Vaccination Strategy Analysis with SIRV Epidemic Model Based on Scale-free Networks with Tunable Clustering

Xueyu Meng¹, Zhiqiang Cai¹,*, Hongyan Dui² and Huiying Cao¹

¹ Department of Industrial Engineering, Northwestern Polytechnical University, Xi’an, Shanxi, 710072, P.R. China
² School of Management Engineering, Zhengzhou University, Zhengzhou, Henan, 450001, P.R. China

*E-mail: caizhiqiang@nwpu.edu.cn

Abstract. In this paper, we propose an SIRV (susceptible, infected, recovered, vaccination) evolutionary game model for infectious disease vaccination strategies based on the scale-free networks with tunable clustering. This model takes into account factors such as vaccination effectiveness, vaccination cost, treatment cost after illness, government subsidy rate and treatment discount rate. First of all, we use the idea of evolutionary game to make each individual in the network get two strategies, including vaccination and non-vaccination. Meanwhile, in each propagation process, according to the policy update rule (PUR), each individual updates its game strategy according to the benefit relationship with the adjacent nodes. Then, we analyze the compulsory and voluntary vaccination on the scale-free network with considering the influence of vaccination efficiency, the cost of vaccination, the cost of treatment after illness, the government subsidy rate, the treatment discount rate on vaccination. The results indicate that when the vaccination effectiveness is about 0.9, it is a better value for the evolution of vaccination strategy. For government decision making, choosing appropriate values of \( s \) and \( d \) can make the overall benefit of society higher.

1. Introduction
At present, the new coronavirus pneumonia (COVID-19) has formed a pandemic in the world, and related vaccine research is also in progress. Then, for infectious diseases such as Ebola virus, atypical pneumonia, and new coronavirus pneumonia, research on vaccination strategies is very important. Most of our real-life social networks are scale-free networks, which are proposed by Barabasi and Albert [1]. But scale-free networks lack the description of other structural attributes of complex social networks in real life, such as aggregation. Therefore, some researchers established a scale-free network model with tunable clustering to understand the generation mechanism of these
social network structural characteristics [2].

In the method of preventing the spread of infectious diseases, vaccination is one of the important means. Random immunization, targeted immunization, acquaintance immunization and other methods have been proposed one after another. Random immunization refers to randomly selecting people to vaccinate in the general population with a certain probability [3]. Acquaintance immunization is based on random immunization, randomly selecting an individual and vaccinating the acquaintances [4]. Targeted immunization refers to the vaccination of a specific group of people in the population. For example, for influenza, middle-aged people, children, health care workers and high-risk contacts are all target groups, and target immunization is carried out [5]. However, the premise of these strategies is compulsory immunization, that is, regardless of whether the individual is willing to be vaccinated, forcing the individual to be vaccinated [6]. But we all know that vaccination should be voluntary. We have to consider the subjective factors of human beings. We cannot force vaccination, which will violate the rights of individuals.

Regarding the strategy of voluntary immunization, many scholars have done a lot of research in order to be able to achieve the effect of a group of immunizations [7,8]. The so-called community immunity means that when the current number of people vaccinated exceeds a certain amount, the infectious disease cannot continue to spread in the social network. At this time, even if the individual is not vaccinated, it will not be infected because they are indirectly protected by the vaccinated population [9]. In this way, unvaccinated individuals benefit from community immunity without making any effort. They avoid the cost of vaccination and the possible side effects of vaccination. Because without any incentives, there may be a certain risk of vaccination. In this case, there will be a free rider problem. [10]. Therefore, the researchers have analyzed the spread of infectious diseases on different networks. Saumell-Mendiola et al. analyzed an epidemic spreading process taking place on top of two interconnected complex networks [11]. In addition, Si et al. summarized the reliability optimization problems and methods of complex systems [12].

However, scale-free networks lack a description of other structural attributes of complex social networks in real life, such as aggregation. Therefore, we use scale-free aggregation network to analyze the SIRV (susceptible, infected, recovered, vaccination) model. The characteristic of the scale-free network model with adjustable aggregation is that it can produce the same scale-free distribution properties, but the network's aggregation level can be changed. Meanwhile, in the SIRV model, we consider factors such as vaccine effectiveness, government subsidy rate and treatment discount rate.

In this paper, we analyze the influence of factors such as the ways to get vaccinated, vaccination costs, disease treatment costs, government subsidy rate, and treatment discount rate by proposing an evolutionary game SIRV model of infectious disease vaccination strategies. First of all, we use the idea of evolutionary game to make each individual in the network get two strategies, including vaccination and non-vaccination. Meanwhile, in each propagation process, according to the policy update rule, each individual updates its game strategy according to the profit relationship with the adjacent nodes. Then we analyze the compulsory and voluntary vaccination on the scale-free network.
2. Evolutionary game model of infectious disease vaccination strategies

Currently, vaccination is the most effective method of disease prevention. When the number of individuals vaccinated in the population exceeds a certain number, the infectious disease cannot spread on the network, thereby indirectly protecting the unvaccinated individuals and achieving the effect of social group immunity. But in the face of new diseases, people will predict the risks of the disease, the risks and costs of vaccination. Individuals often have a herd mentality and free-riding phenomenon in the choice of immunization strategy. Individuals' attitudes to risk and past decision-making benefits will also have a greater impact on their immune behavior.

Therefore, we use the evolutionary game method to deal with whether the individual is vaccinated. The individual's vaccination strategy will change with the game revenue of adjacent nodes, which will affect the spread of infectious diseases. We take into account factors such as the effectiveness of the vaccine, the cost of vaccination, and the cost of disease treatment, and comprehensively analyze the SIRV model of the scale-free aggregation network in the case of vaccination.

2.1. Vaccination decision process

The classic SIR model divides the population into three populations: S means the susceptible individual which is not infected and has no immunity. I means the infected individuals and they are infectious at this time. R means the recovered individuals, that is, the infected person is cured and has certain immunity. Here we suppose that some susceptible people have immunity through vaccination. And the population will increase a class of vaccinators. So the four states of SIRV are finally presented, the constraint relationships they satisfy are as

\[ S(t) + I(t) + R(t) + V(t) = 1 \]

where \( S(t), I(t), R(t) \) and \( V(t) \) represent the proportion of the four individual states of \( S, I, R, V \) in the population at time \( t \).

The idea of the SIRV model is as follows:

**Step1:** Initialize the infection source \( N_I \) and initialize the parameters and the states of each node in the network. \( S = [1,0,0,0] \) represents the susceptible individuals, \( I = [0,1,0,0] \) represents the infected individuals, \( R = [0,0,1,0] \) represents the removed individuals, and \( V = [0,0,0,1] \) represents the vaccinated individuals.

**Step2:** Determine the state of the node.

If node \( i \) is a susceptible individual, check whether its adjacent node contains an infected individual. If not, the state of node \( i \) in the next transmission process is still susceptible. If there is an infected individual in the adjacent nodes, node \( i \) will be infected with a certain probability during the propagation process and becomes an infected state.

If node \( i \) is an infected individual, it will be transformed into a removed individual with a certain probability. Before an outbreak of an infectious disease or ongoing, each individual will choose whether to vaccinate. When there are individuals vaccinated in the network, before the next infectious disease cycle, each individual will update his own vaccination strategy. Each individual's attitude toward vaccination will be affected by many aspects, such as personal preferences, the cost of the vaccine, the cost of treatment, the side effects of the vaccine and so on. In this paper, we assume that the vaccination has no side effects, but the vaccination is not completely effective,
which means that the vaccination may fail. We assume that the benefit of individual \( i \) is \( U^v_i \) when vaccinated, and \( U^I_i \) if it is not vaccinated. Meanwhile, the cost of vaccination for individual \( i \) is \( C_v \), including the capital required for vaccination, time cost and so on. If the individual \( i \) is infected by an infectious disease, the cost of treatment is \( C_I \). The probability of individual \( i \) being infected in the network is \( \lambda_i \). Traditional research on vaccination strategies based on evolutionary games often assumes that the vaccination will not fail, so the benefits of the two strategies are as follows.

\[
\begin{align*}
P^v_i &= -C_v \\
P^I_i &= -C_I \lambda_i
\end{align*}
\]

Without loss of generality, we set \( C_I = 1 \) and set relative consumption \( \varphi = C_v / C_I \). In order to promote vaccination, in general, the cost of vaccination will be less than the cost of treatment after being infected. Otherwise, there is no need for people to get vaccinated, because if an individual don’t get vaccinated, it may be not sick and successfully hitchhike to maximize its benefits. Another possibility is that sickness requires treatment, even if the cost of treatment is less than the cost of the vaccine in this case, it will inevitably cause everyone not to be vaccinated. So generally speaking, the cost of vaccination is less than the cost of treatment after being infected, that is, the value range of \( c \) is \( 0 < c < 1 \). When \( c = 0 \), individual can receive the vaccination for free. When \( c = 1 \), the vaccination cost is equal to the treatment cost after being infected. After the introduction of \( c \), the benefits of the two strategies become

\[
\begin{align*}
P^v_i &= -c \\
P^I_i &= -\lambda_i
\end{align*}
\]

The \( \lambda_i \) represents the probability of individual \( i \) being infected. We assume that the rate of transmission of infectious diseases in the network is \( \beta \), that is, the number of susceptible individuals that an infected individual can infect is directly proportional to the number \( S \) of susceptible individuals in the network at this time, and this proportionality coefficient is \( \beta \). Suppose that at a certain moment, there are \( k \) unvaccinated individuals in the neighbor of individual \( i \). Then the probability of individual \( i \) being infected is

\[\lambda_i = 1 - (1 - \beta)^k\]

Through the above formula, we can find that the greater the number of unvaccinated individuals in an individual’s neighbors, the greater the probability that it will be infected.

In this paper, the situation of vaccination failure will be considered on the basis of traditional research. And subsidies and treatment incentive strategies are added to promote the probability of individual vaccination. We assume that the vaccination effective rate is \( e \), and the proportion of individuals vaccinated is \( x \), then the effective number of vaccinated people is \( w = ex \). After the individual's vaccination fails, the cost of treatment will be discounted and the discounted ratio is \( d \). Meanwhile, if the individual actively participates in the vaccination, the government will subsidize it at a certain rate. So the actual cost of vaccination for individual \( i \) is \( (1 - s)C_v \). Without vaccination, the probability of an individual being infected is represented by \( f(w) \). Obviously, \( f(w) \) is a decreasing function. So the more individuals who are effectively vaccinated, the easier it is for individuals who are not vaccinated to hitchhike. After being infected, the cost of treatment of individual \( i \) is \( C_I \).
Because the vaccination will be ineffective, there will be two situations after vaccination: one is healthy and the other is infected. The probability of keeping the body healthy from infection after vaccination is 
\[ x[e + (1 - e)(1 - f(w))] \]. And the individual is effectively vaccinated, but the vaccination is ineffective and not infected. So the income of this part of individuals is 
\[ U_1^p = -(1 - s)C_v \]. In a similar way, the proportion of infected individuals is 
\[ x(1 - e)f(ex) \] after vaccination. So the income of this part of individuals is 
\[ U_2^p = -(1 - s)C_v - dC_I \]. The proportion of individuals who are not vaccinated and safe is 
\[ (1 - x)(1 - f(ex)) \], and the income of this part of individuals is 
\[ U_1^i = 0 \]. The last one is that the proportion of individuals who are not vaccinated and infected is 
\[ (1 - x)f(ex) \], and the income of this part of individuals is 
\[ U_2^i = -C_I \].

An infectious disease vaccination SIRV model in the scale-free networks with tunable clustering considers vaccination effectiveness, government subsidy rate, and treatment discount rate.

**Step1:** Initialize parameters such as efficiency, government subsidy rate, treatment discount rate, and relative cost.

**Step2:** Determine whether node \( i \) is vaccinated.

If the node \( i \) is vaccinated, it is determined whether the vaccination is effective. If the vaccine is effective, the vaccination fee is paid. If the vaccine is invalid, it is determined whether the infection is infected. If there is no infection, you only need to pay for the vaccination fee subsidized by the government.

If node \( i \) is not vaccinated and infected, it will pay the full cost of treatment. If it is not vaccinated and not infected, it will pay for the risk of illness and other losses.

**Step3:** Update the vaccination strategy according to the policy update rules.

**Step4:** Conduct the next round of spread of infectious diseases.

### 2.2. Formation of the scale-free networks with tunable clustering

The construction of BA (Barabasi-Albert) scale-free network is based on the following two internal mechanisms: growth and priority connection. However, the network generated by the BA scale-free network model shows a very low aggregation coefficient, which is inconsistent with the actual complex network. In view of this, a scale-free network model with tunable aggregation coefficient is proposed, as show in figure 1.

The specific steps of the scale-free network with tunable clustering model algorithm are as follows.

**Step1:** At the initial moment \( t = 0 \), there are \( m_0 \) isolated individuals in the network.

**Step2:** Growth. At each moment, a new individual with \( m \) edges joins the network and connects to \( m(m \leq m_0) \) existing individuals in the network.

**Step3:** Priority connection. The first edge of the \( m \) edges of the individual \( i \) is connected to the existing individual \( j \) by the preferential connection mechanism in the BA scale-free network model. The remaining \( m - 1 \) edges are connected in the following two different ways.

1. Randomly connect to \( m - 1 \) neighbors of individual \( j \) with probability \( q \). If the number of neighbors of individual \( j \) is \( k_j < m - 1 \), after the individual \( i \) connects all the neighbors of \( j \), the remaining \( m - 1 - k_j \) edges are connected to other individuals with the priority connection mechanism in the BA model. Then we have 
\[ q_j = \frac{k_j}{\sum_{i=1}^{N} k_i} \] where \( N \) is the total number of nodes in
the current networks.

(2) Connect to other $m - 1$ individuals with probability $1 - q$ according to the priority connection mechanism in the BA model. Such a network generation process can produce scale-free networks with degree distribution $p(k) \sim k^{-3}$ and tunable clustering.

Figure 1. The scale-free network with tunable clustering

2.3. Policy update rules
Considering that individuals with limited rationality can make certain mistakes when making strategy adjustments, Femi update rule allows irrational probabilistic imitation, which introduces noise parameters into strategy adjustments to characterize the irrational choices of individuals. The dynamic mechanism is as follows. When an individual $i$ wants to update its game strategy, it randomly chooses its own neighbor $j$ to compare the returns. The probability that the individual $i$ will adopt the strategy of neighbor $j$ in the next game is

$$p_j(s_i \leftarrow s_j) = \frac{1}{1 + e^{(U_i - U_j)/\epsilon}}$$

where $U_i$ and $U_j$ represent the benefits of individuals $i$ and $j$ in this game respectively. $\epsilon(\epsilon \geq 0)$ represents the noise effect, which allows individuals to make irrational choices, which makes those strategies with lower returns still get a small probability of being adopted by individuals with higher returns.

The pseudo-code for evolutionary game model of infectious disease vaccination strategies is listed as follows.
Algorithm: Evolutionary game model of infectious disease vaccination strategies

Input: the scale-free network and vaccination strategy
Output: S(t), I(t), R(t), V(t)

(1) for $t$ ← 1 to $t_0$
    initialize: $s_i, e, d, s, N, N_V$
(2) for $i$ ← 1 to $N$: $\lambda_i ← 1 - (1 - \beta)^{k_i}$
(3) if $S_i ← S$ & i vaccinated
(4) if $rand < e$
(5) $U_i ← -C_V$, $S_i ← V$
(6) else if $rand < \lambda_i$
(7) $U_i ← -(1 - s)C_V - dC_i$, $S_i ← I$
(8) else
(9) $U_i ← -(1 - s)C_V$
(10) end
(11) end
(12) end if $rand < \lambda_i$
(13) $U_i ← -C_i$, $S_i ← I$
(14) else
(15) $U_i ← -\lambda_i C_i$
(16) end
(17) end for $i$ ← 1 to $N$
(18) if $S_i ← S$ & $rand < \frac{1}{1 + e(1 - V_i)/e}$
(19) $i$ vaccinated
(20) end
(21) end
(22) end

3. Simulation of evolutionary vaccination game model

At time $t = 0$, the infectious disease begins to spread. At the same time, the vaccine begins to be vaccinated. For the source of infectious diseases, we randomly select one from the list where the degree of nodes in the scale-free network is equal to the average degree $<k>$. We build a scale-free network with tunable clustering composed of 1000 nodes.

3.1. The effect of vaccine effectiveness

This paper considers the failure of the vaccine, so the individual is not permanently fully immunized after vaccination. The individual who is vaccinated will also fail due to the factors of the vaccine itself. So it is necessary to consider the effectiveness of vaccination. First of all, this is more in line with the actual living environment. In actual medical treatment, it is impossible for any vaccination individual to be fully immunized. Secondly, the effectiveness of vaccination does have a certain impact on the area covered by the vaccine. We set the effective values as $e = 0.8, e = 0.85, e = 0.9, e = 0.99, e = 0.999$, and then observe the change curve of the proportion of infected
individuals and voluntary vaccination individuals in the scale-free network.

![Figure 2. The effect of vaccine effectiveness](image)

We can find from figure 2 that, the proportion of infected individuals decreases as the effectiveness increases, and the proportion of vaccinated individuals increases as a whole. However, when the effectiveness rate is 0.9, the proportion of infected persons is the lowest, and the proportion of individual vaccinated is the highest when the evolution is stable. So we think that the effective rate about 0.9 is a better value for the evolution of vaccination strategies.

3.2. Influence of vaccination government subsidy rate $s$ and treatment discount rate $d$

In order to analyze the influence of vaccination government subsidy rate and treatment discount rate on evolution, we take 50 values from 0 to 1 for $s$ and $d$ respectively, and display it with a heat map.

![Figure 3. Influence of vaccination government subsidy rate $s$ and treatment discount rate $d$](image)

We can find from figure 3 that with the change of $s$ and $d$, both the removed individuals and the vaccinated individuals formed small aggregates, which indicate that there exists many local optimal values.

3.3. Ways to get vaccinated

For voluntary vaccination, we randomly set the proportion of individuals who voluntarily vaccinated in the network to be 0.1%, 0.2%, 0.3%, and 0.4% at the initial moment.
Figure 4. Voluntary vaccination

From figure 4, we can find that as the number of initial voluntary vaccination individuals increases, the number of infected individuals gradually decreases, and the number of vaccinated individuals gradually increases and the peak of the proportion curve of infected persons also gradually decreases.

4. Conclusion and discussion

In this paper, we propose an SIRV evolutionary game model for infectious disease vaccination strategies with scale-free networks with tunable clustering. It takes into account factors such as vaccination effectiveness, vaccination cost, treatment cost after illness, government subsidy rate and treatment discount rate. We apply the idea of evolutionary game to make each individual in the network have two strategies, including vaccination and non-vaccination. Meanwhile, in each propagation process, according to the policy update rule, each individual updates its game strategy according to the profit relationship with the adjacent nodes. Then we analyze the compulsory and voluntary vaccination on the scale-free network. The study indicates that choosing appropriate values of $s$ and $d$ can make the overall social benefit higher. Although we have established the evolutionary game model of vaccination strategy from the perspective of evolutionary game, the intimacy between people is different in real life social networks. The intimacy between people in social networks corresponds to the weight of the edges in complex networks. So it will become more meaningful to study the spreading mechanism of epidemic in weighted networks.
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