Thermal instability in a ferrimagnetic resonator strongly coupled to a loop-gap microwave cavity

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We study nonlinear response of a ferrimagnetic sphere resonator (FSR) strongly coupled to a microwave loop gap resonator (LGR). The measured response in the regime of weak nonlinearity allows the extraction of the FSR Kerr coefficient and its cubic damping rate. We find that there is a certain range of driving parameters in which the system exhibits instability. In that range, self-sustained modulation of the reflected power off the system is generated. The instability is attributed to absorption-induced heating of the FSR above its Curie temperature.

I. INTRODUCTION

Ferromagnetic and ferrimagnetic resonators are widely employed in a variety of microwave (MW) devices, including narrow band oscillators, filters, and parametric amplifiers. These resonators exhibit a variety of intriguing physical effects, including Bose-Einstein condensation and magneto-optical coupling. Here we study a strongly coupled hybrid system composed of a loop gap resonator (LGR) integrated with a ferrimagnetic sphere resonator (FSR) made of yttrium iron garnet (YIG). We focus on the regime of nonlinear response. In section III below we explore the effect on nonlinear damping in the region of relatively weak microwave driving. An instability, which is observed with a much stronger driving, is reported in section IV below, and a theoretical model, which attributes the instability to a driving-induced heating, is presented.

Many nonlinear dynamical effects have been observed before in FSRs, including auto-oscillations, optical cooling, frequency mixing and bistability. The Suhl instability (of both first and second orders) has been observed with transverse microwave driving, whereas parallel pumping instability has been observed with longitudinal driving. Applications of nonlinearity for quantum data processing have been explored in.

Heating a YIG sphere from room temperature to 400K by microwave driving having power of 450mW has been reported in. At a Curie temperature given by \(T_c = 560\) K, YIG undergoes a phase transition between an ordered ferrimagnetic state (FS) and a disordered paramagnetic state (PS). Thermal instability was observed in a cavity magneto-mechanical system. Microwave oscillations induced by injecting spin-polarized current into a magnetic-multilayer structure have been reported in. Self-excited oscillations induced by ohmic heating in a \(Y_3Fe_5O_{12}/Pt\) bilayer nanowire have been investigated in. Imaging of heating induced by the spin Peltier effect has been demonstrated in.

![Fig. 1: FSR-LGR coupling: (a) A sketch of the FSR made of YIG having radius of \(R_e = 1\ mm\) that is integrated inside the aluminum cylindrical LGR having gap width of 0.3 mm. The sphere is held by ceramic ferrules (CFs). A sapphire wafer (labeled as S) is inserted into the gap to increase the capacitance. (b) The numerically calculated magnetic field energy density distribution (normalized with respect to the maximum value) corresponding to driving at the resonance frequency \(\omega_e/(2\pi) = 3.3\ GHz\). (c) A VNA reflectivity \(|S_{11}|^2\) measurement as a function of magnon frequency \(\omega_s\) (proportional to the externally applied magnetic field). The coupling coefficient \(g_{eff}\) is extracted from the theoretical fit (white dashed lines) following Eq. (2).](image)

II. LOOP GAP RESONATOR

With relatively low input power, the main mechanisms responsible for FSR nonlinear response are magnetic anisotropy and exchange interaction. Consider a MW cavity mode having angular frequency \(\omega_e\) and an integrated FSR having radius \(R_s\). It is assumed that the applied static magnetic field \(\mathbf{H}_0\) is parallel to the easy axis. In the Holstein-Primakoff approximation (which assumes that magnetization is nearly saturated), the Hamiltonian of the system \(\mathcal{H}_D\) is expressed as

\[
\hbar^{-1}\mathcal{H}_D = \omega_e N_e + \omega_s N_s + K_{eff} N_s^2 + g_{eff} (A_e^\dagger A_e + A_e A_e^\dagger) ,
\]

where \(N_e = A_e^\dagger A_e\) and \(N_s = A_e^\dagger A_s\) are cavity modes for FSR and Kittel mode number operators, \(\omega_s = \gamma_s H_s\) is the Kittel mode angular frequency, \(\gamma_s/(2\pi) = 27.98\ GHz T^{-1}\) is
the gyromagnetic ratio, $K_M = \hbar c^2 K_{c1} / (V_e M_s^2)$ is the anisotropy-induced Kerr frequency, $K_{c1}$ is the first-order anisotropy constant, $V_e = 4 \pi R_1^3 / 3$ is the volume of the sphere, $M_s$ is the saturation magnetization, and $g_{c\ell}$ is the cavity-FSR coupling coefficient. For YIG at room temperature, $M_s = 140$ kA/m and $K_{c1} = -610$ J/m$^3$, hence $K_M = -2.4 \times 10^{-8}$ Hz $\times (R_e / (100 \mu m))^{-3}$.

In the linear regime, where the Kerr nonlinearity can be disregarded, the Hamiltonian $H_\Delta$ can be diagonalized. The angular frequencies $\omega_\pm$ of the two hybrid photon-magnon eigen modes are given by [43]

$$\omega_\pm = \omega_e + \omega_s \pm \left( \frac{\omega_e - \omega_s}{2} \right)^2 + g_{c\ell}^2 \delta.$$

Both angular frequencies $\omega_\pm$ are positive provided that $g_{c\ell} < \sqrt{\omega_e \omega_s}$. Note that the super-radiance Dicke instability occurs in the ultra-strong coupling region where $g_{c\ell} > \sqrt{\omega_e \omega_s}$ [44]. In the rotating wave approximation (RWA) the Kerr coefficients $K_\pm$ of the hybrid modes having angular frequencies $\omega_\pm$ are given by Eqs. (A9) and (A10) of appendix A (see Eq. (A8)).

In the current experiment, we explore the response for a wide range of the MW input powers $P_p$. We find that the response is well described by the Hamiltonian $H_\Delta$ provided that $P_p$ is sufficiently small. However, with sufficiently high $P_p$, the FSR temperature $T$ may exceed the Curie temperature $T_c$ due to MW absorption-induced heating. We study the response of the FSR-LGR system to an injected monochromatic pump tone having a frequency close to resonance. The off reflected power is measured using a spectrum analyzer (SA). We find that there is a certain zone in the pump frequency - pump amplitude plane, in which the resonator exhibits limit-cycle (LC) response resulting in self-sustained modulation of the reflected power. The observed LC is attributed to thermal instability (TD) [43].

A MW cavity made of an LGR allows achieving a relatively large coupling coefficient $g_{c\ell}$ [46, 17]. The MW LGR schematically shown in Fig. (1a), is made of a hollow concentric aluminium tube having an inner and outer radii of $R_{LGR} = 1.7$ mm and 3 mm, respectively, and a height of $H_{LGR} = 12$ mm. A sapphire strip of 260 $\mu$m thickness has been inserted into the gap in order to increase its capacitance, which in turn reduces the frequency $f_e$ of the LGR fundamental mode [$f_e = \omega_e / (2\pi) = 3.3$ GHz with sapphire [48]. An FSR made of YIG having radius of $R_e = 1$ mm is held by two ferrules inside the LGR. The static magnetic field $H_s$ is applied perpendicularly to the LGR axis. The LGR-FSR coupled system has been encapsulated in a metallic rectangular shield made of aluminium. The cavity is weakly coupled to a loop antenna (LA).

The numerically calculated magnetic energy density distribution corresponding to the LGR fundamental mode is shown in Fig. (1b). The calculated density is homogeneous ($\approx 95\%$) over the FSR volume, and it is well confined inside the LGR inner volume. Note that for our device, the LGR inner volume, which is given by $\pi R_{LGR}^2 H_{LGR}$, is 4 orders of magnitude smaller than the volume $\lambda^3$, where $\lambda = c / f_e$ is the free space wavelength corresponding to the LGR frequency $f_e$, and $c$ is the speed of light in vacuum. Consequently, the coupling coefficient $g_{c\ell}$ can be made much larger than typical values obtained with the commonly employed rectangular cavities [28], for which the mode volume commonly has the same order of magnitude as $\lambda^3$.

Based on Eq. (2) of Ref. [28], together with the evaluated energy density shown in Fig. (1b), the calculated value of the coupling coefficient is found to be $g_{c\ell} = 176$ MHz for the LGR fundamental mode of frequency $f_e = 3.3$ GHz. Alternatively, $g_{c\ell}$ can be extracted from measurements of MW reflection coefficient $|S_{11}|^2$ as a function of the Kittel mode frequency $\omega_s / (2\pi)$ and driving frequency $\omega_{NA} / (2\pi)$. Fitting $|S_{11}|^2$, which is measured at temperature of 3 K using a vector network analyzer (VNA), with Eq. (2) [see Fig. (1c)] yields the value $g_{c\ell} = 200$ MHz, which is pretty much close to the value obtained from simulation. Note that $g_{c\ell}$ is only one order of magnitude smaller than the threshold value corresponding to the super-radiance Dicke instability [14].

### III. Kerr Coefficient and Nonlinear Damping

Cavity driving having amplitude $\Omega_p$ and angular frequency $\omega_p$ is taken into account by adding a term given by $\hbar \Omega_p (A_p e^{i\omega_p t} + A_p e^{-i\omega_p t})$ to the Hamiltonian $H_\Delta$ [44]. Steady state solution of the driven system was calculated in Ref. [10] for the case where damping is taken into account to first order only. For that case the solution is found by solving a cubic equation for the FSR dimensionless energy $E_s = \langle N_s \rangle$ [given by Eq. (36) of [40]]. We find, however, that the calculated steady state yields only a moderate agreement with experimental data. Better agreement can be obtained by taking into account nonlinear damping to cubic order [41]. In this approach the cubic equation for $E_s$ becomes

$$(\delta_s^2 + \gamma_s^2) E_s = \eta |\Omega_p|^2,$$

where $\delta_s' = \delta_s - \eta \delta_s - 2K_M E_s$, $\delta_s = \omega_s - \omega_p$ and $\delta_s = \omega_s - \omega_p$ are driving detuning angular frequencies, $\eta = g_{c\ell}^2 / (\delta_s^2 + \gamma_s^2)$, $\gamma_s = \gamma_{1e} + \gamma_{2e}$ with $\gamma_{1e}$ ($\gamma_{2e}$) being the external (intrinsic) cavity damping rate, $\gamma_{3s} = \gamma_{3s} + \gamma_{7s} E_s$, $\gamma_s$ is the FSR linear damping rate and $\gamma_{3s}$ is the FSR cubic nonlinear damping coefficient. Note that $|\Omega_p|^2$ is proportional to the driving power $P_p$ injected into the LA. Note also that when nonlinear damping is disregarded (i.e. when $\gamma_{3s} = 0$) Eq. (3) becomes identical to Eq. (36) of [40].

VNA measurements of the reflection coefficient $|S_{11}|^2$ for three different values of $P_p$ are shown in Fig. (2a-c). For the data presented in both Fig. (2) and Fig. (3) the radius of the FSR is $R_e = 0.1$ mm. The theoretical fit
shown in Fig. 2(d-f) is based on the cubic equation [3], which allows the calculation of the dimensionless energy \( E_s \), and on Eq. (3) of Ref. [28], which evaluates the reflection coefficient \( |S_{11}|^2 \) as a function of \( E_s \). The values of parameters assumed for the calculations are listed in the caption of Fig. 2. Note the driving-induced blue shift observed in the magnetic resonance frequency [see Fig. 2(a-c)]. This shift cannot be accurately reproduced theoretically when nonlinear damping is disregarded.

**IV. THERMAL INSTABILITY**

Further insight can be gained by measuring the spectral density \( I_{SA} \) of the signal reflected off the LA using a SA (see Fig. 3). We find that for \( P_p > P_c = 42.5 \) dBm, and for sufficiently small detuning from resonance, the measured spectral density \( I_{SA} \) contains equally-spaced side-bands (SB) on both sides of the driving frequency \( f_p = \omega_p / (2\pi) \) [see Fig. 3(a)]. We measure the SB spacing frequency \( \omega_{SM} / (2\pi) \) as a function of the driving frequency \( f_p \) and driving power \( P_p \) [see Fig. 3(c)].

The observed equally spaced SBs are attributed to a thermal instability mechanism that is discussed in Ref. [45]. The phase transition occurring at the Curie temperature \( T_c \) between the FS and the PS gives rise to a sharp change in the resonance modes of the hybrid cavity-FSR system. Consider the case where the frequency of the externally applied driving is tuned very close to the frequency of one the hybrid system modes. With sufficiently high driving amplitude the temperature \( T \) of the FSR may exceeds the Curie temperature \( T_c \) due to driving-induced heating. For that case no steady state with \( T < T_c \) (i.e. FS) exists. The transition from the FS to the PS occurring at \( T_c \) is expected to give rise to a resonance frequency shift. Consequently the driving-induced heating is expected to abruptly drop down, since above \( T_c \) the frequency detuning between the continuous wave external driving and the resonance frequency becomes larger (in absolute value). Consider the case where the reduced heating gives rise to a temperature drop below \( T < T_c \). For this case, a steady state with \( T > T_c \) (i.e. PS) also becomes impossible. In the region where no steady state is possible, the temperature is expected to oscillate around \( T_c \). The frequency of temperature oscillation can be determined from the spacing between the measured SBs.

For the measurements presented in Fig. 3 the driving angular frequency \( \omega_p \) is tuned close to \( \omega_c \). The analysis is greatly simplified by disregarding the other hybrid eigen mode having angular frequency \( \omega_\perp \). This approximation is applicable in the strong coupling regime, for which the resonances having angular frequencies \( \omega_\perp \) do not overlap [see Eq. (2)]. In this approach the FSR-cavity system is treated as a single mode having angular frequency \( \omega_+ = 2\pi \times 3.32 \) GHz, and Kerr coefficient \( K_+ = K_M \sin^4 (\theta_s/2) \) [see Eq. (A9)]. The mode damping rate \( \gamma_+ = 30 \) MHz is expressed as \( \gamma_+ = \gamma_1^+ + \gamma_2^+ \), where \( \gamma_1^+ \) is the coupling coefficient between the driven mode and the LA, and \( \gamma_2^+ \) is the mode intrinsic damping rate (note that \( \gamma_1^+ = \gamma_2^+ \) for critical coupling).

To account for the observed SB, we consider the ef-
f
defect of driving-induced heating on the FSR magnetic ordering. The externally applied driving gives rise to a heating power $Q$ given by $Q = 2\hbar \omega_{+} \gamma_{+} |B|^{2}$, where $B$ is the complex amplitude of the driven mode (note that nonlinear damping is disregarded here). It is assumed that the FSR temperature $T$ is uniform, and that the cooling power due to the coupling between the FSR and its environment at a base temperature of $T_{0}$ is given by $H (T - T_{0})$, where $H$ is the heat transfer coefficient. The thermal heat capacity of the FSR is denoted by $C$. It is assumed that all the parameters characterizing the mode abruptly change at a critical temperature given by $T_{c}$. In the adiabatic (diabatic) region, the mode linear damping rate $\gamma_{+}$ is much smaller (larger) than the thermal decay rate $H/C$.

In dimensionless form, system’s time evolution is governed by
\begin{equation}
\dot{\Theta} = \sigma |B|^{2} - w_{T} \Theta .
\end{equation}

Overdot denotes a derivative with respect to a dimensionless time $\tau$, which is related to the time $t$ by $\tau = \gamma_{0} t$, where $\gamma_{0}$ is a constant rate. The dimensionless complex frequency $w$ is given by $w = \left( i \left( \omega_{p} - \omega_{+} - K_{+} |B|^{2} \right) - \gamma_{+} \right) / \gamma_{0}$, the dimensionless driving amplitude $w_{1}$ is given by $w_{1} = i \gamma_{0}^{-1} \sqrt{2 \gamma_{+} + T}$, the dimensionless temperature $\Theta$ is given by $\Theta = (T - T_{0}) / (T_{c} - T_{0})$, the dimensionless heating coefficient $\sigma$ is given by $\sigma = 2 \hbar \omega_{+} \gamma_{+} \gamma_{0}^{-1} C^{-1} (T_{c} - T_{0})^{-1}$, and the dimensionless thermal rate $w_{T}$ is given by $w_{T} = (H/C) / \gamma_{0}$.

The normalized parameters $w, w_{1}, \sigma$ and $w_{T}$ are assumed to have a step function dependence on the temperature. Below (above) the critical temperature $T_{c}$, i.e. for $\Theta < 1 (\Theta > 1)$, they take the values $w_{F}, w_{1F}, \sigma_{F}$ and $w_{TF} (w_{1F}, \sigma_{F}$ and $w_{TF}$), respectively. A steady state (i.e. time independent) solution below (above) the critical temperature $T_{c}$, i.e. in the region $\Theta < 1 (\Theta > 1)$, is possible provided that $E_{F} < E_{cF}$ ($E_{F} > E_{cF}$), where $E_{F} = |w_{1F}/w_{F}|^{2}$ and $E_{cF} = w_{TF}/\sigma_{F}$ ($E_{cF} = |w_{1P}/w_{P}|^{2}$ and $E_{cF} = w_{TP}/\sigma_{P}$) [see Eqs. (4) and (5) and Fig. 3(b)]. Note that both $E_{F}$ and $E_{cF}$ represent steady state values.
of Eq. 2 for $|B|^2$, whereas both $E_{CF}$ and $E_{CP}$ represent values of $|B|^2$, for which $\Theta = 1$ is a steady state value of Eq. 5.

Heat can be removed from the FSR by radiation, exchange with the surrounding air, and exchange with the supporting ferrules, which hold the FSR inside the LGR. The contributions to the total heat transfer coefficient $H$ due to radiation, air and the ferrules are denoted by $h_{rad}S_s$, $h_{air}S_s$, and $H_{ferr}$, respectively, where $S_s = 4\pi R_s^2$ is the FSR surface area. The coefficient $h_{rad}$ is roughly given by $h_{rad} \approx \alpha_{YIG}\sigma_{SB} (T_s^2 - T_d^2) / (T_c - T_0)$, where $\alpha_{YIG}$ is the averaged FSR absorption coefficient in the spectral band corresponding to room temperature $T_0 \approx 300K$ radiation (wavelength $\lambda \approx 10\mu m$), $\sigma_{SB} = \pi k_B^4 / (60h^3c^2)$ is the Stefan-Boltzmann constant, $k_B$ is the Boltzmann’s constant, $h$ is Plank’s constant, and $T_c = 560K$ is the YIG Curie temperature. The absorption coefficient value $\alpha_{YIG} \approx 10^{-1}$ [50] yields $h_{rad} \approx 2Wm^{-2}K^{-1}$. For ambient temperature and pressure $h_{air} \approx 15Wm^{-2}K^{-1}$, hence $(h_{rad} + h_{air}) S_s (T_c - T_0) \approx 0.6mW$ for a FSR having radius $R_s = 0.1mm$. In the region where SB are observed the induced heating power applied to the FSR is about 3 orders of magnitude larger, hence $H \approx H_{ferr}$, i.e. both radiation and air have negligibly small contributions, and thus heat is mainly removed by the ferrules.

The thermal heat capacity of a FSR having radius $R_s = 0.1mm$ and volume $V_s = 4\pi R_s^3 / 3$ is given by $C = 2.9 \times 10^6JK^{-1}m^{-3} \times V_s = 1.2 \times 10^{-2}JK^{-1}$ [51], hence the thermal decay rate is roughly given by $H/C \approx 320Hz \times (Q_c/W) (260K) \approx 1^{-1}$, where $Q_c$ is the heating power applied to the FSR, for which the steady state temperature is $T_c$. Hence for the current device $(H/C) / \gamma_s \approx 10^{-5}$, and thus the diabatic approximation is applicable.

A typical limit cycle (LC) in the diabatic regime is shown in Fig. 4. The LC is calculated by numerically integrating the equations of motion 4 and 5. The blue (red) cross shown in Fig. 4(a) indicates the steady state value $w_l/w$ of $B$ corresponding to the FS (PS), i.e. for $\Theta < 1$ ($\Theta > 1$), and the blue (red) circle represents the relation $|B|^2 = E_{CF}$ ($|B|^2 = E_{CP}$). In the plane of driving frequency and driving amplitude, which is shown in Fig. 4(b), the blue and red curves are derived from the relations $E_F = E_{CF}$ and $E_P = E_{CP}$, respectively. In the region labeled as A, no steady state solution to Eqs. 4 and 5 exists. The LC period time $t_{LC}$ can be calculated by integrating Eqs. 4 and 5 over a single period. In the diabatic limit, one finds that $t \approx |w_P|^{-1} + |w_P|^{-1}$. The measured value of LC frequency roughly agrees with this theoretical estimation.

V. SUMMARY

In summary, we demonstrate that relatively large coupling coefficient $g_{eff}$ can be obtained by employing an LGR having mode volume much smaller than $\lambda^3$. The response of the system in the weak nonlinear regime allows the extraction of the Kerr coefficient $K_M$ and the cubic nonlinear damping rate $\gamma_3$. An instability is revealed by driving the system with a relatively high input power. Above the instability threshold the response of the system to an externally applied monochromatic driving exhibits self-modulation. The instability, which is attributed to driving-induced heating, occurs in a region where the response has no steady state value. Further study will be devoted to developing sensors that exploit this instability for performance enhancement.

VI. ACKNOWLEDGMENTS

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Appendix A: Rotating wave approximation

The Hamiltonian 11 can be expressed as

$$h^{-1}\mathcal{H}_D = (A_e^\dagger A_s^\dagger M (A_e A_s) + K_M N_s^2, \quad (A1)$$

where the $2 \times 2$ matrix $M$ is given by

$$M = \begin{pmatrix} \omega_e & g_{eff} \\ g_{eff} & \omega_s \end{pmatrix}. \quad (A2)$$

The eigenvalues $\omega_\pm$ of the matrix $M$ are given by $\omega_\pm = \omega_m \pm \sqrt{\omega_m^2 + g_{eff}^2}$ [see Eq. (2)], where $\omega_m = (\omega_e + \omega_s) / 2$ and $\omega_d = (\omega_e - \omega_s) / 2$. The matrix $M$ can be expressed as

$$M = \omega_m \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} \omega_d^2 + g_{eff}^2 & \cos \theta & \sin \theta \\ \sin \theta & \cos \theta & -\cos \theta \end{pmatrix}, \quad (A3)$$

where

$$\tan \theta = g_{eff} / \omega_d. \quad (A4)$$

The transformation

$$\begin{pmatrix} A_e \\ A_s \end{pmatrix} = U \begin{pmatrix} A_+ \\ A_- \end{pmatrix}, \quad (A5)$$

where

$$U = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}, \quad (A6)$$

which diagonalizes the linear part of $\mathcal{H}_D$, yields

$$h^{-1}\mathcal{H}_D = \omega_+ N_+ + \omega_- N_- + K_M (A_+^\dagger A_s)^2, \quad (A7)$$

where $A_s = A_+ \sin (\theta / 2) + A_- \cos (\theta / 2)$, and where $N_\pm = A_\pm^\dagger A_\pm$. 

In the rotating wave approximation (RWA) the Hamiltonian \( \hat{H}_D \) becomes
\[
\hbar^{-1} \hat{H}_D = \omega_+ N_+ + \omega_- N_- + K_+ N_+^2 + K_- N_-^2 + K_1 N_+ N_-,
\]
where the Kerr coefficients \( K_\pm \) are given by
\[
K_+ = K_M \sin^4 \frac{\theta}{2}, \quad \text{and} \quad K_- = K_M \cos^4 \frac{\theta}{2},
\]
and the inter-mode Kerr coefficient \( K_1 \) is given by \( K_1 = K_M \sin^4 \theta \).

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