On modeling of three-layered thin bodies

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Abstract. Some questions about the new parameterization of three-dimensional thin body with one small size are given. The vector parametric equations of the multilayered thin body are given. The geometric characteristics are considered. The representations of several differential operators, the equations of motion, the boundary conditions and the constitutive relations (CR) of classic and micropolar theories of elasticity are given. Some interlayer contact conditions are obtained. The definition of the k-order moment of any quantity with respect to the system of Legendre polynomials is given. The problems of the classic and micropolar theories of one-, two- and three-layered prismatic elastic thin bodies in moments of displacement and rotation vectors with respect to the system of Legendre polynomials are formulated. The problem of one-layered two-dimensional rectangular plate with two pinched edges under the distributed load are solved.

Keywords: new parametrization, micropolar theory, thin body, theory of thin bodies, prismatic body, Legendre polynomials.

1. Introduction

Many authors, for example, [1–10], as well as one of the authors of this work [11–19] used an analytical method by applying systems of orthogonal polynomials (Legendre and Chebyshev) to construct single-layer and multilayer theories of thin bodies. In addition, Nikabadze used this method to construct a theory of thin bodies with two small sizes [20,21].

In this work, we construct the micropolar theory of multilayer thin bodies using the systems of orthogonal polynomials. Note that any problem of the thin body theory can be considered in a three-dimensional statement, which is more accurate than the two-dimensional one. However, it is not always possible to implement this approach in practice because of the high complexity of solving three-dimensional problems and a large variety of statements of problems being practically necessary. Due to the wide use of thin bodies (single-layer, two and more layer structures) in mechanical engineering, aircraft engineering, and rocket production, it becomes necessary to create new theories of the thin bodies within the framework of the classical as well as the micropolar theories and to improve the methods for their calculation. Therefore, the construction of complex theories of thin bodies and the development of effective methods for calculating them are important and relevant problems.

2. Parametrization of the multilayered thin body

We consider multilayered thin domain consists of at most countable set of layers. The layers are numbered in ascending order. There are two base regular surfaces on each layer. \( S_{\alpha}^{(-)} \) and \( S_{\alpha}^{(+)} \) is
an inner and outer base surfaces of layer $\alpha$. Thus surfaces $S_\alpha^-$ and $S_\alpha^+$ are the same.

For each layer $\alpha$ we introduce the new parametrization of thin body with one small size. Thus, for each layer $\alpha$ three vectors $(\tilde{r}_1^\alpha, \tilde{r}_2^\alpha, \tilde{r}_3^\alpha)$ defined in the considered points $M \in S_\alpha$ form three-dimensional (spatial) covariant bases, where $\sim \in \{-,0,+,\}$. It is well known that based on these bases [21–23], one can construct the corresponding contravariant bases $(r_1^\alpha, r_2^\alpha, r_3^\alpha)$. By the relation obtained for a new parametrization [21,23] we can obtain relation for each layer $\alpha$ by the next simple rule: if the quantity is belong to the layer $\alpha$ quantity. For example, the vector parametric equation of the layer $\alpha$ follows

$$r_\alpha^\alpha(x^1, x^2) = r_\alpha^- + x^3 h_\alpha(x^1, x^2) = (1 - x^3)(r_\alpha^-) + x^3(\tilde{r}_\alpha^\alpha)x^2, \forall x^3 \in [0, 1].$$

The vector $h_\alpha(x^1, x^2) = (\tilde{r}_\alpha^\alpha - r_\alpha^-)$ performing topological mapping of the inner base $S$ on the surface $S_\alpha$, in general is not perpendicular to the base surface. The end point of each vector $h_\alpha(x^1, x^2)$ is the start point of the vector $h_{\alpha+1}(x^1, x^2)$, $\forall \alpha$. So we have

$$h_{\alpha+\delta} = h_\alpha + \sum_{\nu=\alpha}^{\alpha+\delta} h_\nu, \forall \alpha, \delta.$$ 

Let the multilayer body consists of $K$ layers. Thus we have following relations

$$(+)^\alpha = (-)^\alpha + \sum_{\nu=\alpha}^{\alpha+K} (-)^\nu, h = \sum_{\nu=1}^{K} (+)^\nu - (-)^\nu.$$ 

3. Basic relations of the theory of thin elastic bodies

As is known, motion equations of micropolar deformable rigid body are represented in the following form [24–26]

$$\nabla \cdot \mathbf{P} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad \nabla \cdot \mathbf{\mu} + \mathbf{C} \otimes \mathbf{P} + \rho \mathbf{m} = \mathbf{J} \cdot \partial_t^2 \mathbf{\varphi}, \quad (1)$$

where $\mathbf{P}$, $\mathbf{\mu}$ are stress and moment stress tensors, $\mathbf{C}$ – discriminant tensor, $\mathbf{u}$ – displacement vector, $\mathbf{\varphi}$ – rotation vector, $\mathbf{J}$ – moment of inertia (inner property of the medium), $\mathbf{F}$ – mass force density, $\mathbf{m}$ – mass momentum density, $\rho$ – density of the medium, $\nabla$ – Hamilton nabla operator, $\partial_t = \partial / \partial t$, $t$ – time, $\otimes$ – inner 2-product [21–23]. In the case of thin bodies theory with the new parametrization one can write these equations (1) in the following form [21]

$$g_P^P N_P \mathbf{P} + \partial_3 \mathbf{\tilde{P}} + \rho \mathbf{F} = \rho \partial_t^2 \mathbf{u}, \quad g_P^\mu \mathbf{\mu} + \partial_3 \mathbf{\tilde{\mu}} + \mathbf{C} \otimes \mathbf{P} + \rho \mathbf{m} = \mathbf{J} \cdot \partial_t^2 \mathbf{\varphi}, \quad (2)$$

where $\mathbf{P}^M = r^m \cdot \mathbf{P}$, $\mathbf{\mu}^M = r^m \cdot \mathbf{\mu}$, $N_P = \partial P - g_\alpha^3 \partial_3$. As is known [21], for thin bodies $g_P^P = \sum_{s=0}^{\alpha} A_s^P (x^3)^s$. We consider only the first series term (the 0th order approximation)

$$g_P^P = \sum_{s=0}^{\alpha} A_s^P (x^3)^s = \delta_M^M. \quad (2)$$

Thus for each layer $\alpha$ from (2) for the 0th order approximation we have the equilibrium equations in the following form
\[ N_1 \bar{P}^I + \partial_3 \bar{P}^I + \mu F = 0, \quad N_1 \bar{\mu}_I^+ + \partial_3 \bar{\mu}^+ + C \otimes P + I m = 0. \]  

(3)

CR of micropolar linear elasticity theory for materials without a center of symmetry have a look [21, 26]

\[ \mathbf{P} = \mathbf{A} \otimes \gamma + \mathbf{B} \otimes \kappa, \quad \mathbf{\mu} = \mathbf{C} \otimes \gamma + \mathbf{D} \otimes \kappa, \]  

where \( \gamma = \nabla \mathbf{u} - \mathbf{C} \cdot \varphi \) is the strain tensor, \( \kappa = \nabla \varphi \) is the torsion-bending tensor, \( \mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D} \) – the fourth rank material tensors. If the material has a center of symmetry, then \( \mathbf{B} = 0 \), \( \mathbf{C} = 0 \). Using denotion \( \tilde{\alpha} = \alpha \otimes \tilde{r} \tilde{r} \) similarly to (3) from (4) for each layer \( \alpha \) with a center of simmetry, one has

\[ \tilde{\mathbf{P}} = \mathbf{A} \tilde{M} \tilde{N}_l - \tilde{\mu}_l - \mathbf{A}^{-1} \tilde{q} \tilde{C} \tilde{q}^{-1} \tilde{\varphi}, \quad \tilde{\mu} = \mathbf{D} \tilde{M} \tilde{N}_l - \tilde{\mu}_l. \]  

4. Boundary conditions and interlayer contact conditions

Let the multilayered body consists of \( K \) layers and \( \mathbf{P}_1^{(-)} = \mathbf{P}_1^{(-)} |_{x^3 = 0}, \quad \mathbf{P}_K^{(-)} = \mathbf{P}_K^{(-)} |_{x^3 = 0}, \quad \mathbf{\mu}^{(+)} = \mathbf{\mu}^{(+)} |_{x^3 = 1}. \) Than static boundary conditions on the inner and outer surfaces of a multilayered body are [21]

\[ \tilde{\mathbf{P}}_1^{(-)} = \tilde{\mathbf{P}}_1^{(-)} \tilde{\mathbf{P}}_K^{(-)} = \tilde{\mathbf{P}}_K^{(-)} \tilde{\mathbf{P}}_1^{(-)}, \quad \tilde{\mu}_1^{(-)} = \tilde{\mu}_1^{(-)} \tilde{\mu}_K^{(+)} = \tilde{\mu}_K^{(+)} \tilde{\mu}_1^{(-)}. \]

Interlayer contact conditions for a full contact take the form

\[ \tilde{\mathbf{P}}_\alpha^{(+)} = - \tilde{\mathbf{P}}_\alpha^{(-)} \tilde{\mu}_\alpha^{(+)} = - \tilde{\mu}_\alpha^{(-)} \tilde{u}_\alpha^{(+)} = - \tilde{u}_\alpha^{(-)} \tilde{\varphi}_\alpha^{(+)} = \tilde{\varphi}_\alpha^{(-)} \alpha = 1, \ldots, K - 1. \]

5. Equations and CR in moments with respect to the Legendre polynomials

We expand all unknown functions in a series

\[ P_{ij}(x^1, x^2, x^3) = \sum_{k=0}^{\infty} \binom{k}{k} P_{ij}(x^1, x^2, x^3), \quad \mu_{ij}(x^1, x^2, x^3) = \sum_{k=0}^{\infty} \binom{k}{k} \mu_{ij}(x^1, x^2, x^3), \quad u_i(x^1, x^2, x^3) = \sum_{k=0}^{\infty} \binom{k}{k} u_i(x^1, x^2, x^3), \quad \varphi_i(x^1, x^2, x^3) = \sum_{k=0}^{\infty} \binom{k}{k} \varphi_i(x^1, x^2, x^3), \]

where \( P_k \) are the Legendre polynomials with transformed argument orthogonal on segment \( [0, 1] \). Here, if we consider only the first \( N + 1 \) terms of these series we say that we deal with \( (0, N) \) order approximation [21].

Using the motion equations in the form (3), the definition of the \( k \)th-order moment of tensor quantity with respect to the system of Legendre polynomials, recurrence relations for Legendre polynomials, one can obtain motion equations of the micropolar body in moments of unknown functions with respect to the system of Legendre polynomials. Similarly, one can obtain CR in moments with respect to the system of Legendre polynomials [21].
6. The problem of one-layered two-dimensional rectangular isotropic plate with two pinched edges

Formulation of the problem of the classic theory of elasticity for one-layered two-dimensional rectangular plate with two pinched edges (Figure 1.) of $(0, N)$ order approximation in moments of displacement and rotation vectors with respect to the Legendre polynomials consists of:

1) the motion equations of $(0, N)$ order approximation of the classic theory of elasticity

\[
\frac{\partial}{\partial t} P_{I1} + \frac{2k + 1}{h} (P_{I2} - (-1)^k P_{I2}) - \frac{2k + 1}{h} \sum_{p=0}^{k} (1 - (-1)^{k+p}) P_{I2} = 0, \quad I = 1, 2, \quad k = 0, N, \tag{5}
\]

2) the system of CR of $(0, N)$ order approximation of the classic theory of elasticity

\[
P_{1I}^{(k)} = (\lambda + 2\mu) \frac{\partial}{\partial x} P_{1I}^{(k)} + \frac{2(2k+1)\lambda}{h} \sum_{p=0}^{N} \frac{(k+2p+1)}{u_2} \quad \text{and} \quad P_{12}^{(k)} = \mu \frac{\partial}{\partial x} P_{12}^{(k)} + \frac{2(2k+1)\mu}{h} \sum_{p=0}^{N} \frac{(k+2p+1)}{u_1},
\]

\[
P_{2I}^{(k)} = \lambda \frac{\partial}{\partial x} P_{2I}^{(k)} + \frac{2(2k+1)(\lambda + 2\mu)}{h} \sum_{p=0}^{N} \frac{(k+2p+1)}{u_2}, \quad k = 0, N, \tag{6}
\]

3) the system of boundary conditions on the base surfaces $S$ and $S$

\[
P_{12}^{(k)} = 0, \quad P_{22}^{(k)} = 0, \quad P_{12}^{(k)} = 0, \quad P_{22}^{(k)} = -q, \tag{7}
\]

4) the systems of boundary conditions of $(0, N)$ order approximation on the side surface

\[
u_{I1}^{(k)} |_{x_1 = 0} = 0, \quad \nu_{I1}^{(k)} |_{x_1 = L} = 0, \quad k = 0, N. \tag{8}
\]

7. The problem of three-layered two-dimensional rectangular isotropic plate with two pinched edges

Formulation of the problem of the classic theory of elasticity for three-layered two-dimensional rectangular plate with two pinched edges of $(0, N)$ order approximation in moments of displacement and rotation vectors with respect to the Legendre polynomials consists of:

1) the systems of motion equations of $(0, N)$ order approximation of the classic theory of elasticity for each layer

\[
\frac{\partial}{\partial t} P_{11}^{(k)} + \frac{2k + 1}{h} (P_{12}^{(k)} - (-1)^k P_{12}^{(k)}) - \frac{2k + 1}{h} \sum_{p=0}^{k} (1 - (-1)^{k+p}) P_{12}^{(k)} = 0, \quad I = 1, 2, \quad \alpha = 1, 2, 3, \quad k = 0, N, \tag{9}
\]

2) the systems of CR of $(0, N)$ order approximation of the classic theory for each layer

\[
P_{11}^{(k)} = (\lambda_{\alpha} + 2\mu_{\alpha}) \frac{\partial}{\partial x} P_{11}^{(k)} + \frac{2(2k+1)\lambda_{\alpha}}{h} \sum_{p=0}^{N} \frac{(k+2p+1)}{u_2^{(k)}} \quad \text{and} \quad P_{12}^{(k)} = \mu_{\alpha} \frac{\partial}{\partial x} P_{12}^{(k)} + \frac{2(2k+1)\mu_{\alpha}}{h} \sum_{p=0}^{N} \frac{(k+2p+1)}{u_1^{(k)}},
\]

\[
P_{22}^{(k)} = \lambda_{\alpha} \frac{\partial}{\partial x} P_{22}^{(k)} + \frac{2(2k+1)(\lambda_{\alpha} + 2\mu_{\alpha})}{h} \sum_{p=0}^{N} \frac{(k+2p+1)}{u_2^{(k)}}, \quad k = 0, N, \tag{10}
\]

3) the system of boundary conditions on the base surfaces $S$ and $S$

\[
P_{12}^{(k)} = 0, \quad P_{22}^{(k)} = 0, \quad P_{12}^{(k)} = 0, \quad P_{22}^{(k)} = -q, \tag{11}
\]
Figure 1. Rectangular plate with two pinched edges under the distributed load

4) the systems of boundary conditions of \((0, N)\) order approximation on the side surface

\[
\frac{\partial}{\partial x_1} \mathbf{u}_I |_{x_1=0} = 0, \quad \frac{\partial}{\partial x_1} \mathbf{u}_I |_{x_1=L} = 0, \quad \alpha = 1, 2, 3, \quad k = 0, N,
\]

5) interlayer contact conditions in case of full contact

\[
(+) P_{12} = (-) P_{12}, \quad (+) \mu_{12} = (-) \mu_{12}, \quad \sum_{p=0}^{N}(\mu_{p})_{I} = \sum_{p=0}^{N}(-1)^{p}(\mu_{p})_{I}, \quad \sum_{p=0}^{N}(\mu_{p})_{I} = \sum_{p=0}^{N}(-1)^{p}(\mu_{p})_{I}.
\]

8. The problem of one-layered two-dimensional rectangular micropolar isotropic plate with two pinched edges

Formulation of the problem of the micropolar theory of elasticity for one-layered two-dimensional rectangular plate with two pinched edges (Figure 1.) of \((0, N)\) order approximation in moments of displacement and rotation vectors with respect to the Legendre polynomials consists of:

1) The motion equations of \((0, N)\) order approximation of the micropolar theory of elasticity

\[
\partial_t (\varphi)_{I} + \frac{2k+1}{h} [P_{12} - (-)^k P_{12}] = - \frac{2k+1}{h} \sum_{p=0}^{k} [1 - (-)^k p]^2 \mathbf{u}_I, \quad I = 1, 2,
\]

\[
\partial_t \mu_{13} + \frac{2k+1}{h} \sum_{p=0}^{k} (\mu_{12} - (-)^k \mu_{12}) = - \frac{2k+1}{h} \sum_{p=0}^{k} [1 - (-)^k p] \mu_{23} + P_{21} - P_{12} = 0, \quad k = 0, N,
\]

2) The system of CR of \((0, N)\) order approximation of the micropolar theory of elasticity

\[
(\varphi)_{I} = (\lambda + 2\mu) \partial_t \mathbf{u}_I + \frac{2(2k+1)(\lambda - \alpha)}{h} \sum_{p=0}^{k} \mathbf{u}_I, \quad (\mu)_{22} = \lambda \partial_t \mathbf{u}_I + \frac{2(2k+1)(\lambda + 2\mu)}{h} \sum_{p=0}^{k} \mathbf{u}_I,
\]

\[
(\mu)_{12} = (\mu + \alpha) \partial_t \mathbf{u}_I + \frac{2(2k+1)(\mu - \alpha)}{h} \sum_{p=0}^{k} \mathbf{u}_I, \quad (\mu)_{23} = \frac{2(2k+1)(\delta + \beta)}{h} \sum_{p=0}^{k} \mathbf{u}_I, \quad k = 0, N,
\]

3) The system of boundary conditions on the base surfaces \((+), (-)\)

\[
P_{21} = 0, \quad P_{22} = 0, \quad \mu_{23} = 0, \quad P_{21} = 0, \quad P_{22} = -q, \quad \mu_{23} = 0,
\]

4) The systems of boundary conditions of \((0, N)\) order approximation on the side surface

\[
\mathbf{u}_I |_{x_1=0} = 0, \quad \mathbf{u}_I |_{x_1=L} = 0, \quad \varphi |_{x_1=0} = 0, \quad \varphi |_{x_1=L} = 0, \quad k = 0, N.
\]
Table 1

| Method | (0,0) approx | (0,1) approx | (0,2) approx | (0,3) approx | (0,4) approx | (0,5) approx | (0,6) approx | (0,7) approx | (0,8) approx |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| FEM    | -2.57        | -0.77        | -2.16        | -2.44        | -2.55        | -2.56        | -2.57        | -2.57        | -2.57        |

Table 2

| Method | (0,0) approx | (0,1) approx | (0,2) approx | (0,3) approx | (0,4) approx | (0,5) approx |
|--------|--------------|--------------|--------------|--------------|--------------|--------------|
| Classic theory | -0.01 | -0.47 | -0.74 | -0.74 | -0.75 | -0.75 |
| Micropolar theory | -0.01 | -0.74 | -0.30 | -0.30 | -0.30 | -0.30 |

Figure 2. Numerical solution of the problem for (0,2) order approximation.

9. Numerical solution of the problem of one-layered two-dimensional rectangular isotropic plate with two pinched edges

Solving the problem (5) – (8) for classic elastic steel plate (Young’s modulus $E = 21000 \text{ kg/mm}^2$, Poisson’s ratio $\nu = 0.3$) at thickness $h = 10\text{ mm}$, length $L = 100\text{ mm}$, pressure $q = 100 \text{ kg/mm}$ we obtained the solutions for $(0,0)\text{th} - (0,8)\text{th}$ approximations. The problem also was solved by finite element method. The maximum values of the deflection of a plate are presented in the Table 1. Solution of the problem for $(0,2)\text{th}$ approximation is presented at the Figure 2.

For the problem (9) – (12) for micropolar isotropic plate ($E = 300\text{ MPa}$, $\nu = 0.4$, $l_b = 0.33\text{ mm}$, $N^2 = 0.02$) [27] at thickness $h = 1\text{ mm}$, length $L = 100\text{ mm}$, pressure $q = -0.01 \text{ kg/cm}$ we obtained solution for $(0,0)\text{th} - (0,5)\text{th}$ approximations. Also we obtained solution for analogous isotropic classic plate ($E = 300\text{ MPa}$, $\nu = 0.4$). The maximum values of the deflection of a plate are in the Table 2.

Note also that of great interest are works on the thermomechanics of composite structures under high temperature [28–32], which can be used when considering problems of thermomechanics for thin structures using the method described above.

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