Variational Tracking and Prediction with Generative Disentangled State-Space Models

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Abstract

We address tracking and prediction of multiple moving objects in visual data streams as inference and sampling in a disentangled latent state-space model. By encoding objects separately and including explicit position information in the latent state space, we perform tracking via amortized variational Bayesian inference of the respective latent positions. Inference is implemented in a modular neural framework tailored towards our disentangled latent space. Generative and inference model are jointly learned from observations only. Comparing to related prior work, we empirically show that our Markovian state-space assumption enables faithful and much improved long-term prediction well beyond the training horizon. Further, our inference model correctly decomposes frames into objects, even in the presence of occlusions. Tracking performance is increased significantly over prior art.

1 INTRODUCTION

Perception of the present and prediction of the future are key requirements for the deployment of autonomous systems in the physical world. Many relevant and concrete perception tasks can be solved given sufficient engineering efforts (Pulford, 2005; Cadena et al., 2016). Adaptation of conceptually simple frameworks to specific scenarios requires the exploitation of constraints to achieve satisfying performance. In tracking, e.g., different target representations (point, bounding box), observations (depth, color), and partial models (appearance, motion) need to be incorporated.

In recent years, learning methods and in particular deep neural networks have enhanced or even replaced hand-crafted perception pipelines, promising competitive performance in the presence of rich data sets. These approaches can loosely be put into three categories. First, components of existing pipelines are replaced by neural components, leaving major parts untouched (Schulter et al., 2017; Dosovitskiy et al., 2015; Yang et al., 2018). Second, complete pipelines are replaced with learnable counterparts, often inspired by the previously dominant solutions (Krizhevsky et al., 2012; Kosiorek et al., 2017; Parisotto et al., 2018; Gordon et al., 2018; Kahou et al., 2017). Third, the data generating process is formulated as a latent variable model and the task of interest expressed as Bayesian inference.

The benefit of the latter is the principled quantification of uncertainty, inclusion of domain knowledge and the applicability of unsupervised and semi-supervised learning algorithms (Eslami et al., 2016; Mirchev et al., 2018). Our work places itself in this category: we tackle multiple-object tracking as approximate Bayesian inference in variational state-space models (Krishnan et al., 2015; Archer et al., 2015; Fraccaro et al., 2016; Karl et al., 2017), a class of models that provides efficient latent representations of sequences of observations.

We adopt Attend, Infer, Repeat (AIR), a model for scene decomposition into disentangled objects. Our contributions are the following:

1. We modify and stabilize AIR and extend it to sequences by adding state-space dynamics.
2. We derive an inference algorithm, Variational Tracking State-Space Inference (VTSSI), to reflect the extended generative model. VTSSI explicitly and efficiently exploits temporal consistency.
3. We verify that our model significantly improves tracking and prediction performance compared to original AIR, as well as two related baselines. Our
The resulting glimpses are implemented as a sequence of binary decisions, position \( p_i \) of objects in a canvas \( \mathbb{R}^{x \times y} \). Objects are cropped and resized to a glimpse \( x_i \times s_i \times y_i \). The element of the list are used to multiply the glimpses \( y_i \) in the generative part, cf. fig. 1b. In contrast to rounding, this forces the counting variable to take on values close to an integer in order to minimize reconstruction error. Optimizing on the fractional remainder is challenging scenarios where objects overlap.

Overall, VTSSI provides a flexible, more interpretable framework for multi-object tracking and prediction. The proposed models converge much faster with significant performance gains over state-of-the-art baselines.

## 2 ATTEND, INFER, REPEAT

Eslami et al. (2016) introduced Attend, Infer, Repeat (AIR), a structured variational autoencoder (VAE) for scene understanding. In contrast to the original VAE (Kingma and Welling, 2014; Rezende et al., 2014) it imposes structure on the generative latent-variable model: it assumes scenes of \( n \in \mathbb{N}_0 \) conceptually similar objects defined by a set of properties \( z_i = \{p_i, s_i, d_i\} \), comprised of the position \( p \in \mathbb{R}^2 \), size of the object \( s \in \mathbb{R}^2 \), and a content description vector \( d \in \mathbb{R}^d \).

Figure 1a illustrates inference and generation in AIR. During inference, an LSTM (Hochreiter and Schmidhuber, 1997) determines the amount of objects \( n \) (implemented as a sequence of binary decisions), positions \( p_i \), and extents \( s_i \) of objects in a canvas \( x \in \mathbb{R}^{x \times y} \). Objects are cropped and resized to a glimpse \( x_i \in \mathbb{R}^{y \times y} \) of fixed extent via a spatial transformer (Jaderberg et al., 2015). The resulting glimpses are fed into a VAE-style encoder to obtain a fixed-size description vector \( d_i \). During generation, the \( d_i \) are decoded into fixed-sized glimpses \( y_i \in \mathbb{R}^{y \times y} \). An inverse spatial transformer conditioned on size \( s_i \) and position \( p_i \) pastes the glimpse back to an empty scene. By summing over all sets \( z_i \), we obtain the full scene. The model is trained by stochastic gradient descent on the evidence lower bound (ELBO; Jordan et al., 1999).

## 3 METHODS

### 3.1 Modifications to AIR

We use both the generative and the inference model of AIR as building blocks of our sequential model. In particular, we attempt to be faithful to one of its central properties: the latent space decomposes into a set of distinct objects, each with a set of structured and partially interpretable properties.

In comparison to vanilla AIR, we applied two modifications described below (and in more detail in appendix B.1), leading to increased training stability, as AIR is known to be hard to train (Kosiorek et al., 2018).

**Continuous Counting** Defying the low-variance gradient estimates of reparameterized random variables, discrete counting variables are difficult to integrate in VAE-flavored models. They typically require variance reduction techniques (Mnih and Rezende, 2016; Mnih and Gregor, 2014) or continuous relaxations (Jang et al., 2017; Maddison et al., 2017).

Instead of recurrent binary one-step decisions as suggested by Eslami et al. (2016), we suggest a feed-forward block that returns a real-valued variable \( c \), which is turned into a sequence of ones equal in length to the integer part of \( c \), followed by the remaining fractional part, followed by an appropriate number of zeroes up until the maximum number of objects; e.g. \( c = 2.4 \) is turned into the sequence \( [1, 1, 0.4, 0, \ldots] \). The elements of the list are used to multiply the glimpses \( y_i \) in the generative part, cf. fig. 1b. In contrast to rounding, this forces the counting variable to take on values close to an integer in order to minimize reconstruction error. Optimizing on the fractional remainder is
Inspired by Graves (2016), where this technique regulates the number of computation steps in a recurrent neural network.

**Centering Objects in Bounding Boxes**
We found the inference model of AIR to struggle with centering objects within bounding boxes. In the static case, this is not sufficiently detrimental to reconstruction performance. However, if the position is not reliably detected in the center of an object, position prediction in our dynamic scenario is difficult.

We countered this phenomenon with a simple regularization: before being pasted onto the canvas, each glimpse $y^{(i)}$ is multiplied by a mask of values in $(0, 1)$ that fades out towards the edges like a bell curve, highlighting on the center. The procedure is depicted in fig. 1c, as well as exemplary bounding boxes from models trained without and with the regularization. Over training, all mask values increase monotonically to 1.

### 3.2 Sequential Components

Applying AIR independently to every frame neglects temporal consistency. Closer analysis reveals three core challenges extending AIR to sequential data, exemplified in fig. 3. We discuss each challenge in the subsequent sections and target each with a respective architecture component, culminating in a sequential generative model and inference framework we call Variational Tracking State-Space Inference (VTSSI). Extensive implementation details can be found in appendix B.

#### 3.2.1 Prevent Label Switching

The order of attention in AIR is arbitrary. Empirically, it learns a spatial policy for attention order, e.g. left-to-right, top-to-bottom (Eslami et al., 2016). With moving objects, this inevitably leads to permutations in object discovery order between frames.

The first component aims at preventing label switches. Rather than independently discovering objects with AIR in every frame, we start from an object description $d^{(i)}$ obtained from the first frame and try to find the corresponding object in subsequent frames. In comparison to AIR, this reverses the inference order of object position $p^{(i)}$ and description $d^{(i)}$. This prevents label switches, while reducing the number of applications of the computationally expensive AIR component from $T$ to one. The implementation is inspired by the fast-
we compute convolution kernels from $d^{(i)}$. From the resulting features of frame $x_t$ and the previous position $p_t^{(i)}$, the updated position $\hat{p}_t^{(i)}$ is inferred. Since its task is to find a previously seen object, we call this component FIND. It is depicted schematically in fig. 2a.

### 3.2.2 Inference for Overlapping Objects

In a single frame, AIR cannot distinguish between multiple overlapping objects and non-overlapping regular objects, since it is not equipped with a semantic understanding of the difference between the two or any other prior information as to the appearance of the objects it is supposed to detect.

If we can assume non-overlapping objects in the first frame, AIR can provide a concise object description $d^{(i)}$, and the FIND module will maintain consistent object order throughout the sequence. We introduce the second component RECT (for rectification) to relax this assumption: rather than relying on AIR’s object description from the first frame, a recurrent neural network (RNN) processes the inference output on the first $K$ frames. This net reaches a consensus $z^{(i)}$ from the $K$ sets $\tilde{z}_k^{(i)}$ of latent variables from applications of AIR on the first $K$ frames, e.g. by means of weighted averaging. Finally, we use the more robust consensus $d^{(i)} \in z^{(i)}$ as the input to the FIND module. This procedure is depicted in fig. 2b.

### 3.2.3 State-space Modeling of Motion

Operating on individual frames, AIR cannot incorporate the governing motion law, hence fails to predict likely future paths from an object’s history.

FIND and RECT are designed to deal with label switches and object overlap. This is largely achieved by improving the inference of object positions across time compared to vanilla AIR. The third component introduces a dynamical system to the position variable. In contrast to FIND and RECT, this affects both the generative and the inference model: the state-space model (SSM) assumption requires us to add an explicit motion random variable $m_t^{(i)}$ to the latent space. It captures higher-order motion description, e.g. velocities, accelerations, or curve radii. This allows us to define Markov transition priors $p(p_t^{(i)}, m_t^{(i)} | p_{t-1}^{(i)}, m_{t-1}^{(i)})$ for state prediction in the next frame given the current state. As we will show empirically—cf. section 5—this allows faithful multi-step object-level prediction. To infer the motion variable, we feed the object position proposals $\hat{p}_t^{(i)}$ from FIND to an RNN. After $M$ frames, where $M$ is at least the order of dynamics assumed, the RNN provides inferred motion proposals $\hat{m}_{M+1:T}^{(i)}$. Both position and motion proposals are fused with prior predictions $p_t^{(i)}$ and $m_t^{(i)}$ from the transition prior. The fusion is achieved by averaging. The procedure is depicted in fig. 2c.

### 3.3 Variational Tracking State-Space Inference

Combining all suggested modules, we arrive at the full architecture, which we call Variational Tracking State-Space Inference (VTSSI). It processes initial frames $x_{1:K}$ separately with AIR; reaches a consensus with RECT; uses this consensus in FIND to determine positions; refines the position estimates with dynamic information by exploiting MOT. This procedure is depicted in fig. 2d. For the generative model, the major change towards AIR is the Markovian evolution of positions $p_t$ and motion descriptions $m_t$ over time.

The model is trained with stochastic gradient descent on the sequential evidence lower bound (ELBO)

$$
\mathbb{E}_q \left[ \ln p(x_{1:T}, p_t^{(i)}, \tilde{m}_{M:T}^{(i)}, d^{(i)}, s^{(i)}) | q(n, p_t^{(i)}, \tilde{m}_{M:T}^{(i)}, d^{(i)}, s^{(i)}) \right]
\lesssim \ln p(x_{1:T}).
$$

Factorizations for $p$ and $q$ can be found in appendix A, implementation details in appendix B.
An interesting feature of VTSSI is its modularity: rather than using the full model with all suggested components, we can choose to use only some of them depending on the downstream task for a more efficient model. We will investigate this in the following section.

3.4 Evaluating Components of VTSSI

We study five models corresponding to the architectures depicted in figs. 1a and 2:

1. AIR (with modifications from section 3.1),
2. FIND (based on AIR),
3. RECT/FIND (i.e. VTSSI without MOT),
4. FIND/MOT (i.e. VTSSI without RECT) and
5. full VTSSI.

We trained these variants on four flavors of Moving MNIST—we use several variants with different features to perform targeted studies of the components of VTSSI: the data show either linear or elliptic motion, and either the first frame is guaranteed to contain only non-overlapping digits or not. We evaluated object counting accuracy as a proxy for robustness towards overlapping digits. Further, we report the accuracy of the position inference against ground truth, as well as prediction accuracy for the two models that make use of MOT (all other models cannot generate coherent sequences by design). The results can be found in table 1.

We make several interesting observations:

FIND drastically improves the inference accuracy when the first frame is sufficiently clean to identify objects. In fact, FIND is on a par with VTSSI in these scenarios, despite being much more lightweight. AIR suffers from label switches and recounting every frame. The results for FIND drop significantly when the assumption of non-overlapping objects in the first frame is removed. This can be mitigated by the introduction of RECT. We hypothesize that the slight drop in performance compared to FIND on non-overlapping first frames hints at room for improvement with the consensus mechanism of RECT.

RECT is very robust w.r.t. overlapping objects, as fig. 4 highlights. FIND, SQAIR (Kosiorek et al., 2018), and the RECT-based VTSSI successfully tackle the sequence on the left side with a clean first frame. When these models do not get access to the first five frames, but start with the cluttered frames 6 and higher, FIND and SQAIR are unable to recover from the wrong count in the first frame. This is a consequence of AIR’s inability to deal with overlapping frames. We note that VTSSI succeeds despite RECT only accessing $K = 5$ frames (i.e. frames 6–10 in this case), all of which have overlapping objects.

MOT by itself generally does not lead to improved inference over FIND. When combined with RECT to form VTSSI, however, generative accuracy increases, even for scenarios where RECT is not strictly necessary—being able to predict helps inference. The full VTSSI handles all variants equally well. It performs well on linear and non-linear motion, with slight advantage on the non-linear, but smooth elliptic movements compared to discontinuous bouncing behavior, which is more difficult to predict.

We conclude that each component fulfills its designated purpose: in the absence of FIND, we observe label switching; in the absence of RECT, overlapping objects cannot be disentangled reliably; in the absence of MOT, prediction is impossible, but even inference performance drops slightly.

Our evaluation also suggests that we can take advantage of the modular composition of VTSSI. For inference, FIND and RECT are the decisive factors. If prediction is not necessary, we can reliably train and use a simpler model.

4 RELATED WORK

Multi-object tracking has been the primal concern of many works (Pulford, 2005). Bewley et al. (2016) propose using a detector and a subsequent state-space model, showing the promise of such methods outside a deep learning context. Neiswanger and Wood (2012) formulate tracking as a mixture of Dirichlet processes operating on top of a feature extraction pipeline without the need for supervision signals. A series of works considers tracking via end-to-end supervised learning (Kahou et al., 2017; Kosiorek et al., 2017; Gordon et al., 2018; Ning et al., 2017), showing that it is possible to represent trackers with neural architectures when annotated data is available.

In video prediction the central concern is the prediction of future frames in a video stream (Srivastava et al., 2015; Babaeizadeh et al., 2018; Denton and Fergus, 2018; Lee et al., 2018). This can be expressed as inference in the underlying generative model, but without a focus on tracking. This is the starting point of our method, which is based on variational sequence models (Bayer and Osendorfer, 2014; Chung et al., 2015). We rely on a state-space formulation where the graphical model has Markov properties (Särkkä, 2013); this has been pioneered in a neural variational context by Krishnan et al. (2015), Archer et al. (2015), Fraccaro et
Table 1: Quantitative tracking and prediction results with variants of Variational Tracking State-Space Inference (VTSSI) on 10000 test set trajectories. Counting accuracy refers to the average percentage of frames for which the amount of present objects is determined correctly. Inference and prediction errors refer to the average per-frame Euclidean distance (unit: pixels) from the inferred or predicted object center to the ground truth, respectively.

| motion overlap in 1st frame | AIR | FIND | RECT/FIND | FIND/MOT | VTSSI |
|-----------------------------|-----|------|-----------|----------|-------|
| linear                      |     |      |           |          |       |
| ✓                           | 97.63% | 5.953 | n/a       | 99.98% | 1.019 | n/a | 99.99% | 1.131 | n/a | 99.99% | 1.035 | 3.442 |
| ✓                           | 97.22% | 5.620 | 91.53% | 2.793 | n/a | 99.70% | 1.225 | n/a | 92.67% | 3.002 | 5.197 | 99.90% | 1.109 | 3.544 |
| elliptic                    |     |      |           |          |       |
| ✓                           | 97.34% | 5.160 | n/a       | 99.98% | 0.973 | n/a | 99.98% | 1.095 | n/a | 99.99% | 0.846 | 2.836 | 99.95% | 1.028 | 2.583 |
| ✓                           | 96.72% | 4.833 | 90.95% | 2.130 | n/a | 99.48% | 1.194 | n/a | 89.55% | 2.282 | 4.365 | 99.54% | 1.076 | 2.676 |

Figure 4: Qualitative example of challenging overlap. In the left half, FIND, VTSSI, and SQAIR successfully infer object properties. The right half shows the same sequence, but the first five frames were dropped so that the initial frame is cluttered. FIND and SQAIR, relying on AIR for discovery, only recognize one object, and are unable to correct. VTSSI recognizes both digits, despite overlap in all $K = 5$ first frames.

4.1 Relation to DDPAE and SQAIR

Two related approaches to ours have been suggested in the literature: Decompositional Disentangled Predictive Auto-Encoders (DDPAE; Hsieh et al., 2018) and Sequential AIR (SQAIR; Kosiorek et al., 2018). Both approaches use attention-based amortized inference to decompose video sequences of moving objects into per-object latent state sequences. Like VTSSI, both approaches borrow the likelihood model $p(x_t \mid \{z_t^{(i)}\})$ of AIR, cf. section 2.

DDPAE focuses on faithful prediction of the tail $x_{K+1:T}$ of a sequence from its head $x_{1:K}$. As a consequence, it is trained on a lower bound to the conditional $p(x_{K+1:T} \mid x_{1:K})$ rather than the joint $p(x_{1:T})$. This also leads to architectural differences: in contrast to VTSSI and SQAIR, DDPAE does not auto-encode the entire sequence, but follows a seq2seq-inspired approach (Sutskever et al., 2014). The sequence head $x_{1:K}$ is only used for inference and never reconstructed. Conversely, the latent states $z_{K+1:T}$ of the sequence tail $x_{K+1:T}$ are never inferred from data, but predicted from the head. Both inference and prediction are implemented by RNNs. DDPAE further models interactions between objects by means of another recurrence that connects inference of individual objects.

SQAIR introduces two inference components: PROP and DISC. PROP handles object propagation between frames. Two recurrent cells update the position, then (based on the new position) update description and presence. DISC discovers new objects. It works much akin to inference in AIR, except that the inference of a new object is informed by the latent states of existing objects from propagation to avoid duplicate discovery. Relying on AIR to this extent, SQAIR inherits its inability to handle overlapping objects in inference for the first time step and assumes non-overlapping first frame. SQAIR can, in principle, support entering and exiting object at arbitrary frames.

Contrasting DDPAE and SQAIR with VTSSI, we conclude that all models share the same ancestor AIR, specifically the non-dynamic part of latent space design and resulting likelihood model. A major distinctive feature of this work is enhancing the state space with an explicit motion variable $m$, capturing the dynamics of motion. This extra variable turns the position transition fully Markov and the overall model into a proper state-space model. In contrast, both DDPAE and SQAIR use recurrent cell states in the transition model, which need to capture the motion information. This reduces the interpretability of the latent state, as the role of the recurrent state is unclear for each specific model, and rules out regularization via priors.

All three models implement significantly different infer-
Figure 5: Test set prediction errors of DDPAE vs. VTSSI on data used in the original publication. Details in section 5.1.

Figure 6: Test set prediction of SQAIR vs. VTSSI on its data set and our data set. The models perform inference on three observations (first vertical line), the observation horizon. After that, object trajectories are sampled generatively without access to further observations and beyond training sequence length (second vertical line).

5 EXPERIMENTS

On top of our ablation studies in section 3.4, we study VTSSI against the baselines DDPAE and SQAIR. The experiments investigate the robustness in inference/tracking and prediction, particularly over longer horizons. We build upon of the Moving MNIST data sets previously studied with the baselines.

All models are trained with stochastic gradient descent on the evidence lower bound. We used the Adam optimizer (Kingma and Ba, 2015). We borrow the curriculum schedule from SQAIR, where the length of the training sequence is increased over training time. For details on the training procedure, the experimental setup, and additional results, see appendices C to E.

5.1 Prediction

Hsieh et al. (2018) provide a data generation process for DDPAE. We build a training and test set from this process to ensure fair comparability, cf. appendix D.1. We train both models with \( T = 20 \) and \( K = M = 10 \), as in the original publication. Starting from inferences with \( K = 10 \), we tested the position prediction error for \( T > 20 \), probing the generalization of the learned predictions. The average performance across a test set of 10000 sequences can be seen in fig. 5. DDPAE and VTSSI are equally faithful to ground truth within the training horizon. However, the recurrent prediction cell of DDPAE is unable to generalize beyond the training horizon, it seems to severely overfit on the training horizon. This is particularly remarkable given DDPAE’s loss is tailored towards prediction.

As with DDPAE, we tried to compare VTSSI to SQAIR on its original data set. Kosiorek et al. (2018) also provide a data generation process. We used the same data generation process, except we removed the noise, which turned prediction comparisons in the confined frames futile. We train both models with \( T = 10 \) and \( K = M = 3 \), as in the original publication. On these data, we find SQAIR and VTSSI to perform equally well, with slight advantage for VTSSI within the training horizon, and for SQAIR outside the training horizon, cf. fig. 6a. We noticed a subtle, but crucial difference in the generation process of these data against the data we used for the results in e.g. table 1: the data generation implements bouncing of the walls in terms of the top left corner of the tight bounding box (i.e. an object bounces in-frame on the top and left border and out-of-frame otherwise).

To examine the effect, we trained SQAIR on the linear data set suggested in section 3.4 with clean first frames, i.e. not SQAIR’s original data set, but well within SQAIR’s assumptions. This data does not generate bouncing behavior in terms of bounding boxes, but the actual object appearance. The result can be seen in fig. 6b. When required to model bouncing behavior,
SQAIR falls short of VTSSI. An example highlighting this observation can be found in fig. 9 in appendix E. SQAIR defines object positions in terms of bounding box corners, not the center (as DDPAE and VTSSI do). We believe that this generally makes it harder to learn accurate object dynamics except when the data set reflects this model assumption. This may lead to instabilities in the recurrent motion propagation cell of SQAIR. Using the object center makes it easier to use a simpler Markov transition. Specific motion behavior of an object can be saved into the motion variable $\mathbf{m}$.

### 5.2 Inference and Tracking

With the same models and data as in our evaluation of prediction, we also examined tracking performance of SQAIR and VTSSI. The results can be seen in fig. 7. On both data sets, we see that the tracking performance of SQAIR drops drastically after around 20 steps. In contrast, VTSSI keeps a constant error over long horizons.

We also added one of the models discussed in section 3.4, VTSSI without the MOT component, which is not necessary for pure tracking. Rather than training a new, reduced model separately, this model is achieved by using the full VTSSI model. At test time, the outputs of its FIND component are directly evaluated. Unaffected by prediction errors, this model achieves even more reliable tracking performance.

An example highlighting this observation can be found in fig. 8 in appendix E.

### 5.3 Discussion

In the previous analysis, we found that VTSSI performs much more robustly in both prediction and inference, especially over long horizons. We speculate that this can be attributed to the relative simplicity of our model: where SQAIR and DDPAE use recurrent cells in a black-box fashion, particularly in motion prediction, we use a state-space model in feed-forward fashion with explicit representation of the dynamic state of the object.

Further, our inference model, specifically the FIND component, is also of a feed-forward nature (with adaptive convolution kernels). We believe that this leads to more stable model components, even for long horizons. Moreover, it drastically reduces the number of applications of the AIR component, particularly compared to SQAIR, which we found to be very beneficial to the robustness. As a side effect, VTSSI trains significantly faster than SQAIR. Using reference implementations of the original authors, our model required at least an order of magnitude less wall clock time until convergence. The amount of parameters was roughly equal and most were used by the AIR base model.

DDPAE and SQAIR each provide orthogonal features not covered by VTSSI—object interaction and vanishing objects, respectively. The flexible modular nature of VTSSI allows to add suitable model components for these purposes. In this work, we chose not to focus on such scenarios, as the presumably simpler scenarios we presented already proved challenging for related models. We plan to add these features in future work.

### 6 CONCLUSION

We introduced Variational Tracking State-Space Inference (VTSSI), a generative disentangled state-space model inspired by Attend, Infer, Repeat (AIR) with a modular neural inference procedure. VTSSI successfully decomposes sequences by describing the objects that make up the scene, and learns a state-space model of the observations that is able to predict faithfully over long horizons. Our experiments show that the inference components that define VTSSI form a modular framework that may be tailored to the task at hand. We further showed that our inference model can overcome limiting assumptions of AIR. In comparison to related state-of-the-art baselines, we significantly improved performance in prediction and tracking.
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Appendix to Variational Tracking and Prediction with Generative Disentangled State-Space Models

A GENERATIVE AND INFERENCE MODEL

The full generative process is

\[
p(x_{1:T}, n, \{p_{1:T}, m_{M:T}, d^{(i)}, s^{(i)}\}_{i=1}^n) = p(n) \prod_{i=1}^n p(d^{(i)})p(s^{(i)}) \prod_{t=1}^T p(x_t | \{p_t^{(i)}, d^{(i)}, s^{(i)}\}) \\
\cdot p(p_t^{(i)}) \prod_{t=2}^T p(p_t^{(i)} | p_{t-1}^{(i)})p(m_{M}^{(i)}) \\
\cdot \prod_{t=M+1}^T p(p_t^{(i)}, m_t^{(i)} | p_{t-1}^{(i)}, m_{t-1}^{(i)}).
\]

The inference procedure can be described as

\[
q(n, \{p_{1:T}, m_{M:T}, d^{(i)}, s^{(i)}\} | x_{1:T}) = q_{\text{RECT}}(n, s^{(i)}, d^{(i)} | \hat{n}_{1:K}, \hat{s}_{1:K}, \hat{d}_{1:K}) \\
\cdot \prod_{k=1}^K q_{\text{AIR}}(\hat{n}_k, \hat{s}_k^{(i)}, \hat{d}_k^{(i)} | x_k) \\
\cdot \prod_{t=1}^{M} q_{\text{FIND}}(\hat{p}_t^{(i)} | p_{t-1}, x_t, d^{(i)}) \\
\cdot \prod_{t=1}^T q(p_t^{(i)} | p_t^{(i)})q(m_t^{(i)}) | m_{M}^{(i)}) \\
\cdot \prod_{t=M+1}^T q_{\text{MOT}}(p_t^{(i)} | p_{t-1}^{(i)}, m_{t-1}^{(i)}) \\
\cdot q_{\text{MOT}}(m_t^{(i)} | \hat{m}_t^{(i)}(p_{t-1}, d^{(i)}, s^{(i)}), p_{t-1}^{(i)}, m_{t-1}^{(i)}),
\]

where \(q_{\text{AIR}}, q_{\text{RECT}}, q_{\text{FIND}}, \) and \(q_{\text{MOT}}\) work as described in sections 2 and 3.2.1 to 3.2.3. Variables with hats are intermediate quantities for keeping the model description reasonably short. They can be interpreted as point mass distributions without priors.

B IMPLEMENTATION DETAILS

Detailed algorithmic design of different components of VTSSI and the model as a whole is listed in algorithms 1 to 6. Several points should be taken into account to facilitate the reading of the pseudo code listings:

- All random variables mentioned in the listings are Normal, hence parameterized by location (or mean) \(\mu\) and scale \(\sigma\) (sometimes collectively referred to as “parameters”). All multivariate Normal distributions are parameterized with diagonal covariance matrices.

- When a random variable instance results from a computational block, e.g., \(d^{(i)} = VAE_{\text{enc}}(x^{(i)}_{\text{att}})\), this denotes that the block’s output carries the parameters of this random variable. Unless specified otherwise, a computational block produces a concatenated vector of the appropriate size that is then split into required parameter values.

- Unless specified otherwise, when a random variable is shown as an argument of a computational block (e.g., \(y_{\text{att}} = VAE_{\text{dec}}(d^{(i)})\)), a sample taken from that random variable is meant to be fed as an input to the block. The sampling step is not shown in the algorithms to avoid notational clutter.

- If a sample from the same random variable is used in different parts of an algorithm or different algorithms, in implementation this is a single sample taken once and used multiple times at different parts of a computational flow.

- Square brackets denote concatenation of multiple vectors inscribed in them.

- The arguments of the recurrent nets are shown with the running \(t\) index that distinguishes between inputs at different time steps to the RNN.

- The symbol \(\odot\) denotes point-wise multiplication.

B.1 Details of AIR Implementation

The implementation of AIR may be broken down into two major parts: inference model and generative model. Here we present the details of our extended AIR implementation, including the modifications described in the main text: namely, position regularization and continuous counting.

Inference Model The inference model of AIR is shown in algorithm 1. First, the count latent variable \(c\) is inferred from the frame \(x\) by the counting CNN\(_{\text{cnt}}\). The sample from \(c\) is squashed by a sigmoid and multiplied by the maximum number of objects \(N\) to arrive at the float number of objects \(\hat{n} \in (0, N)\). \(\hat{n}\) is rounded up to the integer upper bound on the number of objects.
Algorithm 1: AIR Inference

\begin{algorithm}
\textbf{Input} : x - single frame, N - maximum number of objects
\begin{algorithmic}[1]
\State $c = \text{CNN}_{\text{cnt}}(x)$ \hfill // object count latent variable
\State $\tilde{n} = N \ast \text{sigmoid}(c)$ \hfill // float number of objects
\State $n = \lceil \tilde{n} \rceil$ \hfill // int ceiling number of objects
\State $f = \text{CNN}_{\text{pre}}(x)$ \hfill // frame pre-processing
\State $\{s(i), p(i)\}_{i=1}^{n} = \text{LSTM}_{\text{loc}}(\{f\}_{i=1}^{n})$ \hfill // size and position latent variables
\For{$i \in [1, \ldots, n]$}
\State $x_{\text{att}} = \text{ST}(x, s(i), p(i))$ \hfill // inferred object glimpse
\State $d(i) = \text{VAE}_{\text{enc}}(x_{\text{att}})$ \hfill // description latent variable
\EndFor
\State \textbf{Output} : $c, \{s(i), p(i), d(i)\}_{i=1}^{n}$
\end{algorithmic}
\end{algorithm}

$n$ that allows limiting the downstream computation (during test time, $\tilde{n}$ is rounded properly to obtain the inferred integer number of objects). Next, the frame $x$ is preprocessed by the $\text{CNN}_{\text{pre}}$ and then fed at $n$ time steps to the localization $\text{LSTM}_{\text{loc}}$ that outputs the size and position latent variables for every one of the $n$ objects. For every object, a fixed-size glimpse is cropped by the spatial transformer ST from the frame in accordance with the object’s inferred size and position. The inferred glimpse is then encoded by the encoder $\text{VAE}_{\text{enc}}$ into a description latent variable of the object. The inference results in a single count variable $c$ and size, position, and description variables $s(i), p(i), d(i)$ for each of the $n$ objects. All latent variables except $d(i)$ have a distinct interpretation.

Generative Model The generative model of AIR is shown in algorithm 2. The count variable $c$ is converted into $\tilde{n}$ and $n$ the same way as in the inference model. Next, the float number of objects $\tilde{n}$ is split into a list of step values: consecutive 1’s totaling to the integer part of $\tilde{n}$ followed by its single fractional remainder (e.g., $\tilde{n} = 2.4$ is split into the list $[1, 1, 0.4]$). Next, a generative glimpse of each object is decoded from its description $d(i)$ by the decoder $\text{VAE}_{\text{dec}}$. The glimpse then undergoes two consecutive transformations corresponding to position regularization and continuous counting. For position regularization, the glimpse is multiplied by a zero-mean Gaussian bell curve sampled at a uniform grid corresponding to the glimpse pixels in the region $[0, 1]^2$. The scale of the bell curve $\sigma_L$ is a hyperparameter of the model. The intuition behind the position regularization is that the intensities of the pixels at the center of a glimpse start having more effect on the final generation than the ones closer to the borders of a glimpse. This effect prompts the model to infer the object positions (corresponding to the glimpse centers) closer to the centers of the object "pixel mass", in the attempt to place the majority of object pixels in the high-influence zone of an object glimpse. For continuous counting, each of the glimpses in a row is multiplied by the respective step value resulting from splitting $\tilde{n}$. These step values are used to modulate the effect of each consecutive object on the generated frame: every object except the last one makes it fully into the generation, whereas the effect of the last object is partial as determined by the magnitude of the last fractional step. As the fractional remainder of $\tilde{n}$ is differentiable, guided by the gradient signal through the remainder, the model learns to infer the appropriate number of objects in the frame. Lastly, the resulting glimpse is back-transformed into the original frame dimensions by inverse spatial transformer $\text{ST}^{-1}$ using the object size $s_i$ and position $p_i$, and pasted onto the cumulative likelihood mean $\mu_L$. The final mean $\mu_L$ and the fixed scale $\sigma_L$ parameterize the output likelihood $\tilde{x}$ of the generative model. The scale $\sigma_L$ is a model hyperparameter.

Hyperparameters $\text{CNN}_{\text{cnt}}$ consists of three conv. layers with 16 5x5, 4x4, and 3x3 kernels respectively, with ReLU non-linearity applied after convolution. 2x2 max-pooling with strides of 2 is applied after the first and the second conv. layer. The result is flattened and processed by two dense layers with 256 and 128 units and ReLU non-linearity before being linearly transformed to the location and scale of $c$. Before being fed to $\text{CNN}_{\text{cnt}}$, a frame is zero-padded with three pixels from each side. $\text{CNN}_{\text{pre}}$ consists of two conv. layers with 16 3x3 kernels with ReLU non-linearity, each followed by a 2x2 max-pooling layer with stride 2. The result of $\text{CNN}_{\text{pre}}$ is flattened and repetitively fed to $\text{LSTM}_{\text{loc}}$ at $n$ steps. $\text{LSTM}_{\text{loc}}$ has 256 units. Dropout with the rate of $0.4$ is applied at training time to the output of $\text{LSTM}_{\text{loc}}$, which is then post-processed by four separate dense layers with 64 units and ReLU non-linearity to arrive at the location and scale of $s(i)$ and $p(i)$ for every object. The four dense layers are shared between different objects (at $n$ time steps). $\text{VAE}_{\text{enc}}$ and $\text{VAE}_{\text{dec}}$ are implemented as feed-forward nets with two
### Algorithm 2: AIR Generation

| Line | Description |
|------|-------------|
| 1    | $\tilde{n} = N \ast \text{sigmoid}(c)$ // float number of objects |
| 2    | $n = \lceil \tilde{n} \rceil$ // int ceiling number of objects |
| 3    | $\{\text{step}(i)\}_i^n = \text{split}(\tilde{n})$ // split $\tilde{n}$ into $n$ 1-steps (e.g. $\tilde{n} = 2.4$ into $[1, 1, 0.4]$) |
| 4    | $\mu_L = 0$ // likelihood mean |
| 5    | $k = N(\text{meshgrid}([-1, 1]^2) | (0, 0), (\sigma_K, \sigma_K))$ // discrete regularization kernel, discretization such that $k$ and $y_{\text{att}}(i)$ are of equal size. |
| 6    | $k = k / \max(k)$ // normalize kernel to only scale down |
| 7    | for $i \in [1, \ldots, n]$ do |
| 8    | $y_{\text{att}}^{(i)} = \text{VAE}_{\text{dec}}(d^{(i)})$ // generated object glimpse |
| 9    | $\hat{y}_{\text{att}}^{(i)} = y_{\text{att}}^{(i)} \odot k$ // position regularization |
| 10   | $\check{y}_{\text{att}}^{(i)} = \hat{y}_{\text{att}}^{(i)} + \text{step}(i)$ // continuous counting |
| 11   | $\mu_L = \text{ST}^{-1}(\check{y}_{\text{att}}^{(i)}, s^{(i)}, p^{(i)})$ // partial likelihood mean |
| 12   | $\mu_L = \mu L + \mu_L^{(i)}$ |
| 13   | $\check{x} = N(\mu_L, \sigma_L)$ // likelihood |

**Output**: $\check{x}$

---

dense layers with ReLU non-linearity. VAE_{enc}’s layers have 256 and 128 units, whereas VAE_{dec}’s layers have 128 and 256. The output of VAE_{dec} is reshaped to the fixed-sized glimpse and taken through sigmoid (to constrain pixel intensities into $[0, 1]$).

The models are trained with the maximum number of objects $N = 2$ (training with $N = 3$ did not make a difference when there are no more than 2 objects in each frame). The glimpse shape is fixed to 25x25 pixels. The size and position variables ($s^{(i)}$ and $p^{(i)}$) have 2 dimensions (corresponding to X and Y axis). The spatial transformer ST assumes the size range of $[0, 1]$ (1 corresponds to the whole frame) and the position range of $[-1, 1]$ (–1 and 1 correspond to the edges of the frame). To comply with this assumption, the means of $s^{(i)}$ and $p^{(i)}$ resulting from LSTM_loc are taken through sigmoid and tanh respectively. The description variable $d^{(i)}$ is 20-dimensional. The likelihood scale $\sigma_L$ is set to 0.3. The prior $p(s^{(i)})$ is a Normal with the location (0.3, 0.4) and the scale 0.1. The priors $p(p^{(i)})$ and $pd^{(i)}$ are standard normals. The prior $p(c^{(i)})$ has initial location of $-2.0$ linearly annealed to $-3.0$ between 100k and 200k gradient steps, and the scale of 1.0 (negative locations of $p(c^{(i)})$ are necessary to mitigate the observed over-counting tendency of AIR). The scale of the position regularization bell curve $\sigma_K$ is initially set to 0.5, but the initial bell curve $K$ is gradually flattened at 1 during the training. The flattening schedule is $K(t) = (K + p)/(1 + p)$ with the flattening parameter $p$ being linearly annealed from 0.0 to 100.0 at the increments of 0.1 after every 1k gradient steps. At test time, the position regularization is not applied.

### B.2 Details of VTSSI Implementation

VTSSI relies on the inference and generative models of AIR described in the previous section as basic building blocks. The components of VTSSI are introduced into the architecture between the inference and generative model of AIR. Below we first describe the details of each of the components and then the whole model formulated in terms of those components.

#### B.2.1 FIND

FIND is aimed at tracking observed objects at future frames. To this end, the latent object description $d^{(i)}$ inferred from the past frame(s), together with the object’s previous position $p_{i-1}^{(i)}$, are used to discover the object in the current frame $x_t$. Algorithm 3 depicts an implementation of FIND applied to a sequence of consecutive frames, but it is also straightforward to formulate FIND applied to a single frame (as shown in the figs. 2b and 2c). In contrast to the inference model of AIR that infers object position followed by description, FIND infers new position given description.

**Architecture** The object description $d^{(i)}$ is translated to a bank of convolutional kernels $k^{(i)}$ by MLP_{ker}. The output of MLP_{ker} is sized and reshaped in accor-
dance with the required number, height, width, and channels in the kernels (which, except channels, are model hyperparameters). As the conv. kernels depend only on the object description that is assumed to be static (and not dynamic), they are computed only once for efficiency and reused with different frames afterwards. The intuition behind MLP\(_{ker}\) is that it translates the object description from the latent space to the image space, so that translated description can then be used to find the object at its new position in a new image. Next, the frames of the input sequence \(x_{1:T}\) are taken through CNN\(_{find}\) with the first conv. layer parameterized by the conv. kernels \(k^{(i)}\), derived from the object description. CNN\(_{find}\) may also have one or more subsequent conv. layers with globally learned weights. Finally, the conv. features \(f^{(i)}\) extracted from the frame \(x_t\) are concatenated with the object’s position at the previous frame \(p^{(i)}_{t-1}\) and the result is fed through MLP\(_{pos}\) to arrive at the object position variable at the current frame \(p^{(i)}_t\). This process is repeated for every input frame in a row.

**Hyperparameters** MLP\(_{ker}\) consists of two dense layers with 128 and 256 units and ReLU non-linearity. The output of MLP\(_{ker}\) is reshaped into 8 10x10 kernels with a single channel. CNN\(_{find}\) consists of a conv. layers parameterized with the kernels derived by MLP\(_{ker}\) following by two globally learned conv. layers with 16 5x5 and 32 3x3 kernels respectively. 2x2 max-pooling with stride 2 is applied after the first and second conv. layers. The result is flattened and processed by two dense layers with 128 and 64 units and ReLU, then linearly transformed into a 50-dimensional feature vector. MLP\(_{pos}\) consists of two 64-unit dense layers with tanh non-linearity followed by two separate 32-unit dense layers, also with tanh non-linearity, and 2-dimensional linear layers to compute the location and scale of the position variable at the current step \(p^{(i)}_t\). The prior position at the current frame \(P\left(\tilde{p}^{(i)}_t\right)\) is a Normal centered at the (sampled) previous position with the fixed scale of 0.1. The idea behind this prior is to incorporate an inductive bias of coherent object motion: i.e., the next object position is assumed to be in the neighborhood of the previous one. During training the gradients are not flown through the previous position sample used as a prior mean.

**B.2.2 RECT**

When objects in a frame are substantially overlapping or partially present, AIR fails to infer an adequate latent representation of the objects. RECT is aimed at rectifying potentially incomplete or contaminated latent variables inferred by AIR from multiple frames into a robust object representation. Algorithm 4 shows the details of RECT implementation.

**Architecture** As an input, RECT receives count, size, and description latent variables inferred by AIR from each of the first \(K\) frames \(x_{1:K}\) individually. The goal is to arrive from those \(K\) sets of intermediate variables to a single robust set. RECT solves this tasks by weighted averaging of each variable over the \(K\) sets. The \(K\) scalar weights used to average every variable are computed by Bi-LSTM\(_{rect}\), to which the concatenated parameters (locations and scales) of all variables in each of the \(K\) intermediate sets are fed at \(K\) time steps. It is worth mentioning that the input dimensionality of Bi-LSTM\(_{rect}\) at each time step must be fixed to the same number by design. As a consequence, an input at each time step must be concatenated from the same number of latents, which is problematic given the different number of objects that AIR can infer from different frames. To overcome this, either AIR can infer the maximum possible number \(N\) of objects from each frame (together with the count variable \(\hat{c}\) controlling the effective number of objects), or the parameters of the missing objects variables at different time steps can be replaced by zeros. In our experiments, we adopted the former approach.

The resulting normalized scalar weights \(w_{1:K}\) are used first to rectify the intermediate count variables \(\hat{c}_{1:K}\).
Algorithm 4: RECT

Input: $\hat{c}_{1:K}, \{\hat{s}_{i:1:K}, \hat{d}_{i:1:K}\}_{i=1}^N$ - intermediate object count, size, and description latent variables inferred by AIR from the first $K$ frames $x_{1:K}$ individually

// feeding concatenated parameters of all intermediate latent variables inferred from the frame $x_t$ at the $t$-th time step (variables are shown as arguments instead of parameters to avoid notational clutter)

1. $o_{1:K} = \text{Bi-LSTM}_{\text{rect}} \left( \left\{ \left[ \hat{c}_t, \hat{s}_t^{(1)} , \ldots , \hat{s}_t^{(N)} , \hat{d}_t^{(1)} , \ldots , \hat{d}_t^{(N)} \right] \right\}_{t=1}^K \right)$

2. $w_{1:K} = \text{softmax} (o_{1:K})$ // rectification weights

3. $c = \mathcal{N} \left( \sum_{t=1}^K w_t * \hat{c}_t , \mu , \sum_{t=1}^K w_t^2 * \hat{c}_t^2 \right)$ // rectified count

4. $n = \lceil N * \text{sigmoid} (c) \rceil$ // rectified ceiling number of objects

for $i \in [1, ..., n]$ do

5. $s^{(i)} = \mathcal{N} \left( \sum_{t=1}^K w_t * \hat{s}_t^{(i)} , \mu , \sum_{t=1}^K w_t^2 * \hat{s}_t^{(i)}^2 \right)$ // rectified size

6. $d^{(i)} = \mathcal{N} \left( \sum_{t=1}^K w_t * \hat{d}_t^{(i)} , \mu , \sum_{t=1}^K w_t^2 * \hat{d}_t^{(i)}^2 \right)$ // rectified description

Output: $c, \{s^{(i)}, d^{(i)}\}_{i=1}^n$

into $c$. Having determined the count, we can proceed with defining the ceiling number of objects $n$ (as in inference and generative models of AIR described above). Finally, we rectify the size and description variables of the $n$ objects by averaging over the respective intermediate variables in the $K$ sets. It is important, that we perform weighted averaging of random variables and not their samples. Resulting single set of rectified latent variables $c, \{s^{(i)}, d^{(i)}\}_{i=1}^n$ forms the output of RECT. At test time, one may opt for turning the weights $w_{1:K}$ into a one-hot representation, which amounts to picking a single frame and using exactly AIR-inferred latents from that frame as the rectified ones. However, we have noticed that allowing RECT to combine partial information from different frames leads to more robust rectification (cf. fig. 4).

Hyperparameters The forward and backward parts of Bi-LSTM$_{\text{rect}}$ both have 128 hidden units. The forward and backward hidden states at each time step are concatenated and the result is post-processed by two 64-unit dense layers with ReLU non-linearity (the dense layers are shared among different time steps). After being linearly transformed to scalars, the $K$ outputs are taken through softmax to arrive at the normalized weights $w_{1:K}$. Weighted average of a set of $K$ intermediate Normal random variables is obtained by weighting the means by $w_{1:K}$ and weighting the variances (squared scales) by $w_{1:K}^2$ (an independence assumption is made). AIR’s priors are used for the rectified latent variables.

B.2.3 MOT

The components described so far are targeted at inferring the latent representation from the available observations. AIR is capable of understanding a scene, FIND can reliably track the objects seen before, RECT can disentangle object representations. But none of those components is able to predict the future given the observed past. MOT is introduced to fill in this gap, as it includes a state-space model of object motion.

Architecture To model the motion, MOT introduces a new latent variable $\mathbf{m}^{(i)}$ – describing the motion of $i$-th object at the $t$-th frame. Albeit not interpretable, this motion description can in principle carry information about object velocity or other higher-order characteristics of the motion. Architecturally, MOT consists of two components: one aimed at inferring $\mathbf{m}^{(i)}$ from a sequence of past object positions and the other being able to predict the future object position and motion variables given those at the current time step.

As an input, MOT receives a sequence of object positions $\mathbf{p}^{(i)}_{1:T}$ inferred from all frames of the sequence (e.g., by FIND), alongside the size $s^{(i)}$ and the description $d^{(i)}$ of the object. As the first step, MOT infers the motion variables $\mathbf{m}^{(i)}_{M:T}$ at all time steps starting from $M$-th (the positions at the first $M$ steps $\mathbf{p}^{(i)}_{1:M}$ are used to gain initial awareness of the motion pattern, hence the motion variables are inferred starting from the $M$-th step). This is achieved by feeding the positions at the time steps from 1 to $T$, each concatenated with the object size and description, at $T$ time
Algorithm 5: MOT

Input: \( s^{(i)}, d^{(i)} \) - the size and description latent variables of an object, \( \hat{p}_{1:T}^{(i)} \) - the position latent variables of the object at all frames \( x_{1:T} \) inferred by AIR and/or FIND, 
\( M \) - seed motion prefix length, \([w_{\text{min}}, w_{\text{max}}]\) - averaging weight interval

1. \( \hat{m}_{M:T}^{(i)} = \text{LSTM}_{\text{mot}} \left( \left\{ \left[ s^{(i)}, d^{(i)}, p_{1:T}^{(i)} \right] \right\}_t^{T} \right) \) // inferred motion latent variables
2. \( \hat{p}_{1:M}^{(i)} = \hat{p}_{1:M}^{(i)} \) // seed position latent variables
3. \( \hat{m}_{1:M}^{(i)} = \hat{m}_{1:M}^{(i)} \) // seed motion latent variable
4. for \( t \in [M+1, \ldots, T] \) do
   5. \( \hat{p}_{t}^{(i)} = \text{TR}_{\text{pos}} \left( \left[ \hat{p}_{t-1}^{(i)}, \hat{m}_{t-1}^{(i)} \right] \right) \) // position prediction (transition)
   6. \( \hat{m}_{t}^{(i)} = \text{TR}_{\text{mot}} \left( \left[ \hat{p}_{t-1}^{(i)}, \hat{m}_{t-1}^{(i)} \right] \right) \) // motion prediction (transition)
   7. \( w \sim \text{Uniform}(w_{\text{min}}, w_{\text{max}}) \)
   8. \( \hat{p}_{t}^{(i)} = \mathcal{N} \left( w \cdot \hat{p}_{t}^{(i)} + (1 - w) \cdot \hat{p}_{t}^{(i)} \cdot \mu, w^2 \cdot \hat{p}_{t}^{(i)} \cdot \sigma^2 + (1 - w)^2 \cdot \hat{p}_{t}^{(i)} \cdot \sigma^2 \right) \)
   9. \( \hat{m}_{t}^{(i)} = \mathcal{N} \left( w \cdot \hat{m}_{t}^{(i)} + (1 - w) \cdot \hat{m}_{t}^{(i)} \cdot \mu, w^2 \cdot \hat{m}_{t}^{(i)} \cdot \sigma^2 + (1 - w)^2 \cdot \hat{m}_{t}^{(i)} \cdot \sigma^2 \right) \)

Output: \( \hat{p}_{1:T}^{(i)}, \hat{m}_{M:T}^{(i)}, \hat{p}_{M+1:T}^{(i)}, \hat{m}_{M+1:T}^{(i)} \)

steps to \( \text{LSTM}_{\text{mot}} \). The result is a sequence of inferred motion variables \( \hat{m}_{M:T}^{(i)} \) (due to the reasons described above, the \( \text{LSTM}_{\text{mot}} \) outputs at the steps before \( M \) are ignored).

The inferred position variables \( \hat{p}_{1:M}^{(i)} \) and motion variable \( \hat{m}_{1:M}^{(i)} \) are treated as the final position and motion variables at those steps \( \hat{p}_{1:M}^{(i)} \) and \( \hat{m}_{1:M}^{(i)} \) respectively. The final position and motion variables at the steps from \( M + 1 \) to \( T \) are obtained through the remaining part of \( \text{MOT} \): prediction-averaging loop. At every iteration of this loop, starting from the time step \( M + 1 \), the concatenated final position and motion variables at the previous step \( \hat{p}_{t-1}^{(i)} \) and \( \hat{m}_{t-1}^{(i)} \) are taken through the position transition network \( \text{TR}_{\text{pos}} \) and motion transition network \( \text{TR}_{\text{mot}} \) to arrive at the position prediction \( \hat{p}_{t}^{(i)} \) and the motion prediction \( \hat{m}_{t}^{(i)} \) variables at the current time step respectively. Finally, each prediction variable \( \hat{p}_{t}^{(i)} \) and \( \hat{m}_{t}^{(i)} \) is weighted-averaged with the corresponding inferred variable \( \hat{p}_{t}^{(i)} \) and \( \hat{m}_{t}^{(i)} \) to obtain the final variable at time step \( t \) (\( \hat{p}_{t}^{(i)} \) and \( \hat{m}_{t}^{(i)} \)). Averaging weight \( w \in [0, 1] \) is sampled from a uniform distribution with the predefined minimum and maximum bounds. Those bounds can be changed in the course of training to regularize and/or control the relative effect of prediction and inference on the final position and motion variables.

Position and motion transition networks – \( \text{TR}_{\text{pos}} \) and \( \text{TR}_{\text{mot}} \) – jointly comprise the state-space model of \( \text{MOT} \). By applying the transition networks repetitively, one can perform fully generative sampling of future object positions, hence predict future object motion conditioned on the past.

Hyperparameters \( \text{LSTM}_{\text{mot}} \) has 64 hidden units. The hidden state at each time step is post-processed by two separate 32-unit dense layers with tanh non-linearity, followed by linear transformations to compute the location and scale of the inferred motion variables \( \hat{m}_{M:T}^{(i)} \). Each of the two transition networks consists of two 64-unit dense layers with tanh non-linearity, followed by two separate 32-unit dense layers with tanh non-linearity, followed by linear transformations to compute the location and scale of the predicted variable.

The motion variable \( m_{t}^{(i)} \) is a 10-dimensional Normal random variable. The prior \( \mathcal{N}(m_{t}^{(i)} | 0, 0, 0) \) is a standard Normal. At the steps from \( M + 1 \) to \( T \), the position prediction \( \hat{p}_{t}^{(i)} \) and motion prediction \( \hat{m}_{t}^{(i)} \) variables are used as priors for the final position \( p_{t}^{(i)} \) and final motion \( m_{t}^{(i)} \) variables respectively. During training, the averaging weight \( w \) is sampled from the Uniform \([0.01, 0.99]\); during test time the weight is fixed to 0.5.

B.2.4 VTSSI

VTSSI relies on the components described above for accomplishing higher-level task. First, AIR inference model is run on the first \( K \) frames \( x_{1:K} \) separately to infer intermediate object counts, sizes, and descriptions. Next, the intermediate variables are rectified into the
Algorithm 6: VTSSI

Input : $x_{1:T}$ - sequence of frames, $N$ - maximum number of objects, $K$ - rectification prefix length, $M$ - seed motion prefix length

1. $\hat{c}_{1:K}, \{s_{1:i}^{(i)}, d_{1:i}^{(i)}\}_{i=1}^{N} = \{\text{AIR}_{inf}(x_i, N)\}_{i=1}^{K}$ \(\text{// intermediate variables}\)
2. $c, \{s^{(i)}, d^{(i)}\}_{i=1}^{n} = \text{RECT}(\hat{c}_{1:K}, \{s_{1:i}^{(i)}, d_{1:i}^{(i)}\}_{i=1}^{N})$ \(\text{// rectified variables}\)
3. for $i \in [1, \ldots, n]$ do
4. \(\hat{p}_{1:T}^{(i)} = \text{FIND}(x_{1:T}, d^{(i)}, \hat{p}_{0}^{(i)} = 0)$ \(\text{// position inference}\)
5. \(p_{1:T}, m_{M:T}, \hat{p}_{M+1:T}, m_{M+1:T} = \text{MOT}(s^{(i)}, d^{(i)}, \hat{p}_{1:T}^{(i)}, M)$ \(\text{// state-space model}\)
6. $\hat{x}_{1:T} = \{\text{AIR}_{gen}(c, \{s^{(i)}, d^{(i)}, \hat{p}_{1:T}^{(i)}, m_{M:T}, \hat{p}_{M+1:T}, m_{M+1:T}\}_{i=1}^{n})\}_{t=1}^{T}$ \(\text{// likelihood}\)
Output : $\hat{x}_{1:T}, c, \{s^{(i)}, d^{(i)}, \hat{p}_{1:T}^{(i)}, m_{M:T}, \hat{p}_{M+1:T}, m_{M+1:T}\}_{t=1}^{n}$

C TRAINING

The model is trained by maximizing ELBO using Adam optimizer with $\beta_1 = 0.5$ and mini-batches of 64 sequences. The training lasts for 1k epochs, which approximately corresponds to 780k gradient steps. On our hardware setup, this amounts to a wall-clock time of roughly 83 hours. The learning rate is initialized at $1e - 4$ and smoothly annealed down to $1e - 5$ starting after 200k gradient steps at the rate of 0.9 per 20k gradient steps. Gradients are clipped (by global norm) at 5.0 for higher training stability.

C.1 Curriculum

Curriculum learning is used to progressively increase the complexity of task as the model trains. For all models except VTSSI, curriculum starts with the sequences of length 1 (effectively, training AIR on the first frames), with the length being incremented by 1 every 20k gradient steps. VTSSI is trained with the curriculum starting at 6 and with the increments of 1 after every 30k gradient steps.

While training VTSSI, we found it beneficial for overall training stability to train AIR on the first $K$ frames jointly with the full model (i.e., adding AIR ELBO to the VTSSI ELBO in the loss function) during the first several steps of the curriculum. In our experiments, AIR is trained in parallel with VTSSI during the first 3 steps. Starting from the 4-th curriculum step, the AIR ELBO term is dropped from the loss function.

C.2 Baselines

There were minor differences in the hyperparameter configuration of the models trained for comparison with the baselines (with the results reported in fig. 6). We list those differences here.

DDPAE The glimpse shape was set to $[32, 32]$ instead of $[25, 25]$. The size of the computed conv. kernels of FIND were set to $[12, 12]$ instead of $[10, 10]$. The averaging weight of MOT was fixed at 0.5 during training (no random sampling). Learning rate was annealed down to $5e - 5$ instead of $1e - 5$. Curriculum started at the sequences of length 2 instead of 6. $K = M = 10$ instead of 5.
**SQAIR** RECT component was not used, as the first frame was clean. The mean of the position variables inferred by FIND and predicted by MOT were scaled with by factor of 1.5 (after taking through tanh) to allow placing objects partially out of frame. The averaging weight of MOT was sampled from Uniform[0.1, 0.9] instead of Uniform[0.01, 0.99] at training time. \( K = 3 \) instead of 5. \( T = 10 \) instead of 20.

## D EXPERIMENTS

### D.1 Data set details

Our datasets consist of 50,000 training, 10,000 validation, and 10,000 test sequences with variable number of MNIST digits moving within 50x50 frames. The length of the sequences is 20. The number of digits in each sequence is sampled uniformly at random from \{0, 1, 2\}, but is fixed for each sequence. MNIST’ digits for each sequence are sampled uniformly at random from the original MNIST dataset. The MNIST digits in our test set are sampled only from the MNIST test set, whereas the ones in our training and validation sets are sampled only from the MNIST training set.

Four versions of our dataset are determined by combination of two factors:

- whether digit motion is linear or elliptic
- whether two digits in the first frame are allowed to overlap

The digits are placed at random position in the initial frame with the conditions of residing within the frame. In non-overlapping first frame dataset two digits are not allowed to overlap in the first frame: i.e., they may not share non-zero intensity pixels (but may still overlap in further frames).

In the dataset with linear motion, random velocity vector is sampled for each digit and kept constant during motion, except flipping the components of the velocity at the edges of the frame: when at least one pixel of the digit goes out of frame after a motion step, the digit bounces off the edge.

In the dataset with elliptic motion, random elliptic trajectory is sampled for each digit such that a digit stays within the frame while moving along it. Angular velocity of each individual object is also sampled randomly and kept constant throughout the sequence.

As the velocity magnitudes are sampled from uniform distributions, while objects are moving, their positions take real values. Instead of rounding the position to the nearest integer pixel and pasting the same constellation of pixels as in the original digit at a new discrete position, we maintain the real position values and through bilinear interpolation smoothen the digit motion. We believe that this makes our datasets closer to real video sequences, where object motion is typically smooth.

### D.2 Evaluation details

The accuracies reported in table 1 are computed by dividing the number of sequences, where the number of objects is correctly inferred by the total number of sequences in the test set. AIR’s accuracy is computed per-frame, as it may infer different numbers of objects from different frames of a single sequence (e.g., when the objects are highly overlapping).

The position error reported in table 1 and figs. 5 and 6 is computed as a distance in pixels between the ground truth object position (part of the dataset meta-data) and the positions inferred or predicted by the model. Ground truth object positions in all datasets correspond to the geometric centers of the tight bounding box.

**SQAIR** SQAIR and VTSSI models with the prediction performance reported in fig. 5 were trained on the data generated by the script from the official DDPAE repository\(^1\). The test set was also generated by the DDPAE script, because the original Moving MNIST dataset lacks ground truth position annotation. It is worth mentioning that VTSSI was trained on 50,000 20-frame sequences, whereas DDPAE was trained on streaming data (with every batch being randomly generated). The performance of both models reported in fig. 5 is evaluated on the test set.

**DDPAE** DDPAE and VTSSI models with the prediction performance reported in fig. 6 were trained on three different datasets corresponding to the two figures. SQAIR data corresponding to fig. 6a was generated by the data generation script from the official SQAIR repository\(^2\), without noise and acceleration in digit motion. Our linear data corresponding to fig. 6b is comprised of 10-frame sequences structurally similar to our non-overlapping linear dataset, with the exception of all frame edges being virtually shifted 3 pixels away from the center. This is to allow the digits going deeper out of frame before bouncing (for higher similarity with SQAIR’s data). Model performance reported in fig. 6 is evaluated on hold-out test sets.

\(^1\)https://github.com/jthsieh/DDPAE-video-prediction
\(^2\)https://github.com/akosiorek/sqair
boxes around the object. The positions inferred or predicted by the models are translated into pixel coordinates before being compared with the ground truth positions. The position error is computed per inferred object and not per sequence: i.e., if there are two objects in one sequence, those are treated as two different subjects of comparison. When there are multiple possible matchings between ground truth and inferred objects, we pick the matching that minimizes the summed distance error on a prefix of a sequence. Observation horizons of the models are used as the length of matching-determining prefixes (e.g., 10 in VTSSI vs. DDPAE and 3 in VTSSI vs. SQAIR evaluation).

At test time, DDPAE and VTSSI replace random variables in the computational graph by their modes. This proves to yield more accurate one-shot long-term predictions of object motion. As the SQAIR code from the official repository samples generative trajectories randomly, this would give a comparative disadvantage to SQAIR. For this reason, during evaluation we have modified SQAIR code to replace all random variables by their modes, the same way as DDPAE and VTSSI do. This modification substantially improved the prediction performance metrics of SQAIR. We also modified the configuration of the trained SQAIR models to avoid dropping the objects from the sequence, even when they disappear behind an edge of a frame. After this change SQAIR always preserved the objects inferred from the first frame throughout the sequence.

E FURTHER RESULTS

Figures 8 and 9 show an example of prediction and tracking of a long sequence, highlighting the findings discussed in sections 5.1 and 5.2.

Figures 10 and 11 show further sequences with flavors of VTSSI, comparable to fig. 3.

Figure 12 shows inference of VTSSI on random sequences. The top 12 are elliptic, the bottom 12 are with linear motion.

Figure 13 shows seeded generative prediction on the same sequences as fig. 12. The inference seed horizon is $K = M = 5$. The depicted frames are ground truth, the bounding boxes are superimposed from the predictions.

Figure 14 shows the same predictions, but with generated frames instead of ground truth.
Figure 8: Tracking performance of SQAIR vs. VTSSI on the same sequence (from the SQAIR dataset).

Figure 9: Prediction performance of SQAIR vs. VTSSI on the same sequence (from our dataset).

Figure 10: Sequences with non-overlapping initial frames evaluated an all flavors of VTSSI.
Figure 11: Sequences with overlapping initial frames evaluated all flavors of VTSSI.
Figure 12: Inference of VTSSI superimposed on ground truth frames of random sequences.
Figure 13: Generative predictions of VTSSI superimposed on ground truth frames of the same random sequences as in fig. 12. Generation is seeded with $K = M = 5$ frames of the ground truth.
Figure 14: Same as fig. 13, but displaying generated frames instead of superimposing on ground truth frames.