Spinodal Instability at the Onset of Collective Expansion in Nuclear Collisions

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Using transport theory to model central Au + Au collisions in the energy region of 20 - 110 MeV/u, at impact parameters $b \leq 5$ fm, we predict a measurable impact of spinoidal instability as the collective expansion sets in with energy. Two transport models are employed, the pBUU model solving a Boltzmann-Uehling-Uhlenbeck equation, and the Brownian Motion (BM) model solving a set of Langevin equations to describe the motion of individual nucleons in a noisy nuclear medium. We find, for the first time, that a combination of delayed equilibration, onset of collective expansion and the spinodal instability produces a pair of transient ring structures, made of the projectile and target remnants, with spectator nucleons predicted to become the 'stones' of the rings. These structures, calculated in the configuration space and mapped onto the velocity space, could be detected in experimental analysis of the collective flow.

During the last decades, the thermodynamics of matter created in heavy-ion (HI) collisions has been studied in a variety of theoretical approaches, see e.g. [1–4], and in numerous experiments, e.g. [5–7], at a wide range of beam energies, impact parameters and projectile-target combinations. In drawing conclusions from the data for matter densities in excess of the normal density $\rho_0 = 0.16$ fm$^{-3}$, the ability to model the collisions in transport theory has been critical for relating the Equation of State (EOS), and other bulk properties of nuclear matter, to measurable signals coming from the collisions.

This effort has been complicated both by the involved physics of the collisions, which is not yet fully understood, and by the unavoidable simplifications in theoretical tools used in their description. The limited duration of the collisions is likely to lead to an incomplete equilibration and the finite system sizes require careful treatment of surface effects.

Different types of collective motion, particularly radial (uniform in azimuth around the beam), sideways (dipole anisotropy in azimuth pointing in the reaction plane) and elliptic (quadrupole in azimuth) [5], identified in the systematics of particle emission from the reaction, serve as diagnostics of the collision mechanism. At high incident energies, of few hundred MeV/u or more, the sideways and elliptic flows exhibit characteristics indicative of hot and dense matter, called participant, expanding under pressure in the vicinity of a much colder matter at density close to that of normal nuclei, called spectator [8, 9] which face no opposing matter when the impact parameter is finite. According to measurements and simulations, at high energies when the motion is supersonic, the spectators are clearly separated from the rest of matter and progress along the beam at the original projectile/target velocities.

As the incident energy decreases, both flows weaken and eventually change character to reflect the primarily attractive nature of nuclear interactions at low energies [7, 10]. Radial collective expansion weakens too with a decrease in incident energy [11] and the strong distinction between participant and spectator matter fades out. The changes in the relative strengths of the sideways and elliptic motion with the incident energy in the higher energy domain have been used to constrain nuclear pressure as a function of density at supranormal nuclear densities [8, 9].

Yields of different species, including nucleons, light clusters (mass number $A \leq 4$) and intermediate mass fragments (IMF) (charge number $Z \gtrsim 3$) from the reaction also contain important information about the dynamics. Increased emission of IMF is usually seen as a fingerprint of a possible liquid-gas phase transition, predicted to occur in equilibrated nuclear matter at subnormal densities [2, 12]. However, definitive experimental evidence of the transition has been lacking [6, 13, 14] because the increased production of IMF could be also explained by mechanisms not invoking the phase transition, such as mechanical fractures, sequential compound system decays and coalescence.

One possibility of an unambiguous demonstration of the transition results from examining the effect of the spinodal instability (see below) in the transition region [11, 15]. Once matter enters the unstable region (see Fig. 2.1 in Ref. [1]), non-uniformities in the order parameter for the transition, such as density, are rapidly amplified until stable coexisting phases are reached. Such a scenario is applicable in a wide range of physical phenomena, for example binary fluids or solids or hadronization of quark-gluon plasma. In HI collisions, the excited system evolves rapidly
and may be driven to spinodal conditions which are then responsible for multifragmentation. Early models [16, 19] predicted that in head-on collisions the conditions of negative adiabatic compressibility combined with negative pressure would cause formation of a bubble- or ring-like structure after nuclei stopped on each other, matter splashed to the side and expansion stalled [18]. At finite impact parameters, the ring would be elongated and inclined relative to the beam axis, rather than perpendicular. Results of the calculations were followed by experimental searches for unusual structures in IMF emissions [20]. Studies of the spinodal decomposition in uniform or symmetrically expanding matter suggested looking for equal-size fragments signifying a growth of a specific spinodal mode in the unstable expanded matter [21]. However, systematic studies of the stopping in energetic collisions of heavy nuclei revealed that the nuclei fail to completely stop even in the most central collision [22, 23], i.e., the step preceding ring formation in the simulations is not happening.

Theoretical challenges remain in transport models. As a fully quantal transport approach is beyond current means, semiclassical theory had to adopt physics and numerical compromises. To account for the non-equilibrium, the input to transport models has been adjusted to match experiments and an extrapolation to equilibrium has been used [8, 9]. The recent comparison of transport codes against each other and the known constraints, strengthened their predictive power [23, 24]. The transport codes moved to a new level of sophistication such as including collective oscillations around ground states [25]. Still, specific physics limitations remain. Models based on solving the semiclassical Boltzmann equation cannot describe production of IMF, and molecular dynamics models cannot describe production of light clusters. Hybrid approaches are employed to overcome these shortcomings [9, 26].

In this work we revisit the impact of a liquid-gas phase coexistence and spinodal instability in the collision (see also [27]), looking for a unique experimental signal that would demonstrate the presence of these effects. We study Au+Au collisions in two contemporary transport models, pBUU [23, 28, 29], which solves the semiclassical Boltzmann-Uehling-Uhlenbeck equation, and the Brownian Motion (BM) model [30], in which the beyond-mean-field dynamics in collision is reformulated in terms of a one-body Brownian motion of nucleons in the nuclear medium, in contrast to the traditional two-body scattering. The major difference between the two models is that in pBUU correlations are perceived to relax quickly while in BM a possible lasting impact of the correlations is simulated with fluctuations.

The incident energies in the simulation are selected to best test the expected spinodal conditions at the onset of collective expansion accompanied by low entropy production. For central collisions in the Au+Au system, the regions of \( \sim 25, 50-60 \) and \( \sim 100 \text{ MeV/u} \) were determined as onsets for the growth of, respectively, radial [11], sideways [10, 31] and elliptic [7] flows.

Fig. 1 shows contour plots of the baryon density \( \rho \), as predicted by pBUU, in the reaction plane at different times in the center of mass frame, during head-on Au + Au collisions at 40, 60 and 100 MeV/u and at 60 MeV/u (\( b=2\text{fm} \)) incident energies. The arrows illustrate the collective velocity field in the reaction plane. As nuclei overlap in the collision, the particle density increases above typical of the projectile and target nuclei. The matter becomes excited and expansion and eventual emission into the vacuum set in. While the equilibration progresses and the energy of the relative motion turns into excitation, the nuclei fail to stop even in the head-on collisions, consistently with data [22], i.e., the equilibration is incomplete during the nuclear overlap. As the collision progresses, the remnants of the original nuclei move with a much reduced velocity as compared to the original nuclei. These remnants are expanding and highly flattened following the impact against the opposing nucleus. The expansion slows down as the matter thins and cools down and negative pressure is developing at densities below \( \rho_0 \). At densities \( \rho \lesssim 2\rho_0/3 \), the matter dives into the region of adiabatic spinodal instability with \( \left( \frac{\partial P}{\partial \rho} \right)_{S/A} < 0 \), where \( S/A \) is entropy per nucleon. The matter at higher density depletes the matter at lower density, i.e., effectively the matter accumulates at the edges [18], so the remnants of the original nuclei turn into rings. Although there is a similarity with predictions of the early models [16–19], the major difference is that the present simulation predicts both flattened pieces of matter to form a ring.

Details and further fate of the rings depend on the incident energy and centrality of the collision. In head-on \(( b = 0 \) collisions, the rings are symmetric and perpendicular to the beam axis. At low energies the expansion is slow enough so that negative pressure can stall it to a halt. Surface tension can further pull the incipient ring structures in and gradually evolve the shape of the reaction products towards spherical. At the highest energies the expansion is vigorous, the matter thins out quickly and the rings fail to form a pattern that might dominate over fluctuations expected in the final stages of collisions and missing so far from the simulations. At intermediate energies around 60 MeV/u and above the expansion just gains strength. It gets then, on the one hand, strong enough to prevent the rings from collapsing back onto a single compact shapes but, on the other hand, weak enough so that the nuclear density has time to grow over significantly large distances and values, making the rings persist rather than fleet.

At finite but low impact parameters, the rings continue to form at 60 MeV/u and above, but are tilted with respect to the beam axis and are asymmetric in shape, with the matter that plays the spectator role at much higher energies contributing to the thicker leading portions or ‘stones’ of the rings. If the impact parameter is increased further, the back sides of the rings continue thinning in the same fashion as the \( b = 0 \) rings when the energy is increased.
FIG. 1. Contour plots of the baryon density in the reaction (z,x) plane at different times in the center of mass of the Au + Au system colliding at different beam energies at two centralities, as predicted by the pBUU simulation. The z-axis is the collision (beam) axis. The top, second and bottom rows represent results of a head-on collision at 40, 60, 100 MeV/u incident energy. The third row demonstrate the effect of a non-zero impact parameter at 60 MeV/u. The columns indicate different times. In each panel, the outer, dashed contour represents 0.1ρ₀, and the subsequent solid contours are at the increments of 0.2ρ₀, starting with 0.2ρ₀. The arrows illustrates collective velocity for the matter at selected locations.

above threshold for the ring appearance. This is important because it is easier to isolate a broader range of impact parameters experimentally - the rings with grossly thinned out backs in the pBUU calculations may realistically survive as croissant shapes in the final stages of the collision.

Next we turn to results of the BM model and similarities and differences with pBUU. At early times, until \( t \sim 140 \text{ fm}/c \), the density evolves in BM in a similar manner as in pBUU. As the reaction progresses, the comparison with pBUU depends on the incident energy. At higher incident energies, growth of non-uniformities due to fluctuations competes with a more rapid expansion. At late times there, the BM density evolves towards smoother shapes, more like in pBUU simulations [30].

Fig. 2 shows density isosurfaces at \( t = 200 \text{ fm}/c \) in Au + Au collisions at 60 MeV/u and \( b = 2 \text{ fm} \). In panel (a) the density distribution in the pair of rings, predicted to form in pBUU (see Fig. 1), is shown in 3D. In contrast, a single BM event at the same time and beam energy, in panel (b), yields separated fragments, each moving at somewhat different velocity, reflecting impact of the fluctuations inserted with a physical justification. Averaged over 100 events these structures consolidate in a compact form, shown in panels (c) and (d), analogous to pBUU.

Relevance of predictions of any transient shapes forming in HI due to instabilities depends on the ability to demonstrate their presence experimentally. Due to the expansion involved in the ring formation, we observe some mapping of the ring pattern from the configuration space onto the velocity space. In measurements, patterns correlated with the reaction plane in the velocity space are quantified with transverse energy, momentum and direction vector moments studied vs rapidity [5, 7–9], such as next illustrated in Fig. 3. To arrive at the figure we weight local velocities at the late stages of collisions (\( t \gtrsim 220 \text{ fm}/c \)), with local baryon densities. The plotted distributions freeze, i.e., become independent of the evaluation time \( t \).

In our current simulations we have two rings, tilted relative to the beam axis, that are moving forward and backward and expanding. In Fig. 3(c), the leading edges of the rings in space end up as leading edges in the rapidity distribution \( dP_x/dy_R \). As the rings expand in their plane, the leading part of each ring yields a transverse momentum in one direction and a trailing edge in the opposite, hence a changing sign for \( dP_x/dy_R \) when rapidity spans the ring extension in \( y_R \) in Fig. 3(b). Overall, the transverse momentum in the reaction plane executes two oscillations along rapidity, contrasting with a single oscillation observed at both higher and lower energies [10, 33]. In the energy asymmetry, a notch appears for each ring, Fig. 3(a), consistent with a transverse expansion of the rings.

The proposed experiment is to measure the changes in the rapidity distributions of observables depicted in Fig. 3.
Detection of the change of the sign of the transverse in-plane momentum might seem as a challenge due to difficulties in determining the reaction plane, but this may be overcome by diagonalizing the momentum correlation matrix, with the additional gain that recoil effects may be isolated [33, 34]. More of the practical issue is that the fragments of the forward ring, easier to measure than those originating from the back ring, emerge at 5–7° in the laboratory frame, requiring forward detectors.

In summary, we find that a combination of stalled equilibration, the spinodal instability at low densities in the matter, reflected in its EoS, and the rise of collective expansion with incident energy in central collisions can lead to formation of transient ring-like structures both in the projectile and target residual systems. The application of both models addresses the question of the role of statistical fluctuations, absent in pBUU and present in BM. The transient structures, predicted in the configuration space, correlate with patterns in the velocity space thus allowing their experimental confirmation. Our simulations provide specific guidance for the experimental strategy to investigate IMFs and thus demonstrate the presence of the liquid-gas transition and the spinodal instabilities experimentally.

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FIG. 3. Distribution of the asymmetry in the net transverse $E^x - E^y$ (in-out reaction plane) energy [panel (a)], the in-plane net momentum $P^x$ [panel (b)] and of the baryon number $A$ [panel (c)] as a function of rapidity $y_R$, normalized to beam, predicted for all particles in pBUU (solid lines), and for IMFs in BM (dashed). In the BM model, the transverse energy, averaged over 100 events, was too noisy to allow a meaningful inspection of the energy asymmetry in the ring region.

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