Nonlinear treatment of many-body effects in double-barrier resonant tunneling structures

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We introduce a simple, solvable model of double-barrier resonant tunneling structure which includes the effects of electron-electron and electron-phonon scattering. The model is based on a generalized effective-mass equation where a nonlinear coupling is introduced to account for those inelastic scattering phenomena. The nonlinear term depends on one parameter which is the only constant in the model to be determined phenomenologically from experimental data. As an example of the application of the model, we discuss GaAs-Ga$_{1-x}$Al$_x$As double-barrier structures. When the nonlinear term is considered, a sideband is observed at an energy of the order of the LO phonon energy in GaAs in addition to and below the main transmission peak. Fitting of the computed spectrum to known experimental results allows to find the value of the nonlinear coupling. As a consequence, we can estimate the electron-phonon coupling from the shift of the main transmission peak in comparison to the linear resonant tunneling. We obtain good agreement with values coming from more sophisticated treatments, which supports our claim that the nonlinear term consistently includes electron-phonon effects. Finally, we use our simple model to study magnitudes of interest for applications, namely $I - V$ characteristics, which are shown to present two negative differential resistance peaks arising from the main transmission peak and the sideband.

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I. INTRODUCTION

Resonant tunneling (RT) through double-barrier structures (DBS) make these systems very promising candidates for a new generation of ultra-high speed electronic devices. For instance, a GaAs-Ga$_{1-x}$Al$_x$As DBS operating at THz frequencies has been reported in the literature. The basic reason for RT to arise in DBS is a quantum phenomenon whose fundamental characteristics are by now well understood: There exists a dramatic increase of the electron transmittivity whenever the energy of the incident electron is close to one of the unoccupied quasi-bound-states inside the well. In practice, a bias voltage is applied to shift the energy of this quasi-bound-state of nonzero width so that its center matches the Fermi level. Consequently, the $I - V$ characteristics present negative differential resistance (NDR).

In actual samples, however, the situation is much more complex than this simple picture. This is so even in good-quality heterostructures, when scattering by dislocations or surface roughness is negligible. In particular, inelastic scattering is always present in real devices. Examples of inelastic scattering events are electron-phonon and electron-electron interactions, in which the energy of the tunneling electron changes and the phase memory is lost. The influence of these many-body effects on DBS has recently attracted considerable attention. Several approaches have been carried out to model electronic-optical-phonon scattering in RT devices. One thing that is currently clear is that electron-phonon interaction shifts downwards the RT peak in energy by $\Delta E = g \hbar \omega_0$, where $g$ is the electron-phonon coupling and $\hbar \omega_0$ is the LO phonon energy. It has also been noted that sideband transmission peaks arise, which are readily observable in $I - V$ characteristics, and correspond to sequential rather than coherent tunneling. On the other hand, the effects of Coulomb repulsion in RT have also been considered in tight-binding models of quasi-one-dimensional systems. Results showed that the conductance exhibits several doublet structures depending upon the $U$ coupling in the on-site Coulomb interaction between two electrons. Also, each peak shifts upwards to higher energies as $U$ increases. Therefore, we may summarize the available knowledge by saying that those inelastic processes split the main RT peak and shift its energy downwards or upwards, depending on the particular scattering channel.

Even with its rather satisfactory degree of success, many-body calculations have difficulties that, in some cases, may complicate the interpretation of the underlying physical processes. For this reason, an alternative approach to inelastic scattering effects above discussed has recently emerged, based on nonlinear dynamics of single excitations in solids. Loosely speaking, this kind of treatment could be regarded as similar to Hartree-Fock and other self-consistent techniques, which substitute many-body interactions by a nonlinear effective potential. In this paper we present one of such a nonlinear model, intended to account for many-body effects in RT through DBS within the one-particle framework, by in-
cluding nonlinear interaction terms in the equation of motion. We want to make it clear from the beginning that we neither search for nor claim quantitative agreement between our one-particle model and existing experiments. We realize that this is a shortcut also found in more elaborated many-body theories and we cannot hope to cure this problem with a much simpler, phenomenological theory: It is obvious that if we try to describe electron-electron and electron-phonon interactions within the same one-particle framework, quantitative accuracy is no longer possible. On the contrary, what we do claim is that our model captures the essential physics of inelastic effects in RT in a very simple way. The virtue of such an approach is that it allows to gain insight on the features of DBS without the burden of intensive computations providing, in an inexpensive way, a qualitative picture of what is to be expected in particular devices. This is so because our model can be easily extended to other systems in spite of the fact that we here present it in the context of a specific DBS. Therefore, our model can give hints leading to the development of sophisticated, possibly many-body calculations with much better accuracy. In addition, the way we connect effective nonlinearity with the physical ingredients it tries to substitute may be helpful for other nonlinear models in condensed matter physics.

The paper is organized as follows. In Sec. II, we present our model, obtained by including a nonlinear coupling in a generalized effective-mass equation. We particularize it for a GaAs-Ga$_{1-x}$Al$_x$As but we insist that the model is quite general and applicable in different contexts. The nonlinear coupling can be regarded as an adjustable parameter to fit as well as possible the available experimental results. We discuss in detail the physics underlying our choice for the coupling. We sketch the exact solution of this phenomenological model, as well as the way to obtain the transmission coefficient as a function of the nonlinear couplings and the applied voltage $V$. For completeness, we include a brief discussion of the range of applicability of the equation and its connection with physical interpretations. Afterwards, Sec. III contains the main results and discussions of our analysis concerning electron transmission and $I − V$ characteristics. We find the correct value of the coefficient of the nonlinear coupling fitting available data and, as a function of it, we discuss the corresponding value of the electron-phonon coupling. We compute the $I − V$ characteristics of the device and discuss its main features, the most salient of them being the existence of two peaks of NDR. Finally, Sec. IV concludes the paper with a brief survey of the results, and some prospects on the application of the ideas we have discussed.

II. MODEL

A. Physical grounds and definitions

To describe our model we have chosen a typical system: A GaAs-Ga$_{1-x}$Al$_x$As DBS under an applied electric field. The thickness of the whole structure is $L$ and the thickness of the well is $d$. The barriers are assumed to be of the same thickness (symmetric case) but as will be evident below this is not a restriction of our approach. The structure is embedded in a highly doped material acting as contact, so that the electric field is applied only in the DBS. We focus on electron states close to the bandgap and thus we can neglect nonparabolicity effects hereafter. Then the one-band effective-mass framework is completely justified to get accurate results. For the sake of simplicity, we will further assume that the electron effective-mass $m^*$ is constant through the whole structure. This hypothesis is related to the fact that we are not interested in high quantitative accuracy, although we note that the spatial dependence of the effective mass can be taken into account if necessary.

Within this approach, the electron wave function is written as a sum of products of band-edge orbitals with slowly varying envelope-functions. Therefore the envelope-function $\psi(z)$ satisfies a generalized effective-mass equation (we use units such that energies are measured in effective Rydberg (Ry*) and lengths in effective Bohr radius (a*), being 1 Ry* = 5.5 meV and 1 a* = 100 Å in GaAs) given by

$$-\psi_{zz}(z) + [V(z)\chi_b(z) - eFz] \psi(z) = E \psi(z),$$

(1)

where $V(z)$ is the potential term which we discuss below, $F$ is the electric field, and $\chi_b(z)$ is the characteristic function of the barriers,

$$\chi_b(z) = \begin{cases} 1, & \text{if } 0 < z < (L-d)/2, \\ 1, & \text{if } (L+d)/2 < z < L, \\ 0, & \text{otherwise}. \end{cases}$$

(2)

We now specify our model by choosing what is the potential term $V(z)$. In order to do that, let us first consider the physics we are trying to represent with this term. The DBS can be regarded as an effective medium which reacts to the presence of the tunneling electron, leading to a feedback mechanism by which inelastic scattering processes change the RT characteristics of the device. It thus follows that $V(z)$ must contain nonlinear terms if it is to summarize the medium reaction which comes from the electron-electron and electron-phonon interaction. The simplest candidate to contain this feedback process is the charge density of the electron, which is proportional to $|\psi(z)|^2$. In our model, we neglect higher order contributions and postulate that the potential in Eq. (1) has the form

$$V(z) = V_b \left[1 + \tilde{\alpha}|\psi(z)|^2\right],$$

(3)

where $V_b$ is the conduction band-offset at the interfaces, and all the nonlinear physics is contained in the coefficient $\tilde{\alpha}$ which we discuss below.
There are two factors that configure the medium response to the tunneling electron. First, it goes without saying that there are repulsive electron-electron Coulomb interactions, which should enter the effective potential with a positive term proportional to the charge, i.e., the energy is increased by local charge accumulations, leading to a positive sign for $\tilde{\alpha}$. On the other hand, in polar semiconductors, the electron polarizes the surrounding medium creating a local, positive charge density. Hence the electron reacts to this polarization and experiences an attractive potential (which implies $\tilde{\alpha}$ negative). This happens, for instance, in the polaron problem in the weak coupling limit, which becomes valid in most semiconductors, the electron polarizes the surrounding medium creating a local, positive charge density. Hence, we can see that the lowest band energy state decreases. It is then clear that in principle any sign would be equally possible for this coefficient if $\tilde{\alpha}$ is to represent the combined action of the polarization of the lattice along with repulsive electron-electron interactions. Intuitively, however, it is most realistic to think that $\tilde{\alpha}$ will be negative, because a positive nonlinear interaction would arise from negative charge accumulation in the barriers, which is not likely to occur. In fact, we discuss below mathematical reasons imposing that $\tilde{\alpha}$ has to be negative as expected, allowing for it to be positive only if it is small and the barriers themselves are very narrow. It can be argued following this line of reasoning that charge accumulation is expected inside the well, and that we should include a nonlinear term in the well to account for that. We have studied this possibility as well and we found that such a term is irrelevant, as we explain in detail in Sec. III.

B. Analytical results

In the preceding subsection, we have defined our model, and the only remaining thing is to fix the value of $\tilde{\alpha}$. This we address in the next section. We now work starting from Eq. (3) with the definition in Eq. (3) to cast the equations in a more tractable form. For simplicity, and because we are interested in intrinsic DBS features, we consider that the contacts in which the structure is embedded behave linearly. Therefore, the solution of Eq. (3) is a linear combination of traveling waves. As usual in scattering problems, we assume an electron incident from the left and define the reflection $r$ and transmission $t$ amplitudes by the relationships

$$\psi(z) = \begin{cases} A (e^{i k_0 z} + r e^{-i k_0 z}) & z < 0, \\ A t e^{i k_L z} & z > L, \end{cases}$$

where $k_0^2 = E$, $k_L^2 = E + eF L$, and $A$ is the incident wave amplitude. Now we define $\psi(z) = A \phi(z)$, and $\alpha = \tilde{\alpha} |A|^2$. Notice that $\alpha$ is a dimensionless parameter. Using Eq. (3) we get

$$-q_{zz}(z) + \frac{1}{q^3(z)} + V_b \chi_b(z) \left[ 1 + \alpha |\phi(z)|^2 \right] - eFz - E \right] \phi(z) = 0,$$

To solve the scattering problem in the resonant structure we develop a similar approach to that given in Ref. [1]. Since $\phi(z)$ is a complex function, we take $\phi(z) = q(z) \exp[i \gamma(z)]$, where $q(z)$ and $\gamma(z)$ are real functions. Inserting this factorization in Eq. (3) we have

$$\gamma_z(z) = q^{-2}(z)$$

$$- q_{zz}(z) + \frac{1}{q^3(z)} + [V_b \chi_b(z) - eFz - E] q(z) + \alpha V_b \chi_b(z) q^3(z) = 0.$$

This nonlinear differential equation must be supplemented by appropriate boundary conditions. However, using Eq. (9) this problem can be converted into an initial conditions equation. In fact, it is straightforward to prove that

$$q(L) = k_L^{-1/2}, \quad q_z(L) = 0,$$

and that the transmission coefficient is given by

$$\tau = \frac{4 k_0 q^2(0)}{1 + 2 k_0 q^2(0) + k_0^2 q^4(0) + q^4\chi_1(0)}.$$

Hence, we can integrate numerically (3) with initial conditions (3) backwards, from $z = L$ up to $z = 0$, to obtain $q(0)$ and $q_z(0)$, thus computing the transmission coefficient for given nonlinear coupling $\alpha$, incoming energy $E$ and applied voltage $V = FL$.

Once the transmission coefficient has been computed, and recalling that contacts are linear media, the tunneling current density at a given temperature $T$ for the DBS can be calculated within the stationary-state model from

$$j(V) = \frac{m^* e k_B T}{2 \pi^2 \hbar^3} \int_0^\infty \tau(E, V) N(E, V) dE,$$

where $N(E, V)$ accounts for the occupation of states to both sides of the device, according to the Fermi distribution function, and it is given by

$$N(E, V) = \ln \left( \frac{1 + \exp[(E_F - E)/k_B T]}{1 + \exp[(E_F - E - eV)/k_B T]} \right),$$

where $k_B$ is the Boltzmann constant.

C. Sign of the nonlinear term

At this point, we are in a position to give mathematical reasons why $\alpha$ (or $\tilde{\alpha}$) should be negative by using Eq. (3). This equation has the form

$$- q_{zz}(z) + \frac{1}{q^3(z)} + f_1(z) q(z) + f_2(z) q^3(z) = 0,$$

where $f_1(z)$ and $f_2(z)$ are well-behaved functions. Let us now consider the discrete version of this equation, with
the second derivative discretized in the usual way; denoting \( q_n = q(z = n\Delta z) \) and \( f_{in} = f_i(n\Delta z) \), \( i = 1, 2 \), with \( \Delta z \) being the integration step, Eq. \( \text{(10)} \) can be rewritten as

\[
q_{n+1} = 2q_n - q_{n-1} + (\Delta z)^2 \left[ \frac{1}{q_n^3} + f_{1n}q_n + f_{2n}q_n^3 \right]. \tag{11}
\]

Notice that in this expression the sign of \( f_{2n} \) is the same as the sign of \( \alpha \) and \( \tilde{\alpha} \). Let us now consider Eq. \( \text{(11)} \) for large \( q_n \); this is general, because if \( q \) is small the term \( q_n^{-3} \) will make it grow quite quickly. In this limit, Eq. \( \text{(11)} \) can be approximately replaced by

\[
\Delta q_{n-1} = \Delta q_n + (\Delta z)^2 f_{2n}q_n^3, \tag{12}
\]

where \( \Delta q_n = q_n - q_{n+1} \), and we have cast the equation in this fashion because it is to be integrated backwards. Recalling the initial conditions \( \text{(6)} \), if \( N \) is the total number of grid points, we have \( q_N = q_{N-1} = k_L^{-1/2} > 0 \). Therefore, we see that \( \Delta q_{N-1} = 0 \), and

\[
\Delta q_{N-2} = (\Delta z)^2 f_{2n}k_L^{-3/2}. \tag{13}
\]

We thus see that if \( f_{2n} \) is positive (and hence \( \alpha \)) the first increment is positive, and so are the subsequent ones, leading to an exponential divergence of \( q \), whereas if \( f_{2n} \) and \( \alpha \) are negative, the increment is negative, and \( q \) decreases until the \( q_n^{-3} \) starts being relevant again. In this last case, it is possible that \( q \) reaches an equilibrium due to the balance of the two cubic terms, which was not for \( \alpha > 0 \). This is in fact seen in the numerical integration of Eq. \( \text{(6)} \), where \( q \) rapidly diverges if \( \alpha \) is positive, unless, of course, \( \alpha \) is positive but very small and/or the barrier (the region for \( \alpha \) to influence \( q \)) is very narrow. In this last situation, however, the effect of \( \alpha \) becomes negligible.

Physically, this can be understood as follows. Electrons impinge on the barrier from outside, and begin to tunnel through it, their wavefunction being real and exponentially increasing or decreasing. If \( \alpha \) is positive, then if there were any charge density in the BDS, this would become even more repulsive, and the wavefunction would diverge even faster (numerically one always see the exponentially growing part, of course). This instability is not present in the opposite case, where a negative \( \alpha \) helps the electron tunnel across the barrier.

III. RESULTS AND DISCUSSIONS

In our calculations we have considered a double-barrier GaAs-Ga\(_{0.65}\)Al\(_{0.35}\)As structure with \( L = 3d = 150 \) Å. The conduction-band offset is \( V_b = 250 \) meV. In the absence of applied electric field and nonlinearities, there exist a single, very narrow resonance \( \tau \sim 1 \) below the top of the barrier, with an energy of 80.7 meV.

![FIG. 1. Transmission coefficient \( \tau \) as a function of the electron energy for \( \alpha = -7.6 \times 10^{-3} \), for different values of the applied voltage (a) 0 volts, (b) 0.029 volts, and (c) 0.105 volts. For comparison, dashed lines indicate the results for \( \alpha = 0 \).](attachment:fig1.png)
Hence the well supports a single quasi-bound state. When there exists an applied voltage, the energy of the quasi-bound state level is lowered and a strong enhancement of the current arises whenever the Fermi level matches this resonance, thus leading to the well-known RT phenomenon. This picture changes dramatically if a small amount of nonlinearity appears in the structure.

Figure 2 shows the transmission coefficient as a function of the incoming energy for different values of the applied bias, when $\alpha = -7.6 \times 10^{-3}$. This value is the one that best fits the experimental splitting that we discuss now. At zero bias, shown in Fig. 2(a), there exist two well differentiated peaks, the main one centered at 78.2 meV and the sideband centered at 41.5 meV. The main peak not only shifts to lower energies but also broadens, in a similar fashion to what happens in RT when inelastic effects arise. The energy separation between the main peak and the sideband is 36.7 meV, very close the energy of LO phonons in GaAs ($\hbar \omega_0 = 36.3$ meV), which was our requirement to choose $\alpha$. Therefore, the self-interaction we introduce in our model gives results similar to those found in dealing with the full electron-phonon Hamiltonian in DBS, where the sideband is due to phonon effects at inelastic channels. Interestingly, at the sideband the transmission probability is close to 0.2, which agrees with more sophisticated treatments. This agreement reinforces our hope to be treating in a proper way these inelastic scattering effects. Furthermore, to check the validity of our model, we can estimate the electron-phonon scattering rates at inelastic channels. Furthermore, to check the validity of our model, we can estimate the electron-phonon coupling from the downwards shift of the main peak when nonlinearities are introduced. This shift is $\Delta E = 2.8$ meV, and using the value of $\hbar \omega_0 = 36.7$ meV obtained previously we get $g = 0.076$, in rather good agreement with the value $g = 0.10$ proposed by Cai et al. We note, however, that this result is not far from $g = 0.03$ given by Wingreen et al. In fact, both many-body results enclose ours, and our calculation is not clearly favorable to any of them.

We now consider the effect of a bias imposed on the DBS. When there exists an applied voltage, the transmission peaks also shifts to lower energies due to the presence of the linear potential, as shown in Fig. 2(b). This behavior is similar to that found in linear (coherent) RT. For instance, these peaks are located at 62.9 meV and 15.2 meV for $V = 0.029$ volts. A further increase of the applied voltage makes the sideband to disappear [see Fig. 2(c)]. The reasons for that will be explained below.

In order to gain insight on the nonlinear RT in DBS, we can rewrite Eq. (1) as follows

$$-\phi_{zz}(z) + V_{\text{eff}}(z,E) \phi(z) = E \phi(z),$$

where we have defined an effective potential as follows

$$V_{\text{eff}}(z,E) = V_0 \chi(z) [1 + \alpha q(z)] - eFz.$$  

Thus (14) is a Schrödinger-like equation for an effective potential due to nonlinearity plus the linear potential and the built-in potential of the DBS. This effective potential depends not only on $z$ but also on the incoming energy $E$ through the function $q(z)$. Since the envelope function changes under RT conditions, and also $q(z)$ accordingly, it should be expected that $V_{\text{eff}}(z,E)$ undergoes severe variations whenever $E$ is close to one of the RT peaks. This is indeed the case, as shown in Fig. 2(b) where $V_{\text{eff}}(z,E)$ is displayed at zero bias. Notice that nonlinear effects have negligible effects on the shape of the effective potential in the right barrier, besides a slight band bending at the interface $z = (L + d)/2$ at low energies. However, the potential in the left barrier region differs significantly from the original square-barrier shape. Out of resonances, the effective barrier height at $z = 0$ is lower than $V_b$, whereas at resonances it takes the value $\sim V_b$. Hence the effective potential presents two local maxima in the plane $z = 0$ as a function of the incoming energy $E$, matching the values of the main resonance and the sideband above discussed. From Fig. 2(b) it is clear that at the energy of the main resonance the effective potential is quite similar to the built-in DBS potential, just producing a small shift of the quasi-bound-state in comparison to the linear RT process. Concerning the sideband, the effective potential presents a deep minimum at the interface $z = (L - d)/2$, which causes the lowering of the quasi-bound-state, thus explaining the origin of this lower RT peak. Indeed, the fact that this added well is responsible for the peak is confirmed by the observation that the disappearance of the peak as the field increases coincides with the vanishing of the well due to the bias; before that, the resonance responsible for the peak is moving towards $E = 0$ where it can be seen that the depth of the secondary well vanishes. We thus provide a complete, coherent picture of the tunneling phenomenology.

![Effective potential](image_url)

**FIG. 2.** Effective potential $V_{\text{eff}}$ as a function of $z$ and the incoming electron energy $E$ at zero bias, for $\alpha = -7.6 \times 10^{-3}$. We now have to comment the $j - V$ characteristics, computed from (14). We have set two different temperatures (77 K and room temperature) and compared these curves with those obtained in linear RT. The Fermi energy was $E_F = 27.5$ meV. Results are shown in Fig. 3.
In all cases we obtained clear NDR signatures. The linear RT shows a single NDR peak whereas self-interaction causes the occurrence of a second peak at a lower voltage, clearly related to the sideband in the transmission coefficient. On increasing temperature both peaks merge into a single one because of the broadening of the Fermi-Dirac distribution function. It is important to notice that the $j - V$ characteristics collect all the main features found in the transmission coefficient, namely shift downwards of the main RT peak, its corresponding broadening due to inelastic effects and the appearance of the sideband, shown in Fig. 3 as a smaller NDR signature at $V = 0.05$ volts.

Finally, some words are in order about possible nonlinearities in the well, as we announced in Sec. II. In principle, charge may accumulate in the well depending upon doping degree, and that is expected to induce a self-repulsive interaction of the electronic cloud, overcoming the attractive part due to polarization of the medium. So, if we are considering that nonlinearities are zero in the well we could argue that one effect compensates the other and then there is no nonlinear term to be added. Admittedly, that would be a very rare phenomenon, because exact compensation of both forces (or any interaction in nature) would be serendipitous. The question then arises as to whether our model is a good one in the sense that it is robust against new terms accounting for inelastic scattering in the well. We checked that by introducing another term in the potential, given by $V'(z) = \beta |\psi|^2$ for $(L - 2)/2 < z < (L + d)/2$. The calculations can be carried out much in the same way we have discussed, and the results obtained by numerical integration of an equation very close to Eq. (3). We studied a range of values for $\beta \equiv \beta |A|^2$ of the same order as those of $\alpha$: $\beta$ positive would indicate that Coulomb effects dominate, whereas $\beta$ negative would be the same as in the barriers, a representation of the effects of polarization. The results were very much satisfactory, because the features of the model did not change, and only minor quantitative modifications were found. We can then conclude that leaving the possible nonlinearity of the well out of our model is not a limitation and that it is not necessary to take it into account.

**IV. CONCLUSIONS**

In conclusion, we have presented a nonlinear effective-mass equation to take into account in a simple way electron-electron and electron-phonon interaction in RT through DBS. Our model represents those effects by including a self-coupling of the wavefunction with the charge cloud inside the barriers, whose strength is the only adjustable parameter. By comparing the results of the model to experimental data on the distance between the main transmission peak and the sideband we fix the value of that constant.

With the same value we found transmission probabilities and electron-phonon coupling values very close to those arising from many-body calculations, which indicates that our model consistently reproduces their features. We have also been able to discard any possible nonlinearity in the well, which shows that our understanding of the physics of this problem is in the right direction. We have to stress that the low values obtained for the nonlinear coupling, $\alpha = -7.6 \times 10^{-3}$, do agree with our purpose of offering an alternative description: Had we obtained large values of $\alpha$, it would be very difficult to understand how those magnitudes could arise from what in principle are perturbative effects of the linear description. In addition, large values of $\alpha$ would have
immediately posed the question about the role that solitonic formation and transmission could be playing along with RT in the DBS (see Refs. 10 and 11 as well as references therein). Therefore, our model satisfactorily achieves the goals we had in mind, a great success if its simplicity is taken into account.

Further extensions of the present work to study non-linear dynamical response of DBS on external ac bias would be of great interest to shed light on related problems like bistability, noise characteristics, and RT at far infra-red frequencies under the influence of inelastic scattering channels as those described here. Besides that, the above mentioned highly nonlinear limit could also be interesting, for nonlinearity is always susceptible to give rise to new and unexpected features. If these new features were seen in our model it would be a very exciting development. If materials with suitable characteristics were found, and that is to be expected, experiments could be made to check the predictions: If the model were wrong, that would establish its range of validity, whereas if the predictions were correct, this work could pave the way to a new family of devices and applications.

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1 T. C. L. G. Sollier, W. D. Goodhue, P. E. Tannenwald, C. D. Parker, and D. D. Peck, Appl. Phys. Lett. 43, 588 (1984).
2 B. Ricco and M. Ya. Azbel, Phys. Rev. B 29, 1970 (1984).
3 N. S. Wingreen, K. W. Jacobsen, and J. W. Wilkins, Phys. Rev. Lett. 61, 1396 (1988).
4 W. Cai, T. F. Zheng, P. Hu, B. Yudanin, and M. Lax, Phys. Rev. Lett. 63, 418 (1989).
5 J. A. Stavnes, E. H. Hauge, P. Lipavský, and V. Špička, Phys. Rev. B 44, 13595 (1991).
6 T. Figielski, A. Makosa, T. Wosiński, P. C. Harness, and K. E. Singer, Solid State Commun. 91, 913 (1994).
7 S. Nonoyama, A. Oguri, Y. Asano, and S. Maekawa, Phys. Rev. B 50, 2667 (1994).
8 A. S. Davidov, Solitons in Molecular Systems (Kluwer Academic Publishers, Dordrecht, 1991).
9 J. Callaway, Quantum Theory of the Solid State (Academic Press, CA, 1991), p 711.
10 R. Knapp, G. Papanicolaou, and B. White, J. Stat. Phys. 63, 567 (1991).
11 A. Sánchez and L. Vázquez, Int. J. Mod. Phys. B 5, 2825 (1991).
12 P. Hawrylak and M. Grabowski, Phys. Rev. B 40, 8013 (1989).
13 A. L. Levy Yeyati and F. Flores, Phys. Rev. B 47, 10 543 (1993).
14 V. A. Chitta, C. Kutter, R. E. M. de Bekker, J. C. Maan, S. J. Hawksworth, J. M. Chamberlain, M. Henini, and G. Hill, J. Phys. Condens. Matter 6, 3945 (1994).