Indirect study of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ reaction

F Hammache$^1$, N Oulebsir$^{1,2}$, P Roussel$^1$, MG Pellegriti$^{1,6}$, L Audouin$^1$, D Beaumel$^1$, A Bouda$^2$, P Descouvemont$^3$, SFortier$^1$, LGaudefroy$^{4,7}$, JKienere$^4$, ALefebvre-Schuhl$^5$ and VTatischeff$^5$

$^1$ Institut de Physique Nucléaire d’Orsay, UMR8608, IN2P3-CNRS, Université Paris sud 11, 91406 Orsay, France
$^2$ Université Abderrahmane Mira, 06000 Bjaal ALGERIE
$^3$ Physique théorique et Mathématique, ULB CP229, B-1050 Brussels, Belgium
$^4$ GANIL/CEA/DSM-CNRS/IN2P3,Bd Henri Becquerel,BP 55027,F-14076 Caen Cedex 5, France
$^5$ CSNSM, UMR 8609, CNRS/IN2P3 and Université Paris Sud 11, F-91405 Orsay, France
E-mail: hammache@ipno.in2p3.fr

Abstract. The radiative capture reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ plays an important role in helium burning in massive stars and their subsequent evolution [1]. However, despite various experimental studies, the cross section of this reaction at stellar energies remains highly uncertain. The extrapolation down to stellar energy ($E_{cm} \sim 300$ keV) of the measured cross sections at higher energies is made difficult by the overlap of various contributions of which some are badly known such as that of the $2^+$ ($E_x = 6.92$ MeV) and $1^-$ ($E_x = 7.12$ MeV) sub-threshold states of $^{16}\text{O}$. Hence, to further investigate the contribution of these two sub-threshold resonances to the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ cross section, a new determination of their $\alpha$-reduced widths and so their $\alpha$-spectroscopic-factors was performed using $^{12}\text{C}(^7\text{Li},t)^{16}\text{O}$ transfer reaction measurements at two incident energies and a detailed DWBA analysis of the data [2]. The measured and calculated differential cross sections are presented as well as the obtained spectroscopic factors and the $\alpha$-reduced widths as well as the asymptotic normalization constants (ANC) for the $2^+$ and $1^-$ sub-threshold states. Finally, the results obtained from the R-matrix calculations of the $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$ cross section using our obtained $\alpha$-reduced widths for the two sub-threshold resonances are presented and discussed.

1. Introduction

During He burning phase in massive stars, the two main and important reactions through which He is consumed are the triple-$\alpha$ reaction and $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. The ratio of these two reaction rates determine the $^{12}\text{C}/^{16}\text{O}$ abundance ratio in stars at the end of their helium burning phase. This ratio has important consequences on further hydrostatic burning stages and so on nucleosynthesis in massive stars and consequently has an effect on the final fate of the stars [1]. The rate of the triple-$\alpha$ reaction is well determined but it is not the case of $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$. The low-energy cross section of the latter remains highly uncertain despite the various experiments.

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6 Present address: Dipartimento di Fisica e Astronomia, Università di Catania and Laboratori Nazionali del Sud - INFN, Catania, Italy
7 Present address: CEA, DAM, DIF, F-91297 Arpajon, France
performed this last four decades. At the Gamow peak of 300 keV where this reaction occurs during the He burning stage, the expected cross section is about $10^{-8}$ barn which makes it impossible to measure directly. Hence, direct measurements were performed down to 900 keV in center-of-mass system and then extrapolated to the energy of interest. The extrapolation in case of this reaction is made difficult by the presence of several contributions, the most important ones being the E1 and the E2 transitions to the ground state via the low energy tail of the $1^-$ broad resonant state at 9.58 MeV of $^{16}$O and the high energy tails of the $2^+$ and $1^-$ sub-threshold resonant states at 6.92 and 7.1 MeV of $^{16}$O. Unfortunately, the effect of these two sub-threshold states is badly known because their measured alpha spectroscopic factors $S_{\alpha}$ and so their corresponding reduced alpha width are spread over a large range of values [2]. So, in view of the importance of $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction and the large uncertainties surrounding the $S_{\alpha}$ and the $\gamma_{2}^2$ of the two sub-threshold states, we performed a new measurement of these quantities via the transfer reaction $^{12}$C($^7$Li,t)$^{16}$O [3].

2. Experiment description

The experiment was carried out at two incident energies, 28 and 34 MeV, using a $^7$Li$^{3+}$ beam provided by the Orsay TANDEM. The used target consisted of a self-supporting enriched $^{12}$C which thickness was of $80 \pm 4$ $\mu$g/cm$^2$. The reaction products were detected with a position-sensitive gas chamber and a $\Delta$E proportional gas-counter located in the focal plane of the Enge Split-pole magnetic spectrometer.

The tritons were detected at angles ranging from 0 to 31° corresponding to angles up to 43° in the center of mass frame. The beam and $^{12}$C amount were continuously monitored with a telescope of silicon detectors mounted inside the scattering chamber at $\theta_{lab}=35^\circ$.

A typical excitation energy spectrum of $^{16}$O obtained at 11.5° is displayed in Figure 1.

![Triton spectrum obtained at 11.5°](image)

**Figure 1.** Triton spectrum obtained at 11.5° [3] with the 34 MeV $^7$Li beam on $^{12}$C target in the excitation energy region from 6 to 11 MeV. The excitation energy (MeV) of $^{16}$O levels are indicated.

One can notice in the spectrum the strong population of the $\alpha$-cluster states, the 6.92 and 10.35 MeV states which indicates that the data are consistent with a direct $\alpha$-transfer mechanism. The other feature one can also notice is the weak population of the non-natural parity state 2$^-$, the 8.87 MeV state of $^{16}$O which can not be populated by direct transfer mechanism. It is probably populated by the compound nucleus mechanism. This is used to evaluate the contribution of the compound nucleus mechanism in this transfer reaction [3].
3. Results

3.1. DWBA analysis and results

The experimental $^{12}$C($^7$Li,t)$^{16}$O differential cross sections of the 6.05, 6.13, 6.92, 7.12, 8.87, 9.58, 9.85 and 10.35 were measured at the two incident energies of 28 and 34 MeV [3] but only those of 6.13, 6.92, 7.12 and 8.87 states are displayed in Figure 2 together with the Finite-range DWBA and HF calculations.

![Figure 2](image_url)

**Figure 2.** Experimental differential cross sections of the $^{12}$C($^7$Li,t)$^{16}$O reaction obtained at 34 MeV (left column) and 28 MeV (right column) for the 6.13, 6.92, 7.12 and 8.87 MeV states [3], compared with FRDWBA calculations (dashed red curve) normalized to the data, Hauser-Feshbach (HF) calculations (dashed-dotted pink line) and the sum HF+FRDWBA (blue solid line).

The Finite-range DWBA (FRDWBA) calculations were performed using the FRESCO code [4]. For the entrance channel, we used the optical potential parameters of Schumacher et al. [5] who performed $^7$Li elastic scattering measurements on $^{12}$C at the energy of 34 MeV. Concerning the exit channel, the optical model potentials used were taken from Garrett et al. [6]. The optical potential parameters finally selected are those giving the best fit for all the studied transitions in the ($^7$Li,t) reaction.

For the $\alpha$ wave function in $^{16}$O, an $\alpha+^{12}$C Wood-Saxon potential was used. A range of radius ($3.5$ fm $\leq$ $R$ $\leq$ $4.5$ fm) and diffuseness ($0.53$ fm $\leq$ $a$ $\leq$ $0.93$ fm) was selected by using the maximum likelihood function (set at $3\sigma$ level) on the angular distributions of all measured levels except the non-natural parity 8.87 MeV state and the 9.85 MeV state which displays a quasi-flat distribution [3]. Within this radius and diffuseness range, the boundary values $R=4.5$ fm and $a=0.73$ fm provide the best fit for the angular distributions of all the studied states at both incident energies (fig.2) except the 8.87 and the 9.85 MeV states. Details in the DWBA analysis
procedure is given in [3]. The displayed FRDWBA calculations in Figure 2 are normalized to the data.

Except for the 8.87 MeV state, the good agreement observed between the DWBA calculations and the measured differential cross sections of the different populated states of $^{16}$O at the two bombarding energies of 28 MeV and 34 MeV respectively, gives strong evidence of the direct nature of the ($^7$Li,t) reaction populating these levels and confidence in our DWBA analysis. However, as one can see in Figure 2, a disagreement at angles smaller than 10° is observed for the 7.12 MeV state and for both incident energies. This discrepancy is not understood and it was also observed in $^{12}$C($^7$Li,t)$^{16}$O experiment of Becchetti et al. [7] at 34 MeV. To try to understand if the decrease of the cross section at angles smaller than 10° is due to a multi-step effect mechanism, coupled channel calculations are needed and they are in progress.

From a $\chi^2$ minimization of the DWBA differential cross sections to the measured ones, $S_\alpha$ mean values of 0.15±0.05 and 0.07±0.03 are deduced for the states of interest at 6.92 MeV and 7.12 MeV of $^{16}$O respectively. The uncertainty on the extracted $\alpha$ spectroscopic factors for the states of interest was evaluated from the dispersion of the deduced $S_\alpha$ values at the two incident energies and using different sets of optical potentials in the entrance [5] and exit channels [6] and different $\alpha$-$^{12}$C well geometry parameters selected above.

Once the $S_\alpha$ of the states of interest are determined, we deduced their $\alpha$-reduced widths using the expression, $\gamma_\alpha^2 = \frac{k^2 R}{\mu} S_\alpha |\varphi(R)|^2$ [8] where $\mu$ is the reduced mass and $\varphi(R)$ is the radial part of the $\alpha$-$^{12}$C wave function. The calculations of $\gamma_\alpha^2$ were performed at the radius $R=6.5$ fm where $\varphi(R)$ reaches its asymptotic behavior. The obtained values are 26.7±10.3 keV and 7.8±2.7 keV for the 6.92 MeV and 7.12 states respectively.

The asymptotic normalisation constants (ANC) [9] of the two sub-threshold states were also deduced. The obtained values $C^2= (2.07\pm0.80) \times 10^{10}$ fm$^{-1}$ and $C^2= (4.00\pm1.38) \times 10^{28}$ fm$^{-1}$ for the 6.92 and 7.12 MeV states respectively were found in excellent agreement with those obtained by Brune et al. [10] from a sub-coulomb ANC measurement.

3.2. R-matrix calculations and results

R-matrix calculation of the E1 and E2 astrophysical S-factor of $^{12}$C($\alpha$, $\gamma$)$^{16}$O reaction, using P. Descouvemont code, were performed using our deduced $\alpha$-reduced widths for the two sub-threshold states. In the R-matrix calculations, both the $^{12}$C($\alpha$, $\gamma$)$^{16}$O astrophysical S-factors obtained by direct measurements at higher energies and the phase shifts data from elastic scattering $^{12}$C($\alpha$, $\alpha$) measurements were fitted and the two components were fitted separately.

For the E2 component, four $2^+$ states were considered in the calculation: the 6.92 MeV state for which we determined the $\gamma_\alpha^2$, the 9.85 MeV, the 11.52 MeV and a background equivalent state which takes into account the tails of other higher-lying $2^+$ states. In the R-matrix fitting procedure, the resonance parameters of all states except the background state are kept fixed [3]. From the best fits displayed in figure 3, we deduced an E2 S-factor at 300 keV of 50±19 keV-b. The same fitting procedure was applied for the E1 component. The $1^-$ states considered are the 7.12 MeV state, the broad resonant state at 9.58 MeV and a background equivalent state which takes into account the tails of other higher-lying $1^-$ states and the direct component. The only free parameters are those of the $1^-$ background equivalent state. From the best fits shown in Figure 4, an E1 S-factor at 300 keV of 100±28 keV-b was deduced.

To validate furthermore our R-matrix fits and results, especially for the E1 component, we performed a p-wave calculation of the $\beta$-delayed $\alpha$-spectrum of $^{16}$N. For the calculation, we used equation 3 of ref [20] and our R-matrix parameters while we considered the $\beta$-feeding amplitudes, $A_M$ (see equation 3 of ref. [20]), as free parameters. As one can see in Figure 5, our calculation describes well the measured data of Tang et al. [20] and this gives strong confidence in our R-matrix calculations. The disagreement between the calculation and the data in the energy
Figure 3. Left: Phase shifts for $^{12}\text{C}(\alpha,\alpha)^{12}\text{C}$ elastic diffusion reaction with $R$-matrix calculations of the E2 component [3]. Data points are from [11] (black points) and [12] (blue triangles). The solid line correspond to our best $R$-matrix fit with $\chi^2=1.02$. Right: Astrophysical S-factor for the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction with $R$-matrix calculations of the E2 component [3]. Experimental data are from [13, 14, 15, 16, 17, 18]. The solid line is our best $R$-matrix fit using our deduced $\gamma_2^\alpha$ for the 6.92 MeV state and the dashed lines when using our upper and lower values for $\gamma_2^\alpha$.

Figure 4. Left: Phase shifts for $^{12}\text{C}(\alpha,\alpha)^{12}\text{C}$ elastic diffusion reaction with $R$-matrix calculations of the E1 component [3]. Data points are from [11] (black points) and [12] (blue triangles). The solid line correspond to our best $R$-matrix fit with $\chi^2=5.4$. Right: Astrophysical S-factor for the $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ reaction with $R$-matrix calculations of the E1 component [3]. Experimental data are from [13, 14, 15, 16, 17, 18, 19]. The solid line is our best $R$-matrix fit using our deduced $\gamma_2^\alpha$ for the 7.12 MeV state and the dashed lines when using our upper and lower values for $\gamma_2^\alpha$.

region between 1.3 and 1.5 MeV is due to the f-wave contribution which was not considered in the calculation as it is not contributing to the E1 component.

A comparison of our results for the E1 and E2 S-factors at 300 keV with those of some previous works are given in table 1.
Figure 5. R-matrix calculation [3] (see text) of the β-delayed α-spectrum of $^{16}$N together with data obtained in [20]. Only the p-wave was considered in the calculation.

Table 1. Comparison of the astrophysical S-factor at 300 keV obtained in various experiments including this work for the E1 and E2 component as well as the total.

| Experiment            | $S_{E1}(0.3 \text{ MeV})$ (keV-barn) | $S_{E2}(0.3 \text{ MeV})$ (keV-barn) | $S_{\text{total}}(0.3 \text{ MeV})$ (keV-barn) |
|-----------------------|--------------------------------------|--------------------------------------|-----------------------------------------------|
| This work [3]         | 100±28                               | 50±19                                | 175$^{+63}_{-62}$                              |
| Brune et al. [10]     | 101±17                               | 44$^{+19}_{-23}$                     | 170$^{+52}_{-55}$                              |
| Tischhauser et al. [12]| -                                    | 53±13                                | -                                             |
| Tang et al. [20]      | 84±21                                | -                                    | -                                             |
| Kunz et al. [14]      | 76±20                                | 85±30                                | 186$^{+66}_{-65}$                              |
| NACRE [21]            | 79±21                                | 120±60                               | 224$^{+97}_{-96}$                              |

Our deduced values for the E1 and E2 S-factors are in excellent agreement with those of Brune et al. [10] and Tischhauser et al [12] and in good agreement with those of Tang et al. [20] and Kunz et al. [14] within the error bars. Concerning NACRE recommended values, our E1 S-factor is in agreement with NACRE result within the error bars while for the E2 component, our central value is two times smaller than NACRE recommended value and our error bar is much smaller.

If we consider for the cascade transition, the value of 25±16 keV-b obtained by Matei et al. [22] which is in total disagreement with the recent value ($< 1$ keV-b) obtained in Schürmann et al. work [23], we obtain a total S-factor of 175±16 keV-b. This results is in good agreement with Brune’s et al result (see table 1) and some previous works [3].

Note that in our work as well as in Brune’s et al one, the alpha-reduced widths of the two sub-threshold states were fixed in the R-matrix calculations at the measured values which is not
the case of all other works where they are considered as free parameters. It is interesting to notice that the two works where the $\gamma_\alpha^2$ of the 6.92 and 7.12 MeV states are kept fixed obtain very close results. This is likely due to the additional constraint provided by this fixed parameters in the R-matrix calculations.

4. Conclusion

The $\alpha$-spectroscopic factors $S_\alpha$ and the reduced $\alpha$-widths $\gamma_\alpha^2$ of the two sub-threshold $2^+$ ($E_x=6.92$ MeV) and $1^-$ ($E_x=7.12$ MeV) states of $^{16}$O were determined from the analysis of the transfer reaction $^{12}$C($^7$Li,t)$^{16}$O measurement performed recently at two incident energies. The uncertainties on the $S_\alpha$ and $\gamma_\alpha^2$ of the 6.92 and 7.12 MeV states of interest are now well and carefully determined thanks to the detailed finite range DWBA analysis of the measured data. The obtained $\gamma_\alpha^2$ for the $2^+$ and $1^-$ sub-threshold resonances were introduced in R-matrix calculations in order to determine the E2 and E1 S-factor at the energy of 300 keV. The result for the E1 S-factor at 300 keV confirms the values obtained in various direct and indirect measurements as well NACRE compilation [3] while for the E2 component, the central value of our result is found to be nearly two times smaller than NACRE recommended value. Our results are in excellent agreement with Brune’s et al. [10] ones and in both works, the $\gamma_\alpha^2$ or the ANC’s of the two sub-threshold states were fixed in the R-matrix calculation at the measured values leading to a bigger constraint in the fitting procedure.

Nevertheless, more precise cascade transitions measurements as well as more precise E2 direct data at higher energies are needed to have a better determination of the total S-factor of $^{12}$C($\alpha, \gamma$)$^{16}$O at 300 keV.

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