Calculation of some properties of the vacuum

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In this article, we calculate the dressed quark propagator with the flat bottom potential in the framework of the rain-bow Schwinger-Dyson equation, which is determined by mean field approximation of the global colour model lagrangian. The dressed quark propagator exhibits a dynamical symmetry breaking phenomenon and gives a constituent quark mass about 392 MeV, which is close to the value of commonly used constituent quark mass in the chiral quark model. Then based on the dressed quark propagator, we calculate some properties of the vacuum, such as quark condensate, mixed quark condensate \( g_0 |\bar{q}G_{\mu\nu}\sigma^{\mu\nu}q|0 \rangle \), four quark condensate \( g_0 |\bar{q}G_T q|0 \rangle \), tensor, \( \pi \) vacuum susceptibilities. The numerical results are compatible with the values of other theoretical approaches.

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I. INTRODUCTION

To investigate the long distance properties of quantum chromodynamics (QCD), many approaches are proposed, such as lattice gauge theory, QCD sum rules, chiral perturbation theory and phenomenological quark models. Each of these approaches has both advantages and disadvantages. For example, lattice calculations are rigorous from the point of view of QCD, but they suffer from lattice artifacts and uncertainties connected with the necessary extrapolation to the physical quark masses. The QCD sum rules approach, introduced by Shifman, Vainshtein and Zakharov [1,2] in 1979, tries to incorporate nonperturbative elements of full QCD. As the starting point, operator product expansion (OPE) method was used to expand the time ordered currents into a series of quark and gluon condensates which parameterize the long distance properties, while the short distance effects are incorporated in the Wilson coefficients. However, these elements (condensates) can not yet be calculated rigorously in QCD, and they are determined from experimental data in the one or other way (For example, the quark condensate can be determined from experimental data with the help of Gell-Mann-Oakes-Renner relation [1,3]); moreover, although for medium and asymptotic momentum transfers the OPE method can be applied for form factors and the moments of wave functions [4–6], at low momentum transfer, the standard OPE method cannot be consistently applied \(^1\), as pointed out in the early work on photon couplings at low momentum for the nucleon magnetic moments [7,8]. In Ref. [7], the problem was solved by using a two point correlator in an external electromagnetic field, with vacuum susceptibilities introduced as parameters for nonperturbative propagation in the external field. In fact, the existence and magnitude of the vacuum susceptibilities are by themselves an important physical information about the structure of the vacuum and have many applications [9–12]. As nonperturbative vacuum properties, the susceptibilities can be introduced for both small and large momentum transfers in the external fields. In Ref. [8], with the long distance effects treated by bilocal power corrections and assumption of \( \rho \) meson dominance, the authors circumvent the problem using a three point

\(^1\)The OPE (Wilson expansion) is valid as long as there is a clear distinction between short distances (\( \propto \frac{1}{\mu^2} \)) which determine the Wilson coefficients and long distances (\( \propto \frac{1}{\mu}, \mu \sim \Lambda_{QCD} \)) which govern the condensates. When the transformed momentum \( Q^2 \) is small, the standard expansion in \( \frac{1}{Q^2} \) would lead to a divergent result.
formulation. For detailed discussion of the relationship between three point and two point external field treatments and the origin of the susceptibilities, one can see Ref. [13]. The two point method, however, has two main problems: it cannot be used to extend the coupling to medium and high momentum transfer and there are additional parameters to be determined, the vacuum susceptibilities. In Ref. [14], by comparing terms appearing in the two point external field expression with those in hybrid expansion of the three point function, the authors obtain a relationship between the nonperturbative elements in the two methods. From these relationships, one can express the induced susceptibilities of the two point method in terms of well-defined four quark vacuum matrix elements. These susceptibilities may play an important role in the QCD sum rules approach. In particular, the strong and parity-violating pion-nucleon coupling depends crucially on the $\pi$ vacuum susceptibility ($\chi_{\pi}^a$), while the tensor vacuum susceptibilities are relevant for the determination of the tensor charge of the nucleon, which is connected through deep-inelastic sum rules to the leading-twist nucleon structure functions and transversity distribution [15–17]. Although in principle the pion susceptibility can be estimated using PCAC, in practice there are inconsistencies with the values needed phenomenologically [14,12].

On the other hand, as it has been pointed out in Ref. [18], that treatment of the vacuum tensor susceptibility is subtle and different treatments can lead to very different results for the tensor charge. Therefore, it is interesting to calculate these vacuum condensates and vacuum susceptibilities within the framework of coupled flat-bottom potential (FBP) model [19] and global colour model (GCM) [20–23].

The global colour model (GCM), a NJL-like theory [24], has provided a lot of successful descriptions of the long distance properties of strong interaction and the QCD vacuum as well as hadronic phenomena at low energy based on the theoretical foundation that the quark propagator contains valuable information about the nonperturbative properties of QCD [20–23]. The flat-bottom potential is a sum of Yukawa potentials, which not only satisfies gauge invariance, chiral invariance and fully relativistic covariance, but also suppresses the singular point which the Yukawa potential has. It works well in understanding the meson structure, such as electromagnetic form factor, radius, decay constant [19].

In this article, we combine the GCM with the FBP to calculate the quark condensate, $g_s \langle 0 | \bar{q} G_{\mu \nu} \sigma^{\mu \nu} q | 0 \rangle$, tensor, pion vacuum susceptibilities in the framework of the rainbow Schwinger-Dyson (SD) equation. The article is arranged as follows: in section 2, we brief out line the GCM and in section 3, introduce the FBP, SD equation; in section 4, we calculate the vacuum condensates and vacuum susceptibilities; in section 5, conclusion and discussion.

II. GLOBAL COLOUR MODEL

Here we brief out line the main skeleton of the GCM. The global colour model, based upon an effective quark-quark interaction which is approximated by the effective flat bottom potential in this article, is defined through a truncation of QCD lagrangian while maintaining all global symmetries.

The generating function for QCD in Euclidean space can be written as

$$Z[\eta, \bar{\eta}] = \int \mathcal{D}q \mathcal{D}\bar{q} \mathcal{D}A \exp \left\{ -S + \int d^4x (\bar{q} \eta + \bar{\eta} \q) \right\}$$

(1)

where

$$S = \int d^4x \left\{ \bar{q} \left[ \gamma_\mu \left( \partial_\mu - ig \frac{\lambda^a}{2} A_\mu^a \right) + m \right] q + \frac{1}{4} G^{a}_{\mu \nu} G^a_{\mu \nu} \right\},$$

and $G^a_{\mu \nu} = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A^b_\mu A^c_\nu$. We leave the gauge fixing term, the ghost field term and its integration measure to be understood. Here we introduce
\[ e^{W[J]} = \int \mathcal{D} A e^{\int d^4x (-\frac{1}{4} G^a_{\mu\nu} G^a_{\mu\nu} + J^a_{\mu} A^a_{\mu})}. \]  

(2)

So the generating function can be written as

\[ Z[\bar{\eta}, \eta] = \int \mathcal{D} q \mathcal{D} \bar{q} e^{-\int d^4x [\bar{q}(\gamma \cdot \partial + m)q - \bar{q}q - g \bar{q}\eta - \bar{q}\bar{\eta}]} e^{W[J]} . \]

(3)

The functional \( W[J] \) can be formally expanded in the current \( J^a_\mu \).

\[ W[J] = \frac{1}{2} \int d^4x d^4y J^a_\mu(x) D^{ab}_{\mu\nu}(x, y) J^b_\nu(y) + \frac{1}{3!} \int J^a_\mu J^b_\nu J^c_\rho D^{abc}_{\mu\nu\rho} + \cdots . \]

(4)

The GCM is defined through a truncation of the functional \( W[J] \) in which the higher order \( n(\geq 3) \)-point functions are neglected, and only the gluon 2-point function \( D^{ab}_{\mu\nu}(x, y) \) is retained. This model maintains all global symmetries of QCD and permits a \( 1/N_c \) expansion, however local SU(3) gauge invariance is lost by the truncation.

Using the functional integration approach, the generating function of the truncation is given by

\[ Z[GCM] = \int \mathcal{D} \mathcal{S} e^{-S[\mathcal{B}]}, \]

(6)

where the action is given by

\[ S[\mathcal{B}] = -\text{Tr} \ln[\mathcal{G}^{-1}] + \int d^4x d^4y \frac{\mathcal{B}^a(x, y) \mathcal{B}^a(y, x)}{2g^2 D(x - y)} + \int d^4x d^4y \eta(x) G(x, y; [\mathcal{B}^a]) \eta(y), \]

(7)

and the inverse quark’s Green function \( G^{-1} \) is defined as

\[ G^{-1}(x, y) = \gamma \cdot \partial \delta(x - y) + \Lambda^a \mathcal{B}^a(x, y). \]

(8)

Here the quantity \( \Lambda^a \) arises from Fierz reordering of the current-current interaction term in Eq.(5),

\[ \Lambda^a_m \Lambda^a_m = (\gamma_\mu \frac{\lambda^a}{2})_{ij} (\gamma_\mu \frac{\lambda^a}{2})_{mn} \]

(9)

and is the direct product of Dirac, flavor SU(3) and color matrices,

\[ \Lambda^a = \frac{1}{2} \left( 1_D, \gamma_5, \frac{i}{\sqrt{2}} \gamma_\mu, \frac{i}{\sqrt{2}} \gamma_\mu \gamma_5 \right) \otimes \left( \frac{1}{\sqrt{3}} 1_F, \frac{1}{\sqrt{3}} \lambda^a_F \right) \otimes \left( \frac{4}{3} 1_c, \frac{i}{\sqrt{3}} \lambda^a_c \right). \]

(10)

The vacuum configurations are defined by minimizing the bilocal action: \( \frac{\delta S[\mathcal{B}]_{[\bar{\eta}, \eta = 0]}}{\delta \mathcal{B}} = 0 \), following this mean field approximation, we obtain

\[ \mathcal{B}^a_0(x - y) = g^2 D(x - y) \text{tr} [\Lambda^a G_0(x - y)]. \]

(11)

These configurations provide self-energy dressing of the quarks through the definition \( \Sigma(p) = \Lambda^a \mathcal{B}^a_0(p) = i \gamma \cdot p [\mathcal{A}(p^2) - 1] + \mathcal{B}(p^2) \). In fact, equation (11) is the rain-bow SD equation. In terms of \( A \) and \( B \), the quark’s Green function at \( \mathcal{B}^a_0 \) is given by

\[ G_0(x, y) = G_0(x - y) = \int \frac{d^4p}{(2\pi)^4} \frac{-i \gamma \cdot p A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} e^{ip(x - y)}. \]

(12)
With the GCM generating function Eq.(6), it is straightforward to calculate the vacuum expectation value of any quark operator of the form

\[ O_n \equiv (\bar{q}_i \Lambda^{(1)}_{i,j_1} q_{j_1})(\bar{q}_{i_2} \Lambda^{(2)}_{i_2,j_2} q_{j_2}) \cdots (\bar{q}_{i_n} \Lambda^{(n)}_{i_n,j_n} q_{j_n}), \]  

(13)
in the mean field vacuum. Here the \( \Lambda^{(i)} \) represents an operator in Dirac, flavor and color space.

Taking the appropriate number of derivatives with respect to the external sources \( \eta_i \) and \( \bar{\eta}_j \) of Eq.(6) and putting \( \eta_i = \bar{\eta}_j = 0 \) [25], we obtain

\[ \langle 0 | : O_n : | 0 \rangle = (-)^n \sum_p \left[ (-)^p \Lambda^{(1)}_{i,j_1} \cdots \Lambda^{(n)}_{i_n,j_n} (G_0)_{j_1,i_{p(1)}} \cdots (G_0)_{j_n,i_{p(n)}} \right], \]  

(14)

where \( p \) stands for a permutation of the \( n \) indices. From Eq.(14), we can easily obtain the expression for the nonlocal four quark condensate, which is another important vacuum condensate in the QCD sum rules approach beside quark condensate.

\[ \langle 0 | : \bar{q}(x)\Lambda^{(1)} q(x)\bar{q}(y)\Lambda^{(2)} q(y) : | 0 \rangle = -tr\gamma_C[G_0(y,x)\Lambda^{(1)} G_0(x,y)\Lambda^{(2)}] + tr\gamma_C[G_0(x,x)\Lambda^{(1)}]tr\gamma_C[G_0(y,y)\Lambda^{(2)}]. \]  

(15)

### III. FLAT BOTTOM POTENTIAL AND SCHWINGER-DYSON EQUATION

The phenomenological FBP is a sum of Yukawa potentials which is an analogous to the exchange of a series of particles and ghosts with different masses,

\[ G(k^2) = \sum_{j=0}^{n} \frac{a_j}{k^2 + (N + j\rho)^2}, \]  

(16)

where \( N \) stands for the minimum value of the mass, \( \rho \) is their mass difference, and \( a_j \) is their relative coupling constant. In its three dimensional form, the FBP takes the following form:

\[ V(r) = -\sum_{j=0}^{n} a_j \frac{e^{-(N+j\rho)r}}{r}. \]  

(17)

In order to suppress the singular point at \( r = 0 \), we take the following conditions:

\[ \frac{dV(0)}{dr} = \frac{d^2V(0)}{dr^2} = \cdots = \frac{d^nV(0)}{dr^n} = 0. \]  

(18)

So we can determine \( a_j \) by solve the following equations, inferred from the flat bottom condition Eq.(18),

\[ \sum_{j=0}^{n} a_j = 0, \quad \sum_{j=0}^{n} a_j(N + j\rho) = V(0), \quad \sum_{j=0}^{n} a_j(N + j\rho)^2 = 0, \quad \cdots \quad \sum_{j=0}^{n} a_j(N + j\rho)^n = 0. \]  

(19)

As in previous literature [19], \( n \) is set to be 9. In Ref. [19] (Phys. Rev. D47 (1993) 2098), the authors solve the Bethe-Salpeter equation based on expansion in terms of Gegenbauer polynomials with FBP, then use the Bethe-Salpeter amplitudes as input to calculate the \( \pi \) meson electromagnetic form factor. Among many other values, the phenomenological value 9 can give stable solution and best fit to experimental data\(^\ddagger\).

\[^\ddagger\]Private communication.
In the rainbow approximation, the SD equation takes the following form:

\[
S^{-1}(p) = \gamma \cdot p - \hat{m} + \frac{16 \pi i}{3} \int \frac{d^4 k}{(2\pi)^4} \gamma_\mu S(k) \gamma_\nu G^{\mu\nu}(k - p),
\]

where

\[
S^{-1}(p) = A(p^2) \gamma \cdot p - B(p^2) \equiv A(p^2)[\gamma \cdot p - m(p^2)],
\]

\[
G^{\mu\nu}(k) = -g^{\mu\nu}G(k^2),
\]

and \( \hat{m} \) stands for an explicit quark mass-breaking term. With the explicit small mass term, we can preclude the zero solution for \( B(p) \) and in fact there indeed exist a bare current quark mass. In order to keep consistently with section two, here we take Feynman gauge. This dressing comprises the notation of constituent quarks by providing a mass \( m(p^2) = B(p^2)/A(p^2) \), which is corresponding to dynamical symmetry breaking. Because the form of the gluon propagator \( g^2 G(p) \) in the infrared region is unknown, one often uses model forms as input in the previous studies of the rainbow SD equation [20,22,23,36].

As a nonperturbative phenomenological potential, the FBP may have intrinsic connections with other nonperturbative phenomena, such as confinement, instanton, vacuum condensate and renormalon, glueball. In this article, we concern mainly the quark condensates, while the vacuum condensates play an important role in determining the properties of the mesons, glueballs and their mixing as indicated by the low energy theorem [28]. However, the realization of gluonic freedom in hadronic models is more ambiguous and difficulty then that for quarks, the glueball spectra obtained from two main nonperturbative methods (lattice QCD and QCD sum rules ) have some contradictions [30]. Here we just replace the gluon propagator with FBP and suppose the nonperturbative effects are incorporated in it, the possible effects on glueball will be our next work. § Although there may be debates against doing that, the quark propagator obtained with the FBP does lead to satisfactory dynamical symmetry breaking phenomenon and quark confinement based on the theoretical foundation that there no singularities on the real timelike \( p^2 \) axial [29].

In this article, we assume that a Wick rotation to Euclidean variables is allowed, and perform a rotation analytically continuing \( p \) and \( k \) into the Euclidean region where them can be denoted by \( \bar{p} \) and \( \bar{k} \), respectively. The Euclidean SD equation can be projected into two coupled integral equations for \( A(\bar{p}^2) \) and \( B(\bar{p}^2) \). For simplicity, here ignore the bar on \( p \) and \( k \). Numerical values for \( A(p^2) \), \( B(p^2) \) and \( m(p^2) \) are shown in Fig.[1].

IV. VACUUM CONDENSATES AND VACUUM SUSCEPTIBILITIES

In this section, we calculate the quark condensate \( \langle 0 | \bar{q} q | 0 \rangle \), the mixed quark gluon condensate \( g_s \langle 0 | i \bar{q} G_{\mu\nu} \sigma^{\mu\nu} q | 0 \rangle \), the four quark condensate \( \langle 0 | i \bar{q} \Gamma \bar{q} \Gamma q | 0 \rangle \) and tensor, pion vacuum susceptibilities. The numerical results are compared with other theoretical works.

Here we take a short digression to discussing gauge invariance. The propagation of a colored quark in the background gluon field results in a phase factor given by the exponential of a line integral,

\[
\hat{P}(x, y; A, C) = P exp \{ i g \int_C A_\mu(z) dz^\mu \},
\]

§For example, we can obtain the gluon condensate from the nonperturbative gluon propagator (the effective potential), then based on the gluon condensate and low energy theorem, we can study the lowest lying glueball mass, their mixing, and so on. There is another way to study the flat bottom potential to glueball spectrum through the Bethe-Salpeter equation.
where $P$ is a path order operator, stands for that the color matrices should be ordered along the path $C$ connecting the point $x$ and $y$. The gauge dependence of the quark propagator can be factored out as an "A-dependent" phase before taking the vacuum expectation value. In operator expanding of the correlators of colorless currents, these phase factors can be canceled out with each other, one can facilitate the calculation by imposing Fock-Schwinger gauge condition \[31,32\],

\[(x^\mu - x_0^\mu)A_\mu(x) = 0,\]

or in the other word, let $\hat{P} = 1$. To gauge dependent quantities, (for example, $\langle 0|\bar{q}(x)q(0)|0\rangle$), we can insert phase factors $\hat{P}$ to produce the corresponding gauge invariant ones ($\langle 0|\bar{q}(x)\hat{P}(x,0;A,C)q(0)|0\rangle$). The gauge invariant quantity $\langle 0| :\bar{q}(x)\hat{P}(x,0;A,C)q(0) :|0\rangle$ calculated in a special gauge, for example, Fock-Schwinger gauge, will not change its value. In the following, the gauge dependent quantities can be taken as their corresponding invariant ones in the special gauge, Fock-Schwinger gauge. Furthermore, although the gauge dependent quantities, such as $\langle 0| :\bar{q}(x)q(0) :|0\rangle$, are dependent on the special gauge we choose, they are important informations in describing the nonperturbative QCD vacuum. In a sense, the gauge dependence is fictitious, since the vacuum field, after averaging, is clearly invariant under transformations.

### A. Vacuum Condensates

The quark propagator is defined as

\[S(x) = \langle 0|T[q(x)\bar{q}(0)]|0\rangle.\]  

(25)

where $q(x)$ is the quark field and $T$ the time-ordering operator. For the physical vacuum consists of both perturbative and nonperturbative parts, so the quark propagator $S(x)$ can be divided into a perturbative and a nonperturbative part as the following:

\[S(x) = S_{PT}(x) + S_{NP}(x).\]  

(26)

In the nonperturbative vacuum, the normal-ordered product $S_{NP}(x)$ does not vanish. For short distance, the OPE for the scalar part of $S_{NP}(x)$ gives

\[\langle 0| :\bar{q}(x)q(0) :|0\rangle = \langle 0| :\bar{q}(0)q(0) :|0\rangle - \frac{x^2}{4}\langle 0| :\bar{q}(0)\sigma \cdot G(0)q(0) :|0\rangle + \cdots\]  

(27)

in which the local operators of the expansion are the quark condensate, the mixed condensate, and so forth. In Ref. \[34\] the nonlocal condensate is put in the following form:

\[\langle 0| :\bar{q}(x)q(0) :|0\rangle = g(x^2)\langle 0| :\bar{q}(0)q(0) :|0\rangle,\]  

(28)

where $g(x^2)$ is the vacuum non-locality of the nonlocal quark condensate. The nonlocal quark condensate $\langle 0| :\bar{q}(x)q(0) :|0\rangle$ is given then by the scalar part of the Fourier transformed inverse quark propagator, which can be inferred from Eq.\[(14).\]

\[\langle 0| :\bar{q}(x)q(0) :|0\rangle = (-)tr_{\gamma C}[G_0(x,0)]\]

\[= (-4N_c)\int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{B(p^2)}{p^2A_4^2(p^2) + B_2^2(p^2)}e^{ipx}\]

\[= (-)\frac{12}{16\pi^2} \int_0^\mu ds \frac{B(s)}{sA_4^2(s) + B_2^2(s)}\left[2J_1(\sqrt{s}x^2)\right]\]  

(29)

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At \( x=0 \) the expression for the local condensate \( \langle 0 | : \bar{q}(x)q(x) : |0 \rangle \) is recovered, 
\[
\langle 0 | : \bar{q}(0)q(0) : |0 \rangle = -\frac{12}{16\pi^2}\int_0^\mu dss\frac{B(s)}{sA^2(s) + B^2(s)}.
\] (30)

The non-locality \( g(x^2) \) can be obtained immediately by dividing Eq.(29) through Eq.(30). Our analysis ignores effects from hard gluonic radiative correction to the condensates which are connected to a possible change of the renormalization scale \( \mu \) at which the condensates are defined. As in Ref. [36], \( \mu \) is taken to be 1 GeV\(^2\). Final we obtain the nonlocal quark condensate which shown in Fig.[2] and local quark condensate which shown in Table 1. From Fig.[2], we can see that the nonlocality is compatible with other theoretical calculations, including instanton liquid model [37], effective quark-quark interaction model [34], GCM [38], dipole fit approach [14], and other potential model [41]. The curves obtained in Ref. [34] and in Ref. [14] are similar to each other, here we choose the curve in Ref. [34] to compare with our result. For simplicity, in Fig.2, we list three curves.

In [44], we calculate the nonlocality of the quark condensate; in calculation, we find that there are other parameters can give correct nonlocality curve. In the present work, we re-fit the parameters to incorporate the mixed condensate and the vacuum susceptibilities. Furthermore, in the present work, we change the coefficient of FBP inserted in the SD equation by a factor \( \pi \), the present formulation is better than the old one [44,19].

By means of Eq.(15), one can calculate all kinds of nonlocal four quark condensates. If we take \( \Lambda^{(1)} = \Lambda^{(2)} = \gamma_\mu \frac{\Lambda^2}{2} \) then
\[
\langle 0 | : \bar{q}(x)\gamma_\mu \frac{\lambda^\alpha}{2} q(x)\bar{q}(0)\gamma_\mu \frac{\lambda^\beta}{2} q(0) : |0 \rangle = (-) \int_0^\mu \int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{d^4q}{(2\pi)^4} e^{ix \cdot (p-q)} \left[ 4^4 \frac{B(p^2) A(p^2) q^2 + B^2(p^2)}{A^2(p^2) p^2 + B^2(p^2)} B(q^2) 
   + 2 \times 4^2 \frac{A(p^2) A(q^2)}{A^2(p^2) p^2 + B^2(p^2)} A^2(q^2) q^2 + B^2(q^2) \right] q^p \cdot q.
\] (31)

Similarly, at \( x=0 \) we obtain the expression for the local four quark condensate \( \langle 0 | : \bar{q}\gamma_\mu \frac{\lambda^\alpha}{2} q\bar{q}\gamma_\mu \frac{\lambda^\beta}{2} q : |0 \rangle \),
\[
\langle 0 | : \bar{q}\gamma_\mu \frac{\lambda^\alpha}{2} q\bar{q}\gamma_\mu \frac{\lambda^\beta}{2} q : |0 \rangle = (-4^4)[ \int_0^\mu \frac{d^4p}{(2\pi)^4} \frac{B(p^2)}{A^2(p^2) p^2 + B^2(p^2)} ]^2 = (-)\frac{4}{9} \langle 0 | : \bar{q}q : |0 \rangle^2.
\] (32)

As far as the mixed condensate \( g_\alpha \langle \bar{q}G_\mu,\sigma^{\mu\nu}q \rangle \) is concerned, the evaluation is somewhat lengthy, one can use the method described by Ref. [36] to obtain the mixed condensate in Euclidean space,
\[
g_\alpha \langle \bar{q}G_\mu,\sigma^{\mu\nu}q \rangle = (-) \frac{N_c}{16\pi^2} \left( \frac{27}{4} \int_0^\mu dsB \frac{2A(\frac{1}{2}A - 1)s + B^2}{A^2s + B^2} + 12 \frac{1}{4} \int_0^\mu ds^2 \frac{B(2A - A)}{A^2s + B^2} \right). \] (33)

In Table 1, we display our results for \( \langle \bar{q}q \rangle \) and \( g_\alpha \langle \bar{q}G_\mu,\sigma^{\mu\nu}q \rangle \) and compare them with the corresponding values obtained from other theoretical approaches, such as QCD sum rules [26], quenched lattice QCD [33], the instanton liquid model [35], the model of confining gluon propagator for GCM [36] and other potential model [39–41]. From Table 1, we can see that the values are compatible with other theoretical works.

**B. Vacuum Susceptibilities**

In the external field of QCD sum rule two–point method, one often encounters the quark propagator in the presence of a external current \( J^\Gamma(y) = \bar{q}(y)\Gamma q(y) \) (\( \Gamma \) stands for the appropriate combination of Dirac, flavor and colour matrices).
\[
S_{\alpha\beta}^{cc}^\Gamma(x) = \langle 0|T[q_\alpha^\dagger(x)q_\beta(x)]|0\rangle_{J\Gamma} = S_{\alpha\beta}^{cc}^{\Gamma,PT}(x) + S_{\alpha\beta}^{cc}^{\Gamma,NP}(x),
\] (34)
where $S_{q}^{c\Gamma, PT}(x)$ is the quark propagator coupled perturbatively to the current and $S_{q}^{c\Gamma, NP}(x)$ is the nonperturbative quark propagator in the presence of the external current $J^{\Gamma}$. (To be explicitly, here we keep all the indexes).

The vacuum susceptibility $\chi^{T}$ in the QCD sum rule two–point external field treatment can be defined as [14]

$$S_{\alpha\beta}^{c\Gamma, NP}(x) = \langle 0 | : q_{c}^{\alpha}(x)\bar{q}_{c}^{\beta}(0) : | 0 \rangle_{J^{\Gamma}} = -\frac{1}{12} \Gamma_{\alpha\beta} \delta_{c\epsilon} \chi^{T} H(x)(0) : \bar{q}(0)q(0) : | 0 \rangle,$$

(35)

where the phenomenological function $H(x)$ represents the nonlocality of the two quark nonlocal condensate. Obviously $H(0)=1$.

By comparing terms appearing in the two point external field expression with those in hybrid expansion of the three point function, one can obtain a relationship between the nonperturbative elements in the two methods. From this relationship, one can express the induced susceptibilities of the two point method in terms of well-defined four quark vacuum matrix element [14].

The presence of external field implies that $S_{\alpha\beta}^{c\Gamma}(x)$ is evaluated with an additional term $\Delta L \equiv -J^{\Gamma} \cdot \phi_{T}$ added to the usual QCD Lagrangian, where $\phi_{T}$ is the value of external field. In three point method of QCD sum rule, if one takes only a linear external field approximation, the $S_{\alpha\beta}^{c\Gamma, NP}(x)$ in Euclidean space is given by

$$S_{\alpha\beta}^{c\Gamma, NP}(x) = \int d^{4}y \ e^{-iqy} \langle 0 | : q_{c}^{\alpha}(x)\bar{q}(y)\Gamma q(y)\bar{q}_{c}^{\beta}(0) : | 0 \rangle. \tag{36}$$

Using Eq.(35) and Eq.(36) we obtain

$$-\frac{1}{12} \delta_{c\epsilon} \Gamma_{\alpha\beta} \chi^{T}(0) : \bar{q}(0)q(0) : | 0 \rangle = \int d^{4}y \ e^{-iqy} \langle 0 | : q_{c}^{\alpha}(0)\bar{q}(y)\Gamma q(y)\bar{q}_{c}^{\beta}(0) : | 0 \rangle. \tag{37}$$

Multiplying Eq.(37) by $\Gamma_{\beta\alpha} \delta_{c\epsilon}$, we get the result:

$$\chi^{T}a = -\frac{16\pi^{2}}{tr_{\gamma}(1T)} \int d^{4}y \ e^{-iqy} \langle 0 | : \bar{q}(0)\Gamma q(0)\bar{q}(y)\Gamma q(y) : | 0 \rangle. \tag{38}$$

In the case of tensor current ($\Gamma = \sigma_{\mu\nu}$), according to Eq.(14), we calculate the tensor matrix element explicitly,

$$\langle 0 | \bar{q}(0)\sigma_{\mu\nu}q(0)\bar{q}(y)\sigma_{\mu\nu}q(y) | 0 \rangle = \text{tr}_{\gamma} \int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}k}{(2\pi)^{4}} \frac{i}{\mu} \cdot p A(p^{2}) + B(p^{2}) - i\gamma \cdot k A(k^{2}) + B(k^{2})}{A(k^{2})k^{2} + B(k^{2})} \text{tr}_{\gamma} \int \frac{d^{4}k}{(2\pi)^{4}} \frac{i}{\mu} \cdot k A(k^{2}) + B(k^{2})}{A(k^{2})k^{2} + B(k^{2})} \tag{39}$$

$$- \int \int \frac{d^{4}p}{(2\pi)^{4}} \frac{d^{4}k}{(2\pi)^{4}} e^{-i(p-k)_{\mu}y} \left[ \sigma_{\mu\nu} - \frac{-i\gamma \cdot p A(p^{2}) + B(p^{2})}{A(p^{2})p^{2} + B(p^{2})} \sigma_{\mu\nu} - \frac{-i\gamma \cdot k A(k^{2}) + B(k^{2})}{A(k^{2})k^{2} + B(k^{2})} \right] \frac{B(p^{2})}{A(p^{2})p^{2} + B(p^{2})} \frac{B(k^{2})}{A(k^{2})k^{2} + B(k^{2})}.$$\]

where $N_{c} = 3$ is the number of colors.

In the limit $q \to 0$, we obtain

$$\chi^{T}a = 3 \int_{0}^{\mu} ds \left[ \frac{B(s)}{A^{2}s + B^{2}(s)} \right]^{2} = 0.0757 GeV^{2}. \tag{40}$$

From Eq.(36) we can see that the vacuum susceptibility originates from the nonlocal four quark condensate contribution. In Ref. [43], the authors get the same conclusion based on completely different viewpoint of duality.

The vacuum susceptibility $\chi^{T}a$ defined in Eq.(38) (same as Ref. [39]) has an opposite sign and a factor $4\pi^{2}$ larger than the definition in Refs. [15,18,43]. In order to compare with those estimations, we make a redefinition

$$\chi^{T}a \to -\frac{\chi^{T}a}{4\pi^{2}} = -0.0019 GeV^{2}. \tag{41}$$

8
In Table 2, we compare the result with the values of other theoretical estimations of the vacuum tensor susceptibility.

In the following, we calculate the \( \pi \) vacuum susceptibility which is crucial to determine the strong and particle-violating pion-nucleon coupling. In the case of pseudoscalar current, we take \( \Lambda^{(1)} = \Lambda^{(2)} = \gamma^5 \) in Eq.(15), and get the \( \pi \) vacuum susceptibility \( \chi^\pi a \):

\[
\chi^\pi a = 12\pi^2 \int dq \, \text{tr} \left[ \int \frac{d^4 p}{(2\pi)^4} \frac{-i\gamma \cdot p A + B(p^2)}{A^2 p^2 + B^2(p^2)} e^{-ip \cdot y} \gamma_5 \int \frac{d^4 q}{(2\pi)^4} \frac{-i\gamma \cdot q A + B(q^2)}{A^2 q^2 + B^2(q^2)} e^{iq \cdot y} \gamma_5 \right]
\]

\[
= 3 \int_0^\mu ds \frac{|qA|^2}{A^2 s + B^2(s)} = 1.06 \text{ GeV}^2, \quad (42)
\]

which is below the range \( \chi^\pi a \simeq (1.7 - 3.0) \text{ GeV}^2 \) obtained within a phenomenological approach [14]. In Ref. [14], the author made a simple estimation of the value based on the space time structure of the nonlocal quark condensate extracted from experimental data on sea-quark distributions but with a crude assumption of vacuum saturation for the intermediate states.

**V. CONCLUSION AND DISCUSSION**

In this article, we calculate the dressed quark propagator with the FBP in the framework of the rainbow SD equation, which is determined by mean field approximation of the GCM lagrangian. The dressed quark propagator exhibits a dynamical symmetry breaking phenomenon and gives a constituent quark mass about 392 MeV, which is close to the value of commonly used constituent quark mass 350 MeV in the chiral quark model. Then based on the dressed quark propagator, we obtain the quark condensate \( \langle 0|\bar{q}q|0 \rangle \), the mixed quark gluon condensate \( g_\mu \langle 0|\bar{q}G_{\mu\nu}q|0 \rangle \), the four quark condensate \( \langle 0|\bar{q}\Gamma q\bar{q}\Gamma q|0 \rangle \) and tensor, pion vacuum susceptibilities at the mean field level, which are compared with other theoretical calculations. The values of the nonlocality of quark condensate, local quark condensate, mixed quark condensate are compatible with other theoretical results. While the value of \( \pi \) susceptibility is below that estimated in Ref. [14], which is based on the space-time structure of the nonlocal quark condensate and a crude assumption of vacuum saturation for the intermediate states.

In Table 3, we also compare with other theoretical results. In Table 2, we can see that the numerical value of the tensor vacuum susceptibility varies with theoretical approach. It has been pointed out in Ref. [15] that experimental measurement of the tensor charge of the nucleon is possible, and one might be able to test the theoretical predictions of the tensor susceptibility in the future.

In solving the rainbow SD equation, the parameters are taken as \( \Lambda_{QCD} = 200 \text{MeV}, N = 3.0\Lambda_{QCD}, \ V(0) = -12.0\Lambda_{QCD}, \ \rho = 2.0\Lambda_{QCD}, \ m_u = m_d = 8\text{MeV} \) and the large momentum cut-off \( L = 630(\Lambda_{QCD}) \).

In calculation, the coupled integral equations for the quark propagator functions \( A(p^2) \) and \( B(p^2) \) are solved numerically by simultaneous iterations. The iterations converge rapidly to a unique stable solution of propagator functions and independent the initial guesses. The propagator functions \( A(p^2) \) and \( B(p^2) \) are shown in Fig.[1], at small \( p^2 \), \( A(p^2) \) differs from the value 1 appreciably, while it tends to 1 for large \( p^2 \). We find that at small \( p^2 \), \( m(p^2) \) is greatly re-normalized, while at large \( p^2 \), it takes asymptotic behaviour. For \( u \) and \( d \) quark, \( m(0) = 392 \text{MeV} \), which is close to the constituent quark masses, the connection of \( m(p) \) to constituent masses is somewhat less direct and is precise only for heavy quarks. For heavy quarks, \( m_{\text{constituent}}(p) = m(p) = 2m_{\text{constituent}}(p) \), for light quarks, it only makes a crude estimation [42]. At about \( p = 1\text{GeV} \), the mass function grows rapidly as the momentum decreases, that is an indication of dynamical symmetry breaking.

The results show that the phenomenological FBP works well with the rainbow SD equation in calculating nonlocal quark vacuum condensate, mixed condensate and vacuum susceptibilities. This can be extend to other system, such
as \( q\bar{q} \) system, \( qQ \) system, vector meson and \( qqq \) system or by applying the results to the calculation of quantities and processes requiring detailed knowledge of the quark propagator.

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Fig. 1 SD functions

\[ \frac{B(p)}{\Lambda_{QCD}} \]

\[ \frac{A(p)}{\Lambda_{QCD}} \]

\[ \frac{m(p)}{\Lambda_{QCD}} \]
Fig. 2 Nonlocality

- **This Work**
- **Ref. [34]**
- **Ref. [38]**

The graph shows the function \( g(x^2) \) as a function of \( x^2 \) with GeV\(^{-2}\) on the x-axis and the function values on the y-axis. The curves represent the nonlocality for different works and references.
Table 1: The values of $\langle 0 | \bar{q} q | 0 \rangle$, $g \langle 0 | \bar{q} \sigma G q | 0 \rangle$.

| Reference | $-\langle \bar{q} q \rangle^{\frac{1}{2}}$ [MeV] | $-g \langle \bar{q} \sigma G q \rangle^{\frac{1}{2}}$ [MeV] |
|-----------|---------------------------------------------|---------------------------------------------|
| 26        | 210-230                                     | 375-395                                     |
| 33        | 225                                         | 402-429                                     |
| 35        | 272                                         | 490                                         |
| 36        | 150-180                                     | 400-460                                     |
| 39        | 217                                         | 429                                         |
| 40        | 178                                         | 456                                         |
| 41        | 171                                         | 447                                         |
| This Work | 182                                         | 422                                         |
Table 2: The values of $\chi^T a$

| Reference | $\chi^T a$ [GeV] |
|-----------|------------------|
| 15        | 0.002            |
| 18        | -0.008           |
| 27        | 0.009-0.017      |
| 43        | -0.0055          |
| 40        | -0.0014          |
| 39        | -0.0058          |
| 41        | -0.0018          |
| This Work | -0.0019          |

Table 3: The values of $\chi^\pi a$

| Reference | $\chi^\pi a$ [GeV] |
|-----------|-------------------|
| 14        | 1.7-3.0           |
| 11        | 1.88              |
| 39        | 2.56              |
| 41        | 0.68              |
| This Work | 1.06              |