ON A NEW TYPE MANNHEIM CURVE

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ABSTRACT. In this paper, we define a new type curve as $V$–Mannheim curve, $V$–Mannheim partner curve and generating curve of Mannheim curve. We give characterization of these curve. In addition, we study a relation between Mannheim curve and spherical curve. Eventually, with Salkowski method, we give an example of the Mannheim curve.

1. Introduction

Regle surface plays an important role in the applied science. The condition that the principal normal of the based curve of a one regle surface may be the principal normal of the based curve of a second regle surface. This problem proposed by Saint-Venant and solved by Bertrant. The generalized of the Saint-Venant and Bertrant problem have been studied by many geometers. In this space, we can study six cases to be considered([7]). The other important condition that the principal normal of $(\alpha)$ curve may be the binormal of $(\beta)$ curve. If the condition satify between corresponded point, then it is said that $(\alpha)$ is Mannheim curve and $(\beta)$ is Mannheim partner curve ([10]). The curve $(\alpha)$ is Mannheim curve if and only if $\kappa = \lambda (\kappa^2 + \tau^2)$ where $\kappa$, $\tau$ are curvature of the curve $(\alpha)$ and $\lambda$ is nonzero constant([7]). The curve $(\beta)$ is Mannheim partner curve if and only if $\frac{d\tau}{ds} = \frac{\kappa}{\lambda} (1 + \lambda^2 \tau^2)$ where $\kappa$, $\tau$ are curvature of the curve $(\beta)$ and $\lambda$ is nonzero constant([7],[10]). After Liu and Wang paper ([10]), many geometer have studied a Mannheim

2000 Mathematics Subject Classification. Primary 53C15; Secondary 53C25.
Key words and phrases. Sasakian Space, curve.
In this paper, we define $V$–Mannheim curve and $V$–Mannheim partner curve we give characterization of the $V$–Mannheim curve and the $V$–Mannheim partner curve.

2. Preliminaries

In 3-Euclidean spaces, let $\gamma : I \rightarrow \mathbb{R}^3(s \rightarrow \gamma(s))$ be a regular curve with unit speed coordinate neighborhood $(I, \gamma)$. Derivation of the Serret-Frenet vectors field given by

$$\begin{pmatrix}
T \\
N \\
B
\end{pmatrix} = \begin{pmatrix}
0 & \kappa & 0 \\
-\kappa & 0 & \tau \\
0 & -\tau & 0
\end{pmatrix} \begin{pmatrix}
T \\
N \\
B
\end{pmatrix}$$

where $\{T, N, B\}$ is orthonormal Serret-Frenet frame of the curve and $\kappa$ and $\tau$ are curvatures of the curves ([8], [9]). Let $\gamma$ be a regular spherical curve. Hence we can define a curve $\alpha : I \rightarrow \mathbb{R}^3(s \rightarrow \alpha(s))K$ as

$$(2.1) \quad \alpha(s) = \int S_M(s)\gamma(s)ds$$

where $S_M : I \rightarrow \mathbb{R} (s \rightarrow S_M(s))$ is differentiable function ([2]). The curve $\alpha$ is spherical curve if and only if

$$S_M(s) = \|\gamma'(s)\| \cos \left( \int_0^s \frac{\det(\gamma(s), \gamma'(s), \gamma''(s))}{\|\gamma'(s)\|^2} ds + \theta_0 \right).$$

(2.1). So, there is $S_T : I \rightarrow \mathbb{R}$ differentiable function such that

$$\left\| \int S_T(s)\gamma'(s)ds \right\| = 1$$

where

$$(2.2) \quad S_T(s) = \kappa(s) \cos \left( \int_0^s \tau(u)du + \theta_0 \right)$$

If we define a curve $K$ with coordinate neighborhood $(I, \beta)$ such that

$$\beta'(s) = \int S_T(s)\gamma'(s)ds$$
then we have

\( (2.3) \quad \beta''(s) = S_T(s)\gamma'(s) \)

Arc-length parameter of \( M \) and \( K \) are same. Let \( \{T, N, B, \pi, \tau\} \) be Serret-Frenet apparatus of the curves where

\( (2.4) \quad S_T(s) = \pi(s) = \kappa(s) \cos \left( \int_0^s \tau(u)\,du + \theta_0 \right) \)

and

\( (2.5) \quad \tau(s) = \kappa(s) \sin \left( \int_0^s \tau(u)\,du + \theta_0 \right) \).

From \((2.3)\), we have

\( (2.6) \quad N(s) = \varepsilon T(s) \)

where \( \varepsilon = \pm 1 \). From equation \((2.3)\) and \((2.6)\), we can see that principal normal of \( K \) and tangent of \( M \) is colinear.

3. **V-Mannheim Curve**

In 3-Euclidean spaces, let \( \gamma : I \to \mathbb{R}^3(s \to \gamma(s)) \) be a regular curve with unit speed coordinate neighborhood \((I, \gamma)\) and \( \{T, N, B\} \) be orthonormal frame of the curve and \( \kappa \) and \( \tau \) be curvatures of the curves. Choi and Kim defined a unit vector field \( V \) given by

\[ V(s) = u(s)T(s) + v(s)N(s) + w(s)B(s) \]

Integral curve of \( V \) is defined by \( \gamma_V(s) = \int V(s)\,ds \) where \( u, v, w \) are functions from \( I \) to \( \mathbb{R}([1]) \). So we can defined a curve \( \beta \) as

\( (3.1) \quad \beta(s) = \int V(s)\,ds + \lambda(s)N(s) \)

where \( \lambda : I \to \mathbb{R} \ (s \to \lambda(s)) \) is differentiable function. Let \( \{\overline{T}, \overline{N}, \overline{B}\} \) be orthonormal frame of the curve and \( \overline{\pi} \) and \( \overline{\tau} \) be curvatures of the curves \( \beta \). Under the above notation, we can give the following theorems and definitions.
Definition 3.1. If \{N, B\} is lineer dependent \( (B = \epsilon N, \epsilon = \pm 1) \), then it is said that the curve \( \gamma \) (respectively \( \beta \)) is said that V–Mannheim curve (V–Mannheim partner curve). By similar method, Camci defined a V–Bertrant curve \((3)\).

Theorem 3.1. The curve \( \gamma \) is a V–Mannheim curve if and only if it is satisfy that

\[
(3.2) \quad u\kappa - w\tau = \lambda (\kappa^2 + \tau^2)
\]

and

\[
(3.3) \quad \lambda(s) = - \int v(s)ds
\]

where \( \lambda_0 \) is constant.

Proof. If \( M \) is a V–Mannheim curve, then V–Mannheim partner curve of \( M \) is equal to

\[
(3.4) \quad \beta(s) = \int_0^s V(u)du + \lambda(s)N(s)
\]

and \{N, B\} is a lineer dependent. If we derivate the equation \((3.4)\), we have

\[
\frac{d}{ds}T = (u - \lambda\kappa) T + (\lambda' + v) N + (w + \lambda\tau) B.
\]

Since \{N, B\} is a lineer dependent, we have

\[
\lambda(s) = - \int v(s)ds
\]

hence we get

\[
(3.5) \quad T = \frac{ds}{ds}(u - \lambda\kappa) T + \frac{ds}{ds}(w + \lambda\tau) B = \cos \theta(s)T + \sin \theta(s)B
\]

where \( \cos \theta(s) = \frac{ds}{ds}(u - \lambda\kappa) \) and \( \sin \theta(s) = \frac{ds}{ds}(v + \lambda\tau) \). So we have

\[
(3.6) \quad \tan \theta(s) = \frac{v + \lambda\tau}{u - \lambda\kappa}
\]

If we derivate the equation \((3.5)\), we have

\[
(3.7) \quad \frac{d}{ds}\kappa N = -\theta' \sin \theta T + (\kappa \cos \theta - \tau \sin \theta) N + \theta' \cos \theta B
\]
Since \( \{N, B\} \) is a linear dependent, we get

\[
\kappa \cos \theta - \tau \sin \theta = 0
\]

(3.8)

From equation (3.6) and (3.8), we have

\[
\kappa u - \tau w = \lambda \left( \kappa^2 + \tau^2 \right).
\]

(3.9)

Conversely, we define a curve as

\[
\beta(s) = \int V(s) ds + \lambda(s) N(s)
\]

(3.10)

where \( \lambda : I \rightarrow \mathbb{R} \) is differentiable function such that \( \lambda(s) = -\int v(s) ds \). If we derivate the equation (3.10), we have

\[
\frac{d\beta}{ds} = (u - \lambda \kappa) T + (w + \lambda \tau) B.
\]

(3.11)

From equation (3.11), we get

\[
\frac{d\beta}{ds} T = (u - \lambda \kappa) T + (w + \lambda \tau) B.
\]

(3.12)

where \( \cos \theta(s) = \frac{du}{ds} \) and \( \sin \theta(s) = \frac{dv}{ds} \). From equation (3.4) and (3.7) we have

\[
\tan \theta(s) = \frac{v + \lambda \tau}{u - \lambda \kappa}
\]

(3.13)

Using equation (3.12), we obtain

\[
\frac{d\beta}{ds} N = -\theta' \sin \theta T + (\kappa \cos \theta - \tau \sin \theta) N + \theta' \cos \theta B
\]

(3.14)

From equation (3.13) and (3.14), we get

\[
\kappa \cos \theta - \tau \sin \theta = \frac{\cos \theta}{u - \lambda \kappa} \left( u \kappa - v \tau - \lambda \left( \kappa^2 + \tau^2 \right) \right) = 0
\]

So, we have

\[
N = -\sin \theta T + \cos \theta B
\]

(3.15)

and

\[
d\Theta = \theta' ds = d\theta
\]

(3.16)

From equation (3.12) and (3.16), we see that \( \{N, B\} \) is a linear dependent.

\( \square \)
Corollary 3.1. If \( u(s) = 1, v(s) = w(s) = 0 \), we have Mannheim (T-Mannheim) curve. From equation (3.3), \( \lambda \) is constant. From equation (3.2), we have \( \kappa = \lambda (\kappa^2 + \tau^2) \). If \( v(s) = 1, u(s) = w(s) = 0 \), we have B-Mannheim curve. Using equation (3.2), we have \( \tau = -\lambda (\kappa^2 + \tau^2) \) where \( \lambda \) is constant.

Theorem 3.2. In \( \mathbb{R}^3 \) Euclidean spaces, let \( M \) be a regular curve with coordinate neighborhood \( (I, \gamma) \) and \( "s" \) be arcparameter of the curve. Let \( \{T, N, B\} \) be orthonormal frame of the curve and \( \kappa \) and \( \tau \) be curvatures of the curves. \( M \) is a \( V-Mannheim \) curve if and only if it is satisfy that
\[
2u\kappa(s) = \frac{1}{\lambda} \left[u^2 + u\sqrt{u^2 + w^2} \cos (2\theta + \theta_0)\right]
\]
and
\[
2w\tau(s) = \frac{1}{\lambda} \left[-w^2 + w\sqrt{u^2 + w^2} \sin (2\theta + \theta_0)\right]
\]
where \( \cos \theta_0 = \frac{u}{\sqrt{u^2 + w^2}} \) and \( \sin \theta_0 = \frac{w}{\sqrt{u^2 + w^2}} \).

Proof. If \( M \) is a \( V-Mannheim \) curve, From equation (3.2), then we have \( u\kappa - w\tau = \lambda (\kappa^2 + \tau^2) \). So we have
\[
\kappa(s) = \sqrt{\frac{u\kappa - w\tau}{\lambda}} \cos \theta
\]
and
\[
\tau(s) = \sqrt{\frac{u\kappa - w\tau}{\lambda}} \sin \theta
\]
From equation (3.20) and (3.21), we have
\[
u\kappa - w\tau = \frac{1}{\lambda} (u \cos \theta - w \sin \theta)^2
\]
and
\[
u\kappa + w\tau = \frac{1}{\lambda} (u^2 \cos^2 \theta - w^2 \sin^2 \theta)
\]
From equation (3.22) and (3.23), we have equation (3.18) and (3.19) \( \square \)

Corollary 3.2. If \( u(s) = 1, v(s) = w(s) = 0 \), we have Mannheim (T-Mannheim) curve. From equation (3.3), \( \lambda \) is constant. In this case, we have
\[
\kappa(s) = R(\cos \theta)^2
\]
where $R = \frac{1}{\lambda}$ is constant. From equation (3.3), we have $\kappa = \lambda (\kappa^2 + \tau^2)$. If $w(s) = 1, u(s) = v(s) = 0$, we have B-Bertrant curve. From equation (3.3), $\lambda$ is constant. In this case we have

$$\kappa(s) = R \cos \theta \sin \theta$$

and

$$\tau(s) = R (\cos \theta)^2$$

where $R = \frac{1}{\lambda}$ is constant.

In 3-Euclidean spaces, let $M, K$ be a regular curve with unit coordinate neighborhood $(I, \alpha)$ and $(I, \beta)$ and "$s$" and "$\beta$" be arcparameter of $M$ and $K$, respectively. Let $(T, N, B, \kappa, \tau)$ and $(\overline{T}, \overline{N}, \overline{B}, \overline{\pi}, \overline{\tau})$ be Frenet apparatus of $M$ and $K$, respectively. Let $V = uT + vN + wB$ be unit vector field where $u, v, w$ are constant.

**Theorem 3.3.** Under the above notation, the curve $(\beta)$ is a $V$–Mannheim partner curve if and only if

$$\frac{d\tau}{ds} = \frac{u \tau \sqrt{1 + \lambda^2 \tau^2}}{\lambda \sqrt{1 - v^2}} - \frac{\overline{\pi}}{\lambda} (1 + \lambda^2 \overline{\tau}^2)$$

where $\pi, \tau$ are curvature of the curve $(\beta)$ and $\lambda(s) = - \int v(s) ds$.

**Proof.** Let $(\beta)$ be $V$–Mannheim partner curve of $(\alpha)$. So we can write $\overline{B} = N$. From equation (3.1), we can write

$$\int_0^s V(u) du = \beta(s) - \lambda \overline{B}(s)$$

If we derivate the equation (3.29), we have

$$\frac{ds}{ds} (uT + vN + wB) = \overline{T} + \lambda \overline{\pi} N - \frac{d\lambda}{ds} B(s)$$
where $\overline{B} = N$ and $\lambda(s) = -\int v(s) ds$. So we obtain

\begin{equation}
\frac{ds}{ds}(uT + wB) = \overline{T} + \lambda \overline{N}
\end{equation}

where

\begin{equation}
\overline{T} = \frac{ds}{ds}(u - \lambda \kappa) T + \frac{ds}{ds}(w + \lambda r) B = \cos \theta T + \sin \theta B
\end{equation}

and

\begin{equation}
\overline{N} = \sin \theta T - \cos \theta B
\end{equation}

From equation (3.30), (3.31) and (3.32), we have

\begin{equation}
\frac{ds}{ds} u = \cos \theta + \lambda \overline{N} \sin \theta
\end{equation}

and

\begin{equation}
\frac{ds}{ds} w = \sin \theta - \lambda \overline{N} \cos \theta
\end{equation}

Using equations (3.33) and (3.34), we obtain

\begin{equation}
\frac{w}{u} = \frac{\sin \theta - \lambda \overline{N} \cos \theta}{\cos \theta + \lambda \overline{N} \sin \theta}
\end{equation}

From equation (3.35), we can easily see that

\begin{equation}
\overline{r} = \frac{1}{\lambda} \frac{u \sin \theta - w \cos \theta}{w \sin \theta + u \cos \theta}
\end{equation}

where

\[(u \sin \theta - w \cos \theta)^2 + (w \sin \theta + u \cos \theta)^2 = u^2 + w^2 = 1 - v^2\]

So, there is $\phi : I \longrightarrow \mathbb{R}$ ($s \rightarrow \phi(s)$) differentiable function such that

\begin{equation}
\sqrt{1 - v^2} \cos \phi = w \sin \theta + u \cos \theta
\end{equation}

and

\begin{equation}
\sqrt{1 - v^2} \sin \phi = u \sin \theta - w \cos \theta
\end{equation}

From equation (3.37) and (3.38), we have

\begin{equation}
\overline{r} = \frac{1}{\lambda} \tan \phi
\end{equation}
and

(3.40) \quad \frac{d\theta}{ds} = \frac{d\phi}{ds} = -\kappa

From equation (3.39) and (3.40), we obtain

(3.41) \quad \frac{d\tau}{ds} = \frac{v}{\lambda} \sqrt{1 + \lambda^2 \tau^2} - \frac{\pi}{\lambda} \left(1 + \lambda^2 \tau^2\right).

Conversely, we can define a curve as

(3.42) \quad \int_0^s V(u) du = \beta(s) - \lambda \beta(s)

If we derivate the equation (3.42), we have

(3.43) \quad \frac{ds}{ds} (uT + vN + wB) = \tau \sqrt{1 + \lambda^2 \tau^2} - \frac{\lambda}{\tau} \frac{d\lambda}{ds} B

From equation (3.43), we obtain

(3.44) \quad \frac{ds}{ds} = \sqrt{1 + \lambda^2 \tau^2} \left(1 - \frac{v^2}{1 - v^2}\right)

and

(3.45) \quad \frac{ds}{ds} (uT + wB) = \sqrt{1 + \lambda^2 \tau^2}

If we derivate the equation (3.45), we have

(3.46) \quad \frac{d^2s}{ds^2} \left(\tau \sqrt{1 + \lambda^2 \tau^2}\right) + \left(\frac{ds}{ds}\right)^2 \left(\frac{d\lambda}{ds}\right) \frac{\lambda}{\tau} T + \left(\frac{d\tau}{ds}\right) N = -\lambda \sqrt{1 + \lambda^2 \tau^2} \left(\tau \sqrt{1 + \lambda^2 \tau^2} + \left(\lambda \frac{d\tau}{ds}\right) N + \lambda \tau^2 B\right)

Using (3.43), we obtain

(3.47) \quad \frac{d^2s}{ds^2} \frac{d^2s}{ds^2} = -\lambda \sqrt{1 + \lambda^2 \tau^2}

and

(3.48) \quad \tau + \left(\frac{d\tau}{ds}\right) = -\lambda^2 \tau^2 \kappa

From equation (3.46), (3.47) and (3.48), we easily can see that \{N, B\} is a linear dependent. \qed
**Corollary 3.3.** If \( v = 0 \), then we can see that \( \lambda \) is non zero constant and \( u^2 + w^2 = 1 - v^2 = 1 \). From equation (3.28), \((\beta)\) is a Mannheim partner curve.

If \( u, v, w \) are functions from \( I \) to \( \mathbb{R} \), then we can give following theorem.

**Theorem 3.4.** The curve \((\beta)\) is a \( V \)–Mannheim partner curve if and only if

\[
\frac{d\varpi}{ds} = \frac{v\tau\sqrt{1 + \lambda^2\tau^2}}{\lambda\sqrt{1 - v^2}} + \frac{1}{\lambda} \left( \frac{d\left(\arctan\left(\frac{w}{u}\right)\right)}{ds} - \kappa \right) \left(1 + \lambda^2\tau^2\right)
\]

where \( \kappa, \varpi \) are curvature of the curve \((\beta)\) and \( \lambda(s) = -\int v(s)ds \).

4. Generating Curve Of The Mannheim curve.

In 3-Euclidean spaces, let \( M, K \) be a regular curve with unit coordinate neighborhood \((I, \gamma)\) and \((I, \beta)\) and "s" be arcparameter of \( M \) and \( K \). Let \((T, N, B, \kappa, \tau)\) and \((\bar{T}, \bar{N}, \bar{B}, \bar{\kappa}, \bar{\tau})\) be Frenet apparatus of \( M \) and \( K \), respectively. For all \( s \in I \),

\[
\beta''(s) = \kappa\gamma'(s)
\]

and

\[
\begin{align*}
\kappa(s) &= \kappa(s) \cos \left( \int_0^s \tau(u)du + \varphi_0 \right) \\
\tau(s) &= \kappa(s) \sin \left( \int_0^s \tau(u)du + \varphi_0 \right)
\end{align*}
\]

**Theorem 4.1.** In this case, \( K \) is a \( V \)–Mannheim curve if and only if

\[
\kappa(s) = F(s) \cos(\varphi(s) + \phi(s))
\]

where \( F(s) = \lambda\sqrt{u^2 + w^2} \), \( \cos \phi(s) = \frac{w}{\sqrt{u^2 + w^2}} \), \( \sin \phi(s) = \frac{u}{\sqrt{u^2 + w^2}} \) and

\[
\varphi(s) = \int_0^s \tau(u)du + \varphi_0.
\]
Proof. i) Let $K$ be a Mannheim curve. In this case, there exist $\lambda \in \mathbb{R}$ such that $(\kappa(s))^2 + (\tau(s))^2 = \lambda \tilde{\kappa}(s)$, where "s" is arc parameter of $K$. From equation (4.1), we have

$$u\kappa(s) \cos \varphi(s) - w\kappa(s) \sin \varphi(s) = \lambda (\kappa(s))^2$$

From equation (4.3), we obtain

$$\kappa(s) = F(s) \cos (\varphi(s) + \phi(s))$$

where $F(s) = \sqrt{\frac{u^2 + w^2}{\lambda}}$, $\cos \phi(s) = \frac{u}{\sqrt{u^2 + w^2}}$, $\sin \phi(s) = \frac{w}{\sqrt{u^2 + w^2}}$ and $\varphi(s) = \int_0^s \tau(u)du + \varphi_0$. Conversely, From equation (4.2), we have

$$\lambda (u\kappa(s) \cos \varphi(s) - w\kappa(s) \sin \varphi(s)) = (\kappa(s))^2$$

From equation (4.1), we get $\lambda (\kappa(s))^2 + (\tau(s))^2 = \tilde{\kappa}(s)$. $\square$

**Theorem 4.2.** i) $K$ is a Mannheim curve if and only if

$$\kappa(s) = R \cos \left( \int_0^s \tau(u)du + \varphi_0 \right)$$

ii) $K$ is a $B$–Mannheim curve if and only if

$$\kappa(s) = R \sin \left( \int_0^s \tau(u)du + \varphi_0 \right)$$

where $R$ is constant.

Proof. i) Let $K$ be a Mannheim curve. In this case, there exist $\lambda \in \mathbb{R}$ such that $(\kappa(s))^2 + (\tau(s))^2 = \lambda \tilde{\kappa}(s)$, where "s" is arc parameter of $K$. From equation (4.1), we have $\lambda \left( (\kappa(s))^2 + (\tau(s))^2 \right) = \lambda (\kappa(s))^2 = \tilde{\kappa}(s)$. So we have

$$\kappa(s) = R \cos \left( \int_0^s \tau(u)du + \varphi_0 \right)$$
where $R = \frac{1}{\lambda} = const$. Conversely, Let $M$ be a curve satisfy that $\kappa(s) = R \cos \varphi(s)$ where $R = \frac{1}{\lambda} = const$ and $\varphi(s) = \int_0^s \tau(u)du + \varphi_0$. From equation (4.1), we have

\begin{equation}
(4.4) \quad \pi(s) = R \cos^2 \varphi
\end{equation}

and

\begin{equation}
(4.5) \quad \tau(s) = R \cos \varphi \sin \varphi
\end{equation}

From equation (4.2), we get

\begin{equation}
(\pi(s))^2 + (\tau(s))^2 = R^2 (\cos \varphi(s))^2 = R \pi(s)
\end{equation}

So $K$ is a Mannheim curve.

ii) Let $K$ be a $B-$Mannheim curve. In this case, there exist $\lambda \in \mathbb{R}$ such that $(\pi(s))^2 + (\tau(s))^2 = \lambda \pi(s)$, where "s" is arcparameter of $K$. From equation (4.1), we have $(\pi(s))^2 + (\tau(s))^2 = (\kappa(s))^2 = \lambda \pi(s)$. So we have

$$
\kappa(s) = R \sin \left( \int_0^s \tau(u)du + \varphi_0 \right)
$$

where $R = \epsilon \lambda = const$. Conversely, Let $M$ be a curve satisfy that $\kappa(s) = R \sin \varphi(s)$ where $R = \frac{1}{\lambda} = const$ and $\varphi(s) = \int_0^s \tau(u)du + \varphi_0$. From equation (4.1), we have

$$
\pi(s) = R \cos \varphi \sin \varphi
\quad \tau(s) = R (\cos \varphi)^2
$$

From equation (4.3), we get

\begin{equation}
(\pi(s))^2 + (\tau(s))^2 = R^2 (\cos \varphi(s))^2 = R \pi(s)
\end{equation}

So $K$ is a $B$-Mannheim curve.
**Definition 4.1.** In 3-Euclidean spaces, let $K$ be a curve satisfy that

$$\pi(\tau) = R \cos \left( \int_0^\tau \tau(u) du + \theta_0 \right).$$

We said that $K$ is a generating curve of the Mannheim curve.

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