Tachyon condensation and D-branes in generalized geometries

Pascal Grange $^a$ and Ruben Minasian $^{b,c}$

$^a$ School of Natural Sciences, Institute for Advanced Study  
Princeton, NJ 08540, USA

$^b$ Service de Physique Théorique, CEA/Saclay  
91191 Gif-sur-Yvette, France

$^c$ Centre de Physique Théorique, École Polytechnique  
91128 Palaiseau Cedex, France

Abstract

In generalized complex geometry, D-branes can be seen as maximally isotropic spaces and are thus in one-to-one correspondence with pure spinors. When considered on the sum of the tangent and cotangent bundles to the ambient space, all the branes are of the same dimension and the transverse scalars enter on par with the gauge fields; the split between the longitudinal and transverse directions is done in accordance with the type of the pure spinor corresponding to the given D-brane. We elaborate on the relation of this picture to the T-duality transformations and stability of D-branes. A discussion of tachyon condensation in the context of the generalized complex geometry is given, linking the description of D-branes as generalized complex submanifolds to their K-theoretic classification.
1 Introduction

In three years since its appearance, the generalized complex geometry (GCG) \cite{1,2} has had a variety of applications of string theory. The formalism is distinguished by a number of features, such as the parity between $B$-field and diffeomorphisms, and the resulting interpolation between complex and symplectic geometries; there is also a natural twisting by a three-form $H$-flux. All this makes GCG a very natural framework for addressing questions like classification of supersymmetric flux backgrounds or extending the notion of mirror symmetry beyond Calabi–Yau manifolds.

The question of the proper definition of D-branes in the context of generalized complex geometry has also received some attention, and some proposals and results have appeared, based on inner geometric consistency and on physical realizations of generalized complex geometry through localization in supersymmetric sigma-models \cite{3,4}. Generalized submanifolds have indeed been proposed as Abelian generalized complex D-branes by Gualtieri with the motivation of covariance with respect to the $B$-field transformation. Furthermore, studies of $(2,2)$ theories with boundaries have shown the relevance of these objects from the point of view of sigma-models. Different aspects of D-branes in GCG have been discussed in \cite{5,6,7,8,9,10}.

More relevantly for the present discussion, the development of generalized complex geometry has led to some unification between the A and B models of topological strings \cite{6,11}. Since as mentioned the framework naturally incorporates the $B$-field, the inclusion of D-branes can be inspired by the gauge-equivalent picture of a field strength. In particular, it was shown \cite{12} that stable B-branes \cite{13} are mapped by mirror symmetry to stable A-branes, including those supported on non-Lagrangian submanifolds. Such an incorporation of Fourier–Mukai transform into the mapping between pure spinors \cite{14} (see also \cite{15,16}) illustrates the unifying power of the generalized geometry.

One characteristic feature of describing the D-branes as generalized submanifolds is that the gauge fields and the transverse scalars enter on the same footing (just as forms and vectors, or $B$-field and diffeomorphisms) and there is no distinction between big (higher-dimensional) and small (lower-dimensional) branes. Thus the understanding of the D-branes in generalized geometries should eventually lead to natural incorporation of phenomena typically described via T-duality (see e.g. \cite{17}). These will be in the focus of the attention of the present paper.

As such the discussion of D-branes here is decoupled from issues related to preservation of supersymmetry; yet the integrability of a given generalized structure is intimately connected with supersymmetry of flux backgrounds \cite{18}. See \cite{19,20} for a related discussion of supersymmetric generalized D-branes. Generalized submanifolds are locally graphs over gauge bundles. It is interesting to inquire about the consistency and possible relations of this description of D-branes with e.g. K-theoretic classification of D-brane charges. As a first step in this direction, we will concentrate here on the transformation properties involving a change of dimension, either by duality or by some dynamical process. So after a discussion of T-duality transformations of generalized complex branes in section 2, we will
address in section 3 the question how they can be subject to tachyon condensation [21].
We will have to face the lack of a definition for tachyon fields in this framework. We expect
that compatibility conditions such as the Hermitian Yang–Mills equations can be gener-
alized in a way that the symmetry between the ordinary gauge bundles and the one-form
coordinates is restored, corresponding to the symmetry of the winding states and momenta
on the D-brane worldvolume. The two questions of T-duality and tachyon condensation
are tied together in discussing the mirror symmetry of stable triples in section 4, where the
role of winding numbers in generalized geometries is also illustrated.

2 Generalized complex submanifolds, T-duality and D-branes

In this section, we will collect a few basic ingredients for describing D-branes in generalized
complex geometry. The proposal by Gualtieri [2] for branes as generalized submanifolds
will be our starting point.

Let us first review the generalized linear algebra. Consider an $n$-dimensional vector
space $V$ and its dual $V^*$. Going from linear algebra to differential geometry, these will
eventually be the local tangent and cotangent spaces of an $n$-dimensional manifold $M$.
The pairing defined on $V \oplus V^*$ by
\[
\langle X + \xi, Y + \eta \rangle = \frac{1}{2}(\iota X \eta + \iota Y \xi),
\]  
with $X, Y \in V$ and $\eta, \xi \in V^*$, has signature $(n, n)$. A generalized almost complex structure
on $V \oplus V^*$ is an almost complex structure that is orthogonal with respect to this pairing.
A null space with respect to this pairing, or isotropic subspace of $V \oplus V^*$, has dimension
$n$ at most. An $n$-dimensional isotropic subspace is therefore called maximally isotropic.

Examples of maximally isotropic subspaces are given by graphs over a subspace $E$ of $V$ in the following form:
\[
L(E, F) := \{X + \xi \in E \oplus V^*, \xi|_E = \iota_X F\},
\]  
where $F$ is a two-form in $\Lambda^2 E$, or equivalently a map from $E$ to $E^*$. Indeed by virtue of
(2.1), such a graph is isotropic
\[
\langle X + \iota_X F, Y + \iota_Y F \rangle = \frac{1}{2}(\iota_X \iota_Y + \iota_Y \iota_X)F = 0,
\]
and it is maximally isotropic since it is defined by $\dim E$ equations in $V^*$ over the basis $E$.
Moreover, it was shown in [2] that every maximally isotropic space is a graph of this form.

The $B$-field transformation acts on $V \oplus V^*$ (without transforming the vector part), as
\[
X + \xi \mapsto X + \xi + \iota_X B.
\]  
This reads as the action of the exponential $e^B$, which in a basis adapted to the sum $V \oplus V^*$,
takes the form
\[
e^B = \begin{pmatrix} 1 & 0 \\ B & 1 \end{pmatrix}
\]  
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The $B$-field transformation induces a transformation of maximally isotropic subspaces that preserves the projection on $V$, as

$$e^B L(E, F) = L(E, B + F), \quad (2.6)$$

in which sense maximally isotropic subspaces are covariant w.r.t. the $B$-field transformation. The codimension of $E$ is called the type of the maximally isotropic subspaces $L(E, F)$, and it is invariant under both diffeomorphisms and $B$-field transformations. The subspace $L(E, 0)$ is the direct sum of $E$ and its annihilator in $V^*$:

$$L(E, 0) = E \oplus \text{Ann}E. \quad (2.7)$$

The exterior algebra $\Lambda^*V^*$ carries a representation of Clifford$(n, n)$ through the action of $V \oplus V^*$ on a sum $\phi$ of differential forms:

$$(X + \xi)_{\phi} = \iota_X \phi + \xi \wedge \phi, \quad (2.8)$$

$$(X + \xi).((X + \xi)_{\phi}) = \langle X + \xi, X + \xi \rangle_{\phi}. \quad (2.9)$$

Given a spinor, that is, a sum of differential forms, one can associate to it its null space in $V \oplus V^*$. Maximally isotropic subspaces are therefore in one-to-one correspondence with pure spinors $[22]$. Moreover, pure spinors can be represented in a way$^1$ that makes the correspondence manifest:

$$L(E, F) \sim \text{det}(\text{Ann}E) \wedge e^F, \quad (2.10)$$

where we have used the symbol $\sim$ to indicate that the RHS is a representative of a pure spinor line, that is defined up to a multiplicative factor. The differential form $\text{det}(\text{Ann}E)$ is the wedge product of one-forms of any basis of $\text{Ann}E$; as mentioned its dimension (type) is an invariant of the maximal isotropics.

Given this geometric set-up, there is a natural definition for generalized complex branes (GC branes). These objects must be supported on submanifolds of a GC manifold, subject to a compatibility condition with the ambient generalized complex structure $J$. Extending from the linear algebra above to the sum of the tangent and cotangent bundles of an $n$-dimensional manifold $M$, let the generalized tangent bundle of a submanifold of $M$, still denoted by $E$, carrying a $U(1)$-bundle with gauge curvature $F$, be

$$\tau^F_E := \{ X + \xi \in TE \oplus T^*M \mid_{E}, \xi = \iota_X F \}. \quad (2.11)$$

This is a maximally isotropic subspace, the $B$-transform (with $B = F$) of $\tau^0_E$. Demanding that it is stable under the action of the GC structure makes it the tangent bundle of a GC brane. The covariance property (2.6) w.r.t. the $B$-field transformation is what motivated the definition of generalized complex branes in $[2]$. As in the linear case, it can be shown that every maximally isotropic subspace is of this form.

$^1$This is seen by first noting that $L(E, 0)$ is the null space of $\text{det}(\text{Ann}E)$, and then by $B$-transforming both objects, with $B = F$. 

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This definition is in accordance with the more conventional definitions of D-branes. Indeed, the two cases where $J$ is diagonal or anti-diagonal in a basis adapted to the direct sum of tangent and cotangent spaces, corresponding to the cases where $J$ comes from a complex structure (on $M$) and from a symplectic structure (on $M$) respectively, were worked out in chapter 7 of [2]. The generalized complex submanifolds yield respectively the holomorphic bundles corresponding to B-branes, and A-branes of all possible forms, either of Lagrangian or non-Lagrangian, that had been previously derived by world-sheet techniques [23].

Whereas $B$-transformations cannot change the type of a pure spinor, T-duality should, since it acts on pure spinors so as to give the usual changes of dimension on the corresponding branes. Having identified $E$ with a D-brane worldvolume, we can see that the type (i.e. $\text{dim}(\text{Ann}E)$) gives the dimension of the transverse space. This is more than a numerical coincidence and shortly we will elaborate the relation of $\det(\text{Ann}E)$ with the characteristic polynomial of transverse displacements.

2.1 T-duality and generalized complex branes: the D6-brane case

Let us specialize to the case $n = 6$, and consider the Chern character encoding a space-filling D6-brane together with lower brane charges [24] smeared along its worldvolume $E^{(6)}$:

$$e^F = 1 + F + \frac{1}{2}F \wedge F + \frac{1}{6}F \wedge F \wedge F.$$  \hfill (2.12)

Naturally, we expect that T-duality will yield branes of lower dimensions.

Let us consider the simplest case of T-duality in a single (the sixth) direction. This will be enough to see the main idea, and the extension of the result to more general geometrical situations is not difficult. With the standard rule of permutation between transverse scalars and gauge fields,

$$\Phi^6 \longleftrightarrow A_6,$$  \hfill (2.13)

we end up with a D5-brane equipped with a connection $A'_\mu$, where $\mu$ takes values in the remaining directions $\mu = 1, \ldots, 5$. That is to say, connection and curvature $F' := dA'$ are supported on the T-dual worldvolume, and $d$ in the definition of $F'$ only comprehends the tangent directions to the T-dual world-volume, $d = \sum_{\mu=1}^5 dx^\mu \frac{\partial}{\partial x^\mu}$.

However, if we follow these rules to transform the Chern character, the result is not the Chern character $e^{F'}$. Of course curvatures are wedged with differentials of the transverse scalars, thus making odd forms enter the expression, but the zero-order term remains untouched:

$$e^F = 1 + F + \frac{1}{2}F^2 + \frac{1}{3!}F^3$$  \hfill (2.14)

$$\longleftrightarrow 1 + d\Phi^6 + F' + d\Phi^6 \wedge F' + \frac{1}{2}F'^2 + \frac{1}{2}d\Phi^6 \wedge F'^2 + \frac{1}{3!}F'^3.$$  \hfill (2.15)
The Chern character is therefore not covariant with respect to T-duality of the six-dimensional object

\[(\Phi^a, A_{\mu})\]  

(2.16)

describing transverse scalars and gauge fields on the branes.

Consider instead, as in (2.10) the wedge product of the Chern character with the determinant of the conormal bundle of the brane. Of course this modification is immaterial in the case of the D6-brane, since the annihilator has dimension zero, and the corresponding factor is a differential form with degree zero:

\[e^F|_{E^{(6)}} = \det(\text{Ann}E^{(6)}) \land e^F.\]  

(2.17)

But the unit factor is not inert under T-duality. The expansion of the T-dual of \(e^F\) is now weighted by the T-dual of the determinant of the annihilator of the six-dimensional space, which is the one-form \(d\Phi^6\). In particular, the zero-order term coming from the expansion of \(e^{F'}\) or from \((e^F)'\) is weighted by a form whose kernel automatically encodes the location of the brane. Higher-order terms from the expansion of \((e^F)'\) either contain factors of \(d\Phi^6\) (and therefore do not contribute to an overall wedge product with \(d\Phi^6\)), or only contain the reduced field strength \(F'\), thus reproducing the expansion of \(e^{F'}\):

\[\left(\det(\text{Ann}E^{(6)}) \land e^F\right)' = d\Phi^6 \land (e^F)' = d\Phi^6 \land e^{F'} = \left(\det(\text{Ann}E^{(6)})\right)'^\land e^{F'}.\]  

(2.18)

This object is the T-dual of \(L(E^{(6)}, F)\) by the correspondence (2.10), and it is still a pure spinor, precisely the one that we would associate to the T-dual of the D6-brane.

We have therefore shown from ordinary Buscher rules that the product of annihilator and Chern character transforms as D-branes under T-duality\(^3\). This object makes use all the six-dimensional data \((\Phi^a, A_{\mu})\) in any of its T-dual descendants, whereas the Chern character only made use of the gauge part. Thus, starting with a maximally isotropic space for the \(O(6, 6)\) pairing (2.1) of type zero, corresponding to the whole six-dimensional space equipped with a two-form, T-duality acts within maximally isotropic subspaces. Thus, T-duals of D6-branes are in one-to-one correspondence with pure spinors, the parity of the type of which is dictated by the number of T-dualities.

The important point is that the transverse displacements enter symmetrically with the gauge fields on the brane and as already mentioned the dimension of the generalized brane is always six \(^2\). What changes in passing from \(Dp\)-brane to \(Dp'\) is the split between the longitudinal and transverse directions, or windings and momenta along the worldvolume, as encoded in the changes of the type of the corresponding pure spinors.

\(^2\) which we normalize to one; this choice of normalization corresponds to the fact that the maximal isotropics are associated with pure spinor lines.

\(^3\) When the brane is non-Abelian or when more than one T-duality is performed, the formula should be properly covariantized. The exponentiated field strength should be modified to include the commutators of the scalars, and the connection on the normal bundle needs to be included. We will concentrate mostly on the simplest case of Abelian D-branes, and the importance of inclusion of \(\det(\text{Ann}E^{(6)})\) should be already clear in this case.
2.2 Stability conditions and pure spinors

The properties of generalized submanifolds worked out in [2] correspond to equations of motion (as D-branes can be considered as instantons), regardless of the conditions for existence of solutions of these equations. Stability equations imply the equations of motion and replace their analytical solving by a topological problem [26].

For example, let $F$ be the curvature of a holomorphic line bundle,

$$F^{(0,2)} = 0,$$

(2.19)

deriving from an antihermitian connection. In complex dimension $n$, the self-duality equation of the curvature reads [27] as the Hermitian Yang–Mills equation

$$F \wedge \frac{\omega^{n-1}}{(n-1)!} = c\frac{\omega^n}{n!} \text{Id},$$

(2.20)

where $c$ is a constant proportionality factor. It turned out in [13] that demanding that a D-brane preserves supersymmetry induces deformations of the stability equation (2.20). These stringy deformations, for a D-brane wrapping a complex $n$-dimensional submanifold $E$ of a Calabi–Yau manifold, take the form

$$\text{Im}(e^{i\theta} e^{\omega + F})|_E = 0.$$

(2.21)

Taking the limit of small field strength gives back (2.20) with the identification $c = -i \tan \theta$.

The constant phase $\theta$ was shown in [28] to be obtained by mirror symmetry from the constant phase contained in the special Lagrangian condition for an A-brane wrapping a submanifold $L$:

$$\omega|_L = 0,$$

(2.22)

$$\text{Im}(e^{i\theta} \Omega|_L) = 0.$$

(2.23)

Stability conditions should be rephrased in terms of the generalized tangent bundle. Indeed, the stability conditions for D-branes of the A and B models [13] [29] can be encoded in terms of pure spinors, and the conditions are exchanged exchanged by mirror symmetry [12]. This exchange incorporates non-Lagrangian A-branes into mirror symmetry, and includes the mirror transformation that is inverse to the one considered in [28]. We will come back to this point in section 4, after generalizing the notion of stable triples. Stable triples consist of holomorphic line bundles $E_1$ and $E_2$, together with a map $T$ between them. As spaces of one-forms are fibered over bundles in GCG by the definition (2.11) of generalized tangent bundles, stability conditions on triples can be generalized by requiring $T$ to be compatible with this fibered structure.
3 Generalized stable triples from the B-model

3.1 Lower D-brane charges and tachyon condensation

D-branes wrapped on generalized complex submanifolds, in the case of zero three-form H-flux, should admit a description in terms of elements in K-theory of the spacetime, consistently with tachyon condensation. However, we do not have a low-energy effective action for a generalized brane-antibrane pair, such as the Abelian Yang–Mills–Higgs model

\[ S = \int_E dx \left( \frac{1}{4} \left( F^{(1)} \right)^2 + \frac{1}{4} \left( F^{(2)} \right)^2 + \nabla T \nabla T^* + \lambda (TT^* - \alpha^2) \right). \]  

(3.1)

We must instead study maps between generalized tangent bundles. These maps are the only candidates for the tachyon fields in generalized complex geometry. We shall first stick to the ordinary complex case of the B-model, in order to be able to ensure consistency with previously-studied brane condensates in the B-model [30]. Mirror symmetry will be used in the next section to investigate pairs of topological brane-antibranes of the A-model as generalized complex submanifolds subject to tachyon condensation.

We will encounter topological constraints of charge conservation, as in the ordinary case. The charges of brane condensates are classified by fundamental groups [31, 32]. Consider a brane-antibrane pair wrapping some p-dimensional submanifold. Its decay into the closed-string vacuum is inconsistent with charge conservation, as soon as the brane and the antibrane carry bundles with different first Chern classes. There exists a non-zero net \((p - 2)\)-brane charge [24]. This is an example of a situation where the K-theoretic description of D-branes gives an accurate description of the charges that are not necessarily captured by homology.

The connection between the net \((p - 2)\)-brane charge and the tachyon field goes as follows. As the condensate has codimension two in the original submanifold, the tachyon field has condensed (to the non-zero minimum \(\alpha\) of its potential) only along two dimensions, which means that it stays at the zero value along the condensate. The equation \(T = 0\) now serves as an equation for the submanifold wrapped by the condensate, and the tachyon can be seen as a map from the transverse space into the gauge group. In the present case, the gauge group is \(U(1)\) and the transverse space is surrounded by a circle. The behavior of the tachyon at transverse infinity must be encoded in a map from this circle into \(U(1)\). The net charge is therefore classified by \(\pi_1(U(1))\). The non-Abelian cases correspond [33] to higher codimensions, say \(2k\), together with a winding number in \(\pi_{2k-1}(U(2^{k-1}))\). We will consider only the case of codimension two and Abelian gauge fields.

3.2 Morphisms between generalized complex branes

Let us review the analysis of stable triples given in [30]. Let \(E_1\) and \(E_2\) be two holomorphic line bundles supported on a two-plane. They carry connections \(A^{(1)}\) and \(A^{(2)}\) with field strengths \(F^{(1)}\) and \(F^{(2)}\) corresponding to different first Chern classes. We may assume for definiteness that \(E_1\) is trivial, but it will be appear that the field \(T\) only depends on the
difference between the two Chern classes

\[ [F^{(1)}] - [F^{(2)}] = [dx^1 \wedge dx^2]. \]  

(3.2)

The two bundles, considered as a brane and an antibrane, cannot condense into the vacuum since this would violate the conservation of D0-brane charge. The two-dimensional model is exactly solvable, in the sense that holomorphicity, together with Hermitian conditions, or vortex equations

\[ \bar{\partial}T + TA^{(2)} - A^{(1)}T = 0, \]  

(3.3)

\[ ig^{\mu\bar{\nu}} F^{(1)}_{\mu\bar{\nu}} + TT^* \sim \text{Id}^{(1)}, \]  

(3.4)

\[ ig^{\mu\bar{\nu}} F^{(2)}_{\mu\bar{\nu}} - TT^* \sim \text{Id}^{(2)}, \]  

(3.5)

determine the tachyon profile\(^4\). Integrating the holomorphicity equation (3.3) yields

\[ \partial \ln T = A^{(1)} - A^{(2)}. \]  

(3.6)

Due to the unit difference between first Chern classes (3.2), one can gauge the phase of \(T\) at infinity as

\[ T \sim f(r)e^{i\theta}, \]  

(3.7)

where \(\theta\) and \(r\) are the polar angle and the radial coordinate on the plane. At some point \(p\) however \(\theta\) becomes ill-defined, and \(f\) must have a zero at \(p\). The position of \(p\) is interpreted as the position of the condensate D0-brane, corresponding to the difference between the two charges in K-theory associated to the two bundles. This is summarized by the exact sequence

\[ 0 \rightarrow O(0) \xrightarrow{T} O(1) \rightarrow O_p \rightarrow 0. \]  

(3.8)

The three equations (3.3), (3.4) and (3.5) therefore play the same role for the Yang–Mills–Higgs model as the Hermitian Yang–Mills equations (2.20) for holomorphic bundles. They imply the equations of motion derived from the action functional (3.1), hence the name stable triple for \((E_1, E_2, T)\).

In order to describe condensates in generalized geometries, we will have to look for the zeroes of (generalized) tachyon fields. Before investigating maps between generalized complex branes, let us recall a formal relationship between transverse scalars and tachyon fields that was noted in [34]. It is based on the similarity between the vortex equations (3.4), (3.5) and the one (called Hitchin equation) obtained from dimensional reduction of Hermitian Yang–Mills equation for non-Abelian gauge group. In two complex dimensions the Hitchin equation reads

\[ F_{11} + [X, X^\dagger] = c \text{Id}. \]  

(3.9)

The relationship is formal in the sense that \(T\) and \(X\) are different objects for dimensional reasons: the tachyon in the vortex equation makes sense for space-filling D-branes, whereas there is no transverse scalar (no Hitchin equation) in that case. Moreover, both vortex

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\(^4\)The dimension of the identity matrices on the RHS of the vortex equations is the rank of the gauge group.
equations and deformed HYM equations we are studying vortex systems with Abelian gauge fields, whereas non-zero commutators in the Hitchin equation are typical of non-Abelian gauge theory. However the generalized geometry does make a difference due to all branes having the same dimension, allowing us to write down tachyon profiles in terms of transverse displacements, or characteristic polynomials of some submanifold. Transverse displacements are taken as arguments by one-forms spanning the annihilator space, and the form do not commute, because of the exterior algebra, that still exists in the Abelian case. The determinant of the annihilator of a submanifold \( E \) has the same zeroes as the characteristic polynomial
\[
p_X(x) = \det(X - x),
\]
where \( X^\mu = x^\mu \) are the equations that define the submanifold. Transformations of transverse scalars under T-duality induce transformations of annihilators, and by the same token of tachyon profiles of the form \( T = \det(X - x) \). Zeroes of the T-dual of \( T \) are T-dual to the zeroes of \( T \). This fact will be used in the next section when working out mirror images of stable triples. For the time being we may note that any pure spinor written in the form \( (2.10) \) can be regarded as the result of condensation involving a tachyon profile whose zeroes are on \( E \). What remains to be checked is that the graph condition defining the generalized tangent bundle is compatible with such maps.

To this end, let us consider the embedding of the two-dimensional model of \([30]\) in GCG. The bundles \( TM \) and \( T^*M \) are therefore separately stable under the action of the diagonal generalized complex structure \( J \)
\[
J = \begin{pmatrix} J & 0 \\ 0 & -J^* \end{pmatrix}
\]  
(3.11)
that comes from an ordinary complex structure \( J \) on \( M \). Consider a GC D2-brane \( \mathcal{E}_1 \) and a GC anti-D2 brane \( \mathcal{E}_2 \). As reviewed above, the generalized complex branes \( \mathcal{E}_1 \) and \( \mathcal{E}_2 \) are locally graphs over holomorphic \( U(1) \)-bundles \( E_1 \) and \( E_2 \) sharing a base denoted by \( N \), which is a submanifold of \( M \). The generalized tangent bundles are respectively
\[
\tau_{N}^{F(1)} = \{ X + \xi \in TB_1 \oplus T^*M |_{N}, \xi = \iota_X F^{(1)} \},
\]
(3.12)
\[
\tau_{N}^{F(2)} = \{ X + \xi \in TB_2 \oplus T^*M |_{N}, \xi = \iota_X F^{(2)} \}.
\]
(3.13)
Let \( T \) be a morphism between the two generalized tangent bundles \( \mathcal{E}_1 \) and \( \mathcal{E}_2 \). As we are considering Abelian gauge fields, it has to act on the \( U(1) \) fibers of \( E_1 \) and \( E_2 \) through fiberwise multiplication by a scalar function, still denoted by \( T \). As usual in the geometry of principal bundles, we consider a change from local chart \( U_\alpha \) to local chart \( U_\beta \), with the data consisting of the local multiplicative function \( T \), together with the local gauge potentials on the intersection of the two charts. Local gauge potentials are pull-backs of the connection one-forms \( a^{(1)} \), \( a^{(2)} \) by local sections of the bundles:
\[
A^{(1)} = s_1^* a^{(1)}, \quad A^{(2)} = s_2^* a^{(2)}.
\]
(3.14)
Let \( g_1 \) and \( g_2 \) be transition functions of \( \mathcal{E}_1 \) and \( \mathcal{E}_2 \) respectively between the two local charts \( U_\alpha \) and \( U_\beta \). As multiplication by \( T \) maps the \( U(1) \)-fiber of \( E_1 \) to the one of \( E_2 \) above the
same point \( x \), we can obtain a transition function relating \( A^{(2)} \) to \( A^{(1)} \), because sections of the two bundles are exchanged as follows:

\[
s_2(x) = s_1(x)g_1(x)T(x)g_2(x),
\]

(3.15)

\[
A^{(2)} = g_2^{-1}T^{-1}g_1^{-1}A^{(1)}g_1Tg_2 + g_2^{-1}T^{-1}g_1^{-1}\partial (g_1Tg_2),
\]

(3.16)

so that there is a holomorphicity condition on \( T \) since the gauge potentials have definite type, say \((1,0)\):

\[
\partial T = TA^{(1)} - A^{(2)}T.
\]

(3.17)

The curvatures \( F^{(1),(2)} \) transform in the usual fashion, by adjoint \( U(1) \) actions performed on the two bundles separately. The two Abelian field strengths are therefore invariant under change of charts, and so are the defining equations of \( \tau^{F^{(1)}}_N \) and \( \tau^{F^{(2)}}_N \):

\[
\xi = \iota_X \left( g_i^{-1}F^{(i)}g_i \right) = \iota_X F^{(i)}, \quad i = 1,2.
\]

(3.18)

Once supplemented with Hermitian conditions such as (3.4), (3.5), the holomorphicity condition for the tachyon gives again the exactly solvable profile of \([30]\). These Hermitian conditions, with \( T \) the characteristic polynomial of some submanifold, make sense in GCG as reduction of Hermitian Yang–Mills equations on that submanifold.

The generalization of the stable triple analyzed in \([30]\) induces a map \( T \) acting fiberwise on the \( U(1) \)-bundles carried by \( E \), such that the following sequence is exact:

\[
0 \rightarrow \tau^0_N \xrightarrow{T} \tau^C_N \xrightarrow{T^{-1}(0)} 0,
\]

(3.19)

where \( C \) is the difference between the first Chern classes of the bundles \( E_1 \) and \( E_2 \). Consistently with K-theory, we could add the same \( U(1) \)-bundle with curvature \( F \) to \( E_1 \) and \( E_2 \), and would still have a sequence, with the same \( T \), but involving \( \tau^F_E \) and \( \tau^{C+F}_E \). The sequence (3.19) embeds stable triples of the topological B-model into generalized complex geometry.

Given the earlier discussed T-duality rules, we know that this low-dimensional situation can be mapped to brane-antibrane pairs of higher (even) dimensions by applying even numbers of T-dualities, provided the product of tachyon condensation still has codimension two, corresponding to Abelian gauge fields. In particular, generalized D4 brane-antibrane pairs with different first Chern classes carry a generalized D2-brane charge.

## 4 More generalized brane-antibrane pairs

### 4.1 Special Lagrangian branes as tachyon condensates

Since the generalized complex structure (3.11) we have considered so far is diagonal (i.e. corresponds to the ordinary complex structure), we have just given a generalized description of a situation already known in the topological B-model. As already mentioned, taking
an anti-diagonal GC structure should give branes of the A-model [2]. Here we describe the latter by applying T-duality, and ask whether we recover expected features of stable A-branes. As we shall see, different dimensions of A-branes, corresponding to different terms in the expansion of $e^F$ in the pure spinor involved in a stability condition, are tied together in the mirror images of particular stable triples.

The open-string version of the mirror correspondence between pure spinors $e^{i\omega}$ and $\Omega$ was derived in [12] as:

$$e^{i\omega + F} \leftrightarrow \Omega \wedge e^{F'}.$$  \hspace{1cm} (4.1)

Picking the term of the right degree in the expansion of the RHS in powers of the field strength, one finds either the special Lagrangian condition

$$\text{Im}(e^{i\Omega|_L}) = 0,$$  \hspace{1cm} (4.2)

or the stability condition with non-trivial field strength that occurs in the case of a stable five-dimensional non-Lagrangian brane $Y$ [29]:

$$\text{Im}(e^{i\hat{\Omega}|_Y \wedge F}) = 0.$$  \hspace{1cm} (4.3)

We are going to see that both terms in the expansion of the RHS of (4.1) contribute, in the situation where a D4 brane-antibrane pair is mapped by mirror symmetry to a brane-antibrane pair of non-Lagrangian type.

If one starts with a brane-antibrane pair wrapping a two-dimensional submanifold, the mirror bound state will be made of three-dimensional branes, as is seen by a Fourier–Mukai transformation [35, 12] in a local coordinate patch, where $x^\mu$ and $y^\mu$, with $\mu = 1, 2, 3$, are respectively coordinates on the base and fibers of a $T^3$ fibration. However, starting with a D4 brane-antibrane system carrying a net D2-brane charge, mirror symmetry can yield a five-dimensional brane-antibrane pair that can condense into a special Lagrangian brane of the A-model.

In order to explain this better, let us pass to local complex coordinates adapted to a $T^3$-fibration, written as

$$z^\mu = x^\mu + iy^\mu.$$  \hspace{1cm} (4.4)

and consider a field strength of type $(1, 1)$,

$$F = F_{\mu\nu}dz^\mu d\bar{z}^\nu,$$  \hspace{1cm} (4.5)

which can be rewritten in terms of symmetric and antisymmetric matrices $S_{\mu\nu}$ and $A_{\mu\nu}$:

$$F = F_{\mu\nu}(dx^\mu \wedge dx^\nu - dy^\mu \wedge dy^\nu) + iF_{\mu\nu}(dx^\mu \wedge dy^\nu - dy^\mu \wedge dx^\nu)$$  \hspace{1cm} (4.6)

$$=: A_{\mu\nu}(dx^\mu \wedge dx^\nu + dy^\mu \wedge dy^\nu) + iS_{\mu\nu}(dx^\mu \wedge dy^\nu).$$  \hspace{1cm} (4.7)

It can be observed that the dimensionality of the mirror A-brane is induced by the rank of $A$, due to the two field strengths on the A and B side being exchanged by Fourier–Mukai
transform. When this rank is zero, Gaussian integration over the $y^\mu$-coordinates leads to a delta-function with three-dimensional support weighting the result, thus leading to a Lagrangian brane. When this rank is two, only the kernel of $A$, say $y^1$ up to a change of coordinates, is integrated out in the form of a delta-function,

$$\int T^3 e^{dy^\mu d\bar{y}_\mu} e^{F} = \delta(\bar{y}_1 - \mathcal{S}_{\mu\nu} y^\nu) e^{F'},$$

and the mirror A-brane is five-dimensional.

A source of concern whenever non-Lagrangian A-branes in Calabi–Yau three-folds are studied, is the lack of five-cycles to support them: there are no five-dimensional charges in homology if the ambient Calabi–Yau three-fold is simply connected. However, $b_1 = b_5 = 0$ is a key point in the argument of [35], ensuring that gauge bundles carried by D6-branes are trivial and therefore have no moduli. It is therefore questionable whether the assumption of the existence of a $T^3$-fibration, used to perform mirror-symmetry transformations in [12] to obtain objects encoding the stability condition of non-Lagrangian A-branes, is eventually consistent with the very existence of these branes.

A way to escape this puzzling situation is provided by the interpretation of D-branes as charges in K-theory rather than homology. Mirror symmetry applied to a D4 brane-antibrane pair with different first Chern classes as in (3.2), predicts the following configuration of A-branes: the D2-brane condensate maps to a Lagrangian D3-brane, carrying trivial field strength. This is associated to a charge in homology. It has to come from tachyon condensation between a brane-antibrane pair of higher dimension. This pair is the mirror of the pair of stable D4-branes. It can consist of two stable A-branes of dimension higher than three. These branes are coisotropic and carry Chern characters subject to the stability condition (4.3). The Lagrangian three-brane $L$ must correspond to the locus where the tachyon vanishes.

The T-duality transformation of the transverse scalars, once the characteristic polynomial of the transverse scalars is identified with tachyons as suggested in [31], dictates the T-duality transformation rule of the tachyon. In particular, the mirror image of the locus $T^{-1}(0)$ of the B-model is the set of zeroes of the characteristic polynomial of the transverse scalars of the Lagrangian image of the D2-brane condensate. Since this set has codimension two inside the brane-antibrane pair, the brane $L$ is still compatible with the $U(1)$ gauge theory and classification of charges by $\pi_1(U(1))$. If we call $F^{(1)}$ and $F^{(2)}$ the Fourier–Mukai transforms of the two field strengths we have been considering in the B-model, then the mirror image of the sequence (3.19)

$$0 \rightarrow \tau^{F^{(1)}}_Y \stackrel{T}{\rightarrow} \tau^{F^{(2)}}_Y \rightarrow \tau^{[0]}_L \rightarrow 0. \quad (4.9)$$

Generalized stable triples of the B-model therefore correspond by mirror symmetry to pairings between stable non-Lagrangian branes in K-theory, further unifying all possible types of A-branes.

In either of the two cases, mapping stability conditions according to (4.11) shows that, starting with a two-dimensional (resp. four-dimensional) stable B-brane, one ends up with
a special Lagrangian (resp. a stable non-Lagrangian) A-brane. A configuration with a stable D2-brane condensate (wrapped on a holomorphic cycle $C$) from stable four-dimensional brane and anti-brane, with first Chern classes differing by $[C]$, is of the type studied in the previous sections. This shows that the two mappings between pure spinors in the presence of gauge fields, namely T-duality \(2.18\) and mirror-symmetric spinors \(4.1\) ensure that some stable triples of the B-model are transformed by mirror symmetry into a special Lagrangian condensate obtained from a non-Lagrangian brane-antibrane pair.

4.2 Generalized D0 brane-antibrane pairs on a torus

In the generalized set-up, variations of dimension of a brane, either by T-duality or tachyon condensation, can be captured by taking into account a non-vanishing first Chern class, thus giving rise to a generalized submanifold. So far we have disregarded any topological charge that would arise from the global geometry of the dual space fibered over ordinary bundles. This restriction ensured the equation \(3.18\) and the simple lift of the vortex equations to GC branes. Moving to the opposite case of generalized D0-branes, were there can be no net first Chern class for dimensional reasons, we are now going to generalize D0 brane-antibrane systems on a six-torus, taking care of the topological effects coming from winding numbers.

Winding numbers of strings stretched between D0-branes on tori have been shown in \([17]\), using a description of D0-branes by matrices, to give rise to non-vanishing commutators that correspond to a field strength on a T-dual brane. This is a manifestation of T-duality at the level of D-brane field theory. The covering space $\mathbb{R}^6$ of a six-torus is paved by such tori, and a D0-brane localized on $T^6$ can be equivalently described by copies of the brane, one on each torus patch, collectively described by a matrix whose blocks $X^i_{n_1,\ldots,n_6}$ are labeled by space-time direction $i$ and patch numbers $n_1,\ldots,n_6$.

Even in the Abelian case, when blocks have size one, some commutators of matrices have a nonzero value, due to winding numbers. Consider a pair of D0 brane-antibranes sitting at the same point $p$ on the six torus, but between which a string stretches once along two of the circles, corresponding to, say, coordinates $x^1$ and $x^2$. That is to say, we are looking at the matrix configuration

$$X^i_{n_1,n_2,\ldots,n_6} = \delta^{i,1} \prod_{k \neq 1} \delta_{n_k,0} + \delta^{i,2} \prod_{k \neq 2} \delta_{n_k,0}$$ \hspace{1cm} (4.10)

corresponding to a single winding sector. There is a term in the dimensional reduction of ten-dimensional super Yang–Mills to 0+1 dimension, proportional to the square of the matrix

$$Q^{12} = 2\pi R_1 + 2\pi R_2,$$ \hspace{1cm} (4.11)

that can be encoded in the field strength along the D2-brane obtained by T-dualizing along the first two cycles,

$$F := \theta^1 \wedge \theta^2.$$ \hspace{1cm} (4.12)
We single out two coordinates in order to keep within the codimension-two case upon generalization, as will become clear in what follows.

In a generalized description we would like to describe this equivalence as a result of tachyon condensation. This involves trading two cycles on the dual torus (two form-coordinates), for two vector directions supporting a gauge curvature equal to $F$. The forms $\theta^1$ and $\theta^2$ would be traded for gauge fields. This indeed makes sense because the graph conditions, up to a factor of one half that can be absorbed in the projective definition of the corresponding pure spinor $\theta^3 \wedge \cdots \wedge \theta^6 \wedge e^F$, read in a coordinate patch as

$$\xi\big|_{T^{-1}(0)}(X) = (\iota_X F)(X) = X_1 \theta^2 - X_2 \theta^1 =: A$$

where $A$ is a one-form potential whose field strength is equal to $F$. Trading two form-directions $\theta^1$ and $\theta^2$ for vector coordinates is just a modification of the splitting between gauge fields and transverse scalars. The corresponding phase transition would come from the tachyon profile

$$T = \det \left( \left( \phi^1 - p^1 \right) \left( \phi^2 - p^2 \right) \right),$$

where $p^1$ and $p^2$ are the coordinates of the D0-branes on the first two circles of the six-torus. In order to put these fields together, we must look for the generalization of the vortex equations to the present case. In particular, tachyons now have to act on the dual torus to capture the topological charge associated to the winding numbers.

The graph condition in the definition of $\tau_p^{[0]}$, which is

$$\xi|_p = 0,$$

is immaterial because $p$ has dimension zero; $p$ corresponds to the pure spinor $\theta^1 \wedge \cdots \wedge \theta^6$. However, describing the D0-branes as generalized objects, we must not forget that there is a topological charge on the dual torus. Winding numbers around the first two directions prevent from writing the one-forms $\theta^1$ and $\theta^2$ globally as exact forms:

$$\int_{T^2} \theta^1 \wedge \theta^2 = 1.$$  \hspace{1cm} (4.16)

This topological charge is not captured by the local graph equations of the generalized tangent bundle. This has consequences on generalized tachyons. In the situation where the topological charge came from the $U(1)$ fibers, we considered tachyons first as maps into the $U(1)$ fibers, then checked that the graph condition was intact, and concluded that no action on the form-coordinates of the GC brane was induced by the topological charge. Now we are in the extreme situation where the tachyon can only act on the form-coordinates, because those are the only coordinates on the GC branes. We have a dual torus fibered above the point $p$, with the topological charge given by (4.16). Encoding this charge in the two-form

$$\mathcal{F} := \theta^1 \wedge \theta^2,$$

we can consider $\mathcal{F}$ as the field strength deriving from a connection $\mathcal{A}$ on the dual torus:

$$\mathcal{F} =: [D_\mathcal{A}, D_\mathcal{A}],$$

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What made the stable triple a solvable system was the net topological charge \([F^{(1)}] - [F^{(2)}]\), together with holomorphicity and Hermitian conditions. We have just encoded the winding number in a cohomology charge on the dual torus. As for holomorphicity and Hermitian conditions, we are still considering the diagonal GC structure (3.11). All the structures we enjoyed on the original torus are therefore transported on the dual torus, and we can require the tachyon field to be holomorphic and to act by multiplication on the circles of the dual torus in a way that satisfies the vortex equations written in terms of the winding charge, namely

\[
\bar{\partial} T = TA, \tag{4.19}
\]

\[
F + 2TT^* = 0, \tag{4.20}
\]

so that we end up with the same system as in section 3, but written in the dual torus. The fact that the number of objects of dimensions zero and two are exchanged does not have to appear in the equations, since the solution for \(T\) only depends on the difference between the two vortex equations. The same gauge transformations can now be used to obtain \(T\) as the profile (4.14) with a single pole on \(p\), the position the D0-branes. Since \(T\) acts on the dual torus, its counterpart in real space as a map between the D0-branes must be the determinant of the Hodge dual of \((\phi^1 - p^1) \wedge (\phi^2 - p^2)\), which is the annihilator of a D2-brane carrying field strength \(F\).

Having studied the two extreme cases where the topological charge comes entirely from the Chern classes or from a winding sector, we can address more general cases where both kinds of charges are present. Consider a generalized D2 brane-antibrane system wrapping a two-torus inside a six-torus with strings stretching between them such that there is a winding number on a dual two-torus, together with a difference between the first Chern classes \([F^{(1)}]\) and \([F^{(2)}]\). The tachyon acts by multiplication both in the \(U(1)\) and the dual-torus fibers, and the vortex equations split between the tangent and cotangent parts. We expect a condensate that corresponds to a generalized D0-D4 system. Note that just as other products of tachyon condensation, this system is supersymmetric and thus stable.

5 Conclusions

We have considered two simple systems embedded in GCG, namely stable triples (together with their mirror images) and D0 brane-antibrane pairs, whose condensation (or rather expansion) is dictated by topological charges and is consistent with T-duality.

The correspondence between GC branes and pure spinors appears to reproduce T-duality properties expected from D-branes. Moreover, the definition of GC branes by a graph condition over a base which is a \(U(1)\)-bundle by itself allows for defining tachyons as fiberwise multiplication. T-duality performed in three directions can then map holomorphic stable triples (solutions to the vortex plus holomorphicity equations) to a realization of special Lagrangian branes as a condensate of a non-Lagrangian brane-antibrane pair. This is an example of the unifying power of generalized geometries.
Since all GC branes are of the same dimension \textit{(from the generalized viewpoint)}, the momenta and windings along the brane worldvolume enter in a symmetrical way and a map between the tachyon field and the scalars on the D-brane worldvolume arises rather naturally. This symmetry predicts condensation of brane-antibrane pairs into a brane that is higher-dimensional \textit{(from the ordinary viewpoint)} and carries a charge that is predicted with T-duality. In the case of stable triples, the holomorphicity and the difference between Chern classes were the key elements, whereas here the crucial role is played by the flux associated with winding numbers. Indeed, starting from a D0 brane-antibrane pair with non-trivial windings on a torus, the action of the generalized tachyon yields the system of vortex plus holomorphicity equations on the dual torus, corresponding to an expansion into a D2-brane. In a similar fashion, a D2 brane-antibrane pair with both Chern class difference and nontrivial windings can produce via generalized tachyon condensation a stable D4-D0 system. While all this is somewhat reminiscent of the dielectric effect of \cite{40}, we note that the branes here are perfectly Abelian.

In this note, we have concentrated on the simplest situations in order to illustrate the way GCG incorporates the basic features of D-branes. In particular we have not explored any complications due to the non-Abelian dynamics (thus avoiding both multiple branes and condensation to codimension higher than two), or the $\alpha'$ corrections in B-model corresponding to the deformed HYM \cite{221}. Also we have not considered the effects of the NS two-form, neither when talking about the K-theory classes classifying D-brane charges nor when discussing T-duality. Considering the twisted K-theory of \cite{41} and the incorporation of (generic) $B$-field in the discussion of T-duality presents a big challenge. We feel however that these complications should enrich but not invalidate the picture developed here.

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