Development of Site Index Curves and Height-DBH Growth Model of *Larix kaempferi* for Deogyu Mountain in South Korea

Hyun-Jun Kim, Hyun-Soo Kim, Se-Ik Park, Hee-Jung Park and Sang-Hyun Lee

*BK21 Plus Eco-Leader Education Center, Korea University, Seoul 02841, Republic of Korea; Department of Environmental Science & Ecological Engineering, Korea University, Seoul 02841, Republic of Korea; Department of Forest Environmental Science, Chonbuk National University, Jeonju 54896, Republic of Korea*

**ABSTRACT**

In this study, we aimed to develop site index curves and a height growth model using forest data with the measured height, diameter at breast height (DBH), and age targeting a *Larix kaempferi* stand located on Mt. Deogyu in Muju, Jeollabuk-do. The site index estimation was calculated using Chapman-Richards equation. The coefficients were found to be $\alpha = 20.7837$, $\beta = 0.0556$, $\gamma = 1.6607$, and MSE = 1.2034. The height growth model of the dominant trees depending on the DBH was determined to be $\alpha = 0.4872$, $\beta = 0.3687$, and MSE = 1.8212 using the Petterson growth model. The Mean residual values for above two models were analyzed as 0.00004 and −0.00007, respectively. When comparing the skewness and kurtosis of the residual, the skewness values were 0.1659 and −0.1272, and the kurtosis values were −0.2310 and −0.5543. Therefore, the fitness was considered to be good because they were close to the normal distribution. The results of Shapiro-Wilk test showed $P$-values (0.77 for the height growth model and 0.62 for the height-DBH model) significantly higher than the alpha level of 0.05, and the residual form of the estimated values for the measured values showed uniform distribution indicating that the fitness of the two models was excellent.

**Introduction**

Currently, Korean forests are primarily composed of 30–60 year-old trees by large-scale forestation projects completed in the 1960s–1970s. As a result, the owners of these mountain areas were able to create profits through wood production centered forest management. Therefore, forest management methods that can be applied to private and public forests are urgently required for more rational forest management. For the development of forest management plans for these private forests, the gradual development of low cost and high efficiency methods should be made by operating profit creation models suitable for each local forest status on a trial basis in national forests. These developed methods could play a pioneering role in private forest management for the development of forest management plans and management skills, and applying them to public and private forests (Min et al. 2013).

In particular, the Korea Forest Service introduced a responsibility management system in 2012 that manages various tasks from seeds and seedlings, to lumbering, to use by electing management experts. Six management complexes, sized slightly more or less than 5 thousand ha in national forests, were designated to manage forests systematically and expertly due to the rising importance of national forest management.

This management involves the protection of forest ecosystems, sustainable forest management, and the optimal display of various forest functions.

In addition, the Korea Forest Service established an economic forest development plan in 2002 to strengthen the role of lumber storage sites, which is one of the major functions of national forests. This efficient and systematic management has achieved more stable lumber production and an increase in lumber self-sufficiency, by designating 450 complexes sized at approximately 292 million ha as economic forest development complexes in 2005.

Currently, *Larix kaempferi* has the most potential as timber in Korea. *L. kaempferi* was a main afforestation species in the 1960s to 1970s, and approached its final age (60 years for national forest, 40 years for private forest), it is expected that the final cutting area will expand continuously and reforestation projects targeting the *L. kaempferi* final cutting areas will also increase significantly (Yang et al. 2013). Recently, Lee et al. (2017) developed the stem volume equations for *L. kaempferi* of eastern part of Korea. Therefore, the target tree species was determined as *L. kaempferi* and this study was carried out targeting the *L. kaempferi* forest in the Deogyu Mountain Trial Management Plan Area under the jurisdiction of the Muju National Forest Management Office in Korea.
The diameter at breast height (DBH) and height are the most important research subjects in forest resources surveys, and they are essential components in the interpretation of the stem volume, growth, and stand structure of forest trees. Since height measurements are relatively difficult to obtain and require a lot of time and money, DBH of all of the sample trees is measured, but it is common to measure only a portion of the heights when examining a fixed sample area and temporary sample area (Curtis 1967; Huang and Titus 1994; Moore et al. 1996; Huang et al. 2000; Peng et al. 2001, Avery and Burkhart 2002; Korea Forest Research Institute 2008).

Therefore, this study aimed to develop a height growth model and site index curves using the measured DBH, height, and age data. This study was carried out to provide basic data for developing rational forest management plans and management skills through more accurate prediction of the growth amount by using the developed model.

Materials and Methods

Study materials

In this study, we developed a height growth model and site index curves targeting the *L. kaempferi* stands located on Mt. Deogyu in Jeollabuk-do, Korea (Figure 1). 10 temporary sample plots of 20 m × 20 m were installed in three compartments where *L. kaempferi* is growing (Table 1). The survey of the trees was carried out to measure the DBH and height in April, 2014. In addition, a total of 100 sample trees (10 trees per plot) were selected by considering the diameter, and a core was collected at breast height to analyze stand age measuring the tree rings for each individual sample tree.

Annual mean temperature and precipitation of the study site is 10.8 °C and 1,251 mm, respectively. There is a mountainous climate with severe day and night temperature differences. For the study site conditions, soil texture is sandy loam, and number of trees is approximately 360 trees per ha. Means of stand age, DBH, and height were 47 years, 27 cm, and 18 m, respectively, and altitude is about 510 m.

Study methods

(1) Site index assessment

For the site assessment, the statistics program SAS ver. 9.3 was used for the height growth of the dominant trees depending on their age using the Modified Chapman-Richards equation. Richard (1959) published the Chapman-Richards growth equation for animal growth, and Pienaar and Turnbull (1973) applied this equation to forestry. This equation has become a...
popular model for describing variables of individual tree and forest stand growth such as height, DBH, basal area, and volume (Zeide 1993). As a primary standard statistical method, the non-linear least squares regression method of the PROC NLIN procedure was used. In the case of the applied dominant tree height regression method of the PROC NLIN procedure was standard statistical method, the non-linear least squares basal area, and volume (Zeide 1993). As a primary tree and forest stand growth such as height, DBH, popular model for describing variables of individual

$$Ho = \alpha \cdot (1 - EXP(-\beta \cdot age))^\gamma$$  \hspace{1cm} (1)  

where, $Ho$ is the dominant tree height and $age$ is the stand age. If substituting the standard age for the derived age in the dominant tree height curve, then a site index equation, such as Equation (2), can be derived. If dividing Equation (1) into Equation (2), then a site index classification curve equation that can be used to derive various height curves using the site index (SI), standard age ($A_SI$) and current age (age), such as Equation (3), can be derived.

$$SI = \alpha \cdot (1 - EXP(-\beta \cdot A_SI))^\gamma$$  \hspace{1cm} (2)  

$$Ho = SI \cdot \left(1 - \frac{1 - EXP(-\beta \cdot age)}{1 - EXP(-\beta \cdot A_SI)}\right)^\gamma$$ \hspace{1cm} (3)  

(2) Height curve equation development of dominant trees depending on the DBH

Of the forest growth factors, diameter is known to be the most greatly affected by density, while height, especially the height of the dominant trees, is barely affected by density (Kim 2015). However, the growth relationship between the diameter and height is very important because the forest growth is basically affected by the density and the site. Therefore, we analyzed the relationship between the diameter and height of the dominant trees in this study in order to find a correlation between the density and the site. In general, the Petterson, Kennel, and Michailow equations are the most commonly used in models in order to predict the height growth depending on the growth of the DBH. Among them, the Petterson equation was analyzed to be relatively good for fit index (FI) values and the standard errors from the results of previous study (Kim 2015) as below equation (4).

$$H = 1.2 + \left(\frac{DBH}{\alpha + \beta \cdot DBH}\right)^3$$ \hspace{1cm} (4)  

(3) Evaluation of height growth models

The Shapiro-Wilk normality test was used to evaluate the fitness of the height growth models developed from this study. This test is a test of normality in frequentist statistics and its statistic is as below equation (5) (Shapiro and Wilk 1965).

$$W = \frac{\left(\sum_{i=1}^{n} d(x(i))\right)^2}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$ \hspace{1cm} (5)  

where, $x(i)$ is the $i$th order statistic, and $\bar{x}$ is the sample mean.

Results and Discussions

Coefficient estimation of the functional formula

In order to analyze the growth of the *L. kaempferi* stand located on Mt. Deogyu in Muju, Jeollabuk-do, the dominant trees height ($Ho$) growth pattern was examined. The site index (SI), showing the production capacity of the forest land as figures, was estimated from the height of the dominant trees. The height growth curve equation of the dominant trees depending on the DBH was estimated in order to indirectly examine the relationship between the density and the site. The resulting coefficients of each functional formula are shown as Table 2.

In addition, the basic statistics for the residual plots of each functional formula were analyzed and are shown in Table 3. The uncertainty of the height growth model of the dominant trees depending on age and DBH was found to be very low because the averages of the residuals were close to 0, 0.00004 and $-0.00007$, respectively, and the Shapiro-Wilk values were determined to be 0.9798 and 0.9849, respectively. The residual distribution statistics were determined to be close to the normal distribution.

Assessment of the site index

The site index estimation of the *L. kaempferi* forest on Mt. Deogyu in Muju, Jeollabuk-do, was calculated using Chapman-Richards equation. In Korea, a mean age of 20 or 30 years is used as the standard age for the site index, but there are no separately determined criteria for the standard age (Kim 2015). In addition,
the mean age of the *L. kaempferi* forests in Korea is slightly more or less than 50 years old. Thus far, there has not been a problem in using the standard age of the site index as 20 or 30 years, but the Korean mean age is currently increasing and the forest area ratio of V age-class is increasing. Hence, it was determined that the standard age of the site index also needed to be increased. Therefore, given that the mean age of the study area was calculated as 46 years, the site index standard age of *L. kaempferi* forest in Mt. Deogyu Mountain was determined to be 50 years old.

For Chapman-Richards equation, regression analysis using the statistics program SAS ver. 9.3 was carried out for the raw data of the sample trees and the results were analyzed as shown in Figure 2. The Ho growth model was accurate with $R^2 = 0.84$ and $P < 0.0001$ (Figure 2a), and also there was a strong correlation between the model’s predictions and its actual results (Figure 2b). However, the prediction for the initial growth of this model could not be considered to be correct because the age of the sample trees was generally more than 20 years. Therefore, it was seemed that studies on the early growth of *L. kaempferi* or studies with stem analysis should be carried out to make the growth model more accuracy for the initial growth. Generally, the growth pattern of young trees has a different shape compared to that of mature trees (Kim 2015).

If the standard age was substituted for the age in the height growth curve equation, the height of the dominant trees’ mean site index could be estimated. If the height growth curve equation and the site index curve equation were expressed as an algebraic difference equation, then a site index classification curve equation that could predict the site index, standard age, and height of the dominant trees by site index by age could be derived. The mean site index was analyzed through the height growth model of the dominant trees depending on the age, and as a result, the site index of the target area in this study was found to be 18.2. Therefore, the site index of the *L. kaempferi* forest on Mt. Deogyu in Muju, Jeollabuk-do, was determined to be 18 and the site index classification curves were derived into 5 classes such as 14, 16, 18, 20, and 22 (Figure 3).

In the Figure 3, the site index of the older ages, such as 80, 90, and 100, can be predicted with a function (3), but it is not realistic predicting the future values with the function where does not compass real data. It means that more future data will be required for the entire site index. In this study, the basic Chapman-Richards function was used to predict the height of dominant trees and site index the *L. kaempferi* stand. However, to increase precision of the model, the environmental factors, such as temperature, altitude, rainfall and soil types should be contained as independent variables. These factors will affect the change of asymptote ($a$) and shape ($b$) parameters.

**Height-DBH growth model estimation of dominant trees**

By performing a non-linear regression analysis with the NLIN procedure of the statistics program SAS ver.
9.3, we were able to analyze the relationship between the DBH growth and the height growth of the dominant trees. A height curve equation for the DBH of the Petterson equation was derived and as a result, the pattern was shown in Figure 4. This figure shows a typical height curve form in which the height growth is rapidly increasing when the DBH is small and the height growth rate is decreasing as the DBH is increasing (Figure 4a). Therefore, the growth status of the stand was determined to be stable because the height growth was gradually slowing down as the age was increasing, while the diameter growth was continuously promoted and the Ho/DBH value of the stand was lowered. The predicted values for Ho showed a strong correlation with its actual values (Figure 4b). The linear regression equation describing the relationship between the predicted and actual values for Ho had of $\alpha = 13.6041$ and $\beta = 0.2553$.

In these plots (Figure 4c, d), the actual values for DBH and the predicted values for Ho made by the Ho vs DBH model are on the x-axis, and the accuracy of the prediction is on the y-axis. The distance from the line at 0 is how bad the prediction was for that value. Since the residual means the observed values minus the predicted values for Ho, the prediction was within 3-m compared with the actual values for Ho. Also, they were pretty symmetrically distributed, tending to cluster towards the middle of the plot, and did not show any clear pattern or trend in the residuals. Thus, this model can be considered as appropriate.

The result of this species in the Muju regions showed that a part of the present management situation of the national forest in Korea which is occupied of about 24% of total forest areas. That means the national forest is well managed and maintained for the production of economically superior timbers and ecologically stabilized forests through the project of forest tending in Korea.

Conclusions

In this study, we estimated the height growth model of the dominant trees depending on the age and the DBH in order to provide basic data for rational and effective forest management targeting the L. kaempferi stands growing on Mt. Deogyu in Muju, Jeollabuk-do. We analyzed the site quality of Mt. Deogyu for L. kaempferi by deriving site index classification curves from the height growth model.

The height growth model of the dominant trees depending on the age was estimated by using Chapman-Richards equation, and the site index classification curve equation was derived by substituting the standard age for the age in the dominant trees’ height curve using the alternative-form method. At this time, the coefficients were found to be $\alpha = 20.7837$, $\beta = 0.0566$, $\gamma = 1.6607$, and MSE = 1.2034. When the standard age of the study area was 50 years, the site index of the L. kaempferi was determined to be 18. As the height growth model of the dominant trees was dependent on the DBH, the Petterson stand growth model was used and the coefficients were determined to be $\alpha = 0.4872$, $\beta = 0.3687$, and MSE = 1.8212.

The residual distribution and statistical values for these two models were analyzed and as a result, the means of the residuals were determined to be 0.00004 and −0.00007, respectively, and the height could be estimated in the scope of a 0.001 mean error. When comparing the skewness and kurtosis of the residuals, the skewness values were determined to be 0.1659 and 0.1272, and the kurtosis values were −0.2310 and −0.5543. These values were close to the normal.
distribution and therefore, the fitness was considered to be good. In addition, P-values of the Shapiro-Wilk normality test were found as 0.77 and 0.62. The normal distribution and the residual form of the estimated values for the actually measured values showed homogeneous distribution, indicating that the fitness of the two models was excellent, especially for the *L. kaempferi* stand in Mt. Deogyu.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

**ORCID**

Hyun-Jun Kim [http://orcid.org/0000-0002-7373-1643](http://orcid.org/0000-0002-7373-1643)

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