation arises for a two-dimensional ballistic constriction with harmonic confinement. In this case, \( S_r \) and \( S_c \) perfectly compensate each other, leading to \( S = \eta = 0 \). This is due to the fact that, in that case (assuming an energy-independent collision time), the transport function \( \Phi(\epsilon) \) has exactly the same energy dependence than that of the density of states in the reservoirs \( \sim \epsilon^2 \). While this perfect cancellation relies on specific assumptions of our simplified modelling, the qualitative conclusion is expected to be robust: in this particular case the thermoelectric effects are expected to be quite small.

To summarize, we have shown that thermoelectric effects can be measured in cold atomic gases within the setup of Ref. [20]. In the current experimental temperature regime, offdiagonal transport properties arise from a combination of the thermoelectric properties of the constriction and the finite dilatation coefficient of the Fermi gas in the reservoirs. Fundamental questions about high-temperature transport can be addressed in this framework.

Inter-particle interactions have been neglected here, implicitly assuming that they have been turned off using a Feschbach resonance. A natural extension of this work is thus to include interactions, and to go beyond the simple Drude description of transport in the constriction. One can also think of reintroducing phonons in a controlled manner by simulating their action via a bosonic bath, an important ingredient for establishing contact with thermoelectric properties of solid-state materials. Other possible directions include the effect of a lattice distortion, or a modification of the geometry of the constriction, which would provide a fruitful relation to recent developments on thermoelectric devices in the mesoscopic physics context [31, 32].

Note added: As the writing of manuscript was being completed, we became aware of a recent preprint by H. Kim and D. Huse [33] considering spin and heat transport in a cold Fermi gas.

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[1] H. J. Goldsmid, *Introduction to Thermoelectricity*, Springer Series in Materials Science (Springer, Dordrecht, 2009)

[2] D. MacDonald, *Thermoelectricity: An Introduction to the Principles*, Dover books on physics (Dover Publications, 2006) ISBN 9780486456030

[3] G. Snyder and E. Toberer, Nat. Mater. 7, 105 (2008)

[4] J. Heremans and M. Dresselhaus, in *Nanomaterials handbook*, edited by Y. Gogotsi (CRC Press, 2006) Chap. 27, pp. 739–770

[5] D. S. Jin, J. R. Ensher, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. 77, 420 (Jul 1996), http://link.aps.org/doi/10.1103/PhysRevLett.77.420

[6] M.-O. Mewes, M. R. Andrews, N. J. van Druten, D. M. Kurn, D. S. Durfee, C. G. Townsend, and W. Ketterle, Phys. Rev. Lett. 77, 988 (Aug 1996), http://link.aps.org/doi/10.1103/PhysRevLett.77.988

[7] M. Ben Dahan, E. Peik, J. Reichel, Y. Castin, and C. Salomon, Phys. Rev. Lett. 76, 4508 (Jun 1996), http://link.aps.org/doi/10.1103/PhysRevLett.76.4508

[8] H. Ott, E. de Mirandes, F. Ferlaino, G. Roati, G. Modugno, and M. Inguscio, Phys. Rev. Lett. 92, 160601 (Apr 2004), http://link.aps.org/doi/10.1103/PhysRevLett.92.160601

[9] J. Billy, V. Josse, Z. Zuo, A. Bernard, B. Hambrecht, P. Lugan, D. Clément, L. Sanchez-Palencia, P. Bouyer, and A. Aspect, Nature 453, 891 (2008)

[10] G. Roati, C. D’Errico, L. Fallani, M. Fattori, C. Fort, M. Zaccanti, G. Modugno, M. Modugno, and M. Inguscio, Nature 453, 895 (Jun. 2008), ISSN 0028-0836, http://dx.doi.org/10.1038/nature07071

[11] S. S. Kondov, W. R. McGehee, J. J. Zirbel, and B. DeMarco, Science 334, 66 (2011), http://www.sciencemag.org/content/334/6052/66.abstract

[12] U. Schneider, L. Hackermüller, J. Ronzheimer, S. Will, S. Braun, T. Best, I. Bloch, E. Demler, S. Maudt, D. Rasch, et al., Nature Physics(2012), doi:10.1038/nphys2205

[13] A. P. Chikkatur, A. Gölitz, D. M. Stamper-Kurn, S. Inouye, S. Gupta, and W. Ketterle, Phys. Rev. Lett. 85, 483 (Jul 2000), http://link.aps.org/doi/10.1103/PhysRevLett.85.483

[14] S. Palzer, C. Zipkes, C. Sias, and M. Köhl, Phys. Rev. Lett. 103, 150601 (Oct 2009), http://link.aps.org/doi/10.1103/PhysRevLett.103.150601

[15] J. Catani, G. Lamporesi, D. Naik, M. Gring, M. Inguscio, F. Minardi, A. Kuntian, and T. Giannarchi, Phys. Rev. A 85, 023623 (Feb 2012), http://link.aps.org/doi/10.1103/PhysRevA.85.023623

[16] A. Sommer, M. Ku, G. Roati, and M. W. Zwierlein, Nature 472, 10 (2011), http://arxiv.org/abs/1101.0780

[17] K. K. Das and S. Aubin, Phys. Rev. Lett. 103, 123007 (Sep 2009), http://link.aps.org/doi/10.1103/PhysRevLett.103.123007

[18] B. T. Seaman, M. Krämer, D. Z. Anderson, and M. J. Holland, Phys. Rev. A 75, 023615 (Feb 2007), http://link.aps.org/doi/10.1103/PhysRevA.75.023615

[19] M. Bruderer and W. Belzig, Phys. Rev. A 85, 013623 (Jan 2012), http://link.aps.org/doi/10.1103/PhysRevA.85.013623

[20] J.-P. Brantut, J. Meineke, D. Stadler, S. Krinner, and T. Esslinger, Science 337, 1069 (2012), http://www.sciencemag.org/content/337/6098/1069.abstract

[21] P. M. Chaikin and G. Beni, Phys. Rev. B 13, 647 (Jan 1976), http://link.aps.org/doi/10.1103/PhysRevB.13.647

[22] R. Heikes and R. Ure, *Thermoelectricity: science and engineering* (Interscience Publishers, 1961)

[23] The definition of the currents differs by a factor two compared to [20].

[24] H. B. Callen, *Thermodynamics* (John Wiley & Sons, Inc., New York, N.Y., 1960)

[25] L. Onsager, Phys. Rev. 38, 2265 (Dec 1931), http://link.aps.org/doi/10.1103/PhysRev.38.2265

[26] L. Onsager, Phys. Rev. 37, 405 (Feb 1931), http://link.aps.org/doi/10.1103/PhysRev.37.405

[27] N. Ashcroft and N. Mermin, *Solid State Physics* (Saunders College, Philadelphia, 1976)

[28] Y. Imry and R. Landauer, Rev. Mod. Phys. 71, S306 (Mar 1999), http://link.aps.org/doi/10.1103/RevModPhys.71.S306

[29] M. Böttiker, IBM J. Res. Dev. 32, 317 (May 1988), ISSN 0018-8646, http://dx.doi.org/10.1147/rd.323.0317

[30] S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge Studies in Semiconductor Physics and Microelectronic Engineering) (Cambridge University Press, 1997) ISBN 0521599431

[31] J. R. Prance, C. G. Smith, J. P. Griffiths, S. J. Chorley, D. Anderson, G. A. Jones, I. Farrer, and D. A. Ritchie, Phys. Rev. Lett. 102, 146602 (Apr 2009), http://link.aps.org/doi/10.1103/PhysRevLett.102.146602

[32] J.-H. Jiang, O. Entin-Wohlman, and Y. Imry, Phys. Rev. B 85, 075412 (Feb 2012), http://link.aps.org/doi/10.1103/PhysRevB.85.075412

[33] D. Kim, H. & Huse, arXiv preprint 1208.6580 http://arxiv.org/abs/1208.6580