Bilman, Deniz; Nabelek, Patrik; Trogdon, Thomas

Computation of large-genus solutions of the Korteweg-de Vries equation. (English)

Physica D 449, Article ID 133715, 26 p. (2023)

Summary: We consider the numerical computation of finite-genus solutions of the Korteweg-de Vries equation when the genus is large. Our method applies both to the initial-value problem when spectral data can be computed and to dressing scenarios when spectral data is specified arbitrarily. In order to compute large genus solutions, we employ a weighted Chebyshev basis to solve an associated singular integral equation. We also extend previous work to compute period matrices and the Abel map when the genus is large, maintaining numerical stability. We demonstrate our method on four different classes of solutions. Specifically, we demonstrate dispersive quantization for “box” initial data and demonstrate how a large genus limit can be taken to produce a new class of potentials.

MSC:
65-XX  Numerical analysis
35-XX  Partial differential equations

Keywords:
Korteweg-de Vries equation; Riemann-Hilbert problem; spectral method; hyperelliptic Riemann surface; finite-genus solutions

Full Text: DOI arXiv

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