THE AXIS-RATIO DISTRIBUTION OF GALAXY CLUSTERS IN THE SDSS-C4 CATALOG AS A NEW COSMOLOGICAL PROBE

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ABSTRACT

We analyze the C4 catalog of galaxy clusters from the Sloan Digital Sky Survey (SDSS) to investigate the axis-ratio distribution of the projected two-dimensional cluster profiles. We consider only those objects in the catalog whose virial mass is close to $10^{14} h^{-1} M_\odot$, with member galaxies within the scale radius 1000 kpc. The total number of such objects turns out to be 336. We also derive a theoretical distribution by incorporating the effect of projection onto the sky into the analytic formalism proposed recently by Lee, Jing, & Suto. The theoretical distribution of the cluster axis ratios is shown to depend on the amplitude of the linear power spectrum ($\sigma_8$) as well as the density parameter ($\Omega_m$). Finally, fitting the observational data to the analytic distribution with $\Omega_m$ and $\sigma_8$ as two adjustable free parameters, we find the best-fitting value of $\sigma_8 = (1.01 \pm 0.09)(\Omega_m/0.6)^{0.07 \pm 0.02} + 0.11\Omega_m$. It is a new $\sigma_8-\Omega_m$ relation, different from the previous one derived from the local abundance of X-ray clusters. We expect that the axis-ratio distribution of galaxy clusters, if combined with the local abundance of clusters, may put simultaneous constraints on $\sigma_8$ and $\Omega_m$.

Subject headings: cosmology: theory — galaxies: clusters: general — large-scale structure of universe

1. INTRODUCTION

The standard theory of structure formation based on the cold dark matter (CDM) paradigm explains that the large-scale structures of the universe like galaxies, groups, and clusters of galaxies are originated from the primordial fluctuations of the matter density field through gravitational instability. In this scenario, the observed properties of the large-scale structures can be used to extract crucial information on the initial conditions of the early universe.

Among the various properties of the large-scale structures, it is the abundance of galaxy clusters that has attracted most cosmological attentions so far. Since the clusters are relatively young still in the quasi-linear regime, their abundance can be inferred from the linear theory (e.g., Press & Schechter 1974). Furthermore, being rare, the evolution of galaxy clusters depends sensitively on the background cosmology, especially on the amplitude of the linear power spectrum on scale of $8 h^{-1}$ Mpc ($\sigma_8$) and the density parameter ($\Omega_m$). It has been shown that by comparing the linear theory prediction with the cluster abundance from X-ray or Sunyaev-Zel’dovich (SZ) effect observations, one can constrain $\sigma_8$ and $\Omega_m$ (White et al. 1993; Bond & Myers 1996; Eke et al. 1996; Pen 1998; Henry 2000; Fan & Chiuhe 2001; Pierpaoli et al. 2001; Seljak 2002). For instance, Eke et al. (1996) found a relation of $\sigma_8 = (0.52 \pm 0.04)\Omega_m^{-0.52+0.13}\Omega$ (for a flat universe with nonzero cosmological constant, $\Lambda$) by comparing the Press-Schechter prediction with the local abundance of the X-ray clusters compiled by Henry & Arnaud (1991).

To break the degeneracy between $\sigma_8$ and $\Omega_m$, however, it is necessary to find another relation obtained from another observable property of galaxy clusters. One candidate is the shape of galaxy clusters. It has long been known observationally as well as numerically that in general the shapes of galaxy clusters are not so much spherical as elliptical (Schipper & King 1978; Binggeli 1982; West et al. 1989; Jing & Suto 2002). Furthermore, it was shown by recent numerical simulations that the ellipticity distribution of galaxy clusters depend strongly on the background cosmology (Jing & Suto 2002; Hopkins et al. 2005). This numerical finding implies that it may be possible to find a new independent relation between $\Omega_m$ and $\sigma_8$ with the ellipticity distribution of galaxy clusters.

Most of the previous approaches to the cluster ellipticity distribution were numerical. In order to use the cluster ellipticity distribution as a cosmological probe, however, it is highly desirable to have an analytic model derived from physical principles. Although Bardeen et al. (1986) made a first analytic attempt in the frame of the density peak formalism, their model was applicable only in the asymptotic limit of low ellipticity.

Very recently, Lee et al. (2005, hereafter LJS05) derived a new analytic expression for the halo axis-ratio distribution by combining the density peak formalism and the Zel’dovich approximation. They demonstrated that the analytic model produces the characteristic behaviors of the halo axis-ratio distribution found in simulations, which gives a hope that it may provide a theoretical footing for the use of the cluster ellipticity distribution as a cosmological probe. Our goal here is to find a new functional form of the $\sigma_8-\Omega_m$ relation, independent of the previous one derived from the cluster abundance, by comparing the LJS05 analytic prediction to the axis-ratio distribution of galaxy clusters determined from the recent observational catalog.

The plan of this paper is as follows: In § 2, we provide a brief description of the observational data. In § 3, we explain how to derive an analytic axis-ratio distribution of two-dimensional projected profiles of galaxy clusters from the LJS05 formalism. In § 4, we find a new functional form of relation of $\sigma_8-\Omega_m$ relation through fitting the model to the observational data. In § 5, we discuss the success and the limitation of our work, and we draw a final conclusion.

2. DATA FROM SDSS-C4 CATALOG

The C4 catalog contains 748 objects (Miller et al. 2005) identified in the Second Data Release (DR2) of the Sloan Digital Sky Survey (SDSS; Strauss et al. 2002). The sky coverage and the redshift range of the catalog are $\sim 2600$ deg$^2$ and $0.02 \leq z \leq 0.17$, respectively. The majority of the objects in this catalog are
clusters of galaxies with the mean membership of 36 galaxies. In the catalog, however, are also included smaller groups with the membership of at most 10 galaxies as well as larger structures with the membership of over 100 clusters. The mass range of the catalog is thus quite broad, from \(10^{12}\) to \(10^{16} \, h^{-1} M_\odot\) (where \(h\) is the dimensionless Hubble parameter).

The SDSS-C4 catalog provides various spectroscopic properties of galaxy clusters, among which the following data are used for our study: the axis-ratio of a projected two-dimensional image \((q)\), the virial mass \((M)\), and the redshift \((z)\). As above, we briefly describe how these quantities were measured.

The axis ratio, \(q\), is derived from an elliptical fit to a projected two-dimensional image of a cluster. The elliptical fit was conducted by the diagonalization of the covariance of the positions of all galaxies within 1000 kpc with respect to the cluster center. The center of a cluster is also measured using all galaxies within 1000 kpc.

The virial mass, \(M\), of each cluster was computed (assuming the isothermality and the spherical shape) as

\[
M = \frac{3}{5} \frac{\sigma^2}{G} r_s,
\]

where \(\sigma_1\) is the line-of-sight velocity dispersion and \(r_s\) is the deprojected three-dimensional virial radius of a cluster (Carlberg et al. 1996). The value of \(r_s\) is related to the “ringwise projected harmonic mean radius,” \(R_h\) (see e.g., [3] in Carlberg et al. 1996), as \(r_s = \pi R_h / 2\) (Limber & Mathews 1960). The value of \(R_h\) depends on which scale radius is used in selecting the member galaxies. In the catalog, there are five different criteria used to measure the value of \(R_h\): \(w = 500, 1000, 1500, 2000,\) and \(2500 \, \text{kpc} \).

In this paper, consistently, we use the criterion \(w = 1000 \, \text{kpc}\). Using the fixed scale radius \(w = 1000 \, \text{kpc}\) does not mean that the virial radius of a cluster is fixed to be 1000 kpc for all clusters. What it means is that the ringwise projected harmonic mean radius, \(R_h\), was determined using those galaxies within the scale radius \(w = 1000 \, \text{kpc}\) from the cluster center.

But note that using the fixed scale radius of \(w = 1000 \, \text{kpc}\) may systematically affect the determination of the cluster mass, \(M\). Since the catalog contains various objects in the wide mass range, using the fixed scale radius of \(w = 1000 \, \text{kpc}\) can cause either underestimate or overestimate of the value of \(M\), depending on how massive an object is. To reduce this kind of possible systematic effect, we focus only on the narrow cluster mass range of \(10^{13.5} \, h^{-1} M_\odot \leq M \leq 10^{14.5} \, h^{-1} M_\odot\). We find a total of 336 such clusters in the catalog, with the median redshift of \(z_m = 0.08\).

3. PHYSICAL ANALYSIS

3.1. Overview of the Analytic Model

The LJS05 analytic model for the axis-ratio distribution of dark halos adopts two classical theories as its background: the Zel’dovich approximation (Zel’dovich 1970) and the density peak formalism (Bardeen et al. 1986). By applying the Zel’dovich approximation, LJS05 assumed that the three principal axes of a triaxial halo, \(\{a, b, c\}\) (with \(a \leq b \leq c\)) are related to the three eigenvalues \(\{\lambda_1, \lambda_2, \lambda_3\}\) (with \(\lambda_1 \geq \lambda_2 \geq \lambda_3\)) of the linear deformation tensor, \(d_{ij} \equiv \partial_i \partial_j \phi\), where \(\phi\) is the perturbation potential:

\[
a \propto \sqrt{1 - D_+ \lambda_1}, \quad b \propto \sqrt{1 - D_+ \lambda_2}, \quad c \propto \sqrt{1 - D_+ \lambda_3},
\]

where \(D_+ = D_+(z)\) is the growth rate of the linear density field. Here, the sum of the three eigenvalues equals the linear density contrast, \(\delta \equiv \Delta \rho / \bar{\rho}\) (where \(\bar{\rho}\) is the mean density): \(\lambda_1 + \lambda_2 + \lambda_3 = \delta\).

By employing the density peak formalism, LJS05 also assumed that a region in the linear density field will condense out a virialized halo if it satisfies the following two conditions:

\[
\delta = \delta_c = \delta_{0,0} D_+(0)/D_+(z), \quad \lambda_3 > \lambda_0 = \lambda_{0,0} D_+(0)/D_+(z),
\]

where \(\delta_c\) and \(\lambda_c\) represent the threshold values of \(\delta\) and \(\lambda\), respectively, with \(\delta_{0,0} \equiv \delta(z = 0)\) and \(\lambda_{0,0} \equiv \lambda(z = 0)\). The value of \(\delta_{0,0}\) can be theoretically evaluated with the help of the top-hat spherical collapse model. For instance, \(\delta_{0,0} = 1.686\) for a flat universe with closure density (e.g., Kitayama & Suto 1996).

As for the value of \(\lambda_{0,0}\), it is precisely 0 in the original density peak formalism. Instead of using this theoretical value, however, LJS05 used a positive value of \(\lambda_{0,0} = 0.37\), which was determined empirically through fitting. Their claim was that the breakdown of the linear theory in the highly nonlinear regime is responsible for the deviation of \(\lambda_{0,0}\) from the theoretical value of 0 in practice.

Here we use the theoretical value of \(\lambda_{0,0} = 0\) rather than the empirical value of \(\lambda_{0,0} = 0.37\) for the following reason. The value of \(\lambda_{0,0} = 0.37\) was found empirically by comparing the analytic model with the numerical result for the concordance cosmology only. Remember that our purpose is to use the analytic distribution as a probe of cosmology without having any bias. Hence, we set \(\lambda_{0,0}\) at the original theoretical value 0 in the rest of this paper.

Defining two real variables \(\mu_1\) and \(\mu_2\) as

\[
\mu_1 \equiv \frac{b}{c}, \quad \mu_2 \equiv \frac{a}{c},
\]

LJS05 derived analytically the joint probability density distribution of \(\mu_1\) and \(\mu_2\), which depends on the cluster mass \(M\) and the formation epoch \(z_f\):
The relation between \( \{\mu_1, \mu_2\} \) and \( \{\lambda_1, \lambda_2\} \) is given as

\[
\lambda_1 = \frac{1 + (D_f \delta_c - 2) \mu_1^2 + \mu_1^2}{D_f (\mu_1^2 + \mu_1^2 + 1)},
\]

(8)

\[
\lambda_2 = \frac{1 + (D_f \delta_c - 2) \mu_1^2 + \mu_2^2}{D_f (\mu_1^2 + \mu_2^2 + 1)},
\]

(9)

and the Jacobian \( |(\partial \lambda_1/\partial \lambda_2)/(\partial \mu_1/\partial \mu_2)| \) was found to be

\[
\left| \frac{(\partial \lambda_1/\partial \lambda_2)}{(\partial \mu_1/\partial \mu_2)} \right| = \frac{4(D_f \delta_c - 3)^2 \mu_1 \mu_2}{D_f^2 (\mu_1^2 + \mu_2^2 + 1)^3}.
\]

(10)

Equation (6) gives the probability distribution of the two axis ratios of a triaxial galaxy cluster with mass \( M \) formed at \( z_f \). In practice, however, what we measure is not the formation epoch, \( z_f \), but the observation epoch, \( z \), i.e., the redshift at which a cluster is observed. Hence, LJS05 rederived an analytic expression for \( p(\mu_1, \mu_2; M; z) \) from equation (6) as

\[
p(\mu_1, \mu_2; M; z) = \int_{z}^{\infty} dz_f \frac{\partial p_f(z_f; 2M, z)}{\partial z_f} p(\mu_1, \mu_2; 2M; z_f),
\]

(11)

where \( \partial p_f/\partial z_f \) represents the formation epoch distribution of galaxy clusters. LJS05 used the fitting formula given by Kitayama & Suto (1996), which is an approximation to the analytic expression found by Lacey & Cole (1994).

3.2. Projection onto the Sky

Although equations (6)–(11) provide an analytic expression for the axis-ratio distribution of the three-dimensional shape of a triaxial cluster with mass \( M \) observed at \( z \), it is hard to compare it directly with the observational data. In observation, as we noted in § 2, what is measured is only a two-dimensional projected profile of a galaxy cluster. Therefore, for a proper comparison with observational data, it is necessary to incorporate the effect of projection along the line of sight into the analytic model.

To incorporate into equation (11) the projection effect onto the plane of the sky, we follow Binney (1985): Let \( (\theta, \phi) \) be the usual polar coordinates of the line-of-sight vector in the principal axes of a triaxial cluster, and let \( q \) be the axis-ratio of a two-dimensional cluster profile projected along the line of sight direction. Then the probability density distribution of \( q \) can be obtained by performing an integration of equation (6) over \( \mu_1, \mu_2, \theta, \) and \( \phi \) as

\[
p(q; M, z) = \frac{1}{4\pi} \int_{0}^{2\pi} d\phi \int_{0}^{\pi} \sin \theta d\theta \int_{1}^{1} d\mu_1 \times \int_{0}^{1} d\mu_2 \delta_D(q - q'(\theta, \phi, \mu_1, \mu_2)) p(\mu_1, \mu_2; M, z),
\]

(12)

where

\[
q'(\theta, \phi, \mu_1, \mu_2) = \left\{ \frac{U + V - [(U - V)^2 + W^2]^{1/2}}{U + V + [(U - V)^2 + W^2]^{1/2}} \right\}^{1/2}
\]

(13)

\[
U = \cos^2 \theta \left( \frac{\sin^2 \theta + \cos^2 \phi}{\mu_1^2} \right) + \sin^2 \theta \mu_2^2,
\]

(14)

\[
W = \cos \theta \sin 2\phi \left( 1 - \frac{1}{\mu_1^2} \right) \frac{1}{\mu_2^2},
\]

(15)

\[
V = \left( \frac{\sin^2 \phi + \cos^2 \phi}{\mu_1^2} \right) \frac{1}{\mu_2^2}.
\]

(16)

Through equations (6)–(16), one can evaluate analytically the probability density that the projected profile of a galaxy cluster of mass \( M \) is observed to have an axis ratio of \( q \) at redshift \( z \). This theoretical axis-ratio distribution varies with the background cosmology, since it depends on the initial power spectrum, \( P(k) \). In this paper, we adopt the following approximation formula for \( P(k) \) (Bardeen et al. 1986):

\[
P(k) \propto \left[ \ln \left( 1 + 2.34q \right) \right]^2 \frac{1}{2.34q} \times \left[ 1 + 3.89q + (16.1q)^2 + (5.46)^3 + (6.71q)^4 \right]^{-1/2},
\]

(17)

where \( q \equiv k/H^{-1} \) Mpc. Here the shape factor, \( \Gamma \), is related to \( \Omega_m, h \), and the baryon density parameter, \( \Omega_b \) (Peacock & Dodds 1994; Sugiyama 1995), as

\[
\Gamma = \Omega_m h \left( \frac{T_0}{2.7 \text{ K}} \right) \exp \left[ -\Omega_b \left( 1 + \sqrt{2h}\Omega_m^{-1} \right) \right],
\]

(18)

where \( T_0 \) is the temperature of the cosmic microwave background radiation.

Figure 1 plots equation (12) on a cluster scale, \( M_b \) (see § 4 for a definition of \( M_b \)), at present redshift (\( z = 0 \)) for four different
cosmological models: ΛCDM (solid line); OCDM (dotted line); SCDM (dashed line); τCDM (White et al. 1995; long-dashed line). The cosmological parameters used to characterize each model are listed in Table 1.

As one can see, equation (12) depends sensitively on the background cosmology. More elliptical clusters are predicted by the ΛCDM and OCDM models with high $\sigma_R$ and low $\Omega_m$ than by the SCDM and τCDM models with low $\sigma_R$ and high $\Omega_m$. It can be understood by the following logic: the SCDM and τCDM models predict more small scale powers, and thus should result in rounder shapes of galaxy clusters in the end.

We compare equation (12) with the observational distribution from the 336 SDSS-C4 clusters, assuming the ΛCDM. Figure 2 plots together the analytic (solid line) and the observational (filled circles) axis-ratio results for comparison. The errors of the observational points are Poissonian. The values of $M$ and $z$ in the analytic axis-ratio distribution are set to be $M_6$ (see § 4.1) and $z_{med} = 0.08$, to be consistent with the observational data. As one can see, the overall agreement between the two is quite good.

4. APPLICATION OF THEORY TO OBSERVATIONS

4.1. Characteristic Cluster Mass

The analytic axis-ratio distribution (eq. [12]) depends on the cluster mass $M$ and redshift $z$. Therefore, to compare it with the observational data, one has to first specify the values of $M$ and $z$. As for $z$, to be consistent with the observational data, we set it at the median redshift of the selected 336 clusters, $z_m = 0.08$, given that the redshift range of the selected clusters is quite narrow.

As for $M$, there is a subtle point that has to be taken into consideration. The virial mass, $M_v$, of each cluster in the C4 catalog was computed under the assumption of a concordance cosmology with $\Omega_m = 0.3$, $\sigma_8 = 0.9$, $h = 0.7$. In other words, the cluster mass $M$ given in the catalog is biased toward the concordance cosmology, which has to be avoided in the parameter estimate. Thus, we set $M$ at the "characteristic mass scale," $M_6$, defined as

$$M_6 = \frac{4\pi}{3} \rho R^3,$$

where $R_8 = 8 \, h^{-1} \, \text{Mpc}$ is the top-hat spherical radius on which scale the rms density fluctuation is observed to be very close to unity, and $\rho = 2.78 \times 10^{11} \Omega_m h^2 M_\odot \, \text{Mpc}^{-3}$. Since the galaxy clusters are the largest collapsed objects in the universe, the characteristic mass scale $M_6$ is believed to represent the typical mass of galaxy clusters. Note that this characteristic cluster mass, $M_6$, by its definition (eq. [19]), depends on the value of $\Omega_m$. Therefore, putting $M = M_6$ into equation (12), will optimize the parameter estimation without bearing any bias toward a certain cosmology.

4.2. Constraint on $\Omega_m$ and $\sigma_8$ for a Flat Universe

The analytic distribution (12) depends not only on $\Omega_m$ and $\sigma_8$ but also on $\Omega_\Lambda$, $h$, and $\Gamma$ as well. Here we focus mainly on constraining $\Omega_m$ and $\sigma_8$, assuming that the other parameters are already known as priors. We use the following priors from the CMB observation (e.g., Lange et al. 2001), the HST Key Project (Freedman et al. 2001), and the big bang nucleosynthesis (Olive et al. 2000): $\Omega_\Lambda = 1 - \Omega_m$ (a flat universe); $h = 0.7; \Omega_m = 0.044$. Once the values of $h$ and $\Omega_\Lambda$ are given, the shape factor, $\Gamma$, will depend only on the value of $\Omega_m$ (see eq. [18]).

Now that all parameters except for $\Omega_m$ and $\sigma_8$ in equation (12) are prescribed, we can fit the observational data points obtained

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**TABLE 1**

| Model | $\Omega_m$ | $\Omega_\Lambda$ | $\sigma_8$ | $h$ | $\Gamma$ |
|-------|-------------|------------------|-------------|-----|--------|
| ΛCDM  | 0.3         | 0.7              | 0.9         | 0.7 | 0.168  |
| OCDM  | 0.3         | 0                | 0.87        | 0.83| 0.25   |
| SCDM  | 1           | 0                | 0.52        | 0.5 | 0.5    |
| τCDM  | 1           | 0                | 0.52        | 0.5 | 0.25   |

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**Fig. 3.**—Best-fit values of $\sigma_8$ vs. $\Omega_m$ through $\chi^2$-statistics. The solid and the dot-dashed lines represent the 1 standard deviation contour and its approximation. The dashed lines represent the previous relation found from the local abundance of X-ray clusters.
in § 2 to equation (12) with adjusting $\Omega_m$ and $\sigma_8$ in ranges of $(0, 1]$ and $(0, 2]$, respectively. We pick as the best-fit values of $\Omega_m$ and $\sigma_8$ those that minimize $\chi^2$:

$$\chi^2 = \sum_{i=1}^{N_p} \left( \frac{p_o(q_i) - p(q_i; \Omega_m, \sigma_8)}{\sigma_o(q_i)} \right)^2,$$

where $N_p$ is the total number of observational data points, {$q_i, p_o(q_i)$}, and $\sigma_o(q_i)$ is the observational standard variation of $q_i$.

The single best-fit values, however, are unreliable given the uncertainty in $\chi^2$ due to the strong correlation between $\Omega_m$ and $\sigma_8$. A more general best fit is defined as the full range of the outer limit of one standard deviation contour. Figure 3 plots the one standard deviation contours as solid lines. We find that this contour is well approximated by the relation $\sigma_8 = (1.01 \pm 0.09)(\Omega_m/0.6)^{0.07 \pm 0.02 \pm 0.10}$. This approximation formula is also plotted as dot-dashed line in Figure 3. The previous relation of $\sigma_8 = (0.52 \pm 0.04)\Omega_m^{0.52 \pm 0.10}$ obtained by Eke et al. (1996) from the cluster abundance is also plotted for comparison as dashed line. The shaded area surrounded by the solid and the dashed lines in Figure 3 represents the simultaneously constrained best-fit values of $\Omega_m$ and $\sigma_8$: $\Omega_m^{\text{cons}} = 0.31 \pm 0.07$ and $\sigma_8^{\text{cons}} = 0.94 \pm 0.07$, which is in good agreement with the results from the first-year WMAP (Wilkinson Microwave Anisotropy Probe) measurements (Spergel et al. 2003).

5. DISCUSSION AND CONCLUSIONS

We have found a new $\sigma_8-\Omega_m$ relation by comparing the LJS05 analytic model for the axis-ratio distribution of galaxy clusters with the observational data from the SDSS-C4 catalog. The success of our final result, however, is subject to a couple of caveats. The first caveat comes from our assumption that the principal axes of the SDSS-C4 clusters derived from the elliptical fits to the galaxy distributions are directly comparable to that of the analytically derived shapes of dark halos. This is an obvious simplification of the reality. Notwithstanding, the following observational and numerical clues should provide a justification for this simplified assumption. Many observations revealed that the spatial distribution of cluster galaxies and the major axes of their host clusters are arranged in a collinear way (West & Blakeslee 2000; Plionis & Basilakos 2002; Plionis et al. 2003; Pereira & Kuhn 2005). Recent numerical simulations also demonstrated that the dark halo substructures are preferentially located along the major axes of their host halos (Kang et al. 2005; Libeskind et al. 2005; Zentner et al. 2005; Lee & Kang 2006). Very recently, Lee & Kang (2006) showed by $N$-body simulations that the triaxial shapes of dark matter halos can be reconstructed from the anisotropic spatial distribution of their substructures. Given these empirical findings, one may expect that the cluster principal axis derived from the elliptical fit to the galaxy distribution is comparable to the analytical one derived from the smooth dark matter distribution.

The second caveat lies in the limited validity of the analytic model. As shown by LJS05, the prediction of our analytic model on the cluster ellipticity-mass relation disagrees with the numerical finding. Our model predicts that the cluster ellipticity decreases with mass, while many different numerical simulations (Bullock 2002; Jing & Suto 2002; Kasun & Evrard 2005; Hopkins et al. 2005; Bailin & Steinmetz 2005) found that the cluster ellipticity actually increases with mass.

In spite of this limitation of the LJS05 analytic model, it is still true that LJS05 tested their model against $N$-body simulations and showed that it works quite well, reproducing the characteristic behaviors of the numerical result, on a single mass scale (or in a narrow mass range). In other words, although the LJS05 model fails in predicting the "change" of the axis-ratio distribution in the wide range of halo mass, it can still be used to predict the shape of the axis-ratio distribution of a dark halo with a given mass. As we consider here only those objects in the C4 catalog whose mass lies in a narrow range $\sim 10^{14} h^{-1} M_\odot$ for the comparison with the analytic model, the LJS05 model is expected to be a useful approximation.

Definitely, however, it will be desirable and necessary to refine our model more realistically before applying it to a wide range of cosmic structure masses. Finally, we conclude that it is possible in principle to constrain both $\Omega_m$ and $\sigma_8$ concurrently from the observation of galaxy clusters alone, as the cluster shape and abundance distributions provide two different $\sigma_8-\Omega_m$ relations. The amplitude of the linear power spectrum obtained from the shape and abundance distribution of the galaxy clusters is consistent with the high value estimated from the recent WMAP measurements.

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