Impact zone analysis of p-cycle

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Abstract—Pre-configured cycle (p-Cycle) method has been studied in literature extensively for optical network protection. A large p-cycle has high capacity efficiency and can protect a large number of nodes against the single link failure scenarios. All the links protected by such a p-cycle lose protection when the p-cycle is consumed to restore traffic after a failure. As the probability of multiple link failure is high for a large network, it also means that with higher probability, on the second failure, protection may not be there for the failed link. Thus, if the number of links protected by a p-cycle is large, it makes the network unprotected with high probability on the advent of the second failure. In this paper, we study the impact zone due to a first link failure in the various configurations of the p-cycles. The study gives insight into how to choose the p-cycle configuration to reduce the impact zone while using minimum spare capacity. We propose new methods and compare them to show how the impact zone analysis can be used to improve the fault tolerance in an optical network.

1. INTRODUCTION

Survivability schemes are an integral part of optical network design. Survivability plays an essential role in an optical network by keeping it functional without interruption in the presence of failures. It involves protection and restoration. The protection is provisioning of redundant resources in the network, to be used during failures. Restoration is the process of establishing the failed connection again after failure, using redundant resources. An optical network which can survive a high number of simultaneous failures with minimum resources is desirable. The number of resources required increases with the increase in the failure protection capability of the network. Therefore, research efforts are mostly directed towards efficiently using the redundant resources to achieve resilience. Various network protection schemes have been proposed in the literature [1] [2] [3] [4]. Ring based schemes have fast recovery (50-60 ms) whereas mesh based protection schemes are more capacity efficient. P-cycles possess both, the speed of ring and efficiency of mesh protection. Grover and Stamatelakis proposed the concept of p-cycles in 1998 [5]. There has been significant amount of research done on p-cycles since then [6] [7] [8] [9] [10] [11] [12]. P-cycles are pre cross-connected ring-like structures formed from the spare capacity of the network. One of the main difference which distinguishes p-cycles from the ring is their ability to restore on-cycle as well as the straddling link failures. On-cycle links of a p-cycle are the links which form the p-cycle. Straddling links are the chords with there end nodes on the p-cycle.

Two-link failure protection schemes are desirable due to a significant probability of frequent two-link failures. Maximizing the number of deployed p-cycles improves dual failure restorability of a network [13]. Dual failure restorability also improves on minimizing the maximum coverage of working capacity by the p-cycles in a network [14]. In [15], Akpuh et al introduced a method to design a network with specified dual failure restorability. In [16], dual failure protection is provided to the links of the network depending on the quality of service of the link. Apart from these, there are some studies with the deterministic approach. Feng et al [17] proposed a method in which each working link in a network is protected by two p-cycles with link disjoint segments. The concept of a pre-configured ball (p-ball) is proposed by Chen Ma et al [18]. P-Ball is a subgraph with ring set which uses minimum backup links for dual link failure protection. Wang et al [19] introduced MFS (multi failure survivability scheme) to provide multi-failure survivability based on p-cycles. Huang et al [20] proposed pre-configured polyhedron (PCP) based protection structure to protect both simultaneous and sequential random link failures with improved spare resource efficiency. The scheme requires searching backup paths within the PCP after any failure to restore the traffic.

The studies in the literature focus mainly on the reduction of the spare capacity. There is no study which quantifies the effect on working capacity after the restoration of any fault. In this paper, we analyze the impact of a link failure on the protection of working capacity of the optical network protected by p-cycles. We define impact zone which mathematically represents the effect of the link failure on protection capacity of the network. Various methods are proposed to reduce the impact zone, to increase the double link failure tolerance of the network. In Section 2, we define and illustrate mathematical formulation of the impact zone. In Section 3, integer linear program (ILP) for optimizing spare capacity for single link failure has been formulated. We also introduced and discussed limiting constraints which can reduce impact zone in Section 3. In Section 4, simulation results have been discussed, and Section 5 concludes the work presented in this paper.

2 IMPACT ZONE

Protection capacity of a p-cycle is shared among a set of links which can be protected by it. Therefore, full protection of only one of these links can be ensured. The other links
may lose full or partial protection after the traffic disrupted by the first failure is restored. Following links will lose protection on the restoration of traffic on the failed link:

1) The links which share the same p-cycle with the failed link for protection.
2) The links which are protected by the p-cycles having the failed link as the on-cycle link.

We can define impact zone of a link as the working capacity which loose protection due to the utilization of p-cycle to protect the failed link. The impact zone has been formulated to quantify the double fault tolerance for a network which is deterministically 100% protected against single link failure. Therefore, we consider networks that have 100% single link failure protection.

In this paper, the considered optical networks have been represented by bidirectional graphs. For a network to survive any two link failures, it should be 3-connected [20] [21] [22]. A 3-connected graph is a graph that has three links disjoint path between any pair of nodes [23]. Since we are investigating double fault tolerance, we have used 3-connected graphs for simulation and analysis. Non 3-connected graphs have been modified to 3-connected graphs by applying merge operation on them. A graph can be modified to a 3-connected graph by identifying the pair of nodes having less than three disjoint paths and performing following merge operations on them.

1) A node in a graph with degree two can be merged to a neighboring node. As an example, in Fig. 1(i), node b with degree two is merged with node a.
2) Node-pairs in a graph having less than three disjoint paths can be merged as illustrated in Fig. 1(ii) to modify the graph to a 3-connected graph.

Notations and variables used for the optical network are as follows:

S=set of spans.
P=set of simple cycles.
w=working capacity on span i.
\(n_p\)=number of copies of p-cycle p.
\(n_{i,p}\)=number of copies of p-cycle p required to protect link i.
\(c_k\)=Overall protection capacity available for link k.

We define \(x_{i,p}\) as follows:

\[
\begin{align*}
&x_{i,p} = \\
&\begin{cases}
1 & \text{if span } i \text{ is on-cycle span of p-cycle } p, \\
2 & \text{if span } i \text{ is straddling link of p-cycle } p, \\
0 & \text{otherwise}.
\end{cases}
\end{align*}
\]

We define an indicator function \(I_A(x)\) as follows:

\[
I_A(x) = \begin{cases}
|x| & \text{if } x < 0, \\
0 & \text{if } x \geq 0.
\end{cases}
\]

The impact on the protection capacity depends on whether the failed link and the protected links are on-cycle or straddling the p-cycle/p-cycles affected by the failed link. The impact for these cases is formulated below.

1) The failed link \(i\) and other protected link \(k\) both are on-cycle on p-cycle \(p\). When the link \(i\) fails, the p-cycle is broken and the whole of the protection given by p-cycle \(p\) will be lost. The protection capacity provided by the p-cycle \(p\) to link \(k\) which is lost due to the failure of the link \(i\) is given as:

\[
I_{i,k,p} = n_{k,p}
\]

2) The failed link \(i\) is on-cycle and other protected link \(k\) is straddling on p-cycle \(p\). Impact due to the failure of link \(i\) on protection capacity of link \(k\) (loss of protection) is given as:

\[
I_{i,k,p} = I_A((n_p - n_{i,p}) - 2n_{k,p}).
\]

3) If link \(i\) is straddling link on p-cycle \(p\), the other protected link \(k\) can be on-cycle or straddling on p-cycle \(p\), and \(n_{i,p} > 0\), the loss of protection for link \(k\) is

\[
I_{i,k,p} = A(x_{i,p}((n_p - n_{i,p}) - n_{k,p})).
\]

\[
I_{i,k} = \sum_{p \in \mathcal{P}} I_{i,k,p}.
\]

The capacity available to protect link \(k\) after failure of link \(i\) is thus \(c_k - I_{i,k}\). Since we have considered deterministic single link protection \(c_k \geq w_k\). If \(c_k - I_{i,k} < w_k\) then, link \(i\) gets full protection even after failure of link \(i\). But, if \(c_k - I_{i,k} < w_k\) then, \(w_k - (c_k - I_{i,k})\) capacity of link \(k\) remains unprotected in case of failure of link \(i\). Thus, the impact zone \(I_i\) of link \(i\) is given as:

\[
I_i = \sum_{k \in S} I_A(c_k - I_{i,k} - w_k).
\]

2.1 Example to calculate impact zone

Consider the network shown in Fig. 1, we calculate impact zone for link \(i\) in Fig. 2. Let \(p(1-2-3-4-5-1), q(1-2-3-4-1)\) and \(r(1-4-5-1)\) are the p-cycles associated with link \(i\) which protect links \((e_1, e_2,i),(e_3, e_4,i)\) and \((e_5,e_6)\) respectively. When link \(i\) fails, p-cycle \(p\) and \(q\) will be used to protect the link \(i\). Links \(e_1,e_2\) will lose protection fully or partially depending on the number of copies of p-cycle \(p\) used to restore link \(i\). The remaining copies of p-cycle \(p\) will be \(n_p - n_{i,p}\). Protection lost by \(e_1\) and \(e_2\) will be \(n_{e_1,p} + n_{e_2,p} - 2(n_p - n_{i,p})\). Link \(e_3\) is
on-cycle on p-cycle q therefore, \( n_{c,i,q} \) capacity of \( e_3 \) will lose protection. Link \( e_6 \) is a straddling link on p-cycle q. p-Cycle q protects \( 2n_{e,6} \) working capacity of link \( e_6 \). The protection lost by \( e_6 \) will be \( 2n_{e,6} - (n_i - n_{i,q}) \). Link \( e_4 \) and \( e_5 \) are on-cycle on p-cycle r and, lose protection by \( n_{e,4,r} \) and \( n_{e,5,r} \), respectively. The impact zone of link \( i \) will be sum of protection capacity lost by links \( e_1 - e_5 \) is given by

\[
I_i = n_{e_1,p} + n_{e_2,p} - 2(n_p - n_{i,p}) + n_{e_3,q} + 2n_{e,6,q} - (n_i - n_{i,q}) + n_{e_4,r} + n_{e_5,r} \tag{6}
\]

### 3 Improving double fault tolerance

An optical network protected for single link failure will have some inherent double link failure tolerance. Reducing impact zone of the network results in the increase in its double fault tolerance capability. In the following subsection, we state an ILP for single link failure protection [5] where additional constraints have been incorporated to reduce its impact zone thereby increasing its tolerance to double link failure, i.e. failure of one more link after the first one.

#### 3.1 ILP for single link protection

In this section, we formulate and describe the ILP for single link failure protection (SLP) in an optical network. The ILP minimizes the total spare capacity required to protect the optical network from single link failure.

**Notations**

In addition to the notation defined in section 2, we define following notation to formulate the ILP for single link failure protection of optical network.

\( c_i \)=cost of unit capacity of span \( k \).

\( s_i \)=spare capacity on span \( k \).

\( a_{i,p} \) is a binary variable associated with link \( i \) and p-cycle \( p \). \( M \) is an arbitrary large positive number.

\[
\delta_{i,p} = \begin{cases} 
1 & \text{if span } i \text{ is on-cycle on p-cycle } p, \\
0 & \text{otherwise}. 
\end{cases}
\]

Minimize:

\[
\sum_{i \in S} c_i s_i \tag{7}
\]

Subject to:

\[
\sum_{p \in P} \sum_{j \in S} n_{i,p} \geq w_i \quad \forall i \in S 
\]

\[
n_p \geq n_{i,p} \delta_{i,p} \quad \forall p; i \in S \tag{9}
\]

\[
s_i \geq \sum_{p \in P} n_{i,p} \delta_{i,p} \quad \forall i \in S 
\]

\[
n_p \geq 0, n_{i,p} \geq 0; \quad \forall i \in S; \forall p \tag{10}
\]

We need to solve the ILP to find minimum spare capacity to provide 100% single link failure protection. The objective function given by equation [7] minimizes the total spare capacity on each link of the optical network. Constraint [8] ensures that the number of copies of the p-cycles are sufficient to protect the working capacity of the links. Constraint [9] selects the number of copies of p-cycle required to protect the maximum working capacity link among all the links protected by the p-cycle. Constraint [10] ensures that spare capacity on each link is sufficient to form the p-cycles. It should be noted that variable \( n_{i,p} \) is required to calculate the impact zone. Therefore, additional variable \( n_{i,p} \) and constraint [5] are required in the formulated ILP.

### 3.2 Minimizing impact zone

Solution of ILP given by eqn. [2] to [11] for a given optical network provides required optimum spare capacity and p-cycles to protect the network from single link failure. The impact zone for the links in the network is computed using the variables obtained by solving the ILP (eqns [2] to [11]) in eqn. [4]. A lower value of the impact zone indicates better double link failure tolerance.

We can limit the value of impact zone by adding limiting-constraints to the ILP of single link protection (eqns [7] to [11]). The limiting-constraints control the variables involved in the impact zone. This approach reduces the impact zone by reducing the sharing of p-cycles among links.

#### 3.2.1 Limiting-constraint 1

Constraints [8] to [11] ensures 100% single link failure protection in a network. We add limiting-constraint 1 (L1) given by eqn. [12] to reduce the value of impact zone. The left hand side of L1 contains all the variables associated with link \( i \) which can affect the protection capacity of other links.

\[
\sum_{p \in P} \sum_{j \in S} n_{i,p} + \sum_{p \in P} \sum_{j \in S} \left[ n_{i,p} - (n_p - n_{i,p}) \right] + \sum_{p \in P} \sum_{j \in S} \left[ n_{i,p} - (n_p - n_{i,p}) \right] \\
+ \sum_{p \in P} \sum_{j \in S} \left[ n_{i,p} - (n_p - n_{i,p}) \right] \leq K_i \quad \forall i \in S. \tag{12}
\]

Here, \( K \) is a constant. For a sufficiently large value of \( K \) the solution of ILP will converge to the solution of pure single link protection ILP. Therefore, impact zone is also same for a large value of \( K \), as for the ILP without any limiting constraint. Impact zone decreases as we reduce the value of \( K \).
3.2.2 Limiting-constraint 2

Another possible limiting-constraint 2 (L2) is given by eqn. 13. It is applied to the ILP of single link protection (eqn. 7 to 11) to reduce the sharing of p-cycles among links.

\[ \sum_{i \in S} n_{i,p} \leq n_p \quad \forall p \in P. \]  

Eqn. 9 along with eqn. 13 reduces the number of links which share a p-cycle. Thus the spare capacity required will increase and impact zone will decrease.

3.2.3 Limiting-constraint 3

We can also add the another additional limiting-constraints 3 (L3), represented by eqn. 14 to 16 to the ILP eqn. 7 to 11. L3 ensures that each p-cycle protects only one link. Thus, there is no sharing of p-cycles among links. We use a variable \( a_{i,p} \) with the conditions such that it can be either zero or one. \( M \) is a sufficiently large number \( (M > \sum_{i \in S} w_i) \).

\[ a_{i,p} \geq \frac{n_{i,p}}{M} \quad \forall i \in S. \]  

\[ a_{i,p} \leq 1 \quad \forall i \in S. \]  

\[ \sum_{i \in S} a_{i,p} \leq 1 \quad \forall p \in P. \]  

Constraint 14 and 15 ensure that the variable \( a_{i,p} \) can take binary value (0 or 1). Constraint 15 ensures that each p-cycle protects only one link.

4 Results and discussion

In this section, we analyze the effect of limiting-constraints L1, L2 and L3 in terms of spare capacity and impact zone. Computation is performed for the networks shown in Fig. 3. The networks net2m, Cost239 (Pan European network), net3mod, Net4, USA LHM (USA long Haul) are taken from [24, 25] and [26]. The networks net2 (also known as NSFNET), USA long Haul and net3 are modified to 3-connected graphs by performing the merge operation in the network as explained in section 2. Scilab 5.5 has been used for ILP formulation. ILOG CPLEX 9 has been used to solve the ILP on AMD Opteron(tm) 1.8 GHz CPU. One of the measure of performance of p-cycle based protection is Spare capacity efficiency (SE). It is defined as follows.

**Definition 1.** Spare capacity efficiency (SE) is the ratio of spare capacity required for protection and the working capacity in the network. It is given by

\[ SE = \frac{\sum_{i \in S} s_i}{\sum_{i \in S} w_i} \]  

where \( s_i \) and \( w_i \) are spare and working capacity on link \( i \) respectively.

Low value of SE is desirable in any protection scheme.

4.1 Performance of L1, L2 and L3

Single link protection ILP eqn. 7 to 11 with L1 (eqn. 12) is solved for different values of \( K \). L1 given by eqn. 12 controls the variable associated with a link which can affect the protection capacity of other links. The value of impact zone increases while, SE decreases with increase in \( K \) (Fig. 4). A lower value of impact zone implies greater fault tolerance capability which in turn requires higher spare capacity. Hence there is a trade-off between SE and impact zone. The value of \( K \) can be chosen to obtain the desired value of SE and impact zone. It should be noted that the value of impact zone does not reduce to zero for \( K = 0 \). \( K \) does not represent the true value of impact zone. For \( K = 0 \), the expression on the left-hand side of eqn. 12 reduces to zero. The expression for impact zone may not contain all the variables present in L1, and therefore it has nonzero value.

Various protection methods described in section 3, SLP, SLP with L1, SLP with L2, SLP with L3 have been compared in Fig. 5 and table 1. In Fig. 5, L1 _K0 represents the result with L1 and \( K = 0 \) while, L1 _Km represents the result with L1 and value of \( K \) for which SE and impact zone for the given network saturates. We observed that impact zone is lowest with L2 and L3 constraint in all the networks. The value of SE is higher for L1 _Km, L2, and L3 as compared to SLP and L1 _K0. L2 and L3 are comparable and give better double link failure tolerance than SLP, L1 _K0 and L1 _Km. L1 with a suitable value of \( K \), L2 and L3 can be useful for the networks that need higher double fault tolerance.

5 Conclusion

The impact zone is a measure of protection lost by a network after a failure and restoration of the working capacity passing through the failed link. Double link failure protection capability of a network can be improved by various techniques to reduce the impact zone. Different limiting constraints can be added to single fault protection ILP to minimize the impact zone. The value of impact zone and SE depends on the value of \( K \) in the limiting constraint L1. L1 is able to provide some control over the value of impact
### TABLE 1: Simulation Results

| Network       | SE | Impact Zone | SE | Impact Zone | SE | Impact Zone | SE | Impact Zone |
|---------------|----|-------------|----|-------------|----|-------------|----|-------------|
| net3mod       | 0.59 | 155.5      | 2.79 | 18.45      | 0.59 | 134.3      | 2.85 | 0          | 2.85 | 0          |
| net2m         | 0.8 | 106        | 3.62 | 21.5       | 0.8 | 80.1       | 3.50 | 1.2        | 3.59 | 0          |
| n7e14B        | 0.54 | 39         | 2.18 | 7.9        | 0.54 | 32         | 2.29 | 0          | 2.29 | 0          |
| Net4          | 0.47 | 106.8      | 2.06 | 15.5       | 0.47 | 88.7       | 2.34 | 0.2        | 2.36 | 0          |
| USA LHM       | 0.71 | 532        | 3.36 | 51.7       | 0.71 | 414        | 3.22 | 3.3        | 3.24 | 0          |
| cost239       | 0.43 | 106        | 1.90 | 17.8       | 0.43 | 99.9       | 2.31 | 0.08       | 2.31 | 0          |

Fig. 4: Effect of varying $K$ on impact zone and SE.
Fig. 5: Effect of different limiting constraints on Impact zone and SE

zone and SE. Limiting constraint L1 can be used to tune to the desired value of impact zone and SE but, L1 does not provide the lowest possible value of impact zone. L2 and L3 reduce the impact zone by limiting the sharing of p-cycles among links. We observed that L2 and L3 give better double fault tolerance as compared to L1 and SLP in terms of impact zone while sacrificing SE.

There is a trade-off between impact zone and SE. Both SE and impact zone should be considered while designing a survivable optical network. Choice of the method might depend on the design requirement of the network protection.

REFERENCES

[1] R. Ramaswami, K. N. Sivarajan, and G. H. Sasaki, “chapter 9 - network survivability,” in Optical Networks (Third Edition), third edition ed., Boston: Morgan Kaufmann, 2010, pp. 511 – 572. [Online].

[2] J. M. Simmons, Optical Protection. Cham: Springer International Publishing, 2014, pp. 277–347.

[3] D. Zhou and S. Subramaniam, “Survivability in optical networks,” IEEE Network, vol. 14, no. 6, pp. 16–23, Nov 2000.

[4] M. F. Habib, M. Tornatore, F. Dikbiyik, and B. Mukherjee, “Disaster survivability in optical communication networks,” Computer Communications, vol. 36, no. 6, pp. 630 – 644, 2013.

[5] W. D. Grover, “Cycle-oriented distributed preconfiguration: Ring-like speed with mesh-like capacity for self-planning network restoration,” Proceedings of IEEE ICC ‘98, Atlanta, vol. 54, no. 17, pp. 537 – 543, 1998.

[6] R. Asthana, Y. N. Singh, and W. D. Grover, “p-cycles: An overview,” IEEE Communications Surveys Tutorials, vol. 12, no. 1, pp. 97–111, First 2010.

[7] W. D. Grover, J. Doucette, A. Kodian, D. Leung, A. Sack, M. Clouqueur, and G. Shen, Design of Survivable Networks Based on p-Cycles. Boston, MA: Springer US, 2006, pp. 391–434.

[8] M. Eigler, H. Luss, and D. Shallcross, “Network restoration under dual failures using path-protecting preconfigured cycles,” Telecommunication Systems, vol. 49, no. 3, pp. 271–286, 2012.

[9] W. Wang and J. Doucette, “Dual-failure availability analysis of span-restorable mesh networks,” Journal of Network and Systems
[10] D. Stamatelakis and W. D. Grover, “IP layer restoration and network planning based on virtual protection cycles,” IEEE Journal on Selected Areas in Communications, vol. 18, no. 10, pp. 1938–1949, Oct 2000.

[11] M. Ju, F. Zhou, Z. Zhu, and S. Xiao, “Distance-adaptive, low capex cost p-cycle design without candidate cycle enumeration in mixed-line-rate optical networks,” J. Lightwave Technol., vol. 34, no. 11, pp. 2663–2676, Jun 2016.

[12] M. Ju, S. F. Zhou, and H. Wu, “Leveraging spectrum sharing and defragmentation to p-cycle design in elastic optical networks,” IEEE Communications Letters, vol. 21, no. 3, pp. 508–511, March 2017.

[13] D. Schupke, “The tradeoff between the number of deployed p-cycles and the survivability to dual fiber duct failures,” Communications, 2003. ICC ’03. IEEE International Conference on, vol. 2, pp. 1428 – 1432, May 2003.

[14] D. Schupke, “Multiple failure survivability in WDM networks with p-cycles,” Circuits and Systems, 2003. ISCAS ’03. Proceedings of the 2003 International Symposium on, vol. 5, no. 17, pp. III – III, May 2003.

[15] J. A. Bondy and U. S. R. Murty, Graph theory with applications. Macmillan London, 1976.

[16] G. Shen and W. D. Grover, “Extending the p-cycle concept to path segment protection for span and node failure recovery,” IEEE Journal on Selected Areas in Communications, vol. 21, no. 8, pp. 1306–1319, Oct 2003.

[17] A. Sack and W. D. Grover, “Hamiltonian p-cycles for fiber-level protection in semi-homogeneous homogeneous and optical networks,” IEEE Network, vol. 18, no. 2, pp. 49–56, Mar 2004.