Sagnac interference in Carbon nanotubes

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The Sagnac interference mode arises when two interfering counterpropagating beams traverse a loop, but with their velocities detuned by a small amount \(2\alpha\), with \(v_{R/L} = v_p \pm \alpha\). In this paper we perform a perturbative non-equilibrium calculation of Sagnac interference in single channel wires as well as armchair nanotube loops. We study the dependence of the Sagnac conductance oscillations on temperature and interactions. We find that the Sagnac interference is not destroyed by strong interactions, but becomes weakly dependent on the velocity detuning \(\alpha\). In armchair nanotubes with typical interaction strength, \(0.25 \leq g \leq 0.5\), we find that the necessary temperature for observing the interference effect, \(T_{SAG}\), is also only weakly dependent on the interaction, and is enhanced by a factor of 8 relative to the temperature necessary for observing Fabry-Perot interference in the same system, \(T_{FP}\).

I. INTRODUCTION

One of the most tantalizing effects predicted by quantum mechanics is the appearance of interference fringes when two matter beams come together. These fringes provided the ultimate testimony to the pertinence of quantum mechanics and the Schrödinger equation. Interferometry of light is employed in many precision measurement devices. The Mach-Zehnder interferometer produces interference between two beams traversing two distinct paths, one of which passes through a test chamber containing, for instance, a dilute gas (see Fig. 1); this setup was originally used to measure the refraction index of the gas in the chamber. Fabry-Perot interferometer recombines a series of beams, where the n'th beam traverses the optical path between two mirrors or through a loop \(n\) times. The narrowness of the resulting interference peaks allows a precise measurement of a light beam's wave length, and is commonly used to measure the Zeeman splitting of an atom in a magnetic field. The most sensitive of all interference constructs, however, is the Sagnac interferometer. In this setup, a light beam is split into two beams, which traverse the interferometer’s loop both clock wise and counter clock wise, before being recombined. In this case, the interference fringes arise due to an absolute rotation, and provide the most accurate measure of the angular velocity of the device. This was used by Michelson to measure the absolute rotation of the Earth. More recently, the Sagnac interference effect was cleverly used to measure time reversal symmetry breaking in superconductors.

Quantum mechanics opened the way for matter-wave interferometry. Electron interferometry is a powerful probe of interaction effects on low-energy phases of quantum matter, as demonstrated by numerous examples. Mach-Zehnder interferometers reveal Aharonov-Bohm oscillations and quantum hall effect channels, and can probe exotic fractional quantum Hall states. Similarly, two-path Mach-Zehnder interferometers can probe correlated states of quantum dots. Of particular interest to us are metallic carbon nanotubes. The Luttinger liquid behavior in these systems was partially verified through the observation of Fabry-Perot interference in finite sections of the nano-tube. The Fabry-Perot interference should, in principle, allow the observation of spin-charge separation and determination of the interaction parameters of the Luttinger liquid. But the similar energy scales of the spin and charge modes’ interference patterns has made such experimental observation challenging.

The most sensitive interferometer of all, however, the Sagnac interferometer, has not been seriously explored yet in the context of interacting electronic systems. In Ref. we proposed that this interference naturally occurs in metallic armchair nanotube loops (Fig. 2). Instead of rotation, the Sagnac interference arises due to the band velocity difference between right- and left-moving electrons about each Dirac node. This velocity difference is present whenever the electronic Fermi surface is tuned away from the Dirac points at half-filling, as shown in Fig. 3(a). The operating principle of the electronic Sagnac effect has the same origin as the universal conductance fluctuations, and weak-localization effects in disordered two-dimensional electron gases. In nanotubes, it can also appear due to band-scattering in a pair of impurities.

Because the Sagnac effect involves electrons traversing the same path in two different directions, rather than repeating the same path as in Fabry-Perot interference, the phase accumulation is extremely small. Therefore Sagnac interference exhibits large-period conductance fluctuations as a function of gate- and source-drain voltages, and is expected to persist to high temperatures in comparison to Fabry-Perot interference, which is more sensitive to thermal dephasing. This interference mode should thus be able to reveal much more precise information about the unique state of interacting electrons in thin quantum wires.

Our goal in this manuscript is to thoroughly explore the range and robustness of the Sagnac interference mode, concentrating on armchair Carbon nanotubes. The questions we will ask concern the amplitude of this interference mode as a function of the temperature, gate and source-drain voltage, and Luttinger parameter of the nanotube.
we analyze the behavior of the oscillating conductance as a function of gate and temperature. In Section II A, we demonstrate this in the simplest form, we first study the above steps for the physically relevant case of carbon nanotubes, including spin and node degeneracies.

The paper is organized as follows. In Section II A as a warm-up, we analyze the simpler case of Sagnac interference in a single channel of right- and left-moving electrons. In II B and II C we set up the non-equilibrium perturbative calculation of the conductance in the presence of cross-loop tunneling, and in II D and II E we analyze the behavior of the oscillating conductance as a function of gate and bias voltages and temperature. In Section III we repeat the above steps for the physically relevant case of carbon nanotubes, including spin and node degeneracies in the calculation, and remark on the similarities and differences from the single channel case. Finally we conclude with a discussion of the experimental implications of our calculations.

II. SAGNAC INTERFERENCE IN A SINGLE CHANNEL

As discussed in the introduction, the Sagnac interference in the loop geometry is due to the asymmetry between the velocities of the left and right moving electrons. To demonstrate this in the simplest form, we first study in this section a single channel with a single type of left and right movers. In a carbon nanotube, there will be four such channels due to spin and node degeneracies.

A. The Model

We start with a single one dimensional channel of electrons and a linearized spectrum, with different left and right mover velocities, and a density-density interaction. The Hamiltonian density for this system is:
\[ H_{1ch} = -i\hbar v_{FR}\psi_R^\dagger \partial_x \psi_R + i\hbar v_{FL}\psi_L^\dagger \partial_x \psi_L + \lambda \left(\psi_R^\dagger \psi_R + \psi_L^\dagger \psi_L\right)^2 \]  

(1)

where the operator \( \psi_{R/L}^\dagger \) creates a right/left moving electron, with the velocity:

\[ v_{R/L} = v_F \pm u. \]  

(2)

The scattering we are interested in is the one which takes a right moving electron at one side of the loop, point \( X \) is Figure 2 and scatters it to a left moving electron at the other side of the loop, point \( X' \), and vice versa. This process has been dubbed Cross-Loop scattering in Ref. [15]. The same effect can also be obtained without the loop geometry by inter-node tunneling, since right movers at node 1 move with the same velocity as left movers at node 2. This inter-node tunneling is shown in Figure 3(a). If we choose our coordinate along the loop such that the point \( X \) corresponds to \( x = 0 \) and the point \( X' \) corresponds to \( x = L \), then this scattering process is described by the Hamiltonian:

\[ H_{bs} = \Gamma_1 \psi_R^\dagger (0) \psi_L (L) + h.c. + \Gamma_2 \psi_L^\dagger (0) \psi_R (L) + h.c. \]  

(3)

In the presence of the quartic density-density interactions in the Hamiltonian, Eq. (11), it is useful to use the standard bosonization procedure, since the Hamiltonian is quadratic in terms of the bosonic fields. The electron fields are bosonized as follows:

\[ \psi_{R/L} \sim e^{i(\phi \pm \theta)} \]  

(4)

where \( \theta \) and \( \phi \) are bosonic fields that satisfy the commutation relations \([\theta(x), \phi(x')] = i(\pi/2)sgn(x - x')\); also, the total density and the current density are given by \( \frac{1}{\beta} \nabla \theta = \rho_R + \rho_L \), and \( \frac{1}{\beta} \nabla \phi = \rho_R - \rho_L \), respectively. The Hamiltonian in terms of the bosonic fields becomes:

\[ H_{1ch} = \frac{\hbar v}{2\pi} \int dx \left[ \frac{1}{g} (\nabla \theta)^2 + g(\nabla \phi)^2 + 2\frac{u}{v} \nabla \theta \nabla \phi \right] \]  

(5)

where \( g = \left(1 + \frac{2\Lambda}{\pi \hbar v_F}\right)^{-1/2} \) is the Luttinger interaction parameter and \( v = v_F/g \). This is the familiar Hamiltonian of a 1D interacting electron system, with the addition of the \( u \) term which gives left and right moving particles different velocities. Indeed, this Hamiltonian can be easily diagonalized and the left and right velocities turn out to be for a general value of the interaction parameter \( g \):

\[ v_{R/L} = v \pm u = \frac{v_F}{g} \pm u. \]  

(6)

Our goal is to calculate the effects of the Sagnac interference as seen in the conductance as a function of the applied bias and gate voltages, and as a function of temperature. Due to the applied voltages the system is not in equilibrium, and we must turn to the Keldysh non-equilibrium formalism23,24. Below we carry out this analysis first for the simplified electron gas with the scattering Hamiltonian \( H_{bs} \), Eq. (3), as a perturbation.

### B. Non-Equilibrium correlation functions and conductance

The response of the loop to a bias source-drain voltage can be analyzed using the non-equilibrium Keldysh formalism. Following Ref.17, we assume that in the distant past, before turning on the backscattering, the left and right moving electrons separately had well defined thermal distributions set by separate chemical potentials. The density matrix corresponding to this initial distribution at temperature \( T = 1/\beta \) is:

\[ \hat{\rho}_V = \frac{1}{Z_V} e^{-\beta H_V}, \]  

(7)

with \( Z_V = Tr[e^{-\beta H_V}] \) and the Hamiltonian which takes into account the applied voltages is:

\[ H_V = H_{1ch} - e\frac{V_{sd}}{2} (N_R - N_L) - \alpha eV_g (N_R + N_L) \]  

\[ = H_{1ch} - e\frac{V_{sd}}{2} \int dx \frac{\nabla \phi}{\pi} - \alpha eV_g \int dx \frac{\nabla \theta}{\pi} \]  

(8)

The gate voltage, \( V_g \), simply couples to the total charge density, with \( \alpha \) being a geometrical factor of the system, while the source-drain voltage, \( V_{sd} \), induces the imbalance in the chemical potentials of the left and right movers.

As explained in Ref.17, both \( V_{sd} \) and \( V_g \) can be eliminated from the unperturbed action by an appropriate unitary transformation, which is equivalent to shifting the bosonic fields by a function of space and time; this is easy to see if one writes down the Lagrangian including the voltages18. The equivalent shifts for the case at hand are:

\[ \theta \to \theta + \frac{\alpha g^2 eV_g}{\hbar v_F} \frac{1}{1 - g^2 u^2/v_F^2} x - \frac{eV_{sd}}{2\hbar} t, \]  

\[ \phi \to \phi - \frac{\alpha g^2 eV_g}{\hbar v_F} \frac{u/v_F}{1 - g^2 u^2/v_F^2} x, \]  

(9)

These shifts remove the voltages from the Hamiltonian \( H_V \) and therefore all the correlations to appear in the calculation will be equilibrium correlation functions with respect to \( H_{1ch} \). The dependence on the applied voltages now appears in the scattering Hamiltonian, \( H_{bs} \), due to the shifted bosonic fields.

Let us now focus our attention at the charge current, which in the bosonic language is \( I = \langle e/\pi \rangle \partial_t \theta \). After performing the unitary transformation described above
we can write the formal expression for the expectation value of the current in the usual interaction picture:

\[ \langle I \rangle = I_0 + \frac{1}{Z_{V=0}} Tr \left( e^{-\beta H_{1ch}} \hat{T}_K \left\{ \hat{I}_K(x,t) e^{-i \int_0^t dt' H_{bs}(t')} \right\} \right) \]

(10)

\( \hat{T}_K \) is the time ordering operator along the Keldysh contour shown in Fig. 4 and \( \hat{I}_K(x,t) \) is the symmetrized current operator with respect to the two branches of the contour. The current \( I_0 = e^2 V_{sd}/h \) is the ideal current that would flow in the absence of backscattering in a completely transmitting channel, and it explicitly appears due to the shift of the \( \theta \) field. The Hamiltonian \( H_{bs}' \) denotes the scattering Hamiltonian \( H_{bs} \) with the properly shifted bosonic fields. The expression for the current can be expanded in powers of \( H_{bs}' \) and all the correlation functions to appear in this expansion are equilibrium correlation functions at temperature \( 1/\beta \). If we denote by \( \theta^+ \) and \( \theta^- \) the fields on the forward branch and backward branch of the Keldysh contour respectively, then time ordering along the contour means that \( \theta^+ \theta^- \) correlations have the usual time ordering, \( \theta^- \theta^- \) are anti-time ordered, and \( \theta^+ \theta^+ \) is always earlier in time that \( \theta^- \theta^- \). The same applies for all the fields.

It is useful to apply a Keldysh rotation to the fields, \( \theta^\pm = \theta \pm 1/2 \), and similarly for \( \phi \). The correlation function \( \langle \hat{T}_K(t)\hat{T}_K(t') \rangle \) vanishes by construction, and we define:

\[ C^\theta(x,t;x',t') = \langle \hat{T}_K(\theta(x,t)\theta(x',t')) \rangle = \frac{1}{2} \langle \{\hat{\theta}(x,t), \hat{\theta}(x',t')\} \rangle \]

(11)

\[ R^\theta(x,t;x',t') = \langle \hat{T}_K(\theta(x,t)\hat{\theta}(x',t')) \rangle = -i \Theta(t-t') \langle \{\hat{\theta}(x,t), \hat{\theta}(x',t')\} \rangle \]

and similarly for the \( \phi \) fields, and for the mixed correlations:

\[ C^{\theta\phi}(x,t;x',t') = \langle \hat{T}_K(\theta(x,t)\phi(x',t')) \rangle = \frac{1}{2} \langle \{\hat{\theta}(x,t), \hat{\phi}(x',t')\} \rangle \]

(12)

\[ R^{\theta\phi}(x,t;x',t') = \langle \hat{T}_K(\theta(x,t)\hat{\phi}(x',t')) \rangle = -i \Theta(t-t') \langle \{\hat{\theta}(x,t), \hat{\phi}(x',t')\} \rangle \]

where operators with a hat are simply the time dependent operators with no time ordering. As explained above, these correlation function are to be evaluated in equilibrium, and therefore are easily explicitly calculated (Appendix B). Due to translational invariance in time and space, these correlations are functions of \( x-x' \) and \( t-t' \), for example:

\[ C^{\theta\phi}(x,t;0,0) = \frac{1}{4} \left[ \log \left( \frac{v_R \sinh \left( \frac{v_R (t-x) \pi}{\beta v_R} \right)}{\beta v_R} \right) - \log \left( \frac{v_L \sinh \left( \frac{v_L (t+x) \pi}{\beta v_L} \right)}{\beta v_L} \right) \right] \]

(13)

C. Perturbation Theory

The Sagnac interference fringes occur already with weak backscattering at the base of the loop, and can be deduced from a perturbation analysis of the tunneling Hamiltonian, Eq. 3. As outlined above, to calculate the current, \( I_{1ch} = \langle \hat{\alpha}^\dagger \hat{\alpha} \rangle \), we absorb the gate and bias voltages, \( V_g \) and \( V_{sd} \), respectively, in the shifts in \( H_{bs} \) which allow us to move the voltages from the unperturbed Hamiltonian \( H_{1ch} \) to the backscattering perturbation, \( H_{bs} \). Then, we expand the formal expression we found for the current using the Keldysh technique, Eq. 10, in powers of \( H_{bs} \), and use Wick’s theorem to evaluate the resulting contributions.

To lowest nontrivial order, which is second order in \( H_{bs} \), we obtain after a lengthy calculation:

\[ I_{1ch} = \frac{e^2 V_{sd}}{h} + I_{co} + I_{inco} \]

(14)

The first term is simply the current that would flow through the system in the absence of backscattering. The coherent current, \( I_{co} \), oscillates with the gate voltages \( V_g \), and is given by:

\[ I_{co} = \frac{g}{2 \Gamma_1 \Gamma_2} \cos \left( \frac{2 \mu g^2 L \alpha}{h^2 v_f^2 (1 - g^2 u^2/v_f^2)} V_g \right) \times \right] \int dt \sin \left( \frac{e V_{sd}}{h} t \right) e^{-C_{inco}(L,t)} \sin(R_{inco}(L,t)) \]

(15)

where \( c \) is a constant of order unity, and we assume that \( \Gamma_i \) are real for simplicity. The incoherent current, \( I_{inco} \), is independent of the gate voltage, and is given by:

\[ I_{inco} = \frac{c}{\Gamma_1^2} \sum_{n=\pm} \int dt \sin \left( \frac{e V_{sd}}{h} t \right) e^{-C_{inco}^{n}(L,t)} \sin(R_{inco}(L,t)) \]

(16)

The functions \( C_{co}, C_{inco}, R_{co} \) and \( R_{inco} \) are complicated combinations of the correlation functions defined in section 7.3 and are given explicitly in Appendix B. These functions do not simplify, partly due to the fact that the correlation functions in this problem are not symmetric under \( x \rightarrow -x \) since left and right movers have different velocities.
cillation will be present even for no velocity detuning, acquire different phases traversing the loop. This oscillation is due to the only two voltage oscillation frequencies in the problem, $\Omega_{\text{up}} = \frac{eE_{\text{up}}}{hL}$ and $\Omega_{\text{down}} = \frac{eE_{\text{down}}}{hL}$, where $v = v_F/g$. The voltage is in units of $h v_F/eL$. The shorter voltage oscillation periods is $\Delta V_{\text{sd}} = 2\pi \left( \frac{\Omega_{\text{up}} + \Omega_{\text{down}}}{2} \right)^{-1} \approx 12.5 h v_F/eL$, and the large oscillation period is $\Delta V_{\text{sd}} = 2\pi \left( \frac{\Omega_{\text{up}} - \Omega_{\text{down}}}{2} \right)^{-1} \approx 250 h v_F/eL$.

D. Voltage dependence of the single-mode Sagnac interference

The voltage current characteristics given in Eqs. (14) - (15) can be evaluated numerically to obtain the voltage and temperature dependence of the single-mode Sagnac interference. The period of the interference as a function of the gate voltage ($I_{\text{co}}$) is easily observed to be (for small $u/v_F$):

$$\Delta V_{\text{sd}}^{\text{Sagnac}} \approx \frac{v_F}{u} \frac{\pi h^2 v_F}{a q^2 L} = \frac{v_F}{u} \Delta V_{\text{sd}}^{FP} \tag{17}$$

where $\Delta V_{\text{sd}}^{FP}$ is the period in gate voltage for Fabry-Perot interference. Fabry-Perot interference occurs whenever part of the wave’s trajectory can be repeated. Since the Sagnac interference involves traversing the same path in two different directions, the phase difference accumulated in the process is much smaller than the difference incurred by repeating part of the path, and therefore the period of the Sagnac interference is much larger than the period of the Fabry-Perot interference. Such large period oscillations have been experimentally observed in Carbon nanotubes, in the loop geometry, as reported in Ref. 18, in addition to the shorter period Fabry-Perot oscillations.

For a given gate voltage, both the coherent and incoherent parts of the current oscillate with the bias voltage $V_{\text{sd}}$. This oscillation is due to the fact that in the presence of bias voltage, the Fermi energy of the left- and right- moving electrons are different by $V_{\text{sd}}$, and hence their Fermi wavevectors are different also and they would acquire different phases traversing the loop. This oscillation will be present even for no velocity detuning, $u = 0$. When the detuning is finite, $u \neq 0$, the differential conductance $G_{1\text{ch}} = \partial I_{\text{ch}}/\partial V_{\text{sd}}$ will show a beating pattern due to the two different left and right moving excitation velocities. Here we are only considering the Sagnac oscillations arising from the cross-loop tunneling, Eq. (3). Figure 5 shows the oscillations of the differential conductance at a fixed gate voltage. For non-interacting electrons, the beating pattern corresponds to the addition of two harmonics with two different frequencies in voltage, $\sin(\Omega_{\text{up}} V_{\text{sd}})$ and $\sin(\Omega_{\text{down}} V_{\text{sd}})$, with $\Omega_{\text{up}}/L = \frac{eV_{\text{up}}}{hL}$ and $\Omega_{\text{down}}/L = v_F \pm u$. The beating pattern will then display fast oscillations with voltage period $\Delta V_{\text{sd}}^{\text{fast}} = 2\pi \left( \frac{\Omega_{\text{up}} + \Omega_{\text{down}}}{2} \right)^{-1}$, and slow voltage oscillations with period $\Delta V_{\text{sd}}^{\text{slow}} = 2\pi \left( \frac{\Omega_{\text{up}} - \Omega_{\text{down}}}{2} \right)^{-1}$. For interacting fermions, $g \neq 1$, the oscillations will not be simple harmonic oscillations. Still, the periods will be evident and will have the same functional form, in terms of $v_{\text{up}}/L$, as the frequencies in the non-interacting case. The periods $\Delta V_{\text{sd}}^{\text{fast}}$ and $\Delta V_{\text{sd}}^{\text{slow}}$ do depend on $g$ through velocities $v_{\text{up}}$, and $v_{\text{up}}/L = \frac{e}{g} \pm u$ (Eq. 19).

These oscillation, generally, lie atop a powerlaw behavior of the differential conductance as a function of $V_{\text{sd}}$, as expected from the known behavior of the conductance in the presence of impurity backscattering. For backscattering from an impurity in a Luttinger liquid, the backscattered current, for low temperature, behaves as $I \propto V_{\text{sd}}^{2g-1}$. We chose to plot the Sagnac oscillations as a function of $V_{\text{sd}}$ (Fig. 5) for the interaction parameter $g = 0.5$ since for that value the corresponding power law would be $I \propto V_{\text{sd}}^{0.5}$, and the contribution of such a powerlaw to the differential conductance would vanish, making the oscillation atop this powerlaw more visible.

E. Temperature dependence of the single-mode Sagnac interference

Next we consider the temperature dependence of the gate-voltage driven oscillations in the coherent part of the current. As argued in Ref. 18, the large period Sagnac oscillations are expected to be observed at much higher temperature than the shorter period Fabry-Perot oscillations. This difference in temperature behavior can be easily understood by examining the phase giving rise to the interference in both cases. In the Fabry-Perot case for a loop, the lowest order interference is between a beam of electrons which is not scattered, and a beam of electrons which, due to scattering at the base of the loop, does a roundtrip between the the two scattering points. The phase difference between these two beams at energy $E$ is $\Delta \phi_{FP} = k_R L = \frac{v_F}{u} L \xi$. Finite temperature effectively causes uncertainty of order $T$ in the energy $E$, and the interference pattern will be washed out when the uncertainty of the the phase $\Delta \phi_{FP}$ is of order $2\pi$, which happens at a temperature $T_{FP} = \frac{2\pi}{v_F} L \xi$. In the Sagnac case, the interference is between a beam that traverses the loop moving left and one which tra-
verses the loop moving right. The phase difference between these two beams at energy $E$ is $\Delta \phi_{SAG} = k_L L - k_R L = \left( \frac{1}{v_L} - \frac{1}{v_R} \right) \frac{E}{h}^2$, and this interference will be washed out at temperature $T_{SPAG} = \frac{2\pi \hbar}{L} \left( \frac{1}{v_L} - \frac{1}{v_R} \right)^{-1}$.

For non-interacting electrons the right and left moving velocities are $v_{R/L} = v_F \pm u$. Thus to lowest order in $u/v_F$, the highest temperatures for observing interference according to the argument above are:

$$T_{FP} \approx \frac{\pi \hbar v_F}{L}, \quad T_{SAG} \approx \frac{\pi \hbar v_F v_F}{L u} = T_{FP} \cdot \frac{v_F}{u} \quad (18)$$

For non-interacting electrons, we expect the Sagnac interference to survive to a temperature higher by a factor of $v_F/u$ than the corresponding Fabry-Perot temperature. We will show through explicit calculation that this is indeed true for the non-interacting case. For interacting electrons, we will see that $T_{SAG}$ will still be considerably larger than $T_{FP}$, but their ratio is less than the dramatic $v_F/u$ ratio.

To explore the Sagnac temperature range, we evaluate the amplitude of the coherent oscillations (the oscillations in $V_g$) as a function of temperature, for different interaction parameters $g$ and different ratios of $u/v_F$. For non-interacting electrons, $g = 1$, we find that the Sagnac oscillations indeed survive up to a high temperature, which is a factor of $v_F/u$ higher than the corresponding Fabry-Perot oscillations. Figure 6 plots the oscillation amplitude as a function of temperature, normalized by its zero-temperature value, and for different values of $u/v_F$. The functional dependence on temperature is given approximately by:

$$\frac{G_{co}(T)}{G_{co}(T = 0)} = \frac{2\pi k_B L T}{\hbar v_F} \left( \frac{u}{v_F} \right) \frac{1}{\sinh(2\pi k_B T / \hbar v_F)} \quad (19)$$

This result is similar to the exact form of the temperature dependence of the Fabry-Perot interference amplitude, with the only difference being the factor of $u/v_F$. Therefore, the Sagnac oscillations of non-interacting electrons indeed survive up to temperatures which are a factor of $v_F/u$ larger than the Fabry-Perot oscillations.

For interacting electrons, $g \neq 1$, the Sagnac interference still survives up to temperatures significantly higher than the corresponding Fabry-Perot temperature scales, but the enhancement is suppressed compared to that of non-interacting electrons. Figure 7 shows the Sagnac temperature scale $T^*$ vs. $u/v_F$ for three different values of the interaction parameter $g$, where we define $T^*$ to be the temperature at which the amplitude of the oscillations reaches $e^{-1}$ of its amplitude at zero temperature. For non-interacting electrons $T^*$ is strongly dependent on the ratio $u/v_F$ as discussed above. For the interaction parameter values $g = 0.5$ and $g = 0.25$ (Dashed lines), the temperature $T^*$ is only weakly dependent on the ratio $u/v_F$. As an example for the resulting enhancement of the Sagnac compared to the Fabry Perot interference, consider $g = 0.25$, where the $T^*$ temperature scale for the

Sagnac oscillations is roughly $1.6 \hbar v_F / k_B L$, a factor of 4 enhancement over $T^*$ of the non-interacting Fabry-Perot oscillations which is $0.42 \hbar v_F / k_B L$, despite the suppression of the Sagnac $T^*$ due to interactions. As can be seen in the figure, for $g = 0.5$ the enhancement is about 7. While it is difficult to extract the analytic dependence of the temperature on the interaction parameter, one can repeat our calculation for any value of $g$. 

FIG. 6: Coherent Sagnac oscillation amplitude vs. temperature for different values of $(v_F/u)$, for noninteracting electrons $(g = 1)$. 

The slowest decaying plot corresponds to $v_F/u = 100$, and the fastest decaying plot corresponds to $v_F/u = 10$. Temperature is given in units of $\hbar v_F / k_B L$.

FIG. 7: $T^*$ vs. $u/v_F$, where $T^*$ is the temperature at which the coherent differential conductance (the part of the conductance which oscillates with gate voltage) reaches $e^{-1}$ of its zero temperature value. For a non-interacting system, $g = 1$, the single channel case gives the same temperature dependence as the case with spin and node degeneracies, $T^* \propto v_F/u$. The single channel temperature dependence is given for $g = 0.5$ (squares, dashed), and $g = 0.25$ (diamonds, dashed). The Carbon nanotube temperature dependence is given for $g = 0.5$ (triangles), and $g = 0.25$ (inverted triangles). Temperature is given in units of $\hbar v_F / k_B L$. For reference, the $T^*$ corresponding to the $g = 1$ Fabry-Perot oscillations is also plotted.
III. INTERFERENCE IN NANOTUBES

Equipped with our understanding of the single-channel Sagnac interference, we can now consider the likely physical system where it may be observed: a metallic Carbon nanotube with four different Dirac nodes. We now add the spin and node degeneracies of a Carbon nanotube, and examine their effect on the Sagnac interference pattern voltage and temperature dependence.

A. The Model

The energy spectrum of a Carbon nanotube is shown in figure 3a. This spectrum is usually linearized around the Fermi surface, which yields four chiral modes, two left moving and two right moving (not including spin), with linear dispersion. These modes can be bosonized and treated within the Luttinger Liquid theory framework, as we have done in the single channel case in the previous sections. All these modes are usually assumed to have the same velocity, the Fermi velocity $v_F$. For the purposes of this paper, it is important to notice that when the Fermi surface is away from the degeneracy points where the upper and lower bands meet, linearizing the spectrum actually gives two different velocities which we shall note $v_{\pm} = v_F \pm u$. The linearized Hamiltonian density is, then:

$$\mathcal{H}_{4ch} = i \sum_{a=1}^{2} \sum_{\sigma=\uparrow,\downarrow} \left( v_R a_{R \sigma} \partial_x \psi_{R \sigma} - v_L a_{L \sigma} \partial_x \psi_{L \sigma} \right) + \lambda \left[ \sum_{a=1}^{2} \sum_{\sigma=\uparrow,\downarrow} \left( \psi_{R \sigma} \psi_{R \sigma} + \psi_{L \sigma} \psi_{L \sigma} \right) \right]^2$$

where $\psi_{R/La}$ stands for a right/left moving electron at node $a$ with spin $\sigma$, and we added a total charge density interaction term. The velocities that appear in the Hamiltonian are:

$$v_{R/L1\sigma} = v_F \pm u = v_{\pm}$$

$$v_{R/L2\sigma} = v_F \mp u = v_{\mp}.$$  

Thus for $u > 0$, at node 1 right movers are faster than left movers, while at node 2 the opposite is true. Now, the nonlinearity of the electronic spectrum in a Carbon nanotube needs to be taken into account when considering the velocity difference, $u$; it depends on the detuning of the chemical potential away from the degeneracy points.

The scattering process we are interested in is very similar to the one we had in the single channel case. We need to consider a term that scatters a right mover at one end of the loop to a left mover at the other end of the loop, conserving spin and node quantum numbers,

$$H_{bs} = \sum_{\sigma,a=1,2} \left[ \Gamma_1 \psi_{R \sigma}^{\dagger} (0) \psi_{L \sigma} (L) + h.c. \right] + \Gamma_2 \psi_{L \sigma}^{\dagger} (0) \psi_{R \sigma} (L) + h.c.$$  

Next we bosonize the electron field operators in the nanotube. The slowly oscillating parts can be written as:

$$\psi_{R/La\sigma} \sim e^{i (\theta_{a\sigma} \pm \phi_{a\sigma})}$$

where $\theta_{a\sigma}$ and $\phi_{a\sigma}$ are bosonic fields that satisfy the commutation relations $[\theta_{a\sigma} (x), \phi_{a'\sigma'} (x')] = i (\pi / 2) \delta_{a,a'} \delta_{\sigma,\sigma'} \text{sgn}(x - x')$. The Hamiltonian in terms of the bosonic fields is:

$$H_{4ch} = \frac{\hbar v_F}{2\pi} \sum_{\sigma,a=1,2} \int dx \left[ (\nabla \theta_{a\sigma})^2 + (\nabla \phi_{a\sigma})^2 \right] + \lambda \left( \sum_{\sigma,a=1,2} \frac{1}{2} \nabla \theta_{a\sigma}^2 \right)^2$$

If the velocities of all branches of the spectrum were equal, i.e. $u = 0$, then the Hamiltonian $H_{4ch}$ would be diagonalized by the spin and node symmetric and antisymmetric combinations of the $\theta$'s and $\phi$'s. By diagonalizing we mean a linear mapping of the $\phi$ and $\theta$ fields such that the Hamiltonian takes the form of four independent channels, each resembling of $H_{4ch}$, Eq. 5. When $u \neq 0$, there still exists a local transformation $\theta_{a\sigma} = \sum_{j=1,4} (A_{a\sigma j}^0 \theta_j + B_{a\sigma j}^0 \phi_j)$ that diagonalizes the Hamiltonian, but it is a more complicated combination of the fields that depends on $u$ and $\lambda$, and mixes the $\theta$ and $\phi$ fields, which makes the conductance calculation quite cumbersome. While the details of this transformation are given in appendix A, the diagonal Hamiltonian is:

$$H_{4ch} = \sum_{j=1,4} \frac{\hbar v_j}{2\pi} \int dx \left[ \frac{1}{g_j} (\nabla \theta_j)^2 + g_j (\nabla \phi_j)^2 \right] + \sum_{j=1,2} \frac{\hbar v_F}{2\pi} \int dx \left[ (\nabla \theta_j)^2 + (\nabla \phi_j)^2 \right] + (-1)^{j+1} \frac{\mu}{v_F} \nabla \phi_j \nabla \theta_j.$$  

The fields $\theta_{1/2}$ and $\phi_{1/2}$ are the spin antisymmetric combinations of $\theta_{1/2\sigma}$ and $\phi_{1/2\sigma}$ respectively. Since the interaction term in Eq. 20 involves only the spin symmetric combinations, the spin antisymmetric combinations are untouched and still have the left and right moving velocities as in Eq. 21. On the other hand, the fields $\theta_{3/4}$ and $\phi_{3/4}$ are not simply the remaining symmetric combination and mix the remaining $\theta$'s and $\phi$'s. These fields have the same left and right moving velocity, which is:

$$v_{3/4} = \frac{v_F}{\sqrt{2}} \left[ 1 + \frac{1}{g^2} + 2 \frac{u^2}{v_F^2} \pm \sqrt{ \left( 1 - \frac{1}{g^2} \right)^2 + 8 \frac{u^2}{v_F^2} \left( 1 + \frac{1}{g^2} \right)^2 } \right]$$

where $g = \left( 1 + \frac{\lambda}{\pi \hbar v_F} \right)^{-1/2}$ is the Luttinger parameter $\lambda$. 

Fortunately, for the region of parameters which is of interest, namely strong interactions, $g \leq 0.5$ and...
$u/v_F \leq 0.1$, the exact change of basis required to diagonalize the spin symmetric part of the Hamiltonian is very close to the usual node symmetric/antisymmetric basis. This can be explicitly seen, for example, from the velocities of these modes. For this entire range of parameters, the velocities of the diagonal fields, given by Eq. (26), are at most 1% different from the values we expect for the left-right symmetric system, which are $v_F/g$ and $u/v_F$. Due to the strong interactions in this spin symmetric sector, the velocity asymmetry is unimportant, and it is for this reason that we choose to still use the node symmetric-antisymmetric basis and treat these fields as the diagonal ones. In Appendix A we elaborate on and justify this approximation. Note that the velocity asymmetry is still apparent in the non-interacting spin antisymmetric modes labeled by $j = 1$ and $j = 2$ in Eq. (26).

B. Perturbation Theory

Using the diagonal form of the Hamiltonian with the above approximation, we proceed to calculate the current, $I = (e/\pi)(\sum a_\sigma \partial_\sigma a_\sigma^\dagger)$, as in Section II. The applied voltages now couple to the total density and total number of left movers and right movers:

$$H_V = H_{4ch} - eV_{sd}\left(N_R - N_L\right) - \alpha eV_g \left(N_R + N_L\right)$$

$$= H_{4ch} - e\frac{V_{sd}}{2} \int dx \sum_{\sigma,a} \frac{\nabla \phi_{\sigma a}}{\pi} - eV_g \int dx \sum_{\sigma,a} \frac{\nabla \theta_{\sigma a}}{\pi}$$

The external voltages can be removed from the Hamiltonian by the appropriate shift of the bosonic fields:

$$\theta_{\alpha a} \rightarrow \theta_{\alpha a} + \frac{\alpha g^2 eV_g}{\hbar v_F} \frac{1}{1 - g^2 u^2/v_F^2} x - \frac{eV_{sd}}{2\hbar} t,$$

$$\phi_{\alpha a} \rightarrow \phi_{\alpha a} + (-1)^a \frac{\alpha g^2 eV_g}{\hbar v_F} u/v_F \frac{1}{1 - g^2 u^2/v_F^2} x.$$  (28)

Again we use the Keldysh contour to write the formal expression for the current, as in Eq. (11), and expand it to lowest order in the appropriate $H_{bs}^\dagger$, which contains the voltage dependence due to the shifts of the fields. The approximation we made above, namely that it is the node symmetric/antisymmetric combination which diagonalize the Hamiltonian, allows us to write the current in a very similar form to the single channel case:

$$I_{4ch} = 4\frac{e^2V_{sd}}{\hbar} + I_{co} + I_{inco}$$  (29)

The first term on the right hand side of Eq. (29) is the current that would flow in the nanotube in the absence of backscattering. The second term is the coherent current which oscillates with the gate voltage:

$$I_{co} = e\Gamma_1 \Gamma_2 \cos \left(\frac{2ug^2La}{\hbar^2 v_F^2 (1 - g^2 u^2/v_F^2)} V_g\right) \times \int dt \sin\left(\frac{eV_{sd}}{\hbar} t\right) e^{-\tilde{C}_{inco}(L,t)} \sin(\tilde{R}_{inco}(L,t))$$  (30)

and the third term is the incoherent current, which is independent of the gate voltage:

$$I_{inco} =$$

$$c \left(\Gamma_1^2 \sum_{\eta = \pm} \int dt \sin\left(\frac{eV_{sd}}{\hbar} t\right) e^{-\tilde{C}_{inco}^\eta(L,t)} \sin(\tilde{R}_{inco}(L,t))\right) +$$

$$c \left(\Gamma_1 \rightarrow \Gamma_2, L \rightarrow -L\right)$$  (31)

The function $\tilde{C}_{inco}, \tilde{C}_{inco}^\pm, \tilde{R}_{inco}$ and $\tilde{R}_{inco}$ are related to the single channel correlation functions as explained in Appendix B.
C. Temperature and Voltage Dependence in Carbon Nanotubes

As in the single channel case, we find there is a coherent part of the interference current which oscillates as a function of the gate voltage with a large period, much larger than the Fabry-Perot oscillation period, as seen explicitly from the voltage dependence of $I_{co}$.

The differential conductance $\partial I_{4ch}/\partial V_{sd}$, on the other hand, displays a beating pattern, but a more complicated one than in the single-channel case, since there are four different velocities in the problem now: $v_F \pm u$, $v_3 \approx v_F$ and $v_4 \approx v_F/g$. Figure 8 shows the differential conductance of the nanotube, $\partial I_{4ch}/\partial V_{sd}$, and its Fourier transform. From the Fourier analysis we see that clearly there are four dominant frequencies, which correspond to the four different velocities of the collective modes in the nanotube. Thus a careful observation of the large-period, and robust, Sagnac interference allows, in principle, to extract the nanotube parameters, namely the interaction strength $g$ and the velocity mismatch $u$ from the Fourier transform of the conductance as a function of bias voltage, up to temperatures much higher than the Fabry-Perot oscillations temperatures.

The temperature dependence of the Sagnac interference in the nanotube case is qualitatively similar to the single channel case. In the absence of interactions ($g = 1$), the interference can be observed to the scale $T^*$ proportional to $v_F/u$; in the presence of strong interactions, however, $T^*$ becomes only weakly dependent on $u$. Unlike the single channel case, $T^*$ in the nanotube case is also only very weakly dependent on $g$ in the range $g \leq 0.5$. This is due to the fact that only one of the four modes which diagonalize the Hamiltonian are interacting and depend on $g$. For the same reason, $T^*$ is higher in the case of the nanotube than in the single channel case, i.e. the reduction of $T^*$ due to interactions is not as severe in the nanotube case. The temperature dependence on $u$ and $g$ is plotted in Figure 7. In the range mentioned above, $T_{SAG}^* \approx 2.8 v_F/\pi k_B \approx 7 T_{FP}$.

IV. SUMMARY AND CONCLUSIONS

In this paper we investigated the conductance oscillations in carbon nanotubes due to Sagnac interference. In addition to theoretical interest in this large-period interference mode, the motivation for our study also comes from a recent experimental realization of carbon nanotube loops. The same interference mode can arise also without the loop geometry in the presence of internode backscattering in the nanotube, as pointed out in Ref. 22.

The source of the Sagnac conductance oscillations is the difference in the velocities of left and right moving excitations in a carbon nanotube when the chemical potential is tuned away from half filling. Compared to the more familiar Fabry-Perot oscillations, Sagnac oscillations are expected to have a much larger period in gate voltage, and, as we show, in non-interacting wires survive to a temperature a factor of $v_F/u$ higher than that required to observe Fabry-Perot oscillations.

In interacting electronic wires, the above temperature estimation for free fermions does not apply. Our results for a single channel Luttinger liquid are that $T_{SAG}$ becomes only weakly dependent on $v_F/u$, although still strongly dependent on $g$. From our $g = 0.5$, results, the enhancement of relative to the FP interference is roughly: $T_{SAG} \approx 15 gT_{FP}$ in the range $u/v_F < 0.1$.

For a strongly interacting armchair nanotube, $g \leq 0.5$, we find that $T_{SAG}$ becomes not only weakly dependent on $v_F/u$, but also nearly independent of $g$. The Sagnac interference is expected to survive up to $T_{SAG}^* \approx 3 v_F/\pi k_B \approx 7 - 8 T_{FP}$. Considering that Fabry-Perot oscillations have been observed in nanotubes up to $T = 10K$, Sagnac oscillations should be observed up to about $70K$ in nanotubes, despite the strong interactions.

There is also something to be learnt from examining the behavior of the conductance as a function of the applied voltages. We saw that Sagnac oscillations would have a large period of oscillations in the applied gate voltage $V_g$; this period itself is a function of the gate voltage, through the dependence of the velocity difference $v_R - v_L = 2u$. Using typical values of a nanotube parameters (e.g. Ref. 18), $v_F = 8 \cdot 10^5 m/s$, $L = 7 \mu m$, $g = 0.3$ and $\alpha = 1/30$, the period of oscillation in the gate voltage would be $\Delta V_g = \frac{2 \pi k_B}{\alpha} \left( \frac{v_F}{u} \right) \approx 17V$, consistent with the observed oscillations in Ref. 18.

On the other hand, oscillations of the conductance as a function of the applied bias voltage $V_{sd}$ depend not only on the bare velocities, but also on the interaction strength. A Fourier transform of the Sagnac oscillations as a function of $V_{sd}$, we show, contains four different frequencies corresponding to the four different velocities in the problem, which are roughly $v_F \pm u, v_F$ and $v_F/g$. Using the same parameters as above we get $\Delta V_{sd} = \frac{2 \pi k_B}{\alpha} \frac{v_F}{u} \approx 0.5 mV$. This period is much smaller than the bandwidth of a nanotube which is a few eV, so in principle many oscillation periods can be observed and the longer period oscillations should also be measurable, allowing the slower frequency oscillations to appear in the Fourier transform. Observation of these frequencies would allow us to read off the parameters of the nanotube, $v_F$, $u$ and $g$, at temperatures up to $T_{SAG} \approx 70mK$, which is higher than the temperatures associated with Fabry-Perot oscillations.

In the single channel case, for non-interacting electrons, we were able to extract an analytic expression for the temperature behavior of the conductance gate voltage oscillations:

$$G_{co}(T) = \frac{G_{co}(T = 0)}{(2\pi k_B L T/hv_F)} u \left( \frac{v_F}{u} \right) \frac{1}{\sinh(2\pi k_B T/hv_F v_F)}$$

and it is apparent how the ratio $v_F/u$ directly enters
the temperature scale. Unfortunately, we were so far unable to extract analytic expression for $T_{SAG}$ in terms of $g$ and $u/v_F$ for the interacting single channel or interacting nanotube cases, inspite of the progress on the qualitative understanding our numerical results allow. Such an analytical understanding should be the focus of a future effort.

As can be observed in Figs. 1 and 2, the paths giving rise to the Sagnac interference are similar to the paths that give rise to weak localization phenomena in 2d disordered conductors. In this work we also essentially show that even in the presence of strong interactions, the interference survives. It is tempting to extrapolate from our results that weak localization should also survive even in the presence of strong interactions, this, however, is presumably true so long that scattering events are dominated by small momentum transfer. Nevertheless, our results suggest that a Luttinger liquid with charge and spin modes will still exhibit weak-localization effects, but suppressed, and only weakly dependent on the detuning between counter propagating electrons. Therefore the magnetoresistance should also be strongly suppressed at low fields.

Acknowledgments

We are indebted to Jinseong Hu, Chetan Nayak, Yuval Oreg, Leonid Pryadko, and Jan von Delft for illuminating discussions. MB is grateful for support by the ONR.

APPENDIX A: DIAGONALIZING THE HAMILTONIAN WITH NODE AND SPIN DEGENERACIES

In this appendix we show how to diagonalize the Hamiltonian $H_{4ch}$ of Eq. (24), where diagonalization entails finding the appropriate change of basis that will transform $H_{4ch}$ to the sum of four Hamiltonians, each having a form resembling the single channel $H_{1ch}$ of Eq. (5). We also explain here the approximations we have used in our calculation.

The first step in the diagonalization of $H_B$ is to change the basis from the spin up/down to the spin symmetric/antisymmetric basis at each node:

$$\theta_{a\pm} = \frac{\theta_{a1} \pm \theta_{a2}}{\sqrt{2}} \quad (A1)$$

applying the same transformation to the $\phi$’s as well. We notice that the density-density interaction term involves only the spin symmetric fields $\theta_{a+}$, hence the spin antisymmetric fields decouple and appear as two non-interacting ($g = 1$) copies of the single channel problem, described by the Hamiltonian $H_{1ch}$, with right moving velocity of $v_F \pm u$ and left moving velocities of $v_F \mp u$. These are the fields labeled with $j = 1$ and $j = 2$ in Eq. (25).

The Hamiltonian for the spin symmetric fields has a similar form to our starting point Hamiltonian, $H_{1ch}$:

$$H_+ = \frac{\hbar v_F}{2\pi} \sum_{a=1,2} \int dx \left[ (\nabla \theta_{a+})^2 + (\nabla \phi_{a+})^2 \right] + (\nabla \phi_{a+}) + \int dx 2\lambda \left( \sum_{a=1,2} \frac{1}{\pi} \nabla \theta_{a+} \right)^2$$

In the absence of the $u$ term, $H_+$ is easily diagonalized by taking the node symmetric and antisymmetric combinations of the fields:

$$\theta_{3/4} = \frac{\theta_{1+} \pm \theta_{2+}}{\sqrt{2}} \quad (A3)$$

The resulting diagonal Hamiltonian would be:

$$H_+|_{u=0} = \frac{\hbar v_3}{2\pi} \int dx \left[ \frac{1}{g_3} (\nabla \theta_3)^2 + g_3 (\nabla \phi_3)^2 \right] + \frac{\hbar v_4}{2\pi} \int dx \left[ \frac{1}{g_4} (\nabla \theta_4)^2 + g_4 (\nabla \phi_4)^2 \right] \quad (A4)$$

with $v_3 = v_F$, $v_4 = v_F/g$, $g_3 = 1$ and $g_4 = g$.

When we consider $u \neq 0$, it is still possible to apply a $g$ and $u$ dependent transformation to the fields, that will restore $H_+$ to the form in Eq. (A3), with velocities $v_{3/4}$ given by Eq. (29). The field mixing this transformation entails, however, considerably complicates the book keeping in our perturbative calculation. Fortunately, we can show that a good approximation is to simply set $u$ to zero in $H_+$, when the interactions are strong, and simply use the transformation given by Eq. (A3). The first indication that this approximation is valid is that the exact velocities $v_{3/4}$ differ from the $u = 0$ velocities by at most 1% in the entire range of parameters we are interested in, which is $u/v_F \leq 0.1$ and $g \leq 0.5$.

Another indication that this approximation is valid comes from the analysis of the single channel problem in Section II. In the single channel case we derived exact expressions for the Sagnac interference, and found that for $g = 0.5$ and $g = 0.25$, the temperature dependence is only weakly dependent on $u/v_F$: furthermore $u$ only enters directly in the expression for the oscillation period of the conductance as a function of gate voltage, the dependence we have explicitly in our expression for the coherent current $I_{co}$, Eq. (18).

Finally, we can also calculate the exact combination of fields that diagonalizes $H_+$, and verify that indeed they are very close to the node symmetric/antisymmetric combinations for the range of $g$ and $u$ of interest. As an example, the explicit change of basis from the node symmetric/antisymmetric basis to the diagonalizing basis for $g = 1/2$, to second order in $u/v_F$, is:

$$\Pi_{4 \times 4} = \begin{pmatrix}
-\frac{71}{10\pi} (u/v_F)^2 & 0 & 0 & \frac{2\sqrt{7}}{3} (u/v_F) \\
0 & -\frac{89}{10\pi} (u/v_F)^2 & \frac{5}{3\pi\sqrt{2}} (u/v_F) & 0 \\
0 & -\frac{2\sqrt{7}}{3} (u/v_F) & -\frac{29}{36} (u/v_F)^2 & 0 \\
-\frac{5}{3\sqrt{2}} (u/v_F) & 0 & 0 & -\frac{11}{36} (u/v_F)^2
\end{pmatrix} \quad (A5)$$
We see that the is matrix is close to the identity matrix \( I_{4 \times 4} \), since \( \frac{C}{g} \ll 1 \). The deviation from the identity becomes even smaller for smaller \( g \). Note that for \( g \approx 1 \) the corresponding change of basis matrix is not close to the identity matrix and our approximation fails.

We stress that setting \( u \) to zero in \( H_u \) is simply a good numerical approximation which simplifies the calculation, and not equivalent to setting \( u \) still zero in the entire problem, as \( u \) still appears in spin anti-symmetric part of the Hamiltonian (where \( g = 1 \), and also in the gate voltage dependence.

**APPENDIX B: CORRELATION FUNCTIONS**

Let us now connect the explicit expressions for the coherent and incoherent currents given in Section III C and Section III. Equations \([15]\) and \([16]\), using the correlation functions defined in Section [11B].

It is useful to define the following combination of \( C^\theta \):

\[
\overline{C^\theta}(x, t) = C^\theta(0, 0) - C^\theta(x, t) \tag{B1}
\]

and similarly for \( \overline{C^\phi} \).

In the single channel case discussed in Section III there are only a single \( \theta \) field and a single \( \phi \) field, with the Hamiltonian given by Eq. [5]. Since the Hamiltonian is quadratic we can easily evaluate all the equilibrium correlation functions at finite temperature, paying attention to the different time orderings that appear as a result of the two branches of the Keldysh contour. The results for finite temperature is:

\[
C^\theta(x, t) = \frac{g}{4} \left[ \log \left( \frac{\beta v_L}{\pi \delta} \sinh \left( \frac{\pi(1 + v_L t - i \delta)}{\beta v_L} \right) \right) + \log \left( \frac{\beta v_R}{\pi \delta} \sinh \left( \frac{\pi(1 - v_L t + i \delta)}{\beta v_R} \right) \right) + (x \rightarrow -x, t \rightarrow -t) \right]; \tag{B2}
\]

and

\[
R^\theta(x, t) = -\frac{\pi}{2} \theta \left[ (t) \theta(x - v_R t) + (t) \theta(-x) \theta(|x| - v_L t) \right]. \tag{B3}
\]

The currents are expressed in integrals over complicated combinations of such correlation functions. For example, the coherent part of the current, given by Eq. [12], involves the following combinations:

\[
C_{co}(L, t) = 2C^\theta(L, t) - 2C^\theta(-L, t) \tag{B5}
\]

\[
+ 2C'^\theta(0, t) - 2C'(L, 0) + C'(L, t) + C'(0, t) \tag{B6}
\]

In a Carbon nanotube there are four channels, rather than a single one. In the non-interacting case, \( g = 1 \), all these channels are independent and we would recover the results of the single channel. Equations \([B5]\) and \([B6]\) still apply for this case. When \( g \neq 1 \), the different channels are coupled through the interaction, and we must find the correct combinations of the fields \( \theta_{i\sigma} \) and \( \phi_{i\sigma} \).
which decouple and therefore diagonalize the Hamiltonian. These combinations are discussed in Appendix A. This change of basis is in general a function of $u/v_F$ and $g$, and it mixes the $\theta$ and $\phi$ fields, which in turn complicates the functions $C_{co}$ and $R_{co}$ further. Luckily, the interactions in Carbon nanotubes are strong, $g \approx 0.3$, and in that range, the change of basis is very close to the usual spin/node symmetric/antisymmetric change of basis. If we approximate the diagonalizing fields by these symmetric/antisymmetric combinations, then equations (B5) and (B6) would apply provided we make the following substitutions:

$$C^g(x, t) \rightarrow \frac{1}{4} \sum_{j=1..4} C^{g_j}(x, t) \quad (B8)$$

Where each $\theta_j$ has a different set of values for $v_R$, $v_L$ and $g$ to be used in Eq.(B2). The fields $\theta_1$ and $\theta_2$ correspond to the spin asymmetric combinations, which decouple from the interaction, and hence have $g = 1$, and velocities $v_R = v_F \pm u$ and $v_L = v_F \mp u$. The fields $\theta_3$ and $\theta_4$ both have the same left and right mover velocities, $v_3$ and $v_4$ respectively, given by Eq. (20), and interaction parameters $1$ and $g$, respectively.

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