Reduction of Cd in circular cylinder using three passive control at Re=500

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Abstract. Passive control is the addition of a small object to an object to reduce the drag force of the object. This study focuses on three passive controls. Numerically, the problem of three passive controls for drag reduction of circular cylinder is studied. In this case, three passive controls are placed in front of and in the rear of the main object. A passive control type-I is in front of circular cylinder while the others are in rear. The distance of the passive controls is varies to the circular cylinder. In this study, we want to find an effective distance of the main object to three passive controls from the shape and the angle position of the passive controls.

1. Introduction

Nowadays, there is a lot of research on fluid. One of them is the study of fluid flow through objects with the aim of reducing the inhibitory properties of objects. That study was studied for industrial and technical development. Applications in this study are usually found in industrial chimneys, offshore structures, flyover structures and others.

Drag can be reduced by two methods. They are active control or passive control. In general, passive control is easier than active control. Some researchers used passive control placed in front of various shapes, such as cylindrical cylinders, type-I cylinders, type-D cylinders etc. Several previous studies were carried out by Igarashi and Tsusui (1992) to test passive controls with a small cylindrical shape placed in front of circular cylinder with variation of distance between passive control with cylinder that is 3 mm to 6 mm and using the Reynolds number to $5.1 \times 10^3$ to $5.1 \times 10^4$\cite{10}. Furthermore, a study by Imron, C. et al (2011), with a passive control cylinder type-I placed in front of the circular cylinder. The experiments have done by observing the effect of variation of distance between passive control and circular cylinder $0.6 \leq S/D \leq 3$ and using the Reynolds number $7 \times 10^3$\cite{2}. Lately, two passive controls were used to reduce drag coefficient. Widodo, B. et al (2017) use two passive controls. The first passive control is cylinder type I with a cutting angle $53^\circ$ is placed in front of the cylinder, while the second passive control of the type I cylinder with the horizontal position is placed behind the cylinder\cite{12}.

In this paper, a research was conducted which aims to study the drag coefficient by using three passive controls with a circular cylinder as the main object in the Reynold number $Re = 500$. 
A passive control in front of main object is type-I while the others in rear can be one of type-I, elliptical or circular cylinder with the same shape. In this study, the passive controls behind makes a certain angle to the horizontal line, besides the passive control distance in front of S and the passive control behind to the main object are varies.

Figure 1. Schematic of three passive controls and circular cylinder.

2. Numerical Method
The previously described problem can be solved by using the unsteady incompressible fluid equation and the Navier-Stokes equation:

\[ \nabla \cdot \mathbf{v} = 0, \tag{1} \]

\[ \frac{\partial \mathbf{v}}{\partial t} + \nabla \cdot (\mathbf{v} \mathbf{v}) = -\nabla P + \frac{1}{Re} \nabla^2 \mathbf{v} \tag{2} \]

where Re is the Reynolds number, \( \mathbf{v} \) is the velocity, and \( P \) is the pressure. The Navier-Stokes equation can be solved by using SIMPLE algorithms and numerical methods. The first thing to do is give to the initial value for each variable. By ignoring the pressure components, we will find the velocity component of the momentum equation, so equation (1) becomes

\[ \frac{\partial \mathbf{v}}{\partial t} = -\nabla \cdot (\mathbf{v} \mathbf{v}) + \frac{1}{Re} \nabla^2 \mathbf{v} \tag{3} \]

by using the finite difference method, we have

\[ (f_x)_i = \frac{2f_{i+1} + 3f_i - 6f_{i-1} + f_{i-2}}{6dx} \quad \text{and} \quad (f_y)_j = \frac{2f_{j+1} + 3f_j - 6f_{j-1} + f_{j-2}}{6dy} \]

\[ (f_{xx})_i = f_{i+1} - 2f_i + f_{i-1} \quad \text{and} \quad (f_{yy})_j = f_{j+1} - 2f_j + f_{j-1} \]

and afterwards

\[ \frac{\partial \mathbf{v}}{\partial t} = \frac{\mathbf{v}^{**} - \mathbf{v}^{*}}{\Delta t} = -\nabla P \tag{4} \]

because of equation (2), then equation (4) becomes

\[ \frac{\nabla \cdot \mathbf{v}^{**}}{\Delta t} = -\Delta P \tag{5} \]

by using SOR (Successive Over Relaxation)

\[ (P_n)_{i,j} = (1-\epsilon)(P_{n-1})_{i,j} + \epsilon(P^*)_{i,j} \tag{6} \]

with \( \epsilon > 1 \).[4] The last is the correction of velocity component over pressure component as follows:

\[ \frac{\partial \mathbf{v}}{\partial t} = -\nabla P \tag{7} \]
3. Main Result

In this study, we used three passive controls. The first passive control is a cylinder of type-I placed in front of a circular cylinder. The second and the third passive control are elliptical cylinders. The second and the third passive control is placed in the rear of the circular cylinder at varying distance, i.e. $T/D = 1.8$ and $C/D = 2.4$ as shown in Figure 1. It creates certain angle to the horizontal line. The angle is also various

| Table 1. Passive Control Cylinder Ellipse Shape. |
|-----------------------------------------------|
| $C_d$  | Angle |
|       | 15°   | 30°   | 45°   | 60°   | 90°   |
| S/D   |       |       |       |       |       |
| 1.8   | 0.859 | 0.834 | 0.884 | 0.932 | 0.991 |
| 2.1   | 0.851 | 0.842 | 0.891 | 0.934 | 0.979 |
| 2.4   | 0.872 | 0.846 | 0.894 | 0.942 | 1.000 |
| 2.1   | 0.864 | 0.854 | 0.901 | 0.945 | 0.990 |

3.1. Drag coefficient

The drag coefficient of a circular cylinder with three passive controls at the front and in the rear. Based on Table 3, drag coefficient on a circular cylinder has the least drag coefficient. The least drag coefficient is $C_d = 0.834$ with configuration $S/D = 1.8$, $T/D = 1.8$ and angle $30^\circ$. By using three passive controls, the drag coefficient is reduced up to 39.70%. The result can be seen as in Figure 2(a) and 2(b).

![Figure 2. Real data plot of drag coefficient with fixed (a) $T/D = 2.1$, (b) $S/D = 1.8$](image)

3.2. Mathematical Model of The Drag Coefficient

The simulation result of the drag coefficient with three passive controls was interpolated to obtain the mathematical model of the drag coefficient. The mathematical model is made by Lagrange trilinear interpolation. The variables are $S/D$ assumed as $x$, $T/D$ assumed as $y$ and angle assumed as $z$. The value of $C_d$ is assumed as $f(x, y, z)$. Based on Lagrange trilinear
By using Lagrange trilinear interpolation, if \( x_1, x_2, \ldots, x_p, y_1, y_2, \ldots, y_q \) and \( z_1, z_2, \ldots, z_r \) are distinct numbers, then Lagrange trilinear interpolation is given by

\[
\hat{f}(x, y, z) = \sum_{k=1}^{r} \left( \sum_{j=1}^{q} \left( \sum_{i=1}^{p} f(x_i, y_j, z_k) L_{p,a}(x) \right) H_{q,b}(y) \right) G_{r,c}(z)
\]

where, for each \( a = 1, 2, \ldots, p \), \( b = 1, 2, \ldots, q \) and \( c = 1, 2, \ldots, r \)

\[
L_{p,a} = \prod_{u=1}^{p} \frac{x - x_u}{x_a - x_u}, \quad H_{q,b} = \prod_{v=1}^{q} \frac{y - y_v}{y_b - y_v}, \quad G_{r,c} = \prod_{w=1}^{r} \frac{z - z_w}{z_c - z_w}
\]

with remainder

\[
R(x, y, z) = \frac{f(\xi(x, y, z))^{p+q+r}}{p!q!r!}(x - x_1)(x - x_2) \ldots (x - x_p)(y - y_1)(y - y_2) \ldots (y - y_q)
\]

\[
(z - z_1)(z - z_2) \ldots (z - z_r)
\]

Taking data from Table 1 and substituting \((x, y) = (T/D, S/D)\) into \( f(x, y, z) \), we can obtain the mathematical model of the drag coefficient as follows:

\[
f(x, y, z) = \frac{xyz^4}{546750000} - \frac{xyz^3}{36450000} - \frac{y^4z}{60750000} + \frac{23xz^4}{1215000000} + \frac{7xyz^2}{486000} - \frac{xyz}{3240} + \frac{13yz^3}{3037500}
\]

\[
- \frac{83xz^3}{24300000} + \frac{1589xz^4}{1012500000} + \frac{71xz^2}{360000} - \frac{113yz^2}{27000} + \frac{xy}{450} - \frac{5821z^3}{20250000} + \frac{89yz}{5400} - \frac{493xz}{108000}
\]

\[
+ \frac{17699x^2}{90000} + \frac{169x}{300} - \frac{29y}{150} - \frac{52757z}{9000} + \frac{17817}{2500}
\]

with remainder

\[
R(x, y, z) = \frac{f(\xi(x, y, z))^{2+2+5}}{2!2!5!}(x - 1.8)(x - 2.4)(y - 1.8)(y - 2.1)
\]

\[
(z - 15)(z - 30)(z - 45)(z - 60)(z - 90)
\]

By using Equation 9, drag coefficient on a circular cylinder has the least drag coefficient, the least drag coefficient is \( C_d = 0.828 \) with configuration \( S/D = 2.4, T/D = 1.8 \) and angle 15°. By using mathematical model, The drag coefficient is reduced up to 40.30%. The mathematical model of drag coefficient using three passive controls can be seen as in Figure 3(a) and 3(b).

4. Conclusion

By using Lagrange trilinear interpolation then one can make the mathematical model of drag coefficient. A mathematical model can be formed for \( C_d \) of circular cylinder using three passive controls at Re = 500. The mathematical model can be written as follows:

\[
f(x, y, z) = \frac{xyz^4}{546750000} - \frac{xyz^3}{36450000} - \frac{y^4z}{60750000} + \frac{23xz^4}{1215000000} + \frac{7xyz^2}{486000} - \frac{xyz}{3240} + \frac{13yz^3}{3037500}
\]

\[
- \frac{83xz^3}{24300000} + \frac{1589xz^4}{1012500000} + \frac{71xz^2}{360000} - \frac{113yz^2}{27000} + \frac{xy}{450} - \frac{5821z^3}{20250000} + \frac{89yz}{5400} - \frac{493xz}{108000}
\]

\[
+ \frac{17699x^2}{90000} + \frac{169x}{300} - \frac{29y}{150} - \frac{52757z}{9000} + \frac{17817}{2500}
\]
Figure 3. Mathematical modeling of drag coefficient with fixed (a) $T/D = 2.1$, (b) $S/D = 1.8$

In addition, a reduction of the drag coefficient in circular cylinder can be done by adding passive control. We use Three passive controls because it is very efficient to reduce drag coefficient on a circular cylinder. It is reduced up to 39.70%. The configuration is $S/D = 1.8$, $T/D = 1.8$ and angle $30^\circ$. Then, by using mathematical model. We has the least drag coefficient. The least drag coefficient is 0.828 with configuration $S/D = 2.4$, $T/D = 1.8$ and angle $15^\circ$. It is reduced up to 40.30%.

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