Asymptotically de Sitter dilatonic space-time, 
holographic RG flow and conformal anomaly from 
(dilatonic) dS/CFT correspondence

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ABSTRACT

The number of asymptotically de Sitter (non-singular) solutions of 5d dilatonic gravity with positive cosmological constant is found. These solutions are similar to the previously known asymptotically AdS spaces where dilaton may generate the singularity. Using these solutions the consistent $c$-function is proposed in the same way as in AdS/CFT. The consistency of RG flow gives further support for dS/CFT correspondence. From holographic RG flow equations we calculate the holographic 4d conformal anomaly with dilatonic contributions. This conformal anomaly turns out to be the same as in AdS/CFT correspondence.

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The universality of holographic principle manifests itself in the form of AdS/CFT correspondence [1] as well as recently proposed dS/CFT correspondence [2, 3]. Despite the numerous attempts (for not complete list of refs. on dS/CFT correspondence, see [4, 5, 6]) dS/CFT set-up is mainly studied from the gravitational side and it is not well understood. One of the fundamental problems in the development of dS/CFT is the lack of the suitable dual CFT which is presumably Euclidean and not unitary. Probably it is related with the fact that de Sitter space unlike to AdS is very difficult to realize as vacuum space of string theory. Nevertheless, it is became clear that most of relations discussed in AdS/CFT have their analogs also in dS/CFT [5].

In the present letter we extend the study of holographic RG flows developed in AdS/CFT to asymptotically de Sitter dilatonic spaces. In particular, the asymptotically de Sitter (non-singular) solution of 5d dilatonic gravity is found. Using this solution (with non-trivial dilaton) the definition of $c$-function in dS/CFT correspondence is proposed. The proposed $c$-function which is similar (up to the sign of the potential) to the AdS $c$-function gives further support for dS/CFT set-up. The holographic RG flow equations are written. Using them the dilatonic conformal anomaly which turns out to be the same as in AdS/CFT is found.

We start from the following action of dilatonic gravity in $d+1$ dimensions:

$$S = -\frac{1}{16\pi G} \int d^{d+1}x \sqrt{-G} \left( R - \Lambda - \alpha G^{\mu\nu} \partial_{\mu}\phi \partial_{\nu}\phi \right).$$

(1)

We now assume $\lambda^2 \equiv \Lambda$ to be positive.

From the variation of the action (1) with respect to the metric $G^{\mu\nu}$, one obtains:

$$0 = R_{\mu\nu} - \frac{1}{2} G_{\mu\nu} R + \frac{\Lambda}{2} G_{\mu\nu} - \alpha \left( \partial_{\mu}\phi \partial_{\nu}\phi - \frac{1}{2} G_{\mu\nu} G^{\rho\sigma} \partial_{\rho}\phi \partial_{\sigma}\phi \right)$$

(2)

and from that of dilaton $\phi$

$$0 = \partial_{\mu} \left( \sqrt{-G} G^{\mu\nu} \partial_{\nu}\phi \right).$$

(3)

The conventions of curvatures are given by

$$R = G^{\mu\nu} R_{\mu\nu},$$

$$R_{\mu\nu} = -\Gamma_{\mu\lambda,\kappa}^{\lambda} + \Gamma_{\mu\kappa,\lambda}^{\lambda} - \Gamma_{\mu\lambda}^{\kappa} \Gamma_{\kappa\eta}^{\lambda} + \Gamma_{\mu\kappa}^{\eta} \Gamma_{\lambda\eta}^{\lambda},$$

$$\Gamma_{\mu\lambda}^{\eta} = \frac{1}{2} G_{\mu\nu} \left( G^{\nu\lambda,\eta} + G_{\lambda\nu,\mu} - G_{\mu\lambda,\nu} \right).$$

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3 The conventions of curvatures are given by
We assume that $\phi$ depends only on the one of the coordinate, say $y \equiv x^d$ and we also assume that $G_{\mu\nu}$ has the following form

$$ds^2_{d+1} = \sum_{\mu,\nu=1}^{d+1} G_{\mu\nu}dx^\mu dx^\nu = f(y)dy^2 + y \sum_{i,j=1}^{d} g_{ij}dx^i dx^j$$  \hspace{1cm} (4)

Here $g_{ij}$ is the metric in the Einstein manifold, which is defined by

$$r_{ij} = kg_{ij}.$$  \hspace{1cm} (5)

$r_{ij}$ is the Ricci tensor given by $g_{ij}$ and $k$ is a constant, especially $k > 0$ for sphere and $k = 0$ for the flat Minkowski space and $k < 0$ for hyperboloid.

The equations of motion (2) and (3) take the following forms:

$$0 = \frac{1}{2} \frac{rf}{y} - \frac{d(d-1)}{8} \frac{1}{y^2} - \frac{\lambda^2}{2} f + \frac{\alpha}{2} (\phi')^2$$  \hspace{1cm} (6)

$$0 = -\left( r_{ij} - \frac{1}{2} r g_{ij} \right) \frac{f}{y}$$
$$+ \left\{ \frac{d-1}{4} \frac{f'}{f} - \frac{(d-1)(d-4)}{8} \frac{1}{y^2} - \frac{\lambda^2}{2} f - \frac{\alpha}{2} (\phi')^2 \right\} g_{ij}$$  \hspace{1cm} (7)

$$0 = \left( \sqrt{\frac{y^d}{f} \phi'} \right)'.$$  \hspace{1cm} (8)

Here $'$ expresses the derivative with respect to $y$ and $r \equiv g^{ij}r_{ij} = kd$. Eq.(3) corresponds to $(\mu, \nu) = (d, d)$ in (2) and Eq.(4) to $(\mu, \nu) = (i, j)$. The case of $(\mu, \nu) = (0, i)$ or $(i, 0)$ is identically satisfied. Integrating (8), one finds

$$\phi' = c \sqrt{-\frac{f}{y^d}}.$$  \hspace{1cm} (9)

Here $c$ is some constant. Substituting (9) into (4), we can solve it algebraically with respect to $f$:

$$f = -\frac{d(d-1)}{4y^2\lambda^2} \left( 1 - \frac{\alpha c^2}{\lambda^2 y^d} - \frac{kd}{\lambda^2 y^d} \right).$$  \hspace{1cm} (10)

Then we find from (9) and (10),

$$\phi = c \int dy \sqrt{\frac{d(d-1)}{4y^{d+2}\lambda^2} \left( 1 - \frac{\alpha c^2}{\lambda^2 y^d} - \frac{kd}{\lambda^2 y^d} \right)}.$$  \hspace{1cm} (11)
The EMT $T_{\mu\nu}^\phi$ coming from $\phi$ is, as one can find it from (2), given by

$$T_{\mu\nu}^\phi = \alpha \left( \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} G_{\mu\nu} G^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi \right).$$

Using (9) one gets

$$T_{yy} = -\frac{\alpha c^2 f}{2y^d}, \quad T_{ij} = \frac{\alpha c^2}{2y^{d-1}} g_{ij}.$$ (12)

The pressure corresponding to $T_{ij}$ is always positive and the energy density corresponding to $T_{yy}$ is positive as long as $f$ (10) is negative. Then the strong and weak energy conditions are satisfied as long as $f$ is positive.

When $y$ is small, $f(y)$ (10) behaves as

$$f(y) \sim \frac{d(d-1)y^{d-2}}{4\alpha c^2},$$ (13)

which makes a curvature singularity at $y = 0$. The scalar curvature behaves when $y \sim 0$ as

$$R \sim \alpha c^2 y^{-d}.$$ (14)

The curvature singularity can be generated by the singular behavior of the dilaton $\phi$ when $y \sim 0$:

$$\phi(y) \sim \text{sgn}(c) \sqrt{\frac{d(d-1)}{4\alpha}} \ln y.$$ (15)

Here $\text{sgn}(c)$ expresses the sign of $c$:

$$\text{sgn}(c) = \begin{cases} +1 & \text{if } c > 0 \\ -1 & \text{if } c < 0 \end{cases}.$$ (16)

On the other hand, when $y \to +\infty$, by using Eq. (10), one finds that $f(y)$ behaves as

$$f = -\frac{d(d-1)}{4y^2\lambda^2}.$$ (17)

Then if define the length parameter $l$ by

$$l^2 = \frac{d(d-1)}{\lambda^2},$$ (18)

the spacetime metric (4) has the following form:

$$ds_{d+1}^2 \sim -\frac{l^2 dy^2}{y^2} + y \sum_{i,j=1}^d g_{ij} dx^i dx^j.$$ (19)
Thus, the spacetime becomes asymptotically de Sitter one. In fact, if we define a time coordinate $t_{\text{asym}}$ by $y = e^{\frac{2t_{\text{asym}}}{\lambda}}$, the metric (19) has a warped form:

$$ds^2_{d+1} \sim -dt_{\text{asym}}^2 + e^{2t_{\text{asym}}} \sum_{i,j=1}^{d} g_{ij}dx^idx^j. \quad (20)$$

Similar solution, which is asymptotically AdS, has been found in [7]. The solution [7] exists when the cosmological constant $\Lambda$ in the action (1) is negative. The solution in (10) becomes asymptotically de Sitter in the time-like infinity but the solution in [7] becomes asymptotically anti-de Sitter in the space-like infinity. It would be interesting to construct the solution interpolating between asymptotically de Sitter and asymptotically Anti-de Sitter ones as in ref.[8].

We should note that $f(y)$ (10) diverges when

$$1 - \frac{\alpha \epsilon^2}{\lambda^2 y^4} - \frac{kd}{\lambda^2 y} = 0. \quad (21)$$

There are three cases that the above equation has two solutions, only one solution and no solution. When Eq.(21) has two solutions ($k \neq 0$), let denote the larger solution by $y_0$. When $y \sim y_0$, $f(y)$ behaves as

$$f(y) \sim -\frac{f_0}{y - y_0} \quad (f_0 > 0). \quad (22)$$

Then if one defines a new (time) coordinate $t$ by

$$t = 2\sqrt{f_0(y - y_0)}, \quad (23)$$

the spacetime metric in (4) behaves as

$$ds^2_{d+1} \sim -dt^2 + \left(\frac{t^2}{4f_0} + y_0\right) \sum_{i,j=1}^{d} g_{ij}dx^idx^j. \quad (24)$$

Then there is obviously no curvature singularity at $y = y_0 \ (t = 0)$. Since $t \to \pm \infty$ corresponds to $y \to +\infty$, the solution connects two asymptotically dS regions corresponding to $t \to \pm \infty$. We also note that since the curvature singularity corresponds to $y \to 0$ in (14), the solution is totally non-singular.

Let us consider the case that Eq.(21) has only one solution when $k \neq 0$ and $f(y)$ behaves as

$$f(y) \sim -\frac{f_1^2}{(y - y_0)^2}. \quad (25)$$
Then if we define a new (time) coordinate $\tau$ by
\[
\tau = f_1 \ln (y - y_0) ,
\] (26)
the spacetime metric (3) behaves as
\[
d s^2_{d+1} \sim -d\tau^2 + \left( e^{\frac{\tau}{f_1}} + y_0 \right) \sum_{i,j=1}^{d} g_{ij} dx^i dx^j .
\] (27)
Then the radius of the universe approaches to a constant $\sqrt{y_0}$ when $\tau \to -\infty$ ($y \to y_0$). The curvature singularity corresponding to $y \to 0$ in (14) does not appear again.

What happens when $k = 0$ and $\alpha > 0$? For simplicity, we consider $d = 4$ case. Then Eq.(21) gives
\[
y_0^4 = \frac{\alpha c^2}{\lambda^2} .
\] (28)
When $y \sim y_0$, $f(y)$ behaves as (22). If we change the coordinate $y$ to $\theta$ by
\[
\cos \theta = \frac{y_0^2}{y^2} ,
\] (29)
Eq.(11) can be integrated explicitly:
\[
\phi = \phi_0 + \theta \sqrt{\frac{3}{\alpha}} .
\] (30)
Eq.(29) tells that $\theta = 0$ corresponds to $y = y_0$ and $\theta \to \frac{\pi}{2}$ to $y \to +\infty$. Then we find that $\phi = \phi_0$ at $y = y_0$ and $\phi = \phi_\infty \equiv \phi_0 + \frac{\pi}{2} \sqrt{\frac{3}{\alpha}}$. Since the string coupling $g_s$ is generally defined by
\[
g_s = e^{\gamma \phi} \quad (\gamma \text{ is a constant}) ,
\] (31)
the beta function, which may be defined by analogy with AdS/CFT as
\[
\beta(\phi) = 2(y - y_0) \frac{d g_s}{dy} ,
\] (32)
vanishes at $y = y_0$ and in the limit of $y \to +\infty$. Then $y = y_0$ and $y \to +\infty$ correspond to a fixed point of the renormalization group from the viewpoint of the dS/CFT correspondence. Then the solution of $k = 0$ and $\alpha > 0$ would connect two fixed points where $g_s = e^{\gamma \phi_0}$ and $g_s = e^{\gamma \phi_\infty}$.
The reason why AdS/CFT can be expected is the isometry of $d+1$-dimensional anti-de Sitter space, which is $SO(d, 2)$ symmetry. It is identical with the conformal symmetry of $d$-dimensional Minkowski space. We should note, however, $d+1$-dimensional de Sitter space has the isometry of $SO(d+1, 1)$ symmetry, which can be a conformal symmetry of $d$-dimensional Euclidean space. Then it might be natural to expect the correspondence between $d+1$-dimensional de Sitter space and $d$-dimensional euclidean conformal symmetry (dS/CFT correspondence\[3\]). In fact, the metric of $D = d+1$-dimensional anti de Sitter space (AdS) is given by

$$ds^2_{AdS} = dr^2 + e^{2r} \left(-dt^2 + \sum_{i=1}^{d-1} \left(dx^i\right)^2\right).$$

(33)

In the above expression, the boundary of AdS lies at $r = \infty$. If one exchanges the radial coordinate $r$ and the time coordinate $t$, we obtain the metric of the de Sitter space (dS):

$$ds^2_{dS} = -dt^2 + e^{2t} \sum_{i=1}^{d} \left(dx^i\right)^2.$$

(34)

Here $x^d = r$. Then there is a boundary at $t = \infty$, where the Euclidean conformal field theory (CFT) can live and one expects dS/CFT correspondence as one more manifestation of holographic principle. Strominger [3] has conjectured that dual CFT in dS/CFT may be non-unitary.

Using the above indication to dS/CFT correspondence, one can speculate on the properties of our asymptotically dS solutions in terms of RG flow. Indeed, the above analysis seems to tell that there is a renormalization group flow. In case of the AdS$_d$/CFT$_4$ correspondence, the curvature radius $l_{AdS}$, by which the scalar curvature is given by $R = -\frac{20}{l_{AdS}^2}$, can be expressed by the string coupling and the integer $N$ which parametrizes the gauge group $SU(N)$ or $U(N)$ of the CFT as follows

$$l = (4\pi g_s N)^{\frac{1}{4}}.$$

(35)

Then it might be one of natural ways to define the $c$-function, in the situation under consideration in this paper, by the scalar curvature:

$$c = (R)^{\frac{3}{4}}.$$

(36)
Since Eq. (2) gives, for \( d = 4 \),

\[
R = \frac{5}{3} \Lambda + \alpha (\partial_y \phi)^2 ,
\]  

(37)

by using the solution (10) with \( d = 4 \) and \( k = 0 \), one has

\[
c = \left( \frac{5}{3} \lambda^2 + \frac{3y_0^4}{y^6} \right)^{-\frac{3}{2}} .
\]  

(38)

Here \( y_0 \) is defined by (28). Then \( c \) is the monotonically increasing function with respect to \( y \) and it vanishes when \( y = y_0 \). From the viewpoint of the dual field theory, the vanishing \( c \) means that all the fields become massive. In the AdS/CFT correspondence, the dilaton gravity with the constant potential, which can be identified with the cosmological constant, as in (1), corresponds to the deformation of the conformal field theory by the marginal operator but the above non-trivial flow would tell that the deformation is not exactly marginal.

We can extend the definition of the \( c \)-function (36) to the case that there is a non-trivial potential \( V(\phi) \) instead of the cosmological term, that is, \( \Lambda \) in (1) is replaced by \( V(\phi) \). If we assume the metric has the form of (4) and \( \phi \) only depends on \( y \), we have

\[
R = \frac{5}{3} V(\phi) + \alpha (\partial_y \phi)^2 ,
\]  

(39)

instead of (37) or (2) and the equation corresponding to (3) has the form

\[
2\alpha \partial_y^2 \phi = V'(\phi) .
\]  

(40)

By using (39) and (11), one finds

\[
\partial_y R = \left( \frac{d + 1}{d - 1} V'(\phi) + 2\alpha \partial_y^2 \phi \right) \partial_y \phi = \frac{2}{d - 1} \partial_y (V(\phi)) = \frac{2\alpha}{d - 1} \partial_y \left( (\partial_y \phi)^2 \right) .
\]  

(41)

Then the \( c \)-function defined by (36) becomes stationary when \( V'(\phi) = 0 \). We also note that when \( \partial_y \phi = 0 \), the \( c \)-function defined by (36) has the following form

\[
c \propto (V(\phi))^{-\frac{3}{2}} ,
\]  

(42)

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which is similar to the standard form $c_{\text{AdS}} = (-V(\phi))^{-\frac{1}{2}}$ in the AdS/CFT correspondence \cite{9,10}. If we choose the metric for $k = 0$ and $d = 4$ in the following form, instead of (4),

\[ ds_{d+1}^2 = -dt^2 + e^{2A(t)} \sum_{i=1}^{4} dx_i dx_i \] (43)

the Einstein equations are:

\[ \frac{d^2 A}{dt^2} = -\frac{\alpha}{3} \left( \frac{d\phi}{dt} \right)^2 , \] (44)

\[ \left( \frac{dA}{dt} \right)^2 = \frac{V(\phi)}{12} + \frac{\alpha}{12} \left( \frac{d\phi}{dt} \right)^2 . \] (45)

Eq. (44) tells that $\frac{dA}{dt}$ is a monotonically decreasing function of $t$. We also note that the region where $\frac{dA}{dt} \sim 0$ is asymptotically de Sitter space. It is natural, as in the case of AdS/CFT correspondence \cite{11}, to assume that $c$-function is given by

\[ c \propto \left( \frac{dA}{dt} \right)^{-\frac{3}{2}} . \] (46)

Eq. (44) indicates that at the critical point, where $\frac{dA}{dt} = 0 \left( \frac{d^2 A}{dt^2} = 0 \right)$, we have $\frac{d\phi}{dt} = 0$, as is assumed in (42). Then Eq. (45) tells that

\[ \left( \frac{dA}{dt} \right)^2 = \frac{V(\phi)}{12} \] (47)

at the critical point, what proves that Eq. (42) and therefore the definition of the $c$-function here (36) is consistent. Of course, in the absence of the explicit example of dual CFT all above discussion on $c$-function is purely speculation. Nevertheless, the existence of consistent $c$-function for dual RG flow gives further support to dS/CFT correspondence.

In \cite{12}, it has been shown that there is an $S^{8-p}$ compactification of the $D = 10$ supergravity theory to an effective gauged $(p+2)$-dimensional supergravity with the action

\[
S = N^2 \int d^{p+2} x \sqrt{-g} \left( N e^{\phi} \right)^{2(p-3)} \left[ R + \frac{4(p-1)(p-4)}{(7-p)^2} \partial_{\mu} \phi \partial^{\mu} \phi \right. \\
\left. + \frac{1}{2} (9 - p)(7 - p) \right]. \] (48)
\[ S = N^2 \int d^{p+2}x \sqrt{-g} \left[ R - \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi + \frac{1}{2} (9 - p)(7 - p) N^{\frac{4(p-3)}{p(p-7)}} e^{-\sqrt{2} \frac{p-3}{p(p-7)} \phi} \right]. \quad (49) \]

Then \( p = 3 \) case is similar to the action (1) with \( d = 4 \) except the sign of the cosmological term. It has been shown by Hull [2] that if we consider Type IIB\(^*\) string theory, which is given by the time-like T-duality from Type II string theory, D-brane is transformed into E-brane and its near light-cone limit gives the de Sitter space instead of the AdS space, which is the near horizon limit of D-brane. In the time-like T-duality, the sign in front of the kinetic term of \((8-p)\)-form is changed. Then in the effective theory as in (48) or (49), the sign of the cosmological constant is changed, what corresponds to the action (1).

As the solution (10) is asymptotically de Sitter space, one can consider the renormalization flow as in [3] in an analogy with the holographic renormalization group formulation for AdS/CFT developed in [13, 14]. Take \( D = d + 1 \) dimensional dS-like metric in the following form

\[ ds^2 = G_{\mu\nu} dX^\mu dX^\nu = -dt^2 + \hat{g}_{ij}(x,t) dx^i dx^j. \quad (50) \]

where \( X^\mu = (x^i,t) \) with \( i, j = 1, 2, \cdots, d \). We consider the action which is the sum of the action \( S \) (1) on a \((d+1)\) dimensional manifold \( M_{d+1} \) and the action on the boundary \( \Sigma_d = \partial M_{d+1}, \)

\[ S_{d+1} = S + 2 \int_{\Sigma_d} d^d x \sqrt{\hat{g}} K \equiv \int d^d x dt \sqrt{-G} \mathcal{L}_{d+1}. \quad (51) \]

where \( K_{\mu\nu} \) is the extrinsic curvature on \( \Sigma_d \). The action is taken in the Minkowski signature. Since we are considering the de Sitter background instead of the AdS background, the cosmological constant \( \Lambda \) is positive and parametrized by the parameter \( l \), which is the radius of the asymptotic dS\(d+1\)

\[ \Lambda = \frac{d(d-1)}{l^2}. \quad (52) \]
In the metric (50), $K_{\mu\nu}$ is given as
\[ K_{ij} = \frac{1}{2} \partial_t \hat{g}_{ij} , \quad K = \hat{g}^{ij} K_{ij} \] (53)

In the canonical formalism, $\mathcal{L}_{d+1}$ is rewritten by using the canonical momenta $\Pi_{ij}$ and $\Pi$ and Hamiltonian density $\mathcal{H}$ as
\[ \mathcal{L}_{d+1} = \Pi_{ij} \partial_t \hat{g}_{ij} + \Pi \partial_t \phi + \mathcal{H} , \]
\[ \mathcal{H} \equiv \frac{1}{d-1} (\Pi^i)^2 - \Pi_{ij} \Pi^{ij} - \frac{1}{2\alpha} \Pi^2 + \hat{R} - \Lambda - \alpha \hat{g}^{ij} \partial_i \phi \partial_j \phi . \] (54)

The equation of motion for $\Pi^{\mu\nu}$ leads to
\[ \Pi_{ij} = K_{ij} - \hat{g}_{ij} K , \quad \Pi = \alpha \partial_t \phi . \] (55)

The Hamilton constraint $\mathcal{H} = 0$ leads to the Hamilton-Jacobi equation (flow equation)
\[ \{ S, S \}(x) = \sqrt{\hat{g}} \mathcal{L}_d(x) \] (56)
\[ \{ S, S \}(x) \equiv \frac{1}{\sqrt{g}} \left[ -\frac{1}{d-1} \left( \frac{\hat{g}_{ij} \delta S}{\delta \hat{g}_{ij}} \right)^2 + \frac{\delta S}{\delta \hat{g}_{ij}} \frac{\delta S}{\delta \hat{g}_{ij}} + \alpha \left( \frac{\delta S}{\delta \phi} \right)^2 \right] , \] (57)
\[ \mathcal{L}_d(x) \equiv \hat{R}[\hat{g}] - \Lambda - \alpha \hat{g}^{ij} \partial_i \phi \partial_j \phi . \] (58)

One can decompose the action $S$ into a local and non-local part as discussed in ref.[13] as follows
\[ S[\hat{g}(x)] = S_{loc}[\hat{g}(x)] + \Gamma[\hat{g}(x)] , \] (59)

Here $S_{loc}[\hat{g}(x)]$ is tree level action and $\Gamma$ contains the higher-derivative and non-local terms. In the following discussion, we take the systematic method of ref.[14], which is weight calculation. The $S_{loc}[\hat{g}]$ can be expressed as a sum of local terms
\[ S_{loc}[\hat{g}(x)] = \int d^d x \sqrt{\hat{g}} \mathcal{L}_{loc}(x) = \int d^d x \sqrt{\hat{g}} \sum_{w=0,2,4,} \left[ \mathcal{L}_{loc}(x) \right]_w \] (60)

The weight $w$ is defined by following rules;
\[ \hat{g}_{ij}, \Gamma : \text{weight} \ 0 \ , \ \partial_\mu : \text{weight} \ 1 \ , \ \hat{R}, \ \hat{R}_{ij} : \text{weight} \ 2 \ , \ \frac{\delta \Gamma}{\delta \hat{g}_{ij}} : \text{weight} \ d \ . \] (61)
Using these rules and (56), one obtains the equations, which depend on the weight as

\[ \sqrt{\hat{g}} \mathcal{L}_d = \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_0 + \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_2 \]  
\[ 0 = \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_w \quad (w = 4, 6, \ldots, d - 2), \]  
\[ 0 = 2 \left[ \{ S_{\text{loc}}, \Gamma \} \right]_d + \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_d \]  
\[ (62) \]

The above equations which determine \( [\mathcal{L}_{\text{loc}}]_w \). \( [\mathcal{L}_{\text{loc}}]_0 \) and \( [\mathcal{L}_{\text{loc}}]_2 \) are parametrized by

\[ [\mathcal{L}_{\text{loc}}]_0 = W, \quad [\mathcal{L}_{\text{loc}}]_2 = -\Phi \hat{R} + M \hat{g}^{ij} \partial_i \phi \partial_j \phi \]  
\[ (65) \]

Thus one can solve (62) as

\[ \Lambda = \frac{d}{4(d-1)} W^2, \quad 1 = \frac{d-2}{2(d-1)} W \Phi \quad \alpha = -\frac{d-2}{4(d-1)} WM \]  
\[ (66) \]

The case of \( d = 2 \) is special and instead of Eqs.(62,63,64), we obtain

\[ \sqrt{\hat{g}} \mathcal{L}_2 = 2 \left[ \{ S_{\text{loc}}, \Gamma \} \right]_2 + \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_0 + \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_2 \]  
\[ (67) \]

When \( d = 2 \), the second equation in (66) is irrelevant but by using (52), we obtain

\[ W_2 = -\frac{2}{l}. \]  
\[ (68) \]

When \( d > 2 \), by using (52), one obtains \( W \) and \( \Phi \) as

\[ W = -\frac{2(d-1)}{l}, \quad \Phi = -\frac{l}{d-2}, \quad M = \frac{2l\alpha}{d-2} \]  
\[ (69) \]

Note that there is an ambiguity in the choice of the sign but the relative sign of \( W \) and \( \Phi \) is different from AdS case.

When \( [\mathcal{L}_{\text{loc}}]_4 = 0 \), one gets for \( d = 4 \) and \( 2\alpha = 1 \)

\[ \frac{1}{\sqrt{\hat{g}}} \left[ \{ S_{\text{loc}}, S_{\text{loc}} \} \right]_4 \]
\[ = \frac{l^2}{12} \hat{R}^2 - \frac{l^2}{4} \hat{R}_{ij} \hat{R}^{ij} - \frac{l^2}{12} \hat{R} \hat{g}^{ij} \partial_i \phi \partial_j \phi + \frac{1}{4} \hat{R}^{ij} \partial_i \phi \partial_j \phi \]
\[ - \frac{1}{24} \left( \hat{g}^{ij} \partial_i \phi \partial_j \phi \right)^2 - \frac{1}{8} \left( \frac{1}{\sqrt{\hat{g}}} \partial_i \left( \sqrt{\hat{g}} \hat{g}^{ij} \partial_j \phi \right) \right)^2. \]  
\[ (70) \]
Then 4d holographic conformal anomaly is obtained as follows:

\[ \kappa^2 \mathcal{W}_4 = \frac{l}{2\sqrt{g}} \left[ \{S_{\text{loc}}, S_{\text{loc}} \} \right]_4 \]

\[ = l^3 \left[ \frac{1}{24} \hat{R}^2 - \frac{1}{8} \hat{R}_{ij} \hat{R}^{ij} - \frac{t^2}{24} \hat{R}^{ij} \partial_i \phi \partial_j \phi + \frac{1}{8} \hat{R'}^{ij} \partial_i \phi \partial_j \phi - \frac{1}{48} \left( \hat{g}^{ij} \partial_i \phi \partial_j \phi \right)^2 \right. \]

\[ \left. - \frac{1}{16} \left\{ \frac{1}{\sqrt{g}} \partial_i \left( \sqrt{g} \hat{g}^{ij} \partial_j \phi \right) \right\}^2 \right]. \quad (71) \]

The obtained form of the anomaly is identical with that in [15]. Thus as an extension of ref. [6] in the dS/CFT correspondence, we reproduced the dilatonic conformal anomaly, which was obtained from the AdS/CFT framework in [15]. (There exists extensive list of refs. related with the study of holographic anomaly in AdS/CFT [16]). The Weyl anomaly coming from the multiplets of \( \mathcal{N} = 4 \) supersymmetric \( U(N) \) or \( SU(N) \) Yang-Mills coupled with \( \mathcal{N} = 4 \) conformal supergravity was calculated in [17]. The expression in [15] as well as the one (71) corresponds to the anomaly of the supergravity in the background where only gravity and the real part of the scalar field \( \phi \) from the \( \mathcal{N} = 4 \) conformal supergravity multiplet are non-trivial and other fields vanish. Thus, holographic RG flow equations permit to calculate 4d conformal anomaly from 5d asymptotically dS space. This maybe considered as further support of proposed dS/CFT correspondence.

Finally, let us make several remarks on possibility of dS supergravities. The dS/CFT can be conjectured from the Type II\(^*\) string [3], which is obtained from Type II string theory by the time-like \( T \)-duality. A solution of Type II supergravity is given by

\[ ds^2 = H^{-\frac{1}{2}} \left( -dt^2 + dx_1^2 + \ldots + dx_p^2 \right) + H^{\frac{1}{2}} \left( dx_{p+1}^2 + \ldots + dx_9^2 \right) \]

\[ H = c + \frac{q}{r^{7-p}}, \quad r^2 \equiv \sum_{i=p+1}^{9} x_i^2. \quad (72) \]

If we consider near horizon limit \( (r^2 \to 0) \) in case \( p = 3 \), which corresponds to D3-brane, the metric (72) has the following form:

\[ ds^2 = \frac{r^2}{\alpha^2} \left( -dt^2 + dx_1^2 + \ldots + dx_3^2 \right) + \frac{\alpha^2}{r^2} dr^2 + a^2 d\Omega_5^2. \quad (73) \]

Here \( d\Omega_5^2 \) is the metric of the 5-dimensional unit sphere. Then the 10 dimensional spacetime becomes AdS\(_5\)×S\(_5\). Taking the time-like \( T \)-duality, the
component $g_{tt}$ of the metric tensor is replaced by $\frac{1}{g_{tt}}$. Then from (72), one can obtain a solution in Type II* supergravity:

$$ds^2 = H^{-\frac{1}{2}} \left( dx_1^2 + \ldots + dx_p^2 \right) + H^{\frac{1}{2}} \left( -dt^2 + dx_{p+1}^2 + \ldots + dx_9^2 \right)$$

(74)

Considering near light-cone limit ($\tau^2 \to 0$), instead of the near horizon limit, and changing the coordinate by $t = \tau \cosh \beta$ and $r = \tau \sinh \beta$, assuming $\tau^2 = t^2 - r^2 > 0$, we obtain the following metric

$$ds^2 = \frac{\tau^2}{a^2} \left( -dt^2 + dx_1^2 + \ldots + dx_3^2 \right) - \frac{a^2}{\tau^2} d\tau^2 + a^2 d\tilde{\Omega}_5^2,$$

$$d\tilde{\Omega}_5^2 = d\beta^2 + \sinh^2 \beta d\Omega_4^2.$$  

(75)

Here $d\tilde{\Omega}_5^2$ is the metric of 5-dimensional hyperboloid. Then the spacetime in (75) is the product of the hyperboloid and the 5-dimensional de Sitter space, whose metric is $\frac{a^2}{\tau^2} \left( -dt^2 + dx_1^2 + \ldots + dx_3^2 \right) - \frac{a^2}{\tau^2} d\tau^2$. Even in case $\tau^2 = -\sigma^2 = t^2 - r^2 < 0$, by the similar calculation, we obtain the spacetime which is the product of the hyperboloid and the 5-dimensional de Sitter space. The expression (71), therefore, might indicate that supergravity obtained from Type II* string has the common structure with the supergravity obtained from Type II string.

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