Impact of Line-of-Sight and Unequal Spatial Correlation on Uplink MU-MIMO Systems

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Abstract—Closed-form approximations of the expected per-terminal signal-to-interference-plus-noise-ratio (SINR) and ergodic sum spectral efficiency of a multiuser multiple-input multiple-output system are presented. Our analysis assumes spatially correlated Ricean fading channels with maximum-ratio combining on the uplink. Unlike previous studies, our model accounts for the presence of unequal correlation matrices, unequal Rice factors, as well as unequal link gains to each terminal. The derived approximations lend themselves to useful insights, special cases and demonstrate the aggregate impact of line-of-sight (LoS) and unequal correlation matrices. Numerical results show that while unequal correlation matrices enhance the expected SINR and ergodic sum spectral efficiency, the presence of strong LoS has an opposite effect. Our approximations are general and remain insensitive to changes in the system dimensions, signal-to-noise-ratios, LoS levels and unequal correlation levels.

Index Terms—Ergodic sum spectral efficiency, expected SINR, line-of-sight, MU-MIMO, unequal correlation.

I. INTRODUCTION

The lack of rich scattering and insufficient antenna spacing at a cellular base station (BS) leads to increased levels of spatial correlation [1]. For multiuser multiple-input multiple-output (MU-MIMO) systems, this is known to negatively impact the signal-to-interference-plus-noise-ratio (SINR) of a given terminal, as well as the sum spectral efficiency of the system. Numerous works have investigated the SINR and spectral efficiency performance of MU-MIMO systems with spatial correlation (see e.g., [2–4] and references therein). However, very few of the above mentioned studies consider the effects of line-of-sight (LoS) components, likely to be a dominant feature in future wireless access with the rise of smaller cell sizes [5]. Thus, understanding the performance of such systems with Ricean fading is of particular importance.

The uplink Ricean analysis presented in [6] does not consider the effects of spatial correlation at the BS. On the other hand, the related literature (see e.g., [3, 7]) routinely assumes that on the uplink, all terminals are seen by the BS via the same set of incident directions, resulting in equal correlation structures. In reality, a different set of incident directions are likely to be observed by multiple terminals, due to their different geographical locations, leading to variations in the local scattering. This gives rise to wide variations in the correlation patterns across multiple terminals [4]. Hence, we consider unequal correlation matrices from each terminal.

Motivated by this, with a uniform linear array (ULA) and maximum-ratio combining (MRC) at the BS, we present insightful closed-form approximations of the expected perterminal SINR and ergodic sum spectral efficiency of an uplink MU-MIMO system. Unlike previous results, for both microwave and millimeter-wave (mmWave) propagation parameters, the closed-form expressions consider unequal correlation matrices, Rice (K) factors and link gains for each terminal. The approximations are shown to be extremely tight for small and large system dimensions, as well as, arbitrary signal-to-noise-ratios (SNRs). To the best of our knowledge, this level of accuracy over such a general channel model capturing a wide range of scenarios has not been achieved previously. Numerical results show the aggregate impact of LoS and unequal spatial correlation. Special cases are presented for Rayleigh fading channels with equal and unequal correlation matrices, as well as, for Ricean fading channels with equal correlation matrices.

II. SYSTEM MODEL

The uplink of a MU-MIMO system operating in an urban microcellular environment (UMi) is considered. The BS is located at the center of a circular cell with radius $R_c$, and is equipped with a $M$ element ULA simultaneously communicating with $L$ single-antenna terminals $(M \gg L)$. Channel knowledge is assumed at the BS, as the prime focus of the manuscript is on performance analysis with general fading channels and not on system level imperfections.

The composite $M \times 1$ received signal at the BS is given by $y = \rho D^2 s + n$, where $\rho$ is the average uplink transmit power, $G$ is the $M \times L$ fast-fading channel matrix between the $M$ BS antennas and $L$ terminals, $D$ is an $L \times L$ diagonal matrix of link gains, where the link gain for terminal $l$ is given by $|D|_{l,l} = \beta_l$. The large-scale fading effects for terminal $l$ in geometric attenuation and shadow-fading are captured in $\beta_l = g_\zeta (r_0/r_l)^\alpha$. In particular, $g$ is the unit-less constant for geometric attenuation at a reference distance of $r_0$, $r_l$ is the distance between the $l$-th terminal and the BS, $\alpha$ is the attenuation exponent and $\zeta$ captures the effects of shadow-fading, modeled via a log-normal density, i.e., $10 \log_{10}(\zeta) \sim \mathcal{N}(0, \sigma^2_\zeta)$. Moreover, $s$ is the $L \times 1$ vector of uplink data symbols from $L$ terminals to the BS, such that the $l$-th entry of $s$, $s_l$ has an expected value of one, i.e., $E[s_l^2] = 1$. The $M \times 1$ vector of additive white Gaussian noise at the BS is denoted by $n$, such that the $l$-th entry of $n$, $n_l \sim \mathcal{C}\mathcal{N}(0, \sigma^2)$. We assume that $\sigma^2 = 1$. Hence, the average uplink SNR is defined as $\rho/\sigma^2 = \rho$. The $M \times 1$ channel vector from terminal $l$ to the BS is denoted by $g_l$, which forms the $l$-th column of $G = [g_1, \ldots, g_L]$. 

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More specifically,
\[ g_l = \gamma_l \hat{h}_l + \gamma_l R_l^\delta h_l. \]  
(1)
The \( M \times 1 \) LoS and the non \( \text{LoS} \) (\( \text{NLoS} \)) components of the channel are denoted by \( h_l \) and \( \hat{h}_l \). Note that \( \gamma_l = (1/(1 + K_l))^1/2 \) and \( \gamma_l = (K_l/(K_l + 1))^{1/2} \), with \( K_l \) being the Ricean \( K \)-factor for the \( l \)-th terminal. \( R_l \) is the receive correlation matrix specific to terminal \( l \), \( \hat{h}_l \sim \mathcal{CN}(0, I_M) \) and \( h_l = [e^{j2\pi d_k \cos(\phi_l^k)}, \ldots, e^{j2\pi d(M-1) \cos(\phi_l^k)}] \). Here, \( d \) is the equidistant inter-element antenna spacing normalized by the carrier wavelength and \( \phi_l^k \sim U[0, 2\pi] \) is the azimuth angle-of-arrival of the \( \text{LoS} \) component for the \( l \)-th terminal.

We employ a linear receiver at the BS array in the form of an MRC filter, where \( G^H = L \times M \) filter matrix used to separate \( y \) into \( L \) data streams by \( r = G^H y = \rho^{1/2} G^H G^D 1/s + G^H n \). Hence, the combined signal from terminal \( l \) is given by \( r_l = \rho^{1/2} \beta_l^2 g_l^H s_l + \rho^{1/2} \sum_{k=1, k \neq l}^L \beta_l^2 g_l^H s_k + g_l^H n_l \). Thus, the corresponding SINR for terminal \( l \) is given by
\[ \text{SINR}_l = \frac{\rho_l \beta_l^2 ||g_l||^2}{||g_l||^2 + \rho \sum_{k=1, k \neq l}^L \beta_k ||g_l^H g_k||^2}. \]  
(2)
As such, the instantaneous uplink spectral efficiency for the \( l \)-th terminal (measurable in bits/sec/Hz) is given by \( R_l^\delta = \log_2 (1 + \text{SINR}_l) \). From here, the ergodic sum spectral efficiency over all \( L \) terminals is given by
\[ E [R^\text{sum}] = E \left[ \sum_{l=1}^L R_l^\delta \right], \]  
(3)
where the expectation is performed over the fast-fading.

III. EXPECTED PER-Terminal SINR AND ERGODIC SUM SPECTRAL EFFICIENCY ANALYSIS
The expected SINR of terminal \( l \) can be obtained by evaluating the expected value of the ratio in (2). Exact evaluation of this is extremely cumbersome, as shown in [6]. Hence, we resort to the first-order Delta method expansion, as shown in the analysis methodology of [6]. This gives
\[ E [\text{SINR}_l] \approx \frac{\rho_l \beta_l^2 E [||g_l||^4]}{E [||g_l||^2] + \rho \sum_{k=1, k \neq l}^L \beta_k E [||g_l^H g_k||^2]}. \]  
(4)

**Remark 1.** The approximation in (4) is of the form of \( \frac{E[X|Y]}{E[Y]} \). The accuracy of such an approximation relies on \( Y \) having a small standard deviation relative to its mean. This can be seen by applying a multivariate Taylor series expansion of \( \frac{X}{Y} \) around \( \frac{E[X]}{E[Y]} \), as shown in the methodology of [6]. Both \( X \) and \( Y \) are well suited to this approximation as \( M \) and \( L \) start to increase. This is evident from the presented numerical results in Section V.

In Lemmas 1, 2 and 3 which follow, we derive the expected values in the numerator and denominator of (4).

**Lemma 1.** For a ULA with \( M \) receive antennas at the BS, considering a correlated Ricean fading channel, \( g_l \), from the \( l \)-th terminal to the BS
\[ \delta_l = E [||g_l||^4] = (\gamma_l)^4 \left( M^2 + \text{tr} \left[ (R_l)^2 \right] \right) + 2 (\gamma_l)^2 (\eta_l)^2 (\gamma_l)^2 + 2 (\gamma_l)^2 (\eta_l)^2 \left[ h_l^H R_l h_l \right] + (\gamma_l)^4 M^2, \]  
(5)
where each parameter is defined after (1).

**Proof:** See Appendix A.

**Lemma 2.** Under the same conditions as Lemma 1,
\[ \varphi_{l,k} = E \left[ ||g_l^H g_k||^2 \right] = (\gamma_l)^2 (\gamma_k)^2 \text{tr} [R_l R_k] + (\gamma_l)^2 (\gamma_k)^2 \text{tr} [h_l h_k^H R_l R_k] + (\gamma_l)^2 (\gamma_k)^2 ||h_l h_k^H R_l R_k||^2. \]  
(6)

**Proof:** See Appendix A.

**Lemma 3.** Under the same conditions as Lemma 1,
\[ \chi_l = E [||g_l||^2] = M \left( (\gamma_l)^2 + (\eta_l)^2 \right) = M. \]  
(7)

**Proof:** We begin by recognizing that \( \chi_l = E [||g_l||^2] = E [g_l^H g_l] \). Substituting the definition of \( g_l \) into (7) and performing the expectations in with respect to \( \hat{h} \) yields the desired result. Only a sketch of the proof is given here, as it relies on straightforward algebraic manipulations.

**Theorem 1.** With MRC and a ULA at the BS, the expected uplink SINR of terminal \( l \) undergoing spatially correlated Ricean fading can be approximated as
\[ E [\text{SINR}_l] \approx \frac{\rho_l \beta_l \delta_l}{\chi_l + \rho \sum_{k=1, k \neq l}^L \beta_k \varphi_{l,k}}, \]  
(8)
where \( \delta_l, \varphi_{l,k} \) and \( \chi_l \) are given by (5), (6) and (7), respectively.

**Proof:** Substituting the results from Lemmas 1, 2 and 3 for \( \delta_l, \chi_l \) and \( \varphi_{l,k} \) yields the desired expression.

**Remark 2.** Further algebraic manipulations allows us to express (8) as (10), shown on top of the next page for reasons of space. Note that (10) can be used to approximate the ergodic sum spectral efficiency of the system by stating
\[ E [R^\text{sum}] \approx \sum_{l=1}^L \log_2 \left( 1 + E [\text{SINR}_l] \right). \]  
(9)
While the accuracy of (10) and (9) is demonstrated in Section V, in the sequel, we present the implications and special cases of (10) to demonstrate its generality.

IV. IMPLICATIONS AND SPECIAL CASES

**A. Implications of (10)**

Both the numerator and the denominator of (10) contain quadratic forms of the type \( h^H R h \). Via the Rayleigh quotient result, such quadratic forms are maximized when \( \tilde{h} \) is parallel (aligned) to the maximum eigenvector of \( R \). From this, an interesting observation can be made: Alignment of \( \hat{h}_l \) and \( R_l \) amplifies the expected signal power, while alignment of \( \hat{h}_k \) with \( R_l \), \( \hat{h}_l \) with \( R_k \) and \( \hat{h}_l \) with \( h_k \) increases the expected interference power, leading to a lower SINR. Likewise, if \( R_k \) and \( R_l \) become similar, then \( \text{tr} [R_l R_k] \) increases, degrading the SINR. The global observation is that the SINR reduces by virtue of channel similarities of various types (LoS and correlation) and increases if the channels are more diverse.

**B. Special Cases of (10)**

**Corollary 1.** In pure \( \text{LoS} \) conditions (i.e., Rayleigh fading) with unequal correlation matrices, (10) reduces to
\[ E [\text{SINR}_l] \approx \frac{\rho \beta_l \left\{ M^2 + \text{tr} [R_l^2] \right\}}{M + \rho \sum_{k=1, k \neq l}^L \beta_k \left\{ \text{tr} [R_l R_k] \right\}}. \]  
(11)

**Proof:** Substituting \( K_l = K_k = 0, \forall l, k = \{1, \ldots, L\} \) in (10) yields the desired result.

**Corollary 2 (Proposition 1 in [3]).** In pure Rayleigh fading with equal correlation matrices, (10) collapses to
\[ E [\text{SINR}_l] \approx \frac{\rho \beta_l \left\{ M^2 + \text{tr} [R_l^2] \right\}}{M + \rho \sum_{k=1, k \neq l}^L \beta_k \left\{ \text{tr} [R_l^2] \right\}}. \]  
(12)
The expected SINR for all cases is seen to saturate with ρ for all cases. The approximations can also be seen to remain tight for the special case of Rayleigh fading with unequal correlation matrices in (11). Furthermore, the expected SINR in each case is seen to saturate with ρ, as the MRC filter is unable to mitigate multiuser interference.

Considering the special cases in (12) and (13), we now examine the aggregate impact of LoS, as well as equal and unequal correlation on the ergodic sum spectral efficiency, as shown in Fig. 2. With M = 256 and L = 32, using the same propagation parameters as in Fig. 1, at ρ = 10 dB, we compare the cumulative distribution functions (CDFs) of the derived ergodic sum spectral efficiency approximation in (9) with its simulated counterparts. Each CDF is obtained by averaging over the fast-fading, with each value representing the variations in the link gains and the K-factors. The derived approximations remain tight with changes in the system size. Moreover, irrespective of the underlying propagation characteristics, unequal correlation matrices result in higher ergodic sum spectral efficiency, allowing the ULA to leverage more spatial diversity. This is noticed when comparing the K1 = 5 dB curves with a fixed φ1 = π/16 (equal correlation) and variable φ1 (unequal correlation) for each terminal. In contrast to the correlated Rayleigh case, a dominant LoS component is again seen to be detrimental to system performance.

VI. CONCLUSION

We have presented a general, yet insightful approximation to the expected per-terminal SINR and ergodic sum spectral efficiency of an uplink MU-MIMO system. With a ULA and MRC at the BS, the approximation is robust to equal and unequal correlation matrices, unequal levels of LoS, unequal link gains, unequal operating SNRs and system dimensions.
Expanding (15) allows us to write
\[ v \]
the definition of \( E \)

After noting that \( \delta = \gamma_i R_i h_i \) and \( q_i = \gamma_i h_i \) yields \( \varphi_{i,k} = \mathbb{E} \left[ (v^H + q^H) (v_k + h_k) \right] \). Expanding and simplifying further gives

\[
\varphi_{i,k} = \mathbb{E} \left[ (v_i^H + q_i^H) (v_k + h_k) \right] + \mathbb{E} \left[ q_i^H v_k^H q_k + q_i^H q_k \right].
\]

(19)

Recognizing that \( \mathbb{E} [v_i v_i^H] = \mathbb{E} [\eta_i R_i^2 h_i \eta_i h_i^H] = (\eta_i)^2 R_i, \) substituting back the definitions of \( v_i, v_k, q_i \) and \( q_k \) in (19) and extracting the relevant constants yields

\[
\varphi_{i,k} = (\eta_i)^2 (\eta_k)^2 \mathbb{E} \left[ \text{tr} (R_k R_l) \right] + (\eta_i)^2 (\eta_k)^2 \mathbb{E} \left[ \text{tr} (R_k^2 h_i \eta_i h_i^H R_k^2 h_i \eta_i h_i^H) \right] + (\eta_i)^2 (\eta_k)^2 \mathbb{E} \left[ \text{tr} (R_k^2 h_i \eta_i h_i^H R_k^2 h_i \eta_i h_i^H) \right].
\]

(20)

Taking the trace and simplifying yields (6).

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