Numerical investigation on added mass and damping force coefficient of an underwater vehicle in cavitating flows

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Abstract. The objective of this paper is mainly to study the influence of cavitation on the added mass and damping force coefficient. Based on Reynolds averaged Navier-Stokes equations, the dynamic mesh is used to calculate the added mass, and the rotating coordinate frame method is applied to research on the damping force coefficient. In order to obtain fluid damping force coefficients, the movement pattern is set as a uniform circular motion. Then the additional force coefficient and pitch damping moment coefficient could be obtained using the method of least squares. The result shows that the method to calculate added mass is reliable by comparing with the analytical solution. With the cavitation number decreasing, the absolute value of the added mass of $\lambda_{22}$ decreases and $\lambda_{32}$ increases. What’s more, both the absolute value of damping force and moment coefficient decrease substantially with the development of cavity when the cavitation number is larger than 0.45. However, with the cavitation number less than 0.45, the un-symmetric cavity is more prominent, the absolute value of damping force and moment coefficient increase slightly. This is probably caused by the strengthened pressure peak at the suction side induced by the re-entrant flow.

1. Introduction
Cavitation occurs in liquid flows if locally drops below the vapor pressure [1]. This can be observed in hydraulic systems at high speed. In order to forecast the dynamic characteristic, it is important to research on the hydrodynamic force of a cavity underwater vehicle. The hydrodynamic force[2] of underwater vehicle design is usually classified into three parts, which are location force, inertial force and damping force. The damping force, such as pitch damping force and momentum, effects the trajectory stability and maneuverability dramatically[3][4]. Therefore, the calculation of damping force is relatively very important during hydrodynamic design.

As the development of computational technique, the inertial force and damping force calculation using CFD method based on Reynolds averaged Navier-Stokes equations has been greatly improved. The dynamic mesh method[5][6][7] is used to calculate the added mass. A method to determine the added mass of running vehicle with cavitation is proposed based on calculating hydrodynamic forces of the vehicle swaying of rolling in numerical tunnel[8]. In hydrodynamics calculation, the unsteady method based on dynamic mesh has been applied to get the fluid damping force and moment coefficients[9][10].

In this paper, the dynamic mesh is used to research on the added mass and the rotating coordinate frame method has been applied to calculate damping force coefficient of an underwater vehicle in the cavitating flows.
2. Numerical Method
The commercial software CFD is used for modeling the hydrodynamics of a cavity underwater vehicle. The Navier-Stokes equations in their conservative form governing a Newtonian fluid without body forces and heat transfers in the Cartesian coordinates are presented below:

\[
\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m u_j)}{\partial x_j} = 0,
\]

\[
\frac{\partial (\rho_m u_j)}{\partial t} + \frac{\partial (\rho_m u_j u_i)}{\partial x_j} = - \frac{\partial p}{\partial x_j} + \nabla \cdot \left( \mu_m + \mu_t \right) \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \frac{\partial u_k}{\partial x_k} \delta_{ij} \right),
\]

\[
\frac{\partial \rho_l}{\partial t} + \frac{\partial (\rho_l u_j u_i)}{\partial x_j} = \dot{m}^+ + \dot{m}^-,
\]

where \( \rho_m \) is the mixture density, \( u \) is the velocity, \( p \) is the pressure, \( \mu_m \) is the mixture laminar viscosity, \( \mu_l \) and \( \mu_t \) are respectively the liquid and vapor dynamic viscosities, and \( \mu_t \) is the turbulent viscosity. The subscripts \( i, j, k \) are the axes directions. The source term \( \dot{m}^+ \) and the sink term \( \dot{m}^- \) which is closure by the Kubota model [11] in this study represent the condensation and evaporation rates, respectively. Moreover, the standard \( k-\varepsilon \) turbulence model[12] is used here.

3. Numerical setup and description
During the numerical damping force calculation procedure, the added mass is ineluctably included and can not be separated effectively. In order to obtain damping force, added mass has to be calculated firstly to eliminate its effect. Thus the added mass calculation method based on dynamic mesh is introduced, and then the rotating coordinate frame method to calculate fluid damping force is presented.

3.1. Add mass calculation method
In this study, the underwater vehicle is simplified as a cylinder with a cone forehead. The cone radius \( R=0.04m \), length \( L=0.532m \). Figure 1 shows the computational domain and boundary condition. The mesh is shown in Figure 2. A no-slip boundary condition is imposed on the vehicle surface, and opening conditions are imposed on the side boundaries of the tunnel. The inlet velocity is set to be
V=4 m/s and the outlet pressure is set to vary according to the cavitation number, defined as
\[ \sigma = \frac{(p_v - p_i)}{(0.5 \rho V^2)} \] where \( p_v \) is the tunnel pressure.

The added mass is known to playing a significant role in defining the hydrodynamic forces acting on moving bodies due to the fact that the movement of the surrounding fluid requires an additional force over and above to accelerate the body itself. So the added mass can be obtained by transient motion with the dynamic mesh method. The movement pattern is shown in Figure 3.

![Figure 3. Curve of velocity](image)

The added mass \( \lambda_{11}, \lambda_{22}, \lambda_{26} \) can be obtain by the following equation.

\[ \lambda_{11} = \frac{\Delta F}{a_x} = \frac{1}{2} \rho V^2 S \Delta C_d / a_x \] (6)
\[ \lambda_{22} = \frac{\Delta F}{a_y} = \frac{1}{2} \rho V^2 S \Delta C_l / a_y \] (7)
\[ \lambda_{26} = \frac{\Delta M}{a_z} = \frac{1}{2} \rho V^2 S L \Delta C_l / a_y \] (8)

where \( \rho \) is the fluid density, \( S \) is the maximum cross-sectional area, \( a_i \) is the acceleration, \( C_d \) is the drag coefficient, \( C_l \) is the lift coefficient.

3.2. Fluid damping force calculation method

In order to obtain fluid damping force coefficients. The movement pattern is circumferential motion, and the rotating coordinate frame method is adopted to conduct steady numerical simulation.

Make the vehicle rotate uniform around the specified center, the distance between vehicle centroid and circular motion center is \( R \), the rotating speed is defined as \( \omega_{z1} \), and the velocity at vehicle centroid is \( V = R \omega_{z1} \). The hydrodynamic force is defined as follows.

\[ qSC_{x1}(\alpha) + qSC_{y1}(\omega_{z1}) = qSC_{x1}^o \alpha + qSC_{x1}^{\alpha z_{1}} \bar{\alpha}_{z_{1}} = N + \lambda_{11} V \omega_{z1} \cos \alpha \] (9)
\[ qS\lambda_{m1}(\alpha) + qS\lambda_{m1}(\omega_{z1}) = qS\lambda_{m1}^o \alpha + qS\lambda_{m1}^{\alpha z_{1}} \] \[ = M + \lambda_{26} V \omega_{z1} \cos \alpha + (\lambda_{22} - \lambda_{11}) V^2 \sin \alpha \cos \alpha \] (10)

The normal force \( N \) and pitch moment \( M \) could be obtained based on the numerical calculation of the vehicle with circumferential motion. Then make sure that the attack angle and the velocity is constant, just change the angular velocity. Then the damping force coefficient \( C_{x1}^{\alpha z_{1}} \) and pitch damping moment coefficient \( C_{m1}^{\alpha z_{1}} \) could be calculated by least squares method.

4. Results and discussion
4.1. Numerical result verification

4.1.1. Added mass calculation. To verify the calculation method, the calculation for the vehicle with the defined movement in the condition of without cavitation is conducted. The time evolution of the lift coefficient is shown in Figure 4, where the acceleration is \( a_y = 40 \text{m/s}^2 \).

![Figure 4](image)

**Figure 4. Curve of the lift coefficient**

From the Eq(7) and (8), the added mass \( \lambda_{22} \) and \( \lambda_{26} \) could be obtained. And the \( \lambda_{11} \) can be obtained by the same method. Comparisons of the measured and predicted value are listed in Table 1. The numerical results showed a good agreement with the measured values.

|                | \( \lambda_{22} \) | \( \lambda_{26} \) |
|----------------|---------------------|---------------------|
| predicted value| 2.855227            | -0.02105            |
| calculation value| 2.74857           | -0.02095            |
| error          | 3.74%               | 0.475%              |

4.1.2. Added mass in cavitation flow. The reliability of the added mass calculation method is validated above. In this section, added mass in the different cavitation numbers will be calculated.

In order to present the variation of the added mass in different cavitation number, the corresponding graphs are shown in Figure 5. The trend of the curves show that the value of \( \lambda_{22}, \lambda_{26} \) decreases when the cavitation number decreases.

![Figure 5](image)

**Figure 5. The variation of the added mass in different cavitation number**

To further analyze the influence of cavitation on added mass, Figure 6 shows the pressure contours.
of different cavitation numbers at two typical time (corresponding to the two moments in Figure 4). The cavity length increases with the cavitation number decreasing. From the pressure distributions, it is found that the difference between the up side and the down side in the longitudinal plane lies in the area without cavitation, as the saturated pressure is in the cavity. So the value of $\lambda_{22}$ decreases when the cavitation number decreases. Figure 7 shows the corresponding pressure distributions on the both surfaces. In figure 7, the center of pressure moves toward the tail of the vehicle, when the cavitation number decreases. That causes the arm of force increases. So the absolute value of $\lambda_{26}$ increases with the decrease of cavitation number.

![Pressure contour in different cavitation numbers](image)

**Figure 6.** Pressure contour in different cavitation numbers

![Curves of pressure coefficient in different cavitation numbers at the same time](image)

**Figure 7.** Curves of pressure coefficient in different cavitation numbers at the same time

### 4.2. The effect of cavitation on damping force coefficient

In the computation, the natural cavity number is decreased by the reduction of the operation pressure. Figure 8 shows the cavity shapes in different cavitation regimes.

![Cavity shape with different cavitation number](image)

**Figure 8.** Cavity shape with different cavitation number
In order to obtain fluid damping force coefficients, the defined movement in the condition of without cavitation is conducted In this research, the attack angle $\alpha=0^\circ$. The corresponding normal force $N$ and pitch moment $M$ can be obtained at different angular velocity. From the Eq.(9) and(10), the coefficients can be calculation. Then the additional force coefficient and pitch damping moment coefficient could be obtained by using least squares method.

Figure 9 and 10 present the damping force and moment coefficient at different cavitation numbers, respectively. When the cavitation number decreases, the cavity coverage area increases, the increment of total normal force decreases. So the damping force coefficient decreases. While the size of cavity grows to a certain stage with the decrease of cavitation number, the cavity dissymmetry due to rotating motion becomes obvious. The cavity dissymmetry will cause larger normal force at closure zone. It could also be observed that the cavity dissymmetry become more obvious with the increment of angular velocity, and the corresponding normal force become larger and increases no-linear. When the cavitation number less than 0.45, the damping force coefficient increases with the decrement of cavitation number.

To be similar, as the cavitation number decreases, the increment of pitch moment decreases. With the decrease of cavitation number, the absolute value on pitch damping moment coefficient increases. When the cavitation number is less than 0.45, the influence of cavity dissymmetry is more obvious. With the increment of cavitation number, the absolute value of pitch damping moment coefficient become larger.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig9.png}
\caption{Fluid damping force with different cavitation number}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig10.png}
\caption{Pitch damping moment with different cavitation number}
\end{figure}

5. Conclusions
In this study, numerical simulation by CFD are applied to investigate the added mass and damping force coefficient of an underwater vehicle in cavitation flow. The main conclusions are below.

1) The numerical method of transient motion with the dynamic mesh method is used to research on the added mass of a vehicle with different cavitation number. The result shows that the method to calculate added mass is reliable by comparing with the analytical solution. With the cavitation number decreasing, the absolute value of the added mass of $\lambda_{22}$ decreases and $\lambda_{26}$ increases.

2) The numerical method of rotating coordinate frame method is used to realize the circumferential motion to research on damping force coefficient. It is shown that both the absolute value of damping force and moment coefficient decrease substantially with the development of cavity when the cavitation number is larger than 0.45. However, with the cavitation number of less than 0.45, the cavity dissymmetry is more obvious, the absolute value of damping force and moment coefficient increase slightly. This is probably caused by the strengthened pressure peak at the suction side which is induced by the re-entrant flow.

Acknowledgements
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