The Role of Progressive Mathematics in Geometry Learning

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Abstract. The purpose of this study is to analyze the learning trajectory that can help students understand the concept of volume cubes and beams with filling and arrangement boxes using dice context in grade VIII of junior high school (SMP). This research is a design research type of validation study that is implemented through Realistic Mathematics Education (RME). Design research is carried out in three stages, namely: preliminary design, teaching experiment consisting of cycle one and cycle 2, and retrospective analysis. The study was conducted in grade VIII of SMP at Tangerang City. The results of the study show that students can understand the volume of cubes and beams through progressive mathematical identified at the situational stage, recognizing the volume unit with the arrangement of unit cubes; referential stage, calculates the number of unit cubes that can be made on cubes and beams; general stage, found the capacity of objects in the form of cubes or beams based on known ribs and blocks; formally stage, calculate the volume of cubes and beams if the ribs are known.

1. Introduction

Geometry is one of the subject matter of mathematics with a large and vital portion both at the elementary and junior high school levels [1]. One part of geometry is the matter of building cubic space and beams by emphasizing the ability to identify the properties and elements and determine the volume of building cubes and blocks. At the level of junior high school, the discussion of geometry, especially the matter of building space, faces several problems, both determining the surface area of the building and the volume of building space. It is not only because the object is abstract; a little algebraic calculation involving connections between objects and size variability is a big problem for students [2, 3].

The view of space in space geometry is a significant problem in learning. It is because the understanding of geometry is always focused on the context of algebra. A contradictory situation in which the understanding of geometry should begin with a visual representation. In van Hiele's Geometry, visuals are the first stage that must be passed. Thus students are facilitated in a real situation where objects have been known to students in everyday life [1]. To create a situation like this, it takes learning to enable students to be directly involved in finding and building mathematical models based on their knowledge, one of which is Realistic Mathematics Education (RME) [4]. This activity involves calculating the building volume of a room where students must understand the unit of
volume and calculate the volume of building space in specific units.

In RME, students are facilitated to use mathematical models that have been previously formed through continuous mathematical processes [4-6]. It is by the characteristics of RME that using models, schemes, diagrams, symbols for progressive mathematics as crucial in the discovery and development of mathematical concepts by students [6-7]. Another term for using models is mathematical. Traffer distinguishes two types of mathematics, namely horizontal mathematical and vertical mathematical [8-9]. Horizontal Mathematics is a mathematical process in which students use mathematics to solve problems in real situations, whereas vertical mathematics is the process of reorganizing using mathematics itself. The model, in this case, relates to situation models and mathematical models developed by students themselves.

Furthermore, Koeno Gravemeijer [9] states that "in RME, models are placed at intermediate levels between situated and formal knowledge." The use of models used by students themselves serves as a bridge for students from real situations to ideal situations or from formal informal-mathematical mathematics. According to Koeno Gravemeijer [9], Fauzan et al. [10], Wijaya [6], and Koeno Gravemeijer [11], this process includes four levels, namely: situational levels, referential level, general level, and formal level. Progressive mathematical analysis based on four stages in the development of a mathematical model, namely determining the volume of cubes and beams.

The research has been determining the volume unit using dice and determining the volume of cubes and beams using dice using RME can help students understand the volume unit and the concept of cube and beam volumes with learning trajectories: 1) find cube volume units and beams using dice; 2) find the contents of cubes and blocks with dice; 3) find the concept of cube and beam volume by filling dice in various ways; 4) find the concept and formula for the volume of cubes and beams.

2. Method
This research was designed using design research methods. While the research stages refer to the stages of design research [12-14], including the stages of preparation for the experiment or preliminary design, the stage of implementation of a design experiment, or teaching experiment, and retrospective analysis. This research was conducted in the odd semester of the 2016/2017 academic year. As a subject were six students of class VIII D, 36 students of class VIII C SMP at Tangerang City. The design research stage is explicitly illustrated in the following figure.

![Cycle Design Research Gravemeijer](image)

**Figure 1.** Cycle Design Research Gravemeijer [15]

The first stage is preparing for the experiment, and researchers conduct observations into schools, interview with classroom teachers, and review literature on cube and beam volume material. The process of obtaining local instructional theory arises from a cyclical learning design process, so the results are used to design a series of learning activities that contain the hypothetical learning trajectory (HLT). The second stage is the implementation of the design experiment includes two stages, namely: cycle 1 (pilot experiment) and cycle 2 (teaching experiment). In cycle 1, the researcher acted as the teacher to conduct initial research as many as six students with different abilities. The results of the first cycle are used to revise the HLT, which is then used in the second cycle. In the second cycle, learning is done by involving as many as 36 students of class VIII. The third stage is a review analysis based on data obtained from the teaching experiment stage by developing designs in the next learning activity. The purpose of retrospective analysis, in general, is to develop Local Intrumentional Theory.
(LIT). Because the theory developed is empirical, the theory constructed from the results of this research is in the form of a local instructional theory that provides general answers to a topic being taught.

HLT that has been designed is then compared with the actual student learning trajectory during the implementation of learning to be carried out retrospectively. The results of the HLT analysis are then poured into the HLT table, whose columns contain activities, learning objectives, description of activities, and conjectures of students' thinking.

**Table 1. Hypothetical Learning Trajectory (HLT)**

| Activity        | Learning Objectives                                                                 | Activity description                                                                 | Student Thinking Conjecture                                                                 |
|-----------------|-------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------|
| Arrange units   | Students can understand the volume unit for the construction of cuboid or beam space objects | In groups, students are asked to arrange dice in a certain amount so that various forms of cubes or beams are formed. | ▪ Students arrange several dice so that they precisely form a cube                      |
| squares         |                                                                                      |                                                                                      | ▪ Students compile several dice but only fulfill a portion of the stack that constructs a cube |
|                 |                                                                                      |                                                                                      | ▪ Students arrange dice to form various beams                                             |

- *Filling box*

| Students can understand the volume of a cube or beam object | In groups, students are asked to insert dice into a cuboid or block box | ▪ Students fill the dice into boxes until they are full                              |
| ----------------------------------------------------------- | ----------------------------------------------------------------------- | ------------------------------------------                                    |
|                                                             |                                                                        | ▪ Students fill dice in boxes layer by layer                          |
|                                                             |                                                                        | ▪ Students fill dice in the box through rows of length, width, and height |

3. Results and Discussion

3.1. First Activity: Compile the Unit Box

In the first activity, each group gets several dice of 30 dice. This dice is represented as a unit cube. Students are asked to do exploration to form the shape of a cube or beam from the dice that has been given. The following activities carried out by each group in this activity can be seen in full in Figure 2 and Figure 3 below.

Figure 2. Arrange the cube from the unit dice

In Figure 2, students arrange cubes with 27 dice units and three remaining dice units. In the cube arrangement 2(a), the arrangement of cubes formed consists of 3 horizontal dice, three dice are sideways, and three dice are up so that the number of dice that make up the cube is 27 dice and the other three dice become the remainder. In the drawing structure 2(b), students compile a unit cube with a composition consisting of 2 horizontal dice, two dice sideways, and two dice up so that the
number of dice that compose the cube is eight dice. Then another cube is arranged with the same cube so that three cubes are formed. If all the cubes are summed, the number of dice that builds the cube is 24 so that in this group, there are still 6 unit dice. In Figure 2(c), students can arrange cubes with 27 unit dice.

In Figure 3, students are seen in compiling a unit box with a beam shaped dice. In Figure 3(d), students arrange blocks with unit dice with a horizontal arrangement of 5 pieces, two dice sideways, and high as many as 3 unit dice. The idea of composing these beams initially tried by arranging the first layer of 10 dice with a horizontal arrangement of 5 dice and sideways two dice, then rearranging the dice in the second and third layers each with ten dice to form a beam with a 5-dice length, width two dice and height three dice. In Figure 3(e), students use the idea of arranging blocks with the first layer with 15 dice with a composition of 15 large and full dice one dice. Then the second set of 15 dice is stacked again in the layer above the first layer the beamformed consists of 15 elongated dice with one dice sideways and two dice high. In Figure 3(f), students can arrange blocks by using the first layer arrangement of 15 dice consisting of 5 dice length and three dice width. Then the second arrangement is stacked 15 dice above the first arrangement. The beamformed consists of dice with an elongated arrangement of 5 dice, sideways three dice, and height two dice.

To find out the process of compiling cubes and blocks of unit boxes, a conversation was held with several students. In this conversation, the researcher had a conversation with three students consisting of students 1, students 2, and students 3. The following is a discussion quotation about determining a cuboid and block unit box.

**Dialog 1 (specifies a cube unit box):**

Researcher: "I will give each group 30 dice. Try to represent group 1; how to arrange cubes with existing dice!"

Student 1: "I just arranged the dice?" (student 1 is representative of group 1)

Researcher: "Try what do students 1 do?"

Student 1: "This is the stack (there is a cube shape with three dice in the front, 3 in the side, and 3 in the top), and there are still three remaining chests."

Researcher: "You sure is this a cube?"

Student 1: "Yes sir, because the sides are all the same"

Researcher: "I will check group 2. what kind of cube arrangement should you try?"

(students two representatives from group 2).

Student 2: "I arrange the initial layer with four dice."

Researcher: "Is the initial layer cuboid?"

Student 2: "Not yet. So I stacked the layers with four dice."

Researcher: "Is it cube-shaped?"

Student 2: "Yes, sir... because the shape is the same between the bottom, side, and top."

Researcher: "Try stacking again with the same cube arrangement. How many cubes do you arrange?"
Student 2: "there are three cubes of the same size as the first shape, and there are the remaining six dice."
Researcher: "As a conclusion, how many dice do students form into cubes?"
Student 1: "(counting) ... there are 27 dice, sir."
Researcher: "For students 2, how many dice are there in one of the cubes that are stacked?"
Student 2: "Just one, sir."
Researcher: "Yes ... one of your three cubes."
Student 2: "There are eight dice, sir."
Researcher: "If I replaced the dice with other items with the same shape, for example, bread, then how much bread is arranged into a cube?"
Student 1: "27 bread pack."
Student 2: "8 bread, sir?"

From the conversation above, it can be seen that student 1 (group 1) and student 2 (group 2) can determine the number of dice that can form or form cubes. The role of progressive mathematics can be illustrated from the results of conversations with students one and students 2 in the stages of formulating formal mathematical concepts. At the situational stage, this activity arises when students one answer will arrange the cube with the given dice. While students two will arrange the initial layer with four dice. The stage of referential can be seen from the ability of students to compose a cube with the arrangement of three dice in the front, three side dice, and three dice upwards. While students two will rearrange four dice in the second layer. In the global stage, student activity 1 appears by giving the cube conclusions that have the same side while students two conclude the sides are the same. In the formal phase, the activities of students 1 and 2 can form another cube in units according to the object used, for example, bread. Based on the results of student activities determining the volume of the cube, the role of continuous mathematical processes looks based on stages in the development of mathematical models.

Dialog 2 (specifies a beam-shaped unit box):
Researcher: "I will ask from group 3. How do you arrange blocks like in Figure (d)?"
Student 3: "I try to pack the dice into boxes?" (students 3 are representatives of group 3)
Researcher: "What was imagined first before composing the beam?"
Student 3: "Boxes, like a gift or box."
Researcher: "Next, what do you do?"
Student 3: "Arrange dice with more length than the width of the chest."
Researcher: "Has the beam been formed in the first arrangement?"
Student 3: "Already. Because there are remaining dice, then I continue to compile the next dice."
Researcher: "How many dice are there in Figure (d)?"
Student 3: "There are 30 dice. With a great arrangement of 5 dice, there are three widths of dice so that the initial layer of 15 dice is arranged. The remaining 15 dice are topped with them."
Researcher: "As a conclusion, how many dice do students form into blocks?"
Student 4: "(while counting) ... there are 30 dice, sir." (student 4 is representative of group 4)
Researcher: "What is the arrangement of the blocks?"
Student 4: "I made 15 dice long, then I stacked another 15 dice."
Researcher: "Check, how long, width, and height with dice?"
Student 4: "(While counting) ... there is 15 dice length, there is one dice width, and two dice height."
Researcher: "Just like the initial question, if I replaced the dice with other items with the same shape, for example, bread, then how much bread is arranged into blocks?"
Student 4: "30 loaves of bread."

The role of progressive mathematics at the situational stage, this activity arises when students will try to arrange blocks from the given dice. In the stage of referential, it appears on the activities of students who will compile blocks with inspiration from beam shapes such as gifts or boxes. In the
global stage, student activities emerge based on the knowledge gained. Namely, students arrange blocks with a concept of length that is more than the width. The formal stage, seen from the activities of students who were able to deduce the beam concept, namely in the first arrangement with 15 dice students were able to conclude the arrangement was formed by a beam and a combination of 15-second dice formed a block with 30 dice. Also, students four can conclude that if the compilation of unit cubes from bread, then the volume of the block is 30 loaves.

3.2. Second Activity: Filling Box
In the second stage, each group of students is asked to put the dice into a cuboid or block box. Activities carried out by students include: students fill dice into boxes until they are full, students fill dice in boxes layer by layer, and students fill dice inboxes through rows of length, width, and height. The following activities carried out by students are presented in Figures 4 and Figure 5.

![Figure 4. Dice Filling Process into Cubes](image1)

The filling process carried out in Figure 4 (a) is, the students fill the bottom to the brim, then continue the top layer until the cube is full. In Figure 4 (b), students fill dice into the cube in an upright position, starting on the side and hold until the cube is filled. In Figure 4 (c), students fill the cube with positions with long, comprehensive, and high rows until they are filled with cubes with dice. Then students fill the beam using unit dice. The processing activity of determining the number of dice that fills the beam is shown in Figure 5.

![Figure 5. The Process of Filling the Dice in Beams](image2)

Based on Figure 5 (d), student activity determines the volume of the beam by filling the beam with the dice starting from the upright position starting from the front side until the beam is filled to the brim. In Figure 5 (e), student activity determines the volume of the beam by filling the dice from the position of the lower layer, continuing the layer above until the beam is filled. In Figure 5 (f), student activity fills the beam with a position with an upright row starting from the side and continuing until the beam is filled. In the dice filling activity, students can fill blocks with dice until they are full, and students are also able to count the dice that fills the beam.
To find out the process of compiling cubes and blocks of unit boxes, interviews were conducted with several students. In this interview, the researcher interviewed three students consisting of 5 students, six students, and seven students. The following is a discussion quotation about determining cubes and blocks of unit boxes.

**Dialog 3 (specify the volume of a cube):**

Researcher: "What is the volume of this cube (while pointing to a hollow cube of cardboard), try to explain the students?"

Student 5: "I fill the cube with dice until it is full."

Researcher: "How many dice did you fill into the cardboard box to the brim?"

Student 5: "27, dice, sir."

Researcher: "The group of students 6, I see you fill it a little differently. How do you fill the cube to the brim?"

Student 6: "I begin to fill the side of the cube in a standing position until the cube is filled."

Researcher: "What is the number of one-layer dice standing?"

Student 6: "for a moment, pack count first (while counting) ... there are nine dice."

Researcher: "how many times is the layer, so the curbs are full."

Student 6: "3 layers."

Researcher: "How many dice fill the cube to the brim?"

Student 6: "27 dice."

**Dialog 4 (determine the volume of the beam):**

Researcher: "How do you determine the volume with this dice with the shape of a beam?"

Student 7: "I just fill this block with dice until it is full."

Researcher: "Is there another way to determine the volume of the beam?"

Student 7: "I fill the bottom layer to the brim, continue to fill the next layer until it is full."

Researcher: "Are there people who fill this block to the full?"

Student 7: "There are 30 dice, sir."

Researcher: "how can you arrange another block with this 30 dice?"

Student 7: "I will stack the length of the chest for 30."

Researcher: "Are there other arrangements?"

Student 7: "I arrange the bottom chart with as many as ten dice with arrangements 5 and 2, then I stack up to 3 times the stack."

Researcher: "Look at beam 1, block 2, each of which has 30 dice. What are your conclusions about beams?"

Student 7: "Beam one consists of a 5-dice length arrangement, three dice width, and two dice height; block two consists of 5 dice in length, two dice width, and three dice height, each illustrating 30 dice contents."

Researcher: "So what are the conclusions with volume or content of blocks."

Student 7: "The contents or volume of the beam are the same as the side length times the side width times the height of the side of the beam."

Based on researchers between students with students 5, students 6, and students 7, it seems they can arrange the volume of cubes and beams appropriately. The progressive role of mathematics in drafting the concept of cube and beam is drawn from the stages [9, 16]. At the situational stage, the activities of students comprise the volume of cubes and beams that can be seen from the answers given, namely, they can be seen from the given dice. In the stage of reference, it appears that students who will arrange the volume of cubes and beams by filling a standing position or starting from the base are filled to the full. On the global stage, students can arrange various forms of cubes and beams. The formal stage, seen from activities of students who can deduce the concept of the volume of cubes and beams, is multiplied by the width of the side multiplied by the height.
3.3. Retrospective analysis
At this stage, the HLT that has been made is compared to the actual student learning process, which shows learning by HLT that is designed, and students understand the material of volume cubes and beams. Based on the role of progressive mathematicians, the process of building mathematical concepts from real contexts to form formal mathematics. In this case, the teacher as a learning facilitator in building students' knowledge to develop mathematical models and concepts with the stages of model formation, namely situational, referential, general, and formal stages [9, 11, 16]. At the situational stage, students can carry out activities given based on particular situations; then, students do simulations by filling in transparent cubes with dice or unit cubes. The learning process with RME starts from a context to discover mathematical concepts by constructing mathematical models themselves through mathematical processes, then using mathematical models to solve contextual problems.

In the reference stage, to find the volume of cubes and beams, students do other simulations with various shapes of cubes or blocks with unit cubes from dice. The simulation results of cubes and beams are recorded about the length, width, height of the cube, and beam shapes by counting the number of unit dice. In the global stage, students calculate the volume of cubes and beams using the prepared student worksheets, where the unit cubes are arranged in the form of verbal and visual representations. In the formal stage, students can formulate the concept of volume building space by using mathematical symbols based on the conclusions of the global stage. Namely, the volume of the cube or beam is the area of the base times the height of the cube or beam.

4. Conclusion
Progressive mathematical analysis based on four stages in the development of a mathematical model, namely determining the volume of cubes and beams. The activities of the 4 levels of model development include: (1) At the situational level, it appears that students do activities to arrange dice to determine the contents of cubes and beams to determine the unit volume of cubes and beams using a unit dice situation as a form of situation to be measured; (2) At the reference level, students have started using dice-composing strategies that will be arranged based on knowledge about cubes and beams with the number of dice available; (3) At the general level, it can be seen that students can mention the number of arrangement of cubes that form cubes and blocks with dice units; (4) At the formal level, students can conclude that objects or objects composing cubes and beams are in the form of volume units with a formula length high times.

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