Rotating D0-branes and consistent truncations of supergravity

Andrés Anabalón\textsuperscript{a,b}, Thomas Ortiz\textsuperscript{b}, and Henning Samtleben\textsuperscript{b}

\textsuperscript{a} Departamento de Ciencias, Facultad de Artes Liberales y Facultad de Ingeniería y Ciencias, Universidad Adolfo Ibáñez, Av. Padre Hurtado 750, Viña del Mar, Chile.

\textsuperscript{b} Université de Lyon, Laboratoire de Physique, UMR 5672, CNRS École Normale Supérieure de Lyon 46, allée d’Italie, F-69364 Lyon cedex 07, France

Abstract

The fluctuations around the D0-brane near-horizon geometry are described by two-dimensional $SO(9)$ gauged maximal supergravity. We work out the $U(1)^4$ truncation of this theory whose scalar sector consists of five dilaton and four axion fields. We construct the full non-linear Kaluza-Klein ansatz for the embedding of the dilaton sector into type IIA supergravity. This yields a consistent truncation around a geometry which is the warped product of a two-dimensional domain wall and the sphere $S^8$. As an application, we consider the solutions corresponding to rotating D0-branes which in the near-horizon limit approach $\text{AdS}_2 \times \mathcal{M}_8$ geometries, and discuss their thermodynamical properties. More generally, we study the appearance of such solutions in the presence of non-vanishing axion fields.
1 Introduction

The standard AdS/CFT correspondence [1], is based on maximally supersymmetric string/brane configurations whose near horizon geometry factorizes into the direct product of and Anti de Sitter (AdS) space and a sphere. Its generalization to non-conformal $D_p$-branes [2] proposes a holographic description for the maximally supersymmetric $(p + 1)$-dimensional Yang-Mills theory (non-conformal for $p \neq 3$). In this latter case, the relevant near-horizon geometry is the warped product of a $(p + 2)$-dimensional domain wall and the sphere $S^{8-p}$. Relatively few tests of these non-conformal dualities have been carried out, and only more recently the techniques of holographic renormalization have been developed systematically also in the non-conformal context [3, 4].

In the supergravity approximation, these so-called domain wall/quantum field theory (DW/QFT) correspondences [5, 6, 7] imply the duality between certain gauged supergravities supporting half-maximal domain-wall solutions, and certain subsectors of the non-conformal quantum field theories. A key issue for the applicability of the lower-dimensional effective actions in this context is the question of their consistent embedding into the full ten-dimensional theory. Consistent truncations of higher-dimensional supergravities are highly constrained and relatively rare. For the maximal (AdS) cases, consistency of the truncations on $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$ have been shown in [8, 9] and [10], respectively, whereas the consistent truncation on $\text{AdS}_5 \times S^5$ so far has only been shown for certain subsectors [11, 12, 13]. For the non-conformal cases, in which the geometry is the warped product of a domain wall and a sphere, the results about consistent truncations are more sporadic. They include most notably [14] in which consistent truncations are obtained for domain-wall supergravities obtainable from particular limits of AdS supergravities, and [15] for reductions on small spheres.

The subject of this letter are compactifications of the IIA theory on the eight-sphere $S^8$, describing fluctuations around the D0 brane near-horizon geometry. The low-energy effective theories are truncations of the two-dimensional maximally supersymmetric $SO(9)$ gauged supergravity, recently constructed in [16]. The dual field theory in this case is the supersymmetric matrix quantum mechanics [17] which itself has been proposed as a non-perturbative definition of M-theory [18]. More precisely, we study the (non-supersymmetric) truncation of $SO(9)$ supergravity to singlets under its Cartan subgroup $U(1)^4$. Its scalar sector consists of five dilaton and four axion fields. In analogy to the well-known results for the AdS supergravities [11], we construct the non-linear Kaluza-Klein ansatz for the embedding of the full dilaton sector into type IIA supergravity. Contrary to the AdS case, this ansatz requires a non-constant ten-dimensional dilaton field which gives rise to an additional dilaton in two dimensions with non-vanishing contribution to the domain wall ground state. Moreover, this $p = 0$ case is special in that there is no Einstein frame in two dimensions such that many of the generic reduction ansaetze [11, 15] in fact degenerate.

As an application, we construct the solutions corresponding to rotating D0-branes
which in the limit of large brane charges approach AdS$_2 \times \mathcal{M}_8$ geometries. More generally, we study the appearance of such solutions (in particular in the presence of non-vanishing axion fields). Their existence is due to non-vanishing flux of the two-dimensional vector fields.

The appearance of an AdS$_2$ in the near extremal limit allows to use Sen’s entropy function formalism [27] to compute the two-dimensional entropy of this configuration. We find that the entropy is proportional to the value of the two-dimensional dilaton (which precludes the existence of the Einstein frame as otherwise the Einstein-Hilbert Lagrangian would be a total derivative). The result exactly matches the ten-dimensional result given by the Bekenstein-Hawking area law. Remarkably, the fact that two-dimensional black holes have zero-dimensional horizons is not an obstruction for Sen formalism to work properly.

The outline of the paper is as follows: in the second section the $SO(9)$ theory is briefly reviewed and the $U(1)^4$ truncation discussed. The third section displays the explicit embedding of the truncation in type IIA supergravity. The fourth section recalls the rotating D0-brane solution of [11] as well as its thermodynamical properties from the ten-dimensional point of view. In the fifth section the two-dimensional entropy function formalism is applied to the limit of large brane charges of the rotating D0-brane. The attractor mechanism is shown to work and the ten-dimensional result of the previous section is exactly recovered. In the last section, the axions are included in the truncation and its ten-dimensional embedding is sketched.

2 $U(1)^4$ truncation of $SO(9)$ supergravity

$SO(9)$ supergravity is a maximally supersymmetric theory in two dimensions. It has been constructed in [16] by gauging the $SO(9)$ subgroup of the global $SL(9)$ symmetry that is manifest after reduction of eleven-dimensional supergravity on a nine-torus $T^9$. A consistent truncating of the theory is given by truncating the theory to singlets under the $U(1)^4$ Cartan subgroup of $SO(9)$. This is in complete analogy to the truncations of higher-dimensional maximal supergravities considered in [11]: the $U(1)^4$ truncation of $D = 4$, $SO(8)$ supergravity and the $U(1)^3$ and $U(1)^2$ truncations of $D = 5$, $SO(6)$ and $D = 7$, $SO(5)$ supergravity, respectively. For $D = 2$, $SO(9)$ supergravity the truncation to $U(1)^4$ singlets reduces the coordinates of the scalar target space to five dilaton and four axion fields. Moreover, the bosonic Lagrangian carries four abelian gauge fields and four auxiliary scalar fields.

$SO(9)$ Action We start from the action given in [16], formula (4.2). Its bosonic part is a dilaton-gravity coupled non-linear sigma model with 128-dimensional target space ($SL(9) \times T_{k4}$)/$SO(9)$ and Wess-Zumino term. Thus 80 scalar fields are parametrized by group-valued $SL(9)$ matrices $V_m^\alpha$ with coset redundancy

$$V_m^\alpha \to V_m^\beta \Lambda_{\beta}^\alpha, \quad \Lambda \in SO(9). \quad (2.1)$$
The remaining 84 scalars fields are labeled as $\phi^{kln} = \phi^{[kln]}$ with the indices running from 1 to 9. The $SO(9)$ symmetry acting by left multiplication on $\mathcal{V}_m^{\alpha}$ and rotating the scalars $\phi^{kln}$ in the $\mathbf{84}$ representation is gauged by 36 vector fields $A^k_\mu = A^k_\mu$. In turn, these couple to 36 auxiliary fields $Y^{kl} = Y^{[kl]}$ in the Lagrangian. Finally let us precise that our signature is $(+-)$. The explicit form of the Lagrangian is

$$
\mathcal{L} = -\frac{1}{4} e^2 P + \frac{1}{4} e P^{\alpha\beta} \mathcal{P}_\mu^{\alpha\beta} + \frac{1}{12} e^2 v^{\alpha\beta\gamma} \mathcal{V}_{kln}^{[\alpha\beta\gamma]} \mathcal{V}^{[\alpha\beta\gamma]}_n p q D_\mu \phi^{kln} D_\mu \phi^{nnpq} + \frac{1}{648} e^{\alpha\beta\gamma} \mathcal{V}_{kln}^{[\alpha\beta\gamma]} \mathcal{V}^{[\alpha\beta\gamma]}_n p q D_\mu \phi^{kln} D_\mu \phi^{nnpq} - \frac{1}{4} e^{\alpha\beta\gamma} Y^{[\alpha\beta\gamma]}_k Y^{[\alpha\beta\gamma]}_l - \mathcal{P}_\mu^{(\alpha\beta)} + \mathcal{Q}_\mu^{(\alpha\beta)} , \quad (2.2)
$$

with the scalar currents defined by

$$
J^{\alpha\beta}_k = \mathcal{V}^{-1} = (\partial_k \mathcal{V}^{\alpha}_{\beta}) = \mathcal{Q}_\mu^{(\alpha\beta)} + \mathcal{P}_\mu^{(\alpha\beta)} , \quad (2.3)
$$

The first term of the Lagrangian carries the two-dimensional Ricci scalar $R$, the determinant of the zweibein $e = \sqrt{\det g_{\mu\nu}}$, and the dilaton field $\rho$. $F^{kl}_{\mu\nu}$ denotes the standard non-abelian $SO(9)$ Yang-Mills field strength of the vector fields. Also, we have introduced an explicit coupling constant $g$. We refrain from spelling out the somewhat lengthy explicit expression for the scalar potential $V_{\text{pot}}$ which has been given in [16]. Let us just mention, that it can be expanded as an eighth order polynomial in the scalars $\phi^{kln}$ with the lowest order given by

$$
V_{\text{pot}} = \frac{g^2}{8} \rho^{5/9} \left( 2 \text{tr}[MM] - (\text{tr} M)^2 \right) + g^2 \rho^{13/9} M^{kln} Y_{k} Y_{l} + \mathcal{O}(\phi^2) \quad (2.4)
$$

with the matrix $M$ defined by $M \equiv (\mathcal{V}^{T})^{-1}$. Modulo the dilaton prefactor, the first term of the potential is precisely the contribution expected from a sphere compactification, cf. [12, 21].

$U(1)^4$ truncation We will now consider the consistent truncation of the Lagrangian [2.2] to singlets under the $U(1)^4$ Cartan subgroup of the gauge group $SO(9)$. Explicitly, we choose a parametrization in which gauge fields and auxiliary scalars reduce to

$$
A_\mu^k = A_\mu^a T_a^{kl} , \quad Y^{kl} = \frac{\rho}{4} y^a T_a^{kl} , \quad a = 1, \ldots, 4 , \quad (2.5)
$$

with generators

$$
T_1^{kl} \equiv 2 \delta_k^\parallel \delta_l^\parallel , \quad T_2^{kl} \equiv 2 \delta_k^\perp \delta_l^\perp , \quad T_3^{kl} \equiv 2 \delta_k^\perp \delta_l^\parallel , \quad T_4^{kl} \equiv 2 \delta_k^\parallel \delta_l^\perp . \quad (2.6)
$$

Correspondingly, the scalar matrix $\mathcal{V}$ reduces to

$$
\mathcal{V} = \exp (\nu_a h^a) , \quad (2.7)
$$

$$
h^a \equiv \text{diag} \left( 1, 1, 0, 0, 0, 0, 0, 0, -2 \right) , \quad h^2 \equiv \text{diag} \left( 0, 0, 0, 1, 0, 0, 0, 0, 2 \right) , \quad h^3 \equiv \text{diag} \left( 0, 0, 0, 1, 1, 0, 0, 0, -2 \right) , \quad h^4 \equiv \text{diag} \left( 0, 0, 0, 0, 0, 1, 1, -2 \right) ,
$$
parametrized by four (dilaton) scalar fields $v_a$. From the 84 (axion) scalars $\phi^{klm}$, four survive the truncation:

$$\phi^1 \equiv \phi^{129}, \quad \phi^2 \equiv \phi^{349}, \quad \phi^3 \equiv \phi^{569}, \quad \phi^4 \equiv \phi^{789}, \quad (2.8)$$

with all other components vanishing. The resulting action is obtained by plugging this truncation into (2.2) and takes the form

$$\mathcal{L} = -\frac{1}{4} \varepsilon \rho R + \frac{1}{2} \varepsilon \rho \sum_a \partial_\mu u_a \partial^\mu u_a + \frac{1}{2} \varepsilon \rho^{1/3} X_0^{-1} \sum_a X_a^{-2} (\partial_\mu \phi^a) (\partial^\mu \phi^a) - \frac{\rho}{8} g e^{\mu\nu} F^a_{\mu\nu} y^a - e \mathcal{V}_{\text{pot}}, \quad (2.9)$$

where we have defined

$$X_a \equiv e^{-2v_a} \equiv e^{-2A_{ab}u_b}, \quad X_0 \equiv (X_1 X_2 X_3 X_4)^{-2}, \quad (2.10)$$

with the matrix

$$A = \begin{pmatrix} 1/6 & -1/\sqrt{2} & -1/\sqrt{6} & -1/(2\sqrt{3}) \\ 1/6 & 0 & 0 & \sqrt{3}/2 \\ 1/6 & 0 & \sqrt{2}/3 & -1/(2\sqrt{3}) \\ 1/6 & 1/\sqrt{2} & -1/\sqrt{6} & -1/(2\sqrt{3}) \end{pmatrix},$$

and the abelian field strengths $F^a_{\mu\nu} \equiv 2 \partial_\mu A^a_{\nu}$. The scalar potential $\mathcal{V}_{\text{pot}}$ in (2.9) can be obtained upon evaluating the explicit expressions given in [16] for the truncation (2.7), (2.8). After some computation, this leads to the expression

$$\mathcal{V}_{\text{pot}} = g^2 \rho^{5/9} \left[ \frac{1}{8} \left( X_0^2 - 8 \sum_{a<b} X_a X_b - 4X_0 \sum_a X_a \right) + \frac{1}{2} \rho^{-2/3} \sum_a X_a^{-2} (X_0 - 4X_a) (\phi^0)^2 \right.$$  

$$+ 2 \rho^{-4/3} \sum_{a<b} X_a^{-2} X_b^{-2} (\phi^a)^2 (\phi^b)^2 + \frac{1}{8} \rho^{-2} \sum_a X_a \left( \rho y^a + 8 \prod_{b \neq a} \phi^b \right)^2 \right.$$  

$$+ \frac{1}{2} \rho^{-8/3} X_0^{-1} \left( \sum_a \rho y^a \phi^a + 8 \prod_a \phi^a \right)^2 \right], \quad (2.11)$$

as a fourth order polynomial in the scalars $\phi^a$. Upon further field redefinition

$$X_a \equiv H_a X_0, \quad \phi^a \equiv \frac{1}{2} \rho^{1/3} \eta_a X_a X_0^{1/2}, \quad (2.12)$$

the potential takes the more compact form

$$\mathcal{V}_{\text{pot}} = \frac{g^2}{8} \rho^{5/9} H_0^{-4/9} \left[ 1 - 8 \sum_{a<b} H_a H_b - 4 \sum_a H_a + \sum_a (1 - 4H_a) \eta_a^2 + \sum_{a<b} \eta_a^2 \eta_b^2 \right.$$  

$$+ \sum_a \eta_a^2 (y^a H_a \eta_a + \eta_0)^2 + \left( \eta_0 + \sum_a y^a H_a \eta_a \right)^2 \right], \quad (2.13)$$

with $H_0 \equiv H_1 H_2 H_3 H_4, \quad \eta_0 \equiv \eta_1 \eta_2 \eta_3 \eta_4.$

---

1 We use the occasion to correct a crucial typo in [16] in the definition of the scalar tensors which are the building blocks of the scalar potential. In equation (4.18), the second term on the r.h.s. in the definition of the tensor $b^a$ should carry a factor of $-\frac{1}{288}$ instead of $\frac{1}{144}$. 
Integrating out the auxiliary fields The auxiliary scalar fields $y^a$ can be eliminated from the Lagrangian by virtue of their field equation

$$y^a = - \sum_b O^{-1} \left( \frac{1}{2} (ge)^{-1} \rho^{4/9} \varepsilon^{\mu\nu} F_{\mu\nu}^b + 8 O_{bb} \prod_{c \neq b} \phi^c \right),$$

with the matrix $O_{ab} \equiv X_a X_b (\delta_{ab} + \eta_a \eta_b) \equiv X_a X_b m_{ab}.$ (2.14)

Then, the vector fields acquire a two-dimensional Maxwell term together with another coupling linear in the field strengths.

$$L = - \frac{1}{4} e\rho R + \frac{1}{2} e\rho \sum_a (\partial_\mu u_a) (\partial^\mu u_a) + \frac{1}{2} e\rho^{1/3} H_0^{2/3} \sum_a H_a^{-2} (\partial_\mu \phi^a) (\partial^\mu \phi^a)$$

$$- \frac{e}{16} \rho^{13/9} H_0^{4/9} \sum_{a,b} H_a^{-1} H_b^{-1} m^{-1}_{ab} F_{\mu\nu}^a F_{\mu\nu}^b$$

$$+ g \rho \eta_0 \sum_{a,b} \varepsilon^{\mu\nu} F_{\mu\nu}^a H_a^{-1} \eta_b^{-1} (1 + \eta_a^2) m^{-1}_{ab} - e \widehat{V}_{pot},$$ (2.15)

with the modified scalar potential given by

$$\widehat{V}_{pot} = \frac{g^2}{8} \rho^{5/9} H_0^{-4/9} \left( 1 - 8 \sum_{a<b} H_a H_b - 4 \sum_a H_a + \frac{9 \eta_0^2}{1 + \sum_a \eta_a^2} \right.$$ 

$$\left. + \sum_a (1 - 4 H_a) \eta_a^2 + \sum_{a<b} \eta_a^2 \eta_b^2 \right).$$ (2.16)

Dilaton sector In the following, we will mainly study the truncation of the Lagrangian (2.15) to the dilaton fields $\{X_a, \rho\}$, i.e. set the axions $\phi^a \equiv 0$. The form of the Lagrangian (2.15) shows that this is a consistent further truncation of the model. This is in contrast to the analogous model in four dimensions, where the $U(1)^4$ of $SO(8)$ supergravity gives rise to a scalar sector of 3 dilators and 3 axions. In that case, the axions are sourced by the field strengths as $F \wedge F$, such that in general they may not consistently be truncated. Under this further truncation, the Lagrangian (2.15) reduces to

$$L = - \frac{1}{4} e\rho R + \frac{1}{2} e\rho \sum_a \partial_\mu u_a \partial^\mu u_a - \frac{1}{16} e\rho^{13/9} \sum_a X_a^{-2} F_{\mu\nu}^a F^{\mu\nu a}$$

$$- \frac{1}{8} e g^2 \rho^{5/9} \left( (X_1 X_2 X_3 X_4)^{-4} - 8 \sum_{a<b} X_a X_b - 4 \sum_a \frac{X_a}{(X_1 X_2 X_3 X_4)^2} \right).$$ (2.17)

This form of the Lagrangian closely resembles the analogous truncations of the maximal AdS supergravities in $D = 4, 5, 7$, with the non-trivial dilaton couplings in $\rho$ exposing the fact that this theory supports a domain wall solution. To analyze the dynamics of the system, it is more convenient to pass over to the Lagrangian with...
auxiliary fields $y^a$, obtained by truncation of (2.9)
\[ \mathcal{L} = -\frac{1}{4} e^\rho R + \frac{1}{2} e^\rho \sum_a \partial_\mu u_a \partial^\mu u_a - \frac{\rho}{8} g \sum_a \varepsilon^{\mu\nu} F_{\mu\nu}^a y^a \]
\[ - \frac{e g^2}{8} \rho^{-4/9} H_0^{-4/9} \left( 1 - 8 \sum_{a<b} H_a H_b - 4 \sum_a H_a + \sum_a (y^a H_a)^2 \right) , \] (2.18)
in terms of the variables (2.12). Now the vector field equations state that the products $(\rho y^a)$ are constant, while variation of the Lagrangian w.r.t. $y^a$ expresses the field strengths as functions of the scalar fields according to
\[ F_{\mu\nu}^a = g e \varepsilon_{\mu\nu} \rho^{-4/9} H_0^{-4/9} H_a^2 y^a . \] (2.19)
The remaining equations of motion are given by the scalar field equations
\[ \sum_b \left( \rho^{-1} \nabla^\mu \left( \rho \partial_\mu u_b \right) A^{-1}_{ba} \right) \]
\[ + g^2 \rho^{-4/9} H_0^{-4/9} \left( 1 + 2 H_a \sum_{b \neq a} H_b + H_a - 2 \sum_b H_b - \frac{1}{2} (y^a H_a)^2 \right) = 0 , \] (2.20)
the traceless part of the Einstein equations
\[ \rho^{-1} \nabla_\mu \partial_\mu \rho + 2 \sum_a \partial_\mu u_a \partial_\mu u_a = \frac{1}{2} g_{\mu\nu} \left( \rho^{-1} \nabla_\mu \partial_\mu \rho + 2 \sum_a \partial_\mu u_a \partial_\mu u_a \right) , \] (2.21)
and suitable combinations of the dilaton and the trace part of the Einstein equations
\[ R = 2 \sum_a \left( \partial_\mu u_a \right) \left( \partial_\mu u_a \right) \]
\[ - \frac{5}{18} g^2 \rho^{-4/9} H_0^{-4/9} \left( 1 - 8 \sum_{a<b} H_a H_b - 4 \sum_a H_a - \frac{13}{5} \sum_a (y^a H_a)^2 \right) , \]
\[ \rho^{-1} \nabla^\mu \partial_\mu \rho = \sum_{a,b} \left( \rho^{-1} \nabla^\mu \left( \rho \partial_\mu u_b \right) A^{-1}_{ba} \right) + \frac{9}{2} g^2 \rho^{-4/9} H_0^{-4/9} \left( 1 - 2 \sum_a H_a \right) . \] (2.22)

**Particular solutions** The simplest solutions of the dilaton sector described above are given by assuming constant scalars $H_a$ and a domain wall ansatz
\[ ds^2 = e^{2 A(r)} dt^2 - dr^2 , \] (2.23)
for the two-dimensional metric. In this case, equations (2.20) determine the auxiliary scalars $y^a$ (therefore the field strengths) in terms of the remaining scalars as
\[ (y^a)^2 = 2 H_a^{-2} (1 + H_a) - 4 + 4 \sum_b \frac{H_b (H_a - 1)}{H_a^2} . \] (2.24)
The remaining equations (2.21), (2.22) then determine $\rho$ and the two-dimensional metric. As special cases, we identify
• the case of vanishing field strengths, i.e. $y^a = 0$. Equations \((2.24)\) then imply that all scalar fields are equal $H_1 = H_2 = H_3 = H_4 \equiv H$, (recall that $H_a > 0$), with two distinct solutions

$$H = 1, \quad \text{or} \quad H = \frac{1}{6}. \quad (2.25)$$

The first choice describes the half-supersymmetric domain-wall solution, completed as

$$\rho = (gr)^{9/2}, \quad A(r) = \frac{7}{2} \ln r, \quad R = \frac{35}{2} r^2, \quad (2.26)$$

corresponding to the ten-dimensional D0-brane near-horizon geometry. The choice $H = \frac{1}{6}$ corresponds to a non-supersymmetric domain-wall with more complicated function $A(r)$.

• the AdS case: imposing a constant dilaton field $\rho$, equation \((2.22)\) implies

$$\sum_a H_a = \frac{1}{2}, \quad (2.27)$$

and the remaining equations of motion are solved by a two-dimensional AdS metric

$$\begin{align*}
\text{ds}^2 &= f(r) \, dt^2 - \frac{1}{f(r)} \, dr^2 \\
f(r) &= -C + g^2 \frac{(1 + 8 \sum_{a<b} H_a H_b) \, r^2}{2 \rho^{4/9} H_0^{4/9}} \\
F_{\mu\nu}^a &= 2 g \rho^{-4/9} \left( \frac{H_a^2 \sqrt{H_a^{-1} - 1}}{H_0^{4/9}} \right) e_{\mu\nu} \\
r_{\text{AdS}} &= \sqrt{2} \rho^{2/9} H_0^{2/9} \left( 1 + 8 \sum_{a<b} H_a H_b \right)^{-1/2}
\end{align*} \quad (2.28)$$

Where $C$ is an integration constant. We obtain a three-parameter family of pure AdS$_2$ solutions. The Killing spinor equations corresponding to the Lagrangian \((2.2)\) quickly show that these solutions break all supersymmetries. While this metric is locally AdS it clearly resembles the $(r-t)$ section of non-rotating BTZ black hole \([19, 20]\) with $C$ being the mass of the spacetime.

### 3 Embedding into IIA Supergravity

We can now describe one of our main results: the reduction ansatz for the embedding of the two-dimensional model \((2.17)\) into type IIA supergravity. The relevant part of
the ten-dimensional Lagrangian is given by

\[
4\pi G L = -\frac{1}{4} e R + \frac{1}{2} e \partial_{M} \phi \partial^{M} \phi - \frac{1}{16} e e^{3\phi} F_{MN} F^{MN} .
\] (3.1)

We split the ten-dimensional coordinates into \(\{x^{M}\} \rightarrow \{x^{\mu}, \mu_{a}, \sigma_{a}\}\), with \(\mu = 0, 1, a = 1, \ldots, 4\), in accordance with the conventions of the previous section. The non-linear Kaluza-Klein ansatz for the proper embedding of the two-dimensional fields is given by generalizing the AdS reduction ansatze from [11] to non-constant dilaton and two-dimensional external space. The ten-dimensional metric then is given by

\[
d s_{10}^{2} = \rho^{-7/36} \Delta^{7/8} d s_{2}^{2} - g^{-2} \rho^{1/4} \Delta^{-1/8} \left( X_{0}^{-1} d \mu_{0}^{2} + \sum_{a} X_{a}^{-1} \left( d \mu_{a}^{2} + \mu_{a}^{2} (d \sigma_{a} + g A^{a})^{2} \right) \right) ,
\] (3.2)

with

\[
\Delta \equiv \sum_{a=0}^{4} X_{a} \mu_{a}^{2} , \quad X_{0} \equiv (X_{1} X_{2} X_{3} X_{4})^{-2} , \quad \mu_{0}^{2} \equiv 1 - \sum_{a} \mu_{a}^{2} .
\] (3.3)

Dilaton and two-form field strength in ten dimensions are given by

\[
\phi = \frac{1}{3} \log \left( \rho^{-7/4} \Delta^{-9/8} \right) ,
\] (3.4)

\[
F = \left( 2 \rho^{5/9} g \sum_{a=0}^{4} \left( X_{a}^{2} \mu_{a}^{2} - \Delta X_{a} \right) + \rho^{5/9} g \Delta X_{0} \right) \varepsilon_{2}
\]

\[
+ \frac{\rho^{13/9}}{2g^{2}} \sum_{a} X_{a}^{-2} d(\mu_{a}^{2}) \wedge (d \sigma_{a} + g A^{a}) (\ast_{2} F^{a}) + \frac{\rho}{2g} \sum_{a=0}^{4} X_{a}^{-1} \ast_{2} d X_{a} \wedge d(\mu_{a}^{2}) ,
\]

with the two-dimensional volume form \(\varepsilon_{2}\) and a two-dimensional Hodge star \(\ast_{2}\) defined with respect to the two-dimensional metric \(g_{\mu \nu}\). All sums over \(\alpha\) run from 0 to 4, the sums over \(a\) run from 1 to 4. Indeed, we have verified explicitly that with this reduction ansatz, the field equations of IIA supergravity (3.1) reduce to the field equations for the two-dimensional fields \(\{g_{\mu \nu}, X_{a}, \rho, A_{a}^{a}\}\), derived from the Lagrangian (2.17) found in the previous section. A convenient way to derive the proper form of the ansatz is by first embedding the solutions (2.27), (2.28) with constant scalars and dilaton \(\rho\) found above. This fixes the ansatz up to the last term in the two-form field strength in (3.4) and the scaling symmetries of the ten-dimensional Lagrangian (3.1)

\[
g_{MN} \rightarrow \lambda^{2} g_{MN} , \quad F_{MN} \rightarrow \lambda \mu F_{MN} , \quad \phi \rightarrow \phi - \frac{2}{3} \log \mu ,
\] (3.5)

with constant \(\lambda, \mu\). Upon proper identification of the scaling parameters \(\lambda, \mu\) as functions of the dilaton \(\rho\), one arrives at the reduction ansatz (3.2), (3.4), which can then be confirmed by explicit calculation.
With (3.2)–(3.4) we have obtained the full ten-dimensional embedding of the dilaton sector of the two-dimensional model (2.2). This is an indispensable tool for further holographic computations around the domain-wall background (2.26) and other backgrounds in this truncation. Such applications to holography of matrix quantum mechanics will be addressed in [22].

4 Rotating branes and Domain-Wall Black Holes

Rotating D0 brane As a further application, let us consider the rotating brane solutions, constructed in the appendix of [11], see also [23]. The case of relevance here, are the rotating 0-brane solutions of (3.1). In the limit of large brane charges, the ten-dimensional solution takes a form that falls into the parametrization (3.2)–(3.4), explicitly given by

\[ ds^2_2 = (gr)^7 h(r)^{-7/9} f(r) \, dt^2 - h(r)^{2/9} f(r)^{-1} \, dr^2 , \]
\[ A^a(r) = \frac{1 - H_a(r)}{l_a} \sqrt{2mg^5} \, dt , \]
\[ \rho(r) = (gr)^{9/2} h(r)^{-1/2} , \]
\[ X_a(r) = h(r)^{-2/9} H_a(r) , \quad (4.1) \]

with free constants \( g, m, l_a \), and the functions

\[ h(r) \equiv \prod_a H_a(r) , \quad H_a(r) \equiv \left( 1 + \frac{l_a^2}{r^2} \right)^{-1} , \quad f(r) \equiv 1 - \frac{2m h(r)}{r^7} . \quad (4.2) \]

One may explicitly verify that the ansatz (4.1) describes a solution of the field equations of the two-dimensional theory derived in section 2. In the massless limit \( m \to 0 \), it preserves half of the supersymmetries. W.r.t. the two-dimensional model, the metric \( ds^2_2 \) in (4.1) describes a domain-wall black hole whose asymptotic curvature approaches (2.26)

\[ R = \frac{35}{2r^2} + \mathcal{O} \left( r^{-23/9} \right) , \quad \text{for} \ r \to \infty , \quad (4.3) \]

whereas at \( r = 0 \) it behaves like

\[ R = -\frac{7}{6} r^{-34/9} \prod_a (l_a)^{4/9} + \mathcal{O} \left( r^{-26/9} \right) . \quad (4.4) \]

We may consider the near-horizon behavior of the two-dimensional metric around \( r = r_0 \), where \( r_0 \) is the highest root of the equation \( f(r) = 0 \). To this end, we expand the coordinates according to

\[ r \to r_0 + \epsilon r , \quad t \to \rho_0^{-7/9} h_0^{-1/9} \frac{t}{\epsilon} , \quad (4.5) \]
where
\[
\rho_0 \equiv (gr_0)^{9/2} h_0^{-1/2}, \quad h_0 \equiv \prod_{a=1}^{4} H_{a0}, \quad H_{a0} \equiv \left(1 + \frac{l_a^2}{r_0^2}\right)^{-1}.
\] (4.6)

In the limit \( \epsilon \to 0 \), we then obtain the near-horizon AdS\(_2\) configuration
\[
ds^2 = f_0 dt^2 - \frac{1}{f_0} dr^2, \quad F_{tr} = 2 g \rho_0^{-4/9} \left(\frac{H_{a0}^2 \sqrt{H_{a0}^{-1} - 1}}{h_0^{4/9}}\right),
\] (4.7)

with
\[
f_0 \equiv g^2 \left(1 + \frac{1}{2 \rho_0^{4/9}} \sum_{a<b} H_{a0} H_{b0}\right) r^2,
\] (4.8)

provided the constants \(H_{a0}\) satisfy the following further condition
\[
\sum_{a=1}^{4} H_{a0} = 1/2.
\] (4.9)

This is the \( C = 0 \) case of the solution (2.28) found above. According to the embedding (3.2)–(3.4), this solution corresponds to a ten-dimensional warped product geometry AdS\(_2\) × \( \mathcal{M}_8 \).

**Thermodynamics of non-rotating D0-branes** Let us consider first the case when the angular momentum vanishes \( l_a = 0 \). The solution then has \( A_a = 0, H_a = 1, h(r) = 1, X_a = 1, \Delta = 1 \) and simplifies to
\[
ds_{10}^2 = \rho^{-7/36} \left[(gr)^7 f(r) dt^2 - f(r)^{-1} dr^2\right] - g^{-2} \rho^{1/4} d\Omega_8,
\] (4.10)
\[
\exp(\phi) = \rho^{-7/12}, \quad F = -7 \rho^{5/9} g \varepsilon_2, \quad \rho = (gr)^{\frac{9}{7}}, \quad f(r) = 1 - \frac{2m}{r^2},
\] (4.11)

where, again, the two-dimensional volume form \( \varepsilon_2 \) is defined as: \( \varepsilon_2 = e dt \wedge dr \). The corresponding string frame metric
\[
\alpha' e^\phi ds_{10}^2 = \alpha' \left(\rho^{-7/9} \left[(gr)^7 f(r) dt^2 - f(r)^{-1} dr^2\right] - g^{-2} \rho^{-1/3} d\Omega_8\right),
\] (4.12)

coincides with eq. (2.3) of [24] \(^2\).

The original asymptotically flat \( p \)-branes were constructed in [25] and their thermodynamics, as \( Dp \)-branes, in the context of the AdS/CFT correspondence was analyzed in [2]. The entropy of this configuration is given by the area law
\[
S = g^{-\frac{7}{2}} (2m)^{\frac{9}{2}} \frac{\pi}{4G} \text{Vol}(S^8).
\] (4.13)

Its temperature it is given by the inverse of the Euclidean period,
\[
(2\pi T)^2 = - g^{\mu\nu} (\partial_\mu N) (\partial_\nu N) \bigg|_{r^2 = 2m} = \frac{49 g^{7} (2m)^{\frac{7}{2}}}{4}.
\] (4.14)

\(^2\)It exactly coincides after the relabeling \( r = U \) and \( g^{-7} = a_0 \lambda \) where \( a_0 = 2^7 \pi \frac{5}{2} \Gamma(\frac{7}{2}) \).
where $N = g_{\mu\nu}K^\mu K^\nu$ and $K = \partial_t$ is the relevant timelike Killing vector. The first law of black hole thermodynamics is

$$\delta M = T \delta S = \frac{9 \text{Vol}(S^8)}{16\pi G} \delta m . \quad (4.15)$$

The energy of the spacetime is then given by the integration of the first law

$$M = \frac{9 \text{Vol}(S^8)}{16\pi G} m . \quad (4.16)$$

Using (4.14) and (4.16) it is possible to see that

$$M \sim T^{\frac{14}{5}}, \quad (4.17)$$

in accordance with the results in the literature [24], [2].

**Thermodynamics of rotating D0-branes** Let us now proceed to the analysis of the rotating solution (4.1). The metric (3.2) is in Boyer-Lindquist form and the eight sphere is round when $r = \infty$. Therefore is possible to identify the angular velocities of the horizon

$$\Omega_i = -g A^a_i(r_+) , \quad (4.18)$$

therefore, the Killing vector that vanishes at the horizon is

$$K = \partial_t - g A^a(r_+) \partial_{\phi_a} , \quad (4.19)$$

where $2m = r_+^7 h(r_+)$ is the location of the horizon. Note that

$$2\delta m = r_+^7 h(r_+) \left( \frac{7}{r_+} + \frac{h'(r_+)}{h(r_+)} \right) \delta r_+ + \frac{r_+^5 h(r_+)}{H_a(r_+)} \delta l_a^2 . \quad (4.20)$$

The temperature is

$$(2\pi T)^2 = \frac{g^7 r_+^7 h(r_+)}{4} \left( \frac{14m}{r_+^8 h(r_+)} + \frac{2m h'(r_+)}{r_+^7 h^2(r_+)} \right)^2 = \frac{g^7 r_+^7 h(r_+)}{4} \left( \frac{7}{r_+} + \frac{h'(r_+)}{h(r_+)} \right)^2 . \quad (4.21)$$

The entropy is straightforward to compute

$$S = \frac{g^{-8} \rho(r_+)}{4G} \text{Vol}(S^8) = \frac{g^{-7/2} r_+ \sqrt{2m}}{4G} \text{Vol}(S^8) . \quad (4.22)$$

It follows that

$$T \delta S = \frac{9}{16\pi G} \text{Vol}(S^8) \delta m + \frac{\text{Vol}(S^8)}{8\pi G} \Omega_a \delta \left( g^{-7/2} l_a \sqrt{2m} \right) , \quad (4.23)$$

\footnote{A very good discussion about thermodynamical aspects of the Kerr black hole is [26].}
which allows to identify the energy $M$ and angular momenta $J_a$ of the spacetime

$$M = 9m \frac{\text{Vol}(S^8)}{16\pi G}, \quad J_a = -g^{-7/2}l_a \sqrt{2m} \frac{\text{Vol}(S^8)}{8\pi G}. \quad (4.24)$$

It is possible to appreciate that the simple relation between the energy density in the CFT and the temperature (4.17) does no longer hold. Our expressions exactly coincide with the thermodynamics in the near horizon limit of D0-branes given by eqs. (3.4a) and (3.4b) of [23].

5 Entropy function

We would like to understand the thermodynamics of the rotating D0 branes, in the extremal limit, from a two-dimensional point of view. To this end we shall use the entropy function formalism introduced by Sen in [27] which also applies to the asymptotically AdS black holes in higher-dimensional gauged supergravity [28]. The idea is to take the scalar and gauge fields at some fixed value and the metric to be AdS$_2$

$$ds^2 = \rho^2 \left( r^2 dt^2 - \frac{dr^2}{r^2} \right), \quad F_{tr} = Q^a, \quad (5.1)$$

which, when evaluated in (2.17), yield the following Lagrangian

$$\mathcal{L} = -\frac{\rho}{2} + \frac{v^2}{8} \rho^{13/9} Q^a Q^a X^{-2} - \frac{v^2}{8} g^2 \rho^{5/9} V(X). \quad (5.2)$$

The entropy of the configuration is given by the extremal value of the Legendre transform of the Lagrangian respect to $Q$

$$S = 2\pi \kappa (q^a Q^a - \mathcal{L}), \quad (5.3)$$

where the $2\pi$ factor is universal, the $\mathcal{L}$ is the two dimensional Lagrangian (2.17), $q^a$ are the physical charges and $\kappa$ is an overall multiplication constant defined by the relation between the ten-dimensional action and the two dimensional action. Indeed, the ten-dimensional Lagrangian, (3.1), has the canonical normalization in front of the Ricci scalar $(16\pi G)^{-1}$, which allowed us to define the entropy in the standard form (4.13)

$$S = \frac{A}{4G}, \quad (5.4)$$

where $A$ is the black hole area. It follows that the constant $\kappa$ is determined by the integral over the eight sphere of the ten dimensional Ricci scalar

$$\kappa \rho (Re)_{2D} = \int_{S^8} (Re)_{10D}. \quad (5.5)$$

---

4 It is necessary to make the identification $h = g^{-1}$, $r_0^2 = 2m$ and $V_0 = 1$.

5 For the sake of simplicity and consistency with the original work [10], the two-dimensional Lagrangian has been written without due normalization.
In this way we get

\[ \kappa = \frac{\text{Vol}(S^8) g^{-8}}{4\pi G} . \]  

(5.6)

The extremal value of the entropy function functional is given by the equations

\[ \kappa^{-1} \frac{\partial S}{\partial Q^a} = q^a - \frac{v^2}{4} \rho^{13/9} g^{a} X^{-2} , \]  

(5.7)

\[ \kappa^{-1} \frac{\partial S}{\partial \rho} = \frac{1}{2} - \frac{13v^2}{72} \rho^{4/9} g^a X^{-2} + \frac{5v^2}{72} g^2 \rho^{-4/9} V(X) , \]  

(5.8)

\[ \kappa^{-1} \frac{\partial S}{\partial v} = \frac{v^{-3}}{4} \rho^{13/9} g^a X^{-2} + \frac{v}{4} g^2 \rho^{5/9} V(X) , \]  

(5.9)

\[ \kappa^{-1} \frac{\partial S}{\partial X^b} = \frac{v^{-2}}{4} \rho^{13/9} g^b X^{-3} + \frac{v^2}{8} g^2 \rho^{5/9} \frac{\partial V(X)}{\partial X^b} . \]  

(5.10)

Equation (5.7) defines the electric charge \( q^a \). Equations (5.8) and (5.9) are not independent. Indeed, is easy to see that

\[ v^{-3} \rho^{13/9} g^a X^{-2} = 2v^2 \rho v^{-2} = -g^2 V(X) \implies 2v^2 \rho^{-4/9} = Q^a g^a X^{-2} . \]  

(5.11)

The field equations for the scalar fields (5.10) can be interpreted as defining the value of the scalar fields as function of the charges \( q^a \) which is the version of the attractor mechanism relevant for our setting. From these equations, we get

\[ 0 = -V(X) + \frac{1}{2} X_b \frac{\partial V(X)}{\partial X^b} , \]  

(5.12)

which is the condition (2.27) for the \( H_a \) adding up to 1/2. Together it is easy to get that the entropy of the two dimensional, near extremal D0 branes is

\[ S = 2\pi \kappa \rho \frac{\rho}{2} = \frac{g^{-8}}{4G} \text{Vol}(S^8) , \]  

(5.13)

in perfect agreement with our ten-dimensional result (4.22). This result is very interesting, as it shows that the entropy function formalism works very well even for objects that have no area. Indeed, as the two-dimensional configurations considered here have a zero-dimensional horizon, the value of their entropies is far from clear. The fact that one can embed the geometries in ten dimensions give us a prescription to compute its thermodynamics, which turns out to coincide, in the near extremal limit, with the entropy function formalism. This is a good indication that the entropy function formalism can be safely used in two dimensions, and could be extended even to cases where no ten-dimensional embedding is known.

6 AdS\(_2 \times \mathcal{M}_8\) solutions with non-vanishing axions

Above, we have found within the pure dilaton-sector of the two-dimensional model the most general solution with constant dilatons, thus lifting to ten-dimensional product
geometries $\text{AdS}_2 \times \mathcal{M}_8$. This results in the three-parameter family of solutions (2.27)–(2.28), with the metric on $\mathcal{M}_8$ given by (3.2). Here, we sketch how the full $U(1)^4$ truncation (2.9) allows for many more solutions of this type (with non-vanishing axions $\phi^a$), although the structure of the equations and their explicit solutions become much more complicated.

Let us start from the Lagrangian (2.9) and consider the equations of motion assuming all scalar fields to be constant. The equations of motion for the auxiliary fields $y^a$ determine the field strengths $F_{\mu\nu}^a$. In turn, the vector fields equations state that the $y^a$ are constants. The field equation for $\rho$ fixes the AdS$_2$ radius. Finally the remaining equations are

$$V_{\text{pot}} = 0, \quad \frac{\partial V_{\text{pot}}}{\partial H_a} = 0, \quad \frac{\partial V_{\text{pot}}}{\partial \eta_a} = 0,$$

for the scalar potential $V_{\text{pot}}$ from (2.13). In total, these are nine algebraic equations for the 12 parameters $H_a$, $y_a$, $\eta_a$ to be determined. Moreover, we can explicitly solve the four last equations of (6.1) for the $H_a$, leaving us with five equations for eight unknowns. The counting suggests that there are various families of solutions to these equations and even though the explicit equations are too complicated to allow for the general explicit solution (collection of algebraic equations that are up to eighth order polynomials), first numerical results support this idea. Just as an example, we mention that an explicit solution can be found upon further truncation the system

$$H_1 \equiv H_3, \quad H_2 \equiv H_4, \quad y_1 \equiv y_3, \quad y_2 \equiv y_4, \quad \eta_1 \equiv \eta_3, \quad \eta_2 \equiv \eta_4.$$  

(6.2)

In this truncation, the equations can further be reduced to quadratic equations and allow for the explicit solution

$$H_1 = \frac{1}{128} (43 - 5 \sqrt{33}), \quad H_2 = \frac{1}{64} (25 + 9 \sqrt{33}),$$

$$(y_1)^2 = 12 \left(6 + \sqrt{33}\right), \quad (y_2)^2 = 2 \left(-1 + \sqrt{33}\right),$$

$$(\eta_1)^2 = \frac{1}{8} (9 + \sqrt{33}), \quad (\eta_2)^2 = \frac{1}{16} (1 + \sqrt{33}).$$

(6.3)

with Ricci scalar given by

$$R = \frac{2^{2/3} 3 (3815 + 759 \sqrt{33})}{\left(-205 + 131 \sqrt{33}\right)^{8/9}} \frac{g^2}{\rho^{4/9}} \approx 143.27 \frac{g^2}{\rho^{4/9}}.$$  

(6.4)

It would certainly be interesting to get better control over the solutions of this type in the full model (2.9) as well as on their higher-dimensional origin. The latter may represent a major step towards finding the Kaluza-Klein ansatz for the model with non-vanishing axion fields into the ten-dimensional theory.

### 7 Acknowledgments.

A.A. would like to thank Toby Wiseman and Dumitru Astefanesei for insightful discussions. Research of A.A. is supported in part by the Fondecyt Grant 11121187 and
by the CNRS project “Solutions exactes en présence de champ scalaire”.

References

[1] J. M. Maldacena, The large $N$ limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231–252, [hep-th/9711200].

[2] N. Itzhaki, J. M. Maldacena, J. Sonnenschein, and S. Yankielowicz, Supergravity and the large $N$ limit of theories with sixteen supercharges, Phys. Rev. D58 (1998) 046004, [hep-th/9802042].

[3] T. Wiseman and B. Withers, Holographic renormalization for coincident Dp-branes, JHEP 0810 (2008) 037, [0807.0755].

[4] I. Kanitscheider, K. Skenderis, and M. Taylor, Precision holography for non-conformal branes, JHEP 0809 (2008) 094, [0807.3324].

[5] H. J. Boonstra, K. Skenderis, and P. K. Townsend, The domain wall/QFT correspondence, JHEP 01 (1999) 003, [hep-th/9807137].

[6] K. Behrndt, E. Bergshoeff, R. Halbersma, and J. P. van der Schaar, On domain wall / QFT dualities in various dimensions, Class.Quant.Grav. 16 (1999) 3517–3552, [hep-th/9907006].

[7] E. Bergshoeff, M. Nielsen, and D. Roest, The domain walls of gauged maximal supergravities and their M-theory origin, JHEP 07 (2004) 006, [hep-th/0404100].

[8] B. de Wit and H. Nicolai, The consistency of the $S^7$ truncation in $D = 11$ supergravity, Nucl.Phys. B281 (1987) 211.

[9] H. Nicolai and K. Pilch, Consistent truncation of $d = 11$ supergravity on $\text{AdS}_4 \times S^7$, JHEP 1203 (2012) 099, [1112.6131].

[10] H. Nastase, D. Vaman, and P. van Nieuwenhuizen, Consistency of the $\text{AdS}_7 \times S^4$ reduction and the origin of self-duality in odd dimensions, Nucl. Phys. B581 (2000) 179–239, [hep-th/9911238].

[11] M. Cvetic, M. Duff, P. Hoehna, J. T. Liu, H. Lu, J. Lu, R. Martinez-Acosta, C. Pope, H. Sati, and T. A. Tran, Embedding AdS black holes in ten-dimensions and eleven-dimensions, Nucl.Phys. B558 (1999) 96–126, [hep-th/9903214].

[12] M. Cvetic, S. Gubser, H. Lu, and C. Pope, Symmetric potentials of gauged supergravities in diverse dimensions and Coulomb branch of gauge theories, Phys.Rev. D62 (2000) 086003, [hep-th/9909121].
[13] M. Cvetic, H. Lu, C. Pope, A. Sadrzadeh, and T. A. Tran, Consistent SO(6) reduction of type IIB supergravity on $S^5$, Nucl. Phys. B586 (2000) 275–286, [hep-th/0003103].

[14] M. Cvetic, J. T. Liu, H. Lu, and C. Pope, Domain wall supergravities from sphere reduction, Nucl. Phys. B560 (1999) 230–256, [hep-th/9905096].

[15] M. Cvetic, H. Lu, and C. Pope, Consistent sphere reductions and universality of the Coulomb branch in the domain wall / QFT correspondence, Nucl. Phys. B590 (2000) 213–232, [hep-th/0004201].

[16] T. Ortiz and H. Samtleben, $SO(9)$ supergravity in two dimensions, JHEP 1301 (2013) 183, [1210.4266].

[17] B. de Wit, J. Hoppe, and H. Nicolai, On the quantum mechanics of supermembranes, Nucl. Phys. B305 (1988) 545.

[18] T. Banks, W. Fischler, S. Shenker, and L. Susskind, M theory as a matrix model: A Conjecture, Phys. Rev. D55 (1997) 5112–5128, [hep-th/9610043].

[19] M. Banados, C. Teitelboim and J. Zanelli, “The Black hole in three-dimensional space-time,” Phys. Rev. Lett. 69 (1992) 1849 [hep-th/9204099].

[20] M. Banados, M. Henneaux, C. Teitelboim and J. Zanelli, “Geometry of the (2+1) black hole,” Phys. Rev. D 48 (1993) 1506 [gr-qc/9302012].

[21] M. Cvetic, H. Lu, and C. N. Pope, Consistent Kaluza-Klein sphere reductions, Phys. Rev. D62 (2000) 064028, [hep-th/0003286].

[22] T. Ortiz, H. Samtleben, and D. Tsimpis, work in progress.

[23] T. Harmark and N. Obers, Thermodynamics of spinning branes and their dual field theories, JHEP 0001 (2000) 008, [hep-th/9910036].

[24] T. Wiseman, On black hole thermodynamics from super Yang-Mills, JHEP 1307 (2013) 101, [1304.3938].

[25] G. T. Horowitz and A. Strominger, Black strings and $p$-branes, Nucl. Phys. B360 (1991) 197–209.

[26] M. M. Caldarelli, G. Cognola, and D. Klemm, Thermodynamics of Kerr-Newman-AdS black holes and conformal field theories, Class. Quant. Grav. 17 (2000) 399–420, [hep-th/9908022].

[27] A. Sen, Black hole entropy function and the attractor mechanism in higher derivative gravity, JHEP 0509 (2005) 038, [hep-th/0506177].

[28] J. F. Morales and H. Samtleben, Entropy function and attractors for AdS black holes, JHEP 0610 (2006) 074, [hep-th/0608044].