Research on Analysis and Evaluation Method of Vulnerability of Water Resources System

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Abstract: The water resources system is composed of water resource utilization system, water environmental quality system and water disaster system. This paper analyzes the process of vulnerability formation of water resources utilization system from the aspects of natural endowment of water resources, development and utilization of water resources and water use efficiency. The vulnerability model of water resources utilization system was constructed, and the research on vulnerability level was carried out. Based on the trapezoidal fuzzy number of the difference coefficient, an evaluation method based on the number of trapezoidal fuzzy numbers is proposed. The application results show that the principle of the method is clear and the calculation process is simple. It avoids the error that may exist when the value of the component coefficient has a wide peak distribution, and can more accurately reflect the objective actual situation.

1. Introduction

The water resources system is composed of water resource utilization system, water environmental quality system and water disaster system. This paper analyzes the vulnerability of water resources utilization system through the analysis of vulnerability formation process. The impact of climate change and human activities on the water resources system is increasing, which seriously affects the sustainable development of social, economic and ecological environments. The vulnerability of water resources systems has gradually become a hot issue in water research. Purpose of this article is to conduct an in-depth study of the vulnerability level. The set pair analysis method has been widely used in the analysis of water resources systems and has achieved good results[1-4]. In practical applications, the difference coefficient has obvious ambiguity in the process of interval [-1, 1] continuous value, and the triangular fuzzy number approximated by the normal distribution can effectively deal with this ambiguity information. According to the continuity of the difference coefficient, the triangular fuzzy number of the difference coefficient can be constructed and an evaluation model based on the number of triangular fuzzy numbers can be established[5-7]. This method is widely used in water resources and water conservancy projects. Further analysis of the correspondence between the value of the contact component coefficient and the threshold of the grading standard reveals that when the value of the connected component coefficient has a wide distribution characteristic, the number of links based on the triangular fuzzy number may have a large error. To this end, this paper uses trapezoidal fuzzy numbers to represent the difference coefficient, and proposes an evaluation method based on the number of trapezoidal fuzzy numbers, which is applied to the evaluation of the vulnerability of actual water resources utilization systems[8-15].
2. Introduction of the vulnerability of water use systems

2.1 Water use system vulnerability concept

Intergovernmental Panel on Climate Change focuses on water vulnerability research in a changing environment, which refers to the extent to which natural or social systems are vulnerable or unable to cope with the adverse effects of climate change. It is a function of climate change rate and its sensitivity and adaptability. Vulnerability research on generalized water resources systems includes both natural vulnerability and socio-economic vulnerability. For water use systems, natural vulnerability comes from the natural endowment of water resources. Socio-economic vulnerability mainly refers to the development and utilization of water resources, which includes both the degree of development and utilization of water resources and the efficiency of utilization. Based on existing vulnerability research results, it is proposed that the vulnerability of water resources utilization system refers to the sensitivity of the water resources system to natural, man-made and other factors, and the ability of the system to adapt to the above factors. It includes three aspects: natural endurance vulnerability, development and utilization vulnerability and water efficiency vulnerability.

2.2 Water resources utilization system vulnerability process mechanism

The vulnerability of water resource utilization systems is firstly constrained by the amount of water resources (natural endowment of water resources) and secondly by the degree of development and utilization of water resources and water use efficiency. The process mechanism of water resource utilization system vulnerability from the aspects of water resources utilization factors can be analyzed. The water resources system is disturbed and destroyed by natural and human factors, which causes sensitive changes in the water resources system and makes the water resources utilization system sensitive. At this time, the water resources system has an adaptive response to the above-mentioned natural and human factors, and this adaptive ability is mainly manifested by human factors.

The natural factors of the vulnerability of water resources utilization system mainly refer to the natural endowment of water resources. The human factors mainly include the degree of water resources development and utilization and the efficiency of water use. Natural endowments of water resources mainly represent the total amount of water resources, space possession, per capita possession, etc. We select annual precipitation, water resources modulus, and per capita water resources as natural endowment indicators for water resources. The precipitation is abundant, the larger the water resources modulus, the more water resources per capita, the more the water resources system can withstand the impact of the social economy and the ecological environment, the less vulnerable the water resources system is. The degree of water resources development and utilization mainly represents the degree of water resources development, spatial water intensity, and residents' water use. We select water resource utilization rate, water use modulus, water consumption rate, and per capita water consumption as indicators of water resources development and utilization. Water efficiency mainly represents the maximum benefit of water resources utilization. We select indicators for industrial, agricultural and domestic water use, including industrial production water consumption, industrial water reuse rate, agricultural production water consumption, water-saving irrigation area rate, irrigation water consumption per unit area, daily water consumption per capita, etc.

Therefore, from the three aspects of water resources natural endurance vulnerability, development and utilization vulnerability and water efficiency vulnerability, the water resource utilization system vulnerability index system can be constructed.

Table 1 The water resource utilization system vulnerability index system

| index                     | attributes                      | indicator type |
|---------------------------|---------------------------------|----------------|
| Annual precipitation      | natural endowment vulnerability | facade         |
| Water resources modulus   |                                 | facade         |
### Per capita water resources

| Water resource utilization | development and utilization of vulnerability |
|---------------------------|-----------------------------------------------|
| Water modulus             | obverse                                       |
| Water consumption rate    | obverse                                       |
| Per capita water consumption | obverse                                   |
| Industrial production water consumption | obverse                                    |
| Industrial water reuse    | obverse                                       |
| Agricultural output water consumption | obverse                                   |
| Water saving irrigation area rate | obverse                                 |
| Farmland irrigation water per unit area | obverse                                  |
| Per capita daily water consumption | obverse                                 |

#### 3. Evaluation method based on trapezoidal fuzzy number connection number

In practical applications, the difference coefficient has obvious ambiguity in the process of interval [-1, 1] continuous value, and the triangular fuzzy number approximated by the normal distribution can effectively deal with this ambiguity information[16-18]. To this end, through further analysis of the continuity of the difference coefficient, the evaluation of the standard threshold value is used to construct the ladder fuzzy number to represent the difference degree coefficient, and the evaluation method based on the trapezoidal fuzzy number connection number is obtained.

#### 3.1 Basic method theory

Set pair is the basic of the theory of SPA. It is composed by two collections with some contact. Under the specific issues involved, through analyzing the IDC (identity, discrepancy and contrary) of the set pair, we can construct the expression of the contact number. It can describe the certainty and uncertainty of things. The basic expression of the contact number is generally referred to as:

$$
\mu = a + bi + cj
$$

In the formula, \(a\), \(b\), \(c\) denote the degree of identity, discrepancy and contrary, \(a, b, c \in [-1, 1]\), and \(a + b + c = 1\); \(i\) is the coefficient of the discrepancy, \(i \in [-1,1]\); \(j\) is the coefficient of the contrary, taking a given value of -1, and sometimes it only plays the role of opposition mark.

Suppose there is a fuzzy number \(A\) on the real number field, defining a membership function for it. The membership function is the following formula(1).
In the formula: \( a \leq \frac{b-a}{x-a} < b \) \( a < x < b \) \( b \leq \frac{b-a}{x-a} < a \) \( a \leq \frac{d-x}{x} < c \) \( c < x < d \) \( d \leq \frac{d-c}{x} < c \) \( c \leq \frac{d-c}{x} < d \) \( d \geq x \geq d \)

(1)

Then A is a trapezoidal fuzzy number. In the formula: \( a \leq b \leq c \leq d \); when \( a = b = c = d \), A is a triangular fuzzy number, and when \( a = b = c = d \), A is a real number.

### 3.2 Method step procedure

According to the basic steps of the same-synchronous analysis and the general steps of the systematic evaluation method, based on the evaluation method of the trapezoidal fuzzy number connection number, the research set is constructed. According to the analysis of the vulnerability process mechanism of the above-mentioned water resources utilization system, referring to the research results of the existing water resources system vulnerability, combined with the social development level of the research area, the vulnerability assessment standard of the water resources utilization system is divided into five levels, which are in turn micro-fragile, light fragile, moderately fragile, strong fragile, extremely fragile. See Table 2 below for details.

| index | turn micro-fragile | light fragile | moderately fragile | strong fragile | extremely fragile |
|-------|-------------------|--------------|--------------------|---------------|------------------|
| A (mm) | [1000,1500] | [700,1500] | [300,700] | [100,300] | [0,100] |
| B(10^3/m²) | [70,100] | [50,70] | [30,50] | [10,30] | [0,10] |
| C (m³/ha) | [3000,8000] | [2200,3000] | [1600,2200] | [900,1600] | [100,900] |
| D (‰) | [0,010] | [0,10,25] | [25,40] | [40,60] | [60,100] |
| E(10³/m³/km²) | [0,10] | [10,15] | [15,30] | [30,35] | [35,60] |
| F (%) | [10,30] | [30,45] | [45,55] | [55,70] | [70,90] |
| G(m³/ha) | [100,200] | [200,300] | [300,400] | [400,600] | [600,1300] |
| H (m³/10⁶ ha) | [30,150] | [150,250] | [250,350] | [350,450] | [450,550] |
| J (%) | [90,100] | [80,90] | [70,80] | [50,70] | [5,50] |
| K (m³/10⁶ ha) | [50,500] | [500,1000] | [1000,1500] | [1500,2000] | [2000,3000] |
| L (%) | [80,100] | [60,80] | [40,60] | [20,40] | [0,20] |
| M(m³/h m³) | [150,320] | [320,560] | [560,800] | [800,1200] | [1200,1800] |
| N(L/m²) | [10,120] | [120,180] | [180,220] | [220,320] | [320,560] |

In Table 2, for convenience. Annual precipitation can be recorded as A. Water resources modulus is recorded as B. Per capita water resources is recorded as C. Water resource utilization is recorded as D. Water modulus is recorded as E. Water consumption rate is recorded as F. Per capita water consumption is recorded as G. Industrial production water consumption is recorded as H. Industrial production water consumption is recorded as J. Industrial water reuse is recorded as K. Agricultural output water consumption is recorded as L. Water saving irrigation area rate is recorded as M.

The sample data set of each indicator is recorded as \( \{x_{ij}\} = 1, 2, ..., n; j = 1, 2, ..., m \}. The rating criteria is recorded as \( \{y_{ij}\} = 1, 2, ..., G; j = 1, 2, ..., m \} \). In which n, m and G are the number of grades of the number of evaluation samples, the number of evaluation indicators and the evaluation level respectively. The sample i of the evaluation object and the sample value \( x_{ij} \) of the index number j constitute a set \( A_j \), and the standard value \( y_{ij} \) of the index level \( g \) constitutes the set \( B_j \); then the sets \( A_j \) and \( B_j \) are constructed as a set pair \( H(A_j, B_j) \). The single indicator contact number \( u_{ij} \) of the evaluation object is constructed by SPA. The basic idea of SPA is to quantitatively compare and analyze the properties of the two sets discussed under the given problem background, and use the number of contacts for quantitative analysis. In the specific calculation of the single index contact number \( \tilde{u}_{ij} \),
the "same and the opposite level method" can be used to calculate the number expression. Taking \( G = 5 \), the expression of the single index contact number \( u_{ij} \) is the following formula (2).

\[
\mu_{ij} = \begin{cases} 
\frac{x_{ij} - y_{ij}}{2(y_{0j} - y_{ij})} + 0.5 + \frac{y_{0j} - x_{ij}}{2(y_{0j} - y_{ij})} I_1 & x_{ij} \in [y_{0j}, y_{1j}] \\
\frac{x_{ij} - y_{2j}}{2(y_{1j} - y_{2j})} + 0.5I_1 + \frac{y_{1j} - x_{ij}}{2(y_{1j} - y_{2j})} I_2 & x_{ij} \in [y_{1j}, y_{2j}] \\
\frac{x_{ij} - y_{3j}}{2(y_{2j} - y_{3j})} I_1 + 0.5I_2 + \frac{y_{2j} - x_{ij}}{2(y_{2j} - y_{3j})} I_3 & x_{ij} \in [y_{2j}, y_{3j}] \\
\frac{x_{ij} - y_{4j}}{2(y_{3j} - y_{4j})} I_2 + 0.5I_3 + \frac{y_{3j} - x_{ij}}{2(y_{3j} - y_{4j})} J_1 & x_{ij} \in [y_{3j}, y_{4j}] \\
\frac{x_{ij} - y_{5j}}{2(y_{4j} - y_{5j})} I_3 + 0.5I_4 + \frac{y_{4j} - x_{ij}}{2(y_{4j} - y_{5j})} J_2 & x_{ij} \in [y_{4j}, y_{5j}] 
\end{cases}
\]

(2)

The fuzzy number is constructed with the evaluation level standard threshold as a parameter. For example, the trapezoidal fuzzy number \( A(I_{1j}) = (y_{0j}, y_{1j}, y_{2j}, y_{3j}) \) is constructed with the gradation standard thresholds \( y_{0j}, y_{1j}, y_{2j}, y_{3j} \) as parameters. The membership function of the trapezoidal fuzzy number is used to represent the partial difference coefficient and the partial difference coefficient. The membership function of the piecewise triangular fuzzy number is used to determine the value of the difference coefficient. Taking the diagonality coefficient \( J_{1j} = J_{2j} = 1 \), the calculation formula of the number of contacts based on the trapezoidal fuzzy number is as shown in the following equation (3).

\[
\tilde{\mu}_{ij} = \begin{cases} 
\frac{x_{ij} - y_{ij}}{2(y_{0j} - y_{ij})} + 0.5 + \frac{(y_{0j} - x_{ij})^2}{4(y_{0j} - y_{ij})} & x_{ij} \in [y_{0j}, y_{1j}] \\
\frac{x_{ij} - y_{2j}}{2(y_{1j} - y_{2j})} + 0.25 + \frac{(y_{1j} - x_{ij})^2(y_{1j} - y_{2j})}{4(y_{1j} - y_{2j})^2} I_2 & x_{ij} \in [y_{1j}, y_{2j}] \\
\frac{(x_{ij} - y_{3j})^2 - (y_{2j} - x_{ij})^2}{4(y_{2j} - y_{3j})^2} & x_{ij} \in [y_{2j}, y_{3j}] \\
\frac{(x_{ij} - y_{4j})^2 - (y_{3j} - x_{ij})^2}{4(y_{3j} - y_{4j})^2} - 0.25 - \frac{y_{3j} - x_{ij}}{2(y_{3j} - y_{4j})} & x_{ij} \in [y_{3j}, y_{4j}] \\
\frac{(x_{ij} - y_{5j})^2}{4(y_{4j} - y_{5j})^2} - 0.5 - \frac{y_{4j} - x_{ij}}{2(y_{4j} - y_{5j})} & x_{ij} \in [y_{4j}, y_{5j}] 
\end{cases}
\]

(3)

The method for determining the weight of sample connection number index by comprehensive evaluation index function method mainly includes subjective weighting method, objective weighting method, combined weighting method and variable weight method. The coefficient of variation method is a method for determining the weight of an evaluation index based on the degree of variation of the values of each evaluation index, and it can avoid the influence of the dimension and magnitude of the indicator[19、20]. At present, the additive weighted synthesis method is a widely used system evaluation method, and the sample contact number \( \tilde{\mu}_{ij} \) can be calculated by using equation (4). In the formula, \( w_j \) is the weight of the indicator \( j \).

\[
\tilde{\mu}_j = \sum_{j=1}^{m} w_j \ast \tilde{\mu}_{ij}
\]

(4)

The number of contacts is calculated by equation (3). The scatter plots for the contact number \( \tilde{\mu}_{ij} \) and the evaluation level \( h_{ij} \) are obtained by mathematical model analysis. The projection function of
the number of contacts and the evaluation level can be established from the scatter plot.

\[ h_{ij} = \begin{cases} 
6 - 3e^{-0.5\mu_i} & (\mu_i \geq 0) \\
3e^{-0.5\mu_i} & (\mu_i < 0)
\end{cases} \quad (5) \]

From the calculation formula (5), the evaluation index value of the water resources utilization system vulnerability evaluation index \( h_{ij} \) can be calculated, and the vulnerability status of the water resource utilization system can be further analyzed.

4. Actual case analysis

Anhui Province is an inland province near the Lancang River in eastern China, spanning the middle and lower reaches of the Yangtze and Huaihe Rivers. The per capita water resources in Anhui Province is 1125 m\(^3\), which is close to half of the national average and only 1/8 of the world average. The province's water resources are relatively poor. According to Anhui Water Resources Bulletin, the total water resources in 2014 was 778.48×10\(^8\) m\(^3\), and the total water consumption was 272.09×10\(^8\) m\(^3\). The province's water use contradictions are prominent. The water resources situation is grim. The vulnerability of water resources utilization systems cannot be ignored[21]. It is necessary to analyze the vulnerability of regional water use systems. This can provide a guarantee for the sustainable use of regional water resources.

According to the vulnerability index system of water resources utilization system, the sample data of Anhui Province from 2006 to 2014 were collected. According to the above basic steps of the evaluation method based on the number of trapezoidal fuzzy numbers, the vulnerability of the water resources utilization system in Anhui Province is comprehensively analyzed. According to the sample data of Anhui Province from 2006 to 2014, the single-index contact number and the evaluation single-index evaluation value based on the trapezoidal fuzzy number can be calculated by formulas (3), (4), and (5).

Using the additive weighted synthesis method to calculate the number of sample contacts, the sample evaluation results can be calculated. See Table 3 for details.

| particular year | contact number | evaluation value |
|-----------------|----------------|------------------|
| 2006            | -0.15          | 3.23             |
| 2007            | 0.08           | 2.88             |
| 2008            | 0.08           | 2.81             |
| 2009            | 0.04           | 2.90             |
| 2010            | 0.25           | 2.58             |
| 2011            | 0.13           | 2.80             |
| 2012            | 0.22           | 2.60             |
| 2013            | 0.18           | 2.70             |
| 2014            | 0.31           | 2.50             |

5. Results analysis

In 2014, the vulnerability of water use systems was light and fragile, which was significantly improved compared with the past. Therefore, we should continuously improve the regional ecological environment, improve water conservation capacity, actively implement water conservation measures, and improve the efficiency of industrial and agricultural water use. It is necessary to carry out the process control of the vulnerability of water resources utilization systems and promote the sustainable development of water resources.

6. Conclusion

This paper analyzes the process of vulnerability formation of water resources utilization system from the aspects of natural endowment of water resources, development and utilization of water resources and water use efficiency. The vulnerability model of water resources utilization system was constructed, and the research on vulnerability level was carried out. Based on the trapezoidal fuzzy number of the difference coefficient, an evaluation method based on the number of trapezoidal fuzzy
numbers is proposed. The application results show that the principle of the method is clear and the calculation process is simple. It avoids the error that may exist when the value of the component coefficient has a wide peak distribution, and can more accurately reflect the objective actual situation. It has application value in the analysis of water resources system.

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References
[1] HAO L, WANG J A. Evaluate to water resources vulnerability using SWAT-WEAP model in tributary of Xiliaohe River. Journal of Natural Resources, 2012, 27(3): 450-490.
[2] XIA J, CHEN J X, WENG J W, et al. Advances and future prospects of research in water resources vulnerability under climate change. Progressus Inquisitiones de Mutatione Climatis, 2012, 8(6): 380-400.
[3] Li Ruzhong. Progress and trend analysis of theoretical models of water quality assessment. Journal of Hefei University of Technology, 2005,28(4):369–373(in Chinese)
[4] PAN Z W, JIN J L, WU K Y, et al. Research on the indexes and decision method of regional water environmental system vulnerability. Resources and Environment in the Yangtze Basin, 2014, 23(4): 510-530.
[5] Li Ruzhong. Progress and trend analysis of theoretical models of water quality assessment. Journal of Hefei University of Technology, 2005,28(4):369–373(in Chinese)
[6] Chen Shouyi, Xiong Deqi. Fuzzy set theory and model of lake eutrophication. Journal of Lake Sciences, 1993,5(2):140–155(in Chinese)
[7] LEE Chih-sheng, CHANG Shui-Ping. Interactive fuzzy optimization for an economic and environmental balance in a river system[J]. Water research, 2005, 39:200-240.
[8] YANG X H, HE J, DI C L, et al. Vulnerability of assessing water resources by the improved set pair analysis [J]. Thermal Science, 2104, 18(5): 1500-1560.
[9] JIAO S X, WANG L C, LI J, et al. Study on the application of set pair analysis method in the evaluation of regional water utilization level. Journal of Natural Resources, 2009, 24(4):730-750.
[10] NAN C Y, SU X L. Comprehensive evaluation for soil-water resources carrying capacity in Guanzhong area based on improved set pair analysis. Journal of Natural Resources, 2012, 27(1): 100-120.
[11] Ronald EG, Robert EY. Analysis of the error in the standard approximation used for multiplication of triangular and trapezoidal fuzzy numbers and the development of a new approximation [J]. Fuzzy Sets and Systems, 1997, 91(1):1-13.
[12] Wu Kaiya, Jin Juliang, Pan Zhengwei. Urban flood vulnerability contact number assessment model based on triangular fuzzy number cut set[J]. Journal of Hydraulic Engineering, 2010, 41(6) : 710-728.
[13] MICHAEL H. On the implementation of fuzzy arithmetical operations for engineering problems[A]. Proceedings of 18th International Conference of the North American Fuzzy Information Processing Society—NAFIPS’99, New York, USA, 1999:460-468.
[14] Silvert W. Ecological impact classification with fuzzy sets[J]. Ecological Modelling, 1997,96(2):1-20.
[15] KENTEL E, AREL M M. 2D Monte Carlo versus 2D Fuzzy Monte Carlo health risk assessment[J].Stochastic Environmental Research and Risk Assessment, 2005,19(1):80-100.
[16] YANG Q X, CHEN N X. A coupling model of set pair analysis and triangular fuzzy numbers for evaluation of groundwater environmental quality. China Rural Water and Hydropower, 2012(8): 40-45.
[17] PENG T, CHEN X H, WANG G X, et al. Assessment of coastal wetland ecosystem health based on set pair analysis and triangular fuzzy numbers. Ecology and Environmental Sciences, 2014, 23(6): 917-922.

[18] LI R Z, TONG F, ZHOU A J, et al. Fuzzy assessment model for the health risk of heavy metals in urban dusts based on trapezoidal fuzzy numbers. Acta Scientiae Circumstantiae, 2011, 31(8): 1790-1798.

[19] CHEN N X, YANG Q X. Set pair analysis of watershed water resources carrying capacity based on game theory combination weighting. Journal of Irrigation and Drainage, 2013, 32(2): 81-85

[20] ZHAO K Q. The application of SPA-based identical-discrepancy-contrary system theory in artificial intelligence research. CAAI Transactions on Intelligent Systems, 2007, 2(5): 21-35

[21] Water Resources Department of Anhui Province. 2014 Anhui water resources bulletin. (201508-21) [2015-11-03]. http://guanli.wswj.net/news_file.