Structure and decay pattern of linear-chain states in neutron-rich Carbon isotopes

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Abstract. The linear-chain states of $^{14}$C are studied by using the antisymmetrized molecular dynamics. The calculated properties of the linear-chain states were compared with the observed data. The results for the $\pi$-bond linear-chain states reasonably agree with the observation. It is also shown that the linear-chain states decay to the excited states of $^{10}$Be. Hence, we regard that this unique decay pattern is a strong evidence of the linear-chain formation.

1. Introduction
The alpha clustering strongly influences the structure of light nuclei and many different types of $\alpha$ cluster structure have been studied for a long time. The linear-chain (LC) configuration of 3$\alpha$ clusters [1] in which $\alpha$ particles are linearly aligned is a famous and exotic cluster structure. The interest in the linear-chain configuration is further reinforced by the physics of unstable nuclei, because the valence neutrons may stabilize it by their glue-like role [2, 3, 4, 5, 6, 7, 8, 9, 10].

Recently, rather promising candidates of LC configuration in $^{14}$C were independently reported by several groups [11, 12]. However, there are several discrepancies between the theory [6] and experiment which must be resolved to establish the LC formation in $^{14}$C. For this purpose, we performed AMD calculation. From the obtained wave functions, we estimated the $\alpha$ decay widths of the LC states as well as those of other cluster and non-cluster states.

2. Theoretical Framework
In this study, we uses the Gogny D1S interaction as an effective interaction. We employ the parity-projected wave function $\Phi^\pi$ as the variational wave function,

$$\Phi^\pi = \frac{1 + \pi \hat{P}_x}{2} \Phi_{\text{int}}, \quad \pi = \pm, \quad \Phi_{\text{int}} = A\{\varphi_1, \varphi_2, \ldots, \varphi_A\}. \quad (1)$$

Here $\varphi_i$ is the single particle wave packet which is a direct product of the deformed Gaussian spatial part [13], spin ($\chi_i$) and isospin ($\xi_i$) parts,

$$\varphi_i(\mathbf{r}) = \exp\left\{-\sum_{\sigma=x,y,z} \nu_\sigma \left(r_\sigma - \frac{Z_{i\sigma}}{\sqrt{\nu_\sigma}}\right)^2\right\} \otimes (a_i \chi_\uparrow + b_i \chi_\downarrow) \otimes (\text{proton or neutron}). \quad (2)$$

The variational parameters $Z_i$, $a_i$, $b_i$ and $\nu_\sigma$ are determined by the constrained energy variation. After the energy variation, we perform the angular momentum projection and generator coordinate method (GCM) to obtain the excitation spectrum of $^{14}$C. To estimate the $\alpha$ reduced widths, we used an approximate method suggested in Ref. [14].
3. Results

3.1. Intrinsic wave functions obtained by the constrained energy variation.

Fig. 1 shows the intrinsic density distributions of $^{14}$C obtained by the quadrupole constrained energy variation. The panel (a) shows the the ground state without any clustering. At deformed region, we found several cluster states with different valence neutron configurations. In the oblate deformed region, we found the triangular configuration of 3α particles with valence neutrons in sd-shell. In the prolate deformed region, we found two different LC configurations. The panel (c) shows the configuration which we call π-bond LC, while the panel (d) shows the σ-bond LC. As clearly seen in there valence neutron distributions, they have two valence neutrons in the π and σ molecular orbits [2].

![Figure 1](image_url)

Figure 1. The density distribution of the positive states. The contour show the proton densities, while the color plots show the single particle orbits occupied by the most weakly bound neutron.

By superposing those wave functions and performing GCM calculation, we obtained the excitation spectrum of the positive-parity states shown in Fig. 2. In addition to the ground band, the cluster configurations shown in Fig. 1 (b)-(d) generate three rotational bands. The triangular band is built on the 0$^+$ state at 7.5 MeV and dominantly composed of the cluster configuration shown in Fig. 1 (b). The linear chain configurations (Fig. 1 (c) and (d)) generate two rotational bands named π-bond and σ-bond LC which are respectively built on the 0$^+$ states at 14.6 and 22.2 MeV. We see that the energy of the calculated π-bond LC is very close to the candidates observed by the resonant scattering [11, 12], while the states reported by von Oertzen [4] are close to the triangular band that is composed of the configuration shown in Fig. 1 (b).

![Figure 2](image_url)

Figure 2. The calculated and observed LC candidates. Open boxes show the observed LC reported by Refs. [4, 11, 12]. Filled symbols show the present result [].

Figure 3 (a) shows the α reduced widths (RW) for the decays to the ground band of $^{10}$Be (0$^+_1$, 2$^+_1$ and 4$^+_1$ states). It is clear that only the π-bond LC has large RW and reasonably agrees with the observed decay width to the $^{10}$Be(0$^+_1$). Since both of the observed and calculated excitation energies and decay widths agree to each other, we consider that the formation of the π-bond LC in $^{14}$C is rather convincing. We also note that the present calculation predicts that π-bond LC
In summary, we have performed the AMD calculations to identify the LC bands in $^{14}$C. We found three different cluster configurations which we call triangular and $\pi$- and $\sigma$-bond LC. We have shown that $\pi$-bond LC band is built on the $0^+$ state at 14.6 MeV, while the $\sigma$-bond LC band is built on the $0^+$ state at 22.2 MeV. The excitation energy and decay width of the $\pi$-bond LC reasonably agree with the observation, and hence, we conclude that the formation of the $\pi$-bond LC is rather convincing. We predict that the $\pi$-bond LC also decays to the $^{10}$Be($2^+_1$), while the $\sigma$-bond LC will decay to the $^{10}$Be($0^+_2$). The observation of these decay pattern will establish the assignment of the LC bands in $^{14}$C.

4. Summary
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