DISCOVERY OF THREE WIDE-ORBIT BINARY PULSARS: IMPLICATIONS FOR BINARY EVOLUTION AND EQUIVALENCE PRINCIPLES

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ABSTRACT

We report the discovery of three binary millisecond pulsars during the Parkes Multibeam Pulsar Survey of the Galactic Plane. The objects are highly recycled and are in orbits of many tens of days about low-mass white-dwarf companions. The eccentricity of one object, PSR J1853+1303, is more than an order of magnitude lower than predicted by the theory of convective fluctuations during tidal circularization. We demonstrate that, under the assumption that the systems are randomly oriented, current theoretical models of the core-mass–orbital-period relation for the progenitors of these systems likely overestimate the white-dwarf masses, strengthening previous concerns about the match of these models to the data. The new objects allow us to update the limits on violation of relativistic equivalence principles to 95% confidence upper limits of $5.6 \times 10^{-3}$ for the Strong Equivalence Principle parameter $|\Delta|$ and $4.0 \times 10^{-20}$ for the Lorentz-invariance/momentum-conservation parameter $|\alpha|$.

Subject headings: pulsars: individual: PSR J1751–2857, PSR J1853+1303, PSR J1910+1256 — stars: binaries — relativity

1. INTRODUCTION

Millisecond radio pulsars are the product of an extended period of mass and angular momentum transfer to a neutron star (NS) from an evolving companion star. This “recycling” scenario was proposed by Bisnovatyi-Kogan & Komberg (1974) shortly after the discovery of the first binary and recycled pulsars (Hulse & Taylor 1975; Backer et al. 1982). Overviews of the mass transfer process are given in several places (e.g., Bhattacharya & van den Heuvel 1991; Phinney & Kulkarni 1994; Tauris & van den Heuvel 1999). It has become clear that there are in fact several sub-classes of recycled pulsars, the most obvious distinction being between those that have NS versus white-dwarf (WD) companions. Even within the latter category and focusing on Galactic field binaries only, a wide range of companion masses and evolutionary histories are represented. One subclass is the “intermediate-mass binary pulsars” (e.g., Camilo et al. 2001; Edwards & Bailes 2001) which have spin periods of tens of milliseconds and/or companions that are likely massive (CO or ONeMg) WDs. Many of these systems may have experienced an ultra-high mass transfer rate, or else undergone a period of common-envelope evolution.

van den Heuvel 1994; Taam et al. 2000; Tauris et al. 2000). Among pulsars with lower-mass Helium WD companions, there is another split. Systems whose initial orbital periods were less than the “bifurcation” period of about 1 or 2 days (Poyner & Savonije 1988; Ergma et al. 1998) will see their orbits shrink through magnetic braking and gravitational radiation, ultimately forming a low-mass binary pulsar in a tight orbit. Systems with longer initial orbital periods undergo stable, long-lived mass transfer via an accretion disk, once the companion star has evolved onto the giant branch. The resulting systems have long orbital periods of several days or more. The prototype of this “wide-orbit binary millisecond pulsar” (WBMSP) group, PSR B1953+29, was one of the first recycled pulsars discovered (Boriakoff et al. 1983); over the years the number of these pulsars with orbital periods greater than 4 days has grown to 18.

The WBMSPs are the best-understood class of pulsar–WD binaries. For instance, the companion mass and orbital separation are thought to follow the “core-mass–orbital-period” ($P_0-m_2$) relation, in which the mass of the core which eventually forms the white dwarf is directly related to the size of the envelope of the Roche-Lobe-filling giant star and hence the orbital radius (e.g., Rappaport et al. 1994; Tauris & Savonije 1999). Measurements of companion masses to date (via pulsar timing or optical spectroscopy) indicate that this relation is fairly well satisfied (Kaspi et al. 1994; van Straten et al. 2001; Splaver et al. 2005; van Kerkwijk et al. 2005). However, we still do not discuss this issue further below. Furthermore, although the orbit becomes tidally circularized during the companion’s giant phase, fluctuations of convective cells in the giant’s envelope force the eccentricity to a non-zero value (Phinney 1992); thus, to within an order of magnitude or so, the eccentricities of these WBMSP systems can be predicted from the orbital periods.

More examples of WBMSP pulsars are needed.
to further test these predictions, and also to provide additional constraints for population synthesis efforts (e.g., Portegies Zwart & Yungelson 1998; Willems & Kolb 2002, 2003; Pfahl et al. 2003). These objects are also valuable for constraining departures from general relativity (GR) in the form of equivalence principle violations (e.g., Damour & Schäfer 1991). Finally, some pulsar-WD binaries permit measurement of NS and WD masses through relativistic or geometric timing effects; thus new systems potentially add to the pool of objects that can be used to constrain theories of the NS interior and/or the amount of matter transferred in different evolutionary processes (e.g., Stairs 2004). In this paper we report the discovery of three more WBMSPs during the Parkes Multibeam Pulsar Survey. In § 2 we describe the search and follow-up timing observations. In § 3 we describe the characteristics of the new pulsars and relate them to evolutionary theory including a comparison with the predictions of the $P \propto m^2$ relation. In § 4 we use the ensemble of pulsars with white-dwarf companions to set new, stringent limits on equivalence principle violations. Finally, in § 4 we summarize our results and look to the future.

2. OBSERVATIONS AND DATA ANALYSIS

The Parkes Multibeam Pulsar Survey (e.g., Manchester et al. 2001) used a 13-beam receiver on the 64-m Parkes telescope to search the Galactic Plane ($|b| < 5^\circ$, $260^\circ < l < 50^\circ$) for young and recycled pulsars. Observations were carried out at 1374 MHz with 96 channels across a 288 MHz bandpass, using 35-minute integrations and 0.25 ms sampling. More than 700 pulsars have been discovered (e.g., Hobbs et al. 2003), nearly doubling the previously known population. The survey processing includes dedispersion of the data at numerous trial dispersion measures, followed by a periodicity search using Fast Fourier Transforms and harmonic summing (Manchester et al. 2001). Recently, the entire data volume has been reprocessed using “acceleration” searches for pulsars in fast binary orbits, as well as improved interference excision techniques. This has resulted in a number of new binary and millisecond pulsars (Faulkner et al. 2004, 2005), including two of the objects described in this paper.

PSR J1751−2857 has been observed since MJD 51972 with Parkes at 1390 MHz using a 512-channel filterbank acquiring 256 MHz bandwidth and 0.25 ms sampling and with the 76 m Lovell Telescope at Jodrell Bank Observatory at 1396 MHz using a 64-channel filterbank across 64 MHz with 0.13 ms sampling.

PSRs J1853+1303 and J1910+1256 were discovered in observations taken on MJDs 52321 and 52322, respectively, and identified as candidates using the REAPER program selection procedure (Faulkner et al. 2003). Both were confirmed as pulsars on MJD 52601. Subsequently, they have been timed at Parkes and Jodrell Bank using the systems described above. Both have also been regularly observed using the 305 m Arecibo telescope in Puerto Rico. These observations initially used one 100 MHz Wideband Arecibo Pulsar Processor (WAPP) in a fast-sampled “search” mode, and, since 2004 Feb., 3 WAPPs in an online folding mode, in all cases with 256 lags and 64 μsec sampling. The single-WAPP observations were centered at 1400 MHz and when 3 WAPPs were used they were centered at frequencies of 1170, 1370 and 1470 MHz. Some of the single-WAPP observations display timing systematics similar in both pulsars relative to the contemporaneous Parkes data; as this likely reflects instrumental errors in the fairly new WAPP, these data points have been left out of the timing analysis. Some of the Arecibo observations were flux calibrated using a pulsed noise diode of known strength; the resulting calibrated profiles are used to determine the flux densities for PSRs J1853+1303 and J1910+1256. The flux density for PSR J1751−2857 was determined using the procedure outlined in Hobbs et al. 2004.

The data from each telescope were dedispersed and folded modulo the predicted topocentric pulse period; this was accomplished off-line for the Parkes data and Arecibo search-mode data and on-line for the Jodrell Bank and Arecibo folding-mode data. A Time-of-Arrival (TOA) was determined for each observation by cross-correlation with a high signal-to-noise standard template (Taylor 1993). The timestamp for each observation was based on the observatory time standard, corrected by GPS to Universal Coordinated Time (UTC). The JPL DE200 ephemeris (Standish 1994) was used for barycentric corrections. The timing solutions were found using the standard pulsar timing program TEMPO8, with uncertainties containing a small telescope-dependent amount added in quadrature and scaled to ensure $\chi^2 \approx 1$. The resulting pulsar parameters are shown in Table 1 while the timing residuals and standard pulse profiles are shown in Figure 1. We note that the correctly folded Arecibo data have rms residuals on the order of 1 μsec per WAPP for PSR J1910+1256 and 1–2 μsec per WAPP for PSR J1853+1303 for roughly 30-minute integrations.

3. DISCUSSION

http://pulsar.princeton.edu/tempo
by triangles. The solid and dashed curves illustrate the eccentricity
95% of pulsars should fall within this range and 90% of the observed
systems do. The dotted line has
\[ P_0 \]
\[ m_2 \]
merit for tests of the Strong Equivalence Principle (SEP).

Fig. 2.— Eccentricity \( e \) vs. orbital period \( P_0 \) for the pulsar-WD
systems thought to be described by stable mass transfer and the
\( P_0 - m_2 \) relation. The three new pulsars are labeled and indicated
by triangles. The solid and dashed curves illustrate the eccentricity
ranges predicted by \textit{Phinney} (1992) as a function of orbital period;
95% of pulsars should fall within this range and 90% of the observed
systems do. The dotted line has \( P_0^\ast \propto e \), indicating the figure-of-
merit for tests of the Strong Equivalence Principle (SEP).

3.1. Pulsar Parameters and Evolution

Each of the new pulsars is in an orbit of several tens of days with a companion of minimum mass of 0.2 to
0.25 \( M_\odot \). The eccentricities of PSRs J1751−2857 and
J1910+1256 agree well with the predictions of \textit{Phinney}
(1992); however, that of PSR J1853+1303 is lower than
the predictions by more than an order of magnitude (Fig. 2). As it has a low-mass companion and its spin
period indicates that it is highly recycled, there is little
other reason to believe that its evolution proceeded in
any unusual fashion, so its low eccentricity may simply
reflect the natural scatter in the population.

There are now 21 pulsars with low-mass WD companions
whose orbital characteristics should be determined by the
\( P_0 - m_2 \) relation (Table 2). To date, however, there are
only a handful of observational tests of this relation:
three Shapiro-delay timing measurements and one optical
WD spectrum for systems with orbital periods greater
than 2 days \textit{SPLAVE et al.} (2003, \textit{van Kerkwijk et al.}
2005). In order to judge the agreement of the whole
population with the theory, therefore, we need to use
statistical arguments based on the observed mass functions.
We follow \textit{Thorsett & Chakrabarty} (1999) in con-
sidering only those pulsars with orbital periods greater
than about 4 days, excluding those in the 2−4 day range
as being too close to the limits of applicability of the
relation. With this many pulsars, it becomes possi-
bile to examine whether different subgroups are equally
well described by the relation. To do this, we adopt the
approach of \textit{Thorsett & Chakrabarty} (1999) (Fig. 4
and related discussion) by assuming a range of pulsar
TABLE 2
The 21 pulsars with low-mass WD companions and orbital periods of more than 4 days

| Pulsar       |   |   |   |   |   |   |
|--------------|---|---|---|---|---|---|
|              |   |   |   |   |   |   |
| J0407−1607   | 25.702 | 669.0704 | 0.00009368(6) | 192.74(2) |   |   |
| J0437−4715   | 7.577   | 5.7410   | 0.000019166(5) | 1.20(5) |   |   |
| J1045−4509   | 7.474   | 4.0835   | 0.000019713(8) | 243(4) |   |   |
| J1455−3330   | 7.987   | 76.1746   | 0.00001697(3) | 223.8(1) |   |   |
| J1640+2224   | 3.163   | 175.4606   | 0.00079272(14) | 50.7308(10) |   |   |
| J1643−1222   | 4.622   | 147.0174   | 0.0005058(1) | 321.81(1) |   |   |
| J1709+2313   | 4.531   | 22.7119   | 0.00001878(2) | 321.81(1) |   |   |
| J1713+0747   | 4.570   | 67.8255   | 0.000749406(13) | 176.1915(10) |   |   |
| J1732−5049   | 5.313   | 5.2630   | 0.00000928(20) | 287(12) |   |   |
| J1751−2857   | 3.915   | 110.7465   | 0.0001283(5) | 45.21(3) |   |   |
| J1809−2717   | 9.498   | 58.4667   | 0.00023022(16) | 106.001(11) |   |   |
| J1833+1303   | 4.092   | 115.6538   | 0.00002369(9) | 346.63(8) |   |   |
| B1855+09     | 5.362   | 12.3272   | 0.00002170(3) | 276.39(4) |   |   |
| J1910+1256   | 4.984   | 58.4667   | 0.00001878(2) | 321.81(1) |   |   |
| J1918−0642   | 7.646   | 10.9132   | 0.000023022(16) | 106.001(11) |   |   |
| J1934−29     | 6.133   | 117.3491   | 0.0003300(1) | 29.55(2) |   |   |
| J2019+1498   | 64.040  | 65.039    | 0.00128(16) | 90(5) |   |   |
| J2019+2425   | 3.935   | 76.5116   | 0.00011090(4) | 159.03(2) |   |   |
| J2033+1734   | 5.949   | 56.3078   | 0.00012876(6) | 78.23(3) |   |   |
| J2129−5721   | 3.726   | 6.6255   | 0.00000608(22) | 178(12) |   |   |
| J2229+2643   | 2.978   | 93.0159   | 0.00025562(6) | 14.42(5) |   |   |

References:— 1. Lorimer et al. (2004), 2. Johnston et al. (1993), 3. van Straten et al. (2001), 4. Bailes et al. (1994), 5. Toscano et al. (1999), 6. Lorimer et al. (1995), 7. Foster et al. (1996), 8. Lohmer et al. (2004), 9. Lewandowski et al. (2004), 10. Foster et al. (1995), 11. Splaver et al. (2004), 12. Edwards & Bailes (2001), 13. Lorimer et al. (1996), 14. Spergel et al. (1995), 15. Splaver (2000), 16. Boriakoff et al. (1983), 17. Wolszczan et al. (1990), 18. Navarro et al. (2003), 19. Nice et al. (1993), 20. Nice et al. (2001), 21. Ray et al. (1999), 22. Camilo et al. (1994).

Pulsar $P$ (ms) $P_b$ (days) $\epsilon$ $\omega(\circ)$ PM RA (mas/yr) PM Dec (mas/yr) $f_1 (M_\odot)$ References

Fig. 3.— Cumulative probability distributions for the measured mass functions $p(f_1 < f)$ (solid lines) and the median predicted values of $\cos i$ (dashed lines) for the 21 binary systems listed in Table 2 after Thorsett & Chakrabarty (1999), Fig. 4. In each panel, the thinnest lines incorporate all systems, the medium-weight lines only those systems with $P_b < 50$ days, and the heaviest lines only those systems with $P_b > 50$ days. Panel (a) assumes $m_2$ is drawn uniformly from the range predicted by the Tauris & Savonije (1999) $P_b$–$m_2$ relation and that $m_1$ is drawn from a Gaussian distribution centered on $1.5M_\odot$ with width $0.04M_\odot$, Thorsett & Chakrabarty (1999). Panel (b) draws $m_1$ from the same range, but assumes $m_2$ is drawn from the range predicted by the Tauris & Savonije (1999) relation. Panel (c) as (a), but drawing $m_1$ from a Gaussian distribution centered on $1.75M_\odot$ with width $0.04M_\odot$, Panel (d) as (b), but with $m_2$ limits given by the Tauris & Savonije (1999) fits to the Rappaport et al. (1999) results. The straight lines indicate the cumulative probability for a uniform distribution.

masses (e.g., 1.35±0.04 $M_\odot$), which was a good match to the set of pulsar masses measured at the time of Thorsett & Chakrabarty (1999) and the (uniform) $P_b$–$m_2$ relation specified by either Tauris & Savonije (1999) or Rappaport et al. (1995). For the first test, for each pulsar we assume a uniform distribution in $\cos i$ and simulate a large number of systems, finding the probability $p(f_1 < f)$ that the simulated mass function $f$ is above the observed value $f_1$. For the second test, we assume the observed $f_1$ and find the median predicted value of $\cos i$, again simulating a large number of systems. The cumulative probability distributions for both $\cos i$ (dashed lines) and $p(f_1 < f)$ (solid lines) are displayed in Fig. 3 for all 21 pulsars, and for those with orbital periods less than (8 systems) and greater than (13 systems) 50 days. For comparison, the straight lines indicate the cumulative probability for a uniform distribution.

The $m_2$ limits from the Tauris & Savonije (1999) relation (panel (a) of Fig. 3) are given by the Pop. I and Pop. II fits to their simulation results; they find a spread in $P_b$ of a factor of about 1.4 around their median value for a given $m_2$. Rappaport et al. (1995) find a spread of about 2.4 in $P_b$ for any given $m_2$, but consider this to cover roughly the full range of possible values; we assume a range of $\sqrt{2} \sim 1.6$ will be comparable to the Tauris & Savonije (1999) ranges and show the corresponding results in panel (b) of Fig. 3. Tauris & Savonije (1999) also provide their own fits to the Rappaport et al. (1995) simulations, and we evaluate these fits in panel (d) of Fig. 3. It is important to note that these Tauris & Savonije (1999) fits do not cover the full spread of the Rappaport et al. (1995) orbital periods, favoring the lower periods at any given $m_2$. Thus it...
is perhaps not surprising that panel (d) indicates higher mass estimates in general than panel (b).

We find that the Tauris & Savoninic (1999) \( P_b - m_2 \) relation is incompatible at the 99.5% level (according to a KS-test) with a uniform distribution of \( \cos i \) if the pulsar masses are drawn from a Gaussian distribution centered on \( 1.35M_\odot \) with width \( 0.04M_\odot \). Better agreement with uniformity in \( \cos i \) (at the 50% level) can be reached if the pulsar masses are very large on average (e.g., \( 1.75\pm0.04M_\odot \)); panel (c) of Fig. 3, a situation not supported by observational evidence. The Rappaport et al. (1995) relation appears to be in slightly better agreement with uniformity in \( \cos i \), though it is clear from Fig. 3 that this occurs because of a tendency to underestimate the companion masses for short-period systems and overestimate those for long-period systems.

We note that, although Thorsett & Chakrabarty (1999) favor using the Rappaport & Joss (1997) version of the \( P_b - m_2 \) relation for \( m_2 < 0.25M_\odot \), this predicts extremely low masses (\(~0.10M_\odot\) for the shortest-\( P_b \) systems, which is in conflict with the observed companion mass of \( 0.236\pm0.017M_\odot \) for PSR J0437−4715 (van Straten et al. 2001). Using this revised relation would lower the estimates of the companion masses in the short-\( P_b \) systems even further; this in combination with the higher estimates for the long-\( P_b \) systems appears to have been responsible for the overall good agreement that Thorsett & Chakrabarty (1999) found for the combined relation with NS masses of \( 1.35\pm0.04M_\odot \) and a uniform distribution in \( \cos i \). Thus, assuming that the cosines of the system inclination angles are in fact uniformly distributed and that most NS masses are near \( 1.35M_\odot \), it appears that the existing forms of the \( P_b - m_2 \) relation tend to underestimate companion masses for long-period systems, while providing conflicting results for the short-period systems.

The tendency to underestimate masses in the short-\( P_b \) case was in fact noted by Rappaport et al. (1997), although they included in their analysis systems now considered to be “intermediate-mass” binaries (such as PSR J2145−0750) having different evolutionary histories. Tauris (1996) and Tauris & Savoninic (1999) comment on the poor match of the (then 5) known WBMSPs to the higher theoretical predictions of \( m_2 \). With the larger number of systems now known, the conclusion of a poor match seems inescapable. Rappaport et al. (1995) note that while the relationship between core mass and luminosity for red giants is well understood, the relation between mass or luminosity and radius is looser (see also Thorsett & Chakrabarty 1999), with uncertainties in the companion’s initial chemical composition and the convective mixing-length parameter; this may explain our results. It appears more theoretical work will be required to derive models that better match the data. The only long-orbital-period system with a timing test of the \( P_1 - m_2 \) relation is PSR J1713+0747 (Splaver et al. 2005), and the measured companion mass is in fact slightly lower than the Tauris & Savoninic (1999) prediction. Mass measurements or constraints in more systems, by timing or by optical spectroscopy of the WD companions, will be needed to confirm or refute our present conclusions. The new systems will likely lend themselves to observations of geometrical effects such as the change in apparent semi-major axis due to motion of the pulsar and/or the Earth (Kopeikin 1995, 1996, and J1910+1256 in particular has sufficient timing precision that it may be possible to measure Shapiro delay in this system.

3.2. Equivalence Principle Violations

The WBMSPs are the best objects for setting limits on violations of the Strong Equivalence Principle (SEP) and the Parametrized Post-Newtonian parameter \( \alpha_3 \), which describes Lorentz invariance and momentum conservation. These tests use the fact that the gravitational self-energy of the NS will be much higher than that of the WD, and therefore, if the equivalence principles are violated, the two objects will accelerate differently in an external gravitational field or under the self-acceleration induced by the velocity relative to a preferred reference frame. The net effect on orbits that are nearly circular will be to force the eccentricity into alignment with this acceleration vector (Damour & Schäfer 1991). The prototype of these tests is the search for orbital polarization in the Earth-Moon system (Nordtvedt 1968), which currently sets a limit on the weak-field violation parameter \( |\eta| \) of 0.001 (Dicke et al. 1994; Will 2001). The pulsar versions of these experiments test the strong-field limit of SEP violation (parameter \( \Delta \)) and Lorentz invariance/momentum conservation (parameter \( \alpha_3 \)), and are thoroughly described in the literature (Damour & Schäfer 1991; Wex 1997, 2000; Bell & Damour 1999; Wex 2001; Stairs 2003; Splaver et al. 2003). Both parameters are identically zero in GR, and \( \alpha_3 \) is predicted to be zero by most theories of gravity.

We now examine the impact of the recently discovered binary systems on these tests. The traditional figures of merit for choosing systems to test \( \Delta \) and \( \alpha_3 \) are \( P_b^2/\dot{P}_b \) and \( P_b^2/(P_e) \), respectively. The other requirements for \( \Delta \) are that each system must be old enough (i.e., have characteristic age large enough) and must have \( \dot{\omega} \) large enough that the longitude of periastron can be assumed to be randomly oriented; and that each system must have \( \dot{\omega} \) larger than the rate of Galactic rotation, so that the projection of the Galactic acceleration vector onto the orbit can be considered constant (Damour & Schäfer 1991; Wex 1997). Similar requirements hold for \( \alpha_3 \). With its extraordinarily low eccentricity, PSR J1853+1303 is a prime candidate to help strengthen these tests. The last few years have seen the discovery of several other systems with comparable or longer-period orbits, notably PSRs J2016+1948 (Navarro et al. 2003) and J0407+1607 (Lorimer & Freire 2003; Lorimer et al. 2005). We therefore find it worthwhile to update the multi-pulsar analysis of Wex (1997, 2000), finding much lower limits on each parameter. In keeping with the spirit of Wex (2000), we use all 21 pulsars listed in Table 2 as these are all thought to have evolved with similar extended accretion periods and therefore represent the overall population of such objects. Some of these systems have quite small values of \( P_b^2/\dot{P}_b \) but need to be included nonetheless, as possible examples of violation. Our calculation will find a median likelihood value of \( |\Delta| \) for each pulsar that corresponds to an induced eccentricity roughly comparable to its observed eccentricity, and the combined limit fairly represents the limits derivable from the known population.

We use the following Bayesian analysis. For the SEP
$\Delta$ test, we are interested in finding the probability density function (pdf) $p(\Delta | D, I)$, where $D$ represents the relevant data on the 21 pulsars (namely, their eccentricities and longitudes of periastron and associated measurement errors) and $I$ represents prior information. The unknown parameters for each system include the two stellar masses, the distance $d$ to the system, and the position angle $\Omega$ of the Line of Nodes on the sky. Given any set of these parameters, a “forced” eccentricity vector $e_F$ may be derived for any given value of $\Delta$, up to a sign ambiguity which amounts to flipping the direction of the vector. This can be written (Damour & Schäfer 1991):

$$|e_F| = \frac{1}{2} F G (m_1 + m_2)(2\pi/P_b)^2,$$

where $c$ is the speed of light, and, in general relativity, $F = 1$ and $G$ is Newton’s constant. Here $g_\perp$ is the projection of the Galactic acceleration vector onto the plane of the orbit, and is given by [Damour & Schäfer 1991].

$$|g_\perp| = |g|[1 - (\cos i \cos \lambda + \sin i \sin \lambda \sin(\phi - \Omega))^2]^{1/2},$$

where $\phi$ is the position angle of the projection of the gravitational acceleration vector $g$ onto the plane of the sky, and $\lambda$ is the angle between the line from pulsar to Earth and $g$. Deriving $e_F$ requires knowledge of the Galactic acceleration at the pulsar position; we assume the vertical potential given by [Kuijken & Gilmore 1989] and a flat rotation curve with velocity of 222 km s$^{-1}$. A prediction for the observed eccentricity $e_{\text{obs,pred}}$ is then the vector sum of $e_F$ and a “natural” eccentricity $e_N$. Thus the magnitude of $e_F$ and the angle $\theta$ between $e_F$ and $-e_F$ are additional parameters.

For any one $j$ of the 21 pulsars, we may then use Bayes’ theorem to write:

$$p(\Delta | i, m_2, \Omega, d, e_N, \theta | D, I) \propto p(D_i | \Delta, i, m_2, \Omega, d, e_N, \theta) p(\Delta, i, m_2, \Omega, d, e_N, \theta | I).$$

We compute the integral over $e_N$ and $\theta$ separately, effectively calculating the marginal $p(\Delta | i, m_2, \Omega, d | D, I)$ for a particular set of parameters $i$, $m_2$, $\Omega$, and $d$. For each such set of parameters, the angle between $e_F$ and the true $e_{\text{obs}}$, and hence the possible values of $e_N$ for which the likelihood is significantly non-zero, will always be very tightly constrained, due to the small measurement errors on $e_{\text{obs}}$ and $\omega$. We therefore determine the four points representing 3-$\sigma$ ranges in both $e_{\text{obs}}$ and $\omega$, and use these to find minimum and maximum values of $e_N$ and $\theta$. The likelihood for the set of parameters $i$, $m_2$, $\Omega$, and $d$ is then set to 1 for values of $e_N$ and $\theta$ fall between their minimum and maximum values and 0 otherwise. Thus the integral will be roughly proportional to $(\theta_{\max} - \theta_{\min})(\log e_{N,\max} - \log e_{N,\min})$, assuming a uniform prior on $\theta$ and a log prior on $e_N$. We set the integral to zero if $e_{N,\min} > 0.05$ and use $1 \times 10^{-6}$ as a lower bound on possible values of $e_{N,\min}$; this conservatively allows each pulsar much more than the eccentricity ranges permitted by [Phinney 1992]. This approximation to the likelihood is necessary as both numerical integration over or (equivalently) Monte Carlo sampling of the full allowed ranges of $e_N$ and $\theta$ would be computationally prohibitive.

For the remaining nuisance parameters $i$, $m_2$, $\Omega$, and $d$ we perform a Monte Carlo simulation, drawing $m_2$ uniformly from twice the [Tauris & Savonije 1999] $P_b - m_2$ range, $\cos i$ from a uniform distribution between 0 and 1 (combining these two parameters and the mass function yields a value of the pulsar mass $m_1$; we use only those systems for which $m_1$ is between 1.0 $M_\odot$ and 2.5 $M_\odot$, or other limits set by timing of the individual pulsar), $\Omega$ from a uniform distribution between 0$^\circ$ and 360$^\circ$ and $d$ from a Gaussian distribution about the best [Cordes & Lazio 2002] distance, assuming a distance uncertainty of 25%, or from a Gaussian distribution in par-
allax, where measured. For PSR J1713+0747, we restrict the parameters \( i, \Omega, m_1 \) and \( m_2 \) to the region constrained by the recent measurement of the orientation of the orbit \cite{Splaver2003}, while for PSR J0437–4715 we assume Gaussian distributions about the parameters given in \cite{vanStraten2001}. We repeat this procedure for values of \( \Delta \) from 0.00002 to 0.1, in steps of 0.00002, then normalize; this results in the posterior pdf \( p(\Delta | D, I) \) for each pulsar \( j \). For PSR J1713+0747 alone, the resulting 95\% confidence limit on \( |\Delta| \) is about 0.0158, similar to the value derived in \cite{Splaver2003}. The pulsar data sets are independent, and thus we multiply the pdfs to derive \( p(\Delta) \) for each pulsar.

Figure 4 shows the pdf curves for this test. We note that while a logarithmic prior on \( |\Delta| \) would result in an upper limit a few orders of magnitude smaller, we have chosen a uniform prior on \( |\Delta| \) in order to be as conservative as possible and more consistent with previous work.

For the \( \alpha_3 \) test, we proceed in a similar fashion. Here the forced eccentricity is \cite{Bell1996}:

\[
|e_F| = \alpha_3 c_p |w| D_b^2 \frac{P_d^2}{P G (m_1 + m_2)} \frac{c^2}{24\pi} \sin \beta
\]  

(4)

where \( c_p \) is the gravitational self-energy fraction of “compactness” of the pulsar, approximated by \( 0.21 m_1 \), \cite{DamourEsposito1992, Bell1996}, and \( \beta \) is the (unknown) angle between the pulsar’s absolute velocity \( w \) (relative to the reference frame of the Cosmic Microwave Background) and its spin vector. For this test, we also need the 3-dimensional velocity of the system. Where proper motion measurements are available, we draw from Gaussian distributions for the proper motion to get the transverse velocities; in other cases, and always for the unknown radial velocities, we draw from Gaussian distributions in each dimension centered on the Galactic rotational velocity vector at the pulsar location and with widths of 80 km s\(^{-1}\) \cite{Lyne1998}. We sample uniform steps of \( \alpha_3 \) ranging from \( 1 \times 10^{-22} \) to \( 5 \times 10^{-19} \). We find a 95\% confidence upper limit on \( |\alpha_3| \) of \( 4.0 \times 10^{-20} \). Figure 5 shows the pdf curves for this test.

The 95\% confidence limits we derive of \( 5.6 \times 10^{-3} \) for \( |\Delta| \) and \( 4.0 \times 10^{-20} \) for \( |\alpha_3| \) are considerably better than previous limits of \( 9 \times 10^{-3} \) and \( 1.5 \times 10^{-19} \), respectively \cite{Wex2000}, while still taking into account the contribution from all pulsars with similar evolutionary histories. The SEP test appears weaker than the best solar-system tests of \(|\eta| < 0.001\) \cite{Dickey1994, Will2001} but pulsars test the strong-field regime inaccessible to the solar-system measurements and are therefore qualitatively different. The \( |\alpha_3| \) test is nearly thirteen orders of magnitude better than tests derived from the perihelion shifts of Earth and Mercury \cite{Will1993}, and again tests the strong-field regime of gravity.

4. CONCLUSIONS

Of the three new WB MSP systems presented here, at least two can be timed at the microsecond level with current instrumentation at large telescopes. These objects thus show promise for measurement of geometrical and/or relativistic timing phenomena in future, giving us an idea of the system inclination angles and perhaps the masses of the objects. This information will be extremely useful in evaluating the validity of the \( P_3 - m_2 \) relations for estimations of companion masses. All three systems should provide proper motion measurements within a few years; these will add to our understanding of millisecond pulsar velocities throughout their population. Finally, in combination with the other low-mass circular-orbit systems discovered in recent years, the new pulsars set firmer limits on violations of relativistic equivalence principles in the strong-field regime of \( 5.6 \times 10^{-3} \) for \( |\Delta| \) and \( 4.0 \times 10^{-20} \) for \( |\alpha_3| \). A better understanding of the low-mass population as a whole will be necessary for further improvement of these tests.

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