Finite field-dependent symmetries in perturbative quantum gravity

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In this paper we discuss the absolutely anticommuting nilpotent symmetries for perturbative quantum gravity in general curved spacetime in linear and non-linear gauges. Further, we analyze the finite field-dependent BRST (FFBRST) transformation for perturbative quantum gravity in general curved spacetime. The FFBRST transformation changes the gauge-fixing and ghost parts of the perturbative quantum gravity within functional integration. However, the operation of such symmetry transformation on the generating functional of perturbative quantum gravity does not affect the theory on physical ground. The FFBRST transformation with appropriate choices of finite BRST parameter connects non-linear Curci–Ferrari and Landau gauges of perturbative quantum gravity. The validity of the results is also established at quantum level using Batalin-Vilkovisky (BV) formulation.

I. INTRODUCTION

The study of the structure of spacetime at Planck scale, where the quantum effects of gravity cannot be neglected, is a great challenge for fundamental physics. It is very essential to understand the perturbative quantum gravity for those who want to proceed towards any kind of non-perturbative approach [1]. The perturbative quantum gravity in curved spacetime as a gauge theory is a subject of interest form many respects [2–4]. The mode analysis and Ward identities for a ghost propagator for perturbative quantum gravity in de Sitter space has been discussed iteratively [5, 6]. The Feynman rules and propagator for gravity in the presence of a flat Robertson–Walker background in the physically interesting cases of inflation have been analyzed [7]. Such models of gravity have founded great attempts to unify gravity with Maxwell theory [8]. The gravity models with gauge invariance have their relevance in string theories also [9–11].

The quantum theory of gravity in general curved spacetime has general coordinate (gauge) invariance and hence cannot be quantized without getting rid of the redundant degrees of freedom. This can be achieved by imposing a suitable gauge conditions. The Landau and non-linear Curci–Ferrari gauge conditions play a pivotal role in the analysis of gauge and ghost condensation of the perturbation theory [12, 13]. These gauge conditions can be incorporated to the theory of gravity at quantum level by adding the suitable gauge-fixing and ghost terms to the classical action which remains invariant under the fermionic rigid BRST invariance [14, 15]. The BRST symmetry plays an important role to study the unitarity and renormalizability of the gauge theories [16, 17]. However, BV formulation to quantize the more general gauge theories with open gauge algebra is more fundamental approach to study the supergravity and topological field theories [16, 22]. The BRST and the anti-BRST symmetries for perturbative quantum gravity in flat spacetime dimensions have been studied by many people [23–25] and their work has been summarized by N. Nakanishi and I. Ojima [26]. The BRST symmetry in two dimensional curved spacetime has been thoroughly studied [27–29]. Recently, the BRST formulation in the theory of perturbative quantum gravity has been analyzed [30, 31]. The BRST symmetry transformations of the gauge theories in flat spacetime have been generalized by making the parameter finite and field-dependent which is known as FFBRST transformations [32]. The FFBRST transformations have found several applications in gauge field theories in flat spacetime [33, 34]. However, so far FFBRST formulation has not been developed for any theory of curved spacetime. This provides a motivation to develop FFBRST transformation in curved spacetime. We develop such a formulation for the first time.
for the theory of quantum gravity in the curved spacetime.

In this paper we discuss the BRST and anti-BRST invariance of gravity theory in Landau and massless Curci–Ferrari gauges. Further, we investigate the FFBRST transformation for perturbative quantum gravity. The FFBRST transformation is constructed by replacing the infinitesimal field-independent BRST parameter with a finite field-dependent global parameter. The formal aspects of such FFBRST formulation are discussed with full generality, in which we show that the FFBRST transformation is symmetry of the action, however, it does not leave the path integral measure of functional integral invariant. The explicit form of the non-trivial Jacobian of the path integral measure is calculated for the theory of quantum gravity. The non-trivial Jacobian changes the gauge-fixing and ghost terms within the functional integral of perturbative quantum gravity. We explicitly show that for a proper choice of field-dependent parameter the FFBRST transformation connects the linear and non-linear gauges within the functional integration of perturbative quantum gravity. The results are also tested at quantum level using BV formulation of perturbative quantum gravity.

This paper is organized as follows. In Sec. II, we discuss the different gauge conditions in perturbative gravity with BRST invariance. In Sec. III, we develop the field-dependent BRST symmetry in the theory of curved spacetime and show that Landau and non-linear gauges can be connected with suitable choices of finite parameter. The result is also established at quantum level in section IV. In the last section, we summarize the results with some discussion on future investigations.

II. THE PERTURBATIVE QUANTUM GRAVITY

We start with the classical Lagrangian density for gravity in general curved spacetime

\[ \mathcal{L}_c = \frac{\sqrt{-g}}{16\pi G} (R - 2\lambda), \]  

where \( R \) is Ricci scalar curvature and \( \lambda \) is a cosmological constant.

In this theory the full metric \( g'_{ab} \) is written in terms of a fixed metric of background spacetime \( g_{ab} \) and small perturbations around it. The small perturbation around the fixed background metric, denoted by \( h_{ab} \), is considered as a field that is to be quantized. So, we can write

\[ g'_{ab} = g_{ab} + h_{ab}. \]  

The Lagrangian density given in Eq. (1) remains invariant under the following general coordinate transformation, which is infinitesimal in nature,

\[ \delta_{\Lambda} h_{ab} = \nabla_a \Lambda_b + \nabla_b \Lambda_a + \mathcal{L}_{(\Delta)} h_{ab}. \]  

The Lie derivative of \( h_{ab} \) with respect to the vector field \( \Lambda_a \) is given by

\[ \mathcal{L}_{(\Lambda)} h_{ab} = \Lambda^c \nabla_c h_{ab} + h_{ac} \nabla_b \Lambda^c + h_{cb} \nabla_a \Lambda^c. \]  

As the theory for perturbative quantum gravity is gauge invariant it contains some redundant degrees of freedom. These redundancy of degrees of freedom give rise to constraints in the canonical quantization and divergences in the generating functional in the path integral quantization. In order to remove the redundancy in degrees of freedom we restrict the gauge by following gauge-fixing condition

\[ G[h]_a = (\nabla^b h_{ab} - k \nabla_a h) = 0, \]  

where \( k \neq 1 \). For \( k = 1 \) the conjugate momentum corresponding to \( h_{00} \) vanishes and therefore the partition function diverges again. For this reason sometimes \( k \) is written in terms of an arbitrary finite constant \( \beta \) as \( 1 + \beta^{-1} \). To ensure the unitarity of the perturbative quantum gravity a Faddeev–Popov ghost term is also needed.
The effects of above gauge condition can be incorporated in the theory by adding suitable gauge-fixing and corresponding ghost terms in the classical Lagrangian density given in Eq. (1). For this theory the Landau gauge-fixing and corresponding Faddeev–Popov ghost terms have the following form:

\[
\mathcal{L}_{gf} = \sqrt{-g}i\delta^a (\nabla^b h_{ab} - k\nabla_a h),
\]

\[
\mathcal{L}_{gh} = i\sqrt{-g}c^a \nabla^b (\nabla_a c_b + 2k g_{ab} \nabla^c c + \mathcal{L}_{(c)} h_{ab} - kg_{ab}g^{ef} \mathcal{L}_{(c)} h_{ef}),
\]

which remain invariant under following BRST variations of fields,

\[
h_{ab} = (\nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}), \quad s c^a = -c_b \nabla^a c, \quad s \bar{c}^a = b^a, \quad s b^a = 0.
\]

This effective action is also invariant under the following anti-BRST transformations where the roles of ghost and anti-ghost fields are interchanged,

\[
h_{ab} = (\nabla_a \bar{c}_b + \nabla_b \bar{c}_a + \mathcal{L}_{(\bar{c})} h_{ab}), \quad \bar{s} c^a = -\bar{c}_b \nabla^a c, \quad \bar{s} \bar{c}^a = -b^a, \quad \bar{s} b^a = 0.
\]

The above BRST and anti-BRST transformations are nilpotent in nature and satisfy absolute anticommutativity, i.e.

\[
s^2 = 0, \quad \bar{s}^2 = 0, \quad \{s, \bar{s}\} = 0.
\]

Now, we express the gauge-fixing and ghost part of the complete Lagrangian density as follows,

\[
\mathcal{L}_g = \mathcal{L}_{gf} + \mathcal{L}_{gh},
\]

\[
= is\sqrt{-g}[\delta^a (\nabla^b h_{ab} - k\nabla_a h)],
\]

\[
= -is\sqrt{-g}[c^a (\nabla^b h_{ab} - k\nabla_a h)],
\]

\[
= 1/2 is\bar{s}\sqrt{-g}(h^{ab} h_{ab}),
\]

\[
= 1/2 is\bar{s}\sqrt{-g}(h^{ab} h_{ab}).
\]

In the BV formalism, the gauge-fixing and ghost part of the Lagrangian density is generally expressed in terms of BRST variation of a gauge-fixed fermion. It is straightforward to write the \(\mathcal{L}_g\) given in Eq. (13) in terms of gauge-fixed fermion \(\Psi\) as

\[
\mathcal{L}_g = s\Psi,
\]

where the expression for \(\Psi\) is

\[
\Psi = i\sqrt{-g}[c^a (\nabla^b h_{ab} - k\nabla_a h)],
\]
In non-linear Curci–Ferrari gauge condition the gauge-fixing and ghost terms can be expressed as

\[ \mathcal{L}_g' = \mathcal{L}_{gf}' + \mathcal{L}_{gh}', \]

\[ = \sqrt{-g} \left[ ib^a(\nabla^b h_{ab} - k \nabla_a h) - i c^b \nabla_b c^a (\nabla^b h_{ab} - k \nabla_a h) + c^a M_{ab} c^b + \frac{1}{2} b^b \nabla_b \bar{c}^a u a \right. \]

\[ - \frac{1}{2} \bar{c}^2 \nabla_c b^a \bar{c}^a c_a - \frac{1}{2} \bar{c}^b \nabla_b b^a c_a - \frac{1}{2} c^a \nabla_b \bar{c}^a c_d \nabla^d c_a - \frac{1}{2} c^a b^a + \alpha c^b b^b \nabla_b c_a \]

\[ + \alpha \bar{c}^a c^b d^d \nabla_b \nabla_d c_a, \quad (16) \]

where \( \alpha \) is a gauge parameter. With these gauge-fixing and Faddeev–Popov ghost terms the effective action of perturbative quantum gravity in non-linear gauge is written as

\[ S_{NL} = \int d^4 x (\mathcal{L}_c + \mathcal{L}_g'). \quad (17) \]

The BRST transformations for perturbative quantum gravity in Curci–Ferrari gauge are given by

\[ s h_{ab} = \nabla_a c_b + \nabla_b c_a + \mathcal{L}_{(c)} h_{ab}, \]

\[ s c^a = - c_b \nabla^b c^a, \]

\[ s \bar{c}^a = b^b - \bar{c}^b \nabla_b c^a, \]

\[ s b^a = - b^b \nabla_b c^a - \bar{c}^b c^d \nabla_b \nabla_d c^a, \quad (18) \]

and the anti-BRST symmetry transformations for this theory are constructed as

\[ \bar{s} h_{ab} = \nabla_a \bar{c}_b + \nabla_b \bar{c}_a + \mathcal{L}_{(\bar{c})} h_{ab}, \]

\[ \bar{s} \bar{c}^a = - c_b \nabla^b \bar{c}^a, \]

\[ \bar{s} c^a = b^b - \bar{c}^b \nabla_b c^a, \]

\[ \bar{s} b^a = - b^b \nabla_b \bar{c}^a + \bar{c}^b c^d \nabla_b \nabla_d \bar{c}^a. \quad (19) \]

These BRST and anti-BRST transformations are also absolutely anticommuting and nilpotent in nature. Now, we are able to write the non-linear gauge-fixing and ghost part of the effective Lagrangian density given in Eq. (16) as

\[ \mathcal{L}_g' = i s \sqrt{-g} \left[ c^a \left( \nabla^b h_{ab} - k \nabla_a h - \frac{i}{2} \nabla_a c^b c_b + \frac{i}{2} b^a + \frac{i}{2} \bar{c}^b \nabla_b c_a \right) \right], \]

\[ = - \frac{i}{2} \bar{s} \sqrt{-g} \left[ h^{ab} h_{ab} - i \alpha \bar{c}^a c_a \right], \]

\[ = \frac{i}{2} \bar{s} \sqrt{-g} \left[ h^{ab} h_{ab} - i \alpha \bar{c}^a c_a \right]. \quad (20) \]

We will study the generalization of such nilpotent symmetries in the next section.

**III. FFBRST FORMULATION FOR PERTURBATIVE QUANTUM GRAVITY**

To develop the FFBRST formulation for theory of quantum gravity in curved spacetime we start with the usual BRST transformation written in terms of infinitesimal and field-independent Grassmann parameter \( \delta \Lambda \) as

\[ \delta_{\phi} \phi(x) = s \phi(x) \delta \Lambda, \quad (21) \]

where \( \phi(x) \) is the generic notation of fields \( (h,c,\bar{c},b) \) involved the theory of quantum gravity. The properties of the such BRST transformation do not depend on whether the parameter \( \delta \Lambda \) is (i) finite or infinitesimal, (ii) field-dependent or not, as long as it is anticommuting and spacetime independent.
These observations give us a liberty to generalize the BRST transformation by making the parameter, \( \delta \Lambda \) finite and field-dependent without affecting its properties. To do so, we start by interpolating a continuous parameter, \( \kappa \) \((0 \leq \kappa \leq 1)\), in the theory to make the infinitesimal parameter field-dependent. We allow the fields, \( \phi(x, \kappa) \), to depend on \( \kappa \) in such a way that \( \phi(x, \kappa = 0) = \phi(x) \) the initial fields and \( \phi(x, \kappa = 1) = \phi'(x) \), the transformed fields.

We consider the intermediate fields \( \phi(x, \kappa) \), \((0 \leq \kappa \leq 1)\) satisfying following infinitesimal field-dependent BRST transformation \[d\phi(x, \kappa) = s[\phi(x)]\Theta'[\phi(\kappa)]d\kappa,\] (22)
where the \( \Theta'[\phi(\kappa)]d\kappa \) is the infinitesimal but field-dependent parameter. The FFBRST transformation with the finite field-dependent parameter then can be constructed by integrating such infinitesimal transformation from \( \kappa = 0 \) to \( \kappa = 1 \), to obtain
\[
\phi' \equiv \phi(x, \kappa = 1) = \phi(x, \kappa = 0) + s[\phi(x)]\Theta[\phi],
\] (23)
where
\[
\Theta[\phi] = \int_0^1 d\kappa' \Theta'[\phi(\kappa')],
\] (24)
is the finite field-dependent parameter. Such transformations with finite field-dependent parameter are the symmetry of the effective action but not of the functional integral \[32\] as the path integral measure is not invariant under such transformations. Thus the Jacobian of path integral measure gives some non-trivial contribution to the generating functional of the theory.

The Jacobian of the path integral measure for such transformations is then evaluated for some particular choices of the finite field-dependent parameter, \( \Theta[\phi(x)] \), as
\[
D\phi = J(\kappa) D\phi(\kappa).
\] (25)
We substitute the Jacobian, \( J(\kappa) \), within the functional integral as
\[
J(\kappa) \rightarrow \exp[iS_1[\phi(x, \kappa), \kappa]],
\] (26)
where \( S_1[\phi(x), \kappa] \) is local functional of fields. This imposes the following condition to the theory to be satisfied \[32\]
\[
\langle \langle \frac{1}{J} \frac{dJ}{d\kappa} - i \frac{dS_1[\phi(x, \kappa), \kappa]}{d\kappa} \rangle \rangle_{\kappa} = 0.
\] (27)
In this method we calculate the infinitesimal change in the \( J(\kappa) \) with the help of following condition
\[
\frac{1}{J} \frac{dJ}{d\kappa} = \int d^4 y \left[ \pm s(\phi(y, \kappa)) \frac{\delta \Theta'[\phi]}{\delta \phi(y, \kappa)} \right],
\] (28)
where sign + is used for bosonic fields \( \phi \) and – sign is used for fermionic fields \( \phi \).

A. From non-linear gauge to Landau gauge

The FFBRST transformation for perturbative quantum gravity in the massless Curci–Ferrari gauge is constructed as
\[
f h_{ab} = (\nabla_a c_b + \nabla_b c_a + L_c h_{ab}) \Theta[\phi],
\]
\[
f e^a = -\epsilon_b \nabla^b c_a \Theta[\phi],
\]
\[
f \bar{e}^a = (b^a - \epsilon^a \nabla b^a) \Theta[\phi],
\]
\[
f b^a = (-b^b \nabla b^a - \epsilon^a \epsilon^c \nabla b \nabla c^a) \Theta[\phi],
\] (29)
where \( \Theta[\phi] \) is finite field-dependent parameter. To connect the Landau and non-linear Curci–Ferrari gauge we construct the finite parameter obtained from following infinitesimal field-dependent parameter

\[
\Theta'[\phi] = -i\frac{\alpha}{2}\sqrt{-g} \int d^4y \left( \bar{c}_a \nabla^b c^a_c - c^a b_a - \bar{c}^a_c \nabla^b c_a \right). \tag{30}
\]

For this expression of \( \Theta' \) and BRST given in Eq. (29) the change in Jacobian, using Eq. (28), is calculated as follows

\[
\frac{1}{J(\kappa)} \frac{dJ(\kappa)}{d\kappa} = \frac{\alpha}{2}\sqrt{-g} \int d^4x \left[ -b_b \nabla^b c^a c_a + \bar{c}^d \nabla_d c_b \nabla^b c^a c_a + \bar{c}_b \nabla^b c^a c_a + \bar{c}_b c^a c_d \nabla^d c_a \\
+ b_b b^a - 2\bar{c}_a b_b \nabla^b c_a - 2\bar{c}^a \bar{c}_a c_d \nabla^d c_a \right]. \tag{31}
\]

The local functional \( S_1 \) appearing in the Eq. (20) is written to have the following explicit form

\[
S_1[\phi(\kappa), \kappa] = \int d^4x \left[ \xi_1 b_b \nabla^b c^a c_a + \xi_2 \bar{c}^d \nabla_d c_b \nabla^b c^a c_a + \xi_3 \bar{c}_b \nabla^b c^a c_a + \xi_4 \bar{c}_b c^a c_d \nabla^d c_a \\
+ \xi_5 b_b b^a + \xi_6 \bar{c}^d b_b \nabla^b c_a + \xi_7 \bar{c}_b b^a c_d \nabla^d c_a \right], \tag{32}
\]

where all fields and \( \xi_i (i = 1, 2, \ldots, 7) \), involved in the above expression, depend on parameter \( \kappa \). Now we have to identify the exact values of \( \xi_i \) in terms of \( \kappa \). For this purpose, we calculate the change in \( S_1 \) with the help of Eq. (22) as

\[
\frac{dS_1[\phi(\kappa), \kappa]}{d\kappa} = \int d^4x \left[ \xi_1 b_b \nabla^b c^a c_a + \xi_2 \bar{c}^d \nabla_d c_b \nabla^b c^a c_a + \xi_3 \bar{c}_b \nabla^b c^a c_a + \xi_4 \bar{c}_b c^a c_d \nabla^d c_a \\
+ \xi_5 b_b b^a + \xi_6 \bar{c}^d b_b \nabla^b c_a + \xi_7 \bar{c}_b b^a c_d \nabla^d c_a \right] \\
\left[ \xi'_1 + \frac{\alpha}{2}\sqrt{-g} = 0, \quad \xi'_2 - \frac{\alpha}{2}\sqrt{-g} = 0, \quad \xi'_3 - \frac{\alpha}{2}\sqrt{-g} = 0, \quad \xi'_4 - \frac{\alpha}{2}\sqrt{-g} = 0, \right. \\
\left. \xi'_5 + \frac{\alpha}{2}\sqrt{-g} = 0, \quad \xi'_6 + \alpha\sqrt{-g} = 0, \quad \xi'_7 + \alpha\sqrt{-g} = 0 \right] \tag{33}
\]

satisfying the relations

\[
\xi_1 + \xi_2 = 0, \quad \xi_1 + \xi_3 = 0, \quad \xi_1 + \xi_4 = 0, \quad \xi_2 - \xi_3 = 0, \quad \xi_2 - \xi_4 = 0, \\
\xi_3 - \xi_4 = 0, \quad 2\xi_5 + \xi_6 = 0, \quad 2\xi_5 + \xi_7 = 0, \quad \xi_6 - \xi_7 = 0. \tag{34}
\]

The solutions of the differential equations given in Eq. (34), fulfilling the boundary conditions \( \xi_i(\kappa = 0) = 0 \), are

\[
\xi_1 = -\frac{\alpha}{2}\sqrt{-g}\kappa, \quad \xi_2 = \frac{\alpha}{2}\sqrt{-g}\kappa, \quad \xi_3 = \frac{\alpha}{2}\sqrt{-g}\kappa, \quad \xi_4 = \frac{\alpha}{2}\sqrt{-g}\kappa, \\
\xi_5 = \frac{\alpha}{2}\sqrt{-g}\kappa, \quad \xi_6 = -\alpha\sqrt{-g}\kappa, \quad \xi_7 = -\alpha\sqrt{-g}\kappa. \tag{36}
\]

These identifications of \( \xi_i(\kappa) \) also satisfy the conditions in Eq. (35). Therefore, the FFBRST transformation (29) with parameter \( \Theta \) obtainable from Eq. (30) changes the effective action within functional integration as

\[
S_{NL} + S_1(\kappa = 1) = \int d^4x \left[ \mathcal{L}_c + i\sqrt{-g} b^a (\nabla^b h_{ab} - k \nabla_a h) - i\sqrt{-g} c^a \nabla_b c^a (\nabla^b h_{ab} - k \nabla_a h) \\
+ \sqrt{-g} e^a M_{ab} e^b \right]. \tag{37}
\]
After shifting the Nakanishi–Lautrup field by \( \bar{c} b \nabla b c^a \), the above expression reduces to

\[
S_{NL} + S_1(\kappa = 1) = \int d^4 x \left[ L_c + i \sqrt{-g} b^a (\nabla^b h_{ab} - k \nabla_a h) + \sqrt{-g} c^a M_{ab} c^b \right],
\]

which is nothing but the effective action for perturbative quantum gravity in Landau gauge.

We end this section by making the following conclusion that the finite field-dependent BRST with appropriate choice of finite parameter connects two different gauges in the theory of quantum gravity in curved spacetime. We show these results also at quantum level using BV formulation in the next section.

IV. BV FORMULATION AND FFBRST SYMMETRY

In the BV formulation the generating functional of quantum gravity (in Landau gauge) in curved spacetime, by introducing antifields \( \phi^* \) corresponding to the all fields \( \phi(\equiv h, \bar{c}, c, b) \) with opposite statistics, is given by

\[
Z_L = \int \mathcal{D}\phi \ e^{i \int d^4 x (L_c + \mathcal{L}_g[\phi, \phi^*])},
\]

This can further be written in compact form as

\[
Z_L = \int \mathcal{D}\phi \ e^{i W^L[\phi, \phi^*]},
\]

where \( W^L[\phi, \phi^*] \) is an extended quantum action in Landau gauge. The generating functional does not depend on the choice of gauge-fixing fermion \([10]\). The extended quantum action for perturbative quantum gravity, \( W^L[\phi, \phi^*] \), satisfies the following mathematically rich relation, called the quantum master equation \([17]\),

\[
\Delta e^{i W^L[\phi, \phi^*]} = 0 \quad \text{with} \quad \Delta = \frac{\partial_r}{\partial\phi} \frac{\partial_r}{\partial\phi^*} (-1)^{r+1}.
\]

The antifields get identified with gauge-fixing fermion \( \Psi \), given in Eq. \([15]\), as follows

\[
\begin{align*}
  h^*_{ab} &= \frac{\delta \Psi}{\delta h_{ab}} = i \sqrt{-g} (-\nabla_b \bar{c}_a + k g_{ab} \nabla^c \bar{c}_c), \\
  \bar{c}^*_a &= \frac{\delta \Psi}{\delta \bar{c}^a} = i \sqrt{-g} (\nabla^b h_{ab} - k \nabla_a h), \\
  c^*_a &= \frac{\delta \Psi}{\delta c^a} = 0, \quad b^*_a = \frac{\delta \Psi}{\delta b^a} = 0.
\end{align*}
\]

Similarly, the generating functional for quantum gravity in a non-linear gauge is defined, compactly, as

\[
Z_{NL} = \int \mathcal{D}\phi \ e^{i \int d^4 x (L_c + \mathcal{L}_g[\phi, \phi^*])},
\]

\[
= \int \mathcal{D}\phi \ e^{i W^L_{NL}[\phi, \phi^*]}.
\]

The following expression for antifields in the case of a non-linear gauge are obtained

\[
\begin{align*}
  h^*_{ab} &= i \sqrt{-g} (-\nabla_b \bar{c}_a + k g_{ab} \nabla^c \bar{c}_c), \\
  \bar{c}^*_a &= i \sqrt{-g} \left( \nabla^b h_{ab} - k \nabla_a h - i \frac{\alpha}{2} \nabla_a \bar{c}_b + i \frac{\alpha}{2} b_a + i \frac{\alpha}{2} \nabla_b \bar{c}_a \right), \\
  b^*_a &= i \frac{\alpha}{2} \sqrt{-g} c_a, \quad c^*_a = i \frac{\alpha}{2} \sqrt{-g} \left[ \nabla^b \bar{c}_a + \nabla_b (\bar{c}_a \bar{c}^b) \right].
\end{align*}
\]
To connect linear and non-linear gauges in BV formulation we construct the following finite field-dependent parameter $\Theta[\phi]$

$$\Theta[\phi] = - \int_0^1 d\kappa \int d^4y [\epsilon^a_c e^a - b^a_b].$$

(45)

The Jacobian of the path integral measure in the generating functional for this FFBRST parameter can be replaced by $e^{iS_1[\phi, \phi^*]}$ iff condition (27) is satisfied. The factor $e^{iS_1[\phi, \phi^*]}$ changes the quantum action as

$$W_{NL}[\phi, \phi^*] \xrightarrow{\text{FFBRST}} W_{F}[\phi, \phi^*].$$

(46)

This reflects the validity of results up to quantum levels also. Hence, we conclude that the finite field-dependent transformations connect two different solutions of quantum master equation also in the case of quantum gravity in curved spacetime.

V. CONCLUSIONS

In this work we have analyzed the general coordinate invariance of perturbative theory of quantum gravity with different gauge conditions. It has been shown that the theory of perturbative quantum gravity in general curved spacetime dimensions in linear and non-linear gauges has supersymmetric BRST and anti-BRST invariance. Further we have developed the FFBRST transformation for quantum gravity by constructing a general finite and field-dependent parameter. The finite and field-dependent structure of BRST transformation changes the path integral of quantum gravity non-trivially as in the case of gauge theory in the flat spacetime. We have observed that the results of FFBRST formulation of usual gauge theory also hold in the theory of gravity on curved spacetime. In this context we have shown that the non-trivial Jacobian appearing in the functional integration is responsible for differences in the effective action of perturbative quantum gravity. The FFBRST transformation with suitable choice of finite parameter connects the linear Landau and non-linear Curci–Ferrari gauges in the theory of curved spacetime. The validity of this result at quantum level is also established by explicit calculations in BV formulation. We expect several other application of this formulation for the theory of perturbative quantum gravity. It would be very interesting to analyze the ghost and graviton condensation for theory of quantum gravity in the Landau and massive Curci–Ferrari gauges.

Acknowledgments

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