Spatial 4U quadrilateral rolling mechanism

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Abstract. A novel of rolling mechanism constructed by four U joints is proposed. The shape of the mechanism is quadrilateral, which consists of four U joints and four links. The straight and turning rolling can be achieved by two actuators. The DOF (degree of freedom) of the mechanism is computed by using screw theory. The rolling motion analysis and the trajectory analysis are carried out. In addition, a prototype is manufactured and a series of experiments are performed to verify the rolling capability of the mechanism.

1. Introduction

Recently, rolling-based locomotion systems are receiving more attention in the literature. The rolling motion is considered as an efficient way to move in some circumstances. Koshiyama A et al. describes a new type of mobile robot that has a spherical wheel, an arched body, and motion-controlling mechanisms installed inside the wheel [1]. Joshi V A et al. proposed a spherical mobile robot which is rolling on a plane with the help of two internal rotors and working on the principle of conservation of angular momentum [2]. Tafreshi et al. proposed a novel fluid actuated system for a spherical mobile robot which is rolled by displacing the cores in the pipes filled with fluid [3]. Rolling gait is a common gait of spherical mobile robot [4-6]. The spherical mobile robot always moves the center of gravity to the contact area where the sphere falls and rolls in that direction [7]. These spherical mobile robots usually do not change their configurations while rolling. Liu and Yao et al. put forward a family of rolling 4R linkages with changeable configuration [8]. Tian and Guo et al. proposed a single-DOF mobile linkage with possibility orientation movements, and the variation of the orientation obeys the Bernoulli distribution [9]. Wang and Yao proposed a ground mobile Bennett mechanism which can move in a constant direction and change its moving direction only by one actuator [10]. Although all these rolling robots are capable of moving functions, they have limited ability to change moving directions.

In this paper, we proposed a novel rolling quadrilateral mechanism based on a spatial 4U mechanism. The spatial 4U quadrilateral rolling mechanism can achieve straight rolling and turning rolling driven by two actuators. In mobility analysis, the spatial 4U mechanism is equivalent to the parallel mechanism and the DOF of the mechanism is calculated. The DOF and kinematic analysis method of parallel mechanism have been extensively studied [11-15]. The rolling motion analysis and the trajectory analysis are carried out, and a prototype is manufactured to verify the rolling capability of the mechanism.
2. Concept Description and Rolling Analysis

2.1. Concept Description
The spatial 4U quadrilateral rolling mechanism proposed in this paper is illustrated in figure 1. The mechanism consists of four U joints \( J_{U1}, J_{U2}, J_{U3}, J_{U4} \) and four links, where the \( J_{Ui} \) \((i=1,2,3,4)\) is split into two revolute joints, and each revolute joint is denoted as \( J_{Ri} \). The axis of \( J_{R1} \) is perpendicular to link 1, the axis of \( J_{R2} \) is along the link 1. Links 1 and 2 are connected by \( J_{U1} \); Links 2 and 3 are connected by \( J_{U2} \); Links 3 and 4 are connected by \( J_{U3} \); Links 4 and 1 are connected by \( J_{U4} \). The 3D model is shown in figure 1(a), and the corresponding schematic diagram in figure 1(b).

The shape of the links is designed as a cross, which makes the landing of the link more stable. The shape and size of links 1, 2, 3 and 4 are identical.

![Figure 1. The spatial 4U quadrilateral rolling mechanism.](image)

2.2. Mobility Analysis
To analyze the motion screws, set the local coordinate system \( o-xyz \) at the center of \( J_{U1} \), the \( x \)-axis is along the axis of \( J_{R11} \), and the \( z \)-axis is along the axis of \( J_{R12} \), as shown in figure 2. So, the screw system of the mechanism can be obtained.

\[
\begin{align*}
S_{1} &= (1 \ 0 \ 0; \ 0 \ 0 \ 0) \\
S_{2} &= (0 \ 0 \ 1; \ 0 \ 0 \ 0) \\
S_{21} &= (1 \ 0 \ 0; \ 0 \ L \ 0) \\
S_{22} &= (0 \ 1 \ 0; \ -L \ 0 \ 0) \\
S_{31} &= (1 \ 0 \ 0; \ 0 \ L \ -L) \\
S_{32} &= (0 \ 0 \ 1; \ L \ 0 \ 0) \\
S_{41} &= (1 \ 0 \ 0; \ 0 \ 0 \ -L) \\
S_{42} &= (0 \ 1 \ 0; \ 0 \ 0 \ 0)
\end{align*}
\]

Where \( L \) is the length of the links 1, 2, 3 and 4. The rank of the eight screws is six. Therefore, according to the modified Griibler–Kutzbach criterion, the DOF of the mechanism is

\[
M = d(n - g - 1) + \sum_{i=1}^{g} f_i + v = 6(4 - 4 - 1) + 8 + 0 = 2
\]
2.3. Rolling Motion Analysis

According to the mobility analysis, the DOF of the 4U mechanism is two, so two actuators are needed for the mechanism to have a defined movement. In this 4U rolling mechanism, two actuators labeled as M1 and M2 in figure 3 are used to drive the four links to leave and land on the ground in turn to realize rolling gaits. The M1 installed at the joint which is perpendicular to the link is used to drive the mechanism to roll. The M2 installed at the joint which is along the link is used to drive the mechanism to turn. In order to balance the center of mass, we select $J_{R21}$ as M1 and $J_{R32}$ as M2.

There are two motion modes when the mechanism is rolling, one is straight rolling mode and the other is turning rolling mode. The straight rolling and the turning rolling can be planned and described in detail as follows.

2.3.1. The straight rolling mode. When the M2 is locked and the axes of $J_{R11}$, $J_{R21}$, $J_{R31}$ and $J_{R41}$ are parallel to each other, the mechanism is in straight rolling mode. At this time, the mechanism is a planar quadrilateral. The 4U rolling mechanism can scroll with all position by an actuator M1 as shown in figure 4, figure 4(a) expresses the link 1 is stationary on the ground and the links 2 and 4 are perpendicular to the link 1. When the rotating direction of M1 is clockwise, the links 2 and 4 are moving in the same direction (Figure 4(b)). Then the link 1 is lifted and the link 2 will land on the ground (Figure 4(c)). Successively, the actuator M1 rotate clockwise, the links 1 and 3 are moving in the same direction to the position as shown in figure 4(d). By successively repeating these sequences, the rolling motion of the rolling 4U mechanism is achieved and the rolling trajectory of the mechanism is a straight line.
2.3.2. The turning rolling mode. In the mode of the turning rolling, the M2 is a turning actuator. The rolling 4U mechanism can turn the direction by two actuators as shown in figure 5. Figure 5(a) expresses the link 1 is stationary on the ground and the links 2 and 4 are perpendicular to the link 1, the M1 and M2 are locked. When the turning rolling mode is performed, the M1 remains locked and the actuator M2 begin to rotate to a certain angle (Figure 5(b)). According to the motion analysis, the mechanism has two DOFs, when a DOF is locked, the mechanism becomes a single DOF spatial four-link mechanism, in this state as show in figure 5(b), the mechanism is a Bennett mechanism. Subsequently, the M2 is locked and the actuator M1 rotates clockwise, the link 1 is lifted and the link 2 will land on the ground (Figure 5(c)-5(d)). And then the actuator M1 continue to rotate clockwise, the M2 still remains locked (Figure 5(e)). Finally, the M1 is locked and the actuator M2 begins to rotate back to that certain angle, the links 1 and 3 are moving in the same direction to the position as shown in figure 5(f). By successively repeating these sequences, the turning rolling mode of the 4U rolling mechanism is achieved and the trajectory in the turning rolling mode is a broken line.

3. Rolling Trajectory Analysis
According to the rolling analysis, the trajectory of the straight rolling mode is a line and the turning rolling is a broken line. To analyze the trajectory of the turning rolling mode, set the coordinate system $o_1x_1y_1z_1$ on the ground, the $y_1$-axis is along the link1; the $x_1$-axis is in the ground and vertical to the $y_1$-axis, as shown in figure 6. Angle $\chi$ is the acute angle between $y_1$-axis and the projection of the link 2...
on the ground. The links 1 and 2 are connected by $J_{R21}$ and $J_{R22}$. The axis of the $J_{R21}$ is perpendicular to the link1, and has an angle $\alpha$ with $x_1$-axis.

![Image: Figure 6. The path analysis.]

The axis vector of the $J_{R21}$ is $[\cos \alpha, 0, \sin \alpha]^T$, the link 2 is perpendicular to the axis. Suppose that the vector before the motion of the link 2 is $R = [-\sin \alpha, 0, \cos \alpha]^T$ in the vertical plane. When the link 2 turns the angle $\beta$ around the revolute axis, its vector $R'$ can be obtained by using the rotation matrix.

$$E = \begin{bmatrix}
\cos \gamma + \lambda^2(1 - \cos \gamma) & \lambda \mu(1 - \cos \gamma) - \nu \sin \gamma & \nu \lambda(1 - \cos \gamma) + \mu \sin \gamma \\
\lambda \mu(1 - \cos \gamma) + \nu \sin \gamma & \cos \gamma + \mu^2(1 - \cos \gamma) & \mu \nu(1 - \cos \gamma) - \lambda \sin \gamma \\
\nu \lambda(1 - \cos \gamma) - \mu \sin \gamma & \mu \nu(1 - \cos \gamma) + \lambda \sin \gamma & \cos \gamma + \nu^2(1 - \cos \gamma)
\end{bmatrix}
$$

In the formula, the vector $[\lambda, \mu, \nu]^T$, that is, the vector of the rotating auxiliary axis $[\cos \alpha, 0, \sin \alpha]^T$.

When the link 2 turns angle $\beta$, the vector is

$$R' = E \cdot R = \begin{bmatrix}
-sin \alpha \cos \beta \\
-sin \beta \\
\cos \alpha \cos \beta
\end{bmatrix}
$$

So the tangent of the angle $\chi$ is

$$\tan \chi = \frac{\sin \alpha}{\tan \beta}
$$

4. Prototype

A prototype is built as shown in Figures 7-8 and the rolling experiments are carried out. A straight line is marked on the ground to compare the trajectory of the mechanism when it is rolling and the start point is indicated by a red dot on the straight line. Figures 7(a)-7(d) show the straight rolling mode process. Figures 8(a)-8(f) show the turning mode rolling process.

![Image: Figure 7. The straight rolling experiment.]
5. Conclusion
A novel spatial 4U quadrilateral rolling mechanism is proposed in this paper. Its concept is described and the mobility is analyzed by using screw theory. The rolling motion and trajectory analysis are carried out. At last, a prototype is manufactured and the proposed rolling mechanism is proved to have the ability of straight and turning rolling through corresponding experiment.

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