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Quantum driving of a two level system: quantum speed limit and superadiabatic protocols – an experimental investigation

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Abstract. A fundamental requirement in quantum information processing and in many other areas of science is the capability of precisely controlling a quantum system by preparing a quantum state with the highest fidelity and/or in the fastest possible way. Here we present an experimental investigation of a two level system, characterized by a time-dependent Landau-Zener Hamiltonian, aiming to test general and optimal high-fidelity control protocols. The experiment is based on a Bose-Einstein condensate (BEC) loaded into an optical lattice, then accelerated, which provides a high degree of control over the experimental parameters. We implement generalized Landau-Zener sweeps, comparing them with the well-known linear Landau-Zener sweep. We drive the system from an initial state to a final state with fidelity close to unity in the shortest possible time (quantum brachistochrone), thus reaching the ultimate speed limit imposed by quantum mechanics. On the opposite extreme of the quantum control spectrum, the aim is not to minimize the total transition time but to maximize the adiabaticity during the time-evolution, the system being constrained to the adiabatic ground state at any time. We implement such transitionless superadiabatic protocols by an appropriate transformation of the Hamiltonian parameters. This transformation is general and independent of the physical system.

1. Introduction
Controlling the dynamics of quantum systems is a key issue in many fields of science [1] (for instance quantum computation [2], metrology [3], atomic and molecular physics [4]) and could be a valuable instrument for testing fundamental issues in quantum mechanics.

A perfect tool for studying quantum control is the “simplest not-simple quantum system” [5], i.e. the evolution of a two level system governed by a time dependent Landau-Zener Hamiltonian [6, 7, 8]

\[ \mathcal{H} = \Gamma(t)\sigma_z + \omega(t)\sigma_x, \]  

written in the basis of the two diabatic states \(|0\rangle, |1\rangle\) with the Pauli matrices \(\sigma_x, \sigma_z\) acting on the two-dimensional basis. Here \(\Gamma(t)\) and \(\omega(t)\) are respectively the energy and the coupling term.
between these states. It is possible to identify an adiabatic basis \(|\psi_{g,e}(t)\rangle\) which instantaneously diagonalizes the Hamiltonian at any time \(t\).

The present paper reports the basis of the experimental realization of this quantum system together with its dynamics and control protocols in specific regimes, for a detailed analysis see \([9, 10]\).

The central idea is to drive an initial state \(|\psi_{in}\rangle\) for a sweep time \(T\) through an avoided crossing shown in Fig. 1, to a chosen final state \(|\Psi_{fin}\rangle\) as close as possible to the adiabatic ground state \(|\psi_g(T)\rangle\), realizing a fidelity \(F = |\langle \Psi_{fin} | \psi_g(T) \rangle|^2\) close to unity. An infinite number of protocols, i.e., paths in the Hilbert space, connects the two states. We are interested in two specific ones. The first protocol minimizes the transition time \(T\) (keeping the coupling term \(\omega\) constant) between the two states, reaching the quantum speed limit imposed by quantum mechanics and related to the Heisenberg uncertainty principle on energy and time. On the other hand, by lifting the constraint on the coupling term of the first protocol we realise protocols that allow a perfect following of the adiabatic ground state, \(|\psi(t)\rangle = |\psi_g(t)\rangle\) for any \(0 \leq t \leq T\). In the following, we will call such protocols ‘superadiabatic’, inspired by Berry’s work on the superadiabatic basis \([11]\). For the case of superadiabatic protocols, also called counter-adiabatic or transitionless, the construction of an auxiliary Hamiltonian that cancels the non adiabatic terms has been widely studied theoretically \([11, 12, 13, 14]\).

As a paradigmatic and benchmark protocol for other protocols, we choose the well-known Landau-Zener problem, where the energy term \(\Gamma(t)\) in the Hamiltonian is linear in time. Before addressing the quantum speed limit or superadiabaticity, we considered a generalized version of the Landau Zener problem (general LZ) in order to test the experimental technique and theoretical models. We experimentally investigated protocols where the diagonal term is governed by a power law in time \([10]\), which have been theoretically studied in the past \([15, 16]\).

Figure 1. Band structure of an effective two level system. The states \(|0\rangle\) and \(|1\rangle\) are the diabatic basis (dashed line), \(|\psi_{g,e}\rangle\) are the adiabatic basis states (solid line) and \(\omega = V_0/4\) is the energy gap between the bands where \(V_0\) is the lattice depth. \(|\Psi_{ini}\rangle\) and \(|\Psi_{fin}\rangle\) represent the initial state and the final state of a protocol, respectively.
Both the quantum speed limit problem and the superadiabatic problem are connected to the optimal control of a quantum system which has a long history in different fields [17] such as nuclear magnetic resonance (NMR) [18], adiabatic quantum computation [19] and reverse engineering using the Lewis-Riesenfeld invariants [20, 21]. Experimentally a number of quantum protocols have been tested in NMR [22] for quantum computation algorithms. For atomic systems an experiment on transforming a non-adiabatic transition into an adiabatic one has been performed by [23], while in the field of Bose-Einstein Condensation in optical lattices the nearly perfect preparation of a quantum state by non-adiabatic transformation was achieved [24].

The case of the quantum speed limit has been extensively studied theoretically [25, 26, 27, 28, 29] but has not been explicitly experimentally tested before our work.

2. Experimental realization and results

In our experiment a two-level system is realized using a Bose-Einstein condensate of rubidium atoms loaded into an optical lattice [30, 31]. The optical lattice is created by two-counterpropagating linearly polarized laser beams controlled by two independent acousto-optic modulators. Bose-Einstein condensates of around $5 \times 10^4$ rubidium-87 atoms are loaded into a periodic potential by ramping up the power of the lattice beams in 100 ms. The lattice constant is given by $d_L = \lambda/2 = 421\text{nm}$ with $\lambda = 842\text{nm}$ the laser wavelength.

By the Bloch theorem a periodic potential generates an energy level structure which can be described by energy bands. Under specific conditions on the optical lattice potential depth, the energy structure can be restricted to the ground state band and the first excited band.

Using acousto-optic modulators we introduced time-dependent frequency differences between the two counterpropagating beams, leading to a moving interference pattern and hence to a moving periodic potential which, in the rest-frame of the lattice, can be interpreted as a time dependent force on the atoms $F_{LZ} = M a_{LZ}$ with $a_{LZ} = d_L \frac{d\omega(t)}{dt}$. Close to the edge of the Brillouin zone, where an energy gap between the bands is present, the system is well approximated by the Landau-Zener Hamiltonian, and the time dependent force can be used to explore the band structure. Varying the laser power changes the lattice depth $V_0$ and the time dependent force changes the quasi-momentum $q$. The Hamiltonian elements $\Gamma(t)$ and $\omega(t)$ are related to the lattice parameters as

$$\Gamma(t) = 2\Gamma_0 \left( \frac{q(t)}{\hbar k} - \frac{1}{2} \right)$$

$$\omega = \frac{V_0}{4}$$

In the following all the physical quantities are written in the natural energy ($E_{\text{rec}} = \hbar \omega_{\text{rec}} = \pi^2 \hbar^2 / 2Md_L^2$) and time units $\tau_{\text{rec}} = 1/\omega_{\text{rec}}$ of the system.

After loading the BEC into the lattice, we manipulated the system by applying time dependent sweeps on the control parameters and we performed a projection measurement on the diabatic or adiabatic basis. Measurements in the free-particle diabatic basis were performed by instantaneously switching off the lattice beams within less than $1\mu$s. The number of atoms in the $q = 0$ and $q = 2p_{\text{rec}}$ ($p_{\text{rec}} = \hbar \pi / d_L$) momentum classes were measured after a time-of-flight by absorption imaging and the transition probability $P_{\text{diab}}(\tau)$ was given by the ratio between the number of atoms in the $q = 2p_{\text{rec}}$ momentum class and the total number of atoms. In order to project on the adiabatic basis the lattice depth was adiabatically decreased to zero by reducing the laser power within $400\mu$s. The ratio between the fraction of atoms in $q = 0$, (the atoms in the lower band) and the total number of atoms gave an experimental measurement in the adiabatic basis and hence of the protocol fidelity. The noise in our imaging system does not
Figure 2. Fidelity as a function of the duration $T$ of the power law sweep for different $\alpha$: red diamonds $\alpha = 4$, blue circles $\alpha = 2$ and green squares for the linear LZ with $\alpha = 1$. The experimental results are compared with numerical simulations, given by the dotted lines (red, blue and green). The dotted black line is the threshold fidelity determining the transition time $T_{thr}$.

allow us to reliably measure fidelities $F$ larger than 0.98. In addition, the non-adiabaticity of the preparation and measurement protocols contributes with an infidelity on the order of few percent.

2.1. Generalized Landau-Zener: power law

As a first step of our investigation we considered a generalized LZ sweep with the following form:

$$
\Gamma(\tau) = \begin{cases} 
-2^{\alpha+1}(1/2-\tau)^{\alpha} & \text{for } 0 \leq \tau \leq 1/2 \\
2^{\alpha+1}(\tau-1/2)^{\alpha} & \text{for } 1/2 \leq \tau \leq 1, 
\end{cases}
$$

where $\tau = t/T$ is the rescaled time and $T$ the sweep time. The boundary conditions $\Gamma(0) = -2$ and $\Gamma(1) = 2$ are satisfied, $\alpha$ is the power law parameter and $\omega(\tau) = \omega$ is constant during the sweep. For $\alpha = 1$ the sweep is linear, corresponding to the standard Landau-Zener sweep.

In Fig. 2, together with a numerical simulation, we show the measured fidelity as function of total time $T$ for different power law parameters $\alpha = 1$, $\alpha = 2$ and $\alpha = 4$. The numerical simulation and the experimental data are in good agreement. Since for Linear Landau Zener the time to reach fidelity $F = 1$ diverges, in order to compare the speed of different sweeps, we fix a threshold fidelity at 0.9 and from experimental results as those in Fig. 2 we derive the time $T_{thr}$ at which that fidelity is reached (see Fig. 3). The dependency of $T_{thr}$ on the parameter $\alpha$ is well demonstrated. For increasing $\alpha$ the limiting time decreases, approaching the minimum time of the quantum speed limit (the dashed red line, $T_{QSL} = 2.75$ for the experimental parameters).

An intuitive explanation for that behavior is that for large $\alpha$ the power laws sweep approaches the composite pulse protocol which, as shown below, reaches the quantum speed limit.
2.2. Quantum speed limit: composite pulse protocol

As shown experimentally and theoretically in [9], we have found that the minimum time to perform the transition with fidelity 1 is reached by a composite pulse protocol of the form:

$$\Gamma(t) = \begin{cases} 
-\Gamma_0 & \text{for } t = 0 \\
\Gamma_M & \text{for } t \in [0, t_0] \\
0 & \text{for } t \in [t_0, T - t_0] \\
-\Gamma_M & \text{for } t \in [T - t_0, 1] \\
+\Gamma_0 & \text{for } t = T,
\end{cases}$$

where $\omega$ remains constant during the sweep; $\Gamma_M$ and $t_0$ satisfy the relation $\Gamma_M t_0 = \pi/4$ while $\Gamma_M$ is asymptotically large and $t_0$ small. This protocol is analogous to the NMR composite protocol consisting of half a Rabi Oscillation with frequency $\omega$ and $\Gamma = 0$ which is preceded and followed by two short pulses of area $\pi/4$.

In Fig. 4 we show experimental results and numerical calculations of the fidelity as a function of the total sweep time $T$ for the composite pulse protocol (black triangles) and for a linear Landau-Zener protocol (red squares). For the composite pulse protocol the minimum time $T_{thr}$ for reaching optimal fidelity is extracted from the position of the maximum of the fidelity $F$ (Fig. 4), typically $0.95 \pm 0.03$. For the linear Landau Zener protocol we measure the threshold time $T_{thr}$ for reaching 0.9 fidelity since the needed time for reaching $F = 1$ diverges. In figure 5 we show the minimum time for the composite pulse (black triangles) and the threshold time for the linear Landau Zener protocol (red squares) as a function of the coupling parameter $\omega$.

While the threshold time for the linear Landau-Zener sweep is always larger than the optimal minimum time for the composite pulse protocol (by an order of magnitude for small $\omega$), the minimum time for the composite pulse protocol approaches the quantum speed limit (Fleming’s or Bhattacharyya’s bound [26]) $T_{qsl}$ given by:

$$T_{qsl} = \frac{\arccos |\langle \Psi_{\text{fin}} | \Psi_{\text{ini}} \rangle|}{\omega}. \quad (7)$$
2.3. Superadiabatic protocol

We now turn to the task of engineering a protocol that drives the system through the anticrossing point in such a way that it remains in the adiabatic ground state at any time $t$. The existence of transitionless or superadiabatic protocols follows directly from the properties of quantum mechanics. For any given time dependent Hamiltonian $H(t)$ it is always possible to write a new
Figure 6. Fidelity as a function of the normalized time $\tau$, for a superadiabatic linear sweep (black squared) and a linear Landau-Zener sweep (red circles). The horizontal line at $\mathcal{F} = 0.98$ denotes a fit through the superadiabatic protocol data.

Hamiltonian

$$H'(t) = H(t) + H_{\text{aux}}(t)$$

by constructing an (ad-hoc) additional control $H_{\text{aux}}$ that cancels the non-adiabatic terms of $H$

$$H_{\text{aux}} = i\hbar \sum_n (|\partial_n(t)\rangle\langle n(t)| - |n(t)\rangle\langle \partial_n(t)|)/2$$

where $|n(t)\rangle$ are the eigenstates of the original Hamiltonian $H$ [11]. In the case of the two level system Hamiltonian of Eq. 1, $H_{\text{aux}}$ can be cast into the form:

$$H_{\text{aux}}(t) = \frac{\hbar}{2} \frac{\partial \psi}{\partial t} \sigma_y$$

where $\psi = \arctan(\omega(t)/\Gamma(t))$.

As we have shown in [9], for any given time dependent Hamiltonian $H$ satisfying the above prescription, we can find a transformation $\{\Gamma(t), \omega(t)\} \rightarrow \{\Gamma'(t), \omega'(t)\}$ so that the new Hamiltonian $H'(\Gamma'(t), \omega'(t))$ realizes a superadiabatic protocol. Applying that transformation to the linear Landau-Zener protocol $\Gamma(t)$, we obtain:

$$\Gamma'(\tau) = \Gamma(\tau) - \frac{4(\tau - \frac{1}{2})}{T^2[(\tau - \frac{1}{2})^2 + \frac{1}{2}\omega^2]^2 + 1},$$

$$\omega'(\tau) = \omega \sqrt{1 + \frac{1}{T^2(8(\tau - \frac{1}{2})^2 + \frac{1}{2}\omega^2)^2}}.$$  

(11)

with $\tau = t/T$ and where the discontinuities in the derivative of $\Gamma(t)$ at the beginning and at the end of the sweep are realised experimentally by introducing a large finite correction $\Delta \Gamma_M$ for a
short time $\Delta \tau$, satisfying:

$$\Delta \tau \Delta \Gamma_M = \mp \frac{1}{2} \arctan \left( \frac{\Gamma \omega - \omega \dot{\Gamma}}{2 \omega (\Gamma^2 + \omega^2)} \right)$$

(12)

where the sign $\mp$ is for the correction at the beginning or at the end of the protocol. From Eq. 11 it follows that the coupling term $\omega'$ is now time dependent. This time dependence is experimentally achieved by varying the laser power.

Fig. 6 reports a time resolved measurement of the fidelity $F(\tau)$ during the sweep for the two protocols. For the superadiabatic linear protocol the fidelity is overall constant with $F = 0.98 \pm 0.02$ indicating that the system remains in the ground adiabatic state during the entire protocol. In the case of the usual linear Landau-Zener protocol, at the band gap (around $\tau = 0.5$) we observe tunneling between the bands.

We have numerically tested the sensitivity of the superadiabatic tangent protocol to the variations in the control parameters by varying both $\omega$ and $T$ around the optimum value and measuring the fidelity $F$ in each case [9]. We clearly showed that the superadiabatic tangent protocol is extremely robust with respect to an increase in $T$ or $\omega$, while for large (more than twenty percent) reductions in $T$ or $\omega$ the fidelity naturally drops sharply as otherwise the quantum speed limit would be violated.

3. Conclusions
We have reported the development of experimental techniques, based on Bose-Einstein condensates in optical lattices, for the investigation of high fidelity quantum control protocols. For the case of a two level system, we have shown the possibility of implementing protocols for reaching the quantum speed limit on the one hand or superadiabatic following on the other hand. For the latter case the generality of the transformation on the Hamiltonian suggests the possibility of applying this technique to other fields of research.

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