STABLE CLOCKS AND GENERAL RELATIVITY

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ABSTRACT

We survey the role of stable clocks in general relativity. Clock comparisons have provided important tests of the Einstein Equivalence Principle, which underlies metric gravity. These include tests of the isotropy of clock comparisons (verification of local Lorentz invariance) and tests of the homogeneity of clock comparisons (verification of local position invariance). Comparisons of atomic clocks with gravitational clocks test the Strong Equivalence Principle by bounding cosmological variations in Newton’s constant. Stable clocks also play a role in the search for gravitational radiation: comparison of atomic clocks with the binary pulsar’s orbital clock has verified gravitational-wave damping, and phase-sensitive detection of waves from inspiralling compact binaries using laser interferometric gravitational observatories will facilitate extraction of useful source information from the data. Stable clocks together with general relativity have found important practical applications in navigational systems such as GPS.
1. Introduction

Stable clocks have long played an important role in the field of general relativity. In addition to the use of the notion of the “ideal clock” in thought experiments that have become part of the conceptual foundation of special and general relativity, stable clocks realized in the laboratory have been employed extensively in experimental tests of relativity. They have been used as direct tools to measure or test relativistic effects, such as in tests of the gravitational redshift. But they have also been used as indirect, supporting tools, such as in the measurement of gravitational-radiation damping of the orbit of the binary pulsar. In fact, from one point of view, most experimental tests of special or general relativity amount to nothing more than comparisons of stable clocks. Finally, stable clocks, together with general relativity, have also found recent practical use in high-precision navigational systems, such as the Global Positioning System (GPS).

In this paper, we survey some of the varied uses of stable clocks in general relativity. In Section II we describe the Einstein Equivalence Principle, which is the foundation for the idea that spacetime is curved (and thereby a foundation for general relativity), and discuss the clock experiments that support it. Many of the experiments involve various kinds of intercomparisons of clocks — comparing identical clocks at different locations; comparing different clocks at the same location but with varying orientation; or comparing different clocks at a single location that moves relative to distant matter. In Section III we discuss the Strong Equivalence Principle and intercomparisons between atomic and gravitational clocks. Section IV discusses the use of stable clocks in the verification of and search for gravitational radiation, and Section V describes the use of stable clocks and general relativity in GPS.

2. Stable Clocks and Metric Gravity

2.1 The Einstein Equivalence Principle

The Einstein Equivalence Principle is the foundation for all metric theories of gravity, such as general relativity, Brans-Dicke theory and many others (for a review of topics and concepts discussed in this paper see Ref. 1). It states, roughly, that all test bodies fall in a gravitational field with the same acceleration (Weak Equivalence Principle), and that in local, freely falling or inertial frames, the outcomes of non-gravitational experiments are independent of the velocity of the frame (Local Lorentz Invariance) and the location of the frame (Local Position Invariance). A consequence of this principle is that the non-gravitational interactions must couple only to the symmetric spacetime metric $g_{\mu\nu}$, which locally has the Minkowski form $\eta_{\mu\nu}$ of special relativity. Because of this local interaction only with $\eta_{\mu\nu}$, local nongravitational physics is immune from the influence of distant matter, apart from tidal effects. Local physics is Lorentz invariant (because $\eta_{\mu\nu}$ is), and position invariant (because $\eta_{\mu\nu}$ is constant in space and time).

How could violations of EEP arise? From the viewpoint of field theory, violations of EEP would generically be caused by other long-range fields additional to $g_{\mu\nu}$ which also couple to matter, such as scalar, vector and tensor fields. Such theories are called...
non-metric theories. A simple example of a non-metric theory is one in which the matter action for charged particles is given by

$$I = -\sum_a m_a \int (g_{\mu\nu} v_a^\mu v_a^\nu)^{1/2} dt + \sum_a e_a \int A_\mu(x_a^\mu) v_a^\mu dt$$

$$- (16\pi)^{-1} \int \sqrt{-h} h^{\mu\alpha} h^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} d^4x ,$$  

(1)

where $m_a, e_a, x_a^\nu, v_a^\nu = dx_a^\nu/dt$ are the mass, charge, world-line and ordinary velocity, respectively, of the $a$-th body, $A_\mu$ and $F_{\mu\nu}$ are the electromagnetic vector potential and Maxwell field, $g_{\mu\nu}$ is the metric, $h_{\mu\nu}$ is a second, second-rank tensor field, and repeated Greek indices imply summation over four spacetime values. Locally one can always find coordinates (local inertial frame) in which $g_{\mu\nu} \to \eta_{\mu\nu}$, but in general $h_{\mu\nu} \neq \eta_{\mu\nu}$, instead, $h_{\mu\nu} \to h^0_{\mu\nu}$, where $h^0_{\mu\nu}$ is a tensor whose values are determined by the cosmological or nearby matter distribution. In the rest frame of the distant matter distribution, $h^0_{\mu\nu}$ will have specific values, and there is no reason a priori why those should correspond to the Minkowski metric (unless $h_{\mu\nu}$ were identical to $g_{\mu\nu}$ in the first place, in which case one would have a metric theory). The values of $h^0_{\mu\nu}$ could also vary with the location of the local frame in space or time relative to the distant matter. This can lead to violations of Lorentz invariance or position invariance in the local physics of electromagnetic systems.

A number of explicit theoretical frameworks have been developed to treat a broad range of non-metric theories, of which this was just one example. They include the $TH\epsilon\mu$ framework of Lightman and Lee,\(^2\) the $\chi-g$ framework of Ni,\(^3\) the $c^2$ framework of Haugan and coworkers,\(^4,5\) and the extended $TH\epsilon\mu$ framework of Vucetich and colleagues.\(^6\)

2.2 Local Lorentz Invariance

Tests of Local Lorentz Invariance are most profitably discussed using the $c^2$ Framework. This is a special case of the $TH\epsilon\mu$ formalism, adapted to situations in which one can ignore the variation with space and time of the external fields that couple to matter, and instead focus on their dependence on the velocity of the local frame. It assumes a class of non-metric theories in which the matter part of the action of Eq. (1) can be put into the local special relativistic form, using units in which the limiting speed of neutral test bodies is unity, and in which the sole effect of any non-metric fields coupling to electrodynamics is to alter the effective speed of light. The result is the action

$$I = -\sum_a m_a \int \sqrt{1-v_a^2} dt + \sum_a e_a \int A_\mu v_a^\mu dt + (8\pi)^{-1} \int (E^2 - c^2 B^2) d^4x ,$$  

(2)

where $E$ and $B$ are the usual electric and magnetic fields defined using components of $F_{\mu\nu}$. Because the action is explicitly non-Lorentz invariant if $c^2 \neq 1$, it must be defined in a preferred universal rest frame (presumably that of the 3K microwave background); in this frame, the value of $c^2$ is then determined by the cosmological values of the non-metric field. Even if the non-metric field coupling to electrodynamics is a tensor field, the homogeneity and isotropy of the background cosmology in the preferred frame is likely to
collapse its effects to that of the single parameter $c^2$. Because this action violates Lorentz invariance, systems moving through the universe will exhibit explicit effects dependent upon the velocity of motion. Detailed calculations of a variety of experimental situations show that those effects depend on the magnitude of the velocity through the preferred frame ($\sim 300$ km/sec), and on the parameter $\delta \equiv c^{-2} - 1$. Those effects also depend in general on the internal structure or dynamics of the system (clock) under study. In any metric theory or theory with local Lorentz invariance, $\delta = 0$, and no such effects occur, regardless of the internal structure of the system.

One can then set observable upper bounds on $\delta$ using a variety of experiments. Modest bounds on $\delta$ can be set by the “standard” tests of special relativity, such as the Michelson-Morley experiment and its descendents,\cite{7,8} or the Brillet-Hall\cite{9} interferometry experiment. In these examples the two clocks are the two arms of the interferometers, and the comparison is of their rates (round-trip time of flight of light) as the arms’ orientation varies relative to the velocity of the Earth through the universe. Other tests of special relativity involve comparison of identical atomic clocks separated in space, as the orientation of their baseline varies; communication between the clocks is by light propagation. These include a test of time-dilation using radionuclides on centrifuges,\cite{10} tests of the relativistic Doppler shift formula using two-photon absorption (TPA),\cite{11} and a test of the isotropy of the speed of light using one-way propagation of light between hydrogen maser atomic clocks at the Jet Propulsion Laboratory (JPL).\cite{12}

Very stringent bounds $|\delta| < 10^{-21}$ have been set by “mass isotropy” experiments of a kind pioneered by Hughes and Drever.\cite{13,14} The idea is simple: in a frame moving relative to the preferred frame, the non-Lorentz-invariant electromagnetic action of Eq. (2) becomes anisotropic, dependent on the direction of the velocity $\vec{V}$. Those anisotropies then are reflected in the energy levels of electromagnetically bound atoms and nuclei (for nuclei, we consider only the electromagnetic contributions). For example, the three sublevels of an $l = 1$ atomic wavefunction in an otherwise spherically symmetric atom can be split in energy, because the anisotropic perturbations arising from the electromagnetic action affect the energy of each substate differently. One can study such energy anisotropies by first splitting the sublevels slightly using a magnetic field, and then monitoring the resulting Zeeman splitting as the rotation of the Earth causes the laboratory $\vec{B}$-field (and hence the quantization axis) to rotate relative to $\vec{V}$, causing the relative energies of the sublevels to vary among themselves diurnally. Using nuclear magnetic resonance techniques, the original Hughes-Drever experiments placed a bound of about $10^{-16}$ eV on such variations. This is about $10^{-22}$ of the electromagnetic energy of the nuclei used. Since the magnitude of the predicted effect depends on the product $V^2\delta$, and $V^2 \approx 10^{-6}$, one obtains the bound $|\delta| < 10^{-16}$. Energy anisotropy experiments were improved dramatically in the 1980s using laser-cooled trapped atoms and ions.\cite{15,16,17} This technique made it possible to reduce the broadening of resonance lines caused by collisions, leading to improved bounds on $\delta$ shown in Figure 1 (experiments labelled NIST, U. Washington and Harvard, respectively).
Figure 1. Selected tests of Local Lorentz Invariance showing bounds on the parameter $\delta$, which measures the degree of violation of Lorentz invariance in electromagnetism. Michelson-Morley, Joos, and Brillet-Hall experiments test isotropy of the round-trip speed of light in interferometers, the later experiment using laser technology. Two-photon absorption (TPA) and JPL experiments test isotropy of the speed of light in one-way configurations. The remaining four experiments test isotropy of nuclear energy levels. Limits assume the speed of the Earth is 300 km/s relative to the mean rest frame of the cosmic microwave background.

2.3 Local Position Invariance

Violations of EEP can also lead to time- and position-dependence of local physics. In the model example of Eq. (1), the values of $h^0_{\mu\nu}$ imposed by cosmology or by nearby matter could vary, resulting, for example, in variations of the effective fine-structure constant, or of the relative rates of atomic clocks. For example, in the quantum dynamics of an atomic clock based on the hyperfine structure of hydrogen (hydrogen maser clock), the components of $h^0_{\mu\nu}$ in Eq. (1) will play a different role than they would say, in the dynamics of a clock based on the resonant frequency of a microwave cavity, because the role of electromagnetism is different in the two cases. If one type of clock is chosen as a reference standard, then the relative rates of other types of clocks measured against the standard in local freely falling frames will generally depend on the location of the frame in space or time. It is straightforward to show from this that the frequency shift $\Delta f$ in the comparison of
two identical clocks at different heights in a gravitational potential $U$ will be given by
\[ \Delta f/f = (1 + \alpha) \Delta U/c^2, \]
where $\alpha$ generally depends on the type of clock being used. If EEP is satisfied, $\alpha = 0$ for all clocks, and one has the standard gravitational redshift prediction of Einstein, indeed of all metric theories of gravity. In the first gravitational redshift experiment, the 1960-1965 Pound-Rebka-Snider experiments,\textsuperscript{18,19} two identical clocks (gamma-ray emitting iron nuclei) at different heights were intercompared, leading to a one percent test ($|\alpha_{\text{Fe}}| < 10^{-2}$). The best bound to date, $|\alpha_{\text{H-maser}}| < 2 \times 10^{-4}$, comes from a 1976 gravitational redshift experiment using a Hydrogen maser clock launched on a Scout rocket to an altitude of 10,000 km, and compared with an identical clock on the ground.\textsuperscript{20}

Another class of experiments compares two different clocks side by side, as the Earth’s orbital motion and rotation moves the laboratory in and out of the Sun’s gravitational field, causing annual and diurnal variations in $U$. In one experiment, a hydrogen maser clock (actually a pair of masers) was compared with a set of oscillator clocks stabilized by superconducting microwave cavities (called SCSO clocks), resulting in the bound $|\alpha_{\text{H-Maser}} - \alpha_{\text{SCSO}}| < 10^{-2}$.\textsuperscript{21} A recent comparison of a cesium standard against a magnesium fine-structure standard over a 430-day period placed a bound on an annual relative variation at the level $|\alpha_{\text{Cs}} - \alpha_{\text{Mg}}| < 7 \times 10^{-4}$.\textsuperscript{22} A comparison involving a hydrogen maser and a trapped mercury ion standard is also planned.\textsuperscript{23}

The effective fundamental non-gravitational constants of physics can vary with cosmological time if EEP is violated. Bounds on such variations have been obtained from a variety of geological, laboratory, and astronomical observations. The best bound, especially for the fine-structure constant, comes from the Oklo natural fission reactor in Gabon, Africa, where the natural occurrence of sustained fission about two billion years ago permits a comparison of the values of various constants affecting nuclear reactions then with the current values. For the fine structure constant, the bound is better that one part in $10^5$ per 20 billion years.\textsuperscript{24} Clock comparison experiments have yielded a bound of $7 \times 10^{-4}$ per 20 billion years.\textsuperscript{23}

### 2.4 The Weak Equivalence Principle

The third element of EEP is the Weak Equivalence Principle, which states that bodies fall with the same acceleration, independently of their internal structure or composition. Although tests of WEP do not generally involve stable clocks, they are worth mentioning here, if only because of the important role played by the Rencontres de Moriond during the period 1987-92 as an annual meeting for investigators working on “fifth-force” experiments. Many of those experiments were also tests of WEP. The current bounds on the fractional difference in acceleration in the solar or terrestrial gravitational fields between bodies of different composition are between $10^{-11}$ and $10^{-12}$.\textsuperscript{25-28} Further improvements in fifth-force experiments are likely to yield bounds tighter by a few orders of magnitude; a satellite test of the equivalence principle has also been proposed that could yield a test at the $10^{-17}$ level.
3. Stable Clocks and the Strong Equivalence Principle

3.1 The Strong Equivalence Principle

The Strong Equivalence Principle (SEP) is a generalization of EEP which states that in local “freely-falling” frames that are large enough to include gravitating systems (such as planets, stars, a Cavendish experiment, a binary system, etc.), yet that are small enough to ignore tidal gravitational effects from surrounding matter, local gravitational physics should be independent of the velocity of the frame and of its location in space and time. Also all bodies, including those bound by their own self-gravity, should fall with the same acceleration. General relativity satisfies SEP, whereas most other metric theories do not (eg. the Brans-Dicke theory).

It is straightforward to see how a gravitational theory could violate SEP. Most alternative metric theories of gravity introduce auxiliary fields which couple to the metric (in a metric theory they can’t couple to matter), and the boundary values of these auxiliary fields determined either by cosmology or by distant matter can act back on the local gravitational dynamics. The effects can include variations in time and space of the locally measured effective Newtonian gravitational constant $G$ (preferred-location effects), as well as effects resulting from the motion of the frame relative to a preferred cosmic reference frame (preferred-frame effects). Theories with auxiliary scalar fields, such as the Brans-Dicke theory and its generalizations, generically cause temporal and spatial variations in $G$, but respect the “Lorentz invariance” of gravity, i.e. produce no preferred-frame effects. The reason is that a scalar field is invariant under boosts. On the other hand, theories with auxiliary vector or tensor fields can cause preferred-frame effects, in addition to temporal and spatial variations in local gravitational physics. For example, a timelike, long-range vector field singles out a preferred universal rest frame, one in which the field has no spatial components; if this field is generated by a cosmic distribution of matter, it is natural to assume that this special frame is the mean rest frame of that matter.

General relativity embodies SEP because it contains only one gravitational field $g_{\mu\nu}$. Far from a local gravitating system, this metric can always be transformed to the Minkowski form $\eta_{\mu\nu}$ (modulo tidal effects of distant matter and $1/r$ contributions from the far field of the local system), a form that is constant and Lorentz invariant, and thus that does not lead to preferred-frame or preferred-location effects.

3.2 Cosmological Variation of Newton’s Constant

In metric theories of gravity that violate SEP, $G$ may also vary with the evolution of the structure of the universe, via the cosmologically imposed boundary values on the auxiliary fields. In fact, a cosmic variation in $G$ was a consideration that partly motivated Dicke to develop the scalar-tensor theory. Varying $G$ is common in the various generalized scalar-tensor theories developed recently for inflationary cosmology. On the other hand, in a wide class of such theories, the variations can be large in the early universe (leading to the desired cosmological consequences), but damp out as the present epoch is approached. In many such theories, general relativity is a natural “attractor” to which the cosmic evolution naturally leads the theory as the conditions of the present universe are reached.
Such a variation of $G$ can be tested by comparing a gravitational clock, such as a planetary orbit, whose period is governed by $G$, with a stable atomic clock, whose period is governed by atomic constants. The best current observational bound, $|\dot{G}/G| < 4 \times 10^{-12} \text{ yr}^{-1}$, comes from long-term observations of the orbit of Mars via Viking ranging data \(^{31,32}\). A similar bound, $|\dot{G}/G| < 3 \times 10^{-11} \text{ yr}^{-1}$, comes from timing of the binary pulsar PSR 1855+09.\(^{33}\)

4. Stable Clocks and Gravitational Radiation

4.1 The Binary Pulsar

In 1974, the discovery of the binary pulsar PSR 1913+16 by Hulse and Taylor\(^{34}\) provided a new laboratory for studying general relativistic effects, and a new arena for the use of stable clocks. The system consists of a 59 ms period pulsar in an eight-hour orbit with a companion that has not been seen directly, but that, on evolutionary grounds, is generally believed to be another neutron star (actually a dead pulsar). The basic measurement technique involves comparing the pulsar “clock” with Earth-based atomic clocks, specifically by comparing the phases of arriving radio pulses with those of local oscillators. The unexpected stability of the pulsar clock, the cleanliness of the orbit, and the use of stable atomic clocks and time-transfer using GPS, allowed radio astronomers to determine the orbital and other parameters of the system to extraordinary accuracy. Furthermore, the system is highly relativistic ($v_{\text{orbit}}/c \approx 10^{-3}$). Observation of the relativistic periastron advance ($4^\circ.22662 \pm 0^\circ.00001 \text{ yr}^{-1}$) and of the effects on pulse arrival times of the gravitational redshift caused by the companion’s gravitational field and of the special relativistic time dilation caused by the pulsar’s orbital motion (0.05% accuracy) have been used, assuming that general relativity is correct, to constrain the nature of the system. In general relativity, these two effects depend in a known way on measured orbital parameters and on the unknown masses $m_p$ and $m_c$ of the pulsar and companion (assuming that the companion is sufficiently compact that tidal and rotational distortion effects can be ignored), and consequently the two masses may be calculated with these two pieces of data, with the result $m_p = 1.4410 \pm 0.0007 \, M_\odot$ and $m_c = 1.3874 \pm 0.0007 \, M_\odot$.\(^{35}\)

A second clock comparison, that of the orbital “clock” against atomic time, gave the first evidence for the effects of gravitational radiation damping. General relativity provides a formula, which is a generalization of one first derived by Einstein in 1916, known as the quadrupole formula, which determines the loss of energy and the consequent orbital damping due to gravitational-wave emission from binary systems such as this. The result is a decrease in the orbital period. Using the measured orbital elements and the two masses, one can obtain the predicted rate $dP/dt = -2.40243 \pm 0.00005 \times 10^{-12}$. The observations are now better than 0.3 percent in accuracy, and it is necessary to take into account a small effect due to the relative acceleration between the binary pulsar and the solar system caused by galactic rotation.\(^{36}\) With this effect subtracted, the result is $dP/dt_{\text{observed}} = -(2.410 \pm 0.009) \times 10^{-12}$, agreeing completely with the prediction.\(^{35}\)

Several new binary pulsars have been discovered in the past few years. Two of these,
PSR 1534+12 in our galaxy\(^{37}\) and PSR 2127+11C in the globular cluster M15,\(^{38}\) are particularly promising as relativity laboratories. Combined observations of these systems may yield an even more accurate determination of \(dP/dt\) than did PSR 1913+16 alone.

### 4.2 Search for Gravitational Waves

Clock comparisons also play a role in the direct search for gravitational radiation. Observations of millisecond pulsars such as PSR 1937+21 have shown some to be at least as stable as the ensemble of the world’s atomic clocks; the residual timing noise in the comparison between pulsars and clocks cannot be allocated conclusively to one or the other. That residual noise sets a significant upper bound on a stochastic background of gravitational waves with wavelengths on the order of light years whose effect would be to cause a fluctuating relative gravitational redshift between pulsar and Earth times.\(^{39,33}\) The resulting bound has ruled out a substantial region of parameter space for cosmological scenarios involving cosmic strings.

The detection and study of gravitational radiation using large-scale laser-interferometric observatories, such as the U.S. LIGO or the European VIRGO project, will also rely heavily on clock comparisons, although not in a manner that will challenge the stability of atomic standards.\(^{40}\) In this case, the broad-band laser systems will have the ability to track the phase of the incoming GW signal. For an inspiralling binary system of compact objects (neutron stars or black holes), one of the most promising detectable sources, the GW phase evolves non-linearly in time because of gravitational-radiation damping of the orbit. That evolution depends on the physical parameters of the system, such as the masses and spins of the bodies, and on general relativity. Using a matched filtering of theoretical waveform templates against the outputs of the detectors, it will be possible, because of the intrinsic high precision of phase comparisons, to determine many of the system parameters to high accuracy.\(^{41}\) However, to achieve that accuracy requires knowing the general relativistic prediction for the evolution of the orbital phase of the inspiralling binary to high accuracy, which translates into a knowledge of gravitational-wave damping to many orders of approximation beyond the normal quadrupole formula of standard textbooks.

Written as a formula for the rate of energy loss, it has the general form

\[
\frac{dE}{dt} = \frac{1}{5} \left( \frac{d^3}{dt^3} M^{ij} \right)^2 \left( 1 + O(\epsilon) + O(\epsilon^{3/2}) + O(\epsilon^2) + \ldots \right),
\]

(3)

where \(M^{ij}\) is the trace-free mass quadrupole moment of the system, and \(\epsilon \sim v^2 \sim m/r\). Using post-Newtonian and post-Minkowskian techniques, several workers have succeeded in deriving this formula for general binary systems through second post-Newtonian order \((O(\epsilon^2))\), including the effects of spin,\(^{42-44}\) and calculations to even higher orders are in progress. For the special case of a test body inspiralling onto a black hole, perturbation methods have yielded corrections to Eq. (3) through \(O(\epsilon^4 \ln \epsilon)\).\(^{45,46}\) It is ironic that, for high-precision atomic frequency work, high-order approximations to solutions of the Schrödinger equation must be used to compare theory with experiment, while here, it is general relativity that must be approximated to very high-order.
5. Stable Clocks, General Relativity and GPS

Recently, stable clocks together with general relativity have begun to find applications in practical everyday life, through the Global Positioning System. This navigation system, based on a constellation of 24 satellites carrying atomic-clocks, uses precise time transfer to provide accurate absolute positioning anywhere on Earth to 30 meters, and differential or relative positioning to the level of centimeters (a Russian system called GLONASS has similar capabilities). It relies on clocks that are stable, run at the same or well calibrated rates, and are synchronized. However, the difference in rate between GPS satellite clocks and ground clocks caused by the gravitational redshift and time dilation is around 38,000 ns per day. Consequently, general relativity must be taken into account in order to achieve the 100 ns time transfer accuracy required for 30 m positioning. In addition, the kinematical Sagnac effect must be taken into account in order to have a consistent clock synchronization scheme on the rotating Earth (for a discussion of relativity in GPS, see.47) GPS is a classic example of the unexpected and unintended benefits of basic research — general relativity early in the century, and masers and atomic beams in the 1950s.48)

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