Coordinate structure of feature geometrical location specifications

V I Glukhov1, L G Varepo1, V V Shalay1, K V Simonenko1, P S Belyaev2
1 Omsk State Technical University, 11, Mira ave., Omsk, 644050, Russia
2 Tambov State Technical University, 106, Sovetskaya St, Tambov, 392000, Russia

E-mail: larisavarepo@yandex.ru, bps800@yandex.ru

Abstract. The paper proposes a systemic transition from the location and orientation of tolerance product features to linear and angular coordinates in the coordinate system of the product electronic model. The coordinate system is formed by designing datums set with a total informativeness content of six constrains on the object degrees of freedom, three of them being linear and three of them being angular constrains. The main design datums determine the object position in the product and form a generalized coordinate system. Auxiliary design datums determine the position of the attached object on the considered object and form an auxiliary coordinate system. Each element of the product has a certain informativeness content to limit the degrees of freedom, both linear and angular. When forming a Cartesian coordinate system, the datum element transfers piece or all of its informativeness content to it. As a result, the coordinate planes acquire different informativeness of three, two and one, and the coordinate axes acquire that of four, two and zero. The coordinate axes and planes informativeness content determines the number of linear and angular coordinates that can be set relative to them. Similarly, the feature datums informativeness means the number of linear and angular coordinates that must be specified in the coordinate system for its single-task location and orientation. The transition to a coordinate system for standardizing location and orientation deviations demonstrates compliance with a systematic approach that improves the quality of products in terms of the geometric specifications accuracy.

Keywords. Technical products, datums and datum systems, coordinate system, linear and angular coordinates

1. Introduction
The high quality of technical products is laid down in the design on the basis of a systematic approach to ensuring the accuracy of the part geometric specifications. Position tolerances provide accuracy location and angular orientation of each feature in a part. To perform this most important function and implement a systematic approach, location tolerances require a reference system for the linear and angular coordinates of the features. The coordinate system of the part should be such a reference system. However, the technical committee ISO / TC213 "Geometrical product specifications and their control" continues to only complicate its standard for form and location tolerances [1] and does not plan to switch to coordinate systems of parts [2] with the next update. Only datums and datum systems are proposed for setting position tolerances [3], although the standard is more than half a century old, and other ISO technical committees use coordinate systems in standards for machine tool design [4] and in industrial automation [5]. Moreover, the standard applies rectangular Cartesian coordinate systems [6], which were proposed by Descartes four centuries ago, and used by Monge for the drawing projections three centuries ago. Electronic models of products are also executed in coordinate systems.
Some authors note the complexity of ISO standards, difficulties in their application [7]. Some authors introduce coordinate systems in their studies [8]. Many authors show the need for geometric tolerances in additive manufacturing [9], in the design of machining centers [10] and technological devices [11]. Others develop the concept of tolerances based on digital technologies [12] and new CAD - models taking into account the geometrical tolerance in the design [13], compare the tolerance results with the measurement data after the manufacture of the parts [14].

Thus, the problem of the geometric tolerances accuracy including location tolerances is urgent. This is evidenced by the facts of opening departments in organizations and firms in industrial centers for setting datums, geometrical specifications and their tolerances in design drawings, in technological documentation and in measurement techniques.

2. Problem statement
Positional deviations, together with form deviations and the intrinsic dimensions of the features, participate in the joints of the parts and form fits. Consequently, the reference system for the numerical values of the location deviations, size and shape of the feature surfaces should be common for all geometrical specifications.

In order to improve the quality of technical products through a systematic approach to the setting of geometric specifications, an alternative coordinate reference system is proposed in the form of a rectangular Cartesian coordinate system for a datum set and for a part (product) as a whole. The introduction of a coordinate system in drawings, technological documentation and in methods for measuring geometrical specifications will increase the quality of project documentation. To achieve this goal in the work, it is necessary to solve the following tasks.

1. To develop a theory of materialization of coordinate systems on the functional purpose of design datum sets.
2. Build alternative models of real parts for illustrating the transition from ISO standardized position tolerances to linear and angular coordinate dimensions.
3. Propose tolerance systems for coordinating linear dimensions and coordinating angular dimensions with their introduction into the geometrical product specifications GPS matrix.

3. Parts coordinate systems materialization
Geometric features are pieces of the part bounded by one or more surfaces. The most common features in technical parts are cylindrical features with one surface and prismatic ones with two to six or more flat surfaces. Features of one part fulfill the functional purpose of basic, actual and free features. Basic features, or datums, determine the position of part in the product or the position of the attached part to the considered one. In the first case, they are the main design datums, in the second case they are auxiliary design datums. The actuating features perform the working functions of the part, mainly kinematic, for example, a toothed involute feature. Free features connect the basic and actuating features in the part material.

Each feature has a certain informativeness content from zero 0inf to six 6inf by the number of constrained linear and angular degrees of freedom of the part as a function of the datum. A set of basic design datums with a total informativeness content of six, limiting three linear t and three angular degrees of freedom of the part 6inf = 3t + 3r, materializes the Cartesian rectangular generalized coordinate system of the part. A set of auxiliary datums with the informativeness of six for the attached part forms an auxiliary coordinate system 6inf = 3t + 3r. A set of actuation surfaces can also form an auxiliary coordinate system. Both generalized and auxiliary coordinate systems are generally rectangular Cartesian systems. Datums initially materialize three coordinate planes, transferring their informativeness content to them (3 + 2 + 1) inf. Then the coordinate planes intersect at right angles. With their intersection lines they form three coordinate axes with information content (4 + 2 + 0) inf, and the origin with informativeness content 3inf. Expanding the content of information content we establish that the first coordinate axis has information content 4inf, which is the sum of two constrains on the linear degrees of freedom 2t and two constrains on the angular degrees of freedom 2r (4inf = 2t + 2r). The second axis with informativeness content 2inf limits two degrees of freedom, namely, one linear 1t and one angular 1r (2inf = 1t + 1r). The third axis has zero informativeness content 0inf, since
the total informativeness content of the first two coordinate axes is six, which is sufficient to set the position of any features using six coordinates. They are three linear and three angular ones: 6inf = 4inf + 2inf = (2t + 2r) + (1t + 1r) = 3t + 3r. In a rectangular coordinate system, all linear and angular coordinates are differentiated values. The axes with informativeness content 4inf and 2inf lie in the coordinate plane with informativeness content 3inf. These axes intersect at the coordinate origin 0, delegating to it three origins of linear coordinates, and give it informativeness content 3inf = 2t + 1t = 3t. Consequently, the three linear coordinates of the actuating and auxiliary features must be specified in the part generalized coordinate system by the coordinates of the origin point of the auxiliary coordinate system. Three angular coordinates are set between the coordinate axes with information content 4inf (2 angles) and informativeness content 2inf (1 angle) for each auxiliary system relative to the coordinate axes with the same informativeness content 4inf and 2inf of the part generalized coordinate system. Similarly, 6 coordinates (3 linear and 3 angular) of the generalized coordinate system of the part are specified in the global coordinate system of the product.

Linear and angular coordinating dimensions also arise between the datums of the set materializing the coordinate system to constrain the degrees of freedom unused by the datums as coordinates of the datums with less informativeness content relative to the datums with more informativeness content.

The linear coordinates of the feature center and the angular coordinates of the axis or the main section plane relative to the coordinate system in which the elements fulfill their functional purpose set the coordinating dimensions of single elements. The form of coordinates coincides with the features informativeness content in the datum function (t, r), i.e. linear and angular. Descriptive geometry is based on three linear coordinates of a point in a rectangular coordinate system.

4. Adequate models of part features geometrical position deviations

Let us consider the structure of geometrical location tolerances in the coordinate system of basic design datum set (Figure 1). In the figure, the main design datums are indicated by a triangle with a flat datum, and the auxiliary ones are the vertices of the triangle. The feature dimensions are indicated by double-sided arrows, the coordinating dimensions are indicated by one-sided arrows and light points on the coordinate axes from which the coordinates are measured. The datum informativeness content is indicated in the datum frame after its letter designation with two numbers: the first number is linear informativeness content; the second number is angular informativeness content.

The datum set includes three main datums that determine the position of the part in the product: flat datum A3 with informativeness content 3inf = 1t + 2r, datum B2 is the axis of a cylindrical hole with informativeness content 2inf = 2t + 0r, and datum G1 is the symmetry plane of the keyway with information content 1inf = 0t + 1r. The informativeness indicator (b) confirms that the datums set A3B2G1 constrains the part of six degrees of freedom 6inf = 3t + 3r without duplicating them: (1t + 2r) + (2t + 0r) + (0t + 1r) = 3t + 3r.

The first A3 datum has the maximum informativeness content 3inf = 1t + 2r in the datum set and is a flat datum. The A3 datum spends all three degrees of freedom on the XOY coordinate plane formation of the generalized coordinate system of the part: one linear 1t limits the translational displacement along the normal to the datum, i.e. along the Z axis and two angular 2r limit two rotations around the X and Y axes located in this coordinate plane. Thus, the datum A3 materializes the coordinate plane XOY3, and transfers its informativeness content to it 3inf = 1t + 2r.

The second coordinate plane XOZ is materialized by a perpendicular plane to the coordinate plane XOY3 and passing through the center point C2 of the datum axis B2 of the cylindrical hole with information content 2inf = 2t + 0r and through point C3 of the datum symmetry plane of the datum G1 with informativeness content 1inf = 0t + 1r. The XOZ plane acquires the informativeness content 2inf = 1t + 1r, obtained from the datums B2 and G1. The intersection line of the coordinate surface XOZ2 with informativeness content 2inf = 1t + 1r and with the coordinate plane XOY3 materializes the first coordinate axis X4 with informativeness content 4inf = (1t + 2r) + (1t + 1r) − 1r = 1t + 2r, where one the angular degree of freedom 1r with a minus sign is spent on the right angle between the planes.

The third coordinate plane YOZ is materialized by a perpendicular plane to the coordinate planes XOY3 and XOZ2, and point C2 of the datum axis B2, which has only one linear degree of freedom 1t. This degree of freedom is transferred to the plane YOZ1 and determines its informativeness value 1t.
The intersection line of the coordinate planes X0Y3 and Y0Z1 materializes the Y2 axis with informativeness content $2\text{inf} = (1t + 2r) + (1t + \theta r) - 1r - 1t = 1t + 1r$. Here minus 1r is spent on the right angle between the planes, and minus 1t is spent on the intersection of the X4 and Y2 coordinate axes, and the formation of the point origin 0. The Y0Z1 coordinate plane intersects the X0Z2 coordinate plane at right angles, and their intersection line materializes the Z axis with information content $\theta \text{inf}: (1t + 1r) + (1t + \theta r) - 1r - 2t = \theta t + \theta r$. Here minus 1r is spent on the right angle between the planes, and minus 2t is spent on aligning Z0 with the point origin 0 along the X4 and Y2 axes.

Figure 1. Location deviation adequate model in the three main design datums set for a prismatic part (a) and the datum informativeness content indicator (b).

Thus, the coordinates origin 0 has three zero coordinates: x\(\theta\), y\(\theta\), z\(\theta\) and is essentially not the letter 0, but the digit zero \(\theta\) in the rectangular coordinate system \(0X4Y2Z0\). Different informativeness have the coordinate axes: X4 (2t + 2r), Y2 (1t + 1r) and Z0 (\(\theta t + \theta r\)), and the coordinate planes: X0Y3 (1t + 2r), X0Z2 (1t + 1r) and Y0Z1 (1t + \theta r). The coordinate axes and planes informativeness of the part generalized coordinate system has an important property: it indicates the number and type of linear (t) and angular (r) coordinates, which can be used to set the position and orientation of the part auxiliary and actuating features in the drawing. With equal opportunities, the coordinates are set from the coordinate axes or planes with the greatest informativeness content.

Linear coordinates are set for the features centers, i.e. for points located on the centerline features in the middle or main sections of features, at the diagonals intersection. The futures angular coordinates are specified for the axes and symmetry planes.

We list the location deviations for the three considered main design datums set. The primary flat datum A3 spent all the information content $3\text{inf} = (1t + 2r)$ to form on the main coordinate plane X0Y, transferring all the informativeness content to it: X0Y3 (1t + 2r). Therefore, the datum A3 will not
have a single location deviation and it does not need any coordinates: NC = (1t + 2r) – (1t + 2r) = 0t + 0r. Secondary cylindrical datum B2 with informativeness content 2inf = 2t + 0r spent two linear informativeness content on materializing two other coordinate planes X0Z2 and Y0Z1. However, the maximum informativeness content of the axis with a cylindrical datum is 4inf = 2t + 2r, i.e. unused informativeness content is 2r, which requires the specifications of two angular coordinates: NC = (2t + 2r) – (2t + 0r) = 2r. These coordinates are two right angles: 90º ± AEY2 in the X0Z coordinate plane relative to the X4 axis and 90º ± AEX2 in the Y0Z coordinate plane relative to the Y2 axis. To limit the deviation from a right angle, two square deviations are required: EPRY2 and EPRX2. To find the total spatial deviation from the perpendicularity of EPR2, you need to find the geometric sum of the deviations around the X4 and Y2 axes

\[ EPR_2 = [(EPRX_2)^2 + (EPRY_2)^2]^{0.5} \]

and additionally set the second coordinate angle γ2 ± AEZ2 for the inclination plane of the datum axis B2 relative to the coordinate axis X4 with the maximum informativeness content 4inf. The upper and lower sections projections of the datum hole B2 onto the main projection X0Y form a torus figure, which has an informative value of 5inf = 3t + 2r. This can explain an additional, third angular coordinate γ2 ± AEZ2 for the B2 datum axis with an informativeness content 2inf. The Z0 axis of the rectangular coordinate system is the datum axis of the maximum material cylinder of diameter D2MM, which will serve as the actual datum B2 when assembling the product. This diameter is less than the intrinsic diameter D2 by the total deviation from the perpendicularity of EPR2, and the diameter of the minimum material D3LM is greater than the intrinsic diameter D2 by the same deviation of EPR2. Consequently, location deviations should be normalized in diametric terms.

The third main design datum G1 has an informativeness value 1inf = 0t + 1r in three datums set, it determines the direction of the X4 coordinate axis and limits the coordinate system θX4Y2Z0 rotation around the Z0 axis. The datum function is performed by the datum center, namely, point C3 on the symmetry plane of keyway. The symmetry plane has the maximum information content 2inf = 1t + 1r due to the small depth of the keyway. The datum G1 contacts during assembly by its point C3 by the protruding head of the cylindrical pin in the function of an auxiliary datum with informativeness content 1t on the basic part. Therefore, the datum G1 limits the rotation of the coordinate system due to the linear informativeness 1t, and the angular informativeness remains unused for it: NC = (1t + 1r) – 1t = 1r. This informativeness content is limited by the angular zero coordinate θ2 ± AEZ3 relative to the X4 axis. The parallelism tolerance relative to the X4 coordinate axis normalizes the angular deviation AEZ of the zero coordinate from zero degrees (0º). Thus, all location deviations of the basic design datums set are coordinating dimensions that are absent in the classification of the geometric product specifications ISO. The point C3 of the datum G1 is located on the X4 coordinate axis and has a horizontal X3 coordinate with a general tolerance, since informativeness of the G1 datum does not limit the displacement of the coordinate system along the X4 axis. This function is performed by the B2 datum.

Let us consider the location deviation geometric specifications for the auxiliary datums set K4M1N1 on the adequate model for a cylindrical part shown in Figure 2. The part generalized rectangular coordinate system 0Z4X2Y0 is materialized by the main design datums set A4B1 with a total informativeness content of 5inf = (2t + 2r) + (1t + 0r) = 3t + 2r. In order to increase the informativeness content of the main datums set up to 6inf, an auxiliary datum N (0t + 1r) is involved, complementing the missing angular information content 1r to identify the complete part generalized coordinate system 0Z4X2Y0. As a result, the Z4 coordinate axis is materialized by the main datum A4 which is the outer cylindrical feature axis. The coordinate origin 0 is materialized by the point of intersection of the Z4 axis with the flat datum B1. The coordinate axis X2 is materialized by the origin 0 and the auxiliary datum N1 with informativeness 1r, and the coordinate axis Y0 is materialized perpendicular to the axes X4 and X2 from origin point 0. The datum N1 is the symmetry plane of the prismatic keyway with the maximum informativeness content 2inf = 1t + 1r.

The auxiliary coordinate system 0’Z’4X’2Y’0 is materialized by an auxiliary design set K4M1N1 according to the rules for the main design datums discussed above. The auxiliary coordinate system has a total informativeness content 6inf = (2t + 2r) + (1t + 0r) + (0t + 1r) = 3t + 3r and is intended for
basing the attached part. Coordinate axis $Z'$ is materialized by the axis of the cylindrical datum $K_4$ with informativeness content $4_{inf} = 2t + 2r$.

**Figure 2.** Adequate model for geometrical deviation specifications of auxiliary datums set arrangement for the cylindrical part (a), the part generalized coordinate system with the informativeness indicators of the main datums set (b), and the auxiliary coordinate system with the informativeness indicators of the auxiliary datums set (c).

The coordinates origin $0'$ is materialized by the intersection point of the $Z'$ axis with the flat datum $M_1$ with informativeness content $1_{inf} = 1t + \theta r$, the $X'$ axis is materialized by the datum $N_1$ with informativeness content $1_{inf} = \theta t + 1r$ perpendicular to the $Z'$ axis from the coordinates origin $0'$. The $Y'$ axis is materialized perpendicular from the origin $0'$ to the $Z'$ and $X'$ axes. The axes of the auxiliary coordinate system in the space of the generalized coordinate system never intersect.

As a result of geometric modeling, we find that the auxiliary coordinate system $0'Z'4X'2Y'0$ will have six location coordinates in the generalized coordinate system $0Z4X2Y0$, namely, three linear and three angular:

- $-Z'0' \pm E ZO$ is the linear coordinate with a minus sign along the $Z4$ axis;
- $\theta \pm EX0'$ is zero linear coordinate along the $X2$ axis of the point origin $0'$;
- $180^\circ \pm AEX_1$ is the angular coordinate of the $Z'$ axis relative to the $Z4$ axis in the $Z0Y$ plane;
- $180^\circ \pm AEY_1$ is the angular coordinate of the $Z'$ axis relative to the $Z4$ axis in the $ZOX$ plane;
- $\theta^\circ \pm AEZ$ is zero angular coordinate of the $X'$ axis relative to the $X2$ axis in the $X0Y$ plane.

In addition, auxiliary datums will have three more location deviations within the auxiliary coordinate system:

- $90^\circ \pm AEY^2_2$ is the angular coordinate for the right angle of the $M_1$ datum relative to the $Z'$ axis in the $Z'0'X'$ plane;
90° ± AEX₂ is the angular coordinate for the right angle of the M1 datum relative to the Z’4 axis in the Z’0’Y’ plane; 
0° ± AEX₃ is the angular coordinate for the zero datum angle N1 relative to the Z’4 axis in the Z’0’X’ plane.

If we go from the differentiated linear coordinates θ ± EX₀’ and θ ± EY₀’ of the auxiliary system origin θ’ to the complex vector displacement E₀’, then the number of location deviations will not decrease, since for the vector, we should specify the second angular coordinate γ ± AEZ relative to the coordinate axis X₂:

\[ E₀’ = \sqrt{(EX₀’)² + (EY₀’)²} \]

Thus, the auxiliary coordinate system has nine coordinates. Three of them are between the datums in its own system and six of them are in the part generalized coordinate system. In other words, the ISO positioning tolerance system will not be used if coordinate systems and coordinate dimensions are introduced into part and assembly drawings. This results in great gains in positioning and orientation accuracy.

5. Coordinating dimensions tolerance systems

The dimensional basis of linear prismatic and cylindrical features, as well as angular prismatic features, are coordinating dimensions [15]. Therefore, for coordinating dimensions, it is recommended to use the previously developed standardized systems of tolerances for linear and angular dimensional features: ISO286-2 – for linear coordinating dimensions [16] and ISO 2538-2 for angular coordinating dimensions [17].

The accuracy class of the linear coordinating dimension is selected depending on the numerical value of the nominal size and is one class more accurate than the linear dimension of the coordinated feature. The linear coordinate dimension tolerance is symmetrical: Js for coordinated holes and js for coordinated shafts.

The degree of accuracy of the angular coordinating dimension is selected depending on the length of the coordinated feature and is one degree of accuracy more accurate than the coordinated element. The angular coordinate dimension tolerance is usually symmetrical for the nominal angular coordinate: ± ATα / 2 is for the angular expression in angle units, and ± ATh / 2 is for the linear expression in length units.

6. Summary and conclusion

1. The main geometric specification of the modern electronic model for a technical product is the coordinate system. In the coordinate system, the assembly product and each part of the product are designed.

2. The coordinate system is formed by a design datums set with a total informativeness of six constraints on the degrees of freedom from an object, three of them are linear and three of them are angular. The main datums determine the position of the object in the product and form a generalized coordinate system. Auxiliary design datums determine the position of the attached object on the considered object and form an auxiliary coordinate system.

3. Each feature of the product has a certain informativeness content to limit the linear and angular degrees of freedom. When forming a Cartesian coordinate system, the datums feature transfers part or all of its informativeness content to it. As a result, the coordinate planes acquire different informativeness of three, two and one, and the coordinate axes acquire that of four, two and zero. The informativeness content of the coordinate axes and planes determines the number of linear and angular coordinates that can be set relative to them. Similarly, the informativeness of the feature datums means the number of linear and angular coordinates that must be specified in the coordinate system for its unique location and orientation.

4. Each auxiliary coordinate system must have six coordinates in the generalized coordinate system, namely, three linear and three angular.

5. A datum set can have linear and angular coordinates within the system to limit the informativeness of the datum that are not used for the formation of the coordinate system.
6. Tolerances of linear and angular coordinates, as a rule, are symmetrical and for their tolerancing one should use tolerance systems for linear and angular dimensions of size features.

7. Conclusion. The transition to a coordinate system for standardizing location and orientation deviations demonstrates compliance with a systematic approach that improves the quality of products in terms of the geometric specifications accuracy. The coordinate system should be included in the GPS geometry matrix model as the first one. The coordinate system should be common to standardize all geometric specifications of products: coordinates, dimensions of size features and deviations in the form of surfaces.

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