Concealed nuclear material identification via combined fast-neutron/$\gamma$-ray computed tomography (FNGCT): a Monte Carlo study

M. Licata and M.J. Joyce

Department of Engineering, Lancaster University, Bailrigg, Lancaster, LA1 4YW United Kingdom

E-mail: m.joyce@lancaster.ac.uk

ABSTRACT: The potential of a combined and simultaneous fast-neutron/$\gamma$-ray computed tomography technique using Monte Carlo simulations is described. This technique is applied on the basis of a hypothetical tomography system comprising an isotopic radiation source (Americium-beryllium) and a number (13) of organic scintillation detectors for the production and detection of both fast neutrons and $\gamma$ rays, respectively. Via a combination of $\gamma$-ray and fast neutron tomography the potential is demonstrated to discern nuclear materials, such as compounds comprising plutonium and uranium, from substances that are used widely for neutron moderation and shielding. This discrimination is achieved on the basis of the difference in the attenuation characteristics of these substances. Discrimination of a variety of nuclear material compounds from shielding/moderating substances (the latter comprising lead or polyethylene for example) is shown to be challenging when using either $\gamma$-ray or neutron tomography in isolation of one another. Much-improved contrast is obtained for a combination of these tomographic modalities. This method has potential applications for in-situ, non-destructive assessments in nuclear security, safeguards, waste management and related requirements in the nuclear industry.

KEYWORDS: Computerized Tomography (CT) and Computed Radiography (CR); Inspection with gamma rays; Inspection with neutrons; Neutron detectors (cold, thermal, fast neutrons)

1Corresponding author.
1 Introduction

Computed Tomography (CT) is a well-known, non-destructive technique that is used to investigate the internal characteristics of materials. X-rays or \( \gamma \) rays have often been used as the form of radiation, particularly for medical purposes, whilst neutrons have also been used extensively, predominantly for materials characterisation [1]. Neutrons and photons interact differently in matter, thus providing sources of information related to the structure of materials under investigation that can be complementary. Depending on their energy, photons interact by means of the photoelectric effect, Compton scattering and pair production. The probability of each interaction also depends on the atomic number \( Z \) of the element in which the photons are interacting. On the other hand, when neutrons interact, they may undertake a variety of nuclear processes, including elastic scattering, inelastic scattering and radiative neutron capture, induced fission and several other types of nuclear reactions, such as \((n, p)\), \((n, d)\), \((n, \alpha)\), etc. For some elements moreover, the interaction cross section, especially for elastic scattering and absorption, is relatively high for low-\( Z \) elements. Due to these differences, neutron and \( \gamma \)-ray tomographies often yield different results when imaging the same objects because two different materials with attenuation coefficients that are similar for photons may have contrasting coefficients for neutrons, and vice versa. For instance, photon-based tomographies are unlikely to discriminate between different materials of similar density or atomic number \((Z)\) and underperform when imaging low-\( Z \) materials; neutron-based techniques, on the other hand, are more suited to highlighting structural contrast in low-\( Z \), porous materials and substances that have high reaction cross sections for neutron scattering and absorption [2].
Simultaneous and combined neutron-$\gamma$ imaging is a relatively unexplored technique. Some works have investigated the potential of the combined neutron-photon imaging technique when used for non-destructive tests [3–10]. In particular, Monte-Carlo simulations have been carried out in [11] in order to examine the feasibility of a combined neutron-photon CT of a model container. The simultaneous n-$\gamma$ technique is often applied making use of two different imaging facilities for both neutron and $\gamma$X-rays as well as separate detector systems for each types of radiation. Research reactors, spallation facilities or neutron generators are often used as neutron sources, and the limited transportability of these facilities can constrain the potential of a tomography system for large-scale industrial applications and/or ex-situ assessments. Moreover, the majority of neutron techniques involve the use of thermal neutrons. These are relatively easy to detect and can yield high resolution images but have the drawback of having poor penetration capabilities when the requirement is to probe high density materials [12]. The use of fast neutrons can be preferable in such applications.

The purpose of this study is to demonstrate the potential of a simultaneous neutron-$\gamma$ CT technique that is able to achieve a reasonable image resolution and an acceptable level of contrast of materials relevant to scenarios in which nuclear materials might be concealed, using an isotopic radiation source. The identification of special nuclear materials is often based on the detection of the radiation they emit, either passively or following stimulus by an external source of radiation. Due to this fact, it is anticipated that were they to be smuggled they would be concealed by significant quantities of shielding materials, in order to prevent their interception, particularly because the emissions from natural decay processes are relatively weak. In-situ, non-destructive, real-time assessment methods to overcome the challenges posed by such scenarios are therefore needed, particularly where the requirement to investigate suspicious objects is widespread, such as at territorial borders, transport hubs and shipping terminals.

The CT system presented in this work has been designed and simulated in such a way as to allow a combined neutron-$\gamma$ scan with a single source of fast neutrons and $\gamma$ rays. The system is based on the same concept of the fast-neutron tomography system reported in [13], which was based on the use of a real-time pulse-shape discrimination (PSD) technique in 7 organic liquid scintillation detectors. In this prior report, fast neutron tomography was demonstrated to discern voids, corners and inhomogeneity in concrete samples and the potential to apply this system for fast neutron assay was discussed. The PSD technique [14] makes both experimental fast-neutron and $\gamma$-ray data available but only neutron data were used in the subsequent off-line data analysis.

In this paper the feasibility to perform combined fast-neutron/$\gamma$-ray tomography will be described, highlighting the potential of this method to discern materials used widely in the nuclear industry, especially in scenarios by which they might be hidden or concealed. The novelty of this research lies in the use of a common source and real-time detection system for neutrons and $\gamma$ rays, designed to operate simultaneously. In addition, the use of fast neutrons allows high-density objects and materials to be probed. This system as modelled is compatible with assembly in a small laboratory consistent with the need to demonstrate its potential for industrial applications and for security inspection.

2 Methods and procedures

Four features fix the geometry of a computed tomography system: the source-to-object distance, the source-to-detector distance, the area to be imaged and the solid angle covered by the detectors.
In this particular study, a 20 MBq americium-beryllium (AmBe) source has been simulated. This source emits a mixed radiation field comprising both fast neutrons (from thermal energies to 10 MeV) and $\gamma$ rays of 4.4 MeV. Californium-252 is also a candidate that might be used as a source, emitting neutrons and $\gamma$ rays as a result of spontaneous fission. The radiation produced by AmBe is collimated into a fan beam directed towards the phantoms to be scanned. The fan beam geometry allows a volume to be reconstructed as a stack of different single slices consistent with the type of configuration that is in widespread use in CT scanning machines. The use of this particular beam arrangement requires the phantom to be moved in order to obtain a range of projections with which to optimise image resolution. Parallel beam and cone beam arrangements are also widespread in tomography and radiography applications. Cone beams, in particular, produce a radiographic image with only one projection and are particularly suitable when pixelated detectors such as charge coupled devices (CCDs) are used. The fan beam geometry has been selected in this study as it is considered that which is most compatible with the type and arrangement of the detectors that have been used in the related prior art [13] and that have been selected for the simulation. Since one of the main purposes of this research is to exploit the same detection system for fast neutrons and $\gamma$ rays, the choice of detectors is restricted to organic scintillation materials that, in a possible experimental scenario, would be read out with real-time pulse-shape discrimination systems [14], discriminating and recording neutrons and $\gamma$ rays simultaneously. The constraints associated with their physical dimensions, sensitive volume and shape limits the possibility of using parallel and cone beams. Also, in the context of a there being a small sample under scrutiny (i.e. relative to the size and mass of the detector system), it is easier to manipulate the position of the sample than it is to move the detector system. Depending on the specific application in mind, the converse might be the case; for example, where the requirement is to assess the integrity of a fixed object such as a freight container or transport vessel.

Schematic diagrams of the plan, elevation and perspective of the computed tomography system are presented in figure 1. The whole system and a generic laboratory space in which it is anticipated that it might be required to function have been simulated with MCNP6 [15].

### 2.1 Collimator

The collimator is designed to produce fan beams of both the neutrons and $\gamma$ rays emitted by the AmBe source and therefore it comprises: two polyethylene blocks of thickness 24 cm and four tungsten blocks of 3-cm thickness, separated by a 4-mm gap. Lead might also be used in place of tungsten, potentially reducing the cost of the system in the event that this is a requirement. The total length of the collimator is 30 cm. The first tungsten layer is included to attenuate the $\gamma$ rays, followed by high-density polyethylene to attenuate the neutrons. A second tungsten layer, located after the polyethylene block is included to shield any residual $\gamma$ radiation arising for example due to scatter in the environment, background or as a result of neutron capture on the hydrogen present in the polyethylene. The 4-mm gap between this arrangement of materials with complementary attenuation properties yields the desired, mixed-field fan beam.

A fundamental requirement in tomography when designing the collimator is the $L/D$ ratio: this is the ratio of the source-to-object-distance ($L$) to the source diameter ($D$). In this particular case, $D$ corresponds to the 4-mm collimator gap [16]. The objects analysed in the simulations have been located at 39 cm from the source; hence $L/D = 97.5$. The $L/D$ ratio provides a quantitative
indication of the beam divergence and infers the quality of a projection. The correspondence of different $L/D$ ratios on the tomographic reconstruction is described in [17].

2.2 Detectors

The detectors have to be positioned as close as possible to the object [18] if detection sensitivity is to be optimised. However, the area to be imaged is constrained by the dimensions of the object itself, as well as by the dimensions of the detectors. For example, if the detectors are located too close to the object, the area subtended by some of them at the position of the object will be beyond the imaging area (this is illustrated in figures 2a and 2b). On the other hand, if the detectors are placed too far from the sample, whilst all of them then benefit from having a clear line-of-sight of the imaging area, the radiation flux subtended at them by the source would be too weak for most practical purposes. The effect of the latter is to increase the required imaging time, potentially rendering the system unsuitable for time-constrained applications. Therefore, a suitable compromise between number of detectors (the larger the number the better) and sample-detector distance (the smaller the better) must be struck.

The choice adopted for this research comprised a twin array of thirteen organic liquid scintillation detectors, with seven and six located at 90 cm and at 93.5 cm from the source, respectively (as depicted in figures 1a, 1b and 2c). This specific arrangement has been selected in order to remove
the gap in the imaging area that arises between each pair of detectors in the scenario where a single row is used. Gaps between detectors produce ring artefacts after the image reconstruction and limit the extent of the final spatial resolution. This can be reduced by placing the detectors as close to one another as possible but even in this case there is a gap in between their sensitive volumes of neighbouring detectors, due to the size of the scintillator cover and its photomultiplier. The double-row array configuration is more suitable to ensure that the total imaging area covered under the view of the detectors is a quasi-continuous space (as depicted in figure 3). It is also anticipated that cross-talk between detectors will be a less significant effect for the twin-row arrangement relative to a single row of closely-packed detectors.

The detectors simulated in this research were of type VS-0653-2 (Scionix, Netherlands [19]) containing the EJ-301 liquid scintillant (Eljen Technologies, U.S.); alternatives such as EJ-309 or the plastic EJ-299 are feasible. The detectors are placed vertically, as shown in figure 1b, equidistant from the source with each row forming an arc such that the balance of the radiation flux at each detector is maintained, as per the inverse-square dependence with distance.

2.3 Modus operandi

The interaction probability of $\gamma$ rays and neutrons is parameterised by the linear attenuation coefficient of the material ($\mu$). The intensity of the radiation exhibits an exponential dependence with material thickness described by the Beer-Lambert law, as per, 

$$ I = I_0 e^{-\mu x}; \quad x \equiv y; $$

(2.1)

where $I_0$ is the initial intensity of the radiation, whereas $I$ is the final intensity and $x$ is the distance travelled by the radiation. The logarithmic ratio $\log_e(I_0/I)$ is called the one-dimensional projection.
of the body in the direction of incidence of the radiation quantum. Scanning the object with a series of rotations and translations, the attenuation coefficient can be treated as function \( \mu(x, y) \) representing the radiating section of the object (figure 4). This particular function has to be determined by a reconstruction algorithm.

**Figure 4.** A schematic representation of bulk radiation attenuation de-convolved into its constituent attenuation coefficients.

Under the condition that the imaging area is the same for both components of the mixed field, two different attenuation indices for neutrons \( \mu_n \) and \( \gamma \) rays \( \mu_\gamma \) can be extracted for a fixed position of the object.

In this research, the tomography arrangement has been simulated with MCNP6, recording both neutrons and \( \gamma \) rays interacting with the detectors after placing the phantom in 624 different positions obtained by its incremental translation and rotation: 26 translations have been made and for each one of these the phantom has been rotated in the horizontal plane through successive 15° steps, 24 rotations per translation. Moreover, an ideal laboratory room, i.e. neglecting background radioactivity and assuming homogeneous material composition, has been reproduced and the residual scatter induced by the simulated AmBe source in the room measured by the detectors. The initial flux intensities, \( I_0_n \) and \( I_0_\gamma \), have been determined by measuring the flux at the detector without any object and the intensity of the neutron and \( \gamma \) flux (\( I_n \) and \( I_\gamma \)) at the detectors has been determined for each projection.

The background (denoted \( bkg \)) has also been evaluated by simulating a laboratory room in which the tomography system would be located. The level of the background then has been subtracted from both intensities. The attenuation coefficient has then been determined for each projection for both neutrons and \( \gamma \) rays, according to equations (2.2) and (2.3), as per,

\[
\mu_n = \log \frac{I_0_n - bkg}{I_n - bkg} \tag{2.2}
\]

\[
\mu_\gamma = \log \frac{I_0_\gamma - bkg}{I_\gamma - bkg} \tag{2.3}
\]

Since 13 detectors have been used, the total number of projections is 8112 (624×13). This number is far greater than the minimum number of projection \( (N_{proj}) \) necessary to yield a satisfactory degree of sampling of the object, defined by the Nyquist-Shannon theorem [18] in equation (2.4):

\[
N_{proj} = \pi O_{pix} \tag{2.4}
\]

where \( O_{pix} \) is the number of pixels defined in the horizontal dimension of the total imaging area. As is shown in the next section, an algebraic image reconstruction algorithm has been used, therefore
a larger number of projections improves the solution for the unknown image pixels. In this specific study, after a detailed analysis considering both sampling time and computer processing time, \( O_{\text{pix}} \) is 128 pixels and the imaging area 13\( \times \)13 cm. This means that the dimension of each single pixel corresponds approximately to 1 mm.

### 2.4 Phantoms tested

In general, the phantoms studied in this research comprise small cylinders of compounds containing uranium or plutonium placed in a variety of arrangements with lead and polyethylene, designed to conceal their presence from view primarily in terms of the radiation they emit and that might otherwise enable them to be detected. The cylinders are 5-mm thick and have a diameter of 2.5 cm (plutonium) and 3 cm (uranium). Hypothetically, the plutonium compounds have been concealed inside a lead box of dimension 6 cm, located in turn inside a 10-cm polyethylene box (as depicted in figure 5a) in order to shield the emission of both neutrons and \( \gamma \) rays in view of the likelihood of neutron emission from spontaneous fission (SF) of constituent quantities of \(^{240}\text{Pu}\). Similarly, uranium compounds have been hidden inside a 7-cm lead cuboid (figure 5b), in order to shield the residual \( \gamma \)-ray emission from the decay of \(^{238}\text{U}\) etc. The small amount of SF in \(^{238}\text{U}\) and the related neutron yield was neglected for the purposes of these measurements. Tables 1 and 2 summarise the materials used for the 6 different samples and configurations, in detail.

![Figure 5](image.png)

**Figure 5.** Schematic pictures of the objects simulated in this research: (a) the configuration used to conceal the plutonium samples, whilst in (b) the lead box used to conceal uranium samples is depicted.

### 3 Results

#### 3.1 Qualitative observations

The CT results are presented in figures 6 and 7 in terms of attenuation index as a function of dimension in \( x \) and \( y \). For each sample the results of its fast-neutron tomography, \( \gamma \)-ray tomography and the combination of neutron and \( \gamma \)-ray tomography are shown.
Table 1. Description of the phantoms containing plutonium-based samples.

| Sample name | Material a               | Material b                  | Material c                        |
|-------------|--------------------------|-----------------------------|-----------------------------------|
| Pu1         | High-density polyethylene| Lead                        | Plutonium metal                   |
| Pu2         | Lead                     | High density polyethylene   | Plutonium metal                   |
| Pu3         | High-density polyethylene| Lead                        | Plutonium oxide (PuO)             |

Table 2. Description of the phantoms containing different uranium-based samples.

| Sample name | Material a | Material b               |
|-------------|------------|--------------------------|
| U1          | Lead       | Highly-enriched uranium (HEU) |
| U2          | Lead       | U₃O₈                     |
| U3          | Lead       | UC₂                      |

The image reconstruction algorithm used to reproduce the neutron and the γ-ray tomography is the Simultaneous Multiplicative Algebraic Reconstruction Technique (SMART). The appendix to this work describes the motivation for this particular choice. In order to simulate combined neutron/γ-ray tomography, it is necessary to reconstruct the γ-ray and neutron images separately and then to normalize them to the maximum value of the attenuation coefficient in each case. This is a crucial process since both the neutron and γ-ray tomography data originate from a mixed radiation source, with each radiation type having a different intensity and energy spectrum. Once they are normalized, the combined neutron/γ-ray tomography \(Q_{ny}\) can be obtained according to equation (3.1),

\[
Q_{ny} = \left(\frac{1}{2} - \alpha\right) Q_n + \left(\frac{1}{2} + \alpha\right) Q_\gamma
\]

where \(Q_n\) and \(Q_\gamma\) are the neutron and γ-ray images, respectively, and \(\alpha\) is a scalar parameter associated with the contrast of the image and with the condition that \(-1/2 < \alpha < 1/2\). Thus, for \(\alpha = 1/2\), \(Q_{ny} = Q_\gamma\), or if \(\alpha = -1/2\), \(Q_{ny} = Q_n\). The results presented here are for the specific case \(\alpha = 0\), and the combined tomography is simply the average of the neutron and γ-ray images. However, this depends on the materials that are imaged. The \(\alpha\) parameter can be varied to highlight features either from the neutron image or from the γ-ray alternative. A similar technique can be applied using the data fusion methodology reported in [20].

Concerning the data for the plutonium samples (as shown in figure 6), the polyethylene and the lead boxes have similar attenuation indices when using the neutron CT only, even when the polyethylene and the lead box are swapped with each other (configurations Pu1 against Pu2). In contrast, the γ-ray CT highlights the lead box and the plutonium metal cylinder (sample Pu1 and Pu2) but, conversely, it does not show evidence of the polyethylene box (sample Pu1, Pu2 and Pu3), which appears instead as a void of similar attenuation index to that of the air. Moreover, the γ-ray CT does not discern lead from plutonium oxide (sample Pu3). Combining the two tomographies,
Figure 6. Tomographic results corresponding to the normalized attenuation index as a function of $x$ and $y$, obtained for plutonium-based samples concealed in polyethylene and lead. See table 1 for a summary of the different materials used in each of the three cases.

the polyethylene box and the inner lead cuboid with plutonium metal and plutonium oxide are all clearly discernible (samples Pu1, Pu2 and Pu3).

Regarding the uranium-based samples (figure 7), the neutron CT does not discriminate uranium trioxide from lead (sample U2), and neither does the $\gamma$-ray CT discriminate between uranium carbide and lead (sample U3). When the combined neutron/$\gamma$-ray CT is applied, all the three uranium samples (U1, U2 and U3) are clearly discernible.

For the purposes of these simulations the samples (both plutonium and uranium) have deliberately not been located in the centre of the rotation imaging area because when the shape is a
Figure 7. Tomographic results corresponding to the normalized attenuation index as a function of $x$ and $y$, obtained for uranium-based samples concealed in a lead box. See Table 2 for a summary of the different materials used in each of the three cases.

cylinder or a sphere, ring artefacts may appear as a result of the reconstruction process. Finally, in the prospect of a real experiment, with a source activity in the order of a few MBq, the estimated time needed for the entire tomography experiment is estimated to be of the order of three to four hours, depending on the choice of the sampling duration.

3.2 Spatial resolution

Spatial resolution has been calculated by measuring the attenuation index profile along a fixed projection. The variation of the attenuation index of a given material from another has been fitted
with a distribution function of a Fermi-Dirac form on an entirely empirical basis, as per,

$$f(x) = \frac{A}{1 + \exp \left[ \left( \frac{x - B}{C} \right) \right]}$$  \hspace{1cm} (3.2)

where $A$, $B$ and $C$ are constants that depend on the profile being fitted. This particular function has been chosen in order to base the spatial resolution on the 10–90% edge response technique.

In figure 8, the results of sample Pu1, namely plutonium metal, are shown. The images on the left refer to neutron-CT (8a), $\gamma$-CT (8b), and neutron/$\gamma$-ray CT (8c), which are also presented in 3D for clarity. The plots on the right, by contrast, show both the attenuation coefficient profile and the respective Fermi-Dirac type fit. The attenuation profile has been extracted along the red, dashed, horizontal line highlighted on the left side and passing through the centre of plutonium cylinder. This line, which extends from one border to the other, covers 128 pixels along the $x$ dimension. For each sample, the minimum and maximum spatial resolution was measured using the 10–90% edge response method. For instance, in the case study of the Pu1 sample, the best resolution obtained was 3 mm, and this has been achieved with the combined neutron/$\gamma$-ray CT. The uncertainty of the resolution measurements presented is 1 mm, since, as mentioned in the previous section, 1 mm corresponds approximately to the dimension of 1 pixel. The spatial resolution results concerning the other samples are shown in Table 3. In five cases out of six, neutron/$\gamma$-ray CT shows a better degree of resolution, with the exception of the U1 sample (namely highly-enriched uranium) for which a better resolution with neutron-CT alone is achieved.

**Table 3.** Spatial resolution estimation using the 10–90% edge response method. Values are presented in millimetres. Each value has an uncertainty of $\pm$1 mm.

| Sample | neutron-CT $\min$ | neutron-CT $\max$ | $\gamma$-CT $\min$ | $\gamma$-CT $\max$ | neutron/$\gamma$-CT $\min$ | neutron/$\gamma$-CT $\max$ |
|--------|------------------|------------------|------------------|------------------|------------------|------------------|
| Pu1    | 6                | n.a.             | 4                | 7                | 3                | 5                |
| Pu2    | 4                | n.a.             | 6                | 8                | 3                | 5                |
| Pu3    | 5                | n.a.             | 6                | 9                | 4                | 6                |
| U1     | 4                | 4                | 5                | 8                | 5                | 7                |
| U2     | n.a.             | n.a.             | 5                | 5                | 4                | 4                |
| U3     | 4                | 4                | 7                | 8                | 4                | 5                |

It has to be highlighted that spatial resolution varies depending on the materials being analysed, due to this fact, the interaction with some materials by neutrons is better than it is with $\gamma$ rays and *vice versa*, thus yielding different levels of contrast, which in turn means better or worse spatial resolution. For instance, in the case of sample U2 (U$_3$O$_8$) and with respect to neutron-CT, the minimum spatial resolution cannot be determined because the attenuation coefficient profile does not allow discrimination between uranium trioxide and lead. The same applies to the neutron-CT analyses of Pu1, Pu2 and Pu3, which do not distinguish polyethylene from lead. On the other hand, it is indeed possible to estimate a minimum resolution exploiting the discrimination of plutonium from either polyethylene or lead.

Shapes, borders and position of the samples are instead recognized with a precision of 2 mm. Using the combined neutron/$\gamma$-ray CT, all the samples of uranium and plutonium are identified
Figure 8. On the left: tomographic results corresponding to the normalized attenuation index as a function of $x$ and $y$, concerning the sample Pu1 (plutonium metal). On the right: the attenuation index profile as a function of the pixel number along the red dotted line traced within the tomographic result on the left.
inside the shielding materials that they are concealed by, with a spatial resolution that ranges from a minimum of 3 mm (sample Pu1 and Pu2) to a maximum of 7 mm (sample U1).

4 Conclusion

This study contributes to the development of neutron/\(\gamma\) based inspection technologies. The vast majority of the existing neutron non-destructive techniques exploit the production of either prompt or delayed neutrons and \(\gamma\) rays for the inspection/recognized of special nuclear materials (SNM) [21]. However, these techniques are often applied to the identification of other hazardous materials such as explosives and contraband (especially illegal drugs) rather than to recognize substances of significance in a nuclear context [22].

The current state-of-the-art denotes that combined neutron- and \(\gamma\)-ray tomography is always undertaken using two different facilities for neutron-CT and \(\gamma\)-CT, therefore comprising separate radiation sources, instrumentation, detectors and image reprocessing techniques. A major advantage of the approach and technique described in this research is that it can provide quick and reliable information about objects under investigation, using the same radiation source and instrumentation, with a significantly-reduced requirement for off-line analysis yielding data in a few minutes and carried out subsequent to the estimated three-to-four hours’ measurement time. In particular, this technique has been applied intentionally in a nuclear framework since the majority of neutron- and \(\gamma\)-radiography/tomography techniques in this environment are often focused on the investigation of nuclear fuels [23]. Only a few papers report tomography as a way for the identification of concealed nuclear materials [24–26]. In addition, on the one hand several works use radioactive sources such as \(^{60}\text{Co}\) or \(^{137}\text{Cs}\) to carry out \(\gamma\)-CT, whilst on the other, only [13] and [24] exploit an isotopic neutron radiation source such as \(^{252}\text{Cf}\) to perform fast-neutron CT. Conversely, \(^{252}\text{Cf}\) is used mostly to undertake thermalized neutron analysis (TNA). It has also to be highlighted that few if any reports focus on the use of an AmBe source for fast-neutron tomography, as done in this study. The technique presented in this paper could be investigated further and improved using a neutron generator as a source, exploiting relatively recent developments in terms of the portability of these systems [27, 28]. A neutron generator would provide a higher-energy, monochromatic neutron flux and therefore potentially a reduction in terms of sampling time.

This research also indicates that materials used in the nuclear industry are discernible from materials that might be used for shielding purposes to conceal these substances. The simulated system in this work is compatible with use in an ordinary laboratory room and has potential for application for security inspection purposes or, from a more general perspective, for non-destructive assessments for which \(\gamma\)-ray or neutron tomography methods in isolation of one another are not able to provide information with which to discern the composition of the objects under scrutiny. The system can recognize the position and the physical size of the sample to a 2-mm level of precision and an uncertainty of \(\pm 1\) mm; the minimum spatial resolution achieved identifying the different samples tested has been of \((3 \pm 1)\) mm. Finally, a comparison and a quantitative result on the choice of using the simultaneous multiplicative algebraic reconstruction technique rather than one another, has been provided.
A On the choice of the image reconstruction approach

Image reconstruction from projections is a problem that has been studied extensively over several decades. A significant number of works have treated the two most-known groups of reconstruction algorithms in detail, namely the filtered back projection approach (FBP) and the algebraic reconstruction method (Gordon [29]). The former requires a large number of projection data taken over a large number of angles [30, 31]. In some cases, it is not possible to acquire the amount of data necessary, often due to time constraints, costs and the wide variety of experimental issues that can constrain acquisition flexibility. When geometries dealing with limited data (i.e. data coming from an incomplete set of projections) are used, algebraic reconstruction techniques (ART) tend to show the best results compared to the most common FBP techniques [30, 32].

The system geometry and the number of projections acquired in this work fall within the category of reconstruction with limited data. Since ART results depend strongly on the geometry of the system, three different algebraic reconstruction algorithms have been compared and applied to the geometry used in this particular research. Four test-phantoms have been used to test ART (Gordon), SIRT (simultaneous iterative reconstruction technique, Gilbert [33]) and SMART, adapted specifically to the simulated system. The algebraic method most suited to the particular system geometry used in this work is yet to be determined.

Other algebraic algorithms, such as Maximum Entropy and Minimum Energy have not been taken into account since this goes beyond the focus of this study at this stage. Two of the four phantoms have been selected deliberately to be the same as presented in [30] and described mathematically in equations (A.1) and (A.2),

\[
\text{Cosine}(x, y) = 0.25 \left(1 - \cos \left[2\pi(x + 0.5)^{4/5}\right]\right) \times \left(1 - \cos \left[2\pi(y + 0.5)^{2/3}\right]\right); \quad (A.1)
\]

for \(|x, y| < 0.5,\)

\[
\text{CosGauss} = 1.09 \left\{0.3 \text{Cosine}(x, y) + \text{Gauss}_1(x, y) + \text{Gauss}_2(x, y)\right\}; \quad (A.2)
\]

where Gauss_1 is,

\[
\text{Gauss}_1(x, y) = 0.8 \exp \left\{-9(x - 0.2)^2 - (y - 0.1)^2\right\};
\]

and Gauss_2 is,

\[
\text{Gauss}_2(x, y) = \exp \left\{-8(x - 0.2)^2 - (30(y + 0.35))^2\right\}.
\]

The other two test objects of relevance are the Shepp-Logan phantom and spikes made of three square peaks with other small peaks distributed randomly across the image, configured in order to reproduce noise over the image. The noise level in the first three phantoms instead is kept to zero.

The original phantoms and the results are shown in figure 9. The imaging area has been discretized in 128\times128 pixels and projection data have been generated calculating the integral of each projected ray analytically. As in the case of the aforementioned MCNP simulations, phantoms have been translated in 26 steps, and for each step have been rotated in turn by 15° each time, obtaining in this way 24 angles over a 360° total angle of view. In this way there are 624 projections for each of the 13 viewing angles corresponding to the 13 detectors.
Figure 9. A comparison of the results of three different algebraic image reconstruction algorithms applied to four different test-phantoms.

Table 4. RMS for each of the algorithms tested.

| Phantom                | ART | SIRT | SMART |
|------------------------|-----|------|-------|
| Shepp Logan            | 0.44| 0.44 | 0.37  |
| CosGauss               | 0.18| 0.15 | 0.17  |
| Cosine                 | 0.19| 0.19 | 0.18  |
| Spikes and random noise| 0.59| 0.57 | 0.53  |

In order to understand which algebraic algorithm is more suited to this particular configuration the Root Mean Square (RMS) error between the original and the reconstructed phantom has been calculated. The error, shown in table 4 for each algorithm, is defined as:

\[
\text{RMS} = \sqrt{\frac{\sum_{i=1}^{N} [S(i) - S'(i)]^2}{\sum_{i=1}^{N} S(i)}}
\]  

(A.3)
where $S(i)$ is the pixel value of the original phantom and $S'(i)$ is the reconstructed pixel value; $N$ is the total number of pixels in the image.

In three cases out of four the SMART algorithm shows better performance than SIRT and ART. This is the reason why it has been chosen to analyse the Monte Carlo simulation data in this research and shows promise for the analysis of the data from future experimental measurements.

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