Nuclear effects in neutrinoproduction of pions

Iván Schmidt, M. Siddikov
Departamento de Física, Universidad Técnica Federico Santa María,
y Centro Científico - Tecnológico de Valparaíso, Casilla 110-V, Valparaíso, Chile

In this paper we study nuclear effects in the neutrinoproduction of pions. We found that in a Bjorken kinematics, for moderate $x_B$ accessible in ongoing and forthcoming neutrino experiments, the cross-section is dominated by the incoherent contribution; the coherent contribution becomes visible only for small $|t| \lesssim 1/R_A^2$, which requires $x_B \lesssim 0.1$. Our results could be relevant to the kinematics of the ongoing MINERvA experiment in the middle-energy (ME) regime. We provide a code which could be used for the evaluation of the $\nu$DVMP observables using different parametrizations of GPDs and different models of nuclear structure.

I. INTRODUCTION

Today one of the key objects used to parametrize the nonperturbative structure of the target are the generalized parton distributions (GPDs). For kinematics where the collinear factorization is applicable [1,2], they allow to evaluate cross-sections for a wide class of processes. Right now all the information on GPDs comes from electron-proton and positron-proton measurements done at JLAB and HERA, in particular the deeply virtual Compton scattering (DVCS) and deeply virtual meson production (DVMP) [1-10,12-17]. A planned CLAS12 upgrade at JLAB [17] and ongoing experiments at COMPASS [18] will help to improve our understanding of the GPDs, and in particular the ability to polarize both the beam and the target will allow to measure a large number of polarization asymmetries, providing various constraints for phenomenological GPD parametrizations. However, in practice the extraction of GPDs from modern experimental data is still aggravated by uncertainties, such as large BFKL-type logarithms in next-to-leading order (NLO) corrections [19] at HERA kinematics, higher-twist components of GPDs and pion distribution amplitudes (DAs) at JLAB kinematics [20-23], and vector meson DAs in the case of $\rho$- and $\phi$-meson production.

From this point of view, consistency checks of the GPD extraction from experimental data, especially of their flavor structure, are important. Earlier we proposed to study the GPDs in deeply virtual neutrino production of pseudo-Goldstone mesons ($\pi, K, \eta$) [24] with high-intensity NuMI beam at Fermilab, which recently switched to the so-called middle-energy (ME) regime [25], with an average neutrino energy of about 6 GeV. The $\nu$DVMP measurements with neutrino and antineutrino beams in this kinematics are complementary to the electromagnetic DVMP measured at JLAB. In the axial channel, due to chiral symmetry breaking, we have an octet of pseudo-Goldstone bosons, which act as a natural probe of the flavor content. Due to the $V-A$ structure of the charged current, in $\nu$DVMP one can access simultaneously the unpolarized GPDs, $H, E$, and the helicity flip GPDs, $\tilde{H}$ and $\tilde{E}$. Besides, using chiral symmetry and assuming closeness of pion and kaon parameters, the full flavor structure of the GPDs may be extracted. We found [24] that the higher-twist corrections in neutrino production are much smaller than in the electroproduction, which gives an additional appeal to the neutrino-production channel.

Unfortunately, in modern neutrino experiments, for various technical reasons, nuclear targets are much more frequently used than liquid hydrogen. By analogy with neutrino-induced deep inelastic scattering ($\nu$DIS) on nuclei, one can expect that $\nu$DVMP on nuclei could be sensitive to many nuclear phenomena such as shadowing, antishadowing, EMC-effect and Fermi motion. In inclusive processes, all these effects give contributions of order $\lesssim 10\%$ in the $0.1 \lesssim x_B \lesssim 0.8$ region relevant for the ongoing and forthcoming $\nu$DVMP experiments [27]. However, there are indications [28] that in the off-forward kinematics ($t \neq t_{\min}$) they could be enhanced. For this reason, in order to be able to test reliably various GPD models, one should take into account nuclear effects. We study them in a handbag approach, since in the regime of $x_B > 0.1$, relevant for the current and forthcoming $\nu$DVMP experiments, multiparticle corrections should be negligible.

The paper is organized as follows. In Section II we discuss the framework used for evaluation of nuclear effects. In Section III for sake of completeness we list briefly the parametrizations of GPDs used for our analysis. In Section IV we present numerical results and draw conclusions.

II. NUCLEAR EFFECTS

There are two types of processes on nuclear targets, coherent (without nuclear breakup) and incoherent (with nuclear breakup into fragments). The former contribution is enhanced due to coherence as $\sim A^2$, but this effect is relevant only at very small values of $t$. At larger values of $|t| \sim 1/r_A^2$, where $r_A$ is the nuclear radius, this contribution vanishes rapidly and eventually gets covered by the incoherent contribution.
The typical values of $x_B$ accessible in the modern and forthcoming $\nu$DVMP measurements are $x_B \gtrsim 0.1$, and for this reason one can neglect multinucleon coherence effects and describe the process by single-nucleon interactions. Combining this with the weak binding of the nucleons inside nuclei, we may use the Impulse Approximation (IA) and write the amplitude of the process as,

$$A_{\text{coh}} = \int d^3\vec{k}_p \rho_p \left( \vec{k} - \frac{\Delta}{2}, \vec{k} + \frac{\Delta}{2} \right) A_p \left( \vec{k} - \frac{\Delta}{2}, \vec{k} + \frac{\Delta}{2}, q \right) + \int d^3\vec{k}_n \rho_n \left( \vec{k} - \frac{\Delta}{2}, \vec{k} + \frac{\Delta}{2} \right) A_n \left( \vec{k} - \frac{\Delta}{2}, \vec{k} + \frac{\Delta}{2}, q \right),$$

where $\vec{k} - \frac{\Delta}{2}$ and $\vec{k} + \frac{\Delta}{2}$ are the momenta of the incoming and outgoing nucleons respectively, $\rho_p$ and $\rho_n$ are the density matrices of the protons and neutrons inside nuclei, and $A_p,n$ are the amplitudes of the process on free protons and neutrons \[29\]. In \[1\] we ignore a poorly known and essentially model-dependent contributions of the so-called non-nucleonic degrees of freedom, which are sometimes added to the rhs of \[1\]. Also, we don’t include the contribution of processes in which a final nucleus remains in an excited isomer state: we expect that such processes are suppressed both at large-$t$ (due to the nuclear formfactor) and small-$t$ (due to additional factor $\sim t^n$ in multipole transitions between different shells).

In a Bjorken kinematics region, a collinear factorization theorem tells us that the scattering amplitude both on the nucleons and nuclei has a form of the convolution of the GPD of the baryon $H_A$ with a process-dependent hard coefficient function $C(x, \xi)$ \[1\],

$$A_{\text{coh}} \sim \int dx \, C(x, \xi) \, H_A(x, \xi, t),$$

which, combined with \[1\], yields a convolution relation for the GPDs of the nucleus \[2\],

$$H_{q/A}(x, \xi, t) = \int_0^1 dy \, H_{p/A}(y, \xi, t) \, H_p \left( \frac{x}{y}, \frac{\xi}{y}, t \right) + \int_0^1 dy \, \rho_n(y, \xi, t) \, H_{n/A} \left( \frac{x}{y}, \frac{\xi}{y}, t \right),$$

where $y$ is the light-cone fraction of the nuclear momentum carried by the nucleon, and we introduced the so-called light cone nucleon distributions $H_{p/A}, H_{n/A}$ related to the densities $\rho_{p,n}$ as

$$H_{i/A}(y, \xi, t) = m_N \int d^2k_\perp \rho_i \left( m_N(y + \xi), k_\perp - \frac{\Delta}{2}; m_N(y - \xi), k_\perp + \frac{\Delta}{2} \right), \quad i = p, n.$$

The two equations \[23\] may be schematically illustrated with the so-called double handbag diagram in the left pane of the Figure \[1\] as a two-stage process. This approximation has been used e.g. in \[28,33\], and describes $e A$ data reasonably well.

For the incoherent processes, we may assume completeness of the final states (the so-called closure approximation), and using unitarity, as schematically shown by the diagram in the right pane of the Figure \[1\], get a similar expression for the cross-section of the process \[29\],

$$\sigma_{\text{inc}} = \int d^3\vec{k} \sum_{i=p,n} \rho_i \left( \vec{k}, \vec{k} \right) \sigma_i \left( \vec{k}, \vec{k} + \Delta, q \right) \approx \int \frac{d^3y}{y} \sum_{i=p,n} H_{i/A}(0, 0, 0) \sigma_i \left( \vec{k}, \vec{k} + \Delta, q \right) e = y P_A,$$

where $P_A$ is the momentum of the nucleus, and the last equality in \[5\] is valid in a collinear approximation.

Since the binding energy of a nucleon in the nucleus is very small compared to a mass of the free nucleon, the distributions $H_{p/A}, H_{n/A}$ are strongly peaked functions, and in the first approximation may be approximated as \[28\]

$$H_{p/A}(y, \xi \approx 0, t \approx 0) = Z \frac{\sqrt{\alpha}}{\pi} \exp \left[ -\alpha (y - 1)^2 \right] \frac{F_A(t)}{F_A(0)},$$

$$H_{n/A}(y, \xi \approx 0, t \approx 0) = (A - Z) \frac{\sqrt{\alpha}}{\pi} \exp \left[ -\alpha (y - 1)^2 \right] \frac{F_A(t)}{F_A(0)},$$

---

1 Explicit expressions for the leading twist coefficient functions for various $\nu$DVMP processes may be found in \[24\].

2 In what follows we assume for the sake of simplicity that the spin of the nucleus is zero, which is true for most frequently used nuclear targets like $^{12}$C, $^{40}$Ca, $^{40}$Ar, $^{56}$Fe, $^{132}$Xe.
where \( Z \) is the atomic number, \( A \) is the mass number, and the parameter \( \alpha \approx k_F^2/m_N^2 \approx 200 \text{ MeV} \) is the Fermi momentum inside the nucleus, which controls the width of the distribution. In the extreme limit \( \alpha \to 0 \), the product of the exponent in (6,7) and a prefactor \( \sqrt{\alpha/\pi} \) reduce to a \( \delta \)-function, and instead of a convolution we end up with a mere sum of amplitudes (for the coherent case) or cross-sections (for the incoherent case) on separate nucleons. However, such a factorized form is an oversimplification, since it cannot describe the \( A \)-dependence of the first moment of the so-called \( D \)-term \( d_A(0) \), for which there are estimates based on very general assumptions [34].

A more realistic approach is to use the functions \( H_{p/A}, H_{n/A} \), evaluated in the shell model of the nuclear structure. One of the most popular choices for the evaluation of the nucleon dynamics inside a nucleus is a QHD-I model proposed in [35–37]. The lagrangian of this model, in its simplest form describes the interaction of the nucleons with effective vector and scalar fields,

\[
L = \bar{\psi} \left( i \slashed{\partial} - M - g_v \slashed{V} + g_s \phi \right) \psi + \frac{1}{2} (\partial_\mu \phi \partial^\mu \phi) - \frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{m_V^2}{2} V_\mu V^\mu. \tag{8}
\]

where we used a shorthand notation \( V_{\mu\nu} = \partial_\mu V_\nu \), \( V_\mu \) and \( \phi \) are the fields of vector and scalar mesons respectively. The mean field models based on a Lagrangian of type (8), have been successful in the description of various characteristics of nuclei. The simplest version of the model used in this work consists of baryons and isoscalar scalar and vector mesons. The pseudoscalar pion degrees of freedom are neglected because their contribution to the ground state of \( 0^+ \)-nuclei essentially averages to zero [36]. In the literature one may find extensions of the model (8), which have additional mesonic degrees of freedom and give better quantitative description of nuclei, especially with nonzero spin and isospin. The corresponding explicit expression for the distribution functions \( H_{p/A}, H_{n/A} \) were calculated in the model (8) in [28], yielding

\[
H_{p,n/A}(y,\xi,t) = \sum_i \int \frac{d^2 k_\perp}{(2\pi)^2} \Phi_i^\dagger \left( y - \xi, \vec{k}_\perp + \vec{A}_\perp/2 \right) \gamma_\perp \Phi_i \left( y + \xi, \vec{k}_\perp - \vec{A}_\perp/2 \right), \tag{9}
\]

where \( \Phi_i \) is the wave function of the nucleon inside the nucleus, the summation index \( i \) runs over the proton or neutron shells respectively.

### III. GPD PARAMETRIZATION

For numerical estimates of the nuclear effects, one should use a particular parametrization of GPDs available from the literature [7,13,38,44]. For the sake of definiteness, in what follows we use the parametrization of Kroll-Goloskokov [38,45,46], which succeeded to describe HERA [47] and JLAB [38,45,46] data on electroproduction of different mesons, and therefore it should provide a reasonable description of neutrino-induced DVMP. The parametrization is based on the Radyushkin’s double distribution ansatz, in which the skewness is introduced separately for sea and valence quarks,

\[
H(x,\xi,t) = H_{\text{val}}(x,\xi,t) + H_{\text{sea}}(x,\xi,t), \tag{10}
\]

Figure 1: (color online) **Left**: Double handbag diagram for the amplitude of the coherent pion production on the nucleus. **Right**: Closure approximation and its relation to distribution of the cross-section of the incoherent process.
In order to quantify the size of the nuclear effects, we consider a ratio

\[ R_A = \frac{d\sigma_A}{d\nu dQ^2} \left( \frac{Z}{\mu} \frac{d\sigma_p}{d\nu dQ^2} + (A-Z) \frac{d\sigma_n}{d\nu dQ^2} \right), \tag{13} \]

which takes into account differences in isotopic content of different nuclei. For the case of self-conjugate nuclei, this ratio up to a coefficient \( A/2 \) coincides with a deuteron-normalized cross-section \(^3\) used in the presentation of experimental data.

\(^3\) The nuclear effects in the deuteron are small. The shadowing corrections are also negligible since we consider the kinematics \( x_B \gtrsim 0.1 \).
In the left pane of Figure 3, we have plotted the ratio $R_A$ for several spin-0 nuclei. In the regime of small-$x_B$ the ratio $R_A$ is close to $A$ due to the coherence of contributions of separate nucleons. For higher values of $x_B$, the behavior of the cross-section is similar to that of a formfactor: it decreases rapidly and has nodes, with average distance between the nodes $∼ 1/r_A$, where $r_A$ is the nuclear radius. However, the positions of the nodes do not coincide with those of a nuclear formfactor, a type of behavior which cannot be reproduced by a simple model (6,7).

In neutrino experiments this kinematic region is hardly accessible experimentally, since it is covered by the incoherent contribution if the final nucleus breakup is not detected (see the right pane of the same Figure). In the lower pane of each figure, we’ve shown the asymmetry $A_{\pi} = \frac{d\sigma_{\nu A} - d\sigma_{\nu A}}{d\sigma_{\nu A} + d\sigma_{\nu A}}\sim \int d^3k \left( \rho_p(k,k) - \rho_n(k,k) \right) \left( d\sigma_{\nu p} - d\sigma_{\nu p} - d\sigma_{\nu n} - d\sigma_{\nu n} \right)$, which is sensitive to an isospin-1 GPD combination $H^u - H^d$. For self-conjugate nuclei, this asymmetry is exactly zero since in the model (36, 37) the difference between proton and neutron distributions is negligible. For $^{90}$Zr and $^{208}$Pb, the asymmetry in general is small and does not exceed 10%, although increases slightly near the nodes of $\pi^+$ and $\pi^−$.

The $t$-dependence of the cross-section is shown in Figure 4. At small-$t$, the coherent ratio $R_A$ scales as $R_A \propto A \exp(t_{\text{min}}(x_B) r_A^2/4)$, where $r_A$ is the nuclear radius, but decreases rapidly at large-$t$. The incoherent cross-section shown schematically in the right pane of Figure 3 is close to unity, as expected. Its suppression at small-$|t| ∼ |t_{\text{min}}|$ comes from $\xi_p \leq 1$ and onshellness conditions in the convolution integral in (3).

In order to understand the sensitivity of the cross-sections to a choice of GPD parametrization, in the left pane of the Figure 4 we compared the predictions of Kroll-Goloskokov model discussed in Section III with a simple zero-skewness model $H^{q} = q(x) F_N(t)$. As we can see, there is an up to a factor of two difference between the two models.

Frequently the targets in neutrino experiments are organic scintillators with a general atomic structure $\text{CH}_n$. As one can see from the right pane of the Figure 3 in the region $x_B \gtrsim 0.3$ there are two dominant contributions, from hydrogen atoms and from incoherent cross-sections, which cannot be separated unless a final nucleus is detected. A coherent cross-section is strongly suppressed in this kinematics.

V. CONCLUSIONS

In this paper we studied the nuclear effects in the coherent and incoherent pion production. We found that the former has a complicated structure, with coherent enhancement in the region of small-$x_B$, small-$t$, and strong nuclear

---

4 The nodes of $\pi^+$ and $\pi^−$ don’t exactly match due to differences in proton and neutron distributions $\rho_p, \rho_n$. 

suppression outside this kinematics. Similar to a formfactor, the leading twist contribution has nodes. For the incoherent case, the nuclear dependence is quite mild, and for $|t| \gtrsim 3|t_{\min}(x_B)|$ the nuclear effects are negligible, i.e. the full cross-section is a mere sum of contributions of separate nucleons. This is a model-independent result, and from a practical point of view, this allows to get rid of extra uncertainties related to nuclear structure. Our approach is applicable in the regime $x_B \gtrsim 10^{-2}$. For smaller values of $x_B$, this picture is modified due to coherence [48] and saturation [39] effects. For further practical applications, we provide a code, which can be used for the evaluation of nuclear cross-sections with different parametrizations of GPDs and models of nuclear structure. The modular structure of the code allows to easily consider different parametrizations of GPDs and nuclear distributions. For illustration, we provide with this package the libraries for the Kroll-Goloskokov GPD model and the QHD-I nuclear structure model distribution used in this paper. Also, we provide detailed instructions how to build and use new libraries.

Finally, we would like to stop briefly on the recent results of the MINERvA collaboration [25]. Albeit those results are for coherent pion production on nuclei, here we do not make any comparison, because the conditions of applicability of collinear factorization ($Q^2 \gg m^2_N$) are not met in that kinematics. Models based on extrapolation of the Adler relation [50] are more appropriate for that kinematics.

Acknowledgments

This work was supported in part by Fondecyt (Chile) grants No. 1140390 and 1140377.

[1] X. D. Ji and J. Osborne, Phys. Rev. D 58 (1998) 094018 [arXiv:hep-ph/9801260].
[2] J. C. Collins and A. Freund, Phys. Rev. D 59, 074009 (1999).
[3] D. Mueller, D. Robaschik, B. Geyer, F. M. Dittes and J. Horejsi, Fortsch. Phys. 42, 101 (1994) [arXiv:hep-ph/9812448].
[4] X. D. Ji, Phys. Rev. D 55, 7114 (1997).
[5] X. D. Ji, J. Phys. G 24, 1181 (1998) [arXiv:hep-ph/9807358].
[6] A. V. Radyushkin, Phys. Lett. B 380, 417 (1996) [arXiv:hep-ph/9604317].
[7] A. V. Radyushkin, Phys. Rev. D 56, 5524 (1997).
[8] A. V. Radyushkin, arXiv:hep-ph/0101225.
[9] J. C. Collins, L. Frankfurt and M. Strikman, Phys. Rev. D 56, 2982 (1997).
[10] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Mueller and M. Strikman, Phys. Rev. D 50, 3134 (1994).
[11] K. Goede, M. V. Polyakov and M. Vanderhaegen, Prog. Part. Nucl. Phys. 47, 401 (2001) [arXiv:hep-ph/0106012].
[12] K. Goede, M. V. Polyakov and M. Vanderhaegen, Prog. Part. Nucl. Phys. 47, 401 (2001) [hep-ph/0106012].
[13] M. Diehl, T. Feldmann, R. Jakob and P. Kroll, Nucl. Phys. B 596, 33 (2001) [Erratum-ibid. B 605, 647 (2001)] [arXiv:hep-ph/0009255].
[14] A. V. Belitsky, D. Mueller and A. Kirchner, Nucl. Phys. B 629, 323 (2002) [arXiv:hep-ph/0112108].
[15] M. Diehl, Phys. Rept. 388, 41 (2003) [arXiv:hep-ph/0307382].
[16] A. V. Belitsky and A. V. Radyushkin, Phys. Rept. 418, 1 (2005) [arXiv:hep-ph/0504030].
[17] V. Kubarovsky [CLAS Collaboration], Nucl. Phys. Proc. Suppl. 219-220, 118 (2011).
[18] N. d'Hose [COMPASS Collaboration], EPJ Web Conf. 73, 01010 (2014).
[19] D. Y. Ivanov, arXiv:0712.3193 [hep-ph].
[20] S. Ahmad, G. R. Goldstein and S. Liuti, Phys. Rev. D 79 (2009) 054014 [arXiv:0805.3568 [hep-ph]].
[21] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 65, 137 (2010) [arXiv:0906.0460 [hep-ph]].
[22] S. V. Goloskokov and P. Kroll, Eur. Phys. J. A 47, 112 (2011) [arXiv:1106.4897 [hep-ph]].
[23] G. R. Goldstein, J. O. G. Hernandez and S. Liuti, arXiv:1201.6088 [hep-ph].
[24] B. Z. Kopeliovich, I. Schmidt and M. Siddikov, Phys. Rev. D 86 (2012), 113018 [arXiv:1210.4825 [hep-ph]].
[25] A. Higuera et al. [MINERvA Collaboration], arXiv:1409.3835 [hep-ex].
[26] B. Z. Kopeliovich, I. Schmidt and M. Siddikov, Phys. Rev. D 89, 053001 (2014) [arXiv:1401.1547 [hep-ph]].
[27] P. Amaudruz et al. [New Muon Collaboration], Nucl. Phys. B 441 (1995) 3 [hep-ph/9503291].
[28] V. Guzey and M. Siddikov, J. Phys. G 32, 251 (2006) [hep-ph/0509158].
[29] V. Guzey, A. W. Thomas and K. Tsushima, Phys. Lett. B 673, 9 (2009) [arXiv:0806.3288 [hep-ph]].
[30] V. Guzey and M. Strikman, Phys. Rev. C 68, 015204 (2003) [hep-ph/0301216].
[31] S. Liuti and S. K. Taneja, Phys. Rev. C 72, 032201 (2005) [hep-ph/0505123].
[32] M. Rinaldi and S. Scopetta, Few Body Syst. 55, 861 (2014) [arXiv:1401.1350 [nucl-th]].
[33] I. A. Schmidt and R. Blankenbecler, Phys. Rev. D 15, 3321 (1977).
[34] M. V. Polyakov, Phys. Lett. B 555, 57 (2003) [hep-ph/0210165].
[35] Chin S. A. and Walecka J. D., Phys. Lett. B 52 (1974), 24.
[36] Serot B. D. and Walecka J. D., Adv. Nucl. Phys. 16 (1986), 1.
[37] Serot B. D. and Walecka J. D., Int. J. Mod. Phys. E 6 (1997), 515 [arXiv:nucl-th/9701058].
[38] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 59 (2009) 809 [arXiv:0809.4126 [hep-ph]].
[39] K. Kumericki, D. Muller and A. Schafer, JHEP 1107, 073 (2011) [arXiv:1106.2808 [hep-ph]].
[40] M. Guidal, Phys. Lett. B 693, 17 (2010) [arXiv:1005.4922 [hep-ph]].
[41] M. V. Polyakov and K. M. Semenov-Tian-Shansky, Eur. Phys. J. A 40, 181 (2009) [arXiv:0811.2901 [hep-ph]].
[42] M. V. Polyakov and A. G. Shuvaev, hep-ph/0207153.
[43] A. Freund, M. McDermott and M. Strikman, Phys. Rev. D 67, 036001 (2003) [hep-ph/0208160].
[44] G. R. Goldstein, J. O. G. Hernandez and S. Liuti, arXiv:1311.0483 [hep-ph].
[45] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 50, 829 (2007) [hep-ph/0611290].
[46] S. V. Goloskokov and P. Kroll, Eur. Phys. J. C 53, 367 (2008) [arXiv:0708.3569 [hep-ph]].
[47] P. D. Aaron et al. [H1 Collaboration], JHEP 1005 (2010) 032 [arXiv:0910.5831 [hep-ex]].
[48] B. Z. Kopeliovich, J. G. Morfin and I. Schmidt, Prog. Part. Nucl. Phys. 68, 314 (2013) [arXiv:1208.6541 [hep-ph]].
[49] A. H. Rezaeian, M. Siddikov, M. Van de Klundert and R. Venugopalan, Phys. Rev. D 87, no. 3, 034002 (2013) [arXiv:1212.2974].
[50] S. L. Adler, Phys. Rev. 135, B963 (1964).