Domain Walls and Vortices with Non-Symmetric Core

Minos Axenides

Institute of Nuclear Physics,
N.C.S.R. Demokritos
153 10, Athens, Greece

Leandros Perivolaropoulos

Department of Physics,
University of Crete
71 003 Heraklion, Greece

We review recent work on a new class of topological defects which possess a nonsymmetric core. They arise in scalar field theories with global symmetries, $U(1)$ for domain walls and $SU(2)$ for vortices, which are explicitly broken to $Z_2$ and $U(1)$ respectively. Both of the latter symmetries are spontaneously broken. For a particular range of parameters both types of defect solutions are shown to become unstable and decay to the well known stable walls and vortices with symmetric cores.

---

1 Talk presented at the "Particle Physics and the Early Universe" Conference, April 7-11, Univ. of Cambridge.
2 mailto: axenides@gr3801.nrcps.ariadne-t.gr
3 http://www.edu.physics.uch.gr/~leandros
1 Introduction

Topological defects\cite{1,2} are stable field configurations (solitons\cite{3}) that arise in field theories with spontaneously broken discrete or continuous symmetries. Depending on the topology of the vacuum manifold $M$ they are usually identified as domain walls\cite{2} (kink solutions\cite{3}) when $M = \mathbb{Z}_2$, as strings\cite{4} and one-dimensional textures (ribbons\cite{5,6}) when $M = S^1$, as monopoles (gauged\cite{7,8,9} or global\cite{10,11}) and two dimensional textures ($O(3)$ solitons \cite{12,3}) when $M = S^2$ and three dimensional textures \cite{13} (skyrmions\cite{14}) when $M = S^3$. They are expected to be remnants of phase transitions \cite{15} that may have occurred in the early universe. They also form in various condensed matter systems which undergo low temperature transitions \cite{16}. Topological defects appear to fall in two broad categories. In the first one the topological charge becomes non-trivial due to the behavior of the field configuration at spatial infinity. The symmetry of the vacuum gets restored at the core of the defect. Domain walls, strings and monopoles belong to this class of symmetric defects.

In the second category the vacuum manifold gets covered completely as the field varies over the whole of coordinate space. Moreover its value at infinity is identified with a single point of the vacuum manifold. Textures \cite{13} (skyrmions \cite{14}), $O(3)$ solitons \cite{12} (two dimensional textures \cite{13}) and ribbons \cite{5} belong to this class which we will call for definiteness texture-like defects. The objective of the present discussion is to present examples of defects which belong to neither of the two categories, namely the field variable covers the whole vacuum manifold at infinity with the core remaining in the non-symmetric phase. For definiteness we will call these non-symmetric defects.

Examples of nonsymmetric defects have been discussed previously in the literature. Everett and Vilenkin \cite{17}, in particular, pointed out the existence of domain walls and strings with non-symmetric cores which are unstable though to shrinking and collapse due to their string tension. A particular case of non-symmetric gauge defect was recently considered by Benson and Bucher \cite{18} who pointed out that the decay of an electroweak semilocal string leads to a gauged ”skyrmion” with non-symmetric core and topological charge at infinity. This skyrmion however, rapidly expands and decays to the vacuum.
In the present talk we review recent work where we presented more examples of topological defects that belong to what we defined as the "non-symmetric" class. We will study in detail the properties of global domain walls in section 2 and of global vortices in section 3. In both cases we will identify the parameter ranges for stability of the configurations with either a symmetric or a non-symmetric core. For the case of a domain wall wall we will discuss results of a simulation for an expanding bubble of a domain wall.

Finally, in section 4 we conclude, summarize and discuss the outlook of this work.

## 2 Domain Walls with NonSymmetric Core

We consider a model with a $U(1)$ symmetry explicitly broken to a $Z_2$. This breaking can be realized by the Lagrangian density \[ \mathcal{L} = \frac{1}{2} \partial_\mu \Phi^* \partial^\mu \Phi + \frac{2}{2} |\Phi|^2 + \frac{m^2}{2} \text{Re}(\Phi^2) - \frac{h}{4} |\Phi|^4 \]

where $\Phi = \Phi_1 + i\Phi_2$ is a complex scalar field. After a rescaling

\[ \Phi \rightarrow \frac{m}{\sqrt{h}} \Phi \]  
\[ x \rightarrow \frac{1}{m} x \]  
\[ M \rightarrow \alpha m \]

The corresponding equation of motion for the field $\Phi$ is

\[ \ddot{\Phi} - \nabla^2 \Phi - (\alpha^2 \Phi + \Phi^*) + |\Phi|^2 \Phi = 0 \]

The potential takes the form

\[ V(\Phi) = -\frac{m^4}{2h} (\alpha^2 |\Phi|^2 + \text{Re}(\Phi^2) - \frac{1}{2} |\Phi|^4) \]

For $\alpha < 1$ it has the shape of a "saddle hat" potential i.e. at $\Phi = 0$ there is a local minimum in the $\Phi_2$ direction but a local maximum in the $\Phi_1$ (Fig 1). For this range of values of $\alpha$ the equation of motion admits the well known static kink solution

\[ \Phi_1 = \Phi_R \equiv \pm (\alpha^2 + 1)^{1/2} \tanh((\frac{\alpha^2 + 1}{2})^{1/2} x) \]
\[ \Phi_2 = 0 \]
Figure 1: (a) The domain wall potential has a local maximum at $\Phi = 0$ in the $\Phi_1$ direction. (b) For $\alpha > 1$ ($\alpha < 1$) this point is a local maximum (minimum) in the $\Phi_2$ direction.

It corresponds to a *symmetric* domain wall since in the core of the soliton the full symmetry of the Lagrangian is manifest ($\Phi(0) = 0$) and the topological charge arises as a consequence of the behavior of the field at infinity ($Q = \frac{1}{2} (\Phi(-\infty) - \Phi(+\infty))/(\alpha^2 + 1)^{1/2}$).

For $\alpha > 1$ the local minimum in the $\Phi_2$ direction becomes a local maximum but the vacuum manifold remains disconnected, and the $Z_2$ symmetry remains. This type of potential may be called a "Napoleon hat" potential in analogy to the Mexican hat potential that is obtained in the limit $\alpha \to \infty$ and corresponds to the restoration of the $S^1$ vacuum manifold.

The form of the potential however implies that the symmetric wall solution may not be stable for $\alpha > 1$ since in that case the potential energy favors a solution with $\Phi_2 \neq 0$. However, the answer is not obvious because for $\alpha > 1$, $\Phi_2 \neq 0$ would save the wall some potential energy but would cost additional gradient energy as $\Phi_2$ varies from a constant value at $x = 0$ to 0 at infinity. Indeed a stability analysis was performed by introducing a small perturbation about the kink solution reveals the presence of negative modes for $\alpha > \alpha_{crit} = \sqrt{3} \simeq 1.73$ For the range of values $1 < \alpha < 1.73$ the potential takes the shape of a "High Napoleon hat". We study the full non-linear static field equations obtained from (6) for a typical value of $\alpha = 1.65$ with boundary conditions

\begin{align}
\Phi_1(0) &= 0 \quad \lim_{x \to \infty} \Phi_1(x) = (\alpha^2 + 1)^{1/2} \\
\Phi_2'(0) &= 0 \quad \lim_{x \to \infty} \Phi_2(x) = 0
\end{align}

Using a relaxation method based on collocation at gaussian points \cite{19} to solve the system (6) of second order non-linear equations we find that for $1 < \alpha < \sqrt{3}$ the
solution relaxes to the expected form of (7) for $\Phi_1$ while $\Phi_2 = 0$ (Fig. 2). For $\alpha > \sqrt{3}$ we find $\Phi_1 \neq 0$ and $\Phi_2 \neq 0$ (Fig. 3) obeying the boundary conditions (13), (14) and giving the explicit solution for the non-symmetric domain wall. In both cases we also plot the analytic solution (7) stable only for $\alpha < \sqrt{3}$ for comparison (bold dashed line). As expected the numerical and analytic solutions are identical for $\alpha < \sqrt{3}$ (Fig. 2).

We now proceed to present results of our study on the evolution of bubbles of a domain wall. We constructed a two dimensional simulation of the field evolution of domain wall bubbles with both symmetric and non-symmetric core. In particular we solved the non-static field equation (6) using a leapfrog algorithm \cite{19} with reflective boundary conditions. We used an $80 \times 80$ lattice and in all runs we retained $\frac{dt}{dx} \simeq \frac{1}{3}$ thus satisfying the Cauchy stability criterion for the timestep $dt$ and the lattice spacing $dx$. The initial conditions were those corresponding to a spherically symmetric bubble with initial field ansatz

$$\Phi(t_i) = (\alpha^2 + 1)^{1/2}\tanh[\left(\frac{\alpha^2 + 1}{2}\right)^{1/2}(\rho - \rho_0)] + i \ 0.1 \ e^{-||x| - \rho_0| \ |x|}$$

where $\rho = x^2 + y^2$ and $\rho_0$ is the initial radius of the bubble. Energy was conserved to
Figure 4: Initial field configuration for a non-symmetric spherical bubble wall with \( \alpha = 3.5 \).

within 2\% in all runs. For \( \alpha \) in the region of symmetric core stability the imaginary initial fluctuation of the field \( \Phi(t_i) \) decreased and the bubble collapsed due to tension in a spherically symmetric way as expected.

For \( \alpha \) in the region of values corresponding to having a non-symmetric stable core the evolution of the bubble was quite different. The initial imaginary perturbation increased but even though dynamics favored the increase of the perturbation, topology forced the \( \text{Im}\Phi(t) \) to stay at zero along a line on the bubble: the intersections of the bubble wall with the y axis (Figs. 4, 5). Thus in the region of these points, surface energy (tension) of the bubble wall remained larger than the energy on other points of the bubble. The result was a non-spherical collapse with the x-direction of the bubble collapsing first (Fig. 5).

3 Vortices with Nonsymmetric Core

We have generalized our analysis for domain walls to the case of a scalar field theory that admits global vortices. We consider a model with an SU(2) symmetry explicitly
Figure 5: Evolved field configuration \((t = 14.25,\ 90\ \text{timesteps})\) for a non-symmetric initially spherical bubble wall with \(\alpha = 3.5\).

broken to \(U(1)\). Such a theory is described by the Lagrangian density:

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} \Phi^{\dagger} \partial^{\mu} \Phi + \frac{M^2}{2} \Phi^{\dagger} \Phi + \frac{m^2}{2} \Phi^{\dagger} \tau_3 \Phi - \frac{h}{4} (\Phi^{\dagger} \Phi)^2
\]

where \(\Phi = (\Phi_1, \Phi_2)\) is a complex scalar doublet and \(\tau_3\) is the \(2 \times 2\) Pauli matrix. After rescaling as in equations (2)-(4) we obtain the equations of motion for \(\Phi_{1,2}\)

\[
\partial_{\mu} \partial^{\mu} \Phi_{1,2} - (\alpha^2 \pm 1) \Phi_{1,2} + (\Phi^{\dagger} \Phi) \Phi_{1,2} = 0
\]

where the \(+\ (\ -\) corresponds to the field \(\Phi_1\ (\Phi_2)\).

Consider now the ansatz

\[
\Phi = \begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} = \begin{pmatrix} f(\rho) e^{i\theta} \\ g(\rho) \end{pmatrix}
\]

with boundary conditions

\[
\lim_{\rho \to 0} f(\rho) = 0, \quad \lim_{\rho \to 0} g'(\rho) = 0
\]

\[
\lim_{\rho \to \infty} f(\rho) = (\alpha^2 + 1)^{1/2}, \quad \lim_{\rho \to \infty} g(\rho) = 0
\]

This ansatz corresponds to a global vortex configuration with a core that can be either in the symmetric or in the non-symmetric phase of the theory. Whether the
core will be symmetric or non-symmetric is determined by the dynamics of the field equations. As in the wall case the numerical solution of the system (21) of non-linear complex field equations with the ansatz (22) for various values of the parameter $\alpha$ reveals the existence of an $\alpha_{cr} \approx 2.7$. For $\alpha < \alpha_{cr} \approx 2.7$ the solution relaxed to a lowest energy configuration with $g(\rho) = 0$ everywhere corresponding to a vortex with symmetric core (Fig. 6).

For $\alpha > \alpha_{cr} \approx 2.7$ the solution relaxed to a configuration with $g(0) \neq 0$ indicating a vortex with non-symmetric core (Fig. 7). Both configurations are dynamically and topologically stable and consist additional paradigms of the defect classification discussed in the introduction.

4 Conclusion

We have studied the existence and stability properties of defects (domain walls and vortices) with non-symmetric core and non-trivial winding at infinity. These defects arise in scalar field theories that exhibit an explicit breaking of a global symmetry, $U(1)$ for domain wall and $SU(2)$ for vortices. In their spectrum and for a particular range of parameters topologically stable and unstable defects appear with either symmetric
or nonsymmetric cores. Possible implications for the cosmology of the early universe are the following: With regard to the case of domain walls with a symmetric core (saddle and high Napoleon hat potentials) a possible embedding of such configurations in a realistic 2Higgs electroweak model may realize a new mechanism for baryogenesis at the electroweak phase transition. Defect mediated baryogenesis has been so far only successfully implemented at scales introduced near or above the electroweak one [21]. These mechanisms are based on unsuppressed B+L violating sphaleron transitions taking place in the symmetric core of the defects during scattering processes [22]. As a result of our work the question of existence of electroweak domain walls with a symmetric core now translates to whether in the most general electroweak lagrangian with two Higgs doublets potential energies of the ”saddle hat” or ”high napoleon hat” type exist for an appropriate range of parameters. As the parity symmetry in these models is broken both spontaneously an explicitly the expected domain walls in their spectrum are certainly of the nontopological type. Moreover it would be of interest to see if such defects arise at a second order electroweak phase transition. Our observation of non-spherical collapse of wall bubbles with non-symmetric core may imply that the domain wall network simulations need to be re-examined for parameter ranges where a non-symmetric core in energetically favored.

With regard to our demonstration of existence of vortices with nonsymmetric core it becomes immediately suggestive the existence of a new kind of a bosonic supercon-
ducting string, possessing massive charge carriers. This would be the case if our model is properly coupled to a $U(1)$ gauge field. The physics of fermions introduced to such a system is also open for investigation. The astrophysical and cosmological role of superconducting strings has been extensively investigated in the literature.

5 Acknowledgements

This work was supported by the E.U. grants $CHR X - CT93 - 0340$, $CHR X - CT94 - 0621$ and $CHR X - CT94 - 0423$ as well as by the Greek General Secretariat of Research and Technology grants $95E \Delta 1759$ and $\Pi EN E \Delta 1170/95$. We are particular thankful to D.A.M.T.P. of the U.of Cambridge and Anne Davis for their hospitality and during our stay at Cambridge.

References

A. Vilenkin, E.P.S. Shellard, in "Cosmic Strings and other Topological Defects" Cambridge U. Press, 1994.

A. Vilenkin, Phys.Rep. 121, 263, 1985;

J. Preskill, Ann.Rev.Nucl.Part. Sci. 34, 461, 1984.

R. Rajaraman, 'Solitons and Instantons' North Holland Pub., 1987.

H.B. Nielsen, P. Olesen, Nucl. Phys. B61, 45, 1973.

C. Bachas, T.N. Tomaras, Nucl.Phys. B428, 209, 1994.

C. Bachas, T.N. Tomaras, Phys.Rev. D51, 5356, 1995.

G. tHooft, Nucl. Phys. B79, 276, 1974.

A. Polyakov, JETP Lett. 20, 194, 1974.

C. P. Dokos, T. N. Tomaras, Phys.Rev. D21, 2940, 1980.

M. Barriola, A. Vilenkin, Phys. Rev. Lett. 63, 341, 1989.
L. Perivolaropoulos, *Nucl. Phys.* **B375**, 665, 1992.

A. Belavin, A. Polyakov *JETP Lett.* **22**, 245, 1975.

N. Turok, *Phys. Rev. Lett.* **63**, 2625, 1989.

T. Skyrme, *Proc. R. Soc.* **A262**, 233, 1961.

T.W.B. Kibble, *J. Phys.* **A9**, 1387, 1976;

E. Kolb, M.S. Turner in *"The Early Universe"* (Addison-Wesley Pub. Co. 1990)

W. H. Zurek, *Nature* **317**, 505, 1985; *ibid* **382**, 296, 1996.

A. Vilenkin, A. E. Everett, *Phys. Rev. Lett.* **48**, 1867 1982.

K. Benson, M. Bucher, *Nucl. Phys.* **B406**, 355, 1993.

W. Press et. al., *Numerical Recipes*, Cambridge U. Press, 2nd ed., 1993.

M. Axenides, L. Perivolaropoulos, *"Topological Defects with Non-symmetric Walls"*, 

to appear in *Phys. Rev. D*.

R. Brandenberger, A. Davis, T. Prokopec, M. Trodden, *Phys. Rev.* **D53**, 4257, 1996.

A. Cohen, D. Kaplan and A. Nelson, *Annu. Rev. Nucl. Part. Sci.* **43**, 27, 1993.

E. Witten, *Nucl. Phys.* **B249**, 557, 1985.

J. P. Ostriker, C. Thompson and E. Witten *Phys. Lett.* **B180**, 231, 1986.

R. L. Davis and E. P. S. Shellard, *Phys. Lett.* **B209**, 485, 1988;

R. L. Davis and E. P. S. Shellard, *Nucl. phys.* **B323**, 209, 1989.