Correlation induced memory effects in the transport properties of low dimensional systems

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We demonstrate the remnant presence of initial correlations in the steady-state electrical current flowing between low-dimensional interacting leads. The leads are described as Luttinger liquids and electrons can tunnel via a quantum point-contact. We derive an analytic result for the time-dependent current and show that ground-state correlations have a large impact on the relaxation and long-time behavior. In particular, the I-V characteristic cannot be reproduced by quenching the interaction in time. We further present a universal formula of the steady-state current $j_s$ for an arbitrary sequence of interaction quenches. It is established that $j_s$ is history dependent provided that the switching process is non-smooth.

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Non-equilibrium phenomena in open nanoscale systems offer a formidable challenge to modern science[1]. Controlling the electron dynamics of a molecular device is the ultimate goal of nanoelectronics and quantum computation[2]; its microscopic description a problem at the forefront of statistical quantum physics[3]. Resorting to approximate methods is inevitable to progress.

Standard many-body techniques consider an initial state with no interaction and with no contact between the system and the baths (leads from now on), and then switch them on in time[4–6]. In fact, it is plausible to believe that starting from the true interacting and contacted state the long-time results would not change. To what extent, however, such belief is actually the truth? This question is of both practical and fundamental interest. It has been shown by us [7] and others [8] that for non-interacting electrons the initial contact plays no role at the steady-state [9] (theorem of equivalence). Allowing for interactions in the system only (non-interacting leads) Myöhänен et al. found that steady-state quantities are not sensitive to initial correlations either[10]. It is the purpose of this Letter to show that interacting leads change dramatically the picture: the switching process can indeed have a large impact on the relaxation and the steady-state behavior.

We consider two one-dimensional interacting leads described as Luttinger Liquids (LL)[11], see Fig. 1. It is known that a LL does not relax to the ground state after a sudden quench of the interaction [12–14] (thermalization breakdown). The implications of such important result in the context of time-dependent (TD) transport are totally unknown and will be here explored for the first time. We compare the dynamics of initially (a) contacted ($\eta_T = 1$) versus uncontacted ($\eta_T = 0$) and (b) interacting ($\eta_I = 1$) versus non-interacting ($\eta_I = 0$) LL when driven out of equilibrium by an external bias. Our main findings are that in (a) the system relaxes towards the same steady-state although with a different power law decay. In (b) the sudden quench of the interaction when $\eta_I = 0$ alters the steady-current $j_s$ as well. This remains true for an arbitrary sequence of interaction quenches. We are able to write $j_s$ as an explicit functional of the switching process and to establish that $j_s$ is history dependent for non-smooth switchings.

The equilibrium Hamiltonian for the system of Fig. 1 reads

$$H_0 = H_R + H_L + \eta_I H_I + \eta_T H_T.$$  (1)

The one-body part of the left ($L$) and right ($R$) leads is $H_{R/L} = \mp iv_F \int_{-\infty}^{\infty} dx \psi^\dagger_{R/L}(x) \partial_x \psi_{R/L}(x)$, where the fermion field $\psi_{R/L}$ describes right/left moving electrons in lead $R/L$ with Fermi velocity $v_F$ (chiral leads). We take a density-density interaction of the form $H_I = \frac{1}{2} \int_{-\infty}^{\infty} dx [2g_2 \rho_R(x)\rho_L(x) + g_4 (\rho^2_R(x) + \rho^2_L(x))]$, where $\rho_{R/L} = \psi_{R/L}^\dagger \psi_{R/L}$ is (in standard notation) the fermionic density operator relative to the Fermi sea, and $g_{2/4}$ are the forward scattering couplings, corresponding to inter/intra lead interactions respectively. The two chiral liquids are linked at $x = 0$ via the tunneling term $H_T = \lambda \psi_{R}^\dagger(0) \psi_{L}(0) + H.c.$, which does not commute with

![FIG. 1. Sketch of the device. Two interacting leads hosting L and R movers are connected at $x = 0$ via a weak link. A bias voltage $V_L - V_R$ can be applied between the leads.](image-url)
the total number of electrons $N_{R/L}$ of each lead.

If a bias $V = V_L - V_R$ is applied at, say, time $t = 0$, a finite current $j(t)$ starts flowing across the link. The current operator (in atomic units) $J = dN_L/dt = -dN_R/dt$ reads $J = i\lambda\psi_R^\dagger(0)\psi_L(0) + \text{H.c.}$. At zero temperature the current $j(t)$ is the TD average of $J$ over the ground state $|\Psi_0\rangle$ of $H_0$, i.e.,

$$j(t) = \langle \Psi_0|J_{H_1}(t)|\Psi_0\rangle,$$

where $J_{H_1}(t)$ is the $J$ operator in the Heisenberg representation with respect to the interacting, contacted and biased Hamiltonian $H_1 = H_L + H_R + H_I + H_T + H_V$, $H_V = V_R N_R + V_L N_L$. Note that the factors $\eta_I, \eta_T$ refer to times $t < 0$ and different values $\eta_I, \eta_T = 0, 1$, respectively different initial states $|\Psi_0\rangle$. At positive times the Hamiltonian is the same in all cases.

The exact non-interacting solution. We start our analysis by calculating $j(t)$ when $\eta_T = 0$ (initially uncontacted) and $g_2 = g_4 = 0$ (always non-interacting). In terms of the Fourier transform $\psi_{R/L}^\dagger$ of the original fermion fields, the current operator reads $J = (i\lambda/\alpha) \sum_{k,k'} \psi_{k'}^\dagger \psi_{k} + \text{H.c.}$, with $\alpha$ the usual short-distance cutoff. Its expectation value is then

$$j(t) = \lambda \alpha m \sum_{\alpha_0} \int_\pi \Gamma_{p\alpha}^\dagger(t) f_p \Gamma_{p\alpha}^\dagger(t) \prod_{\alpha=0} \prod_{\alpha_0} \sqrt{\pi} \gamma_\alpha(t)/\gamma_{\alpha_0}(t)$$

where the sum runs over $\alpha = R, L$, $f_p^{R/L} = f(\pm v_F p)$ is the Fermi function of lead $R/L$, and $\Gamma_{p\alpha}^\dagger(t) = -i\alpha f \int_\pi \psi_{\alpha_0} e^{-itH_1(t)\psi_{\alpha_0}} |\Psi_0\rangle$ is the sum of the probability amplitudes (retarded Green’s functions) for the transition $p \beta \rightarrow k_0$. From the Dyson equation it is straightforward to find $\Gamma_{p\alpha}^\dagger(t) = -ie^{i(\alpha v_F^p + V_c) t}/(1 + c^2)$ and $\Gamma_{p\alpha}^\dagger(t) = -ie^{i\gamma_\beta(t)/2}$, with $c = \lambda/(2v_F)$, and hence

$$j(t) = \frac{2e^2}{\pi(1 + c^2)^2} V.$$

The current is discontinuous in time; the steady-state value is reached instantaneously. This is due to the unbounded (relativistic) energy spectrum[5] and the lack of interactions, as discussed in detail in Ref. 15. As we shall see, when $H_I \neq 0$ the transient regime is more complex.

Current to lowest order in $\lambda$. The problem does not have an exact solution when both $H_I$ and $H_T$ are present. Below, we calculate $j(t)$ to lowest order in $\lambda$. In general, perturbative treatments in the tunneling amplitude are a delicate issue[16]. In our case $j(t)$ has a Taylor expansion with convergence radius $\lambda < 2v_F$ for $H_I = 0$, see Eq. (3). We, therefore, expect a finite convergence radius at least for small interaction strengths. Let the unperturbed Hamiltonian be $H_0 = H_R + H_L + \eta_I H_I$ in equilibrium ($t < 0$) and $H_I = H_R + H_L + \eta_I H_I$ at positive times. At zero temperature and lowest order in $\lambda$

$$j(t) = i\langle \tilde{\Psi}_0| \int_0^t ds \left[H_{T,R_0}(s), J_{R_0}(t)\right] - i\eta_I \int_0^{\infty} d\tau \left[H_{T,R_0}(\tau), J_{R_0}(t) + J_{R_0}(t)H_{T,R_0}(-\tau)\right]|\tilde{\Psi}_0\rangle,$$

with $\tilde{\Psi}_0$ the ground state of $\tilde{H}_0$. The first term in the r.h.s. is the standard Kubo formula. Such term alone describes the transient response when the contacts are switched on at $t = 0$ ($\eta_T = 0$). If, however, the equilibrium system is already contacted ($\eta_T = 1$) we must account for a correction; this is the physical content of the second term[17]. At any finite time initial correlation effects are visible in both terms due to the ground state dependence on $\eta_I$. When $t \rightarrow \infty$ only the Kubo term survives, which yields the steady-current $j_s$. The dependence of $j_s$ on the ground state ($\eta_I = 0, 1$) will be addressed below.

The averages in Eq. (4) can be explicitly calculated by resorting to the bosonization method[11]. We introduce the scalar fields $\phi$ and $\theta$ from $\rho_R(x) + \rho_L(x) = \sqrt{\pi} \eta_I \phi(x) + \sqrt{\pi} \eta_I \theta(x)$, with $\kappa^{R/L}$ the anticommuting Klein factors. The scalar fields obey the commutation relation $[\phi(x), \theta(x')] = \text{sign}(x-x')/2$. In terms of $\phi$ and $\theta$ the Hamiltonian $H = H_R + H_L + H_I$ is a simple quadratic form $H = \frac{1}{2} \int_\pi \int_\pi \text{d}x \text{d}y (K^{-1}(\partial_x \phi(x))^2 + K\partial_y \phi(x)^2)$, where $v = \sqrt{(2\pi v_F - g_2)^2/\pi}$ the renormalized velocity and $K = \sqrt{2(2\pi v_F + g_4 + g_2)(2\pi v_F + g_4 + g_2)}$ a parameter which measures the interaction strength. Note that $0 < K \leq 1$ for repulsive interactions; $K = 1$ corresponds to the noninteracting case while small values of $K$ indicate a strongly correlated regime.

By employing the gauge transformation[18] $\psi_{L,R} \rightarrow \psi_{L,R} e^{iV_c L/R}$ the problem of evaluating Eq. (4) is reduced to the calculation of different bosonic vacuum averages[11]. After some tedious algebra one finds

$$j(t) = \xi \text{Re} [\eta_T A_{\eta_I}(t) + B_{\eta_I}(t)],$$

where

$$A_0(t) = \sin(Vt) \int_0^{\infty} d\tau \gamma^2(t + i\tau),$$

$$B_0(t) = \int_0^t ds \sin[V(s-t)] \gamma^{2K}(s-t) \times |\gamma(s-t)|^{(1-K)^2} \left[\gamma^2(s-t)/\gamma(2t)\gamma(2s)\right]^{1-K^2},$$

for $\eta_I = 0$ and

$$A_1(t) = \sin(Vt) \int_0^{\infty} d\tau \gamma^{2K}(t + i\tau),$$

$$B_1(t) = \int_0^t ds \sin[V(s-t)] \gamma^{2K}(s-t)$$

for $\eta_I = 1$, and where $\gamma(z) = a/(a - ivz)$ and $\xi = \lambda^2/(16a^2)$. In all cases ($\eta_I, \eta_T = 0, 1$) $j(t)$ is an odd function of $V$, as it should be. We also notice that for noninteracting systems ($K = 1$) we recover the expected result $A_1 = A_0$ and $B_1 = B_0$. In this case the function $\xi \text{Re}[B_{1,a}]$ coincides with the current in Eq. (3) to lowest
order in $\lambda$. We can now provide a quantitative analysis of the TD current response for different preparative configurations.

**Contacted versus uncontacted ground state.** We consider an initially contacted ($\eta_T = 1$) and uncontacted ($\eta_T = 0$) correlated ground state ($\eta_L = 1$) and compare the corresponding TD currents $j_{T1} \equiv \xi \Re[A_1 + B_1]$ and $j_{T0} \equiv \xi \Re[B_1]$. The current $j_{T0}(t)$ has been recently computed in Ref. [19]. In the long time limit it returns the well known steady-state result

$$j_S(\beta) = \sin(\pi K)\kappa(\beta)\text{sgn}(V)|V|^\beta$$  \hspace{1cm} (8)

with $\kappa(\beta) = -\xi(a/v)^{\beta+1}\Gamma(-\beta)\sin(\beta\pi/2)$ and the exponent $\beta = 2K - 1$, obtained long ago by Kane and Fisher[20]. Since $A_1(t \to \infty) = 0$, $j_{T1}$ approaches the same steady state. Note that the small bias limit is ill-defined for $K < 1/2$ due to the break down of the perturbative expansion in powers of $\lambda[18, 21]$. Even though $j_{T0}(t \to \infty) = j_{T1}(t \to \infty)$ the relaxation is different in the two cases, see Fig. 2. The function $j_{T0}(t)$ approaches the asymptotic limit with transient oscillations of frequency $V$ and damping envelope proportional to $t^{-2K}$ [19]. The more physical current $j_{T1}$, instead, decays much slower. The integral in $A_1(t)$ can be calculated analytically and yields

$$j_{T1}(t) - j_{T0}(t) = \xi a^{2K} \frac{\sin(Vt) \cos[(2K - 1)\arctan(vt/a)]}{2v(2K - 1)(a^2 + v^2t^2)^{K-1/2}},$$  \hspace{1cm} (9)

which for long times decays as $t^{1-2K}$. (Equation (9) provides an independent, TD evidence that the perturbation treatment breaks down for $K < 1/2$. Thus, an initially contacted state changes the power-law decay from $\sim t^{-2K}$ to the slower $\sim t^{1-2K}$. The amplitude of the transient oscillations is also significantly different, due to the factor $(2K - 1)^{-1}$ in Eq. (9). For $K = 0.75$, $j_{T1}$ oscillates with an amplitude about 10 times larger than that of $j_{T0}$, see Fig. 2. The magnification of the oscillations was unexpected since for $j_{T1}$ we only switch a bias while for $j_{T0}$ also the contacts. This effect is not an artifact of the perturbative treatment: to support the validity of our results we checked that for $\eta_L = \eta_T = 1$ and zero bias the density matrix $\rho(t) = \langle \Psi_0 | \psi_R(t) \psi_{L,H}(t) | \Psi_0 \rangle$ does not evolve in time to first order in $\lambda$ (this is obvious for the exact density matrix). The constant value $\rho(t) = \rho(0)$ is the result of a subtle cancellation of TD functions similar to $A_1(t)$ and $B_1(t)$.

**Correlated versus uncorrelated ground state.** Next we consider the effects of correlations in the ground state. We take $\eta_T = 1$ and compare the TD currents $j_{T1}$ and $j_{T0}$ resulting from Eq. (5) when $\eta_L = 1$ and $\eta_L = 0$ respectively. Note that $j_{T1} \equiv j_{T1}$ (already calculated above). The current $j_{T0} = \xi \Re[B_0]$ is the response to a sudden bias switching and interaction quench; at $t > 0$ the electrons start tunneling from $L$ to $R$ and at the same time forming interacting quasiparticles. The interaction quench has a dramatic impact on the transport properties, both in the transient and steady-state regimes. From Fig. 3 we clearly see that the relaxation behavior is different. The damping envelope of $j_{T0}(t)$ is proportional to $t^{-K^2 - 1} \sim t^{1-2K}$ of $j_{T1}(t)$. Notice that the exponent $-K^2 - 1 < 0$ for all $K$ (first-order perturbation theory in $\lambda$ is meaningful for all $K$).

In the long-time limit we find the intriguing result that $j_{T0}(t \to \infty)$ is exactly given by Eq. (8) with exponent $\beta = K^2$, thus suggesting that the structure of the formula (8) is universal. Below we will prove that this is indeed the case and that $\beta$ is an elegant functional of the switching process. For now, we observe that ground state correlations are not reproducible by quenching the interaction. The system remembers them forever and steady-state quantities are inevitably affected. This behavior is reminiscent of the thermalization breakdown enlightened by Cazalilla[12] and others[13, 14]. Here, however,
above solution to systems initially interacting with the quenching times. We have been able to extend the and with \( \eta = \eta_T = 0 \) for the quench 1 \( \rightarrow K \) (dashed) and the quench sequence 1 \( \rightarrow \frac{1}{2}K \rightarrow K \) (dotted-dashed). Here \( K = 0.75 \) and \( V = 10^{-2} \) and the second quench occurs at \( t_1 = 1 \); same units as in Fig. (2).

we are neither in equilibrium nor close to it (the bias is treated to all orders). The non-equilibrium exponents \( \beta = 2K - 1 \) and \( \beta = K^2 \) refer to current-carrying states as obtained from the full TD Schrödinger equation with different initial states.

**History dependence.** We now address the question whether or not the physical steady-current \( j_S(2K - 1) \) of Eq. (8) is reproducible by more sophisticated switching processes of the interaction like, e.g., an adiabatic switching. Preliminary insight can be gained by calculating \( j(t) \) for a double quench: we first quench an interaction with \( K_1 = (1 + K)/2 \), let the system evolve, and then change suddenly \( K_1 \rightarrow K_2 = K \). The current is calculated along the same line of reasoning of Eq. (4), although the formulas become considerably more cumbersome. In Fig. 4 we compare the TD currents for initially uncontacted leads resulting from an interaction \( K \) (solid), a single quench 1 \( \rightarrow K \) (dashed), and the aforementioned double quench (dotted-dashed). We clearly see that in the latter case the steady-current is larger than \( j_S(K^2) \) (single-quench) and gets closer to \( j_S(2K - 1) \). Strikingly, the double-quench steady-current is again given by \( j_S(\beta) \) of Eq. (8) with \( \beta = \frac{1}{2}((1 + K_1^2)(1 + (\frac{K}{2})^2) - 1. This value of \( \beta \) depends only on the \( K \)-sequence and is independent of the quenching times. We have been able to extend the above solution to systems initially interacting with \( K_0 \) and then subject to an arbitrary sequence of quenches \( K_0 \rightarrow K_1 \rightarrow \ldots \rightarrow K_N = K \). We found the remarkable result that the formula (8) is universal, with the sequence dependent \( \beta \) given by

\[
\beta[K_n] = \frac{K_0}{2^{N-1}} \prod_{n=0}^{N-1} \left[ 1 + \left( \frac{K_{n+1}}{K_n} \right)^2 \right]^2 - 1. \tag{10}
\]

This formula yields the correct values of \( \beta \) for the single and double quench discussed above. Note that for a sequence of increasing interactions \( K_{n+1} \leq K_n \) it holds \( \beta \geq 2K - 1 \) with the equality valid only for \( K_0 = K_1 = \ldots = K_N = K \).

We now show that the special value \( \beta = 2K - 1 \) is also reproducible by an arbitrary (not necessarily adiabatic) **continuous \((N \to \infty)\)** sequential quenching. In this limit the variable \( x_n = n/N \) becomes a continuous variable and we can think of the \( K_n \) as the values taken by a differentiable function \( K(x) \) in \( x = x_n \), with \( K(0) = K_0 \) and \( K(1) = K \). Then, the quantity \( \beta \) becomes a functional of \( K(x) \) that we now work out explicitly. Approximating \( K(x_n+1) = K(x_n + \frac{1}{N}) \approx K(x_n) + \frac{1}{N}K'(x_n) \) and taking the logarithm of Eq. (10) we can write

\[
\log \left( \frac{\beta[K(x)] + 1}{2K(0)} \right) = \lim_{N \to \infty} \sum_{n=0}^{N-1} \log \left( 1 + \frac{1}{N} \frac{K'(x_n)}{K(x_n)} \right)
= \int_0^1 dx \frac{K'(x)}{K(x)} = \log \frac{K(1)}{K(0)}, \tag{11}
\]

from which it follows the history independent result

\[
\beta[K(x)] = 2K - 1. \tag{12}
\]

The above result can easily be generalized to discontinuous switching functions \( K(x) \) for which, instead, the exponent \( \beta \) is history dependent.

**Conclusions.** In conclusion we studied the role of different preparative configurations in TD quantum transport between LLs. By using bosonization methods we showed that a sudden switching of the contacts does not change the steady-state but alters significantly the transient behavior, changing the damping envelope from \( \sim t^{-2K} \) to \( \sim t^{1-2K} \) and magnifying the amplitude of the oscillations. The effects of a sudden interaction quench is even more striking. Besides a different power law decay \( \sim t^{-2K} \) versus \( \sim t^{-K^2-1} \) damping envelope) the steady-current is also different; the I-V characteristic \( j_S \propto V^\beta \) changes from \( \beta = 2K - 1 \) to \( \beta = K^2 \). More generally we proved that for a sequence of interaction quenches the steady-current is a universal function of the exponent \( \beta \) which, in turn, is a functional of the switching process. It is only for smooth switchings that \( \beta \) is history independent and equals the value \( 2K - 1 \) of the initially interacting LL. The explicit \( \beta \) functional derived in this Letter establishes the existence of intriguing memory effects that point to a complex entanglement between equilibrium and non-equilibrium correlations in strongly confined systems.

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