Evolution of cooperation driven by majority-pressure based interdependence

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Abstract
The evolution of cooperation on interdependent networks is arousing increasing concern based on the fact that more and more complex systems in the real-world have been proven to be organized in the form of multi-layer networks rather than single-layer networks. In this study, we examine the effects of self-organized interdependence on the evolution and stabilization of cooperation with social dilemmas depicted by the Prisoner’s Dilemma Game (PDG) and the Public Goods Game (PGG) in which agents with the most common strategy have the chance to be rewarded proportionally to the fitness of corresponding agents belonging to the other network. We show that such a type-free rewarding rule, independent of game strategy, establishes a time-varying interdependence between two initially independent populations whereby cooperation is highly promoted as well as stabilized both in the two-player PDG and in the multi-player PGG. Majority-pressure based interdependence at stake has proven pretty neutral in regard to game strategy because it is contingent on strategy configuration rather than strategy itself, which thus gives birth to homologous communities, including cooperative as well as non-cooperative, and thereby an enhanced spatial reciprocity between non-identical networks is triggered. Of particular interest is the double-edged sword effect of network interdependence on cooperation although in most instances the heavier the interdependence, the better the evolution of cooperation. Furthermore, interpretations of the nontrivial relationship between cooperation and benchmark threshold measuring the strategy’s local popularity highlight that rewarding the minimum majority is optimal for the evolution of cooperation in such scenario. Finally, we claim our observations are also quite robust with respect to mutation.

1. Introduction

In accordance with theoretical predictions by the theory of survival of fittest, selfish defection should have been the only outcome of natural selection considering that cooperators always benefit their opponents at a cost to themselves, which stands in sharp contrast to realistic observations emphasizing cooperation is ubiquitous [1] and also builds up a dilemma where individual optimization does not lead to a social optimum [2]. It remains largely unclear how non-selfish or costly cooperative behavior survives and also expands territories although such altruistic activity has made substantial contribution to the evolution of human as well as animal society [3–6]. In order to explore principles and explanations lurking beneath this puzzle, scientists resort to evolutionary game theory that creatively introduces the notion of evolution into solutions to aforementioned social dilemmas and has indisputably become one of the most powerful and commonly utilized tools [7, 8].

As a discipline closely related to evolutionary game theory, network science has amazingly penetrated into various realms due to growing dependence of modern society on the intricate web of critical infrastructure systems whose backbones are perfectly mapped by complex networks [9–20]. Equally striking is the fact that...
spatial games concerning structured populations also have attracted considerable attention in the past few decades [21–40]. And network reciprocity, inspired by the seminal work of Nowak et al [41], has long since been regarded as a most well-known mechanism to encourage the evolution of cooperation, which probably signifies network structure indeed has a talent for boosting cooperation by means of insulating cooperators from exploitation by noncooperators. Most of the studies in this area have been well considered with the application of single-layer networks.

Our motivation in this paper comes mainly from the following two aspects. On the one hand, despite the widespread application of single-layer models, issues concerning multi-layer topologies are capturing the current interest of scientists in the light of the fact that many a real system is best described by multi-layer or multi-domain structure rather than by single-layer structure [42–55], among which how to build up proper interdependent relationship between different networks remains of great interest and significance [56–81]. On the other side of the coin, it is reported in real experiments that individuals are readily confronted with challenges, in the decision-making procedure, from group pressure exactly established by unanimous majority, which generally leads to the significant distortion of perception, judgment as well as action [82, 83]. Aside from this real observation, it is often the case that conforming to the majority is able to guarantee cooperation a promising evolutionary fate [84–87]. From a psychological perspective, it is often the case in reality that being majority can give agents a certain sense of belonging and make them become more confident in their behavior and strategic choices. These materials, therefore, might highlight a fact that once agents are arranged to make decisions in grouped patterns, the majority of the population or the group who are carrying most common strategy (MCS), could possibly have a certain number of advantages (including but not limited to psychological) in terms of strategy-spreading and opinion-conveying, no matter whether or not they are really doing right. However, it remains unclear how to quantify this widespread social phenomenon in networked systems, as well as what role such majority-pressure might play in the evolution of strategy and also in the establishment of network interdependence. As a possible solution to these problems, in this study we associate the perceived advantage of being majority with the evolutionary fitness of corresponding agents on the other network and obtain a robust self-organized intercorrelation between two initially independent lattices. Specifically, if the strategy popularity of an agent exceeds the benchmark threshold (no matter she is a cooperator or defector), she is likely to be rewarded proportionally to the fitness of the homologous agent on the other network. Majority-pressure based interdependence at stake does matter strategy’s local configuration rather than strategy itself. From this point of view, such type-free interdependence has no declared or intentional bias in terms of treating non-identical social behavior. Not only can it benefit cooperation, it can also help defection. And it creates homologous or co-evolving communities including cooperative and defective on two networks in the evolution, however, we elucidate that in most cases only the cooperative community is favored by selection and is consistent with the prevalence of cooperation. Furthermore, by comparison, we demonstrate that rewarding the minimum majority is always able to induce optimal promotion as well as stabilization of cooperation both in the two-player PDG and in the multi-player PGG.

The rest of this study is arranged as follows. We first present the PDG and PGG on interdependent networks. Then, we shall report the primary results in the second section. Finally, we show relevant discussion extracted from this work.

2. The evolution of cooperation on interdependent networks

The population is structured by two \( L \times L \) square lattices, termed \( A \) and \( B \), with von Neumann neighborhood and periodic boundary conditions. \( N \) denotes the population size, and \( N = N_A + N_B \), where \( N_A \) represents the size of \( A \), \( N_B \) the size of \( B \), respectively. Each agent is initially designated either as a cooperator (\( C \)) or a defector (\( D \)) with equal probability. For the PDG, each game involves three possible pairwise interactions such as mutual cooperation yielding the reward \( R \), mutual defection leading to the punishment \( P \), and mixed choice endowing the cooperator the sucker’s payoff \( S \) and the defector the temptation \( T \). The payoff matrix mapping agents’ interactions is given by:

\[
\begin{pmatrix}
C & D \\
C & F \end{pmatrix}
\]

where \( R = 1 \), \( T = b \), \( S = P = 0 \), constituting a weak Prisoner’s Dilemma [41]. \( b \) is the dilemma strength, such that increasing \( b \) implies a stronger attraction into a full-\( D \) phase.

Then for the PGG, each agent launches a PGG involving all her direct partners and herself. During this process, the cooperator contributes a constant amount \( c = 1 \) to the common pool, and the defector contributes nothing. The total contributions will be multiplied by a synergy factor \( \epsilon \) and allocated evenly among all
participants irrespective of their contributions. Accordingly, each participant receives a payoff given by the following expression:

$$\Pi_{ix} = \sum_{i,j} \delta * s_{ix} - s_{ij}; \quad i_x, j_x \in \Omega_{ix}$$  \hspace{1cm} (2)$$

where $\delta = \frac{c}{k_i + 1}$ is the normalized enhancement factor. $k_i$ and $\Omega_{ix}$ characterize the degree and neighborhoods of agent $i$ on the network $X$, respectively. $s_{ix} \in \{G, D\}$ represents the strategy of agent $i_x$, and $s_{ix} = 1$ (0) if $i_x$ is a cooperator (defector).

We stimulate the evolutionary process on both networks in accordance with the standard Monte Carlo simulations procedure comprising the following elementary steps. First, each agent plays the PDG or PGG with all direct neighbors and calculates the payoff $\Pi_{ix}$.

Next, testing whether an agent’s strategy is a MCS with the Heaviside step function

$$\phi_{ix} = \Theta(n_{sx} - M) = \begin{cases} 
0, & n_{sx} < M \\
1, & n_{sx} \geq M,
\end{cases}$$  \hspace{1cm} (3)$$

where $n_{sx}$ denotes the number of agents adopting strategy $s_{ix}$ within $i_x$’s neighborhoods. $M \in \{1, ..., k_i + 1\}$ serves as the benchmark that defines a MCS and quantifies the local popularity of strategies. Intuitively, it means $s_{ix}$ is a MCS only provided that $\phi_{ix} = 1$, and vice versa.

In order to quantify majority-effect emerging in human society and also to link it with the interdependence of networks, we assume that if $i_x$ adopts a MCS she is going to be rewarded with an extra payoff proportional to the payoff of the corresponding (homologous) agent $i_x$ on the other network with the probability $\gamma \in [0, 1]$. In this case, the final utility of $i_x$ should be rewritten as

$$U_{ix} = \Pi_{ix} + \alpha \Pi_{ix^*},$$  \hspace{1cm} (4)$$

where $\alpha$ quantifies coupling strength imposed by the homologous agent of the other network and the ingredient $\alpha \Pi_{ix^*}$ stresses agents’ perceived advantage of being majority. It is worth noting that in this report rewarding is undirected in terms of layers unlike in [65], and also that whether or not the rewarding event occurs is exactly determined both by the parameter $\gamma$ and the above Heaviside step function.

Lastly, each agent updates strategy by learning a randomly drawn neighbor’s strategy with the probability given by the Fermi function:

$$T(s_{ix} \rightarrow s_{ix^*}) = \frac{1}{1 + \exp[(U_{ix} - U_{ix^*})/K]},$$  \hspace{1cm} (5)$$

where $K$ quantifies the uncertainty or noise related to the strategy transmission [21] and provides a measure of the strength of natural selection. All Monte Carlo simulations are performed for sufficiently long time steps and we average the final results over up to 100 independent runs.

### 3. Results

In this study, cross-network rewarding probability, fitness-coupling intensity, and together with benchmark threshold testing MCS constitute the primary determinants determining interdependence between different evolutionary dynamics proceeding on non-identical networks. Thus our analysis is conducted mainly surrounding these three parts. First, we begin with checking the impact that $\gamma$ imposes on the evolution race between cooperators and noncooperators. Figure 1 reveals that interdependence, catalyzed by type-free rewarding probability, $\gamma$, on cooperative behavior. That is, in spite of its most common enhancement effect on cooperation compared to the situation without interdependence (see $\gamma = 0$ in figure 1), there does exist an opposite and long-term inhibition (e.g., refer to $\gamma = 0.2$ in figure 1). By comparison, it is easy to find when $\gamma = 0.2$ cooperation is always inhibited for the PDG in figure 1(a), and similarly for scenarios with small synergy factors in the PGG (refer to figure 1(b)). The outcome outlines a minimum cross-network rewarding frequency is potentially needed for facilitation of cooperation in the two-player PDG and also in the multi-player PGG taking place on intercorrelated networks.

Different from the complicated relationship between $\gamma$ and cooperation, the influence of fitness-coupling intensity seems much clearer as depicted in figure 2. Typically, cooperation is substantially impelled compared with the case without fitness entanglement (i.e., the curve $\alpha = 0$ in figure 2) and it demonstrates an increasing improvement of cooperation with a monotonous rise of $\alpha$, no matter in the PDG or in the PGG. This observation means a heavy inter reliance on evolutionary fitness of homologous pairs is more apt to favor non-selfish cooperators rather than those who fail to cooperate. Critical thresholds of $b$ in the PDG beyond which the full cooperation phase and cooperation phenomenon vanish are highly enlarged as $\alpha$ creeps up (see figure 2(a)).
With regard to the PGG, in turn, cut-offs of $\delta$ marking analogous key points of dynamics are also remarkably mini- figured in regard to the same upward trend of $\alpha$ (check figure 2(b)). These traits manifest that entangling evolutionary fitness of different layers is probably capable of inspiring cooperation against selfish non-cooperation.

It is worth pointing out that type-free rewarding here in fact nourishes three well-known ingredients such as pro-social [88], anti-social [89], and as well as neutral reward [90] in accordance with differences in strategic actions of the target agents. As we mainly focus on the differences that cross-layer rewarding brings about at the level of strategy pair rather than a single strategy on two networks, thus we make the following assumption. Pro-social (anti-social) reward describes the case rewarding a pair of cooperators (defectors), i.e., rewarding helps cooperation (defection). It is very easy to understand both pro-social and anti-social rewarding. However, neutral reward in this study refers to rewarding a pair within which one is a cooperator (defector) and the other should always be a defector (cooperator) from the other network. It means that neutral reward simultaneously helps cooperation and defection so that rewarding is unbiased at the level of game strategy. Figure 3 pictures the time course of fraction of cooperation (left panel) as well as abundance of type-free reward (right panel) in the PDG. Pro-social, anti-social, and neutral reward are simultaneously observed in one complete evolutionary course. Quite interestingly, the plot also gives prominence to the fact that an evolutionary race should not only be a tournament between different strategies but also be a competition for different sorts of rewarding in this work. More importantly, the war of strategies and the combat concerning reward mostly co-exist until the system ultimately converges to a strategic consensus state. It illustrates that, at the very beginning, random initialization incurs a sharp drop in the frequency of cooperators. In fact, once for a while there are less than 0.73 (close to extinction) percent of cooperators in the entire population (check the left panel in figure 3). This
induces and also exacerbates mounting imbalance among three incentives. As most cooperators die out fast in this stage, the abundance of pro-social and neutral rewarding rapidly decreases and their curves exhibit almost the same slope within this time frame (refer to the first half of the curve in figure 3). As a consequence, it fuels an apparent growth of anti-social rewarding. In fact, majority-pressure based interdependence in this study is pretty neutral in terms of game strategy because it is contingent on strategy configuration rather than on strategy itself. It thus helps both cooperation and defection simultaneously. The rapid spreading of defection at the beginning of figure 4 has verified this point and countless defective co-evolving communities are forming on two networks. Ironically, the climb of defection level, in turn, harms the welfare of defectors. The main reason for this probably lies in the following two aspects. On the one hand, the increase of proportion of defective neighbors often weakens defector’s superiority in fitness just as the payoff matrix indicates. On the other hand, for defectors the extra fitness imposed by majority-effect is also supposed to decline as a defector embraced by other defectors usually gains nothing from and contributes nothing to their homologous defectors on the other network. Quite crucially, however, for cooperators the opposite is often the case with respect to above two key points. Finally, a very small number of cooperative survivors struggle to succeed in forming homologous communities that co-evolve with each other on two layers (see clusters outspreading within the circled area in figure 4). Once these allied communities emerge by means of self-organization they are going to be guaranteed to become pretty robust in line with the above analysis. In most cases they are fed not only by neighboring cooperators on their own network, but also by homologous cooperators on the other interdependent network. The spatial reciprocity reported in [41] has therefore been heavily amplified. Then after a long depression, cooperation begins to revive.
with the aid of such improved reciprocity and to enlarge territories by means of replacing noncooperators who reside on the borders and sever as the boundary line bridging different communities in figure 4. Despite the fact that majority-pressure driven interdependence helps cooperators (defectors) to form cooperative (defective) co-evolving communities as shown in figure 4, the outcome stresses natural selection often favors the cooperative co-evolving community.

As regards how to define a MCS, different people might have different opinions. Without loss of generosity, we therefore employ a benchmark $M$ to quantify the local popularity of agents' strategies. In this way, for a given agent if the number of her neighbors who adopt the same strategy with her exceeds $M$, then her strategy is identified as a MCS, and vice versa. We carry out a throughout study on influences of $M$ as shown in figure 5 although in reality majority rule generally corresponds to the situations that meet requirements $\frac{k-1}{2} \leqslant M \leqslant \bar{k}$, where $\mathbb{Z}$ represents the integer set and $\bar{k}$ is the average degree of two interdependent networks. At first glance, what catches our eyes is the different impacts of dynamics in the PDG and PGG. For instance, unlike the obstructive effect upon cooperation in the PDG, we instead witness a slight facilitation of cooperative behavior in the PGG when $M=4$ (check figures 5(a) and (b)). In addition, it suggests there exists an optimal benchmark threshold $M_{\text{opt}}$, at which cooperation is maximally boosted. That is $M_{\text{opt}}=\frac{k}{2}+1=3$ for both types of interactions (see figures 5(a) and (b)). Furthermore, we have verified such relationship between $M_{\text{opt}}$ and $\bar{k}$ also applies to the case with the application of Moore neighborhood in figures 5(c) and (d), i.e., $M_{\text{opt}}=\frac{k}{2}+1=5$ in this situation. Amazingly, promotion effect on cooperation with Moore neighborhood, in fact, appears even far stronger than that for von Neumann neighborhood in this figure. This phenomenon differs apparently from previous observations concerning dynamically organized partnerships [91–94] and also yeast societies [95], where authors argue that denser networks often exhibit apparent propensity to deter the establishment of cooperation. The existence of $M_{\text{opt}}$ emphasizes that rewarding agents whose strategy popularity slightly oversteps half size of their neighborhoods (referred to as the minimum majority) most benefits the evolution of cooperation no matter in which kind of games. Any departure from this $M_{\text{opt}}$ will definitely cause a quick decline in cooperation level. The outcome not only enriches our comprehension about the role of network
density in the evolution of cooperation but also delivers very impressively an inspiration to us that rewarding the minimum majority is probably consistent with the prevalence of pro-social behavior in such scenario.

In view of non-negligible effect of spatial patterns to our co-evolutionary protocol, we finally examine the robustness of the designed mechanism in regard to mutation as mutation itself is always able to disorder the microscopic spatial distribution of strategy. Here we embed a mutation by which their strategies are randomly reassigned with a non-zero probability \( \zeta \) when agents plan to update strategies. The related outcome presented in figure 6 shows that the introduction of mutation does not qualitatively change our results apart from some almost negligible fluctuations in cooperation level. Furthermore, the stabilization of cooperation, generally investigating what causes the collapse of cooperation, is considered as much crucial as its emergence. We study this issue in figure 7 by seeding at random a pair of coordinated monomers (first column), two-tuples (second column), and triads (third column) comprising solely defectors on two interdependent lattices, respectively, with the rest being cooperators. They are three representative invasions and others can be realized by their rich combinations. It has been reported that in such configurations (often referred as noncooperation’s Paradise) the
evolution of cooperation is supposed to be highly challenged [96], thus testing anti-invasion stability of cooperation against defection in such circumstance is more convincing. Under the conditions shown in figure 7, we find each subsystem (i.e., corresponds to each lattice) always settles in absorbing states where each subpopulation is taken over by only one type of strategies, i.e., C or D. Consequently, the ultimate states of the whole system can be accurately categorized into full cooperation, full defection, as well as the mixed state where one layer is full cooperation and simultaneously the other is full defection. And we count the frequency by which the system resides in each state. As figures 7(a1)–(a3) shows, notably, with respect to $\gamma = 1$ cooperation has even been often stabilized by a probability over 62% for the maximum $b = 2$ (an extremely harsh environment for cooperation). And it statistically explains majority-pressure driven interdependence in most circumstances is readily to stabilize the evolution of cooperation. Uncommonly, for $b \leq 1.85$, defection always fails to invade in this condition. Although the challenge for stabilization of cooperation surges with the growth of $b$ in this period, the mixed phase has taken over almost half of experiments in which the phase full cooperation fails to maintain just as this plot displays, that is, corresponding to $b > 1.85$ curves for the mixed equilibrium and full defection nearly coincide with each other (refer to figures 7(a1)–(a3)). As $\gamma$ decreases, yet, the maintenance of cooperation begins to become more difficult (check figures 7(b1)–(b3)). These details thus spotlight no doubt the notable capability and high efficiency of newly designed co-evolution mechanism in stabilizing the evolution of cooperation. In general, the higher the chance of rewarding, the stronger this capability as well as efficiency.

4. Discussion

In human society, it has been reported in real experiments that in many circumstances people are often confronted with majority-pressure when they make decisions or express opinions [82, 83]. In view of its importance for the evolution of human behavior, we here investigate the evolution of cooperation with social dilemmas comprising the PDG and PGG, in which on the basis of such majority-pressure or majority-effect a self-organized interdependence of non-identical networks is well established by means of type-free cross-network rewarding. We demonstrate that such widespread majority-pressure phenomenon has a profound impact on the emergence as well as maintenance of cooperation.

Agents with the MCS are endowed with the chance to be rewarded by an extra payoff related to the evolutionary fitness of homologous agents (also called corresponding agents in other studies) on the other network. In this study, such type-free cross-network rewarding always depends exactly on strategy’s spatial distribution rather than on strategy itself, so it is therefore obviously unbiased at the level of game strategy. In view of this point, it is not only beneficial to cooperation but also conducive to defection, thus it often leads to emergence of countless co-evolving (homologous) communities among which some are cooperative and others are defective. What’s more important is, however, in most instances only cooperative co-evolving communities are favored by natural selection and are consistent with the prevalence of cooperation. In accordance with specific strategies of the target agents, incentives such as pro-social [88], anti-social [89], and neutral rewarding [90] simultaneously emerge and compete with each other during the evolution. As a consequence, in this work the evolution of strategies, in fact, is often a race among positive, negative, and neutral incentives.

We have also shown that majority-pressure driven interdependence is highly capable of triggering the optimal promotion of cooperation and stabilizing cooperation provided that frequent cross-network rewarding as well as heavy fitness entanglement between homologous agents of intercorrelated layers is possible. This, together with the negative effect of small cross-network rewarding probability, constitutes a double-edged sword for cooperative behavior. Bridging majority-pressure or majority-effect with cross-network rewarding not only establishes an adaptive interdependence between non-identical networks, but also introduces diversity into network interdependence at stake. Rewarding agents who carry MCS, to some extent, captures the nature of fitness entanglement between homologous agents of intercorrelated layers. This, notably, with respect to $\gamma = 1$ cooperation has even been often stabilized by a probability over 62% for the maximum $b = 2$ (an extremely harsh environment for cooperation). As $\gamma$ decreases, yet, the maintenance of cooperation begins to become more difficult (check figures 7(b1)–(b3)). These details thus spotlight no doubt the notable capability and high efficiency of newly designed co-evolution mechanism in stabilizing the evolution of cooperation. In general, the higher the chance of rewarding, the stronger this capability as well as efficiency.

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