Finite temperature effects on monopole and dipole excitations

Y.F. Niu$^{1,2}$, N. Paar$^2$, D. Vretenar$^2$, J. Meng$^{3,1,4}$

$^1$State Key Laboratory of Nuclear Physics and Technology, School of Physics, Peking University, Beijing 100871, China
$^2$Physics Department, Faculty of Science, University of Zagreb, Croatia
$^3$School of Physics and Nuclear Energy Engineering, Beihang University, Beijing 100191, China
$^4$Department of Physics, University of Stellenbosch, Stellenbosch, South Africa

E-mail: nyfster@gmail.com

Abstract. The relativistic random phase approximation based on effective Lagrangian with density dependent meson-nucleon couplings has been extended to finite temperature and employed in studies of multipole excitations within the temperature range $T = 1 - 2$ MeV. The model calculations showed that isoscalar giant monopole and isovector giant dipole resonances are only slightly modified with temperature, but additional transition strength appears at low energies because of thermal unblocking of single-particle orbitals close to the Fermi level. The analysis of low-lying states shows that isoscalar monopole response in $^{132}$Sn results from single particle transitions, while the isovector dipole strength for $^{60}$Ni, located around 10 MeV, is composed of several single particle transitions, accumulating a small degree of collectivity.

1. Introduction

Microscopic modeling of stellar core-collapse, supernovae explosions and r-process nucleosynthesis necessitates description of excitations in nuclei at finite temperature [1]. For example, the low-lying dipole excitations influence the neutron capture rate in r-process nucleosynthesis, especially if located in the vicinity of the neutron separation energy [2]. Stellar electron capture rates crucially depend on the nature of charge-exchange excitations at finite temperature [3]. Therefore, it is of great interest to explore in detail the temperature effects on various excitations in nuclei.

Over the past years, several theoretical studies of multipole excitations in hot nuclei have been conducted, e.g., random phase approximation (RPA) based on schematic interactions [4], linear response theory [5], and RPA based on Skyrme functional [6]. Finite temperature quasiparticle RPA [7, 8] has been formulated using the single-particle basis defined in finite-temperature Hartree-Fock Bogoliubov (HFB) theory [9]. Extensive reviews of theoretical, as well as experimental studies of the multipole response in hot nuclei, including thermal damping of giant resonances, are given in Refs. [10, 11].

Recently the relativistic random phase approximation (RRPA) based on effective Lagrangian with density dependent meson-nucleon couplings has been extended to include finite temperature effects [12]. In this work, by employing the finite temperature RRPA, we study in detail how monopole and dipole excitations evolve with temperature. Of particular interest is to explore possible occurrence of excitation modes that are not present in nuclei at zero temperature [12].
2. Finite temperature relativistic random phase approximation

The fully self-consistent finite temperature relativistic random phase approximation (FTRRPA) is formulated in the single-nucleon basis of the relativistic mean-field (RMF) model at finite temperature (FTRMF). Effective interactions are implemented in a self-consistent way, i.e., both the FTRMF equations and the matrix equations of FTRRPA are based on the same relativistic energy density functional with medium-dependent meson-nucleon couplings [13].

The FTRRPA represents the small amplitude limit of the time-dependent relativistic mean-field theory at finite temperature. Starting from the response of the time-dependent density energy density functional with medium-dependent meson-nucleon couplings [13].

The FTRRPA matrices $A$ and $B$ are composed of matrix elements of the residual interaction $V$, derived from the effective Lagrangian with DD-ME2 parameter set [15], as well as certain combinations of thermal occupation factors $n_k$. The diagonal matrix elements contain differences of single particle energies $\epsilon_m - \epsilon_i$, where $\epsilon_i < \epsilon_m$. Because of finite temperature, the configuration space includes particle-hole (ph), particle-particle (pp), and hole-hole (hh) pairs. In addition to configurations of positive energy, the FTRRPA configuration space must also contain pair-configurations formed from the fully or partially occupied positive-energy states and the empty negative-energy states from the Dirac sea. The inclusion of configurations built from occupied positive-energy states and empty negative-energy states is essential for current conservation, decoupling of spurious states, and for a quantitative description of excitation energies of giant resonances [14]. The full set of FTRRPA equations is solved by diagonalization, yielding the occupation factors of single-particle states $n_k$. The resulting FTRRPA equations read

$$i\hbar\dot{\rho} = [\hat{H}[\rho] + \hat{f}(t), \rho],$$

In the small amplitude limit the density matrix is expanded to linear order

$$\rho(t) = \rho^0 + \delta \rho(t),$$

where $\rho^0$ denotes the stationary ground-state density.

$$\rho_{kl}^0 = \delta_{kl} n_k = \left\{ \begin{array}{ll} [1 + \exp(\frac{\epsilon_k - \mu}{kT})]^{-1} & \text{for states in the Fermi sea (index } k, l) \\ 0 & \text{for unoccupied states in the Dirac sea (index } \alpha) \end{array} \right.,$$

which includes the thermal occupation factors of single-particle states $n_k$. The resulting FTRRPA equations read

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar \omega \begin{pmatrix} X \\ Y \end{pmatrix},$$

where

$$A = \left( \begin{array}{cc} (\epsilon_m - \epsilon_i) \delta_{ii}' \delta_{mm'} & (\epsilon_m - \epsilon_i) \delta_{aa'} \delta_{ii}' \\ (\epsilon_a - \epsilon_i) \delta_{aa'} \delta_{ii}' & (\epsilon_a - \epsilon_i) \delta_{aa'} \delta_{ii}' \end{array} \right) + \left( \begin{array}{cc} (n_{i'} - n_{m'}) V_{mi'jm'} & n_{i'} V_{mi'ia'} \\ (n_{i'} - n_{m'}) V_{ai'jm'} & n_{i'} V_{ai'ia'} \end{array} \right),$$

and

$$B = \left( \begin{array}{cc} (n_{i'} - n_{m'}) V_{mm'ii'} & n_{i'} V_{mm'ii'} \\ (n_{i'} - n_{m'}) V_{am'ii'} & n_{i'} V_{aa'ii'} \end{array} \right).$$

The FTRRPA matrices $A$ and $B$ are composed of matrix elements of the residual interaction $V$, derived from the effective Lagrangian with DD-ME2 parameter set [15], as well as certain combinations of thermal occupation factors $n_k$. The diagonal matrix elements contain differences of single particle energies $\epsilon_m - \epsilon_i$, where $\epsilon_i < \epsilon_m$. Because of finite temperature, the configuration space includes particle-hole (ph), particle-particle (pp), and hole-hole (hh) pairs. In addition to configurations of positive energy, the FTRRPA configuration space must also contain pair-configurations formed from the fully or partially occupied positive-energy states and the empty negative-energy states from the Dirac sea. The inclusion of configurations built from occupied positive-energy states and empty negative-energy states is essential for current conservation, decoupling of spurious states, and for a quantitative description of excitation energies of giant resonances [14]. The full set of FTRRPA equations is solved by diagonalization, yielding the occupation factors of single-particle states $n_k$. The resulting FTRRPA equations read

$$i\hbar\dot{\rho} = [\hat{H}[\rho] + \hat{f}(t), \rho],$$

In the small amplitude limit the density matrix is expanded to linear order

$$\rho(t) = \rho^0 + \delta \rho(t),$$

where $\rho^0$ denotes the stationary ground-state density.

$$\rho_{kl}^0 = \delta_{kl} n_k = \left\{ \begin{array}{ll} [1 + \exp(\frac{\epsilon_k - \mu}{kT})]^{-1} & \text{for states in the Fermi sea (index } k, l) \\ 0 & \text{for unoccupied states in the Dirac sea (index } \alpha) \end{array} \right.,$$

which includes the thermal occupation factors of single-particle states $n_k$. The resulting FTRRPA equations read

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \hbar \omega \begin{pmatrix} X \\ Y \end{pmatrix},$$

where

$$A = \left( \begin{array}{cc} (\epsilon_m - \epsilon_i) \delta_{ii}' \delta_{mm'} & (\epsilon_m - \epsilon_i) \delta_{aa'} \delta_{ii}' \\ (\epsilon_a - \epsilon_i) \delta_{aa'} \delta_{ii}' & (\epsilon_a - \epsilon_i) \delta_{aa'} \delta_{ii}' \end{array} \right) + \left( \begin{array}{cc} (n_{i'} - n_{m'}) V_{mi'jm'} & n_{i'} V_{mi'ia'} \\ (n_{i'} - n_{m'}) V_{ai'jm'} & n_{i'} V_{ai'ia'} \end{array} \right),$$

and

$$B = \left( \begin{array}{cc} (n_{i'} - n_{m'}) V_{mm'ii'} & n_{i'} V_{mm'ii'} \\ (n_{i'} - n_{m'}) V_{am'ii'} & n_{i'} V_{aa'ii'} \end{array} \right).$$

Discrete spectra are averaged with a Lorentzian distribution of arbitrary width ($\Gamma = 1$ MeV in the present calculation).
3. Results

3.1. Evolution of isoscalar monopole excitations with temperature

In Fig. 1, we display the FTRRPA transition strength distributions obtained with isoscalar monopole operator for $^{132}$Sn at temperatures $T = 0, 1, 2$ MeV. At zero temperature, the strength distribution is clearly dominated by the pronounced collective isoscalar giant monopole resonance (ISGMR) with the centroid energy at $E = 14.5$ MeV. With temperature increased up to $T=2$ MeV, the main giant resonance peaks are only slightly shifted to lower energies. However, it is interesting to notice that at temperatures $T = 1 − 2$ MeV, new transition strength develops in the energy region below 10 MeV, emphasized by the yellow area at $T = 2$ MeV in Fig. 1. The low-energy strength at $T = 2$ MeV is mainly composed of two excited states at $E = 5.45$ MeV and $E = 7.02$ MeV. The formation of these two states can be explained from the analysis of involved FTRRPA configurations and FTRMF single-particle spectra. In Fig. 2, the neutron single particle spectrum at $T = 2$ MeV is shown for $^{132}$Sn. The occupation probability of each single particle level is shown by the blue curve. It has the form of Fermi-Dirac distribution, which leads to partial occupations of the single particle levels above the Fermi surface. Therefore, the single particle transitions $3p_{3/2} \rightarrow 4p_{3/2}$ and $2f_{7/2} \rightarrow 3f_{7/2}$ become possible at $T = 2$ MeV (denoted by red arrows in Fig. 2). The FTRRPA analysis shows that these two unblocked transitions are responsible for the appearance of excited states at $E = 5.45$ and $7.02$ MeV, respectively.

3.2. Evolution of isovector dipole excitations with temperature

In Fig. 3, the evolution of isovector dipole transition strength with temperature is illustrated in the example of $^{60}$Ni. In the case of zero temperature, the strength distribution is calculated with the relativistic QRPA [14], where the pairing correlations are described by the pairing part of the finite-range Gogny interaction. At finite temperatures, $T = 1$ and $2$ MeV, the FTRRPA is employed. The isovector giant dipole resonance (IVGDR) at $T=0$ MeV is located at about $E = 18$ MeV. With temperature increased toward 2 MeV, the main IVGDR peak is lowered by about 1 MeV, while its strength becomes enhanced. Similar as in the case of monopole
excitations, we also find new dipole transition strength in the energy region below 10 MeV, now appearing already at lower temperature, $T = 1$ MeV. At $T = 2$ MeV the main low-energy peak is located at 9.71 MeV, and exhausts 1.54% of the Thomas-Reiche-Kuhn (TRK) sum rule. We have explored in detail the underlying structure of this low-lying state.

Figure 3. Isovector dipole transition strength distributions for $^{60}\text{Ni}$, calculated with the relativistic QRPA at $T = 0$ MeV and FTRRPA at $T = 1, 2$ MeV with parameterization DDME2.

Figure 4 shows the proton and neutron single particle spectra for $^{60}\text{Ni}$ at temperature $T = 2$ MeV. Fermi surfaces are denoted by red dotted lines. Four neutron and three proton single-particle transitions give the main contributions to the dipole state at $E = 9.71$ MeV (represented by the arrows in Fig. 4). These transitions appear due to thermal population of single-particle levels around the Fermi surface. The number on each arrow in Fig. 4 shows the contribution of respective configuration to the total sum of FTRRPA amplitude: $\sum m_i (X_{mi}^2 - Y_{mi}^2)(n_i - n_m)$, illustrating the relative importance of the corresponding configuration. The figure includes only configurations with contribution $>2\%$. Rather rich RPA structure shows that low-energy state at $E = 9.71$ MeV accumulates a small degree of collectivity, i.e. it has different nature than the monopole one discussed in Sec. 3.1.

4. Conclusions
We have employed fully self-consistent FTRMF+FTRRPA based on effective Lagrangians with density dependent meson-nucleon couplings in studies of finite temperature effects on nuclear multipole excitations. Illustrative calculations of isoscalar monopole and isovector dipole response in nuclei reveal pronounced effects in the low-energy transition strength at temperatures $T = 1 - 2$ MeV due to thermal unblocking of single particle levels around the Fermi surface. The relevance of these results in modeling astrophysical neutron capture rates contributing in r-process nucleosynthesis should be further explored. The ongoing research based on FTRMF+FTRRPA framework includes description of charge-exchange excitations at finite temperature and electron capture rates for nuclei of relevance in modeling supernova evolution.

Acknowledgements
This work was supported by NSF of China under Grant Nos. 10775004, 10947013, and 10975008, the Major State 973 Program 2007CB815000 of China, the Unity through Knowledge Fund (UKF
Figure 4. Proton and neutron single particle spectra for $^{60}$Ni at temperature $T = 2$ MeV. The red dotted lines denote the Fermi surfaces. The arrows represent transitions with dominant contributions to the dipole state at $E = 9.71$ MeV, and the number on each arrow denotes the contribution of respective configuration to the total sum of FTRRPA amplitude.

Grant No. 17/08) and Ministry of Science, Education and Sports of the Republic of Croatia (project No. 1191005-1010).

References
[1] Janka H-Th, Langanke K, Marek A, Martínez-Pinedo G and Müller B 2007 Phys. Rep. 442 38
[2] Goriely S 1998 Phys. Lett. B 436 10
[3] Paar N, Colo G, Khan E, and Vretenar D 2009 Phys. Rev. C 80 055801
[4] Civitarese O, Broglia R and Dasso C 1984 Ann. Phys. 156 142
[5] Faber M, Egidio J and Ring P 1983 Phys. Lett. B 127 5
[6] Sagawa H and Bertsch G 1984 Phys. Lett. B 146 138
[7] Sommermann H 1983 Ann. Phys. 151 163
[8] Khan E, Van Giai N and Grasso M 2004 Nucl. Phys. A 731 311
[9] Khan E, Van Giai N and Sandulescu N 2007 Nucl. Phys. A 789 94
[10] Bortignon P, Bracco A and Broglia R 1998 Giant Resonances: Nuclear Structure at Finite Temperature (Amsterdam: Harwood Academic)
[11] Santonocito D and Blumenfeld Y 2006 Eur. Phys. J. A 30 183
[12] Niu Y, Paar N, Vretenar D and Meng J 2009 Phys. Lett. B 681 315
[13] Nikšić T, Vretenar D, Finelli P and Ring P 2002 Phys. Rev. C 66 024306
[14] Paar N, Ring P, Nikšić T and Vretenar D 2003 Phys. Rev. C 67 034312
[15] Lalazissis G, Nikšić T, Vretenar D and Ring P 2005 Phys. Rev. C 71 024312