COLLABORATIVE ENVIRONMENTAL MANAGEMENT FOR TRANSBOUNDARY AIR POLLUTION PROBLEMS: A DIFFERENTIAL LEVIES GAME

DAVID W. K. YEUNG*  
SRS Consortium for Advanced Study in Dynamic Cooperative Games  
Hong Kong Shue Yan University, Hong Kong  
Center of Game Theory  
St Petersburg State University, St Petersburg, 198904, Russia

YINGXUAN ZHANG  
SRS Consortium for Advanced Study in Dynamic Cooperative Games  
Hong Kong Shue Yan University, Hong Kong  
Decision Sciences and Modelling Program  
Victoria University, Australia

HONGTAO BAI  
College of Environmental Science and Engineering  
Nankai University, China  
MOE Key Laboratory of Pollution Processes and Environmental Criteria  
Nankai University, China

SARDAR M. N. ISLAM  
Decision Sciences and Modelling Program  
Victoria University, Australia

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Abstract. This paper develops a new cooperative dynamic time consistent model for studying regional air pollution management issues in a cooperative game framework for formulating pollution control policies and dynamically consistent compensation mechanisms. As air pollution is a transboundary issue, unilateral response on the part of one region is generally ineffective. Regional cooperation is essential to resolve serious environmental problems. In addition, the long-term environmental impacts are closely related to the building up existing air pollution stocks in Sulfur Dioxide (SO2), Nitrogen Dioxide (NO2),

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* Corresponding author: David W. K. Yeung.
Respirable suspended particulates (RSP) and Ozone (O3). A cooperative dynamic game with different types of pollutants is developed. We characterize the non-cooperative outcomes, and examine the cooperative arrangements, group optimal actions, and individually rational imputations. In particular, an air pollution levy consisting of four components involving damage charges on emissions of sulfur dioxide, nitrogen dioxide, respirable suspended particulates and ozone depletion materials. Cooperative games offer the possibility of socially optimal and group efficient solutions to the lack of cooperation among different regions involving decision problems among strategic actors. This paper makes a valuable contribution to the literature as this is the first cooperative dynamic time consistent model for regional management of different types of air pollutants.

1. Introduction. Air pollution is a transboundary issue. Unilateral response on the part of one region is often ineffective (e.g. Kaitala, et al. 1991, 1992a and 1992b). While regional collaboration is the key to resolve the serious environmental problems, there are various hurdles in cooperation among regions (Finus, et al., 2017). Existing international and regional joint initiatives could hardly be expected to offer any long-term solution, because there is no guarantee that an original cooperative agreement would be effective throughout the duration of collaboration (Yeung, et al., 2017). In particular, there is no assurance that the participants will always be better off or will remain in the cooperative agreement over the relevant time period, as economic conditions change over time.

There is an abundance of studies in transboundary air pollution abatement. As early as the 1970s, Lakshmanan and Lo (1972) described the development and demonstration of an operational regional economic model for the assessment of economy-wide effects of air pollution abatement strategies in ninety-one major metropolitan areas in the United States. Måler (1989) formulated an acid rain game, considering the case of sulfur emissions, using a full information approach. Other contributions of simulations carried out under the assumption of full information can be found in Kaitala et al. (1991, 1992a and 1992b). Tahvonen et al. (1993) analyzed cost effectiveness of environmental cooperation, i.e. sulfur abatement between Finland and the former Soviet Union. Caplan and Silva (1999) presented a two-stage game to analyze Federal environmental policy which was designed to control acid rain. Nagase and Silva (2007) studied transboundary pollution between China and Japan, and examined simultaneous and sequential non-cooperative games to illustrate the shortcomings of decentralized policy making. Other contributions can be found in Tulkens (1979), Kaitala et al. (1995) and so on.

Cooperative games suggest the possibility of socially optimal and group efficient solutions to environmental management problems, involving strategic interests and actions. Cooperative games in environmental issues were studied by Dockner and Long (1993), Jorgensen and Zaccour (2001), Fredj et al. (2004), Breton et al. (2005 and 2006), Petrosyan and Zaccour (2003), Yeung (2007), Germaina et al. (2010), Yaron and Ratner (1990), Dinar and Wolf (1994), Yeung (2008), Yeung and Petrosyan (2005), Courtois and Tazdait (2014), Vasin and Gusev (2012) and Vasin (2014). Inter-regional environmental collaboration can be found in Yeung (2007, 2008 and 2014), Yeung and Petrosyan (2008a), Yeung and Zhang (2012), and Yeung et al. (2017).

Cooperative dynamic game models in environmental management with dynamically consistent solution have been provided in Yeung (2014), Yeung and Petrosyan (2012) and Yeung et al. (2017). In particular, it is possible to construct a cooperative time consistent solution of a cooperative dynamic game model where every
party would commit to from the beginning until the end, any proposed arrangement, must guarantee that every participant will be better-off and the originally agreed upon arrangement remains effective at any time within the cooperative period for any feasible state brought about by prior optimal behavior.

In this paper, following the analytical framework of Yeung (2007), Yeung and Petrosyan (2008a) and Yeung (2014), we present a game model of transboundary air pollution problem. The long-term environmental impacts are caused by the building up existing air pollution stocks in Sulfur Dioxide (SO$_2$), Nitrogen Dioxide (NO$_2$), Respirable suspended particulates (RSP) and Ozone (O$_3$). Our objective is to characterize the non-cooperative outcomes, and examine the cooperative arrangements, group optimal actions, and individually rational imputations. In particular, an air pollution levy consisting of four components involving damage charges on emissions of sulfur dioxide, nitrogen dioxide, respirable suspended particulates and ozone depletion materials.

The paper is organized as follows. Following the introduction in Section 1, in Section 2, this paper will first identify the problems in environmental cooperation to be resolved. Exploration of the need for dynamical stability in environmental cooperation is presented in Section 3. Section 4 provides the game formulation. Policy implementation and dynamically consistent compensation are derived and scrutinized in Section 5. Concluding remarks are presented in Section 6.

2. Problems in transboundary Air pollution control. Any ecological damage and environmental pollution of the ecosystem can have an impact not only on a local area, but on the ecosystem as an integral whole. Dynamical systems of the environment can be found in Weber et al. (2008) and Weber et al. (2009). Air pollution dispenses and transfers in a complex way. Air pollutants emitted from a region cause damages to the environment and the society not only in the vicinity of that region, but also in distant areas. To understand the situation of air pollution in a region, one should consider the emissions, both from the region itself and from the neighboring regions, i.e. the spillover.

The fragmentation of cross-border environmental governance makes it impossible to internalize external issues of current transboundary pollution, while governance efficiency is low. Transboundary collaborative environmental management refers to some collaborative activities to meet the environmental interests of all or most of the members within a geographical space (Hu, 2010). These activities are led by different stakeholders and would finally benefit these stakeholders. However, the environmental costs are considered “external” and how to allocate the environmental costs among the stakeholders is an important issue. More discussion about environmental costs can be found in Paksoy and Ozceylan (2014) and Paksoy et al. (2012). Moreover, Usta et al. (2018) showed that cooperative game theory helps to define a fair cost allocation.

Worldwide, extensive research and practice on collaborative environmental management have been carried out, and a great deal of theoretical and practical experience have been accumulated, e.g. Prager (2015a and 2015b); Westerink et al. (2017); Zhang et al. (2018); Zaldivar-Jimenez et al. (2017); Liu et al. (2018); Hildebrand et al. (2002) and so on. The design and practice of inter-regional cooperation systems and mechanisms have been continuously improved, and have shown the following characteristics: 1) cooperation is government-led and often mandatory; 2) cooperation is usually an external demand-oriented crisis response; 3) the different
levels of political economy and social development of various participants lead to differences in environmental goals.

The collection of environmental taxes has always been considered to be the most effective method to solve the external problems of transboundary environmental pollution. Markusen (1975) applied static model to analyze transboundary pollution, and suggested that punitive tariffs should be levied on the relevant products of importing countries that caused the pollution; while Copeland (1996) believed that the importer of a commodity should also bear the pollution caused by the commodity. Antoci et al. (2012) proposed a taxonomy of different structural changes on the basis of distributive, environmental and economic outcomes. They studied a two-sector model with environmental externalities to identify under which conditions each structural change could occur. Lera-Lopez et al. (2012) examined the willingness-to-pay of people living in a number of villages in Navarre, in the Spanish Pyrenees to reduce noise and air pollution. Other studies can be found in Chander and Tulkens (1995 and 1997).

There are five main problems in transboundary collaborative environmental management: first, the main structure of collaboration is irrational; second, the integration of collaborative governance resources did not do well; third, the collaborative governance consultation mechanism is imperfect; fourth, the collaborative governance system and legal protection is not well organized and fifth, the process of collaborative governance is not systematic and orderly. With regards to the restrictions of cross-border environmental governance in transboundary air pollution control, we propose to construct a collaborative environmental management model. In such a mode, spillover effect of air pollutions from other regions will be considered; environmental cost will be incorporated; environmental tax will be calibrated; and payoff allocated to a region under cooperation will be computed to ensure that participants will always remain in the cooperative agreement over the relevant time period.

3. Dynamically stable cooperation and the notion of consistency. In dynamic cooperation, an important condition on cooperation and agreement is required for achieving dynamically stable cooperation over time. In the solution of dynamic games, the agreed upon optimality principle must remain optimal throughout the cooperation period. This condition is known as dynamic stability or time consistency. In other words, dynamic stability of solutions to any cooperative scheme involved the property that, as cooperation proceeds, the same optimality principle is maintained at each stage, and hence the participants do not possess incentives to deviate from the previously adopted optimal behavior (Yeung and Petrosyan, 2006a and 2006b; Yeung, 2008). In particular, it ensures that: (i) the continuation of the agreed-upon policy to a later stage would remain optimal; (ii) all participants have no incentive to deviate from the initial plan.

Existing inter-regional joint initiatives in environmental collaboration can hardly be expected to offer any long-term solution. Policy initiatives may be well-intentioned and plans may be innovative, but there is no guarantee that participants will always be better off within the entire duration of the agreement. There are several dynamically stable cooperative schemes in the literature (see Yeung and Petrosyan (2008b). However, an interesting dynamically stable cooperative scheme concerning inter-regional environmental collaboration can be found in Yeung (2007, 2008 and 2014), Yeung and Petrosyan (2008a and 2008b) and Li (2014).
4.1. Pollution dynamics and payoffs. Suppose that in a jurisdiction (a country or a pact of neighbouring countries) there are regions. Each region’s economic growth creates long-term environmental impacts by building up existing air pollution stocks in Sulfur Dioxide (SO$_2$), Nitrogen Dioxide (NO$_2$), Respirable suspended particulates (RSP) and Ozone (O$_3$). In addition, air pollutants emitted from a region cause damages to the environment and the society, not only in the vicinity of that region, but also in distant areas. Air pollutants are in the trajectory of pollutants dispersion. To understand the situation of air pollution in a region, one should consider the emissions, both from the region itself and from the neighboring regions.

For notational convenience, we designate $\{SO_2, NO_2, RSP, O_3\}$ by $\{1,2,3,4\}$. We use $x_i^t$ to denote the level of sulfur dioxide at stage $t$, $x_i^t$ to denote the level of nitrogen dioxide, $x_i^t$ to denote the level of respirable suspended particulates, and $x_i^t$ to denote the ozone level. The dynamics of pollution accumulation of Sulfur Dioxide (SO$_2$), Nitrogen Dioxide (NO$_2$), Respirable suspended particulates (RSP) and Ozone (O$_3$) are governed by the following dynamic equations:

$$x_{t+1}^h = x_t^h + \sum_{i \in N} a_i^{i(h)} q_i^t - \sum_{i=1}^{N} b_i^{i(h)} u_i^{i(h)}(x_t^h)^{1/2} - \delta_h x_t^h, \quad x_1^h = x_1^0, \quad (1)$$

for $h \in \{1,2,3,4\}$, $t \in \{1,2, ..., T\}$, and $i \in N$, where $a_i^{i(h)}$ is the amount of air pollutant $h$ created by a unit of region $i$’s output, $q_i^t$ denote the output of region $i$, $u_i^{i(h)}$ is the pollution abatement effort of region $i$, $b_i^{i(h)}$ is the amount of pollution removed by $u_i^{i(h)}$ units of abatement effort of region $i$, $\delta_h$ is the natural rate of decay of the pollutant $h$.

The profit of region $i$’s industrial sector in stage $t$ can be expressed as

$$\pi_i^t = [\alpha_i^t - \sum_{j \in N} \beta_j^i q_j^t - \beta_i^i q_i^t] q_i^t - c_i^t q_i^t - v_i^t q_i^t, \quad (2)$$

where $[\alpha_i^t - \sum_{j \in N} \beta_j^i q_j^t - \beta_i^i q_i^t] q_i^t$ is economic revenue from industrial production, $c_i^t$ is the unit cost of production, and $v_i^t$ is the pollution levy.

The industrial sector of region $i$ seeks to maximize (2). The first order condition for a market equilibrium in stage $t$ yields

$$\alpha_i^t - \sum_{j \in N} \beta_j^i q_j^t - \beta_i^i q_i^t = c_i^t + v_i^t, \quad (3)$$

which shows that the industrial sectors will produce up to a point where marginal revenue (the left-hand side of the equations) equals the cost plus tax of a unit of output produced (the right-hand-side of the equations).

The governments of each region have to promote business interests and at the same time bear the costs brought about by pollution. In particular, each government...
maximizes the net gains in the industrial sector plus tax revenues minus the sum of expenditures on pollution abatement and damages from pollution. The payoff of region \( i \) at stage \( t \) can be expressed as:

\[
[\alpha^i_t - \sum_{j \in N} \beta^i_j q^i_t q^j_t - c^i q^i_t - \sum_{h=1}^4 RC^i_t[u^i(h)]^2 - \sum_{h=1}^4 EC^i_t x^i_t],
\]

where the cost of \( u^i(h) \) units of pollutant reduction effort is \( RC^i_t[u^i(h)]^2 \), and the damage (cost) of \( x^i_t \) amount of pollution to region \( i \) is \( EC^i_t x^i_t \), with \( RC^i_t \) and \( EC^i_t \) being positive numbers.

The regions’ planning horizon involves \( T \) stages. Region \( i \in N \) seeks to maximize the sum of its instantaneous objective specified in (3) over the planning horizon,

\[
q^i_t, u^i(1), u^i(2), u^i(3), u^i(4),
\]

\[
\max \left\{ \sum_{t=1}^{T} \left[ \alpha^i_t - \sum_{j \in N} \beta^i_j q^i_t q^j_t - c^i q^i_t - \sum_{h=1}^4 RC^i_t[u^i(h)]^2 - \sum_{h=1}^4 EC^i_t x^i_t \right] \right\}
\]

subject to pollution dynamics (1), where \( r \) is the interest rate and \( g^i(\hat{Q} + Q^{i(1)}x^1_{T+1} + Q^{i(2)}x^2_{T+1} + Q^{i(3)}x^3_{T+1} + Q^{i(4)}x^4_{T+1}) \) is the terminal payoff of region \( i \) in stage \( T + 1 \).

In particular, \( Q^{i(1)}, Q^{i(2)}, Q^{i(3)} \) and \( Q^{i(4)} \) are negative reflecting the damage or penalty that region \( i \) has to bear when the game finishes.

4.2. Non-cooperative outcomes. We will first consider the non-cooperative game where each region acts on its own with the behaviour of the other region taken as given. For notational simplicity, we denote the vector \((x^1_t, x^2_t, x^3_t, x^4_t)\) by \( x_t \). A feedback Nash equilibrium solution can be characterized by the following theorem.

**Theorem 4.1.** 4.1 A set of strategies \( \{q^*_i = \phi^i_i(x), u^*_i(h) = v^i_i(h)(x)\} \) for \( i \in N \), \( h \in \{1, 2, 3, 4\} \), and \( t \in \{1, 2, ..., T\} \), provides a feedback Nash equilibrium solution to the game (1) and (4) if there exist functions \( V^i(t, x) : R \to R \), \( i \in N \), and \( t \in \{1, 2, ..., T\} \), such that the following recursive relations are satisfied:

\[
V^i(T+1, x) = g^i(\hat{Q} + Q^{i(1)}x^1_{T+1} + Q^{i(2)}x^2_{T+1} + Q^{i(3)}x^3_{T+1} + Q^{i(4)}x^4_{T+1}) \left( \frac{1}{1+r} \right)^T,
\]

\[
V^i(t, x) = q^i_t, u^i(1), u^i(2), u^i(3), u^i(4),
\]

\[
\max \left\{ \alpha^i_t - \sum_{j \in N} \beta^i_j q^i_t - \sum_{j \neq i} \beta^i_j \phi^i_j(x) q^j_t - c^i q^i_t - \sum_{h=1}^4 RC^i_t[u^i(h)]^2 \right\},
\]
for $t \in \{1, 2, ..., T\}$, where $x_{t+1}^h = x_t^h + A_t^{i(h)} q_t^i + \sum_{j \in N} a_t^{j(h)} \phi_t^j(x) - u_t^{i(h)} - \sum_{j \notin i} b_t^{i(h)} v_t^{j(h)}(x)^{1/2} - \delta^h x_t^h$, for $h \in \{1, 2, 3, 4\}$.

**Proof.** The value function $V_t^i(t, x)$ satisfies the optimality conditions in dynamic programming given the optimal strategies of the other $n - 1$ regions, for all $i \in N$. Hence a feedback Nash equilibrium results. \hfill \Box

Performing the indicated maximization in (6) yields:

$$a_t^i - \sum_{j \in N} \beta_t^j \phi_t^j(x) - \beta_t^i \phi_t^i(x) = c^i - \sum_{h=1}^4 a_t^{i(h)} \frac{\partial V_t^i(t + 1, x_{t+1})}{\partial x_t^h} (1 + r)^{t-1},$$

and

$$v_t^{i(h)}(x) = -\frac{b_t^{i(h)}}{2RC_t^{i(h)}} \frac{\partial V_t^i(t + 1, x_{t+1})}{\partial x_t^h} (1 + r)^{t-1} (x^h)^{1/2}.$$

Recalling the market equilibrium condition in (3), a pollution levy equaling

$$v_t^i = -\sum_{h=1}^4 a_t^{i(h)} \frac{\partial V_t^i(t + 1, x_{t+1})}{\partial x_t^h} (1 + r)^{t-1}.$$

have to be imposed in the industrial sector of region $i$ so that the producers would produce up to point of the regional optimal condition in (7). Worth-noting is that the levy in (9) has four components – levies on Sulfur Dioxide ($SO_2$), Nitrogen Dioxide ($NO_2$), Respirable suspended particulates (RSP) and Ozone ($O_3$). The game equilibrium payoffs of the regions can be obtained as:

**Proposition 1.** 4.1 The game equilibrium payoffs of the region $i \in N$ can be obtained as:

$$V_t^i(t, x) = (A_t^{i(1)} x_t^1 + A_t^{i(2)} x_t^2 + A_t^{i(3)} x_t^3 + A_t^{i(4)} x_t^4 + C_t^i) \left(\frac{1}{1 + r}\right)^{t-1},$$

where $A_{T+1}^{i(1)} = Q_t^{i(1)}$, $A_{T+1}^{i(2)} = Q_t^{i(2)}$, $A_{T+1}^{i(3)} = Q_t^{i(3)}$, $A_{T+1}^{i(4)} = Q_t^{i(4)}$ and $C_{T+1}^i = \hat{Q}^i$;

$$A_t^{i(h)} = -\frac{[b_t^{i(h)} A_{t+1}^{i(h)} (1 + r)^{-1}]^2}{4RC_t^{i(h)}},$$

and

$$A_t^{i(h)} \left[1 - \delta^h + \sum_{j \in N} \frac{[b_t^{i(h)}]^2}{2RC_t^{i(h)} A_{t+1}^{i(h)} (1 + r)^{-1}}\right],$$

for $h \in \{1, 2, 3, 4\}$ and $t \in \{1, 2, ..., T\}$; and $C_t^i$ is a constant term involving the model parameters and the values of $A_t^{i(h)}$, for $j \in N$, which have been determined through the backward induction process in the preceding stage $t + 1$ in system (11).
Proof. Invoking Proposition 4.1 and (8), one can obtain
\[ u_t^i(x) = -\frac{h_t^{i(h)}}{2RC_t^{i(h)}}A_t^{i(h)}(1 + r)^{-1}(x^h)^{1/2}. \]

Substituting (12) into (6) of Theorem 4.1, one can obtain a system of equations with both the left-hand-side and the right-hand-side are linear functions in \((x_t^1, x_t^2, x_t^3, x_t^4)\). In particular, the left-hand-side is \((A_t^{i(1)}x_t^1 + A_t^{i(2)}x_t^2 + A_t^{i(3)}x_t^3 + A_t^{i(4)}x_t^4 + C_t^i)\left(\frac{1}{1 + r}\right)^{t-1}\). One can obtain (11) from system (6). Hence Proposition 4.1 follows.

Given that \(Q^{i(1)}, Q^{i(2)}, Q^{i(3)}\) and \(Q^{i(4)}\) in stage \(T + 1\) are negative, system (11) would generate negative values of \(A_t^{i(h)}\), for \(i \in N, h \in \{1, 2, 3, 4\}\) and \(t \in \{1, 2, ..., T\}\). A regional pollution levy consisting of four component levies on pollution \(h\) can obtained explicitly as:
\[ u_t^i = -\sum_{h=1}^{4} a_t^{i(h)} A_{t+1}^{i(h)} (1 + r)^{-t-1}. \]

4.3. Cooperative arrangements in pollution control. Now we consider the case when all the regions want to cooperate in air pollution reduction and agree to act so that an optimum solution could be achieved. For the cooperative scheme to be upheld throughout the game horizon, both group rationality and individual rationality are required to be satisfied at any time based on the following two reasons:

1) group optimality ensures that all potential gains from cooperation are captured;
2) individual rationality is required to hold so that the payoff allocated to a region under cooperation will be no less than its non-cooperative payoff. If any of the regions refuses to act accordingly at any time within the game horizon, cooperation will cease. Failure to guarantee individual rationality leads to a condition where concerned participants would deviate from the agreed upon solution.

4.3.1. Group Optimality and Cooperative State Trajectory. Consider the case when all the regions agree to act cooperatively so that the joint payoff will be maximized. To maximize the joint payoff, we have to solve the following optimal control problem:

\[
\begin{align*}
&\max \left\{ \sum_{i=1}^{N} \sum_{t=1}^{T} \left[ \left( \alpha_t^i - \sum_{j \in N} \beta_{ij}^t q_t^j \right) q_t^i - \sum_{h=1}^{4} R_t^{i(h)} [u_t^{i(h)}]^2 - \sum_{h=1}^{4} E_t^{i(h)} x_t^{i(h)} \right] \\
&\quad \left( \frac{1}{1 + r} \right)^{(t-1)}, \right. \\
&\quad \left. + \sum_{i=1}^{N} g_t^{i} (Q_t^{i(1)} x_t^{i(1)} + Q_t^{i(2)} x_t^{i(2)} + Q_t^{i(3)} x_t^{i(3)} + Q_t^{i(4)} x_t^{i(4)}) \left( \frac{1}{1 + r} \right)^{T} \right\}
\end{align*}
\]  

subject to pollution dynamics (1).

A solution to the optimal control problem can be characterized by the following theorem.
Theorem 4.2. A set of strategies \( \{q_i, u_{i(t)}^{(1)}, u_{i(t)}^{(2)}, u_{i(t)}^{(3)}, u_{i(t)}^{(4)}\} \), \( i \in N, h \in \{1, 2, 3, 4\} \) and \( t \in \{1, 2, ..., T\} \), provides an optimal solution to the control problem (1) and (14) if there exist functions \( W(t, x) : R \rightarrow R^4 \), and \( t \in \{1, 2, ..., T\} \), such that the following recursive relations are satisfied:

\[
W(T+1, x) = \sum_{i=1}^{N} q_i^{(T)} x_{T+1}^{4} + Q^{(1)} x_{T+1}^{3} + Q^{(2)} x_{T+1}^{2} + Q^{(3)} x_{T+1}^{1} + Q^{(4)} x_{T+1}^{0} \left( \frac{1}{1+r} \right)^T
\]

\[
W(t, x) = \sum_{i=1}^{N} q_i^{(t)} x_{t+1}^{4} + Q^{(1)} x_{t+1}^{3} + Q^{(2)} x_{t+1}^{2} + Q^{(3)} x_{t+1}^{1} + Q^{(4)} x_{t+1}^{0} \left( \frac{1}{1+r} \right)^{t-1}
\]

\[
\max \left\{ \sum_{i=1}^{N} \left( \alpha_i^{(t)} - \sum_{j \in N} \beta_j^{(t)} q_j^{(t)} x_{t+1}^{4} + \sum_{h=1}^{4} RC_t^{(h)} u_{i(t)}^{(h)}(x_{t+1})^h \right) + W(t+1, x_{t+1}) \right\}
\]

for \( t \in \{1, 2, ..., T\} \), where \( x_{t+1}^{4} = x_{t}^{4} + \sum_{j \in N} a_{j}^{(t)} q_j^{(t)} x_{t}^{4} - \sum_{h=1}^{4} b_{j}^{(t)} u_{j}^{(h)}(x_{t}^{h})^1/2 - \delta h x_{t}^{h} \), for \( h \in \{1, 2, 3, 4\} \).

Proof. The value function \( W(t, x) \) satisfies the optimality conditions in dynamic programming. Hence Theorem 4.2 follows.

Performing the indicated maximization in (16) yields:

\[
\alpha_i^{(t)} - \sum_{j \in N} \beta_j^{(t)} q_j^{(t)} x_{t+1}^{4} + \sum_{h=1}^{4} a_{i(t)}^{(h)} \frac{\partial W(t+1, x_{t+1})}{\partial x_{t+1}^{h}} (1+r)^{t-1} = c_i^{(t)} - \sum_{h=1}^{4} a_{i(t)}^{(h)} \frac{\partial W(t+1, x_{t+1})}{\partial x_{t+1}^{h}} (1+r)^{t-1},
\]

and

\[
\varphi_{i(t)}^{(h)}(x) = - \frac{b_{i(t)}^{(h)}}{2RC_t^{(h)}} \frac{\partial W(t+1, x_{t+1})}{\partial x_{t+1}^{h}} (1+r)^{t-1} (x_{t}^{h})^1/2.
\]

for \( i \in N \) and \( h \in \{1, 2, 3, 4\} \).

Recalling the market equilibrium in (3), a pollution levy equaling

\[
v_{i}^{(t)} = - \sum_{h=1}^{4} a_{i(t)}^{(h)} \frac{\partial W(t+1, x_{t+1})}{\partial x_{t+1}^{h}} (1+r)^{t-1}.
\]

have to be imposed in the industrial sector of region \( i \) so that the producers would produce up to point of the optimal condition under cooperation in (17). Again, the levy in (19) has four components – levies on pollutant \( h \).

The cooperative payoffs of all the participating regions can be obtained as:

**Proposition 2.** 4.2. The cooperative payoffs of all the participating regions can be obtained as:

\[
W(t, x) = (A_1^{(1)} x_1^{4} + A_2^{(2)} x_2^{4} + A_3^{(3)} x_3^{4} + A_4^{(4)} x_4^{4} + C_t) \left( \frac{1}{1+r} \right)^{t-1},
\]

where \( A_1^{(1)}, A_2^{(2)}, A_3^{(3)}, A_4^{(4)}, C_t \) are constants specific to each region.
where \( A_{T+1}^{(1)} = \sum_{i=1}^{N} Q_{i(1)}^{(1)} \), \( A_{T+1}^{(2)} = \sum_{i=1}^{N} Q_{i(2)}^{(2)} \), \( A_{T+1}^{(3)} = \sum_{i=1}^{N} Q_{i(3)}^{(3)} \), \( A_{T+1}^{(4)} = \sum_{i=1}^{N} Q_{i(4)}^{(4)} \) and 
\[
C_{T+1} = \sum_{i=1}^{N} Q_i^T; \text{ and } \] 
\[
A_t^{(h)} = -\frac{N}{2RC_t^{i(h)}} \sum_{i=1}^{N} \left[ b_t^{i(h)} A_{i+1}^{(h)} (1 + r)^{-1} \right]^2 - \frac{N}{2RC_t^{j(h)}} \sum_{i=1}^{N} \left( 1 - \delta^h \right) + \sum_{j=1}^{n} \frac{b_t^{j(h)}}{2RC_t^{j(h)}} A_{i+1}^{(h)} (1 + r)^{-1} , \] (21)

for \( h \in \{1, 2, 3, 4\} \) and \( t \in \{1, 2, ..., T\} \); and \( C_t \) is a constant term involving the model parameters and the values of \( A_{i+1}^{(h)} \), for \( h \in \{1, 2, 3, 4\} \), which have been determined through the backward induction process in the preceding stage \( t+1 \) in system (21).

Proof. Invoking Proposition 4.2 and (18), one can obtain the group optimal pollution abatement efforts

\[
\varpi_t^{i(h)}(x) = -\frac{b_t^{i(h)}}{2RC_t^{i(h)}} A_{i+1}^{(h)} (1 + r)^{-1} (x^h)^{1/2}, h \in \{1, 2, 3, 4\}. \] (22)

Substituting (22) into (16) of Theorem 4.2, one can obtain a system of equations with both the left-hand-side and the right-hand-side are linear functions in \((x_t^1, x_t^2, x_t^3, x_t^4)\). In particular, the left-hand-side is \((A_t^{(1)} x_t^1 + A_t^{(2)} x_t^2 + A_t^{(3)} x_t^3 + A_t^{(4)} x_t^4 + C_t) \left( \frac{1}{1 + r} \right)^{t-1} \). One can obtain (21) from system (16). In addition, Using (21), all \( A_t^{(h)} \) for \( t \in \{1, 2, ..., T\} \) are obtained backwardly starting from stage \( T + 1 \). Hence Proposition 4.2 follows.

Given that \( \sum_{i=1}^{N} Q_i^{(h)} \), for \( h \in \{1, 2, 3, 4\} \), in stage \( T + 1 \) are negative, system (21) would generate negative values of \( A_t^{(h)} \), for \( i \in N \), \( h \in \{1, 2, 3, 4\} \), and \( t \in \{1, 2, ..., T\} \). A group optimal pollution levy consisting of four component levies on \( h \) can obtained explicitly as:

\[
v_t^i = -\sum_{h=1}^{4} a_t^{i(h)} A_{i+1}^{(h)} (1 + r)^{t-1} . \] (23)

Using Proposition 4.2, equation (17) can be expressed as:

\[
\alpha_t^i - \sum_{j \in N} \beta_t^j \psi_t^j(x) - \beta_t^i \psi_t^i(x) = c^i - \sum_{h=1}^{4} a_t^{i(h)} A_{i+1}^{(h)} (1 + r)^{-1} . \] (24)

System (24) is a linear equation system in \( \psi_t^j(x) \), for \( j \in N \). Therefore, \( \psi_t^j(x) \) in stage \( t \) can be solved as a function \( A_{t+1}^{(h)} \), for \( h \in \{1, 2, 3, 4\} \), which is independent of \( x \). Substituting \( \psi_t^j(x) \) and \( \varpi_t^{j(h)}(x) \) from (22) into dynamics (1) we obtain the dynamics of the cooperative trajectory of the pollution as:

\[
x_{t+1}^h = x_t^h + \sum_{j \in N} a_t^{(h(j))} \psi_t^j(x_t) + \sum_{j=1}^{N} \frac{b_t^{(j(h))}}{2RC_t^{(h(j))}} A_{t+1}^{(h)} (1 + r)^{-1} x^h - \delta^h x_t^h . \] (25)
The dynamics in (25) is first order linear difference equation which can be readily solved by standard techniques. We use \( \{x_i^t\}_{t=1}^T = \{x_i^{(1)*}, x_i^{(2)*}, x_i^{(3)*}, x_i^{(4)*}\}_{t=1}^T \) to denote the solution path of (25).

4.3.2. Individual Rationality and Subgame Consistent Imputation. In a cooperation framework, individual rationality has to be maintained at every instant of time within the cooperative duration, otherwise players will deviate from the cooperation arrangement and abandon their cooperative strategies. Individual rationality along the cooperative trajectory requires that the cooperative payoff allocated to a region under cooperation will be no less than its non-cooperative payoff, that is

\[
\xi^i(t, x_i^t) \geq V^i(t, x_i^t)
\]

for along all \( t \in \{1, 2, ..., T\} \).

Since the regions are asymmetric and the number of regions may be large, a reasonable optimality principle for gain distribution is to share the gain from cooperation proportionally to each respective region’s relative size of non-cooperative payoffs. To offer a long-term solution for cooperation, firstly, there has to be a guarantee that the participating regions will always be better off within the entire duration of the agreement. To create a cooperative solution that every region would commit to from the beginning to the end, the proposed arrangement must remain optimal throughout the cooperation period. This is the ‘classic’ game-theoretic problem of dynamical (or subgame) consistency (see Yeung and Petrosyan (2010) and (2012)). A dynamically consistent compensation satisfying the share of cooperation gain proportionally to the region’s relative size of non-cooperative payoffs will be

\[
\xi^i(t, x_i^t) = \frac{V^i(t, x_i^t)}{\sum_{j \in N} \xi^j(t, x_j^t)} W(t, x_i^t)
\]

for \( i \in N \), in all stages \( t \in \{1, 2, ..., T\} \).

Finally, for group optimality to be maintained, the total payoffs received by all regions under cooperation must equals the total cooperative payoff, that is \( \sum_{j \in N} \xi^j (t, x_i^t) = W(t, x_i^t) \).

5. Policy implementation and dynamically consistent compensation. Using the above analysis as a policy guide, a coalition of regions should be formed to pursue a comprehensive cooperative scheme of air pollution abatement. First, to induce the industrial sector to produce up to the group optimal level, a pollution levy including four components levies on pollutant \( h \) equaling

\[
v_i^h = - \sum_{h=1}^{4} a_i^{(h)} A_i^{(h)} (1 + r)^{t-1} .
\]

for \( i \in N \), has to be imposed in the industrial sector of region \( i \).

The differential levy in (27) provides a measurement of the sum of damages from the increments in the four pollutants caused by a unit of region \( i \)’s output. Worth-noting is that the levy allows the differentiation of each type of damage. For instance, if region \( i \)’s output does not contribute to the building up of sulfur dioxide, then \( a_i^{(1)} = 0 \) and the levy in (27) would not include any levy \( SO_2 \) component.

In cooperative pollution abatement, region \( i \) will put up the group optimal abatement efforts in (22) to reduce sulfur dioxide, nitrogen dioxide, respirable suspended particulates and ozone depletion materials.
Finally, a dynamically consistent compensation plan will be designed so that the agreed upon subgame consistent imputation in (26) can be achieved. Following Yeung and Petrosyan (2010), we let 

\[ B_i^t(x_i^t) \]

denote the payment that region \( i \) will receive at stage \( t \) under the cooperative agreement given the state \( x_i^t \) at stage \( t \). Therefore, we have

\[ \xi^i(t, x_i^t) = \sum_{\zeta=t}^T B_i^\zeta(x_i^\zeta) \left( \frac{1}{1 + r} \right)^{\zeta-t} \]

and dynamically consistent compensation mechanisms are derived. It is the first study on managing different pollutants in a cooperative game framework and therefore this paper makes a valuable contribution to the literature.

Theorem 5.1. A payment given to nation \( i \in N \) at stage \( t \in \{1, 2, \ldots, T \} \) equaling

\[ B_i^t(x_i^t) = (1 + r)^{t-1}[\xi^i(t, x_i^t) - \xi^i(t + 1, x_{i+1}^t)], \]  

for \( i \in N \), would lead to the realization of the subgame consistent imputation (26), where \( x_{i+1}^t = (x_{i+1}^t, x_{i+1}^t, x_{i+1}^t, x_{i+1}^t) \) and

\[ x_{i+1}^h = x_i^h + \sum_{j \in N} (\sum_{j \in N} [b_i^j(x_i^j)]^2 + \sum_{j \in N} \frac{[b_i^j(x_i^j)]^2}{2RC_i^j} + \sum_{j \in N} \frac{[b_i^j(x_i^j)]^2}{2RC_i^j} A_i^{j}(1 + r)^{-1} x_i^h - \delta_i x_i^h, \]

\[ t \in \{1, 2, \ldots, T \}. \]

Proof. From (28) one can obtain

\[ B_i^t(x_i^t)(\frac{1}{1 + r})^{t-1} = \xi^i(t, x_i^t) - \xi^i(t + 1, x_{i+1}^t). \]  

Hence,

\[ B_i^t(x_i^t) = (1 + r)^{t-1}[\xi^i(t, x_i^t) - \xi^i(t + 1, x_{i+1}^t)]. \]  

for \( i \in N \). \( \square \)

Theorem 5.1 implies that the cooperative payoff for each region is always maintained to be proportional to the relative size of each region’s non-cooperative GDP net of environmental cost throughout the cooperation period. With equation (31) being held, each region will obtain an addition payoff (proportional to each region’s non-cooperative status) in each of the collaborating years and hence would have no incentive to leave the plan. A compensation scheme will be used to distribute the cooperative payoff so region \( i \in N \) will receive a payment of in \( B_i^t(x_i^t) \) stage \( t \in \{1, 2, \ldots, T \}. \)

6. Concluding remarks. In this paper, we present a cooperative game of trans-boundary air pollution problem with different types pollutants. Non-cooperative outcomes have been characterized; the cooperative arrangements, group optimal actions, and individual rationality imputations have been examined. Policy implementation and dynamically consistent compensation mechanisms are derived. It is the first study on managing different pollutants in a cooperative game framework and therefore this paper makes a valuable contribution to the literature.

The new model constructed in this paper is expected to provide practical policy choices for environmental collaboration schemes and transboundary air pollution problems. It will facilitate the exploration of hitherto intractable problems in
transboundary air pollution control issues and establish time consistent long term solution plans for solving transboundary air pollution problems

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E-mail address: dwkyeung@hksyu.edu
E-mail address: yxzhang@hksyu.edu
E-mail address: baiht@nankai.edu.cn
E-mail address: Sardar.Islam@vu.edu.au