Spontaneous emission in the linear problem of shock wave disturbance in a non-classical case

E V Semenko$^{1,2}$ and T I Semenko$^{1,2}$

$^1$ Novosibirsk State Technical University, Novosibirsk, Russia
$^2$ Novosibirsk State Pedagogical University, Novosibirsk, Russia

E-mail: semenko54@gmail.com

Abstract. The isentropic solutions of the linear problem of the shock wave disturbance are considered for a special (non-classical) case, where the both pre-shock and post-shock flows are subsonic. As it is known, only neutral stability is possible here, hence the spontaneous emission appears. The spontaneously emitted waves are studied and their general form is determined both for damped and oscillating initial values. For oscillating initial values both non-resonant and resonant cases are considered. Some interesting distinctions from the classical case are found. First, the spontaneously emitted acoustic waves are evanescent with respect to the variable normal to the shock but oscillate with respect to time. Second, only initial pre-shock vorticity wave may cause the resonance, while the resonance is impossible for initial acoustic both pre-shock and post-shock waves.

1. Introduction

A classical problem of the shock wave, i.e., of a discontinuous solution of the Euler equations, is considered, see for example the classical works [1, 2, 3, 4, 5, 6]. If the equation of state satisfies the condition \((\partial^2 V/\partial p^2)_s > 0\), where \(V = 1/\rho\) is the specific volume, \(p\) is the pressure and \(s\) is entropy, then the entropy, pressure, and density increase across the shock (i.e., only a compression wave is possible), the pre-shock flow is supersonic, and the post-shock flow is subsonic, which in turn ensures the well-posedness of the Cauchy problem [1]. This case is called the classical one.

The case where the above-mentioned condition is not satisfied is considered as a non-classical problem. These non-classical cases (known as “anomalous” or “exotic” shock wave behavior), and also the existence (admissibility) of rarefaction shock waves, were studied by many researches, e.g. [6, 7, 8, 9, 10, 11, 12, 13], the list is far from being complete. Generally they investigate the existence of the piecewise-constant discontinuous solutions. The problem of disturbance of these solutions, and particularly their linear stability, usually isn’t studied. The problem of shock wave disturbance for isentropic flows was studied in papers [11, 12]. As it was clarified, only neutral stability is possible there. At last, in author’s paper [13] the isentropic solution of linear problem of shock wave disturbance was constructed and its properties were investigated.

The so-called spontaneous emission, i.e. the existence of refracted or reflected (emitted) plane waves at the absence of the incident plane waves, occurs in the case of the neutral stability. In the present paper, we announce some results of the spontaneous emission in the mentioned non-classical case. All investigations are based on the solution constructed in [13]. On the other
hand, all considerations are absolutely similar to the papers [14, 15, 16] devoted to the classical case, and we use the same terms and notations as there.

2. Problem formulation

Since we consider the isentropic flow, the sought-for functions are density \( \rho \), velocity \( U = (U_x, U_y) \), pressure \( p \), specific (per unit volume) internal energy \( \epsilon \), enthalpy \( w \), and specific energy \( e \). All sought-for values are considered as functions of \((x, y, t)\). These functions satisfy the relations \( p = p(\rho) \) (equation of state), \( \epsilon = \epsilon(\rho) \), \( w = \epsilon + p/\rho \), \( e = \rho \epsilon + p/\rho \) and Euler equations. We are looking for the discontinuous solutions, so the standard Rankine–Hugoniot conditions take place on the surface of discontinuity \( x = f(y, t) \), which is the sought-for function too. According to the common terminology, the region \( x < f(y, t) \) is called the pre-shock zone, and the region \( x > f(y, t) \) is the post-shock zone.

In our (isentropic) case, outside the shock the energy conservation law is equivalent to the entropy conservation law \( ds/dt = \partial s/\partial t + (U, \nabla s) = 0 \) [1]; thus, it is automatically satisfied. Therefore, only three Euler equations remain outside the shock:

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho U) = 0, \quad \frac{dU_x}{dt} + \frac{\partial p}{\partial x} = 0, \quad \frac{dU_y}{dt} + \frac{\partial p}{\partial y} = 0, \quad x \neq f(y, t). \tag{1}
\]

In turn, for discontinues functions, i.e., those on the shock, the energy conservation law is not equivalent to the entropy conservation law. That means we have four Rankine–Hugoniot conditions in the form

\[
[J] = 0, \quad J[U_x] + [p] = 0, \quad [U_y + f'_y U_x] = 0, \quad \left[ \frac{|U|^2}{2} + w \right] = f'_x[U_x], \tag{2}
\]

where

\[
J = J(y, t) = \rho(U_x - f'_x - f'_y U_y) \bigg|_{x=f(y,t)}
\]

is the flux through the shock and, as usual, the square brackets denote the jump of a value across the discontinuity.

3. Basic solution, linear problem

We are looking for a basic solution as a piecewise-constant solution in the form

\[
f_0(y, t) \equiv 0; \quad \rho_0 \equiv \rho_0^\pm = \text{const}, \quad U_0 \equiv U_0^\pm = (u_x^\pm, u_y) = \text{const}, \quad \pm x < 0
\]

(hereinafter we use the plus and minus superscripts for the pre-shock and post-shock zones respectively). As it is shown in [13] for van der Waals gas, if the flux \( J_0 = \rho_0^\pm u_x^\pm \) is small enough then such solutions exist and the pre-shock and post-shock flows are subsonic: \( u_x^\pm < c^\pm \), where \( c^\pm = \sqrt{p'(\rho^\pm)} \) is the sound velocity.

For the problem, linearized at the basic solution, we use the following notations: the equation of the shock perturbation is \( x = f(y, t) \) and the sought-for vector of disturbed quantities is

\[
G(x, y, t) = \begin{pmatrix} \frac{\delta \rho}{\rho_0} & \frac{\delta U_x}{\rho_0} / c^\pm & \frac{\delta \rho}{\rho_0} & \frac{\delta U_y}{\rho_0} / c^\pm \end{pmatrix} = G^\pm,
\]

where

\[
\begin{align*}
\frac{\delta \rho}{\rho_0} & = \frac{\delta U_x}{\rho_0} / c^\pm, \\
\frac{\delta \rho}{\rho_0} & = \frac{\delta U_y}{\rho_0} / c^\pm.
\end{align*}
\]
±x ≤ 0. Note that we reduce the space of sought-for values to one dimension (density). Then the linearized system (1) takes the form

\[ B^±G^± = 0, \quad B = \begin{pmatrix} \frac{d}{dt} & 0 & \frac{\partial}{\partial x} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial x} & 0 & \frac{d}{dt} & 0 \\ \frac{\partial}{\partial y} & 0 & 0 & \frac{d}{dt} \end{pmatrix}, \]

\[ \frac{d}{dt} = \frac{\partial}{\partial t} + (u, \nabla), \quad u = (u^+_x, u_y), \quad x \neq 0, \]

and the linearized system of the Rankine–Hugoniot conditions (2) is

\[ A^+ A^+_0 G^+_\Gamma = A^- A^-_0 G^-_\Gamma + F_0 f, \]

where \( G^\pm_\Gamma = G^\pm(0, y, t), \)

\[ A^+_0 = \begin{pmatrix} u_x^+ & c_x^+ & 0 \\ c_x^+ & u_x^+ & 0 \\ 0 & 0 & u_x^+ \end{pmatrix}, \quad A^+ = \begin{pmatrix} 1 & 0 & 0 \\ u_x^+ & 0 & 0 \\ 0 & u_x^+ c_x^+ & 0 \end{pmatrix}, \quad F_0 = \begin{pmatrix} -[\rho_0](\partial/\partial t + u_y \partial/\partial y) \\ 0 \\ J_0[u_x](\partial/\partial t + u_y \partial/\partial y) \end{pmatrix}. \]

The initial conditions are \( G(x, y, 0) = G_0(x, y) = G^\pm_0(x, y), \pm x < 0. \)

4. General form of the solution

To solve the problem, we use the so-called one-sided Fourier transform \([13, 14, 15, 16]\). We denote the dual (Fourier) variables for \((x, y, t)\) as \((\xi, \eta, \omega)\) respectively. With respect to these Fourier variables the problem became algebraic, and all sought and given quantities become analytic (in generalized sense) with respect to \(\omega\) in the upper half-plane \(\Im \omega > 0\) and with respect to \(\xi\) in the upper half-plane \(\Im \xi > 0\) for the pre-shock zone and in the lower half-plane \(\Im \xi < 0\) for the post-shock zone. The full solution of that problem is described in \([14]\) for classical case and in \([13]\) for introduced non-classical one. We briefly describe its qualitative properties.

The whole solution is a sum of two terms. The first term is straightforwardly determined by the initial perturbations. The second term and the shock disturbance \(f\) are determined by initial perturbations by means of the solution of the boundary (i.e. on the shock) system of linear equations with respect to the Fourier variables \((\eta, \omega)\). Further we call it as the basic system. So this second term together with shock disturbance are the result of the interaction between the initial (incident upon the shock) waves and the shock itself. These waves induced by the interaction with the shock are refracted and reflected waves.

The matrix of basic system depends on the basic solution, and right hand side of the system depends on the basic solution and initial perturbations. The determinant of that system for any \(\eta\) has two real zeros \(\omega_{\pm} = \eta(u_y \pm \alpha_0)\), where

\[ \alpha_0 = \sqrt{\frac{A(B^+ + B^-) + \sqrt{A^2(B^+ + B^-)^2 + 4A(1 - A)AB^+B^-}}{2(1 - A)}}, \]
The roots where the branch of the square root is chosen under the following condition: for
the solution of the basic system contains additional singular terms, e.g. in a form $I_1(\omega, \omega_{\pm})$ (see
Appendix). Just these singular terms produce the spontaneously emitted waves similar to the
classical case [16].

Beside that, each solution naturally decomposes into the acoustic wave $G_\alpha$ (it is the solution
of wave equation $d^2 G_\alpha /dt^2 - \Delta G_\alpha = 0$ and it represents the density and potential component of
the velocity field) and vorticity wave $G_v$ (it is the solution of transfer equation $d G_v /dt = 0$ and
represents the solenoidal component of the velocity field) [13, 14, 15, 16]. That decomposition
takes place both for initial and for induced (refracted and reflected) waves. Since the induced
wave in the pre-shock zone is acoustic only [13], we denote it as $G_\alpha^+$. The induced wave in the
post-shock zone contains both terms $G_\alpha^-$ and $G_v$.

We describe the general form of spontaneously emitted waves. To formulate the results, we
use the zeros of symbols of the transfer operator $-i \partial /\partial t$ (the symbol $Q^\pm = \xi u_x^\pm + \eta u_y - \omega$) and
the wave operator $-d^2 /dt^2 + \Delta$ (the symbol $P^\pm = (Q^\pm)^2 - (c^\pm)^2 (\xi^2 + \eta^2)$).

5. Allocation of zeros; some terms and notations

In this sections, we drop the superscripts $\pm$ for brevity. We introduce the roots of $P$ and $Q$
directly for the complex $\omega$, $\text{Im} \omega \geq 0$.

We express the roots of $P$ as

$$
\xi_1(\eta, \omega) = \frac{u_x (\omega - \eta u_y) + c \sqrt{(\omega - \eta u_y)^2 - \eta^2 (c^2 - u_x^2)}}{u_x^2 - c^2},
$$

$$
\xi_2(\eta, \omega) = \frac{u_x (\omega - \eta u_y) - c \sqrt{(\omega - \eta u_y)^2 - \eta^2 (c^2 - u_x^2)}}{u_x^2 - c^2},
$$

where the branch of the square root is chosen under the following condition: for $\omega = \eta u_y + i \alpha$
and $\alpha > 0$

$$
\sqrt{(\omega - \eta u_y)^2 - \eta^2 (c^2 - u_x^2)} = i \sqrt{\alpha^2 + \eta^2 (c^2 - u_x^2)}.
$$

The roots $\xi_{1,2}$ are analytic with respect to $\omega$ at $\text{Im} \omega > 0$, at that $\text{Im} \xi_1(\eta, \omega) < 0$ and
$\text{Im} \xi_2(\eta, \omega) > 0$.

The root of $Q$ is marked by the subscript 3: $\xi_3(\eta, \omega) = (\omega - \eta u_y) / u_x$. Obviously, $\text{Im} \xi_3 > 0$
at $\text{Im} \omega > 0$.

Now we are ready to consider (formulate the results) the spontaneous emission for different
kinds of initial perturbations. For simplicity, we consider the solution at constant $\eta$, i.e. we take
the initial perturbation in a form $G_0^\pm(x, y) = G_0^\pm(x) e^{i \eta y}$, $\eta = \text{const}$, $\pm x < 0$. Further we denote
the linear functionals as

$$
\varphi(G_0) = \int_{-\infty}^{\infty} \varphi(x) G_0(x) \, dx,
$$

where $\varphi(x)$ is some vector or scalar function determined by basic solution.

6. Damped initial value

Let the initial values be damped with respect to $x$, i.e. $G_0^\pm(x) \to 0$, $x \to \infty$. Then the right
hand side of the basic system is regular with respect to $\omega$ and (see Appendix) the solution of
this system contains only two singular terms in a form $A_+(\eta) I_1(\omega - \omega_+) + A_-(\eta) I_1(\omega - \omega_-)$
(remember that $\eta = \text{const}$), where $A_\pm = \varphi_\pm(\gamma_0)$. So the straightforward computations show that (we display the singular terms only)

$$f(\eta, \omega) = \Phi(0, \gamma_0)(\omega - \omega) + \Psi(0, \gamma_0)(\omega - \omega),$$

$$G^+_a = \Phi(0, \gamma_0)(\omega - \omega) + \Psi(0, \gamma_0)(\omega - \omega),$$

$$G^-_a = \Phi(0, \gamma_0)(\omega - \omega) + \Psi(0, \gamma_0)(\omega - \omega),$$

$$G^-_v = \Phi(0, \gamma_0)(\omega - \omega) + \Psi(0, \gamma_0)(\omega - \omega).$$

After inverse Fourier transform, we obtain spontaneously emitted waves in a form

$$f(y, t) = \Phi(0, \gamma_0)e^{i\eta(y-tu_y+ta_0)} + \Psi(0, \gamma_0)e^{i\eta(y-tu_y+ta_0)},$$

$$G^+_a = \Phi(0, \gamma_0)e^{i\eta(y-tu_y+ta_0)}e^{i\xi_1^+(\eta, \omega)} + \Psi(0, \gamma_0)e^{i\eta(y-tu_y+ta_0)}e^{i\xi_1^+(\eta, \omega)},$$

but

$$\xi_1^+(\eta, \omega) = \mp \alpha_1 \eta + \beta_1 |\eta|, \quad \alpha_1 = \frac{u^2 - u_0^2}{(c^2)(c^2 - u_0^2)} > 0,$$

$$\beta_1 = \frac{c^2 \sqrt{(c^2 - u_0^2)^2 - (u_0^2)}}{(c^2)(c^2 - u_0^2)} > 0,$$

since $\alpha_0^2 < (c^2)(c^2 - u_0^2)$ [13], and then

$$G^+_a = e^{\beta_1 \eta} \left( \Phi(0, \gamma_0)e^{i\eta(-\alpha_1 x + y - tu_y + ta_0)} + \Psi(0, \gamma_0)e^{i\eta(\alpha_1 x + y - tu_y + ta_0)} \right).$$

Absolutely similar

$$G^-_a = e^{\beta_2 \eta} \left( \Phi(0, \gamma_0)e^{i\eta(-\alpha_2 x + y - tu_y + ta_0)} + \Psi(0, \gamma_0)e^{i\eta(\alpha_2 x + y - tu_y + ta_0)} \right),$$

$$\alpha_2 = \frac{u^{-2} - u_0^{-2}}{(c^2)(c^2 - u_0^2)} > 0,$$

$$G^-_v = \Phi(0, \gamma_0)e^{i\eta(\alpha_2 x + y - tu_y + ta_0)} + \Psi(0, \gamma_0)e^{i\eta(-\alpha_2 x + y - tu_y + ta_0)},$$

$$\alpha_3 = u_0 \alpha_0 / u_0.$$

Thus, as we see, the spontaneously emitted acoustic pre- and post-shock waves $G^\pm_a$ are evanescent with respect to $x$ but oscillate with respect to $t$. The spontaneously emitted post-shock vorticity wave $G^-_v$ is usual plane wave.

7. Oscillating initial value, resonance

Let now $\gamma_0(x) = e^{i\xi_0 x}$, so its Fourier transform is $\hat{I}_1(\xi, \xi_0) / 2\pi i$ for the pre-shock zone and $\hat{I}_1(\xi, \xi_0) / 2\pi i$ for the post-shock zone. Then the right hand side of the basic system has a singular term $C_1(\omega, \omega_0)$, where $C$ is a constant vector and $\omega_0$ depends on $\eta$ and $\xi_0$ (differently for acoustic and vorticity waves and also for pre- and post-shock zones). If $\omega_0 \neq \omega_\pm$ (non resonant case), then the solution of basic system has three singular terms (Appendix)

$C_1 I_1(\omega, \omega_0) + C_2 I_1(\omega, \omega_0) + C_3 I_1(\omega, \omega_0)$. The first term $C_1 I_1(\omega, \omega_0)$ leads to the terms of the form $e^{i(\omega x + \eta y - \omega_0 t)}$ in the induced waves. They are the forced oscillations (caused by the initial ones). The last terms $C_2 I_1(\omega, \omega_0) + C_3 I_1(\omega, \omega_0)$ produce spontaneously emitted waves of the same form as in previous section. Note that the coefficients before $I_1(\omega, \omega_0)$, $I_1(\omega, \omega_0)$ has a factor $\omega_0 - \omega_\pm$ in denominator (Appendix) and so, generally speaking, the amplitudes of the forced oscillations and spontaneously emitted waves approaches infinity when $\omega_0 \to \omega_+$ or $\omega_0 \to \omega_0$. 

5
When $\omega_0 = \omega_+$ or $\omega_0 = \omega_-$, the resonance occurs, i.e. the solution of basic system contains the term like $I_2(\omega, \omega_0)$ (Appendix). But as we have seen, Im $\xi_j(\eta, \omega_\pm) \neq 0$, $j = \frac{1}{2}, 2$ for both pre- and post-shock zones, i.e. the resonance is impossible for initial acoustic perturbations (and $\omega_0(\xi_0, \eta)$ never approaches $\omega_\pm$ at any $\xi_0$). The resonance is possible for the initial vorticity perturbations, when $\xi_0 = \pm \eta_0 u_x / u_y$. But initial post-shock vorticity wave is not incident upon the shock, i.e. does not generate shock wave disturbance and induced waves \cite{13}, therefore the resonance takes place only for the initial pre-shock vorticity waves. So the waves caused the resonance have a form

$$G^+_{\eta}(x, y, t) = \left( \begin{array}{c} 0 \\ \mp \alpha_0 u_y \\ \mp \alpha_0 u_y \end{array} \right) e^{i\eta(y-tu_y \pm \alpha_0(1+tu_y/u_y^+))}, \quad x < 0,$$

and corresponding induced (forced and spontaneously emitted) waves have a form (we take $\omega_0 = \omega_+$ to be more specific)

$$f(y, t) = (\tilde{\Phi}_{01} t + \tilde{\Phi}_{02}) e^{i\eta(y-tu_y - t\alpha_0)} + \tilde{\Psi}_0 e^{i\eta(1+tu_y + t\alpha_0)},$$

$$G^+_{a} = e^{\beta_1 t} \eta |x| (\tilde{\Phi}_{11} t + \tilde{\Phi}_{12} t + \tilde{\Phi}_{13}) e^{i\eta(-\alpha_1 x + y - tu_y - t\alpha_0)} + \tilde{\Psi}_1 e^{i\eta(-\alpha_2 x + y - tu_y + t\alpha_0)},$$

$$G^-_{a} = e^{-\beta_2 t} \eta |x| (\tilde{\Phi}_{21} t + \tilde{\Phi}_{22} t + \tilde{\Phi}_{23}) e^{i\eta(-\alpha_3 x + y - tu_y - t\alpha_0)} + \tilde{\Psi}_2 e^{i\eta(-\alpha_3 x + y - tu_y + t\alpha_0)},$$

$$G^-_{v} = (\tilde{\Phi}_{31} x + \tilde{\Phi}_{32} t + \tilde{\Phi}_{33}) e^{i\eta(-\alpha_3 x + y - tu_y - t\alpha_0)} + \tilde{\Psi}_3 e^{i\eta(-\alpha_3 x + y - tu_y + t\alpha_0)},$$

where $\tilde{\Phi}$, $\tilde{\Psi}$ are specific constant vectors depending on the basic solution and $\eta$.

8. Summary
We introduce some results of spontaneous emission for isentropic solutions of problem of shock wave disturbance in non-classical case. Namely, we indicate the general form of spontaneously emitted waves both for damped and oscillating initial values. For oscillating initial values we consider both non-resonant and resonant cases.

Some interesting distinctions from the classical case are found. First, in the non-classical case the spontaneously emitted acoustic waves are evanescent with respect to the variable $x$ but oscillate with respect to $t$. Second, only initial pre-shock vorticity wave may cause the resonance. The resonance is impossible for initial acoustic both pre- and post-shock waves.

9. Appendix
Here we give some necessary details for the singular solutions of linear equations in the class of analytic (in generalized sense) functions.

The one-sided Fourier transform for functions $f_1(t) = 2\pi e^{-i\omega_0 t}$ and $f_2(t) = 2\pi t e^{-i\omega_0 t}$ (at $t > 0$) is

$$I_1(\omega, \omega_0) = \begin{cases} \frac{1}{\omega - \omega_0}, & \text{Im} \omega > 0, \\ \frac{1}{\omega - \omega_0} - \pi i \delta(\omega - \omega_0), & \text{Im} \omega = 0, \end{cases}$$

$$I_2(\omega, \omega_0) = \frac{\partial I_1(\omega, \omega_0)}{\partial \omega_0} =$$

$$= \begin{cases} \frac{1}{(\omega - \omega_0)^2}, & \text{Im} \omega > 0, \\ \frac{1}{(\omega - \omega_0)^2} + \pi i \delta'(\omega - \omega_0), & \text{Im} \omega = 0, \end{cases}$$
respectively. These functions, though they are singular, are analytic (in generalized sense) in the upper half-plane [13, 14, 16].

Let \( Z(\omega) \) is given regular (smooth) and analytic in upper half-plane function, \( Y(\omega) \) is sought-for analytic in upper half-plane function. The solutions of linear equations have a form (everywhere \( \varphi(\omega) \) is regular):

\[
(\omega - \omega_0)Y(\omega) = Z(\omega) \iff Y(\omega) = Z(\omega) I_1(\omega, \omega_0) + \varphi;
\]

\[
(\omega - \omega_0)Y(\omega) = Z(\omega) I_1(\omega, \omega_1) \iff Y(\omega) = \frac{Z(\omega_1)}{\omega_1 - \omega_0} I_1(\omega, \omega_1) + \frac{Z(\omega_0)}{\omega_0 - \omega_1} I_1(\omega, \omega_0) + \varphi,
\]

\( \omega_1 \neq \omega_0 \) (non resonant case);

\[
(\omega - \omega_0)Y(\omega) = Z(\omega) I_1(\omega, \omega_0) \iff Y(\omega) = Z(\omega_0) I_2(\omega, \omega_0) + Z'(\omega_0) I_1(\omega, \omega_0) + \varphi,
\]

(resonance).

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