Principles, Detections and Applications of Cosmological Gravitational Waves

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Abstract. Contemporarily, the gravitational wave has played an important role in the field of astronomy. A systematic review of the gravitational wave is presented in this paper. Primarily, the principle of gravitational waves is introduced in both Newton and Einstein’s concept of gravity. Besides, we deduced the solution of Einstein’s field equation. Moreover, we described the detection of gravitational waves with the mechanisms of the two gravitational wave detectors LIGO and Virgo. Finally, the applications of gravitational waves are discussed in detecting black hole mergers, neutron star mergers, gamma-ray burst, and core-Collapse Supernova explosion. The gravitational wave could contribute to more scientific findings in the future and solve more mysteries of the Universe.

1. Introduction
Since the dawn of civilization, stars and the universe are the most popular topics investigated by human beings. Still, the properties of the universe were only revealed until the twentieth century [1]. Gravitational wave, discovered by the predictions of many renowned scientists (e.g., Einstein [2]), was one of the properties that massive objects have on objects on a universal scale [3]. Gravitational waves are considered one of the effects on spacetime, generated by interactions of celestial bodies as a form of radiation [4].

Although still in progress, recent theories developed were mostly based on the original Einstein’s general relativity theory [5]. With the precise analytical deduction, the ‘amount’ of gravitational wave could be derived and used to solve and understand the origination of wave [6]. Detection for gravitational waves had become more and more accurate with LIGO’s installation and its successful operations [7]. Applications had become wider with the further hypothesis of gravitational wave generation, e.g., a binary pulsar in 1974 [3]. In sum, the future of gravitational waves does seem bright and open.

In this perspective, we will systematically review the progress of gravitational waves into three parts. First of all, the principles and the derivation of gravitational waves are introduced in Sec. 2. Besides, the developments of the detection approaches have been demonstrated in Sec. 3. Moreover, the feasible applications of gravitational waves are discussed in Sec. 4. Finally, a brief summary is given in Sec. 5.
2. Principles of Gravitational Wave

2.1 Newton's Concept of Gravitation
For centuries, the common view of modern quantitative science of gravitation was discussed under the Newtonian mechanic system. Based on his law of inertia, an object without any force applied moves in a straight line with a uniform velocity, i.e., there is a gravitational attraction between all objects [8]. This kind of gravitation can spread instantaneously, i.e., the change of gravitation of mass object will affect the state of motion of surrounding objects immediately [9]. Afterward, several scientists questioned Newton's theoretical model and successfully confirmed their views both theoretically and observationally. Einstein had proven that the Newtonian gravitation did not fit in the special relativity [10]. Eddington's 1919 solar eclipse expedition also opposed it by detecting the bending of light by gravitational source [11].

2.2 Einstein's Theory of Gravitational Waves
In 1905, Albert Einstein published the Electrodynamics of Moving Bodies, which introduced the concept of special relativity for the first time [12]. One of the assumptions of special relativity is that nothing can travel faster than the speed of light, \( c \) [13], French scientist Jules Poincaré speculated that gravity was propagated in the form of gravitational waves and that its velocity was equal to the speed of light [14]. Subsequently, Einstein proposed general relativity and gravitational waves. The theory revealed that curved spacetime itself could express the direction and size of the gravitational action on the object. Thus, one can determine the trajectory of the object in spacetime [15].

Einstein's field equation is the most classical and core description of general relativity and the foundation of later derivation of the gravitational wave. The equation can be written as:

\[
R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \frac{8\pi G}{c^4} T_{\mu \nu}
\]  

(1)

The left side of the equation is a geometric parameter showing the curvature of spacetime; \( g_{\mu \nu} \) represents the metric tensor, which is the curvature scale of the background spacetime (or can be simply understood as the gravitational potential). Normally, the potential has only one component. But \( \mu \) and \( \nu \) each can be selected from the numbers 0, 1, 2, 3. Here, the number 0 refers to the time dimension, and 1, 2, 3 refers to the space dimension [9]. \( R \) is the curvature scalar and \( R_{\mu \nu} \) is Ricci tensor (a symmetric tensor that arises naturally in pseudo-Riemannian geometry [16]).

The right side of the equation is a material item. \( T_{\mu \nu} \) is energy momentum tensor, which is composed of energy, momentum, energy flow and momentum flow. Moreover, \( G \) is the gravitational constant, and \( C \) is the speed of light in vacuum.

This field equation illustrates the existence and movement of matter that affect the curvature of time and space. According to the theory of general relativity, there is no Euclidean space composed of gravitation force, one-dimensional time and three-dimensional space. In any given reference system, the time space curvature caused by gravitation force exists. The spacetime is a four-dimensional none-Euclidean Riemannian space. Therefore, the distribution of matter energy density and momentum density affects the spacetime and curvature structure. Vice versa, the spacetime curvature structure also exerts influence on the matter motional orbits [17].

This equation can be reasonably inferred that when there is little gravitation force and less time space curvature, the gravitation described in general relativity theory is very close to Newton's law of gravitation. However, these two theories have big differences when gravitation force is strong, and space curvature should be considered [17].

2.3 Deduction
Starting from the field equation's far-field radiation solution, it can be assumed that the gravitational wave propagates in a certain spacetime background. Besides, its energy and momentum can be ignored, i.e., not affect its own propagation. Then \( g_{\mu \nu} \) can be separated to \( h_{\mu \nu} \) and \( \eta_{\mu \nu} \):

\[
g_{\mu \nu} = h_{\mu \nu} + \eta_{\mu \nu}
\]  

(2)
where $\eta_{\mu\nu}$ is the Minkowski metric for flat spacetime, and $h_{\mu\nu}$ is the disturbance. Further, consider the gravitation wave in general relativity, the gravitation amount can be described as the following:

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} R$$

(3)

After the variation of metric, the Einstein field equation can be written as:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0$$

(4)

Under the linear condition, Riemann tensor can be described as:

$$R_{\mu\nu\alpha\beta} \approx \frac{1}{2} \left( h_{\nu\alpha,\mu\beta} + h_{\mu\beta,\nu\alpha} - h_{\mu\alpha,\nu\beta} - h_{\nu\beta,\mu\alpha} \right)$$

(5)

For the convenience of calculation, D Alembert operator is introduced:

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} h$$

(6)

Under the linear approximation, $G_{\mu\nu}$ can be written as:

$$G_{\mu\nu} = \frac{1}{2} \left( \partial_{\mu} \partial_{\alpha} \bar{h}_{\nu}^{\alpha} + \partial_{\nu} \partial_{\alpha} \bar{h}_{\mu}^{\alpha} - \Box \bar{h}_{\mu\nu} - \eta_{\mu\nu} \partial_{\alpha} \partial_{\beta} \bar{h}^{\alpha\beta} \right)$$

(7)

Taking $\partial_{\mu} \bar{h}^{\mu\nu} = 0$ into equation (2.7):

$$\Box \bar{h}_{\mu\nu} = 0$$

(8)

The solution of the above equation is [6]:

$$\bar{h}_{\mu\nu} = \epsilon_{\mu\nu} \exp(ik_{\mu}x^\mu) + c. c.$$

(9)

Thus, gravitation is transmissible, and its propagation speed is the ultimate speed of light [6]. This is basically the pattern to deduce the gravitational wave equation with the equation of general relativity.

3. Detection of Gravitational Wave

3.1 Structure and Detection Principles of LIGO

3.1.1 Origin of LIGO: The Michelson Interferometer

Before looking at the actual LIGO, the Michelson Interferometer, which severs as the origination, must be introduced. The Michelson Interferometer was invented by Albert Abraham Michelson intended to gauge small measurement through the span of interference fringes by the methodology of optical interferometry [18].

![Figure 1. The configuration of the Michelson Interferometer in 2D [19].](image)
The configuration of the Michelson Interferometer is schematically shown in Fig. 1. The light source first produces monochromatic light. As the monochromatic light encounters G1, the first half-silvered mirror, it will be partially reflected with the same incident angle as the normal on G1 [19]. This partial reflection is due to a principle proposed in thin film interference theory. In such a theory, a portion of the light will be reflected out of phase when the wave (light wave) travels through a medium to another medium with a higher refractive index [20]. After the reflection, the reflected beam will then travel to another movable mirror, A1, and reflected to G1 again. Besides the partial reflection beam, another refracted beam will pass through the half-silvered mirror and continue traveling in a direction parallel to the original light source [18]. G2 here serves for a significant purpose for producing clear and distinct fringes for the observer when the light source is polychromatic (white light) but doesn’t affect fringes with the source of monochromatic light [19]. The refracted light beam will be reflected by A2, a fixed mirror, back to the half-silvered mirror G1. At this point, both the partially reflected light beam and the refracted light beam will merge and recombine with each other after travelling to the different optical paths. As A1 is moved a distance d, half the length of the wavelength, and by counting the number of fringes m passing through a particular point as A1 is moved, the observer will be able to measure displacement with the precision of fractions in a wavelength [18]. The relationship between the variables above are shown in Eq. (10).

\[
d = m \frac{\lambda_0}{2}
\]  

(10)

3.1.2 Initial LIGO in 2002

Starting from the early 1900s, ideas of detecting gravitational waves had already begun to form. With the confirmation of the formula of general relativity for gravitational wave in 1918 by Einstein, scientists start envisioning an observatory for detecting gravitational wave. After the failure of the Michelson-Morley experiment (try to detect ether wind), astronomers replicated the apparatus in this experiment but advanced the Michelson Interferometer with higher preciseness [21]. Now, time jumping towards the 1970s, the initial LIGO (abbreviation for Laser Interferometer Gravitational-Wave Observatory) was funded by the National Science Foundation and construction was on the way [22]. In 2002, the construction of the initial LIGOs, placed at Hanford and Livingston, was announced to be complete. Each interferometer had an arm span of 4km long and the interferometers were constructed 3000km away to detect and eliminate local interference. The 4km arm span was intended to enhance the interferometer’s sensitivity, which was able to detect and measure a change in the difference of 10-16m.

Figure 2. The Initial Schematic of the Construction for LIGO [23].

Fig 2. presents the rough configuration of the initial LIGO. The hanged mirrors in the Michelson Interferometer now serve as gravitational test masses. The basic principles still align with the original
interferometer. Light is reflected and refracted to recombine and form interference patterns. However, taking the effects of gravitational waves into considerations, the reflected/refracted light waves will now be compressed or stretched, causing the light waves to switch phase. This will then lead to an interference pattern with the recombined light beam, which could provide the information of the gravitational waves by calculations [24, 25].

3.1.3 Advanced LIGO in 2015

Even with the high precision of $10^{-16}$m, the initial LIGO project had failed to detect any gravitational wave from 2002 to 2010. Nevertheless, scientists had brought up plans for updating the LIGO into the aLIGO (seen from Fig. 3), standing for advanced LIGO. With experience learned from operating and constructing the initial LIGO, advanced LIGO takes the sensitivity of gravitational waves to another level. For example, increasing the detection zone of a gravitational wave by a thousand, improved the sensitivity of any wave frequency from its predecessors by a factor of ten [26].

![Figure 3. The improvements made to the advanced LIGO by increasing the sensitivity of the initial LIGO with a factor of 10 subsequently increased the detection volume by 1000 [27].](image)

The improvements are demonstrated as the following. Primarily, as shown in Fig. 3, the mirrors (the half-silvered mirror in Michelson Interferometer or the test masses in LIGO) were increased by size and mass, and the suspension had updates on components to silence inner LIGO noise (quadruple pendulum and glass fiber wires). Besides, the seismic isolator had additional improvements instead of passively decreasing local noise. Therefore, it will now also actively send out signals to deliberately counteract the external noise by using principles of superposition [28]. The increase in the size of the mirrors means it is now able to resist more of the momentum and spread out more of the heat that photons bring when they are released from the laser. Compared to the initial LIGO with only two metal strings and a small mirror, it is better to make measurements with higher precision. Same with the suspension and the seismic isolator, both based on past experiences, updated with important and newer features, making the advanced LIGO a lot more precise than the initial LIGO [26].
3.2 **Observations of LIGO and VIRGO**

3.2.1 **Additional Methods Leading to Confirmation**
Since detecting gravitational waves needs significant precision, claiming a detection for a gravitational wave needs confirmation after confirmation before it was claimed to be confirmed. In other words, many checking and verifications needed to be done before announcing any detection of gravitational waves. One of the first things to confirm is to use the advantage of the double-observatory. Considered a distance of 3000km in between the Hanford and Livingston observatory, the delay of detecting gravitational waves should only be up to ten milliseconds as gravitational waves are predicted to travel close to the speed of light [29]. The waves detected in both observatories should have the same properties, e.g., the same amplitude, wavelength and frequency. This is only a first pass, and the second pass is at Hanford Observatory, an additional detector with only 2km arm span, different from the 4km that of the original interferometer also needs to observe the gravitation wave. Moreover, with the same sensitivity, the gravitational wave should also trigger this interferometer but with only half of its amplitude detected in 4km detector. Besides, researchers worldwide collaborated to do the old-fashioned ‘multiple trials’ to further strengthen the confirmation of a gravitational wave. For example, one of LIGO’s sister facilities, VIRGO (shown in Fig. 4), located in the rural areas of Pisa, Italy, was built to assist LIGO in confirming the existence of a gravitational wave. With a 3km arm span, VIRGO was still able to detect with high sensitivity [30]. All in all, the three passes mean there will be a high precision of detecting any gravitational waves and that is how the first gravitational wave was detected.

![VIRGO observatory](image)

**Figure 4.** An aerial view of the VIRGO observatory, located at a rural area [31].

3.2.2 **Data and Interpretations of Detections**
On September 14, 2015, the first detection of gravitational waves was done by the twin LIGO in Livingston and Hanford, just after the aLIGOs were updated and put into practice runs. The data’s precision arrived 5.1 $\sigma$, or a false alarm rate smaller than $2\times10^{-7}$, thanks to the high sensitivity of the observatories.
The first and second graphs in Fig. 5 show that the correlation of the predicted gravitational waves using general relativity is fairly high. Subsequently, when the gravitational waves become more significant, the predictions almost matched completely. The third graph presents the high correlation of data received from the two observatories, providing more realistic data as data from 0.38 to 0.42 almost overlap each other.

3.2.3 Significance of the GW150914

One important piece of information that this detection gave is the origin of the gravitational waves. Calculated from the wave’s amplitude, the luminosity distance is given as 410Mpc with errors of +160 and -180 [32]. The information of the origin is also given. Through general relativity-based models, black holes were indicated as responsible for the gravitational wave as general relativity predicts an escape in energy as forms of gravitational waves when a black hole to black hole collision happens at the speed of light [33]. The initial and final mass of black holes are calculated to be $36M_{\odot}$ (+5, -4), $29M_{\odot}$ (+4, -4), and the final merged black hole as $62M_{\odot}$ (+4, -4) with 0.67 spin rate (+0.05, -0.07) [32].

There are also many other implications for further astronomical research. For example, there still exists a question of how exactly binary black hole systems form and evolve. Researchers suspect two possible origins, one is close to the process of black hole merging mentioned in GW150914, where the binary black holes came from an isolated system of massive stars. The other origin might be from a dense environment. The gravitational forces force the gas clusters to merge. The implications of the investigation into gravitational waves could well detect the scarcely positioned primordial black holes, which gave more insights into the origin of our universe [34].

In general, the future of gravitational waves does look bright. Until 2021, many gravitational waves have been spotted and analyzed, proving the accuracy of the test apparatus used, the LIGO and its sisters. It will only be a matter of time until more phenomena of our universe will be detected and observed directly by humans.

4. Applications of Gravitational Wave

4.1 Investigation of astrophysics phenomenon using gravitational waves

4.1.1 The merging of black holes
Black holes, the supermassive remnant of the star, even light cannot escape the massive gravity of black holes. We cannot observe black holes directly. For years, astronomers have been overserving the surroundings to deduce the existence of black holes. Nevertheless, gravitational wave allows direct observation of black holes with no "intermediate" as a messenger of space-time [35].

LIGO first detected the gravitational waves from the formation of space-time ripples of two merging black holes in 2015 [7]. On April 12, 2019, a unique event was seen by LIGO and Virgo's third overserving run: a binary black hole system with unequal mass. When the two black holes merge, one of the black holes' mass is about 8 times the Sun, the other black hole's mass is about 31 times the Sun [36], approximately 3.75 times the first one. The difference in the mass resulted in subtle variations in the gravitational wave signal: higher "harmonics" in the waveform [37].

Fig. 6 depicts an image from the simulation of two black holes with unequal mass and orbital precession merging. The unbalance caused the larger black hole distorted the space around it. Thus, the other black hole's orbital trajectory deviated from the perfect spiral. The gravitational wave that GW190412 created is not like any wave produced in the past merger events. In detail, it has a "chirp" shape as the frequency and intensity of the wave increases until the moment of collision, but more complex than the intensity increases as in a chirp [36].

4.1.2. The merging of neutron stars
When a giant star explode in a supernova and its core collapses, neutron star forms are very dense objects spinning at high velocities consisting of neutrons [39]. When two neutron stars spin together in a spiral shape, they start to merge violently, which sends gravitational shudders [40]. They will spew fragments with the light of all wavelengths when they collide [41].

| Source Properties for GW190425 [42] |
|-----------------------------------|
| **Low-spin Prior**               |
| **$\chi < 0.05$**               |
| Primary mass $m_1$               | $1.60-1.87 M_\odot$ |
| Secondary mass $m_2$             | $1.46-1.69 M_\odot$ |
| Chirp mass $M$                   | $1.44^{+0.02}_{-0.02} M_\odot$ |
| Detector-frame chirp mass        | $1.4868^{+0.0003}_{-0.0003} M_\odot$ |
| Mass ratio $m_2/m_1$             | $0.8-1.0$ |
| Total mass $m_{tot}$             | $3.3^{+0.1}_{-0.0} M_\odot$ |
| Effective inspiral spin parameter $\chi_{eff}$ | $0.012^{+0.01}_{-0.01}$ |
| Luminosity distance $D_L$        | $159^{+\frac{5}{2}}_{-\frac{5}{2}} Mpc$ |
| Combined dimensionless tidal deformability $\Lambda$ | $\lesssim 600$ |
| **High-spin Prior**             |
| **$\chi < 0.89$**               |
| Primary mass $m_1$               | $1.61-2.52 M_\odot$ |
| Secondary mass $m_2$             | $1.12-1.68 M_\odot$ |
| Chirp mass $M$                   | $1.44^{+0.02}_{-0.02} M_\odot$ |
| Detector-frame chirp mass        | $1.4873^{+0.0008}_{-0.0006} M_\odot$ |
| Mass ratio $m_2/m_1$             | $0.4-1.0$ |
| Total mass $m_{tot}$             | $3.4^{+0.3}_{-0.2} M_\odot$ |
| Effective inspiral spin parameter $\chi_{eff}$ | $0.058^{+0.11}_{-0.05}$ |
| Luminosity distance $D_L$        | $159^{+\frac{5}{2}}_{-\frac{5}{2}} Mpc$ |
| Combined dimensionless tidal deformability $\Lambda$ | $\lesssim 1100$ |
The second observation of the neutron star merger was on April 25, 2019. Then, the LIGO observatory observed the gravitational waves from the collision of two neutron stars. The event is named "GW190425". In the first observation, both light and gravitational waves are detected. However, only the gravitational wave has been detected this time. Through the analysis of the data, researchers found out that the product of this collision is an object that has an unusually high mass, as shown in Table 1 [40].

In the high-spin prior, the luminosity distance is given in 159Mpc with errors of +69 and -71. The primary mass of the first neutron star is 1.61-2.52M⊙, while the second neutron star is 1.12-1.68 M⊙. The mass of the final merged neutron star is calculated as 3.4M⊙ with errors of +0.3 and -0.1 within spin prior of χ < 0.89, approximately 3.4 times of the Sun.

4.1.3 Gamma-ray burst
Gamma-ray burst (GRB), a flash of highly energetic light produced by the death event of the most energetic star in the Universe, could glow for many months [43]. Short GRBs could happen when two neutron stars collide and last for one or two seconds; long GRBs stay for thousands of seconds and are considered to occur when the core of a massive star collapse. However, the information that we gain using a telescope to observe GRB, such as the energy of the event that produces them and what we have left behind, is not comprehensive. Light is easy to be obscured by other objects in the space. Thus, the light from the GRB could also be blocked by the substance that they burst out during the explosion. The only way to see them is when they are directly facing us. Nevertheless, the afterglow fades quickly and makes it even harder to detect. Gravitational waves are directly related to the dynamic motion of energy and mass inside an object. Therefore, it is almost impossible to be obscured. It can detect GRBs in a way that light could not [43]. The GRB is still a mystery, and astronomers thought the origin of this event is neutron star mergers [44].

On August 17, 2017, NASA's Fermi Gamma-ray Space Telescope detected a short GRB: GRB170817A from an explosion that produced a pulse of high-energy light. The LIGO observatory picked up a gravitational wave event GW170817 from a pair of colliding neutron stars and related to GRB. This is the first time that astronomers detected both light and gravitational waves from the same event. The merging of the two neutron stars produced gravitational waves and launched jets (streaming out in near light speed), therefore created GRB [45]. Fig. 7 presents the results, where the top panel shows the gamma ray detected by Fermi while the bottom panel gives the gravitational wave detected by LIGO at the same time.

Figure 7. The gamma-ray burst and gravitational wave are detected on August 17, 2017 [46].
4.1.4. Core-Collapse Supernova explosion

Core-Collapse Supernova (CCSN) occurs when a massive star dies in the explosion. Gravitational waves are produced during the burst [47]. However, CCSN events are infrequent. The direct observations using an optical telescope cannot penetrate the core, and the light from the center of a star cannot penetrate the surrounding materials. Therefore, information about the origin of the explosion is missing. The only way is through gravitational waves, which will escape from the core without being absorbed by the outer layers of the star, to learn about the dynamics of the core's collapse [43].

However, when the researchers in LIGO and Virgo observatories started to analyze the interferometer data from past CCSN events and calculate the possible gravitational wave energy that two supernovas could emit, no gravitational wave is found. Due to the lack of data (CCSN event only happens 2 to 3 times per century in our Milky Way galaxy), astronomers could not exclude or confine any CCSN models [48]. In the future, with more advanced gravitational wave detectors, the detection of gravitational waves in a core-collapse supernova event might be possible.

![Predicted signal for the gravitational wave emitted by a core collapse supernova](image)

Figure 8. Predicted signal for the gravitational wave emitted by a core collapse supernova [49]

In 2001, a group of scientists in Max Planck Institute for Astrophysics had simulated a core-collapse supernova, including the effects of general relativity. The red line in Fig. 8 shows the predicted signal for the gravitational wave in Newtonian simulation. In contrast, the blue line is the expected signal for the gravitational wave in improved general relativistic simulation. The significant change in the curve means an impact on the gravitational wave that may be detected in the future [49].

4.2 Evidence of the theory of relativity

In 1916, Albert Einstein's theory of relativity suggested that massive objects can distort space and time, which is gravity. He is the first person who predicted that gravitational waves should be a natural outcome of the general theory of relativity [50]. When massive accelerating objects such as black holes or neutron stars orbiting each other, the space-time has been distorted that waves will spread out to all directions from the source. These space-time ripples carry information about the origin of itself and gravity and travel at the speed of light [3].

The first evidence that proves the existence of gravitational waves was in 1974, Arecibo Radio Observatory in Puerto Rico discovered a binary pulsar (a sketch is exhibited in Fig. 9). This system could emit gravitational waves, as the theory of general relativity predicted. They are approaching each other at the speed as predicted too [3].
Figure 9. A sketch of binary pulsar [51].

The detection of gravitational waves has opened a new window in the astronomy field. Gravitational wave has carried important information about the source that emits it, allowed us to observe the universe uniquely, and brought us closer to the universe's mysterious origin [52].

In the future, with more advanced detectors built-in space or under the ground to reduce the noise, further research should focus on detecting gravitational waves from the very early stage of the Universe and simulating the Universe before the Big Bang.

5. Conclusion
In summary, we systematically review the basic principle, the detection method, and the application of gravitational waves. Specifically, the core of the principle of the gravitational wave is explained, where we derive the gravitational wave formula from Einstein's field equations in general relativity. Besides, the detection methods and progress of aLIGO have been demonstrated, making the accurate observation of gravitational waves possible. Moreover, the applications of gravitational waves are discussed. With the continuous development and progress in this field, the concept of the gravitational wave will make outstanding contributions to the exploration of the universe and the tracing of the origin of our mysterious space. These results will offer a guideline for the development of gravitational waves in detection and application.

Reference
[1] Cosmology, Astronomy General: Cosmology, Encyclopedia.com Cengage, (2018), available at: www.encyclopedia.com/science-and-technology/astronomy-and-space-exploration/astronomy-general/cosmology.
[2] Steinicke W., (2005) Einstein and the gravitational waves. Astronomische Nachrichten, 326: 640-641.
[3] LIGO Lab | Caltech, (2015) What Are Gravitational Waves?, available at: www.ligo.caltech.edu/page/what-are-gw. Accessed April 8 2021.
[4] Drake N, Greshko M, (2017) What Are Gravitational Waves, and Why Do They Matter? National Geographic, 16.
[5] Weinstein, Galina, (2016) Einstein's Discovery of Gravitational Waves 1916-1918, arXiv preprint arXiv:1602.04040, available at: arxiv.org/pdf/1602.04040.pdf.
[6] Gao Q, Gong Y, Liang D., (2018) The polarizations of gravitational waves, Chinese Science Bulletin, 63(9): 801-815.
[7] Caltech, Gravitational Waves, As Einstein Predicted, available at: www.ligo.caltech.edu/image/ligo20160211a.
[8] Faller, James E., and et.al, (1998), Newton's Law of Gravity. Gravity, Encyclopædia Britannica, Inc., available at: www.britannica.com/science/gravity-physics/Newtons-law-of-gravity.
[9] Zhao, Zheng, and et. al., (2016) Gravitational Wave and General Relativity, College Physics (Chinese), available at: max.book118.com/html/2018/0410/160928038.shtml
[10] Toth, Viktor T.,(2018) What's Wrong With Newtonian Gravity?, Forbes Magazine, available at:www.forbes.com/sites/quora/2018/02/08/whats-wrong-with-newtonian-gravity/?sh=61f86639c604.
[11] Eddington A S., (1919) Solar eclipse, Photograph collection-Einstein—EB Series-barn collection21-EB PHOTOS-AE other (1 of 1).
[12] Einstein A, (1905) On the electrodynamics of moving bodies, Annalen der physik, 17(10): 891-921.
[13] Utah State University. Gravitational Waves, available at: www.physics.usu.edu/Wheeler/GenRel2013/Notes/GravitationalWaves.pdf.
[14] Roger, (2016) Poincare Predicted Gravitational Waves, available at: blog.darkbuzz.com/2016/02/poincare-predicted-gravitational-waves.html#:~:text=Poincare%20proved%20that%20in%20a%20relativistic%20theory%20of,fHe%20was%20a%20decade%20ahead%20of%20everyone%20else.
[15] Wang, Shaqin, and Xuefeng Wu, (2018), Gravitational Wave - A Search That Lasted a Century(Chinese), available at: www.ixueshu.com/document/a1a89e49bc6b803540f3877f7633d0318947a18e7f9386.html.
[16] Rmilson, Ricci tensor, PlanetMath, (2013). available at: https://www.planetmath.org/RicciTensor
[17] The explanation of general relativity, (2016), available at: https://wenku.baidu.com/view/cb3eeaf6c5da50e2534d7f54.html
[18] Ling S J, Sanny J, Moews W., (2016) Relativistic Momentum, University Physics Volume 3,, available at: opentextbc.ca/universityphysicsv3openstax/chapter/the-michelson-interferometer/.
[19] University of Toronto, Interferometers, University of Toronto, available at: www8.physics.utoronto.ca/~phy293lab/interferometers.pdf.
[20] Boston University, Diffraction: Thin-Film Interference,, available at: physics.bu.edu/~duffy/PY106/Diffraction.html.
[21] Ethan Siegel, (2016), The Failed Experiment That Changed the World, available at: medium.com/starts-with-a-bang/the-failed-experiment-that-changed-the-world-3c9c575d72ca.
[22] Caltech, (2012), The Detection of Gravitational Waves, available at: www.pmaweb.caltech.edu/Courses/ph136/yr2012/1227.1.K.pdf.
[23] Barish B C, Weiss R., (1999), LIGO and the detection of gravitational waves, Physics Today, 52: 44-50.
[24] Abramovici A, Althouse W E, Drever R W P, et al, (1992), LIGO: The laser interferometer gravitational-wave observatory, Science, 256(5055): 325-333.
[25] Caltech, What Is an Interferometer?, available at: www.ligo.caltech.edu/page/what-is-interferometer.
[26] Caltech, About 'ALIGO.', available at: www.ligo.caltech.edu/page/about-aligo.
[27] Caltech, Facts, available at: www.ligo.caltech.edu/page/facts.
[28] Adhikari R, Fritschel P, Waldman S., (2006) Enhanced ligo, LIGO document, LIGO-T060156-01, available at: http://www.ligo. ligo. caltech. edu/docs.
[29] Irion, Robert, (2000), LIGO's Mission of Gravity, JSTOR, available at: www.jstor.org/stable/3074976.
[30] Caltech, Our Sister Facilities, available at: www.ligo.caltech.edu/page/ligo-sister-facilities.
[31] VIRGO, The Virgo Collaboration Observatory Gallery, The Virgo Collaboration, available at: public.virgo-gw.eu/index.php?gmedia=5vp4v&t=g.
[32] Abbott B P, Abbott R, Abbott T D, et al., (2016), Observation of gravitational waves from a binary black hole merger, Physical Review Letters, 116(6): 061102.
[33] Królak A, Patil M., (2017), The first detection of gravitational waves, Universe, 3(3): 59.
[34] APPEC. Gravitational Wave Detection GW190521 – an Unexpected Discovery, available at: www.appec.org/news/gravitational-wave-detection-gw190521-an-unexpected-discovery.
[35] Horizon: The EU Research & Innovation Magazine, (2015), Gravitational Waves Helping to Expose Black Holes, Dark Matter and Theoretical Particles, available at: horizon-magazine.eu/article/gravitational-waves-helping-expose-black-holes-dark-matter-and-theoretical-particles.html.
[36] Castelvecchi, Davide., (2020), This Black-Hole Collision Just Made Gravitational Waves Even More Interesting, Nature, available at: www.nature.com/articles/d41586-020-01153-7, 10.1038/d41586-020-01153-7.
[37] LIGO Lab | Caltech, (2020), GW190412: The Merger of Two Black Holes with Unequal Masses, available at: www.ligo.caltech.edu/news/ligo20200420?highlight=GW190412.
[38] LIGO Lab | Caltech, (2020), Simulation of a Black Hole Merger with Asymmetric Masses and Orbital Precession, Consistent with GW190412, available at: www.ligo.caltech.edu/image/ligo20200420a.
[39] Nola Taylor Redd, (2018), Neutron Stars: Definition & Facts, available at: www.space.com/22180-neutron-stars.html. Accessed April 8 2021.
[40] LIGO Lab | Caltech, (2020), LIGO-Virgo Network Catches Another Neutron Star Collision, available at: www.ligo.caltech.edu/news/ligo20200106. Accessed April 8 2021.
[41] Cho A., (2017), Merging Neutron Stars Generate Gravitational Waves and a Celestial Light Show, Science, 16., available at: www.sciencemag.org/news/2017/10/merging-neutron-stars-generate- gravitational-waves-and-celestial-light-show.
[42] Abbott, B. P., et al. "GW190425: Observation of a Compact Binary Coalescence with Total Mass $\sim 3.4 \, M_{\odot}$." The Astrophysical Journal, vol. 892, no. 1, March 19. 2020, p. L3, available at: iopscience.iop.org/article/10.3847/2041-8213/ab75f5, 10.3847/2041-8213/ab75f5. Accessed 8 Apr. 2021.
[43] Ligo.org, (2019), LIGO Scientific Collaboration - the Science of LSC Research. available at: www.ligo.org/science/Publication-O2GRB/index.php.
[44] Of Particular Significance, (2017), A Scientific Breakthrough! Combining Gravitational and Electromagnetic Waves, available at: profmattstrassler.com/2017/10/16/a-scientific-breakthrough-combining-gravitational-and-electromagnetic-waves/.
[45] NASA, (2011), NASA Missions Catch First Light from a Gravitational-Wave Event, available at: www.nasa.gov/press-release/nasa-missions-catch-first-light-from-a-gravitational-wave-event.
[46] NASA, (2018), "August 17, 2018.", available at: fermi.gsfc.nasa.gov/fermi10/fridays/08172018.html.
[47] Hensley K., (2019), Can We Detect Gravitational Waves from Core-Collapse Supernovae?, AAS Nova Highlights: 5405.
[48] Ligo.org, (2016), LIGO Scientific Collaboration - the Science of LSC Research, available at: www.ligo.org/science/Publication-SNSearchS5A5S6/index.php.
[49] Max-Planck institute, (2021), Gravitational Wave Bursts from Core Collapse Supernovae, available at: www.mpa-garching.mpg.de/HIGHLIGHT/2001/highlight0111_e.html.
[50] Drake N, Greshko M., (2017), What Are Gravitational Waves, and Why Do They Matter?, National Geographic, 16.
[51] Admin, (2019), Part 4 – the Binary Pulsar – 10CC Science Club, available at: www.10ccscienceclub.com/2019/01/19/part-4-the-binary-pulsar/.
[52] LIGO Lab | Caltech, (2021) Why Detect Them?, available at: www.ligo.caltech.edu/page/why-detect-gw.