Restrictions on the Hadronic Contribution to the Muon \((g - 2)\)-factor and to the Pion Electromagnetic Formfactor from Analytical Properties of the Pion Electromagnetic Formfactor

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Abstract

We investigate restrictions on the hadronic contribution to the muon \((g - 2)\) factor and to the pion electromagnetic formfactor from the analytical properties of the pion formfactor and the experimental data of the pion formfactor in the space-like [1] region. The values of the pion formfactor of [1] have been improved (see Table 1) and the values of the pion formfactor have been calculated at 20 additional points (see Table 2).
1. Introduction

The aim of this work is to obtain restrictions on the hadronic vacuum-polarization contributions to the anomalous magnetic moment $a^F_\mu$ and to the pion electromagnetic formfactor (f.f.) from the analytical properties of the pion formfactor and from the measurements of the pion formfactor in the space-like region at 45 points $^{[1]}$.

The contribution of the pion electromagnetic formfactor to the anomalous magnetic moment can be written as $^{[2]}$

$$a^F_\mu = \frac{\alpha^2}{12\pi^2} \int_{4m^2_\pi}^{\infty} \frac{ds}{s} K(s) \left( 1 - \frac{4m^2_\pi}{s} \right)^{3/2} |F_\pi(s)|^2$$  \hspace{1cm} (1)

where

$$K(s) = x^2 (1 - x^2/2) + (1 + x)^2 (1 - x^{-2}) \left[ \ln(1 + x) - x + x^2/2 \right] +$$

$$+ \frac{1 + x}{1 - x} x^2 \ln x, \quad x = \frac{1 - (1 - 4m^2_\pi/s)^{1/2}}{1 + (1 - 4m^2_\pi/s)^{1/2}}$$ \hspace{1cm} (2)

$m_\mu$ is the muon mass, $m_\pi$ is the pion mass.

Function $F_\pi(s)$ has the following analytical properties:

1. $F_\pi(s)$ is an analytical function with the cut $[4m^2_\pi, \infty]$.
2. $F_\pi(s)$ is a real function on the real axis $[-\infty, 4m^2_\pi]$.
3. $F_\pi(0) = 1$.

The article is organized as follows:

In Sec.2 we find the minimum of $a^F_\mu$ from analytical properties $F_\pi(s)$ only. In Sec.3 the formula for the minimum $a^F_\mu$ is obtained for the ease when $F_\pi(s)$ has given values $b_k$ at given points $s_k$

$$F_\pi(s_k) = b_k, \quad k = 0, 1, ...N$$ \hspace{1cm} (3)

In Sec.4 we show that if $F_\pi(s_k)$ is exactly equal to the central value of $F_\pi(s_k)$ for 45 points from $^{[1]}$, the minimal value of $a^F_\mu$ will be by $10^{100}$ times larger than the conventional value $a^F_\mu \approx 4.10^{-8}$ (see Table 1). Sec.5 shows the way to resolve the contradiction of Sec.4. It is necessary to allow $F_\pi(s_k)$ to vary within the limits of experimental errors. In this case $a^F_\mu \approx 10^{-8}$.

In Sec.6 it is shown that the use of the analiticity of $F_\pi(s)$ and of the requirement $\min_a F_\mu \leq 4.10^{-8}$ makes it possible to diminish the errors in the values of $F_\pi(s_k)$ given in $^{[1]}$.

In Sec.7 we calculate the values of $F_\pi(s_k)$ at 20 additional points $0.263GeV^2 \leq s_k \leq 0.463GeV^2$.

2. Derivation of Minimum $a^F_\mu$ Using the Analytical Properties of $F_\pi(s)$ only.

Let us map conformally the plane with a cut into interior of the circle $z$ by the formula
The value $a_\mu^F$ takes the form

$$a_\mu^F = \frac{\alpha^2}{12\pi} \Phi$$

where

$$\Phi = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) | F_\pi(e^{i\theta}) |^2 \, d\theta$$  \hspace{1cm} (5a)$$

and

$$f(\theta) = K(x) \sin^4 \theta / 2 / \cos \theta / 2$$

$$x = \frac{1 - \sqrt{1 - (m_\mu/m_\pi)^2 \cos^2 \theta / 2}}{1 + \sqrt{1 - (m_\mu/m_\pi)^2 \cos^2 \theta / 2}}$$

Let us find minimum $\Phi$. Expand the function $F_\pi(z)$ in series in orthogonal polynomials with the weight function $f(\theta)$:

$$F_\pi(z) = \sum_{n=0}^{\infty} a_n P_n(z)$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) P^*_n(z) P_m(z) \, d\theta = \delta_{nm}, \quad z = e^{i\theta}$$

Substituting expansion (6) into the formula (5a) and into the normalization condition $F_\pi(0) = 1$ we get

$$\Phi = \sum_{n=0}^{\infty} a_n^2$$

$$\sum_{n=0}^{\infty} a_n P_n(0) = 1$$

The coefficients $a_n$ are real because the function $F_\pi(s)$ is real on the real axis $[-\infty, 4m_\pi^2]$. Minimum $\Phi$ at the additional condition (9) can be found using the method of Lagrange multipliers.

$$\tilde{\Phi} = \sum_{n=0}^{\infty} a_n^2 - \lambda \sum_{n=0}^{\infty} a_n P_n(0)$$

From equation $\frac{\partial \Phi}{\partial a_n} = 0$ we obtain $a_n = \frac{1}{2} \lambda P_n(0)$ and from eq.(9) it follows

$$\lambda = \frac{1}{\sum_{n=0}^{\infty} P_n^2(0)}$$

and $\Phi_{\text{min}} = 1 / \sum_{n=0}^{\infty} P_n^2(0)$. 3
Let us use the formula from the theory of orthogonal polynomials [3]:

\[ A_{km} \equiv \sum_{n=0}^{\infty} P_n^*(z_k)P_n(z_m) = \frac{1}{1-z^*_kz_m} \frac{1}{D^*(z_k)D(z_m)} \]  

(11)

where

\[ D(z) = \exp \left\{ \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{1 + z e^{-i\theta}}{1 - z e^{-i\theta}} \ln f(\theta) d\theta \right\} \]  

(12)

Because of \( f(-\theta) = f(\theta) \) we obtain

\[ D(z_k) = \exp \left\{ \frac{1}{2\pi} \int_{0}^{\pi} \frac{1 - z_k^2}{1 - 2z_k \cos \theta + z_k^2} \ln f(\theta) d\theta \right\} \]  

(13)

and

\[ \Phi_{min} = D^2(0) = \exp \left\{ \frac{1}{\pi} \int_{0}^{\pi} \ln f(\theta) d\theta \right\} \]  

(14)

After taking the integral (14) we obtain the minimum of \( a^F_\mu \) following from analytical properties f.f. \( F_\pi(s) \) only:

\[ \Phi_{min} = 1.13 \cdot 10^{-13}, \quad (a^F_\mu)_{min} = 1.60 \cdot 10^{-9} \]  

(15)

3. Derivation of Minimum \( a^F_\mu \) Using the Analytical Properties of \( F_\pi(s) \) and the Conditions (3)

To derive the minimum of \( a^F_\mu \) if f.f. \( F_\pi(s) \) satisfies the conditions (3) we use the method of Lagrange multipliers:

\[ \tilde{\Phi} = \sum_{n=0}^{\infty} a_n^2 - \sum_{i=0}^{N} \lambda_i \sum_{n=0}^{\infty} a_n P_n(z_i) \]  

(16)

From condition of minimum \( \tilde{\Phi} \frac{\partial \tilde{\Phi}}{\partial a_n} = 0 \) we find

\[ a_n = \frac{1}{2} \sum_{i=1}^{N} \lambda_i P_n(z_i) \]  

(17)

The Lagrange multipliers are found from the conditions

\[ \frac{1}{2} \sum_{n=0}^{\infty} \lambda_i P_n(z_k)P_n(z_i) = b_i \quad (i = 0, 1, \ldots N) \]  

(18)

Minimum of \( \Phi \) under the conditions (3) is equal to

\[ \Phi_{min} = \frac{1}{2} \sum_{i=0}^{N} \lambda_i b_i \]  

(19)

The Lagrange multipliers \( \lambda_i \) are found from a system of linear equations.
\[ \sum_{k=0}^{N} A_{ki} \lambda_k = 2b_k, \quad i = 0, 1, \ldots N \]  

Substituting \( \lambda_k \) from (20) into formula (19) we obtain

\[ \Phi_{\text{min}} = -\left| \begin{array}{ccc} 0 & b_1 & \ldots & b_N \\ b_1 & A_{11} & \ldots & A_{1N} \\ \vdots & \vdots & \ddots & \vdots \\ b_N & A_{N1} & \ldots & A_{NN} \end{array} \right| \cdot \frac{1}{\text{Det}(A_{ik})} \]  

Formula (21) may be simplified by the substitution

\[ \bar{b}_i = b_i D(z_i) \]  

\[ \bar{A}_{ik} = A_{ik} D(z_i)D(z_k) \]  

\( \Phi_{\text{min}} \) may be written in new variables in the form

\[ \Phi_{\text{min}} = \sum_{i,k=0}^{N} C_{ik} \bar{b}_i \bar{b}_k \]  

where

\[ C_{ik} = (-1)^{i+k} \frac{\bar{\Delta}_{ik}}{\text{Det}(\bar{A}_{ik})} \]  

\( \bar{\Delta}_{ik} \) is the \((i,k)\) minor of the matrix \( \bar{A}_{ik} \).

\( \text{Det}(\bar{A}_{ik}) \) may be transformed into the form

\[ \text{Det}(\bar{A}_{ik}) = (-1)^{i+k} \frac{(1-z_iz_k)}{(1-z_i^2)(1-z_k^2)} \prod_{m=0}^{N} \frac{z_i - z_m}{1 - z_i z_m} \prod_{l=0}^{N} \frac{z_k - z_l}{1 - z_k z_l} \bar{\Delta}_{ik} \]  

Prime above the products implies that the factors with \( m = i \) and \( l = k \) should be omitted. As a result we get

\[ C_{ik} = \frac{1}{1 - z_iz_k} f_i f_k \]  

\[ f_i = (1 - z_i^2) \prod_{m=0}^{N} \frac{1 - z_i z_m}{z_i - z_m} \]  

For \( |z_i| \ll 1 \) formula (24) can be transformed into the following form, which is more convenient for numerical calculation:

\[ \Phi_{\text{min}} = \sum_{k=0}^{\infty} \left( \sum_{i=1}^{N} f_i \bar{b}_i z_i^k \right)^2 \]  

Table I presents the values \( Q_k \) in which f.f. \( F_\pi(Q_k^2) \) is measured\(^1\), the values of f.f. at these points, the values of \( z_n, f_n \) and of min \( a_\mu^F(n) \) if \( F_\pi(z_i) \) are equal to the central values f.f. at the points \( Q_i^2, i = 0, 1, \ldots n \). It is seen from Table 1 that even min \( a_\mu^F(3) > 4.10^{-8} \). If we will use all 46 points, min \( a_\mu^F(46) \) will be larger than expected one by a factor of \( 10^{100} \).
4. Resolution of the Contradiction

The problem under investigation belongs to the type of the so-called ”incorrect” problems. The reason is that \( \Phi_{\text{min}} \) has gigantic sensitivity to the measured values of f.f. \( F_\pi(Q_k^2) \). The values \( f_i \) have the order of \( 10^{36} \sim 10^{52} \) but all the sum in \( \Phi_{\text{min}} \) (29) has the order \( 3 \cdot 10^{-3} \). There is a gigantic cancellation in the sum of \( \Phi_{\text{min}} \).

The contradiction may be resolved if we permit \( F_\pi(s_k) \) to vary within the limits of experimental errors. Let us write \( \Phi_{\text{min}} \) in the form

\[
\Phi_{\text{min}} = \sum_{i,k=0}^{N} \frac{1}{1 - z_i z_k} x_i x_k
\]

where \( x_i = f_i \cdot D(z_i) F_\pi(z_i), \quad i = 1, 2, \ldots, N, \quad x_0 = f_0 D(0) \).

The value \( x_0 \) is known exactly, the rest of \( x_i \) varies so that the values \( F_\pi(s_i) \) does not contradict to experiment. Let us introduce the functional

\[
\tilde{\Phi}_{\text{min}} = x_0^2 + 2x_0 \sum_{i=1}^{N-1} x_i + \sum_{i,k=1}^{N-1} \frac{1}{1 - z_i z_k} x_i x_k + C \sum_{i=1}^{N-1} \frac{(x_i - \bar{x}_i)^2}{(\Delta x_i)^2}
\]

where \( \bar{x}_i = f_i D(z_i) \bar{F}_\pi(z_i), \quad \bar{F}_\pi \) is the value of f.f. \( F_\pi(z_i) \) in the central point, \( \Delta F_\pi^2(z_i) \) is the experimental error in \( F_\pi^2(z_i) \).

The constant factor \( C \) is chosen in the way that \( \chi^2 = \frac{1}{N-1} \sum_{i=1}^{N-1} \frac{(x_i - \bar{x}_i)^2}{(\Delta x_i)^2} \) is close to 1.

Let us write the condition of minimum \( \tilde{\Phi}_{\text{min}}, \quad \frac{\partial \tilde{\Phi}_{\text{min}}}{\partial x_i} = 0 \) in the form

\[
\sum_{k=1}^{N-1} \frac{1}{1 - z_i z_k} x_k + a_i x_i = -x_0 + a_i \bar{x}_i
\]

where \( a_i = C/(\Delta x_i)^2 \). The value \( x_0 ^2 \) is very large and the values \( a_i \) are very small and for this reason we will solve first the system of equations

\[
\sum_{k=1}^{N-1} \frac{1}{1 - z_i z_k} x_k^{(0)} = -x_0
\]

The solution of eqs.(33) is

\[
x_k^{(0)} = f_k \cdot D(0)
\]

This solution corresponds to f.f. \( F_\pi(z) \) minimizing \( \Phi \) at one condition \( F_\pi(0) = 1 \), it corresponds to the f.f. \( F(z) = D(0)/D(z) \). Due to this we make the substitution

\[
x_k = x_k^{(0)} + y_k
\]

For \( y_k \) we get the system of linear equations:

\[
\sum_{k=1}^{N-1} \frac{1}{1 - z_i z_k} y_k + a_i y_i = a_i (\bar{x}_i - x_i^{(0)}), \quad i = 1, 2, \ldots, N - 1
\]

The system of equations was solved numerically. Taking into account that \( x_k \) is the solution of the system of eqs.(32) we obtain
$$\min(a^h_{\mu})^{\text{var}} = \frac{\alpha^2}{12\pi^2} \left\{ x_0 \left( \sum_{i=1}^{N-1} x_i + x_0 \right) + \sum_{i=1}^{N-1} a_i (\bar{x}_i - x_i) x_i \right\} = 1.02 \cdot 10^{-8} \quad (37)$$

5. Refinements of the values of f.f. obtained in [1]

Let us fix one of the 45 values of f.f. (e.g., $F_\pi(s_k)$) within the limits of experimental errors. The rest values of f.f. will be varied within the limits of experimental errors (eqs.(31)-(36)). Let us scan $F_\pi^2(Q_l)$ and then vary $F_\pi^2(Q_l)$ $l \neq k$ until $\min a^h_{\mu}$ becomes $4 \cdot 10^{-8}$. The constant $C$ changes in a way that $\chi^2 = 1$.

As a result we obtain the refined values of f.f. $-(F_\pi^2(Q^2))_{\text{ref.}}$ (see Table 1).

6. Calculation of the Values $F_\pi^2(Q_k)$ at the additional points

$Q_k^2 = 0.263 + 0.01 \cdot (k - N), \ N + 1, ..., N + 20$

Let us scan $F_\pi^2(Q_k), \ k = N + 1, ... N + 20$ and vary $F_\pi^2(Q_k^2), k = 1, ... N - 1$, so that $\chi^2 = 1$ and $\min a^F_{\mu} \leq 4.10^{-8}$. As a result, we obtain $F_\pi^2(Q_k)$ ($k = N + 1, ... N + 20$) (see Table 2).

7. Conclusions

In conclusion we make some additional comments:

1) There is very large correlation between the values of f.f. at different $Q_k$. As a rule, if we fix two values $F_\pi(Q_k)$ and $F_i(Q_i)$ within the limits of experimental errors and will vary the rest of the $F_\pi(Q_\mu)$ values (eqs.(31)-(36)), then the $\min a^F_{\mu}$ will be by several orders of magnitude larger than $4.10^{-8}$. Thus, the use of the analytical properties of f.f. in analysis of the experiments on f.f. measurements makes it possible to improve significantly the accuracy of the experimental results.

2) In many theoretical works calculations are made in Euclidean space. One of the results of this work is that the transition from Euclidean space to Minkowski space is a nontrivial problem.

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References

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[3] G.Szego, Orthogonal polynomial 1959.
Table 1

| n | $Q^2$ | $-z_n$ | $(F^2_n(Q^2))_{\text{Exp}}$ | $f_n$ | $\text{min} \alpha^\ell_\mu(n)$ | $(F^2_n(Q^2))_{\text{ref.}}$ |
|---|-----|-----|-----------------|-----|----------------|-----------------|
| 0 | 0   | 0   | 1               | 2.43 $\cdot 10^{36}$ | 1.60 $\cdot 10^{-9}$ | |
| 1 | 0.015 | 0.044 | 0.944 ± 0.007 | $-1.25 \cdot 10^{45}$ | 2.84 $\cdot 10^{-9}$ | |
| 2 | 0.017 | 0.0493 | 0.921 ± 0.006 | 4.61 $\cdot 10^{46}$ | 2.07 $\cdot 10^{-6}$ | 0.936 ± 0.007 |
| 3 | 0.019 | 0.0545 | 0.933 ± 0.006 | $-8.15 \cdot 10^{47}$ | 0.0803 | 0.932 ± 0.005 |
| 4 | 0.021 | 0.0596 | 0.926 ± 0.006 | 9.12 $\cdot 10^{48}$ | 805.02 | |
| 5 | 0.023 | 0.0646 | 0.914 ± 0.007 | $-7.22 \cdot 10^{49}$ | 3.13 $\cdot 10^{6}$ | |
| 6 | 0.025 | 0.0695 | 0.905 ± 0.007 | 4.28 $\cdot 10^{50}$ | 5.98 $\cdot 10^{9}$ | |
| 7 | 0.027 | 0.0742 | 0.898 ± 0.008 | $-1.96 \cdot 10^{51}$ | 6.32 $\cdot 10^{-12}$ | |
| 8 | 0.029 | 0.0789 | 0.884 ± 0.008 | 7.03 $\cdot 10^{51}$ | 4.73 $\cdot 10^{15}$ | 0.899 ± 0.003 |
| 9 | 0.031 | 0.0835 | 0.884 ± 0.009 | $-1.99 \cdot 10^{52}$ | 6.74 $\cdot 10^{18}$ | 0.885 ± 0.008 |
| 10 | 0.033 | 0.0881 | 0.890 ± 0.009 | 4.44 $\cdot 10^{52}$ | 2.66 $\cdot 10^{22}$ | 0.884 ± 0.003 |
| 11 | 0.035 | 0.0925 | 0.866 ± 0.01 | $-7.58 \cdot 10^{52}$ | 9.22 $\cdot 10^{25}$ | 0.872 ± 0.004 |
| 12 | 0.037 | 0.0968 | 0.876 ± 0.011 | 9.43 $\cdot 10^{52}$ | 1.62 $\cdot 10^{29}$ | 0.869 ± 0.004 |
| 13 | 0.039 | 0.101 | 0.857 ± 0.011 | $-7.26 \cdot 10^{52}$ | 9.26 $\cdot 10^{31}$ | 0.861 ± 0.007 |
| 14 | 0.042 | 0.107 | 0.849 ± 0.009 | 3.80 $\cdot 10^{52}$ | 1.45 $\cdot 10^{43}$ | 0.853 ± 0.005 |
| 15 | 0.046 | 0.115 | 0.837 ± 0.009 | $-2.71 \cdot 10^{52}$ | 1.56 $\cdot 10^{48}$ | 0.841 ± 0.005 |
| 16 | 0.050 | 0.123 | 0.83 ± 0.01 | 2.96 $\cdot 10^{52}$ | 3.22 $\cdot 10^{44}$ | 0.829 ± 0.005 |
| 17 | 0.054 | 0.131 | 0.801 ± 0.011 | $-3.65 \cdot 10^{52}$ | 2.88 $\cdot 10^{44}$ | 0.818 ± 0.006 |
| 18 | 0.058 | 0.138 | 0.800 ± 0.012 | 4.54 $\cdot 10^{52}$ | 1.57 $\cdot 10^{47}$ | 0.807 ± 0.005 |
| 19 | 0.062 | 0.145 | 0.809 ± 0.012 | $-5.39 \cdot 10^{52}$ | 6.09 $\cdot 10^{49}$ | 0.800 ± 0.003 |
| 20 | 0.066 | 0.152 | 0.785 ± 0.014 | 5.85 $\cdot 10^{52}$ | 1.78 $\cdot 10^{52}$ | 0.785 ± 0.006 |
| 21 | 0.070 | 0.159 | 0.785 ± 0.015 | $-5.59 \cdot 10^{52}$ | 4.11 $\cdot 10^{54}$ | 0.775 ± 0.005 |
| 22 | 0.074 | 0.165 | 0.777 ± 0.016 | 4.46 $\cdot 10^{52}$ | 7.79 $\cdot 10^{56}$ | 0.766 ± 0.005 |
| 23 | 0.078 | 0.172 | 0.769 ± 0.017 | $-2.64 \cdot 10^{52}$ | 1.24 $\cdot 10^{59}$ | 0.757 ± 0.005 |
| 24 | 0.083 | 0.179 | 0.757 ± 0.01 | 1.13 $\cdot 10^{52}$ | 1.63 $\cdot 10^{61}$ | 0.746 ± 0.004 |
| 25 | 0.089 | 0.188 | 0.715 ± 0.016 | $-5.15 \cdot 10^{52}$ | 1.73 $\cdot 10^{65}$ | 0.727 ± 0.004 |
| 26 | 0.095 | 0.197 | 0.724 ± 0.018 | 2.97 $\cdot 10^{51}$ | 1.61 $\cdot 10^{65}$ | 0.715 ± 0.007 |
| 27 | 0.101 | 0.205 | 0.680 ± 0.017 | $-1.81 \cdot 10^{51}$ | 1.25 $\cdot 10^{67}$ | 0.704 ± 0.008 |
| 28 | 0.107 | 0.213 | 0.696 ± 0.019 | 1.10 $\cdot 10^{60}$ | 8.49 $\cdot 10^{68}$ | 0.691 ± 0.009 |
| 29 | 0.113 | 0.220 | 0.688 ± 0.020 | $-6.30 \cdot 10^{59}$ | 5.09 $\cdot 10^{60}$ | 0.679 ± 0.009 |
| 30 | 0.119 | 0.228 | 0.676 ± 0.021 | 3.31 $\cdot 10^{61}$ | 2.72 $\cdot 10^{62}$ | 0.667 ± 0.009 |
| 31 | 0.125 | 0.235 | 0.665 ± 0.023 | $-1.53 \cdot 10^{60}$ | 1.31 $\cdot 10^{64}$ | 0.656 ± 0.009 |
| 32 | 0.131 | 0.242 | 0.651 ± 0.024 | 5.28 $\cdot 10^{49}$ | 5.74 $\cdot 10^{55}$ | 0.644 ± 0.010 |
| 33 | 0.137 | 0.248 | 0.646 ± 0.027 | $-1.76 \cdot 10^{49}$ | 2.3 $\cdot 10^{77}$ | 0.634 ± 0.010 |
| 34 | 0.144 | 0.256 | 0.616 ± 0.023 | 3.22 $\cdot 10^{48}$ | 8.33 $\cdot 10^{68}$ | 0.621 ± 0.011 |
| 35 | 0.153 | 0.265 | 0.654 ± 0.023 | $-3.97 \cdot 10^{47}$ | 2.69 $\cdot 10^{68}$ | 0.605 ± 0.013 |
| $n$ | $Q^2$ | $F^2_{\pi}(Q^2)$ | $n$ | $Q^2$ | $F^2_{\pi}(Q^2)$ |
|-----|-------|-----------------|-----|-------|-----------------|
| 1   | 0.263 | 0.448 ± 0.043   | 11  | 0.363 | 0.357 ± 0.077   |
| 2   | 0.273 | 0.439 ± 0.046   | 12  | 0.373 | 0.349 ± 0.082   |
| 3   | 0.283 | 0.428 ± 0.049   | 13  | 0.383 | 0.342 ± 0.085   |
| 4   | 0.293 | 0.417 ± 0.052   | 14  | 0.393 | 0.336 ± 0.086   |
| 5   | 0.303 | 0.407 ± 0.056   | 15  | 0.403 | 0.330 ± 0.089   |
| 6   | 0.313 | 0.398 ± 0.060   | 16  | 0.413 | 0.324 ± 0.093   |
| 7   | 0.323 | 0.389 ± 0.064   | 17  | 0.423 | 0.318 ± 0.096   |
| 8   | 0.333 | 0.381 ± 0.067   | 18  | 0.433 | 0.312 ± 0.099   |
| 9   | 0.343 | 0.373 ± 0.070   | 19  | 0.443 | 0.306 ± 0.102   |
| 10  | 0.353 | 0.365 ± 0.073   | 20  | 0.453 | 0.301 ± 0.105   |