Field-induced meron and skyrmion superlattice in chiral magnets on the honeycomb lattice

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Magnetic skyrmions are stable topological spin textures with significant potential for spintronics applications. Merons, as half-skyrmions, have been discovered by recent observations, which have also raised the upsurge of research. The main purpose of this work is to study further the lattice forms of merons and skyrmions. We study a classical spin model with Dzyaloshinskii-Moriya interaction, easy-axis, and in-plane magnetic anisotropies on the honeycomb lattice via Monte Carlo simulations. This model could also describe the low-energy behaviors of a two-component bosonic model with a synthetic spin-orbit coupling in the deep Mott insulating region or two-dimensional materials with strong spin-orbit coupling. The results demonstrate the emergence of different sizes of spiral phases, skyrmion and vortex superlattice in absence of magnetic field, furthered the emergence of field-induced meron and skyrmion superlattice. In particular, we give the simulated evolution of the spin textures driven by the magnetic field, which could further reveal the effect of the magnetic field for inducing meron and skyrmion superlattice.

I. INTRODUCTION

Magnetic skyrmions are localized topological spin textures corresponding to highly stable particle-like excitations, which have triggered enormous interest due to their significant potential for spintronics applications. The mechanism of skyrmion formation is the competition between Dzyaloshinskii-Moriya (DM) interaction and ferromagnetic exchange interaction with an external field. The DM interaction arises from the relativistic spin-orbit coupling (SOC) in the presence of broken inversion symmetry, such as in chiral magnets (MnSi, FeGe, Fe1−xCo2Si, Cu2OSeO3, etc) [5–12], the synthetic spin-orbit coupling bilayers or bulks with artificial DM interaction [13–17], ultra-cold systems with a synthetic SOC. In the strong coupling limit, we derive the results from the bosonic model on the honeycomb lattice with a symmetric off-diagonal interaction [41–44]. The results demonstrate the emergence of different sizes of spiral phases, skyrmion superlattice (SkL) and vortex superlattice (VL) in absence of magnetic field, furthered the emergence of field-induced meron superlattice (ML), incommensurate skyrmion superlattice (IC-SkL), and z-vortex superlattice (Z-VL). Most of all, we give the simulated evolution of the spin textures driven by the magnetic field, which could further reveal the effect of the magnetic field for inducing ML and SkL. These rich varieties of topological spin textures and their lattice forms should stimulate further investigation of emergent electromagnetic properties.

The paper is organized as follows. In Sec. II, we start from the bosonic model on the honeycomb lattice with a synthetic SOC. In the strong coupling limit, we derive the spin model with DM interaction, easy-axis and in-plane magnetic anisotropies via Monte Carlo simulations. This model could be derived from the system of a two-component bosonic model with a synthetic SOC in the strong coupling limit, where the SOC could induce both the DM interaction and the symmetric off-diagonal interaction. The results demonstrate the emergence of different sizes of spiral phases, skyrmion superlattice and vortex superlattice in absence of magnetic field, furthered the emergence of field-induced meron superlattice, incommensurate skyrmion superlattice (IC-SkL), and z-vortex superlattice (Z-VL). Most of all, we give the simulated evolution of the spin textures driven by the magnetic field, which could further reveal the effect of the magnetic field for inducing ML and SkL. These rich varieties of topological spin textures and their lattice forms should stimulate further investigation of emergent electromagnetic properties.

Here we study further the lattice forms of merons and skyrmions. We study a classical spin model including DM interaction, easy-axis, and in-plane magnetic anisotropies via Monte Carlo simulations. This model could be derived from the system of a two-component bosonic model with a synthetic SOC in the strong coupling limit, where the SOC could induce both the DM interaction and the symmetric off-diagonal interaction. The results demonstrate the emergence of different sizes of spiral phases, skyrmion superlattice (SkL) and vortex superlattice (VL) in absence of magnetic field, furthered the emergence of field-induced meron superlattice (ML), incommensurate skyrmion superlattice (IC-SkL), and z-vortex superlattice (Z-VL). Most of all, we give the simulated evolution of the spin textures driven by the magnetic field, which could further reveal the effect of the magnetic field for inducing ML and SkL. These rich varieties of topological spin textures and their lattice forms should stimulate further investigation of emergent electromagnetic properties.

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II. MODEL AND METHOD

A. model

The low-energy behavior of bosons on the honeycomb lattice with a synthetic SOC can be described with the following Hamiltonian:

\[
\hat{\mathcal{H}}_{\text{boson}} = -t \sum_{\langle ij \rangle} (c_{i\sigma}^\dagger \mathcal{R}_{ij} c_{j\sigma} + \text{H.c.}) + \hat{\mathcal{H}}_U,
\]

\[
\mathcal{R}_{ij} = \exp \left[ i A \cdot (r_{ij} - r_j) \right],
\]

\[
\hat{\mathcal{H}}_U = U/2 \sum_i \hat{n}_i \hat{\tau}_i - 1 + U' \sum_i \hat{n}_{\uparrow i} \hat{n}_{\downarrow i},
\]

where the first term in \(\hat{\mathcal{H}}_{\text{boson}}\) describes the hopping term between nearest-neighbor sites, with \(t\) representing the hopping amplitude and \(c_{i\sigma}^\dagger (\hat{c}_{i\sigma})\) is the creation (annihilation) operator at site \(i\), with subscripts \(\uparrow\) and \(\downarrow\) representing two internal hyperfine states of bosons. The \(A = (\theta \sigma_y, -\theta \sigma_x, 0)\) in matrix \(\mathcal{R}_{ij}\) is a non-Abelian gauge field by the bosons, which is the well-known Rashba term. The second term \(\hat{\mathcal{H}}_U\) describes on-site interactions between bosons, where \(U\) is intracomponent interaction and \(U'\) is the intercomponent one. \(\hat{n}_i = \hat{c}_i^\dagger \hat{c}_i\) is the boson number operator with spin \(\tau\) at site \(i\).

Such a Hamiltonian has been widely studied on one-dimensional chain and two-dimensional square lattice system. Especially, in the deep Mott insulating region, it can be approximated \(^{21-25}\) as a spin model with DM interaction, easy-axis and in-plane magnetic anisotropies, \(\mathcal{S}^\nu_{ij} = \sum_{\tau \tau'} c_{\tau \tau'}^\dagger \mathcal{\hat{\sigma}}_{\tau \tau'} c_{\tau \tau'}\), the pseudo-spin operators, with \(\hat{\sigma}\nu\) Pauli matrix and \(\nu = x, y, z\). A variety of magnetic structures including spirals, skyrmions, and vortex phases were revealed via Monte Carlo simulations.

At unit filling and in the strong coupling limit \(U/t \gg 1\), to the order \(O(t^2/U)\), we obtain the effective low-energy spin Hamiltonian:

\[
\hat{\mathcal{H}}_{\text{spin}} = J \sum_{\langle ij \rangle} \{ J_0 \mathcal{S}_i \cdot \mathcal{S}_j + D_{ij} \cdot (\mathcal{S}_i \times \mathcal{S}_j) \}
\]

\[
+ A_c \left[ \mathcal{S}_i \cdot (z \times r_{ij}) \right] \left[ \mathcal{S}_j \cdot (z \times r_{ij}) \right]
\]

\[
+ J \sum_{\langle i \rangle} \Delta (\mathcal{S}_{i\uparrow}^2),
\]

\[
J = -4t^2/U, \quad J_0 = \cos 2\theta, \quad A_c = 2 \sin^2 \theta, \quad \triangledown \]

where the first term is the nearest-neighbor Heisenberg exchange \(J_0\). The second term is DM interaction, which is anisotropic antisymmetric interaction. SOC also induces another anisotropic exchange interaction, \(A_c\) term, which is symmetry-allowed even in the presence of inversion symmetry. On the square lattice, the \(A_c\) term includes only a symmetric diagonal compass term. However, on the honeycomb lattice, the \(A_c\) term includes a symmetric diagonal compass term and an extra symmetric off-diagonal term due to the lattice anisotropy, corresponding to the in-plane magnetic anisotropy. The last term in \(\hat{\mathcal{H}}_{\text{spin}}\) is easy-axis anisotropy (\(\Delta > 0\)), which is derived from intercomponent interaction \(U'\) between bosons.

We study on a honeycomb lattice with \(L \times L\) \((L = 60)\) unit cells placed in \(xy\)-plane, each unit cell contains two sites, colored with blue and red points shown in Fig. 1. We set the lattice constant \(a = 1\) and the magnitude of the spins is fixed by the normalization condition \(|\mathcal{S}_i| = 1\). The periodic boundary conditions are imposed. Due to the anisotropy of the honeycomb lattice, there are three types of \(r_{ij}\) pointing from blue site to red one: \(r_{ij}^1 = (1, 0), r_{ij}^2 = (-1/2, \sqrt{3}/2)\) and \(r_{ij}^3 = (-1/2, -\sqrt{3}/2)\) and the three types of DM vectors \((D_{\alpha}, D_{\beta}, D_{\gamma})\) are shown with green arrows. We use the convention \(D_{ij} = (\mathcal{S}_i \times \mathcal{S}_j)\) in which the first spin in the cross product \(\mathcal{S}_i\) is always on the sublattice color with blue. The lattice is constructed from different sets of primitive vectors \((a_1, a_2)\) shown with yellow arrows in Fig 1.

![Fig. 1. Sketch of the honeycomb lattice, with the primitive vectors \(a_1, a_2, D_{\alpha}\), \(D_{\beta}, D_{\gamma}\) correspond to the DM vectors in \(\alpha, \beta\) and \(\gamma\) bonds, respectively.](image-url)

B. Monte Carlo method

To reveal the true classical ground states of \(\hat{\mathcal{H}}_{\text{spin}}\) in Eq. (2), we begin with paralleling-annealing Monte Carlo (MC) simulation on 40 replicas, with temperature \(T/|J|\) ranging from 0.001 to 1.0. For each replica, we sample it with a combination of heat-bath and over-relaxation methods. A whole MC step consists of a single heat-bath sweep and subsequent 10 over-relaxation sweeps over the entire lattice. We perform \(2 \times 10^5\) MC steps per replica, then, we copy out the spin configuration from the lowest-T replica and sample it with a combination of zero-temperature heat-bath and over-relaxation method to obtain the ground states. The zero-temperature heat-bath sampling is simply aligning the spins according to
their local fields:

\[
S_i = \frac{\mathbf{h}_{i}^{\text{loc}}}{|\mathbf{h}_{i}^{\text{loc}}|} S_i,
\]

(3)

with

\[
\mathbf{h}_{i}^{\text{loc}} = \sum_{\langle j \rangle} J_{0} S_j + \sum_{\langle j \rangle} D_{ij} \times S_j + \sum_{\langle j \rangle} A_c [S_j \cdot (z \times r_{ij})] + \Delta \sum_{\langle j \rangle} S_i z.
\]

(4)

Due to several competing states, we start from 20 different initial configurations to obtain the correct classical ground states. Such methods for systems with complicated ground states were also used in Ref. [23].

C. Characterization of spin configurations

From the magnetic configurations of ground states, we compute the spin structure factors given by:

\[
F_k = \frac{1}{N} \sum_{ij} \langle S_i \cdot S_j \rangle e^{i\mathbf{k} \cdot (\mathbf{r}_i - \mathbf{r}_j)},
\]

(5)

which is one of key characterizes to identify magnetic phases.

For the topological number, we adopt the definition on lattice version induced by Berg [45, 46]. Firstly, we divide the entire lattice into elementary triangles. Then we calculate the solid angle \(\Omega_l\) for each triangle \(l\) through

\[
e^{\frac{i\mathbf{\Omega}_l}{2\pi}} = \frac{1 + \langle S_1 \cdot S_2 + S_2 \cdot S_3 + S_3 \cdot S_1 + iS_1 \cdot (S_2 \times S_3)\rangle}{\sqrt{2(1 + S_1 \cdot S_2)(1 + S_2 \cdot S_3)(1 + S_3 \cdot S_1)}},
\]

(6)

with \(\Omega \in (-2\pi, 2\pi)\), and the topological number can be obtained as: \(Q_{\text{topo}} = \sum_l \Omega_l / 4\pi\). For elementary skyrmions and merons, the topological number \(|Q_{\text{topo}}|\) equals to 1 and 1/2, respectively [34].

We also calculate the vector chirality \(\kappa_{\text{chi}}\) for each triangle:

\[
\kappa_{\text{chi}} = \frac{2}{3\sqrt{3}} (S_1 \times S_2 + S_2 \times S_3 + S_3 \times S_1),
\]

(7)

where three spins \(S_1, S_2\) and \(S_3\) are chosen anti-clockwise on the triangle.

III. PHASE DIAGRAM OF GROUND STATES

In this section, by using the MC simulation, we obtain the ground states properties of \(\mathcal{H}_{\text{spin}}\) in Eq. (2), with tuning the strength of SOC \(\theta\) and the easy-axis anisotropy \(\Delta\). We get the following phases characterized by the spin structure factors \(F_k\). In Fig. 2(a), we give the phase diagram of ground states, which includes ferromagnetic phase (Z-FM), antiferromagnetic phase (Z-AFM), spiral phases and two topological phases (3×3 SkL and \(\sqrt{3} \times \sqrt{3}\) VL). The 3×3 SkL and \(\sqrt{3} \times \sqrt{3}\) VL phases mean the 3×3 unit cell skyrmion superlattice and \(\sqrt{3} \times \sqrt{3}\) unit cell vortex superlattice, respectively. In Fig. 2(b)-(d), we give the real-space spin configurations of spiral phases, 3×3 SkL and \(\sqrt{3} \times \sqrt{3}\) VL phases. In Fig. 2(e)-(f), we give the spin structure factors of two topological phases. The details of different phases are as follows.

A. Z-FM phase and Z-AFM phase

Firstly, we introduce the ferromagnetic phase and antiferromagnetic phase, viz Z-FM and Z-AFM phases in Fig. 2(a), which result from the ferromagnetic or antiferromagnetic exchange interaction and easy-axis anisotropy. Z-FM phase is the ferromagnetic phase where spins orient along the \(\pm z\) axis. The spin structure factor exhibits a peak at \(k = (0, 0)\), which is the \(\Gamma\) point in the first Brillouin region. Analogously, the Z-AFM phase is the antiferromagnetic phase where spins pointing along with the \(\pm z\) axis. The spin structure factor exhibits peaks at \((\pm 2\pi/3, \pm 2\sqrt{3}\pi/3)\) and \((\pm 4\pi/3, 0)\), which are the vertices of the second Brillouin region.

At \(\theta = 0\) and \(\Delta = 0\), the \(\mathcal{H}_{\text{spin}}\) just recover the classical isotropic Heisenberg model, which has a FM ground state with U(1) symmetry. When \(\Delta > 0\), easy-axis anisotropy results in spins orient along with the \(\pm z\) axis, which leads to the Z-FM ground state.

At \(\theta\) is close to 0.5\(\pi\), DM interaction is almost to zero, the \(\mathcal{H}_{\text{spin}}\) includes the antiferromagnetic Heisenberg exchange, \(A_c\) term, and easy-axis anisotropy. When \(\Delta > 0.2\), easy-axis anisotropy increases dramatically, which leads to a Z-AFM ground state.

B. Spiral phases

The largest region in the phase diagram is spiral phases, which mainly result from the DM interaction. Spiral phases are coplanar states, where spins are all parallel at a plane and their direction rotates by a constant angle from one plane to a neighboring plane along the helical axis. There are several different periods of spiral phases, we describe these spiral phases in detail as follows.

Firstly, when \(\theta\) and \(\Delta\) are small, the DM interaction is very weak and the ferromagnetic exchange interaction is very strong, which leads to the IC-Spiral phase. IC-Spiral are incommensurate spiral phases, which may have several sizes of spiral states. Due to the size effect and difficulty of numerical calculation for this model, we do not give a clear real-space spin configuration of IC-Spiral phase.

As \(\theta\) increases, the emergence of \(N^* \times 1\) spiral phases in phase diagram. \(N^* \times 1\) spiral phases are different sizes of \(N^* \times 1\) unit cell spiral states, where \(N^* = 2, 3, 4, 5, 6\) and \(N^*\) decreased with \(\theta\) increasing. The real-space spin
Fig. 2. (a) Phase diagram from Monte Carlo method of the $\hat{H}_{\text{spin}}$ in Eq. (2). Different phases are abbreviated as described in the text. (b)-(d) The snapshots of real-space spin configurations of different phases, in-plane components (arrows) and out-of-plane components (colour) of spins. (b) Different sizes of commensurate spiral phases. (c) The $3 \times 3$ Skyrmion superlattice phase. (d) The $3 \times 3$ vortex superlattice phase. (e) The spin structure factor of $3 \times 3$ SkL phase, which has peaks at $k = (\pm 2\pi/9, \pm 2\sqrt{3}\pi/9)$ and $(\pm 4\pi/9, 0)$. (f) The spin structure factor of $\sqrt{3} \times \sqrt{3}$ VL phase, which has peaks at $k = (\pm 2\pi/3, \pm 2\sqrt{3}\pi/9)$ and $(0, \pm 4\sqrt{3}\pi/9)$.

configurations are shown in Fig. 2(b), where we only pick two unit cells for each $N^* \times 1$ spiral state. Spins spiral in the $z$-$k$ plane along $a_1$, $a_2$, or $x$ direction, which are triple degeneracy because of $C_3$ symmetry. The three sets of spin structure factors are $k_1 = (-2\pi/3N^*, 2\sqrt{3}\pi/3N^*)$, $(2\pi/3N^*, -2\sqrt{3}\pi/3N^*)$; $k_2 = (2\pi/3N^*, 2\sqrt{3}\pi/3N^*)$, $(-2\pi/3N^*, -2\sqrt{3}\pi/3N^*)$; and $k_3 = (\pm 4\pi/3N^*, 0)$. 
C. Topological phases

1. $3 \times 3$ skyrmion superlattice

At $\theta$ is close to 0.2$\pi$ and $\Delta < 0.5$, an extremely strong competition between DM interaction and $A_c$ term has arisen, which leads to a non-coplanar topological phase, the $3 \times 3$ skyrmion superlattice (SkL) phase.

$3 \times 3$ SkL is a non-coplanar state, where the spins form a $3 \times 3$ unit cell skyrmion superlattice. In Fig. 2(c), we give the real-space spin configuration of the $3 \times 3$ SkL. The elementary skyrmion contains a large hexagon with $3 \times 3$ unit cell. In the hexagon of $3 \times 3$ unit cell, a spin with negative $S^z$ magnetization at the core, spins from the core lie in the $xy$ plane rotating $2\pi$ and six outermost spins with positive $S^z$ magnetization. The structure factor of $3 \times 3$ SkL phase has peaks at $(\pm 2\pi/9, \pm 2\sqrt{3}/9)$ and $(\pm 4\pi/9, 0)$ shown in Fig. 2(c).

We also calculated the topological number of the $3 \times 3$ SkL phase by Eq. (9). The elementary skyrmion was marked with the dashed line in Fig. 2(c), where the topological number $Q_{\text{topo}} = -1$ for each skyrmion.

2. $\sqrt{3} \times \sqrt{3}$ vortex superlattice

At $\theta$ is close to 0.5$\pi$, DM interaction is almost to zero and the $A_c$ term reaches its maximum. When $\Delta$ is close to 0, the spins coplanar in $xy$ plane because of the $A_c$ term, which lead to another topological spin texture, the $\sqrt{3} \times \sqrt{3}$ vortex superlattice (VL) phase.

$\sqrt{3} \times \sqrt{3}$ VL is a coplanar state, with spins in the $xy$ plane form a $\sqrt{3} \times \sqrt{3}$ unit cell vortex superlattice, where the real-space spin configuration is shown in Fig. 2(d). The spins wind anti-clockwise $2\pi$ around each hexagon corresponding to $\kappa_{\text{chi}}$ equal 1. The structure factor has peaks at $(\pm 2\pi/3, \pm 2\sqrt{3}/9)$ and $(0, \pm 4\sqrt{3}/9)$ in Fig. 2(d), which are the vertexes of the first Brillouin region.

IV. PHASE DIAGRAM WITH MAGNETIC FIELD

In this section, we induce the out-of-plane magnetic field in the $\hat{H}_{\text{spin}}$. We obtain different phase diagrams corresponding to different easy-axis anisotropy by using the Monte Carlo simulation. Three phase diagrams (see Fig. 3) are correspond to different easy-axis anisotropy ($\Delta = 0.2, 0.5, 0.8$), which are marked with dashed lines in Fig. 2(a). Most of the phases in Fig. 2(a) are still stable when inducing the magnetic field. It is wondrous that three field-induce topological phases have emerged, incommensurate skyrmion superlattice (IC-SkL), $2 \times 2$ meron superlattice (2 × 2 ML) and $\sqrt{3} \times \sqrt{3}$ vortex superlattice ($\sqrt{3} \times \sqrt{3}$ Z-VL), which have not been focused on in previous work, especially the $2 \times 2$ ML phase. We investigate these new topological phases in detail as follows.

A. Incommensurate skyrmion superlattice

In the phase diagram of ground states (Fig. 2(a)), the IC-Spiral states and larger size of spiral states are not stable when inducing the magnetic field. As the magnetic field increases, the IC-Spiral states and large-size spiral states are superseded by incommensurate skyrmion superlattice (IC-SkL) phases gradually.

IC-SkL phases are non-coplanar states, which have incommensurate skyrmions and the number of skyrmions change with the different parameters. In Fig. 4(a)-(b), we give the spin configurations of IC-SkL states corresponding to different parameter $\theta$ and the spin texture of skyrmion with negative $S^z$ magnetization at the core, and in-plane spin component pointing radially to the core, where the topological number $Q_{\text{topo}} = -1$ for per skyrmion. From Fig. 4(a) to Fig. 4(b), the $\theta$ increases from 0.10 to 0.13, an increase in the magnitude of the spin wave vector, which leads to the number of skyrmions increases and the size of skyrmion lattice decreases.
FIG. 4. The snapshots of real-space spin configurations of new topological phases. In-plane components (arrows) and out-of-plane components (colour) of spins. (a) IC-SkL state at $\Delta = 0.2, H = 0.2, \theta = 0.10$. (b) IC-SkL state at $\Delta = 0.2, H = 0.2, \theta = 0.13$. (c) $2 \times 2$ ML phase and the whole meron marked with the dashed line. (d) $\sqrt{3} \times \sqrt{3}$ Z-VL phase.

B. $2 \times 2$ meron superlattice

We get the $2 \times 2$ ML in the region of $2 \times 1$ spiral phase when inducing the out-of-plane magnetic field, $2 \times 2$ ML is another non-coplanar state, where the spins form a $2 \times 2$ unit cell meron. The magnetic anisotropy leads to the transformation from the $2 \times 1$ spiral phase to $2 \times 2$ ML phase and the $2 \times 2$ ML phase is favored by the competition between $A_c$ term, DM interaction, and magnetic anisotropy.

In Fig. 4(c), we give the real-space spin configuration of the $2 \times 2$ ML phase. The whole meron contains a large hexagon with $2 \times 2$ unit cell. In the hexagon of $2 \times 2$ unit cell, the in-plane spin component rotating $2\pi$ in the center hexagon and six vertices of the outer hexagon spins point down or up. The structure factor of the $2 \times 2$ ML phase has peaks at $(\pm \pi/3, \pm \sqrt{3}\pi/3)$ and $(\pm 2\pi/3, 0)$.

We also calculated the topological number of the $2 \times 2$ ML phase by Eq. (6). The whole meron is marked with the dashed line in Fig. 4(c), where the topological number $Q_{\text{topo}} = -1/2$ for per meron.

C. $\sqrt{3} \times \sqrt{3}$ z-vortex superlattice

We get the $\sqrt{3} \times \sqrt{3}$ Z-VL phase in the region between $\sqrt{3} \times \sqrt{3}$ VL phase and Z-AFM phase with the magnetic field, which mostly results from the competition between $A_c$ term and magnetic anisotropy. The $A_c$ term is favorable to form the VL phase, whereas, with the inducing of magnetic anisotropy, the out-of-plane components of spins appear ferromagnetic order.

$\sqrt{3} \times \sqrt{3}$ Z-VL phase is also a non-coplanar state, where the in-plane components of spins form a $\sqrt{3} \times \sqrt{3}$ unit cell VL state and the out-of-plane components of spins appear ferromagnetic order. The spins wind anti-clockwise $2\pi$ around each hexagon, where the real-space spin configuration is shown in Fig. 4(d). The structure factor of $\sqrt{3} \times \sqrt{3}$ Z-VL phase is similar to $\sqrt{3} \times \sqrt{3}$ VL phase, which also has peaks at $(\pm 2\pi/3, \pm 2\sqrt{3}\pi/9)$ and $(0, \pm 4\sqrt{3}\pi/9)$. However, there has an extra peak at $(0, 0)$, corresponding to the ferromagnetic order of out-of-plane components.

V. FIELD-INDUCED MERON AND SKYRMION SUPERLATTICE

In this section, we give the hysteresis loops and $\text{dm}/\text{dH}$ (see Fig. 5) of typical $3 \times 3$ SkL and $2 \times 1$ spiral phases. By simulating the evolution of these two phases in response to the out-of-plane magnetic field, we further reveal the effect of magnetic field for inducing different sizes of meron and skyrmion superlattice. The evolution of the spin textures of these phases in response to the out-of-plane magnetic field is depicted in Fig. 6 and Fig. 7, which counts several phases and metastable states. The magnetization curves of other phase regions are depicted in Appendix A.

The magnetization processes in $3 \times 3$ SkL phase are great complex, which results from the intense competition between DM interaction, $A_c$ term and magnetic anisotropy. The magnetization process of $3 \times 3$ SkL phase is shown in Fig. 6, where the magnetic field is from $H/|J|=1.5$ to $-1.5$.

In Fig. 6(a), we give the mean topological number for per $3 \times 3$ unit cell of different phases in the magnetization processes of $3 \times 3$ SkL phase. In the magnetization
FIG. 6. The evolution of $3 \times 3$ SkL phase in response to out-of-plane magnetic field. (a) Different phases and their mean topological number for per $3 \times 3$ unit cell in the magnetization processes. (b)-(g) Snapshots of spin textures at $H/|J| = 0.90$, 0.85, 0.80, 0.60, 0.12, 0.10 in the right part of the magnetization processes. (h)-(l) Snapshots of spin configurations at $H/|J| = -0.10, -0.60, -0.85, -1.35, -1.40$ in the left part of the magnetization processes. In-plane components (arrows) and out-of-plane components (colour) of spins. (b) FM polarized state, $Q_{\text{topo}} = 0$. (c) $3 \times 3$ ML, $Q_{\text{topo}} = -1/2$. (d) $3 \times 3$ SkL, $Q_{\text{topo}} = -1$. (e) $3 \times 1$ Spiral state, $Q_{\text{topo}} = 0$. (f) metastable state including spiral and skyrmions state, $-1 < Q_{\text{topo}} < 0$. (g) $3 \times 3$ SkL, $Q_{\text{topo}} = -1$. (h) $3 \times 3$ SkL, $Q_{\text{topo}} = +1$. (i) $3 \times 1$ Spiral, $Q_{\text{topo}} = 0$. (j) $3 \times 3$ ML, $Q_{\text{topo}} = +1/2$. (k) isolated merons, $0 < Q_{\text{topo}} < +1/2$. (l) FM polarized state, $Q_{\text{topo}} = 0$.

Processes of $3 \times 3$ SkL phase, when the magnetic field decreases, polarized states evolve into $3 \times 3$ ML, form metastable $3 \times 3$ SkL gradually, and then form $3 \times 1$ spiral state, which is shown in Fig. 6(b)-(c). When the magnetic field is close to zero, $3 \times 1$ spiral state evolve into $3 \times 3$ SkL state, where the emergence of metastable state including spiral and skyrmions shown in Fig. 6(c)-(g). When the magnetic field goes from positive to negative near zero, the topological number of $3 \times 3$ SkL change opposite in sign shown in Fig. 6(g)-(h). As the magnetic field increases, the $3 \times 3$ SkL phase firstly has the continuous deformation of the spin texture in response to the change of magnetic field, where the total topological number do not change. Then the $3 \times 3$ SkL state evolves into $3 \times 1$ spiral state. A decrease in the magnitude of the spiral wave vector, which leads to the $3 \times 1$ spiral state evolve into $3 \times 3$ ML, a constriction of merons and the number of merons decreases, finally end up with polarized states, shown in Fig. 6(h)-(l).

We also give the simulated evolution of the $2 \times 1$ spi-
FIG. 7. The evolution of $2 \times 1$ Spiral phase in response to out-of-plane magnetic field. (a) Different phases and their mean topological number for per $2 \times 2$ unit cell in the magnetization processes. (b)-(d) Snapshots of spin textures at $H/|J| = 2.10$, $1.90$, $0.70$, in the right part of the magnetization processes. (e)-(h) Snapshots of spin configurations at $H/|J| = 0.0$, $-0.50$, $-0.80$, $-1.50$ in the left part of the magnetization processes. In-plane components (arrows) and out-of-plane components (colour) of spins. (b) FM polarized state, $Q_{\text{topo}} = 0$. (c) metastable state between FM state and $2 \times 2$ ML, $Q_{\text{topo}} = 0$. (d) $2 \times 2$ ML, $Q_{\text{topo}} = -1/2$. (e) $2 \times 1$ Spiral state, $Q_{\text{topo}} = 0$. (f) $2 \times 2$ ML, $Q_{\text{topo}} = +1/2$. (g) metastable state including merons and spiral state, $0 < Q_{\text{topo}} < +1/2$. (h) FM polarized state, $Q_{\text{topo}} = 0$.

VI. DISCUSSIONS AND CONCLUSIONS

In this paper, we find that the different sizes of field-induced meron superlattice are all fully meron superlattice, which has not been reported in previous studies. Based on our results, these novel topological spin textures could be reproduced in real chiral magnets and ultra-cold atoms by inducing the magnetic field. Moreover, we obtain the hysteresis loops of different phases, which could provide guidance to distinguish the different phases in experimental observations.

In conclusion, we study a classical spin model for chiral magnets that include DM interaction, easy-axis, and in-plane magnetic anisotropies on the honeycomb lattice via the Monte Carlo annealing method. We obtain several different sizes of spiral phases and novel topological spin textures, such as $3 \times 3$ SkL phase, and $\sqrt{3} \times \sqrt{3}$ VL phase in absence of the magnetic field. As the magnetic field induced, we obtain new topological spin textures, IC-SkL phase, $2 \times 2$ ML phase, and $\sqrt{3} \times \sqrt{3}$ Z-VL phase. In particular, we give the simulated evolution of the typical $3 \times 3$ SkL phase and $2 \times 1$ Spiral phase in response to the out-of-plane magnetic field. By simulating the evolution of these phases in response to magnetic fields, we reveal that the magnetic field could induce different sizes of meron and skyrmion superlattice. Overall, our results of these topological spin textures and their lattice forms should stimulate further investigation of emergent electromagnetic properties.

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Appendix A: Magnetization curve

In this section, we give the hysteresis loops of different phase region in the phase diagram of ground states, further reveal the effect of magnetic field for inducing different sizes of meron and skyrmion superlattice.

1. Magnetization curves of Z-FM phase, Z-AFM phase and $\sqrt{3} \times \sqrt{3}$ VL phase

Firstly, we give the hysteresis loops in the Z-FM phase (Fig. 8(b)), where the significant magnetic hysteresis in the magnetization process of Z-FM phase.

We find that here appears Z-VL states in the hysteresis loop of Z-AFM phase and $\sqrt{3} \times \sqrt{3}$ VL phase, which lead to the oblique lines in the magnetization curve of Fig. 8(d) and Fig. 8(f). The platform near zero field in Fig. 8(d) is the Z-AFM state, which is the ground state in absence of field. In Z-AFM and $\sqrt{3} \times \sqrt{3}$ VL phase regions, DM interaction is close to zero and compass term reaches its maximum, which leads to the vortex structure in $xy$ plane. As the magnetic field increases, the enhanced polarization Field in $S_z$ leads to the Z-VL states, which is corresponding to the platform in Fig. 8(c) and Fig. 8(e).

2. Magnetization curves of Spiral phases

In Fig. 9, we give the hysteresis loops in the regions of different spiral phases. The magnetization processes in different spiral phases are great difference but there are internal connections, which mostly result from the intense competition between DM interaction and magnetic anisotropy. The hysteresis loops of different spiral phases are presented respectively and these magnetization processes are discussed as follows, which indicate the transformation between meron and skyrmion superlattice.

The magnetization processes in $6 \times 1$ and $5 \times 1$ spiral phases are very similar. There have been lots of metastable states domain wall that IC-SkL order and spiral order coexist when the magnetic field decreases. As the magnetic field increases, IC-SkL states formed and a constriction of skyrmions, then end up with polarized states.

The magnetization processes in $4 \times 1$ spiral phase are extremely complicated. When the magnetic field decreases, polarized states evolve into metastable states that merons and skyrmions coexist. When the magnetic field is close to zero, there has been $4 \times 1$ spiral states. As the magnetic field increases, a decrease in the magnitude of the spiral wave vector, which leads to the $4 \times 1$ spiral states evolve into $4 \times 4$ meron lattice, a constriction of merons and the number of merons decreases gradually, then end up with polarized states.

The magnetization processes in $3 \times 1$ spiral phase is similar to $3 \times 3$ SkL phase. When the magnetic field decreases, polarized states evolve into $3 \times 3$ meron su-
perlattice, form metastable $3 \times 3$ skyrmion superlattice gradually, then form $3 \times 1$ spiral state near zero field. As the magnetic field increases, a decrease in the magnitude of the spiral wave vector, which leads to the $3 \times 1$ spiral states evolve into $3 \times 3$ meron lattice, a constriction of merons and the number of merons decreases gradually, then end up with polarized states.

The magnetization processes in $2 \times 1$ spiral phase are extremely complicated. When the magnetic field decreases, polarized states evolve into metastable state that merons and FM state coexist, then form $2 \times 2$ ML state. When the magnetic field is close to zero, there has been $2 \times 1$ spiral states. As the magnetic field increases, a decrease in the magnitude of the spiral wave vector, which lead to the $2 \times 1$ spiral states evolve into $2 \times 2$ meron lattice, a constriction of merons and the number of merons decreases gradually, then end up with polarized states.

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