Research Article

Grey Multiattribute Emergency Decision-Making Method for Public Health Emergencies Based on Cumulative Prospect Theory

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In view of the different risk preferences of decision-makers and the information gap in the timing of the subjects under evaluation, a multiattribute dynamic decision-making model that considers decision risk in addition to the priority of time is proposed. First, to account for the uncertainty of the evaluation information, the interval grey number method was applied to perform numerical abstraction and key data feature extraction on the scene of public health emergencies and to obtain the information after attributes were unified. Second, acknowledging the bounded rational behavior of actual decision-makers, the decision-makers’ risk preference was introduced into emergency decision-making. In combination with the time degree, the multiattribute unified decision matrix was transformed into an expected dynamic cumulative prospect decision matrix. Furthermore, based on cumulative prospect theory (CPT), the prospect value function and the weight function are constructed; a new optimization model is proposed to obtain the attribute weight. And according to the comprehensive prospect value of each alternative, the priority of the alternatives was determined. Finally, with the COVID-19 epidemic serving as an example, the results confirm that the emergency decision-making method proposed in this paper is feasible and effective.

1. Introduction

A public health emergency refers to a major infectious disease, a mass disease of unknown cause, a major food poisoning or occupational poisoning, natural disasters, accidents, or sudden social safety events that cause or may cause serious damage to the public’s physical and mental health [1]. Such occurrences have caused huge losses and impacts to human life, property, and social development, seriously hindering the development of human society.

Emergency decision-making is essential for an appropriate management of public health emergencies in which decisions have to be taken quickly and the best emergency alternative has to be chosen in real time and in response to the development of incidents to control the spread of the public health emergency and reduce losses. Because public health emergencies are characterized by uncertainty, derivation, dynamics, and so on [2], multiattribute, multi-scenario, and multistage factors need to be considered in formulating emergency response alternatives. Uncertain multiattribute information and the dynamic evolution of public health emergencies may increase the risk of emergency decision-making. Therefore, most emergency decisions in public health emergencies are regarded as multiattribute risk emergency decisions.

Scholars have carried out a number of studies on multiattribute risk emergency decision-making. Sha et al. used intuitionistic fuzzy numbers to represent prospect decision-making information for multiattribute decision-making problems and proposed a probabilistic hesitant fuzzy TOPSIS emergency decision-making method based on the cumulative prospect theory [3]. Gupta et al. pointed out that decision-makers describe the initial decision-making information in the interval-valued intuitionistic fuzzy environment, which is conducive to retaining the inherent uncertainty in the multiattribute group decision-making process [4]. Zhang and Chen proposed a group decision-making method with hesitant fuzzy linguistic preference relations for the situation where the attribute weight is not completely determined [5]. In consideration of bounded rational behavior, Shi et al.
proposed a neutral cross-efficiency evaluation method based on interval reference points that comprehensively considers the uncertainty and randomness of evaluation information [6]. Several scholars conducted in-depth studies on the multicriteria decision-making problem in which the criterion value is or can be transformed into an interval grey number. These studies were carried out mainly when the probability is a certain real number [7–18].

In the existing studies, many methods are still proposed with reference to interval number sorting, and interval grey numbers are converted into interval numbers or real numbers for calculation. There have been few studies reported on the problem of grey random multicriteria emergency decision-making with both probability and criterion values undetermined. However, in the actual emergency decision-making during public health emergencies, it is difficult for decision-makers to accurately predict the possibility of occurrence of an event or various natural states. Besides, it is often difficult to accurately determine the profit and loss values of alternative alternatives under various criteria. Therefore, solving the problem of emergency decision-making on multiattribute risk public health emergencies is an important research topic.

In this paper, a dynamic decision-making method based on cumulative prospect theory and interval grey numbers is proposed to address the emergency decision-making problem of dynamic multiattribute risk public health emergencies under consideration of decision-makers’ different risk preferences, weights at different time points, and standardization of multi-attribute information. First, the interval grey number method is applied to perform numerical abstraction and key data feature extraction on a scene of public health emergencies and to describe the information after the unification of attributes. Second, considering that different risk psychology and profit and loss considerations will affect the final decisions of decision-makers, their risk preference is introduced into the emergency decision-making process, and by applying cumulative prospect theory, the comprehensive cumulative prospect value of each alternative is calculated to select the best emergency alternative. Finally, based on the emergency decision-making model proposed in this paper, the COVID-19 epidemic is used as an example to analyze and discuss its emergency decision-making process in detail, which verifies that the emergency decision-making method proposed in this article is feasible and effective.

The remainder of this paper is organized as follows. Section 2 first introduced theoretical framework, including cumulative prospect theory and interval grey numbers. In Section 3, the proposed method is introduced in detail. Section 4 applies the proposed model to an application example to illustrate its feasibility and rationality. Section 5 conducts a comparative analysis and discusses the results. Finally, Section 6 concludes this paper.

2. Theoretical Framework

2.1. Cumulative Prospect Theory. Cumulative prospect theory (CPT) is a decision-making method that considers the risk preference of decision-makers proposed by Tversky and Kahneman [19] on the basis of improving prospect theory. It mainly contains three basic points: (1) profits and losses are relative to a reference point; (2) decision-makers are inclined to risk aversion when earning profits, while they are inclined to risk preference when facing losses; (3) decision-makers tend to overestimate the possibility of small-probability events and underestimate the probability of high-probability events. Under the framework of CPT, each alternative corresponds to the respective value function and the decision-making weight function, and their product is the comprehensive prospect value of the alternative. According to CPT, decision-makers use the prospect value of an alternative as the basis for decision-making. To calculate the prospect value of an alternative, three basic elements are needed, namely, value function, attribute value, and decision-making weight function.

The value function of an alternative is closely related to its attribute value, which can be expressed by the following formula:

\[
g(T_k^w) = \begin{cases} 
(T_k^w - u^w)^m, & T_k^w \geq u^w \\
-\lambda(u^w - T_k^w)^n, & T_k^w < u^w
\end{cases}
\]

where \(m > 0\) and \(n \leq 1\) represent the decision-makers’ sensitivity to value profits and losses; \(\lambda \geq 1\) represents the decision-maker’s loss aversion coefficient; \(u^w\) is the reference point of the attribute value selected by decision-makers; and \(T_k^w\) is defined as the outcome.

The endogenous method is used to determine the reference point of the attribute value, that is, the minimum value of the value income budget in all alternatives for the reference point:

\[
u_0^w = \min_{k \in K} \{ T_k^w \}.
\]

To ensure that the weight function is a monotonically increasing function with regard to the probability of the decision-making alternative, the following form of the decision weight function given by Prelec [20] is used in this paper:

\[
\omega(\rho) = \exp[(-\ln \rho)^\gamma], 0 < \gamma < 1.
\]

Following the derivation process of Connors and Sumalee [21], let \(F(T_k^w)\) denote the probability distribution function of \(T_k^w\) to obtain the prospect value of the alternative:

\[
g_{k}^w = \int_{\omega_k^L}^{\omega_k^U} \frac{\omega(\rho)}{g(T_k^w)} g(T_k^w) dT_k^w
\]

where \(\omega_k^L\) and \(\omega_k^U\) are the lower and upper bounds of the key attribute feature values in the decision-making process. The values of the upper and lower bounds will affect the calculation of the prospect value of an alternative. In a more reasonable evaluating method, \(\omega_k^L\) is the attribute weight in the decision-making process of an alternative and can be expressed as a decision weight function:
\[
\overline{u}_k = E(T_k^n) + 3\sqrt{\text{var}(T_k^n)}. \tag{5}
\]

2.2. Interval Grey Numbers. A number with its approximate range known but exact value unknown is called a grey number [22]. In practical applications, a grey number refers to an uncertain number that takes values in a certain interval or a general number set. It can be represented by \( \oplus \). Generally, a grey number with both lower bound \( a \) and upper bound \( \bar{a} \) is called an interval grey number, which is denoted as \( \oplus \in [a, \bar{a}] \).

In the arithmetic dealing with real numbers and interval grey numbers in this paper, the real number is regarded as an interval grey number with its lower bound equal to its upper bound, and the operation is performed according to the algorithm of interval grey numbers. Let \( \overline{r}_{ij} \) represent the value for the \( i \)th alternative satisfying the \( j \)th attribute. In combination with the multiattribute decision-making model of interval grey numbers, the calculation process is as follows:

1. First, to eliminate the influence of different physical dimensions on the decision-making results, the decision matrix needs to be standardized.
   
   \[
   \overline{r}_{ij} = \frac{\overline{a}_{ij}}{\overline{a}_{ij}} \quad i \in N, j \in |I_1|, \tag{6}
   \]
   
   \[
   \bar{r}_{ij} = \frac{1}{\overline{a}_{ij}} \quad i \in N, j \in |I_2|, \tag{7}
   \]

   According to formulas (6) and (7), decision matrix \( \overline{A} \) is transformed into a standardized matrix:
   
   \[
   \overline{R} = (\overline{r}_{ij})_{nm \times m}. \tag{8}
   \]

2. According to the multiattribute weight determination method of the Analytical Hierarchy Process (AHP) [23] scale, all decision attributes \( U_n \) are compared in pairs, and the judgment matrix \( A \) is formulated as follows:
   
   \[
   A = \begin{pmatrix}
   u_{11} & u_{12} & \ldots & u_{1n} \\
   u_{21} & u_{22} & \ldots & u_{2n} \\
   \vdots & \vdots & \ddots & \vdots \\
   u_{n1} & u_{n2} & \ldots & u_{nn}
   \end{pmatrix}. \tag{9}
   \]

   The eigenvector is obtained as follows:
   
   \[
   \xi = (\xi_1, \xi_2, \ldots, \xi_n)^T. \tag{10}
   \]

   Then, the weight vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_m) \) of the attribute is calculated using formula (8):
   
   \[
   \omega_j = \frac{\xi_j}{\sum_{j=1}^n \xi_j}, \quad j \in M. \tag{11}
   \]

   Finally, fuzzy set theory is used for processing.

3. Consider the following two formulas:
   
   \[
   \mu_{r_{ij}} = r_{ij}^L - r_{ij}^U \in [0, 1], \tag{12}
   \]
   
   \[
   \mu_{\omega_j} = (\omega_j^U - \omega_j^L) \in [0, 1].
   \]

   The standardized formula above is
   
   \[
   \overline{r}_{ij} = [r_{ij}^L, r_{ij}^U]. \tag{13}
   \]

   Additionally, the weight internal number \( \bar{\omega}_j \) is rewritten in the form of a correlate.

4. The comprehensive attribute value \( \overline{z}_i(\omega) (i \in N) \) of alternative \( x_i (i \in N) \) is obtained using formula (11).
   
   \[
   \overline{z}_i(\omega) = \sum_{j=1}^m \omega_j \overline{r}_{ij}. \tag{14}
   \]

5. With the comprehensive evaluation result of \( x_i \) denoted as \( \overline{z}_i(\overline{\omega}) \), the following can be obtained:
   
   \[
   \overline{z}_i(\overline{\omega}) = \overline{m} \sum_{j=1}^m \bar{\omega}_j \overline{r}_{ij}. \tag{15}
   \]

6. \( \bar{\omega}_j \) and \( \overline{r}_{ij} \) in formula (12) are converted to corresponding correlates to obtain a comprehensive evaluation correlate model:
   
   \[
   \overline{z}_i(\overline{\omega}) = \overline{m} \sum_{j=1}^m \bar{\omega}_j \overline{r}_{ij}. \tag{16}
   \]

7. According to the characteristics of uncertainty, the comprehensive evaluation values of each alternative in the cases of \( \bar{r} = 0, \bar{t} = 0.5 \), and \( \bar{t} = 1 \) are analyzed to confirm all possible sorting numbers \( q_{ij} \). The best is denoted as 1, the second best is denoted as 2, ..., and the worst is denoted as \( n \). Then, the sum of different sorting numbers of each alternative under different conditions is calculated using formula (14), and finally, the optimal alternative is obtained after sorting.
   
   \[
   M(\overline{z}_i(\overline{\omega})) = \sum_{i=0,0.5,1}^n \sum_{i=1}^m q_{ij}. \tag{17}
   \]

3. Methodology

The response to public health emergencies involves the components of multiple participating roles and multiple forms of relief supplies, personnel, and financial affairs. These components form a variety of attributes of the emergency decision-making model, which contribute at different levels and to different degrees to the emergency decisions taken by the decision-makers. Thus, it can be seen that in the emergency decision-making process of public health emergencies, decision-makers should first clarify the set of factors that affect the decision-making, and then abstract the relation of mutual influence degrees between
factors, which can be further defined by attribute weights. On this basis, the attribute state relation is used to conduct matrix transformation of eigenvalues to obtain the corresponding matrix calculation value. Subsequently, a priority of emergency alternatives is generated through the value function and the corresponding weight function. Based on the above ideas, the specific decision-making process is designed as described below.

3.1. Decision-Making Model Building. Let the set of emergency decision-making alternatives of dynamic multi-attribute risk public health emergencies be $A = \{a_1, a_2, \ldots, a_n\}$; the attribute set be $B = \{b_1, b_2, \ldots, b_m\}$; the corresponding attribute weight be $\omega_j$ where $\xi_j \leq \omega_j \leq \zeta_j$, $\xi_j \leq \zeta_j$ and $\xi_j, \zeta_j \in [0, 1]$. The time series used to examine the feasible alternative is the time series used to examine the feasible alternative. The corresponding time weight is $\lambda_l$ with $\sum_{j=1}^{m} \lambda_l = 1$. $u_{ij}(\theta)$ is the sample value of the effect of alternative $a_i$ under attribute $b_j$ at time $t_i$. This sample value is an interval grey number denoted as $u_{ij}(\theta) = [u_{ij}^{l}, u_{ij}^{u}](0 \leq u_{ij}^{l} \leq u_{ij}^{u})$. Then, the decision matrix at time $t_i$ can be obtained as $U^i = (u_{ij}(\theta))_{nm}$.

3.2. Attribute Determination. The most common attributes of samples include cost-based and benefit-based attributes. To increase the comparability in the process of dealing with the decision-making model for public health emergencies, the decision-making process matrix should be standardized. The relevant attributes can be divided to form an attribute set $U = \{U_1, U_2, \ldots, U_n\}$.

In the scope of attribute cognition, according to the development law or influence factors of events, each attribute can be refined into multiple states. These states have mutual event relations that can represent the key links of the events or denote their prominent state points; for example, $U_1 = \{s_1, s_2, \ldots, s_l\}$.

After logical sorting of each state, according to the information of the basic data, the empirical probability of each state point is obtained. These probabilities can be scored based on the experience database or by experienced experts. The relationships among the probabilities need to be coordinated and arranged.

At the same time, treatment alternatives for each state of public health emergencies are given for specific execution processes after decision-making. These processes can be descriptive qualitative information, as shown in Table 1, or formal quantitative analysis information. For decision-making model processing in this paper, the alternative information needs to be processed in intervals to form the grey data subordination range value dominated by the attribute state. The value serves as the data amount in the decision-making process, making the final decision more quantifiable and rational.

| Attribute | State | Probability | Alternatives |
|-----------|-------|-------------|--------------|
| $U_1$     | $s_1$ | $p_1^1$     | $x_{12}^l(\otimes)$ | $x_{12}^m(\otimes)$ | $\ldots$ | $x_{1n}^m(\otimes)$ |
|           | $s_2$ | $p_1^2$     | $x_{22}^l(\otimes)$ | $x_{22}^m(\otimes)$ | $\ldots$ | $x_{2n}^m(\otimes)$ |
|           | $\ldots$ | $\ldots$     | $\ldots$     | $\ldots$     | $\ldots$ | $\ldots$     |
| $U_2$     | $s_1$ | $p_2^1$     | $x_{12}^l(\otimes)$ | $x_{12}^m(\otimes)$ | $\ldots$ | $x_{1n}^m(\otimes)$ |
|           | $s_2$ | $p_2^2$     | $x_{22}^l(\otimes)$ | $x_{22}^m(\otimes)$ | $\ldots$ | $x_{2n}^m(\otimes)$ |
|           | $\ldots$ | $\ldots$     | $\ldots$     | $\ldots$     | $\ldots$ | $\ldots$     |
| $U_n$     | $s_1$ | $p_n^1$     | $x_{12}^l(\otimes)$ | $x_{12}^m(\otimes)$ | $\ldots$ | $x_{1n}^m(\otimes)$ |
|           | $s_2$ | $p_n^2$     | $x_{22}^l(\otimes)$ | $x_{22}^m(\otimes)$ | $\ldots$ | $x_{2n}^m(\otimes)$ |
|           | $\ldots$ | $\ldots$     | $\ldots$     | $\ldots$     | $\ldots$ | $\ldots$     |

3.3. Value Function. According to CPT [19], the prospect value of an alternative is mainly determined by value function $V(\Delta x)$ as shown in the following formula:

$$V(\Delta x_i) = \begin{cases} (\Delta x)^{\alpha}, & \Delta x \geq 0 \\ -\theta(-\Delta x)^{\beta}, & \Delta x < 0 \end{cases},$$

where $\Delta x$ describes loss ($\Delta x < 0$) or profit ($\Delta x \geq 0$) relative to the reference point of decision-makers in a decision-making alternative; $\alpha$ ($\alpha > 0$) and $\beta$ ($\beta < 1$) indicate risk aversion coefficients and risk preference coefficients; and $\theta$ ($\theta > 1$) indicates that decision-makers are more sensitive to losses than profits.

3.4. Weight Function and Its Processing. In the multiattribute emergency decision-making in public health emergencies, the traditional method of maximizing the deviation is most
commonly used to obtain the attribute weight [24]. The larger the total deviation of correlation coefficients of all the alternatives with respect to the positive and negative ideal alternatives under attribute \( u_j \) are, the more important is the role of attribute \( u_j \) in the decision-making process, and the greater is the attribute weight. Conversely, the smaller the total deviation of correlation coefficients of all the alternatives with respect to the positive and negative ideal alternatives under attribute \( u_j \) is, the less important is the role of attribute \( u_j \) in the decision-making process, and the smaller is the attribute weight [25]. In particular, if there is no difference between the correlation coefficients of all the decision-making alternatives with respect to the positive and negative ideal alternatives under attribute \( u_j \), this attribute will have no effect on the decision-making on an alternative, and its weight can be set to 0.

Based on the above ideas, \( d_+^*(\omega_j) \) and \( d_-^*(\omega_j) \) represent the deviations of the correlation coefficients of alternative \( A_i \) and of all other alternatives under attribute \( A_j \) with respect to the positive and negative ideal alternatives, which are defined as follows:

\[
d_+^*(\omega_j) = \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j, \\
d_-^*(\omega_j) = \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j.
\]  

Let

\[
d^*(\omega_j) = \sum_{i=1}^{m} d_+^*(\omega_j) = \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j, \\
d^*(\omega_j) = \sum_{i=1}^{m} d_-^*(\omega_j) = \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j.
\]  

In formulas (18) and (19), \( d^*(\omega_j) \) represents the total deviation between correlation coefficients of all decision-making alternatives and of other decision alternatives with respect to the positive ideal alternative under attribute \( u_j \). The value \( d^- (\omega_j) \) represents the total deviation between correlation coefficients of all decision-making alternatives and of other decision alternatives with respect to the negative ideal alternative under attribute \( u_j \).

Therefore, for the selection of weight vector \( \omega = (\omega_1, \omega_2, \ldots, \omega_p)^T \), one should maximize the sum of the total deviations of correlation coefficients of all decision-making alternatives with respect to the positive ideal alternative and to the negative ideal alternative under all attributes as expressed in formulas (20) and (21), respectively.

\[
d^*(\omega) = \sum_{j=1}^{n} d^*(\omega_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j, \\
d^-(\omega) = \sum_{j=1}^{n} d^-(\omega_j) = \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j.
\]  

To this end, the following optimization model can be established:

\[
\begin{aligned}
\text{max} \; d(\omega) &= \mu d^*(\omega_j) + (1 - \mu) d^-(\omega_j) \\
&= \mu \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j + (1 - \mu) \sum_{j=1}^{n} \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| \omega_j,
\end{aligned}
\]

\[
s.t. \sum_{j=1}^{n} \omega_j = 1, \; \omega_j \geq 0.
\]

(24)

The Lagrangian function is formulated to solve this model, and normalization processing is performed. The calculation formula for the attribute weight is obtained as follows:

\[
\omega_j = \frac{\mu \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| + (1 - \mu) \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}|}{\sum_{j=1}^{n} [\mu \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}| + (1 - \mu) \sum_{i=1}^{m} \sum_{k=1}^{n} |\xi_{ij} - \xi_{kj}|]}. 
\]

(25)

In general, the correlation coefficient of each alternative under attribute \( u_j \) with respect to the positive and negative ideal alternatives has the same important effect on the decision-making result and is therefore assigned the value of \( \mu = 0.5 \).

The faster the actual response to public health emergencies is, the more the spread of losses can be prevented. How to reasonably and accurately determine the time weights at different stages of an emergency response based on the existing public health emergency information is important to the evaluation and sorting of various alternative emergency emergencies. Therefore, the time degree can be reasonably used to obtain the time weight vector [26]. The closeness of the time weight vector to the ideal time weight vector is shown in the following formula:

\[
C(V) = \frac{d(\eta(t_k), \eta(t_k^-))}{d(\eta(t_k), \eta(t_k^-)) + d(\eta(t_k), \eta(t_k^-))}. 
\]

(26)

Since the closeness under a fixed time weight can reach the maximum in a certain time degree, the formula can be transformed into the following model:

\[
\begin{aligned}
\max C(V) &= \frac{d(\eta(t_k), \eta(t_k^-))}{d(\eta(t_k), \eta(t_k^-)) + d(\eta(t_k), \eta(t_k^-))} \\
\text{s.t.} \lambda &= \sum_{k=1}^{p} \frac{p - k}{p - 1} \eta(t_k), \sum_{k=1}^{p} \eta(t_k) = 1, \\
\eta(t_k) \in [0, 1], k = 0, 1, 2, \ldots, p.
\end{aligned}
\]

(27)

This nonlinear model is solved to obtain the weight vector function for the problem of multiattribute risk decision-making: \( \eta(t) = (\eta(t_1), \eta(t_2), \ldots, \eta(t_p))^T \).

3.5. Calculation of the Comprehensive Prospect Value. In view of the characteristics of the decision-making process of public health emergencies, under the premise that the general grey number algebraic operation system has not
resulted in a satisfactory answer, converting the general grey number equivalently to a grey number set aids achieving scientific decision-making and improving decision accuracy [27]. In addition, calculation problems such as comparison between general grey numbers and standardization of indicators can be avoided with this method.

In the specific calculation process for obtaining the comprehensive prospect value, the distance $d(b_{ij}(t), r_j)$ between each attribute value $b_{ij}(t)$ and reference point $r_j$ is compared through the following formula:

$$d(b_{ij}(t), r_j) = \frac{1}{\sqrt{n}} \left( (\mu_1 - \mu_2)^2 + (v_1 - v_2)^2 + (\pi_1 - \pi_2)^2 \right).$$

The distance reflects the profit or loss state of the decision-making alternative relative to the reference point. The corresponding profit and loss matrix $F(b_{ij}(t))$ is obtained as shown in the following formula:

$$F = \left[ F(b_{ij}(t)) \right]_{mon} = \begin{cases} d(b_{ij}(t), r_j), b_{ij}(t) \geq r_j, \\ -d(b_{ij}(t), r_j), b_{ij}(t) > r_j. \end{cases}$$

In actual decision-making, there are differences in the sensitivity of decision-makers regarding losses and profits. The inclusion of the decision-makers’ risk preference can improve the rationality of the selection of emergency alternatives on public health emergencies. Therefore, the different risk preferences for profits and losses are incorporated into the decision-making process, and the corresponding cumulative prospect decision matrix $V(b_{ij}(t))$ is established through the profit-and-loss matrix $F(b_{ij}(t))$.

$$V(b_{ij}(t)) = \begin{cases} (F(b_{ij}(t)))^\theta, b_{ij}(t) \geq r_j, \\ -\theta(-F(b_{ij}(t)))^\theta, b_{ij}(t) < r_j. \end{cases}$$

Attribute weight $\omega_j$ and time weight $\eta(t)$ are comprehensively considered, and according to the weighting principle, the final comprehensive cumulative prospect value for each alternative is calculated by the following formula:

$$U(X_i) = \sum_{j=1}^{n} \sum_{t=1}^{p} \omega_j \eta(t) V(b_{ij}(t)).$$

In summary, based on the preceding introduction, the procedure of the proposed method can be summarized in Figure 1.

The steps of the grey multiattribute emergency decision-making method for public health emergencies based on cumulative prospect theory are as follows.

**Step 1.** Based on the construction principle of the interval grey number possibility function, in combination with the background of the actual problem with public health emergencies, construct the grey number possibility function.

**Step 2.** Convert the decision-making information represented by the general grey number equivalently to a number set represented by the interval grey number.

**Step 3.** Convert the grey number set of attribute indicators in the decision-making process of each public health emergency into a grey number.

**Step 4.** Calculate the optimal ideal alternative in the decision-making model and the grey number value of the corresponding public health emergency attribute.

**Step 5.** Obtain the comprehensive prospect values of each alternative and the optimal ideal alternative and sort them; the smaller the distance value, the better the alternative.

### 4. Case Analysis

Since the first case of Novel Coronavirus-Infected Pneumonia (NCIP) with 2019-nCoV, also called COVID-19, was discovered in Wuhan, China, in December 2019, the epidemic has developed rapidly. According to the data released by the World Health Organization, as of 16:33 on August 28, 2020 (Central European Time), the number of confirmed cases of COVID-19 and the number of associated deaths worldwide were reported as 24,299,923 and 827,730, respectively [28]. Judged from the characteristics of public data, COVID-19 is characterized by high incidence as well as a durable and high infectiousness. If emergency decisions can be made in a timely and effective manner for the emergency response, both the harm caused by the epidemic and its expansion can be greatly reduced.

Many social factors influence the emergency decision-making process during the COVID-19 epidemic. In the process of decision-making modeling, it is necessary to clarify the influence factors applicable to the quantitative analysis of the model and to ensure the retention and accuracy of information in the quantification process. Among the quantifiable influence factors, obvious factors like the...
number of facilities available for emergency rescue, the number of medical staff, the number of infected people, and the infection area range are important characteristic data that need to be determined at any time. Based on the emergency decision-making modeling ideas in this paper, the data can serve as the basis for formulating related data sets and representing state association. The level-related evaluation information construction method is used to obtain the corresponding correlation matrix construction mode so that these data can be calculated and accurately used in the specific decision-making process to formulate corresponding emergency rescue alternatives or form an emergency rescue alternative system. In the following part, abstract data are used to facilitate the understanding of the application of an actual decision-making process by elaborating and analyzing how decision-makers formulate and implement the corresponding rescue alternative through the data on specific public health emergencies.

It is assumed that a COVID-19 epidemic command center responds to the epidemic mainly based on five indicators. One day, a case of COVID-19 occurred in a community with the information shown in Table 2. To solve this emergency, the decision center needs to choose from three alternative alternatives: x1, dispatch a medical team and an emergency ambulance; x2, on the basis of x1, send a medical expert and appropriate emergency medical treatment equipment; x3, on the basis of x2, send a medical emergency protection team. To determine the best rescue alternative, three indicators of these alternatives are evaluated: c1, health status of the case; c2, the number of rescuers; c3, rescue equipment and labor cost. While c1 and c2 are benefit indicators, c3 is a cost indicator.

Assuming that emergency assistance for the epidemic is divided into four time periods, \( t_1 \sim t_4 \), the attribute weight of the indicator is \( \omega = (0.4, 0.45, 0.15) \), and the expectation vector of the attribute is \( \theta = (6, 5, 11, 7, 12) \). The initial evaluation information is shown in Table 3.

Note. The evaluation information of attribute \( c_3 \) is represented by interval linguistic information; that is, \( \{\text{VeryPoor (VP)}, \text{Poor (P)}, \text{ModeratelyPoor (MP)}, \text{Moderate (M)}, \text{ModeratelyGood (MG)}, \text{Good (G)}, \text{VeryGood (VG)}\} \).

The expectation vector of the attribute and the initial decision matrix are standardized and unified. Taking the time \( t_1 \) as an example, the initial decision matrix and the expectation vector of the attribute are standardized as follows:

\[
B(t_1) = b_{ij}(t_1)_{\text{norm}} = \begin{bmatrix}
(0,1) & (0,0.667) & (0,0.429) \\
(0,1) & (0.222,0.444) & (0.143,0.571) \\
(0.5,0.5) & (0.111,0.556) & (0.429,0)
\end{bmatrix}
\]

\[
\bar{R} = \left[ (1,0) (0.333,0) (0.143,0.286) \right].
\]

According to the above calculation formula for the time degree, the time weight is determined as follows:

\[
\eta(t) = (0.228,0.108,0.664)^T.
\]

Based on this result, a profit-and-loss decision matrix is obtained as follows:

\[
F = \left[ F(b_{ij}(t)) \right]_{\text{norm}} = \begin{bmatrix}
0.816 & 0.472 & 0.117 \\
0.816 & 0.327 & 0.233 \\
0.408 & 0.396 & 0.234
\end{bmatrix}.
\]

The profit-and-loss matrix is converted to the corresponding cumulative prospect decision matrix:

\[
V = \left[ V(b_{ij}(t)) \right]_{\text{norm}} = \begin{bmatrix}
-1.881 & -1.162 & -0.341 \\
-1.881 & -0.841 & -0.624 \\
-1.022 & -0.996 & 0.387
\end{bmatrix}.
\]

The experimental reference data are set as \( \alpha = \beta = 0.88, \theta = 2.25 \). The indicator weight is comprehensively considered with

\[
\omega = (0.4,0.45,0.15),
\]
and the comprehensive cumulative prospect value of each alternative is calculated as follows:

\[ U(x_1(t_1)) = -1.3265, U(x_2(t_1)) = -1.2245, U(x_3(t_1)) = -0.7990. \]  

(37)

In view of the time weight \( \eta(t) = (0.228, 0.108, 0, 0.664)^T \) and

\[ U(X_i) = \sum_{t=1}^{p} \eta(t) U(X_i(t)), \]

(38)

the final cumulative prospect value can be obtained as follows:

\[ U(x_1) = -1.3367, U(x_2) = 0.0091, U(x_3) = -0.0901. \]  

(39)

From the above results, the best emergency alternative order for the epidemic based on the applied decision-making process can be determined as follows: \( x_2 > x_1 > x_3 \).

The final cumulative prospect values of \( x_1 \) and \( x_3 \) are negative, indicating that the emergency decision-making alternative is at a loss relative to the reference point. The emergency alternative \( x_1 \) characterized by fewer emergency reserve resources and weaker emergency response capabilities cannot guarantee a timely emergency response; the alternative \( x_3 \) with sufficient emergency resources and strong emergency response capabilities guarantees a timely emergency response but greatly increases emergency costs. The emergency alternative \( x_2 \), as the best choice, can save part of the emergency cost under the premise of meeting the decision-making requirement for a timely emergency response.

5. Comparative Analysis

In this section, to demonstrate the characteristics of the proposed method and provide guidance for practical application, comparative analyses are implemented based on the effect of time weight and the effect of the decision-makers’ risk preference.

5.1. Effect of Time Weight. In this section, comparison analyses are conducted to confirm the effectiveness of the proposed method. In order to verify the effect of time weight on decision-making problems, this section only changes the time weight and observes the results and ranking of the decision-making alternatives.

Via the same calculation process, comprehensive cumulative prospect value of each alternative is obtained as shown in Section 4. Here, only the time weight is changed. The comparison of decision results with different time weights is shown in Table 4.

| Number | Weight of subgroup | Ranking result of alternatives |
|--------|--------------------|-------------------------------|
| 1      | (0.1, 0.3, 0.6)    | \( x_2 > x_3 > x_1 \)        |
| 2      | (0.3, 0.3, 0.4)    | \( x_2 > x_3 > x_1 \)        |
| 3      | (0.5, 0.3, 0.2)    | \( x_2 > x_3 > x_1 \)        |
| 4      | (0.7, 0.2, 0.1)    | \( x_2 > x_3 > x_1 \)        |
| 5      | (0.1, 0.5, 0.4)    | \( x_2 > x_3 > x_1 \)        |
| 6      | (0.1, 0.7, 0.2)    | \( x_2 > x_3 > x_1 \)        |
| 7      | (0.6, 0.1, 0.3)    | \( x_2 > x_3 > x_1 \)        |
| 8      | (0.4, 0.1, 0.5)    | \( x_2 > x_3 > x_1 \)        |
| 9      | (0.2, 0.1, 0.7)    | \( x_2 > x_3 > x_1 \)        |

(40)

From the above results, the best emergency alternative order for the epidemic based on the applied decision-making process can be determined as \( x_3 > x_1 > x_2 \). The main reasons are as follows.

(1) In emergency decision-making, decision-makers have different understanding and cognition of decision issues, and they take different measures to risk. Therefore, decision-makers often have different risk preferences, which will affect the decision results.

(2) In emergency decision-making, the irrational subjective factors of decision-makers often have a great influence on the scheme optimization decision. Moreover, due to the change of decision time, the irrational subjective factors of decision-makers vary dynamically at different time points. The grey model based on cumulative prospect theory not only considers the dynamic influence of time change on decision-making situation, but also considers decision-makers’ risk preference, so the alternative optimization result is closer to the objective reality.

6. Conclusions

To address the emergency decision-making problem of dynamic multiattribute risk public health emergencies, a dynamic decision-making model based on cumulative prospect theory and interval grey numbers is reported in this paper. Representing the unified decision-making information by the conversion of the model matrix can not only retain the uncertainty of the initial decision-making information but also reduce the loss of decision-making information to a certain extent. In view of the bounded rational behavior of actual decision-makers, their psychological expectation vector for the attribute was taken as a reference point, and a multiperiod cumulative prospect decision-making results, in this section, the decision results are analyzed without decision-makers’ risk preference. That is, this subsection does not consider cumulative prospect theory.

The attribute weight, time weight, and initial evaluation information are the same as the proposed method. Based on the calculation process in Section 4, the final cumulative prospect value can be obtained as

\[ U(x_3) = 0.8732, U(x_2) = 0.7543, U(x_1) = 0.9421. \]

(41)

From the above results, the best emergency alternative order for the epidemic based on the applied decision-making process can be determined as \( x_3 > x_1 > x_2 \).
matrix relative to the reference point was established to calculate the comprehensive cumulative prospect value of each emergency alternative for selecting the best alternative. Finally, with the COVID-19 epidemic serving as an example, the risk preference of decision-makers was introduced into the emergency decision analysis process. The total deviation of the grey correlation coefficients of all alternatives with respect to the positive and negative ideal alternatives was maximized to solve the attribute weight. Based on this, the weight function and the prospect value function of the attribute were constructed. Finally, the priority of the alternatives was determined according to the calculated comprehensive prospect value of each alternative.

As a more realistic approach to decision-making, the emergency decision-making method proposed in this paper integrates the different risk preferences of decision-makers when facing profits or losses. This new method and idea might solve the emergency decision-making challenges in grey multiattribute risk public health emergencies. However, in the actual decision-making process, with the deepening of the decision-making process, decision-makers will change their attitude towards risks when facing gains and losses. Therefore, how to depict the dynamic psychological change process of decision-makers is a problem to be solved in the future.

Data Availability

The data are obtained from data published by the World Health Organization: Weekly Operational Update on COVID-19 August 28, 2020, https://www.who.int/docs/default-source/coronaviruse/situation-reports/wou-28-august-approved.pdf?sfvrsn=d9e49c20_2.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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