Effect of the $\delta$-meson on the instabilities of nuclear matter under strong magnetic fields

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We study the influence of the isovector-scalar meson on the spinodal instabilities and the distillation effect in asymmetric non-homogenous nuclear matter under strong magnetic fields, of the order of $10^{18} - 10^{19}$ G. Relativistic nuclear models both with constant couplings (NLW) and with density dependent parameters (DDRH) are considered. A strong magnetic field can have large effects on the instability regions giving rise to bands of instability and wider unstable regions. It is shown that for neutron rich matter the inclusion of the $\delta$ meson increases the size of the instability region for NLW models and decreases it for the DDRH models. The effect of the $\delta$ meson on the transition density to homogeneous $\beta$-equilibrium matter is discussed. The DDRH$\delta$ model predicts the smallest transition pressures, about half the values obtained for NL$\delta$.

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Neutron stars with very strong magnetic fields of the order of $10^{14} - 10^{15}$ G are known as magnetars [1–3] and they are believed to be the sources of the intense gamma and X rays detected in 1979 (for a review refer to [4]). To date, 16 magnetars have been identified as short $\gamma$-ray repeaters or anomalous X-ray pulsars, although some are still unconfirmed candidates [5]. However, according to Ref. [6], a fraction as high as 10% of the neutron star population could be magnetars. These neutron stars are hot, young stars, $\sim$1 kyear old. In [7], Thompson and Duncan have considered turbulent dynamo amplifications in young neutron stars as a mechanism for generating the strong magnetic fields.

In magnetars the crust is stressed by very strong forces which deform the crust and may crack it. Once the surface cracks, the violent motions blast particles along the magnetic fields, triggering gamma rays and x-rays. In particular, scientists believe that the giant burst of energy experienced by the magnetar SGR 1806-20 in 2004 was triggered by a "starquake" in the neutron star’s crust that caused a catastrophic disruption in the magnetar’s magnetic field. SGR 1806-20 is the most magnetic object observed and has on the surface a magnetic field of intensity over $10^{15}$ G [5].

In [8] the authors have shown how to obtain both the moment of inertia of a neutron star as well as the fractional moment of inertia of its crust only in terms of the mass and the radius of the star. For the fractional moment of inertia of the crust an additional dependence of the equation of state (EOS) which enters through the values of the pressure and the density at the crust core transition [8, 9]. The transition density enters just as a correction to the fractional moment of inertia, while the transition pressure is the main EOS parameter which defines that quantity. Using this result, Link et al. [9] have obtained a lower limit for the neutron star radius with a given mass, from the glitches occurring in the Vela pulsar and in other six pulsars. This constraint will put severe restrictions on the acceptable equations of state of stellar matter if the radius and mass of a neutron star is measured.

Spin-up glitches have also been observed in all known persistent AXP [10, 11]. However, different from the conventional low-field radio pulsar, the glitches in AXPs are accompanied by a significant recovery of the spin down rate of the pulsar. It would be interesting if information of the crust and star properties could also be obtained from these glitches.

It was recently shown in Ref. [12] that a strong magnetic field, of the order of $10^{18}$, $10^{19}$G, has large effects on the instability regions of nuclear matter. Relativistic nuclear models both with constant couplings and with density dependent parameters were considered. It was shown that a strong magnetic field can have large effects on the thermodynamic spinodal instabilities zones giving rise to bands of instability and wider unstable regions. As a consequence, it was predicted larger transition densities at the inner edge of the crust of compact stars with a strong magnetic field. The direction of instability gives rise to a very strong distillation effect if protons occupy only partially a Landau level. However, for almost full Landau levels an anti-distillation effect may occur.

In this paper, which completes Ref. [12], we study the influence of the isovector scalar meson on the low density instabilities of asymmetric nuclear matter under strong magnetic fields. We estimate the density and pressure at the transition from the non-homogeneous to the homogeneous phase in stellar matter from the crossing of the

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EOS in β-equilibrium matter with the spinodal and discuss the effect of the magnetic field on the direction of instability. We consider two kinds of relativistic mean-field approaches: non-linear Walecka models (NLW) models with constant coupling parameters and density dependent relativistic hadronic (DDRH) models with density-dependent coupling parameters. The last models seem to give more realistic results at subsaturation densities [13]. The inclusion of the δ-meson brings to the isovector channel the same symmetry existing already in the isoscalar channel with the meson pair (σ, ω) responsible for saturation in RMF models [14, 15]. The presence of the δ-meson softens the symmetry energy at subsaturation densities and hardens it above saturation density, giving rise to stable compact stars with larger masses [16]. In [13] it was shown that the instability region in the isovector direction becomes smaller if the δ meson is included. Therefore, it is expected that in the presence of a strong magnetic field these differences become larger.

In the present paper we consider the NLW models NLρ and NLδ [15] and the DDRH models TW [17] and DDRHδ [18]. Only NLδ and DDRHδ include the δ-meson.

For the description of the neutron star matter, we employ the standard mean-field theory (MFT) approach. A complete set of the equations and the description of the method can be found in Ref. [12, 19]. The Lagrangian density of TW [17, 20] and DDRHδ [18, 21] models reads:

\[
\mathcal{L} = \bar{\Psi}_b \left( i \gamma_\mu \partial^\mu - g_\sigma \gamma_\mu A^\mu - m_b + \Gamma_\sigma \sigma + \Gamma_\delta \tilde{\sigma} \cdot \tilde{\delta} \right. \\
- \left. \Gamma_\rho \gamma_\mu \rho^\mu + \frac{1}{2} \gamma_\sigma \partial^\mu \sigma + \frac{1}{2} \gamma_\omega \partial^\mu \omega + \frac{1}{2} \gamma_\rho \partial^\mu \rho + \frac{1}{2} \gamma_\delta \partial^\mu \delta + \frac{1}{2} \gamma_\sigma \partial^\mu \sigma + \frac{1}{2} \gamma_\omega \partial^\mu \omega + \frac{1}{2} \gamma_\rho \partial^\mu \rho + \frac{1}{2} \gamma_\delta \partial^\mu \delta + \frac{1}{2} \gamma_\gamma \partial^\mu \gamma + \frac{1}{2} \gamma_\gamma \partial^\mu \gamma + \frac{1}{2} \gamma_\gamma \partial^\mu \gamma + \frac{1}{2} \gamma_\gamma \partial^\mu \gamma \right) \Psi_b \\
+ \frac{1}{4} \partial_\mu \sigma \partial^\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{4} \partial_\mu \omega \partial^\mu \omega - \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{4} \partial_\mu \rho \partial^\mu \rho - \frac{1}{2} m_\rho^2 \rho^2 \\
- \frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} m_\rho^2 \rho^2 + \frac{1}{2} P_{\mu \nu} P^{\mu \nu},
\]  

(1)

where \( \Psi_b \) are the baryon (b=n, p) Dirac fields; \( \sigma, \omega, \rho, \) and \( \delta \) represent the scalar, vector, isovector-vector and isovector-scalar meson fields, which are exchanged for the description of nuclear interactions and \( A^\mu = (0, 0, Bx, 0) \) refers to a constant external magnetic field along the z-axis. The nucleon mass and isospin projection for the protons and neutrons are denoted by \( m_b \) and \( m_\sigma \), \( m_\omega \), \( m_\rho \), \( m_\delta \), respectively. The mesonic and electromagnetic field strength tensors are given by their usual expressions:

\[
\Omega_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad P_{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu, \quad F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\]

The nucleon anomalous magnetic moments (AMM) are introduced via the coupling of the baryons to the electromagnetic field tensor with \( \sigma_{\mu \nu} = \frac{1}{2} \left[ \gamma_\mu, \gamma_\nu \right] \) and strength \( \kappa_n \) with \( \kappa_n = g_n / 2 = -1.91315 \) for the neutron and \( \kappa_p = (g_p / 2 - 1) = 1.79285 \) for the proton, respectively. The electromagnetic field is assumed to be externally generated (and thus has no associated field equation), and only frozen-field configurations will be considered. The density dependent strong interaction couplings are denoted by \( \Gamma \), the electromagnetic couplings by \( g \) and the nucleon, mesons masses by \( m \). The density dependent coupling parameters are adjusted in order to reproduce some of the nuclear matter bulk properties using the following parametrization

\[
\Gamma_i(\rho) = \Gamma_i(\rho_{sat}) f_i(x), \quad i = \sigma, \omega, \rho, \delta
\]  

(2)

where \( \rho = \rho / \rho_{sat} \), with

\[
f_i(x) = a_i \frac{1 + b_i (x + d_i)^2}{1 + c_i (x + d_i)^2}, \quad i = \sigma, \omega,
\]

(3)

for TW,  

\[
f_\rho(x) = \exp \left[ -a_\rho (x - 1) \right],
\]

(4)

and

\[
f_i(x) = a_i \exp \left[ -b_i (x - 1) \right] - c_i (x - d_i), \quad i = \rho, \delta
\]

(5)

for DDRHδ, with the values of the parameters \( m_i, \Gamma_i, a_i, b_i, c_i \) and \( d_i \), \( i = \sigma, \omega, \rho, \delta \) given in Table I.

In the sequel, we define the magnetic field in units of the electron critical field \( B_c^\text{e} = 4.414 \times 10^{13} \) G, so that \( B = B^\text{e} B_c^\text{e} \).

For the DDRHδ model we use the parametrization given in [18, 21] except for the parameter \( \Gamma_\rho(\rho_{sat}) \) which we increase so that the symmetry energy is 31 MeV at saturation density and not 25 MeV as in [18], see Table I. The inclusion of the δ meson reduces the symmetry energy at subsaturation densities but makes it much stiffer at supra-saturation densities because the δ meson field reaches saturation and the ρ-meson field increases always with density [14, 15]. This is seen in Fig. 1 where the symmetry energy is given for all models we are studying.

For the NLW models, i.e. NLρ and NLδ, we add to the Langrangian density, Eq. (1), with \( g_i = \Gamma_i \), the scalar
without magnetic field intensities and for two NL W models: NL\(\rho\) with respect to the size, but also on the shape of the spinodal zone. Unlike lines). As discussed in [12], due to the Landau quantization of the first three and two Landau levels, respectively. We define the thermodynamic spinodal at \(T = 0\) as the curve on the \((\rho_n, \rho_p)\) plane for which the determinant of the \(F_{ij}\) is zero. The eigenvalues of the stability matrix are given by

\[
\lambda_{\pm} = \frac{1}{2} \left( \text{Tr}(F) \pm \sqrt{\text{Tr}(F)^2 - 4\text{Det}(F)} \right)
\]

and the eigenvectors \(\delta p_{\pm}\) by

\[
\frac{\delta p_{\pm}}{\delta \rho_j} = \frac{\lambda_{\pm} - F_{ji}}{F_{ji}}, \ i, j = p, n.
\]

In Fig. 2 we show the spinodal sections for several magnetic field intensities and for two NLW models: NL\(\rho\) without \(\delta\)-meson (thin lines) and NL\(\delta\) including \(\delta\) (thick lines). As discussed in [12], due to the Landau quantization, the magnetic field has a strong effect not only on the size, but also on the shape of the spinodal zone. Unlike the \(B = 0\) case, the instability zone is no longer symmetric with respect to the \(\rho_n = \rho_p\) line. For the proton-rich matter, the inclusion of the \(\delta\)-meson has only a small effect on the spinodal, namely a small reduction similar to the effect already described for the \(B = 0\) [24]. For neutron-rich matter in the presence of a strong magnetic field, the effect of \(\delta\)-meson is stronger: the NL\(\delta\) model has a spinodal region larger than NL\(\rho\). This effect is similar whether the AMM is included or not.

The DDRH\(\delta\) models behave in a different way due to density dependence of the coupling parameters. In Fig. 3, we present the results of the thermodynamic spinodal instability region for TW (thin lines) and DDRH\(\delta\) (thick lines). For \(B^* = 10^5\) and \(2 \times 10^5\) the spinodal has three and two bands, see Fig. 3, corresponding to the occupation of the first three and two Landau levels, respectively...

### Table I: Parameters for the TW and DDRH\(\delta\) models. These two models have the same parametrization for the \(\sigma\) and \(\omega\) mesons.

| \(i\) | \(m_i\) | \(\Gamma_i\) | \(a_i\) | \(b_i\) | \(c_i\) | \(d_i\) |
|---|---|---|---|---|---|---|
| \(\sigma\) | 550 | 10.72854 | 1.365469 | 0.226961 | 0.409704 | 0.901995 |
| \(\omega\) | 783 | 13.29015 | 1.402488 | 0.172577 | 0.344293 | 0.983955 |
| TW | 763 | 7.32196 | 0.515 | |
| DDRH\(\delta\) | 763 | 12.7530 | 0.095268 | 2.171 | 0.05336 | 17.8431 |

### Table II: Predicted density, proton fraction and pressure at the inner edge of the crust of a compact star at zero temperature, as defined by the crossing between the thermodynamic instability region of np matter and the \(\beta\)-equilibrium EOS for homogeneous, neutrino-free stellar matter in the \((p_B, p_n)\) plane. The AMM is not included.

| \(B^*\) | Model | \(\rho_{\text{cross}}\) \(\text{fm}^{-3}\) | \(Y_p\) | \(P_m\) \(\text{MeVfm}^{-3}\) |
|---|---|---|---|---|
| 0 | NL\(\rho\) | 0.067 | 0.013 | 0.255 |
| | NL\(\delta\) | 0.063 | 0.010 | 0.175 |
| | TW | 0.085 | 0.037 | 0.523 |
| | DDRH\(\delta\) | 0.086 | 0.036 | 0.269 |
| \(3 \times 10^4\) | NL\(\rho\) | 0.071 | 0.043 | 0.284 |
| | NL\(\delta\) | 0.071 | 0.040 | 0.264 |
| | TW | 0.071 | 0.056 | 0.342 |
| | DDRH\(\delta\) | 0.077 | 0.056 | 0.259 |
| \(5 \times 10^4\) | NL\(\rho\) | 0.084 | 0.068 | 0.467 |
| | NL\(\delta\) | 0.084 | 0.064 | 0.467 |
| | TW | 0.082 | 0.086 | 0.461 |
| | DDRH\(\delta\) | 0.084 | 0.086 | 0.306 |
| \(10^5\) | NL\(\rho\) | 0.105 | 0.121 | 0.822 |
| | NL\(\delta\) | 0.105 | 0.117 | 0.896 |
| | TW | 0.101 | 0.146 | 0.673 |
| | DDRH\(\delta\) | 0.096 | 0.148 | 0.404 |
| \(2 \times 10^5\) | NL\(\rho\) | 0.128 | 0.207 | 1.203 |
| | NL\(\delta\) | 0.130 | 0.203 | 1.447 |
| | TW | 0.128 | 0.236 | 1.016 |
| | DDRH\(\delta\) | 0.118 | 0.237 | 0.644 |
| \(5 \times 10^5\) | NL\(\rho\) | 0.162 | 0.367 | 1.275 |
| | NL\(\delta\) | 0.170 | 0.369 | 1.830 |
| | TW | 0.198 | 0.402 | 2.331 |
| | DDRH\(\delta\) | 0.228 | 0.422 | 4.460 |
the EoS for $\beta$-equilibrium stellar matter with the thermodynamic spinodal gives a reasonable prediction of the transition density $[25, 26]$ of the crust to an homogeneous phase in stellar matter. It was shown in $[9]$ that the transition pressure and, at a second level, the transition density define the fraction of the star’s moment of inertia contained in the solid crust. In the sequel we will estimate for the different models and magnetic fields the transition density and transition pressure at the inner edge of a compact star in $\beta$-equilibrium. We will discuss only the cases for which the crossing is occurring at the first LL. For fields $B^* < 3 \times 10^4$ at densities above the crossing with the first LL, there may occur other crossings with other LL. For these cases the determination of the extension of the non-homogeneous phase should be carried within a more precise method.

The values of the transition density $\rho_{\text{cross}}^c$, and respective proton fraction $Y_p$ and pressure $P_m$ are given for stellar matter under different magnetic field intensities in Tables II (without AMM) and III (with AMM). For convenience we have plotted the results without AMM in Fig. 4.

For $B = 0$, we can see that the inclusion of the $\delta$-meson reduces the transition density, and the corresponding proton fraction and pressure for NLW models, while for DDRH models the transition density increases and the corresponding proton fraction and pressure decrease. For DDRH models the crossing is occurring at three times larger proton fractions and twice the pressure.

For a finite magnetic field it was shown in $[12]$ for the models TW and TM1 (a parametrization similar to NL$\rho$) that the transition density increases with the magnetic field. We confirm this result when the $\delta$ meson is included and summarize the conclusions:
a) the proton fraction at the crossing increases monotonically with the magnetic field intensity, and for $B^* = 5 \times 10^5$ it reaches the values 0.36 and 0.4 respectively for NLW and DDRH models. The proton fraction does not
which the symmetry energy for DDRH δ
curvature of the symmetry energy at
for the larger fields seems to be related with the change of
δ pressure. For DDRH models, DDRH
this may explain the transition density behaviour. How-
two models start to differ at the saturation density and
the Fig. 1 it is seen that the symmetry energy for these
β ρL
are almost coincident and only above
B 5
ρL
δ
3
NL
for NLW models and below B∗ < 3 × 105, it is the model including δ which has the smallest pressure, about half the value calculated with NLδ.

For B∗ = 5 × 105, the transition density is above the
saturation density, ρ ∼ 0.16 – 0.17 fm–3 for NLW models
and to ρ ∼ 0.19 – 0.23 fm–3 for DDRH models. For this
high field both models with the δ meson predict larger transition densities, pressures and proton fractions due to
the stiffness of the symmetry energy at those densities.

| B∗ (fm–3) | Model | ρ0 CROSS (fm–3) | Yp | Pm (MeV fm–3) |
|-----------|-------|----------------|-----|--------------|
| 5 × 104   | NLρ  | 0.079          | 0.079 | 0.570      |
|           | NLδ  | 0.079          | 0.075 | 0.558      |
|           | TW   | 0.078          | 0.097 | 0.606      |
|           | DDRHδ | 0.080         | 0.098 | 0.498      |
| 105       | NLρ  | 0.093          | 0.152 | 1.274      |
|           | NLδ  | 0.094          | 0.149 | 1.329      |
|           | TW   | 0.091          | 0.178 | 1.281      |
|           | DDRHδ | 0.088         | 0.183 | 1.108      |
| 2 × 105   | NLρ  | 0.091          | 0.153 | 1.206      |
|           | NLδ  | 0.108          | 0.260 | 1.819      |
|           | TW   | 0.092          | 0.305 | 1.355      |
|           | DDRHδ | 0.088         | 0.313 | 1.198      |
| 5 × 105   | NLρ  | 0.134          | 0.434 | 1.693      |
|           | NLδ  | 0.143          | 0.442 | 2.245      |
|           | TW   | 0.149          | 0.488 | 2.772      |
|           | DDRHδ | 0.170         | 0.504 | 4.356      |

In Table III we show the same data given in Table II
but including the AMM in the calculation. The conclu-
sions are similar: for NLW models, the transition den-
sity and the corresponding pressure increase when the
δ-meson is included, whereas for DDRH models the inclu-
sion of the isovector-scalar meson decreases the transition density and the corresponding pressure for B∗ < 5 × 105.

In summary, the δ meson gives rise to a larger crust for
NLW models and, except to very large fields, to a smaller crust with DDRH models.

The eigenvector associated with the eigenvalue of the
free energy curvature matrix defines the direction of the
instability and tells us how does the system separate into
a dense liquid and a gas phase. It was shown in [13, 27]
that in the absence of the magnetic field the direction of
instability favors the reduction of the isospin asymmetry
of the dense clusters of the system, and increases the
isospin asymmetry of the gas surrounding the clusters,
the so called distillation effect. This effect is represented
in Fig. 5 where it is seen that for the \( B = 0 \) curve the
fraction \( \delta \rho_p / \delta \rho_n \) is larger than \( \rho_p / \rho_n \) below \( y_p = 0.5 \) and
the other way round above. In this figure we show,
respectively for NLW models (left) and DDRH models
(right), the fraction \( \delta \rho_p / \delta \rho_n \) as a function of \( y_p \) for a
fixed baryonic density, \( \rho = 0.06 \ \text{fm}^{-3} \), chosen inside the
instability region.

For NLW models and for the two largest fields consid-
ered the spinodal region contains a single Landau level
and the curve varies smoothly starting at \( \delta \rho_p / \delta \rho_n \sim 1.5 \)
for NL\( \rho \) and \( \sim 1.62 \) for NL\( \delta \). We point out the very
large value of this fraction, always above 1. The mag-
netic field favors a strong increase of the proton fraction.
For \( y_p > 0.5 \), NL\( \rho \) and NL\( \delta \) behave in a similar way,
while below this value the main difference is the larger
\( \delta \rho_p / \delta \rho_n \) for NL\( \delta \) corresponding to a stronger distillation
effect.

For \( B^* = 10^5 \) the spinodal has two bands, see Fig. 2,
 corresponding to the occupation of the first two Landau
levels. The transition from one to the other is clearly
seen with a large discontinuity of \( \delta \rho_p / \delta \rho_n \) at \( y_p \sim 0.7 \).
Above this \( y_p \) value the curve behaves like the previous
ones. However for \( y_p < 0.7 \) the behavior is quite differ-
ent: the curve decreases from the value at \( y_p = 0 \), which is
independent of the magnitude of the magnetic field, to a
value much smaller than the corresponding value of the
fraction \( \rho_p / \rho_n \). The same behaviour occurs for NL\( \rho \) and
NL\( \delta \). The fluctuations will not drive the system out of
the first Landau level and therefore the larger the pro-
ton fraction, the closer the system comes to the top of the
band and the smaller are the allowed proton fluctuations.
For \( y_p > 0.7 \) or for the larger magnetic fields the Landau
levels are only partially filled and the fluctuations will
never drive the system out of the corresponding Landau
level. In summary, the effect of the \( \delta \)-meson on the
instability region of NLW models is to reduce the strength
of the distillation effect of neutron-rich matter.

For DDRH models the inclusion of the isovector scalar
meson reduces the strength of the distillation effect,
which is not so dramatic in these models. Including the
AMM similar conclusions are drawn. The AMM favors
still larger proton fluctuations because neutron polariza-
tion stiffens the EOS.

In conclusion, we have studied the influence of the
isovector scalar meson on the instabilities of stellar mat-
ter under very strong magnetic fields. The fields consid-
ered are much stronger than the strongest field measured
at the surface of a magnetar which is \( B^* \sim 10^2 \) for SGR
1806-20 [5]. However, the magnetic fields in the interior
of neutrons stars could be larger and the present work
shows how fields of the order of \( B^* = 10^{18} - 10^{19} \) could
affect the inner crust of a compact star. According to
the scalar virial theorem [28] the maximum magnetic energy
could be comparable to the gravitational energy in an
equilibrium configuration, which would correspond to a
value of the order of \( \sim 10^{18} \ \text{G} \).

We have considered four relativistic nuclear models:
two models with constant couplings (NL\( \rho \) and NL\( \delta \)) and
two models with density dependent couplings (TW and
DDRH\( \delta \)). Two of the models include the \( \delta \)-meson. For
all the models, we have determined the spinodal surface
from the curvature matrix of the free energy for different
magnitudes of the magnetic field. It had already been
shown [12] that the instability region could be divided in
to several bands according to magnitude of the magnetic
field and the number of the Landau levels occupied and
that the presence of the magnetic field would generally
increase the instability region.

We have seen that the inclusion of the \( \delta \) meson in-
creases (reduces) the size of the thermodynamic insta-
Bility zone for very neutron-rich matter for the NLW
(DDRH) models when compared with the spinodal ob-
tained without the \( \delta \)-meson. These results reflect them-
selves on the extension of the crust of a compact star
under a strong magnetic field.

By making a rough estimation of the transition den-
sity at the inner crust of a compact star under a strong
magnetic field from the crossing of the EOS with the
thermodynamic spinodal, we have shown that the transi-
tion density and associated pressure increases, for NLW
models, with the inclusion of the \( \delta \)-meson. On the other
hand, for DDRH models the influence of the \( \delta \)-meson is
to decrease the transition density and associated pressure
for fields \( B^* < 5 \times 10^5 \). DDRH\( \delta \) is predicting the small-
est transition densities, almost half the value obtained
for NL\( \delta \).

If we consider fields \( \sim 10^{18} \ \text{G} (B^* \sim 5 \times 10^4) \), as
indicated by the scalar virial theorem, we may take the
following conclusions: a) for conventional pulsars TW
would predict a larger transition pressure and, there-
fore, larger fractional moment of inertia in the crust
than all the other models, about twice as large; b) for
\( B^* \sim 5 \times 10^4 \), and taking into account AMM, TW would

FIG. 5: (Color online) \( \delta \rho_p / \delta \rho_n \) plotted as a function of
the proton fraction with \( \rho = 0.06 \ \text{fm}^{-3} \) for the NLW models (NL\( \rho \)
(thin lines) and NL\( \delta \) (thick lines)) (left) and DDRH models
(TW (thin lines) and DDRH\( \delta \) (thick lines)) (right) and for
several values of the magnetic fields without AMM. The frac-
tion \( \rho_p / \rho_n \) is given by the thin dotted line.
predict just a small increase of the fractional moment of inertia in the crust, ∼ 15%, the NLW models predict an increase of 120% and 220%, respectively, with and without δ-meson, while DDRHδ would predict an increase of 80%. If the AMM are not considered, within TW the fractional moment of inertia in the crust would be smaller than for conventional pulsars, while for DDRHδ there would be a slight increase of 20% and for the NLW models an increase of ∼ 100%. The different behaviours in magnetars and conventional pulsars predicted by the different models might be a possibility to impose stronger constraints on the EOS of nuclear matter.

We have also investigated the direction of instability. If the first Landau level is only partially occupied the density fluctuations are such that the system evolves for a state with dense clusters very proton rich immersed in a proton poor gas. A larger proton fraction is favored energetically due to the degeneracy of the Landau levels. The δ meson will only reduce slightly the strength of this distillation effect for DDRH models and increase it for NLW models. For particles occupying an almost complete Landau level, proton fluctuations are smaller or forbidden and an anti-distillation effect results with a decrease of the proton fraction of the dense clusters.

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[1] V. V. Usov, Nature 357, 472 (1992).
[2] B. Paczyński, Acta Astron. 42, 145 (1992).
[3] Christopher Thompson and Robert C. Duncan, Astrophys. J. L9, 392 (1992).
[4] A. K. Harding and D. Lai, Rep. Prog. Phys. 69, 2631 (2006).
[5] SGR/APX online Catalogue, http://www.physics.mcgill.ca/~pulsar/magnetar/main.html
[6] C. Kouveliotou, S. Dieter, T. Strohmayer, J. van Paradijs, G.J. Fishman, C.A. Meegan, K. Hurley, Nature 393, 235 (1998).
[7] Christopher Thompson and Robert C. Duncan, Astrophys. J. 408, 194 (1993).
[8] D. G. Ravenhall and C. J. Pethick, Astrophys. J. 424, 846 (1994).
[9] Bennett Link, Richard I. Epstein, and James M. Lattimer, Phys. Rev. Lett. 83, 3362 (1999).
[10] Victoria M. Kaspi, Astrophys. Space Sci. 308, 1 (2007).
[11] Rim Dib, Victoria M. Kaspi, and Fotis Gavriil, Astrophys. J. 673, 1044 (2008).
[12] A. Rabhi, C Providência, and J. da Providência, Phys. Rev C 79, 015804 (2009).
[13] Camille Ducoin, Constança Providência, Alexandre M. Santos, Lucília Brito, Philippe Chomaz, Phys. Rev. C 78, 055801 (2008).
[14] S. Kubis and M. Kutschera, Phys. Lett. B399, 191 (1997).
[15] B. Liu, V. Greco, V. Baran, M. Colonna, and M. Di Toro, Phys. Rev. C 65, 045801 (2002).
[16] D. P. Menezes and C. Providência, Phys. Rev. C 70, 058801 (2004).
[17] S. Typel and H. H. Wolter, Nucl. Phys. A656, 331 (1999).
[18] T. Gaitanos, M. Di Toro, S. Typel, V. Baran, C. Fuchs, V. Greco, and H. H. Wolter, Nucl. Phys. A732, 24 (2004).
[19] A. Rabhi, C Providência, and J. da Providência, J. Phys. G: Nucl. Part. Phys. 35,125201 (2008).
[20] C. Fuchs, H. Lenske, and H. H. Wolter, Phys. Rev. C 52, 3043 (1995).
[21] S. S. Avancini, L. Brito, D. P. Menezes, and C. Providência, Phys. Rev. C 70, 015203 (2004).
[22] H. Müller and B. D. Serot, Phys. Rev. C 52, 2072 (1995).
[23] J. Margueron and P. Chomaz, Phys. Rev. C 67, 041602(R) (2003).
[24] S.S. Avancini, L. P. Brito, J.R. Marinelli, D.P. Menezes, M.M.W. Moraes, C. Providência and A. M. Santos, Phys. Rev. C 79, 035804 (2009).
[25] S.S. Avancini, D.P. Menezes, M.D. Alloy, J.R. Marinelli, M.M.W. Moraes and C. Providência, Phys. Rev. C 78, 015802 (2008).
[26] J. Xu, L.W. Chen, B.A. Li and H.R. Ma, arXiv:0807.4477v1 [nucl-th].
[27] S. S. Avancini, L. Brito, Ph. Chomaz, D. P. Menezes, and C. Providência, Phys. Rev. C 74, 024317 (2006).
[28] Dong Lai and Stuart L. Shapiro, Astrophys. J. 383, 745 (1991).