Stress intensity factors of three parallel edge cracks under bending moments

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Abstract. This paper reports the study of stress intensity factors (SIF) of three edge cracks in a finite plate under bending moments. The goal of this paper was to analyze the three edge crack interactions under such loading. Several studies can be found in literature discussing on mode I SIF. However, most of these studies obtained the SIFs using tensile force. Lack of SIF reported discussing on the SIFs obtained under bending moments. ANSYS finite element program was used to develop the finite element model where singular elements were used to model the cracks. Different crack geometries and parameters were utilized in order to characterize the SIFs. According to the present results, crack geometries played a significant role in determining the SIFs and consequently induced the crack interaction mechanisms.

1. Introduction
Parallel edge cracks are one of the most common flaws especially in industrial failures [1]. Accurate stress analysis of these components in the presence of cracks and evaluation of fracture design parameters are needed for reliable estimation of the crack growth rates and residual fracture strength [2]. This paper discusses three parallel cracks in a sheet with finite width where the sheets are subjected to the bending moment. The crack will be appeared on the surface component which can reduce the component durability [3-8]. According to the literature survey [9], most of the work conducted to analyze the crack with the assumption that only a single crack can be used to represent such problems. In many cases the multiple cracks also can be carried out and must be studied to increase the safety factor of the component structure [10]. This paper discusses three parallel cracks in a sheet with finite width. This sheet is subjected to bending moment in the longitudinal direction remote from the cracks. Then, crack interactions are analyzed and discussed.

2. Displacement Extrapolation Method
The finite element method is an appropriate approach to calculate the stress intensity factor (SIF) for linear elastic fracture mechanics problems. In order to determine the SIFs, a displacement extrapolation method [11] is used in this study. Several other works have implemented a similar method are also available [12-13]. After obtaining the elastic finite element solution of the particular problem, nodal displacements between two crack faces are determined and used to compute the SIFs as follows.
\[ K_I = \frac{2G\sqrt{2\pi}}{1 + \kappa} \frac{|v_b - v_f|}{\sqrt{r}} = \frac{2G\sqrt{2\pi}}{1 + \kappa} \frac{|\Delta v|}{\sqrt{r}} \]  
(1)

\[ K_{II} = \frac{2G\sqrt{2\pi}}{1 + \kappa} \frac{|u_b - u_f|}{\sqrt{r}} = \frac{2G\sqrt{2\pi}}{1 + \kappa} \frac{|\Delta u|}{\sqrt{r}} \]  
(2)

\[ K_{III} = 2G\sqrt{2\pi} \frac{|w_b - w_f|}{\sqrt{r}} = 2G\sqrt{2\pi} \frac{|\Delta w|}{\sqrt{r}} \]  
(3)

where, \( K_I, K_{II} \) and \( K_{III} \) are the respective mode I, II and III SIFs, \( \Delta v, \Delta u \) and \( \Delta w \) are the relative nodal displacements between two crack faces in the direction of \( y \)-axis, \( x \)-axis and \( z \)-axis, respectively, and \( G \) is the modulus of rigidity. For plain strain condition, \( \kappa = 3 - 4\nu \), where, \( \nu \) is the Poisson’s ratio. All the SIFs obtained from the analysis are converted into normalized values in order to ensure the generality of the results. A normalized SIF, \( F \), can be defined as follows [14-15]

\[ F_{I,a} = \frac{K_{I,a}}{\sigma_a \sqrt{\pi a}} \]  
(4)

\[ F_{I,b} = \frac{K_{I,b}}{\sigma_b \sqrt{\pi a}} \]  
(5)

\[ F_{II} = \frac{K_{II}}{\tau_{xy} \sqrt{\pi a}} \]  
(6)

\[ F_{III} = \frac{K_{III}}{\tau_{xy} \sqrt{\pi a}} \]  
(7)

where, \( \sigma_a, \sigma_b \) and \( \tau_{xy} \) are the axial, bending and shear stresses, respectively and \( a \) is a crack depth.

3. Methodology and Model Validation

ANSYS commercial finite element (FE) analysis software is used to model the parallel edge cracks through the use of ANSYS Parametric Design Language (APDL) as in figure 1(a). Due to the symmetrical problem, half finite element model is sufficiently used to analyze the problem as in figure 1(b) where singular finite element is used around the crack tips as revealed in the enlarged area of Figure 1(b). Two important parameters are used in this work such as \( b/a \) and \( a/W \). Both parameters are in the range of 0.25 to 2.00 and 0.1 to 0.5, respectively. The calculation of stress intensity factor is based on the theory of displacement extrapolation method [10]. In order to validate the results, the present model must be compared with the previous results as produced by Jiang et al. [3, 4]. Unfortunately, there are unavailable stress intensity factors under bending moment, especially for multiple parallel edge cracks. Therefore, the SIFs obtained under tensile force are used for validation purposes. Figure 2(a) shows the meshed edge crack model subjected to axial stress and Figure 2(b) represents the stress distribution due to the crack interactions. Figure 5 shows the model validations where it is showed that the present model is well in agreement with the previous model. Therefore, it is assumed that the present model can be validated significantly using the results obtained from Jiang et al. [3, 4]. For the present analysis, it is assumed that plain strain finite element model is used where PLANE83 element type in ANSYS library is utilized. In order to bend the plate remotely, an
independent node is created on the top of the plate. Then, all nodes situated on the top edge of plate are connected to this newly created node. Bending moment is applied to this node and then consequently creating mode I crack mechanism as shown in figure 2(c). Due to symmetrical problem, stress intensity factor (SIF) for crack 3 is not accounted since it is similar to crack 1 (outer crack). Middle crack in this work is called crack 2. All the SIFs are normalized according to Eqs. (4)-(7). The normalized SIF is also called geometrical correction factor. It is important to normalize the SIFs for the sake of analysis generalization.

Figure 1(a). Parallel edge cracks in a finite plate.

Figure 1(b). Half symmetrical finite element model of parallel edge cracks with enlarged crack tip area.
4. Results and Discussion

Tables 1 and 2 list the geometrical correction factors for outer and central multiple edge cracks under bending moments, respectively. Figure 4 and 5 show the plot of SIF against $a/W$ when $b/a$ is varied for cracks 1 and 2, respectively when compared with non-interacting cracks. It is showed that the SIF increased when the cracks become deeper. These characteristics are widely reported in literature [3, 4]. Figure 4 shows the stress intensity factors (SIFs) of single edge crack under bending moment. It is found that the effects of crack eccentricities are played an important role in increasing the SIFs. However, the increment of SIFs occurred for the cracks situated within $b/a \leq 0.50$. For the cracks with $b/a \geq 2.0$, it is seemed that the SIFs decreased as a function of $a/W$ compared with central crack ($b/a = 0.0$). The decrement of SIFs is due to the fact when the location of crack is away from the central line of finite plate, bending moment or stress decreased.
Table 1. Geometrical correction factor, $F_{I,b,o}$ for outer edge multiple cracks.

| $b/a$ | 0.10  | 0.15  | 0.20  | 0.25  | 0.30  | 0.35  | 0.40  | 0.45  | 0.50  |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.25  | 0.7980| 0.8110| 0.8360| 0.8770| 0.9400| 1.0130| 1.1130| 1.2410| 1.4060|
| 0.50  | 0.8400| 0.8760| 0.9150| 0.9690| 1.0410| 1.1300| 1.2420| 1.3810| 1.5560|
| 1.00  | 0.9440| 0.9780| 1.0260| 1.0880| 1.1630| 1.2530| 1.3580| 1.4940| 1.6470|
| 2.00  | 1.0750| 1.1050| 1.1430| 1.1860| 1.2390| 1.3040| 1.3910| 1.4980| 1.6520|

Table 2. Geometrical correction factor, $F_{I,b,c}$ for central edge multiple cracks.

| $b/a$ | 0.10  | 0.15  | 0.20  | 0.25  | 0.30  | 0.35  | 0.40  | 0.45  | 0.50  |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 0.25  | 0.5290| 0.5290| 0.5410| 0.5680| 0.6000| 0.6780| 0.7710| 0.8990| 1.0750|
| 0.50  | 0.6000| 0.6280| 0.6620| 0.7180| 0.7990| 0.9080| 1.0490| 1.2310| 1.4420|
| 1.00  | 0.7600| 0.8010| 0.8640| 0.9620| 1.0720| 1.1930| 1.3260| 1.4780| 1.6440|
| 2.00  | 0.9950| 1.0540| 1.1180| 1.1790| 1.2390| 1.3060| 1.3930| 1.4980| 1.6540|

Figure 4. Stress intensity factors (SIFs) of single edge cracks under bending moment for crack 1.

Figure 5 reveals the SIFs distribution for multiple edge cracks subjected to bending moment for cracks 1 and 2, respectively. Figure 5(a) shows the SIFs of multiple edge cracks for crack 1. It is found that the SIF increased in the similar pattern due to the interactions and stress relaxations among the cracks. As the results, the SIFs for multiple cracks are lower than the offset single cracks. This is due to the stress field redistribution when the present of cracks. As expected, the SIFs of multiple cracks reduced when the cracks interacted to each others as in figure 5(b). It is also found that greater amount of interactions occurred when closer crack intervals are used. However, such interactions are diminished when the neighbouring crack is situated greater than $b/a \geq 2.0$.

5. Conclusion
This paper present the calculation of stress intensity factors of three parallel edge cracks in a finite plate subjected to remote bending moment. Several main conclusions can be drawn from this work are as follows:
(a) The results show that due to the presence of multiple crack edge cracks, the stress distribution is relaxed and therefore, the stress intensity factors for all cracks decreased.
When the distance between the cracks is increased, interaction is seemed to be diminished and it is can be neglected especially for swallow crack.

Figure 5. Stress intensity factors (SIFs) of multiple edge cracks under bending moment for (a) crack 1 and (b) crack 2.

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