Active vibration reduction of a flexible structure bonded with optimised piezoelectric pairs using half and quarter chromosomes in genetic algorithms

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Abstract. The optimal placement of sensors and actuators in active vibration control is limited by the number of candidates in the search space. The search space of a small structure discretized to one hundred elements for optimising the location of ten actuators gives \(1.73 \times 10^{13}\) possible solutions, one of which is the global optimum. In this work, a new quarter and half chromosome technique based on symmetry is developed, by which the search space for optimisation of sensor/actuator locations in active vibration control of flexible structures may be greatly reduced. The technique is applied to the optimisation for eight and ten actuators located on a 500×500mm square plate, in which the search space is reduced by up to 99.99%. This technique helps for updating genetic algorithm program by updating natural frequencies and mode shapes in each generation to find the global optimal solution in a greatly reduced number of generations. An isotropic plate with piezoelectric sensor/actuator pairs bonded to its surface was investigated using the finite element method and Hamilton’s principle based on first order shear deformation theory. The placement and feedback gain of ten and eight sensor/actuator pairs was optimised for a cantilever and clamped-clamped plate to attenuate the first six modes of vibration, using minimization of linear quadratic index as an objective function.

1. Introduction
Genetic algorithm is a powerful random search method guided by a fitness function, which may be used to detect efficient locations for discrete sensors and actuators on smart structure for active vibration control. The complexity of the genetic algorithm for the placement problem is defined by the number of candidate solutions in the search space, which is the statistical combination of the number of sensors and actuators and the total number of possible locations on the structure.

Flexible structure bounded with distributed sensor and actuator pair investigated thoroughly for vibration attenuation [1-3]. The dynamic equation of piezoelectric electroelasticity is formulated by Allik and Hughes and found to be reducible in form to the well-known equations of structural dynamics [1]. Optimal placement of discrete piezoelectric sensors and actuators was investigated for flexible structures by many researchers using the genetic algorithm based on different types of objective functions. Optimal placement of two actuators was obtained by Sadri et al for a thin plate based on modal [4] and grammian controllability [5] as an objective function using a binary number coded chromosome. Placement of two actuators and piezofilm sensors were optimised used by Han and Lee using controllability, observability and spillover as an objective function to suppress the first three modes of vibration using a binary number coded chromosome [6]. Peng et al studied optimal placement of four sensor/actuator pairs to control the first five modes of vibration based on grammian controllability index maximization as an objective function [7]. Kumar and Narayanan investigated...
optimal placement of ten sensor/actuator pairs using minimization of linear quadratic index as an objective function to suppress the first six modes of vibration [8]. Roy and Chakraborty studied a composite shell to place six piezoelectric sensor/actuator pairs using a modified genetic algorithm based on maximization of closed loop damping with limited feedback voltage value as an objective function to suppress the first eight modes of vibration [9]. Various published works have investigated symmetrical dynamic structures and variously found asymmetrical [4-6, 8, 9] and symmetrical [7] optimal configurations.

Daraji and Hale have investigated global optimal locations for two, four, six, eight and ten piezoelectric actuators for flat plates and plates stiffened by beams with both symmetrical and asymmetrical geometries and boundary conditions. They have found symmetrical configurations of actuators for symmetrical structures and asymmetrical actuators configurations for asymmetrical structures using minimization of linear quadratic index as an objective function and the symmetrical piezoelectric configuration gave higher vibration attenuation than published asymmetrical configuration [10-13]. Hale and Daraji [14] have researched global optimal piezoelectric sensor/actuator configurations for a cantilever plate based using new fitness function based on H_infinity and found a symmetrical piezoelectric actuator distribution which gives higher vibration reduction than published.

This paper complements the published works [10-15] by reducing the genetic algorithm search space and obtaining the global optimal sensor and actuator locations with few generations using a half and quarter chromosome technique. In this paper, an isotropic plate with a number of discrete piezoelectric sensor/actuator pairs bonded to its surface is modelled using finite element and Hamilton’s principle based on first order shear deformation theory taking account of piezoelectric mass, stiffness and electromechanical coupling effects. A matlab m-code computer program has been built for this new technique to locate ten and eight piezoelectric actuators, respectively, for a cantilever and clamped-clamped plate, based on minimization of optimal linear quadratic control as an objective function.

2. Modelling

The plate with number of piezoelectric patches bonded to its surfaces is modelled based on finite element, Hamilton’s principle and first shear deformation theory using isoparametric four nodes element, the modal dynamic equation for the plate with a number of discrete piezoelectric pairs bonded to its surfaces was written in state space form as follows[11];

\[
\begin{bmatrix}
\dot{X}
\end{bmatrix} = \begin{bmatrix}
0 & \omega  \\
-\omega & -2\xi_\omega
\end{bmatrix} \begin{bmatrix}
X
\end{bmatrix} + \begin{bmatrix}
0 \\
-\phi^T K_{u\phi} a
\end{bmatrix} \begin{bmatrix}
\phi_a
\end{bmatrix}
\]

\[
\dot{X} = [A]X + [B] u, \quad [\phi_s] = [C] X, \quad X = \omega \eta \quad \eta^T
\]

\[
[A_i] = \begin{bmatrix}
0 & \omega_i \\
-\omega_i & -2\xi_i\omega_i
\end{bmatrix}, \quad [B_i] = \begin{bmatrix}
0 \\
-\phi_i^T K_{u\phi} a
\end{bmatrix}, \quad [C_i] = \begin{bmatrix}
-\phi_i^T K_{\phi\phi} s^{-1} K_{u\phi} s \omega_i^{-1} 0
\end{bmatrix}
\]

Where\([A_i], [B_i]\) and \([C_i]\) are state, actuator and sensor matrices in which subscript \(i\) refers to mode number. \([K_{u\phi}]\) and \([K_{\phi\phi}]\) refer to piezoelectric electromechanical coupling and capacitance matrices.

3. Control law and objective function

Linear quadratic optimal controller design is based on minimization of a performance index \(J\). Values of positive-definite weighted matrices \([Q]\), of dimension \((2n_m \times 2n_m)\), and \([R]\) of dimension \((r_a \times r_a)\) are controlled the value of the performance index, where \(n_m, r_a\) represent the number of modes and actuators, respectively. Ogata has shown it is possible to follow this derivation to design a linear quadratic controller [16], which leads to the following Riccati equation:
\[
\begin{align*}
[A]^T[P] + [P][A] - [P][B][R]^{-1}[B]^T[P] + [Q] &= 0 \quad (4) \\
[K] &= [R]^{-1}[B]^T[P], \quad \{u\} = -[K]\{X\} \quad (5)
\end{align*}
\]

Solution of the Reduced Riccati equation (4) gives the value of matrix \([P]\); if matrix \([P]\) is positive definite then the system is stable or the closed loop matrix \([A] - [B][K]\) is stable. Feedback control gain can be obtained after substitution of \([P]\) in equation (5). Minimization of the linear quadratic cost function \(J\) is proposed by Kondoh et al as an objective function to optimise gain and piezoelectric actuator location [17].

\[
J(x,y) = \text{Trace}([P(x,y)])
\]

\[
J_{opt}(x,y) = \min(\text{Trace}([P(x,y)]), [K])
\]

Where \(x, y \in\) plate dimension 50 × 50 cm

4. Genetic algorithm

In 1975, Holland invented the genetic algorithm, a heuristic method based on “survival of the fittest” or the principle of natural evolution. It has been continuously improved and is now a powerful method for searching optimal solutions. The published works [10-15], for flat and stiffened plate by beams with different boundary conditions based on minimization of linear quadratic index and H_infinity and using the full length chromosome, has shown that the global optimal distribution of piezoelectric sensors and actuators has the same axes of symmetry as the structure. In this work, the chromosome length is reduced to quarter and half based on symmetries of the structure, using quarter chromosome length for a structure having two axes of symmetry and half length for a structure having one axis of symmetry. The placement strategy using genetic algorithm comprising conditional filter proposed by Daraji and Hale is used in this work [10].

5. Results and discussion

5.1. Research problem

A flat plate dimensioned 500 × 500 × 1.9mm was mounted rigidly from the left hand edge to form a cantilever and from left and right hand edges to form a clamped-clamped plate, respectively. The plate is discretised to one hundred elements 10 × 10 sequentially from left to right and down to up as shown in Figure 2. Table 1 shows plate and piezoelectric material properties.

| Properties                  | Plate | Piezoelectric PIC225 |
|-----------------------------|-------|----------------------|
| Modulus, GPa                | 210   | --------             |
| Density, kg/m³              | 7810  | 7180                |
| Poisson’s ratio             | 0.3   | --------             |
| Thickness, mm               | 1.9   | 0.5                  |
| Length, width, mm           | 500   | 50                   |
| \(e_{31}, e_{32}, e_{33}\), \(C/m²\) | -------- | -7.15, -7.15, 13.7 |
| \(C_{11}^E, C_{12}^E, C_{13}^E, C_{55}^E\), GPa | -------- | 123, 76.7, 70.25, 22.226 |
| \(\mu_{33}^E\) F/m          | -------- | 1.5 × 10^{-8}       |

\(C^E, e\) and \(\mu^E\) refer to elasticity, piezoelectric and permittivity constants

5.2. Natural frequencies

The first six natural frequencies and mode shapes for the cantilever plate were determined using a Matlab m-code program based on the present model and validated by the ANSYS finite element package using two dimensional SHELL63 and three dimensional SOLID45 elements, and also
experimentally. The results converged with mesh refining to constant values, showing that the mesh of
10 × 10 shell63 elements gives good accuracy for the first six natural frequencies compared with finer
meshes and with three dimensional solid45 elements, as well as with experimental results as shown in
table2.

Table2. ANSYS natural frequencies compared with present model and experimental results

| Element type                  | Mode (Hz) |
|-------------------------------|-----------|
|                               | 1<sup>st</sup> | 2<sup>nd</sup> | 3<sup>rd</sup> | 4<sup>th</sup> | 5<sup>th</sup> | 6<sup>th</sup> |
| ANSYS Shell63 (10 × 10)       | 6.59       | 16.17       | 41.32       | 52.37       | 59.79       | 104.70       |
| ANSYS Solid45 (50 × 50)       | 6.59       | 16.15       | 40.44       | 51.68       | 58.86       | 103.18       |
| Present model                 | 6.59       | 16.14       | 40.62       | 51.78       | 58.99       | 103.29       |
| Experimental Frequency        | 5.90       | 16.90       | 37.30       | 51.60       | 58.20       | 101.00       |

5.3. Actuators optimization using half chromosome length

The genetic algorithm described in section 4 based on reference [10] was used to find optimal
locations for ten piezoelectric actuators on a 0.5m square cantilever plate mounted rigidly from the left
hand edge using half chromosome length. The progressive convergence of the population onto an
optimal solution is shown in figure 1, in which the population is distributed around the circle with
radius (r) representing fitness value to be minimised.

At the first generation figure 1(a) the population is very diverse with representatives of high and
low fitness and the range in between. After fifteen generations figure 1(b) the population is much less
diverse, made up of individuals of high, though not yet optimal, fitness. After 100 generations
figure 1(c) the population has completely converged to a level of fitness higher than any individual in
the first or fifteenth generations.

Figure 2 shows optimal piezoelectric configuration for cantilever and clamped-clamped plate using
half and quarter chromosome (figures 2(b) and 2(d)) compared with published results using the full
chromosome (figures 2(a) and 2(c)). It can observed from figure 2 that same optimal configuration of
sensors and actuators was obtained with a smaller number of generations.
5.4. Genetic algorithm problem reduction

Percentage genetic algorithm problem reduction is calculated using the half and quarter chromosomes compared with the full chromosome. The results are represented in Table 3. Two measures of reduction are calculated: size of search space and average number of fitness calculations required to get an optimal solution. The total search space (the number of candidate chromosomes) is the statistical combination of the number of actuators to be optimised and the number of possible locations. This gives a reduction in the number of candidates equal to 99.9999878% for half and 99.99999983% for quarter chromosome length compared with the full chromosome.

The second reduction is calculated on the basis of the actual computational effort required to obtain the optimal solutions. Table 3 shows that the half chromosome length technique requires 700 fitness solutions compared with 31900 for the full chromosome, giving a 97.80564263% reduction in computer calculation. Similarly, Table 3 shows that the quarter chromosome requires only 25 calculations, compared with 33450 for the full chromosome, which gives 99.2537313% reduction in computer calculation.

Table 3. Percentage genetic algorithm problem reduction obtained using half and quarter chromosomes

| Case                        | Number of candidates | Number of fitness calculations |
|-----------------------------|----------------------|-------------------------------|
| cantilever plate (one axis of symmetry) | Full Chromosome [10] | $1.73 \times 10^{13}$ | 31900 |
|                             | Present half chrom.  | $2.1 \times 10^6$             | 700   |
|                             | Reduction obtained   | $>99.99\%$                    | 97.80\% |
| clamped-clamped plate (two axes of symmetry) | Full chromosome [13] | $1.86 \times 10^{11}$ | 3350   |
|                             | Present quarter chrom. | 300                     | 25    |
|                             | Reduction obtained   | $>99.99\%$                    | 99.25\% |

6. Conclusion

Half and quarter chromosomes are developed to optimise discrete piezoelectric locations for a flat plate fixed as a cantilever and clamped-clamped, respectively, giving one and two axes of symmetry. Minimization of linear quadric index is used as an objective function to locate ten and eight discrete piezoelectric actuators and attenuate the first six modes of vibration.

An isotropic plate with these discrete actuators bonded to its surface is modelled using finite element and Hamilton’s principle based on first order shear deformation theory taking account of piezoelectric mass, stiffness and electromechanical coupling effects. The first six natural frequencies
are determined by this model and validated using the Ansys finite element package and experimentally. It has been observed in this work that the configuration of optimal sensor/actuator pairs is in agreement with the modal strain and electric charge distribution over the plate and piezoelectric surface respectively.

A Matlab m-code genetic algorithm program has been built incorporating the new half and quarter chromosome technique and the results for optimal placement of ten and eight actuators are given, validated with published work. The new technique gives >99.99% reduction in genetic algorithm search space and >97.8% reduction in computer calculations compared with the conventional full length chromosome.

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