Proposal for quantum many-body simulation and torsional matter-wave interferometry with a levitated nanodiamond

Yue Ma,1,2 Thai M. Hoang,3 Ming Gong,4,5,* Tongcang Li,3,6,7,8,† and Zhang-qi Yin1,‡

1 Center for Quantum Information, Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing 100084, China
2 Department of Physics, Tsinghua University, Beijing 100084, China
3 Department of Physics and Astronomy, Purdue University, West Lafayette, IN 47907, USA
4 Key Laboratory of Quantum Information, University of Science and Technology of China, Hefei, 230026, Anhui, China
5 Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei, 230026, P.R. China
6 School of Electrical and Computer Engineering, Purdue University, West Lafayette, IN 47907, USA
7 Purdue Quantum Center, Purdue University, West Lafayette, IN 47907, USA
8 Birck Nanotechnology Center, Purdue University, West Lafayette, IN 47907, USA

Hybrid spin-mechanical systems have great potentials in sensing, macroscopic quantum mechanics, and quantum information science. In order to induce strong coupling between an electron spin and the center-of-mass motion of a mechanical oscillator, a large magnetic gradient is usually required, which is difficult to achieve. Here we show that strong coupling between the electron spin of a nitrogen-vacancy (NV) center and the torsional vibration of an optically levitated nanodiamond can be achieved in a uniform magnetic field. Thanks to the uniform magnetic field, multiple spins can strongly couple to the torsional vibration at the same time. We propose to utilize this new coupling mechanism to realize the Lipkin-Meshkov-Glick (LMG) model by an ensemble of NV centers in a levitated nanodiamond. The quantum phase transition in the LMG model and finite number effects can be observed with this system. We also propose to generate torsional superposition states and realize torsional matter-wave interferometry with spin-torsional coupling.

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I. INTRODUCTION

Micro and Nano-mechanical resonators in the quantum regime, based on light-matter interaction, have wide applications in quantum metrology and quantum information science [1]. It is one of the best test-beds for generating macroscopic quantum superpositions, and studying quantum-classical boundaries [2, 3]. To this end, mechanical resonators need to be coupled to other systems, such as atoms [4], superconducting circuits [5, 6], cavity modes [1], nitrogen-vacancy (NV) centers [7–9], etc. Among these systems, NV centers attract a great attention [10] due to the extraordinary long coherence time (ms) even at room temperature [11] as well as its high manipulation and detection efficiency. The center-of-mass motion can even be coupled to the electron spins in magnetic field with a large gradient [7, 17]; typically this gradient should be of the order of $10^7$ T/m, which is difficult to achieve in stat-of-art experiments. Furthermore, this large magnetic gradient prevents collective coupling between NV electron spin ensemble and the mechanical oscillator.

The motion of the nanoparticles can behave in a totally different way when levitated in a high quality vacuum by optical trapping [18–20]. In this case the nanoparticles can vibrate along different directions controlled by the external optical field. In recent years this torsional vibration for a nonspherical nanodiamond in an optical trap in vacuum was observed [21]. It was also proposed that a torsional mode can be cooled down to the ground state by a linearly polarized cavity mode [21, 22]. In this work, we investigate the coupling between the torsional vibration of a levitated nanodiamond and NV center electron spins (see Fig. 1). The orientation of an NV center will change together with the torsional vibration of the nanodiamond [23], thus even in a uniform magnetic field the energy levels of the NV electron spins can still depend strongly on its orientation as well as its displacement from the origin, which induce coupling between the torsional vibration and the NV spin. We find that strong coupling can be reached with a modest uniform magnetic field (for example 0.05 T), thus can circumvent the technical difficulty mentioned above [7, 17].

We also propose several applications of spin-torsional coupling. We show how to realize matter-wave interferometry, and propose to use the collective coupling between an ensemble of NV electron spins and the torsional mode to realize the Lipkin-Meshkov-Glick (LMG) model [24–26]. The LMG model was first introduced in nuclei physics for phase transitions, and has been found to be relevant to a large number of quantum systems such as Bose-Einstein condensates in different traps [27–30], the Bardeen-Cooper-Schrieffer superconducting model [31], the radiation-matter Dicke model [32–35] and cavity QED [36]. Up to now the special case of LMG model was realized [37, 38] in ultracold atoms and the
corresponding transition from Rabi dynamics to Josephson dynamics has been reported, yet the full LMG model has never been experimentally realized. We show that the quantum phase transition in the LMG model and finite number effects can be observed in our proposed system.

II. SPIN-TORSIONAL COUPLING

We consider a non-spherical nanodiamond with one long axis and two short axes optically trapped in high vacuum [21, 39] in a static uniform magnetic field (Fig. 1). The direction of the nanodiamond can be manipulated and aligned with the laser field [21, 23, 40]. We consider the torsional vibration of the nanodiamond along \( \theta \) direction around the polarization direction of the laser beam. The torsional Hamiltonian is \( H_{\text{tor}} = \omega b b \) (with natural unit \( \hbar = 1 \)). Typically \( \omega_b \) is of the order of MHz.

We first consider a single NV center in the nanodiamond. The direction of the magnetic field is denoted as \( z \) while the intrinsic quantization direction of the NV center is denoted as \( z' \) (see Fig 1). The Hamiltonian of the NV center is \( H_{\text{NV}} = DS_z^2 + \mu_B B \cdot S \), where \( D = 2.8 \) GHz for typical NV center. \( S_z^{(z')} \) is the spin-1 operator along \( z(z') \) direction. If we use the eigenvectors of \( S_z \) to expand \( H_{\text{NV}} \), and define \( \Delta = -g \mu_B B \), the Hamiltonian becomes [41]

\[
H_{\text{NV}} = \begin{pmatrix}
D - \Delta \cos \theta & \Delta z' \sin \theta & 0 \\
\Delta z' \sin \theta & 0 & \Delta \sin \theta \\
0 & \Delta \sin \theta & D + \Delta \cos \theta 
\end{pmatrix} \quad z'.
\]

If the gradient of \( B \) is large, the strong coupling between the translational motion of the diamond and the spin \( S \) could be achieved [9, 17].

Here we suppose that \( B \) is homogeneous, and \( \theta \) changes with the torsional motion. The angles \( \theta_0 \) and \( \phi_0 \) denote the equilibrium orientation. The eigenvalues for \( H_{\text{NV}} \) is determined by the following cubic function,

\[
z^3 - 2Dz^2 + (D^2 - \Delta^2)z + D\Delta^2 \sin^2(2\theta) = 0,
\]

which is independent of phase \( \phi \). The calculated eigenvalues \( z = E_i \) \( (i = -1, 0, 1) \) for a modest magnetic field as a typical example is shown in Fig 2(a). We only deal with two spin states, \( |S_{z''} = -1 \rangle \) and \( |S_{z''} = 0 \rangle \), where \( S_{z''} \) is the spin operator in whose representation \( H_{\text{NV}}(\theta_0, \phi_0) \) is diagonal. We change the energy zero point to \( E_0 \) and define the energy of \( |S_{z''} = -1 \rangle \) as \( E(\theta, \phi) = E(\theta) = E_{-1}(\theta) - E_0(\theta) \).

The total Hamiltonian for the NV center and the torsional oscillator reads

\[
H = \omega_b b b + \frac{E(\theta_0)}{2} \sigma_{z''} + \frac{g_N}{2} \sigma_{z''} (b^\dagger + b),
\]

where \( \sigma_{z''} = |S_{z''} = -1 \rangle \langle S_{z''} = -1| - |S_{z''} = 0 \rangle \langle S_{z''} = 0| \). We let \( |S_{z''} = -1 \rangle \equiv | -1 \rangle \) and \( |S_{z''} = 0 \rangle \equiv | 0 \rangle \). This is the representation that we will focus on in the rest of this paper. The coupling between the torsional mode and NV center electron spin is

\[
g_N = \sqrt{\frac{1}{2I\omega_b}} \left. \frac{\partial E(\theta)}{\partial \theta} \right|_{\theta = \theta_0},
\]

where \( I \) is the moment of inertia, and \( \omega_b \) is the angular frequency of the torsional mode [21]. As shown in Fig. 2(b), \( g_N / 2\pi \) could be about 300 kHz at 0.05 T, which is much larger than both torsional mode decay (\( < \) kHz) [21] and the NV center decay (\( < \) kHz) and dephasing (\( \sim \) kHz) rates [42]. Therefore the strong coupling condition is fulfilled. In experiments the value of \( g_N \) can be tuned in a wide range by controlling either the trap potential or the external uniform magnetic field. The typical energy scales used in this paper are summarized in Table I.
III. THE LMG MODEL WITH LEVITATED NV CENTERS

Here we show that this novel platform provides an excellent opportunity to simulate the LMG model [24–26]. For the nanodiamond considered here, the mean separation between NV centers is assume to be $d > 15$ nm and the direct dipole-dipole interactions between NV centers ($J_{d-d} < 20$ kHz [44]) are much smaller than the spin-torsional coupling ($\sim 300$ kHz at 0.05 T). The coherence time of NV centers in nanodiamond at such low concentration could be around ms [11, 42, 43]. As multiple NV centers are coupled to the same torsional mode coherently in a uniform magnetic field, the torsional mode mediating coupling between NV centers can be strong using a proper experimental scheme. To realize the LMG model, a microwave driving field with frequency $\omega_0$ and Rabi frequency $\Omega$ is added. The Hamiltonian of the system that contains multiple NV centers and the microwave drive is,

$$H = \omega_0 b^\dagger b + E_0 \sum_{\alpha} S_{\alpha}^{zz} + \frac{g N}{2} \sum_{\alpha} S_{\alpha}^{xx} (b^\dagger + b) + \frac{\Omega}{2} e^{-i\omega_0 t} S^+ + \text{h.c.},$$

(5)

where $S_{\alpha} = \sum_{j=1}^{N} \sigma^{\alpha}_{\alpha,j}$ for $\alpha = x', y'$ and $z'$ are the total spin operators for a system with $N$ NV centers in $\alpha$' direction. $S^+ = (S_{x'} + iS_{y'})/2$ and $S^- = (S_{x'} - iS_{y'})/2$. NV centers along other directions (see Fig 1) can be neglected as they have very different energy levels. In our model, the torsional mode mediated spin flip is forbidden in the $x'' - y'' - z''$ coordinate. Thus the last driving term should be included to realize the long-range LMG model. The above time-dependent model, in which $E_0$ and $\omega_0$ are the dominant frequencies, can be transformed to the low-frequency stationary Hamiltonian $H_{\text{eff}} = U^\dagger H U - U^\dagger i \partial_t U$ via a unitary rotation $U = e^{-i\Omega_{\text{eff}} S_{z'} / 2}$. We obtain

$$H_{\text{eff}} = \omega_0 b^\dagger b + E_0' \sum_{\alpha} S_{\alpha}^{xx} + \frac{E_0'}{2} \sum_{\alpha} S_{\alpha}^{zz} (b^\dagger + b),$$

(6)

where $E_0' = E_0 - \omega_0$. So the microwave driving condition fulfilled with $E_0' = 0$, in which case an isotropic LMG model can be realized.

To adiabatically eliminate the influence of the torsional mode, we need to go into a rotating frame. Let us define $H_{\text{eff}} = H_0 + V$. Here $H_0 = \omega_0 b^\dagger b + \frac{\Omega}{2} S_{\text{z'}}$ defines which the rotating frame we use, where $\Omega_{0}$ is the frequency of the rotating frame for NV centers. In an experiment, it could be adjusted by changing the frequency of a local reference oscillator. $V$ is treated as perturbation, which is a quite good approximation regarding that $\Omega \sim \Omega_0 \sim \omega_0$ is the leading term in Eq. (6), while all the other coefficients are much smaller than these energy scales (see Table I). After the transformation the Hamiltonian $H_{\text{eff}}$ is represented by the interaction term

$${V_{\text{int}}} = \frac{g N}{2} [\tilde{S}_x b^\dagger e^{i(\Omega_0 + \omega_0) t} + \tilde{S}_y b e^{i(\Omega_0 - \omega_0) t} + \text{h.c.}] + \frac{h_0}{2} \tilde{S}_z^2,$$

(7)

where $\tilde{S}_x = (\tilde{S}_x + i\tilde{S}_y)/2, \tilde{S}^- = (\tilde{S}_x - i\tilde{S}_y)/2$ and $h_0 = \Omega - \Omega_0$. Notice that here we have adopted the notations: $S_{x'}^+ = S_z, S_{y'}^+ = -S_y, S_{z'}^+ = S_x$.

Here we consider many NV centers in a single nanodiamond. The single spin-torsional coupling strength $g_N \propto 1/\sqrt{\omega_0}$ depends on the torsional frequency $\omega_0$ and the size of the nanodiamond. We suppose the concentration of NV center is kept as a constant. As the size of nanodiamond increases, both $I$ and $N$ will increase. However, the coupling strength $g_N$ decreases. We suppose that $g_N \propto 1/\sqrt{N}$. In the following discussion, we define $g = \sqrt{N g_N}$ as the collective spin-torsional coupling strength. The effective Hamiltonian can be derived using the method in Refs. [45–47]. In the limit that $\Omega_0 \pm \omega_0 \gg g_N, h_0$, we get

$$H_{\text{LMG}} = \frac{\lambda}{N} (\tilde{S}_x \tilde{S}_x + \tilde{S}_y \tilde{S}_y) + h \tilde{S}_z,$$

(8)

where $\lambda = g^2 - \frac{\omega_0}{\Omega_0^2 - \omega_0^2}, h = \frac{h_0}{2} + \frac{\Omega_0}{\Omega_0^2 - \omega_0^2} b^\dagger b + \frac{1}{2} g^2/2N$.

We mainly focus on the condition of ferromagnetic coupling ($\lambda < 0$) when $\omega_0 > \Omega_0$. The effective Zeeman field is consisted of two parts: the contribution from the external Zeeman field inherent from the original Hamiltonian and the contribution from phonons. Notice that the second part is due to the fact that the interaction $V_{\text{int}}$ breaks the time-reversal symmetry. This term will become unimportant if the total number of NV centers $N$ is large enough. The first term $h_0$ will play primary role for the phase transition near torsional ground state or when $N$ is much larger than the torsional thermal phonon number $b^\dagger b$.

| $(E_0', \omega_0)$ | Size | $\omega_0$ | $g_N$ | $\Omega_0$ | $g$ | $\lambda$ | $|h_0|$ |
|------------------|------|-----------|------|----------|-----|----------|-------|
| ~2000 (60, 40) nm | 2.52 | 0.1 | 1.0 | 0.42 | 0.01 | ~0.01 |

In the thermodynamic limit, the LMG model hosts phase transition at $h = \lambda$. In realistic systems, the num-

TABLE I. Typical parameters. All the frequencies are in unit of $2\pi \times$ MHz.
meander of NV centers in a diamond nanocrystal is always finite and can be controlled by doping. For a 80 nm-diameter diamond, the total number of NV color centers can be in the range of $N \sim 0 - 100$, which is sufficient to observe the phase transitions \([50, 51]\). In this model the order parameters are defined as $m_{xy} = \frac{2}{\lambda} \sqrt{\langle S_x^2 + S_y^2 \rangle}$ for the in-plane polarization, and $m_z = \frac{2}{\lambda} \sqrt{\langle S_z^2 \rangle}$ for the out-of-plane polarization\([52-54]\). For a finite system, we find $m_z = 1$ when $|h/\lambda| > 1$, and otherwise,

$$m_z = \begin{cases} \frac{1}{N} + \frac{|h/2N|}{2}, & N \text{ is odd} \\ \frac{1}{2N} + \frac{|h/2N|}{2}, & N \text{ is even} \end{cases}$$

where $|x|$ takes the integer part of $x$. The in-plane polarization can be determined via $m_{xy} = \sqrt{1 + \frac{4}{N} - m_z^2}$. We see that the discontinuous jump of $m_z$ and $m_{xy}$ can be observed at $\lambda = \frac{\lambda}{2N}$ for $k = 1, 2, 3, \cdots \left\lfloor \frac{N}{2} \right\rfloor$ when $N$ is odd and $\lambda = \frac{\lambda}{4N}$ for $k = 1, 2, 3, \cdots \left\lfloor \frac{N}{4} \right\rfloor$ when $N$ is even. The jump of these quantities arises from the quantization of the spin. These two polynomials will collapse to the well-known continue limit, $m_{xy} = \sqrt{1 - (h/\lambda)^2}$ and $m_z = \frac{1}{N}$ for $|h/\lambda| < 1$, when $N \to \infty$.

Both analytical results and numerical results for $m_{xy}$ and $m_z$ (see Fig. 3) suggest that the strong signature of quantum phase transition can be observed even in a small system. When $|h/\lambda| > 1$, the out-of-plane polarization $m_z$ exactly equals to one, while the in-polarization can still be finite ($m_{xy} = \sqrt{1 - \frac{1}{N} \approx 0.3}$ when $N = 18$). This is different from the proposal for classical phase transition in Ref. 48, where an experimentally observable effect requires an extraordinary large number of NV centers.

To observe the phase transition, we can measure $m_z$ as a function of $h/\lambda$. We prepare the NV centers in ground state, then adiabatically tune the parameter $h/\lambda$. The exact degeneracy at the jumping point should be removed by a modest in-plane Zeeman field. Experimentally, we measure the population of the spin in the intrinsic quantization direction $z'$. We can first apply a microwave pulse that rotates the state $S_z$ to the state $S_{\psi}$, and then measure the spin state in the intrinsic frame.

IV. SCHRÖDINGER’S CAT STATE AND TORSIONAL MATTER-WAVE INTERFEROMETRY

Schrödinger’s cat state is generally considered as an entangled state between a microscopic quantum system and a macroscopic system. It can be prepared with an optically levitated nanodiamond using the coupling between the center-of-mass motion and the electron spin with a strong magnetic gradient \([17]\). Here we show how to realize Schrödinger’s cat state with torsional motion and spin in a uniform magnetic field.

First we need to cool the torsional motion near ground state $|0\rangle_\theta$ by sideband cooling \([21, 55, 56]\). Then we adiabatically lower the trapping frequency from $\omega_\theta$ to $\omega'_\theta$, so that $|0\rangle_{\omega_\theta}$ evolves to a new vacuum state $|0\rangle_{\omega'_\theta}$. The spin is initialized to $|0\rangle_\theta \oplus |1\rangle_\theta$ by sideband cooling \([17]\) and a macroscopic system. It can be prepared with an optical field.

We can use matter-wave interferometry to verify the creation of the Schrödinger’s cat state. First, we impose pulses to disentangle the spin and the oscillation \([17]\). With the displacement operator $D(\beta) = \exp(\beta b^\dagger - \beta^* b)$, we get $D(\beta)|0\rangle_{\omega'_\theta} = |\beta\rangle_{\omega'_\theta}$, and $D(-\beta)|0\rangle_{\omega'_\theta} = |\beta\rangle_{\omega'_\theta}$.

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The torsional motion state becomes $|\psi(t_0)\rangle = \frac{1}{\sqrt{2}} \left( e^{\frac{i E_t}{h/\lambda} t_0} - 1 \right) \otimes e^{-\frac{\lambda}{2} (|\beta\rangle_{\omega'_\theta} - |\beta\rangle_{\omega'_\theta})}$.

which is the Schrödinger’s cat state. For convenience we define $\beta = gN/\omega'_\theta$. With the displacement operator $D(\beta) = \exp(\beta b^\dagger - \beta^* b)$, we get $D(\beta)|0\rangle_{\omega'_\theta} = |\beta\rangle_{\omega'_\theta}$, and $D(-\beta)|0\rangle_{\omega'_\theta} = |\beta\rangle_{\omega'_\theta}$.

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2π angle, the interference fringe becomes very complex. We only need to focus on the region without the complicated pattern.

V. CONCLUSION

We discuss the strong coupling mechanism between the torsional motion of a nanodiamond and the spin of built-in NV centers under a homogeneous magnetic field. This novel system can be used for simulating LMG model and the many-body phase transitions. Strong evidence for this phase transition can even be observed for a small nanodiamond containing only a few tens of NV centers. The system may also be used to realize Schrödinger cat state and the corresponding torsional matter-wave interferometry.

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Appendix A: Generation of Schrödinger’s cat state

In this part, we provide a derivation of the Eq. (10). Suppose the system evolves under the Hamiltonian

\[ H' = \frac{\omega_0}{2} b^\dagger b + \frac{E(\theta_0)}{2} \sigma_{z''} + \frac{g_N}{2} \sigma_{z''}(b^\dagger + b) \]  

What we would like to do is calculating \( e^{-iH't}(|0\rangle + |-1\rangle)/\sqrt{2} \otimes |\omega_b'\rangle \), but this cannot be done directly. Instead, it is better to find a way to consider the effect of the parts in \( H' \) separately. In a rotating frame, \( H' \) becomes

\[ H'_1 = \frac{g_N}{2} \sigma_{z''}(be^{-i\omega_b't} + be^{-i\omega_b't}) \]  

\( H'_1 \) is time-dependent, and its corresponding time evolution operator is \( U_{t} = Te^{-i\int_{0}^{t} H'_1(t)dt} \), \( T \) is the time-ordering operator. \( U_{t} \) can be expanded by Magnus expansion, \( U_{t} = e^{\sum_{k=1}^{N} U_{k}(t)} \). The first order is

\[ \Omega_1 = -i \int_{0}^{t} H'_1(t_1)dt_1 \]

\[ = \frac{g_N}{2} \left( \frac{e^{-i\omega_b't} - 1}{\omega_b'} \right) \]

The second order is

\[ \Omega_2 = \frac{1}{2} \frac{1}{(-i)^2} \int_{0}^{t} \int_{0}^{t_1} \left[ H'_1(t_1), H'_1(t_2) \right] dt_1 dt_2 \]

\[ = \frac{g_N^2}{4} \left( \frac{t}{\omega_b'} - \sin(\omega_b't) \right) \]

As \( \Omega_2 \) does not contain any operators, it is just a global phase of the state and can be neglected in our situation. Moreover, \( \Omega_3 = \Omega_4 = \ldots = 0 \). So \( U_{t} = e^{\beta(t)b^\dagger - \beta(t)'b} \), where \( \beta(t) = \frac{g_N}{2\omega_b'} \). After we return to the original frame, the quantum state at time \( t \) can be expressed as

\[
\begin{align*}
|\psi(t)\rangle &= e^{-i\omega_b't}b^\dagger b|0\rangle e^{-i\frac{E(\theta_0)t}{2}}e^{\frac{g_N}{2} \sigma_{z''}}\left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right) \\
&= e^{-i\omega_b't}b^\dagger b|0\rangle e^{-i\frac{E(\theta_0)t}{2}}e^{\frac{g_N}{2} \sigma_{z''}}\left(e^{i\omega_b'(t-1)}b - (e^{-i\omega_b'(t-1)}b)^\dagger\right) \\
&= e^{-i\omega_b't}b^\dagger b \frac{1}{\sqrt{2}} \left(|-1\rangle \otimes e^{-i\frac{E(\theta_0)t}{2}}\sigma_{z''} e^{\frac{g_N}{2} \sigma_{z''}} \right) |0\rangle_{\omega_b'} \\
&= e^{-i\omega_b't}b^\dagger b \frac{1}{\sqrt{2}} \left(|-1\rangle \otimes e^{-i\frac{E(\theta_0)t}{2}} \sigma_{z''} e^{\frac{g_N}{2} \sigma_{z''}} |0\rangle_{\omega_b'} \right) \\
&= e^{-i\omega_b't}b^\dagger b \frac{1}{\sqrt{2}} \left(|-1\rangle \otimes e^{-i\frac{E(\theta_0)t}{2}} \right) |0\rangle_{\omega_b'} \\
&= e^{\frac{E(\theta_0)t}{2}} |0\rangle \otimes e^{-\frac{g_N}{\omega_b'} \left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right)} |0\rangle_{\omega_b'} \\
&= e^{-\frac{g_N}{\omega_b'} \left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right)} |0\rangle_{\omega_b'} \otimes e^{\frac{E(\theta_0)t}{2}} |0\rangle \otimes e^{-\frac{g_N}{\omega_b'} \left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right)} |0\rangle_{\omega_b'} \\
&= e^{\frac{E(\theta_0)t}{2}} |0\rangle \otimes e^{-\frac{g_N}{\omega_b'} \left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right)} |0\rangle_{\omega_b'} \\
&= e^{\frac{E(\theta_0)t}{2}} |0\rangle \otimes e^{-\frac{g_N}{\omega_b'} \left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right)} |0\rangle_{\omega_b'} \\
&= e^{\frac{E(\theta_0)t}{2}} |0\rangle \otimes e^{-\frac{g_N}{\omega_b'} \left(e^{-i\omega_b'(t-1)}b - (e^{i\omega_b'(t-1)}b)^\dagger\right)} |0\rangle_{\omega_b'}
\end{align*}
\]
While \( \exp \left( \frac{2 \pi}{\omega_\theta} \left( e^{-i \omega_\theta b^t} - 1 \right) b - \left( e^{i \omega_\theta b^t} - 1 \right) b^\dagger \right) \) and \( \exp \left( - \frac{2 \pi}{\omega_\theta} \left( e^{-i \omega_\theta b^t} - 1 \right) b - \left( e^{i \omega_\theta b^t} - 1 \right) b^\dagger \right) \) are likely displacement operators, the effect of \( e^{-i \omega_\theta b^t b^\dagger} \) cannot be seen directly. In fact, if \( e^{-i \omega_\theta b^t b^\dagger} \) first acts on the state \(|0\rangle_{\omega_\theta} \), it will become just a time-dependent phase factor. So we now develop how to “interchange” the first and second factors.

Suppose \( e^X e^Y = K e^X \), \( K \) is the operator we need to find out.

\[
K = e^X e^Y e^{-X} = e^X (1 + Y + \frac{1}{2!} Y^2 + \frac{1}{3!} Y^3 + \cdots) e^{-X} = 1 + e^X Y e^{-X} + \frac{1}{2!} e^X Y e^{-X} e^X Y e^{-X} + \cdots
\]

\[
= 1 + e^X Y e^{-X} + \frac{1}{2!} (e^X Y e^{-X})^2 + \frac{1}{3!} (e^X Y e^{-X})^3 + \cdots = e^X Y e^{-X} = e^Y [X, Y] + \frac{1}{2!} [X, [X, Y]] + \frac{1}{3!} [X, [X, [X, Y]]] + \cdots \tag{A6}
\]

In the last equality, we use the Baker-Hausdorff lemma [57]. First we calculate \( e^{-i \omega_\theta b^t b^\dagger} \exp \left( \frac{2 \pi}{\omega_\theta} \left( (e^{-i \omega_\theta b^t} - 1) b - (e^{i \omega_\theta b^t} - 1) b^\dagger \right) \right) |0\rangle_{\omega_\theta} \).

We let \( X = -i \omega^l \omega_\theta b^t b^\dagger \) and \( Y = \frac{2 \pi}{\omega_\theta} \left( (e^{-i \omega_\theta b^t} - 1) b - (e^{i \omega_\theta b^t} - 1) b^\dagger \right) \) to use Equation (A6). Then we can get

\[
e^{-i \omega_\theta b^t b^\dagger} e^{-i \omega_\theta b^t b^\dagger} \exp \left( \frac{2 \pi}{\omega_\theta} \left( (e^{-i \omega_\theta b^t} - 1) b - (e^{i \omega_\theta b^t} - 1) b^\dagger \right) \right) |0\rangle_{\omega_\theta} = e^{-i \omega_\theta b^t b^\dagger} e^{-i \omega_\theta b^t b^\dagger} |0\rangle_{\omega_\theta} \tag{A7}
\]

The second term in Eq. (A5) can be dealt with under similar process. So finally the state in Eq. (A5) is changed to

\[
|\psi(t)\rangle = \frac{1}{\sqrt{2}} \left( e^{-i \beta(t_0)} |1\rangle - 1 \right) \otimes e^{-i \omega_\theta b^t b^\dagger} e^{-i \omega_\theta b^t b^\dagger} |0\rangle_{\omega_\theta} + e^{-i \beta(t_0)} |0\rangle \otimes e^{-i \omega_\theta b^t b^\dagger} e^{-i \omega_\theta b^t b^\dagger} |0\rangle_{\omega_\theta} \tag{A8}
\]

From the definition of displacement operator \( \hat{D}(\beta) = e^{\beta b^\dagger - \beta^* b} \) which displaces \( b \) to \( b + \beta \) and \( b^\dagger \) to \( b^\dagger + \beta^* \), we can see clearly that in our system when \( t_0 = \frac{\pi}{\omega_\theta} \), the displacement of the equilibrium orientation is the maximum. Thus we get

\[
|\psi(t_0)\rangle = \frac{1}{\sqrt{2}} \left( e^{-i \beta(t_0)} |1\rangle - 1 \right) \otimes e^{-i \omega_\theta (b^\dagger - b)} |0\rangle_{\omega_\theta} + e^{-i \beta(t_0)} |0\rangle \otimes e^{-i \omega_\theta (b^\dagger - b)} |0\rangle_{\omega_\theta} \tag{A9}
\]

This is Eq. (10) in the main text.

**Appendix B: Torsional matter waver interference**

In this part, we discuss how to get the interference fringes as shown in Fig. 4 in the main text. Initially, the wavefunction expressed by the orientation \( \theta \) is \( \psi_0(\theta, t = 0) = (\varphi_+(\theta) + \varphi_-(\theta)) / \sqrt{2} \).

\[
\varphi_+(\theta) = (\theta | \beta) \quad \varphi_-(\theta) = (\theta | - \beta) \tag{B1}
\]

\( \theta_0 \) is the original equilibrium orientation. Normally the normalization of a Gaussian distribution requires the argument to range from \(-\infty \) to \(+\infty \). But here \( \theta \) can only take values between 0 and \( 2\pi \). However, for a Gaussian distribution with mean value \( \mu \) and standard deviation \( \sigma \), the probability is almost 0 if the argument is out of the range \((\mu - 3\sigma, \mu + 3\sigma)\). So the lower and upper limits can be extended to \(-\infty \) and \(+\infty \), respectively. We will show later that this requirement is very easy to be fulfilled in experiment.

As in the standard quantum solutions, if we want to find the time evolution of a wave function, we should decompose it to the linear superposition of the eigenfunctions of the Hamiltonian, because we know the time evolution of the Hamiltonian’s eigenfunctions is just the original state multiplied by a factor \( e^{-i E_n t} \), \( E_n \) is the eigenvalue. This applies to all time-independent Hamiltonian, so it is also suitable for our \( H_{free} \) here.

We expand \( \varphi_+(\theta) \) with the eigenfunctions of \( L_{zm} \) in spherical coordinates, \( \frac{1}{\sqrt{2\pi}} e^{im\theta} \) (\( m = 0, \pm 1, \pm 2, \cdots \)).

\[
\varphi_+(\theta) = \sum_{m=-\infty}^{+\infty} A_m \frac{1}{\sqrt{2\pi}} e^{im\theta} \tag{B2}
\]
According to the orthonormal property of \( e^{im\theta} \), we have

\[
A_m^+ = \int_0^{2\pi} \phi_+(\theta) \frac{1}{\sqrt{2\pi}} e^{-i\theta m} d\theta = \int_0^{\pi} I_{\omega_0}^{\pi/2} \int_{-\infty}^{+\infty} \frac{e^{-i\theta m}}{2\sqrt{\pi}} e^{-\frac{m^2}{\omega_0^2}} e^{-i\theta m} \left( \theta + \sqrt{\frac{m^2}{\omega_0^2}} \right) d\theta
\]

We have used the assumption above to extend the integral interval. Then we can use two formulae of integral for Gaussian distributions: \( \int_{-\infty}^{+\infty} e^{-kx^2} dx = \sqrt{\pi/k} \) for \( k > 0 \), \( \int_{-\infty}^{+\infty} e^{-i\theta x} dx = \int_{-\infty}^{+\infty} e^{-i\theta x} dx = \sqrt{2\pi} \) for arbitrary real number \( \kappa \). The equation above can finally be simplified to

\[
A_m^+ = \sqrt{\frac{1}{\pi I_{\omega_0}}} e^{-\frac{m^2}{2I_{\omega_0}}} e^{-i\theta m} \left( \theta + \sqrt{\frac{m^2}{\omega_0^2}} \right)
\]

(B3)

So we get the expansion of \( \phi_+(\theta) \).

\[
\phi_+(\theta) = \sqrt{\frac{1}{\pi I_{\omega_0}}} \sum_{m=-\infty}^{+\infty} e^{-\frac{m^2}{2I_{\omega_0}}} \frac{1}{\sqrt{2\pi}} e^{-i\theta m} \left( \theta + \sqrt{\frac{m^2}{\omega_0^2}} \right)
\]

(B5)

Similarly,

\[
\phi_-(\theta) = \sqrt{\frac{1}{\pi I_{\omega_0}}} \sum_{m=-\infty}^{+\infty} e^{-\frac{m^2}{2I_{\omega_0}}} \frac{1}{\sqrt{2\pi}} e^{-i\theta m} \left( \theta - \sqrt{\frac{m^2}{\omega_0^2}} \right)
\]

(B6)

For every fixed \( m \), the time evolution factor is \( e^{-iE_m t} \), \( E_m = \frac{m^2}{2I_{\omega_0}} \). So the time evolution of the superposition state becomes

\[
\psi_0(\theta, t) = \sqrt{\frac{1}{16\pi^3 I_{\omega_0}}} \sum_{m=-\infty}^{+\infty} \left( e^{i\theta - (\theta_0 + \sqrt{\frac{m^2}{\omega_0^2}})} + e^{i\theta - (\theta_0 - \sqrt{\frac{m^2}{\omega_0^2}})} \right) e^{-\frac{m^2}{2I_{\omega_0}}} e^{-i\theta m t}
\]

(B7)

The probability distribution \( \psi_0^*(\theta, t)\psi_0(\theta, t) \) should form an interference pattern around the \( 2\pi \) circle. As the state is not the Hamiltonian’s eigenstate, the probability distribution will change with time. So the interference fringe will evolve with time, too. But to deal with this expression, we need to truncate the summation of \( m \) at some finite value. This can be determined from Equation (B5) or (B6). Each term in the summation has a Gaussian part, \( e^{-\frac{m^2}{2I_{\omega_0}}} \). As a property of Gaussian distribution, if \( m \) is larger than \( 3 \times \sqrt{2I_{\omega_0}} \), the contribution of \( e^{-\frac{m^2}{2I_{\omega_0}}} \) will be only around \( 10^{-4} \). So this can be a criterion to truncate the summation.

Now we can discuss an example and show the fringe numerically (Fig. 4 in the main text). Consider a nanodiamond with long semi-axis \( a = 40 \) nm and short semi-axis \( b = 20 \) nm. If the external magnetic field is taken as \( B = 0.05 \) T, the maximum coupling strength will be \( g_B = 2\pi \times 3.31 \times 10^5 \) Hz, and the corresponding equilibrium angle is \( \theta_0 = \pi/4 \). We relax the angular frequency of the trap to \( \omega_0 = 2\pi \times 50000 \) Hz.

When the angular separation of the two Gaussian states is the largest, we remove the trap, and take this time as \( t = 0 \). At this time, the center orientations of the two peak are \( 45^\circ - 1.01^\circ \), \( 45^\circ + 1.01^\circ \). The width of the distribution is very small (as can be seen from Fig. 5(a)). So our approximation above is valid. The maximum of \( m \) should be larger than 2000, we take the limit as 3000 here.

The whole evolution process of the interference fringe is shown in Fig 5.

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FIG. 5. Time evolution of interference fringe for a nano-diamond with long semi-axis 40 nm and short semi-axis 20 nm. The external magnetic field is 0.05 T. The coupling strength is $g_N = 2\pi \times 331$ kHz with the equilibrium orientation $\theta_0 = \pi/4$. The trapping frequency is $\omega_N = 2\pi \times 50$ kHz. When $t = 0$, two original peaks are there, but no interference. At time $t = 0.0005$ s, two peaks start to interfere, but the distribution of fringes is still limited. At time $t = 0.001$ s, wavefunction continues to expand, but they still do not occupy the whole circle. At time $t = 0.0025$ s, the wavefunction starts to occupy the whole circle. At around 225°, the “unnamed” meeting happens. The fringes become unclear around that region. But other regions are not affected yet. When $t = 0.003$ s, wide regions are affected by the superposition effect on a circle. We can estimate that the spacing between two fringes is around 30°, which is large enough to be detected experimentally.

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