Euclidean Path Integral of the Gauge Field

--- Holomorphic Representation ---

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Basing on the canonical quantization of a BRS invariant Lagrangian, we construct holomorphic representation of path integrals for Faddeev-Popov (FP) ghosts as well as for unphysical degrees of the gauge field from covariant operator formalism. A thorough investigation of a simple soluble gauge model with finite degrees will explain the metric structure of the Fock space and constructions of path integrals for quantized gauge fields with FP ghosts. We define fermionic coherent states even for a Fock space equipped with indefinite metric to obtain path integral representations of a generating functional and an effective action. The same technique will also be developed for path integrals of unphysical degrees in the gauge field to find complete correspondence, that insures cancellation of FP determinant, between FP ghosts and unphysical components of the gauge field. As a byproduct, we obtain an explicit form of Kugo-Ojima projection, $P^{(n)}$, to the subspace with $n$-unphysical particles in terms of creation and annihilation operators for the abelian gauge theory.

§1. Introduction

Path integral quantization of a system with local gauge invariance is beautifully formulated by the Faddeev-Popov (FP) procedure.\textsuperscript{1} The characteristic feature of systems with such symmetries is the presence of redundant gauge variant degrees of freedom required to maintain manifest gauge invariance and also other symmetries such as space-time covariance. Abelian and non-abelian gauge theories will be typical examples of such systems. Physical contents of such theories can be extracted immediately if we choose some gauge fixing condition to be able to eliminate unphysical degrees of freedom at the cost of manifest covariance. The covariant perturbation theory in terms of Feynman path integral for quantum gauge theories therefore involves this price to be paid. The FP procedure and its reinterpretation by the BRS invariant formulation of a path integral of such systems (BFV-BRS approach)\textsuperscript{2,3} play the role for this purpose. We shall call a path integral formulated in this way, including original one by FP, as a “conventional” path integral in this paper. An important remark on the conventional path integral of gauge theories is that the FP determinant appears in this formalism must be understood as the absolute value of the determinant because otherwise, in non-perturbative region of the field configuration, the positivity of a field dependent functional determinant is not guaranteed.\textsuperscript{4} Therefore a path integral that involves the FP determinant without taking its absolute value may yield a meaningless result.

However beautiful, there exist several open questions in the formalism of quantization by means of such prescription. Among them typical issue will be the Gribov ambiguities.\textsuperscript{5–8} If we need to analyze gauge theory beyond perturbation, the van-
ishing or sign changes of the FP determinant will arise to result in obstructions for the formulation of the theory in terms of path integral. A prescription to modify the original FP path integral, concerning the locality of the gauge fixing factor and the compensating factor in the gauge space, was suggested to recast the Gribov complication to the breaking of one to one correspondence between the gauge parameter and the variable of the functional integration of the normalization factor for a path integral; but essential solution seems to be still far away from our current knowledge about formulating gauge theories.

Some alternatives for FP path integral have been proposed by a few authors. In Ref. 10), Kashiwa and Sakamoto formulated path integrals from operator formalism on the reduced phase space for physical degrees then performed c-number gauge transformations under a path integral with the aid of identities with respect to delta functions or Gaussian integrations to find manifestly covariant path integrals. On the other hand, in Ref. 11), Arisue, Fujiwara, Inoue and Ogawa dealt with the resolution of unity for vector space equipped with indefinite metric as a fundamental ingredient. Then a path integral based on the canonical formalism in the Feynman gauge for electromagnetic field was derived. There seems to have been a disagreement between the results of Ref. 10) and Ref. 11) concerning the domain of integration with respect to the time component of the vector field.

In addition to these attempts, another approach for constructing path integrals within canonical formalism for systems with gauge invariance have been proposed by others. In Ref. 12), the authors made use of projection to the physical subspace, since naive application of Faddeev-Senjanovic(FS) formula had seemed to fail to yield a well-defined path integral for some Hamiltonian systems with first class constraints on compact phase spaces. The others assert that the projection method provides a prescription to formulate path integrals avoiding the Gribov problem since it does not rely on any gauge fixing condition. The feature of the projection method being free from gauge fixing will also help us to sidestep on the use of path integral with FP ghosts. Therefore it will be best to formulate path integrals for systems with gauge symmetry in terms of the projection method if it can maintain the space-time covariance entirely, because the connection to the operator formalism is very clear in this method. As far as the present author knows, however, there seems to be no satisfactory prescription for dealing with situations we meet in quantization of gauge theories with a covariant gauge conditions within this formalism. Hence it will be difficult for the projection method to keep covariance manifest.

Another issue to be asked on the FP path integral may be the precise definition of the path integral over ghost fermions. The original definition of the ghost fermions is given by a formal path integral representation of a functional determinant. If we adopt covariant type gauge fixing, we will find a second order Lagrangian for the path integral of ghost fermions. As is well-known, however, from coherent state path integral, that is formulated basing upon canonical formalism, for usual physical fermions, there does not seem to exist any room for such Lagrangian in the action of path integrals for fermions. Furthermore, the boundary condition for such fermion path integral is not clear from the formal definition as a determinant. From very definition, it may be naively expected to integrate whole of fermion variables
including those at both initial and final boundaries of a Feynman kernel, otherwise the fermion path integral becomes a kernel that contains wave function factor for ghosts instead of yielding a determinant. Even so it is still unclear if we should integrate a Feynman kernel with periodic or anti-periodic boundary condition. From the BRS invariance point of view, it may be natural to expect the periodic boundary conditions on all fields.\cite{18,19} This immediately conflicts with the anti-periodic one for the determinant from path integrals of usual fermions.\cite{16} If it requires any boundary conditions other than the familiar anti-periodic one, the path integral of ghost fermions then differs significantly from the usual trace formula. In this sense the issue itself shall be interesting as a subject of path integral technique. These problems were solved by Kashiwa\cite{20} by constructing a path integral of FP ghosts in the field diagonal representation. He also clarified the relation of Kugo-Ojima's quartet mechanism\cite{21} and the Gaussian identity in the path integral. There may remain, however, a room for considering alternatives for the path integral given by Ref. 20) in terms of another representation.

Our main purpose in this article is therefore to formulate a path integral of ghost fermions as well as the corresponding one of unphysical degrees of the gauge field in a unified manner without making use of FP prescription. To achieve this we introduce coherent states for these unphysical variables. We shall try such procedure working with a simple gauge invariant model in the BRS invariant treatment. In the next section we explain our model and its quantization on representation spaces with indefinite metric. Canonical quantization and path integral representation for ghost fermions in terms of coherent states will be investigated in section 3 in detail. In section 4 we generalize the method of coherent state to include unphysical degrees of the gauge field. Generating functionals and effective actions both for FP ghosts and unphysical degrees of the gauge field will then be evaluated in section 5. Our construction of coherent states and their use in gauge model requires complete analysis of the vector space on which we formulate the quantum theory of the model. Performing such a detailed investigation brings us a nice understanding of BRS quartet\cite{21} and Kugo-Ojima projection. We will show an explicit form of Kugo-Ojima projection expressed as an integration to yield a projection to an eigenspace of a number operator. This will be given in section 6. Application to the quantization of a free gauge field will be then shortly discussed in section 7. We will confirm there the applicability and reliability of our method by observing that our prescription reproduces zeroth order results for covariant perturbation of the quantized gauge field. Considerations on the relation of the results of Ref. 10) and Ref. 11) will be made in section 8 by comparing our prescription in this paper with the standard “Euclidean Technique”.

§2. Simple models for the quantized gauge field

In quantization of a free gauge field, considerations on the following two Lagrangian systems are quite useful. For a fixed 3-dimensional vector $k \neq 0$, we take
a Lagrangian
\[ L_{\text{phys}} = \frac{1}{2} \dot{A}^2 - \frac{1}{2} k^2 A^2, \quad k = |k|, \quad k \cdot A = 0 \] (2.1)
as a model for physical variables and
\[ L_{\text{unphys}} = \frac{k^2}{2} (\dot{A} - A_0)^2 \] (2.2)
for unphysical degrees. The former for physical variables is understood as a model for quantization in the Coulomb gauge while the latter represents gauge variant degrees. This can be seen by observing that the Lagrangian Eq. (2.2) has a gauge symmetry under a gauge transformation
\[ A \mapsto A + \theta, \quad A_0 \mapsto A_0 + \dot{\theta} \] (2.3)
while there exists no such symmetry in the Lagrangian Eq. (2.1). Note that, in the above Lagrangians, the zero-modes both in physical and unphysical degrees are excluded from our consideration. This corresponds to a regularization for infrared singularities in quantization of the massless gauge field.

If we introduce two independent vectors \( e_s(k)(s = 1, 2) \) such that
\[ k \cdot e_s(k) = 0, \quad e_s(k) \cdot e_{s'}(k) = \delta_{ss'}, \] (2.4)
the physical variables are parametrized as
\[ A(t) = \sum_{s=1,2} q_s(t; k)e_s(k) \] (2.5)
by two real variables \( q_s(t; k)(s = 1, 2) \). Then the Lagrangian becomes
\[ L_{\text{phys}} = \sum_{s=1,2} {\frac{1}{2}} \{ \dot{q}_s^2(t; k) - k^2 q_s^2(t; k) \} \] (2.6)
to be found as a set of two symmetric harmonic oscillators. Hence the quantizations of physical variables are straightforward. We thus omit detailed description of this procedure and concentrate the quantization of unphysical degrees by writing \( L_{\text{unphys}} \) as \( L \) for brevity.

\section{The BRS formalism}

In this section we consider canonical quantization of variables appear in the BRS Lagrangian obtained from Eq. (2.2) by adding terms both for gauge fixing and ghost fermions.

\subsection{BRS invariant Lagrangian}

The BRS transforms of the original variables in Eq. (2.2) are given by
\[ \delta_B A(t) = c(t), \quad \delta_B A_0(t) = \dot{c}(t), \] (3.1)
or by
\[ \delta_B A(t) = c(t)\theta, \quad \delta_B A_0(t) = \dot{c}(t)\theta, \quad (3.2) \]
with a Grassmann parameter \( \theta \). Defining the BRS transform of the ghost \( c(t) \) to be
\[ \delta_B c(t) = 0, \quad (3.3) \]
we can see the nil-potency of the BRS transformation on these variables. Next we introduce the anti-ghost \( \bar{c}(t) \) to satisfy
\[ \delta_B \bar{c}(t) = -iB(t). \quad (3.4) \]
Then \( \delta_B B(t) = 0 \) ensures the nil-potency of the BRS transformation on all variables. Taking these into account, the gauge fixing part of the Lagrangian is defined by
\[ L_{\text{GF+FP}} = i\delta_B \left\{ \bar{c} \left( \dot{A}_0 + k^2 A + \frac{\alpha}{2} B \right) \right\}, \quad (3.5) \]
for a covariant gauge condition
\[ \dot{A}_0 + k^2 A + \alpha B = 0. \quad (3.6) \]
The reason why we call this as a covariant condition will be made clear in the application of the present model to the gauge field in later sections.

Explicitly, \( L_{\text{GF+FP}} \) is given by
\[ L_{\text{GF+FP}} = B(\dot{A}_0 + k^2 A) + \frac{\alpha}{2} B^2 + i\bar{c}(\ddot{c} + k^2 c) \quad (3.7) \]
to yield a gauge fixed Lagrangian
\[ \tilde{L} = L + L_{\text{GF+FP}} = \frac{k^2}{2}(\dot{A} - A_0)^2 + B(\dot{A}_0 + k^2 A) + \frac{\alpha}{2} B^2 + i\bar{c}(\ddot{c} + k^2 c). \quad (3.8) \]
Abelian nature of the symmetry under consideration allows us to separate this total Lagrangian into two parts, the Lagrangian \( L_G \) that describes unphysical degrees of the gauge field with the multiplier field \( B \), given by
\[ L_G = \frac{k^2}{2}(\dot{A} - A_0)^2 + B(\dot{A}_0 + k^2 A) + \frac{\alpha}{2} B^2, \quad (3.9) \]
and the one \( L_{\text{FP}} \) for ghost fermions:
\[ L_{\text{FP}} = i\bar{c}(\ddot{c} + k^2 c). \quad (3.10) \]
The BRS transform of each Lagrangian is given by
\[ \delta_B L_G = B(\dot{c} + k^2 c) \quad (3.11) \]
and
\[ \delta_B L_{\text{FP}} = -B(\dot{c} + k^2 c) \quad (3.12) \]
respectively to result in the BRS invariance of the whole Lagrangian \( \tilde{L} \) by their cancellation.
3.2. Coherent state path integral of ghost fermions

If we take care the BRS invariance of the total system, integration by part on the kinetic term of the Lagrangian \( L_{FP} \) for FP ghosts (See section \( \Box \) for the consequence of this on the corresponding part in the Lagrangian for the gauge field) can be done to rewrite \( L_{FP} \) as

\[
L_{FP} = -i\dot{\bar{c}}c + ik^2 \bar{c}c. \tag{3.13}
\]

The corresponding Hamiltonian is then given by

\[
H_{FP} = ip_c\dot{p}_c - ik^2 \bar{c}c \quad \tag{3.14}
\]

in which \( p_c(p_c) \) being the canonical conjugate of \( c(\bar{c}) \). The canonical structure for the system in consideration is defined by the following Poisson brackets:

\[
\{c, p_c\} = \{\bar{c}, p_c\} = 1, \quad \{c, p_c\} = \{\bar{c}, p_c\} = \{c, \bar{c}\} = \{p_c, p_c\} = 0. \tag{3.15}
\]

From equations of motion, we can set \( c(t), \bar{c}(t), p_c(t) \) and \( p_\bar{c}(t) \) as

\[
c(t) = \frac{1}{\sqrt{2k}}(e^{-ikt}b + e^{ikt}b^*), \quad \bar{c}(t) = \frac{1}{\sqrt{2k}}(e^{-ikt}d + e^{ikt}d^*),
\]

\[
p_c(t) = -\sqrt{\frac{k}{2}}(e^{-ikt}d - e^{ikt}d^*), \quad p_\bar{c}(t) = \sqrt{\frac{k}{2}}(e^{-ikt}b - e^{ikt}b^*). \tag{3.16}
\]

Combining Eq. \((3.15)\) and Eq. \((3.16)\) together, we obtain Poisson brackets among \( b, b^*, d, \) and \( d^* \):

\[
\{b(t), d^*(t)\} = 1, \quad \{b^*(t), d(t)\} = -1, \quad \{b(t), b^*(t)\} = \{d(t), d^*(t)\} = 0,
\]

\[
\{b(t), b(t)\} = \{b^*(t), d^*(t)\} = \{d(t), d(t)\} = \{d(t), d^*(t)\} = 0, \tag{3.17}
\]

in which \( b(t) = e^{-ikt}b, \ b^*(t) = e^{ikt}b^*, \ d(t) = e^{-ikt}d \) and \( d^*(t) = e^{ikt}d^* \).

Upon quantization we replace these Poisson brackets by the following anticommutators of Schrödinger operators:

\[
\{\hat{b}, \hat{d}\} = i, \quad \{\hat{d}, \hat{b}\} = -i, \quad \text{others} = 0 \tag{3.18}
\]

or the equal-time anticommutators for Heisenberg operators

\[
\{\hat{b}(t), \hat{d}^\dagger(t)\} = i, \quad \{\hat{d}(t), \hat{b}^\dagger(t)\} = -i, \quad \text{others} = 0. \tag{3.19}
\]

To construct a representation of this algebra, let us introduce two sorts of number operator, \( \hat{N}_i (i = 1, 2) \), by

\[
\hat{N}_1 = ib^\dagger\hat{d}, \quad \hat{N}_2 = -i\hat{d}\hat{b}. \tag{3.20}
\]

to find

\[
[\hat{N}_1, \hat{b}] = \hat{b}^\dagger, \quad [\hat{N}_1, \hat{d}] = -\hat{d}, \quad [\hat{N}_1, \hat{b}] = [\hat{N}_1, \hat{d}] = 0, \tag{3.21}
\]

\[
[\hat{N}_2, \hat{b}] = -\hat{b}, \quad [\hat{N}_2, \hat{d}] = \hat{d}^\dagger, \quad [\hat{N}_2, \hat{b}] = [\hat{N}_2, \hat{d}] = 0. \tag{3.22}
\]
Apparently \( \{ \hat{N}_1, \hat{b}^\dagger, \hat{d} \} \) and \( \{ \hat{N}_2, \hat{d}^\dagger, \hat{b} \} \) form their own closed algebra. Hence we obtain the following basis vectors of the representation space:

\[
\hat{N}_i \{ n_2 n_1 \} = n_i \{ n_2 n_1 \}, \quad n_i = 0, 1 \quad (i = 1, 2)
\]  

(3.23)

where the order of \( n_i \) in \( \{ n_2 n_1 \} \) has meaning and the vacuum \( \{ 00 \} \) is defined by

\[
\hat{b} \{ 00 \} = \hat{d} \{ 00 \} = 0,
\]  

(3.24)

upon which the rest of the basis are generated as

\[
\{ 01 \} = \hat{b}^\dagger \{ 00 \}, \quad \{ 10 \} = \hat{d}^\dagger \{ 00 \}, \quad \{ 11 \} = \hat{d}^\dagger \hat{b}^\dagger \{ 00 \}.
\]  

(3.25)

We may naturally expect the vacuum \( \{ 00 \} \) to have positive norm and to be normalized as

\[
\langle \{ 00 \} | \{ 00 \} \rangle = 1.
\]  

(3.26)

Then by writing the hermitian conjugation of a state vector without taking the metric into account as

\[
\langle \{ n_2 n_1 \} | = (\{ n_2 n_1 \}^\dagger),
\]  

(3.27)

we find the norm of others given by

\[
\langle \{ 01 \} | \{ 01 \} \rangle = \langle \{ 10 \} | \{ 10 \} \rangle = 0, \quad \langle \{ 11 \} | \{ 11 \} \rangle = -1.
\]  

(3.28)

Though one particle states, \( \{ 01 \} \) and \( \{ 10 \} \) are zero-normed, they have non-zero inner products with each other

\[
\langle \{ 01 \} | \{ 10 \} \rangle = i \quad \text{and} \quad \langle \{ 10 \} | \{ 01 \} \rangle = -i
\]  

(3.29)

to yield off diagonal elements to the metric of this vector space. Taking this metric structure into account, we find the identity operator on this vector space given by

\[
1 = \langle \{ 00 \} | \{ 00 \} \rangle + i \langle \{ 01 \} | \{ 10 \} \rangle - i \langle \{ 10 \} | \{ 01 \} \rangle = \langle \{ 11 \} | \{ 11 \} \rangle.
\]  

(3.30)

If we introduce a conjugation defined by

\[
| \{ n_2 n_1 \} \rangle \leftrightarrow \langle \{ n_2 n_1 \} | = e^{\pi i (\hat{b}^\dagger + i\hat{d}^\dagger)(\hat{b} - i\hat{d})/2} \quad (\hat{\eta}_{FP}^{-1} = \hat{\eta}_{FP}^\dagger = \hat{\eta}_{FP})
\]  

(3.31)

corresponding to the above metric structure, the identity operator can be expressed in a concise form by

\[
1 = \sum_{n_i=0,1} | \{ n_2 n_1 \} \rangle \langle \{ n_2 n_1 \} |.
\]  

(3.32)

Note here that the conjugation introduced above generates the following transformation on operators:

\[
(\hat{b}, \hat{d}) \rightarrow \hat{\eta}_{FP}^\dagger (\hat{b}, \hat{d}) \hat{\eta}_{FP} = (i\hat{d}, -i\hat{b})
\]  

(3.33)

together with their hermitian conjugations.
We regard the representation space to be a Grassmann valued vector space in order to construct coherent states\(^{16}\) of the above fermions. Taking the algebra above into account, we define a coherent state
\[
|\xi\rangle = e^{i(b^{\dagger}\xi_2 - d^{\dagger}\xi_1)}|\{00\}\rangle = |\{00\}\rangle - i\xi_2|\{01\}\rangle + i\xi_1|\{10\}\rangle + \xi_2\xi_1|\{11\}\rangle \tag{3.34}
\]
to be a simultaneous eigenvector of \(\hat{b}\) and \(\hat{d}\):
\[
\hat{b}|\xi\rangle = \xi_1|\xi\rangle, \quad \hat{d}|\xi\rangle = \xi_2|\xi\rangle. \tag{3.35}
\]
Taking hermitian conjugation of the above equations, we find that \(\hat{b}^{\dagger}\) and \(\hat{d}^{\dagger}\) have common left eigenvectors defined by
\[
\langle \xi^*| = \langle \{00\}|e^{-i(\xi^*_2\hat{b} - \xi^*_1\hat{d})} = \langle \{00\}| + i\langle \{01\}|\xi^*_2 - i\langle \{10\}|\xi^*_1 + \langle \{11\}|\xi^*_1\xi^*_2 \tag{3.36}
\]
to yield
\[
\langle \xi^*|\xi'\rangle = e^{i(\xi^*_1\xi'_2 - \xi^*_2\xi'_1)}. \tag{3.37}
\]
The set of coherent states thus defined is complete:
\[
\int d^2\xi^* d^2\xi e^{-i(\xi^*_1\xi_2 - \xi^*_2\xi_1)}|\xi\rangle\langle \xi^*| = 1, \tag{3.38}
\]
where use has been made of our convention of integration with respect to Grassmann numbers\(^{16}\)
\[
\int \xi d\xi = \int \xi^* d\xi^* = i, \quad \int d\xi = \int d\xi^* = 0. \tag{3.39}
\]
The order of variables in the measure will be important for integrations over Grassmann numbers; Explicit definition of the measure in Eq. (3.38) is given by
\[
d^2\xi^* = d\xi^*_1 d\xi^*_2, \quad d^2\xi = d\xi_1 d\xi_2. \tag{3.40}
\]
If we make use of another version of the dual vector to \(|\xi\rangle\), given by
\[
\langle \xi| = \langle \{00\}|e^{\xi_1\hat{b} + \xi_2\hat{d}} = \langle \xi^*|\tilde{\eta}_{\text{FP}}, \tag{3.41}
\]
the inner product Eq. (3.37) and the expression for the resolution of unity Eq. (3.38) given above are replaced by
\[
\langle \xi|\xi'\rangle = e^{\xi^*\xi'} \tag{3.42}
\]
and to
\[
\int (d\xi d\xi^*)^2 e^{-\xi^*\xi}|\xi\rangle\langle \xi| = 1, \quad (d\xi d\xi^*)^2 = d\xi_1 d\xi^*_1 d\xi_2 d\xi^*_2 \tag{3.43}
\]
respectively. Though above two expressions of the resolution of unity are equivalent, it will be natural to use the latter because it reflects the metric structure and yields a Gaussian path integral below. Furthermore, it is this latter form that has an analogue in the quantization of unphysical degrees of gauge fields. We thus utilize the resolution of unity Eq. (3.43) in the following to obtain a holomorphic representation of a path integral for ghost fermions.
Path Integral of the Gauge Field

Now we proceed to formulate a path integral for the system in terms of coherent states defined above. The Hamiltonian operator
\[
\hat{H}_{\text{FP}} = i k (\hat{b} \hat{d} - \hat{d}^\dagger \hat{b}) - k = k (\hat{N}_{\text{FP}} - 1), \quad \hat{N}_{\text{FP}} = \hat{N}_1 + \hat{N}_2
\]
for the quantum system is obtained by replacing canonical variables with corresponding operators:
\[
\hat{c} = \frac{1}{\sqrt{2k}} (\hat{b} + \hat{b}^\dagger), \quad \hat{\bar{c}} = \frac{1}{\sqrt{2k}} (\hat{d} + \hat{d}^\dagger), \quad \hat{p}_c = -\sqrt{\frac{k}{2}} (\hat{d} - \hat{d}^\dagger), \quad \hat{p}_{\bar{c}} = \sqrt{\frac{k}{2}} (\hat{b} - \hat{b}^\dagger)
\]
in the classical one given by Eq. (3.14). A Feynman kernel for an imaginary time \(\beta\) is defined by
\[
\langle \xi^F | e^{-\beta \hat{H}_{\text{FP}}} | \xi^I \rangle = \lim_{n \to \infty} \langle \xi^F | (1 - \epsilon \hat{H}_{\text{FP}})^n | \xi^I \rangle, \quad \epsilon = \frac{\beta}{n}.
\]
Repeated use of the resolution of unity given by Eq. (3.43) and the evaluation of the matrix element
\[
\langle \xi(j) | (1 - \epsilon \hat{H}_{\text{FP}}) | \xi(j-1) \rangle = \exp \left\{ (1 - \epsilon k) \xi^\dagger(j) \xi(j-1) + \epsilon k \right\}
\]
will bring us a path integral formula
\[
K(\xi^F, \xi^I; \beta) = e^{\beta k} \lim_{n \to \infty} \int \prod_{i=1}^{n-1} (d\xi(i) \, d\xi^\ast(i))^2
\]
\[
\times \exp \left\{ - \sum_{j=1}^{n-1} \xi^\dagger(j) \xi(j) + (1 - \epsilon k) \sum_{j=1}^{n} \xi^\dagger(j) \xi(j-1) \right\}
\]
for the Euclidean kernel
\[
K(\xi^F, \xi^I; \beta) = \langle \xi^F | e^{-\beta \hat{H}_{\text{FP}}} | \xi^I \rangle.
\]
Note that it is the expression in Eq. (3.43) for the resolution of unity that brings a Gaussian path integral above even for the present system equipped with indefinite metric.

Gaussian integrations are carried out by use of the standard technique for Grassmann variables to yield
\[
K(\xi^F, \xi^I; \beta) = e^{\beta k} \lim_{n \to \infty} \exp \left\{ (1 - \epsilon k)^n \xi^\dagger F \xi^I \right\}
\]
\[
= \exp \left( \beta k + e^{-\beta k} \xi^\dagger F \xi^I \right).
\]
The validity of our construction of Euclidean path integral for FP ghosts can be checked by an observation
\[
\lim_{\beta \to \infty} e^{-\beta k} K(\xi^F, \xi^I; \beta) = 1 = \langle \xi^F | \{00\} \rangle \langle \{00\} | \xi^I \rangle.
\]
We thus formulated a holomorphic representation of path integral for ghost fermions.
§4. Path integral of unphysical degrees of the gauge field

Our task in this section is to perform path integral of the Lagrangian

\[ L_G = \frac{k^2}{2} (\dot{A} - A_0)^2 - \dot{B}A_0 + k^2AB + \frac{\alpha}{2}B \]  

(4.1)

for unphysical components, \( A \) and \( A_0 \), of the gauge field and the multiplier field \( B \). Here, a comment is in order. In the above Lagrangian, we have performed an integration by parts for the term \( B\dot{A}_0 \) in the original Lagrangian Eq. (3.9). It is necessary to preserve the BRS invariance of the Lagrangian because we have already done the same on the term \( i\bar{c}\dot{c} \) in the FP ghost part. Changing \( i\bar{c}\dot{c} \) to \( -i\dot{\bar{c}}\dot{c} \) requires addition of \( \frac{d}{dt}(i\bar{c}\dot{c}) \) which is a half of the BRS exact total derivative \( \frac{d}{dt}\{\delta_B(-i\bar{c}A_0)\} \). We must therefore add this total derivative to the total Lagrangian in order to maintain the BRS invariance.

4.1. Canonical formulation for covariant gauge conditions

Canonical momenta of the gauge and multiplier fields are given by

\[ P = \frac{k^2}{2}(\dot{A} - A_0), \quad P_0 = 0, \quad P_B = -A_0 \]  

(4.2)

and the Hamiltonian for this system reads

\[ H_G = \frac{1}{2k^2}P^2 + A_0P - k^2AB - \frac{\alpha}{2}B^2, \]  

(4.3)

where \( A \) and \( B \) are regarded as coordinate variables with their canonical conjugates \( P = -\dot{B} \) and \( P_B = -A_0 \) as a consequence of the Dirac-Bergmann prescription. By use of equations of motion, \( P \) can be always replaced by \(-\dot{B}\). Hence we have fundamental commutation relations

\[ [\dot{A}(t), -\dot{B}(t)] = i, \quad [\dot{B}(t), -\dot{A}_0(t)] = i \]  

(4.4)

as well as

\[ [\dot{A}(t), \dot{A}_0(t)] = [\dot{A}(t), \dot{B}(t)] = [\dot{A}_0(t), \dot{B}(t)] = [\dot{B}(t), \dot{B}(t)] = 0 \]  

(4.5)

together with the Heisenberg equations

\[ \ddot{A}(t) - \ddot{A}_0(t) - \dot{B}(t) = 0, \]  

(4.6)

\[ k^2(\dot{A}(t) - \dot{A}_0(t)) + \dot{B}(t) = 0, \]  

(4.7)

\[ \dot{A}_0 + k^2\ddot{A}(t) + \alpha\dot{B}(t) = 0, \]  

(4.8)

for quantization. As can be seen easily, \( \dot{B}(t) \) obeys a free field equation:

\[ \dddot{B} + k^2\ddot{B}(t) = 0 \]  

(4.9)

while \( \dot{A}(t) \) and \( \dot{A}_0(t) \) in general involves dipole ghost and satisfy

\[ \dddot{A}(t) + k^2\ddot{A}(t) = (1 - \alpha)\dot{B}(t), \quad \dddot{A}_0(t) + k^2\ddot{A}_0(t) = (1 - \alpha)\dot{B}(t). \]  

(4.10)
In order to accomplish the quantization of the system, we need to find a representation for the above algebra. To this aim, we first study formal solutions of the Heisenberg equations for $\hat{A}(t)$, $\hat{A}_0(t)$ and $\hat{B}(t)$ by making use of the quantum mechanical version of the formulation for quantizing vector fields with indefinite metric.\(^{23)}\) If we introduce Fourier transforms of Heisenberg operators by

$$\hat{A}(t) = \int_{-\infty}^{\infty} dp \{ \hat{a}(p)e^{-ipt} + \hat{a}^\dagger(p)e^{ipt} \}, \quad (4.11)$$

$$\hat{A}_0(t) = \int_{-\infty}^{\infty} dp \{ \hat{a}_0(p)e^{-ipt} + \hat{a}_0^\dagger(p)e^{ipt} \}, \quad (4.12)$$

$$\hat{B}(t) = \int_{-\infty}^{\infty} dp \{ \hat{\beta}(p)e^{-ipt} + \hat{\beta}^\dagger(p)e^{ipt} \}, \quad (4.13)$$

equations of motion will be formally solved by putting

$$\hat{a}(p) = -i\theta(p) \left\{ \delta(p^2 - k^2) || e^{ipt}, \hat{A}(t) || + (1 - \alpha)\delta'(p^2 - k^2) || e^{ipt}, \hat{B}(t) || \right\}, \quad (4.14)$$

$$\hat{a}_0(p) = -i\theta(p) \left\{ \delta(p^2 - k^2) || e^{ipt}, \hat{A}_0(t) || + (1 - \alpha)\delta'(p^2 - k^2) || e^{ipt}, \hat{B}(t) || \right\}, \quad (4.15)$$

$$\hat{\beta}(p) = -i\theta(p)\delta(p^2 - k^2) || e^{ipt}, \hat{B}(t) || \quad (4.16)$$

in which $|| F(t), G(t) ||$ being the Wronskian of $F(t)$ and $G(t)$.

As the most fundamental ingredient for the canonical formulation, we require the existence of the vacuum state $|0\rangle$ to fulfill

$$\hat{a}(p)|0\rangle = \hat{a}_0(p)|0\rangle = \hat{\beta}(p)|0\rangle = 0 \quad (4.17)$$

or equivalently

$$\hat{A}^{(+)}(t)|0\rangle = \hat{A}_0^{(+)}(t)|0\rangle = \hat{B}^{(+)}(t)|0\rangle = 0, \quad (4.18)$$

where positive and negative frequency parts of the Heisenberg operators are defined by

$$(\hat{A}^{(+)}(t), \hat{A}_0^{(+)}(t), \hat{B}^{(+)}(t)) = \int_{-\infty}^{\infty} dp \{ \hat{a}(p), \hat{a}_0(p), \hat{\beta}(p) \} e^{-ipt}, \quad (4.19)$$

and by

$$(\hat{A}^{(-)}(t), \hat{A}_0^{(-)}(t), \hat{B}^{(-)}(t)) = \int_{-\infty}^{\infty} dp \{ \hat{a}^\dagger(p), \hat{a}_0^\dagger(p), \hat{\beta}^\dagger(p) \} e^{ipt}, \quad (4.20)$$

respectively.

By making use of the equal-time commutators in Eq. (4.4) and Eq. (4.5), we can find the following commutation relations for the Fourier coefficients:

$$[\hat{a}(p), \hat{a}_0^\dagger(q)] = \theta(p)\delta(p - q) \frac{1}{k^2} \{ \delta(p^2 - k^2) - (1 - \alpha)k^2\delta'(p^2 - k^2) \}, \quad (4.21)$$

$$[\hat{a}_0(p), \hat{a}_0^\dagger(q)] = -\theta(p)\delta(p - q) \{ \delta(p^2 - k^2) + (1 - \alpha)k^2\delta'(p^2 - k^2) \}, \quad (4.22)$$

$$[\hat{\beta}(p), \hat{\beta}^\dagger(q)] = 0, \quad (4.23)$$

$$[\hat{a}(p), \hat{\beta}^\dagger(q)] = [\hat{\beta}(p), \hat{a}^\dagger(q)] = -\theta(p)\delta(p - q)\delta(p^2 - k^2), \quad (4.24)$$

$$[\hat{a}_0(p), \hat{\beta}^\dagger(q)] = -[\hat{\beta}(p), \hat{a}_0^\dagger(q)] = i\theta(p)\delta(p - q)\delta(p^2 - k^2), \quad (4.25)$$

$$[\hat{a}(p), \hat{a}_0^\dagger(q)] = -[\hat{a}_0(p), \hat{a}^\dagger(q)] = -i(1 - \alpha)\theta(p)\delta(p - q)\delta(p^2 - k^2). \quad (4.26)$$
These relations can be utilized for determination of arbitrary commutators of Heisenberg operators and Green’s functions.

4.2. Coherent states for unphysical degrees of the gauge field

We here define coherent states for unphysical degrees of the gauge field. To this aim, we rewrite Heisenberg operators in Eq. (4.19) as follows

\[ \hat{A}^{(+)}(t) = \frac{1}{\sqrt{2k}} \hat{a}(t), \quad \hat{A}_0^{(+)}(t) = \frac{1}{\sqrt{2k}} \hat{a}_0(t), \quad \hat{B}^{(+)}(t) = \frac{1}{\sqrt{2k}} \hat{b}(t), \]

and their hermitian conjugates in Eq. (4.20) as well to find the following equal-time commutation relations

\[ [\hat{a}(t), \hat{a}^{\dagger}(t)] = \frac{1 + \alpha}{2k^2}, \quad [\hat{a}_0(t), \hat{a}_0^{\dagger}(t)] = -\frac{1 + \alpha}{2}, \quad [\hat{a}(t), \hat{a}_0(t)] = 0, \]

\[ [\hat{a}(t), \hat{b}^{\dagger}(t)] = -1, \quad [\hat{a}_0(t), \hat{b}^{\dagger}(t)] = ik \]

from Eq. (4.26). For constructing a representation of this algebra, it is convenient to introduce the following Heisenberg operators:

\[ \hat{B}(t) = \frac{1}{k} \hat{b}(t), \quad \hat{D}(t) = \frac{i}{2} (k\hat{a}(t) + i\hat{a}_0(t)), \quad \hat{\bar{D}}(t) = \frac{i}{2} (k\hat{a}(t) - i\hat{a}_0(t)) \]

with their hermitian conjugates. By use of the commutation relations in Eq. (4.28), we obtain

\[ [\hat{B}(t), \hat{D}^{\dagger}(t)] = i, \quad [\hat{D}(t), \hat{B}^{\dagger}(t)] = -i, \]

as well as

\[ [\hat{B}(t), \hat{B}^{\dagger}(t)] = [\hat{D}(t), \hat{D}^{\dagger}(t)] = [\hat{B}(t), \hat{D}(t)] = 0 \]

for \( \hat{B}(t) \) and \( \hat{D}(t) \) and their conjugates. As for the operator \( \hat{D}(t) \), we can utilize the relation:

\[ \hat{D}(t) = -\frac{i}{4} (1 + \alpha) \hat{B}(t) \]

by observation that equal-time commutators of \( \hat{D}(t) \) and \( \hat{D}^{\dagger}(t) \) with other operators vanish excepting

\[ [\hat{D}(t), \hat{D}^{\dagger}(t)] = -[\hat{D}(t), \hat{D}^{\dagger}(t)] = \frac{1 + \alpha}{4}. \]

Note that the relation Eq. (4.22) can be also obtained directly by considering the Fourier transform of

\[ -ip\hat{a}_0(p) + k^2\hat{a}(p) = -\alpha\hat{\beta}(p) \]

which is a consequence of the equations of motion (Eq. (4.18)). Inversion of the definition of the operators \( \hat{B}(t) \) and \( \hat{D}(t) \) can be done to express original Heisenberg operators in terms of them if we also make a use of Eq. (4.32). We thus obtain

\[ \hat{A}(t) = -\frac{1}{\sqrt{2k^3}} \left\{ \frac{1 + \alpha}{4} (\hat{B}(t) + \hat{B}^{\dagger}(t)) + i(\hat{D}(t) - \hat{D}^{\dagger}(t)) \right\}, \]

\[ \hat{A}_0(t) = -\frac{i}{\sqrt{2k}} \left\{ \frac{1 + \alpha}{4} (\hat{B}(t) - \hat{B}^{\dagger}(t)) - i(\hat{D}(t) + \hat{D}^{\dagger}(t)) \right\}, \]

\[ \hat{B}(t) = \sqrt{\frac{k}{2}} (\hat{B}(t) + \hat{B}^{\dagger}(t)), \quad \hat{\bar{B}}(t) = -i\sqrt{\frac{k^3}{2}} (\hat{B}(t) - \hat{B}^{\dagger}(t)). \]
As must be clear from definition, \( \hat{B}(t) \) is essentially the positive frequency part of \( \hat{\mathcal{B}}(t) \) and hence BRS invariant. On the other hand, though the form of the definition is independent of \( \alpha \), the operator \( \hat{D}(t) \) is a gauge dependent objects through the \( \alpha \)-dependence of \( \hat{a}(t) \) and \( \hat{a}_0(t) \).

Interestingly, commutation relations above closely resemble the anticommutation relations Eq. (3.18) for ghost fermions. The similarity is not limited only to the algebra but also seen in the form of Hamiltonian for both systems. If we express the Hamiltonian \( \hat{H}_G \) in terms of above operators by making use of Eq. (4.35), we find

\[
\hat{H}_G = \hat{H}_G' + \frac{1 - \alpha}{2} k \hat{B}(t) \hat{B}(t),
\]

\[
\hat{H}_G' = i k (\hat{B}(t) \hat{D}(t) - \hat{D}^\dagger(t) \hat{B}(t)) + k.
\]

Combining this Hamiltonian together with that for ghost fermions, we see that total Hamiltonian for the system under consideration is expressed as

\[
\hat{H} = i k \{ (\hat{B}(t) \hat{D}(t) - \hat{D}^\dagger(t) \hat{B}(t)) + (\hat{b}^\dagger(t) \hat{d}(t) - \hat{d}^\dagger(t) \hat{b}(t)) \} + \frac{1 - \alpha}{2} k \hat{B}(t) \hat{B}(t),
\]

where \( \hat{b}(t), \hat{d}(t) \) and their hermitian conjugates designate Heisenberg operators for ghost fermions. The last term, proportional to \( 1 - \alpha \), in the Hamiltonian can be eliminated by shifting \( \hat{D}(t) \) and its conjugate according to

\[
\hat{D}(t) \mapsto \hat{D}'(t) = \hat{D}(t) - \frac{i}{4} (1 - \alpha) \hat{B}(t), \quad \hat{D}^\dagger(t) \mapsto \hat{D}'^\dagger(t) = \hat{D}^\dagger(t) + \frac{i}{4} (1 - \alpha) \hat{B}(t)
\]

(4.38)

to allow us to rewrite the total Hamiltonian as

\[
\hat{H} = i k \{ (\hat{B}(t) \hat{D}'(t) - \hat{D}'^\dagger(t) \hat{B}(t)) + (\hat{b}^\dagger(t) \hat{d}(t) - \hat{d}^\dagger(t) \hat{b}(t)) \}.
\]

(4.39)

Although the Hamiltonian becomes simple, the above shifts in \( \hat{D}(t) \) and \( \hat{D}^\dagger(t) \) also changes the commutation relation of them to

\[
[\hat{D}(t), \hat{D}^\dagger(t)] \mapsto [\hat{D}'(t), \hat{D}'^\dagger(t)] = \frac{1 - \alpha}{2}.
\]

(4.40)

This makes the construction of a representation complicated compared with the one given below. We thus take the \( \alpha \)-dependent form the Hamiltonian as well as the commutators in Eq. (4.30) and Eq. (4.31).

Observing the analogy in the algebra as well as in the structure of Hamiltonian between those of the gauge field and ghost fermions, we now construct the basis of the vector space on which the operators are represented. It will be easy to see that the operator \( \hat{H}_G' \) in the Hamiltonian \( \hat{H}_G \), expressed here by Schrödinger operators, has eigenvectors defined by

\[
|n_2 n_1\rangle = \frac{1}{\sqrt{n_1!n_2!}} (\hat{D}^\dagger)^{n_2} (\hat{B}^\dagger)^{n_1} |0\rangle, \quad |00\rangle = |0\rangle, \quad n_1, n_2 = 0, 1, 2, \ldots
\]

(4.41)

to satisfy

\[
\hat{H}_G' |n_2 n_1\rangle = k(n_1 + n_2 + 1) |n_2 n_1\rangle.
\]

(4.42)
Since taking hermitian conjugate changes the place of \(i\mathbf{B}^\dagger \mathbf{D}\) and \(-i\mathbf{D}^\dagger \mathbf{B}\) to each other, their right eigenvectors will be also brought to the left ones of the other. Hence, for \(n_1, n_2 = 0, 1, 2, \ldots\),

\[
\langle [n_2 n_1] | \hat{H}_{G'} \rangle = k(n_1 + n_2 + 1)\langle [n_2 n_1] \rangle, \quad \langle [n_2 n_1] \rangle = \frac{1}{\sqrt{n_1! n_2!}} \langle 0 | \hat{B}^{n_1} \hat{D}^{n_2}.
\]

An inner product of these eigenvectors is given by

\[
\langle [n_2 n_1] | [n'_2 n'_1] \rangle = i^{n_1-n_2} \delta_{n_2 n_1} \delta_{n_1 n'_2}.
\]

This determines the metric structure of the vector space under consideration. Then, to be consistent with this metric, we can construct an expression for the resolution of unity:

\[
\sum_{n_1, n_2=0}^\infty \langle [n_2 n_1] | i^{n_2-n_1} \langle [n_1 n_2] \rangle = 1.
\]

Thus we have found that the vector space for representation of quantum theory of our model is again equipped with indefinite metric (See Eq. (4.32)). Similar to the fermionic case, the inner product given above is brought to

\[
\langle [n_2 n_1] | [n'_2 n'_1] \rangle = \delta_{n_1 n_1'} \delta_{n_2 n_2'}
\]

by introducing a conjugate \(\langle [n_2 n_1] \rangle\) for \(\langle [n_2 n_1] \rangle\) defined by

\[
| [n_2 n_1] \rangle \leftrightarrow \langle [n_2 n_1] | \propto \langle [n_2 n_1] | \eta_G \rangle, \quad \eta_G = e^{\pi i (\hat{B}^\dagger \hat{D}) (\hat{B} - i \hat{D})^2 / 2} \quad (\eta_G^{-1} = \eta_G^\dagger = \eta_G).
\]

The conjugation introduced above generates the following transformation on operators:

\[
(\hat{B}, \hat{D}) \rightarrow \eta_G^\dagger (\hat{B}, \hat{D}) \eta_G = (i \hat{D}, -i \hat{B})
\]

together with their hermitian conjugations. In terms of the dual vectors defined above, we can rewrite the identity operator as

\[
\sum_{n_1, n_2 = 0}^\infty \langle [n_2 n_1] | \langle [n_1 n_2] \rangle = 1.
\]

The analogy between ghost fermions and unphysical components of the gauge field still continues to bring us the following definition of a coherent state:

\[
| z \rangle = e^{i(\hat{B}^\dagger z_2 - \hat{D}^\dagger z_1)} | 0 \rangle, \quad z_1, z_2 \in \mathbb{C}.
\]

It will be straightforward to see

\[
\hat{B} | z \rangle = z_1 | z \rangle, \quad \hat{D} | z \rangle = z_2 | z \rangle,
\]

together with their hermitian conjugates. The inner product of these coherent states will be given by

\[
\langle z^\ast | z' \rangle = e^{i(z^\ast z' - z^\ast z')} , \quad \langle z^\ast | = (| z \rangle)^\dagger.
\]
It would be beautiful if we were able to give some meaning to the integral

$$\int \left(\frac{dz dz^*}{\pi}\right)^2 e^{-i(z_1^*z_2 - z_2^*z_1)} |z\rangle \langle z^*|, \quad (dz dz^*)^2 = d\mathbb{R}(z_1) d\mathbb{R}(z_2) d\mathbb{R}^*(z_2)$$

(4.53)

as the analogue of the resolution of unity for ghost fermions given by Eq. (3.38). A possible way to make the above integral well-defined may be treating $-iz_2^*$ and $iz_1^*$ as complex conjugate to $z_1$ and $z_2$, respectively. But it is actually equivalent to replacing $\langle z_2 |$ by $\langle z_1 | = \langle 0 | e^{z_1^* H + z_2^* D} = \langle z_2^* | \eta_G$ (4.54) to yield

$$\int \left(\frac{dz dz^*}{\pi}\right)^2 e^{-z_1^* z_2} |z\rangle \langle z^*| = 1.$$  

(4.55)

The analogy thus terminates here; we find only one formula of resolution of unity in terms of coherent states for the case of the gauge field while two formulas were possible for ghost fermions. The reason for such difference is evident. In integrations with respect to Grassmann numbers, we can treat $\xi$ and its conjugate entirely independent to allow us the change of variables such that $(-i\xi_2^*, i\xi_1^*) \mapsto (\xi_1^*, \xi_2^*)$ but this is not allowed for usual c-numbers. It will be, therefore, natural that we cannot find complete resemblance between bosonic and fermionic degrees. Rather, we should be surprised at the existence of a formula like Eq. (4.55) as an analogue of Eq. (3.43).

4.3. Coherent state path integral for unphysical degrees of the gauge field

Acquiring the basic ingredient we now formulate a path integral for the Hamiltonian $\hat{H}_G$ in terms of the coherent state. First, consider an infinitesimal version of the Euclidean kernel defined by

$$\langle z(j)| (1 - \epsilon \hat{H}_G) |z(j-1)\rangle = \exp \left\{ z^\dagger(j)(1 - \epsilon h)z(j-1) - \epsilon k \right\},$$  

(4.56)

where $\epsilon = \beta/n$ and

$$h = \begin{pmatrix} 1 & 0 \\ -i(1 - \alpha)/2 & 1 \end{pmatrix}.$$  

(4.57)

Then repeated convolution of these infinitesimal kernels will bring us a discretized path integral

$$\langle z_F| e^{-\beta \hat{H}_G} |z_I\rangle = e^{-\beta k} \lim_{n \to \infty} \prod_{i=1}^{n-1} \left(\frac{dz(i) dz^*(i)}{\pi}\right)^2$$

$$\times \exp \left\{ -\sum_{j=1}^{n-1} z^\dagger(j)z(j) + \sum_{j=1}^{n} z^\dagger(j)(1 - \epsilon h)z(j-1) \right\}.$$  

(4.58)

This can be evaluated in a straightforward way to be

$$\langle z_F| e^{-\beta \hat{H}_G} |z_I\rangle = \exp \left\{ -\beta k + e^{-\beta k} z_F^\dagger \gamma(\beta)z_I \right\},$$  

(4.59)
where
\[ \gamma(t) = \begin{pmatrix} 1 & 0 \\ -i(1 - \alpha)kt/2 & 1 \end{pmatrix}. \] (4.60)

Combining this result with Eq. (3.50) for ghost fermions, we obtain for the total Hamiltonian \( \hat{H} = \hat{H}_G + \hat{H}_{FP} \)
\[ \langle z_F, \xi_F | e^{-\beta \hat{H}} | z_I, \xi_I \rangle = \exp \left\{ e^{-\beta k} z_F^\dagger \gamma(\beta) z_I + e^{-\beta k} \xi_F^\dagger \xi_I \right\}, \] (4.61)
where \( |z, \xi\rangle \) designates a direct product given by
\[ |z, \xi\rangle = |z\rangle \otimes |\xi\rangle. \] (4.62)

Since \( \gamma(t) \) becomes unity for \( \alpha = 1 \), we can see the complete correspondence between FP ghosts and unphysical degrees of the gauge field in the Feynman gauge. Indeed there exists a trivial symmetry, for \( \alpha = 1 \), in the total Hamiltonian given by Eq. (4.37) under exchanging the corresponding degrees between bosonic and fermionic part. It must be, however, rather artificial that the symmetry is restricted only to the Feynman gauge because a Feynman kernel is a gauge dependent object. If we calculate a trace of time evolution operator to remove the gauge dependence, we will find
\[ \text{Tr}(e^{-\beta H}) = \int \left( \frac{dz dz^*}{\pi} \right)^2 (d\xi d\xi^*)^2 e^{-z^\dagger z - \xi^\dagger \xi} \langle z, \xi | e^{-\beta \hat{H}} | z, \xi \rangle = 1 \] (4.63)
regardless of the value of gauge parameter \( \alpha \). Here we have adopted the periodic boundary condition for taking a trace of the time evolution operator. It will be interesting to see that we can generalize the periodic boundary condition for a trace formula to a twisted boundary condition in the following way
\[ \text{Tr}_\theta(e^{-\beta H}) = \int \left( \frac{dz dz^*}{\pi} \right)^2 (d\xi d\xi^*)^2 e^{-z^\dagger z - \xi^\dagger \xi} e^{i\theta} \langle z, \xi | e^{-\beta \hat{H}} | e^{i\theta} z, e^{i\theta} \xi \rangle = 1 \] (4.64)
without breaking the cancellation of determinants from both bosonic and fermionic Gaussian integrals. Taking a trace over unphysical degrees will not cause any effect on the physical partition function. Hence we are free to make any choice for boundary conditions for unphysical degrees. But the autonomy of unphysical degrees will be strongly restricted by other reason, such as space-time symmetry for example. Therefore if we require Lorentz covariance for the quantized gauge field the unphysical degrees must obey the same boundary condition with physical ones. This returns to the unique choice of the boundary condition for FP ghosts because of the need for cancellation of Gaussian determinants. Hence the periodic boundary condition even for FP ghosts will be preferred in the calculation of a trace of a physical quantity.

Another point that should be remarked on our definition of the trace formula is that, through the definitions of left eigenvectors of creation operators, we have already included the metric structure of the vector space under consideration. We have defined the coherent states to yield the resolution of unity, taking the metric structures into account, given by Eq. (3.30) or Eq. (3.32) for FP ghosts and by
Eq. (4.49) for the gauge field. We may define an operator given by

$$\hat{\eta} = \hat{\eta}_G \hat{\eta}_{FP} = \hat{\eta}^\dagger = \exp \left[ \frac{i\pi}{2} \left\{ (\hat{B}^\dagger + i\hat{D}^\dagger)(\hat{B} - i\hat{D}) + (\hat{b}^\dagger + i\hat{d}^\dagger)(\hat{b} - i\hat{d}) \right\} \right]$$

(4.65)

to find

$$\langle z, \xi | \hat{\eta} \rangle = (| z, \xi \rangle^\dagger) = \langle z^*, \xi^* | \hat{\eta} \rangle$$

(4.66)

and also

$$\hat{\eta}^\dagger (\hat{B}, \hat{D}, b, d) \hat{\eta} = (i\hat{D}, -i\hat{B}, i\hat{d}, -ib).$$

(4.67)

Note here that $\hat{A}$ and $\hat{A}_0$ transform under the action of $\hat{\eta}$ as

$$\hat{\eta}^\dagger \hat{A} \hat{\eta} = \hat{A}, \quad \hat{\eta}^\dagger \hat{A}_0 \hat{\eta} = -\hat{A}_0.$$

(4.68)

By virtue of this relation, $\hat{A}_0$ becomes an hermitian-like operator on this indefinite metric vector space. Further, we see that multiplication of $\hat{\eta}_G$ is just the conventional one for dealing with the negative-norm property of $\hat{A}_0$ in covariant quantization for electromagnetic field in Feynman gauge.26)

Thus we understand that we need to multiply this operator to complete the hermitian conjugation on the vector space we are working with. We may call the operator defined by Eq. (4.65) as conjugating operator hereafter. Hermiticity of operators must also be defined to be consistent against this hermitian conjugation. It will be then clear that left eigenvectors of creation operators are indeed the hermitian conjugate to the right ones of annihilation operators.

§5. Generating functional and effective action

We will find the effective actions for FP ghosts and unphysical degrees of the gauge field by making use of coherent state path integral in this section. To this aim we first consider the generating functionals for these subsystems separately. Since the model we are working with is a toy model of the free gauge field, the effective action found in this analysis should have some analogue in the tree level calculation of usual formulation of the conventional path integral for the gauge field and hence expected to be trivial. However, it will be significant to confirm such fundamental aspects of the method under development to make it reliable.

To begin with, for a finite imaginary time $\beta = t_F - t_I$, we consider a Feynman kernel of FP ghosts under the influence of external Grassmann fields $\eta^\dagger(t)$ and $\eta(t)$ defined by

$$K_{(ex)}(\xi_F, t_F; \xi_I, t_I) = \langle \xi_F; t_F | T \exp \left[ - \int_{t_1}^{t_2} dt \ \left\{ \eta^\dagger(t) \left( \frac{\hat{b}(t)}{\hat{d}(t)} \right) + (\hat{b}^\dagger(t), \hat{d}^\dagger(t)) \eta(t) \right\} \right] | \xi_I; t_I \rangle$$

(5.1)

in which $\hat{b}(t), \hat{d}(t)$ and their conjugate designate Heisenberg operators for ghost fermions and coherent states are the left and right eigenvectors of the Heisenberg operators at $t = t_F$ and $t = t_I$. The lower and upper limits of the integration in the exponent defined by T-product should be assumed to satisfy $t_F > t_2 > t_1 > t_I$. 

Dividing $\beta$ into $n$ pieces and making use of the resolution of unity Eq. (5.13) repeatedly, we obtain a path integral

$$K_{(ex)}(\xi_F, t_F; \xi_I, t_I) = \lim_{n \to \infty} \int \prod_{i=1}^{n-1} (d\xi(i) d\xi^*(i))^2$$

$$\times \exp \left\{ - \sum_{j=1}^{n-1} \xi^\dagger(j) \xi(j) + (1 - \epsilon_k) \sum_{j=1}^{n} \xi^\dagger(j) \xi(j - 1) \right\}$$

$$\times \exp \left\{ - \epsilon \sum_{j=1}^{n} \eta^\dagger(j) \xi(j - 1) - \xi^\dagger(j) \sigma_2 \eta(j) \right\}, \quad (5.2)$$

where $\sigma_2$ is the Pauli matrix. Here and in the following we will drop the constant $\mp k$ in the Hamiltonians for FP ghosts and unphysical degrees of the gauge field because they cancel each other as was already stated. The Gaussian integration can be performed easily and we can take continuum limit to find

$$K_{(ex)}(\xi_F, t_F; \xi_I, t_I) = \exp \left\{ e^{-k(t_F-t_I)} \xi^\dagger \eta \right\}$$

$$\times \exp \left\{ \int_{t_1}^{t_2} dt \ e^{-k(t_F-t)} \xi^\dagger \sigma_2 \eta(t) - e^{-k(t-t_1)} \eta^\dagger(0) \xi(0) \right\}$$

$$\times \exp \left\{ - \int_{t_1}^{t_2} dt \ dt' \ \theta(t-t') e^{-k(t-t')} \eta^\dagger(0) \sigma_2 \eta(t') \right\}, \quad (5.3)$$

where $\theta(t)$ is the step function. By taking limits $t_F, -t_I \to \infty$ and then $-t_1, t_2 \to \infty$ in this order, we can find a generating functional

$$Z^{(FP)}[\eta, \eta^\dagger] = e^{-W^{(FP)}[\eta, \eta^\dagger]}$$

$$= \langle\{00\} | T \ exp \left\[ - \int_{-\infty}^{\infty} dt \ \left\{ \eta^\dagger(t) \left( \hat{b}(t) \hat{d}(t) + \hat{b}^\dagger(t) \hat{d}^\dagger(t) \right) \eta(t) \right\} \right] |\{00\} \rangle$$

$$= \exp \left\{ - \int_{-\infty}^{\infty} dt \ dt' \ \theta(t-t') e^{-k(t-t')} \eta^\dagger(0) \sigma_2 \eta(t') \right\}. \quad (5.4)$$

From the generating functional, we can read the propagator of FP ghosts

$$\langle\{00\} | T \left( \hat{b}(t) \hat{d}(t) \right) \left( \hat{b}^\dagger(t'), \hat{d}^\dagger(t') \right) |\{00\}\rangle = -\theta(t-t') e^{-k(t-t')} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (5.5)$$

If we put

$$\eta^\dagger(t) = \frac{1}{\sqrt{2k}} \left( \bar{\psi}(t), -\psi(t) \right), \quad \eta(t) = \frac{1}{\sqrt{2k}} \begin{pmatrix} -\bar{\psi}(t) \\ \psi(t) \end{pmatrix} \quad (5.6)$$

to render the external sources couple to $\hat{c}(t)$ and $\hat{c}^\dagger(t)$, we will find a generating functional of connected Green’s function given by

$$W^{(FP)}[\bar{\psi}, \psi] = -\frac{1}{2} \int_{-\infty}^{\infty} dt \ dt' \ \frac{1}{i} \Delta_F(t-t') \left\{ \bar{\psi}(t) \psi(t') - \psi(t) \bar{\psi}(t') \right\} \quad (5.7)$$
in which $\Delta_F(t)$ being the Feynman propagator

$$\Delta_F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp_0 e^{-ip_0t} \frac{1}{p^2}, \quad p^2 = p_0^2 + k^2. \quad (5.8)$$

We may write expectation values of $c(t)$ and $\bar{c}(t)$ under the external sources as

$$c(t) = \frac{\delta W^{(FP)}[\bar{\psi},\psi]}{\delta \bar{\psi}(t)}, \quad \bar{c}(t) = \frac{\delta W^{(FP)}[\bar{\psi},\psi]}{\delta \psi(t)}, \quad (5.9)$$

where functional derivatives are defined by right action, to find that a Euclidean effective action of FP ghosts in the tree level is given by

$$\Gamma^{(FP)}_E[\bar{c},c] = c \cdot \bar{\psi} + \bar{c} \cdot \psi - W^{(FP)} = i \int_{-\infty}^{\infty} dt \left\{ \dot{\bar{c}}(t) \dot{c}(t) + k^2 \bar{c}(t)c(t) \right\}. \quad (5.10)$$

Here and in the following we may often use a notation

$$f \cdot g \equiv \int_{-\infty}^{\infty} dt f(t)g(t). \quad (5.11)$$

The Euclidean effective action in Eq. (5.10) is translated into the corresponding one for a Minkowski(real) time by inverse Wick rotation, $t \rightarrow it$;

$$\Gamma^{(FP)}_E[\bar{c},c] \xrightarrow{t \rightarrow it} i\Gamma^{(FP)}[\bar{c},c] = \int_{-\infty}^{\infty} dt \left\{ \dot{\bar{c}}(t) \dot{c}(t) - k^2 \bar{c}(t)c(t) \right\} \quad (5.12)$$

which is nothing but the classical action(times $i$) for the FP ghosts. We thus see the validity of our method of constructing Euclidean path integral in terms of coherent states for ghost fermions.

In the same way we can calculate a generating functional for unphysical degrees of the gauge field by means of coherent state path integral. The process of calculation is quite familiar one hence we omit the detail but simply list some results below. The generating functional with Feynman’s boundary condition is defined and given by

$$Z^{(G)}[\bar{j},j^\dagger] = e^{-W^{(G)}[\bar{j},j^\dagger]}$$

$$= \langle [00] | \mathcal{T} \exp \left[ - \int_{-\infty}^{\infty} dt \left\{ \bar{j}^\dagger(t) \left( \frac{\partial}{\partial \bar{j}^\dagger(t)} \hat{\mathcal{B}}(t) \right) + (\hat{\mathcal{B}}^\dagger(t), \hat{\mathcal{D}}^\dagger(t)) \bar{j}(t) \right\} \right] | [00] \rangle$$

$$= \exp \left\{ - \int_{-\infty}^{\infty} dt dt' \theta(t-t')e^{-k(t-t')} \bar{j}^\dagger(t)\gamma(t-t') \sigma_2' \bar{j}'(t') \right\}. \quad (5.13)$$

where $\gamma(t)$ is the one defined in Eq. (4.60). From the generating functional we read the propagator

$$\langle [00] \mathcal{T} \left( \hat{\mathcal{B}}(t), \hat{\mathcal{D}}^\dagger(t') \right) | [00] \rangle = -\theta(t-t')e^{-k(t-t')} \begin{pmatrix} 0 & -i \\ i & (1-\alpha)k(t-t')/2 \end{pmatrix}. \quad (5.14)$$

The existence of a component that is linear in $t-t'$ in the propagator clearly exhibits the effect of dipole ghost in the unphysical degrees of the gauge field.
By putting
\[
j^j(t) = \frac{1}{\sqrt{2k}} \left( -\frac{1 + \alpha}{4k} (J(t) + ikJ_0(t)) + kJ_B(t), \quad \frac{i}{k} (J(t) - ikJ_0(t)) \right),
\]
\[
j(t) = \frac{1}{\sqrt{2k}} \left( -\frac{1 + \alpha}{4k} (J(t) - ikJ_0(t)) + kJ_B(t) \right),
\]
(5.15)
to change the source term to \( J \cdot \hat{A} + J_0 \cdot \hat{A}_0 + J_B \cdot \hat{B} \), we find a generating functional of connected Green’s functions for unphysical components of the gauge field given by
\[
W^{(G)}[J, J_0, J_B] = -\frac{1}{2} \int_{-\infty}^{\infty} dt \int_{-\infty}^{t} dt' \mathbf{J}^T(t)D_F(t - t')\mathbf{J}(t'), \quad \mathbf{J}^T(t) = (J(t), J_0(t), J_B(t)),
\]
in which the Feynman propagator \( D_F(t) \) for the gauge field is defined by
\[
D_F(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dp_0 e^{-ip_0t} \hat{D}_F(p_0),
\]
\[
\hat{D}_F(p_0) = \frac{1}{p^2} \begin{pmatrix}
\frac{1}{k^2} \left( 1 - (1 - \alpha)\frac{k^2}{p^2} \right) & (1 - \alpha)\frac{p_0}{p^2} & -1 \\
-\frac{(1 - \alpha)}{p^2} & -1 + (1 - \alpha)\frac{p_0^2}{p^2} & -p_0 \\
1 & -1 & 0
\end{pmatrix},
\]
(5.17)
where \( p^2 = p_0^2 + k^2 \). The effective action for unphysical degrees of the gauge field is then found to be
\[
\Gamma^{(G)}_E[A, A_0, B] = A \cdot J + A_0 \cdot J_0 + B \cdot J_B - W^{(G)}
\]
\[
= \frac{1}{2} \int_{-\infty}^{\infty} dt \left\{ k^2(i\dot{A}(t) - A_0(t))^2 - 2i\dot{B}(t)A_0(t) + 2k^2A(t)B(t) + \alpha B^2(t) \right\},
\]
(5.18)
where
\[
A(t) = \frac{\delta W^{(G)}}{\delta J(t)}, \quad A_0(t) = \frac{\delta W^{(G)}}{\delta J_0(t)}, \quad B(t) = \frac{\delta W^{(G)}}{\delta J_B(t)}.
\]
(5.19)
Again by inverse Wick rotation, we obtain the classical action for unphysical degrees of the gauge field
\[
\Gamma^{(G)}_E \xrightarrow{t \to -it} i\Gamma^{(G)}
\]
\[
= i \int_{-\infty}^{\infty} dt \frac{1}{2} \left\{ k^2(\dot{A}(t) - A_0(t))^2 - 2\dot{B}(t)A_0(t) + 2k^2A(t)B(t) + \alpha B^2(t) \right\}
\]
(5.20)
as an effective action of zeroth order calculation in the perturbative expansion. We thus confirmed that Euclidean path integral for generating functional of FP ghosts and unphysical components of the gauge field in terms of coherent states constructed in preceding sections can reproduce classical actions for these variables in the leading order of perturbation theory. Hence we may regard our method to be reliable.
§6. BRS quartet and Kugo-Ojima projection

In this section we will classify the state vectors appear in the vector space of the representation for the quantum system under consideration. Then we will find an explicit form of the Kugo-Ojima projection in term of the field variables. From the BRS invariance of the Lagrangian there follows a conserved Noether charge (BRS charge) $Q_B$ given by

$$Q_B = \hat{P}(t)\dot{c}(t) - i\hat{B}(t)\dot{\phi}(t) = -\dot{\hat{B}}(t)\dot{c}(t) + \hat{B}(t)\dot{c}(t). \quad (6.1)$$

This can be expressed in terms of the creation and annihilation operators as

$$Q_B = \hat{k}\hat{Q}_B, \quad \hat{Q}_B = i(\hat{B}\hat{b} - \hat{B}\hat{b}) \quad (6.2)$$

where we have employed the Schrödinger picture. Among basic vectors only the vacuum state

$$|0\rangle = |\{00\}\rangle \otimes |\{00\}\rangle \quad (6.3)$$

is classified as a BRS singlet because it has a positive norm and BRS invariant. Other state vectors, given by

$$|_{\{m_2m_1\}\{n_2n_1\}}\rangle = |\{m_2m_1\}\rangle \otimes |\{n_2n_1\}\rangle, \quad m_i = 0, 1, 2, \ldots, n_i = 0, 1, \quad (6.4)$$

have zero-norm excepting the diagonal ones, specified by

$$|_{\{mm\}\{nn\}}\rangle \quad (6.5)$$

if $m_1 + m_2 + n_1 + n_2 \geq 1$. Though norms of basic vectors diagonal both within bosonic and fermionic sectors themselves do not vanish, they are arranged to have zero-norm in a pair wise manner within a BRS quartet $^{21)}$ as will be shown below.

For a given pair of $m$ and $n (m, n = 0, 1, 2, \ldots)$ we can see the following cyclic sequence of finding BRS-quartet. First, take $|_{\{mn\}\{10\}}\rangle$ and make BRS transform by multiplying $\hat{Q}_B$ to obtain a BRS-exact state vector:

$$\hat{Q}_B|_{\{mn\}\{10\}}\rangle = \sqrt{m}|_{\{m - 1n\}\{11\}}\rangle + \sqrt{n + 1}|_{\{mn + 1\}\{00\}}\rangle. \quad (6.6)$$

It is then brought to its partner with respect to the inner product by a multiplication of $\hat{\eta}$, defined by Eq. (4.65),

$$\hat{\eta}\hat{Q}_B|_{\{mn\}\{10\}}\rangle = i^{m-n-1}(\sqrt{m}|_{\{nm - 1\}\{11\}}\rangle + \sqrt{n + 1}|_{\{n + 1m\}\{00\}}\rangle). \quad (6.7)$$

BRS transform again on this vector will produces another BRS-exact vector given by

$$\hat{Q}_B\hat{\eta}\hat{Q}_B|_{\{mn\}\{10\}}\rangle = i^{m-n-1}(m + n + 1)|_{\{mn\}\{10\}}\rangle \quad (6.8)$$

and finally conjugation again to find

$$\hat{\eta}\hat{Q}_B\hat{\eta}\hat{Q}_B|_{\{mn\}\{10\}}\rangle = (m + n + 1)|_{\{mn\}\{10\}}\rangle \quad (6.9)$$
and close the cycle. We thus obtain the following cyclic diagram

\[
\begin{align*}
|[mn]\{10\} & \xrightarrow{\hat{Q}_B} \sqrt{m}[m-1n]\{11\} + \sqrt{n+1}[mn+1]\{00\} \\
\hat{\eta} & \downarrow \quad \hat{\eta} \\
|[nm]\{01\} & \xleftarrow{\hat{Q}_B} -\sqrt{m}[nm-1]\{11\} + \sqrt{n+1}[n+1m]\{00\}
\end{align*}
\]  

(6.10)

for a BRS quartet. From Eq. (6.9), we are naturally lead to define another quantity, though it is not conserved in general, \(\hat{Q}_D\) by

\[
\hat{Q}_D = \hat{\eta}\hat{Q}_B\hat{\eta} = i(\hat{D}\hat{d}^\dagger - \hat{D}^\dagger\hat{d})
\]  

(6.11)

and to rewrite above BRS cyclic diagram as

\[
\begin{align*}
|[mn]\{10\} & \xleftarrow{\hat{Q}_D} \sqrt{m}[m-1n]\{11\} + \sqrt{n+1}[mn+1]\{00\} \\
\hat{\eta} & \downarrow \quad \hat{\eta} \\
|[nm]\{01\} & \xrightarrow{\hat{Q}_D} -\sqrt{m}[nm-1]\{11\} + \sqrt{n+1}[n+1m]\{00\}
\end{align*}
\]  

(6.12)

to be viewed as a cycle of \emph{BRS-inversion} in a quartet. Here, the meaning of BRS-inversion will be clear from above diagrams. It will be interesting to see that BRS variant members of a BRS quartet are exact under the BRS-inversion hence zero-normed.

Since a member of BRS quartet has its partner with respect to the inner product only within the same quartet, in which only two members are physical states satisfying

\[
\hat{Q}_B|\text{Phys}\rangle = 0,
\]  

(6.13)

the BRS quartet spans a four dimensional subspace of the total vector space. A projection to this subspace may be expressed as

\[
P_{mn} = |B_{mn}^{(0)}\rangle\langle A_{mn}^{(0)}| + |B_{mn}^{(+)}\rangle\langle A_{mn}^{(-)}| + |A_{mn}^{(0)}\rangle\langle B_{mn}^{(0)}| + |A_{mn}^{(-)}\rangle\langle B_{mn}^{(+)}|,
\]  

(6.14)

where

\[
\begin{align*}
|A_{mn}^{(-)}\rangle & = |[mn]\{10\}\rangle, \\
|A_{mn}^{(0)}\rangle & = \frac{1}{\sqrt{m+n+1}}\hat{\eta}\hat{Q}_B|[mn]\{10\}\rangle, \\
|B_{mn}^{(0)}\rangle & = \frac{1}{\sqrt{m+n+1}}\hat{Q}_B|[mn]\{10\}\rangle, \\
|B_{mn}^{(+)}\rangle & = \frac{1}{m+n+1}\hat{Q}_B\hat{\eta}\hat{Q}_B|[mn]\{10\}\rangle.
\end{align*}
\]  

(6.15)

The total vector space under consideration is then decomposed into a direct sum of these subspaces in addition to the one dimensional really physical subspace spanned by the vacuum state. Hence we recognize that a sum of all \(P_{mn}\) given above is expressed as

\[
\sum_{m,n=0}^{\infty} P_{mn} = 1 - |0\rangle\langle 0|.
\]  

(6.16)
If we partially sum $P_{mn}$ in the above entire summation by putting $m + n + 1 = l$ with a positive integer $l$, we obtain a projection to a $4l$ dimensional subspace given by

$$P^{(l)} = \sum_{m+n+1=l} P_{mn}. \quad (6.17)$$

We thus obtain

$$\sum_{l=0}^{\infty} P^{(l)} = 1, \quad P^{(0)} \equiv |0\rangle\langle 0|. \quad (6.18)$$

Let us now give an explicit form of the projection $P^{(l)}$, which is nothing but the Kugo-Ojima projection \(^{21}\) in terms of the creation and annihilation operators introduced before. The Kugo-Ojima projection $P^{(l)}$ for the system in consideration is given by

$$P^{(l)} = \frac{1}{2\pi} \int_0^{2\pi} d\theta \exp \left\{ i\theta (\hat{N} - l) \right\}, \quad \hat{N} = i \left\{ \hat{B}^{\dagger} \hat{D} - \hat{D}^{\dagger} \hat{B} + (\hat{b}^{\dagger} \hat{d} - \hat{d}^{\dagger} \hat{b}) \right\}. \quad (6.19)$$

The proof will be quite simple because $\hat{N}$ is the operator that counts the total number of excitations in a state vector $|m_2 m_1 \{ n_2 n_1 \}\rangle$ so that

$$\hat{N}|m_2 m_1 \{ n_2 n_1 \}\rangle = (m_1 + m_2 + n_1 + n_2)|m_2 m_1 \{ n_2 n_1 \}\rangle \quad (6.20)$$

while $m + n + 1$ being the common eigenvalue of $\hat{N}$ on all members of a BRS quartet of which $|mn\{10\}\rangle$ being a member. It will be worth noting that the operator $\hat{N}$ introduced above is essentially the Hamiltonian for the total system given by Eq. \(^{4.37}\). In particular, when we employ the Feynman gauge by choosing $\alpha = 1$, the gauge parameter dependent part disappears from the Hamiltonian to result in $\hat{H} = k\hat{N}$. It holds, however, that two parts of the Hamiltonian Eq. \(^{4.37}\) are expressed as a BRS transform separately for any gauge parameter $\alpha$. Actually and indeed surprisingly, the operator $\hat{N}$ can be expressed by the anticommutator

$$\hat{N} = \{ \hat{Q}_B, \hat{Q}_D \}. \quad (6.21)$$

Hence $\hat{N}$ itself is BRS exact and BRS-inversion exact simultaneously. It is also true for the Hamiltonian for the Feynman gauge. The fact that $\hat{N}$ can be expressed in a BRS exact form explains the reason why Kugo-Ojima projection $P^{(l)}$ can be written as an anticommutator with $R^{(l)}$ by explicit construction (See Eq.\(^{3.29}\) of Ref.\(^{21}\)).

An immediate application of the formula Eq. \(^{6.19}\) for Kugo-Ojima projection will be found in the calculation of projection inserted Feynman kernel defined by

$$K^{(l)}(z_F, \xi_F; z_I, \xi_I; \beta) \equiv \langle z_F, \xi_F | e^{-\beta \hat{H}} P^{(l)} | z_I, \xi_I \rangle. \quad (6.22)$$

By use of the formula Eq. \(^{6.19}\), the calculation of this kernel reduces to

$$K^{(l)}(z_F, \xi_F; z_I, \xi_I; \beta) = \frac{1}{2\pi} \int_0^{2\pi} d\theta \exp \left[ -i\beta k + i\theta \left\{ \hat{z}^+_F \gamma(\beta) \hat{z}_I + \hat{z}^+_F \hat{\xi}_F \hat{\xi}_I \right\} \right]. \quad (6.23)$$
because the existence of $e^{i\theta N}$ causes changes to the path integral only in the shift of $\beta k$ to $\beta k + i\theta$ excepting the $\alpha$ dependent part. If we apply the same technique to the calculation of trace formula Eq. (6.63), we will obtain

$$Z^{(l)}(\beta) = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{-i\theta} \int \left( \frac{dz \, d\xi^*}{\pi} \right)^2 (d\xi \, d\xi^*)^2 e^{-z \dagger z - \xi \dagger \xi} \times \exp \left[ e^{-\beta k + i\theta} \left\{ z \dagger \gamma(\beta) z + \xi \dagger \xi \right\} \right]$$

$$= \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{-i\theta}$$

$$= \delta_{l0}$$

(6.24)

to observe that only the vacuum state can contribute to the partition function. On the other hand, we may perform the integration with respect to $\theta$ ahead, that is equivalent to insert the projection $P^{(l)}$ in the form given by Eq. (6.17), to find

$$Z^{(l)}(\beta) = e^{-i\beta k} \int \left( \frac{dz \, d\xi^*}{\pi} \right)^2 (d\xi \, d\xi^*)^2 e^{-z \dagger z - \xi \dagger \xi} \left\{ z \dagger \gamma(\beta) z + \xi \dagger \xi \right\}$$

(6.25)

For this case $Z^{(l)} = 0 (l \geq 1)$ must be checked term by term. This clearly exhibits the usefulness of the formula Eq. (6.19): The set of infinitely many identities in Eq. (6.24) as a consequence of the BRS invariance must be checked directly for each $l = 0, 1, 2, \ldots$ if we do not have the expression of $P^{(l)}$ in Eq. (6.19). Therefore it should be stressed again that the knowledge of the explicit form of Kugo-Ojima projection, that is having the formula given by Eq. (6.19) in hand, is quite significant.

§7. Application to the free gauge field

Having completed the thorough study of the toy model, let us consider the application of our technique developed in preceding sections to quantum theory of the abelian gauge field. We first rewrite the Lagrangian for a free gauge field

$$\mathcal{L}_0(x) = -\frac{1}{4} F_{\mu \nu}(x) F^{\mu \nu}(x), \quad F_{\mu \nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x)$$

(7.1)

by parameterizing spatial components as

$$A(x) = -\nabla A(x) + A_T(x), \quad \nabla \cdot A_T(x) = 0$$

(7.2)

to find

$$\mathcal{L}_0(x) = \frac{1}{2} \left\{ \nabla(\dot{A}(x) - A_0(x)) \right\}^2 + \frac{1}{2} \dot{A}_T^2(x) - \frac{1}{2} (\nabla \times A_T(x))^2.$$ 

(7.3)

Clearly the second term together with the third one in this Lagrangian describes two physical degrees of the abelian gauge field in the Coulomb gauge and its properties including quantum theory on a Fock space with positive definite metric are quite well-known. Therefore our target in this section is the first term. Let us write it as

$$\mathcal{L}_0'(x) = \frac{1}{2} \left\{ \nabla(\dot{A}(x) - A_0(x)) \right\}^2$$

(7.4)
and make an investigation on the quantization of this Lagrangian with a covariant gauge condition
\[ \dot{A}_0(x) - \nabla^2 A(x) + \alpha B(x) = 0 \] (7.5)
in which Nakanishi-Lautrup\textsuperscript{24} field \( B(x) \) has been introduced. Although we are dealing with this covariant type gauge condition, the space-time covariance has already been lost by the decomposition of the original Lagrangian into two non-covariant parts. The covariance will, however, be restored if we recombine them together after quantization by explicit calculation of an effective action for entire system. Hence we discard the absence of space-time covariance in the subsystem we are working with for a while.

We introduce gauge fixing and FP ghosts according to the BRS formalism by adding
\[ L_{\text{GF+FP}}(x) = i\delta_B \left\{ \dot{c}(x)(\dot{A}_0(x) - \nabla^2 A(x) + \alpha B^2(x)/2) \right\} \] (7.6)
to \( L'(x) \) to find
\[ L(x) = L_G(x) + L_{\text{FP}}(x), \]
\[ L_G(x) = L_0'(x) - B(x)A_0(x) + \nabla B(x) \cdot \nabla A(x) + \frac{\alpha}{2} B^2(x), \] (7.7)
\[ L_{\text{FP}}(x) = -i\partial_\mu \bar{c}(x) \partial^\mu c(x). \]

From variation of this Lagrangian we can immediately obtain equations of motion for field variables. As is naturally expected, unphysical degrees of the gauge field \( A(x) \) and \( A_0(x) \) satisfy
\[ \Box A(x) = (1 - \alpha) B(x), \] (7.8)
\[ \Box A_0(x) = (1 - \alpha) \dot{B}(x), \] (7.9)
while \( B(x), c(x) \) and \( \dot{c}(x) \) are obeying massless free field equations
\[ \Box B(x) = \Box c(x) = \Box \dot{c}(x) = 0. \] (7.10)

We thus quantize FP ghost and anti-ghost as free fields in a quite similar manner as has been done for \( \dot{c}(t) \) and \( \dot{c}(t) \) in the subsection \([22]\) for each Fourier components in
\[ \dot{c}(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2|k|}} \left\{ \hat{b}(k)e^{-ikx} + \hat{b}^\dagger(k)e^{-ikx} \right\}, \]
\[ \dot{c}(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2|k|}} \left\{ \hat{d}(k)e^{-ikx} + \hat{d}^\dagger(k)e^{-ikx} \right\}, \] (7.11)
with the following anticommutation relations
\[ \{ \hat{b}(p), \hat{d}^\dagger(q) \} = i\delta^3(p - q), \quad \{ \hat{b}^\dagger(p), \hat{d}(q) \} = -i\delta^3(p - q), \quad \text{others} = 0, \] (7.12)
together with the Hamiltonian
\[ \hat{H}_{\text{FP}} = \int d^3k \, |k| \left\{ i(\hat{b}^\dagger(k)\dot{d}(k) - \hat{d}^\dagger(k)\dot{b}(k)) - 1 \right\}. \] (7.13)
According to the first-ordered nature of the kinetic part for $A_0(x)$ and $B(x)$, we meet two second class constraints in obtaining the Hamiltonian. We treat them in the similar way as we have done for the toy model to find

$$
\mathcal{H}_G(x) = -\frac{1}{2} \dot{B}(x) \left( \frac{1}{\nabla^2} \dot{B}(x) \right) - \dot{B}(x) \dot{A}_0(x) - \nabla \dot{B}(x) \cdot \nabla \dot{A}(x) - \frac{\alpha}{2} \dot{B}^2(x) \quad (7.14)
$$

from $\mathcal{L}_G$. Here a comment will be in order. In Eq. (7.14), there appears the inverse of the Laplacian that is usually defined by

$$
\frac{1}{\nabla^2}(x, y) = \frac{1}{4\pi} \frac{1}{|x - y|} \quad (7.15)
$$

hence causes non-locality and behaves singular at $x = y$ but we must remember that our initial Lagrangian $\mathcal{L}_0'$ does not includes the zero-modes of $A(x)$ and $A_0(x)$ from very definition. Hence we should understand it by a regularized one, with an infrared cut off parameter $\epsilon$, given explicitly in Fourier expansion by

$$
-\frac{1}{\nabla^2}(x) = \lim_{\epsilon \to +0} -\frac{1}{\nabla^2_\epsilon}(x) = \int_{(2\pi)^3} \frac{d^3k}{k^2} e^{ik \cdot x}, \quad (7.16)
$$

in which $\epsilon$ as the lower limit of integration designates the condition $|k| \geq \epsilon$. We may also need this cut off prescription when defining creation and annihilation operators for fields under consideration in the following. As was already stated in section 2, we also need the same regularization for physical variables because zero-modes of massless fields cannot be quantized in a Fock space. Hence above prescription for infrared problem applies to both physical and unphysical degrees of the gauge field.

Keeping these in mind, we consider Hamiltonian

$$
\hat{H}_G = \int d^3x \mathcal{H}_G(x) \quad (7.17)
$$

with the following equal-time commutators

$$
\left[ \hat{A}(x_0, x), -\dot{B}(x_0, y) \right] = \left[ \hat{A}_0(x_0, x), \dot{B}(x_0, y) \right] = i\delta^3(x - y), \text{ others } = 0. \quad (7.18)
$$

Heisenberg equations obtained from the Hamiltonian $\hat{H}_G$ is identical to the Euler-Lagrange equation and reduces, by Fourier transform, to the one we have studied in subsection 4.1. Hence we put

$$
\dot{A}(x; \epsilon) = \int \frac{d^3p}{\sqrt{(2\pi)^32|p|}} \left\{ \hat{a}_p(x_0; \epsilon)e^{ip \cdot x} + \hat{a}^\dagger_p(x_0; \epsilon)e^{-ip \cdot x} \right\},
$$

$$
\dot{A}_0(x; \epsilon) = \int \frac{d^3p}{\sqrt{(2\pi)^32|p|}} \left\{ \hat{a}_{0p}(x_0; \epsilon)e^{ip \cdot x} + \hat{a}_{0p}^\dagger(x_0; \epsilon)e^{-ip \cdot x} \right\}, \quad (7.19)
$$

$$
\dot{B}(x; \epsilon) = \int \frac{d^3p}{\sqrt{(2\pi)^32|p|}} \left\{ \hat{b}_p(x_0; \epsilon)e^{ip \cdot x} + \hat{b}_p^\dagger(x_0; \epsilon)e^{-ip \cdot x} \right\},
$$

$$
\dot{B}(x; \epsilon) = \int \frac{d^3p}{\sqrt{(2\pi)^32|p|}} \left\{ \hat{b}_{0p}(x_0; \epsilon)e^{ip \cdot x} + \hat{b}_{0p}^\dagger(x_0; \epsilon)e^{-ip \cdot x} \right\}.
$$
to find the following equal-time commutation relations of creation and annihilation operators

\[
[A_p(x; \epsilon), \hat{a}_q^\dagger(x; \epsilon)] = \frac{1 + \alpha}{2|p|^2} \delta_\epsilon^\Delta(p - q),
\]

\[
[A_0p(x; \epsilon), \hat{a}_0q^\dagger(x; \epsilon)] = -\frac{1 + \alpha}{2} \delta_\epsilon^\Delta(p - q),
\]

\[
[A_p(x; \epsilon), \hat{b}_q^\dagger(x; \epsilon)] = -\delta_\epsilon^\Delta(p - q),
\]

\[
[A_0p(x; \epsilon), \hat{b}_0q^\dagger(x; \epsilon)] = i|p|\delta_\epsilon^\Delta(p - q),
\]

\[
[A_p(x; \epsilon), \hat{a}_q^\dagger(x; \epsilon)] = 0,
\]

where \(\delta_\epsilon^\Delta(p - q)\) is a delta function accompanied with a step function to achieve the above-mentioned regularization:\(^{(25)}\)

\[
\delta_\epsilon^\Delta(p - q) \equiv \theta(|p| - \epsilon)\delta_\epsilon(p - q).
\]

We then define following operators and their hermitian conjugates

\[
\hat{B}_\epsilon(x_0, p) = \frac{1}{|p|}\hat{b}_p(x_0; \epsilon), \quad \hat{D}_\epsilon(x_0, p) = \frac{1}{2}\{i|p|\hat{a}_p(x_0; \epsilon) - \hat{a}_0p(x_0; \epsilon)\}
\]

\[
\hat{D}_\epsilon(x_0, p) = \frac{1}{2}\{i|p|\hat{a}_p(x_0; \epsilon) + \hat{a}_0p(x_0; \epsilon)\} = -\frac{i}{4}(1 + \alpha)\hat{B}_\epsilon(x_0, p)
\]

together with

\[
\hat{D}_\epsilon(x_0, p) = \frac{1}{2}\{i|p|\hat{a}_p(x_0; \epsilon) + \hat{a}_0p(x_0; \epsilon)\} = -\frac{i}{4}(1 + \alpha)\hat{B}_\epsilon(x_0, p)
\]

as the analogue of Eq. \((7.23)\) and Eq. \((7.24)\).

Inversion of Eq. \((7.22)\) for finite \(\epsilon\) with the aid of Eq. \((7.23)\) bring us

\[
\hat{a}_p(x_0; \epsilon) = -\frac{1}{|p|}\left(\frac{1 + \alpha}{4}\hat{B}_\epsilon(x_0, p) + i\hat{D}_\epsilon(x_0, p)\right),
\]

\[
\hat{a}_0p(x_0; \epsilon) = -i\left(\frac{1 + \alpha}{4}\hat{B}_\epsilon(x_0, p) - i\hat{D}_\epsilon(x_0, p)\right),
\]

\[
\hat{b}_p(x_0; \epsilon) = |p|\hat{B}_\epsilon(x_0, p).
\]

Then we are able to rewrite \(\hat{A}(x; \epsilon), \hat{A}_0(x; \epsilon)\) and \(\hat{B}(x; \epsilon)\) in terms of these creation and annihilation operators. We thus obtain, by putting \(\epsilon \to +0\) after all calculation, that the Hamiltonian \(\hat{H}_G\) can be expressed as

\[
\hat{H}_G = \int d^3p \left\{ i\left(\hat{B}_\epsilon(x_0, p)\hat{D}_\epsilon(x_0, p) - \hat{D}_\epsilon(x_0, p)\hat{B}_\epsilon(x_0, p)\right) + 1 + \frac{1 - \alpha}{2}\hat{B}_\epsilon(x_0, p)\hat{B}_\epsilon(x_0, p)\right\}.
\]

\[
\hat{H} = \int d^3p \left\{ i\left(\hat{B}_\epsilon(p)\hat{D}(p) - \hat{D}_\epsilon(p)\hat{B}(p)\right) + i(\hat{b}_\epsilon(p)\hat{d}(p) - \hat{d}_\epsilon(p)\hat{b}(p)) + \frac{1 - \alpha}{2}\hat{B}_\epsilon(p)\hat{B}(p)\right\}
\]
in which we have made use of operators in Schrödinger picture. The observation here is that we can regard the system of unphysical degrees of the gauge field with FP ghosts under a covariant gauge condition as a collection of infinitely many copies of the system we studied in the preceding sections through the toy model. Hence we already know the structure, including its metric, of the Fock space equipped with this system and how to define coherent states for constructing a path integral in terms of them. We may, therefore, list here main results obtained in this study:

1. The Fock space is spanned by state vectors given by
\[
\prod_{k \neq 0} |[m_1 m_2 | n_1 n_2 \rangle; k\rangle, \quad m_i = 0, 1, 2, \ldots, n_i = 0, 1 \text{ for each } k. \tag{7.27}
\]

2. The conjugation operator defined in Eq. (4.65) is generalized to
\[
\hat{\eta} = \prod_{k \neq 0} \exp \left[ \frac{i\pi}{2} \left\{ (\hat{B}^+ (k) + i\hat{D}^+ (k))(\hat{B} (k) - i\hat{D} (k)) \right\} \right]
\times \exp \left[ i \frac{\pi}{2} (\hat{b}^+ (k) + i\hat{d}^+ (k))(\hat{b} (k) - i\hat{d} (k)) \right], \tag{7.28}
\]
which brings above basic vectors to their conjugates with respect to the inner product.

3. A coherent state, that is a simultaneous eigenvector of annihilation operators \( \hat{B}(k), \hat{D}(k), \hat{b}(k) \) and \( \hat{d}(k) \) for all \( k \), can be defined by
\[
|\{z, \xi\}\rangle \equiv \prod_{k \neq 0} \exp \left[ i \left\{ (\hat{B}^+ (k)z_1 (k) + \hat{D}^+ (k)z_2 (k)) \right\} \right]
\times \exp \left[ i \left\{ (\hat{b}^+ (k)\xi_1 (k) + \hat{d}^+ (k)\xi_2 (k)) \right\} \right]|0\rangle. \tag{7.29}
\]

4. Left eigenvectors of creation operators are obtained by usual hermitian conjugation followed by a multiplication of the conjugation operator:
\[
\langle \{z, \xi\} | = (|\{z, \xi\}\rangle)\dagger \hat{\eta} \tag{7.30}
\]
to yield an inner product
\[
\langle \{z, \xi\} | \{z', \xi'\} \rangle = \exp \left[ \int d^3k \left\{ z^\dagger (k)z' (k) + \xi^\dagger (k)\xi' (k) \right\} \right]. \tag{7.31}
\]

5. The set of coherent states gives a resolution of unity
\[
\int \prod_{k \neq 0} \left[ \left( \frac{dz(k) dz^* (k)}{\pi} \right)^2 \left( d\xi (k) d\xi^* (k) \right)^2 \right]
\times \exp \left[ - \int d^3k \left\{ z^\dagger (k)z (k) + \xi^\dagger (k)\xi (k) \right\} \right] |\{z, \xi\}\rangle \langle \{z, \xi\}| = 1. \tag{7.32}
\]

6. Only the vacuum specified by
\[
|0\rangle \equiv \prod_{k \neq 0} |00\rangle; k\rangle \tag{7.33}
\]
can be a positive normed physical state, i.e. BRS singlet.
7. The normalized BRS charge is given by
\[ \hat{Q}_B = \int d^3k i \left\{ \hat{B}(k)\hat{b}^\dagger(k) - \hat{B}^\dagger(k)\hat{b}(k) \right\} \]  
(7.34)
and accompanied with another operator, that is conserved only in the Feynman gauge, \( \hat{Q}_D = \hat{\eta}\hat{Q}_B\hat{\eta} \) that generates the BRS-inversion and is given by
\[ \hat{Q}_D = \int d^3k i \left\{ \hat{D}(k)\hat{d}^\dagger(k) - \hat{D}^\dagger(k)\hat{d}(k) \right\}. \]  
(7.35)

8. BRS variant member of a BRS quartet is a partner state of BRS exact member and is BRS-inversion exact. In other word, BRS daughter state is BRS-inversion parent state.

9. An operator that counts total number of excitations in all BRS quartet is found to be
\[ \hat{N} = \int d^3p \left\{ i \left( \hat{B}^\dagger(p)\hat{D}(p) - \hat{D}^\dagger(p)\hat{B}(p) \right) + i(\hat{b}^\dagger(p)\hat{d}(p) - \hat{d}^\dagger(p)\hat{b}(p)) \right\}. \]  
(7.36)

Another expression for this operator is given by
\[ \hat{N} = \{ \hat{Q}_B, \hat{Q}_D \}. \]  
(7.37)

10. Kugo-Ojima projection is constructed explicitly as
\[ P^{(n)} = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{i\theta(\hat{N} - n)} , \quad n = 0, 1, 2, \ldots . \]  
(7.38)

It can be expressed in terms of coherent states as
\[
P^{(n)} = \frac{1}{2\pi} \int_0^{2\pi} d\theta e^{-in\theta} \int \prod_{k\neq 0} \left[ \frac{(dz(k)dz^*(k))}{\pi} \right]^2 (d\xi(k)d\xi^*(k))^2 \times \exp \left[ -\int d^3k \left\{ z^\dagger(k)z(k) + \xi^\dagger(k)\xi(k) \right\} \right] |\{e^{i\theta}z, e^{i\theta}\xi}\rangle\langle\{z, \xi}|. \]  
(7.39)

Besides these fundamental properties, we should confirm the usefulness and reliability of our method of constructing a path integral by means of coherent states. As will be expected from the facts we have seen in this section, generating functionals, Eq. (5.7) for FP ghost and Eq. (5.16) for unphysical degrees of the gauge field, can be immediately generalized to the current case: generating functionals for the system under consideration is defined by inserting sources \( \bar{\eta}(x) \), \( \eta(x) \), \( J(x) \), \( J_0(x) \) and \( J_B(x) \) against \( \hat{c}(x) \), \( \hat{\bar{c}}(x) \), \( \hat{A}(x) \), \( \hat{\bar{A}}_0(x) \) and \( \hat{B}(x) \) in the same manner as has been done in section 5 to be evaluated as
\[ W^{(FP)}[\eta, \eta^\dagger] = -\frac{1}{2} \int d^4x d^4x' \frac{1}{i} \Delta_F(x - x') \left\{ \bar{\eta}(x)\eta(x') - \eta(x)\bar{\eta}(x') \right\} , \]  
(7.40)
\[ W^{(G)}[J, J_0, J_B] = -\frac{1}{2} \int d^4x d^4x' J_T(x)D^{(G)}_F(x - x')J(x') , \]  
(7.41)
\[ J_T(x) = (J(x), J_0(x), J_B(x)), \]
in which propagators, $-i\Delta_F(x)$ for FP ghosts and $D_F^{(G)}(x)$ for the gauge field, are defined by

$$-i\Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} -i \frac{1}{p^2},$$

$$D_F^{(G)}(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \tilde{D}_F^{(G)}(p),$$

$$\tilde{D}_F^{(G)}(p) = \frac{1}{p^2} \begin{pmatrix}
1 & (1 - \alpha)\frac{p^2}{p^2} & -1 \\
-(1 - \alpha)\frac{p_0}{p^2} & -1 & (1 - \alpha)\frac{p_0^2}{p^2} - p_0 \\
-1 & (1 - \alpha)\frac{p_0}{p^2} & p_0
\end{pmatrix},$$

(7.42)

where $p^2 = p_0^2 + p^2$. The generating functional for FP ghosts given by Eq. (7.40) will be immediately brought to its corresponding effective action

$$\Gamma_{E}^{(FP)} = \int d^4x i\partial_\mu \bar{c}(x)\partial^\mu c(x) \stackrel{x_0 \to ix_0}{\longrightarrow} i\Gamma^{(FP)} = i \int d^4x \{-i\partial_\mu \bar{c}(x)\partial^\mu c(x)\}$$

(7.43)

to yield the classical action of FP ghosts in the Minkowski space-time. We must next combine generating functional for unphysical part of the gauge field with the one from physical degrees. To this aim we rearrange external sources for $\hat{A}^\mu(x)$ to be $j_\mu(x)$. Since $J_0(x)\hat{A}^0(x)$ is already included, we just set $j_0(x) = J_0(x)$ and concentrate on the spatial components $-j(x) \cdot \hat{A}(x)$. From our definition, this term is rewritten as

$$-j(x) \cdot \hat{A}(x) = -(\nabla \cdot j(x))\hat{A}(x) - j(x) \cdot \hat{A}_T(x).$$

(7.44)

Hence we put $J(x) = -\nabla \cdot j(x)$ in Eq. (7.41) and combine it with the contribution from physical degrees, given by

$$W^{(T)}[j] = -\frac{1}{2} \int d^4x d^4x' J^T(x)D_F^{(T)}(x - x')j(x'),$$

(7.45)

where $D_F^{(T)}(x - x')$ is defined by

$$D_F^{(T)}(x - x') = \int \frac{d^4p}{(2\pi)^4} e^{-ipx} \frac{1}{p^2} \left(1 - \frac{pp^T}{p^2}\right).$$

(7.46)

Thus total generating functional for the gauge field is obtained as

$$W[j_\mu, J_B] = W^{(G)}[j_\mu, J_B] + W^{(T)}[j]$$

$$= -\frac{1}{2} \int d^4x d^4x' J^T(x)D_F^{(T)}(x - x')J(x'),$$

(7.47)
in which \( \mathbf{J}^T(x) = (j^T(x), j_0(x), J_B(x)) \) and the propagator is given by

\[
\tilde{D}_F^{(\alpha)}(p) = \frac{1}{i} \left( \begin{array}{ccc}
1 - (1 - \alpha) \frac{pp^T}{p^2} & -i(1 - \alpha) \frac{p_0 p}{p^2} & ip \\
-i(1 - \alpha) \frac{p_0 p^T}{p^2} & -1 + (1 - \alpha) \frac{p_0^2}{p^2} & -p_0 \\
-ip^T & p_0 & 0
\end{array} \right).
\]

If we perform the inverse Wick rotation, \( x_0 \rightarrow ix_0, x_0' \rightarrow ix_0' \) in Eq. (7.47) in addition to \( p_0 \rightarrow -ip_0 \) in the propagator in Eq. (7.48), we can immediately find that the Euclidean generating functional given by Eq. (7.47) yields the corresponding one for the abelian gauge field with a covariant gauge condition in Minkowski space-time. The Euclidean effective action for the gauge field is also found from Eq. (7.47) to be

\[
\Gamma_E = \int d^4x \left\{ \frac{1}{2} \left\{ (i\dot{\mathbf{A}}(x) + \nabla A_0(x))^2 - (\nabla \times \mathbf{A}(x)) \right\}
\right.
\left. -2i\dot{B}(x)A_0 + 2B(x)\nabla \cdot \mathbf{A}(x) + \alpha B^2(x) \right\}
\]

which can be translated into Minkowski effective action by inverse Wick rotation:

\[
\Gamma_E \xrightarrow{x_0 \rightarrow ix_0} i\Gamma = \int d^4x \left\{ (\dot{\mathbf{A}}(x) + \nabla A_0(x))^2 - (\nabla \times \mathbf{A}(x)) \right\}
\left. -2i\dot{B}(x)A_0 + 2B(x)\nabla \cdot \mathbf{A}(x) + \alpha B^2(x) \right\}.
\]

Hence restoration of Lorentz covariance is evident. We thus confirmed that our prescription for constructing a path integral of a free gauge field from manifestly covariant operator formalism in terms of coherent states works quite fine to provide essential ingredients for perturbative expansion in its zeroth order in entirely covariant manner.

§8. Field diagonal representation

Our considerations has been so far restricted to the construction and its use of the coherent state for unphysical degrees of a gauge field in a unified manner with those for FP ghosts. As for ghost fermions, there is a path integral in terms of field eigenvectors. It will be, therefore, beautiful if we can formulate a path integral in terms of eigenvectors of field operators and their canonical conjugates in entirely covariant way. To discuss such problem as a whole is, however, beyond the scope of this paper. We here consider a field diagonal representation only for the Feynman gauge to fill the discrepancy of the results in Ref. 10 and Ref. 11.

In the covariant path integral for the gauge field of Ref. 10, \( A_0(A_4) \) is treated as an auxiliary field that disappears once from the formulation but comes back into the path integral by use of the Gaussian identity. The technique utilized there is the one for functional analysis on a Hilbert space with positive definite metric. On
the other hand, the basic ingredient of the construction in Ref. 11) is the use of eigenvector of $\hat{A}_0$ defined on a representation space with indefinite metric. So let us reconsider the prescription of Ref. 11) from the viewpoint of our standpoint in this paper.

Returning back to the toy model again and putting here $\alpha = 1$ to restrict ourselves to the Feynman gauge, let us consider an operator

$$C \int d\mu d\nu e^{i\mu(\hat{A} - q) + \nu(\hat{A}_0 - q_0)}$$

(8.1)

in which ranges of integrations with respect to $\mu$ and $\nu$ together with the normalization factor $C$ should be determined so that integrations with respect to $q$ and $q_0$ of above introduced operator yield an expression of the resolution of unity. Note that the construction of such an expression in terms of Schrödinger operators will be immediately translated to that of Heisenberg operators. Making use of relations $\hat{A}$, $\hat{A}_0$ and $\hat{B}$ with the creation and annihilation operators $\hat{B}$, $\hat{D}$, $\hat{B}^\dagger$ and $\hat{D}^\dagger$, we can easily obtain

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (2\pi)^2 \langle z| e^{i\mu(\hat{A} - q) + \nu(\hat{A}_0 - iq_0)} | z' \rangle = k^2 \pi \exp \left[ -k^2 \left\{ q - \frac{i}{\sqrt{2k}} \left( \frac{1}{2} (z_2^* + iz_1') - i(z_1^* - iz_2') \right) \right\}^2 \right]$$

(8.2)

for an arbitrary pair of coherent states. From this calculation, we read

$$\int_{-\infty}^{\infty} dq dq_0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (2\pi)^2 \langle z| e^{i\mu(\hat{A} - q) + \nu(\hat{A}_0 - iq_0)} | z' \rangle = \langle z| z' \rangle$$

(8.3)

which implies

$$\int_{-\infty}^{\infty} dq dq_0 \langle q, q_0 | q, q_0 \rangle = 1,$$

(8.4)

$$|q, q_0 \rangle \langle q, q_0 | = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} (2\pi)^2 e^{i\mu(\hat{A} - q) + \nu(\hat{A}_0 - iq_0)}.$$

(8.5)

To find the explicit form of the eigenvector $|q, q_0 \rangle$ and its conjugate, we calculate the projection $|q, q_0 \rangle \langle q, q_0 |$ defined above by

$$\int \left( \frac{dz dz^*}{\pi} \right)^2 \left( \frac{dz' dz'^*}{\pi} \right)^2 e^{-z^*z - z'z'} |z \rangle \langle z| q, q_0 \rangle |q, q_0 \rangle \langle q, q_0 | z' \rangle \langle z'|.$$

(8.6)
It is straightforward to obtain
\[
|q, q_0\rangle = \frac{k}{\sqrt{\pi}} \exp \left\{ -\frac{k}{2}(k^2 q^2 + q_0^2) \right\} \times \exp \left[ -\frac{k}{2}(kq - q_0)B^\dagger + i\sqrt{2k}(kq + q_0)D^\dagger - \frac{1}{4}B^\dagger B + D^\dagger D \right] |0\rangle
\]
(8.7)
and its conjugate
\[
\langle q, q_0| = \frac{k}{\sqrt{\pi}} \exp \left\{ -\frac{k}{2}(k^2 q^2 + q_0^2) \right\} \times \langle 0| \exp \left[ -i\sqrt{2k}(kq - q_0)D - \sqrt{2k}(kq + q_0)B + \frac{1}{4}D^2 - B^2 \right].
\]
(8.8)
Thus we have found a simultaneous eigenvectors of \( \hat{A} \) and \( \hat{A}_0 \) because they satisfy
\[
\hat{A}|q, q_0\rangle = q|q, q_0\rangle, \quad \hat{A}_0|q, q_0\rangle = iq_0|q, q_0\rangle,
\]
\[
\langle q, q_0| \hat{A} = q\langle q, q_0|, \quad \langle q, q_0| \hat{A}_0 = iq_0\langle q, q_0|
\]
by definition. It is easy to see that an inner product of these eigenvectors are given by
\[
\langle q, q_0| q', q'_0\rangle = \delta(q - q')\delta(q_0 - q'_0).
\]
(8.9)
Turning now to the eigenvectors of canonical conjugates to \( \hat{A} \) and \( \hat{A}_0 \), we may examine
\[
\int d\mu d\nu e^{i\mu(\vec{p} - \vec{p}_0) + \nu(\vec{p}_0 - \vec{p})}
\]
(8.11)
in the same way we have done above. It will immediately fail, however, to yield simultaneous eigenvectors of \( \hat{P} \) and \( \hat{P}_0 \) because they are expressed solely by \( \hat{B} \) and \( \hat{B}^\dagger \) hence no Gaussian normalization, which was found for \( \hat{A} \) and \( \hat{A}_0 \) as is seen in Eq. 8.2, will be available. We can overcome this difficulty by introducing \( \vec{H} = \hat{P} + k^2 \hat{A}_0 \) and \( \vec{H}_0 = -\hat{P}_0 - k^2 \hat{A} \) instead of \( \hat{P} \) and \( \hat{P}_0 \) to observe
\[
\int_{-\infty}^{\infty} dp \, dp_0 \langle p, p_0| p, p_0\rangle = 1,
\]
(8.12)
\[
|p, p_0\rangle \langle p, p_0| = \int_{-\infty}^{\infty} \frac{d\mu d\nu}{(2\pi)^2} e^{i\mu(\vec{H} - \vec{p}) + \nu(\vec{H}_0 - \vec{p}_0)},
\]
(8.13)
\[
\langle p, p_0| p', p'_0\rangle = \delta(p - p')\delta(p_0 - p'_0),
\]
(8.14)
and the inner products
\[
\langle q, q_0| p, p_0\rangle = \frac{1}{2\pi} e^{ipq + ip_0q_0}, \quad \langle p, p_0| q, q_0\rangle = \frac{1}{2\pi} e^{-ipq - ip_0q_0},
\]
(8.15)
as well. Similar to the eigenvectors of $\hat{A}$ and $\hat{A}_0$ above, we have
\[
\hat{H}|p,p_0\rangle = p|p,p_0\rangle, \quad \hat{H}_0|p,p_0\rangle = i\epsilon_0|p,p_0\rangle,
\]
\[
\langle p,p_0|\hat{H} = p\langle p,p_0|, \quad \langle p,p_0|\hat{H}_0 = i\epsilon_0\langle p,p_0|.
\]

(8.16)

It should be noted here that our definition of these eigenvectors is precisely same as the one in Ref. 11) excepting that we accept eigenvalues of $\hat{A}_0$ and $\hat{H}_0$ being pure imaginary. This causes an artificial contradiction to the hermiticity of these operators. But we must take the metric into account in considering hermiticity of operators on the indefinite metric vector space. Indeed, spectral representations of these operators satisfy
\[
\tilde{\eta}_G^\dagger \hat{A}_0^\dagger \tilde{\eta}_G = \tilde{\eta}_G^\dagger \left( \int_{-\infty}^{\infty} dq dq_0 i\epsilon q q_0 \langle q, q_0 | \langle q, q_0 \rangle \right) \tilde{\eta}_G = \hat{A}_0,
\]
\[
\tilde{\eta}_G^\dagger \hat{H}_0^\dagger \tilde{\eta}_G = \tilde{\eta}_G^\dagger \left( \int_{-\infty}^{\infty} dp dp_0 i\epsilon_0 p p_0 \langle p, p_0 | \langle p, p_0 \rangle \right) \tilde{\eta}_G = \hat{H}_0.
\]

(8.17)

If we put $\alpha = 1$ for the Feynman gauge in Eq. (4.13), the Hamiltonian $\hat{H}_G$ becomes
\[
\hat{H}_G^{(F)} = \frac{1}{2k^2} \hat{H}^2 + \frac{k^4}{2} \hat{A}^2 - \frac{1}{2} \hat{H}_0^2 - \frac{k^2}{2} \hat{A}_0^2
\]

(8.18)

to yield a path integral
\[
\int_{-\infty}^{\infty} dq dq_0 \langle q, q_0 | \hat{t}_F \hat{T} \exp \left\{ - \int_{t_1}^{t_2} dt J(t) \hat{A}(t) + J_0(t) \hat{A}_0(t) \right\} | q, q_0 | \hat{t}_I \rangle
\]
\[
= \lim_{n \to \infty} \prod_{i=1}^{n} \frac{d^2 p(i) d^2 q(i)}{(2\pi)^2} \exp \left\{ \sum_{k=1}^{n} \left\{ \frac{i\epsilon(k\Delta q(k) + i\epsilon_0(k\Delta q_0(k))}{2} \right\} - \frac{\epsilon}{2} \left( \frac{1}{k^2} p^2(k) + k^4 q^2(k) + p_0^2 + k^2 q_0^2 \right) - \epsilon(J(k)q(k) + iJ_0(k)q_0(k)) \right\},
\]

(8.19)

where $\epsilon = (\hat{t}_F - \hat{t}_I)/n$ and the limiting procedure, $\hat{t}_F, -\hat{t}_I \to \infty$ followed by $t_2, -t_1 \to \infty$ should be expected. Eq. (8.19) is nothing but another path integral representation of the generating functional for unphysical components of the gauge field in the toy model. On performance of the same prescription as has been done in section 4, we will obtain the same generating functional and the effective action again. Hence it reproduces Eq. (5.20) through the same procedure.

We can immediately extend the above construction to the field theoretical situation to obtain a Euclidean path integral
\[
\langle 0| \hat{T} \exp \left\{ - \int d^4 x J_\mu(x) \hat{A}_\mu(x) \right\} | 0 \rangle
\]
\[
= \mathcal{N} \int D\{A\} \exp \left\{ -\frac{1}{2} \int d^4 x \left\{ \left( \partial_\mu A_\mu \right)^2 + J(x) \cdot A(x) + iJ_0(x)A_0(x) \right\} \right\},
\]

(8.20)

where $\mathcal{N} = \{\text{Det}(\partial_\mu^2)\}^2$ and summations over repeated indices are expected by assuming the Euclidean metric. This is the generalization to the result of Arisue et
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al in Ref. 11) by the introduction of the source terms. If we put $iJ_0 = J_4$ and write $A_0$ as $A_4$, our result coincides with $\tilde{Z}[J_\mu; \infty]_{\text{covariant}}$ given by Kashiwa and Sakamoto in Ref. 10). Then we may apply the “Euclidean Technique” to the above formula. There exist, however, another prescription to deal with this generating functional; We may leave $iJ_0$ as it is and carry out the same calculation as has been done in the previous section to find

$$W[J_\mu] = -\frac{1}{2} \int d^4x \left\{ J^T(x) \frac{1}{-\partial_\mu^2} J(x) - J_0(x) \frac{1}{-\partial_\mu^2} J_0(x) \right\} \quad (8.21)$$

and the effective action

$$\Gamma_E[A_\mu] = \frac{1}{2} \int d^4x \left\{ A^T(x) \partial_\mu^2 A(x) - A_0(x) \partial_\mu^2 A_0(x) \right\}. \quad (8.22)$$

Then, by performing the inverse Wick rotation, we obtain the effective action for the gauge field in Minkowski space-time:

$$\Gamma[A_\mu] = -\frac{1}{2} \int d^4x (\partial_\mu A^\nu(x))(\partial^\mu A_\nu(x)), \quad (8.23)$$

which is nothing but the classical action of the gauge field in the Feynman gauge. In this way we see that we need only to perform Wick rotation and its inverse to obtain a covariant result through a well-defined path integral although we meet a non-covariant action in the exponent of the path integral.

Let us add some remarks on the path integral given above. First, we have not taken care about boundary conditions of the path integral excepting the periodic one for the time axis. In order to give a precise meaning to the above manipulation, we must define the theory in a box of finite volume then a boundary condition must be assigned for the spatial boundaries. In doing so we must also treat the infrared singularity in a proper manner because we are dealing with a massless field. These issues were beautifully resolved at once in Ref. 10) by Kashiwa and Sakamoto by means of $b$-boundary prescription. Secondly, if we consider the Minkowski version of the Eq. (8.20) by inserting the vacuum wave-functionals, we will obtain again a non-covariant action in the exponent of the path integral because of the negative metric for $A_0$. Furthermore, the source term of $\hat{A}_0$ contributes $J_0 A_0$ while spatial components bring to us $iJ \cdot A$ in the action of the path integral. Since the quadratic part of the action is treated as Fresnel integral for this case, we must regard $J_0$ as pure imaginary for the convergence of the path integral. Then we may be possible to obtain the effective action above again. It should be stressed, however, that the Minkowski path integral thus obtained from the covariant canonical formalism does not possess the manifest covariance. We may replace $iA_0$ in the path integral by $A_0$ to recover the covariant appearance. This is the meaning of the integration along an imaginary curve by Arisue et al for $A_0$ in Ref. 11). The conventional path integral of gauge fields and the result of Ref. 10) for the Minkowski space-time also have manifest covariance and integrate $A_0$ along real axis. To understand this disagreement let us recall that even for a negative definite Hamiltonian

$$\hat{H} = -\frac{1}{2} p^2 - \frac{1}{2} q^2, \quad (8.24)$$
there exist a unitary representation of $e^{-it\hat{H}}$ for a real $t$ on the representation space of CCR $[\hat{q},\hat{p}] = i$, that is a positive metric Hilbert space, though the Euclidean path integral for $it = \beta > 0$ becomes ill-defined. Manifestly covariant path integrals for gauge fields in Minkowski space-time can be viewed as the one obtained in this way by use of positive metric Hilbert space even for the unphysical degrees. The Euclidean versions of these path integrals will be then obtained on substitutions:

$$ix^0 = x^4, \ A_0 = iA_4$$

as path integrals of a field theory on the Euclidean space-time. It must be remembered, however, that a path integral obtained in this way does not have an immediate connection to the covariant canonical formalism. This prescription will be applied to the negative oscillator above to be seen as

$$\int Dp \, Dq \, e^{i \int dt \{pq + p^2 + q^2/2\}} = \int Dq \, e^{-i \int dt \{(q^2 - q^2)/2\}} \mapsto \int Dq_4 \, e^{-\int d\tau \{(q_4^2 + q_4^2)/2\}}.$$  

(8.26)

On the other hand, the prescription yields the same Euclidean path integral as the one obtained through usual “Euclidean Technique” for a normal harmonic oscillator:

$$\int DQ \, e^{i \int dt \{(Q^2 - Q^2)/2\}} \mapsto \int DQ \, e^{-\int d\tau \{(\dot{Q}^2 + Q^2)/2\}} = \text{Tr}(e^{-\tau(\hat{P}^2 + \hat{Q}^2)/2}).$$

(8.27)

This clearly explains why Kashiwa and Sakamoto could obtain covariant path integrals in the Minkowski space-time as well as the Euclidean formulas by treating both integrations with respect to $A_0$ and $A_4$ along the real axis.

§9. Conclusion

We have made a thorough investigation on a toy model which explains the structure of Fock space of the quantized gauge field with a covariant gauge condition by means of BRS formalism. A prescription for defining coherent states both for FP ghosts and unphysical degrees of the gauge field has been developed to achieve a construction of path integrals for them by use of these coherent states. Coherent state path integral constructed in this way has a concrete relation to the canonical formulation of the theory. Hence we can always go and back between both formulations easily. This is in sharp contrast with the situation of conventional formulation of path integrals, that is formal functional integrations with classical action in the exponent of its integrand, for gauge theories.

Although our considerations were restricted to an abelian gauge theory, our approach will hold even for non-abelian gauge fields as far as the zeroth order of perturbation or renormalized asymptotic fields are concerned. Therefore we may expect, at least formally, that there exist same structures of Fock spaces and BRS-quartets will be formulated entirely in the same manner as for the abelian case if we express the Lagrangian of such systems in terms of renormalized asymptotic fields. Then it immediately follows that Kugo-Ojima projection expressed in terms of these asymptotic fields possesses the same form as the one constructed in this paper for
the abelian case, even though practical use of such a formula in terms of Heisenberg operators may be difficult.

As for the practical use in perturbation theory, both our approach in this paper and the conventional functional method for construction of path integrals yield same results as far as for the zeroth order of perturbation, i.e., free fields. Since they share same generating functional and propagators for fundamental fields, they yield entirely same results for any perturbative calculations. In this sense they are equivalent. On this point, we should recall that a path integral is a definite integration and if two definite integrations share a same answer they are equivalent. Hence the existence of a changes of variables that brings our formulation to the conventional one will be expected. Nevertheless, when we need to extract some information on the state vectors, the advantage of our prescription is evident because it is built upon manifestly covariant canonical formalism.

In addition to the formulation of path integral, we have found an interesting operator that yields BRS-inversion and also plays the key for obtaining an explicit expression of Kugo-Ojima projection in terms of field variables or creation annihilation operators. The operator $\hat{Q}_D$, that is conserved only in the Feynman gauge, together with the BRS charge $\hat{Q}_B$ provides a nice understanding for BRS-quartet and their anticommutator is the essential part for the operator expression of Kugo-Ojima projection. The existence of such an operator and the solubility of the model will be a consequence of the topological nature of the quantized system because the genuine physical state in our model is the vacuum alone. From the view point of explicit construction, however, it was the viewpoint of quantization according to the decomposition of the gauge field into physical and unphysical degrees that made it easy to classify the state vectors in the Fock space as has been shown in section 6 because otherwise we had never met an idea to make use of creation and annihilation operators, $\hat{B}$, $\hat{D}$ and their conjugates. There will be no other approaches that exhibits in such a clear way the existence of almost complete analogy between unphysical degrees of the gauge field and FP ghosts. Hence we may conclude our prescription for quantization of these variables will be fundamental for understanding a BRS invariant system. In this regard, non-abelian generalization of such decomposition will be desired for non-perturbative analysis of non-abelian gauge theories in terms of Heisenberg operators.

Although we have only dealt with the Feynman gauge, considerations on the discrepancy between the results reported in Ref. 10) and Ref. 11) were made to confirm that a path integral constructed from covariant canonical formalism does not appear to be manifestly covariant. Hence differs from the conventional manifestly covariant path integral. It seems that we should adopt the prescription given by Ref. 10) to preserve a clear relation to the operator formalism while having a manifestly covariant expression at the same time. The significance may not be, however, always in the appearance of a path integral; The covariance can be restored from path integrals with non-covariant actions. We must recognize the importance of the role played by external sources in this regard as is emphasized by Kashiwa in the first of Ref. 10).
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