Equivalent method of coarsening permeability in low permeability reservoirs

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Abstract. In view of considering reservoir segregation, it is preferred that the permeability coarsening method is of great importance. Based on the mesh coarsening technique, the conceptual model of low permeability reservoir segregation distribution is established, and different permeability coarsening methods are selected to simulate the daily production of oil field and other parameters and the error analysis. The most appropriate permeability equivalent method and the grid step size are selected at last. It is concluded that the harmonic-arithmetic mean method is the best method to simulate errors of the parameters, and it is also the best way to calculate the equivalent permeability in low permeability reservoir considering the reservoir segregation distribution. After the coarsening, the grid step on horizon is 20 m or so and on vertical is 4-6m.

1. Introduction
Reservoir numerical simulation is widely used in studying the reasonable development of various types of reservoirs. It can be used to study the key technical problems, optimize the development plan, analyze and forecast the development dynamics, and optimize the economic scheme. Therefore, numerical simulation in the development of modern oil and gas fields is essential [1]. Reservoir heterogeneity research is the most imperative content of reservoir description. It plays an important role in controlling the distribution and exploitation of remaining oil, which is a basis and key factor to improve oil and gas reserves and production [2]. Reservoir segregation is one of the main factors to form reservoir heterogeneity. The reservoir segregation is the rock stratum which is substantially impermeable to the vertical. It can separate the upper and lower reservoirs. The thickness is generally large and the lateral continuity is good.

A complete reservoir description of the number of grid nodes sometimes up to tens of millions, such a large grid data cannot be directly used for reservoir numerical simulation software. Therefore, it is necessary to coarsen the fine reservoir description data, reduce the amount of grid data and include as much as possible the rich geological information in the original model. The permeability is the tensor parameter related to the flow direction. The other parameters are the scalar parameters of the grid point, and it can be combined by the volume weighting method. Therefore, the key step to coarsen the mesh is permeability coarsening, and the mesh coarsening technique is also directed at it [3].
In view of the coarsening method of reservoir permeability, a lot of researches have been conducted. Initially, a simple mathematical average method was used to coarsen permeability. For example, Liu Fuping [4-6] and others suggested that the fine grid should be used automatically, and the pressure distribution should be solved by direct solution in the area with severe change of permeability. In the region with little change in permeability, the equivalent permeability can be calculated by using the arithmetic mean and harmonic mean weighted averaging algorithm. Based on the REV (Representative Elementary Volume) theory, Qi Dasheng [7] proposed a new concept, a representative elementary volume grid, which can be best described in a given scale range reservoir heterogeneity. Considering the influence of fractal spatial distribution and property on flow, Yao Jun [8] proposed a mathematical model to solve the equivalent permeability of fractured porous media and gave the boundary element method of mathematical model. Based on the finite analysis algorithm, Liu Zhifan [9] proposed a numerical method for calculating the coarsening permeability of nonuniform porous media. The method can calculate the complete permeability tensor and has high computational efficiency.

From the above, the authors obtained the equivalent method based on different theories. However, the effect of the segregation on reservoir heterogeneity has not been studied systematically. In this paper, the corresponding conceptual model for the segregation is established and different equivalent methods and grid steps are used to reservoir numerical simulation. Eventually, the most accurate simulation method will be obtained.

2. Mesh coarsening method and error analysis

2.1. Mesh coarsening method

The mesh coarsening method can be divided into three categories: simple average method, compound average method and flow method based on three micro-solution. Simple averaging methods include arithmetic averaging (A is indicated below), geometric averaging (G is indicated below), harmonic averaging (H is indicated below) and power averaging (P is indicated below). Compound averaging methods include harmonic - arithmetic mean method (H-A is indicated below), arithmetic - harmonic averaging method (A-H is indicated below) and RMS coarsening algorithm (RMS is indicated below). The flow method based on the three-dimensional numerical solution not only takes into account the coarsening of the static properties, but also the fluidity of the fluid. At present, this kind of grid method mainly includes non-flow boundary method, linear flow boundary method and adjoint matrix method.

2.2. Error analysis

Error analysis methods include average cumulative error analysis and single point error analysis.

2.2.1. Average cumulative error analysis. The results of the rough model at different time points and the results of the fine model are analyzed as follows:

$$E_{rx} = \frac{1}{m} \sqrt{\sum_{i=1}^{m} (X_{icoarse}^i - X_{ifine}^i)^2} \quad X_{ifine}^i$$

$$E_r = \frac{1}{n} \sum_{j=1}^{n} E_{rxj} \quad (1)$$

Where Er,X is X parameter average cumulative error, m is the X parameter’s number of outputs, Xicoarse is the Xth parameter value of rough model in the i-th time, Xifine is the Xth parameter value of refine model in the i-th time, n is the number of error analysis parameters, Er is the average error of n parameters.
2.2.2. Single point error analysis. The results of the rough model at a certain point in time and the results of the fine model are analyzed as follows:

\[ E_r = \frac{1}{n} \sum_{i=1}^{n} \sqrt{(X_{\text{coarse}}(i) - X_{\text{fine}}(i))^2} \]

3. Model establishment and model parameters

In this paper, Petrel and CMG reservoir numerical simulation software were used to establish the sand and shale interbed by using the phased modeling method of sequential indication simulation and five-point modeling method. Figure 1 is the model well position plan. The injection wells B1 is in the middle, surrounded by production wells A1, A2, A3, A4. Figure 2 is cross-section of stratigraphy. The depth is located in the ground 1400m. Sandstone thickness is 6m and mudstone thickness is 4m, which the total thickness is 36m. Initial formation pressure is 140 MPa. Crude oil density is 867 kg/m³. Water density is 1000 kg/m³.

![Figure 1. Model well position plan.](image)

![Figure 2. Cross-section of stratigraphy.](image)

4. Coarse method optimization and error analysis

The fine model (5m×5m×2m) was subjected to the transverse coarsening (10m × 10m × 2m) for error analysis. The error analysis includes nine aspects: cumulative oil production, daily oil production, daily liquid production, crude oil geological reserves, oil saturation, oil recovery, average formation pressure, water saturation and oilfield water intrusion. For example, the results of cumulative oil production are showed in Figure 3 and the cumulative oil production error analysis is showed in Figure 4. Core samples of three-dimensional CT image file into the Mimics software can get the core of three-dimensional reconstruction models. Pressure boundary conditions set the model for the import and export, to import and export the z distribution, the bottom surface of the model is set to import and export pressure boundary, respectively, the other four set to a surface does not penetrate the border.

It can be seen from Figure 4 that the roughing result error of the harmonic-arithmetic mean method is the smallest and the roughing result error of the RMS roughing method is the biggest. The average cumulative error analysis is shown in Table 1 and Figure 5.
Table 1. Porosity of rock core image.

| Item                      | A (arithmetic averaging) | A-H (arithmetic-harmonic averaging method) | G (geometric averaging) | H (harmonic averaging) | H-A (harmonic-arithmetic mean method) | P (power averaging) | RMS (RMS coarsening method) |
|---------------------------|--------------------------|-------------------------------------------|-------------------------|------------------------|----------------------------------------|---------------------|------------------------------|
| cumulative oil production | 0.1254                   | 0.1277                                    | 0.135                   | 0.1219                 | 0.1252                                 | 0.1332              | 0.1414                       |
| daily oil production      | 0.3321                   | 0.3100                                    | 0.3524                  | 0.3144                 | 0.2921                                 | 0.3606              | 0.3457                       |
| daily liquid production   | 0.3475                   | 0.3500                                    | 0.3516                  | 0.3614                 | 0.3509                                 | 0.382               | 0.3464                       |
| crude oil geological reserves | 0.1292                  | 0.1438                                    | 0.1343                  | 0.1347                 | 0.1269                                 | 0.1362              | 0.1333                       |
| oil saturation            | 0.1303                   | 0.1346                                    | 0.1341                  | 0.134                  | 0.1264                                 | 0.1361              | 0.1345                       |
| oil recovery              | 0.1224                   | 0.1259                                    | 0.1331                  | 0.1232                 | 0.1257                                 | 0.1314              | 0.1348                       |
| average formation pressure| 0.0456                   | 0.0441                                    | 0.0464                  | 0.0423                 | 0.0379                                 | 0.1008              | 0.1021                       |
| water saturation          | 0.0947                   | 0.0955                                    | 0.0921                  | 0.0971                 | 0.0945                                 | 0.1011              | 0.098                        |
| oilfield water intrusion  | 0.0917                   | 0.0921                                    | 0.0974                  | 0.0935                 | 0.092                                 | 0.0984              | 0.0952                       |
| average error             | 0.1577                   | 0.1582                                    | 0.1641                  | 0.1581                 | 0.1524                                 | 0.1755              | 0.1702                       |

Figure 3. Digitization of rock core images.
As can be seen from Table 1 and Figure 5, the method of minimizing the average cumulative error is the harmonic-arithmetic method, and the method of maximum error is the power averaging method. The fifth year error is used as an example to the single point error analysis. Table 2 shows the results of single point of error analysis.

**Table 2.** Single point error analysis.

| Item                        | Item A | Item A-H | Item G | Item H | Item H-A | Item P | RMS  |
|-----------------------------|--------|----------|--------|--------|----------|--------|------|
| cumulative oil production   | 0.1069 | 0.1066   | 0.1056 | 0.1056 | 0.1066   | 0.1151 | 0.1240|
| daily oil production        | 0.3698 | 0.3301   | 0.3858 | 0.3858 | 0.3087   | 0.3558 | 0.3558|
| daily liquid production     | 0.3374 | 0.3374   | 0.3319 | 0.3519 | 0.3439   | 0.3685 | 0.3319|
| crude oil geological reserves| 0.1169 | 0.1254   | 0.1254 | 0.1254 | 0.116    | 0.1254 | 0.1254|
| oil saturation              | 0.1198 | 0.1254   | 0.1198 | 0.1254 | 0.116    | 0.1228 | 0.1254|
| oil recovery                | 0.1056 | 0.1066   | 0.1066 | 0.1056 | 0.1069   | 0.124  | 0.1151|
| average formation pressure  | 0.0378 | 0.0252   | 0.0252 | 0.0252 | 0.0195   | 0.0882 | 0.0252|
| water saturation            | 0.0851 | 0.0851   | 0.0851 | 0.0864 | 0.085    | 0.0879 | 0.0879|
| oilfield water intrusion    | 0.0541 | 0.0523   | 0.0541 | 0.0559 | 0.0521   | 0.0533 | 0.0523|

**Figure 4.** Error analysis of accumulated oil production.

**Figure 5.** Average cumulative error analysis results.
Through the above analysis, the six factors including daily oil production, crude oil geological reserves, oil saturation, average formation pressure, water saturation and oil field water invasion have the smallest error by using harmonic - arithmetic average method.

In summary, the model established in this paper adopted the harmonic-arithmetic mean method as the optimal coarsening algorithm.

5. Roughening step size optimization
The model for setting up different steps is shown in Table 3.

Table 3. Different step size coarsening model.

| Model    | X-direction step length (m) | Y-direction step length (m) | Z-direction step length (m) | X-direction grid | Y-direction grid | Z-direction grid | Total grids number |
|----------|-----------------------------|-----------------------------|-----------------------------|------------------|------------------|------------------|--------------------|
| Refine model | 5                           | 5                           | 2                           | 70               | 70               | 18               | 88200              |
| Model 1  | 10                          | 10                          | 4                           | 35               | 35               | 9                | 11025              |
| Model 2  | 17.5                        | 17.5                        | 4                           | 20               | 20               | 9                | 3600               |
| Model 3  | 23.33                       | 23.33                       | 6                           | 15               | 15               | 6                | 900                |
| Model 4  | 35                          | 35                          | 6                           | 10               | 10               | 6                | 600                |
| Model 5  | 50                          | 50                          | 12                          | 7                | 7                | 3                | 147                |
| Model 6  | 70                          | 70                          | 36                          | 5                | 5                | 1                | 25                 |

Taking crude oil geological reserves and cumulative oil production as an example, the simulation results are shown in Figure 6 to Figure 7.

Figure 6. Coarse model of crude oil geological reserve simulation results.
Figure 7. Coarse model of cumulative oil production simulation results.

The crude oil geological reserves and cumulative oil production of the average cumulative error for model 1-6 are shown in Table 4.

Table 4. The oil geological reserves and cumulative oil production of the average cumulative error.

| Error                        | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 | Model 6 |
|------------------------------|---------|---------|---------|---------|---------|---------|
| crude oil geological reserves| 5       | 5       | 2       | 70      | 70      | 18      |
| cumulative oil production    | 10      | 10      | 4       | 35      | 35      | 9       |

As can be seen from Figure 7 and Table 4, consider the calculation cost and the computer reserved, in this case, the optimal horizontal grid step length is about 20m and the vertical step length of 4-6m as 2 to 3 times as the original step length. This result will guide the selection of the model coarsening method of the segregation and the step size.

6. Conclusions
(1) Different geometric models have different coarsening algorithms. Through the comparison and error analysis of the coarsening method, it is concluded that the optimal algorithm is the harmonic-arithmetic mean method considering the reservoir segregation.

(2) The grid step size has a great influence on the numerical simulation of the reservoir. According to the example of this study, when the reservoir model is coarsened, the optimal horizontal grid step is about 20m and the vertical is 4-6m.

(3) The calculation time of the refine model is 10 minutes and 45 seconds. The time of the coarsening model which used the optimal algorithm and the best step length is 1 minute and 12 seconds, which greatly improves the computational efficiency and reduces the computational complexity. It reduces time and economic costs for engineering.

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