E and B families of the Stokes parameters in the polarized synchrotron and thermal dust foregrounds

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Abstract. Better understanding of Galactic foregrounds is one of the main obstacles to detection of primordial gravitational waves through measurement of the B mode in the polarized microwave sky. We generalize the method proposed in [1] and decompose the polarization signals into the E and B families directly in the domain of the Stokes $Q, U$ parameters as $(Q, U) \equiv (Q_E, U_E) + (Q_B, U_B)$. This also enables an investigation of the morphology and the frequency dependence of these two families, which has been done in the WMAP K, Ka (tracing synchrotron emission) and Planck 2015 HFI maps (tracing thermal dust). The results reveal significant differences in spectra between the E and B families. The spectral index of the E family fluctuates less across the sky than that of the B family, and the same tendency occurs for the polarization angles of the dust and synchrotron channels. The new insight from WMAP and Planck data on the North Polar Spur and BICEP2 zones through our method clearly indicates that these zones are characterized by very low polarization intensity of the B family compared to the E family. We have detected global structure of the B family polarization angles at high Galactic latitudes which cannot be attributed to the cosmic microwave background or instrumental noise. However, we cannot exclude instrumental systematics as a partial contributor to these anomalies.

Keywords: CMBR experiments, gravitational waves and CMBR polarization

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1 Introduction

The next generation of cosmic microwave background (CMB) experiments will focus on detection of the B mode of polarization, which is a unique indicator of the presence of cosmological gravitational waves in the universe [2–5]. The theory of the generation of the CMB polarization predicts that in the absence of cosmological gravitational waves, the B mode is equal to zero except for the lensing effect. The gravitational waves in a cosmological plasma are usually characterized by the so-called tensor-to-scalar ratio $r$, which is constrained to $r < 0.07$ at a 95% confidential level in [6]. This constraint reflects the present status of our knowledge of Galactic and extra-galactic foregrounds, including synchrotron radiation, thermal dust emission (TDE), free-free radiation, anomalous dust emission (AME), as well as contamination from systematic effects. Bearing in mind the target of $r \sim 10^{-4}$ to $10^{-3}$, we should understand the properties of the strongly polarized foregrounds (synchrotron and TDE) at least 2–3 orders of magnitude better than now. Besides, we should not forget the potentially weak, but not negligible polarization of the free-free and AME components, which are poorly understood.

Better understanding of the polarized sky emission requires more detailed analysis of the morphology of the foregrounds, including local and global features of the synchrotron and TDE signals. Efficient removal of the foregrounds after cleaning depends on the method

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1 Introduction

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Better understanding of the polarized sky emission requires more detailed analysis of the morphology of the foregrounds, including local and global features of the synchrotron and TDE signals. Efficient removal of the foregrounds after cleaning depends on the method
applied and is limited not only by the absolute value of polarized amplitudes, but also by the variation of spectral indices. This problem is closely related to the proper usage of sky masks, which are implemented in WMAP [7] and Planck [8] data analysis. These masks are designed to remove the brightest zones of the polarization intensity from the analysis, in order to minimize the leakage from the foregrounds to the derived polarized CMB.

Implementation of the E-B decomposition for the pure cosmological CMB signal is practically useful, since in absence of primordial gravitational waves the B mode vanishes. However, in reality the polarized CMB sky consists of a mix of the primordial CMB, the Galactic and extra-galactic foregrounds, the E mode lensing, and residuals of systematic effects (uncertainties of the antenna beam shape, possible bandpass leakage from temperature anisotropy and the foregrounds, etc.). This is why a priori it is not clear what kind of representation (Q-U or E-B) will be most suitable for cleaning of the data for different frequency bands in order to extract the cosmological signal.

In this paper we would like to highlight a novel method based on decomposition of the Stokes parameters $Q$ and $U$ into the $(Q_E, U_E)$ and $(Q_B, U_B)$ families, associated with the E and B modes respectively,\(^1\) which satisfy $(Q, U) = (Q_E, U_E) + (Q_B, U_B)$. Based on this representation we will investigate the polarization intensity maps for the E family and the B family,

$$P_E = \sqrt{Q_E^2 + U_E^2}, \quad P_B = \sqrt{Q_B^2 + U_B^2}, \quad (1.1)$$

in combination with the corresponding polarization angles $\theta_E = 0.5 \arctan(U_E, Q_E)$ and $\theta_B = 0.5 \arctan(U_B, Q_B)$. We show that these estimators reveal new properties of the polarized synchrotron emission and TDE in the WMAP K and Ka maps and the Planck 217 and 353 GHz maps.\(^2\)

We will show that both the E and B family foregrounds have a relatively stable structure of polarization angles across the K, Ka and 217, 353 GHz frequencies, as well as confirm their existence against random noise with high significance. For the whole sky the global structure of the synchrotron and TDE polarization angles, $\theta_E^{\text{sync}}$ and $\theta_E^{\text{dust}}$, reveals the existence of large angular structure, which correlates with the Galactic and interstellar magnetic field.

Namely, we introduce the ratio $\rho(n) = P_E(n)/P_B(n)$ in each sky direction $n$ and look at the distribution functions $F(\rho)$ for the whole sky in the WMAP K, Ka and Planck 30, 44 GHz maps. We find remarkable similarity of $F(\rho)$ for the WMAP and Planck polarization maps when $\rho \gg 1$, which allows us to determine the specific zones of the sky where $P_E(n) \gg P_B(n)$. These zones are localized along the northern part of the Loop I, and include the BICEP2 zone.

We will investigate the frequency dependence of the E and B families via their spectral indices and identify some distinct patterns of variation of $\beta$, $\beta_E$, and $\beta_B$ corresponding to the total intensity $P$, $P_E$, and $P_B$. The $\beta_E$ index is significantly less varied across the full sky, and especially in the NPS and BICEP2 zones. For the B family the variations of the spectral index in the same angular domains are much more pronounced. This makes the removal of the E family foregrounds (either synchrotron or dust) more stable, unlike the B family foregrounds. This is also potentially important for investigation of the E to B excess problem [9, 10].

Our method is sensitive enough for detection of the systematic anomalies of the Planck 30, 44 GHz maps compared to the low frequency WMAP K, Ka maps. These anomalies

\(^1\)We use “family” for the $(Q_E, U_E)$ and $(Q_B, U_B)$ vector maps to distinguish them from the scalar E or B components of polarization.

\(^2\)Obviously, 217 and 353 GHz are not the best tracers of TDE. It would be much better to use 353 and 545 GHz maps, or even include 857 GHz. However, only the 217 and 353 GHz maps are presented with polarization data in the Planck public data release.
Figure 1. An illustration of the E and B modes. For each mode, only two polarization directions (arms of the cross) are allowed in respect to the normal vector (arrow). Only one radial direction is plotted for example, and a complete E or B mode is made up of all its rotations. Therefore both E and B modes are rotationally invariant by design.

manifest themselves as a systematic shift of the distribution function $F(\rho)$ for $\rho \geq 2$ in both the 30 and 44 GHz maps related to the bandpass leakage correction of the Planck polarization.

In this work, we use $N_{\text{side}} = 128$ resolution and $2^\circ$ smoothing by default. The outline of our paper is the following: in section 2 we will define the E and B families of the Stokes parameters as a result of linear convolution of $Q$ and $U$ through the projection operator $W(n, n')$ and present the relationship $Q_E, U_E \rightarrow E$, $Q_B, U_B \rightarrow B$. Section 3 is devoted to investigation of the morphology of the E and B families of the synchrotron and the thermal dust foregrounds. We discuss the angular distribution of the polarization angles and the corresponding intensity across the sky, and identify the peculiar zones with low ratio $\rho$. In section 4 we analyze the frequency dependence of the E and B family foregrounds and show that the E family is characterized by lesser variation of the spectral indices compared to the B family, and in section 5 the frequency dependence is represented by the spectral index, and we focus on two particular zones of the sky, related to the North Polar Spur and BICEP2 zone, and derive the angular distribution and variation of the spectral indices in them. We summarize our results in section 6.

2 The E and B families of the Q and U Stokes parameters

2.1 Stokes parameters and basic definitions

The state of polarization is described by the Stokes parameters $Q$ and $U$. Since Thompson scattering does not generate circular polarization, $Q$ and $U$ are sufficient to describe the CMB polarization [11]. In a sky observation, the $Q$ and $U$ Stokes parameters are separated by a relative and constant $45^\circ$ rotation around the line of sight (LOS), but the reference frame is allowed to rotate freely around the LOS. However, if one chooses to bind the reference frame with a family of normal vectors of a given rotation, then $Q$ and $U$ are directly related to the E and B modes, as illustrated in figure 1 in the pixel domain.
Rotation of $Q$ and $U$ by an angle $\psi$ on the plane perpendicular to the direction of $\hat{n}$ is given by \cite{12, 13}

$$
(Q \pm iU)'(\hat{n}) = e^{\mp 2i\psi}(Q \pm iU)(\hat{n}).
$$

(2.1)

The separation into E and B modes can be derived from the Stokes parameters \cite{14–17}. Below we briefly review the standard approach.

In the context of rotation, $Q$ and $U$ can be decomposed into spin $\pm 2$ spherical harmonics \cite{12, 13} as follows:

$$
Q(\hat{n}) \pm iU(\hat{n}) = \sum_{l,m} a_{\pm 2,lm} \pm 2 Y_{lm}(\hat{n}),
$$

(2.2)

where $\pm 2 Y_{lm}(\hat{n})$ are the spin $\pm 2$ spherical harmonics, and the coefficients $a_{\pm 2,lm}$ are given by

$$
a_{\pm 2,lm} = \int (Q(\hat{n}) \pm iU(\hat{n})) \pm 2 Y_{lm}^*(\hat{n}) d\hat{n}.
$$

(2.3)

The E and B modes in harmonic space are then formed by

$$
E_{lm} = \frac{1}{2} \left( \sum_{lm} a_{2,lm} Y_{lm} + \sum_{lm} a_{-2,lm} Y_{lm} \right),
$$

$$
B_{lm} = \frac{i}{2} \left( \sum_{lm} a_{2,lm} Y_{lm} - \sum_{lm} a_{-2,lm} Y_{lm} \right).
$$

(2.4)

(2.5)

As we have pointed out in section 1, it is not clear a priori what kind of representation of the polarized signal ($Q, U$ or $E, B$) will be most suitable for cleaning of the data in different frequency bands in order to extract the cosmological component. The properties of the E and B modes derived from the following decomposition can help to improve cleaning using multi-frequency methods.

### 2.2 Polarization intensity of the E and B modes

For analysis of the polarized foregrounds, it is particularly important to study the polarization intensity

$$
P = \sqrt{Q^2 + U^2}
$$

(2.6)

(for convenience, $\hat{n}$ is suppressed from now on). Consequently, it is also important to study the polarization intensity due to the E and B modes respectively. However, the EB decomposition provided in eq. (2.5) can not be directly transformed to a polarization intensity, thus an alternative way is introduced as follows.

Eq. (2.2) can be solved for $Q$ and $U$, giving

$$
Q = \frac{1}{2} \left( \sum_{l,m} a_{2,lm} 2 Y_{lm} + \sum_{l,m} a_{-2,lm} -2 Y_{lm} \right),
$$

$$
U = \frac{1}{2i} \left( \sum_{l,m} a_{2,lm} 2 Y_{lm} - \sum_{l,m} a_{-2,lm} -2 Y_{lm} \right).
$$

(2.7)
In the particular case when $a_{B,lm} = 0$ or $a_{E,lm} = 0$, one gets the contribution to $Q$ and $U$ from only the E or B mode respectively. According to eq. (2.4), when $a_{B,lm} = 0$, one gets

$$-a_{E,lm} = a_{2,lm} = a_{-2,lm},$$

(2.8)

and when $a_{E,lm} = 0$, one gets

$$-ia_{B,lm} = a_{2,lm} = -a_{-2,lm}.$$  

(2.9)

We define $F_{+,lm}$ and $F_{-,lm}$ as

$$F_{+,lm} = -\frac{1}{2} (2Y_{lm} + 2Y_{lm}), \quad F_{-,lm} = -\frac{1}{2i} (2Y_{lm} - 2Y_{lm}).$$

(2.10)

Then, by setting $a_{B,lm} = 0$ and combining eqs. (2.7) and (2.8), one gets the $Q$ and $U$ parameters due to the E mode:

$$Q_E = \sum_{l,m} a_{E,lm} F_{+,lm}, \quad U_E = \sum_{l,m} a_{E,lm} F_{-,lm}.$$  

(2.11)

Similarly, the Stokes parameters that are only from the B mode are

$$-Q_B = \sum_{l,m} a_{B,lm} F_{-,lm}, \quad U_B = \sum_{l,m} a_{B,lm} F_{+,lm}.$$  

(2.12)

Since the E and B modes are components of linear decomposition of the input map, the corresponding Stokes parameters satisfy

$$Q = Q_E + Q_B, \quad U = U_E + U_B.$$  

(2.13)

Therefore, one can calculate the polarization intensity that is only from the E or B mode:

$$P_E = \sqrt{Q_E^2 + U_E^2}, \quad P_B = \sqrt{Q_B^2 + U_B^2}.$$  

(2.14)

The E and B decomposition for the $Q$ and $U$ Stokes parameters is linear and orthogonal while the polarization intensity is quadratic and consequently nonlinear. Combining eqs. (2.6), (2.13) and (2.14), one gets

$$P^2 = Q^2 + U^2 = P_B^2 \left(1 + \rho^2 + 2\rho \cos(2\theta_E - 2\theta_B)\right),$$

(2.15)

where $\rho = P_E/P_B$, and

$$\theta_{E,B} = \frac{1}{i} \arctan(U_{E,B}, Q_{E,B})$$

(2.16)

are the polarization angles for the E and B families. Defining a quadratic difference of the polarization intensity as

$$\Delta = P^2 - (P_E^2 + P_B^2) = 2(Q_EQ_B + U_EU_B),$$

(2.17)

and substituting eqs. (2.11)–(2.12), one gets:

$$\Delta = 2 \sum_{lml'm'} a_{E,lm} a_{B,l'm'}^* G_{lml'm'},$$

(2.18)

where

$$G_{lml'm'} = F_{-,lm} F_{+,l'm'}^* - F_{+,lm} F_{-,l'm'}^*.$$  

(2.19)

Therefore, $\Delta$ is completely determined by the cross quadratic term $a_{E,lm} a_{B,l'm'}^*$ between the E and B modes.
2.3 E/B family decomposition in the pixel domain

The method presented above for decomposing \( Q \) and \( U \) into the E and B families \( Q_E, U_E \) and \( Q_B, U_B \) is applicable for the full sky analysis. However, for data sets with partial sky coverage or defects (stripes, missing data in the pixels, and others), the E and B families can be defined locally in the pixel domain through linear convolution of the Stokes parameters. In this subsection we will derive the corresponding approach from the full sky approach, using some general relations presented in the previous section. Then, the final representation of the linear filter in the pixel domain can be easily generalized for the incomplete sky model.

By combining eqs. (2.3)–(2.4) with eq. (2.10), we get

\[
    a_{E,lm} = -(a_{2,lm} + a_{-2,lm})/2 = \int (Q(\hat{n})F^{*}_{+,lm}(\hat{n}) - U(\hat{n})F^{*}_{-,lm}(\hat{n})) \, d\hat{n},
\]

\[
    a_{B,lm} = i(a_{2,lm} - a_{-2,lm})/2 = \int (Q(\hat{n})F^{*}_{-,lm}(\hat{n}) + U(\hat{n})F^{*}_{+,lm}(\hat{n})) \, d\hat{n}. \tag{2.20}
\]

Then we substitute eqs. (2.11)–(2.12), giving

\[
    Q_E(\hat{n}) = \int (Q(\hat{n}')G_1(\hat{n}, \hat{n}') - U(\hat{n}')G_2(\hat{n}, \hat{n}')) \, d\hat{n}',
\]

\[
    U_E(\hat{n}) = \int (Q(\hat{n}')G_3(\hat{n}, \hat{n}') - U(\hat{n}')G_4(\hat{n}, \hat{n}')) \, d\hat{n}',
\]

\[
    -Q_B(\hat{n}) = \int (Q(\hat{n}')G_4(\hat{n}, \hat{n}') + U(\hat{n}')G_3(\hat{n}, \hat{n}')) \, d\hat{n}',
\]

\[
    U_B(\hat{n}) = \int (Q(\hat{n}')G_2(\hat{n}, \hat{n}') + U(\hat{n}')G_1(\hat{n}, \hat{n}')) \, d\hat{n}', \tag{2.21}
\]

where we have defined

\[
    G_1(\hat{n}, \hat{n}') = \sum_{l,m} F_{+,lm}(\hat{n})F^{*}_{+,lm}(\hat{n}'), \quad G_2(\hat{n}, \hat{n}') = \sum_{l,m} F_{+,lm}(\hat{n})F^{*}_{-,lm}(\hat{n}'),
\]

\[
    G_3(\hat{n}, \hat{n}') = \sum_{l,m} F_{-,lm}(\hat{n})F^{*}_{+,lm}(\hat{n}'), \quad G_4(\hat{n}, \hat{n}') = \sum_{l,m} F_{-,lm}(\hat{n})F^{*}_{-,lm}(\hat{n}'). \tag{2.22}
\]

which are pure real-space functions, whose physical meaning is the combined two-point correlations for all spin-2 harmonics. A more simple matrix form of eq. (2.21) can then be given as:

\[
    \begin{bmatrix}
        Q_E \\
        U_E \\
        Q_B \\
        U_B
    \end{bmatrix}(\hat{n}) = \int \begin{bmatrix}
        +G_1, & G_2 \\
        G_3, & +G_4
    \end{bmatrix}(\hat{n}, \hat{n}') \begin{bmatrix}
        Q \\
        -U
    \end{bmatrix}(\hat{n}') \, d\hat{n}', \tag{2.23}
\]

According to eqs. (2.21) and (2.23), the E/B family decomposition of the Stokes parameters can be done purely in the pixel space over the area of the data available. This provides an important direction for a pure real space determination of the E and B modes from partial sky coverage. However, in the case of an incomplete sky, there is inevitably the problem of E/B leakage, and the decomposition is partially ambiguous. In order to alleviate E/B leakage, the window function for the cut sky can be smoothed along its edge (see, for example, [17]).
Figure 2. Maps of the polarization angles. From top to bottom: the WMAP K-band, Ka-band (synchrotron), and the Planck 217, 353 GHz (dust). From left to right: no E/B separation, the E family ($\theta_E$), and the B family ($\theta_B$). The color-to-angle mapping is given by the lower-left color disc. The black contour lines mark the North Polar Spur and the BICEP2 zones.

3 Morphology of the E and B foreground families

In this section we work with the polarization angles of the E and B families. These polarization angles are plotted in figure 2. They are shown for the WMAP K and Ka bands (representing synchrotron emission) and the Planck 217 GHz and 353 GHz bands (representing TDE). For each band we calculate the total polarization angle and also the polarization angles $\theta_E$ and $\theta_B$ associated with only the E and B modes respectively (see eq. (2.16)).

3.1 Polarization angles of the E/B families: simple difference

In order to characterize the residuals of the polarization angles between the K, Ka and 217 GHz, 353 GHz bands we use the absolute polarization angle difference, defined by

$$\Delta \theta = |0.5 \arctan[\sin(2\theta_1 - 2\theta_2), \cos(2\theta_1 - 2\theta_2)]|.$$  \hspace{1cm} (3.1)

The result of the calculation is presented in figures 3 and 4 for K-Ka and 217–353 GHz respectively. All the cases show remarkable similarity of the polarization angles for the
Figure 3. The differences of the polarization angles between the K and Ka bands for the BICEP2 zone (upper row) and the NPS zone (lower row). Note that the Ka band is apparently affected by systematics. From left to right: no separation for Q and U, only E family ($Q_E$ and $U_E$), and only B family ($Q_B$ and $U_B$). The inset histograms show the distributions of values for the pixels inside each zone.

synchrotron and TDE bands (note that the upper limit of the color map of the plots is only $30^\circ$), which is consistent with table 1 and figure 6.

A flattening of the distribution of values, visible for example in the inset histograms in the right panels of figure 3, indicates the existence of higher amplitude residuals after subtraction of the polarization angles.

From the upper panel of figure 4, one can see that for the 217 and 353 GHz bands, the polarization angles are highly consistent for the E family and the B family individually, but less consistent for the total angle without separation. This fact clearly indicates that the E and B families have different spectral indices in this region.

Finally we would like to present the maps of differences for the polarization angles $\theta_{\text{dust}} = \theta_{353} - \theta_{217}$ and $\theta_{\text{sync}} = \theta_{K} - \theta_{Ka}$, both with and without separation into the E and B families. From figure 5 we see the importance of the separation: $\theta_{\text{dust}}$ has very strong fluctuations in peripheral zones of the full sky map outside the Loop I region, while for the E and B families these differences reveal significantly lesser variations across the sky. This result clearly illustrates the advantage of the method proposed over the standard approaches. Due to its definition, the B family is acting as “noise” added to the E family, and in superposition it makes the polarization angles $\theta_{353}, \theta_{217}$ fluctuate more than each component. We have also confirmed that this result is not sensitive to the removal of the best-estimated CMB signal.
Figure 4. Similar to figure 3 but for 217 and 353 GHz.

Figure 5. Simple differences of the polarization angles for K-Ka (upper) and 353–217 GHz (lower). From left to right: no separation for Q and U, only E family ($\theta_E$), and only B family $\theta_B$. Apparently, in either case, the E mode gives higher band-to-band polarization angle consistency than the total polarization intensity.

3.2 Polarization angles of the E/B families: angular correlation and significance

The properties of the polarization angles can be understood if we consider eq. (2.13):

$$\tan(2\theta) = \frac{P_E \sin(2\theta_E) + P_B \sin(2\theta_B)}{P_E \cos(2\theta_E) + P_B \cos(2\theta_B)} = \frac{\rho \sin(2\theta_E) + \sin(2\theta_B)}{\rho \cos(2\theta_E) + \cos(2\theta_B)}.$$ (3.2)
Thus, if $\rho \gg 1$ (E family dominates), then $\theta \simeq \theta_E$, and if $\rho \ll 1$ we have $\theta \simeq \theta_B$. In figure 2 we can see strong similarity between $\theta$ and $\theta_E$, which indicates a general dominance of the E family. For $\rho \gg 1$, the B family acts like a perturbation of $\theta_E$:

$$\theta \simeq \theta_E - \frac{1}{2\rho} \sin(2\theta_E - 2\theta_B).$$

In the domain of the North Polar Spur, the polarization angle of the total signal is remarkably stable and homogeneous, and is dominated by the E family of polarization. We will discuss this phenomenon in detail in a separate paper.

In figure 2, there is remarkable similarity between the two synchrotron bands (K, Ka) and the two dust bands (217, 353 GHz) in the BICEP2 zone for $\theta_E$. For $\theta_B$ more differences can be seen in the same zone. However, in both maps we can clearly see similarities in global structure that cannot be reproduced by noise, which will be confirmed both by table 1 and in section 3.3. In the Ka band, the structure consists of some stripe-like features across the Galactic plane, which can be associated with the corresponding foregrounds and/or residuals of systematic effects.

In order to characterize the similarity between these maps, we define the cross-correlation coefficient between two polarization angle maps $\theta_1(n)$ and $\theta_2(n)$ as follows:

$$C(\theta_1, \theta_2) = \frac{1}{N_s} \sum_{i=0}^{N_s} \cos(2\theta_1(n_i) - 2\theta_2(n_i)),$$

where $\theta_j(n_i)$ is the polarization angle for $j$-th component in the pixel $i$, $N_s$ is the size of the sample, and the factor 2 comes from the definition of polarization angles. Therefore, $C(\theta_1, \theta_2)$ takes values between $-1$ and $1$, and two 100% correlated polarization angle maps will give $C(\theta_1, \theta_2) = 1$. The values of the cross-correlation coefficient for different pairs of maps are shown in table 1, for the full sky, BICEP2, and NPS regions.

From table 1 it is evident that for the K and Ka bands, the polarization angles for the E family have very high cross-correlations in the NPS and BICEP2 zones (0.98 and 0.95 respectively), and even the synchrotron-TDE correlations (K-353 GHz) reach 0.82 and 0.88. For the TDE bands 217-353 GHz these coefficients are both equal to 0.97 for E family, and 0.87, 0.95 for the B family. This clearly indicates that the structure of the maps in figure 2 is associated with intrinsic properties of the Stokes components ($Q_E, U_E$) and ($Q_B, U_B$) and is not an artifact of the noise.

|                | K-Ka | K-Ka (E) | K-Ka (B) | K-217 | K-217 (E) | K-217 (B) |
|----------------|------|----------|----------|-------|-----------|-----------|
| Full sky       | 0.73 | 0.85     | 0.67     | 0.50  | 0.62      | 0.37      |
| NPS            | 0.96 | 0.98     | 0.60     | 0.89  | 0.81      | 0.24      |
| BICEP2         | 0.87 | 0.95     | 0.45     | 0.66  | 0.93      | 0.62      |
| K-353          |      |          |          |       |           |           |
| K-353 (E)      |      |          |          |       |           |           |
| K-353 (B)      |      |          |          |       |           |           |
| 353-217        |      |          |          |       |           |           |
| 353-217 (E)    |      |          |          |       |           |           |
| 353-217 (B)    |      |          |          |       |           |           |
| Full sky       | 0.52 | 0.63     | 0.40     | 0.79  | 0.94      | 0.93      |
| NPS            | 0.90 | 0.82     | 0.25     | 0.88  | 0.97      | 0.87      |
| BICEP2         | 0.85 | 0.88     | 0.42     | 0.82  | 0.97      | 0.95      |

Table 1. List of $C(\theta_1, \theta_2)$ for various 2-band combinations. Three regions are listed: full sky, Bicep2 and the NPS region.
Figure 6. Left: the histogram of $C(\theta_1, \theta_2)$ from simulations assuming no band-to-band correlation. All values of $C(\theta_1, \theta_2)$ in table 1 are plotted as vertical lines, and they are all outside the range allowed by the simulations. Right: the variation of the cross-correlation values as a function of the band combination. For the right panel: the BICEP2 region is shown with a thick solid line, the NPS region with a solid line, and the full sky with a dashed line.

To estimate the significance of these values, we randomize the phases of the K and Ka bands to get 1,000 simulated noise maps. The distribution of CC coefficients is shown in figure 6, together with vertical lines for each value in table 1. One can see that all values in table 1 deviate significantly from the simulations. The simulations assume no band-to-band correlation, yet the band-to-band polarization angle correlation is very significant between synchrotron and dust bands and for all selected sky regions.

3.3 Polarization intensities of the E/B families

In this section, using the definitions of $P_E$ and $P_B$ from eq. (2.14) two foreground families are identified: one in which the E mode dominates the polarization, and one in which the B mode dominates. For illustration of our method we will use the WMAP K-band and the Planck 353 GHz maps, from which $P_E$ and $P_B$ are derived and presented in figure 7. The signal in WMAP K-band map is mainly synchrotron emission, while the Planck 353 GHz map consists of thermal dust emission. The ratio between $P_E$ and $P_B$, denoted by $\rho = P_E/P_B$, is presented in figure 8, with the arches listed in [18] marked for comparison.

In figure 7, all loop-like structures are clearly visible in the $P_E$ maps, but are missing in the $P_B$ maps. This tendency is further confirmed in figure 8 by higher amplitudes of $\rho$ near the loops and arches. Furthermore, it can be seen from figure 8 that the B mode tends to be subdominant ($\rho > 1$) in the BICEP2 zone, NPS zone, and loops, which makes these parts of the sky peculiar.

3.4 Significance of the E/B families by polarization intensities

We define the cross-correlation coefficient for the $P$ and $\rho$ maps as

$$C(P, \rho) = \frac{\sum_i (P_i - \langle P_i \rangle)(\rho_i - \langle \rho_i \rangle)}{\sqrt{\sum_i (P_i - \langle P_i \rangle)^2 \sum_i (\rho_i - \langle \rho_i \rangle)^2}},$$

where $(X_i) = \frac{1}{N} \sum_i X_i$, the sums are taken over all pixels $i$, and $N$ is the total number of pixels in the maps. As was pointed out in [9], at 353 GHz, the E mode is stronger than the
Figure 7. The polarization intensity maps and its EB decomposition, in K-band (upper) and 353 GHz (lower). From left to right: total polarized intensity $P$, $P_E$, and $P_B$. The loop-like structures are marked by arches.

Figure 8. The ratio $\rho = P_E/P_B$ in K-band (upper) and 353 GHz (lower), both full sky (left) and around the arches (right). The outline of the BICEP2 zone is also plotted.

B mode, which is also the case for the WMAP K band (23 GHz). For an input map with a stronger E mode, pixels with higher $\rho$ are more affected by the E mode and thus tend to have higher $P_E$ in comparison to $P_B$. Therefore we may expect to get positive cross correlation $C(P, \rho)$ for $\rho < 1$ and $\rho > 1$.

However, the sign of correlations depends on the mean values of $P_B^2$ and $\rho$, which makes the shape of $C(P, \rho)$ more complicated. In figure 9 we plot the relationship between $P$ and $\rho$ which shows anti-correlation in the $\rho < 1$ region, and the histograms of $C(P, \rho)$. To illustrate the significance of $C(P, \rho < 1) < 0$, we generate $10^5$ realizations of the difference $\theta_E - \theta_B$ from a random uniform distribution in $[0, 2\pi]$ and plot the distribution $H(C_{\text{rand}})$ of the corresponding values of $C_{\text{rand}}(P, \rho)$ in figure 9. Contrary to the actual distribution of $C(P, \rho)$, the result from the simulation shows that $H(C_{\text{rand}})$ is consistent with $C_{\text{rand}} \geq 0$, and strongly disfavoured at $\rho < 1$. Thus, the anti-correlation $C(P, \rho < 1) < -(0.1 - 0.2)$ cannot be produced by noise.
Figure 9. The relationship between total polarization intensity $P$ and the ratio $\rho = P_E/P_B$. Upper panels show the K-band and lower panels show 353 GHz. From left to right: real data, one random realisation, the histograms of $C(P, \rho)_{\rho<1}$ and for $\rho > 1$ for $10^5$ simulations. The vertical lines for the two panels on the left mark $\rho = 1$, and the vertical lines on the two right panels mark $C(P, r)$ of the real data. A negative value of $C(P, \rho)_{\rho<1}$ is an indicator for the B mode family. The positive $C(P, \rho)_{\rho>1}$ indicate the E family.

In order to evaluate the significance of this B mode foreground family, $10^5$ random simulations are generated with the same angular power spectrum to the WMAP K-band foreground map in use, and for each simulation, $P$ and $r$ are calculated for the $|b| > 10^\circ$ and $\rho < 1$ region. Under the same conditions, the K-band map gives $C(P, \rho) = -0.1$, which is not reached by any of the 100,000 simulations, as presented by the histogram in figure 9. Therefore, the existence of a B mode family is confirmed at a 99.999% confidence level. One possible candidate for this B mode family is the asymmetry structure of the loops, which is briefly discussed in section 6. The above analysis is also repeated for the 353 GHz band and the results are included in figure 9, which are similar to the K band. The existence of a B mode family with $\rho < 1$ can be an important source of contamination for future primordial gravitational wave detection, which depends critically on the level of residual B mode foreground.

In addition to the B family the existence of an E mode family can also be identified from figure 9 by an apparent clustering of pixels in the $\rho > 1$ domain localized in the $P < 40 \mu K$ region of the WMAP K band. In fact, this family was already discovered as the loops in figure 7. The significance of detection of the E mode family is evaluated by the number of pixels in the $\rho > 1$ and $P < 40 \mu K$ region of the WMAP K band, which covers 57% of the sky for the K band, whereas in 100,000 simulations, this ratio is exceeds the threshold 45%. This corresponds to a confidential level of 99.999% against chance correlations. The same results are obtained for the TDE in the Planck 353 GHz map.

4 Frequency dependence of the E and B families

In the previous sections, we used the two most representative frequency maps — the WMAP K-band as an indicator of synchrotron emission, and the Planck 353 GHz map as an indicator of thermal dust emission. Below we will extend this analysis for all the WMAP and Planck frequency bands in order to trace the properties of the E and B foreground families, and more importantly, to investigate their frequency dependence. This analysis is very important for future implementation of component separation tools, for verification of the residuals of data...
Figure 10. Left: histogram of the ratio $\rho = P_E/P_B$ for 4 example frequency bands (K, Ka, 30 GHz and 353 GHz in black, blue, red, and green respectively) without correction to 30 GHz (note that the uncorrected 30 GHz data is used only in this panel). Middle: similar to left but 30 GHz is corrected. Right: the median values of $\rho$ for each WMAP and Planck frequency band. The vertical lines in left and middle mark $\rho = 1$.

Cleaning (for instance, the bandpass leakage correction), and for identification of different masks that can help to avoid peculiar zones of the maps where there is strong variation of the spectral indices of the components.

4.1 Asymptotic of E mode excess for high E/B ratio

As we have shown in section 3.3–3.4, the ratio $\rho = P_E/P_B$ is a very useful tool for determining the morphological features of the synchrotron and TDE maps, especially because it is unaffected by the absolute amplitude of the emission. In this section we will extend this analysis for WMAP K, Ka and Planck 30-353 GHz maps in terms of the E and B families. Needless to say, unlike the K-band and 353 GHz maps, which are dominated by synchrotron emission and TDE respectively, for other bands (44–217 GHz) the common signal is given by a superposition of these two basic components, and also primordial CMB.

Our primary goal is to derive the corresponding maps of $\rho(n)$ for each frequency band and plot the histograms $H(\rho)$ for each map under investigation. In figure 10 we show these histograms (as a number of counts vs $\rho$) for the K, Ka, 30 and 353 GHz maps. From figure 10 we come to the very important conclusion that in spite of very different physical mechanisms underlying the synchrotron and polarized thermal dust generation, the corresponding distributions of $\rho$ reveal remarkable similarity (convergence), especially when $\rho > 2$.

Surprisingly, the $H(\rho)$ estimator is very sensitive to the systematic effects, namely a bandpass mismatch leakage in the Planck 30 GHz band [19, 20]. On the left panel of figure 10 we show the $H(\rho)$ histograms for the K, Ka and 30, 353 GHz maps, where the 30 GHz signal is not corrected for the bandpass mismatch leakage. One can see that for $\rho > 2$ the $H(\rho)$ histogram decays more rapidly than the other histograms. At the same time, in the domain $0.7 < \rho < 1.5$, the uncorrected 30 GHz signal is more dominant than the WMAP K and Ka bands. Considering that the Planck 30 GHz signal is close in frequency to the WMAP Ka (33 GHz) signal, it is obvious that this very different asymptotic behavior is due to the bandpass mismatch leakage in the Planck 30 GHz map.

In the middle plot of figure 10 we show the same bands as the left panel, but with correction of the 30 GHz map. Now we see the convergence of the Planck 30 GHz signal to the 353 GHz distribution for $\rho > 2$. In this region the histograms closely follow an exponential distribution, marked by an orange line in the same panel. Meanwhile, for $0.7 < \rho < 1.5$, the amplitude of $H(\rho)$ for the 30 GHz bandpass leakage corrected signal is closer to the
corresponding distributions for K and Ka bands, but not exactly equal to them. Although
the 30 GHz band is expected to give results similar to the K and Ka bands rather than the
353 GHz band, this is not achieved even after correction, which confirms the conclusion of [21]
that the bandpass mismatch leakage correction needs further improvements, and reminds us
that the 30 GHz map should always be used with great caution.

The most pronounced part of the distributions in figure 10 is in the domain $0.7 \leq \rho \leq 2$,
where the synchrotron channels have a plateau, and TDE channels have a point of maximum.
To compare these distributions we will use the median value of $\rho$ for all WMAP and Planck
23–353 GHz maps, which are plotted in the right panel of figure 10. One can see that almost
all bands show E mode excess with the median value of $\rho$ between 1.1 and 1.8, except for
the WMAP W band, where the median $\rho_W \simeq 0.75$. In addition to this feature, it is needless
to point out the convergence of all Planck 100–353 GHz signals to the asymptotic $\rho \simeq 1.4$,
which coincides with the Planck 30 GHz median (which might still be abnormal).

4.2 Real space distribution of the E/B ratio

The important question now arises: how much does the similarity of the $H(\rho)$ distributions
reflect similarity of the morphology of the $\rho(n)$ sky maps? To answer this question, in
figure 11 we show the maps of $\rho(n)$ for different thresholds of $\rho$. The left column of this
figure shows in blue those parts of the sky where $\rho \leq 0.7$ in the K, Ka and 353 GHz signals
(from top to bottom). There is well pronounced similarity between the K and Ka maps, but
the 353 GHz map has a different pattern. Note that this threshold indicates the angular
distribution of the B mode family. Thus, for synchrotron emission, this family is largely
localized in the right half of the map, while for TDE at 353 GHz this family is closer to the
Galactic plane (on the left part of the map).

We would like to remind the reader that some of the most interesting features of the
polarization intensity maps are the NPS and Loop I regions. In all the maps presented in
figure 11, we indicate the position of Loop I from [22–24] together with two others lines,
which indicate ±10° circles around it. As one can see, for the synchrotron signals, the B family of polarization is relatively very weak in the NPS zone.

The same tendency persists in column 2 in figure 11, showing the zones where 0.7 ≤ ρ ≤ 1.5. Again, the NPS domain is empty in all the synchrotron maps, and the only few spots are visible in the 353 GHz map. Note that the threshold 0.7 ≤ ρ ≤ 1.5 includes the median value of ρ ≃ 1.4 discussed above. The WMAP K and Ka maps significantly depart from this value (see figure 10). Nevertheless, even for these maps one can see that the NPS domain is empty or almost empty for the 353 GHz map.

Columns 3–4 of figure 11 show the thresholds 1.5 ≤ ρ ≤ 7 and ρ > 7 respectively, which both correspond to the E family of the polarization. An important feature of these maps is the significant difference between the synchrotron and TDE signals. The most interesting feature for the threshold ρ > 7 occurs for the K band in the upper part of the NPS region. There we find a wide zone dominated by the E family in the synchrotron maps, which does not exist in the dust emission. In the BICEP2 zone we can see the peak of ρ for the K band, which is less visible in the Ka band, and most pronounced again in 353 GHz map. On the other hand, for the ρ > 7 threshold, the northern spot at NPS occurs for the K and Ka maps, while the E family does not exist in the BICEP2 zone for this threshold.

5 Spectral indices for the E and B families

A critical point for extraction of the primordial CMB polarization is the frequency dependence of the synchrotron and thermal dust emission [25]. In addition to blind methods, such as NILC [26], others (Commander, SMICA) use well-formulated assumptions about the transition of the linear combination of the foreground templates from one frequency band to others. Assuming a power law for synchrotron emission, we can convert the frequency dependence of the polarized intensity to the corresponding spectral index, defined as

\[ \beta(n) = \frac{\log(P_1(n)/P_2(n))}{\log(\nu_1/\nu_2)}, \tag{5.1} \]

where \( P_1 \) and \( P_2 \) are the polarization intensities (either total or associated with one of the E/B families) at frequencies \( \nu_1 \) and \( \nu_2 \) respectively. The spectral index is defined independently at each pixel specified by \( n \), and can vary across the sky.

In this section we will address the question of whether the E and B families of polarized emission have the same frequency dependence, as measured by their spectral indices \( \beta_E(n) \) and \( \beta_B(n) \). If they are different, then the transition of Q and U, or E and B, in the frequency domain will be more complicated.

5.1 Spectral indexes for the full sky

We consider the low frequency WMAP K and Ka bands for synchrotron emission, and the high frequency Planck 217 and 353 GHz bands for TDE. The spectral indices for these map pairs are presented in figure 12, and the corresponding histograms \( H(\beta) \) are shown in figure 13. Note that a center value of \( \beta = -3 \) is subtracted from the synchrotron band results and \( \beta = 4 \) is subtracted from the dust band results, and for the dust emission a power law is assumed for simplification (see appendix A for details). From figure 12 we can see that variation of the synchrotron spectral index is strong in the upper right corner of the polarized intensity map \( P(n) \). For the E family we can see significant homogenization of the spectral index \( \beta_E \), and great non-uniformity for \( \beta_B \). We illustrate the corresponding distributions
Figure 12. The spectral index maps derived from K and Ka bands (upper panels) and 217, 353 GHz (lower panels). From left to right: the total polarization intensity, only E-mode family, only B mode family, and the mask for statistics that covers only the highest 40% polarization intensities (the red zone is used).

Figure 13. The histograms of the spectrum index maps shown in figure 12. Upper: K and Ka bands. Lower: 217, 353 GHz. Left: full sky. Right: only the high-intensity zone shown in figure 12.

$H(\beta)$ in figure 13 for the full sky and for selected high polarization intensity zones, and in table 2 we show the mean values and standard deviations for spectral indices in each case.

Apparently, for the sky region with higher polarization intensity, included in the masks in figure 12, the B mode foreground has much wider distribution of the spectral index; however, the mean values of the spectral indices do not deviate significantly from the center value. This rules out noise as an explanation for the broadening of the distribution, because the noise will broaden and shift the distributions at roughly same level.
Table 2. The mean values and standard deviation $\sigma$ for $P$, $P_E$ and $P_B$ spectral indexes after monopole subtraction (the first, the second and the last values in the boxes) for figure 13.

|                | K/Ka, no mask | K/Ka with mask | 217/353, no mask | 217/353 with mask |
|----------------|---------------|----------------|------------------|-------------------|
| Mean           | 0.14, −0.03, 0.16 | −0.06, −0.02, 0.04 | 0.17, 0.34, 0.13 | 0.22, 0.26, 0.16  |
| $\sigma$       | 1.21, 0.81, 1.31 | 0.43, 0.37, 0.98 | 1.14, 0.59, 0.71 | 0.19, 0.36, 0.70  |

Figure 14. The histograms of the spectrum index variations for the NPS zone (upper) and the BICEP2 zone (lower). Left: for K-Ka. Right: for 217-353 GHz.

5.2 Spectral indices for BICEP2 and NPS zones

In section 3 we have shown that the E family dominates in the NPS and BICEP2 zones of the synchrotron K band map and TDE 353 GHz map (see figure 8). The BICEP2 zone has attracted very serious attention after BICEP2 experiment [27, 28] as well as the corresponding comments in [6]. Following the analysis presented above, we would like to discuss here the properties of the spectral indices of the E and B families in the NPS and BICEP2 zones.

In figure 14 we show the histograms $H(\beta)$ for the NPS and BICEP2 zones for $P$, $P_E$ and $P_B$, similar to figure 12. From this figure we can see that the spectral index for the E family deviates significantly from that of the B family. In the NPS zone, the B family is sub-dominant with respect to the E family, and the spectral index for the total polarization intensity $P$ is given mainly by the E-component. This result is consistent with figure 8. At the same time, for the NPS zone we can see significant differences in variation of the B family synchrotron spectral index with respect to the $\beta$ for the total polarization intensity $P$, and the E family.
We also calculate the noise level for the $Q$ and $U$ maps with $N_{\text{side}} = 128$ and $2^\circ$ smoothing using simulations based on the released Planck $QQ$ and $UU$ covariances, which gives about $n_1 \approx 0.6 \mu K$ for the 217 GHz band and $n_2 \approx 2.4 \mu K$ for the 353 GHz band. In figure 15, we show the regions with total polarization intensity $P > \sqrt{2}n_{1,2}$ for the two bands (without separation). We can see that most of the region has a reasonably high signal-to-noise ratio. We also note that the region with $P > \sqrt{2}n_{1,2}$ is even larger — covering almost the full sky — for the E and B families separately than for the total signal, which indicates an E-B anti-correlation. We have also confirmed that there are no apparent differences with or without CMB removal.

The properties of the $\Gamma$-parameter, presented in the last row of figure 15, reflect the coupling between the total intensity $P(n)$, $P_E(n)$, $P_B(n)$, and the polarization angles $\theta_E, \theta_B$ (see eq. (2.15)). We have

$$\Gamma^2 = \frac{P_E^2 + P_B^2}{P^2} = \frac{1 + \rho^2}{1 + 2\rho \cos(2\theta_E - 2\theta_B) + \rho^2}. \quad (5.2)$$

In all zones in the second row of figure 15 with $\Gamma > 1$, we have $\cos(2\theta_E - 2\theta_B) < 0$, which indicates E-B anti-correlation.

In the BICEP2 zone the variation of the B family synchrotron spectral index has significantly different behavior than the NPS zone. Firstly, the mean value of $\beta$ is 3.27 with standard deviation $\sigma \approx \pm 0.38$. However, for the E family we get $\beta_E \approx 2.76 \pm 0.24$, and $\beta_B \approx 2.77 \pm 1.1$. The strongest variations of $\beta_B$ occurs in TDE, where $\beta \approx 5.48 \pm 0.73$, $\beta_E \approx 4.26 \pm 0.18$, but $\beta_B \approx 3.56 \pm 0.53$. In spite of very narrow distribution of $\beta_E$, the distributions of $\beta$ and $\beta_B$ are almost opposites of each other (see figure 14). We summarize these results for spectral index variations in NPS and BICEP2 zones in table 3.

An important question related to figure 14 is the morphology of fluctuation causing the corresponding departure of spectral indexes $\beta, \beta_E$ and $\beta_B$ from their central (mean) values.
Table 3. The mean values and standard deviations $\sigma$ for the $P$, $P_E$ and $P_B$ spectral indices (the 1st, 2nd and 3rd values in the boxes), after subtraction of the spectrum index offset for NPS and BICEP2 zones.

The most obvious (but not only) explanation of that effect is the presence of noise or systematic effects in the corresponding K, Ka, 217 and 353 GHz maps. If the instrumental noise is a source of spectral index variation, we may expect small scale fluctuations of the polarization angles, coinciding with noise patterns at North and South ecliptic poles. For systematic effects we should see some peculiarities adjusted to the Galactic and ecliptic planes [29].

6 Conclusion

The separation of the Stokes parameters $Q$ and $U$ into E and B families reveals very important properties of these components, valuable for determination of peculiarities of the synchrotron and TDE polarization. Since the E/B family representation is linear, all information stored in $Q$ and $U$ is preserved in $(Q_E, U_E)$ and $(Q_B, U_B)$, but redistributed between them. This redistribution allows to detect the following features of the foreground polarization:

1. We have shown that for the synchrotron emission the local features of polarization in the loops and arches are mainly associated with the E family with much less influence from the B family.

2. We have applied the E/B family decomposition for determination of the polarization angles for each component: $\theta$, $\theta_E$ and $\theta_B$. We have found remarkable stability of $\theta_E$ for synchrotron emission and TDE in the whole sky, and in the NPS and BICEP2 zones as well. We have also discovered well pronounced large sky patterns at 217–353 GHz for the B family well above and below the Galactic plane, highly contaminated by the residuals of systematic effects. The origin of this structure needs further investigation.

3. We have shown that the ratio of the corresponding intensities $\rho = P_E/P_B$ is very useful tool for identification of the synchrotron and TDE dominated zones, which is important for construction of the corresponding masks for future CMB analysis. For the synchrotron emission and TDE, the asymptotic of $\rho$ with $\rho > 7$ indicate the NPS and BICEP2 zones with low amplitude of the B family. Both these zones are anomalous in respect to other zones and the properties of the derived B mode of polarization (not the B family!) needs to be taken with care. As we have shown, the $\rho$-test is very sensitive to the residuals of systematics. The number of pixels with $\rho > 7$ is significantly different for the Planck 30 GHz map with and without the bandpass leakage correction, relative to the WMAP K and Ka distribution of $\rho$.

4. We have investigated the dependency of the spectral indices over frequency for synchrotron emission and TDE, using the K and Ka bands for synchrotron and 217 and
353 GHz bands for dust emission. We have detected different patterns for variation of \( \beta \), \( \beta_E \) and \( \beta_B \) for total intensity \( P \), \( P_E \) and \( P_B \). The important feature of the \( \beta_E \) index is that it is significantly less fluctuated across the full sky, and especially in the NPS and BICEP2 zones. For the B family the variations of the spectral index are much more pronounced in the same zones. Taking into account, that \( (Q,U) \equiv (Q_E,U_E) + (Q_B,U_B) \), for total intensity \( P \) the variation of the TDE spectral index \( \beta \) is mainly associated with B family. This effect dominates the BICEP2 zone, where the \( \beta \) spectral index variation is given by variation of \( \beta_B \). This means the removal of the E family in this domain by multi-frequency methods will be extremely effective, while for the B family it will be quite problematic. For future ground based and space CMB experiments, devoted to determination of the primordial B mode from cosmological gravitational waves, these variations of \( \beta_B \) will act as an additional “noise”. From the ground the highest frequency bands are limited by the opacity of the Earth’s atmosphere (200–250 GHz). At these frequencies synchrotron emission is already sub-dominant to cosmic thermal dust emission. However, since we are interested in very small tensor-to-scalar ratio \( r \leq 0.01 – 0.0001 \), even small synchrotron components require more specific attention.

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A Simplification of the dust spectrum index

Note that in section 5.1, the definition of \( \beta \) for TDE is different in respect to the gray-body model for non-polarized spectral energy density \( I_d \): in the optically thin limit, the model of spectral energy distribution \( I_d(\nu) \) (SED) in the direction \( n \) and the frequency \( \nu \) for the dust grains is given by [30]:

\[
I(\nu) = \tau(n) \left( \frac{\nu}{\nu_0} \right)^{\beta_d(n)} B_\nu(T_d(n)) \tag{A.1}
\]

where: \( \tau_\nu = \tau(n) \left( \frac{\nu}{\nu_0} \right)^{\beta_d(n)} = \sigma_d(\nu) N_H(n) \) is the dust optical depth, \( \sigma_d(\nu) \) is the dust opacity, \( N_H \) is the gas column density,

\[
B_\nu(T_d(n)) = \frac{2h\nu^3}{c^2} \left( e^{\frac{h\nu}{kT_d(n)}} - 1 \right)^{-1} \tag{A.2}
\]

is the Planck spectral function with the temperature \( T_d(n) \), \( \beta(n) \) is the spectral index, and \( \nu_0 \) is a reference frequency. For any two frequency bands \( \nu_1 \) and \( \nu_2 \), the ratio \( I(\nu_1)/I(\nu_2) \) is given by

\[
\frac{I(\nu_1)}{I(\nu_2)} = \left( \frac{\nu_1}{\nu_2} \right)^{\beta_d(n)} \frac{B_{\nu_1}(T_d)}{B_{\nu_2}(T_d)}. \tag{A.3}
\]
Thus, the spectral index $\beta$ related to $\beta_d$ through the following equation:

$$\beta(n) = \beta_d(n) + \ln \left( \frac{e^{\frac{h\nu_2}{kT_d(n)}} - 1}{e^{\frac{h\nu_1}{kT_d(n)}} - 1} \right) / \ln \left( \frac{\nu_1}{\nu_2} \right) + 3. \quad (A.4)$$

As one can see from eq. (A.4), for $\nu_1, \nu_2 \ll kT_d/h$, we have $\beta \simeq \beta_d + 2$. In this case the variation of the spectral index for $\beta$ and $\beta_d$ is almost identical. If we take $\nu_1 = 353$ GHz and $\nu_2 = 217$ GHz, and use $T_d = 20$ K, then eq. (A.4) simplifies to $\beta = \beta_d + 1.62$.

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