Students’ perceptions of mathematics writing and its impact on their enjoyment and self-confidence

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There have been universal endorsements of the benefits of writing as an effective medium of communicating mathematically. Writing and learning are seen as isomorphic to each other and writing can facilitate the comprehension of mathematical thinking through intrapersonal communication. Through a short writing intervention, this study investigates students’ perceptions on the use of writing in the mathematics classroom and explores the impact of writing on students’ affective domains of self-confidence and enjoyment levels in mathematics. A mixed-methods approach was employed using a pre-test, intervention, post-test design for the study. Quantitative data were collected through a questionnaire adapted from the Attitudes Towards Mathematics Inventory (Tapia & Marsh, 2004), which was administered before and after the intervention. An analysis of the quantitative data revealed a significant increase in students’ mean scores for both enjoyment and self-confidence. Qualitative data collected in the form of students’ reflections of the writing intervention indicated that, overall, students had a positive perception of writing as a means of communicating in the mathematics classroom.

1. Background to the study
In India, the National Curriculum Framework (2005) describes mathematical communication as an important feature of any mathematical undertaking. It is recommended that mathematical communication which uses unambiguous and precise language is crucial for developing an appreciation of the subject. Matsuura et al. (2013) determined that such endeavours can be effective in the development of mathematical thinking and habits of mind. ‘These habits are not about particular definitions, theorems, or algorithms that one might find in a textbook; instead, they are about the thinking, mental habits, and research techniques that mathematicians employ to develop such definitions, theorems, or algorithms’ (Matsuura et al., 2013, p. 736). Despite such recognized importance, there has been no research to date investigating the issue of mathematical writing in India. This present study is the first of its kind to be carried out in the Indian context.
Mathematics has evolved significantly over the past number of decades and so have the teaching practices associated with it. Various reforms have been implemented worldwide from time to time, with a focus on differing strategies to enhance deeper mathematical understanding and engagement among students. One feature of mathematical reform that has been under continuous scrutiny is ‘mathematical communication’. It has been suggested by the educational policies of almost all countries that it is necessary for students to be able to communicate mathematically and that mathematical communication should be at the heart of mathematical teaching. For example, in the USA, the National Council of Teachers of Mathematics (1989, 2000) called for mathematical literacy for all, stressing the need for mathematical communication. Similarly, India’s National Curriculum Framework (2005) states that ‘Children see mathematics as something to talk about, to communicate through’ (p. 43). In Ireland, a recent reform of lower secondary level education has identified communication as one of the key skills of the entire curriculum. More specifically, communication also underpins a unifying strand across the Irish mathematics syllabus. The syllabus states that ‘Students should be able to communicate mathematics effectively in verbal and written form’ (NCCA, 2018, p. 9).

With specific reference to writing as the mode of communication, there has been increasing interest in recent years towards its role in mathematics classrooms. While participation in discourses and debates of mathematics for improved mathematical learning has been frequently emphasized (Burton & Morgan, 2000), formal recommendations for mathematical writing are less explicitly available (Casa et al., 2016). Despite this, studies and interventions have consistently reported the benefits of writing in learning mathematics (Fry & Villagomez, 2012; Knox, 2017; Kostos & Shin, 2010; Kuzle, 2013; Pugalee, 2001).

1.1 Mathematical writing—the perceived benefits

Bangert-Drowns et al. (2004) determine that writing and learning are isomorphic to each other. In all aspects of life, writing serves as a psychologically powerful instrument, providing a vent for thoughts through deep reflection and understanding (Hacker et al., 2009). In the classroom, writing assists students in constructing new knowledge through activities such as exploration, representation, investigation and justification (Countryman, 1992). Being a planned and conscious process, it strengthens current knowledge while building new connections at the same time (Kenney et al., 2013; Kuzle, 2013). Pugalee (2001) notes that in order to express one’s thoughts through writing, inner speech has to be compressed maximally by connecting current knowledge to the new knowledge gained. With specific reference to mathematics, various studies have focused on the effects of mathematical writing in the mathematics classroom. Such research has revealed enhanced metacognitive thinking, self-confidence and enjoyment levels, which leads to increased mathematical achievement (Knox, 2017; Kostos & Shin, 2010; Kuzle, 2013; Pugalee, 2001).

1.2 Writing and the affective domains

Emotions are fundamental to learning and can affect students’ thought processes as well as memory (Hinton et al., 2008; Westen, 1999). For example, threatening situations, such as peer competition, parental pressure, exam stress, image in front of teacher, etc., can affect learning in a negative way (Wolfe & Brandt, 1998). Writing aids in dealing with such issues and may help bring positive changes in the affective constructs such as self-confidence and enjoyment (Countryman, 1992).

Several studies have revealed that mathematics learning is highly influenced by learners’ mathematics-related beliefs, especially self-confidence (Hannula & Malmivuori, 1997; Hannula et al., 2004). In fact,
self-confidence has been observed as the greatest non-cognitive predictor for academic achievement among other self-belief measures such as self-efficacy and self-concept (Stankov et al., 2014). On the other hand, a lack of self-confidence may negatively impact students’ motivation to learn (Boekaerts & Rozendaal, 2010). Engaging students to communicate their mathematical ideas through writing may instil in them a higher level of self-confidence and critical thinking skills (Quitadamo & Kurtz, 2007).

Another important construct is students’ enjoyment while learning. Many studies have evidenced a positive relationship between the level of enjoyment while performing a task and students’ attitudes and self-efficacy beliefs (Ahmed et al., 2010; Lorsbach & Jinks, 1999; Sakiz et al., 2012). These effects are greatly enhanced if students enjoy the tasks they are working on (Bramlett & Herron, 2009). Mathematical writing is one such task that may prove to be an enjoyable class activity, while also adhering to formal curricular demands.

However, little research has been conducted to examine the role of writing in relation to these affective domains (Miller & Meece, 1997). With such a dearth of research in mind, this paper investigates students’ attitudes towards mathematics with regards to their enjoyment and self-confidence after a short writing intervention.

This study also aimed to capture students’ experiences of the writing intervention. It has been argued in the literature that learners’ perceptions of classroom events strongly influence their classroom behaviour and response to teaching approaches (Marx, 1983; Roese & Sherman, 2007; Struyven et al., 2005). Therefore, capturing students’ perceptions may provide useful insights for successful reform efforts. However, at the moment, such insights are not given due consideration (Levin, 2000). This paper thus explores participants’ written reflections of the intervention and investigates their perceptions of it.

2. Theoretical perspectives

Keeping up with the goal of promoting mathematical communication, the Elementary Writing Mathematical Task Force from the University of Connecticut in the USA proposed the following four types of mathematical writing, with their purposes described as follows:

1. Exploratory (to personally make sense of a problem, situation or one’s own ideas)
2. Explanatory/informative (to describe, to explain)
3. Argumentative (to construct or critique an argument)
4. Mathematically creative (to document original ideas, problems and/or solutions, to convey fluency and flexibility in thinking, to elaborate on ideas)

(Casa et al., 2016, p. 4)

It has been suggested by the Task Force that all students should have exposure to all types of writing. However, there is an ambiguity in comprehending the context of writing in relation to mathematics (Bossé & Faulconer, 2008). The Task Force highlighted two categories of writing that take place in mathematics classrooms—‘writing about mathematics’ and ‘mathematical writing’. Writing about mathematics stresses on the learning of literacy skills e.g. a mathematics autobiography, while mathematical writing emphasizes the use of mathematical symbols and vocabulary with an aim to develop mathematical reasoning.

The present study focused on the ‘explanatory’ type of mathematical writing where the purpose is to describe or explain ideas for mathematical reasoning. The study also explored the perceptions of participants for its use in mathematics class. The study aims to address the following research questions:
(1) What effect, if any, does a short mathematical writing intervention have on students’ self-confidence and enjoyment levels in mathematics?
(2) What are students’ perceptions about the use of mathematical writing in the classroom?

3. Methodology

The study used a pre-test, intervention, post-test design to explore students’ perceptions and investigated the impact of a short writing intervention on students’ enjoyment and self-confidence in mathematics. A convergent mixed-methods approach was employed to look into the research phenomena from different viewpoints (Creswell & Plano Clark, 2011), with both quantitative and qualitative data being collected. The instruments used for addressing the research questions were questionnaires and participants’ reflections. The analysis and triangulation of this data enabled a ‘complete, holistic and contextual portrayal’ of the research (Clark & Creswell, 2008, p. 109).

3.1 Study sample

The study was conducted in a secondary co-educational school in New Delhi, India, which is affiliated with the country’s Central Board of Secondary Education. This school caters for approximately 1000 students from Kindergarten to Grade 10. It was selected through a purposive sampling method as its location was in close proximity to one of the researchers and they had existing contacts in the school. Students from a Grade 7 and Grade 8 (aged 12–15 years old) class group were invited to partake in the study and 55 students agreed to participate. The sample was made up of 38 males (69%) and 17 females (31%), with a mean age of 13 years. For all the students, Hindi was their first language and English was their second language. However, the language of instruction for all students in the school was English.

3.2 The intervention

Realizing that students are the primary stakeholders in the field of education and that any educational reform affects them foremost, an intervention was designed to measure the impact of writing on students’ attitudes, especially the affective domains of enjoyment and self-confidence. Six sessions were administered for the intervention, in addition to an introductory session which detailed the purpose of the study. The methodology of the study was also shared at this opening session, ensuring that each student understood the research instruments that would be used. Each session was conducted for a duration of 40 min. These sessions took place during regular school study hours and in the periods allocated for extracurricular activities. This meant that participants’ formal studies were not interrupted by any activities related to this research.

Tasks for the intervention were selected from the Trends in International Mathematics and Science Study (TIMSS) 2011 grade 8 mathematics assessment items. These items manifest various ways of measuring students’ understanding in several content and cognitive domains (see Appendix A for sample tasks). Six tasks were selected, one for each session of the intervention. As this study focused particularly on the ‘explanatory’ genre of mathematical writing, tasks based on cognitive domains of reasoning were specifically selected. The criteria for selection of tasks were based on students’ previous knowledge and their current class group curriculum. Two tasks were based on identification of patterns, one was a geometry problem and the remaining three tasks were multiple choice based (see Appendix A). Each session started with class discussion of a specific task wherein students came up with different ways
of solving the problem at hand. This was then followed by students’ explanation of their understanding and thought processes through writing. The researcher was present throughout each session to offer any assistance if required. No formal assessment was done for the submitted writings. Oral feedback was provided, and suggestions given to improve their writing for the subsequent sessions. One of the tasks from the intervention is specified in Fig. 1.

3.3 Data collection
Quantitative data were collected in the form of questionnaires which were administered both before and after the intervention. The questionnaires were adopted from the Attitudes Towards Mathematics Inventory (ATMI; Tapia & Marsh, 2004). The subscales of enjoyment and self-confidence were selected for the present study. The enjoyment subscale consisted of a total of 10 statements and the self-confidence subscale comprised of 15 statements (see Appendix B). Each statement had alternative response options which were based on a 5-point Likert-type scale. Every positive statement in the questionnaires was scored from 5 to 1, ranging from 5 = ‘strongly agree’ to 1 = ‘strongly disagree’. Negatively worded items were scored in the reverse direction ranging from 1 = ‘strongly agree’ to 5 = ‘strongly disagree’. The maximum score for enjoyment subscale was 50 and for the self-confidence subscale was 75, giving an overall total score of 125.
After the final session of the intervention, qualitative data were collected in the form of participants’ written reflections. This enabled participants to share their experience regarding the use of mathematical writing as a tool for learning. Participants were prompted to reflect and write about whether they liked the intervention and to include their reasoning. Varied opinions emerged as they expressed their perceptions about the use of writing in the mathematics classroom.

3.4 Data analysis

The quantitative data for the study were analysed using SPSS (Statistical Package for the Social Sciences) software. Mean scores for the pre- and post-intervention tests were compared by descriptive analysis. In addition, t-tests were conducted to determine the statistical significance of the findings.

In order to ensure the internal consistency of the quantitative scales, Cronbach alpha coefficients were calculated. The Cronbach alpha coefficients were recorded as 0.83 for the self-confidence subscale and 0.84 for the enjoyment subscale. The alpha coefficient for the overall scale was 0.95. High values of Cronbach alpha coefficients (suggested values greater than 0.70—Nunnally and Bernstein, 1994) indicate good internal consistency and ensure content validity for the data.

The qualitative data were analysed thematically by one of the authors. Following this, a convergence model of triangulation design (Creswell et al., 2007) was used to interpret the results from both sets (qualitative and quantitative) of analysis. This enabled the authors to examine the convergence, consistency or inconsistency of the overall data (Ary et al., 2010). Findings for both types of data are detailed in subsequent sections.

4. Findings

As discussed previously, both quantitative and qualitative data were collected in order to address the research questions. Findings from the quantitative data will be presented first followed by those from the qualitative data.

4.1 Quantitative findings

Descriptive statistics revealed an increase in the mean scores for both the enjoyment and self-confidence scales, from ‘pre-intervention’ to ‘post-intervention’. Figure 2 illustrates a comparison of mean scores on the enjoyment (EN) and self-confidence (SC) subscales.

As illustrated from Fig. 2, the mean enjoyment scores increased from 35 (before the intervention) to 40 (after the intervention) from a total score of 50. The mean self-confidence scores rose from 49 (before the intervention) to 54 (after the intervention) from a total score of 75.

In addition, a paired-samples t-test was conducted to assess the statistical significance of the findings. The differences in enjoyment scores from pre-intervention ($M = 35$, $SD = 7$) to post-intervention ($M = 40$, $SD = 5$), with $t(54) = -10.8$, $p < 0.05$ (two-tailed), were statistically significant. The mean increase in enjoyment scores was recorded as 5 with a 95% confidence interval ranging from 5.9 to 4.1. For the effect size, eta square was calculated and found to be 0.6. As suggested by Cohen (1988), values greater than 0.1 indicate a large effect size. Therefore, it can be concluded that there was a large effect size with a significant increase in enjoyment mean scores recorded from before and after the intervention.

Similarly, for self-confidence, the increase in mean scores from the pre-intervention test ($M = 49$, $SD = 9$) to the post-intervention test ($M = 54$, $SD = 9$), with $t(54) = -9.4$, $p < 0.05$ (two-tailed),
was statistically significant. The 95% confidence interval ranged from 5.9 to 3.8 with an increase of 5 in the self-confidence mean scores. The eta square statistic (0.6) also indicated a large effect size, implying that participants felt greater levels of self-confidence in learning mathematics aided by the writing intervention.

4.1.1 Further analysis. A statement-wise analysis of the two subscales for both the pre- and post-intervention revealed some noteworthy findings. Six out of 10 statements on the enjoyment subscale showed noticeable changes in the responses marked by the participants at both testing points. As an example, for the statement ‘I really like mathematics’, 20 out of 55 students (36% of the participants) recorded their response as ‘strongly agree’ in the post-test as compared to 8 students (14%) in the pre-test.

In the self-confidence questionnaire analysis, 5 out of 15 statements showed clear changes in terms of responses obtained from participants. For example, for the statement ‘I am always under a terrible strain in a mathematics class’, 23 students (42% of the participants) recorded the response ‘strongly disagree’ in the post-test as compared to 9 participants (16%) in the pre-test. Interestingly, for the statement ‘When I hear the word mathematics, I have a feeling of dislike’, there were no responses in favour of ‘strongly agree’ or ‘agree’ in the post-test questionnaire as compared to 9 responses (2 for ‘strongly agree’ and 7 for ‘agree’) in the pre-test.

4.1.2 Gender-based findings. A descriptive analysis was performed to explore for any gender-based differences in the mean scores. Table 1 compares the mean scores of male and female participants in both the pre- and post-intervention tests and these are further illustrated by the line graph in Fig. 3.

A general look at the line graph determines that females scored slightly higher on both the EN and SC subscales for the pre-intervention tests. While this trend continued in the post-intervention tests, the scores between both groups were closer.
To test the statistical significance of these gender-based differences, an independent-samples \( t \)-test was conducted on the mean scores of both males and females for each of the subscales, both before and after the intervention.

Regarding the enjoyment subscale, the tests recorded \( t(53) = -0.5, p > 0.05 \) (two-tailed) for the pre-intervention and \( t(53) = -0.3, p > 0.05 \) (two-tailed) for the post-intervention. For the self-confidence subscale, the tests showed the values as \( t(53) = -0.9, p > 0.05 \) (two-tailed) for pre-intervention and \( t(53) = -0.4, p > 0.05 \) (two-tailed) for the post-intervention. These findings revealed that differences between the scores obtained by the two gender groups were not statistically significant.

4.2 Qualitative findings

An analysis of participants’ reflections provided an interesting glimpse of their thoughts and opinions about writing. Based on responses that emerged from the qualitative data, participants’ reflections were coded into the six themes which are outlined in Table 2. This table also shows the percentage of responses under each theme in the data sample.

As evident from Table 2, 29% of the participants related the writing activity to increased content knowledge and a greater understanding of the mathematics (either directly or indirectly). Many of these responses signalled the importance of connections to prior knowledge using phrases such as ‘I need to...’
**Table 3. Sample responses for participants’ attitude towards writing**

| Theme                                                                 | N = 55 | Sample responses                                                                                                                                 |
|----------------------------------------------------------------------|--------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| Increase in content knowledge/understanding                         | 16     | ‘...by writing, my understanding of the question was clear to me.’  
|                                                                      |        | ‘...good for understanding more...’  
|                                                                      |        | ‘...it is like when you write you understand what you think and where is mistake in your thinking’  
| Support for non-gradation of the writing assignments                | 11     | ‘...no marks ...so I am happy to write’  
|                                                                      |        | ‘...A good thing is that this is not a test...’  
|                                                                      |        | ‘...there are no marks and no report card and also I am learning new’  
|                                                                      |        | ‘...for other tests I am scared, for this I am not’  
| Ease of written explanation as opposed to oral                      | 8      | ‘...speaking to the whole class is difficult...this is good’  
|                                                                      |        | ‘...writing is better than speaking’  
|                                                                      |        | ‘...no one can read your answers. They will not laugh for wrong answer’  
| Improved efficiency in writing                                      | 6      | ‘...i can improve my writing through this’  
|                                                                      |        | ‘...if I learn how to write my thoughts, I will improve my written skills. This is good for me because I want to become a novelist’  
|                                                                      |        | ‘...I learn new words to write’  
|                                                                      |        | ‘...writing maths in English is good as it improves your English also’  
|                                                                      |        | ‘...lengthy activity’  
|                                                                      |        | ‘It takes a long time to write’  
|                                                                      |        | ‘solving is quicker than writing’  
| More time consuming                                                  | 9      | ‘...My English is weak. Solving by maths is good’.  
|                                                                      |        | ‘...if teacher is there to guide me how to write and explain’  
|                                                                      |        | ‘...Writing in English is difficult...’  
| Linguistic barriers                                                  | 5      | ‘know first what is average’ and ‘you have to remember what you have done before’ e.g. a student wrote the following: ‘for solving, I just need to remember the formula but for writing I have to remember the work of [the] previous class also and how it comes’  

Support for non-gradation of the writing assignments ranked second in the list of themes that emerged from the data and was noted in 20% of the respondents’ reflections. Furthermore, 15% of participants expressed that it was easier to explain through writing as opposed to an oral explanation. For example, one student expressed ‘I like it. I know all [the] answers, but I am shy to speak it to all my class. When [the] teacher tells me to explain, I cannot do it. But by writing, I can explain. So, I like it’. Moreover, 11% of participants also noted an ‘improved efficiency in writing’ in general.

However, the lengthy nature of writing activities was a cause of concern for 16% of participants, as could be found in their responses for feedback which included phrases like ‘lengthy activity’, ‘It takes a long time to write’ and ‘solving is quicker than writing’.

Finally, 9% of the participants found the task of writing challenging due to a lack of vocabulary to express themselves. They mentioned that they were more comfortable in solving questions mathematically as opposed to writing explanations using the English language.

Sample responses from participants’ reflections under the above-mentioned themes are presented in Table 3.

There were also some responses where the participants liked the activity provided some conditions were met. Their responses marked the presence of ‘If’, ‘but’, etc. For example,

‘...if teacher is there to guide me how to write and explain’
‘...but not for big questions. You have to write a lot’
‘...only when it is for activity, not for exam’
A full excerpt from a student’s similar response is cited here for reference:

‘I think writing helps to understand more by explaining and also it is a good brain exercise when we have to remember what we done before in the previous class. But in the [examination] paper, we have to solve only and then we get full marks. So, we should not waste time in writing because in this time we can solve many questions if we remember the formula’.

Although this student acknowledged the benefit of writing in terms of understanding more, they preferred giving a procedural solution or stating the answer directly. This student recognized that memorizing formulae is enough for getting good marks in an examination (owing to the particular marking scheme adopted). This raises the question of whether current assessment systems and education policies are really in favour of developing students’ understanding or are merely ranking procedures that reward rote learning.

A detailed discussion of these findings with illustrative examples and links to the research questions and relevant literature is included in the next section.

5. Discussion

In this section, each research question is addressed, and findings discussed with relevant supporting references from the literature.

5.1 Research question 1: what effects, if any, does a short mathematical writing intervention have on students’ self-confidence and enjoyment levels in mathematics?

5.1.1 Impact of the intervention on students’ enjoyment. The analysis of the quantitative data revealed that participants’ mean enjoyment scores increased from 34.79 (prior to the intervention) to 39.82 (after the intervention). Results of a paired-samples \( t \)-test confirmed the statistical significance of this increase. In addition, these findings were supported through participants’ reflections of their perceptions of the writing intervention. Responses such as ‘I like it’, ‘It was enjoyable’, ‘... I like my maths class as this’ indicate participants’ perceived enjoyment and positive attitude towards the writing intervention. A few students even expressed a desire for writing to be a regular feature of their mathematics lessons. For instance,

‘...Can we have it in our daily class also?’
‘... I think it should be weekly or on alternate days’
‘...We should have it in our daily class also’

This enjoyment was facilitated in a number of ways. For example, the role of a comfortable and interactive classroom environment was emphasized in the literature (Firmender et al., 2017; Hidi, 2000) and this was given due consideration throughout this study. The aim was to make students feel stress-free while writing their solution strategies.

5.1.2 Impact of the intervention on students’ self-confidence. Writing can develop greater confidence in mathematics by providing students with opportunities to grapple with mathematical ideas (Powell, 1997). This assertion is particularly relevant to the present study, given the results of the self-confidence questionnaire. The increase in participants’ mean scores for self-confidence from 49.48 (pre-test) to
54.40 (post-test) is statistically significant as confirmed by the paired-samples t-test. The results are also supported by participants’ reflections of the writing intervention.

Participants felt an increase in confidence for a variety of reasons, for example, a greater understanding, improved efficiency in writing, clarity of thoughts, etc. Some excerpts from participants’ reflections are provided in Fig. 4 to highlight the findings specifically in relation to confidence.

The use of phrases such as ‘was sure’, ‘understood more’ and ‘can improve’ in these excerpts are indicative of students’ increased confidence through writing and help to reaffirm the quantitative findings.

The reflection process which occurs during writing provides learners with an opportunity to look at their own thoughts and refine them in accordance with the information to be conveyed. ‘Such acquisition of control and monitoring capabilities engenders in students’ feelings of accomplishment...students develop faith in themselves as learners who are capable of doing and understanding mathematics’ (Powell, 1997, p. 23). According to Bandura (1977), performance accomplishments are the most powerful contributors to one’s self-efficacy beliefs. The confidence gained through these small accomplishments leads to motivation which brings further enjoyment for the task at hand.

5.2 Research question 2: what are students’ perceptions about the use of mathematical writing in the classroom?

A variety of student perceptions about the use of mathematical writing emerged from participants’ reflections. As detailed in Section 4.2, these responses were categorized into themes which will now be discussed considering similar studies from the literature.

5.2.1 Increase in content knowledge/understanding. As mentioned previously, 29% of participants noted an increase in their content knowledge and/or understanding. One participant wrote the following:

‘I don’t want to do maths for marks. This activity I enjoyed because you are doing it for your understanding, not as a paper (exam). I understood more when I solved my answers by writing’.

Participants’ feedback included statements such as ‘able to understand more by writing’, ‘writing helps to understand in a good way’, ‘by writing, there is less confusion for the answer’ and that ‘it will
Fig. 5 Sample of participants’ work.
also clear any doubts that you had since you have to provide reasons on why your answer is correct’.

**Figure 5** presents a students’ work from the six sessions of the intervention, along with the feedback for how the student felt about the writing intervention.

These findings resonate with the results from many other studies which have reported the instrumental role of writing in a greater acquisition of content knowledge through a deeper engagement with the subject (for example, Borasi & Rose, 1989; Craig, 2016; Porter & Masingila, 2000; Pugalee, 2004).

A noteworthy observation was the reflection of thoughts through writing, as shown by the following response.

‘When we write, we can revise it many times and we can know if we get a right or wrong answer. If I see that I am thinking wrong, I can start again with some other method but if we are just explaining orally, you have said all [the] words, and you cannot go back and change your answer. So, I like this part of writing that you can see what you are thinking and change it any time before submitting [the] final answer’.

This excerpt is indicative of the importance of the reflection process that occurs while writing. Even though students may not recognize this on-going process, it is one of the many potential benefits of writing (Craig, 2011; Ray Parsons, 2011). Effective learning occurs while resolving the cognitive conflicts in writers’ minds and results in metacognitive development (Kuzle, 2013; Pugalee, 2004).

### 5.2.2 Support for non-gradation of the writing assignments

In the qualitative data, many participants (20%) attributed the non-grading criterion of the written tasks as one of the reasons that they were comfortable with writing. They commented that while writing, they did not experience fear or anxiety of mathematics and enjoyed the activity without any stress as ‘it was not an exam’. Some participants’ reflections relevant to this are provided in **Fig. 6**.

The literature provides evidence that anxiety regarding grading and assessment not only disrupts students’ capability to reason and understand but also causes a dislike for the subject (Wells, 1994). In other words, the fear of being assessed may obstruct learning from taking place naturally and may hold back students from even attempting various mathematical tasks. This raises concerns regarding the adequacy of current assessment systems which often fail to assess the process and are more focused on the product of learning (Little et al., 2017). Another point of importance here is that participants were more comfortable because their wrong answers or mistakes were not highlighted. This is noteworthy, especially in the domain of mathematics where making mistakes can be an integral part of the learning experience. The fear of making mistakes may inhibit the brain’s growth and capacity to learn and understand (Boaler & Dweck, 2016).
5.2.3 Ease of written explanation as opposed to oral. Among the various benefits of writing is its efficacy to reach out to diverse learners (Bakewell, 2008). This assertion proved to be particularly true for the current study where approximately 15% of participants expressed being more comfortable with written explanations as opposed to oral. There were varied reasons for this response with many signalling a lack of confidence for class interactions. Some excerpts are provided in Fig. 7 to illustrate this.

These findings, as well as others from the literature, highlight that writing is a useful medium for empowering students who feel too shy to take part in class discussions. For example, a year-long study by Fry and Villagomez (2012) in the USA showed that writing helped introverts who seldom took part in class interactions. Students who participated in the writing-to-learn activities of that study showed an increased engagement with the course content. Furthermore, other research (for example, Pugalee, 2004) also notes the benefits of written explanations over oral, thus providing a rationale for writing to be an important vehicle for learning.
5.2.4 Improved efficiency in writing. Another feature perceived by participants in favour of writing was improved competence in their writing skills. Some students (11%) commented that they felt an improvement as they learned new words and gained confidence for writing. A few participants also expressed that they expect to improve further if they keep practicing. Figure 8 provides some of the participants’ reflections under this theme.

These findings are supported by existing literature in the field. It has been reported that an improved use of vocabulary (both in terms of formal mathematical vocabulary as well as the usage of complete sentences and linking words) is an associated advantage of providing reasoning in mathematical writing (Cohen et al., 2015). Rubenstein (2007) contends that in order to communicate mathematically, students must learn how to use correct mathematical language and this learning is supported through writing. Although participants in this study did not report an improvement in mathematics vocabulary, in particular, it could be expected that enjoyment and confidence gained through an improved comprehension in general may enthuse them with a liking for mastering discipline-specific language.

5.2.5 More time consuming. There were mixed responses from respondents in relation to the time-factor. Nine out of 55 students commented that writing takes a longer time and that they preferred giving a direct answer to the problem. Some of these contended that the aim of solving a mathematical task is getting the correct answer and thus viewed writing as a ‘waste of time’. A smaller number also felt that even though writing results in improved learning and is a good brain exercise, the lengthy nature of this activity trumps its benefits and thus, it may be ‘good for some problems but not for all’. Excerpts from some participants’ responses that fall into this category are presented in Fig. 9.

The time-consuming nature of writing, as reported by the participants of this study, has also been reported by many others. In fact, the constraint of time acts as a potential drawback to the implementation of writing in regular teaching (Baxter et al., 2005; McIntosh & Draper, 2001). On the contrary, Porter and Masingila (2000) assert that the success of writing in promoting a deeper mathematical understanding might be primarily due to the increased time that is spent on writing for a given task. In fact, they consider whether the primary contributor in the process is the time spent on the task or the writing itself.

5.2.6 Linguistic barriers. Although English was the language of instruction in the school where this study was carried out, 5 out of 55 students manifested a difficulty in using the English language for their explanations. For these students, a lack of language proficiency hindered their ability to explain
Fig. 10 Sample example related to language.

their reasoning. For example, one student wrote the following: ‘...I know the maths of answer but not English words’. Findings from other studies (for example, Craig, 2011; Porter & Masingila, 2000) confirm the prevalence of such linguistic difficulties for students.

The following reflection (Fig. 10) is noteworthy and is worth mentioning with respect to this theme.

In this instance, although there were no rewards or incentives for the participants, this child wanted to be a good writer. She started learning new words to be more able to express herself. This one excerpt sets an example of how writing may instil a desire to learn more and implies that writing is a beneficial medium for inter-disciplinary learning.

6. Conclusion

This study sought to examine students’ perceptions and explore the impact of mathematical writing on students’ affective constructs of enjoyment and self-confidence. An analysis of the quantitative data revealed an increase in the mean scores for both enjoyment and self-confidence. Results of t-tests confirmed that these increases were statistically significant. A further analysis revealed there to be no gender-related differences.

A thematic analysis of participants’ reflections of the writing intervention also signalled a positive perception towards such activities. Overall, participants gave a positive response towards the intervention and reported an increase in content knowledge/understanding as the main reason. As well as supporting the non-grading of the tasks, some students also noted the ease of written explanations as opposed to oral and an improved efficiency in writing. At the same time, the time-consuming nature of the activity and a lack of proficiency in English language emerged as the factors of concern for a few participants.

In conclusion, findings from this study indicated a progressive shift in students’ attitude post-intervention. Hence, although this was a short intervention with a relatively small cohort, it can be inferred that mathematical writing has the potential to increase students’ enjoyment and self-confidence in mathematics and has a positive impact on their learning. In contrast with the traditional methods of teaching, writing activities in mathematics may serve as an effective medium for transforming students’ mindsets and fostering positive attitudes towards the subject.

However, the benefits of writing are contingent on a host of factors such as the nature of the writing tasks allotted to students, the intensity of intervention by instructors, the students’ ability to exploit its benefits, etc. It is necessary to keep these factors in mind as these may neutralize the positives that can be gained from writing. Furthermore, students and teachers may hold different views about mathematical writing which may affect the quality and nature of writing in a mathematics classroom. In addition, time-bound learning also constrains the integration of writing into classrooms. Future research might gauge the effect of other contextual factors, for example, classroom environment, motivational and constructive feedback, etc. that may aid in bringing out positive changes in students’ affective domains.
while learning. Additional research is also required to investigate how writing, if incorporated into the regular curriculum, may change learners’ as well as teachers’ beliefs about the nature of mathematics.

It is important to keep in mind that the absence of a comparison or control group may affect the generalizability of the results of this study and that the positive results may not be solely due to the intervention. Several other factors such as the non-routine nature of the mathematics tasks, the activity-based sessions, non-grading of work, absence of teacher, etc., could have contributed to the findings. Nonetheless, the results of the study point to the potential of mathematical writing to be used as an effective scaffolding tool for students’ mathematics learning.

Additionally, the effect of various other contextual factors cannot be ignored. There is a chance that participants’ enhanced enjoyment and self-confidence in this study could have been affected by the nature of the research context itself. For example, participants may be keen to impress and please the researcher which might also have acted as a restraining factor affecting the results of the study in a positive or negative manner. Activity-based intervention with no assessment involved may also have contributed to the heightened enjoyment and reduced levels of anxiety in participants.

Finally, the study has emphasized a domain of mathematics education where there is a dearth of research. It has thus added value to the existing body of research and is particularly enriching from an Indian context. In terms of future educational policy, it may prove to be helpful and act as a starting point for further research in India and indeed in other education systems around the world.

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A. Sample tasks from the writing intervention

(All tasks were selected from TIMSS (2011) 8th-Grade Mathematics Concepts and Mathematics Items
SOURCE: TIMSS 2011 Assessment. Copyright © 2013 International Association for the Evaluation of
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Task 1
Here is a pattern:
3–3 = 0
3–2 = 1
3–1 = 2
3–0 = 3
What will the next line in the pattern be? (Item number M042186, Cognitive Domain—Reasoning).

Task 2
The results of a long jump competition were reported as follows:

| Team   | Average length |
|--------|----------------|
| Team A | 3.6 m          |
| Team B | 4.8 m          |

There was the same number of students in each team. Which statement about the competition MUST
be true?

A. Each student in team B jumped farther than any student in team A.
B. After every student in team A jumped, there was a student in team B who jumped farther.
C. As a group, team B jumped farther than team A.
D. Some students in team A jumped farther than some students in team B.

(Item number M042269, Cognitive Domain—Reasoning)

B. Student questionnaire

EN1. I get a great deal of satisfaction out of solving a mathematics problem.
EN2. I have usually enjoyed studying mathematics in school.
EN3. Mathematics is dull and boring.
EN4. I like to solve new problems in mathematics.
EN5. I would prefer to do an assignment in mathematics than to write an essay.
EN6. I really like mathematics.
EN7. I am happier in a mathematics class than in any other class.
EN8. Mathematics is a very interesting subject.
EN9. I am comfortable expressing my own ideas on how to look for solutions to a difficult problem
in mathematics.
EN10. I am comfortable answering questions in mathematics class.
SC1. Mathematics is one of my most dreaded subjects.
SC2. My mind goes blank and I am unable to think clearly when working with mathematics.
SC3. Studying mathematics makes me feel nervous.
SC4. Mathematics makes me feel uncomfortable.
SC5. I am always under a terrible strain in a mathematics class.
SC6. When I hear the word mathematics, I have a feeling of dislike.
SC7. It makes me nervous to even think about having to do a mathematics problem.
SC8. Mathematics does not scare me at all.
SC9. I expect to do fairly well in any mathematics class I take.
SC10. I am always confused in my mathematics class.
SC11. I have a lot of self-confidence when it comes to mathematics.
SC12. I am able to solve mathematics problems without too much difficulty.
SC13. I feel a sense of insecurity when attempting mathematics.
SC14. I learn mathematics easily.
SC15. I believe I am good at solving mathematics problems.

(ATMI; Tapia & Marsh, 2004)