Studies of nonresonant Higgs pair production at electron-proton colliders

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Abstract

The measurement of the Higgs quartic coupling modifier between a Higgs boson pair and a vector boson pair, $\kappa_{2V}$, is expected to be achieved from vector-boson fusion (VBF) production of a Higgs boson pair. However, this process involves another unmeasured parameter, the trilinear Higgs self-coupling modifier $\kappa_1$. A sensitivity analysis should target both parameters. Since the LHC cannot avoid the gluon fusion pollution, which becomes severe for non-SM $\kappa_1$, an electron-proton collider is more appropriate for the comprehensive measurement. In this regard, we study the VBF production of a Higgs boson pair in the $bb\bar{b}\bar{b}$ final state at the LHeC and FCC-he. Performing detailed analysis using the simulated dataset, we devise the search strategy specialized at the LHeC and FCC-he and give a prediction for the sensitivity to both $\kappa_{2V}$ and $\kappa_1$. We find that the two electron-proton colliders have high potential: the LHeC has similar exclusion prospects as the HL-LHC; the FCC-he is extremely efficient, excluding the parameter space outside $\kappa_{2V} \in [0.8, 1.2]$ and $\kappa_1 \in [1, 2.5]$ at 95\% C.L. for the total luminosity of 10 ab$^{-1}$ and 10\% uncertainty on the background yields.

Keywords: $HHVV$ coupling, trilinear Higgs self-coupling, LHeC, FCC-he

1. Introduction

Albeit the absence of any signatures of the physics beyond the Standard Model (BSM), the journey to the final theory of the Universe will never stop. One important task to achieve the goal is to measure every coupling among the SM particles precisely, especially to the Higgs boson $H$. The Higgs coupling modifiers associated with a single Higgs boson have been observed to be SM-like at the LHC [1,2]. Their future projections at the high luminosity LHC (HL-LHC) expect the precisions at or below the percent level [3]. However, coupling modifiers involving a pair of Higgs bosons remain unmeasured, such as $\kappa_1$ for the trilinear Higgs self-coupling and $\kappa_{2V}$ for the quartic coupling between a Higgs boson pair and a vector boson pair. The $\kappa_1$ shall be probed mainly from nonresonant Higgs boson pair (HH) production via gluon fusion: the triangle diagram mediated by the Higgs boson in the s-channel gives access to $\kappa_1$. It is found that if $\kappa_1 \neq 1$, non-trivial changes occur on both the shape and rate of the main kinematic distributions [4]. The current observed interval at the 95\% confidence level (C.L.) is $-5.0 < \kappa_1 < 12.0$ in the ATLAS analysis [5] and $-11.8 < \kappa_1 < 18.8$ in the CMS analysis [6]. Several studies of the prospects for measuring $\kappa_1$ at the HL-LHC and future colliders have been performed [7] [8] [9] [10] [11] [12] [13] [14], which expect more stringent bounds.

The quartic coupling modifier $\kappa_{2V}$ is much more challenging to measure at the LHC since the most efficient process, nonresonant $HH$ production via vector boson fusion (VBF), has very small cross-section of $\sigma^{SM}_{VBF}(pp \rightarrow HHjj)_{\text{NNLO}} = 1.73$ fb at $\sqrt{s} = 13$ TeV [15] in addition to the huge SM backgrounds. The ATLAS collaboration performed the first search and excluded $\kappa_{2V} < -0.76$ and $\kappa_{2V} > 2.90$ at the 95\% C.L. for $\kappa_{2V} = 1$ and $\kappa_1 = 1$ [16]. The assumption of $\kappa_{2V} = 1$ is well motivated by the Higgs precision measurements at the LHC, but $\kappa_1 = 1$ is questionable. The VBF production of $HH$, which also depends on $\kappa_1$ via the $H$-mediated s-channel diagram, is susceptible to anomalous Higgs self-coupling ($\kappa_1 \neq 1$). For example, the cross-section for $\kappa_1 = 5$ at the 14 TeV LHC is about twenty times that for $\kappa_1 = 1$. More serious is the pollution from the gluon fusion production of $HH$ associated with two jets, $gg \rightarrow HHjj$ [17] [18]. This pollution also has the contribution from $\kappa_1$ and greatly increases for $\kappa_1 \neq 1$. Considering huge QCD uncertainties in the gluon fusion pollution [19], we expect an inevitable limitation to the precision measurement of $\kappa_{2V}$ at the LHC.

Targeting the measurements of $\kappa_1$ and $\kappa_{2V}$ without the assumption about $\kappa_1$ and thus the ambiguity of the gluon fusion pollution, we turn to two electron-proton colliders, the Large Hadron electron Collider (LHeC) [20] [21] [22] and the Future Circular Collider (FCC-he) [23]. The development of the energy recovery linac for the electron beam makes it possible to simultaneously operate the $pp$ and $e^+p$ collisions. In particular, the LHeC has a bright outlook for its working group recently announced the default configuration and staging based on the cost estimation [22]. We find the following advantages of electron-proton colliders in probing rare BSM events:

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• The pileup, which degrades the quality of the data for physics analyses, is very small even at the high luminosity option (~ 10^{34}/cm^2/s): one expects about 0.1 (1) pileup collisions per event at the LHeC (FCC-he) while ≳ 150 at the LHC.

• The QCD backgrounds and the higher-order corrections are suppressed, providing a clean environment.

• The charged-current (CC) and neutral-current (NC) processes can be disentangled by tagging the outgoing neutrino (as large missing transverse energy) or electron.Independent measurements of κ_{2W} and κ_{2Z} are possible.

• The asymmetric initial state allows us to distinguish the forward and backward directions, which can increase the signal significance.

• High polarization of the electron beam, P_e, is feasible, as large as ±80% [22]. The CC production cross-section increases by the factor of (1 − P_e), while the NC cross-section does not change much.

For the configurations of

\[ \text{LHeC: } E_e = 50 \text{ GeV, } E_p = 7 \text{ TeV}, \]

\[ \text{FCC-he: } E_e = 60 \text{ GeV, } E_p = 50 \text{ TeV}, \]

we shall analyze the sensitivity of the LHeC and FCC-he to κ_{2W} and κ_{1} via the VBF production of HH through the CC channel. Taking full advantage of the characteristics of the electron-proton collider, we shall propose a search strategy which we believe is optimal for measuring κ_{2W} and κ_{1}. Finally, we will present the 95% C.L. exclusion in the (κ_{2W}, κ_{1}) space, based on the detector-level analysis of the signals and the relevant backgrounds. The remainder of this letter is organized as follows. In section 2 we discuss the formalism of Higgs boson pair production in e+e− collisions within the k-framework along with a discussion of the modeling of the signal and background processes. In section 3 we discuss the analysis strategy and present our results. We conclude in section 4.

2. Formalism and modeling for the signal and backgrounds

Based on the observed Higgs precision data via single Higgs production at the LHC, we assume that all the couplings to a single Higgs boson are the same as in the SM:

\[ \kappa_{Hi} = 1, \]

where i and j are the SM particles. For renormalizable couplings to a Higgs boson pair, we consider

\[ \mathcal{L} \supset \kappa_{2W} \frac{g^2}{4} W^\mu W^\nu H^2 - \kappa_{1} \frac{3 m_{H}^2}{v} H^3, \]

where \( v \approx 246 \text{ GeV} \). Note that κ_{2W} and κ_{1} parameterize the BSM interactions within the context of the non-linear effective field theory given by the electroweak chiral Lagrangian [26, 27, 28, 29, 30].

Aiming at the precision measurement of κ_{2W} and κ_{1} together, we focus on the pair production of Higgs bosons through the CC VBF interaction in the bbbc final state,

\[ p^e \rightarrow HH + j_1V_e \rightarrow bbbc + j_1V_e, \]

where \( j_1 \) is a forward jet. There are three kinds of Feynman diagrams for this process, the contact one involving HHW\(^+\)W\(^-\) coupling, the s-channel involving HHH coupling, and the t, u-channels with the square of HHW\(^+\)W\(^-\) coupling. The scattering amplitudes of W\(^+\)W\(^-\) \rightarrow HH help us to understand the characteristics of the signal. As explicitly shown in Ref. [36], the longitudinally polarized W\(^+\) and W\(^-\) make an overwhelmingly dominant contribution. The corresponding amplitude, \( M_{LL} \), in the limit of \( \sqrt{s} \gg m_H \) satisfies

\[ \frac{1}{g^2} M_{LL} = \left( \kappa_{2W} - \kappa_1^2 \right) \frac{s}{4 m_W^2} + \kappa_1 \frac{3 m_H^2}{4 m_W^2} \]

\[ \left( 1 - \frac{m_H^2}{2 m_W^2} - \frac{2}{\sin^2\theta^*} - \frac{\kappa_{2W}}{2} + O \left( \frac{m_{W,HH}}{s} \right) \right), \]

where we keep the notation of κ_{2W} to show its effects and \( \theta^* \) is the scattering angle in the center-of-mass frame of W\(^+\)W\(^-\). Note that the effect of κ_{1} dominates in the small \( m_{HH} = \sqrt{s} \) region while that of κ_{2W} does in the high m_{HH} region.

First, at the parton level, we calculate the total cross-sections of the signal by varying both κ_{2W} and κ_{1}. The calculations have been performed at leading order (LO) using MadGraph at MC@NLO with a modified UFO model file for the Lagrangian in Eq. [1]. Based on the current experimental bounds, we consider \(-1 \leq \kappa_{2W} \leq 3 \) [16] and \(-6 \leq \kappa_1 \leq 12 \) [5, 6]. The SM cross-section of the process \( p^e \rightarrow HHj_1V_e \) is very small: with the unpolarized electron beam, it is \( \sigma_{SM} \sim 28, 29, 30 \) at the FCC-he. Despite tiny SM signals, it is promising that the total cross-section rapidly increases when either κ_{2W} or κ_{1} deviates from their SM values: the two electron-proton colliders can exclude a large portion of the parameter space (κ_{2W}, κ_{1}). To show this behavior, we present \( \sigma/\sigma_{SM} \) in Fig. [1]. It is clear to see that the deviation from κ_{2W} = 1 greatly increases the cross-section because it invalidates the cancellation of the longitudinal polarization enhancement, the first term of Eq. [5]. The hypothesis of κ_{1} ≠ 1 also increases the signal cross-section, though less than that of κ_{2W} ≠ 1. Quantitatively, we have a tenfold increase of \( \sigma/\sigma_{SM} \) if \(|\kappa_{2W} - 1| = |\kappa_{1} - 1| = 1\). We also note that the same-sign κ_{2W} and κ_{1} yield constructive interference, explaining the negative slopes of the \( \sigma/\sigma_{SM} \) contours: see the first two terms of Eq. [5]. In detail, the LHeC and FCC-he show different shapes.

3Note that concrete BSM scenarios can accommodate large values of κ_{1} at the quantum level [31, 32, 33, 34, 35]. However, it is not straightforward to construct a BSM model that can have a large κ_{2W} without significantly affecting κ_{2W}. 

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of the contours. As shall be demonstrated, the LHeC is more sensitive to $\kappa_4$ than to $\kappa_2$. The FCC-he has enough sensitivity to probe both.

For the $b\bar{b}b\bar{b}$ decay mode, the final state of the signal comprises at least four $b$-tagged jets, one light untagged jet, and large missing transverse energy ($E_{T\text{miss}}$). The main backgrounds are the QCD multi-jets, diboson, $t\bar{t}$, and single Higgs processes, all of which are associated with a forward jet $j_f$ and an electron neutrino $\nu_e$. In Table 1, we show the calculation of the LO cross-sections for the backgrounds at parton level using MadGraph version 2.6.7. Parton luminosities were modeled with the NNPDF31Lo PDF set with $\alpha_s(m_Z^2) = 0.118$. Setting the direction of the proton beam as forward, we convolute the partonic cross-sections with the PDFs in the LHAPDF6 library. The decays of $H$, $Z$, and the top quark are modeled with MadSpin. We confirmed that various kinematic distributions from on-shell samples using MadSpin well agree with those from the off-shell samples. For the parton-showering and hadronization, we rely on Pythia6 since Pythia8 does not support the LHE input in electron-proton collisions yet. To correctly model hadronization of the events, we modified the default Pythia6 setup. First, we switch off the lepton PDF by setting MSTP(11)=0. Second, we also switch off the QED initial state radiation for the electron beam by setting MSTP(61)=0. Finally, we switch off the negligible multiple-parton interactions, which restrict the parton-level process.

The total cross-section of all the CC backgrounds is about 100 fb (751 fb) at the LHeC (FCC-he). The most dominant is the QCD production of $b\bar{b}jj$ where $j$ refers to a light quark (including a charm quark) or a gluon. The second dominant backgrounds are from the production of a $Z$ boson associated with another $Z$ boson, the QCD $b\bar{b}$, or a Higgs boson. The QCD production of four $b$ quarks follows, and the production of a Higgs boson in association with $b\bar{b}$ is less critical. Finally, the contribution of a top quark pair production is smaller than the QCD 4b at the LHeC, but similar at the FCC-he. Important theoretical uncertainties arise from the scale variations, as shown in Table 1. PDF uncertainties are of order 1-2% for all the backgrounds.

We close this section by summarizing the Monte Carlo event generation procedure. Initially, events for the signal and backgrounds are generated at LO using MadGraph with NNPDF31Lo version 2.6.7. Parton luminosities were modeled with the NNPDF31Lo PDF set with $\alpha_s(m_Z^2) = 0.118$. Setting the direction of the proton beam as forward, we convolute the partonic cross-sections with the PDFs in the LHAPDF6 library. The decays of $H$, $Z$, and the top quark are modeled with MadSpin. We confirmed that various kinematic distributions from on-shell samples using MadSpin well agree with those from the off-shell samples. For the parton-showering and hadronization, we rely on Pythia6 since Pythia8 does not support the LHE input in electron-proton collisions yet. To correctly model hadronization of the events, we modified the default Pythia6 setup. First, we switch off the lepton PDF by setting MSTP(11)=0. Second, we also switch off the QED initial state radiation for the electron beam by setting MSTP(61)=0. Finally, we switch off the negligible multiple-parton interactions, which restrict the parton-level process.

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Figure 1: $\sigma/\sigma_{\text{SM}}(p^+e^- \rightarrow HHj\nu_e)$ projected on the plane of $(\kappa_2, \kappa_3)$ at the LHeC (upper panel) and FCC-he (lower panel). The SM cross-section $(\kappa_2 = \kappa_3 = 1)$ with the unpolarized electron beam is $\sigma_{\text{SM}} = 5.97$ ab at the LHeC and $\sigma_{\text{SM}} = 233.77$ ab at the FCC-he.

| Process          | LHeC $\sigma_{\text{CC}}$ [ab] | FCC-he $\sigma_{\text{CC}}$ [ab] |
|------------------|-------------------------------|----------------------------------|
| $b\bar{b}jj + j\nu_e$ | $1.00 \times 10^5 \pm 5.79\% - 33.9\%$ | $7.18 \times 10^5 \pm 51.1\% - 31.6\%$ |
| $ZZ + j\nu_e$     | $9.37 \times 10^4 \pm 3.45\% - 3.2\%$ | $2.24 \times 10^4 \pm 3.65\% - 2.9\%$ |
| $Zbb + j\nu_e$    | $5.38 \times 10^2 \pm 2.75\% - 19.9\%$ | $4.77 \times 10^3 \pm 22.6\% - 17.1\%$ |
| $ZH + j\nu_e$     | $1.23 \times 10^2 \pm 7.29\% - 6.7\%$ | $3.45 \times 10^3 \pm 3.99\% - 3.58\%$ |
| $b\bar{b}b + j\nu_e$ | $1.82 \times 10^2 \pm 53.8\% - 32.4\%$ | $7.11 \times 10^2 \pm 50.9\% - 31.4\%$ |
| $Hb\bar{b} + j\nu_e$ | $4.53 \times 10^1 \pm 29.4\% - 20.4\%$ | $4.77 \times 10^2 \pm 23.7\% - 17.8\%$ |
| $t\bar{t} + j\nu_e$ | $2.00 \times 10^1 \pm 29.6\% - 21.2\%$ | $7.49 \times 10^2 \pm 22.9\% - 17.4\%$ |

Table 1: Parton level cross-sections in attobarn (ab) for the charged-current background processes at the LHeC with $E_p = 50$ GeV and $E_e = 7$ TeV and at the FCC-he with $E_e = 60$ GeV and $E_p = 50$ TeV. The electron beam is unpolarized. A small correction of scale variations is included in the cross-sections. The uncertainties correspond to scale variations around the nominal scale defined in Eq. (6). PDF uncertainties are at the percent level and hence are not shown here. SysCalc was used to compute these uncertainties.
saves a considerable amount of computing time. We use the default PDF at the Pythia6 level, CTEQ6L[43]. Fast detector simulation was performed using Delphes version 3.4.2[45]. To match the particle efficiencies, momentum smearing, and isolation parameters with the default values in the Concept Design Report of the LHeC[22], we have performed minor modifications on the Delphes cards in the GitHub repository https://github.com/delphes/delphes/tree/master/cards. Jets are clustered using the anti-kT algorithm[46] with a jet radius $R=0.4$ in FastJet version 3.3.2[47]. The b-tagging efficiency is set to be 70%. For the mistagging rates of the light and charm jets as a b jet, we adopted the default values in the above Delphes cards: at the LHeC, $P_{j\rightarrow b} = 0.001$ and $P_{c\rightarrow b} = 0.05$; at the FCC-he, $P_{j\rightarrow b} = 0.001$ and $P_{c\rightarrow b} = 0.04$ for $|\eta| < 2.5$, $P_{j\rightarrow b} = 0.00075$ and $P_{c\rightarrow b} = 0.03$ for $2.5 < |\eta| < 4$.

3. Results and Discussion

3.1. Event selection

In this section, we update the ATLAS analysis strategy for the VBF production of $HH$[16], to optimize the signal significance at the LHeC and FCC-he. As summarized in Table 2, the event selection takes the following steps:

- **Initial:**
  The initial number of events, $n_0$, is obtained from the full detector-level simulation. We consider the decays of $H \rightarrow b\bar{b}, Z \rightarrow b\bar{b}$, and both the semi-leptonic and hadronic decays of a top quark pair.

- **4b-tag:**
  We require the presence of at least four $b$-tagged jets with $p_T^{b} > 20$ GeV and $|\eta|^b < 5$. The acceptance times efficiency for the signal processes is around 10-16%, depending on the values of $\kappa_1$ and $\kappa_{2Y}$.

- **Forward jet:**
  We demand that at least one jet, untagged as a $b$ jet, has $p_T^{j} > 20$ GeV and $1.5 < |\eta|^j < 7$. Note that the definition of being forward at asymmetric $e^+p$ colliders is different from that at the LHC. This selection reduces the signal events by about 20%, irrespective of the hypothesis of $\kappa_{2Y}$ and $\kappa_1$.

- **Lepton veto:**
  We veto the events which contains an isolated lepton ($\ell = e^+, \mu^+$) with $p_T^{\ell} > 10$ GeV and $|\eta|^\ell < 5$. The criteria of lepton isolation is required so that charged leptons from heavy hadron decays are not subject to this selection but their momenta are added to the hadronic jet if $\Delta R(\ell,j) < 0.2$. Here $\Delta R \equiv \sqrt{\Delta\phi^2 + \Delta\eta^2}$. This selection is very effective in suppressing the NC backgrounds. The event yields for the signal and CC backgrounds remain almost the same.

- **$E_{T}^{miss}$:**
  This selection consists of two requirements, $E_{T}^{miss} > 40$ GeV and $|\phi_{j_1} - \phi_{E_{T}^{miss}}| > 0.4$. The latter removes the backgrounds with incorrectly measured $E_{T}^{miss}$. At this stage, the signal event yield is reduced by about 10%.

- **Minimum $D_{HH}$:**
  The mission here is to find two Higgs boson candidates from four $b$-tagged jets. There are three possible combinations for pairing two $b$-jets out of four, called the dijet. In each combination, we order two dijets according to their transverse momentum, and call them the ‘leading’ dijet and the ‘sub-leading’ dijet. Computing the angular separation of two $b$-jets inside each dijet system, $\Delta R_{lead}$ and $\Delta R_{sublead}$, we require

$$\Delta R_{lead} < \begin{cases} \frac{653 \text{ GeV}}{M_{ab}} + 0.475 & \text{if } M_{ab} < 1250 \text{ GeV}, \\ 1.0 & \text{if } M_{ab} > 1250 \text{ GeV}, \end{cases}$$

$$\Delta R_{sublead} < \begin{cases} \frac{875 \text{ GeV}}{M_{ab}} + 0.35 & \text{if } M_{ab} < 1250 \text{ GeV}, \\ 1.0 & \text{if } M_{ab} > 1250 \text{ GeV}, \end{cases}$$

where $M_{ab}$ is the invariant mass of the four $b$-tagged jets. Among the pairings that satisfy Eq. [7], we choose the pairing with the smallest value of $D_{HH}$ as the final $HH$ candidate. Here $D_{HH}$ is[16]:

$$D_{HH} = \sqrt{M_{lead}^2 + M_{sublead}^2} \times \left| \sin^{-1} \frac{M_{lead}^{dijet}}{M_{lead}^{dijet} - \tan^{-1} 116.5 \text{ GeV}} \right|,$$

where $M_{lead}^{dijet}$ ($M_{sublead}^{dijet}$) is the invariant mass of the leading (sub-leading) dijet system. The values of 116.5 GeV and 123.7 GeV are adopted to properly treat the energy loss in the semi-leptonic decays of the $b$-hadrons.

- **$X_{HH}$-cut:**
  Finally, the signal region is defined by the following variable[16]:

$$X_{HH} = \begin{cases} \frac{(M_{lead}^{dijet} - 123.7 \text{ GeV})^2}{11.6 \text{ GeV}} + \frac{(M_{sublead}^{dijet} - 116.5 \text{ GeV})^2}{18.1 \text{ GeV}} & \text{if } X_{HH} < 1.6 \text{ at } D_{HH} \text{ for the LHeC and FCC-he in Fig. 2}. \end{cases}$$

The ATLAS collaboration required $X_{HH} < 1.6$ to maximize the LHC signal significance. To optimize the search at the LHeC and FCC-he, we present the differential cross-sections as a function of $X_{HH}$ for the LHeC and FCC-he in Fig. 2. The histograms in gray represent the total background distributions. We also show the signal results in six different hypotheses of $(\kappa_1, \kappa_{2Y}) = \{(-6,-1), (12,3), (0,3), (1,1), (-3,0), (5,0)\}$ in green, blue, olive, red, purple, and cyan respectively. It is clear to see that the backgrounds are distributed in the high $X_{HH}$ region. We have calculated the signal significance, to be defined below, for different values of the upper-cut on $X_{HH}$. We found that $X_{HH} < 3$ ($X_{HH} < 2$) at the LHeC (FCC-he) maximizes the signal significance, by which we
Table 2: Cut-flow chart of the number of events of the signal and backgrounds at the LHeC and the FCC-he with the unpolarized electron beam. The background processes are denoted as omitting $\nu$ for simplicity. The numbers inside the parentheses show the acceptance times efficiency after the selection step $i$ with respect to the initial number of events $m_i$, i.e. $\epsilon_i = n_i/m_i$.

| Cut                | $bbjj/bbbb$ | ZZ/HZ | $(Z/H)bb$ | $t\bar{t}$ | total backgrounds | Signal ($\kappa = \kappa_{2\nu} = 1$) |
|--------------------|-------------|-------|-----------|-------------|------------------|--------------------------------------|
| LHeC with $L_{tot} = 1 \text{ ab}^{-1}$ |             |       |           |             |                  |                                      |
| Initial            | 100167.05   | 32.10 | 107.41    | 17.65       | 100324.21 (100%) | 1.98 (100%)                        |
| 4b-tag             | 4.36        | 2.77  | 2.26      | 0.02        | 9.41 (0.0094%)   | 0.25 (12.34%)                      |
| Forward jet        | 1.80        | 2.15  | 1.24      | 0.01        | 5.20 (0.0052%)   | 0.17 (8.88%)                       |
| Lepton veto        | 1.80        | 2.15  | 1.24      | 0.01        | 5.20 (0.0052%)   | 0.17 (8.88%)                       |
| $E_T^{\text{miss}} > 40 \text{ GeV}$ | 1.33        | 1.52  | 0.95      | 0.01        | 3.81 (0.0038%)   | 0.074 (3.73%)                      |
| Minimum $D_{HH}$   | 1.27        | 1.48  | 0.91      | 0.01        | 3.66 (0.0037%)   | 0.064 (3.25%)                      |
| $X_{HH} < 3.0$     | 0.15        | 0.23  | 0.17      | 0.00        | 0.55 (0.00043%)  | 0.04 (2.04%)                       |
| FCC-he with $L_{tot} = 10 \text{ ab}^{-1}$ |             |       |           |             |                  |                                      |
| Initial            | 7180161     | 8141.8| 9989.6    | 6673.6      | 7204970.0 (100%) | 779.70 (100%)                     |
| 4b-tag             | 934.2       | 745.6 | 274.7     | 24.2        | 1978.7 (0.026%)  | 94.05 (12.06%)                     |
| Forward jet        | 562.6       | 637.1 | 185.6     | 21.9        | 1407.2 (0.018%)  | 74.71 (9.58%)                      |
| Lepton veto        | 562.5       | 637.0 | 185.6     | 18.8        | 1403.9 (0.018%)  | 74.71 (9.58%)                      |
| $E_T^{\text{miss}} > 40 \text{ GeV}$ | 492.3       | 497.7 | 153.0     | 16.2        | 1159.2 (0.015%)  | 42.11 (5.40%)                      |
| Minimum $D_{HH}$   | 412.9       | 458.8 | 129.9     | 13.9        | 1015.5 (0.014%)  | 29.71 (3.81%)                      |
| $X_{HH} < 2.0$     | 29.8        | 32.6  | 10.2      | 1.5         | 74.1 (0.00098%)  | 10.99 (1.41%)                      |

3.2. Results

In this section, we discuss the results of our analysis. After the full selection, the signal efficiency is about 2.0% for the LHeC and about 1.4% at the FCC-he, while the background efficiency is about $\mathcal{O}(10^{-3}-10^{-1})\%$ (see Table 2). To obtain the discovery potential, we compute the signal significance including the background uncertainty [33], defined by

$$S = \left[2(N_s + N_b) \log \left(\frac{(N_s + N_b)(N_s + \delta_s^2)}{N_s + (N_s + N_b)\delta_b^2}\right) - \frac{2N_b^2}{\delta_b^2} \log \left(1 + \frac{\delta_b^2 N_s}{N_b(N_b + \delta_b^2)}\right)^{1/2}\right],$$

where $N_s$ is the number of signal events, $N_b$ is the number of background events, and $\delta_b = \Delta_b N_b$ is the uncertainty in the background yields. The numbers of the signal and background events are

$$N_s = L_{tot} \times \epsilon_{HHH} B_{HHH}^2 H_{-bb},$$

$$N_b = L_{tot} \times \epsilon_{bbjj} B_{bbjj}^2 + \epsilon_{ZZZ} B_{ZZZ}^2 + \epsilon_{HHH} B_{HHH}^2 + 2\epsilon_{HZZ} B_{HZZ}^2 + 2\epsilon_{H} B_{H}^2 + \epsilon_{b\nu} B_{b\nu}^2 \epsilon_{b\nu},$$

where $L_{tot}$ is the total integrated luminosity, $\epsilon_s$ is the acceptance times efficiency for the process $X$ in the signal region, and $B_X$ is the branching ratio of the decay $X$. Brief comments on the error estimation for the backgrounds are in order here.

In principle, the background errors show different variation according to jet energy scale, the momentum smearing, $b$-tagging efficiency, jet energy resolution, and theoretical uncertainties. Since the detailed study is beyond the scope of this work, we take two simple cases, $\Delta_{bg} = 10\%$ and $\Delta_{bg} = 50\%$ [3].

In Fig. 3, we display the expected exclusions on the plane of $\kappa_{2\nu}$ and $\kappa_4$ at the LHeC (upper panel) and the FCC-he (lower panel), corresponding to $S > 2$. We consider the electron beam polarization of $P_e = -80\%$ and two cases of the background uncertainty, $\Delta_{bg} = 10\%$ (solid) and $\Delta_{bg} = 50\%$ (dashed). For the total integrated luminosity $L_{tot}$, we take $1 \text{ ab}^{-1}$ (olive) and $10 \text{ ab}^{-1}$ (orcid) at the LHeC, and 0.1 $\text{ ab}^{-1}$ (olive), 1 $\text{ ab}^{-1}$ (orcid), and $10 \text{ ab}^{-1}$ (blue) at the FCC-he. The common result of the LHeC and FCC-he is that the same-sign $\kappa_{2\nu}$ and $\kappa_4$ region is more strongly constrained because of the constructive interference discussed before.

In detail, the LHeC and FCC-he have different exclusion potential. In general, the LHeC has limitations in constraining $\kappa_{2\nu}$ and $\kappa_4$ because of its lower center-of-mass energy. Nevertheless, it can produce some meaningful results. If $\kappa_{2\nu} = 1$, the LHeC data with $L_{tot} = 1 \text{ ab}^{-1}$ and $\Delta_{bg} = 10\%$ can constrain $\kappa_4$ as $-3 \lesssim \kappa_4 \lesssim 6$, which is weaker than the HL-LHC prospect. If $\kappa_4 = 1$, the LHeC data with $L_{tot} = 1 \text{ ab}^{-1}$ and $\Delta_{bg} = 10\%$ can exclude $\kappa_{2\nu} \lesssim -1$ and $\kappa_{2\nu} \gtrsim 3.2$, which is compatible with the current bound on $\kappa_{2\nu} \in [-0.66, 2.89]$ at 95% C.L. [10].

6In this study, we adopted a conservative approach for the background uncertainties. Considering the expected improvement of various precisions, e.g., the PDF precision at the LHeC, we expect that $\Delta_{bg} = 10\%$ can be obtained in the future.
Considering the feasibility of the concurrent operation of the HL-LHC and LHeC, two colliders shall play a complementary role in probing $k_{2V}$. In terms of the ratio of the cross-section of the CC VBF production of $HH$ to the SM value, the LHeC with $L_{tot} = 1$ ab$^{-1}$ and $\Delta_{bg} = 10$ (50)% can limit $\sigma/\sigma_{SM} \lesssim 30$ (35).

On the other hand, the FCC-he has high potential in probing both $k_{2V}$ and $k_{1}$. For $k_{j} \in [0.1, 2.3]$ suggested by the HL-LHC prospect study [3], $|k_{2V}| \gtrsim 0.2$ is to be excluded by the FCC-he data with $L_{tot} = 10$ ab$^{-1}$ and $\Delta_{bg} = 10\%$. Two important reasons for this high precision are higher signal cross-section and similar rejection rates of the SM backgrounds (see Table 2). At the FCC-he with $\Delta_{bg} = 10\%$ (50\%), we estimated conservative bounds on the ratio of the Higgs pair production cross section to the SM value as follows:

$$\frac{\sigma}{\sigma_{SM}}|_{\text{FCC-he}} < \begin{cases} 
11 (14) & \text{for } L_{tot} = 0.1 \text{ ab}^{-1}, \\
3.5 (8) & \text{for } L_{tot} = 1 \text{ ab}^{-1}, \\
1 (7) & \text{for } L_{tot} = 10 \text{ ab}^{-1}.
\end{cases} \quad (12)$$

Final comments on the role of higher electron beam energy in probing the $HH$ process are in order here. Although it is practical for the LHeC working group to choose $E_{e} = 50$ GeV due to the cost issues, the physics gain from higher $E_{e}$ is more important than anything else. We found that setting $E_{e} = 120$ GeV increases the background cross sections by a factor of 2.32 (1.82) at the LHeC (FCC-he). For the signal cross sections, the enhancement factor is 2.1 – 2.7 at the LHeC and 4.4 – 5.9 at the FCC-he, depending on the values of $k_{j}$ and $k_{2V}$. Assuming similar efficiencies for both the signal and backgrounds to those in Table 2, we expect that the significance increases by a factor of $1.4 - 1.8 \times 3.3 - 4.4$ at the LHeC (FCC-he). At the FCC-he, increasing $E_{e}$ into 120 GeV has almost the same effect as increasing the total luminosity tenfold. We strongly suggest that the FCC-he working group seriously consider the higher $E_{e}$ option.
4. Conclusions

Upon the current status where both the trilinear Higgs self-coupling modifier ($\kappa_3$) and the quartic coupling modifier between a Higgs boson pair and a vector boson pair ($\kappa_{2Y}$) are unmeasured, we consider two electron-proton colliders, the LHeC and FCC-he, in probing $\kappa_3$ and $\kappa_{2Y}$ simultaneously. As a proton-proton collider, the LHC cannot avoid the gluon fusion process in the VBF production of a Higgs pair, which becomes much worse for $\kappa_3 \neq 1$. At electron-proton colliders, the gluon fusion process is absent, and thus the charged-current VBF production of a Higgs boson pair can be solely measured if there is enough signal significance. With this motivation, we study the detailed phenomenology of production of a Higgs boson pair can be solely measured if there is enough signal significance. To this end, we suggest a search strategy at the LHeC and FCC-he.

We have completed the analysis with full simulations to devise an optimal strategy. We found that most of the current ATLAS search strategies for $HH$ via VBF production apply to those at the LHeC and FCC-he. The key difference of ours is the cut on $X_{HH}$ defined in Eq. (9). We found that the signal significance at the LHeC (FCC-he) is maximized by $X_{HH} < 3$ ($X_{HH} < 2$). A larger upper bound on $X_{HH}$ than the one for the LHC, $X_{HH} < 1.6$, increases the signal significance. As the final result, we calculated the expected exclusions on $(\kappa_{2Y}, \kappa_3)$. The LHeC can play an meaningful role in probing $\kappa_{2Y}$: the data with the total integrated luminosity of $L_{\text{tot}} = 1 \text{ ab}^{-1}$ and the background uncertainty of $\Delta_{bg}$ = 10% can constrain $-1 \lesssim \kappa_{2Y} \lesssim 3.2$ for $\kappa_3 = 1$. The FCC-he has immense power in constraining both $\kappa_{2Y}$ and $\kappa_3$. If $\kappa_3 \in [0.1, 2.3]$ as the HL-LHC prospect, $|\kappa_{2Y}| \geq 0.2$ is to be excluded with $L_{\text{tot}} = 10 \text{ ab}^{-1}$ and $\Delta_{bg} = 10\%$. We hope that this study would provide input to strongly support the future programs of electron-proton colliders which are capable of measuring two fundamental couplings, $\kappa_{2Y}$ and $\kappa_3$.

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