An Improved Partheno-Genetic Algorithm for Open Path Multi-Depot Multiple Traveling Salesmen Problem

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Abstract. The Multiple traveling salesmen problem (MTSP) is a complex combinatorial optimization problem, which is an extension of the well-known traveling salesmen problem (TSP). Compared to TSP, MTSP is more suitable to model real-life problems. In this paper, an open path multi-depot multiple traveling salesmen problem (OPMDMTSP) is studied. For the problem studied, two different objectives are considered: minimizing the total cost of all sales staff and minimizing the longest travel length. For the OPMDMTSP, an improved partheno-genetic algorithm (IPGA) is proposed in this paper. In IPGA, a new selection operator that combining roulette selection and elitist selection is implemented. In addition, a more comprehensive mutation operation that introduces the propagation mechanism of invasive weed optimization algorithm is used. Extensive experiment that compares the proposed method with some state of the art methods shows that the IPGA is outperform other methods in terms of both solution quality and convergence ability.

1. Introduction

The traveling salesmen problem is a classical optimization problem. Some practical problems can be modeled as TSP, such as routing [1] and maintenance network [2]. However, with the rise of technology, many new problems emerge and the classic TSP is not suitable for solving these problems. Addressing such problem, the MTSP was raised. In MTSP, more than one salesman are present to travel the cities and each city must be visited exactly once by only one salesman. Depending on different constraints, such as number of depots or access time limit, there are many variants of MTSP. Thus, compared to TSP, many real world problems are better modeled with MTSP. In this sense, research on MTSP is necessary.

Many scholars have done related research on MTSP and many available methods were proposed. These methods are divided into two categories: exact solution algorithm and heuristic algorithm. The exact solution algorithms is to solve the MTSP mathematically. It use integer linear programming approaches with constraints. Nobert et al. [3] proposed the cutting plane algorithm to solve the MTSP without transformation. Gavish et al. [4] proposed a branch-and-bound algorithm to solve MTSP symmetric. Heuristic algorithm is a method of searching based on empirical rules. Chen et al [5] proposed a modified two-part wolf pack search algorithm to solve MTSP and proved its effectiveness. Venkatesh et al. [6] used a hyper-heuristic algorithm to present the k-IMDMTSP. Chao et al. [7] proposed a hybrid genetic algorithm to solve MTSP, which combining partheno-genetic algorithm (PGA)
and ant colony algorithm (ACO). Raulcézar et al [8] presented an improved GA to reduce both the total distance and the longest distance. Lo et al. [9] proposed a new local search operator for MTSP and proved the effectiveness of the improved GA.

Although these methods are feasible in some cases, there still have some limitations. The exact solution algorithm has rigorous mathematical foundation, but the scale of MTSP that can be solved by exact method is very limited. Heuristic algorithm can deal with large-scale MTSP, but it also have inevitable problems such as premature convergence, poor searching ability etc. In addition, the previous studies have mostly focused on classical close path single-depot MTSP and most of them only consider the objective of minimizing the total cost. However, in real life, many problems need to be solved by a variant of MTSP, and considering more than one objective. In this paper, the OPMDMTSP that consider two objectives, minimizing the sum of total distance (minsum) and minimizing the maximum distance (minmax), is studied. The minmax objective are often used in scheduling, task allocation and problems that consider workload balancing etc. On the other hand, an improved partheno genetic algorithms is proposed to solve the OPMDMTSP. A hybrid selection operator and a more comprehensive mutation operator are proposed in the IPGA.

The structure of this paper is organized as follows. In section 2, the definition of OPMDMTSP is introduced. In section 3, the improved partheno-genetic algorithm is introduced. In section 4, the experiment and results are provided. In section 5, the conclusion of this paper is given.

2. The formal description of the problem
The OPMDMTSP considered in this paper has fixed multiple depots. In the case of OPMAMTSP, there are \( m \) salesmen in fixed \( m \) depots respectively to visit \( n \) cities. In this problem, every cities should be visited only once. The objectives considered have two criteria, minsum and minmax. The minsum criteria is to reduce the total cost of the solution, and the minmax is to ensure the workload balance of the salesmen.

The problem described can be modeled as follows: Consider an undirected graph \( G=(V,A) \), where \( V \) is a set of \( m+n \) nodes. The first \( m \) nodes of \( V \) are respectively fixed depots of \( m \) salesmen and the follows \( n \) nodes are city nodes. \( A \) is the set of arcs that join any pair of nodes in \( V \). Let \( C=(c_{ij}) \) be a distance matrix that associated with \( A \). In this paper, the considered MTSP is symmetric, so \( C \) is also a symmetric matrix. To concisely describe the problem, the following variables are defined:

\[
x_{ij}^k = \begin{cases} 1, & \text{the } k\text{th salesman from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}
\]

\[
y_{i}^k = \begin{cases} 1, & \text{the } k\text{th salesman visit node } i \\ 0, & \text{otherwise} \end{cases}
\]

(1)

(2)

Through the above variables, the route cost of kth salesman \( z_k \) can calculated to be:

\[ z_k = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}^k, \quad k=1,2,...,m \]

(3)

The total cost of all salesmen \( \text{obj1} \) and longest tour distance \( \text{obj2} \) were calculated to be:

\[ \text{obj1} = \sum_{k=1}^{m} z_k \]

(4)

\[ \text{obj2} = \max(z_1,z_2,...,z_m) \]

(5)

In addition, some standard MTSP constraints are the same as that described in [10].
3. Improved partheno-genetic algorithms for OPMDMTSP

3.1. Chromosome representation schemes

For apply GA to an optimization problem, a proper chromosome representation is indispensable. The two-part chromosome technique, a kind of sequence coding method, is adopted for OPMDMTSP. In detail, the first part of the chromosome is a sequence of n numbers corresponding to each city node, which represents the order of cities on the path. The second part is the number of cities each salesman visit. In OPMDMTSP, the depot for each salesman is fixed, so it is not considered in the coding method. For instance, assume two salesmen visit nine cities. The possible assignment are as follows:

- The order of cities visited by salesman 1: City 7, City 4, City 5, City 1, City 9.
- The order of cities visited by salesman 1: City 2, City 8, City 6, City 3.

This assignment can be represented by the chromosome in figure 1. The city sequence is sorted and divided into two parts by this coding method, corresponding to the city sequence of the two salesman.

![Figure 1. Two-part chromosome representation.](image)

3.2. Fitness function

In genetic algorithm, fitness function is used to represent the adaptability of individual. In this paper, two different objectives are considered for the OPMDMTSP. Thus, the fitness function are designed as equation (6), where $a$ and $b$ correspond to the weighted coefficient for the objectives.

$$ f = \frac{1}{a \cdot obj1 + b \cdot obj2} $$

(6)

3.3. Hybrid selection operator

The selection operation is to select individuals with better adaptability from the population. In previous studies, roulette selection and elitist selection are the most commonly used operator. However, they all have certain drawbacks. The roulette selection is a probabilistic method. For each individual in a population, the greater the fitness, the greater the probability of being selected. But there is randomness in this method, which may cause highly adaptive individuals to be eliminated. The elitist selection is to rank the individuals in the population according to fitness, and then directly select the individuals with top $N$ fitness. However, the method of directly eliminating poor fitness individuals will reduce the diversity of the population and even lead to a locally optimal solution.

![Figure 2. The hybrid selection operator.](image)
Therefore, a hybrid selection operator that combining the above two operator is proposed in this paper. As shown in figure 2, the main ideas of the hybrid selection operator are as follows: First, all individuals in the population are sorted according to their fitness. Then, select the first $N \cdot \xi$ individuals according to the elite selection method and keep them directly to the next generation. Finally, for the remaining individuals in the population, the roulette wheel is used to select $N \cdot (1-\xi)$ individuals. Through this hybrid selection operator, the global convergence property of elitist selection is utilized and the diversity of population is also guaranteed.

3.4. Mutation operator based on reproduction mechanism

The traditional PGA is based on a random mutation parameter. But this random mutation has some drawbacks. Firstly, the determination of mutation parameter threshold is also an optimization problem. Secondly, some individuals containing useful information not mutate, which leads to the inadequacy of local search ability of the algorithm. In addition, many unnecessary mutation operations occur in those individuals with low fitness. Therefore, a mutation operator based on individual fitness is proposed in this paper. This operator incorporates the seed propagation mechanism of IWO algorithm [11]. For individuals with high fitness, more mutations occur to improve the local search ability of the algorithm, and for those with low fitness, a small amount of mutation occur to ensure population diversity.

According to IWO algorithm, the number of offspring produced by each individual are ranged linearly from a minimum $I_{\min}$ to a maximum $I_{\max}$. In this paper, nine different mutation operations are designed for each individual to produce new offspring. Thus, $I_{\max}$ is set to nine. Considered the two-part chromosome representation, the nine mutation operations are designed as follow:

1. Perform the FlipInsert to the first part of the chromosome of the individual.
2. Perform the SwapInsert to the first part of the chromosome of the individual.
3. Perform the LSlideInsert to the first part of the chromosome of the individual.
4. Perform the RSlideInsert to the first part of the chromosome of the individual.
5. Regenerate the second part of the chromosome of the individual.
6. Perform the FlipInsert to the first part of the chromosome and regenerate the second part.
7. Perform the SwapInsert to the first part of the chromosome and regenerate the second part.
8. Perform the LSlideInsert to the first part of the chromosome and regenerate the second part.
9. Perform the RSlideInsert to the first part of the chromosome and regenerate the second part.

Where the FlipInsert, SwapInsert, LSlideInsert and RSlideInsert are four mutation operations proposed in [12]. In addition, to guarantee the diversity of population, $I_{\min}$ is set to one, which ensures that even individuals with low fitness can produce offspring. The concrete steps of the proposed mutation operator are as follows:

**Step 1:** According to the fitness of the individuals, sort the population from large to small.

**Step 2:** Divide the sorted population into nine groups evenly with group index from 1 to 9.

**Step 3:** Decide the mutation operations to each individual according to their group index. Actually, the operations that need to executed on each individual is $\{(9),...,(i\xi)\}$, where $i\xi$ is the group index of the individual. For instance, an individual with a group index of 7 needs to perform the following operations: $\{(9),(8),(7)\}$, which means that it will produce three offspring.

**Step 4:** Perform Step3 until execute mutation to all individual.

Through the above mutation operator, for high fitness individuals, more mutation offspring are produced to improve the local search ability of the algorithm. For low fitness individuals, there still be a small amount of mutation offspring to preserve the information that may be useful in their genes.

4. Experiments and result analysis

The effectiveness of proposed method in OPM:MTSP is evaluated by a series of comparative experiments with IWO [13], MTWPS [5], GA2C (GA with two-part chromosome) and PGA. The algorithms is encoded and implemented in MATLAB R2018b. The experiment platform is a computer with i5-4460 CPU@3.20 GHz and 8 GB RAM under MS Windows 10.
4.1. Convergence ability
In this part, the convergence ability of the algorithm is tested using eil51 benchmark instance of TSPLIB. In the test, the number of salesman is three, and the maximum iterations is 5000. Each algorithm was tested 30 times under this configuration. The average optimal solution curve of the five algorithms is recorded in figure 3. The vertical axis is the inverse of the fitness of the solution.

As shown in figure 3, PGA and the proposed IPGA converge fastest. Then IWO and MTWPS enter the convergence state respectively. GA2C has the worst convergence ability, converging at the end of 5000 generations. It can be seen from the final convergence result of the curve that the proposed IPGA obtain the best solution in five algorithms. The optimal solution obtained by MTWPS is slightly better than that of IWO. Although PGA has similar convergence ability as our algorithm, the solution obtained by it is not as good as IPGA. Finally, GA2C not only converges slowly, but also results in poor fitness of the optimal solution. Through the above analysis, it can be concluded that the proposed algorithm not only has strong convergence ability, but also has better fitness of the obtained solutions.

Figure 3. Average optimal solution curve of five methods.

4.2. Solution quality
Although the test in the previous part has shown that the IPGA leads to better solutions, the objectives considered in OPMDMTSP is to minimize the total cost and the longest tour length. Thus, an experiment considered the both objectives of the solution are carried out in this part. The eil51, eil76 and eil101 of TSPLIB are used in this part. For each algorithm, ten times test are executed on each benchmark instance. The results, the best solution obtained by each algorithms in terms of the two objectives considered, are recorded in table 1. As can be seen from the table, for all benchmark, the best solutions obtained by IPGA are superior to that of other algorithms. This proves that the quality of solution obtained by IPGA is excellent in terms of the both objectives.

Table 1. Best solution obtained by various method.

| Objective | Instance | IPGA | IWO | MTWPS | GA2C | PGA |
|-----------|----------|------|------|--------|------|-----|
| total sum | eil51    | 409  | 624  | 539    | 751  | 726 |
|           | eil76    | 526  | 707  | 757    | 1013 | 887 |
|           | eil101   | 597  | 903  | 837    | 1417 | 1013|
| longest tour | eil51    | 159  | 261  | 209    | 297  | 257 |
|           | eil76    | 201  | 307  | 351    | 407  | 375 |
|           | eil101   | 231  | 351  | 297    | 601  | 507 |
5. Conclusions
In this paper, the OPMDMTSP that considered two different objectives was defined. An improved partheno-genetic algorithm was proposed to solve the OPMDMTSP. In the proposed algorithm, an operator combining roulette selection and elite selection was used. Furthermore, considering the existing problems of PGA, a more comprehensive mutation operator based on the propagation mechanism of invasive weed optimization algorithm is used. The proposed method is evaluated through a series of comparative experiments with four state-of-the-art approaches. The experiment results show that the proposed algorithm is superior to other state-of-the-art algorithms in convergence ability and solution quality.

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