Parton-Parton Elastic Scattering and Rapidity Gaps
at SSC and LHC Energies

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ABSTRACT

The theory of the perturbative pomeron, due to Lipatov and collaborators, is used to compute the probability of observing parton-parton elastic scattering and rapidity gaps between jets in hadron collisions at SSC and LHC energies.

1. Introduction

At the SSC and LHC hadron colliders events predicted by the Standard Model, like Higgs-boson production via electroweak boson fusion, will be experimentally accessible. A characteristic signature of this process is that in the $t$ channel no color is exchanged between the scattering hadrons, the color exchange being confined to the fragmentation region between the struck and spectator partons. On a Lego plot in azimuthal angle and rapidity, the signal will present, at the parton level, a rapidity gap between the struck partons. In order, though, for the gap to survive at the hadron level, it is necessary that no color radiation is exchanged between the spectator partons in the hadronization process. Bjorken has first defined and estimated the survival of the rapidity gap in the presence of soft spectator interactions. A study of the event characteristics of Higgs-boson production via electroweak-boson fusion has then been recently undertaken.

The Higgs boson may be produced also via gluon-gluon fusion. This will usually have color all over the $t$ channel, since the gluons, on their way to fuse together, will emit gluon radiation. A fraction of events in this process, though, may simulate Higgs-boson production via electroweak-boson fusion even in its characteristic signature, namely the Higgs boson may be produced by the fusion of two pairs of gluons in color singlet configurations. Then no color is exchanged in the $t$ channel between the struck partons. To understand the dynamics of these background events, it is better to undertake the propedeutic study of hadron-hadron scattering with exchange in the $t$ channel of a pair of gluons in a color singlet configuration. Such a study can be already done experimentally at the energies of the Tevatron collider, and indeed the first data on rapidity gaps in hadron collisions starts being
2. Rapidity Gaps

In this talk I illustrate a way of computing the probability of observing parton-parton elastic scattering and rapidity gaps between jets in hadron collisions at very high energies\(^7\), and use it to make predictions on rapidity-gap production at SSC and LHC energies. In order to obtain quantitative predictions of jet production in the very high energy limit and separate it from the uncertainty involving the small \(x\) dependence of parton distributions\(^8\), Mueller and Navelet\(^9\) proposed to measure the two-jet inclusive cross section in hadron collisions by tagging two jets at a large rapidity interval \(y\) and with transverse momentum of order \(m\). These tagging jets are produced in a nearly forward scattering of gluons or quarks with large center-of-mass energy \(\sqrt{s}\). Lipatov and collaborators\(^{10–13}\) (BFKL) have shown that, in this regime, the rapidity interval \(y = \ln(s/m^2)\) between the scattered partons is filled in by the radiation of additional gluons, roughly uniformly spaced in rapidity, all with transverse momenta of order \(m\). The BFKL theory systematically corrects the lowest-order QCD result by summing the leading logarithms of \(s\). The result is to replace the gluons exchanged in the \(t\) channel with effective, reggeized gluons, with an infrared-sensitive trajectory\(^{11}\). Then one uses this resummed, effective gluon exchange to compute the elastic amplitude in the Regge limit \(s \gg -\hat{t}\) with color singlet exchange in the \(t\)-channel. This is known as the BFKL pomeron\(^{11–13}\). The imaginary part of the forward amplitude is the parton-parton total cross section.

To leading order in rapidity, the parton-parton total cross section and the related 2-jet inclusive cross section exhibit the energy dependence \(\exp[(\alpha_P - 1)y]\) with

\[
\alpha_P = 1 + 4 \ln 2 \frac{\alpha_s C_A}{\pi},
\]

where \(C_A = N_c = 3\) is the number of colors in QCD.

The elastic scattering amplitude with color singlet exchange in the \(t\) channel is a higher order \((\alpha_s^4)\) process\(^{13–14}\) but with energy dependence \(\exp[2(\alpha_P - 1)y]\), and it leads to a final state which, at the parton level, contains two jets with a rapidity gap in gluon production between them. Some fraction of these states may produce the experimental signatures of a large rapidity gap in secondary particle production.

To understand the relation between rapidity gaps in hard-gluon and hadron production, we must discuss the potential backgrounds to these signals at the parton and hadron level. To analyze the parton-level background, assume that we cannot detect partons with transverse momentum smaller than a fixed parameter \(\mu\). In this case, there is an additional contribution to elastic scattering from color octet exchange in the \(t\) channel. According to BFKL, this proceeds via the exchange of a reggeized gluon, which contains all the leading virtual radiative corrections and it has the form of a Sudakov form factor. The parameter \(\mu\) fixes the scale below which soft gluon radiation is allowed. As \(\mu \to 0\), the contribution of the color octet exchange vanishes, since it is impossible to have scattering with exchange of a gluon,
without allowing for the emission of soft gluon radiation.

In order to use perturbative QCD, the parameter $\mu$ must be larger than $\Lambda_{QCD}$. Thus we have two options: first, we can consider $m \gg \mu \gg \Lambda_{QCD}$, that is, we define a rapidity gap to be present if there are no jets of transverse momentum larger than $\mu$ between the tagging jets. We will call this case quasi elastic scattering, since it allows gluon radiation below the scale $\mu$. The ratio $R$ of the quasi-elastic to the total cross section is given by

$$R(\mu) = \frac{\sigma_{\text{singlet}} + \sigma_{\text{octet}}}{\sigma_{\text{tot}}}.$$  

(2)

where all the cross sections in (2) have been convoluted with the appropriate parton distributions. Alternatively, we can consider $\mu = O(\Lambda_{QCD})$. Then at the parton level the color octet exchange is strongly suppressed, and only the color singlet exchange contributes to the cross section for producing rapidity gaps.

At the hadron level, the interaction between spectator partons may produce hadrons across the rapidity interval, spoiling the rapidity gap. Thus in order to compute the cross section for producing a rapidity gap at the hadron level, we need a non-perturbative model which describes the hadron interaction and estimates the survival of the rapidity gap in the presence of soft spectator interactions\(^2\). The rapidity-gap survival probability $<S^2>$ is defined as the probability that in a scattering event no other interaction occurs beside the hard collision of interest. $<S^2>$ is expected to depend on the hadron-hadron center of mass energy, but only weakly on the size of the rapidity gap. Then to obtain the probability of a scattering event with a large rapidity gap at the hadron level, we must compute the ratio $R$ at $\mu = 0$, that is, using only the singlet elastic cross section, and multiply it by the survival probability $<S^2>$:

$$R_{\text{gap}} = <S^2> R(\mu = 0).$$  

(3)

In this contribution, we compute $R(\mu)$ at the SSC and LHC center-of-mass energies $\sqrt{s}$ of 40 TeV and 16 TeV respectively, and at different values of the minimum transverse momentum of the tagging jets $m$ and the elastic scale $\mu$.

3. Jet Cross Sections

We consider the scattering of two hadrons of momenta $k_A$ and $k_B$ in the center-of-mass frame and we imagine to tag two jets at the extremes of the Lego plot, with the rapidity interval between them filled with jets. The tagging jets can be characterized by their transverse momenta $p_{A\perp}$ and $p_{B\perp}$ and by their rapidities $y_A$ and $y_B$. The inclusive cross section for producing two tagging jets with transverse momenta greater than a minimum value $m$ is then\(^9\)

$$\frac{d\sigma}{dydy}(AB \rightarrow j(x_A)j(x_B) + X) =$$

$$\int dp_{A\perp}^2 dp_{B\perp}^2 \prod_{i=A,B} \left[ G(x_i, m^2) + 4/9 \sum_f [Q_f(x_i, m^2) + \bar{Q}_f(x_i, m^2)] \right] \frac{d\sigma_{\text{tot}}}{dp_{A\perp}^2 dp_{B\perp}^2}.$$  

(4)
where \( x_A \simeq e^{y_A p_{A \perp}}/\sqrt{s}, \) \( x_B \simeq e^{-y_B p_{B \perp}}/\sqrt{s} \) are the light-cone momentum fractions of the tagging jets with respect to their parent hadrons, \( y = |y_A - y_B| \) is the rapidity difference and \( \bar{y} = (y_A + y_B)/2 \) is the rapidity boost, \( \bar{s} = 2k_A \cdot k_B x_A x_B \) is the parton-parton squared center-of-mass energy, and

\[
\frac{d\hat{\sigma}_{\text{tot}}}{dp_{A \perp}^2 dp_{B \perp}^2} = \frac{(\alpha_s C_A)^2}{2p_{A \perp}^2 p_{B \perp}^2} \int_0^\infty d\nu e^{\omega(\nu)y} \cos \left( \nu \ln \frac{p_{A \perp}^2}{p_{B \perp}^2} \right) \tag{5}
\]

is the BFKL total cross section for gluon-gluon scattering within an impact distance of size \( 1/m, \) and

\[
\omega(\nu) = \frac{2\alpha_s C_A}{\pi} \left[ \psi(1) - \text{Re} \psi\left(\frac{1}{2} + i\nu\right) \right], \tag{6}
\]

with \( \psi(z) \) the logarithmic derivative of the Gamma function. In eq. (4) we use the large-\( y \) effective parton distribution functions\(^{15} \), computed at the factorization scale \( Q^2 = m^2 \).

The high-energy elastic cross section for two tagging jets, with color singlet exchange in the \( t \) channel, is

\[
\frac{d\hat{\sigma}_{\text{sing}}}{dydy} (AB \to j(x_A)j(x_B)) = \int dt \left[ \prod_{i=A,B} \left[ G(x_i, m^2) + (4/9)^2 \sum_f [Q_f(x_i, m^2) + \bar{Q}_f(x_i, m^2)] \right] \right] d\hat{\sigma}_{\text{sing}} \times \frac{dt}{dt}, \tag{7}
\]

where \( \hat{t} \simeq -p_{\perp}^2 \), with \( p_{\perp} \) the transverse momentum of the tagging jets. The gluon-gluon elastic scattering cross section, with the tagging jets collimated and with minimum transverse momentum \( m, \) is\(^ {14} \)

\[
\frac{d\hat{\sigma}_{\text{sing}}}{dt} = \frac{(\alpha_s C_A)^4}{4\pi^2} \left( \int_{-\infty}^{\infty} d\nu \frac{\nu^2}{(\nu^2 + \frac{4}{9})^2} e^{\omega(\nu)y} \right)^2. \tag{8}
\]

Since two reggeized gluons are involved in the color singlet exchange in the \( t \) channel, in keeping into account in (7) the possibility that the scattering is initiated by quarks we obtain the suppression factor \((4/9)^2 \). The background to the color singlet exchange comes from the exchange of a reggeized gluon. This contribution is given by

\[
\frac{d\hat{\sigma}_{\text{octet}}}{dydy} (AB \to j(x_A)j(x_B)) = \prod_{i=A,B} \left[ G(x_i, m^2) + 4/9 \sum_f [Q_f(x_i, m^2) + \bar{Q}_f(x_i, m^2)] \right] d\hat{\sigma}_{\text{octet}} \times \frac{dt}{dt}, \tag{9}
\]

where the gluon-gluon elastic scattering cross section in the color octet channel is

\[
\frac{d\hat{\sigma}_{\text{octet}}}{dt} = \frac{\pi(\alpha_s C_A)^2}{2\ell^2} \exp \left( -\frac{\alpha_s C_A}{\pi} \frac{y}{\sqrt{1 + 4\mu^2/p_{\perp}^2}} \ln \frac{\sqrt{1 + 4\mu^2/p_{\perp}^2} + 1}{\sqrt{1 + 4\mu^2/p_{\perp}^2} - 1} \right), \tag{10}
\]

For \( m \gg \mu \) the exponential reduces to\(^ {14} \) \( \exp(-\alpha_s C_A/\pi \ln(p_{\perp}^2/\mu^2) y) \) and has the typical form of a Sudakov form factor. As \( \mu \to 0, \) or \( y \) becomes large, the contribution of the octet to the gluon-gluon elastic cross section vanishes.

4. The Numerical Evaluation of the Ratio \( R(\mu) \)

\( R(\mu) \) is the probability of having elastic scattering at the parton level, as de-
fined in (2), and is obtained by summing (7) and (9), and dividing by (4). To evaluate it, we scale the running coupling constant from $\alpha_s(m(Z)) = 0.12$ using the 1-loop evolution with 5 flavors, and use the CTEQ set-5 parton distribution functions$^{16}$. We plot $R(\mu)$ at LHC and SSC energies as a function of $y$, at rapidity boost $\bar{y} = 0$, and with $m = 30, 60, 100$ GeV and elastic scale $\mu = 0, 0.5, 2, 5, 10$ GeV. The rapid growth of $R(\mu)$ at the kinematical upper boundary in $y$ is due to the energy dependence of the pomeron trajectory (1) enhanced by the scaling behavior at $x$ near 1 of the distribution functions integrated over transverse momentum.
The ratio $R(\mu)$ as a function of $y$, at $\bar{y} = 0$, at $\sqrt{s} = 16$ TeV in the left column and $\sqrt{s} = 40$ TeV in the right column.

The growth of $R(\mu)$ due to the pomeron trajectory only is more apparent in the plots with $m = 30$ GeV where the largest kinematically accessible values of $y$ can be probed. At $\mu = 0$, the octet exchange does not contribute to $R(\mu)$. The value of $R(\mu = 0)$, multiplied by the survival probability $\langle S^2 \rangle$, gives the probability of having a collision with a rapidity gap in secondary particle production (3). Since $\langle S^2 \rangle$ is estimated in ref. 2 to be $\approx 0.1$ and in ref. 17 to be in the range of 0.05 to 0.2, we expect that, at the LHC and SSC, a few tenths percent of events with tagging jets will show rapidity gaps. $\langle S^2 \rangle$, though, is expected to decrease with $\sqrt{s}$, so fewer rapidity-gap events should be found at LHC and SSC energies than at Tevatron energies, at comparable values of $y$\textsuperscript{18}. The probability of finding a gap increases with the rapidity interval between the tagging jets. This prediction is peculiar of the radiative corrections to $R(\mu)$, since $R(\mu)$ at the lowest order in $\alpha_S$ does not depend on $y$.

Since all of the analysis above is in the leading logarithmic approximation, there is ambiguity in the choice of the proper scale in rapidity for which this analysis is valid, and so the exact value of the normalization and thus of $R(\mu)$ cannot be determined precisely. However, the slope of the curves in the asymptotic regime is free from this scale uncertainty and thus the experimental measurement of the ratio $R(\mu)/R_{\text{gap}}$ in the large rapidity-gap regime should give us an unambiguous determination of the survival probability $\langle S^2 \rangle$.

Work supported by the Department of Energy, contract DE-AC03-76SF00515.

I wish to thank Bj Bjorken, Jerry Blazey, Andrew Brandt, Jeff Forden, Michael Peskin, Carl Schmidt, Michael Sotiropulos, George Sterman, Wai-Keung Tang and Harry Weerts for many stimulating discussions and suggestions.

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