Reconstruction of complex temperature field based on improved Tikhonov regularization

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Abstract. In order to improve the reconstruction accuracy of complex temperature field by acoustic CT, an improved Tikhonov regularization temperature field reconstruction algorithm is proposed. In this method, the filtering factor of standard Tikhonov regularization method is changed, so that no correction is made for larger singular values, while the correction for smaller singular values will increase with the decrease of singular values. The reconstruction results of typical temperature fields based on simulation data and experiment data verify the effectiveness of the proposed algorithm on reducing reconstruction errors of temperature fields.

1. Introduction

Acoustic CT temperature field measurement[1-2] is an inverse problem solving process[3]. This technique uses the propagation time of sound waves passing through the measured area from multiple directions and an appropriate reconstruction algorithm to calculate the temperature distribution of the measured area. It has the advantages such as non-contact measurement, low cost, convenient maintenance.

Typical acoustic tomography reconstruction algorithms can be divided into two categories. One is based on the length of the sound ray in each grid, such as the least square method[4], and the other is based on the basis function approximation of the sound velocity, such as the Tikhonov regularization method & Markov radial basis function approximation[5], for short, call it the Tikhonov regularization method. The advantage of the Tikhonov regularization method is that the number of grids contained in the measured area can be much larger than the number of sound rays, thus it is more suitable for reconstructing complex temperature fields. But the reconstruction effect of this method at the edge of the measured area is not good.

In order to improve the reconstruction ability of complex temperature field in acoustic CT, the filter factor of standard Tikhonov regularization method is improved to effectively reconstruct complex temperature field distribution, and the reconstruction ability of the algorithm is verified by using three model temperature fields and actual temperature field.

2. Basic principle of acoustic CT temperature field reconstruction

The relationship between the velocity of sound propagation in the gas medium \( c (m/s) \), the absolute temperature of the gas medium \( T(k) \), and the \( Z \) of the sound constant determined by the composition of the gas medium can be expressed as[6],

\[
c = Z \sqrt{T}
\]
In order to measure the temperature distribution of a measured area, the measured area needs to be divided into \( M \) grids, and a certain number of acoustic transceivers are placed at the boundary of the measured area. Each acoustic transceiver sounds in sequence, forming \( N \) acoustic propagation paths through the measured area. Sound time-of-flight (TOF) data along these paths can be measured by a time delay estimation method, for example, cross-correlation method. Then the temperature distribution in the measured area can be reconstructed by using an appropriate reconstruction algorithm and the relationship between sound velocity and temperature.

3. Reconstruction algorithm

3.1. Tikhonov regularization method

The flight time of sound wave on the the \( k \) propagation path \( p_k \) can be expressed as\[6\]:

\[
t_k = \int_{\mathcal{P}_k} s(x,y) dl, k = 1,2,\ldots,N
\]  

(2)

Where \( s(x,y) \) is the reciprocal of sound velocity field. \( s(x,y) \) can be approximate by a linear combination of \( M \) Markov basis functions as follows.

\[
s(x,y) = \sum_{i=1}^{M} \beta_i e^{-\alpha[(x-x_i)^2 + (y-y_i)^2]^{1/2}}
\]

(3)

Where \( \beta_i \) is the undetermined coefficient, \((x_i, y_i)\) is the coordinate value of the i-th grid center, and \( \alpha \) is the shape parameter of Markov radial basis function. Substituting equation (3) into equation (2), we have:

\[
t_k = \sum_{i=1}^{M} \beta_i l_{ki}, t_k = \int_{\mathcal{P}_k} e^{-\alpha[(x-x_i)^2 + (y-y_i)^2]^{1/2}} dl
\]

(4)

Let \( A=(l_{ki})_{k=1,\ldots,N,i=1,\ldots,M} \), \( t=(t_1 \ldots t_N)^T \), \( \beta=(\beta_1 \ldots \beta_M)^T \), we have

\[
A\beta = t
\]

(5)

Using the singular value decomposition of matrix \( A \) and the standard Tikhonov regularization technique, the regularization solution of equation (5) can be expressed as follows:

\[
\beta = \sum_{j=1}^{p} f_j \frac{u_j^T t}{\sigma_j} = \frac{\sigma_j}{\sigma_j^2 + \mu} f_j
\]

(6)

Which \( f = \frac{\sigma}{{\sigma_j}^2 + \mu} \) is the filtering factor, \( \sigma_1 \geq \sigma_2 \geq \sigma_3 \geq \cdots \geq \sigma_p > 0 \) is the singular value of the reconstruction matrix, \( p \) is the total number of non-zero singular values, \( \mu \) is the regularization parameter which is used to control the stability of solutions, The reconstruction matrix \( A \) can be obtained after the layout of the acoustic transceiver, the meshing method and the shape parameter \( \alpha \) of the radial basis function are determined.

3.2. Improved Tikhonov regularization method

The standard Tikhonov regularization method makes no difference correction to all singular values. However, a more reasonable correction method is that the larger the singular value, the smaller the correction amplitude, or even not to correct.

In this paper, an improved Tikhonov regularization method is proposed. Firstly, the singular values are divided into large singular values and small singular values by using the threshold singular value \( \sigma_j \). The larger singular value is not modified, but the smaller singular value is modified with
increasing amplitude. In this paper, the maximum singular value of equation (7) is set as the threshold singular value $\sigma_j$.

$$\sigma_j \geq k\sigma_j, j = 2, 3, \cdots p$$  \hspace{1cm} (7)

Where $k$ is the threshold constant obtained by experience.

In conclusion, the filtering factors of improved Tikhonov regularization are as follows:

$$f=\begin{cases} 
1, & 1<i<t \\
\sigma_j^2 + \frac{\sigma_j}{\sigma_t} & t \leq j \leq p 
\end{cases}$$  \hspace{1cm} (8)

4. Simulation verification of reconstruction algorithm

In this paper, the measured area is set as $1.3 \times 1.3$ m, which is divided into $10 \times 10 = 100$ grids. Twelve sensors are evenly arranged around the measured area to form 54 sound lines, as shown in Figure 1. The threshold constant $k = 10^{-7}$, shape parameter $\alpha = 10^{-4}$, regularization parameter $\mu = 10^{-10}$. The matrix condition number of the corresponding reconstruction matrix $A$ is $154530677.7905$.

![Acoustic transceiver and its acoustic path.](image)

Figure 1. Acoustic transceiver and its acoustic path.

The Tikhonov regularization method and the improved Tikhonov regularization method proposed in this paper are used to reconstruct the temperature field of typical single peak symmetry, bimodal symmetry and four peak symmetry models.

Figure 2 shows the model temperature field and the reconstructed temperature field without noise. Table 1 shows the corresponding reconstruction error. $R_{\text{max}}$, $R_{\text{ave}}$ and $R_{\text{rms}}$ denote maximum relative error, average relative error and root mean square error respectively. In order to test the noise resistance of the reconstruction algorithm, Gaussian white noise with standard deviation of 0.0001 is added to the noiseless acoustic flight time. The corresponding reconstruction errors are given in Table 2.
Figure 2. Model temperature field and reconstructed temperature field (noise free).

Table 1. Noise free reconstruction error.

| Method                     | Reconstruction error | Unimodal symmetry | Bimodal symmetry | Four peak symmetry |
|----------------------------|----------------------|-------------------|-----------------|-------------------|
| Tikhonov regularization    | $R_{\text{max}}$    | 13.2735%          | 26.0617%        | 25.714%           |
|                            | $R_{\text{ave}}$    | 2.197%            | 3.9345%         | 5.2886%           |
|                            | $R_{\text{rms}}$    | 2.129%            | 5.5949%         | 6.0229%           |
| Improved Tikhonov          | $R_{\text{max}}$    | 11.2514%          | 18.9285%        | 20.6361%          |
| regularization             | $R_{\text{ave}}$    | 1.7099%           | 2.4589%         | 3.3822%           |
|                            | $R_{\text{rms}}$    | 1.8309%           | 3.5116%         | 3.9559%           |

Table 2. Reconstruction error with noise.

| Method                     | Reconstruction error | Unimodal symmetry | Bimodal symmetry | Four peak symmetry |
|----------------------------|----------------------|-------------------|-----------------|-------------------|
| Tikhonov regularization    | $R_{\text{max}}$    | 14.107%           | 26.291%         | 26.733%           |
|                            | $R_{\text{ave}}$    | 2.265%            | 3.854%          | 5.259%            |
|                            | $R_{\text{rms}}$    | 2.335%            | 5.364%          | 5.975%            |
| Improved Tikhonov          | $R_{\text{max}}$    | 12.276%           | 18.736%         | 21.869%           |
| regularization             | $R_{\text{ave}}$    | 1.968%            | 2.700%          | 3.543%            |
|                            | $R_{\text{rms}}$    | 2.419%            | 3.880%          | 4.169%            |
From table 1, table 2 and Figure 2, it can be seen that both the Tikhonov regularization method and the improved Tikhonov regularization method can reconstruct the characteristics of the temperature field correctly, and have better anti noise ability. Compared with the reconstructed images of Tikhonov regularization method, the reconstructed images of improved Tikhonov regularization method are closer to the real temperature fields, with more obvious hot spots. In most cases the reconstruction errors of the improved Tikhonov regularization method are better than those of the Tikhonov regularization method.

5. Verification of the reconstruction algorithm with measured data
A sound time-of-flight measurement system was developed for measuring the actual acoustic TOF data. 12 acoustic transceivers are evenly arranged around a space of 1.3m × 1.3m as shown in Figure 3. The measured area is evenly divided into 10×10 = 100 grids. The threshold constant \( k = 10^3 \), the shape parameter \( \alpha = 10^4 \), and the regularization parameter \( \mu = 10^{-10} \). A single peak temperature field generated by a electric heater is reconstructed by the Tikhonov regularization method and the improved Tikhonov regularization method respectively. Figure 4 shows the reconstructed images. From Figure 4, it can be seen that the improved Tikhonov regularization method can reconstruct feature of the temperature field better.

6. Conclusion
In this paper, an improved Tikhonov regularization algorithm for temperature field reconstruction by acoustic tomography is proposed. In this paper, an improved Tikhonov regularization algorithm for temperature field reconstruction by acoustic tomography is proposed. In which, the filtering factor of standard Tikhonov regularization method is changed, no correction is made for larger singular values,
while the correction range for smaller singular values gradually increases. The reconstruction results show that compared with the Tikhonov regularization algorithm, the reconstruction errors of the improved Tikhonov regularization algorithm are reduced effectively. The requirement of estimation reliability and estimation accuracy can be satisfied better by the improved Tikhonov regularization method.

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