LIFETIME OF SURFACE FEATURES AND STELLAR ROTATION: A WAVELET TIME-FREQUENCY APPROACH

WILLIE SOON, 1 PETER FRICK, 2 AND SALLIE BALIUNAS 1

Received 1998 June 30; accepted 1998 November 6; published 1998 December 10

ABSTRACT

We explore subtle variations in disk-integrated measurements spanning \( \pm 18 \) yr of stellar surface magnetism by using a newly developed time-frequency gapped wavelet algorithm. We present results based on analysis of the Mount Wilson Ca \( \text{II} \) \( \text{H} \) and \( \text{K} \) emission fluxes in four, magnetically active stars (HD 1835 [G2 V], HD 82885 [G8 IV–V], HD 149661 [K0 V], and HD 190007 [K4 V]) and sensitivity tests using artificial data. When the wavelet basis is appropriately modified (i.e., when the time-frequency resolution is optimized), the results are consistent with the existence of spatially localized and long-lived Ca \( \text{II} \) features (assumed here as activity regions that tend to recur in narrowly confined latitude bands), especially in HD 1835 and HD 82885. This interpretation is based on the observed persistence of relatively localized Ca \( \text{II} \) wavelet power at a narrow range of rotational timescales, enduring as long as \( \pm 10 \) yr.

Subject headings: methods: numerical — stars: chromospheres — stars: rotation

1. INTRODUCTION

At Mount Wilson Observatory (MWO), the Ca \( \text{II} \) \( \text{H} \) and \( \text{K} \) (396.8 nm) and \( \text{K} \) (393.4 nm) emission fluxes have been observed as a proxy for surface magnetism since 1966 in many lower main-sequence stars. Since 1980, observations have been obtained several times per week over periods as long as several months. These densely sampled records are suited for studying changes several times per week over periods as long as several months. The admissibility condition can be broken when the wavelet basis is appropriately modified (i.e., when the time-frequency resolution is optimized), the results are consistent with the existence of spatially localized and long-lived Ca \( \text{II} \) features (assumed here as activity regions that tend to recur in narrowly confined latitude bands), especially in HD 1835 and HD 82885. This interpretation is based on the observed persistence of relatively localized Ca \( \text{II} \) wavelet power at a narrow range of rotational timescales, enduring as long as \( \pm 10 \) yr.

2. METHOD OF ANALYSIS: THE GAPPED WAVELET ALGORITHM

Because of mathematical refinements of the wavelet transform (i.e., self-similarity of the wavelet basis function, time-frequency localization), it has become increasingly popular as a tool for extracting local-frequency information (e.g., Farge 1992; Kumar & Foufoula-Georgiou 1997). The wavelet transform differs from traditional Fourier analysis because of its ability to efficiently detect multiscale, nonstationary processes.

We applied a newly introduced gapped wavelet algorithm (Frick et al. 1997) to the MWO records of four stars to study time variations of rotational modulation of the Ca \( \text{II} \) fluxes. The algorithm alleviates two constraints in stellar activity records that complicate traditional methods of period analysis: limited duration and sampling gaps. This goal is achieved by fine correction (adaptation) of the wavelet for a given time and frequency while still satisfying the admissibility condition (for which the mean value of the wavelet must be zero: \( \overline{\psi} = 0 \)). The admissibility condition can be broken when the wavelet overlaps data gaps in, or the edges of, data series.

In this analysis, we use the Morlet wavelet with an adjustable parameter \( c \), which can be fine-tuned to yield optimal resolutions of time and frequency:

\[
\psi(t) = e^{-\pi t^2/2} e^{i 2 \pi ct}.
\]

In that case, the resolutions of the wavelet for a given characteristic timescale \( T \) are

\[
\delta t = c \kappa T, \quad \delta \omega = \frac{c}{\kappa T},
\]

where \( c \) is a constant of order unity. Small values of \( \kappa \) give better time resolution, while large values of \( \kappa \) improve frequency resolution. The key step in any application of the wavelet transform is to deduce the optimum trade-off between frequency and time resolution, and the art is to choose \( \kappa \) carefully so that it fits the physical phenomena of interest.

The commonly adopted value of \( \kappa \) is 1; the limit \( \kappa \to \infty \)

1 Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, MA 02138.

2 Institute of Continuous Media Mechanics, Koltsoyov str. 1, 614061 Perm, Russia.

3 The algorithm was first introduced under the name adaptive wavelet, but because this term has been widely used for different wavelet techniques, it has been renamed gapped wavelet (see Frick, Grossmann, & Tchamitchian 1998 for mathematical motivations and proofs).
corresponds to the Fourier transform. But the choice of $\kappa$ is highly restricted by the admissibility condition ($\psi = 0$), so that only a finite range and discrete values of $\kappa$ are allowed. Under the application of the standard Morlet wavelet algorithm, the admissibility condition breaks down when $\kappa$ is below unity because the real parts of the Morlet wavelet do not vanish. In contrast, because of the renormalization performed in the gapped wavelet algorithm, that problem is avoided automatically, and a wide range of $\kappa$ (provided $\kappa$ is not too small) can be used.

3. RESULTS

We examine two aspects of stellar variability in the records: (1) the presence of any dominant surface magnetic features and signatures of rotating features and (2) the nature of time variations of any dominant rotational signals. Results are presented for a wide range of $\kappa$ and illustrate that aspect 1 is best studied with lower values of $\kappa$, while aspect 2 is more optimally addressed with larger values.

3.1. Optimum Time-Frequency Resolutions and Beat Phenomena: Tests from Simple Oscillator Models

The MWO time series of Ca ii emission fluxes are assumed to be a superposition of short-lived oscillations with random, initial phases and individual frequencies, which have a mean value $\Omega$ (or period, assumed to be rotation, $T = 2\pi/\Omega$) and characteristic width $\Delta \Omega$. The choice for time-frequency resolution is set by two factors: the typical time of an individual oscillation and the band width $\Delta \Omega$. If one is interested in studying variations in frequencies caused by the emergence of fluxes drifting in latitude, a wavelet with $\delta t$ on the order of typical lifetime of individual features should be used.

From a detailed analysis of the magnetic flux budget on the Sun, Schrijver & Harvey (1994) deduced that the surface-averaged lifetime of photospheric magnetic flux (which included contributions from magnetic bipolar and plage regions and other smaller scale features) varies from $\sim 130$ days at cycle maximum to $340$ days at cycle minimum. These timescales yield estimates of $\kappa$ from about 4 to 10. For the Ca ii records of young and active stars, the values for $\kappa$ may need to be smaller. But small values of $\kappa$ produce frequency beating because of reduced wavelet spectral/frequency resolution (or increased time resolution). We illustrate this phenomenon with two examples. First, we consider the simple superposition of two harmonic oscillators with nearby frequencies $\Omega$ and $\Omega + \Delta \Omega$. Beating arises if the wavelet frequency resolution $\delta \omega \approx \Delta \Omega$. In Figure 1a, the wavelet power with $\kappa = 1$ is shown for the case of two superposed sinusoids with periods of 9 and 11 days. The maximum wavelet power for time $t$ displays a beating with characteristic frequency $\Omega_b = \Delta \Omega$.

Figure 1b shows the modulus of wavelet coefficients for another artificial signal, one that is produced by the superposition of short-lived variations with random phases and frequencies ($9 \leq T \leq 11$ days). Again, the local maxima in wavelet power for each time oscillates around the mean value of the dominant periods. The period of the beating yields an estimate of the average width of a band of frequencies. The beat frequency $\Omega_b$ suggests $\Delta T = \Omega_b T^2 / 2 \pi \approx 1$ day, which represents the mean difference between the periods of any two individual oscillations.

Figure 1c shows the results from the artificial series of Figure 1b analyzed with a larger value of $\kappa = 10.0$. The beat phenomenon disappears, and only discrete periods are resolved at a given time interval. Thus, appropriate choices of $\kappa$ allow different kinds of information to be extracted.

3.2. Ca ii H and K Wavelet Spectra: HD 1835, HD 82885, HD 149661, and HD 190007

Figure 2 shows the time-averaged wavelet spectra for the four active stars. Table 1 summarizes the values of rotation periods determined from the gapped wavelet technique. The dominant timescales found here correspond well with results

| Star          | $T_r$ | $T_b$ | $\Delta T$ | $\Delta T/T_r$ | $\kappa$ |
|---------------|------|------|-----------|--------------|---------|
| HD 1835 (G2 V) | 7.7  | 47   | 1.3       | 0.15         | 1       |
| HD 82885 (G8 IV–V) | 18   | 96   | 3.4       | 0.19         | 0.9     |
| HD 149661 (K0 V) | 21   | 210  | 2.1       | 0.10         | 1.6     |
| HD 190007 (K4 V) | 28   | 120–230 | 7–3     | 0.25–0.1     | 0.7–1.5 |

Note.—$T_r$ is the rotation period deduced from time-averaged wavelet spectra (see Fig. 2); $T_b$ is the beat period found from the time variations of the rotational timescales (see Fig. 3); $\Delta T$ is the characteristic range of rotation periods deduced from values of the beat period interpreted as a simple model of oscillators with a range of $T_r$; $\kappa$ is the characteristic $\kappa$ that distinguishes lower values for which the beat phenomenon is observed from higher values for which time resolution becomes too poor to capture the beat among dominant periods.
Fig. 2.—Time-averaged Ca ii wavelet spectra \( W(T) \) for four stars: HD 1835, HD 82885, HD 149661, and HD 190007. Peaks in the spectra corresponding to rotation and are located around 7.7, 18.0, 21.0, and 28 days, respectively (Table 1).

Fig. 4.—Distribution of the modulus of Ca ii wavelet coefficients \( W(T, t) \) for the four stars. Lighter shading corresponds to increasing values of the wavelet modulus (except for the white regions of data gaps). The dominant scales of Ca ii wavelet power are traced by filled circles.

To study the time variations of the rotation periods, we started the analysis with \( \kappa = 1 \). We found significant oscillations of the primary period in the time-frequency plane of the wavelet modulus for each of the records of the four stars. An example of these oscillations is shown in Figure 3a for HD 1835.

The maxima of wavelet power (traced by filled circles) are themselves a time series \( T_m(t) \) from which a wavelet spectrum is also calculated. Those spectra for all four stars are shown in Figure 3b. Each spectrum displays a pronounced peak corresponding to the main beat frequency \( \Omega_b \) (Table 1). The test examples suggest that the beat frequency contains information on the range of unresolved frequencies in each record. The dominant but unresolved periods range from 1 to a few days and are also listed in Table 1. The beat frequency yields an estimate of the characteristic \( \kappa^* \) (Table 1) and suggests the choice of a larger \( \kappa \) that would be useful for studying, e.g., the change in dominant periods over extended times.

Figure 4 shows the time variations of the dominant timescales for each star calculated with optimal wavelet time-frequency resolution. The values of the wavelet resolution parameter \( \kappa \) are 2.5, 2.0, 4.0, and 4.0, respectively. These values were chosen to be about 2.5 times larger than \( \kappa^* \) in order to (1) avoid the beat phenomena shown in Figure 3 and (2) investigate the relative long-term stability of the dominant peaks in the spectra. Figure 4 shows that the dominant rotational
periods for HD 1835, HD 82885, and HD 149661 are relatively narrow in range and remarkably stable throughout the full interval of observations. This pattern of time-frequency localization may be interpreted as recurring Ca II emissions from localized surface features that continued for $\approx 10$ yr. The dominant rotational peaks for HD 190007 seem less localized, and it may or may not be dominated by narrowly distributed, persistent or recurring surface features. The range of each dominant scale is assumed to be the wavelet frequency resolution $\delta \omega$; the values for the records of the four stars are $0.5, 1.5, 0.5$, and $0.5$ days, respectively.

4. DISCUSSION

Previous analyses of the MWO Ca II records used Fourier methods for studying surface rotation and the pattern of surface differential rotation (e.g., Baliunas et al. 1985; Donahue et al. 1996). Two interconnected assumptions were made in those studies: First, that by binning the data series into individual observing seasons ($\approx$several months), time resolution for studying the underlying rotational modulation is automatically optimum. Second, once any dominant rotational signal is detected, systematic variations with time of the rotation period can be studied by simply considering the change in rotational signal from one seasonal bin to another. Because the wavelet transform adopts localized basis function, information on the time dependence of spectral properties is preserved, so it avoids the assumptions used in the Fourier studies. However, although the wavelet method provides information on spectral changes with time, there is a practical limit to the physical interpretation of the results because the spatial location of the stellar features is unknown. That means that proving that rotational timescales associated with specific surface features are changing with time is difficult.

On the other hand, one can invert the question and ask if a stationary, rotational signal exists. This is an easier question to answer because the wavelet method can establish clearly the presence of a persistent signal. The results in Figure 4 suggest that the Ca II-emitting regions marking the rotation periods last $\approx 10$ yr, for at least two of the four active stars analyzed.

Thus, the idea of using the wavelet time-frequency approach to establish stationarity of the rotational signal is a useful strategy in gaining understanding of the surface Ca II activity.

Because the Ca II features in HD 1835 and HD 82885 seem to persist so much longer than the lifetime of individual magnetic features (e.g., active regions) on the Sun, our results might caution against indiscriminate application of the stellar knowledge to the Sun. This would seem to be a conclusion that opposes any hope of applying the solar-stellar connection in a two-way, quantitative manner. On the other hand, an analysis of sunspot groups from 1940 to 1956 suggests that there are recurrent clusters of sunspot nests that can persist and keep their rotation rate for up to several years, while at the same time showing systematic meridional drift toward equator with velocity of about 1 m s$^{-1}$ (e.g., van Driel-Gesztelyi, van der Zalm, & Zwaan 1992). Those authors also noted that components in large active nests tend to overlap in time with their mean latitudes differing by less than 25°, while the difference in longitude may extend up to 55°. Thus, the observed persistence of Ca II features in our active star sample may not be entirely incompatible with characteristics observed on the Sun.

We thank Cristina Christian and Robert Donahue for their valuable contributions to the MWO HK project. We are also grateful for the dedicated efforts of our colleagues, especially Mike Bradford, Laura Woodard-Eklund, Jim Frazer, and Kirk Palmer at Mount Wilson Observatory. The comments by the referee led to improvement of this manuscript.

This work was supported by the Electric Power Research Institute, the Richard C. Lounsbery Foundation, and MIT Space grant 5700006633. Additional support from the US Civilian Research and Development Foundation grant 171600 is also gratefully acknowledged. This research was made possible by a collaborative agreement between the Carnegie Institution of Washington and the Mount Wilson Institute.

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