Teleparallel Energy-Momentum Distribution of Lewis-Papapetrou Spacetimes

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Abstract

In this paper, we find the energy-momentum distribution of stationary axisymmetric spacetimes in the context of teleparallel theory by using Möller prescription. The metric under consideration is the generalization of the Weyl metrics called the Lewis-Papapetrou metric. The class of stationary axisymmetric solutions of the Einstein field equations has been studied by Galtsov to include the gravitational effect of an external source. Such spacetimes are also astrophysically important as they describe the exterior of a body in equilibrium. The energy density turns out to be non-vanishing and well-defined and the momentum becomes constant except along θ-direction. It is interesting to mention that the results reduce to the already available results for the Weyl metrics when we take ω = 0.

Keywords: Teleparallel Theory, Energy-Momentum, Lewis-Papapetrou Spacetimes

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1 Introduction

Localization of energy-momentum is one of the oldest and thorny problems [1] in General Relativity (GR) which is still without any acceptable answer in general. Being a natural field, it is expected that gravity should have its own local energy-momentum density. However, it is usually asserted that such a density cannot be locally defined on the basis of the equivalence principle [1]. As a consequence, a massive number of attempts have been made in GR starting from Einstein who constructed his locally conserved energy-momentum complex [2]. Following the same lines, Landau-Lifshitz [3], Papapetrou [4], Bergman [5], Tolman [6] and Weinberg [7] constructed their own energy-momentum which are also locally conserved. The major drawback of these complexes is that one can get acceptable results only by using Cartesian coordinate system. Möller removed this drawback and proposed a new energy-momentum complex [8], which is not coordinate dependent.

In spite of some doubts [1], different authors [9-12] continue work to explore this problem further. In particular, the idea of quasi-local mass seems to be more clear [13]. According to this approach, a Hamiltonian boundary term is associated to each gravitational energy-momentum pseudo-tensor and consequently energy-momentum complexes are quasi-local. One can use any coordinate system in this formalism.

Many authors [14-20] defined energy of the gravitational field by introducing the Hamiltonian approach in the framework of Schwinger’s condition [21]. According to this formalism, energy is given by an integral of a scalar density in the form of total divergence which appears as the Hamiltonian constraint of this theory. Andrade et al. [15,22-24] constructed Lagrangian density of teleparallel equivalent of GR, which provides an alternative geometrical framework for the Einstein field equations (EFEs).

Virbhadra et al. [25-29] explored the energy-momentum distribution of several spacetimes, such as, Kerr-Newmann, Kerr-Schild classes, Einstein-Rosen, Vaidya and Bonnor-Vaidya spacetimes. They conclude that different energy-momentum prescriptions provide the same results which agree with those obtained by Penrose [30] and Tod [31] in the framework of quasi-local mass. On the other hand, it is found [28-29, 32-42] that Möller prescription yields different results from the other prescriptions by considering particular examples including Schwarzschild spacetime.

Teleparallel gravity is an alternate description of gravitation which corresponds to a gauge theory for the translation group [23]. The energy local-
The problem was reconsidered in the framework of this theory by many authors [43-49]. They showed that energy may be localized in this formalism and found results which are quite consistent with the available results in the framework of GR. Vargas [45] found that the total energy of the closed Friedmann-Robertson-Walker spacetime is zero by using teleparallel version of Einstein and Landau-Lifshitz complexes. This exactly coincides with that obtained by Rosen [50] in the framework of GR.

According to Lessner [51], Möller prescription is a powerful concept of energy and momentum in GR and also some authors [39-43, 52-56] concluded that it is a good tool for evaluating energy distribution in a given spacetime. The use of Möller prescription thus seems to be more interesting, useful and appropriate while finding the energy distribution. In this paper, we use teleparallel version of Möller prescription and find the components of energy-momentum densities of the Lewis-Papapetrou spacetimes.

The paper is organized as follows. Section 2 will provide some basic concepts of teleparallel theory of gravity and the Möller energy formalism in tetrad theory. Section 3 is devoted to evaluate the components of the energy-momentum densities of Lewis-Papapetrou spacetimes. The last section provides summary and discussion of the results obtained.

2 Teleparallel Gravity

The theory of teleparallel gravity (TPG) is described by the Weitzenböck connection given by [23]

\[ \Gamma^a_{\mu\nu} = h_a^\theta \partial_\nu h^a_{\mu} , \]  

(1)

where the non-trivial tetrad \( h^a_{\mu} \) and its inverse field \( h_a^{\nu} \) satisfy the relations

\[ h^a_{\mu} h_a^{\nu} = \delta^\nu_\mu; \quad h^a_{\mu} h^b_{\mu} = \delta^a_b . \]  

(2)

We shall use the Latin alphabet \( (a, b, c, ... = 0, 1, 2, 3) \) to denote the tangent space indices and the Greek alphabet \( (\mu, \nu, \rho, ... = 0, 1, 2, 3) \) to denote the spacetime indices. The Riemannian metric in TPG arises as a by product [23] of the tetrad field given by

\[ g_{\mu\nu} = \eta_{ab} h^a_{\mu} h^b_{\nu} , \]  

(3)
where $\eta_{ab}$ is the Minkowski spacetime such that $\eta_{ab} = \text{diag}(+1,-1,-1,-1)$. In TPG, the gravitation is attributed to torsion [57], which plays the role of force here. For the Weitzenböck spacetime, the torsion is defined as [23]

$$T^\theta_{\mu\nu} = \Gamma^\theta_{\nu\mu} - \Gamma^\theta_{\mu\nu}, \quad (4)$$

which is antisymmetric in nature. Due to the requirement of absolute parallelism, the curvature of the Weitzenböck connection vanishes identically. The Weitzenböck connection also satisfies the relation given by

$$\Gamma^0_{\mu\nu} = \Gamma^\theta_{\mu\nu} - K^\theta_{\mu\nu}, \quad (5)$$

where

$$K^\theta_{\mu\nu} = \frac{1}{2} [T^\theta_{\mu\nu} + T^\theta_{\nu\mu} - T^\theta_{\mu\nu}], \quad (6)$$

is the contortion tensor and $\Gamma^0_{\mu\nu}$ are the Christoffel symbols.

Mikhail et al. [43] defined the superpotential (which is antisymmetric in its last two indices) of the Möller tetrad theory as

$$U^\mu_{\nu\beta} = \sqrt{-g} P^{\nu\beta}_{\chi\rho\sigma} [\Phi^\rho g^{\alpha\chi}g_{\mu\sigma} - \lambda g_{\nu\mu}K^{\chi\rho\sigma} - (1 - 2\lambda)K^{\sigma\rho\chi}], \quad (7)$$

where

$$P^{\nu\beta}_{\chi\rho\sigma} = \delta^\nu_{\delta} \delta^\beta_{\tau} g^{\rho}_{\sigma\chi} + \delta^\delta_{\rho} \delta^\tau_{\chi} g^{\nu}_{\sigma\chi} - \delta^\chi_{\rho} \delta^\nu_{\tau} g^\nu_{\sigma\chi}, \quad (8)$$

and $g^{\nu\beta}_{\rho\sigma}$ is a tensor quantity defined by

$$g^{\nu\beta}_{\rho\sigma} = \delta^\nu_{\rho} \delta^\beta_{\sigma} - \delta^\nu_{\sigma} \delta^\beta_{\rho}. \quad (9)$$

$K^{\sigma\rho\chi}$ is contortion tensor given by Eq.(6), $g$ is the determinant of the metric tensor $g_{\mu\nu}$, $\lambda$ is free dimensionless coupling constant of teleparallel gravity, $\kappa$ is the Einstein constant and $\Phi_{\mu}$ is the basic vector field given by

$$\Phi_{\mu} = K^\nu_{\nu\mu}. \quad (10)$$

We define the energy momentum density as

$$\Xi^\nu_{\mu} = U^{\nu\rho}_{\mu\rho}, \quad (11)$$

where comma means ordinary differentiation. The momentum 4-vector of Möller prescription can be expressed as

$$P_{\mu} = \int_\Sigma \Xi^0_{\mu} dx dy dz, \quad (12)$$
where $P_0$ gives the energy and $P_1$, $P_2$ and $P_3$ are the momentum components while the integration is taken over the hyper-surface element $\Sigma$, which is described by $x^0 = t = \text{constant}$. The energy may be given in the form of surface integral [8] as

$$E = \lim_{r \to \infty} \int_{r=\text{constant}} U_0^0 u_\rho dS,$$

(13)

where $u_\rho$ is the unit three-vector normal to the surface element $dS$.

### 3 Energy-Momentum Densities of Lewis-Papapetrou Spacetimes

Due to the non-linearity of EFEs, frame dragging effect and the presence of black hole, the study of rotating black holes with external source has been a favorite research topic of many scientists. The gravitational effect of the external source can be studied appropriately in the framework of stationary axisymmetric spacetimes. Rotation is a universal phenomenon which seems to be shared by all objects, at all possible scales [58]. All rotating regular solutions have been found within the usual Lewis-Papapetrou ansatz [59] for a stationary axially symmetric spacetimes with two Killing vector fields $\partial_t$ and $\partial_\theta$. The class of stationary axisymmetric solutions of the EFEs has been studied by Galtsov [60] to include the gravitational effect of an external source. On the other hand, such spacetimes are also astrophysically important because they describe the exterior of a body in equilibrium [61]. The Lewis-Papapetrou metric is given by

$$ds^2 = e^{2\psi}(dt - \omega d\theta)^2 - e^{2(\gamma-\psi)}(d\rho^2 + dz^2) - \rho^2 e^{-2\psi} d\theta^2,$$

(14)

where $\omega$ is the angular velocity and $\gamma, \psi, \omega$ are arbitrary functions of $\rho$ and $z$ only. It is mentioned here that this metric reduces to static axially symmetric spacetime [62] for $\omega = 0$ and satisfies the following constraints

$$\ddot{\psi} + \frac{1}{\rho} \dot{\psi} + \psi'' = 0,$$

(15)

$$\dot{\gamma} = \rho(\dot{\psi}^2 - \psi'^2), \quad \gamma' = 2\rho \dot{\psi} \psi',$$

(16)

where dot and prime denote the derivatives w.r.t. $\rho$ and $z$ respectively. Following the same procedure as in [63, 24], one can find the tetrad components
for the Lewis-Papapetrou metric as follows

\[
h^a_{\mu} = \begin{bmatrix}
e^\psi & 0 & -\omega e^\psi & 0 \\
0 & e^{\gamma - \psi} \cos \theta & -\rho e^{-\psi} \sin \theta & 0 \\
0 & e^{\gamma - \psi} \sin \theta & -\rho e^{-\psi} \cos \theta & 0 \\
0 & 0 & 0 & e^{\gamma - \psi}
\end{bmatrix}
\]  \hspace{1cm} (17)

with its inverse

\[
h_\mu^a = \begin{bmatrix}
e^{-\psi} & 0 & 0 & 0 \\
-\omega \rho^{-1} e^\psi \sin \theta & e^{-\gamma + \psi} \cos \theta & -\rho^{-1} e^\psi \sin \theta & 0 \\
\omega \rho^{-1} e^\psi \cos \theta & e^{-\gamma + \psi} \sin \theta & -\rho^{-1} e^\psi \cos \theta & 0 \\
0 & 0 & 0 & e^{-\gamma + \psi}
\end{bmatrix}.
\]  \hspace{1cm} (18)

We see that Eqs.(2) and (3) can be easily verified with the help of Eqs.(17) and (18). Using Eqs.(17) and (18) in Eq.(1), we obtain the following non-vanishing components of the Weitzenböck connection

\[
\Gamma_{01}^0 = \dot{\psi}, \quad \Gamma_{03}^0 = \psi', \quad \Gamma_{12}^0 = \omega \rho^{-1} e^\gamma,
\]
\[
\Gamma_{21}^0 = \omega \rho^{-1} (\dot{\omega} + 2 \omega \dot{\psi}), \quad \Gamma_{23}^0 = -(\omega' + 2 \rho \psi'),
\]
\[
\Gamma_{11}^1 = \dot{\gamma} - \dot{\psi} = \Gamma_{31}^3, \quad \Gamma_{22}^1 = -\rho e^{-\gamma}, \quad \Gamma_{13}^1 = \gamma' - \psi' = \Gamma_{33}^3,
\]
\[
\Gamma_{21}^2 = \rho^{-1} e^\gamma, \quad \Gamma_{21}^3 = \rho^{-1} (1 - \rho \dot{\psi}), \quad \Gamma_{23}^2 = -\psi'.
\]  \hspace{1cm} (19)

The corresponding non-vanishing components of the torsion tensor are obtained by using Eq.(19) in Eq.(4). These are given by

\[
T_{01}^0 = -\dot{\psi} = -T_{10}^0, \quad T_{03}^0 = -\psi' = -T_{30}^0,
\]
\[
T_{12}^0 = \omega \rho^{-1} (1 - e^\gamma) - (\dot{\omega} + 2 \omega \dot{\psi}) = -T_{21}^0,
\]
\[
T_{23}^0 = \omega' + 2 \omega \psi' = -T_{32}^0, \quad T_{13}^1 = -\gamma' + \psi' = -T_{31}^1,
\]
\[
T_{12}^2 = \rho^{-1} (1 - e^\gamma) - \dot{\psi} = -T_{21}^2, \quad T_{23}^2 = \psi' = -T_{32}^2,
\]
\[
T_{31}^3 = -\gamma + \dot{\psi} = -T_{32}^3.
\]  \hspace{1cm} (20)

Substituting these values in Eq.(10), we obtain the following non-vanishing components of the basic vector field

\[
\phi_1 = \dot{\psi} - \dot{\gamma} - \rho^{-1} (1 - e^\gamma),
\]
\[
\phi_3 = \psi' - \gamma',
\]  \hspace{1cm} (21)
\[
\phi_3 = e^{2(\psi - \gamma)} (\dot{\gamma} + \rho^{-1} (1 - e^\gamma) - \dot{\psi}),
\]
\[
\phi_3 = e^{2(\psi - \gamma)} (\gamma' - \psi').
\]  \hspace{1cm} (23)

which can also be written as

\[
\phi_1 = e^{2(\psi - \gamma)} \{ \dot{\gamma} + \rho^{-1} (1 - e^\gamma) - \dot{\psi} \},
\]
\[
\phi_3 = e^{2(\psi - \gamma)} (\gamma' - \psi').
\]  \hspace{1cm} (24)
Using Eq.(20) in Eq.(6), it yields the following non-zero components of the contorsion tensor

\[ K^{010} = \frac{\dot{\psi}}{\rho^2} \left\{ \rho^2 e^{-2\gamma} - \omega^2 e^{2(2\psi - \gamma)} \right\}, \quad K^{030} = \frac{\psi'}{\rho^2} \left\{ \rho^2 e^{-2\gamma} - \omega^2 e^{2(2\psi - \gamma)} \right\}, \]

\[ K^{131} = e^{4(\psi - \gamma)} (\psi' - \gamma'), \quad K^{212} = \frac{1}{\rho^3} (\rho \dot{\psi} + e^\gamma - 1) e^{2(2\psi - \gamma)}, \]

\[ K^{232} = \frac{\psi'}{\rho^2} e^{2(2\psi - \gamma)}, \quad K^{313} = e^{4(\psi - \gamma)(\psi - \gamma')}, \]

\[ K^{120} = K^{021} = K^{102} = \frac{1}{2\rho^3} \{ \omega (1 - e^\gamma) - \rho (\omega + 2\omega \dot{\psi}) \} e^{2(2\psi - \gamma)}, \]

\[ K^{320} = K^{023} = K^{302} = -\frac{1}{2\rho^2} (\omega' + 2\omega \psi') e^{2(2\psi - \gamma)}. \] (25)

It is worth mentioning here that the contorsion tensor is antisymmetric w.r.t. its first two indices. Making use of Eqs.(23)-(25) in Eq.(7), we get the required independent non-vanishing components of the superpotential in Möller tetrad theory as

\[ U^{01}_0 = \frac{1}{\kappa} \left\{ 3\rho \dot{\psi} - 2\rho \gamma + 2e^\gamma - 2 + \frac{\omega e^{4\psi}}{2\rho^2} (\lambda - 1)(\rho \omega + 2\rho \omega \dot{\psi} - \omega + \omega e^\gamma) \right\}, \]

\[ U^{03}_0 = \frac{1}{\kappa} \left\{ 3\rho \psi' - 2\rho \gamma' + \frac{\omega e^{4\psi}}{2\rho} (\lambda - 1)(\omega' + 2\omega \psi') \right\}, \]

\[ U^{12}_0 = \frac{e^{4\psi}}{\rho^2 \kappa} [\omega (\rho \dot{\psi} - e^\gamma - 1) + \frac{1}{2} (1 + \lambda) \{ \omega (1 - e^\gamma) - \rho (\omega + 2\omega \dot{\psi}) \}], \]

\[ U^{23}_0 = \frac{e^{4\psi}}{\rho \kappa} [ -\omega \psi' + \frac{1}{2} (1 + \lambda)(\omega' + 2\omega \psi')], \]

\[ U^{10}_1 = \frac{1}{2\rho^2 \kappa} e^{2\gamma} (1 - \lambda) [\omega (e^\gamma - 1) + \rho (\omega + 2\omega \dot{\psi})], \]

\[ U^{01}_2 = \frac{1}{\kappa} [-\rho \omega \psi + \frac{1}{\rho} \omega^3 \dot{\psi} e^{4\psi} + \frac{1}{2} (\lambda - 1) \{ \omega (e^\gamma - 1) + \rho (\omega + 2\omega \dot{\psi}) \}], \]

\[ U^{03}_2 = \frac{1}{\kappa} [-\rho \omega \psi' + \frac{1}{\rho} \omega^3 \psi' e^{4\psi} + \frac{2}{5} (\lambda - 1)(\omega' + 2\omega \psi')], \]

\[ U^{30}_2 = \frac{1}{2\rho \kappa} e^{2\gamma} (1 - \lambda)(\omega' + 2\omega \psi'). \] (26)
Substituting these values in Eq.(11), it follows that the energy-momentum density components

\[ \Xi^0_0 = \frac{1}{\kappa} \left[ 3 \rho (\ddot{\psi} + \psi''') - 2 \rho (\dot{\gamma} + \gamma''') + 3 \dot{\psi} + 2 \dot{\gamma} (e^\gamma - 1) + \frac{1}{2 \rho^2} e^{4\psi} (\lambda - 1) \right] \]

\[ \{ \rho (\ddot{\omega} + \omega^'') + 8 \rho \omega (\dot{\omega} \dot{\psi} + \omega' \psi') + 8 \rho \omega^2 (\dot{\psi}^2 + \psi'^2) + \rho \omega (\dot{\omega} + \omega''') \}

\[ + 2 \rho \omega^2 (\ddot{\psi} + \psi''') - 2 \omega^2 \dot{\psi} + 2 \rho \omega^2 \dot{\psi} - 3 \omega \dot{\omega} + \frac{2 \omega^2}{\rho} + \omega e^\gamma (\omega \dot{\gamma} + 2 \dot{\omega}) + 4 \omega \dot{\psi} - \frac{2 \omega}{\rho} \}, \]

\[ \Xi^0_2 = \frac{1}{\kappa} \left[ - \omega \dot{\psi} - \rho (\dot{\omega} \dot{\psi} + \omega' \psi') - \rho \omega (\ddot{\psi} + \psi''') + \frac{\omega^2}{\rho} e^{4\psi} \{ 3 (\dot{\omega} \dot{\psi} + \omega' \psi') \}

\[ + \omega (\ddot{\psi} + \psi''') + 4 \omega (\dot{\psi}^2 + \psi'^2) \frac{\omega}{\rho} \dot{\psi} \} + \frac{1}{2} (1 + \lambda) \{ e^\gamma (\omega + \omega \dot{\gamma}) \}

\[ + 2 \rho (\ddot{\omega} \dot{\psi} + \omega' \psi') + 2 \rho \omega (\ddot{\psi} + \psi''') + \rho (\dot{\omega} + \omega''') + 2 \omega \dot{\psi} \}. \]  

(27)

We see that the energy-momentum turns out to be non-vanishing and well-defined quantities. The component of the momentum density is non-zero only along the \( \theta \)-direction which is due to the cross term \( dtd\theta \) involving in the given metric. If we take \( \omega = 0 \) and use Eqs.(15) and (16), the above results take the following form

\[ \Xi^0_0 = \frac{2}{\kappa} (2 \rho \dot{\psi}^2 + \dot{\gamma} e^\gamma), \]

\[ \Xi^0_i = 0, \quad (i = 1, 2, 3). \]  

(28)

It is worth mentioning that these results are exactly the same as found by us in the case of static axisymmetric spacetimes [64].

### 4 Summary and Discussion

The debate of localization of energy-momentum has been an open issue since the time of Einstein when he formulated the well-known relation between mass and energy. Misner et al. [2] concluded that energy can only be localized in spherical coordinates. But, soon after, Cooperstock and Sarracino [65] demonstrated that if the energy is localizable in spherical systems then it can be localized in any system. Bondi [66] rejected the idea of non-localization
of energy in GR due to the reason that there should be any form of energy which contributes to gravitation and hence its location can, in principle, be found. Many authors believed that a tetrad theory should describe more than a pure gravitation field [67]. In fact, Möller [68] considered this possibility in his earlier attempt to modify GR. Recently, Salti et. al. [46-48, 69-70] explored many examples in the context of both GR and TPT and found consistent results.

In this work, we have evaluated the energy-momentum density components of stationary axisymmetric Lewis-Papapetrou spacetimes. For this purpose, we use the teleparallel version of Möller prescription. It is found that the energy density of the Lewis-Papapetrou spacetime turns out to be non-zero and well-defined and the momentum becomes constant except along $\theta$ direction. This is due to the fact that the Lewis-Papapetrou metric contains a cross term $dtd\theta$. Further, we note that for $\omega = 0$, these results exactly match with those of the Weyl metrics in the context of teleparallel theory of gravity [64]. It is mentioned here that our results of TPT do not coincide with those of GR.

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