Teachers content knowledge in generalizing mind maps of quadrilateral

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Abstract. This study aims to explain teacher content knowledge in generalizing mind maps of quadrilateral. This research use descriptive qualitative approach. The research subjects were 26 mathematics teachers (9 men and 17 women) who had taught at least 5 years and fulfilled the requirements for the teacher certification program. Knowledge of teacher content in generalizing quadrilateral concept maps was analyzed using De Villiers's framework on three types of definitions in geometry. The results showed that there were three types of knowledge of teacher content in defining rectangle, namely (1) fulfill the necessary requirements and sufficient conditions for minimal definition, (2) fulfill the necessary and sufficient condition but excessive requirements, and (3) not meet the necessary requirements and sufficient conditions. The mistakes shown by teachers are: incorrectly stating the nature or attributes; does not state sufficient conditions of definition; necessary conditions and sufficient conditions are excessive. The results show that there are two types of teachers content knowledge in generalizing hierarchically and partially mind maps.

1. Introduction
Mathematics teachers, who are the main actors in instructional process, should have mathematical content knowledge to support effective learning. However, many studies conclude that mathematics teachers’ content knowledge is inadequate for effective learning [1-3]. Teachers who understand the concept should be able to answer students’ questions about the meaning behind a symbol manipulation, explain why certain algorithms can be used and why others cannot, and explain the relationships among the concepts. On the other hand, teachers who lack knowledge tend to emphasize facts, rules (formulas), and procedures [4].

Knowledge of the content is used to transform the material owned by the teacher into the form of representation or presentation that can help students to develop their knowledge and skills. Content knowledge is very pivotal for the teacher to know the position of the taught mathematics material, the competence to handle the mathematical concepts, and the sources of knowledge needed to recognize the various mathematical knowledge to teach [3,5].

Mathematics is a universal language. Accordingly, its conceptualized concept exists in every context [6]. However, the concept of mathematics is defined differently based on logical relationships in different mathematical terms that are related to concepts [7]. Thus, the mathematical concept needs to be understood by the teacher so that mathematical concepts and other concepts can also be taught well to students. A definition is an expression to limit a concept [8]. By definition, people can make
illustrations, images or symbols of a defined concept. There are two kinds of definitions in geometry, namely analytic definitions and genetic definitions [8]. The analytic definition is the definition by mentioning the proximal genus (close family) and the specific deference (special distinction). A genetic definition is a definition that shows or reveals the way in which the occurrence or formation of a defined concept exists. The elements of definition are background, genus, defined terms, and attributes.

The definition has a crucial role in mathematics. The activity of constructing a definition is a scientific activity that is no less important than other processes such as problem-solving, making conjectures, generalizations, specializations, and proofs. Thus it would be unfair if it is ignored in mathematics teaching [9]. In fact, definitions in mathematics play a role in: (1) introducing theoretical objects and capturing the essence of a concept by conveying characteristic properties, (2) being fundamental components of concept formation, (3) establishing the foundation for evidence and problem solving, creating uniformity in terms of concepts, and making it possible to communicate mathematical ideas easily [10].

Referring to the suggestion [11], the knowledge of teacher content in generalizing quadrilateral needs to be known because in defining the quadrilateral formally pay attention to the necessary requirements and sufficient requirements for a minimal definition. This is also supported by the findings [6] that some teachers are incomplete making a quadrilateral mind maps relationship.

The content knowledge owned by the teachers in constructing the definition and classifying the quadrilateral can provide good guidance to find out whether or not the teacher struggle to generalize the mind map of quadrilateral. Therefore, teachers who can generalize the quadrilateral concept will prevent students from misconceptions and difficulties in studying the quadrilateral. Based on the description that has been proposed, this study aims to reveal and explore the teachers’ content knowledge to generalize the mind map of quadrilateral.

2. Methods
This study uses a qualitative approach with a type of descriptive-explorative research. The research subjects consisted of 26 mathematics teachers who had taught and fulfilled the requirements to participate in the subrayon 126 teacher certification program in 2017. All participants were graduates of mathematics education with a tenure of 5 years at least junior high school (junior) consisting of 9 men (35%) and 17 women (65%). All Participants is a graduate of Mathematics Education. 85% of subjects teach in public junior high schools and 15% of subjects teach in private junior high schools in Southeast Sulawesi Province. From 26 teachers, three volunteer subjects S1, S2, and S3 were taken for an interview. The selection of these three subjects is based on their success in defining and making quadrilateral mind maps.

This research begins by giving test to all subject of research. The subject's answers are then analyzed to reveal how the teacher defines quadrilateral. The research instruments are the researchers who collected and analyzed the research data [12], and a test as a secondary instrument. The researcher acts as the main instrument meaning that the existence of the researcher cannot be replaced by someone else or something else. The researcher also does not manipulate another variable. The test has been validated by two senior lecturers of Surabaya State University. The questions of the test are: (1) How do you explain the rectangle, parallelogram, rhombus, and square to your students? (2) Draw a diagram (mind maps) to link all the rectangular constructions: rectangle, parallelogram, rhombus, and square!

The collected data is then transcribed, reviewed and categorized. Response responses are categorized into three, namely: economic, right and wrong. Definitions are analyzed by examining the terms/attributes, dedicating one of the terms/attributes of the other attributes, and developing a new build of reduction of the necessary conditions and sufficient conditions [9,12]. Categorization includes wrong definition, either some or all of the attributes; states the requirement is necessary but does not state sufficient conditions. Definition is correct that states the necessary and sufficient conditions. The economical definition is to state sufficient conditions and conditions as minimal as it might be [9,12].

Data categorization and classification results were interpreted descriptively.
3. Results and Discussion

3.1. Results

Teacher's answers to the attributes and definitions of quadrilateral including side, angle, side-angle, side-diagonal and others are shown in Table 1.

| Quadrilaterals | Side   | Angle  | Side-Angle | Side-Diagonal | Side-Angle-Diagonal | Others  |
|----------------|--------|--------|------------|---------------|---------------------|---------|
| Rectangular    | 6(23%) | 15(58%)| -          | -             | 5(19%)              |         |
| Parallelogram  | 7(27%) | 12(46%)| -          | 3(12%)        | 4(15%)              |         |
| Rhombus        | 4(15%) | 4(15%) | 9(35%)     | 1(4%)         | 7(27%)              |         |
| Square         | 7(27%) | 19(73%)| -          | -             | -                   |         |

Table 1 indicates that 58% of the attributes use the angular attribute which is two pairs of parallel sides and four right angles'. 23% use side attributes, 'two pairs of opposite sides with the same length', 'parallel and perpendicular'. 19% use other attributes, 'two pairs of edges with each of which are equal in length and parallel to one another which has four right angles'.

Table 1 also indicates that 46% of the attributes use side-angle attributes which is 27% use side attributes, 'four sides and each opposite side has the same length'. 15% use diagonal-angle attributes,' in which the opposite parallel sides have the same length and two pairs of angles which is equal to the angle in front of them, the number of adjacent angles 180° and the two diagonals intersect each other in the middle of the parallelogram surface. 15% use another attribute, 'two parallel edges have two same-size-angles'.

The teachers admitted on the definition of the rhombus that 'the four sides have the same length and the diagonals intersect each other perpendicularly'. Each of the 15% uses side attributes such as 'the four sides are the same length' and the angular attribute, 'has four sides of equal length and the corners are opposite equally, and the corners are not right angled'. 4% use angular attributes, 'square rotated 90°, and side-angle-diagonal,' have four sides of equal length and have two diagonals with four equal angles', 27% use other attributes,' formed by four edges which have the same length and has two pairs of angles with does not form right edge, each of which is equal to the angle in front of it'.

| Quadrilaterals | Categories | Definitions                                                                 |
|---------------|------------|-----------------------------------------------------------------------------|
| Rectangle     | Incorrect  (15%) | Rectangle is a two-dimentional figure, both sides facing the same length.   |
|               | Correct    (85%) | Rectangle is a flat build that has two pairs of sides that are equal and parallel; and has 4 angles that are 90° (right angles).   |
|               | Economical (0%) |                                                                           |
| Parallelogram | Incorrect  (19%) | Parallelogram : a quadrilateral whose sides are parallel, not perpendicular, angles facing are equal and not perpendicular. |
|               | Correct    (73%) | Parallelogram is a two-dimentional figure that has a pair of sides that are parallel and equal in length.   |
|               | Economical (8%) | Parallelogram is a quadrilateral whose sides are equal and parallel.       |
Table 2. Cont.

| Quadrilateral | Incorrect (%) | Correct (%) | Economical (%) |
|---------------|---------------|-------------|----------------|
| **Rhombus**   | 8%            | 88%         | 4%             |
| Incorrect     | Rhombus is a square that is rotated 90°. |
| Correct       | Rhombus is a two-dimensional figure that has four sides. |
| Economical    | Rhombus is a rectangle whose four sides are the same length. |
| **Square**    | 12%           | 84%         | 4%             |
| Incorrect     | Square is a rectangle with all sides equal in length. |
| Correct       | Square is a two-dimensional figure that has four equal sides and all four angles are right. |
| Economical    | Square is a rectangle whose four sides are the same length. |

Table 2 indicates that most of the teachers do not understand the definition of the quadrilateral which is eligible based on necessary and sufficient conditions (or called economic term). One of the influential factors is the teachers' teaching experience on this topic, so they struggle to define the definition of quadrilateral [13, 14].

Furthermore, the teachers were asked to create a diagram (mind maps) to connect all the quadrilaterals. The analysis result indicates that there are three types of mind maps made by the teachers. First, there is no relationship between the rectangle, parallelogram, square, and rhombus which is called type 1. Second, the relationship is classified into (i) parallelogram which includes rectangle, square, and (ii) subsequent rhombus is called type 2. Third, the relationship is classified into parallelogram which is divided into rectangle and rhombus, then rectangle and rhombus are classified as quadrilateral called type 3.

Based on Figure 1, 69% create mind maps that square as a special kind of rectangle and rhombus (type 3). It is a fact that teachers interpret based on the long side and the angle. There is a 19% make a mind map that square as part of the rectangle and rhombus as part of a parallelogram (type 2). This condition results from the fact that teachers interpret wake quadrilateral partially, based on attributes of the next based on attributes corner. 12% could not make a mind map quadrilateral properly (type 1). The condition results from the fact that teachers interpret the rectangle, parallelogram, rhombus, and square each one stands alone.

To investigate more, the teachers did an in-depth interview concerning the definition and classification of the quadrilateral. They were interviewed separately. The research subject S1 incorrectly
defined the quadrilateral's definition and classified it into type 1. The research subject S2, they correctly defined the definition of quadrilateral and classified it into type 2. While the research subject S3 defined the definition economically and grouped it into type 3.

3.1.1. Insufficient characteristics. S1 does not understand the necessary conditions and the conditions are sufficient for the employer. They define the level of hierarchy based only on the characteristics seen. Examples of responses in this category:

S1: Parallelogram is a quadrilateral whose two sides are parallel not perpendicular, the angle opposite is as large and not perpendicular.

Another thing that is stated is that S1 is not able to connect the type of quadrilateral, for example: rectangle is not a long distance. Examples of responses in this category:

S1: It's already available individually. Parallelogram has two pairs of parallel sides and the opposite angle is equal. While the rectangle has two equal and parallel long sides, the two short sides are equal and parallel, and all angles are 90.

3.1.2. More than the characteristics needed. S2 says that the square must have four sides of equal length and the four corners are right. Both of these characteristics are true for square but this statement is more than necessary to claim quadrilateral as a square.

S2: Square is a flat building that has four equal sides and all four corners are right.

Even though it is excessive, but according to the master's degree, the rectangular and the pervasive subject is a long distance.

S2: Rectangle, has two pairs of sides facing the same length and one angle right. Parallelogram also has two pairs of opposite sides. Rhombus, having four adjacent sides is the same length. So, it can be said that the rectangle and the border are the long lines.

3.1.3. Characteristics of necessary conditions and conditions are minimal. To define the scope, S3 use the necessary conditions and the requirements are minimal. Examples of response to this category:

S3: Split is a square whose four sides are the same length.

In addition, S3 can connect rectangles and stripes to be a long distance, along with the response.

S3: Because the properties of long distance are also possessed by chips and rectangles. One of the properties of the skull, having four adjacent sides is the same length. One of the rectangular properties, has two pairs of parallel facing sides and one right angle. So, rectangles and chips are included in the ranks.

3.2. Discussion

Most mathematics teachers define parallelogram as a quadrilateral with two pairs of opposite sides which are parallel to each other. They also define a rectangle as a 'rectangle having two pairs of opposite sides, equal length and all right angles'. Also, they define rhombus as 'square with all sides together', square as 'quadrilateral with all sides and same angles'. According to this definition, mathematics teachers emphasize parallels in the definition of a parallelogram, and they do not mention it in the definition of a rhombus, square and rectangular. These results are in line with [15-19].

According to the definition of the mathematics teachers, the square, rhombus and rectangle are included in the category of partition definitions; whereas parallelogram is included in the hierarchical definition. This suggests that what the mathematics teacher understands is to write down all the attributes of a shape [20].

Student text books adopt the definition of a parallelogram which is as a 'quadrilateral with two pairs of opposite and parallel sides with the same length'. The reason why most teachers adopt this because it also has the previous learning experience in elementary, high school and university [21]. Teachers are required to have extensive knowledge, able to define a quadrilateral concept with its language, not only adopt the definitions on the textbooks and teacher handbooks.

The interview result with S1 indicates that the subject does not understand the terms and conditions
of sufficient definition. In contrast, The research subject S2 are less thorough with the necessary conditions definition. In defining geometrical shapes in mathematics, teachers are required to know the sufficient terms of shape. The definition has been sufficiently satisfactory, yet it does not comply with the conditions of necessary terms where the attributes do not need to have right angles for its angles. Based on the terms, it may only have one right angle.

Most teachers build a hierarchical classification well. Teachers recognize square as a special shape of rhombus and rectangle. Teachers also recognize rhombus and rectangles as a special shape of the parallelogram. The teacher takes into account the similarity of quadrilateral properties, recognizing the square relationship as a special occurrence of rhombus and rectangles [13,21]. The interview result with S2 and S3 on this classification, the two subjects know the relationship between rectangular shapes. This implies that the teacher has sufficient knowledge in defining the concept of the quadrilateral with its own language and knowing the equivalent form of a definition.

Teachers who do not exactly build a hierarchical classification simply recognize the square as a special shape of a rectangle. Teachers classify rhombus as the special shape of a parallelogram. Some teachers do not classify the square into rhombus because it is still affected by the orientation of different forms of display. This is in line with the findings [13,21] that the bullet is affected by the orientation and visual appearance of the quadrilateral.

Teachers who construct a partition classification classify rectangle, parallelogram, rhombus, and square separately. The interview result with S1 indicates that the subject chooses geometry based only on one particular nature and ignores the other. This shows that S1 is still lacks of quadrilateral knowledge. The teachers possess only little knowledge to connect the quadrilateral attributes and their logical nature.

4. Conclusion
Based on the above discussion, the teacher uses a side-angle attribute to define all quadrilaterals, except for rhombus using a side-diagonal attribute. Teachers content knowledge in defining quadrilateral is categorized as high enough when it comes to giving the correct definition, while still low for the economical definition. Some mathematics teachers do not yet have enough knowledge in defining quadrilateral economically. Teachers struggle to exact state the nature or attributes; define the requirements; state sufficient conditions of definition; state the conditions are necessary or sufficient, and good definition but redundant. The teacher has sufficient knowledge of the quadrilateral relationship and defines the quadrilateral with the language itself. Teachers can understand the equivalent form of a definition and subset of the quadrilateral, but the teacher struggles to make the mind map of the quadrilateral. This difficulty is due to the limited knowledge of the teacher's orientation and the visual appearance of the quadrilateral and the lack of knowledge about the properties of the quadrilateral.

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