Density dependent relativistic hadron theory for hadron matter with inclusion of pentaquark $\Theta^+$

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Abstract

The density dependent relativistic hadron field theory (DDRH) is extended to inclusions of pentaquark $\Theta^+$. We investigate in-medium properties of $\Theta^+$ and nucleons. As well as the neutron and proton, the effective $\Theta^+$ mass decreases as the baryon density increases, and remains larger than nucleons'. The effective mass of $\Theta^+$ is calculated, $M_{\Theta^+}^* \simeq 0.71 M_{\Theta^+}$ at normal nuclear density with the fraction of $\Theta^+ 0.2$. The effective masses of all baryons increase with the increasing fraction of $\Theta^+$. The binding energy is also studied. We find the binding energy is much larger than it in nuclear matter without $\Theta^+$, and then discuss the stability of the system with conclusions of pentaquark $\Theta^+$. It is shown that the system becomes bounder added small fraction of $\Theta^+$ finally we discuss the condition when the system is the most stable and calculate the minimum of binding energy and the fractions of all baryons as functions of baryon density.

PACS: 14.20.-c; 24.10.Nz

Key words: $\Theta^+$, density dependent relativistic hadron, binding energy, effective mass

1 Introduction

The possibility of the existence of pentaquark $\Theta^+(uudd\bar{s})$ has been theoretically discussed for many years. In 1997, Diakonov, Petrov, and Polyakov, in the framework of the chiral soliton model, predicted a exotic baryon $\Theta^+$ with mass $M \sim 1.53 GeV, S = +1, spin \ 1/2$ and isospin 0$\Pi$. However, the subject has been thrust to the forefront during the past two years by the experimental discovery of an exotic baryon$[2\ 3\ 4\ 5\ 8\ 9\ 10\ 11]$. It has $K^+ n$ quantum numbers($B = +1$, $Q = +1$, $S = +1$), and its minimal quark content should be $uudd\bar{s}$. The remarkable features of

*Supported by Chinese Academy of Sciences Knowledge Innovation Project (KJCX2-SW-No2), National Natural Science Foundation of China (10435080)
the \( \Theta^+ \) are its small mass (1540MeV) and very narrow width (<25MeV)[3]. The discovery of the exotic baryon \( \Theta^+ \) with positive strangeness opens new possibilities of forming exotic \( \Theta^+ \) hypernuclei as same as in the case of negative strangeness \( \Lambda, \Sigma, \Xi \) hypernuclei. D. Cabrera et al have suggested that \( \Theta^+ \) could be bound in nuclei and calculated self energy of the \( \Theta^+ \) pentaquark in nuclei[12],they shown that the in-medium renormalization of the pion in the two meson cloud of the \( \Theta^+ \) leads to a sizable attraction, enough to produce a large number of bound and narrow \( \Theta^+ \) states in nuclei.

On other hand, the relativistic mean field approximation have been a widely used and successful approach to describe properties of nuclear matter and finite nuclei[13].the Density Dependent Relativistic Hadron (DDRH) theory was introduced previously as an effective field theory for isospin nuclei[14, 15].In DDRH theory the meson-baryon interactions is described by meson-nucleon vertices which are functionals of the fermion field operators. The coupling coefficients are not constants but field operators. DDRH theory reduces to a Hartree description with density dependent coupling constants similar to the initial proposal of Brockmann and Toki in mean-field approximation and its development has been achieved a significant progress in recent years[16]. For including the scalar isovector \( \delta \) meson, DDRH is successfully applied to asymmetric matter at extreme neutron-to-proton ratios.[17].

In the present paper we attempt to extend DDRH theory to inclusions of pentaquark \( \Theta^+ \) and investigate properties of matter with inclusions of \( \Theta^+ \). We study medium modifications within the mean field theory and get the mass modifications of baryons in-medium. We calculate the binding energy at different baryon fractions. We find the system becomes more bound with inclusions of \( \Theta^+ \) but when too much \( \Theta^+ \) added it becomes less bound. The critical fraction of \( \Theta^+ \) is 0.56 in isospin symmetric matter and it becomes lower in isospin symmetric matter. The isospin effects on the effective mass and the binding energy are also discussed. Finally, we calculate the minimum of binding energy and the fractions of all baryons as functions of baryon density when the system is the most stable.

2 The density dependent relativistic hadron field

The density dependent relativistic hadron field theory has been proven to give a good description of nuclear matter in bulk and of the properties of finite nuclei[17]. Assuming the interaction between nucleons and mesons is similar to \( \Theta^+ \) and meson, we start from the effective lagrangian density which includes pentaquark \( \Theta^+ \)

\[
\mathcal{L} = \mathcal{L}_B + \mathcal{L}_M + \mathcal{L}_{int},
\]

\[
\mathcal{L}_B = \sum_{i=N, \Theta^+} \bar{\Psi}_i (i\gamma_\mu \partial^\mu + M_i) \Psi_i,
\]

\[
\mathcal{L}_M = \frac{1}{2} \sum_{i=\sigma, \delta} (\partial_\mu \Phi_i \partial^\mu \Phi_i - m_i^2 \Phi_i^2) - \frac{1}{2} \sum_{j=\omega, \rho} (F^{(j)}_{\mu\nu} F^{(j)\mu\nu} - m_j^2 A^{(j)}_{\mu} A^{(j)\mu})
\]
\[ \mathcal{L}_{int} = \sum_{i=N,\Theta^+} (\bar{\Psi}_i \Gamma_i^\dagger(\rho) \Psi_i \Phi_\sigma - \bar{\Psi}_i \Gamma_{\omega_\tau}(\rho) \gamma_\mu \Psi_i A^{(\omega)\mu} + \bar{\Psi}_N \Gamma^N_\delta(\rho) \tau_\alpha \Psi_N \Phi_\delta^a - \bar{\Psi}_N \Gamma^N_\rho(\rho) \gamma_\mu \tau_\alpha \Psi_N A^{(\rho)\mu}) \]

where

\[ F^{(j)}_{\mu\nu} = \partial_\mu A^{(j)}_\nu - \partial_\nu A^{(j)}_\mu. \]

Here, \( \mathcal{L}_B \) and \( \mathcal{L}_M \) are the free baryonic and the free mesonic Lagrangians, respectively, and interactions are defined by \( \mathcal{L}_{int} \). The meson fields are denoted by \( \phi_\sigma, \phi_\delta, A^\omega, \) and \( A^\rho \), and their masses by \( m_\sigma, m_\delta, m_\omega \) and \( m_\rho \), respectively. The isospin Pauli matrices are written as \( \tau^a, \tau^3 \) being the third component of \( \tau^a \). For exotic baryon \( \Theta^+ \), as we assume isospin \( I = 0, \tau^a = 0 \), there is no coupling to \( \rho \) and \( \delta \) mesons. The main difference to DDRH theory in Ref. [17] is that the lagrangian density includes the interactions of pentaquark \( \Theta^+ \) and mesons. In contrast to standard QHD models [18, 19, 20], the meson-baryon vertices \( \hat{\Gamma}_\alpha (\alpha = \sigma, \omega, \delta, \rho) \) are not constant numbers but the baryon field operators \( \Psi \) dependent.

In the mean field approximation (MFA), the meson field operators can be replaced by their expectation values, which are classical fields [18, 19, 20]

\[ \phi_i \rightarrow \langle \phi_i \rangle = \phi_{i0} \]  
\[ A^i \rightarrow \langle A^i \rangle = A^i_0 \]

The field equations are reduced to

\[ (-\nabla^2 + m^2_\sigma) \Phi_\sigma = \Gamma^N_\sigma(\rho) \rho_S^N + \Gamma^{\Theta^+}_\sigma(\rho) \rho_S^{\Theta^+}, \]
\[ (-\nabla^2 + m^2_\delta) \Phi_{\delta0} = \Gamma^N_\delta(\rho) \rho_S^3, \]
\[ (-\nabla^2 + m^2_\omega) A_0^{(\omega)} = \Gamma^N_\omega(\rho) \rho_S^N + \Gamma^{\Theta^+}_\omega(\rho) \rho_S^{\Theta^+} \]
\[ (-\nabla^2 + m^2_\rho) A_0^{(\rho)} = \Gamma^N_\rho(\rho) \rho_S^3 \]

where the densities are values of the following ground state expectation

\[ \rho_S^N = \langle \bar{\Psi}_N \Psi_N \rangle \]  
\[ \rho^i = \langle \bar{\Psi}_i \Psi_i \rangle \]
\[ \rho_S^3 = \langle \bar{\Psi}_N \tau_3 \Psi_N \rangle = \rho_S^P - \rho_S^N \]
\[ \rho^3 = \langle \bar{\Psi}_N \tau_3 \Psi_N \rangle = \rho_P - \rho^N \]

The Dirac equation, separated in isospin, is the only remaining operator field equation

\[ [i \gamma_\mu \partial^\mu - \Gamma^N_\omega \gamma^0 A_0^\omega - \Gamma^N_\rho \gamma^0 \tau_3 A_0^\rho - (M_N - \Gamma^N_\sigma \Phi_\sigma - \tau_3 \Gamma^N_\delta \Phi_\delta)] \Psi_N = 0 \]
\[ [i \gamma_\mu \partial^\mu - \Gamma^{\Theta^+}_\omega \gamma^0 A_0^\omega - \Gamma^{\Theta^+}_\rho \gamma^0 \tau_3 \Phi_\delta^{\Theta^+}] \Psi_{\Theta^+} = 0 \]
The in-medium mass of nucleons and pentaquark $\Theta^+$ is then given by

\[
M^*_P = M_N - \Gamma^N_\sigma(\rho) - \Gamma^N_\delta(\rho) \Phi_\delta
\]

(18)

\[
M^*_N = M_N - \Gamma^N_\sigma(\rho) + \Gamma^N_\delta(\rho) \Phi_\delta
\]

(19)

The field equations can be further simplified assuming translational invariance and neglecting the electromagnetic field. Solutions of the stationary Dirac equation

\[
[\gamma_\mu k^*_b - m_b^*] u_b^*(k) = 0
\]

(20)

The usual plane wave Dirac spinors is given by

\[
u_b^*(k) = \sqrt{E_b^* + m_b^*} \left( \frac{\sigma_k^* \sigma}{E_b^* + m_b^*} \right) \chi_b,
\]

(21)

where $\chi_b$ is a two-component Pauli spinor and the index $b$ distinguishes between neutrons, protons and pentaquark $\Theta^+$. Due to the inclusion of the $\delta$ meson, the effective mass differs for neutrons and protons. The kinetic 4-momenta $k_b^*$ and the energy $E_b^*$ of the particle are related by the in-medium on-shell condition, so it is easily to get $k_b^* = m_b^*$. Integrating over all states $k \leq k_{F_b}$ inside the Fermi sphere and introducing $E_{F_b} = \sqrt{k_{F_b}^2 + m_b^*}$ the scalar and vector densities in infinite nuclear matter lead to

\[
\rho_b = \frac{2}{(2\pi)^3} \int_{|k| < k_{F_b}} d^3k \frac{k_{F_b}^3}{3\pi^2}
\]

(22)

\[
\rho_b^s = \frac{2}{(2\pi)^3} \int_{|k| < k_{F_b}} d^3k \frac{m_b^*}{E_b^*} k_{F_b} E_{F_b} + m_b^* \ln \frac{k_{F_b} E_{F_b}}{m_b^*}.
\]

(23)

The density-momentum tensor is defined

\[
T^{\mu\nu} = \sum_i \frac{\partial \mathcal{L}}{\partial (\partial^{\mu}\Phi_i)} \partial_\nu \Phi_i - g^{\mu\nu} \mathcal{L}
\]

\[= \sum_i \bar{\Psi}_i \gamma_\mu \partial_\nu \Psi_i - g^{\mu\nu} \mathcal{L} - \frac{1}{2} \sigma_{\omega A} F_{\omega A}^2 - \frac{1}{2} \rho_{\omega A} F_{\omega A}^2 - \frac{1}{2} \sigma_{\delta A} F_{\delta A}^2 - \frac{1}{2} \rho_{\delta A} F_{\delta A}^2
\]

(24)

\[
\varepsilon = \langle T^{00} \rangle = \frac{2}{(2\pi)^3} \int_0^{K_{F_n}} d^3k (k^2 + M_n^*)^{1/2}
+ \frac{2}{(2\pi)^3} \int_0^{K_{F_p}} d^3k (k^2 + M_p^*)^{1/2}
+ \frac{2}{(2\pi)^3} \int_0^{K_{F_{\Theta^+}}} d^3k (k^2 + M_{\Theta^+}^*)^{1/2}
+ \frac{1}{2} \left[ m_\sigma^2 \Phi_\sigma^2 + m_\omega^2 \Phi_{\omega A}^2 + m_\delta^2 \Phi_{\delta A}^2 + m_\rho^2 A_0^2 \right]
\]

(25)

Here the meson-baryon vertices $\tilde{\Gamma}_\alpha (\alpha = \sigma, \omega, \delta, \rho)$ are dependent on the baryon field operators $\Psi$ rather than constant numbers. Relativistic covariance requires that the vertices are functions
$\hat{\Gamma}_\alpha(\hat{\rho})$ of Lorenz-scalar bilinear forms $\hat{\rho}(\bar{\Psi}, \Psi)$ of the field operators. In mean-field approximation they reduce to density dependent coupling. Hofmann got a rational approximation as follows by mapping DB calculations\cite{17}.

$$\Gamma^N_\alpha(\rho) = a_\alpha \left[ 1 + b_\alpha \left( \frac{\rho}{\rho_0} + d_\alpha \right)^2 \right] \left[ 1 + c_\alpha \left( \frac{\rho}{\rho_0} + e_\alpha \right)^2 \right]$$

Table 1: The DDRMF model parameter sets are taken from Ref.\cite{17}

| meson $\alpha$ | $\sigma$ | $\omega$ | $\delta$ | $\rho$ |
|----------------|---------|---------|---------|-------|
| $m_\alpha$ [MeV] | 550     | 783     | 983     | 770   |
| $a_\alpha$     | 13.1334 | 15.1640 | 19.1023 | 12.8373 |
| $b_\alpha$     | 0.4258  | 0.3474  | 1.3653  | 2.4822 |
| $c_\alpha$     | 0.6578  | 0.5152  | 2.3054  | 5.8681 |
| $d_\alpha$     | 0.7914  | 0.5989  | 0.0693  | 0.3671 |
| $e_\alpha$     | 0.7914  | 0.5989  | 0.5388  | 0.3598 |

$\rho_0 = 0.16 \text{ [fm}^{-3}]$

The theta-meson coupling constants can be determined with the quark meson model developed in Ref.\cite{22}. For isospin $I = 0$, we only consider $\sigma$ and $\rho$ meson. The equation of motion for meson field operators are as follows

$$\partial_\mu \partial^\mu \hat{\sigma} + m^2_\sigma \hat{\sigma} = g^q_\sigma \bar{q}q,$$

$$\partial_\mu \partial^\mu \hat{\omega}^\nu + m^2_\omega \hat{\omega}^\nu = g^q_\omega \bar{q}\gamma^\nu q,$$

where $g^q_\sigma$ and $g^q_\omega$ are the quark-meson coupling constants for $\sigma$ and $\omega$, respectively. The mean fields are defined as the expectation values

$$\langle A|\hat{\sigma}(t, r)|A \rangle = \sigma(r),$$

$$\langle A|\hat{\omega}^\nu(t, r)|A \rangle = \delta(\nu, 0)\omega(r),$$

Where $|A\rangle$ is the ground state of the nucleus and $(t, r)$ are the coordinates in the rest frame of the nucleus. In the mean field approximation the sources are the sum of the sources created by each nucleon - the latter interacting with the meson fields. Thus

$$\bar{q}q(t, r) = \sum_{i=1,A} \langle \bar{q}q(t, r) \rangle_i,$$

$$\bar{q}\gamma^\nu q(t, r) = \sum_{i=1,A} \langle \bar{q}\gamma^\nu q(t, r) \rangle_i,$$

where $\langle (...) \rangle_i$ denotes the matrix element in the nucleon $i$ located at $R_i$ at time $t$. Finally for the strange baryon, X.H. Zhong etc have got the relations\cite{23}

$$g^S_\sigma = \frac{n_q}{3} g^N_\sigma \Gamma_{S/B}, g^S_\omega = \frac{n_q}{3} g^N_\omega, g^S_\rho = g^N_\rho.$$
Where \( n_q \) is the total number of valence \( u \) and \( d \) quarks in the baryon \( S \). For \( \Theta^+ \), \( n_q = 4 \). \( \Gamma_{S/B} \approx 1 \) for all hyperons in practice \([24] \).

In our calculation, we use the relations as follows

\[
\Gamma_\Theta^+(\rho) = \frac{4}{3} \Gamma_\alpha(\rho) \tag{34}
\]

The baryon density is given by

\[
\rho = \rho^n + \rho^p + \rho^{\Theta^+}
\]

\[
= \frac{1}{(3\pi)^2} K_{Fn}^3 + \frac{1}{(3\pi)^2} K_{Fp}^3 + \frac{1}{(3\pi)^2} K_{F\Theta^+}^3 \tag{35}
\]

Where \( K_{Fn}, K_{Fp} \), and \( K_{F\Theta^+} \) are the Fermi momenta for \( n, p \), and \( \Theta^+ \), respectively. The baryon fraction is

\[
Y_i = \frac{\rho^i}{\rho^n + \rho^p + \rho^{\Theta^+}}, i = n, p, \Theta^+ \tag{36}
\]

The energy per baryon for multi-Theta matter is defined

\[
\frac{E}{B} = \frac{\varepsilon}{\rho} - Y_nM_n - Y_pM_p - Y_{\Theta^+}M_{\Theta^+} \tag{37}
\]

### 3 Results and discussion

We first calculate the effective baryonic masses with different \( \Theta^+ \) fractions in isospin symmetric matter and isospin asymmetric matter. The results are presented in Fig.1(a) and Fig.1(b). It is shown that the effective masses all decrease as the baryon density increases, whether the nucleons or the \( \Theta^+ \). The isospin effects on the effective nucleon mass can not be neglected due to the interaction between the scalar isovector \( \delta \), especially at high density. The neutron-proton (n/p) effective mass splits. But the effects on the \( m^{*+}_{\Theta^+} \) is negligible. The effective Theta masses always decrease more slowly than the nucleons. The fraction \( Y_{\Theta^+} \) can also affect the baryonic masses. At the same baryon density, the baryonic masses all increase with more conclusion of pentaquark \( \Theta^+ \).

Fig.2(a) shows the binding energy in isospin symmetric matter. We plot the binding energy with different \( \Theta^+ \) fractions. Our calculation show that the binding energy is sensitive to the variation of the \( \Theta^+ \) fraction. The absolute minimum of binding energy increases until the \( \Theta^+ \) fraction reaches about 0.23, which is up to nearly 24Mev at the baryon density 1.4\( \rho_0 \). But when the fraction of \( \Theta^+ \) reaches to 0.56, the system becomes not bound anymore.

The binding energy of the system without proton is presented in Fig.2(b). The system with inclusion of a small fraction of pentaquark \( \Theta^+ \) becomes stable. The absolute minimum reaches the peak at the fraction 0.3. More \( \Theta^+ \) added, system becomes less bound. When the percent of pentaquark \( \Theta^+ \) reaches to half an whole, system becomes unstable again. It shows that the binding is also sensitive to the isospin by comparing with the curves in Fig.2(a).

Minimizing the \( E/B \) with respect to \( Y_{\Theta^+} \) and \( Y_p \), we get the minimum of the binding energy. The results are given in in Fig.3. It is shown that the minimum of binding energy occurs at the density about 1.5\( \rho_0 \). In low-density region \((0 \leq \rho \leq 1.5\rho_0)\), the binding energy increases with the increasing...
baryon density. The opposite dependence occurs in high-density region ($\rho_0 \geq 1.5 \rho_0$). Fig. 3(b) displays the fractions of neutron, proton and pentaquark $\Theta^+$ as calculated by minimizing the energy. The curves display the neutron-to-proton ratio is near 1:1. It is easily to comprehend since the asymmetric nuclear matter is more stable than symmetric matter.

In summary, we calculate the Multi-Theta matter could be bound in the framework of density dependent relativistic hadron field theory. It seems possible that there is a large fraction of pentaquark $\Theta^+$. Whether nucleons or pentaquark $\Theta^+$, it’s effective mass in-medium is decreasing with the increasing baryon density. But effective mass of pentaquark $\Theta^+$ is larger than that of the $N, \Theta^+$.

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Figure 1: effective baryonic masses.
Figure 2: binding energy.
Figure 3: FIG.3(a) shows the minimum of the binding energy. FIG.3(b) shows the fractions of neutrons, protons and pentaquark $\Theta^+$