A NEW EQUATION OF STATE FOR NEUTRON STAR MATTER WITH NUCLEI IN THE CRUST AND HYPERONS IN THE CORE

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ABSTRACT

The equation of state for neutron stars in a wide-density range at zero temperature is constructed. The chiral quark–meson coupling model within relativistic Hartree–Fock approximation is adopted for uniform nuclear matter. The coupling constants are determined so as to reproduce the experimental data of atomic nuclei and hypernuclei. In the crust region, nuclei are taken into account within the Thomas–Fermi calculation. All octet baryons are considered in the core region, while only $\Xi^-$ appears in neutron stars. The resultant maximum mass of neutron stars is 1.95 $M_\odot$, which is consistent with the constraint from the recently observed massive pulsar, PSR J1614-2230.

Key words: dense matter – equation of state – stars: neutron

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1. INTRODUCTION

Understanding of the equation of state (EOS) for dense matter is important to clarify compact astrophysical phenomena. For instance, neutron stars are composed of a core with supra-nuclear densities and a crust with subnuclear densities (e.g., Glendenning 2002; Lattimer & Prakash 2004, 2006). The mass and radius of neutron stars are mainly determined by the EOS at the core. Nevertheless, the crust is also important to account for some phenomena such as pulsar glitches (Anderson & Itoh 1975; Link et al. 1999; Andersson et al. 2012) or quasi-periodic oscillations in giant flares emitted by highly magnetized neutron stars (Strohmayer & Watts 2006; Sotani et al. 2012). Therefore, the range of densities required to investigate neutron stars is enormous. Furthermore, to study supernova explosions, proton-neutron stars (nascent neutron stars from supernovae), or binary neutron star mergers, we need an EOS with not only a wide density range but also finite temperatures.

At present, there are a few types of nuclear EOSs that have been widely used in astrophysical simulations. One of the most popular EOSs is calculated with the relativistic mean-field (RMF) theory for uniform nuclear matter and the Thomas–Fermi model for nonuniform matter (Shen et al. 1998a, 1998b). The EOS of Lattimer & Swesty (1991), which is based on Skyrme-type nuclear interactions and the compressible liquid-drop model, has also been used over the past decade. Moreover, recently, several EOSs based on the RMF model have been opened to the public (e.g., Hempel & Schaffner-Bielich 2010; Shen et al. 2010). Incidentally, there is an attempt to establish the EOS with the cluster variational method for nuclear matter (Kanawa et al. 2007, 2009). Unfortunately, however, these EOSs do not include hyperons. According to terrestrial nuclear experiments such as heavy-ion collisions in J-PARC and GSI-FAIR, there is much evidence of existing hyperons (e.g., Nagae 2010; Botta et al. 2012). Roughly speaking, since the mass differences between hyperons and nucleons are comparable to the nucleon Fermi energy in neutron stars, hyperons should be considered in an EOS for neutron stars.

These days, thanks to advances in observations, it will become possible astrophysically to obtain some information on the properties of nuclear matter. In particular, the discovery of a so-called two-solar-mass neutron star (PSR J1614-2230, 1.97 ± 0.04 $M_\odot$) puts a severe constraint on the EOS for nuclear matter (Demorest et al. 2010). Generally speaking, the inclusion of other degrees of freedom, such as hyperon admixture, meson condensation, and quark matter, softens an EOS, and the maximum mass of neutron stars is thus reduced (e.g., Glendenning & Moszkowski 1991; Glendenning & Schaffner-Bielich 1998; Weber 2005; Schaffner-Bielich 2008). In fact, the mass of J1614-2230 cannot be accounted for extending the EOS of Shen et al. (1998a, 1998b) to a hyperon inclusion (Ishizuka et al. 2008; Shen et al. 2011) or a hadron–quark phase transition (Nakazato et al. 2008). Thus, it is urgent to construct an EOS that is consistent with both terrestrial nuclear experiments and the astrophysical observations.

To solve the “hyperon puzzle,” one of us (T.M.) introduced a framework based on the Hartree–Fock calculation in RMF theory (Miyatsu et al. 2012; Katayama et al. 2012). An EOS becomes stiff due to the exchange (Fock) term, which has not been taken into account in previous studies (Ishizuka et al. 2008; Shen et al. 2011). They showed that it is possible to construct an EOS with hyperons that is consistent with the mass of J1614-2230, using the hyperon interaction based on the experimental hyperon potentials (Schaffner-Bielich & Mishustin 1996; Ishizuka et al. 2008; Botta et al. 2012) and hyperon–nucleon scattering data (Rijken et al. 2010). Among the models proposed in Katayama et al. (2012), they recommended the chiral quark–meson coupling (CQMC) model that was illustrated as case (d) in Table 8 of their paper. In this model, the internal structure of baryons is considered based on the quark degrees of freedom and the quark–quark hyperfine interactions, and its modification in the nuclear medium is effectively taken into account as the density dependence of an interaction. Hereafter, we denote this model as KMS12.

We extend the EOS of KMS12 for use in astrophysics. As a first step, in this paper, we fix the model that is consistent with the experimental data on atomic nuclei. Here we readjust the coupling constants in KMS12 so as to reproduce the gross feature of nuclear mass data with the Thomas–Fermi calculation, as done in Kanawa et al. (2009). Lately, finite...
nuclei have been studied within the framework of RMF model. Nevertheless, our final goal is an EOS for a wide range of parameters including finite temperatures. The Thomas–Fermi approximation is applicable for this purpose (Shen et al. 1998b), and we self-consistently adopt it also for the calibration of the coupling constants. Moreover, in this paper, we construct the EOS for low-density nonuniform matter at zero temperature and apply the result to neutron stars including hyperons. Consequently, we can account for a neutron star with 1 coupling constants. The Thomas–Fermi limit on the mass of J1614-2230 (Demorest et al. 2010).

This paper is arranged as follows. In Section 2, a brief review of the formalism for the Hartree–Fock calculation in RMF theory with the CQMC model is presented. We show the method to determine the coupling constants from the experimental data of atomic nuclei and hypernuclei in Section 3. The application to determine the coupling constants from the experimental data theory with the CQMC model is presented. We show the method of the formalism for the Hartree–Fock calculation in RMF approximation. Furthermore, we consider the baryon structure variation due to the interaction in matter. In the RMF approximation, the meson field vanishes. Nevertheless, our final goal is an EOS for a wide range of nuclei have been studied within the framework of RMF model.

2. HARTREE–FOCK CALCULATION IN RELATIVISTIC MEAN-FIELD THEORY

In this section, we summarize the formulations of our EOS for uniform nuclear matter (Miyatsu et al. 2012; Katayama et al. 2012). Here we adopt the RMF theory, in which baryons interact via the exchange of mesons. In order to calculate the interactions, we take into account not only the tadpole (Hartree) but also exchange (Fock) diagrams. We call this method relativistic Hartree–Fock (RHF) approximation. Furthermore, we consider the baryon structure variation due to the interaction in matter using the CQMC model. In this model, the properties of nuclear matter can be self-consistently calculated by the coupling of scalar and vector fields to the quarks within nucleons (Guichon 1988; Saito & Thomas 1994) and the quark–quark hyperfine structures due to the exchanges of gluon and pion are included based on chiral symmetry (Guichon et al. 2008; Nagai et al. 2008; Miyatsu & Saito 2010). In the CQMC model, we assume that the baryon structure variation in matter is reflected in the σ-field dependence of the mean-field values: \( \bar{\sigma} \), \( \bar{\omega} \), and \( \bar{\rho} \) (the \( \rho^0 \) field). Note that the mean-field value of the pion vanishes.

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We start the descriptions of our EOS with the Lagrangian density for hadronic matter:

\[
L_H = L_B + L_M + L_{\text{int}}.
\]

The first term is the baryon term and is given by

\[
L_B = \sum_B \bar{\psi}_B \left( i \gamma \mu \partial^\mu - M_B \right) \psi_B,
\]

where \( \bar{\psi}_B \) is the baryon field and \( M_B \) is the baryon mass in vacuum. The sum \( B \) runs over the octet baryons, \( \rho \), \( n \), \( \Lambda \), \( \Sigma^{0-} \), \( \Xi^{0-} \). Protons and neutrons are denoted collectively as \( N \). For the free baryon masses, we take \( M_\rho = 939 \text{ MeV} \), \( M_n = 1116 \text{ MeV} \), \( M_\Lambda = 1193 \text{ MeV} \), and \( M_\Sigma = 1313 \text{ MeV} \) (Pal et al. 1999). Taking into account the isoscalar (\( \sigma \) and \( \omega \)) mesons and the isovector (\( \pi \) and \( \rho \)) mesons, we write the meson term as

\[
L_M = \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2 \right) + \frac{1}{2} m_\omega^2 \omega \partial^\mu \omega \partial_\mu \omega
\]

\[
+ \frac{1}{4} m_\rho^2 \rho \partial^\mu \rho \partial_\mu \rho
\]

\[
+ \frac{1}{4} \left( \partial_\mu \pi \cdot \partial^\mu \pi - m_\pi^2 \pi^2 \right),
\]

with

\[
W_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu,
\]

\[
R_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu,
\]

where the meson masses are chosen to be \( m_\sigma = 550 \text{ MeV} \), \( m_\omega = 783 \text{ MeV} \), \( m_\pi = 138 \text{ MeV} \), and \( m_\rho = 770 \text{ MeV} \) (Pal et al. 1999).

The interaction Lagrangian is given by

\[
L_{\text{int}} = \sum_B \bar{\psi}_B \left[ g_{\sigma B}(\sigma) \sigma - g_{\rho B} \gamma_\mu \omega^\mu + \frac{f_{\rho B}}{2 M} \sigma^\mu \partial_\mu \omega^\nu \omega^\nu
\]

\[
- g_{\rho B} \gamma_\mu \rho^\mu \cdot I_B + \frac{f_{\rho B}}{2 M} \sigma^\mu \partial_\mu \rho^\nu \cdot I_B
\]

\[
- \frac{f_{\pi B}}{m_\pi} \gamma_5 \gamma_\mu \partial_\mu \pi \cdot I_B \right] \psi_B,
\]

where the common scale mass \( M \) is taken to be the free nucleon mass (Rijken et al. 2010) and the commutation operator for iso-singlet baryons. While \( f_{\pi B} \) and \( f_{\rho B} \) are the isoscalar-tensor and isovector-tensor coupling constants, the terms with \( g_{\sigma B} \), \( g_{\rho B} \), and \( f_{\pi B} \) correspond to \( \sigma \), \( \omega \), \( \rho \), and \( \pi \) couplings, respectively. We set \( g_{\sigma B} \), \( g_{\rho B} \), \( f_{\rho B} \), and \( f_{\pi B} \) to be constants. Nevertheless, it is not the case for \( g_{\sigma B} \), as is explained later. In the RMF approximation, the meson field values are replaced by the constant mean-field values: \( \bar{\sigma} \), \( \bar{\omega} \), and \( \bar{\rho} \) (the \( \rho^0 \) field).

In the CQMC model, we assume that the baryon structure variation in matter is reflected in the \( \sigma \)-field dependence of the isoscalar field (Guichon 1988; Saito & Thomas 1994). For simplicity, we parameterize it in the linear form (Tsushima et al. 1998)

\[
g_{\sigma B}(\bar{\sigma}) = g_{\sigma B} b_B \left[ 1 - \frac{a_B}{2} g_{\sigma N} \bar{\sigma} \right],
\]

where \( g_{\sigma N} \) is the \( \sigma \)-\( N \) coupling constant at zero density and \( a_B \) and \( b_B \) are parameters listed in Table 1 (Miyatsu et al. 2012). The parameter \( b_B \) represents the effect of the quark–quark hyperfine interaction due to the exchanges of gluon and pion. In the case of \( a_B = 0 \) and \( b_B = 1 \), the CQMC model is identical to the ordinary RMF model (Serot & Walecka 1986; Bouyssy et al. 1987). Note that the several non-linear self-interaction terms of the \( \sigma \) (or vector \( \omega \), \( \rho \)) mesons (Sugahara & Toki 1994; Lalazissis et al. 1997; Todt-Rutel & Pietrzwicz 2005; Fattoyev et al. 2010) are included to soften the EOS and reproduce experimental value for the incompressibility. In contrast, there is no need to introduce the additional meson terms using the CQMC model. Therefore the EOS is determined by \( g_{\sigma B} \), \( g_{\rho B} \), \( f_{\rho B} \), \( f_{\pi B} \), and \( f_{\pi B} \). We investigate the setting for the coupling constants and its validity in Section 3.

In the following, we derive the total energy density of hadronic matter in the RHF approximation. In order to sum up all orders of the tadpole (Hartree) and exchange (Fock) diagrams in the baryon Green’s function, \( G_B \), we use the Dyson’s equation

\[
G_B(k) = G^0_B(k) + G^0_B(k) \Sigma_B(k) G_B(k),
\]
with the four momentum of baryon $k^\mu$, the baryon self-energy $\Sigma_B^\mu$, and the Green’s function in free space $G^0_B$. The baryon self-energy in matter is generally given by

$$\Sigma_B(k) = \Sigma_B^0(k) - \gamma_0 \Sigma_B^\mu(k) + (\gamma \cdot \hat{k}) \Sigma_B^\nu(k),$$

(9)

where $\hat{k}$ is the unit vector along the (three) momentum $k$ and $\Sigma_B^\mu$, $\Sigma_B^0$, and $\Sigma_B^\nu$ are the scalar part, the time component of the vector, and the space component of the vector of the self-energy, respectively. Here we define the effective baryon mass, the vector, and the space component of the vector of the self-energy in matter as (Serot & Walecka 1986; Bouyssy et al. 1987)

$$M_B^\nu(k) = M_B + \Sigma_B^\nu(k),$$

(10)

$$k_B^\mu = (k_B^0, k_B),$$

(11)

$$E_B^\nu(k) = \left[ k_B^2 + M_B^2(k) \right]^{1/2}.$$

(12)

The baryon self-energies in Equation (9) are calculated introducing a form factor and their expressions can be seen in Katayama et al. (2012). Using them, the total baryon number density $n$ and the total energy density of hadronic matter $\varepsilon_H$ are expressed as

$$n = \sum_B \frac{k_{FB}^2}{3\pi^2},$$

(13)

$$\varepsilon_H = \sum_B \frac{1}{\pi^2} \int_0^{k_{FB}} k^2 dk \left[ T_B(k) + \frac{1}{2} V_B(k) \right],$$

(14)

with

$$T_B(k) = \frac{M_B M_B^\nu(k) + kk_B^\nu}{E_B^\nu(k)},$$

(15)

$$V_B(k) = \frac{M_B^\nu(k) \Sigma_B^\nu(k) + k_B^2 \Sigma_B^\mu(k)}{E_B^\nu(k)} - \Sigma_B^\nu(k),$$

(16)

where $k_{FB}$ is the Fermi momentum of baryon $B$.

### 3. COUPLING CONSTANTS

In this section, we study the coupling constants shown in the previous section. First of all, among the coupling constants, whichever are not related with hyperons are determined so as to reproduce the gross feature of nuclear mass data with Thomas–Fermi calculation. Then we examine the saturation properties of the resultant EOS. As a next step, we study the coupling constants related to hyperons from the recent analysis of hypernuclei and hyperon production reactions.

#### 3.1. Thomas–Fermi Calculations for Atomic Nuclei

In the RHF approximation, the EOS without hyperons is determined by $g_{NN}$, $g_{NN}$, $g_{NN}$, $f_{NN}$, $f_{NN}$, and $f_{NN}$. We set them to reproduce the experimental data on atomic nuclei. The saturation properties of symmetric nuclear matter are mainly described by the $\sigma$–$N$ and $\omega$–$N$ coupling constants, $g_{NN}$ and $g_{NN}$. The $\rho$–$N$ coupling constant $g_{NN}$ is related to the symmetry energy. Instead of optimizing all the parameters, as for the isoscalar- and isovector-tensor coupling constants, we fix the fractions of $f_{NN}$ to $g_{NN}$ and $f_{NN}$ to $g_{NN}$, for simplicity. According to the Nijmegen extended-soft-core (ESC) model (Rijken et al. 2010) based on the $NN$- and $NN$-scattering data, the fractions are $f_{NN} / g_{NN} = -0.8070 / 3.5452 = -0.2276$ and $f_{NN} / g_{NN} = 3.9298 / 0.6918 = 5.6805$. As for $f_{NN}$, we use the value suggested by the ESC model. Hereafter, we adopt these values and $g_{NN}$, $g_{NN}$, and $g_{NN}$ are determined. Note that in the case of the RHF calculation, not only the symmetry energy but also the saturation energy are sensitive to $f_{NN}$ (Bouyssy et al. 1987). In contrast, the contribution of $f_{NN}$ is tiny near the saturation density.

In the optimization, the nuclear masses evaluated from our EOS are compared with the experimental mass data. Within the framework of a simplified version of the extended Thomas–Fermi theory (Oyamatsu & Iida 2003), the binding energy $B(Z, N)$ of a nucleus with the proton number $Z$ and neutron number $N$ is given by

$$- B(Z, N) = \int \epsilon_H (n_p(r), n_p(r)) d^3r + F_0 \int |\nabla n(\mathbf{r})|^2 d^3r + \epsilon^2 \int \frac{n_p(r)n_p(r')}{|\mathbf{r} - \mathbf{r}'|} d^3r d^3r'$,$$

(17)

where $n_p(r)$ and $n_p(r')$ are neutron and proton number densities, respectively, and they satisfy their number conservation laws:

$$N = \int n_p(r)d^3r,$$

(18a)

$$Z = \int n_p(r)d^3r,$$

(18b)

and $n(r)$ is a total nucleon number density defined as $n(r) = n_p(r) + n_n(r)$. The first term of the right-hand side of Equation (17) is a bulk term and $\varepsilon_H$ is an energy density of the uniform nuclear matter, which is given by the Hartree–Fock calculation as a function of $n_p$ and $n_n$. The second term corresponds to the gradient energy due to the density inhomogeneity and $F_0$ is the gradient coefficient. While one can treat $F_0$ as an adjustable parameter, we here choose $F_0 = 68$ MeV fm$^3$ from previous studies (Oyamatsu & Iida 2003; Kanzawa et al. 2009). It is confirmed that the result shown below is not very sensitive to this choice. The third term is Coulomb energy and $e$ is the elementary electric charge. The goal is to determine the density distributions $n_p(r)$ and $n_p(r)$, which maximize $B(Z, N)$ under the constraints (18a) and (18b) for given $N$ and $Z$.

For simplicity, we assume spherical nuclei and the nucleon distributions $n_i(r)$ $(i = n, p)$ to be (Oyamatsu & Iida 2003)

$$n_i(r) = \begin{cases} n_i^0 \left[ 1 - \left( \frac{r}{R_i} \right)^{3/2} \right]^3, & r < R_i, \\ 0, & r \geq R_i, \end{cases}$$

(19)

where $r$ is the distance from the center of the nucleus. In this expression, $R_i$, $t_i$, and $n_i^0$ $(i = n, p)$ are adjustable parameters. Here, $R_i$ roughly represents the nucleon radius, $t_i$ corresponds to the relative surface diffuseness, and $n_i^0$ is the central nucleon number density. These six parameters are computed for each give nucleus, e.g., $^{208}$Pb $(Z = 82, N = 126)$, by the maximization of $B(Z, N)$.

Here, we determine $g_{NN}$, $g_{NN}$, and $g_{NN}$ so that the binding energies evaluated for 2226 nuclei in the above Thomas–Fermi calculations reproduce the gross feature of their nuclear mass data (Audi et al. 2003). Moreover, in this study, we set the saturation energy to $\varepsilon_0 = -16.1$ MeV according to the previous
past studies (Oyamatsu & Iida 2003; Kanzawa et al. 2009), for the rms deviation is 2.93 MeV, which is comparable with be the optimal set. As a result, we find that the minimum value deviation of the calculated masses from the experimental data to also rms charge radii (Oyamatsu & Iida 2003; Kanzawa et al. 2009). Therefore, we regard the combination of A

\[ g_{\sigma N} / \sqrt{4\pi} \]

and the resultant coupling constants are shown in Table 2 with those of KMS12. In Figure 1, the differences between the masses from each other while, as shown later, the difference of their our case, the neutron drip lines of this study and KMS12 differ

\[ \Delta \text{mass} = M_{TF} - M_{exp} \]

for KMS12 may not follow the trend of

\[ L \]

is not so large. The neutron drip line for KMS12 is different from 84.8 MeV, the result shown as case (d) in Table 8 of Katayama et al. (2012), because the definitions are different. We define the slope of the symmetry energy L by Equation (21), while KMS12 defined it by the density derivative of \( E_{\text{sym}}(n) \). With their definition, the model in this study gives \( L = 75.8 \text{ MeV} \).

Figure 1. Mass deviation \( \Delta M = M_{TF} - M_{\text{exp}} \) of 2226 nuclei (Audi et al. 2003), where \( M_{TF} \) is the mass by the Thomas–Fermi calculation and \( M_{\text{exp}} \) is the experimental data, for (a) this study and (b) KMS12. The horizontal axes represent the mass number A.

Thomas–Fermi studies with nuclear data of not only masses but also rms charge radii (Oyamatsu & Iida 2003; Kanzawa et al. 2009). Therefore, we regard the combination of \( g_{\sigma N} \), \( g_{\omega N} \), and \( g_{\rho N} \), which satisfies \( w_0 = -16.1 \text{ MeV} \) and minimizes the rms deviation of the calculated masses from the experimental data to be the optimal set. As a result, we find that the minimum value for the rms deviation is 2.93 MeV, which is comparable with previous studies (Oyamatsu & Iida 2003; Kanzawa et al. 2009), and the resultant coupling constants are shown in Table 2 with those of KMS12. In Figure 1, the differences between the masses by Thomas–Fermi calculations and the experimental data are plotted. We can see that the Thomas–Fermi calculations with the optimal coupling constants in this study well reproduce the gross feature of nuclear mass except shell effects. On the other hand, the rms deviation given by the Thomas–Fermi calculations with the coupling constants in KMS12 is 36.33 MeV and the mass difference becomes roughly larger as the mass number A increases.

The \( \beta \)-stability line and neutron drip line obtained from the Thomas–Fermi calculations with the EOS of this study are shown in Figure 2. We can see that the resultant \( \beta \)-stability line well traces empirically known stable nuclei (Tachibana et al. 2010). As for the neutron drip line, it is confirmed that our result is consistent with the sophisticated atomic mass formula constructed by Koura et al. (2005). We also show the results for the EOS of KMS12. Recently, Oyamatsu et al. (2010) pointed out that the neutron drip line depends on the slope of the symmetry energy, \( L \), using EOSs whose Thomas–Fermi calculations reproduce the gross feature of nuclear mass data. In our case, the neutron drip lines of this study and KMS12 differ from each other while, as shown later, the difference of their \( L \) is not so large. The neutron drip line for KMS12 may not follow the trend of \( L \) because KMS12 is not consistent with mass data. Therefore, the calibration by mass data is important.

### 3.2. Saturation Properties

Some of the quantities shown in Table 2 are key parameters for characterizing the saturation properties of uniform bulk nuclear matter. They correspond to the coefficients of the power-series expansions of the energy per baryon \( \varepsilon_H(n_n, n_p)/n \). The energy per baryon of symmetric nuclear matter is written as

\[ \varepsilon_H(n/2, n/2) / n = w_0 + \frac{K_0}{18n_0^2}(n - n_0)^2 + \mathcal{O}((n - n_0)^3), \tag{20} \]

and that of neutron matter is written as

\[ \varepsilon_H(n, 0) / n = \frac{E_H(n/2, n/2)}{n} + S_0 + \frac{L}{3n_0^2}(n - n_0) + \mathcal{O}((n - n_0)^2). \tag{21} \]

Both this study and KMS12 provide reasonable values for the saturation density \( n_0 \) and saturation energy \( w_0 \). The

### Table 2

| EOS       | \( g_{\sigma N} / \sqrt{4\pi} \) | \( g_{\omega N} / \sqrt{4\pi} \) | \( g_{\rho N} / \sqrt{4\pi} \) | \( w_0 \) (MeV) | \( n_0 \) (fm\(^{-3}\)) | \( S_0 \) (MeV) | \( E_{\text{sym},0} \) (MeV) | \( K_0 \) (MeV) | \( L \) (MeV) |
|-----------|---------------------------------|---------------------------------|---------------------------------|---------------|-----------------|----------------|----------------|----------------|----------------|
| this study | 1.94                            | 2.39                            | 0.596                           | -16.1         | 0.155           | 33.6           | 32.7           | 274            | 77.1           |
| KMS12      | 1.81                            | 2.42                            | 0.692                           | -15.7         | 0.150           | 36.5           | 35.5           | 261            | 86.0           |

Notes. The value of \( L \) for KMS12 is different from 84.8 MeV, the result shown as case (d) in Table 8 of Katayama et al. (2012), because the definitions are different. We define the slope of the symmetry energy \( L \) by Equation (21), while KMS12 defined it by the density derivative of \( E_{\text{sym}}(n) \). With their definition, the model in this study gives \( L = 75.8 \text{ MeV} \).
incompressibility $K_0$ is related to the stiffness of the EOS. While the range of $K_0 = 240 \pm 10$ MeV is derived from the isoscalar giant monopole resonance (Piekarewicz 2010), we think that the value of $K_0$ in this study as well as KMS12 is still reasonable.

The symmetry energy $S_0$ is the difference between the energies per baryon of symmetric nuclear matter and neutron matter at the saturation density. Incidentally, some authors define the symmetry energy $E_{\text{sym}}(n)$ as

$$\frac{\varepsilon_H(n_\sigma, n_\rho)}{n} = \frac{\varepsilon_H(n/2, n/2)}{n} + E_{\text{sym}}(n) \delta^2 + O(\delta^4),$$

(22)

where $\delta$ is a neutron–proton asymmetry defined as $\delta \equiv (n_\sigma - n_\rho)/n$, and use the value at the saturation density, $E_{\text{sym},0} \equiv E_{\text{sym}}(n_0)$. Generally, $E_{\text{sym},0}$ differs from $S_0$ and both values are shown in Table 2. Recently, the constraint $30 \text{ MeV} \lesssim E_{\text{sym},0} \lesssim 34 \text{ MeV}$ is suggested by intermediate-energy heavy-ion collisions (Tsang et al. 2009) and other works also predict values around $E_{\text{sym},0} \approx 32 \text{ MeV}$ (Li et al. 2013). The symmetry energy of this study is consistent with the current implications, whereas that of KMS12 is somewhat higher.

In Equation (21), the parameter $L$ gives the slope of the symmetry energy. It is known that the possible range of $L$ depends on $S_0$ and nuclear models with large values of $S_0$ tend to predict large values of $L$. In particular, a relation $S_0 \approx 28 \text{ MeV} + 0.075 L$ is given by the systematic Thomas–Fermi calculations (Oyamatsu & Iida 2003). The value of $L$ in this study satisfies these conditions. According to the recent analyses of terrestrial experiments, the constraints on $L$ favor a somewhat smaller value (Li et al. 2013). However, they have yet to converge and our result is not far from them. Note that some model dependences remain in extracting the above constraints for the key parameters, and further studies are important.

### 3.3. Hyperon Potentials in Symmetric Nuclear Matter

We determine the coupling constants of hyperons in this subsection. The couplings with the $\sigma$ meson are determined from the potential depths of hyperons in symmetric nuclear matter at the saturation density $n_0$ (Schaffner-Bielich & Mishustin 1996; Ishizuka et al. 2008). According to experimental results on the single particle energies of many $\Lambda$ hypernuclei, the potential depth of $\Lambda$ in nucleons ($N$) is well estimated as $U^{(N)}_\Lambda \simeq -30 \text{ MeV}$. On the other hand, $\Sigma$ hyperons are thought to feel repulsive potential in nuclear matter as $U^{(N)}_\Sigma \sim +30 \text{ MeV}$ from the recently observed quasi-free $\Sigma$ production spectra (Harada & Hirabayashi 2005, 2006). It is suggested that a potential depth of $\Xi$ is around $U^{(N)}_\Xi \sim -15 \text{ MeV}$ by analyses of the twin hypernuclear formation (Aoki et al. 1995) and $\Xi$ production spectra (Khaustov 2000).

In this study, we set $(U^{(N)}_\Lambda, U^{(N)}_\Sigma, U^{(N)}_\Xi) = (-30 \text{ MeV}, +30 \text{ MeV}, -15 \text{ MeV})$. The potential depth of hyperons in nuclear matter is written as (Jaminon et al. 1981)

$$U^{(N)}_Y = -g_{\sigma Y}(\delta^{(N)} \sigma^{(N)} + \bar{\omega}^{(N)} \bar{\omega}^{(N)}) + \frac{1}{2M_Y} \left[ -g_{\sigma Y}(\delta^{(N)} \sigma^{(N)} + \bar{\omega}^{(N)} \bar{\omega}^{(N)}) \right]^2,$$

(23)

where $\Lambda, \Sigma$, and $\Xi$ are denoted collectively as $Y$. Meanwhile, $\delta^{(N)}$ and $\bar{\omega}^{(N)}$ are the mean-field values of $\sigma$ and $\omega$ mesons in the symmetric nuclear matter, respectively. So as to fit the potential depth for each hyperon in symmetric nuclear matter at $n_0$, we determine $g_{\sigma \Lambda}, g_{\sigma \Sigma},$ and $g_{\sigma \Xi}$. For this, as in the ESC model

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$B$ & $N$ & $\Lambda$ & $\Sigma$ & $\Xi$ \\
\hline
$g_{\sigma N}/\sqrt{4\pi}$ & 1.94 & 2.15 & 1.67 & 1.50 \\
$g_{\omega N}/\sqrt{4\pi}$ & 2.39 & 2.82 & 2.82 & 2.09 \\
$f_{\rho N}/\sqrt{4\pi}$ & -0.545 & -3.39 & -0.261 & -4.40 \\
$g_{\rho N}/\sqrt{4\pi}$ & 0.596 & 0.171 & 0.596 & \\
$f_{\rho N}/\sqrt{4\pi}$ & 3.39 & 2.94 & -0.446 & \\
$f_{\omega N}/\sqrt{4\pi}$ & 0.268 & 0.190 & -0.0772 & \\
\hline
\end{tabular}
\caption{Coupling Constants used in the Reference Model of This Study}
\end{table}

(Rijken et al. 2010), the $\rho – Y$ couplings $g_{\rho Y}$ are determined through the SU(6) spin-flavor relations:

$$g_{\rho N} = \frac{1}{2} g_{\rho \Xi} = g_{\rho \Sigma}, \quad g_{\rho \Lambda} = 0,$$

(24)

where the $\rho – N$ coupling $g_{\rho N}$ is fixed in Section 3.1. The tensor couplings of the isovector meson $f_{\rho Y}$ are determined again from the fractions of $f_{\rho Y}$ to $g_{\rho Y}$ ($f_{\rho Y}/g_{\rho Y}$) in the ESC model. For the other coupling constants ($g_{\sigma \sigma}, f_{\sigma \rho}$, and $f_{\omega \sigma}$), we use values in the ESC model for our reference model. Nevertheless, there may be an ambiguity in the $\omega – Y$ coupling constants, and we investigate other values of $g_{\omega \sigma}$ and $f_{\omega \sigma}$. The resultant coupling constants of the reference model are summarized in Table 3.

### 4. APPLICATION TO NEUTRON STARS

In this section, we apply our model to neutron stars. The inside of a neutron star is divided into two parts, a crust and a core. In the crust, nucleons distribute nonuniformly while matter is uniform in the core. First, we study the neutron star crust with the model described in the former sections to determine the nucleon distributions. Next, we investigate the composition and structure of neutron stars. Here we discuss the impacts of not only the core EOS but also the crust EOS.

### 4.1. Neutron Star Crust

In the neutron star crust, the lattice of proton clusters is structured and the bcc lattice is preferred to minimize the Coulomb energy (Oyamatsu et al. 1984). Neutrons accompany the proton distribution and form “nuclei.” In a high-density regime, some of the neutrons drip out of the “nuclei.” The region where the neutron drip occurs is called the inner crust. The low-density region without the neutron drip is called the outer crust. Electrons exist so as to achieve charge neutrality and $\beta$-equilibrium, and distribute almost uniformly due to their high Fermi energy.

In this paper, we study the nucleon distribution in the crust-extending Thomas–Fermi model described in Section 3.1 (Oyamatsu 1993; Shen et al. 1998a; Kanzawa et al. 2009). We treat the bcc lattice approximately with a spherical Wigner–Seitz cell. Each unit cell has the same volume $a^3$ and we refer to $a$ as the lattice constant. The total energy in the cell is written as

$$W = W_N + W_e + W_{\text{Coul}},$$

(25)

The first term $W_N$ is the nuclear energy and is expressed, as in Equation (17), in the density functional form:

$$W_N = \int_{\text{cell}} \{ \varepsilon_H(n_\sigma(r), n_\rho(r)) + \delta M_n n_n(r) \}
+ M_p n_p(r) + F_0 |\nabla n(r)|^2 \, d^3r,$$

(26)

where $M_n$ and $M_p$ are the neutron mass and proton mass, respectively. Note that the terms $M_n n_n(r)$ and $M_p n_p(r)$ correspond to...
the rest mass energy of nucleons. The second term $W_e$ is the electron energy. In this paper, we regard electrons to be uniform relativistic Fermi gas and $W_e$ is calculated as

$$\frac{m_e^3 a^3}{8 \pi^2} \left[ x_e (2x_e^2+1)(x_e^2+1)^{1/2} - \ln \left[ x_e + (x_e^2+1)^{1/2} \right] \right],$$

with

$$x_e = \frac{1}{m_e} \left( \frac{3 \pi^2 n_e}{} \right)^{1/3},$$

where $m_e$ and $n_e$ are the electron mass and electron number density, respectively. Note that $n_e$ is determined so as to satisfy the charge neutrality condition,

$$a^3 n_e = Z = \int_{\text{cell}} n_p(r) d^3r,$$

with the proton number in the cell, $Z$. The last term $W_{\text{Coul}}$ is the Coulomb energy in the Wigner–Seitz cell and is written as

$$W_{\text{Coul}} = \frac{e^2}{2} \int \int \frac{[n_p(r) - n_e] [n_p(r') - n_e]}{|r - r'|} d^3r d^3r' + c_{\text{bcc}} \frac{(Ze)^2}{a},$$

where the second term on the right-hand side of Equation (30) is the correction for the bcc lattice with $c_{\text{bcc}} = 0.006562$ (Oyamatsu 1993).

For nucleon distributions in the Wigner–Seitz cell, we assume spherical symmetry and again utilize the parameterization:

$$n_i(r) = \begin{cases} (n_i^{\text{in}} - n_i^{\text{out}}) \left[ 1 - \left( \frac{r}{R_i} \right)^3 \right] + n_i^{\text{out}}, & r < R_i, \\ n_i^{\text{out}}, & R_i \leq r \leq R_{\text{cell}}. \end{cases}$$

where $R_i$, $n_i^{\text{in}}$, and $n_i^{\text{out}}$ ($i = n, p$) are adjustable parameters. The radius of the Wigner–Seitz cell $R_{\text{cell}}$ is related to the lattice constant $a$ as $R_{\text{cell}} = (3/4\pi r)^{1/3} a$. Note that $n_i^{\text{out}}$ corresponds to the number density of dripped neutrons and we set $n_p^{\text{out}} = 0$. Here the ground state of the system at a given (average) baryon number density,

$$n_B = \frac{1}{a^3} \int_{\text{cell}} n(r) d^3r,$$

is determined by minimizing the total energy density $\epsilon = W/a^3$. When the minimized total energy density is higher than that of uniform matter at the same $n_B$, we regard the uniform matter as the ground state.

We show the resultant nucleon distributions in Figure 3. The distance between the nearest nuclei becomes smaller with increasing density. Above the critical density $n_{\text{drip}}$, neutron drip occurs. For our model, it is evaluated to be $n_{\text{drip}} = 2.70 \times 10^{-4}$ fm$^{-3}$, which corresponds to $4.48 \times 10^{11}$ g cm$^{-3}$ in baryon mass density. At a higher density, nuclei melt and the phase transition from nonuniform matter to uniform matter takes place. The transition density is $n_{\text{uni}} = 6.85 \times 10^{-2}$ fm$^{-3}$ (1.14 \times 10^{14}$ g cm$^{-3}$ in baryon mass density) for our model. Note that for a region with somewhat lower density than $n_{\text{uni}}$, nuclei are thought to deform to rod-like and slab-like shapes, which are often called nuclear pasta (Ravenhall et al. 1983; Hashimoto et al. 1984). The pasta nuclei may further reduce the energy density. While we do not take into account pasta nuclei, they would not significantly change the EOS of the matter. Moreover, since the slope of the symmetry energy is somewhat high, $L = 77.1$ MeV, for our model, the density region containing pasta nuclei would not be so large (Oyamatsu & Iida 2007).

In Figure 4, we show the results of our Thomas–Fermi calculations for the proton number $Z$ determined by Equation (29) and the average proton fraction $Y_p$ given by

$$Y_p = \frac{\int_{\text{cell}} n_p(r) d^3r}{\int_{\text{cell}} n(r) d^3r}.$$
for the inner crust region, the value of $Z$ strongly depends on the slope of the symmetry energy, $L$. The discrepancy between BBP and our model would be within this uncertainty. Incidentally, the frequency of quasi-periodic oscillations discovered in the decaying tail of the giant flares, which are bursty gamma-ray emissions from neutron stars with strong magnetic fields, is sensitive to $Z$. Recently, Sotani et al. (2012) pointed out that low-$Z$ models, or EOSs with $L \gtrsim 50$ MeV, would be preferred to account for the observed frequency from SGR 1806-20. Note that the results for $Y_p$ are again similar between BBP and this study.

### 4.2. Composition and Structure of Neutron Stars

Nuclei melt into the uniform matter of neutrons and protons in the neutron star core. In this region reside not only electrons but also muons, achieving charge neutrality and $\beta$-equilibrium under weak processes. Here we treat them as non-interacting fermions. Furthermore, hyperons are also generated at higher densities. In Figure 5, we show the particle fractions, $Y_i = n_i/n_B$, from the crust region to the core region as functions of the total baryon number density, $n_B$, where $n_i$ represents the number density of particle $i$. As already stated, there are dripped neutrons in the inner crust. Note that the fraction of nuclei (A) increases artificially near the transition density to the uniform matter, $n_n$, because the radius of nuclei, $R_n$, gets closer to that of the Wigner–Seitz cell, $R_{\text{cell}}$, in Equation (31). Not the spherical nuclei assumed here but the pasta nuclei may reside in this region.

We show the particle fractions of the uniform neutron star matter in Figure 6. In the RHF calculation of this study, since the Fock contribution enhances the repulsive effect mainly through $\omega$ mesons at supra-nuclear densities, the hyperon creation is suppressed. Therefore the threshold densities of hyperons rise. In contrast, the creation of $\Xi^-$ is promoted by the inclusion of the tensor coupling with $\rho$ mesons (Rikovska-Stone et al. 2007; Whittenbury et al. 2012). In fact, the critical density of $\Xi^-$ creation in this study is higher than KMS12. Since the value of $f_{\rho \Xi}/R_{\rho N}$ is fixed and the coupling relation in Equation (24) is assumed both in our model and KMS12, the value of $f_{\rho \Xi}/R_{\rho N}$ is kept. Therefore, our model has a smaller absolute value of $f_{\rho \Xi}$ compared to KMS12 (see also Table 2), which is consistent with the difference in the critical density between the two models. Note that $\Lambda$ hyperons do not interact with $\rho$ mesons due to $I_\Lambda = 0$ in Equation (6). In addition, we assume that the $\Sigma$ hyperons feel a repulsive interaction at high densities and, hence, they cannot appear. As a result, among the hyperons taken into account, only $\Xi^-$ appears and the others are not produced below 1.2 fm$^{-3}$.

In Figure 7, we show the EOS for neutron star matter from the crust region to the core region. The pressure is given as

$$P = n_B^2 \frac{\partial}{\partial n_B} \left( \frac{\varepsilon}{n_B} \right).$$

In this study, the boundary between the crust and the core is determined by self-consistently comparing the energy density.
of nonuniform matter with that of uniform matter. Therefore, our EOS continues smoothly from the crust to the core. In contrast, the previous EOS of KMS12 adopts the BPS model for the outer crust and the BBP model for the inner crust. In the inner crust region, the size of nuclei is quite different between our model and the BBP model as shown in Figure 4, and it is reflected in the EOS. On the other hand, in the core region, our model is stiffer than the EOS of KMS12 while they are barely distinguishable in the figure. This is consistent with the fact that, as shown in Table 2, our EOS has larger incompressibility $K_0$ than the EOS of KMS12. In the higher density region, we can see the softening of the EOS due to the mixture of $\Xi$ hyperons. Note that our EOS satisfies the causality even at high densities because it is based on a relativistic framework. Incidentally EOSs constructed by nonrelativistic theories (e.g., Kanzawa et al. 2007, 2009) sometimes cause a causality violation, which corresponds to the region above the line of $P = \varepsilon$ in Figure 7.

In Figure 8, the neutron star radii $R$ are shown as functions of masses $M$. The result of KMS12 is drawn for comparison, as well as that of the model in which the core EOS is the same as our model, and the BPS and BBP models are adopted as the crust EOS. It is known that the inclusion of hyperons generally reduces the maximum mass of neutron stars drastically (Glendenning & Moszkowski 1991; Schaffner-Bielich 2008). Nevertheless, the Fock contribution prevents the hyperon appearance in our model, and the resultant maximum mass is high as $1.95M_\odot$. This value is within the mass range of the recently observed massive neutron star J1614-2230, $1.97 \pm 0.04M_\odot$ (Demorest et al. 2010).\(^3\) Incidentally, the maximum mass of our model is higher than that of KMS12 ($1.93M_\odot$) for two reasons. The first one is because the critical density of $\Xi^-$ creation is higher for our model. The second reason, as already stated, is that our EOS is stiffer than the EOS of KMS12. In our model, $\Xi^-$ hyperons are included in the core of neutron stars more massive than $1.77M_\odot$, and the maximum-mass neutron star has a baryon number density of $1.01\text{ fm}^{-3} (1.67 \times 10^{15} \text{ g cm}^{-3} \text{ in baryon mass density})$ at the center.

\(^3\) After the first submission of this paper, Antoniadis et al. (2013) reported that the mass of PSR J0348+0432 was evaluated as $2.01 \pm 0.04M_\odot$ in the 1σ limit. Their analysis depends on the model of the companion white dwarf and further investigation is important.
In our model, the radii of neutron stars are around 12–13 km for typical values of the neutron star mass. The difference between our EOS and that of KMS12 comes mainly from the core region. On the other hand, as seen in Figure 8, the crust EOS also affects the radius at about 1%. When a neutron star radius is observationally determined at high precision, we may be able to study not only the core EOS but also the crust EOS. Furthermore, for the crust thickness $\Delta R_{\text{crust}}$, the impact of the crust EOS is comparable to that of the core EOS. To show this more clearly, we show $\Delta R_{\text{crust}}$ as functions of neutron star masses in Figure 10. The variation is about 10% among the different EOS models. Incidentally, the pulsar glitches are thought to associate with the superfluidity of dripped neutrons in the inner crust and the average rate of spin-reversal due to glitches depends on the moment of inertia of superfluid neutrons (Anderson & Itoh 1975; Link et al. 1999; Andersson et al. 2012). To determine the neutron star structure and composition precisely, the crust EOS as well as the core EOS is important.

5. SUMMARY

We have constructed the EOS for neutron star matter at zero temperature including nuclei in the crust and hyperons in the core. For the EOS of uniform nuclear matter, the framework of the CQMC model based on the RMF theory, which was proposed in Katayama et al. (2012), has been adopted. In this study, the $\sigma$–$N$, $\omega$–$N$, and $\rho$–$N$ couplings have been determined so as to reproduce the gross feature of nuclear mass data with the Thomas–Fermi calculation. The $\sigma$–$Y$ couplings have been determined to fit the potential depth in symmetric nuclear matter from the recent analyses of hypernuclei and hyperon production reactions. The $\omega$–$Y$ and $\pi$–$B$ couplings have been taken from the ESC model based on the hyperon–nucleon scattering data. The $\rho$–$Y$ couplings have been determined through the SU(6) spin-flavor relations. The tensor couplings have been determined from scaling the ESC model. Due to the calibration from the mass data, the resultant symmetric energy is consistent with the current implications.

To construct the neutron star EOS, we have dealt with not only high-density uniform matter in the core but also low-density nonuniform matter in the crust within the Thomas–Fermi approximation. In our method, the neutron drip and transition to matter undescription are made self-consistently. Although all octet baryons are taken into account in the core region, only $\Xi^-$ appears and the other hyperons are not generated below 1.2 fm$^{-3}$. The resultant maximum mass of neutron stars is 1.95 $M_\odot$, which is consistent with the mass range of the recently observed massive neutron star J1614-2230, 1.97 $\pm$ 0.04 $M_\odot$ (Demorest et al. 2010). In a future work, we will extend this study to finite temperatures for use in astrophysics.

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APPENDIX

EOS TABLE

The results for the reference model in this study are publicly available on the Web at http://asphwww.ph.noda.tus.ac.jp/yn/. This data set is open for general use in any numerical simulations for astrophysics. The EOS table and the data on particle fractions and properties of nuclei in the crust region are provided.

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