IS RADIATION OF QUANTIZED BLACK HOLES OBSERVABLE?

I.B. Khriplovich
Budker Institute of Nuclear Physics
11 Lavrentjev pr., 630090 Novosibirsk, Russia,
and Novosibirsk University

N. Produit
INTEGRAL Science Data Center
16, Chemin d’Ecogia, CH-1290 Versoix, Switzerland

Abstract
If primordial black holes (PBH) saturate the present upper limit on the dark matter density in our Solar system and if their radiation spectrum is discrete, the sensitivity of modern detectors is close to that necessary for detecting this radiation. This conclusion is not in conflict with the upper limits on the PBH evaporation rate.

\[^1\text{khriplovich@inp.nsk.su}\]
\[^2\text{Nicolas.Produit@obs.unige.ch}\]
1. We discuss in the present note the possibility to detect the radiation of primordial black holes (PBH), under the assumption that they constitute a considerable part of dark matter. We consider the situation when this radiation has a discrete spectrum, as predicted in some models of quantized black holes.

The idea of quantizing the horizon area of black holes was put forward by Bekenstein in the pioneering article [1]. The idea is quite popular now, but the list of references on the subject is too long to be presented in this note.

We will essentially rely here on the results of [2]. It was demonstrated therein, under quite natural general assumptions, that the spectrum of black hole radiation is discrete and fits the Wien profile (see also in this relation earlier article [3]), and that the natural widths of the lines are much smaller than the distances between them. This spectrum starts with a line of a typical frequency close to the Hawking temperature \( T = 1/(8\pi k m) \) \((k\text{ is the Newton gravitational constant, } m\text{ is the black hole mass})\), and due to the exponential fall-down, the spectrum consists effectively of 2 – 3 lines only, separated by intervals also close to \( T \). However, the total intensity of these few lines, situated around the maximum of the Planck profile, is about the same as that of the Hawking thermal radiation, the latter being saturated essentially just by this region.

2. The analysis of the observational data for the secular perihelion precession of Earth and Mars results in the following upper limit on the density \( \rho_{\text{dm}} \) of dark matter in the Solar system [4]:

\[
\rho_{\text{dm}}^{\text{ss}} < 3 \times 10^{-19} \text{ g/cm}^3.
\]

This limit is based on the precision EPM ephemerides constructed in [5], and on the possible deviations [6] of the results of theoretical calculations from the observational data for the planets obtained from about 250000 high-precision American and Russian ranging to planets and spacecraft. Of course, limit (1) is quite modest as compared to the galactic dark matter density \( \rho_{\text{dm}}^{\text{g}} \simeq 0.5 \times 10^{-24} \text{ g/cm}^3 \), to say nothing of the cosmological one \( \rho_{\text{dm}}^{\text{c}} \simeq 0.4 \times 10^{-29} \text{ g/cm}^3 \). However, in the absence of better observational data on \( \rho_{\text{dm}}^{\text{ss}} \), we will rely below on limit (1) (having in mind, in particular, the huge difference of scales between \( \rho_{\text{dm}}^{\text{g}} \) and \( \rho_{\text{dm}}^{\text{c}} \)).

Our quantitative estimates for the expected signal from PBHs are performed under the optimistic assumption that their density \( \rho \sim 10^{-19} \text{ g/cm}^3 \). The results of these estimates are presented in Table 1. We confine here to four values for the black hole mass \( m \), starting with \( 5 \times 10^{14} \text{ g} \). It is well-known that a PBH with the initial mass \( m_0 \lesssim 5 \times 10^{14} \text{ g} \) just could not survive till our time due to the same radiation. On the other hand, for masses essentially larger than \( 10^{17} \text{ g} \), the signal gets hopelessly small. In Table 1, \( n = \rho/m \) is the density of number of primordial black holes, \( \bar{r} = n^{-1/3} \) is the typical distance between two of them;

\[
N \simeq \frac{1}{64\pi^4} \frac{c}{r_g}
\]

is the number of quanta per second emitted in a line by a black hole with a gravitational radius \( r_g \) (here we use the estimate from [2], and retain for clarity the velocity of light \( c \)); \( \nu \) is the typical expected flux of quanta in a line at the distance \( \bar{r} = n^{-1/3} \). We have mentioned already that the temperature \( T \) roughly corresponds to the energy of the first line in the...
Table 1: Predictions for radiation of primordial black holes in Solar system

| $m$, g   | $n$, cm$^{-3}$ | $\bar{r}$, cm | $T$, MeV | $N$, ph s$^{-1}$ | $\nu$, ph cm$^{-2}$ s$^{-1}$ |
|----------|----------------|---------------|----------|------------------|-----------------------------|
| $5 \times 10^{14}$ | $2 \times 10^{-34}$ | $1.7 \times 10^{11}$ | 20 | $6 \times 10^{19}$ | $1.6 \times 10^{-4}$ |
| $2 \times 10^{15}$ | $5 \times 10^{-35}$ | $2.7 \times 10^{11}$ | 5 | $1.5 \times 10^{19}$ | $1.6 \times 10^{-5}$ |
| $10^{16}$ | $10^{-35}$ | $0.5 \times 10^{12}$ | 1 | $3 \times 10^{18}$ | $10^{-6}$ |
| $10^{17}$ | $10^{-36}$ | $10^{12}$ | 0.1 | $3 \times 10^{17}$ | $2 \times 10^{-8}$ |

discrete spectrum of a black hole. With 2—3 relatively strong lines in a spectrum, one may expect gamma lines with energies about 20, 40, 60 MeV in the spectra of PBHs with mass $5 \times 10^{14}$ g; about 5, 10, 15 MeV in the spectra of PBHs with mass $2 \times 10^{15}$ g; about 1, 2, 3 MeV in the spectra of PBHs with mass $10^{16}$ g; about 100, 200, 300 KeV in the spectra of PBHs with mass $10^{16}$ g. The presence of 2 — 3 roughly equidistant narrow lines in the spectrum of a source of radiation is essential for its identification with a quantized black hole.

Experimentally, to look for such a source we need a gamma ray telescope that is able to do fine spectroscopy for point sources. One candidate here could be the IBIS imager on board INTEGRAL, its point spread function being 12 arcmin. With the typical distance to the nearest source $10^{12}$ cm and typical speed of object in the solar system $10^{6}$ cm s$^{-1}$, such a source will displace itself relative to the observing satellite at an angular velocity of $10^{-6}$ rad s$^{-1}$ in an unpredictable direction. This limits the useful integration time of the search for such a source to $10^4$ s. Degrading the angular resolution of the images would allow to stack the images longer, but will decrease the contrast by increasing the noise, so that the limit will not improve. Using the last official IBIS sensitivity data [7], and recalculating them for a $10^4$ s exposure, we obtain the sensitivity curve of Figure 1. The best sensitivity of IBIS for an unresolved line of a point source amounts to $1.7 \times 10^{-4}$ ph cm$^{-2}$ s$^{-1}$ at 100 KeV. Clearly, the IBIS sensitivity is insufficient for our purpose, by about 4 orders of magnitude at 100 KeV, and by about 3 orders of magnitude at 1 Mev. We note that IBIS never detected any convincing point source with discrete lines.

Another possibility is to use the SPI spectrometer on board INTEGRAL, which has better sensitivity for discrete spectrum [8]. Its imaging resolution being worse, one can stack here images up to $10^6$ seconds without loss of sensitivity. The sensitivity curve of SPI varies wildly with energy due to the presence of numerous gamma ray emission lines in the background
Figure 1: Sensitivity of IBIS imager for narrow line of point-like source exposed for $10^4$ s and of the SPI spectrometer for point-like source exposed for $10^6$ s spectrum, but stays always within the shaded area in Figure 1. Thus, for all energies SPI is more sensitive than IBIS. In particular, the SPI sensitivity is sufficient to observe the line around 5 MeV which belongs to a PBH with $m \sim 2 \times 10^{15}$ g. This would require to analyze very deep sky images in a large set of fine energy bands, which is time consuming, but doable in some sky direction with existing data. If a signal will be found, still one should exclude the existence of an unexpected background feature.

One may wonder also about the quantized radiation of galactic primordial black holes. However, the typical distances to PBHs of the Galaxy are much larger than distances to PBHs of the Solar system. Then, even the galactic dark matter density $\rho_{\text{dm}}^g \simeq 0.5 \times 10^{-24}$ g/cm$^3$, to say nothing of its possible PBH component, is certainly much lower than $\rho \sim 10^{-19}$ g/cm$^3$ assumed above. Therefore, one can measure here only a collective effect of many PBHs, but not the radiation of a single PBH. In this case, it does not look realistic at all to resolve the discrete spectrum of a single black hole.

3. Let us discuss now whether our assumption

$$\rho \sim 10^{-19} \text{ g/cm}^3$$  

for the density of primordial black holes in the Solar system is compatible with other searches for PBHs. The direct searches for the bursts of gamma rays expected from their evaporation...
result in upper limits on the PBH evaporation \cite{9} on the level \cite{11}
\[ 10^6 \text{ pc}^{-3} \text{ yr}^{-1}. \tag{4} \]

These results constrain directly only the number density of very light PBHs with typical life
time about one year, which was the typical observation time. To relate it to the number
density of much heavier (but still light) black holes of interest to us, we need to know the
PBH mass distribution. For the estimates we assume, following \cite{12}, that it is as follows:
\[ dn = \delta \rho_{\text{dm}} \frac{1}{2} m_0^{1/2} m_1^{-5/2} dm_1, \quad m_1 > m_0. \tag{5} \]

Here the factor $\delta$ is the relative contribution of PBHs to the dark matter (we put above
effectively $\delta \sim 1$). Distribution (5) is normalized in such a way that, being integrated with
the weight $m_1$, it gives $\rho = \delta \rho_{\text{dm}}$, the mass density of PBHs. We have labelled the mass
in this distribution with index 1 to demonstrate that it refers in fact not to the present mass
distribution, we are interested in, but to the mass distribution after the formation of PBHs.
The cut-off at $m_1 = m_0 = 0.5 \times 10^{15} \text{ g}$ reflects the mentioned fact that a PBH with a smaller
mass will not survive till present. The behavior of distribution (5) at the masses less than
$m_0$, including the effective cut-off of the divergence at small $m_1$, is not directly related to our
problem.

The initial mass $m_1$ of a black hole is related to its contemporary one $m$ by relation
\[ m_1^3 = m^3 + m_0^3; \tag{6} \]

it follows immediately from the differential equation $dm/dt \sim -1/m^2$ that describes the
evolution of the mass of a black hole due to its radiation. When rewritten in terms of $m$,
distribution (5) transforms into
\[ dn = \delta \rho_{\text{dm}} \frac{1}{2} m_0^{1/2} m^2 (m_3 + m_0^3)^{-3/2} dm, \quad m > 0. \tag{7} \]

Though this mass spectrum extends formally to $m = 0$, it decreases rapidly for small masses.
Still, its maximum is at $m = (4/5)^{1/3} m_0 \simeq 0.9 m_0$, i.e. lies somewhat below $m_0$.

Let us estimate now the number density of PBHs exploding during a year under the
assumption of mass distribution (7). The radiative life-time of a black hole with mass $m$
is well known to be proportional to $m^3$. Since for $m = m_0$ the life-time is $\sim 10^{10}$ years, for the
mass $\mu$ of the very light PBH of interest, we obtain
\[ \left( \frac{\mu}{m_0} \right)^3 \sim 10^{-10}. \]

Then, with distribution (7), the number density of these light PBHs is at present
\[ n_\mu = \delta \rho_{\text{dm}} \frac{1}{2} m_0^{1/2} \int_0^\mu dm m^2 (m_3 + m_0^3)^{-3/2} = \frac{1}{6} \delta \frac{\rho_{\text{dm}}}{m_0} \left( \frac{\mu}{m_0} \right)^3 \sim 10^{-11} \delta \frac{\rho_{\text{dm}}}{m_0}. \tag{8} \]

\[ ^3 \text{According to} \cite{11}, \text{these limits are the most model–independent ones.} \]
With $\rho_{\text{dm}} \sim 10^{-19} \text{ g/cm}^3$ and $m_0 = 0.5 \times 10^{15} \text{ g}$, to comply with the upper limit (4), we should put here $\delta \sim 2 \times 10^{-5}$, instead of our assumption $\delta \sim 1$. However that upper limit (4) corresponds to typical distances between evaporating black holes

$$\bar{r}_1 \sim 10^{-2} \text{ pc} \sim 2 \times 10^3 \text{ au}. \quad (9)$$

But such distances are too large. There are no serious reasons to expect that our initial assumption $\rho \sim 10^{-19} \text{ g/cm}^3$ for the PBH density, being valid for the distances about 1 au from the Sun, should be true for much larger distances $\sim 2 \times 10^3 \text{ au}$. It looks quite natural that at such large distances the PBH density is lower by orders of magnitude than $\rho \sim 10^{-19} \text{ g/cm}^3$.

On the other hand, even with our assumption $\rho \sim 10^{-19} \text{ g/cm}^3$ for the PBH density, we arrive with equation (8) at typical distances between evaporating black holes

$$\bar{r}_2 \sim 10^2 \text{ au}, \quad (10)$$

which is still much larger than the distances about 1 au from the Sun, we are interested in.

Therefore, the observational results of [9] – [11] do not exclude the possibility of existence of the point-like sources of radiation with discrete spectrum, i.e. of quantized PBHs, in the Solar system.

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