Dimensional Reduction and Hadronic Processes

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Abstract. We consider the application of regularization by dimensional reduction to NLO corrections of hadronic processes. The general collinear singularity structure is discussed, the origin of the regularization-scheme dependence is identified and transition rules to other regularization schemes are derived.

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INTRODUCTION

The LHC era necessitates the computation of next-to-leading order (NLO) predictions in the standard model and extensions such as supersymmetry. One element of such computations is the choice of a regularization scheme. In particular, in supersymmetric theories, regularization by dimensional reduction is often advantageous compared to dimensional regularization, which breaks supersymmetry. In recent years, progress on the understanding of dimensional reduction has been achieved in three directions. A consistent definition and proofs of various supersymmetry relations have been obtained [2,3]; multiloop applications have been pioneered [4], and the factorization problem found in Refs. [6, 7, 8] has been resolved [5].

In these proceedings we present our study [2], where we reconsider the factorization problem in a more general context. We consider real and virtual NLO QCD corrections to arbitrary hadronic $2 \to (n-2)$ processes with massless or massive partons. We discuss the infrared singularity structure and the associated regularization-scheme dependence of all these corrections, provide transition rules between the schemes and show that all singularities factorize. In this way we show that the framework of dimensional reduction is completely consistent with factorization, and we show how this scheme can be used to compute hadronic processes in practice.

FOUR REGULARIZATION SCHEMES

In a first step we need to precisely define the regularization schemes. As it turns out, in the literature two different versions of dimensional reduction with different factorization behaviour have been used, and it is crucial to distinguish between them.

In both dimensional regularization and dimensional reduction, space-time and momenta are continued from 4 to $D = 4 - 2\varepsilon$ dimensions. Gluon fields (and other vector fields) are basically treated as $D$-dimensional in dimensional regularization and 4-dimensional in dimensional reduction. In the consistent definition of dimensional reduction [2] three spaces are distinguished: the original 4-dimensional Minkowski space, the $D$-dimensional space for regularized momenta and space-time coordinates, and a formally 4-dimensional space for the regularized gluon fields. The associated metric tensors are denoted as $\bar{g}^{\mu\nu}$, $\hat{g}^{\mu\nu}$, and $g^{\mu\nu}$, respectively. The dimensionalities of the spaces are expressed by the following equations:

$$g^{\mu\nu} g_{\mu\nu} = 4, \quad \bar{g}^{\mu\nu} \bar{g}_{\mu\nu} = D, \quad \hat{g}^{\mu\nu} \hat{g}_{\mu\nu} = 4.$$  (1)

The following projection relations express the hierarchical structure of the three spaces:

$$g^{\mu\nu} \bar{g}_{\nu}^{\rho} = \bar{g}^{\mu\rho}, \quad \bar{g}^{\mu\nu} \bar{g}_{\nu}^{\rho} = \bar{g}^{\mu\rho}, \quad \hat{g}^{\mu\nu} \hat{g}_{\nu}^{\rho} = \hat{g}^{\mu\rho}. \quad (2)$$

It is useful to introduce the orthogonal complement to the $D$-dimensional space. This is a $4 - D = 2\varepsilon$-dimensional space with metric tensor $\bar{g}^{\mu\nu}$, which satisfies

$$g^{\mu\nu} = \bar{g}^{\mu\nu} + \hat{g}^{\mu\nu}, \quad \bar{g}^{\mu\nu} \bar{g}_{\mu\nu} = 2\varepsilon, \quad (3)$$

$$g^{\mu\nu} \bar{g}_{\nu}^{\rho} = \bar{g}^{\mu\rho}, \quad \bar{g}^{\mu\nu} g_{\nu}^{\rho} = 0, \quad \hat{g}^{\mu\nu} \hat{g}_{\nu}^{\rho} = 0. \quad (4)$$

It is not strictly necessary to regularize all gluons. Gluons which appear inside a closed loop or inside a singular region (soft or collinear) of a phase space integral are called “internal”, and these need to be regularized. All other gluons are called “external”; they require no regularization. As a result it is possible to distinguish two versions CDR and HV of dimensional regularization and two versions of dimensional reduction DRED and FDH, depending on whether external gluons are treated in the same way as internal ones or not. The following table defines the four schemes by specifying which metric ten-
sor is to be used for the gluons in the gluon propagators/polarization sums:

|             | CDR | HV | FDH | DRED |
|-------------|-----|----|-----|------|
| internal gluon | $\hat{g}^{\mu\nu}$ | $\hat{g}^{\mu\nu}$ | $\hat{g}^{\mu\nu}$ | $\hat{g}^{\mu\nu}$ |
| external gluon | $\bar{g}^{\mu\nu}$ | $\bar{g}^{\mu\nu}$ | $\bar{g}^{\mu\nu}$ | $\bar{g}^{\mu\nu}$ |

Note that the FDH version of dimensional reduction [10] has been denoted as DR e.g. in Refs. [11, 12, 14] (for the one-loop equivalence see e.g. Refs. [11, 15]). The scheme DRED is the one defined in e.g. [1, 16, 2].

**INFRARED STRUCTURE AND FACTORIZATION IN THE FOUR SCHEMES**

**Factorization problem**

In Refs. [6, 8] an apparent non-factorizing behaviour of DRED has been found in the real corrections to the process $gg \rightarrow t\bar{t}g$. The reason for this apparent problem has been identified in Ref. [5]. On the regularized level in DRED, the splitting gluon cannot be treated as a single, formally 4-dimensional gluon, but it should be decomposed into its $D$- and $(4 - D)$-dimensional parts, according to $g^{\mu\nu} = \hat{g}^{\mu\nu} + \bar{g}^{\mu\nu}$. The $\hat{g}$-part behaves as a $D$-dimensional gauge field, while $\bar{g}$ behaves as $2\varepsilon$ scalar fields. If this decomposition is taken into account factorization holds in DRED just as expected in a theory with two different partons $\hat{g}$ and $\bar{g}$.

**Splittings**

The regularization-scheme (RS) dependence of real and virtual corrections to arbitrary processes is related to ultraviolet, soft, and collinear singularities. In these proceedings we only take into account collinear singularities, assuming fully renormalized and thus ultraviolet finite amplitudes and noting that the soft singularities only lead to a trivial RS dependence. The collinear singularities of real corrections factorize into products of lowest-order matrix elements and splitting functions, and the associated RS dependence is expressed in terms of the RS dependence of the splitting functions.

The splitting functions $P_{r \rightarrow ik}^{\text{RS}}(z)$ describe the splitting of a parton $i$, which is slightly off-shell, into collinear partons $j$, $k$, where the momenta of $j$, $k$ are given by $z$ and $(1 - z)$ times the momentum of $i$ (for $z < 1$). Most interesting for our purposes are the splitting functions involving gluons, which are RS dependent owing to the different gluon prescriptions.

Figure 1 shows the four different gluon prescriptions for $P_{r \rightarrow gg}^{\text{RS}}$. According to the definition given above, the two collinear gluons $j$ and $k$ are treated as “internal”, and the virtual gluon $i$ as “external”.

In order to understand the RS dependence it should first be noted that the projection from $\hat{g}$ onto $\bar{g}$ does not change the result of the splitting functions since $\bar{g}$ is simply a part of the $D$-dimensional gauge field and thus behaves in the same way as $\hat{g}$. Then Fig. 1 shows how the results change in going from CDR to HV, FDH, and DRED:

- **CDR**: $P_{r \rightarrow gg}^{\text{CDR}} = P_{r \rightarrow \bar{g}g}^{\text{DRED}}$ (5)
- **HV**: $P_{r \rightarrow gg}^{\text{HV}} = P_{r \rightarrow \bar{g}g}^{\text{DRED}}$ (6)
- **FDH**: $P_{r \rightarrow gg}^{\text{FDH}} = P_{r \rightarrow \bar{g}g}^{\text{DRED}} + P_{r \rightarrow \bar{g}g}^{\text{DRED}}$ (7)
- **DRED**: $P_{r \rightarrow gg}^{\text{DRED}} = P_{r \rightarrow \bar{g}g}^{\text{DRED}} + P_{r \rightarrow \bar{g}g}^{\text{DRED}}$ (8)

In words, in FDH there is a new final state, $\bar{g}\bar{g}$, which modifies the splitting function, and in DRED there is a new initial state of the splitting, $\bar{g}$, which gives rise to an independent splitting function. This reflects the discussion of the previous subsection. Splitting functions involving quarks are related in a similar way.

The splitting functions $P_{r \rightarrow q\bar{q}}^{\text{RS}}$ for $r \rightarrow \text{anything}$ defined for $z < 1$ give rise to RS dependent constants $\gamma_{\text{RS}}(i)$

$$\gamma_{\text{RS}}(i) = -\sum_{k,l} \int_0^1 dz \frac{\left(1 - z\right)}{(1 - z)} P_{r \rightarrow kl}^{\text{RS}}(z).$$

Via unitarity, these constants $\gamma_{\text{RS}}(i)$ are the origin of the RS dependence of the virtual corrections [11, 12]. As above, the RS dependence of these constants is easily understood: $\gamma_{\text{CDR}}(i) = \gamma_{\text{VD}}(i)$, the differences $\gamma_{\text{DD}}(i) - \gamma_{\text{DD}}(i)$ are non-zero due to the possible splittings $\hat{g} \rightarrow \bar{g}\bar{g}$ and $q \rightarrow q\bar{g}$, and in DRED there is a new constant $\gamma_{\text{DRED}}(\bar{g})$, but otherwise the results in DRED and FDH are the same, $\gamma_{\text{DRED}}(\bar{g}) = \gamma_{\text{DD}}(\bar{g})$ and $\gamma_{\text{DRED}}(q) = \gamma_{\text{DD}}(q)$.

The results in DRED are new, while the results in the other schemes have already been obtained in Refs. [11, 12].

**Results for squared matrix elements**

The collinear singularities of real corrections in $\text{RS}^* \in \{\text{CDR, HV, FDH}\}$ are well known. In these schemes, if two outgoing partons $\bar{a}_k$ and $\bar{a}_l$ become collinear, the squared matrix element for a process involving $\bar{a}_k$ and $\bar{a}_l$ satisfies

$$\mathcal{M}^{(0)}_{\text{RS}}(a_1, a_2; \ldots; \bar{a}_k(p_l) \ldots; \bar{a}_l(p_k) \ldots) \propto \frac{2 \hat{s}^2}{\bar{s}_{kl}} \times (11)$$

$$P_{r \rightarrow kl}^{\text{RS}}(z) \mathcal{M}^{(0)}_{\text{RS}}(a_1, a_2; \ldots; \bar{a}_k(p_l) \ldots; \bar{a}_l(p_k) \ldots).$$
Here \((kl)\) denotes the (uniquely determined) flavour of the splitting \((kl) \rightarrow k.l\). Our new result for DRED can be written in the same form,

\[
\mathcal{M}^{(0)}_{\text{DRED}}(a_1, a_2; \ldots \tilde{a}_i(p_l) \ldots \tilde{a}_k(p_k) \ldots) p_k p_l \frac{2 g_s^2}{s_{kl}} \sum_{\tilde{a}_i(kl)} \mathcal{M}^{(0)}_{\text{DRED}}(a_1, a_2; \ldots \tilde{a}_i(kl) (p_k + p_l) \ldots).
\]

(12)

Here the split \(g = \hat{g} + \hat{g}\) becomes essential and therefore there is a sum over all possible splittings \(\sum_{\tilde{a}_i(kl)}\), where \(\tilde{a}_i(kl) \in \{\hat{g}, \hat{g}\}\) if \((kl)\) is a gluon, and \(\tilde{a}_i(kl) \in \{q\}\) if \((kl)\) is a quark.

The collinear singularities of virtual corrections in \(\text{RS} \neq \text{DRED}\) are given by \([13, 14]\)

\[
\mathcal{M}^{(1)}_{\text{RS}(i)}(a_1 \ldots a_n) = \frac{\alpha_s}{2\pi} \times \sum_i \mathcal{M}^{(0)}_{\text{RS}(i)}(a_1 \ldots a_n) \left( -\frac{1}{\varepsilon} \gamma_{\text{RS}(i)}(a_i) \right) + \ldots
\]

(13)

where \(\mathcal{M}^{(1)}\) denotes the fully renormalized one-loop squared matrix element, the dots denote finite terms and soft singularities, and the sum over \(i\) is over all external legs. In DRED, the result has a similar form,

\[
\mathcal{M}^{(1)}_{\text{DRED}}(a_1 \ldots a_n) = \frac{\alpha_s}{2\pi} \times \sum_i \mathcal{M}^{(0)}_{\text{DRED}(i)}(a_1 \ldots a_i \ldots a_n) \left( -\frac{1}{\varepsilon} \gamma_{\text{DRED}(i)}(\tilde{a}_i) \right) + \ldots
\]

(14)

Again, the only difference is the additional sum over the two possibilities \(\tilde{a}_i(kl) \in \{\hat{g}, \hat{g}\}\) if \((kl)\) is a gluon.

From Eqs. \((13)\) and \((14)\) and the explicit results for the \(\gamma(i)\) \([9]\) one can obtain explicit rules for translating the results in one scheme into results in any of the other schemes.

**CONCLUSIONS FOR PRACTICAL APPLICATIONS**

Based on the main results on the singularity structure in DRED discussed above, one can derive two further crucial consequences \([9]\): (1) it is possible to realize the \(\overline{\text{MS}}\)-factorization scheme, even if e.g. DRED is used, and (2) even in DRED no parton distribution functions for the unphysical \(\varepsilon\)-scalars are required. Hence, even in DRED the standard, \(\overline{\text{MS}}\)-PDF can be used.

There are explicit, simple rules on how to transform the various parts (real and virtual corrections, collinear counterterm) from DRED to other RS or vice versa. It is thus possible to use different RS for different parts of an NLO computation, depending on which is most practical. Since DRED is better compatible with supersymmetry, it might be a simplification to apply DRED in particular to the computation of virtual corrections.

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