Finite-Size Bosonization of 2-Channel Kondo Model: a Bridge between Numerical Renormalization Group and Conformal Field Theory

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We generalize Emery and Kivelson’s (EK) bosonization-refermionization treatment of the 2-channel Kondo model to finite system size and on the EK-line analytically construct its exact eigenstates and finite-size spectrum. The latter crosses over to conformal field theory’s (CFT) universal non-Fermi-liquid spectrum (and yields the most-relevant operators’ dimensions), and further to a Fermi-liquid spectrum in a finite magnetic field. Our approach elucidates the relation between bosonization, scaling techniques, the numerical renormalization group (NRG) and CFT. All CFT’s Green’s functions are recovered with remarkable ease from the model’s scattering states.

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A dynamical quantum impurity interacting with metallic electrons can cause strong correlations and sometimes lead to non-Fermi-liquid (NFL) physics. A prototypical example is the 2-channel Kondo (2CK) model, in which a spin-1/2 impurity is “overscreened” by conduction electrons, leaving a non-trivial residual spin object even in the strong-coupling limit. Many theoretical treatments of this model have been developed \[^\text{1} \text{4}\] including Wilson’s numerical renormalization group (NRG) \[^\text{1} \text{4}\] for the crossover from the free to the NFL regime, Affleck and Ludwig’s (AL) conformal field theory (CFT) \[^\text{5} \text{4}\] for exact thermodynamic and transport quantities, valid only near the NFL fixed point, and Emery and Kivelson’s (EK) bosonization-refermionization mapping onto a resonant-level model \[^\text{1}\] for the free and NFL fixed points. In this Letter we elucidate the well-known yet remarkable fact that these three approaches, despite tremendous differences in style and technical detail, yield mutually consistent results: we show that EK bosonization in a system of finite size \(L\) yields NRG-like finite-size spectra, and reproduces all known CFT results.

Our method requires no knowledge of CFT, only that we bosonize and refermionize with care: Firstly, we construct the boson fields \(\phi\) and Klein factors \(F\) in the bosonization relation \(\psi \sim \Phi e^{-i\phi}\) explicitly in terms of the model’s original fermion operators \(\{c_{\alpha j}\}\). Secondly, we clarify how the Klein factors for EK’s refermionized operators act on the original Fock space. Thirdly, we keep track of the gluing conditions on all allowed states. This enables us (i) to explicitly construct the model’s finite-size eigenstates; (ii) to analytically obtain NRG-like finite-size spectra that cross over from free to CFT’s universal NFL spectra; (iii) to describe magnetic-field-induced crossovers exactly; and (iv) for \(L \to \infty\) to easily recover all AL’s CFT results \[^\text{4}\].

The model.— We consider the standard anisotropic 2CK model with a linearized energy spectrum \[^\text{1} \text{1}\] defined by

\[
H = H_0 + H_z + H_L + H_h \quad (h = v_F = 1): \\
H_0 = \sum_{k \alpha j} c_{k \alpha j}^\dagger c_{k \alpha j}; \quad H_h = h_t S_z + h_s N_s, \\
H_z + H_L = \Delta_L \sum_{k k' \alpha \alpha' j a} \lambda_a : c_{k \alpha j}^\dagger \frac{1}{2} \sigma_{x \alpha \alpha'} S_a c_{k' \alpha' j a} :.
\]

Here \(c_{k \alpha j}^\dagger\) creates a free-electron state \(|k \alpha j\rangle\) with spin \(\alpha = (\uparrow, \downarrow)\), flavor \(j = (1, 2)\), radial momentum \(k \equiv |\vec{p}| - p_F\), and normalization \(c_{k \alpha j}^\dagger c_{k' \alpha' j a} = \delta_{k k'} \delta_{\alpha \alpha'} \delta_{j j'}\). We let the large-\(|k|\) cut-off go to infinity and quantize \(k\) by defining 1-D fields with, for simplicity, antiperiodic boundary conditions at \(x = \pm L/2\); \[^\text{2}\]

\[
\psi_{\alpha j}(x) \equiv \sqrt{\Delta_L} \sum_{n_k \in \mathbb{Z}} e^{-i k x} c_{k \alpha j},
\]

where \(k = \Delta_L (n_k - 1/2)\) and \(\Delta_L = 2 \pi / L\) is the mean level spacing. By : : we denote normal ordering relative to the Fermi ground state \(|0\rangle\). \(H_z + H_L\) is the Kondo coupling (with dimensionless \(\lambda_x \neq \lambda_L \equiv \lambda_y \equiv 0\)) to a local spin-1/2 impurity \(S_a\) (with \(S_z\)-eigenstates \(|\uparrow\rangle, |\downarrow\rangle\)), and \(H_h\) describes magnetic fields \(h_t\) and \(h_s\) coupled to the impurity spin and the total electron spin \(N_s\).

Conserved quantum numbers.— Diagonalizing \(H\) requires choosing a suitable basis. Let any (nonunique) simultaneous eigenstate of \(\hat{N}_{\alpha j} \equiv \sum k ; c_{k \alpha j}^\dagger c_{k \alpha j} ;\), counting the number of \((\alpha j)\) electrons relative to \(|0\rangle\), be denoted by \(|\hat{N}\rangle \equiv |N_{11}\rangle \otimes |N_{12}\rangle \otimes |N_{22}\rangle \otimes |N_{12}\rangle\), with \(\hat{N} \in \mathbb{Z}^4\). Since \(H\) conserves charge, flavor and total spin, it is natural to define new counting operators, \(\hat{N}_y (y = c, s, f, x)\), \(^\text{4}\)

\[
\begin{pmatrix}
\hat{N}_c \\
\hat{N}_s \\
\hat{N}_f \\
\hat{N}_x
\end{pmatrix}
= \frac{1}{2}
\begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & 1 & -1 & 1 \\
1 & -1 & -1 & 1
\end{pmatrix}
\begin{pmatrix}
\hat{N}_{11} \\
\hat{N}_{12} \\
\hat{N}_{22} \\
\hat{N}_{12}
\end{pmatrix},
\]

which give the total electron number, the electron spin, flavor, and spin difference between channels, respectively.
Equation (3) implies that the eigenvalues $\tilde{N}$ are either all integers or all half-integers (i.e. $\tilde{N} \in (\mathbb{Z} + P/2)^\dagger$, with $P = (0, 1)$ for even/odd total electron number), and that they obey the free gluing condition

$$N_c \pm N_f = (N_s \pm N_z) \mod 2.$$  \hspace{1cm} (3)

All non-zero matrix elements of $H_\perp$ have the form $\langle N_c, S_T - \frac{1}{2}N_f, N_\perp | H_\perp | N_c, S_T + \frac{1}{2}N_f, N_\perp \rangle \equiv |H_\perp|N_c, S_T + \frac{1}{2}N_f, N_\perp \pm 1; \psi\rangle$, and since the total spin $S_T = N_s + S_z$ is conserved, the $\tilde{N}_c$-eigenvalue flips only between $S_T \equiv \frac{1}{2}$, i.e. it fluctuates only “mildly”. In contrast, the $\tilde{N}_c$-eigenvalue fluctuates “wildly” [an appropriate succession of spin flips can produce any $\tilde{N}_c$ satisfying (3)]; this will be seen below to be at the root of the 2CK model’s NFL behavior (in revealing contrast to the 1CK model, which has no wildly fluctuating quantum number, and lacks NFL behavior). For given $(N_c, S_T, N_f)$ it thus suffices to solve the problem in the corresponding invariant subspace $\sum_{n\in \mathbb{Z}} \langle N_c, S_T - \frac{1}{2}N_f, N_\perp ; \psi\rangle \equiv |N_c, S_T + \frac{1}{2}N_f, N_\perp + 1; \psi\rangle$, to be denoted by $S$, where the prime on the sum indicates its restriction to $\tilde{N}_c$-values respecting (3).

Bosonization. — To bosonize (3) the model in terms of the original $c_{k\alpha j}$’s (4), we define bosonic fields through

$$b_{q\alpha j} \equiv \frac{i}{\sqrt{n_q}} \sum_{n_q \in \mathbb{Z}} C_{k+q\alpha j}^{\dagger} c_{k\alpha j}^\dagger, \quad \phi_{\alpha j}(x) \equiv \frac{1}{\sqrt{n_q}} (\psi_{q\alpha j}^b + \psi_{q\alpha j}^d) e^{-i\phi_{\alpha j}/2},$$

which account for particle-hole excitations (the $b$’s by construction satisfy $[b_{q\alpha j}, b_{q'\alpha' j'}^\dagger] = \delta_{qq'}\delta_{\alpha\alpha'}\delta_{jj'}$ and $[b_{q\alpha j}, \tilde{N}_{\alpha' j'}] = 0$). Then the usual bosonization relation

$$\psi_{\alpha j}(x) = a^{-1/2} F_{\alpha j} - i(N_{\alpha j}) \tau_2 \pi L e^{-i\phi_{\alpha j}/2}$$

holds as operator identity, where the Klein factors $F_{\alpha j}$ satisfy (see (3)) $[F_{\alpha j}, \tilde{N}_{\alpha' j'}] = \delta_{\alpha\alpha'}\delta_{jj'} F_{\alpha j}$, $[F_{0}, \phi] = 0$ and $\{F_{0}, F_{\alpha j}\} = 2\alpha\phi_{\alpha j}$. Thus $F_{\alpha j}$, $F_{\alpha j}^\dagger$ ladder between the $N_{\alpha j}$, $\tilde{N}_{\alpha j} \equiv 1$ Hilbert spaces without creating particle-hole excitations, and ensure proper $\psi$, $\phi$’s anticommutation relations.

To exploit the conserved quantities in the $N_y$ basis, we now use the transformation (3) to define new boson fields $b_{q\alpha j} \rightarrow b_{q\alpha j}$ and $\phi_{\alpha j} \rightarrow \phi_{\alpha j}$. Writing $H$ in terms of these [via (3)], only $\phi_{\alpha j}$ and $\phi_{\alpha j}$ couple to the impurity (3):

$$H_0 = \Delta_L \sum_y \frac{\tilde{N}_y^2}{2} + \sum_{y, q > 0} q b_{qy}^\dagger b_{qy},$$
$$H_z = \lambda_z \Delta_L S_z \tilde{N}_z + \lambda_z \Delta_L S_z \sum_{n_q > 0} i (b_{qy} - b_{qy}^\dagger),$$
$$H_{\perp} = \frac{\lambda_{\perp}}{2\alpha} e^{i\phi_{\alpha j}(0)} S_z \sum_{y, j, q > 0} \tilde{F}_{y j}^\dagger F_{y j} e^{ij\phi_{\alpha j}(0)} + H.c.$$  \hspace{1cm} (7)

To eliminate $H_{\perp}$, make the UK 3 unitary transformation

$$H' = UHU^\dagger,$$

with $U(\lambda_z) \equiv e^{i\lambda_z S_z \phi_{\alpha j}(0)}$. This yields

$$H'_z = H_0 + H_z', \quad H'_0 = H_0 + \lambda_z \Delta_L \tilde{N}_z S_z + \text{const.,}$$

and $\phi_{\alpha j}$ incurs a phase shift

$$U(\varphi_{\alpha j})U^\dagger = \varphi_{\alpha j}(x) - \lambda_z \pi S_z \text{sgn}(x) \equiv \tilde{\varphi}_{\alpha j}(x).$$

Next we define a pseudofermion field $\tilde{\psi}_{\alpha j}(x)$ by

$$\tilde{\psi}_{\alpha j}(x) = a^{-1/2} F_{\alpha j} - i(N_{\alpha j} - 1/2) \pi \tau_2 L e^{-i\tilde{\phi}_{\alpha j}}$$

and expand it as $\sqrt{\Delta_L} \sum_k e^{-ikx} c_{k\alpha j}$, by analogy with (3) and (4), which imply $c_{k\alpha j}^\dagger = \delta_{kk'} c_{k'\alpha j}$ in the $c_{k\alpha j}$ basis, the $c_{k\alpha j}$’s create highly non-linear combinations of electron-hole excitations, as is clear from their explicit definition, via $\tilde{\psi}_{\alpha j}$ and $F_{\alpha j}$, in terms of the $c_{k\alpha j}$’s. Since $\tilde{N}_c \in \mathbb{Z} + \frac{1}{2}$, we note that $\psi_{\alpha j}$ has a P-dependent boundary condition, implying $\tilde{k} = \Delta_L (n_q - \frac{1}{4} + P)$, and further that $\Delta_L \left( \tilde{N}_c^\dagger / 2 + \sum_{q > 0} n_q b_{qy}^\dagger b_{qy} \right) = H_{0x} + H_{0z} / 8$, where $H_{0x} \equiv \sum_k \tilde{k} : c_{k\alpha j}^\dagger c_{k\alpha j} :$ and $n_q$ denotes normal ordering of $c_{k\alpha j}$’s, with $\sum_k c_{k\alpha j} c_{k\alpha j} \equiv \tilde{N}_c - \frac{P}{2}$. We further define the “local pseudofermion” $c_{\alpha j} \equiv F_{\alpha j} S_z$, implying $c_{\alpha j}^\dagger c_{\alpha j} = S_z + \frac{1}{2}$. Eliminating $\tilde{N}_z$ in the subspace $S$ using $\tilde{N}_c = S_T + \frac{1}{2} - c_{\alpha j}^\dagger c_{\alpha j}$ we can rewrite $H'$ as $H_{\text{eff}}(b_{\alpha j}, b_{\alpha j}^\dagger, n_x, N_c, N_f) + H_{\text{G}}$, where $H_{\text{eff}}$ has a trivial spectrum and $H_x$ is quadratic:

$$H_x = \varepsilon_c c_{\alpha j}^\dagger c_{\alpha j} + H_{0x} + \sqrt{\Delta_L} \sum_k (c_{k\alpha j}^\dagger c_{k\alpha j} + c_{k\alpha j} c_{k\alpha j}^\dagger) - c_{\alpha j}^\dagger c_{\alpha j},$$
$$E_G = \Delta_L \left( \frac{1}{2} S_T^2 - \frac{1}{4} + P / 8 \right) - \frac{1}{2} h_x + h_x (S_T + \frac{1}{2}).$$

Here $\Gamma \equiv \lambda_{\perp}^2 / 4\alpha$ and $\varepsilon_c \equiv h_x - h_e$ is the spin flip energy cost. As first noted by EK (3), who derived $H'$ for $L \rightarrow \infty$, impurity properties show NFL behavior since “half the pseudofermion”, $(c_{\alpha j} + c_{\alpha j}^\dagger)$, decouples.
Diagonalizing $\mathcal{H}_x$.—To study the NFL behavior of electron properties, caused by nonconservation of $N_e$, we diagonalize $\mathcal{H}_x$. First define pseudofermions having all non-negative energies: $\alpha_k = \frac{1}{\sqrt{2}}(c_{kx} + c_{kx}^\dagger)$ and $\beta_k = \frac{1}{\sqrt{2}}(c_{kx} - c_{kx}^\dagger)$ for $\mathcal{H}_x$. Then the $\beta_k$'s decouple in $\mathcal{H}_x$, and a Bogoliubov transformation $\tilde{a}_k^\dagger = \sum_{n,d,k} \sum_{\nu = \pm} B_{\nu n \nu'} (\alpha_n^\dagger + \nu \alpha_n)$ yields

$$
\mathcal{H}_x = \frac{\varepsilon_d}{2} + \sum_{\varepsilon > 0} \left( \alpha_k^\dagger \alpha_{-\varepsilon} - \frac{1}{2} \right) + \sum_{k > 0} \left( \beta_k^\dagger \beta_{-k} + \frac{1}{2} \right),
$$

$$
4\pi \Gamma \varepsilon^2 \left( \varepsilon^2 - \varepsilon_d^2 \right) = -\cot \pi \varepsilon / (\Delta_L - P/2).
$$

Equation (11) for the pseudofermion eigenenergies $\varepsilon$ implies that each $\tilde{E}$ smoothly evolves into a corresponding $\varepsilon(k)$ as $\Gamma$ is turned on. Since $\varepsilon(k) \approx \tilde{k} + \Delta_k' \Gamma$ (or $\approx k$) for $\varepsilon < 0$, we see very nicely that the spectrum's low- and high-energy parts are strongly and weakly perturbed, respectively, with crossover scale $T_K \approx \Gamma^{-1}$.

As mentioned above, the pseudofermions act on an extended Fock space. To identify which eigenstates $|\tilde{E}\rangle$ of $H'$ are physical, note that each has to adiabatically develop, as $\Gamma$ increases from 0, from some state obeying the free gluon condition (11). The latter can be shown to develop into the general gluon condition (GCC) (12) that $|\tilde{E}\rangle \sum_{\varepsilon > 0} \tilde{a}_k^\dagger \tilde{a}_{-\varepsilon} + \sum_{k > 0} \tilde{b}_k^\dagger \tilde{b}_{-k} \bmod 2 |\tilde{E}\rangle$ must be equal to $[N_e + N_d - (S_T + 1) + \frac{d}{2} - P_d] \bmod 2$, where $P_d = 0(1)$ for $\varepsilon_d > 0 \leq 0$. The GCC and Eqs. (11) together constitute an exact analytical solution of the 2CK model at the EK-line for arbitrary $\lambda_\perp$, $h_i$, and $h_c$.

Relation to RG methods.—Our exact solution allows us to implement Anderson’s “poor man’s scaling” and Wilson’s NRG treatments of the Kondo problem analytically, thus illustrating the main idea behind both, namely to try to uncover the low-energy physics via an RG transformation. In the first, the RG is generated by reducing (at fixed $L$, usually $\approx \infty$) the bandwidth while adjusting the couplings to keep the dynamical properties invariant. Since the cut-off used when bosonizing is $1/\alpha \sim \pi p_F$ and a occurs in $H'$ only through $\Gamma$, the scaling equations (1)

$$
d_{\ln \lambda}/d_{\ln \Gamma} = 0, \quad d_{\ln \lambda}/d_{\ln \Gamma} = 1/2, \quad \text{which imply that } \lambda_\perp \text{ grows}
$$

under rescaling \([13];\), are exact along the EK-line. Renormalizing the spin-flip vertex, possible only approximately in the original $c_{kx}$ basis by summing selected diagrams, thus becomes trivial after bosonizing and refermionizing, which in effect resums all diagrams into a quadratic form.

Wilson’s NRG (13) is, in effect, a finite-size scaling method which increases (at fixed bandwidth and couplings) the system size, thus decreasing the mean level spacing and pushing ever more eigenenergies down into the spectrum’s strongly-perturbed regime below $T_K$. Each RG step enlarges the system by order $\Delta > 1$ by including an extra “onion-skin shell” of electrons, then rescales $H \to \Lambda H$ to measure energy in units of the new reduced level spacing. We can mimick this by transforming $L \to L' = \Lambda L$ (thus $\Gamma/\Delta_L \to \Lambda \Gamma/\Delta_L$) and plotting the spectrum in units of $\Delta_L' = \frac{\lambda_\perp}{\lambda_\perp'}$.

Figure (1) shows the evolution of the spectrum toward the EK-line for $\lambda_\perp \in [0,1]$ at $\Gamma = \varepsilon_d = 0$ (i.e. free fermions, phase-shifted by $\pm \pi/2$ in the spin sector, see (16)). Figure (1b) shows its further evolution on the EK-line for $\Gamma/\Delta_L \in [0, \infty]$ at $\lambda_\perp = 1$, $\varepsilon_d = 0$. Decreasing $\Delta_L$ at fixed $\Gamma$ yields an NRG-like crossover spectrum that for $\Delta_L \to 0$ indeed reproduces the NRG’s universal NFL fixed point spectrum (2, 3) (irrespective of the specific $\Gamma$ value, illustrating the irrelevance of spin anisotropy (3)). This NFL spectrum also agrees with that found by AL using a so-called fusion hypothesis (4), which our GCC thus proves simply and directly (in contrast to the CFT proof of Ref. (14b)).

Next we illustrate Wilson’s program of extracting the most relevant operator’s dimensions from the $L$ dependence of the finite-size corrections, $\delta \tilde{E}(L) \equiv \tilde{E}(\Gamma/\Delta_L) - \tilde{E}(\infty)$, to the universal NFL spectrum. For $\varepsilon_d = 0$, Eq. (11) gives $\delta \tilde{E} \sim \tilde{F}_\perp$, thus on the EK line the least irrelevant operator has dimension 1 but perturbative corrections in $\lambda_\perp - 1$ yield $\delta \tilde{E} \sim (\lambda_\perp)^{-1} \sim (h_i)^{-1}$, thus the general leading irrelevant operators (absent on the EK line) have dimension $\frac{1}{2}$. Next, turning on a local field $\varepsilon_d = h_i$, we find from (11) that for $h_i \ll h_i = \sqrt{\Gamma/\pi}$ the NFL spectrum is only slightly affected, while for $h_i \ll h_i = \Gamma$ the spectrum has three distinct regions: It is Fermi-liquid-like (3) (with uniform level spacing) for $\varepsilon < h_K \equiv \lambda_\perp \frac{1}{\lambda_\perp}$, and $\varepsilon \gg \Gamma$, and NFL-like (nonuniform level spacings) for $h_K \ll \varepsilon \ll \Gamma$. Both the $L$ dependence of $h_i$ and the $h_i$ dependence of the crossover scale $h_K$ show that the local magnetic field is relevant, with dimension $-\frac{1}{2}$; it causes a crossover, shown in Fig. (1c), to a Fermi-liquid spectrum for all states with $\varepsilon < h_K$.

For $\Gamma/\Delta_L \to \infty$, $h \to 0$, we find logarithmic divergences for the susceptibility, $\chi \approx \frac{1}{\pi e^2} \ln(\Gamma L)$, and the

![Figure 1](image_url)
fluctuations, $\langle \hat{N}_x^2 \rangle \approx \frac{1}{6} \ln (\Gamma L)$ (with $\langle \hat{N}_x \rangle = 0$). Both are clear signs of 2CK NFL physics: the first shows that no spin singlet is formed due to “overscreening”, the second how strongly this perturbs the electron sea.

Relation to CFT.— Recent CFT [3] and scaling [3] arguments showed that the NFL regime can be described by free boson fields. This can be confirmed very easily by finding the scattering state operators $c^\dagger_{kx}$ [and field $\tilde{\psi}^\dagger(x)$] into which the free $c^\dagger_{kx}$’s [$\tilde{\psi}^\dagger(x)$] develop when $\Gamma$ is turned on adiabatically as $e^{i\theta^\Gamma} (at \xi = 0)$, and deducing from these the behavior of the $\tilde{\varphi}$ fields. In the continuum limit [as $L \to \infty$, then $\langle \Delta_L \rangle \to \eta \to +1$], the $c^\dagger_{kx}$’s obey [10], the Lippmann-Schwinger equation $[H_x, c^\dagger_{kx}] = \bar{k} c^\dagger_{kx} + i\theta^\dagger_{kx} - c^\dagger_{kx}$, which gives $[10]$

$$c^\dagger_{kx} = \frac{\bar{k}}{\sqrt{\Delta_L}} \int \frac{dk}{4\pi} e^{ikx} \tilde{\psi}^\dagger_{x} \left( \frac{\theta^\dagger(x) + \theta^\dagger(-x)}{2} \right).$$

To find the asymptotic behavior ($|x| \to \infty$) of $\tilde{\psi}^\dagger_{x}$, we write $\tilde{\psi}^\dagger_{x} \sim \tilde{\psi}^\dagger_{y} \sim \tilde{\psi}^\dagger_{y}$. To translate this into “boundary conditions” on the $\varphi$ boson fields, we write $\tilde{\psi}^\dagger_{x/L}(x) \sim \tilde{\psi}^\dagger_{y} \sim {\tilde{\psi}^\dagger_{y}}$. This thus free and scattering boson fields are asymptotically related (with $\eta_c, \eta_s, \eta_f = 1 = -\eta_x$) by

$$\left( \eta_y \tilde{\psi}_{y} - \pi S_{z} \delta_{y} \right) \sim \left( \tilde{\psi}_{y} + \pi S_{z} \delta_{y} \right) \sim \varphi_y,$$

while $\eta_y \tilde{\psi}_{y} \sim \tilde{\psi}_{y} = \tilde{\psi}_{y}$ for $y = s, f, x$. This central result, first found in Ref. [3] (with different phases since Klein factors were neglected), shows that the NFL regime can be described by boson fields $\tilde{\varphi}_{y/L}$ that are, asymptotically, free, with only a trivial $S_z$ dependence.

Next we consider the 16 bilinear fermion currents $j^A_{y} :\tilde{\psi}_{y} \tilde{\psi}_{y}^\dagger$ ($j^A_{y} :\tilde{\psi}_{y} \tilde{\psi}_{y}^\dagger$) for $y = s, f, x$. This central result, first found in Ref. [3] (with different phases since Klein factors were neglected), shows that the NFL regime can be described by boson fields $\tilde{\varphi}_{y/L}$ that are, asymptotically, free, with only a trivial $S_z$ dependence.

All asymptotic NFL behavior of electron Green’s functions arises from the fact that $\eta_y = -1$, combined with relations like [13]; it directly yields, e.g., the so-called “unitarity paradox” $\tilde{\psi}_{y/L}(x) \tilde{\psi}_{y/L}^\dagger(x) \sim 0$ (for $L \to \infty$, then $|x' - x| \to \infty$). Note, though, that probability is not lost during scattering: $\tilde{\psi}_{y}^\dagger(x)$ shows that each pseudoparticle $c^\dagger_{kx}$ incident from $x > 0$ is “Andreev-scattered”, emerging at $x < 0$ as pseudohole $c_{-k^\dagger x}$, orthogonal to what was incident; this very NFL-like behavior dramatically illustrates the effects of $\hat{N}_x$ nonconservation.

To find AL’s boundary operators in terms of the $\tilde{\varphi}$’s $[11]$, one calculates the operator production expansion of $\psi_{R\alpha} \sim \tilde{\psi}_{L\alpha}$. Since $\eta_y = -1$, all terms contain a factor $e^{ix\varphi_y}$ (with $y = s, f$ or $x$) dimension $\frac{1}{2}$; this ultimately causes the famous $T^{1/2}$ in the resistivity [4].

In conclusion, finite-size bosonization allows one (i) to mimic, in an exact way, the strategy of standard RG approaches, and (ii) to recover with remarkable ease all exact results known from CFT for the NFL fixed point. It thus constitutes a bridge between these theories.

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