Analysis of Coupling Bending Vibration Characteristics of Shafting and Hull under Propeller Excitation

Yuan-bo LI\textsuperscript{1,2,3}, Rui HUO\textsuperscript{1,2}, Wei-ke WANG\textsuperscript{4}, Yuan-bo SUN\textsuperscript{1,2} and Ze-kun ZHANG\textsuperscript{1,2}

\textsuperscript{1}School of Mechanical Engineering, Shandong University, Ji'nan; 250061, China
\textsuperscript{2}Key Lab. of High Efficiency and Clean Mechanical Manufacture, Shandong University, Ministry of Education, Ji'nan; 250061, China
\textsuperscript{3}92143 Force of PLA, San'ya; 572000, China
\textsuperscript{4}32128 Force of PLA, Ji'nan; 250061, China

*Corresponding author

Keywords: Propeller lateral vibration force, Bending vibration, Power flow calculation, Argument variables, Characteristic analysis.

Abstract. Affected by some difficult factors such as bearing stiffness and lubricant film stiffness and so on, the calculation of the lateral vibration response of the ship shafting is difficult to guarantee high calculation accuracy. A mathematical model of the propulsion system-submarine hull coupling bending vibration was established by mathematical physics, and the bearing excitation of the propeller was calculated. A mathematical model of the bending vibration response of the propulsion shaft system and the transmission of vibrational power to the hull was established to calculate the transmitted power flow. The vibration response and bending vibration power flow of the shafting under the lateral excitation of the propeller were solved. Through MATLAB simulation calculation and analysis, the influence of system parameter variation on the bending vibration flow transmission of each node in the power flow transmission path was obtained.

Introduction

The ship propeller is operated by the fluid excitation force according to the leaf frequency cycle in the uneven flow field of the ankle, which causes the transverse vibration resonance of the shaft system [1]. In recent years, the research on dynamic modeling and vibration characteristics analysis of propulsion shafting is mainly based on modeling the propulsion shafting as an independent system. The research on torsional vibration and longitudinal vibration is more mature [2], while bending vibration The research is relatively rare. The power flow includes the force and velocity parameters of the vibration and is therefore theoretically and practically applicable to describe the vibration characteristics [3]. Based on the background of submarine stealth technology research, this paper analyzes the bending vibration characteristics of the coupled vibration of the system from the viewpoint of vibration power flow transmission from the background of the propulsion shaft-propeller-hull lateral vibration coupling. The form and mechanism of noise radiation generated by the ship's propulsion system to excite the hull.

Establish, Solution and Power Flow Calculation of Shafting Lateral Vibration Model

Lateral Vibration Model of Propulsion Shafting of a Submarine

The bending vibration of the propulsion shafting is due to the lateral exciting force [4]. Since the lateral restraint stiffness of the thrust bearing of a submarine is much larger than the lateral restraint stiffness of the adjacent elastic coupling, the elastic coupling to the main thrust motor part of the propulsion shaft system is less affected by the lateral excitation of the propeller, so - Mechanical modeling of the shaft-hull coupling lateral vibration, mainly for the shaft section between the propeller and the thrust bearing. Figure 1 is a diagram showing the power transmission relationship.
between the above subsystems. In order to simplify the analysis, the axial width of each bearing is neglected in Figure 1, and the contact between each bearing and the drive shaft and the hull is regarded as point contact; at the same time, each bearing is regarded as a massless linear elastic element.

\[ J_{P2} \dot{\theta}_{2} = \ddot{\bar{M}}_{B} - c_{P2} \dot{\theta}_{2}, \quad m_{B} \ddot{\bar{w}}_{P} = -F_{P2} - F_{T} - c_{P} \bar{W}_{P} \]  

(1)

Transmission shaft: The vibration characteristics of the drive shaft are described by the transverse vibration Euler equation of the beam:

\[ \rho_{2} S_{B} \ddot{\bar{w}}_{B}(x, t) + (EI_{B})_{2} \dddot{\bar{w}}_{B,xxx}(x, t) = 0 \]  

(2)

Note that equation (2) applies to the three shaft segments in Figure 1, simplifying the coupling of each bearing to the drive shaft to point contact, we can get:

\[
\begin{align*}
\ddot{\bar{w}}_{B}(x, t) &= \ddot{\bar{w}}_{MB}(x, t) \quad (0 \leq x \leq L_{MB}) \\
\ddot{\bar{w}}_{B}(x, t) &= \ddot{\bar{w}}_{FB}(x, t) \quad (L_{MB} \leq x \leq L_{MB} + L_{FB}) \\
\ddot{\bar{w}}_{B}(x, t) &= \ddot{\bar{w}}_{BH}(x, t) \quad (L_{MB} + L_{FB} \leq x \leq L_{2})
\end{align*}
\]  

(3)

Sliding bearing lubrication film:

- Expression of power transmission characteristics of rear bearing lubrication film (\( \ddot{\bar{w}}_{BH} \) and \( \ddot{\bar{w}}_{BH} \)) indicates vibration displacement and vibration deflection angle; \( \ddot{\bar{M}}_{B} \) and \( \ddot{\bar{F}}_{B} \) represent the alternating components in M Band F B, \( \ddot{\bar{M}}_{BH} \) and \( \ddot{\bar{F}}_{BH} \) represent the alternating components in M BH and F BH:

\[
\begin{bmatrix}
\ddot{\bar{M}}_{B} \\
\ddot{\bar{F}}_{B}
\end{bmatrix} =
\begin{bmatrix}
\ddot{\bar{M}}_{BH} \\
\ddot{\bar{F}}_{BH}
\end{bmatrix} =
\begin{bmatrix}
k_{B} & 0 \\
0 & k_{B}
\end{bmatrix}
\begin{bmatrix}
\ddot{\bar{w}}_{BH} - \ddot{\bar{w}}_{B}(0, t) \\
\ddot{\bar{w}}_{BH} - \ddot{\bar{w}}_{B}(0, t)
\end{bmatrix} =
k_{B}
\begin{bmatrix}
\ddot{\bar{w}}_{BH} - \ddot{\bar{w}}_{BH}(0, t) \\
\ddot{\bar{w}}_{BH} - \ddot{\bar{w}}_{BH}(0, t)
\end{bmatrix}
\]  

(4)

- Expression of power transmission characteristics of tail intermediate bearing lubricating film:
The bending vibration energy of the shaft and its coupling structure with the hull:

\[ \tilde{\psi}_s(\sigma, t) = \sum_{n=1}^{\infty} W_{Hn}(\sigma) q_n(t) \]  

Where \( \sigma \) represents the position coordinate of the response pickup point in the hull structure coordinate system; \( W_{Hn} \) is the modal function; \( q_n \) is the modal coordinate, and satisfies:

\[ \ddot{q}_n + \omega_n^2 (1 + j \xi) q_n = \frac{-1}{M_n} (Q_{Bn} + Q_{Mn} + Q_{Fn} + Q_{Ln}) \]  

Where \( M_n, \omega_n \) and \( \xi_n \) are the modal mass, natural frequency and damping loss factor of each order; \( \sigma_B, \sigma_M, \sigma_F, \sigma_L \) represent the rear bearing of the ankle, the intermediate bearing of the ankle, the front bearing of the ankle and the thrust. The coordinates of the bearing housing in the hull structure coordinate system.

**Solving the Differential Equation of Motion**

Perform Fourier transform on each of the 2.2 sections, \( \lambda_{B2} = \rho S_2 \omega^2 l(EI)_2 \). Find the general solution:

\[ W_2(x_2\omega) = A \cdot \exp(-j \lambda_{B2} x_2) + B \cdot \exp(-\lambda_{B2} x_2) + C \cdot \exp(j \lambda_{B2} x_2) + D \cdot \exp(\lambda_{B2} x_2) \]  

\[
W_{MB} = \left[ \begin{array}{l}
\lambda_{MB} \\
\lambda_{MB}
\end{array} \right] A_{MB} + \left[ \begin{array}{l}
\lambda_{MB} \\
\lambda_{MB}
\end{array} \right] B_{MB} \\
\lambda_{MB} \left[ \begin{array}{l}
\lambda_{MB} \\
\lambda_{MB}
\end{array} \right] C_{MB} + \left[ \begin{array}{l}
\lambda_{MB} \\
\lambda_{MB}
\end{array} \right] D_{MB} \\
0 \leq x_2 \leq l_{MB}
\]

\[
W_{FM} = \left[ \begin{array}{l}
\lambda_{FM} \\
\lambda_{FM}
\end{array} \right] A_{FM} + \left[ \begin{array}{l}
\lambda_{FM} \\
\lambda_{FM}
\end{array} \right] B_{FM} \\
\lambda_{FM} \left[ \begin{array}{l}
\lambda_{FM} \\
\lambda_{FM}
\end{array} \right] C_{FM} + \left[ \begin{array}{l}
\lambda_{FM} \\
\lambda_{FM}
\end{array} \right] D_{FM} \\
l_{MB} \leq x_2 \leq l_{MB} + l_{FM}
\]

\[
W_{LF} = \left[ \begin{array}{l}
\lambda_{LF} \\
\lambda_{LF}
\end{array} \right] A_{LF} + \left[ \begin{array}{l}
\lambda_{LF} \\
\lambda_{LF}
\end{array} \right] B_{LF} \\
\lambda_{LF} \left[ \begin{array}{l}
\lambda_{LF} \\
\lambda_{LF}
\end{array} \right] C_{LF} + \left[ \begin{array}{l}
\lambda_{LF} \\
\lambda_{LF}
\end{array} \right] D_{LF} \\
l_{MB} + l_{FM} \leq x_2 \leq L_2
\]

\( \lambda_{MB}, \lambda_{FM}, \lambda_{LF} \) indicates the wavenumber of the three axis segments; assuming that each axis segment has the same material and cross-sectional shape, \( \lambda_{MB} = \lambda_{FM} = \lambda_{LF} = \lambda_{B2} \). Taking the
partial derivative of equation (11) on $x_2$, apply it to equation (12), and solve the equation $a_{MB}^+, a_{MB}^-, a_{FM}^+, a_{FM}^-, a_{LF}^+, a_{LF}^-$. 

$$
\begin{bmatrix}
\Gamma_1 \\
\Gamma_2 \\
\Gamma_3 \\
\Gamma_4 \\
\Gamma_5 \\
\Gamma_6
\end{bmatrix}
= \begin{bmatrix}
a_{mb}^+ \\
a_{mb}^- \\
a_{fm}^+ \\
a_{fm}^- \\
a_{lf}^+ \\
a_{lf}^-
\end{bmatrix}
\begin{bmatrix}
a_{mb}^+ \\
a_{mb}^- \\
a_{fm}^+ \\
a_{fm}^- \\
a_{lf}^+ \\
a_{lf}^-
\end{bmatrix}
= \Gamma
$$

(13)

In the middle, $O = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$; $\alpha = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$;

$$
\Gamma_{11} = \frac{f_{MB}^+ + (-Z_{FB} + R_{Z_{FB}1})w_{MB}^+ + R_{Z_{FB}2}w_{MB}^+ e_{MB}^+ (l_{MB})}{1 + \frac{n}{n}}
$$

(14)

Calculation of Power Flow of Shaft Bending Vibration Transmission

Propeller lateral excitation $\tilde{r}_I$ vibration power flow input for the shafting:

$$
P_{pi} = \frac{\alpha t}{2\pi} \int_{0}^{2\pi} \text{Re}\{\tilde{r}_I^*\} \text{Re}\{-\tilde{w}_I\} dt = \frac{1}{2} \text{Re}\{\tilde{F}_I^* \cdot (-j\omega W_p^*)\} = \frac{1}{2} \text{Re}\{\tilde{F}_I^* \cdot j\omega W_p^* (0, \omega)\}
$$

Average power of vibration dissipation of fluid damping, etc. during propeller bending vibration:

$$
P_{cp} = \frac{\alpha t}{2\pi} \int_{0}^{2\pi} \text{Re}\left\{\tilde{r}_I^* \tilde{\theta}_p \right\} \text{Re}\left\{\tilde{\theta}_p^* \tilde{w}_p \right\} + \text{Re}\left\{\tilde{r}_I^* \tilde{\omega}_p \right\} \text{Re}\left\{\tilde{\omega}_p^* \tilde{w}_p \right\} dt
$$

(15)

Transfer power flow at the junction of the propeller and the shafting:

$$
P_{th} = \frac{\alpha t}{2\pi} \int_{0}^{2\pi} \text{Re}\left\{(EI)_{MB} \tilde{w}_{MB,x} (0, t)\right\} \text{Re}\left\{-\tilde{w}_{MB,x} (0, t)\right\} + \text{Re}\left\{(EI)_{MB} \tilde{w}_{MB,x} (0, t)\right\} \text{Re}\left\{\tilde{w}_{MB} (0, t)\right\} dt
$$

(16)

Use complex stiffness and complex wave number concepts: $(EI)_{MB} = (EI)_{MB}^R (1+jn_{MB})$
The power flow transmitted to the bearing housing and the hull structure by the rear bearing is contributed by the bearing radial excitation force \( \vec{F}_{\text{BH}} \) and the bending excitation \( \vec{M}_{\text{BH}} \), denoted \( P_{\text{FBH}} \) and \( P_{\text{MBH}} \).

\[
P_{\text{FBH}} = \frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} \text{Re}\{\vec{F}_{\text{BH}}(t)\} \cdot \text{Re}\{\omega \vec{\omega}_{\text{BH}}(\sigma_{B}, t)\} \, dt = \frac{1}{2} \text{Re}\{\vec{F}_{\text{BH}} \cdot (j \omega \vec{\omega}_{\text{BH}})\}
\]

\[
P_{\text{MBH}} = \frac{\omega}{2\pi} \int_{0}^{2\pi/\omega} \text{Re}\{\vec{M}_{\text{BH}}(t)\} \cdot \text{Re}\{\omega \vec{\omega}_{\text{BH}}(\sigma_{B}, t)\} \, dt = \frac{1}{2} \text{Re}\{\vec{M}_{\text{BH}} \cdot (j \omega \vec{\omega}_{\text{BH}})\}
\]

The total power flow transmitted to the bearing housing and the hull structure by the rear bearing:

\[ P_{\text{BH}} = P_{\text{FBH}} + P_{\text{MBH}}. \]

According to the above calculation method, the radial excitation force \( \vec{F}_{\text{BH}} \) and the bending moment excitation \( \vec{M}_{\text{BH}} \) at the intermediate bearing, the radial excitation force \( \vec{F}_{\text{FH}} \) and the bending moment excitation \( \vec{M}_{\text{FH}} \) at the front bearing, the radial excitation force \( \vec{F}_{\text{FH}} \) at the thrust bearing, and the bending moment excitation \( \vec{M}_{\text{FH}} \) are sequentially obtained. The power flow input for the bearing housing and the hull structure is as follows:

\[
P_{\text{FBH}} = \frac{1}{2} \text{Re}\{j \omega \vec{\alpha}_{\text{BH}}^H \vec{W}_{\text{BH}}^H \vec{H}_F^2 \cdot [1 \ 0 \ 0 \ 0] \cdot \vec{Z}_{\text{BH}} \cdot \vec{\alpha}_{\text{BH}}\};
\]

\[
P_{\text{MBH}} = \frac{1}{2} \text{Re}\{j \omega \vec{\alpha}_{\text{BH}}^H \vec{W}_{\text{BH}}^H \vec{H}_F^2 \cdot [0 \ 0 \ 0 \ 1] \cdot \vec{Z}_{\text{BH}} \cdot \vec{\alpha}_{\text{BH}}\};
\]

\[
P_{\text{FBH}} = \frac{1}{2} \text{Re}\{j \omega \vec{\alpha}_{\text{BH}}^H \vec{W}_{\text{BH}}^H \vec{H}_F^3 \cdot [1 \ 0 \ 0] \cdot \vec{Z}_{\text{BH}} \cdot \vec{\alpha}_{\text{BH}}\};
\]

\[
P_{\text{MBH}} = \frac{1}{2} \text{Re}\{j \omega \vec{\alpha}_{\text{BH}}^H \vec{W}_{\text{BH}}^H \vec{H}_F^3 \cdot [0 \ 0 \ 0 \ 1] \cdot \vec{Z}_{\text{BH}} \cdot \vec{\alpha}_{\text{BH}}\};
\]

The same reason: \( P_{\text{MH}} = P_{\text{FMH}} + P_{\text{MMH}}; P_{\text{FH}} = P_{\text{FFH}} + P_{\text{MFH}}; P_{\text{LH}} = P_{\text{FLH}} + P_{\text{MLH}}. \)

In combination with the above various equations, the sum of the transmitted power flows in the bending vibration of the shafting is:

\[
P_{\text{HS}} = P_{\text{BH}} + P_{\text{MBH}} + P_{\text{FH}} + P_{\text{LH}} = 0.5 \text{Re}\{j \omega \vec{\alpha}_{\text{BH}}^H \vec{W}_{\text{BH}}^H \vec{H}_F^2 \vec{Z}_{\text{BH}} \vec{W}_{\text{BH}}^H \vec{\alpha}_{\text{BH}}\}
\]

### Simulation Analysis of Power Flow Transfer of System Bending Vibration

Referring to the relevant information [3], the radial stiffness of the rear bearing, the intermediate bearing, the front bearing and the thrust bearing are sequentially set to \( K_{R}=3.5\times10^7 \text{ N/m} \), \( K_{M}=2\times10^7 \text{ N/m} \), \( K_{F}=2\times10^7 \text{ N/m} \), \( K_{L}=1\times10^8 \text{ N/m} \). The deflection stiffness is set to \( K_{B}=3.5\times10^4 \text{ N-m/rad} \), \( K_{M}=2\times10^4 \text{ N-m/rad} \), \( K_{L}=1\times10^5 \text{ N-m/rad} \). The damping loss factor is set to \( \eta=0.1 \). The parameters of the remaining subsystems can be found in the references[8].

Through MATLAB numerical simulation, the influence law of various subsystem characteristic parameters in the bending vibration power flow transmission path on power flow transfer characteristics is analyzed. Figure 2 changes the propeller moment of inertia by changing the propeller radius and mass.
Figure 2. Effect of propeller moment of inertia.

Figure 2(a)–(d), the variation of the propeller deflection moment of inertia has a great influence on the low-frequency power flow transmission characteristics. The larger propeller deflection moment of inertia increases the input power of the hull power in the low frequency band.

Using the same simulation method, the experimental results of the No. 2 axis parameters, the radial stiffness of each bearing, and the change of the hull parameters can be obtained: the change of the propeller mass will cause the movement of the main peak position of the spectrum curve.

In the frequency band, the bending vibration transmission power flow increases, and the main peak decreases in the frequency band.

Moderately reducing the bending stiffness of the transmission shaft (axis 2) can reduce the power flow of the low frequency band input from each bearing to the hull structure.

The radial stiffness of the thrust bearing can significantly reduce the hull power flow input in the higher frequency domain after the main formant frequency.

The stronger hull structure rigidity is beneficial to reduce the low frequency band through each The bending vibration power flow of the bearing input; increasing the hull damping loss factor mainly leads to an increase in the bending vibration power flow transmitted to the hull.

Conclusion

In this paper, the influence law of the power flow transfer characteristics of the characteristic parameters of each subsystem in the bending vibration power flow transmission path was summarized. The form and influence mechanism of the noise radiation generated by the part of the ship propulsion system were found.

a) the power flow spectrum of the bending vibration of the shafting under the lateral excitation of the propeller gradually increases with the increase of the frequency in the low frequency range, and
gradually decreases with the increase of the frequency after crossing the main peak of the system resonance.

However, due to the transmission shaft, the coupling of multiple bearings and the hull is higher in the higher frequency band after the main peak of the system resonance, and the power flow level transmitted to the hull through each bearing is higher. From the viewpoint of transmission power flow control, the following conclusions were drawn:

b) The smaller transmission shaft (axis 2) bending stiffness is beneficial to reduce the power flow input of the low-frequency hull structure.

c) The larger radial rear bearing radial stiffness is beneficial to reduce the transmission of power flow in the low frequency band.

d) Reducing the radial stiffness of the thrust bearing can significantly reduce the hull power flow input in the higher frequency domain after the main formant frequency.

e) The stronger hull structure rigidity is beneficial to reduce the low frequency boat body power flow input.

f) Smaller propeller deflection moment of inertia reduces the hull power flow input in the low frequency range.

The stronger hull structure rigidity is beneficial to reduce the low frequency boat body power flow input.

g) Increasing the propeller mass causes the vibration power flow to increase in the low frequency band before the main peak.

References

[1] Dongliang L 2012 Shafting—Analysis and experimental study of vibrational acoustic radiation of the hull coupling system [D] Shanghai Jiaotong University P 119.

[2] Yong D, Lin M 2017 Research on vibration control technology of propulsion shafting of an underwater vehicle [D] 16th Symposium on Shipborne Underwater Noise Guiyang, Guizhou, China P 7.

[3] Chen L 2014 Research on coupled vibration and hydroacoustic characteristics of the ship's shafting-tail structure [D] China Ship Research Institute P 79.

[4] Liaoyuan L 2016 Research on hull structural vibration and underwater radiation noise caused by propeller excitation force [D] Harbin Engineering University P 217.

[5] Hoshino T 1989 Hydrodynamic Analysis of Propellers in Steady Flow Using a Surface Panel Method, 2nd Report: Flow Field Around Propeller [J] Journal of the Society of Naval Architects of Japan 1989(166) P 79-92.

[6] Xiangxin L 2016 Research on the influence of double-blade disturbance on the exciting force of propeller [D] Harbin Institute of Technology P 84.

[7] Peng Zh 2016 Research on the Influence of Lubricating Oil Film Characteristics on the Bearing System of Ship Rear Tube [D] 2016 Dalian University of Technology P 57.

[8] Weike W 2016 Research on vibration power flow transfer characteristics of propulsion shaft-hull coupling system [D] Shandong University P 105.