Nonlinear Enhancement of the Multiphonon Coulomb Excitation in Relativistic Heavy Ion Collisions

M.S. Hussein, A.F.R. de Toledo Piza, O.K. Vorov

Instituto de Física, Universidade de São Paulo
Caixa Postal 66318, 05315-970,
São Paulo, SP, Brasil

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Abstract

We propose a soluble model to incorporate the nonlinear effects in the transition probabilities of the multiphonon Giant Dipole Resonances based on the SU(1,1) algebra. Analytical expressions for the multi-phonon transition probabilities are derived. Enhancement of the Double Giant Resonance excitation probabilities in relativistic ion collisions scales as \((2^k + 1)(2^k)^{-1}\) for the degree of nonlinearity \((2^k)^{-1}\) and is able to reach values 1.5 – 2 compatible with experimental data. The enhancement factor is found to decrease with increasing bombarding energy.

[KEYWORDS: Relativistic Heavy Ion Collisions, Double Giant Resonance]
Coulomb Excitation in collisions of relativistic ions is one of the most promising methods in modern nuclear physics [1,2]. One of the most interesting applications of this method to studies of nuclear structure is the possibility to observe and study the multi phonon Giant Resonances [1]. In particular, the double Dipole Giant Resonances (DGDR) have been observed in a number of nuclei [3,4]. The “bulk properties” of the one- and two-phonon GDR are now partly understood [1] and they are in a reasonable agreement with the theoretical picture based on the concept of GDR-phonons as almost harmonic quantized vibrations.

Despite that, there is a persisting discrepancy between the theory and the data, observed in various experiments [5,6] that still remains to be understood: the double GDR excitation cross sections are found enhanced by factor $1.3 - 2$ with respect to the predictions of the harmonic phonon picture [1,3,11,12]. This discrepancy, which almost disappears at high bombarding energy, has attracted much attention in current literature [1,3,7,22-24]; among the approaches to resolve the problem are the higher order perturbation theory treatment [7], and studies of anharmonic/nonlinear aspects of GDR dynamics [4,22,23,24]. Recently, the concept of hot phonons [15,16] within Brink-Axel mechanism was proposed that provides microscopic explanation of the effect. These seemingly orthogonal explanations deserve clarification which we try to supply here.

The purpose of this work is to examine, within a a soluble model the role of the nonlinear effects on the transition amplitudes that connect the multiphonon states in a heavy-ion Coulomb excitation process. Most studies of anharmonic corrections [22,24] concentrated on their effect in the spectrum [25,26]. Within our model, the nonlinear effects are described by a single parameter, and the model contains the harmonic model as its limiting case when the nonlinearity goes to zero. We obtain analytical expressions for the probabilities of excitation of multiphonon states which substitutes the Poisson formula of the harmonic phonon theory. For the reasonable values of the nonlinearity, the present model is able to describe the observed enhancement of the double GDR cross sections quoted above.

Having in mind to show how analytical results follows from the nonlinear model, and to explain how the model works, we restrict ourselves here to its simplest version (transverse approximation, or SU(1,1) dynamics) and keep numerics up to minimum level. We postpone till further publications detailed numerical analysis and comparison with the data. Microscopic origins of the nonlinear effects (which are considered here phenomenologically) are also beyond the scope of this presentation.

We work in a semiclassical approach [11] to the coupled-channels problem, i.e., the projectile-target relative motion is approximated by a classical trajectory and the excitation of the Giant Resonances is treated quantum mechanically [13]. The use of this method is justified due to the small wavelengths associated with the relative motion in relativistic heavy ion collisions. The separation coordinate is treated as a classical time dependent variable, and the projectile motion is assumed to be a straight line [4].

The intrinsic dynamics of excited nucleus is governed by a time dependent quantum Hamiltonian (see Refs. [11,12]). The intrinsic state $|\psi(t)\rangle$ of excited nucleus is the solution of the time dependent Schr"{o}dinger equation

$$i\frac{\partial}{\partial t}|\psi(t)\rangle = [H_0 + V(t)]|\psi(t)\rangle$$

where $H_0$ is the intrinsic Hamiltonian and $V$ is the channel-coupling interaction. We use
the system of units where $\hbar = 1$, $c = 1$. The standard coupled-channel problem for the amplitudes $a_n(t)$ reads

$$|\psi(t)\rangle = \sum_{n=0}^\infty a_n(t) |n\rangle \exp\left(-iE_n t\right), \tag{2}$$

where $E_n$ is the energy of the state $|n\rangle$ in the wave packet $|\psi\rangle$. In our treatment, the nuclear states are specified by the numbers of excited GDR phonons, $N$ or $n$. Taking scalar product with the states $<N|$, we get the set of coupled equations for the amplitudes $a_n$ as functions of impact parameter $b$

$$i\dot{a}_N(t) = \sum_{n=0}^\infty <N|V|n\rangle e^{i(E_N-E_n)t}a_n(t). \tag{3}$$

We assume the colliding nuclei to be in their ground states before the collision. The amplitudes obey the initial condition $a_n(t \to -\infty) = \delta(n,0)$ and they tend to constant values as $t \to \pm\infty$ (the interaction $V(t)$ dies out at $t \to \pm\infty$). The excitation probability of an intrinsic state $|N\rangle$ in a collision with impact parameter $b$ is given as

$$W_N(b) = |a_N(\infty)|^2. \tag{4}$$

and the total cross section for excitation of the state $|N\rangle$ is given by the integral over the impact parameter

$$\sigma_N = 2\pi \int_{b_{gr}}^\infty b W_N(b) \, db \tag{5}$$

with the grazing value $b_{gr} = 1.2(A_{exc}^{1/3} + A_{sp}^{1/3})$ as the lower limit. Hereafter, the labels $exc$ ($sp$) refer to the excited (spectator) partner in a colliding projectile-target pair. We neglect the here nuclear contribution [21] to the excitation process.

In the following, it is convenient to treat the coupled channel equations (1),(3) in terms of the unitary operator $U_I$ (in the interaction representation) that acts in the reference basis of multiphonon states (including the ground state $|0\rangle$):

$$i\frac{d}{dt}U_I(t) = V_I(t)U_I(t), \quad V_I(t) = e^{iH_0 t}V(t)e^{-iH_0 t}, \quad U_I(t = -\infty) = I, \tag{6}$$

and the time-dependent Hamiltonian $H(t) = H_0 + V(t)$ that acts in the intrinsic multi-GDR states is given by

$$H_0 = \omega N_d, \quad N_d \equiv \sum_m d^+_m d_m,$$

$$V(t) = v_1(t)[(E1_{-1})^\dagger - (E1_{+1})^\dagger] + v_0(t)(E1_0)^\dagger + \text{Herm.Conj.} \tag{7}$$

where $E1_m^\dagger$ and $E1_m$ are the dimensionless operators acting in the internal space of the multi-GDR states. In the harmonic approximation, they are given by the GDR phonon creation and destruction operators of corresponding angular momentum projection $m$, $E1_m^\dagger = d^+_m$. The function $v_1(t)$ is given [2] by
\[ v_1(t) = \frac{w}{[1 + (\frac{v}{b} t)^2]^{3/2}}, \quad w = \frac{Z_{sp} e^2 \gamma}{2b^2} \sqrt{\frac{N_{exc} Z_{exc}}{A_{exc}^2 m_N \cdot 80 \text{MeV}}}, \tag{8} \]

(the corresponding expression for \( v_0(t) \) can be found in Ref. \[2\]). Here, \( m_N \) and \( e \) are the proton mass and charge, \( Z, N \) and \( A \) denote the nuclear charge, the neutron number and the mass number of the colliding partners, \( \gamma = (1 - v^2)^{-1/2} \) is relativistic factor, \( v \) is the velocity and the parameter \( \rho \) is the deal of the strength absorbed by the collective motion (usually assumed to be close to unity) \[1\].

The harmonic approximation (ideal bosons) yields the transition probabilities between the states with numbers of GDR phonons differing by unity to grow linearly \( \propto N \). This model of ideal bosons has well known exact solution (see, e.g. \[12\]) that is given by the Poisson formula for the excitation probabilities

\[ W_N = e^{-|\alpha_{\text{harm}}|^2} \frac{|\alpha_{\text{harm}}|^{2N}}{N!}, \]

\[ |\alpha_{\text{harm}}|^2 = \sum_{m=0, \pm 1} |\alpha_m^{\text{harm}}|^2 = 2|\alpha_1^{\text{harm}}|^2 + |\alpha_0^{\text{harm}}|^2 \tag{9} \]

where the amplitudes \( \alpha_m^{\text{harm}} \) are expressed in terms of the modified Bessel functions. At the colliding energies sufficiently high, the longitudinal contribution (\( \propto |\alpha_0^{\text{harm}}|^2 \)) is suppressed by a factor proportional to \( \gamma^{-2} \). In the following, we will work in the “transverse approximation” dropping the longitudinal term for the sake of simplicity. This is a good approximation at high energies and the results are still qualitatively valid at lower energies.

Our idea is to keep the spectrum of GDR system harmonic with the Hamiltonian \( H_0 = \omega N \). That is supported by the systematics of the observed DGDR energies, \( E_2 \), which yields \( E_2 \simeq (1.75 - 2)\omega \) \[1\]. The conclusion on the weak anharmonicity in the spectrum follows also from theoretical considerations \[25\], \[26\].

The transition operators \( E_1^\dagger, E_1 \) can however include nonlinear effects. (In particular, this could be a result of nonlinearities in the phonon Hamiltonian obtained in higher orders of perturbation theory). A reasonable accounting for these nonlinear effects that we adopt in this work consists of generalization of the transition operators

\[ E_1^\dagger_m = d_m^\dagger \sqrt{1 + \frac{1}{2kN_d}}, \quad E_1_m = \sqrt{1 + \frac{1}{2kN_d}} d_m^\dagger, \tag{10} \]

where the parameter \( k \neq 0 \) determines the strength of the nonlinear effects, i.e., the problem reduces to the harmonic oscillator with linear coupling when

\[ \frac{1}{2k} \rightarrow 0. \tag{11} \]

If \( k < 0 \), the transition probabilities are suppressed as \( N \) grows. Positive values of \( k \) that we will consider here correspond to the enhancement of the matrix elements.

It is convenient to introduce the three following operators \( D^+ \), \( D^- \) and \( D^0 \)

\[ D^+ = \frac{1}{2^{1/2}} (d_{+1}^+ - d_{-1}^+) \sqrt{2k + N_d}, \quad D^- = \frac{1}{2^{1/2}} \sqrt{2k + N_d} (d_{+1} - d_{-1}), \]

\[ D^0 = \frac{1}{4} \left[ (d_{+1}^+ - d_{-1}^+) (d_{+1} - d_{-1}) + 2(2k + N_d) \right], \tag{12} \]
with $N_d \equiv d_+^* d_{+1} + d_-^* d_{-1}$. It is easy to check that they obey the standard commutation relation for the noncompact $SU(1,1)$ algebra

$$[D^-, D^0] = D^-, \quad [D^+, D^0] = -D^+, \quad [D^-, D^+] = 2D^0.$$  \quad (13)

The dynamics of our system, in transverse approximation, can be written in terms of the operators $D^+, D^-$ and $D^0$ (12) only. In the interaction representation, the evolution equation (6) takes the form

$$i \frac{d}{dt} U_I(t) = \left[ \frac{v_1(t)}{\sqrt{k}} e^{i\omega t} D^+ + \frac{v_1(t)}{\sqrt{k}} e^{-i\omega t} D^- \right] U_I(t),$$  \quad (14)

for $U_I(t)$ (the last equation follows from (1), (2) after using the commutation relations (13) and $[N_d, D^\pm] = \pm D^\pm$.

From purely mathematical viewpoint, the problem described by the last equation drops into the universality class of the systems with $SU(1,1)$ dynamics that can be analyzed by means of generalized coherent states [29] for the $SU(1,1)$ algebra. In particular, the problem of the parametric excitation of a one-dimensional harmonic oscillator (29, 30) belongs to the same class. The physical meaning of the algebra generators (12) in our case is of course quite different from that of [29,30]. (For other algebraic approaches to scattering problems, see Refs. [19], [20]).

The formal solution of our problem is given by the expression for the unitary operator $U_I(t)$ as a time-ordered exponential (see, e.g., [31])

$$U_I(t) = T \exp \left( -i \int_{-\infty}^{t} dt' v_1(t') \right)$$  \quad (15)

Due to closure of the commutation relations between the operators $D^+, D^-$ and $D^0$ that enter the exponential in Eq.(12), the time-ordered exponential can be represented in another equivalent form that involve ordinary operator exponentials only (see, e.g., [18]). The operator $U_I(t)$ can be expressed as

$$U_I(t) = \exp \left[ \frac{\alpha(t)}{\sqrt{k}} D^+ \right] \exp \left[ \log \left( 1 - \frac{\alpha(t)^2}{k} \right) - i\phi(t) \right] D^0 \exp \left[ -\frac{\alpha^*(t)}{\sqrt{k}} D^- \right]$$  \quad (16)

where one has the product of ordinary exponentials [29] which contain some time-dependent complex number $\alpha(t)$ (star means complex conjugation) and real number $\phi(t)$ (phase). The functions $\alpha(t)$ and $\phi(t)$ can be found from simple differential equations which relate the unknown $\alpha(t)$ and $\phi(t)$ with the function $v_1(t)$ in the Hamiltonian $H(t)$. These equations (see below) can be restored after substituting the right hand side of Eq.(16) into the left hand side of the Schrödinger equation for the operator $U_I(t)$ (14) and collecting the terms which have the same operator structure.

Proceeding this way, we obtain, after some algebraic manipulations, from (14) with using the commutation relations (13), the formula

$$\frac{d}{dt} e^A = \int_0^t d\tau e^{\tau A} \left( \frac{d}{dt} e^{(1-\tau)A} \right) e^{(1-\tau)A}$$
for differentiating the operator exponential and the Baker-Hausdorf relations for the operator exponentials \[31\], the following Riccati-type equation for the complex amplitude \(\alpha\):

\[
i \frac{d}{dt} \alpha = v_1(t)e^{i\omega t} + v_1(t)e^{-i\omega t}\frac{\alpha^2}{k}.
\]

(17)

The expression for the phase \(\phi(t)\) is given by a simple integral \(\phi(t) = (2/k) \int_{-\infty}^{t} dt_1 \text{Re}(v_1(t_1)\alpha(t_1)e^{-i\omega t_1})\) (as the phase does not contribute to the squared absolute values of amplitudes that enter the excitation probabilities and the cross sections, we will not be interested in it in the following).

Once the solution for the differential equation (17) is found, the expression for the amplitudes \(a_N(t)\) which we are interesting in follows from (16) immediately after projection of the state

\[
|\psi(t)\rangle = U_I(t)|0\rangle
\]

onto the intrinsic states with definite number of GDR phonons, \(N\). For example, we have

\[
U_I(t)|0\rangle = e^{-ik\phi(t)} \left(1 - \frac{|\alpha(t)|^2}{k}\right)^k e^{\frac{\alpha(t)}{\sqrt{k}} D^+}|0\rangle.
\]

We thus obtain the expression for the amplitude and the probabilities of the transitions from the ground state to the excited states with \(N\) phonons:

\[
|a_N(\infty)| = \left(1 - \frac{|\alpha(\infty)|^2}{k}\right)^k \left(\frac{\Gamma(2k + N)}{N!\Gamma(2k)}\right)^{1/2} \left(\frac{|\alpha(\infty)|^2}{k}\right)^{N/2}
\]

\[
W_N = |a_N(\infty)|^2.
\]

(18)

Here, the quantity \(\alpha(\infty)\) is the asymptotic solution to the Riccati equation (17) at \(t \rightarrow \infty\) subject to the initial condition \(\alpha(-\infty) = 0\). The simple nonlinear equation (17) abbreviates all orders of quantum perturbation theory for the problem Eqs.(6), (7), (14). It is also seen from Eq.(16) that unitarity is automatically preserved within present formalism \((U_I^\dagger = U_I^{-1})\).

The harmonic limit of these results corresponds to the case \(k \rightarrow \infty\), when the nonlinearity disappears in the transition operators (11) and the coupling to electromagnetic field becomes linear (harmonic approximation). Then at \(k \rightarrow \infty\) the last nonlinear term drops from the equation (17), and the solution to the equation \(|\alpha(\infty)|\) reduces to its harmonic value \(|\alpha_{\pm 1}^{\text{harm}}|\) given by modified Bessel functions [12], [1]. At the same time, the expression for the probabilities \(W\), Eq.(18), reduces at \(k \rightarrow \infty\) to the Poisson formula (9), thus the harmonic results [12], [1] are restored.

The simple nonlinear equation (17) for the amplitude \(\alpha(t)\) can be readily solved numerically. One can construct the solution by means of successive approximations, provided that the nonlinearity is small enough to keep the first terms in the expansion to powers of \(|v_1|/\sqrt{k}\),

\[
\alpha(t) = -i \int_{-\infty}^{t} v_1(t_1)e^{i\omega t_1}dt_1 + i \int_{-\infty}^{t} \frac{v_1(t_1)}{k}e^{-i\omega t_1}dt_1 \left[\int_{-\infty}^{t_1} v_1(t_2)e^{i\omega t_2}dt_2\right]^2 + ...
\]

(19)
In fact, the first term of this expansion is an excellent approximation in many cases, especially for light projectiles/targets when $|v_1|$ is small. We will use this approximation below as a good reference point to analyze the enhancement factor for excitation of DGDR. Keeping thus the first term in Eq. (19) we have

$$|\alpha(\infty)| \simeq -i \int_{-\infty}^{\infty} v_1(t) e^{i\omega t} \, dt = \frac{2w}{\omega} \xi^2 K_1(\xi)$$  \hspace{1cm} (20)$$
i.e., the well known expression expression in terms of the modified Bessel function. While the amplitude $\alpha$ is thereby reduced to the harmonic value in this approximation, the excitation probabilities $W_N$ are still given by the formula for the nonlinear model, Eq. (18). Using (18) and (20), one has therefore the approximate expression for the excitation probabilities

$$W_N = \left(1 - \frac{|K_1(\xi)|^2 s^2}{k} \right)^{2k} \frac{\Gamma(2k + N)}{N!\Gamma(2k)} \left(\frac{2|K_1(\xi)|^2 s^2}{2k}\right)^N N!$$

(21)

here, $s = \frac{2w}{\omega} \xi^2$ and $\xi \equiv \frac{\omega_0}{v}$. The cross sections for the excitation of N-phonon GDR are

$$\sigma_N = 2\pi \frac{\Gamma(2k + N)}{N!\Gamma(2k)(2k)^N} \int_{bgr} \left(1 - \frac{|K_1(\xi)|^2 s^2}{k} \right)^{2k} (2|K_1(\xi)|^2 s^2)^N$$

(22)

At nonzero nonlinearity $1/(2k) > 0$, the excitation probabilities $W_N$ for Multiple GDR turn out to be enhanced as compared to their values $W_N^{\text{harm}}$ in the harmonic limit. In general, deviation in the excitation probabilities for the N-phonon states $W_N$ from their harmonic values $W_N^{\text{harm}}$ are given by the ratio

$$\frac{W_N}{W_N^{\text{harm}}} = \prod_{i=0}^{N-1} \frac{2k + i}{2k} \left(1 - \frac{|\alpha(+\infty)|^2}{k}\right)^{2k} \frac{|\alpha(+\infty)|^{2N}}{|\alpha_1^{\text{harm}}(+\infty)|^{2N}}$$

In perturbative regime, when the amplitudes $\alpha$ are close to the harmonic solutions, the two last factors in this equation are close to unity, and the deviations are basically due to the factor $\prod_{i=0}^{N-1} \frac{2k+i}{2k}$. It is convenient to introduce the following ratio $R_2$ that characterizes enhancement of the excitation probability of the two-phonon GDR due to nonlinear effects with respect to the transition probability in the harmonic limit

$$R_2 = \frac{W_2}{W_2^{\text{harm}}} / \frac{W_1}{W_1^{\text{harm}}}$$

From equations (9), (18) we obtain the result

$$R_2 = \frac{2k + 1}{2k} \frac{|\alpha(+\infty)|^2}{|\alpha_1^{\text{harm}}(+\infty)|^2}$$  \hspace{1cm} (24)$$

The first factor in this expression, $\frac{2k+1}{2k}$, results from the kinematic enhancement of the transition probabilities due to nonlinear effects considered here. When the amplitude $|\alpha(+\infty)|$
is small, the second and the third factors in Eq.(24) are close to unity and we obtain the result

\[ R_2 \simeq \frac{2k + 1}{2k}. \]

(25)

The cross sections can be obtained from the usual formula (5). For the ratio of the cross sections of excitation of the double GDR calculated within present nonlinear model \( \sigma_2 \) and within the harmonic phonon model \( \sigma_2^{\text{harm}} \), we obtain in the same scaling regime

\[ r_2 = \frac{\sigma_2}{\sigma_2^{\text{harm}}} \simeq \psi \frac{2k + 1}{2k} \]

(26)

where the exact numerical factor \( \psi \) that stems from equations (18), (21), (22), is close to unity when \(|\alpha| \simeq |\alpha^{\text{harm}}| \ll 1\). It contains dependence on the bombarding energy and describes deviations from the scaling behavior which are valuable at both low and high energies (we discuss these deviations below).

The interesting feature of these results is that they are rather insensitive to many of the parameters that describe the colliding system. In the scaling regime, the enhancement factor \( r_2 \) depends on the parameter \( 1/k \) only, which is assumed to be an intrinsic property of the excited partner. As a consequence, the enhancement must be roughly independent of the properties of the spectator partner, once we study excitation of double GDR in the same nucleus.

This is just what has been observed in experiments: the values of \( r_2^{\text{exp}} \) found for DGDR in \(^{208}\text{Pb} \) projectile using different targets \(^{120}\text{Sn}, ^{165}\text{Ho}, ^{208}\text{Pb}, ^{238}\text{U} \) are very close to each other and they correlate, within the error bars, with the value

\[ r_2^{(208\text{Pb})} \simeq 1.33. \]

(27)

According to the scaling prediction, this corresponds to the nonlinearity parameter \( k \) equal to

\[ k^{(208\text{Pb})} \simeq 1.5 \]

(28)

Numerical evaluation shows that the value of \( \psi \) (see Eq.(26)) is close to 0.88, thus, the value of the required nonlinearity is a bit bigger, corresponding to \( 1/k \simeq 1 \).

The same picture was found in experiments on Coulomb desintegration of \(^{197}\text{Au} \) target using various projectiles \(^{20}\text{Ne}, ^{86}\text{Kr}, ^{197}\text{Au}, ^{209}\text{Bi} \). Also, the similar conclusion of nearly constant value of \( r_2 \) in \(^{208}\text{Pb} \) target with various projectiles has been made in work \([10]\) (though for differing experimental set-up).

Below, we present the exact results for the cross sections calculated according to Eqs.(5), and (18) and with solving Eq.(17) numerically.

The dependence of the enhancement factor \( r_2 = \sigma_2/\sigma_2^{\text{harm}} \) for the DGDR excitation on the strength of the nonlinearity \( k \) is shown on Fig. 1. for the process \(^{208}\text{Pb} + ^{208}\text{Pb} \) (we use the case of bombarding energy \( \varepsilon = 0.64\text{GeV}/\text{per nucleon} \). It is seen that the enhancement factor drops to unity at big values of \( k \) (harmonic limit) and grows at stronger nonlinearity. The scaling value of \( r_2 \) is also shown for comparison.

It is interesting to trace energy dependence of the enhancement factor.
For typical projectile-target pairs and for moderate bombarding energies ($\gamma \approx 1.3$), both $|\alpha|$ and $|\alpha_{1}^{\text{harm}}|$ are small as compared to unity and $|\alpha| \simeq |\alpha_{1}^{\text{harm}}|$. The value of $R_{2}$ is well approximated by $\frac{2k+1}{2k}$, and this results in approximate scaling behavior for the cross sections: one has $r_{2} = \frac{\sigma_{2}}{\sigma_{2}^{\text{harm}}} \simeq \frac{2k+1}{2k}$. For reasonable values of $k$, the enhancement factor $r_{2}$ of the cross-section of Double Giant Resonance excitation can reach values $\sim 1.5$ that is compatible with experimental data [1].

Deviations from this simple scaling rule occur at both low and high energies. At $\gamma \rightarrow 1$, adiabatic approximation is valid, and this yields to $|\alpha| > |\alpha_{1}^{\text{harm}}|$. Thus, $R_{2} > R_{2}^{\text{scaling}} = \frac{(2k + 1)}{(2k)}$. By contrast, at higher energies, the dynamical nonlinear effects tend to reduce the magnitude of $|\alpha|$, thus $|\alpha|/|\alpha_{1}^{\text{harm}}| < 1$, and $R_{2} < R_{2}^{\text{scaling}}$. To sum up, the enhancement factor for the DGDR excitation cross section, $r_{2} = \frac{\sigma_{2}}{\sigma_{2}^{\text{harm}}}$ drops from $2 - 2.5$ (for low bombarding energies $\varepsilon \sim 100 \text{MeV per nucleon}$) to $1.2 - 1.3$ (for $\varepsilon \sim 640 - 700 \text{MeV per nucleon}$) while fixed value of $k$ is used. On Fig.2., we plotted the value of the enhancement factor calculated numerically for the case of $^{208}\text{Pb} + ^{208}\text{Pb}$ process. The magnitude of nonlinearity is kept fixed, $k = 1.3$.

These results are is in correspondence with the experimentally observed trends and with microscopic models based on Axel-Brink concept [14].

To conclude, we presented here a simple model that accounts for the nonlinear effects in the transition probabilities for the excitation of multi-phonon Giant Dipole Resonances in Coulomb excitation via relativistic heavy ion collisions. The model is based on the group theoretical properties of the boson operators. It allows to construct the solution for the dynamics of the multi-phonon excitation within coupled-channel approach in terms of the generalized coherent states of the corresponding algebras. The well known exactly solvable harmonic phonon model appears to be a limiting case of the present model when the nonlinearity parameter $1/k$ goes to zero. The main advantages of the limiting harmonic case (unrestricted multiphonon basis, preservation of unitarity and possibility of analytical treatment) remain present in our nonlinear scheme. Therefore, the model can be viewed as a natural extension of the harmonic phonon model to include the nonlinear effects in a consistent way while keeping the model solvable.

At high enough projectile energies, the problem is simplified and the dynamics is governed by the generators of SU(1,1) group. The problem of the excitation amplitudes is reduced to single Riccati-type equation for the complex amplitude $\alpha$. Its solutions can be obtained by means of successive approximations. In most cases relevant to applications, the perturbation is weak enough to be considered in the first order; the explicit solution for the amplitude is then given by the modified Bessel functions. The probabilities to excite double-phonon GDR appear to be enhanced by an approximately universal factor $\frac{(2k + 1)}{(2k)}$; the same scaling is roughly valid for the cross sections. This can be viewed as a hint that the discrepancy between the measured cross-sections of double GDR and the harmonic phonon calculations can be resolved within present nonlinear model by means of using an appropriate value of the nonlinear parameter $1/k$ for a given nucleus. The experimental values of enhancement of $\sigma_{2}$ with respect to the harmonic results for the excited $^{208}\text{Pb}$ nucleus are almost insensitive to the details of the collision process. On the other hand, the enhancement factor drops as the bombarding energy grows. Though the present results are obtained in the “transverse approximation” (neglecting the longitudinal response), they approximately hold in general case. This is consistent with the data and gives results similar to those recently obtained in

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a possibly different context, with a theory based on the concept of fluctuations (damping) and the Brink-Axel mechanism [13, 16, 28]. It would be certainly worthwhile to establish possible connections between the two approaches.

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Figure Captions

Fig.1. Enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ for the Double GDR excitation in $^{208}\text{Pb} + ^{208}\text{Pb}$ process at bombarding energy $\varepsilon = 640\text{MeV}$/per nucleon as a function of the parameter $k$ (symbols, solid curve is to guide the eye). Dashed curve corresponds to the scaling $r_2 \simeq \frac{2k+1}{2k}$.

Fig.2. Enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ for the Double GDR excitation in $^{208}\text{Pb} + ^{208}\text{Pb}$ process as a function of relativistic factor $\gamma$ (symbols, solid curve is to guide the eye). The value of the nonlinear parameter is kept to be equal to $k = 1.3$. The scaling value (constant $\frac{2k+1}{2k}$) is shown by dashed curve.
$r_2 = \sigma_2 / \sigma_2^{\text{harm}}$

$\epsilon = 0.64$ GeV per nucl.
$r_2 = \sigma_2 / \sigma_2^{\text{harm}}$

$$2k + 1 \over 2k$$