One of the fundamental problems with the interpretation of Quantum Mechanics, according to Bohr [1], is the fact that “our usual description of physical phenomena is based entirely on the idea that the phenomena concerned may be observed without disturbing them appreciably”. Furthermore in his articles [1, 2] discussing the subject Bohr argues that the action of the probe will be affected by the system and inasmuch the system will be affected by the probe. Specifically in *Gedanken* experiments he tests the wave–particle duality of the system and implicitly assumes that the probe is also a quantum system. A universal character can only be attributed to Quantum Mechanics provided a complementarity relation is also valid for the probe. As a consequence the system–probe becomes entangled as extensively discussed by N. Bohr, W. Heisenberg, A. Einstein, E. Schrödinger, de Broglie and W. Pauli among others starting in the famous Solvay conference [3]. Soon after entanglement has been brought to discussion [4, 5] and since then it has always been present in several contexts [6]. In the past forty years complementarity tests have been proposed [7–11], cavity interferometry [12, 13], and experiments involving which-way detection [14, 15], to quote some. However, much less attention has been paid to the study of an arbitrary probe system. In the present contribution, we fill in this gap and show that the key ingredient for the quantum–classical transition is not necessarily the information generated by the system–probe interaction but rather by its accessibility. Our results have been successfully tested in the interferometric experiment by [13]. Our results also allow for a simple physical interpretation of the physics of Ramsey Zones [16] where one photon (average) interacts with a two level atom in a classical manner, *i.e.*, no entanglement is generated.

Let us consider a Mach–Zehnder interferometer (see [19] for a revision) with a probe system capable of obtaining and storing the which–way information provided by a particle sent through it, as shown in Figure 1. In the process of obtaining and storing which–way information, the probe and particle will entangle. This results in a kind of Einstein–Podolsky–Rosen [5] pair. The configuration is that of a bipartite system defined in the Hilbert space $\mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)}$, where the subsystem in $\mathcal{H}^{(1)}$ has two levels and is entangled to another system in $\mathcal{H}^{(2)}$. In general one can describe in the Schmidt base [21]

$$|\Psi\rangle = \sum_{i=1}^{2} \sqrt{\lambda_i} |v^{(1)}_i\rangle |v^{(2)}_i\rangle$$

where $\{|v^{(1)}_i\rangle\}$ and $\{|v^{(2)}_i\rangle\}$ are orthonormal bases in $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$, respectively, and the $\sqrt{\lambda_i}$ are the Schmidt coefficients. The state of the system in $\mathcal{H}^{(1)}$ will be $\rho^{(1)} = \sum_{i=1}^{2} \lambda_i |v^{(1)}_i\rangle \langle v^{(1)}_i|$. Since $\lambda_1 + \lambda_2 = 1$ it follows immediately that $(\lambda_1 - \lambda_2)^2 + 4\det(\rho^{(1)}) = 1$, where $4\det(\rho^{(1)})$ is the concurrence $C_{(1,2)}$ squared [22] between particle–probe.

The system in $\mathcal{H}^{(2)}$ is a two level system and therefore can be written in terms of Pauli matrices $\{\sigma_2, \sigma_1, \sigma_3\}$. The equation above becomes $(\sigma_2^{(1)})^2 + 4(\sigma_3^{(1)})^2 + C_{(1,2)}^2 = 1$, where $\langle \sigma_3^{(1)} \rangle = \text{tr}(\rho^{(1)} \sigma_3)$. Note that as we identify the $\sigma_3^{(1)}$ eigenstates with the arms of the interferometer, $|\sigma_3^{(1)}\rangle$ represents the probability of finding the particle in one of the arms, usually called *predictability* $P^{(1)}$.
On the other hand, the term $|\sigma_{(1)}^{(1)}|$ represents the magnitude of the coherence between the interferometers arms. The interference fringes will exhibit a visibility $V_1 = 2|\sigma_{(1)}^{(1)}|$ [23]. Having these two results it follows that $S_{(1)}^2 + C_{(1,2)}^2 = 1$, result first found in [23], where $S_{(1)}^2 = \mathcal{T}_0^2 + V_1^2$ represents wave–particle duality. Given the symmetry of the Schmidt decomposition the wave–particle duality of the other system will be the same,

$$S_{(2)}^2 = S_{(1)}^2.$$  \hspace{1cm} (2)

In our problem the second system is the probe. Let us consider that the particle has been prepared in the state $|\psi\rangle = (|+\rangle + |0\rangle)/\sqrt{2}$ and sent through the interferometer. The probe, prepared in $|m\rangle$, evolves to $|m\pm\rangle$. We may at this point ask for the probability that a measurement of the state of the probe will yield $|m\pm\rangle$. This probability will be $|\langle m|\pm\rangle|^2$, for both cases and may be interpreted as the imperfection of the probe. As a consequence we may define $Q = 1 - |\langle m|\pm\rangle|^2$ a quantity which represents the quality of the probe. The quantity $D = \sqrt{Q}$ is a quantitative measure of the distinguishability of the probe states [14] i.e., of the which–way information available in the probe system. In this notation we can say that a perfect probe with $D = 1$ is the one which makes the which–way information completely available. Thus a perfect probe ($Q = 1$) will have completely distinguishable probe states ($|\langle m|\pm\rangle|^2 = 0$) and the particle state will be a statistical mixture showing thus no interference pattern. On the other hand, when the probe is completely imperfect ($Q = 0$) the probe states will be indistinguishable ($|\langle m|\pm\rangle|^2 = 1$) and differ at most by phase. This phase is not contain which–way information. As a consequence the global state is factorized. For a probe with $0 < Q < 1$ we will have information generation however its accessibility is only partial, proportional to $Q$. In this case the particle will present an intermediate visibility. This means that generating the which–way information is not enough to destroy its interference pattern.

**The probe:** A general initial state of the system can be written as $\rho_0 = (1 + u_0 \cdot \sigma^{(1)}) \otimes |m\rangle\langle m|/2$ where, $\sigma^{(1)} = \{\sigma_x^{(1)}, \sigma_y^{(1)}, \sigma_z^{(1)}\}$ are Pauli matrices, $|u_0| = 1$, $u_0 = \{x_0, y_0, z_0\}$ is the Bloch vector of the particle state and $|m\rangle\langle m|$ the initial state of the probe. The particle state immediately before the second BS will be $\rho_{(1)} = \text{tr}_{(2)} \{U_F U_U \rho_0 U_U U_F^\dagger \}$ and its predictability $P_{(1)} = |\langle x_0|\rangle$ which is identical to the a priori predictability $P_0$ [24]. Now if the particle is detected $D_1$ and the phase difference $\Phi$ varies we will obtained an interference fringe with visibility

$$V_{(1)} = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$

where $I_{\text{max}}(I_{\text{min}})$ is the maximum (minimum) fringe intensity, respectively. After second BS the particle state will be $\rho_{(1)} = \text{tr}_{(2)} \{U_B U_F U_U \rho_0 U_B^\dagger U_U U_F^\dagger \}$ and the intensity $I$ will be proportional to $I \propto (\langle 0|\rho_{(1)}|0\rangle)^2 = 1 + (z_0 + i y_0)(m_+ + m_-)e^{-i\Phi} + (z_0 - i y_0)(m_- - m_+)e^{i\Phi}$. This yields the visibility $V_{(1)} = V_0 \sqrt{1 - D^2}$, where $V_0 = \sqrt{y_0^2 + z_0^2}$ is the a priori visibility [23]. Observe that $V_{(1)}$ directly depends on the distinguishability of the information contained in the probe. This means that which–way information generation is a necessary condition to destroy the interference pattern of the particle. It is however not enough. In order to destroy interference it is necessary not only to generate information but also to make it accessible. The concurrence [22] between the particle and the probe is given by

$$C_{(1,2)} = V_0 D$$ \hspace{1cm} (3)

and it is easy to the verify that $S_{(1)}^2 + C_{(1,2)}^2 = 1$.

**The probe:** From the preceding discussion its easy to see that the complementarity relation for the probe system entangled with a particle has the same form as that for the first system $S_{(2)}^2 + C_{(2)}^2 = 1$, where the duality $S_{(2)}$ must respect equation (2). In order to have separated expressions to characterize the particle or wave like nature of the probe system we will have to construct, e.g., a quantity $P$ which reflect the particle character of the probe. Its wave like character can then be extracted from the equality $S_{(2)}^2 = S_{(2)}^2$.

**Physical conditions for constructing $P$:** After the probe interacts with the particle it will unitarily evolve to $|m\rangle$. The operators $\Pi_\pm = |m\rangle\langle m|/2$ are projectors and their averages $\langle \Pi_\pm \rangle = \text{tr}(\rho_{(2)} \Pi_\pm)$ are the probabilities to find the probe in $|m\rangle$, respectively. Similarly to the particle predictability we would like to have $P = 0$ when $\langle \Pi_\pm \rangle = 1/2$ and $P = 1$ when $\langle \Pi_\pm \rangle = 1$. Another interesting condition is that $P$ has the interpretation of the predictability when the probe is perfect. All these conditions will be satisfied when we define $P = \left|1 - 2\langle \Pi_\pm \rangle\right|$ (and $\pm x_0 \to |x_0\rangle$). The probe state for the evaluation of $P$ will be $\rho_{(2)} = \text{tr}_{(1)} \{U_B U_F U_U \rho_0 U_B^\dagger U_B^\dagger U_F^\dagger \}$. From the above construction we get

$$P = 1 - \left|1 - P_{(1)}\right| D^2$$ \hspace{1cm} (4)

The factor $(1 - P_{(1)})$ quantifies our uncertainty as to the path chosen by the particle. On the other hand, $D^2$ measures the distinguishability of the final states of the probe. For a value of $D$ fixed, the power to predict the final state of the probe decreases as this uncertainty grows. The product $(1 - P_{(1)})D^2$ quantifies the contribution due to the wave–like character of the probe and the non–local correlations between the two subsystems produced by the interaction.

**Probe wave like character:** As discussed before it is now immediate to obtain the wave–like characteristics of the probe. One substitutes equation (4) in (2) and assuming that $S_{(2)}^2 = P^2 + V^2$, we get

$$V = (1 - P_{(1)}) D \sqrt{1 - D^2}$$ \hspace{1cm} (5)
Since $0 \leq D \leq 1$, the product $D\sqrt{1-D^2}$ varies between 0 and 1/2. Two different values of $D$ are associated to a particular value of this product. Hence, we have $V = 0$ both for the perfect probe (which we associate with the extreme quantum regime) as for imperfect one (extreme classical regime). When the probe is perfect it entangles with the particle and the global state of the system is an authentic Einstein–Podolsky–Rosen pair [5]. In the opposite limit the probe state remains unaltered and therefore no quantum features of any sort will be present. Interesting to note that one will always have $V \leq C_{(1,2)}$ where the equality is only reached in trivial case $P = 1$. Of course this is a particular feature of the problem in question, since there is no local unitary operation on the probe to enhance $V$. Thus particle characteristics and its ability to entangle will assume an important role in the complementarity relation. Resorting to the complementarity relation for the probe, $S_2 + C_{(1,2)} = 1$, and to equation (1), it is now clear that the product $(1 - P_0)D^2$ represents the sum of the contributions related to the wave–like behavior of the probe, quantified by $V$, and the non–local correlations due to the interaction, measured by $C_{(1,2)}$.

From the quantum to the classical limit: For an arbitrary particle state, the availability of the which–way information will be minimum when $S_2 = |x_0\rangle$, i.e., the probe is completely imperfect and devoid of quantum behavior. In this case, the extreme classical regime is reached and the probe exhibits corpuscular behavior. On the other hand, it will be maximum when $S_2 = \mathbb{P} = |x_0\rangle$ when its behavior will be completely quantum. Note that $D = C_{(1,2)}$ when the particle is prepared in a state which satisfies $P_0 = 0$. In this case the probe complementarity relation only depends on its own characteristics. We can thus define a good probe when $D > 1/\sqrt{2}$, i.e., its imperfection will be smaller than 1/2. The opposite limit, i.e., $D < 1/\sqrt{2}$ is a weak condition since there will be a region $2/(1 + \sqrt{5}) < D < 1/\sqrt{2}$ where $C_{(1,2)} \geq V \geq 0$, as shown in figure 2. Thus a better definition for a bad probe is $P > C_{(1,2)} \rightarrow D < 2/(\sqrt{5} + 1)$.

The physics of Ramsey Zones: To display the generality of our findings we will discuss in their light an intriguing question. Ramsey Zones are microwave cavities with low quality factor. It serves the purpose of rotating effectively two level Rydberg atoms. The process consists in sending atoms through the cavity where there is a coherent field of approximately one average photon maintained by an external source. The interesting conceptual issue is: Why do Ramsey Zones work as if they contain classical fields and no entanglement between atom and photon is generated? A full microscopic calculation has been performed [29], however a convincing transparent physical picture has not emerged. For typical values of the atomic flight velocity the time $t_i$ of the atom field interaction is that of a few $\mu$s [13]. Let us consider $t_i$ sufficiently small so that one can assume a unitary evolution during the interaction. Under this hypothesis, the global state immediately after the interaction will be

$$|\phi\rangle = \frac{1}{\sqrt{2}}(|e\rangle|\alpha_+\rangle + |g\rangle|\alpha_-\rangle)$$

with $|\alpha_+\rangle = \sqrt{2}\sum_n C_n \cos(\Omega\sqrt{n + t_i})|n\rangle$ and $|\alpha_-\rangle = \sqrt{2}\sum_n C_n \sin(\Omega\sqrt{n + t_i})|n\rangle$, where $C_n = e^{-n^2/2}/\sqrt{n!}$, $\Omega$ is the Rabi frequency in vacuum and $t_i$ is the necessary time for the atom to suffer a $\pi/2$ pulse defined by $\sum_n |C_n|^2 \cos^2(\Omega\sqrt{n + t_i}) = 1/2$. If we have $|\alpha|^2 = 1$ we will have $P \approx 0.3271$, $V \approx 0.4691$ and $C_{(1,2)} \approx 0.8203$. So one should expect that the field in the Ramsey Zone would have measurable quantum characteristics and behave as a good which–way discriminator, since the information available is large $D \approx 0.8203$. Why don’t the expected features appear?

Immediately after the atom leaves the cavity the dynamics of the feeding source along with strong cavity dissipation act on the field state. Since the Ramsey Zone possesses a relaxation times very short, i.e., smaller than $T_\gamma = 10\mu$s [13,18], then in a time interval of the order $T_r$ the photons inside the cavity are renewed and the state $|\alpha\rangle$ is restored resulting $D = 0$ with the information no longer available. It is important to note that this not mean that the which–way information disappeared. It has become stored in a system with infinite degrees of
freedom which renders it inaccessible. Therefore atom–
field disentangle. In other words the field goes almost
instantaneously from a prevailing quantum regime where
it was, a priori, a good probe, to a situation where there
will be only corpuscular characteristics $P = 1$. This is a
classical regime as we defined. This suggests the follow-
ging general picture: systems which do not store neither
make the information about its interaction available, it
can be by intrinsic properties of the systems or by aux-
iliaries dynamics, may be consider classical because the
act on the quantum system without get entangled. This
might explain why quantum objects like photons cross
macroscopic apparatuses and suffer unitary evolution in
the complete absence of entanglement, e.g.: photon plus
beam splitters or photons plus quarter wave plates, to
quote some.

The present work, simple as it is, led us to believe
that further exploiting Bohr’s complementarity principle
may lead to very simple and solid physical explanation
for important conceptual problems which at first sight
appear to be of high mathematical intricacy.

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