$B$ to light meson transition form factors calculated in perturbative QCD approach

Cai-Dian Lü and Mao-Zhi Yang *

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China;
Institute of High Energy Physics, P.O. Box 918(4), Beijing 100039, China†

November 2, 2018

Abstract

We calculate the $B \rightarrow P$, $B \rightarrow V$ ($P$: light pseudoscalar meson, $V$: light vector meson) form factors in the large-recoil limit in perturbative QCD approach, including both the vector (axial vector) and tensor operators. In general there are two leading components $\phi_B$ and $\bar{\phi}_B$ for $B$ meson wave functions. We consider both contributions of them. Sudakov effects ($k_\perp$ and threshold resummation) are included to regulate the soft end-point singularity. By choosing the hard scale as the maximum virtualities of internal particles in the hard $b$-quark decay amplitudes, Sudakov factors can effectively suppress the long-distance soft contribution. Hard contribution can be dominant in these approaches.

PACS: 13.25.Hw, 11.10.Hi, 12.38.Bx,

*Email address: lucd@mail.ihep.ac.cn (C.D.Lü), yangmz@mail.ihep.ac.cn (M.Z.Yang)
†Mailing address
1 Introduction

The most difficult task in calculating $B$ meson decay amplitude is to treat the hadronic matrix element $\langle M_1 M_2 | Q_i | B \rangle$, which is generally controlled by soft non-perturbative dynamics of QCD. Here $Q_i$ is one of the effective low energy transition operators of $b$ quark decays[1], $M_1$ and $M_2$ are the final state mesons produced in $B$ decays. In the earlier years, this hadronic matrix elements of $B$ decays were treated by an approximate method, which is called factorization approach [2]. In factorization approach the hadronic matrix element of four-fermion operator is approximated as a product of the matrix elements of two currents, $\langle M_1 M_2 | Q_i | B \rangle \simeq \langle M_1 | j_{1\mu} | 0 \rangle \langle M_2 | j_{2\mu} | B \rangle$, where $j_{1\mu}$ and $j_{2\mu}$ are the two relevant currents which can be related to $Q_i$ through $Q_i = j_{1\mu}j_{2\mu}$. The matrix element of $j_{1\mu}$ sandwiched between the vacuum and meson state $M_1$ directly defines the decay constant of $M_1$. For example, if $M_1$ is a pseudoscalar and $j_{1\mu}$ is $V-A$ current, the relation between the matrix element and the decay constant will be $\langle M_1 | j_{1\mu} | 0 \rangle = i f_M p_\mu$, where $f_M$ and $p_\mu$ are the decay constant and the four-momentum of $M_1$, respectively. The other matrix element $\langle M_2 | j_{2\mu} | B \rangle$ can be generally decomposed into transition form factors of $B \rightarrow M_2$ due to its Lorentz property. The explicit definition of $B$ meson transition form factors through such matrix elements can be found in section 4 and 5.

In semi-leptonic decays of $B$ meson, the decay amplitude can be directly related to $B$ meson transition form factors without the factorization approximation. For example, for $B \rightarrow \pi \ell \bar{\nu}_\ell$, the decay amplitude can be written in the form:

$$A(p_B, p_\pi) = \frac{G_F}{\sqrt{2}} V_{ub}(\bar{\ell} \gamma_\mu(1-\gamma_5)\nu_\ell) \langle \pi(p_\pi) | \bar{u} \gamma^\mu b | \bar{B}(p_B) \rangle,$$

where the form factors $F_1(q^2)$ and $F_0(q^2)$ are defined through the $B \rightarrow \pi$ transition matrix element $\langle \pi(p_\pi) | \bar{u} \gamma^\mu b | \bar{B}(p_B) \rangle$ in eq.(14) of section 4. In general the form factors are functions of momentum transfer squared $q^2 = (p_B - p_\pi)^2$. In the region of small recoil, where $q^2$ is large and/or the final particle is heavy enough, the form factors are dominated by soft dynamics, which is out of control of perturbative QCD. However, in the large recoil region where $q^2 \rightarrow 0$, and when the final particle is light (such as the pion), $5\text{GeV}$ ($m_B = 5\text{GeV}$) of energy is released. About half of the energy is taken by the light final particle, which suggests that large momentum is transferred in this process and the interaction is mainly short-distance. Therefore
perturbative QCD can be applied to $B$ to light meson transition form factors in large recoil region.

Before applying perturbative method in this calculation, one must separate soft dynamics from hard interactions. This is called factorization in QCD. The factorization theorem has been worked out in ref.[3] based on the earlier works on the applications of perturbative QCD in hard exclusive processes [4], where the soft contributions are factorized into wave functions or distribution amplitudes of mesons, and the hard part is treated by perturbative QCD. Sudakov resummation has been introduced to suppress the long-distance contributions. Recently this approach has been well developed and extensively used to analyze $B$ decays [5, 6, 7, 8, 9, 10, 11, 12, 13]. There is also another direction to prove factorization in the soft-collinear effective theory [14], which shows correctly the power counting rules in QCD. In this work we shall calculate a set of $B \to P$ and $B \to V$ ($P$: light pseudoscalar meson, $V$ light vector meson) transition form factors in perturbative QCD (PQCD) approach. We use the $B$ wave functions derived in the heavy quark limit recently [15], and include Sudakov effects from transverse momentum $k_\perp$ and threshold resummation [9, 16]. In general there are two Lorentz structures for $B$ wave functions. If they are appropriately defined, only one combination gives large contribution, the other combination contributes 30%.

A direct calculation of the one-gluon-exchange diagram for the $B$ meson transition form factors suffers singularities from the end-point region of the light cone distribution amplitude with a momentum fraction $x \to 0$ in longitudinal direction. In fact, in the end-point region the parton transverse momenta $k_\perp$ are not negligible. After including parton transverse momenta, large double logarithmic corrections $\alpha_s n^2 k_\perp$ appear in higher order radiative corrections and have to be summed to all orders. In addition to the double logarithm like $\alpha_s n^2 k_\perp$, there are also large logarithms $\alpha_s n^2 x$ which should also be summed to all orders. This is called threshold resummation [16]. The relevant Sudakov factors from both $k_\perp$ and threshold resummation can cure the end-point singularity which makes the calculation of the hard amplitudes infrared safe.

We check the perturbative behavior in the calculation of the $B$ meson transition form factors and find that with the hard scale appropriately chosen, Sudakov effects can effectively suppress the soft dynamics, and the main contribution comes from the perturbative region.

The content of this paper is as follows. Section 2 is the kinematics and the framework of
the PQCD approach used in the calculation of $B \to P$ and $B \to V$ transition form factors. Section 3 includes wave functions of $B$ meson and the light pseudoscalar and vector mesons. We give the results of $B \to P$ transition form factors in section 4, and the $B \to V$ transition form factors in section 5. Section 6 are the numerical results and discussion. Finally section 7 is a brief summary.

2 The Framework

Here we first give our conventions on kinematics. In light-cone coordinate, the momentum is taken in the form $k = (k^+, k^-, \vec{k}_\perp)$ with $k^\pm = k^0 \pm k^3$ and $\vec{k}_\perp = (k^1, k^2)$. The scalar product of two arbitrary vectors $A$ and $B$ is $A \cdot B = A_\mu B^\mu = \frac{1}{2}(A^+ B^- + A^- B^+) - \vec{A}_\perp \cdot \vec{B}_\perp$. Our study is in the rest frame of $B$ meson. The mass difference of $b$ quark and $B$ meson is negligible in the heavy quark limit and we take $m_b \simeq m_B$ in our calculation. The masses of light quarks $u$, $d$, $s$ and light pseudoscalar mesons are neglected, while the masses of light vector mesons $\rho$, $\omega$, $K^*$ are kept in the first order. The momentum of light meson is chosen in the “+” direction. Under these conventions, the momentum of $B$ meson is $P_B = \frac{1}{\sqrt{2}}(m_B, m_B, \vec{0}_\perp)$, and in the large recoil limit $q^2 \to 0$, the momentum of the light pseudoscalar meson is $P_P = (\frac{m_P}{\sqrt{2}}, 0, \vec{0}_\perp)$. For the case of light vector meson, its momentum is $P_V = \frac{m_V}{\sqrt{2}}(1, r^2_V, \vec{0}_\perp)$ with $r_V$ defined as $r_V \equiv m_V/m_B$. The longitudinal polarization of the vector meson is $\varepsilon_L = \frac{1}{\sqrt{2}}(\frac{1}{r_V} - r_V, \vec{0}_\perp)$, its transverse polarization $\varepsilon_T = (0, 0, \vec{1}_\perp)$. The light spectator momenta $k_1$ in the $B$ meson and $k_2$ in the light meson are parameterized as $k_1 = (0, x_1 \frac{m_B}{\sqrt{2}}, k_{1\perp})$ and $k_2 = (x_2 \frac{m_P}{\sqrt{2}}, 0, k_{2\perp})$, where $k_{2\perp}$ is dropped because of its smallness (In the meson moving along the ‘plus’ direction with large momentum, the minus component of its parton’s momentum $k_{2\perp}$ should be very small). We also dropped $k_1^+$ because it vanishes in the hard amplitudes, which can be simply shown below.

The lowest-order diagrams for $B$ to light meson transition form factors are displayed in Fig.1. The hard amplitude $H$ are proportional to the propagator of the gluon, i.e., $H \propto 1/(k_2 - k_1)^2 \simeq 1/(2k_2 \cdot k_1) \simeq 1/(k_2^+ k_1^-)$. It is obvious that only $k_1^-$ left in the hard amplitude.

Factorization is one of the most important part of applying perturbative QCD in hard exclusive processes, which separates long-distance dynamics from short-distance dynamics. The
factorization formula for $B \to P, V$ transition matrix element can be written as

$$
\langle P, V(P_2) | \bar{b} \Gamma_\mu q' | B(p_1) \rangle = \int dx_1 dx_2 d^2 k_1 d^2 k_2 \frac{dz^+ d^2 z_\perp dy^+ d^2 y_\perp}{(2\pi)^3 (2\pi)^3} 
\times e^{-i k_2 y} \langle P, V(P_2) | \bar{q}(y)_{\alpha} q'_\beta(0) | 0 \rangle H_{\mu}^{\beta\alpha; \sigma \rho} e^{ik_1 \cdot z} \langle 0 | \bar{b}(0)_{\rho} q_\sigma(z) | B(P_1) \rangle,
$$

where the matrix elements $\langle P, V(P_2) | \bar{q}(y)_{\alpha} q'_\beta(0) | 0 \rangle$ and $\langle 0 | \bar{b}(0)_{\rho} q_\sigma(z) | B(P_1) \rangle$ define the wave functions of light pseudoscalar (vector) meson and $B$ meson, which absorb all the soft dynamics. $H_{\mu}^{\beta\alpha; \sigma \rho}$ denotes the hard amplitude, which can be treated by perturbative QCD. $\beta, \alpha, \sigma$ and $\rho$ are Dirac spinor indices. Both the wave functions and the hard amplitude $H$ are scale dependent. This scale is usually called factorization scale. Above this scale, the interaction is controlled by hard dynamics, while the interaction below this scale is controlled by soft dynamics, which is absorbed into wave functions. The factorization scale is usually taken the same as renormalization scale. In practice it is convenient to work in transverse separation coordinate space ($b$-space) rather than the transverse momentum space ($k_\perp$-space). So we shall make a Fourier transformation $\int d^2 k_\perp e^{-i k_\perp \cdot b}$ to transform the wave functions and hard amplitude into $b$-space. $1/b$ will appear as a typical factorization scale. As the scale $\mu > 1/b$, the interactions are controlled by hard dynamics, and as $\mu < 1/b$ soft dynamics dominates which is absorbed into wave functions.

Higher order radiative corrections to wave functions and hard amplitudes generate large double logarithms through the overlap of collinear and soft divergence. The infrared divergence is absorbed into wave functions. The double logarithms $\alpha_s \ln^2 P b$ have been summed to all orders to give an exponential Sudakov factor $e^{-S(x, b, P)}$, here $P$ is the typical momentum transferred in
the relevant process, \(x\) is the longitudinal momentum fraction carried by the relevant parton. The resummation procedure has been analyzed and the result has been given in [3], we do not repeat it here.

In addition to double logarithms \(\alpha_s \ln^2 P b\) in \(b\)-space (or say \(k_\perp\)-space equivalently), radiative corrections to hard amplitudes also produce large logarithms \(\alpha_s \ln x\). These double logarithms should also be summed to all orders. This threshold resummation leads to

\[
S_t(x) = \frac{2^{1+2c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} [x(1 - x)]^c,
\]

where the parameter \(c = 0.3\). This function is normalized to unity. \(S_t(x)\) vanishes very fast at the end-point region \(x \to 0\) and \(x \to 1\). Therefore the factors \(S_t(x_1)\) and \(S_t(x_2)\) suppress the end-point region of meson distribution amplitudes.

### 3 The Wave Functions

In the resummation procedures, the \(B\) meson is treated as a heavy-light system. In general, the \(B\) meson light-cone matrix element can be decomposed as [17, 18]

\[
\langle 0| \bar{q}_\alpha(z) b_\beta(0)| B(p_B) \rangle = \int_0^1 \frac{d^4z}{(2\pi)^4} e^{ik_1 \cdot z} \langle 0| \bar{q}_\alpha(z) b_\beta(0)| \bar{B}(p_B) \rangle
\]

\[
= \frac{i}{\sqrt{2N_c}} \left\{ \left( \phi_B + m_B \right) \gamma_5 \left[ \frac{\not{k_1}}{\sqrt{2}} \phi_B^\dagger(k_1) + \frac{\not{n}}{\sqrt{2}} \phi_B^\dagger(k_1) \right] \right\}_{\beta\alpha},
\]

where \(n = (1, 0, 0, T)\), and \(v = (0, 1, 0, T)\) are the unit vectors pointing to the plus and minus directions, respectively. From the above equation, one can see that there are two Lorentz structures in the \(B\) meson wave function. In general, one should consider both these two Lorentz structures in calculations of \(B\) meson decays. The light cone distribution amplitudes \(\phi_B^\dagger\) and \(\phi_B^\dagger\) are derived by Kawamura et al. in the heavy quark limit [15],

\[
\phi_B^\dagger(x, b) = \frac{f_B x}{\sqrt{6}\Lambda_0^2} \theta(\Lambda_0 - x) J_0 \left[ m_B b \sqrt{x(\Lambda_0 - x)} \right],
\]

\[
\phi_B^\dagger(x, b) = \frac{f_B(\Lambda_0 - x)}{\sqrt{6}\Lambda_0^2} \theta(\Lambda_0 - x) J_0 \left[ m_B b \sqrt{x(\Lambda_0 - x)} \right],
\]

with \(\Lambda_0 = 2\bar{\Lambda}/M_B\), and \(\bar{\Lambda}\) is a free parameter which should be at the order of \(m_B - m_b\).
The relation between $\phi_B$, $\bar{\phi}_B$, $\phi_B^+$, $\phi_B^-$ are

$$\phi_B = \phi_B^+, \quad \bar{\phi}_B = \phi_B^+ - \phi_B^-.$$  

(6)

The normalization conditions for these two distribution amplitudes are:

$$\int d^4k_1 \phi_B(k_1) = \frac{f_B}{2\sqrt{2}N_c}, \quad \int d^4k_1 \bar{\phi}_B(k_1) = 0.$$  

(7)

From eqs.(5) and (6), we can see that when $x \to 0$, $\bar{\phi}_B \not\to 0$, while $\phi_B \to 0$. The behavior of $\phi_B$ with the definition (6) is similar to the one defined in previous PQCD calculations \[5, 6, 7, 8, 9, 10, 11, 12, 13\]. Note that our definition of $\phi_B$, $\bar{\phi}_B$ are different from the previous one in the literature \[8\].

$$\int_0^1 \frac{dz}{(2\pi)^4} e^{i(k_1 \cdot z)} \langle 0|\bar{s}_\alpha(z)b_\beta(0)|\bar{B}(P_B)\rangle$$

$$= \frac{i}{\sqrt{2}N_c} \left\{ \left[ \hat{\gamma}_5 \left( \frac{\hat{P}}{\sqrt{2}} \phi_B^+(k_1) + \frac{\hat{P}}{\sqrt{2}} \phi_B^-(k_1) \right) \right] \right\}_{\beta\alpha}$$

$$= -\frac{i}{\sqrt{2}N_c} \left\{ \left[ \hat{\gamma}_5 \left( \phi_B^+(k_1) + \frac{\hat{P}}{\sqrt{2}} \phi_B^-(k_1) \right) \right] \right\}_{\beta\alpha},$$  

(8)

with

$$\phi_B' = \frac{\phi_B^+ + \phi_B^-}{2}, \quad \bar{\phi}_B' = \frac{\phi_B^+ - \phi_B^-}{2}.$$  

(9)

This definition is equivalent to eqs.(4, 6) in the total amplitude. Although the final numerical results should be the same, the form factor formulas are simpler using our new definition.

Another outcome is that our new formula shows explicitly the importance of the leading twist contribution $\phi_B$, which will be shown later. However, if $\phi_B'$ and $\bar{\phi}_B'$ are defined as in eqs.(9), both of their contributions are equivalently important (see the numerical results in Table 2 of Ref.[13]). It is easy to check that both $\phi_B'$ and $\bar{\phi}_B'$ here have non-zero endpoint at $x \to 0$. In this case, $\phi_B'$ does not correspond to the one defined in the previous PQCD calculations \[8, 9\], where $\phi_B \to 0$, at endpoint, when $x \to 0$ or 1.

The $\pi$, $K$ mesons are treated as a light-light system. At the $B$ meson rest frame, the $K$ meson (or pion) is moving very fast, one of $k_1^+$ or $k_1^-$ is zero depending on the definition of the $z$ axis. We consider a kaon (or $\pi$ meson) moving in the plus direction in this paper. The $K$ meson distribution amplitude is defined by \[20\]

$$\langle K^-(P)|\bar{s}_\alpha(z)u_\beta(0)|0\rangle$$

$$= \frac{i}{\sqrt{2}N_c} \int_0^1 dx e^{ixP_z} \left[ \gamma_5 \gamma_\mu P\phi_K(x) + m_0 \gamma_5 \left( \gamma_\mu - m_0 \gamma_\nu \gamma_\mu z_\nu \phi_\sigma(x) \right) \right]_{\beta\alpha}$$  

(10)
For the first and second terms in the above equation, we can easily get the projector of the distribution amplitude in the momentum space. However, for the third term we should make some effort to transfer it into the momentum space. By using integration by parts for the third term, after a few steps, eq. (10) can be finally changed to be

\[ < K^{-}(P)|s_{\alpha}(z)u_{\beta}(0)|0 > = \frac{i}{\sqrt{2N_c}} \int_{0}^{1} dx e^{ixPz} \left[ \gamma_{5} P\phi_{K}(x) + m_{0}\gamma_{5}\phi_{P}(x) + m_{0}[\gamma_{5}(\gamma_{\mu}Pz - 1)]\phi_{K}^{T}(x) \right]_{\beta\alpha} \] (11)

where \( \phi_{K}^{T}(x) = \frac{1}{6} \frac{d}{dx} \phi_{\sigma}(x) \), and vector \( n \) is parallel to the \( K \) meson momentum \( p_{K} \). And \( m_{0K} = \frac{m_{K}^{2}}{(m_{u} + m_{s})} \) is a scale characterized by the Chiral perturbation theory. For \( \pi \) meson, the corresponding scale is defined as \( m_{0\pi} = \frac{m_{\pi}^{2}}{(m_{u} + m_{d})} \).

For the light vector meson \( \rho, \omega \) and \( K^{*} \), we need distinguish their longitudinal polarization and transverse polarization. If the \( K^{*} \) meson (so as to other vector mesons) is longitudinally polarized, we can write its wave function in longitudinal polarization [8, 21]

\[ < K^{*-}(P, \epsilon_{L})|\bar{d}_{\alpha}(z)u_{\beta}(0)|0 > = \frac{1}{\sqrt{2N_c}} \int_{0}^{1} dx e^{ixPz} \left\{ \epsilon \left[ \phi_{K}^{T} \phi_{K^{*}}(x) + m_{K}\phi_{K^{*}}(x) \right] + m_{K}\phi_{K^{*}}(x) \right\}. \] (12)

The second term in the above equation is the leading twist wave function (twist-2), while the first and third terms are sub-leading twist (twist-3) wave functions. If the \( K^{*} \) meson is transversely polarized, its wave function is then

\[ < K^{*-}(P, \epsilon_{T})|\bar{d}_{\alpha}(z)u_{\beta}(0)|0 > = \frac{1}{\sqrt{2N_c}} \int_{0}^{1} dx e^{ixPz} \left\{ \epsilon \left[ \phi_{K}^{T} \phi_{K^{*}}(x) + m_{K}\phi_{K^{*}}(x) \right] + i m_{K}\epsilon_{\mu\nu\rho\sigma} \gamma_{5}\gamma^{\mu}\epsilon^{\nu}\epsilon^{\rho}\epsilon^{\sigma} \phi_{K^{*}}(x) \right\}. \] (13)

Here the leading twist wave function for the transversely polarized \( K^{*} \) meson is the first term which is proportional to \( \phi_{K^{*}}^{T} \).

### 4 B \rightarrow P Form Factors

The \( B \rightarrow P \) form factors are defined as following:

\[ \langle P(p_{1})|\bar{q}\gamma_{\mu}b|B(p_{B}) \rangle = \left[ (p_{B} + p_{1})_{\mu} - \frac{m_{B}^{2} - m_{P}^{2}}{q^{2}} q_{\mu} \right] F_{1}(q^{2}) + \frac{m_{B}^{2} - m_{P}^{2}}{q^{2}} q_{\mu} F_{0}(q^{2}). \] (14)
where \( q = p_B - p_1 \). In order to cancel the poles at \( q^2 = 0 \), we must impose the condition

\[
F_1(0) = F_0(0).
\]

That means in the large recoil limit, we need only calculate one independent form factor for the vector current. For the tensor operator, there is also only one independent form factor, which is important for the semi-leptonic decay \( B \to K\ell^+\ell^- \):

\[
\langle P(p_1)|\bar{q}\sigma_{\mu\nu}b|\bar{B}(p_B)\rangle = i \left[p_{1\mu}q_\nu - q_\mu p_{1\nu}\right] \frac{2F_T(q^2)}{m_B + m_P},
\]

(15)

\[
\langle P(p_1)|\bar{q}\sigma_{\mu\nu}\gamma_5b|\bar{B}(p_B)\rangle = \epsilon_{\mu\nu\alpha\beta}p_1^\alpha q^\beta \frac{2F_T(q^2)}{m_B + m_P}.
\]

(16)

In the previous section we have discussed the wave functions of the factorization formula in eq.(4). In this section, we will calculate the hard part \( H \). This part involves the current operators and the necessary hard gluon connecting the current operator and the spectator quark. Since the final results are expressed as integrations of the distribution function variables, we will show the whole amplitude for each diagram including wave functions and Sudakov factors.

There are two types of diagrams contributing to the \( B \to K \) form factors which are shown in Fig.1. The sum of their amplitudes is given as

\[
F_1(q^2 = 0) = F_0(q^2 = 0) = 8\pi C_F m_B^2 \int_0^1 dx_1dx_2 \int_0^\infty b_1db_1b_2db_2 \left\{ \langle h_e(x_1, x_2, b_1, b_2) (\phi_B(x_1, b_1) \\
[ (1 + x_2)\phi_A^K(x_2, b_2) + r_K(1 - 2x_2) (\phi^P_K(x_2, b_2) + \phi^t_K(x_2, b_2)) ] - \bar{\phi}_B(x_1, b_1) \\
[\phi^A_K(x_2, b_2) - r_K x_2 (\phi^P_K(x_2, b_2) + \phi^t_K(x_2, b_2))] \right\} \alpha_s(t^1_\epsilon) \exp[-S_{ab}(t^1_\epsilon)] +
\]

(17)

\[
2r_K \phi^P_K(x_2, b_2) \phi_B(x_1, b_1) \alpha_s(t^2_\epsilon) h_e(x_2, x_1, b_1, b_2) \exp[-S_{ab}(t^2_\epsilon)]
\],

where \( r_K = m_{0K}/m_B = m_K^2/[m_B(m_s + m_d)] \); \( C_F = 4/3 \) is a color factor. The function \( h_e \), the scales \( t^1_\epsilon \) and the Sudakov factors \( S_{ab} \) are displayed at the end of this section.

For \( B \to \pi \) form factors, one need only replace the above K meson distribution amplitudes \( \phi^t_K \) by pion distribution amplitudes \( \phi^t_\pi \) and replace the scale parameter \( r_K \) by \( r_\pi = m_{0\pi}/m_B = m_\pi^2/[m_B(m_u + m_d)] \), respectively.

For the tensor operator we get the form factor formulas as

\[
F_T(q^2 = 0) = 8\pi C_F m_B^2 \int_0^1 dx_1dx_2 \int_0^\infty b_1db_1b_2db_2 \left\{ h_e(x_1, x_2, b_1, b_2) (\phi_B(x_1, b_1)
\]
\[
\begin{align*}
&\left[\phi_K^A(x_2, b_2) - x_2 r_K \phi_K^P(x_2, b_2) + r_K(2 + x_2)\phi_K^A(x_2, b_2)\right] - \bar{\phi}_B(x_1, b_1) \\
&\phi_K^A(x_2, b_2) - r_K \phi_K^P(x_2, b_2) + r_K \phi_K^A(x_2, b_2)\right] \alpha_s(t_1^e) \exp[-S_{ab}(t_1^e)] \\
+ 2r_K h_e(x_2, x_1, b_2) \alpha_s(t_2^e) \phi_B(x_1, b_1) \phi_K^P(x_2, b_2) \exp[-S_{ab}(t_2^e)] \right) . \quad (18)
\end{align*}
\]

In the above equations, we have used the assumption that $x_1 << x_2$. Since the light quark momentum fraction $x_1$ in $B$ meson is peaked at the small region, while quark momentum fraction $x_2$ of K meson is peaked around 0.5, this is not a bad approximation. The numerical results also show that this approximation makes very little difference in the final result. After using this approximation, all the diagrams are functions of $k_1^- = x_1 m_B / \sqrt{2}$ of B meson only, independent of the variable of $k_1^+$. The function $h_e$, coming from the Fourier transform of the hard amplitude $H$, is

\[
h_e(x_1, x_2, b_1, b_2) = K_0(\sqrt{x_1 x_2 m_B} b_1) [\theta(b_1 - b_2)K_0(\sqrt{x_2 m_B} b_1) I_0(\sqrt{x_2 m_B} b_2) + \theta(b_2 - b_1)K_0(\sqrt{x_2 m_B} b_2) I_0(\sqrt{x_2 m_B} b_1)] S_1(x_2) , \quad (19)
\]

where $J_0$ is the Bessel function and $K_0$, $I_0$ are modified Bessel functions.

The Sudakov factors used in the text are defined as

\[
S_{ab}(t) = s\left(x_1 m_B / \sqrt{2}, b_1\right) + s\left(x_2 m_B / \sqrt{2}, b_2\right) + s\left((1 - x_2) m_B / \sqrt{2}, b_2\right) - \frac{1}{b_1} \left[ \ln \frac{\ln(t/\Lambda)}{-\ln(b_1 \Lambda)} + \ln \frac{\ln(t/\Lambda)}{-\ln(b_2 \Lambda)} \right] , \quad (20)
\]

where the function $s(q, b)$ are defined in the Appendix A of ref.\[3\]. The hard scale $t_i$’s in the above equations are chosen as the largest scale of the virtualities of internal particles in the hard $b$-quark decay diagrams,

\[
t_1^e = \max(\sqrt{x_2 m_B}, 1/b_1, 1/b_2) , \\
t_2^e = \max(\sqrt{x_1 m_B}, 1/b_1, 1/b_2) . \quad (21)
\]

5 $B \to V$ Form Factors

For the $B \to K^*$ form factors, we first define the axial vector current part

\[
\langle K^*(p_1)|q\gamma_\mu\gamma_5 b|B(p_B)\rangle = i \left( \epsilon_{\mu}^{*} - \frac{\epsilon_{\mu}^{*} \cdot q}{q^2} q_\mu \right) (m_B + m_{K^*}) A_1(q^2)
\]
$-i \left( (p_B + p_1)_\mu - \frac{(m_B^2 - m_{K^*}^2)}{q^2} q_\mu \right) (\epsilon^* \cdot q) \frac{A_2(q^2)}{m_B + m_{K^*}}$

\[ T_{\mu\nu} = \frac{2m_{K^*}(\epsilon^* \cdot q)}{q^2} q_\mu A_0(q^2), \]  

where $\epsilon^*$ is the polarization vector of $K^*$ meson. To cancel the poles at $q^2 = 0$, we must have

\[ 2m_{K^*}A_0(0) = (m_B + m_{K^*})A_1(0) - (m_B - m_{K^*})A_2(0). \]  

For the vector current, only one form factor $V$ is defined

\[ \langle K^*(p_1)|\bar{q}\gamma_\mu b|\bar{B}(p_B)\rangle = \epsilon^{\mu\nu\alpha\beta}p_B^\alpha p_1^\beta \frac{2V(q^2)}{(m_B + m_{K^*})}. \]  

And for the tensor operators, three form factors are defined:

\[ \langle K^*(p_1)|\bar{q}\sigma_{\mu\nu}b|\bar{B}(p_B)\rangle = -i\epsilon^{\mu\nu\alpha\beta}p_B^\alpha p_1^\beta T_1(q^2) - i\epsilon^{\mu\nu\alpha\beta}p_B^\alpha p_1^\beta T_2(q^2) \]

\[ -iT_3(q^2) \frac{(p_B \cdot \epsilon^*)}{p_B \cdot p_1} \epsilon^{\mu\nu\alpha\beta}p_B^\alpha p_1^\beta, \]  

\[ \langle K^*(p_1)|\bar{q}\sigma^{\mu\nu}\gamma_5 b|\bar{B}(p_B)\rangle = (p_\mu^\mu p_B^\nu - p_B^\mu p_1^\nu) \frac{(p_B \cdot \epsilon^*)}{p_B \cdot p_1} T_3(q^2) + (\epsilon^\nu p_B^\nu - p_B^\nu \epsilon^\nu) T_2(q^2) \]

\[ + [\epsilon^\nu p_1^\nu - p_1^\nu \epsilon^\nu] T_1(q^2). \]  

Another frequently used set of tensor form factors are defined as below [24]:

\[ \langle K^*(p_1)|\bar{q}\sigma_{\mu\nu}q^\nu \frac{(1 + \gamma_5)}{2} b|\bar{B}(p_B)\rangle = 2i\epsilon^{\mu\nu\alpha\beta}p_B^\alpha p_1^\beta T'_1(q^2) \]

\[ + \left[ \epsilon^\mu(p_B^2 - m_{K^*}^2) - (\epsilon \cdot q)(p_1 + p_B)_\mu \right] T'_2(q^2) \]

\[ + (q \cdot \epsilon) \left[ q_\mu - \frac{q^2}{m_B^2 - m_{K^*}^2} (p_1 + p_B)_\mu \right] T'_3(q^2). \]  

They are useful for the discussion of the flavor changing neutral current decay $B \rightarrow K^*\gamma$ and $B \rightarrow K^*\ell^+\ell^-$. The relation between the two set of form factors are

\[ T'_1(q^2) = \frac{1}{4} \left[ T_1(q^2) + T_2(q^2) \right] \]  

\[ T'_2(q^2) = \frac{1}{4} \left[ T_1(q^2) + T_2(q^2) + \frac{q^2}{m_B^2 - m_{K^*}^2} (T_2(q^2) - T_1(q^2)) \right] \]

\[ T'_3(q^2) = \frac{1}{4} \left[ T_1(q^2) - T_2(q^2) - \frac{m_B^2 - m_{K^*}^2}{p_B \cdot p_1} T_3(q^2) \right]. \]  

It is easy to see from the above that at large recoil limit $q^2 = 0$, $T'_1(0) = T'_2(0)$.  

11
As for $B \to \rho$, $B \to \omega$ form factors, the definition is similar to the above, just replacing $K^*$ by $\rho$ and $\omega$ respectively.

Calculating the corresponding amplitude for Fig.1(a) and (b), we get the formulas for the form factors at large recoil as

$$A_0(q^2 = 0) = 8\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \times \{ \alpha_s(t_e) \exp[-S_{ab}(t_e)] [\phi_B(x_1, b_1) \\
(1 + x_2)\phi_{K^*}(x_2, b_2) + (1 - x_2)r_{K^*} (\phi^t_{K^*}(x_2, b_2) + \phi^s_{K^*}(x_2, b_2)) - \bar{\phi}_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) - x_2 r_{K^*} (\phi^t_{K^*}(x_2, b_2) + \phi^s_{K^*}(x_2, b_2))] h_e(x_1, x_2, b_1, b_2) + 2r_{K^*} \phi_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) h_e(x_1, x_2, b_1, b_2) \exp[-S_{ab}(t_e)] \},$$

where $r_{K^*} = m_{K^*}/m_B$. The form factor $A_0$ is the one contributing to the non-leptonic $B$ decays $B \to PV$, where the vector meson is longitudinally polarized. This is shown in the above equation (31) that the formula depends only on longitudinal wave functions. On the other hand, the form factor $A_1$ contributing to the $B \to VV$ decays depends only on transverse wave functions, which is shown below

$$A_1(q^2 = 0) = 8\pi C_F m_B (m_B - m_{K^*}) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \{ h_e(x_1, x_2, b_1, b_2) \exp[-S_{ab}(t_e)] \alpha_s(t_e) \phi_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) + (2 + x_2)r_{K^*} \phi^s_{K^*}(x_2, b_2) - r_{K^*} x_2 \phi^t_{K^*}(x_2, b_2) - \phi_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) + r_{K^*} \phi^t_{K^*}(x_2, b_2) - r_{K^*} \phi^s_{K^*}(x_2, b_2)] + r_{K^*} \phi_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) \phi^s_{K^*}(x_2, b_2) \exp[-S_{ab}(t_e)] \},$$

The form factor $A_2$ can be calculated from eq.(23), using the above eqs.(31, 32) for $A_0$ and $A_1$. It depends on both transverse and longitudinal wave functions.

The vector form factor $V$ depending only on transverse wave functions, is expressed as

$$V(q^2 = 0) = 8\pi C_F m_B (m_B + m_{K^*}) \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \{ \alpha_s(t_e) h_e(x_1, x_2, b_1, b_2) \\
(\phi_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) + (2 + x_2)r_{K^*} \phi^s_{K^*}(x_2, b_2) - r_{K^*} x_2 \phi^t_{K^*}(x_2, b_2) - \phi_B(x_1, b_1) \phi^t_{K^*}(x_2, b_2) + r_{K^*} \phi^s_{K^*}(x_2, b_2) \exp[-S_{ab}(t_e)] \\
+ r_{K^*} h_e(x_1, x_2, b_1, b_2) \phi^t_{K^*}(x_2, b_2) + \phi^s_{K^*}(x_2, b_2) \phi_B(x_1, b_1) \exp[-S_{ab}(t_e)] \},$$

As for the tensor form factors, $T_1$ and $T_2$ contributing to the $B \to K^{*\gamma}$ decay, depend only
on transverse wave functions

\[ T_1(q^2 = 0) = 16\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \left\{ \alpha_s(t_e) h_e(x_1, x_2, b_1, b_2) (\phi_B(x_1, b_1) \\
(1 + x_2)\phi_{K^*}(x_2, b_2) + 2(1 - x_2) r_{K^*}\phi_{K^*}(x_2, b_2) - 2x_2 r_{K^*}\phi_{K^*}(x_2, b_2) \right\} - \\
\bar{\phi}_B(x_1, b_1) \left[ \phi_{K^*}(x_2, b_2) + (1 - x_2) r_{K^*}\phi_{K^*}(x_2, b_2) - (1 + x_2) r_{K^*}\phi_{K^*}(x_2, b_2) \right] \times \exp[-S_{ab}(t_e^1)] + r_{K^*} h_e(x_2, x_1, b_1, b_1) [\phi_{K^*}(x_2, b_2) + \phi_{K^*}(x_2, b_2)] \right] \right\} . (34) \]

While form factor \( T_3 \) depends on both longitudinal and transverse wave functions

\[ T_2(q^2 = 0) = 16\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \alpha_s(t_e) h_e(x_1, x_2, b_1, b_2) r_{K^*} \left[ \phi_{K^*}(x_2, b_2) - \phi_{K^*}(x_2, b_2) \right] \left( \phi_B(x_1, b_1) - \bar{\phi}_B(x_1, b_1) \right) \exp[-S_{ab}(t_e^1)] . (35) \]

\[ T_3(q^2 = 0) = 16\pi C_F m_B^2 \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 db_1 db_2 \left\{ \alpha_s(t_e) h_e(x_1, x_2, b_1, b_2) (\phi_B(x_1, b_1) \\
\phi_{K^*}(x_2, b_2) + (2 + x_2) r_{K^*}\phi_{K^*}(x_2, b_2) - x_2 r_{K^*}\phi_{K^*}(x_2, b_2) \right\} - \\
\bar{\phi}_B(x_1, b_1) \left[ \phi_{K^*}(x_2, b_2) + r_{K^*}\phi_{K^*}(x_2, b_2) - r_{K^*}\phi_{K^*}(x_2, b_2) \right] \times \exp[-S_{ab}(t_e^1)] + 2r_{K^*} h_e(x_2, x_1, b_1, b_1) \phi_{K^*}(x_2, b_2) \right\} r_{K^*} - 2r_{K^*}^2 T_1 - T_2 . (36) \]

6 Numerical Calculations and Discussions

In the numerical calculations we use

\[ \Lambda_{MS}^{(f=4)} = 250 MeV, \quad f_\pi = 132 MeV, \quad f_K = 160 MeV, \]
\[ f_B = 190 MeV, \quad m_0 = 1.4 GeV, \quad m_0K = 1.7 GeV, \]
\[ M_B = 5.2792 GeV, \quad f_{K^*} = 220 MeV, \quad f_{K^*}^T = 180 MeV, \]
\[ M_W = 80.41 GeV, \quad f_\rho = 217 MeV, \quad f_\rho^T = 160 MeV, \]
\[ f_\omega = 195 MeV, \quad f_\omega^T = 160 MeV. \]

(37)

The distribution amplitudes \( \phi_i^l(x) \), \( \phi_i^s(x) \), \( \phi_i^p(x) \) (\( \phi_i^w(x) \)) and \( \phi_i^{K^*}(x) \) of the light mesons used in the numerical calculation are listed in Appendix A.
Figure 2: Contributions to $B \to \pi$ transition form factors ($F^{B\pi}(0)$) from different ranges of $\alpha_s/\pi$,
(a) with the hard scale chosen as virtualities of internal particles including both quarks and gluons;
(b) the hard scale chosen as virtualities of only internal gluons.

Fig.2(a) displays the contributions to $B \to \pi$ transition form factor at the large recoil limit
$q^2 \to 0$ from different ranges of $\alpha_s/\pi$, where the hard scale $t$ is chosen as eq.(21), i.e., the
maximum virtuality of both internal quarks and gluons in the hard $b$-quark decay diagrams. It
shows that most of the contribution comes from the range $\alpha_s/\pi < 0.3$, implying that the average
scale is around $\sqrt{\Lambda_{QCD} m_B}$. For other $B$ to light meson transition form factor calculations, we
have very similar results. It is observed that with the hard scale chosen in eq.(21), PQCD is
applicable to $B \to$ light meson transition form factors. Recent study shows that PQCD is even
applicable to $B \to D^{(*)}$ form factors [9]. However, different perturbative percentage distribution
over $\alpha_s/\pi$ was observed in [19, 22]. We check the reason which causes this difference and find
that the most important reason is the way of choosing the hard scale $t$. If the hard scale is
chosen as the maximum virtuality of only the gluon and other transverse momentum scales,
i.e., $t \equiv \max(\sqrt{x_1 x_2 m_B}, 1/b_1, 1/b_2)$, the perturbative percentage distribution will be similar to
ref.[19, 22], as shown in Fig.2(b). Therefore the way of choosing the hard scale is one of the
important ingredients in the PQCD approach, which deserves more concern. [1] Provided that
the virtuality of the internal quark momentum (longitudinal) must appear as a characteristic
scale in the hard diagram, in general it should be taken into account. Therefore we think that

---

[1] By numerical check we find that these two different choices of the hard scale only slightly affect the mag-
nitude of the form factors. For example it can only change the $B \to \pi$ form factor by a few percent.
it is reasonable to choose both the virtualities of internal quarks and gluons as the hard scale. Certainly the most powerful proof of this point should be performed under the help of numerical calculation of higher order loop corrections. However, such a deeper discussion of this problem is beyond the scope of this paper, which shall be left to other attempts.

Table 1: $B$ meson transition form factors at $q^2 = 0$ with the hard scale chosen in eq.(21), and the numbers in parentheses are results without the contribution of $\bar{\phi}_B$.

| process | $F_0(0) = F_1(0)$ | $F_T(0)$ |
|---------|-------------------|----------|
| $B \rightarrow \pi$ | $0.292 \pm 0.030$ | $0.278 \pm 0.028$ |
|          | $(0.199)$ | $(0.189)$ |
| $B \rightarrow K$ | $0.321 \pm 0.036$ | $0.311 \pm 0.033$ |
|          | $(0.231)$ | $(0.223)$ |

| process | $V(0)$ | $A_0(0)$ | $A_1(0)$ | $A_2(0)$ | $T_1(0)$ | $T_2(0)$ | $T_3(0)$ |
|---------|--------|----------|----------|----------|----------|----------|----------|
| $B \rightarrow \rho$ | $0.318 \pm 0.032$ | $0.366 \pm 0.036$ | $0.25 \pm 0.02$ | $0.21 \pm 0.01$ | $0.56 \pm 0.05$ | $0.013 \pm 0.001$ | $0.06 \pm 0.01$ |
|          | $(0.226)$ | $(0.256)$ | $(0.17)$ | $(0.14)$ | $(0.41)$ | $(0.004)$ | $(0.05)$ |
| $B \rightarrow \omega$ | $0.305 \pm 0.030$ | $0.347 \pm 0.036$ | $0.24 \pm 0.02$ | $0.20 \pm 0.02$ | $0.53 \pm 0.05$ | $0.012 \pm 0.001$ | $0.06 \pm 0.01$ |
|          | $(0.212)$ | $(0.250)$ | $(0.16)$ | $(0.13)$ | $(0.38)$ | $(0.003)$ | $(0.05)$ |
| $B \rightarrow K^*$ | $0.406 \pm 0.042$ | $0.455 \pm 0.047$ | $0.30 \pm 0.03$ | $0.24 \pm 0.02$ | $0.69 \pm 0.08$ | $0.007 \pm 0.001$ | $0.09 \pm 0.01$ |
|          | $(0.293)$ | $(0.336)$ | $(0.21)$ | $(0.16)$ | $(0.51)$ | $(-0.001)$ | $(0.07)$ |

The results of $B \rightarrow P$, $V$ light meson transition form factors are given in Table 1 with the hard scale chosen in eq.(21). Compared with previous PQCD calculations on some $B \rightarrow P$, $V$ transition form factors [8, 9, 23], the current work is different from them mainly on two points:

1. The $B$ meson wave function used here is what derived from the equation of motion in Heavy Quark Effective Theory [15]. There is only one free parameter in the functions of distribution amplitudes, $\bar{\Lambda}$. We show the results for $\bar{\Lambda} = (700 \pm 50)$MeV in table 1. All the form factors are sensitive to this parameter, i.e. sensitive to the shape of the B meson distribution amplitudes.

2. Two Lorentz structure terms of $B$ meson wave function, both $\phi_B$ and $\bar{\phi}_B$ defined in
eqs. (4, 8), are taken into account in this work. To see how large the $\bar{\phi}_B$ term contributes, we give the results without the contribution of $\bar{\phi}_B$ in the parentheses in Table 1. They show that the contribution of $\bar{\phi}_B$ is about 30%. The dominant contribution comes from $\phi_B$ term. This result shows that simply dropping the contribution of $\bar{\phi}_B$ is not a good approximation.

Table 2: Form factors at $q^2 = 0$ for $B \to \pi$ and $B \to \rho$ transitions calculated in this work and UKQCD.

|               | $F_0(0) = F_1(0)$ | $V(0)$   | $A_0(0)$ | $A_1(0)$ | $A_2(0)$ |
|---------------|-------------------|----------|----------|----------|----------|
| UKQCD \[25\] | 0.27 ± 0.11       | 0.35$^{+0.06}_{-0.05}$ | 0.30$^{+0.06}_{-0.04}$ | 0.27$^{+0.05}_{-0.04}$ | 0.26$^{+0.05}_{-0.03}$ |
| this work     | 0.292             | 0.318    | 0.366    | 0.250    | 0.210    |

We compare some of the results calculated in this work with lattice calculation by UKQCD collaboration \[25\] in Table 2. It shows that our results are consistent with theirs.

The $B \to K^*$ form factors are useful for the calculation of flavor changing neutral current process $B \to K^*\gamma$ and $B \to K^*\ell^+\ell^-$, which have been discussed many times \[26\]. We show some of them in Table 3 for comparison. It is easy to see that our results agree with lattice calculations \[25\] and the results calculated using lattice-constrained dispersion quark model \[29\].

7 Summary

We have calculated $B \to P$ and $B \to V$ transition form factors in the PQCD approach. We not only calculate the $B$ to light meson transition form factors defined in vector and axial vector currents, but also form factors defined in tensor currents $\bar{q}\sigma_{\mu\nu}b$ and $\bar{q}\sigma_{\mu\nu}\gamma_5b$, which can be used to study semi-leptonic and radiative $B$ decays induced by magnetic penguin operators $\bar{q}\sigma_{\mu\nu}(1+\gamma_5)bF_{\mu\nu}$. With the hard scale appropriately chosen, Sudakov effects can effectively suppress long-distance dynamics, which makes short-distance contribution dominate. The characteristic scale in $B$ to light meson transition processes is around $\sqrt{\Lambda_{QCD}m_B}$. 
Table 3: Some form factors at $q^2 = 0$ for $B \to K^*$ transitions calculated in this work and some other works.

| Method                        | $T_1'(0)$ | $T_2'(0)$ | $A_0(0)$ | $A_1(0)$ |
|-------------------------------|-----------|-----------|----------|----------|
| quark model [27]              | 0.155     | 0.32      | 0.26     |          |
| QCD sum rule [24]             | 0.19 ± 0.03 | 0.3 ± 0.03 | 0.37 ± 0.03 |          |
| light cone sum rule [28]      | 0.18      | 0.27      | 0.36     |          |
| Lattice QCD [25]              | 0.16$^{+0.02}_{-0.01}$ | 0.33 | 0.29 |          |
| dispersion quark model [24]   | 0.177     | 0.44      | 0.33     |          |
| this work                     | 0.175     | 0.455     | 0.297    |          |

We considered both of the two Lorentz structures of $B$ meson wave functions, and found that the contribution of $\bar{\phi}_B$ defined in eqs. (4, 3) is about 30%.

Finally we compared our results with Lattice calculation, some quark model and QCD sum rule calculations, we found that they are consistent with our results.

Acknowledgments

We thank T. Kurimoto, H.n. Li, S. Mishima and D. Pirjol for reading the manuscript and making helpful comments. This work is supported in part by National Science Foundation of China with contract No.10205017, 90103013 and 10135060.

A Wave Functions of Light Mesons Used in the Numerical Calculation

For the light meson wave function, we neglect the $b$ dependence part, which is not important in numerical analysis.

The distribution amplitude $\phi^A_\pi$ for the twist-2 wave function and the distribution amplitudes $\phi^P_\pi$ and $\phi^t_\pi$ of twist-3 wave functions are taken from [20],

$$
\phi^A_\pi(x) = \frac{3f_\pi}{\sqrt{6}}x(1 - x)\left[1 + 0.44C_2^{3/2}(t) + 0.25C_4^{3/2}(t)\right],
$$

(38)
\[
\phi_P^\pi(x) = \frac{f_\pi}{2\sqrt{6}} \left[ 1 + 0.43C_2^{1/2}(t) + 0.09C_4^{1/2}(t) \right], \\
\phi_T^\pi(x) = \frac{f_\pi}{2\sqrt{6}} t \left[ 1 + 0.55(10x^2 - 10x + 1) \right],
\]

where \( t = 1 - 2x \). The Gegenbauer polynomials are defined by

\[
C_2^{1/2}(t) = \frac{1}{2}(3t^2 - 1), \quad C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3), \\
C_2^{3/2}(t) = \frac{3}{2}(5t^2 - 1), \quad C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1),
\]

whose coefficients correspond to \( m_{0\pi} = 1.4 \text{ GeV} \).

We choose the different distribution amplitudes of \( \rho \) meson longitudinal wave function as

\[
\phi_\rho^\pi(x) = \frac{3f_\rho}{\sqrt{6}} x(1 - x) \left[ 1 + 0.18C_2^{3/2}(t) \right], \\
\phi_T^\pi(x) = \frac{f_T^\rho}{2\sqrt{6}} \left\{ 3t^2 + 0.3t^2 \left[ 5t^2 - 3 \right] + 0.21 \left[ 3 - 30t^2 + 35t^4 \right] \right\}, \\
\phi_\rho^\pi(x) = \frac{3f_\rho}{2\sqrt{6}} t \left[ 1 + 0.76(10x^2 - 10x + 1) \right].
\]

For the transverse \( \rho \) meson we use [21]:

\[
\phi_\rho^T(x) = \frac{3f_\rho}{\sqrt{6}} x(1 - x) \left[ 1 + 0.2C_2^{3/2}(t) \right], \\
\phi_T^\rho(x) = \frac{f_T^\rho}{2\sqrt{6}} \left\{ \frac{3}{4}(1 + t^2) + 0.24(3t^2 - 1) + 0.12(3 - 30t^2 + 35t^4) \right\}, \\
\phi_\rho^T(x) = \frac{3f_\rho}{4\sqrt{6}} t \left[ 1 + 0.93(10x^2 - 10x + 1) \right].
\]

For the \( \omega \) meson, we use the same as the above \( \rho \) meson, except changing the decay constant \( f_\rho \) with \( f_\omega \).

We use \( \phi_A^K \) of the K meson twist-2 wave function and \( \phi_P^K \) and \( \phi_T^K \) of the twist-3 wave functions from [20, 21, 30],

\[
\phi_A^K(x) = \frac{3f_K}{\sqrt{6}} x(1 - x) \left[ 1 + 0.51t + 0.3\{5t^2 - 1\} \right], \\
\phi_P^K(x) = \frac{f_K}{2\sqrt{6}} \left[ 1 + 0.12(3t^2 - 1) - 0.12(3 - 30t^2 + 35t^4)/8 \right], \\
\phi_T^K(x) = \frac{f_K}{2\sqrt{6}} t \left[ 1 + 0.35(10x^2 - 10x + 1) \right],
\]

whose coefficients correspond to \( m_{0K} = 1.7 \text{ GeV} \).
We choose the light cone distribution amplitudes of K* meson longitudinal wave function as [21],

\[
\phi_{K^*}(x) = \frac{3f_{K^*}}{\sqrt{6}} x(1-x) \left[ 1 + 0.57t + 0.07C_2^{3/2}(t) \right], \quad (51)
\]

\[
\phi_{K^*}^T(x) = \frac{f_{K^*}^T}{2\sqrt{6}} \left\{ 0.3t(3t^2 + 10t - 1) + 1.68C_4^{1/2}(t) + 0.06t^2(5t^2 - 3) \\
+ 0.36[1 - 2t - 2t \ln(1 - x)] \right\}, \quad (52)
\]

\[
\phi_{K^*}^s(x) = \frac{f_{K^*}^s}{2\sqrt{6}} \left\{ 3t \left[ 1 + 0.2t + 0.6(10x^2 - 10x + 1) \right] - 0.12x(1 - x) \\
+ 0.36[1 - 6x - 2 \ln(1 - x)] \right\}. \quad (53)
\]

The light cone distribution amplitudes of K* transverse wave function are used as

\[
\phi_{K^*}^T(x) = \frac{3f_{K^*}^T}{\sqrt{6}} x(1-x) \left[ 1 + 0.6t + 0.04C_2^{3/2}(t) \right], \quad (54)
\]

\[
\phi_{K^*}^v(x) = \frac{f_{K^*}^v}{2\sqrt{6}} \left\{ \frac{3}{4}(1 + t^2 + 0.44t^3) + 0.4C_2^{1/2}(t) + 0.88C_4^{1/2}(t) \\
+ 0.48[2x + \ln(1 - x)] \right\}, \quad (55)
\]

\[
\phi_{K^*}^a(x) = \frac{f_{K^*}^a}{4\sqrt{6}} \left\{ 3t \left[ 1 + 0.19t + 0.81(10x^2 - 10x + 1) \right] - 1.14x(1 - x) \\
+ 0.48[1 - 6x - 2 \ln(1 - x)] \right\}. \quad (56)
\]

References

[1] For a review, see G. Buchalla, A.J. Buras and A.E. Lautenbacher, Rev. Mod. Phys. 68 (1996) 1125.

[2] D. Fakirov and B. Stech, Nucl. Phys. B133 (1978) 315; N. Cabibbo and L. Maiani, Phys. Lett. B73 (1978) 418; ibid. 76 (1978) 663 (E).

[3] H.N. Li and H.L. Yu, Phys. Rev. Lett. 74 (1995) 4388; H.N. Li and H.L. Yu, Phys. Lett. B353 (1995) 301; H.N. Li, Phys. Rev. D52 (1995) 3958; H.N. Li and H.L. Yu, Phys. Rev. D53 (1996) 2480; C.H. Chang and H.N. Li, Phys. Rev. D55 (1997) 5577; T.W. Yeh and H.N. Li, Phys. Rev. D56 (1997) 1615; M. Nagashima, H.N. Li, hep-ph/0202127 and hep-ph/0210173.

[4] S. Brodsky and G. Lepage, Phys. Rev. Lett. 43 (1979) 545; S. Brodsky and G. Lepage, Phys. Lett. B87 (1979) 359; S. Brodsky and G. Lepage, Phys. Rev. D22 (1980) 2157;
A. Efremov and A. Radyushkin, Phys. Lett. B94 (1980) 245; J. Botts and G. Sterman, Nucl. Phys. B225 (1989) 62; H.N. Li and G. Sterman, Nucl. Phys. B381 (1992) 129.

[5] Y.Y. Keum, H.-n. Li, A.I. Sanda, Phys. Lett. B504, (2001) 6; Phys. Rev. D63, (2001) 054008; Y.Y. Keum and H.N. Li, Phys. Rev. D63 (2001) 074006.

[6] C.D. Lü, K. Ukai, M.Z. Yang, Phys. Rev. D63, 074009 (2001); C.D. Lü and M.Z. Yang, Eur. Phys. J. C23, (2002) 275.

[7] C.H. Chen and H.N. Li, Phys. Rev. D63, (2000) 014003; C.H. Chen, Y.Y. Keum, and H.N. Li, Phys. Rev. D64 (2001) 112002; Phys. Rev. D66 (2002) 054013.

[8] T. Kurimoto, H.N. Li and A.I. Sanda, Phys. Rev. D65 (2002) 014007.

[9] T. Kurimoto, H.-n. Li, A.I. Sanda, hep-ph/0210289.

[10] C.D. Lü, K. Ukai, Phys. J. C24 (2002) 121; C.D. Lü and K. Ukai, hep-ph/0210206.

[11] E. Kou and A.I. Sanda, Phys. Lett. B525 (2002) 240.

[12] C.H. Chen, Y.Y. Keum and H.N. Li, Phys. Rev. D66 (2002) 054013.

[13] S. Mishima, Phys. Lett. B521 (2001) 252.

[14] C.W. Bauer, S. Fleming, D. Pirjol, I.W. Stewart, Phys. Rev. D63, 114020 (2001); C.W. Bauer, D. Pirjol and I.W. Stewart, Phys. Rev. D65, 054022 (2002); Phys. Rev. D66, 054005 (2002); hep-ph/0211069.

[15] H. J. kawamura, J. Kodaira, C.F. Qiao and K. Tanaka, Phys. Lett. B 523 (2001) 111; preprint hep-ph/0112174.

[16] H.N. Li, hep-ph/0102013.

[17] A.G. Grozin, M. Neubert, Phys. Rev. D55 (1997) 272; M. Beneke, T. Feldmann, Nucl. Phys. B592 (2001) 3.

[18] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B591 (2000) 313.

[19] Z.T. Wei and M.Z. Yang, Nucl. Phys. B642 (2002) 263.
[20] V.M. Braun and I.E. Filyanov, Z Phys. C48 (1990) 239; P. Ball, J. High Energy Phys. 01 (1999) 010.

[21] P. Ball, V.M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. B529 (1998) 323.

[22] S. Descotes-Genon and S.T. Sachrajda, Nucl. Phys. B625 (2002) 239.

[23] C.H. Chen and C.Q. Geng, [hep-ph/0203003].

[24] P. Colangelo et. al., Phys. Rev. D53, 3672 (1996).

[25] L.D. Debbio, J.M. Flynn, L. Lellouch and J. Nieves, Phys. Lett. B416 (1998) 392.

[26] For recent discussions, see for example, C.S. Kim, Y.G. Kim, C.-D. Lü, T. Morozumi, Phys. Rev. D62, 034013 (2000); Phys. Rev. D64, 094014 (2001); A. Ali, E. Lunghi, C. Greub, G. Hiller, Phys. Rev. D66, 034002 (2002); A. Ali, A.S. Safir, Eur. Phys. J. C25, 583-601 (2002).

[27] W. Jaus and D. Wyler, Phys. Rev. D41, 3405 (1990).

[28] T.M. Aliev et al. Phys. Rev. D56, 4260 (1997); Phys. Lett. B400, 194 (1997).

[29] D. Melikhov, N. Nikitin and S. Simula, Phys. Rev. D57, 6814 (1998).

[30] P. Ball, JHEP 09 (1998) 005.