Mixing dipolar condensates: a new opportunity for enhancing superfluid pairing in a spin-polarized Fermi gas

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Abstract. The recent experimental progress in realizing Bose-Fermi mixtures and dipolar quantum gases has created the exciting possibility of creating dipolar Bose-Fermi mixtures with intriguing and unique properties. In a dipolar condensate, the dipole-dipole interaction represents a control knob inaccessible to nondipolar Bosons. Thus, mixing dipolar bosons with fermions may open up new possibilities for creating superfluids with unconventional pairings. We consider a mixture of a spin-polarized Fermi gas and a dipolar Bose-Einstein condensate in a three-dimensional setting in which s-wave scattering between fermions and the quasiparticles of the dipolar condensate can result in an effective attractive Fermi-Fermi interaction anisotropic in nature and tunable by the dipolar interaction. We develop a procedure which allows us to determine the parameters such as densities and scattering lengths that are required to achieve optimal critical temperature before the system starts to phase separate. We perform a systematic investigation, finding that a superfluid with a critical temperature, that is orders of magnitude higher than achievable in nondipolar mixtures, can be realized in dipolar mixtures at experimentally accessible densities and scattering lengths before the system phase separates.

In this proceeding paper, we report our recent work concerning the p-wave (or triplet, to be precise) Bardeen-Cooper-Schrieffer (BCS) superfluid pairing in a spin-polarized Fermi gas mixed with a dipolar condensate. To begin with, we note that experimentally accessible samples of quantum gases or liquids used to be limited only to a few isotopes such as $^3$He and $^4$He because most substances in nature solidify before the temperature can reach the regime where the macroscopic quantum nature of gases or liquids can be manifested. The rapid technological advancement in cooling and trapping of neutral atoms has completely turned the situation around. The experimentally realized Bose-Fermi mixtures now include $^7$Li - $^6$Li [1, 2], $^{23}$Na - $^6$Li [3], $^{87}$Rb - $^{40}$K [4, 5, 6, 7], $^6$Li - $^{87}$Rb [8], and $^{23}$Na - $^{40}$K [9]. The possible quantum gas mixtures under this new technology seem endless in view of the rich existence of atomic elements and their isotopes in nature.
More recently even mixtures containing two-component Fermi gases were realized in cold-atom experiments [10, 11]. Another development that motivates our interest in cold-atom mixtures involving dipolar condensates has been the recent upsurge of efforts aimed at expanding an ever-growing list of the experimentally realized dipolar quantum gases. Included are the heteronuclear $^{40}$K$^{87}$Rb molecules with electric dipoles (led by Deborah Jin and Jun Ye at JILA [12, 13, 14]) and spinor $^{87}$Rb condensates with magnetic dipoles (led by Dan Stamper-Kurn at Berkeley [15]). This list has now expanded from alkali elements to the transition element, $^{52}$Cr, with magnetic dipoles [16]. The latest members that joined this list are $^{164}$Dy [17] and $^{168}$Er [18], two lanthanide elements with such large magnetic moment and large atomic mass that promise to open a new frontier in the study of strongly dipolar quantum gases. Thus, it is conceivable that the dipolar Bose-Fermi mixtures can be realized in cold-atom experiments not too far down the road.

![Figure 1.](image)

A distinctive advantage of ultracold atomic systems over solid-state systems is that with the ultracold atomic systems, key system parameters, including the dimensionality and the interaction between particles of same and different species, can be tuned precisely. In a dipolar condensate, the dipole-dipole interaction represents a control knob inaccessible to nondipolar Bosons. Thus, mixing dipolar bosons with fermions opens up new possibilities. Of particular relevance to this proceeding paper is the question whether mixing dipolar bosons can help to enhance the opportunity for two fermions of same spin to form highly correlated p-wave Cooper pairs. For a typical pair of ground state atoms in a partial wave with a finite angular momentum $l\hbar$, the two-body scattering amplitude $f_l$ scales, according to the celebrated Wigner threshold law [19], as $(\hbar k_F^l)^{21}$, where $\hbar k_F$ is the Fermi momentum, so that the cross section for the p-wave scattering is vanishingly small in the ultracold limit where $k_F^l r_0 \ll 1$ with $r_0$ the range of interparticle potential. Thus, for ultracold quantum gases consisting of typical ground state atoms, the critical temperature for p-wave BCS superfluid is at such a low level that it becomes essentially inaccessible to current technology. Numerous schemes have been proposed, using, for example, dipolar Fermi gases [20, 21, 22, 23, 24, 25] and...
p-wave Feshbach resonance \[26, 27, 28\], in order to overcome the limitation set by the Wigner threshold law (the latter appears not feasible since Feshbach molecules are short lived \[29, 30, 31, 32\]).

In our recent studies \[33, 34\], we have paid particular attention to the schemes that seek to achieve the same goal but using cold-atom quantum gas mixtures (see, references \[35, 36, 37, 38\] for several examples). In particular, we have considered, in Ref. \[34\], a uniform mixture of a spin-polarized Fermi gas and a dipolar condensate, shown in Fig. 1(a), in a three-dimensional (3D) setting, with the goal of extending the work by Dutta and Lewenstein \[36\] from 2D to 3D dipolar Bose-Fermi mixture. In our model, a boson of mass \(m_B\) interacts with a fermion of mass \(m_F\) via a contact interaction of strength \(U_{BF} = 4\pi \hbar^2 a_{BF}/m_{BF}\), and two bosons interact via both a short-range s-wave interaction of strength \(U_{BB} = 4\pi \hbar^2 a_{BB}/m_B\) and a long-range dipole-dipole interaction \[Fig. 1(b)\] of the form \(U_{DD}(k) = 8\pi d^2 P_2(\cos k)\) in momentum space, where \(a_{BF}\) and \(a_{BB}\) are the corresponding s-wave scattering lengths, \(d\) the dipole-dipole interaction strength, \(P_2(x)\) the second-order Legendre polynomial, \(\theta_k\) the polar angle of wavevector \(k\), and \(m_{BF} = 2m_Bm_F/(m_B+m_F)\).

At very low temperatures, spontaneous symmetry breaking of the global gauge \([U(1)]\) symmetry results in an ordered phase, known as the Bose-Einstein condensate (BEC), where the order parameter \(\langle \hat{\psi}_B \rangle\) has a finite value \(\langle \hat{\psi}_B \rangle \neq 0\), where \(\hat{\psi}_B\) is the field operator for annihilating a dipolar boson and \(n_B = |\langle \hat{\psi}_B \rangle|^2\) is the number density for the dipolar condensate. Nambu and Goldstone theorem then asserts that the condensate must contain a gapless excitation. Using the Bogoliubov perturbative ansatz and diagonalization procedure, one can show that this gapless branch of the excitation corresponds simply to the phonon spectrum given by

\[
E_{k,B} = v_B \hbar k \sqrt{1 + (\xi_B k)^2 + 2\varepsilon_{dd} P_2(\cos \theta_k)}
\]

where \(\varepsilon_{dd} = 4\pi d^2/(3U_{BB})\) \[39\] measures the dipolar interaction relative to the s-wave collision, \(v_B = \sqrt{n_B U_{BB}/m_B}\) is the phonon speed, and \(\xi_B = \hbar/\sqrt{4m_B U_{BB}}\) is the healing length. In a 3D dipolar condensate, the side-to-side arrangement where the dipole-dipole interaction is the most repulsive coexists with the head-to-tail configuration where the dipole-dipole interaction is the most attractive \[see Fig. 1(c)\]. This renders the phonon spectrum anisotropy and for the same reason, \(\varepsilon_{dd} = 4\pi d^2/(3U_{BB}) < 1\) has to be less than 1, meaning that the short-range repulsive interaction has to be sufficiently large, in order for the condensate to be stable against the collapse due to the attractive part of the dipole-dipole interaction.

In a spin-polarized model, although the s-wave contact interaction is inaccessible to two identical atoms, two identical fermions can interact indirectly by exchanging virtual phonons with the help of the intermediate states containing one phonon without violating the Pauli exclusion principle. One atom can emit a virtual phonon which is later captured by another atom or vise versa. This phonon-induced Fermi-Fermi interaction in momentum space is given by

\[
U(k) = -\frac{U_{BF}^2}{U_{BB}} \frac{1}{1 + (\xi_B k)^2 + 2\varepsilon_{dd} P_2(\cos \theta_k)}.
\]
Figure 2. (Color online) (a) A polar plot of the scaled $|U(k)|$ [Eq. (2)] when $k/k_F = 1$ and $\delta(=\xi_B k_F^0) = 0.4$ shows that $\theta_k = 90^\circ$ is the most attractive direction. (b) The largest positive eigenvalue $\omega$ (scaled to $-U_B^2/U_{BB}$) of the triplet interaction potential matrix, $U_t(k,k')$, on the Fermi surface as a function of $\delta$ [34]. The dashed curve corresponds to $\varepsilon_{dd} = 0$. The solid curves, from bottom to top, corresponds to $\varepsilon_{dd} = 0.1, 0.2, 0.5$, and 0.8. The superfluid critical temperature, $T_s$, is proportional to $\exp(-1/\omega \lambda)$ according to Eq. (7). (c) $T$ (scaled to the Fermi temperature $\epsilon_F^0/k_B$) is plotted as a function of $\delta$ [34] when, from bottom to top, $\varepsilon_{dd} = 0.6, 0.7$ and 0.8. Throughout, $\lambda = 0.87$. $T$ in the absence of the dipolar interaction is too low to be visible in the scale of Fig. 2(c)]

in the static limit [40, 35, 41]. A polar plot of the induced interaction at a fixed momentum, Fig. 2(a), indicates that in the absence of dipolar interaction, this induced interaction is isotropic, and it becomes increasingly anisotropic as the dipolar interaction, $\varepsilon_{dd}$, increases with $\theta_k = 90^\circ$ being the most attractive direction.

Integrating out the phonon degrees of freedom and taking the Hartree-Fock mean-field approach, we arrive at the mean-field Hamiltonian

$$\hat{H}_F^M = \sum_k \xi_k \hat{a}_k^\dagger \hat{a}_k + \frac{1}{2} \sum_k \left[ \Delta(k) \hat{a}_k^\dagger \hat{a}_{-k} + h.c. \right],$$

where $\xi_k = \epsilon_k - \mu_F + \Sigma(k)$, $\hat{a}_k$ is the fermionic field operator, $\mu_F$ is the chemical potential. By diagonalizing the Hamiltonian in Eq. (3) using the Bogoliubov transformation, we obtain the equation for the Fock potential

$$\Sigma(k) = -\frac{1}{V} \sum_{k'} U(k-k') \frac{1}{2} \left[ 1 - \tanh \frac{\beta E_{k'}}{2} \right],$$

and the equation for the gap parameter,

$$\Delta(k) = -\frac{1}{V} \sum_{k'} U_t(k,k') K(k') \Delta(k'),$$

which are to be solved simultaneously with the particle number equation

$$n_F = \frac{1}{V} \sum_{k'} \frac{1}{2} \left( 1 - \tanh \frac{\beta E_{k'}}{2} \right),$$
to determine $\Sigma(k), \mu_F$ and $\Delta(k)$ self-consistently for a given set of Fermi density $n_F$ and temperature $T$. In Eq. (5), the integration kernel $K(k) = \tanh(\beta E_k/2)/(2E_k) - 1/(2\epsilon_k)$ is defined in terms of the temperature via $\beta = 1/k_BT$ and the quasi particle spectrum $E_k = \sqrt{\epsilon_k^2 + \Delta(k)^2}$, and $U_1(k, k') = [U(k + k') - U(k - k')]/2$ is the triplet interaction potential symmetrized so that the gap parameter becomes the antisymmetric function of the relative position or momentum.

The external field introduced to establish the dipole direction breaks the spherical symmetry so that the matrix, $U_1(k, k')$, is not fully diagonalized in the space spanned by the spherical harmonic functions. However, the system under the external field still possesses the cylindrical symmetry, which means that $U_1(k, k')$ remains diagonalized with respect to the magnetic quantum number but not the angular momentum quantum number. As a result, the superfluid resulting from anisotropic attractive interactions like the one shown in Fig. 2(a) is a triplet with a gap parameter characterized with a coherent superposition of all the odd partial waves [22, 34]. In the weak-coupling limit, one can single out the states near the Fermi surface as the dominating contribution to the gap equation. The critical temperature (correct up to a preexponential factor) then becomes

$$T = \frac{8\epsilon_0^F e^{\gamma-2}}{\pi k_B} \exp \left(-\frac{1}{\omega \lambda}\right),$$

where $\gamma$ is the Euler’s constant, $k_B$ the Boltzmann gas constant, $\epsilon_0^F$ the Fermi energy of a non-interacting Fermi gas with number density $n_F$, $\lambda = N(\epsilon_0^F)U_{BF}^2/UBB$ a unitless parameter with $N(\epsilon_0^F)$ being the density of states, and finally $\omega$ the largest positive eigenvalue of the triplet potential matrix, $U_1(k, k')$, on the Fermi surface. Figure 2(b) shows how the largest positive eigenvalue $\omega$ changes with $\delta$ for different dipolar interaction $\varepsilon_{dd}$. The dotted curve represents the value of $\omega$ in the absence of the dipolar interaction. As $\delta$ decreases (or equivalently the Fermi momentum reduces), $\omega$ begins to decrease to zero after reaching its peak at $\delta \approx 1$. In contrast, when the dipolar interaction is sufficiently strong, $\omega$ not only asymptotes to a finite value but this value increases appreciably with $\varepsilon_{dd}$ in the limit $\delta \rightarrow 0$, in clear defiance of Wigner’s threshold law. As a result, we see from Fig. 2(c) that the critical temperature increases dramatically with the increase in the dipolar interaction.

There is an issue of whether bosons and fermions can stay miscible or stay phase separated. An illuminating way to examine this issue is to construct the phase diagram in the chemical potential space from the grand canonical thermodynamical potential density:

$$\Omega = \frac{1}{2}UBBn_B^2 + U_{BF}n_Bn_F + \frac{3}{5}An_F^{5/3} - \mu_Bn_B - \mu_Fn_F.$$

where $\mu_B$ ($\mu_F$) is the chemical potential of bosons (fermions), $n_B$ ($n_F$) the boson (fermion) number density, and $A = \hbar^2(6\pi^2)^2/2m_F$. Phase separation manifests itself in the chemical potential space as a first-order phase transition line where different phases share the same energy. Figure 3 is the phase diagram in the chemical potential. Here, the space is divided into a pure Bose phase, a pure Fermi phase, and a Bose-Fermi mixture. Our proposal works only in the region indicated as BS where the bosons and fermions can coexist in the same spatial volume. Notice that there exists a first-order phase transition indicated by the dashed line between the mixed phase and the pure
Figure 3. The phase diagram in the chemical potential [42] where the scaled chemical potentials are defined as $	ilde{\mu}_B = U^2_{BF} \mu_B / U^3_{BB} A^3$ and $	ilde{\mu}_F = U^4_{BF} \mu_F / U^2_{BB} A^3$ with $A = \hbar^2 (6\pi^2)^{2/3} / 2m_F$. B, S, and BS stand for pure Bose, pure Fermi, and Bose-Fermi mixed phases, respectively.

Fermi phase. This suggests that at sufficiently high densities the system will phase separate into a pure Fermi phase and a mixed Bose-Fermi system.

Phase separation limits the densities and therefore limits the achievable critical temperature. In our study, we have developed a procedure which allows us to determine the parameters such as densities and scattering lengths that are required to achieve the optimal critical temperature before the system starts to phase separate. A systematic investigation that takes into consideration the phase separation [42] finds that a superfluid with a critical temperature, that is orders of magnitude higher than achievable in nondipolar mixtures, can be realized in dipolar mixtures at experimentally accessible densities and scattering lengths before the system phase separates [34]. In conclusion, the recent experimental progress in realizing Bose-Fermi mixtures and dipolar quantum gases has created the exciting possibility of creating dipolar Bose-Fermi mixtures with intriguing and unique properties. In a dipolar condensate, the dipole-dipole interaction represents a control knob inaccessible to nondipolar Bosons. Thus, mixing dipolar bosons with fermions may open up new possibilities for creating superfluids with unconventional pairings. In this proceeding paper, we have reported our recent work and have shown that mixing dipolar bosons can induce an effective Fermi-Fermi attractive interaction which, owing to its long-range and anisotropic nature, can significantly increase the prospect of realizing a superfluid with a gap parameter which is a coherent superposition of all odd partial waves.

Acknowledgments

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