The Smectic $A$ to $C$ Phase Transition in Isotropic Disordered Environments

Leiming Chen

College of Science, The China University of Mining and Technology, Xuzhou Jiangsu, 221116, P. R. China

John Toner

Department of Physics and Institute of Theoretical Science, University of Oregon, Eugene, OR 97403

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We study theoretically the smectic $A$ to $C$ phase transition in isotropic disordered environments. Surprisingly, we find that, as in the clean smectic $A$ to $C$ phase transition, smectic layer fluctuations do not affect the nature of the transition, in spite of the fact that they are much stronger in the presence of the disorder. As a result, we find that the universality class of the transition is that of the "Random field XY model" (RFXY).

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The effect of quenched disorder on condensed matter systems has been widely studied for many years [1-3], both for practical reasons (since disorder is always present in real systems) and fundamental ones. Disorder can destroy many types of long ranged order (e.g., ferromagnetic order in systems with quenched random fields [4]), and it can radically change the critical behavior of many phase transitions [3].

Such effects have been found in, e.g., superconductors [5], charge density waves [6, 7], Josephson junction arrays [8], superfluid helium in aerogel [9], and ferromagnetic superconductors [10].

Some of the most novel and dramatic effects of quenched disorder are found in liquid crystals confined in random porous media [11, 12]. These intriguing systems exhibit a variety of exotic "Bragg Glass" phases. They also undergo unique types of phase transitions [13], one of which, the Smectic $A$ to Smectic $C$ (hereafter, $AC$) transition [14, 15], is the subject of this paper.

In the high temperature phase (the "$A$" phase), the nematic director $\hat{n}$ (which points along the axis of alignment of the constituent long molecules that make up the smectic material), and the normal $\hat{N}$ to the smectic layers, are parallel. In the low temperature phase (the "$C$" phase), $\hat{n}$ and $\hat{N}$ tilt away from each other.

The $AC$ transition in clean systems was first considered by deGennes [16], who showed that, if fluctuations of the smectic layers could be neglected, the $AC$ transition should be in the universality class of the ferromagnetic $XY$ model [17].

The effect of layer fluctuations on this result was considered later by Grinstein and Pelcovits [14], who showed that their effect on the $AC$ transition can, in fact, be neglected, and that, therefore, the $AC$ transition in clean systems is $XY$-like.

Unfortunately, for reasons not yet well understood, the critical region of the $AC$ transition in clean systems appears to be extremely small; as a result, most experimental systems exhibit a transition that is accurately described by mean-field theory [18]. As a result, no definitive experimental test of the above predictions has yet been made.

Recently the nature of the $AC$ transition has been studied for a liquid crystal confined in uniaxial [13] and biaxial [19] disordered environments. In these systems, the anisotropy essentially freezes the direction of the molecular axes, and the $AC$ transition can be described entirely in terms of the configuration of the smectic layers [13, 19].

In an isotropic quenched random environment (which can be realized most simply by putting the smectic in aerogel [20]), which we study in this paper, the problem is in many ways more difficult, since now both fluctuations of the molecular direction and those of the layers must be addressed. Indeed, it is not even obvious that the two phases between which the transition we wish to study occurs even exist in $d = 3$; the stability of the $A$ phase in the presence of even arbitrarily weak disorder remains an open question both theoretically [12] and experimentally [11]. Presumably, similar issues arise with the $C$ phase.

However, if we assume that both the $A$ and $C$ phases are stable, then we are able to completely determine the nature of the transition between them. We find that, if this stability assumption is correct, the layer fluctuations do not affect the universality class of this transition, which proves to be just that of the random field $XY$ model [21, 22].

This implies a substantial quantitative change in the universal critical exponents from their values in the clean problem. It is known [21] that the coefficients in the $\epsilon = 6 - d$ expansion are the same as those for the $\epsilon = 4 - d$ expansion of the clean (i.e., no random field) problem. However, since $\epsilon = 3$ in the physical case $d = 3$ for the random field problem, the $\epsilon$-expansion is not quantitatively reliable. It is clear, however, that the exponents will be quite different from those for the clean $XY$ model, as even the first order in $\epsilon$ terms change by a factor of 3.

From a quantitative standpoint, the most useful feature of our result is that it connects the exponents of the $AC$ transition in an isotropic disordered environment to those of a random field $XY$ model, as can be experi-
mentally realized in, e.g., anti-ferromagnets with substitutional disorder \[23\].

The remainder of this paper is devoted to demonstrating that the AC transition in the presence of isotropic disorder is in the random field XY universality class.

Our starting model is a modification of the model for clean smectics near a Smectic A-Smectic C transition \[14\], the Hamiltonian \( H = H_u + H_c + H_{ac} \) for which consists of three parts:

\[
H_u = \frac{1}{2} \int d^d r \left[ K (\nabla \perp^2 u) + B \left( \partial_z u - \frac{1}{2} |\nabla u|^2 \right) \right],
\]

\[
H_c = \frac{1}{2} \int d^d r \left[ K_1 \left( \nabla \cdot \hat{e} \right)^2 + K_2 \left( \nabla \times \hat{e} \right)^2 \right. \\
\left. + K_3 \left( \frac{\partial \hat{e}}{\partial z} \right)^2 + D c^2 + 2 \omega c^4 \right],
\]

\[
H_{ac} = \frac{1}{2} \int d^d r \left[ g_1 c^2 \left( \partial_z u - \frac{1}{2} |\nabla u|^2 \right) + g_2 (\nabla \perp^2 u) \times \right. \\
\left. \left( \nabla \cdot \hat{e} \right) + g_3 \left( \frac{\partial \hat{e}}{\partial z} \right) \cdot \left( \partial_z \nabla \perp u \right) + D \left( \hat{e} \cdot \nabla \perp u \right)^2 \right],
\]

where we have defined the direction parallel to the averaged layer normal in the A phase as the \( \hat{z} \)-axis, and the random field \( \delta r = (\delta x, \delta y, \delta z) \) is taken to have Gaussian distributions of zero mean, with anisotropic short-ranged correlations:

\[
\tilde{h}_i (\delta r) \tilde{h}_j (\delta r') = \Delta \delta_{ij} \delta^d (\delta r - \delta r'),
\]

\[
\tilde{h}_i^c (\delta r) \tilde{h}_j^c (\delta r') = \Delta \delta_{ij} \delta^d (\delta r - \delta r'),
\]

\[
\tilde{h}_i^c (\delta r) \tilde{h}_j^c (\delta r') = \Delta' \delta_{ij} \delta^d (\delta r - \delta r').
\]

The first term in equation (4) has been treated in the earlier work \[12\] on the smectic A phase in isotropic disordered environments, where it leads to strong-power-law anomalous \[12\]. The second term is just the random field disorder present in the RFXY model \[4, 22\].

To cope with the quenched disorder we employ the replica trick \[2\]. We assume that the free energy of the system for a specific realization of the disorder is the same as that averaged over many realizations. To calculate the averaged free energy \( F = \ln Z \), where \( Z \) is the partition function, we use the mathematical identity \( \ln Z = \lim_{n \to 0} \frac{\ln Z^n}{n} \). When calculating \( Z^n \), we can first compute the average over the random fields \( \tilde{h}(\delta r) \), whose statistics have been given earlier. Implementing this procedure gives a replicated Hamiltonian \( H^r = H_u^r + H_c^r + H_{ac}^r \) with the effect of the random fields transformed into couplings between \( n \) replicated fields, with the limit \( n \to 0 \) corresponding to the original quenched disorder problem:

\[
H_u^r = \frac{1}{2} \int d^d r \sum_{\alpha=1}^n \left[ B \left( \partial_z u_\alpha - \frac{1}{2} |\nabla u_\alpha|^2 \right) \right. \\
\left. + K (\nabla \perp^2 u_\alpha)^2 \right] - \frac{\Delta}{2k_BT} \int d^d r \sum_{\alpha=1}^n \nabla \perp u_\alpha \cdot \nabla \perp u_\beta,
\]

\[
H_c^r = \frac{1}{2} \int d^d r \sum_{\alpha=1}^n \left[ K_1 \left( \nabla \cdot \hat{e}_\alpha \right)^2 + K_2 \left( \nabla \perp \times \hat{e}_\alpha \right)^2 \right. \\
\left. + K_3 \left( \frac{\partial \hat{e}_\alpha}{\partial z} \right)^2 + D c^2 + 2 \omega c^4 \right],
\]

\[
H_{ac}^r = \frac{1}{2} \int d^d r \sum_{\alpha=1}^n \left[ g_1 c^2 \left( \partial_z u_\alpha - \frac{1}{2} |\nabla u_\alpha|^2 \right) \\
\left. + g_2 (\nabla \perp^2 u_\alpha) \times \left( \nabla \cdot \hat{e}_\alpha \right) + g_3 \left( \frac{\partial \hat{e}_\alpha}{\partial z} \right) \cdot \left( \partial_z \nabla \perp u_\alpha \right) \\
\left. + D \left( \hat{e}_\alpha \cdot \nabla \perp u_\alpha \right)^2 \right] \right].
\]

If we set \( u_\alpha = 0 \), the entire Hamiltonian reduces to Eq. \( 4 \), which reduces to the RFXY model if \( K_1 = K_2 = K_3 \). An RG analysis shows that departures from this “one constant approximation” (i.e., \( K_{1,2,3} = K \)) are irrelevant \[23\]; hence, in the absence of the \( u \) field, the transition is in the RFXY universality class.

The piece \( H_u^r \) Eq. \( 6 \) of \( H \) which involves \( u \) alone is precisely the model for smectics A in isotropic aerogel
studied in reference 12. From the analysis of that reference, we know that the critical dimension of Eq. (4), below which the anharmonic terms in Eq. (6) become important, is 5. On the other hand, the critical dimension of \( H' \) Eq. (7) is well known 4, 21, 22 to be 6. Because of this discrepancy between the two critical dimensions, a standard \( \epsilon \)-expansion study of the entire model Eqs. (6-8) is impossible. Our solution to this quandary is to integrate out only the \( u_\alpha \) fields perturbatively in a momentum shell RG approach, which is controlled in an \( \epsilon = 5 - d \)-expansion, to obtain an effective model that only involves \( \tilde{c}_\alpha \). While unorthodox, this approach is very much in the spirit of more conventional RG’s: we are performing a partial trace over some degrees of freedom to obtain a more tractable Hamiltonian in terms of the degrees of freedom remaining after the trace.

The momentum shell RG procedure consists of tracing over the short wavelength Fourier modes of \( u_\alpha(\vec{r}) \) followed by a rescaling of the length. We initially restrict wavevectors to lie in a bounded Brillouin zone followed by a rescaling of the length. We initially re-

\[
\frac{d\tilde{g}}{dl} = \left( d + 1 - \omega - \frac{3g}{16d} \right) \tilde{g},
\]

where \( g \) is a dimensionless coupling:

\[
g \equiv \Delta \left( \frac{B}{K^2} \right)^{\frac{1}{5}} C_{d-1} \Lambda^{d-5},
\]

where \( C_d \) is the surface area of a \( d \)-dimensional sphere with radius one divided by \((2\pi)^d\).

Note that the graphical corrections inside the parenthesis in Eqs. (9) and (12) are the same. This is not just an approximation to one-loop order, but exact to arbitrary loop order. This can be easily understood by analyzing the structures of the Feynman graphs. In Fig. 1 the upper graph summaries all the possible graphical corrections to \( (\partial z u_\alpha)^{\frac{1}{2}} \tilde{\nabla} \cdot \tilde{c}_\alpha \); the lower one does for \( (\partial_z u_\alpha) \tilde{c}_\alpha \). The parts inside the two square boxes are the same no matter how complicated they are and how many loops they have.

There are no graphical corrections to \( \tilde{\nabla}^2 u_\alpha(\tilde{\nabla} \cdot \tilde{c}_\alpha) \), which is also exact to arbitrary-loop order. This is because both terms have one power of \( c_0 \) while all anharmonic terms have even powers of \( c_0 \). Therefore, under renormalization both \( g_{2,3} \) flow only as a result of length and field rescaling.

The recursion relations for \( B, K, \) and \( \Delta \) are identical with those found for a smectic \( A \) in an isotropic disordered medium in reference 12. This is also exact to all orders, since we have not, in our unusual approach, integrated out the \( \tilde{c} \) fluctuations. This means that all of the results obtained in 12 for the long-wavelength behavior of these quantities also hold here. We will also make use of this fact later.

To analyze these flow equations we introduce an additional dimensionless coupling: \( g_3 = \frac{g^2}{Bv} \). Combining flow
FIG. 2: RG flows of the dimensionless couplings \(g\) and \(g_3\) from equations (15) and (16). All initial models starting to the left of the stability limit \(g_3 = 8\) flow into the \(g = g^*\), \(g_3 = 0\) fixed point, which therefore controls the AC transition. All models starting to the right of the stability limit are unstable.

Eqs. (11)-(13) with the definitions of \(g\) and \(g_3\) we find
\[
\frac{dg}{dl} = \epsilon g - \frac{5}{32} g^2, \quad (15)
\]
\[
\frac{dg_3}{dl} = 3g(-8 + g_3) g_3, \quad (16)
\]
where \(\epsilon = 5 - d\). These flow equations have four fixed points: \(g^* = 0\) or \(g^*_3 = 0\) or \(g^*_3 = 8\). The RG flows of \(g\) and \(g_3\) around these fixed points are illustrated in Fig. 2. Note that \(g^*_3 = 8\) corresponds to the stability limit of the system. Linearizing Eqs. (15) (16) around the only stable fixed point \(g^* = 8\), \(g^*_3 = 0\), we find the graphical corrections to \(v\) vanish exponentially as \(l \to \infty\). This implies that integrating out \(u_\alpha\) only gives a finite correction to \(v\), even at arbitrarily long wavelengths. Hence, these corrections to \(v\) coming from the \(u_\alpha\) fields do not affect the nature of the AC transition.

During each RG cycle the integration over \(u^\perp_\alpha\) also generates terms which do not exist in \(H_r\). The most relevant ones are produced in the second cumulant by \((\partial_\perp u_\alpha)c^2_{\alpha\alpha}\) and \((\nabla^2 u_\alpha)(\nabla \cdot c_\alpha)\). Elementary power counting shows that the terms generated by \((\partial_\perp u_\alpha)c^2_{\alpha\alpha}\) are less relevant.

We’ll now show that these terms also do not affect the nature of the AC transition. We start with the terms generated by \((\partial_\perp u_\alpha)c^2_{\alpha\alpha}:
\[
\sum_{\alpha, \beta} \sum_{q_1, q_2} g^2_{1}(\vec{k}) \left[ k_B T k^2 G(\vec{k}) \delta_{\alpha\beta} + \Delta(\vec{k}) k^2 z^2 G^2(\vec{k}) \right]
\]
\[
\times c_{\alpha, i}(\vec{q}_1)c_{\alpha, i}(-\vec{q}_1 + \vec{k})c_{\beta, j}(\vec{q}_2)c_{\beta, j}(-\vec{q}_2 - \vec{k}) \quad (17)
\]
where \(G(\vec{k}) = 1/[B(\vec{k})k^2_z + K(\vec{k})k^4_z]\). The \(k\)-dependences of \(B, K, \Delta,\) and \(g_1\) arise due to the the nonzero graphical corrections in the recursion relations Eqs. (12) (13). Because, as mentioned earlier, Eqs. (12) (13) are identical, to all orders, with those for a smectic A in an isotropic disordered environment, we can simply use the results of [12] for the wavevector dependences of these quantities. Furthermore, since, as noted earlier, there is an exact relation between the renormalization of \(g_1\) and that of \(B,\) the wavevector dependence of \(g_1\) is identical to that of \(B,\) up to an overall multiplicative constant.

Using the just noted connections to the work of [12], we can simply quote \(k\)-dependences of \(B, K, \Delta,\) and \(g_1:\)
\[
B(\vec{k}), g_1(\vec{k}) \propto \begin{cases} 
\begin{align*}
& k^8_{\perp} \kappa, \quad k_z \ll k^\zeta, \\
& k^9_{\perp} k^\zeta, \quad k_z \gg k^\zeta,
\end{align*}
\end{cases} \quad (18)
\]
\[
K(\vec{k}), \Delta(\vec{k}) \propto \begin{cases} 
\begin{align*}
& k^2_{\perp} k^\zeta, \quad k_z \ll k^\zeta, \\
& k^2_{\perp} k^\zeta, \quad k_z \gg k^\zeta,
\end{align*}
\end{cases} \quad (19)
\]
where the anisotropy scaling exponent \(\zeta = 2 - \frac{n_B + n_K}{2}\), and \(\eta_{B,K,\Delta} > 0\). Another result of [12] is that the exponents \(\eta_{B,K,\Delta}\) are not fully independent, but connected by the exact scaling relation:
\[
5 - d + \eta_{\Delta} = \frac{n_B}{2} + \frac{5}{2} n_K, \quad (20)
\]
which is implied by the fact that \(g\) flows to a nonzero stable fixed point [12]. Furthermore, there are certain bounds on the values of \(n_B\) that must be satisfied in order for the smectic A phase in an isotropic random environment to be stable, which is a prerequisite condition for the existence of a sharp smectic A-C transition [12] in such environments. It is only meaningful within these bounds to discuss the relevance of the terms in formula (17). These bounds are
\[
\eta_K + \eta_B < 2, \quad \eta_B < 1, \quad \eta_B + 5 n_K > 4. \quad (21)
\]
The first two bounds come from the requirement of long-ranged orientational order and the condition for dislocations to remain confined, respectively. The third bound is obtained by combining \(\eta_{\Delta} > 0\) with the exact scaling relation (20) in \(d = 3\).

Using expressions (18) (19) we can write equation (17) in a scaling form:
\[
\sum_{\alpha, \beta} \sum_{q_1, q_2} \left[ k^8_{\perp} f_1 \left( \frac{k^8_{\perp}}{k^1_{\perp}} \right) \delta_{\alpha\beta} + k^8_{\perp} \eta_{\beta,\Delta} / 2 f_2 \left( \frac{k^8_{\perp}}{k^1_{\perp}} \right) \right]
\]
\[
\times c_{\alpha, i}(\vec{q}_1)c_{\alpha, i}(-\vec{q}_1 + \vec{k})c_{\beta, j}(\vec{q}_2)c_{\beta, j}(-\vec{q}_2 - \vec{k}), \quad (22)
\]
where \(f_{1,2}(x)\) are scaling functions. Clearly, as \(\vec{k} \to 0\) the replica-diagonal term (i.e., the one which contains \(\delta_{\alpha\beta}\) in (22) is irrelevant compared to the quartic \((\nu)\) term in \(H^r\), since its coefficient vanishes like \(k^8_{\perp}\).

To decide whether the off-diagonal piece is relevant, we treat it as a perturbation and calculate its contributions to \(D\):
\[
\delta D = \int d^d k \left[ k_{\perp} \eta_{\beta,\Delta} / 2 f_2 \left( \frac{k_{\perp}}{k^1_{\perp}} \right) \right] \frac{1}{ck^2 + D}
\]
\[
= \int d^d k \left[ k_{\perp} \eta_{\beta,\Delta} / 2 f_2 \left( \frac{k_{\perp}}{k^1_{\perp}} \right) \right] \frac{1}{ck^2} \left( 1 - \frac{D}{ck^2} \right).
\]
It is readily shown that this integral converges for $d$ near 6 if the exponents $\eta_{B,K}$ satisfy the bounds [21]. Therefore, this off-diagonal piece is also irrelevant.

Now we discuss the terms generated by $(\nabla^2_\perp u_{\alpha})(\nabla^3_\perp c_{\alpha})$, which also have a diagonal and an off-diagonal part:

$$
\sum_{\alpha,\beta} \sum_{q} \frac{g^2}{q^2} \left[ k_B T q^2_{\perp} G(q) \delta_{\alpha,\beta} + \Delta(q) q^0_{\perp} G^2(q) \right] \times q_i q_j c_{\alpha,i}(q) c_{\beta,j}(-q). \tag{23}
$$

Here, unlike $g_1$, $g_2$ has no dependence on $q$ since there are no graphical corrections to $(\nabla^2_\perp u_{\alpha})(\nabla^3_\perp c_{\alpha})$. Again we can rewrite Eq. (23) in a scaling form:

$$
\sum_{\alpha,\beta} \sum_{q} \frac{g^2}{q^2} \left[ q^2_{\perp} f_3 \left( \frac{q_{\perp}}{q_{\perp}} \right) \delta_{\alpha,\beta} + q^2_{\perp} (\eta a + 3 \eta / 2) f_4 \left( \frac{q_{\perp}}{q_{\perp}} \right) \right] \times q_i q_j c_{\alpha,i}(q) c_{\beta,j}(-q), \tag{24}
$$

where $f_{3,4}(x)$ are scaling functions similar to $f_{1,2}(x)$. Clearly, both terms are subdominant to the quadratic terms in $H_6^c$ as $q \to 0$ provided that $\eta_{B,K}$ are within the stability bounds.

Therefore, we conclude that integrating out $u_\alpha$ only gives minor corrections to $H_6^c$, which do not affect the nature of the transition. Therefore, the universality class of the transition is just that of the random field $XY$ model, as it would be were the full Hamiltonian just $H_6^c$.

In summary, in this paper we’ve studied the smectic $A$ to $C$ phase transition in isotropic disordered environment. Our analysis shows that if the smectic phases are stable against fluctuations and unbinding of dislocations, the universality class of the transition is that of the “Random Field XY Model”. Surprisingly, in spite of the fact that the smectic layer fluctuations are large due to the disorder, they have no effect on the nature of the transition; that is, if the layers can be frozen by some experimental means the universality class of the transition still remains the same. During this study we developed a “partial renormalization group” strategy which proves to be very successful. We expect this strategy to be useful in dealing with many problems with anharmonic Hamiltonians which involve multiple fields with different critical dimensions.

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[24] The RG analysis is similar to the pure case [14]. We find that, in an $\epsilon = 6 - d$ expansion, the eigenvalue $\lambda_{iK}$ of this perturbation is $\frac{n^2}{3(n+2)}$, where $n$ denotes the number of components of $\vec{c}$. 