Coupled Resonance Mechanism of Interface Stratification of Thin Coating Structures Excited by Horizontal Shear Waves

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Abstract: The coupled resonance mechanism of interface stratification of thin coating structures excited by horizontal shear waves is investigated by the forced vibration solution derived from the global matrix method, the integral transformation method, and the plane wave perturbation method. The interface shear stress reaches the peak at coupling resonance frequencies which are an inherent property of the structure, and decreases with the increase of coating thickness or the increase of shear wave velocity difference between the substrate and coating. At the coupling resonance frequency, the thin coating structure is more easily stratified at the interface. The result could provide a theoretical basis for the popularization and application of ultrasonic deicing/defrosting/de-accretion technology.

Keywords: coupled resonance mechanism; interface stratification; thin coating structures; horizontal shear waves; the forced propagation solution

1. Introduction

In industry, elastic waves sometimes are performed to disassemble coating structures at the interface [1–3]. Therefore, it is with far-reaching significance to investigate the mechanism of interface delamination of accretion structures.

The propagation of elastic waves is one of the main contents in the field of solid mechanics [4–7]. The particle displacement in the medium is controlled by wave equations which are usually partial differential equations formulated by variational principle, differential principle, higher-order shear deformation principle, and so on [8–11]. Analytical solutions could be obtained by solving partial differential equations. But it is usually very difficult to solve the partial differential equation, especially the inhomogeneous partial differential equation. Fortunately, some simplified wave equations by mechanical assumptions could be solved by separation of variables methods, the Green’s function method, integral transformation methods, displacement potential function methods, partial wave methods, variational iteration methods, homotopy perturbation methods, and so on [12–18].

If the displacement and the stress are continuous at interfaces, and the traction is zero on surfaces, the analytical solution of laminated structures with the boundary conditions could be solved by the transfer-matrix method, the stiffness-matrix method, the global-matrix method, and so on [19–24]. The free vibration solution and the dispersion equation could be obtained by solving the homogeneous wave equation, while the forced vibration solution could be obtained by solving the inhomogeneous wave equation with external forces [25–27]. So far, the displacement solution and the stress solution have received great attention, but the interfacial shear stress solution has received little attention. Due to
the lack of analytical solutions to interfacial shear stress, it is impossible to further study the problem.

The study of interface stratification excited by elastic waves begins with the observation of experimental phenomena [28,29]. For example, the ice layer attached to a metal plate is instantly completely stratified by ultrasonic transducers [30,31]. Then, it is proved by the finite element method that the interfacial shear stress directly leads to interfacial delamination [32–44]. Furthermore, it is found that the interfacial shear stress of double layer structures, sandwich structures, and thin internal layer structures excited by the horizontal shear wave has coupling resonance characteristics [45–47]. Coating structures have widely existed in the field of industrial technology [48]. However, the mechanism of interface stratification of thin coating structures excited by elastic waves is still not clear at present.

The solutions of wave equations often contain complex integrals which are solved directly explicitly. But the complex integrals could be solved by the numerical integration method such as the Simpson method, the trapezoidal method, the newton-cotes method, and so on, in which the Gauss–Kronrod quadrature numerical integration method in the Simpson method is particularly valid for integrating oscillating functions [49–52].

To solve the above problems, the forced vibration solution of the interfacial shear stress of thin coating structures excited by horizontal shear waves is derived. The mechanism of interfacial stratification is studied by analytical methods. The effects of structural inherent properties on interface stratification are analyzed and discussed.

2. The Forced Vibration Solution and the Necessary Condition of Interfacial Stratification of Thin Coating Structures Excited by Horizontal Shear Waves

The forced vibration solution of interfacial stratification of thin coating structures excited by horizontal shear waves is derived from plane wave perturbation methods, integral transformation methods, and global matrix methods. The necessary condition of interfacial stratification is analyzed and discussed by the interfacial shear fatigue failure theory.

2.1. The Forced Vibration Solution of Interface Shear Stress of Thin Coating Structures Excited by Horizontal Shear Waves

λ, μ, ρ and h are used to represent the First Lamé constant, the Second Lamé constant, the density, and thickness, respectively. In Cartesian coordinates, the Navier wave equation of particle displacement u of the elastic medium could be written as [8]

$$(\lambda + \mu)u_{ij,ij} + \mu u_{ij,jj} + F = \rho \frac{\partial^2 u_i}{\partial t^2}, \quad (i, j = 1, 2, 3)$$

(1)

Here, i and j represent coordinate axes, and F is the body force.

If $x_1$ and $x_3$ are used to represent the propagation direction and the polarization direction of horizontal shear waves, according to the plane strain hypothesis, $u_1 = 0$, $u_2 = 0$ and $u_3$ is independent of $x_3$,

$$u_3(x_1, x_2, t) = U_3(x_1, x_2)e^{-i\omega t}$$

(2)

The above displacement $u_1$, $u_2$, and $u_3$ are substituted in (1), which is simplified as

$$\mu \nabla^2 u_3 + F = \rho \frac{\partial^2 u_3}{\partial t^2}$$

(3)

Here,

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}$$

(4)

The thin coating structure consists of a thin coating and a semi-infinite matrix, which are rigidly connected together, as shown in Figure 1.
Mark (1) and (2) in the upper right corner represent coating and matrix, respectively. A horizontal shear plane wave perturbation is applied to the thin coating, which is written as

$$ F = f_0 \delta(x_1) e^{-i \omega t} \quad (5) $$

Here, $f_0$ is used to represent the excitation source amplitude,

$$ \delta(x_1) = \begin{cases} 1, & x_1 = 0 \\ 0, & x_1 \neq 0 \end{cases} \quad (6) $$

$$ \omega = 2 \pi f \quad (7) $$

Here, $f$ is used to represent the excitation frequency.

Consequently, for the structure illustrated in Figure 1, the particle displacements of horizontal shear waves in the thin coating and matrix satisfy the following formula respectively (For simplicity, the time-harmonic factor is omitted in the following derivation),

$$ \mu^{(1)} \nabla^2 u_3^{(1)} + F = -\rho^{(1)} \omega^2 u_3^{(1)} \quad (8) $$

$$ \mu^{(2)} \nabla^2 u_3^{(2)} = -\rho^{(2)} \omega^2 u_3^{(2)} \quad (9) $$

For thin coating structures, displacements and stress are continuous at the interface, the surface of the coating is free of tractions and the displacement component is zero as $x_2$ approaches infinity. The boundary conditions are written as

$$ \sigma_{23}^{(1)} (x_2 = -h) = 0 \quad (10) $$

$$ \sigma_{23}^{(1)} (x_2 = 0) = \sigma_{23}^{(2)} (x_2 = 0) \quad (11) $$

$$ u_3^{(1)} (x_2 = 0) = u_3^{(2)} (x_2 = 0) \quad (12) $$

$$ u_3^{(2)} (x_2 \to +\infty) = 0 \quad (13) $$

The relationship between stress and displacement is

$$ \sigma_{23}^{(n)} = \mu^{(n)} \frac{d u_3^{(n)}}{dx_2}, \quad n = 1, 2 \quad (14) $$

The problem is reduced to solving two second-order partial differential equations with four boundary conditions. There are many methods to solve the problem such as the separation of variables method, Green’s function method, integral transformation method, variation method, and so on. The integral transformation method is very effective by which partial differential equations could be eliminated conveniently.

If $\tilde{u}_3(s, x_2)$ is the integral transform of $u_3(x_1, x_2)$, they could be expressed as

$$ \tilde{u}_3(s, x_2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} u_3(x_1, x_2) e^{-isx_1} dx_1 \quad (15) $$
\[ u_3(x_1, x_2) = \int_{-\infty}^{\infty} \hat{u}_3(s, x_2) e^{isx_1} ds \] (16)

Taking Fourier transformation of wave Equations (8) and (9), boundary conditions (10)–(13), and the relation (14) on \( x_1 \) by (15), we obtain

\[ \frac{d^2 \hat{u}_3^{(1)}}{dx_2^2} + q^{(1)} \hat{u}_3^{(1)} = - \frac{f_0}{\mu} \] (17)

\[ \frac{d^2 \hat{u}_3^{(2)}}{dx_2^2} + q^{(2)} \hat{u}_3^{(2)} = 0 \] (18)

\[ \hat{u}_3^{(1)}(x_2 = -h) = 0 \] (19)

\[ \hat{u}_3^{(1)}(x_2 = 0) = \hat{u}_3^{(2)}(x_2 = 0) \] (20)

\[ \hat{u}_3^{(1)}(x_2 = 0) = \hat{u}_3^{(2)}(x_2 = 0) \] (21)

\[ \hat{u}_3^{(2)}(x_2 \to +\infty) = 0 \] (22)

\[ \hat{u}_3^{(n)} = \mu^{(n)} \frac{\partial \hat{u}_3^{(n)}}{\partial x_2}, \quad n = 1, 2 \] (23)

Here,

\[ q^{(n)2} = k^{(n)2} - s^2, \quad n = 1, 2 \] (24)

\[ k^{(n)} = \frac{\omega}{c^{(n)}}, \quad n = 1, 2 \] (25)

\[ c^{(n)} = \sqrt{\frac{\mu^{(n)}}{\rho^{(n)}}}, \quad n = 1, 2 \] (26)

Here, \( s \) is used to represent the integrating factor.

By the integral transformation method, the problem is further simplified to two ordinary differential equations with four boundary conditions.

Considering the propagation characteristics of elastic waves in the thin coating and matrix, the following conditions are assumed.

\[ q^{(1)2} > 0, \quad q^{(2)2} < 0 \] (27)

General solutions of the ordinary differential Equations (17) and (18) are assumed as follows.

\[ \hat{u}_3^{(1)} = A^{(1)} \cos(q^{(1)}x_2) + B^{(1)} \sin(q^{(1)}x_2) + C \] (28)

\[ \hat{u}_3^{(2)} = A^{(2)} e^{-q^{(2)x_2}} + B^{(2)} e^{q^{(2)x_2}} \] (29)

Here, \( A^{(1)}, A^{(2)}, B^{(1)}, \) and \( B^{(2)} \) are unsolved constants.

\[ C = - \frac{f_0}{\mu^{(1)}(k^{(1)2} - s^2)} \] (30)

By global matrix methods, the global matrix controlling the whole system could be obtained by substituting the general solution (28)–(29) into the boundary conditions (19)–(22).

\[
\begin{bmatrix}
-q^{(1)} \sin(-q^{(1)}h) & 0 & 0 & q^{(1)} \cos(-q^{(1)}h) & 0 \\
0 & \mu^{(2)} q^{(2)} & \mu^{(1)} q^{(1)} & 0 & -\mu^{(2)} q^{(2)} \\
-1 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
A^{(1)} \\
A^{(2)} \\
B^{(1)} \\
B^{(2)} \\
C
\end{bmatrix}
= \begin{bmatrix} 0 \\ 0 \\ C \\ 0 \end{bmatrix}
\] (31)
Finally, the problem is reduced to solving an inhomogeneous system of equations. By solving the system (31), $A^{(1)}, A^{(2)}, B^{(1)},$ and $B^{(2)}$ are obtained, which are functions of excitation amplitude, excitation frequency, thin coating thickness, and material parameters including the shear modulus and density of the thin coating and matrix.

According to hypothesis (27), the integrating factor $s$ must satisfy the following conditions.

$$-k_T^{(1)} < s < k_T^{(1)}, \quad s > k_T^{(2)}, \quad s < -k_T^{(2)}$$  \hfill (32)

Therefore, the condition for the existence of $s$ is obtained.

$$k_T^{(1)} > k_T^{(2)}$$  \hfill (33)

Further, it reduces to

$$c_T^{(1)} < c_T^{(2)}$$  \hfill (34)

The necessary condition for the existence of the integral factor $s$ is that the shear wave velocity of the thin coating is less than that of the matrix, which is consistent with that of the existence of the Love wave in classical literature. On the other hand, the conclusion also proves the correctness of the method.

According to (32) and (33), the range of the integrating factor $s$ is as follows.

$$-k_T^{(1)} < s < -k_T^{(2)}, \quad k_T^{(2)} < s < k_T^{(1)}$$  \hfill (35)

Taking the inverse Fourier transformation of the general solutions (28)–(29) on $x_1$ by (16), the forced vibration solution of displacement components of thin coating structures excited by the horizontal shear wave is obtained.

$$u_3^{(n)} = \int_{-k_T^{(2)}}^{-k_T^{(1)}} u_3^{(n)} e^{i k x_1} ds + \int_{k_T^{(2)}}^{k_T^{(1)}} u_3^{(n)} e^{i k x_1} ds, \quad n = 1, 2$$  \hfill (36)

Then, by the relation (14), the forced vibration solution of the interfacial shear stress of thin coating structures excited by horizontal shear waves is obtained finally, which is a complex integral determined by excitation amplitude, excitation frequency, thin coating thickness, material parameters including the density and the shear modulus of thin coating and matrix, and the phase factor $e^{i k x_1}$. The complex integral could be solved by numerical integration methods.

### 2.2. The Necessary Condition of Interface Delamination of the Thin Coating Structure Excited by Horizontal Shear Waves

The interface excited by the dynamic shear stress may be fatigue stratified. According to the maximum stress criterion of interfacial shear fatigue failure, the necessary condition for interfacial stratification excited by the time-harmonic symmetric alternating shear stress is that the stress amplitude is greater than the allowable fatigue shear stress $[\sigma_{-1}]$ which is a constant related to interfacial material parameters and environmental conditions [52].

$$\sigma_{23}^{(1)}(x_2 = 0) > [\sigma_{-1}]$$  \hfill (37)

From the analysis in Section 2.1, it is easy to know that the excitation amplitude $f_0$ is only the coefficient of the interfacial shear stress $\sigma_{23}^{(1)}(x_2 = 0)$. Therefore, both sides of (37) are divided by $f_0$ to normalize,

$$\tilde{\sigma}_{23}^{(1)}(x_2 = 0) > \tilde{[\sigma_{-1}]}$$  \hfill (38)

Here,

$$\tilde{\sigma}_{23}^{(1)}(x_2 = 0) = \frac{\sigma_{23}^{(1)}(x_2 = 0)}{f_0}$$  \hfill (39)
The analytical result and the finite element result of normalized interfacial shear stress amplitude is the largest at the first coupling resonance frequency 130 kHz. With \([\tilde{\sigma}_{-1}] = 2\), around the frequency of 130 kHz of coupling resonance, the interfacial shear delamination interval 110 kHz–190 kHz is formed.

As illustrated in Figure 3, with the increase of coating thickness, the normalized interfacial shear stress amplitude also fluctuates and eventually tends to a fixed value. There are many coupling resonance cladding thicknesses such as 8.5 mm, 24 mm, 39 mm, and so on, but the amplitude of normalized interfacial shear stress is the largest at the first coupling resonance coating thickness of 8.5 mm. With \([\tilde{\sigma}_{-1}] = 2\), around the coupling resonance coating thickness of 8.5 mm, the interval of 6.5 mm–12 mm of interfacial shear delamination is formed.

As illustrated in Figures 2 and 3, the analytical results are in good agreement with the finite element results. The coupling resonance characteristic is an inherent property of thin coating structures, which depends on structural parameters including the coating thickness and material parameters including the shear modulus and density of coating and substrate.

Finally, (38) is the necessary condition for interface stratification of thin coating structures excited by horizontal shear waves, which shows that the normalized amplitude of interface shear stress is greater than the normalized allowable fatigue shear stress.

3. Coupled Resonance Mechanism of Interfacial Delamination of Thin Coating Structures Excited by Horizontal Shear Waves

Taking the thin frost coating on an aluminum matrix as an example, the analytical solution of the normalized interfacial shear stress amplitude is calculated by the adaptive Gauss–Kronrod quadrature numerical integration method, which is verified by finite element solutions, see Figures 2–4. The interval of interfacial shear stratification is obtained. The material parameters are shown in Table 1.

As illustrated in Figure 2, with the increase in excitation frequency, the normalized interfacial shear stress amplitude fluctuates and eventually tends to a fixed value. There are many coupling resonance frequencies such as 130 kHz, 380 kHz, 625 kHz, and so on, but the normalized interface shear stress amplitude is the largest at the first coupling resonance frequency 130 kHz. With \([\tilde{\sigma}_{-1}] = 2\), around the frequency of 130 kHz of coupling resonance, the interfacial shear delamination interval 110 kHz–190 kHz is formed.

\[
[\tilde{\sigma}_{-1}] = \frac{[\sigma_{-1}]}{f_0}
\] (40)
Figure 3. The analytical result and the finite element result of normalized interfacial shear stress amplitude in the coating thickness domain when the excitation frequency is 80 kHz and the coating on the aluminum substrate is frost, and the interfacial shear delamination interval with $[\tilde{\sigma}_{-1}] = 2$.

Figure 4. The finite element model (a), the displacement component $u_y$ in the $x$-$z$ plane (b), the horizontal shear stress $\sigma_{zy}$ in the $x$-$z$ plane (c), and the horizontal shear stress $\sigma_{zy}$ at the interface (d) at 130 kHz as the frost coating thickness on the aluminum substrate is 5 mm.

Table 1. Material parameters.

| Material      | Young's Modulus (GPa) | Poisson's Ratio | Density (kg/m³) | Shear Modulus (GPa) | Shear Wave Velocity (m/s) |
|---------------|-----------------------|-----------------|-----------------|---------------------|---------------------------|
| Frost         | 7.759                 | 0.343           | 800             | 2.661               | 1900                      |
| Rime Ice      | 8.759                 | 0.337           | 900             | 3.276               | 1907                      |
| Glaze Ice     | 9.759                 | 0.331           | 950             | 3.666               | 1964                      |
| Aluminum      | 69.4                  | 0.337           | 2700            | 23.4                | 3140                      |
| Titanium      | 102                   | 0.35            | 4600            | 37.778              | 2866                      |
| Steel         | 205.0                 | 0.28            | 7850            | 80                  | 3194                      |
| Molybdenum    | 297.3                 | 0.297           | 10,200          | 114.3               | 3350                      |
As illustrated in Figure 4, the middle model is divided by regular tetrahedral meshes, which are ideally connected at the interface. The perfect matching layer (the blue area) around the model could absorb incident and reflected waves to simulate infinite boundary conditions. The displacement component $u_y$ is mainly distributed near the surface in the coating while the horizontal shear stress $\sigma_{xy}$ is mainly concentrated near the interface.

4. The Influence of Thin Coating Material on Coupling Resonance Characteristic and Interfacial Shear Delamination Interval

The amplitude of normalized interfacial shear stress is calculated in the excitation frequency domain and the coating thickness domain when the substrate is aluminum and the coating is glaze ice, rime ice, and frost in order, see Figure 5. The interval of interfacial shear stratification with $[\tilde{\sigma}_{-1}] = 1.55$ is determined, see Figure 6.

As illustrated in Figure 5, the coupling resonance frequency decreases with the increase of coating thickness. As the coating is glaze ice, rime ice, and frost in order, the coupling resonance frequency decreases, and the normalized interface shear stress amplitude at coupling resonance frequencies increases. The microstructure of the coating becomes more compact, the coating is more firmly bonded to the substrate, and the excitation of interfacial shear stratification becomes easier as the coating is glaze ice, rime ice, and frost in order. The smaller the shear wave velocity difference between the coating and the substrate, the higher the coupling resonance frequency of interfacial shear delamination, and the larger the excitation source strength required for interfacial shear delamination. The frost is more easily sheared and stratified from the aluminum matrix surface than glaze ice.

![Figure 5](image-url)

**Figure 5.** The normalized amplitude of interfacial shear stress in the excitation frequency domain and the coating thickness domain when the substrate is aluminum and the coating is glaze ice (a), rime ice (b) and frost (c).
As illustrated in Figure 6, in the interfacial shear delamination interval, the excitation frequency increases with the decrease of coating thickness. Interfacial shear delamination intervals widen when the coating is glaze ice, rime ice, and frost in order. The lower the frequency of the elastic wave, the easier it is to propagate through the thicker coating. As the coating is glaze ice, rime ice, and frost in order, with the coating thickness increasing from 2 mm to 20 mm, the first coupling resonance frequency decreases from 300 kHz to 50 kHz, 45 kHz, and 40 kHz, respectively.

Analyzing Figures 5 and 6, it is found that when the shear wave velocity of the coating increases, the coupling resonance frequency increases, the normalized interface shear stress amplitude at coupling resonance frequencies decreases, and interfacial shear delamination intervals narrow. With the increase of coating thickness, the decreasing rate of coupling resonance frequency decreases.

5. The Influence of Matrix Material on Coupling Resonance Characteristic and Interfacial Shear Delamination Interval

As shown in Figure 7, the coupling resonance frequency decreases with the increase of coating thickness. As the substrate is molybdenum, steel and titanium in order, the coupling resonance frequency increases, and the normalized amplitude of interface shear stress at
coupling resonance frequencies decreases. The excitation of interfacial shear stratification becomes more difficult as the substrate is molybdenum, steel, and titanium in order. The higher the shear wave velocity difference between the coating and the substrate, the lower the coupling resonance frequency of interfacial shear delamination, and the smaller the excitation source strength required for interfacial shear delamination. The coating on the surface of the molybdenum matrix is easier to be sheared and stratified than that on the surface of the titanium matrix.

As shown in Figure 8, in the interfacial shear delamination interval, the excitation frequency increases with the decrease of coating thickness. Interfacial shear delamination intervals become narrower and narrower when the substrate is molybdenum, steel, and titanium in order. The higher the frequency of the elastic wave, the easier it is to propagate through the thinner coating. As the substrate is molybdenum, steel, and titanium in order, with the coating thickness increasing from 2 mm to 20 mm, the first coupling resonance frequency decreases from 300 kHz to 40 kHz, 43 kHz, and 46 kHz, respectively.

Analyzing Figures 7 and 8, it is found that when the shear wave velocity of the substrate increases, the coupling resonance frequency decreases, the normalized amplitude of interface shear stress at coupling resonance frequencies increases, and interfacial shear delamination intervals become wider and wider. With the increase of coating thickness, the decreasing rate of coupling resonance frequency decreases.

![Figure 7](attachment:figure7.png)

Figure 7. The normalized amplitude of interfacial shear stress in the excitation frequency domain and the coating thickness domain as the coating is frost and the substrate is molybdenum (a), steel (b) and titanium (c).
Figure 8. Intervals of interfacial shear delamination in the excitation frequency domain and the coating thickness domain as $\tilde{\sigma}_{-1} = 1.8$, the coating is frost and the substrate is molybdenum (a), steel (b), and titanium (c).

6. Conclusions

The forced vibration solution of interface shear stress of thin coating structures excited by horizontal shear waves is derived. The interfacial stratification necessary condition excited by the alternating shear stress is analyzed and investigated. The normalized amplitude of interfacial shear stress in the excitation frequency domain and the coating thickness domain is calculated by the adaptive Gauss–Kronrod quadrature numerical integration method. The coupling resonance mechanism of interface stratification is revealed. The coupling resonance frequency increases as the coating is frost, rime ice, and glaze ice in order or the substrate is molybdenum, steel, and titanium in order, and decreases with the increase of coating thickness. Of all the material combinations, the thin frost layer on the molybdenum matrix is the most easily stratified.

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