Economic dispatch in a power system considering environmental pollution using a multi-objective particle swarm optimization algorithm based on the Pareto criterion and fuzzy logic

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Abstract In recent years, many studies have studied economic dispatch problem in power systems. However, most of them have not considered the environmental pollution caused by fossil fuels. In this study, the use of an evolutionary search algorithm called multi-objective particle swarm optimization algorithm is proposed to solve the economic dispatch problem in power systems while considering environmental pollution. The proposed method is validated in terms of its accuracy and convergence speed based on comparisons with the results obtained using the classic nonlinear programming method. The proposed strategy is applied to a realistic power system under various conditions. Overall, six generating units are investigated along the corresponding constraints. The results obtained reveal that costs of operation and pollution with/without power loss are reduced significantly by the proposed approach. Obtained results show a good compromise can be established between two contradicting functions of exploitation cost and pollution by optimizing them simultaneously. Values of these function without considering their loss is $46,112.09$ $$/h and $682.32$ kg/h, respectively. And if losses are considered, these values would be $48,381.09$ $$/h and $726.52$ kg/h, respectively.

Keywords Economic dispatch · Multi-objective optimization · Multi-objective particle swarm optimization (MOPSO) algorithm · Pareto criterion · Power plant environmental pollution

List of symbols

- $a_i$, $b_i$, and $c_i$ i
dth generating unit coefficients
- $F(P_i)$ i
dth generating unit cost function
- $N$ Number of generating units in operation
- $P_i$ i
dth generating unit output power
- $x_i$, $y_i$, and $z_i$ Emissions coefficients for the i
dth generating unit
- $P_{loss}$ System power loss
- $P_{min}$ and $P_{max}$ The minimum and maximum power levels generated by each generating unit
- $RDR_i$ and $RUR_i$ Ramp-down and ramp-up rates for a generating unit
- $X$ Vector includes the output power of the generating units
- $\mu_i$ Membership function that shows the i
dtective objective function's optimality
- $f_{max}$ and $f_{min}$ Upper and lower boundaries of the i
djective function
- $n$ Number of non-dominated solutions
- $m$ Number of objective functions

Introduction

Emissions from the fossil fuels consumed by power plants lead to increased operational costs, as well as requiring much attention to minimize pollution. The economic dispatch (ED) problem has been considered since 1990, when ED was enacted by the Clean Air Organization to control environmental pollution, including SO$_2$, NO$_x$, and CO$_2$. Various methods have been utilized to reduce emissions and the different strategies utilized to decrease emissions can be divided into three groups [1], as follows:
• Installation of pollution removal devices in power plant sites;
• Replacement of old devices with new ones;
• Operation of power plants by considering environmental pollutants.

Various approaches have been proposed that consider emissions from power plants to address the ED problem. Finnegan and Fouad considered the emissions from power plants for the first time in 1974 [2], where they treated emissions as a constraint within a permissible range. Later, this strategy was used to control pollution in related studies [3]. The EMO algorithm was employed in [5] to speed up the convergence of an operational cost function. Many other solutions have been suggested to address this problem in previous studies. Analytical methods [6], a Lagrangian method [7], and the Newton–Raphson method [8] have all been employed as initial approaches. Evolutionary methods have also been employed for this purpose, such as a genetic algorithm [9], particle swarm optimization (PSO) [10], simulated annealing [11], artificial immune system [12], differential evolution [13], and the frog algorithm [14]. In [15], the PSO-SIL algorithm was used to obtain an economical power flow with the optimum costs.

These methods differ in terms of their speed and accuracy. The ED problem without consideration of environmental issues leads to increased costs. In addition, concerns over environmental pollutions are increasing constantly. However, the aforementioned techniques only consider the costs related to systems. In this study, we propose an analytical strategy for simultaneously minimizing costs and the emissions from power plants. This multi-objective problem is solved using an analytical PSO (MOPSO) algorithm. We applied the proposed strategy to a realistic six-bus test system. The results obtained were validated based on comparisons with those produced using the classic nonlinear programming (NLP) method.

The remainder of this paper is organized as follows. In Sect. 2, the ED problem is modeled for a power system by considering an emissions function. Additional constraints such as the ramp-rate limit and prohibited operating zone for generators, as well as network security are also considered. Furthermore, the problem-solving method is described in this section. In Sect. 3, MOPSO algorithm is proposed as a method for solving the problem. In Sect. 4, we present the simulations and the numerical results are discussed. Finally, we give our conclusions in Sect. 5.

Economic dispatch considering emissions

In power distribution systems, electrical engineers attempt to improve the power system efficiency by increasing the number of generating units and making profits to obtain the maximum benefits with the least cost. In addition, they should satisfy the total load requirements of the network and observe all the operating constraints on power plants, as well as transmission lines. The ED problem is an optimization problem regarding environmental emissions and the operating costs for generating units. The aim when solving this problem is to meet the load demand with the least cost and emissions, while also satisfying the constraints on the problem.

Operating cost function

The operating cost function comprising the constant cost and output power cost is given by Eq. (1). Quadratic operating cost function is the most common and simplest one because of neglecting valve-point effect that is expressed in ED problems.

\[
\text{Min } F_1(x) = \sum_{i=1}^{N} 10^{-2}(x_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2)
\]

(1)

Emission function

To minimize environmental deterioration due to emissions from fossil fuels in power plants, the amount of pollutants should depend on the output power from the generating power plant. Thus, an objective function for minimizing environmental pollution can be described by Eq. (2) [3, 4]:

\[
\text{Min } F_2(x) = \sum_{i=1}^{N} 10^{-2}(x_i + \beta_i P_{Gi} + \gamma_i P_{Gi}^2)
\]

(2)

Constraints

The power system considered operates subject to technical constraints.

Power balance constraint

The total generating power by all generating units is equal to the total load demand:

\[
\sum_{i=1}^{N} P_i - P_D - P_{loss} = 0
\]

(3)

The system power loss is obtained by Eq. (4) based on matrix B, as follows.

\[
P_{\text{loss}} = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{i0} P_i + B_{00}
\]

(4)

Generating units operating constraint

According to its technical characteristics, each generating unit can operate in an appropriate range. If the generating
units violate the specified range, this does not result in cost-effective operation. Thus, upper and lower boundary power levels exist for each generating unit, as specified by Eq. (5):

\[ P_i^{\text{min}} \leq P_i \leq P_i^{\text{max}} \]  

(5)

\( P_i^{\text{min}} \) and \( P_i^{\text{max}} \) are generated by each generating unit, respectively.

**Ramp-rate limit**

Thermal power plants cannot increase or decrease their output power instantaneously. Instead, the changes occur with a limited rate. Violating this rate will lead to damage to the rotor, thereby increasing the operating costs. The output power of active units should observe the following constraints:

\[ P_i^{\text{max}} (t) = \min(P_i(t-1) + RUR_i, P_i^{\text{min}}) \]  

(6)

\[ P_i^{\text{min}} (t) = \max(P_i(t-1) - RDR_i, P_i^{\text{min}}) \]  

(7)

To comply with the aforementioned limitations, it is necessary to know the initial generation level for each unit. Figure 1 shows how to apply these limitations.

**MOPSO algorithm**

Multi-objective optimization problems have several conflicting objective functions as well as equality and inequality constraints that should be optimized simultaneously, as follows.

\[ \text{Min } F(\vec{X}) = [f_1(\vec{X}), f_2(\vec{X}), \ldots, f_N(\vec{X})]^T \]  

Subject to:

\[ g_i(X) < 0 \Rightarrow i = 1, 2, \ldots, N_{\text{eq}} \]  

\[ h_i(X) = 0 \Rightarrow i = 1, 2, \ldots, N_{\text{eq}} \]  

(8)

The space where the objective function is defined is called the objective space. In multi-objective optimization, both solutions have two states: one solution dominates another solution or none is dominated by another, as described by Eq. (9).

\[ \forall j \in \{1, 2, \ldots, n\}, \Rightarrow f_j(\vec{X}_1) \leq f_j(\vec{X}_2) \]  

\[ \exists k \in \{1, 2, \ldots, n\}, \Rightarrow f_k(\vec{X}_2) < f_k(\vec{X}_2) \]  

(9)

By applying the Pareto criterion to the PSO algorithm, we can use this algorithm to solve multi-objective problems [16]. The MOPSO algorithm uses an external memory where dominant solutions are stored, i.e., an archive. First, the algorithm starts with initial random particles. Next, all the particles in the population are compared with each other in an iterative process and the positions of the dominant particles are stored in the archive. The velocity and position of \( i \)th particle in the \( d \)th dimension for the \( t + 1 \)th iteration are updated using Eqs. (10) and (11). For detailed information, readers are referred to [16].

\[ x^{t+1}_d = x^t_d + v^{t+1}_d \]  

(10)

\[ v^{t+1}_d = w \times v^t_d + c_1 r_1 \times (P_{\text{best}} - x^t_d) + c_2 r_2 \times (P_{\text{best}} - x^t_d) \]  

(11)

The following steps are required to apply the algorithm to the problem considered in this study.

Step 1) Enter the input data

First, the input data required by the program are entered in detail: the power system configuration, operating characteristics of the generating units, and pollution coefficients for each generating unit.

Step 2) Define the initial population.

An initial populations and velocities are formed by considering the problem constraints using Eq. (12):

\[ \text{Population} = [X_1, X_2, \ldots, X_{\text{NSwarm}}]^T \]  

\[ X_0 = [x^0_1, x^0_2, \ldots, x^0_n] \]  

\[ x^0_i = \text{rand}(0) \times (x^{\text{max}}_i - x^{\text{min}}_i) + x^{\text{min}}_i \]  

\[ j = 1, 2, \ldots, n \]  

Velocity = [\( V_1 \ V_2 \ \ldots \ V_{NSwarm} \)]^T

\[ V_i = [v_i]_{1 \times n} \]  

\[ v_i = \text{rand}(0) \times (v^{\text{max}} - v^{\text{min}}) + v^{\text{min}} \]  

\[ i = 1, 2, \ldots, NSwarm \]  

where, \( n \) is the number of state variables, \( v_i \) and \( x_i \) are the velocity and position of the \( i \)th state variable, respectively, \( \text{rand}(0) \) is a random number between 0 and 1.

Step 3) Apply the power dispatch algorithm, as shown in Fig. 2, to each generated population. In addition, calculate

![Fig. 1 Various constraints on generating unit](image-url)
Fig. 2 Power dispatch algorithm flowchart

Step 1) Initialize a fitness function based on the equations, i.e., the objective functions.

Step 4) Determine the non-dominated solutions. The non-dominated solutions are determined by Eq. (8).

Step 5) Separate the non-dominated solutions and store them in an archive. To access non-dominated solutions, they should be stored.

Step 6) Select the best particle from the archive of non-dominated solutions as a leader. The leader selection process is as follows. First, the search space is divided into equal parts. Next, a probability distribution is assigned to each search space division. Finally, the best particle is selected as the leader by a roulette wheel method.

Step 7) Obtain the new velocity and position for each particle using Eqs. (10) and (11).

Step 8) Update the best position for each particle by comparing each particle’s new position with the previous positions.

\[
P_{\text{best},i}(t+1) = \begin{cases} 
    P_{\text{best},i}(t) & P_{\text{best},i}(t) < X_i(t+1) \\
    X_i(t+1) & X_i(t+1) < P_{\text{best},i}(t) \\
    \text{select randomly} & (P_{\text{best},i}(t) \text{or } X_i(t+1)) \text{ otherwise} 
\end{cases}
\]  

(13)

Step 9) Integrate the current non-dominated solutions into the archive.

Step 10) Eliminate dominated solutions from the archive. Dominated-solutions are eliminated from solution circle of the archive.

Step 11) If the number of solutions in the archive exceeds a predefined value, eliminate extra solutions.

Step 12) Check the program termination criterion. If the maximum number of iterations is reached, the optimization process is terminated; otherwise, the previous generation is replaced by the new current generation and the algorithm jumps to Step 6. Figure 3 shows a flowchart illustrating the proposed algorithm for solving the optimization problem.

Step 13) Select the best interactive solution. To select the best solution among the optimum solutions obtained, a fuzzy decision function with a membership function is employed to include the exact values of variables, where the membership function \( \mu_i^k \) shows the \( i \)th objective function’s optimality among the \( k \)th Pareto optimal solutions calculated by Eq. (14):

\[
\mu_i^k = \begin{cases} 
    1 & f_i \leq f_i^{min} \\
    \frac{f_i^{max} - f_i}{f_i^{max} - f_i^{min}} & f_i^{max} < f_i < f_i^{min} \\
    0 & f_i \geq f_i^{max} 
\end{cases}
\]  

(14)

These limits are calculated separately in the proposed method using the optimization results for each objective.
function. \( \mu_i^k \) Ranges between 0 and 1 such that \( \mu_i^k = 0 \) indicates the incompatibility of the solution with the specified objectives, whereas \( \mu_i^k = 1 \) indicates full compatibility. For each Pareto k’s optimum solution, the normalized membership function is obtained by Eq. (15):

\[
\mu^k = \frac{\sum_{i=1}^{m} \mu_i^k}{\sum_{k=1}^{n} \sum_{i=1}^{m} \mu_i^k}
\]  

(15)

The solution with the highest membership function is selected as the best compatible solution. Figure 4 illustrates the optimization problem and the application of the fuzzy method to select the best solution from the Pareto solution set.

**Case study and simulation results**

To demonstrate the effectiveness of the proposed algorithm, a six-bus test system was used for simulation purpose. This power system is utilized to solve economic load dispatch considering air pollution under safety constraint by MOPSO algorithm. The proposed approach was applied to the system under various load conditions and the results obtained were compared with those produced by the classic NLP method (for detailed information readers are referred to [17]). The system considered has been in use for 25 years. The average increased cost of generating units was 45 $/MWh. The system is shown in Fig. 5.

The system considered in the case study comprised three power plants and six generating units. The fuel costs and pollution coefficients for the generating units are given in

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**Fig. 3 MOPSO flowchart**

![MOPSO flowchart](image-url)
Table 1 and 2, respectively. For simplicity, we only considered one pollutant, i.e. NO\(_x\). Table 3 shows the coefficients for the system’s transmission power loss. The method used in this study is based on the fundamentals algorithm as a multi-objective function. Thus, using the Pareto optimization principle, we formulated a MOPSO algorithm to solve the problem. In addition, the results obtained were compared with those produced by the classic NLP to demonstrate the superior performance of our method in terms of accuracy.
In this section, the results obtained in terms of minimizing the operating cost function are presented in two modes: with and without system power loss. The optimum power dispatch levels for the generating units using PSO and NLP are shown in Tables 4 and 5, respectively. The results in Tables 4 and 5 indicate that generating units 5 and 6, which had the lowest start-up cost among all the generating units, contributed enormously to meeting the

**Fig. 6** Convergence trend of the operating costs: (a) without transmission power loss; and (b) with transmission power loss

**Table 6** Economic dispatch of emissions without transmission system power loss

| Generating unit (MW) | PSO   | NLP   |
|----------------------|-------|-------|
| 1                    | 116.99| 116.99|
| 2                    | 116.98| 116.99|
| 3                    | 135.69| 135.69|
| 4                    | 135.69| 135.66|
| 5                    | 197.31| 197.31|
| 6                    | 197.31| 197.31|
| Total fuel costs ($/h)| 48051.22| 48051.51|
| Total emissions (kg/h)| 646.12  | 646.18 |

**Fig. 7** Convergence trend in the emission costs: (a) without transmission power loss; and (b) with transmission power loss

**Table 7** Economic dispatch of emissions with transmission system power loss

| Generating unit (MW) | PSO   | NLP   |
|----------------------|-------|-------|
| 1                    | 124.60| 122.75|
| 2                    | 127.51| 122.75|
| 3                    | 140.27| 139.22|
| 4                    | 140.30| 141.96|
| 5                    | 204.11| 206.64|
| 6                    | 204.19| 207.78|
| Total fuel costs ($/h)| 50223.97| 50262.99|
| Total emissions (kg/h)| 697.03  | 701.12 |
| Transmission loss (MW)| 40.99   | 41.12  |

**Operating cost function**

In this section, the results obtained in terms of minimizing the operating cost function are presented in two modes: with and without system power loss. The optimum power dispatch levels for the generating units using PSO and NLP are shown in Tables 4 and 5, respectively.

The results in Tables 4 and 5 indicate that generating units 5 and 6, which had the lowest start-up cost among all the generating units, contributed enormously to meeting the
load demand. Generating unit-5 produces 2,871,629 MW and generating unit-6 generates 2,828,004 MW without considering power losses. Obviously, these values increase when power losses are considered (see Table 5).

Figure 6 shows the convergence trend with the proposed method when minimizing the operating cost function with/without system power loss. Based on the results obtained and Fig. 6, we can state that when transmission system loss was included, the operating cost increased from 45,463.49 $/h to 47,170.92 $/h.

Emission cost function

In this section, we present the results obtained by minimizing the emission cost function in two modes: with and without system power loss. The optimum power dispatch levels for the generating units using PSO and NLP are shown in Tables 6 and 7, respectively.

Based on the results in Tables 6 and 7, we can state that generating units 1 and 2, which had the lowest emissions, operated near their maximum value and these units made major contributions to meeting the load demand. Generating unit-1 produces total power of 116.99 MW and generating unit-2 generates 116.98 MW without power loss. This will definitely increase when power loss is considered (See Table 7).

Figure 7 shows the convergence trend with the proposed method when minimizing the emission cost function with/without system power loss. According to the results obtained, when transmission system loss was included, the emission costs increased from 646.12 to 697.03 kg/h.

Simultaneous minimization of the operating cost and emission cost functions

In this section, we present the results obtained by simultaneously minimizing the operating and emission cost functions in two modes: with and without system power loss. The optimum power dispatch levels for the generating units using MOPSO and NLP are shown in Tables 8 and 9, respectively.

According to the results in Tables 8 and 9, when we simultaneously optimized the operating and emission cost functions, there was a trade-off between the two

| Table 8 | Simultaneous minimization of the operating cost and emission cost functions without transmission system power loss |
|---------|--------------------------------------------------------|
|         | Unit | MOPSO | NLP |
| Generating unit (MW) | | | |
| 1 | 68.86 | 36.02 |
| 2 | 66.77 | 16.66 |
| 3 | 143.77 | 147.79 |
| 4 | 156.01 | 146.54 |
| 5 | 244.55 | 278.73 |
| 6 | 220.01 | 274.24 |
| Total fuel costs ($/h) | 46112.09 | 46248.23 |
| Total emissions (kg/h) | 682.32 | 775.48 |

| Table 9 | Simultaneous minimization of the operating cost and emission cost functions with transmission system power loss |
|---------|--------------------------------------------------------|
|         | Unit | MOPSO | NLP |
| Generating unit (MW) | | | |
| 1 | 81.71 | 37.53 |
| 2 | 79.57 | 18.81 |
| 3 | 152.51 | 154.58 |
| 4 | 162.82 | 151.32 |
| 5 | 232.58 | 286.6 |
| 6 | 228.84 | 289.4 |
| Total fuel costs ($/h) | 48381.09 | 48181.49 |
| Total emissions (kg/h) | 726.52 | 843.19 |
| Transmission loss (MW) | 38.1 | 38.34 |

Figure 8 Pareto criterion distribution based on the operating and emission costs: a without transmission power loss; and b with transmission power loss
conflicting functions to operate the system at the optimum point.

Figure 8 shows the convergence trend with the proposed method when we simultaneously minimized the operating cost and emission cost functions with/without system power loss.

Conclusions

In this study, the MOPSO algorithm was used to solve the ED problem considering emissions in a power system with various constraints. To obtain satisfactory results, the problem was solved by taking into account the operating and emission costs in a separate mode. Next, the problem was solved by trading off between two contrasting objective functions, i.e., operating and emission cost functions, for simultaneous minimization. This process was analyzed in two modes, without and with transmission system power loss. As results indicate when two objective functions are optimized simultaneously the related costs are highly reduced. Comparing the results obtained with those produced by the NLP method demonstrated that the MOPSO algorithm outperformed the NLP method in terms of its accuracy and convergence speed as results show, if two pollution and exploitation cost functions are optimized simultaneously, without considering losses, their values would be 46,112.09 $/h and 682.32 $/h and if losses are considered, these values would be 48,381.09 $/h and 726.52 $/h.

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References

1. Ebrahimian, H., Taheri, B., Yousefi, N.: Optimal operation of energy at hydrothermal power plants by simultaneous minimization of pollution and costs using improved ABC algorithm. Front. Energy 9(4), 426–432 (2015). doi:10.1007/s11708-015-0376-4
2. Finnigan, O.E., Fouad, A. (eds.): Economic dispatch with pollution constraints. IEEE Transactions on Power Apparatus and Systems; 1974: IEEE-Inst Electrical Electronics Engineers Inc 345 E 47th St, New York, NY 10017-2394
3. Brodsky, S.F., Hahn, R.W.: Assessing the influence of power pools on emission constrained economic dispatch. Power Syst. IEEE Trans. 15(1), 57–62 (1986)
4. Roy, P.K., Hazra, S.: Economic emission dispatch for wind-fossil-fuel-based power system using chemical reaction optimisation. Int. Trans. Electr. Energy Syst. (2014). doi:10.1002/etep.2033
5. Ghobbar, N., Vakili, S., Babaei, E., Sakhavati, A.: Particle swarm optimization with smart inertia factor for solving non-convex economic load dispatch problems. Int. Trans. Electr. Energy Syst. 24, 1120–1133 (2014)
6. Anantasate, S., Bhasaputra, P.: A multi-objective bees algorithm for multi-objective optimal power flow problem, Electrical Power Engineering and Power System. The 8th Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI) Association of Thailand—Conference 2011
7. El-Keib, A., Ma, H., Hart, J.: Environmentally constrained economic dispatch using the Lagrangian relaxation method. Power Syst. IEEE Trans. 9(4), 1723–1729 (1994)
8. Chen, S.-D., Chen, J.-F.: A direct Newton-Raphson economic emission dispatch. Int. J. Electr. Power Energy Syst. 25(5), 411–417 (2003)
9. Yang, Z., Li, K., Niu, Q., Zhang, C., Foley, A. (eds): Non-convex dynamic economic/environmental dispatch with plug-in electric vehicle loads. Comput. Intell. Appl. Smart Grid (CIASG), 2014 IEEE Symposium on; 2014: IEEE
10. Lu, S., Sun, C., Lu, Z.: An improved quantum-behaved particle swarm optimization method for short-term combined economic emission hydrothermal scheduling. Energy Convers. Manage. 51(3), 561–571 (2010)
11. Basu, M.: A simulated annealing-based goal-attainment method for economic emission load dispatch of fixed head hydrothermal power systems. Int. J. Electr. Power Energy Syst. 27(2), 147–153 (2005)
12. Geetha, R., Bhuvaneswari, R., Subramanian, S., (eds.): Artificial immune system based combined economic and emission dispatch. TENCON 2008-2008 IEEE Region 10 Conference; 2008: IEEE
13. Pandit, M., Srivastava, L., Sharma, M.: Environmental economic dispatch in multi-area power system employing improved differential evolution with fuzzy selection. Appl. Soft Comput. 28, 498–510 (2015)
14. Anita, J.M., Raglend, I.J.: Power flow constrained unit commitment problem using improved shuffled frog leaping algorithm. Power Electr. Renew. Energy Syst., p. 1545–55, Springer (2015)
15. Roy, P.K., Blui, S.: A multi-objective hybrid evolutionary algorithm for dynamic economic emission load dispatch. Int. Trans. Electr. Energy Syst. 2015:etep.2066
16. Tarafdar Hagh, M., Ebrahimian, H., Ghadimi, N.: Hybrid intelligent water drop bundled wavelet neural network to solve the islanding detection by inverter based DG. Front. Energy 9(1), 75–90 (2015)
17. Nanda, J., Hari, L., Kothari, M.: Economic emission load dispatch with line flow constraints using a classical technique. IEEE Proc. Gener. Trans. Distrib. 141, 1–10 (1994)