On interplay between flavour anomalies and neutrino properties

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A minimal extension of the Standard Model (SM) featuring two scalar leptoquarks, an SU(2) doublet with hypercharge 1/6 and a singlet with hypercharge 1/3, is proposed as an economical benchmark model for studies of an interplay between flavour physics and properties of the neutrino sector. The presence of such type of leptoquarks radiatively generates neutrino masses and offers a simultaneous explanation for the current B-physics anomalies involving $b \rightarrow c\ell\nu$ decays. The model can also accommodate both the muon magnetic moment and the recently reported $W$ mass anomalies, while complying with the most stringent lepton flavour violating observables.

I. INTRODUCTION

The Standard Model (SM) of particle physics is our current guide towards a consistent description of the subatomic phenomena, able to withstand a series of most stringent tests [1–7]. However, the SM does not resemble a fundamentally complete theory. It cannot explain various observations such as neutrino masses, dark matter relic density or the baryon asymmetry of the Universe. Apart from these limitations, recent anomalies have emerged in significance as of late. Specifically, the anomalous magnetic moment of the muon [8] and hints for lepton flavour universality (LFU) violation in B meson decays, such as $R_{D^{(*)}}$ [9–14], defined as

$$R_{D^{(*)}} \equiv \frac{\text{Br} (\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}_\tau)}{\text{Br} (\bar{B} \rightarrow D^{(*)}\ell\bar{\nu}_\ell)}, \text{ with } \ell = \mu, e$$  \hspace{1cm} (1)

as well as tensions regarding decays of the $B_0/B_s$ mesons into a pair of muons, showcasing a 2.3σ deviation from the SM prediction [15]. Some previous results on $R_{K^{(*)}}$ [16–19] indicated a tension, but recently [20] it was shown to be consistent with the SM. There is also the recently reported CDF-II precision measurement of the $W$ mass indicating a 7.0σ deviation from the SM prediction [21], whose new physics (NP) effects can be parameterized in a modification to the oblique $T$ parameter [22]. Attempts to address these anomalies have been extensively reported in the literature (see, e.g. [23–37]) but are often treated in isolation rather than being simultaneously resolved in the same model. In a recent article [38], the B-physics anomalies and the anomalous magnetic moment of the muon were shown to be simultaneously accommodated in an economical framework solely featuring a leptoquark (LQ) and a charged scalar singlet. An explanation for neutrino properties is also well known to be a tantalizing possibility in LQ models as discussed in [39–63]. Particularly relevant are [42–48] where a minimal two-LQ scenario featuring a weak-singlet $S \sim (\mathbf{3}, 1)_{1/3}$ and a doublet $R \sim (\mathbf{3}, 2)_{1/6}$, offers the simplest known framework for radiative neutrino mass generation. However, a complete analysis of such an economical setting in the light of current flavour anomalies is lacking.

Furthermore, while minimal models often imply that fits to experimental data can become rather challenging, they also represent an opportunity for concrete and falsifiable predictions. In this letter, we then propose an inclusive study where B-physics, the muon $a_\mu \equiv \frac{1}{2}(g-2)_\mu$ and the CDF-II $W$ mass anomalies are simultaneously explained alongside neutrino masses and mixing while keeping lepton flavour violating (LFV) observables under control. We further inspire our model on the flavoured grand unified framework first introduced by some of the authors in [64, 65].

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in order to motivate the presence of a baryon number parity defined as \( P_B = (-1)^{3B+2S} \), with \( B \) being the baryon number and \( S \) the spin. Such a parity forbids di-quark type interactions for the \( S \) LQ otherwise responsible for fast proton decay.

In this model, the \( R_{D,D} \)- observables are explained via the tree-level exchange of the \( S \) LQ as in diagram (a). Noteworthy, the mixing between the \( S \) and \( R \) doublet induces radiative generation of neutrino masses at one-loop level, while a splitting between the two components of the \( R \) doublet can modify the \( W \) mass.

The \( b \to s\ell\ell \) observables are impacted via both tree-level and box diagrams involving the virtual exchange of the \( S \) LQ. We show in Fig. 1, the possible contributions to such observables described in terms of the Wilson operators \( O^{\mu}_{\alpha} \propto C^{\text{had}}_{\mu}(s^\gamma_{\mu}P_Lb)(\ell\gamma_{\mu}\ell) \) and \( O^{5\mu}_{\alpha} \propto C^{\text{had}}_{\mu}(s^\gamma_{\mu}P_Lb)(\ell\gamma_{\mu}) \) for diagrams (e) and (f), and \( O^{\mu}_{\alpha} \propto C^{\text{had}}_{\mu}(s^\gamma_{\mu}P_Lb)(\ell\gamma_{\mu}) \) and \( O^{5\mu}_{\alpha} \propto C^{\text{had}}_{\mu}(s^\gamma_{\mu}P_Lb)(\ell\gamma_{\mu}) \) for diagrams (b), (g) and (h). As usual, the \( C \)-factors are the Wilson coefficients and \( \ell = e, \mu \).

In what follows, we present the model and demonstrate how the fields contribute to each of the relevant observables and the main experimental constraints that affect the allowed parameter space. We then discuss the regions of parameter space where all anomalies and constraints are realized within experimental bounds. Finally, we summarize our results.

II. THE MINIMAL LQ MODEL

The interactions of the singlet and doublet LQs with the SM fermion sector invariant both under the gauge symmetry and the \( P_B \) parity are described by the following terms

\[
\mathcal{L}_V = \Theta_{ij} \bar{Q}^c_i L_i S + \Omega_{ij} \bar{L}_i d_j R^i + \Upsilon_{ij} \bar{u}^c_j e_i S + \text{h.c.},
\]

As usual, \( Q \) and \( L \) are the left-handed quark and lepton SU(2) doublets, respectively, whereas \( d \) and \( e \) are the right-handed down quark and charged lepton SU(2) singlets. All Yukawa couplings, \( \Theta, \Omega \) and \( \Upsilon \), are complex \( 3 \times 3 \) matrices. Here, SU(2) contractions are also left implicit. For example, \( \bar{Q}^c_i L \equiv \epsilon_{\alpha\beta} \bar{Q}^c \epsilon^{\alpha\beta} L \), with \( \epsilon_{\alpha\beta} \) being the Levi-Civita symbol in two dimensions and \( \epsilon \) indicating charge conjugation. These interactions contain the necessary ingredients to change the SM prediction for both the \( R_{D,D} \) anomalies, diagram (a) in Fig. 1, and the anomalous magnetic moment of the muon, diagram (c) of the same figure, while preserving \( R_{K^0} \) and \( B_s/B_0 \to \mu \mu \) SM-like, diagrams (b) and (e)-(h).

The relevant part of the scalar potential reads as

\[
V \supset -\mu^2 |H|^2 + \mu_3^2 |S|^2 + \mu_4^2 |R|^2 + \lambda (H^\dagger H)^2 + g_{HR}(H^\dagger H)(R^\dagger R) + g'_{HR}(H^\dagger R)(R^\dagger H) + g_{HS}(H^\dagger H)(S^\dagger S) + (a_1 RSH^\dagger + \text{h.c.}).
\]

Once the electroweak symmetry is broken by a non-zero vacuum expectation value \( v \) of the Higgs field, one of the components of the \( R \) doublet mixes with the \( S \) field, via the \( a_1 \) interaction term in Eq. (3), resulting in two physical LQs with electric charge \( 1/3e \), denoted as \( S_{1/3}^1 \) and \( S_{2/3}^2 \). The other \( R \) component, which has electric charge \( 2/3e \), does not mix and is named as \( S_{2/3}^2 \) in what follows. The quartic parameters \( g_{HR} \) and \( g'_{HR} \) are responsible for generating a mass shift between the two components of the \( R \) doublet, providing a contribution to the CDF-II \( W \) mass discrepancy.

The mixing parameter \( a_1 \) is also responsible for enabling radiative generation of neutrino masses at one-loop level via the diagram (d) in Fig. 1. For simplicity one assumes a flavour diagonal basis for the up-type quarks such that the Cabibbo–Kobayashi–Maskawa (CKM) mixing resides entirely within the down-quark sector. Therefore, one can express the components of the neutrino masses as

\[
(M_\nu)_{ij} = \frac{3}{16\pi^2(m^2_{S_{2/3}^2} - m^2_{S_{1/3}^1})} \sqrt{2} v a_1 \sum_{m,a} (m_d)_a \langle V_{am}(\Theta_{jm} \Omega_{ja} + \Theta_{jm} \Omega_{la}) \rangle,
\]

where \( V \) is the CKM matrix and \( (m_d)_a \) are the down-type quark masses. In the limit of vanishing LQ mixing, i.e. \( a_1 \rightarrow 0 \), the loop contribution goes to zero. Indeed, mixing between the doublet and singlet LQs is a necessary aspect for a viable phenomenology. We work in the basis of a flavour diagonal charged lepton mass matrix such that the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) mixing matrix is entirely within the neutrino sector. The flavour democratic nature of the PMNS matrix, as opposed to an hierarchical CKM structure, implies that a generic texture for both \( \Theta \) and \( \Omega \) Yukawa matrices is preferred. Indeed, employing a minimal flavour ansatz as e.g. in [38] would result in a non-viable neutrino phenomenology. Sizeable chirality flipping contributions proportional to e.g. \( \Theta_{et} \Upsilon_{\mu \ell} \) or \( \Theta_{et} \Upsilon_{\ell \mu} \) can efficiently generate large contributions to tightly constrained LFV observables such as \( \mu \to e\gamma \) or \( \tau \to e\gamma \). However, a competition between neutrino, B-physics, \( a_\mu \) and LFV contributions renders the model rather constrained, thus falsifiable in future measurements.
FIG. 1: Feynman diagrams responsible for explaining all studied observables in this letter. (a) corresponds to the tree-level contribution to the $R_{D,D^*}$ anomaly, (b) is a tree-level contribution to the B-physics observables, (c) is the dominant one-loop contribution to the anomalous magnetic moment of the muon and, (d), radiatively generates neutrino masses and mixing. Box graphs from (e) to (h) are responsible for generating contributions to $R_{K,K^*}$ and $B_s/B_0 \to \mu^+\mu^-$.

III. NUMERICAL RESULTS

We perform a parameter space scan considering a plethora of different observables as listed in Appendix A. The experimental limits were taken from the latest PDG review [66]. We also consider a generic complex parameterization of the Yukawa couplings, and as such, the imaginary components of $\Theta$, $\Omega$ and $\Upsilon$ can generate non-zero contributions to the electric dipole moments (EDM) of the charged leptons, also constrained by experiment. Additionally, the $R_{\nu\nu}^{K,K^*}$ observable, defined as the ratio of the branching fraction $\text{BR}(B^{+(0)} \to K^{*(0)}\nu\nu)$ between the NP and SM predictions [67], is induced at tree-level by the diagram (a) in Fig. 1 if $\Upsilon$ is replaced with $\Theta$. In turn, maximizing $R_{D^*}$ can also result in larger contributions to $R_{\nu\nu}^{K,K^*}$, in particular, if $\Theta$ contains additional sizeable entries (see Fig. 3 (a)).

Direct searches for LQs at collider experiments also pose limits on their allowed masses. Constraints coming from pair production channels at the ATLAS and CMS experiments [68–71] provide a lower bound, approximately, between 1 and 1.5 TeV, considered in this work.

FIG. 2: Preferred sizes for each of the LQ Yukawas couplings: (a) $\Omega$, (b) $\Theta$ and (c) $\Upsilon$. The radius of the circumference represents the size of the absolute value of the coupling while the color gradation describes how frequent such a magnitude appears in the scan, i.e. darker shades indicate more preferred sizes, according the probability $P(X) = \frac{N(X)}{N_{\text{tot}}}$ with $N(X)$ the number of points with order $X$ in our data.
FIG. 3: Scatter plots of selected observables analysed in this work. In (a) we plot the $R_{D^*}$ as a function of $R_D$ with $R_{K^*}^T$ in the color scale. In (b), the branching ratios of $B_0 \rightarrow \mu^- \mu^+$ and $B_s \rightarrow \mu^- \mu^+$, with $R_K[1.1; 6.0]$ in the color scale. In (c) we plot $R_{K^*}$ vs. $R_K$ with BR($B_s \rightarrow \mu \mu$) in the color scale. In (d) the product of the real and imaginary parts of $\Upsilon_{\mu \mu} \Theta_{\mu \mu}$ are shown with $a_\mu$ in the color gradation while in (e) the $T$ parameter as function of the logarithm of the mass difference between the masses of $S_{1/3}^1$ and $S_{2/3}^2$ is presented and in (f) we show BR($Z^0 \rightarrow \mu \tau$) versus $R_K^{\mu \nu}$ with $\log_{10} \text{BR} (\tau \rightarrow \mu \mu \mu)$ in the colour axis. The color scale of the latter represents the fraction of $S_{1/3}^1$ in doublet $R$. Areas of phenomenological interest lie inside the contours. For the $T$ parameter, we show the areas of interest for both the scenario where $T$ is NP-like or SM-like. For $a_\mu$, the relevant range lies within $(251 \pm 59) \times 10^{-11}$ (1σ range), $\text{BR}(Z^0 \rightarrow \mu \tau) < 1.2 \times 10^{-5}$, $R_K^{\mu \nu} < 4.35$ and BR($\tau \rightarrow \mu \mu \mu$) < $2.1 \times 10^{-8}$. The three best fit points are marked as blue square (scenario a), cyan circle (scenario b) and a red diamond (scenario c).

For an extensive analysis featuring a large number of observables we have implemented the model in SARAH [72], where interaction vertices and one-loop contributions relevant for such observables were determined. Outputs were then generated for numerical evaluation in SPheno [73], where the particle spectrum and the necessary Wilson coefficients to be used in flavio [74] were calculated. With this in consideration, we have constructed a $\chi^2$ function,
The data as well as some auxiliary codes are publicly available in one of author’s github page https://github.com/Mrazi09/ Leptoquark-project---Data.
IV. CONCLUSIONS

In this letter, we have studied the most economical extension of the SM with just two scalar LQs, representing the minimal scenario capable of addressing all measured flavour anomalies as well as explaining neutrino masses and their mixing structure. Additionally, the model can accommodate the measured value of the muon anomalous magnetic moment as well as opening the door for, or at least alleviating, the CDF-II W mass anomaly, if both observables are confirmed to be inconsistent with the SM predictions. For the best-fit points the lightest LQ can have a mass around 1.6 TeV, which should be accessible at the high-luminosity phase of the LHC. In this regard, our numerical results have highlighted the preferred sizes for the LQ Yukawa couplings which will be relevant in pinpointing the direction for future searches.

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Appendix A: Numerical benchmarks

If we take that both the W mass and the anomalous magnetic moment of the muon are SM-like, scenario a), then the best fit point found in the scan is

\[
\begin{bmatrix}
-2.686 \times 10^{-7} + 3.303 \times 10^{-7}i \\
0.0004216 - 0.0006776i \\
-2.448 \times 10^{-7} - 1.462 \times 10^{-7}i \\
0.0496 + 0.1055i \\
3.359 \times 10^{-6} + 9.47 \times 10^{-7}i \\
0.0189 - 0.0062i \\
0.000555 - 0.003117i \\
-0.0001547 + 0.000106i \\
0.0093 + 0.02506i
\end{bmatrix}
\]

This point correspond to the blue diamond in the scatter plots plots of the main text. On the other hand, if we assume the W boson mass to take the SM value, but the muon $a_\mu$ anomaly requires a NP explanation, scenario b), the following best fit point is obtained

\[
\begin{bmatrix}
-2.319 \times 10^{-7} + 6.392 \times 10^{-7}i \\
0.000526 - 0.002488i \\
-6.79 \times 10^{-8} - 2.811 \times 10^{-7}i \\
0.0447 + 0.1871i \\
5.394 \times 10^{-6} + 3.39 \times 10^{-7}i \\
0.006595 + 0.006782i \\
0.006683 - 0.002639i \\
-6.09 \times 10^{-5} + 0.0003344i \\
0.02939 - 0.00698i
\end{bmatrix}
\]

(A1)
This point correspond to the cyan diamond in the scatter plots of the main text. Last but not least, if we assume that both the $W$ mass and $a_\mu$ require a NP explanation, scenario c), then the best fit point is

$$\Upsilon = \begin{pmatrix} 
-5.9 \times 10^{-7} + 8.26 \times 10^{-7}i & 4.842 \times 10^{-6} + 4.7 \times 10^{-8}i & -1.352 \times 10^{-8} - 3.261 \times 10^{-8}i \\
0.000319 - 0.002549i & -3.34 \times 10^{-8} + 1.449 \times 10^{-7}i & -0.00759 - 0.01239i \\
-1.524 \times 10^{-7} - 9.06 \times 10^{-8}i & -0.15 + 2.51i & 0.0002995 + 0.0004214i 
\end{pmatrix},$$

$$\Omega = \begin{pmatrix} 
0.0567 + 0.1529i & -0.0032 + 0.000344i & -5.832 \times 10^{-7} - 6.36 \times 10^{-8}i \\
3.488 \times 10^{-6} + 2.24 \times 10^{-7}i & 2.34 \times 10^{-6} - 3.97 \times 10^{-7}i & 1.354 \times 10^{-6} - 1.34 \times 10^{-7}i \\
-0.02495 - 0.00425i & -0.00879 - 0.01723i & 1.72 \times 10^{-6} + 7.294 \times 10^{-6}i 
\end{pmatrix},$$

$$\Theta = \begin{pmatrix} 
0.00135 - 0.002976i & 1.444 \times 10^{-7} - 9.32 \times 10^{-8}i & 7.894 \times 10^{-7} + 2.147 \times 10^{-7}i \\
-5.44 \times 10^{-5} + 0.0002732i & -1.592 \times 10^{-8} + 1.964 \times 10^{-8}i & 0.5828 + 0.0578i \\
0.02236 + 0.00576i & -0.01707 - 0.02831i & 0.408 + 2.085i 
\end{pmatrix}.\tag{A3}$$

This point correspond to the red diamond in the scatter plots of the main text. For each of these cases, the LQ masses are indicated in the main text. These benchmarks were determined by minimizing the $\chi^2$ function in Eq. (B12), whose input observables are showcased in Tab. I. In Tabs. II and III we indicate the predictions for the observables for each of the benchmark scenarios.
| Observable          | Experimental measurement          |
|---------------------|-----------------------------------|
| $(g-2)_\mu$         | $\left(251 \pm 59\right) \times 10^{-11}$ [8] |
| $\hat{T}$           | $(0.88 \pm 0.14) \times 10^{-3}$ [22] |
| $R_K[1.1, 6.0]$     | $0.949^{+0.024+0.022}_{-0.041-0.022}$ [20] |
| $R_K'[1.1, 6.0]$    | $1.027^{+0.072+0.027}_{-0.068-0.026}$ [20] |
| $R_K[0.1, 1.1]$     | $0.994^{+0.030+0.029}_{-0.022-0.027}$ [20] |
| $R_K'[0.1, 1.1]$    | $0.927^{+0.039+0.036}_{-0.027-0.035}$ [20] |
| $R_D$               | $0.340 \pm 0.027 \pm 0.013$ [82] |
| $R_{D^*}$           | $0.295 \pm 0.011 \pm 0.008$ [82] |
| BR($h \to e\mu$)    | $< 6.1 \times 10^{-5}$ [95% CL] [66] |
| BR($h \to e\tau$)   | $< 4.7 \times 10^{-3}$ [95% CL] [66] |
| BR($h \to \mu\tau$) | $< 2.5 \times 10^{-3}$ [95% CL] [66] |
| BR($\mu \to e\gamma$)| $< 4.2 \times 10^{-13}$ [90% CL] [66] |
| BR($\mu \to \epsilon e\epsilon$) | $< 1.0 \times 10^{-12}$ [90% CL] [66] |
| BR($\tau \to e\gamma$)| $< 3.3 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \mu\gamma$) | $< 4.4 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \epsilon e\epsilon$) | $< 2.7 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \epsilon\nu\nu$) | $< 2.7 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \epsilon\nu\nu$) | $< 1.5 \times 10^{-9}$ [90% CL] [66] |
| BR($\tau \to \mu\mu$) | $< 2.1 \times 10^{-8}$ [90% CL] [66] |
| BR($Z \to \mu\mu$) | $< 7.5 \times 10^{-7}$ [95% CL] [66] |
| BR($Z \to \tau\tau$) | $< 9.8 \times 10^{-6}$ [95% CL] [66] |
| BR($Z \to \mu\tau$) | $< 1.2 \times 10^{-5}$ [95% CL] [66] |
| BR($\tau \to \epsilon\nu\nu$) | $< 8.0 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \epsilon e\epsilon$) | $< 1.1 \times 10^{-7}$ [90% CL] [66] |
| BR($\tau \to \phi\epsilon$) | $< 3.1 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \phi\mu\mu$) | $< 8.4 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \phi\tau\tau$) | $< 1.8 \times 10^{-8}$ [90% CL] [66] |
| BR($\tau \to \phi\mu\tau$) | $< 1.2 \times 10^{-8}$ [90% CL] [66] |
| $d_\epsilon$        | $< 1.1 \times 10^{-29}$ e.cm [90% CL] [66] |
| $d_\mu$             | $< 1.8 \times 10^{-19}$ e.cm [95% CL] [66] |
| $d_\tau$            | $< (1.15 \pm 1.70) \times 10^{-17}$ e.cm [95% CL] [83] |
| BR($B^0 \to \mu\mu$)| $(0.56 \pm 0.70 \times 10^{-10})$ [15] |
| BR($B_s \to \mu\mu$) | $(2.93 \pm 0.35 \times 10^{-9})$ [15] |
| $R(B \to \chi_{\epsilon}\chi_{\epsilon}$) | $1.009 \pm 0.075$ |
| $R_d^{\ell\nu}$     | $< 4.65$ [95% CL] [84] |
| $R_d^{K*}$          | $< 3.22$ [95% CL] [84] |
| $|\text{Re } \delta g_R|$ | $\leq 2.9 \times 10^{-4}$ [39, 85] |
| $|\text{Re } \delta g_L|$ | $\leq 3.0 \times 10^{-4}$ [39, 85] |
| $|\text{Re } \delta g_R'$ | $\leq 1.3 \times 10^{-3}$ [39, 85] |
| $|\text{Re } \delta g_L'$ | $\leq 1.1 \times 10^{-3}$ [39, 85] |
| $|\text{Re } \delta g_R''|$ | $\leq 6.2 \times 10^{-4}$ [39, 85] |
| $|\text{Re } \delta g_L''|$ | $\leq 5.8 \times 10^{-4}$ [39, 85] |
| $R(\epsilon_k)$     | $1.234 \pm 0.144$ |
| $R(\Delta M_d)$     | $0.838 \pm 0.109$ |
| $R(\Delta M_s)$     | $0.935 \pm 0.054$ |
| $R(Re'\epsilon/e)$  | $0.868 \pm 0.496$ |
| $Q_W(p^+)$          | $0.0719 \pm 0.045$ |
| $Q_W(Cs^{133})$     | $-72.82 \pm 0.42$ |

**TABLE I:** Set of observables used as input for the $\chi^2$ function, as well as the experimental measured value. The observables $F_L$, $A_{FB}$, $S_t$, $P_t$ and $P_L$ are relative to the $[0.10, 0.98]$ GeV$^2$ $q^2$ bins. Observables starting with "R" are defined as the ratio between the experimental value for the observable, taken from [66] and the SM prediction, determined in flavio. The total uncertainty is taken by error propagation, taking into account both the experimental and theoretical errors, with the latter determined also in flavio.
| Observable | Theoretical prediction: (A1) | Theoretical prediction: (A2) | Theoretical prediction: (A3) |
|------------|----------------------------|----------------------------|----------------------------|
| $a_\mu$    | $-2.748 \times 10^{-10}$  | $2.649 \times 10^{-9}$     | $1.879 \times 10^{-9}$     |
| $\bar{T}$ | $3.393 \times 10^{-5}$    | $0.0001631$                | $0.0009105$                |
| $R_K[1.1,6.0]$ | 1.005                  | 1.006                      | 1.006                      |
| $R_K[1.1,6.0]$ | 0.9995                 | 1.001                      | 1.001                      |
| $R_K[0.1,1.1]$ | 0.9976                 | 0.9984                     | 0.999                      |
| $R_K[0.1,1.1]$ | 1.005                  | 1.006                      | 1.006                      |
| $R_D$      | 0.358                   | 0.3434                     | 0.334                      |
| $R_D^*$    | 0.2800                  | 0.2875                     | 0.283                      |
| $\mathrm{BR}(h \to e\mu)$ | $6.045 \times 10^{-18}$ | $9.688 \times 10^{-19}$   | $5.978 \times 10^{-19}$   |
| $\mathrm{BR}(h \to e\tau)$ | $6.653 \times 10^{-17}$ | $1.822 \times 10^{-17}$   | $5.912 \times 10^{-18}$   |
| $\mathrm{BR}(h \to \mu\tau)$ | $2.871 \times 10^{-6}$  | $1.039 \times 10^{-6}$    | $9.904 \times 10^{-7}$    |
| $\mathrm{BR}(\mu \to e\gamma)$ | $2.931 \times 10^{-16}$ | $2.958 \times 10^{-15}$   | $1.42 \times 10^{-15}$   |
| $\mathrm{BR}(\mu \to eee)$ | $2.121 \times 10^{-18}$ | $1.981 \times 10^{-17}$   | $9.36 \times 10^{-18}$   |
| $\mathrm{BR}(\tau \to e\gamma)$ | $1.042 \times 10^{-17}$ | $1.185 \times 10^{-17}$   | $1.489 \times 10^{-17}$   |
| $\mathrm{BR}(\tau \to \mu\gamma)$ | $4.793 \times 10^{-9}$  | $3.294 \times 10^{-9}$    | $4.864 \times 10^{-9}$    |
| $\mathrm{BR}(\tau \to eee)$ | $8.267 \times 10^{-17}$ | $2.8 \times 10^{-16}$     | $1.57 \times 10^{-15}$    |
| $\mathrm{BR}(\tau \to e\mu\mu)$ | $5.649 \times 10^{-17}$ | $1.895 \times 10^{-16}$   | $1.077 \times 10^{-15}$   |
| $\mathrm{BR}(\tau \to \mu\mu)$ | $7.315 \times 10^{-30}$ | $8.357 \times 10^{-29}$   | $2.898 \times 10^{-28}$   |
| $\mathrm{BR}(\tau \to \mu\mu)$ | $1.115 \times 10^{-11}$ | $7.66 \times 10^{-12}$    | $1.132 \times 10^{-11}$   |
| $\mathrm{BR}(Z \to \mu\tau)$ | $1.01 \times 10^{-22}$  | $9.718 \times 10^{-21}$   | $1.074 \times 10^{-20}$   |
| $\mathrm{BR}(Z \to \tau\tau)$ | $1.378 \times 10^{-17}$ | $3.49 \times 10^{-18}$    | $2.317 \times 10^{-17}$   |
| $\mathrm{BR}(Z \to \mu\tau)$ | $3.941 \times 10^{-8}$  | $2.244 \times 10^{-8}$    | $4.64 \times 10^{-8}$    |
| $\mathrm{BR}(\tau \to \pi\pi)$ | $2.621 \times 10^{-13}$ | $1.15 \times 10^{-13}$    | $1.645 \times 10^{-12}$   |
| $\mathrm{BR}(\tau \to \pi\pi)$ | $1.555 \times 10^{-20}$ | $3.619 \times 10^{-19}$   | $2.152 \times 10^{-19}$   |
| $\mathrm{BR}(\tau \to \pi\pi)$ | $2.936 \times 10^{-15}$ | $3.547 \times 10^{-16}$   | $1.534 \times 10^{-15}$   |
| $\mathrm{BR}(\tau \to \phi\phi)$ | $1.335 \times 10^{-12}$ | $9.174 \times 10^{-13}$   | $1.354 \times 10^{-12}$   |
| $\mathrm{BR}(\tau \to \phi\phi)$ | $3.249 \times 10^{-13}$ | $2.739 \times 10^{-13}$   | $4.817 \times 10^{-12}$   |
| $\mathrm{BR}(\tau \to \rho\rho)$ | $1.205 \times 10^{-11}$ | $8.283 \times 10^{-12}$   | $1.223 \times 10^{-11}$   |
| $d_e$      | $3.303 \times 10^{-34}$ | $1.076 \times 10^{-33}$   | $5.591 \times 10^{-34}$   |
| $d_s$      | $1.847 \times 10^{-22}$ | $5.388 \times 10^{-23}$   | $1.094 \times 10^{-22}$   |
| $d_s$      | $1.816 \times 10^{-23}$ | $1.638 \times 10^{-23}$   | $1.999 \times 10^{-24}$   |
| $\mathrm{BR}(B_d \to \mu\mu)$ | $1.127 \times 10^{-10}$ | $1.14 \times 10^{-10}$    | $1.148 \times 10^{-10}$   |
| $\mathrm{BR}(B_s \to \mu\mu)$ | $3.674 \times 10^{-9}$  | $3.685 \times 10^{-9}$    | $3.679 \times 10^{-9}$    |
| $R(B \to \chi^\pm)$ | $1.000$                 | $1.000$                    | $1.000$                    |
| $R_{K^\star}^{\mu\nu}$ | 1.497                  | 1.227                      | 1.476                      |
| $R_{K^\star}^{\mu\nu}$ | 1.497                  | 1.227                      | 1.476                      |
| $|\mathrm{Re} \delta g_{K^\star}|$ | $9.304 \times 10^{-8}$  | $8.8 \times 10^{-8}$      | $7.077 \times 10^{-8}$    |
| $|\mathrm{Re} \delta g_{K^\star}|$ | $1.991 \times 10^{-8}$  | $5.182 \times 10^{-8}$    | $6.254 \times 10^{-8}$   |
| $|\mathrm{Re} \delta g_{K^\star}|$ | $6.432 \times 10^{-8}$  | $6.549 \times 10^{-7}$    | $1.454 \times 10^{-7}$   |
| $|\mathrm{Re} \delta g_{K^\star}|$ | $0.002597$              | $0.002659$                 | $0.002321$                 |
| $|\mathrm{Re} \delta g_{K^\star}|$ | $0.0004998$             | $0.0005697$                | $0.0005943$                |
| $|\mathrm{Re} \delta g_{K^\star}|$ | $1.959 \times 10^{-8}$  | $1.049 \times 10^{-8}$    | $6.591 \times 10^{-9}$    |
| $R(\epsilon)$ | $1.229$                 | $1.326$                    | $1.107$                    |
| $R(\Delta M_3)$ | $0.8161$               | $0.7878$                   | $1.08$                     |
| $R(\Delta M_s)$ | $0.8719$               | $1.01$                     | $0.9177$                   |
| $R(Re\epsilon'/\epsilon)$ | $1.165$                | $1.013$                    | $1.425$                    |
| $Q_W(p^+)$ | $0.071$                 | $0.071$                    | $0.071$                    |
| $Q_W(Cs^{133})$ | $-73.33$               | $-73.33$                   | $-73.33$                   |

**TABLE II:** Theoretical predictions for each of the benchmarks.
| Observable | Theoretical prediction: (A1) | Theoretical prediction: (A2) | Theoretical prediction: (A3) |
|------------|-----------------------------|-----------------------------|-----------------------------|
| \( F_L(B^+ \to K\mu\mu) \) | 0.3039 | 0.304 | 0.3042 |
| \( S_L(B^+ \to K\mu\mu) \) | 0.01081 | 0.0108 | 0.01081 |
| \( S_L(B^+ \to K\mu\mu) \) | 0.08928 | 0.08907 | 0.08927 |
| \( S_L(B^+ \to K\mu\mu) \) | 0.2593 | 0.2597 | 0.2593 |
| \( A_{FB}(B^+ \to K\mu\mu) \) | -0.09705 | -0.09717 | -0.09707 |
| \( S_L(B^+ \to K\mu\mu) \) | -0.01791 | -0.01793 | -0.01792 |
| \( S_L(B^+ \to K\mu\mu) \) | -0.01216 | -0.01215 | -0.01216 |
| \( S_L(B^+ \to K\mu\mu) \) | -0.0007136 | -0.0007131 | -0.0007135 |
| \( P_1(B^+ \to K\mu\mu) \) | 0.0454 | 0.04538 | 0.04541 |
| \( P_2(B^+ \to K\mu\mu) \) | -0.1359 | -0.1361 | -0.136 |
| \( P_3(B^+ \to K\mu\mu) \) | 0.001499 | 0.001498 | 0.001499 |
| \( P_4'(B^+ \to K\mu\mu) \) | 0.2347 | 0.2341 | 0.2346 |
| \( P_5'(B^+ \to K\mu\mu) \) | 0.6817 | 0.6826 | 0.6816 |
| \( P_6'(B^+ \to K\mu\mu) \) | -0.04707 | -0.04713 | -0.04709 |
| \( P_6''(B^+ \to K\mu\mu) \) | -0.03196 | -0.03193 | -0.03196 |
| \( F_L(B^0 \to K\mu\mu) \) | 0.297 | 0.2971 | 0.2973 |
| \( S_L(B^0 \to K\mu\mu) \) | 0.01083 | 0.01082 | 0.01082 |
| \( S_L(B^0 \to K\mu\mu) \) | 0.0957 | 0.09549 | 0.0957 |
| \( S_L(B^0 \to K\mu\mu) \) | 0.2607 | 0.2611 | 0.2608 |
| \( A_{FB}(B^0 \to K\mu\mu) \) | -0.09674 | -0.09686 | -0.09675 |
| \( S_L(B^0 \to K\mu\mu) \) | -0.02057 | -0.02059 | -0.02057 |
| \( S_L(B^0 \to K\mu\mu) \) | -0.002191 | -0.002182 | -0.002197 |
| \( S_L(B^0 \to K\mu\mu) \) | -0.0006989 | -0.0006985 | -0.0006988 |
| \( P_1(B^0 \to K\mu\mu) \) | 0.04447 | 0.04444 | 0.04447 |
| \( P_2(B^0 \to K\mu\mu) \) | -0.1324 | -0.1326 | -0.1325 |
| \( P_3(B^0 \to K\mu\mu) \) | 0.001435 | 0.001434 | 0.001436 |
| \( P_4'(B^0 \to K\mu\mu) \) | 0.2516 | 0.2511 | 0.2516 |
| \( P_5'(B^0 \to K\mu\mu) \) | 0.6856 | 0.6864 | 0.6855 |
| \( P_6'(B^0 \to K\mu\mu) \) | -0.05408 | -0.05413 | -0.05408 |
| \( P_7'(B^0 \to K\mu\mu) \) | -0.005762 | -0.005738 | -0.005776 |
| \( C_{\beta\mu\mu}^{gg} \) | 0.01718 | 0.0142 | 0.02012 |
| \( C_{\beta\mu\mu}^{gg} \) | 0.003933 | -0.00568 | -0.002635 |
| \( C_{10}^{gg} \) | 7.005 \times 10^{-8} | 3.501 \times 10^{-8} | 6.651 \times 10^{-8} |
| \( C_{10}^{gg} \) | -6.996 \times 10^{-8} | -3.496 \times 10^{-8} | -6.645 \times 10^{-8} |
| \( C_{20}^{ee} \) | -5.692 \times 10^{-7} | -1.607 \times 10^{-7} | -5.1 \times 10^{-7} |
| \( C_{20}^{ee} \) | 6.537 \times 10^{-7} | 1.424 \times 10^{-7} | 4.972 \times 10^{-7} |
| \( R(K^+ \to \pi^0 \mu^+ \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K^+ \to \pi^0 e^+ \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K_L^0 \to \pi^+ \mu^- \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K_L^0 \to \pi^+ e^- \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K_L^0 \to \pi^0 \mu^- \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K_L^0 \to \pi^0 e^- \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K_L^0 \to e^+ e^-) \) | 1.097 | 1.058 | 1.021 |
| \( BR(K_L^0 \to e^+ \mu^+) \) | 1.806 \times 10^{-15} | 6.363 \times 10^{-16} | 3.129 \times 10^{-15} |
| \( BR(K_S^0 \to \mu^+ \mu^-) \) | 5.173 \times 10^{-12} | 5.172 \times 10^{-12} | 5.169 \times 10^{-12} |
| \( BR(K_S^0 \to e^+ e^-) \) | 1.73 \times 10^{-16} | 1.637 \times 10^{-16} | 1.762 \times 10^{-16} |
| \( R(K^+ \to \mu^+ \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K^+ \to e^+ \nu) \) | 1.000 | 1.000 | 1.000 |
| \( R(K^+ \to \pi^+ \nu) \) | 1.69 | 0.8588 | 1.939 |
| \( BR(K_L^0 \to \pi^0 \nu \bar{\nu}) \) | 2.004 \times 10^{-10} | 4.059 \times 10^{-11} | 2.184 \times 10^{-10} |

**TABLE III:** Theoretical predictions for each of the benchmarks. The numerical values of Wilson coefficients \( C_9 \) and \( C_{10} \) are shown in the same basis as [15].
Appendix B: Inversion procedure and scan methodology

Once the Higgs doublet gains a vacuum expectation value (VEV), which in the unitary gauge corresponds to $\langle H \rangle = [0 \ (v + h)/\sqrt{2}]^T$ and $v \approx 246$ GeV, the mass for the Higgs field remains identical to that of the Standard model (SM), $m_h^2 = 2\lambda v^2$. The S leptoquark (LQ) and the second component of the R doublet mix (corresponding to the LQs with an electrical charge of 1/3e), leading to the squared mass matrix

$$M^2_{LQ^{1/3}} = \begin{bmatrix} \mu_S^2 + \frac{gH_S v^2}{2} & \frac{v a_1}{\sqrt{2}} \\ \frac{v a_1}{\sqrt{2}} & \mu_R^2 + \frac{G v^2}{2} \end{bmatrix}$$

(B1)

where $G = (g_{HR} + g'_{HR})$ and we assume that $a_1$ is a real parameter. The eigenvalues of the mass matrix read

$$m^2_S^{1/3} = \frac{1}{4} \left( 2\mu_R^2 + 2\mu_S^2 + (G + gH_S)^2 - \sqrt{\left(2\mu_R^2 - 2\mu_S^2 + (G - gH_S)v^2\right)^2 + 8a_1^2 v^2} \right),$$

$$m^2_S^{2/3} = \frac{1}{4} \left( 2\mu_R^2 + 2\mu_S^2 + (G + gH_S)^2 + \sqrt{\left(2\mu_R^2 - 2\mu_S^2 + (G - gH_S)v^2\right)^2 + 8a_1^2 v^2} \right),$$

(B2)

where we adopt the notation for the mass eigenstates of $S^{1/3}_1$ and $S^{2/3}_2$. Do note that one can diagonalise the matrix in Eq. (B1) via a bi-unitary transformation, that is,

$$M^\text{diag}_{LQ^{1/3}} = Z^H M^2_{LQ^{1/3}} Z^{H\dagger},$$

(B3)

where $Z^H$ is an unitary matrix and $M^\text{diag}_{LQ^{1/3}}$ is the LQ mass matrix in the diagonal form. Since this is a $2 \times 2$ matrix, the mixing can be parameterized by a single angle, which in terms of the mass eigenstates it is given by

$$\sin(2\theta) = v a_1 / (m^2_S^{1/3} - m^2_S^{2/3}),$$

(B4)

where $\theta$ is a mixing angle. This relation necessarily implies the condition $-1 \leq (\sqrt{2}a_1 / (m^2_S^{1/3} - m^2_S^{2/3})) \leq 1$. The remainder LQ does not mix with the others and its tree level mass reads as

$$m^2_{S^{2/3}} = \mu_R^2 + \frac{g_{HR} v^2}{2},$$

(B5)

where we adopt the nomenclature for the $2/3e$ one as $S^{2/3}$. The relations in (B2) can be inverted such that the physical masses of the LQ can be given as input in the numerical scan. Solving with the system of equations with respect to $\mu_R^2$ and $\mu_S^2$, one obtains

$$\mu_S^2 = \frac{1}{2} \left( m^2_{S^{1/3}} + m^2_{S^{2/3}} - gH_S v^2 + \sqrt{(m^2_{S^{1/3}} - m^2_{S^{2/3}})^2 - 2a_1^2 v^2} \right),$$

$$\mu_R^2 = \frac{1}{2} \left( m^2_{S^{1/3}} + m^2_{S^{2/3}} - (g_{HR} + g'_{HR}) v^2 - \sqrt{(m^2_{S^{1/3}} - m^2_{S^{2/3}})^2 - 2a_1^2 v^2} \right).$$

(B6)

Note that the mass of the $(2/3)e$ LQ is not given as input and is determined from the input $g_{HR}$ and the calculated value of $\mu_R^2$.

A similar analysis can be conducted in both the quark and lepton sectors. In the gauge basis, the up and down quark mass matrices can be cast as

$$M_u = \frac{v}{\sqrt{2}} \begin{bmatrix} (Y_u)_{11} & (Y_u)_{12} & (Y_u)_{13} \\ (Y_u)_{21} & (Y_u)_{22} & (Y_u)_{23} \\ (Y_u)_{31} & (Y_u)_{32} & (Y_u)_{33} \end{bmatrix}, \quad M_d = \frac{v}{\sqrt{2}} \begin{bmatrix} (Y_d)_{11} & (Y_d)_{12} & (Y_d)_{13} \\ (Y_d)_{21} & (Y_d)_{22} & (Y_d)_{23} \\ (Y_d)_{31} & (Y_d)_{32} & (Y_d)_{33} \end{bmatrix},$$

(B7)

where $Y_u$ and $Y_d$ are Yukawa couplings, which originate from the operators $(Y_u)_{ij} Q^i_L u^j_R H$ and $(Y_d)_{ij} Q^i_L d^j_R H$, respectively. Here, we have defined $H = i \sigma_2 H^\dagger$ with $\sigma_2$ being the second Pauli matrix. The diagonalization results in the mass of the quarks as

$$M_u^\text{diag} = U_{u,L} M_u U_{u,R}^\dagger, \quad M_d^\text{diag} = U_{d,L} M_d U_{d,R}^\dagger,$$

(B8)
such that the Cabibbo–Kobayashi–Maskawa (CKM) matrix is given by $V_{\text{CKM}} = U_{u,L}^\dagger U_{d,L}$. For simplicity of the numerical analysis, the up sector will be made diagonal, such that the CKM is generated from the down sector, that is, $V_{\text{CKM}} \equiv U_{d,L}$. The charged lepton matrix can be written as

$$M_e = \frac{v}{\sqrt{2}} \begin{bmatrix} (Y_e)_{11} & (Y_e)_{12} & (Y_e)_{13} \\ (Y_e)_{21} & (Y_e)_{22} & (Y_e)_{23} \\ (Y_e)_{31} & (Y_e)_{32} & (Y_e)_{33} \end{bmatrix}$$  \hspace{0.5cm} (B9)$$

where $Y_e$ is a Yukawa coupling for the interaction term $(Y_e)_{ij} \bar{L} e R H$, and can be diagonalized in a similar fashion as in the quarks, i.e. $M_e^{\text{diag}} = U_{e,L} M_e U_{e,R}^\dagger$. For the numerical analysis that follows, we consider the diagonal basis, such that $U_{e,R} = U_{e,L} = \mathbb{1}_{3 \times 3}$ and therefore, the entirety of the Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix is located in the neutrino sector. Similarly to the LQs, these mass relations can be inverted such that the fermion masses are reproduced by

$$Y_d = \frac{\sqrt{2}}{v} V_{\text{CKM}}^\dagger M_d^{\text{diag}} U_{d,R}, \hspace{0.5cm} Y_u = \frac{\sqrt{2}}{v} M_u^{\text{diag}}, \hspace{0.5cm} Y_e = \frac{\sqrt{2}}{v} M_e^{\text{diag}}. \hspace{0.5cm} (B10)$$

The neutrino mass matrix is one-loop generated and is given by the equation (5) in the main text,

$$(M_\nu)_{ij} = \frac{3}{16\pi^2 m_s^{2/3} - m_{\pi}^{2/3} v^2} \ln \left( \frac{m_{\pi}^{2/3}}{m_s^{2/3}} \right) \sum_{m,a} (m_a)_{ij} V_{am} (\Theta_{im} \Omega_{ja} + \Theta_{jm} \Omega_{ia}), \hspace{0.5cm} (B11)$$

and can be diagonalized via the PMNS matrix $M_\nu^{\text{diag}} = U_{\text{PMNS}} M_\nu U_{\text{PMNS}}^\dagger$. As in the previous cases, it can be inverted such that the neutrino mass differences as well as the mixing angles can be given as input. In this case, however, we do not obtain a closed-form formula for the inversion in terms of the physical input parameters and instead we numerically invert equation (B11).

With this in mind, a numerical scan over all relevant parameters of the model is then conducted. In particular, we perform an inclusive logarithmic scan over the various parameters within the ranges shown in Tab. IV.

| $m_{s_1^{1/3}}$, $m_{s_2^{1/3}}$ (TeV) | $|g_{HS}:g_{H\bar{R}},g_{\bar{H}R}|$, $|Y|$, $|\Theta|$, $|\Omega|$ | $\alpha_1$ (GeV) |
|-----------------------------------|--------------------|------------------|
| [1.5, 10]                         | $[10^{-8}, 4\pi]$ | $[10^{-8}, 4\pi]$ | $[10^{-8}, 100]$ |

TABLE IV: Ranges used for the free parameters during the numerical scan. The values for the masses of the SM fields and corresponding mixings were varied within the allowed experimental ranges.

Once valid solutions are found within the first initial random scan, we then use these points as seeds for finding new solutions in subsequent runs, by perturbing around the valid couplings/masses in order to find new consistent points. Do note that not all $\Theta$ and $\Omega$ Yukawas are free parameters, with some being calculated through the inversion procedure of the neutrino mass matrix. In this regard, within the GitHub page (https://github.com/Mrazi09/Leptoquark-project---Data) one can find auxiliary jupyter notebooks, which demonstrate how to numerically implement the inversion procedure for the neutrinos/quark/charged leptons and LQs (named Neutrino_inversion.ipynb) as well as how to utilise the data to extract the relevant neutrino observables (named Read_neutrino_v3.ipynb).

To analyse the quality of the fit, we consider the following $\chi^2$ function

$$\chi^2 = (\mathcal{O}_{\text{exp}} - \mathcal{O}_{\text{th}})^T (\Sigma_{\text{th}} + \Sigma_{\text{exp}})^{-1} (\mathcal{O}_{\text{exp}} - \mathcal{O}_{\text{th}}) \hspace{0.5cm} (B12)$$

using the observables indicated in the tables of the previous section. In (B12) $\mathcal{O}_{\text{exp}}$ and $\mathcal{O}_{\text{th}}$ represent vectors of experimental values and the model prediction, respectively, while $\Sigma_{\text{exp}}$ is the experimental covariance and $\Sigma_{\text{th}}$ is the theoretical one. Both covariance matrices can be computed using well-known formulas

$$\Sigma_{\text{th}} = \sigma_{\text{th}} \rho_{\text{th}} \sigma_{\text{th}}, \hspace{0.5cm} \Sigma_{\text{exp}} = \sigma_{\text{exp}} \rho_{\text{exp}} \sigma_{\text{exp}}, \hspace{0.5cm} (B13)$$

where $\sigma_{\text{th}}$ ($\sigma_{\text{exp}}$) are diagonal matrices whose entries are the $1\sigma$ theoretical (experimental) errors and $\rho_{\text{th}}$ ($\rho_{\text{exp}}$) are the theoretical (experimental) correlation matrices. For the experimental inputs, the experimental uncertainties can be easily extracted from literature, while for the experimental correlations, we extract those that are available and neglect if those do not exist. The various uncertainties and correlations were taken from the references inside Tab. I.
For the theoretical inputs, the errors can be computed inside `flavio` [74], with the function `flavio.np_uncertainty` for each of the observables of interest. This also takes into account potential hadronic uncertainties that exist for observables sensitive to these. As for the theoretical correlations, those can be computed from our entire dataset using standard methods available in statistics libraries. In our case, we have use Pearson’s algorithm through the `pandas` package [88].

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