Vertical vibration analysis for elevator compensating sheave

Seiji Watanabe¹, Takeya Okawa², Daisuke Nakazawa¹ and Daiki Fukui¹

¹ Advanced Technology R&D Center, Mitsubishi Electric Corp., Hyogo, Japan
² Inazawa Works, Mitsubishi Electric Corp., Aichi, Japan
E-mail: Watanabe.Seiji@ay.MitsubishiElectric.co.jp

Abstract. Most elevators applied to tall buildings include compensating ropes to satisfy the balanced rope tension between the car and the counter weight. The compensating ropes receive tension by the compensating sheave, which is installed at the bottom space of the elevator shaft. The compensating sheave is only suspended by the compensating ropes, therefore, the sheave can move vertically while the car is traveling. This paper shows the elevator dynamic model to evaluate the vertical motion of the compensating sheave. Especially, behavior in emergency cases, such as brake activation and buffer strike, was investigated to evaluate the maximum upward motion of the sheave. The simulation results were validated by experiments and the most influenced factor for the sheave vertical motion was clarified.

1. Introduction

Conventional elevators move vertically by the traction machine, which suspends the car and the counter weight by the rope. The rope weight which is installed in high-rise buildings becomes so heavy that the car and the counterweight have to hang the compensating rope, to keep the rope tension balance, at the traction machine. The compensating sheave applies the tension to the compensating rope, and the sheave is guided at the bottom of the elevator shaft. As the compensating sheave is suspended by the compensating ropes, therefore, the sheave weight is equivalent to the rope tension. The applied tension prevents the rope from sway motion while the car is running.

Conventionally, a lot of research concerning elevator ride comfort, or evaluation of rope resonance condition, have been investigated. Some papers show the compensating rope vibration due to the building sway motion. However, there is not much research focusing on the dynamic behavior of the compensating sheave.

This paper shows the compensating sheave model which includes the car, counter weight and the compensating rope. The proposed model is verified by the experiment, and the model can clarify the mechanism of the sheave vertical motion. The simulation model can also show the influence of each elevator parameter against the sheave vertical motion.

2. Simulation model

Figure 1 shows the total elevator configuration, which includes the compensating sheave. The main rope and the compensating rope are the continuous body, and they are modeled by
the divided mass and the connecting spring. However, the rope model in figure 1 is simply represented by a single spring, \( k_{01} \sim k_{23} \).

The total kinetic energy and the potential energy are given by the following equations.

\[
T = \frac{1}{2} J_0 \dot{\theta}_0^2 + \frac{1}{2} J_3 \dot{\theta}_3^2 + \sum_{i=1}^{3} \frac{1}{2} M_i \dot{x}_i^2
\]

\[
U = \frac{1}{2} k_{01} (r_0 \theta_0 + x_1)^2 + \frac{1}{2} k_{02} (r_0 \theta_0 - x_2)^2
\]

\[
+ \frac{1}{2} k_{13} (x_1 - x_3 + r_3 \theta_3)^2 + \frac{1}{2} k_{23} (x_2 - x_3 - r_3 \theta_3)^2
\]

where, \( r_0 \) and \( r_3 \) are the rotation radii of the traction machine and the compensating sheave.

The equation of motion is derived by the above equations, and it contains the three degrees of translational freedom \( x_1 \sim x_3 \), and the two degrees of rotational freedom \( \theta_0, \theta_3 \).

\[
M \ddot{x} + C \dot{x} + K x = F
\]

\[
x = [x_1, x_2, x_3, \theta_0, \theta_3]^T
\]

In equation (3), two equations of motion concerning the vertical translation \( x_3 \) and the rotation \( \theta_3 \) are given by,

\[
M_3 \ddot{x}_3 + T_{13} + T_{23} = -M_3 g
\]

\[
J_3 \ddot{\theta}_3 + r_3 (-T_{13} + T_{23}) = 0
\]

where, \( T_{13} \) and \( T_{23} \) show each rope tension, and \( g \) is the gravitational acceleration.

\[
T_{13} = k_{13} (x_3 - x_1 - r_3 \theta_3) , \quad T_{23} = k_{23} (x_3 - x_2 + r_3 \theta_3)
\]

\[\text{Figure 1. Configuration of elevator system.}\]

3. Sheave vertical motion during normal operation

Based on the model in the previous section, the vertical motion of the compensating sheave is simulated, while the car is running at normal operation. The simulation results are compared with the experiments. Table 1 shows the simulation conditions.
Figure 2 shows the sheave vertical displacement, while the car moves downward from the highest floor to the lowest one. The zero position in vertical axis represents the initial displacement at the highest floor. The car accelerates from zero seconds to five seconds, and reaches the rated constant speed. Then the car decelerates from 45 seconds to 50 seconds.

The simulation result indicated by the broken line shows the same motion as the experiment in the solid line. Therefore, the proposed simulation model can evaluate the vertical motion of the compensating sheave properly.

### Table 1. Simulation conditions.

|                  |            |
|------------------|------------|
| car weight       | 9300 kg    |
| counter weight   | 10600 kg   |
| traveling distance| 132 m     |
| rated speed      | 180 m/min  |
| car load         | no load    |

The following describes the mechanism of sheave vertical motion during the acceleration and deceleration. The car starts from the highest floor. Therefore, the stiffness of the car side main rope $k_{01}$ in figure 1 is much higher than the counter weight side $k_{02}$. The car can trace the motion of the traction sheave, but the counter weight does not move in the same pattern as the traction sheave, due to the rope stiffness.

The simplified equation of motion for the main rope and the counter weight is derived by,

$$M_2 \ddot{x}_2 + k_{02} (x_2 - r_0 \theta_0) = -M_2 g$$  \(8\)

During acceleration, the counter weight receives upward acceleration, $\alpha > 0$. Therefore, the rope tension of the counter weight side is given by the following equation.

$$T_{02} = k_{02} (x_2 - r_0 \theta_0) = -M_2 (g + \alpha)$$  \(9\)

The above equation shows that the rope tension of the counter weight side increases during the acceleration. As the car position does not change, in spite of the rope stretch due to the
increased tension of the counter weight side, the increased tension induces the vertical motion of the compensating sheave during the acceleration pattern from zero to five seconds.

In the same manner, the main rope tension of the car side during the deceleration near the lowest floor is given by the following equation.

\[ T_{01} = k_{01} (x_1 + r_0 \theta_0) = -M_1 (g + \beta) \]  

(10)

where, \( \beta(>0) \) is the upward deceleration. In equation (10), the rope tension increases during the deceleration, and the main rope of the car side stretches more than the counter weight side. The difference in stretch between both main ropes induces the downward motion of the compensating sheave during the deceleration motion from 45 seconds to 50 seconds.

The next step is the evaluation of the compensating sheave motion during constant speed. Generally, the counter weight is heavier than the car, as shown in table 1. Therefore, the rope tension of the counter weight side is always larger than the car side.

The main rope stretch of the counter weight side at the highest floor is longer than the car side stretch at the lowest floor. When the zero position of the compensating sheave is defined at the highest floor, the compensating sheave moves upward gradually due to the stretch difference. This is the reason why the graph in figure 2 shows the right upward pattern.

When the car weight is equivalent to the counter weight, the graph in figure 2 shows a flat pattern during constant speed conditions. On the other hand, the graph shows right downward pattern, if the car weight is heavier than the counter weight due to the number of rated passengers.

Figure 3 shows the upward motion from the lowest floor to the highest floor. The result in figure 3 is the same pattern as figure 2 by flipping the lateral axis. From the result, we can conclude that the running direction has no effect on the sheave vertical motion.

![Figure 3. Sheave displacement during upward motion.](image)

4. Sheave vertical motion during emergency brake operation

The compensating sheave has a limit switch which prevents the sheave from an excessive upward motion. The switch works during the safety gear operation or the buffer strike, and it should not be applied during the emergency brake operation. Therefore, it is important to evaluate the maximum upward displacement while using the emergency brake, to fulfill the design requirement.

The car deceleration while the emergency brake is engaged, changes according to the car running direction and the car load condition. As the car deceleration is higher, the vertical
motion of the compensating sheave increases more. Therefore, the following conditions are investigated because of the larger deceleration occurrence.

(i) downward motion without car loading
(ii) upward motion with the rated car loading

As described in the previous section, the sheave vertical motion shows the right downward pattern when the car has a rated load. It is important to check the upward motion of the sheave, the proper simulation condition is the condition of (i).

Figure 4 shows the sheave vertical motion while the emergency brake is in operation near the highest floor. Please note that the acceleration in the following simulation is higher than in the previous section.

At the highest floor, the sheave motion is affected by the main rope stretch of the counter weight side. Therefore, equation (9) is used for evaluation. In equation (9), the acceleration $\alpha > 0$ is assigned to the brake deceleration $\gamma > 0$. The rope tension decreases as follows,

$$|T_{02}| = M_2(g - \gamma) < M_2g$$

By the above equation, the compensating sheave moves upward during the emergency brake operation. In figure 4, the car accelerates from zero to four seconds, and then the emergency brake works past five seconds. The sheave motion is exactly the same as described in the above sentences. During the emergency brake operation from six seconds to eight seconds, the compensating sheave keeps the same position. When the car stops finally by the brake operation, the sheave receives a large vertical motion. The sheave shows a residual vibration, and the frequency matches the natural frequency due to the counter weight and the main rope.

Even though the peak residual vibration is larger than the experiment, the overall vibration pattern is the same as the experiment. From figure 4, it is important to check the timing of the emergency brake activation and the car stopping time.

Figure 4. Brake stop near the top floor.

Figure 5 shows the emergency brake operation during the downward motion near the lowest floor. Figure 5 is the magnified scale near the time range of emergency brake. The simulation result matches the experiment.

As the sheave motion at the lowest floor is affected by the main rope of the car side, Equation (10) is used to evaluate the tension change. When the deceleration due to the emergency brake $\gamma$ is applied upward to the car, the rope tension is given by,

$$|T_{01}| = M_1(g + \gamma) > M_1g$$
When the emergency brake works at 43 seconds, the compensating sheave moves downward as shown in figure 5. The car also receives a large vibration at the instant of the car stop time at 45 seconds. The above large vibration is caused by the natural frequency of the car and the main rope.

As the sheave moves downward during the emergency brake operation, we can only focus on the residual vibration at the car stopping time.

![Figure 5. Brake stop near the bottom floor.](image)

Concerning the emergency brake operation, we can say that the sheave moves upward near the highest floor in figure 4, and it moves downward near the lowest floor in figure 5. From these results, we can estimate that the sheave does not move vertically near the middle floor. To evaluate the assumption, figure 6 shows the simulation result during the emergency brake operation near the middle floor.

![Figure 6. Brake stop at the middle floor.](image)

In figure 6, the car speed increases to 540 m/min, and the car continues accelerating for the first eight seconds. After that, the car reaches the middle floor and the emergency brake begins working at 10 seconds. The sheave does not move vertically in the simulation or the experiment between 10 and 16 seconds while the brake is in operation.

From the above results, the highest upward motion occurs when the car moves downward with no car loading, and emergency brakes are working, near the lowest floors. The peak value appears at the instant when the car stops due to the brake activation.
In the following, each elevator parameter is checked to evaluate the relation with the vertical motion of the compensating sheave. As described before, the worst condition occurs near the lowest floors, but the following simulations are evaluated near the highest floors. This is because the relative evaluation is enough to check the parameter sensitivity.

We can guess that the stiffness of the main rope affects the sheave motion. To evaluate the stiffness change, several conditions of different building heights are evaluated and compared with each other. According to the building height, the main rope stiffness changes and the result is shown in figure 7.

![Figure 7. Effect of traveling distance.](image)

In figure 7, the building height of 132 m in solid line is the original pattern, and the other conditions are adjusted to the zero position at the start time. In the case of 82 m shown in the broken line, the rope stiffness increases due to the short rope length. Therefore, the sheave vertical motion is less than the original height. On the other hand, in the case of 182 m shown in the dotted line, the rope stiffness decreases and the sheave shows larger vibration.

From the above result, we can calculate the lowest rope stiffness necessary in the most severe cases, as in the rope parameter study.

In the next step, the sheave weight influence is evaluated. Figure 8 shows the weight variation results by 50% increase and decrease.

![Figure 8. Effect of compensating sheave weight.](image)
As shown in figure 8, the weight change does not affect the sheave’s vertical motion. Therefore, the sheave motion is mostly governed by both ends of the compensating rope.

5. Sheave vertical motion during buffer strike

In the previous section, the emergency brake operation is evaluated for the sheave vertical motion. In this section, the buffer strike operation is investigated.

When the car does not stop at the lowest floor, the car can stop any further downward motion by the buffer, which is installed at the bottom of the elevator shaft. Figure 9 shows the sheave’s vertical motion during the buffer strike. The simulation conditions are given in table 2.

|                |               |
|----------------|---------------|
| car weight     | 4600 kg       |
| counter weight | 5400 kg       |
| traveling distance | 147 m    |
| rated speed    | 210 m/min     |
| car load       | no load       |

In figure 9, the car passes the lowest floor at the start time, and the car strikes the buffer. The sheave weight is decreased and increased by 50%. From the simulation results, we can see that the sheave weight variation does not affect the sheave’s vertical motion.

Figure 10 shows the corresponding car and counter weight motion. When the car strikes the buffer as shown in the solid line, the car stops at 0.5 seconds. On the other hand, the counter weight continues to move upward by its inertia force. Therefore, the compensating sheave also follows the upward motion of the counter weight during the first 0.7 seconds. After that, the counter weight begins falling by gravitational force, and the compensating sheave also moves downward until 1.4 seconds. When the rope tension of the counter weight side recovers due to falling motion, the car and the counter weight cause vibration around 1.5 seconds. At that time, the sheave receives two vibrations; one is induced by the car side motion of 0.9 second time period, and the other is 0.4 second time period due to the counter weight vibration.

![Figure 9. Effect of compensating sheave weight.](image)

From the above simulation results, we can say that the sheave motion is governed by the car and counter weight motion, and the sheave weight does not affect the upward motion.
6. Conclusions
This paper showed the elevator dynamic model to calculate the compensating sheave motion. The derived simulation model can show the same vertical behavior as the experiment results. In the case of normal traveling conditions, we can reach the conclusion that the weight difference between the car and the counter weight affects the transient motion of the sheave vertical direction while the car is traveling.

We can also reach the following conclusions:

(1) The maximum upward motion during emergency brake operation, occurs in the no-loaded condition with the downward traveling near the lowest floors.

(2) Lower rope stiffness of the main rope induces larger upward motion of the compensating sheave.

(3) The weight of the compensating sheave has no influence on its upward motion.