Numerical Simulations of THz Emission from Intrinsic Josephson Junctions

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Abstract. Recently, a strong coherent THz wave emission was observed in the mesa-shaped intrinsic Josephson junction of Bi-2212 with a constant bias current and without any applied magnetic field. We investigate the size-effect of the THz wave emission by use of the numerical simulation. We assume in-phase motions, and the equations for junctions, leads and the vacuum are solved simultaneously in two-dimensional models. The emitted wave has a similar pattern as in the dipole emission, and electromagnetic oscillations inside the two-dimensional junction show creation and annihilation of flux loops, which are quite different patterns from ones in a one-dimensional junction.

1. Introduction

Recently, a strong emission of terahertz waves from the intrinsic Josephson junction (IJJ) of the high $T_c$ superconductor Bi$_2$Sr$_2$CaCu$_2$O$_8$ was observed by use of mesa-shaped samples\cite{1}. Although most of theoretical studies on this subject have been performed with an applied magnetic field \cite{2, 3, 4, 5, 6, 7}, a strong emission is obtained without any magnetic field. The sample size is about $300 \mu m \times (40-100) \mu m$, which is nearly the $c$-axis magnetic penetration depth $\lambda_c$. The largest voltage branch was chosen, and in the bias current decreasing process below the critical current, a strong wave emission is observed near the unstable point of the retrapping current. The emitted electromagnetic waves are highly monochromatic.

The THz wave emission with a constant bias current under no applied field was discussed recently in ref.\cite{8} with the result that a shorter sample size of a few $\mu m$ is preferable, which does not agree with the experiment. In the previous papers\cite{9, 10}, we analyzed the voltage dependence of the emitted power, by assuming in-phase motions of the superconducting phase-differences. The emitted power gradually increases with the decreasing voltage, taking the maximum at the retrapping current. Experimentally, strong emissions occur sharply in quite restricted voltage region. It was pointed out that effects of out-of-phase motions through the interlayer couplings make a resonating nature much sharper\cite{10}. It was argued in refs.\cite{11} and \cite{12} that a careful choice of boundary conditions leads to a solution for vortex-lines to penetrate in an alternative direction layer by layer, having a sharper resonating nature. All those theoretical analysis were based on an array of one-dimensional junctions. In-phase motions show different oscillation patterns depending on the junction length. In shorter junctions, a standing wave is excited with the Josephson frequency, while in longer junctions, a solitonic wave is excited with half of
the Josephson frequency[9, 10]. Then there arise questions what kind of excitation dominates in a two-dimensional rectangular shape of junctions, which length determines the oscillatory frequency, and how thickness of junctions affects the emitted power.

In this paper we investigate the spatial distribution and oscillatory pattern of electromagnetic fields inside and outside of junctions, by taking into account the two-dimensional size effect. We consider two models: 1)xz-model, where junctions and leads are stacked in the z-direction with homogeneity in the y-direction, and 2)xy-model, where junctions have a rectangular shape in the xy-plane with homogeneity in the z-direction. The result of the xz-model shows similar wave patterns as in the dipole emission. The result of the xy-model shows that flux loops are formed and oscillate, going through the inside and outside of the junction and that waves are emitted in all directions of the xy-plane, compensating a local structure and leaving an overall oscillation with the Josephson frequency controlled by a shorter length.

2. Models and equations

2.1. xy-model

Junctions have a rectangular shape of $L_x \times L_y$, and the homogeneity in the z-direction is assumed. In this model, we can investigate two-dimensional oscillatory pattern in the junctions. We solve the equation for the phase-difference $\phi$ and the Maxwell equations for the junctions, leads and vacuum, simultaneously. An in-phase motion is assumed, so that the equation of $\phi$ is essentially the same as one in the single junction. The relevant electromagnetic fields are $\vec{E} = (0, 0, E_z)$ and $\vec{B} = (B_x, B_y, 0)$. Inside the junctions, we have

$$\frac{\partial}{\partial t} \phi = \frac{2ed}{\hbar} E_z, \quad \frac{1}{c} \frac{\partial}{\partial t} \epsilon E_z = \nabla_x B_y - \nabla_y B_x - \frac{4\pi}{c} (\sigma E_z + j_c \sin \phi)$$

and in the vacuum, we have

$$\frac{1}{c} \frac{\partial}{\partial t} \epsilon(x) \vec{E} = \nabla \times \vec{B}, \quad \frac{1}{c} \frac{\partial}{\partial t} \vec{B} = -\nabla \times \vec{E},$$

where constants, $d, \epsilon, \sigma$ and $j_c$, are, respectively, the thickness, dielectric constant, conductivity and critical current of the junctions. The condition of the bias current is given from the Ampere’s law as

$$\int_C d\vec{r} \cdot \vec{B}_{DC} = \frac{4\pi}{c} j_{ext} L_x L_y,$$

where the contour C is taken anticlockwise along the edge of junctions and the subscript 'DC' indicates the DC-component.

2.2. xz-model

The system of junctions, leads and surrounding vacuum is considered. The homogeneity along the y-direction is assumed. In this model, we can investigate thickness effects of the junctions. The relevant fields are $\phi, \vec{E} = (E_x, 0, E_z)$ and $\vec{B} = (B_x, B_y, 0)$. The most of the equations are same as in the xy-model, except that the Maxwell equations Eqs. (3) become in the leads and vacuum as

$$\frac{1}{c} \frac{\partial}{\partial t} \epsilon(x) \vec{E} = \nabla \times \vec{B} - \frac{4\pi}{c} (\sigma(x) \vec{E} + j_{ext}(x)),$$

where $\epsilon(x) = \epsilon_L$ (in the leads), $\epsilon_0$ (in the vacuum), $\sigma(x) = \sigma_L$ (in the leads), 0 (in the vacuum). The bias current is injected in the z-direction homogeneously on the leads, that is, $j_{ext}(x) = (0, 0, j_{ext}(x))$ is given by $j_{ext}(x) = j_{ext}$ for $|x| < L_x/2, |z| > L_{jz}/2$ and otherwise 0.
The emitted power in a $\vec{n}$-direction at a position $\vec{r}$ is obtained by taking the time average of the Poynting vector evaluated from oscillatory parts of electromagnetic fields,

$$ S_{\vec{n}}(\vec{r}) = \frac{c}{4\pi} \left( t_{\text{max}} - t_{\text{min}} \right) \int_{t_{\text{min}}}^{t_{\text{max}}} \left( (\vec{E}(\vec{r}, t) - \vec{E}_{\text{stat}}(\vec{r})) \times (\vec{B}(\vec{r}, t) - \vec{B}_{\text{stat}}(\vec{r})) \right) \, dt \cdot \vec{n}, $$

where the subscript "stat" indicates the static part.

3. Numerical results
We have performed numerical simulation by use of the midpoint method for the spatial differential and the fourth order Runge-Kutta method for the time-development. The vacuum is surrounded by the perfectly matched layers (PML)[13] to avoid the reflection of waves in the finite size simulation. We use the scale $S_p = \frac{c}{4\pi} E_p B_c$, where $E_p = \frac{\omega p}{2\epsilon_0}$ and $B_c = \frac{\hbar c}{2\pi e d}$ with $\omega_p$ and $\lambda_c$ being the angular frequency of the Josephson plasma and the c-axis penetration depth, respectively[9, 10]. We use the damping parameter $\beta$ (the scaled $\sigma$) = 0.05, $\epsilon = 10.0$ and $\sigma_L/\sigma = 2.0 \times 10^3$.

![Figure 1. Results of the xy-model](image1)

In Fig.1, we summarize the results of the xy-model. Fig.1(a) is the voltage-dependence of the emitted power observed in the direction $\vec{n} = (0, 1, 0)$ at the distance $r_{\text{obs}}/\lambda_c = 4.5$. The dotted line is for the one-dimensional model. By the two-dimensional effect, a resonating nature is enhanced at $V/V_p \sim n\pi$. Fig.1(b) is a snapshot of the spatial distribution of $b(= B - B_{\text{stat}})$ at $V/V_p = 4.98$. The height shows the magnitude. Flux loops are formed, which extend outside of the junctions. Fig.1(c) is the angle-dependence of the emitted power. The wave is emitted all the direction, though the intensity is the strongest in the y-direction.

![Figure 2. Results of the xz-model](image2)
In Fig.2, the results of the xz-model are summarized. Fig.2(a) is the voltage-dependence of the emitted power observed in the direction $\vec{n} = (1, 0, 0)$ at the distance $r_{obs} = 2.9$. It strongly resonates at $V/V_p \sim 2\pi n$. Fig.2(b) is a snapshot of the spatial distribution of $\vec{e} (= \vec{E} - \vec{E}_{stat})$ at $V/V_p = 5.83$. The pattern is similar as the dipole emission. Fig.2(c) is the angle-dependence of the emitted power at $r_{obs}/\lambda_c = 3.0$. The solid line is the angle-dependence of the dipole emission, $S \sim \sin^2 \theta/r_{obs}$, which agrees fairly well.

![Figure 3](image)

**Figure 3.** Effects of the substrate in the xz-model

In Fig. 3, we show the effect of the substrate which reflects some of the emitted wave. We choose the length of the substrate (the lower lead) three times longer than that of the upper lead, $L_{sub}/\lambda_c = 3.0$. Fig.3(a) is the voltage-dependence of the emitted power observed in directions $\vec{n} = (1, 0, 1), (1, 0, 0)$ and $(0, 0, 1)$ at the distance $r_{obs}/\lambda_c = 3.0$. Resonating conditions seem to be affected by the reflection from the substrate. Fig.3(b) is a snapshot of the spatial distribution of $\vec{e} (= \vec{E} - \vec{E}_{stat})$ at $V/V_p = 5.84$. We can see the reflection from the substrate. Fig.3(c) is the angle-dependence of the emitted power. The strongest emission is observed with certain angle.

From those analysis we have the conclusions summarized at the end of the introduction. The emitted waves and their resonating conditions are very much influenced by the two-dimensional property and thickness of junctions.

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**References**
[1] Ozyuzer L, Koshelev A E, Kurter C, Gopalsami N, Li O A, Tachiki M, Kadowaki K, Yamamoto T, Tachiki T, Gray K E, Kwok W K and Welp U 2007 Science 318 1291
[2] Koyama T and Tachiki M 1995 Sol. St. Comm. 96 367
[3] Machida M, Koyama T, Tanaka A and Tachiki M 2000 Physica C 330 85.
[4] Machida M, Koyama T, Tanaka A and Tachiki M 2001 Physica C 362 16.
[5] Tachiki M, Iizuka M, Tejima S and Nakamura H 2005 Phys. Rev. B71 134515
[6] Matsumoto H 2006 Physica C437–438 199
[7] Lin S, Hu X and Tachiki M 2008 Phys. Rev. B77 14507
[8] Bulaevskii L N, Koshelev A E 2007 Phys. Rev. Lett. 99 057002
[9] Matsumoto H, Koyama T and Machida M 2008 Physica C468 654
[10] Matsumoto H, Koyama T and Machida M 2008 Physica C (to be published)
[11] Lin S and Hu x 2008 Cond-Mat(arXiv:0803.4244v1)
[12] Koshelev A E 2008 Cond-Mat(arXiv:0804.0146v1)
[13] Berenger J -P 1994 J. Comp. Phys. 114 185