Status of the search for $\eta$-mesic nuclei with particular focus on $\eta$-Helium bound states

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Abstract In this paper the search for $\eta$-mesic nuclei with particular focus on light $\eta$-He bound states is reviewed. A brief description of recent theoretical studies and experimental results is presented.

Keywords mesic nuclei · $\eta$ meson

1 Introduction

Recent meson-nucleon and meson-nuclei interactions studies have been mainly focused on aspects connected to the existence of the bound systems, like mesonic atoms and mesic nuclei. The mesonic atoms, where an electron is replaced by negatively charged meson ($\pi^-$ or $K^-$), predicted already in 1940s [1,2,3] have been discovered [4,5] and investigated in many precise experiments [6,7,8,9,10,11,12,13,14]. The studies performed for deeply bound pionic atoms [6,7] allowed one to deduce the $s$-wave pion-nucleus potential and the effective $\pi^-$ mass in nuclear medium by comparing the experimental binding energies and widths with theoretical values obtained for different optical potential parameters. The most precise measurements for light kaonic atoms (the shift and width of the 1s level, with respect to the purely electromagnetic calculated values), like kaonic hydrogen and kaonic helium, have been performed by the SIDDHARTA experiment at DAΦNE [10,11,12], providing strong constraints on existing theories [15,16,17,18,19,20]. The ongoing high-precision SIDDHARTA-2 experiment aims to measure for the first time...
ever the kaonic deuterium atom and then to extract, using also the kaonic hydrogen measurement result, antikaon-nucleon isospin-dependent scattering lengths, crucial ingredients for models dealing with low-energy QCD in the strangeness sector [15,16,17,18,19,20,21].

In contrary to mesonic atoms where Coulomb effects play a dominant role, mesic nuclei are bound systems with the dominance of the strong interaction or the purely strong interaction objects, for charged (K+, K−) and neutral mesons (η, η′, K0, ω, φ), respectively. The nuclear bound states for a charged meson in the form of “K−pp” clusters has been recently discovered and investigated at J-PARC in the 3He(K−,Λp)n reaction [22]. However, until now no experiment has confirmed the existence of a nucleus bound via the strong interaction to a neutral meson. Therefore, this mesic nuclei issue is currently one of the hottest topic in nuclear and hadronic physics, both among the theoreticians [23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53] and experimentalists [54,55,56,57,58,59,60,61,62,63,64]. The most promising candidates for such bound systems are η(η′)-mesic nuclei. This results from the fact that the η-nucleon interaction was reported to be attractive [65,66] while the real part of η′-nucleus optical potential was found to be negative with modulus significantly higher than the imaginary part [67], though the data indicates that the η′-nucleus interaction is weaker than the η-nucleon interaction [68,69]. In this report we will focus on the η-mesic nuclei issue.

Initially, it was predicted that due to the relatively small value of the ηN scattering length estimated in [65], η-mesic bound state could be formed only with nuclei having masses A ≥ 12. However, recently performed studies on hadron induced and photo-production of η mesons resulting in a range of ηN scattering length values indicating attractive and strong enough η meson-nucleon interaction to create η-mesic nuclei even for light nuclei such as 4He, 3He, tritium and deuterium [33,34,37,38,39,70,71,72]. These bound states have been searched in many experiments but, nevertheless, no experiments have found a clear signature of their existence. The measurements allow only to observe signals which might be interpreted as indications of the hypothetical η-mesic nuclei and to determine the upper limits of the total cross section for the bound state formation [50,54,55,56,57,59,60,63,74,75,76,77,78,79,80,81,82,83,84].

The discovery of the η-mesic bound systems would be crucial for better understanding of the elementary meson-nucleon interaction in nuclear medium for low energy region. Moreover, it would provide information about η meson inside a nuclear matter as well as to explain its structure, since according to Refs. [30,31,32] the η meson binding inside nuclear matter is very sensitive to the singlet component (η-η′ mixing) in the wave function of the η meson. The model predicts η binding of -50 MeV at nuclear matter density without η-η′ mixing, while for -20° mixing angle the binding is increased by factor of 2. The investigations of η-mesic bound states can also be helpful to study N*(1535) resonance properties in medium and to test different theoretical models describing its internal structure [42,46,85,86,87,88,89,90].
This review is aimed at giving an overall status of recent theoretical studies and experimental research on \( \eta \)-mesic nuclei.

2 Theoretical overview

After the discovery of the attractive \( \eta \)-nucleon interaction the \( \eta \)-mesic nuclei issue has been considered and discussed in plenty of theoretical works. Different approaches and predictions concerning this exotic object have appeared. The most current are presented below.

2.1 Investigation of \( \eta \)-nucleon interaction

The \( \eta \) meson-nucleon interaction has been investigated since many years taking into account the possibility of bound states creation. The most proper way to study such interactions would be through investigation of elastic scattering data. However, it is impossible to create the \( \eta \) beams due to the short lifetime of the \( \eta \) (\( \tau \leq 5 \times 10^{-19} \) s). Therefore, the information about interaction between \( \eta \) meson and nucleon is extracted based on experimental data for the processes like \( \pi N \rightarrow \eta N \), \( \gamma N \rightarrow \eta N \) as well as \( NN \rightarrow NN\eta \) \((pp \rightarrow ppp \eta)\) \cite{91}, \( pn \rightarrow pnn\eta \) \cite{92}). The low energy interaction of \( \eta \) meson with recoiling nucleon is dominated by broad (150 MeV) nucleon resonance \( N^*(1535) \) located just 49 MeV above the \( \eta N \) threshold and strongly coupled to both, the \( \eta \) and the \( \pi \) (the coupling is manifested as a steep rise in the pion-nucleon cross section spectra \cite{93,94}). The \( \eta N \) scattering amplitude is then complex with imaginary part corresponding to the \( \eta \) absorption. The scattering amplitude have been determined mostly phenomenologically by performing coupled channel calculations and comparing them to the available experimental data \cite{24}. The first of such coupled channel analysis has been carried out by Bhalerao and Liu \cite{65} (including \( \eta - N \), \( \pi - N \) and \( \Delta - \pi \) channels) showing that interaction between \( \eta \) and nucleon is attractive and strong in the close to threshold region (s-wave). Until now many studies of \( \eta \) production in photon-, hadron-induced reactions have been performed based on different phenomenological and theoretical models delivering a broad spread of the \( a_{\eta N} \) scattering length value from \( a_{\eta N} = (0.18, 0.16i) \) fm to \( a_{\eta N} = (1.03, 0.49i) \) fm \cite{24,34,33}. The calculations are model dependent (huge uncertainty in the \( \eta \)-nucleus scattering amplitude), therefore \( a_{\eta N} \) "is not a useful indicator of whether or not \( \eta \) meson bind in nuclei" \cite{95}. However, the obtained results don’t exclude formation of \( \eta \)-mesic strongly bound systems even in light nuclei \cite{33,34,37,58,59,70,71,72}.

2.2 Recent theoretical/phenomenological approaches and predictions

Motivated by the discovery of Bhalerao and Liu \cite{65} many theoretical and phenomenological studies devoted to \( \eta \)-mesic nuclei have been performed \cite{23,24}.
The investigations have been carried out mostly based on construction of the optical potential used mainly for description of heavy \(\eta\)-mesic nuclei as well as based on models involving few body equations in case of \(\eta\) bound in light nuclei. One can also find other approaches such as the concept of Wigner's time delay presented in Refs. \[24,25\]. Several theoretical works have been summarised in the recent reviews \[24,59,60\].

The performed theoretical studies predict the \(\eta\)-mesic nuclei width in the range from few to about 50 MeV \[23,33,35,39,40,41,47,87,90,96,97,98\], including \(\eta\)-Helium bound states width varying in the range from 1 to 23 MeV \[23,33,39,40,41,47,96,97,99,100\]. Moreover, it is predicted that \(\eta\)-mesic nuclei widths are larger than the binding energies \[33,34,36,96,100,102\].

The most recent theoretical investigations are devoted in particular to the light \(\eta\)-mesic nuclei like \(\eta\)-Helium nuclei \[28,29,33,34,36,39,40,47,48\], which have been intensively searched for by many experimental groups (see Sec. 3). Calculations of \(\eta\)-few-nucleon systems performed by Jerusalem-Prague Collaboration \[36,39,40,41\] based on the Stochastic Variational Method (SVM), considering the Minnesota NN potential and two \(\eta\mathrm{N}\) interaction models - GW (Green & Wycech) and CS (Cieply & Smajkal), result in \(^4\mathrm{He}\)-\(\eta\) bound state for the GW model while \(^3\mathrm{He}\)-\(\eta\) issue remains questionable. In contrast, authors of Refs. \[33,34\], fitting the existed \(pd(dp)\rightarrow 3\mathrm{He}\eta\) and \(dd\rightarrow 4\mathrm{He}\eta\) data with \(\mathrm{He}-\eta\) optical potentials, report weakly bound \(^3\mathrm{He}\)-\(\eta\) state with binding energy \(B_s \approx 0.3\) MeV and a width \(\Gamma \approx 3\) MeV and do not confirm nor rule out the existence of \(^4\mathrm{He}\)-\(\eta\) nuclei. Those and other calculations are model dependent. Therefore it is not possible to resolve whether \(\eta\)-mesic Helium exists and to judge which out of the \(^4\mathrm{He}\) or \(^3\mathrm{He}\) is more probable to form a bound state. There are only experimental indications in favour of the \(^3\mathrm{He}\) system presented in Sec. 3.

Current phenomenological calculations \[26,27,28,29,47,48\] devoted to \(\eta\)-mesic Helium formation in \(dd\) and \(pd\) collisions have been performed for the purpose of recent experimental data analyses \[54,55,56\] and are shown in the next section. Two main mechanisms of hypothetical \(\eta\)-mesic Helium decay are considered: (i) assuming the \(\eta\) meson absorption on one of the nucleons inside helium, and then its possible propagation in the nucleus via consecutive excitations of nucleons to the \(N^*(1535)\) state, until the resonance decays into the \(N^*-\pi\) pair (e.g. \(dd\rightarrow (4\mathrm{He}\eta)_{\text{bound}}\rightarrow N^*\mathrm{He}3\mathrm{He}\rightarrow 4\mathrm{He}\pi\), \(pd\rightarrow (4\mathrm{He}\eta)_{\text{bound}}\rightarrow N^*-d\rightarrow dp\pi\)) and (ii) via \(\eta\) meson decay while it is still "orbiting" around a nucleus (e.g. \(pd\rightarrow (4\mathrm{He}\eta)_{\text{bound}}\rightarrow 3\mathrm{He}2\gamma\)).

In the case of the first mechanism, the kinematics of particles in the final state depends on the momentum of \(N^*\) resonance inside Helium. The first attempt which allowed for determination of the \(N^*\) resonance momentum distribution in the \(N^*-3\mathrm{He}\) and \(N^*-\mathrm{NN}\) systems has been recently performed and described in Refs. \[26,27,28,29\]. The calculations are based on construction of elementary \(NN^*\rightarrow NN^*\) interaction within a \(\pi\) plus \(\eta\) meson exchange model and then folding it with the known nuclear densities, thus leading to evaluation
of the \( N^* \)-nucleus potentials. The \( N^*-^3\text{He} \) and \( N^*-^d \) momentum distributions for two values of binding energy are shown in the left and right panels of Fig. 1 respectively. They are narrower with respect to the distribution of a neutron in \(^4\text{He} \) or proton in \(^3\text{He} \) which is due to the fact that \( N^* \) binding energy is smaller than the energy separation of nucleons in \(^4\text{He} \) and in \(^3\text{He} \). The obtained distributions have been applied in Monte Carlo simulations for recently analysed processes \( dd \rightarrow (^4\text{He}-\eta)^{\text{bound}} \rightarrow ^3\text{He}n\pi^0 \), \( dd \rightarrow (^4\text{He}-\eta)^{\text{bound}} \rightarrow ^3\text{He}p\pi^- \) and \( pd \rightarrow (^3\text{He}-\eta)^{\text{bound}} \rightarrow d\pi^0 \) [55,56,106,107].

Fig. 1 (Left) Momentum distribution of \( N^* \) (black solid and dashed lines) and neutron (red solid line) inside \(^4\text{He} \) nucleus calculated for \( N^*-^3\text{He} \) potential for binding energy \(-3.6 \text{ MeV} \) and \(-4.78 \text{ MeV} \) and \( n^-^3\text{He} \) potential with \( 20.6 \text{ MeV} \) binding energy, respectively. (Right) Momentum distribution of \( N^* \) (black solid and dashed lines) and proton (red solid line) inside \(^3\text{He} \) nucleus calculated for \( N^*-^d \) potential for binding energy \(-0.74 \text{ MeV} \) and \(-0.33 \text{ MeV} \) and \( p^-^3\text{He} \) potential with \( 5.5 \text{ MeV} \) binding energy, respectively. Figures are obtained based on Refs. [27,26,28,29].

The \( dd \rightarrow (^3\text{He}-\eta)^{\text{bound}} \rightarrow ^3\text{He}N\pi \) reactions have been also investigated by Ikeno et al. [47]. These authors provided for the first time the shapes and values of the cross sections for \( dd \rightarrow (^3\text{He}-\eta)^{\text{bound}} \rightarrow ^3\text{He}N\pi \) processes in the excess energy range relevant to the \( \eta \)-mesic nuclei search. The developed phenomenological model, reproducing quite well the data on the \( dd \rightarrow ^4\text{He}\eta \) reaction, allows to determine the total cross sections for a broad range of an \( ^4\text{He}-\eta \) optical potential parameters \( (V_0,W_0) \). The example of the calculated total cross section, for three different sets of the optical potential parameters is presented in Fig. 2. The contour plot of the determined conversion cross section in the \( V_0,W_0 \) plane is shown in Fig. 21 of Ref. [47]. As a comparison, previous calculations based on approximation of the scattering amplitude for two body processes [51] allowed one to estimate the cross section for \( dd \rightarrow (^4\text{He}-\eta)^{\text{bound}} \rightarrow ^3\text{He}p\pi^- \) process to \( \sigma \approx 4.5 \text{ nb} \). In case of \( pd \rightarrow (^3\text{He}-\eta)^{\text{bound}} \rightarrow XN\pi \) only rough estimation of the total cross section was performed based on hypothesis that the creation of the bound state below the threshold is to first order the same as the \( \eta \) meson production cross section close to threshold. It amounts to about 80 nb.
Fig. 2 Calculated total cross section of the $dd \rightarrow (^4\text{He}-\eta)_{\text{bound}} \rightarrow ^3\text{HeN} \pi$ reaction for the formation of the $^4\text{He}-\eta$ bound system plotted as function of the excess energy $E_\eta - m_\eta$ for $\eta$-$^4\text{He}$ optical potential parameters $(V_0, W_0)=-(100,5)$, $-(100,20)$, $-(100,40)$ MeV (solid, dashed and dotted lines, respectively). The figure is adapted from Ref. [47].

The second mechanism of the $\eta$-mesic helium decay, via $\eta$ meson decay while it is still "orbiting" around a nucleus has been considered recently in Ref. [48]. A first theoretical model for the $\eta$-mesic $^3\text{He}$ non-mesonic decay into $^3\text{He}2\gamma(6\gamma)$ channels has been developed describing the bound state structure as a solution of the Klein-Gordon equation. The calculations provided relative $^3\text{He}-\eta$ momentum distribution in the $^3\text{He}-\eta$ bound state as well as in-medium branching ratios of $\eta \rightarrow 2\gamma$ and $\eta \rightarrow 3\pi^0$ for different combinations of optical potential parameters. The momentum distributions determined for different sets of $(V_0, W_0)$ are shown in Fig. 3. The estimated in-medium branching ratios vary from about $2 \cdot 10^{-5}$ to $7 \cdot 10^{-4}$ depending on optical potential parameters. Results obtained in the frame of this work were crucial for the Monte Carlo simulations and interpretation of experimental data collected by the WASA collaboration [54].
Fig. 3 Fermi momentum distribution of the $\eta$ meson in $^3\text{He}-\eta$ bound system estimated for $(V_0, W_0)=-(75,20)$ MeV (thick solid line), $(V_0, W_0)=-(75,1)$ MeV (thick dotted line), $(V_0, W_0)=-(90,20)$ MeV (thin solid line), and $(V_0, W_0)=-(90,1)$ MeV (thin dotted line). The distributions are normalized to be 1 in the whole momentum range. The figure is adapted from Ref. [48].

3 Status of experimental searches for $\eta$-mesic nuclei

In parallel to the theoretical investigations, many experiments have been performed to search for $\eta$-mesic bound states, initially in heavy nuclei region and currently focusing mainly on a light nuclei. The real challenge for experimentalists was an measurement of a very weak signal of $\eta$-mesic nuclei over a large background. The measurements for heavy nuclei region carried out with pion [103,104], photon [75] and hadron beams [63,105] are reviewed in Refs. [24,59,108]. In this section we will concentrate on the recent results obtained for experiments devoted to $\eta$-mesic Helium searches.

Despite considerable effort of many theoretical groups in recent years, there are still no model independent calculations which could indicate whether or not $\eta$-mesic Helium exists. However, there are some experimental observations which may suggest the existence of the bound state, such as a steep rise in the total cross section measured for $dp\rightarrow ^3\text{He}\eta$ [73,74,76,77] and $dd\rightarrow ^4\text{He}\eta$ [109,110,111,112] reactions (Fig. 4) being a sign of very strong final state interaction (FSI). The FSI and thus the $\eta$-nucleus interaction is much stronger in the case of $^3\text{He}\eta$ system indicating that $\eta$ is more likely to bind to $^3\text{He}$ than to $^4\text{He}$. Another argument in favour of $^3\text{He}-\eta$ bound states is the small value and weak energy dependence of the tensor analysing power $T_{20}$ measured by ANKE collaboration in excess energy range $Q\in(0,11)$ MeV which confirms that the very strong variation of the $s$-wave amplitude for $dp\rightarrow ^4\text{He}\eta$ process [73,74,76,77] is associated with the strong $^3\text{He}\eta$ interaction. Moreover, the total cross section for $^3\text{He}\eta$ production is independent of the initial channel. The $^4\text{He}\eta$ hadron [76,77] and photo-production [115,116] cross section shows similar behaviour above threshold which can be assigned to the $^3\text{He}\eta$ interaction.
The first search of direct signal of a light $\eta$-nucleus bound states has been performed for the $\eta$ photoproduction process $\gamma^3\text{He} \rightarrow \pi^0pX$ by the TAPS Collaboration [115]. The difference between the excitation functions measured for two ranges of $\pi^0$-proton opening angles ($170^\circ - 180^\circ$ and $150^\circ - 170^\circ$) has shown enhancement just below the $^3\text{He}\eta$ threshold suggesting possible $^3\text{He}-\eta$ bound state production and then its decay via $\eta$ meson absorption in nucleon, $N^*(1535)$ resonance excitation and its decay into pion-nucleon pair. However, a later experiment carried out with much higher statistics [116] showed that the observed structure is an artefact derived from complicated background behaviour.

Very promising experiments related to $\eta$-mesic Helium nuclei have been performed at the COSY facility in Forschungszentrum Jülich [117]. The COSY-11 group carried out measurements to search for $\eta$-mesic $^4\text{He}$ signature in $dp \rightarrow ppm\pi^-$ and $dp \rightarrow ^4\text{He}\pi^0$ reactions in the vicinity of the $\eta$ production threshold [79,76,118]. The determined excitation functions allowed one to establish the upper limits of the total cross section to be about 270 nb and 70 nb, respectively. The search for $^4\text{He}-\eta$ and $^3\text{He}-\eta$ mesic bound systems has been recently carried out with the WASA-at-COSY detection setup using a deuteron pellet target and deuteron and proton beams, respectively. The measurements have been performed in three dedicated experiments, in 2008, 2010 and 2014, applying a unique ramped beam technique, namely, continuous and slow changes of the beam momentum around the $\eta$ production threshold in each of acceleration cycle allowing for reduction of systematic uncertainties with respect to separate runs at fixed beam energies [57,76].

The search for $^4\text{He}-\eta$ bound states was carried out via studying the excitation functions for $dd \rightarrow ^3\text{He}\pi^-$ [55,56,57] and $dd \rightarrow ^3\text{He}\pi^0$ [55,56] reactions in vicinity of the $^4\text{He}\eta$ production threshold ($Q \in (-70, 30)$ MeV). The
data analysis was performed assuming the formation of an intermediate $N^*$ resonance, and applying in Monte Carlo simulations the $N^*$ momentum distribution calculated in \cite{26,27} (see Sec. 2). The details of analysis procedures leading to determination of the excitation functions are described in Refs. \cite{56,57}. Obtained excitation curves do not show any narrow structure below the $\eta$ production threshold, which could be a signature of the bound state. Therefore, the upper limit for the total cross section for the $\eta$-mesic $^4\text{He}$ formation was determined at the 90\% confidence level by fitting the excitation functions with Breit-Wigner function (signal) with a fixed binding energy and width combined with a second order polynomial (background). For 2010 data set \cite{56} the fit was performed simultaneously for $dd \to ^3\text{He}p\pi^-$ and $dd \to ^3\text{He}n\pi^0$ channel taking into account the isospin relation between $n\pi^0$ and $p\pi^-$ pairs. The analysis allowed for the first time to determine experimentally the upper limit of the total cross section for $dd \to (^4\text{He}-\eta)_{\text{bound}} \to ^3\text{He}n\pi^0$ process, which varies in the range from 2.5 to 3.5 nb. In case of $dd \to (^4\text{He}-\eta)_{\text{bound}} \to ^3\text{He}p\pi^-$ reaction a sensitivity of the cross section of about 6 nb \cite{56} was achieved which is about four times better in comparison with the result obtained in the previous experiment \cite{57}. The obtained upper limits as a function of the bound state width are presented for both of the studied reactions in Fig. 5.

![Fig. 5](image-url)  

**Fig. 5** Upper limit of the total cross-section for $dd \to (^4\text{He}-\eta)_{\text{bound}} \to ^3\text{He}n\pi^0$ (left panel) and $dd \to (^4\text{He}-\eta)_{\text{bound}} \to ^3\text{He}p\pi^-$ (right panel) reaction as a function of the width of the bound state. The binding energy was fixed to 30 MeV. The upper limit was determined via the simultaneous fit for both channels. The green area denotes the systematic uncertainties. The figures are adapted from \cite{56}.

Due to a lack of theoretical predictions for the $dd \to (^4\text{He}-\eta)_{\text{bound}} \to ^3\text{He}N\pi$ reactions cross sections below the $\eta$ production threshold, in the previous data analyses the bound state signal was assumed to have a Breit-Wigner shape (with fixed binding energy and width) \cite{56,57}. However, recently, a phenomenological calculations of the cross sections in the excess energy range relevant to the $\eta$-mesic nuclear search were presented in Ref. \cite{47} (see Sec. 2). Fitting the theoretical spectra (convoluted with the experimental resolution of the excess energy) to experimental excitation functions \cite{56}, the upper limit of the total cross section (CL=90\%) for creation of $\eta$-mesic nuclei in the $dd \to ^3\text{He}N\pi$ reaction was found to vary from about 5.2 nb to about 7.5
Comparison of the experimentally determined upper limits with the cross sections obtained in Ref. [47] allowed one to put a constraint on the $\eta$-$^4\text{He}$ optical potential parameters. As shown in Fig. 6, only extremely narrow and loosely bound states are allowed within the model. Details of the performed studies are presented in Ref. [55].

The last high statistics WASA-at-COSY experimental run (2014) was devoted to the search for $\eta$-mesic $^3\text{He}$ considering three different mechanisms of the $\eta$-mesic bound state decay: (i) via N$^*$ (1535) resonance decay (assumed as well in previous data analysis and interpretations), (ii) via decay of $\eta$-meson which is still “orbiting” around a nucleus (see Sec. 2) and (iii) two-nucleon $\eta$ meson absorption process [51,119].

The $pd \to ^3\text{He}2\gamma$ and $pd \to ^3\text{He}6\gamma$ reactions have been investigated for the first time aiming to search for $\eta$-mesic $^3\text{He}$ with an assumption of the recently developed theoretical model [48] (details in Sec. 2). The obtained excitation functions for both channels show slight sign of the signal from the bound state for width greater than 20 MeV and binding energy in the range from 0 to 15 MeV. However, the observed indication is covered by the systematic error which does not allow one to conclude whether or not the bound state is created in the considered mechanism. Therefore, finally the upper limit of the total cross section at the CL=90% was determined for the $\eta$-mesic $^3\text{He}$ nucleus creation followed by the $\eta$ meson decay, by fitting simultaneously excitation functions for both reactions with a Breit-Wigner + polynomial (signal+background) taking into account branching ratio relation between $\eta \to 2\gamma$ and $\eta \to 3\pi^0$ in vacuum. The estimated upper limit varies between 2 nb to...
15 nb depending on the bound state parameters (binding energy, width) [54]. It is shown in Fig. 7 for the bound state width $\Gamma=28.75$ MeV. The determined upper limit is much lower than the limit obtained in [79,118] for $pd \rightarrow (^{3}\text{He}-\eta)_{\text{bound}} \rightarrow ^{3}\text{He}\pi^0$ (70 nb) and is comparable with upper limits obtained in [54,55].

![Fig. 7](image-url)  
**Fig. 7** Upper limits for the bound state production cross section via $pd \rightarrow (^{3}\text{He}-\eta)_{\text{bound}} \rightarrow ^{3}\text{He}(\eta \text{ decays})$ as function of binding energy for fixed width $\Gamma=28.75$ MeV. The values of the Breit-Wigner amplitude $\sigma$ are shown with statistical uncertainties. The range of possible bound state production cross section obtained based on statistical uncertainty corresponding to 90% confidence level is shown by blue lines. The range of possible bound state production cross section including systematic uncertainty is shown by green lines. Figure is adapted from Ref. [54].

Recently the data analysis for reactions corresponding to mechanism (i) (e.g. $pd \rightarrow dp\pi^0$) is ongoing. Preliminary results are presented in Refs. [106, 107, 120].

4 **Summary and Perspectives**

In this report recent theoretical predictions and experimental results concerning mesic nuclei, and in particular $\eta$-mesic bound states have been discussed. We have focused mostly on the $\eta$-mesic Helium searches presenting the latest experimental data analyses with application of current phenomenological models. The performed measurements result in the valuable upper limits of the total cross sections for $(^{4}\text{He}-\eta)_{\text{bound}}$ and $(^{3}\text{He}-\eta)_{\text{bound}}$ production and decay considering different mechanisms. The analysis of data collected by WASA-at-COSY Collaboration is still in progress giving the hope for the discovery of $^{3}\text{He}-\eta$ bound state.
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