Scales and Cosmological Applications of M Theory

Karim Benakli
Phys. Dept. Texas A & M, College Station
TX 77843, USA.
E-mail: karim@chaos.tamu.edu

I review recent results in three topics of the M-world: (i) Scales. (ii) New dark matter candidates. (iii) Cosmological solutions from p-branes. The three topics are discussed in the framework of Hořava-Witten compactifications. Part (iii) includes comments on cosmological solutions in M-theory describing nucleation of universes through instanton effects and expansions toward asymptotically flat or anti-de-Sitter spaces.

1 Introduction:

Recent works have provided evidence for the existence of a fundamental theory (M-theory) unifying all known vacua of string theory as well as eleven dimensional supergravity. This allows us to hope that a mathematical structure unifying the algebraic structure of quantum mechanics and the geometrical approach of general relativity in a consistent way exists. M-theory in the way it is formulated today assumes that the law of quantum mechanics are valid at length scales far much smaller that those probed experimentally. At very short scales particles cease to be point-like and complicate dynamics of extended objects might lead to modifications of the form of the fundamental laws. However, mathematical consistency will not replace checking experimental predictions of M-theory to prove its relevance to the description of our world. Away from such an ambitious and probably premature program, it is useful to study the different phenomenologies that arise from different vacua of M-theory as they provide a new sight on problems of low energy physics (as the naturality of the structure and parameters of the standard model) and hopefully this will also have a feedback on mathematical physics.

Models with chiral $N = 1$ supergravity effective theory can be obtained by compactification of M-theory on orbifolds. The simplest example is the Hořava-Witten supergravity. In the infrared limit, the geometry is that of an eleven-dimensional space with two boundaries. The low energy effective theory in the bulk contains a graviton supermultiplet formed by the degrees of freedom of graviton, gravitino and a three form $C_{IJK}$ whose four-form field strength we denote by $G$. On each boundary there are nine-branes carrying an $E_8$ Yang-Mills supermultiplet. There are two scales appearing in this model: the eleven-dimensional Planck mass $M_{11}$ and the size of the $S^1/Z_2$ segment.
In section 2 we discuss the scales which govern the phenomenology arising from this theory in the case of compactification on non-singular Calabi-Yau manifolds. Many authors have explicitly discussed in the past the expected sizes for the different scales appearing in the Hořava-Witten model. However, they focused on the standard embedding in which case the segment $S^1/Z_2$ has a maximal length. This critical value depends on details of the compactification considered and thus was never computed explicitly. I believe it is useful to reconsider this issue in a more generic Calabi-Yau context. In section 3 we consider the possibility that dark matter originates on the hidden wall or in the bulk of the theory. We outline the main properties and possible signature. A detailed study will appear elsewhere. Section 4 is devoted to attempts to obtain simple cosmological solutions in M-theory. We first discuss a toy model in the Hořava-Witten supergravity context obtained by rotating an euclidean instanton to Lorentzian signature. This model exhibits two interesting features: spontaneous compactification and difference in early time behavior of the boundaries. We comment at the end on the possibility to generate solutions which describe asymptotically anti-de-Sitter spaces.

2 Scales revisited

One looks for a solution of the equations of motion of M-theory describing an eleven dimensional manifold, with boundaries separated by a segment of length $\pi \rho$, compactified on a Calabi-Yau six-fold $CY$ of volume $V$. Following Witten, this solution can be organized as an expansion in the dimensionless parameter $\rho M^{-3/2}/V^{2/3}$. At the lowest order our vacuum has the geometrical interpretation of $M_4 \times CY \times S^1/Z_2$. At higher orders in $\rho M^{-3}/V^{2/3}$ the factorization is lost and the volume of the Calabi-Yau space becomes a function of the coordinate parametrizing the $S^1/Z_2$ segment. More precisely, within the approximations of the volumes of $CY$ seen by the observable sector $V_o$ and the one on the hidden sector $V_h$ are given by:

$$V_o = V \left( 1 + \left( \frac{\pi}{2} \right)^{4/3} a_o \frac{\rho M^{-3}}{V^{2/3}} \right) \quad (1)$$

and

$$V_h = V \left( 1 + \left( \frac{\pi}{2} \right)^{4/3} a_h \frac{\rho M^{-3}}{V^{2/3}} \right) \quad (2)$$

\(^a\) The lengths are measured using the eleven-dimensional metric. 
\(^b\) We will use the subscripts $o$ for parameters of the observable sector and $h$ for those of the hidden sector.
thus

\[ V_h = V_o + \left( \frac{\pi}{2} \right)^{4/3} (a_h - a_o) M_{11}^{-3} V^{1/3} \rho \]  

(3)

where now \( V \) is the (constant) lowest order value for the volume of the Calabi-Yau manifold and \( a_{o,h} \) are model-dependent constants that we will discuss in some details below.

Due to non-trivial gauge configuration (“instanton” like) in the internal Calabi-Yau manifold, the \( E_8 \) gauge symmetries on the boundaries are replaced by the gauge groups \( G_o \) at \( x^{11} = 0 \) and \( G_h \) at \( x^{11} = \pi \rho \). We would like to consider the possibility of embedding the minimal supersymmetric standard model in this M-theory framework. Under the assumption of a desert scenario LEP data suggest unification at a scale \( M_{GUT} \sim 2 \cdot 10^{16} \text{ GeV} \). In an abuse of language, we will refer to the scale and group of unification as GUT scale and group while bearing in mind that there might be no new unifying gauge symmetry and \( G_o \) could be just the standard model group \( SU(3) \times SU(2) \times U(1) \). If \( M_{GUT} \gg V^{-1/6} \) then the theory becomes higher dimensional before reaching \( M_{GUT} \) and the four dimensional field theoretical prediction of unification is no more valid. In presence of twisted states arising from orbifold singularities, field theory is not reliable and one needs to use fully M-theoretical computations. Typically \( M_{GUT} \) is expected to be roughly of the order of the mass of the lightest heavy mode on the Calabi-Yau manifold. If we suppose that the standard model resides on the \( x^{11} = 0 \) wall, we have:

\[ M_{GUT} \sim cV_o^{-1/6} , \]  

(4)

where \( c \) is a model dependent constant. It might involve geometrical factors as well as contribution from the gauge symmetry breaking (Wilson lines...). However explicit computations in the weakly coupled heterotic string case seem to indicate that the natural value remains \( c \sim 1 \). A similar discussions holds for the scale associated with the hidden group \( G_h \) on the other side of the universe.

Three other quantities of phenomenological interest are the Newton constant \( G_N \) and the values of the coupling constants \( \alpha_o \) of \( G_o \) (that we wish to identify with the GUT group and its coupling constant) and \( \alpha_h \) of \( G_h \). They are given by

\[ G_N = \frac{1}{16\pi^2 M_{11}^2 \rho(V)} , \]  

(5)

\[ \alpha_o = (4\pi)^{2/3} \frac{1}{f_o V_o M_{11}^6} = (4\pi)^{2/3} \frac{1}{\rho} \left( \frac{M_{GUT}}{c M_{11}} \right)^6 = \alpha_{GUT} \]  

(6)
Here $\langle V \rangle$ is the average volume of the Calabi-Yau space. In the linear approximation of (1) and (2), we have:

$$\langle V \rangle = \frac{V_o + V_h}{2}$$  \hspace{1cm} (8)

The constant $f_o$ ($f_h$) is a ratio of normalization of the traces of adjoint representation of $G_o$ ($G_h$) compare to $E_8$ case. For a given compactification, $a_o,h, f_o,h$ and $c$ are taken to be fixed. The theory has then three free parameters $M_{11}, V_6$ and $\rho$ which in principle could be determined as to fit the experimental value of $G_N$ and the theoretical predictions for $M_{GUT}$ and $\alpha_{GUT}$. However some values of these parameters might lead to a negative $V_{h6}$ or a very small value of $\pi \rho$ and thus we will discard them as their physics is outside the domain of validity of our approximations.

From equations (6) and (5), using $\alpha_{GUT} = 1/25$, and $M_{GUT} = 2 \cdot 10^{16}$ GeV, one gets:

$$M_{11} \sim 2.2 \frac{M_{GUT}}{f_o^{1/6}c} = \frac{4.5 \cdot 10^{16}}{f_o^{1/6}c} \text{GeV}$$  \hspace{1cm} (9)

and

$$\rho\langle V \rangle \sim (1.9 \cdot 10^{16} \text{ GeV})^{-7}(f_o^{1/6}c)^9$$  \hspace{1cm} (10)

As $c \leq 1$ and $1.1 \leq f_o^{1/6} \leq 1.35$, this relation implies that $M_{11}$ is of the order of $M_{GUT}$ or a factor 2 larger.

In absence of fivebranes, the parameters $a_o$ and $a_h$ are given by:

$$a_o = \int_{CY} \omega \wedge tr(F_o \wedge F_o) - \frac{1}{2} tr(R \wedge R) \frac{1}{8\pi^2}$$  \hspace{1cm} (11)

and

$$a_h = \int_{CY} \omega \wedge tr(F_h \wedge F_h) - \frac{1}{2} tr(R \wedge R) \frac{1}{8\pi^2}$$  \hspace{1cm} (12)

where $\omega$ is a Kahler two-form of the Calabi-Yau manifold. To compute these quantities we need to specify the vacuum configurations of the gauge fields on $\rho \rho = 2.5, 3.75, 6$ for $E_6$, $SO(10)$ and $SU(5)$ respectively.

$\frac{c}{d} f_0 = 2.5, 3.75, 6$ for $E_6$, $SO(10)$ and $SU(5)$ respectively.
the two boundaries. This is done by specifying the gauge sheaves \( v_o \) and \( v_h \) for the two \( E_8 \)s.

The four form equations lead to the topological constraint that:

\[
\frac{1}{8\pi^2} (trF_o \wedge F_o + trF_h \wedge F_h - trR \wedge R)
\]

must vanish cohomologically. This means that the second Chern numbers associated with a four-cycle of the Calabi-Yau tangent bundle \( TM \) and the two gauge sheaves \( v_o \) and \( v_h \) must satisfy (the \( c_1 \)'s vanish):

\[
c_2(TM) = c_2(v_o) + c_2(v_h)
\]

In the models considered below this implies that:

\[
a_h = -a_o
\]

In this case the average volume of the Calabi-Yau manifold appearing in (3) is identical to the lowest order value \( \langle V \rangle = V \).

A detailed discussion of the different scales involves use of explicit values of the \( a_{o,h} \) parameters \( \epsilon \). As these are model dependent and need to be discussed.

### 2.1 Living on the edge with weakest gauge coupling constant

Let’s first discuss the situation where \( a_o > 0 \). This is the case for example of Calabi-Yau’s with standard and some the non-standard embeddings. In this case \( V_o > V_h \) and thus \( a_o < a_h \): the standard model gauge bosons and matter live on the wall of the universe with weakest coupling at the unification scale. It was observed by Witten that as \( V_o \) is fixed, the relation (3) implies that the coupling constant become strong and the volume of the Calabi-Yau on the hidden world becomes of the order of the Planck length and low energy supergravity approximation breaks down. Let’s estimate the different values of the scales of the theory at this critical point. Taking \( V_h = 0 \) we find that \( < v > = V = \frac{V}{2} \) and from (10) we get for the inverse size of the \( S^1/Z_2 \) segment:

\[
\frac{1}{\pi\rho_c} \sim \frac{M_{GUT}}{9.4c^3f^{3/2}} \sim \frac{2.1 \cdot 10^{15}}{c^3f^{3/2}} \text{GeV}
\]

For \( G_o = E_6 \) and taking \( c = 1 \) we obtain \( \frac{1}{\pi\rho_c} \sim 5.4 \cdot 10^{14}\text{GeV} \). Plugging the value (16) in (1) we get the necessary value for \( a_o \) to solve our set of constraints:

\[
a_o \sim 1.3
\]

\(^e\)All previous numerical studies have assumed \( a_o \sim 1 \) or \( a_0 = 0 \).
Away from the critical point, varying \( V \neq \frac{V_0}{2} \), it is easy to see that it is possible to solve our constraint if:

\[
a_o \leq a_{omax} \sim \frac{48}{(3125\pi^4)^{1/3}} \frac{M_3^{11}V_o^{5/3}}{\rho V} \sim 1.7
\]  

(18)

These are in agreement with previous observation that for \( a_o = 1 \) the desired value for \( \rho \) is close to its maximum \( \rho_c \).

The condition (18) is very constraining for model builders. We were not able to find simple examples satisfying this constraint.

Some words of caution:

(i) This solution with linear variation of the volume was derived assuming supersymmetry and lowest order factorization of the internal space as a segment times a compact internal. It should remain a good approximation as long as supersymmetry breaking scales (hidden and observable) are small compared to the size of the segment.

(ii) In the limit of \( \rho \to \rho_{max} \) the expansion parameter is large \( \rho M_3^{-3}/V^{2/3} \sim 1 \) and our expansion (and thus the numerical values obtained from it) can not be trusted. However, Witten has argued that there is a value of \( \rho \) where the size of the hidden world becomes the order of the Planck length.

(iii) The dynamics of the theory at \( \rho \sim \rho_{max} \) is still unknown.

2.2 Non-standard embedding: living on the edge with strongest coupling

An issue that has not been discussed previously in the litterature is the possibility of having the ordinary particles living on the wall with weakest coupling. Consider embedding the example of Calabi-Yau manifold of \([\mathbb{C}P^4]^{1,1,2,2,3,3}\) in the Ho\'rava-Witten model. This space is defined as the intersection of vanishing loci of two degree six equations in \( WCP_{1,1,1,1,1,1} \), space from which it inherits Kahler form \( J \) of the ambiant \( CP^4 \). The gauge vacuum configuration is specified by the gauge sheaves: \( v_o = (2,2,2;1,1,1,1,1) \) and \( v_h = (7;1,1,1,2,2) \) leading to an \( E_6 \) with three families in the observable sector and \( SU(5) \) with 54 families (!) in the hidden sector. For this model:

\[
a_o = (c_2(v_o) - \frac{1}{2}c_2(TM)) \int J \wedge J \wedge J = \frac{1}{2}(c_2(v_o) - c_2(v_h))) = -8 \ , \quad (19)
\]

and

\[
a_h = (c_2(v_h) - \frac{1}{2}c_2(TM)) \int J \wedge J \wedge J = -\frac{1}{2}(c_2(v_o) - c_2(v_h))) = 8 \ , \quad (20)
\]
The sign of $a_o$ and $a_h$ have been reversed compare to the previous example. The volume of the Calabi-Yau in the observable sector is now smaller that the one on the hidden wall. There is no maximal value of the length of the fifth dimension as we fix $V_o$ but let $G_N$ vary as was done in section 2.1. Some models might lead to very small $\rho$ and we might dismiss them. Solving our constraints leads to:

$$\frac{1}{\pi \rho} \sim 5.5 \cdot 10^{15}\text{GeV} \quad (21)$$

This toy example is not realistic as the the hidden sector coupling constant is small $\alpha_h \sim 0.001$ and the number of generations is huge. However we believe that this possibility merits more attention.

One can investigate the resulting scales for other examples of Calabi-Yau’s with standard and non-standard embedding as well as models with non-trivial $\pi_1$ or possibilities for $a_0 = 0$. Results will be given elsewhere.

2.3 Exotics I: Five-brane impurities:

The two coefficients $a_0$ and $a_h$ have opposite values. This is due to the constraint (13). In the presence of fivebranes this constraint becomes

$$\frac{\text{tr} F_o \wedge F_o + \text{tr} F_h \wedge F_h - \text{tr} R \wedge R}{8\pi^2} + \Sigma_i [C_i] \quad (22)$$

must vanish cohomologically where where $[C_i]$ is the Poincaré dual cohomology of the holomorphic curve $C_i$ on which the fivebrane is located. If on the fifth direction the five-branes are located on the boundaries then the relation (15) is still true. However if there an arbitrary set of fivebranes located in the bulk, the linear dependence in the bulk might change and the equations of motion should be solved for the new configuration.

2.4 Exotics II: Gauge symmetries in the bulk

While generically not present in the bulk, gauge symmetries might arise when the six dimensional internal manifold becomes singular if a small instanton is sitting on the singularity. As discussed there are two ways the $Z_2$ symmetry may act only in the fifth coordinate as it is the case in the rest of this paper. In that case the vector fields are odd. They are projected out and there is no new gauge symmetry at low energies. Neutral chiral multiplets are left in the boundaries and the gauge symmetry act on them as a global symmetry.

$^f$ In this case $\frac{1}{\pi \rho} \sim 4.2 \cdot 10^{15}\text{GeV}$. 

7
Another possibility is that the $Z_2$ projection is supplied with a freely acting $Z_2$ on the CY space one might be able to preserve the gauge symmetry in the bulk. There are three phenomenological scenarios that one might consider: (a) The ordinary matter lives on one boundary. (b) The ordinary matter lives in the bulk. Supersymmetry might be broken by non-perturbative effects on the boundaries. The two potentials might stabilize the moduli because of the difference dependences in the potentials. (c) Part of the ordinary matter is on the boundaries and part in the bulk.

One may also speculate about the possibility that the non-perturbative and the perturbative gauge groups mix. In that case after $Z_2$ projecting out the gauge bosons one may generates a “global” symmetry acting on the ordinary matter that might for example be used to solve the problem of fast proton decay. In the case (a) the gauge bosons in the bulk might play the role of mediators of supersymmetry breaking at low energies. While it is useful to keep in mind these possibilities, in absence of concrete models these remain pure speculations and we do not discuss further these issues.

3 Life on the edges and in the bulk of the M-world

3.1 Dark matter

Many astronomical observations seem to to indicate that most of the mass of the universe is invisible. While part of it might be simply vacuum energy (cosmological constant), a big mass (at least 40 times the one of the observed part) seems to be made of some “dark” matter. Here we would like to discuss the possibility of such matter arises from the hidden wall or the bulk of M-theory on $S^1/Z^2$. The early expansion of universe might have been different on the two boundaries. This anisotropy however disappears as it gets washed away by gravitational forces at late times. We suppose that after a period of inflation we are left with an isotropic universe with a single Hubble “constant” $H_0$ and a standard thermal history of the universe. The inflaton will decay into the ordinary particles and the hidden ones with a widths $\Gamma_o$ and $\Gamma_h$ respectively reheating the boundaries to temperatures $T_o \sim \Gamma_o^{1/2} G^{-1/4}$ and $T_h \sim \Gamma_h^{1/2} G^{-1/4}$.

The gravitational interactions are weak and do not thermalize the two baths. If $\Gamma_h \sim 0$ the hidden universe is cold, empty and (except for supersymmetry breaking) has no influence on the large scale structure of the universe.

\footnote{In the sense of not emitting photons but of course it is visible through gravitational effects.}

\footnote{There is the possibility that part of hidden matter masses keep quickly varying with time leading to anisotropies.}
As pointed out in agreement between the mass of earth computed from seismic data and the one computed from kinematics of its satellites on one hand and on the other hand difficulties to modify solar models, imply that the density of these particles is small in the close neighborhood of earth. However this does not exclude the possibility that hidden worlds provide a large fraction of the dark matter in the universe.

Let’s discuss (in a qualitative way) the properties of the particles generically present in the model and the possibility that they might provide us with dark matter candidates.

3.2 Particles living on the hidden wall:

Light hidden particles with masses less than $O(\text{MeV})$ would contribute to the effective degrees of freedom $g_{\text{eff}}$ that determines the energy density (thus the expansion rate) of the universe during the primordial nucleosynthesis period:

$$g_{\text{eff}} = g_s(1 + \frac{g_{s_h}}{g_s})(\frac{T_h}{T_0})^4|_{t=t_N}$$  \hspace{1cm} (23)

where $g_s$ and $g_{s_h}$ are the degrees of freedom of ordinary particles and hidden ones, $g_s = 10.75$. All the quantities are computed at the Big-Bang nucleosynthesis time $t_N$. Using $g_{\text{eff}} \leq 13$, one could arrive at a constraint on $T_h$ for specific models where $g_{s_h}$ is known or on $g_{s_h}$ if $T_h$ is known. This constraint leads in general to $T_h < T_0$. The possibility of such light particles being a large part of dark matter depends on their abundances which is very model dependent.

Heavy hidden particles might arise as bound states of a non-abelian group confining in the hidden wall. Their mass is expected to be roughly of the order of the scale of confinement:

$$\Lambda \sim V^{-1/6}_h e^{-b/\alpha_h}$$  \hspace{1cm} (24)

where $\alpha_h$ is given in (3).

Let’s suppose that in analogy with the case of proton in the observable world who are stable (or to the “cryptons” of weakly heterotic strings that have been claimed to have a lifetime bigger than the age of the universe), the lightest of the bound states $X$ is cosmologically stable. In it was suggested that discrete symmetries would be responsible of this stability in a similar way to the case of the proton.

If the confining gauge group is the same responsible for supersymmetry breaking then the mass of $X$, $m_X$ is roughly of the order of

$$m_x \sim \Lambda \sim 10^{11-14}\text{GeV}$$  \hspace{1cm} (25)
Consider the possibility that there is such state \( X \) weakly interacting and \( m_x \sim H_0 \). Weakly stands here for \( n_X (\sigma_A |v|) \leq H \), where \( n_X \) is the (conserved) co-moving number density, \( H \) is the Hubble constant and \( |v| \) is the Moller speed for the dark matter particles. Notice that the only constraint that might be hard to satisfy is stability and it is model dependent. Under these conditions \( X \) might have never been and will never go in local thermal equilibrium.

The computation of the abundance of such kind of particles have been recently reconsidered\(^{24, 25, 26}\). The numerical computations of\(^{25, 26}\) seem to indicate that interactions of the vacuum with the gravitational field might lead to the creation of the desired amount of \( X \) particles so that it constitute a large fraction of the mass of the universe.

The \( X \) particles do not decay into ordinary matter and thus can only be detected through their gravitational effects. Their experimental signature would be abundance of black holes and dark massive objects that might be detected in experiment based on gravitational lensing. We will return to these issues\(^4\).

3.3 Particles living in the bulk

In addition to the massless supergravity multiplet one has massive modes. In the models where the fifth dimension is much larger than the other six compactified ones, the lightest massive modes, the Kaluza-Klein (KK) states associated with the fifth dimension, have masses given by:

\[
m_n \sim \frac{n}{\rho} \sim n \cdot 10^{15-16}\text{GeV}
\]

Let’s first discuss the quantum numbers of these particles. Consider the \( N = 2 \) theory obtained through compactification of eleven dimensional supergravity on \( CY \times S^1/Z_2 \). Under the action of \( Z_2 \) the fields are divided into even ones

\[
\psi_{\text{even}} \rightarrow \psi_{\text{even}}
\]

which form multiples of an \( N = 1 \) supergravity four dimensional theory and odd ones

\[
\psi_{\text{odd}} \rightarrow -\psi_{\text{odd}}
\]

for the remaining of the fields. In particular all the vector fields on the bulk are projected out as they originate in eleven dimensions from the three form which is odd under \( Z_2 \).

Now at the level of massive states all the quantum numbers survive\(^5\). This

\(^{4}\)Physics of KK states in orbifolds has been investigated in\(^{27}\).
because translation invariance is broken in the fifth direction and propagating
states do not have a definite momentum but are of the form:

\[
\psi_{\text{even}} \times \left( \left| \frac{n}{\rho} > + \left| \frac{n}{\rho} > \right. \right. \right) \quad \text{and} \quad \psi_{\text{odd}} \times \left( \left| \frac{n}{\rho} > - \left| \frac{n}{\rho} > \right. \right. \right) \quad (29)
\]

We do not expect the bulk states to carry new quantum numbers that are
not present in the twisted sector. KK states are instable and they decay with
a lifetime:

\[ \tau_{\text{KK}} \sim \frac{\rho^3}{G_N} \quad (30) \]

for those which decay through gravitational strength interactions. One expect
a correction to the strength of the coupling constant roughly going as \( e^{-n/\rho^2} \)
due from the Euclidean action necessary to extract momenta from the vacuum
at the fixed points. However such corrections are small for large \( \rho \). For \( \rho \sim 10^{15} \)
GeV we find \( \tau_{\text{KK}} \sim 10^{-40} \) second. In the simplest class\( \mathbb{I} \) of compactifications
of Hořava-Witten theory the bulk does not provide natural candidates for dark
matter.

Finally I will end this section by a comment on extreme cases. If the seven
dimensional internal space is very deformed with the Calabi-Yau characteristic
length becoming large at some point of the segment or anisotropic, the massive
modes are described by localized states as no translation invariance remains
along the fifth direction.

\[ \text{3.4 View from Dr. Jekyll’s and Mr. Hyde’s powerful microscopes} \]

We consider the geometry of space-time when we increase the probe energy as
seen by Dr. Jekyll living on the observable world and Mr. Hyde living in the
hidden world.

At low energies the world is four dimensional. Both Dr. Jekyll and Mr.
Hyde are not aware of existence of higher dimensions. All observations in
nature are described with four dimensional representations and parameters.
But the latter do not seem neither simple nor natural. Interactions between
the two worlds are with gravitational strength. Only at the cosmological scales,
where all the other interactions are screened, that the two universes influence
each other.

With the increase the probe energy and luminosity of his particle colliders
Dr. Jekyll studies some very energetic process where (thanks to the very
good precision of his instruments) he begins to observe deviations from the

\[ \text{\footnote{This is not the case if non perturbative gauge symmetries arise live in the bulk(see section 2.4).}} \]
predictions of the minimal supersymmetric model. To explain the observations
he needs to take into account gravitational forces. If the precision is better
than $10^{-14}$, this happens at energies around $10^{14}$ GeV.

Time to time, if in the hidden world there are particles with lifetime long
enough to escape, he observes some missing energy events. He has created
particles on the hidden wall. Through his “gravitational detectors” he can
observe the new particles.

At energies of the order of $10^{15}$ GeV Dr. Jekyll observes that the strength
of the gravitational interactions grows faster. The rate of production of hidden
particles increases. He realizes that the world is five-dimensional. This puts
some order in the parameters of his models: they look more natural.

At energies of the order $10^{16}$ GeV all the interactions have the same
strength. New massive modes begin to contribute to the interactions, the
world looks eleven dimensional...If the precision of Dr Jekyll’s apparatus is
worse than $\sim 10^{-4}$, it is only very close to $10^{16}$ GeV energy scales that he
finds out that he needs to include gravitational effects and these should be
in a five-dimensional space-time.

Depending of the strength of the interactions weaker or stronger in the
hidden world, Mr. Hyde (if stable) might have been aware of the eleven di-
mensions before or after Jekyll respectively.

4 Cosmological solutions:

4.1 Generating cosmological solutions from p-branes:

A simple way to generate cosmological solutions is to start with p-brane solu-
tions in D-dimensions. These lead to a space with a metric of the form:

$$ds_{D}^{2} = ds_{p+1}^{2} + ds_{D-p-1}^{2}$$

The $ds_{p+1}^{2}$ is the world-volume part: the space-time swept by the brane
when it propagates. It contains a time coordinate. Our first step is to Eu-
clideanize this coordinate and may be (if we like so) compactify this space.
We have then an Euclidean solution in D-dimensions. Next we remark that
the metric contains factors with explicit dependence on the coordinate of the
transverse space. If the p-brane solution has a “spherical” symmetry in a part
of the transverse space then the dependence of the metric will be on the radial
coordinate associate with this space. Thus our second space is to rotate this
radial coordinate into a time-like direction. We have thus generated a solution
with time dependent metric. To get a time which varies from $-\infty$ to $+\infty$ one
needs to make a simple changement of variables. As p-branes metrics interpo-
late between the horizon geometry and some space-time geometry at infinity,
our process generates a cosmological solution that interpolates in time between the two geometries. We would like to discuss briefly some examples of such solutions in M-theory. A long list of references on cosmological solutions can be found in [21].

4.2 An example in M-theory on $S^1/Z_2$

One starts with a five-brane solution of the Hořava-Witten M-theory [22, 29, 30]. The metric on this space is of the form:

$$ds^2 = e^{2A} \left(-dt^2 + dx^\mu dx^\nu \delta_{\mu\nu}\right) + e^{2B} \left(dr^2 + r^2 \left(d\phi^2 + \sin^2(\phi) \left(d\theta^2 + \sin^2(\theta) d\omega^2\right)\right)\right) + e^{2B} (dx^{11})^2,$$

where the functions $A, B$ are related by $A = -B/2 = -C/6$ and the four-form $G$ is given by:

$$G_{mnrs} = \pm \frac{1}{\sqrt{2}} e^{-\frac{2A}{3}} \epsilon_{mnrs} \partial_t e^C$$

(32)

with $m, n, ..., 6, ..., 9, 11$. The function $C$ depends only on $x^{11}$ and $r$. On the boundaries the four-form gets contributions from the gauge vacuum configuration. More precisely:

$$G_{ABCD} \bigg|_{x^{11}=0} = -\frac{3}{\sqrt{2}} \frac{k^2}{\lambda^2} \text{tr}(F^{(1)}_{AB} F^{(1)}_{CD})$$

$$G_{ABCD} \bigg|_{x^{11}=\pi r} = +\frac{3}{\sqrt{2}} \frac{k^2}{\lambda^2} \text{tr}(F^{(2)}_{AB} F^{(2)}_{CD})$$

(34)

Notice that the solution is independent on the time $t$ coordinate and the latter can be taken to be compact. Moreover a rotation to Euclidean space leads to a theory with real parameters in the action. By rotating the radial coordinate into a time-like direction we can achieve the change of variables:

$$t = \sqrt{q} x^0, \quad \phi = \sqrt{q} \chi, \quad r = \frac{\tau}{\sqrt{q}}$$

(35)

on the original solution, where $q$ is taken from $+1$ to $-1$. This leads to a cosmological solution of the form $[k]$. 

$^k \kappa^2 = M_p^2$ and $\frac{2\sqrt{q}}{\lambda} = (4\pi)^3$ 

$^l$This solution corresponds to a strong heterotic string coupling limit of a solution considered in [24].
\[ ds^2 = \Phi^{-1/3} ((dx^0)^2 + dx^\mu dx^\nu \delta_{\mu\nu}) + \Phi^{2/3} (d\tau^2 + \tau^2 d\Omega^2_{3,-1}) + \Phi^{2/3} (dx^{11})^2, \]  

with

\[ d\Omega^2_{3,k} = d\chi^2 + \sin^2 \left( \sqrt{k} \chi \right) \left( d\theta^2 + \sin^2 \theta d\omega^2 \right). \]

In (36) \( \Phi \) is given by:

\[ \Phi = \Phi_0 + \phi, \]

where \( \Phi_0 \) is a function of \( \tau \) only:

\[ \Phi_0 = 1 + \frac{2\kappa^2}{\pi \rho \lambda^2} \left( \frac{2\sigma_1^2 + \tau^2}{(\sigma_1^2 + \tau^2)^2} + \frac{2\sigma_2^2 + \tau^2}{(\sigma_2^2 + \tau^2)^2} \right), \]

and \( \phi \) is the perturbation dependent on \( x^{11} \). It can be written as:

\[ \phi = \sum_{n=0}^{n=\infty} \phi_n \]

The leading order of the expansion is given by:

\[ \phi_0 = 48 \frac{\kappa^2}{\lambda^2} \left( P_0(x^{11}) \frac{\sigma_1^4}{(\sigma_1^2 + \tau^2)^4} + Q_0(x^{11}) \frac{\sigma_2^4}{(\sigma_2^2 + \tau^2)^4} \right) \]

where:

\[ P_0 = -\frac{(x^{11})^2}{4\pi \rho} + \frac{x^{11}}{2} - \frac{\pi \rho}{6}, \quad Q_0 = -\frac{(x^{11})^2}{4\pi \rho} + \frac{\pi \rho}{12} \]

To have a time coordinate which goes from \(-\infty\) to \(+\infty\) one makes the rescaling \( \tau = e^\eta \). The choice \( \tau = e^{-\eta} \) lead to a similar branch disconnected from this one. A four dimensional solution is obtained by considering the six-dimensional transverse space parametrized by \( x^0, \ldots, x^5 \) as compact.

For \( \tau \to 0 \) or equivalently \( \eta \to -\infty \) the size of the compact internal space is finite. However the radius of the pseudo-sphere of the universe goes to zero. Through tunneling effect a bubble with a new expectation value of the field \( \Phi \) (the dual of the four-form \( G \)) is created at \( \eta \to -\infty \) and expands. This event is at finite proper time from any point in the future. If one start with different initial conditions on the boundaries (\( \sigma_1 \neq \sigma_2 \)), the expansion on the two boundaries follows different time dependence. On the \( x^{11} = 0 \) boundary:
\[ \Phi = 1 + \frac{2\kappa^2}{\pi \rho \lambda^2} \left( \frac{2\sigma_1^2 + \tau^2}{(\sigma_1^2 + \tau^2)^2} + \frac{2\sigma_2^2 + \tau^2}{(\sigma_2^2 + \tau^2)^2} \right) \\
+ 8\frac{\kappa^2 \pi \rho}{\lambda^2} \left( \frac{\sigma_1^4}{(\sigma_1^2 + \tau^2)^4} + \frac{\sigma_2^4}{2(\sigma_2^2 + \tau^2)^4} \right) + \ldots, \] (43)

while for the hidden universe \( x_{11} = \pi \rho \):

\[ \Phi = 1 + \frac{2\kappa^2}{\pi \rho \lambda^2} \left( \frac{2\sigma_1^2 + \tau^2}{(\sigma_1^2 + \tau^2)^2} + \frac{2\sigma_2^2 + \tau^2}{(\sigma_2^2 + \tau^2)^2} \right) \\
+ 8\frac{\kappa^2 \pi \rho}{\lambda^2} \left( \frac{\sigma_1^4}{2(\sigma_1^2 + \tau^2)^4} - \frac{\sigma_2^4}{(\sigma_2^2 + \tau^2)^4} \right) + \ldots, \] (44)

where the dots represent higher orders of the expansion in \((\pi \rho)^2/\sigma^2\).

At later times as \( \tau \to +\infty \) or equivalently \( \eta \to +\infty \), we find \( \Phi \to 1 \). Thus the solution describes an expanding four dimensional flat universe. The expansion anisotropy between the two boundaries is washed away by gravitational interactions between them. It is also interesting to notice that the volume of the internal space and the size of the segment remain finite (small) and thus we have a mechanism of spontaneous compactification.

Our field \( \Phi \) (obtained through dualization of the four form) or more precisely \( \Phi_0 \) would the analogous of the scalar field considered in [31].

We have illustrated the possibility of having different initial conditions on the two boundaries of the universe which might lead to different expansions at early time.

4.3 Other examples of nucleation of universe in M-theory

In the previous example the asymptotic geometry as \( \tau \to \infty \) is a flat space-time. Nucleations of universes that are described through rotating instanton solutions in Euclidean space-time into a tunneling effect in spaces with Lorentzian signature have attracted a lot of interest recently [31].

I would like to comment on the possibility to obtain solution which at late time become asymptotically an anti-de-Sitter space. Details will be provided in [32].

To simplify the discussion, consider an extremal p-brane solution in a \( D \)-dimensional Minkowski space. The metric takes the form:

\[ ds^2 = H^\alpha dx_{\mu}^\alpha dx_{\mu} + H^\beta dx_{T}^\alpha dx_{T}^\alpha \] (45)
where $x_1$ and $x_T$ are the coordinates of the parallel and transverse spaces to the world-volume of the p-brane. The harmonic function $H$ depends on the radial coordinate of the space transverse to the p-brane. Generically

$$H \sim h + \frac{c}{r^{\tilde{d}}}$$

(46)

where $\tilde{d} = D - p - 3$, $c$ is related to the charge carried by the p-brane and $h$ is a constant of integration taken to be $h = 1$ so that the asymptotical geometry as $r \to \infty$ is a flat Minkowski space-time.

Recently it has been argued that there is a set of transformations that shifts the value of $h$. Starting with $h = 1$ these transformations lead to the metric with the difference that now $h = 0$. This allows us to get solutions which asymptotically as $r \to \infty$ go to an anti-de-Sitter geometry. After exchange of the role of time and transverse radial coordinate, we obtain cosmological solutions expanding toward anti-de-Sitter space. Some of these solutions have instanton interpretation in a similar way to the example of the previous subsection.

One could have instead started with a solution on a curved background. For instance one could transform the membrane solution and generate a new cosmological solution: at time goes to infinity the space-time geometry goes to four-dimensional anti-de-Sitter space with the membrane at the end of the universe.

5 Conclusions

We have described three issues related to cosmological applications of M-theory. We have investigated in great details the question of scales as it is necessary prior to any discussion of physical phenomena. The geometry of the space-time is a product of a flat four dimensional Minkowski space-time and a seven dimensional compact space. The latter is approximated as a fibration of a Calabi-Yau space on a segment. The volume of the Calabi-Yau varies slowly (as to satisfy the adiabatic principle) and the theory describes strongly coupled limit of heterotic string theory. A linear approximation was used to extract orders of magnitude of the characteristic sizes for the Calabi-Yau space and the fifth dimension. We investigated the possibilities that three possibilities that $\alpha_o < \alpha_h$, $\alpha_o > \alpha_h$ and $\alpha_o = \alpha_h$. Only in the first class we found that (if the value of the Calabi-Yau volume on the observable boundary is fixed) the length of the fifth dimension has a maximal value.

We discussed the possibility that dark matter originates from the bulk or on the hidden wall of the theory. We found that the former is unlikely while
the latter is an open possibility that needs to be investigated in more details.

We discussed some model-independent properties of this particles.

Finally we presented a toy model for a cosmological solution describing an instanton effect in M-theory on $S^1/Z_2$ framework. This has the important features that it exhibits an example for a scenario where internal dimensions remain small while the four dimensional space expands. Moreover the expansion is anisotropic in the fifth direction, a situation we believe is generic to Hořava-Witten class of models.

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