On differential spectra in the reaction $pp \rightarrow K^+\bar{K}^0d$
in the nearthreshold region

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1. The reactions with production of the lightest scalar mesons are presently the subject
of experimental study by the ANKE collaboration at COSY machine at Jülich [1, 2].
Recently the preliminary results on the reaction

$$pp \rightarrow K^+\bar{K}^0d$$

(1)
at the proton-beam energy $T_p = 2.645$ GeV, obtained in the current experiment at ANKE
spectrometer [3, 4], have been reported. The experiment is going on and has the aim to
observe the scalar $a_0^+(980)$ meson ($I^GJ^{PC} = 1^{-0++}$) in the decay channel $K\bar{K}$
to and study its properties. Note that the allowed $K\bar{K}$-mass interval in the reaction (1) at a given
incident energy is rather narrow ($991.4 < M_{K\bar{K}} < 1037.3$ MeV), i.e. approximately two
times smaller than the expected width of $a_0$ meson ($\Gamma_a \sim 80$ MeV). Thus, the phase-volume
limitations should already reproduce a resonance-like bump in the $K\bar{K}$-mass spectrum. In
this situation, the observation of $a_0$ meson in the reaction (1) appears to be not an easy
problem.

The subject of this article is to discuss the possible distributions for different observ-
ables one may expect from the reaction (1) at the incident energy $T_p = 2.645$ GeV. The
modern effective meson-nucleon theories of strong interactions are not able to predict cross
sections for production of heavy mesons ($m_a \sim 1$ GeV) with high accuracy. There exist
theoretical predictions for production rate of $a_0$ meson in the reaction $NN \rightarrow a_0d$, based on
a meson exchange model [5]. There are also estimations, according to which nonresonance
background in the reaction (1) is expected to be strongly suppressed in comparison with
$a_0$ contribution (see, Ref. [1]). However, in the present article we consider the problem in
the model-independent way, making use of the conservation laws for quantum numbers in
the reaction (1).

Below we shall discuss the $K^+\bar{K}^0$- and $\bar{K}^0d$-mass spectra for this reaction with unpolar-
ized particles at different hypotheses for the production amplitude. From the experimental
point of view the $K^+\bar{K}^0$-mass spectrum should be the most sensitive to the possible con-
tribution of $a_0^+$ resonance in this channel. On the other hand the $\bar{K}^0d$-mass spectrum may
be essentially influenced by strong final-state $\bar{K}^0d$ interaction (this question in details was
discussed in Ref. [4]). Some comments on the angular distributions will be given.

2. Note that at the incident kinetic energy $T_p = 2.645$ GeV the the reaction (1) is
rather close to threshold regime ($Q = \sqrt{s} - m_d - m_{K^+} - m_{\bar{K}^0} \approx 45.9$ MeV, where $\sqrt{s}$
is total energy in CMS). Thus, one may expect that contributions of lower partial waves
should dominate. Let us introduce the following notations:
Differential cross section may be written as

\[ K \bar{\phi}_1 \]

Here:

- \( K \) - final kaons;
- \( \bar{\phi}_1 \) - polarizaton vector of the deuteron in CMS;
- \( q, L_q \) - relative 3-momentum of kaons and orbital angular momentum, respectively, in the final \( K \bar{K} \) system;
- \( k_1 \) - relative momenta in the final \( \bar{K}d \) system;
- \( k_2 \) - relative momenta of \( K \) meson with respect to the final \( \bar{K}d \) system in CMS.

In what follows, the final particles in the CMS of the reaction (1) are considered to be nonrelativistic and the corresponding momenta are given by the expressions:

\[
q = \sqrt{\frac{2m_K m_{\bar{K}}}{m_K + m_{\bar{K}}}} (m_{\bar{K}} - m_K - m_{\bar{K}}), \quad k = \sqrt{\frac{2(m_K + m_{\bar{K}}) m_d}{m_K + m_{\bar{K}} + m_d}} (\sqrt{s} - m_d - m_{\bar{K}}),
\]

\[
k_1 = \sqrt{\frac{2m_d m_K}{m_d + m_K}} (m_{\bar{K}} - m_d - m_{\bar{K}}), \quad k_2 = \sqrt{\frac{2(m_d + m_{\bar{K}}) m_K}{m_K + m_{\bar{K}} + m_d}} (\sqrt{s} - m_K - m_{\bar{K}}).
\]

Differential cross section may be written as

\[
d\sigma = N |M|^2 q k q m_{K\bar{K}} d\Omega_k d\Omega_q \quad (N = (4\pi)^{-5} p^{-1} s^{-1}).
\]

Here: \( M \) is matrix element; \( \Omega_k \) and \( \Omega_q \) are solid angles for the directions of the momenta \( k \) and \( q \), respectively.

In the simplest approximation, in which the production amplitude \( M \) is constant, the mass spectra are given only by phase-space limitations for the three final particles, i.e.

\[
\frac{d\sigma}{dm_{K\bar{K}}} = N_0 k q, \quad \frac{d\sigma}{dm_{\bar{K}d}} = N_0 k_1 k_2,
\]

where \( N_0 = (4\pi)^2 N |M|^2 = const \). The distributions (4) are shown in Fig.1a and Fig.1b (dotted curves) and are symmetric.

However, the approximation \( M = const \) corresponds to the forbidden case \( L_k = L_q = 0 \). Note that since the final \( K^+ \bar{K}^0 d \) system in the reaction (1) has isospin 1 the case \( L_k = L_q = 0 \) is forbidden due to parity, angular momentum and isospin conservation laws and Pauli principle. Thus, the possible lowest-partial-wave contributions correspond to the cases: \( L_k = 1, L_q = 0 \) and/or \( L_k = 0, L_q = 1 \). In both these cases the initial \( NN \) system has the total spin \( S = 1 \) and the orbital angular momentum \( L_p = 1 \) or \( L_p = 3 \) (see also Refs. 5, 6).

3. The case \( L_k = 1, L_q = 0 \). Nonresonance production of \( K\bar{K} \)-system. In this case the reaction amplitude should be linear in \( k \) and contain odd powers (\( \leq 3 \)) of the initial relative momentum \( p \). This amplitude may be written in the general form

\[
M = a (p \cdot S) (k \cdot e^*) + b (p \cdot k) (S \cdot e^*) + c (k \cdot S) (p \cdot e^*) + d (p \cdot S) (p \cdot e^*) (k \cdot p).
\]

Hereafter: \( e \) is the polarization vector of the deuteron and \( S = \phi_1^T \sigma_2 \phi_2 \), where \( \phi_1 \) and \( \phi_2 \) are the spinors of the initial nucleons. The coefficients \( a, b, c \) and \( d \) are independent
complex scalar amplitudes. In terms of amplitudes $a$, $b$, $c$, i.e. omitting amplitude $d$, the expression (3) was also discussed in Ref. [2]. The matrix element (4), when squared and averaged (summed) over the polarizations of the initial nucleons (final deuteron), gives

$$\frac{d^2\sigma}{dm_{KK}d\Omega_k} = 4\pi N (A + B\cos^2\theta) k^3 q,$$

(6)

where $\theta$ is the CM polar angle of outgoing $K\bar{K}$ system with respect to the proton beam in the reaction (1) and

$$A = \frac{1}{2}(|a|^2 + |c|^2) p^2, \quad B = \left[|b|^2 + \frac{1}{2}|b + p^2d|^2 + \text{Re} \left(a^*c + (a + c)^*(b + p^2d)\right)\right] p^2.$$

(7)

The values $A$ and $B$ in Eq. (6) are known if any concrete model is used. If $a = c = 0$ in Eq. (3) then $A = 0$ and $d^2\sigma/dm_{KK}d\Omega_k \sim \cos^2\theta$. On the other hand, $d^2\sigma/dm_{KK}d\Omega_k$ should be flat with respect to $\cos\theta$ if $B = 0$. The latter is valid if $a = b = d = 0$ or $c = b = d = 0$ in Eq. (3). The amplitude (3) is always necessarily leads to flat distribution on $\Omega_q$ (angular distribution of outgoing kaon in the rest frame of the $K^+\bar{K}^0$-system. For the $K^+\bar{K}^0$- and $K^0d$-mass distributions one gets

$$\frac{d\sigma}{dm_{KK}} = N_1 k^3 q, \quad \frac{d\sigma}{dm_{Kd}} = N_1 \left[k_1^2 + \left(\frac{m_d}{m_{K^0} + m_d}\right)^2 k_2^2\right] k_1 k_2,$$

(8)

where $N_1 = (4\pi)^2 N (A + B/3) = \text{const}$. The distributions (8) are shown in Fig.1a and Fig.1b by solid curves 1.

4. The case $L_k = 1$, $L_q = 0$. Resonance production of $K\bar{K}$-system. Consider now the case of a pure resonance production of $K\bar{K}$ system through the chain

$$pp \rightarrow a_0^+ d \rightarrow K^+\bar{K}^0 d.$$

(9)

In this case the expression (3) should be considered as the amplitude of $a_0^+$-meson production in the reaction $pp \rightarrow a_0^+ d$. To obtain the amplitude of the reaction (3) we should multiply the expression (4) by the $a_0$-meson propagator and by the constant $g_{aKK}$ of the decay $a_0^+ \rightarrow K^+\bar{K}^0$. Note that angular distributions for resonance production mechanism are identical to those for nonresonance case, discussed above in Section 3. For the mass spectra we have

$$\frac{d\sigma}{dm_{KK}} = N_2 |\Pi_a(m_{KK})|^2 k^3 q, \quad \frac{d\sigma}{dm_{Kd}} = \frac{1}{2} N_2 k_1 k_2 \int k^2 |\Pi_a(m_{KK})|^2 dz'.$$

(10)

Here: $N_2 = (4\pi)^2 N (A + B/3)(g_{aKK}/2m_a)^2 = \text{const}; \; \Pi_a(m) = (m - m_a + i\Gamma(m)/2)^{-1}$ is the nonrelativistic $a_0$ propagator; $m_a$ is the nominal mass of $a_0^+$ meson; $z' = \theta'$ and $\theta'$ is

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1 The maximal value $L_p = 3$ for orbital momentum of the initial nucleons is taken into account in Eq. (1). All the amplitudes $a$, $b$, $c$ and $d$ may be expressed through vertex constants of some effective Lagrangian for the reaction (1). This amplitudes may also depend on the total energy $\sqrt{s}$. One may consider them to be a constants if $\sqrt{s}$ is fixed.
the angle between the directions of outgoing $\bar{K}^0$ and $K^+$ mesons in the rest frame of $\bar{K}^0d$ system. For the width $\Gamma(m)$ in $\Pi_a(m_{\bar{K}K})$ we use the analytic expression (Flatte [4]):

$$\Gamma(m) = g_1 q_1 + g q,$$

(11)

where $q_1$ is relative momentum in the $a_0 \rightarrow \pi + \eta$ decay channel, taken at $m = m_a$, and $q$ is the relative momentum in the $K\bar{K}$ system (see Eqs. (4)). We use the parameters $m_a = 998$ MeV, $g_1 = 0.243$ and $g = 0.221$ for $a_0^+$ meson from Ref [8].

To calculate the integral in the expression for $d\sigma/dm_{\bar{K}d}$ (10) one should express the values $k^2$ and $m_{\bar{K}K}$ in terms of variable $z'$. The effective mass $m_{\bar{K}K}$ can be expressed through the value $q^2$ according to Eqs. (2). The expressions for $k^2$ and $q^2$ in terms of the variable $z'$ are the following:

$$k^2 = k_1^2 + \beta^2 k_2^2 + 2\beta k_1 k_2 z', \quad q^2 = \alpha_1^2 k_1^2 + \beta_1^2 k_2^2 - 2\alpha_1\beta_1 k_1 k_2 z',$$

(12)

$$\beta = \frac{m_d}{m_{\bar{K}} + m_d}, \quad \alpha_1 = \frac{m_K}{m_{\bar{K}} + m_K}, \quad \beta_1 = \frac{m_K (m_{\bar{K}} + m_K + m_d)}{(m_{\bar{K}} + m_K) (m_{\bar{K}} + m_d)}.$$

The distributions (10) are shown in Fig.1a and Fig.1b by dashed curves.

5. The case $L_k = 0$, $L_q = 1$. Nonresonance production of p-wave $K\bar{K}$-system in s wave with respect to deuteron. The amplitude of the reaction (1) in this case may be written in the form (4), where $k$ is substituted by $q$, i.e.

$$M = a (p \cdot S) (q \cdot e^*) + b (p \cdot q) (S \cdot e^*) + c (q \cdot S) (p \cdot e^*) + d (p \cdot S) (p \cdot e^*) (q \cdot p).$$

(13)

The values $a$, $b$, $c$ and $d$ are also taken to be constants. Using this amplitude, one gets

$$\frac{d^2\sigma}{dm_{\bar{K}K} d\Omega_q} = 4\pi N (A + B \cos^2 \theta_1) k q^3,$$

(14)

Here: $A$ and $B$ are the constants, given in Eqs. (7); $\theta_1$ is the angle of outgoing $\bar{K}^0$ meson with respect to the proton beam in the rest frame of $K^0d$ system. If $a = c = 0$ in Eq. (13) then $A = 0$ and $d^2\sigma/dm_{\bar{K}d} d\Omega_q \sim \cos^2 \theta_1$. On the other hand, if $a = b = d = 0$ or $c = b = d = 0$ then $B = 0$ and the distribution (14) should be flat with respect to $\cos \theta_1$. In any case the amplitude (13) leads to flat angular distribution on $\Omega_k$.

The $K^+\bar{K}^0$- and $\bar{K}^0d$-mass distributions are the following:

$$\frac{d\sigma}{dm_{\bar{K}K}} = N_1 k q^3, \quad \frac{d\sigma}{dm_{\bar{K}d}} = N_1 (\alpha_1^2 k_1^2 + \beta_1^2 k_2^2) k_1 k_2,$$

(15)

where $N_1 = (4\pi)^2 N (A + B/3) = const$. The values $\alpha_1$ and $\beta_1$ are given in Eqs. (12). These distributions are shown in Fig.1a and Fig.1b by solid curves 2.

6. Let us here discuss the results. Looking at $K^+\bar{K}^0$-mass distributions in Fig. 1a, one can see that this mass spectrum is very sensitive to the choice of the partial-wave amplitude of the reaction. The cases $L_k = 1$, $L_q = 0$ (solid curve 1 and dashed curve) and
\( L_k = 0, \ L_q = 1 \) (solid curve 2) correspond to strongly different \( K^+\bar{K}^0 \)-mass distributions. Generally the experimental data may correspond to some intermediate case as well as to confirm one of these two limiting cases. The preliminary results \[4\] seem to confirm the variant with \( L_k = 1, \ L_q = 0 \). This variant looks like more as the argument in favour of \( a_0^+ \)-resonance hypothesis than against it. However, since the data \[4\] are preliminary and the experiment is going on, the situation may also change.

Comparing \( K^+\bar{K}^0 \)-mass spectra for resonance (solid curve 1) and nonresonance (dashed curve) hypotheses, one can see that these distributions are not drastically different. That is why to separate \( a_0^+ \)-resonance and nonresonance mechanisms of the reaction (1) at low incident energy \( T_p = 2645 \text{ MeV} \) seems to be not an easy problem.

The \( K^0d \)-mass spectra in Fig. 1b are less sensitive to the choice of the production amplitude. The spectra shown by solid curves 1 and 2 are rather different, but in the case of \( L_k = 1, \ L_q = 0 \) the \( a_0^+ \)-resonance and nonresonance hypotheses correspond to approximately the same distributions (dashed curve and solid curve 1). The \( K^0d \)-mass spectrum is more interesting in connection with effects of final state interaction (FSI) of \( K^0 \) mesons with deuteron. This question was thoroughly studied in Ref. \[7\], and it was found that the calculated mass spectra were essentially influenced by strong \( K^0d \) FSI, when the latter was taken into account. Here the following remark is also possible. Suppose the pure \( a_0^+ \)-production mechanism in the reaction (1). The FSI process \( a_0d \rightarrow K\bar{K}d \) leads to the contribution, which looks like a background for \( a_0 \). This subprocess may also essentially contribute to the partial waves with \( L_k = 0 \) and \( L_q = 1 \) in the total amplitude of the reaction (1) and modify \( K\bar{K} \)-mass spectrum. The observation of the FSI-induced desintegration of \( a_0^+ \) meson should be a serious argument in favour of the molecular hypothesis \[10\] of \( a_0 \) meson.

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Figure 1: Mass spectra of $K^+\bar{K}^0$ (a) and $\bar{K}^0d$ (b) systems in the reaction (1) at $T_p = 2645$ MeV. All distributions are normalized to 1 at the maximal values. Solid curves 1: nonresonance production with $L_k = 1$, $L_q = 0$. Solid curves 2: nonresonance production with $L_k = 0$, $L_q = 1$. Dashed curves: pure $a_0$-meson production ($L_k = 1$, $L_q = 0$). Dotted curves: pure phase-spase distributions (this case corresponds to $L_k = L_q = 0$ and is forbidden).