Optimization of forest road network layout problem

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Abstract. The article describes a three-stage hierarchical method of solving the optimization of forest road network layout problem taking into account road construction and transportation costs using a mathematical model based on the multi-commodity flow model. Optimization of the forest road network layout problem is formulated as an alternative of the Steiner tree problem in graphs. The first stage involves optimization of the forest road network layout within clusters. Many harvesting areas have already been divided into clusters using a clustering optimization model. The second stage involves solution of the similar problem between clusters. At the third stage, the final optimization is performed based on the results obtained at the first two stages. The developed methodology of heuristic solution of optimization of forest road network layout problem allows to improve solving time by 3-6 times with a deviation from the optimal result in the range of 0.5-8%.

1. Introduction

Forestry companies face a complex problem of planning the layout, construction and use of forest roads when making tactical-level management decisions [1]. This task requires significant resources and is one of the main efficiency drivers of the forestry companies in the future. Therefore, a lot of studies have been dedicated to the forest roads network layout optimization problem. Some authors solve this problem together with the optimization of harvesting area selection [2, 3], some authors solve these problems separately [4, 5]. Several approaches may be used for optimization of the forest roads network layout. The problem can be solved using iterative algorithms based on the sequential application of a modified algorithm for searching the shortest path [2, 5], the evolutionary algorithm [6], and other algorithms; using MIP models [7], or dynamic programming [8]. Comparative analysis of heuristic algorithms for finding a problem solution is given in the work of Shirasawa [9]. A similar task of searching for the shortest route in networks based on deep learning method of artificial neural networks is a case of great interest [10].

Complexity of the considered problem is driven by the need for detailed planning of road network layout in large forest areas. The existing approaches applicable to large areas are based on the algorithms of searching for the shortest path and they are not able to design the optimal forest road network and assess the optimality of the provided solution. An optimal or near-optimal solution can be obtained by formulating the problem as an alternative of NP-hard Steiner tree problem in graphs. Stückelberger [4] used the same approach and formulated the problem as a Steiner tree problem alternative. However, his heuristic solution method is not applicable or is limited in application to large forest areas. Solving time
for an area of 3 500 hectares ranged from 0.5 to 12 hours. One of the best practical solutions of a Steiner tree problem in graphs with mathematical programming and preprocessing of a graph is given in [11], which considers graphs up to 300 000 vertices and 400 000 edges. Solution of the optimization problem for the forest road network layout requires creation of a graph with a far greater number of vertices and edges. It should be noted that the authors are not aware of any studies involving assessment of impact of timber transportation costs on the optimal layout of the forest road network.

The developed methodology for solving a forest road network layout optimization problem for large areas with a detailed graph is aimed at finding a near-optimal solution which is based on application of linear programming, is hierarchical, and takes into account wood transportation costs. Within the framework of the methodology, an optimization problem of the forest road network layout is formulated as an alternative of the Steiner tree problem in graphs. Basic block diagram of the developed methodology is given in figure 1.

![Basic block diagram of the developed methodology](image)

**Figure 1. Framework of the optimization of forest road network layout problem solution methodology.**

The method of searching minimum cost paths based on road construction costs between harvest blocks using network flow models (SCF and MCF in figure 1) is described in the paper [12]. To solve the forest road network layout problem for large areas and improve the solving time, it is proposed to use a hierarchical approach based on clustering of the entire set of harvesting areas. The developed method of clustering is a mixed integer linear programming optimization model and is described in the paper [13]. The purpose of the model is to divide the entire set of harvesting areas into groups with the lowest costs of road construction from harvesting areas inside the cluster to its center, which is also a harvesting area, given the restrictions on the minimum and maximum number of harvesting areas in the cluster. The result of clustering is presented as a set of harvesting areas with the lowest road construction costs within the cluster to its center divided into clusters/groups. Below are the examples of clustering with division of the harvesting area set into 4 clusters (with the minimum number of supply-nodes in the cluster equaling 3) (figure 2) and division of the harvesting area set into 3 clusters (with the minimum number of supply-nodes in the cluster equaling 5) (figure 3).
The article focuses on the hierarchical method of searching for an optimal forest road network layout within and between clusters based on a mathematical model of linear programming (highlighted by hatching in Figure 1).

2. Methodology

A weighted, directed mathematical graph with extraverted edges was created to test the methodology. The vertices of the graph are located in the centroids of a squared grid with a 100 m × 100 m resolution. Valence of the graph vertices is limited to 16. The weight of the graph edge represents construction cost of a potential road. Cost calculation of constructing the edges is based on the penalty functions method similar to that described in [2, 5]. The basic cost of construction of one kilometer of forest road is multiplied by penalties depending on the grade, intersection with talwegs, and watersheds. Vertices, grade, talwegs, and watersheds were calculated based on digital elevation model data of the Shuttle Radar Topography Mission (SRTM) [14]. The graph with 11 919 vertices and 183 843 edges is used in this study for consideration. 18 graph vertices are accepted as harvesting areas. Each harvesting area was randomly assigned an inventory from 5 000 to 15 000 m³.

We propose a process of solving an optimization problem of forest road network layout within the clusters and between them hierarchically in 3 stages.

Stage 1: Solve the linear programming optimization model of forest road network layout within each cluster based on the mathematical model of multi-commodity flow.

Stage 2: Solve the linear programming optimization model of forest road network layout between cluster’s centers using the model applied in Stage 1 and results of Stage 1.

Stage 3: Solve the linear programming optimization model of forest road network layout for all harvesting areas using the model applied in Stages 1 and 2 where results of the two previous stages will serve as input data.

2.1. Stage 1

Mathematical model of forest road network layout based on multi-commodity flow model:

\[
\text{Objective function: } \sum_{v_i \in sV, v_j \in sV} v_{R_{v_i, v_j}} pC_{v_i, v_j} + \sum_{v_i \in sV, v_j \in sV, p \in sP} v_{F_{v_i, v_j}} pD_{v_i, v_j} pT_p I_p \rightarrow \min
\]

Table 1. Indices and sets.

| Set    | Index | Description          |
|--------|-------|----------------------|
| sV     | v, v_i, v_j | Vertices of the graph |
| sP     | p     | Products, | |sP| = | |sV| - 1 |

Table 2. Parameters.

| Parameter | Description |
|-----------|-------------|
| pX_v      | Coordinate X of the vertex v |
| pY_v      | Coordinate Y of the vertex v |
| pD_{v_i, v_j} | Distance from the vertex v_i to the vertex v_j, km |
| pU_{vp}   | 1, if the vertex v is a harvesting area (supply-node) and the source of the product p, 0 – otherwise |
| pM_{vp}   | 1, if the vertex v is a demand-node of the product p, 0 – otherwise |
| pB_{vp}   | 0|pU_{vp} = 1; 0|pU_{vp} + pM_{vp} = 0; 0|pM_{vp} = 1; |
| pT        | Roundwood transportation rate, thousand rubles / (m³ * km) |
| pI_p      | Inventory at the harvesting area (supply-node) corresponding to the product p |
| pC_{v_i, v_j} | Construction cost of the edge from the vertex v_i to the vertex v_j, thousand rubles |
Table 3. Variables.

| Variable   | Description                                                                 |
|------------|-----------------------------------------------------------------------------|
| \( vF_{v_i,v_j,p} \) | Product (p) quantity, transported from the vertex \( v_i \) to the vertex \( v_j \), units \( p \) |
| \( vR_{v_i,v_j} \) | 1, if the graph’s edge from the vertex \( v_i \) to the vertex \( v_j \) is built, 0 – otherwise |

Table 4. Variables properties.

| Variable   | Type     | Lower bound | Upper bound | Index domain                                                                 |
|------------|----------|-------------|-------------|------------------------------------------------------------------------------|
| \( vF_{v_i,v_j,p} \) | Continuous | 0           | 1           | \( v_i, v_j \in sV \mid pC_{v_i,v_j}>0 \)                                   |
| \( vR_{v_i,v_j} \) | Continuous | 0           | 1           | \( v_i, v_j \in sV \mid pC_{v_i,v_j}>0 \)                                   |

Constraints:

\[
\sum_{v_i \in sV} vF_{v_i,v_j,p} + pB_{v_i,p} = \sum_{v_j \in sV} vF_{v_j,v_i,p} \quad \forall v_i \in sV, p \in sP \tag{2}
\]

\[
vR_{v_i,v_j} \geq vF_{v_i,v_j,p} \quad \forall v_i, v_j \in sV \mid pC_{v_i,v_j}>0, p \in sP \tag{3}
\]

\[
vR_{v_i,v_j} + vR_{v_j,v_i} \leq l \quad \forall v_i, v_j \in sV \mid pC_{v_i,v_j}>0 \tag{4}
\]

Sets and indices are given in Table 1. Parameters of the model and their description are given in Table 2. Variables of the model and their properties are given in Table 3 and Table 4.

The objective function (1) of the model is to minimize road network construction and wood transportation costs.

The equation (2) is a transshipment constraint where for each vertex the sum of incoming and outgoing flows of each product is equal to 0. Herewith, in order to model the supply and take into account the corresponding costs the following parameters are used: 1 for the vertex representing the harvesting area (supply node) of the corresponding product, -1 for the demand-node of each product and 0 for all other vertices. At Stage 1, we consider the harvesting areas – cluster centers – as demand-nodes. The constraint (3) prohibits transportation of products using unbuilt roads (edges). The constraint (4) partially eliminates symmetry in allowable range of model solutions. Logically, they only allow construction of roads in one direction. The constraints (4) are similar to those (12) in Naderializadeh [2]. As a result of the mathematical model solution for each cluster the optimal network of forest roads was developed, an example of the model solution for four clusters (with the minimum number of supply-nodes in the cluster equaling 3) (figure 4) and three clusters (with the minimum number of supply-nodes in the cluster equaling 5) (figure 5).

2.2. Stage 2
At the second step, the following changes are introduced into the mathematical model described above:

Objective function:

\[
\sum_{v_i \in sV} vR_{v_i,v_j,p}C_{v_i,v_j}^{\text{stage2}} + \sum_{v_i \in sV} vF_{v_i,v_j,p}D_{v_i,v_j}T_{p}I_{p} \rightarrow \min \tag{5}
\]

Table 5. Parameters.

| Parameter | Description                                                                 |
|-----------|-----------------------------------------------------------------------------|
| \( pC_{v_i,v_j}^{\text{stage2}} \) | 0.001 thousand rubles for the edges that were included in the solution of the task of Stage 1; cost of road construction from vertex \( v_i \) to vertex \( v_j \), thousand rubles – for all other edges |

A new parameter for Stage 2 model and its description is given in Table 5.
The results of the first stage were used in the second stage as input data, i.e. the edges included in the solution in the first stage were assigned a minimum weight of one ruble. This was done in order to ensure that the road network between the cluster centers would be partially routed along the edges which were included in the optimal solution in the first stage. A new parameter for road construction cost is introduced. The value of this parameter is equal to 1 ruble for all the edges included in the solution of stage 1, for all other edges the value of the parameter does not change. At Stage 2, a harvesting area (V11688) is considered to be a demand-node of all products appearing in the cluster centers.

As a result of the solution of the mathematical model between the cluster centers, the optimal network of forest roads was developed, an example of the model solution for four clusters (with the minimum number of supply-nodes in the cluster equaling 3) (figure 6) and three clusters (with the minimum number of supply-nodes in the cluster equaling 5) (figure 7).

**Figure 2.** Example of dividing the harvesting area aggregation into 4 clusters (with the minimum number of supply-nodes in a cluster equaling 3).

**Figure 3.** Example of dividing the harvesting area aggregation into 3 clusters (with the minimum number of supply-nodes in a cluster equaling 5).

**Figure 4.** An example of Stage 1 model solution for a set of 4 clusters, where the cluster centers are the vertices (with the minimum number of supply-nodes in a cluster equaling 3).

**Figure 5.** An example of Stage 1 model solution for a set of 3 clusters, where the cluster centers are the vertices (with the minimum number of supply-nodes in a cluster equaling 5).
2.3. Stage 3. Final optimization
At the third stage, the harvesting area (V11688) – is considered as a demand-node of all products appearing in each supply-node. Here, optimization is carried out only among those edges which were
delivered as models solution results at the first two stages (Stage 1 and Stage 2). The following changes are introduced into the mathematical model described in Stage 1.

Table 6. Parameters.

| Parameter       | Description                                      |
|-----------------|--------------------------------------------------|
| $p_{E^1_{v_i,v_j}}$ | 1 – if the edge from the vertex $v_i$ to the vertex $v_j$ is included in the optimal solution at Stage 1, 0 - otherwise |
| $p_{E^2_{v_i,v_j}}$ | 1 – if the edge from the vertex $v_i$ to the vertex $v_j$ is included in the optimal solution at Stage 2, 0 - otherwise |
| $p_{E_{v_i,v_j}}$ | 1 – if $p_{E^1_{v_i,v_j}}+p_{E^2_{v_i,v_j}}\geq 1$, 0 - otherwise |

Table 7. Variables properties.

| Variable            | Type     | Lower bound | Upper bound | Index domain                                      |
|---------------------|----------|-------------|-------------|---------------------------------------------------|
| $v_{F_{v_i,v_j,p}}$ | Continuous | 0           | 1           | $v_i, v_j \in sV \ | p_{C_{v_i,v_j}} > 0 \ & \ p_{E_{v_i,v_j}} = 1$ |
| $v_{R_{v_i,v_j}}$   | Continuous | 0           | 1           | $v_i, v_j \in sV \ | p_{C_{v_i,v_j}} > 0 \ & \ p_{E_{v_i,v_j}} = 1$ |

Constraints:

$$v_{R_{v_i,v_j}} \geq v_{F_{v_i,v_j,p}}, \forall v_i, v_j \in sV \ | p_{C_{v_i,v_j}} > 0 \ & \ p_{E_{v_i,v_j}} = 1, p \in sP$$ (6)

$$v_{R_{v_i,v_j}} + v_{R_{v_j,v_i}} \leq 1, \forall v_i, v_j \in sV \ | p_{C_{v_i,v_j}} > 0 \ & \ p_{E_{v_i,v_j}} = 1$$ (7)

New parameters for Stage 3 model and their description are given in Table 6, new variables and their properties are given in Table 7.

Having solved the model described in Stage 1, provided that the demand-node of each product located in the harvesting area $V11688$, it is possible to find an optimal solution to the forest road network layout problem.

An example of a model solution for Stage 3 for four clusters (with the minimum number of supply-nodes in a cluster equaling 3) (figure 8) and three clusters (with the minimum number of supply-nodes in a cluster equaling 5) (figure 9). Figures in green show the optimal forest road network layout, black color highlights the network layout obtained by the hierarchical approach.

3. Results and discussion

The calculations were performed using a MSI GT72VR 7RE laptop, mathematical models were written in the AIMMS v4.64.1.0 mathematical modeling system, the solutions were found directly using commercial solver CPLEX v12.6 with the standard parameters except for the algorithm of searching for a solution to the linear programming problem where Simplex method was replaced by the method of internal point (LP Solve – Barrier) [15]. The results delivered during the computational experiment are given in Table 8.

The values of the objective function and the time required to find a solution were compared with those of the optimal solution developed based on the computational experiment. The following conclusions can be made on the basis of the analysis:

- in 5 out of 7 cases, the deviation of the objective function value from the optimal value was less than 1%;
- in all cases, it was possible to significantly reduce the solving time by a factor of approximately 3-6.

It should be noted that the value of the objective function obtained at the first stage of the solution, which is the sum of the values of the objective functions of the model solution for each cluster, increases as the minimum number of supply-nodes in a cluster grows, which can be explained by the fact that the smaller the number of harvesting areas in the cluster, the less the length and cost of road network.
construction in the cluster. On the other hand, the smaller number of supply-nodes in the cluster and the higher number of clusters result in expansion of the road network between the clusters and, consequently, to its cost increase.

It is impossible to determine the optimal value of the minimum number of supply-nodes in a cluster based on one computational experiment, which makes it necessary to take further steps:

- develop an approach to determine the optimal minimum number of supply-nodes in one cluster;
- conduct a large number of computational experiments using different data sets to determine the optimal minimum number of supply-nodes in one cluster;
- conduct computational experiments using other clustering methods in order to select the best method;
- test the methodology on a large data set after determining the optimal minimum number of supply-nodes in a cluster and selecting the best harvesting area clustering method.

Table 8. Results.

| Minimum number of supply-nodes in a cluster | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Number of clusters | 4 | 4 | 3 | 3 | 2 | 2 | 2 |
| Stage 1. Value of the objective function (thousand rubles) | 60181 | 64285 | 69405 | 75063 | 75685 | 77040 | 80431 |
| Stage 1. Solving time (sec.) | 151.15 | 156.76 | 220.74 | 196.46 | 400.95 | 377.36 | 312.61 |
| Stage 2. Value of the objective function (thousand rubles) | 31826 | 27256 | 23653 | 18971 | $0^a$ | $0^a$ | 4649 |
| Stage 2. Solving time (sec.) | 50.6 | 29.52 | 14.88 | 14.33 | $0^a$ | $0^a$ | 6.95 |
| Stage 3. Value of the objective function (thousand rubles) | 85020 | 86168.29 | 84400.58 | 84770.5 | 84648.8 | 84493 | 90792 |
| Stage 3. Road construction costs | 74532 | 75834 | 73958 | 74305 | 74244 | 74031 | 80833 |
| Stage 3. Transportation costs | 10488 | 10334 | 10443 | 10466 | 10405 | 10449 | 9959 |
| Stage 3. Solving time (sec.) | 202.02 | 186.34 | 235.7 | 210.85 | 332.56 | 377.36 | 319.62 |
| Optimal value of the objective function (thousand rubles) | 84002.98 |
| Optimal road construction costs | 73784 |
| Optimal transportation costs | 10219 |
| Solving time (sec.) | 746.41 |
| Deviation from the optimum objective function value (%) | 1.21 | 2.58 | 0.47 | 0.91 | 0.77 | 0.58 | 8.08 |
| Time deviation from the optimal solution solving time (%) | 146.69 | 300.56 | 216.68 | 254.00 | 124.44 | 97.80 | 133.53 |

*the Stage 2 model was not solved because the Stage 1 model solution had already provided roads within the clusters connecting the clusters to each other*
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