Generation and stability of discrete gap solitons

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We analyze stability and generation of discrete gap solitons in weakly coupled optical waveguides. We demonstrate how both stable and unstable solitons can be observed experimentally in the engineered binary waveguide arrays, and also reveal a connection between the gap-soliton instabilities and limitations on the mutual beam focusing in periodic photonic structures.

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Suppression of the diffraction-induced beam spreading and generation of spatial optical solitons is usually associated with the transverse confinement of a beam in a self-induced waveguide. As the medium refractive index increases through the nonlinear response, the beam becomes self-trapped in the effective high-index region due to the total internal reflection. In periodically modulated photonic structures such as fiber Bragg gratings and arrays of weakly coupled optical waveguides, nonlinearity-induced localization of light is also possible in the band-gaps which appear due to the Bragg reflection. The later mechanism is responsible for the formation of temporal Bragg solitons and spatial gap solitons.

Temporal Bragg solitons are usually analyzed in the framework of the coupled-mode theory, where the total field is presented as a superposition of nonlinearly coupled counter-propagating waves which experience Bragg reflection. This case corresponds to a narrow gap in the transmission spectrum, and it is well studied in the theory of fiber Bragg gratings.

However, the coupled-mode theory accounts for an isolated gap only, and such a simplified description of the properties of periodic structures is not valid for analyzing deeper modulations of the refractive index such as, for example, those in waveguide arrays and photonic crystals. Deeper gratings demonstrate more complicated dynamics, including instabilities of nonlinear guided modes due to multi-gap coupling effects and a resonant interaction between different bands.

In this Letter, we study the stability properties of discrete gap solitons in engineered binary arrays of optical waveguides, which have many common properties with spatial gap solitons observed experimentally. Our analysis, based on effective discrete equations, captures the main features of discrete solitons, and these results are confirmed by numerical simulations. In particular, we demonstrate how both stable and unstable gap solitons can be observed experimentally, and also reveal a fundamental link between soliton instabilities and limitations on the mutual beam focusing.

We consider the head-on excitation geometry, when optical beams are incident at grazing angles on a one-dimensional periodic structure of weakly coupled optical waveguides. Then, the beam self-action and interaction resulting in soliton formation can be described by the scalar nonlinear Schrödinger equation,

\[ i \frac{\partial \mathcal{E}}{\partial z} + \frac{\partial^2 \mathcal{E}}{\partial x^2} + \nu(x) \mathcal{E} + |\mathcal{E}|^2 \mathcal{E} = 0, \tag{1} \]

where \( x \) and \( z \) are the transverse and longitudinal coordinates, respectively, \( \mathcal{E}(x, z) \) is the normalized electric field envelope, \( \nu(x) = \nu(x + h) \) is the normalized refractive index profile, \( h \) is the spatial period, and medium has a self-focusing nonlinear response.

Recently, it has been suggested that the properties of spatial gap solitons can be engineered in binary waveguide arrays, where the first gap is controlled by the width difference between alternating “thick” and “thin” waveguides. Wave localization and soliton formation in this gap can be described within the framework of a discrete model of the tight-binding approximation, where the total field is decomposed into a superposition of weakly overlapping modes \( \tilde{\varphi}_n(x) \) of the individual waveguides in the form, \( \mathcal{E}(x, z) = \sum_n E_n(z) \tilde{\varphi}_n(x) \exp(i \lambda_n z) \). After substituting this expression into Eq. (1), we can obtain the modified discrete nonlinear Schrödinger (DNLS) equation for the normalized mode amplitudes \( E_n \),

\[ i \frac{d E_n}{dz} + \lambda_n E_n + (E_{n-1} + E_{n+1}) + \gamma_n |E_n|^2 E_n = 0. \tag{2} \]

Here \( \lambda_n \) characterizes the linear propagation constant of the mode guided by the \( n \)-th waveguide, \( \gamma_n \) are the effective nonlinear coefficients, and \( \lambda_{2n+1} \equiv -\rho, \lambda_{2n} \equiv 0 \), where we assume appropriate normalization.

Soliton solutions of Eq. (2) have the form \( E_n = u_n \exp(i \beta z) \), where \( \beta \) is the propagation constant, and \( u_n \) is the soliton profile. Discrete bright solitons should have exponentially decaying tails, \( u_{\pm n+2m} \approx u_{\pm n} \exp(-2m |\kappa|) \) as \( n \), \( m \to +\infty \). This condition is satisfied inside the band-gaps where Re\( \kappa \neq 0 \). It can be demonstrated that \( \kappa = (1/2) \cosh^{-1}(-\eta/2) \), where \( \eta = 2 - \beta(\beta + \rho) \). Then, discrete solitons can appear in the semi-infinite total internal reflection gap, \( \beta > -(\rho/2) + \sqrt{(\rho/2)^2 + 4} \), and gap solitons can form inside the Bragg reflection gap for \( -\rho < \beta < 0 \).

We can find an analytic estimate for the width of the gap soliton, a key parameter which can be controlled directly in experiment. In the total internal reflection gap,
the width of the conventional discrete solitons can vanish for $\beta \to +\infty$, until almost all the energy is confined in a single waveguide. On the other hand, the situation is qualitatively different for discrete gap solitons, where the soliton width is bounded from below, and it can be estimated according to the linear tail asymptotic,

$$\Delta > 4/\max_{\beta, -\rho < \beta < 0} \Re[\kappa(\beta)] = 4/(\cosh^{-1}(1+\rho^2/8)).$$

(3)

Thus, the minimum soliton width $\Delta$ is determined by the gap width ($\rho$), and the strongest localization is achieved at the middle of the gap ($\beta \sim -\rho/2$). For narrow gaps we have $\Delta > 8/\rho \gg 1$, and in this case the soliton dynamics can be described by the coupled-mode equations. However, for larger bandgaps the soliton can be localized at several waveguides ($\Delta \approx 1$), and discreteness effects become very important.

The soliton profiles and associated powers $P = \sum_n |u_n|^2$ are calculated numerically. We find that there exist two families of gap solitons, A and B, which bifurcate from the lower gap edge ($\beta = -\rho$) in a self-focusing medium, see the example in Fig. 1. As the power increases, the soliton width decreases, and it reaches the minimum value inside the gap, in agreement with the analytic estimate. Profiles of strongly localized discrete gap solitons are shown in Fig. 1(bottom).

A practically important question is soliton stability. We present the field in the form

$$E_n = (u_n + v_n e^{-i\Gamma z} + w_n e^{i\Gamma z}) e^{i\beta z},$$

(4)

and perform the linear stability analysis for the evolution of small-amplitude perturbations ($v_n$ and $w_n$) on top of the soliton profile $u_n$. After substituting Eq. (4) into Eq. (2), we obtain a set of linear equations for $v_n$ and $w_n$ which possess localized solutions at discrete values of $\Gamma$. Solutions with $\Im \Gamma > 0$ indicate the soliton instability, as the perturbations grow exponentially.

We find that the gap solitons of the A-type exhibit symmetry-breaking instabilities, and the instability growth rate becomes larger for strongly localized modes. This property is common for the systems with a broken translational symmetry, and it was demonstrated experimentally that such instabilities can strongly affect the soliton generation, and they can be utilized for the soliton steering.

Conventional discrete solitons of “odd” symmetry are always stable. Similarly, we find that the B-type discrete gap solitons do not exhibit symmetry-breaking instabilities. However, our analysis reveals that these gap solitons become unstable above a critical power due to two types of the oscillatory instabilities. First, instability can appear through an external resonance with the mode in a different gap, as illustrated in Fig. 2(a). A similar instability mechanism was discussed earlier for nonlinear defect modes in a layered medium. Second, the gap soliton can lose stability due to an internal resonance...
within the gap, see Fig. 2(b). The latter scenario was first discovered for temporal Bragg solitons within the gap, see Fig. 2(b). The latter scenario was first discovered for temporal Bragg solitons. In both cases, instabilities are related to resonant excitation of the band modes, and the periodic beating is indicated by a non-zero real part of the instability eigenvalue $\Gamma$.

Soliton excitation inside the frequency gaps of the fiber Bragg gratings is always accompanied by reflection of a substantial energy fraction of an incident pulse. The same limitation is known for the side-on excitation of spatial gap solitons: an input beam is almost completely reflected when inclination angle is tuned deep inside the gap, and the soliton formation is observed only in the vicinity of gaps. More efficient energy coupling into spatial gap solitons can be achieved by a head-on excitation, since the backward scattering is negligible.

Spatial gap solitons can be generated by two Gaussian beams, which are tuned to the Bragg resonance and have opposite inclination angles, as schematically illustrated in Fig. 3 insert. If the incident beams are identical, the input field can be written as

$$\psi_0(x) = Ce^{-(x-x_c)^2/d^2} \cos \left[ \pi(x-x_s)/d \right] e^{i(\varphi_1+\varphi_2)/2},$$

where $x_c$ is the beam center, $d$ is width, $C$ is amplitude, and $\varphi_{1,2}$ are the phases of interfering beams. Parameter $x_s = (\varphi_2 - \varphi_1)d/2\pi$ defines a shift of the interference pattern due to the relative phase between the beams.

Our analysis of the gap soliton solutions revealed that the soliton energy is mainly localized at the narrow waveguides, see Fig. 3(b). We perform numerical simulations using Eq. (1) and confirm that, when the interference maxima are at the narrow waveguides, the two Gaussian beams experience mutual focusing and a gap soliton forms, see Fig. 3(a). The optimal input powers and beam widths can be chosen to significantly reduce the emission of radiation. On the other hand, if the interference maxima appear at wide waveguides, nonlinear beam interaction accelerates the beam spreading and break-up [Fig. 3(b)]. The soliton generation by two beams becomes impossible for higher powers [cf. Figs. 3(a) and 3(c)], and the generated beam exhibits periodic beating during its propagation along the array. This happens due to the development of soliton instability which limits mutual beam focusing through a resonant excitation of modes in different bands, in agreement with our linear stability analysis. Finally, we illustrate how the soliton motion can be induced by varying the power imbalance of the input beams [Fig. 3(d)].

In conclusion, we have analyzed the basic properties of discrete gap solitons using engineered binary arrays as an example. For the first time to our knowledge, we have described the families of discrete gap solitons and analyzed their stability. We have revealed two basic mechanisms of the soliton instability and discussed its connection with mutual beam focusing and gap soliton generation in periodic photonic structures.

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