WW DECAYS AND BOSE-EINSTEIN CORRELATIONS

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Various methods of implementing the Bose-Einstein effect into Monte Carlo generators, especially for the process $e^+e^- \rightarrow W^+W^-$, are briefly reviewed and their predictions for the W mass shifts are compared. The weight methods, which yield very similar predictions independent on the detailed prescription for weights, are discussed in more detail. In particular, we advocate a new method, which seems to be practical and reasonably well justified theoretically.

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1 Introduction

With the increasing domination of the models equipped with Monte Carlo generators, the problem of proper description of the Bose-Einstein second order interference effect (called often "HBT effect", and here denoted as the BE effect) has reappeared. This effect allows to learn more about the space-time development of the production processes, especially if results for semi-inclusive samples of data could be compared with predictions resulting from various model assumptions. However, it is non-trivial to implement the quantum interference effect into Monte Carlo generators, which deal with probabilities and not with amplitudes.

The problem became suddenly quite acute, when one realized that the interference effects may result in the mass shifts for W bosons produced pairwise in $e^+e^-$ collisions and decaying into hadrons. Conflicting predictions for this shift were presented, resulting in some confusion about the possibility of using 4-jet events for precise measurements of the W mass.

In this paper we review very shortly different procedures implementing the BE effect into Monte Carlo generators and compare their predictions for W mass shifts. We discuss in more detail the weight methods, presenting a practical algorithm which avoids some of the difficulties inherent for this class of procedures. We conclude with the list of further studies to be performed.

2 How to implement the Bose-Einstein effect in Monte Carlo generators?

The standard discussion of the BE effect starts from the classical space-time source emitting identical bosons with known momenta. Thus the most natural procedure is to treat the original Monte Carlo generator as the model for the source and to symmetrize the final state wave function. This may be done in a more proper way using the formalism of Wigner functions. In any case, however, the Monte Carlo generator should yield both the momenta of produced particles and the space-time coordinates of their creation (or last interaction) points. Even if we avoid troubles with the uncertainty principle by using the Wigner function approach, such a generator seems reliable only for heavy ion collisions. It has been constructed also for the $e^+e^-$ collisions, but localizing the hadron creation point in the parton-based Monte Carlo program for lepton and/or hadron collisions is a rather arbitrary procedure, and it is hard to say what does one really test comparing such a model with data.

The best procedure seems to be taking into account the interference effects before generating events. Unfortunately, this was done till now only for the JETSET generator for a single Lund string, and a generalization for multi-string processes is not obvious. No similar modifications were yet proposed for other generators.

The most popular approach, applied since quite a few years to the description of BE effect in various processes, is to shift the final state momenta of events generated by the PYTHIA/JETSET generators. The prescription for a shift starts from the observation that original generators produce very small correlations in two-particle distributions of like-sign pairs of pions. The standard quantity to measure such correlations is the ratio

$$c_2(Q) = \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} = \frac{\int d^3 p_1 d^3 p_2 \delta(p_1 + p_2) \delta(Q - \sqrt{-m^2})}{\int d^3 p_1 d^3 p_2 \rho_1(p_1) \rho_2(p_2) \delta(Q - \sqrt{-m^2})},$$

which is a function of a single invariant variable $Q$. As noted above, this ratio is close to one for a default generator version, whereas experiments show the "BE enhancement", often parametrized by

$$c_2(Q) = 1 + \lambda \exp(-R^2 Q^2),$$

where $R$ and $\lambda$ are parameters interpreted as the source radius and "incoherence strength", respectively.
Now for all the pairs of identical pions a shift of momenta is calculated to assure such a shift in the value of \( Q = \sqrt{-(p_1 - p_2)^2} \), that the resulting numerator of (1) will be multiplied by the "BE factor" (2). The shift of momenta is made unique by the requirement that the pair’s 3-momentum should not change. After performing the shifts, all the CM 3-momenta of final state particles are rescaled to restore the original energy. In more recent versions of the procedure \( ^1 \) "local rescaling" is used instead of the global one. In any case, each event is modified and the resulting generated sample exhibits now the "BE enhancement": the ratio (1) is no longer close to one, and may be parametrized as in (2).

There is no theoretical justification for the procedure, so it should be regarded as an imitation rather than implementation of the BE effect. Its success or failure in describing data is the only relevant feature. Unfortunately, whereas the method is very useful for the description of two-particle inclusive spectra, it fails to reproduce (with the same fit parameters \( R \) and \( \lambda \)) the three-particle spectra \( ^2 \) and the semi-inclusive data \( ^3 \). This could be certainly cured, e.g., by modifying the shifting procedure and fitting the parameters separately for each semi-inclusive sample of data. However, the fitted values of parameters needed in the input factor (2) used to calculate shifts are quite different from the values one would get fitting the resulting ratio (1) to the same form \( ^4 \). Thus it seems to be very difficult to learn something reliable on the space-time structure of the source from the values of fit parameters in this procedure.

All this has led to the revival of weight methods, known for quite a long time \( ^5 \), but plagued with many practical problems. The method is clearly justified within the formalism of the Wigner functions, which allows to represent (after some simplifying assumptions) any distribution with the BE effect built in as a product of the original distribution (without the BE effect) and the weight factor, depending on the final state momenta \( ^6 \). With an extra assumption of factorization in momentum space, we may write the weight factor for final state with \( n \) identical bosons as

\[
W(p_1, ... p_n) = \sum_{P} \prod_{i=1}^{n} w_2(p_i, p_{P(i)}),
\]

where the sum extends over all permutations \( P \) of \( n \) elements, and \( w_2(p_i, p_k) \) is a two-particle weight factor reflecting the effective source size. A commonly used simple parametrization of this factor for a Lorentz symmetric source is

\[
w_2(p, q) = \exp[-(p - q)^2 R^2 / 2],
\]

The only free parameter is now \( R \), representing the effective source size. In fact, the full weight given to each event should be a product of factors \( ^3 \) calculated for all kinds of bosons; in practice, pions of all signs should be taken into account. As before, only direct pions and \( \rho \) decay products should be taken into account, since for other pairs much bigger \( R \) should be used, resulting in negligible contributions.

There are two problems with using (3) as a prescription for weight to be given to each generated event. First, as the sum contains \( n! \) terms, the time needed for its calculation becomes prohibitive for more than 15 identical particles in the final state. This has been dealt with in different ways: by allowing for permutation only in separate CM hemispheres \( ^7 \), by replacing the sum (3) by other, better or worse justified ansatzes \( ^8, ^9, ^10, ^11 \) or by restricting the class of permutations, over which the sum is performed \( ^6, ^7, ^11 \). It is rather difficult to judge, how precise is the approximation of the sum (3) in each case, although for the last methods some estimates were given \( ^12 \). We will return to this problem later on.

\(^6\)In fact, only the direct pions and \( \rho \) decay products are counted, since for other pairs the effective source size is too large to give the visible enhancement in momentum space.
The second problem concerns side-effects of the weights. Obviously, introducing weights changes not only the ratio \((1)\), but all the distributions obtained from the generated sample of events. In particular, since the values of weight factors \((3)\) will be in average larger for the larger multiplicities, the multiplicity distribution may be seriously distorted by weights. Let us stress that this is by no means a drawback of the weight methods: in the real world there is always a BE effect, and if the free parameters in Monte Carlo were fitted to the data neglecting this effect, their values are simply incorrect. However, the iterative procedure of refitting the parameters taking the weights into account would be very tedious. Thus a simple ansatz may be used: the calculated weights should be rescaled by a simple factor \(cV^n\), where \(n\), as above, is the number of identical bosons in the final state, and \(c, V\) are free parameters. Their values should be fitted to restore the shape and normalization of the original multiplicity distribution. Obviously, this ansatz may be insufficient for a more detailed analysis. For example, since in general different parameters define the distribution of the number of jets \(N_j\) in the \(e^+e^-\) collisions, and the distribution in number of particles \(j\) for a single jet, double rescaling in \(N_j\) and \(j\) may be needed if one wants to analyze the fully inclusive sample. The energy dependence may be also troublesome. Nevertheless, for the particular problem of the \(WW\) pair production the present versions of the weight methods were found to be sufficient.

3 Predictions for the W mass shifts

The four classes of the procedures outlined in the previous section give very different predictions for the \(W\) mass shift in the \(WW \to 4jets\) final state. There are not many really new results since last year, so we may refer the reader for more detailed analysis to the review paper by Webber. Here we give only a very short recapitulation supplemented by a few new developments.

Very large mass shifts (hundreds of MeV) are predicted for symmetrized production from the parton cascade, but this comes mainly from the unorthodox color reconnection effects, and not from the BE effect. This model seems to be already contradicted by the data.

Results for the Lund string with interference has been recently presented. Perhaps not surprisingly, taking into account the BE effect inside each string does not result in any significant mass shift.

Various weight methods, although differing significantly in the prescriptions for weights and their spectrum, predict also negligible mass shifts (below 20 MeV). Let us note that this is not trivial: the weights could be \textit{a priori} correlated with the 2-jet mass value, resulting in a quite substantial mass shift.

The momentum shifting method predicts mass shift about 200 MeV, which was attributed to the global momentum rescaling present in this procedure. The new versions of the procedure, using more local rescaling, give essentially no predictions, as the values obtained range from 0 to 180 MeV, depending on the details of the algorithm.

Thus one can see that the excitement over the subject has been significantly reduced. It seems rather unlikely that the BE effect should damage the possibility of high precision measurement of the \(W\) mass in 4 jet events. On the other hand, obviously none of the methods implementing the BE effect in the Monte Carlo programs is fully satisfactory and really well developed and a lot of work is needed to solve remaining practical problems. In the next section we will shortly discuss some recent improvements in the weight methods.

4 New developments for the weight methods

As already noted, it is difficult to estimate, how well one approximates formula, even if different truncations of the sum seem to give very similar results. Thus it was proposed to use an integral representation of this sum, borrowed from the field theory, and to calculate the
integral in the saddle point approximation. There is, however, a condition for the momentum configuration, for which this method may be applied: each momentum must be close (in the sense of smallness of $Q^2 = -(p_1 - p_2)^2$) to at least one another momentum. Since this is in general not true for the final states in the multiparticle production, one must divide first the final state into clusters fulfilling this condition. The weight factor for each kind of identical particles is then a product of the weight factors for all clusters. It was found that for reasonable values of parameters the clusters contain typically only one or a few particles. Thus in fact the integral representation is not needed: exact calculation of the sum for clusters of less than five particles, and a simple truncation of the sum for larger clusters provides a good approximation for the full weight, although it needs much less computer time than the previous methods.

The weak spot of the weight method seems to be the rescaling procedure. Therefore it is encouraging to note that the shape of BE ratio (1) seems to depend very weakly on this procedure (see fig. 1). The double ratio of "BE ratios" (1) for positive pions with- and without weights generated for $(e^+ e^-)$ collisions at $Z_0$ peak. Diamonds and crosses correspond to the rescaled and unrescaled weights, respectively.

Moreover, we have checked that (at least for the pp collisions) using the rescaling parameters fitted to restore the original multiplicity distribution one restores as well the original inclusive momentum distribution. Thus the simple rescaling seems to work better than expected.

Let us conclude this section with a remark on the average multiplicities in the hadronic $W$ decay. Some preliminary data suggest that the multiplicity for $WW$ final state is more than just twice the multiplicity from a single $W$ decay. Such an effect could not be described by the momentum shifting method, since in this case all events preserve their multiplicities. On the other hand, rescaling of weights which restores the original distributions for a single $W$ decay will in general change the average multiplicity for $WW$ final state, since the weight in this case is not just a product of weights for decay products of two single $W$. Thus more precise measurement of this effect may decide which method is better for implementing the BE effect into Monte Carlo generators.

5 Conclusions and outlook

We have reviewed shortly the methods of implementing the BE effect into Monte Carlo generators. One may conclude that the competition for "best BE in MC" is not yet decided, but
may soon be over. There are already working weight methods which may replace the dominant
momentum shifting method. The future tests should include not only the quantities relevant for
\(WW\) production (as the multiplicity shift mentioned above), but also the problems in which the
momentum shifting method has failed, as the semi-inclusive data and higher order correlations.
One should consider non-symmetric form of two-particle weight factor \(2\), the dependence of
its free parameters on energy and the possibility of different parameter values for various pairs
(e.g. from the same- and from different \(W\)-s). In any case, one should stress that we do not
speak about comparing data with non-existing "world without BE effect". There is a lot of
possible real physical effects due to the BE effect (as the difference between the \(WW\) state and
the superposition of two single \(W\)-s) and investigating them will certainly enlarge our knowledge
on the space-time development of the multiple production processes.

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