Projectile motion in real-life situation: Kinematics of basketball shooting

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Abstract. Basketball shooting is a basic practice for players. The path of the ball from the players to the hoop is projectile motion. For undergraduate introductory physics course student must be taught about projectile motion. Basketball shooting can be used as a case study for learning projectile motion from real-life situation. In this research, we discuss the relationship between optimal angle, minimum initial velocity and the height of the ball before the player shoots the ball for basketball shooting problem analytically. We found that the value of optimal angle and minimum initial velocity decreases with increasing the height of the ball before the player shoots the ball.

1. Introduction

Projectile motion is one topic in an introductory physics course for undergraduate student. The path followed by a projectile is called its trajectory. If we negligible air resistance, projectiles follows a curved trajectory or curved path that is a parabola. One of the classical problems of projectile motion is a basketball shooting problem, What is the optimal arc angle for shooting a ball? Many students would have guessed that the optimal angle is 45 degrees, which not the correct answer. Based on ground-to-ground projectile motion, a projectile launched from the origin and returns to the same horizontal level. The optimal angle that the object can travel from the origin to the maximum horizontal range with a minimum velocity is 45 degrees [1]. But in basketball shooting problem the beginning level is different from final level. Therefore, the optimal angle in basketball shooting problem is different from the optimal angle in ground-to-ground projectile motion.

The basketball shooting problem was studied by many researchers. Tan and Miller [2] studied the kinematics of the two basic styles of shooting in basketball the overhand push shot and the underhand loop shot. It is shown that from a purely kinematics and trajectory point view, the overhand push shot is preferable to the underhand loop shot. The advantages of the underhand loop shot lie in the actual execution of the shot. The optimal arc angles for shooting a basketball were considered by Frank [3], who studied the optimal arc angles that the shooter can send the ball to the center of the hoop with a minimum required amount of speed. He found that the value of this arc angle depends on the shooting height of the player and his distance from the hoop. In another projectile motion situation, Day [4] suggested that the real-life situation of a vehicle vaulting over the side of an elevated roadway and landing on the ground below. Lichtenberg and Will [5] address the problem of the optimum angle at which a putter should release the shot in order to achieve maximum distance.
In this paper discuss the relationship between optimal angle, minimum initial velocity and the height of the ball before the player shoots the ball for basketball shooting problem analytically.

2. Model and Calculations
Consider the case when the ball is released from the player as shown in figure 1. In this work air resistance to movement of the ball and the force due to the spinning motion of the ball will be considered negligible. When the ball is released, the trajectory of the ball is a parabola. If the hoop center is \( x \) m away in horizontal from the player to the hoop is \( h' \) m higher than the released height of the ball \( h \), let us find the minimum initial velocity \( v_0 \) and the associated arc angle \( \theta \) to send the ball from player to the hoop.

![Figure 1 Trajectory of the ball from the player to the hoop](image)

The trajectory of the ball can be defined by the following set of equations.

Horizontal motion:

\[
X = \left( v_0 \cos \theta \right) \left( t_1 + t_2 \right)
\]

(1)

Vertical motion:

\[
v_0 \sin \theta = gt_1
\]

(2)
From eq. (2), we got displacement of the ball in vertical motion \( h' \) as

\[
h' = (v_0 \sin \theta)(t_1 + t_2) - \frac{1}{2} g(t_1 + t_2)^2
\]

\[
= (gt_1)(t_1 + t_2) - \frac{1}{2} g(t_1 + t_2)^2 = \frac{1}{2} g(t_1^2 - t_2^2). 
\]

\[
(3)
\]

Here \( g \) is acceleration due to gravity at the earth’s surface. \( t_1 \) and \( t_2 \) are time interval when the ball is released from the player to the maximum height \( Y \) and the time interval from the maximum height to the hoop respectively.

To find the minimum initial velocity of the ball, the relationship between release angle and initial velocity for a given range must be considered. From eq. (3), we can be rewritten as

\[
h' = \frac{g}{2}(t_1 + t_2)[2t_1 - (t_1 + t_2)].
\]

Substituting eq. (1) and eq. (2) in eq. (4) we go to

\[
h' = \frac{g}{2} \frac{X}{v_0 \cos \theta} \left[ 2t_1 - \frac{X}{v_0 \cos \theta} \right] = X \tan \theta - \frac{gX^2}{2v_0^2 \cos^2 \theta}. \]

\[
(4)
\]

We can rearrange eq. (4) as

\[
v_0 = \sqrt{\frac{gX^2 \left( \tan^2 \theta + 1 \right)}{2(X \tan \theta - h')}}. \]

\[
(5)
\]

To solve the problem, we need to differentiate \( v_0 \) with respect to \( \theta \) and set the derivative equal to zero. Then, we got

\[
\frac{dv_0}{d\theta} = (X \tan \theta - h') \tan \theta - \frac{X}{2} \sec^2 \theta = 0. \]

\[
(6)
\]

The trigonometric identities \( \sec^2 \theta = \frac{1}{\cos^2 \theta} = \tan^2 \theta + 1 \) enable us to write eq. (6) more simply as

\[
\frac{dv_0}{d\theta} = X \tan^2 \theta - 2h' \tan \theta - X = 0. \]

\[
(7)
\]

From quadratic equation \( Ax^2 + Bx + C = 0, x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A} \) where \( A, B \) and \( C \) are constant. We choose appropriate solution as positive solution, then we get optimal angle as

\[
\theta = \tan^{-1} \left( h' + \sqrt{h'^2 + X^2} \right). \]

\[
(8)
\]

Substituting eq. (8) back into eq. (5), gives an expression for the minimum initial velocity consistent with the ball reaching the hoop. The result is

\[
v_0 = \left[ g \left( \frac{h' + \sqrt{h'^2 + X^2}}{X} \right) \right]^{\frac{1}{2}}, \]

\[
(9)
\]

where \( h' = H - h \).
3. Results and Discussions

The relationship between optimal angle \( \theta \), minimum initial velocity \( v_0 \) and the height of the ball before the player shoots the ball \( h \) are shown in figure 2. Here, we use \( H = 3 \text{ m} \) and \( X = 6.75 \text{ m} \) that is the hoop distance above the floor and three point field goal distances respectively. We consider the optimal angle and the minimum initial velocity of 1.5-2.1 m in the height of the ball before the player shoots the ball. Figure 2 shows that the value of optimal angle and the value of the minimum initial velocity decrease with increasing the height of the ball before the player shoot the ball. In other words, for basketball shooting problem the optimal angle depends on the height of the ball before the player shoots the ball not fixed at \( 45^\circ \) as in the case of ground-to-ground projectile motion.

![Figure 2](image)

Figure 2 The relationship between optimal angle \( \theta \), minimum initial velocity \( v_0 \) and the height of the ball before the player shoots the ball \( h \)

4. Conclusions

The relationship between the optimal angle, the minimum initial velocity and the height of the ball before the player shoots the ball for basketball shooting problem are calculated analytically. The effect of air resistance to movement of the ball and the force due to the spinning motion of the ball will be considered negligible. We find that the value of optimal angle and the value of minimum initial velocity decreases with increasing the height of the ball before the player shoot the ball. We can use this result as an example to describe the projectile motion that the beginning level is different from final level. An improvement in this research could come from include the effect of air resistance and the force due to the spinning motion of the ball. Which makes the calculation results in a similar to real-life situation increased or applied this result in other projectile motion in real-life situations.

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