Partial Averaged Navier-Stokes approach for cavitating flow

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Abstract. Partial Averaged Navier Stokes (PANS) is a numerical approach developed for studying practical engineering problems (e.g. cavitating flow inside hydroturbines) with a resonance cost and accuracy. One of the advantages of PANS is that it is suitable for any filter width, leading a bridging method from traditional Reynolds Averaged Navier-Stokes (RANS) to direct numerical simulations by choosing appropriate parameters. Comparing with RANS, the PANS model will inherit many physical nature from parent RANS but further resolve more scales of motion in great details, leading to PANS superior to RANS. As an important step for PANS approach, one need to identify appropriate physical filter-width control parameters e.g. ratios of unresolved-to-total kinetic energy and dissipation. In present paper, recent studies of cavitating flow based on PANS approach are introduced with a focus on the influences of filter-width control parameters on the simulation results.

1. Introduction
For studying practical engineering problems (e.g. cavitating flow, pressure fluctuation and instability inside the hydroturbine), two kinds of approaches is prevalent: (unsteady) Reynolds Averaged Navier-Stokes (RANS) and Large Eddy Simulation (LES). RANS is simple and can predict the large scale vortexes. However, the physical motions induced by small scale vortexes are usually ignored by RANS, leading to inaccuracy of prediction. LES is accurate and can successfully capture the structure of vortexes with variable scales. However, the computational cost of LES is quite intensive and sometimes it is impractical to use it for engineering simulations. Hence, a bridging method named as Partial Averaged Navier Stokes (PANS) was proposed by Girimaji and collaborators [1-3].

According to Girimaji [1], PANS is suitable for any filter width (e.g. from RANS to direct numerical simulations). The development of PANS can be divided into two steps. In the first step, one need to identify appropriate physical filter-width control parameters. Girimaji [1] suggested that the best quantitative evaluation parameters of the degree of filtering are ratios of unresolved-to-total kinetic energy and dissipation. In the second step, the PANS equations are derived from parent RANS closure by resolving more scales of motion. The scale of motion further treated in PANS is highly dependent on the values of the filter-control parameters in the first step. If we set these ratios as unity, RANS equations are obtained. If we set these ratios as zero, full Navier-Stokes equations are recovered from PANS method. Obviously, the PANS model will inherit many physical nature from parent RANS but further resolve more scales of motion in great details, leading to PANS superior to RANS. Comparing with traditional
LES, PANS has distinct characteristics from three aspects [1], e.g., decomposition of velocity filed using kinetic energy content, implicit filter demarcating calculation method and grid-spacing free subfilter scale constitutive relationship.

Various kinds of PANS models have been developed based on the framework of Girimaji [1]. Ma et al. [4] proposed a low Reynolds number variant of PANS model for turbulent flow. Liu et al. [5] proposed a new nonlinear PANS model based on RNG $k$-$\varepsilon$ turbulence model and validated their model through comparing experimental data with simulations (e.g., a NACA hydrofoil, a curved rectangular duct, and a low specific centrifugal pump).

Currently, PANS model has been widely implemented in a wide range of engineering problems. Girimaji and Srinivasan [3] employed PANS model to simulate ramjet/scramjet mixing environment. Liu et al. [6] applied PANS model to the study of the instability of the pump turbine with misaligned guide vanes. There are also numerous studies of employing PANS model with appropriate cavitation model for the study of cavitating flow e.g. around a hydrofoil [7,8] or marine propellers [9].

In present paper, the employment of PANS model on studying of cavitating flow is introduced. The detailed discussions of principles of PANS is out the scope of present paper. For details, readers are referred to Girimaji [1].

2. Selected results of cavitating flow simulation based on PANS approach

In this section, some typical simulation results of the simulation work of cavitating flow based on PANS approach are introduced with details.

Ji et al. [9] studied the cavitating flow around a marine propeller in a uniform wake based on PANS model. Firstly, influences of two important parameters (namely, the ratio of the unresolved-to-total kinetic energy and dissipation, $f_k$ and $f_\varepsilon$, respectively) are discussed through simulating cavitating flow around a NACA66-mod hydrofoil. The $f_k$ varies between 0.2 and 1 in order to fit the experimental data (as shown in Figure 1). It was found that when $f_k=0.2$, the predicted cavity evolution and shedding frequency agree well with the experiments. Comparing with RANS, it was found that the resolution of the simulation results predicted by PANS is better in terms of large cavity volume pulsation when the blade passing through the wake region. The choice of $f_k$ was also found to be a crucial parameter influencing the predictions of cavitating flow. Generally, a smaller value of $f_k$ tends to increase the acceleration of cavity pulsation while a larger value of $f_k$ will overestimate the turbulent viscosity (referring to Figure 2). The PANS approach in Ji et al. [9] was validated by comparing with the experimental data (referring to Figure 3).

![Figure 1](image-url). Influences of $f_k$ on the instantaneous vapor volume fractions. This figure was adapted from Ji et al. [9].
Huang and Wang\cite{7} evaluated the influences of ratio of the unresolved-to-total kinetic energy ($f_k$) on the predictions of cavitating flow over a Clark-Y hydrofoil. A more rigorous comparison between predictions with the variations of $f_k$ and the experimental data (e.g. forces, frequencies, cavity visualization and velocity distributions) are performed. With the increase of $f_k$, a standard k-ε model was recovered. With the decrease of the $f_k$, more smaller scales of the vortexes in the cavitating flow is observed (referring to more peaks in the life coefficient in Figure 4).

Figure 2. Influences of $f_k$ on the time averaged turbulence viscosity. This figure was adapted from Ji et al. \cite{9}.

![Figure 2](image2.png)

Figure 3. Comparisons between the predicted cavity patterns and the experimental evolution. This figure was adapted from Ji et al. \cite{9}.

![Figure 3](image3.png)
Figure 4. Influences of $f_k$ on the variations of lift coefficient. This figure was adapted from Huang and Wang [7].

Recently, Hu et al. [10] modified the $f_k$ as a function of the ratios between water density and mixture density in the flow as follows:

$$
f_k = \tanh(\text{atanh} \frac{\rho_m}{\rho_l} C_2) + 1 - C_2
$$

Here, $\rho_m$ is the density of mixture; $\rho_l$ is the density of liquid; $C_1$ and $C_2$ are two parameters newly introduced to modify the values of $f_k$. Figure 5 shows the influences of $C_1$ and $C_2$ on the variations of $f_k$.

Figure 5. Influences of $C_1$ and $C_2$ on the variations of $f_k$. This figure was adapted from Hu et al. [10].
The modified PANS model was validated through simulating a Clark-Y hydrofoil by fitting the parameters of $C_1$ and $C_2$ with the experimental data (such as hydrofoil lift and frequency as shown in Figure 6). It was found that $C_1=34.05$ and $C_2=0.99$ show satisfactory results [10].

**Figure 6.** Data fitting of hydrofoil lift and frequency using different values of $C_1$ and $C_2$. This figure was adapted from Hu et al. [10].

| Time      | Experiment       | Simulation       |
|-----------|------------------|------------------|
| $t_0$     | attached cavity  |                  |
| $t_0+0.4T$| break-off        |                  |
| $t_0+0.6T$|                  | U-type vortex shedding |
| $t_0+0.8T$|                  |                  |

**Figure 7.** Evolution of cavity shape around the axisymmetric body against time. This figure was adapted from Hu et al. [11].
Hu et al. [11] further applied the modified PANS model coupled with Kubota cavitation model to simulate unsteady cavitating flows around an axisymmetric body with a blunt headform. Figure 7 shows the comparisons between experimental observations and numerical predictions. As shown in Figure 7, the simulations based on modified PANS can successfully predict the paramount phenomenon in cavitating flow (e.g. cavity break-off and U-type shedding).

3. Conclusion
Based on reported work using PANS approach, it was found that the ratio of the unresolved-to-total kinetic energy ($f_k$) plays an important role on the predictions (e.g. cavity pulsations, turbulent viscosity, shedding dynamics). Apart from setting it as a pure constant, variable $f_k$ has been implemented through considering it as a function of ratio of mixture density and liquid density. Due to complex of the phenomenon, much more works on the physical origins of the paramount parameters in PANS approach on the cavitation dynamics should be investigated.

Acknowledgement
The authors acknowledge the financial support from "the Fundamental Research Funds for the Central Universities" (Project No.: 2014ZD09).

References
[1] Girimaji S 2006 ASME J. of Applied Mechanics 73 413
[2] Girimaji S, Jeong E and Srinivasan R 2006 J. of Applied Mechanics 73 422
[3] Girimaji S and Srinivasan R 2009 AIAA 2009-134, 47th AIAA Fluid Dynamics Conference and Exhibit (Orlando, FL, USA, 5–8 January 2009)
[4] Ma J M, Peng S H, Davidson L and Wang F J 2011 Int. J. of Heat and Fluid Flow 32 652
[5] Liu J T, Zuo Z G, Wu Y L, Zhuang B T and Wang L Q 2014 Computers & Fluids 102 32.
[6] Liu J T, Wu Y L and Wang L Q 2013 Advances in Mech. Eng. 2013 710769.
[7] Huang B and Wang G.Y. 2011 J Hydrodyn 23 26
[8] Ji B, Luo X W, Wu Y L and Xu H.Y. 2012 Chinese Phys. Lett. 29 076401
[9] Ji B, Luo X W, Wu Y L, Peng X X and Xu H Y 2012 Int. J. Heat Mass Transfer 55 6582
[10] Hu C, Wang G, Chen G and Huang B 2014 Sci. China Physics, Mechanics & Astronomy 57 1967
[11] Hu C, Wang G, Chen G and Huang B 2014 J. of Mech. Sci. Tech. in press