GPD’S AND SSA’S

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Generalized parton distributions involving transverse polarization are transversely deformed. The deformation of chirally odd GPDs is related to a transversity decomposition of the quark angular momentum. Potential consequences for T-odd single-spin asymmetries (Sivers and Boer-Mulders effects) are discussed.

1. Introduction

Hadron form factors provide information about the Fourier transform of the charge distribution within the hadron. Generalized parton distributions (GPDs) provide a momentum decomposition of the form factor w.r.t. the average momentum fraction \( x = \frac{1}{2}(x_i + x_f) \) of the active quark

\[
\int dx H_q(x, \xi, t) = F^q_1(t) \quad \int dx E_q(x, \xi, t) = F^q_2(t)
\]

where \( F^q_1(t) \) and \( F^q_2(t) \) are the Dirac and Pauli formfactors, respectively. \( x_i \) and \( x_f \) are the momentum fractions of the quark before and after the momentum transfer. The momentum direction of the active quark singles out a direction and it makes a difference whether the momentum transfer is along this momentum or in a different direction. GPDs thus not only depend on \( x \) and the invariant momentum transfer \( t \) but also on the longitudinal momentum transfer through the variable \( 2\xi = x_f - x_i \).

Since GPDs are the form factor of the same operator whose forward matrix elements yield the usual parton distribution functions (PDFs)

\[
\int \frac{dx^-}{2\pi} e^{ix^-p^+x} \left\langle p' \left| \bar{q} \left( -\frac{x^-}{2} \right) \gamma^+ q \left( \frac{x^-}{2} \right) \right| p \right\rangle = H(x, \xi, \Delta^2)\bar{u}(p')\gamma^+ u(p) + E(x, \xi, \Delta^2)\bar{u}(p')\frac{i\sigma^\nu\Delta^\nu}{2M}u(p).
\]

it is possible to develop a position space interpretations for GPDs.
2. Position Space Interpretation for GPDs

Charge distributions in position space are usually measured in the center of mass frame, i.e. relative to the center of mass of the system. For impact parameter dependent PDFs, the analogous reference point is the \( \perp \) center of momentum of all partons (quarks and gluons) \( \mathbf{R}_\perp = \sum_{i=q,g} x_i \mathbf{r}_\perp,i \), where \( x_i \) is the momentum fraction carried by each parton and \( \mathbf{r}_\perp,i \) is their \( \perp \) position. One can form eigenstates of \( \mathbf{R}_\perp \)

\[
| p^+, \mathbf{R}_\perp = 0_\perp, \lambda \rangle \equiv N \int d^2 p_\perp | p^+, p_\perp, \lambda \rangle.
\]

Impact parameter dependent PDFs are defined using the familiar light-cone correlation function in such transversely localized states

\[
q(x, b_\perp) = \int \frac{dx^-}{4\pi} \langle p^+, \mathbf{R}_\perp = 0_\perp | q(-\frac{x^-}{2}, b_\perp) \gamma^+ q(\frac{x^-}{2}, b_\perp) | p^+, \mathbf{R}_\perp = 0_\perp \rangle e^{ixp^- x}(4)
\]

and with an additional \( \gamma_5 \) for the polarized distribution \( \Delta q(x, b_\perp) \). Impact parameter dependent PDFs are Fourier transforms of GPDs for \( \xi = 0 \)

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} H(x, 0, -\Delta_\perp^2) \]

\[
\Delta q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \tilde{H}(x, 0, -\Delta_\perp^2) \]

Due to a Galilean subgroup of \( \perp \) boosts in the infinite momentum frame there are no relativistic corrections to Eq. (5). Furthermore, impact parameter dependent PDFs have a probabilistic interpretation very similar (and with the same limitations) as the usual PDFs\(^4\). For example, for \( x > 0 \) (quarks) one finds \( q(x, b_\perp) \geq |\Delta q(x, b_\perp)| \geq 0 \).

It is important to utilize theoretical constraints when parameterizing these functions to supplement experimental data. One such constraint arises directly from the fact that the reference point for impact parameter dependent PDFs is the \( \perp \) center of momentum. For \( x \to 1 \) the active quark becomes the center of momentum and therefore \( b_\perp \) can never be large, and the \( \perp \) width of \( q(x, b_\perp) \) should go to zero for \( x \to 1 \). For decreasing \( x \) the \( \perp \) width is expected to increase gradually. Although the width in the valence region should still be relatively compact, its size should increase further once \( x \) is small enough for the pion cloud to contribute\(^5\). Therefore the \( t \)-dependence of GPDs should decrease with increasing \( x \). This is consistent with recent lattice results, which showed that higher moments of GPDs have less \( t \) dependence than lower moments\(^6\).
3. Transversely Polarized Target

For a $\perp$ polarized target, impact parameter dependent PDFs are no longer axially symmetric. The deviation from axial symmetry is described by $E(x,0,t)$. For example, the unpolarized quark distribution $q(X,b_\perp)$ for a target polarized in the $+\hat{x}$ direction reads

$$q(X,b_\perp) = q(x,b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta}{(2\pi)^2} E(x,0,-\Delta^2_\perp) e^{-ib_\perp \cdot \Delta}.$$  \hfill (7)

Here $q(x,b_\perp)$ is the impact parameter dependent PDF in the unpolarized case (5). This distortion arises since the virtual photon in DIS couples more strongly to quarks that move towards it than quarks that move away from it (hence the $\gamma^+$ in the quark correlation function relevant for DIS). \cite{7,8} If the orbital motion of the quarks and the spin of the target are correlated then quarks are more likely to move towards the virtual photon on one side of the target than the other and the distribution of quarks in impact parameter space appears deformed towards one side. The details of this deformation for each quark flavor are described by $E_q(x,0,t)$, which is not known yet. However, sign and overall scale can be estimated by considering the mean displacement of flavor $q$ ($\perp$ flavor dipole moment)

$$d^2_q = \int dx \int d^2b_\perp q(x,b_\perp) b_y = \frac{1}{2M} \int dx E_q(x,0,0) = \frac{\kappa_q}{2M}.$$ \hfill (8)

$\kappa_q = O(1-2)$ are the contributions from each quark flavor to the anomalous magnetic moment of the nucleon, i.e. $F_2(0) = \frac{2}{3} \kappa_u - \frac{1}{3} \kappa_d - \frac{1}{3} \kappa_s$, yielding $|d^2_q| = O(0.2fm)$, with opposite signs for $u$ and $d$ quarks (Fig. 1).

The $\perp$ distortion can also be linked to Ji’s relation \cite{9} between the 2nd moment of the GPDs $H_q$ and $E_q$ and the quark angular momentum.

![Figure 1](image-url) Expected impact parameter dependent PDF for $u$ and $d$ quarks ($x_{Bj} = 0.3$ is fixed) for a nucleon that is polarized in the $x$ direction in the model from Ref. For other values of $x$ the distortion looks similar.
Here \( M_{q}^{0jk} = T_{q}^{0k}x^{j} - T_{q}^{0j}x^{k} \) and

\[
T_{q}^{\mu\nu} = i\bar{q}\gamma^{\mu} D_{q}^{+\nu}
\]

(a symmetrization in \( \mu \) and \( \nu \) is implicit). Since the angular momentum is obtained by taking the weighted average of the position, where the weight factor is the momentum density, one would intuitively expect some connection between the transverse center of momentum for the quarks and their angular momentum. Indeed, as has been shown in Ref. 10, one can relate the \( \perp \) shift of the center of momentum for a quarks with flavor \( q \) to the angular momentum carried by these quarks. Using Eq. (7) and taking into account an overall \( \perp \) shift due to boosting to the infinite momentum frame one thus recovers the Ji relation

\[
\langle J_{i}^{q} \rangle = S_{i} \int dx x [H_{q}(x, 0) + E_{q}(x, 0)],
\]

where \( S_{i} \) is the nucleon spin. In combination with measurements of the fraction of the quark spin contribution to the nucleon spin in polarized DIS, Eq. (11) is expected to provide novel information about the orbital angular momentum carried by the quarks.

The deformation of quark distributions in a \( \perp \) polarized target also provides a very physical source for single-spin asymmetries (SSA) in semi-inclusive DIS. The Sivers function \( f_{q/p}^{\perp q} \), which parameterized the left-right asymmetry reads

\[
f_{q/p}^{\perp q}(x, k_{\perp}) = f_{q}^{\perp q}(x, k_{\perp}^{2}) - f_{1T}^{\perp q}(x, k_{\perp}^{2}) \frac{(\hat{P} \times k_{\perp}) \cdot S}{M},
\]

where \( f_{q/p}^{\perp q}(x, k_{\perp}) \) represents the unintegrated parton density for quarks ejected from a \( \perp \) polarized target. The phenomenology of these functions can be found for example in Ref.13 and references therein. Although one may naively expect that these \( T \)-odd functions vanish, they survive the Bjorken limit due to final state interactions. For an (on average) attractive final state interaction, the position space deformation into the \(+\hat{y}\) direction translates into a momentum space asymmetry for the ejected quark that prefers the \(-\hat{y}\) direction and vice versa (Fig. 2) Since the sign of the position space distortion is governed by the sign of the anomalous magnetic moment contribution \( \kappa_{q/p} \) from each quark flavor, this implies that the sign of the SSA is correlated to the sign
of $\kappa_{q/p}$. Following the Trento convention,\textsuperscript{12} this yields a negative Sivers function $f_{1T}^{u}$ in the proton, while $f_{1T}^{d} > 0$.\textsuperscript{17} For neutrons the signs are reversed. These predictions are consistent with recent HERMES data\textsuperscript{18}.

\begin{equation}
\vec{p}_\gamma \quad \vec{q}^1 \quad \pi^+
\end{equation}

Figure 2. The transverse distortion of the parton cloud for a proton that is polarized into the plane, in combination with attractive FSI, gives rise to a Sivers effect for $u$ ($d$) quarks with a $\perp$ momentum that is on the average up (down).

4. Chirally Odd GPDs

The distribution of transversely polarized quarks in impact parameter space is described by the Fourier transform of chirally odd GPDs\textsuperscript{19}. For an unpolarized target the distribution of quarks with transversity $s^i$ reads

\begin{equation}
q_i(x,b_\perp) = -\frac{s^i\xi^j}{2M} \frac{\partial}{\partial b_j} \int \frac{d^2 \Delta}{(2\pi)^2} \left[ 2\tilde{H}_T(x,0,-\Delta_\perp^2) + E_T(x,0,-\Delta^2) \right] e^{-i\mathbf{b}_\perp \cdot \mathbf{\Delta}} \tag{13}
\end{equation}

While Eq. (7) describes the $\perp$ deformation of unpolarized quark distributions in a $\perp$ polarized nucleon, Eq. (13) demonstrates that a similar deformation is present in the distribution of $\perp$ polarized quarks in an unpolarized nucleon — except the latter deformation is described by the chirally odd GPDs $2\tilde{H}_T + E_T$. In Sec. 3, we linked the $\perp$ deformation of the unpolarized quark distributions in a $\perp$ polarized nucleon to the angular momentum carried by those quarks, yielding the Ji relation (11) which tells us how the quark angular momentum is correlated to the nucleon spin. Intuitively, we thus expect that there is some connection between chirally odd GPDs, which describe the $\perp$ deformation of $\perp$ polarized quark distributions in an unpolarized nucleon, and the correlation between the quark spin and the quark angular momentum in an unpolarized nucleon.

In order to investigate the correlation between polarization and angular momentum of the quarks, we decompose $J_q^z$ into transversity components. The projector on $\perp$ spin (transversity) eigenstates $P_{\pm\hat{x}} \equiv \frac{1}{2}(1 \pm \gamma_x^\perp \gamma_5)$ commutes with $\gamma^0$, $\gamma^y$, and $\gamma^z$. Hence all components of the energy momentum tensor that appear in the definition of $J_q^z$ do not mix between transversity (in the $\hat{x}$ direction) states, defined as $q_{\pm\hat{x}} = \frac{1}{2}(1 \pm \gamma^x \gamma_5)q$. It is thus
possible to decompose $J^x_q$ into transversity components
\[ J^x_q = J^x_{q,+\hat{x}} + J^x_{q,-\hat{x}}. \]  (14)

Transversity projections of Eq. (9) yield the transversity components $J^x_{q,\pm \hat{x}}$
\[ J^x_{q,\pm \hat{x}} = \frac{i}{2} \int d^3x \bar{q}(x,\pm \hat{z}) \left[ \gamma^0 \frac{\gamma^x}{D^z + \gamma^z D^0} \right] q(\pm \hat{z}) y - "y \leftrightarrow z" = \frac{1}{2} \left[ J^x_{q,\pm} \right], \]  (15)
where the transversity dependent piece reads
\[ \delta^z J^x_q = \frac{1}{2} \int d^3x \bar{q}(x,z) \left[ \sigma^{0x} \frac{\gamma^z}{D^z + \sigma^{zx} D^0} \right] q(x,z) - "y \leftrightarrow z". \]  (16)

Taking the matrix elements of $J^x_q$ yields the Ji relation (11). In order to examine the contribution from the chirally odd term (16), we consider the form factor of the transversity density with one derivative $^{19,20}$
\[ \langle p'| \tilde{q} \sigma^{\mu\nu} \gamma_5 \frac{\gamma^x}{D^z + \sigma^{zx} D^0} q | p \rangle = \tilde{u} \sigma^{\mu\nu} \gamma_5 u \bar{p} \gamma^x A_{T,20}(t) + \frac{\epsilon^{\mu \alpha \beta \nu} \Delta_\alpha \bar{p} \gamma^x u \bar{u} \bar{\Delta}_\beta }{M^2} \bar{u} \bar{A}_{T,20}(t) \]  (17)
\[ + \frac{\epsilon^{\mu \alpha \beta \nu} \bar{p} \gamma^x u \bar{u} \gamma \beta u \bar{B}_{T,21}(t) + \frac{\epsilon^{\mu \alpha \beta \nu} \bar{u} \gamma \beta u \bar{u} \gamma \beta u \bar{B}_{T,21}(t). \]

Antisymmetrization in $\lambda$ and $\mu$ and symmetrization in $\mu$ and $\nu$ is implied. The form factors in Eq. (17) are the 2nd moments of chirally odd GPDs
\[ A_{T,20}(t) = \int_{-1}^{1} dx x H_T(x,\xi,t) \]  \[ A_{T,20}(t) = \int_{-1}^{1} dx x \tilde{H}_T(x,\xi,t) \]  (18)
\[ B_{T,20}(t) = \int_{-1}^{1} dx x E_T(x,\xi,t) \]  \[ - 2 \xi \tilde{B}_{T,21}(t) = \int_{-1}^{1} dx x \tilde{E}_T(x,\xi,t). \]

The chirally odd GPDs entering Eq. (18) are defined as non-forward matrix elements of light-like correlation functions of the tensor charge
\[ p^+ \int \frac{dz}{2\pi} e^{ixp^+z} \left\langle p' | \tilde{q} \left( \gamma^0 \frac{2}{z} \right) \gamma^x \gamma_5 u \left( \gamma^0 \frac{2}{z} \right) | p \rangle = H_T(x,\xi,t) \tilde{u} \sigma^j \gamma_5 u + \]  (19)
\[ H_T(x,\xi,t) \tilde{u} \sigma^j \gamma_5 u + \frac{\Delta_\alpha \bar{p}_\beta u}{M^2} \bar{u} \sigma^j \gamma_5 u + \]  \[ E_T(x,\xi,t) \tilde{u} \sigma^j \gamma_5 u + \tilde{E}_T(x,\xi,t) \tilde{u} \sigma^j \gamma_5 u \]

Upon taking the expectation value of $\delta^z J^x$ in a delocalized wave packet (rest frame), the factor $y (z)$ projects out terms linear in $\Delta^x$ in Eq. (17)
\[ \langle \delta^z J^x \rangle = \frac{1}{2} \int dx x \left[ H_T(x,0,0) + 2H_T(x,0,0) + E_T(x,0,0) \right], \]  (20)
yielding a decomposition of the Ji relation into transversity components
\[ \langle J^x_{q,\pm \hat{x}} \rangle = \frac{S_x}{2} \int dx x \left[ H(x,0,0) + E(x,0,0) \right] \pm \frac{1}{4} \int dx x \left[ H(x,0,0) + 2H_T(x,0,0) + E_T(x,0,0) \right]. \]  (21)
Here $S^x$ is the spin of the nucleon and for an unpolarized target, only the second term contributes. Although the derivation presented above was for one specific component, it is evident that rotational invariance implies analogous relations for $J^y_{q, \pm y}$ and $J^z_{q, \pm z}$. Similar relations can also be derived for a spinless target, such as a pion. The scale dependence of Eq. (20) is the same as for the second moment of the quark transversity $\int dx x H_T(x, 0, 0)$.

It is instructive to apply our new relations to a point-like spin-$\frac{1}{2}$ particle, where the “quark” spin is always equal to the “nucleon” spin $H(x, 0, 0) = H_T(x, 0, 0) = \delta(x - 1)$ and $E = H_T = E_T = 0$. In this case $\langle \delta^x J^x \rangle = \frac{1}{2}$.

For an unpolarized target there is a 50% probability that $S^x = \frac{1}{2}$ and a 50% probability that $S^x = -\frac{1}{2}$. When a quark has $s^x = \frac{1}{2}$, which occurs with 50% probability, the quark also has $J^x = \frac{1}{2}$, resulting in $\langle J^x(s^x = +1/2) \rangle = 0.5 \times \frac{1}{2} = \frac{1}{4}$, which is consistent with Eqs. (20,21). As a second example, when the same point-like “nucleon” has $S^x = +\frac{1}{2}$, all of its angular momentum is carried by “quarks” with $s^x = +\frac{1}{2}$, while none is carried by “quarks” with $s^x = -\frac{1}{2}$. This is again consistent with Eq. (21). A constituent quark model estimate for Eq. (20) can be found in Ref. 22.

In the general case, when the second moments of the involved GPDs are nontrivial, it is expected that Eqs. (20) and (21) will provide novel insights about the spin structure and spin-orbit correlation for quarks in the nucleon. While experimental results for $H_T(x, 0, 0)$ are expected soon, measuring the other two chirally odd GPDs which enter Eq. (20) will be more challenging. Therefore, initial applications of Eq. (20) will have to rely on lattice QCD simulations. 21.

5. Chirally odd GPDs and the Boer-Mulders Function

The Boer-Mulders function $h^{L\perp q}$ 23 is similar to the Sivers function (12) except that the nucleon spin is replaced by the quark spin $s$

$$f_{q' L/p}(x, k_{\perp}) = \frac{1}{2} \left[ f_1^q(x, k_{\perp}^2) - h^{L\perp q}(x, k_{\perp}^2) \frac{\mathbf{P} \times \mathbf{k}_{\perp} \cdot \mathbf{s}}{M} \right]$$

and describes the correlation between the $\perp$ momentum and the $\perp$ spin of the ejected quark in semi-inclusive DIS from an unpolarized target. In Sec. 3, a mechanism was suggested through which the FSI in semi-inclusive DIS translates a position space asymmetry in the target into a momentum asymmetry for the outgoing quark. Applying the same mechanism here yields again a negative correlation between the sign of the momentum asymmetry
and the sign of the deformation in position space, i.e. we expect

$$\frac{h_{1}^{1g}}{2H_{T} + E_{T}} \sim \frac{f_{1g}}{E} < 0.$$  \hfill (23)

Tests of this qualitative relationship require lattice determinations of $2H_{T} + E_{T}$, while $h_{1}^{1g}$ is accessible in polarized Drell-Yan experiments.\textsuperscript{23}

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