Superloop space

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Abstract – In this paper we will construct and analyse the superloop space formulation of a $\mathcal{N} = 1$ supersymmetric Chern-Simons theory in three dimensions. We will obtain expressions for the connection and curvature in this superloop space in terms of ordinary supergauge fields. This curvature will vanish, unless there is a monopole in the spacetime. We will also construct a quantity which will give the monopole charge in this formalism. Finally, we will show how these results even hold for a deformed superspace.

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Introduction. – In this paper we will construct a superloop space formulation of a $\mathcal{N} = 1$ supergauge theory. To do that we will generalize Polyakov variables to superspace. Polyakov variables have been used to construct a loop space formulation of ordinary gauge theories [1]. In this formulation, the Polyakov variables take values in the Lie algebra and depend on the parameterized loop in the abstract infinite-dimensional loop space. The loops are formed by first parameterizing the full loop space and then constructing functionals on this parameterized loop space. The Polyakov variables act as connection on loop space and are constructed in analogy with the connections in the regular gauge theory. Thus, the Polyakov variables measures the change in phase as one moves from one point in the loop space to a neighboring point. They can also be used to construct a curvature for the loop space [2,3].

This curvature vanishes when the Bianchi identities are satisfied. Thus, it vanishes when no monopoles exist. However, when monopoles exist this curvature does not vanish. A loop can also be constructed in the loop space [4]. This loop in the loop space covers a surface in spacetime and can thus be used to obtain the non-Abelian monopole charge. These results are know to hold for ordinary gauge theories. In this paper we generalize these results to three-dimensional supergauge theories with $\mathcal{N} = 1$ supersymmetry. In order to do that we will first review a superspace formalization of supergauge theories in three dimensions.

Supersymmetric gauge theories in three dimensions have been thoroughly studied in the $\mathcal{N} = 1$ superspace formalism [5–8]. These theories have become very important due to the discovery of the BLG theory [9–12] and the ABJM theory [13–16]. These theories are Chern-Simons matter theories which that are thought to be the low-energy theories for multiple M2-branes. The ABJM theory has only $\mathcal{N} = 6$ supersymmetry, however, by the use of the monopole operators it is possible to obtain the full $\mathcal{N} = 8$ supersymmetry [17–20]. Thus, starting from the supersymmetric Chern-Simons matter theory which is a truncated version of the ABJM theory, a monopole operator can be used to show that there is an additional $\mathcal{N} = 2$ supersymmetry associated with a particular gauge group. This additional supersymmetry can combine with the $\mathcal{N} = 6$ supersymmetry of the original ABJM theory to an enhanced $\mathcal{N} = 8$ supersymmetry with that particular gauge group. Hence, it is important to study to effect of monopoles in three dimensions.

Superspace coordinates for the $\mathcal{N} = 1$ supersymmetry in three dimensions are parameterized by the three-dimensional bosonic coordinates $x^\mu$ and a two-component Grassman number $\theta^a$. The spinor indices are raised and lowered by $C^{ab}$ and $C_{ab}$, respectively. It is useful to define $x^{ab} = (\gamma^\mu x_\mu)^{ab} \partial_{\theta^a} = (\gamma^\mu \partial_\mu)^{ab} \theta^a$ and $\theta^2 = C^{ab} \theta_a \theta_b / 2$.

The $\mathcal{N} = 1$ supersymmetry in three dimensions is generated by $Q_a = \partial_a - \theta^b \partial_{ab}$. This generator of supersymmetry commutes with $D_a = \partial_a + \theta^b \partial_{ab}$. We now start with a scalar superfield which transforms like $\delta \psi = i \Lambda \psi$, where $\psi = \psi^A T_A$, and $[T_A, T_B] = i f_{AB}^C T_C$. Now we can define a covariant derivative for these scalar superfield by requiring the covariant derivative to transforms as the original scalar superfield. This covariant derivative is given by $\nabla_a = D_a - i \Gamma_a$, where $\Gamma_a = \Gamma^A a T_A$ is a spinor superfield which transforms as $\delta \Gamma_a = \nabla_a \Lambda$. We also define a vector covariant
Yang-Mills theory with superloop variables for a three-dimensional super-Yang-Mills theory is given by
\[ \mathcal{L} = \mathcal{L}^A = \frac{1}{2} F_{ab} F^{ab} \]
where \( F_{ab} = \partial_a \Phi_b - \partial_b \Phi_a - i [\Phi_a, \Phi_b] \).

For a geometric viewpoint as \( z^A = (x^{ab}, \theta^a) \), so, it is natural to regard \( \Gamma_a \) and \( \Gamma_{ab} \) as components of a superform \( \Gamma_A = (\Gamma_a, \Gamma_{ab}) \). So, we define \( D_A = (\partial_a, \theta^a) \)
and \( \nabla_A = D_A - i \Gamma_A \). Now we have \( \{\nabla_A, \nabla_B\} = T^C_{AB} \nabla_C - i F_{AB} \).
The Bianchi identities are given by \( \{\nabla_A, \{\nabla_B, \nabla_C\}\} = 0 \), where \( [\cdot, \cdot] \) is the graded antisymmetrization symbol. It is identical to the antisymmetrization symbol but with extra factor of \((-1)^{ab}\) for each pair of interchanged fermionic indices. Thus, the Bianchi identities can be written as \( [\nabla_A, H_{BC}] = 0 \).

It may be noted that for \( \Gamma_{ab} \) to be defined from \( \Gamma_a \), we have set \( F_{ab} = 0 \) as a constraint. We now define \( W_a \) as \( \{\nabla_a, \nabla_{bc}\} = C_{abc} W_c \), and so we get \( W_a = \frac{1}{2} D^b D_a \Gamma_a - \frac{i}{2} \left[ \Gamma_a, \Gamma_b \right] \). Now \( \lambda_a = [W_a] \) and \( f_{ab} = [D_a W_b] = [D_b W_a] \) is the spinor form of the usual field strength \( (F_{\mu
u})^a_{\mu\nu} \). It may be noted that \( \{\nabla_a, \nabla_{bc}\} = -\frac{1}{2} \delta^{(a}_b f^{c)b} \).

Different deformations of the superspace occurs due to various backgrounds in the string theory. The presence of a constant NS-NS background gives rise to noncommutativity [21–26] and a RR background gives rise to non-anticommutativity [27–30]. Also, a graviphoton background give rise to a noncommutativity between spacetime and superspace coordinates [31–34]. Noncommutative deformations generated by the NS-NS and graviphoton backgrounds do not break any supersymmetry. As we are studying \( N = 1 \) supersymmetric theories in three dimensions, any non-anticommutative deformation will break all the supersymmetry. So, we will only analyse noncommutative deformation of the superspace. Thus, in this paper we will only analyse a noncommutative deformation of superloop formalization of supergauge theories.

**Superloop variables.** – In this section we will construct superloop variables for a three-dimensional super-Yang-Mills theory with \( N = 1 \) supersymmetry. The Lagrangian for this theory is formed from a combination of the gauge fields and the fermionic fields. These fields transform under the action of the generator \( Q_a = \partial_a - \theta^a \partial_a \) and these generators satisfy the \( N = 1 \) superalgebra, \( \{Q_a, Q_b\} = 2 \delta_{ab} \). The Lagrangian for this super-Yang-Mills theory is given by \( \mathcal{L} = D^2 |W|^2 \). In component form this Lagrangian is given by \( \mathcal{L} = \frac{1}{2} \left( \partial_a \Phi_b - \partial_b \Phi_a - i [\Phi_a, \Phi_b] \right)^2 \).

In order to capture such effect in super-Yang-Mills theories we need to construct a superloop formalization for them.

In order to do that we first construct the coordinates for each point in the superloop space. These coordinates parameterizing the superloop space are \( \xi^A = (\xi^{ab}, \xi^a) \),

\[ C : \{\xi^A(s) : s = 0 \rightarrow 2\pi, \xi^A(0) = \xi^A(2\pi)\} \]

where \( \xi^A(0) = \xi^A(2\pi) \) is a fixed point in the superloop space. Now we can define a superloop variable as a functional on the set of all such functions

\[ \Phi[\xi] = P_s \exp i \int_0^{2\pi} \Gamma^A(\xi(s)) \frac{d\xi^A}{ds} \]

where \( \Gamma^A(\xi(s)) \) is a scalar superfield from the supersymmetric point of view,

\[ \left[ \Phi[\xi] \right]_a = \phi_a(\xi), \left[ D_a \Phi[\xi] \right]_a = \phi_a(\xi), \]

\[ (D^2 \Phi[\xi])^a_b = \delta^{(a}_b \phi(\xi), \]

where \( \phi_a(\xi), \phi_a(\xi), \phi(\xi) \) are loop variables formed from the component fields of the super-Yang-Mills theory. They are thus regular loops formed from linear combinations of various fields that exists in the super-Yang-Mills theory. Now using \( \Phi[\xi] \), we can define

\[ F_A[\xi|s] = i \Phi^{-1}[\xi]\frac{\delta}{\delta \xi^A(s)} \Phi[\xi]. \]

Here each of these components of \( F_A[\xi|s] = (F_{ab}[\xi,s], F_a[\xi,s]) \) is obtained by taking a vector or a spinor derivative. This equation can be understood as a parallel phase transport first to \( \xi(s) \) along some path, followed by a detour at \( s \), and then backward along the same path. The phase factors generated by first going forward and then going backward from \( \xi(s) \) cancel each other. However, the phase factor generated by taking a detour at \( s \) generates \( H^{\mu
u(AB)} \) because of the transport along the infinitesimal circuit at \( \xi(s) \). Thus, we can write

\[ F_A[\xi|s] = \Phi^{-1}(\xi : s, 0) H^{\mu
u(AB)}(\xi(s)) \Phi(\xi : s, 0) \frac{d\xi^B}{ds} \]

where

\[ \Phi[\xi : s_1, s_2] = P_s \exp i \int_{s_1}^{s_2} \Gamma^A(\xi(s)) \frac{d\xi^A}{ds} \]

is the parallel transport from a point \( \xi(s_1) \) to \( \xi(s_2) \) along the path parametrized by \( \xi \). Thus, we parallel transport first forward to \( s \) and then take a detour at \( s \) and then turn backwards again along the same path. The phase
factor for the segment of the superloop beyond \( s \) cancels and the factor for the remainder does not. The detour at \( s \) gives a truncated phase factor and the infinitesimal circuit generated at \( s \) gives rise to \( H^{AB}(\xi(s)) \). It may be noted that \( F^A[\xi|s] \) is a connection in the superloop space and not in spacetime. It is proportional to the field strength in spacetime. In the next section we will show that this connection is flat as the field strength corresponding to it vanishes due to the Bianchi identity. However, in the presence of a monopole the Bianchi identity does not hold, so this connection is not flat in the presence of a monopole.

Now as \( F^A[\xi|s] \) represents the change in the phase of \( \Phi[\xi] \) as one moves from one point in the superloop space to a neighboring point, we can regard it as a connection in the superloop space. Thus, in analogy with ordinary gauge theories, we can proceed to construct a curvature for the superloop space. The local change in the phase as a point moves around an infinitesimal closed circuit in the superloop space will now be given by the curvature \( G_{AB}[\xi, s_1, s_2] \), where

\[
G_{AB}[\xi, s_1, s_2] = \frac{\delta}{\delta \xi_B(s_2)} F^A[\xi|s_1] - \frac{\delta}{\delta \xi_A(s_1)} F^B[\xi|s_2] + i [F_A[\xi|s_1], F_B[\xi|s_2]].
\]

(8)

**Monopoles.** – In this section we will show that the loop space curvature vanishes unless monopoles are present in the spacetime. To obtain this result, we first have to express the connection in the superloop space in terms of usual field variable. In order to do that, we first evaluate the value of \( \Phi^{-1}[\xi_2] \Phi[\xi_3] - \Phi^{-1}[\xi] \Phi[\xi_1] \), where \( \xi_2^A(s) = \xi_1^A(s) + d\xi^A(s) \), \( \xi_3^A(s) = \xi_1^A(s) + d\xi^A(s) \), and \( \Phi^A(s) = \Phi^A + d\Phi^A \). By repeating the argument used in the derivation of eq. (6), we obtain the following result:

\[
\Phi[\xi_1] = \Phi[\xi] - i \int ds \Phi(\xi, 2\pi, s) H^{AB}(\xi(s)) \frac{d\xi_B(s)}{ds} \delta\xi_A(s) \Phi(\xi, s, 0),
\]

(9)

A similar expression can be obtained for \( \Phi[\xi_2] \). Furthermore, we also have

\[
\Phi[\xi_3] = \Phi[\xi_1] - i \int ds \Phi(\xi_1, 2\pi, s) H^{AB}(\xi_1(s)) \frac{d\xi_B(s)}{ds} \delta\xi_A(s) \Phi(\xi_1, s, 0).
\]

(10)

Now we can write

\[
\Phi(\xi_1, s, 0) = \Phi(\xi, s, 0) - i \int ds' \Phi(\xi, s, s') H^{AB}(\xi(s')) \frac{d\xi_B(s')}{ds'} \delta\xi_A(s') \Phi(\xi, s', 0) + i F^A(\xi(s)) \Phi(\xi, s, 0) \delta\xi_A(s).
\]

(11)

Here the last term is due to the variation of the end-point in the integral for \( \Phi(\xi, s, 0) \). A similar expression can be written for \( \Phi(\xi, 2\pi, s) \). Collecting all the variations, we can write

\[
\delta \frac{d}{d\xi_B(s_2)} F^A[\xi|s_1] = i [F^B[\xi|s_2], F^A[\xi|s_1]] \delta(s_1 - s_2)
\]

\[
+ \Phi^{-1}(\xi, s_1, 0) \nabla A H^{BC}(\xi(s)) \Phi(\xi, s_2, 0)
\]

\[
\times \frac{d\xi_B(s_1)}{ds_1} \delta\xi_C(s_1) \Phi(\xi, s_1, 0) - \Phi^{-1}(\xi, s_2, 0) H^{BC}(\xi(s_2)) \Phi(\xi, s_2, 0)
\]

\[
\times \frac{d}{ds_2} \delta\xi_C(s_1) \delta(s_1 - s_2).
\]

(12)

Thus, we get

\[
G_{AB}[\xi, s_1, s_2] = \Phi^{-1}(\xi, s_1, 0) [\nabla_A, H_{BC}] \Phi(\xi, s_2, 0) \frac{d\xi_C(s_1)}{ds_1} \delta(s_1 - s_2).
\]

(13)

Now if the Bianchi identity holds, \( [\nabla_A, H_{BC}] = 0 \), then, \( G_{AB}[\xi, s_1, s_2] = 0 \). It may also be noted that this curvature is proportional to \( \delta(s_1 - s_2) \).

We have now seen that the curvature of the superloop space vanishes if the Bianchi identity holds. However, if a monopole exists then at places where the superloop space intersects with the world line of a monopole, the Bianchi identity need not hold. Thus, if a monopole exists the curvature tensor of the superloop space will not vanish. So, if monopoles are present then \( [\nabla_A, H_{BC}] \neq 0 \), and thus, \( G_{AB}[\xi, s_1, s_2] \neq 0 \). In other words if \( G_{AB}[\xi, s_1, s_2] \neq 0 \) then the superloop is intersecting word-lines of a monopole.

In order to analyse this further, we define a loop in the superloop space, as follows:

\[
\Sigma : \{ \xi^A(t : s), s = 0 \to 2\pi, t = 0 \to 2\pi \},
\]

(14)

where

\[
\xi^A(t : 0) = \xi^A(2\pi : s), \quad t = 0 \to 2\pi,
\]

\[
\xi^A(0 : s) = \xi^A(2\pi : s), \quad s = 0 \to 2\pi.
\]

(15)

Thus, for each \( t \), \( \xi^A(t : s) \) represents a closed superloop passing through a fixed point. As \( t \) varies a curve in the superloop space is constructed. Now we can define a loop variable for this space as

\[
\Theta(\Sigma) = P_t \exp i \int_0^{2\pi} dt \int_0^{2\pi} ds F^A(\xi(t : s)) \frac{d\xi_A(s)}{dt},
\]

(16)

and \( P_t \) denotes ordering in \( t \) increasing from right to left and the derivative is taken from below. It may be noted that \( \Theta(\Sigma) \) is also a scalar superfield from the supersymmetric point of view,

\[
[\Theta(\Sigma)] = \theta(\Sigma), \quad [D_a \Theta(\Sigma)] = \theta_a(\Sigma),
\]

(17)

where \( \theta(\Sigma), \theta_a(\Sigma), \hat{\theta}(\Sigma) \) are loop variables formed from the component fields of the super-Yang-Mills theory.

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Here $F^A[\xi|s]$ plays the role of connection, with the difference that it is infinite dimensional. Thus, apart from the sum over $\mu$, we have to also integrate over $s$. In spacetime this loop in superloop space is generated by a parametrized two-dimensional surface, enclosing a three-dimensional volume. Now as $F^A[\xi|s]$ is generated by the derivative of $\Phi[\xi]$, so $\Theta(\Sigma)$ measures the total change in $\Phi[\xi:t]$ as $t$ varies from $t=0 \rightarrow 2\pi$. Now $\Theta(\Sigma)$ is again an element of the gauge group, say $SU(2)$. If $\Sigma$ encloses a monopole, then $t=0 \rightarrow 2\pi$ will only trace the curve in $SU(2)$ which winds only half way around the group. If $\Sigma$ does not include a monopole then it traces out a closed curve in $SU(2)$. As $\Theta(\Sigma)$ measures the total change, it is proportional to the monopole charge. Thus, the monopole charge of a $SU(2)$ theory is $\pm 1$.

Deformed superspace. – In this section we shall generalize the results of the previous sections to a noncommutative deformation of the superspace. In order to analyse deformation of the superspace both the Grassman coordinates and the spacetime coordinates are promoted to operators and a deformation of their superalgebra is imposed. Thus, we promote $\theta^a$ and $y^\mu$ to operators $\hat{\theta}^a$ and $\hat{y}^\mu$ which satisfy the following superalgebra:

$$[\hat{y}^\mu, \hat{y}^\nu] = B^{\mu\nu}, \quad [\hat{y}^\mu, \hat{\theta}^a] = A^{a\mu}. \quad (18)$$

This deformation induces the following star product between functions of ordinary superspace [15,16]:

$$\Gamma^A(y, \theta) \ast \Gamma_A(y, \theta) = \exp -\frac{i}{2} \left( B^{\mu\nu}\partial_\mu \partial_\nu + A^{a\mu} (\partial_\mu \partial_a - \partial_a \partial_\mu) \right) \Gamma^A(y_1, \theta_1) \times \Gamma_A(y_2, \theta_2) \bigg|_{y_1=y_2=y, \theta_1=\theta_2=\theta}. \quad (19)$$

It may be noted that if we deform this algebra by $\{\theta_a, \theta_b\} = C_{ab}$, we will break all the supersymmetry of the theory. However, in four dimensions or for $\mathcal{N}=2$ supersymmetry in three dimensions, this deformation can be performed [27–30]. Here we have defined the star product between ordinary functions using the superderivative $D$ rather than $\partial$ because they commute with the generators of the supersymmetry $Q_a$.

In this deformed superspace a deformed version of the Bianchi identity is satisfied $\{\nabla_A, H_{BC}\}$, where $H_{BC} = \nabla_A \nabla_C$. It may be noted that for $1\ast 0$ to be defined from $\nabla_a$, we have to again set $F_{ab} = 0$ as a constraint. Now we have $[\nabla_A, \nabla_{bc}] = C_{abc} \nabla_c$, and so we get $W_a = \frac{i}{2} D_a D_a A - \frac{i}{2} \Gamma_0 \Gamma_0 \Gamma_a A - \frac{i}{2} \Gamma_0 \Gamma_0 A_0 [\Gamma_0, \Gamma_a] A$. The Lagrangian for the deformed super-Yang-Mills theory is given by $L = D^a[W^a \ast W_a]$. After analysing the deformation of the superspace, we can construct a superloop space formalization for the deformed gauge theory on it. Thus, we can define a super variable as

$$\Phi_\ast[\xi] = P_\ast \left[ \exp i \int_0^{2\pi} \Gamma^A(\xi(s)) \frac{d\xi^A}{ds} \right], \quad (20)$$

where

$$C : \{\xi^A(s) : s = 0 \rightarrow 2\pi, \xi^A(0) = \xi^A(2\pi)\}. \quad (21)$$

Here all the products of fields inside the brackets are taken as star products. It may be noted that $\Phi_\ast[\xi]$ is also scalar superfield from the supersymmetric point of view,

$$[\Phi_\ast[\xi]], \quad [D_\mu \Phi_\ast[\xi]] = \phi_\ast[\xi], \quad [D^2 \Phi_\ast[\xi]] = \phi_\ast[\xi], \quad (22)$$

where $\phi_\ast[\xi], \phi_\ast[\xi], \phi_\ast[\xi]$ are loop variables formed from a different combination of the component fields of the super-Yang-Mills theory, as compared to eq. (4). These loop variables exist on noncommutative spacetime. We can now define a connection for this deformed superloop space as

$$F_\ast = \Phi_\ast^{-1}(\xi : s, 0) \ast H_{AB}(\xi(s)) \ast \Phi_\ast^{-1}(\xi : s, 0) \frac{d\xi^B(s)}{ds}, \quad (23)$$

Here again $F_\ast[\xi|s]$ represents the change in the phase of $\Phi_\ast[\xi]$ as one moves from one point in the deformed superloop space to a neighboring point in it.

Now we can again construct a curvature for the deformed superloop space by replacing all the products of fields by star products. Thus, the curvature in this deformed superloop space is given by

$$G_{AB}[\xi, s_1, s_2] = \frac{\delta}{\delta \xi^B(s_1)} F_{A*}[\xi|s_1] - \frac{\delta}{\delta \xi^A(s_2)} F_{B*}[\xi|s_2] + i [F_{A*}[\xi|s_1], F_{B*}[\xi|s_2]]. \quad (24)$$

Repeating the above argument with the star product replacing the ordinary product, we obtain

$$G_{AB}[\xi, s_1, s_2] = \Phi_\ast^{-1}(\xi : s_1, 0) \ast [\nabla_A, H_{BC}] \ast \Phi_\ast(\xi : s_1, 0) \frac{d\xi^C(s_1)}{ds_1} \delta(s_1 - s_2). \quad (25)$$

If there are no monopoles in the spacetime, then the deformed Bianchi identity holds, $[\nabla_A, H_{BC}] = 0$, and thus, $G_{AB}[\xi, s_1, s_2] = 0$. To analyse the effect of monopoles we again define

$$\Theta_\ast(\Sigma) = P_\ast \left[ \exp i \int_0^{2\pi} dt \int_0^{2\pi} ds F_{A*}(\xi(t : s)) \frac{d\xi^A(s)}{dt} \right], \quad (26)$$

where

$$\xi^A(t : 0) = \xi^A(t : 2\pi), \quad t = 0 \rightarrow 2\pi, \quad (27)$$

$$\xi^A(0 : s) = \xi^A(2\pi : s), \quad s = 0 \rightarrow 2\pi.$$ We can obtain the component fields $\theta_a(\Sigma), \theta_a(\Sigma), \phi_\ast(\Sigma)$ from $\Theta_\ast(\Sigma)$, just as we obtained the component fields of $\Phi_\ast[\xi]$. These component fields also exist on noncommutative spacetime. Now by repeating the above argument for a monopole in this deformed theory, we can show that the monopole charge of a $SU(2)$ theory on this deformed superspace is again $\pm 1$. Thus, all the results of the ordinary superloop space hold even after deforming
the superspace, with the only difference that the ordinary product of the fields is converted into a star product.

Conclusion. – In this paper we first constructed a superloop space formulation of the super-Yang-Mills theory. This was done by defining a superloop variable that was a superscalar field from the viewpoint of supersymmetry. This was formed by a linear combination of the component fields. Then a connection on this superloop space was constructed by taking the spinor and vector derivatives of this quantity. Finally, a curvature on this superloop space was also constructed. This curvature vanished if there was no monopole in the spacetime. However, if a monopole existed in the spacetime and the superloop passed through its world lines, then this curvature did not vanish. We also constructed a quantity that would measure the monopole charge by constructing loops of superloop space. This two-dimensional quantity measured the monopole charge. Finally, it was shown that all these results hold even after deforming the superspace.

In Abelian gauge theories a duality exists which is generated by the Hodge star operation. This duality cannot be generalized in a straightforward way to non-Abelian gauge theories. However, in the loop space formulation of Yang-Mills theories, this duality has been generalized and a non-Abelian generalized dual transform has been constructed [39,40]. It will be interesting to generalize these results to super-Yang-Mills theories. This can be done by first using the results of this paper and constructing generalized duality transformations in three dimensions. After that it will be interesting to generalize the results of this paper to \(N = 1\) supersymmetric Yang-Mills theory in four dimensions or a \(N = 2\) supersymmetric Yang-Mills theory in three dimensions and obtain generalized duality transformations for them. Lastly, it will also be interesting to analyse the ABJM theory with monopole operators in this formalism. It may be noted that so far the analysis of a non-Abelian two-form gauge field in loop space has not been performed. It will be interesting to construct a loop space formalization of this field. It could be possible to do so by using the concept of parametrized surfaces. It might also then become possible to study a theory of multiple M5 branes using this formalism [41–44].

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