I. INTRODUCTION

Neutrino oscillations have provided the only particle physics evidence for new physics beyond the standard model (BSM) to date [1, 2], making it an excellent place to probe new physics scenarios. The phenomenology of neutrino oscillations is fairly unique, as it provides an opportunity to observe the accumulation of a relative phase over macroscopic distances, making neutrino oscillations one of the purest probes of quantum mechanics available. During propagation, the environment may also modify the phases due to an interaction. Such an interaction exists in the standard model (SM) and is called the Wolfenstein matter effect [3], wherein a neutrino in the electron state of the flavor basis experiences a potential with the background electrons via a charged-current (CC) interaction.

In the same paper that pointed out the SM matter effect, Wolfenstein also suggested the possibility of a new interaction that provides a matter effect, so-called neutrino non-standard interactions (NSI) [3–5]. Since then, there has been an explosion of interest to probe these new interactions. Numerous UV complete models have been developed [6–11] and the phenomenology has been generalized beyond vector currents [12–14]. In addition, several NSI parameters introduce various interesting degeneracies in oscillation or scattering experiments [4, 15–23]. In the same time, there has been an explosion of interest to probe these new interactions. Numerous UV complete models have been developed [6–11] and the phenomenology has been generalized beyond vector currents [12–14]. In addition, several NSI parameters introduce various interesting degeneracies in oscillation or scattering experiments [4, 15–23], which demonstrates the importance of complementary measurements of the NSI parameters.

One of the most complete ways to probe neutrino oscillations is through long-baseline accelerator experiments with electron (anti)neutrino appearance. While these measurements are extremely challenging experimentally, they provide a wealth of information, as they are sensitive to many oscillation parameters, including those that are the least constrained, like the CP-violating phase \( \delta \) from the leptonic mass mixing matrix. In addition, appearance measurements provide a crucial probe of certain NSI parameters.

The two state-of-the-art long-baseline neutrino experiments are NOvA and T2K [41, 42]. Both are off-axis; therefore, each detects a flux of neutrinos with a relatively narrow energy distribution. The latest results from both experiments [43, 44] show a slight tension at the \( \sim 2 \sigma \) level, depending on how exactly it is quantified. Both experiments prefer the normal mass ordering, but T2K prefers \( \delta \sim 3\pi/2 \) while NOvA does not have much preference and is generally around \( \delta \sim \pi \). While this is not yet significant, it provides an interesting test case for new physics should it persist, as both experiments plan to accumulate additional data.

In this paper, we review NSI and show how to approximate the NSI parameters that describe the NOvA and T2K data in section II. We then describe our treatment of the NOvA and T2K data and show results in the standard oscillation picture in sections III and IV. Then, we show in section V that the NOvA and T2K data can be resolved by the inclusion of NSI with complex CP-violating (CPV) phases with a preference for CPV values over CP-conserving values. Finally, we discuss our results in a broader picture of other neutrino measurements and present some possible plans to improve these results, and we conclude in section VI. All the relevant data files are available at peterdenton.github.io/Data/NOvA+T2K_NSI/index.html.

II. NSI OVERVIEW

NSI in oscillations provides an additional contribution to the matter potential of the neutrino oscillation Hamiltonian in the weak basis

\[
H = \frac{1}{2E} \left[ U^\dagger M^2 U + a \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon_{e\mu}^* & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon_{e\tau}^* & \epsilon_{\mu\tau}^* & \epsilon_{\tau\tau} \end{pmatrix} \right],
\]

where \( E \) is the neutrino energy, \( U \equiv R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta) R_{12}(\theta_{12}) \) is the PMNS mixing matrix [45, 46] that is parameterized in the usual
way [47], $M^2 \equiv \text{diag}(0, \Delta m_{21}^2, \Delta m_{32}^2)$ is the diagonal mass-squared matrix, $a \equiv 2\sqrt{2}G_FN\epsilon$ is the matter potential, and $N\epsilon$ is the electron density. The $\epsilon_{\alpha\beta}$ terms parameterize the size of the new interaction relative to the weak interaction and typically arise from effective Lagrangians of the form

$$L_{\text{NSI}} = -2\sqrt{2}G_F \sum_{\alpha,\beta,f} \epsilon_{\alpha\beta}^f (\bar{\nu}_\alpha \gamma^\mu \nu_\beta)(\bar{f} \gamma_\mu f).$$

(2)

For simplicity, we only consider NSI with vector mediators. The Lagrangian level NSI parameters in eq. 2 are related to the Hamiltonian level terms in eq. 1 by $\epsilon_{\alpha\beta} = \sum_f \frac{N_f}{f} \epsilon_{\alpha\beta}^f$, where $N_f$ is the number density of fermion $f$. In the context of oscillations, it isn’t possible to identify which matter particles (electrons, up quarks, or down quarks) the new physics is coupled to without comparing neutrino trajectories through materials with different neutron fractions, such as the Earth and the sun. Within the context of long-baseline trajectories through the crust, the neutron fraction is close to one. While the NSI parameters are often taken to be real for simplicity, we consider complex NSI, where $\epsilon_{\alpha\beta}$ NSI parameters are often taken to be real for simplicity since the matter effect at T2K is comparably small. Thus, NOvA’s measurements would be a function of the same $\delta_{\text{T2K}} \sim 3\pi/2$ with a correction from NSI such that NOvA infers their best fit value of $\delta_{\text{NOvA}} \sim \pi$ (although NOvA has a broad allowed region in $\delta_{\text{NOvA}}$). NSI phase reduction is possible to a good approximation for $\epsilon_{\mu\mu}$ and $\epsilon_{\tau\tau}$. That is, at leading order the complex phases only appear as $\delta + \phi_{\mu\mu}$ and $\delta + \phi_{\tau\tau}$. Thus, under the assumption that T2K experiences no matter effect and NOvA experiences a sizable matter effect, we find the following relation:

$$\delta_{\text{NOvA}} \approx \delta_{\text{T2K}} + \phi_{\alpha\beta}$$

(3)

for $\beta = \mu, \tau$ (note that the phase reduction does not apply for $\epsilon_{\mu\tau}$). Therefore, we anticipate we will find that $\phi_{\mu\beta} \sim 3\pi/2$ will reduce the tension between the experiments.

In addition, one can estimate the magnitude of the NSI parameter that would resolve different measurements of $\delta$ in experiments experiencing distinct matter potentials. We find that if two experiments at two different matter potentials measure two disparate values of $\delta$ due to $\epsilon_{\alpha\beta}$ NSI for $\beta \in \{\mu, \tau\}$, the magnitude of the NSI in the NOvA is approximately given by

$$|\epsilon_{\alpha\beta}| \approx \frac{s_{12}c_{12}c_{23}s_{23}}{2s_{23}|w_\beta|} \frac{|a_{\text{NOvA}} - a_{\text{T2K}}|}{a_{\text{NOvA}} - a_{\text{T2K}}}$$

(4)

$$\approx \begin{cases} 0.22 & \text{for } \beta = \mu \\ 0.24 & \text{for } \beta = \tau \end{cases}$$

where $w_\beta = s_{23}$ or $c_{23}$ for $\beta = \mu$ or $\tau$ respectively. The preferred value of $\epsilon_{\mu\mu}$ is larger than that for $\epsilon_{\mu\tau}$ since T2K prefers the upper octant and T2K is less affected by NSI than NOvA. The difference between $\epsilon_{\mu\mu}$ and $\epsilon_{\mu\tau}$ makes sense since long-baseline oscillations are dominated by $\nu_\beta$, which contains more $\nu_\mu$ in the upper octant, and thus, not as much NSI affecting $\nu_\mu$ is required to produce a given effect. We also note that the approximations presented here are quite consistent with our numerical results discussed below and shown in fig. 2 and table 1. For more on the approximate derivations in this section, see appendix A.

III. ANALYSIS DETAILS

The appearance channels at NOvA and T2K can be approximated by counting experiments, while for the disappearance channels, the energy distribution of the events is important. This approximation ignores several potentially problematic issues: the energy distributions aren’t exactly delta distributions, there are correlated systematics between the different channels, and the cross section systematics may well be related even between the different experiments. Nonetheless, we find an acceptable reproduction of the results with the simple treatment described below.

NOvA measures neutrinos with $E \sim 1.9$ GeV after traveling 810 km through the Earth with density $\rho = 2.84$ g/cc, while T2K measures neutrinos with $E = 0.6$ GeV after traveling 295 km through the Earth with average density $\rho = 2.3$ g/cc. For the appearance channels, we find that the number of events can be expressed as a constant normalization term and a constant factor which multiplies the oscillation probability in matter (see also [50] for a similar approach). These constant factors can be derived from the provided bi-event plots in [43, 44, 51]. As wrong sign leptons contribute to the flux, especially in antineutrino mode, we parameterize the predicted numbers of events as

$$n(\nu_e) = xP(\nu_\mu \rightarrow \nu_e) + yP(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) + z, \quad \text{ and similarly for the antineutrino channel.}$$

(5)

For NOvA, a good fit is obtained for the neutrino channel without including the wrong sign leptons, so we find

$$n(\nu_e)_{\text{NOvA}} = 31.15 + 1149.7 \times P(\nu_\mu \rightarrow \nu_e), \quad n(\bar{\nu}_e)_{\text{NOvA}} = 13.97 + 472.60 \times P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) + 22.96 \times P(\nu_\mu \rightarrow \nu_e),$$

(6)

(7)

1 We use the standard $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ shorthand.
We can calculate the effective vacuum NSIs as described in eq. 1 is straightforward. For the disappearance channel, we calculate the effective vacuum NSIs as described in eq. 1 is straightforward. For the appearance channel, incorporating the effect of NSI, we find

\[ n(\nu_e)^{\text{T2K}} = 19.80 + 1297.88 \times P(\nu_\mu \rightarrow \nu_e) + 21.21 \times P(\nu_\mu \rightarrow \bar{\nu}_e) , \]
\[ n(\bar{\nu}_e)^{\text{T2K}} = 5.77 + 231.95 \times P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) + 49.15 \times P(\nu_\mu \rightarrow \nu_e) . \]

The disappearance channel cannot be treated as a counting experiment, since the energy distribution of the events is important. At leading order, the oscillation probability for neutrinos and antineutrinos is the same in this channel. However, this changes in the presence of NSI. In the following, we will assume that the results in the disappearance channel are dominated by the neutrino sample, which provides higher statistics than the antineutrino sample. We adapt the results from [50] for the disappearance channel at NOvA, where they found as best fit \(|\Delta m_{32}^2| = (2.41 \pm 0.07) \times 10^{-3} \, \text{eV}^2\) and \(4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2) = 0.99 \pm 0.02\). For T2K, we obtain the test statistic for \(\theta_{23}\) and \(\Delta m_{32}^2\) from the 1D distributions of the test statistics provided by the experiment [43].

For the appearance channel, incorporating the effect of NSIs as described in eq. 1 is straightforward. For the disappearance channels, we calculate the effective vacuum mixing parameters by solving

\[ U^\dagger M^2 U + A + N = \tilde{U}^\dagger \tilde{M}^2 \tilde{U} + A , \]

where \(A \equiv \text{diag}(a, 0, 0)\) and the \(N\) matrix contains the \(\epsilon\)'s and is proportional to the matter potential \(a\). Then, by diagonalizing \(U^\dagger M^2 U + N\), one finds the vacuum parameters that a long-baseline accelerator experiment would extract in the presence of NSI. Various approximate techniques for the diagonalization of matrices in the context of neutrino oscillations in matter have been explored in [52–59]. The approach presented in eq. 10 is exact in the case of constant matter density; it does not apply to solar or atmospheric neutrinos, and additional care is necessary there. Finally, one can compare the effective vacuum mixing parameters extracted from \(M^2\) and \(U\) to the measured oscillation parameters.

To analyze the data, we construct a test statistic using a log likelihood ratio with Poisson statistics for the appearance data and simple \(\chi^2\) pulls for the disappearance constraints.

IV. STANDARD OSCILLATION RESULTS

Before we address new physics in the neutrino sector, we show the preferred regions in the standard oscillation picture in fig. 1. Contours are drawn relative to the best fit point at \(\Delta \chi^2 = \chi^2 - \chi^2_{\text{bf}} = 4.61\). Note that combining the data sets raises the minimum \(\chi^2\) by \(\approx 5.5\) over either experiment individually; this tension can be somewhat alleviated by switching the mass ordering [50, 60].

We show the preferred regions of \(\theta_{23}\) and the Jarlskog invariant where \(J = s_{12} s_{13} c_{13} c_{23} s_{23} c_{23} \sin \delta = \text{the Jarlskog}\) [61], which is a parameterization-independent quantification of CPV in the leptonic mass matrix [62]. Note that the maximum value of the Jarlskog is \(1/6\sqrt{3} \approx 0.096\); we are already quite far from maximal CPV in the leptonic sector due primarily to the fact that \(\theta_{13}\) is fairly small.

For fig. 1 we include a minimization over the four other standard oscillation parameters and the sign of \(\cos \delta\) for the Jarlskog panel. We include priors from KamLAND

\[ J = s_{12} s_{13} c_{13} c_{23} s_{23} c_{23} \sin \delta = \text{the Jarlskog}\]
TABLE I. Best fit values and $\Delta \chi^2 = \chi^2_{SM} - \chi^2_{NSI}$ for a fixed MO considering one complex NSI parameter at a time.

| MO | $|\epsilon_{\alpha \beta}|$ | $\phi_{\alpha \beta}/\pi$ | $\delta/\pi$ | $\Delta \chi^2$ |
|----|-----------------|------------------|--------------|--------------|
| NO | $\epsilon_{e\mu}$ | 0.19             | 1.50         | 1.46         | 4.68         |
|    | $\epsilon_{e\tau}$ | 0.29             | 1.60         | 1.46         | 3.99         |
|    | $\epsilon_{\mu\tau}$ | 0.38             | 0.60         | 1.16         | 1.03         |
| IO | $\epsilon_{e\mu}$ | 0.05             | 1.24         | 1.52         | 0.30         |
|    | $\epsilon_{e\tau}$ | 0.07             | 1.70         | 1.47         | 0.28         |
|    | $\epsilon_{\mu\tau}$ | 0.31             | 0.12         | 1.51         | 2.53         |


We see that in the normal mass ordering (NO), while T2K has some significance to disfavor $J = 0$, the inclusion of NOvA data weakens this, making CPV in the standard oscillation picture an important goal for NOvA and T2K [65, 66] in coming years, as well as upcoming long-baseline accelerator neutrino experiments such as DUNE and T2HK [67, 68]. This weakening of the significance in the NO when the experiments are combined emphasizes the slight tension between the experiments.

Similarly to refs. [50, 60], we also find that while NOvA and T2K both individually prefer the NO, the combination shows a slight preference for the inverted mass ordering (IO) at $\chi^2_{NO} - \chi^2_{IO} = 2.7$. When combined with Super-KamiokaNDE (SK) atmospheric data [69, 70], the best fit mass ordering (MO) remains normal [50, 60]. This MO question is of crucial significance beyond just measuring parameters in the SM. It may provide guidance about the structure of neutrino mass [71] and is a key input for many experimental measurements of neutrinos, including cosmological measurements of neutrino properties, kinematic measurements of neutrinos, and neutrinoless-double-beta decay measurements should neutrinos have a Majorana mass term, see e.g. [72].

In the next section, we find that in the presence of NSI, the long-baseline data is better described by the NO than the IO, so we assume the NO unless otherwise specified. The MO can be confirmed independently of the presence of NSI via JUNO [73].

---

3 SK preferred the NO at $\chi^2_{IO} - \chi^2_{NO} > 5$, but with their latest data release, the significance dropped to $\sim 3.2$, although it is still enough to prefer the NO in total.

---

V. NSI RESULTS

We analyze one complex NSI parameter at a time, using the appearance and disappearance data from NOvA and T2K and assuming the NO. In fig. 2, we present the allowed parameter regions in the $|\epsilon_{\alpha \beta}|$-$\phi_{\alpha \beta}$ plane for $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$. The results for $\epsilon_{\mu\tau}$ can be found in appendix B. For simplicity, we fix $\theta_{13}$, $\theta_{12}$, and $\Delta m^2_{21}$ to the best fit values from Daya Bay and KamLAND as described above and marginalize over $\Delta m^2_{31}$, $\delta$, and $\theta_{23}$, including the pull on $\Delta m^2_{31}$ from Daya Bay. We have verified that including the pulls associated with $\theta_{13}$, $\theta_{12}$, and $\Delta m^2_{21}$ do not significantly affect our results. The best fit values for the parameters for each case of $\epsilon_{e\mu}$, $\epsilon_{e\tau}$, and $\epsilon_{\mu\tau}$ in both MOs are given in table I. Note that while the combination of both experiments raises the $\chi^2$ by about 5.5 as mentioned in the previous section, that can be nearly completely alleviated with the addition of $\epsilon_{e\mu}$ which provides an improvement in the test statistic of 4.7 (compare this to switching to the IO which only improves the test statistic by 2.7 and is in tension with SK data). In the presence of NSI, we still find that the upper octant is preferred with $\sin^2 \theta_{23} = 0.56$ for all three NSI parameters and both MOs.

Consistent with our analytic estimates, we find moderate evidence for CP-violating NSI. The best solution is with the $\epsilon_{e\mu}$ parameter with maximal CP-violating phases for both the standard CP phase and the new NSI CP phase.

The constraints on complex NSI parameters from IceCube [74] slightly disfavor the preferred region for $\epsilon_{e\mu}$, although it is possible to get an improved fit to the NOvA and T2K data while not being in too strong of tension with the IceCube data. In fact, the best fit point to the IceCube data for $\epsilon_{e\mu}$ is at $|\epsilon_{e\mu}| = 0.07$ and $\delta_{e\mu}/\pi = 1.91$, close to the relevant numbers for NOvA and T2K. It is also interesting to note that IceCube slightly disfavors $|\epsilon_{e\mu}| = 0$ at just over 1$\sigma$.

We show the constraints from IceCube on complex NSI from [74] on figs. 2 and 3, which only slightly disfavors this NSI explanation of long-baseline data with $\epsilon_{e\mu}$. The IceCube constraints are comparable to other constraints in the literature on real NSI from oscillation experiments [4, 36, 75].

COHERENT’s measurement of the coherent elastic neutrino nucleus scattering (CEvNS) process [76] provides constraints [13, 36, 39, 77–83] on the NSI parameter space that is also an explanation of the NOvA and T2K data. While the parameters relevant for NOvA and T2K are not strongly ruled out by COHERENT yet, they can be probed by COHERENT in coming years. It should be noted, however, that the NSI constraint derived from COHERENT only applies to NSI governed by mediators heavier than $\sim 10$ MeV [36, 84]. Constraints for lower mediators masses down to $\sim 1$ MeV can be placed with upcoming low-threshold CEvNS experiments at nuclear reactors. Meanwhile, early universe measurements constrain mediators lighter than $\sim 5$ MeV [85, 86].
FIG. 2. The preferred parameter regions for $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$ using the newest appearance and disappearance data from NOvA and T2K and assuming the NO. The gray region is disfavored compared to the SM, and the dark gray region is ruled out by NOvA and T2K data at $\Delta\chi^2 = -4.61$. The blue stars show the best fit points. Each of the orange contours are drawn at integer values of $\Delta\chi^2$. See table I for the best parameters. IceCube disfavors the region to the right of the black dotted curve at 90% [74].

Thus we anticipate that COHERENT or future reactor CEvNS experiments will be able to probe the NSI parameters that could explain the NOvA and T2K data in coming years.

VI. CONCLUSIONS

Measuring and understanding CP violation is of the utmost importance in particle physics. Somewhat confusingly, the weak interaction violates CP while the strong interaction seems to conserve CP. Meanwhile, the quark mass mixing matrix has relatively small CP violation. To better understand the important role that CPV plays in particle physics, we must measure it and understand it in the leptonic sector.

In this manuscript, we have analyzed a new physics explanation for the slight tension in the recent NOvA and T2K data. We performed a fit to the data and showed that this tension can be resolved when introducing complex CP-violating NSI parameters. As an example, we analyzed non-zero $\epsilon_{e\mu}$, $\epsilon_{e\tau}$, $\epsilon_{\mu\tau}$ one at a time and found that the best fit points for the new complex phases of $\epsilon_{\alpha\beta}$ prefers not only maximal CPV in the new interaction around $3\pi/2$ for $\alpha = e$, but also large CPV in the leptonic mass matrix. These NSI parameters are best constrained (not counting long-baseline experiments) by atmospheric oscillation measurements by Super-KamiokaNDE and IceCube. These measurements rule out the favored parameter region for $\epsilon_{\mu\tau}$, whereas the atmospheric constraints only partially disfavor the preferred regions of $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$. We anticipate that improvements from Super-KamiokaNDE and IceCube can further test this hypothesis in the future. Furthermore, experiments that probe coherent elastic neutrino nucleus scattering will provide strong constraints on NSI parameters of a similar order of magnitude, though they only apply to mediators heavier than the $\sim 10$ MeV scale. In addition, while without new physics, the IO is slightly preferred by NOvA and T2K, the inclusion of NSI shifts the preference back to the NO. JUNO’s measurement of the MO, which has almost no dependence on the matter effect, will determine the MO independent of NSI.

We can see clearly from e.g. eq. 10 that in order to measure NSI with long-baseline neutrinos, one needs to either compare two different experiments or use a broad band beam such as that which DUNE will have [67].

To summarize, we have shown that the tension of the recent NOvA and T2K data can be resolved in a BSM scenario with the introduction of CP-violating NSI parameters, which can be further probed with near-future experiments. It would be interesting to see if other new physics models could also explain the discrepancy, such as the presence of sterile neutrinos, decoherence, or neutrino decay.

ACKNOWLEDGMENTS

We acknowledge support from the US Department of Energy under Grant Contract DE-SC0012704. The work presented here that RP did was supported by the U.S. Department of Energy, Office of Science, Office of Workforce Development for Teachers and Scientists, Office of Science Graduate Student Research (SCGSR) program. The SCGSR program is administered by the Oak Ridge Institute for Science and Education (ORISE) for the DOE. ORISE is managed by ORAU under contract number DE-SC0014664. All opinions expressed in this paper are the authors’ and do not necessarily reflect the...
policies and views of DOE, ORAU, or ORISE.

[1] Y. Fukuda et al. (Super-Kamiokande), Phys. Rev. Lett. 81, 1562 (1998), arXiv:hep-ex/9807003.
[2] Q. Ahmad et al. (SNO), Phys. Rev. Lett. 89, 011301 (2002), arXiv:nucl-ex/0204008.
[3] L. Wolfenstein, Phys. Rev. D 17, 2369 (1978).
[4] Y. Farzan and M. Tortola, Front. in Phys. 6, 10 (2018), arXiv:1710.09360 [hep-ph].
[5] Neutrino Non-Standard Interactions: A Status Report, Vol. 2 (2019) arXiv:1907.00991 [hep-ph].
[6] D. V. Forero and W.-C. Huang, JHEP 03, 018 (2017), arXiv:1608.04719 [hep-ph].
[7] P. B. Denton, Y. Farzan, and I. M. Shoemaker, Phys. Rev. D 98, 035003 (2018), arXiv:1811.01310 [hep-ph].
[8] U. K. Dey, N. Nath, and S. Sadhukhan, Phys. Rev. D 96, 055023 (2017), arXiv:1612.00784 [hep-ph].
[9] M. Gonzalez-Garcia, M. Maltoni, and F. Punzi, JHEP 04, 072 (2016), arXiv:1605.05301 [hep-ph].
[10] D. V. Forero and Wi.-C. Huang, JHEP 03, 018 (2017), arXiv:1608.04719 [hep-ph].
[11] K. Babu, A. Friedland, P. Machado, and I. Mocioiu, JHEP 12, 097 (2017), arXiv:1705.01822 [hep-ph].
[12] S.-F. Ge and A. Y. Smirnov, JHEP 10, 138 (2016), arXiv:1607.08513 [hep-ph].
[13] S. K. Agarwalla, S. S. Chatterjee, and A. Palazzo, Phys. Lett. B 762, 64 (2016), arXiv:1607.01745 [hep-ph].
[14] M. Blennow, S. Choubey, T. Ohlsson, D. Pramanik, and S. K. Raut, JHEP 08, 090 (2016), arXiv:1606.08851 [hep-ph].
[15] S. Fukasawa, M. Ghosh, and O. Yasuda, Phys. Rev. D 95, 055005 (2017), arXiv:1611.06141 [hep-ph].
[16] K. Deepthi, S. Goswami, and N. Nath, Phys. Rev. D 96, 075023 (2017), arXiv:1612.00784 [hep-ph].
[17] D. V. Forero and P. Huber, Phys. Rev. Lett. 117, 031801 (2016), arXiv:1601.03736 [hep-ph].
[18] K. Deepthi, S. Goswami, and N. Nath, Nucl. Phys. B 936, 91 (2018), arXiv:1711.04840 [hep-ph].
[19] P. Coloma, B. Denton, M. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, JHEP 04, 116 (2017), arXiv:1701.04828 [hep-ph].
[20] P. Coloma, M. Gonzalez-Garcia, M. Maltoni, and T. Schwetz, Phys. Rev. D 96, 115007 (2017), arXiv:1708.02899 [hep-ph].
[21] J. M. Hyde, Nucl. Phys. B 949, 114804 (2019), arXiv:1806.09221 [hep-ph].
[22] P. Coloma, I. Esteban, M. Gonzalez-Garcia, and M. Maltoni, JHEP 02, 023 (2020), arXiv:1911.09109 [hep-ph].
[23] I. Esteban, M. Gonzalez-Garcia, and M. Maltoni, JHEP 06, 055 (2019), arXiv:1905.05203 [hep-ph].
[24] D. Ayres et al. (NOvA), (2020), 10.2172/935497.
[25] K. Abe et al. (T2K), Nucl. Instrum. Meth. A 659, 106 (2011), arXiv:1106.1238 [physics.ins-det].
[26] P. Dunne, “Latest Neutrino Oscillation Results from T2K,” (2020).
[27] A. Himmel, “New Oscillation Results from the NOvA Experiment.” (2020).
[28] B. Pontecorvo, Sov. Phys. JETP 6, 429 (1957).
[29] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).
[30] M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018).
[31] A. M. Gago, H. Minakata, H. Nunokawa, S. Uchinami, and R. Zukunovich Funchal, JHEP 01, 049 (2010), arXiv:0904.3360 [hep-ph].
[32] M. Gonzalez-Garcia, M. Maltoni, and J. Salvado, JHEP 05, 075 (2011), arXiv:1103.4365 [hep-ph].
[33] K. J. Kelly, P. A. Machado, S. J. Parke, Y. F. Perez Gonzalez, and R. Zukunovich-Funchal, (2020), arXiv:2007.08526 [hep-ph].
[34] M. Baird, “Latest Oscillation Results Combining Neutrino and Antineutrino Data from the NOvA Experiment,” presented at ICHEP2020 (2020).
[35] H. Yokomakura, K. Kimura, and A. Takamura, Phys. Lett. B 496, 175 (2000), arXiv:hep-ph/0009141.
[36] K. Kimura, A. Takamura, and H. Yokomakura, Phys. Rev. D 66, 073005 (2002), arXiv:hep-ph/0205295.
[37] S. K. Agarwalla, Y. Kao, and T. Takeuchi, JHEP 04, 047 (2014), arXiv:1302.6773 [hep-ph].
[38] H. Mimakata and S. J. Parke, JHEP 01, 180 (2016), arXiv:1505.01826 [hep-ph].
[39] P. B. Denton, H. Mimakata, and S. J. Parke, JHEP 06, 051 (2016), arXiv:1604.08167 [hep-ph].
Since the inclusion of NSI allows one, in principle, to exactly map one set of vacuum parameters onto another (see eq. 10), we can write down a system of equations of the form

\[ P(\epsilon = 0, \delta_{\text{meas}}) = P(\epsilon, \delta_{\text{true}}), \]  
\[ \bar{P}(\epsilon = 0, \delta_{\text{meas}}) = \bar{P}(\epsilon, \delta_{\text{true}}), \]

where we require both neutrino and antineutrino modes are equal for a given experiment\(^4\). Here, \(\delta_{\text{meas}}\) is the value of \(\delta\) extracted by the experiment, assuming the standard oscillation picture. That is, the LHS represents the probabilities, while the RHS represents the probabilities in terms of the “true” parameters.

We can use approximate expressions for NSI in long-baseline experiments to determine the relationship among the measured values of \(\delta\), the true value of \(\delta\), and the magnitude and phase of the NSI. From refs. [17, 87] after some

\[^4\] We assume that the effect of NSI is completely absorbed in the CP phase; in principle, the other parameters are also modified, specifically \(\theta_{23}\) and \(\Delta m^2_{31}\), but we assume that the effect from those parameters are small since many different measurements of \(\Delta m^2_{31}\) tend to agree, and the precision on \(\theta_{23}\) is still relatively poor.
manipulation, we find

\[-s_1 c_2 \pi \Delta m^2_{21} \sin \delta + a_{\text{NOvA}} |\epsilon_{e}\| \left[ w_{\beta} s_{23} \cos(\delta + \phi_{e\beta}) - v_{\beta} c_{23} \pi \sin(\delta + \phi_{e\beta}) \right] \approx -s_1 c_2 \pi \Delta m^2_{21} \sin \delta_{\text{NOvA}}, \tag{A3}\]

\[s_1 c_2 \pi \Delta m^2_{21} \sin \delta - a_{\text{NOvA}} |\epsilon_{e}\| \left[ w_{\beta} s_{23} \cos(\delta + \phi_{e\beta}) + v_{\beta} c_{23} \pi \sin(\delta + \phi_{e\beta}) \right] \approx s_1 c_2 \pi \Delta m^2_{21} \sin \delta_{\text{NOvA}}, \tag{A4}\]

where \(w_{\beta} = s_{23} (c_{23})\), \(v_{\beta} = c_{23} (-s_{23})\) for \(\beta = \mu, \tau\), and we have assumed that the NO is correct and that both experiments measure the NO. A similar expressions exists for T2K, as well. This confirms that phase reduction is valid [17].

From the requirement that the probabilities in the neutrino and antineutrino channel should both be satisfied with the same parameters, one immediately finds that \(\sin(\delta + \phi_{e\beta}) = 0\). This means \(\delta + \phi_{e\beta} = 0\) or \(\pi\) and that either \(\cos(\delta + \phi_{e\beta}) = 1\) or \(\cos(\delta + \phi_{e\beta}) = -1\), respectively. Plugging this in and subtracting the NOvA and T2K equations, we find

\[|\epsilon_{e}\| \approx \frac{s_1 c_2 \pi \Delta m^2_{21} (\sin \delta_{\text{T2K}} - \sin \delta_{\text{NOvA}})}{2 s_{23} w_{\beta} (a_{\text{NOvA}} - a_{\text{T2K}}) \cos(\delta + \phi_{e\beta})}. \tag{A5}\]

Given that \(a_{\text{NOvA}} > a_{\text{T2K}}\) and that the data suggests that \(\sin \delta_{\text{T2K}} < \sin \delta_{\text{NOvA}}\), we find that \(\cos(\delta + \phi_{e\beta}) = -1\), and thus, \(\delta + \phi_{e\beta} = \pi\). In any case, we can write down the general result using absolute values, as shown in eq. 4.

We can instead divide the NOvA and T2K equations to find

\[\sin \delta \approx \frac{\sin \delta_{\text{NOvA}} a_{\text{T2K}} - \sin \delta_{\text{T2K}} a_{\text{NOvA}}}{a_{\text{T2K}} - a_{\text{NOvA}}}. \tag{A6}\]

Plugging in the numbers, we find that the true value of \(\delta\) one would expect is \(\sin \delta = -1.7\). This means that for an NSI explanation of NOvA and T2K, we would expect \(\sin \delta = -1\), and T2K would infer \(\sin \delta_{\text{T2K}}\) slightly larger than \(-1\). In addition, the effect of eq. A6 in our situation of \(\sin \delta_{\text{T2K}} \sim -1\) is somewhat alleviated by changes in \(\theta_{23}\) due to NSI which we have not accounted for. Given that we have \(\cos(\delta + \phi_{e\beta}) = -1\) in our scenario, in the limit where \(a_{\text{T2K}} \rightarrow 0\), we see from eq. A6 that \(\sin \delta \approx \sin \delta_{\text{T2K}}\) as expected and that \(\delta_{\text{T2K}} + \phi_{e\beta} = \pi\), and thus \(\phi_{e\beta} = 3\pi/2\), consistent with our numerical results.

All of these results are derived assuming the approximate expressions from ref. [87], that the experiments are at the first oscillation maximum, and that the matter potentials are small relative to \(\Delta m^2_{31}\) (for NOvA (T2K) we have \(a/\Delta m^2_{31} \approx 1/6\ (1/20))\).

Appendix B: Results for \(\epsilon_{\mu\tau}\)

It is expected that \(\epsilon_{\mu\tau}\) will not easily address the NOvA and T2K tension. Moreover, there are very strong constraints on \(\epsilon_{\mu\tau}\) from atmospheric data [74, 75, 88]. While these were generally derived under the assumption of real NSI, the relaxation to complex NSI should not significantly weaken the constraints. Nonetheless, we show the preferred region in fig. 3 for NOvA and T2K data while marginalizing over \(\theta_{23}\), \(\Delta m^2_{31}\) (including a pull from Daya Bay), and \(\delta\) while the other three standard oscillation parameters were set to their best fit values from Daya Bay and KamLAND.
FIG. 3. The preferred parameter region for $\epsilon_{\mu\tau}$ using the newest appearance and disappearance data from NOvA and T2K and assuming the NO (left) or the IO (right). The gray region is disfavored compared to the SM and the blue star shows the best fit point. The orange contours are drawn at integer values of $\Delta\chi^2$. See table I for the best parameters. IceCube disfavors the region to the right of the black dotted curve at 90% [74].