The model of turbulent mixing zone evolution, which accounts for the deviation from spherical symmetry of laser thermonuclear target compression

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Abstract. The present report presents an analytical model destined to describe the development of hydrodynamic instabilities and their influence on the neutron yield under laser target compression. A number of specific numerical simulations formed the basis of the analysis. The plane and spherical tasks with certain initial conditions were considered. In a plane case the two pairs of gases typical for the shock tube experiments were studied. The spherical task was the modeling one and involved a small number of harmonics (the calculation parameters corresponded to the regime of laser compression). A notable difference in the calculation parameters of the gases in shock tubes and the laser compressed targets is not an obstacle for the construction of a unified model, since in gas dynamic problems the fundamental properties of gas-dynamic similarity are fulfilled.

Basing on the simulations [1] and theory [2], the authors constructed a model of mixing zone development, which takes into account the initial conditions of perturbation. In addition to the earlier studied problems the report presents 2D-simulations of spherical target compression with account for the mixing processes. The simulations were aimed at determination of the degree of the mixing zone influence on the neutron yield.

1. Numerical results (plane geometry)

To study systematically the influence of different initial conditions on the development of turbulence caused by the Rayleigh-Taylor and Richtmyer-Meshkov instabilities in case of plane geometry a series of calculations using the NUT code [3] was performed.

The results from this part of the research are given in [1] and may be formulated as follows:

- on the basis of numerical simulations the authors created a large database covering the development of instabilities and mixing of two gases in plane geometry under constant acceleration for the regimes different in the type of instability (RT or RM), Atwood number, number of the accounted modes, initial amplitudes \(a_0\) and their dependence on the mode number \((a_0 = \text{const}/k \text{ and } a_0 = \text{const})\);
- at the stage considered the mixing zone width strongly depends on the amplitude of initial perturbation, and changes with time by the law close to the linear one;
- the mixing zone width weakly depends on the contribution from the higher modes (slightly decreases if the higher modes are involved).

Figure 1 illustrates the calculation data for the gas pair He-Xe with 6 harmonics and initial amplitude given by the law \(a_0 = a_0^* \lambda / 2\pi\).
2. Model describing the mixing zone width

A theoretical model for the description of the width and range of the mixing zone was constructed. The model takes into account the influence of the initial conditions.

Basing on correct asymptotic for the beginning of the process and the later stage, one can make several conclusions: at an earlier stage under the presence of high modes the mixing zone grows with time in accordance with a quadratic law (this follows from the evolutionary model of instability development \[2\]); at a later stage the rate of the mixing zone development tends to a constant limit value (the Layzer model \[4\]). The \(i\)-th harmonic enlarges the mixing zone up to the value:

\[
L_i(t)=2a_0^+\frac{\lambda_i}{2\alpha_{\text{eff}}}\left(\sqrt{1+\left(\frac{\alpha_{\text{eff}}\gamma_i t}{2\pi}\right)^2}-1\right)
\]  

(1)

Here \(\gamma_i=\frac{2\pi}{\lambda_i}gA\), \(A=\frac{\rho_2-\rho_1}{\rho_2+\rho_1}\), \(\alpha_{\text{eff}}=\frac{\alpha_0}{\lambda_i}\), \(\alpha_{\text{eff}}=\frac{2\pi}{\lambda_i}a_{\text{eff}}\).

According to \[2\], \(\alpha^*\) defines the amplitude at which an exponential growth of the instability decelerates, since the mushroom-like structures are formed. Such structures are easily seen in Fig. 1.

Below the full width of the mixing zone will be given in the following form:

\[
L(t)=\sum_{i}L_i(t)w_i(t),
\]

(2)

where \(w_i(t)\), is the weighting coefficient responsible for the \(i\)-th harmonic contribution. At the initial time moment \(w_i(0)\) is defined by a random phase of perturbation:

\[
w_i(0)=\cos(k_iz_1+\phi_i)−\cos(k_iz_2+\phi_i)
\]

(3)

Here \(z_1\) defines the maximal depth of the heavy gas penetration into the light one; \(z_2\), the maximal depth of the light gas penetration into the heavy one. Later \(w_i(t)\) decreases, and this corresponds to the destruction of the given instability mode due to the development of the Kelvin-Helmholtz instability \[2\]. At later stages the contribution from higher harmonics is not equal to zero. The following formula for the \(i\)-th harmonic weight is proposed:

\[
w_i(t)=w_i(0)[\frac{2}{p_i}+2(1-\frac{2}{p_i})\frac{1}{1+\exp(\frac{\lambda^2_{\text{max}}}{p_i^2}w_i^2(0)\cdot\gamma_i^2t^2)}]^{-1}
\]

(4)

The main contribution into the mixing zone width comes from the longwave perturbations emerging by this moment. Figure 2 shows the diagrams of time dependence of the mixing zone width (as seen from the figure the theory is in good agreement with the calculation results \[1\]).
3. Numerical results (spherical geometry)

The next step in the investigation of mixing processes within the problem of inertial thermonuclear fusion turned to be the numerical simulation of a spherical target compression maximally close to the real laser compression, and an attempt to describe them by the proposed above theoretical model [5].

In our task we considered the compression processes taking place during the last three nanoseconds (the spherical target collapse stage). The density and radius of different layers were as follows: the ablator, $\rho_{CH}=12$ g/cm$^3$ and $R_{CH}=926$ $\mu$m (not shown in the figure); DT-ice layer, $\rho_{DT}=0.5$ g/cm$^3$ and $R_{DT}=916$ $\mu$m; the inner layer (DT-gas), $\rho_{in}=3.5*10^{-5}$ g/cm$^3$ and $R_{in}=768$ $\mu$m. The instability was set at the DT-ice – inner gas boundary [1]. At the time moment $t$ (in our case 0 ns) the CH and DT layers started to move towards the centre with the rate of $V_0=300$ km/s and compressed the target.

Figure 3 illustrates the calculation results for the 6-th and 15-th harmonics (the initial amplitudes of 4.5 $\mu$m and 18 $\mu$m), and the 48-th harmonic (3$\mu$m amplitude).

Figure 4 illustrates a comparison of the calculated mixing zone widths with those given by formulas (1)-(4).
The formulas (1)-(4), which define the mixing zone width, may be easily introduced into 1D coded destined for the calculation of target compression and thermonuclear burning.

4. Account for the mixing zone influence on the neutron yield

The model should involve a possibility to define the neutron yield basing on the data on the initial conditions of mixing. For this purpose we started the simulations of target compression with account for mixing and its effect on the thermonuclear burning and neutron yield. As an example we performed the calculations of compression of the targets close to the experimental ones used in Iskra-5 facility [6].

Figures 5 and 6 present the calculation results for the targets with the DT initial radius of 200 μm.

**Figure 5.** The target consists of the inner DT layer and CH shell, the external layer – the CH gas. High-frequency perturbations of the shell are absent.

**Figure 6.** The target consists of the inner DT layer and CH shell, the external layer – CH gas. The shell is perturbed by the 24-th harmonic having a 6μm amplitude.

The neutron yield for the target shown in Fig. 5 constitutes $1.1 \times 10^{10}$, for the target shown in Fig. 6 – $8.0 \times 10^9$. It is seen that under present conditions the high-frequency perturbations do not essentially influence the neutron yield.

Our simulations contain detailed information on the state of the interacting gases, the size of the mixing zone, etc. So, in future one can hope to define more accurately the influence of mixing on the efficiency of a thermonuclear reaction.

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