Deterministic secure direct communication using GHZ states and swapping quantum entanglement

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Abstract
We present a deterministic secure direct communication scheme via entanglement swapping, where a set of ordered maximally entangled three-particle states (GHZ states), initially shared by three spatially separated parties, Alice, Bob and Charlie, functions as a quantum information channel. After ensuring the safety of the quantum channel, Alice and Bob apply a series local operations on their respective particles according to the tripartite stipulation and the secret message they both want to send to Charlie. By three Alice, Bob and Charlie’s Bell measurement results, Charlie is able to infer the secret messages directly. The secret messages are faithfully transmitted from Alice and Bob to Charlie via initially shared pairs of GHZ states without revealing any information to a potential eavesdropper. Since there is not a transmission of the qubits carrying the secret message between any two of them in the public channel, it is completely secure for direct secret communication if perfect quantum channel is used.

PACS numbers: 03.67.Dd, 03.67.Hk

1. Introduction
Cryptography is the art of enabling two parties to communicate in private. Effective cryptosystems make it easy for parties who wish to communicate to achieve privacy, but make it very difficult for third parties to ‘eavesdrop’ on the content of the conversation. A simple, yet highly effective private key cryptosystem is the Vernam cipher, sometimes called a one time pad. The great feature of this system is that as long as the key strings are truly secret, it is provably secure. The major difficulty of private key cryptosystems is secure distribution of key bits, since a malevolent third party may be eavesdropping on the key distribution, and then use the intercepted key to decrypt some of the message in transmission.

One of the earliest discoveries in quantum computation and quantum information was that quantum mechanics can be used to do key distribution in such a way that Alice and Bob’s security can not be compromised. This procedure is known as quantum cryptography or quantum key distribution (QKD). The basic idea is to exploit the quantum mechanical principle that observation in general disturbs the system being observed. In 1984, Bennett and Brassard proposed the first quantum cryptography protocol [1] using quantum mechanics to distribute keys between Alice and Bob, without any possibility of a compromise. Since then numerous QKD protocols have been proposed, such as Ekert 1991 protocol (Ekert91) [2], Bennett-Brassard-Mermin 1992 protocol (BBM92) [3], B92 protocol [4] and other protocols [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

Recently, Shimizu and Imoto [23, 24] and Beige et al. [25] presented novel quantum secure direct communication (QSDC) schemes, in which the two parties communicate important messages directly without first establishing
a shared secret key to encrypt them and the message is deterministically sent through the quantum channel, but can be read only after a transmission of an additional classical information for each qubit. Boström and Felbinger put forward a communication scheme, the "ping-pong protocol", which also allows for deterministic communication. This protocol can be used for the transmission of either a secret key or a plaintext message. Wójcik discussed the security of the "ping-pong protocol" in a noisy quantum channel. Deng et al. suggested a two-step quantum direct communication protocol using Einstein-Podolsky-Rosen pair block. However in all these QSDC schemes it is necessary to send the qubits with secret messages (message-coding sequence) in the public channel. Therefore, Eve can attack the qubits in transmission and make the communication interrupt.

More recently, Yan and Zhang presented a QSDC scheme using Einstein-Podolsky-Rosen pairs and teleportation. By means of controlled quantum teleportation we proposed two controlled QSDC protocols. Since in these protocols there are no particles carrying secret messages to be transmitted in public channel, so the communication can not be interrupted by any eavesdropper. Therefore, they are completely secure for direct secret communication as long as perfect quantum channel is used.

Entanglement swapping is a method that enables one to entangle two quantum systems that do not have direct interaction with one another. Based on entanglement swapping, we presented a QSDC scheme. In this paper, we introduce another QSDC scheme achieved by swapping quantum entanglement, in which a set of ordered three-particle Greenberger-Horne-Zeilinger (GHZ) states initially shared by three spatially separated parties, Alice, Bob and Charlie, functions as a quantum information channel. The proposed QSDC scheme is simultaneous mutual communications among different pairs of parties, one for Alice and Charlie and another for Bob and Charlie. After ensuring the safety of the quantum channel, Alice and Bob encode secret classical bits by applying predetermined unitary operations on GHZ triplets. The secret messages encoded by local operations are faithfully transmitted from two distant senders (Alice and Bob) to a remote receiver (Charlie) without revealing any information to a potential eavesdropper.

2. A simultaneous mutual quantum secure direct communication protocol between the central party and other two parties

In this section we propose a simultaneous mutual quantum secure direct communication scheme which utilizes shared GHZ states and entanglement swapping between communicating parties, in the form three people (Alice, Bob and Charlie).

2.1 Notation

Let us start by illustrating entanglement swapping. We first define four Bell states (EPR pairs) as

\[ \Phi^\pm = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle), \quad \Psi^\pm = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle), \]

and eight GHZ states as

\[
|P^\pm\rangle = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle), \quad |Q^\pm\rangle = \frac{1}{\sqrt{2}}(|001\rangle \pm |110\rangle), \\
|R^\pm\rangle = \frac{1}{\sqrt{2}}(|010\rangle \pm |101\rangle), \quad |S^\pm\rangle = \frac{1}{\sqrt{2}}(|011\rangle \pm |100\rangle).
\]

Suppose three parties, Alice, Bob and Charlie, share two GHZ triplets \(|P^+\rangle_{123}\) and \(|P^+\rangle_{456}\) where Alice has qubits 1 and 4, Bob possesses 2 and 5, and particles 3 and 6 are in Charlie's side. Two operations are performed on qubits 1 and 4, 2 and 5 with the Bell basis, \(\Phi^\pm\) and \(\Psi^\pm\), by Alice and Bob, respectively, then the total state \(|P^+\rangle_{123} \otimes |P^+\rangle_{456}\) is projected onto \(\Phi^+_1 \otimes \Phi^+_4 \otimes \Phi^+_3, \Phi^+_1 \otimes \Phi^-_4 \otimes \Phi^-_3, \Phi^-_1 \otimes \Phi^+_4 \otimes \Phi^-_3, \Phi^-_1 \otimes \Phi^-_4 \otimes \Phi^+_3, \quad \Psi^+_1 \otimes \Psi^+_4 \otimes \Psi^+_3, \Psi^+_1 \otimes \Psi^-_4 \otimes \Psi^-_3, \Psi^-_1 \otimes \Psi^+_4 \otimes \Psi^-_3, \Psi^-_1 \otimes \Psi^-_4 \otimes \Psi^+_3 \) and \(\Psi^+_1 \otimes \Psi^-_4 \otimes \Psi^-_3, \Psi^-_1 \otimes \Psi^+_4 \otimes \Psi^+_3 \) with equal probability of 1/8 for each. Previous entanglement of qubits 1, 2 and 3, and 4, 5 and 6 are now swapped into entanglement between 1 and 4, 2 and 5, and 3 and 6. Although we considered entanglement swapping with the initial state \(|P^+\rangle_{123} \otimes |P^+\rangle_{456}\), similar results can be achieved with other GHZ states. For example, when Alice, Bob and Charlie originally share \(|S^+\rangle_{123}\) and \(|S^+\rangle_{456}\), there are eight possible measurement outcomes, \(\Psi^+_1 \otimes \Phi^+_4 \otimes \Phi^+_3, \Psi^+_1 \otimes \Phi^-_4 \otimes \Phi^-_3, \Psi^-_1 \otimes \Phi^+_4 \otimes \Phi^-_3, \Psi^-_1 \otimes \Phi^-_4 \otimes \Phi^+_3, \quad \Phi^+_1 \otimes \Psi^+_4 \otimes \Psi^+_3, \Phi^+_1 \otimes \Psi^-_4 \otimes \Psi^-_3, \Phi^-_1 \otimes \Psi^+_4 \otimes \Psi^-_3, \Phi^-_1 \otimes \Psi^-_4 \otimes \Psi^+_3 \) and \(\Phi^+_1 \otimes \Psi^-_4 \otimes \Psi^-_3, \Phi^-_1 \otimes \Psi^+_4 \otimes \Psi^+_3 \) with equal probability.
2.2 Preparing quantum channel

Suppose that three spatially separated parties wish to realize simultaneous mutual communications in secret among different pairs of parties, one for Alice and Charlie and another for Bob and Charlie. In order to achieve tripartite communications between one party and the other two parties in private at the same time, the first step is to establish quantum channel (GHZ triplets). Obtaining these GHZ triplets could have come about in many different ways, such as Charlie prepares a sequence of GHZ triplets and then share each triplet with Alice and Bob; or they could have met a long time ago and shared them, storing them until the present. Alice, Bob and Charlie then choose randomly a subset of GHZ triplets, do some appropriate tests of fidelity. Passing the test certifies that they continue to hold sufficiently pure, entangled quantum states. However, if tampering has occurred, they throw out the GHZ triplets and reconstruct them. We will discuss the details in Section 3.

2.3 The direct communication scheme by shared GHZ states and entanglement swapping

After ensuring the security of the quantum channel (GHZ states), Alice, Bob and Charlie begin secure direct communication. The QSDC scheme works as follows:

1. Alice, Bob and Charlie randomly divide all pure GHZ triplets into $N$ ordered groups $\{\xi(1)_{123}, \eta(1)_{456}\}$, $\{\xi(2)_{123}, \eta(2)_{456}\}, \ldots, \{\xi(N)_{123}, \eta(N)_{456}\}$, where $\xi(i)_{123}$ and $\eta(i)_{456}$ denote two GHZ states of Alice’s particles 1 and 4, Bob’s particles 2 and 5, and Charlie’s 3 and 6 in the $i$-th group. For simplicity, let us suppose that these GHZ triplets are in the state $|P^+\rangle$.

2. Alice, Bob and Charlie agree on a local encoding for their respective messages (secret classical bits) on GHZ triplets $\xi(i)_{123}$, and Bob by local operations
   \[
   \begin{align*}
   \sigma_{00} &= I = |0\rangle\langle 0| + |1\rangle\langle 1|, \quad \sigma_{01} = \sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|, \\
   \sigma_{10} &= \sigma_y = |0\rangle\langle 1| - |1\rangle\langle 0|, \quad \sigma_{11} = \sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|.
   \end{align*}
   \]

3. Alice and Charlie, and Bob and Charlie assign secretly two bits and one bit to Alice and Bob’s respective operations as following encoding
   \[
   \begin{align*}
   \sigma_{00} &\rightarrow 00, \quad \sigma_{01} \rightarrow 01, \quad \sigma_{10} \rightarrow 10, \quad \sigma_{11} \rightarrow 11,
   \end{align*}
   \]
   and
   \[
   \begin{align*}
   \sigma_0 &\rightarrow 0, \quad \sigma_1 \rightarrow 1.
   \end{align*}
   \]

4. Alice and Bob encode their respective messages (secret classical bits) on GHZ groups. Explicitly, both Alice and Bob apply a predetermined unitary operation on each of their particles 1 and 2 according to their respective secret message sequence, respectively.

   Suppose Alice, Bob, and Charlie initially share GHZ state $|P^+\rangle_{123}, |P^+\rangle_{456}$, then the originally total state of them is
   \[
   |P^+\rangle_{123} \otimes |P^+\rangle_{456} = \frac{1}{2\sqrt{2}} \left[ \Phi^+_1 \otimes \Phi^+_2 \otimes \Phi^+_3 + \Phi^+_4 \otimes \Phi^-_2 \otimes \Phi^-_3 + \Phi^-_1 \otimes \Phi^+_2 \otimes \Phi^-_3 + \Phi^-_1 \otimes \Phi^-_2 \otimes \Phi^+_3 \\
   + \Phi^+_1 \otimes \Phi^+_2 \otimes \Phi^+_3 + \Phi^-_1 \otimes \Phi^-_2 \otimes \Phi^-_3 + \Phi^-_1 \otimes \Phi^-_2 \otimes \Phi^-_3 + \Phi^-_1 \otimes \Phi^-_2 \otimes \Phi^-_3 \right].
   \]

If Alice wishes to transmit 11 to Charlie and Bob wants to send 1 to Charlie, then Alice performs a local operation $\sigma_{11}$ on particle 1 and Bob applies $\sigma_1$ on his particle 2, thus the state $|P^+\rangle_{123}$ is turned into $|R^-\rangle_{123}$.

5. Alice and Bob make a Bell measurement on particles 1 and 4, and 2 and 5, respectively. We can see the effects of measurements by Alice and Bob on Charlie’s particles 3 and 6 if we express the product of GHZ states
\[
|R^-\rangle_{123} \otimes |P^+\rangle_{456} \text{ in the following equation: }
\]
\[
\begin{align*}
|R^-\rangle_{123} \otimes |P^+\rangle_{456} = & \frac{1}{2\sqrt{2}} \left[ \Phi_{14}^+ \otimes \Psi_{25}^- \otimes \Phi_{36}^+ \Phi_{14}^- \otimes \Psi_{25}^+ \otimes \Phi_{36}^- + \Phi_{14}^+ \otimes \Psi_{25}^+ \otimes \Phi_{36}^- - \Phi_{14}^- \otimes \Psi_{25}^- \otimes \Phi_{36}^+ \\
& + \Psi_{14}^- \otimes \Phi_{25}^+ \otimes \Phi_{36}^+ - \Psi_{14}^+ \otimes \Phi_{25}^- \otimes \Phi_{36}^- + \Psi_{14}^+ \otimes \Phi_{25}^+ \otimes \Psi_{36}^- - \Psi_{14}^- \otimes \Phi_{25}^- \otimes \Psi_{36}^+ \right].
\end{align*}
\]

If Alice and Bob get measurement outcomes \( \Phi_{14}^+ \) and \( \Psi_{25}^- \), respectively, then Charlie’s two particles 3 and 6 will have the state \( \Phi_{36}^+ \).

(5) Alice and Bob inform Charlie that they have made a Bell measurement on particles 1 and 4, and 2 and 5 over a classical channel, respectively, but do not tell the results of their measurements.

(6) Charlie performs a Bell measurement on his particles 3 and 6 and deduces the outcomes of both Alice and Bob’s measurements.

From the calculation of entanglement swapping (Eq.8) and his measurement outcome \( \Phi_{36}^+ \), Charlie could calculate that the initially whole state \( |P^+\rangle_{123} \otimes |P^+\rangle_{456} \) should collapse to \( \Phi_{14}^+ \otimes \Phi_{25}^+ \otimes \Phi_{36}^+ \) or \( \Phi_{14}^- \otimes \Phi_{25}^- \otimes \Phi_{36}^- \) without Alice and Bob’s local operations.

(7) Charlie asks and gets Alice and Bob’s measurement results publicly.

(8) Charlie can read out Alice and Bob’s secret message by comparing his calculation result with Alice and Bob’s practical measurement outcomes.

From the measurement results announced by Alice and Bob, and his calculation result, Charlie can infer that Alice and Bob have applied local operations \( \sigma_{11} \) and \( \sigma_1 \) on particles 1 and 2, respectively, such that \( \Phi_{14}^+ \otimes \Phi_{25}^- \otimes \Phi_{36}^+ \) turns into \( \Phi_{14}^+ \otimes \Psi_{25}^- \otimes \Phi_{36}^+ \), since it is impossible for Alice and Bob to change \( \Phi_{14}^+ \otimes \Phi_{25}^- \otimes \Phi_{36}^+ \) into \( \Phi_{14}^+ \otimes \Phi_{25}^+ \otimes \Phi_{36}^+ \) by applying unitary operation \( \sigma_{k_1,k_2} = \sigma_{k_1} \otimes \sigma_{k_2} \) \((k_1,k_2 \in \{0,1\})\) on particles 1 and 2, thus he obtains Alice’s message 11 and Bob’s 1. Finally, three spatially separated parties have realized deterministic secure direct communication between one party and the other two parties.

Remark 1: We should point out that the encoding schemes of Eq.5 and Eq.6 are secrete, i.e. only Alice and Charlie know the encoding scheme Eq.5, and only Bob and Charlie know Eq.6. The reason is as follows. After Charlie performs a Bell measurement on his qubits 3 and 6, he asks Alice and Bob to declare their Bell measurement results on the qubits 1 and 4, and 2 and 5. This public declaration step is crucial. However, an eavesdropper who knows that the original initial state is \( |P^+\rangle_{123} \otimes |P^+\rangle_{456} \) will do her calculation the same as Charlie. When she hears that, Alice and Bob, respectively, obtained measurement results \( \Phi_{14}^+ \) and \( \Psi_{25}^- \), the eavesdropper looks at Eq.7 and can easily deduce that such a measurement result can be obtained by applying one-qubit unitary operators on the following four cases: \( \Phi_{14}^+ \otimes \Phi_{25}^- \otimes \Phi_{36}^+ \), \( \Phi_{14}^+ \otimes \Phi_{25}^+ \otimes \Phi_{36}^+ \), \( \Phi_{14}^+ \otimes \Phi_{25}^- \otimes \Psi_{36}^- \), and \( \Phi_{14}^+ \otimes \Psi_{25}^+ \otimes \Phi_{36}^+ \). Since Charlie’s measurement result is secret (not publicly declared), the eavesdropper may pick the correct state only with a probability of 1/4. If the information on the encoding scheme is not available to the eavesdropper, there is no way for the eavesdropper to find the correct classical bits. So it is necessary for the two pairs, Alice and Charlie, and Bob and Charlie, to keep their respective encoding schemes Eq.5 and Eq.6 in private. In order to achieve privacy safely, Alice and Charlie, and Bob and Charlie may use secret keys generated by shared GHZ states to communicate the encoding method with each other. Since Alice, Bob and Charlie want to achieve simultaneous mutual communications in secret among different pairs of parties, one for Alice and Charlie and another for Bob and Charlie, they three must be trustworthy and cooperative. Two communication parties Alice and Charlie (Bob and Charlie) can generate secret key used to transmit their encoding scheme via initially shared pairs of GHZ states with the help of the third pair Bob (Alice). The details of generating secret keys are as follows. Suppose Alice wants to send Charlie her encoding scheme. Each of Alice, Bob and Charlie performs a Bell measurement on their respective particles 1 and 4, 2 and 5, and 3 and 6, obtaining one of four possible results, \( \Phi^+, \Phi^-, \Psi^+ \) and \( \Psi^- \). Bob tells Alice and Charlie of his measurement outcome. Depending on Bob’s information, Alice and Charlie can infer the measurement result of each other. Alice and Charlie agree on that each of the four Bell
states carry two bits classical message (there are $4! = 24$ kinds of encoding methods, they can choose one kind at random) and regard the information carrying by either Alice’s measurement results or Bob’s measurement results as secret key bits used to transmit their encoding scheme. For instance, if the original state is $|R^−⟩_{123}|P^+⟩_{456}$, and the outcome of Bob’s measurement is $Ψ^+_{25}$, then according to her measurement result $Φ^+_{14}$, Alice can infer that the outcome of Charlie’s measurement must be $Φ^−_{36}$. Similarly, Charlie can deduce Alice’s measurement result $Φ^+_{14}$ from his measurement outcome $Φ^−_{36}$. If Alice and Charlie encode $Φ^+_{14}$, $Φ^+_{14}$, $Ψ^+_{14}$, and $Ψ^−_{14}$ as 00, 01, 10, and 11, then they share two classical bits 01. Alice and Charlie sacrifice some randomly selected bits to test the “error rate”. If the error rate is too high, they abort this QKD protocol. Otherwise, they perform information reconciliation and privacy amplification \[36, 37, 38, 39, 40, 41, 42\] on the remaining bits to obtain secure final key bits for Alice informing Charlie of the encoding scheme Eq.(5). Using the same method, Bob and Charlie get secret key for Bob sending the encoding scheme Eq.(4) to Charlie. Thus, in our QSDC scheme, the eavesdropper can not get the encoding scheme of the classical bits on the unitary operators. That is, if the eavesdropper understands that the operator Alice has applied is $σ_z$, she does not know that this corresponds to the classical bits 11. Therefore, our scheme is a deterministic QSDC scheme.

Remark 2: In order to protect the transmitting information from the eavesdropper, Alice, Bob, and Charlie can make use of classical error correction protocol \[38\]. That is, Alice and Bob encode their secret message and Charlie decode these message according to a pre-determined classical error correction protocol.

Note: (A) The above protocol is also a quantum key distribution (QKD) scheme based on GHZ states and entanglement swapping. If the communication parties want to distribute keys, Alice and Bob randomly generate their respective classical bit strings $a$ and $b$, and then Alice divides her string $a$ in length of two bits and encode by applying the unitary operators on her qubits and in the same way Bob applies his operator for each classical bit of $b$. Alice, Bob, and Charlie agree upon in advance that each of the four Bell states can carry two bits classical information and encode $Φ^+, Φ^−, Ψ^+, and Ψ^−$ as 00, 01, 10, and 11, respectively. Protocol then follows before. By Alice’s measurement result $Φ^+_{14}$, only both Alice and Charlie derive $Φ^+_{14} → Φ^+_{14}$, i.e. Alice and Charlie obtain $σ_{11}$ and $Φ^−_{14}$ secretly. Since Alice’s operator $σ_{11}$ is certain and her measurement result $Φ^+_{14}$ is random, Alice and Charlie share two certain bits 11 and two random bits 01 in private. Similarly, from Bob’s measurement outcome $Ψ^−_{25}$, Bob and Charlie obtain $Φ^−_{25} → Ψ^−_{25}$ and share one certain bit 1 and two random bits 01 privately. Therefore, in our proposed protocol, Alice and Bob perform one local operation on their respective particles 1 and 2, Charlie shares 2 certain bits and 2 random bits with Alice, and 1 certain bit and 2 random bits with Bob secretly.

(B) Bob can also apply unitary operator $I = |0⟩⟨0| + |1⟩⟨1|$ and $iσ_y = |0⟩⟨1| − |1⟩⟨0|$, and he and Charlie agree beforehand as the encoding: $I → 0$, $iσ_y → 1$, instead of that in the above protocol.

(C) Particles 1 and 2 play symmetric and equal role. That is, Alice (Bob) can use one local operation in Eq.(4) (Eq.(3)) and transmit one bit (two bits) information to Charlie.

(D) There are $4! = 24$ ( $2! = 2$ ) kinds of encoding method for one assigning two bits to local operations $I, σ_z, iσ_y$ and $σ_z$ ( one bit to $I$ and $σ_z$ ). Two communication parties can choose randomly one kind as their encoding scheme.

3. Security

The security of these schemes are limited with the quality of the quantum channel between the parties. We base our argument of security on perfect quantum channel (that is, the shared GHZ states between the parties are maximally entangled and free of noise). Since the communication parties are spatially separated, and one can not distinguish the noise introduced by the eavesdropper and the noise induced during the preparation and distribution phases, after generating and distributing such states, the parties may share an ensemble of noisy GHZ states. In order to share perfect quantum information channel, they first purify noisy GHZ states and then test the security of quantum channel. Suppose that the three communication parties share an ensemble of $N$ identical mixed
multi-partite states, they can obtain perfect GHZ states by using efficient multipartite entanglement distillation protocol—multi-party hashing method [13] and its improvement [14]. After that, the parties verify if they share perfect maximally entangled GHZ states. They can utilize the similar method in [32] to do the tests. In fact, as long as the states taking as quantum information channel are the eigenvector of $\sigma_x \otimes \sigma_x \otimes \sigma_x$, $\sigma_z \otimes \sigma_z \otimes I$ and $\sigma_z \otimes I \otimes \sigma_z$, then the quantum channel is perfect [14].

The procedure obtaining perfect GHZ states by using efficient multipartite entanglement distillation protocol—multi-party hashing method [13] and its improvement [14] is as follows. Suppose three parties Alice, Bob, and Charlie share an ensemble of $N$ identical mixed tripartite states $\rho$ and they would like to distill out perfect GHZ states $|P^+\rangle$. The GHZ state $|P^+\rangle$ is the +1 eigenstate of the following set of commuting observables:

$$
S_0 = \sigma_x \otimes \sigma_x \otimes \sigma_x,
$$
$$
S_1 = \sigma_z \otimes \sigma_z \otimes I,
$$
$$
S_2 = \sigma_z \otimes I \otimes \sigma_z.
$$

Denote GHZ states in Eqs. (2) by

$$
|\text{GHZ}_{p,i_1,i_2}\rangle_{ABC} = \frac{1}{\sqrt{2}}(|0\rangle |i_1\rangle |i_2\rangle + (-1)^p |1\rangle |\bar{i}_1\rangle |\bar{i}_2\rangle),
$$

(10)

where $p$ and the $i$'s are zero or one and a bar over a bit value indicates its logical negation. Here, the three labels $(p, i_1, i_2)$ correspond to the eigenvalues of the 3 stabilizer generators $S_0, S_1, S_2$ by correspondence relation:

- eigenvalue $1$ $\rightarrow$ label 0,
- eigenvalue $-1$ $\rightarrow$ label 1.

According to [15] [16], Alice, Bob, and Charlie can depolarize 3-party density matrix $\rho$ by the following steps. They three perform the operator $\sigma_x \otimes \sigma_x \otimes \sigma_x$ with a probability 1/2, and then apply $\sigma_z \otimes \sigma_z \otimes I$ with a probability 1/2. Finally, they also apply $\sigma_z \otimes I \otimes \sigma_z$ with a probability 1/2. The overall operation corresponds to

$$
\rho \rightarrow \rho_{ABC} = \frac{1}{8} (\rho + (\sigma_x \otimes \sigma_x \otimes \sigma_x)\rho(\sigma_x \otimes \sigma_x \otimes \sigma_x) + (\sigma_z \otimes \sigma_z \otimes I)\rho(\sigma_z \otimes \sigma_z \otimes I) + (\sigma_y \otimes \sigma_y \otimes \sigma_y)\rho(\sigma_y \otimes \sigma_y \otimes \sigma_y) + (I \otimes \sigma_z \otimes \sigma_z)\rho(I \otimes \sigma_z \otimes \sigma_z) + (\sigma_x \otimes \sigma_y \otimes \sigma_y)\rho(\sigma_x \otimes \sigma_y \otimes \sigma_y)).
$$

(11)

The overall operation makes $\rho$ diagonal in the basis Eq. (10) by the following form:

$$
\rho_{ABC} =
\begin{pmatrix}
    p_{000} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & p_{100} & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & p_{011} & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & p_{111} & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & p_{010} & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & p_{101} & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & p_{001} & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{101}
\end{pmatrix},
$$

(12)

without changing the diagonal coefficients. Thus, three party Alice, Bob, and Charlie share a large ensemble of a density matrix, $\rho_{ABC}$, that is GHZ-diagonal. They can estimate its matrix elements reliably by using local operations and classical communications (LOCCs) only. Measuring along $X, Y, Z$ basis and comparing the results of their local measurements, they can estimate the diagonal matrix elements in [12] by applying classical random sampling theory. [This is due to the commuting observable argument in [17].] By definition, any GHZ-basis vector in Eq. (10) is a simultaneous eigenvector of the 7 non-trivial stabilizer group elements $\sigma_x \otimes \sigma_x \otimes \sigma_x$, $\sigma_z \otimes \sigma_z \otimes I$, $\sigma_z \otimes I \otimes \sigma_z$. [This is due to the commuting observable argument in [17].]
\( \sigma_z \otimes I \otimes \sigma_z, -\sigma_y \otimes \sigma_y \otimes \sigma_z, I \otimes \sigma_z \otimes \sigma_z, -\sigma_y \otimes \sigma_x \otimes \sigma_y, \) and \( -\sigma_x \otimes \sigma_y \otimes \sigma_y. \) If the error rates for all of the 7 non-trivial group elements are denoted \( s_1, \ldots, s_7, \) then

\[
\begin{align*}
    p_{000} &= 1 - \frac{1}{4}(s_1 + s_2 + s_3 + s_4 + s_5 + s_6 + s_7), \\
    p_{100} &= \frac{1}{4}(s_1 - s_2 - s_3 + s_4 - s_5 + s_6 + s_7), \\
    p_{011} &= \frac{1}{4}(-s_1 + s_2 + s_3 + s_4 - s_5 + s_6 - s_7), \\
    p_{111} &= \frac{1}{4}(s_1 + s_2 + s_3 - s_4 - s_5 - s_6 + s_7), \\
    p_{010} &= \frac{1}{4}(-s_1 + s_2 - s_3 + s_4 + s_5 - s_6 + s_7), \\
    p_{110} &= \frac{1}{4}(s_1 + s_2 - s_3 - s_4 + s_5 + s_6 - s_7), \\
    p_{001} &= \frac{1}{4}(-s_1 - s_2 + s_3 - s_4 + s_5 + s_6 + s_7), \\
    p_{101} &= \frac{1}{4}(s_1 - s_2 + s_3 + s_4 + s_5 - s_6 - s_7).
\end{align*}
\]

(13)

Since \( s_1, \ldots, s_7 \) can be determined by local operations and classical communications (LOCCs) by Alice, Bob and Charlie, the above equations relate the diagonal matrix element of the density matrix, \( \rho_{ABC} \), to experimental observables.

Maneva and Smolin [43] constructed an efficient multi-partite entanglement distillation protocol—multi-party hashing method and showed that its yield (per input mixed state):

\[
D_h = 1 - \max_{j>0} \{|H(b_j)|\} - H(b_0).
\]

(14)

Here \( b_0 \) is formed by concatenating the unknown phase bits of all \( \rho_{ABC} \) while \( b_j \) are formed by concatenating the \( j \)th amplitude bits, and

\[
\begin{align*}
    H(b_0) &= -\sum_{b_0=0,1} (\sum_{b_1,b_2=0,1} p_{b_0 b_1 b_2}) \log_2(\sum_{b_1,b_2=0,1} p_{b_0 b_1 b_2}), \\
    H(b_1) &= -\sum_{b_1=0,1} (\sum_{b_0,b_2=0,1} p_{b_0 b_1 b_2}) \log_2(\sum_{b_0,b_2=0,1} p_{b_0 b_1 b_2}), \\
    H(b_2) &= -\sum_{b_2=0,1} (\sum_{b_0,b_1=0,1} p_{b_0 b_1 b_2}) \log_2(\sum_{b_0,b_1=0,1} p_{b_0 b_1 b_2}).
\end{align*}
\]

(15)

Therefore, If \( D_h > 0 \), using Maneva and Smolin’s multi-party hashing method, Alice, Bob, and Charlie can distill out \( N'D_h \) perfect (generalized) GHZ states \( |P^+\rangle \). Chen and Lo [43] presented an improved hashing protocol and proved that its yield can be increased to:

\[
D_h' = 1 - \max \{ H(b_1), H(b_2|b_1) \} - H(b_0) + I(b_0; b_1, b_2).
\]

(16)

With the improved random hashing method of Chen and Lo, Alice, Bob, and Charlie can distill out \( N'D_h' \) perfect (generalized) GHZ states \( |P^+\rangle \) if \( D_h' > 0 \).

The only place an eavesdropping can affect the system is the distribution phase of the GHZ states between the communication parties. If the eavesdropper couples her ancilla states during preparing or distribution GHZ state, the communication parties can find out her by the method of [42], and remove the entanglement between eavesdropper’s particles and GHZ tripartite by multi-party hashing method [43]. That is, by testing the security of quantum channel, the eavesdropper can be detected, and as long as \( D_h > 0 \) (\( D_h' > 0 \)), the three communication parties can get perfect GHZ states. However, by testing the security of quantum channel, if \( D_h = 0 \) (\( D_h' = 0 \)), Alice, Bob and Charlie discard the quantum channel and construct it again. In one word, under any case, as long
as an eavesdropper exists, we can find her and ensure the safety of quantum channel. Once the security of the quantum channel is assured, which means that Alice, Bob and Charlie share pure GHZ triplets (perfect quantum channel), then no information is leaked to Eve. Hence our proposed protocol is secure, even if the shared quantum channels are public.

4. Summary

We present a new deterministic secure method for direct communication by GHZ states and swapping quantum entanglement, where the three spatially separated parties faithfully transmit secret messages and detect eavesdroppers by the correlations of entanglement swapping results. In our scheme the secret messages can be encoded directly and are faithfully transmitted from two senders Alice and Bob to a remote receiver Charlie at the same time via initially shared GHZ states without revealing any information to a potential eavesdropper. The distributed entangled particles shared by Alice, Bob, and Charlie function as a quantum information channel for faithful transmission. Using $2N$ GHZ states, Alice can send $2N$ bits secret messages to Charlie, meanwhile, Bob can also transmit $N$ bits information to Charlie. Since there is not a transmission of the qubit carrying the secret message between Alice and Bob, and Charlie in the public channel, it is completely secure for direct secret communication if perfect quantum channel is used. That is, simultaneous many mutual QSDC of the central party and other two parties can be realized.

Acknowledgments

This work was supported by Hebei Natural Science Foundation under Grant No. A2004000141 and No. A2005000140, and Natural Science Foundation of Hebei Normal University of China.

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