Thermodynamics, magnetization and pressure anisotropy for magnetized quark-gluon plasma using the the extended self-consistent quasiparticle model

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The thermomagnetic behavior of quark-gluon plasma has recently received a lot of attention. In this work we make use of the extended self-consistent quasiparticle model to study the thermodynamic properties of magnetized (2+1) flavor quark-gluon plasma. The system is considered as a non-interacting system of quasiparticles with masses depending on both temperature and magnetic field. This allows to obtain the equation of state of the system and the other thermodynamic properties such as the speed of sound. We use the extended self-consistent model to obtain the magnetization and show that QGP has a paramagnetic nature. In addition we study the pressure anisotropy and calculate the transverse pressure. The obtained anisotropic pressure may be used in hydrodynamic studies of magnetized QGP produced in heavy-ion collisions.

I. INTRODUCTION

Quark-gluon plasma (QGP), the state of matter believed to have existed shortly after the big bang, has been successfully created in high energy collisions [1]. The charged ions can produce large magnetic fields reaching up to $eB \approx (1 - 5)m_{q}^{2}$ during off-central collisions. Magnetic fields created in this manner may exist only for a short while but can be stationary during this time [4–7]. The theoretical tools used to study QGP need modifications to incorporate effects of external magnetic fields and there has been flurry of research activity in this area [8–36]. Measurements at the LHC [37] along with those at RHIC [38] are capable of providing new insights that can constrain the theoretical modelling. The equation of state has a significant impact on QGP evolution [39]. The study of the equation of state of magnetized QGP is relevant in the contexts of cosmology [40] and strongly magnetised neutron stars too [41]. The investigation of the behavior of magnetized QGP is, therefore, of importance [12, 13].

II. THE EXTENDED SELF-CONSISTENT QUASIPARTICLE MODEL

In the extended self-consistent quasiparticle model [42, 43], the thermal mass is defined to be proportional to the plasma frequencies as,

$$m_{q}^{2} = \frac{3}{2} \omega_{p}^{2}$$

and

$$m_{g}^{2} = 2m_{f}^{2}, \tag{1}$$

for massless particles. For massive quarks $m_{q}^{2}$ is written as,

$$m_{q}^{2} = (m_{a} + m_{f})^{2} + m_{f}^{2}. \tag{2}$$

The plasma frequencies are calculated from the density dependent expressions [50]

$$\omega_{p}^{2} = a_{q}^{2} g^{2} \frac{\beta_{q}}{T} + a_{f}^{2} g^{2} \frac{\beta_{g}}{T}, \tag{3}$$

for gluons and,

$$m_{f}^{2} = \frac{a_{q}^{2} g^{2}}{T} \beta_{g}. \tag{4}$$

for quarks. Here $n_{q}$ and $n_{g}$ are the quark and gluon number density respectively. $g^{2} = 4\pi\alpha_{s}$ is the QCD running coupling constant. The coefficients $a_{g}, a_{q}, b_{q}$ are determined by demanding that as $T \rightarrow \infty$, $\omega_{p}$ and $m_{f}$ both go to the corresponding perturbative results. The motivation for choosing such an expression for plasma frequency is that the plasma frequency for electron-positron
plasma is known to be proportional to $n/T$ in the relativistic limit [52, 53]. Since the thermal masses appear in the expression for the density, we need to solve the density equation self-consistently to obtain the thermal mass, which may be used to evaluate the thermodynamic quantities of interest. The result obtained have shown a good fit with lattice data even at temperatures near $T_c$ [51].

In the presence of magnetic fields the energy eigenvalue equations is a thermomagnetic coupling, a coupling that in-prications is a thermomagnetic coupling, a coupling that in-

The expression for number density for gluons remains unchanged in the presence of magnetic fields as gluons are chargeless and the thermomagnetic mass for gluons is obtained by solving equation (3) in a self-consistent manner. Note that, even though the expression for gluon density remains unchanged in the presence of magnetic fields, they acquire a thermo-magnetic mass through the quark number density. Using the thermomagnetic mass we can obtain the thermodynamics and study the thermomagnetic properties of magnetized QGP.

### A. Thermomagnetic Coupling

The only ingredient we need in order to make calculations is a thermomagnetic coupling, a coupling that in-corporates the effect of both temperature and magnetic fields. To this end, throughout this work, we make use of the one-loop running coupling constant that evolves with both the momentum transfer and the magnetic field [59] as,

$$\alpha_s(\Lambda^2, |eB|) = \frac{\alpha_s(\Lambda^2)}{1 + b_1 \alpha_s(\Lambda^2) \log \left( \frac{\Lambda^2}{\Lambda^2 + |eB|} \right)}.$$  

The one-loop running coupling in the absence of a magnetic field at the renormalization scale is given by,

$$\alpha_s(\Lambda^2) = \frac{1}{b_1 \log(\Lambda^2/\langle M^* \rangle)},$$

where, $b_1 = (11N_c - 2N_f)/12\pi$ and following [4], $\langle M^* \rangle = 1.176 GeV$ at $\alpha_s(1.5 GeV) = 0.326$ for $N_f = 3$.

It is to be noted that the above thermomagnetic coupling has been obtained using the Lowest Landau Level approximation and hence may not be completely appropriate for calculations involving higher Landau Level contributions. Besides, the coupling is obtained in the one loop order and so this may be appropriate only at very high temperature. We use this coupling as an approximation and so our results are bound to be qualitative in nature. A two-loop thermomagnetic coupling which includes the contribution from higher Landau Levels is expected to give results that are quantitatively reliable.

### III. THERMODYNAMICS OF (2+1) FLAVOR QGP IN THE PRESENCE OF MAGNETIC FIELDS

In this section we study the thermodynamics of (2+1) flavor QGP. We are interested in the thermomagnetic correction and hence we will drop the pure-field contributions [60] from our calculations.

#### A. Thermodynamic pressure

For quarks, the grand canonical potential, within the self-consistent quasiparticle model is,

$$\Phi_q = -P_q = -T \frac{g_f q_f |eB|}{2\pi^2} \sum_{l=1}^{\infty} \frac{1}{1} \sum_{j=0}^{\infty} (2 - \delta_{0j}) \left[ \frac{T}{l^2} m_{q,l} K_1 \left( \frac{m_{q,l}}{T} \right) + \int_{T_0}^{T} \frac{dT}{T} m_{q,l} \partial \frac{m_{q,l}}{\tau} K_0 \left( \frac{m_{q,l}}{\tau} \right) \right]$$

Here we have taken $\mu = 0$. Note that in the self-consistent quasiparticle model the grand canonical potential is not equal to $-KT \log Z$, where $Z$ is the grand partition function, due to the temperature dependence of masses. This is the reason why the expression for pressure in equations [10] and [11] do not match the corresponding expressions for an ideal gas even though in the quasiparticle model the system is considered as an ideal gas with temperature dependent masses. There is an extra term which ensures thermodynamic consistency as shown in reference [17]. The temperature dependence of mass in quasiparticle models has lead to a whole lot of
discussion about thermodynamic inconsistency problem and introduction of extra terms like $B(T)$ whose physical meaning is not obvious. The self-consistent quasiparticle model avoids this problem by starting with the expressions energy density and number density and calculating everything else from them.

The contribution from gluons is,

$$
\Phi_g = -P_g = -T \frac{g_f}{2\pi^2} \sum_{l=1}^{\infty} \sum_{j=1}^{l} \left( m_{gl} \right) K_2 \left( \frac{m_{gl}}{T} \right) \left( m_{gl} \right) \frac{3}{\tau} K_1 \left( \frac{m_{gl}}{\tau} \right)
$$

The contribution from gluons is,

$$
\Phi_g = -P_g = -T \frac{g_f}{2\pi^2} \sum_{l=1}^{\infty} \frac{1}{T^3} \left( m_{gl} \right)^2 K_2 \left( \frac{m_{gl}}{T} \right)
$$

Similarly, we obtain the expression for magnetization of gluons from equation (11) as,

$$
\mathcal{M}_g = \frac{T g_f}{2\pi^2} \sum_{l=1}^{\infty} \left( \frac{m_{gl}}{T} \right) \frac{3}{\tau} K_1 \left( \frac{m_{gl}}{\tau} \right) \frac{\partial}{\partial (eB)} \left( \frac{m_{gl}}{T} \right)
$$

The variation of magnetization with temperature for different magnetic fields is plotted in Fig. 3. We see that the magnetization has a positive value for all values of temperature above $T_c$. This shows that QGP has a paramagnetic nature. The small deviation in the behavior of the graph for $eB = 0.2 GeV^2$ towards higher temperatures is because the contribution from even higher Landau Levels becomes relevant at this magnetic field. The small deviation in the behavior of magnetization with magnetic field for different temperatures. It is seen that the magnetization increases with magnetic field. The behavior of magnetization of QGP as seen in our work is qualitatively consistent with lattice QCD results and with results from HTL perturbation theory. We have summed over 20 Landau Levels whereas, in our work the Lowest Landau Level approximation has been used in the description of hot QCD medium. The velocity of sound is a fundamental quantity that is used in the description of hot QCD medium. The velocity of sound square $c_s^2$ is given by,

$$
c_s^2 = \frac{\partial P}{\partial \epsilon} = \frac{dP}{dT} \frac{d\epsilon}{dT},
$$

where, $\epsilon$ is the energy density which can be obtained from pressure using the thermodynamic relation,

$$
\epsilon = T \frac{P}{\partial T} - P.
$$

In Fig. 3, we have plotted $C_s^2$ as a function of temperature, for different magnetic field values. The speed of sound is seen to reach the Stefan-Boltzmann limit of 1/3, asymptotically. This behavior is consistent with the behavior of $p/\epsilon$ in and with the behavior of sound velocity in.

**IV. MAGNETIZATION**

Magnetization can be obtained from the Grand canonical potential $\Phi$.

$$
\mathcal{M} = \frac{1}{V} \frac{\partial \Phi}{\partial (eB)}.
$$

We confine our calculation to the region where $eB$ is greater than zero. Note that the equation for magnetization in the self-consistent quasiparticle model is not related to the partition function as in lattice QCD results. This is because of the extra terms in and which ensure thermodynamic consistency.

Using equation (9), we get, for quarks,

$$
\mathcal{M}_q = \frac{T g_f q_f}{2\pi^2} \sum_{l=1}^{\infty} \sum_{j=1}^{l} \left( \frac{m_{ql}}{T} \right) \frac{3}{\tau} K_1 \left( \frac{m_{ql}}{\tau} \right) \frac{\partial}{\partial (eB)} \left( \frac{m_{ql}}{T} \right)
$$

Similarly, we obtain the expression for magnetization of gluons from equation (11) as,

$$
\mathcal{M}_g = \frac{T g_f}{2\pi^2} \sum_{l=1}^{\infty} \left[ \frac{\partial}{\partial (eB)} \int_{T_0}^{T} \frac{T}{m_g} \frac{3}{\tau} K_1 \left( \frac{m_{gl}}{\tau} \right) \left( \frac{m_{gl}}{T} \right) \frac{\partial}{\partial (eB)} \left( \frac{m_{gl}}{T} \right) \right]
$$

The variation of magnetization with temperature for different magnetic fields is plotted in Fig. 3. We see that the magnetization has a positive value for all values of temperature above $T_c$. This shows that QGP has a paramagnetic nature. The small deviation in the behavior of the graph for $eB = 0.2 GeV^2$ towards higher temperatures is because the contribution from even higher Landau Levels become relevant at this magnetic field. In Fig. 3, we have plotted the variation of magnetization with magnetic field for different temperatures. It is seen that the magnetization increases with magnetic field. The behavior of magnetization of QGP as seen in our work is qualitatively consistent with lattice QCD results and with results from HTL perturbation theory. We have summed over 20 Landau Levels whereas, in our work the Lowest Landau Level approximation has been used.
FIG. 1. Thermodynamic Pressure as a function of temperature different magnetic fields.

FIG. 2. Thermodynamic Pressure for different temperatures as a function of magnetic field.

used. We see that in our model the contribution from higher Landau Levels cannot be neglected.

V. PRESSURE ANISOTROPY

There has been some discussion in the literature regarding the existence of a pressure anisotropy and it has been suggested that the anisotropy is scheme dependent[60, 61, 66, 68–73]. In the \( \phi \) scheme, the presence of magnetic fields breaks rotational symmetry due to magnetization of the system in the direction of the magnetic field, resulting in a pressure anisotropy. Thus, in this scheme, the pressure has a transverse component different from the longitudinal component.

The transverse pressure is related to the longitudinal pressure as,

\[
P_T = P - eB \cdot \mathcal{M}. \tag{16}\]

Using equations \([9]\) and \([14]\) the contribution from quarks to transverse pressure becomes,

\[
\frac{(P_T)_q}{T} = \frac{g_f q_f(eB)^2}{2\pi^2} \sum_{l=1}^{\infty} \sum_{j}(2 - \delta_{0j}) \left[ T \left( \frac{m_{ql} l}{T} \right) K_0 \left( \frac{m_{ql} l}{T} \right) \frac{\partial}{\partial(eB)} \left( \frac{m_{ql} l}{T} \right) \right] - \int_{T_0}^{T} \frac{dT}{T} m_{qj} \frac{\partial}{\partial(eB)} \left( \frac{m_{ql} l}{T} \right) \left( \frac{m_{ql} l}{T} \right) . \tag{17}\]

Similarly, for gluons,

\[
\frac{(P_T)_g}{T} = \frac{g_f}{2\pi^2} \sum_{l=1}^{\infty} \frac{1}{l^4} \left[ T^3 \left( \frac{m_{gl} l}{T} \right)^2 K_2 \left( \frac{m_{gl} l}{T} \right) + \int_{T_0}^{T} \frac{dT}{T} m_{gj} \frac{\partial}{\partial(eB)} \left( \frac{m_{gl} l}{T} \right)^3 K_1 \left( \frac{m_{gl} l}{T} \right) - eB \left[ \int_{T_0}^{T} \frac{dT}{T} m_{gj} \frac{\partial}{\partial(eB)} \left( \frac{m_{gl} l}{T} \right)^3 K_1 \left( \frac{m_{gl} l}{T} \right) - T^3 \left( \frac{m_{gl} l}{T} \right)^2 K_1 \left( \frac{m_{gl} l}{T} \right) \right] \right] . \tag{18}\]

In Fig. 6 we have plotted the variation of transverse pressure with temperature with different magnetic fields.
FIG. 3. Velocity of sound as a function of temperature for different values of magnetic fields.

FIG. 4. Magnetization for different magnetic fields as a function of temperature.

In Fig. (7) we show the variation of transverse pressure with magnetic fields for different temperatures. Since magnetization increases with temperature, the transverse pressure tends to decrease with increase in magnetic field. It may also go to negative values, indicating that the system may shrink in the transverse direction [67]. This behavior too is qualitatively consistent with the results from [6, 66] and [67].

VI. CONCLUSION

We studied the thermodynamics, magnetization and pressure anisotropy of (2 + 1) flavor, magnetized QGP using the self-consistent quasiparticle model extended to include the effects of external magnetic field. The equation of state of magnetized (2 + 1) flavor QGP thus obtained show the same qualitative behavior as the EoS obtained from other works.

We calculate the magnetization and see that QGP has a paramagnetic nature. It has a small but positive magnetization at all temperatures above the transition temperature. The variation of magnetization with temperature and magnetic field are plotted. The presence of magnetization makes the system anisotropic, causing different pressures in directions parallel and perpendicular to the magnetic field. We evaluate the transverse pressure and plot its variation with respect to both magnetic fields and temperature. Our results show the same quali-
tative behavior as those obtained from Lattice QCD calculations and Hard Thermal Loop (HTL) perturbation theory approach. The equation of state and anisotropic pressure calculated here can be used as an input for magnetohydrodynamic calculations and analysis of elliptic flow of QGP formed in heavy-ion collisions.

We see that the extended quasiparticle model is quite effective in studying various thermodynamic and thermomagnetic properties of the de-confined QCD matter in the presence of magnetic fields. It is an advantage to this model that the higher Landau Level contributions can be incorporated very easily. The present results could be improved with a two-loop order thermomagnetic coupling which also incorporates the contributions from higher Landau Levels. Such a coupling would allow us to make predictions which are quantitatively reliable.

It would be interesting to study the transport coefficients of magnetized QGP with the equation of state obtained using the extended self-consistent quasiparticle model. Another area where the model, with appropriate modifications, can be applied is QGP at finite temperature and density. This would require the knowledge of the how the coupling strength depends on temperatures, magnetic fields and the chemical potential. Such a parametrization of the coupling strength would let us study the interior of a strongly magnetized neutron star using this model.

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