NEUTRINOS AND DARK MATTER IN GALACTIC HALOS

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Abstract

One of the most important problems in astrophysics concerns the nature of the dark matter in galactic halos, whose presence is implied mainly by the observed flat rotation curves in spiral galaxies. Due to the Pauli exclusion principle it can be shown that neutrinos cannot be a major constituent of the halo dark matter. As far as cold dark matter is concerned there might be a discrepancy between the results of the N-body simulations and the measured rotation curves for dwarf galaxies. A fact this, if confirmed, which would exclude cold dark matter as a viable candidate for the halo dark matter.

In the framework of a baryonic scenario the most plausible candidates are brown or white dwarfs and cold molecular clouds (mainly of $H_2$). The former can be detected with the ongoing microlensing experiments. In fact, the French collaboration EROS and the American-Australian collaboration MACHO have reported the observation of altogether $\sim 10$ microlensing events by monitoring during several years the brightness of millions of stars in the Large Magellanic Cloud. In particular, the MACHO team announced the discovery of 8 microlensing candidates by analysing their first 2 years of observations. This would imply that the halo dark matter fraction in form of MACHOs (Massive Astrophysical Compact Halo Objects) is of the order of 45-50% assuming a standard spherical halo model. The most accurate way to get information on the mass of the MACHOs is to use the method of mass moments, which leads to an average mass of $0.27 M_\odot$.

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1 Introduction

One of the most important problems in astrophysics concerns the nature of the dark matter in galactic halos, whose presence is implied by the observed flat rotation curves in spiral galaxies \[1, 2\], the X-ray diffuse emission in elliptical galaxies as well as by the dynamics of galaxy clusters. Primordial nucleosynthesis entails that most of the baryonic matter in the Universe is nonluminous, and such an amount of dark matter falls suspiciously close to that required by the rotation curves. Surely, the standard model of elementary particle forces can hardly be viewed as the ultimate theory and all the attempts in that direction invariably call for new particles. Hence, the idea of nonbaryonic dark matter naturally enters the realm of cosmology and may help in the understanding of the process of galaxy formation and clustering of galaxies.

The problem of dark matter started already with the pioneering work of Oort \[3\] in 1932 and Zwicky \[4\] in 1933 and its mystery is still not solved. Actually, there are several dark matter problems on different scales ranging from the solar neighbourhood, galactic halos, cluster of galaxies to cosmological scales. Dark matter is also needed to understand the formation of large scale structures in the universe.

Many candidates have been proposed, either baryonic or not, to explain dark matter. It is beyond the scope of this lecture to go through all of these candidates. Here, we restrict ourself to the dark matter in galactic halos, and in particular in the halo of our own Galaxy. First, we review the evidence for dark matter near the Sun and in the halo of spiral galaxies. In Section 3 we discuss the constraint due to the Pauli exclusion principle on neutrinos as a candidate for halo dark matter. Based on that argument neutrinos can practically be excluded as a major constituent of the dark galactic halos. In Section 4 we present the baryonic candidates and in Section 5 we elaborate in some detail on the detection of MACHOs (Massive Astrophysical Compact Halo Objects) through microlensing as well as on the most recent observations. Section 6 is devoted to a scenario in which part of the dark matter is in the form of cold molecular clouds (mainly of H\(_2\)).

2 Evidence for dark matter

In this Section we briefly outline the evidence for dark matter in the solar neighbourhood and in the halos of spiral galaxies. Moreover, we discuss also the total mass of our Galaxy, which can be inferred from studies of the proper motion of the satellites of the Milky Way.

2.1 Dark matter near the Sun

The local mass density \[5\] in main sequence and giant stars, stellar remnants (directly observed or inferred from models of galactic and stellar evolution), gas and dust yields a lower limit to the total density of \(\rho \simeq 0.114 \ M_\odot \ {\rm pc}^{-3}\). Correspondingly, one finds a mass-to-light ratio of

\[\Upsilon \simeq 1.7 \Upsilon_\odot\]  

(\(\Upsilon_\odot = M_\odot/L_\odot\), where \(M_\odot\) is the mass and \(L_\odot\) the luminosity of the Sun, respectively). The local mass density is determined from carefully selected star samples by analyzing the
velocity dispersion and density profile in the direction normal to the galactic plane. This yields a total density \( \rho = 0.18 \pm 0.03 \, M_\odot \, \text{pc}^{-3} \) for the local matter, or equivalently

\[
\Upsilon \simeq 2.7 \Upsilon_\odot .
\]  

Therefore, at least 0.03 \( M_\odot \, \text{pc}^{-3} \) is the contribution from dark matter. The presence of disk dark matter has long been suspected \cite{3} and it is most likely baryonic.

Recently, at least 8 brown dwarfs have been detected within a distance of about 100 light years from the Sun. One of these brown dwarfs is about 70 million years old and has an estimated mass of 0.065 \( M_\odot \). Moreover, some brown dwarfs have been found orbiting brighter compaognons, and other as free flying in the Pleiades cluster. It is still premature to infer on their contribution to the local dark matter, although it is plausible that they may make up an important fraction, if not all.

### 2.2 Rotation curves of spiral galaxies

The best evidence for dark matter in galaxies comes from the rotation curves of spirals. Measurements of the rotation velocity \( v_{\text{rot}} \) of stars up to the visible edge of the spiral galaxies and of \( HI \) gas in the disk beyond the optical radius (by measuring the Doppler shift in the 21-cm line) imply that \( v_{\text{rot}} \approx \) constant out to very large distances, rather than to show a Keplerian falloff. These observations started around 1970 \cite{6}, thanks to the improved sensitivity in both optical and 21-cm bands. By now there are observations for over thousand spiral galaxies with reliable rotation curves out to large radii \cite{7}. In almost all of them the rotation curve is flat or slowly rising out to the last measured point. Very few galaxies show falling rotation curves and those that do either fall less rapidly than Keplerian have nearby companions that may perturb the velocity field or have large spheroids that may increase the rotation velocity near the centre.

One of the best exemple for the measurement of the rotation velocity is the spiral galaxy NGC 3198 \cite{8} (see Fig. 1). Its halo density can be fitted by the following formula

\[
\rho(r) = \frac{\rho_0}{1 + (r/a)^\gamma},
\]  

where \( \rho_0 = 0.013 h_0^2 \, M_\odot \, \text{pc}^{-3} \) (\( h_0 \) being the Hubble constant in units of \( H_0 = 100 h_0 \, \text{km s}^{-1} \, \text{kpc}^{-1} \), and \( 0.4 \leq h_0 \leq 1 \)), \( a = 6.4 h_0^{-1} \) kpc, and \( \gamma = 2.1 \). The total mass inside the last measured point of the rotation curve is \( 1.1 \times 10^{11} h_0^{-1} M_\odot \), which yields a total mass-to-light-ratio \( \Upsilon = 28 h_0 \Upsilon_\odot \). This has to be considered as a lower limit, since there is certainly still a lot of dark matter beyond the last measured point on the rotation curve. The dark halo is at least four times as massive as the disk. Such a value for the mass-to-light-ratio is typical for spiral galaxies. Similar conclusions hold also for elliptical galaxies, although one cannot measure rotation velocities.

### 2.3 Mass of the Milky Way

There are also measurements of the rotation velocity for our Galaxy. However, these observations turn out to be rather difficult, and the rotation curve has been measured only
up to a distance of about 20 kpc. Without any doubt our own galaxy has a typical flat rotation curve. A fact which imply that it is possible to search directly for dark matter characteristic of spiral galaxies in our own Milky Way.

In order to infer the total mass one can also study the proper motion of the Magellanic Clouds and of other satellites of our Galaxy. Recent studies [9, 10, 11] do not yet allow an accurate determination of \( v_{\text{rot}}(\text{LMC})/v_0 \) \( (v_0 = 210 \pm 10 \text{ km/s} \) being the local rotational velocity). Lin et al. [10] analyzed the proper motion observations and concluded that within 100 kpc the Galactic halo has a mass \( \sim 5.5 \pm 1 \times 10^{11} M_\odot \) and a substantial fraction \( \sim 50\% \) of this mass is distributed beyond the present distance of the Magellanic Clouds of about 50 kpc. Beyond 100 kpc the mass may continue to increase to \( \sim 10^{12} M_\odot \) within its tidal radius of about 300 kpc. This value for the total mass of the Galaxy is in agreement with the results of Zaritsky et al. [9], who found a total mass in the range 9.3 to 12.5 \( \times 10^{11} M_\odot \), the former value by assuming radial satellite orbits whereas the latter by assuming isotropic satellite orbits.

The results of Lin et al. [11] suggest that the mass of the halo dark matter up to the Large Magellanic Cloud (LMC) is roughly half of the value one gets for the standard halo model (with flat rotation curve up to the LMC and spherical shape), implying thus the same reduction for the number of expected microlensing events. Kochanek [11] analysed the global mass distribution of the Galaxy adopting a Jaffe model, whose parameters are determined using the observations on the proper motion of the satellites of the Galaxy, the Local Group timing constraint and the ellipticity of the M31 orbit. With these observations Kochanek [11] concludes that the mass inside 50 kpc is \( 5.4 \pm 1.3 \times 10^{11} M_\odot \). This value becomes, however, slightly smaller when using only the satellite observations and the disk rotation constraint, in this case the median mass interior to 50 kpc is in the interval 3.3 to 6.1 (4.2 to 6.8) without (with) Leo I satellite in units of \( 10^{11} M_\odot \). The lower bound without Leo I is 65% of the mass expected assuming a flat rotation curve up to the LMC.

### 3 Neutrinos as halo dark matter

For stable neutrinos (with mass < 1 MeV) one gets the following cosmological upper bound on the sum of their masses [12, 13]

\[
\sum_{\nu} m_{\nu} < 200 h_0^2 \text{ eV} .
\]  

(4)

If neutrinos make up the dark matter in the galactic halos, we may describe them as forming a bound system which resembles in the central regions to an isothermal gas sphere. The core radius of such an isothermal sphere is

\[
r_c = \left( \frac{9 \sigma^2}{4 \pi G \rho_0} \right)^{1/2} ,
\]  

(5)

where \( \sigma \) is the one-dimensional velocity dispersion and \( \rho_0 \) is the central density. The velocity distribution of the neutrinos is Maxwellian and the maximum phase-space density is

\[
n_{\nu} = \frac{4.5}{(2 \pi)^5 G r_c^2 \sigma m_{\nu}^4} .
\]  

(6)
The requirement that the maximum phase-space density does not violate the Pauli exclusion principle \( n_c < g_\nu/h^3 \), where \( g_\nu \) is the number of helicity states and \( h \) Planck’s constant) leads then to the following lower limit for the neutrino mass \[14\] 

\[ m_\nu > 120 \, eV \left( \frac{100 \, \text{km} \, \text{s}^{-1}}{\sigma} \right)^{1/4} \left( \frac{1 \, \text{kpc}}{r_c} \right)^{1/2} g_\nu^{-1/4}. \] \text{(7)}

Typical values for spiral galaxies are \( \sigma \simeq 150 \, \text{km} \, \text{s}^{-1} \) and \( r_c \simeq 20 \, \text{kpc} \). This way we get a lower bound \( m_\nu > 25 - 30 \, \text{eV} \) \[14, 15\], which is still consistent with the cosmological bound eq.(4). See also refs. \[14, 17\] for a discussion of more precise bounds for spirals and ellipticals by considering different visible and dark matter distributions as well as the case where the neutrinos are not fully degenerate. However, when considering dwarf galaxies for which \( r_c < 2 \, \text{kpc} \) and \( \sigma \sim 10 - 30 \, \text{km} \, \text{s}^{-1} \) one gets \( m_\nu > 100 - 500 \, \text{eV} \) \[15\], which is clearly in contradiction with the cosmological bound, excluding thus neutrinos as dark matter candidate for the halo of dwarf galaxies and in turn of spiral galaxies.

This latter point follows also from considering the dwarf galaxies Draco and Ursa Minor, which are both satellites of our Galaxy \[19\] and, therefore, are located in its halo. In fact, if their dark matter halo consist of neutrinos with mass \( \sim 30 \, \text{eV} \), then \( r_c \sim 10 \, \text{kpc} \) and the total mass would be \( \sim 4 \times 10^{11} \, M_\odot \). However, such a high value for the total mass can be excluded by the requirement that the dynamical friction time for such a satellite galaxy moving in the halo of our Galaxy has to be longer than the age of Galaxy \( \sim 10^{10} \, \text{yr} \). The upper value for the total mass one infers this way is of the order of \( 10^{10} \, M_\odot \). Therefore, one gets a lower limit of \( \sim 80 \, \text{eV} \) for the neutrino mass.

### 4 Baryonic dark matter

Before discussing the baryonic dark matter we would like to mention that another class of candidates which is seriously taken into consideration is the so-called cold dark matter, which consists for instance of axions or supersymmetric particles like neutralinos \[20\]. Here, we will not discuss cold dark matter in detail. However, recent studies seem to point out that there is a discrepancy between the calculated (through N-body simulations) rotation curve for dwarf galaxies assuming an halo of cold dark matter and the measured curves \[21\, 22\]. If this fact is confirmed, this would exclude cold dark matter as a major constituent of the halo of dwarf galaxies and possibly also of spiral galaxies.

From the Big Bang nucleosynthesis model \[23\, 24\] and from the observed abundances of primordial elements one infers: \( 0.010 \leq h_0^2 \Omega_B \leq 0.016 \) or with \( h_0 \simeq 0.4 - 1 \) one gets \( 0.01 \leq \Omega_B \leq 0.10 \) (where \( \Omega_B = \rho_B/\rho_{\text{crit}} \), and \( \rho_{\text{crit}} = 3H_0^2/8\pi G \)). Since for the amount of luminous baryons one finds \( \Omega_{\text{lum}} \ll \Omega_B \), it follows that an important fraction of the baryons are dark. In fact the dark baryons may well make up the entire dark halo matter.

The halo dark matter cannot be in the form of hot ionized hydrogen gas otherwise there would be a large X-ray flux, for which there are stringent upper limits. The abundance of neutral hydrogen gas is inferred from the 21-cm measurements, which show that its contribution is small. Another possibility is that the hydrogen gas is in molecular form clumped

\[2\] One gets a slightly higher bound, by a factor \( 2^{1/4} \), using the fact that neutrinos behaves practically as collisionless particles and thus by applying Liouville’s theorem \[14\].
into cold clouds, as we will discuss in some detail in Section 6. Baryons could otherwise have been processed in stellar remnants (for a detailed discussion see [23]). If their mass is below $\sim 0.08 \, M_\odot$ they are too light to ignite hydrogen burning reactions. The possible origin of such brown dwarfs or Jupiter like bodies (called also MACHOs), by fragmentation or by some other mechanism, is at present not understood. It has also been pointed out that the mass distribution of the MACHOs, normalized to the dark halo mass density, could be a smooth continuation of the known initial mass function of ordinary stars [26]. The ambient radiation, or their own body heat, would make sufficiently small objects of H and He evaporate rapidly. The condition that the rate of evaporation of such a hydrogenoid sphere be insufficient to halve its mass in a billion years leads to the following lower limit on their mass [26]:

$$M > 10^{-7} M_\odot (T_S/30 \, K)^{3/2} (1 \, g \, cm^{-3}/\rho)^{1/2}$$

($T_S$ being their surface temperature and $\rho$ their average density, which we expect to be of the order $\sim 1 \, g \, cm^{-3}$).

Otherwise, MACHOs might be either M-dwarfs or else white dwarfs. As a matter of fact, a deeper analysis shows that the M-dwarf option look problematic. The null result of several searches for low-mass stars both in the disk and in the halo of our galaxy suggest that the halo cannot be mostly in the form of hydrogen burning main sequence M-dwarfs. Optical imaging of high-latitude fields taken with the Wide Field Camera of the Hubble Space Telescope indicates that less than $\sim 6\%$ of the halo can be in this form [27]. Also a substantial component of neutron stars and black holes with mass higher than $\sim 1 \, M_\odot$ is excluded, for otherwise they would lead to an overproduction of heavy elements relative to the observed abundances. A scenario with white dwarfs as a major constituent of the galactic halo dark matter has been explored [28]. However, it requires a rather ad hoc initial mass function sharply peaked around 2 - 6 $M_\odot$. Future Hubble deep field exposures could either find the white dwarfs or put constraints on their fraction in the halo [29].

## 5 Detection of MACHOs through microlensing

It has been pointed out by Paczyński [30] that microlensing allows the detection of MACHOs located in the galactic halo in the mass range [26] $10^{-7} < M/M_\odot < 1$. In September 1993 the French collaboration EROS [31] announced the discovery of 2 microlensing candidates and the American–Australian collaboration MACHO of one candidate [32]. In the meantime the MACHO team reported the observation of altogether 8 events (one of which is a binary lensing event) analyzing 2 years of their data by monitoring about 8.5 million of stars in the LMC [33]. Their analysis leads to an optical depth of $\tau = 2.9^{+1.4}_{-1.0} \times 10^{-7}$ and correspondingly to a halo MACHO fraction of the order of 45-50% and an average mass $0.5^{+0.3}_{-0.2} M_\odot$, under the assumption of a standard spherical halo model. It may well be that there is also a contribution of events due to MACHOs located in the LMC itself or in a thick disk of our galaxy, the corresponding optical depth is estimated to be $\tau = 5.4 \times 10^{-8}$ [33]. EROS has also searched for very-low mass MACHOs by looking for microlensing events with time scales ranging from 30 minutes to 7 days [34]. The lack of candidates in this range places significant constraints on any model for the halo that relies on objects in the range $5 \times 10^{-8} < M/M_\odot < 5 \times 10^{-4}$. Similar conclusions have been reached also by the MACHO team [33]. Moreover, the Polish-American team OGLE [35], the MACHO [36] and the French DUO [37] collaborations found altogether more than $\sim 100$ microlensing events by monitoring stars
located in the galactic bulge. The inferred optical depth for the bulge turns out to be higher than previously thought. These results are very important in order to study the structure of our Galaxy.

In the following we present the main features of microlensing, in particular its probability and the rate of events [38]. An important issue is the determination from the observations of the mass of the MACHOs that acted as gravitational lenses as well as the fraction of halo dark matter they make up. The most appropriate way to compute the average mass and other important information is to use the method of mass moments developed by De Rújula et al. [39], which will be briefly discussed in Section 5.4.

5.1 Microlensing probability

When a MACHO of mass $M$ is sufficiently close to the line of sight between us and a more distant star, the light from the source suffers a gravitational deflection (see Fig. 2). The deflection angle is usually so small that we do not see two images but rather a magnification of the original star brightness. This magnification, at its maximum, is given by

$$A_{\text{max}} = \frac{u^2 + 2}{u(u^2 + 4)^{1/2}}.$$  

Here $u = d/R_E$ ($d$ is the distance of the MACHO from the line of sight) and the Einstein radius $R_E$ is defined as

$$R_E^2 = \frac{4GM_D}{c^2}x(1 - x)$$

with $x = s/D$, and where $D$ and $s$ are the distance between the source, respectively the MACHO and the observer.

An important quantity is the optical depth $\tau_{opt}$ to gravitational microlensing defined as

$$\tau_{opt} = \int_0^1 dx \frac{4\pi G}{c^2} \rho(x)D^2x(1 - x)$$

with $\rho(x)$ the mass density of microlensing matter at distance $s = xD$ from us along the line of sight. The quantity $\tau_{opt}$ is the probability that a source is found within a radius $R_E$ of some MACHO and thus has a magnification that is larger than $A = 1.34$ ($d \leq R_E$).

We calculate $\tau_{opt}$ for a galactic mass distribution of the form

$$\rho(\vec{r}) = \frac{\rho_0(a^2 + R_{GC}^2)}{a^2 + \vec{r}^2},$$

$|\vec{r}|$ being the distance from the Earth. Here, $a$ is the core radius, $\rho_0$ the local dark mass density in the solar system and $R_{GC}$ the distance between the observer and the Galactic centre. Standard values for the parameters are $\rho_0 = 0.3 \text{ GeV/cm}^3 = 7.9 \times 10^{-3} M_\odot/\text{pc}^3$, $a = 5.6 \text{ kpc}$ and $R_{GC} = 8.5 \text{ kpc}$. With these values we get, for a spherical halo, $\tau_{opt} = 0.7 \times 10^{-6}$ for the LMC and $\tau_{opt} = 10^{-6}$ for the SMC [40].

The magnification of the brightness of a star by a MACHO is a time-dependent effect. For a source that can be considered as pointlike (this is the case if the projected star radius at
the MACHO distance is much less than $R_E$) the light curve as a function of time is obtained by inserting
\[ u(t) = \frac{(d^2 + v_T^2 t^2)^{1/2}}{R_E} \] (12)
into eq.(8), where $v_T$ is the transverse velocity of the MACHO, which can be inferred from the measured rotation curve ($v_T \approx 200 \text{ km/s}$). The achromaticity, symmetry and uniqueness of the signal are distinctive features that allow to discriminate a microlensing event from background events such as variable stars.

The behaviour of the magnification with time, $A(t)$, determines two observables namely, the magnification at the peak $A(0)$ - denoted by $A_{\text{max}}$ - and the width of the signal $T$ (defined as being $T = R_E/v_T$).

5.2 Microlensing rates

The microlensing rate depends on the mass and velocity distribution of MACHOs. The mass density at a distance $s = xD$ from the observer is given by eq.(11). The isothermal spherical halo model does not determine the MACHO number density as a function of mass.

A simplifying assumption is to let the mass distribution be independent of the position in the galactic halo, i.e., we assume the following factorized form for the number density per unit mass $dn/dM$,
\[ \frac{dn}{dM} dM = \frac{dn_0}{d\mu} \frac{a^2 + R_{GC}^2}{a^2 + R_{GC}^2 + D^2 x^2 - 2D R_{GC} x \cos \alpha} d\mu = \frac{dn_0}{d\mu} H(x) d\mu, \] (13)
with $\mu = M/M_\odot$ ($\alpha$ is the angle of the line of sight with the direction of the galactic centre), $n_0$ not depending on $x$ and is subject to the normalization $\int d\mu \frac{dn_0}{d\mu} M = \rho_0$. Nothing a priori is known on the distribution $dn_0/dM$.

A different situation arises for the velocity distribution in the isothermal spherical halo model, its projection in the plane perpendicular to the line of sight leads to the following distribution in the transverse velocity $v_T$
\[ f(v_T) = \frac{2}{v_H^2} v_T e^{-v_T^2/v_H^2}. \] (14)
($v_H \approx 210 \text{ km/s}$ is the observed velocity dispersion in the halo).

In order to find the rate at which a single star is microlensed with magnification $A \geq A_{\text{min}}$, we consider MACHOs with masses between $M$ and $M + \delta M$, located at a distance from the observer between $s$ and $s + \delta s$ and with transverse velocity between $v_T$ and $v_T + \delta v_T$. The collision time can be calculated using the well-known fact that the inverse of the collision time is the product of the MACHO number density, the microlensing cross-section and the velocity. The rate $d\Gamma$, taken also as a differential with respect to the variable $u$, at which a single star is microlensed in the interval $d\mu du dv_T dx$ is given by [39, 41]
\[ d\Gamma = 2v_T f(v_T) D r_E [\mu x (1 - x)]^{1/2} H(x) \frac{dn_0}{d\mu} d\mu du dv_T dx, \] (15)
with
\[ r_E^2 = \frac{4GM\odot D}{c^2} \sim (3.2 \times 10^9 \text{km})^2. \] (16)

One has to integrate the differential number of microlensing events, \( dN_{ev} = N_\star t_{\text{obs}} d\Gamma \), over an appropriate range for \( \mu, x, u \) and \( v_T \), in order to obtain the total number of microlensing events which can be compared with an experiment monitoring \( N_\star \) stars during an observation time \( t_{\text{obs}} \) and which is able to detect a magnification such that \( A_{\text{max}} \geq A_{TH} \). The limits of the \( u \) integration are determined by the experimental threshold in magnitude shift, \( \Delta m_{TH} \):

we have \( 0 \leq u \leq u_{TH} \).

The range of integration for \( \mu \) is where the mass distribution \( dn_0/d\mu \) is not vanishing and that for \( x \) is \( 0 \leq x \leq D_h/D \) where \( D_h \) is the extent of the galactic halo along the line of sight (in the case of the LMC, the star is inside the galactic halo and thus \( D_h/D = 1 \)).

The galactic velocity distribution is cut at the escape velocity \( v_e \approx 640 \text{ km/s} \) and therefore \( v_T \) ranges over \( 0 \leq v_T \leq v_e \). In order to simplify the integration we integrate \( v_T \) over all the positive axis, due to the exponential factor in \( f(v_T) \) the so committed error is negligible.

However, the integration range of \( d\mu du dv \) does not span all the interval we have just described. Indeed, each experiment has time thresholds \( T_{\text{min}} \) and \( T_{\text{max}} \) and only detects events with: \( T_{\text{min}} \leq T \leq T_{\text{max}} \), and thus the integration range has to be such that \( T \) lies in this interval.

The total number of micro-lensing events is then given by
\[ N_{ev} = \int dN_{ev} \Theta(T - T_{\text{min}})\Theta(T_{\text{max}} - T), \] (17)

where the integration is over the full range of \( d\mu du dv \). \( T \) is related in a complicated way to the integration variables, because of this, no direct analytical integration in eq.(17) can be performed.

To evaluate eq.(17) we define an efficiency function \( \epsilon_0(\mu) \) which measures the fraction of the total number of microlensing events that meet the condition on \( T \) at a fixed MACHO mass \( M = \mu M_\odot \). A more detailed analysis [39] shows that \( \epsilon_0(\mu) \) is in very good approximation equal to unity for possible MACHO objects in the mass range of interest here. We now can write the total number of events in eq.(17) as
\[ N_{ev} = \int dN_{ev} \epsilon_0(\mu). \] (18)

Due to the fact that \( \epsilon_0 \) is a function of \( \mu \) alone, the integration in \( d\mu du dv \) factorizes into four integrals with independent integration limits.

In order to quantify the expected number of events it is convenient to take as an example a delta function distribution for the mass. The rate of microlensing events with \( A \geq A_{\text{min}} \) (or \( u \leq u_{\text{max}} \)), is then
\[ \Gamma(A_{\text{min}}) = \tilde{\Gamma} u_{\text{max}} = D r_E u_{\text{max}} \sqrt{\pi} v_H \rho_0 \frac{1}{M_\odot} \int_0^1 dx [x(1-x)]^{1/2} H(x). \] (19)

Inserting the numerical values for the LMC (\( D=50 \text{ kpc} \) and \( \alpha = 82^0 \)) we get
\[ \tilde{\Gamma} = 4 \times 10^{-13} \frac{1}{s} \left( \frac{v_H}{210 \text{ km/s}} \right) \left( \frac{1}{\sqrt{D/\text{kpc}}} \right) \left( \frac{\rho_0}{0.3 \text{ GeV/cm}^3} \right) \frac{1}{\sqrt{M/M_\odot}}. \] (20)
For an experiment monitoring \( N \) stars during an observation time \( t_{\text{obs}} \) the total number of events with a magnification \( A \geq A_{\text{min}} \) is: \( N_{\text{ev}}(A_{\text{min}}) = N_{\star} t_{\text{obs}} \Gamma(A_{\text{min}}) \). In the following Table 1 we show some values of \( N_{\text{ev}} \) for the LMC, taking \( t_{\text{obs}} = 10^7 \text{ s (} \sim 4 \text{ Months)} \), \( N_{\star} = 10^6 \) stars and \( A_{\text{min}} = 1.34 \) (or \( \Delta m_{\text{min}} = 0.32 \)).

Table 1

| MACHO mass in units of \( M_{\odot} \) | Mean \( R_E \) in km | Mean microlensing time | \( N_{\text{ev}} \) |
|-----------------------------------|---------------------|-----------------------|--------|
| \( 10^{-1} \) | \( 0.3 \times 10^9 \) | 1 month | 1.5 |
| \( 10^{-2} \) | \( 10^8 \) | 9 days | 5 |
| \( 10^{-4} \) | \( 10^7 \) | 1 day | 55 |
| \( 10^{-6} \) | \( 10^6 \) | 2 hours | 554 |

Gravitational microlensing could also be useful for detecting MACHOs in the halo of nearby galaxies \([42, 43]\) such as M31 or M33, for which experiments are under way. In fact, it turns out that the massive dark halo of M31 has an optical depth to microlensing which is of about the same order of magnitude as that of our own galaxy \( \sim 10^{-6} \) \([42, 44]\). Moreover, an experiment monitoring stars in M31 would be sensitive to both MACHOs in our halo and in the one of M31. One can also compute the microlensing rate \([44]\) for MACHOs in the halo of M31, for which we get

\[
\bar{\Gamma} = 1.8 \times 10^{-12} \frac{1}{s} \left( \frac{v_H}{210 \text{ km/s}} \right) \left( \frac{1}{\sqrt{D/kpc}} \right) \left( \frac{\rho(0)}{1 \text{ Gev/cm}^3} \right) \frac{1}{\sqrt{M/M_{\odot}}}.
\]

(\( \rho(0) \) is the central density of dark matter.) In the following Table 2 we show some values of \( N_{\text{ev}}^a \) due to MACHOs in the halo of M31 with \( t_{\text{obs}} = 10^7 \text{ s and } N_{\star} = 10^6 \) stars. In the last column we give the corresponding number of events, \( N_{\text{ev}} \), due to MACHOs in our own halo. The mean microlensing time is about the same for both types of events.

Table 2

| MHO mass in units of \( M_{\odot} \) | Mean \( R_E \) in km | Mean microlensing time | \( N_{\text{ev}}^a \) | \( N_{\text{ev}} \) |
|-----------------------------------|---------------------|-----------------------|--------|--------|
| \( 10^{-1} \) | \( 7 \times 10^5 \) | 38 days | 2 | 1 |
| \( 10^{-2} \) | \( 2 \times 10^8 \) | 12 days | 7 | 4 |
| \( 10^{-4} \) | \( 2 \times 10^7 \) | 30 hours | 70 | 43 |
| \( 10^{-6} \) | \( 2 \times 10^6 \) | 3 hours | 700 | 430 |

Of course these numbers should be taken as an estimate, since they depend on the details of the model one adopts for the distribution of the dark matter in the halo.

### 5.3 Most probable mass for a single event

The probability \( P \) that a microlensing event of duration \( T \) and maximum amplification \( A_{\text{max}} \) be produced by a MACHO of mass \( \mu \) (in units of \( M_{\odot} \)) is given by \([15]\)

\[
P(\mu, T) \propto \frac{\mu^2}{T^4} \int_0^1 dx (x(1-x))^2 H(x) \exp \left( -\frac{r_E^2 \mu x (1-x)}{v_H^2 T^2} \right),
\]

(22)
which does not dependent on $A_{max}$ and $P(\mu,T) = P(\mu/T^2)$. The measured values for $T$ are listed in Table 3, where $\mu_{MP}$ is the most probable value. We find that the maximum corresponds to $\mu r_2^2/v_H^2 T^2 = 13.0 \pm 4.6$. The 50% confidence interval embraces for the mass $\mu$ approximately the range $1/3\mu_{MP}$ up to $3\mu_{MP}$. Similarly one can compute $P(\mu,T)$ also for the bulge events (see [16]).

Table 3: Values of $\mu_{MP}$ (in $M_\odot$) for eight microlensing events detected in the LMC ($A_i$ = American-Australian collaboration events ($i = 1$,..,6); $F_1$ and $F_2$ French collaboration events). For the LMC: $v_H = 210$ km s$^{-1}$ and $r_E = 3.17 \times 10^9$ km.

| $T$ (days) | $A_1$ | $A_2$ | $A_3$ | $A_4$ | $A_5$ | $A_6$ | $F_1$ | $F_2$ |
|------------|-------|-------|-------|-------|-------|-------|-------|-------|
| $\tau(\equiv \frac{\mu}{r_E} T)$ | 0.099 | 0.132 | 0.177 | 0.235 | 0.249 | 0.329 | 0.155 | 0.172 |
| $\mu_{MP}$ | 0.13  | 0.23  | 0.41  | 0.72  | 0.81  | 1.41  | 0.31  | 0.38  |

### 5.4 Mass moment method

A more systematic way to extract information on the masses is to use the method of mass moments [39]. The mass moments $< \mu^m >$ are defined as

$$< \mu^m > = \int d\mu \epsilon_n(\mu) \frac{dn_0}{d\mu} \mu^m. \quad (23)$$

$< \mu^m >$ is related to $< \tau^n > = \sum_{\text{events}} \tau^n$, with $\tau \equiv (v_H/r_E)T$, as constructed from the observations and which can also be computed as follows

$$< \tau^n > = \int dN_{\text{ev}} \epsilon_n(\mu) \tau^n = V u TH \Gamma(2-m) \tilde{H}(m) < \mu^m >, \quad (24)$$

with $m \equiv (n+1)/2$ and

$$V \equiv 2 N_* t_{\text{obs}} D r_E v_H = 2.4 \times 10^3 \text{ pc}^3 \frac{N_* t_{\text{obs}}}{10^6 \text{ star - years}}, \quad (25)$$

$$\Gamma(2-m) \equiv \int_0^\infty \left(\frac{v_T}{v_H}\right)^{1-n} f(v_T) dv_T, \quad (26)$$

$$\tilde{H}(m) \equiv \int_0^1 (x(1-x))^m H(x) dx. \quad (27)$$

The efficiency $\epsilon_n(\mu)$ is determined as follows (see [39])

$$\epsilon_n(\mu) \equiv \frac{\int dN_{\text{ev}}(\bar{\mu}) \epsilon(T) \tau^n}{\int dN_{\text{ev}}(\bar{\mu}) \tau^n}, \quad (28)$$

where $dN_{\text{ev}}^*(\bar{\mu})$ is defined as $dN_{\text{ev}}$ in eq.(17) with the MACHO mass distribution concentrated at a fixed mass $\bar{\mu}$: $dn_0/d\mu = n_0 \delta(\mu - \bar{\mu})/\mu$. $\epsilon(T)$ is the experimental detection efficiency. For a more detailed discussion on the efficiency see ref. [17].
A mass moment $< \mu^m >$ is thus related to $< \tau^n >$ as given from the measured values of $T$ in a microlensing experiment by

$$< \mu^m > = \frac{< \tau^n >}{V u_{TH} \Gamma(2-m) \hat{H}(m)}.$$  \hfill (29)

The mean local density of MACHOs (number per cubic parsec) is $< \mu^0 >$. The average local mass density in MACHOs is $< \mu^1 >$ solar masses per cubic parsec. In the following we consider only 6 (see Table 3) out of the 8 events observed by the MACHO group, in fact the two events we neglect are a binary lensing event and an event which is rated as marginal. The mean mass, which we get from the six events detected by the MACHO team, is

$$\frac{< \mu^1 >}{< \mu^0 >} = 0.27 \, M_\odot.$$  \hfill (30)

(To obtain this result we used the values of $\tau$ as reported in Table 3, whereas $\Gamma(1) \hat{H}(1) = 0.0362$ and $\Gamma(2) \hat{H}(0) = 0.280$ as plotted in Fig. 6 of ref. \[39\]). If we include also the two EROS events we get a value of 0.26 $M_\odot$ for the mean mass. The resulting mass depends on the parameters used to describe the standard halo model. In order to check this dependence we varied the parameters within their allowed range and found that the average mass changes at most by $\pm 30\%$, which shows that the result is rather robust. Although the value for the average mass we find with the mass moment method is marginally consistent with the result of the MACHO team, it definitely favours a lower average MACHO mass.

One can also consider other models with more general luminous and dark matter distributions, e.g. ones with a flattened halo or with anisotropy in velocity space \[48\], in which case the resulting value for the average mass would decrease significantly. If the above value will be confirmed, then MACHOs cannot be brown dwarfs nor ordinary hydrogen burning stars, since for the latter there are observational limits from counts of faint red stars. Then white dwarfs are the most likely explanation. As mentioned in Section 4 such a scenario has been explored recently \[28\]. However, it has some problems, since it requires that the initial mass function must be sharply peaked around $2 - 6 \, M_\odot$. Given these facts, we feel that the brown dwarf option can still provide a sensible explanation of the observed microlensing events \[49\].

Another important quantity to be determined is the fraction $f$ of the local dark mass density (the latter one given by $\rho_0$) detected in the form of MACHOs, which is given by $f \equiv M_\odot/\rho_0 \sim 126 \, pc^3 < \mu^1 >$. Using the values given by the MACHO collaboration for their two years data \[33\] (in particular $u_{TH} = 0.661$ corresponding to $A > 1.75$ and an effective exposure $N_{\star} t_{obs}$ of $\sim 5 \times 10^6$ star-years for the observed range of the event duration $T$ between $\sim 20 - 50$ days) we find $f \sim 0.54$, which compares quite well with the corresponding value ($f \sim 0.45$ based on the six events we consider) calculated by the MACHO group in a different way. The value for $f$ is obtained again by assuming a standard spherical halo model.
Table 4: Values of $\mu_{MP}$ (in $M_\odot$) as obtained by the corresponding $P(\mu, T)$ for eleven microlensing events detected by OGLE in the galactic bulge \[17\]. ($v_H = 30$ km s\(^{-1}\) and $r_E = 1.25 \times 10^9$ km.) ($T$ is in days as above.)

|    | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $T$ | 25.9 | 45  | 10.7| 14  | 12.4| 8.4 | 49.5| 18.7| 61.6| 12  | 20.9|
| $\tau$ | 0.054 | 0.093 | 0.022 | 0.026 | 0.017 | 0.103 | 0.039 | 0.128 | 0.025 | 0.043 |
| $\mu_{MP}$ | 0.61 | 1.85 | 0.105 | 0.18 | 0.14 | 0.065 | 2.24 | 0.32 | 3.48 | 0.13 | 0.40 |

Similarly, one can also get information from the events detected so far towards the galactic bulge. The mean MACHO mass, which one gets when considering the first eleven events detected by OGLE in the galactic bulge (see Table 4), is $\sim 0.29 M_\odot$ \[46\]. From the 40 events discovered during the first year of operation by the MACHO team \[34\] (we considered only the events used by the MACHO team to infer the optical depth without the double lens event) we get an average value of 0.16 $M_\odot$. The lower value inferred from the MACHO data is due to the fact that the efficiency for the short duration events ($\sim$ some days) is substantially higher for the MACHO experiment than for the OGLE one. These values for the average mass suggest that the lens are faint disk stars.

Once several moments $< \mu^m >$ are known one can get information on the mass distribution $d\mu_0/d\mu$. Since at present only few events towards the LMC are at disposal the different moments (especially the higher ones) can be determined only approximately. Nevertheless, the results obtained so far are already of interest and it is clear that in a few years, due also to the new experiments under way (such as EROS II and OGLE II), it will be possible to draw more firm conclusions.

6 Dark clusters of MACHOs and cold molecular clouds

A major problem which arises is to explain the formation of MACHOs, as well as the nature of the remaining amount of dark matter in the galactic halo. We feel it hard to conceive a formation mechanism which transforms with 100% efficiency hydrogen and helium gas into MACHOs. Therefore, we expect that also cold clouds (mainly of $H_2$) should be present in the galactic halo. Recently, we have proposed a scenario \[50, 51\] in which dark clusters of MACHOs and cold molecular clouds naturally form in the halo at galactocentric distances larger than 10-20 kpc, where the relative abundance depends on the distance (similar ideas have also been developed in refs. \[52, 53\]). Our scenario can be summarized as follows.

After its initial collapse, the proto galaxy (PG) is expected to be shock heated to its virial temperature $\sim 10^6$ K. Since overdense regions cool more rapidly than average (by hydrogen recombination), proto globular cluster (PGC) clouds form in pressure equilibrium with diffuse gas. At this stage, the PGC cloud temperature is $\sim 10^4$ K, its mass and size are $\sim 10^6(R/kpc)^{1/2}M_\odot$ and $\sim 10(R/kpc)^{1/2}$ pc, respectively. The subsequent evolution of the PGC clouds will be different in the inner and outer part of the galaxy, depending on the decreasing collision rate and ultraviolet (UV) fluxes as the galactocentric distance increases. Below $10^4$ K, the main coolants are $H_2$ molecules and any heavy element produced in a first chaotic galactic phase. In the central region of the galaxy an Active Galactic Nucleus
and/or a first population of massive stars are expected to exist, which act as strong sources of UV radiation that dissociates the $H_2$ molecules present in the inner part of the halo. As a consequence, cooling is heavily suppressed and so inner PGC clouds remain for a long time at temperature $\sim 10^4$ K, resulting in the imprinting of a characteristic mass $\sim 10^6 M_\odot$. Later on, the cloud temperature suddenly drops below $10^4$ K and the subsequent evolution leads to the formation of stars and ultimately to stellar globular clusters. In the outer regions of the halo the UV-flux is suppressed, so that no substantial $H_2$ depletion actually happens. This fact has three distinct implications: (i) no imprinting of a characteristic PGC cloud mass shows up, (ii) the Jeans mass can now be lower than $10^{-1} M_\odot$, (iii) the cooling time is much shorter than the collision time. PGC clouds subsequently fragment into smaller clouds that remain optically thin until the minimum value of the Jeans mass is attained, thus leading to MACHO formation in dark clusters. Moreover, because the conversion efficiency of the constituent gas in MACHOs could scarcely have been 100%, we expect the remaining fraction of the gas to form self-gravitating molecular clouds, since, in the absence of strong stellar winds, the surviving gas remains bound in the dark cluster, but not in diffuse form as in this case the gas would be observable in the radio band.

6.1 Observational Tests

Let us now address the possible signatures of the above scenario, in addition to the single MACHO detection via microlensing.

We proceed to estimate the $\gamma$-ray flux produced in molecular clouds through the interaction with high-energy cosmic-ray protons. Cosmic rays scatter on protons in the molecules producing $\pi^0$’s, which subsequently decay into $\gamma$’s. An essential ingredient is the knowledge of the cosmic ray flux in the halo. Unfortunately, this quantity is experimentally unknown and the only available information comes from theoretical estimates. More precisely, from the mass-loss rate of a typical galaxy we infer a total cosmic ray flux in the halo $F \simeq 1.1 \times 10^{-4}$ erg cm$^{-2}$ s$^{-1}$. We also need the energy distribution of the cosmic rays, for which we assume the same energy dependence as measured on the Earth. We then scale the overall density in such a way that the integrated energy flux agrees with the above value. Moreover, we assume that the cosmic ray density scales as $R^{-2}$ for large galactocentric distance $R$. Accordingly, we obtain $[50, 51]$

$$
\Phi_{CR}(E, R) \simeq 1.9 \times 10^{-3} \Phi_{\odot}^{\text{CR}}(E) \frac{a^2 + R_{GC}^2}{a^2 + R^2},
$$

where $\Phi_{\odot}^{\text{CR}}(E)$ is the measured primary cosmic ray flux on the Earth, $a \sim 5$ kpc is the halo core radius and $R_{GC} \sim 8.5$ kpc is our distance from the galactic center. The source function $q_\gamma(r)$, which gives the photon number density at distance $r$ from the Earth, is

$$
q_\gamma(r) = \frac{4\pi}{m_p} \rho_{H_2}(r) \int dE_p \Phi_{CR}(E_p, R(r)) \sigma_{\text{in}}(p_{\text{lab}}) < n_\gamma(E_p) > .
$$

Actually, the cosmic ray protons in the halo which originate from the galactic disk are mainly directed outwards. This circumstance implies that the induced photons will predominantly leave the galaxy. However, the presence of magnetic fields in the halo might give rise to a temporary confinement of the cosmic ray protons similarly to what happens in the disk. In addition, there could also be sources of cosmic ray protons located in the halo itself,
as for instance isolated or binary pulsars in globular clusters. As we are unable to give a quantitative estimate of the above effects, we take them into account by introducing an efficiency factor $\epsilon$, which could be rather small. In this way, the $\gamma$-ray photon flux reaching the Earth is obtained by multiplying $q_\gamma(r)$ by $\epsilon/4\pi r^2$ and integrating the resulting quantity over the cloud volume along the line of sight.

The best chance to detect the $\gamma$-rays in question is provided by observations at high galactic latitude. Therefore we find

$$\Phi_\gamma(90^0) \simeq \epsilon f \left(3.5 \times 10^{-6}\right) \text{photons cm}^{-2} \text{s}^{-1} \text{sr}^{-1}. \quad (33)$$

The inferred upper bound for $\gamma$-rays in the 0.8 - 6 GeV range at high galactic latitude is $3 \times 10^{-7}$ photons cm$^{-2}$ s$^{-1}$ sr$^{-1}$ [54]. Hence, we see from eq. (33) that the presence of halo molecular clouds does not lead nowadays to any contradiction with such an upper limit, provided $\epsilon f < 10^{-1}$.

Molecular clouds can be detected via the anisotropy they would introduce in the Cosmic Background Radiation (CBR), even if the ratio of the temperature excess of the clouds to the CBR temperature is less than $\sim 10^{-3}$. Consider molecular clouds in M31. Because we expect they have typical rotational speeds of $50 - 100$ km s$^{-1}$, the Doppler shift effect will show up as an anisotropy in the CBR. The corresponding anisotropy is then [55]

$$\frac{\Delta T}{T_r} = \pm \frac{v}{c} S f \tau_\nu, \quad (34)$$

where $S$ is the spatial filling factor and $T_r$ is the CBR temperature. If the clouds are optically thick only at some frequencies, one can use the average optical depth over the frequency range of the detector $\tau$. We estimate the expected CBR anisotropy between two fields of view (on opposite sides of M31) separated by $\sim 4^0$ and with angular resolution of $\sim 1^0$. Supposing that the halo of M31 consists of $\sim 10^6$ dark clusters and that all of them lie between 25 kpc and 35 kpc, we would be able to detect $10^3 - 10^4$ dark clusters per degree square. Scanning an annulus of $1^0$ width and internal angular diameter $4^0$, centered at M31, in 180 steps of $1^0$, we would find anisotropies of $\sim 2 \times 10^{-5} f \tau$ in $\Delta T/T_r$ (as now $S = 1/25$). In conclusion, the theory does not permit to establish whether the expected anisotropy lies above or below current detectability ($\sim 10^{-6}$), and so only observations can resolve this issue.

An attractive strategy to discover the halo molecular clouds clumped into dark clusters relies upon the absorption lines they would introduce in the spectrum of a LMC star [56].

Let us now turn to the possibility of detecting MACHOs in M31 via their infrared emission. For simplicity, we assume all MACHOs have equal mass $\sim 0.08 M_\odot$ (which is the upper mass limit for brown dwarfs) and make up the fraction $f$ of the dark matter in M31. In addition, we suppose that all MACHOs have the same age $t \sim 10^{10}$ yr [57]. As a consequence, MACHOs emit most of their radiation at the wavelength $\lambda_{\text{max}} \sim 2.6 \mu$m. The infrared surface brightness $I_\nu(b)$ of the M31 dark halo as a function of the projected separation $b$ (impact parameter) is given by

$$I_\nu(b) \sim 5 \times 10^5 \frac{x^3}{e^x - 1} \frac{a^2 f}{D\sqrt{a^2 + b^2}} \arctan \sqrt{\frac{L^2 - b^2}{a^2 + b^2}} \text{Jy sr}^{-1}, \quad (35)$$
where the M31 dark halo radius is taken to be $L \sim 50$ kpc. Some numerical values of $I_{\nu_{\text{max}}}(b)$ with $b = 20$ and 40 kpc are $\sim 1.6 \times 10^3 \, \text{f Jy sr}^{-1}$ and $\sim 0.4 \times 10^3 \, \text{f Jy sr}^{-1}$, respectively. The planned SIRTF Satellite contains an array camera with expected sensitivity of $\sim 1.7 \times 10^3 \, \text{Jy sr}^{-1}$ per spatial resolution element in the wavelength range 2-6 $\mu$m. Therefore, the MACHOs in the halo of M31 can, hopefully, be detected in the near future.

7 Conclusions

The mystery of the dark matter is still unsolved, however, thanks to the ongoing microlensing experiments there is hope that progress on its nature in the galactic halo can be achieved within the next few years. It is well plausible that only a fraction of the halo dark matter is in form of MACHOs, either brown dwarfs or white dwarfs, in which case there is the problem of explaining the nature of the remaining dark matter and the formation of the MACHOs. Before invoking the need for new particles as galactic dark matter candidates for the remaining fraction, one should seriously consider the possibility that it is in the form of cold molecular clouds. A scenario this, for which several observational tests have been proposed, thanks to which it should be feasible in the near future to either detect or to put stringent limits on these clouds.

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Figure Captions

Fig. 1: Rotation curve for NGC 3198 according to van Albada et al. [8]. The dotted line with error bars refers to the optical and 21 cm hydrogen data, while the solid lines are theoretical fits.

Fig. 2: Definition of various quantities describing a microlensing event. The observer is O, the source is S and M is the MACHO.