Implementation of Trained Factorization Machine Recommendation System on Quantum Annealer

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Abstract—Factorization Machine (FM) is the most commonly used model to build a recommendation system since it can incorporate side information to improve performance. However, producing item suggestions for a given user with a trained FM is time-consuming. To address this problem, we propose a quadratic unconstrained binary optimization (QUBO) scheme to combine with FM and apply quantum annealing (QA) computation. Compared to classical methods, this hybrid algorithm provides a fast sub-optimal sampling of good user suggestions. We then demonstrate the aforementioned computational behavior on current noisy intermediate-scale quantum (NISQ) hardware by experimenting with a real example on a D-Wave annealer.

Index Terms—Quantum annealing, Recommendation system, Factorization machine, Quantum machine learning

I. INTRODUCTION

Recommendation systems are widely used to predict the rating or preference of items for arbitrary users, such as suggesting books or movies, and also for many other business applications [1]–[4]. A common principle for constructing a recommendation system assumes that users prefer similar things based on past behavior. Amongst several approaches, the objective of Matrix Factorization (MF) is to find the factor matrices of historical data [5], [6]. Together with MF, other improvements such as SVD++ [7], pairwise interaction tensor factorization (PITF) [8], factorizing personalized Markov chains (FPMC) [9] and Monte Carlo on bipartite graphs [10] has also been comprehensively studied. These methods introduce specialized models to treat specific tasks but with the trade-off of losing generality.

To fix this, the Factorization Machine (FM) [11] is proposed as a more general supervised learning method, unifying MF, SVD++, PITF, and FPMC. With the capability to input arbitrary real-valued feature vectors to the model, including side information (e.g., gender and age), FM can be used for regression, classification, and ranking tasks. Moreover, it can be trained with linear complexity and accurately estimate model parameters from a sparse dataset, which makes FM highly competent for analyzing large datasets. Thus, it is more natural and adequate to utilize FM for real-world applications than the other methods mentioned earlier.

In recent years, quantum computing has become one of the most popular domains, which aims to benefit from the superposition nature of quantum states. Quantum Machine Learning (QML) employs parameterized quantum circuits as a neural network, which can then be applied to different classes of machine learning, such as Quantum Neural Network (QNN) [12]–[14], Quantum Convolutional Neural Network (QCNN) [15]–[17], Quantum Reinforcement Learning (QRL) [18]–[20], and Quantum Generative Adversarial Neural Network (QGAN) [21]–[25]. Besides the structural exploration, several works [26]–[34] have also investigated the learnability and trainability of QNN. Although the data encoding to a Hilbert space may give potential computational advantages, at the current stage of the NISQ era, only low-dimensional or small sample problems can be implemented on real-world hardware. Thus it is hard to justify the advantage experimentally, and this needs to be further studied [35].

The concept of a Quantum Recommendation System (QRS) has also been proposed [36], for which the idea is to sample an approximation of the preference matrix by quantum singular value estimation and quantum projection. In that framework, QRS takes $O(\text{poly}(k)\text{polylog}(mn))$ running time with $k$ the rank of the approximation and $mn$ the matrix dimension, while classically reconstructing an approximation of the preference matrix requires polynomial time $mn$. However, a classical
analogy of QRS, called Quantum-Inspired Recommendation System (QIRS), provides $O(\text{poly}(k)\log(mn))$ running time [37], which closes the gap between the classical and quantum methods. Besides gate-based quantum computing, it is possible to apply quantum annealing for feature selection [38] and use these features to train a classical ItemKNN content-based model [39]. This hybrid QPU solver shows good scalability for larger instances but may not have a significant advantage in the running time.

The performance of QRS or QIRS depends on a good low-rank approximation to the preference matrix [36], [37]. Moreover, since these algorithms do not incorporate side information, they may not be competitive with FM. Thus in the recommendation system domain, an approach to both utilize the side information and attain potential speedup is a crucial target for practical quantum computing applications.

In this work, we formulate a hybrid recommendation system by incorporating a classically trained FM with a quadratic unconstrained binary optimization (QUBO) scheme to be solved quantumly. Applying Quantum Annealing (QA) computation enables us to sample a large amount of sub-optimal solutions quantumly. Applying Quantum Annealing (QA) computation enables us to sample a large amount of sub-optimal suggestions in a short period of time compared to classical methods. Our algorithm will soon provide a computational advantage from the prospects of today’s developing NISQ hardware, such as a D-Wave annealer, for practical problems.

A. Main Results

Our main results in a both theoretical and experimental speedup of a QA-assisted FM recommendation system are summarized as follows:

- **Suggestion process as energy minimizing task:** We propose a QUBO scheme for the suggestion process of the FM recommendation system. The energy minimization task of the corresponding Ising problem is equivalent to finding the highest score (rating) candidates in the recommendation system.
- **NISQ advantage of Quantum Annealing in recommendation system:** Ideally, the quantum annealing process aims to yield the ground state of the Hamiltonian. However, the presence of noise and the limitation of annealing time on current NISQ hardware often leads to a multitude of sub-optimal solutions for the Hamiltonian. Interestingly, these sub-optimal solutions align quite well with the objectives of the recommendation system, which is designed to provide multiple suggestions rather than just the absolute best one.
- **Scalability:** The capability of solving fully-connected size-$N_p$ Ising problems of the most advanced D-Wave Advantage 4.1 system is $N_p = 145$. At the same time, the corresponding QUBO size of suggesting $N_m$ candidates is $\lceil \log_2 N_m \rceil$ in our formulation. Hence, theoretically, the solvable problem size $N_m$ scales up to $2^{145} \approx 10^{43}$ in today’s quantum hardware.

B. Related Work

Several works apply the QUBO scheme to the machine learning study. Date et al. [40] proposes QUBO formulations of linear regression, support vector machine (SVM), and balanced $k$-means clustering, where the required qubit number usually scales as $O(N_{\text{data}}^2)$, with $N_{\text{data}}$ the number of the data points of the training data. However, $N_{\text{data}}$ could go easily beyond $10^4$ in most machine learning applications, which is higher than the capability of the most advanced quantum system for solving an Ising problem: $N_{\text{data}} = 145$ from D-Wave [41]. One can transform an FM into a QUBO problem by binary encoding with a size equivalent to the length of the input vector. The works [42]–[45] have applied this idea in structural design problems to maximize the acquisition function associated to an FM. In this manner, the size of the QUBO problem is usually within a range that today’s hardware could reach. These examples open the possibility of applying such a QUBO-FM scheme to any FM-related research.

II. PRELIMINARIES

In this preliminary section, we review the key ingredients of our work: FM, QUBO, and Quantum Annealing (QA). FM is the most commonly used model in recommendation systems and machine learning. The QUBO scheme can apply to various combinatorial optimization problems, and QA is an effective optimization method for solving QUBO problems.

A. Factorization Machine

A factorization machine (FM) is a supervised learning algorithm that can be used for classification, regression, and ranking tasks [11]. Here we consider FM with degree $D = 2$, which involves only single and pairwise interactions of items. The model equation for an FM of degree $D = 2$ with variables

$$x = (x_1, \ldots, x_d)$$

is defined as:

$$y_{\text{fm}}(x) := w_0 + \vec{u} \cdot \vec{x} + \sum_{i<j} v_{ij} x_i x_j,$$

where $w_0 \in \mathbb{R}$ is the global bias, $\vec{u} = (u_1, \ldots, u_d) \in \mathbb{R}^d$ refers to the weights of each variables and

$$V = \begin{bmatrix} v_1 & \cdots & v_d \end{bmatrix} \in \mathbb{R}^{k \times d}$$

(2)

denotes the feature embeddings with $k$ the dimensionality of latent factors. The term $v_{ij}$ represents the interaction between the $i$-th and $j$-th terms.

We can use FM to construct a recommendation system: Separate the variables $\vec{x}$ into $x_i = u_i$ for $1 \leq i \leq n_u$ and $x_{i+n_u} = m_i$ for $1 \leq i \leq n_m$, so that the vector $\vec{u} = (u_1, \ldots, u_{n_u})$ denotes the information of a user and $\vec{m} = (m_1, \ldots, m_{n_m})$ represents the information of an item to be recommended, with the dimension relation $n_u + n_m = d$, this then turns $y_{\text{fm}}(\vec{x})$ into a predictor that estimates the score or rating for such a $(\vec{u}, \vec{m})$ pair.
The solution to a QUBO problem is a binary vector \( \bar{x}^* \) that minimizes the objective value \( y_{\text{QUBO}} \):

\[
\bar{x}^* = \arg \min_{\bar{x}} y_{\text{QUBO}}(\bar{x}).
\]

Note that one can transform a minimization problem into a maximization problem by adding a minus sign to the objective value. Moreover, it is possible to map a QUBO problem into an Ising problem [48] with the relation \( \sigma^z_i = 2x_i - 1 \). In particular, a QUBO problem is computationally equivalent to the following form:

\[
H_p = \sum_{i=1}^{n} h_i \sigma^z_i + \sum_{i<j} J_{ij} \sigma^z_i \sigma^z_j,
\]

where \( h_i \in \mathbb{R} \) and \( J_{ij} \in \mathbb{R} \) are the corresponding external field and coupling terms, and \( \sigma^z \) is the Pauli-\( z \) operator. Thus, we can solve a QUBO problem by solving its corresponding Ising form. This approach is achievable by energy minimization through adiabatic quantum computation, such as quantum annealing.

C. Quantum Annealing

Quantum Annealing (QA), formulated in its current form by T. Kadowaki and H. Nishimori [49], is a meta-heuristic approach capable of solving combinatorial optimization problems by utilizing the quantum mechanical nature of the associated physical systems. The QA process evolves from the ground state \( |\psi_i\rangle \) of an initial Hamiltonian \( H_i \) to the ground state \( |\psi_p\rangle \) of the problem Hamiltonian \( H_p \) (eq. (5)), where presumably one can easily prepare \( \{H_i, |\psi_i\rangle\} \). Mathematically, this adiabatic evolution can be described by the Hamiltonian in the following form, with \( s(t) \) evolving from 1 to 0 during a QA process:

\[
H(t) = s(t)H_i + (1 - s(t))H_p.
\]

For example, it is set to be \( H_i \propto \sum_{i=1}^{n} \sigma^z_i \) in D-Wave’s implementation [50]. With a proper annealing schedule \( s(t) \), probability of finding the ground state of \( H_p \) is close to 1 [51]. Thus, QA can search for the solution to a QUBO problem by studying the corresponding Ising form (eq. (5)). This approach has a wide range of applications for different problems mentioned in Sec. II-B [47].

III. TRAINED FACTORIZATION MACHINE AS QUBO

In this section, we derive the QUBO formulation of the suggestion process with a given trained FM and a fixed user. The key component here is that we will specialize the \( d \)-dimensional input vector \( \bar{x} = (\bar{u}, \bar{m}) \) in eq. (1) to be in the binary form where \( \bar{u} \in \{0, 1\}^{n_u} \) encodes a user and \( \bar{m} \in \{0, 1\}^{n_m} \) an item in the dataset, with the dimension relation \( n_u + n_m = d \).

Consider an FM recommendation system modeled by eq. (1), where the training process described in Sec. II-A determines the global bias \( w_0 \in \mathbb{R} \), the weight \( \bar{w} \in \mathbb{R}^d \) and the feature embedding \( \mathbf{V} \in \mathbb{R}^{k \times d} \). When assuming the input vector \( \bar{x} \) to be binary, we view eq. (1) as a QUBO
By summing over like terms, we further reduce this equation where the “offset” term in the last step is neglected as it does not affect the underlying optimization problem. Finally, we can map eq. (9) into an Ising model on the \( n_m \)-dimensional lattice as discussed in Sec. II-B:

\[
y_{\text{QUBO}}(\vec{x})|_{\vec{a}=\vec{a}^0} = \sum_{i=1}^{n_m} \tilde{h}_i \sigma_i + \sum_{i<j}^{n_m} \tilde{J}_{ij} \sigma_i \sigma_j. \tag{10}
\]

By calculating the ground states and possibly some low-lying excited states of the Ising problem with QA or other algorithms, we can transform the quest of producing high-rating recommendations for the trained FM into the task of finding low-lying energy states for the corresponding Ising problem.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

We use the well-known MovieLens-20M dataset [52] to demonstrate the effectiveness and behaviors of our hybrid algorithm, with \( \vec{u} \) and \( \vec{v} \) the binary encoding of userID and movieID, and \( y_{\text{fm}}(\vec{x}) \) the prediction of rating. Note that even though FM can utilize side information of users and movies, here we only use userID and movieID to train an FM. The dataset MovieLens-20M contains 20 million ratings of 27000 movies by 138000 users, and we use different fractions of this dataset and up to 6 million ratings to observe behaviors of our method.

A. Set Up

In our experiment, we first train an FM model with latent factors of dimension \( k = 200 \) and then transform it into the corresponding Ising problem for the suggestion process. However, there is an issue with binary encoding: the dimension of the solution space in quantum computing is typically more than the number of items (all possible movies for us) from the dataset. For the direct method, the solution space of the suggestion process is the number \( N_m \) of all the movies, while the dimension of the solution space for the QUBO method is \( 2^{\log_2 N_m} \). In this case, QUBO method may consider \( 2^{\log_2 N_m} - N_m \) binary vectors that is not in the movie list as solutions. To deal with this issue, we map all the computational basis vectors of the solution Hilbert space to our movie list (item dataset) by encoding the \( 2^{\log_2 N_m} - N_m \) higher-rating movies (items) in the dataset to 2 different binary vectors. Thus, any solution sampled by quantum annealing will produce one of the existing movies.

The FM in this case is trained by 6 million ratings for 41305 users and 21011 movies, so that the lengths for the user and movie part of the input vector are respectively \( n_u = \lceil \log_2 41305 \rceil = 16 \) and \( n_m = \lceil \log_2 21011 \rceil = 15 \). In particular, the QUBO/Ising problem size of the suggestion process for a fixed user is 15. We use the quantum annealer device D-Wave DW_2000Q_6, which has 2000 qubits and 6000 couplers.

B. Execution time of Quantum Annealing

By using different fractions of MovieLens-20M dataset as training data, Fig. 2(b) shows the execution time for direct method and QA for \( N_m \in \{2090, 8227, 11719, 13950, 16715\} \), where each
data point represents the average results of 5 randomly chosen users. We perform QA on the D-Wave DW\textsubscript{2000Q 6} QPU and the direct method on an Apple M1 Max chip with a 10-core CPU. Note that the upper limit of total execution time for quantum annealing is set to 1 second in the platform we used.

The execution time shows that QA is an efficient sampling method for suggesting candidates in the FM recommendation system. With some sacrifice of solution quality (Fig. 2(a)), this could be useful if \(N_m\) is extremely large and there is no efficient algorithm in the classical approaches.

C. Solution Quality

We use the overlapping rate (in percentage) as the standard of our search results. Here the overlapping rate in the top-\(k_s\) experiment is defined as the proportion of QA-captured overlapped movies in the top-\(k_s\) list from the direct method. Fig. 2(a) shows the overlapping rate of QA movie recommendation for both 100 measurement shots and 4000 measurement shots.

In Fig. 2(a), each data point represents the average of QA suggesting results from 100 randomly picked users, while a vertical strip represents the overlapping rate of each QA 4000/100-shot results for \(k_s \in \{10, 30, 50\}\). We observe that QA captures only a small amount of targeting movies from the 100-shot result but relatively more movies from the 4000-shot result.

On average, the overlapping rates are 51.4\%, 39.07\%, and 32.96\% from the top-10, top-30, and top-50 suggestions from the 4000-shot results. The overlapping rate is higher for a smaller \(k_s\), which is consistent with the principle that QA samples are more frequently lowly-excited states.

To the best of the authors’ knowledge, we can not use a typical Adiabatic Theorem (AT) to estimate our overlapping rates date. Denote \(H_{t_f}(t)\) the Hamiltonian governing a quantum annealing process with total evolution time \(t_f\) and \(H(s) = H_{t_f}(st_f)\) the normalized Hamiltonian with the dimensionless parameter \(s = t/t_f \in [0, 1]\). A typical AT asserts roughly that for all \(s \in [0, 1]\)

\[
\|P_{t_f}(s) - P(s)\| \leq O\left(\frac{C_H}{t_f \Delta}\right)
\]

where \(P_{t_f}(s)\) and \(P(s)\) are the spectral projections at time \(s\) to the eigenspace of the lowest \(m\) eigenvalues, with the spectrum separated by a gap \(\Delta = \min_s \{\Delta(s)\} > 0\), and \(C_H\) is a constant involving smoothness of \(H(s)\), cf. \cite[Section II]{51}. Consequently, one can conclude that the result of QA approximates nicely to the lowest \(m\) eigenstates when \(t_f \gg 0\). But note that \(t_f = 1\) second in the reality of applying the D-Wave platform, where the upper bound in eq. (11) is always larger than one and provides no information.

Due to the limitation of the execution time (=1 sec.), this brings up the necessity again to improve eq. (11). Our experiment provides possible evidence that the upper bound in eq. (11) under suitable conditions, as in the applications to the recommendation systems, can be significantly improved.

D. NISQ Advantage of Quantum Annealing in FM recommendation system

In the realm of quantum annealing, the ultimate goal is to achieve the ground state of the Hamiltonian. This state represents the optimal solution sought after for the given problem. However, noise and the inherent constraint of annealing time on present-day NISQ hardware pose significant challenges. Consequently, quantum annealing often generates a diverse set of sub-optimal solutions instead of obtaining a single, flawless solution.

Interestingly, these sub-optimal solutions align with the fundamental objectives of recommendation systems. Rather than merely providing the absolute best solution, recommendation systems are designed to present multiple suggestions to users. Although solutions generated by quantum annealing in NISQ hardware are not the absolute best, the sub-optimal ones offer a valuable range of choices for the recommendation system to propose to users. This assortment of suggestions can cater...
to individual preferences, diverse contexts, and the inherent subjectivity often present in decision-making processes.

Therefore, despite the inherent limitations and imperfections introduced by noise and annealing time constraints in NISQ hardware, the sub-optimal solutions provided by quantum annealing are unexpectedly well-suited for fulfilling the objectives of recommendation systems. This convergence of goals opens up exciting possibilities for leveraging quantum annealing in recommendation systems.

E. Effects of Annealing Parameters

Since QA involves several factors that could affect our experiment, to understand our algorithm’s properties better, we investigate different tunable parameters of the QA process to see the behavior of the results, where the energy scale of the quantum annealing process is in the order of $10^{-24}$ joule according to D-Wave [53]. We select four tunable parameters to investigate: “Annealing time”, “Programming thermalization”, “Readout thermalization” and “Shots”. According to the solver parameters document from D-Wave [54]: “Annealing time” is the time duration of the quantum annealing; “Programming thermalization” is the time to wait after programming the quantum annealer for it to cool back to a base temperature; “Readout thermalization” is the time to wait after each state is read from the quantum annealer for it to cool back to the base temperature; “Shots” is the number of states to read from the solver. We use the number of overlapping movies between top-100 suggestions from the direct method and QA to analyze the performance.

Fig. 3(a) investigates different parameters of programming thermalization and annealing time, where the number of overlapping movies shows no significant difference among these variations of parameters. One can also observe similar behavior in Fig. 3(b): the readout thermalization and annealing time could result in the thermal noise in the system, which is in the same order of magnitude as the thermal variation before and after the waiting time. For annealing time, these results show that the default value 20 µs is already sufficient for our QA process. Thus no improvement would be found with larger values. In Fig. 3(c), it is no surprise that results with more shots perform better since we have more chances to hit the high score candidates in these cases.

V. CONCLUSION AND FUTURE WORK

In this work, by transforming a trained FM from user dataset into a QUBO formulation, we construct a hybrid recommendation system based on solving the associated Ising problem via quantum annealing.

Theoretically, we first derive the QUBO formulation for the suggestion process and then describe the potential advantage of NISQ hardware quantum annealing in recommendation systems.

Experimentally, we apply our hybrid algorithm to the MovieLens-20M dataset to demonstrate the applicability of current NISQ hardware and analyze the solution quality. By encoding the whole solution space of QUBO to the movie list, we use the D-Wave quantum annealer to solve the corresponding optimization problem of the suggestion process. Our experiment shows that the top-10 suggestion has 51.4% average overlapping rate with that from the direct method and has the scaling behavior of the execution time matching our theoretical results.

Our future work could be searching for other use cases of FM and studying whether the quantum annealing approach is advantageous. Moreover, the limitation of the execution time of the current NISQ hardware leads to the necessity of further developing a finer math theory for improving the Adiabatic Theorem, which under suitable conditions, applies to the recommendation systems. Our numerical experiment provides evidence for the existence of such a theorem.

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REFERENCES

[1] J. Lu, D. Wu, M. Mao, W. Wang, and G. Zhang, “Recommender system application developments: A survey,” Decision Support Systems, vol. 74, pp. 12–32, 2015. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0167923615000627
[2] Y. Shi, M. Larson, and A. Hanjalic, “Collaborative filtering beyond the user-item matrix: A survey of the state of the art and future challenges,” ACM Comput. Surv., vol. 47, no. 1, may 2014. [Online]. Available: https://doi.org/10.1145/2536270
[3] F. Yang, X. Hu, and H. Liu, “Social recommendation: a review,” Social Network Analysis and Mining, vol. 3, no. 4, pp. 1113–1133, 2013. [Online]. Available: https://doi.org/10.1007/s13278-013-0141-9
[4] X. Yang, Y. Guo, Y. Liu, and H. Steck, “A survey of collaborative filtering based social recommender systems,” Computer Communications, vol. 41, pp. 1–10, 2014. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S0140366413001722
[5] F. Ricci, L. Rokach, and B. Shapira, Recommender Systems Handbook, F. Ricci, L. Rokach, B. Shapira, and P. B. Kantor, Eds. Boston, MA: Springer US, 2011.
[6] Y. Koren, R. Bell, and C. Volinsky, “Matrix factorization techniques for recommender systems,” Computer, vol. 42, no. 8, pp. 30–37, 2009.
[7] Y. Koren, “Factorization meets the neighborhood: A multifaceted collaborative filtering model,” in Proceedings of the 14th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, ser. KDD ’08. New York, NY, USA: Association for Computing Machinery, 2008, p. 426–434.
[8] S. Rendle and L. Schmidt-Thieme, “Pairwise interaction tensor factorization for personalized tag recommendation,” in Proceedings of the Third ACM International Conference on Web Search and Data Mining, ser. WSDM ’10. New York, NY, USA: Association for Computing Machinery, 2010, p. 81–90. [Online]. Available: https://doi.org/10.1145/1718487.1718498
[9] S. Rendle, C. Freudenthaler, and L. Schmidt-Thieme, “Factorizing personalized markov chains for next-basket recommendation,” in Proceedings of the 19th International Conference on World Wide Web, ser. WWW ’10. New York, NY, USA: Association for Computing Machinery, 2010, p. 811–820.
Fig. 3. The number of overlapping movies between top-100 suggestions from the direct method and QA for different annealing parameters. (a) Different programming thermalization and annealing time with measurement shots $= 2500$ and readout thermalization $= 0 \mu s$. (b) Different readout thermalization and annealing time with measurement shots $= 2500$ and programming thermalization $= 1000 \mu s$. (c) Different measurement shots and annealing time with readout thermalization $= 0 \mu s$ and programming thermalization $= 1000 \mu s$.

[10] Y. Rong, X. Wen, and H. Cheng, “A monte carlo algorithm for cold start recommendation,” in Proceedings of the 23rd International Conference on World Wide Web, ser. WWW ‘14. New York, NY, USA: Association for Computing Machinery, 2014, p. 327–336.

[11] S. Rendle, “Factorization machines,” in 2010 IEEE International Conference on Data Mining, 2010, pp. 995–1000.

[12] M. Schuld, I. Sinayskiy, and F. Petruccione, “The quest for a quantum neural network,” Quantum Information Processing, vol. 13, no. 11, pp. 2567–2586, 2014. [Online]. Available: https://doi.org/10.1007/s11128-014-0809-8

[13] M. V. Altaisky, “Quantum neural network,” 2001. [Online]. Available: https://arxiv.org/abs/quant-ph/0107012

[14] S. C. Kak, “Quantum neural computing,” Advances in Imaging and Electron Physics, vol. 94, pp. 259–313, 1995. [Online]. Available: https://www.sciencedirect.com/science/article/pii/S1076567008701472

[15] I. Cong, S. Choi, and M. D. Lukin, “Quantum convolutional neural networks,” Nature Physics, vol. 15, no. 12, pp. 1273–1279, 2019. [Online]. Available: https://doi.org/10.1038/s41567-019-0648-8

[16] Y. Li, R.-G. Zhou, R. Xu, J. Luo, and W. Hu, “A quantum deep convolutional neural network for image recognition,” Quantum Science and Technology, vol. 5, no. 4, p. 044003, jul 2020. [Online]. Available: https://doi.org/10.1088/2058-9565/ab993

[17] S. Y.-C. Chen, T.-C. Wei, C. Zhang, H. Yu, and S. Yao, “Quantum convolutional neural networks for high energy physics data analysis,” Phys. Rev. Research, vol. 4, p. 013231, Mar 2022. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevResearch.4.013231

[18] D. Dong, C. Chen, H. Li, and T.-J. Tarn, “Quantum reinforcement learning,” IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), vol. 38, no. 5, pp. 1207–1220, 2008.

[19] J.-Y. Hsiao, Y. Du, W.-Y. Chiang, M.-H. Hsieh, and H.-S. Goan, “Untangled quantum reinforcement learning agents in the openai gym,” 2022. [Online]. Available: https://arxiv.org/abs/2203.14348

[20] V. Dunjko, J. M. Taylor, and H. J. Briegel, “Advances in quantum reinforcement learning,” in 2017 IEEE International Conference on Systems, Man, and Cybernetics (SMC), 2017, pp. 282–287.

[21] P.-L. Dallaire-Demers and N. Killoran, “Quantum generative adversarial networks,” Phys. Rev. A, vol. 98, p. 012324, Jul 2018. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevA.98.012324

[22] S. Lloyd and C. Weedbrook, “Quantum generative adversarial learning,” Phys. Rev. Lett., vol. 121, p. 040502, Jul 2018. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevLett.121.040502

[23] J. Tian, X. Sun, Y. Du, S. Zhao, Q. Liu, K. Zhang, W. Yi, W. Huang, C. Wang, X. Wu, M.-H. Hsieh, T. Liu, W. Yang, and D. Tao, “Recent advances for quantum neural networks in generative learning,” 2022. [Online]. Available: https://arxiv.org/abs/2206.03066

[24] Y. Du, M.-H. Hsieh, and D. Tao, “Efficient online quantum generative adversarial learning algorithms with applications,” 2019. [Online]. Available: https://arxiv.org/abs/1904.09602

[25] H.-L. Huang, Y. Du, M. Gong, Y. Zhao, Y. Wu, C. Wang, S. Li, F. Liang, J. Lin, Y. Xu, R. Yang, T. Liu, M.-H. Hsieh, H. Deng, H. Rong, C.-Z. Peng, C.-Y. Lu, Y.-A. Chen, D. Tao, X. Zhu, and J.-W. Pan, “Experimental quantum generative adversarial networks for image generation,” Phys. Rev. Applied, vol. 16, p. 024051, Aug 2021. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevApplied.16.024051

[26] Y. Du, M.-H. Hsieh, T. Liu, S. You, and D. Tao, “Learnability of quantum neural networks,” PRX Quantum, vol. 2, p. 040337, Nov 2021. [Online]. Available: https://link.aps.org/doi/10.1103/PRXQuantum.2.040337

[27] M. Soltanolkotabi, A. Javanmard, and J. D. Lee, “Theoretical insights into the optimization landscape of over-parameterized shallow neural networks,” IEEE Transactions on Information Theory, vol. 65, no. 2, pp. 742–769, 2019.

[28] Y. Du, M.-H. Hsieh, T. Liu, and D. Tao, “A grover-search based quantum learning scheme for classification,” New Journal of Physics, vol. 23, no. 2, p. 023020, feb 2021. [Online]. Available: https://dx.doi.org/10.1088/1367-2630/abdefa

[29] ——, “Expressive power of parametrized quantum circuits,” Phys. Rev. Research, vol. 2, p. 033125, Jul 2020. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevResearch.2.033125

[30] K. Zhang, M.-H. Hsieh, L. Liu, and D. Tao, “Toward trainability of quantum neural networks,” 2020. [Online]. Available: https://arxiv.org/abs/2011.06258

[31] ——, “Gaussian initializations help deep variational quantum circuits escape from the barren plateau,” 2022. [Online]. Available: https://arxiv.org/abs/2203.09376

[32] ——, “Quantum gram-schmidt processes and their application to efficient state readout for quantum algorithms,” Phys. Rev. Research, vol. 3, p. 043095, Nov 2021. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevResearch.3.043095

[33] Y. Du, M.-H. Hsieh, T. Liu, S. You, and D. Tao, “Quantum differentially private sparse regression learning,” IEEE Transactions on Information Theory, vol. 68, no. 8, pp. 5217–5233, 2022.

[34] Y. Du, M.-H. Hsieh, T. Liu, D. Tao, and N. Liu, “Quantum noise protects quantum classifiers against adversaries,” Phys. Rev. Research, vol. 3, p. 023153, May 2021. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevResearch.3.023153

[35] R. Zhao and S. Wang, “A review of quantum neural networks: Methods, models, dilemma,” 2021. [Online]. Available: https://arxiv.org/abs/2109.01840

[36] I. Kerenidis and A. Prakash, “Quantum recommendation systems,” 2016. [Online]. Available: https://arxiv.org/abs/1603.08675

[37] E. Tang, “A quantum-inspired classical algorithm for recommendation systems,” in Proceedings of the 51st Annual ACM SIGACT Symposium on Theory of Computing, ser. STOC 2019. New York, NY, USA: Association for Computing Machinery, 2019, p. 217–228.

[38] R. Nembrini, M. Ferrari Dacrema, and P. Cremonesi, “Feature selection
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for recommender systems with quantum computing,” *Entropy*, vol. 23, no. 8, 2021.

[39] M. Deshpande and G. Karypis, “Item-based top-$i$ recommendation algorithms,” *ACM Trans. Inf. Syst.*, vol. 22, no. 1, p. 143–177, 2004.

[40] P. Date, D. Arthur, and L. Pusey-Nazzaro, “Qubo formulations for training machine learning models,” *Scientific Reports*, vol. 11, no. 1, p. 10029, 2021. [Online]. Available: https://doi.org/10.1038/s41598-021-89461-4

[41] D-Wave, “Upper limit for an arbitrary ising problem on d-wave,” https://github.com/aws/amazon-braket-examples/blob/main/examples/quantum_annealing,” n.d.

[42] K. Kitai, J. Guo, S. Ju, S. Tanaka, K. Tsuda, J. Shiomi, and R. Tamura, “Designing metamaterials with quantum annealing and factorization machines,” *Phys. Rev. Research*, vol. 2, p. 013319, 2020.

[43] ——, “Designing metamaterials with quantum annealing and factorization machines,” *Phys. Rev. Research*, vol. 2, p. 013319, Mar 2020. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevResearch.2.013319

[44] T. Matsumori, M. Taki, and T. Kadowaki, “Application of qubo solver using black-box optimization to structural design for resonance avoidance,” *Scientific Reports*, vol. 12, no. 1, p. 12143, 2022. [Online]. Available: https://doi.org/10.1038/s41598-022-16149-8

[45] K. Hatakeyama-Sato, T. Kashikawa, K. Kimura, and K. Oyaizu, “Tackling the challenge of a huge materials science search space with quantum-inspired annealing,” *Advanced Intelligent Systems*, vol. 3, no. 4, p. 2000209, 2021. [Online]. Available: https://onlinelibrary.wiley.com/doi/abs/10.1002/aisy.202000209

[46] S. Rendle, “Factorization machines with libfm,” *ACM Trans. Intell. Syst. Technol.*, vol. 3, no. 3, May 2012. [Online]. Available: https://doi.org/10.1145/2168752.2168771

[47] A. Lucas, “Ising formulations of many np problems,” *Frontiers in Physics*, vol. 2, 2014. [Online]. Available: https://www.frontiersin.org/articles/10.3389/fphy.2014.00005

[48] Z. Bian, F. Chudak, W. Macready, A. Roy, R. Sebastiani, and S. Varotti, “Solving sat and maxsat with a quantum annealer: Foundations, encodings, and preliminary results,” 2018. [Online]. Available: https://arxiv.org/abs/1811.02524

[49] T. Kadowaki and H. Nishimori, “Quantum annealing in the transverse ising model,” *Phys. Rev. E*, vol. 58, pp. 5355–5363, Nov 1998. [Online]. Available: https://link.aps.org/doi/10.1103/PhysRevE.58.5355

[50] D-Wave, “The Hamiltonian and the eigenspectrum,” https://docs.dwavesys.com/docs/latest/c\_gs\_2.html#the-hamiltonian-and-the-eigenspectrum,” n.d.

[51] T. Albash and D. A. Lidar, “Adiabatic quantum computation,” *Rev. Mod. Phys.*, vol. 90, p. 015002, Jan 2018. [Online]. Available: https://link.aps.org/doi/10.1103/RevModPhys.90.015002

[52] “Movielens-20m dataset,” https://grouplens.org/datasets/movielens/20m/,” 2015.

[53] D-Wave, “Annealing implementation and controls,” https://docs.dwavesys.com/docs/latest/c\_gpu\_annealing.html,” n.d.

[54] D-Wave, “Solver parameters,” https://docs.dwavesys.com/docs/latest/c\_solver\_parameters.html,” n.d.