Evaluating $k-\epsilon$ with One–Equation Turbulence Model

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Abstract
An extended version of the isotropic one–equation model is proposed to account for the
distinct effects of low-Reynolds number (LRN) and wall proximity. The turbulent kinetic energy
$k$ and the dissipation rate $\epsilon$ are evaluated using the $R (= k^2/\epsilon)$ transport equation together with
some empirical relations. The eddy viscosity formulation maintains the positivity of normal
Reynolds stresses and the Schwarz’ inequality for turbulent shear stresses. The model coeffi-
cients/functions preserve the anisotropic characteristics of turbulence in the sense that they
are sensitized to rotational and nonequilibrium flows. The model is validated against a well-
documented flow case, yielding predictions in good agreement with the direct numerical simu-
lation (DNS) data. Comparisons indicate that the present model offers some improvement over
the Spalart–Allmaras one–equation model.

Keywords: One–equation model, turbulence anisotropy, realizability, nonequilibrium flow.

1. Introduction
One-equation turbulence model enjoys a wide popularity due to its simplicity of implementa-
tion and less demanding computational requirements, compared with the standard two-equation
$k-\epsilon$ and $k-\omega$ models. The algebraic model such as Baldwin–Lomax model [1] is efficient from
a numerical point of view but lacks generality for not having transport and diffusion effects.
However, one-equation model includes transport effects and can be considered as a good com-
promise between algebraic and two-equation models.

Considerable research is devoted to improving the accuracy of one-equation models, com-
prising the equilibrium and non-equilibrium flows [2–8]. The Baldwin–Barth (BB) model [2]
derived using the $k-\epsilon$ closure is among the first one-equation models to be self-consistent by
avoiding the use of algebraic length scales. Nevertheless, in the course of transformation some
other major assumptions are made that weaken the link with its parent $k-\epsilon$ models. As a re-
result, the BB model performs very differently from the underlying $k-\epsilon$ model, even in simple
equilibrium flows [4]. To a larger extent, the failure of the BB model lies in the destruction
term. Besides, it is sensitive to the free-stream value of the turbulent Reynolds number and yields unexpected results in predicting the separation in attached flows with mild to strong adverse pressure gradients. In addition, the diffusive term that is not directly connected to the $k-\epsilon$ model renders the model ill-conditioned near the edge of the shear layers. However, the BB model has good near-wall benign properties like the linear behavior of its transport property, which in turn does not require a finer grid than an algebraic model does [5]. Spalart and Allmaras (SA) [3] derive their model using empirical criteria and arguments from dimensional analysis, having no link to the $k-\epsilon$ equations. The motivation for this approach is that the BB model is constrained by assumptions inherited from the $k-\epsilon$ model. Note that the SA model is a modified version of the BB model.

Using the Bradshaw–relation [8] (i.e., the shear stress in the boundary layer is proportional to the turbulent kinetic energy), Menter [4], in his transformation from the $k-\epsilon$ closure to the one-equation model shows a closer connection than the BB model. Menter also mentions that using the Bradshaw–relation seems to be more effective in nonequilibrium flows. However, transforming the $k-\epsilon$ closure may carry many of its deficiencies, such as the bad performance in wall-bounded flows in the presence of mild adverse pressure gradients. It should be emphasized that Menter aims at establishing a firm bond between the one- and two-equation models rather than endorsing a new model for general use. Further modifications to one-equation models based on Menter’s transformation of the $k-\epsilon$ and $k-\omega$ closures are proposed by Elkhoury [5] that have no effect for zero pressure gradient flows. However, they improve the predictive capabilities of the models in wall-bounded nonequilibrium flows compared with the SA model and retain their wall-distance-free feature. Fares and Schröder [6] devise a one-equation model using the findings of the SA and $k-\omega$ models that predicts a wide range of flows especially jets and vortical flows more accurately than the SA model while retaining the same quality of results for near-wall flows, and to be more efficient than the $k-\omega$ model. Nagano et al. [7] propose a low-Reynolds number (LRN) one-equation model derived from an LRN two-equation $k-\epsilon$ model using the modified Bradshaw–relation that accounts for near-wall turbulence. The model provides good results for simple flows and the flow with separation and reattachment.

In the present study, an LRN extension of the BB one-equation turbulence model is proposed and evaluated. This version has several desirable attributes relative to the original BB model: (a) It revives the link between the BB and $k-\epsilon$ models via the source/sink and diffusion terms, using the turbulence structure parameter $a_1 = |\frac{\bar{u}v}{k}|$ (Bradshaw–relation); (b) an eddy damping function $f_\mu$, the length scale of which is explicitly influenced by the mean flow and turbulent variables, is devised to suppress the excessive eddy viscosity in near-wall regions; (c) a physically appropriate time scale is used that never falls below the Kolmogorov (dissipative eddy) time scale; (d) the turbulent Prandtl number $\sigma$ is adjusted such as to provide substantial turbulent diffusion in the near-wall region; (e) source/sink term coefficients $C_{1,2}$ and $C_\mu$, that depend nonlinearly on both the rotational and irrotational strains are proposed based on the realizability constraints and appropriate experiments. Consequently, the model extends the ability of the BB model to account for nonequilibrium and anisotropic effects, a feature that is missing in the single equation models developed so far.

The performance of the new model is demonstrated through the comparison with the DNS data such as fully developed channel flows. The test case is selected such as to justify the ability of the model to replicate the combined effects of LRN, near-wall turbulence. Since the SA model is not transformed from the $k-\epsilon$ closure, it would be interesting to compare the present model predictions with those of the SA model.
2. Turbulence modeling

The principal assumption in deriving the one-equation model is that the turbulent shear stress \(-\overline{\tau_{ij}}\) is proportional to the turbulent kinetic energy \((k)\), which is equivalent to the assumption of Production \((P) = \text{Dissipation} (\epsilon)\) in standard two-equation \(k-\epsilon\) models. The second assumption is that \(\sigma = \sigma_k = \sigma_\epsilon\). A more detailed derivation can be found in Menter [4]. In collaboration with the Reynolds-averaged Navier-Stokes (RANS) equations, the proposed model determines \(R\) by the following transport relation. \(R = k^2/\dot{\epsilon}\) can be enumerated as an undamped tentative eddy viscosity, where the reduced dissipation rate \(\dot{\epsilon} \rightarrow 0\) as the wall is approached, while \(\epsilon\) remains finite. The one-equation turbulence model for high-Reynolds number wall-bounded flows developed by Baldwin and Barth [2] is modified to evaluate \(R\) as:

\[
\frac{\partial \rho R}{\partial t} + \frac{\partial \rho u_j R}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_T}{\sigma} \right) \frac{\partial R}{\partial x_j} \right] + C_1 \rho \sqrt{\overline{\nu R}} - C_2 \rho \left( \frac{\partial \tilde{\rho}}{\partial x_i} \right)^2 \tag{1}
\]

subjected to \(R_w = 0\) at solid walls. Herein, \(\rho\) is the density, \(\mu\) denotes the molecular viscosity, \(\sigma\) is the appropriate turbulent Prandtl number, the production term \(P = -\overline{\tau_{ij} \partial u_i / \partial x_j}\), and the undamped actual eddy viscosity \(\tilde{R} = kT_i\), where \(T_i\) is the hybrid time scale. Compared with the original Baldwin–Barth (BB) model, the new model replaces \(R\) by \(\tilde{R}\) from \(\mu_T\) (eddy-viscosity/diffusion), \(C_1\) (production) and \(C_2\) (destruction) terms that renders the direct coupling between \(R\), \(k\) and \(T_i\) (i.e., \(\epsilon\) since \(T_i\) contains both \(k\) and \(\epsilon\)), thus reducing the free–stream sensitivity. Equation (1) presents a closure problem with three unknowns and therefore, in order to close it, \(k\) and \(\epsilon\) are evaluated using the \(R\)–transport equation together with the Bradshaw [8] and other empirical relations. Alternatively, \(k\) and \(\epsilon\) are represented in terms of \(R\) in section 2.5.

The Reynolds stresses \(\rho \overline{\dot{u}_i \dot{u}_j}\) are related to the mean strain rate tensor \(S_{ij}\) through the Boussinesq approximation:

\[
-\rho \overline{\dot{u}_i \dot{u}_j} = 2 \mu_T \left( S_{ij} - \frac{1}{3} S_{kk} \delta_{ij} \right) - \frac{2}{3} \rho k \delta_{ij}, \quad S_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \tag{2}
\]

The turbulent viscosity is evaluated from

\[
\mu_T = C_\mu f_\mu \rho \tilde{R} = C_\mu f_\mu \rho k T_i \tag{3}
\]

where the eddy viscosity damping function \(f_\mu\) is obtained by solving the elliptic \(f_\mu\) equation that envisages LRN and wall proximity effects. However, \(f_\mu\) relaxes to 1 (one) far from the wall. The model coefficient \(C_\mu\) is in general a scalar function of the invariants formed on the strain rate \(S_{ij}\) and vorticity \(W_{ij}\) tensors in question [9]. The vorticity tensor \(W_{ij}\) is defined as

\[
W_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) \tag{4}
\]

The invariants of mean strain rate and vorticity tensors are defined by \(S = \sqrt{2S_{ij}S_{ij}}\) and \(W = \sqrt{2W_{ij}W_{ij}}\), respectively. The detailed functional form of \(C_\mu\) is determined relying on the constraints such as realizability and appropriate experiments. The formulation of the model coefficients and associated relevant aspects are discussed in some detail in subsequent sections.
2.1. Hybrid time scale $T_t$

The standard argument to introduce a specific time scale is that near a wall the flow is not turbulent anymore, and hence the use of the dynamic time scale $k/\epsilon$ is not appropriate. Employing $k/\epsilon$ results in that the time scale vanishes when approaching a wall, where $k \rightarrow 0$ and $\epsilon$ is non-zero. To avoid this, the Kolmogorov time scale $\sqrt{\nu/\epsilon}$ is used as a lower bound, where the viscous dissipation is dominant. In $k-\epsilon$ models, this approach prevents the singularity in the dissipation equation down to the wall. To interpolate smoothly between Kolmogorov and dynamic scales, a hybrid time scale is formed as

$$T_t = \sqrt{\frac{k^2}{\epsilon^2} + C_T^2 \frac{\nu}{\epsilon}} = \frac{k}{\epsilon} \sqrt{1 + \frac{C_T^2}{Re_T}}, \quad Re_T = \frac{k^2}{\nu \epsilon}$$

where $\nu$ denotes the kinematic viscosity and $Re_T$ is the turbulence Reynolds number. Equation (5) warrants that the eddy time scale never falls below the Kolmogorov time scale $C_T \sqrt{\nu/\epsilon}$, dominant in the immediate neighborhood of the solid wall. Alternatively, the turbulence time scale is $k/\epsilon$ at large $Re_T$ but approaches the Kolmogorov limit $C_T \sqrt{\nu/\epsilon}$ for $Re_T \ll 1$. The empirical constant $C_T = \sqrt{2}$ associated with the Kolmogorov time scale is estimated from the behavior of $k$ in the viscous sublayer [10].

2.2. Coefficient $C_{\mu}$

The new model appears with recourse to the realizability constraints, reflecting physically necessary conditions for developing a compatible turbulence model. The realizability conditions represent the minimal requirement to prevent a turbulence model from producing nonphysical results [11]. The commonly used isotropic eddy viscosity model with a constant $C_{\mu} = 0.09$ becomes unrealizable in the case of a large mean strain rate parameter $T_t S$ (when $T_t S > 3.7$), producing negative normal stresses in question and realizability is violated. To ensure realizability, the model coefficient $C_{\mu}$ cannot be a constant. It must be related with the mean flow deformation rate. Accordingly, a new formulation for $C_{\mu}$ as suggested by Gatski and Speziale [9] is adopted:

$$C_{\mu} = \frac{\alpha_1}{1 - \frac{2}{3} \eta^2 + 2 \xi^2}, \quad \eta = \alpha_2 T_t S, \quad \xi = \alpha_3 T_t W$$

(6)

The coefficients $\alpha_1-\alpha_3$ associated with Eq. (6) are given by

$$\alpha_1 = g \left( \frac{1}{4} + \frac{2}{3} \Pi_b^{1/2} \right), \quad \alpha_2 = \frac{3}{8 \sqrt{2}} g, \quad \alpha_3 = \frac{3}{\sqrt{2}} \alpha_2, \quad g = \left( 1 + 2 \frac{P_k}{\epsilon} \right)^{-1}$$

(7)

Note that the constants associated with $g$ are slightly modified to reproduce the data of DNS and experiments. The invariant of the Reynolds stress $\Pi_b$ and production to dissipation ratio $P_k/\epsilon$ in Eq. (7) are modeled such that they depend nonlinearly on both the rotational and irrotational strains [12]:

$$\Pi_b = C_{\nu} \frac{P_k}{\epsilon}, \quad \frac{P_k}{\epsilon} = C_{\nu} \zeta^2$$

(8)

with

$$C_{\nu} = \frac{1}{2 \left( 1 + T_t S \sqrt{1 + \Re} \right)}, \quad \zeta = T_t S \max(1, \Re)$$

(9)
where $\mathcal{R} = |W/S|$ is a dimensionless parameter that is very useful to characterize the flow. For instance, for a pure shear flow $\mathcal{R} = 1$, whereas for a plane strain flow $\mathcal{R} = 0$. It is appropriate to emphasize herein that the calibrated relations for $\Pi_b$ and $P/\epsilon$ can assist the coefficients ($\alpha_1-\alpha_3$) in responding to both the shear and vorticity dominated flows that are far from equilibrium. Detailed analysis of the model realizability is available elsewhere [12].

2.3. Damping function

The eddy viscosity damping function $f_\mu$ faces the distinct effects of LRN and wall proximity in near-wall regions. Alternatively, the primary objective of introducing $f_\mu$ to turbulence models is to represent the kinematic blocking by the wall, and is devised pragmatically as

$$f_\mu = 1 - \exp \left( -\frac{y}{L} \right), \quad L^2 = 2 \zeta \left( 6 + C_\mu R_T \right) \sqrt{\frac{\nu^3}{\epsilon}}$$

where $(\nu^3/\epsilon)^{1/4}$ signifies the Kolmogorov length scale. A plot of $C_\mu f_\mu$ against the DNS data for fully developed turbulent channel flows is shown in Fig. 1 and good correlation is obtained for $y^+ > 1$. For $y^+ \leq 1.5$, $C_\mu f_\mu$ seems likely to increase proportionally to $y$ (i.e., like a single $f_\mu$) in the very near-wall region as evinced by Fig. 1 in comparison with the DNS data [13]. Overall, the adopted form of $C_\mu f_\mu$ converges to replicate the influences of LRN and wall proximity. The product $C_\mu f_\mu \approx 0.09$ (the standard choice for $C_\mu = 0.09$, pertaining to the linear $k-\epsilon$ model) remote from the wall to ensure that the model is compatible with the high-Reynolds number turbulence model.

2.4. Other model coefficients

The model coefficients $C_1$ and $C_2$ are related to the $k-\epsilon$ constants by [2]

$$C_1 = C_{c_2} - C_{c_1} = 0.48, \quad C_2 = (C_{c_2} - C_{c_1}) \frac{C_\mu s}{\kappa^2} \approx 0.08$$

where $\kappa = 0.41$ is the von Karman constant. The coefficients $C_1$ and $C_2$ are calculated based on the values of the standard $k-\epsilon$ closure where $C_{c_1} = 1.44$, $C_{c_2} = 1.92$ and $C_\mu^* = 0.09$. 

Figure 1: Variations of eddy viscosity coefficient with wall distance in channel flow.
Figure 2: Mean velocity profiles of channel flow.

However, the necessity to account for changes in $C_1$ and $C_2$ is desirable in order to include the local anisotropy of turbulence as is practised in the $k$-$\epsilon$ turbulence model [12]. To explore the anisotropic situation, $C_1$ and $C_2$ are devised as a function of mean shear and vorticity parameters (i.e., $T_t S$ and $T_t W$, respectively):

$$C_1 = 2 \sqrt{\Pi_b^*} \left(1 - \sqrt{\Pi_b^*}\right), \quad C_2 = 2 \left(\tilde{C}_\mu - \frac{|\tilde{C}_\mu| - \tilde{C}_\mu}{C_T^2}\right)$$

where $\tilde{C}_\mu = C^{*}_\mu - C_\mu$, $C^{*}_\mu = 0.09$ and $\sqrt{\Pi_b^*} = C_\mu \zeta$ is essentially identical to Eq. (9), however, with the exception that $C_\nu$ is replaced by $C_\mu$. It can be stressed that the shear/vorticity parameter certainly induces compatible changes in $C_{1,2}$ which account for the anisotropy of turbulence. Remarkably, $C_1 \approx 0.42$ and $C_2 \approx 0.18$ in the log layer of a channel flow with $\zeta(R = 1) \approx 3.3$. However, at some value of $C_\mu = C_\mu(T_t S, T_t W)$, $C_2$ will reach $0.08$ as given in Eq. (11). In principle, the reconstruction of $C_{1,2}$ assists qualitatively in predicting turbulent flows with separation and reattachment as shown in the computation section.

The budgets of $k$ and $\epsilon$ from the DNS data suggest that the role of turbulent diffusion in the near-wall region is substantial. Accordingly, the Prandtl number $\sigma$ is modeled, rather than being assigned constant value (unlike the commonly adopted practice with $\sigma \approx 1$):

$$\sigma = C_\mu + f_\mu / C_T$$

The model coefficients $\sigma$ is developed so that sufficient diffusion is obtained in the vicinity of the wall. This contrivance tends to successfully predict the kinetic energy and dissipation rate profiles from the $R$–transport equation. Nevertheless, $C_\mu \approx 0.3$ and $f_\mu = 1.0$ in the free-stream region and therefore, $\sigma \approx 1$ is recovered therein.

2.5. Evaluation of $k$ and $\epsilon$

The professed interest herein is to represent $k$ and $\epsilon$ in term of $R$ in order to evaluate $\tilde{R}$ (and therefore $\mu_T$) in Eq. (3). Probably, it is the most essential step, since the generality of the reconstructed $k$ and $\epsilon$ must be guaranteed through a wide range of flows. The most appropriate
assumption concerning such a reconstruction is the Bradshaw hypothesis \cite{8} implemented directly into many turbulence models \cite{6}. With the Bradshaw–relation, \( k \) may be expressed using the tentative eddy viscosity \( (C_\mu f_\mu R) \) through the turbulence structure parameter:

\[
\frac{|-uv|}{k} = a_1 = C_\mu f_\mu R \frac{S}{k}
\]  

(14)

where the turbulence structure parameter \( a_1 = \sqrt{C_\mu} \). The exponent \( n \) of \( f_\mu \) is chosen to be \( n = 0.8 \) to fit DNS/experimental data and sensibly, without the loss of generality. To avoid the implicit formulation, Eq. (3) is not used to form Eq. (14) and the purpose herein is to revive the link between the BB and \( k-\epsilon \) models via the source/sink and diffusion terms utilizing \( a_1 \).

Recent DNS and experimental data indicate that the Bradshaw–hypothesis is neither exactly valid in the viscous sublayer of the turbulent boundary layer nor in the free shear layers \cite{6,7}. However, it is to be expected that the introduction of Eq. (14) with the one–equation model will actually lead to improved predictions of nonequilibrium flows \cite{4}. Therefore, \( k \) can be determined from Eq. (14) as

\[
k = \sqrt{C_\mu R S f_\mu^{0.8}}
\]  

(15)

Since \( S \rightarrow 0 \) away from the wall (i.e., free-stream region), \( k \) given by Eq. (15) is insufficient there. In fact, the region where \( S \) is locally zero is bridged mutually by the diffusion and convection terms in the \( k-\epsilon \) turbulence model. With the assistance of \cite{7}, the mean strain rate correction \( S_\alpha \) away from the wall is determined by numerical optimization:

\[
S_\alpha = \frac{2C_\alpha f_\alpha}{3\nu} \left( \frac{\sqrt{u_i^2/2}}{1 + \mu_T/\mu} \right)^2
\]  

(16)

with

\[
C_\alpha = \sqrt{C_\mu^2 + \frac{\nu}{\nu + R}}, \quad f_\alpha = 1 - \exp \left( -\frac{\mu_T}{36\mu} \right)
\]  

(17)
where \( \mathbf{u} = \sqrt{u^2 + v^2 + w^2} \) is the velocity magnitude and \((u,v,w)\) is the velocity vector in Cartesian coordinates. The expression \( C_\alpha \) uses \((\nu + R)\) to avoid the singularity in the near–wall region since \( R \to 0 \) there.

Note that \( C_\mu \) depends nonlinearly on both the shear and vorticity parameters and therefore, the structure parameter \( a_1 = \sqrt{C_\mu} \) used in reconstructing \( k \) is no longer constant. However, the Bradshaw–relation Eq. (14) has no meaning for flows without shear. To extend the predictive capability, a modification is proposed to account for the effect of mean rotation rate on the mean strain rate:

\[
\tilde{S} = S - \frac{[\eta_1] - \eta_1}{C_T}, \quad \eta_1 = S - W
\]  

(18)

The advantage of this formulation is that \( k \) (and therefore, the turbulence eddy viscosity) is reduced in the regions where the magnitude of the vorticity exceeds that of the strain rate, such as in the vortex core. Nevertheless, the overwhelming majority of applications of turbulence models is for shear dominated flows, where the one–equation model is probably well suited.

Thus, Eq. (15) can be reconstructed as follows:

\[
k = f_\mu^{0.8} \sqrt{C_\mu R S_k}, \quad S_k = \sqrt{\tilde{S}^2 + S^2}
\]  

(19)

The value of \( \epsilon \) plays an important role in evaluating the hybrid time scale \( T_t \) accompanied by the turbulence eddy viscosity \( \mu_T \), and is reconstructed as follows:

\[
\epsilon = \sqrt{\epsilon_w^2 + \bar{\epsilon}^2}, \quad \bar{\epsilon} = \frac{k^2}{\nu + \bar{R}}
\]  

(20)

where \( \epsilon_w \) signifies the wall-dissipation rate that equals to the viscous-diffusion rate [14] and is modeled as

\[
\epsilon_w = 2A_\epsilon \nu \left( \frac{\partial u}{\partial y} \right)_w^2 \approx 2A_\epsilon \nu S^2
\]  

(21)

where \( A_\epsilon \) is a function of the Reynolds number. Experimental and DNS data of flat plate and channel flows indicate that \( 0.05 < A_\epsilon < 0.11 \), with a preference for higher values at larger Reynolds numbers [13]. In the current work, \( A_\epsilon = C_\mu^* = 0.09 \) is adopted. Apparently, the contribution of \( \epsilon_w \) to \( \epsilon \) is confined within the wall layer.

### 3. Computations

To validate the generality and efficacy of the proposed model, fully developed channel flows are considered. To evaluate the model reliability and accuracy, the present model predictions are compared with those from the SA model [3]. However, compared with the SA model, the new model is additionally sensitized to nonequilibrium and anisotropic effects (i.e., anisotropic model coefficients, depending nonlinearly on both the rotational and irrotational strains). A cell centered finite-volume scheme combined with an artificial compressibility approach is employed to solve the flow equations [15, 16].

The computation is carried out for fully developed turbulent channel flows at \( Re_\tau = 180 \) and 395 for which turbulence quantities are available from the DNS data [13]. The calculation is conducted in the half-width of the channel, using one–dimensional RANS solver. The computation involving a \( 1 \times 64 \) nonuniform grid refinement is considered based on the grid
independence test. To ensure the resolution of the viscous sublayer the first grid node near the wall is placed at $y^+ \approx 0.3$. Comparisons are made by plotting the results in the form of $u^+ = u/u_\tau$, $k^+ = k^2/u_\tau^2$, $\overline{w^3} = \overline{w^2}/u_\tau^2$ and $\epsilon^+ = \nu \epsilon/u_\tau^4$ versus $y^+$.

Figure 2 shows the velocity profiles for different models. Predictions of the present and SA models agree well with the DNS data. However, at $Re_\tau = 180$ the relative errors on the prediction of $Re_\tau$ are evaluated as $+2\%$ (averaged value) and $-1.7\%$ for the present and SA models, respectively. Profiles of turbulent shear stresses are displayed in Figure 3. Agreement of all model predictions with the DNS data is fairly good. It seems likely that the present model returns superior predictions in near-wall regions relative to the SA model.

Further examination of the model performance is directed to the $k^+$ profiles as portrayed in Fig. 4. As is evident, $k^+$ is somewhat overpredicted in the near-wall region. This is probably due to the improper behavior of the Bradshaw–relation employed to evaluate $k$. Figure 5 exhibits the profiles of $\epsilon^+$ from the present computations that provides a maximum $\epsilon^+$ at the wall which is more in line with the experimental and DNS data. Nevertheless, $\epsilon^+$ is over-
predicted/underpredicted in near-wall regions. The observed discrepancy might be due to the limitation of the proposed near-wall correction $\epsilon_w$ in Eq. (21). Figure 6 shows the turbulent eddy viscosity profiles. As notable from the figure, both the SA and present models reproduce the correct near-wall behavior, comparable with the DNS data. However, both model predictions are inaccurate beyond $y^+ = 50$. Surprisingly, this inaccuracy has little impact on the mean flow and other turbulent parameters since they are reasonably predicted.

4. Conclusions

The present study reconstructs the Baldwin–Barth model to be more consistent with the $k–\epsilon$ models. Contrasting the predicted results with DNS data demonstrates that the new model returns predictions comparable with the SA model. Compared with the SA model, the new model is additionally sensitized to nonequilibrium and anisotropic effects. In particular, the present model may be a good choice for engineering applications, since it can easily be extended to a nonlinear eddy viscosity model.

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