A new 3-D chaotic system with four quadratic nonlinear terms, its global chaos control via passive control method and circuit design

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Abstract. This paper reports the finding of a new three-dimensional chaotic system with four quadratic nonlinear terms. The paper starts with a detailed dynamic analysis of the properties of the system such as phase plots, Lyapunov exponents, Kaplan-Yorke dimension and equilibrium points. Our new chaotic system is obtained by modifying the dynamics of the Zhu chaotic system (2010), and it has complex chaotic properties. As an engineering application, passive control method is applied for the global chaos control of the new chaotic system. Finally, an electronic circuit implementation of the new chaotic system is designed and implemented in MultiSIM. A good qualitative agreement has been shown between the MATLAB simulations of the new chaotic system and the MultiSIM results.

1. Introduction

Chaotic dynamical systems have applications in several areas of science and engineering [1-4]. Chaotic systems are highly complex and sensitive to even small changes in the initial conditions of the systems. Chaotic systems find applications in areas such as oscillators [5-10], ecology [11-12], biology [13-15], fuzzy logic [16-17], artificial neural networks [18-20], weather systems [21-22], chemical reactors [23-25], robotics [26-27], communication systems [28-30], circuits [31-36], finance systems [37-38], jerk systems [39-42], hyperjerk systems [43-45], etc.

In this research work, we have derived a new chaotic system by modifying the dynamics of the Zhu chaotic system ([46], 2010). Specifically, our new chaotic system has four quadratic nonlinear terms and it exhibits more chaotic properties than the Zhu chaotic system. By deriving Lyapunov exponents of the new chaotic system, we have done a detailed qualitative study of the new chaotic system such as its phase plots, maximum Lyapunov exponent, Kaplan-Yorke dimension, equilibrium points, etc.

As an engineering application, we have used passive control [47-48] for the global chaos control of the new chaotic system. We have established our main result using passivity-based control via Lyapunov stability theory.
The organization structure of this paper is as follows. Section 2 announces the new chaotic system with four quadratic nonlinear terms and details its qualitative properties. Section 3 details the global chaos control of the new chaotic system via passive control. Section 4 describes an electronic circuit design of the new chaotic jerk system using MultiSIM. Section 5 draws the main conclusions.

2. A new chaotic system with four quadratic nonlinearities

In 2010, Zhu, Lio and Guo [46] proposed a 3-D chaotic system given by the dynamics

\[
\begin{align*}
    \dot{x}_1 &= -x_1 - ax_2 + x_2 x_3 \\
    \dot{x}_2 &= bx_2 - x_1 x_3 \\
    \dot{x}_3 &= -cx_3 + x_1 x_2
\end{align*}
\]

(1)

In Eq. (1), \(x_1, x_2, x_3\) are the states and \(a, b, c\) are positive parameters. In [46], it was shown that the Zhu system (1) is chaotic for the choice of parameters \((a, b, c) = (1.5, 2.5, 4.9)\).

Using Wolf’s algorithm [49], the Lyapunov exponents of the Zhu system (1) are calculated for \((a, b, c) = (1.5, 2.5, 4.9)\) and \(X(0) = (0.2, 0.2, 0.2)\) for \(T = 1E4\) seconds as

\[
\begin{align*}
    L_1 &= 0.6616, & L_2 &= 0, & L_3 &= -4.0662
\end{align*}
\]

(2)

The Zhu system (1) is chaotic as it possesses a positive Lyapunov exponent in (2).

The Kaplan-Yorke fractal dimension of the Zhu system (1) is calculated as

\[
D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.1627
\]

(3)

The value \(D_{KY} = 2.1627\) describes the chaotic nature of the Zhu system (1).

In this paper, we propose a new chaotic system by modifying the dynamics of the Zhu system (1) and obtaining the following system:

\[
\begin{align*}
    \dot{x}_1 &= -a(x_1 + x_2) + x_2 x_3 \\
    \dot{x}_2 &= bx_2 - x_1 x_3 - dx_1^2 \\
    \dot{x}_3 &= -cx_3 + x_1 x_2
\end{align*}
\]

(4)

In Eq. (4), \(x_1, x_2, x_3\) are the states and \(a, b, c\) are positive parameters.

We show that the system (4) is chaotic for the parameter values

\[
    a = 1.2, \quad b = 3, \quad c = 5, \quad d = 0.1
\]

(5)

Using Wolf’s algorithm [49], the Lyapunov exponents of the new chaotic system (4) are obtained for the parameter values as in (7) and \(X(0) = (0.1, 0.1, 0.1)\) for \(T = 1E4\) seconds as

\[
\begin{align*}
    L_1 &= 0.7248, & L_2 &= 0, & L_3 &= -3.9289
\end{align*}
\]

(6)

Clearly, the new system (4) is chaotic since it has a positive Lyapunov exponent \(L_1\).

By adding all the Lyapunov exponents in (6), we get their sum as
Thus, the new chaotic system (4) is dissipative and it has a strange chaotic attractor.

The Kaplan-Yorke fractal dimension of the new chaotic system (4) is calculated as

$$D_{KY} = 2 + \frac{L_4 + L_2}{|L_3|} = 2.1845$$

(8)

The maximal Lyapunov exponent (MLE) of the new chaotic system (4) is found as $L_1 = 0.7248$, which is greater than the maximal Lyapunov exponent (MLE) of the Zhu chaotic system (1) found as $L_1 = 0.6616$. Similarly, the Kaplan-Yorke dimension of the new chaotic system (4) is seen as $D_{KY} = 2.1845$, which is greater than the Kaplan-Yorke dimension of the Zhu chaotic system (1) given by $D_{KY} = 2.1627$.

The equilibrium points of the new chaotic system (4) are calculated by solving the equations

$$-a(x_1 + x_2) + x_2x_3 = 0$$

(9a)

$$bx_2 - x_1x_3 - dx_1^2 = 0$$

(9b)

$$-cx_3 + x_1x_2 = 0$$

(9c)

Solving the system (9) for the chaotic case $(a, b, c, d) = (1.2, 3, 5, 0.1)$, we get five equilibrium points of the new chaotic system (4), listed as follows:

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, 
E_1 = \begin{bmatrix} 3.6173 \\ 3.4154 \\ 2.4709 \end{bmatrix}, 
E_2 = \begin{bmatrix} 4.5229 \\ -1.8744 \\ -1.6956 \end{bmatrix}, 
E_3 = \begin{bmatrix} -3.4097 \\ 1.7229 \\ -1.1749 \end{bmatrix}, 
E_4 = \begin{bmatrix} -4.2087 \\ -3.2639 \\ 2.7474 \end{bmatrix}$$

(10)

It is easy to verify that $E_0$ is a saddle point and other equilibrium points are saddle-foci.

Thus, all equilibrium points of the new chaotic system are unstable.

Figures 1-3 show the 2-D phase portraits of the new chaotic system (4), while Figure 4 shows the 3-D phase portrait. Figure 5 shows the Lyapunov exponents of the new chaotic system (4).
Figure 1. Two-dimensional phase plot of the new chaotic system (4) in the \((x_1,x_2)\) plane for 
\((a,b,c,d) = (1.2,3,5,0.1)\) and \(X(0) = (0.2,0.2,0.2)\)

Figure 2. Two-dimensional phase plot of the new chaotic system (4) in the \((x_2,x_3)\) plane for 
\((a,b,c,d) = (1.2,3,5,0.1)\) and \(X(0) = (0.2,0.2,0.2)\)

Figure 3. Two-dimensional phase plot of the new chaotic system (4) in the \((x_1,x_3)\) plane for 
\((a,b,c,d) = (1.2,3,5,0.1)\) and \(X(0) = (0.2,0.2,0.2)\)
Figure 4. Strange attractor of the new chaotic system (4) in $\mathbb{R}^3$ for $(a, b, c, d) = (1.2, 3, 5, 0.1)$ and $X(0) = (0.2, 0.2, 0.2)$

Figure 5. Lyapunov exponents of the new chaotic system (4) for $(a, b, c, d) = (1.2, 3, 5, 0.1)$ and $X(0) = (0.2, 0.2, 0.2)$

3. Global chaos control of the new chaotic system via passive control method

In this section, we exhibit an engineering application of the new chaotic system (4), namely global chaos control of the new chaotic system (4) via passive control method. The main result in this section is established via Lyapunov stability theory [50].

Thus, we consider the controlled chaotic system given by

$$
\begin{align*}
\dot{x}_1 &= -a(x_1 + x_2) + x_2 x_3 \\
\dot{x}_2 &= b x_2 - x_1 x_3 - d x_1^2 + u \\
\dot{x}_3 &= -c x_3 + x_1 x_2
\end{align*}
$$

(11)

In (11), $x_1, x_2, x_3$ are states and $u$ is a passive control to be determined.

We suppose that $x_2$ is the output of the new chaotic system (11), i.e. $y = x_2$.

We suppose also that $z_1 = x_1$ and $z_2 = x_3$.

Then the control system (11) can be expressed in normal form as follows:

$$
\begin{align*}
\dot{z}_1 &= -a z_1 + (z_2 - a) y \\
\dot{z}_2 &= -c z_2 + z_1 y \\
\dot{y} &= b y - z_1 z_2 - d z_1^2 + u
\end{align*}
$$

(12)

We express the system (12) in the standard form of passive control theory [48] as
\begin{equation}
\begin{aligned}
\dot{z} &= f_0(z) + p(z, y)y \\
\dot{y} &= \beta(z, y) + \alpha(z, y)u 
\end{aligned}
\end{equation}

where

\begin{equation*}
f_0(z) = \begin{bmatrix} -az_1 \\ -cz_2 \end{bmatrix}, \quad p(z, y) = \begin{bmatrix} z_2 - a \\ z_1 \end{bmatrix}, \quad \alpha(z, y) = 1, \quad \beta(z, y) = by - z_1z_2 - dz_1^2
\end{equation*}

(14)

Now, we consider the quadratic positive definite function defined by

\begin{equation*}
W(z) = \frac{1}{2}(z_1^2 + z_2^2)
\end{equation*}

(15)

We claim that \(W(z)\) is a Lyapunov function for the dynamics

\begin{equation}
\dot{z} = f_0(z)
\end{equation}

(16)

Taking the time-derivative of \(W\) along the dynamics (16), we get

\begin{equation}
\dot{W}(z) = \frac{\partial W}{\partial z} f_0(z) = -az_1^2 - cz_2^2
\end{equation}

(17)

this is a negative definite function on \(R^2\).

As in [48], we choose the storage function as

\begin{equation*}
V(z, y) = W(z) + \frac{1}{2}y^2
\end{equation*}

(18)

We note that \(V\) is a quadratic and positive definite function on \(R^3\).

Taking the time-derivative of \(V\) along the trajectories of the system (13), we get

\begin{equation}
\dot{V}(z) = \frac{\partial W}{\partial z} f_0(z) + \frac{\partial W}{\partial z} p(z, y)y + \left[ \beta(z, y) + \alpha(z, y)u \right]y
\end{equation}

(19)

A simple calculation gives

\begin{equation}
\dot{V} = -az_1^2 - cz_2^2 + y\left[ z_1(z_2 - a) - dz_1^2 + by + u \right]
\end{equation}

(20)

We take the feedback control \(u\) as

\begin{equation}
u = -z_1(z_2 - a) + dz_1^2 - (b + k)y
\end{equation}

(21)

where \(k > 0\) is a gain constant.

Substituting (21) into (20), we get

\begin{equation}
\dot{V} = -az_1^2 - cz_2^2 - ky^2
\end{equation}

(22)

this is a quadratic and negative definite function on \(R^3\).
Substituting $y = x_2$, $\dot{z}_1 = x_1$ and $\dot{z}_2 = x_3$, we can also express the passive control (21) as

$$u = -x_1(x_3 - a) + dx_1^2 - (b + k)x_2$$

(23)

By Lyapnov stability theory [50], we have established the following main result.

**Theorem 1.** The new chaotic system (11) is globally and exponentially stabilized for all initial conditions $x(0) \in R^3$ by the passive control $u$ defined by Eq. (23), where $k > 0$ is a gain constant.

For numerical simulations, we take the gain constant as $k = 20$. We take the parameter values as in the chaotic case, $v_i$.(a, b, c, d) = (1.2, 3, 5, 0.1).

As the initial conditions, we take $(x_1(0), x_2(0), x_3(0)) = (5.4, 12.8, 3.9)$.

Figure 6 shows the time-history of the trajectories of the controlled system (11), when the passive control $u$ given by Eq. (23) is activated.

![Figure 6. Time-history of the controlled chaotic system (11) for the initial state $X(0) = (5.4,12.8,3.9)$, parameter values $(a,b,c,d) = (1.2,3,5,0.1)$ and $k = 20$](image)

4. Circuit realization of the new chaotic system

In this section, we will design an electronic circuit for new three-dimensional chaotic system with four quadratic nonlinear terms. The circuit in Figure 7 is designed using an approach based operational amplifiers where the state variables $x_1$, $x_2$ and $x_3$ of new chaotic system (4) are associated with the voltages across the capacitors $C_1$, $C_2$ and $C_3$, respectively. Four multipliers (AD633JN), resistors, capacitors and five amplifiers (TL082CD) are used to design the system dynamics in the circuit.

In accordance with the circuit schematic, the new chaotic system (4) can be rewritten as follows:
\[
\begin{align*}
\dot{x}_1 &= -\frac{1}{C_1 R_1} x_1 - \frac{1}{C_1 R_5} x_2 + \frac{1}{10C_1 R_3} x_2 x_3 \\
\dot{x}_2 &= \frac{1}{C_2 R_4} x_2 - \frac{1}{10C_2 R_5} x_1 x_3 - \frac{1}{10C_2 R_6} x_1^2 \\
\dot{x}_3 &= -\frac{1}{C_3 R_7} x_3 + \frac{1}{10C_3 R_8} x_1 x_2
\end{align*}
\] (24)

MultiSIM results are shown in Figure 8 by choosing convenient values for resistors and capacitors: \(R_1 = R_2 = 333.33\ \text{k}\Omega, R_3 = R_5 = R_6 = 40\ \text{k}\Omega, R_4 = 133.3\ \text{k}\Omega, R_7 = 400\ \text{k}\Omega, R_8 = 80\ \text{k}\Omega, R_9 = R_{10} = R_{11} = R_{12} = 100\ \text{k}\Omega, C_1 = C_2 = C_3 = C_4 = 3.2\ \text{nF}\). Figure 8 presents MultiSIM results of the circuit, which show its chaotic behavior.

5. Conclusions

A new three-dimensional chaotic system with four quadratic nonlinear terms has been proposed in this paper. Numerical simulations have been used to illustrate that this system has a chaotic behavior. The dynamical properties of this new chaotic system were analyzed. As an engineering application, we have derived new results for the global chaos control of the new chaotic system by passivity based control. The designed circuit has been implemented and examined using the MultiSIM software to verify the simulation results.
Figure 7. The electronic circuit schematic of the new chaotic system (4)
Figure 8. Phase portraits of the MultiSIM simulation in the
(a) $x_1 - x_2$ plane, (b) $x_2 - x_3$ plane and (c) $x_1 - x_3$ plane

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