Asymmetric surface brightness structure of caustic crossing arc in SDSS J1226+2152: a case for dark matter substructure

Liang Dai 1, Alexander A. Kaurov, Keren Sharon 2, Michael Florian, Jordi Miralda-Escudé, Tejaswi Venumadhav, Brenda Frye, Jane R. Rigby and Matthew Bayliss

1 Institute for Advanced Study, 1 Einstein Drive, Princeton, NJ 08540, USA
2 Department of Astronomy, University of Michigan, 1085 S. University Ave, Ann Arbor, MI 48109, USA
3 Observational Cosmology Lab, NASA Goddard Space Flight Center, 8800 Greenbelt Rd., Greenbelt, MD 20771, USA
4 Institut Catalana de Recerca i Estudis Avançats, Passeig Lluís Companys 23, E-08010 Barcelona, Catalonia, Spain
5 Institut de Ciències del Cosmos, Universitat de Barcelona, Martí i Franquès 1, E-08028 Barcelona, Catalonia, Spain
6 Department of Astronomy/Steward Observatory, University of Arizona, 933 N. Cherry Ave., Tucson, AZ 85721, USA
7 Department of Physics, University of Cincinnati, Cincinnati, OH 45221, USA
8 MIT Kavli Institute for Astrophysics and Space Research, 77 Massachusetts Ave., Cambridge, MA 02139, USA

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ABSTRACT
We study the highly magnified arc SGAS J122651.3+215220 caused by a star-forming galaxy at zs=2.93 crossing the lensing caustic cast by the galaxy cluster SDSS J1226+2152 (zl=0.43), using Hubble Space Telescope observations. We report in the arc several asymmetric surface brightness features whose angular separations are a fraction of an arcsecond from the lensing critical curve and appear to be highly but unequally magnified image pairs of underlying compact sources, with one brightest pair having clear asymmetry consistently across four filters. One explanation of unequal magnification is microlensing by intracluster stars, which induces independent flux variations in the images of individual or groups of source stars in the lensed galaxy. For a second possibility, intracluster dark matter subhaloes invisible to telescopes effectively perturb lensing magnifications near the critical curve and give rise to persistently unequal image pairs. Our modelling suggests, at least for the most prominent identified image pair, that the microlensing hypothesis is in tension with the absence of notable asymmetry variation over a six-year baseline, while subhaloes of $\sim 10^6-10^8 M_\odot$ anticipated from structure formation with cold dark matter typically produce stationary and sizable asymmetries. We judge that observations at additional times and more precise lens models are necessary to stringently constrain temporal variability and robustly distinguish between the two explanations. The arc under this study is a scheduled target of a Director’s Discretionary Early Release Science program of the James Webb Space Telescope, which will provide deep images and a high-resolution view with integral field spectroscopy.

Key words: gravitational lensing: micro – gravitational lensing: strong – galaxies: clusters: individual: SGAS J122651.3+215220; SDSS J1226+2152 – dark matter.

1 INTRODUCTION
Massive clusters of galaxies are powerful cosmic gravitational lenses that often make outstanding magnified images of background galaxies, usually referred to as ‘arcs’. The most dramatic phenomenon takes place when a background galaxy happens to straddle the caustic cast by the foreground gravitational lens, offering an opportunity to closely study an elongated image of the galaxy with a zoomed-in view near the caustic. Smoothly distributed mass in the foreground lens universally produces symmetric pairs of merging images near a fold lensing critical curve. However, intracluster microlensing of individual superluminous stars in the source galaxy (Miralda-Escudé 1991; Venumadhav, Dai & Miralda-Escudé 2017; Diego et al. 2018; Oguri et al. 2018) or subgalactic dark matter (DM) substructure predicted to exist in cluster haloes (Press

* E-mail: ldai@ias.edu
Asymmetry of arc in SDSS J1226+2152

2 DATA

The caustic-straddling lensed arc SGAS J122651.3+215220 was discovered in 2007 December at the 2.5-m Nordic Optical Telescope (Koester et al. 2010) and is in the catalog of SGAS (Hennawi et al. 2008). The arc redshift is determined to be \( z_s = 2.9260 \pm 0.0002 \) from nebular lines (Rigby et al. 2018). The spectroscopic redshift of the galaxy cluster lens SDSS J1226+2152 is \( z_l = 0.43 \) as reported by Bayliss et al. (2014). Data were used to perform lens modelling with the LENSTOOL software package (Jullo et al. 2007). It approximates the cluster mass distribution as a linear superposition of mass haloes and then uses Markov Chain Monte Carlo sampling to determine the best-fitting set of parameters for the lens model. The fitting procedure for analogous clusters is described in Sharon et al. (2020).
Table 1. Archival HST imaging of SDSS J1226+2152 used in this work. Multiple visits in individual filters are combined into single drizzled science images sampled at a pixel scale of 0.03 arcsec.

| Instrument | Filter | Exposure (ks) | Date       | Program number | PI   |
|------------|--------|--------------|------------|----------------|------|
| ACS        | F606W  | 2.0          | 2011-04-14 | 12368          | Morris |
| ACS        | F606W  | 2.0          | 2011-04-15 | 12368          | Morris |
| ACS        | F814W  | 2.0          | 2011-04-14 | 12368          | Morris |
| ACS        | F814W  | 2.0          | 2011-04-15 | 12368          | Morris |
| WCF3       | F110W  | 1.2          | 2017-12-18 | 15378          | Bayliss |
| WCF3       | F160W  | 1.3          | 2017-12-18 | 15378          | Bayliss |

The arc S1226 was imaged in multiple HST filters. We use for our analysis images taken with the Advanced Camera for Surveys (ACS) in two optical filters, F606W and F814W, and with the Wide Field Camera 3 (WFC3) in two IR filters, F110W and F160W. The images in the optical filters were obtained 6 yr prior to the images obtained in the IR filters. Table 1 summarizes the observations. In Fig. 1, we show a composite false-colour image of the cluster and a zoomed-in view of the arc.

The arc will be observed with the Near Infrared Spectrograph (NIRSpec) and the Mid-Infrared Instrument (MIRI) integral field unit (IFU) and imaged with the Near Infrared Camera (NIRCam) in multiple filters aboard JWST, as part of the DD-ERS program (see Table 2 for details).

3 ASYMMETRIES IN THE ARC

According to our macrolens model, the portion of the critical line that passes through the arc is nearly a straight line and is approximately perpendicular to the direction of arc elongation (shown as the white curve in Fig. 1), except that a small cluster member galaxy to the north-west (also shown in Fig. 1 with the small white curve) slightly deforms the surface brightness profile of the arc. The focus of this study is on the brightest part of the arc on the south which is not significantly influenced by the cluster member galaxy (see discussion in Section 7.1). Images of the arc in four HST filters are shown in Fig. 2, after rotation to make the principal axis of elongation of the arc horizontal. Our five slits are shown in the bottom panel, on the same image of the arc as in Fig. 1. If the lens surface mass distribution is locally smooth, a symmetric appearance across the critical curve is expected.

In Section 3.1, we first qualitatively show evidence for departure from symmetry using a simple method which was previously applied to a similar caustic-straddling arc in MACS J0416.1–2403 (Kaurov et al. 2019). In this method, we align multiple slits with the direction of arc elongation, and identify prominent bright features on both sides of the critical curve that are likely to be image pairs of major underlying compact sources. We then undertake a rigorous measurement of their flux inequality that is described in Section 3.2.

3.1 Slits

We define five 1 arcsec × 0.17 arcsec slits along the critical curve, each of which is positioned to align with the direction of arc elongation and to include a pair of major surface brightness features, as shown in Fig. 2. For every HST filter, we sum the flux along the perpendicular direction, and plot the summed flux as a function of the position along the slit direction in Fig. 3.

In Slit D, a major pair of flux peaks in both F606W and F814W is clear, with sizable asymmetries that are compatible between the two filters. The pair of peaks is also present in F110W and F160W, but are broadened by the wider point spread functions (PSFs). In each of Slits B, C, and E, a major pair of unequal peaks are also visible in F606W and F814W, but other minor peaks make the pair identification less clear. Asymmetric double-peaked profiles in rough correspondence are also visible in the two IR filters. The profiles in Slit A are more complex with multiple peaks of comparable heights and are harder to interpret.

To further explore the presence of asymmetric image pairs, we apply in Appendix A a non-physical technique to search for asymmetric features which flexibly accounts for small deviations from perfect fold symmetry. As shown there, this method highlights the asymmetric features in Slits C and D in F606W and F814W, as well as the feature in Slit E in F606W. The most outstanding feature in Slit D is also clear from the results for F110W and F160W.

In each filter separately, and for each asymmetric feature under study, we first create a suitable cutout which consists of $N_{pix}$ pixels and contains the presumptive image pair. The cutout is chosen to be large enough to capture most information provided by the PSFs of the image pair, but cropped to exclude complex neighbouring surface brightness structures. We evaluate a likelihood function $L = \exp(-\chi^2/2)$, where we define the chi-squared function:

$$\chi^2 = \sum_{i=1}^{N_{pix}} \frac{1}{\sigma_i^2} \left[ d_i - \sum_{j=1}^{2} F^{(j)} \cdot p^{(j)}(x^{(j)}) - \sum_{j=1}^{n_{bg}} c^{(j)} b^{(j)}(p^{(j)}) \right]^2. \quad (1)$$

Here $d_i$ is the flux in pixel $i$, which we fit to the superposition of two PSFs representing a pair of unresolved images centred at $x^{(j)}$ with fluxes $F^{(j)}$ for $j = 1, 2$ respectively, and $n_{bg}$ components of diffuse surface brightness background with profiles $b^{(j)}$, parametrized by an additional set of parameters $p^{(j)}$, for $j = 1, 2, \ldots, n_{bg}$ respectively. We introduce the inverse weights $\sigma_i$ to quantify flux errors, making the simplifying assumption that errors are uncorrelated among pixels. In this work, we set all $\sigma_i$’s to be a uniform number $\sigma_F$ (i.e. the white noise approximation). Refer to Appendix B for our choices for the diffuse background profiles $b^{(j)}$ and $\sigma_F$.

In equation (1), the noise $\sigma_i$ should not be interpreted as a photometric error only. In fact, the HST images are deep enough that detector noise and photon shot noise are subdominant. We adopt the philosophy that $\sigma_F$ mainly reflects our ignorance about...
Asymmetry of arc in SDSS J1226+2152

Figure 1. False colour RGB images in the field of the galaxy cluster SDSS J1226+2152, assembled from the HST images in F110W, F814W, and F606W filters respectively. In the left-hand panel, the lensing critical curves corresponding to a source redshift $z = 2.93$ are drawn as the white curves, which pass through the magnified arc of interest. The right-hand panel is a 4.5 arcsec $\times$ 4.5 arcsec image cutout centred at (RA, Dec.) = (12:26:51.31, +21:52:19.62). The exact location of intersection between the critical curve and the arc is subject to uncertainty in the macrolens model. The arc forms as two images of the source galaxy merge at the position of the critical curve, which results in the mirror-symmetric appearance of the arc. In fact, the highly magnified arc constrains the local position and shape of the critical curve better than other far away lensed sources used to construct the macro model.

Table 2. Forthcoming JWST observations with proposed filters/methods and exposure times (number 1335, PI: Rigby, Rigby et al. 2017).

| Instrument | Filters/methods | Exposure (ks) |
|------------|-----------------|---------------|
| NIRSPEC    | F170LP, G235H (IFU) | 8.3          |
| MIRI       | F560W, MRS (IFU)  | 3.6          |
| NIRCAM     | F115W, F277W     | 0.3          |
| NIRCAM     | F150W, F356W     | 0.3          |
| NIRCAM     | F200W, F444W     | 0.3          |

the details of an underlying population of minor surface brightness structures on the arc, acting as an effective noise background when we fit the image pairs. To avoid overfitting due to overconstrained background templates used in equation (1), we set an empirical value for $\sigma_F$ by requiring that the $\chi^2$ per effective degree of freedom in equation (1), for the maximal likelihood solution, should be around unity. In this way, we include modelling uncertainties about fainter surface brightness structures into our quoted error bars.

The likelihood evaluation is then coupled to the sampler PYMULTINEST (Buchner et al. 2014) to derive the joint posterior distribution of the model parameters. Flat priors are adopted for all parameters over conservatively wide sampling ranges.

We apply the above methodology to the asymmetric image pairs in Slits C, D, and E, which are visually discernible in all four HST filters. From the measured normalizations for the two fitted PSFs, we compute the fractional flux asymmetry

$$a_F = 2 \frac{F^{(1)}_v - F^{(2)}_v}{F^{(1)}_v + F^{(2)}_v},$$  \hspace{1cm} (2)

in accord with the fractional magnification asymmetry that will be defined later in equation (4), and the absolute flux difference $\Delta F_v = F^{(1)}_v - F^{(2)}_v$, where $F^{(1)}_v$ ($F^{(2)}_v$) is the flux of the brighter

Figure 2. Asymmetric surface brightness features in the arc. Top four panels separately show four HST filters. Bottom panel shows the same false-colour image as in Fig. 1 with the five 1 arcsec $\times$ 0.17 arcsec slits, defined to be roughly perpendicular to the cluster critical curve. The brightness profiles along these slits are shown in Fig. 3. The cluster critical curve is nearly vertical in this figure, and the slits are parallel to the direction of arc elongation.
Flux asymmetries of features in Slits C and E are also robustly detected in the F606W and F814W filters, with probable values in the range \( \alpha_F = 0.2 \text{--} 1.0 \). Asymmetries and absolute flux measurements in F110W and F160W are subject to large uncertainties because centroiding of point sources is affected by blending in these two filters with wide PSFs. The asymmetry is nevertheless compatible with that seen in F606W and F814W, and a source colour consistent with star-forming regions. We therefore find evidence that even the features in Slits C and E exhibit significant asymmetry in optical filters, although consistency in IR filters remains unproven.

Having demonstrated that asymmetric image pairs arise in several places along the critical curve, with at least one bright pair (Slit D) being well measured, in the next sections we will explore the physical interpretation of this phenomenon.

### 4 Macro-lens Model near Critical Curve

At the location of arc, we infer from the cluster lens model a ratio \( \kappa_0 = \Sigma_0/\Sigma_{\text{crit}} = 0.8 \) between the coarse-grained total surface mass density \( \Sigma_0 \) of the lens at the line of sight and the critical surface mass density (Blandford & Narayan 1986) \( \Sigma_{\text{crit}} = (c^2/4\pi G)(D_L/D_s D_{DL}) \approx 2 \times 10^6 M_\odot \) kpc\(^{-2} \), where \( D_L \), \( D_s \), and \( D_{DL} \) are the angular diameter distances to the cluster lens at \( z_L = 0.43 \), to the source galaxy at \( z_s = 2.93 \), and from the lens to the source. This is the local coarse-grained value for the lensing convergence of the macrolens. Near the critical curve, a compact source appears as a pair of macro images, each of which has an unsigned magnification factor

\[
\mu = \frac{1}{2|\Delta\theta| (1 - \kappa_0) d |\sin \alpha|}
\]

where \( \Delta\theta \) (signed) equals to half the angular separation between the image pair, \( d \) is the gradient of inverse magnification in the critical curve vicinity, and \( \alpha \) is the angle between the critical curve and the direction of arc elongation. Numerically, we measure from our cluster lens model \( d \approx 7.5 \) arcmin\(^{-1} \) and \( \alpha \approx 90^\circ \), which gives \( \mu \approx 140/(|\Delta\theta|/0.14 \text{ arcsec})^{-1} \). While the lensed arc considered here at \( z_s = 2.92 \) is substantially more distant than the caustic straddling lensed galaxies studied in MACS J1149.5+2223 (Kelly et al. 2018) and MACS J0416.1–2403, the magnification factor as a function of image separation across the critical curve is similar in order of magnitude to the values found in those systems.

### 5 intracluster stars

The line of sight to the arc is at a projected distance of \( B \approx 50 \) kpc to the brightest cluster galaxy (BCG). Intervening intracluster stars can originate from several sources: (i) diffuse light extending from the BCG (Zwicky 1951; Lin & Mohr 2004; Zibetti et al. 2005); (ii) diffuse light extending from a major cluster member galaxy \( \sim 4 \) arcsec south of the arc; and (iii) diffuse light extending from a minor cluster member galaxy just \( \sim 2 \) arcsec away from the arc to the north-west. The minor member galaxy acts as a substructure lens, causing a compact and bright star-forming source to appear in multiple images, as can be seen in the right-hand panel of Fig. 1. The colours measured from the four HST filters are consistent between components (i) and (ii), while it is difficult to reliably determine the colours of component (iii) due to arc contamination. Inferred from the image taken in the reddest filter F160W, component (iii) can be safely neglected, and component (ii) is subdominant compared to the diffuse light halo of the BCG, if not entirely negligible.
The major axis of the BCG diffuse light nearly intersects the lensed arc. We fit the isophotes in each *HST* filter to ellipses centred at the BCG and adopt the same ellipticity for all four filters. We find consistent colours wherever contamination from other sources are negligible. The surface brightness at the location of the arc calculated from the fitted isophote ellipses is within a factor of 2 when compared to a few nearby places that are not on top of the arc.

We use the population synthesis code Flexible Stellar Population Synthesis (FSPS) to model intracluster stars (Conroy, Gunn & White 2009; Conroy & Gunn 2010) assuming the initial mass function of Kroupa (2001). We find that the colours are consistent with an old simple stellar population, with a single age and metallicity. The age ranges from $t_{\text{top}} = 7$ Gyr for low metallicity log ($Z/Z_\odot$) $= -1.0$ to $t_{\text{top}} = 1.5$ Gyr for high metallicity log ($Z/Z_\odot$) $= 0.3$. Varying log ($Z/Z_\odot$) from $-1.0$ to $0.3$ correspond to a conservative range $0.002 < \kappa < 0.009$. While breaking the age–metallicity degeneracy necessitates spectroscopic analysis, we think the most likely age is $t_{\text{top}} \approx 3$ Gyr with metallicity log ($Z/Z_\odot$) $\approx -0.3$, a value found typical for intracluster light (ICL) at similar cluster-centric radii in many cluster lenses of similar redshifts (Montes & Trujillo 2018). This translates into a local surface density of intracluster stars $\Sigma_{\text{ICL}} \approx 10^4 M_\odot$ kpc$^{-2}$ and hence a convergence $\kappa_* = \Sigma_{\text{ICL}}/\Sigma_{\text{crit}} \approx 0.005$, which we use hereafter as our fiducial value. This value is comparable to previous estimates carried out for caustic straddling lensed galaxies in MACS J1149.5+2223 (Oguri et al. 2018) and in MACS J0416.1−2403 (Kaurov et al. 2019).

In calculating $\kappa_*$, we have included into the synthesized stellar population white dwarfs in addition to damped lines (MSs) and evolved stars, but have neglected neutron stars and black holes. The latter group are in any case remnants of massive stars $M \gtrsim 8 M_\odot$, which account for only a subdominant fraction $\lesssim 14$ per cent of the initial star-forming mass. In our model, intracluster stars have masses in the range $0.08$–$1.4 M_\odot$. Insensitive to modelling uncertainty, the most abundant microlenses are MS dwarfs whose masses are around $\sim 0.2 M_\odot$.

6 INTRACLUSTER MICROLENSING

The observed flux of a compact source is affected by stellar microlenses near the line of sight, which render the two macro images unequally bright at a given epoch. One possibility is that each observed asymmetric pair of images reflects the microlensing phenomenon acting on either a single superluminous source star or a compact group of bright source stars in the lensed galaxy. Under this hypothesis, no substructure in the lens mass distribution other than that in individual stellar microlenses needs to be invoked to explain the asymmetries.

Intracluster stars should break the smooth macro critical curve into an interconnected network of micro critical curves, whose full width is $2 \kappa_*/d = 0.03$–0.14 arcsec, with our best estimate being 0.08 arcsec. This network band has one edge at the expected location of the macro critical curve, and the other edge lies on its interior (Vennumadhav et al. 2017). Under the assumption that the macro critical curve roughly bisects the asymmetric pairs of surface brightness features in Fig. 2, none of the features lie within this band. This implies that microlensing variability is infrequent and occurs over a long time-scale for any single-lensed star.

6.1 Microlensing of a single source star

To corroborate this, in Fig. 5 we simulate random realizations of the total microlensing magnification factor for one of the macro images across a small region on the source plane, assuming a macrolens model appropriate for S1226, and stellar microlens masses randomly drawn from a simple stellar population model with age $t_{\text{top}} = 3$ Gyr and log ($Z/Z_\odot$) = $-0.3$. The resolution of these magnification maps is $\sim 1$ au. For $\kappa_*$ = 0.002–0.009 and $\Delta \theta = \pm 0.14$ arcsec, appropriate, for example for the prominent asymmetries in Slits D and C, the range of source-plane length-scales indicates that the magnification may vary by order unity on a time-scale of several years for a typical effective source–lens relative velocity $v_r$ (see equation 12 of Vennumadhav et al. 2017) of $\sim 100$ au yr$^{-1}$.

If the asymmetric fluxes observed in several slits (e.g. Slits C, D, and E, and especially Slit D) are due to microlensing of a single source star, the pairs of macro images must have large differences in their magnification factors. At $z_s = 2.93$, only the most luminous source stars in the rest-frame ultraviolet (UV) undeniably high magnification can explain the observed flux asymmetries reported in Fig. 4. The best constraints are obtained from the F814W filter because the asymmetric features are brighter than in F606W, and sharper than in F110W and F160W. In F814W, the image pair flux difference is safely brighter than magnitude $m = 28.5$ in Slits C.
Figure 5. Magnification maps of random source-plane realizations for one of the two macro images of a point source appropriate for the caustic straddling arc S1226. The source-plane region in each panel maps to the image-plane vicinity of a macro image that is $\Delta \theta = \pm 0.14$ arcsec away (appropriate for image pairs in Slits C and D) from the smooth cluster critical curve. We do not consider DM substructure lensing in these plots. The macrolens model near the intersection of the arc and the cluster critical curve is assumed to be a simple fold model (Schneider, Ehlers & Falco 1992) with parameters $\kappa_0 = 0.8$, $d = 7.5$ arcmin$^{-1}$, and $\alpha = 90^\circ$. The three columns correspond to three likely values for the total convergence from intracluster microlenses: $\kappa_* = 0.002, 0.005, \text{and } 0.009$. The top (bottom) row corresponds to the macro image on the interior (exterior) of the smooth critical curve with $\Delta \theta = 0.14$ arcsec ($\Delta \theta = -0.14$ arcsec). The coordinate system is oriented such that $y_1$-axis and $y_2$-axis map to the two eigendirections of the lensing Jacobian matrix of the macrolens model on the image plane, with $y_1$-axis corresponding to the direction of nearly vanishing eigenvalue. Scales along the $y_1$ and $y_2$ axes are highly different for better visualization.

There is no evidence for rare stars (e.g. zero metallicity and extremely massive stars) that might be much brighter than those listed in Table 3. The asymmetric image pairs, for example in Slits C and D, appear barely resolved in the F814W image. Given the angular distance and D, and is likely as bright as $m = 28.5$ in Slit E. Assuming $m_{F814W} \leq 28.5$ mag, which is conservative for Slits C and D, we list in Table 3 the required minimum magnification differences, $|\Delta \mu|_{\text{min}}$, needed to explain the measured flux asymmetries for several candidate types of stars.

The best candidate source stars are low-metallicity blue supergiants (BSG), but even these require a magnification asymmetry $|\Delta \mu| \gtrsim 600$. The most luminous MS stars require much higher magnifications of $|\Delta \mu| \gtrsim 3000$, which are much less likely. This is under the optimistic assumption of zero dust reddening, while Chisholm et al. (2019) report $E(B-V) = 0.13$ mag measured for S1226. Since we have conservatively compute for $m_{F814W} \leq 28.5$ mag, the magnification difference needs to be another factor of four larger in order to explain the asymmetry in Slit D.

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The required values of $|\Delta \mu|$ in Table 3 clearly exceed the expected macro magnification $\bar{\mu} \approx 140$ by a large factor, and are rarely reached as seen in Fig. 5. This large magnification difference is only possible when the source star undergoes a microcaustic transit. Fig. 6 indicates that $|\Delta \mu| \gtrsim 600$ randomly occurs for less than a few percent of the time. Microlensing of a single source star is therefore unlikely to be the cause of the flux asymmetries seen in the arc. The main difference between the case of S1226 and the caustic straddling galaxies lensed by MACS J1149.5+2223 and MACS J0416.1–2403, which have confirmed microlensed individual stars, is the lower redshift of the latter sources, $z = 1.49$ and 0.94, making it much more likely for a microlensed luminous star to reach the faintest observable fluxes.

6.2 Microlensing of a group of source stars

The large asymmetric fluxes are more naturally explained by a group of luminous stars (possibly co-existing with many more fainter stars). Indeed, hot luminous stars commonly cluster in compact star-forming regions or open clusters. The age of the arc stellar population, estimated to be $\sim 20–26$ Myr from rest-frame UV spectroscopy (Chisholm et al. 2019), is longer than that of the BSGs in Table 3, which implies that the stellar light is dominated by giant B stars that are individually less massive and luminous than the BSGs in Table 3.

The asymmetric image pairs, for example in Slits C and D, appear barely resolved in the F814W image. Given the angular distance
Table 3. Examples of ultraluminous stars and their properties as generated by FSPS: (1) stellar age $t_{\text{age}}$; (2) metallicity $\log (Z/Z_{\odot})$; (3) stellar mass $M_*$; (4) effective temperature $T_{\text{eff}}$; (5) stellar radius $R_*$; (6) bolometric luminosity $L_{\text{bol}}$; and (7) minimum magnification difference $|\Delta \mu|_{\text{min}}$ for an image pair flux asymmetry $m_{\text{F814W}}=28.5$ as seen from $z_s=2.93$ in the absence of dust reddening (conservative lower bound for the measured flux asymmetry of image pairs in Slits C and D; see Fig. 4). The flux asymmetry measured in F814W for the image pair in Slit D is larger by another magnitude, requiring even higher magnifications. Redshifted from $z_s=2.93$, only very massive hot O-type MS stars or evolved BSGs are sufficiently bright in the F814W filter; smaller or colder stellar types would require implausibly large $|\Delta \mu|_{\text{min}} \gtrsim 10^4$. Even for hot O-type MSs or BSGs, $|\Delta \mu|_{\text{min}}$ is much greater than the predicted macro magnification factor in equation (3).

| Stellar type        | $t_{\text{age}}$ (Myr) | $\log (Z/Z_{\odot})$ | $M_*$ ($M_{\odot}$) | $T_{\text{eff}}$ (K) | $R_*$ ($R_{\odot}$) | $L_{\text{bol}}$ ($10^6 L_{\odot}$) | $|\Delta \mu|_{\text{min}}$ |
|---------------------|-------------------------|-----------------------|----------------------|----------------------|---------------------|-----------------------------------|-----------------|
| Blue supergiant     | 3                       | 0.0                   | 42                   | 22 000               | 64                  | 0.88                             | 1869            |
| Blue supergiant     | 3                       | -0.3                  | 47                   | 13 000               | 274                 | 1.9                              | 1064            |
| Blue supergiant     | 3                       | -1.0                  | 66                   | 16 000               | 205                 | 2.4                              | 655             |
| Blue supergiant     | 3                       | -2.0                  | 66                   | 16 000               | 200                 | 2.4                              | 645             |
| Main sequence       | 0.3                     | 0.0                   | 115                  | 50 000               | 17.5                | 1.7                              | 3317            |
| Main sequence       | 0.3                     | -0.3                  | 117                  | 52 000               | 16                  | 1.7                              | 3565            |
| Main sequence       | 1                       | -1.0                  | 107                  | 53 000               | 15                  | 1.6                              | 4127            |
| Main sequence       | 0.3                     | -2.0                  | 118                  | 60 000               | 12                  | 1.7                              | 4085            |

Figure 6. Numerically derived cumulative distribution for the magnification difference $\Delta \mu$ between the two macro images of a single source star. The assumption of a point source is valid for either MS or giant star across the range of $|\Delta \mu|$ shown here. Curves are plotted for $k_*=0.002$, 0.005, and 0.009. The grey vertical line marks the required minimum magnification asymmetry $|\Delta \mu| \geq 600$ for the example BSGs considered in Table 3. The tail of the distribution at $|\Delta \mu| \gtrsim 10^3$ steepens due to finite pixels in numerical ray shooting. The choice $\Delta \theta = \pm 0.14$ arcsec is appropriate for Slits C and D. The grey vertical line indicates the required magnification difference for the most luminous BSGs available in our model stellar population, and suggests that a single source star is unlikely to account for the observed flux asymmetry.

Figure 7. Cumulative distribution for the magnification asymmetry $\Delta \mu_g$ for $k_* = 0.005$. This is similar to Fig. 6, but for a group of $N_*$ identical source stars subject to uncorrelated microlensing. The grey shaded band corresponds to 10 per cent–50 per cent magnification asymmetry estimated for the asymmetric features shown in Fig. 3. For each value of $N_*$ (colour coded), a vertical line is drawn to indicate $|\Delta \mu| = 600/N_*$. The location of the grey shaded band in this plot favours $N_* \sim 10$–30 brightest stars.

from the cluster critical curve, our macrolens model constrains their physical sizes to be $\lesssim 5$ pc. Neglecting closely bound multiple stars for the time being, member stars within the group should have mutual separations on the order of parsecs, which is much greater than the source-plane correlation lengths $\sim 10^2$–$10^3$ au of microlensing magnifications as shown in Fig. 5.

This implies that at any given time individual luminous stars have statistically independent microlensing magnifications. Under this assumption, we show in Fig. 7 the probability distribution of $\Delta \mu_g$ for a star group, where $\mu_g$ is the (microlensing affected) magnification averaged over $N_*$ stars for one (unresolved) macro image of the star group on one side of the macro critical curve. As $N_*$ increases, the distribution of $\Delta \mu_g$ approaches a Gaussian centred at zero with decreasing width. For simplicity, we assume that all $N_*$ stars are identical. A small number $N_*$ is disfavored for the same reason as in the previous subsection (typical values of $|\Delta \mu_g|$ are insufficient to explain the flux asymmetries). The minimum $|\Delta \mu_g|$ to account for the flux asymmetries is reduced as $\propto 1/N_*$. We find that a value $N_* \sim 10$–30 is most consistent with the level of macro image asymmetry seen in Fig. 3. This number yields a typical magnification difference $|\Delta \mu_g|$ induced by uncorrelated microlensing of $\sim 10$ per cent–50 per cent of $\mu_g \approx 140$, shown as the shaded vertical band in Fig. 7. A number $N_* \sim 100$–$500$ of stars of comparable brightness might still produce a ratio $|\Delta \mu_g|/\mu_g$ compatible with Fig. 7, but each star should then be fainter in order not to overproduce the observed flux of either macro image. For $N_* \gtrsim 500$, $\Delta \mu_g$ would be highly diluted and a flux asymmetry as
large as ~10 per cent–50 per cent would be implausible. For a more accurate analysis, one could replace the simplifying assumption of identical source stars with a model of a continuous luminosity function. We leave such analysis for future investigation.

If \( N_\star \) ultraluminous stars form a system more compact than \( \sim 10^2 \) au, their microlensing magnifications will be highly correlated. They may simultaneously have a magnification asymmetry \( |\Delta \mu| \sim 600/N_\star \). For \( N_\star \sim 3–6 \), the value for \( |\Delta \mu| \sim |\Delta \mu| \) would be typical of the distribution shown in Fig. 6. The lower required magnification may then be more likely to occur, but the tight star clusters required may not be abundant enough to explain the multiple observed asymmetric image pairs.

To summarize, by measuring unequal image pairs at just one random epoch per HST filter, we conclude that while the hypothesis of microlensing of a single ultraluminous source star is hardly viable, the asymmetric effects might be explained by a group of stars, under one of the following three scenarios: uniform microlensing of \( N_\star \sim 3–6 \) extremely luminous BSGs tightly bound within \( \sim 10^2 \) au, or uncorrelated microlensing of a group of \( N_\star \sim 10–30 \) such BSGs within a region of size \( \lesssim O(\text{pc}) \), or uncorrelated microlensing of a cluster of \( N_\star \sim 100–500 \) less luminous stars (most likely B-type giants) within a region of size \( \lesssim O(\text{pc}) \). As we will see next, frequent temporal variability of the flux asymmetry is generally expected from these microlensing scenarios.

6.3 Temporal variability of flux asymmetry

The flux asymmetry between the two macro images induced by microlensing is expected to vary with time as each source star slowly traverses the microlensing magnification pattern on the source plane (as in Fig. 5). As a result, the asymmetry can frequently change its sign.

As the star number \( N_\star \) of a group increases, the overall flux asymmetry should vary more rapidly because the flux asymmetry from each member star varies independently and there are more frequent micro caustic crossings, but the fractional size of the asymmetry is more diluted. This is clearly reflected in Fig. 8, where we show numerical examples of flux asymmetry variation from randomly drawn microlens realizations, up to the uncertainty in the time-scale that scales inversely with the unknown effective transverse velocity parameter \( v_t \).

Since the arc has not been imaged at more than one epoch in any single filter, we cannot rule out temporal variation in the flux asymmetries with complete certainty. Still, the asymmetries in Slit D seen in F606W and F814W are very similar to those seen in F110W and F160W, if not identical, even though the two optical filters were used at two different epochs \( F_110W \) and \( F_160W \), if not identical, even though the two optical filters were used at two different epochs.

\[ a_\mu \equiv 2 (|\mu| - |\mu'|)/(|\mu| + |\mu'|). \]  

(4)

Let \( M_b \) be the mass of a faraway perturber, \( \theta_p \) its Einstein angular scale, and \( b \) the angular impact parameter from the image pair. We estimate that \( |a_\mu| \) is of the order (derived in Appendix C)

\[ |a_\mu| \sim \frac{0.04}{(\Delta \theta / 0.14 \text{arcsec})} \times \left( \frac{5d}{10^4 M_\odot} \right) \left( 7.5 \text{ arcmin}^{-1} \right) \left( \frac{2 \text{ arcsec}}{b} \right)^4. \]  

(5)

where we assume \( b \gg \Delta \theta \) and that the perturber is at the cluster redshift \( z_\star = 0.43 \). The perturber galaxy would therefore have to enclose a mass \( \sim 10^{12} M_\odot \) within \( \lesssim 10 \) kpc to produce tens of percent asymmetry. In fact, a blue compact source, probably a yellow star cluster associated with the arc at \( z_\star = 2.93 \), is lensed by this foreground galaxy into multiple images separated by a critical curve loop of size \( \sim 1 \) arcsec. Taking into account that a macro magnification \( \sim 30 \) should be acting to enhance the effect of the perturber, we find the small size of the critical curve loop inconsistent with the central part of the foreground galaxy (where star light is detected) enclosing \( \sim 10^{11} M_\odot \).

In Fig. 9, we further confirm that the galaxy perturber alone could not have caused the magnification asymmetry. For every image point, we locate the counter image point and calculate \( a_\mu \) as defined in equation (4).

In our smooth lens model, \( a_\mu \) only reaches a few percent within \( \sim 0.2 \) arcsec of the critical curve, which is compatible with our order of magnitude estimate in equation (5).

7 SUBSTRUCTURE LENSING

In Section 6, we have studied flux asymmetries from intracluster microlensing operating on miniscule angular scales far beyond telescope resolution. In this section, we consider substructure lensing from either small galaxies or star-free DM subhaloes inside the cluster halo, which also cause flux asymmetries as we show below. The characteristic lensing angular scales of \( \sim 0.01–0.1 \) arcsec are marginally resolved in diffraction limited exposures. Unlike microlensing, asymmetric patterns resultant from substructure lensing do not exhibit noticeable variability over time-scales of decades to centuries.

7.1 Galaxy perturbers

The smooth lens model inevitably deviates from the idealized fold model (Schneider et al. 1992) away from the macro critical curve. Large perturber lenses far away in projection can induce curvature in the macro critical curve on arcsecond scales. Indeed, the right-hand panel of Fig. 1 shows that our smooth lens model predicts such an example of curvature due to a minor foreground galaxy \( \sim 2 \) arcsec to the north-west. However, these large and distant perturbers are usually unable to generate magnification asymmetries larger than \( \sim 10 \) per cent between image pairs with small separation \( \Delta \theta \).

Consider an image of a point source on one side of the critical curve, with magnification \( \mu_1 \) and its counter image with magnification \( \mu_2 \). Define the signed fractional magnification asymmetry

\[ a_\mu \equiv 2 (|\mu| - |\mu'|)/(|\mu| + |\mu'|). \]  

(4)

Let \( M_b \) be the mass of a faraway perturber, \( \theta_p \) its Einstein angular scale, and \( b \) the angular impact parameter from the image pair. We estimate that \( |a_\mu| \) is of the order (derived in Appendix C)

\[ |a_\mu| \sim \frac{0.04}{(\Delta \theta / 0.14 \text{arcsec})} \times \left( \frac{5d}{10^4 M_\odot} \right) \left( 7.5 \text{ arcmin}^{-1} \right) \left( \frac{2 \text{ arcsec}}{b} \right)^4. \]  

(5)

where we assume \( b \gg \Delta \theta \) and that the perturber is at the cluster redshift \( z_\star = 0.43 \). The perturber galaxy would therefore have to enclose a mass \( \sim 10^{12} M_\odot \) within \( \lesssim 10 \) kpc to produce tens of percent asymmetry. In fact, a blue compact source, probably a young star cluster associated with the arc at \( z_\star = 2.93 \), is lensed by this foreground galaxy into multiple images separated by a critical curve loop of size \( \sim 1 \) arcsec. Taking into account that a macro magnification \( \sim 30 \) should be acting to enhance the effect of the perturber, we find the small size of the critical curve loop inconsistent with the central part of the foreground galaxy (where star light is detected) enclosing \( \sim 10^{11} M_\odot \).

In Fig. 9, we further confirm that the galaxy perturber alone could not have caused the magnification asymmetry. For every image point, we locate the counter image point and calculate \( a_\mu \) as defined in equation (4). In our smooth lens model, \( a_\mu \) only reaches a few percent within \( \sim 0.2 \) arcsec of the critical curve, which is compatible with our order of magnitude estimate in equation (5).
Asymmetry of arc in SDSS J1226+2152

Figure 8. Numerical random realizations of the microlensing induced temporal variability for the fractional magnification asymmetry \( \Delta \mu_{lg}/\bar{\mu} \) between the two macro images over a baseline of decades. The panels correspond to different values of \( N_* \), the number of identical source stars. The time-scales indicated on the horizon axis can be rescaled with the effective source–lens relative transverse velocity \( v_t \) along the direction of high elongation, for which we use a fiducial value \( v_t = 400 \text{ km s}^{-1} \). The vertical axis shows the logarithm \( -2.5 \log(|\Delta \mu_{lg}|/\bar{\mu}) \), which measures change in the flux asymmetry in units of photometric magnitude. In each light curve, the red (blue) portion corresponds to the macro image to the interior (exterior) of the cluster critical curve being brighter (fainter) than the macro image to the exterior (interior) of the cluster critical curve. The horizontal shaded band in grey corresponds to fractional flux asymmetry \(|\Delta \mu_{lg}|/\bar{\mu}| = 10 \text{ per cent–50 per cent.}\) We note that oscillations of tiny amplitude on very short time-scales are pixelization artefacts in our ray-shooting simulation (most visible in the \( N_* = 1 \) curve).

7.2 Dark matter subhaloes

While no evidence supports that any minor foreground galaxy significantly breaks the symmetry near the cluster critical curve, we hypothesize that a population of abundant and non-luminous DM subhaloes may be the reason. Unlike the larger but rarer perturbers, subgalactic subhaloes are predicted to be numerous enough to be frequently found close to the critical curves, where their perturbing effects are greatly enhanced (Minor, Kaplinghat & Li 2017; Dai et al. 2018).

To numerically assess the effect of subhaloes expected from the standard CDM theory, we populate the cluster DM halo with randomly generated subhaloes, which are superimposed on top of an ideal fold model with parameters appropriate for the arc S1226. We follow the method outlined in Dai et al. (2018) using an extrapolation of the semi-analytic model of Han et al. (2016) down to subhalo-host mass ratios as small as \( m_{sh}/M_{host} \sim 10^{-8} \). For simplicity, we model the host DM halo of SDSS J1226+2152 using a spherical Navarro–Frenk–White (NFW) profile (Navarro, Frenk & White 1996; Navarro, Frenk & White 1997), with a characteristic mass \( M_{200} = 1.5 \times 10^{14} \text{ M}_\odot \) and a concentration parameter \( C_{200} \simeq 13 \). These broadly reproduce a measured Einstein radius \( \theta_E \simeq 10 \text{ arcsec} \) (Oguri et al. 2012), a local convergence \( \kappa_0 = 0.8 \) at the line of sight as inferred from our macrolens model, as well as an empirical mass–concentration relation found for a catalogue of strong- and weak-lensing SGAS clusters (Bayliss et al. 2011; Oguri et al. 2012). The host DM halo has an NFW scale radius \( R_s = 75 \text{ kpc} \). Subhaloes are assumed to follow a power-law mass function \( \text{d}n(m_{sh})/\text{d} \log m_{sh} \propto m_{sh}^{-0.9} \) (Mo, Van den Bosch & White 2010), and are approximated as spherical but tidally truncated NFW profiles (Baltz, Marshall & Oguri 2009; Cyr-Racine et al. 2016).

We refer interested readers to appendix B of Dai et al. (2018) for technical details.

Similarly to Fig. 9, we calculate the magnification asymmetry \( a_{\mu} \) between a pair of images by ray shooting: for every chosen image position, we first find the corresponding source position, and then locate the counter images. When more than one counter image...
is present, we pick the brightest counter image and evaluate $a_\mu$ according to equation (4).

In Fig. 10, we show the pattern of $a_\mu$ influenced by one random realization of subhaloes for a compact source, within a 1 arcsec × 1 arcsec vicinity of the cluster critical curve. The pattern of signed magnification is also shown. The influence of a subhalo much more massive than $10^{10} M_\odot$ lying on top of the lensed arc should be large enough to be noticeable. But the expected number density of such massive subhaloes is low enough that having any one of them within ~1 arcsec of the cluster critical curve is improbable. We generate smaller subhaloes with $10^9 M_\odot < m_{sh} < 10^{10} M_\odot$. The model of Han et al. (2016) we adopt here predicts about one subhalo with $m_{sh} \gtrsim 10^9 M_\odot$ within a disc of 1 arcsec radius centred on the cluster critical curve. On the other hand, typically many subhaloes with $m_{sh} \sim 10^9 M_\odot$ are present just a fraction of an arcsecond away from the cluster critical curve. Unable to host star formation, these haloes cannot be observed except for gravitational probes sensitive to small-scale structures, like the lensing perturbations causing asymmetries in highly magnified image pairs.

As shown in Fig. 10, even one subhalo can induce substantial magnification asymmetry if it is sufficiently close to one of the image pair. A subhalo of mass $m_{sh} \approx 10^9 M_\odot$ located ~0.4 arcsec from the cluster critical curve can cause $a_\mu \gtrsim 0.1$ within ~0.1–0.3 arcsec from its centre. At ~0.1 arcsec from the cluster critical curve, small subhaloes with $m_{sh} \sim 10^9 M_\odot$ appear common enough to produce $a_\mu \gtrsim 0.1$ across a significant portion of the image plane. Even smaller subhaloes should be more numerous and may contribute to sizable $a_\mu$ very close to the critical curve. Our simulation of tiny subhaloes $m_{sh} < 10^6 M_\odot$ is limited by computational cost. Their effects may be important for extremely compact sources such as individual bright stars, but are likely to dilute away for sources of size larger than ~1 pc.

The precise distribution of magnification asymmetry naturally fluctuates among realizations of subhaloes, and mildly depends on host halo structural parameters $M_{200}$ and $C_{200}$. The typical magnification asymmetry scales with the normalization of the subhalo mass function. Despite these uncertainties, it seems plausible from our simulations that a population of DM subhaloes in the mass range $10^6 – 10^8 M_\odot$ induces ubiquitous asymmetry at the level $a_\mu \gtrsim 10$ per cent for pairs of highly magnified images within ~0.2 arcsec from the cluster critical curve. Subhaloes in this mass range are also capable of imprinting astrometric effects detectable with future imaging efforts (Dai et al. 2018). It is therefore theoretically possible that the flux asymmetries in S1226, such as those in Slits C, D, and E, are entirely or partially caused by subhalo lensing. Regarding the ambiguous identification of any image pair in Slit A, for example, we surmise that it could be due to a subhalo creating additional lensed images of the same source.

7.3 Satellite galaxies and globular clusters

If subgalactic perturber lenses are in fact responsible for the observed image pair asymmetries, could DM subhaloes be confused by dwarf galaxies in the cluster or along the line of sight, globular clusters (GCs) or other known stellar systems that are too faint to be detected by HST? Such star clusters or dwarf galaxies would have to be much smaller than, for example, the minor perturber galaxy to the north of S1226 so as not to induce obvious deformation of the arc, and would have to be numerous enough to be found near the critical curve with reasonable likelihood. Our ICL estimate at the location of the arc $\kappa_0 \approx 0.005$ implies that the 1 arcsec × 1 arcsec field of view (FoV) in Fig. 10 on average only encloses $\sim 10^4 M_\odot$ of stars, leaving little mass budget to have many stellar mass clumps comparable to the DM subhaloes we consider. It is also well known that the general dwarf galaxy luminosity function is much less steep at the faint end than the predicted CDM subhalo mass function at the low-mass end, so dwarf galaxy abundances should be much smaller than required for producing the perturbations illustrated in Fig. 10.

GCs populate the intracluster medium due to tidal stripping from their host galaxies (White 1987; West et al. 1995). In recent years, intracluster GCs have been studied in several galaxy clusters up to $z \approx 0.3$ (Lee, Park & Hwang 2010; Peng et al. 2011; West et al. 2011; Alamo-Martínez et al. 2013; D’Abrusco et al. 2016; Lee & Jang 2016). In the core of rich clusters, the GC surface number density is found to be around few $\times 10^{-2}$ kpc$^{-2}$ (Lee et al. 2010; Alamo-Martínez et al. 2013; D’Abrusco et al. 2016; Lee & Jang 2016), which would correspond to no more than a few GCs within the same FoV in Fig. 10. Even in the extreme case of the Coma Cluster, for which a GC surface density of ~1 kpc$^{-2}$ was reported by Peng et al. (2011), this number would be ~30, an order of magnitude smaller than DM subhaloes with $m_{sh} \gtrsim 10^9 M_\odot$. Furthermore, these GCs have masses $\sim 10^5–10^6 M_\odot$ (probably without any significant DM), smaller than those of the DM subhaloes we have considered. Hence, interloper GCs are also not abundant enough to be the major cause of the observed image asymmetries.

Satellite galaxies are thought to reside in DM haloes. Therefore, satellites should be no more numerous than the DM subhaloes that match their host DM halo size, which are already included in the DM subhalo population model we adopt. DM subhaloes small enough to be common within the FoV of Fig. 10 probably lack any significant stellar component. In the cores of nearby rich galaxy clusters, an extrapolation of the observed satellite luminosity...
8 CONCLUSIONS

We have investigated HST data showing asymmetric surface brightness features within ≲0.3 arcsec from the lensing critical curve in a $z_s = 2.93$ magnified arc behind the galaxy cluster SDSS J1226+2152. We have identified highly magnified image pairs of compact star-forming regions, many of which are inconsistent with being perfectly symmetric. Careful measurement of the brightest of these image pairs robustly constrains the flux ratio to be ~1.2 in two optical filters, and indicates a most probable flux ratio as large as ~1.3–1.8 in all four HST filters.

Intracluster microlensing inevitably introduces uncorrelated magnification fluctuations in different images of the same source, causing unequal fluxes at any given epoch. Due to the high source redshift $z_s = 2.93$, we have found that the sources of several asymmetric image pairs are too luminous to be just one or a few supergiants even if each star is as bright as $L_{bol} \simeq 10^6 L_\odot$, but are more likely to be clusters of more than ~10 bright stars. Our modelling suggests that having $N_\star \sim 10–30$ comparably luminous stars shortens the time-scale of asymmetry variability to just ~1 or 2 yr. In this case, the asymmetry should even flip its sign several times over six years. This appears, in the case of the brightest image pair, at odds with the consistent sign and degree of asymmetry between optical and IR exposures taken six years or 2 yr. In this case, the asymmetry should even flip its sign.

Asymmetry of arc in SDSS J1226+2152

Figure 10. DM subhalo effects on the magnification $\mu$ (bottom panels) and the asymmetry $a_\mu$ (top panels; see equation 4 and the text). A population of $\sim 10^6–10^8 M_\odot$ subhaloes render it rare to have magnified image pairs with highly equal fluxes $|a_\mu| \lesssim 1$ per cent. We zoom into a 1 arcsec × 1 arcsec field, which centres on the cluster critical curve and is oriented such that the $x_1$-axis aligns with the direction of arc elongation. Following the model described in the main text, subhaloes less massive than $10^{10} M_\odot$ are randomly generated. From left to right, we lower the minimum allowed subhalo mass to $10^8$, $10^7$, and $10^6 M_\odot$, respectively. Within a projected radius 1 arcsec from the coordinate origin, the number of subhaloes we generate is $N_{sh} = 1$, 27, and 196, in the first, second, and third columns, respectively. These in total contribute no more than 1 per cent to the local (coarse-grained) surface mass density. In panels of the first column, the only subhalo to the left of the cluster critical curve has a mass $m_{sh} \approx 10^7 M_\odot$. In the region where subhaloes are added, we include a negative uniform convergence to enforce mass conservation. Top panels: contours of $a_\mu = \pm 0.1 (a_\mu = \pm 0.3)$ are drawn in black (magenta); with the solid (dashed) line style indicating $a_\mu > 0 (a_\mu < 0)$. Regions having $|a_\mu| < 10^{-2}$ are artificially coloured according to $a_\mu = \pm 10^{-2}$. Our simulation can have numerical artifacts either when $\mu$ diverges on top of critical curves or when multiple images with very small separations render the solutions to the ray equation inaccurate, but only a negligible fraction of each panel suffers from this problem. Bottom panels: contours of constant signed magnification factor $\mu = \pm 50$, $\pm 150$, $\pm 300$ are drawn in black, with the solid (dashed) line style indicating $\mu > 0 (\mu < 0)$.
induced asymmetry should be severely diluted, while variability should occur even more rapidly. Since HST images in 2011 and in 2017 were not taken in the same filters, we have not been able to derive a tight constraint on variability, and in fact, variability at some level, however small, is unavoidable. An additional HST epoch in both UV and IR filters would tighten the constraint on any asymmetry component of microlensing origin, giving helpful guidance to upcoming JWST observations.

A population of ~10^6–10^8 M_☉ intervening DM subhaloes perturbing the magnification symmetry in the proximity of the critical curve is the other viable hypothesis we have considered, for which persistent asymmetries are natural outcomes. Using a reasonable model for subhalo abundance and mass function derived for the CDM paradigm, we have found it plausible that image pairs having more than a ~10 per cent fractional difference in magnifications can be common within ~0.2 arcsec of the cluster critical curve in the case of S1226.

Precise characterization of the cluster halo of SDSS J1226+2152 and improved substructure modelling will enable us to draw a more robust conclusion on whether asymmetric image pairs neighbouring the critical curve in S1226 are caused by substructure lensing. Deeper images taken at many epochs with HST and JWST will help reduce the uncertainty in the flux asymmetry measurement, separate any time-varying component from the persistent component, and rule out possible interloper satellite galaxies or GCs. In particular, in IR filters JWST will have good spatial resolution on par with HST in optical filters.

Nebular lines from H II regions surrounding hot stars, if spatially compact enough, might provide additional evidence for asymmetries insensitive to microlensing thanks to large source sizes. More generally, we expect an increased number of caustic straddling arcs with clumpy star-forming features in the proximity of lensing critical curves to be discovered in the future. Imaging and spectroscopy follow-ups of the source-lens systems will allow us to examine many such magnified image pairs and establish whether asymmetric magnifications are ubiquitous.

Detecting the population of small-scale subhaloes through lensing near caustics will be unprecedented and will complement other efforts to uncover the invisible DM structure on subgalactic scales, including flux ratio measurements of multiply imaged quasars (Nierenberg et al. 2014, 2017; Gilman et al. 2019) and scales, including flux ratio measurements of multiply imaged quasars (Nierenberg et al. 2014, 2017; Gilman et al. 2019). Thereby, improved constraints will be derived for alternative DM models such as warm dark matter (Kusenko 2000; Shoemaker & Kusenko 2009; Abazajian 2017) or ‘fuzzy’ DM (Press, Ryden & Spergel 1990; Sin 1994; Goodman 2000; Hu, Barkana & Gruzinov 2000; Peebles 2000; Amendola & Barbieri 2006; Schive, Chiu & Broadhurst 2014; Hui et al. 2017).

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APPENDIX A: NON-RIGID REGISTRATION

The method of non-rigid registration allows to smoothly deform one surface brightness pattern to match another. In Kaurov et al. (2019), this method was applied to the arc in MACS J0416.1−2403. In that case, asymmetries were marginally observed in several slits using deeper co-added HST exposures. Thus, in that case non-rigid registration relied on matching arc surface brightness features seen on both sides of the critical curve to account for imperfections of the ideal fold lens model. The main finding from this method in Kaurov et al. (2019) was that asymmetries appear more prominent as one examines the portion of the image plane closer to the critical curve, a trend broadly expected from both intracluster microlensing and DM substructure lensing. For S1226, asymmetries are significant in many of the slits we define in the current analysis.

The method of non-rigid registration is presented in Fig. A1, where we plot the speculated morphology of the (perhaps perturbed) critical curve as the white dashed curve. The figure highlights many inconsistencies along the critical line, which is in line with what we have found in Section 3.1. In Fig. A2, we show the average amplitude of match residuals as a function of distance to the critical curve, similar to what was done in Kaurov et al. (2019). While some surface brightness mismatches are mitigated or eliminated by the non-rigid transformation, others cannot be perfectly cured, especially at smaller distances to the critical curve.

Figure A1. The left-hand column presents the same HST images as in Fig. 2 for F606W, F814W, F110W, and F160W filters. The middle column shows the right-hand part of the arc flipped under non-rigid transformation. The right-hand column shows the surface brightness difference between the left-hand and the middle columns. The white dashed curve indicates the estimated position of the critical curve.
APPENDIX B: MEASURING FLUXES OF LENSED IMAGE PAIRS

For our analysis, we used reduced science product images drizzled to a uniform pixel scale of 0.03 arcsec, one image per filter. The pixel resolution improves upon those of individual exposures, namely a detector pixel scale 0.05 arcsec for ACS (F606W and F814W) and 0.128 arcsec for WFC3 IR (F110W and F160W).

We use the publicly available software package TINYTIM (Krist, Hook & Stoehr 2011). To allow subpixel centroiding, we need subsampled PSFs output by TINYTIM, which neglect the charge diffusion effect (Krist 2004). Only precisely applicable to individual exposures, those neither include further PSF broadening in drizzled images (Hook & Fruchter 2000). Using stars in the FoV for calibration, we empirically find that convolving TINYTIM PSFs with a 2D isotropic Gaussian kernel whose sigma equals 0.5 pixel scale on the detector provides good fits, with a 2D isotropic Gaussian kernel whose sigma equals 0.5 pixel scale on the detector provides good fits, with a 2D isotropic Gaussian kernel whose sigma equals 0.5 pixel scale on the detector provides good fits.

In each filter and for each of the significant asymmetric features in Slits C, D, and E, we explore the χ² in equation (1) within a cutout of suitable size which contains the presumptive lensed image pair. We centroid each lensed image within a 0.2 arcsec × 0.2 arcsec search region in (RA, Dec.) centring at the pixel of peak flux. Some of the relevant parameters adopted in our analysis can be found in Table B1.

We adopt simple and reasonable parametrized models for the diffuse background based on the surface brightness properties within each cutout. For the image pairs in Slits C and E, we adopt the gradient model which is simply a 2D linear function with three coefficients:

\[ S_0 + S_1 \text{RA} + S_2 \text{Dec}. \]

This form captures any smooth ICL and sky backgrounds.

The brighter image pair in Slit D happen to be located at a boundary separating two distinct sections of the arc with high and low surface brightness, respectively. The arc is bright enough that ICL and sky backgrounds are unimportant. Near the lensing critical curve, this boundary is physically constrained to be parallel to the direction of arc elongation. We therefore introduce the smooth step model for the diffuse background, which is the sum of a uniform component and a 1D profile describing a smooth transition in the surface brightness:

\[ S_0 + S_1 \tanh \left( \frac{\cos \theta \text{RA} - \sin \theta \text{Dec.} - \Delta}{b} \right). \]

The smooth step model has a total of five parameters, which include two constants \( S_0 \) and \( S_1 \), and three other parameters \( (\theta, \Delta, b) \) characterizing the orientation, position, and width of the surface brightness transition, respectively.

For the inverse variance weights \( \sigma_i^2 \)'s in equation (1), we adopt the white noise approximation by setting all \( \sigma_i^2 \)'s to be a constant \( \sigma_f \), which models fit residuals due to both flux measurement errors and uncertainties in the intrinsic surface brightness inhomogeneities on the arc. Its value is tuned such that for the maximum likelihood solution the χ² per effective degree of freedom is around unity. In the two optical filters F606W and F814W, the total number of effective degrees of freedom is taken to be the total number of cutout pixels \( N_{\text{pix}} \) minus the total number of fitting parameters \( n_{\text{dim}} \). The drizzled 0.03 arcsec pixel⁻¹ images for F110W and F160W are oversampled, which causes residuals to show correlation across many pixels (see bottom rightmost panel of Fig. B1). We estimate a factor of 4 for oversampling in resolution, and hence reduce the effective number of pixels by a factor of 16 in those two filters.

As an example, in Fig. B1 we show results of our aforementioned fitting procedure for the most prominent asymmetric image pair feature in Slit D, in one optical filter F814W and one IR filter F110W. The fitting residuals for F814W show only moderate departure from white noise, mainly due to the presence of secondary underlying compact sources on the arc. We do not expect the white noise approximation leads to significant misestimation of the error bars. The fitting residuals for F110W clearly exhibit correlation as expected from oversampling, justifying the reduction in the effective number of pixels used to tune \( \sigma_f \).

For a cross check, we reperform the analysis using HST images of individual visits (two visits each for F606W and F814W with 0.05 arcsec pixel⁻¹; one visit each for F110W and F160W with 0.128 arcsec pixel⁻¹) produced by the standard reduction pipeline and downloadable from the Mikulsky Archive for Space Telescopes (https://mast.stsci.edu/portal/Mashup/Clients/Mast/Portal.html), properly treating astrometric misalignment between images taken from different visits. The results obtained are in good agreement with what we conclude with our own drizzled images.
Table B1. Detailed choices for $\chi^2$ minimization of equation (1) for different asymmetric features in various filters. $N_{\text{pix}}$ is counted without applying oversampling reduction. $n_{\text{dim}}$ is the total number of dimensions of the parameter space to explore.

| Image pair | HST filter | $N_{\text{pix}}$ | $n_{\text{bkg}}$ | Diffuse background model | $n_{\text{dim}}$ | $\sigma_F$ (nJy) |
|------------|------------|-------------------|-------------------|--------------------------|------------------|------------------|
| Slit C     |            |                   |                   |                          |                  |                  |
|            | F606W      | 195               | 2                 | gradient                 | 9                | 0.60             |
|            | F814W      | 188               | 2                 | gradient                 | 9                | 0.80             |
|            | F110W      | 264               | 2                 | gradient                 | 9                | 1.20             |
|            | F160W      | 264               | 2                 | gradient                 | 9                | 1.70             |
| Slit D     |            |                   |                   |                          |                  |                  |
|            | F606W      | 210               | 2                 | smooth step              | 11               | 0.50             |
|            | F814W      | 210               | 2                 | smooth step              | 11               | 0.65             |
|            | F110W      | 383               | 2                 | smooth step              | 11               | 1.00             |
|            | F160W      | 383               | 2                 | smooth step              | 11               | 1.60             |
| Slit E     |            |                   |                   |                          |                  |                  |
|            | F606W      | 339               | 2                 | gradient                 | 9                | 0.40             |
|            | F814W      | 253               | 2                 | gradient                 | 9                | 0.54             |
|            | F110W      | 406               | 2                 | gradient                 | 9                | 0.60             |
|            | F160W      | 406               | 2                 | gradient                 | 9                | 1.50             |

Figure B1. Example fitting results for the image pair in Slit D in the F814W filter (upper row) and in the F110W filter (lower row). The maximum likelihood solutions are shown. The four columns, from left to right, show the drizzled data (0.03 arcsec pixel$^{-1}$), the fitted lensed image pair modelled as a pair of PSFs, the data after the image pair are subtracted, and residuals, respectively. For panels in the first three columns, fluxes are colour coded in units of nJy per pixel. For panels showing the residuals, fluxes are colour coded in units of $\sigma_F$, whose values can be found in Table B1. Note the different cutout sizes used for F814W and F110W.

APPENDIX C: MAGNIFICATION ASYMMETRY ACROSS CRITICAL CURVE

In this appendix, we estimate the degree of uneven fluxes between a close pair of highly magnified images on both sides of the critical curve. We aim to derive the parametric dependence on the perturber parameters.

Consider a pair of images at image-plane positions $x_{1,i}$ and $x_{2,i}$. Demanding that they map to the same source point, we obtain from the lens equation

$$x_{2,i} - x_{1,i} = \alpha_i(x_2) - \alpha_i(x_1),$$

where $\alpha_i(x)$ is the deflection field as a function of the 2D image-plane vector $x$. Let us write $x_{1,i} = x_{0,i} - \Delta x_i$ and $x_{2,i} = x_{0,i} + \Delta x_i$, where $x_{0,i}$ is the mid-point between the image pair.
Assuming that $\alpha_i(x_j)$ is smooth, we Taylor expand in powers of $\Delta x_i$:

$$
\Delta x_i = [\nabla_j \alpha_i]_{x_0} \Delta x_j + \frac{1}{6} [\nabla_k \nabla_j \nabla_i \alpha_i]_{x_0} \Delta x_k \Delta x_j + O[\Delta x^3],
$$

(C2)

where $\nabla_i$ denotes image-plane derivative with respect to $x_i$, and $[\cdots]_{x_0}$ stands for evaluation at $x_0$. Ignoring terms at cubic and higher orders, the Jacobian matrix $J_{ij}(x) = \delta_{ij} - \nabla_i \alpha_j(x)$ at $x_0$ satisfies $[J_{ij}]_{x_0} \Delta x_j \approx 0$. This means that $\Delta x_i$ is an eigenvector with a vanishing eigenvalue.

The inverse of the signed magnification is

$$
\frac{1}{\mu(x)} = \det[\delta_{ij} - \nabla_i \alpha_j(x)],
$$

(C3)

where we again Taylor expand around $x = x_0$:

$$
\delta_{ij} - \nabla_i \alpha_j(x_{1,2}) = [J_{ij}]_{x_0} \pm \frac{1}{2} [\nabla_k \nabla_j \nabla_i \alpha_i]_{x_0} \Delta x_k
$$

$$
- \frac{1}{6} [\nabla_l \nabla_k \nabla_j \nabla_i \alpha_i]_{x_0} \Delta x_l \Delta x_k + O[\Delta x^3].
$$

(C4)

In a suitably oriented coordinate system, $\Delta x_i$ aligns with the first axis, and the only nonzero element of $[J_{ij}]_{x_0}$ is the 22 element being order unity.

If we neglect terms of $O[\Delta x^2]$ in equation (C4), the determinant is at the leading order proportional to the $O[\Delta x]$ term in the 11 element of the full Jacobian matrix. Since this term has equal magnitudes but opposite signs at both image positions $x_1$ and $x_2$, we recover the familiar result that a pair of images across an ideal fold are equally magnified.

Departure from perfect magnification symmetry is induced by two types of correction in equation (C4): (i) the $O[\Delta x^2]$ term in the 11 element; and (ii) the product of $O[\Delta x]$ terms from two matrix elements. Having the same signs at $x_1$ and at $x_2$, these correct the leading magnification factor in opposite directions at the two images and hence generate asymmetry.

The lowest order derivative of the deflection field is usually determined by the large-scale lens, while the higher order derivatives can be dominated by small perturber lenses. By definition, $\Delta x$ is on the order of the image separation $\Delta \theta$. Neglecting numerical prefactors, we assume $[\nabla_k \nabla_j \nabla_i \alpha_i]_{x_0}$ is of order $d$ from the large-scale mass distribution. We assume that a perturber lens dominates the derivative at the next order,

$$
[\nabla_l \nabla_k \nabla_j \nabla_i \alpha_i]_{x_0} \sim \theta_p^2 / b^4,
$$

(C5)

as long as the angular impact parameter $b$ is larger than the characteristic Einstein radius $\theta_p$ of the perturber. We also require $b \gtrsim \Delta \theta$ so that Taylor expansion in $\Delta x$ is justified. Fractional correction to magnification from terms of type (i) is therefore

$$
|a_{\mu}| \sim \left( \frac{\theta_p^2 \Delta \theta}{d b^4} \right)
$$

(C6)

The fractional correction to magnification from terms of type (ii) is of order $d / \Delta \theta$, independent of the perturber. This is comparable to the inverse of the leading order magnification factor. For image pairs of interest in this work, the magnification factor is as large as $\gtrsim 100$. Hence, these terms only induce insignificant fractional magnification asymmetry at (sub-)percent levels.

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