Isospin effects on the mass dependence of balance energy

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We study the effect of isospin degree of freedom on balance energy throughout the mass range between 50 and 350 for two sets of isotopic systems with N/Z = 1.16 and 1.33 as well as isobaric systems with N/Z = 1.0 and 1.4. Our findings indicate that different values of balance energy for two isobaric systems may be mainly due to the Coulomb repulsion. We also demonstrate clearly the dominance of Coulomb repulsion over symmetry energy.

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I. INTRODUCTION

The investigation of the system size effects in various phenomena of heavy-ion collisions has attracted a lot of attention. For example, in the low-energy regime where phenomena like fusion, fission, cluster radioactivity, formation of super heavy nuclei, etc.,\textsuperscript{1} take place, the contribution of the Coulomb force toward barrier has been reported to scale with the mass and charge of the colliding nuclei\textsuperscript{2}. Similarly, the system size dependences have been reported in various other phenomena like particle production, multifragmentation, collective flow (of nucleons/fragments), density, temperature and so on. For instance, in Ref.\textsuperscript{3} the power law scaling ($\propto A^\tau$) of pion/kaon production with the size of the system has been reported. Similar power law behavior for the system size dependence has been reported for the multiplicity of various types of fragments also\textsuperscript{4}. The collective transverse in-plane flow which reflects the competition between attractive and repulsive interactions has been investigated extensively during the past three decades and has been found to depend strongly on the combined mass of the system\textsuperscript{5,6,7} as well as colliding geometry\textsuperscript{7}. The energy dependence of collective transverse in-plane flow has led us its disappearance. It has now been well established that there exists a particular incident energy at which the attractive and repulsive parts of nuclear interactions counterbalance each other and the net flow disappears. This energy has been termed as the balance energy ($E_{bal}$) or the energy of vanishing flow (EVF)\textsuperscript{8,9}. $E_{bal}$ has been found to depend strongly on the combined mass of the system. A power law mass dependence ($\propto A^\tau$) of $E_{bal}$ also has been reported\textsuperscript{10,12,13}. Earlier power law parameter $\tau$ was supposed to be close to -1/3 (resulting from the interplay between attractive mean field and repulsive nucleon-nucleon (nn) collisions)\textsuperscript{10}, whereas recent studies showed a deviation from the above-mentioned power law\textsuperscript{11,14} where $\tau$ was close to -0.45. Recently Sood and Puri\textsuperscript{12}, reported the power law mass dependence $\propto 1/\sqrt{A}$ for heavier nuclei which suggested the increasing importance of the Coulomb interactions. Another interesting study of the mass dependence of density and temperature reveals that the maximum temperature is insensitive toward the combined mass of the system, whereas maximum density scales with the size of the system\textsuperscript{13}. However, Sood and Puri\textsuperscript{16} recently suggested the power law mass dependence for the temperature at $E_{bal}$, which was later confirmed experimentally by Wang et al.\textsuperscript{17}. In another study Sood and Puri\textsuperscript{18} studied the effect of momentum-dependent interactions (MDI) on the collective flow as well as its disappearance through out the mass range (from $^{12}\text{C}$ + $^{12}\text{C}$ to $^{197}\text{Au}$ + $^{197}\text{Au}$). They found that the impact of MDI differs in lighter nuclei as compared to the heavier ones.

With the availability of high intensity radioactive beams at many facilities\textsuperscript{18}, the effects of isospin degree of freedom in nuclear reactions can be studied in more details over a wide range of masses at different incident energies and colliding geometries. Such studies can help to isolate the isospin dependent part of the nuclear matter equation of state (EOS) which is vital to understand the astrophysical phenomena such as neutron stars, supernovae explosions, and so on. In our recent communication\textsuperscript{19}, we have studied the $E_{bal}$ for $^{58}\text{Ni}$ + $^{58}\text{Ni}$ and $^{58}\text{Fe}$ + $^{58}\text{Fe}$ data\textsuperscript{20}. Our calculations are able to reproduce the experimental data within 3% on the average over all colliding geometries (guided by Ref.\textsuperscript{20}). The good agreement of our calculations with the data motivated us to study the isospin effects on the $E_{bal}$ throughout the mass range. It is worth mentioning here that the isospin-dependent quantum molecular dynamics (IQMD) model has also been able to reproduce the other data (e.g. high-energy proton spectra, gamma production) in the incident energies relevant in this article\textsuperscript{21,22}. The present aim is atleast twofold.

1) To study the effect of isospin degree of freedom on the $E_{bal}$ throughout the mass range.

2) As reported in literature, the isospin dependence of collective flow has been explained as the competition
among various reaction mechanisms, such as nucleon-nucleon collisions, symmetry energy, surface property of the colliding nuclei, and Coulomb force. The relative importance among these mechanisms is not yet clear. In the present study, we aim to shed light on the relative importance among the above-mentioned reaction mechanisms by taking pairs of isotopic as well as isobaric systems throughout the mass range. Section II describes the model in brief. Section III explains the results and discussion and Sec. IV summarizes the results.

II. THE MODEL

The present study is carried out within the framework of IQMD model. In the IQMD model, each nucleon propagates under the influence of mutual two- and three-body interactions. The propagation is governed by the classical equations of motion:

\[ \ddot{r}_i = \frac{\partial H}{\partial \dot{r}_i}, \]  
where \( H \) stands for the Hamiltonian which is given by:

\[ H = \sum_i \frac{\dot{p}_i^2}{2m_i} + \sum_i (V_i^{Sky} + V_i^{Yuk} + V_i^{Coul} + V_i^{MDI} + V_i^{sym}). \]  

The \( V_i^{Sky}, V_i^{Yuk}, V_i^{Coul}, V_i^{MDI}, \) and \( V_i^{sym} \) are, respectively, the Skyrme, Yukawa, Coulomb, momentum dependent interactions (MDI), and symmetry potentials. The MDI are obtained by parameterizing the momentum dependence of the real part of the optical potential. The final form of the potential reads as [22]

\[ U_{MDI} \approx t_4 n^2 |t_5 (p_1 - p_2)^2 + 1| \delta(r_1 - r_2). \]  

Here \( t_4 = 1.57 \) MeV and \( t_5 = 5 \times 10^{-4} \) MeV\(^{-2} \). A parameterized form of the local plus MDI potential is given by

\[ U = \alpha \left( \frac{\rho}{\rho_0} \right)^\gamma + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma + \delta \ln^2 \left( \epsilon (\rho/\rho_0)^{2/3} + 1 \right) \frac{\rho}{\rho_0}. \]  

The parameters \( \alpha, \beta, \gamma, \delta, \) and \( \epsilon \) are listed in Ref. [22].

III. RESULTS AND DISCUSSION

For the present study, we simulate the various reactions in the incident energy range between 40 and 150 MeV/nucleon. In particular, we simulate the reactions \( ^{26}\text{Mg} + ^{26}\text{Mg}, ^{65}\text{Zn} + ^{65}\text{Zn}, ^{91}\text{Mo} + ^{91}\text{Mo}, ^{117}\text{Xe} + ^{117}\text{Xe}, ^{164}\text{Os} + ^{164}\text{Os} \) having \( N/Z = 1.16 \) and reactions \( ^{28}\text{Mg} + ^{28}\text{Mg}, ^{70}\text{Zn} + ^{70}\text{Zn}, ^{98}\text{Mo} + ^{98}\text{Mo}, ^{126}\text{Xe} + ^{126}\text{Xe}, \) \( ^{177}\text{Os} + ^{177}\text{Os} \) having \( N/Z = 1.33 \), respectively, at semicentral impact parameter range 0.35 - 0.45. The \( N/Z \) for a given pair is varied by adding the neutron content only keeping the charge fixed, so that the effect of the Coulomb potential is same for a given mass pair. However this will lead to the increase in mass for systems with higher neutron content. In Ref. [21], there is evidence that \( N/Z \) is an order parameter. We use a soft equation of state with MDI labeled SMD. We also use anisotropic standard isospin- and energy dependent nucleon-nucleon cross section \( \sigma = 0.8 \sigma_{N/F}^{free} \). The details about the elastic and inelastic cross sections for proton-proton and proton-neutron collisions can be found in Ref. [24, 28]. The cross sections for neutron-neutron collisions are assumed to be equal to the proton-proton cross sections. Some studies even took constant cross section [29]. It is worth mentioning that the results with the above choice of equation of state and cross section were in good agreement with the data [19]. The choice of reduced cross section has also been motivated by Ref. [30] as well as many previous studies [31]. In Ref. [19], we found the effect of different cross sections on the \( E_{bal} \) to be negligible. Recently, Sood and Puri [14] have discussed the role of different cross sections on the \( E_{bal} \) throughout the mass range between 47 and 394. They found the effect of different cross sections to be consistent throughout the mass range. The largest cross section gives the more positive flow (hence smaller \( E_{bal} \)) followed by the second largest cross section. Similar results were obtained in Ref. [19]. The reactions are followed until the transverse flow saturates. The saturation time varies form 100 fm/c for lighter masses to 300 fm/c for heavier masses. For the transverse flow, we use the quantity “directed transverse momentum \( p_{x}^{d\text{ir}} \)” which is defined as [12, 13, 14, 32]

\[ \langle p_{x}^{d\text{ir}} \rangle = \frac{1}{A} \sum_{i=1}^{A} \text{sign}(y(i)) p_{x}(i), \]  

where \( y(i) \) is the rapidity and \( p_{x}(i) \) is the momentum of \( i \)th particle. The rapidity is defined as

\[ Y(i) = \frac{1}{2} \ln \frac{E(i) + p_{z}(i)}{E(i) - p_{z}(i)}, \]  

where \( E(i) \) and \( p_{z}(i) \) are, respectively, the energy and longitudinal momentum of \( i \)th particle. In this definition, all the rapidity bins are taken into account.

In Fig. 1, we display the \( E_{bal} \) as a function of combined mass of the system for two sets of isotopic systems with different neutron content. Closed (open) circles represent systems with less (more) neutron content. Lines are the power law fit \( \propto A^{\tau} \). As expected, both the sets of isotopic masses follow a power law behavior \( \propto A^{\tau} \), where \( \tau = -0.44 \pm 0.01 \) for systems with less neutron content (labeled as \( \tau_{1.16} \)) and \(-0.42 \pm 0.01 \) for systems with more neutron content (labeled as \( \tau_{1.33} \)). Note that the values of \( \tau_{1.16} \) and \( \tau_{1.33} \) are very close to the previous values of
$\tau$ in Ref. [11]. Interestingly, isospin effects are not visible for any mass system. As reported in literature [20, 23], system with more neutron content has higher $E_{\text{bal}}$ which has been attributed mainly to the above-mentioned reaction mechanisms and to the fact that neutron-neutron or proton-proton cross section is a factor of 3 lower than the neutron-proton cross section. However, in our case system size effects seem to dominate the isospin effects throughout the mass range. It is worth mentioning that the effect of the Coulomb force is almost the same for a given pair of masses.

As a next step, we take the pairs of isobars with $N/Z = 1.0$ and 1.4. We simulate the reactions $^{24}\text{Mg} + ^{24}\text{Mg}, ^{58}\text{Cu} + ^{58}\text{Cu}, ^{72}\text{Kr} + ^{72}\text{Kr}, ^{96}\text{Cd} + ^{96}\text{Cd}, ^{120}\text{Nd} + ^{120}\text{Nd}, ^{135}\text{Ho} + ^{135}\text{Ho}$, having $N/Z = 1.0$ and reactions $^{24}\text{Ne} + ^{24}\text{Ne}, ^{58}\text{Cr} + ^{58}\text{Cr}, ^{72}\text{Zn} + ^{72}\text{Zn}, ^{96}\text{Zr} + ^{96}\text{Zr}, ^{120}\text{Sn} + ^{120}\text{Sn}$, and $^{135}\text{Ba} + ^{135}\text{Ba}$, having $N/Z = 1.4$, respectively. Here $N/Z$ for a given pair is changed by varying both the proton and neutron content keeping the mass fixed. In Fig. 2(a), we display the $E_{\text{bal}}$ as a function of combined mass of the system for the two sets of isobars. We also display the experimental data [20] and our corresponding theoretical calculations [19] (both displaced horizontally for clarity) in the range of colliding geometry used in the present study. Solid (open) stars represent the experimentally measured $E_{\text{bal}}$ for the reaction $^{58}\text{Ni} + ^{58}\text{Ni} (^{58}\text{Fe} + ^{58}\text{Fe})$ having $N/Z = 1.07 (1.23)$ in the impact parameter bin $0.28 < b/b_{\text{max}} < 0.39$ (guided by Ref. [20]). Pentagons represent the corresponding results of our theoretical calculations. Clearly our results are in good agreement with the data. Our theoretical calculations are able to reproduce the experimentally measured $E_{\text{bal}}$ for both the systems. Solid and open green circles represent the $E_{\text{bal}}$ for systems with less and more neutron content, respectively. Lines are power law fit $\propto A^{k}$. Interestingly, throughout the mass range, a more neutron-rich system has a higher $E_{\text{bal}}$. The calculated $E_{\text{bal}}$ fall on the line that is a fit of power law nature $(\propto A^{k})$, where $\tau = -0.45 \pm 0.01$ and $-0.50 \pm 0.01$ for $N/Z = 1.4$ and 1.0 (labeled by $\tau_{1.4}$ and $\tau_{1.0}$), respectively. The difference of two curves can be attributed to the larger role of Coulomb force in the case of systems with more proton (less neutron) content. Our value of $\tau_{1.4}$ is equal/close to the value $-0.45/-0.42$ in Ref. [11] both of which show deviation from the standard value $\simeq -1/3$ where analysis was done for lighter mass nuclei only ($\leq 200$). However, when heavier systems like $^{139}\text{La} + ^{139}\text{La}$ and $^{197}\text{Au} + ^{197}\text{Au}$ were included, $\tau$ increased to $-0.45$ [11], suggesting the increasing importance of Coulomb repulsion. A further analysis in Ref. [12] showed that when only heavier nuclei were taken into account, the value of $\tau$ increased to $-0.53$ which is very close to our value of $\tau_{1.0} (-0.50)$ in the present case. Although the mass range in the present study and in Ref. [12] are differ substantially the present power law parameter $\tau_{1.0}$ is very close to the power law parameter of Ref. [12] which can be attributed to the extra Coulomb repulsion in case of systems with more proton (less neutron) content as well as Ref. [12]. This indicates that the difference in the $E_{\text{bal}}$ for a given pair of isobaric systems may be dominantly due to the Coulomb potential, which is further supported by the fact that since both asymmetry energy and nucleon-nucleon cross section add to the repulsive interactions, so both lead to the reduction of $E_{\text{bal}}$. In systems with more neutron content, role of asymmetry energy could be larger, whereas effects due to isospin-dependent cross section could play a dominant role in systems with less neutron content (more proton content). Therefore, there is a possibility that the combined effect of asymmetry energy and cross section could be approximately the same for two isobaric systems with different neutron and proton content.

To demonstrate the dominance of Coulomb, we have also calculated the $E_{\text{bal}}$ throughout the mass range in

FIG. 1: $E_{\text{bal}}$ as a function of combined mass of system. Various symbols and lines are explained in the text.

FIG. 2: (a) $E_{\text{bal}}$ as a function of combined mass of system. (b) The percentage difference $\Delta E_{\text{bal}}(\%)$ as a function of combined mass of system. Solid (open) symbols are for $N/Z = 1.0 (1.4)$. Various symbols and lines are explained in the text.
The present study for isobaric pairs with Coulomb being reduced by a factor of 100. The results are displayed in Fig. (a) with solid and open (orange) diamonds representing systems with less and more neutron content, respectively. Lines represent power law fit \( \propto A^\gamma \). One can clearly see the dominance of Coulomb repulsion in both the mass dependence as well as in isospin effects. The value of \( \tau_{1,4} \) and \( \tau_{1,0} \) are now, respectively, -0.28 ± 0.02 and -0.25 ± 0.02. There is large enhancement in the \( E_{\text{bal}} \) for medium (e.g., \( ^{58}\text{Cu} + ^{58}\text{Cu} \)) and heavy \( (^{120}\text{Nd} + ^{120}\text{Nd}) \) mass systems thus reducing the value of both \( \tau_{1,0} \) and \( \tau_{1,4} \). The effect is small in lighter masses \( (^{24}\text{Mg} + ^{24}\text{Mg}) \). One should also note that when we have full Coulomb included in our calculations, for medium and heavy mass systems, \( E_{\text{bal}} \) is less for systems having more proton (less neutron) content. However, the trend is reversed when we reduce the Coulomb. Now the systems with more neutron content have less \( E_{\text{bal}} \). This is because of the fact that the reduced Coulomb repulsion leads to higher \( E_{\text{bal}} \). As a result, the density achieved during the course of the reaction will be more due to which the impact of the repulsive symmetry energy will be more in neutron-rich systems, which in turn leads to less \( E_{\text{bal}} \) for neutron-rich systems and hence to the opposite trend for \( \tau_{1,4} \) and \( \tau_{1,0} \) for two different cases (Coulomb full and reduced). To check this point we have also calculated the \( E_{\text{bal}} \) by reducing the strength of both symmetry energy as well as Coulomb potential for isobaric pairs with combined mass of the system = 116 and 270 (shown by blue triangles in Fig. 2(a)). We find that both the systems of a given isobaric pair have same \( E_{\text{bal}} \). The above discussion clearly points towards the dominance of Coulomb repulsion over asymmetry energy for medium as well as heavy mass systems, whereas their impact is small in lighter masses.

In Fig. 2(b), we display the percentage difference \( \Delta E_{\text{bal}}(\%) \) between the systems of isobaric pairs as a function of combined mass of system where \( \Delta E_{\text{bal}}(\%) = \frac{E_{\text{bal}}^{1,4} - E_{\text{bal}}^{1,0}}{E_{\text{bal}}^{1,4}} \times 100 \). Superscripts to the \( E_{\text{bal}} \) represent different N/Z. Half filled (green) circles are for full Coulomb and half filled (orange) diamonds are for reduced Coulomb. Negative (positive) values of \( \Delta E_{\text{bal}}(\%) \) shows that the \( E_{\text{bal}}^{1,0} \) is more (less) than \( E_{\text{bal}}^{1,4} \). From Fig. 2(b) (circles), we see that the percentage difference between the two masses of a given pair is larger for heavier masses as compared to the lighter ones. In lighter masses, the magnitude of Coulomb repulsion is small so there is small difference, whereas in heavier masses, due to the large magnitude of Coulomb repulsion, there is a large difference in the \( E_{\text{bal}} \) for a given pair of isobars. However, this trend is not visible when we reduce the Coulomb (diamonds). The values of \( \Delta E_{\text{bal}}(\%) \) is almost constant for medium and heavy masses.

In Fig. 3, we display the \( E_{\text{bal}} \) as a function of impact parameter for the system having mass 240 with N/Z = 1.0. In the present case, the \( E_{\text{bal}} \) seems to decrease with increase in impact parameter (in contrast to earlier studies where \( E_{\text{bal}} \) increases with increase in impact parameter \( \pi_{1b}^{2b} \), since at a higher impact parameter there will be large transverse flow of nucleons due to the dominant Coulomb repulsion. This could also play a part for larger difference in the \( E_{\text{bal}} \) at higher colliding geometries \( \pi_{19}^{20} \) for systems with different neutron and proton content. It is worth mentioning that dominance of Coulomb repulsion in isospin effects has also been reported by Ref. [30].

IV. SUMMARY

We have studied the effect of isospin degree of freedom on the \( E_{\text{bal}} \) throughout the mass range for two sets of isotopic systems with N/Z = 1.16 and 1.33 as well as isobaric systems with N/Z = 1.0 and 1.4. Our results indicate that the difference between the \( E_{\text{bal}} \) for the two isobaric systems may be mainly due to the Coulomb repulsion. We have also shown clearly the dominance of Coulomb repulsion over symmetry energy. Our findings also point that the larger magnitude of isospin effects in \( E_{\text{bal}} \) at peripheral collisions as compared to central collisions may be dominantly due to the Coulomb repulsion.

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