Review

Probes of Lepton Flavor Universality in $b \to u$ Transitions

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Abstract: Anomalies recently observed in semileptonic $b \to c \ell^- \bar{\nu}_\ell$ and $b \to s \ell^+ \ell^-$ transitions point to violation of Lepton Flavour Universality. Strategies for new analyses of different modes are required, in particular for the modes induced by the $b \to u$ transition. We describe the purely leptonic $B$ decay, the $B \to \pi \ell^- \bar{\nu}_\ell$ channel and the $B$ semileptonic modes to $\rho(770)$ and $a_1(1260)$ in extensions of the Standard Model involving Lepton Flavour Universality violating $b \to u$ operators. In particular, we review the observables in the four-dimensional angular $B \to \rho(\pi\pi) \ell^- \bar{\nu}_\ell$ and $B \to a_1(\rho\pi) \ell^- \bar{\nu}_\ell$ distributions, suitable to pin down deviations from the Standard Model. We discuss the complementarity among the various modes for New Physics searches.

Keywords: Lepton Flavour Universality; semileptonic $B$ decays

1. Introduction

Several anomalies have recently emerged in tree-level $b \to c \ell^- \bar{\nu}_\ell$ and in loop-induced $b \to s \ell^+ \ell^-$ semileptonic transitions. In tree-level modes, the measured ratios $R(D^{(*)}) = \frac{B(B \to D^{(*)} \tau^- \bar{\nu}_\tau)}{B(B \to D^{(*)} \ell^- \bar{\nu}_\ell)}$ (with $\ell = e, \mu$) [1–9] exceed the Standard Model (SM) predictions at 3.1 $\sigma$ [10]. This is remarkable, since the hadronic uncertainties largely cancel out in the ratios [11,12]. The measurement of $R(J/\psi) = \frac{B(B_s^+ \to J/\psi \pi^+ \nu_\tau)}{B(B_s^+ \to J/\psi \mu^+ \nu_\mu)}$ [13] is also above the (more uncertain) SM expectation [14–17]. For neutral current $b \to s$ semileptonic modes, the ratios $R_{K^+} = \frac{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \frac{d^2}{dq^2} \frac{B(B \to K^{(*)} \mu^+ \mu^-)}{dq^2}}{\int_{q_{\text{min}}^2}^{q_{\text{max}}^2} dq^2 \frac{d^2}{dq^2} \frac{B(B \to K^{(*)} e^+ e^-)}{dq^2}}$, measured in various ranges of the $q^2$ dilepton invariant mass [18–20], also deviate from the SM predictions, which are close to one. Tests involving $\Lambda_b$ decay modes are also carried out [21]. The anomalies point to violation of lepton flavour universality (LFU), which is an accidental SM symmetry, broken by the Yukawa interactions (a review is in Reference [22]).

There are other tensions, namely the difference in the value of $|V_{cb}|$, an element of the Cabibbo-Kobayashi-Maskawa quark mixing matrix, when determined using inclusive $B \to X_c \ell^- \bar{\nu}_\ell$ observables (width and moments) and exclusive modes, mainly $B \to D^* \ell^- \bar{\nu}_\ell$. The tension persists after recent experimental analyses [23,24]. The latest measurement using $B_s^0 \to D_s^{(*)-} \mu^+ \nu_\mu$ agrees with the inclusive determination, but it is affected by a sizable error [25]. Even though its solution within SM is possible [26–29], a connection with the $b \to c$ semileptonic anomalies can also be envisaged in frameworks incorporating LFU violation [12,30]. Experimental signatures of Beyond
the Standard Model (BSM) effects in such modes have been scrutinized, in particular using the full $B \to D^*(D\pi,D\gamma) \ell^- \nu_\ell$ decay distributions for different lepton species [31–35].

Although the semileptonic $b \to u$ transitions are CKM suppressed with respect to the $b \to c$ ones, new precision measurements are foreseen by LHCb and Belle II Collaborations. At present, there is a tension between the inclusive determination of $|V_{ub}|$ and the exclusive measurement from the $B \to \pi \ell^- \nu_\ell$ decay width. The $B^-$ purely leptonic and the semileptonic $B \to \pi$ mode have been analyzed in References [36–43], showing their sensitivity to BSM effects. Other decay modes can be exploited to pin down possible deviation from the Standard Model, in particular those involving several hadrons in the final state. This is the case of $B$ to vector $\rho(770)$ and axial-vector $a_1(1260)$ channels, for which the fully differential angular distributions, when $\rho$ decays in two pions and $a_1$ decays into $\rho \pi$, produce a wealth of correlated observables, with coherent patterns within and beyond SM.

We extend the semileptonic $b \to u$ effective Hamiltonian with the inclusion of additional scalar $(S)$, pseudoscalar $(P)$, vector $(V)$ and tensor $(T)$ operators, and study the effects of such operators on $B$ transitions to leptons and to $\pi \ell \ell$. Then, we provide the expressions of the fully differential decay distributions for the $B \to \rho(\pi \pi) \ell^- \nu_\ell$ and $B \to a_1(\rho \pi) \ell^- \nu_\ell$ modes (Section 4). In the Large Energy Limit for the light mesons, the distributions are expressed in terms of a small number of hadronic form factors. Observables in $B \to \rho(\pi \pi) \ell^- \nu_\ell$ are studied at a benchmark point in the parameter space of the new effective couplings, scrutinizing their sensitivity to the different operators (Section 5). For the $a_1(1260)$ mode, analyzed in Section 06, the uncertainties from the form factors are large; nevertheless, also in this channel observables particularly sensitive to New Physics (NP) can be identified [44].

2. Extended $b \to u \ell^- \nu_\ell$ Effective Hamiltonian

Beyond the Standard Model (BSM) effects in beauty hadron decays can be analyzed using the Standard Model Effective Field Theory. For a NP scale $\Lambda_{NP}$ much larger than the EW scale the new massive degrees of freedom can be integrated out, recovering an effective Hamiltonian involving SM fields and invariant under the SM gauge group. This Hamiltonian contains additional operators with respect to SM, suppressed by powers of $\Lambda_{NP}$: dimension-six four-fermion operators appear at $O\left(1/\Lambda_{NP}^2\right)$ [45]. We consider the general expression of the effective $b \to u$ Hamiltonian:

$$H_{\text{eff}}^{b \to u \ell \nu} = \frac{G_F}{\sqrt{2}} V_{ub} \left\{ \left(1 + \epsilon_1^\ell\right) \left( a_2 \gamma_\mu (1 - \gamma_5) b \right) \left( \bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell \right) + \epsilon_2^\ell (ab) \left( \bar{\ell} (1 - \gamma_5) \nu_\ell \right) \right\} + \text{h.c.,}$$

consisting in the SM term and in NP operators with complex lepton-flavour dependent couplings $\epsilon_1^\ell_{V,S,P,T}$, $V_{ub}$ and $\epsilon_2^\ell$ are independent parameters. A purely left-handed lepton current is assumed. Moreover, the quark right-handed vector current is excluded, since the only four-fermion operator of this type, invariant under the SM group, is non-linear in the Higgs field [46–48]. See References [37,38,40] for investigations of right-handed currents. The Hamiltonian (1) governs decays of $B$ mesons, of $B_s$ (such as $B_s \to K^{(*)+} \ell^- \nu_\ell$), $B_c$ (e.g., $B_c \to D^{(*)+} \ell^- \nu_\ell$ [49]), $\Lambda_b$ (e.g., $\Lambda_b \to p \ell^- \nu_\ell$ [50]), and so forth. Here we focus on $B$ meson modes, for which precise measurements are available.

3. $B^- \to \ell^- \nu_\ell$ and $B \to \pi \ell^- \nu_\ell$

The couplings of the NP operators in (1) are constrained by the purely leptonic $B^-$ and semileptonic $B \to \pi \ell^- \nu_\ell$ modes. The $B^- \to \ell^- \nu_\ell$ decay width obtained from Equation (1),

$$\Gamma(B^- \to \ell^- \nu_\ell) = \frac{G_F^2 |V_{ub}|^2 f_B^2 m_B^3}{8\pi} \left( 1 - \frac{m_\ell^2}{m_B^2} \right)^2 \left( \frac{m_\ell}{m_B} \right) \left( 1 + \epsilon_1^\ell \right) + \frac{f_B}{m_B + m_\ell} \epsilon_2^\ell,$$

is determined by the SM parameters $m_B, f_B, m_\ell, V_{ub}$, and $\epsilon_1^\ell, \epsilon_2^\ell$. This expression includes the dominant contribution from the $S$-wave and can be further improved by including the contributions from the $P$-, $V$-, and $T$-waves, as well as by exploring the full angular distribution.
with $f_B$ defined as $\langle 0|\bar{u}\gamma_\mu\gamma_5b|\bar{B}(p)\rangle = if_Bp_\mu$, shows that this mode is insensitive to the scalar and tensor operators. The pseudoscalar operator lifts the helicity suppression, which is effective for light leptons. This produces stringent bounds to $\epsilon_\mu^e$.

The $\bar{B} \to \pi\ell^-\bar{\nu}_\ell$ decay distribution in the dilepton mass squared $q^2$, obtained using (1) and adopting the definition of the form factors $f_i(q^2) = f_i^{\bar{B}\to\pi}(q^2)$ as in Reference [44],

$$\frac{d\Gamma}{dq^2}(\bar{B} \to \pi\ell^-\bar{\nu}_\ell) = \frac{G_F^2|V_{ub}|^2\lambda^{1/2}(m_B^2,m_\pi^2,q^2)}{128m_B^3\pi^3q^2} \left(1 - \frac{m_\ell^2}{q^2}\right)^2 \times \left\{\left|m_\ell(1 + \epsilon_\nu^f) + \frac{q^2\epsilon_\nu^S}{m_B - m_\ell}\right|^2 (m_B^2 - m_\pi^2)^2 f_0^2(q^2) + \lambda(m_B^2,m_\pi^2,q^2) \left[\frac{1}{3}m_\ell(1 + \epsilon_\nu^f) f_+(q^2) + \frac{4q^2}{m_B + m_\pi}\epsilon_\nu^T f_T(q^2)\right]^2 + \frac{2q^2}{3} \left|(1 + \epsilon_\nu^f) f_+(q^2) + 4\frac{m_\ell}{m_B + m_\pi}\epsilon_\nu^T f_T(q^2)\right|^2 \right\}, \tag{3}$$

(with $\lambda$ the triangular function), shows that the operator $P$ does not contribute in this case. In Equations (2) and (3) the CKM matrix element $V_{ub}$ appears in the combination $V_{ub}(1 + \epsilon_\nu^f)$. The lepton-flavour dependence of the effective couplings $\epsilon_\nu^f$ would produce different values of an effective $|V_{ub}|$ when measured in channels involving different lepton generations.

In the $B$ rest-frame the energy $E$ of the emitted pion is obtained by the relation $q^2 = m_B^2 + m_\pi^2 - 2m_BE$, and is in the range $m_\pi \leq E \leq \frac{m_B}{2}\left(1 + \frac{m_\pi^2}{m_B^2} - \frac{m_\pi^2}{m_B^2}\right)$. In the Large Energy Limit, $E \approx \frac{m_B}{2}$, the $\bar{B} \to \pi\ell^-\bar{\nu}_\ell$ distribution involves a single form factor $\xi_\pi$ [51,52]. The consequence is that, in this limit, the ratios

$$\frac{dR(\pi)_{\ell\nu}}{dE} = \frac{d\Gamma}{dE}(\bar{B} \to \pi\ell^-\bar{\nu}_\ell)/\frac{d\Gamma}{dE}(\bar{B} \to \pi\ell^-\bar{\nu}_\ell) \tag{4}$$

are free of hadronic uncertainties and only involve combinations of $\epsilon_{\nu}^{f,i,S,T}$ [44] .

4. $\bar{B} \to \rho(\to \pi\pi)\ell^-\bar{\nu}_\ell$ and $\bar{B} \to a_1(\to \rho\pi)\ell^-\bar{\nu}_\ell$ Full Angular Distributions

For the modes $\bar{B} \to \rho(\to \pi\pi)\ell^-\bar{\nu}_\ell$ and $\bar{B} \to a_1(\to \rho\pi)\ell^-\bar{\nu}_\ell$, the main sensitivity to the new operators in (1) is in the 4-dimensional differential decay distribution in $(q^2, \theta, \theta_\nu, \phi)$, with angles defined in Figure 1.

**Figure 1.** Definition of the angles in the fully differential $\bar{B} \to \rho(\pi\pi)\ell^-\bar{\nu}_\ell$ distribution. The $\rho \to \pi\pi$ and $W^- \to \ell^-\bar{\nu}_\ell$ decay planes are shown.
For the $\rho$ mode the distribution reads:

$$
\frac{d^4\sigma(\bar{B} \to \rho(\to \pi \pi)\ell^- \bar{\nu}_\ell)}{da^2 \cos \theta \, d \phi \, d \cos \theta_V} = \mathcal{N}_\rho |\vec{p}_f| \left(1 - \frac{m_T^2}{q^2}\right)^2 \left\{ I_{1s}^\rho \sin^2 \theta_V + I_{1c}^\rho \cos^2 \theta_V \right.
\left. + \left( I_{2s}^\rho \sin^2 \theta_V + I_{2c}^\rho \cos^2 \theta_V \right) \cos 2\theta
\right.
\left. + I_{3s}^\rho \sin^2 \theta_V \sin^2 \theta \cos 2\phi + I_{3c}^\rho \sin 2\theta_V \sin 2\theta \cos \phi \right.
\left. + I_{4s}^\rho \sin 2\theta_V \sin \theta \cos \phi + \left( I_{6s}^\rho \sin^2 \theta_V + I_{6c}^\rho \cos^2 \theta_V \right) \cos \theta
\right. 
\left. + I_{5s}^\rho \sin 2\theta_V \sin \theta \sin \phi \right\},
$$

with $\mathcal{N}_\rho = \frac{3G_F^2|V_{ub}|^2 B(\rho \to \pi \pi)}{128(2\pi)^4 m_B^2}$. Other angular structures appear in the differential distribution if the quark right-handed vector operator is included in (1). The same expression holds for $B_s \to K^{*-}(K \pi)\ell^- \bar{\nu}_\ell$. The relations of the angular coefficient functions in Equation (5) to the hadronic matrix elements are collected in Reference [44], obtained in the narrow width approximation which factorizes the production and decay amplitude of the intermediate vector meson. A discussion on this procedure, together with references to studies of non-resonant contributions, can also be found in Reference [44].

For the $a_1(\rho \pi)$ channel with the final $\rho$ transversely ($\rho_\perp$) or longitudinally ($\rho_\parallel$) polarized, the 4-dimensional distribution has the same expression as in (5) with the substitution $I_i^\rho \to I_i^{a_1(\rho \pi), \perp}$ and $\mathcal{N}_\rho \to \mathcal{N}_{a_1}^{\rho,(I)}(\perp) = \frac{3G_F^2|V_{ub}|^2 B(a_1 \to \rho_\perp(\rho \pi))}{128(2\pi)^4 m_B^2}$. The subscripts $\perp$ and $\parallel$ refer to the $\rho$ polarizations. The expressions of the coefficients functions in terms of $B \to a_1$ hadronic matrix elements are in Reference [44]. The separation of the polarizations of the final $\rho$ is justified by the different sensitivity of the distribution to the NP operators.

The angular functions $I_i^\rho$ ($I_i^{a_1(\rho \pi)}$) in Equation (5) have the general expression

$$
I_i = |1 + e_V|^2 I_i^{SM} + |e_X|^2 I_i^{NP,X} + |e_T|^2 I_i^{NP,T} + 2 \text{Re} \left[ e_X (1 + e_V) \right] I_i^{\text{INT},X},
$$

$$
I_i = 2 \text{Re} \left[ e_T (1 + e_V) \right] I_i^{\text{INT},T} + 2 \text{Re} \left[ e_X e_T^* \right] I_i^{\text{INT},XT},
$$

$$
I_7 = 2 \text{Im} \left[ e_X (1 + e_V) \right] I_7^{\text{INT},X} + 2 \text{Im} \left[ e_T (1 + e_V) \right] I_7^{\text{INT},T} + 2 \text{Im} \left[ e_X e_T^* \right] I_7^{\text{INT},XT},
$$

with $X = P$ in case of $\rho$, and $X = S$ for $a_1$, and functions $I_i^{SM}$, $I_i^{NP}$ and $I_i^{\text{INT}}$ collected in Reference [44]. All angular coefficient functions, with the exception of $I_7$, do not vanish in SM and are sensitive to $e_V$. Structures able to disentangle the other $S$, $P$ and $T$ operators can be identified. For $B \to \rho \ell \bar{\nu}_\ell$, the functions $I_1^{3s}, I_2^{3s}, I_2^{3c}, I_4^{a_1(\rho \pi)}$ do not depend on $e_\rho$ and are only sensitive to the tensor operator. The corresponding quantities in $B \to a_1(\rho_\parallel, \pi)\ell \bar{\nu}_\ell$ do not depend on $e_\rho$. In $B \to a_1(\rho_\perp, \pi)\ell \bar{\nu}_\ell$ the functions $I_1^{a_1,1}, I_2^{a_1,1}, I_2^{a_1,2}, I_3^{a_1,3}, I_4^{a_1,4}, I_5^{a_1,6}, I_6^{a_1,6}$ are insensitive to the scalar operator. There are angular coefficient functions related only to the helicity amplitudes corresponding to the transversely polarized $W$, hence to transverse $\rho (a_1)$ in $B \to \rho \ell \bar{\nu}_\ell (B \to a_1 \ell \bar{\nu}_\ell)$. Such observables depend on $e_T$, not on $e_\rho$ (in the $\rho$ channel) or $e_\rho$ (in $a_1$).

In the Large Energy Limit of the light meson, the $B \to \rho (a_1)$ weak matrix elements can be written in terms of two form factors, $\xi_\rho^0 (\xi_\perp^0)$ and $\xi_\rho^0 (\xi_\parallel^0)$ [44]. In this limit, several angular coefficients depend only on $\xi_\perp$, others involve both $\xi_\perp$ and $\xi_\parallel$. The coefficients depending only on $\xi_\perp (E)$ are, in $B \to \rho$, $I_1^{3s}, I_2^{3c}, I_4^{a_1(\rho \parallel)}$ and in $B \to a_1$, $I_1^{a_1(\rho \perp)} I_2^{a_1(\rho \parallel)}$ and $I_4^{a_1(\rho \parallel)}$ (for longitudinally polarized $\rho$), and $I_1^{a_1(\rho \perp)} I_2^{a_1(\rho \parallel)}$ and $I_4^{a_1(\rho \parallel)}$ (for transversely polarized $\rho$). When a single form factor is involved, ratios of coefficient functions are free of hadronic uncertainties.
5. Observables in $\bar{B} \to \rho (\pi \pi) \ell^- \bar{\nu}_\ell$

To give examples of the effects of NP operators in (1) in $B \to \rho \ell^- \bar{\nu}_\ell$, we constrain the space of the effective couplings using a set of hadronic quantities and the available data. The couplings $\epsilon^\mu_P, \epsilon^\mu_T$ are constrained by the measurements of $B(B^0 \to \pi^+ \ell^- \bar{\nu}_\ell)$ and $B(B^0 \to \rho^+ \ell^- \bar{\nu}_\ell)$ \cite{53}, together with $B(B^- \to \mu^- \bar{\nu}_\mu)$ \cite{54} and $B(B^- \to e^- \bar{\nu}_e), B(B^- \to \tau^- \bar{\nu}_\tau)$ \cite{53}. An upper bound $B(B^0 \to \pi^+ \tau^- \bar{\nu}_\tau)$ has been established \cite{55}. We use the $B \to \pi$ form factors in Appendix A and the $B \to \rho$ form factors in Reference \cite{56}. The parameter space for the NP couplings, together with the chosen benchmark point, is shown in Figure 2 in the case of $\mu$ and $\tau$. The $\bar{B} \to \pi \mu^+ \nu_\mu$ decay distribution, using the form factors in Appendix A, is depicted in Figure 3 for SM and NP at the benchmark point, compared to Belle \cite{57} and BaBar measurements \cite{58,59}.

![Figure 2](image.png)

**Figure 2.** Allowed regions (at 68% CL) for the couplings $\epsilon^\mu_P$ and $\epsilon^\mu_T$, (first row plots), $\epsilon^\mu_S$ and $\epsilon^\mu_T$ (second row plots). The stars are the benchmark points: $(\text{Re}[\epsilon^\mu_P], \text{Im}[\epsilon^\mu_P]) = (0, 0), (\text{Re}[\epsilon^\mu_T], \text{Im}[\epsilon^\mu_T]) = (-0.03, -0.02), (\text{Re}[\epsilon^\mu_S], \text{Im}[\epsilon^\mu_S]) = (0.12, 0)$ and $(\text{Re}[\epsilon^\mu_T], \text{Im}[\epsilon^\mu_T]) = (-0.04, 0)$, with $|V_{ub}| = 3.5 \times 10^{-3}$. The plots in the third row show the allowed regions for the couplings $\epsilon^\mu_P$ and $\epsilon^\mu_T$, with the stars indicating the benchmark points $\epsilon^\mu_P = 0, \epsilon^\mu_S = 0, (\text{Re}[\epsilon^\mu_P], \text{Im}[\epsilon^\mu_P]) = (0.01, 0)$ and $(\text{Re}[\epsilon^\mu_T], \text{Im}[\epsilon^\mu_T]) = (0.12, 0)$. The various colors indicate the different couplings.
Figure 3. $B \to \pi \mu^- \bar{\nu}_\mu$ distribution using the form factors in Appendix A and parameters in Table A1, in Standard Model (SM) and in New Physics (NP) at the benchmark point. The red and brown data are from Babar [58,59], the blue ones from Belle Collaboration [57].

The functions $I^{\rho}_{1\sigma}$, $I^{\rho}_{2\sigma}$, $I^{\rho}_{2c}$, $I^{\rho}_{4}$ and $I^{\rho}_{6c}$, which are independent of $\epsilon_P$, are shown in Figure 4 for SM and NP. The bands are obtained varying the form factor parameters within their estimated ranges. Notice the zero in $I^{\rho}_{2\sigma}(q^2)$, absent in SM.

Figure 4. $B \to \rho(\pi \pi) \mu^- \bar{\nu}_\mu$: angular coefficient functions $I^\rho_i(q^2)$ in Equation (5), for SM and NP at the benchmark point. The bands for SM and NP, with the colors indicated in each plot, are obtained varying the form factor parameters within the estimated ranges. A zero in $I^{\rho}_{2\sigma}(q^2)$ appears in NP.

The other coefficient functions are in Figure 5, and there is a zero in $I^{\rho}_{6c}(q^2)$ not present in SM. The function $I^\rho_7$ in Figure 6 vanishes in SM, and is only sensitive to the imaginary part of the NP couplings. For the $\tau$ modes, the angular functions are in Figures 7 and 8, and also for this mode $I^{\rho}_{6c}$ has a zero in NP, not in SM.
Figure 5. $B \to \rho(\pi\pi) \mu^- \bar{\nu}_\mu$: angular coefficient functions $I_{1c}^R(q^2)$ (left), $I_{5s}^R(q^2)$ (middle) and $I_{6c}^\rho(q^2)$ (right).

Let us consider the ratios

$$R_{2s/1s}^\rho(q^2) = \frac{I_{5s}^R(q^2)}{I_{1c}^R(q^2)},$$

(7)

$$R_{2s/1s}^{a_1}(q^2) = \frac{I_{5s}^{2s_\perp}(q^2)}{I_{1c}^{2s_\perp}(q^2)},$$

(8)

and $R_{2s/1s}^{a_1} = R_{2s/1s}^{a_1 \perp}$. $R_{2s/1s}^\rho$ is form independent in SM. In NP it is still form factor independent in the Large Energy Limit, since $I_{5s}^R$ and $I_{1c}^R$ depend on $q^2$. As shown in Figure 9, the ratio (7) has a zero in NP, not in SM, whose position in the Large Energy Limit, since $I_{5s}^R$ and $I_{1c}^R$ depend on $q^2$. As shown in Figure 9, the ratio (7) has a zero in NP, not in SM, whose position $q_{0,\rho}^2$ depends only on $|e_T^\rho|$ with a small form factor effect. In the Large Energy Limit we have

$$|e_T^\rho|^2 = \frac{q_{0,\rho}^2 \lambda(m_B^2, m_{\rho}^2, q_{0,\rho}^2) + 2m_B^2 m_\rho^2}{16m_B^2 \lambda(m_B^2, m_\rho^2, q_{0,\rho}^2) + 2q_{0,\rho}^2 m_\rho^2}.$$  

(9)

Analogously, for the $(a_1)_{||}$ channel (for $(a_1)_{\perp}$ considering $R_{2s/1s}$ we have:

$$|e_T^{a_1}|^2 = \frac{q_{0,a_1}^2 \lambda(m_B^2, m_{a_1}^2, q_{0,a_1}^2) + 2m_B^2 m_{a_1}^2}{16m_B^2 \lambda(m_B^2, m_{a_1}^2, q_{0,a_1}^2) + 2q_{0,a_1}^2 m_{a_1}^2}.$$  

(10)

The positions of the zeros in two modes are related, and their measurement would provide access to the tensor operator.

Figure 6. $B \to \rho(\pi\pi) \mu^- \bar{\nu}_\mu$: angular coefficient function in NP at the benchmark point.
Figure 7. $\bar{B} \rightarrow \rho(\pi\pi) \tau^- \bar{v}_\tau$ mode: angular coefficient functions $I_{\rho}^\rho(q^2)$ in Equation (5). The bands for SM and NP, with colors indicated in each plot, are obtained varying the form factor parameters.

Figure 8. $\bar{B} \rightarrow \rho(\pi\pi) \tau^- \bar{v}_\tau$: angular coefficient functions $I_{\rho\mu}^\rho(q^2)$ (left), $I_{\rho T}^\rho(q^2)$ (middle), $I_{\rho T}^\rho(q^2)$ (right).

Figure 9. Ratio $R_{2e/1s}^\rho$ in (7) for $\bar{B} \rightarrow \rho(\pi\pi) \mu^- \bar{v}_\mu$ (left) and $\bar{B} \rightarrow \rho(\pi\pi) \tau^- \bar{v}_\mu$ (right). The dashed lines are the Large Energy result (extrapolated to the full $q^2$ range).

The function $I_{6c}^\rho$, shown for SM and NP in the right panel of Figure 5, is sensitive to $\epsilon_\mu$, $\epsilon_\rho$, $\epsilon_T$. Different possibilities can be envisaged [44]:

1. If there is no NP ($\epsilon_\rho = \epsilon_T = 0$), $I_{6c}^\rho$ has no zero (right panel of Figure 5).

2. For NP with $\epsilon_T = 0$ and $\epsilon_\rho \neq 0$, $(I_{6c}^\rho)_{\epsilon_T=0}$ has a zero at $q_0^2 = -\frac{m_b + m_\rho}{m_\rho} \frac{1}{\text{Re}[\epsilon_\rho]}$, whose measurement would determine $\text{Re}[\epsilon_\rho]$. We show $I_{6c}^\rho$ in the left panel of Figure 10 in the region where the zero is present, and $q_0^2$ versus $\text{Re}[\epsilon_\rho]$ in the middle panel of the same figure.

3. For NP with $\epsilon_\rho = 0$ and $\epsilon_T \neq 0$, $(I_{6c}^\rho)_{\epsilon_\rho=0}$ has a zero if $\text{Re}[\epsilon_T] > 0$, its position has a form factor dependence (right panel of Figure 10).
Finally, if there is NP with both $\epsilon_P \neq 0$ and $\epsilon_T \neq 0$, both real and imaginary parts of $\epsilon_P$ and $\epsilon_T$ are involved, and two zeroes are possible.

Figure 10. $\bar{B} \to \rho(\pi\pi)\mu^-\bar{\nu}_\mu$ mode: $I_{6c}^\rho(q^2)$ (left), and position of the zero $q_0^2$ varying $\text{Re}(\epsilon_P)$ with $\epsilon_T = 0$ (middle panel) and $\text{Re}(\epsilon_T)$ with $\epsilon_P = 0$ (right).

Integrating the 4-dimensional distribution several observables can be constructed. The $q^2$-dependent forward-backward (FB) lepton asymmetry

$$A_{FB}(q^2) = \left[ \int_0^1 d\cos \theta \frac{d^2\Gamma}{dq^2 d\cos \theta} - \int_{-1}^0 d\cos \theta \frac{d^2\Gamma}{dq^2 d\cos \theta} \right] / d\Gamma/dq^2$$

is given by

$$A_{FB}(q^2) = \frac{3(I_{6c}^\rho + 2I_{6s}^\rho)}{6I_{1c}^\rho + 12I_{1s}^\rho - 2I_{2c}^\rho - 4I_{2s}^\rho}.$$  \hspace{1cm} (11)

The transverse forward-backward (TFB) asymmetry, that is, the FB asymmetry for transversely polarized $\rho$, is given by

$$A_{TFB}(q^2) = \frac{3I_{6s}^\rho}{6I_{1s}^\rho - 2I_{2s}^\rho}.$$  \hspace{1cm} (12)

$A_{FB}$ and $A_{TFB}^\ell$ are shown in Figure 11 for $\ell = \mu$ and $\ell = \tau$. In NP the zero of $A_{FB}$ is shifted in the $\tau$ mode. $A_{TFB}^\ell$ is sensitive to the new operators, and in the case of $\tau$ it has a zero absent in SM.

There are observables which depend on the $\rho$ polarization, namely the differential branching ratio for longitudinally (L) and transversely (T) polarized $\rho$, as a function of $q^2$ or of one of the angles $\theta, \theta_V$: $dB_{L(T)}/dq^2$, $dB_{L(T)}/d\cos \theta$ and $dB_{L(T)}/d\cos \theta_V$. They are shown in Figure 12 for $\ell = \mu$ and $\ell = \tau$. 

![Figure 10](image-url)
Figure 11. Forward-backward (FB) lepton asymmetries (11) (left) and (13) (right) for $\bar{B} \to \rho^- \bar{\nu}_\mu$ (top line) and $\bar{B} \to \rho^- \bar{\nu}_\mu$ (bottom line).
Integrating the distributions we obtain for the \( \rho \) mode: \( F_L(\bar{B} \to \rho \mu^− \bar{\nu}_\mu)|_{SM} = 0.52 \pm 0.15 \), \( \mathcal{B}(B^0 \to \rho^+ \mu^- \bar{\nu}_\mu)|_{SM} = (3.37 \pm 0.52) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2 \), and for the \( \tau \) mode \( F_L(\bar{B} \to \rho \tau^- \bar{\nu}_\tau)|_{SM} = 0.50 \pm 0.13, \mathcal{B}(B^0 \to \rho^+ \tau^- \bar{\nu}_\tau)|_{SM} = (1.80 \pm 0.25) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2 \). For \( B \to \pi \) we have: \( \mathcal{B}(B^0 \to \rho \pi^- \bar{\nu}_\pi)|_{SM} = \)
\[ \pi^+ \mu^- \nu_\mu |_{SM} = (1.5 \pm 0.1) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2 \] and \[ B(B^0 \to \pi^+ \tau^- \bar{\nu}_\tau) |_{SM} = (0.92 \pm 0.06) \times 10^{-4} \times \left( \frac{|V_{ub}|}{0.0035} \right)^2. \] As shown in Table 1, the ratios

\[
R_\pi = \frac{B(B \to \pi \tau^- \bar{\nu}_\tau)}{B(B \to \pi e^- \bar{\nu}_e)}, \quad R_\rho = \frac{B(B \to \rho \tau^- \bar{\nu}_\tau)}{B(B \to \rho e^- \bar{\nu}_e)} \tag{14}
\]

are modified by the NP operators in (1). The deviations are correlated, and large effects are possible varying the effective couplings in their allowed ranges (Figure 13).

Table 1. \( R_\pi \) and \( R_\rho \) in Equation (14) in SM and in NP at the benchmark point.

|               | SM            | NP (Benchmark Point) |
|---------------|---------------|----------------------|
| \( R_\pi \)   | 0.60 \pm 0.01 | 0.75 \pm 0.02        |
| \( R_\rho \)  | 0.53 \pm 0.02 | 0.49 \pm 0.02        |

Figure 13. \( R_\rho \) vs \( R_\pi \) correlation when only the tensor operator is included in (1). The colors correspond to the different signs of \( \text{Re}[\epsilon_T^{\mu}] \) and \( \text{Re}[\epsilon_T^{\tau}] \) in the range of the parameter space. The red and brown points correspond to SM and NP at the benchmark point.

6. \( \bar{B} \to a_1(1260) \ell^- \bar{\nu}_\ell \)

We analyze the \( \bar{B} \to a_1(\rho \pi) \ell^- \bar{\nu}_\ell \) mode in SM and in the NP extension Equation (1) at the same benchmark points for \( \epsilon_{\ell \pi S,T} \). Exclusive hadronic \( B \) decays into \( a_1(1260) \) have been studied at the \( B \) factories considering the dominant \( a_1 \to \rho \pi \) mode, and the measurements of the semileptonic \( B \to a_1 \) mode are within the experimental reach, in particular at Belle II. The study of \( \bar{B} \to a_1 \ell^- \bar{\nu}_\ell \) requires an assessment of the accuracy of the hadronic quantities. We use the set in Reference [60], for which an uncertainty of about 20% is quoted. The angular coefficient functions, for the \( \mu \) and \( \tau \) modes and for both the \( \rho \) polarizations are in Figure 14, Figure 15 and in Figure 16, Figure 17, respectively. In general, the hadronic uncertainties obscure the effect of the NP operators. Nevertheless, there are coefficient functions in which deviations from SM can be observed, namely \( I_{2\mu \perp}^{a_1} (q^2) \), \( I_{6c \perp}^{a_1} (q^2) \) (Figure 14) and \( I_{2\tau \perp}^{a_1} (q^2) \) (Figure 16) for the \( \mu \) channel, \( I_{12s}^{a_1} (q^2) \), \( I_{6c \perp}^{a_1} (q^2) \) (Figure 15) and \( I_{12c \perp}^{a_1} (q^2) \), \( I_{6c \perp}^{a_1} (q^2) \) (Figure 16) for the \( \tau \) mode. The forward/backward lepton asymmetry shows sizable deviations from SM in the case of \( \tau \) (Figure 18).
Figure 14. Angular coefficient functions $I^i_t(q^2)$ in SM and NP (at the benchmark point) for $\bar{B} \to a_1(\rho/\pi) \mu^- \bar{\nu}_\mu$ using the form factors in Reference [60]. The band widths for SM and NP, with colors indicated in each plot, are due to the form factor uncertainty.

Figure 15. Angular coefficient functions for $\bar{B} \to a_1(\rho/\pi) \tau^- \bar{\nu}_\tau$ (same notations as in Figure 14).
Figure 16. Angular coefficient functions for $\bar{B} \to a_1 (\rho, \pi) \mu^- \bar{\nu}_\mu$ (same notations as in Figure 14).

Figure 17. Angular coefficient functions for $\bar{B} \to a_1 (\rho, \pi) \tau^- \bar{\nu}_\tau$ (same notations as in Figure 14).
For the ratio \( R_{a_1} = \frac{B(\bar{B} \to a_1(1260)\tau^-\bar{v}_\tau)}{B(\bar{B} \to a_1(1260)\ell^-\bar{v}_\ell)} \) we obtain, in the SM and for NP at the benchmark point,

\[
R_{a_1}^{SM} = 0.44 \pm 0.07, \quad R_{a_1}^{NP} = 0.67 \pm 0.12,
\]

with individual branching fractions in SM \( B(\bar{B} \to a_1 \mu^-\bar{v}_\mu) = (3.0 \pm 1.7) \times 10^{-4} \) and \( B(\bar{B} \to a_1 \tau^-\bar{v}_\tau) = (1.3 \pm 0.6) \times 10^{-4} \) \cite{60}.

Summarizing the complementarity between the various \( B \) modes to the NP search, we remark that the presence of the tensor operator in (1) can be established independently of the other operators, from deviations of the observables that depend only on \( \epsilon_T \), for example, those involving transversely polarized \( \rho \) and \( a_1 \). \( |\epsilon_T| \) is constrained looking at the zero of the ratios in Equations (7) and (8). A correlation between the position of the zero in the \( \rho \) and \( a_1 \) channels should be observed.

If the pseudoscalar operator is present, without other NP structures, deviations should be observed in leptonic \( B \) decays and in the semileptonic decay to \( \rho \), not in \( \pi \) and \( a_1 \) modes. Determining the position of the zero in \( I_{6\pi}^\rho \) would constrain \( \text{Re}[\epsilon_P] \). Zeros should not be present in \( I_{6\pi}^{a_1} \). On the other hand, if the scalar operator is present without additional NP structures, deviations should be observed in semileptonic \( B \) decays to \( \pi \) and \( a_1 \). A zero would be present in \( I_{6\pi}^{a_1} \), not in \( I_{6\pi}^\rho \). All operators would manifest in correlated patterns of deviations.

Precise measurements of modes with final \( \tau \) provide new important tests of LFU. \( R_\mu \) and \( R_\tau \) give information on the relative sign of \( \text{Re}[\epsilon_T^{\ell\ell}] \) and \( \text{Re}[\epsilon_T^{\ell\tau}] \). The \( a_1(1260) \) channel plays a significant role in the searches.

7. Outlook

The anomalies observed in \( b \to c \) semileptonic modes also require new precision analyses of the CKM suppressed \( b \to u \) processes. We have discussed the impact of an extended effective \( b \to u \) Hamiltonian on purely leptonic \( B \) decays, on \( \bar{B} \to \pi^\ell^-\bar{v}_\ell \) and, in details, on \( \bar{B} \to \rho(\pi\pi)\ell^-\bar{v}_\ell \) and \( \bar{B} \to a_1(\rho\pi)\ell^-\bar{v}_\ell \), which involve more than one hadron in the final state. For these two modes we have studied the sensitivity of the 4-dimensional angular distribution to different NP operators. The quantum numbers of the light mesons select the contributions of the NP terms, and complementary information can be gained on the new operators in the effective Hamiltonian. Some observables have zeroes absent in SM. Comparing the modes into \( \mu \) and \( \tau \), LF universality in \( b \to u \) weak transitions is probed. The error connected to the hadronic matrix elements represents a sizable uncertainty, in particular for the \( a_1 \) mode. The Large Energy Limit, in which the number of hadronic form factors is reduced, is therefore useful for the analyses.

There are exciting perspectives for NP searches in heavy hadron decays in the coming years, in particular investigating \( b \to u \) induced semileptonic \( B \) modes.
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Appendix A. Parametrization of the $B \to \pi$ form Factors

The $B \to \pi$ form factors, in the decomposition of the weak matrix element as given in Reference [44], can be expressed in the form

$$
 f_{+T}(t) = \frac{1}{1 - \frac{t}{m_B^2}} \sum_{n=0}^{N-1} a_n \left[ z(t)^n - \frac{n}{N} (-1)^n - N z(t)^N \right],
$$

$$
 f_0(t) = \sum_{n=0}^{N-1} a_n z(t)^n, \tag{A1}
$$

with

$$
 z(t) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}} \tag{A2}
$$

and different parameters $a_n$ for each form factor. In this expression $t_+ = (m_B + m_\pi)^2$, and $t_0$ is chosen at $t_0 = (m_B + m_\pi) (\sqrt{m_B^2 - m_\pi^2})^2$. The values for $a_n$ in Table A1 for $f_{+0,T}$, satisfying the condition $f_{+}(0) = f_{0}(0)$, are obtained using Light-Cone QCD sum rule results in the range $m_\pi^2 \leq t \leq 12$ GeV$^2$ [61,62] and lattice QCD results for 16 GeV$^2 \leq t \leq (m_B - m_\pi)^2$ [63]. Other quantities used in the above analysis are the quark masses $m_u = 2.16_{-0.26}^{+0.49}$ MeV (in the $\overline{MS}$ scheme at $\mu = 2$ GeV) and $m_b (m_b) = 4.18_{-0.02}^{+0.04}$ GeV [53]. The $B^- \to \bar{\tau} \nu_\tau$ decay constant is set to $f_B = 188 \pm 7$ MeV [63].

| $f_{+0,T}$ | $f_0$ | $f_B$ |
|------------|------|------|
| $a_0$      | 0.416 (20) | 0.492 (20) | 0.400 (21) |
| $a_1$      | -0.430 | -1.35 | -0.50 |
| $a_2$      | 0.114 | 2.50 | 0.00076 |
| $a_3$      | 0.534 | | |

Table A1. Parameters of $B \to \pi$ form factors in Equation (A1).

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