Asymptotically Lifshitz wormholes and black holes for Lovelock gravity in vacuum

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Abstract

Static asymptotically Lifshitz wormholes and black holes in vacuum are shown to exist for a class of Lovelock theories in $d = 2n + 1 > 7$ dimensions, selected by requiring that all but one of their $n$ maximally symmetric vacua are AdS of radius $l$ and degenerate. The wormhole geometry is regular everywhere and connects two Lifshitz spacetimes with a nontrivial geometry at the boundary. The dynamical exponent $z$ is determined by the quotient of the curvature radii of the maximally symmetric vacua according to $n(z^2 - 1) + 1 = \frac{l^2}{L^2}$, where $L^2$ corresponds to the curvature radius of the nondegenerate vacuum. Light signals are able to connect both asymptotic regions in finite time, and the gravitational field pulls towards a fixed surface located at some arbitrary proper distance to the neck. The asymptotically Lifshitz black hole possesses the same dynamical exponent and a fixed Hawking temperature given by $T = \frac{z}{2\pi l}$. Further analytic solutions, including pure Lifshitz spacetimes with a nontrivial geometry at the spacelike boundary, and wormholes that interpolate between asymptotically Lifshitz spacetimes with different dynamical exponents are also found.
I. INTRODUCTION

Exotic gravitational configurations, when consistent, naturally attract the attention of theoretical physicists. Wormhole solutions, describing handles in the spacetime topology, barely fall within this category. Indeed, as pointed out by Morris, Thorne and Yurtsever [1, 2], static wormholes in General Relativity necessarily lead to the violation of the null energy condition localized around the neck (for a nice review see Ref. [3]), and the picture does not change neither in higher dimensions nor in the presence of cosmological constant (see e.g. [1, 5]). However, in higher dimensions, the presence of terms with higher powers in the curvature provided by certain class of Lovelock theories, allows to circumvent this obstacle even in vacuum [6–8]. Thus, the possibility of violating energy conditions is then clearly removed since the whole spacetime is devoid of any kind of stress-energy tensor. Further wormholes solutions in Lovelock theories with matter fields that do not conflict with energy conditions have been found in Refs. [9–14]. Besides, spacetimes with unusual asymptotic behaviour, possessing anisotropic scaling symmetries at infinity, can be obtained from General Relativity once endowed with “unfamiliar and contrived” matter fields (see e.g. [15]). Although they obstruct the possibility of defining all the possible conserved charges and stress-energy fluxes as in the case asymptotically maximally symmetric spacetimes, they become relevant due to their potential applications aimed to describe condensed matter models in the strong coupling regime along the lines of the AdS/CFT correspondence (see e.g. [16–19]). A concrete example in this vein was first provided by the so-called Lifshitz spacetimes in Ref. [20] (see also [21]), and thereafter a wide class of asymptotically Lifshitz solutions have been found, either analytic [15, 22–37] or numerical [38–44].

One of the main results reported here is that certain class of Lovelock theories admits exact analytic solutions exhibiting at once all of the unusual features described above, i.e., asymptotically Lifshitz wormholes in vacuum.

Hereafter we will focus on Lovelock theories in \( d = 2n + 1 \geq 5 \) dimensions, selected by requiring that all but one of their \( n \) maximally symmetric vacua are AdS spacetimes of radius \( l \) and degenerate. In the absence of torsion, the field equations can then be written as (see e.g. [15])

\[
\mathcal{E}_a := \epsilon_{a_2 a_3 \ldots a_d} \bar{R}^{a_2 a_3} \ldots \bar{R}^{a_{d-3} a_{d-2}} \left( \bar{R}^{a_{d-1} a_d} + \frac{1}{\bar{L}^2} e^{a_{d-1}} e^{a_d} \right) = 0 ,
\]  

(1)
where $R^{ab} := R^{ab} + \frac{1}{l^2} e^a e^b$, and $L^2$ stands for the radius of the non degenerate vacuum whose sign is not fixed a priori. Here $R^{ab} = d\omega^{ab} + \omega^a \omega^{cb}$ is the curvature 2-form for the spin connection $\omega^{ab}$, and $e^a = e^a_\mu dx^\mu$ is the vielbein.

The plan of the paper is as follows. In the next section the asymptotically Lifshitz wormhole solution is discussed, including some of their causal and geometrical properties. An asymptotically Lifshitz black hole with the same dynamical exponent is found in section III. Further analytic solutions, including pure Lifshitz spacetimes with a nontrivial geometry at the spacelike boundary, and wormholes that interpolate between asymptotically Lifshitz spacetimes with different dynamical exponents are described in section IV. Section V is devoted to the summary and discussion of the results, and an appendix that concerns with a subset of the theories defined by Eq. (1) is also included.

II. ASYMPTOTICALLY LIFSHITZ WORMHOLES IN VACUUM

The field equations (1) admit the following exact solution

$$ds^2 = l^2 \left[ - \cosh^2(z(\rho - \rho_0)) dt^2 + d\rho^2 + \cosh^2(\rho) d\Sigma_{d-2}^2 \right],$$

where $z$ is determined by the quotient of the curvature radii of the maximally symmetric vacua according to

$$n(z^2 - 1) + 1 = \frac{l^2}{L^2},$$

and $\rho_0$ is an integration constant. The coordinates range as $-\infty < t < \infty$, $-\infty < \rho < \infty$, and the line element $d\Sigma_{d-2}^2$, being independent of $t$ and $\rho$, stands for the metric of the base manifold $\Sigma_{d-2}$ which is assumed to be compact and smooth. The spacetime (2) is geodesically complete and regular everywhere. It also possesses two disconnected boundaries so that it describes a static wormhole with a neck of radius $l$ located at $\rho = 0$. As one approaches to both asymptotic regions, i.e., at $\rho \to \pm\infty$, the metric (2) reads

$$ds^2 \to \frac{l^2}{4} \left[ -e^{2z\rho} dt^2 + 4d\rho^2 + e^{2\rho} d\Sigma_{d-2}^2 \right],$$

which under the coordinate transformation defined through $r = \frac{l}{2} e^\rho$, $\tau = lt$, explicitly acquires the form of a Lifshitz spacetime in Schwarzschild-like coordinates, given by

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1 Making $z \to -z$ just amounts to a total reflection in the radial coordinate.
2 Spacetimes whose asymptotic region approaches (5), such that the metric of the spacelike boundary is not flat, were recently dubbed as “asymptotically locally Lifshitz” in [46].
\[ ds^2 \rightarrow -\frac{r^{2z}}{l^{2z}}d\tau^2 + \frac{l^2}{r^2}dr^2 + r^2d\Sigma^2_{d-2}, \]  

possessing anisotropic scaling symmetry of the form \( \tau \rightarrow \lambda^z \tau, r \rightarrow \lambda^{-1}r \), provided the metric of the spacelike boundary conformally rescales as \( d\Sigma^2_{d-2} \rightarrow \lambda^2 d\Sigma^2_{d-2} \).

Therefore, the wormhole connects two Lifshitz spacetimes of dynamical exponent \( z \) with a nontrivial geometry at the spacelike boundary. For \( z > 0 \) the causal structure is similar to the one of AdS spacetime in two dimensions (see Fig. 1a).

FIG. 1: Causal structure for the wormhole. Figs. (a) and (b) correspond to the cases \( z > 0 \) and \( z = 0 \), respectively.

Note that when all the maximally symmetric vacua coincide, i.e., for \( L^2 = l^2 \), according to Eq. (3) the dynamical exponent is given by \( z = 1 \) and one then recovers the asymptotically AdS wormhole solution found in Ref. [8]. In this sense, the dynamical exponent \( z \) measures the deviation of the non degenerate maximally symmetric vacuum with respect to the \((n-1)\)-degenerate AdS ones.

One can also see that the wormhole metric (2) shares many properties with its asymptotically AdS cousin. For instance, it is simple to check that timelike curves can go forth and back from the neck, and it is amusing to verify that radial null geodesics are able to connect both asymptotic regions in finite time. Indeed, as it can be seen from (2), a photon that travels radially from one asymptotic region to the other, i.e., starting from \( \rho = -\infty \)
towards $\rho = +\infty$, performs the entire trip in a coordinate time given by

$$\Delta t = \int_{-\infty}^{+\infty} \frac{d\rho}{\cosh (z(\rho - \rho_0))} = \frac{2}{z} \left[ \arctan (e^{\rho - \rho_0}) \right]_{-\infty}^{+\infty} = \frac{\pi}{z},$$

which does not depend on $\rho_0$. A static observer located at $\rho = \rho_0$, who actually stands on a timelike geodesic, then says that this occurred in a proper time given by $\pi lz^{-1}$. Furthermore, perturbations along $\rho$ makes this observer to oscillate around $\rho = \rho_0$, which means that gravity is pulling towards the fixed surface defined by $\rho = \rho_0$, being at finite proper radial distance of the neck. Indeed, radial timelike geodesics are confined since they fulfill

$$i - \frac{E}{l^2 \cosh^2 (z(\rho - \rho_0))} = 0, \quad (6)$$

$$l^2 \dot{\rho}^2 - \frac{E^2}{l^2 \cosh^2 (z(\rho - \rho_0))} + \sigma = 0, \quad (7)$$

where dot stands for derivatives with respect to proper time, the velocity is normalized as $u_\mu u^\mu = -\sigma$, and the integration constant $E$ corresponds to the energy. Note that the position $\rho(\tau)$ of a radial geodesic, in proper time behaves as a particle in a Pöschl-Teller potential. It also follows that null and spacelike radial geodesics connect both asymptotic regions in finite time.

The solution (2) is well defined provided

$$z^2 \geq 0 \rightarrow \frac{l^2}{E^2} \geq 1 - n, \quad (8)$$

and according to the range of $z$ different remarks are worth to be pointed out:

- **$z^2 > 1$**: This case becomes relevant within the context of non-relativistic holography (see e.g. \[23\]), since the effective speed of light of the –interacting– dual theories at each boundary goes to infinity. For the wormhole in eq. (2), this condition is fulfilled provided $0 < L^2 < l^2$, which means that a non-relativistic dual picture could be obtained if the theory described by (1) admits a single nondegenerate vacuum being AdS spacetime of radius $L$ necessarily smaller than the one of the degenerate AdS vacua, given by $l$.

- **$z^2 = 1$**: As mentioned above, in this case the wormhole is asymptotically AdS and reduces to the solution found in Ref. \[8\]. Since $L^2 = l^2$, the theory admits a unique maximally symmetric AdS vacuum \[47\], and the Lagrangian can be written as a Chern-Simons theory for the AdS group \[48\]. Furthermore, one can verify that the wormhole also
provides a solution for the corresponding locally supersymmetric extensions in five \[49\] and higher odd dimensions \[50, 51\].

- \(1 - \frac{1}{n} < z^2 < 1\): For this range the theory has to admit a single nondegenerate vacuum being AdS spacetime of radius \(\mathcal{L}\) necessarily of greater radius than the one of the degenerate AdS vacua of radius \(l\), i.e. \(\mathcal{L}^2 > l^2\).

- \(z^2 = 1 - \frac{1}{n}\): In this case the nondegenerate maximally symmetric vacuum of the theory has vanishing curvature and it is so given by Minkowski spacetime \((\mathcal{L}^2 \to \infty)\). Therefore, the volume – or cosmological– term in the action, proportional to \(\sqrt{-g}\), has to be absent in the action.

- \(0 < z^2 < 1 - \frac{1}{n}\): This range requires the theory described by (1) to admit a single nondegenerate dS vacuum, fulfilling \(\mathcal{L}^2 < \frac{l^2}{1-n} < 0\). It is worth mentioning that for the special values of the dynamical exponent, given by

\[
z^2 = 1 - \frac{1}{n-k},
\]

the \(k\)-th power of the curvature is absent in the field equations, and hence in the action (see Appendix). Note that the previous case, \(z^2 = 1 - \frac{1}{n}\), is then consistently recovered for \(k = 0\).

- \(z^2 = 0\): In this case the theory (1) also admits a single nondegenerate dS vacuum, whose curvature radius is fixed as \(\mathcal{L}^2 = \frac{l^2}{1-n}\) and, according to (9), is such that the \(k = (n-1)\)-th power of the curvature is absent in the field equations. The metric (2) acquires a very simple form, that reads

\[
ds^2 = l^2 \left[ -dt^2 + d\rho^2 + \cosh^2(\rho) d\Sigma_{d-2}^2 \right],
\]

connecting two static universes at \(\rho \to \pm \infty\). Its causal structure looks like the one of Minkowski spacetime in two dimensions (see Fig. 1b).

\[3\] Wormholes in vacuum, similar in form as compared with eq. (10), have been previously found for conformal gravity in \[52\] and for different three-dimensional gravity theories in Refs. \[53\] and \[54\]. This is also the case for the Einstein-Gauss-Bonnet theory with matter \[12\], as well as for compactified Lovelock theories in eight dimensions \[13\].
A. Nontrivial spacelike boundary geometries

Finding the explicit form of the base manifold metric $\Sigma_{d-2}$, which determines the geometry of the neck as well as the one of the spacelike boundary is not a simple task. The conditions that the metric $d\Sigma^2_{d-2}$ has to fulfill can be obtained as follows:

For the metric given by eq. (2) the vielbein can be chosen as

$$e^0 = l \cosh(z(\rho - \rho_0)) dt ; \quad e^1 = l d\rho ; \quad e^m = l \cosh(\rho) \tilde{e}^m,,$$

where $\tilde{e}^m$ is the vielbein of $\Sigma_{d-2}$. The nonvanishing components of $\bar{R}^{ab} = R^{ab} + \frac{1}{l^2} e^a e^b$ then read

$$\bar{R}^{01} = \frac{1 - z^2}{l^2} e^0 \wedge e^1,$$
$$\bar{R}^{0m} = \frac{1}{l} \left[ \cosh(\rho) - z \sinh(\rho) \tanh(z(\rho - \rho_0)) \right] e^0 \wedge \tilde{e}^m,$$
$$\bar{R}^{mn} = \bar{R}^{mn} + e^m \wedge \tilde{e}^n,$$

where $\bar{R}^{mn}$ stands for the curvature two-form of $\Sigma_{d-2}$. Then, the component $E_0 = 0$ of the field equations (1) reduces to the following scalar condition on $\Sigma_{d-2}$:

$$(z^2 - 1) \epsilon_{m_3 m_4 \ldots m_d} \bar{R}^{m_3 m_4} \ldots \bar{R}^{m_{d-2} m_{d-1}} \tilde{e}^{m_d} = 0. \tag{11}$$

Analogously, the combination $E_0 e^0 - E_1 e^1 = 0$ gives a different scalar condition, which reads

$$\epsilon_{m_3 \ldots m_d} \left[ n \bar{R}^{m_3 m_4} \ldots \bar{R}^{m_{d-2} m_{d-1}} + (n - 1)(z^2 - 1) \cosh^2(\rho) \bar{R}^{m_3 m_4} \ldots \bar{R}^{m_{d-4} m_{d-3}} \tilde{e}^{m_{d-2}} \tilde{e}^{m_{d-1}} \right] \tilde{e}^{m_d} = 0, \tag{12}$$

while the projection of the field equations along $\Sigma_{d-2}$, $E_m = 0$, reduces to

$$\epsilon_{m_3 m_4 \ldots m_d} \left[ \left( \frac{l^2}{E^2} - 1 - n(z^2 - 1) \right) \bar{R}^{m_4 m_5} \ldots \bar{R}^{m_{d-1} m_d} + A(\rho) \bar{R}^{m_4 m_5} \ldots \bar{R}^{m_{d-3} m_{d-2}} \tilde{e}^{m_{d-2}} \tilde{e}^{m_{d-1}} \right] \tilde{e}^{m_d} = 0, \tag{13}$$

with $A(\rho) := (1 - n)(z^2 - 1) \left[ (z^2 - 3) \cosh^2(\rho) + z \sinh(2\rho) \tanh(z(\rho - \rho_0)) \right]$. It can be seen from eq. (13) that at least one of the components of the $(d - 3)$-form

$$\epsilon_{m_3 m_4 \ldots m_d} \bar{R}^{m_4 m_5} \ldots \bar{R}^{m_{d-1} m_d} \tag{14}$$

must not vanish, otherwise the dynamical exponent $z$ would be arbitrary. This is just a reflection of the fact that if (14) vanished, actually the metric would be undetermined since
in this case the $g_{tt}$ component becomes an arbitrary function. Therefore, the degeneracy in the metric is removed requiring the dynamical exponent to be fixed as in eq. (3). Thus, eq. (12) is fulfilled by virtue of eqs. (13) and (11).

In sum, for $z^2 \neq 1$, the metric of the base manifold $\Sigma_{d-2}$ fulfills the field equation

$$\epsilon_{m_3 m_4 \ldots m_d} \bar{R}^{m_4 m_5} \ldots \bar{R}^{m_d-3 m_d-2} \bar{e}^{m_d-1} \bar{e}^{m_d} = 0 ,$$

with an additional scalar condition

$$\epsilon_{m_3 m_4 \ldots m_d} \bar{R}^{m_3 m_4} \ldots \bar{R}^{m_d-2 m_d-1} \bar{e}^{m_d} = 0 ,$$

provided at least one of the components of (14) does not vanish.

The case $z^2 = 1$ allows much more freedom for the choice of base manifold $\Sigma_{d-2}$, since its metric only fulfills the scalar condition (16), where at least one of the components of (14) is different from zero. This is in agreement with the results found in [8]. Note that for $z = 1$, if the base manifold were chosen as being locally isomorphic to the hyperbolic space of radius one, the scalar condition (16) would be trivially fulfilled, but since (14) also vanishes in this case the metric turns out to be undetermined. Both conditions are satisfied for any compact and smooth base manifold whose metric is given by the one of $\Sigma_{d-2} = S^1 \times H_{d-3}/\Gamma$, where the radius of the hyperbolic space $H_{d-3}$ is given by $(2n-1)^{-1/2}$, and $\Gamma$ is a freely acting discrete subgroup of $O(d-3,1)$.

In $d = 5$ and 7 dimensions the solution of the form (2) holds only for $z = 1$. In five dimensions this is because the field equations defined by eq. (1) correspond to the ones of the Einstein-Gauss-Bonnet theory possessing a single maximally symmetric vacua of squared radius given by $l^2$—which cannot be degenerate unless $L^2 = l^2$, and hence $z = 1$. Indeed, as explained in Refs. [6–8], $L^2 = l^2$ is a necessary condition for the existence of static asymptotically AdS wormholes in vacuum for the Einstein-Gauss-Bonnet theory in $d \geq 5$ dimensions. In seven dimensions the reason is different. The field equations (1) correspond to a cubic Lovelock theory. In this case, the field equation of the base manifold metric given by (15) means that $\Sigma_5$ has to be an Euclidean Einstein manifold of negative scalar curvature, while the scalar condition (16) further restricts its geometry so that $\Sigma_5$ has to be of constant curvature $-1$, i.e., $\bar{R}^{mn} = 0$. Therefore, the $(d-3)$-form in eq. (14) vanishes, and hence the
The $g_{tt}$ component of the metric becomes undetermined. As explained in the previous paragraph, the conditions on $\Sigma_5$ are different for $L^2 = l^2$ so that the asymptotically AdS solution (2) with $z = 1$ exists and it is well defined in seven dimensions.

Obstructions of the sort mentioned above do not apply for spacetimes of the form (2) with $0 \leq z^2 \neq 1$ in $d \geq 9$ dimensions. Nonetheless, finding an explicit metric for the base manifold that fulfills the required conditions is not a straightforward task. This is left as an open problem.

### III. ASYMPHTOTICALLY LIFSHITZ BLACK HOLE

The theory described by (1) also admits a different solution, whose metric is given by

$$ds^2 = l^2 \left[ -\sinh^2(z\rho)dt^2 + d\rho^2 + \cosh^2(\rho)d\Sigma_{d-2}^2 \right] , \quad (17)$$

where $z$ is fixed in terms of the quotient of the curvature radii of the maximally symmetric vacua as in eq. (3). The solution is well defined provided the bound (8) is not saturated, i.e.,

$$z^2 > 0 \rightarrow l^2 \frac{L^2}{L^2} > (1 - n) . \quad (18)$$

The base manifold metric $d\Sigma_{d-2}^2$ is independent of the coordinates $t, \rho$ and fulfills the same conditions as the ones for the wormhole described in the previous chapter; i.e., for $z^2 \neq 1$ the metric of $\Sigma_{d-2}$ must solve the field equation (15) with the additional scalar condition (16) provided at least one of the components of (14) does not vanish. The metric (17) possesses an event horizon located at $\rho = 0$, and it describes an asymptotically Lifshitz black hole with dynamical exponent $z$, since for $\rho \to +\infty$, the line element reduces to the one in eq. (4). In terms of Schwarzschild-like coordinates, $r = l \cosh(\rho)$ the metric reads

$$ds^2 = \frac{-4^{-z}}{l} \left( \frac{r}{l} + \sqrt{\frac{r^2}{l^2} - 1} \right)^{\frac{z}{2}} \left[ \frac{r}{l} + \sqrt{\frac{r^2}{l^2} - 1} \right]^{\frac{z}{2}} dr^2 + \frac{dr^2}{r^2 - 1} + r^2 d\Sigma_{d-2}^2 , \quad (19)$$

where the time coordinate has been rescaled as $\tau = 2^{z-1}lt$ in order to fit the standard form of Lifshitz spacetime (5) in the asymptotic region. The horizon is now located at $r = l$ and encloses the singularity at the origin, $r = 0$. Its Hawking temperature can be found demanding regularity of the Euclidean solution at the horizon, and it is given by

$$T = \frac{z}{2^z \pi l} . \quad (20)$$
A number \( m \) of additional singularities that shield the one at the origin, are developed at \( r = l \cos(q\pi z^{-1}) < l \), where \( q \leq m \) is a positive integer, provided the dynamical exponent fulfills \( z > \frac{2}{m} \). This can be seen as follows. The inner region, \( r < l \) is suitably covered by the patch defined through \( r = l \cos(\theta) \) with \( 0 < \theta \leq \frac{\pi}{2} \), so that the metric (19) reads

\[
\begin{align*}
  ds^2 &= l^2 \left[ -d\theta^2 + \sin^2(z\theta)dt^2 + \cos^2(\theta)d\Sigma_{d-2}^2 \right].
\end{align*}
\]

(21)

It is then apparent that the horizon (\( \theta = 0 \)) not only surrounds the singularity at the origin (\( \theta = \frac{\pi}{2} \)), but also the additional ones developed at \( \theta = q\pi z^{-1} \). The singularity at the origin is generically spacelike unless the dynamical exponent \( z \) is an even integer so that it becomes null\(^4\).

In the case of \( z^2 = 1 \), i.e., for \( \mathcal{L}^2 = l^2 \), the solution is asymptotically locally AdS and it can be extended to admit an integration constant \( r_+ \) that parametrizes the horizon radius, so that the metric is given by\(^5\)

\[
\begin{align*}
  ds^2 &= -\left( r^2 - r_+^2 \right) \frac{dr^2}{l^2} + \frac{l^2}{(r^2 - r_+^2)} dr^2 + r^2 d\Sigma_{d-2}^2.
\end{align*}
\]

(22)

It would then be desirable exploring whether the asymptotically Lifshitz black hole (17) could also be extended so as to admit an integration constant.

IV. FURTHER EXACT SOLUTIONS

The field equations (1) admit a wider class of asymptotically Lifshitz solutions in vacuum, whose metric is given by

\[
\begin{align*}
  ds^2 &= -\left( a \left( \frac{r}{l} + \sqrt{\frac{r^2}{l^2} + \gamma} \right)^z + b \left( \frac{r}{l} + \sqrt{\frac{r^2}{l^2} + \gamma} \right)^{-z} \right)^2 dt^2 + \frac{dr^2}{r^2 + \gamma} + r^2 d\Sigma_{d-2}^2,
\end{align*}
\]

(23)

where the dynamical exponent \( z \) is again fixed as in eq. (3). Here \( a \) and \( b \) are integration constants, and \( \gamma \) can always be rescaled as \( \gamma = \pm 1, 0 \). The case of \( \gamma = 1 \) generically leads

\(^4\) Curiously, requiring the singularity at the origin to be null, quantizes the quotient \( \frac{l^2}{L^2} \) of the curvature radii of the maximally symmetric vacua to be and even or odd integer for odd and even \( n \), respectively.

\(^5\) As explained in [55], in eq. (22) the geometry of \( \Sigma_{d-2} \) is arbitrary, but fixed by the boundary conditions (see also [56]), and it becomes further restricted requiring (22) to admit Killing spinors. This is an extension of the solutions previously discussed in [56–58]. Further aspects of this spacetime have been discussed in Refs. [60–68].
to solutions with naked singularities, and so the remaining cases of interest are discussed in what follows.

- $\gamma = -1$:

In this case, the black hole (19) is recovered from (23) with $a = -b = 4^{-\frac{2}{3}}$. The wormhole metric (2) can also be recovered from (23) in the case of $a = \frac{1}{2}e^{-z\rho_0}$ and $b = \frac{1}{2}e^{z\rho_0}$, followed by a change of coordinates given by $r \to l \cosh(\rho)$, and $t \to lt$. This means that the region $r < l$ in (23) can be consistently excised, so that two copies of the exterior region $r > l$, with parameters $a = \frac{1}{2}e^{-z\rho_0}$, $b = \frac{1}{2}e^{z\rho_0}$ ($\rho > 0$), and $a = \frac{1}{2}e^{z\rho_0}$, $b = \frac{1}{2}e^{-z\rho_0}$ ($\rho < 0$) can be smoothly matched at $r = l$ ($\rho = 0$) in vacuum, without the need of introducing any kind of stress energy tensor at the neck, as it would be necessary in the case of General Relativity. This is a known feature of Lovelock gravity theories (see e.g., Refs. [69–73]).

A different solution is recovered from the metric (23) with $b = 0$, which after changing the coordinates as $r \to l \cosh(\rho)$, and $t \to \frac{l}{a}t$, reads

$$ds^2 = l^2 \left[-e^{2z\rho} dt^2 + d\rho^2 + \cosh^2(\rho) d\Sigma^2_{d-2}\right],$$

where the base manifold $\Sigma_{d-2}$ fulfills the same conditions as the solutions described above. This geometry describes a static wormhole with a neck of radius $l$ located at $\rho = 0$, and interpolates between asymptotically Lifshitz spacetimes with different dynamical exponents given by $z$ and $-z$, for $\rho \to \infty$, and $\rho \to -\infty$, respectively. As it can be seen from (24), the gravitational field pulls towards the asymptotic region $\rho \to -\infty$. In spite of the fact that the curvature invariants are regular everywhere, it is simple to verify that this spacetime is not geodesically complete, since null radial geodesics can also reach $\rho = -\infty$ in a finite affine parameter, and the warp factor of the base manifold blows up there.

- $\gamma = 0$:

Making $b = 0$ and $a = 2^{-z}$ in (23) the metric reads

$$ds^2 = -\frac{r^{2z}}{l^{2z}} dt^2 + \frac{l^2}{r^2} dr^2 + r^2 d\Sigma^2_{d-2},$$

The case $a = 0$ corresponds to $b = 0$ with $z \to -z$. Furthermore, it can be assumed that $z > 0$, since as for the previous wormhole solution, making $z \to -z$ amounts to a total reflection in the radial coordinate.
which corresponds to a Lifshitz spacetime with a nontrivial base manifold. In the case of \( z^2 \neq 1 \), the geometry of \( \Sigma_{d-2} \) fulfills the field equation

\[
\epsilon_{m_3 m_4 \cdots m_d} \tilde{R}^{m_3 m_5} \cdots \tilde{R}^{m_{d-3} m_{d-2}} e^{m_{d-1}} e^{m_d} = 0 ,
\]

(26)

with an additional scalar condition, which reads

\[
\epsilon_{m_3 m_4 \cdots m_d} \tilde{R}^{m_3 m_4} \cdots \tilde{R}^{m_{d-2} m_{d-1}} e^{m_d} = 0 ,
\]

(27)

provided at least one of the components of the \((d-3)\)-form given by

\[
\epsilon_{m_3 m_4 \cdots m_d} \tilde{R}^{m_4 m_5} \cdots \tilde{R}^{m_{d-1} m_d}
\]

(28)
does not vanish; else the \( g_{tt} \) component of the metric becomes undetermined, as it would the case if \( \Sigma_{d-2} \) were chosen as a locally flat spacetime.

For \( z^2 = 1 \) the base manifold metric has less restrictive conditions, since it has to fulfill a scalar condition given by

\[
\epsilon_{m_3 m_4 \cdots m_d} \tilde{R}^{m_3 m_4} \cdots \tilde{R}^{m_{d-2} m_{d-1}} e^{m_d} = 0 ,
\]

(29)

where at least one of the components of (28) does not vanish.

An additional curious solution is recovered once making \( a = 2^{-z} \), and \( b = b_0 2^z \) in (23). The metric is given by

\[
\text{ds}^2 = - \left( \frac{r^z}{l^z} + \frac{b_0 l^z}{r^z} \right)^2 dt^2 + \frac{l^2}{r^2} \frac{dr^2}{r^2} + r^2 d\Sigma_{d-2}^2 ,
\]

(30)

where the base manifold fulfills the same conditions as the pure Lifshitz spacetime described above. This spacetime interpolates between an asymptotically Lifshitz spacetime with dynamical exponent \( z \), for \( r \to \infty \), and another Lifshitz spacetime of dynamical exponent \(-z\) at \( r = 0 \), as it can be seen after a time rescaling given by \( t \to b_0^{-1} t \). For \( z > 0 \), gravity pulls towards the surface defined by \( r = l b_0^\frac{z}{2} \), and timelike geodesics turn out to be bounded, since they neither reach the (repulsive) singularity at the origin nor the asymptotic region.

V. SUMMARY AND DISCUSSION

A class of Lovelock theories selected by requiring that all but one of their \( n \) maximally symmetric vacua are AdS of radius \( l \) and degenerate was considered. The field equations
were shown to admit exact static asymptotically Lifshitz wormholes and black holes in vacuum, given by eqs. (2) and (17), respectively, for $d = 2n + 1 > 7$ dimensions. The wormhole exists provided the curvature radius of the nondegenerate vacuum is outside the range $\frac{l^2}{1-n} < L^2 \leq 0$, and it connects two Lifshitz spacetimes with a nontrivial geometry at the spacelike boundary. The dynamical exponent $z$ is determined by the quotient of the curvature radii of the maximally symmetric vacua as in eq. (3), and then measures the deviation of the non-degenerate maximally symmetric vacuum of radius $L$ with respect to the $(n-1)$-degenerate AdS ones. The wormhole geometry is geodesically complete and regular everywhere, and hence, as one approaches to the inner region, the potentially divergent tidal forces appearing around the origin of pure Lifshitz spacetime [20, 74, 75] are circumvented by the presence of a throat.

In the case of $z = 0$, the wormhole metric reduces to (10) and it connects two static universes at $\rho \to \pm \infty$. The corresponding causal structures for $z > 0$, and $z = 0$ are depicted in Figs. 1a and 1b, respectively.

It is worth pointing out that for General Relativity, as explained in [1], static wormhole solutions necessarily violate the standard energy condition around the neck, while asymptotically Lifshitz spacetimes also do for $z < 1$ [76, 77]. Remarkably, since the wormhole solution (2) solves the field equations (1) in vacuum for $z \geq 0$, the spacetime is devoid of any kind of stress-energy tensor everywhere, and hence no energy conditions can be violated. Therefore, the results found for General Relativity do not apply for Lovelock gravity. It would then be interesting to explore whether this spacetime, for a generic dynamical exponent, could also be stable against scalar field perturbations, as it is the case of its asymptotically AdS cousin for $z = 1$ [78].

It is known that wormholes raise some interesting puzzles in the context of the gauge/gravity correspondence [79–81]. Intriguing results along this line have been found in Refs. [82–84].

As explained in Sec. II A, the geometry of the spacelike boundary $\Sigma_{d-2}$ fulfills a curious set of conditions, which for $z^2 \neq 1$, reduce to the field equation (15) corresponding to the class of theories discussed in [47], together with the additional scalar condition (16). An additional condition, which requires that at least one of the components of (14) does not vanish, has to be imposed; otherwise, the $g_{tt}$ component of the metric would be an arbitrary function, as it is the case of asymptotically Lifshitz solutions for Lovelock theories.
in vacuum previously found in the literature. Finding an explicit Euclidean metric that fulfills the required conditions turned out to be not a simple task and it is then left as an open problem. Indeed, product manifolds of constant curvature solving both (15) and (16) for certain fixed radii, simultaneously make the $d-3$-form (14) to vanish, making the $g_{tt}$ component of the metric to be undetermined. Different classes of solutions with nontrivial geometries at the boundary for Lovelock theories in vacuum have also been considered in e.g. Refs. [6–8, 57–59, 85–87].

The asymptotically Lifshitz black hole solution whose metric is given by (19) possesses the same dynamical exponent (3), with a fixed event horizon that surrounds the singularities, and a Hawking temperature given by (20). It would also be interesting to explore the stability of this solution, its entropy, as well as whether it could be extended so as to admit an integration constant parametrizing the horizon radius.

The wormhole (2) and the black hole (17) were found to belong to a wider class of asymptotically Lifshitz solutions in vacuum, given by the metric (23). This class was shown to include pure Lifshitz spacetimes with a nontrivial geometry at the spacelike boundary, given by (25), wormholes that interpolate between different asymptotically Lifshitz spacetimes with dynamical exponents given by $z$ and $-z$, as in (24), and the solution (30) that interpolates between an asymptotically Lifshitz spacetime with dynamical exponent $z$, and another Lifshitz spacetime of dynamical exponent $-z$ at the origin.

As a final remark, it would be worth exploring whether the class of Lovelock theories considered here could be widened so as to admit well-behaved asymptotically Lifshitz solutions in vacuum.

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Appendix A: Absence of the $k$-th power of the curvature in the action and a special class of dynamical exponents

Let us consider the solutions in vacuum discussed above in the case for which the dynamical exponent lies within the range $0 \leq z^2 \leq 1 - 1/n$. Here we show that there is a special class, defined by

$$z^2 = 1 - \frac{1}{n-k}, \quad (A1)$$

where $k < n$ is a non negative integer, that corresponds to Lovelock theories of the form $(1)$ being such that the $k$-th power of the curvature is absent in the field equations, and hence in the action. By virtue of $(3)$, the field equations $(1)$ read

$$\epsilon_{a_2 a_3 \cdots a_d} \bar{R}^{a_2 a_3} \cdots \bar{R}^{a_{d-3} a_{d-2}} \left( \bar{R}^{a_{d-1} a_d} + \frac{n(z^2 - 1)}{l^2} e^{a_{d-1}} e^{a_d} \right) = 0, \quad (A2)$$

which is equivalent to

$$\epsilon_{a_2 a_3 \cdots a_d} \sum_{k=0}^{n} \frac{n!}{k!(n-k)!} l^{2(k-n)} [(n-k)(z^2 - 1) + 1] \bar{R}^{a_2 a_3} \cdots \bar{R}^{a_{2k} a_{2k+1}} e^{a_{2k+2}} \cdots e^{a_d} = 0. \quad (A3)$$

Therefore, it is apparent that the $k$-th power of the curvature is absent provided

$$z^2 = 1 - \frac{1}{n-k}.$$

In particular, for $z^2 = \frac{n-2}{n-1}$, the Einstein-Hilbert term is absent in the action.

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