Gauge Principles for Multi-superparticles

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Abstract

We formulate new gauge principles for \( n \) supersymmetric particles in a worldline formalism with \( N \) supersymmetries. The models provide realizations of the more general supersymmetries that are encountered in sectors of S-theory or Matrix theory, with a superalgebra of the form \( \{ Q^A, Q^B \} = \delta^{AB} \gamma_\alpha^{\mu_1 \ldots \mu_n} (p^\mu_1 \cdots p^\mu_n) \). Due to the local gauge and kappa symmetries the \( n \) superparticle momenta \( p^\mu_i \) and \( N \) supercharges \( Q^A_\alpha \) are constrained by \( p^i \cdot p^j = 0 \) and \( \gamma \cdot p^i Q^A = 0 \). The constraints have solutions only in a space with \( n \) timelike dimensions and \( \text{SO}(d + n - 2, n) \) spacetime symmetry. The cases \( \text{SO}(9,1) \), \( \text{SO}(10,2) \) and \( \text{SO}(10,3) \), with one, two and three timelike dimensions respectively, are of special interest. In each case, due to the constraints, the classical motion and quantum theory of each superparticle are equivalent to the physics with a single time-like dimension in an effective 10D superspace with \( \text{SO}(9,1) \) Lorentz symmetry.

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1 Multi-particles and multi-times

Relativistic particles with spacetime symmetry \( \text{SO}(d - 1, 1) \) which move freely in \( d \) dimensions can be described by an action for \( n \) worldlines \( x_i^\mu(\tau) \)

\[
S = \frac{1}{2} \int_0^T d\tau \sum_{i=1}^n [e_i^{-1} \dot{x}_i^2 - e_i m_i^2].
\]

(1)

The einbeins \( e_i(\tau) \) impose the mass-shell constraints \( p_i^2 + m_i^2 = 0 \) on the canonical momenta \( p_i^\mu = e_i^{-1} \dot{x}_i^\mu \) which are conserved according to the equations of motion \( \partial_\tau p_i^\mu = 0 \). The constraints emerge because the action is gauge invariant under the following \( n \) independent infinitesimal reparametrizations

\[
\delta x_i^\mu(\tau) = \varepsilon_i(\tau) \partial_\tau x_i^\mu(\tau), \quad \delta e_i(\tau) = \partial_\tau (\varepsilon_i e_i)(\tau).
\]

(2)

In recent papers [1][2] a generalization of the gauge symmetry (2) was introduced. The purpose of the present paper is to propose an elegant and more satisfactory formulation of the gauge symmetries and supersymmetries in [1][2][3]. Before we present the new formalism let us recall the role of the gauge symmetry by reviewing the simplest two particle case. The new symmetry requires orthogonal on shell momenta

\[
p_1^2 + m_1^2 = 0, \quad p_2^2 + m_2^2 = 0, \quad p_1 \cdot p_2 = 0.
\]

(3)

This set of constraints have a solution provided there are at least two time-like dimensions with \( \text{SO}(d - 1, 2) \) spacetime symmetry if the particles are massive, or \( \text{SO}(d, 2) \) if they are massless. However, in the background of the other particle, each particle effectively moves in a subspace with a single timelike dimension with \( \text{SO}(d - 1, 1) \) spacetime symmetry [1].

The motivation for a higher geometrical space, with more than one time-like dimension, comes from theoretical attempts to explain the supersymmetry and duality symmetries among p-branes [4]-[8]. As argued in [1][2], the presence of two or more time-like dimensions is compatible with standard physics. The essential point is that each particle moves effectively as if there is a single time-like dimension, and satisfies the standard energy-momentum relation. When the particles (or p-branes) of type 2,3,\( \cdots \) are frozen to one energy-momentum eigenvalue (i.e. Kaluza-Klein type reduction) the remaining theory describes the physics for particles (or p-branes) of type 1. This
leads to a cosmological scenario for deriving a universe with an effective single timelike dimension from a pre-Big-Bang universe with multi-timelike dimensions [1] [2]. In this scenario, the multi-times could be physically relevant before the Big-Bang in an era of unified symmetries, including supersymmetry and dualities of various p-D-branes [3]. After the Big Bang phase transition, the symmetries are broken, and one species of particles propagate in the background of all the others. Thus, the particles that make up our present visible universe behave effectively as if there is a single time-like dimension. Nevertheless, the compactified part of the universe, with its multi-times, has an effect in determining the properties and quantum numbers of low energy physics in new ways, as illustrated in [4].

2 Multiparticle gauge symmetry

The two particle constrained system can be generalized to an \(n\) particle constrained system of the form

\[
 p^i \cdot p^j + m_i^2 \delta^{ij} = 0. \tag{4}
\]

To have a solution with an effective SO\( (d - 1, 1) \) symmetry for each particle, one adds \((0, 1)\) dimensions for each additional massive particle and \((1, 1)\) dimensions for each additional massless one. Then there are \(n\) timelike dimensions with spacetime symmetry that ranges from SO\( (d - 1, n) \) if all particles are massive, to SO\( (d + n - 2, n) \) symmetry if all particles are massless. An action that gives this set of constraints can be constructed by using \(\tau\)-independent global Lagrange multipliers, as in [1] [2]. Here we will propose yet another approach using \(\tau\)-dependent local gauge functions, bringing the formulation closer to standard concepts of local gauge and reparametrization symmetries. The approach introduces new types of gauge symmetries for multi-particles as well as new actions.

Consider the following action for \(n\) particles described by \(x_i^\mu (\tau)\)

\[
 S = \frac{1}{2} \int_0^T d\tau \left( \dot{x}_i^\mu e^{ij} \dot{x}_j^\nu \eta_{\mu\nu} - \left( m^2 \right)^{ij} e_{ij} \right), \tag{5}
\]

where \(e_{ij} (\tau)\) is a symmetric matrix that generalizes the einbein, and \(e^{ij} (\tau) = (e^{-1})^{ij}\) is its inverse. This action is invariant under the following gauge
symmetry that replaces the standard $\tau$-reparametrizations of (2)

$$\delta x_\mu^i = \varepsilon_{ij}(\tau) e^{jk}(\tau) \partial_\tau x_\mu^k(\tau), \quad \delta e_{ij} = \partial_\tau \varepsilon_{ij}(\tau).$$

There are $n(n + 1)/2$-parameters $\varepsilon_{ij}(\tau)$ which mix positions with velocities. The inverse $e^{ij}$ transforms as $\delta e_{ij} = -e^{ik}(\partial_\tau \varepsilon_{kl}) e^{lj}$. The Lagrangian transforms into a total derivative given by

$$\delta L = \frac{1}{2} \partial_\tau \left( \dot{x}_i^\mu \varepsilon_{ij} e^{jk} \dot{x}_j^\nu \eta_{\mu\nu} - \left( m^2 \right)^{ij}_{\varepsilon_{ij}} \right).$$

The action is invariant provided $\varepsilon_{ij}(0) = \varepsilon_{ij}(T) = 0$. This is a new gauge symmetry that cannot be thought of as $\tau$ reparametrization for $n > 1$.

A first order formalism may also be given by defining the canonical momenta $p^i_\mu \equiv e^{ij} \dot{x}_j^\mu$, and replacing all velocities in the Hamiltonian by the momenta

$$S = \int_0^T d\tau \left( \dot{x}_i^\mu p^i_\mu - \frac{1}{2} \left[ p^i_\mu p^j_\nu \eta_{\mu\nu} + \left( m^2 \right)^{ij}_{\varepsilon_{ij}} \right] \varepsilon_{ij} \right).$$

This action is invariant under a local multi-particle gauge transformation given by

$$\delta x_\mu^i = \varepsilon_{ij} e^{jk} \partial_\tau x_\mu^k, \quad \delta e_{ij} = \partial_\tau \varepsilon_{ij}, \quad \delta p^i_\mu = e^{ij} \varepsilon_{jk} \partial_\tau p^k_\mu,$$

since the variation of the Lagrangian is a total derivative

$$\delta L = \partial_\tau \left( p^i_\mu \varepsilon_{ij} e^{jk} \dot{x}_j^\nu \eta_{\mu\nu} - \frac{1}{2} p^i_\mu p^j_\nu \varepsilon_{ij} \right).$$

We emphasize that, in the first order formalism, the symmetry is valid when each function $x, e, p$ is varied independently according to (3), without using any equation of motion (such as the relation between momenta and velocities). There is a true symmetry in the off shell path integral.

According to the equations of motion the canonical momenta are conserved, $\partial_\tau p^i_\mu = 0$. Furthermore, they are constrained as in (4). One can choose a basis in which the mass matrix is diagonal ($m^2)^{ij}_{\varepsilon_{ij}} = \delta^{ij} m^2_i$. Thus,

\[1\text{It is possible to combine } \varepsilon_{ij}(\tau) e^{jk}(\tau) \text{ into a new local parameter } \tilde{\varepsilon}_{ij}^k(\tau). \text{ Then for } n = 1 \text{ the new transformation becomes the standard } \tau \text{ reparametrization. However, for } n > 1 \text{ it cannot be written as reparametrizations of a single } \tau, \text{ showing that this is a new gauge symmetry for multi-particles.} \]
the new formulation reproduces the set of constraints that were obtained in the previous formulations.

The gauge symmetry permits the gauge fixing $e_{ij} = \delta_{ij}$, bringing the system to the form of a "gas" of particles that move freely except for the fact that their conserved momenta are mutually orthogonal. In this sense there is an interaction among them.

### 3 Superparticles, global and local new SUSY

For multi-superparticles there are $n$ positions $x_i^\mu (\tau)$ and momenta $p_i^\mu (\tau)$, $i = 1, 2, \cdots, n$, and $N$ spinors $\theta_{\alpha A} (\tau)$ labelled by $A = 1, 2, \cdots, N$, where $N$ is the number of supersymmetries, and the index $A$ is unrelated to the index $i$. The indices $\mu, \alpha$ are classified as the vector and spinor in $d+2n-2$ dimensions, with spacetime symmetry $SO(d+n-2, n)$. We propose an action directly in the first order formalism

$$S = \int_0^T d\tau \left( \dot{x}_i^\mu p_i^\mu - \frac{1}{2} p_\mu^i p_\nu^j \eta^{\mu\nu} e_{ij} + \bar{\theta}_A \Gamma (p) \dot{\theta}_A \right).$$

where

$$\Gamma_{\alpha\beta} (p) = \gamma^{\mu_1 \cdots \mu_n} (p^1_\mu \cdots p^n_\mu).$$

The second order formalism, in which the Lagrangian is written only in terms of velocities is extremely non-linear, and fortunately not needed.

This action is invariant under the following bosonic gauge symmetry that generalizes the local multi-particle symmetry of (3) to superparticles

$$\delta x_i^\mu = \varepsilon_{ij} e^{jk} \left( \partial_\tau x_k^\mu + \bar{\theta}_A V_k^\mu (p) \partial_\tau \theta_A \right),$$

$$\delta p_i^\mu = e^{ij} \varepsilon_{jk} \partial_\tau p_k^\mu, \quad \delta e_{ij} = \partial_\tau (\varepsilon_{ij}), \quad \delta \theta_A = 0,$$

where

$$V_k^\mu = \frac{\partial}{\partial p_k^\mu} \Gamma (p) = \gamma^{\mu_1 \cdots \mu_n} \frac{\partial}{\partial p_k^\mu} (p^1_\mu \cdots p^n_\mu).$$

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2We are discussing dimensions $d+2n-2$ for which $\gamma^{\mu_1 \cdots \mu_n}$ is symmetric in $(\alpha\beta)$.

For $n = 1$ start with $d = 10$. Then, for $n = 2$, twelve dimensions with signature $(10, 2)$, and for $n = 3$, fourteen dimensions with signature $(11, 3)$ satisfy this property. These are the dimensions of greatest interest in our program. For lower dimensions see footnote in 2.
Note that $\delta \theta_A = 0$, unlike the standard $\tau$ reparametrization, illustrating once again that the multi-particle symmetry is different than $\tau$ reparametrization, as in the purely bosonic case, there are $\frac{1}{2} n(n+1)$ local parameters $\varepsilon_{ij}(\tau)$. Under these variations the Lagrangian transforms into a total derivative
\[
\delta L = \partial_\tau \left[ \varepsilon_{ij} e^{jk} \left( p^i \cdot \dot{x}_k + \tilde{\theta}_A \left( p^j \cdot V_k \right) \dot{\theta}_A \right) - \frac{1}{2} p^i \cdot p^j \varepsilon_{ij} \right].
\]
(15)
Therefore the action is invariant provided $\varepsilon_{ij}(0) = \varepsilon_{ij}(T) = 0$.

Our action is also invariant under global and local supersymmetries. The global supersymmetry transformation is of the new type, involving the product of momenta which appear in $V^\mu_i(p)$,
\[
\begin{align*}
\delta_\varepsilon \theta_{A\alpha} &= \varepsilon_{A\alpha}, \\
\delta_\varepsilon x^i_\mu &= -\bar{\varepsilon} A V^\mu_i(p) \theta_A,
\end{align*}
\]
(16)
where $\varepsilon_{A\alpha}$ are $\tau$ independent constant spinors. The $V^\mu_i(p)$ which appear in the transformation of particle $i$ is independent of the momentum of the particle $i$, but depends on the product of the momenta of all other particles. Thus, when the momenta of all other particles are frozen, this supersymmetry transformation reduces effectively to the standard supersymmetry transformation in the single particle sector. Examples of this mechanism have been discussed in more detail in earlier papers. Under (16) the Lagrangian transforms into a total derivative
\[
\delta L = (1 - n) \partial_\tau (\bar{\varepsilon} A \Gamma(p) \theta_A). \tag{17}
\]
To show this, we have used $\partial_\tau \Gamma(p) = V^\mu_i \dot{p}^i_\mu$ and $V^\mu_i \dot{p}^j_\mu = n \Gamma(p)$. The closure of the multi-particle superalgebra is obtained by considering $[\delta_{\varepsilon_1}, \delta_{\varepsilon_2}]$ on the various fields, and is given by
\[
\{Q_{A\alpha}, Q_{B\beta}\} = (\gamma^{\mu_1 \cdots \mu_n})_{\alpha\beta} \left( p^1_{\mu_1} \cdots p^n_{\mu_n} \right), \tag{18}
\]
where the right hand side coincides with $\Gamma(p)$ given in (12).

Next we consider local $\kappa$ supersymmetry with the transformations
\[
\begin{align*}
\delta_\kappa x^i_\mu &= \delta_\kappa \bar{\theta}_A V^\mu_i \theta_A, \\
\delta_\kappa p^i_\mu &= 2 e^{ij} \bar{\kappa}_A (V^\mu_j \Gamma) \dot{\theta}_A, \\
\delta_\kappa \bar{\theta}_A &= -e^{ij} \left[ \dot{x}_{i\mu} + \bar{\theta}_A V_{i\mu} \dot{\theta}_A \right] \left( \bar{\kappa}_A V^\mu_j \right)_\alpha, \\
\delta_\kappa \varepsilon_{ij} &= -(-1)^{n(n-1)/2} 4 \text{Cof}(M)_{ij} \bar{\kappa}_A \dot{\theta}_A, \tag{19}
\end{align*}
\]
where we have used
\[(\Gamma (p))^2 = (-1)^{n(n-1)/2} \det(M),\]
and \(M\) is the \(n \times n\) matrix with entries \(M^{ij} = (p^i \cdot p^j)\), determinant \(\det(M)\), inverse \(M^{-1}\), and cofactor matrix
\[Cof(M)_{ij} = \frac{1}{(n-1)!} \varepsilon_{i_1 i_2 \ldots i_n} \varepsilon_{j_1 j_2 \ldots j_n} M^{i_2 j_2} \ldots M^{i_n j_n} = \det M M^{-1}.\]

With these \(\kappa\) supersymmetry transformations the \(\kappa\) variation of the Lagrangian is a total derivative. To verify this invariance we need the following steps
\[\delta_\kappa L = \partial_\tau (\delta_\kappa x_i) \cdot p + \cdots = \partial_\tau (\delta_\kappa x_i \cdot p) - \delta_\kappa x_i \cdot \partial_\tau p + \cdots = n \partial_\tau \left( \delta_\kappa \tilde{\theta}_A \Gamma(p) \theta_A \right) - \delta_\kappa \tilde{\theta}_A (\partial_\tau \Gamma(p)) \theta_A + \cdots,\]
where we have used the form of \(\delta_\kappa x_i\) given in (19) and the identities \(V^\mu_i p^i_\mu = n\Gamma(p)\) and \(\partial_\tau \Gamma(p) = V^\mu_i \dot{p}^i_\mu\). Combining this with the rest of the \(\kappa\) variation, and using \(\tilde{\theta}_A \Gamma \partial_\tau (\delta_\kappa \theta_A) = -\partial_\tau \left( \delta_\kappa \tilde{\theta}_A \right) \Gamma \theta_A\) due to the symmetry of \(\Gamma_{\alpha \beta}\), gives
\[\delta_\kappa L = (n-1) \partial_\tau \left( \delta_\kappa \tilde{\theta}_A \Gamma(p) \theta_A \right) + 2\delta_\kappa \tilde{\theta}_A \Gamma(p) \dot{\theta}_A - \frac{1}{2} (p_i \cdot p_j) \delta_\kappa e_{ij} + (\dot{x}_{i\mu} + \tilde{\theta}_A V_{i\mu} \dot{\theta}_A - e_{ij} p^i_\mu) \delta_\kappa p^j_\mu.\]

The first term is just a total derivative, and the other terms cancel each other after substituting the \(\kappa\) transformations (19) and using \(\{\Gamma, V^\mu_i\} \dot{p}^i_\mu = \partial_\tau (\Gamma^2)\) with the relations (20), (21). We emphasize that each field \(x, p, \theta, e\) is independently transformed, without using any equation of motion that relates them. In particular the equations of motion that relate the velocities and momenta are not used. So, the local \(\kappa\) invariance is valid in the fully off-shell path integral, as are all other symmetries discussed in this paper.

This model provides a realization of the generalized supersymmetry (18) in agreement with previous suggestions [7] [12]-[16][6][3][2]. The current model improves on [1][2] by not having global Lagrange multipliers, and unlike [3] avoids using equations of motion in symmetry transformations.

\[^3\]In [3] another formulation involving multiple \(\tau_i\) parameters was suggested. But, to have consistent \(\tau_i\) dependence in the transformations laws of various fields, the momenta of the particles had to be taken as constants (i.e. on shell, \(\partial_\tau p^i_\mu = 0\).
4 Superparticle constraints and quantization

The equations of motion for $p^i_\mu$ are

$$e_{ij}p^j_\mu = \dot{x}^\mu + \bar{\theta}_A \gamma^{\mu_1 \cdots \mu_n} \hat{\theta}_A \frac{\partial}{\partial p^i_\mu} \left( p^1_\mu \cdots p^n_\mu \right). \quad (24)$$

This gives a non-linear set of equations from which velocities are determined in terms of momenta. The equations of motion for $x^\mu_i$ indicate that the canonical conjugate momenta are conserved $\dot{p}^i_\mu = 0$. Similarly, the canonical conjugate for $\theta_A$ is

$$\bar{\xi}^A = \bar{\theta}_A \Gamma(p), \quad (25)$$

and the equation of motion for $\theta_A$ indicates that it is time independent $\partial_\tau \bar{\xi}^A = 0$.

A straightforward application of Noether’s theorem for the global supersymmetry ($16$) shows that $\xi_A^\alpha$ is the conserved supercharge

$$\xi_A^\alpha = Q_A^\alpha. \quad (26)$$

Let us derive its commutation rules. According to canonical quantization, it commutes with the momenta $p^i_\mu$. Furthermore, ignoring constraints, the naive anticommutation rules are $\{\theta_A^\alpha, \bar{\xi}_B^\beta\} = \delta^B_A \delta^\beta_\alpha$. Multiplying from the left with $\Gamma(p)$ we obtain

$$\{\xi_A^\alpha, \bar{\xi}_B^\beta\} = \delta^B_A \left( \Gamma(p) \right)^\beta_\alpha. \quad (27)$$

This is identical to the anticommutation rules of the supercharges given in ($18$). The supercharges are gauge invariant $\delta_\kappa Q_A^\alpha = 0$ under the on-shell $\kappa$ transformations. This is easily seen after using ($24$) in ($19$) and applying the constraints below. Therefore, the commutation rules ($27$) or ($18$) are gauge invariant and consistent with the constraints, even though the naive anticommutation rules for $\{\theta_A^\alpha, \bar{\xi}_B^\beta\}$ need modification due to the constraints.

Applying $\gamma \cdot p^j$ on $\xi^A$ or $Q^A$ and using the momentum constraints $p^i \cdot p^j = 0$ we find that there are also fermionic constraints

$$\gamma \cdot p^j Q^A = 0. \quad (28)$$
These constraints are due to local $\kappa$-symmetries and they help remove fermionic degrees of freedom. As explained below, each superparticle, in the background of all other superparticles, effectively has the same bosonic and fermionic degrees of freedom as the standard superparticle in 10 dimensions.

Let us recall the constraints for a simple superparticle in 10 dimensions with $\text{SO}(9,1)$ symmetry (specialize our equations to $n = 1$). They are

$$p^2 = 0, \quad \hat{p}Q^A = 0,$$

where the Majorana-Weyl spinors $\theta_{\alpha A}$ or $Q^{\alpha A}$ have 16 spinor components $\alpha = 1, \cdots, 16$. The fermionic constraint is solved by

$$Q^A = \hat{p}\theta^A.$$  \hspace{1cm} (30)

$Q$ has only 8 unrestricted components because the lightlike $p^\mu$ projects out half of the components of $\theta$. Thus the constraints are solved by 8 unrestricted transverse bosonic components in $p^\mu$ and 8 unrestricted fermionic components in $Q^A_\alpha$ or $\theta^A_\alpha$ (for each $A$). The 8 unrestricted fermions $Q$ satisfy a Clifford algebra which follows from (27) or (18). For one supersymmetry (one $A$), the quantum Hilbert space is labelled by the Clifford states $|a, p>$ consisting of $2^4 = 8_{\text{bosons}} + 8_{\text{fermions}}$. This is the Yang-Mills supermultiplet in 10 dimensions. It corresponds to a short representation of the 10-dimensional superalgebra $\{Q_\alpha, Q_\beta\} = \gamma^\mu_\alpha\beta p_\mu$.

Next consider the $n = 2$ case in 12 dimensions, with $\text{SO}(10,2)$ symmetry. The Majorana-Weyl spinors $Q^A_\alpha$ have 32 components $\alpha = 1, \cdots, 32$ for each $A$, while the momenta $p^\mu_i$ have two timelike and 10 spacelike components for each $i$. Two timelike dimensions are necessary to solve the constraints

$$p^2_1 = p^2_2 = p_1 \cdot p_2 = 0, \quad \hat{p}_1Q^A = \hat{p}_2Q^A = 0.$$  \hspace{1cm} (31)

From (27) we may write the solution of the fermionic constraint

$$Q^A = \hat{p}_1\theta^A_1 = \hat{p}_2\theta^A_2 = \hat{p}_1 \hat{p}_2\theta^A,$$  \hspace{1cm} (32)

where we have defined $\theta^A_1 \equiv \hat{p}_2\theta^A$ and $\theta^A_2 \equiv -\hat{p}_1\theta^A$. Consider the degrees of freedom of particle 1 in the background of particle 2. For a fixed $p_{2\mu}$ that satisfies $p_1 \cdot p_2 = p^2_2 = 0$, the momentum $p_{1\mu}$ generally has 10 components in an effective $\text{SO}(9,1)$ subspace orthogonal to the lightlike direction of $p_{2\mu}$. Furthermore, for the most general $\theta^A$ we see that $\theta^A_1 \equiv \hat{p}_2\theta^A$ can have only
16 components that form the spinor representation in the same SO(9,1) subspace. The remaining 10 components of $p_1^\mu$ and 16 components of $\theta_1^A$ (for each $A$) are further restricted by

$$p_1^2 = 0, \quad Q^A = p_1 \theta_1^A, \quad p_1 Q^A = 0. \quad (33)$$

This is the same set of SO(9,1) covariant equations satisfied by the superparticle in 10 dimensions, as above. Hence, we conclude that superparticle 1, in the background of superparticle 2, behaves just like the usual 10 dimensional superparticle with a single timelike dimension. Of course, the same argument holds for superparticle 2 in the background of superparticle 1. The treatment is democratic for either particle. This is the result we wished to have in order to obtain consistency with standard physics. The cosmological scenario of [1][2] may also be applied.

The remaining degrees of freedom in $Q^A$ or $\theta^A$ are still 8 unrestricted fermions (for each $A$). These satisfy a Clifford algebra that follows from (27) or (18). Therefore, the Clifford states for the full system are $|a, p_1, p_2>$, which corresponds again to $2^4 = 8_{\text{bosons}} + 8_{\text{fermions}}$ (for one supersymmetry). These can be expressed covariantly in 12 dimensions, as the states that form short supermultiplets of the 12-dimensional superalgebra

$$\{Q_\alpha, Q_\beta\} = (\gamma^{\mu\nu})_{\alpha\beta} Z_{\mu\nu} + (\gamma^{\mu_1...\mu_6})_{\alpha\beta} Z_{\mu_1...\mu_6}^+, \quad (34)$$

with $Z_{\mu\nu} = \frac{1}{2} (p_{1\mu} p_{2\nu} - p_{1\nu} p_{2\mu})$ and $Z_{\mu_1...\mu_6}^+ = 0$. This general form was first given in [4] as a BPS solution of the general 12D superalgebra. The model presented here provides an explicit realization of this symmetry in a short representation.

The arguments are similar for higher values of $n$. The bigger $n$, the bigger the spinor. For example, for $n = 3$ the spacetime group is SO(11,3) in 14 dimensions and the Majorana-Weyl spinors $\theta, Q$ have 64 components [8]. For three orthogonal lightlike momenta the solution of the fermionic constraints is $Q = p_1 \not{p_2} \not{p_3} \theta$. In the background of particles 2,3 the effective degrees of freedom of particle 1 are $p_1, \theta_1$ with the constraint $\not{p_1} Q = 0$, where $\theta_1 = \not{p_2} \not{p_3} \theta$ and $Q = \not{p_1} \theta_1$. Since the $\not{p_2} \not{p_3}$ projector reduces the spinor degrees of freedom by a factor of 1/4, the number of independent components in $\theta_1$ is $64/4 = 16$. This is the right number for particle 1 to move effectively in 10 dimensions. The description is fully democratic for every superparticle. The overall number of unrestricted fermions is again 8. The Clifford algebra...
that they form is realized on quantum states $|a, p_1, p_2, p_3>$ which describe $2^4 = 8_{\text{bosons}} + 8_{\text{fermions}}$ (for one supersymmetry). This is a short multiplet of the SO$(11, 3)$ covariant superalgebra in 14 dimensions \[8\]

$$\{Q_\alpha, Q_\beta\} = \left(\gamma^{\mu\nu\lambda}\right)_{\alpha\beta} Z_{\mu\nu\lambda} + \left(\gamma^{\mu_1\cdots\mu_7}\right)_{\alpha\beta} Z^+_{\mu_1\cdots\mu_7}, \quad (35)$$

such that $Z_{\mu\nu\lambda} = p^1_{\mu} p^2_{\nu} p^3_{\lambda}$ and $Z^+_{\mu_1\cdots\mu_7} = 0$. It can be regarded as a BPS state of a bigger S-theory \[8\].

5 Remarks

It is also possible to formulate multi-particle actions with fewer constraints, such that instead of constraining the momenta of each particle, only the total momenta of groups of particles are constrained. In this sense, the action given in this paper contains $n$ groups, with a single particle in each group. We will not discuss the generalization in detail to groups with many particles here, but instead refer the reader to \[2\] for a realization of this idea.

Also, a generalization of our current approach to multi-strings or multi p-branes may be considered. There will be many strings or p-branes, but there will be only one set of p-volume parameters $(\tau, \vec{\sigma})$. The string or p-brane version naturally comes with interactions through the geometry of the p-brane volume. Progress along those lines will be reported in a future publication.

We think that the ideas in this paper can be connected to Matrix theory \[10\], by combining them with the outline in \[8\] \[11\], to provide a matrix formulation that is spacetime covariant, duality covariant and gauge invariant. As a hint, we can show that our superalgebra (18) fits into the framework of Matrix theory. This point can be illustrated by rewriting (18) for $n = 3$ in matrix language

$$\{Q_\alpha, Q_\beta\} = \frac{1}{N} Tr \left( X_{\mu_1} X_{\mu_2} X_{\mu_3} \right) + \cdots \quad (36)$$

with the $N \times N$ matrix $X_\mu = J_1 p^1_\mu + J_2 p^2_\mu + J_3 p^3_\mu$. The $J_i$ correspond to the $N \times N$ representation of the generators of SU$(2)$ (embedded in the Lie algebra of SU$(N)$), i.e. $[J_i, J_j] = i \varepsilon_{ijk} J_k$. Then (36) reduces to (18). This raises the hope that we should be able to express our ideas in a matrix formulation...
and also find a more covariant formulation which explain the successes of Matrix theory so far. A covariant formulation of matrix theory or S-theory is expected to include the representations of the generalized superalgebra (35) of the type described in this paper, in addition to those that follow from the non-covariant matrix sector formulated in [10].

There should be a multi-local super Yang-Mills type theory that corresponds to the field theory version of the present first quantized multi-particle theory. The number of physical components of the fields is $2^4 = 8_{\text{bosons}} + 8_{\text{fermions}}$, but the fields generally depend on $n$ locations $A_a(x^\mu_1, \ldots, x^\mu_n)$. For $n = 1$ the theory can be formulated covariantly as the standard SO(9,1) super Yang-Mills theory in 10 dimensions $A_a \rightarrow (A_\mu, \psi_\alpha)$. For 12 or 14 dimensions the covariant theory has not been constructed so far. However, hints of its existence have been provided in references [12]-[16]. We interpret the work of these authors as a non-democratic description of particle 1 in the background of the other particles, such that the other particles have been frozen to constant momenta. Because of the freezing of the momenta the covariance in [12]-[16] is really SO(9,1) rather than SO(10,2) or SO(11,3). The freezing corresponds to a Kaluza-Klein type reduction of the covariant multi-local field theory. What has been missing in the field theory version is the democratic treatment of all $n$ particles in terms of fields that depend on $n$ locations. The field theory must have supersymmetry of the new type as in eq.(18), and must have some generalized gauge invariance that imitates a Yang-Mills theory. Its generalization to gravity with $2^8 = 128_{\text{bosons}} + 128_{\text{fermions}}$, as suggested in [7], would be the 12 or 14 dimensional multi-local generalization of supergravity that would reduce to local 11D supergravity upon the multi-local Kaluza-Klein reduction. The construction of such a multi-local field theory remains as a challenge. However, a first attempt with the new supersymmetry and two times, with partial reduction appropriate to post Big-Bang physics, has been successful for free fields [4].

References

[1] I. Bars and C. Kounnas, “Theories with two times”, hep-th/9703060, Phys. Lett. B402 25 (1997); and “String and Particle with Two Times”, hep-th/9705205, Phys. Rev. D56 3664 (1997).
[2] I. Bars and C. Deliduman, “Superstring and New Supersymmetry in (9,2) and (10,2) Dimensions”, [hep-th/9707215], to appear Phys. Rev.D.

[3] I. Rudychev and E. Sezgin, “Superparticles in $d > 11$”, [hep-th/9704057].

[4] I. Bars, “Duality and hidden dimensions”, [hep-th/9604200], in the proceedings of the conference *Frontiers in Quantum Field Theory*, Toyonaka, Japan, Dec. 1995; and Phys. Rev. **D54**, 5203 (1996).

[5] C. Vafa, Nucl. Phys. **B469**, 403 (1996); D.R. Morrison and C. Vafa, Nucl. Phys. **B473**, 74 (1996); E. Witten, Nucl. Phys. **B471**, 195 (1996); A. Kumar and C. Vafa, Phys. Lett. **B396**, 85 (1997).

[6] D. Kutasov and E. Martinec, Nucl. Phys. **B477**, 652 (1996); Nucl. Phys. **B477**, 675 (1996); Class. Quant. Grav. **14**, 2483 (1997).

[7] I. Bars, “S-Theory”, Phys. Rev. **D55**, 2373 (1996); and I. Bars, “Algebraic Structures in S-Theory”, [hep-th/9608061].

[8] I. Bars, “A case for 14 dimensions”, [hep-th/9704054], Phys. Lett. **B403** 257 (1997).

[9] I. Bars and C. Kounnas, “A new supersymmetry”, [hep-th/9612119].

[10] T. Banks, W. Fishler, S. Shenker and L. Susskind, Phys. Rev. **D55**, 5122 (1997).

[11] I. Bars, “Duality covariant type IIB supersymmetry and non-perturbative consequences”, [hep-th/9706185], to appear in Phys. Rev.D.

[12] H. Nishino and E. Sezgin, Phys. Lett. B **388**, 569 (1996).

[13] E. Sezgin, “Super Yang-Mills in ( 11,3) dimensions”, [hep-th/9703123], Phys. Lett. **B403** 265 (1997).

[14] H. Nishino, “Supergravity in 10+2 dimensions ...”, [hep-th/9703214].

[15] H. Nishino, “Supersymmetric Yang-Mills Theories in $d \geq 12$”, [hep-th/9708064].

[16] V. Periwal, Phys. Rev. **D55**, 1711 (1997).