Research on the construction of stable sandcastle structure

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Abstract. We model and analyze the optimal 3d geometry of sandcastle, discuss the optimal sand-water ratio of sandcastle, and analyze whether it is still the optimal 3d geometry in the case of rainfall. First of all, we assume that the sandcastle is a solid, the waves are fluid, through the establishment of fluid-solid interaction model, using the deformation variable of the sandcastle as indicators of sandcastle collapse or instability, and then we apply the ALE method, namely Eulerian method and LaGrange method, list the relevant differential equations by calculating the transport volume between grids, the simulation results show that the shape of semi ellipsoid is the best. At the same time, we analyze the phenomenon such as the stress on the arc slope, the movement of sand grains along the arc and the percolation during rainfall using simplified Bishop method and obtain the expression of the safety factor of the landslide in the rainfall. Through iteration, the results show that the semi ellipsoidal shape is still the best. Finally, we establish the relationship between the shear strength and water content of the foundation in the unsaturated state, and the optimal water content is calculated between the interval [1.8%, 2.1%].

1. Introduction
At present, the fluid-solid interaction simulation based on the ALE method has been applied in many aspects of life, for example, Honghui Shi studied the influence of the shape of the elastic cylindrical shell head on the head deformation and pressure distribution in the process of water entry[1], Xiaoni Yang studied the stress distribution on the wall when the vehicle is at rest in pipe flow with different flow rates[2]. The accurate prediction of landslides and the reduction of landslide hazards have been an important issue of concern to governments around the world, especially the stability analysis of rainfall-induced landslides is a hot issue in landslide research [3]. But very few studies have combined these two aspects to study the impact on something. Different from the above study, we consider the erosive effect of sea waves and rainfall on sandcastles by combining the fluid-solid interaction and slope stability analysis, the optimal three-dimensional geometry of the sandcastle is obtained. In addition, we further explore the optimal sand-water mixture ratio to make the sandcastle more stable.
2. Method

2.1. The model of fluid-solid interaction

2.1.1. Confirm coordinate system. The ALE method observes the motion of fluid on a reference point (i.e. grid point) that can move arbitrarily. Therefore, in order to determine the position of the reference point, the reference point coordinates $\eta$ introduced. And any particle point $X$ corresponds to the reference point coordinate in the reference system is:

$$\eta = \eta(\vec{X}, t)$$  \hspace{1cm} (1)

Reference point $\eta$ at $t$ time the space position:

$$x = x(\eta, t)$$  \hspace{1cm} (2)

2.1.2. Fluid model

- Momentum equation

The momentum equation states that the overall rate of change of the total momentum of an object occupying the reference domain at time $t$ is equal to the sum of the external forces applied to the object.

$$\frac{\partial p u}{\partial t} + \frac{1}{r} \frac{\partial r p u^2}{\partial x} + \frac{\partial p u v}{\partial y} = - \frac{\partial (p + q)}{\partial x} + \frac{1}{r} \frac{\partial r \pi_{xx}}{\partial x} + \frac{\partial \pi_{xy}}{\partial y} - \frac{\pi_{\theta}}{r} + \rho g_x$$

$$D = \frac{1}{r} \frac{\partial r u}{\partial x} + \frac{\partial v}{\partial y}$$  \hspace{1cm} (3)

The pressure of the fluid can be derived from the equation of state, $p = p(\rho, I)$, is the pressure of the fluid; Considering the calculated discontinuous solution, the increased artificial viscosity is:

$$q = \lambda_0 \rho d A r e a D \min(0, D)$$  \hspace{1cm} (4)

The artificial viscosity is only used in the compressible region, that is, $(D < 0)$, and it's proportional to the unit area $\lambda_0$.

- Continuous equation

We regard fluid as a continuous medium, so the particles of the fluid must be connected to each other and no gaps appear during the flow of the fluid. Therefore, any flow problem must satisfy the conservation of mass that the increase in mass per unit time is equal to the net mass per unit time. According to the conservation of mass, we can derive the mass conservation equation for fluid flow, which is continuity equation:

$$\frac{\partial p}{\partial t} + \frac{1}{r} \frac{\partial r p u}{\partial x} + \frac{\partial p v}{\partial y} = 0$$  \hspace{1cm} (5)

- Energy conservation equation

If the fluid system has a thermal effect in motion, and there is heat transfer inside and outside the system, then we need to establish the energy equation. According to the law of conservation and conversion of energy, the time rate of change of energy in the fluid system should be equal to the work done to the system by the mass and surface force per unit time, plus the heat exchanged between the environment and the system per unit time. The energy conservation equation is as follows:

$$\frac{\partial p l}{\partial t} + \frac{1}{r} \frac{\partial r p l u}{\partial x} + \frac{\partial p l v}{\partial y} = -(p + q)D + \pi_{xx} \frac{\partial u}{\partial x} + \pi_{xy} \frac{\partial u}{\partial y} + \frac{u \pi_{\theta}}{r} + \pi_{xy} \frac{\partial v}{\partial x} + \pi_{yy} \frac{\partial v}{\partial y}$$  \hspace{1cm} (6)

Where, $l$ is the kinetic energy of the fluid.
In short, the momentum equation, continuity equation and energy conservation equation mentioned above are the governing equations of ALE method.

2.2. The best sand-water mixing ratio

In order to determine the best sand-water mixture ratio of sandcastle foundation, that is, the most stable sand-water mixture ratio of sandcastle foundation, we decided to establish a model to analyze the influence of sand moisture content on shear strength. Shear strength reflects its ability to resist shear failure. We made an analogy between sand and soil. By consulting the literature, we obtained the currently widely accepted formula for shear strength of soil:

\[
\tau_f = c' + (\sigma_n - u_a) \tan \varphi' + (u_a - u_w) \tan \varphi^b
\]  

(7)

Where, \( c' \) is effective cohesion, \( \varphi' \) is effective angle of internal friction, \( u_a \) is pore pressure and \( u_w \) is pore-water pressure, \( \varphi^b \) is the Angle of internal friction varying with suction. It can be seen that shear strength is also related to suction.

Taking this part of the strength caused by suction as \( \tau_s \), the formula can be re-listed as:

\[
\tau_f = c' + \tau_s + (\sigma_n - u_a) \tan \varphi'
\]  

(8)

\( \tau_s \) is a shear strength directly related to suction, which is called suction strength. Here is the empirical formula:

\[
\tau_s = \frac{u_s}{1.85 + \frac{0.85}{p_{at}}}
\]  

(9)

Where, \( u_s \) is suction, \( p_{at} \) is atmospheric pressure.

Suction in a sand-water mixture usually consists of two parts, matric suction and osmotic suction. The existing research results show that the osmotic suction is less than the matric suction in terms of the magnitude and the influence on the engineering properties of the soil, and it does not change significantly with the water content, so we ignore it. By referring to the literature, the expression of matric suction is obtained as:

\[
u_a - u_w = a(1 + w)^{-8.2089}\rho_d^{2.5} + \frac{b}{(w - 2)^{6.5} + 1}
\]

\[
a = 10^4 \cdot (50.28\rho_d^{15.5} + 2.997\rho_d^2 - 8.64\rho_d + 6.187)
\]

\[
b = 145.8\rho_d - 192.5
\]  

(10)

Where, \( w \) is water content, \( \rho_d \) is dry density (\( g/cm^3 \)). We take the empirical value of sand dry density of 1.56 \( g/cm^3 \). We use the matrix suction value \( (u_a - u_w) \) instead of suction \( (u_s) \), simultaneous equations (8)(9)(10)(11), shear strength can be obtained \( \tau_f \) relationship with water content \( w \).

2.3. Slope stability analysis

During atmospheric rainfall, rainwater infiltration will increase the saturation of sandcastle foundation, decrease the suction and cause a significant decrease in shear strength. Therefore, when the duration and intensity of rainfall exceed a certain limit, rainwater infiltration will reach a certain depth and cause slope instability. Simplified bishop method is one of the limit equilibrium methods for Slope Stability Analysis. Due to the consideration of the horizontal force between the bars, the obtained safety factor is higher. Therefore, we choose this method to explore it.

A simple homogeneous saturated sand slope AODC is provided. Slope OD is a single straight slope, slope height is \( h \), slope I =1: m. Bulk density of \( \gamma \), internal friction Angle \( \phi \), cohesive force to \( c \). In order to simplify the boundary equation of sand slope, a plane rectangular coordinate system is established and the origin is set at the toe of the slope. When the sand slope slides, the sliding surface \( ABC \) is the circle arc surface, its center coordinate is \( O(x_0,y_0) \), and the radius is \( R \). Take the differential
sand bar of any unit thickness from the arc sliding body. The width of the sand bar is \( dx \). The force acting on the differential sand bar is shown in the figure:

In order to facilitate the solution, according to the assumption of the simplified Bishop method, \( dX = 0 \), that is, only the horizontal force acts on the thrust \( E \) between the differential sand bars, and there is no vertical shear force \( X \). The formula of \( F_s \) is:

\[
F_s = \frac{\int_{x_0}^{x_0+\sqrt{R^2-(h-y_0)^2}} \int_{x_0}^{x_0+\sqrt{R^2-(h-y_0)^2}} \frac{1}{\mu} \left[ c \, dx + dW \tan \phi \right] \, (x - x_0) \, dW}{\int_{x_0}^{x_0+\sqrt{R^2-(h-y_0)^2}} \int_{x_0}^{x_0+\sqrt{R^2-(h-y_0)^2}} (x - x_0) \, dW}
\]

(11)

3. Experiment

(1) We simulated the deformation of geometry with different cross-sections when they are washed by the waves (from left to right).

The result is shown below:

Figure 2. Simulation of geometry which cross-section is flat
Figure 3. Simulation of geometry which cross-section is arc

The simulation results intuitively show the magnitude and deformation of stress when the cross-section is arc and the cross-section is flat. After comparison, we find that the geometrical shape of the cross-section is arc when the force of sea wave scouring is minimal.

Therefore, we reasonably speculate that the best sandcastle shape is the semi ellipsoidal shape.
Figure 4. Stable sandcastle structure

(2) We simulate the relationship curve between $\tau_f$ and $\omega$, as shown in the figure below:

![Graph showing the relationship between shear strength and water content.]

Figure 5. Relationship between shear strength and water content

It can be seen from Figure 5 that when the volumetric water content is within the range of [1.8%, 2.1%], the shear strength reaches the maximum, and the model is most stable.

(3) Through the Slope Stability Analysis, we get that:

| Angle of inclination | Slope Face  | Factor of safety |
|----------------------|------------|------------------|
| 30°                  | Straight   | 1.508            |
|                      | Arcuate    | 2.014            |

It can be seen from the table that the arc-shaped slope has a greater safety factor than the straight slope. Therefore, we believe that the semi ellipsoidal shape is still the best three-dimensional geometry under rainfall conditions.

4. Result

In order to investigate the optimal three-dimensional geometry of sandcastle, we put the erosion of the rainfall and the waves to the sandcastle consider jointly, by using the ALE method-based fluid-structure interaction simulation, conclude that the semi ellipsoidal shape is the best three-dimensional geometry under the influence of ocean waves. Then, the slope stability analysis is carried out, and the simplified Bishop method is adopted. It is found that when the slope of a sandcastle is curved, the safety factor is higher than the straight face, it is further confirmed that the semi ellipsoidal shape is the most stable sandcastle shape. Finally, we get the best sand-water ratio by setting out the relationship between the shear strength and the water content.

References

[1] Shi, H.H., Zhou, D., Wen, J.S., Jia, H.X. (2020) Fluid-structure interaction simulation of elastic cylindrical shells with water entry based on Ale Method. Journal of Ballistics, 2020,3201:9-14 + 46.

[2] Yang, X.N., Ma, J.J., Li, Y.Y., Sun, X.O. (2020) Distribution of wall stress of pipeline vehicle resting in pipe flow with different flow rate based on Ale fluid-structure Interaction. Journal
of the Taiyuan University of Technology, 2020, 5102:298-309.

[3] Dou, H.Q. (2015) Study on stability of soil slope under rainfall infiltration. (Zhejiang University).

[4] Cui, B.Y. (2006) Two-dimensional numerical simulation calculation based on ALE method. (Mongolia Industrial University).

[5] Wang, X.F. (2003) Study on the effect of rainfall infiltration on the stability of unsaturated soil slopes. (Xi’an University of Architecture and Technology).

[6] Liao, L.C., Yin, Z.Z. (1999) Shear strength of unsaturated soil. Geotechnical mechanics. 1999(03):1-6.

[7] Jia, D., Sun, D., Zhang, L. (2018) Effects of salt solution and sand content on the strength of Gaomiaozi bentonite. Journal of Shanghai University (Natural Science Edition).

[8] Wang, G.R., Dong, C., Duan, Z., Ma, J.Q., Tang, H. (2020) Effect of Coarse Particle Content on Shear Strength of Medium and Fine Sand. Journal of Xi’an University of Science and Technology.

[9] Dai, Z.H., Sheng, P.S. (2002) Numerical solution of simplified Bishop method for slope stability analysis. Rock and soil mechanics. 2002(06):760-764.