Perfect Conversion of a TM Surface Wave Into a TM Leaky Wave by an Isotropic Periodic Metasurface Printed on a Grounded Dielectric Slab

Svetlana N. Tcvetkova, Enrica Martini, Senior Member, IEEE, Sergei A. Tretyakov, Fellow, IEEE, and Stefano Maci, Fellow, IEEE

Abstract—This article presents an exact solution for a perfect conversion of a TM-polarized surface wave (SW) into a TM-polarized leaky wave (LW) using a reciprocal and lossless penetrable metasurface (MTS) characterized by a scalar sheet impedance, located on a grounded slab. On contrary to the known realizations of LW antennas, the optimal surface reactance modulation which is found here ensures the absence of evanescent higher-order modes of the field Floquet-wave expansion near the radiating surface. Thus, all the energy carried by the SW is used for launching the single inhomogeneous plane wave into space without accumulation of reactive energy in the higher-order modes. It is shown that the resulting penetrable MTS exhibits variation from an inductive to a capacitive reactance passing through a resonance. The present formulation complements a previous article of the authors, in which a perfect conversion from TM-polarized SW to TE-polarized LW was found for impenetrable boundary conditions. Here, the solution takes into account the grounded slab dispersion, and it is convenient for practical implementation.

Index Terms—Antennas, Floquet waves (FW), leaky waves (LWs), metasurface (MTS), surface waves (SWs).

I. INTRODUCTION

SURFACE wave (SW) to leaky wave (LW) conversion is one of the classical problems in antennas, plasmonics, and nanophotonics [1]–[15]. In recent antenna applications [16]–[22], this conversion is obtained by the interaction of a cylindrical SW excited by a point source, with a curvilinear type modulation obtained by printing subwavelength patches on a grounded slab. In particular, in [23], the local interaction is studied by assuming a local plane SW wavefront interacting with a local boundary value problem (BVP) with unidirectional periodic modulation. This periodic BVP can be extended by an infinite Floquet mode expansion [23] using an extension of the Oliner–Hessel method for impenetrable (Leontovitch type) boundary condition [5] which accounts for the slab Green’s function (GF). Therefore, the periodic BVP assumes a strategic importance in the design of a very flexible class of flat antennas. In any of these periodic BVP solutions, the vicinity of the reactive boundary contains the fields of an infinite number of higher-order Floquet harmonics, which store reactive energy, producing a dispersive effect that limits the antenna bandwidth. This happens even if the penetrable metasurface (MTS) on top of the slab exhibits a gentle periodic variation of parameters, for instance, of a sinusoidal form. Therefore, there is an interest in finding a particular functional form of the modulated MTS reactance that allows for limiting as much as possible storage of reactive energy close to the surface, still having a “perfect” SW-to-LW conversion, where only the 0 and −1 modes are present in the exact expansion.

Recently, new methods have been proposed for efficient conversion of a homogeneous plane wave into an SW using MTSs [9], [24]. In [16], an MTS system able to route space waves via SWs has been introduced. The system is synthesized based on a momentum transfer approach using phase-gradient MTSs and tested experimentally.

In [24], the problem of perfect conversion has been faced by using an impenetrable impedance model. Similarly to that work, the problem of the perfect conversion for the case of a single harmonic of an SW into a single harmonic of an inhomogeneous plane wave was considered in [25]. Despite this model does not describe the slab dispersion properly, it allows to find a simple analytical closed-form solution which is useful as a guideline. However, the results for the point-wise lossless implementation can be obtained only for TM-SW to TE-LW conversion with an anisotropic MTS. Abdo-Sánchez et al. [26] described a solution with only one propagating plane wave radiating from a partially transparent wall of a waveguide realized using a bianisotropic MTS. However, the structure proposed for realization is rather complicated and bulky compared to the conventional LW antennas used in practice.
In this article, we introduce a theoretically perfect wave converter based on a single-sheet penetrable MTS, which can be practically realized in a simple way as an array of metal patches or slots in a metallic plate over a grounded dielectric slab. We have found that using an appropriate functional form of the isotropic MTS leads to a perfect TM–TM conversion solution. This solution is presented in Section II.

II. PROBLEM GEOMETRY AND PERIODIC BVP

Let us consider an MTS characterized by a periodic lossless impedance (Fig. 1). The MTS can be constituted by lossless periodically arranged subwavelength patches printed at z = 0 level over a lossless grounded dielectric slab of relative permittivity εr and thickness h [Fig. 1(a)]. However, for the purpose of this article, the MTS will be considered as a homogenized surface with x-axis periodic dependence [Fig. 1(b)]. In the following, k0 = ω/ε0μ0 and ζ = ωε0/μ0 denote the free-space wavenumber and impedance, respectively.

A. Isotropic Boundary Condition

The MTS can be modeled by isotropic continuous impedance BCs. If the metallic patch layer has an infinitesimal thickness, one may assume Ez|x=0 = Ez|x=0 = E0. Assuming and suppressing time dependence exp(jωt), the MTS is modeled by using “transparent” isotropic BCs, defined by [23]

\[ E_z = jX(H_y|_{x=0} - H_y|_{x=0}) = jXJ_x \]  \tag{1}

where X is the surface reactance, and Jx is the average electric surface current density flowing in the reactive sheet. In the absence of losses, X is a real number. The model of transparent reactance in (1) is accurate when applied to a general wave field since it is almost independent of the x-component of the complex wavenumber kx in a quite large frequency range [27].

B. TM-SW in the Absence of Modulation

In the presence of a uniform (i.e., nonmodulated) impedance of value X = X0, the structure supports an SW with currents

\[ J_{s0} = J_0e^{-j\beta_{sw}x}. \]

The dispersive wavenumber βsw is the TM solution of the local transverse resonance equation over X0 given by

\[ \left[ \frac{1}{X^+(\beta_{sw})} + \frac{1}{X^-(\beta_{sw})} + \frac{1}{X_0} \right] = 0 \]  \tag{2}

where

\[ X^+(k_x) = -\zeta \sqrt{k_x^2 - k_0^2} / k_0 \]  \tag{3}

and

\[ X^-(k_x) = \zeta \sqrt{\varepsilon_r k_0^2 - k_x^2} / k_0 \tan(\zeta \sqrt{\varepsilon_r k_0^2 - k_x^2}) \]  \tag{4}

are the TM reactances of a z-directed transmission line toward free space and toward the ground, respectively (see Fig. 2). We note that in (2), the functions in (3) and (4) are evaluated for kx = βsw. In Sections III and IV, the same functions will be evaluated at different complex wavenumbers.

III. REACTANCE SYNTHESIS FOR TWO MODES ONLY

In the presence of a periodic modulation of the reactance X(x) = X(x – d) for x > d, where d is the period of modulation, the energy transported by the SW leaks from the surface, transforming the bounded SW into an LW. The dominant component of the currents is substantially determined by the average transparent reactance, and the global currents can be thought of as a result of interference between the SW over X0 and the oscillation of the reactance. The 0-indexed mode of the Floquet expansion is given by

\[ J^{(0)} = J_0e^{-jk^{(0)}x}. \]  \tag{5}

We denote J^{(0)} as “0-mode” current, where J0 is its constant amplitude, k^{(0)} = k^{(0)}_x = β^{(0)}_sw - j\alpha_x, is the complex wavenumber of the SW (0-mode), and x is the axis along the surface. The attenuation constant \alpha_x is associated with the transfer of energy from the 0-mode to the radiating mode during the propagation path.

We would like to synthesize a periodic reactance X that allows for the presence of only two modal currents in the Floquet wave (FW) expansion, i.e., the 0-mode and the –1-mode (leaky mode). Defining K = 2\pi/d, we, therefore, assume that the exact current distribution is given by

\[ J_x = J^{(0)} + J^{(-1)}. \]  \tag{6}
with \( J^{(0)} \) defined in (5) and
\[
J^{(-1)} = J_{-1} e^{-j(k_x^{(0)} - K)} \tag{7}
\]
where \( J_{-1} \) is the constant amplitude of the \(-1\) indexed mode, and \( k_x = k_x^{(-1)} = k_x^{(0)} - K \) is the complex wavenumber of the LW (\(-1\)-mode).

The FW electric field mode is locally defined through the spectral GF of the grounded slab evaluated at the wavevector \( k_x^{(0)} \) and \( k_x^{(-1)} \) (see Fig. 3), i.e.,
\[
\begin{align*}
E_x &= E^{(0)} + E^{(-1)} \tag{8} \\
E^{(0)} &= jX_{GF}^{(0)} \cdot J^{(0)}; \quad E^{(-1)} = jX_{GF}^{(-1)} \cdot J^{(-1)} \tag{9} \\
X_{GF}^{(0)} &= -\frac{X^{+}(k_x^{(0)}) X^{-}(k_x^{(0)})}{X^{+}(k_x^{(0)}) + X^{-}(k_x^{(0)})} \tag{10} \\
X_{GF}^{(-1)} &= -\frac{X^{+}(k_x^{(-1)}) X^{-}(k_x^{(-1)})}{X^{+}(k_x^{(-1)}) + X^{-}(k_x^{(-1)})} \tag{11}
\end{align*}
\]
in which \( X^{\pm}(k_x) \) are defined in (3) and (4). These reactances are parallel to the sheet reactance.

The exact solution of the BVP with two waves only is found by introducing (6) and (8) in (1), thus obtaining
\[
X(J^{(0)} + J^{(-1)}) = X^{(0)} \cdot J^{(0)} + X^{(-1)} \cdot J^{(-1)} \tag{12}
\]
which is the transverse resonance equation for the problem. Fig. 3 shows the interpretation of (12). Due to the absence of forced excitation, the net active power balance is zero, therefore
\[
\frac{1}{2} \text{Re}[jX(J^{(0)} + J^{(-1)})(J^{(0)} + J^{(-1)})^*] = 0 \tag{13}
\]
where the left-hand side is the time average power totally furnished by the currents; this power should be zero since the power goes from the SW to the LW in any spatial period, and there are no other modes in the expansion. Equation (13) provides an implicit evidence that \( X \) is real on the dispersion curve in the absence of losses.

A two-wave solution exists if we find a solution of (12) with a real-valued reactance \( X \). To this end, we substitute the explicit values from (7) into (12) and simplify by dividing both the sides by \( e^{-jk_x^{(0)}} \), thus obtaining
\[
X(J_0 + J_{-1} e^{jKx}) = X^{(0)}_{GF} J_0 + X^{(-1)}_{GF} J_{-1} e^{jKx}. \tag{14}
\]

This equation can be rearranged as
\[
X = \frac{X^{(0)}_{GF} e^{-jKx/2} + X^{(-1)}_{GF} J_{-1} e^{jKx/2}}{e^{-jKx/2} + \frac{j}{2} e^{jKx/2}}. \tag{15}
\]
It may be seen that \( X \) assumes real values if
\[
J_{-1} = J_0 e^{-j\gamma} \tag{16} \\
X^{(-1)}_{GF} = X^{(0)*}_{GF} \tag{17}
\]
where \( \gamma \) is a constant phase. Indeed, denoting \( X^{(0)}_{GF} = a + jb \)
(15) leads to
\[
X = \frac{(a+jb)e^{-j(Kx/2-\gamma/2)} + (a-jb)e^{j(Kx/2-\gamma/2)}}{2 \cos(Kx/2 - \gamma/2)} \tag{18}
\]
and therefore, one obtains the real solution
\[
X = a + b \tan(Kx/2 - \gamma/2). \tag{19}
\]
Defining \( Kx_0 = \gamma \), one has
\[
X = \text{Re}X^{(0)}_{GF} + \text{Im}X^{(0)}_{GF} \tan \left( \frac{2\pi}{d} (x - x_0)/2 \right) \tag{20}
\]
where \( \text{Re}X^{(0)}_{GF} = X_0 \) plays the role of the average value of \( X \). We note that the reactance in (20) is defined up to an arbitrary shift \( x_0 \) of the reference system, and therefore, we can limit the investigation to \( x_0 = 0 \). The expression (20) depends on the complex parameter \( k_x^{(0)} \). Therefore, the solution of the perfect TM-to-TM conversion exists if one finds \( k_x^{(0)} \) such that \( X^{(-1)}_{GF} = X^{(0)*}_{GF} \). The physical meaning of this condition, together with (16), is substantiated in Section V.

The exact solution of the two-wave problem is, therefore, determined if one finds a complex value of \( k_x^{(0)} \) that respects the condition \( X^{(-1)}_{GF} = X^{(0)*}_{GF} \). This condition in explicit form reads
\[
\frac{1}{X^{+}(k_x^{(0)})} + \frac{1}{X^{-}(k_x^{(0)})} = \left[ \frac{1}{X^{+}(k_x^{(-1)}) - K} + \frac{1}{X^{-}(k_x^{(-1)}) - K} \right]^* \tag{21}
\]
This can be rewritten as
\[
f(k_x^{(0)}) = f^*(k_x^{(-1)} - K) \tag{22}
\]
where \( f(k_x) = \xi X^{+}(k_x) + \xi X^{-}(k_x) \), that is
\[
f(k_x) = \frac{\varepsilon_r \cot(k_0 h \sqrt{\varepsilon_r - (k_x/k_0)^2})}{\sqrt{\varepsilon_r - (k_x/k_0)^2}} - \frac{1}{j \sqrt{1 - (k_x/k_0)^2}}. \tag{23}
\]
The solution for a desired pointing angle \( \theta \) can be found as zero of a real positive function, i.e.,
\[
\left| f(\beta^{(0)}_s - j\alpha_s) - f^*(k_0 \sin \theta - j\alpha_s) \right| = 0 \tag{24}
\]
where \( \beta^{(0)}_s - K = k_0 \sin \theta \). The branch of the square root in (23) is taken in the appropriate way for the forward and backward LW, as described in Section IV.
The dispersion equation in (24) subjected to (28) and (29) is found from (20) with the use of (10) and (11) with the choice in (27) has to be as follows

\[
\frac{(\beta_x^{(0)})^2 - \alpha_z^{(0)} + (\beta_x^{(1)})^2 - \alpha_z^{(-1)}}{k_0^2} = k_0^2 \quad \text{(25)}
\]

\[
\frac{(\beta_x^{(-1)})^2 - \alpha_z^{(-1)} + (\beta_x^{(0)})^2 - \alpha_z^{(0)}}{k_0^2} = k_0^2 \quad \text{ (26)}
\]

where \(\alpha_z^{(n)} = -\text{Im}(k_x^{(n)}), \beta_x^{(n)} = \text{Re}(k_x^{(n)})(n = 0, -1)\), with

\[
k_x^{(0)} = \sqrt{k_0^2 - (k_x^{(0)})^2} \quad \text{and} \quad k_x^{(-1)} = \sqrt{k_0^2 - (k_x^{(-1)})^2} \quad \text{ (27)}
\]

Equations (25) and (26) allow for univocally defining all the parameters of the SW and LW as the functions of \(k_0^{(0)}\). According to the well-known concepts and terminology, when the signs of \(\beta_x^{(0)}\) and \(\beta_x^{(-1)}\) are the same (i.e., the wavefront propagation has the same direction as the power flow), the LW is called “forward LW” (FLW), and when they are opposite, it is called “backward LW” (BLW). While the BLW is “proper”, namely, it attenuates toward the positive \(z\)-axis (\(\alpha_z^{(-1)} = -\text{Im}(k_x^{(-1)}) > 0\)), the FLW is “improper”, namely (\(\alpha_z^{(-1)} = -\text{Im}(k_x^{(-1)}) < 0\)).

To visualize the two cases, Fig. 4 shows a sketch of the orientation of \(\text{Re}(k^{(n)}) = \beta_x^{(n)}x + \beta_z^{(n)}z\), and \(\text{Im}(k^{(n)}) = -\alpha_x^{(n)}x - \alpha_z^{(n)}z\) for the BLW [Fig. 4(a)] and FLW [Fig. 4(b)]. Accordingly, the branch of the imaginary part of the last term in (27) has to be as follows:

\[
\text{Im} \sqrt{1 - \left(\sin \theta - j\frac{\alpha_z}{k_0}\right)^2} < 0 \quad \text{ (\(\theta \leq 0\), BLW)} \quad \text{(28)}
\]

\[
\text{Im} \sqrt{1 - \left(\sin \theta - j\frac{\alpha_z}{k_0}\right)^2} > 0 \quad \text{ (\(\theta > 0\), FLW).} \quad \text{(29)}
\]

The 0-mode is obviously proper, namely

\[
\text{Im} \sqrt{1 - \left(\beta_x^{(0)} - j\alpha_z\right)^2} / k_0^2 < 0.
\]

The procedure for designing the reactance is summarized here for convenience. First, one fixes the value of the dielectric permittivity and thickness of the slab, the operating frequency, and the pointing angle \(\theta\) of the beam. Next, one finds the solution of (22) or (24) which provides \(k_x^{(0)}\). Finally, the reactance is found from (20) with the use of (10) and (11) with the choice of the branch defined in (28) and (29).

**IV. FORWARD AND BACKWARD LW**

The local dispersion equation of an inhomogeneous plane wave in the free space yields

\[
(\beta_x^{(0)})^2 - \alpha_z^{(0)} + (\beta_x^{(1)})^2 - \alpha_z^{(-1)} = k_0^2
\]

\[
(\beta_x^{(-1)})^2 - \alpha_z^{(-1)} + (\beta_x^{(0)})^2 - \alpha_z^{(0)} = k_0^2
\]

where \(\alpha_z^{(n)} = -\text{Im}(k_x^{(n)}), \beta_x^{(n)} = \text{Re}(k_x^{(n)})(n = 0, -1)\), with

\[
k_x^{(0)} = \sqrt{k_0^2 - (k_x^{(0)})^2} \quad \text{and} \quad k_x^{(-1)} = \sqrt{k_0^2 - (k_x^{(-1)})^2} \quad \text{ (27)}
\]

Equations (25) and (26) allow for univocally defining all the parameters of the SW and LW as the functions of \(k_0^{(0)}\). According to the well-known concepts and terminology, when the signs of \(\beta_x^{(0)}\) and \(\beta_x^{(-1)}\) are the same (i.e., the wavefront propagation has the same direction as the power flow), the LW is called “forward LW” (FLW), and when they are opposite, it is called “backward LW” (BLW). While the BLW is “proper”, namely, it attenuates toward the positive \(z\)-axis (\(\alpha_z^{(-1)} = -\text{Im}(k_x^{(-1)}) > 0\)), the FLW is “improper”, namely (\(\alpha_z^{(-1)} = -\text{Im}(k_x^{(-1)}) < 0\)).

To visualize the two cases, Fig. 4 shows a sketch of the orientation of \(\text{Re}(k^{(n)}) = \beta_x^{(n)}x + \beta_z^{(n)}z\), and \(\text{Im}(k^{(n)}) = -\alpha_x^{(n)}x - \alpha_z^{(n)}z\) for the BLW [Fig. 4(a)] and FLW [Fig. 4(b)]. Accordingly, the branch of the imaginary part of the last term in (27) has to be as follows:

\[
\text{Im} \sqrt{1 - \left(\sin \theta - j\frac{\alpha_z}{k_0}\right)^2} < 0 \quad \text{ (\(\theta \leq 0\), BLW)} \quad \text{(28)}
\]

\[
\text{Im} \sqrt{1 - \left(\sin \theta - j\frac{\alpha_z}{k_0}\right)^2} > 0 \quad \text{ (\(\theta > 0\), FLW).} \quad \text{(29)}
\]

The 0-mode is obviously proper, namely

\[
\text{Im} \sqrt{1 - \left(\beta_x^{(0)} - j\alpha_z\right)^2} / k_0^2 < 0.
\]

The procedure for designing the reactance is summarized here for convenience. First, one fixes the value of the dielectric permittivity and thickness of the slab, the operating frequency, and the pointing angle \(\theta\) of the beam. Next, one finds the solution of (22) or (24) which provides \(k_x^{(0)}\). Finally, the reactance is found from (20) with the use of (10) and (11) with the choice of the branch defined in (28) and (29).

**V. POWER BALANCE**

Since (16) and (17) ensure \(X\) to be real-valued, (13) implies that the currents \(J^{(0)}\) and \(J^{(-1)}\) do not complexively produce active power but only exchange power with each other in each period. In particular, the power density transported by the SW and LW contribution per unit surface is given by

\[
P_{sw} = -(1/2)\text{Re}\left\{J^{(0)*}E^{(0)}\right\} = -(1/2)\text{Im}\left\{X_{GF}^{(0)}J_0\right\}^2
\]

and

\[
P_{lw} = -(1/2)\text{Re}\left\{J^{(-1)*}E^{(-1)}\right\} = -(1/2)\text{Im}\left\{X_{GF}^{(-1)}J_{-1}\right\}^2
\]

respectively, and the reactive power stored per unit surface area is given by

\[
W_{sw} = -\frac{1}{2}\text{Re}\left\{X_{GF}^{(0)}J^{(0)*}\right\}^2
\]

and

\[
W_{lw} = -\text{Re}\left\{X_{GF}^{(-1)}J^{(-1)*}\right\}^2
\]

respectively. The conditions (16) and (17) are, therefore, equivalent to

\[
P_{sw} + P_{lw} = 0 \quad \text{(30)}
\]

\[
W_{sw} = W_{lw}. \quad \text{(31)}
\]

Equation (30) means that there is a perfect conversion of power from the SW to the LW and (31) means that there is an equal amount of energy stored in the fields of the SW and LW.

**VI. DISPERSION EQUATION**

The dispersion equation in (24) subjected to (28) and (29) can be solved numerically using the conventional MATLAB routines. In the following examples, the dielectric permittivity of the slab assumes the three values \(\varepsilon_r = 3, 6, 15\). We observe that for a fixed \(\varepsilon_r\), the solution \(k_x^{(0)}/k_0\) depends only on \(k_0h\) and \(k_0d\). Fig. 5 shows, for each permittivity, the real and imaginary part of the solution \(k_x^{(0)}/k_0\) for two cases, denoted by continuous and dashed lines, relevant to broadside radiation and a backward LW.
The solution is found for the propagating 0-mode toward the positive \( x \); the numerical process also allows for finding the solution \(-k_x^0/k_0\) which corresponds to propagation toward the negative \( x \), not plotted in the figure. The broadside-beam solution is obtained by setting \( K = \beta_x^{(0)} \), namely \( \beta^{(1)} = 0 \) in (22), solving
\[
\left| f(\beta_x^{(0)} + j\alpha_x) - f^{\ast}(j\alpha_x) \right| = 0. \tag{32}
\]
The backward LW solution, radiating at 30° direction (dashed lines), where the angle is counted counterclockwise from the normal to the surface, is obtained by setting \( \beta^{(1)} = -k_0 \sin(30^\circ) \), namely, \( \beta^{(0)} - K = -k_0 \sin(30^\circ) \). From (24), this is equivalent to finding a solution of
\[
\left| f(\beta_x^{(0)} + j\alpha_x) - f^{\ast}(-\frac{1}{k_0} + j\alpha_x) \right| = 0 \tag{33}
\]
with the choice of the branch in (28). The blue and red curves in Fig. 5 denote Re\((k_x^{(0)}/k_0) = \beta_x^{(0)}/k_0\) and \(-\text{Im}(k_x^{(0)}/k_0) = \alpha_x/k_0\), respectively, as the functions of the thickness of the slab within the range \( h/\lambda \in (0.02 - 0.12) \). The calculations have been carried out at 10 GHz; it is understood, however, that one can rescale the solution for a different frequency. The period which allows for the broadside beam is \( d = 2\pi/\text{Re}(k_x^{(0)}) \).

The results show that, by changing the thickness of the slab, one can change the leakage rate and, eventually, the pattern beamwidth and directivity of the antenna. It is interesting to note that, in the case \( \varepsilon_r = 15\), for a fixed period corresponding to a certain desired radiation direction and a desired decay rate \( \alpha \), the surface can be realized by two different reactance profiles using different thicknesses \( h \) and, therefore, different average impedances \( X_0 = \text{Re}X_{GR}^{(0)} \). For instance, for the broadside case in Fig. 5, the same \( d \) and \( \alpha \) are found at \( h/\lambda = 0.1 \) and \( h/\lambda = 0.02 \) with two different periodic MTS reactances. For both the cases in Fig. 5, the broadside one and the backward propagation at the angle \( \theta = 30^\circ \), we observe that for \( h/\lambda < 0.06 \), the average impedance is inductive, and for \( h/\lambda > 0.06 \), it is capacitive.

Fig. 6 shows the reactances as obtained from (20) (with \( x_0 = 0 \)). In the two cases, the average reactance \( \text{Re}X_{GR}^{(0)} = X_0 \) is negative (capacitive) or positive (inductive).

VII. CONSIDERATIONS ON PRACTICAL IMPLEMENTATION

In the exact solution (20), a tangent function appears which exhibits periodic singularities. These singularities can be approximated by printing elements which exhibit resonance of an equivalent LC circuit of parallel type, for instance, printed slots close to the resonance. As an example, we take the reactance solution of Fig. 6 for \( h/\lambda = 0.02 \) as shown in Fig. 7 together with a layout of slots that emulate the required impedance. We note that, in this case, the poles and zeros of the solution are very close to each other. The slots are orthogonal to the propagation direction, and therefore, the model is valid only for propagation along \( x \). At the beginning of the period, the slots are very small and gradually become longer. When the slot becomes resonant, the impedance becomes infinite. At the end of the period, they become small again with a discontinuity. It is worth noting that implementing a solution with capacitive average values (e.g., the one in Fig. 6 for \( h/\lambda = 0.1 \)) requires printed dipoles, which actually exhibit a series-type resonant behavior at the first resonance; therefore, the printed dipoles should be used at the second resonance. Furthermore, the dipoles should be directed for this TM case in the direction of propagation; therefore, reproducing the impedance profile is critical. We are investigating a way to do it by using the convoluted dipoles.

It is also worth noting that for the slot implementation, the approximation of homogeneous reactance is not a worry, while it could be significant to assess how sensitive the solution is to the accuracy of the representation of the desired profile close to the singularities. To this end, a sensitivity analysis to the deviation of the modulation profile with respect to the ideal one is studied in Section VIII-B on the basis of the full-wave analysis carried out in Section VIII-A.

VIII. FULL-WAVE ANALYSIS AND VALIDATION

A. Full-Wave Analysis of Arbitrary Periodic Reactance

In order to test the solution and perform the sensitivity analysis, we have developed a full-wave code for arbitrary homogenized penetrable reactance \( X(x) \) with period \( d \). The algorithm has been formulated by a periodic method of moments, in which both the currents and fields are represented.
through FW basis functions. This analysis is a generalization to the penetrable impedance and the arbitrary periodic profile of the Oliner’ method, which was presented in [5] for sinusoidal periodic Leontovitch BCs. In the Appendix, it is shown that this generalization leads to a solution of the following linear algebraic infinite system of equations for the infinite number of unknowns $J_m$, which are the coefficients of the FW expansion of the currents that flow on the MTS impedance

$$\sum_m \chi_{n,m} J_m = 0 \quad n = 1, 2 \ldots \infty$$  \hspace{1cm} (34)

where $k_n^{(m)} = k_n^{(0)} - 2\pi n/d$, $\delta_{n,m}$ is the Kronecker delta, and $X_p$ are the coefficients of the Fourier series expansion of the arbitrary periodic impedance. Equation (34) should be truncated with a sufficiently high number of terms in order to reduce to a finite algebraic system, which admits solution only if $\det[\chi_{n,m}] = 0$; the latter is the dispersion equation which identifies the complex value $k_n^{(0)}$ for the particular periodic reactance $X(x)$. Once the complex solution $k_n^{(0)}$ is found, one can derive the coefficients of the Floquet mode expansion as the entries of the eigenvector associated with the vanishing eigenvalue of the matrix $[\chi_{n,m}]$.

### B. Sensitivity Analysis

The solution of (34) is used here to investigate the distribution of modal coefficients when the singularity of the tangent function profile is not reproduced well by practical elements close to the singularity. The analysis has been performed for a slab of relative permittivity $\varepsilon_r = 15$ and $h/\lambda = 0.08$, and the LW direction is broadside. Assuming a reactance of type $X(x) = a + b \tan(\pi x/d)$ and analyzing it through the full-wave code, we have found first that the solution of $\det[\chi_{n,m}] = 0$ is equivalent to imposing $X_{GF}(k_n^{(0)}) = X_{GF}^*(k_n^{(-1)}) = a + jb$ and then that the eigenvector of the vanishing eigenvalue has only two components ($n = 0$ and $n = -1$). The details of this demonstration are given in the Appendix. The amplitude of the FW current coefficients is shown in Fig. 8(a), with the reactance profile in the inset. Fig. 8(a)–(c) show the results when $X(x) = a + b \tan(\pi x/d)$ is approximated using the Taylor expansion around $x = 0$ of the tangent function, truncating the result in the period and repeating periodically the resulting function.

The tangent function is approximated with $N$ terms of the Taylor expansion with $N = 2, 4, 6$ [Fig. 8(a)–(c)] and next compared with the ideal profile [Fig. 8(d)] outcome from the full-wave analysis. This approximation emulates the case in which the resonance is not sampled properly by the elements. It is seen that decreasing the accuracy of the approximation close to the resonance, the amplitudes of the coefficients of the higher-order FWs increase, and correspondingly, the amplitude of the $(-1)$ mode coefficient decreases with respect to the amplitude of the $(0)$ mode. This means that the energy per unit cell transferred from the 0 mode to the $(-1)$ mode decreases.

The same exercise is repeated for $\varepsilon_r = 3$ and $h/\lambda = 0.08$ in Fig. 9 (the perfect approximation case is not presented since it gives the exact two-mode approximation similar to the one in Fig. 8(d)). The results show that the situation appears less critical with respect to the case with higher permittivity in Fig. 8. It should be observed that changing the approximation around
coupling in the absence of losses. The found impedance can be synthesized in order to ensure a certain beam direction and decay rate. Along the direction of propagation, the solution of transparent impedance exhibits the alternation of positive and negative values around a certain average, passing through a resonance at each period. This suggests that a possible realization of this impedance can consist of a periodic distribution of printed subwavelength elements, where some are close to resonance, inside a distribution of nonresonant subwavelength printed elements.

It is also worth noting that the achievement of the two-mode only solution can have some drawbacks in terms of design flexibility. In fact, changing the leakage parameter to control only solution can have some drawbacks in terms of design flexibility. In changing the leakage parameter to control the singularity also changes the beam direction. Although this change can be an inaccuracy in sampling the ideal profile of the MTS impedance on a grounded dielectric slab, which has been found by imposing a reactive balance of the wave only solution can have some drawbacks in terms of design flexibility. In fact, changing the leakage parameter to control the singularity also changes the beam direction. Although this change can be an inaccuracy in sampling the ideal profile of the MTS impedance on a grounded dielectric slab, which has been found by imposing a reactive balance of the wave

IX. Conclusion

We have presented a solution of a periodic isotropic transparent MTS impedance on a grounded dielectric slab, which corresponds to a perfect conversion from a TM-polarized SW to a TM-polarized LW without higher-order Floquet harmonics in the exact expansion of the field solution. This solution has been found by imposing a reactive balance of the wave

\[ X(x) \sum_q J_q e^{-jk_{q,x}x} = \sum_p X_{GF}(k_{q,p}) J_p e^{-jk_{q,p}x} \]  

(36)

where \( X_{GF}(k_x) \) is the GF defined in (10), and \( k_{q,p} = k_{q,0} - (2\pi p/d) \) are the Floquet-mode wavenumbers. Expanding the reactance in Fourier series \( X(x) = \sum_m X_m e^{-j2\pi m/d}x \) in (36) leads to

\[ \sum_{m,q} J_q X_m e^{-j2\pi m+q}x = \sum_p X_{GF}(k_{q,p}) J_p e^{-j2\pi x}. \]  

(37)

Here, the factor \( k_{q,p}^{(0)} = k_{q,0} - 2\pi p/d \) has been deleted as present on both the sides of (36). We note that both sides of (37) are periodic functions of a period \( d \). The left-hand side term can be, therefore, expanded in Fourier series by projecting on the term \( \exp(j2\pi n/d)x \). This leads to Fourier expansion coefficient equal to \( \sum_m X_m X_{n-m} \), namely the convolution between the coefficient expansion between reactance and currents. Since these coefficients must be equal to the ones at right-hand side of (37), one obtains

\[ \sum_m J_m X_{n-m} = X_{GF}(k_{q,n}) J_n \]  

(38)

which is a system with an infinite number of equations and unknowns. The above can be easily rearranged as in (34) and (35). Equation (38) can be also used to check the validity of the solution when \( X(x) = a + b \tan(\pi x/d) \). In this case, the expression of the Fourier coefficients is \( X_p = a\delta_{p,0} - jb(-1)^p \text{sgn}(p) \), where \( \delta_{p,0} \) is the Kronecker delta and \( \text{sgn}(p) = 1 \) for \( p \) positive, \(-1 \) for \( p \) negative, and \( 0 \) for \( p = 0 \). We can specialize (38) for \( n \) and \( n - 1 \) and add the two equations, thus obtaining

\[ \sum_m (X_{n-m} + X_{n-1-m}) J_m = X_{GF}(k_{n,0}) I_n + X_{GF}(k_{n-1}) J_{n-1}. \]  

(39)

With the defined values of \( X_p \), all the terms of the summation exactly cancel each other except for the values \( n = m \) and

\[ \beta_{n,m}^{(0)} k_0 = 1.48 - j0.6 \]  

for \( \varepsilon_r = 15 \), and \( \beta_{n,m}^{(1)} k_0 = 1.15 - j0.08 \) for \( \varepsilon_r = 3 \).

The spectrum of FW amplitude of currents flowing in the metasurface when the reactance deviates from \( (x) = a + b \tan(\pi x/d) \) is shown in Fig. 9. Only two cases of Taylor approximations are shown. The value of the perfect case \( k_0 = 1 \) for the cases \( \varepsilon_r = 3 \) and \( \varepsilon_r = 15 \) leads to

\[ \frac{\beta_{n,m}^{(0)}}{k_0} \]  

for \( \varepsilon_r = 3 \) and \( \varepsilon_r = 15 \).
\[n - 1 = m,\] thus leading to
\[(a + jb)J_0 + (a - jb)J_{n-1} = X_{GF}(k_x^{(n)})J_0 + X_{GF}(k_x^{(n-1)})J_{n-1}.\] (40)

The latter can be rewritten as
\[X_{GF}(k_x^{(n)}) - (a + jb)J_0 + X_{GF}(k_x^{(n-1)}) - (a - jb)J_{n-1} = 0\] (41)

which must be satisfied for any \(n\). Specializing the latter expressions for \(n = 0\), one has
\[X_{GF}(k_x^{(0)}) - (a + jb)J_0 + X_{GF}(k_x^{(-1)}) - (a - jb)J_{-1} = 0\] (42)

which is identically satisfied when
\[X_{GF}(k_x^{(0)}) = X_{GF}(k_x^{(-1)}) = (a + jb).\] (43)

This result recovers the expression found in Section III. Furthermore, specializing (41) for \(n = 1\) and \(n = -1\) and using (43), one gets \(J_1 = 0\) and \(J_{-2} = 0\), respectively. Proceeding iteratively for all \(n\), one finds that the \(I_n\) vanish for \(n \neq 0\) and \(n \neq -1\). This demonstrates that equations (40) implies only two FWs in the solution for the electric currents.

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Enrica Martini (Senior Member, IEEE) was born in Spilimbergo (PN), Italy, in 1973. She received the Laurea degree (cum laude) in telecommunication engineering from the University of Florence, Florence, Italy, in 1998, and the Ph.D. degree in informatics and telecommunications from the University of Florence and the Ph.D. degree in electronics from the University of Nice-Sophia Antipolis, Nice, France, under joint supervision.

She worked under a one-year research grant from the Alenia Aerospazio Company, Rome, Italy, until 1999. In 2002, she was a Research Associate at the University of Siena, Siena, Italy. In 2005, she received the Hans Christian Ørsted Postdoctoral Fellowship at the Technical University of Denmark, Lyngby, Denmark, and she joined the Electromagnetic Systems Section of the Ørsted DTU Department until 2007. From 2007 to 2017, she was a Post-Doctoral Fellow at the University of Siena. From 2016 to 2018, she was the CEO of the start-up Wave Up Srl, Siena, that she co-founded in 2012. She is currently an Assistant Professor with the University of Siena. Her research interests include metasurfaces, metamaterial characterization, electromagnetic scattering, antenna measurements, finite-element methods, and tropospheric propagation.

Dr. Martini was a co-recipient of the 2016 Schellkunoff Transactions Prize Paper Award and of the Best Paper Award in Antenna Design and Applications at the 11th European Conference on Antennas and Propagation.

Sergei A. Tretyakov (Fellow, IEEE) received the Dipl. Engineer-Physicist, the Candidate of Sciences (Ph.D.), and the Doctor of Sciences degrees in radiophysics from the St. Petersburg State Technical University, St. Petersburg, Russia, in 1980, 1987, and 1995, respectively.

From 1980 to 2000, he was with the Radiophysics Department, St. Petersburg State Technical University. He is currently a Professor of radio science with the Department of Electronics and Nanoengineering, Aalto University, Espoo, Finland. He has authored or coauthored five research monographs and more than 300 journal articles. His main scientific interests are electromagnetic field theory, complex media electromagnetics, metamaterials, and microwave engineering.

Dr. Tretyakov served as a President of the Virtual Institute for Artificial Electromagnetic Materials and Metamaterials (“Metamorphose VI”), as a General Chair, International Congress Series on Advanced Electromagnetic Materials in Microwaves and Optics (Metamaterials), from 2007 to 2013, and as a Chairman of the St. Petersburg IEEE ED/MTT/AP Chapter from 1995 to 1998.

Stefano Maci (Fellow, IEEE) received the Laurea degree (cum laude) from the University of Florence, Florence, Italy, in 1987.

Since 1997, he has been a Professor with the University of Siena, Siena, Italy. The research activity of him is documented in 160 articles published in international journals, (among which 100 on IEEE journals), 10 book chapters, and about 400 articles in proceedings of international conferences. These articles have received around 7100 citations with h index 42. His research interest includes high-frequency and beam representation methods, computational electromagnetics, large phased arrays, planar antennas, reflector antennas and feeds, metamaterials, and metasurfaces.

Mr. Maci was a member of the Technical Advisory Board of 12 international conferences and has been a member of the Review Board of six International Journals, since 2000. He organized 25 special sessions in international conferences, and he held ten short courses in the IEEE Antennas and Propagation Society (AP-S) Symposia about metamaterials, antennas and computational electromagnetics. In 2004–2007, he was a WP leader of the Antenna Center of Excellence (ACE, FP6-EU) and in 2007–2010, he was International Coordinator of a 24-institution consortium of a Marie Curie Action (FP6). He has been a Principal Investigator since 2010 of six cooperative projects financed by the European Space Agency. In 2004, he was the Founder of the European School of Antennas (ESoA), a postgraduate school that presently comprises 34 courses on Antennas, Propagation, Electromagnetic Theory, and Computational Electromagnetics and 150 teachers coming from 15 countries.

Since 2004, he has been the Director of ESoA. He has been a former member of the AdCom of IEEE Antennas and Propagation Society (AP-S), an Associate Editor of AP-Transaction, the Chair of the Award Committee of IEEE AP-S, and a member of the Board of Directors of the European Association on Antennas and Propagation (EurAAP). From 2008 to 2015, he was the Director of the Ph.D. program in information engineering and mathematics with the University of Siena, and from 2013 to 2015, he was a member of the National Italian Committee for Qualification to Professor. He has been a former member of the Antennas and Propagation Executive Board of the Institution of Engineering and Technology (IET, UK). He is the Director of the consortium FORESEEN, presently involving 48 European Institutions, and principal investigator of the Future Emerging Technology project “Nanarchitelectronics” of the 8th EU Framework program. He was a co-founder of two Spin-off companies. He was a Distinguished Lecturer of the IEEE Antennas and Propagation Society (AP-S), and the EuRAAP Distinguished Lecturer in the ambassador program. He was a recipient of the EurAAP Award in 2014, the IEEE Schellkunoff Transaction Prize 2015, the Chen-To Tai Distinguished Educator Award 2016, and the URSI Dellinger Gold Medal in 2020. He is a TPC Chair of the METAMATERIAL 2020 conference. In the last ten years, he has been invited 25 times as keynote speaker in international conferences.