The Case of the Curved Universe: Open, Closed, or Flat?

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Determination of the geometry of the Universe has been a central goal of cosmology ever since Hubble discovered its expansion seventy-five years ago. Is it a multidimensional equivalent of the two-dimensional surface of a sheet of paper (“flat”), a sphere (“closed”), or a saddle (“open”)? The geometry determines whether the Universe will expand forever or eventually recollapse, and it may also shed light on its origin. Particle theories suggest that in the extreme temperatures prevalent in the very early Universe, gravity may have briefly become a repulsive, rather than attractive, force. If so, the ensuing period of “inflation” could account for some of the most fundamental features of the Universe, such as the remarkable smoothness of the cosmic microwave background (CMB), the afterglow of the big bang (see schematic timeline).

Until now, most astronomers have pursued the geometry by attempting to measure the mass density of the Universe. According to general relativity, if the density is equal to, larger than, or smaller than a “critical density” fixed by the expansion rate, then the Universe is flat, open, or closed, respectively. Several measurements currently seem to suggest a density only a fraction $\Omega \approx 0.3$ of the critical density (as opposed to $\Omega = 1$ predicted by inflation). However, most of these probe only the mass that clusters with galaxies. If a significant amount of some more diffuse component of matter exists, such as neutrinos and/or “vacuum energy” (Einstein’s cosmological constant), then the measurements do not necessarily tell us the geometry of the Universe. The research article by Gawiser and Silk on page 1405 of this issue and an accompanying commentary on page 1398 by Primack tell this side of the story.

Another possibility is to look directly for the effects of a curved Universe. As an analogy, consider geometry on a two-dimensional surface. On a flat surface, the interior angles of a triangle sum to 180 degrees and the circumference of a circle is $2\pi$ times its radius. However, when drawn on the surface of a sphere, the interior angles of a triangle sum to more than 180 degrees, and the circumference of a circle is less than $2\pi$ times the radius. Similar lines of reasoning show that in an open (closed) Universe, objects of some fixed size will appear to be smaller (larger) than they would in a flat Universe.

The problem, then, is to find distant objects in the Universe of known size (“standard rulers”). It was recently proposed that features at the CMB surface of last scatter could provide such standard rulers. The photons that make up the CMB last scattered roughly 10 to 15 billion years ago, when the Universe was only 300,000 years old. Therefore, when we

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FIG. 1. From smooth to structured. Schematic history of the Universe. The big bang may have been followed by a period of rapid inflation, with the resulting “soup” of particles coalescing into nucleons and lighter elements. Matter and radiation eventually became decoupled, the former gravitationally clumping into the structure of the modern Universe and the latter yielding the microwave background we see today. The seeds from which galaxies grew should be apparent in the variations in the radiation background.

look at the CMB, we see a spherical surface in the early Universe 10 to 15 billion light-years away. Although galaxies and clusters of galaxies had not yet formed, the seeds which later grew into these structures existed, and we know the distribution of their intrinsic sizes. By measuring the distribution of their apparent sizes on the sky, we can determine the geometry of the Universe.

More precisely, one must measure the angular power spectrum of the CMB: Suppose we measure the temperature $T(\vec{\theta})$ as a function of direction $\vec{\theta}$ on the sky over some approximately square region of the sky. We may then compute the Fourier transform $\tilde{T}(\vec{\ell})$ of this temperature map. The power spectrum is then given by the set of multipole moments $C_\ell = \langle \tilde{T}(\vec{\ell})\tilde{T}^*(\vec{\ell}) \rangle$, where the angle brackets denote an average over all wavevectors $|\vec{\ell}| = \ell$. Roughly speaking, each $C_\ell$ measures the mean-square temperature difference between two points separated by an angle $(\theta/\text{deg}) \simeq (200/\ell)$, so larger-$\ell$ modes measure temperature fluctuations on smaller angular scales. Increasingly accurate measurements of the $C_\ell$’s requires mapping larger portions of the sky to reduce the sampling error. Precise temperature measurements are also required. Good angular resolution is needed to determine the larger-$\ell$ moments.

If galaxies and clusters grew from gravitational instability of tiny primordial density perturbations, then the CMB power spectrum (the $C_\ell$) should look like the curves shown in the graph. The bumps in the curves are due to physical processes that lead to large-scale structures. If $\Omega$ is smaller than unity, then the Universe is open and the structure in the CMB is shifted to smaller angular scales, or equivalently, larger $\ell$’s. Therefore, the location of the peaks (primarily the first peak) in the CMB spectrum determines $\Omega$ and therefore the
FIG. 2. Bumps in the background. Power spectrum of the cosmic microwave background as a function of angle $\theta$ or wavenumber $\ell$. Curves show spectral behavior expected for different mass densities, $\Omega$. Future MAP data (simulated, red) should permit better constraints on which curve actually represents the cosmic microwave background. Even better constraints should be produced by the future Planck Surveyor mission (simulated, black).
geometry of the Universe [4].

The blue points are current measurements from balloon-borne and ground-based experiments. Several groups [5] have recently found a value of $\Omega$ consistent with unity by fitting these data to the theoretical curves. Although these results are intriguing and perhaps suggestive, even a cursory glance demonstrates that the current data cannot robustly support a flat Universe.

However, a new generation of experiments will soon provide significant advances. As indicated by the red points in the Figure, the Microwave Anisotropy Probe (MAP), a NASA satellite mission scheduled for launch in the year 2000, should confirm the peak structure suggested by the gravitational-instability paradigm (if it is correct) and make a precise determination of the geometry. The Planck Surveyor, a European Space Agency mission scheduled for launch in 2005, should improve on MAP’s precision and may also illuminate the nature of the missing mass.

If the peak structure of gravitational instability is confirmed and the measurements are precisely consistent with the inflationary prediction of a flat Universe, then new avenues of inquiry will be opened to provide clues to the new particle physics responsible for inflation. As one example, the polarization of the CMB may probe a stochastic background of gravitational waves predicted by inflation [6].

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