Effect of the momentum dependence of nuclear symmetry potential on $\pi^-/\pi^+$ ratio in heavy-ion collisions

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In the framework of the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, effect of the momentum dependence of nuclear symmetry potential on $\pi^-/\pi^+$ ratio in the neutron-rich reaction $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon is studied. We find that the momentum dependence of nuclear symmetry potential affects the compressed density of colliding nuclei, numbers of produced $\pi^-$ and $\pi^+$, as well as the value of $\pi^-/\pi^+$ ratio. The momentum dependent nuclear symmetry potential increases the compressed density of colliding nuclei, numbers of produced resonances $\Delta(1232), N^*(1440)$, $\pi^-$ and $\pi^+$, as well as the value of $\pi^-/\pi^+$ ratio.

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Nowadays pion production in heavy-ion collisions has attracted much attention in the nuclear physics community. The reason for this is that pion production is connected with the high-density behavior of nuclear symmetry energy. The latter is crucial for understanding many interesting issues in both nuclear physics and astrophysics. The high-density behavior of nuclear symmetry energy, however, has been regarded as the most uncertain property of dense neutron-rich nuclear matter. Many microscopic and/or phenomenological many-body theories using various interactions predict that the symmetry energy increases continuously at all densities. However, other models predict that the symmetry energy first increases to a maximum and then may start decreasing at certain suprasaturation densities. Thus, currently the theoretical predictions on the symmetry energy at suprasaturation densities are extremely diverse. Therefore, what is crucially needed for the first step is a qualitative observable to probe whether the symmetry energy at high densities is soft or stiff. This has been done originally by qualitatively comparing the n/p ratio of light and heavy preequilibrium emitting clusters. Making further progress in determining the n/p ratio of light and heavy preequilibrium emitting clusters needs some guidance from dialogues between experiments and transport models, which have been done extensively in the studies of nuclear symmetry energy at low densities.

To use $\pi^-/\pi^+$ to probe the high-density behavior of nuclear symmetry energy has evident advantage within both the $\Delta$ resonance model and the statistical model. Several hadronic transport models have quantitatively shown that $\pi^-/\pi^+$ ratio is indeed sensitive to the symmetry energy, especially around pion production threshold. These transport models, however, usually use different momentum dependent interactions among nucleons. The importance of the momentum dependence of nuclear symmetry potential was seldom mentioned in other transport models except our transport model used in the present studies. In the framework of the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, as an example, we studied the effect of the momentum dependence of nuclear symmetry potential on $\pi^-/\pi^+$ in the neutron-rich reaction of $^{132}\text{Sn}+^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon. It is found that the momentum dependence of nuclear symmetry potential affects the compressed density of colliding nuclei, numbers of produced $\pi^-$ and $\pi^+$, as well as the value of $\pi^-/\pi^+$ ratio.
by recent studies based on FOPI experimental data [1].
The main characteristic of the present single particle is
the momentum dependence of nuclear symmetry potential,
which has evident effect on energetic free n/p ratio
in heavy-ion collisions [14]. But the momentum depen-
dence of nuclear symmetry potential on π⁻/π⁺ ratio
was seldom reported. In this note we study the momentum
dependence of nuclear symmetry potential on π⁻/π⁺ ra-
tio. We keep the isoscalar part of nuclear symmetry potential to
momentum independent symmetry potential and keep the symmetry energy fixed [44].
The reaction channels related to pion production and ab-
sorption are

\[
\begin{align*}
NN &\rightarrow NN, \\
NR &\rightarrow NR, \\
NR &\rightarrow NN, \\
R &\rightarrow \Delta \pi ,
\end{align*}
\]

where \( R \) denotes \( \Delta \) or \( N^* \) resonances. In the present
work, we use the isospin-dependent in-medium reduced
NN elastic scattering cross section from the scaling
model according to nucleon effective mass [46][49]. For
in-medium NN inelastic scattering cross section, we use
the forms in free space since it is quite controversial.

![FIG. 1: (Color online) Evolution of the central baryon den-
sity for the central reaction \( \text{132Sn}^+\text{124Sn} \) at a beam energy of
400 MeV/nucleon with and without momentum dependence
of nuclear symmetry potential, signed with MDI and MIDI,
respectively.](image1.png)

Fig. 1 shows the effect of the momentum dependence of
nuclear symmetry potential on the central baryon density of
colliding nuclei. It is seen that the maximum baryon
density is about 2 times normal nuclear matter density.
Moreover, the compression is sensitive to the momentum
dependence of nuclear symmetry potential. The momentum
dependence of nuclear symmetry potential makes

the nuclear matter less compressed whereas the momentum
dependence of nuclear symmetry potential causes a
larger compression.

![FIG. 2: (Color online) Evolution of \( \pi^- \), \( \pi^+ \) and \( \Delta(1232) \),
\( N^*(1440) \) multiplicities in the central reaction \( \text{132Sn}^+\text{124Sn} \) at
a beam energy of 400 MeV/nucleon with and without mo-
mentum dependence of nuclear symmetry potential, respec-
tively.](image2.png)

To see more clearly effect of the momentum depen-
dence of nuclear symmetry potential on pion produc-
tion, we show in Fig. 2 the multiplicities of \( \pi^+ , \pi^- \) and
\( \Delta(1232) \), \( N^* \) as a function of time. We can first see that,
owing to small compression with momentum indepen-
dent symmetry potential shown in Fig. 1, the momentum
independence of nuclear symmetry potential decreases
the productions of resonances, especially \( \Delta(1232) \). More
\( \Delta(1232) \) resonances are produced than \( N^* \). This is be-
because \( N^*(1440) \) is related to more energetic collisions.
Second, we see that numbers of produced charged pi-
os are also reduced with the momentum independent
symmetry potential, especially for \( \pi^- \). In the studies,
the usage of momentum dependence of nuclear symmetry
potential increases charged pions about 30%. We
also made simulations of changing nuclear incompress-
ibility (\( \delta K \sim 20 \)) and find that the isoscalar part of the
Equation of State has little effects on charged pion yields.
But the momentum dependence of the isoscalar part of the
Equation of State also has evident affection on pion
yields [50]. All these results indicate the importance of
the momentum dependence of nuclear potential on the
studies of pion production.

Shown in Fig. 3 is effect of the momentum dependence of
nuclear symmetry potential on the (\( \pi^- / \pi^+ \) like) ratio as
a function of time in the central reaction \( \text{132Sn}^+\text{124Sn} \) at
a beam energy of 400 MeV/nucleon. In the dynamics of
pion resonance productions and decays the (\( \pi^- / \pi^+ \) like)
reaction ratio reads

$$\frac{\pi^{-}/\pi^{+}}{\text{like}} = \frac{\pi^{-} + \Delta^{-} + \frac{2}{3}\Delta^{0} + \frac{2}{3}N^{0} + \frac{2}{3}N^{+}}{\pi^{+} + \Delta^{+} + \frac{2}{3}\Delta^{0} + \frac{2}{3}N^{0} + \frac{2}{3}N^{-}} \quad (3)$$

This ratio naturally becomes $\pi^{-}/\pi^{+}$ ratio at the freeze-out stage. From Fig. 3 we can first see that sensitivity of $(\pi^{-}/\pi^{+})_{\text{like}}$ ratio to the effect of the momentum dependence of nuclear symmetry potential (MDI), $\pi^{-}/\pi^{+}$ ratio is higher than that with the momentum independent symmetry potential (MIDI), the effect of the momentun dependence of nuclear symmetry potential in this study is about 7.4%. The $\pi^{-}/\pi^{+}$ ratio with the momentum dependent symmetry potential is higher than that with the momentum independent symmetry potential is consistent with the free n/p ratio's momentum dependence of nuclear symmetry potential [44]. With the momentum dependent symmetry potential, free n/p ratio is lower than that with the momentum independent symmetry potential. The n/p ratio of dense matter thus has a opposite situation, i.e., with the momentum dependent symmetry potential, dense matter's n/p ratio is higher than that with the momentum independent symmetry potential. According to both the $\Delta$ resonance model and the statistical model [39, 40], with the momentum dependent symmetry potential, $\pi^{-}/\pi^{+}$ ratio is higher than that with the momentum independent symmetry potential.

Therefore, if the published PRL paper [1] of Xiao et al. uses a momentum independent symmetry potential, according to the experimental data, the resulting symmetry energy will be more soft. The whole physical result of that PRL paper [1] still does not change, just more soft. To study the momentum dependence of nuclear symmetry potential, high p$_{\text{t}}$ or kinetic energy’s neutron to proton ratio or light over heavy cluster’s n/p ratio may be useful [33, 41].

In conclusion, based on the isospin-dependent Boltzmann-Uehling-Uhlenbeck transport model, effect of the momentum dependence of nuclear symmetry potential on $\pi^{-}/\pi^{+}$ ratio in the neutron-rich reaction $^{132}\text{Sn} + ^{124}\text{Sn}$ at a beam energy of 400 MeV/nucleon is studied. It is found that momentum dependent nuclear symmetry potential increases the compressed density of colliding nuclei, numbers of produced resonances $\Delta(1232)$, $N^{*}(1440)$, $\pi^{-}$ and $\pi^{+}$, as well as the value of $\pi^{-}/\pi^{+}$ ratio. It is therefore necessary to consider the momentum dependence of nuclear symmetry potential while studying the effect of nuclear symmetry energy by using heavy-ion collisions.

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