The puzzle of excessive non-$D\bar{D}$ component of the inclusive $\psi(3770)$ decay and the long-distant contribution

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In this letter we suggest that the obvious discrepancy between theoretical prediction on the non-$D\bar{D}$ decays of $\psi(3770)$ and data is to be alleviated by taking final state interaction (FSI) into account. By assuming that $\psi(3770)$ overwhelmingly dissociates into $D\bar{D}$, then the final state interaction induces a secondary process, we calculate the branching ratios of $\psi(3770) \to D\bar{D} \to J/\psi \eta, \rho, \omega, K^* K$. Our results show that the branching ratio of $\psi(3770) \to \text{non-}D\bar{D}$ can reach up to $2^{\text{FSI}}_{\text{non-}D\bar{D}} = (0.2 \pm 1.1)\%$ while typical parameters $I = 0.4\ GeV^{-2}$ and $\alpha = 0.8 \sim 1.3$ are adopted. This indicates that the FSI is obviously non-negligible.

Obviously, physics about Charmonium is still not a closed book yet, instead, this field is full with challenges and opportunities [1]. Due to developments and improvements of facility and technique, the detection precision is greatly enhanced in the past, which is still three times smaller than 15% non-radiative decay contribution with that of charm decay modes, such as $\psi(2S)$ and $\psi(3S)$, respectively [2, 3, 4, 5, 6] in the past three years. Since $\psi(3770)$ MeV is a bit above the threshold of $D\bar{D}$ production, such a bound state may dissolve into open charms which eventually hadronize into $D\bar{D}$. Therefore, before observing sizable non-$D\bar{D}$ decay rates, $\psi(3770)$ was supposed to dominantly decay into $D\bar{D}$, including $D^0\bar{D}^0$ and $D^+\bar{D}^-$. There could be some possible non-$D\bar{D}$ modes [3, 10, 11, 12, 13, 14], especially the hidden charm decay modes, such as $J/\psi\pi\pi$ and $J/\psi\eta$ with $\mathcal{B}(\psi(3770) \to J/\psi\pi\pi) = (1.93 \pm 0.28) \times 10^{-3}$, $\mathcal{B}(\psi(3770) \to J/\psi\eta) = (8.0 \pm 3.0) \times 10^{-4}$ and $\mathcal{B}(\psi(3770) \to J/\psi\eta) = (9 \pm 4) \times 10^{-4}$ respectively [15], and E1 radiative decays $\gamma\chi_{cJ}$ with decay widths $172 \pm 30\ keV, 70 \pm 17\ keV$ and $< 21\ keV$ for $J = 0, 1, 2$ respectively [10, 14]. The sum of all the branching ratios of these hidden charm decay modes is less than 2%, so all these measurements support the allegation that $\psi(3770)$ overwhelmingly decays into $D\bar{D}$.

However, the BES collaboration investigated the inclusive decays of $\psi(3770)$ and found that the branching ratio of $\psi(3770) \to D\bar{D}$ is about $85 \pm 5\%$ [3, 4]. This is later verified by the measurements of non-$D\bar{D}$ inclusive processes with the branching fraction $\mathcal{B}(\psi(3770) \to \text{non-}D\bar{D}) = (13.4 \pm 5.0 \pm 3.6)\%$ [5] and $\mathcal{B}(\psi(3770) \to \text{non-}D\bar{D}) = (15.1 \pm 5.6 \pm 1.8)\%$ [6] respectively by adopting two different methods. The CLEO measurements indicate $\sigma(e^+e^- \to \psi(3770) \to \text{hadrons}) = (6.38 \pm 0.08^{+0.41}_{-0.30})\ nb$ [16] and $\sigma(e^+e^- \to \psi(3770) \to D\bar{D}) = (6.57 \pm 0.04 \pm 0.01)\ nb$ [17], which together make a $\mathcal{B}(\psi(3770) \to D\bar{D}) = (103.0 \pm 1.4^{+6.8}_{-5.6})\%$. Notice that the error on the high side is about 6.8\% [16], by this error tolerance, there could be a large ($10 \sim 15\%)$ fraction of $\psi(3770)$ to non-$D\bar{D}$ decays. The CLEO and BES results are inconsistent at $>2\sigma$ level, and we would employ both of them as inputs to our numerical computations and an obvious difference is explicitly noticed.

Kuang and Yan [18] calculated the $\psi(3770) \to J/\psi\pi\pi$ using the QCD multi-expansion, which properly deals with the emission of light hadrons during heavy quarkonia transitions (for a review see an enlightening paper [19]). Their prediction is consistent with the exclusive measurement on hidden charm decays of $\psi(3770)$. It is generally concurred that, the measurements on the well measured channels $J/\psi\pi\pi, J/\psi\eta$ and $\gamma\chi_{cJ}$ are consistent with present theoretical predictions. Thus to understand the experimental results, one should find where $\psi(3770)$ goes besides $J/\psi\pi\pi, J/\psi\eta$ and $\gamma\chi_{cJ}$. Recently He, Fan and Chao [20] introduced the color-octet mechanism and calculated the $\psi(3770) \to \text{light hadrons}$ in the framework of NRQCD by considering next to leading order contribution. The calculation result shows that $\Gamma(\psi(3770) \to \text{light hadrons}) = 467^{+187}_{-338}\ keV$. If combing radiative decay contribution with that of $\psi(3770) \to \text{Light Hadrons}$, the branching ratio of the non-$D\bar{D}$ of $\psi(3770)$ is about 5\% [20], which is still three times smaller than 15\% non-$D\bar{D}$ branching ratio measured by the experiment.
Instead, Voloshin suggested, $\psi(3770)$ is not a pure $c\bar{c}$ state. There exists a sizable four quark component ($u\bar{u} \pm d\bar{d})c\bar{c}$ and the fraction is about $O(10\%)$ in $\psi(3770)$, which results in a measurable rate of $\psi(3770) \rightarrow \pi^0 J/\psi, \eta J/\psi$ [21].

Generally, one can categorize the strong decay modes of $\psi(3770)$ into three types: open charm decay ($DD$), hidden charm decay $J/\psi X$ ($X$ = light mesons) and the decay into light hadrons (L-H decay). One can be more confident that the rates of hidden charm decays are properly evaluated in terms of the QCD multi-expansion, and the L-H decay occurs via three-gluon emission mechanism $c\bar{c} \rightarrow 3g$.

There is an alternative explanation to the puzzle. Twenty years ago, Lipkin proposed that the non-$DD$ strong decays of $\psi(3770)$ realize via $DD$ intermediate states, and further suggested that $\psi(3770)$ does not 100% decay into $DD$ [22]. Later Achasov and Kozhevnikov calculated the non-$DD$ channels of $\psi(3770)$ only considering the contribution from the imaginary part of the decay amplitude [23]. Namely such final state interactions which are involved in the hadronic loop effects, do contribute to both the hidden charm and L-H decays. The essential point of the loop effect is attributed to the coupled channel effects. A quark-level process is explicitly illustrated in the left diagram of Fig. 1. Such a mechanism should exist in all hidden charm and L-H decays of charmonia [24, 25]. As shown in Fig. 1, $\psi(3770) \rightarrow J/\psi X$ and $\psi(3770) \rightarrow$ Light Hadrons processes do not suffer from the Okubo-Zweig-Iizuka (OZI) suppression. Since $\psi(3770) \rightarrow DD$ takes place near the energy threshold, one can expect that the FSI may be significant.

![Fig. 1: Quark-level descriptions of the hadronic loop mechanism for the hidden charm decay (diagram (a)) and L-H decay (diagram (b)) of $\psi(3770)$.](image1)

In this letter, we focus on two-body hidden charm decay modes ($J/\psi \eta$) and two-body L-H decay modes ($\rho \pi, \omega \eta$ and $K^* K$) which obviously are the main ones. Here $K^* K$ denotes $K^* \bar{K} + \bar{K}^* K$.

In order to calculate the hadronic loop effect in strong decays of $\psi(3770)$, we consider the diagrams depicted by Fig. 2, which are an alternative description in the hadron-level language. $\psi(3770)$ first dissolves into two charmed mesons, then by exchanging $D^*$ in t-channel, they turn into two on-shell real hadrons $\mathcal{A}$ and $\mathcal{B}$. Since the dissociation does not suffer from the OZI suppression, one can expect it to be dominant.

![Fig. 2: The hadron-level diagrams depicting hadronic loop effect on $\psi(3770) \rightarrow D^0 \bar{D}^0 \rightarrow J/\psi \eta, \rho^0 \pi^0$. Of course they can be simply replaced by other states $D^{*0} (\bar{D}^{*0}) \rightarrow D^{*+} (\bar{D}^{*-})$ with an isospin transformation and $D^{*0} \rightarrow \bar{D}^{*0}$ with a charge conjugate transformation to constitute new but similar diagrams. By replacing relevant mesons, we can obtain the diagrams for $\omega \eta$ and $K^* K$ channels.](image2)

One can obtain the absorptive part of the decay amplitude of $\psi(3770) \rightarrow D + \bar{D} \rightarrow \mathcal{A} + \mathcal{B}$ ($\mathcal{AB} = J/\psi \eta, \rho^0 \pi^0$ and $K^* K$)

$$
A_{\mathcal{AB}} (M^2_{\psi}) = \frac{|\mathbf{p}|}{32\pi^2 M_{\psi}} \int d\Omega M^2 \left[ \psi(3770) \rightarrow D \bar{D} \right] \times M \left[ D \bar{D} \rightarrow \mathcal{AB} \right] \cdot F^2 \left[ m^2_{\rho^0\pi^0}, q^2 \right],
$$

where $|\mathbf{p}| = \left[ \lambda (M_{\psi}^2, M_D^2, M_{\bar{D}}^2) / (2M_{\psi}) \right]^{1/2}$ is the three-momentum of the intermediate charmed mesons in the center of mass frame of $\psi(3770)$. $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$ is the Källen function. The form factor $F \left[ m^2_{\rho^0\pi^0}, q^2 \right]$ is the key point for
the evaluation of the amplitude. One can use the monopole form factor (FF)

$$\mathcal{F} \left[ m_i^2, q^2 \right] = \frac{\Lambda(m_i)^2 - m_i^2}{\Lambda(m_i)^2 - q^2}$$

which compensates the off-shell effect of exchanged meson and describes the structure effect of the interaction vertex. As a free parameter, $\Lambda$ can be parameterized as $\Lambda = m_i + \alpha_{QCD}$ [26]. $m_i$ denotes the mass of exchanged meson, $\Lambda_{QCD} = 220$ MeV. The range of dimensionless phenomenological parameter $\alpha$ is around $0.8 < \alpha < 2.2$ [26]. As a matter of fact, there are other possible forms for $\mathcal{F} \left[ m_i^2, q^2 \right]$, such as the exponential one etc. in literature. Generally they are equivalent somehow, as long as their asymptotic behaviors are the same.

Since the mass of $\psi(3770)$ is close to the threshold of $D\bar{D}$ production, the dispersive part of the amplitude of $\psi(3770) \rightarrow D + \bar{D} \rightarrow \mathcal{A} + \mathcal{B}$ makes a large contribution to the decay width. By unitarity, one can obtain the dispersive part in terms of the dispersion relation. The total decay amplitude of $\psi(3770) \rightarrow D + \bar{D} \rightarrow \mathcal{A} + \mathcal{B}$ which includes both absorptive and dispersive parts is expressed by [27,28,29]

$$M[\psi(3770) \rightarrow D\bar{D} \rightarrow \mathcal{A}\mathcal{B}] = \frac{1}{\pi} \int_{r_D}^{\infty} \frac{A_{\mathcal{AB}}(r) R(r)}{r - M_\psi^2} dr + A_{\mathcal{AB}}(M_\psi^2),$$

where $r_D^2 = 4m_D^2$. After replacing $M_\psi^2$ in the amplitude in eq. (1) with $r_D$, we get the amplitude $A_{\mathcal{AB}}(r)$. The energy dependent factor $R(r)$ is defined as $R(r) = \exp(-I|q(r)|^2)$, which not only reflects the $|q(r)|^2$-dependence of the interaction between $\psi(3770)$ and $D\bar{D}$ mesons, but also plays the role of ultraviolet cutoff. Meanwhile, $R(r)$ can be understood as the coupled channel effect summing up all the bubbles from the charm meson loops [30]. Here $|q(r)|$ denotes the three momentum of $D$ meson in the rest frame of $\psi(3770)$ with the mass $M_\psi \approx \sqrt{r}$. The interaction length factor $I$ is related to the radius of the interaction by $I = R/6$ [31]. Pennington and Wilson indicated that $I = 0.4\; GeV^{-2}$ corresponding to $R = 0.3\; fm$ is favorable when studying the charmion mass shift [31].

Based on the effective Lagrangian approach, one can formulate $M[\psi(3770) \rightarrow D\bar{D}]$ and $M[DD \rightarrow \mathcal{A}\mathcal{B}]$. The effective Lagrangians related to our calculation are constructed by considering the chiral and heavy quark symmetries [32,33,34].

$$\mathcal{L}_{\psi DD} = ig_{\psi DD} \left[ D_i \partial_\mu D_{j\mu}^i \right] \bar{\Psi} \gamma^\mu \Psi,$$

$$\mathcal{L}_{\psi DD'} = g_{\psi DD'}\epsilon_{\mu\nu\rho\sigma} \partial_\mu D_{\nu j}^i \partial_\rho D_{\sigma j}^i,$$

$$\mathcal{L}_{D\bar{D}} = -i g_{\psi DD} \left[ D_i \partial_\mu D_{j\mu}^i - D^j_i \partial_\mu D^i_\mu \right],$${

$$\mathcal{L}_{D\bar{D}'D'} = -2 f_{\psi DD'}\epsilon_{\mu\nu\rho\sigma} \left( \partial_\mu \partial_\nu \right) \left( \partial^\rho \partial_\sigma \right),$$

where $\Psi$ denotes charmion states $J/\psi$ and $\psi(3770)$. $\bar{\Psi}$ and $\bar{\Psi}$ are the octet pseudoscalar and nonet vector meson matrices, respectively. The values of coupling constants relevant to our calculation are $g_{\psi DD} = 4.70$, $g_{J/\psi DD'} = 4.25\; GeV^{-1}$, $g_{DD' \bar{D}'} = 17.31$ and $f_{\psi DD'} = 2.33\; GeV^{-1}$ determined in Refs. [24,27,28,29].

For the process $\psi(3770) \rightarrow D(k_1) + \bar{D}(k_2) \rightarrow J/\psi(k_3) + \eta(k_4)$ by exchanging $D^{0}$, one formulates its amplitude

$$A_{J/\psi} = g_{J/\psi} Q_{J/\psi} \left| \frac{|p|}{2\pi^2 M_\psi} \right| \rho \partial_\mu \left( D_i \partial_\nu D_{j\mu}^i \right) \bar{\eta} \gamma^\mu \eta \cdot \epsilon_{\rho\sigma} \left( \partial_\rho \partial_\sigma \right),$$

where $\eta$ denotes meson states $\psi(3770)$ and $\eta(9580)$. The amplitude reads

$$A_{J/\psi} = \mathcal{G}_{J/\psi} Q_{J/\psi} \left| \frac{|p|}{2\pi^2 M_\psi} \right| \rho \partial_\mu \left( D_i \partial_\nu D_{j\mu}^i \right) \bar{\eta} \gamma^\mu \eta \cdot \epsilon_{\rho\sigma} \left( \partial_\rho \partial_\sigma \right),$$

where $\epsilon_{\rho\sigma}$ is the polarization vector of $J/\psi$, $Q_{J/\psi}$ is the charge of $J/\psi$, and $\mathcal{G}_{J/\psi}$ is the form factor of $J/\psi$. The absorptive amplitude of $\psi(3770) \rightarrow D(k_1) + \bar{D}(k_2) \rightarrow \mathcal{P}(k_3) + \mathcal{V}(k_4)$ reads

$$A_{\mathcal{P}\mathcal{V}} = \mathcal{G}_{\mathcal{P}\mathcal{V}} Q_{\mathcal{P}\mathcal{V}} \left| \frac{|p|}{2\pi^2 M_\psi} \right| \rho \partial_\mu \left( D_i \partial_\nu D_{j\mu}^i \right) \bar{\eta} \gamma^\mu \eta \cdot \epsilon_{\rho\sigma} \left( \partial_\rho \partial_\sigma \right),$$

where $\mathcal{P}$ and $\mathcal{V}$ matrices results in an extra factor $\mathcal{G}_{\mathcal{P}\mathcal{V}}$ in the above amplitudes, which are $1/\sqrt{6}$, $1/2$, $1$ and $1/\sqrt{2}$ for the amplitudes of $J/\psi\eta$, $\rho\rho\pi^0$, $K^-\pi^K$ $(K^-\pi^+K^-, K^0\pi^0K^0, K^0\pi^0K^0)$, $\omega\eta$ modes, respectively. If considering SU(3)
symmetry, the factor $Q_{RS}$ comes from both the isospin transformation $D_i \rightarrow D_i'$ and the charge conjugate transformation $D_{i}^c \rightarrow D_{i}^c$, which results in $Q_{RS} = 4, 2$ for the amplitudes of $J/\psi \rho \eta, \omega \eta$, $K^* K$ channels, respectively.

The branching ratio of $\psi(3770) \rightarrow J/\psi \rho \eta, \omega \eta, K^* K$ with $\alpha$.

In the left diagram of Fig. 3 we plot the dependence of numerical result on $\alpha$ with several typical value $I = 0.4 \sim 1.0$ GeV$^{-2}$, which is the sum of decay widths of $\psi(3770) \rightarrow J/\psi \rho \eta, \omega \eta, K^* K$ induced by $D\bar{D}$ intermediate states. We set $\Gamma[\psi(3370) \rightarrow D\bar{D} \rightarrow \rho \pi]$ as $3\Gamma[\psi(3370) \rightarrow D\bar{D} \rightarrow \rho^0 \pi^0]$ and $\Gamma[\psi(3370) \rightarrow D\bar{D} \rightarrow K^* K] \approx 4\Gamma[\psi(3370) \rightarrow D\bar{D} \rightarrow K^+ K^-]$ which are determined by the SU(3) symmetry. The dependence of decay widths of each modes $\psi(3770) \rightarrow J/\psi \rho \eta, \omega \eta, K^* K$ induced by long-distant contribution on the parameter $\alpha$ within the range of $0.8 \leq \alpha \leq 2.2$ is shown in the right diagram of Fig. 3 where a typical value $I = 0.4$ GeV$^{-2}$ is adopted.

Our numerical results indicate that the decay widths of $\psi(3770) \rightarrow D\bar{D} \rightarrow \rho \pi, K^* K$ are about one order larger than that of $\psi(3770) \rightarrow D\bar{D} \rightarrow J/\psi \rho \eta, \omega \eta, K^* K$ which are determined by the phase space, factors $G_{RS}$ and $Q_{RS}$. Whereas the amplitudes for $\rho \pi, K^* K$ are comparable, and they are the main non – $D\bar{D}$ decay channels obviously.

The branching ratio of $\psi(3770) \rightarrow J/\psi \rho \eta, \omega \eta, K^* K$ modes with a fixed value $I = 0.4$ GeV$^{-2}$ is shown in Fig. 4 within the range of $\alpha = 0.8 \sim 2.2$. Furthermore, let us compare our result with the BES data [5] and the result of $\psi(3770) \rightarrow J/\psi \rho \eta, \omega \eta, K^* K$ and therefore cannot be determined based on a first principle, one can only phenomenologically fit it by fitting data. We also notice that the amplitudes for $\rho \pi, K^* K$ are comparable, and they are the main non – $D\bar{D}$ decay channels obviously.

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of $\psi(3770) \to \text{non}-D\bar{D}$ caused by the FSI can reach up to $B^{\text{FSI}}_{\text{non}-D\bar{D}} = (0.2 \sim 1.1)\%$ (taking CLEO data of $\psi(3770) \to \rho\pi$ to constrain $\alpha$). It indicates that even though FSI is significant, it cannot make a drastic change as long as $\alpha$ is restricted to be less than 2.1. Furthermore, the upper limit of the total contribution of the NRQCD and FSI is up to 4.6%. The branching ratios of E1 transition $\psi(3770) \to \gamma_{\chi_{cJ}} (J=0,1,2)$ and $\psi(3770) \to \rho\pi, \rho\eta$ are about $1.5 \sim 1.8\%$ [10,14,15]. If summing up all the above non-$D\bar{D}$ contributions, the branching ratio of the channels with non-$D\bar{D}$ final states can be as large as 6.4%, which is still smaller than the experimental value $B[\psi(3770) \to \text{non} - D\bar{D}] = (13.4 \pm 5.0 \pm 3.6)\%$ but near its lower bound [3].

As a short summary, let us emphasize a few points. First, even including contributions of color-octet, the NRQCD prediction on the branching ratio of $\psi(3770) \to \text{non} - D\bar{D}$ which is calculated up to NLO, cannot coincide with the data of BES [20]. At the energy range, the FSI obviously is significant and this allegation has been confirmed by many earlier phenomenological studies on other processes. When the FSI effects are taken into account, the discrepancy between theoretical prediction and data is significantly alleviated, even though not sufficient. Considering the rather large error range in measurements of both inclusive non-$D\bar{D}$ decay of $\psi(3770)$ and the exclusive mode $\psi(3770) \to \rho\pi$, one would still be able to obtain a value for the parameters $\alpha$ which does not conflict with the data, by which the theoretical prediction and data might be consistent. The more accurate measurements which will be conducted in the future will provide more information which can help to make a definite conclusion if the FSI indeed solves the "puzzle" or not. Secondly, our result shows that the FSI can make significant contribution to all the channels of $\psi(3770) \to \rho\pi, K^*K$, and each of them should be searched in future experiments. Thirdly, no doubt, more accurate measurements on $\psi(3770) \to \text{non} - D\bar{D}$, especially $\psi(3770) \to \text{Light Hadrons}$, are necessary. Thanks to the great improvement

![Graph showing the comparison of our result (blue dash-dotted line) with the BES data (dashed line with shadow band) of excessive non-$D\bar{D}$ component of the inclusive $\psi(3770)$ decay and the result of $\psi(3770) \to \text{Light Hadrons}$ (dash-dotted line with shadowed band) by the color-octet mechanism calculated up to next to leading order within the framework of NRQCD [20]. Here the red line with green shadowed band is the total result including the NLO NRQCD effects and FSI contribution. The green shadowed band corresponds to the error tolerance, coming from the NRQCD estimate in Ref. [20]. The orange and light blue shadowed bands are the suitable window for $\alpha$, which is respectively determined by the BES data [3] and CELO data [12] of $\psi(3770) \to \rho\pi$.]

| $\alpha$ | $\Gamma_{\rho\pi}$ (keV) | $\Gamma^{\text{FSI}}_{\text{non}-D\bar{D}}$ (keV) | $B^{\text{FSI}}_{\text{non}-D\bar{D}}$ (%) | $B[\text{NRQCD + Ours}]$ (%) |
|---------|----------------|----------------|----------------|----------------|
| 0.8     | 20            | 48             | 0.2           | 1.4 ~ 3.7(2.2) |
| 0.9     | 32            | 75             | 0.3           | 1.5 ~ 3.8(2.3) |
| 1.0     | 47            | 113            | 0.4           | 1.6 ~ 3.9(2.4) |
| 1.1     | 66            | 160            | 0.6           | 1.8 ~ 4.1(2.5) |
| 1.2     | 94            | 223            | 0.8           | 2.0 ~ 4.3(2.8) |
| 1.3     | 127           | 301            | 1.1           | 2.3 ~ 4.6(3.1) |

TABLE I: The typical values of decay width of $\rho\pi$ channel $\Gamma_{\rho\pi}$, the sum of decay widths $\Gamma^{\text{FSI}}_{\text{non}-D\bar{D}}$ and the branching ratio $B^{\text{FSI}}_{\text{non}-D\bar{D}}$ of all channels discussed in this work. Here we fix $I = 0.4$ GeV$^{-2}$. The branching fraction $B[\text{NRQCD + Ours}]$, which is the sum of our result and the NRQCD result. The values in bracket are the central values.
of facility and technology of detection at the charm-tau energy region, the BESIII [36] will provide much more precise data, by which we may gain more information. Furthermore, along the other lines more theoretical studies which may involve other mechanics, even new physics beyond standard model are badly needed.

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[1] X. Q. Li, X. Liu and Z. T. Wei, arXiv:0808.2587 [hep-ph].
[2] J. Z. Bai et al. [BES Collaboration], Phys. Lett. B 605, 63 (2005).
[3] M. Ablikim et al. [BES Collaboration], Phys. Rev. Lett. 97, 121801 (2006).
[4] M. Ablikim et al. [BES Collaboration], Phys. Lett. B 641, 145 (2006).
[5] M. Ablikim et al., Phys. Rev. D 76, 122002 (2007).
[6] M. Ablikim et al. [BES Collaboration], Phys. Lett. B 659, 74 (2008).
[7] Y. B. Ding, D. H. Qin and K. T. Chao, Phys. Rev. D 44 (1991) 3562.
[8] J. L. Rosner, Phys. Rev. D 64, 094002 (2001).
[9] N. E. Adam et al. [CLEO Collaboration], Phys. Rev. Lett. 96, 082004 (2006).
[10] T. E. Coan et al. [CLEO Collaboration], Phys. Rev. Lett. 96, 182002 (2006).
[11] G. S. Huang et al. [CLEO Collaboration], Phys. Rev. Lett. 96, 032003 (2006).
[12] G. S. Adams et al. [CLEO Collaboration], Phys. Rev. D 73, 012002 (2006).
[13] D. Cronin-Hennessy et al. [CLEO Collaboration], Phys. Rev. D 74, 012005 (2006) [Erratum-ibid. D 75, 119903 (2007)].
[14] R. A. Briere et al. [CLEO Collaboration], Phys. Rev. D 74, 031106 (2006).
[15] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).
[16] D. Besson et al. [CLEO Collaboration], Phys. Rev. Lett. 96, 092002 (2006) [arXiv:hep-ex/0512038].
[17] S. Dobbs et al. [CLEO Collaboration], Phys. Rev. D 76, 112001 (2007) [arXiv:0709.3783 [hep-ex]].
[18] Y. P. Kuang and T. M. Yan, Phys. Rev. D 41, 155 (1990).
[19] Y. P. Kuang, Front. Phys. China 1, 19 (2006).
[20] Z. G. He, Y. Fan and K. T. Chao, Phys. Rev. Lett. 101, 112001 (2008).
[21] M. B. Voloshin, Phys. Rev. D 71, 114003 (2005). M. B. Voloshin, arXiv:0711.4556 [hep-ph].
[22] H. J. Lipkin, Phys. Lett. B 179, 278 (1986).
[23] N. N. Achasov and A. A. Kozhevnikov, Phys. Lett. B 260, 425 (1991); Phys. Rev. D 49, 275 (1994); Phys. Atom. Nucl. 69, 988 (2006) [Yad. Fiz. 69, 1017 (2006)].
[24] X. Liu, X. Q. Zeng and X. Q. Li, Phys. Rev. D 74, 074003 (2006).
[25] X. Liu, B. Zhang and S. L. Zhu, Phys. Lett. B 645, 185 (2007).
[26] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005).
[27] C. Meng and K. T. Chao, Phys. Rev. D 75, 114002 (2007).
[28] X. Liu, B. Zhang, L. L. Shen and S. L. Zhu, Phys. Rev. D 75, 074017 (2007).
[29] B. Zhang, X. Liu and S. L. Zhu, Chin. Phys. Lett. 24, 2537 (2007) [arXiv:0705.3082 [hep-ph]].
[30] E. van Beveren and G. Rupp, Annals Phys. 323, 1215 (2008).
[31] M.R. Pennington and D.J. Wilson, Phys. Rev. D 76, 077502 (2007).
[32] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].
[33] M. B. Wise, Phys. Rev. D 45, R2188 (1992).
[34] R. Casalbuoni, A. Deandrea, N. Di Bartolomeo, R. Gatto, F. Feruglio and G. Nardulli, Phys. Rept. 281, 145 (1997); A. Deandrea, G. Nardulli and A.D. Polosa, Phys. Rev. D 68, 034002 (2003).
[35] M. Ablikim et al. [BES Collaboration], Phys. Rev. D 72, 072007 (2005).
[36] D. M. Asner et al., arXiv:0809.1869 [hep-ex].
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