Modeling and analysis of magneto-Carreau fluid with radiative heat flux: Dual solutions about critical point

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Abstract
In this article, we aim to analyze the dual solutions for the flow of non-Newtonian material (Carreau fluid) over a radially shrinking surface. Magnetohydrodynamics fluid is considered. Concept of Stefan Boltzmann constant and mean absorption coefficient is used in the mathematical modeling of energy expression. Mass transfer is discussed. The upper and lower branch solutions for the Sherwood number, skin friction coefficient, and Nusselt number are calculated for different pertinent flow variables. Appropriate transformation variables are employed for reduction of partial differential equations system into ordinary differential equations. Dual solutions are obtained for the non-dimensional concentration, temperature, velocity, gradient of concentration, gradient of temperature, and gradient of velocity. The critical values for each upper and lower solutions are obtained for the case of gradient of velocity, gradient of temperature, and gradient of concentration. It is formed that concentration and temperature fields display same impact regarding both upper and lower branch solutions for velocity ratio and temperature ratio parameters.

Keywords
Carreau fluid, heat generation/absorption, joule heating, radiative heat flux, stagnation point flow, magnetic field

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Introduction
It is very well recognized that the non-Newtonian materials are more applicable than viscous materials in processes of engineering, geophysics, and biomechanics.¹,² Many non-Newtonian materials exist in nature for their different characteristics. The difference between these materials can be distinguished from the functional relation between shear stress, the force per unit area mandatory to tolerate a constant rate of shear rate and liquid movement, and rate of velocity change when different layers of fluid or one layer passes through an adjacent layer. In terms of the rheological impact, the non-Newtonian materials are also classified as either shear thickening or shear thinning materials. The apparent viscosity of shear thinning materials like foams, solutions, polymer melts and emulsions, and suspensions decay via applied shear stress. Numerous models have been introduced to investigate the rheological impact of such types of materials. The most common ones are the Jeffrey model, Maxwell model, etc.

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second grade, third grade, Williamson fluid model, Carreau model, Power law model, and many others. Here, we study the flow behavior of Carreau model over a stretched and shrinking sheet. The Carreau model is able to capture both shear thickening and shear thinning behaviors; Carreau\textsuperscript{6} gave an innovative constitutive model for the rheological impact of carbon nanotubes (CNTs)/polymer composites. Grabski and Kolodziej\textsuperscript{7} reported flow of Carreau fluid between corrugated plates. Kefayati et al.\textsuperscript{8} worked for fluid flow of Carreau liquid with magnetohydrodynamics (MHD). Kefayati and Tang\textsuperscript{7} discussed three-dimensional (3D) natural convection in entropy optimized flow of Carreau fluid in heated enclosure.

Stretched flows play an important role in mechanical and industrial engineering including crystal growing, glass fiber, hot rolling, extrusion of plastic sheet, wire drawing, and so many others. Initially, Crane\textsuperscript{9} closed from solution for flow over stretching surface. Behavior of viscous liquid over a stretchable surface with radiative flux is addressed by Cortell.\textsuperscript{9} Malvandi et al.\textsuperscript{10} investigated time-dependent slip stagnation point flow of nanomaterial over a stretchable sheet. Khan et al.\textsuperscript{11} studied flow of different species over a stretching surface for Casson fluid subject to stretched sheet. Hayat et al.\textsuperscript{12} inspected Jeffrey fluid flow with variable fluid properties and Cattaneo–Christov model. Previous studies\textsuperscript{13–32} scrutinized flow of Newtonian and non-Newtonian fluids with various aspects and surface conditions.

Motivated by the aforementioned attempts, the main propose here is to address the stagnation point flow when Carreau fluid is taken over a radially shrinking sheet. Dual solutions are obtained. The concepts of Stefan Boltzman constant and mean absorption coefficient is used in the mathematical modeling. Special consideration is given to numerical study of the lower and upper branch solutions for skin friction coefficient, Nusselt number, and Sherwood number. Runge–Kutta Fehlberg integration technique\textsuperscript{13,34} is employed. It is concluded from the obtained outcomes that velocity portrays both decreasing and increasing impact at the upper and lower branch. However, concentration and temperature fields have same impact at both upper and lower branch solutions for velocity ratio and temperature ratio parameters.

Flow modeling and viscosity relation

Here, mathematical modeling is presented for the MHD flow of Carreau fluid over a stretched/shrinking surface. The mathematical form of viscosity for Carreau fluid is described as follows\textsuperscript{6}

\[ \mu = \mu_\infty + (\mu_0 - \mu_\infty)(1 + \Gamma \dot{\gamma})^{n-1} \]  

(1)

in which \( \Gamma \) denotes the material constant, \( n \) is the flow behavior index, and \( \mu_0 \) and \( \mu_\infty \) represent the zero and infinity shear rate viscosities. For simplicity, the infinity shear rate is ignored and the shear rate \( \dot{\gamma} \) is addressed as follows

\[ \dot{\gamma} = \sqrt{\frac{1}{2} \mathbf{M} : \mathbf{M}}, \text{ where } \mathbf{M} = \nabla \mathbf{V} + (\nabla \mathbf{V})^T \]  

(2)

For \( n = 1 \) or \( \Gamma = 0 \), the above relation reduce to viscous fluid.

The governing flow equations are as follows

\[ \nabla \cdot \mathbf{V} = 0 \]  

(3)

\[ \rho (\mathbf{V} \cdot \nabla) \mathbf{V} = -\nabla P + \nabla \cdot \tau + \mathbf{j} \times \mathbf{B} \]  

(4)

where \( \nabla \) is the gradient operator, \( \mathbf{V} \) is the velocity, \( \rho \) is the density, \( P \) is the pressure, \( \tau \) is an extra stress tensor, \( \mathbf{B} \) is the total magnetic field, and \( \mathbf{j} \) is the current density. For present flow analysis, the velocity field is defined as

\[ \mathbf{V} = [u(r, z), 0, w(r, z)] \]  

(5)

where \( u(r, z) \) and \( w(r, z) \) signify the velocity components in radial and axial directions.

The shear rate \( \dot{\gamma} \) for non-Newtonian fluid (Carreau model) is written as

\[ \dot{\gamma} = \left[ 2 \left( \frac{\partial u}{\partial r} \right)^2 + 2 \left( \frac{\partial w}{\partial z} \right)^2 + \left( \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right)^2 + \frac{2u^2}{r^2} \right]^{\frac{1}{2}} \]  

(6)

We now have the following expressions

\[ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \]  

(7)
Flow analysis

Figure 1 depicts the schematic flow geometry. Here, we investigate axisymmetric electrically conducting flow of Carreau fluid by a shrinking stretched sheet at \( z = 0 \). Magnetic field of constant strength \( B_o \) is applied. Impact of induced magnetic field is absent due to small Reynolds number. The flow expressions in components form are mathematically after implementation of boundary layer approximations \( O(u) = 1, \)

\[
O(v) = \delta, \ O(r) = \delta, \ O(z) = \delta, \ O(v) = O(\Gamma^2) = \delta^2 \]

are subject to

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \tag{10}
\]

\[
u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial z} = u_e \frac{\partial u}{\partial r} - \frac{\sigma B_o^2}{\rho} (u - u_e) + r \frac{\partial^2 u}{\partial z^2} \left[ 1 + \Gamma^2 \left( \frac{\partial \phi}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} + \nu (n-1) \Gamma^2 \left( \frac{\partial \phi}{\partial z} \right)^2 \left[ 1 + \Gamma^2 \left( \frac{\partial \phi}{\partial z} \right)^2 \right]^{\frac{n-1}{2}} \tag{11}
\]

\[
\rho c_p \left( \frac{\partial T}{\partial r} + \frac{\partial T}{\partial z} \right) = k \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_r}{\partial r} + \sigma B_o^2 r^2 + Q' (T - T_\infty) \tag{12}
\]

\[
\frac{\partial C}{\partial r} + \nu \frac{\partial C}{\partial z} = D \left[ \frac{\partial^2 C}{\partial r^2} + \frac{1}{r} \frac{\partial C}{\partial r} + \frac{\partial^2 C}{\partial z^2} \right] \tag{13}
\]

with

\[
u = u_e (r) = cr, \quad w = w_\infty, \quad T = T_\infty, \quad C = C_\infty \quad \text{at} \quad z = 0, \quad u \to u_e (r) = ar, \quad T \to T_\infty, \quad C \to C_\infty, \quad \text{and} \ z \to \infty \tag{14}
\]

where \((r, z)\) denotes the coordinates system, \((u, w)\) is the velocity components, \(u_e\) is the free stream velocity, \(\rho\) is the density, \(\nu\) is the kinematic viscosity, \(\Gamma\) is the material parameter, \(n\) is the flow behavior index, \(c_p\) is the specific heat, \(T\) is the temperature, \(k\) is the thermal conductivity, \(q_r\) is the radiative heat flux, \(C\) is the concentration, \(D\) is
the diffusion, \((a, c)\) are the rates, \(T_w\) is the sheet temperature, \(u_w\) is the stretching velocity, \(C_w\) is the sheet concentration, and \(T_\infty\) and \(C_\infty\) are the ambient temperature and concentration.

By Roseland approximation, the radiative heat flux satisfies

\[
We^2 = \frac{a^2 T^2 r^2}{\nu}, \quad M = \sqrt{\frac{aB_0^2}{\rho a}} \quad \Pr = \frac{\mu c_p}{k}, \quad \theta_w = \frac{T_w}{T_\infty}, \quad Nr = \frac{kk^*}{4\sigma T_\infty^3}, \quad s = -\frac{w_0}{2\sqrt{av}}, \quad \lambda = \frac{c}{a}, \quad Ec = \frac{a^2 r^2}{c_p(T_w - T_\infty)}
\]

\[
Q = \frac{Q_0}{apc_p}, \quad Sc = \frac{\nu}{D}
\]

(23)

\[
q_r = -\frac{4 \sigma^*}{3k^*} \frac{\alpha \partial T}{\partial z}
\]

or

\[
q_r = -\frac{16 \sigma^*}{3k^*} T \frac{\partial T}{\partial z}
\]

(15)

(16)

Note that \(We^2\) denote the local Weissenberg number, \(M\) is the magnetic parameter, \(Pr\) is the Prandtl number, \(\theta_w\) is the temperature ratio parameter, \(Ec\) is the Eckert number, \(s\) is the suction/shrinking parameter, \(\lambda\) is the stretching ratio parameter, \(Sc\) is the Schmidt number, \(Q\) is the heat source/sink parameter, and \(Nr\) is the radiation parameter. These dimensionless parameters are mathematically defined as follows

\[
\tau_w, \ q_w, \text{ and } q_m, \text{ respectively, depict the shear stress and heat and mass fluxes, that is}
\]

\[
\tau_w = \mu \left(1 + \frac{\mu}{\lambda} \right) \left[1 + \Gamma^2 \left(\frac{\partial u}{\partial z}\right)^2 \right]^{\frac{n-1}{2}}
\]

\[
q_w = -k \left[1 + \frac{16 \sigma^*}{3k^*} T^3 \right] \left(\frac{\partial T}{\partial z}\right)_{z=0}
\]

\[
q_m = -D \left(\frac{\partial C}{\partial z}\right)_{z=0}
\]

(25)

(26)

(27)

Finally, we obtain

\[
\text{Re}^{1/2} C_f = f''(0) \left[1 + We^2(f''(0))^2 \right]^{\frac{n-1}{2}}
\]

\[
\text{Re}^{1/2} = -\theta'(0) \left[1 + \frac{3}{4Nr} \theta'(0) ((\theta_w - 1) + 1) \right]
\]

\[
Sh = -\phi'(0)
\]

(28)

(29)

(30)

where \(Re = (ar^2/\nu)\) denotes the local Reynolds number.

**Methodology**

Numerical results are constructed for the nonlinear problem by using Runge–Kutta Fehlberg integration. In this algorithm, outcomes are obtained by converting the nonlinear ODEs into first-order ODEs. Thus we set

\[
\begin{align*}
J_1, J_2, J_3, \theta, \phi, J_4, \phi' & = J_7 \\
J_5, \theta' & = J_5
\end{align*}
\]

(22)

(31)
\[ J_0^3 = \frac{J_2 - 2J_1 J_3 - M^2(1 - J_2)}{(1 + nWe^2 J_3)(1 + We^2 J_5^{\frac{n-2}{2}})} \]  
\[ J_1' = \frac{\Pr J_1 J_2 + \frac{12}{27n}(1 + (\theta_w - 1)J_4)(\theta_w - 1)J_5^2 + \Pr m Ec J_2^2 + \Pr Q J_4}{1 + \frac{4}{3n}(1 + (\theta_w - 1))^3} \]  
\[ J_2' = -2Sc J_3 J_7 \]  

For implementing of the numerical algorithm by Runge–Kutta Fehlberg integration, we need three initial conditions \( f''(0), \theta'(0), \) and \( \phi'(0) \) which satisfy the conditions \( f'(\infty) \to 1, \theta'(\infty) \to 0, \) and \( \phi'(\infty) \to 0. \) The initial guesses are taken from domain where the

**Figure 2.** Magnetic parameter effects on heat transfer rate.

**Figure 3.** Magnetic parameter effects on skin friction coefficient.

**Figure 4.** Magnetic parameter effects on Sherwood number.

**Figure 5.** Weissenberg number effects on heat transfer rate.
convergence condition for the Newton Rapshen method is satisfied \( \sum |\psi_j^m - \psi_j^{m-1}| \leq 10^{-6} \).

**Discussion**

In this section, we investigate the effects of magnetic parameter \( M \) and Weissenberg number \( We \) on Nusselt number, skin friction, and Sherwood number with shrinking parameter \( \lambda \) by retaining all other parameters fixed. Dual solutions exist for shrinking parameter \( \lambda \leq 0 \). The fact of the dual solution is studied by Miklavčič and Wang,\(^{35}\) Fang,\(^{36}\) and Wang.\(^{37}\) Godstien essentially gives the concept of backward flow. He stated that new type of flow occurs for shrinking case. There is always a region near the sheet where reverse flow starts. Backflow means that fluid particles starts flowing opposite to the normal direction.

In this article, the dual solution is found due to the irregularity of shrinking sheet flow. Figures 2–7 show the dual solution for skin friction, Nusselt number, and Sherwood number. There are two types of solutions, namely, upper branch solution and lower branch solution. Effect of magnetic parameter \( M \) on Nusselt number, skin friction, and Sherwood number is shown in
Figures 2–4. We noticed that dual solution also increases for higher $M$. Dual solution exists in range $\lambda_c \leq \lambda < -1.2$. Figures 5–7 are portrayed for effect of $\text{We}$ against skin friction, Nusselt number, and Sherwood number. Here, one can easily observe that there are two solutions for two different initial guesses. These figures also show that domain of the dual solution is higher for larger $\text{We}$. Figures 8 and 9 are sketched for velocity and concentration against $M$. We can see that the dual solution exists for both velocity and concentration distribution. By taking higher magnetic field strength, the upper solution of velocity and concentration increases whereas lower solution decays.

The stronger magnetic field accelerates more of the lower solution when compared with upper solution. Effect of suction $s$ and shrinking parameter $\lambda$ on velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $\phi(\eta)$ are discussed in Figures 10–13. Figure 10 sketches the impact of $s$ on velocity. It is observed that upper branch solution enhances for larger $s$ while lower branch solution behaves in opposite manner. Figure 11 displays the upper and lower branch solutions of temperature against $s$. Here, we see that upper branch solution is decreasing and lower branch solution is increasing for higher $s$. Figures 12 and 13 display the effect of $\lambda$ on velocity and concentration. It is witnessed...
that for lower $\lambda$, the upper branch solution of velocity increases while a decay is noticed in lower branch solution (see Figure 12). Figure 13 shows that for lower shrinking parameter $\lambda$, the concentration enhances. We can see that both lower and upper solutions of concentration are increased. Figure 14 displayed Prandtl number Pr effect on temperature. There are two different types of results for upper and lower solutions. For higher Pr, the temperature reduces for upper solution while it shows increasing behavior for lower solution. Temperature ratio parameter effect on temperature is sketched in Figure 15. It is evident from figure that upper and lower solutions are increasing functions of $\theta_w$. Variation in the upper solution is greater than the lower solution. Figure 16 show the effect of thermal radiation parameter $Nr$ on $\theta$. We can conclude opposite behavior of both solutions is possible through $Nr$. Furthermore, for higher thermal radiation parameter, the behavior of the upper solution decreases while lower solution has increasing behavior.

### Concluding remarks

Main results of presented study include the following points:

- Thickness of the upper branch solution is always thinner than the lower branch solution via maximum range of variables.
- Velocity shows both decreasing and increasing impact at the upper and lower branches of solutions for higher values of magnetic parameter, suction parameter, and shrinking variable.
- Temperature and concentration have similar impact at both upper and lower branches subject to higher velocity and temperature ratio parameters.
- Skin friction is more for higher magnetic variable.
- Thermal field increases for lower branch, and decreases for upper branch via higher Prandtl number.

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