Layered Coding of Hidden Markov Sources

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Abstract—The paper studies optimal coding of hidden Markov sources (HMS), which represent a broad class of practical sources obtained through noisy acquisition processes, beside their explicit modeling use in speech processing and recognition, image understanding and sensor networks. A new fundamental source coding approach for HMS is proposed, based on tracking an estimate of the state probability distribution, and is shown to be optimal. Practical encoder and decoder schemes that leverage the main concepts are introduced. An iterative approach is developed for optimizing the system. It also focuses on a significant extension of the optimal HMS quantization paradigm. It proposes a new approach for scalable coding of HMS which accounts for all the available information while coding a given layer. Simulation results confirm that these approaches significantly reduce the reconstructed distortion and substantially outperform existing techniques.

Index Terms—Hidden Markov Sources, Source Coding, Scalable Coding, Packet Loss Concealment

I. INTRODUCTION

THE Hidden Markov model (HMM) is a discrete-time finite state Markov chain observed through a memoryless channel. The random process consisting of the sequence of observations is referred to as a hidden Markov source (HMS). Markov chains are common models for information sources with memory and memoryless channel is among the simplest communication models. HMMs are widely used in image understanding and speech recognition [1], source coding [2], communications, information theory, economics, robotics, computer vision and several other disciplines. Note that most signals modeled as Markov process are in fact captured by imperfect sensors and are hence contaminated with noise, i.e., the resulting sequence is an HMS. Motivated by its modeling capability of practical sources with memory, we consider optimal quantization of the HMS. One conventional approach to design quantizers for the HMS is to employ predictive coding techniques (such as DPCM) to exploit time correlations, followed by standard scalar quantization of the residual. However, any direct prediction method cannot fully exploit the structure of the source process, i.e., the underlying Markov process. Indeed, even if the underlying process is first order Markov (depends on the past only through the previous sample), the HMS is not Markov, i.e., all prior samples are needed to optimally predict the current sample. But a naive infinite order prediction is clearly of impractical complexity. An alternative to predictive coding is encoding by a finite-state quantizer (see eg. [3], [4]). A finite-state quantizer is a finite-state machine used for data compression: each successive source sample is quantized using a quantizer code book that depends on the encoder state. The current encoder state and the codeword obtained by quantization then determine the next encoder state, i.e., the next quantizer codebook. In [5], the authors optimized the finite-state quantizer for a given HMS with known parameters, and analyzed its steady state performance. While a finite state quantizer is, in general, an effective approach, it does not fully exploit the underlying structure of the HMS. In other words, the finite state quantizer is agnostic to the source state, although it updates the encoder state based on the HMS observations. This subtle but important shortcoming motivates a scheme that explicitly considers the source states in the quantization process, which is the approach pursued in our previous work [6].

The fundamental approach is to exploit all available information on the source state, i.e., the probability distribution over the states of the underlying Markov chain. An important distinction of this paradigm from prior work, and specifically from the finite-state quantizer, is that the state probability distribution captures all available information and depends on the entire history of observations. Hence, with each observation both encoder and decoder refine the estimate of state probability distribution and correspondingly update the coding rule. The fundamental approach is therefore optimal. We then specialize to a practical “codebook switching” variant and propose an algorithm that optimizes the decision rule, i.e., we optimize the codebook used for quantization based on the estimate of state probabilities.

Also we focused on a significant extension of the optimal HMS quantization paradigm. Advances in internet and communication technologies, have created an extremely heterogeneous network scenario with data consumption devices of highly diverse decoding and display capabilities, all accessing the same content over networks of time varying bandwidth and latency. Thus it is necessary for coding and transmission systems to provide a scalable bitstream that allows decoding at a variety of bit rates (and corresponding levels of quality), where the lower rate information streams are embedded within the higher rate bitstreams in a manner that minimizes redundancy. That is, we need to generate layered bitstreams, wherein a base layer provides a coarse quality reconstruction and successive layers refine the quality, incrementally. Scalable coding with two quality levels, transmits at rate $R_{12}$ to both decoders and at rate $R_2$ to only the second decoder.

The commonly employed technique for scalable coding of HMS, completely neglects the hidden states and employs a predictive coding approach such as DPCM (i.e., assumes a simple Markov model) at the base layer, and at the enhancement layer, the base layer reconstruction error is compressed and transmitted. That is, the base layer reconstruction is used as an estimate for the original signal, and the estimation error is compressed in the enhancement layer. A review of scalable coding techniques for audio and video signals can be found in [7], [8]. An estimation theoretically optimal scalable predictive coding technique was proposed in [9], which accounts for all the information available from the current base layer and
past reconstructed samples to generate an optimal estimate at the enhancement layer. In another work of our lab [10], [11], we proposed a novel scalable coding technique for HMS, which accounts for all the available information while coding a given layer. At the base layer, we exploit all the available information by employing our previously proposed technique of tracking the state probability distribution at the encoder and the decoder, and using it to update the quantizers for encoding the current observation. At the enhancement layer, we again track the state probability distribution at the encoder and the decoder, but using the higher quality enhancement layer reconstruction for a better estimate, and then the enhancement layer quantizer is adapted to the interval determined by base layer quantization so as to enable full exploitation of all available information.

Beyond what has been done before in our previous works, in this paper we tackle the new problems of the unreliable networks, delayed scalable coding, and designing optimal coding for audio’s LSF attribute.

Multimedia transmission over networks enables a wide range of applications such as online radio, TV and high-definition teleconferencing. These applications are often suffering by the problem of unreliable networking conditions, which leads to the loss of data. Packet loss concealment is an important tool to solve this issue. The packet loss concealment objective is to exploit all available information to approximate the lost packet while maintaining smooth transition with neighboring packets. In this paper we propose an approach, which the state probability distribution tracking unit, considers two scenarios of either having a packet loss or not, and based on that it updates the probability distribution and design the other parameters.

An important feature of scalable coding is that the base layer can be decoded independently of enhancement layers, which allows the enhancement layer coder potential access to information about future base layer reconstruction, at a given coding latency relative to the base layer. In this paper we also consider means to exploit such future information, in addition to the current base layer and prior enhancement layer information, in a scheme that complements the above scalable framework, to further refine the enhancement layer probability distribution, and thereby achieve considerable performance gains on top of the non-delayed approach.

Finally, we tackle the problem of designing coding approach for LSF data in this paper. Linear predictive coding (LPC) is a tool used mostly in speech processing for representing the spectral envelope of a digital signal of speech in compressed form. It is one of the most useful methods for encoding good quality speech at a low bit rate and provides extremely accurate estimates of speech parameters. Transmission of the filter coefficients directly is undesirable, since they are very sensitive to errors. Line spectral frequencies (LSF) are used to represent LPC for transmission over a channel. In this paper we design the best coding approach for LSF coefficients.

The rest of this paper is organized as follows: In Section II, we overview background information. In Section III and IV, the proposed methods are described. Experimental results are presented in Section V and concluding remarks are in Section VI.

II. PRELIMINARIES

The hidden Markov model drives its name from two defining properties: First it assumes that the observation at time \( t \) is generated based on a state which is hidden from observer. Secondly these state sequences are related through a Markov process in which the current state’s dependence on the past is completely determined by the immediately preceding state.

HMMs have become more popular in various important applications. HMMs are widely used in image understanding and speech recognition [11, 12, 13, 14, 15, 16], source coding [2], communications, information theory, economics, robotics, computer vision and several other disciplines. Note that most signals modeled as Markov processes are usually captured by imperfect sensors and are hence contaminated with noise, i.e., the resulting sequence is in fact an HMM. HMS is a special case of the broader family of multivariate Markov sources, which has been a focus of recent research, notably in the context of parameter estimation [17, 18].

Following we will specify the formal definition and the most basic problems associated with the HMS.

A. Definition of Hidden Markov Sources

A hidden Markov source (HMS) is the output of a hidden Markov model (HMM) determined by five parameter sets (see eg., [19]):

1) The number of states in the model \( N \). We denote the set of all possible states as \( S = \{S_1, S_2, \ldots, S_N\} \), and the state at time \( t \) as \( q_t \).
2) The state transition probability distribution \( A = \{a_{ij}\} \), where \( a_{ij} = P[q_{t+1} = S_j | q_t = S_i], 1 \leq i, j \leq N \).
3) Observation symbols, in discrete case we denote the symbols as \( V = \{v_1, v_2, \ldots, v_M\} \), where \( M \) is the number of distinct observation symbols per state.
4) The observation (emission) probability density function (pdf) in state \( j \), \( g_j(x) \), in the continuous case, and the observation (emission) probability mass function (pmf), \( B = \{b_j(v_k)\} \), where \( b_j(v_k) = P(O_t = v_k | q_t = S_j) \) in the discrete case. \( O_t \) denotes the source output at time \( t \).
5) The initial state distributions \( \pi = \{\pi_1\} \) where \( \pi_i = P[q_1 = S_i], 1 \leq i \leq N \).

Fig. 1 depicts an example of a continuous hidden Markov source with \( N \) states. In hidden Markov sources we have no access to the state of the source, only to the emitted observation, \( O_n \).

For convenience we use the notation \( \lambda = (A, B, \pi) \) to indicate the parameter set.

B. Three Basic Problems

To use the hidden Markov model in real-world applications, there are typically three very fundamental problems that need to be solved (see eg., [19]):

1) Given the model \( \lambda = (A, B, \pi) \) what is the probability of generating the observation sequence \( O_{1\rightarrow T} = O_1O_2, \ldots, O_T \), i.e., \( P(O_{1\rightarrow T} | \lambda) \).
Finally probability of the sequence observation \(O\) is:

\[ P(O_{1:T} | \lambda) = \sum_{i=1}^{N} \alpha_T(i) \]

Solution to the First Problem: Evaluation Problem

We want to calculate the probability of the observation \(O_{1:T} = O_1, O_2, \ldots, O_T\) given the model \(\lambda = (A, B, \pi)\). An efficient way to solve the problem is by using the so-called Forward and Backward procedure: Let us define the Forward variable \(\alpha_t(i)\) as:

\[ \alpha_t(i) = P(O_{1:t}, q_t = S_i), \]

i.e., the joint probability of emitting the observed source sequence \(O_1^t\) up to time \(t\) and that the state at time \(t\) is \(S_i\). The forward variables can be computed recursively:

1) \(\alpha_1(i) = \pi_i b_i(O_1)\) for \(1 \leq i \leq N\)
2) \(\alpha_{t+1}(j) = \sum_{i=1}^{N} \alpha_t(i)a_{ij}b_j(O_{t+1})\)

Finally probability of the sequence observation \(O_1^T\) is:

\[ P(O_{1:T} | \lambda) = \sum_{i=1}^{N} \alpha_T(i) \]

Solution to the Second Problem: Optimal Path

There are several ways to define the optimal state. One way is to choose the state \(q_t\) individually such that it is most likely. To implement this solution let us define the variable:

\[ \gamma_t(i) = P(q_t = S_i | O_{1:T}) \]

i.e., the probability of being at state \(i\) at time \(t\) given the observation sequence \(O_1^T\). Using forward and backward variable:

\[ \gamma_t(i) = \frac{\alpha_t(i)\beta_t(i)}{P(O_{1:T})} = \frac{\alpha_t(i)\beta_t(i)}{\sum_{i=1}^{N} \alpha_t(i)\beta_t(i)} \]

Using \(\gamma_t(i)\) we can solve for individually optimal state \(q_t\) at time \(t\):

\[ q_t = \arg \max \gamma_t(i) \]

Solution to the Third Problem: Parameter Training

The third problem is the most difficult HMM problem and there is no way to analytically solve this problem. However we can choose \(\lambda = (A, B, \pi)\), such that \(P(O_1^T | \lambda)\) be locally maximized using an iterative procedure such as Baum-Welch (or any other EM) method as explained in [1].

III. OPTIMAL CODING OF HMS

One important property of the HMS is that the state at time \(t-1\), captures all past information relevant to the emission of the next source symbol. Specifically \(P(O_t = v_k | q_{t-1} = S_j, O_1^{t-1}) = P(O_t = v_k | q_{t-1} = S_j)\) which implies that all observations until time \(t-1\) provide no additional information on the next source symbol, beyond what is obtained by the state of the Markov chain at time \(t-1\). Note further that we can not know with certainty the state of the HMS. Based on this fact the fundamental paradigm for achieving optimal coding is to track the state probability distribution of the HMS. In this approach, each output of the encoder (quantized observation) is sent into a unit called the state probability distribution tracking function. This unit estimates the state probability distribution, i.e., probabilities of the Markov chain being in each of the states, denoted by \(\hat{p}\). These probabilities are used in the encoder adapter unit to redesign the encoder optimally for the next input sample. Fig. 2 shows the fundamental approach for the optimal HMS encoder.

Our objective is to design the Encoder, the state probability distribution tracking function and the Encoder adapter, given a training set of samples, so as to minimize the average reconstruction distortion at a given encoding rate. We first describe tracking of the state probability distribution, then we describe fundamental design approach, then proceed to practical encoder design approach. Finally we describe the approach under lossy network condition.
In fundamental approach, state probability distribution tracking unit finds, \( \hat{p} \), using the reconstructed observation samples \( \hat{O} \), as:

\[
\hat{\mu}_L(t) = \{\hat{p}(t,1), \ldots, \hat{p}(t,N)\} = \{P[q_t = S_1|O_{1\rightarrow(t-1)}], \ldots, P[q_t = S_N|O_{1\rightarrow(t-1)}]\}
\]

(12)

Given \( \hat{\mu}_L(t) \), the best estimate for the source distribution is:

\[
\sum_{j=1}^{N} \hat{p}(t,j)q_j(x)
\]

(13)

Fig. 4 depicts this notion.

In this approach, each output of the encoder (quantized observation) is sent into the state probability distribution tracking function. This unit estimates the state probability distribution, i.e., \( \hat{p} \). These probabilities are used in the encoder adapter unit to design a quantizer for the pdf in Eq. (13) for each source output from the ground up. Finally Encoder encode the symbols.

C. Practical Encoder Design

The fundamental framework requires the encoder be redesigned for each observed sample, which entails high complexity. Therefore, to propose a practical approximation with both low complexity and low memory requirement, we restrict the encoder adapter unit to select a codebook from a prespecified set of codebooks and use it as an initialization. To facilitate this operation, we introduce a module called the ‘next quantizer selector’, which decides the codebook to be used for initialization of the codebooks of the next sample, based on the output of the state probability distribution tracking function. Fig. 5 shows the proposed practical encoding scheme.

For a set of \( T \) representative \( p \), we find the best codebook using Lloyd’s (or other) algorithm offline:

\[
C_n = \{x_{n1}, \ldots, x_{n(2^R)}\} \quad \text{for} \quad n = 1, 2, \ldots, T
\]

Then for a given \( \hat{\mu}_L(t) \) in the process of encoding or decoding, we find the closest representative \( p \) from the set of \( T \). Finally using the codebook of the closest representative as an initialization, we run one iteration of the Lloyd’s algorithm, to find the codebook to be used for current sample.

D. Packet Loss Concealment for HMS

Audio and video transmission over networks enables a wide range of applications such as multimedia streaming, online radio, TV and high-definition teleconferencing. These applications are often plagued by the problem of unreliable networking conditions, which leads to intermittent loss of data. This problem is severe, and the packet loss rate in Internet communications, for example, may reach 20%. Clearly, robustness to packet loss is a crucial requirement. Packet loss concealment forms a crucial tool amongst the various strategies used to mitigate this issue. The packet loss concealment objective is to exploit all available information
For the design procedure, we first estimate the best estimate of the decoder’s probability distribution would translates the state probability as \( \hat{p} \) before it reaches the decoder. In another words, by probability transition with neighboring packets.

\[
\hat{p}(t+1) = \{ \hat{p}_{1}(t+1), \ldots, \hat{p}_{N}(t+1) \} \\
= \{ P[q_{t} = S_{1} | \hat{O}_{1\rightarrow t}], \ldots, P[q_{t} = S_{N} | \hat{O}_{1\rightarrow t}] \} \quad (14)
\]

has two scenarios:

- If there is no packet loss at time \( t \), the decoder would have access to \( \hat{O}_{t} \) and would calculate the \( \hat{p}_{(t+1), no-loss} \) as:

\[
\hat{p}_{(t+1), no-loss} = C \sum_{i=1}^{N} \hat{p}_{(t,i)} a_{ij} b_{i}(\hat{O}_{t})
\]

- If there is packet loss at time \( t \) then the decoder would not have access to \( \hat{O}_{t} \) and would calculate the \( \hat{p}_{(t+1), with-loss} \) as:

\[
\hat{p}_{(t+1), with-loss} = \sum_{i=1}^{N} \hat{p}_{(t,i)} a_{ij}
\]

Since the decoder with probability \( 1 - p_{loss} \) would receive the packet and calculates the state probability as \( \hat{p}_{(t+1), no-loss} \) and with probability \( p_{loss} \) it would miss the packet and calculates the state probability as \( \hat{p}_{(t+1), with-loss} \), the encoder’s best estimate of the decoder’s probability distribution would be:

\[
\hat{p}_{(t+1), lossy} = (1 - p_{loss}) \hat{p}_{(t+1), no-loss} + p_{loss} \hat{p}_{(t+1), with-loss} \quad (15)
\]

For the design procedure, we first estimate \( \hat{p}_{(t+1), lossy} \) for all \( t \), later as before we vector quantize \( p \) and design the best codebook for each class. Also for each class we store the average of the class as the representative.

At the decoder side, depending on whether we had packet loss at time \( t \) or not, the decoder chooses one of the above scenarios to calculate the probability distribution at time \( t+1 \).

If there is no packet loss at time \( t+1 \) the decoder, as usual, finds the correct codebooks and decodes the received symbol. However if there is packet loss at time \( t+1 \) the decoder conceals it by substituting the decoded symbol by a weighted average of the class representatives, where the weights are given by the probability distribution over states at time \( t+1 \).

Let \( p_{loss} \) be the probability of the network’s packet loss before it reaches the decoder. In another words, by probability \( p_{loss} \), the encoded packet will not reach the decoder, and gets lost. Then the decoder calculation of

\[
\hat{p}_{(t+1), no-loss} = \sum_{i=1}^{N} \hat{p}_{(t,i)} a_{ij} b_{i}(\hat{O}_{t})
\]

\[
\hat{p}_{(t+1), with-loss} = \sum_{i=1}^{N} \hat{p}_{(t,i)} a_{ij}
\]

\[
\hat{p}_{(t+1), lossy} = (1 - p_{loss}) \hat{p}_{(t+1), no-loss} + p_{loss} \hat{p}_{(t+1), with-loss} \quad (15)
\]

For the design procedure, we first estimate \( \hat{p}_{(t+1), lossy} \) for all \( t \), later as before we vector quantize \( p \) and design the best codebook for each class. Also for each class we store the average of the class as the representative.

At the decoder side, depending on whether we had packet loss at time \( t \) or not, the decoder chooses one of the above scenarios to calculate the probability distribution at time \( t+1 \).

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\[
\hat{p}_{(t+1), lossy} = (1 - p_{loss}) \hat{p}_{(t+1), no-loss} + p_{loss} \hat{p}_{(t+1), with-loss} \quad (15)
\]

For the design procedure, we first estimate \( \hat{p}_{(t+1), lossy} \) for all \( t \), later as before we vector quantize \( p \) and design the best codebook for each class. Also for each class we store the average of the class as the representative.

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Fig. 5. Proposed encoder for the base and the enhancement layer.

Fig. 6. Overall observation pdf in the enhancement layer given \( \hat{p}(t) = \{0.5, 0.5\} \) and the quantization interval information from the base layer \((I_b, P(O_t|\hat{p}(t)) = \{0.5, 0.5\}, I_b)\).

A. Enhancement Layer Design

The goal here is to design the best codebook for the enhancement layer based on:

- Quantization interval from the base layer, and
- State probability distribution based on the enhancement layer reconstructed observation samples \( \hat{O}_e \).

The enhancement layer state probability distribution tracking unit finds \( \hat{p}_e(t) \), similar to the base layer, but using enhancement layer reconstructed observation samples \( \hat{O}_e \), as

\[
\hat{p}_e(t) = \{\hat{p}_e(t,1), \ldots, \hat{p}_e(t,N)\} = \{P[q_t = S_1|\hat{O}_e^{(t-1)}], \ldots, P[q_t = S_N|\hat{O}_e^{(t-1)}]\}.
\]

Given just \( \hat{p}_e(t) \), the best estimate for the source distribution is:

\[
\sum_{j=1}^{N} \hat{p}_e(t,j) \hat{g}_j(x)
\]

However, in combination with the quantization interval information from the base layer, the best estimate for the source distribution is:

\[
\sum_{j=1}^{N} \hat{p}_e(t,j) \hat{g}_j(x)
\]

where, \( \hat{g}_j(x) \) is the observation pdf in state \( j \) truncated and normalized to the interval determined by quantization at the base layer. Fig. 6 illustrates the best estimate for the source distribution using interval information from the base layer.

We design the quantizer for this pdf of the current sample, by using uniform codebook as an initialization and running one iteration of Lloyd’s algorithm.

Note that we use uniform codebook as an initialization for the enhancement layer quantizer design as observation pdf within the quantization interval of the base layer has lesser variations and is closer to uniform distribution. However, this assumption is not true for the base layer. Using the uniform codebook as the initialization also reduces the memory requirements of the encoder and the decoder. For the first symbol, we assume that the source symbol is from a fixed state (based on \( \pi \)) at both the encoder and decoder, and use this in combination with the quantization interval information from the base layer.

Note that we can generalize our proposed approach to any number of enhancement layers, by combining the refined estimate of state probability distribution based on observation reconstruction of the given layer, with the quantization interval information from its lower layers.

B. Enhancement Layer Design With Access to Future Reconstruction of Base Layer

The goal here is to design the best codebook for the enhancement layer based on:

- Quantization interval from the base layer, and
- State probability distribution based on
The enhancement layer reconstructed observation samples \( \hat{O}_e \) till time \( t-1 \)

- The base layer reconstructed observation samples \( \hat{O}_b \) till time \( t-1+L \), where \( L \) shows the number of the future samples from base layer.

The enhancement layer state probability distribution tracking unit finds \( \hat{p}_e(t) \), similar to the base layer, but with enhancement layer reconstructed observation samples \( \hat{O}_e \) till time \( t-1 \) as well as base layer reconstructed observation samples \( \hat{O}_b \) till time \( t-1+L \), as

\[
\hat{p}_e(t) = \{ \hat{p}_e(i,1), \ldots, \hat{p}_e(i,N) \} = \{ P[q_t = S_1|\hat{O}_1^t, \hat{O}_b^t, q_t = S_i] \}, \quad P[q_t = S_N|\hat{O}_1^t, \hat{O}_b^t, q_t = S_i] \}
\]  

(20)

For finding \( P[q_t = S_i|\hat{O}_1^t, \hat{O}_b^t, q_t = S_i] \), we use the forward-backward approach explained earlier. First we define the forward variable:

\[
\alpha_i(t) = P(\hat{O}_1^t, \hat{O}_b^t, q_t = S_i, \hat{O}_1^t = S_i),
\]

(21)

and the backward variable:

\[
\beta_i(t) = P(\hat{O}_b^t, q_t = S_i | \hat{O}_1^t) = \frac{\alpha_i(t) \beta_i(t)}{\sum_{i=1}^N \alpha_i(t) \beta_i(t)}
\]

(22)

and finally state probability is calculated as:

\[
P[q_t = S_i|\hat{O}_1^t, \hat{O}_b^t, q_t = S_i] = \frac{\alpha_i(t) \beta_i(t)}{\sum_{i=1}^N \alpha_i(t) \beta_i(t)}
\]

(23)

for \( i = 1, 2, \ldots, N \).

Given just \( \hat{p}_e(t) \), the best estimation for the source distribution is:

\[
\sum_{j=1}^N \hat{p}_e(i,j) \hat{g}_j(x)
\]

(24)

However, in combination with the quantization interval information from the base layer, the best estimate for the source distribution is:

\[
\sum_{j=1}^N \hat{p}_e(i,j) \hat{g}_j(x)
\]

(25)

where, \( \hat{g}_j(x) \) is the observation pdf in state \( j \) truncated and normalized to the interval determined by quantization at the base layer.

We design the quantizer for this pdf of the current sample, by using uniform codebook as an initialization and running one iteration of Lloyd’s algorithm.

V. EXPERIMENTAL RESULTS

Let us assume that the encoder arbitrarily uses the last quantizer when it processes the first source symbol. (Any convention can be used as long as it is known to the decoder. Alternatively we can send an initial side information to determine which quantizer is used.) For the rest of the symbols, the encoder first finds \( \hat{p}_e \) based on which it selects the suitable quantizer. The exact same procedure is employed by the decoder.

It should be noted that finding \( \hat{p}_e \) does not impose a major computational burden on the encoder and decoder. They simply update the forward variables (as discussed earlier) and obtain \( \hat{p}_e \).

A lower bound \[5\] on the distortion, when using a switched scalar quantizer is:

\[
\hat{D} = \sum_{m=1}^N \rho_m \min_{Q} \sum_{j=1}^N a_{m,j} E_j (x - Q(x))^2
\]

(26)

where \( E_j \) is expected value using \( j \)th subsource statistics, \( \rho_m \) is the stationary probability of being in state \( m \), and minimization is over all possible quantizers. Note that this bound is very optimistic, as it pretends that the Markov chain state at time \( t-1 \) is known exactly for selecting the next quantizer.

A. Results for Synthetic Random Sources

In the first experiment the HMS has two Gaussian sub-sources, one of them having mean \( \mu_1 = -1.5 \) and variance \( \sigma_1^2 = 1 \) the other one having mean \( \mu_2 = +1.5 \) and variance \( \sigma_2^2 = 1 \). Moreover \( a_{12} = a_{21} = 0.1 \), as depicted in Fig. 7. For the simulation we set \( T = 5 \) (i.e., the number of codebooks is 5) and the coding rate varies from 1 to 6 bits per symbol. We compare the proposed method with finite state quantizer, as well as DPCM. The result is shown in Fig. 8. As is evident, the proposed method offers gains of approximately 1 dB over the finite state quantizer and DPCM. Comparison to the theoretical lower bound shows that the method leaves little room for improvement and performs very close to the bound. Note again that this distortion bound makes the optimistic assumption that the state at time \( t-1 \) is known and used to obtain the best encoder for time \( t \). Obviously, a real system does not have access to the source state at any time. So it is not an achievable lower bound, yet the proposed system performs fairly close to this bound.

The second experiment involves the same source except for varying the transition probability \( a_{12} = a_{21} \) from 0.01 to 0.1 with coding rate of 4 bits per symbol.
The method shows consistent gains over its competitors with somewhat larger gains at low values of $a_{12}$ as depicted in Fig. 9.

**Note:** The lower bound introduced earlier, assumes we know the HMM state at time $t-1$ exactly. However it’s a bold assumption and makes a looser lower bound. A more fair and tighter lower bound could be assuming we know the actual non quantized data till time $t-1$ and based on that we calculate the lower bound. Fig. 10 shows the comparison between our proposed approach and the new lower bound. As we could see at higher rate our proposed approach is almost identical to the lower bound. Since in real design the decoder does not have access to the non quantized data, so the small gap between the proposed approach and the lower bound at lower rate, does not undermine the optimality of our approach.

**B. Results for the Packet Loss Case**

As before in the experiment the HMS has two Gaussian subsources, one of them having mean $\mu_1 = -1.5$ and variance $\sigma^2_1 = 1$ the other one having mean $\mu_2 = +1.5$ and variance $\sigma^2_2 = 1$. Moreover we ran the experiment on 3 different cases:

Case 1: Transition probability $a_{12} = a_{21} = 0.1$
Case 2: Transition probability $a_{12} = a_{21} = 0.05$
Proposed Method

Case 3: Transition probability \( a_{12} = a_{21} = 0.01 \)

As is evident in Tab. I, Tab. II, and Tab. III for all sources and under different loss scenarios, our proposed approach outperforms the competition.

C. Results for Real Audio Data

Linear predictive coding (LPC) is a tool used mostly in audio signal processing and speech processing for representing the spectral envelope of a digital signal of speech in compressed form, using the information of a linear predictive model. It is one of the most useful methods for encoding good quality speech at a low bit rate and provides extremely accurate estimates of speech parameters. Since LPC is frequently used for transmitting spectral envelope information, and as such it has to be tolerant of transmission errors. Transmission of the filter coefficients directly is undesirable, since they are very sensitive to errors. In other words, a very small error can distort the whole spectrum, or worse, a small error might make the prediction filter unstable.

Line spectral frequencies (LSF) are used to represent LPC for transmission over a channel. LSFs have several properties (e.g. smaller sensitivity to quantization noise) that make them superior to direct quantization of LPCs.

In our last experiment we used a real speech dataset. We want to design the best coding approach for LSF coefficients.

Using approaches explained in Section II, we first find the best Gaussian emission HMM, modeling the dataset. Next using the approaches explained in Section III, we design the best codebooks, as well as coding approaches.

As depicted in Fig. 11, for coding rates 3 till 5 bits per samples on average we have 0.4 dB gain compare to DPCM.

D. Results for Standard Enhancement Layer (Without Access to Future Base Layer Reconstructions)

For this experiment, we use an HMS which has two Gaussian subsources, one of them with mean \( \mu_1 = -1.5 \) and variance \( \sigma_1^2 = 1 \) the other one with mean \( \mu_2 = +1.5 \) and variance \( \sigma_2^2 = 1 \), and the transition probabilities are, \( a_{12} = a_{21} = 0.01 \). For the scalable coder, we set the base layer rate to be \( R_{12} = 3 \) bits and the enhancement layer rate varies as \( R_2 = 2, 3, 4, 5 \) bits. We compare the proposed method with the prior approach which assumes a simple Markov model and uses DPCM in the base layer and employs a quantizer designed via Lloyd’s algorithm for encoding the base layer reconstruction error in the enhancement layer. Also we compare our results with the theoretical lower bound for using a switched scalar quantizer \( 5 \) operating at the total rate of \( R_{12} + R_2 \) bits. The rate distortion plots for the different approaches and the lower bound are shown in Fig. 12. The superiority of the proposed method is evident from the plots with substantial gains of around 5 dB over the competition. Note that there is a gap of around 2 dB from the lower bound due to the scalable coding penalty of the hierarchical structure employed, as this source distortion pair is not successively refinable.

In the second experiment, the same source is used, but with varying transition probability, \( a_{12} = a_{21}, \) from 0.01 to 0.1, and fixed coding rate of 3 bits in the base and the enhancement layer. Results for this experiment, as shown in Fig. 13, demonstrate large gains that decrease marginally with values of \( a_{12} \).

Note that calculating \( \hat{p} \) at the base and enhancement layers of both encoder and decoder does not impose a significant computational burden, as forward variables are easily updated recursively for each sample (as given in Section III-A and

| Source | FSQ | DPCM | Proposed Method |
|--------|-----|------|-----------------|
| No Loss | -16.61 dB | -16.59 dB | -17.58 dB |
| 1% Loss | -12.65 dB | -12.63 dB | -13.72 dB |
| 5% Loss | -7.91 dB | -7.43 dB | -8.49 dB |
| 10% Loss | -4.61 dB | -4.59 dB | -5.57 dB |

| Source | FSQ | DPCM | Proposed Method |
|--------|-----|------|-----------------|
| No Loss | -16.63 dB | -16.87 dB | -18.05 dB |
| 1% Loss | -12.68 dB | -12.88 dB | -14.08 dB |
| 5% Loss | -7.97 dB | -7.33 dB | -8.60 dB |
| 10% Loss | -4.63 dB | -4.75 dB | -5.57 dB |

| Source | FSQ | DPCM | Proposed Method |
|--------|-----|------|-----------------|
| No Loss | -16.62 dB | -17.64 dB | -19.04 dB |
| 1% Loss | -12.68 dB | -13.15 dB | -14.16 dB |
| 5% Loss | -7.97 dB | -8.39 dB | -9.74 dB |
| 10% Loss | -4.66 dB | -4.84 dB | -5.63 dB |
then \( \hat{p} \) is obtained with a few more manipulations.

### E. Results for Enhancement Layer With Access to Future Base Layer Reconstructions

We use the same HMS which has two Gaussian subsources, one of them with mean \( \mu_1 = -1.5 \) and variance \( \sigma_1^2 = 1 \) the other one with mean \( \mu_2 = +1.5 \) and variance \( \sigma_2^2 = 1 \), and the transition probabilities are, \( a_{12} = a_{21} = 0.01 \). For the scalable coder, we set the base layer rate at \( R_{12} = 4 \) bits and the enhancement layer rate varies at \( R_2 = 1, 2, 3 \) bits. We set \( L = 1 \), i.e., using only one future base layer sample. We measure the additional gains of the proposed method due to access to future base layer reconstructed sample. As depicted in the Fig. 14, on average the method obtains an additional \( 0.3 \) dB gain by exploiting available future base layer reconstructed samples.

**Note:** For different values of \( L \) we ran the experiment, however the additional gain for \( L \) bigger than one was negligible compare to the gain for \( L = 1 \).

### VI. Conclusion

A new fundamental source coding approach for hidden Markov sources is proposed, based on tracking the state probability distribution, and is shown to be optimal. Practical encoder and decoder schemes that leverage the main concepts are introduced, which consist of three modules: a switchable set of quantizer codebooks, a state probability distribution tracking function, and a next quantizer selector. An iterative approach is developed for optimizing the system given a training set.

The state probability distribution tracking module estimates, at each time instance, the probability that the source is in each possible state, given the quantized observations, and the next quantizer selector determines the codebook to be used, given the state probability distribution it receives at its input. The decoder mimics the encoder and switches to the same codebook. The switching decisions are made while optimally accounting for all available information from the past, and are obtained efficiently by recursive computation of HMM forward variables.

Also we have proposed a novel technique for scalable coding of hidden Markov sources which utilizes all the available information while coding a given layer. In the enhancement layer, the state probability distribution is refined for each sample using past enhancement layer reconstructed observations. Then this information is combined with the quantization interval available from the base layer, and the quantizer for the current sample is updated accordingly. The decoder mimics...
this quantizer updates of the encoder in both the base and enhancement layers.

Experimental results show substantial performance improvements over existing approaches.

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REFERENCES

[1] L. R. Rabiner, S. E. Levinson, and M. M. Sondhi, “On the application of vector quantization and hidden markov models to speaker-independent, isolated word recognition,” Bell System Technical Journal, vol. 62, no. 4, pp. 1075–1105, 1983. [Online]. Available: http://dx.doi.org/10.1002/j.1538-7305.1983.tb03115.x

[2] A. Gerhso and R. Gray, Vector quantization and signal compression. Springer, 1992.

[3] J. Foster, R. Gray, and M. Dunham, “Finite-state vector quantization for waveform coding,” IEEE Transactions on Information Theory, vol. 31, no. 3, pp. 348–359, 1985.

[4] M. Dunham and R. Gray, “An algorithm for the design of labeled-transition finite-state vector quantizers,” IEEE Transactions on Communications, vol. 33, no. 1, pp. 83–89, 1985.

[5] D. Goblirsch and N. Farvardin, “Switched scalar quantizers for hidden markov sources,” IEEE Transactions on Information Theory, vol. 38, no. 5, pp. 1455–1473, September 1992.

[6] M. Salehifar, E. Akyol, K. Viswanatha, and K. Rose, “On optimal coding of hidden markov sources,” IEEE Data Compression Conference, March 2014.

[7] J. Ohm, “Advances in scalable video coding,” Proceedings of the IEEE, vol. 93, no. 1, pp. 42–56, 2005.

[8] T. Painter and A. Spanias, “Perceptual coding of digital audio,” Proceedings of the IEEE, vol. 88, no. 4, pp. 451–515, 2000.

[9] K. Rose and S. Regunathan, “Toward optimality in scalable predictive coding,” IEEE Transactions on Image Processing, vol. 10, no. 7, pp. 965–976, July 2001.

[10] M. Salehifar, T. Nanjundaswamy, and K. Rose, “On scalable coding of hidden markov sources,” IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), March 2016.

[11] M. Salehifar, “Common information and hidden state dependencies in layered source coding,” Ph.D. dissertation, University of California, Santa Barbara, Sept. 2017.

[12] B. Jiang, “On the hidden markov model and dynamic time warping speech recognition,” Art Tech., vol. 63, no. 7, 1984.

[13] B. Jiang and L. Rabiner, “A probabilistic distance measure for hidden markov models,” Art Tech., vol. 64, no. 2, pp. 391–408, 1985.

[14] ———, “Mixture autoregressive hidden markov models for speech signals,” IEEE Trans., vol. 33, no. 6, pp. 1404–1413, December 1985.

[15] B. Jiang, S. Levinson, and M. Sonphi, “Maximum likelihood estimation for multivariate mixture observations of markov chains.” IEEE Transactions on Information Theory, vol. 32, no. 2, pp. 291–298, March 1986.

[16] B. Jiang and L. Rabiner, “The segmental k-means algorithm for estimating the parameters of hidden markov models,” IEEE Trans. Acoust., Speech, Signal Processing, vol. 38, no. 9, pp. 1639–1641, 1990.

[17] Y. Ephraim and B. Mark, “On forward recursive estimation for bivariate markov chains,” 4th Annual Conference on Information Sciences and Systems (CISS), pp. 1–6, March 2012.

[18] ———, “Causal recursive parameter estimation for discrete-time hidden bivariate markov chains,” IEEE Transactions on Signal Processing, vol. 63, no. 8, pp. 2108–2117, April 2015.

[19] L. Rabiner, “A tutorial on hidden markov models and selected applications in speech recognition,” Proceedings of the IEEE, vol. 77, no. 2, pp. 257–286, February 1989.