Perturbative contribution to the $\sin \phi$ asymmetry in inclusive $\pi^+$ electroproduction

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We consider the $\sin \phi$ single target-spin asymmetry in deep-inelastic $\pi^+$ inclusive electroproduction off a longitudinally polarized target. We show that at larger transverse momentum of the outgoing hadron the evaluated asymmetry decreases if one takes into account the first order $\alpha_S$ perturbative contribution to the cross section, integrated over the azimuthal angle. This leads to good agreement with recent HERMES data.

1. Introduction

Single spin asymmetries (SSA) in semi-inclusive deep inelastic scattering (SIDIS) of leptons off polarized target are forbidden in the simplest version of the parton model and in leading order $\alpha_S$ perturbative quantum chromodynamics (pQCD); they vanish in models based on hadrons consisting of non-interacting collinear partons (quarks and gluons). Effects due to finite quark transverse momentum play a significant role in most explanations of non-zero single-spin asymmetries in polarized hadronic reactions [1,2,3]. Especially polarized fragmentation functions with transverse momentum dependence are relevant for the description of single-target spin asymmetries observed by HERMES [4] and SMC [5]. These functions do not vanish even if time reversal invariance is applied, because of final-state interactions between the outgoing hadron and other produced particles.

In this note we calculate the dependence of the $\sin \phi$ single spin asymmetry of inclusive pion production in $e\pi^+$-scattering on the transverse momentum of the detected hadron, $P_{hT}$. Taking into account the first order $\alpha_S$ perturbative contribution to the cross section, integrated over the azimuthal angle, leads to a good description of the HERMES data [4] over the measured $P_{hT}$ range.

2. Formulae

The kinematics of deep-inelastic pion inclusive electroproduction is the following: $k_1 (k_2)$ is the 4-momentum of the incoming (outgoing) charged lepton, $Q^2 = -q^2$, where $q = k_1 - k_2$, is the 4-momentum of the virtual photon. $P (P_h)$ is the momentum of the target (observed hadron), $x = Q^2/(2Pq)$, $y = (Pq)/(Pk_1)$, $z = (PP_h)/(Pq)$, $k_{1T}$ is the incoming lepton transverse momentum with respect to the virtual photon momentum direction, and $\phi$ is the azimuthal angle between $P_{hT}$ and $k_{1T}$ around the virtual photon direction. Note that the azimuthal angle of the transverse (with respect to the virtual photon) component of the target polarization, $\phi_S$, is equal to $0$ ($\pi$) for the target polarized parallel (anti-parallel) to the beam [4].

The $\sin \phi$ moment in the semi-inclusive deep inelastic cross section, which measures the left-right asymmetry of $P_{hT}$ along the $k_{1T}$ direction, is given by

$$\langle \sin \phi \rangle = \frac{\int \sigma^{(NP)} \sin \phi}{\int \sigma^{(NP)} + \int \sigma^{(\alpha_S)}},$$

where $\sigma^{(NP)}$ and $\sigma^{(\alpha_S)}$ are the non-perturba-
tive and the first order $\alpha_S$ perturbative hadronic scattering cross sections, respectively. Note, that the leading perturbative effects in the SSA appear only at second order $\alpha_S$ pQCD which is beyond of the scope of this work.

The integrations in Eq. (1) are over $P_{hT}, \phi, x, y, z$. The azimuthal asymmetry defined above is related to the ones measured by HERMES [4] through the following relation:

$$A_{UL}^{\sin \phi} = 2(\sin \phi),$$

(2)

where the subscripts $U$ and $L$ indicate unpolarized beam and longitudinally polarized target, respectively.

The relevant processes for the first order in $\alpha_S$ are the subprocesses $e(k_1) + i(p_1) \rightarrow e(k_2) + j(p_2) + X$, where $i(j)$ denotes initial (final) quarks/ gluons [7, 8]. $p_1$ ($p_2$) is the incident (scattered) parton momentum and

$$\int d\sigma^{(1)} d\phi = \frac{4\alpha_S\alpha^2}{3Q^2} \frac{1}{y} \int_x^1 \frac{dx_p}{x_p} \int_z^1 \frac{dz_p}{z_p} \sum_q e_q^2 \left[ f_1^q(\xi) AD_1^q(\xi') + f_1^q(\xi) BD_1^q(\xi') + f_1^q(\xi) CD_1^q(\xi') \right],$$

(3)

where $f_1^q(\xi)$ is the probability distribution describing a parton $q$ with a fraction $\xi$ of the target momentum, $p_1^q = \xi P_1^q$, $D_1^q(\xi)$ is the probability distribution for a parton $q$ to fragment producing a hadron with a fraction $\xi'$ of the parton’s momentum, and $e_q^2$ is the charge square of a parton $q$. In Eq. (3)

$$A = \left[ 1 + (1 - y)^2 \right] \frac{x_p^2 + z_p^2}{(1 - x_p)(1 - z_p)} + 2y^2(1 + x_p z_p) + 4(1 - y)(1 + 3x_p z_p),$$

(4)

$$B = \left[ 1 + (1 - y)^2 \right] \frac{x_p^2 + (1 - z_p)^2}{z_p(1 - x_p)} + 2y^2(1 + x_p - x_p z_p) + 4(1 - y)(1 + 3x_p(1 - z_p)),$$

(5)

$$C = \frac{3}{8} \left[ 1 + (1 - y)^2 \right]$$

and $x_p$ and $z_p$ are the parton variables which are related to the hadron variables as $x_p = x/\xi = Q^2/2p_1q$, $z_p = z/\xi' = p_1p_2/p_1q$. These expressions are identical to previous perturbative results in Ref.[7, 8, 9].

The $P_{hT}$-integration of the non-perturbative cross section can be performed analytically assuming that the transverse momentum dependence in the distribution and fragmentation functions can be written in factorized exponential form:

$$f(\vec{p}_T) = \frac{1}{a^2 \pi} e^{-p_T^2/a^2},$$

(7)

$$d\vec{k}_T = \frac{z^2}{b^2 \pi} e^{-z^2k_T^2/b^2},$$

(8)

where $p_T$, $k_T$ are the intrinsic transverse momenta of the initial and final quark, respectively and $a = 2(p_T)/\sqrt{\pi}, b = 2(zk_T)/\sqrt{\pi}$. Then (for more details see Refs. [7, 8])

$$\int d\sigma dP_T = S(P_T)(1 + (1 - y)^2)f_1(x)D_1(z),$$

(9)

$$\int d\sigma \sin \phi d\phi = S(P_{hT}) \left\{ S_L 2(2 - y) \sqrt{1 - y} \frac{P_{hT}^2}{2Q} \left[ R_1 xh_L(x)H_1^+(z) - R_2 g_1(x)H_1^+(z) - R_3 h_1^+(x) \frac{\tilde{H}(z)}{z} \right] + S_T x(1 - y) \frac{P_{hT}^2}{2Q} R_4 h_1(x)H_1^+(z) \right\} \left\{ \right\}$$

(10)

Here

$$S(P) = \frac{4\pi^2\alpha^2}{Q^2 y} \exp(-P^2/(b^2 + a^2z^2)),$$

and

$$R_1 = \frac{M_p b^2}{M_h (b^2 + a^2z^2)^2}, \quad R_2 = \frac{m b^2}{M_h (b^2 + a^2z^2)^2},$$

$$R_3 = \frac{M_h a^2 z^2}{M_p (b^2 + a^2z^2)^2}, \quad R_4 = \frac{b^2}{M_h (b^2 + a^2z^2)^2}.$$
where $M_p$, $M_h$, and $m$ are the proton, final hadron and current quark masses, respectively. In Eq. (10) the components of the longitudinal and transverse target polarization in the virtual photon frame are denoted by $S_L$ and $S_{Tx}$, respectively [8]. Note that to get rid off the divergences at $P_{hT}$ close to zero ($x_p \to 1$, $z_p \to 1$) and to control the integration of the order-$\alpha_s$ cross section [8], we introduce $P_C$ as a lower cutoff for $P_{hT}$. This has no impact on our results because of we are not considering the fully inclusive cross section. We perform all numerical investigations for a series of $P_C$ cutoffs, which approximates the $P_{hT}$ dependence of the asymmetry. Twist-2 distribution and fragmentation functions have a subscript ‘1’: $f_1(x)$ and $D_1(z)$ as already mentioned above are the usual unpolarized distribution and fragmentation functions, $g_1(x)$ is the longitudinally polarized distribution function, while $h^T_{1T}(x)$ and $h_1(x)$ describe the quark transverse spin distribution in longitudinally and transversely polarized nucleons, respectively. The interaction-dependent part of the twist-3 distribution function in the longitudinally polarized nucleon, $h_L(x)$ [10], is denoted by $\tilde{h}_L(x)$ [11, 12]. The spin-dependent twist-2 fragmentation function $H^T_{1}(z)$, describing transversely polarized quark fragmentation [8], correlates the transverse spin of a quark with a preferred transverse direction for the production of the pion. The fragmentation function $\tilde{H}(z)$ is the interaction-dependent part of the twist-3 fragmentation function [3].

3. Numerical Results

For numerical calculations the non-relativistic approximation $h_1(x) = g_1(x)$ is used as a lower limit [8] and $h_1(x) = (f_1(x) + g_1(x))/2$ as an upper limit [17]. For the sake of simplicity, $Q^2$-independent parameterizations were chosen for the distribution functions $f_1(x)$ and $g_1(x)$ [8]. We use the approximation where the twist-2 transverse quark spin distribution in the longitudinally polarized nucleon is zero (i.e. $h_L(x) = h_{1L}(x) = h_1(x)$) which, as shown in Ref. [11, 12], leads to a consistent description of the $A_{UL}^{\sin \phi}$ and $A_{UL}^{\sin 2\phi}$ asymmetries observed by the HERMES collaboration [8]. To estimate the $T$-odd fragmentation function $H^T_{1}(z)$, the Collins type ansatz [1] for the analyzing power of transversely polarized protons evaluated using $M_C = 2m_\pi$ and $\eta = 0.6$ in Eq. (11). Full lines correspond to results where the perturbative contribution is taken into account in the denominator of Eq. (11), while dashed ones are without this contribution. For each case two curves are presented corresponding to $h_1 = g_1$ (lower curve) and $h_1 = (f_1 + g_1)/2$ (upper curve). HERMES data are from Ref. [4].
polarized quark fragmentation was adopted:

\[ A_C(z, k_T) = \frac{|k_T| \, H_L^+(z, k_T^2)}{M_h \, D_1(z, k_T^2)} = \eta \frac{M_C \, |k_T|}{M_C^2 + k_T^2} \]  

(11)

Here \( \eta \) is taken as a constant, although in principle it could be \( z \) dependent and \( M_C \) is a typical hadronic mass whose value ranges from \( 2m_\pi \) to \( M_p \).

For the distribution of the final parton’s intrinsic transverse momentum, \( k_T \), in the unpolarized fragmentation function \( D_1(z, k_T^2) \) a Gaussian parameterization was used [21] with \( \langle z^2 k_T^2 \rangle = b^2 \) (in the numerical calculations \( b = 0.36 \) GeV was taken [22]). For \( D_\pi^+(z) \) the parameterization from Ref. [23] was adopted.

In Fig. 1, the asymmetry \( A_{UL}^{\sin \phi} \) of Eq.(11) for \( \pi^+ \) production on a proton target is presented as a function of \( P_{hT} \) and compared to the HERMES data [4]. The results obtained with and without taking into account the leading order \( \alpha_S \) pQCD term in the denominator of Eq.(1), are denoted by pairs of full and dashed lines, respectively. Each pair of curves corresponds to the two limits chosen for \( h_1(x) \). From Fig. 1 it can be seen that taking into account the first order \( \alpha_S \) perturbative contribution to the cross section (integrated over the azimuthal angle), the asymmetry \( A_{UL}^{\sin \phi} \) goes down at higher \( P_{hT} \). This leads to a good agreement with HERMES data [4].

4. Conclusion

We have discussed the \( P_{hT} \) behavior of the \( \sin \phi \) single-spin asymmetry for \( \pi^+ \) production in semi-inclusive deep inelastic scattering of leptons off longitudinally polarized protons including the pQCD contribution to the \( \phi \)-independent cross section. We have shown that the HERMES data at larger \( P_{hT} \) can be described well if one takes into account the first order \( \alpha_S \) contribution.

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