Oxygen recombination in the nebular phase of supernovae 1998bw and 2002ap

I. Maurer1* and P. A. Mazzali1,2,3
1Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85741 Garching, Germany
2Scuola Normale Superiore, Piazza dei Cavalieri 7, 56126 Pisa, Italy
3National Institute for Astrophysics-OAPd, Vicolo dell’Osservatorio 5, 35122 Padova, Italy

Accepted 2010 May 24. Received 2010 May 21; in original form 2010 February 18

ABSTRACT
Late-time spectra of stripped-envelope core-collapse supernovae (SNe) are dominated by strong [O I] \( \lambda \lambda 6300, 6363 \) emission, caused by thermal electron excitation of forbidden [O I] transitions. The permitted O I \( 7774 \) Å line is also often observed. This line cannot result from thermal electron excitation of the oxygen ground state. In this work tests are performed to verify whether the line can be powered by oxygen recombination alone, using the examples of two of the best studied Type Ic SNe, 1998bw and 2002ap.

Temperature-dependent effective recombination coefficients for neutral oxygen are calculated using available atomic data. Missing atomic data are computed in a temperature range typical for SN nebulae. Core ejecta models for SNe 1998bw and 2002ap are obtained from modelling their nebular emission spectra so that oxygen recombination line formation is computed consistently with oxygen forbidden line emission.

While SN 2002ap can be explained well by a one-dimensional shell model, this seems not to be possible for SN 1998bw, for which a two-dimensional model is found. At very late epochs the formation of the O I \( 7774 \) Å line can be explained by recombination radiation for both SNe, but at earlier epochs strong absorption is present which may determine the strength of this line even at \( \sim 200 \) d.

Key words: line: formation – radiation mechanisms: non-thermal – radiative transfer – supernovae: general.

1 INTRODUCTION
Massive stars (\( >8M_\odot \)) collapse when the nuclear fuel in their central regions is consumed, producing a core-collapse supernova (CC-SN) and forming a black hole or a neutron star. CC-SNe with a H-rich spectrum are classified as Type II (Filippenko 1997). If the star was stripped of at least most of its H envelope prior to the explosion, the SNe are classified according to the degree of stripping as Type IIb (strong He lines, and weak but clear H), Type Ib (strong He lines but no H) and Type Ic (no He or H lines).

Some CC-SNe, called broad-lined SNe (BL-SNe), exhibit very broad absorption lines at early times, caused by the presence of sufficiently massive ejecta expanding at high velocities. These SNe are sometimes called hypernovae, and can be associated with long-duration gamma-ray bursts (GRBs; see Woosley & Bloom 2006, and references therein).

In GRB scenarios a relativistic outflow launched by a central engine deposits some fraction of its energy into the SN ejecta. Since the energy is probably deposited preferentially along the polar axis, the SN might be strongly asymmetric (e.g. Maeda et al. 2002). The nearest, best-studied GRB-SNe are SN 1998bw/GRB 980425 (Galama et al. 1999), SN 2003dh/GRB 030329 (Matheson 2004), SN 2003lw/GRB 031203 (Malesani et al. 2004) and SN 2006aj/GRB/XRF 060218 (Pian et al. 2006). It is not yet fully established whether the GRBs (or X-ray flashes) accompanying nearby CC-SNe share the same properties of high-redshift GRBs. CC-SNe may also be characterized by asphericities, although a jet does not necessarily form (Blondin, Mezzacappa & DeMarino 2003; Kotake et al. 2004; Moiseenko, Bisnovatyi-Kogan & Ardeljan 2006; Burrows et al. 2007; Takiwaki, Kotake & Sato 2009).

Asphericities in the inner and outer ejecta are evident in at least some CC-SNe. Two clear indicators are velocity differences of Fe and lighter element lines (e.g. Mazzali et al. 2001) and polarization measurements (e.g. Höflich 1991). Indirect indications also emerge from a comparison of the inner and outer ejecta velocities (Maurer et al. 2010).

Independent of their type, SNe become increasingly transparent to optical light with time, as the ejecta thin out. At late times (\( >200 \) d after the explosion), the innermost layers of the SN can

*E-mail: maurer@mpa-garching.mpg.de
be observed. This epoch is called the nebular phase, because the spectrum turns from being dominated by absorption to an emission spectrum, mostly showing forbidden lines. In this phase the radiated energy of a SN is provided by the decay of radioactive $^{56}$Ni (which is produced by the earlier decay of $^{56}$Co), as well as the decay of $^{54}$Fe. Decaying $^{56}$Co emits $\gamma$-rays and positrons which are absorbed by the SN ejecta. As the deposition rate of $\gamma$-rays and positrons depends on the density and $^{56}$Ni distribution, the inner parts of the SN dominate the nebular spectra. Therefore the nebular phase is especially suitable for studying the core of SNe.

Several authors have modelled nebular-phase spectra of SNe to derive quantities such as the $^{56}$Ni mass (e.g. Mazzali et al. 2004; Sauer et al. 2006; Stritzinger et al. 2006; Maeda et al. 2007a), ejecta velocities (e.g. Mazzali et al. 2007b; Maurer et al. 2010; Taubenberger et al. 2009), asphericities (e.g. Mazzali et al. 2005, 2007a, 2008; Maeda et al. 2006) and elemental abundances (e.g. Maeda et al. 2007a,b).

To infer SN core ejecta velocities and geometry usually the [O i] $\lambda\lambda 6300, 6364$ doublet is investigated, since this is by far the strongest nebular emission line (SNe of Type Ic), especially at very late epochs. These lines are formed by thermal electron excitation of the 2p($^1$D) state. Higher quantum states ($n \geq 3$) cannot be excited by thermal electrons sufficiently because of the large ratio of excitation energy ($\sim 9$ eV) and electron temperature ($\sim 0.4$ eV). Possible excitation mechanisms are absorption, non-thermal electrons and recombination.

Previous studies of central oxygen have focused almost exclusively on the [O i] $\lambda\lambda 6300, 6364$ doublet. It is useful to investigate whether extending the analysis to other oxygen lines gives a consistent picture. Therefore it is important to understand the formation of these lines in detail.

In Section 2 temperature-dependent effective recombination rates are obtained for the $n = 3$ levels of neutral oxygen and some quantities relevant for oxygen recombination and emission-line formation in CC-SN nebulae are estimated. A one-dimensional shell model for SN 2002ap is described in Section 3 and a two-dimensional model for SN 1998bw in Section 4 emphasizing the role of the O i 7774 Å line. Results are discussed in Section 5.

2 OXYGEN LINES IN THE NEBULAR PHASE OF CC-SNe

In this section temperature-dependent effective recombination rates are obtained for the $3s, 3p$ and $3d$ triplet and quintet states of neutral oxygen. These rates are used in our nebular code to calculate oxygen recombination line emission. These levels are responsible for the $\lambda\lambda 7774, 8446, 9264$ and 11287 Å line emission (see Fig. 1 for illustration). We further investigate the relevant processes for oxygen recombination and absorption line formation in the nebular phase of CC-SNe.

Recombination into the $2p$ singlet states is not important in the CC-SN nebular phase. About 80–90 per cent of the radioactive energy is deposited in thermal electrons, while only 10–20 per cent goes into ionization (see below). Since the $2p$ singlet states can be efficiently excited by thermal electron collisions, the population of these levels is predominantly determined by this process.

2.1 Effective recombination rates for neutral oxygen

Di-electronic recombination is much weaker than radiative recombination [roughly a factor of 5 (Nussbaumer & Storey 1983) at $10^4$ K and decreasing rapidly with temperature] and is not included when calculating the recombination fractions. At high electron densities collisional transitions between excited states can become important. However, for the electron densities expected in the nebular phase ($n_e < 10^{10}$ cm$^{-3}$), this effect is weak. At high electron densities ($n_e \sim 10^{10}$ cm$^{-3}$) and low temperatures ($T \lesssim 3000$ K) collisional recombination can become important (e.g. see Storey & Hummer 1995). However, during the nebular phase only the combinations high density, high temperature (early) and low density, low temperature (late) are important and the contribution of collisional recombination is weak (e.g. $\sim 20$ per cent at $n_e \sim 10^5$ cm$^{-3}$ and $T \sim 5000$ K).

Therefore we only consider radiative recombination. We are interested in which fraction of recomining electrons reaches a certain atomic level, whether by direct recombination or cascading from higher states. It had been noted by Julienne, Davis & Oran (1974) and Chung, Lin & Lee (1991) that high quantum levels ($n \sim 20$) might be important to calculate the recombination cascade. In the literature recombination coefficients and radiative rates are available for neutral oxygen up to maximum quantum numbers $n \sim 10$ (see e.g. TIPTOPbase1 and National Institute of Standards and Technology (NIST)2).

For azimuthal quantum numbers $l \geq 3$ one can use the hydrogenic approximation, however, for the s, p and d orbitals this should be avoided (e.g. Chung et al. 1991). Therefore, we obtained radiative recombination coefficients and radiative rates for all s, p and d levels of oxygen up to $n = 20$ using a quantum defect method (QDM; Bates & Damgaard 1949; Seaton 1958; Burgess & Seaton 1960).

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1 http://cdsweb.u-strasbg.fr/topbase/
2 http://www.nist.gov/index.html
The potential of the atomic core and the inner electrons approaches the asymptotic form $1/r$ quickly. The wave functions of the excited states can be described by a hydrogen-like solution, with the difference that the quantum energy levels are shifted owing to the unknown structure of the core potential. This shift of energy levels is described by the quantum defect (defined as the difference between the real and ‘effective’ principal quantum number) and can be measured in experiments or is determined by more sophisticated calculations.

The wave functions calculated by the QDM are described with sufficient accuracy at radii larger than a few (atomic units) and therefore the QDM is reliable as long as the main contribution of the transition integrals comes from sufficiently large radii and there is no strong cancellation in the integrand. Especially for ground state transitions, where contributions from small radii are important, the QDM is therefore not reliable.

To improve the calculation, we numerically integrate the electron wave equation (e.g. Seaton 1958),

$$\frac{d^2}{dr^2} \frac{l(l+1)}{r^2} - V(r) + E \right] P(E, l; r) = 0,$$

(1)

inwards, starting with the solution obtained by the QDM at large radii and using a Thomas–Fermi potential:

$$V(r) = \frac{2N}{r} \exp(-Z^{1/3}r) + \frac{2(Z - N)}{r},$$

(2)

where $Z$ is the charge of the nucleus and $N$ the number of core electrons. This integration is not part of the QDM. The Thomas–Fermi potential gives the correct asymptotic behaviour of the atomic potential for $r \to 0$ and $r \to \infty$ (Burke & Robb 1975). The result of this integration is a behaviour of the wavefunction for $r \to 0$ different than predicted by the QDM. For most transitions the contribution from small radii is approximately zero and therefore the calculated rates are not influenced by this correction. For the transitions which are influenced by the inner part of the wavefunction we achieve better agreement with oxygen atomic data provided in the literature at low quantum numbers ($n < 10$).

The difference for ground state radiative transitions relative to NIST recommended data is always less than 50 per cent and the agreement increases considerably with the quantum number of the excited states. For ground state transitions from levels $n > 5$ the disagreement becomes less than 10 per cent. For transitions between excited states the maximum difference to NIST recommended data is 40 per cent with most transitions agreeing at the 10 per cent level or better. Whenever the disagreement was worse than 10 per cent we replaced our data with the values from the literature. We compared our direct and effective recombination coefficients to Julienne et al. (1974) and Chung et al. (1991) and found good agreement (better than 10 per cent), as we did for the effective recombination rates at 1160 K. Our total recombination rate agrees with the value given by Chung et al. (1991) to 1 per cent at 10000 K. However compared to Aldrovandi & Pequignot (1973) our total recombination rate is ~25 per cent too low. It was noted by Julienne et al. (1974) that neglecting the $n > 20$ levels might cause an underestimate of ~20 per cent of the total recombination rate (at temperatures around 10000 K; the effect becomes smaller at higher temperatures). More importantly Aldrovandi & Pequignot (1973) used a hydrogenic approximation for all atomic states, which leads to an overestimate of the recombination rate (Chung et al. 1991). A comparison to a recent calculation from Badnell (2006) shows that our total radiative recombination rates agree to their results within 15 per cent. To handle that deviation, we normalize our result.

We calculate the ratio of the effective recombination rates of the $n = 3$ levels to the total recombination rate into excited states and find that this ratio varies only weakly with the number of levels included (it changes by ~1 per cent (3s), 1 per cent (3p) and 8 per cent (3d) when using a maximum quantum number of $n = 20$ instead of $n = 10$) and is not too sensitive to uncertainties in the atomic data (see below), while the total recombination rate is (it increases by 15–30 per cent depending on temperature when using a maximum quantum number $n = 20$ instead of $n = 10$).

Since fitting formulae for the total and ground state recombination rates are available in the literature, we can obtain the total recombination rate into excited states and normalize our effective recombination rates to the total radiative recombination rate. We call this the normalized effective recombination fraction.

The recombination rates used for this normalization are the total recombination rate of Badnell (2006):

$$R_{GS} = \left(1.174 + 0.2463x + 0.2144x^2 - 0.0621x^3\right) \times 10^{-11} T_e^{-0.5} \text{cm}^3 \text{s}^{-1},$$

(4)

with $x = \log(1 + T_e)/\log(2)$ and $T_e = T/10000$ K. These normalized effective recombination fractions are shown in Fig. 2 and are listed in Appendix B. To obtain absolute values one has to multiply by the total radiative recombination rate. Since the total recombination rate contains a contribution from direct ground state recombination and since lower levels contain cascading contributions from higher states, these fractions do not add up to one.

Since there are uncertainties in our atomic data we want to quantify their influence. We randomly vary all the radiative rates obtained

Figure 2. Temperature-dependent normalized effective recombination fractions. The quintet states (3d, 3p, 3s from bottom to top) are shown by solid lines. Triplet states (3p, 3s, 3d from bottom to top; note that 3p and 3s do overlap) for the case of optically thin ground state transitions are shown by dashed lines. Triplet states (3d, 3p, 3s from bottom to top) for the case of optically thick ground state transitions are shown by dotted lines.
by the QDM by a value between ±20 per cent and the recombination rates obtained by the QDM by a value between ±50 per cent several times. This should be larger than the typical error, at least for excited states. The resulting deviations from the standard values are typically a few per cent, often < 5 per cent.

### 2.2 Recombination line formation

With the effective recombination rates shown in Fig. 2 it is possible to calculate the strength of all [O I] \( n = 3 \) lines due to recombining electrons.

The population of excited oxygen levels by recombination is implemented in the one-dimensional (Mazzali et al. 2001, 2007a) and three-dimensional version (Maurer et al. 2010) of our nebular code and will be used in Sections 3 and 4 to calculate synthetic recombination lines for SN 1998bw and 2002ap.

However, to obtain insight into the formation of these lines it is useful to derive some estimates for the recombination line formation. Below, we estimate the strength of the O I 7774 Å line. This is for different clumping factors (3 lines due to recombining \( Y \) 947–960 n I + I ζ Y is the energy lost to thermal electrons per ionization, which is the high effective recombination rate and transition energy [3p(\( ^5 \)P) to 3s(\( ^3 \)S)]. Ground state recombination line observations are usually not available since most spectra reach a minimum wavelength of 3000–4000 Å only.

The ionization rate can be calculated using the concept of ‘work per ion’ (e.g. Axelrod 1980), which describes the energy lost to thermal electrons in order to produce one ionization. The ionization rate per atom and ion is then given by

\[
\frac{Y_{\text{O I}}}{n_{\text{Tot}}} = \frac{L_{\text{Dep}}}{N_{\text{Tot}} W_{\text{O I}}},
\]

where \( L_{\text{Dep}} \) is the total deposited luminosity, \( N_{\text{Tot}} \) is the total number of atoms and \( W_{\text{O I}} \) is the energy lost to thermal electrons per ionization and can be estimated by comparing the ionization cross-section with atomic and plasma loss functions (more details are given in Section 2.5).

During the first several hundred days of a SN, ionization balance is a valid assumption (Axelrod 1980), therefore

\[
\frac{Y_{\text{O I}}(1 + \mathcal{R}) n_{\text{O I}} = Y_{\text{O I}} n_{\text{O I}} = R_{\text{O I}} n_{e} \xi n_{\text{O II}},}
\]

where \( \mathcal{R} \) is the recycling fraction (recombination radiation causes further ionization and \( \mathcal{R} \) is expected to have a value between 0.3 and 0.5; Axelrod 1980) and \( \xi \) is the clumping factor (explained in more detail below), defined as the inverse of the filling factor (Li & McCray 1992).

Since oxygen is the dominant element in CC-SNe cores (∼55 per cent in SN 2002ap and ∼75 per cent in SN 1998bw, as obtained from our modelling in Sections 3 and 4; see also Mazzali et al. 2001, 2007a) and the degree of ionization of oxygen is rather high (roughly 10–50 per cent; Fig. 3) it is a reasonable simplification to neglect all elements besides the iron-group elements and oxygen when calculating the ionization balance and to assume that all electrons are provided by oxygen and by iron, which is assumed to be 100 per cent singly ionized. Therefore

\[
Y_{\text{O I}} n_{\text{O I}} \sim R_{\text{O I}} n_{\text{O II}} + n_{\text{Fe}} \xi n_{\text{O II}},
\]

where \( n_{\text{Fe}} \) is the total number of iron atoms.

### Figure 3. The ratio \( n_{\text{O I}}/n_{\text{Tot}} \) for different clumping factors (\( \zeta = 1, 10, 100 \) from bottom to top) calculated for the test model described in Appendix A. Since clumping increases recombination the fraction of neutral oxygen increases with \( \zeta \).

\[
\frac{n_{\text{O I}}}{n_{\text{Tot}}} \sim 1 - \frac{n_{\text{Fe}}}{n_{\text{Tot}}} = \frac{R_{\text{O I}} n_{\text{Fe}} \xi + Y_{\text{O I}}}{2 R_{\text{O I}} n_{\text{Tot}} \xi} \times \left( -1 + \sqrt{1 + \frac{4 Y_{\text{O I}} R_{\text{O I}} (n_{\text{Tot}} - n_{\text{Fe}}) \xi}{(R_{\text{O I}} n_{\text{Fe}} + Y_{\text{O I}})^2}} \right)
\]

\[
\sim 1 - \frac{n_{\text{Fe}}}{n_{\text{Tot}}} + \frac{R_{\text{O I}} n_{\text{Fe}} \xi + Y_{\text{O I}}}{2 R_{\text{O I}} n_{\text{Tot}} \xi} \times \sqrt{\frac{Y_{\text{O I}} (n_{\text{Tot}} - n_{\text{Fe}})}{R_{\text{O I}} n_{\text{Tot}}^2 \xi}}, \quad \frac{Y_{\text{O I}} (n_{\text{Tot}} - n_{\text{Fe}})}{R_{\text{O I}} n_{\text{Tot}}^2 \xi} \gg 1\]

\[
\sim 1 - \frac{n_{\text{Fe}}}{n_{\text{Tot}}} + \frac{Y_{\text{O I}} (n_{\text{Tot}} - n_{\text{Fe}})}{(R_{\text{O I}} n_{\text{Fe}} + Y_{\text{O I}}) n_{\text{Tot}}}, \quad \frac{Y_{\text{O I}} (n_{\text{Tot}} - n_{\text{Fe}})}{R_{\text{O I}} n_{\text{Tot}}^2 \xi} \ll 1.
\]

In Fig. 3 this ratio is computed for a test model described in Appendix A using clumping factors of 1, 10 and 100. The total luminosity of any recombination line X of oxygen relative to the total deposited luminosity is then given by

\[
\frac{L_X}{L_{\text{Dep}}} = Y_{\text{O I}} n_{\text{O I}} V E_X f_X L_{\text{Dep}}^{-1}
\]

\[
= (1 + \mathcal{R}) \frac{E_X f_X}{W_{\text{O I}}} \left( \frac{n_{\text{O I}}}{n_{\text{Tot}}} \right),
\]

where \( E_X \) is the energy and \( f_X \) is the normalized effective recombination fraction of the line. Usually one can assume \( 4 Y_{\text{O I}} R_{\text{O I}} n_{\text{Tot}} \xi \gg (R_{\text{O I}} n_{\text{Fe}} + Y_{\text{O I}})^2 \); see Appendix A. \( L_{\text{Dep}} \) (e.g. Axelrod 1980) and all other variables (besides the temperature) can be calculated for any ejecta model and equation (9) can be evaluated directly. The temperature can be determined by balancing heating and cooling taking into account several hundred emission lines (therefore an exact estimate of the temperature is difficult) and varies between 8000 and 2000 K at the epochs of interest (100 to 600 d). A sufficiently exact estimate can be obtained assuming a constant temperature of ∼4000 K (see Appendix A).

The luminosity of any oxygen recombination line is directly proportional to the normalized effective recombination fraction \( f_X \) and the energy of the transition \( E_X \) (eV). The recombination line luminosity is influenced only weakly by the clumping factor \( \xi \) (which changes the ionization fraction). The oxygen NGS recombination
lines will be weak in general [taking \( n_{\text{ion}}/n_{\text{Tot}} \sim 0.5, f_{7774} \sim 0.4, E_{7774} \sim 1.6 \text{ eV} \) and \( W_{O I} \sim 75 \text{ eV} \) the strongest NGS oxygen recombination line (7774 Å) carries less than 1 per cent of the total luminosity] and we will neglect all other NGS oxygen recombination lines in the rest of this paper. They are calculated by the nebular code, but in the nebular spectra of SNe 1998bw and 2002ap on oxygen recombination lines other than 7774 Å can be identified.

2.3 Excitation of the O I 7774 Å line

We will find in Sections 3 and 4 that the luminosity provided by recombination is not sufficient to explain the observations of the 7774 Å line earlier than \( \sim 250 \text{ d} \). Therefore, in this section we investigate other excitation mechanisms which might operate simultaneously with recombination.

Ground-state excitation by thermal electrons can clearly be ruled out. All transitions from \( n \geq 3 \) to the ground state have energies larger than 9 eV, which makes thermal excitation by electrons of temperatures \( \sim 0.5 \text{ eV} \) ineffective.

Non-thermal excitation by the same electrons causing ionization (which are produced by Compton scattering of \( \gamma \)-rays emitted by \( ^{56}\text{Co} \) decay) is also too weak. The ratio of the cross-sections for electron impact excitation and ionization (at the high electron energies which are of interest here) can be approximated by (e.g. Rozsnyai, Jacobs & Davis 1980)

\[
\sigma_{\text{el-\alpha\ell}} \sim 1.66 \frac{f_{\text{el-\alpha\ell}} \epsilon_{\text{el}}}{N_{\text{el}} N_{\text{el}}^{\text{el-\alpha\ell}}} \tag{10}
\]

where \( \epsilon_{\text{el}} \) is the transition energy, \( \epsilon_{\text{el}} \) the ionization energy, \( N_{\text{el}} \) is the number of electrons in the shell \( n \), \( l \) and \( f_{\text{el-\alpha\ell}} \) the oscillator strength of the transition. We compare hydrogen and helium non-thermal excitation rates obtained by this approximation to values obtained from more sophisticated calculations (Hachinger, in preparation), solving an energy-balance equation derived from the Spencer–Fano equation (Lucy 1991; Xu & McCray 1991) and find that the agreement is always better than 30 per cent, which suggests that this approximation is also of acceptable accuracy for oxygen.

For allowed ground state transitions of neutral oxygen the oscillator strengths are of the order of \( 10^{-2} \) decreasing with increasing quantum number (Bell & Hibbert 1990). The total excitation rate into triplet states is \( \sim 5 \) per cent of the ionization rate only. The excitation rates of quintet states by non-thermal (high-energetic) electrons is even lower, since the excitation cross-sections of spin-forbidden lines strongly decrease with electron energy, as compared to allowed transitions (e.g. Rachenko et al. 2008).

Therefore, non-thermal excitation is not important compared to recombination for populating excited levels. In addition, if the O I 7774 Å line was excited by non-thermal electrons, one would expect the ratio of the line to the total luminosity to be approximately constant at all epochs, which is clearly not observed (this ratio in fact decreases considerably at late epochs).

Although it is generally assumed that the SN is optically thin during the nebular phase, some lines can still be optically thick (especially ground state transitions). However, as we show here, the optical depth of the 7774 Å transition is still high, even at epochs of \( \sim 200 \text{ d} \) because the 3s(\( ^{2}\text{S} \)) level is populated by recombining electrons.

We estimate the time-dependent optical depth of the 7774 Å line resulting from recombination taking into account the effective recombination rates, resonance scattering of the 3s(\( ^{2}\text{S} \)) to 2p(\( ^{2}\text{P} \)) transition as well as the recycling of ground state recombination radiation.

Based on equations (7) and (8), the number density of the 3s(\( ^{2}\text{S} \)) state relative to the total density is approximately given by

\[
\frac{n_{3s^{2}S}}{n_{\text{Tot}}} \sim \frac{f_{3s^{2}S} \epsilon_{3s^{2}S} R_{3s^{2}S} (n_{3s^{2}S} + n_{\text{ion}}) n_{\text{Tot}}}{(A' + C) W_{\text{Tot}}} \\
\sim \frac{f_{3s^{2}S} \epsilon_{3s^{2}S} n_{\text{Tot}}}{A} \frac{n_{\text{ion}}}{n_{\text{Tot}}}, \quad A' \gg C \\
\sim (1 + R) \frac{f_{3s^{2}S} \epsilon_{3s^{2}S} \tau_{\text{Dep}}}{A V W_{O I}} \left( \frac{n_{\text{ion}}}{n_{\text{Tot}}} \right)^{2} \tau, \tag{11}
\]

defining \( A' \) as the reduced (due to self-absorption) radiative rate

\[
A' = A \left( 1 - \exp\left( -\frac{-\tau}{\tau} \right) \right) \sim A' \tau, \quad \tau \gg 1 \tag{12}
\]

from the 3s(\( ^{2}\text{S} \)) to the ground state, where \( \tau \) is the Sobolev optical depth of this ground state transition:

\[
\tau \sim \frac{\lambda_{1355} g_{3s^{2}S} A_{1355} \sigma_{\text{Dep}}}{8 \pi g_{2p^{2}P_{1}}}, \quad \tau \gg 0, n_{\text{ion}} \gg n_{3s^{2}S}, \tag{13}
\]

with the ground state transition wavelength \( \lambda_{1355} = 1355 \AA \), \( \tau \) the epoch in seconds, \( V \) the volume in cm\(^3\), \( A_{1355} \) the radiative rates from the 3s(\( ^{2}\text{S} \)) to the ground state (note that the ground state is split into three \( j \) sublevels and appropriate weights have to be used; also the radiative rates of these \( j \) substate transitions differ). The weights of the upper and lower states are \( g_{3s^{2}S} g_{2p^{2}P_{1}} f_{3s^{2}S} \) is the normalized effective recombination fraction into the 3s(\( ^{2}\text{S} \)) state and \( C \) is the deexcitation rate due to thermal electrons which is much smaller than \( A' / \tau \). The 3s(\( ^{2}\text{S} \)) to 2p(\( ^{2}\text{P} \)) transition is spin-forbidden but dipole-allowed and therefore collisional rates are weaker than radiative rates by several orders in magnitude (van Regemorter 1962) for typical SN nebular densities. For \( n_{\text{ion}} \gg 10^{5} \text{ cm}^{-3} \) this assumption breaks down since \( C \) and \( \tau \) will increase with density and clumping factor.

Equation (11) can be used to calculate the Sobolev optical depth of the 7774 Å line:

\[
\tau_{3s^{2}S} \sim \frac{\lambda_{7774} g_{3p^{2}P_{1}} A_{7774} n_{3s^{2}S}}{8 \pi g_{3s^{2}S}}, \quad n_{3s^{2}S} \gg n_{3p^{2}P_{1}}, \tag{14}
\]

with the transition wavelength \( \lambda_{7774} = 7774 \AA \), \( \tau \) the epoch in seconds, \( A_{7774} \) the radiative rates from the 3p(\( ^{2}\text{P} \)) to the 3s(\( ^{2}\text{S} \)) state [note that the 3p(\( ^{2}\text{P} \)) state is split into three \( j \) sublevels and appropriate weights have to be used] and \( g_{3p^{2}P_{1}} g_{3s^{2}S} \) the weights of the upper and lower states. Again we compare this estimate with numerical results from our nebular code in Appendix A.

At this point it is important review the concept of clumping in more detail. It is assumed that the ejecta are distributed into small ‘blobs’ with size much smaller than the typical scale of the SN, covering the SN volume homogeneously (e.g. Li & McCray 1992). Therefore, although the optical depth is increased locally in the emitting region, the global optical depth of the SN remains constant. This means that, when scattering radiation from remote regions of the SN, the opacity is not influenced directly by clumping. On the other hand, when scattering radiation in the emission region, the optical depth increases proportionally to the clumping factor.

In addition, there is an increase of the ratio \( n_{3s^{2}S}/n_{\text{Tot}} \) (see equation 11), which results from the increased self-absorption of ground state transition radiation. Therefore, even when scattering radiation from remote regions, clumping increases the optical depth and so does influence the 7774 Å resonance scattering of background radiation.

The 3p(\( ^{2}\text{P} \)) state is separated from the 3s(\( ^{2}\text{S} \)) state by \( \sim 1.6 \text{ eV} \), therefore thermal electron excitation is possible, but the
population of the \(3s(S)\) state by thermal electron excitation is too low. Therefore the state must be populated by recombinating electrons.

The thermal electron excitation coefficient from \(3s(S)\) to \(3p(P)\) is given by

\[
C_{3p\rightarrow 3s} = 8.6 \times 10^{-6} \frac{n_i \zeta}{g_{3s(S)}} \exp \left( -\frac{E_{7774}}{kT} \right)
\]

where all constants have the same meaning as in equation (14), \(E_{7774}\) is the energy corresponding to 7774 Å and \(\Omega_{7774} = 25.1/40.2\) (at 5000/10 000 K; Bhatia & Kastner 1995) is the effective collision strength of the O\(\text{I}\) 7774 Å transition. One can compare the population of the \(3p(P)\) state due to thermal electron collisional excitation and recombination, respectively, by calculating their ratio:

\[
R_{3p\rightarrow 3s} = \frac{C_{3p\rightarrow 3s} n_{3s(S)}}{Y_{O}\Omega_{3p} f_{3p\rightarrow 3s}} = (1 + R) C_i \frac{f_{3s(S)}}{f_{3p\rightarrow 3s}} \frac{y_0}{A} (n_{\text{Tot}} - n_{O}) n_{O} \zeta^2.
\]

The ratio of the O\(\text{I}\) 7774 Å luminosity induced by thermal electron excitation compared to the total deposited luminosity is then given by

\[
\frac{L_{3p\rightarrow 3s}}{L_{\text{Dep}}} \sim (1 + R)^2 C_i \frac{f_{3s(S)}}{f_{3p\rightarrow 3s}} \frac{y_0}{W_{O i}} \frac{y_0}{n_{\text{Tot}}} \left(1 - \frac{n_{O}}{n_{\text{Tot}}} \right) n_{O} \zeta^2
\]

\[
\sim 10^{-21} [d] \left(1 - \frac{n_{O}}{n_{\text{Tot}}} \right) n_{O} \zeta^2, \quad T = 5000 \text{ K}.
\]

This estimate becomes invalid for \(n_{O} \zeta > 10^5 \text{ cm}^{-3}\) since equation (11) had been used for its derivation. Equation (17) is compared to results from our nebular code in Appendix A. Whether collisional excitation of the O\(\text{I}\) 7774 Å line is negligible depends mainly on the temperature, which decreases with time, on the density of neutral oxygen, which decreases with \(r^{-3}\), and on the clumping factor. Therefore, especially for early epochs (high temperature, high density) and for large clumping factors \((\zeta \gg 1)\) the O\(\text{I}\) 7774 Å line may be excited by thermal excitation of recombinating electrons. Around 150 d the core (oxygen) density of CC-SNe is typically of order \(10^{2-3} \text{ cm}^{-3}\), which gives \(L_{3p\rightarrow 3s}/L_{\text{Dep}} \sim (10^{-5} - 10^{-3}) \zeta^2\) assuming a temperature of 5000 K.

From the above considerations it becomes clear that clumping influences the recombination line strength only weakly, but increases the effect of resonance scattering and thermal excitation of the O\(\text{I}\) 7774 Å transition strongly. For both processes the 3s(S) population is provided by recombination and not by thermal electron excitation from the ground level.

### 2.4 Emission versus absorption line shapes

Since the O\(\text{I}\) \(\lambda\lambda 6300, 6363\) doublet has often been used to probe the ejecta velocity and the geometry of CC-SNe cores (see Section 1) it is interesting to study the profile of the O\(\text{I}\) 7774 Å line. If this line were caused by recombination or thermal excitation alone, one would expect that the shape of the 6300, 6363 and 7774 Å lines should be at least approximately similar.

However, as we have shown above, O\(\text{I}\) 7774 Å might result from line scattering at 200 d. At later times the line usually becomes very weak and it is difficult to use it to probe the ejecta geometry. The profile of a scattering dominated line depends on the flux distribution, so one cannot expect that the shape of the 6300, 6363 Å lines and the 7774 Å line will be even approximately similar.

---

**Figure 4.** Left-hand side: flux of a synthetic [O\(\text{I}\)] \(\lambda\lambda 6300, 6363\) doublet for an asymmetric model (double-peak shape). Right-hand side: the 7774 Å line opacity calculated from this asymmetric model was used to calculate the flux of the absorption line at 7774 Å (dashed vertical line). The flux is normalized to an arbitrary constant. The formation of the absorption line strongly depends on the assumed background (indicated by the dashed horizontal line); it not only depends on the absolute background flux but also on the background rest-frame wavelength which determines the emission site and therefore the absorption line shown would look different for a different background radiation field. The asymmetry has some influence on the absorption line, but the clear double-peak shape of the [O\(\text{I}\)] \(\lambda\lambda 6300, 6363\) doublet is not seen.

To demonstrate this, we use a multidimensional Monte Carlo code. A photon background is generated and scattered in an asymmetric oxygen distribution (e.g. taken from a nebular model; the nebular code in its current version cannot simulate scattering processes). We calculate the [O\(\text{I}\)] \(\lambda\lambda 6300, 6363\) doublet for an asymmetric oxygen model and use the 7774 Å line opacity calculated in the nebular code to compute the absorption of a broad-band background (3000–10 000 Å) emitted in the inner region of the model SN. The formation of the absorption line is very sensitive to this background and therefore our results (see Fig. 4) illustrate just one of many possibilities. As expected, the absorption line has quite a different shape than the emission line.

### 2.5 Ionization rates for the nebular code

To improve the accuracy of the recombination line calculation in our nebular code we updated the treatment of ionization for all the ions treated. This is necessary, since the formation of recombination lines is much more sensitive to the ionization rate than the formation of other emission lines, and higher accuracy is needed. In the older version of the nebular code the ionization rates were calculated using a simple analytical form.

To obtain more accurate ionization rates we calculate the average `work per ion` \(W_i\) (see Axelrod 1980), which is the amount of energy lost by a non-thermal electron (with energy \(\geq 1\) keV) to cause one ionization. The ionization rate is then computed dividing the total deposited luminosity by this energy.

The value of \(W_i\) can be calculated averaging the `work per ion` \(W_i\) of all individual ions over the total ionization cross-sections. \(W_i\) can be obtained for any ion integrating the ratio of the energy-dependent ionization cross-section \(\sigma_i\) and the energy loss functions \(L_i\) (Axelrod 1980);

\[
W_i = \frac{E_{\text{max}}}{\int_{E_{\text{max}}}^{E_{\text{min}}} \left( \frac{\sigma_i(E')}{L_i(E')} \right) dE'}.
\]
Highly energetic electrons lose most of their energy to ionization and to the secondary electrons produced in the ionization, but also by non-thermal excitation. At low energies, losses to the electron plasma become important, however, most of the primary energy is lost in atomic collision processes.

Ionization cross-sections and loss functions can be obtained for any ion from the literature at low electron energies (e.g. Lotz 1967, 1970; Lennon et al. 1988). At high electron energies they can be computed by appropriate extrapolations of the low-energy values (Axelrod 1980).

The ionization rate of any ion is then given by
\[
Y_i = \frac{L_{\text{Dep}} \sigma_i}{N_{\text{e,tot}} W(\sigma)} = \frac{L_{\text{Dep}}}{N_{\text{e,tot}} W_i}.
\]

The ionization rates mainly affect the recombination lines but also have some influence on all other emission lines. This will be discussed in Section 5.

3 A SHELL MODEL OF SN 2002ap

SN 2002ap is classified as a broad-lined SN of Type Ic. It had an ejected mass of \(~2.5\, M_\odot\) and a kinetic energy of \(~4 \times 10^{51}\, \text{erg}\) (Mazzali et al. 2007a). The distance and reddening to SN 2002ap are only known approximately. To be consistent with previous work we use \(\mu = 29.50\, \text{mag}\) and \(E(B-V) = 0.09\, \text{mag}\) as done by Mazzali et al. (2002), Yoshii et al. (2003) and Mazzali et al. (2007a). It is not clear whether SN 2002ap was a spherical symmetric event or not. Recently, Maurer et al. (2010) have shown that an asymmetry might be observable in all broad-lined CC-SNe, however, this point is not clear yet.

Mazzali et al. (2007a) found appropriate one zone and shell models of the SN 2002ap nebular ejecta. We quickly summarize the main findings for the shell models here. Using a filling factor of 0.1 (\(\xi = 10\)) in the \(56\,\text{Ni}\)-rich regions, a total ejecta mass of \(~2.5\, M_\odot\) was found, containing roughly \(0.11\, M_\odot\) of \(56\,\text{Ni}\) and \(1.3\, M_\odot\) of oxygen.

In contrast to this previous work, which aimed at modelling each spectrum individually, here we try to find one single \(56\,\text{Ni}/\text{O}\) model which can produce the time evolution of the different spectra at all observed epochs consistently. Special attention is paid to reproducing the exact shape of the \([\text{O} \, \text{i}]\lambda\lambda 6300, 6363\) line profile, which is a tracer of the distribution of oxygen, the most abundant element of the nebula. The spectra of SN 2002ap used in this work are those used by Mazzali et al. (2007a) and were originally presented by Foley et al. (2003).

Using the same distance modulus and reddening as Mazzali et al. (2007a) and clumping factors of 5 and 25 (we had to use a clumping factor of 1 for the innermost shell of the \(\xi = 25\) model to avoid the formation of sharp high-density lines which are not observed; Mazzali et al. 2007a) had used a clumping factor of 10; this will be discussed in Section 5) we obtain models similar to the ones given in Mazzali et al. (2007a) giving reasonable agreement with the observations at all epochs (see Figs 5 and 6). The forbidden oxygen lines are reproduced at all epochs by one and the same oxygen distribution, which provides evidence for the reliability of this model.

Our \(56\,\text{Ni}\) zone with a total mass of \(~1.2\, M_\odot\) extends out to 8000 km s\(^{-1}\), containing \(~0.07\, M_\odot\) of \(56\,\text{Ni}\) (in agreement with Mazzali et al. 2002) and \(~0.7\, M_\odot\) of O. More mass is located at higher velocities, but the nebular modelling becomes inaccurate for the outer regions. The exact values depend on the clumping factor as well as on the \(56\,\text{Ni}\) distribution, which is not known.

The \([\text{O} \, \text{i}]\) \(7774\,\text{Å}\) line is excited by recombination and thermal electron excitation in our nebular modelling (no line scattering) and is too weak to explain the observations at 129 and 163 d using low clumping factors (\(\xi \approx 5\)). The \(\xi = 25\) model can reproduce the observations of the \([\text{O} \, \text{i}]\) \(7774\,\text{Å}\) line at 129 and 163 d better than the \(\xi = 5\) model. This results from thermal electron scattering of 3s(\(^3\text{S}\)) electrons, which are provided by recombination. In general the \(\xi = 5\) model reproduces the formation of the forbidden lines better than...
The nebular phase of SN 1998bw has been well studied by means of nebular modelling (Mazzali et al. 2001; Maeda et al. 2006). While Mazzali et al. (2001) attempted to model individual nebular spectra by one-zone models (concluding that this is insufficient), Maeda et al. (2006) computed two-dimensional synthetic nebular spectra for models obtained from hydrodynamical simulations, finding good agreement with spectra of individual epochs. However no model has so far described the time sequence of nebular spectra of SN 1998bw consistently.

In this section we try to find a single two-dimensional model which is consistent with the full time evolution from 108 to 388 d after explosion. While successful models for individual epochs can be found even in one-dimensional modelling, it seems impossible to model the time sequence of all spectra with a single one-dimensional model (in contrast to SN 2002ap, where this approach works well).

Since SN 1998bw is quite massive compared to other CC-SNe (like SN 2002ap for example), the transition to the nebular phase occurs at rather late times, although the ejecta expand at high velocities. The spectra at 108 and 139 d after explosion are not strictly nebular and therefore large deviations between the model and the observations are to be expected when trying to reproduce these spectra (especially at 108 d) with our nebular code.

When considering two dimensions one is immediately confronted with the problem that the parameter space is much larger (angular distribution of elements, observer inclination). In addition, the computation time to obtain one synthetic spectrum increases dramatically. Since hundreds of spectra have to be computed to obtain a good model, this technical problem can make modelling unfeasible. Therefore some assumptions about the SN geometry have to be introduced.

There are several good arguments to assume that SN 1998bw might consist of some kind of two-dimensional ‘jet-disc’ structure, as described for example by Maeda et al. (2006) (see Section 1). Furthermore, since the SN was accompanied by a GRB, it seems likely that the observer inclination is not too far from polar, although this is uncertain since the opening angle of GRB 980425 is not known.

Therefore we work with a parametrized two-dimensional model which consists of a polar zone (0° to 45°) and an equatorial zone (45° to 90°). This simplification will introduce some error, since most likely it does not represent physical reality. However, it seems sufficiently exact to obtain an acceptable fit at all epochs with a single $^{56}$Ni/O model. A model with more degrees of freedom is highly degenerate anyway, since the information that can be extracted from a series of spectra $F(v, t)$ is limited.

Because of this simplification the reliability of our model is in question, but this kind of uncertainty is inherent to all multidimensional modelling. We performed tests for viewing angles of 0°, 15° and 30°. Using the smaller viewing angles the iron lines which are mainly produced in the jet-like structure become very narrow, which is not observed. Therefore we decided to use a viewing angle of 30°. However this might be a consequence of our simplified geometry and therefore it is not possible to obtain valuable information about the observers inclination from our modelling approach.

Our model is roughly consistent with previous findings (Maeda et al. 2006). The polar zone contains $\sim 0.24 M_\odot$ of $^{56}$Ni at velocities below 12 000 km s$^{-1}$. The equatorial zone also contains $\sim 0.24 M_\odot$ of $^{56}$Ni (note that the equatorial zone has more than twice the surface area of the polar zone), but located at velocities below $\sim 8000$ km s$^{-1}$. The total mass below 12 000 km s$^{-1}$ is estimated to be $\sim 2.7 M_\odot$ containing a total oxygen mass (below 12 000 km s$^{-1}$) of $\sim 2 M_\odot$. There is a lot of material at higher velocities, but the
The two earliest spectra (108 and 139 d) of SN 1999bw are clearly not nebular and the agreement between observations and simulations is expected to be poor. However, the evolution of total and oxygen 6300 Å flux as well as the shape of the [O I] λλ6300, 6363 doublet line are reproduced at all epochs much better than by a one-dimensional model. At epochs of 108 to 349 d, the simulated flux is too low to match the observations of the 7774 Å line. At 388 d, considering the constant offset caused by some background flux which is not reproduced by the synthetic spectrum, the synthetic 7774 Å line seems to be consistent with the observations. Unfortunately the noise level is high and a detailed comparison is not possible.

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We tried to explain the formation of O I 7774 Å at the earlier epochs (108 and 139 d) by increasing the clumping factor, which in turn increases the thermal electron excitation from the 3s(3P) to the 3p(3P) level. However, this does not seem to work. Both [O I] 5577 Å and the Ca II IR-triplet become very strong at clumping factors >10, while the synthetic flux around 7774 Å increases but seems still too weak to explain the observations. Owing to the high density of SN 1998bw the nebular approach might not be suitable for such early epochs (the strong continuum which is observed is not reproduced). In addition there are uncertainties on the atomic data (especially the collision strengths), which may influence our conclusion.

In Figs 10 and 11 we show the opacity of the 7774 Å line for the five innermost shells for clumping factors of ζ = 1 and 5. There will be 7774 Å line absorption up to ~200–300 d depending on clumping.

Again, at least moderate clumping (ζ ~ 5) seems necessary in order to provide enough O I 7774 Å line opacity to reproduce the observations at 214 d.

5 DISCUSSION

In Section 2.1 we have obtained normalized effective recombination fractions using available atomic data and computing missing atomic data. The agreement with atomic data available in the literature
The logarithm of the $\lambda 7774$ Å scattering line comprises SN 1998bw, case of clumping ($\xi = 5$). Solid lines show the equatorial region, dotted lines the polar region. Apart from the innermost shell, the optical depth drops below one at $\sim 310$ d. This optical depth is calculated for scattering remote emission radiation, which has a weaker dependence on the clumping factor than for local scattering.

is good. Effective recombination rates had been given before by Julienne et al. (1974) at a temperature of 1160 K. These results are reproduced well. We extend the temperature range up to 10000 K, which should be an upper limit for temperatures of SN nebular ejecta between 100 and 600 d. Typical uncertainties in the atomic data seem to have small influence.

In Sections 2.2 and 2.3 we have obtained estimates for the formation of oxygen recombination lines and line absorption by the $3s(3S)$ state of neutral oxygen. We have shown that the luminosity of recombination lines is weak at any epoch. The influence of clumping on the recombination line is weak as well. Clumping will increase the recombination rate, but will decrease the number of $O\,^\text{I}$ ions at the same time. The total number of recombinations, which is equivalent to the total number of ionizations, will increase only slightly. Therefore clumping offers no direct way for increasing $\lambda 7774$ Å emission considerably.

We have shown that there is line scattering of $\lambda 7774$ Å photons even at very late epochs. This is possible because of the large number of ground state $O\,^\text{I}$ atoms, which resonantly scatter the ultraviolet (UV) radiation emerging from $3s\to2p$ transitions and slow down the de-population of the $3s$ state. Combined with recombination this leads to a high population of the $3s$ level, which in turn can resonantly scatter $\lambda 7774$ Å radiation. We have shown that, in contrast to the direct recombination line, the scattering line is very sensitive to clumping of the ejecta.

In addition, for high clumping factors, there may be thermal electron collisional excitation of the $O\,^\text{I}\ 7774$ Å line at early epochs. However, it is in question whether these high clumping factors are realistic since high-density lines, which are not observed, may form. Uncertainties in the atomic data, especially the collision strengths, may influence our results.

The considerations regarding the $O\,^\text{I}\ 7774$ Å line are also in principle valid for the $[O\,^\text{I}]\ 8446$ Å line [$3s(3S)\to2p(3P)$] with the difference that the effective recombination rate into the $3s$ triplet state is smaller than into the quintet state. The optical depth of the 8446 Å line is therefore expected to be lower, which is still sufficient to cause line scattering at very early epochs. However, the 8446 Å line is blended with the blue wing of the [Ca II] IR-triplet, which can explain why no $[O\,^\text{I}]\ 8446$ Å line is identified in SNe 1998bw and 2002ap.

We have shown that there is likely to be no other effective excitation mechanism than ‘recombination+thermal excitation’ or ‘recombination+absorption’ for $O\,^\text{I}\ n \geq 3$ levels. Thermal electron excitation of $n \geq 3$ levels from the ground level is ineffective at typical nebular temperatures. Non-thermal electron excitation can also be ruled out. First, it is too weak and second, it would produce a temporal behaviour of the $O\,^\text{I}\ 7774$ Å line which is not observed. As detailed modelling showed (Sections 3 and 4) absorption might be too weak to explain the observations as long as there is no clumping of the ejecta ($\xi \sim 5$). To enable thermal electron excitation even larger clumping factors seem necessary.

The concept of clumping has often been used in stellar wind and SN radiation physics (e.g. Li & McCray 1992, 1993; Mazzali et al. 2001, 2007a; Maeda et al. 2006) without quantitative understanding of the physics behind ejecta clumping, therefore playing the role of a fitting factor. Li & McCray (1992) have modelled the ratio of the $[O\,^\text{I}]\ 5577, 6363$ doublet lines and Li & McCray (1993) the Ca ii emission of SN 1987A using a clumping factor of $\sim 10$. Mazzali et al. (2001, 2007a) and Maeda et al. (2006) have modelled the nebular spectra of SNe 1998bw and 2002ap finding that better fits to observations are obtained using clumping factors of $\sim 10$ than when using no clumping. Our results are similar, but show some difference. We obtain good fits to all forbidden lines using clumping factors of 1 and 5 as well. The abundance of elements will change slightly depending on the value chosen. We even found a slight decrease in the quality of the fit when going from $\xi = 5$ to 10 and therefore we used the lower value. This difference probably results mainly from our more accurate treatment of ionization which increases the ionization rate of carbon and oxygen considerably. This means that the electron density is increased, which can be mimicked by increasing the clumping factor and using lower ionization rates. It might also be partially attributed to the fact that there is no strict criterion for the quality of a fit for nebular spectra.

Still, the constraints we derived for the clumping factor of SNe 1998bw and 2002ap from the $O\,^\text{I}\ 7774$ Å scattering line compare well to previous findings. We found that a clumping factor of $\sim 5$ seems necessary to provide sufficient opacity at 7774 Å. Clumping factors of $\xi > 10$ make it possible to model the $O\,^\text{I}\ 7774$ Å emission at early epochs without line scattering, but may cause the formation of high-density emission lines at early epochs (100–150 d). One example is the $[O\,^\text{I}]\ 5577$ Å singlet line, which is observed to be weak. Since there are uncertainties on the atomic data it is not clear whether thermal electron excitation is important and whether the $O\,^\text{I}\ 7774$ Å line at early epochs is dominated by line scattering or thermal electron excitation of recombining electrons.

At later epochs, between 150 and 250 d, the line is most likely a combination of line scattering and recombination radiation, since thermal electron excitation becomes too weak for any reasonable value of clumping. Around $\sim 250$ d the $O\,^\text{I}\ 7774$ Å line seems to become a true recombination line where the flux is mainly provided by the electrons cascading into the $3p(3P)$ level.

In Section 2.4 we have studied the expected profile of the $[O\,^\text{I}]\ 7774$ Å line in light of our previous findings. As expected, the absorption- and emission-line shapes caused by a certain ejecta distribution can show important differences. Further, the profile of the absorption line depends on the background radiation field and one should be careful when using the shape of a scattering line for any kind of argumentation about ejecta geometry as long as it not clear how this line is formed in detail.

Figure 11. SN 1998bw, case of clumping ($\xi = 5$). The logarithm of the 7774 Å line optical depth of our ejecta model for the inner shells (from top to bottom: 0–1000, 1000–2000, 2000–3000, 3000–4000 and 4000–5000 km s$^{-1}$). Solid lines show the equatorial region, dotted lines the polar region. Apart from the innermost shell, the optical depth drops below one at $\sim 310$ d.
In Sections 3 and 4 we obtained core ejecta models for SNe 1998bw and 2002ap. The primary goal of this modelling was not to re-derive ejecta properties, but to obtain reliable models for studying oxygen recombination consistently with other line formation.

As a by-product we obtained estimates for the $^{56}$Ni and oxygen masses of the cores of these SNe. We find that SN 2002ap can be described very well by a one-dimensional shell model (as already found before), which does not necessarily mean that there is no asymmetry. Our $^{56}$Ni ($\sim$0.07 M$_{\odot}$) and oxygen mass estimates seem consistent with previous work. For SN 1998bw one-dimensional modelling seems to be insufficient to obtain an acceptable fit at all epochs using a single model. An acceptable fit is obtained with a two-dimensional 'jet+disc' model (this had been found before). The fit could certainly be improved by increasing the degree of freedom of the model, but this would become extremely time-consuming. The model found in this work reproduces the observations at all epochs with increasing accuracy at later times, which is expected. The estimate of the $^{56}$Ni ($\sim$0.48 M$_{\odot}$) and oxygen mass are consistent with previous results.

Using these ejecta distributions we were able to reproduce the formation of O I 7774 Å at late epochs. There is enough line opacity at 7774 Å at early and intermediate nebular epochs to produce a strong scattering line. Since the background radiation field is important for the calculation of this absorption line, we were not able to show that our ejecta distribution reproduces the observations at early epochs exactly, however, this seems likely. If there is strong clumping the O I 7774 Å line may be additionally excited by thermal electrons. The oxygen distribution inferred from forbidden line observations allows the reproduction of the allowed oxygen lines at late epochs. Since different physics are involved, this provides a test for the consistency of the nebular modelling approach, which is passed for SNe 1998bw and 2002ap.

Several other ions may produce absorption lines at wavelengths between 4000 and 10 000 Å (the part of the spectrum that is usually observed). Since the abundance of these elements is much lower than that of oxygen, there will be no excited states with sufficient population to cause significant line scattering (an exception might be the Ca II IR-triplet). Allowed ground state transitions with energies between $\sim$1 and 3 eV are interesting since their optical depth can be high owing to the high radiative rates of allowed lines and to the possibly sufficiently high ground state populations of these low-abundance elements. The elements of interest are Na I, Mg I and Ca II. These three elements have low quantum-level transitions with wavelengths of $\sim$5890 Å (Na I), 4570 Å (Mg I) and 3950, 7300 and 8500 Å (Ca II), which can be present in sufficient amounts in SN nebulae. Iron group element lines, especially Fe I, may be optically thick as well. However, since these ions are complex we do not treat them explicitly here.

If 0.01 M$_{\odot}$ of Na I were to be distributed homogeneously within 5000 km s$^{-1}$, the optical depth of the Na I 5890 Å line would be $\sim$6 × 10$^5$ at 200 d (this result is obtained by balancing thermal-electron excitation with radiative and collisional de-excitation analogously to Section 2.2), scaling linearly with Na I mass and with epoch t$^{-2}$. If 0.01 M$_{\odot}$ of Mg I were to be distributed homogeneously within 5000 km s$^{-1}$, the optical depth of the Mg I 4570 Å line would be $\sim$1 at 200 d, scaling linearly with Mg I mass and with epoch t$^{-2}$.

Three Ca II lines might be seen in absorption even at late epochs ($\sim$3950, 7300 and the IR-triplet 8500 Å). To estimate their strength, it is important to know the relative population of the Ca II 3d to 4s states, which can be approximated by

$$\frac{n_{4s}}{n_{3d}} \sim \frac{C_{4s\rightarrow3d}}{A_{3d\rightarrow4s} + C_{3d\rightarrow4s}},$$

where $C_{4s\rightarrow3d}$ and $C_{3d\rightarrow4s}$ are the collisional rates for the 4s to 3d transition and $A'$ is the radiative rate from 3d to 4s, reduced due to resonance scattering.

If 0.01 M$_{\odot}$ of Ca II were to be distributed homogeneously within 5000 km s$^{-1}$, the optical depth of the Ca II 3950 Å and [Ca II] 7300 Å lines would be $\sim$2 × 10$^5$ and $\sim$0.02, respectively, at 200 d, scaling linearly with the Ca II mass and with epoch t$^{-2}$. The optical depth of the Ca II IR-triplet depends on the population of the 3d state. Using the 7300 Å optical depth estimated above, the optical depth of the Ca II 8500 Å transition is $\sim$430 (n$_e$ = 10$^7$ and T = 5000 K) at 200 d, with a stronger dependence on Ca II mass and epoch than the optical depths at 3950 and 8500 Å (see also Li & McCray 1993, for a detailed discussion of Ca II line formation).

The exact mass of Na I, Mg I and Ca II is not well known, since estimates of their masses are not very accurate. However, typical masses in CC-SNe cores for Na, Mg and Ca might be of the order of 0.01 M$_{\odot}$.

Therefore, even at late times (>200 d) there is strong line scattering at wavelengths between ~4000 and 10 000 Å. The Ca II 3950, Na I 5890 and O I 7774 Å lines cause strong scattering in any reasonable scenario. The [Mg I] 4570, O I 8446 and Ca II 8500 Å lines may have sufficient optical depth for line scattering, depending on epoch, degree of ionization, total mass and clumping.

6 SUMMARY AND CONCLUSION

We have computed temperature-dependent effective recombination rates for neutral oxygen in a temperature range suitable for all types of SN nebulae (at epochs between 100 and 600 d). Since oxygen is the most abundant element in stripped-envelope CC-SNe, oxygen lines are of special interest among other recombination lines.

We obtained core ejecta models for CC-SNe 1998bw and 2002ap. Similar models had been derived previously. Using these oxygen profiles, the O I 7774 Å recombination line, which is the strongest observed recombination line, is calculated and compared to observations. We show that up to late epochs pure recombination is too weak to power O I 7774 Å. At earlier epochs the line is powered by scattering and possibly by thermal electron excitation of recombin-
seems to consist of a superposition of several weak lines this could become a difficult task. It is not clear which elements or ions produce the observed background flux in this region. Iron group elements seem to be promising candidates, but currently the accuracy of our calculations is not sufficient.

ACKNOWLEDGMENT
We thank Keith Butler for reading the manuscript and for making useful comments.

REFERENCES
Aldrovandi S. M. V., Pequignton D., 1973, A&A, 25, 137
Akedo T. A., 1980, PhD thesis, California University, Santa Cruz
Badnell N. R., 2006, ApJS, 167, 334
Bates D. R., Damgaard A., 1949, Philos. Trans. R. Soc. Lond. Ser. A: Math. Phys. Sci., 242, 101
Bell K. L., Hibbert A., 1990, J. Phys. B: Atomic Molecular Opt. Phys., 23, 2673
Bhatia A. K., Kastner S. O., 1995, ApJS, 96, 325
Blöndin J. M., Mezzacappa A., DeMarino C., 2003, ApJ, 584, 971
Burgess A., Seaton M. J., 1960, MNRAS, 120, 121
Burke P. G., Robb W. D., 1975, Adv. At. Mol. Phys., 11, 143
Burrows A., Livine E., Dessart L., Ott C. D., Murphy J., 2007, ApJ, 655, 416
Chung S., Lin C. C., Lee E. T. P., 1991, Phys. Rev. A, 43, 3433
Filippenko A. V., 1997, ARA&A, 35, 309
Foley R. J. et al., 2003, PASP, 115, 1220
Galama T. J. et al., 1999, A&AS, 138, 465
Höflich P., 1991, A&A, 246, 481
Höflich P., Wheeler J. C., Wang L., 1999, ApJ, 521, 179
Iwamoto K. et al., 1998, Nat, 395, 672
Julienne P., Davis J., Oran E., 1974, J. Geophys. Res., 79, 2540
Kay L. et al., 1998, BAAS, 30, 1323
Kotake K., Sawai H., Yamada S., Sato K., 2004, ApJ, 608, 391
Lenon M. A., Bell K. L., Gilbody H. B., Hughes J. G., Kingston M. J., Murray M. J., Smith F. J., 1988, J. Phys. Chem. Ref. Data, 17, 1285
Li H., McCray R., 1992, ApJ, 387, 309
Li H., McCray R., 1993, ApJ, 405, 730
Lotz W., 1967, ApJS, 14, 207
Lotz W., 1970, J. Opt. Soc. America, 60, 206
Lucy L. B., 1991, ApJ, 383, 308
Maeda K., Nakamura T., Nomoto K., Mazzali P. A., Patat F., Hachisu I., 2002, ApJ, 565, 405
Maeda K., Nomoto K., Mazzali P. A., Deng J., 2006, ApJ, 640, 854
Maeda K. et al., 2007a, ApJ, 658, L5
Maeda K. et al., 2007b, ApJ, 666, 1069
Malesani D. et al., 2004, ApJ, 609, L5
Matheson T., 2004, in Höflich P., Kumar P., Wheeler J. C., eds, Cosmic Explosions in Three Dimensions. Cambridge Univ. Press, Cambridge, p. 351
Maurer J. et al., 2010, MNRAS, 402, 161
Mazzali P. A., Nomoto K., Patat F., Maeda K., 2001, ApJ, 559, 1047
Mazzali P. A. et al., 2002, ApJ, 572, L61
Mazzali P. A., Deng J., Maeda K., Nomoto K., Filippenko A. V., Matheson T., 2004, ApJ, 614, 858
Mazzali P. A. et al., 2005, Sci, 308, 1284
Mazzali P. A. et al., 2007a, ApJ, 670, 592
Mazzali P. A., Röpke F. K., Benetti S., Hillebrandt W., 2007b, Sci, 315, 825
Mazzali P. A. et al., 2008, Sci, 321, 1185
Moiseenko S. G., Bisnovatyi-Kogan G. S., Ardeljan N. V., 2006, MNRAS, 370, 501
Nakamura T. et al., 2000, in Martens P. C. H., Tsuruta S., Weber M. A., eds, Proc. IAU Symp. 195, Highly Energetic Physical Processes and Mechanisms for Emission from Astrophysical Plasmas. Astron. Soc. Pac., San Francisco, p. 347
Nussbaumer H., Storey P. J., 1983, A&A, 126, 75
Patat F. et al., 2001, ApJ, 555, 900
Pequignot D., 1990, A&A, 231, 499
Pian E. et al., 2006, Nat, 442, 1011
Ralphenko Y., Janev R. K., Tato K., Fursa D. V., Bray L., de Heer F. J., 2008, Atomic Data Nuclear Data Tables, 94, 603
Rozsnyai B. F., Jacobs V. L., Davis J., 1980, Phys. Rev. A, 21, 1798
Sauer D. N., Mazzali P. A., Deng J., Valenti S., Nomoto K., Filippenko A. V., 2006, MNRAS, 369, 1939
Seaton M. J., 1958, MNRAS, 118, 504
Storey P. J., Hummer D. G., 1995, MNRAS, 272, 41
Stritzinger M., Mazzali P. A., Sollerman J., Benetti S., 2006, A&A, 460, 793
Takiwaki T., Kotake K., Sato K., 2009, ApJ, 691, 1360
Taubenberger S. et al., 2009, MNRAS, 397, 677
van Regemorter H., 1962, ApJ, 136, 906
Woosley S. E., Bloom J. S., 2006, ARA&A, 44, 507
Xu Y., McCray R., 1991, ApJ, 375, 190
Yoshii Y. et al., 2003, ApJ, 592, 467

APPENDIX A
In Sections 2.2 and 2.3 we have derived several estimates for quantities relevant for calculating O 1 7774 Å nebular emission. To derive these estimates, we used some approximations. These are justified here by comparison to results from the nebular code, which are not based on these approximations and will show deviations as soon as our approximations fail. An important difference is introduced by the temperature. While the electron temperature is calculated considering hundreds of emission lines in the nebular code, this is not possible for our estimates and we use a temperature of 4000 K for the calculations.

To perform these tests we set up a test one-zone model with a total mass $M_{\text{Tot}} \sim 0.5 M_\odot$, an expansion velocity of 5000 km s$^{-1}$, containing 80 per cent oxygen, 10 per cent $^{56}$Ni, 3 per cent calcium, magnesium and other elements like iron. Considering hundreds of emission lines in the nebular code, this is not possible for our estimates and we use a temperature of 4000 K for the calculations.

Usually one can assume $4Y_{\text{O}}R_{\text{O}}p_{\text{O}}\zeta/(R_{\text{O}}p_{\text{Fe}}\zeta + Y_{\text{O}})^2 \ll 1$, which depends on the iron to oxygen ratio, the deposited luminosity, the electron temperature and the clumping factor. In Fig. A1 we show this ratio for our test model using clumping factors of 1 to 100. At any epoch of interest (100 to 600 d) and for any clumping factor the assumption is valid. The assumption may fail for clumping

![Figure A1](https://academic.oup.com/mnras/article-abstract/408/2/947/1026741/0)

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The solid line is a computation of equation (9) performed for the O 37774 Å line assuming a constant temperature of 4000 K at all epochs for the one-zone model described in the text. The luminosity of this line is compared to the total deposited luminosity. The results are given in per cent. The recycling fraction was set to the ratio of ground state to total recombination rate at 4000 K. The crosses show data points obtained with the nebular code for the same model, where the temperature is computed at all epochs self-consistently and ionization is treated in detail. Considering the large change of the temperature between 100 and 600 d (6800 and 2800 K, respectively) the agreement seems reasonable (note that the temperature calculated by the code drops below 4000 K around 450 d). The minimum around 200 d is present in both curves.

Factors ~1 at lower densities and for higher $^{56}\text{Ni}$ to oxygen ratios. For clumping factors $\zeta \gg 1$ the assumption will hold for any plausible CC-SN scenario.

In Figs A2 and A3 we compare equations (9) and (14) (temperature set to $T = 4000$ K) with the results obtained from our nebular code using our test model. The temperature is kept constant in the calculation, while it is calculated self-consistently in the nebular code, influencing the recombination fraction and recombination rate. Also the ionization of ions other than O I and Fe I has been neglected in the calculation. Despite the approximations, the agreement seems reasonable.

In Fig. A4 we compare the O 37774 Å flux from pure recombination and thermal excitation of recombining electrons relative to the total deposited luminosity in per cent for the test models described in the text using clumping factors between 1 and 100. Solid lines show estimates obtained from equations (9) and (17) for oxygen densities of $10^7$, $10^7.5$ and $10^8$ cm$^{-3}$ from bottom to top. Results from our nebular code are shown by diamonds ($10^7$ cm$^{-3}$), triangles ($10^7.5$ cm$^{-3}$) and squares ($10^8$ cm$^{-3}$). The agreement is good. Small deviations are predominately caused by our rough approximation of the temperature. Filled squares mark $n_0 \zeta \sim 10^8$ cm$^{-3}$ where our estimates are expected to become inaccurate. At $n_0 \zeta > 10^9$ cm$^{-3}$ our approximation breaks down completely, but this regime is probably not important for the nebular phase of stripped CC-SNe. The 7774 Å luminosity increases with the increasing clumping until the density becomes high enough to depopulate the 3s($^3S$) state by thermal electron collisions effectively.

**APPENDIX B**

Normalized effective recombination fractions for triplet and quintet states of O I $n = 3$ and 4 (see Table B1). For the triplet states we show calculations for the cases of optical thin and thick ground states. These numbers have to be multiplied by the total radiative recombination rate to obtain the effective recombination rates. Since the total radiative recombination rate also contains a direct ground state component and since lower levels contain cascade contributions from the higher levels these fractions do not add up to one.
Table B1. Effective normalized recombination fraction of O I for triplet and quintet states of O I \( n = 3 \). The rates are normalized on the total radiative recombination rate of Badnell (2006). States indicated by * show the case of optical thin ground state transitions. Values in brackets indicate a power of 10, e.g. \( a(b) = a \times 10^b \).

| State  | 1000 K  | 2000 K  | 3000 K  | 4000 K  | 5000 K  | 6000 K  | 7000 K  | 8000 K  | 9000 K  | 10000 K |
|--------|---------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| 3s(3S)* | 4.6(−2) | 4.2(−2) | 4.0(−2) | 3.8(−2) | 3.7(−2) | 3.6(−2) | 3.4(−2) | 3.3(−2) | 3.2(−2) | 3.1(−2) |
| 3p(3P)* | 4.3(−2) | 4.0(−2) | 3.8(−2) | 3.6(−2) | 3.5(−2) | 3.4(−2) | 3.2(−2) | 3.0(−2) | 2.9(−2) | 2.8(−2) |
| 3d(3D)* | 1.6(−1) | 1.4(−1) | 1.3(−1) | 1.2(−1) | 1.1(−1) | 1.1(−1) | 1.0(−1) | 9.4(−2) | 8.9(−2) | 8.5(−2) |
| 3s(3S)  | 2.7(−1) | 2.5(−1) | 2.4(−1) | 2.3(−1) | 2.2(−1) | 2.1(−1) | 2.0(−1) | 1.9(−1) | 1.8(−1) | 1.7(−1) |
| 3p(3P)  | 2.6(−1) | 2.4(−1) | 2.3(−1) | 2.1(−1) | 2.0(−1) | 2.0(−1) | 1.9(−1) | 1.8(−1) | 1.7(−1) | 1.6(−1) |
| 3d(3D)  | 1.7(−1) | 1.5(−1) | 1.4(−1) | 1.3(−1) | 1.2(−1) | 1.2(−1) | 1.1(−1) | 1.0(−1) | 9.8(−2) | 9.3(−2) |
| 3s(3S)  | 4.7(−1) | 4.5(−1) | 4.3(−1) | 4.1(−1) | 3.9(−1) | 3.8(−1) | 3.6(−1) | 3.5(−1) | 3.3(−1) | 3.2(−1) |
| 3p(3P)  | 4.6(−1) | 4.3(−1) | 4.1(−1) | 3.9(−1) | 3.8(−1) | 3.6(−1) | 3.5(−1) | 3.3(−1) | 3.2(−1) | 3.0(−1) |
| 3d(3D)  | 2.7(−1) | 2.4(−1) | 2.2(−1) | 2.1(−1) | 1.9(−1) | 1.8(−1) | 1.7(−1) | 1.6(−1) | 1.5(−1) | 1.5(−1) |
| 4s(3S)* | 9.6(−3) | 7.9(−3) | 7.0(−3) | 6.8(−3) | 6.6(−3) | 6.4(−3) | 6.2(−3) | 6.0(−3) | 5.7(−3) | 5.5(−3) |
| 4p(3P)* | 9.0(−3) | 9.1(−3) | 9.0(−3) | 8.7(−3) | 8.4(−3) | 8.1(−3) | 7.8(−3) | 7.5(−3) | 7.1(−3) | 6.8(−3) |
| 4d(3D)* | 4.2(−2) | 4.0(−2) | 3.8(−2) | 3.7(−2) | 3.5(−2) | 3.3(−2) | 3.1(−2) | 2.9(−2) | 2.8(−2) | 2.6(−2) |
| 4d(3F)* | 8.5(−2) | 6.8(−2) | 5.9(−2) | 5.1(−2) | 4.6(−2) | 4.1(−2) | 3.8(−2) | 3.4(−2) | 3.1(−2) | 2.8(−2) |
| 4s(3S)  | 3.8(−2) | 3.8(−2) | 3.7(−2) | 3.6(−2) | 3.4(−2) | 3.3(−2) | 3.2(−2) | 3.0(−2) | 2.9(−2) | 2.7(−2) |
| 4p(3P)  | 5.2(−2) | 5.2(−2) | 5.1(−2) | 4.9(−2) | 4.7(−2) | 4.5(−2) | 4.3(−2) | 4.1(−2) | 3.9(−2) | 3.6(−2) |
| 4d(3D)  | 4.5(−2) | 4.4(−2) | 4.2(−2) | 4.0(−2) | 3.8(−2) | 3.6(−2) | 3.4(−2) | 3.2(−2) | 3.0(−2) | 2.9(−2) |
| 4d(3F)  | 8.6(−2) | 6.9(−2) | 5.9(−2) | 5.2(−2) | 4.7(−2) | 4.2(−2) | 3.8(−2) | 3.5(−2) | 3.2(−2) | 2.9(−2) |
| 4s(3S)  | 6.8(−2) | 6.9(−2) | 6.7(−2) | 6.5(−2) | 6.3(−2) | 6.1(−2) | 5.8(−2) | 5.5(−2) | 5.3(−2) | 5.0(−2) |
| 4p(3P)  | 8.1(−2) | 8.1(−2) | 8.0(−2) | 7.7(−2) | 7.4(−2) | 7.1(−2) | 6.8(−2) | 6.5(−2) | 6.1(−2) | 5.8(−2) |
| 4d(3D)  | 7.1(−2) | 6.8(−2) | 6.5(−2) | 6.2(−2) | 5.9(−2) | 5.6(−2) | 5.3(−2) | 5.0(−2) | 4.7(−2) | 4.4(−2) |
| 4d(3F)  | 1.4(−1) | 1.1(−1) | 0.9(−2) | 0.8(−2) | 0.7(−2) | 0.7(−2) | 0.6(−2) | 0.5(−2) | 0.5(−2) | 0.4(−2) |

At high densities, collisional transitions become important, which are not included in our cascade calculation. At densities \( n_e < 10^{10} \text{ cm}^{-3} \) and temperatures of \( T \sim 5000 \text{ K} \) the error on the recombination rates owing to this effect should be less than 10 percent.

At high temperatures di-electronic recombination becomes important, while at low temperature and high densities collisional recombination can become important. Under nebular conditions both effects should be of the order of 10 percent or less.

To obtain the effective recombination coefficient for a certain line one has to weigh the upper level by the corresponding radiative rates.

\(^3\) http://www.nist.gov/index.html

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