Optical klystron undulator radiation with electromagnetic wave undulator

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Abstract. In the present study we proposed optical klystron in an electromagnet wave undulator. We consider the circularly polarized electromagnetic waves are travelling collinearly in opposite direction formed a beat wave. Spectral properties of the beat wave optical klystron are analysed and investigated using Lienard-Wiechert potential. We observed spikes as well as sidebands on the either side of the fundamental frequency.

1. Introduction
Scientist and researchers have been developed interests in optical klystron for free electron lasers (FEL) due to the enhancement of the small gain. In optical klystron gain of the FEL can improved in weak optical field and small gain [1]. Optical klystron consist of two undulators are separated by drift section[2]. First section is responsible for the bunching of the electrons and these electrons leads to energy modulation. In the drift section the density modulation takes place. The third section worked as a radiator for the emission of electron bunching[3]. Based on the prebunching of the electron beams, theoretical [4] and experimental study carried out [5]. The OK gives a higher signal gain than an undulator. The theory of high gain OK operating in high gain regime has been discussed by several authors[6-7]. Two stage OK undulator can improve gain in weak optical fields[8-9]. There are several studies on OK undulator that explains the modeling of the drift section[10]. During the last several years the undulator technology has been improved substantially and number of new design concepts have been formulated and proposed. Electromagnetic wave undulator has their own advantages of fast switching of the polarization of light over the conventional undulator[11]. The beat frequency electromagnetic wave undulator consist of two circularly polarized electromagnetic wave are travelling opposite to each other. In plane polarized wave beat frequency undulator cannot obtained. In this paper we concentrate on the beat wave optical klystron electromagnetic wave undulator radiation. The construction of the beat wave is very simple due to the circularly polarized electromagnetic wave. Two collinearly circularly polarized waves are travelled in opposite direction are eligible to form an electromagnetic wave undulator[12]. In this system of the optical klystron is obtained from two electromagnetic waves slightly difference between wavelengths are separated by the drift section. Two frequency scheme shows the improvement in the intensity and complex spectra at the higher harmonics. The two frequency scheme is rich in harmonics due to electron motion modulation and generation of sidebands that corresponds to the sum and difference frequencies of the undulator field.
2. OK Undulator Radiation Single Circularly Polarized Wave

The energy radiated per unit solid angle per unit frequency interval i.e. the brightness is given by the Lienard-Wiechert integral[13],

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi c} \left[ n \times (n \times \beta) \right] \exp \left[ i\omega \left( t - \hat{n} \cdot \vec{r} \right) \right] dt$$

(1)

Where $\hat{n}$ is a unit vector determining the direction of observation, $\vec{r}$ refers to the particle position, and $\beta$ to its reduced velocity. Consider the electron motion of a relativistic electron in a field consist of a circularly polarized electromagnetic wave. The magnetic field and electric field of the system is given by,

$$B_{ni}(z,t) = B_{ni} \left[ \hat{x} \cos(k_{ni} z + \omega_{ni} t) - \hat{y} \sin(k_{ni} z + \omega_{ni} t) \right]$$

$$E_{ni}(z,t) = \frac{\omega_{ni}}{k_{ni} c} B_{ni} \left[ \hat{x} \sin(k_{ni} z + \omega_{ni} t) + \hat{y} \cos(k_{ni} z + \omega_{ni} t) \right]$$

(2)

In equation (2) $B_{ni}$ denote the amplitude of the undulator field, $\omega_{ni}$ is the frequency and $k_{ni}$ describes the wave number. The trajectories are obtained by Lorentz force equation,

$$x = \frac{cK_i}{\gamma\Omega_i} \sin(\Omega_i t), \quad y = \frac{cK_i}{\gamma\Omega_i} \cos(\Omega_i t), \quad z = \beta^* ct$$

(3)

Where, $\beta^* = 1 - \frac{1}{2\gamma^2} (1 + K_i^2)$ $K_i = \frac{eB_i \lambda_i}{2\pi mc}$ and $\Omega_i = k_{ni} \gamma + \omega_{ni}$

Oscillating term of Equation (1) can be written as,

$$\exp \left[ i\omega t \frac{c}{\gamma} (\hat{n} \cdot \vec{r}) \right] = \exp \left[ i\omega t - \frac{c}{\gamma} \left\{ \beta^* c + \frac{e\psi \cos\phi K_i}{\gamma\Omega_i} \sin(\Omega_i t) + \frac{\psi c \sin\phi K_i}{\gamma\Omega_i} \cos(\Omega_i t) \right\} \right]$$

(4)

To rewrite Equation (4) we use the generating function involving generalized Bessel function of first kind,

$$\sum e^{-i\omega t} D_m(x, y; z, u) = \exp \left( x \cos \theta + y \cos 2\theta - i \{ z \sin \theta + u \sin 2\theta \} \right)$$

(5)

Then Equation (4) can be re-written in the following form,

$$\exp \left[ i\omega t - \frac{c}{\gamma} (\hat{n} \cdot \vec{r}) \right] = \left[ \sum_{m=-\infty}^{\infty} D_m(\xi_1, 0, \xi_2, 0) \times \exp \left[ i \left\{ \frac{\omega}{2\gamma^2} (1 + K_i^2 + \gamma^2 \psi^2) \right\} - (m\Omega_i) t \right] \right]$$

(6)

Where

$$\xi_1 = \frac{\psi \sin\phi K_i \omega}{\gamma\Omega_i}, \quad \xi_2 = \frac{\psi \cos\phi K_i \omega}{\gamma\Omega_i}$$

The optical klystron consists of two undulators separated by a drift section. In this case, the time integral in Equation (6) is read as,

$$\int_0^T dt(....) = \int_0^T dt(....) + \int_{T(d+D)}^{T(2+D)} dt(....)$$

(7)

Where D is the drift section and it is given by $L_d/L$. $L$ is the length of two undulator sections and $L_d$ is the length of the drift section. Using Equation (7), Equation (1) can be rewritten as,
\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} \left[ \int_0^{T_{(2+D)}} \left( \hat{n} \times (\hat{n} \times \hat{\beta}) \right) \exp \left\{ i\omega \left( \frac{t}{c} \right) \right\} dt \right]^2
\]

(8)

Defining the terms,

\[
T_{\text{st}} = \int_0^{T_{(2+D)}} \left( \hat{n} \times (\hat{n} \times \hat{\beta}) \right) \exp \left\{ i\omega \left( \frac{t}{c} \right) \right\} dt
\]

\[
T_{\text{stll}} = \int_{T_{(1+D)}}^{T_{(2+D)}} \left( \hat{n} \times (\hat{n} \times \hat{\beta}) \right) \exp \left\{ i\omega \left( \frac{t}{c} \right) \right\} dt
\]

(9)

Equation (8) is simplified to read as,

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2}{4\pi^2 c} |T_{\text{st}} + T_{\text{stll}}|^2
\]

(10)

Solving Equation (9) we get

\[
T_{\text{st}} = \frac{T_l}{2} \exp \left( i\nu \left( \frac{\nu}{2} \right) \sin c \left( \frac{\nu}{2} \right) \right) \left[ 1 + \exp \left[ i\left\{ \nu (1+D) \right\} \right] \right]
\]

\[
T_{\text{stll}} = \frac{T_{l'}}{2} \exp \left( i\nu \left( \frac{\nu}{2} \right) \sin c \left( \frac{\nu}{2} \right) \right) \left[ 1 + \exp \left[ i\left\{ \nu (1+D) \right\} \right] \right]
\]

(11)

In Equation (11), the term \((1+D)\) is the dimensionless time which is calculated from entering in the buncher and the drift section to leaving the radiator and the detuning parameters are defined by,

\[
\nu = \left( \frac{\omega}{\Lambda} - m\Omega \right) \frac{T}{2}, \quad \Lambda = \frac{2\gamma^2}{1 + K_i^2 + \gamma^2 \nu^2}
\]

\[
I_1 = \frac{K_1}{2\gamma} \left[ J_{m-1}(0) + J_{m+1}(0) \right]
\]

\[
I_2 = \frac{K_1}{2\gamma} \left[ J_{m-1}(0) - J_{m+1}(0) \right]
\]

Collecting the terms Equation (1) is put in the form as,

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2 T^2 \left( I_1^2 + I_2^2 \right)}{16\pi^2 c} \sin c \left( \frac{\nu}{2} \right) \left[ 1 + \exp \left[ i\left\{ \nu (1+D) \right\} \right] \right]^2
\]

(12)

In the case when \(I_1 = I_2\), Equation (12) we rewrite as,

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2 T^2 I_1^2}{16\pi^2 c} \left[ 2 + 2 \cos \left\{ \nu (1+D) \right\} \right] \sin c \left( \frac{\nu}{2} \right)
\]

(13)

Equation (13) reads the intensity of the optical klystron electromagnetic wave undulator radiation.

3. Beat Frequency OK Undulator Radiation Two Circularly Polarized Wave

Consider the electromagnetic wave system of the electromagnetic wave undulator in V. Huse et.al. [12].
\[
B_{i_1}(z,t) = B_{i_1} \left[ \hat{x} \cos(k_{i_1} z + \omega_{i_1} t) - \hat{y} \sin(k_{i_1} z + \omega_{i_1} t) \right]
\]
\[
B_{i_2}(z,t) = B_{i_2} \left[ \hat{x} \cos(k_{i_2} z + \omega_{i_2} t) - \hat{y} \sin(k_{i_2} z + \omega_{i_2} t) \right]
\]
\[
E_{i_1}(z,t) = \frac{\omega_{i_1}}{k_{i_1} c} B_{i_1} \left[ \hat{x} \sin(k_{i_1} z + \omega_{i_1} t) + \hat{y} \cos(k_{i_1} z + \omega_{i_1} t) \right]
\]
\[
E_{i_2}(z,t) = \frac{\omega_{i_2}}{k_{i_2} c} B_{i_2} \left[ \hat{x} \sin(k_{i_2} z + \omega_{i_2} t) + \hat{y} \cos(k_{i_2} z + \omega_{i_2} t) \right]
\]

(14)

On solving equation (14) and using the Equation (6) and equation (9) we obtain the spectral brightness terms of the electromagnetic undulator optical klystron as follows,

\[
I_1 = -\frac{K_1}{2\gamma} \left[ J_{m-1}(0) + J_{m+1}(0) \right] J_\eta(0) J_\nu(\xi) + \frac{K_2}{2\gamma} \left[ J_{m-1}(0) + J_{m+1}(0) \right] J_\eta(0) J_\nu(\xi)
\]
\[
I_2 = \frac{K_1}{2\gamma} \left[ J_{m-1}(0) - J_{m+1}(0) \right] J_\eta(0) J_\nu(\xi) + \frac{K_2}{2\gamma} \left[ J_{m-1}(0) - J_{m+1}(0) \right] J_\eta(0) J_\nu(\xi)
\]

(15)

Where \( K_1 = \frac{eB_{i_1} \lambda_{i_1}}{2\pi mc^2} \) and \( \xi = -\frac{K_1 \lambda_{i_1} \omega}{\gamma^2 \Delta \Omega} \)

Collecting the terms Equation (6) is put in the form as,

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2 T^2}{16\pi^2 c} \sin^2 \left( \frac{\nu}{2} \right) \left[ 1 + \exp \left[ i \nu (1 + D) \right] \right]^2
\]

(16)

In the case when \( K_1 = K_2 \) we Equation (16) reduces to,

\[
\frac{d^2 I}{d\omega d\Omega} = \frac{e^2 \omega^2 T^2}{16\pi^2 c} \left[ 2 + 2 \cos \left( \nu (1 + D) \right) \right] \sin^2 \left( \frac{\nu}{2} \right)
\]

(17)

Equation (17) is the intensity of the beat frequency optical klystron electromagnetic wave undulator radiation.

4. Result and discussions:

In this paper we have discussed the spectral properties of optical klystron with electromagnetic wave undulator. In figure 1, D is the dispersion section by which two electromagnetic wave undulator of the same length are separated to form the optical klystron. These two electromagnetic waves are circularly travelling collinearly in opposite is direction.

Figure 1. Schematic of the optical klystron with beat frequency electromagnetic wave undulator.
We have explained the spectral properties of the undulator with the help of Lienard–Wiechert potential. We compared the two frequency undulator and beat frequency electromagnetic wave undulator. In the present system two electromagnetic wave gives rise to longitudinal velocity modulation of electron motion at the beat frequency.

The detuning parameter is given by,

\[ \nu = \left( \frac{\omega(1 + K_1^2 + K_2^2 + \gamma^2 \psi^2)}{2\gamma^2} - (m\Omega_1 + n\Omega_2 + p\Delta\Omega) \right) \frac{T}{2} \]

In Equation (16) the function \( \sin^2(\nu/2) \) equal to 1 when \( x \) is close to zero which describes the resonance condition for FEL.

\[ \omega = \left( \frac{2\gamma^2 (m\Omega_1 + n\Omega_2 + p\Delta\Omega)}{1 + K_1^2 + K_2^2 + \gamma^2 \psi^2} \right) \]

Figure 2. Intensity spectrum for optical klystron electromagnetic wave undulator.

In case of the single circularly polarised electromagnetic wave only on axis emission of radiations are possible due to the constant motion of the electrons. In two circularly polarised, oppositely propagated electromagnetic wave electrons having transverse oscillations. We consider the two lasers of slightly difference between their wavelengths having the peak power of 40 TW. We consider the undulator parameter K1 and K2 are equal.

The modulation of electrons longitudinal velocity at \( \Omega_1, \Omega_2 \) and \( \Delta\Omega \). The form factor \( J_p(\xi_5) \) appeared in Equation (15) is responsible for the modification of the optical klystron undulator. We get the sidebands and spikes on the either side of the fundamental frequency. The modifications are appeared in the form of sidebands and spikes on the right and left side of the on axis fundamental frequency. For typical undulator parameters as shown in Figure 2 the fundamental frequency is \( \omega = 7.667 \times 10^{17} \) rad/sec. Denoting the right nearest spike by \( \omega^+ \) and left nearest spike by \( \omega^- \). We define the frequency width by \( \Delta \omega = \omega^+ - \omega^- \). For \( D=2 \) we have \( \Delta \omega = 0.011 \times 10^{17} \) rad/sec and \( \omega^- = 7.656 \times 10^{17} \) rad/sec. Considering that sidebands due to electron oscillations on the either side of the fundamental frequency contain a peak at \( \Omega_1 - \Delta\Omega (p = -1) \) and \( \Omega_2 + \Delta\Omega (p = 1) \) at \( \omega_{p=-1} = 7.574 \times 10^{17} \) rad/sec and \( \omega_{p=+1} = 7.762 \times 10^{17} \) rad/sec. The spikes on the left side of \( \omega_{p=-1} \)
denoted by $\omega_{LP} = 7.750 \times 10^{17}$ rad/sec and right side $\omega_{RP} = 7.775 \times 10^{17}$ rad/sec. The spikes on the left side of $\omega_{P_{-1}}$ denoted by $\omega_{LP} = \omega_{LP} = 7.561 \times 10^{17}$ rad/sec and right side $\omega_{RP} = 7.586 \times 10^{17}$ rad/sec. In Figure 3, as one increases the beat frequency $\Delta \Omega$, frequency width $\Delta \omega$ increases linearly. In Figure 4, there is increment as well as decrement of normalized intensity does not depend on the D, it depends on the form factor $J_p(\xi)$ in Equation (15).

![Figure 3. beat frequency ($\Delta \Omega$) versus $\Delta \omega$](image1)

![Figure 4. Normalized intensity versus $\Delta \lambda_0$ (m)](image2)

**Acknowledgement**

Author is very thankful to Dr. A.M. Garode, Principal, Shri Shivaji Science and Arts College Chikhli.
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