Gravity induced over a smooth soliton

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Abstract

I consider gravity induced over a smooth (finite thickness) soliton. Graviton kinetic term is coupled to bulk scalar that develops solitonic vacuum expectation value. Couplings of Kaluza-Klein modes to soliton-localized matter are suppressed, giving rise to crossover distance $r_c = M_P^2/M_*^3$ between 4D and 5D behavior. This system can be viewed as a finite thickness brane regularization of the model of Dvali, Gabadadze and Porrati.

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I. INTRODUCTION

Brane worlds theories with large or infinite extra dimensions recently provided insight to a number of problems of high energy physics and possible relations between them. Those models can emerge as a part of the fundamental higher dimensional theory described in terms of strings. Even if extra dimensions are not realized in nature, they are valuable testing ground for new ideas in particle physics and gravity.

For some time, it was believed that the extra dimensions can exist only if they are compact or have a finite volume. The problem with the infinite volume extra dimensions was how to obtain a four dimensional force of gravity between objects located at the brane. In work [1], it was shown that the four dimensional Newton’s force arises on the brane due to an induced kinetic term. The model of Dvali, Gabadadze and Porrati [1] is described by the action

\[ S = M_3^3 \int d^4xdy \sqrt{|G|} R_5 + M_P^2 \int d^4xdy \delta(y) \sqrt{|g|} R_4. \]  

Here \( y \) is the coordinate of an extra dimension, \( R_5 \) is 5D curvature, \( G_{MN} \) 5D metric, \( R_4 \) is 4D curvature constructed from metric induced on the brane \( g_{\mu\nu} \equiv G_{MN}(y = 0) \delta^M_{\mu} \delta^N_{\nu} \), \( M_3 \) is the fundamental scale of gravity and \( M_P \) scale that characterizes brane rigidity. In order to obtain the correct value for Newton’s constant on the brane, \( M_P \) has to be taken \( \sim 10^{19} \text{GeV} \). The brane is represented with the delta function i.e. thickness of the brane is taken to be zero. A related model with one infinite extra dimensions was presented and studied in [2]. Phenomenology of the action (1.1) was studied in [3]. There it was shown that the scale of the bulk gravity can be as low as \( M_3 \sim 10^{-3} \text{eV} \). Further properties of the model, cosmological evolution, new manifestations at astronomical scales and string theory realizations were examined in [4–6].

In these notes, I study a similar system of induced (enhanced) gravity over a smooth soliton

\[ S = M_3^3 \int d^4xdy \sqrt{|G|} R_5 (1 + \frac{M_P^2}{M_3^3} \chi^2(M_P y)). \]  

Here \( \chi(M_P y) \) is a localized function representing a smooth tension-less soliton. I find that at the linearized level, features of (1.1) and (1.2) are the same. In short, four dimensional gravity at distance \( r < r_c \equiv M_P^2/M_3^3 \) is mediated by a metastable massless resonance. Width of the resonance is \( \sim r_c^{-1} \). At distances \( r > r_c \), gravity becomes five dimensional. Therefore I conclude that (1.2) is a finite thickness brane regularization of (1.1).

Through the paper I will work with the simpler ‘scalar gravity’ model that captures main features of gravity. A Variation of the strength of the kinetic term can be a consequence of dilaton-like coupling to bulk scalara (\( \chi \)) that exhibits solitonic vev. I neglect the gravitational effect of the soliton, which is equivalent to fine-tuning its tension to zero. In the next section I study properties of the spectrum of Lagrangian in which the strength of kinetic term is a function of the extra coordinate. Results are applicable for scalar and gauge bulk field as well as for linearized gravity. In section 3, I investigate the spectrum of Kaluza Klein modes in system (1.2) and their coupling to soliton-localized matter. I find the crossover distance \( r_c = M_P^2/M_3^3 \), both from the suppression of couplings of KK modes to matter localized on soliton and from an equivalent problem of tunneling through the potential barrier. I find
no new scale \( \sim \sqrt{r_c/a} \) (where \( 1/a \) is the brane thickness), contrary to results of [7], where the different regularization scheme was considered. I conclude that the system (1.2) can be viewed as a finite thickness brane regularization of action (1.1). In the appendix I outline the procedure for finding the spectrum of fermionic modes when the kinetic term of a bulk fermion field is varying through the extra dimension.

II. LOCALIZATION IN MODEL WITH NON-HOMOGENEOUS KINETIC TERM

In this section I investigate general properties of a model in which the strength of a kinetic term for a bulk field is not homogeneous through the infinite extra dimension. Let us take that the bulk field \( \Phi(x_\mu, y) \) is coupled through the kinetic term to a field \( \chi(x_\mu, y) \)

\[
\mathcal{L} = f(\chi) \partial_A \Phi \partial^A \Phi + \partial_A \chi \partial^A \chi - V(\chi).
\]  

(2.1)

Function \( f(y) \) characterizes the coupling of two fields. Field \( \chi(x_\mu, y) \) has potential \( V(\chi) \) and can obtain an expectation value in the form of a soliton \( \chi \rightarrow \chi_{Cl}(y) \). Then, the Lagrangian for the field \( \Phi \) has the following form

\[
\mathcal{L} = f(y) \partial_A \Phi \partial^A \Phi.
\]  

(2.2)

Field \( \Phi \) can represent bulk scalar, gauge field or graviton in linearized approximation.

To find the four dimensional spectrum of (2.2) we decompose field \( \Phi \) into Kaluza Klein modes \( \Phi(x_\mu, y) = \sum_m \Phi_m(y) \sigma_m(x_\mu) \), where \( \Box \sigma_m = -m^2 \sigma_m \). The differential equation for wave functions \( \Phi_m(y) \) is

\[
f(y) \partial_y^2 \Phi_m + \partial_y f(y) \partial_y \Phi_m + f(y)m^2 \Phi_m = 0.
\]  

(2.3)

Taking equations (2.3) for two different eigenvalues \( m, n \), cross-multiplying them with eigenfunctions, subtracting and integrating over the length of extra dimension we get the orthogonality relation

\[
\int f(y) \Phi_n(y) \Phi_m(y) dy = \alpha_m \delta_{m,n}.
\]  

(2.4)

where \( \alpha_m \) is the normalization constant. Combining (2.3) and (2.4) we get

\[
\frac{\int f(y) \partial_y \Phi_n(y) \partial_y \Phi_m(y) dy}{\int f(y) \Phi_n(y) \Phi_m(y) dy} = m^2.
\]  

(2.5)

For non-negative \( f(y) \), both the denominator and numerator of this expression are positive, which means that there are no ghosts, tachions or ghost tachions in effective four dimensional action obtained from (2.2). We can introduce additional coupling of field \( \Phi \) to a 'charge' density \( \Psi(y) \rho(x) \) localized at the soliton (centered at \( y = 0 \)). In 4D effective theory the coefficient of a coupling is determined by the convolution of the matter profile \( \Psi(y) \) and the wave function \( \Phi_m \) (an average of wave function over the soliton-localized matter)

\[
\tilde{\Phi}_m(0) \equiv \int \Psi(y) \Phi_m(y) dy.
\]  

(2.6)
Effective action for modes $\sigma_m$ then reads

$$L = \frac{1}{2} \sum_m \left( \partial_\mu \sigma_m \partial^\mu \sigma_m - m^2 \sigma_m^2 \right) + \sum_m \frac{\Phi_m(0)}{M^3_m \alpha_m} \sigma_m \rho(x), \quad (2.7)$$

where $M_*$ is the fundamental scale. Zero mode $\sigma_0$ will be normalizable for choices of $f(y)$ whose integral is finite, while it will be a single mode in continuum for functions $f(y)$ that are nonzero at $y \rightarrow \pm \infty$. This feature distinguishes between the cases of localization and quasi-localization. In order to find the spectrum and determine localization properties of KK modes, one has to specify the function $f(y)$. To better understand the connection between the shape of $f(y)$ and localization properties, it is useful to transform (2.3) to related Schroedinger problem

$$\Phi_m \equiv \phi_m / \sqrt{f}, \quad \partial_y^2 \phi_m + V(y) \phi_m = m^2 \phi_m, \quad (2.8)$$

where the localizing potential $V$ has the form

$$V(y) = \left( \frac{1}{2} \frac{f''(y)}{f(y)} - \frac{1}{4} \left( \frac{f'(y)}{f(y)} \right)^2 \right). \quad (2.9)$$

Whether this potential will behave as an attractive well or a repulsive barrier depends on the balance between the magnitude of the second and first derivative of the profile $f(y)$.

A. $f(y) = e^{-y^2/\alpha^2}$

Now we consider two choices of function $f(y)$ that lead to the localization of part or all of the spectrum of KK excitations. First example is a Gaussian profile $f(y) = e^{-y^2/\alpha^2}$. Here, $\alpha$ is the scale that determines the brane width. This profile will give rise to a localized tower of KK modes with masses $m \sim \sqrt{n}$, where $n$ is the integer. This was used as a model of localization of gauge fields in [8]. Here we repeat it because we will use it as a starting point for the investigation of soliton induced gravity in the next section.

Potential (2.9) that results from this profile is the potential of a simple harmonic oscillator shifted by a constant so that the zero point energy vanishes

$$V(y) = \alpha^2 (\alpha^2 y^2 - 1). \quad (2.10)$$

Normalized wave functions (2.8) are

$$\phi_n(y) = \frac{1}{\sqrt{2^nn!\pi^{1/2}}} e^{-(ya)^2/2} H_n(ya), \quad (2.11)$$

where the spectrum of masses goes like

$$m_n = \alpha \sqrt{2n}, \quad n = 0, 1, 2, \ldots \quad (2.12)$$

The interaction of those modes with some charge located at the brane is determined by the value of the wave function at the position of the charge. For example, coupling of a charge at $y = 0$ is
\[
L_I = \sum_n \frac{(-)^{n/2} \sqrt{an!}}{2^{n/2} \sqrt{M_*^3 (n/2)!}} \sigma_n \rho_n, \tag{2.13}
\]

for \(n\) even, while it vanishes for odd \(n\). The vanishing of coupling for odd values of \(n\) is also true for a smooth brane that localize charge \(\rho\). The reason is that wave functions of modes with odd \(n\) are odd, while the matter localization profile \(\Psi(y)\) is even and coupling (2.6) vanishes.

**B. \(f(y) = e^{-|ya|}\)**

Other example is the exponentially decaying profile \(f(y) = e^{-|ya|}\). This is presented as an example of a localized profile \(f(y)\) with a finite integral, that doesn’t localize all of the KK modes (as one could naively expect). The Schroedinger potential for this profile is

\[
V(y) = \frac{1}{4} a^2 - a \delta(y). \tag{2.14}
\]

Spectrum of modes consist of a single localized zero mode

\[
\phi_0 = \sqrt{a^2/2} e^{-a|y|^2/2}, \tag{2.15}
\]

and a continuum of scattering states starting at \(m > a/2\). The potential between two charges \(q_1\) and \(q_2\) located at \(y = 0\) due to exchange of field \(\Phi\) would change from a four dimensional law to a five dimensional law at distance \(r \sim 1/a\).

### III. SMOOTHER BRANE REGULARIZATION OF DGP MODEL

In this section I establish the relation between the model of Dvali, Gabadadze and Porrati and action (1.2) at linearized level. Let us consider the five dimensional model with two scales. Fundamental scale \(M_*\) determines the action of the bulk filed \(\Phi\). The potential for the other bulk field \(\chi\) is governed by the much higher brane scale \(a\). Therefore, scale \(a\) determines the width of the brane, which is soliton of field \(\chi\). Field \(\chi\) is coupled quadratically to a kinetic term of \(\Phi\), and the coupling is suppressed in powers of fundamental scale \(M_*\)

\[
\mathcal{L} = \left(1 + \frac{\chi^2}{M_*^2}\right) \partial_A \Phi \partial^A \Phi + \partial_A \chi \partial^A \chi - V(\chi). \tag{3.1}
\]

Let us take that the field \(\chi\) gets expectation value \(\chi_{CI} = a^{3/2} \exp(-(ya)^2/2)\) so that the profile function (2.2) is

\[
f(y) = 1 + \left(\frac{a}{M_*}\right)^3 e^{-(ya)^2} \equiv 1 + A e^{-(ya)^2}. \tag{3.2}
\]

Since we take that the soliton scale \(a\) is much bigger than the bulk scale \(M_*\), coefficient \(A\) is a huge number. The Schroedinger potential (2.9) for this profile is
\[ V(y) = -Aa^2 \frac{A(1 - (ya)^2) + e^{(ya)^2}(1 - 2(ya)^2)}{(A + e^{(ya)^2})^2}. \] (3.3)

Potential (3.3) is shown in Fig. 1 together with the potential of harmonic oscillator (2.10).

![Potential Plot](Image)

**Figure 1:** Potential (3.3) (solid line) together with the harmonic oscillator potential (2.10) (dashed line). \( y \) is in units of inverse \( a \), the potential is in units of \( a^2 \), while \( A = 10000 \).

For small values of \( y \), the potential coincides with the potential of a harmonic oscillator. After certain a distance \( \sim 1/a \), the square well is 'cut' and the potential rapidly drops to zero. From this behavior we can qualitatively predict the spectrum of states. Lowest bound states of the harmonic oscillator will obtain width and will tunnel through the barrier. Since we are interested in the case of gravity we want to find the decay width of a zero mode that will mediate the four dimensional Newton’s force on the brane. Higher modes have mass \( \sim a \) and are irrelevant for low energy physics. Decay width in the WKB approximation for the zero mode is

\[ \Gamma_0 \sim a \exp \left( -2 \int_{y_1}^{y_2} \sqrt{V} dy \right), \] (3.4)

where \( y_1, y_2 \) are classical turning points and \( 1/a \) is the width of the well. For large values of \( A \) condition for WKB validity \( |V'|/(2V)^{3/2} | \ll 1 \) is well satisfied everywhere (except at classical turning points). The integral in (3.4) can be evaluated numerically for the large range of values \( A \). The integral is independent of \( a \) and behaves as \( \sim A^{0.488} \) (power has negligible numerical error). Therefore, in the WKB approximation zero mode has a decay width

\[ \Gamma_0 \sim \frac{a}{A^{0.976}}. \] (3.5)

Wave functions of the continuous Kaluza-Klein spectrum can be found by solving the Schroedinger equation. We find that the couplings of modes (2.6) to matter localized on the soliton are suppressed.

Here, we can explore the question of the universality of the coupling of filed \( \Phi \) (scalar gravity) to brane matter. If different types of matter localized on the soliton have different profiles \( \Psi(y) \), coupling (2.6) of gravity would be non-universal. This is sometimes argued
to be a problem \(^1\) for theories with gravity induced over a smooth brane \([9]\). We adopt the following framework in which the violation of universality is suppressed by soliton thickness scale \(a\). The potential for a scalar field that forms solitonic brane is governed by high mass scale \(a\). Here we take that \(a \sim M_P\). Mass of Standard Model fields is much lower than the cutoff \(a\). We take that SM particles come from the zero mode of another bulk field, while the mass splitting is due to some higher order effect. Zero modes have profiles roughly of a soliton itself \(\Psi_0 \sim a \exp(-(ya)^2)\). Potential well that localizes zero mode can have bound states whose masses would be \(\sim a\). Indeed wave functions of those states \(\Psi_{B.S.}\) are different and lead to the different coupling (2.6). However, because of their large mass they are decoupled from low energy physics. Now let us return to the spectrum of potential (3.3). The Schroedinger equation for (3.3) can be solved numerically. For simplicity, we take that the matter profile is \(\Psi_0 \sim a \exp(-(ya)^2)\) and evaluate coupling (2.6) for the continuum of KK modes. The qualitative behavior of wave functions is as follows. Outside the well, wave functions are plane waves of frequency \(m\). Inside the well, wave functions are roughly Gaussians with mass-dependent height and constant width (\(\sim 1/a\)). Approximately, one can take for the coupling \(\Psi_m(0) \approx \sqrt{\pi} \Psi_m(0)\). Actual coupling was evaluated for each mode numerically. Dependence of coupling (2.6) on the mass of KK mode is shown in Fig.2

![Figure 2](image)

**Figure 2:** Suppression of KK coupling for \(A = 10000\). For \(m = 0\) coupling has value 1 (outside the graph range). Peaks are at positions of resonant states of harmonic oscillator \(m = 0, 2, \sqrt{8}...\). Log-log plot of the same graph (multiplied with \(\sqrt{A}\)) is shown in the inset. The solid line has coefficient -1 to show the suppression of coupling \(\sim 1/m\).

There is a peak located at the mass zero. This is the zero mass graviton resonance that

\(^1\)non-universality of coupling can in fact be a desirable property of models with infinite extra dimensions. It opens the possibility for solving the cosmological constant problem without spoiling general covariance of the resulting four dimensional theory. This was pointed out to me by Gia Dvali.
is responsible for the 4D force at short distances. This resonance starts at coupling 1 and
decays as \( \sim 1/m \) (see the log-log plot). Dependence of the half-width of the peak at \( m = 0 \)
on the magnitude of \( A \) can be evaluated by numerical analysis. By sampling data for the
large range of values \( A \) (10\(^2\) < \( A \) < 10\(^5\)) we find
\[
\Delta m \sim \frac{a}{A^{0.997\pm0.005}}.
\] (3.6)

Besides the metastable ‘massless’ graviton there is no other resonances until we get to
higher states of the harmonic oscillator that have mass \( \sim a \). In particular, nothing unusual
is happening at scale \( \sim a/\sqrt{A} \) contrary to the results of [7] where a different regularization
was used (there advocated mass scale \( a/\sqrt{A} \) would be on position -4.605 on the log-log graph
in Fig.2).

Newtonian potential due to an exchange of field \( \Phi \) between two masses at short distances
is given by
\[
V(r) \approx \int_0^{\infty} \frac{|\tilde{\Phi}_m(0)|^2 e^{-mr}}{M_s^3} \frac{1}{r} dm \approx \frac{a}{M_s^3 A r}, \quad r < A/a.
\] (3.7)

Here the integral is supported over the resonance of width \( a/A \) and height 1. At large
distances \( r \gg A/a \) only the lightest modes \((m < a/A)\) contribute to the exchange. Their
coupling is unsuppressed \((\tilde{\Phi}_m(0) \approx 1)\) and one can approximate
\[
V(r) \approx \int_0^{\infty} \frac{|\tilde{\Phi}_m(0)|^2 e^{-mr}}{M_s^3} \frac{1}{r} dm \approx \frac{1}{M_s^{3/2} r^2}, \quad r \gg A/a.
\] (3.8)

This is the same behavior of Newtonian potential as in the model of Dvali, Gabadadze and
Porrati. A weak four dimensional force is mimicked by the exchange of a continuum of
massive KK gravitons at distances smaller than the crossover distance \( r < r_c \equiv A/a \). At
larger distances, the force becomes five dimensional. Crossover distance in (3.1) coincides
with the crossover distance in [1]. In order to reproduce the correct Newton’s constant one
has to take the scale \( a \) to be of the order of Planck scale \( M_p \)
\[
r_c = \frac{A}{a} = \frac{a^2}{M_s^2} = \frac{M_p^2}{M_s^3}.
\] (3.9)

At the linearized level, an exchange of massive spin two states gives ‘wrong’ tensor struc-
ture of the propagator [10]. It was shown [5,11] that non-linear effects cure this problem in
model [1]. I conjecture that a similar mechanism occurs in the model at hand. Investigation
of that effect as well as the spectrum of induced gravity in codimensions two and three is
currently in progress [12].

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discussion, comments and encouraging me to make this manuscript public.
Here we outline the procedure for finding the KK spectrum for bulk fermion field with kinetic term that is not homogeneous in extra dimension. The action we consider is

\[ S = \int d^5X f(y) \bar{\Psi} i\gamma_A \partial^A \Psi. \]  

(1.1)

Dirac algebra in 5D \(\{\gamma^A, \gamma^B\} = 2g^{AB}\) can be satisfied with a choice of 4D gamma matrices \(\gamma^\mu\) and \(\gamma_5^{(5D)} = -i\gamma_5^{(4D)} \equiv -i\gamma^5\).

The equation of motion is independent of \(f(y)\) and is solved with the ansatz

\[ \Psi(p, y) = \sum_m (g_m(y) \xi^L_m + h_m(y) \xi^R_m) \]  

\[ \gamma^\mu \partial_\mu \xi^L_m = m \xi^R_m; \quad \gamma^\mu \partial_\mu \xi^R_m = m \xi^L_m, \]  

(1.2)

where spinors \(\xi^L_m\) and \(\xi^R_m\) together form a 4D Dirac spinor \(\psi_m\), i.e. \(\frac{1}{2}(1 - \gamma^5)\psi_m = \xi^L_m, \quad \frac{1}{2}(1 + \gamma^5)\psi_m = \xi^R_m\). Wave functions are

\[ g_m(y) = A_m \sin(my) + B_m \cos(my) \]  

\[ h_m(y) = A_m \cos(my) - B_m \sin(my). \]  

(1.3)

Although wave functions are the same as in the case \(f(y) = 1\), four dimensional effective action will be different, since one integrates over the profile \(f(y)\). Plugging (1.3) into (1.1) and integrating over \(y\) we obtain the following 4D action

\[ S = \sum_{m,n} \left( \bar{\psi}_m f_{mn} i\partial \psi_n + \frac{1}{2} \bar{\psi}_m (m + n) f_{mn} \psi_n \right), \]  

(1.4)

where \(f_{mn}\) is an infinite dimensional matrix given by a Fourier transform of the profile

\[ f_{mn} = \int dy f(y) \cos(m - n)y. \]  

(1.5)

Clearly, finding the spectrum of KK fermions (as well as their couplings) is now reduced to a problem of diagonalization of infinite hermitian matrices \(K \equiv f_{mn}\) and \(M \equiv (m + n)f_{mn}/2\). Diagonalization can be done in steps, and we can write schematically

\[ \psi^\dagger i\partial K \psi + \psi^\dagger M \psi \rightarrow \psi^\dagger \lambda_i i\partial \psi_i + \psi^\dagger MU^\dagger \psi \rightarrow \]  

\[ \psi^\dagger i\partial \psi + \sum_{i,j} \psi_i^\dagger V \frac{U^\dagger U}{\sqrt{\lambda_i \lambda_j}} V^\dagger \psi_j \equiv \sum_m (\bar{\psi}_m i\partial \psi_m + m \bar{\psi}_m \psi_m), \]  

(1.6)

where we rotated fermion sequentially as \(\psi \rightarrow U \psi, \psi_i \rightarrow \sqrt{\lambda_i} \psi_i \rightarrow V^\dagger \psi\). Orthogonal matrices \(U, V\) are chosen to diagonalize the kinetic and mass term. The simplest example is \(f(y) = 1\) in which case \(f_{mn} = \delta(m - n)\) and we recover ordinary KK tower of massive fermions. Another simple example is \(f(y) = \delta(y)\) in that case \(f_{mn}\) has 1 at every entry. All eigenvalues of that matrix are zero except one that is \(2N + 1\), where \(N\) is (infinite) number of positive modes in \(\psi\) (modes go from \(-m\) to \(m\)). Thus, only one mode will
have nonzero kinetic term. Matrix $U$ that diagonalizes $f_{mn}$ in this case has entries 1 on the diagonal that goes from the lower left to upper right corner, entries -1 in first column and entries 1 in last row, while all other entries are zero. Matrix $UMU^\dagger$ has zeroes on diagonal. Thus, a single propagating mode has the mass zero. It would be interesting to find the spectrum of induced 4D fermions by diagonalization of the infinite matrix $f_{mn} = \int dy(1 + A\exp(-ya^2)\cos(m-n)y$ and compare it to the results obtained by inducing 4D kinetic term on the brane [13,14].
REFERENCES

[1] G. Dvali, G. Gabadadze and M. Porrati, Phys. Lett. B485, 208 (2000) [hep-th/0005016].

[2] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, Phys. Rev. Lett. 84, 5928 (2000) [arXiv:hep-th/0002072];
C. Csaki, J. Erlich and T. J. Hollowood, Phys. Rev. Lett. 84, 5932 (2000) [arXiv:hep-th/0002161].

[3] G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D 64 (2001) 084004 [arXiv:hep-ph/0102216];
G. R. Dvali, G. Gabadadze, M. Kolanovic and F. Nitti, Phys. Rev. D 65, 024031 (2002) [arXiv:hep-th/0106058].
M. Carena, A. Delgado, J. Lykken, S. Pokorski, M. Quiros and C. E. Wagner, Nucl. Phys. B 609, 499 (2001) [arXiv:hep-ph/0120172];

[4] C. Deffayet, Phys. Lett. B 502, 199 (2001) [arXiv:hep-th/0010186]; R. Dick, Acta Phys. Polon. B 32, 3669 (2001) [arXiv:hep-th/0110162]; C. Deffayet, G. R. Dvali and G. Gabadadze, Phys. Rev. D 65, 044023 (2002) [arXiv:astro-ph/0105068]; G. Kofinas, JHEP 0108, 034 (2001) [arXiv:hep-th/0108013];
E. Ponton and E. Poppitz, JHEP 0106, 019 (2001) [arXiv:hep-ph/0105021];
M. Kolanovic, Phys. Rev. D 65, 124005 (2002) [arXiv:hep-th/0203136];
G. Kofinas, E. Papantonopoulos and V. Zamarias, Phys. Rev. D 66, 104028 (2002) [arXiv:hep-th/0208207];
C. Middleton and G. Siopsis, arXiv:hep-th/0210033.

[5] C. Deffayet, G. R. Dvali, G. Gabadadze and A. I. Vainshtein, Phys. Rev. D 65, 044026 (2002) [arXiv:hep-th/0106001]; A. Lue, arXiv:hep-th/0111168; A. Gruzinov, arXiv:astro-ph/0112246; M. Porrati, arXiv:hep-th/0203014.
G. Dvali, A. Gruzinov and M. Zaldarriaga, arXiv:hep-ph/0212069.
A. Lue and G. Starkman, arXiv:astro-ph/0212083.

[6] S. Corley, D. A. Lowe and S. Ramgoolam, JHEP 0107, 030 (2001) [arXiv:hep-th/0106007];
I. Antoniadis, R. Minasian and P. Vanhove, Nucl. Phys. B 648, 69 (2003) [arXiv:hep-th/0209030].

[7] E. Kiritsis, N. Tetradis and T. N. Tomaras, JHEP 0108, 012 (2001) [arXiv:hep-th/0106050].

[8] G. Dvali and A. Vilenkin, arXiv:hep-th/0209217.

[9] S. L. Dubovsky and V. A. Rubakov, arXiv:hep-th/0212222.

[10] H. van Dam and M. J. Veltman, Nucl. Phys. B 22, 397 (1970);

[11] A. I. Vainshtein, Phys. Lett. B 39, 393 (1972).

[12] G. Dvali, G. Gabadadze and M. Kolanovic, In preparation.

[13] G. Dvali, M. Kolanovic and A. Yu. Smirnov, In preparation.

[14] R. Dick and D. M. McArthur, Phys. Lett. B 535, 295 (2002) [arXiv:hep-th/0203271].