Nonclassicality of Photon Added Bipartite Glauber Lachs State

Sathiyabama Ramanujam and A. B. M. Ahmed
School of Physics, Madurai Kamaraj University, Madurai, India.
E-mail: abmahmed@gmail.com

Abstract. Mixture of coherent state with thermal noise is known as the Glauber Lachs state, which is taken into consideration. Nonlocal photon addition is imposed on the two mode Glauber Lachs state by the application of bosonic creation operators. This nonlocal photon addition induces nonclassicality, revealed in the single mode and two mode squeezing properties. The ratio of the coherent noise to thermal noise is the key factor in observing the nonclassical features. The tolerance limit of the noise for the nonclassicality is discussed in single mode squeezing and two mode squeezing quadrature operators. The results show that the squeezing properties are less pronounced even for the low level of thermal noise.

Keywords: Coherent state, Thermal state, Glauber Lachs State, Photon addition, Squeezing.

1. Introduction

Quantum Information Processing[1] deals with creation, manipulation and storage of information in terms of quantum states. The quantumness present in the physical system act as a resource for implementing quantum information protocols. Hence creation of such quatum states which can play vital role in the quantum information protocols is of paramount interest. Of the physical system available for implementation of quantum information, the radiation states occupy important place because of their versatile nature.

One of the photon level operation which induces nonclassicality is known as photon addition. A single mode coherent state, excited by photon addition shows several nonclassical features [2]. Further nonlocal photon addition on two mode thermal state [3], a classical state is known to introduce not only nonclassicality at single mode but also infuses correlation between two modes. Similar such operation i.e., nonlocal photon addition on two mode coherent state [4], an another classical state, creates nonclassicality at single mode and two mode cases. Nonlocal photon addition on two mode squeezed vacuum[5] and two mode squeezed thermal state[6] are reported. Interesting extension includes nonlocal photon subtraction on two mode squeezed vacuum state reported in [7].
The present paper is concerned with nonlocal Photon Added Bipartite Glauber Lachs State. The layout of the present work is given as follows. In section 2, the coherent states and thermal noise are combined to form a nonclassical state. In section 3, the squeezing properties of such state are calculated according to the uncertainty limit and the effect of thermal noise is also discussed. Finally, the optimal ratio of the coherent noise to thermal noise for the nonclassicality is discussed and conclusions are given in section 4.

2. Frame of work

The coherent state was introduced by Schrodinger as a Gaussian wave packet to describe the evolution of harmonic oscillator. During the evolution the Gaussian nature of the wave packet does not change. Glauber has coined the term coherent state and reintroduced it as the eigenstate of the photon annihilation operator \( \hat{a}|\beta\rangle = \beta|\beta\rangle \). Coherent state is a minimum uncertainty state. The number state expansion is represented as \( |\beta\rangle = e^{-\frac{|\beta|^2}{2}} \sum_{n=0}^{\infty} \frac{\beta^n}{\sqrt{n!}} |n\rangle \). The number state expansion of thermal state is given as \( \rho_{th} = \frac{1}{1+N} \sum_{n=0}^{\infty} \left( \frac{N}{1+N} \right)^n |n\rangle \langle n| \). Where N is the average number of thermal photons. The absence of classical description for some of the radiation states enforces quantum mechanical treatment. Such states are called nonclassical states of radiation. Thermal states and coherent states of radiation are the two important radiation sources which also have a classical description. The mixture of coherent light and thermal noise is given as Glauber lachs state\([9]\). The density operator for the Glauber lachs state is given as \( \hat{\rho}_{GL} = \sum_{m,n=0}^{\infty} P_{mn} |m\rangle \langle n| \), where,

\[
P_{mn} = \sqrt{\frac{n!}{m!}} \left( \frac{N^n \beta^{m-n} \exp \left[-\frac{|\beta|^2}{N+1}\right] L_n^{m-n} \left[-\frac{|\beta|^2}{N(N+1)}\right]}{(N+1)^{m+1}} \right).
\]

Here N represents the mean number of thermal photons, \( \beta \) is the coherence parameter and \( |\beta|^2 \) represents mean number of coherent photons. By extending the photon addition into two modes we get a bipartite Glauber lachs state using bosonic creation creation operators. For simplicity the average number of thermal photons (N) and coherent parameter (\( \beta \)) are taken as same for both modes of the Glauber lachs state. The new state, called nonlocal Photon Added Bipartite Glauber Lachs State (PABGLS) is given as,

\[
\hat{\rho} = Z(\hat{c}^\dagger + \hat{d}^\dagger) \rho_{GL} \otimes \rho_{GL}(\hat{c} + \hat{d}).
\]

Z is the normalization constant and \( \hat{c}^\dagger(\hat{c}), \hat{d}^\dagger(\hat{d}) \) are the creation(annihilation) operators for the mode c and d. The single mode and two mode nonclassical features of the above state are examined in the succeeding section.

3. Nonclassical features

A quantum mechanical description is required if the state considered cannot be described classically. Such nonclassical description is not unique, as there are many metrics
available for measuring the nonclassical features. For the present paper, the nonclassical features are investigated through squeezing properties. In order to explain the squeezing properties, we define the dimensionless quadrature operators which can be written in terms of creation and annihilation operators. Here the squeezing properties studied are single mode squeezing, higher order squeezing, intermodal squeezing and sum squeezing.

3.1. Single mode squeezing

Single mode squeezing is one of the important nonclassical features of the quantum state. Two dimensionless single mode quadrature operators are defined as,

\[
\hat{Q}_1 = \frac{\hat{c} + \hat{c}^\dagger}{\sqrt{2}}, \quad \hat{Q}_2 = \frac{\hat{c} - \hat{c}^\dagger}{i\sqrt{2}}.
\]

Similar such operators for the \(d\) mode can also be constructed. Since the mean no of thermal photons and coherent photons are taken as same in both modes, the properties will not differ. The operators \(\hat{Q}_1\) and \(\hat{Q}_2\) satisfy the commutation relation \([\hat{Q}_1, \hat{Q}_2] = i\).

The corresponding uncertainty relation satisfied is \(\langle (\Delta \hat{Q}_1)^2 \rangle \langle (\Delta \hat{Q}_2)^2 \rangle \geq \frac{1}{4}\).

If variance of the any one of the quadrature operator (\(\hat{Q}_1\) or \(\hat{Q}_2\)) goes below the coherent limit \(\frac{1}{2}\), then the state is said to be squeezed.

![Figure 1](image_url)

**Figure 1.** The graph depicts that the coherent parameter(\(\beta\)) and quadrature variance for lower values of thermal noise.

Figure 1 reveals the single mode squeezing of the PABGLS, the quadrature variance\(\langle \Delta \hat{Q}_1^2 \rangle\) for mode c shows squeezing for range of \(\beta\) values and \(N(= 0.01 \& 0.001)\) values. Higher \(N(\approx 0.1)\) value erases the squeezing induced by the photon addition. For larger \(\beta\) values, the state behaves like a coherent state and the quadrature squeezing vanishes. This is illustrated in the Figure 1. The nonlocal photon addition introduces quadrature squeezing for the coherent and thermal photon ratio of \(\frac{|\beta|^2}{N} \sim 100\) for the moderate \(\beta\) value.
3.2. Higher order squeezing

The higher moments of the single mode quadrature operators also exhibit nonclassicality by violating the coherent limit. Hillery [10] designate it as amplitude squared squeezing and introduced the two dimensionless higher order quadrature operators in single mode case, which is defined in terms of $\hat{c}^2, \hat{c}^\dagger^2$.

$$\hat{P}_1 = \frac{\hat{c}^2 + \hat{c}^\dagger^2}{\sqrt{2}}, \hat{P}_2 = \frac{\hat{c}^2 - \hat{c}^\dagger^2}{i\sqrt{2}}.$$ 

The uncertainty relation is as follows

$$\left(\Delta \hat{P}_1\right)^2 \left(\Delta \hat{P}_2\right)^2 > \frac{1}{4} \left| \left\{ \hat{P}_1, \hat{P}_2 \right\} \right|^2.$$ 

If the condition $\left(\Delta \hat{P}_1\right)^2 - \frac{1}{2} \left| \left\{ \hat{P}_1, \hat{P}_2 \right\} \right| < 0$, then $P_1$ is said to be squeezed and vice versa. The above single mode higher order quadrature variance are calculated for PABGLS and the condition is plotted in Figure 2.

![Figure 2](image_url)

**Figure 2.** The graph depicts that the coherent parameter($\beta$) and condition for lower values of thermal noise.

The above quadrature variances are calculated for the photon added bipartite Glauber lachs state and the results are plotted in the Figure 2. The state exhibits amplitude squared squeezing for the lower value of mean thermal photons ($N \approx 0.01$). The higher values of $N (>0.1)$ do not exhibit the amplitude squared squeezing. The ratio of the coherent photons to thermal photons for the amplitude squared squeezing is same as the quadrature squeezing. Larger $\beta$ values lead to coherent state behavior.

3.3. Intermodal squeezing

The nonclassical correlation between two modes can be gauged by defining appropriate intermodal quadrature operators and the condition derived from the uncertainty
relations. The intermodal quadrature operator[11] for the mode c and d are given as,

\[ \hat{U}_1 = \frac{\hat{c} + \hat{c}^\dagger + \hat{d} + \hat{d}^\dagger}{2\sqrt{2}}, \]

\[ \hat{U}_2 = \frac{\hat{c} - \hat{c}^\dagger + \hat{d} - \hat{d}^\dagger}{i2\sqrt{2}}. \]

The state is said to be squeezed in intermodal sense, if any one of the quadrature variance, \( \langle \Delta \hat{U}_1^2 \rangle \) (or) \( \langle \Delta \hat{U}_2^2 \rangle \) goes below the value \( \frac{1}{4} \).

Figure 3. The graph depicts that the coherent parameter(\( \beta \)) and quadrature variances for the intermodal case for lower values of thermal noise.

The quadrature variance\( \langle \Delta \hat{U}_1^2 \rangle \) between mode c and d shows squeezing for the lower \( N (=0.01 \& 0.001) \) values. Higher values reduce the squeezing induced by the photon addition. For higher values of \( \beta \), the state behaves like a two mode coherent state and the intermodal squeezing vanishes. Figure 3 shows that the nonclassical behaviour is obtained as the noise level is decreased.

3.4. Sum squeezing

Sum squeezing is one of the multimode nonclassical criteria used to study the nonclassical properties. The squeezing operator is defined as[12]

\[ V_0 = \frac{1}{2} [e^{i\theta} \hat{c}^\dagger \hat{d} + e^{-i\theta} \hat{c} \hat{d}]. \]

Here the value of \( \theta \) is taken as zero, and the operator is given as, \( V_0 = \frac{1}{2} [\hat{c}^\dagger \hat{d} + \hat{c} \hat{d}]. \)

If the two mode state is said to be sum squeezed then the quadrature operator must satisfy the following conditions.

\[ S = \frac{4\langle (\Delta V_0)^2 \rangle - \langle \hat{c} \hat{c}^\dagger + \hat{d} \hat{d}^\dagger - 1 \rangle}{\langle \hat{c} \hat{c}^\dagger + \hat{d} \hat{d}^\dagger - 1 \rangle} < 0, \]
\langle (\Delta V_0^2) \rangle < \frac{1}{4} \langle N_c + N_d + 1 \rangle.

Where \( N_c = \hat{c}^\dagger \hat{c} \), \( N_d = \hat{d}^\dagger \hat{d} \).

Figure 4. The graph depicts that the coherent parameter(\( \beta \)) and condition(\( S \)) for lower values of thermal noise.

Figure 4 reveals the sum squeezing properties of the PABGLS, the values of \( S \) go below zero for lower \( N = 0.01 \) values. Higher values of \( N \approx 0.1 \) erases the signature of the sum squeezing in the state.

4. Conclusion

The nonlocal photon addition on two mode Glauber Lachs state induces nonclassical properties, such as single mode squeezing, higher order squeezing, two mode squeezing and sum squeezing. The ratio of coherent noise to thermal noise is vital for the clear exhibition of nonclassical properties. Even lower levels of thermal noise erases the signature of nonclassicality. The results enforces the known result [13] that even small disorder is enough to establish ordered system. Thermal noise is inevitable in the experimental scenario. Hence fixing the signal to noise ratio for the creation of multimode nonclassical states requires further studies. Nonlocal photon subtraction on noisy states may also be considered, provided the operation induces nonclassicality.

5. References

[1] Neilsen M. A and Chuang I. L 2000 Quantum Computation and Quantum Information Cambridge University Press
[2] G.S. Agarwal and K. Tara 1991 Phys. Rev. A 43 492-497.
[3] Xue- xiang xu and Fang-sen Xie 2013 Int. J. Theor. Phys. 52 2784-2795.
[4] Gang Ren and Wenhai Zhang 2018 *Optik* **181** 191-201.
[5] Li-Yun Hu and Zhi-Ming Zhang 2013 *J. Opt. Soc. Am. B* **30** 518-529.
[6] Heng-Mei Li *et al* 2018 *Int. J. Theor. Phys* **57** 941-950.
[7] Cun Jin *et al* 2017 *Front. Phys.* **12** 120307.
[8] R. J. Glauber 1963 *Phys. Rev.* **131** 2766-2788.
[9] G. Lachs 1965 *Phys. Rev.* **138** B 1012-1016.
[10] Mark Hillery 1987 *Phys. Rev. A* **36** 3796-3802.
[11] Leonard Mandel and Emil Wolf 1995 Optical Coherence and Quantum Optics, *Cambridge University Press*.
[12] Mark Hillery 1989 *Phys. Rev. A* **40** 3147-3155.
[13] M. Venkata Satyanarayana *et al* 1992 *Phys. Rev. A* **45** 5301-5304.