Determination of effective atomic numbers from mass attenuation coefficients of tissue-equivalent materials in the energy range 60 keV- 1.33 MeV

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Abstract. The main aim of this study was to establish a cost-effective tissue-equivalent material for phantom fabrication. Effective atomic numbers ($Z_{\text{eff}}$) and effective electron densities ($N_{\text{eff}}$) were calculated based on mass attenuation coefficient values. The linear and mass attenuation coefficients of two samples of paraffin wax and NaCl compositions were measured using Si detector for NaI (Tl) detector of 1.5” resources. Radioactive source was placed in front of detector and the sample was placed between the source and the photomultiplier tube (PMT) of the detector. The real time was set for 6000 seconds. The photopeak, full width at half maximum (FWHM) and net area of photopeak were measured using Meastro software. The attenuation coefficient values obtained from this study were used to calculate $Z_{\text{eff}}$ and $N_{\text{eff}}$ of paraffin wax and NaCl compositions. The measured results were compared with the theoretical values from XCOM and ICRU Report 44. The relative percentage difference of mass attenuation coefficients between experimental and human tissue for both paraffin wax and NaCl mixture are below 5%, whereas the relative percentage difference of $Z_{\text{eff}}$ and $N_{\text{eff}}$ are above 5%. The measured values of $Z_{\text{eff}}$ and $N_{\text{eff}}$ of paraffin wax and NaCl help us to establish the optimal mixtures to fabricate a cost-effective tissue-equivalent material.

1. Introduction

Studies on tissue equivalent materials for anthropomorphic physical phantom fabrication are important for radiation dosimetry, calibration and medical imaging [1]. These tissue-equivalent materials are being constructed from several chemicals [2] that exactly represent the anatomical shape of human organs [3-4] and have similar interaction of ionizing radiation to human body. In order to be accepted as a tissue-equivalent material, the absorbed and the scattered radiations should be as close as possible to those experienced by the irradiated tissue under similar conditions [5]. To understand the physical properties of tissue equivalent materials, a study of parameters such as mass attenuation coefficient ($\mu_a$), linear attenuation coefficient ($\mu_l$), total atomic cross-section ($\sigma_t$), electronic cross-section ($\sigma_e$), effective atomic number ($Z_{\text{eff}}$), electron density ($N_{\text{eff}}$), mean free-path ($\lambda$) are important [6].

Mass attenuation coefficient is defined as the measure of a material’s ability to absorb or scatter electromagnetic radiation in any form (e.g., x-rays, gamma ray, etc.) per unit of mass. The mass attenuation coefficient is then used to derive other photon interaction parameters. Linear attenuation coefficient ($\mu_l$) describes the fraction of a beam of x-rays or $\gamma$- rays that is absorbed or scattered per unit thickness of the absorber. The scattering and absorption of gamma radiations are...
related to the density and atomic number of each element. For composite material, it is related to effective density and effective atomic number. The knowledge of mass attenuation coefficients is prime importance in the determination of effective atomic number [6]. A single number of atomic number therefore, cannot represent the atomic number uniquely across the entire energy range, since the partial interaction cross-sections depend on different element numbers [7].

The $Z_{eff}$ of the materials is one of the most important parameter representing radiation interaction, and this parameter has widely been used in medical imaging, radiotherapy and nuclear medicine. In this study, the mass attenuation coefficients and other photon interaction parameters of composite materials, paraffin wax and NaCl samples were determined at different gamma energies. The measured values were compared with the values obtained using semi empirical relations based on mixture rule, and values from XCOM and ICRU-44 report. Other photon interaction parameters were derived from the mass attenuation coefficient.

2. Experimental Method

2.1. Sample preparation
Two mixtures of paraffin wax and NaCl were prepared to fabricate soft tissue equivalent materials and bone tissue equivalent material. The paraffin wax was melted at 60 °C. Then, the NaCl was added quickly into paraffin wax. The mixtures were then poured into fabricated mould with dimension of 3 cm x 3 cm x 0.5 cm. Two difference samples were prepared to represent soft tissue equivalent material of thyroid and bone tissue equivalent material of spinal cord. Both samples have difference ratio of NaCl added into paraffin wax.

2.2. Measurement of linear attenuation coefficients of paraffin and NaCl compositions
The linear attenuation coefficients of soft-tissue equivalent and bone tissue equivalent samples were measured for energies between 88 keV and 1.33 MeV from exponential attenuation law. These energy range was chosen due to this range are used in medical imaging, radiotherapy and nuclear medicine department. The measurement was performed using a NaI (Tl) detector for gamma spectrometry. The detector was shielded with a lead housing to reduce the scattered radiation coming directly from the source or from the background. The attenuation measurements were measured using multichannel analyser (MCA). The samples were placed between detector and radiation source. The real time which is the total time it takes for a detector to count (maybe several seconds longer than live-time) was set for 6000 seconds. The photopeak, full width at half maximum (FWHM) and net area of photopeak were measured using Meastro software. The radioactive sources used to gained differences in gamma energies were Am-241 (87.83KeV), Na-22 (1274.53KeV), Cs-137 (661.66KeV), Mn-54 (834.85KeV) and Co-60 (1173.24 KeV and 1332.5 KeV). The linear attenuation coefficients of both samples were determined from the XCOM software. XCOM software is a web database used in calculating the photon cross sections and total attenuation coefficients for element, compound or mixture ($Z \leq 100$) at energies from 1 keV to 100 GeV.
2.3. Comparison between measured mass attenuation coefficients, XCOM software and ICRU-44 report

Two comparisons were carried out in this study to verify the soft-tissue and bone tissue equivalent materials. The relative percentage difference of mass attenuation coefficients of soft-tissue and bone tissue equivalent samples between experimental values and XCOM software values were determined. Next, the relative percentage difference of mass attenuation coefficients between samples and ICRU-44 were evaluated. The soft tissue and bone tissue equivalent samples were compared with ICRU-44 soft tissue and ICRU-44 bone tissue respectively for the energies of the radionuclide used.

2.4. Calculating mass attenuation coefficient, effective atomic number and effective electron density of paraffin wax and NaCl compositions.

Determination of mass attenuation coefficients of the samples were performed at difference energies based on Equation (1).

\[ I = I_0 \exp(-\mu_m t) \]  

where \( I_0 \) is unattenuated photon intensity, \( I \) is attenuated photon intensity, \( \mu_m = \mu/\rho \) (cm\(^2\)/g) is the mass attenuation coefficient and \( t \) (g/cm\(^2\)) is sample mass thickness (the mass per unit area).

For any chemical compound or mixture of elements, the total mass attenuation coefficient \( \mu \) is given by mixture rule shown in Equation (2):

\[ \mu_m = \sum w_i \left( \mu_{m_i} \right) \]

where \( w_i \) is the weight fraction, \( \left( \mu_{m_i} \right) \) is the mass attenuation coefficient of \( i \)th element.

For a material composed of multi elements the fraction by weight is given by Equation (3):

\[ W_i = \frac{n_i A_i}{\sum n_i A_i} \]

where \( A_i \) is the atomic weight of the \( i \)th element and \( n_i \) is the number of formula units.

From \( \mu_m \) values measured, the total atomic cross-section (\( \sigma_t \)) for materials can be generated using the following relation shown in Equation (4):

\[ \sigma_i = \frac{\mu_m N}{N_A} \]

where \( N = \sum n_i A_i \) is atomic mass of materials and \( N_A \) is the Avogadro’s number.

Total electronic cross-section (\( \sigma_e \)) for the element is expressed by the following equation (5):

\[ \sigma_e = \frac{1}{N_A} \sum f_i N_i \left( \frac{\mu_{m_i}}{Z_i} \right) = \frac{\sigma_i}{Z_{eff}} \]

where \( f_i \) denotes the fractional abundance of the element \( i \) with respect to the number of atoms such that

\[ f_1 + f_2 + f_3 + f_4 + \ldots = 1 \]

\( Z_i \) is the atomic number of \( i \)th element.

The effective atomic number (\( Z_{eff} \)) of the material can be calculated using equation (6)
The effective electron number or electron density \((N_{\text{eff}})\) (number of electrons per unit mass) can be calculated using the following Equation (7):

\[
N_{\text{eff}} = \frac{N_A Z_{\text{eff}}}{N} \sum n_i = \frac{\mu_m}{\sigma_e}
\]  

The photon mean free path \((\lambda)\) which is the average distance between two successive interactions, is calculated by Equation (8):

\[
\lambda = \frac{1}{\int_0^x \exp(-\mu x) dx} = \frac{1}{\mu_l}
\]

where \(\mu_l\) is linear attenuation coefficient and \(x\) is the absorber thickness.

3. Results and discussion

The relative percentage difference between the mass attenuation coefficients of soft-tissue and bone tissue equivalent materials were evaluated between experimental values and XCOM software. The results are presented in Table 1. For both samples, at energy 88 keV, the relative percentage difference are above 5%. The results also shown below 5% for energies between 662 keV and 1.3 MeV for both samples. Therefore, the use of soft tissue and bone tissue equivalent materials may be considered as a substitute material for soft tissue and bone tissue for radiotherapy and nuclear medicine uses.

| \(E_\gamma\) (KeV) | 88  | 662 | 835 | 1173 | 1275 | 1333 |
|------------------|-----|-----|-----|------|------|------|
| \(\mu_{\text{STE}}\) (cm\(^2\)/g\(^{-1}\)) | 0.1860 | 0.0860 | 0.0750 | 0.0630 | 0.0610 | 0.0590 |
| \(\Delta_\gamma(E_\gamma)_{\text{STE}}\) (%) | 7.2 | 3.49 | 1.33 | 4.76 | 0.21 | 1.69 |
| \(\Delta_\gamma(E_\gamma)\) (%) | 4.49 | 3.24 | 1.35 | 5.12 | 0 | 1.72 |
| \(\mu_{\text{BTE}}\) (cm\(^2\)/g\(^{-1}\)) | 0.195 | 0.079 | 0.070 | 0.062 | 0.057 | 0.055 |
| \(\Delta_\gamma(E_\gamma)_{\text{BTE}}\) (%) | 5.4 | 0.38 | 0.43 | 0.32 | 0.88 | 0.9 |
| \(\Delta_\gamma(E_\gamma)\) (%) | 1.52 | 0.38 | 0.43 | 0.32 | 0.35 | 0.36 |

\(\Delta_\gamma(E_\gamma)\)_{\text{STE}}\); Absolute percentage difference between mass attenuation coefficients of experimental and stimulation

\(\Delta_\gamma(E_\gamma)\)_{\text{BTE}}\); Absolute percentage difference between mass attenuation coefficients of XCOM value and ICRU-44

The relative percentage difference of mass attenuation coefficient between measured values and ICRU-44 report was determined for the gamma energies of common radionuclide used. Differences below 5 % are observed in Table 1 for soft tissue equivalent sample. It shows that soft tissue equivalent has a good relationship in respect to soft tissue ICRU-44, except in the gamma energy of 1173 KeV which is 5.12%. For bone tissue equivalent material and ICRU-44, the differences below 0.5% were obtained of energies range 662 keV and 1.3 MeV and 1.52% for energy 88 keV. It is clear that mass attenuation coefficient depends on photon energy and chemical content. The mass attenuation coefficient of a material decreases due to probability of absorption reduces with increasing incident
photon energies which results in the increase in the transmission of photons through it. The total experimental uncertainty of mass attenuation coefficient values depend on the uncertainties of peak area evaluation, mass thickness measurements, experimental system, counting statistics, and efficiency errors.

The other parameters such as linear attenuation coefficient ($\mu_l$), total atomic cross-section ($\sigma_t$), electronic cross-section ($\sigma_e$), effective atomic number ($Z_{\text{eff}}$), electron density ($N_{\text{eff}}$), mean free-path ($\lambda$) were calculated using mass attenuation coefficient for both samples at different photon energies. The results were displayed in Tables 2 and 3. The photon mean free path ($\lambda$) for soft-tissue and bone tissue equivalent materials increases with the photon energy (Figure 2). This is due to decrease in the probability of interaction of photons in the material with the increase in energy. At low energies, total atomic cross-section ($\sigma_t$) and electronic cross-section ($\sigma_e$) decreases when energies increases (Figures 3 and 4). However, only at low energies, $\sigma_t$ and $\sigma_e$ the dependence of photon energy is dominant. The $Z_{\text{eff}}$ and the $N_{\text{eff}}$ remain constant and are found to be independent of photon energy for a compound except for energy 622 KeV which is slightly higher. Both effective atomic number and electron density have close relations to each other and thus have the same qualitative energy dependence.

The agreement between the calculated values from semi empirical relations, XCOM and experimental results is good. In a real practice, the uniformity of NaCl in the paraffin wax were varied as the mixture of these two elements are inhomogeneous, and the experimental results is good. In a real practice, the uniformity of NaCl in the paraffin wax were varied as the mixture of these two elements are inhomogeneous, and the experimental results is good.

### Table 2. $\mu_l$, $\mu_m$, $\sigma_t$, $\lambda$, $\sigma_e$, $Z_{\text{eff}}$ and $N_{\text{eff}}$ values for soft tissue equivalent sample at difference gamma energies.

| Energy(KeV) | 87.8 | 622.0 | 834.9 | 1173.2 | 1274.5 | 1332.5 |
|-------------|------|-------|-------|--------|--------|--------|
| $\mu_l$ (cm$^{-1}$) | 0.202 | 0.093 | 0.081 | 0.068 | 0.066 | 0.064 |
| $\mu_m$ (cm$^2$/g) | 0.186 | 0.086 | 0.075 | 0.063 | 0.061 | 0.059 |
| $\lambda$ (cm) | 4.95 | 10.75 | 12.35 | 14.71 | 15.15 | 15.63 |
| $\sigma_t$ $(10^{23})$ barn/atoms | 4.05 | 1.96 | 1.63 | 1.37 | 1.33 | 1.28 |
| $\sigma_e$ $(10^{25})$ barn/atoms | 4.14 | 1.82 | 1.67 | 1.40 | 1.36 | 1.31 |
| $Z_{\text{eff}}$ (electrons) | 9.78 | 10.72 | 9.78 | 9.78 | 9.78 | 9.78 |
| $N_{\text{eff}}$ $(10^{23})$ electrons/z | 4.49 | 4.72 | 4.49 | 4.49 | 4.49 | 4.49 |

### Table 3. $\mu_l$, $\mu_m$, $\sigma_t$, $\lambda$, $\sigma_e$, $Z_{\text{eff}}$ and $N_{\text{eff}}$ values for bone tissue equivalent sample at difference gamma energies.

| Energy(KeV) | 87.8 | 622.0 | 834.9 | 1173.2 | 1274.5 | 1332.5 |
|-------------|------|-------|-------|--------|--------|--------|
| $\mu_l$ (cm$^{-1}$) | 0.276 | 0.111 | 0.099 | 0.088 | 0.081 | 0.078 |
| $\mu_m$ (cm$^2$/g) | 0.195 | 0.079 | 0.07 | 0.062 | 0.057 | 0.055 |
| $\lambda$ (cm) | 3.62 | 9.01 | 10.10 | 11.26 | 12.35 | 12.83 |
| $\sigma_t$ $(10^{23})$ barn/atoms | 5.56 | 2.17 | 1.99 | 1.77 | 1.63 | 1.58 |
| $\sigma_e$ $(10^{25})$ barn/atoms | 3.25 | 1.26 | 1.16 | 1.04 | 0.97 | 0.92 |
| $Z_{\text{eff}}$ (electrons) | 17.10 | 17.25 | 17.10 | 17.10 | 17.10 | 17.10 |
| $N_{\text{eff}}$ $(10^{23})$ electrons/z | 6.0 | 6.25 | 6.0 | 6.0 | 6.0 | 6.0 |
4. Conclusion
The above experimental study were carried out to determine the mass attenuation coefficients and other related photon interaction parameters for soft tissue equivalent and bone tissue equivalent samples. In the interaction of photon with matter, the $\mu_m$ values are dependent on the physical and chemical composition of the elements in the sample. The $\mu_m$ values of both samples decrease with increase in photon energy. Both soft tissue equivalent and bone tissue equivalent samples have mass attenuation coefficient values close to human tissue, with percentage difference below 5%. The $\sigma_t$ and $\sigma_e$ decreases as the photon energy increases but the influence dominant at low energies. It can be concluded that the mass attenuation coefficients can be used to generate and determine the $Z_{eff}$ and $N_{eff}$ and other photon interaction parameters for soft tissue equivalent of thyroid and bone tissue equivalent of spinal cord. The $Z_{eff}$ and $N_{eff}$ are independence on photon energy. However, the $Z_{eff}$ and $N_{eff}$ calculated are above 5% of its percentage difference when compare to $Z_{eff}$ and $N_{eff}$ human tissue. All photon interaction parameters of soft tissue equivalent and bone tissue equivalent obtained in this study will enhance the understanding on how the mass attenuation coefficients change with variation of the atomic and electronic number for difference amount of NaCl added into paraffin wax. This study also aimed to establish soft tissue equivalent and bone tissue equivalent as tissue equivalent materials for phantom fabrication.

5. Reference
[1] SM Naderi, S Sina, M Karimipoorfard, F Lotfalizadeh, M Entezarmahdi, H Moradi and R Faghihi. 2015. *Radiation Protection Dosimetry* ,pp. 1–6.
[2] H Hasanzadeh, A Abedelahi. 2011. *Journal of Pramedical Sciences* 2(4): 2008-4978.
[3] White, D. R. 1993. *Radiation Protection Dosimetry* **49** (1/3), 359-369.
[4] Cerqueira, R. A. D. and Maia, A. F. 2014. *Radiat. Phys. Chem* **95**, 174–176.
[5] L.S. Del Lama, L.D.H. Soares et al. 2014. *Nuclear Instruments and Methods in Physics Research* **784** (2015) 597-601.
[6] A.S. Madhusudhan Rao, A Getachew. 2015. *International Journal of Advance Research In Science And Engineering* **4**(01), 2319-8354.
[7] Singh, N., Singh, K.J., Singh, K. and Singh, H., 2006. *Radiat. Meas.* **41**, pp.84-88.
[8] International Commission on Radiation Units and Measurements ICRU. 1992. *ICRU Report 48*, 96–101.
[9] M Kurudirek. 2014. *Applied Radiation and Isotopes* **94** (2014), 1-7.
[10] H Yücel, E Gülüoğlu, Ş Çubukçu and Yiğit Ali Ü. 2016. *Journal of Physical Science*, **27** (1), 111–128.
[11] Chaudhari L. M and Raje D. V. 2013. *Journal of Chemical, Biological and Physical Sciences* **3** (2) 1504-1510
[12] A. Hermosilla, G. Díaz Londono, M. García, F. Ruiz, P. Andrade and A. Pérez. 2014. *Radiation Protection Dosimetry* **162** (4), 508–514
[13] Parisa Akhlaghi, Hashem Miri Hakimabad, Laleh Rafat Motavalli. 2015. *Radiation Physics and Chemistry* **112** (2015), 169–176.
[14] C C Ferreira , R E Ximenes, C A B Garcia, J W Vieira, A F Maia. 2010. *Journal of Physics: Conference Series* **249** (2010) 012029

Acknowledgements
The authors would like to acknowledge the financial support from Ministry of Higher Education through Fundamental Research Grant Scheme (FRGS).