An illustration of mathematical skills: the procept (process-concept) junior high school students in solving mathematical problems

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Abstract. This study is inspired by the result of a previous study which found that student skills in managing mathematical concepts to solve mathematics are very low. Students find difficulties in searching, applying, or choosing the problem-solving procedures. By considering this issue, the researchers aim to discover the process of managing knowledge to find flexible solutions with conceptual basis while understanding the symbols used, both as the process and concept to solve mathematical problems. Procept in this paper is defined as the symbol that expresses process and concept. This study was a qualitative-explorative descriptive research. The research subject was determined by giving mathematics skill test to 42 Junior High School students as the subject, thus, five high-skilled students were acquired, while 22 students fell into the moderate category, and 15 students fell into the low category. The characterization for the illustration of procept skills of each category is formulated. Data collecting was conducted by implementing a task-based interview, and was analysed by applying the qualitative data analysis technique performed through data reduction, data presentation, and conclusion drawing. Research results show that in the formation stages of procept (pre-procedure, procedure, multi-procedure, process, procept), subjects with high and moderate skills of mathematics have reached the procept stage, while subjects with low mathematical skills are in the stage between the procedural and multi-procedural.

1. Introduction
There are three abilities that have to be put into attention in mathematics learning, namely the ability to understand mathematical concepts, the ability to use algorithms (procedures) properly and efficiently, and the ability to use both abilities in solving mathematical problems. This view is consistent with the learning purpose of mathematics at the level of Junior High School/Madrasah Tsanawiyah, including how students understand concepts of mathematics, i.e., capable of explaining the relation among concepts and using concepts or algorithms (procedures) swiftly, accurately, efficiently, and properly to solve problems (the Regulation of the Minister of Education and Culture No.58/2014). According to Akbar [1], problem solving is crucial in mathematics, this can be seen from every Standard of competencies and Basic competencies that asserts the necessity of problem solving skills, thus, it can be said that problem solving is the heart of mathematics.

However, ground facts showed a finding that student’s skill in managing mathematical concepts to solve mathematics was very low. Students are facing difficulties in searching, using, or selecting problem solving procedures. Based on this consideration, the researchers aim to discover the process of managing knowledge so flexible solutions with conceptual basis can be obtained, and symbols used
can be understood, both as the process or concept to solve mathematical problems. Even from the result of TIMSS in 2015 [2], Indonesia got the lowest score, below the average, ranked 44th out of 49 countries. Similar to the results of PISA in 2018, which ranked Indonesia 7th from the bottom with a score of 379, below the OECD average [3]. This condition shows that most students in Indonesia can only recognize a few basis facts, and have not been capable of using the relation between concepts to solve mathematical problems.

This study was focused on discovering the illustration of mathematical skills of Junior High School students in solving mathematical problems on gradient and linear equation materials, which are crucial considering the following reasons. First, students start to learn variables formally at Junior High School, which is an issue [4], unlike the Elementary School students that only focus on arithmetic, i.e., working with numbers, calculation operation and its properties. A study by Jupri, Drijver, and Van den Heivel-Panhuizen [5] found five categories that often become the issue for many Indonesian students in their algebra study, which usually found in 1) implementing arithmetical operations, 2) understanding variable concepts, 3) understanding the expression of algebra, 4) understanding different meanings of the same symbols, and 5) mathematization.

Second, the procept and development as suggested by NCTM [6], there are five components to be developed to achieve mathematical proficiency, namely conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The directly related components are mainly regarding the development of procedural fluency that is supported by conceptual understanding that is used in solving problems as the application of strategic competence.

Third, studies about procept needs to be done so the quantity and quality can be improved in an aim to enrich repertoires and references, either in the forms of textbook or scientific journal. Some studies related to procept are 1) [7], which studied the comprehension of function procept in High School students based on the framework of Gray and Tall [8], 2) Weber [9] applied procept theory in the learning design of Trigonometry Subject for first-year students which were divided into two groups, the first group was taught conventionally while the second group was taught with Tall’s procept theory (procedure-process-concept). A conclusion was drawn through interview and writing test methods in which students taught conventionally have limited comprehensions regarding function while the group that learned by using the procept theory has depth understanding about functions of trigonometry; 3) besides being implemented in informal mathematics, the procept concept is also being implemented in formal mathematics, as the studies conducted by [10] and[11], which implemented the procept concept in geometric proofing informally at Junior High Schools, i.e., proof problems in geometry. Therefore, the researchers are intended to discover the procept of Junior High School students in solving mathematical problems.

Symbols are perceived as the axis between process and concept, which are denoted with precept [12]. Procept is perceived as a cognitive construct, in which symbols can change focus from a process to calculate or manipulate to a concept that can be perceived as a manipulable entity. In other words, procept is the combination of concept and process represented by the same symbol. Procept is a combination of mental objects consisting of processes, concepts generated by the process, and symbols are used to define the concept or process, or both. Thus, procept is simply defined as the process in interpreting symbols in processes and concepts.

The concept of procept began from the concept of number that was started by counting an object, for instance, when a child counts three apples, the action that may be done by him is that the child counts by saying “there are one, two, three apples”. This counting activity is repeated and done every time an object is counted by its number, therefore, such an activity becomes a routine action that can be done in mind. Counting activity is routinely conducted, therefore, the counting process in mind occurs within a concept formation of number 3 through the provision of three apples. The child compresses the counting process done in mind and then mentions that “there are … three apples” or “there are three apples”. So, this method indicates a way of counting so it is compressed into a counting concept. Symbol 3 then defines a counting process or a counting concept. Therefore, this action can be determined as compressing process (counting) to a concept (number) by using symbols, i.e., through procedure, process, and procept. Tall then refined this stage into pre-procedure, procedure, multi-procedure, process and procept as seen in Figure 1. This figure illustrates the formation stage of
procept, from the left side that is procedural to the right side that is flexible or proceptual. Procedural is steps that have to be followed in solving a problem. Students in the pre-procedure stage will be

![Diagram of Spectrum of Outcomes]

**Figure 1.** Stages of procept formation.

highly procedural while students in the procept stage will be flexible (proceptual)[13].

According to Tall et. al. [14], mathematical problem solving will develop student’s flexibility skill, in other words, it can be said that procept can be developed through mathematical problem solving. Hence, there is a relation between student’s capability in searching, using, or choosing procedures and the flexibility level. In solving a problem, students in the pre-procedure stage will still be unable to acquire a solution or maybe only a partial solution is obtained, or even no solution is found at all. Students in the procedure stage is capable of solving a problems through routine measures that they have mastered, so there is a big possibility that students get a solution. Students in the stage of multi-procedure will have many strategies or procedures in solving a problem, thus, they can determine a more efficient solution. Students in the process stage acquire a flexible solution with its conceptual basis. However, at this stage, such students have yet to understand symbols, either as the representation of procedures or processes and concepts. At the final stage, i.e., procept stage, students will acquire a flexible solution with its conceptual basis and understand the symbols used as the process or concept.

Some examples of procept in Elementary School that we may come across include whole number procept, addition procept, subtraction procept, multiplication procept, division procept, and place value procept. The examples of procept in Secondary Schools are algebra expression, function, gradient, linear equation, trigonometry comparison, function limit, derivation, integral, and vector [15]. Generally, in learning mathematics, students are using their procedural understanding more often, by memorizing procedures in solving mathematical problems for non-routine problems, notably the routine problems. It is very rare that the procedures established over the conceptual comprehension basis are used, as study conducted by [16].

There are many factors affecting student’s skills in solving problems, including mathematical skills [17]. Students with high mathematical skills also have high skills in solving mathematical problems; while students with moderate mathematical skills have decent skills in solving problems. Students with low mathematical skills have poor skills in solving problems. Students with high, moderate, and
low skills will be made as the focus in building an illustration of procept in solving mathematical problems.

2. Research Method
This study was an explorative-descriptive research with qualitative approach. The collection of research subjects or the participant selection technique was conducted through purposive sampling, which is based on specific criteria that in this context are learning materials received and mathematical skills [18]. This study was conducted at a Junior High School with VIII class students as the subject, done in January 2020 before the pandemic. The selection of research subjects was based on Mathematical Skill Test given towards 42 subjects, consisting of 12 male students and 30 female students. Then, from the result scores of the mathematical skill test, it has been acquired that 15 potential subjects (six male students, and nine female students) have low mathematical skills (Score < 60), 22 potential subjects (five male students, and 17 female students) have moderate mathematical skills (60 ≤ score< 80), and five potential subjects (one male, and four female students) have high mathematical skills (Score ≥ 80). Next, of each group, the characterization that illustrates the procept of high, moderate, and low groups was made.

There were two instruments in the study, namely the main and supporting instruments. The main instrument is the researchers themselves, because they are the ones who plan, implement, collect data, and report research results. The supporting instrument is the mathematical skill test (TKM), mathematical problem solving skill test (TPM), interview guideline, and audio-visual recording device that was used to record the activities during the study. The instrument of mathematical skill test was used to obtain data of students’ mathematical skills that will be used to select research subjects. The problem-solving test was used to obtain data of gradient and linear equation procept of Junior High School students in solving mathematical problems by referring to the interview guideline. The following is the example of one of the tests of mathematical problem solving.

Based on the following figure, define the equation of line that is perpendicular to the line $k$.

![Figure 2](image)

**Figure 2.** A Question of problem-solving test.

After valid data have been acquired, the researchers analysed the data by reducing the data, presenting the reducted data, interpreting the data, and drawing conclusions.

3. Results and discussion
Based on the procept formation stages of Tall [19], i.e., pre-procedure, procedure, multi-procedure, process and procept, then the indicators for each stage were determined as shown in Table 1.
Table 1. Indicators of procept formation stages.

| No | Stage       | Indicator                                                                 |
|----|-------------|---------------------------------------------------------------------------|
| 1  | Pre-procedure | a. Understanding the problem                                              |
|    |             | b. Having a little information about concepts associated with problem solving |
|    |             | c. Understanding the use of procedures to solve problems                   |
|    |             | d. Un-acquiring solutions or only finding parts of problem solving         |
| 2  | Procedure   | a. Creating problem-solving plans                                          |
|    |             | b. Implementing problem-solving measures in accordance with problem-solving plans |
|    |             | c. Can only solve problems with one solution                               |
|    |             | d. Have not been capable of determining the efficient solution measures of various available solution measures. |
| 3  | Multi Procedure | a. Understanding some different procedures to solve similar problems and capable of solving problems with more than one method. |
|    |             | b. Determining the efficient procedure to solve problems based on logical reasons. |
|    |             | c. Understanding various types of procedure to solve problems, but the conceptual basis is yet to understand. |
| 4  | Process     | a. Obtaining flexible solutions with their conceptual basis                |
|    |             | b. Understanding the use of symbols as the representation of procedures but have yet to understand the use of symbols as the representation of concepts. |
| 5  | Procept     | a. Acquiring flexible solutions with their conceptual basis                |
|    |             | b. Understanding symbols as an entity of procedure and concept to solve problems. |

To determine the description of gradient and linear precept based on [20] regarding procedural knowledge and conceptual gradient and linear equation, these are the types of concept and process of gradient and linear equation used in the task-based interview, in an aim to determine the illustration of procept skills of gradient and linear equation in solving mathematical problems based on their mathematical skills.

The following is the results of mathematical problem solving of students.

3.1 The work result of subjects with high mathematical skills (ST)

Figure 3. Determining the Equation of line $k, l$.  
Figure 4. Drawing line $l$. 

Table 1. Indicators of procept formation stages.
3.2 *The work result of subjects with moderate mathematical skills (SS)*

![Figure 5](image5.png) Determining the Equation of line *k, l.*

![Figure 6](image6.png) Drawing line *l.*

3.3 *The work result of subjects with low skills (SR)*

![Figure 7](image7.png) Determining the Equation of line *k.*

![Figure 8](image8.png) Drawing line *l.*

The following is the summary of research results based on the task-based interview and subjects’ work.

| No. | Description                                | ST                                        | SS                  | SR                                      |
|-----|--------------------------------------------|-------------------------------------------|---------------------|-----------------------------------------|
| 1   | Solving problems                           | Able to solve and the result is correct  | Able to solve and the result is correct | Able to solve but the result is less correct Between procedures and multi-procedures |
| 2   | Procept stage                              | Procept                                  | Procept             |                                         |
| 3   | Gradient as the concept (*m_k*)            | a) sd. d)                                | b) sd. d)           | b) sd. d)                               |
| 4   | Gradient as the process (*m_p*)            | a) sd. d)                                | b) sd. d)           | b) sd. d)                               |
Students understand the gradient procept, namely as the concept \( (m_k) \), which is understood a) geometrically, gradient is the ratio of the change in the vertical side to the change in the length of the horizontal side; b) parametrically, gradient as the coefficient of \( x \) from the equation of \( y = mx + c \); c) algebraically, the gradient concept is associated with the ratio between the change in the ordinate of the abscissa of two points; and d) the correlation among lines, the concepts involved in determining whether the two lines are parallel, perpendicular, intersect, and so on. As the gradient process \( (m_p) \) is understood procedurally as determining the gradient, namely: a) determining the gradient with the ratio of vertical and horizontal components, b) determining the gradient with the ratio of the difference of ordinates and abscissas, c) determining the gradient by manipulating a linear equation in its general form, and d) determining the gradient in solving the equation of the correlation among lines, e) using and utilizing as well as selecting the procedure to determine the linear equation and f) applying concepts or algorithms in problem solving. As the process \( (y_p) \), the linear equation is understood as the subject with procedures used to define the linear equation, namely a) \( y = mx + c \), if the gradient \( (m) \) and the intersection with ordinate axis are known \( (c) \); b) \( y-y_1 = m(x-x_1) \), if the gradient and one of the points in the line are known; c) \( \frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1} \), if two points are known.

Therefore, the results of the study indicate that subjects with high mathematical skills and are in the stage of forming a procept (pre-procedure, procedure, multi-procedure, process, procept) have entered the procedural stage, while subjects with low mathematical skills are in between procedural and multi-procedural stages.

4. Conclusion

From the result and discussion, it can be concluded that subjects with high mathematical skills and are in the stage of forming the procept (pre-procedure, procedure, multi-procedure, process, procept) have entered the procedural stage, while subjects with low mathematical skills are in between procedural and multi-procedural stages. Subjects with high mathematical skills understand the gradient and straight line equations that meet all the criteria, while subjects with high mathematical skills meet most of the criteria and subjects with low mathematical skills meet some of the criteria.

For the next research, the description of the procept can be applied to other Mathematics or Physics materials in order to add scientific insight.

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