Control of atomic motion with an atom-optical diode on a ring

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Abstract
We propose a method to manipulate the motion of an atom on a ring by means of an atom diode—a laser valve for one-way atomic motion. Several applications are numerically modelled and illustrated: the direction of motion is reversed if the atom moves initially counterclockwise and left unaltered otherwise; a net clockwise motion is imparted on a wave packet with zero initial velocity; and, combining the diode with a well, we also demonstrate cooling and trapping, since the atom can be efficiently slowed down at each diode crossing until it is finally captured in the well when its energy is below the well threshold.

1. Introduction

There is currently much interest in controlling the motion of cold atoms motivated by fundamental physical phenomena, quantum information processing, atom laser generation, metrology, interferometry, or the achievement of even lower temperatures. Cold atoms are relatively easy to produce and offer, with respect to other particles, many possibilities for coherent manipulation with lasers, magnetic fields or mechanical interactions. They may be trapped in artificial lattices, can be guided in effectively one-dimensional wires or adopt interesting collective behaviour; also, their mutual interactions can be changed, or suppressed. All this flexibility facilitates the translation of some of the concepts and applications of electronic circuits into the atom-optical realm to implement atom chips, atom circuits or quite generally ‘atomtronics’ [1]. In this context, efficient elementary circuit elements playing the role of diodes or transistors need to be developed. In particular, we have proposed and studied a laser device acting as a one-way barrier for atomic motion [2–6]. Similar ideas have been considered by Raizen and co-workers for atom cooling [7, 8], and have been recently implemented experimentally [9, 10]. These one-way models and experiments rely on atom–laser interactions in the independent atom regime, but there are also complementary proposals making use of interatomic interaction for ‘diodic’ one-way transport [11].

In this paper we propose to confine the atoms on a ring where an atom diode controls the direction of motion, and demonstrate numerically that this device is capable of cooling and trapping the atoms with phase-space compression. Only a few cycles of spontaneous emissions are generally required to achieve this goal, in contrast, say, to optical molasses, so that the dissipation and heating are kept at a minimal level. The process is similar to Sysiphus cooling [12] but in the ring device every cooling step occurs in a more efficient way.

Ring-shaped traps for cold atoms have been put forward or implemented for matter-wave interferometry and highly precise sensors [13], for studying the stability of persistent currents [14, 15], sound waves, solitons and vortices in Bose–Einstein condensates [16], collisions [17], for coherent acceleration [18, 19], production of highly directional output beams [20] or quantum computation [21]. (For further applications see [22].) The ring traps are implemented by magnetic waveguides [20, 13, 23], purely optical dipole forces [24], magnetoelectrostatic potentials [25], overlapping of magnetic and optical dipole traps [26] or misalignment of counterpropagating laser beam pairs in a magneto-optical trap [17].
2. The model

In our model the confining ring has radius $r_0$. We assume tight lateral confinement so that the motion along the ring is effectively one dimensional, $x$ being the arc coordinate $-l/2 < x \leq l/2$, $l = 2\pi r_0$. For the numerical computation a number of simplifications are applied. The explicit internal structure of the atom is limited to three levels: a ground state 1, a stable excited state 2 and a third auxiliary unstable state 3. In practice further levels may be necessary, e.g., for optical confinement in the ring, two-photon pumping, and for well or barrier creation, but their populations will be negligible and can be dropped from the simulation.

The atom diode (optionally extended for cooling applications with a ground-state well) is put on the ring as in figure 1(a). The goal of the atom diode is to let ground-state atoms pass from left to right (clockwise in the ring) but reflect them if they come from the right (arriving anticlockwise) within a broad velocity range. In our application excited atoms in level 2 are reflected from both sides, remaining in 2 for incidence from the left and decaying to 1 for right incidence. The result in the ring setting will be that the atoms end up moving clockwise (in the ground state outside the diode region) irrespective of their initial internal or translational state, within the velocity working range of the device.

There are different possible schemes for realizing such a behaviour and some of them have been explained before [2–4, 6]. The experimental realizations [9, 10] in particular, could be adapted to the diodic behaviour described above, mutatis mutandis. Instead of restricting ourselves to a specific atom and level structure, we shall demonstrate the principles involved with an idealized scheme depicted in figures 1(b) and (c). Without considering first the rightmost quenching part of figure 1, the atom diode consists of a central resonant pumping interaction for the atomic transition $1 \rightarrow 2$ described by a Rabi frequency $\Omega_p$, partially overlapping with two state-selective mirror potentials $W_0/2$ and $W_1/2$ that block the excited state 2 and ground state 1 on the left, respectively right, side of the pumping region. With this scheme an atom incident from the left in state 1 is transmitted in state 2. If the atom arrives from the left in 2 it will be reflected in 2. On the other hand, if it arrives in state 1 from the right, the atom will be reflected in 1. This is almost the desired diodic behaviour, and it is very robust and stable for a velocity range whose upper bound can be increased by increasing laser intensities. The reason for the stability is the adiabatic transfer achieved between levels 1 and 2 as explained in [3]. Reflection $2 \rightarrow 1$ for incidence from the right is not achieved (in fact an excited atom from the right would pass leftwards ending in 1), but we shall show that with an additional quenching operation, see below. The $W_2\Omega_pW_1$ scheme can be realized by a detuned stimulated Raman adiabatic passage (STIRAP) [27] transfer between $1 \rightarrow 2$ (using an additional auxiliary state) with just two overlapping lasers [6], or, by creating the mirrors with blue-detuned lasers coupling the ground, respectively excited, state to other, not shown, levels. Note that this may result in additional effects on the other levels, for example, a laser creating a mirror for level 2 may induce a small well for 1, as in [10], or a barrier for 1 may also lead to a smaller barrier for 2, depending on the specific level configuration and transitions used. In general such secondary effects will not spoil the adiabatic transfer and diodic behaviour if kept small, and will produce lower bounds for the working velocity range. They can be minimized or even suppressed by a careful level and interaction selection, or by compensation schemes [28]. We shall deal with these issues in more detail elsewhere.

As stated, there are other possibilities to implement an atom diode, for example by using an on-resonance STIRAP transfer between $1 \rightarrow 2$ (using state 3 as an additional auxiliary state) and a single state-selective mirror potential for the ground state [4].

Let us now discuss the quenching, right end of the device in figure 1: its purpose is to force the irreversible decay of level 2 into level 1, after the atom has crossed the $W_2\Omega_pW_1$ diode.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Atom diode in a ring, length $l = 200 \mu m$ ($r_0 = 31.831 \mu m$, $-100 \mu m \leq x < 100 \mu m$), \(b\) schematic action of the different lasers on the atom levels for the two-level atom diode plus quenching and \(c\) schematic spatial location of the different laser potentials and their effect on the moving atom. They are all taken as Gaussian functions: $\Omega_\rho(x) = \Omega_\rho \Pi(x, x_r, \sigma_r)$, $W_\rho(x) = W_\rho \Pi(x, x_r, \sigma_r)$, $\Pi(x, x_r, \sigma_r) = e^{-\frac{1}{2} \left(\frac{x-x_r}{\sigma_r}\right)^2}$; $\mu$ mass of neon, $x_\rho = -100 \mu m$, $x_\rho = -45 \mu m$, $x_p = -20 \mu m$, $x_1 = 5 \mu m$, $x_r = 40 \mu m$, $x_Q = 50 \mu m$, $\sigma = 7.5 \mu m$, $\sigma_r = 15 \mu m$, $\sigma_Q = 5/\sqrt{2} \mu m$; $\Omega_\rho = 1.6 \times 10^5 s^{-1}$, $W_1 = W_2 = 1.6 \times 10^5 s^{-1}$; $W_Q = 4 \times 10^5 s^{-1}$.}
\end{figure}
part from left to right, i.e. the atom is left finally in 1, ready for
continuing its clockwise motion in the ring. The quenching also
transfers any atoms that could come in 2 from the right
to 1, then they are reflected in that level by the W₁ mirror and
end up moving clockwise in that level too. The quenching is
modelled with a laser on resonance with the 2–3 transition with
Rabi frequency Ω₂, while state 3 decays rapidly to the ground
state. This quenching is also very efficient and independent of
velocity in a broad velocity interval. In a real setting level 3
might in addition have some probability to decay into level 2,
but the laser will pump the atom back to level 3 where it has a
’sec’ond’ chance to decay to level 1 (see e.g. the experimental
realization of a one-way barrier in [10]). After a few of these
cycles the atom would finally decay to state 1, so we simplify
the description of the process by an effective decay rate γ₃
from 3 → 1.

A novelty in the full scheme of figure 1, with respect to
previous diode models, is the optional addition of a ground-
state well W₁; containing the quenching laser region. The
effect of such a well is twofold: it subtracts kinetic energy
from the ground-state atoms which try to escape from it, and
it also traps eventually the atoms when they are cool enough,
see section 4. Again, the well can be implemented by a red-
detuned laser coupling the ground state to other not shown
levels, and similar considerations apply to those already stated
above for the barriers W₁,₂.

For modelling the quantum dynamics on the ring, usual approximations are applied (dipole and rotating wave
approximations, semiclassical laser fields with wave vectors
perpendicular to the ring, neglecting atom–atom interactions
and Lamb–Dicke regime without transversal excitations), and
the Hamiltonian for the atom using |1⟩ = (1, 0, 0)ᵀ, |2⟩ =
(0, 1, 0)ᵀ, and |3⟩ = (0, 0, 1)ᵀ, where T means ‘transpose’,
in an interaction picture that makes it time independent, may
be written, without decay, as

\[
H = \frac{p_{x}^{2}}{2m} + \frac{h}{2} \begin{pmatrix}
W₁(x) + W₆(x) & \Omega₆(x) & 0 \\
\Omega₆(x) & W₂(x) & \Omega₂(x) \\
0 & \Omega₂(x) & 0
\end{pmatrix},
\]

where \(p_{x}\) is the momentum operator conjugate to the ring
coordinate \(x\). All potentials are chosen as Gaussian functions
according to the caption of figure 1. We examine the time
evolution by means of a one-dimensional master equation
which includes the effect of recoil for laser polarization
perpendicular to the ring,

\[
\frac{\partial}{\partial t} \rho = -\frac{i}{\hbar}[H, \rho] - \frac{\gamma₃}{2} |3⟩⟨3|, \rho⟩,
+ \gamma₁ \int_{-1}^{1} du \frac{3}{8} (1 + u²) \exp \left( \frac{imν_{rec}}{\hbar} ux \right) |1⟩⟨1|ρ⟩|3⟩⟨3|
\times \exp \left(-\frac{i}{\hbar} mν_{rec} ux \right),
\]

where ν_{rec} is the recoil velocity, \(x\) is the ring position operator
and the periodic boundary conditions have to be taken into
account. We are assuming that the excited part |3⟩⟩|3⟩ is
localized in the quenching region such that the curvature of
the ring can be neglected concerning the recoil term and the
usual one-dimensional approximation can be used [30]. The
initial condition is chosen as a pure state \(\rho(0) = |Ψ₀⟩⟨Ψ₀|\),
namely a Gaussian wave packet

\[
Ψ₀(x) = \frac{1}{\sqrt{2π}} \int dk \ Φ₀(k) e^{ikx},
\]

with

\[
Φ₀(k) = \frac{1}{\sqrt{2π} \sqrt{Δk}} (1, 0, 0)ᵀ\langle
\frac{−(k - k₀)²}{4Δk²} − ik(k − k₀) \langle x₀ − \frac{h}{m} Δtk₀ \rangle
− \frac{1}{14} \Delta k^{2} \rangle, \]

where \(k₀ = \frac{m}{\hbar} v₀\) and \(Δk = \frac{m}{\hbar} Δv\).

The master equation (1) is solved by using the quantum
jump approach [31]. A computational simplification is
achieved by approximating the three-level treatment by a two-
level one in the regime of large γ₃. A basic step in the quantum
jump approach is to solve a time-dependent Schrödinger
equation with an effective Hamiltonian \(H_{eff} = H − i\frac{2}{γ₃}|3⟩⟨3|\),
For large γ₃ (see [29]),

\[
⟨3|Ψ(τ)⟩ \approx \frac{−i}{γ₃} Ω₂(x) |2⟩⟨2|Ψ(τ)⟩.
\]

Therefore, the three-level Schrödinger equation can be
approximated by a two-level one with the effective Hamiltonian

\[
H_{approx} = \frac{p_{x}^{2}}{2m} + \hbar \left( \frac{W₁(x) + W₆(x)}{Ω₆(x)} \Omega₆(x) \right)
− W₂(x) + iΩ₂(x) \right),
\]

where \(W₂ = Ω₂(x)∧\). The second element of the quantum
jump approach is the resetting operation at each jump,
exp \(i\frac{mν_{rec}}{\hbar} ux \) \(⟨3|Ψ(τ)⟩ \longrightarrow ⟨1|Ψ(τ)⟩\), where \(u ∈ [-1, 1]\) is
chosen with the probability \(\frac{1}{2}(1 + u²)\), all other
amplitudes are set to zero, and the wavefunction is normalized.
Because of equation (4), this can also be done in the
two-level approach, −i
\(\sqrt{W₂(τ)} \exp \left( \frac{i mν_{rec}}{\hbar} ux \right) |2⟩⟨2|Ψ(τ)⟩ \longrightarrow
⟨1|Ψ(τ)⟩\), then the second level is set to zero and the
wavefunction is normalized. In the following, we will use this
approximate approach with an effective quenching potential
\(W₂ = \frac{Ω₂(x)∧}{γ₃} \) = \(\langle W₂ Π(x, x₀, σ₀) \) (see caption of figure 1).
The evolution of the wavefunction is numerically performed
using the standard fast-Fourier transform method [34].

3. Preparing the direction of motion

As a first, simple application let us examine the effect of the
diode, without the ground-state well \(W₁\), for two different
initial states of the atom, moving clockwise and anticlockwise.
We expect that after some time the atom will end up moving
clockwise for both states and figure 2(a) shows that this is
indeed the case. Note that the modulus of the average velocity
is conserved.

As a further example of motion control, consider initially
a broad Gaussian wave packet ‘at rest’, i.e., with zero mean
velocity, see figure 2(b): the mean velocity increases with time.
until a stationary value is reached, proportional to the initial standard deviation. This ‘spontaneous flow’ on a ring with a one-way diaphragm was envisioned by Zhang and Zhang in a discussion of Maxwell’s demon [32]. Our device realizes this motion without violating the second principle because of the necessary irreversible step of the spontaneous photon emission [33, 35]. In the figure the error bars correspond to $\pm \Delta v_t$, where $\Delta v_t$ is the velocity standard deviation at time $t$. The final mean velocity $v_{1\text{final}}$ and the final standard deviation $\Delta v_{1\text{final}}$ can be estimated by assuming that the diode is simply ‘folding’ the velocity distribution, i.e. mapping the negative velocities into positive ones. Thus we get $v_{1\text{final}} \approx (2/\pi)^{1/2} v \approx 7.98 \text{ cm s}^{-1}$ and a reduction of the velocity width, $\Delta v_{1\text{final}} \approx [(\pi - 2)/\pi]^{1/2} \Delta v \approx 6.03 \text{ cm s}^{-1}$. The numerical results are instead $v_{1\text{final}} = 6.83 \pm 0.05 \text{ cm s}^{-1}$ and $\Delta v_{1\text{final}} = 6.40 \pm 0.10 \text{ cm s}^{-1}$ (the error is defined by the difference between averaging over $N$ and $N/2$ trajectories.) The discrepancy indicates the crudeness of the perfect folding assumption, this can also be seen by looking at the actual final velocity distribution in figure 2(c). The atom diode range for perfect ‘diodic’ behaviour (defined as in [3]) for the parameters used here is $-22 \text{ cm s}^{-1} < v < 22 \text{ cm s}^{-1}$.

4. Atom cooling and trapping

Now we assume the presence of a well $W_T$ for the ground state (see figure 1). The initial state of the ground-state atom will have some velocity and position width, but the anticlockwise moving components are reflected by the diode—which can be crossed only in the direction of the arrow (figure 1(a))—so all atoms will eventually approach the diode clockwise as shown in the previous section. After each crossing of the diode, the atom, which is now excited, is forced to emit a spontaneous photon (quenched), at or very near the bottom of the well; the atom has to lose kinetic energy to leave the well, and it is slowed down at every crossing, until it is finally trapped in the well when its energy is not enough to escape from it. The process is reminiscent of Sysiphus cooling [12], a difference being that both the transfer from ground to excited state in the diode and the quenched decay are here highly controlled, robust and efficient processes.

Before looking at the quantum-mechanical description, we will examine a simple classical model to estimate the timescales and the cooling efficiency for different recoil velocities. In this classical toy model, the diode and the well are reduced to a point at position $x_0$, and the initial particle positions and momenta are distributed according to Gaussian probability distributions corresponding to the initial quantum distributions $|\Psi_0(x)|^2$, respectively $|\Psi_0(k)|^2$, see equations (2) and (3). In each classical trajectory a random recoil kick is imparted at the diode clockwise passage, as in the quantum jump calculations done above. The trajectory is ‘trapped’ (and eliminated from the ensemble) when the energy becomes smaller than the threshold imposed by the well depth; otherwise a fixed amount of kinetic energy corresponding to the well depth $\frac{1}{2}mv_T^2$ is subtracted and the motion continues. The results for the trapping probability are shown in figure 3(a). Random recoil affects the result in two ways: higher recoil velocities accelerate a rapid initial trapping, but they also increase slightly the time necessary for cooling and trapping the complete ensemble. The number of diode crossings to trap the atom for an initial velocity $v > 0$ and no recoil is given by the smallest integer $n_D$ fulfilling $v_{\text{rec}} n_D v_T^2 < 0$. The time required is given by the time to reach the diode at the first crossing, $t_0 = (x_D + |x_0|)/v$, plus the total time for the $n_D - 1$ rounds, $t_{n_D-1} = \sum_{j=1}^{n_D-1} (v^2 - jv_T^2)^{-1/2}$. For the parameters of figure 3(a) and $v = v_0$, we get $n_D = 8$ and $t_0 + t_1 \approx 25.5 \text{ ms}$.

Now we switch to the quantum-mechanical description. The ‘velocity depth’ of the well used is $v_T := \sqrt{\frac{2}{\hbar^2} |W_T|} \approx 3.6 \text{ cm s}^{-1}$ and it corresponds to the well depth used in the classical simulation.

We start by neglecting recoil, $v_{\text{rec}} = 0$, and calculate the trapping probability in coordinate space, $P_{T,x} = \int_{0}^{t_{\text{max}}} dx (x |\rho_{11}| x)$, and in velocity space, $P_{T,v} = \int_{-v_{\text{rec}}}^{v_{\text{rec}}} dv (v |\rho_{11}| v)$. The results are shown in figures 3(b) and (c) (thick dotted green line) averaging over $N = 200$ trajectories; a numerical error defined by the difference of the result between averaging over $N$ and $N/2$ trajectories is also plotted in figures 3(b) and (c). The parameters used

Figure 2. (a) Mean velocity versus time; $v_0 = 10 \text{ cm s}^{-1}$ (thick dotted line), $v_0 = -10 \text{ cm s}^{-1}$ (solid line); $\Delta v = 0.2 \text{ cm s}^{-1}$; (b) mean velocity (line) and velocity standard deviation (error bars) versus time; $v_0 = 0$, $\Delta v = 10 \text{ cm s}^{-1}$; (c) velocity distribution at time $t = 5 \text{ ms}$ of case (b); common parameters: $\Delta t = 0$, $W_T = 0$, $v_{\text{rec}} = 0$, averaged over $N = 200$ trajectories, for other parameters see figure 1.
for the atom diode result in a range for perfect ‘diodic’ behaviour \(-22 \text{ cm s}^{-1} < v < 22 \text{ cm s}^{-1}\) (defined as in [3]). Figure 4(a) shows the evolution of mean velocity and the velocity standard deviation versus time. The final probability densities can be seen in more detail in figure 5 (thick dotted green line). In this figure we may verify the occurrence of cooling and phase-space compression, namely, a narrower distribution both in coordinate and velocity space. The final trapping probabilities are \(P_{T,x} = 0.984 \pm 0.001\) and \(P_{T,v} = 0.981 \pm 0.004\), with the errors calculated as before.

Let us now examine the case with recoil velocity \(v_{\text{rec}} = 2 \text{ cm s}^{-1}\). The trapping probability versus time shown in figures 3(b) and (c) (solid red line) shows a high final trapping probability. Figure 4(b) shows the evolution of mean velocity and the velocity standard deviation versus time. The final probability densities are shown in figure 5 (solid red line) and are quite similar to the case without recoil. We have finally \(P_{T,x} = 0.973 \pm 0.004\) and \(P_{T,v} = 0.969 \pm 0.008\).

5. Summary

In summary, we have put forward and illustrated numerically the manipulation of the motion of an atom on a ring with an atom diode. We have shown that the direction of motion can be controlled, and we have proposed and numerically demonstrated a cooling method with phase-space compression that reduces the velocity of the atom by repeated passages across an atom diode combined with a ground-state well. A common property is that only a few cycles of spontaneous emissions are required to achieve these goals, unlike for
example, in an optical-molasses approach where the number of spontaneously emitted photons is orders of magnitude greater. Another application, namely an increase of the atomic velocity by replacing the well by a barrier is straightforward. We hope that the results will encourage further work, in particular to find optimal parameters for combining the atom ring and the diode experimentally.

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