Excited $D_s$ (and Pentaquarks) in Chiral Perturbation Theory *

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(Dated: March 26, 2022)

Abstract

I present results of a heavy hadron chiral perturbation theory analysis of the decays and masses of the recently discovered excited charm mesons. The present data on the electromagnetic branching ratios are consistent with heavy quark symmetry predictions and disfavor a molecular interpretation of these states. I also discuss model independent predictions for the strong decays of pentaquarks in the $\overline{10}$ representation of $SU(3)$ which can be used to constrain the angular momentum and parity quantum numbers of these states.

* Presented at Cracow Epiphany Conference on Hadron Spectroscopy, 6-8 January 2005, Cracow, Poland
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I. INTRODUCTION

In recent years there has been a resurgence in hadron spectroscopy as many experiments have reported evidence for new hadrons. Examples include excited charm strange mesons $D_s(2317)$ [1] and $D_s(2460)$ [2], their nonstrange partners [3, 4, 5], the exotic pentaquarks $\Theta^+$ [6], $\Xi^-$ [7] and $\Theta_c(3099)$ [8], the new charmonium state $X(3872)$ [9, 10, 11] and doubly charm baryons [12]. The status of these various hadrons varies greatly. For example, the $D_s(2317)$ and $D_s(2460)$ are firmly established [13] while the existence of pentaquarks is quite controversial.

In this talk, effective field theory methods are used to obtain model independent predictions for the properties of the excited charm mesons as well as pentaquarks. These predictions yield qualitative insight into the nature of the novel states. Heavy hadron chiral perturbation theory (HH$\chi$PT) [14, 15, 16], which synthesizes heavy quark symmetry for heavy mesons and the spontaneously broken chiral symmetry which governs the low energy interactions of pions, can be used to make predictions for the electromagnetic and strong decays of the $D_s(2317)$ and $D_s(2460)$. These predictions can be used to test the hypothesis that the $D_s(2317)$ and $D_s(2460)$ are molecular bound states of $DK$ and $D^*K$, respectively [17]. I also discuss the puzzle of the $SU(3)$ splittings of the excited states and attempts to address the problem within HH$\chi$PT [18, 19]. The successful prediction of parity doubling models [20, 21, 22, 23] that the hyperfine splittings of the excited and ground state heavy meson doublets are equal is shown to be robust at the one-loop level [19]. Finally, heavy baryon chiral perturbation theory [24] is extended to include pentaquarks and used to make parameter free predictions for certain ratios of two-body decays which constrain the angular momentum and parity quantum numbers of the exotic states [25].

II. ELECTROMAGNETIC AND STRONG DECAYS OF $D_s(2317)$ AND $D_s(2460)$

The discovery of $D_s(2317)$ [1] and $D_s(2460)$ [2] came as a surprise because quark models [26, 27] as well as lattice calculations [23, 28, 30] predicted that these states would lie above the threshold for kaon decays. If the $J^P = 0^+$ and $J^P = 1^+$ charmed strange mesons were above this threshold, they would have been rather broad resonances. In fact, the $D_s(2317)$ and $D_s(2460)$ are about 40 MeV below the threshold for decay into $DK$ and $D^*K$, respectively. The only kinematically allowed strong decays are $D_s(2317) \to D_s\pi^0$ and $D_s(2460) \to D_s^*\pi^0$, which violate isospin. Therefore, the states are quite narrow: $\Gamma[D_s(2317)] < 4.6$ MeV and $\Gamma[D_s(2460)] < 5.5$ MeV [13]. Allowed electromagnetic decays are

$$D_s(2460) \to D_s^*\gamma, \quad D_s(2460) \to D_s\gamma, \quad D_s(2317) \to D_s^*\gamma,$$

while the decay $D_s(2317) \to D_s\gamma$ is forbidden by angular momentum conservation.

To date only the decay $D_s(2460) \to D_s\gamma$ has been observed. Belle has observed the decay $D_s(2460) \to D_s\gamma$ from $D_s(2460)$ produced in the decays of $B$ mesons [31] and from continuum $e^+e^-$ production [32]. The BaBar experiment has also recently reported observing this decay [33]. The electromagnetic branching ratio obtained by averaging the three experimental measurements is shown in the first column of Table 1 along with upper bounds on the unobserved electromagnetic branching ratios from the CLEO experiment [2]. (The Belle collaboration quotes weaker lower bounds for these ratios [32].)
The low mass of the $D_{s0}(2317)$ and $D_{s1}(2460)$ has prompted speculation that these states are exotic. Possibilities include $DK$ molecules $^{34, 35, 36}$, $D_s\pi$ molecules $^{37}$, and tetraquarks $^{35, 38, 39, 40, 41, 42, 43}$. The proposal that these are $DK$ molecules, in addition to resolving the discrepancy with model predictions of the masses, could potentially explain why the hyperfine splitting between the $D_{s0}(2317)$ and $D_{s1}(2460)$ is equal to the hyperfine splitting of the ground state $D$ meson doublet to within a few MeV. This hypothesis can be tested using chiral perturbation theory.

If the $D_{s0}(2317)$ is a molecular bound state of $D$ and $K$, then the typical three-momentum of its constituents is $p = \sqrt{2\mu B} \approx 190$ MeV, where $\mu$ is the reduced mass and $B$ is the binding energy. This means that both constituents are nonrelativistic. Corrections to the nonrelativistic approximation are $\sim p^2/m_K^2 \sim 0.16$. For a nonrelativistic bound state, the decay rate can be expressed as a product of the wavefunction at the origin and a transition matrix element involving its constituents. For example, if the $D_{s0}(2317)$ is a bound state of $D$ and $K$, then the electromagnetic decay amplitude for the $D_{s0}(2317)$ is

$$\mathcal{M}[D_{s0}(2317) \to D_s^*\gamma] \propto \int d^3\vec{p}|\tilde{\psi}(\vec{p})|^2 \mathcal{M}[D(\vec{p})K(\vec{-p}) \to D_s^*\gamma]$$

$$\propto |\psi(0)|^2 \mathcal{M}[DK \to D_s^*\gamma].$$

Here $\tilde{\psi}(\vec{p})$ is the momentum space wavefunction and $\psi(0)$ is the position space wavefunction at the origin. In the last line the matrix element $\mathcal{M}[DK \to D_s^*\gamma]$ has been expanded to lowest order in $p$. To calculate $\psi(0)$ requires detailed knowledge of the mechanism that binds the $DK$ into a composite hadron. Such a calculation is necessarily nonperturbative. However, this factor cancels out of the ratios in Table 1. The experimentally observed branching ratios are then determined by ratios of the amplitudes for $D^{(*)}K \to D_s^{(*)}\gamma$ and $D^{(*)}K \to D_s^{(*)}\pi^0$ at threshold. These were computed using HH$\chi$PT in Ref. $^{17}$.

The diagrams for electromagnetic and strong decays are shown in Figs. 1 and 2, respectively. The diagrams in Figs. 2b and 2c only contribute to the P-wave channel so the entire contribution to the strong decay comes from the graph in Fig. 2a. Dashed lines are Goldstone bosons, wavy lines are photons and the double lines are heavy mesons. The blob represents the bound state wavefunction and the cross represents the isospin violating $\pi^0 - \eta$ mixing term. The coupling of heavy mesons and Goldstone bosons to photons comes from gauging the kinetic terms in the HH$\chi$PT Lagrangian and the coupling of the heavy mesons to Goldstone bosons is proportional to the axial coupling, $g$, of the heavy mesons. This

| Expression | Expt. | Molecule | HQS |
|------------|-------|-----------|-----|
| $\Gamma[D_{s1}(2460) \to D_s^*\gamma]$ | $< 0.16$ | 3.23 (1.08) | 0.32 ± 0.05 ± 0.10 |
| $\Gamma[D_{s1}(2460) \to D_s^*\pi^0]$ | $0.39 ± 0.06$ | 2.21 (0.74) | 0.39 (fit) |
| $\Gamma[D_{s0}(2317) \to D_s\gamma]$ | $< 0.059$ | 2.96 (0.99) | 0.12 ± 0.02 ± 0.04 |

**TABLE I:** Electromagnetic Branching Fraction Ratios
coupling is known from the strong decay of the $D^*$. At this order the molecular scenario makes predictions for the electromagnetic branching fraction ratios, which are shown in the column labeled “Molecule” in Table 1. The results depend on two parameters: $g$ and the Goldstone boson decay constant, $f$. In this calculation $g = 0.27$\cite{44}. At lowest order, $f = f_\pi = f_K = f_\eta$ but $SU(3)$ breaking leads to different decay constants for pions, kaons and etas. In the calculation of the electromagnetic decays, $f = f_K = 159$ MeV is used since these decays involve kaons only. Two different values of $f$ are used in the calculation of the strong decays. The first number in the second column of Table 1 corresponds to using $f = f_\eta = 171$ MeV and the number in the parenthesis corresponds to using $f = f_\pi = 130$ MeV. The difference gives a crude estimate of the uncertainty due to higher order $SU(3)$ breaking effects. Because the matrix element squared for the strong decay is $\sim f^{-4}$, the magnitude of the branching fraction ratios is highly uncertain. However, even allowing for this considerable uncertainty, the electromagnetic branching fraction ratios are badly over-predicted in the molecular scenario. The branching fraction ratios are proportional to $g^2$, so larger values of $g$ which are sometimes used in the literature will lead to even larger disagreement with experiment. Also, the relative sizes of the branching ratios is qualita-
tively incorrect. The ratio $\Gamma[D_{s1}(2460) \to D_s\gamma]/\Gamma[D_{s1}(2460) \to D_s^*\pi^0]$ is predicted to be the smallest rather than the largest as is experimentally observed. The molecular hypothesis is in disagreement with the data on electromagnetic decays.

An alternative approach is to use heavy quark symmetry to relate the electromagnetic decays and strong decays. At the level of HH$_{\chi}$PT this is implemented by adding the excited $J^P = 0^+$ and $J^P = 1^+$ states to the Lagrangian by hand in a manner consistent with heavy quark symmetry [45]. A single operator in the HH$_{\chi}$PT Lagrangian mediates all three electromagnetic decays and another operator mediates the two strong decays of the excited $D$ mesons, so the electromagnetic branching ratios can be predicted in terms of a single parameter which is fit to the observed value of $\Gamma[D_{s1}(2460) \to D_s\gamma]/\Gamma[D_{s1}(2460) \to D_s^*\pi^0]$ [17]. The heavy quark symmetry prediction appears in the column labeled “HQS” in Table 1. The other two ratios can then be predicted. The first error is due to experimental uncertainty in $\Gamma[D_{s1}(2460) \to D_s\gamma]/\Gamma[D_{s1}(2460) \to D_s^*\pi^0]$, the second error is a 30% uncertainty due to $O(\Lambda_{QCD}/m_c)$ corrections to heavy quark symmetry. The experimental upper bounds on the unobserved branching ratios are below the predicted central values but are within expected errors.

III. CHARMED MESON MASSES IN HH$_{\chi}$PT

Experiments also claim to observe the nonstrange partners of the $D_{s0}(2317)$ and $D_{s1}(2460)$ [3, 4, 5]. The $J^P = 0^+$ and $J^P = 1^+$ nonstrange charm mesons are above the threshold for isospin conserving strong decays into $D\pi$ which makes these states much broader than their nonstrange counterparts. The experimental average for the mass of the $D_{0}^{0}(J^P = 0^+)$ is $2308 \pm 36$ MeV, and the mass of the $D_{1}^{0}(J^P = 1^+)$ is $2438 \pm 31$ MeV. The $SU(3)$ splitting of the excited charm mesons is $9 \pm 36$ MeV for the $J^P = 0^+$ mesons and $21 \pm 31$ MeV for the $J^P = 1^+$ mesons. This is surprising because typically $SU(3)$ splittings between strange and nonstrange particles is $\sim 100$ MeV.

Ref. [13] made the first attempt to address this problem within the framework of HH$_{\chi}$PT. These authors calculated $\Delta m_{u/d} - \Delta m_s$, where $\Delta m_{u/d}$ is the splitting between the spin-averaged mass of the even-parity and odd-parity heavy meson doublets in the nonstrange sector, while $\Delta m_s$ is the analogous quantity in the strange sector. Numerically, $\Delta m_s = 348$ MeV while $\Delta m_{u/d} \approx 430$ MeV. The calculation of Ref. [18] works in the heavy quark limit so all $1/m_c$ suppressed operators are neglected. A linear combination of $SU(3)$ breaking counterterms contributing to $\Delta m_{u/d} - \Delta m_s$ is fixed from lattice calculations of the quark mass dependence of $\Delta m_{u/d} - \Delta m_s$. Ref. [18] then finds $\Delta m_{u/d} - \Delta m_s \approx -100$ MeV, which has the wrong sign!

Recently, Ref. [19] improved upon the calculation of Ref. [18] by systematically including all $O(1/m_c)$ and $SU(3)$ breaking counterterms. Unfortunately this leads to a large number of free parameters appearing in the one loop calculation. These were determined by fitting to the observed spectrum. Two fits were performed in Ref. [19]. The first used the value of $g$ extracted from Ref. [44]. Another axial coupling, $h$, was extracted from a tree level fit to the widths of the excited nonstrange charm mesons [17]. For these values of $g$ and $h$, the fit systematically underpredicts the excited nonstrange meson masses, similar to the result of Ref. [18]. However, the extractions of $g$ and $h$ use calculations that make different approximations than are used in the one loop mass calculations. Therefore, those values of $g$ and $h$ may not be the correct parameters for the mass calculation. In the second fit of Ref. [19], the couplings $g$ and $h$ were treated as free parameters. This fit
is highly underconstrained and it is possible to find regions of parameter space where the observed spectrum can be reproduced. However, in these fits some of the $O(1/m_c)$ suppressed operators have uncomfortably large coefficients. The $SU(3)$ splitting of the excited states remains puzzling in $\chi$PT. An alternative approach is to extend the quark model to include couplings to the $DK$ continuum and try to explain the spectrum via the coupled channel effect.

Parity doubling models of heavy hadrons make a tree level prediction that the axial couplings and hyperfine splittings of the even- and odd-parity doublets are equal. The prediction for the hyperfine splittings of the charm strange mesons is in good agreement with data while the uncertainties in the masses of the nonstrange excited charm mesons are too large to test this prediction. The analysis of Ref. reveals that the region of $\chi$PT parameter space predicted by the parity doubling model is invariant under the renormalization group flow of $\chi$PT at one loop. It is encouraging to see that the parity doubling predictions are robust at the one loop level. Data is currently not accurate enough to test parity doubling model predictions for axial couplings.

IV. STRONG PENTAQUARK DECAYS

In this section I briefly describe the results of Ref. which applied heavy baryon chiral perturbation theory to the strong decays of exotic pentaquarks in the $\mathbf{10}$ representation. The exotic pentaquarks in the $\mathbf{10}$ are the $\Theta^+$, $\Xi^-$ and $\Xi^+$. Various experiments have reported evidence for the $\Theta^+$ while evidence for the exotic cascades comes from the NA49 experiment. There are also several experiments that do not see the pentaquarks. The allowed two-body strong decays for these states are

$$\Theta^+ \rightarrow pK^0, nK^+$$

$$\Xi^- \rightarrow \Xi^-\pi^-, \Sigma^-K^-$$

$$\Xi^+ \rightarrow \Xi^0\pi^+, \Sigma^+K^0.$$ 

All these decays are related by $SU(3)$. Only two-body decays are kinematically allowed for the $\Theta^+$ while the $\Xi$’s should also have multi-body decays. There are experiments claiming to see the $\Theta^+$ in both $pK^0$ and $nK^+$ channels. The NA49 experiments has only seen the $\Xi^-$ state in the $\Xi^-\pi^-$ channel.

Of course the primary experimental problem regarding the $\Theta^+$ and exotic $\Xi$’s is firmly establishing whether or not these states actually exist. If the pentaquarks are confirmed, the most important experimental problem will be to determine their angular momentum and parity quantum numbers, $J^P$, which can distinguish between various pentaquark models. The most commonly discussed method is to measure production of the $\Theta^+$ in polarized $pp$ collisions near the production threshold. Currently, experimental data on this process is unavailable.

The main point of Ref. is that interesting constraints on $J^P$ can be obtained by measuring two-body decays of the exotic members of the $\mathbf{10}$ multiplet. The $J^P$ quantum numbers of the pentaquark determine the angular momentum, $L$, of the pion or kaon emitted in the decay. The rates are proportional to an $SU(3)$ Clebsch-Gordan coefficient times a phase space factor,

$$p^{2L+1} \quad (L \neq 0), \quad E^2 p \quad (L = 0),$$

where $p$ is the three-momentum and $E$ is the energy of the pion or kaon emitted in the decay. Therefore, the ratio $\Gamma[\Xi^- \rightarrow \Sigma^-K^-]/\Gamma[\Xi^- \rightarrow \Xi^-\pi^-]$, for example, is determined
entirely by $SU(3)$ and kinematic factors. Results for some interesting ratios are given in Table 2. Lower bounds on the ratio of the total widths of two exotic pentaquarks are also shown. These are lower bounds because the $\Xi$’s can have multi-body decay modes while the width of the $\Theta^+$ is saturated by two-body decays. The errors quoted are 30%, which is the typical size of $SU(3)$ breaking. The ratios $\Gamma(\Xi^{--}\rightarrow\Sigma^-K^-)/\Gamma(\Xi^{--}\rightarrow\Xi^-\pi^-)$ and $\Gamma(\Xi^0\rightarrow\Sigma^+K^-)/\Gamma(\Xi^0\rightarrow\Xi^-\pi^+)$ can discriminate between $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^+$, which are the most common quantum number assignments that appear in existing pentaquark models. If one finds that $\Gamma(\Xi^{--}), \Gamma(\Xi^+) < 10 \Gamma(\Theta^+)$, then $J^P = \frac{3}{2}^-$ and $J \geq \frac{5}{2}$ can be ruled out.

V. CONCLUSION

In this talk I described applications of chiral perturbation theory to the strong interactions of newly discovered hadrons. HH $\chi$PT was applied to the electromagnetic and strong decays of the $D_{s0}(2317)$ and $D_{s1}(2460)$. Existing data is consistent with heavy quark symmetry predictions and is inconsistent with a molecular interpretation of these states. The $SU(3)$ splitting of the excited even-parity charm mesons is puzzling. The one-loop HH $\chi$PT formulae for the mass spectrum contains a large number of free parameters from $1/m_c$ operators and axial couplings that are not well determined, so it is not possible to make predictions for the spectrum.

Parity doubling models make the prediction that the axial couplings and hyperfine splittings of the even-parity and odd-parity heavy mesons are equal. This was shown to hold at one loop order. The hyperfine splittings of the charm strange mesons are in agreement with this prediction. Currently data is not accurate enough to seriously constrain the axial couplings of the excited states. It would be interesting to obtain lattice calculations of these couplings to test the parity doubling scenario as well as reduce theoretical uncertainty in HH $\chi$PT calculations. It would also be interesting to observe the even-parity excited bottom strange mesons, who are also also predicted to lie below the kaon decay threshold \[19\] and should therefore be quite narrow.

Finally, I discussed $SU(3)$ predictions for the strong decays of exotic pentaquarks and showed how these can be used to constrain their $J^P$ quantum numbers.
VI. ACKNOWLEDGEMENTS

This research has been supported by DOE grants DE-FG02-96ER40945 and DE-AC05-84ER40150. I thank the Institute for Nuclear Theory at the University of Washington for its hospitality during the completion of this work. It is a pleasure to acknowledge collaboration with R. Springer and C. Schat.

[1] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 90, 242001 (2003).
[2] D. Besson et al. [CLEO Collaboration], Phys. Rev. D 68, 032002 (2003).
[3] S. Anderson et al. [CLEO Collaboration], Nucl. Phys. A 663, 647 (2000).
[4] K. Abe et al. [Belle Collaboration], Phys. Rev. D 69, 112002 (2004).
[5] J. M. Link et al. [FOCUS Collaboration], Phys. Lett. B 586, 11 (2004).
[6] For a recent experimental review on the status of the $\Upsilon^+$, see K. Hicks, hep-ex/0504027.
[7] C. Alt et al. [NA49 Collaboration], 92 (2004) 042003.
[8] A. Aktas et al. [H1 Collaboration], Phys. Lett. B 588, 17 (2004).
[9] S. K. Choi et al. [Belle Collaboration], 91, 262001 (2003).
[10] D. Acosta et al. [CDF II Collaboration], Phys. Rev. Lett. 93, 072001 (2004).
[11] V. M. Abazov et al. [D0 Collaboration], 93, 162002 (2004).
[12] M. Mattson et al. [SELEX Collaboration], Phys. Rev. Lett. 89, 112001 (2002).
[13] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592, 1 (2004).
[14] M. B. Wise, Phys. Rev. D 45, 2188 (1992).
[15] G. Burdman and J. F. Donoghue, Phys. Lett. B 280, 287 (1992).
[16] T. M. Yan, H. Y. Cheng, C. Y. Cheung, G. L. Lin, Y. C. Lin and H. L. Yu, Phys. Rev. D 46, 1148 (1992) [Erratum-ibid. D 55, 5851 (1997)].
[17] T. Mehen and R. P. Springer, Phys. Rev. D 70, 074014 (2004).
[18] D. Becirevic, S. Fajfer and S. Prelovsek, Phys. Lett. B 599, 55 (2004).
[19] T. Mehen and R. P. Springer, hep-ph/0503134.
[20] W. A. Bardeen, E. J. Eichten and C. T. Hill, Phys. Rev. D 68, 054024 (2003).
[21] W. A. Bardeen and C. T. Hill, Phys. Rev. D 49, 409 (1994).
[22] M. A. Nowak, M. Rho and I. Zahed, Phys. Rev. D 48, 4370 (1993).
[23] M. A. Nowak, M. Rho and I. Zahed, Acta Phys. Polon. B 35, 2377 (2004).
[24] E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991).
[25] T. Mehen and C. Schat, Phys. Lett. B 588, 67 (2004).
[26] S. Godfrey and N. Isgur, Phys. Rev. D 32, 189 (1985).
[27] S. Godfrey and R. Kokoski, Phys. Rev. D 43, 1679 (1991).
[28] J. Hein et al., Phys. Rev. D 62, 074503 (2000).
[29] P. Boyle [UKQCD Collaboration], Nucl. Phys. Proc. Suppl. 63, 314 (1998).
[30] R. Lewis and R. M. Woloshyn, Phys. Rev. D 62, 114507 (2000).
[31] P. Krokovny et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262002 (2003).
[32] K. Abe et al., [Belle Collaboration] Phys. Rev. Lett. 92, 012002 (2004).
[33] B. Aubert et al. [BABAR Collaboration], hep-ex/0408067.
[34] T. Barnes, F. E. Close and H. J. Lipkin, Phys. Rev. D 68, 054006 (2003).
[35] S. Nussinov, hep-ph/0306187.
[36] Y. Q. Chen and X. Q. Li, Phys. Rev. Lett. 93, 232001 (2004).
[37] A. P. Szczepaniak, Phys. Lett. B 567, 23 (2003).
[38] H. Y. Cheng and W. S. Hou, Phys. Lett. B 566, 193 (2003).
[39] T. E. Browder, S. Pakvasa and A. A. Petrov, Phys. Lett. B 578, 365 (2004).
[40] K. Terasaki, Phys. Rev. D 68, 011501 (2003).
[41] K. Terasaki, AIP Conf. Proc. 717, 556 (2004).
[42] K. Terasaki, hep-ph/0405146.
[43] J. Vijande, F. Fernandez and A. Valcarce, AIP Conf. Proc. 717, 352 (2004).
[44] I. W. Stewart, Nucl. Phys. B 529, 62 (1998).
[45] A. F. Falk and M. E. Luke, Phys. Lett. B 292, 119 (1992).
[46] D. S. Hwang and D. W. Kim, Phys. Lett. B 601, 137 (2004).
[47] I. W. Lee, T. Lee, D. P. Min and B. Y. Park, hep-ph/0412210.
[48] Y. A. Simonov and J. A. Tjon, Phys. Rev. D 70, 114013 (2004).
[49] E. van Beveren and G. Rupp, Phys. Rev. Lett. 91, 012003 (2003).
[50] E. van Beveren and G. Rupp, Eur. Phys. J. C 32, 493 (2004).
[51] E. van Beveren and G. Rupp, Phys. Rev. Lett. 93, 202001 (2004).
[52] A. W. Thomas, K. Hicks and A. Hosaka, Prog. Theor. Phys. 111, 291 (2004).
[53] C. Hanhart, J. Haidenbauer, K. Nakayama and U. G. Meissner, Phys. Lett. B 606, 67 (2005).