Impact of radio sources and cosmic infrared background on thermal Sunyaev–Zel’dovich – gravitational lensing cross-correlation

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ABSTRACT

Cross-correlation with thermal Sunyaev–Zel’dovich (tSZ) effect in cosmic microwave background observation and weak gravitational lensing effect in galaxy imaging survey opens a new window on constraining matter contents in the Universe at redshifts less than 1. In this paper, we study the impact of radio sources and cosmic infrared background (CIB) on observed tSZ-lensing correlation. Assuming the best-fitting model of CIB by the Planck satellite, we estimate that the residual correlation of CIB with large-scale structures will be of the order of 2 per cent of expected tSZ-lensing correlation from intracluster medium in current lensing surveys. On the other hand, despite large model uncertainties, we find that correlation of lensing and radio sources can induce a negative correction for the observed tSZ-lensing correlation with an ∼10–30 per cent level. This is originated from positive cross-correlation with radio sources and lensing at ∼100 GHz frequency, whereas tSZ-lensing correlation should show a negative value in temperature fluctuations at that frequency. We also show that the negative correction by radio-lensing correlation can solve the tension between recent measurements of tSZ-lensing correlation and an expected signal from ‘universal’ gas pressure profile of nearby galaxy clusters, when the radio sources with a flat-spectral index are assumed to populate massive cluster-sized dark matter haloes. Our results indicate that minor source population in radio bands can play an important role in determining observed tSZ-lensing correlation at ∼10 arcmin.

Key words: methods: analytical – large-scale structure of Universe – submillimetre: diffuse background.

1 INTRODUCTION

The observation of cosmic microwave background (CMB) radiation by the Planck satellite is among the most crucial for modern astrophysics and cosmology. A wide frequency coverage in the Planck observation enables not only to have a robust estimate of CMB (e.g. Planck Collaboration IX 2016a), but also study other interesting physical effects printed in CMB radiation. Inverse-Compton scattering of CMB photons by hot relativistic electrons (Zeldovich & Sunyaev 1969), referred to as thermal Sunyaev–Zel’dovich (tSZ) effect, is one of the main science targets in the Planck mission (e.g. Planck Collaboration XXII 2016c). Since the tSZ effect leads to a frequency-dependent distortion of the CMB blackbody spectrum (Zeldovich & Sunyaev 1969), the measurement on different wavelengths is essential as the Planck satellite has done. Previous studies have shown that the tSZ effect by intracluster medium (ICM) in galaxy clusters is of extreme cosmological importance.

The statistics of the tSZ effect can probe the abundance of most massive dark matter haloes at various redshifts and show a strong dependence on the amplitude in linear density fluctuations (e.g. Komatsu & Kitayama 1999; Komatsu & Seljak 2002; Bhattacharya et al. 2012). It is worth noting that the precise measurement of the amplitude in density fluctuations allows us to constrain physics beyond the standard model, including massive neutrinos (e.g. Saito, Takada & Taruya 2008), dark matter (e.g. Smith & Markovic 2011; Enqvist et al. 2015), and cosmic acceleration (e.g. Weinberg et al. 2013).

To use tSZ effect for cosmological analyses, we need accurate modelling of ICM. The tSZ statistics have degeneracy between the ICM property and cosmological parameters in principle (e.g. Battaglia et al. 2010). Hence, the current cosmological constraints by the tSZ statistics hinge on the still poorly understood property of ICM (e.g. Planck Collaboration XXII 2016c; Horowitz & Seljak 2017). Various approaches have been proposed to learn the ICM physics more and break the degeneracy in tSZ statistics with astronomical measurements other than the tSZ effect (e.g. Battaglia et al. 2017; Hill et al. 2018). Among them, the measurement of gravitational lensing effect in galaxy imaging survey can play a central
role in determining the relation between the ICM and large-scale structures.

Gravitational lensing effect causes a small distortion in shape of distant sources and its amplitude depends on surface mass density in the direction of individual sources (e.g. Bartelmann & Schneider 2001, for a review). Therefore, the information of total matter density (including dark matter) can be obtained with the lensing measurement, and those information would be a base for developing an accurate model of ICM. Battaglia, Hill & Murray (2015) have shown that the cross-correlation between tSZ and lensing can probe the ICM over a wide range of halo masses and redshifts and future measurement of the cross-correlation can constrain the relation between ICM and dark matter haloes with a 5–20 per cent precision.

Recently, the measurements of tSZ-lensing cross-correlation have been performed with lensing data in the Canada France Hawaii Lensing Survey2 (CFHTLenS; Van Waerbeke, Hinshaw & Murray 2014) and the Red Cluster Sequence Lensing Survey3 (RCSLenS; Hojjati et al. 2017). The signal-to-noise ratio in the latest measurement already reaches a $13\sigma$–$17\sigma$ level (Hojjati et al. 2017), implying that ongoing and future lensing surveys will be able to present more precise measurements. A natural question then arises: Can we explain the observed tSZ-lensing correlation in CFHTLenS and RCSLenS assuming the ICM properties observed in nearby clusters (e.g. Planck Collaboration V 2013)? This question is still under debate. At least, it seems difficult to explain the tSZ-lensing correlation at an angular scale of $<10$ arcmin by the model with the ICM property from local observation (Ma et al. 2015). To fill the gap, one may need a relatively small amplitude in matter density fluctuations today compared to the constraint by CMB measurements and/or a strong baryonic feedback by active galactic nuclei (AGNs) in galaxy groups (Hojjati et al. 2017). Furthermore, Osato et al. (2018) examined a semi-analytic model of ICM in Shaw et al. (2010) to explain both the tSZ-lensing correlation in Hojjati et al. (2017) and the power spectrum of tSZ effect measured in Planck Collaboration XXII (2016c). The authors found that the tSZ power spectrum prefers $\sim30$ per cent of ICM pressure at outer region in galaxy clusters to be non-thermal, while the tSZ-lensing correlation prefers a much lower fraction of non-thermal pressure with a level of $\sim5$ per cent. At present, the ICM model in Shaw et al. (2010) cannot explain both the statistics simultaneously without reducing gas fraction in group-scaled haloes.

Motivated by the inconsistency between the observed tSZ-lensing signal and the model with the ICM property of nearby clusters constrained by Planck (Planck Collaboration V 2013), we here consider other relevant effects in the cross-correlation measurement. In this paper, we study the impact of the presence of astrophysical sources in brightness temperature maps on the cross-correlation measurement. Except for galactic sources, relevant sources in the cross-correlation measurement are expected to be radio point sources and/or a strong baryonic feedback by active galactic nuclei fluctuations today compared to the constraint by CMB measurements and discuss the possible effects from astrophysical sources at frequency bands of interest. We then summarize the basics of gravitational lensing and the cross-correlation between tSZ and lensing in Section 3. Our model of the tSZ-lensing correlation is summarized in Section 4 and Section 5 presents the outcome of our model and comparison of the observed tSZ-lensing correlation with the model. We conclude the paper in Section 6. Throughout this study, we adopt the flat-geometry ΛCDM (Lambda cold dark matter) model that is consistent with the Planck 2015 results (see the result of ‘TT+lowP + lensing’ in Planck Collaboration XIII 2016b). The cosmological parameters are as follows: the matter density, $\Omega_m = 1 - \Omega_{\Lambda} = 0.308$, the baryon density, $\Omega_B = 0.0484$, the Hubble parameter, $h = H_0/(100\,\text{km\,s}^{-1}\,\text{Mpc}^{-1}) = 0.678$, the present amplitude of density contrast at 8$h^{-1}$ Mpc, $\sigma_8 = 0.8149$, and the spectral index, $n_s = 0.9677$.

## 2 ESTIMATION OF TSZ EFFECT FROM CMB MEASUREMENTS

At frequency $i$, the change in CMB temperature by the tSZ effect is expressed as

$$\Delta T_i = g(x_i) \gamma,$$

where $T_i = 2.725\,\text{K}$ is the CMB temperature (Fixsen 2009), $g(x) = \text{coth}(x/2) - 4$ with $x = h\nu/k_BT_i$, $h$ and $k_B$ are the Planck constant and the Boltzmann constant, respectively. Compton parameter $\gamma$ is computed as the integral of the electron pressure $P_e$ along a line of sight:

$$\gamma(i) = \int_0^{r_i} \frac{d\chi}{1+z} \frac{k_B \sigma_T}{m_e c^2} P_e(r(\chi), z(\chi)),$$

where $\sigma_T$ is the Thomson cross-section, $\chi$ is the comoving radial distance to redshift $z$, $r(\chi)$ is the angular diameter distance, and $z_H$ is the comoving distance up to $z \to \infty$.

Given the frequency dependence in the tSZ effect as equation (1), one can construct an estimator of Compton $\gamma$ map from brightness temperature maps at multiple frequencies as

$$\hat{\gamma}(\theta) = \sum_i w_i \frac{T_i(\theta)}{T_0},$$

where $\hat{\gamma}$ is an estimated Compton parameter $\gamma$, $T_i$ is the observed temperature at $i$-th frequency $\nu_i$, and the sum in equation (3) is over frequency bands of interest. Van Waerbeke et al. (2014) imposed three conditions to construct the Compton $\gamma$ map for the cross-correlation with the tSZ effect and weak lensing effect in galaxy shapes. Those include (i) $\sum g(x_i) w_i = 1$ to produce an unbiased Compton $\gamma$ map on average, (ii) $\sum w_i = 0$ to nullify the primary CMB fluctuations, and (iii) $\sum w_i \cdot e_j v_i^j = 0$ to remove

\[\text{in this paper, we ignore the relativistic correction for } g(x) \text{ that is important for the tSZ effects in most massive galaxy clusters (Itoh, Kohyama & Nozawa 1998; Nozawa, Itoh & Kohyama 1998). Note that the tSZ-lensing correlation can probe the ICM at clusters with masses of } \sim 10^{14} h^{-1} \,M_\odot \text{ (Ma et al. 2015; Osato et al. 2018).}\]
a contribution from dust emission with a spectral index $\beta_d \sim 2$ in antenna temperature units (The factor $c$ is given by the conversion of antenna temperature to thermodynamic temperature. See also Section 4.2). In Van Waerbeke et al. (2014), the authors worked on the Planck temperature maps (Planck Collaboration VI 2014a) at four frequency bands of 100, 143, 217, and 353 GHz to determine the weight $w_i$ for different $\beta_d$. Throughout this paper, we work with the weight in the $y$ map named Planck C in Van Waerbeke et al. (2014) for a representative example.

In addition to estimating the tSZ effect, Van Waerbeke et al. (2014) examined to set different weights in equation (3) by projecting out both the tSZ effect and primary CMB fluctuation (i.e. $\sum g(x_i)wi = \sum wi = 0$). Even after nulling the tSZ effect in equation (3), they found a weak correlation of the observed $\gamma$ and weak lensing (see fig. 4 in Van Waerbeke et al. 2014). This supports that the contamination in $y$ map construction should (partly) arise from some extragalactic sources since weak gravitational lensing effect is relevant for the large-scale structures at $z \sim 0.2$–0.4 (Van Waerbeke et al. 2013, also see Fig. 1). In this paper, we study the correlation of $\gamma$ and gravitational lensing when setting weight in equation (3) so as to remove the tSZ effect.\(^5\) For the weight, so as to null the tSZ effect in $y$ map, we work with the case named as Planck E in Van Waerbeke et al. (2014).

Note that the Planck team has worked on a more sophisticated approach than equation (3) to construct a Compton y map. They allowed a spatially varying weight in equation (3) (see Planck Collaboration XXII 2016c, for the latest map). For a spatially varying weight, we can easily take into account the effect on the cross-correlation measurements by using the formula in appendix A of Planck Collaboration XXI (2014b). The primary purpose in this paper is to follow the analyses in Van Waerbeke et al. (2014), and we leave it to a future work to include the effect of spatially varying weight.

\(^5\) The conditions in Van Waerbeke et al. (2014) for nulling tSZ cannot determine the overall normalization of weights. This degree of freedom does not affect our results since we use the same weights as in Van Waerbeke et al. (2014).

Nevertheless, the following results are expected to be less affected by the choice of the weight in equation (3) or spatially varying weight in Planck Collaboration XXII (2016c). First of all, the spatially varying weights can reduce the contamination from spatially resolved objects (Hurier, Macías-Pérez & Hildebrandt 2013), but it is no longer valid for unresolved sources such as CIB or faint radio sources. Also, the spatially varying weight in Planck Collaboration XXII (2016c) is introduced so as to localize tSZ effects in temperature maps at different frequency channels. Hence, the frequency dependence of such a weight is expected to be similar to one of the weights in equation (3). In addition, the weight in Planck Collaboration XXII (2016c) uses small-scale information of temperature maps to localize the tSZ effects, leaving the impact of faint localized objects on the cross-correlation analysis.

### 3 CROSS-CORRELATION WITH COMPTON $Y$ AND WEAK LENSING

As introduced in Section 2, the actual observable $\hat{\gamma}$ is given by the linear combination of brightness temperature as in equation (3). Here we summarize the basic of the cross-correlation between $\hat{\gamma}$ map and weak gravitational lensing effect in galaxy imaging survey. Lensing convergence $\kappa$ is responsible for the strength of weak gravitational lensing effect. Under the Born approximation, one can express the lensing convergence field as the weighted integral of matter overdensity field $\delta_m(x)$ (Bartelmann & Schneider 2001):

$$\kappa(\theta) = \int_0^{\chi(u)} d\chi \, W(\kappa) \delta_m(r(\theta), z(\chi)),$$

where $W(\kappa)$ is called lensing kernel. For a given redshift distribution of source galaxies, the lensing kernel is expressed as

$$W(\chi) = \frac{3}{2} \left( \frac{H_0}{c} \right)^2 \Omega_{m0} \frac{r(\chi)}{dA(\chi)} \int_0^{\chi(u)} d\chi' \, s(\chi') \frac{r(\chi') - r(\chi)}{r(\chi')} ,$$

where $s(\chi)$ represents the redshift distribution of source galaxies normalized to $\int d\chi' \, s(\chi') = 1$. Hence, the quantity of interest in this paper is defined as

$$\xi_{\gamma - \kappa}(\theta) = \langle \hat{\gamma}(\phi) \delta_m(\phi + \theta) \rangle = \sum_i w_i / T_i \langle C(\theta) \rangle ,$$

and the cross-correlation of $\xi_{\gamma - \kappa}$ can be computed as (e.g. Hojjati et al. 2017)

$$\xi_{\gamma - \kappa}(\theta) = \sum_\ell \left( \frac{2\ell + 1}{4\pi} \right) C_{\gamma - \kappa}(\ell) P_\ell(\cos \theta) b^\gamma_\ell b^\kappa_\ell ,$$

where $P_\ell$ are the Legendre polynomials, and $b^\gamma_\ell$ and $b^\kappa_\ell$ are the smoothing kernel of $\gamma$ and $\kappa$ maps, respectively. The power spectrum of $C_{\gamma - \kappa}(\ell)$ can be decomposed into

$$C_{\gamma - \kappa}(\ell) = \left( \sum_i w_i g(x_i) \right) C_{\gamma - \kappa}(TSZ|\ell) + \sum_i w_i C_{\kappa - \ell}(v_i |\ell) ,$$

where the first term in the right-hand side expresses the correlation of tSZ effect and lensing convergence as studied in the literature, whereas the second term is new contribution arising from the correlation of astrophysical sources in the Planck bands and lensing convergence. Note that the second term in the r.h.s of equation (8) can have different dependence in frequency $v$ from the tSZ effect and CMB blackbody spectrum, and it cannot be vanished in general. We define $C_{\gamma - \kappa}(v_i |\ell)$ to be dimensionless by normalizing the $T$ field with CMB temperature $T_0$ throughout this paper.
4 MODEL

In this section, we describe a theoretical model of equation (8) based on halo-model approach.

4.1 Intracluster medium

We first summarize the halo-model prediction of tSZ-lensing cross-correlation induced by ICM as developed in Hill & Spergel (2014) and Ma et al. (2015). In the halo model, we can decompose the power spectrum into two components as

\[ C_{\gamma-k} = C_{\gamma-k}^{th} + C_{\gamma-k}^{2h}, \tag{9} \]

where the first term in the r.h.s arises from the correlation within single haloes, while the second term represents the correlation due to clustering of neighbouring haloes. For tSZ effect, one can express those terms as

\[ C_{\gamma-k}^{th} (tSZ) = \int_0^{z_{\text{max}}} dz \frac{dV}{dz \, d\Omega} \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} \gamma_i(M, z) \kappa_i(M, z), \tag{10} \]

\[ C_{\gamma-k}^{2h} (tSZ) = \int_0^{z_{\text{max}}} dz \frac{dV}{dz \, d\Omega} P_L(k, \ell/M, z) \times \left[ \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} \gamma_i(M, z) b(M, z) \right] \times \left[ \int_{M_{\text{min}}}^{M_{\text{max}}} dM \frac{dn}{dM} \kappa_i(M, z) b(M, z) \right], \tag{11} \]

where we set \( z_{\text{max}} = 7, M_{\text{min}} = 10^{10} h^{-1} M_\odot, \) and \( M_{\text{max}} = 10^{16} h^{-1} M_\odot, \) with \( P_L(k, \ell/M, z) \) is the linear matter power spectrum, \( \frac{dn}{dM} \) is the halo mass function in Tinker et al. (2008) and linear bias in Tinker et al. (2009). In equations (10) and (11), \( \kappa_i \) is the Fourier transform of Compton y profile in single dark matter halo with the NFW density profile (Navarro, Frenk & White 1996):

\[ \kappa_i(M, z) = \frac{W_i(x(z))}{x^2} \int dx \sqrt{4 \pi r_o^2 \rho_{\text{NFW}}(r, M, z)/\rho_m}, \tag{12} \]

where \( \rho_{\text{NFW}} \) is the NFW profile and \( \rho_m \) is the mean matter density in the Universe. The NFW profile can be characterized by single parameter called halo concentration for a given SO mass. In this paper, we adopt the model of halo mass function in Tinker et al. (2008) and linear bias in Tinker et al. (2010). In equations (10) and (11), \( \gamma_i \) is the Fourier transform of lensing convergence profile of single dark matter halo with the NFW density profile (Navarro, Frenk & White 1996):

\[ \gamma_i(M, z) = \frac{W_i(M, z)}{x^2} \int dx \sqrt{4 \pi r_o^2 \rho_{\text{NFW}}(r, M, z)/\rho_m}, \tag{13} \]

where \( r_o \) is the SO radius with respect to 500 times critical density, we define as \( x = ar_i r_o, \) and \( \xi_{500} = ax r_o. \) When computing \( y_i, \) we adopt the model of 3D electron pressure profile in single halo \( P_{e, h} \) as constrained in Planck Collaboration V (2013),

\[ P_{e, h}(x = r/r_{500}, M, z) = 1.65 \times 10^{-3} \left[ \frac{\text{keV cm}^{-3}}{\text{yr}^{-2} \text{km}^{-2}} \right] E_5^{3/2} \left( \frac{M_{500}}{3 \times 10^{14} h_{70}^{-1} M_\odot} \right)^{2/3+0.12} \frac{P(x) h_{70}^4}{M_{500}}, \tag{14} \]

where \( E(z) = H(z)/H_0, h_{70} = H_0/70, M_{500} = 4/3\pi \cdot 500 \rho_{\text{crit}} r_{200}^3, \) \( \rho_{\text{crit}} \) is the critical density in the Universe, and \( P(x) \) is the so-called universal pressure profile (Nagai, Kravtsov & Vikhlinin 2007). The functional form of \( P(x) \) is given by

\[ P(x) = \frac{P_0}{(c_{500}x)^\gamma [1 + (c_{500}x)\gamma]^\beta - 1/a}, \tag{15} \]

where we adopt the best-fitting values of five parameters \((P_0, c_{500}, \alpha, \beta, \gamma) \) from Planck Collaboration V (2013). Note that the input mass parameter \( M_{500} \) in equation (14) will be affected by hydrostatic mass bias. For a given halo mass of \( M \) (the SO mass w.r.t 200 times mean matter density), we compute \( M_{500} \) by using the halo concentration as in Hu & Kravtsov (2003) and then include the hydrostatic mass bias \( b_{\text{HM}} \) by \( M_{500} \rightarrow (1 - b_{\text{HM}})M_{500} \) and \( r_{500} \rightarrow (1 - b_{\text{HM}})^{3/2} r_{500} \) for equation (14). We set \( 1 - b_{\text{HM}} = 0.833 \) as follows in Dolag, Komatsu & Sunyaev (2016).

It is worth noting that Dolag et al. (2016) have shown the above ICM model can explain the observed tSZ power spectrum (Planck Collaboration XXII 2016c). In fact, one can also explain the tSZ-lensing correlations (Van Waerbeke et al. 2014; Hojjati et al. 2017) with the above ICM model by setting \( P_0 \sim 4.4 \) (the best-fitting value from Planck analysis is 6.41), whereas it turns to be difficult to explain the tSZ power spectrum. In addition, the value of \( P_0 \sim 4.4 \) is inconsistent with the observations of nearby galaxy clusters (Arnaud et al. 2010; Planck Collaboration V 2013).

4.2 Astrophysical sources

Next we consider cumulative emission from astrophysical sources at a frequency of 100–800 GHz and the cross-correlation with lensing convergence field. At the frequency of interest, relevant astrophysical sources include point sources in radio bands, referred to as radio galaxies or/and radio AGNs in the literature, and CIB emission.

Observed specific intensity at a given frequency \( v \) can be expressed as

\[ I_v(\theta) = \int \frac{d\chi}{1+z} j_i(r(\chi),\theta, z(\chi)). \tag{16} \]

where \( j_i(x) \) represents the comoving specific emission coefficient. One can convert the specific intensity to antenna temperature using the CMB blackbody spectrum as

\[ T_v(\theta) = \left( \frac{\partial B_v}{\partial T} \right)^{-1} T_{v0} I_v(\theta), \tag{17} \]

\[ \frac{\partial B_v}{\partial T} \bigg|_{T=T_0} = 99.27 \left[ \frac{\text{Jy str}^{-1}}{\mu K} \right] \frac{x^4 e^x}{(e^x - 1)^2}, \tag{18} \]

where \( x = h v/k T_0 = v/56.84 \text{GHz}. \)

In the halo-model approach, we can relate the emission coefficient with underlying astrophysical sources as follows (e.g. Shang et al. 2012):

\[ j_v = \int dL \frac{dn}{dL} (L, z) \frac{L(1+z)^{\alpha v}}{4\pi}, \tag{19} \]

where \( L \) is the luminosity in infrared or radio for our case. \( \frac{dn}{dL} \) represents the luminosity function, and \( L(1+z)^{\alpha v} \) is related to the flux \( S_v \) from relevant object as

\[ S_v = \frac{(1+z)^{-1} L(1+z)^{\alpha v}}{4\pi x^2}. \tag{20} \]
In the Appendix, we summarize the derivation for cross-power spectra of weak lensing and astrophysical sources based on the halo model.

Radio sources

For radio sources, we work with three-population model as introduced in de Zotti et al. (2005). In this model, extragalactic radio sources consist of flat-spectrum radio quasars (FSRQs), BL Lac objects, and steep-spectrum sources. Their spectral index is assumed to be $\alpha = 0.1, 0.1$, and 0.7 for FSRQs, BL Lac objects, and steep-spectrum sources, respectively (the index is defined in terms of $S_\nu \propto \nu^{-\alpha}$). The radio luminosity function at 1.4 GHz for these three populations has been constrained in Massardi et al. (2010) with local luminosity functions, multifrequency source counts, and redshift distributions. In this paper, we adopt the radio luminosity function in Massardi et al. (2010) and assume that the emission coefficient from extragalactic radio sources can be computed as

$$ j_\nu \simeq \sum_i \int dL_{1.4} \frac{d\nu_i}{dL_{1.4}} \frac{L_{\nu_i}(1+z_i)^{\alpha_i}}{1.4 \text{GHz}} (1 + \delta_i), \quad (21) $$

$$ L_{\nu_i} = \frac{L_{1.4}}{(1+z)^2} (1+z)^{\alpha_i} (1.4 \text{GHz})^{-\alpha_i}, \quad (22) $$

where $L_{1.4}$ is the radio luminosity at 1.4 GHz, and the index of $i$ runs over BL Lac objects and steep-spectrum sources. In equation (21), $\delta_i$ represents the fluctuation in number density of radio sources. In this paper, we predict the term of $\delta_i$ by using the following halo-occupation distribution (HOD):

$$ \delta_i(x, M) \propto \exp \left( -\frac{M_{\text{cut}}(x)}{M} \right) \delta^{(3)}_D(x), \quad (23) $$

where $\delta^{(3)}_D$ is the $n$-dimensional Dirac delta function and we assume all the radio sources locate at the centre of their host halo. The functional form of HOD is motivated by the study in Wake et al. (2008). For steep-spectrum radio sources, we adopt the best-fitting value of $M_{\text{cut}} = 9.65 \times 10^{13} h^{-1} M_\odot$, from Wake et al. (2008), whereas we examine various values of $M_{\text{cut}}$ for BL Lac objects. Note that the steep-spectrum sources dominate the observed flux counts at $\sim$GHz frequency in this model (Massardi et al. 2010). This indicates that the clustering measurements of radio sources in the literature should be mainly determined by the clustering of steep-spectrum sources within our framework, allowing us to vary the typical host halo mass for BL Lac objects.

Given the model as above, we can compute the cross-power spectrum of radio sources and lensing convergence as

$$ C_{\kappa - T}^{\nu}(\ell | \ell) = C_{\kappa - T}^{R, 1b}(\ell | \ell) + C_{\kappa - T}^{R, 2h}(\ell | \ell), \quad (24) $$

$$ C_{\kappa - T}^{R, 1b}(\ell | \ell) = \int_0^{\chi_{\text{H}}^{\nu}} \frac{d\chi}{\chi} W_{\nu}(\chi) W_{\nu}(\chi) \tilde{\rho}_{\nu}^{-1} \left[ \sum_{i} \int_{\ell}^{\ell_{\max}} \frac{d\nu_i}{dL_{1.4}} \left[ \frac{L_{\nu_i}}{(1+z_i)^2} (1+z_i)^{\alpha_i} (1.4 \text{GHz})^{-\alpha_i} \right] \delta_i(x, M) \right], \quad (25) $$

$$ C_{\kappa - T}^{R, 2h}(\ell | \ell) = \int_0^{\chi_{\text{H}}^{\nu}} \frac{d\chi}{\chi} W_{\nu}(\chi) W_{\nu}(\chi) \tilde{\rho}_{\nu}^{-1} \left[ \sum_{i} \int_{\ell}^{\ell_{\max}} \frac{d\nu_i}{dL_{1.4}} \left[ \frac{L_{\nu_i}}{(1+z_i)^2} (1+z_i)^{\alpha_i} (1.4 \text{GHz})^{-\alpha_i} \right] \delta_i(x, M) \right], \quad (26) $$

where $\tilde{\rho}_{\nu}$ is the Fourier transform of NFW profile and the effective window in radio $W_{\nu}$ is defined as

$$ W_{\nu}(\chi, M) = \frac{1}{T_0} \left[ \frac{\partial B_{\nu}}{\partial T} \right]^{-1} \left( T = T_0 \right) \left[ \frac{L_{\nu}}{1+z} \right] \left( 1 + \frac{L_{\nu}}{L_{\text{min}}} \right) \left( 1 + \frac{L_{\nu}}{L_{\text{max}}} \right), \quad (27) $$

where we set $L_{\text{min}} = 10^{30} \text{erg s}^{-1} \text{Hz}^{-1}$ and $L_{\text{max}} = 10^{30} \text{erg s}^{-1} \text{Hz}^{-1}$.

Cosmic infrared background

For CIB, we follow the model developed in Shang et al. (2012). In this model, equation (19) is rewritten as

$$ j_{\nu} = \int dM \frac{dn}{dM} \frac{L_{\nu}}{4\pi} \left[ N_{\text{cen}} L_{\nu}(1+z)^{\nu}, \text{cen}(M, z) \right] + \int dM \frac{dn}{dM} \left[ f_{\nu, \text{cen}}(M, z) + f_{\nu, \text{sat}}(M, z) \right], \quad (28) $$

$$ = \int_{M_{\text{min}}}^{M_{\text{max}}} dM \left[ f_{\nu, \text{cen}}(M, z) + f_{\nu, \text{sat}}(M, z) \right], \quad (29) $$

where $N_{\text{cen}}$ is the HOD of central galaxies, $L$ is the infrared luminosity, $m$ is the subhalo mass, and $dn_{\nu}/dM$ is the subhalo mass function. In this paper, we use the model of subhalo mass function in Tinker & Wetzel (2010). As seen in equation (28), the model assumes the statistical relation between the luminosity $L$ and subhalo mass and terms of $L_{\nu}(1+z)^{\nu}$ are responsible for the $L-M$ relation. For simplicity, we assume there are no differences of the $L-M$ relation between haloes and subhaloes, i.e. $L_{\nu}(1+z)^{\nu}$ and $L_{\nu}(1+z)^{\nu}$, sat. The functional form of $L_{\nu}(1+z)^{\nu}$ is characterized with seven physical parameters. We also assume $N_{\text{cen}} = 1$ for $M > M_{\text{cen}}$ and 0 otherwise. We adopt the best-fitting parameters in the $L-M$ relation and the value of $M_{\text{cen}}$ to the recent CIB measurement by the Planck satellite (Planck Collaboration XXX 2014d).

Hence, we can express the cross-power spectrum between CIB and lensing convergence as

$$ C_{\kappa - T}^{\nu}(\ell | \ell) = C_{\kappa - T}^{\text{CIB}, 1b}(\ell | \ell) + C_{\kappa - T}^{\text{CIB}, 2h}(\ell | \ell), \quad (30) $$

$$ C_{\kappa - T}^{\text{CIB}, 1b}(\ell | \ell) = \int_0^{\chi_{\text{H}}^{\nu}} \frac{d\chi}{\chi} W_{\nu}(\chi) W_{\nu}(\chi) \tilde{\rho}_{\nu}^{-1} \left[ \sum_{i} \int_{\ell}^{\ell_{\max}} \frac{d\nu_i}{dL_{1.4}} \left[ \frac{f_{\nu, \text{cen}}(M, z)}{f_{\nu, \text{sat}}(M, z)} \right] \tilde{\rho}_{\nu}^{-1} \right], \quad (31) $$

We ignore the contributions from FSRQs in the following, since the FSRQs will have much smaller number density than other two sources at the relevant redshift of $z \gtrsim 1$. 

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where \( u_{\text{sat}}(x) \) is the number density profile of satellite galaxies normalized to \( \int u_{\text{sat}} \, dV = 1 \), and \( u_{\text{sat}} \) is the Fourier counterpart. We assume \( u_{\text{sat}} = \rho_{\text{newt}}/M \) throughout this paper. The kernel function of \( W_{\text{CIB}}(\chi) \) is given by

\[
W_{\text{CIB}}(\chi(z)) = \frac{1}{T_0} \left( \frac{\partial B_i}{\partial T} \right)_{T=T_0}^{-1} \frac{1}{1 + z}.
\]

### 4.3 Effective redshifts in cross-correlations of astrophysical sources and lensing

Before detailed computations, we shall show the effective redshift range to be probed by the cross-correlation between astrophysical sources at millimetre wavelengths and lensing convergence. For this purpose, we compute the mean intensity from cumulative emission from radio sources and CIB. For the \( i \)-th radio source (BL Lac object or steep-spectrum source), the mean intensity is given by

\[
I_{\nu,i}^R = \int_0^{\chi_{\nu,i}} \frac{d\chi}{1 + z} \int_{\ell_{\min}}^{\ell_{\max}} \frac{d\ell}{\ell} \frac{d\ell}{d\chi} \frac{L_{1,\nu,i}(\ell,1,4) \nu_{i}^{4}}{4\pi},
\]

while the CIB mean intensity can be computed as

\[
I_{\nu,i}^{\text{CIB}} = \int_0^{\chi_{\nu,i}} \frac{d\chi}{1 + z} \int_{M_{\min}}^{M_{\max}} \frac{dM}{d\chi} \frac{d\ell}{dM} \left[ f_{\text{f,con}}(M, z) + f_{\nu,\text{sat}}(M, z) \right].
\]

The top panel in Fig. 1 shows the redshift dependence in \( I_{\nu,i}^R \) at 100 GHz and \( I_{\nu,i}^{\text{CIB}} \) at 217 GHz and the bottom represents the lensing kernel \( W_{\chi}(\chi) \) for two lensing surveys of CFHTLenS (Van Waerbeke et al. 2013) and RCSLenS (Hojjati et al. 2017). In the top panel, the solid line shows the CIB intensity, while the red and blue lines are the intensity coming from BL Lac objects and steep-spectrum sources, respectively. To compute the coloured lines, we assume the flux cut of \( S_{\text{lim}} = 400 \, \text{mJy} \) at 100 GHz which is taken from table 1 in Planck Collaboration XXX (2014). First of all, the main contribution in CIB intensity will come from star-forming galaxies at higher redshift of \( z \approx 2 \), making the cross-correlation with galaxy lensing irrelevant. In contrast, the radio sources can have a sizable cross-correlation with large-scale structures at \( z \approx 0.2-0.4 \). Interestingly, BL Lac objects, minor population in radio flux counts at \( \approx 1 \, \text{GHz} \), can dominate a possible correlation with gravitational lensing in imaging survey. The model predicts BL Lac objects can be the main contributor to the mean intensity at lower redshifts \(( z < 1 )\) because of the flatness of their spectral index. We also see the impact of flux cut by comparing the red line with the dashed line in the top panel in Fig. 1. The flux cut in the Planck satellite can remove the BL Lac objects at \( z \approx 0.1, 0.1 \), but the objects at \( z \approx 0.2-0.4 \) will still survive in the observed temperature maps.

Furthermore, Fig. 2 represents the redshift derivative of the cross-power spectrum \( d\ln C_{\ell} / dz \), showing which redshift sources the

\[ C_{\ell} = \int_0^{\chi} \frac{d\chi}{\chi^2} W_{\chi}(\chi) W_{\nu,i}(\chi) \rho_{\text{newt}}^{-1} P_L(k = \ell/\chi, z) \]

where \( u_{\text{sat}}(x) \) is the number density profile of satellite galaxies normalized to \( \int u_{\text{sat}} \, dV = 1 \), and \( u_{\text{sat}} \) is the Fourier counterpart. We assume \( u_{\text{sat}} = \rho_{\text{newt}}/M \) throughout this paper. The kernel function of \( W_{\text{CIB}}(\chi) \) is given by

\[
W_{\text{CIB}}(\chi(z)) = \frac{1}{T_0} \left( \frac{\partial B_i}{\partial T} \right)_{T=T_0}^{-1} \frac{1}{1 + z}.
\]

### 5 RESULTS

#### 5.1 Frequency dependence on cross-correlations of astrophysical sources and lensing

The frequency dependence on the cross-power spectrum \( C_{\ell} \) in equation (8) is the key to understanding the observed tSZ-lensing correlation within our framework. Fig. 3 shows the cross-spectrum \( C_{\ell} \) at \( \ell = 500 \) as a function of frequency. Note that \( \ell = 500 \) roughly corresponds to 6–7 arcmin in angular scale and it is relevant for the tension between the observed y-x correlation and an expected signal from tSZ effect in ICM (Ma et al. 2015). In the top panel, the solid line represents the CIB-lensing power spectrum, while the dashed line is for the radio-lensing power spectrum from BL Lac objects. We here assume the HOD parameter \( M_{\text{cut}} = 9.65 \times 10^{13} \, M_\odot \) for BL Lac objects. In the bottom panel, we also plot the weight for construction of Compton y map. The black lines in the bottom panel correspond to the weights for extracting the tSZ effect from observed temperature maps, while the red line is the specific case so as to remove the tSZ effects in observed Compton y map. As shown in the figure, the CIB-lensing correlation has a steep spectrum and it becomes important if high-frequency maps are weighted for y-map construction. On the other
hand, the correlation with BL Lac objects and lensing convergence shows almost a flat spectrum but it will be likely to dominate the cross-correlation at ∼100 GHz. Since one needs a negative weight at 100 GHz with a large amplitude to obtain an unbiased estimate of the tSZ effect in practice (see the black line in the bottom panel), the radio-lensing correlation can induce a negative correction for observed tSZ-lensing correlation. In addition, we expect a non-zero cross-correlation arising from various terms in $C_{y-x}$ even if working with the weight to be $\sum w_i g(x_i) = 0$ as in red in the bottom panel.

5.2 Comparison of observed tSZ-lensing cross-correlation with our model

Let us then make a comparison of the observed tSZ-lensing cross-correlations (Van Waerbeke et al. 2014; Hojjati et al. 2017) with our model prediction. When predicting the $y-\kappa$ correlation in CFHTLenS, we set the Gaussian filter with the beaming size of 6 arcmin for lensing convergence and the FWHM of 9.5 arcmin for Compton $y$ map (Van Waerbeke et al. 2014). Similarly, we adopt the Gaussian filter with the FWHM of 10.0 arcmin for both $y$ and $\kappa$ maps in RCSLenS (Hojjati et al. 2017). Note that we apply the weight defined in CFHTLenS analyses for the RCSLenS prediction. This simplified procedure can induce a 10 per cent-level uncertainty in theoretical model, while it is less problematic under the current statistical uncertainty (Hojjati et al. 2017).

Fig. 4 summarizes the comparison with the observed correlation and our model prediction. The black points with error bar in the top panels show the observed correlation taken from Van Waerbeke et al. (2014) and Hojjati et al. (2017). In the top panels, the green dashed lines represent the expected correlation coming from tSZ effect in ICM, the red lines include the residual correlation from CIB, and the blue lines contain the residuals from CIB and radio sources. In the bottom panels, we show the ratio of the $y-\kappa$ correlations with residuals from CIB and/or radio sources to the tSZ-$\kappa$ correlation expected from ICM. In this figure, we assume that the flat-spectrum radio sources live in a massive host halo with the HOD parameter of $M_{\text{cut}} = 10^{15} \, h^{-1} \, M_\odot$.

Footnote: Planck Collaboration XXI (2014b) and Planck Collaboration XXIX (2014c) have also discussed similar effects; the negative response of the $y$-map weights to radio sources and the positive response due to dust emission.

Figure 3. Top: Our model of cross-power spectrum between brightness temperature in Planck bands and weak lensing as a function of frequency. The solid line in the top panel is the cross-power spectrum between CIB and galaxy weak lensing at multipole $\ell = 500$, while the dashed line shows the cross-power spectrum between flat-spectrum radio source and lensing. We assume the source redshift distribution of galaxy lensing measurement in CFHTLenS (Van Waerbeke et al. 2014). Bottom: Relative contribution from multiple frequencies in construction of Compton $y$ map. The black lines show the weight for construction of Compton $y$ map used for the measurement of tSZ-lensing correlation, while red represents the weight for the evaluation of residual foreground contamination in the tSZ-lensing correlation (Van Waerbeke et al. 2014).

Figure 4. Comparison of $y-\kappa$ correlation with recent measurements and our prediction. In the top three panels, the black points with error bar show the measurements of $y-\kappa$ correlation in CFHTLenS (Van Waerbeke et al. 2014) and RCSLenS (Hojjati et al. 2017). The top left and right-hand panels show the $y-\kappa$ correlations that are expected to be dominated in tSZ-lensing correlation, while the top middle panel represents the $y-\kappa$ correlation in the absence of tSZ effects. In the top panels, the green dashed line shows the expected correlation from the tSZ effects by ICM, the red lines include the residual correlation from CIB, and the blue lines contain the residuals from CIB and radio sources. In the bottom panels, we show the ratio of the $y-\kappa$ correlations with residuals from CIB and/or radio sources to the tSZ-$\kappa$ correlation expected from ICM. In this figure, we assume that the flat-spectrum radio sources live in a massive host halo with the HOD parameter of $M_{\text{cut}} = 10^{15} \, h^{-1} \, M_\odot$. 

(Downloaded from https://academic.oup.com/mnras/article/483/1/342/5203637 by university of winnipeg user on 04 March 2022)
of the tSZ effect, but the model with $M_{\text{cut}} = 10^{15} \, h^{-1} \, M_\odot$ is in better agreement with (see Fig. 5).

Can the radio sources be allowed to live in massive cluster-sized haloes with masses of $\sim 10^{15} \, h^{-1} \, M_\odot$ at redshift of $z = 0.2$–0.4? This is still an open question when we think of flat-spectrum radio sources like BL Lac objects, since the measurements of clustering of radio sources have been performed at frequency of $\sim$100 GHz where steep-spectrum sources should be abundant. When adopting the model with $M_{\text{cut}} = 10^{15} \, h^{-1} \, M_\odot$, we can infer the expected halo bias of flat-spectrum sources to be $\sim 5$–8 at $0 < z < 1$. This is found to be significantly higher than the halo bias of radio galaxies (e.g. Lindsay et al. 2014a; Lindsay, Jarvis & McAlpine 2014b; Allison et al. 2015; Nusser & Tiwari 2015) and radio-loud AGNs (e.g. Shen et al. 2009; Retana-Montenegro & Röttgering 2017). This finding is still in no contradiction to the existing clustering analyses of radio sources, since one can determine the halo bias of steep-spectrum sources alone from the clustering analyses in the literature.

6 CONCLUSION

In this paper, we studied the impact of astrophysical sources at multiple frequencies in the CMB measurement on the cross-correlation between tSZ effect and weak gravitational lensing effect. We developed a halo model to predict possible correlations between astrophysical sources and lensing convergence $\kappa$. Starting from an estimator of Compton $y$ map in the CMB measurement, we found the correlation between astrophysical sources and lensing effect in galaxy imaging survey may affect the observed $y$-$\kappa$ correlation. Assuming the best-fitting model of CIB to the recent observation, we evaluated the CIB-lensing correlation is less important for observed $y$-$\kappa$ correlation in current imaging surveys. In contrast, the radio sources with a flat-spectral index of $\sim0.1$ can be the main contributor to the observed extragalactic intensity at $\sim$100 GHz and the cross-correlation between such flat-spectrum sources and lensing induces a negative correction for the observed $y$-$\kappa$ correlation with a level of 20–30 per cent if the flat-spectrum sources could populate most massive dark matter haloes with masses of $\sim 10^{15} \, h^{-1} \, M_\odot$. Including possible negative corrections from radio-lensing correlation enables us to explain the observed $y$-$\kappa$ correlation without introducing strong AGN feedback or a small amplitude in linear matter density fluctuations.

A caveat in our model is that the model relies on the cross-correlation between lensing convergence and a minor population in radio bands. In fact, there are still large theoretical uncertainties in the modelling of the flat-spectrum radio sources (in particular of their HOD). To improve our understanding of the observed tSZ-lensing correlation, we require additional observational tests to learn about the radio sources at $\sim$100 GHz. The previous measurements of clustering in radio galaxies and AGNs would not be helpful to improve our understanding of the radio-lensing correlation by flat-spectrum sources, since most of clustering measurements are subject to another population called steep-spectrum sources. We expect that the measurement of cross-correlation of galaxy lensing with Compton $y$ map in the absence of tSZ effect [i.e. $\sum w_i g(x_i) = 0$ in equation (3)] is a possible approach to determine the statistical relationship between faint flat-spectrum radio sources and dark matter haloes (see also Shirasaki et al. 2018, for the study of clustering of BL Lac objects at gamma-ray frequencies). Such a measurement has a large statistical uncertainty at present, whereas ongoing and future imaging surveys can change the current situation, allowing to establish a precise theoretical framework to relate flat-spectrum radio sources with large-scale structures. It would be worth noting that future studies should investigate optimal weighting of temperature maps over different frequencies to search for the relationship between radio sources and their host haloes. The clustering analyses of flux-limited sample at $\sim$100 GHz should be a complementary approach, but the number density of flat-spectrum sources is evaluated to be of the order of $\sim 100 \, \text{str}^{-1}$ with the flux cut of 400 mJy at 100 GHz. Deeper observations at $\sim$100 GHz will enable us to study the HOD of flat-spectrum sources in detail by increasing their number density.

It is also important for precise measurement of $y$-$\kappa$ cross-correlation in the future to consider other systematic effects, for instance, the correlation between intrinsic alignment of satellite galaxies in galaxy clusters (e.g. Troxel & Ishak 2015) and the tSZ effect from the same clusters. Furthermore, it is worth exploring if lensing tomography can mitigate the impact of radio sources on observed $y$-$\kappa$ correlation and developing some approach to have an unbiased estimate of tSZ-lensing correlation by fully utilizing the frequency dependence on radio-lensing correlation. We will leave those for our future studies.

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Figure 5. Similar to Fig. 4, but this shows the dependence of the mass parameter in the HOD of flat-spectrum radio sources, denoted by $M_{\text{cut}}$. 

To place a meaningful constraint of $M_{\text{cut}}$, the current measurements would be insufficient due to the degeneracy among parameters in our model. To break the degeneracy among cosmology, the ICM, and residual components from CIB and faint radio sources, the tomographic tSZ-lensing correlation is expected to be essential. We will work on it in the near future.
I

\text{emission coefficient,} \ 
\text{where } d

\text{n}

\text{halo–halo correlation function with mass of } \rho

\text{halo,} \ 
\text{and } \rho

\text{x}

\text{redshift} \ 
\text{can be approximated as}

\sum

\text{M}_i

\text{xi} \ 
\text{being the density profile of a dark matter halo and } \rho

\text{h}

\text{is the halo mass function and two-point correlation of haloes}

\int

\text{d} \chi

\text{M}_x \ 
\text{on a halo-model approach (Cooray & Sheth 2002). The lensing}

\text{convergence } \kappa \ 
\text{and cumulative emission from astrophysical sources}

\text{at a given frequency } \nu \ 
\text{are defined as}

\int

\text{d} \chi

\text{W}_x(\chi) \ 
\text{is the halo mass function and } \xi

\text{M,M}

\text{1} \ 
\text{is the three-dimensional cross-power spectrum between matter overdensity } \delta

\text{x} \ 
\text{and the comoving specific emission coefficient } j_x(\chi) \text{ at redshift of } \chi.

\text{In the halo model, underlying matter density field at a given}

\text{redshift } \chi \text{ can be approximated as}

\rho

\text{x} \ 
\text{is the density profile of a dark matter halo and } \delta

\text{M}_x \ 
\text{is the } n\text{-dimensional Dirac delta function. In the following equations,}

\text{we omit the redshift } \chi \text{ for simplicity. Within the halo-model frame-}

\text{work, the halo mass function and two-point correlation of haloes}

\text{are defined as}

\sum

\text{M}_x \ 
\text{is the halo mass function and } \xi

\text{x}

\text{M,M}

\text{1} \ 
\text{is the three-dimensional cross-power spectrum between matter overdensity } \delta

\text{x} \ 
\text{and the comoving specific emission coefficient } j_x(\chi) \text{ at redshift of } \chi.

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\text{redshift } \chi \text{ can be approximated as}

\rho

\text{x} \ 
\text{is the density profile of a dark matter halo and } \delta

\text{M}_x \ 
\text{is the } n\text{-dimensional Dirac delta function. In the following equations,}

\text{we omit the redshift } \chi \text{ for simplicity. Within the halo-model frame-}

\text{work, the halo mass function and two-point correlation of haloes}

\text{are defined as}
Similar to equation (A4), we can express the comoving specific emission coefficient as
\[ j_\nu(x) = \sum_j J_{h,v}(x - x_j|M_i) \]
\[ = \sum_j \int dM \frac{\delta_0(M - M_i)}{M_i} \int d^3x' \delta_D^{(3)}(x' - x_j) J_{h,v}(x - x'|M), \]
(A7)

where \( J_{h,v} \) is the emission coefficient profile in single halo and we assume it depends on halo mass \( M \).

We then consider the three-dimensional correlation function of \( \rho_m \) and \( j_\nu \). The correlation function is defined as
\[ \tilde{\rho}_m \xi_{m-j_\nu}(x_1 - x_2) = (\rho_m(x_1) j_\nu(x_2)) - \bar{\rho}_m \overline{j_\nu}, \]
where \( \bar{\rho}_m \) is the mean matter density and \( \overline{j_\nu} \) is the spatially averaged emission coefficient. Using equations (A4)–(A7), one can decompose \( \xi_{m-j_\nu} \) into two parts: one is so-called one-halo term arising from the correlation in single haloes and another is two-halo term describing the correlation due to clustering of two distinct haloes.

The one-halo term of equation (A8) is given by
\[ \int dM \frac{dn}{dM} \int d^3y \bar{\rho}_m(x_1 - y) J_{h,v}(x_2 - y|M), \]
(A9)

while the two-halo term is
\[ \int dM \frac{dn}{dM} b(M) \int dM' \frac{dn}{dM'} b(M') \int d^3y \bar{\rho}_m(x_1 - y|M) \]
\[ \times \int d^3y' J_{h,v}(x_2 - y'|M) \xi_L(y - y'), \]
(A10)

where \( b(M) \) is the linear halo bias and \( \xi_L \) is the linear matter correlation function. In equation (A10), we assume \( \xi_L(x|M, M') = b(M) b(M') \xi_L(x) \). Hence, the final expression of equation (A8) is given by
\[ \xi_{m-j_\nu}(x_1 - x_2) = \xi_{m-j_\nu}^{1h}(x_1 - x_2) + \xi_{m-j_\nu}^{2h}(x_1 - x_2), \]
(A11)

\[ \xi_{m-j_\nu}^{1h}(x_1 - x_2) = \int dM \frac{dn}{dM} \int d^3y \frac{\bar{\rho}_m(x_1 - y|M)}{\bar{\rho}_m} J_{h,v}(x_2 - y|M), \]
(A12)

\[ \xi_{m-j_\nu}^{2h}(x_1 - x_2) = \int dM \frac{dn}{dM} b(M) \int dM' \frac{dn}{dM'} b(M') \]
\[ \times \int d^3y \int d^3y' \frac{\bar{\rho}_m(x_1 - y|M) \bar{\rho}_m(x_2 - y'|M)}{\bar{\rho}_m} \]
\[ \times J_{h,v}(x_2 - y|M) \xi_L(y - y'). \]
(A13)

The Fourier transform of equation (A11) is the three-dimensional power spectrum of \( P_{m-j_\nu} \), which is expressed as
\[ P_{m-j_\nu}(k) = P_{m-j_\nu}^{\text{1h}}(k) + P_{m-j_\nu}^{\text{2h}}(k), \]
(A14)

\[ P_{m-j_\nu}^{\text{1h}}(k) = \int dM \frac{dn}{dM} \frac{\tilde{\rho}_m(k|M) \overline{J_{h,v}(k|M)}}{\bar{\rho}_m}, \]
(A15)

\[ P_{m-j_\nu}^{\text{2h}}(k) = \left[ \int dM \frac{dn}{dM} b(M) \tilde{\rho}_h(k|M) \right] \]
\[ \times \left[ \int dM' \frac{dn}{dM'} b(M') \overline{J_{h,v}(k|M)} \right] P_L(k), \]
(A16)

where \( P_L(k) \) is the linear matter power spectrum, and \( \tilde{\rho}_h \) and \( J_{h,v} \) are the Fourier counterparts of \( \rho_h \) and \( J_{h,v} \), respectively.

To obtain the expressions in Section 4.2, we assume radio sources to be point sources and their HOD is expressed as an exponential form (see equation 23). We also assume the HOD is independent of redshift and radio luminosity. In this case, the emission coefficient profile of radio sources is given by
\[ \tilde{J}_\nu(k|M) = \tilde{J}_{\nu,R} S(M), \]
(A17)

\[ \tilde{J}_{\nu,R} = \int dL_{1.4} \frac{dn}{dL_{1.4}} \frac{L_{1.4}(L_{1.4})}{4\pi}, \]
(A18)

\[ S(M) = \exp \left( -\frac{M_{\text{cut}}}{M} \right) \left[ \int dM \frac{dn}{dM} \exp \left( -\frac{M_{\text{cut}}}{M} \right) \right]^{-1}, \]
(A19)

where \( L_{1.4} \) is the radio luminosity at 1.4 GHz, \( dn/dL_{1.4} \) is the luminosity function, and \( L_{1.4} + z_0(L_{1.4}) \) is given by equation (22).

For the CIB, we adopt the model in Shang et al. (2012):
\[ \tilde{J}_\nu(k|M) = f_{\nu,\text{cen}}(M) + f_{\nu,\text{sat}}(M) \tilde{u}_{\text{sat}}(k|M), \]
(A20)

where \( f_{\nu,\text{cen}}(M) \) represents the luminosity-weighted HOD of central galaxies, \( f_{\nu,\text{sat}}(M) \) is the HOD of satellites, and \( \tilde{u}_{\text{sat}}(k|M) \) is the Fourier transform of satellite number density profile. The details of luminosity-weighted HOD are found in Shang et al. (2012).

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