STANDARDIZING DISTANCE AND TIME

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ABSTRACT. Einstein’s Equivalence Principle is used with the electromagnetic spectrum to translate meters and seconds into radians and seconds. Based on a unique geometric relationship, a new transformation of velocities and a changed Lorentz transformation result. The physical angle of parallelism is quantified. The way we measure distance and time is standardized, constructing a theory of time and geometry to the universe.

Key words and phrases. Time, measure, second, meter, metric, radian, parsec, Gudermann, Lobachevskii, Lobachevskii, Lobachevsky, angle of parallelism, distance scale, radius of curvature, hyperbolic geometry, Einstein, relativity, Lorentz transformation, electromagnetic spectrum, velocity, acceleration.
1. INTRODUCTION

In *Speculations in Science and Technology* 21, 213–225 (1999), “Time, gravity and the exterior angle of parallelism,” (available on-line from [http://www.wkap.nl](http://www.wkap.nl)) the author, Russell Clark Eskew, set out to standardize distance and time with the electromagnetic spectrum [5, p. 220]. The number of seconds to a light year illustrated how an infinite number of seconds derive a mathematically convenient scale for the physical angle of parallelism, \(2\tan^{-1} e^{-a/s} = 2\tan^{-1} e^{-1}\). In this paper, we follow up by showing how the wavespeed of light, 299792458 meters per second, with wavelengths in meters and radians per cycle, apply to a frequency \(f\) in cycles per second to create a wavespeed in radians per second

\[
\lambda = \tan \frac{\psi}{2} = \tanh \frac{a}{2},
\]

where \(\psi = \tan^{-1} \sinh \alpha\) [1, pp. 312–313] [5, p. 215], \(0 < \psi < \frac{\pi}{2}\), \(a = \ln f\), and \(s\) is the hyperbolic radius of curvature. The significance of \(\lambda = \tan \frac{\psi}{2}\), as Roger Penrose points out [14, pp. 74–75], is that the stereographic projection of a line drawn with an angle \(\psi\) from the west pole of a sphere, \(P\), and an angle \(\psi\) from the origin, maps the point \(Q\) onto the plane \(x = 1\) at \(\tan \frac{\psi}{2}\). This preserves measuring great circles and angles with a partial metric [2, pp. 92–95].

2. A UNIQUE GEOMETRIC RELATIONSHIP

The geometric relationship is unique in this scheme. With \(\lambda\) we know that the hypotenuse line from the unit circle to the horizontal unit hyperbola, \(x^2 - y^2 = 1\), is

\[
(x^2 + y^2)^{1/2} = (\cosh \frac{a}{2})^2 + (\sinh \frac{a}{2})^2 = \left(\frac{(e^a + 1)^2 + (e^a - 1)^2}{4e^a}\right)^{1/2} = \left(\frac{1 + \lambda^2}{1 - \lambda^2}\right)^{1/2}.
\]

We use the ratio of both hypotenuses to multiply the circular coordinates

\[
\cos \frac{\psi}{2} = \left(\frac{1}{1 + \lambda^2}\right)^{1/2} = \frac{1}{\cosh \sinh^{-1} \lambda} = \frac{1}{e^{\sinh^{-1} \lambda} - \lambda} = e^{i\psi/2} - i \sin \frac{\psi}{2},
\]
\[
\sin \frac{\psi}{2} = \left(\frac{\lambda^2}{1 + \lambda^2}\right)^{1/2} = \tanh \sinh^{-1} \lambda = \frac{\lambda}{e^{\sinh^{-1} \lambda} - \lambda} = e^{i\psi/2} - \cos \frac{\psi}{2}
\]
by (2.3) to derive the hyperbolic coordinates

\[
\cosh \frac{a}{2} = \left(\frac{1}{1 - \lambda^2}\right)^{1/2} = \frac{1}{\cos \sinh^{-1} \lambda} = \frac{1}{e^{i\sinh^{-1} \lambda} - i \lambda} = e^{a/2} - \sinh \frac{a}{2},
\]
\[
\sinh \frac{a}{2} = \left(\frac{\lambda^2}{1 - \lambda^2}\right)^{1/2} = \tan^{-1} \lambda = \frac{\lambda}{e^{i\sinh^{-1} \lambda} - i \lambda} = e^{a/2} - \cosh \frac{a}{2}.
\]
3. A NEW TRANSFORMATION OF VELOCITIES

This paper’s transformation of velocities is different from that of Galileo Galilei (1564–1642)

\[ u = u' + v. \]  

(3.1)

The quantities are the “absolute velocity” \( u \), i.e., a moving particle’s velocity with respect to a fixed reference frame, its “relative velocity” \( u' \), i.e., the particle’s velocity with respect to a moving reference frame, and the “transport velocity” \( v \), i.e., the velocity of the moving reference frame. Albert Einstein (1879–1955) believes when \( u' = c \), \( c \) being the speed of light, that \( u = c \) rather than \( u = c + v \) \[7, pp. 161, 173, 203–212\] \[10\] \[3\]. The transformation of velocities of Einstein and H. A. Lorentz (1853–1928) replaced (3.1) with

\[ u = \frac{u' + v}{1 + u'v/c^2}. \]

(3.2)

This paper’s transformation of velocities is different from that of Einstein, too.

For the derivatives of the wavespeed

\[ \frac{d\lambda}{d\psi} = \frac{d}{d\psi} \tan \frac{a}{2} = \frac{1}{2} \sec^2 \frac{\psi}{2} \]

\[ \frac{d\lambda}{da} = \frac{d}{da} \tanh \frac{a}{2} = \frac{1}{2} \sech^2 \frac{a}{2} = \frac{2f}{(f + 1)^2} \]

make this paper’s transformation of velocities

\[ \frac{d\lambda}{d\psi} = \frac{d\lambda}{da} + \lambda^2 \]

acceleration_{absolute} = acceleration_{relative} + acceleration_{transport}.

If the wavespeed in meters, \( c \), is a constant, then \( dc = 0 \). But since the wavespeed in radians, \( \lambda \), is a variable, then we might have \( d\lambda \neq 0 \). This is why we hereby replace \( c \) with \( \lambda \), and \( v \) with \( \lambda^2 \), which is also known as the accelerationFrame of the gravitational frame force \( F_{\text{frame}} = -\text{mass} \times \text{acceleration}_{\text{frame}} \), in a rotating frame of reference

\[ F_{\psi} = F_a - F_{\text{frame}} \]

\[ m \frac{d\lambda}{d\psi} = m \frac{d\lambda}{da} + m \lambda^2. \]

4. CHANGING THE LORENTZ TRANSFORMATION

With the concept of proper time, a moving particle with instantaneous velocity (i.e., “transport acceleration” of a moving reference frame) \( v(t) = \lambda^2(t) \) relative to some inertial system \( K \) (i.e., with “absolute acceleration” \( d\lambda/d\psi \) with respect to a fixed reference frame) changes its position in a time interval \( dt \) by \( dx = vdt = \lambda^2 dt \). The space and time coordinates in \( K' \), \( (t', z', x', y') = (x_0, x_1, x_2, x_3) = (ct', z', x', y') \), where the system is instantaneously at rest (i.e., with the particle’s “relative acceleration” \( d\lambda/da \) with respect to the moving reference frame), are related to those in \( K \), \( (t, z, x, y) = (x_0, x_1, x_2, x_3) = (ct, z, x, y) \), by the inverse
\textit{Lorentz transformation} \\
(4.1) \quad x_0 = \gamma(x'_0 + x'_1\beta) = (\cosh \alpha)(x'_0 + x'_1\tanh \alpha) \\
x_1 = \gamma(x'_1 + x'_0\beta) = (\cosh \alpha)(x'_1 + x'_0\tanh \alpha) \\
x_2 = x'_2 \\
x_3 = x'_3.

With the boost parameter \(\xi\) it said that \\
(4.2) \quad \beta = \tanh \xi \\
(4.3) \quad \gamma = \cosh \xi \\
(4.4) \quad \gamma\beta = \sinh \xi

It is Einstein's thought that \(c = 1\) with the velocity \(v\) in \(\tanh \xi = \frac{v}{c}\). However, the wavespeed \(\lambda = \frac{c + v}{c - v} = \frac{1 + \beta}{1 - \beta}\) and frequency \(f = \frac{\gamma}{1 - \beta} = \frac{1 + \beta}{1 - \beta}t\) relate differing wavespeeds with a differing number of seconds \(t\). Rather than using the boost parameter \(\xi\) in (4.3), the hyperbolic coordinates are equated with the circular coordinates by \(\lambda = \tan \frac{\psi}{r} = \tanh \frac{\theta}{r}\). The Lorentz time \(t\) equalling the distance over \(c\) of the \(x'_0\) observer becomes \\
(4.5) \quad x'_0 = x_0 \cosh \frac{\alpha}{2} - x_1 \sinh \frac{\alpha}{2} = (\cosh \frac{\alpha}{2})(x_0 - x_1 \tanh \frac{\alpha}{2}) = (1 - \lambda^2)^{-1/2}(x_0 - x_1\lambda) \\
x'_1 = -x_0 \sinh \frac{\alpha}{2} + x_1 \cosh \frac{\alpha}{2} = (\cosh \frac{\alpha}{2})(x_1 - x_0 \tanh \frac{\alpha}{2}) = (1 - \lambda^2)^{-1/2}(x_1 - x_0\lambda).

The moving particle has advanced a distance \(vdt = \lambda^2 dt = dx'_0\). Time is measured with twice the distance \(L\) of the hypotenuse, vs. twice the height \(D\) of the side of the triangle. The \textit{proper time} observer sees \(x_0 = 2D/\lambda\). The \(x'_0\) observer, however, sees \(x'_0 = 2L/\lambda\), where \(L = ((\lambda^2x'_0/2)^2 + D^2)^{1/2}\) and \(D = \lambda x_0/2\), by which

is derivable when \(x_1 = 0\) occurs simultaneously. Moving clocks run slow. Events will be separated by the time interval

(4.6) \quad x'_0 = (1 - \lambda^2)^{-1/2}x_0

since \(x_0 = 0\), although the events are simultaneous in time.

The element of arc length \(ds\) of the particle’s path has \(ds^2 = c^2 dt^2 - |dx|^2\) when \(dx_1 = r \sin \psi \, ds, dx_2 = \cos \psi \, ds, dx_3 = \sin \psi \, ds\) are “increments” of \(x_1, x_2, x_3\) having the angle \(\psi\). The direction of the path curve’s polar-equation-tangent determined by the angle \(\phi\) which this tangent makes with the radius \(r\) or by the angle \(\psi = \theta + \phi\) which it makes with the x-axis thereby constructs \(dr = \cos \phi \, ds, rd\phi = \sin \phi \, ds,\) and \(r \sin \varphi d\theta\) [2, pp. 120–121]. A motionless particle has \(ds^2 = dt^2\). The new distance \(s\) is called \textit{proper time} \(\tau\), and the Lorentz metric is \(d\tau^2 = c^2 dt^2 - r^2 \sin^2 \varphi d\theta^2 - dr^2 - r^2 d\phi^2\).
The square of the corresponding infinitesimal invariant interval \( ds \) is

\[
(4.7) \quad ds^2 = c^2 dt^2 - |dx|^2 = \lambda^2 dt^2(1 - \lambda^2)
\]

where \( \beta = \frac{v}{c} = \tanh a \) or where \( c \) is replaced by \( \lambda = \tanh \frac{a}{2} \) and \( v \) by \( \lambda^2 \). In the coordinate system \( K' \) where the system is instantaneously at rest, the space-time increments are \( dt' = d\tau, dx' = 0 \). Thus the invariant interval is \( ds = c d\tau \) or \( ds = \lambda d\tau \). The increment of time \( d\tau \) in the instantaneous rest frame of the system is an invariant quantity that takes the form

\[
(4.8) \quad d\tau = dt(1 - \beta^2(t))^{1/2} = \frac{dt}{\gamma(t)}
\]

\[
(4.9) \quad d\tau = dt(1 - \lambda^2(t))^{1/2}
\]

where \( \gamma = \cosh a = (1 - \beta^2)^{-1/2} \) and \( \cosh \frac{a}{2} = (1 - \lambda^2)^{-1/2} \). That is the time as seen in the rest frame of the system [8, pp. 524–528].

5. FROM THE METRIC TO THE ANGLE OF PARALLELISM

Further definition of the metric toward the physical angle of parallelism, \( 2 \tan^{-1} e^{-a/\lambda} \), uses the arc length

\[
(5.1) \quad s = \int ds = \int_{x_1}^{x_2} (1 + \left(\frac{dy}{dx}\right)^2)^{1/2} dx = \int_{t_1}^{t_2} \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \frac{dt}{\gamma(t)}
\]

Using the Gudermann

\[
(5.2) \quad \psi = \sin^{-1} \tanh a = \cos^{-1} \text{sech} a = \tan^{-1} \sinh a \\
= \csc^{-1} \coth a = \sec^{-1} \cosh a = \cot^{-1} \csch a
\]

to solve the arc length of a unit circle from \((0, 1)\) to \((\cos \psi, \sin \psi)\), that is, \((\text{sech} a, \tanh a)\), becomes

\[
(5.3) \quad s = \int_0^{\text{sech} a} (1 + \left(\frac{dy}{dx}\right)^2)^{1/2} dx = \int_0^a \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \frac{dt}{\gamma(t)}
\]

\[
= \int_0^a \left(-\text{sech} t \tanh t\right)^2 + \left(\text{sech}^2 t\right)^2 \frac{dt}{\gamma(t)} = \int_0^a \frac{2}{e^t + e^{-t}} dt
\]

\[
= \int_0^{\psi} \frac{2}{u + \frac{1}{u}} du = 2 \tan^{-1} e^{-a} - \frac{\pi}{2} = \psi.
\]

Solving the \( s \) arc length of a horizontal unit hyperbola \( x^2 - y^2 = 1 \) from \((1, 0)\) to \((\cosh a, \sinh a)\) becomes

\[
(5.4) \quad s = \int_1^{\cosh a} (1 + \left(\frac{dy}{dx}\right)^2)^{1/2} dx = \int_0^a \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 \frac{dt}{\gamma(t)}
\]

also known as the hyperbolic radius of curvature.

Lobachevskii’s simpler angle of parallelism [3, pp. 11–45] [4, pp. 376–377] is

\[
(5.5) \quad 2 \tan^{-1} e^{-a} = 2 \tan^{-1} \frac{1}{\sqrt{1}} = \frac{\pi}{2} - \psi = \frac{\pi}{2} - 2 \tan^{-1} \lambda = \theta,
\]
where $0 < \theta < \frac{\pi}{2}$,

\[
\theta = \sin^{-1} \text{sech} \, a = \cos^{-1} \tanh \, a = \tan^{-1} \text{csch} \, a
\]

\[
= \csc^{-1} \cosh \, a = \sec^{-1} \coth \, a = \cot^{-1} \sinh \, a
\]

is of a vertical unit hyperbola $y^2 - x^2 = 1$.

As $\lambda$ approaches 1, $2 \tan^{-1} e^{-a/s}$ approaches 0. With the hyperbolic radius of curvature $s$, however, $2 \tan^{-1} e^{-a/s}$ approaches $\frac{\pi}{2}$ radians, implying parallelism [11, pp. 414, 434] [5, p. 217] [3, p. 315]. The metric is further defined to involve the physical angle of parallelism, the main conclusion of hyperbolic geometry [3, pp. 216–218] [13, p. 77] [12, p. 210]. In the case of infinite seconds, the physical angle of parallelism is $2 \tan^{-1} e^{-1} = 0.705026844...$ radians, in agreement with the physical evidence cited by Martin [11, p. 300] [5, p. 220]. The base $a = \ln f$ of such astronomical asymptotic triangles is an important new kind of curve which is orthogonal to all parallels.

6. AN IMPROVED ELECTROMAGNETIC SPECTRUM

The electromagnetic spectrum is illustrated with the Table, to be read with all eight columns viewed on both facing pages. The eight related categories, along with the electromagnetic nomenclature, are formulated

| wavespeedmeters = wavelengthmeters | \( \frac{\text{meters}}{\text{cycle}} \times \frac{\text{cycles}}{\text{second}} = 299792458 \frac{\text{meters}}{\text{second}} \) |
| wavespeedradians = \( \lambda \) = wavelength | \( \frac{\text{radians}}{\text{cycle}} \times \frac{\text{cycles}}{\text{second}} = \frac{f - 1}{f + 1} \frac{\text{radians}}{\text{second}} \) |
| frequency | \( f = \frac{\text{cycles}}{\text{second}} = \frac{1}{299792458} \frac{\text{cycles}}{\text{meter}} = \frac{\text{seconds} + 1}{\text{seconds} - 1} \) |
| wavelengthradians = wavespeedradians | \( \frac{\text{radians}}{\text{cycle}} \times \frac{1}{\text{seconds}} = \frac{\text{radians}}{\text{cycle}} \) |
| wavelengthmeters = wavespeedradians | \( \frac{\text{meters}}{\text{cycle}} \times \frac{1}{\text{seconds}} = \frac{\text{meters}}{\text{cycle}} \) |
| absolute acceleration = \( \frac{1}{2} \sec^2 (\psi/2) \) |
| relative acceleration = \( \frac{1}{2} \, \text{sech}^2 (a/2) = 2f/(f + 1)^2 \) |
| instantaneous velocity = \( \lambda^2 \text{radians} \) |
| \( \Pi (\frac{a}{s}) = 2 \tan^{-1} e^{-a/s}, \quad a = \ln f, \quad s = \int_0^a ((\sinh t)^2 + (\cosh t)^2)^{1/2} dt. \) |

A second with meters is singular, while seconds with radians increase from one to infinity. You see an infinite number of cycles apply to 1 second per radian, a smaller 3 cycles apply to 2 seconds, 2 cycles apply to 3 seconds, and 1 cycle applies to an infinite number of seconds. Thus an astronomer might observe a star to have a wavelength of 1.67 meters per cycle, with a frequency of $e^{19}$ cycles per second. Reading the equivalent radian values on the same row of the Table, he
could also know that there are \(1.0 + (1.12 \times 10^{-8})\) seconds per radian, a wavelength of \(5.60 \times 10^{-3}\) radians per cycle, a wavespeed of \(\lambda = 1.0 - (1.12 \times 10^{-8})\) radians per second, absolute acceleration of \(0.999999989\), relative acceleration of \(0.000000011\), and instantaneous velocity of \(1.0 - (2.24 \times 10^{-8})\) radians per second, and an angle of parallelism of \(\Pi(\frac{\pi}{4}) = \frac{\pi}{2} - (1.50 \times 10^{-7})\) radians per second [4].

The additional radian information constructs a theory of time and geometry to the universe. All of the values are fixed throughout the Table. The Equivalence Principle is refined with values of the variable \(\lambda\) based upon the speed of light, \(c\) [4, pp. 22–25]. In this manner, the universe may be measured mathematically [5].

7. CONCLUSION

George Martin (1932– ) asks an interesting question [11, pp. 300–302],

“A meter was originally intended to be one ten-millionth of the distance from the earth’s equator to a pole measured along a meridian. . . . [It] turns out to be mathematically convenient to pick a [distance] scale such that \(\Pi(1)\) is \(2 \arctan e^{-1}\). In applying this to the physical world, there is little difficulty in determining \(A, B, C\) such that [angle] \(\angle ABC\) has measure quite close to \(2 \arctan e^{-1}\). However, how long is a segment of length 1? That is, how many meters long is it? Since a meter has nothing to do with the axioms of our geometry, the question is a valid one. Although it may be really neat to have a geometry that provides for a standard angle determining a standard length, all physical experiments indicate that \(\Pi(x)\) could noticeably differ from \(\frac{\pi}{2}\) only for very large astronomical distances \(x\). So a physical segment of length 1 would be very, very long indeed.”

The answer is 299792458 meters long. Fewer meters apply to larger angles of parallelism. Martin continues,

“The largest physical triangles that can be accurately measured are astronomical. Let \(E\) stand for the Earth, \(S\) for the Sun, and \(V\) for the brilliant blue star Vega. \(\angle SEV\) can be measured from the Earth when \(\angle ESV\) is right. Using this measurement and the fact that [the defect of the triangle] \(SEV\) is less than \(\frac{\pi}{2} - \angle SEV\), one obtains [the defect of the triangle] \(SEV < 0.0000004\).”

In our Table, that agrees closely to a Short-wave radio (blue) angle of parallelism. By comparison, one parsec is defined as the distance of the Earth to the Sun (1 astronomical unit) that subtends an angle of 1 second of arc, equivalent to 206265 astronomical units. The comparable \(ES\) base of the asymptotic triangle is about \(a = \ln e^{19} = 19\) radial units. With the angle of parallelism and electromagnetic spectrum, meters and seconds can hereby be exchanged with radians and seconds. We can geometrically measure the universe.
| seconds/radian 1/λ | frequency cycles/second | wavelength r radians/cycle | wavelength m meters/cycle | absolute acceleration dλ/dψ |
|-------------------|-----------------------|---------------------------|--------------------------|-----------------------------|
| 1.0               | ∞                     | 0                         | 0                        | 1.0 (−1.92 × 10⁻²³)         |
| 1.0 + (1.92 × 10⁻²³) | e³³ = 1.04 × 10²³     | 9.60 × 10⁻²⁴              | 2.87 × 10⁻¹⁵             | 1.0 − (1.92 × 10⁻²³)         |
| 1.0 + (1.41 × 10⁻²²) | e³⁴ = 1.40 × 10²²     | 7.09 × 10⁻²³              | 2.12 × 10⁻¹⁴             | 1.0 − (1.41 × 10⁻²²)         |
| 1.0 + (1.04 × 10⁻²¹) | e³⁵ = 1.90 × 10²¹     | 5.24 × 10⁻²²              | 1.57 × 10⁻¹³             | 1.0 − (1.04 × 10⁻²¹)         |
| 1.0 + (7.74 × 10⁻²¹) | e³⁶ = 2.58 × 10²⁰     | 3.87 × 10⁻²¹              | 1.16 × 10⁻¹²             | 1.0 − (7.74 × 10⁻²¹)         |
| 1.0 + (5.72 × 10⁻²⁰) | e³⁷ = 3.49 × 10¹⁹     | 2.86 × 10⁻²⁰              | 5.74 × 10⁻¹²             | 1.0 − (5.72 × 10⁻²⁰)         |
| 1.0 + (4.23 × 10⁻¹⁹) | e³⁸ = 4.72 × 10¹⁸     | 2.11 × 10⁻¹⁹              | 6.34 × 10⁻¹¹             | 1.0 − (4.23 × 10⁻¹⁹)         |
| 1.0 + (3.12 × 10⁻¹⁸) | e³⁹ = 6.39 × 10¹⁷     | 1.56 × 10⁻¹⁸              | 4.68 × 10⁻¹⁰             | 1.0 − (3.12 × 10⁻¹⁸)         |
| 1.0 + (2.30 × 10⁻¹⁷) | e⁴⁰ = 8.65 × 10¹⁶     | 2.30 × 10⁻¹⁷              | 3.46 × 10⁻⁹              | 1.0 − (2.30 × 10⁻¹⁷)         |
| 1.0 + (1.70 × 10⁻¹⁶) | e⁴¹ = 1.17 × 10¹⁶     | 8.53 × 10⁻¹⁷              | 1.71 × 10⁻⁸              | 1.0 − (1.70 × 10⁻¹⁶)         |
| 1.0 + (1.26 × 10⁻¹⁵) | e⁴² = 1.58 × 10¹⁵     | 6.30 × 10⁻¹⁶              | 1.89 × 10⁻⁷              | 1.0 − (1.26 × 10⁻¹⁵)         |
| 1.0 + (9.31 × 10⁻¹⁵) | e⁴³ = 2.14 × 10¹⁴     | 4.65 × 10⁻¹⁵              | 1.39 × 10⁻⁶              | 1.0 − (9.31 × 10⁻¹⁵)         |
| 1.0 + (6.88 × 10⁻¹⁴) | e⁴⁴ = 2.90 × 10¹³     | 3.44 × 10⁻¹⁴              | 0.00000103               | 1.0 − (6.88 × 10⁻¹⁴)         |
| 1.0 + (5.08 × 10⁻¹³) | e⁴⁵ = 3.93 × 10¹²     | 2.54 × 10⁻¹³              | 0.0000762                | 1.0 − (5.08 × 10⁻¹³)         |
| 1.0 + (3.75 × 10⁻¹²) | e⁴⁶ = 5.32 × 10¹¹     | 1.87 × 10⁻¹²              | 0.0005634                | 1.0 − (3.75 × 10⁻¹²)         |
| 1.0 + (2.77 × 10⁻¹¹) | e⁴⁷ = 7.20 × 10¹⁰     | 1.38 × 10⁻¹¹              | 0.0041635                | 1.0 − (2.77 × 10⁻¹¹)         |
| 1.0 + (2.05 × 10⁻¹⁰) | e⁴⁸ = 9.74 × 10⁹      | 1.02 × 10⁻¹⁰              | 0.0307643                | 1.0 − (2.05 × 10⁻¹⁰)         |
| 1.0 + (1.51 × 10⁻⁹)  | e⁴⁹ = 1.31 × 10⁹      | 7.58 × 10⁻¹⁰              | 0.2273194                | 0.999999998                 |
| 1.0 + (6.67 × 10⁻⁹)  | e⁵⁰ = 299792458       | 3.33 × 10⁻⁹               | 1.0                      | 0.999999993                 |
| 1.0 + (1.12 × 10⁻⁸)  | e⁵¹ = 1.78 × 10⁸      | 5.60 × 10⁻⁹               | 1.6796761                | 0.999999989                 |
| 1.0 + (8.27 × 10⁻⁸)  | e⁵² = 2.41 × 10⁷      | 4.13 × 10⁻⁸               | 12.411221                | 0.999999917                 |
| 1.0 + (6.11 × 10⁻⁷)  | e⁵³ = 3.26 × 10⁶      | 3.05 × 10⁻⁷               | 91.707208                | 0.999999388                 |
| 1.0 + (4.52 × 10⁻⁶)  | e⁵⁴ = 442413.39       | 2.26 × 10⁻⁶               | 677.62970                | 0.99995479                  |
| 1.000033403          | e¹¹ = 59874.141       | 0.00001670                | 5067.0439                | 0.999966598                 |
| 1.000256850          | e⁹ = 8103.0839        | 0.00012337                | 36997.328                | 0.999753241                 |
| 1.001825428          | e⁷ = 1096.6331        | 0.00091022                | 273375.333               | 0.998179558                 |
| 1.013567309          | e⁵ = 148.41315        | 0.00664775                | 2.01 × 10⁶               | 0.986703887                 |
| 1.104791392          | e³ = 20.085536        | 0.04506467                | 1.49 × 10⁷               | 0.909646681                 |
| 1.313035285          | e² = 7.3890560        | 0.10307056                | 4.05 × 10⁷               | 0.790012829                 |
| 2                  | 0.31                 | 1/(2 × 3)                 | 9.99 × 10⁷               | 5/8                         |
| 2.163953413          | e¹ = 2.7182818        | 0.17000340                | 1.10 × 10⁸               | 0.606776134                 |
| 3                  | 4/3                  | 2/(3 × 4)                 | 1.49 × 10⁸               | 5/9                         |
| 4                  | 5/3                  | 3/(4 × 5)                 | 1.79 × 10⁸               | 17/32                       |
| 5                  | 6/4                  | 4/(5 × 6)                 | 1.99 × 10⁸               | 13/25                       |
| ∞                  | 1.0                  | 1/∞                       | 299792458                | 1/2                         |
| relative accel $dx/da$ | transport acceleration $\lambda^2$ | angle of parallelism $\Pi(\hat{\beta}) = 2 \tan^{-1} e^{a/\hat{\beta}}$ | Nomenclature |
|------------------------|----------------------------------|---------------------------------|---------------|
| 0                      | 1.0                              | $\pi/2$                         | Cosmic photons |
| $1.92 \times 10^{-23}$ | $1.0 - (3.84 \times 10^{-23})$  | $\pi/2 - (7.19 \times 10^{-22})$ | $\gamma$ - rays |
| $1.41 \times 10^{-22}$ | $1.0 - (2.83 \times 10^{-22})$  | $\pi/2 - (5.11 \times 10^{-21})$ | X-rays         |
| $1.04 \times 10^{-21}$ | $1.0 - (2.09 \times 10^{-21})$  | $\pi/2 - (3.63 \times 10^{-20})$ |               |
| $7.74 \times 10^{-21}$ | $1.0 - (1.54 \times 10^{-20})$  | $\pi/2 - (2.57 \times 10^{-19})$ |               |
| $5.72 \times 10^{-20}$ | $1.0 - (1.14 \times 10^{-19})$  | $\pi/2 - (1.82 \times 10^{-18})$ |               |
| $4.23 \times 10^{-19}$ | $1.0 - (8.46 \times 10^{-19})$  | $\pi/2 - (1.28 \times 10^{-17})$ |               |
| $3.12 \times 10^{-18}$ | $1.0 - (6.25 \times 10^{-18})$  | $\pi/2 - (9.06 \times 10^{-17})$ |               |
| $2.30 \times 10^{-17}$ | $1.0 - (4.61 \times 10^{-17})$  | $\pi/2 - (6.36 \times 10^{-16})$ |               |
| $1.70 \times 10^{-16}$ | $1.0 - (3.41 \times 10^{-16})$  | $\pi/2 - (4.46 \times 10^{-15})$ |               |
| $1.26 \times 10^{-15}$ | $1.0 - (2.52 \times 10^{-15})$  | $\pi/2 - (3.12 \times 10^{-14})$ |               |
| $9.31 \times 10^{-15}$ | $1.0 - (1.86 \times 10^{-14})$  | $\pi/2 - (2.17 \times 10^{-13})$ |               |
| $6.88 \times 10^{-14}$ | $1.0 - (1.37 \times 10^{-13})$  | $\pi/2 - (1.50 \times 10^{-12})$ |               |
| $5.08 \times 10^{-13}$ | $1.0 - (1.01 \times 10^{-12})$  | $\pi/2 - (1.04 \times 10^{-11})$ |               |
| $3.75 \times 10^{-12}$ | $1.0 - (7.51 \times 10^{-12})$  | $\pi/2 - (7.17 \times 10^{-11})$ |               |
| $2.77 \times 10^{-11}$ | $1.0 - (5.55 \times 10^{-11})$  | $\pi/2 - (4.19 \times 10^{-10})$ |               |
| $2.05 \times 10^{-10}$ | $1.0 - (4.10 \times 10^{-10})$  | $\pi/2 - (3.33 \times 10^{-9})$  | Television    |
| $0.000000002$          | $1.0 - (3.03 \times 10^{-9})$   | $\pi/2 - (2.25 \times 10^{-8})$  | FM radio      |
| $0.000000007$          | $1.0 - (1.33 \times 10^{-8})$   | $\pi/2 - (9.20 \times 10^{-8})$  | Short-wave radio |
| $0.000000011$          | $1.0 - (2.24 \times 10^{-8})$   | $\pi/2 - (1.50 \times 10^{-7})$  | AM radio      |
| $0.000000083$          | $1.0 - (1.65 \times 10^{-7})$   | $\pi/2 - (9.95 \times 10^{-7})$  |               |
| $0.000000612$          | $1.0 - (1.22 \times 10^{-6})$   | $\pi/2 - (6.48 \times 10^{-6})$  | Long-wave radio |
| $0.000004521$          | $1.0 - (9.04 \times 10^{-6})$   | 1.57075477                      | Induction heating |
| $0.000033402$          | 0.99993319                      | 1.57053650                      |               |
| $0.000246759$          | 0.99950648                      | 1.56922541                      |               |
| $0.001820442$          | 0.99635911                      | 1.56176230                      |               |
| $0.013296113$          | 0.97340777                      | 1.52289663                      |               |
| $0.090353319$          | 0.81929336                      | 1.35203152                      |               |
| $0.209987171$          | 0.58002565                      | 1.15128526                      |               |
| $3/8$                  | 1/4                             | 0.90301878                      |               |
| $0.393223866$          | 0.21355226                      | 0.87551570                      |               |
| $4/9$                  | 1/9                             | 0.79641144                      |               |
| $15/32$                | 1/16                            | 0.75740822                      |               |
| $12/25$                | 1/25                            | 0.73888970                      |               |
| $1/2$                  | 1/∞                             | 0.70502684                      |               |

\[1\] In the January 2001 Notices of the AMS, Irving Ezra Segal believed that a static universe recessional velocity of $\tan^{-1} e^{a/\hat{\beta}}$, $\rho$ radians, fit the evidence better than Hubble’s linear proportionality.
REFERENCES

1. G. Chrystal, *Algebra*, vol. II, Black, London, 1931.
2. H. S. M. Coxeter, *Introduction to geometry*, 4th ed., Toronto Univ. Press, Toronto, 1961.
3. A. Einstein, *Relativity, the special and the general theory*, 15th ed., Methuen, London, 1957.
4. R. C. Eskew, *Relating a circle and a hyperbola*, J. Undergrad. Math. 21(2) (1989), 49–54.
5. ______, *Time, gravity and the exterior angle of parallelism*, Speculations in Science and Technology 21 (1999), 213–225.
6. S. W. Hawking, *A brief history of time*, Bantam, Toronto, 1988.
7. I. M. Iaglom, *A simple non-euclidean geometry and its physical basis*, Nauka, Moscow, 1969 (Russian), English translation available.
8. J. D. Jackson, *Classical electrodynamics*, 3rd ed., Wiley, New York, 1999.
9. N. I. Lobachevskii, *Geometrical researches on the theory of parallels*, Karzan, Berlin, 1840 (Russian), English translation available.
10. H. A. Lorentz, *The principle of relativity*, Dover, New York, 1923.
11. G. E. Martin, *The foundations of geometry and the non-euclidean plane*, Intext Educational, New York, 1972.
12. R. S. Millman, *Geometry, a metric approach*, Springer-Verlag, New York, 1981.
13. C. W. Misner, *Gravitation*, Freeman, San Francisco, 1970.
14. W. Rindler, *Essential relativity: special, general, and cosmological*, 2nd ed., Van Nostrand Reinhold, New York, 1969.

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