RESONANCES AND WEAK INTERACTIONS IN $D^+ \to \pi^+\pi^-\pi^+$ DECAYS∗

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We describe the $\pi\pi$ S-wave in $D^+ \to \pi^+\pi^-\pi^+$ decays using a unitary model for the $\pi\pi$ Final State Interactions (FSI). The three body decay is treated as a quasi two-body process where, at the weak vertex, the $D$ meson decays into a resonance and a pion. The weak part of the decay amplitude is evaluated using the effective weak Hamiltonian within the factorization approximation.

1. Introduction

The E791 collaboration found a strong evidence for a light and broad scalar-isoscalar resonance in $D^+ \to \pi^+\pi^-\pi^+$ decays, known as the $\sigma(500)$ (or $f_0(600)$) meson[1]. According to their fit, this resonance contributes to approximately half of the decays and has a mass and width given by $m_\sigma = 478^{+24}_{-23} \pm 17$ MeV and $\Gamma_\sigma = 328^{+42}_{-40} \pm 21$ MeV. This presence of the $\sigma(500)$ in $D^+ \to \pi^+\pi^-\pi^+$ has been confirmed by the CLEO collaboration in a very recent paper[2] and the results they obtained are in agreement with those of E791. On the other hand, this resonance was for a long time hidden in scattering experiments, but at present its pole position in the complex energy plane is rather well determined[3], which gives support to the E791 findings.

In our approach, we calculate the weak decay $D^+ \to R \pi^+$ where $R = \sigma(500)$, $\rho(700)$, $f_0(980)$, using the effective weak Hamiltonian within the factorization approximation. The $(\pi\pi)s$-wave final state is then constructed using the scalar form factor previously introduced in the context of $B$ decays to pions and kaons[4]. This

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model for the FSI, based on the framework developed in [6], is unitary and takes into account the coupling to the $K\bar{K}$ channel. The $P$-wave, which mainly corresponds to the $\rho(770)$, and the $D$-wave, arising from the $f_2(1270)$, are described by the usual Breit-Wigner functions. The general idea of the model is depicted in Fig. 1: the $\pi^+$ produced at the weak vertex acts as a spectator of the rescattering between the other two pions. The amplitude is to be symmetrized since there are two identical pions in the final state.

\[ A_{\sigma} = \langle \sigma \pi^+ | H_{\text{eff}} | D^+ \rangle = \frac{G_F}{\sqrt{2}} V_{cd} V_{ut}^* a_1 f_\pi (m_D^2 - m_\sigma^2) F_0^{D-\sigma}(m_\pi^2), \]  

where $f_\pi$ is the pion decay constant, $m_D$, $m_\pi$ and $m_\sigma$ are the $D$ meson, pion and $\sigma$ masses, respectively, $F_0^{D-\sigma}$ is a transition form factor treated as a parameter of the model, $G_F$ is the Fermi constant and $V_{ut}$ is the CKM matrix element for the transition $q_1 \rightarrow q_2$. The coefficient $a_1$ is a combination of Wilson coefficients, $C_1 + C_2/N_c$. It is worth stressing that this expression is simplified by the fact it receives no contribution from the color suppressed diagram since the matrix element $\langle \sigma | j^\mu | 0 \rangle$ with $j^\mu = \bar{q} \gamma^\mu (1 - \gamma^5) q$ vanishes by symmetry. The result for the weak decay $D^+ \rightarrow \rho(980)\pi^+$ is very similar but one must also add the non-zero $s\bar{s}$ component of the $f_0(980)$, which can be done introducing a mixing angle $\theta$ in the spirit of [8]. The $D^+ \rightarrow \rho(770)^0\pi^+$ amplitude does receive a contribution from the color suppressed diagram, proportional to $a_2 = C_2 + C_1/N_c$, since now $\langle \rho | j^\mu | 0 \rangle \neq 0$.

3. Final state interactions

One can construct the $3\pi$ final state using a form factor to describe the propagation and decay into two pions of the resonance $R$. Usually a Breit-Wigner function is used for this purpose [8]. In this work, we employ a model for the $S$-wave in which...

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Fig. 1. $D^+$ decay to $\pi^+\pi^-\pi^+$ intermediated by a resonance $R$. The amplitude is symmetrized.

2. Weak amplitudes

To evaluate the weak decays $D^+ \rightarrow (\sigma, \rho, f_0)\pi^+$, we use an effective weak Hamiltonian within the QCD factorization approximation. In the present case, the contributions from strong penguins are CKM suppressed and can be safely neglected which allows one to work at tree level. Assuming that the $\sigma(500)$ is scalar-isoscalar with a $(u\bar{u} + d\bar{d})$ quark content, the result for the decay $D^+ \rightarrow \sigma\pi^+$ can be cast as

\[ A_{\sigma} = \frac{G_F}{\sqrt{2}} V_{cd} V_{ut}^* a_1 f_\pi (m_D^2 - m_\sigma^2) F_0^{D-\sigma}(m_\pi^2), \]  

where $f_\pi$ is the pion decay constant, $m_D$, $m_\pi$ and $m_\sigma$ are the $D$ meson, pion and $\sigma$ masses, respectively, $F_0^{D-\sigma}$ is a transition form factor treated as a parameter of the model, $G_F$ is the Fermi constant and $V_{ut}$ is the CKM matrix element for the transition $q_1 \rightarrow q_2$. The coefficient $a_1$ is a combination of Wilson coefficients, $C_1 + C_2/N_c$. It is worth stressing that this expression is simplified by the fact it receives no contribution from the color suppressed diagram since the matrix element $\langle \sigma | j^\mu | 0 \rangle$ with $j^\mu = \bar{q} \gamma^\mu (1 - \gamma^5) q$ vanishes by symmetry. The result for the weak decay $D^+ \rightarrow \rho(980)\pi^+$ is very similar but one must also add the non-zero $s\bar{s}$ component of the $f_0(980)$, which can be done introducing a mixing angle $\theta$ in the spirit of [8]. The $D^+ \rightarrow \rho(770)^0\pi^+$ amplitude does receive a contribution from the color suppressed diagram, proportional to $a_2 = C_2 + C_1/N_c$, since now $\langle \rho | j^\mu | 0 \rangle \neq 0$.
Resonances and weak interactions in $D^+ \to \pi^+\pi^\mp\pi^+$ decays.

four scalar form factors are introduced to describe the $(\pi\pi)_S$ and $(K\bar{K})_S$ FSI. However, in the case being considered here, only one of them, namely $\Gamma_{1}^{*}(s)$, enters in the calculations. This model respects unitarity and the coupling to the $K\bar{K}$ channel is properly accounted for. Employing an approximation where all the particles are taken to be on-shell, this form factor is given analytically: it is written in terms of the scalar-isoscalar scattering phase shifts and inelasticities $\delta_{\pi\pi}(s)$, $\delta_{K\bar{K}}(s)$ and $\eta(s)$, as well as of the production functions $R_{1}^{0}(s)$ and $R_{2}^{0}(s)$ obtained in chiral perturbation theory. Below the $K\bar{K}$ threshold Watson’s theorem is fulfilled: the phase of the form factor $\Gamma_{1}^{*}(s)$ is given by $\delta_{\pi\pi}(s)$. The full amplitude $D^+ \to \sigma\pi^+, \sigma \to \pi^+\pi^-$ can then be written as

$$M_{S}(u, t) = A_{S} \chi \left[ \Gamma_{1}^{*}(u) + \Gamma_{1}^{*}(t) \right],$$

where $A_{S}$ is given in Eq. (1). This amplitude is symmetrized as shown in Fig. 1, it is expressed in terms of two Lorentz invariant quantities namely $u = (p_1 + p_{\pi^-})^2$ and $t = (p_2 + p_{\pi^-})^2$, following the labels used in Fig. 1. In Eq. (2), $\chi$ is a normalization constant related to the coupling between the pions and the scalar resonance, estimated to be $\chi \approx 30$ GeV$^{-1}$. The amplitude for the decay $D^+ \to \rho^0\pi^+$, $\rho^0 \to \pi^-\pi^+$ is written similarly

$$M_{P}(u, t) = A_{P} \left[ (t - s)\Gamma_{\rho\pi\pi}(u) + (u - s)\Gamma_{\rho\pi\pi}(t) \right],$$

where $s = (p_1 + p_2)^2$, $\Gamma_{\rho\pi\pi}$ is a relativistic Breit-Wigner and $A_{P}$ the corresponding weak amplitude. The $D$-wave is included in the model by means of another Breit-Wigner function and two additional parameters: a magnitude $a_{f_2}$ and a phase $\delta_{f_2}$. The total amplitude is written as the sum of all the partial waves

$$M_{\text{Total}}(u, t) = M_{S}(u, t) + M_{P}(u, t) + M_{D}(u, t).$$

This expression is in a form suited to obtain a Dalitz plot in terms of $u$ and $t$.

4. Preliminary results

Eq. (1) enables one to perform a fit to the E791 data reproduced as in Suppl. We show in Tab. 1 preliminary results for the normalization constant $\chi$ and for the transition form factor $F_{0}^{D \to \sigma}(m_\pi^2)$. The value for $\chi$ is in agreement with the results from Suppl whereas our value for $F_{0}^{D \to \sigma}(m_\pi^2)$ is smaller than 0.79 $\pm$ 0.15 obtained in Suppl.

| Param. | Result |
|--------|--------|
| $F_{0}^{D \to \sigma}(m_\pi^2)$ | 0.43 $\pm$ 0.03 |
| $\chi$ | 22.5 $\pm$ 0.8 GeV$^{-1}$ |
In Tab. 2 we show our results for the fit fractions (f.f.s.), which measure the contribution of each channel to the total Dalitz plot. Due to interference effects, the sum of the f.f.s. is not necessarily 100%. In our model, the \((\pi\pi)_S\) fit fraction approximatively corresponds to the sum of the f.f.s. of \(\sigma\), NR and \(f_0'\)s of CLEO. Compared to Oller’s work, we do not introduce a non-resonant background. The description of the \(S\)-wave, unified as it is in the \(\Gamma_1^{\pi^+}(s)\) form factor, avoids the use of a sum of Breit-Wigner functions. Use of a unitary model for the \(\pi\pi\) FSI enables us to reproduce the \(S\)-wave two-pion resonance spectrum in \(D^+ \rightarrow \pi^+\pi^-\pi^+\).

| Channel | E791 | CLEO | Oller | This work |
|---------|------|------|-------|-----------|
| \(\sigma\) | 46.3\(\pm\)9.2 | 41.8\(\pm\)1.4 \(\pm\)2.5 | - | - |
| NR | 7.8 \(\pm\)6.6 | <3.5 | 17 | - |
| \(f_0(980)\) | 6.2 \(\pm\)1.4 | 4.1 \(\pm\)0.9 \(\pm\)0.3 | - | - |
| \(f_0(1370)\) | 2.3 \(\pm\)1.7 | 2.6 \(\pm\)1.8 \(\pm\)0.6 | - | - |
| \(f_0(1500)\) | - | 3.4 \(\pm\)1.0 \(\pm\)0.8 | - | - |
| \((\pi\pi)_S\) | - | - | 102 | 53 |
| \(\rho(770)\) | 33.6\(\pm\)3.9 | 20.0\(\pm\)2.3 \(\pm\)0.9 | 36 | 32 |
| \(f_2(1270)\) | 19.4\(\pm\)2.5 | 18.2\(\pm\)2.6 \(\pm\)0.7 | 21 | 10 |
| \(\rho(1440)\) | 0.7\(\pm\)0.8 | <2.4 | 1 | 2 |

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