The Problem of $\alpha_s$ in Supersymmetric Unified Theories

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Abstract

It is shown that in $SO(10)$ there is a general connection between the suppression of higgsino mediated proton decay and the value of $\alpha_s$. However, agreement with the experimental value of $\alpha_s$ can be obtained if there are relatively large negative threshold corrections to $\alpha_s$ coming from superheavy split multiplets. It is shown that such split multiplets can arise in $SO(10)$ without fine-tuning of parameters.

1 Introduction

One of the strongest pieces of evidence for grand unification is the fact that the gauge couplings of the minimal supersymmetric standard model (MSSM) come very nearly together when extrapolated to high energy using the renormalization group equations [1]. If one were to use the best experimental values for $\alpha_s$ and $\alpha$, the experimental value of $\sin^2\theta_W$ and the value predicted by unification and the would be discrepant by less than a percent. However,

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it now customary to use the measured values of $\alpha$ and $\sin^2 \theta_W$, since they are the better known quantities, to “predict” the value of $\alpha_s$. When expressed in terms of $\alpha_s$, the agreement with experiment appears somewhat less dramatic. Indeed, as is well known, there is a bit of a problem.

The prediction of (minimal) supersymmetric grand unified theories (SUSY GUTs) is that $\alpha_s = 0.127 \pm 0.005 \pm 0.002$, according to the analysis of [2], where the first error is the uncertainty in the low-energy sparticle spectrum, and the second is the uncertainty in the top quark and Higgs boson masses. In [3] a more exact treatment of the low-energy thresholds gave slightly larger values of $\alpha_s$. In particular, it was found that $\alpha_s(M_Z) > 0.126$ for no unification thresholds, $m_\tilde{q} \leq 1$ TeV, and $m_t = 170$ GeV. On the other hand, a global fit of low-energy data gives the result: $\alpha_s = 0.117 \pm 0.005$ [4]. One way these numbers can be brought into better agreement is by threshold corrections at the GUT scale. Let us define the threshold correction in $\alpha_s$ at the GUT scale to be $\epsilon_3 \equiv [\alpha_3(M_G) - \tilde{\alpha}_G]/\tilde{\alpha}_G$, where $M_G$ is the scale at which the gauge couplings $\alpha_1$ and $\alpha_2$ meet [5]. Then, to bring the SUSY GUT prediction of the strong coupling into agreement with experiment by high-energy threshold corrections requires the existence of superheavy fields beyond those of minimal $SU(5)$ that contribute $\delta \epsilon_3 \sim -0.02$ to $-0.03$ [5].

Actually, the problem is worse than this because of the need to suppress the Higgsino-mediated proton decay amplitudes [6]. The mechanisms which do this naturally tend to give a positive contribution to $\epsilon_3$, as was noted in [7] and [5]. Typically, for $\tan \beta \sim 1$ this contribution is about $+0.02$, and for $\tan \beta \sim 60$ it is about $+0.05$. Thus, for large $\tan \beta$, the other threshold corrections would have to be about $-0.08$ to bring $\alpha_s$ into agreement.

There are two problems with achieving such large negative threshold corrections. First, the multiplets which are naturally highly split increase rather than decrease $\alpha_s$. Second, it is difficult to get the threshold corrections at the GUT scale to be as large as needed without either having a fine-tuning of the splittings in multiplets, or having large numbers of multiplets that happen all to contribute in the same direction.

Other solutions to the problem have been suggested besides threshold effects at the GUT scale [8]. These include corrections from sparticle thresholds [9], and intermediate-scale gauge symmetry breaking [10]. In this paper, we shall look at the problem in the context of $SO(10)$ with no intermediate gauge symmetry breaking, and we shall assume that high energy threshold effects account for the discrepancy. In section 2, we shall review the problem
in more detail. In particular we shall look at the simplest mechanism for suppressing Higgsino-mediated proton decay and show why it exacerbates the problem. We shall also show why it is hard to get threshold effects of the needed sign and magnitude without large numbers of multiplets or fine-tuning.

In section 3, we shall prove a theorem that shows that there is a close connection between the $\alpha_s$ problem and the proton-decay problem in a wide class of Higgs sectors. This theorem will allow us to see what is necessary to solve the problem without fine-tuning. In particular, we shall see why it is easy to solve the problem in the context of $SO(10)$ models with two or more adjoint Higgs fields.

In section 4, we shall examine the question of whether the needed threshold corrections can be obtained in a natural way with only a single adjoint Higgs. Two ways of doing this will be proposed, one where the VEV of the adjoint is proportional to $B - L$, and one where it is proportional to the third component of $SU(2)_R$. These are the two possibilities which allow a natural doublet-triplet splitting in $SO(10)$.

## 2 The magnitude of the problem

In $SO(10)$ the only natural way to solve the doublet-triplet splitting problem appears to be the Dimopoulos-Wilczek mechanism [11],[7], also known as the missing-VEV mechanism. The heart of the mechanism is an adjoint Higgs multiplet (a 45), which gets a vacuum expectation value (VEV) proportional to the generator $B - L$. That is, calling the adjoint $A$, one has in an obvious notation,

$$
\langle A \rangle = \begin{pmatrix} 0 & 0 & a \\ a & a & 0 \\ 0 & 0 & a \end{pmatrix} \times i\tau_2,
$$

where $a \simeq M_G$. Suppose that this adjoint couples to a pair of 10’s, denoted $T_1$ and $T_2$, as follows,

$$W_{2/3} = g T_1 A T_2 + M(T_2)^2.$$
The $T_i$ contain $(1, 2, \frac{1}{2}) + H.c.$ representations, which we denote $d_i + \overline{d}_i$, and $(3, 1, -\frac{1}{3}) + H.c.$ representations, which we denote $t_i + \overline{t}_i$. These have the following mass matrices due to the terms in Eq. (2):

$$W_2 = (\overline{d}_1, d_2) \begin{pmatrix} 0 & 0 \\ 0 & M \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix},$$

and

$$W_3 = (\overline{t}_1, t_2) \begin{pmatrix} 0 & a \\ -a & M \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}.$$  

The light doublets, which are the Higgs of the MSSM, are then just the $d_1 + \overline{d}_1$. (Eq. (3) shows these to be massless, but they will get weak-scale mass from other effects that are not included in the terms of Eq. (2). However, we will ignore weak-scale effects.) Proton decay can happen through dimension-five operators coming from the exchange of their color-triplet partners, $\overline{t}_1 + t_1$. The relevant propagator, at low momenta, is just given by $(M^{-1})_{11}$, where $M_3$ is the two-by-two mass matrix in Eq. (4). This is just given by $(M^{-1})_{11} = (M/a^2)$. Since, in the minimal SUSY $SU(5)$ model, the analogous color-triplet propagator is just given by $1/a \simeq 1/M_G$, the suppression factor of the proton-decay amplitude relative to the minimal $SU(5)$ model, which we shall call $F^{-1}$, is $F^{-1} = M_G(M^{-1})_{11} = M/a$. For $\tan \beta \sim 1$ typically $F^{-1}$ must be about 1/10, while for $\tan \beta \sim 60$ it needs to be of order 1/600 [6].

In [5] it is shown that the effect of superheavy matter multiplets on $\epsilon_3$ is given by

$$\epsilon_3 = \frac{\alpha_G}{2\pi} \sum_\gamma \left[(b_2^\gamma - b_2^\gamma) - \frac{1}{2}(b_2^\gamma - b_1^\gamma)\right] \ln(\det'(M_\gamma/M_G)),$$

where the index $\gamma$ stands for a specific type of $SU(3) \times SU(2) \times U(1)$ multiplet, and $M_\gamma$ is the mass matrix of the multiplets of that type. $\det'M_\gamma$ is the determinant of $M_\gamma$, unless $\gamma = (1, 2, \frac{1}{2}) + H.c.$, in which case it stands for that determinant with the weak-scale eigenvalue removed. $b_\gamma^\gamma$ is the index of the representation $\gamma$ under $SU(N)$. (That is, $b_\gamma^\gamma \equiv \text{Tr}_\gamma(\lambda_N)^2$, where $\lambda_N$ is a generator of $SU(N)$ normalized so that $b_N = 1/2$ for the fundamental representation.) Let us see what the effect of the vector multiplets $T_1$ and $T_2$ are. From Eq. (4), the determinant of the color-triplet multiplets $t_i + \overline{t}_i$ is just $a^2$; while, from Eq. (3), the determinant (i.e. $\det'$) of the superheavy
weak doublets is just $M$. Thus, Eq. (5) gives for the contribution to $\epsilon_3$ from the doublet-triplet splitting sector

$$
\delta\epsilon_3|_{2/3} = \frac{3\tilde{\alpha}_G}{5\pi} \ln(a^2/MM_G) = \frac{3\tilde{\alpha}_G}{5\pi} \ln F. \tag{6}
$$

Notice that the greater the suppression of proton decay — that is, the larger $F$ — the larger is $\alpha_s$. Every increase in $F$ by a factor of 10 increases $\epsilon_3$ by about 0.02. In minimal SUSY $SU(5)$, one needs $\epsilon_3$ to be about $-0.02$. Thus, if $F = 600$, the matter other than the $T_i$ contribute about $-0.08$ to $\epsilon_3$.

We shall show in the next section that this relation between $\epsilon_3$ and $F$ is fairly general.

The question arises whether there are small sets of multiplets that would give a negative contribution to $\epsilon_3$ as large as is needed. The obvious choice would be additional 10’s of $SO(10)$. For 10’s, as already noted, $\delta\epsilon_3|_{10} = (3\tilde{\alpha}_G/5\pi) \ln(\det M_3/\det M_2)$, where $M_3$ and $M_2$ are the mass matrices of the triplets and doublets in the 10’s respectively. Let the mass of the additional 10’s come from both explicit mass terms and from the VEV of the adjoint $A$. If, for example, there are in addition to $T_1$ and $T_2$, a pair $T$ and $T'$, with mass terms $mT^2 + m'T'^2 + TAT'$, then the determinant of the mass matrix of the doublets in $T$ and $T'$ is just $mm'$, while that of the triplets is $mm' - a^2$. And therefore the logarithm in $\delta\epsilon_3|_{T,T'}$ is $\ln(\det M_3/\det M_2) = \ln |1 - (a^2/mm')|$. If $a^2 \gg mm'$, the logarithm is large, but gives a positive contribution to $\epsilon_3$. On the other hand, if $mm' \gg a^2$, the logarithm is negative but small. The largest negative logarithm one would expect to get, without a fine-tuned cancellation between $mm'$ and $a^2$, is of order one. Therefore, there would have to be of order ten pairs of 10’s, all happening to contribute negatively, to get the needed $\delta\epsilon_3 \sim -0.08$. This itself is equivalent to a fine-tuning.

One can look at other representations besides vectors, but the situation is similar. There has to be either fine-tuning or a large number of multiplets which all happen to contribute negatively to $\epsilon_3$. For example, if there is a spinor pair that gets mass from a term $\overline{16}(45)_{16}$ with $\langle 45 \rangle \propto B - L$, the contribution to $\epsilon_3$ is only $-(3\tilde{\alpha}_G/10\pi) \ln 3 \cong -4 \times 10^{-3}$. 

5
3 Relation of $\alpha_s$ to p-decay in general

The relationship between $\epsilon_3$ and the suppression of Higgsino-mediated proton decay that exists in the simplest situation just analyzed is actually quite general. We shall first illustrate this with another example, and then prove a general statement.

Consider a model where the doublet-triplet splitting comes from the following superpotential terms: $W_{2/3} = m_{10_1}10_2 + m'_{10_2}10_3 + 10_310_3(A)^2/M$. Then the mass matrices of the doublets and triplets in the $10_i$ are given by

$$M_2 = \begin{pmatrix} 0 & m & 0 \\ m & 0 & m' \\ 0 & m' & 0 \end{pmatrix}, \quad M_3 = \begin{pmatrix} 0 & m & 0 \\ m & 0 & m' \\ 0 & m' & a^2/M \end{pmatrix}. \quad (7)$$

Consequently, there is a pair of “massless” doublets, which are the MSSM Higgs, and two supermassive pairs of doublets with determinant $\text{det} M_2 = m^2 + m'^2$. All three pairs of color-triplets are supermassive, with determinant $\text{det} M_3 = m^2a^2/M$. Thus, the contribution to $\epsilon_3$ is $\delta \epsilon_3 = (3\tilde{\alpha}_G/5\pi) \ln \left( \frac{m^2 + m'^2}{m^2a^2/M} \right)$. To compute $F^{-1}$, assume that at the GUT scale only the $5_1$ and $\overline{5}_1$ couple to the quark and lepton multiplets, with Yukawa couplings $Y_1$ and $\overline{Y}_1$. (Here and throughout it will often be convenient to classify fields using $SU(5)$. We are not assuming, however, that $SO(10)$ actually breaks to $SU(5)$.)

From the form of the doublet mass matrix in Eq. (7), it is apparent that the light MSSM doublets are in the linear combinations $(m'5_1 - m\overline{5}_3)/\sqrt{m^2 + m'^2}$ and $(m'\overline{5}_1 - m5_3)/\sqrt{m^2 + m'^2}$. It is the VEVs of these combinations that are the $v_2$ and $v_1$ of the MSSM and give mass to the light quarks and leptons. Hence, the Yukawa couplings $Y_1$ and $\overline{Y}_1$, which control proton decay, are proportional to $(m'/\sqrt{m^2 + m'^2})^{-1}$ times the light fermion mass matrices $M_U$ and $M_D$. Furthermore, the propagator of the superheavy color-triplet Higgsinos is $(M^{-1}_3)_{11} = \frac{m'^2}{m^2a^2/M}$. Consequently

$$F^{-1} = M_G \left( \frac{m^2 + m'^2}{m^2} \right) \left( \frac{m'^2}{m^2a^2/M} \right) = \left( \frac{(m^2 + m'^2)MMG}{m^2a^2} \right).$$

Comparing with the expression for $\epsilon_3$, one sees that the doublet-triplet-splitting sector contributes here, as it does for the simpler model, $\epsilon_3 = (3\tilde{\alpha}_G/5\pi) \ln F$.

This example shows how to generalize the result. Suppose that the doublets and triplets, including the light doublets and the triplets that contribute to proton decay, are contained in a set of $5 + 5$ Higgs fields labelled by the index $k$. Let the weak doublets in these multiplets be denoted by $H_k$ and
\( \overline{H}_k \). Then the masses of the light fermions are given by \( M_U = \sum_k Y_k \langle H_k \rangle \) and \( M_D = M_L = \sum_k \overline{Y}_k \langle \overline{H}_k \rangle \), where \( Y_k \) and \( \overline{Y}_k \) are Yukawa coupling matrices. We assume for simplicity that the color-triplet Higgsinos have the same Yukawa couplings as the doublet Higgs fields, as is the case in minimal SUSY SU(5).

The mass matrices of the doublets and triplets in \( \overline{5}_k + 5_k \) will be denoted \( M_2 \) and \( M_3 \). Let \( M_2 \) be diagonalized by unitary matrices \( \overline{U} \) and \( U \): \( \overline{U}^* M_2 U = \Lambda = \) diagonal. Unification of gauge couplings requires that there be only one light pair of doublets, \( \overline{H}_0 \) and \( H_0 \). The light eigenvalue of \( \Lambda \) is therefore \( \Lambda_0 \), and the masses of the light fermions are given by \( M_U = \sum_k Y_k U_{k0} \langle H_0 \rangle = (\sum_k Y_k U_{k0}) v_2 \), and \( M_D = \sum_k \overline{Y}_k U_{k0} \langle \overline{H}_0 \rangle = (\sum_k \overline{Y}_k U_{k0}) v_1 \).

Consider the inverse of the matrix \( M_2 = \overline{U} \Lambda U^\dagger \). The \( kk' \) element of that inverse is given by \( (M_2^{-1})_{kk'} = (U \Lambda^{-1} U^\dagger)_{kk'} \approx U_{k0} U_{k'0} \Lambda_0^{-1} \). The fact has been used that \( \Lambda_0^{-1} \) is much greater than all the other \( \Lambda_j^{-1} \) and hence dominates the sum. On the other hand, \( (M_2^{-1})_{kk'} = \det(m_2^{kk'})/\det M_2 \), where \( m_2^{kk'} \) is the cofactor of the \( kk' \) element of \( M_2 \). Similarly, \( (M_3^{-1})_{kk'} = \det(m_3^{kk'})/\det M_3 \), where \( m_3^{kk'} \) is the cofactor of the \( kk' \) element of \( M_3 \). Thus,

\[
(M_3^{-1})_{kk'} = (M_2^{-1})_{kk'} \left( \frac{\det M_2}{\det M_3} \right) \left( \frac{\det m_3^{kk'}}{\det m_2^{kk'}} \right).
\]  

(8)

Moreover, the Higgsino-mediated proton decay amplitude is proportional to the expression \( \sum_{kk'} Y_k \overline{Y}_{k'} (M_3^{-1})_{kk'} \). Using the fact that \( \det M_2 = \Lambda_0 \det' M_2 \), as well as the result just obtained that \( (M_2^{-1})_{kk'} \approx U_{k0} U_{k'0} \Lambda_0^{-1} \), and Eq. (8), this expression can be written

\[
\sum_{kk'} Y_k \overline{Y}_{k'} (M_3^{-1})_{kk'} = \sum_{kk'} Y_k \overline{Y}_{k'} U_{k0} U_{k'0} \left( \frac{\det' M_2}{\det M_3} \right) \left( \frac{\det m_3^{kk'}}{\det m_2^{kk'}} \right).
\]  

(9)

Assume that \( \left( \frac{\det m_3^{kk'}}{\det m_2^{kk'}} \right) \equiv r_{kk'} \) is independent of \( k \) and \( k' \), and call it simply \( r \). Then the sum over \( k \) and \( k' \) reduces to \( \sum_{kk'} Y_k \overline{Y}_{k'} U_{k0} U_{k'0} = (M_U/v)(M_D/v') \), which, of course, is just the same combination that appears in the proton decay amplitude in the minimal SUSY SU(5) model. Therefore, the suppression factor \( F^{-1} \) is given by

\[
F^{-1} = r M_G \left( \frac{\det' M_2}{\det M_3} \right).
\]  

(10)
This means that the contribution to $\epsilon_3$ of the doublets and triplets in the fields $\Phi_k + 5_k$ is just

$$
\delta \epsilon_3|_{2/3} = \frac{3\alpha_G}{5\pi} \ln(rF). \tag{11}
$$

Since $F$ must generally be large to sufficiently suppress proton decay, it must be that $r (\equiv \text{det} m^{kk'}_3 / \text{det} m^{kk'}_2)$ is quite small. How can this happen? Some contributions to the mass matrices will be $SU(5)$ invariant, and will therefore contribute in the same way to the matrices $m^{kk'}_3$ and $m^{kk'}_2$. If these are the only contributions, then $r = 1$. On the other hand, a VEV which is non-singlet under the $SU(5)$ subgroup of $SO(10)$, such as the VEV of an adjoint, will generally contribute differently to $m^{kk'}_3$ and $m^{kk'}_2$. Calling this VEV $\langle A \rangle$, one can therefore write $m^{kk'}_3 = m^{kk'} + \Delta^{kk'}_3(\langle A \rangle)$ and $m^{kk'}_2 = m^{kk'} + \Delta^{kk'}_2(\langle A \rangle)$.

If $r_{kk'}$ is to be much less than one, as needed, one of two things obviously has to be the case. (1) There must be a fortuitous relationship between the magnitude of the elements of $m^{kk'}$ and those of $\Delta^{kk'}_3(\langle A \rangle)$ so that the determinant of the sum of those two matrices nearly vanishes. This requires a fine-tuning. Or, (2) there must be enough entries in $m^{kk'}_3$ which get vanishing contributions from both $m^{kk'}$ and $\Delta^{kk'}_3(\langle A \rangle)$ that $m^{kk'}_3$ has vanishing determinant without fine-tuning. However, in this case, $m^{kk'}_2$ will also have vanishing determinant unless certain elements of $\Delta^{kk'}_2(\langle A \rangle)$ are non-zero even though the corresponding elements of $\Delta^{kk'}_3(\langle A \rangle)$ vanish.

The foregoing argument implies that for proton decay to be greatly suppressed without fine-tuning and without giving a contribution to $\epsilon_3$ that contains a large positive logarithm, there must be a superheavy VEV which can contribute to doublet masses while not contributing to the mass of their color-triplet partners. This VEV is presumably that of an adjoint.

It is easily shown that this cannot happen if the only superlarge VEV which breaks the $SU(5)$ subgroup of $SO(10)$ is an adjoint pointing in the $B − L$ direction. However, it can happen if there are two (or more) adjoints, with one of them having a VEV proportional to $B − L$ and the other a VEV proportional to $I_{3R}$ (that is, the third generator of the $SU(2)_R$ subgroup of $SO(10)$). In fact, such a situation was proposed in [7], where the following type of structure was suggested as a possibility: $W_{2/3} = gT_1AT_2 + T_2B^2T_2/M$. (Compare with Eq. (2).) Here $A$ and $B$ are adjoints and $\langle A \rangle \propto B − L$ and $\langle B \rangle \propto I_{3R}$. Then the doublet and triplet mass matrices are of the form
\[ M_2 = \begin{pmatrix} 0 & 0 \\ 0 & b^2/M \end{pmatrix}, \quad M_3 = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}. \]  

(12)

In this case, if only \( T_1 \) couples to the quarks and leptons, \( F^{-1} = M_G(M^{-1}_3)_{11} = 0 \), while the logarithm in \( \delta \epsilon_3 \) is \( \ln(a^2M/b^2M_G) \), which can take any value, and is naturally of order one. The reason this structure achieves the desired result is that the cofactor \( m^{11}_3 = (M_3)_{22} = 0 \), while the cofactor \( m^{11}_2 = (M_2)_{22} = b^2/M \). That is, as our previous reasoning indicated was needed, the VEV of \( B \) contributes to the 22 entry of the doublet matrix without contributing to the 22 element of the triplet matrix.

This shows that one can suppress proton decay without making the problem of \( \alpha_s \) worse than it is in minimal SUSY \( SU(5) \). Moreover, it is also possible to get the contribution of \( \delta \epsilon_3 \approx -0.02 \) which is needed even in minimal SUSY \( SU(5) \) by assuming the ratio \( a^2M/b^2M_G \) to be about \( 10^{-1} \) to \( 3 \times 10^{-2} \). This is the idea suggested in [12].

4 Solution to the problem of \( \alpha_s \) with one adjoint

If the \( SU(5) \) subgroup of \( SO(10) \) is broken at unification scale only by a single adjoint, then it is not as straightforward to solve the problem of \( \alpha_s \) with split multiplets. Nevertheless, there are solutions, both in the case that the sole adjoint points in the \( B-L \) direction, and in the case that it points in the \( I_{3R} \) direction.

(I) \( \langle A \rangle \propto B-L \):

An adjoint VEV that points in the \( B-L \) direction can give mass to triplets that are in 10’s of \( SO(10) \), without giving mass to doublets. This the idea behind the Dimopoulos-Wilczek (or missing VEV) mechanism. But there are no representations in which it does the opposite, giving mass to doublets but not triplets. This means that the suppression of proton decay in this case will necessarily lead to a significant positive contribution to \( \epsilon_3 \). However, it is possible with \( \langle A \rangle \propto B-L \) for there to be other kinds of split multiplets which compensate by giving large negative contributions to \( \epsilon_3 \).

The simplest possibility is an extension of the mechanism proposed in [13]. In that paper the following structure was proposed for coupling pairs
of spinors \((16 + \overline{16})\) to an adjoint:

\[
W_{CA} = \overline{C} (PA/M_1 + Z_1) C' + \overline{C'} (PA/M_2 + Z_2) C.
\]  

(13)

Here \(C + \overline{C}\) are a conjugate pair of spinors which get superlarge VEVs in the \(SU(5)\)-singlet direction, and \(C' + \overline{C'}\) are a conjugate pair of spinors that get no superlarge VEVs. The fields \(P, Z_1\) and \(Z_2\) are gauge singlets which have superlarge expectation values. It is assumed that the quantum numbers of the \(Z_i\) are such that they have no other couplings. (This is the purpose of the presence of the field \(P\). Were it not there, \(Z_i\) and \(A\) would have the same non-\(SO(10)\) quantum numbers, and other couplings of the \(Z_i\)'s would be possible.) The significance of this structure, as explained in [13], is that it allows the coupling of the adjoint \(A\) to the spinors \(C + \overline{C}\), which is required to avoid disastrous colored pseudo-goldstone bosons, without destabilizing the VEV of the adjoint. In particular, these terms do not destabilize the required form of \(\langle A \rangle\) by contributing to \(F_A\), since their contribution to \(F_A\) vanishes due to the vanishing of the VEVs of \(C'\) and \(\overline{C'}\). The \(Z_i\) are effectively "sliding singlets", which adjust to make \(F_{C'}\) and \(F_{\overline{C'}}\) vanish, and thus avoid breaking supersymmetry at the GUT scale.

Consider the equation \(F_{\overline{C'}} = 0\). This implies that \((PA/M_2 + Z_2)C = 0\). Since \(\langle A \rangle = \frac{3}{2}(B - L)\) (see Eq. (1)), it follows that \(\langle Z_2 \rangle = -\frac{3}{2}a(B - L)C)P/M_2\). (Here we mean by \(B - L\) the \(SO(10)\) generator that acts on the quarks and leptons in a spinor like \(B - L\).) The component of \(C\) that gets a VEV has the same \(SO(10)\) quantum numbers as a left-handed antineutrino, and hence \((B - L)_{(C)} = -1\). Thus \((PA/M_2 + Z_2) = \frac{3}{2}a(B - L + 1)P/M_2\).

What is interesting about this for our present purposes is that \((B - L + 1)\) vanishes not only for left-handed antineutrinos but also for left-handed "positrons", that is, for the components of a spinor which are in \((1, 1, +1)\)'s of \(SU(3) \times SU(2) \times U(1)\). As we shall see, this can lead to massless (or at least very light) fields which have those quantum numbers. The virtue of this is that such a field makes a sizable negative contribution to \(\epsilon_3\).

For example, suppose the structure in Eq. (13) is extended in the following way:

\[
W_{CA} = \sum_{i=1}^{2} \overline{C}_i (PA/M_i + Z_i) C'_i + \sum_{i=1}^{2} C'_i (PA/M'_i + Z'_i) C_i,
\]  

(14)

where dimensionless couplings have been suppressed. Here there are four
spinor-antispinor pairs, instead of two. It is easily checked that in addition to the goldstone modes that are eaten by the Higgs mechanism, there is one pair of goldstones with the \( SU(3) \times SU(2) \times U(1) \) quantum numbers \((1, 1, \pm 1)\). These goldstones can get mass from higher-dimensional operators. The mass they get is therefore highly model dependent. Let us call it \( m_1 \).

The contribution of these pseudo-goldstones to \( \epsilon_3 \) is at one loop \( \delta \epsilon_3 = -\frac{\alpha}{16\pi^2} \ln(M_G/m_1) \). If \( m_1 \sim 10^{11} \text{ GeV} \), then this contributes \( \delta \epsilon_3 \cong -0.04 \), while if \( m_1 \sim 10^8 \text{ GeV} \), \( \delta \epsilon_3 \cong -0.08 \).

It is possible that these pseudo-goldstone particles can even be as light as the weak scale. This would be an interesting signal of unification physics. Even if these pseudos are at some intermediate scale they may play a cosmological role.

(II) \( \langle A \rangle \propto I_{3R} \)

Suppose that there is only a single adjoint and its VEV points in the \( I_{3R} \) direction. That is,

\[
\langle A \rangle = \begin{pmatrix} a & a \\ 0 & 0 \end{pmatrix} \times i\tau_2.
\]  

(15)

In this case the Dimopoulos-Wilczek or missing-VEV mechanism cannot work as in Eq. (2), for an adjoint with a VEV proportional to \( I_{3R} \) will give mass to the weak doublets in a \( 10 \) of \( SO(10) \) while leaving the triplets massless. This is the just the reverse of what is needed to solve the doublet-triplet splitting problem. However, in [14] it was pointed out that an adjoint VEV proportional to \( I_{3R} \) can leave massless the weak doublets that are in spinors of \( SO(10) \), while making their color-triple partners superheavy. The reason for this is that those weak doublets have \( I_{3R} = 0 \). This is obvious, since such doublets have the same gauge charges as a lepton doublet, and hence are singlets under \( SU(2) \).

The models suggested in [14] actually have more than one adjoint, but it is a simple matter to modify those models so that only a single adjoint is needed. Consider the following superpotential:
\[ W_{2/3} = m_1 \overline{16}_1 \overline{16}_1 + m_2 \overline{16}_2 \overline{16}_2 + \overline{16}_1 10 \langle 16 \rangle + \overline{16}_2 10 \langle 16 \rangle + \overline{16}_1 \overline{16}_2 \langle 45 \rangle. \] (16)

The sector that contains the light Higgs doublets consists of a vector 10 and two conjugate pairs of spinors \( \overline{16}_1 + \overline{16}_1 + \overline{16}_2 + \overline{16}_2 \). Denote the superlarge VEVs of the \( SU(5) \)-singlet components of \( 16 \) and \( \overline{16} \), by \( \Omega \) and \( \overline{\Omega} \) respectively. Then the mass matrix of the \( SU(5) \) \( 5 \)'s has the following form:

\[
\begin{pmatrix}
\langle A \rangle & m_2 & 0 \\
0 & \Omega & 0 \\
0 & 0 & \overline{\Omega}
\end{pmatrix}
\begin{pmatrix}
m_1 \\
m_2 \\
\Omega
\end{pmatrix}
\begin{pmatrix}
\overline{5}_{16}_1 \\
\overline{5}_{16}_2 \\
\overline{5}_{10}
\end{pmatrix}.
\] (17)

In the doublet mass matrix, \( M_2 \), the entry \( \langle A \rangle \) vanishes, since, as we noted, the doublets in the spinors have \( I_{3R} = 0 \). Thus \( M_2 \) has a zero eigenvalue. Projecting this out, \( \det M_2 = (m_1^2 + \Omega^2)^{1/2} (m_2^2 + \Omega^2)^{1/2} \). On the other hand, all the eigenvalues of the mass matrix of the triplets are superheavy, and \( \det M_3 = a^2 \Omega \Omega \). Thus the desired doublet-triplet splitting is achieved. Note also that the \( SU(5) \) 10's in the spinors are all made superheavy by the VEVs \( \Omega \) and \( \overline{\Omega} \).

Suppose, as is simplest, that only the 10 of Higgs fields couples to the quarks and leptons. Then it is easy to see that the conditions of the theorem proved in section 3 are satisfied, since only one \( r_{kk'} \) is relevant. Thus \( \delta \epsilon_3 = \frac{3 \xi_3}{5 \pi R} \ln F \), where \( F^{-1} \) is the factor by which the proton-decay amplitude is suppressed relative to minimal \( SU(5) \). Thus, so far, we have only seen how doublet-triplet splitting may arise, but have not solved the problem of \( \alpha_s \). Now, however, it is possible to have “upside-down” split multiplets that give the needed large negative contributions to \( \epsilon_3 \). These can arise from the coupling of the adjoint to other vectors of Higgs fields, which we will denote \( T' \) and \( T'' \). If there are terms \( T' A T'' + m'T'^2 + m'' T''^2 \), where \( m'm'' \ll a^2 \), for example, then in these multiplets the doublets are much heavier than the triplets and they contribute \( \delta \epsilon_3 \cong -\frac{3 \xi_3}{5 \pi R} \ln(a^2/m'm'') \). This can easily be made large enough to give a satisfactory agreement for \( \alpha_s \).

One can see that in both the cases with one adjoint Higgs the sector which gives doublet-triplet splitting substantially increases \( \alpha_s \), and that agreement with experiment is achieved by assuming that another sector decreases \( \alpha_s \) by a roughly equal amount. In that case, the fact that the gauge couplings appear to unify so accurately in the MSSM seems to be the result of a
fortuitous cancellation. On the other hand, in the two-adjoint case considered at the end of section 3 the doublet-triplet sector does not tend to give large contributions to $\alpha_s$, and therefore the agreement with experiment can happen in a manner that appears less contrived.

5 Conclusion

We have shown that in a very general class of supersymmetric unified models based on $SO(10)$ the sector that produces the doublet-triplet splitting contributes a positive amount to $\epsilon_3$ and thus worsens the discrepancy in $\alpha_s$ between the simplest models and experiment. We have also shown that with two adjoints, whose VEVs are proportional to $B - L$ and $I_{3R}$, it is easy to arrange that the doublet-triplet-splitting sector does not exacerbate the discrepancy in $\alpha_s$, and even cures it by having “upside-down” split multiplets (that is, split multiplets where the doublets are much heavier than the triplets) that contribute negatively to $\epsilon_3$ and thus bring $\alpha_s$ back into agreement. We have seen why the problem of $\alpha_s$ is not as straightforward to solve if there is only a single adjoint. Nevertheless, we have found that a technically natural solution can be found both in the case where the VEV of the single adjoint points in the $B - L$ direction, and the case where it points in the $I_{3R}$ direction. In these cases, however, the doublet-triplet-splitting sector makes the problem worse, and another sector must happen to contribute in a way that compensates for it.

One of the interesting features of the solution with a single adjoint whose VEV is proportional to $B - L$ is the appearance of pseudo-goldstone fields with the standard model quantum numbers $(1, 1, \pm 1)$. These can conceivably be at the weak scale. The possibility of a single adjoint whose VEV is proportional to $I_{3R}$ deserves further study. Important questions for this case are whether the gauge hierarchy is easily stabilized against the effects of higher-dimensional operators, and whether realistic Yukawa structures for the quarks and leptons are possible.

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