Time-dependent quantum transport with superconducting leads

Gianluca Stefanucci, Enrico Perfetto and Michele Cini
Dipartimento di Fisica, Università di Roma Tor Vergata, Via della Ricerca Scientifica 1, I-00133 Rome, Italy
E-mail: gianluca.stefanucci@roma2.infn.it

Abstract. We present results on the microscopic dynamics of electrons in nanoscale systems coupled to superconducting leads. By solving the time-dependent Bogoliubov-deGennes equations for the Nambu-Gorkov Keldysh Green’s function we are able to calculate the current and the charge density when the system is perturbed by bias voltages. For scattering of electrons across a normal-superconducting surface we provide a time-dependent picture of the Andreev reflections. When the nanoscale system is contacted to two DC biased superconducting leads the amplitude of the current oscillations at even multiple of the bias can be extracted by Fourier transforming the time-dependent results. In the transient regime the dwelling time is inversely proportional to the bias in agreement with the occurrence of multiple Andreev reflections.

1. Introduction
The progressive miniaturization of electronic devices has led to the emerging field of nanoelectronics. Due to the reduced dimensionality, heat losses in integrated circuits of semiconductors or normal conductors will be the main reason of instabilities like, e.g., bond-breakings, electromigrations, etc. [1, 2, 3, 4]. Heat losses are considerably reduced in superconducting devices. Thus, superconducting nanoelectronics offers a promising and timely alternative to the semiconductor nanotechnology [5, 6, 7]. A large theoretical and experimental effort is currently devoted to, e.g., the realization of quantum bits (QUBIT) using atomic-size quantum point contacts or quantum dots coupled to superconducting leads. The state of a QUBIT can then be manipulated using electromagnetic pulses of the duration of few nanoseconds or even faster. The description of the ultrafast electron dynamics in such Josephson nano-junctions is not only of considerable fundamental interest but also of importance for their potential applications in future electronics.

The study of the electronic response to time-dependent ac field or train pulses in nanodevices chemically attached to superconducting leads has so far remained largely unexplored. There are several difficulties related to the construction of a feasible scheme already at a mean-field level. The system is an open system, the electronic energy scales are 2-3 orders of magnitude larger than a typical superconducting gap, the problem is intrinsically time-dependent even for DC biases (Josephson oscillations), the scattering states extends from $z = -\infty$ to $z = +\infty$, with $z$ the longitudinal direction, and the possible formation of Andreev bound states (ABS) give rise to persistent oscillations in the density and current. The time-evolution of localized wave-packets scattering across a superconducting-normal (SN) interface was explored long ago [8, 9].
recently the analysis has been extended to scattering states [10, 11, 12] in standard quantum transport geometries. However, there has been no attempt to calculate the time-dependent response of a superconductor-normal-superconductor (SNS) junction.

In this work we propose and illustrate a general scheme suited to study time-dependent (TD) quantum transport in SNS junctions. The propagation scheme is tested in popular model Hamiltonians that can describe quantum point contacts, atomic contacts, atomic chains, etc. We provide the first ever calculation of the time-dependent transient response of a Josephson nano-junction and illustrate the role played by the multiple Andreev reflections in the on-set of the Josephson regime. We also show the Andreev-reflection space-time pattern in long atomic chains sandwiched between a normal and a superconducting lead.

The plan of the paper is as follows: In the next Section we describe the theoretical framework based on the equation of motion for the Nambu-Keldysh Green’s function in the BCS approximation. The Green’s function projected on the region of interest obeys a Dyson-like equation with an embedding self-energy which accounts for the openness of the system. In Section 3 we briefly illustrate the main ideas of the propagation algorithm while in Section 4 we present the numerical results. Conclusions and outlooks are summarized in Section 5.

2. Equation of motion for the Nambu-Keldysh Green’s function in the BCS approximation

We consider electrons interacting through an attractive Gorkov point-contact interaction $W_g(r)$. Without loss of generality we assume that the system is in equilibrium at negative times, $t < 0$. We are interested in studying the dynamics of the system if an external time-dependent perturbation is switched on at $t = 0$ [13, 14]. The system is described by a Hamiltonian $\hat{H}(t) = \hat{K}(t) + \hat{H}_g$ which is the sum of a quadratic term

$$\hat{K}(t) = \sum_{\sigma} \int dr dr' \psi_{\sigma}^\dagger(r) \langle r|K_{\sigma}(t)|r'\rangle \psi_{\sigma}(r'), \quad \sigma = \uparrow, \downarrow$$ (1)

with $K_{\sigma}(t)$ a hermitean matrix, and of the following interaction operator

$$\hat{H}_g = \int dr \ W_g(r) \ \psi_{\uparrow}^\dagger(r)\psi_{\downarrow}^\dagger(r)\psi_{\downarrow}(r)\psi_{\uparrow}(r).$$ (2)

The BCS approximation consists in discarding the correlations between the motion of two electrons added to the system. In normal systems the BCS approximation is the same as the Hartree-Fock approximation. In superconducting systems we further have to account for anomalous diagrams describing the amplitude of finding the system in the same state after the removal or the addition of two electrons. The normal and anomalous Keldysh-Green’s functions are defined according to

$$\langle r|G_{\sigma}(z,z')|r'\rangle = \frac{1}{i} \langle T_{K} \left\{ \psi_{\sigma}(r,z)\psi_{\sigma}^\dagger(r',z') \right\} \rangle, \quad \sigma = \uparrow, \downarrow$$ (3)

$$\langle r|\mathcal{F}(z,z')|r'\rangle = \frac{1}{i} \langle T_{K} \left\{ \psi_{\downarrow}(r,z)\psi_{\uparrow}(r',z') \right\} \rangle,$$ (4)

$$\langle r|\mathcal{F}(z,z')|r'\rangle = -\frac{1}{i} \langle T_{K} \left\{ \psi_{\uparrow}(r',z')\psi_{\downarrow}(r,z) \right\} \rangle,$$ (5)

where $z, z'$ run on the Keldysh contour $\gamma$ which is an oriented contour composed by an upper branch going from 0 to $\infty$, a lower branch going from $\infty$ to 0 and a purely imaginary (thermal) segment going from 0 to $-i\beta$ (here $\beta$ is the inverse temperature). The operator $T_{K}$ in the above
definitions is the contour ordering operator and move operators with later contour variable to the left (an extra minus sign has to be included for odd permutations). We have introduced the short hand notation

$$\langle T_K \{ \ldots \} \rangle = \frac{\text{Tr} \left[ e^{\beta \mu N} T_K \left\{ e^{-i \int_0^1 d\tau \mathcal{H}(\tau) } \right\} \right]}{\text{Tr} \left[ e^{\beta \mu N} T_K \left\{ e^{-i \int_0^1 d\tau \mathcal{H}(\tau) } \right\} \right]}, \quad (6)$$

for denoting averages. In Eq. (6) the quantity $\mu$ is the chemical potential and $\hat{N}$ is the total number of particle operator. For $z$ on the upper/lower branch of $\gamma$ the Hamiltonian $\hat{H}(z)$ is equal to $\hat{H}(t)$, with $t$ the distance of $z$ from the origin. For $z$ on the thermal segment $\hat{H}(z) = \hat{H}(0)$. Introducing the Nambu-Gorkov Keldysh-Green’s function

$$\mathcal{G}(z; z') = \begin{bmatrix} G_{\uparrow}(z; z') & -\mathcal{F}_T(z'; z) \\ \mathcal{F}(z; z') & -G_{\uparrow}^T(z'; z) \end{bmatrix}, \quad (7)$$

where the superscript $T$ denotes the transpose of the indicated quantity, the equation of motion in the BCS approximation can be cast in a rather elegant form

$$\left\{ -\frac{d}{dz} - \hat{K}(z) \right\} \mathcal{G}(z; z') = \delta(z - z'). \quad (8)$$

In Eqs. (8) the effective BCS Hamiltonian reads

$$\hat{K}(z) = \begin{bmatrix} K_{\uparrow}(z) + \Sigma_{\uparrow}(z) & \Delta(z) \\ \Delta(z) & -K_{\uparrow}^T(z) - \Sigma_{\uparrow}^T(z) \end{bmatrix}, \quad (9)$$

where $\Sigma_{\uparrow/\downarrow}(z)$ is the Hartree-Fock self-energy (which in this case is the same as the Hartree self-energy since the Gorkov interaction is point-like) while

$$\langle \mathbf{r} | \Delta(z) | \mathbf{r}' \rangle = i \delta(\mathbf{r} - \mathbf{r}') W_g(\mathbf{r}) \langle \mathbf{r} | \mathcal{F}(z^+; z) | \mathbf{r} \rangle \quad (10)$$

is the so called pairing potential or order parameter. From the definition of the normal and anomalous Green’s function the equation of motion has to be solved with the boundary conditions

$$\mathcal{G}(0_-; z') = -e^{\beta \mu \sigma} \mathcal{G}(-i\beta; z'), \quad \mathcal{G}(z; 0_-) = -\mathcal{G}(z; -i\beta) e^{-\beta \mu \sigma}, \quad (11)$$

where $\sigma = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is the third Pauli matrix.

Let us now specialize to the transport setup. The one-particle Hilbert space can be partitioned in a L/R electrode and a central region $C$, see Fig. 1. For simplicity we here also consider the case $K_{\uparrow}(t) = K_{\downarrow}(t) = \mathcal{E}(t) + \mathcal{V}$. The matrix $\mathcal{E}(t) = \mathcal{E} + \mathcal{U}(t)$ describes the normal part of the isolated $L$, $R$ and $C$ regions and is composed of an equilibrium time-independent matrix $\mathcal{E}$ and of a time-dependent local perturbation $\mathcal{U}(t)$ which represents the bias (in the leads) and/or a gate voltage (in the central region), $\mathcal{U}_{ij}(t) = \delta_{ij} U_i(t)$. The matrix $\mathcal{V}$ accounts for the hopping between different regions.

Introducing the uncontacted Green’s function $\mathcal{g}$ which obeys the equation of motion (8) with $\mathcal{V} = 0$ and the same boundary conditions as $\mathcal{G}$ the equation of motion for $\mathcal{G}$ projected onto region $CC$ takes the form

$$\left\{ -\frac{d}{dz} - \hat{K}_{CC}(z) \right\} \mathcal{G}(z; z') = \delta(z - z') + \int d\bar{z} \Sigma(z; \bar{z}) \mathcal{G}_{CC}(\bar{z}; z'), \quad (12)$$
where the embedding self-energy is defined according to
\[
\Sigma(\tilde{z}; \tilde{z}') = \sum_{\alpha=L,R} \Sigma_{\alpha}(\tilde{z}; \tilde{z}') = \sum_{\alpha=L,R} \gamma C_{\alpha} q_{\alpha}(\tilde{z}; \tilde{z}') \psi_{\alpha C}. \tag{13}
\]

We here wish to emphasize that the equation of motion is defined on the generalized Keldysh contour. Converting the above equation in equations for real times we obtain a set of coupled equations known as Kadanoff-Baym equations \cite{15, 16, 17, 18} recently implemented to study correlated quantum transport \cite{19, 20}. Below we briefly describe how to solve Eq. (12) with a pure embedding self-energy.

3. Algorithm for the time-propagation

Physically observable quantities like the current and the density can be extracted from the lesser component of the Green function. From Eq. (8) one can show that \(G(\tilde{z}; \tilde{z}') = i \int \gamma q(E) |\psi_q(t)\rangle \langle \psi_q(t')| \) where \(f^>(\omega) = 1/[1 + \exp(\beta \omega)]\) is the Fermi function and \(f^>(\omega) = f^\prime(\omega) - 1\). The states \(|\psi_q(0)\rangle\) are eigenstates of \(K - \mu \sigma\) with eigenvalue \(E_q\) and evolve in time according to the Schrödinger equation \(i \frac{d}{dt} |\psi_q(t)\rangle = K(t) |\psi_q(t)\rangle\), which, in this case, is equivalent to the time-dependent Bogoliubov-deGennes (BdG) equation. For the calculation of the initial states we used the generalized waveguide approach of Ref. \cite{21} properly extended to superconducting leads. We consider homogeneous time-dependent perturbations in electrode \(\alpha = L, R\), i.e., \(K_{t/\alpha}(t) = K_{t/\alpha}(0) + U_\alpha(t) 1_\alpha\) (where \(1_\alpha\) is the identity matrix in region \(\alpha\)). Assuming that the constriction does not alter the equilibrium of the macroscopically large superconducting reservoirs we have
\[
K_{\alpha \alpha}(t) = e^{-i\mu_\alpha(t) \sigma_\alpha} \left( K_{\alpha \alpha}(0) + U_\alpha(t) \sigma_\alpha \right) e^{i\mu_\alpha(t) \sigma_\alpha}, \tag{14}
\]
with \(\sigma_\alpha\) the third Pauli matrix in electrode \(\alpha\) and \(\mu_\alpha(t) = \mu t + \int_0^t d\bar{t} U_\alpha(\bar{t})\). In writing Eqs. (14) we have made use of the fact that a bulk superconductor perturbed by a homogeneous time-dependent potential has a pairing potential with a time-dependent phase which guarantees the gauge invariance of the equations. For region \(C\) there are no restrictions on the time-dependence of the perturbation. The BdG equation for every single one-particle state \(|\Psi\rangle\) is numerically solved using the Crank-Nicholson formula
\[
\left( 1 + i \delta K^{(m)} \right) |\Psi^{(m+1)}\rangle = \left( 1 - i \delta K^{(m)} \right) |\Psi^{(m)}\rangle, \tag{15}
\]
where \(|\Psi^{(m)}\rangle = |\Psi(t_m)\rangle, K^{(m)} \rightleftharpoons \frac{1}{2} [K(t_{m+1}) + K(t_m)]\), \(t_m = 2\delta m\). Eq. (15) is of no practical use since the matrices representing the operators are infinite-dimensional. It is possible, however, to extract from Eq. (15) an equation for the projection of \(|\Psi\rangle\) on region \(C\) only \cite{22, 23}
\[
\left( 1_C + i \delta K^{(m)}_{eff} \right) |\Psi^{(m+1)}\rangle = \left( 1_C - i \delta K^{(m)}_{eff} \right) |\Psi^{(m)}\rangle.
\]
\[- i\delta \sum_{\alpha=L,R} z^{(m)}_{\alpha} K_{C\alpha} n^{m}_{\alpha} (1_{\alpha} + g_{\alpha\alpha}) \mid \Psi^{(0)}_{\alpha} \mid \]
\[- \delta^2 \sum_{\alpha=L,R} \sum_{j=0}^{m-1} z^{(m)}_{\alpha} (Q^{(j+1)}_{\alpha} + Q^{(j)}_{\alpha}) z^{(m-1-j)}_{\alpha} \times \left( \mid \Psi^{(m-j)}_{C} \mid + \mid \Psi^{(m-1-j)}_{C} \mid \right). \] (16)

The effective Hamiltonian $K_{\text{eff}}^{(m)} = K_{C\alpha}^{(m)} - i\delta \sum_{\alpha=L,R} z^{(m)}_{\alpha} Q^{(0)}_{\alpha} z^{(m)}_{\alpha}$ has the size of region $C$ and is expressed in terms of the phase factors $z^{(m)}_{\alpha} = \frac{1}{2} (e^{-i\mu_{\alpha}(m+1)} - e^{-i\mu_{\alpha}(m)} - e^{i\mu_{\alpha}(m)} - 1_{\alpha}$), and embedding matrices $Q^{(m)}_{\alpha} = \frac{1}{2} K_{C\alpha} n^{m}_{\alpha} (1_{\alpha} + g_{\alpha\alpha}) K_{\alpha C}$ with propagator $g_{\alpha\alpha} = (1_{\alpha} - i\delta K_{\alpha\alpha})/(1_{\alpha} + i\delta K_{\alpha\alpha})$. More details on the practical implementation of the numerical algorithm will be given in a future publication.

4. Numerical results

In this Section we present results on NNS and SNS junctions obtained using the time-evolution algorithm described in the previous Section. In a typical superconducting nano-junction set-up the band-width of the leads, $W$, is the largest energy scale. If the leads are connected to a central region with hopping integrals $V_{\text{hop}} \ll W$ the time-dependent density and current in region $C$ depend only on the ratio $\Gamma = 8V_{\text{hop}}^2/W$: the quantity $\Gamma$ provides the broadening of the energy levels of region $C$. The geometrical structure of the leads is, therefore, of no relevance and in the calculations below we model the superconducting leads as one-dimensional semi-infinite tight-binding chains with nearest neighbor hopping $V_{L} = V_{R} = V$; the band-width is then $W = 4|V|$. To illustrate the fundamental physical phenomena occurring in a nano-scale Josephson junction we here consider one-dimensional central regions with nearest neighbor hopping $V_{C}$. Numbering the site of region $C$ from left to right as $1, 2, \ldots, N_{C}$ we further consider that the hopping between $L$ and $C$ occur only via the end-site of $L$ and site 1 of $C$ while the hopping between $R$ and $C$ via the end-site of $R$ and site $N_{C}$ of $C$. The corresponding hopping parameters are denoted by $V_{LC}$ and $V_{RC}$. We wish to observe that these kind of model junctions can describe several physically relevant situations, ranging from quantum-point contacts to quantum dot arrays or atomic chains. The pairing potential is taken to be constant in lead $\alpha = L, R$ and equal to $\Delta_{\alpha}$. In the central region we set the pairing potential to $0$. This means that the pairing potential in the central region is not calculated self-consistently but rather constitutes an input of the problem. All calculations presented below have been performed with the chemical potential $\mu = 0$ and the temperature $T = 1/\beta = 0$.

The first system we study is an NNS junction with $V = 50$, $V_{LC} = V_{RC} = 7$ and $N_{C} = 21$ sites with nearest neighbor hopping $V_{C} = 7.5$. The pairing potential is chosen as $\Delta_{L} = 0$ and $\Delta_{R} = 1$. With these parameters the broadening $\Gamma \sim 2$ and the superconducting gap is therefore $2\Delta_{R} \sim \Gamma$: this is the so called intermediate coupling regime. The generic right-going scattering state can be written as a superposition of a spin-up electron $(c_{kL}^{\dagger} + R_{kL} c_{-kL}^{\dagger})$ and a spin-down hole $c_{-kL}$ wave-functions in lead $L$ and as a superposition of the two independent superconducting quasiparticles, or bogolons, $\alpha_{kR}^{L} = u_{k} c_{kR}^{\dagger} - v_{k} c_{-kR}$ and $\beta_{kR}^{L} = u_{k} c_{-kR}^{\dagger} + v_{k} c_{kR}$ wave-functions in lead $R$. The reflection and transmission coefficients as a function of energy are displayed in Fig. 2. In the energy window considered the probability amplitude $R_{-}$ for an electron to be reflected as a hole is rather small except in the vicinity of the middle of the gap. A perfect Andreev reflection (AR) occurs at zero energy, i.e., the spin-up electron is entirely reflected as a spin-down hole. In perfect Andreev conditions the incident $k \uparrow$ electron is not reflected, so it goes across the boundary and must be found in lead $R$ either as such or as a $(k \uparrow, -k \downarrow)$ Copper pair arising from the incoming electron bound to the one that left the spin-down hole.
in the $L$ lead. By creating a $\beta$ bogolon in lead $R$ one would reach a many-body state where the incident electron is destroyed. This cannot happen since that electron is transmitted with probability 1. Infact such an amplitude may be seen to vanish in the right panel of Fig. 2. The $\alpha$ bogolon amplitude instead is allowed and it corresponds to detecting a Cooper pair. There is considerable structure also at energy $E = -1$, at the bottom of the gap. The AR probability is of the order $1/2$ (left panel). The right panel shows that one has essentially the same chance of observing an $\alpha$ or $\beta$ bogolon. Both events can be understood as incomplete Andreev scatterings.

To provide a time-dependent picture of the AR we consider the system in which the normal lead $L$ is initially uncontacted and filled with electrons up to the chemical potential $\mu = 0$. At time $t = 0$ we switch on the contact $V_{LC}$ as well as a DC bias $U_L$ and propagate in time only the spin-up electron states in lead $L$ with energy in the range $(-1, 0)$. In Fig. 3 we display the contour plot of the change in the spin-up electron density and spin-down hole density in the
$N_C = 21$ sites of the chain for bias $U_L = -0.5$. We observe that the change in density must be entirely attributed to the propagation of spin-up electrons from lead $L$. As expected from Fig. 2 AR’s are strongly suppressed for $U_L = -0.5$ and the electrons are entirely reflected as electrons. Correspondingly, no change in the hole-density is detected, see right panel of Fig. 3. In the long-time limit the density profile remains constant and the period of its corrugation agrees with the period of the Friedel oscillations.

![Figure 4: Same as in Fig. 3 but with bias $U_L = 0$.](onresonance.nb)

The situation is completely different if $U_L = 0$. In this case electrons at the Fermi energy undergo a perfect AR and we should be able to appreciate a change in the spin-down hole density. Fig. 4 shows that this is indeed the case. We notice that the electron density front hits the superconducting lead earlier than in Fig. 3 since the Fermi velocity is larger. No change in the hole density is visible before the electrons have time to scatter. From that time on, however, we can clearly observe a hole density front propagating backward. Due to charge conservation a carrier with charge two times larger than the electron charge, i.e., a Copper pair, is formed in lead $R$ and moves right. Fig. 4 thus provides a time-dependent picture of the normal-current supercurrent conversion at the NS interface.

We next consider an SNS junction with $V = 100$, $V_{CL} = V_{CR} = -7$, $\Delta_L = \Delta_R = 1$ and $N_C = 2$ sites connected with a hopping integral $V_C = -1.5$. The system admits two couple of degenerate ABS at energy $\sim \pm 0.89$. In Fig. 5, left panel, we show the time-dependent change of the density in site 1 and 2 when a DC bias $U_L$ is switched on in the left lead. After a transient the density oscillates in time and the period of the oscillations is $T = 2\pi/(2U_L)$, as expected. The transient time is longer the smaller is the bias. This is caused by the occurrence of multiple AR at the junction. An electron with energy $U_L$ with respect to the top of the valence band has to undergo $\Delta/U_L$ reflections before can be transmitted. To the best of our knowledge Fig. 5 contains the first explicit calculation of the time-dependent response of a SNS junction to a constant DC bias. The transient regime, the on-set of the Josephson oscillations and the Josephson regime are all clearly visible.

An extra merit of performing real-time evolutions is the possibility of calculating the frequency components of the Josephson current by simply Fourier transforming the time-dependent curves in the Josephson regime. At present the calculation of the Fourier components of the current
Figure 5: Left panel: Time dependent density change $\delta n(t) \equiv n(t) - n(0)$ in site 1 [black (thick)] and 2 [red (thin)] when a DC bias $U_L = 0.1, 0.2, 0.3$ is applied to the left superconducting lead. The other parameters are given in the main text. The onset of the Josephson regime is clearly visible. Right panel: Discrete Fourier transform of the time-dependent current flowing between site 1 and 2. The curves corresponding to bias $U_L = 0. n$ are shifted upward by $0.005(n - 2)$ for clarity.

is performed by combining scattering formalisms with Floquet theory and can be shown to be equivalent to the solution of a tridiagonal matrix system\cite{24, 25, 26}. In Fig. 5, right panel, we report the real and imaginary part of the discrete Fourier transform of the current flowing between site 1 and 2. The curves exhibit peaks at frequency $2nU_L$ as expected. We observe that the method here used can also be employed to calculate the frequency spectrum of the current when the system is exposed to an AC bias. Such study will permit us to obtain the intensity of the Shapiro spikes in nanoscale transport experiment.

5. Conclusions and outlooks

We reported preliminary results of a novel time-propagation scheme applied to NNS and SNS nano-junctions. The entire approach is based on the BCS approximation to deal with superconducting correlations in the leads. However, it can be combined with the superconducting extension of density functional theory (DFT) \cite{27} and of TDFFT \cite{28} to go beyond a mean field description and include exchange-correlation effects. We have been able to provide a time-dependent picture of a non-interacting electron gas scattering across a normal-superconducting interface and to observe the occurrence of AR’s. We also calculated the transient response of a SNS junction to a sudden switch on of the bias. The transient regime is dominated by multiple AR’s and is longer the smaller is the bias. After the transient regime a Josephson regime sets in and both current and density oscillate in time. The frequency spectrum of the current can be computed by a simple discrete Fourier transform and reveals the presence of sharp peaks at even multiple of the bias. Our results lend themselves to experimental tests and may be of relevance for technological applications.

References

[1] R. S. Sorbello, Solid State Phys. 51, 159 (1997).
[2] T. N. Todorov, J. Hoekstra, and A. P. Sutton, Phys. Rev. Lett. 86, 3606 (2001).
[3] M. Di Ventra, S. T. Pantelides, N. D. Lang, Phys. Rev. Lett. 88, 046801 (2002).
[4] C. Verdozzi, G. Stefanucci and C.-O. Almbladh, Phys. Rev. Lett. 97, 046603 (2006).
[5] C. J. Lambert and R. Raimondi, J. Phys.: Condens. Matter 10, 901 (1998).
[6] Y. Makhlin, G. Schön and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001).
[7] J.-P. Cleuziou, W. Wernsdorfer, V. Bouchiat, T. Ondarchu, and M. Monthioux, Nature Nanotech. 1, 251 (2006).
[8] R. König, Z. Phys. 218, 472 (1969).
[9] H. De Raedt, K. Michielsen, and T. M. Klapwijk, Phys. Rev. B 50, 631 (1994).
[10] A. Jacobs and R. König, Phys. Rev. B 64, 104515 (2001).
[11] Y. Xing, Q.-f. Sun, and J. Wang, Phys. Rev. B 75, 125308 (2007).
[12] E. Perfetto, G. Stefanucci, and M. Cini, Phys. Rev. B 80, 205408 (2009).
[13] M. Cini, Phys. Rev. B 22, 5887 (1980).
[14] G. Stefanucci, and C.-O. Almbladh, Phys. Rev. B 69, 195318 (2004).
[15] L. P. Kadanoff and G. Baym, Quantum Statistical Mechanics (Benjamin, New York, 1962).
[16] P. Danielewicz, Ann. Phys. (N.Y.) 152, 239 (1984).
[17] N.-H. Kwong and M. Bonitz, Phys. Rev. Lett. 84, 1768 (2000).
[18] N. E. Dahlen and R. van Leeuwen, Phys. Rev. Lett., 98, 153004 (2007).
[19] P. Måhönen, A. Stan, G. Stefanucci, and R. van Leeuwen, Europhys. Lett. 84, 67001 (2008).
[20] P. Måhönen, A. Stan, G. Stefanucci, and R. van Leeuwen, Phys. Rev. B 80, 115107 (2009).
[21] G. Stefanucci, E. Perfetto, S. Bellucci, and M. Cini, Phys. Rev. B 79, 073406 (2009).
[22] S. Kurth, G. Stefanucci, C.-O. Almbladh, A. Rubio, and E. K. U. Gross, Phys. Rev. B 72, 035308 (2005).
[23] G. Stefanucci, E. Perfetto, and M. Cini, Phys. Rev. B 77, 075339 (2008).
[24] Q.-f. Sun, H. Guo, and J. Wang, Phys. Rev. B 65, 075315 (2002).
[25] D. F. Martinez, J. Phys. A 36, 9827 (2003).
[26] G. Stefanucci, S. Kurth, A. Rubio, and E. K. U. Gross, Phys. Rev. B 77, 075339 (2008).
[27] L. N. Oliveira, E. K. U. Gross, and W. Kohn, Phys. Rev. Lett. 60, 2430 (1988).
[28] O.-J. Wacker, R. König, and E. K. U. Gross, Phys. Rev. Lett. 73, 2914 (1994).