A NEW METHOD FOR CALCULATING ARRIVAL DISTRIBUTION OF ULTRA–HIGH-ENERGY COSMIC RAYS ABOVE $10^{19}$ eV WITH MODIFICATIONS BY THE GALACTIC MAGNETIC FIELD

HIROYUKI YOSHIGUCHI,1 SHIGEHIRO NAGATAKI,1,2 AND KATSUHIKO SATO1,2

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ABSTRACT

We present a new method for calculating arrival distribution of ultra–high-energy cosmic rays (UHECRs), including modifications by the Galactic magnetic field. We perform numerical simulations of UHE anti-protons, which are injected isotropically at the Earth, in the Galaxy and record the directions of velocities at the Earth and outside the Galaxy for all of the trajectories. We then select some of them so that the resultant mapping of the velocity directions outside the Galaxy of the selected trajectories corresponds to a given source location scenario, applying Liouville’s theorem. We also consider energy-loss processes of UHE protons in the intergalactic space. Applying this method to the source location scenario that is adopted in our recent study and can explain the Akeno Giant Air Shower Array (AGASA) observation above $4 \times 10^{19}$ eV, we calculate the arrival distribution of UHECRs, including lower energy ($E > 10^{19}$ eV) ones. We find that our source model can reproduce the large-scale isotropy and the small-scale anisotropy on UHECR arrival distribution above $10^{19}$ eV observed by the AGASA. We also demonstrate the UHECR arrival distribution above $10^{19}$ eV, with the event number expected by future experiments in the next few years. The interesting feature of the resultant arrival distribution is the arrangement of the clustered events in the order of their energies, reflecting the directions of the Galactic magnetic field. This is also pointed out by Alvarez-Muniz, Engel, & Stanev. This feature will allow us to obtain some kind of information about the composition of UHECRs and the magnetic field with increasing amount of data.

Subject headings: cosmic rays — galaxies: general — ISM: magnetic fields — large-scale structure of universe — methods: numerical

On-line material: color figures

1. INTRODUCTION

There is no statistically significant large-scale anisotropy in the observed arrival distribution of ultra–high-energy cosmic rays (UHECRs) above $10^{19}$ eV (Takeda et al. 1999). This may imply an extragalactic origin of cosmic rays above $10^{19}$ eV, combined with the change of spectral slope of the observed energy spectrum at $\sim 10^{19}$ eV (Bird et al. 1994; Yoshida et al. 1995; Takeda et al. 1998). Another important feature of the UHECR arrival distribution is the small-scale clusterings of the arrival directions (Takeda et al. 1999, 2001). The current Akeno Giant Air Shower Array (AGASA) data set of 57 events above $4 \times 10^{19}$ eV contains four doublets and one triplet within a separation angle of 2°5. The chance probability to observe such clusters under an isotropic distribution is only about 1% (Hayashida et al. 1999; Takeda et al. 2001).

On the other hand, the cosmic-ray energy spectrum does not show the Greisen-Zatsepin-Kuz’min (GZK) cutoff (Greisen 1966; Zatsepin & Kuz’min 1966) because of photopion production with the photons of the cosmic microwave background (CMB) and extends above $10^{20}$ eV (Takeda et al. 1998). The discrepancy between the AGASA and the High-Resolution Fly’s Eye (HiRes; Wilkinson et al. 1999), which reports the cosmic ray flux with the GZK cutoff around $10^{20}$ eV (Abu-Zayyad et al. 2002), remains one of the major open questions in astroparticle physics. This issue is left for future investigation by new large-aperture detectors now under development such as the South and North Auger project (Capelle, Cronin Parente, & Zas 1998), the EUSO (Benson & Linsley 1982), and the OWL (Cline & Stecker 2000) experiments.

In our recent work (Yoshiguchi et al. 2003a, hereafter Paper I), we perform numerical simulations for propagation of UHE protons in intergalactic space and examine whether the present AGASA observation above $4 \times 10^{19}$ eV can be explained by a bottom-up scenario in which the source distribution of UHECRs is proportional to that of galaxies. We use the Optical Redshift Survey (ORS; Santiago et al. 1995) to construct realistic source models of UHECRs.

In Paper I, we calculate both the energy spectrum and arrival directions of UHE protons and compare the results with the AGASA observation above $4 \times 10^{19}$ eV. We find that the large-scale isotropy and the small-scale anisotropy of the UHECR arrival distribution observed by the AGASA can be reproduced when $\sim 1/50$ of the ORS sample more luminous than $\sim 20.5$ mag are selected as UHECR sources, in the case of weak extragalactic magnetic field (EGMF; $B \sim 1$ nG). In terms of the source number density, this constraint corresponds to $\sim 10^{-6}$ Mpc$^{-3}$.

The small-scale anisotropy cannot be well reproduced in the case of strong EGMF ($B \geq 10$ nG) because the correlation at small scale between events that originate from a single source is eliminated, or the correlation continues to larger angle scale due to large deflection when UHECRs propagate in the EGMF from sources to the Earth.

1 Department of Physics, School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan; hiroyuki@utap.phys.s.u-tokyo.ac.jp.
2 Research Center for the Early Universe, School of Science, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113-0033, Japan.
Although Isola & Sigl (2002) and Sigl, Miniati, & Ensslin (2003) conclude that the expected small-scale anisotropy and large-scale isotropy for local enhancement of UHECR sources in the Local Super Cluster (LSC) in the presence of the strong EGMF (~1 μG) are in marginal agreement with the AGASA, the consistency is somewhat worse than that predicted by our scenario for $B = 1$ nG. Of course, we cannot draw any firm conclusion about the strength of the EGMF, considering the current limited amount of data. However, we assume extremely weak EGMF throughout the paper.

If local enhancement of UHECR sources in the LSC (Isola & Sigl 2002; Sigl, Miniati, & Ensslin 2003) is disfavored from the observations, there is no way to explain the observed extension of the cosmic-ray spectrum beyond the GZK cutoff. Our conclusion in Paper I is that a large fraction of cosmic rays above $10^{20}$ eV observed by the AGASA experiment might originate in the top-down scenarios, or that the energy spectrum measured by the HiRes experiment might be better.

As mentioned above, we obtain the constraint on the source number density as $\sim 10^{-6}$ Mpc$^{-3}$ by comparing our model prediction with the AGASA data only above $4 \times 10^{19}$ eV. It is very important to examine whether our source model can explain the AGASA data, including the lower energy ($\sim 10^{19}$ eV) one. On the other hand, the arrival directions of UHECRs above $10^{19}$ eV are modified by the Galactic magnetic field (GMF) by a few $\sim 10^\circ$. In order to accurately calculate the expected UHECR arrival distribution and compare with the observations, the effect of the GMF should be taken into account.

The first step of the studies of UHECR propagation in the GMF is found in Alvarez-Muniz, Engel, & Stanev (2002). Alvarez-Muniz et al. (2002) calculate the expected arrival distribution of UHECRs above $10^{19.3}$ eV for several source location scenarios. They perform numerical simulations of UHECR propagation in the Galaxy injected from sources toward the Earth. The radius of the Earth (detector) must be small enough that the unavoidable smearing in arrival angle is kept smaller than the accuracy of arrival direction determination $\sim$ a few degrees (Takeda et al. 2001). In this case, the number fraction of injected UHECRs arriving at the Earth is very small. This requires a large number of particles to be propagated, which takes enormous CPU time.

In this paper, we present a new method for calculating UHECR arrival distribution that can be applied to several source location scenarios, including modifications by the GMF. We numerically calculate the propagation of antiprotons from the Earth toward the outside of the Galaxy (in this study, we set a sphere centered around the Galactic center with radius $r_{\text{src}} = 40$ kpc as the boundary condition), including the effects of Lorentz force due to the GMF. The antiprotons are ejected isotropically from the Earth. By this calculation, we can obtain the trajectories and the sky map position of antiprotons that have reached to the boundary at radius $r_{\text{src}} = 40$ kpc.

Next, we regard the obtained trajectories as the ones of protons from the outside of the Galaxy toward the Earth. Also, we regard the obtained sky map position of antiprotons at the boundary as relative probability distribution (per steradian) for protons to be able to reach the Earth for the case in which the flux of the UHE protons from the extragalactic region is isotropic. (In this study, this flux correponds to the one at the boundary $r_{\text{src}} = 40$ kpc.) This treatment is supported by Liouville’s theorem. When the flux of the UHE protons at the boundary is anisotropic (e.g., the source distribution is not isotropic), this effect can be included by multiplying this effect (i.e., by multiplying the probability density of arrival direction of UHE protons from the extragalactic region at the boundary) to the obtained relative probability density distribution mentioned above.

By adopting this new method, we can consider only the trajectories of protons that arrive on the Earth, which, of course, helps us to save the CPU time efficiently and makes calculation of propagation of CRs, even with low energies ($\sim 10^{19}$ eV), possible within a reasonable time. We also consider the energy-loss processes of UHE protons in the intergalactic space, which is not taken into account by Alvarez-Muniz et al. (2002).

With this method, we calculate the UHECR arrival distribution above $10^{19}$ eV for our source scenario, which can explain the current AGASA observation above $10^{19.6}$ eV. Using the harmonic amplitude and the two-point correlation function as statistical quantities, we compare our model prediction with the AGASA observation. We also demonstrate the arrival distribution of UHECRs with the event number expected by future experiments such as the South and North Auger project (Capelle et al. 1998), the EUSO (Benson & Linsley 1982), and the OWL (Cline & Stecker 2000) experiments.

In § 2, we introduce the GMF model. We explain the method for calculating UHECR arrival distribution in § 3. Results are shown in § 4. In § 5, we summarize the main results.

2. GALACTIC MAGNETIC FIELD

In this study, we adopt the GMF model used by Alvarez-Muniz et al. (2002), which is composed of the spiral and the dipole fields. In the following, we briefly introduce this GMF model.

Faraday rotation measurements indicate that the GMF in the disk of the Galaxy has a spiral structure with field reversals at the optical Galactic arms (Beck 2001). We use a bisymmetric spiral field (BSS) model, which is favored from recent work (Han, Manchester, & Qiao 1999; Han 2001). The solar system is located at a distance $r_\odot = 8.5$ kpc from the center of the Galaxy in the Galactic plane. The local regular magnetic field in the vicinity of the solar system is assumed to be $B_\text{GMF} = 1.5 \mu G$ in the direction $l = 90^\circ + p$, where the pitch angle is $p = -10^\circ$ (Han & Qiao 1994).

In the polar coordinates $(r_\parallel, \phi)$, the strength of the spiral field in the Galactic plane is given by

$$B(r_\parallel, \phi) = B_0 \left( \frac{r_\parallel}{r_0} \right) \cos \left( \phi - \beta \ln \frac{r_\parallel}{r_0} \right),$$

where $B_0 = 4.4 \mu G$, $r_0 = 10.55$ kpc, and $\beta = 1/\tan p = -5.67$. The field decreases with galactocentric distance as $1/r_\parallel$, and it is zero for $r_\parallel > 20$ kpc. In the region around the Galactic center ($r_\parallel < 4$ kpc), the field is highly uncertain, and thus assumed to be constant and equal to its value at $r_\parallel = 4$ kpc.

The spiral field strengths above and below the Galactic plane are taken to decrease exponentially with two scale
heights (Stanev 1996),
\[ |B(r||, \phi, z)| = |B(r||, \phi)| \left\{ \begin{array}{ll}
\exp(-z) & |z| \leq 0.5 \text{ kpc}, \\
\exp(-3/8 \cdot z) & |z| > 0.5 \text{ kpc},
\end{array} \right. \]

where the factor \( \exp(-3/8) \) makes the field continuous in \( z \). The BSS field we use is of even parity, i.e., the field direction is preserved at disk crossing.

Observations show that the field in the Galactic halo is much weaker than that in the disk. In this work, we assume that the regular field corresponds to an A0 dipole field as suggested in Han (2002). In spherical coordinates \((r, \theta, \phi)\), the \((x, y, z)\) components of the halo field are given by

\[ B_x = -3 \mu_G \sin \theta \cos \theta \cos \phi/r^3, \]
\[ B_y = -3 \mu_G \sin \theta \cos \theta \sin \phi/r^3, \]
\[ B_z = \mu_G (1 - 3 \cos^2 \theta)/r^3, \]

where \( \mu_G \sim 184.2 \mu G \text{ kpc}^3 \) is the magnetic moment of the Galactic dipole. The dipole field is very strong in the central region of the Galaxy but is only 0.3 \( \mu G \) in the vicinity of the solar system directed toward the north Galactic pole.

There is a significant turbulent component, \( B_{\text{ran}} \), of the Galactic magnetic field. Its field strength is difficult to measure and results found in literature are in the range of \( 0.5-2 B_{\text{reg}} \) (Beck 2001). However, we neglect the random field throughout the paper in order to make it easy to see the effects of the regular field, such as the arrangement of the clustered event in the order of their energies (§4.3). Possible dependence of the results on the random field is discussed in §4.2.

3. NUMERICAL METHOD

3.1. Propagation of UHECRs in the Intergalactic Space

The energy spectrum of UHECRs injected at extrasolar sources is modified by the energy-loss processes when they propagate in the intergalactic space. This subsection provides the method of Monte Carlo simulations for propagation of UHE protons in intergalactic space.

We assume that UHECRs are protons injected with a power-law spectrum within the range of \( 10^{19}-10^{22} \) eV. Ten thousand protons are injected in each of 31 energy bins, i.e., 10 bins per decade of energy. Then UHE protons are propagated, including the energy-loss processes (explained below) over 3 Gpc for 15 Gyr. We take a power-law index as 2.6 in order to fit the calculated energy spectrum to the one observed by the AGASA (De Marco, Blasi, & Olinto 2003).

UHE protons below \( \sim 8 \times 10^{19} \) eV lose their energies, mainly by pair creations and adiabatic energy losses, and above it, by photopion production (Berezinsky & Grigorieva 1988; Yoshida & Teshima 1993) in collisions with photons of the CMB. We treat the adiabatic loss as a continuous loss process. We calculate the redshift \( z \) of source at a given distance using the cosmological parameters \( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1}, \Omega_m = 0.27, \) and \( \Omega_L = 0.73. \) Similarly, the pair production can be treated as a continuous loss process, considering its small inelasticity \((\sim 10^{-4})\). We adopt the analytical fit functions given by Chodorowski, Zdziarske, & Sikora (1992) to calculate the energy-loss rate for the pair production on isotropic photons. The same approach has been adopted in our previous studies (Paper I; Ide et al. 2001; Yoshiguchi, Nagataki, & Sato 2003c).

On the other hand, protons lose a large fraction of their energy in the photopion production. For this reason, its treatment is very important. We use the interaction length and the energy distribution of final protons as a function of initial proton energy, which is calculated by simulating the photopion production with the event generator SOPHIA (Mucke et al. 2000).

In this study, we neglect the effect of the EGMF for the following two reasons. First, numerical simulations of UHECR propagation in the EGMF, including lower energy \((\sim 10^{19} \) eV\) ones, take a lot of CPU time. Second, we show in our previous study that small-scale clustering can be well reproduced in the case of weak EGMF \((B < 1 \text{ nG}; \text{Paper I})\). Isola & Sigl (2002) and Sigl, Miniati, & Ensslin (2003) show that the expected small-scale anisotropy for the local enhancement scenario of UHECR sources in the presence of strong EGMF \((\sim 1 \mu G)\) in the LSC is marginally consistent with the AGASA observation. However, the consistency of small-scale anisotropy is somewhat worse than that predicted by our scenario in the case of weak EGMF (Paper I). Although we cannot draw any firm conclusion because of the limited amount of data, we assume extremely weak EGMF throughout the paper.

3.2. Source Distribution

In this study, we apply the method for calculating the UHECR arrival distribution with modifications by the GMF (§3.3) to our source location scenario, which is constructed by using the ORS (Santiago et al. 1995) galaxy catalog. As mentioned in §1, we show in Paper I that the arrival distribution of UHECRs observed by the AGASA can be reproduced when \( \sim 1/50 \) of the ORS galaxies more luminous than \( M_{\text{lim}} = -20.5 \) are selected as UHECR sources. We consider only this source model throughout the paper. It is unknown how much an ultimate UHECR source contributes to the observed cosmic-ray flux. In Paper I, we thus consider the two cases in which (1) all galaxies inject the same amount of cosmic rays or (2) they inject cosmic rays proportionally to their absolute luminosity. However, we find that the results in the two cases do not differ from each other, as far as we focus on the luminous galaxies as UHECR sources.

Accordingly, we restrict ourselves to the case that all galaxies inject the same amount of cosmic rays.

In order to calculate the energy spectrum and the distribution of arrival directions of UHECRs realistically, there are two key elements of the galaxy sample to be corrected. First, galaxies in a given magnitude-limited sample are biased tracers of matter distribution because of the flux limit (Yoshiguchi et al. 2003b). Although the sample of galaxies more luminous than \( -20.5 \text{ mag} \) is complete within 80 \( h^{-1} \) Mpc (where \( h \) is the Hubble constant divided by 100 \text{ km s}^{-1}, \) and we use \( h = 0.71 \)), it does not contain galaxies outside it for the reason of the selection effect. We distribute sources of UHECRs outside 80 \( h^{-1} \) Mpc homogeneously. Their number density is set to be equal to that inside 80 \( h^{-1} \) Mpc.

We do not take into account luminosity evolution for simplicity.

Second, our ORS sample does not include galaxies in the zone of avoidance \((|b| < 20^\circ)\). In the same way, we
distribute UHECR sources in this region homogeneously and calculate its number density from the number of galaxies in the observed region.

3.3. Calculation of the UHECR Arrival Distribution with Modifications by the GMF

In this subsection, we present the method of calculation of UHECR arrival distribution with modifications by the GMF. We start by injecting antiprotons from the Earth isotropically and follow each trajectory until antiprotons reach a sphere of radius 40 kpc centered at the Galactic center or the total path length traveled by antiprotons is larger than 200 kpc.

By integrating the equations of motion in the magnetic field. It is noted that we regard these antiprotons as protons injected from the outside of the Galaxy toward the Earth. The number of propagated antiprotons is 2,000,000. We have checked that the number of trajectories that are stopped by the limit in the second case above is smaller than 0.1% of the total number. The energy loss of protons can be neglected for these distances. Accordingly, we inject antiprotons with an injection spectrum similar to the observed one $\sim E^{-2.7}$. (Note that this is not the energy spectrum injected at extragalactic sources.)

The result of the velocity directions of antiprotons at the sphere of radius 40 kpc is shown in the right panel of Figure 1 in the Galactic coordinate. From Liouville’s theorem, if the cosmic-ray flux outside the Galaxy is isotropic, one expects an isotropic flux at the Earth, even in the presence of the GMF. This theorem is confirmed by numerical calculations shown in Figure 6 of Alvarez-Muniz et al. (2002), which is the same figure as our Figure 1, except for threshold energy. Thus, the mapping of the velocity directions in the right panel of Figure 1 corresponds to the sources that actually give rise to the flux at the Earth in the case that the sources (including ones that do not actually give rise to the flux at the Earth) are distributed uniformly.

We calculate the UHECR arrival distribution for our source scenario using the numerical data of the propagation of UHE antiprotons in the Galaxy. In detail, the method is as follows. At first, we divide the sky into a number of bins with the same solid angle. The number of bins is taken to be $360(b) \times 200(b)$. We then distribute all the trajectories into each bin according to their directions of velocities (source directions) at the sphere of radius 40 kpc. Finally, we randomly select trajectories from each bin with probability $P_{sel}(j, k, E)$ defined as

$$P_{sel}(j, k, E) \propto \sum_i \frac{1}{d_i^2} \frac{dN/dE(d_i, E)}{E^{-2.7}}.$$  (4)

Here subscripts $j$ and $k$ distinguish each cell of the sky, $d_i$ is distance of each galaxy within the cell of $(j, k)$, and the summation runs over all of the galaxies within it; $E$ is the energy of proton, and $dN/dE(d_i, E)$ is the energy spectrum of protons at our Galaxy injected at a source of distance $d_i$.

The normalization of $P_{sel}(j, k, E)$ is determined so as to set the total number of events equal to a given number, e.g., the event number of the current AGASA data. When $P_{sel} > 1$, we newly generate events with a number of $(P_{sel} - 1) \times N(j, k)$, where $N(j, k)$ is the number of trajectories within the sky cell of $(j, k)$. The arrival angle of newly generated protons (equivalently, the injection angle of antiprotons) at the Earth is calculated by adding a normally distributed deviate with zero mean and variance equal to the experimental resolution $2^\circ/8 (1^\circ/8)$ for $E > 10^{19}$ eV (4 $\times$ 10$^{19}$ eV) to the original arrival angle. We perform this event generation 20 times in order to calculate the averages and variances due to the finite number of the simulated events of the statistical quantities ($\S$ 3.4).

3.4. Statistical Methods

In this subsection, we explain the two statistical quantities, the harmonics analysis for large-scale anisotropy (Hayashida et al. 1999a) and the two-point correlation function for small-scale anisotropy.

The harmonic analysis of the right ascension distribution of events is the conventional method to search for global anisotropy of cosmic-ray arrival distribution. For a ground-based detector like the AGASA, the almost uniform observation in right ascension is expected. The $m$th harmonic amplitude $r$ is determined by fitting the distribution to a sine wave with period $2\pi/m$. For a sample of $n$ measurements of phase, $\phi_1, \phi_2, \ldots, \phi_n \ (0 \leq \phi_i \leq 2\pi)$, it is expressed as

$$r = (a^2 + b^2)^{1/2},$$  (5)

where $a = (2/n)\sum_{i=1}^{n} \cos m\phi_i, \ b = (2/n)\sum_{i=1}^{n} \sin m\phi_i$. We calculate the harmonic amplitude for $m = 1, 2$ from a set of events generated by the method explained in $\S$ 3.3.

If events with total number $n$ are uniformly distributed in right ascension, the chance probability of observing the

![Fig. 1](image-url). Arrival directions of antiprotons with $E > 10^{19}$ eV at the sphere of galactocentric radius of 40 kpc (right) in Galactic coordinates. The antiprotons are injected at the Earth isotropically (left) with an injection spectrum $E^{-2.7}$. 

The current AGASA 775 events above $10^{19}$ eV are consistent with isotropic source distribution within a 90\% confidence level (Takeda et al. 2001). We therefore compare the harmonic amplitude for $P = 0.1$ with the model prediction.

The two-point correlation function $N(\theta)$ contains information on the small-scale anisotropy. We start from a set of generated events. For each event, we divide the sphere into concentric bins of angular size $\Delta \theta$ and count the number of events falling into each bin. We then divide it by the solid angle of the corresponding bin, i.e.,

$$N(\theta) = \frac{1}{2\pi} \cos \theta - \cos(\theta + \Delta \theta) \sum_{\theta < \phi < \phi + \Delta \phi} 1 \, \text{[sr]}^{-1},$$

where $\phi$ denotes the separation angle of the two events. The quantity $\Delta \theta$ is taken to be 1° in this analysis. The AGASA data show correlation at small angles ($\sim 3'$) with 2.3 (4.6) $\sigma$ significance of deviation from an isotropic distribution for $E > 10^{19}$ eV ($E > 4 \times 10^{19}$ eV; Takeda et al. 2001).

4. RESULTS

4.1. Arrival Distribution of UHECRs above $10^{19}$ eV

In this subsection, we present the results of the arrival distribution of UHECRs above $10^{19}$ eV using the method explained in § 3.3. At first, Figure 2 shows the distribution of the sources for a specific source selection when $\sim 1/50$ of the ORS galaxies more luminous than $M_{\text{lim}} = -20.5$ are randomly selected as UHECR sources in the Galactic coordinate. We show only the sources within 300 Mpc from us for clarity as circles of radius inversely proportional to their distances. It is noted that the sources outside 113 (= 80 $h^{-1}$) Mpc are randomly distributed because the ORS sample does not contain any galaxy outside it.

We show in Figure 3 the expected energy spectrum for the source model of Figure 2. The injection spectrum is set to be $E^{-2.6}$. The contributions from sources at different distances are also shown. We also show the observed cosmic-ray spectrum by the AGASA experiments (Hayashida et al. 1999b).

The resultant spectrum is in good agreement with the one observed by AGASA, except for $E > 10^{20}$ eV. As mentioned above, we conclude in Paper I that a large fraction of cosmic rays above $10^{20}$ eV observed by the AGASA experiment might originate in the top-down scenarios. Accordingly, we consider only cosmic rays with $E < 10^{20}$ eV throughout the paper.

As mentioned above, Alvarez-Muniz et al. (2002) do not take the energy-loss processes in the intergalactic space into account. Thus, they can not include the effects of difference between resultant energy spectra injected at different distances into numerical calculations. In our calculations, however, sources at larger distance mainly contribute to the cosmic-ray flux at lower energies, as is evident in Figure 3. This enables us to calculate the arrival distribution of UHECRs under more realistic situations.

Given the source distribution and the resultant energy spectrum as a function of the source distance, we can calculate the right-hand side of equation (4). Then we perform the selection of trajectories according to the probability $P_{\text{select}}$, as explained in § 3.3.

One realization of the event generations is shown in Figure 4. The events are shown by color, according to their energies. This figure corresponds to Figure 1; i.e., the injection directions of antiprotons at the Earth (Fig. 1, left) corresponds to the arrival directions of protons (Fig. 4, left). Similarly, the arrival directions of antiprotons at the sphere of galactocentric radius of 40 kpc (Fig. 1, right) go in the directions of the sources that actually give rise to the cosmic-ray flux (Fig. 4, right). For the source model of Figure 2, the nearest source is located at $(b, l) = (31^\circ, 284^\circ)$ and 64 Mpc from us. A number of the simulated events are clustered in this direction, as seen in Figure 4. Furthermore, these events are aligned in the sky according to the order of
their energies, reflecting the direction of the GMF at this direction. As we will show in the § 4.3, this interesting feature of the UHECR arrival distribution becomes evident with the increasing amount of the event number.

4.2. Statistics on the UHECR Arrival Distribution

In this subsection, we show the results of the statistical quantities on the UHECR arrival distribution above $10^{19}$ eV. In the last section, we showed the results for a specific source scenario when $1/50$ of the ORS galaxies more luminous than $M_{\text{lim}} = -20.5$ are randomly selected as UHECR sources. However, the statistical quantities presented in this section are calculated with not only the statistical error but also the variation between different selections of source from our ORS sample.

The upper panels of Figure 5 show the first and second harmonics predicted by our source model as a function of the cosmic-ray energies for $B_{\text{S}} = 1.5 \, \mu G$, where $B_{\text{S}}$ is the strength of the GMF in the vicinity of the solar system. It is noted that we calculate the harmonic amplitudes for the simulated events within only $-10^\circ \leq \delta \leq 80^\circ$ in order to compare with AGASA data. The error bars represent the statistical fluctuations due to the finite number of the simulated events, which is set to be equal to that observed by the AGASA (775 events for $E > 10^{19}$ eV). The event selections are performed 20 times. The shaded regions represent $1 \sigma$ total error due not only to the statistical error but also the source selections from our ORS sample. The random source selections are performed 100 times. The region below the histogram is expected values for the statistical fluctuation of isotropic source distribution with the chance probability larger than 10%.

It is clear that our source model predicts the large-scale isotropy fully consistent with that expected by uniform source distribution within $1 \sigma$ total error (statistical one plus source selection). We have checked that 27 source distributions out of 100 predict the sufficient large-scale isotropy within $1 \sigma$ statistical error. In order to investigate the effects of the GMF on the large-scale anisotropy, we also calculate the harmonic amplitude for the case of $B_{\text{S}} = 0.0 \, \mu G$. For $B_{\text{S}} = 0.0 \, \mu G$, the predicted arrival distribution is relatively more isotropic than that for $B_{\text{S}} = 1.5 \, \mu G$. We also note that this tendency can be seen at lower energies ($\sim 10^{19}$ eV). Because the deflection angle of cosmic rays with such energies by the GMF is about $\sim a \times 10^\circ$, the harmonic amplitude of the arrival distribution of UHECRs can be affected by anisotropy of the event distributions, which is caused by the events aligned according to the order of their energies, reflecting the direction of the GMF.

In Figure 6, we show a two-point correlation function predicted by our source model for $E > 4 \times 10^{19}$ (left) and $E > 10^{19}$ eV (right). It is noted that we calculate the two-point correlation function for the simulated events within only $-10^\circ \leq \delta \leq 80^\circ$ in order to compare with the AGASA data. The error bars represent the statistical fluctuations due to the finite number of the simulated events, which is set to be equal to that observed by the AGASA (775 events for $E > 10^{19}$ eV). The shaded regions represent $1 \sigma$ total error due not only to the statistical error but also the source selections from our ORS sample. The event selections and the random source selections are performed 20 and 100 times, respectively. The event numbers shown in this figure are averaged over all trials of the event selections and the source selections. The histograms represent the AGASA data. However, the AGASA data for $E > 10^{19}$ eV are fitted to the result of our calculation at a larger angle ($30^\circ$) since we cannot know the normalization of the AGASA data with this energy.

Obviously, the large peak at the small-angle scale is too strong compared with the AGASA observation (Takeda et al. 2001). We have checked that when extremely nearby sources are selected by accident, predicted small-scale anisotropy becomes very strong. This is the reason for a peak that is too large at small-angles scale in Figure 6, where the averages and the variances are calculated including such source distributions.

Provided that extremely nearby sources are selected by accident, not only the small-scale anisotropy but also the large-scale isotropy is inconsistent with the AGASA observation. Accordingly, we calculate the two-point correlation function only for the source distributions that predict the large-scale isotropy consistent with uniform source
distribution within 1 $\sigma$ statistical error. The number of such source distributions is 27 out of all the 100 source selections. The result is shown in Figure 7.

The two-point correlation function in Figure 7 exhibits a structure that is similar to that seen in the AGASA data, i.e., a large peak at small-angle scales followed by a tail at large angles. For $E > 10^{19}$ eV, a peak at small angles is somewhat weaker than that for $E > 4 \times 10^{19}$ eV because of the large deflection by the GMF. This feature is also seen in AGASA data (Takeda et al. 2001). From this result, it can be understood that the source distributions that predict sufficient large-scale isotropy also predict the small-scale anisotropy that is similar to that seen in the AGASA data.

However, we should note that the peak at small-angle scales is still relatively strong compared with the AGASA. There may be two possible explanations for this fact. First, we neglect the effects of the extragalactic magnetic field in this study in order to save CPU time. If we can include this effect in future studies, strong correlations at small scales will be reduced because of the deflection of UHECRs in intergalactic space. Second, we also neglect the random component of the GMF in order to make easy to see the effect of the regular field. This may also relax the large peak at small-angle scales, provided that there is the random component with same level of strength as the regular component. These issues are left for future investigations.

4.3. Future Prospects of UHECR Arrival Distribution

In this subsection, we demonstrate the results of the UHECR arrival distribution above $E > 10^{19}$ eV with the event number expected by future experiments, such as Auger, EUSO, and OWL. The results for the source model of Figure 2 are shown in Figure 8. The events are shown by color according to their energies. It is noted that the expected event rate by the Auger experiment is $\sim 3000$ per year above $10^{19}$ eV. These results are extended versions of our previous study (Yoshiguchi, Nagataki, & Sato 2003c), where we predict the UHECR arrival distribution above $4 \times 10^{19}$ eV without modifications by the GMF.

A remarkable feature is the arrangement of clustered events in the directions of nearby sources (see Fig. 2). The events are aligned according to the order of their energies reflecting the direction of the GMF. We will be able to obtain some kind of information about the GMF and the chemical composition of UHECRs. In forthcoming work,
we will study new statistical quantities that allow us to obtain such invaluable information.

5. SUMMARY AND DISCUSSION

In this paper, we presented a new method for calculating the arrival distribution of UHECRs, which can be applied to several source location scenarios, including modifications by the GMF. We performed numerical simulations of UHE antiprotons, which are injected isotropically at the Earth and in the Galaxy and we recorded the directions of velocities at the Earth and outside the Galaxy for all of the trajectories. It is noted that we regard these antiprotons as protons injected from the outside of the Galaxy toward the Earth. We then selected some trajectories so that the resultant mapping of the velocity directions outside the Galaxy of the selected trajectories corresponds to our source location scenario, applying Liouville's theorem.

There are two points of improvement in our work over the work of Alvarez-Muniz et al. (2002). First, we calculated only the trajectories actually reaching the detectors by propagating antiprotons backward from Earth instead of propagating protons from the source and selecting those reaching the Earth. This helps us to save CPU time and makes calculations of propagation of cosmic rays even with lower energies ($10^{19} \text{eV}$) possible within a reasonable time. Second, we considered energy-loss processes of UHE protons in intergalactic space, which is not taken into account in Alvarez-Muniz et al. (2002). This enables us to include the effects of difference between resultant energy spectra injected at different distances into numerical calculations. We can calculate the arrival distribution of UHECRs under more realistic conditions.

As an application of this method, we calculate the UHECR arrival distribution above $10^{19} \text{eV}$ for the source model that is adopted in our recent study (Paper I) and can explain the current AGASA observation above $4 \times 10^{19} \text{eV}$. We found that the predicted large-scale anisotropy is fully consistent with uniform source distribution in the same manner as the current AGASA data. In order to investigate the effects of the GMF on the large-scale anisotropy, we calculated the harmonic amplitude for the case of $B_{\text{GMF}} = 0.0 \mu G$. For $B_{\text{GMF}} = 0.0 \mu G$, the predicted arrival distribution is relatively more isotropic than that for $B_{\text{GMF}} = 1.5 \mu G$. This would be due to anisotropy of the event distributions caused by the events aligned according to the order of their energies, reflecting the directions of the GMF.

It is also found that the calculated two-point correlation function is similar to that of AGASA data when we restrict our attention to the source distributions that predict sufficient isotropic arrival distribution of UHECRs. There may be effects of the extragalactic magnetic field and the random component of the GMF on the large peak of the twopoint correlation function. These issues are left for future studies.

Finally, we demonstrated the UHECR arrival distribution above $10^{19} \text{eV}$ with the event number expected by future experiments in the next few years. The interesting feature of the resultant arrival distribution is the events aligned according to the order of their energies, reflecting the directions of the galactic magnetic field. This is also pointed out by Alvarez-Muniz et al. (2002). This feature will become clear with the increasing amount of data and allow us to obtain some kind of information about the composition of UHECRs in the future.
UHECRs and the GMF. In forthcoming work, we will study new statistical quantities that allow us to obtain such invaluable information.

In the present work, we calculate the arrival distribution of UHECRs for our source location scenario, which is adopted in our previous study (Paper I). However, it should be mentioned that the same results would be obtained if the sources were truly drawn at random, as far as the source number density is $C_2^{10}/C_0^{6} \text{Mpc}^{-3}$. The results, such as the events aligned according to the order of their energies, are independent of our assumptions about the source distribution.

In this study, we calculate the harmonic amplitude and two-point correlation function, which are the only published quantities on UHECRs observed by the AGASA including lower energy ones ($<10^{19}$ eV). In particular, the AGASA observation has published neither existence nor nonexistence of the events aligned according to the order of their energies. If more detailed event data with $E<_4^{10^{19}}$ eV are published, we may obtain a stronger constraint on our source model other than the source number density using another statistical quantities. This is also one of our future study plans.

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Fig. 8.—Arrival directions of protons with $E > 10^{19}$ eV at the Earth expected for the source model of Fig. 2 in Galactic coordinates. Events are shown by color according to their energies. It is noted that the expected event rate by the Auger experiment is $\sim 3000 \text{per year above } 10^{19}$ eV. [See the electronic edition of the Journal for a color version of this figure.]

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