Instability of subdiffusive spin dynamics in strongly disordered Hubbard chain

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We study spin transport in a Hubbard chain with strong, random, on–site potential and with spin–dependent hopping integrals, $t_{\sigma}$. For the the SU(2) symmetric case, $t_{\uparrow} = t_{\downarrow}$, such model exhibits only partial many-body localization with localized charge and (delocalized) subdiffusive spin excitations. Here, we demonstrate that breaking the SU(2) symmetry by even weak spin–asymmetry, $t_{\uparrow} \neq t_{\downarrow}$, localizes spins and restores full many-body localization. To this end we derive an effective spin model, where the spin subdiffusion is shown to be destroyed by arbitrarily weak $t_{\uparrow} \neq t_{\downarrow}$. Instability of the spin subdiffusion originates from an interplay between random effective fields and singularly distributed random exchange interactions.

Introduction— Many–body localization (MBL) is one of the most challenging phenomena in condensed matter physics [1,2] which has recently stimulated intensive theoretical and experimental studies concerning the low–dimensional strongly disordered many-body systems. Theoretical studies considered and identified MBL predominantly in the chains of interacting spinless fermions (or in equivalent Heisenberg–like spin models) [3,19]. Among distinctive properties of such disordered systems, there is absence of thermalization in the MBL phase [20,40] and, moreover, unusually slow equilibration also beyond the boundaries of the MBL regime. In particular, the subdiffusive dynamics has been found in several one-dimensional models for moderate disorder and has been identified as a precursor to MBL [15,30,41–46].

The qualitative features of MBL have been confirmed in several experimental studies of cold–fermion lattice systems [36,47,51] which, however, address the physics of the disordered Hubbard model with both density (charge) and spin degrees of freedom. The remaining SU(2) spin–symmetry of the latter models poses essential limitations to the existence of the full MBL [52,56]. While the charge degrees of freedom appear to be localized for sufficiently strong disorder, the spin degrees remain delocalized and undergo a subdiffusive dynamics [40,43,54]. This implies that only partial (charge) MBL may occur in the SU(2) symmetric Hubbard chains. This scenario is consistent with the number of local integrals of motion [57] which stays well below the value expected for systems with full MBL. Moreover, one cannot exclude that coupling of localized charges and delocalized spins will eventually delocalize also the charge degrees of freedom [58], even if the latter delocalization will happen at exceedingly long time–scales.

In this paper we reconsider the problem of full/partial MBL and demonstrate that in strongly disordered Hubbard model, the subdiffusive (but ergodic) spin dynamics is unstable against even weak perturbations that break the SU(2) spin symmetry. In particular, we consider Hubbard chain with random on–site potential and with anisotropic (spin–dependent) hopping integrals, $t_{\sigma}$. We study the long–time ($t \to \infty$) behavior of local spin–spin correlations, $C_0 = \lim_{t \to \infty} \langle S_i^z(t) S_i^z(0) \rangle$, representing local spin stiffness being also an indicator of nonergodicity. While $C_0 = 0$ in the SU(2) symmetric case ($t_{\uparrow} = t_{\downarrow}$), in agreement with a subdiffusive dynamics, it is shown to be non-zero even for very weak hopping asymmetry. For asymmetric hopping, it exhibits a power–law dependence $C_0 \propto |t_{\uparrow} - t_{\downarrow}|^\gamma$ indicating that full MBL is restored. In order to explain the instability of the spin subdiffusion, we derive an effective (squeezed) model which describes the dynamics of spin excitations. The latter model takes the form of the Heisenberg chain with random exchange interactions but also with random local magnetic fields. The interplay between random spin interactions (with a singular distribution [43] and random fields appears to be responsible for the spin localization and restoration of full MBL for arbitrarily small difference $t_{\uparrow} - t_{\downarrow}$. The numerical results for the Hubbard model in this regime confirm the simplified model and a general scenario.

Model and method— We study a disordered Hubbard chain

$$H = H_0 + U \sum_{i} n_i^\uparrow n_i^\downarrow, \quad (1)$$

$$H_0 = - \sum_{i,\sigma} t_{\sigma} c_{i\sigma}^\dagger c_{i+1\sigma} + H.c. + \sum_{i} \epsilon_i (n_i^\uparrow + n_i^\downarrow), \quad (2)$$

where $c_{i\sigma}^\dagger$ creates a fermion with spin $\sigma$ at site $i$, $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ and the disorder enters only via random potentials, $\epsilon_i$, which are uniformly distributed in $[-W,W]$. The spin asymmetry is introduced via hopping integrals, where we adopt $t_{\uparrow} = 1$ as the energy unit, while $t_{\downarrow} \leq 1$.

As it follows from the experimental [43] and theoretical [40,54] studies, the charge dynamics in the Hubbard chain [1] is frozen for sufficiently strong disorder, $W \gg 1$. Therefore, it is useful and sufficient to derive a squeezed model which involves only spin degrees of freedom. To this end we diagonalize the single–particle Hamiltonian, $H_0 = \sum_{a,\sigma} \epsilon_{a\sigma} c_{a\sigma}^\dagger c_{a\sigma}$, where $c_{a\sigma}^\dagger = \sum_i \phi_{i a\sigma} c_{i\sigma}^\dagger$ creates
a fermion in the Anderson state and we take all $\phi_{iaa}$ as real. We consider only strong disorder $W \gtrsim 4$, when the single-particle localization length is very short, $\lambda < 1$. For convenience, the Anderson states are sorted according to the maxima of $|\phi_{iaa}|$ in real-space so that $\phi_{ia\uparrow}$ and $\phi_{ia\downarrow}$ are centered in the vicinity of the same lattice site, $i$, despite $\phi_{ia\uparrow} \neq \phi_{ia\downarrow}$. Consequently the quantum number $a$ marks positions of the Anderson states in real space.

In order to obtain the squeezed spin model, we rewrite the Hubbard term in Eq. (1) using the Anderson basis [13]. In view of the frozen charge dynamics, we keep only terms which do not alter the occupancy of the Anderson states, i.e., we keep terms commuting with $n_a = n_{a\uparrow} + n_{a\downarrow}$. Then we can rewrite the effective Hamiltonian using the spin operators, $S^z_a = \frac{1}{2}(n_{a\uparrow} - n_{a\downarrow})$, $S^+_{a\uparrow} = c_{a\uparrow}^\dagger c_{a\downarrow}$ and $S^-_{a\downarrow} = c_{a\downarrow}^\dagger c_{a\uparrow}$,

$$H \simeq -\sum_{a < b} \left[ J^z_{ab} S^z_a S^z_b + \frac{J^z_{ab}}{2} (S^+_a S^-_b + S^-_a S^+_b) \right] + \sum_a h_a S^z_a,$$

where

$$J^z_{ab} = U \sum_i \left[ (\phi_{ia\uparrow} \phi_{ib\downarrow})^2 + (\phi_{ia\downarrow} \phi_{ib\uparrow})^2 \right],$$

$$J^z_{ab} = 2U \sum_i \phi_{ia\uparrow} \phi_{ib\downarrow} \phi_{ia\downarrow} \phi_{ib\uparrow},$$

$$h_a = \Delta \varepsilon_a + \frac{U}{2} \sum_{b \neq a} n_b \left[ (\phi_{ia\uparrow} \phi_{ib\downarrow})^2 - (\phi_{ia\downarrow} \phi_{ib\uparrow})^2 \right],$$

with $\Delta \varepsilon_a = \varepsilon_{a\uparrow} - \varepsilon_{a\downarrow}$. On the one hand, starting from the SU(2) symmetric Hubbard chain ($\phi_{ia\uparrow} = \phi_{ia\downarrow}$) one obtains a SU(2) symmetric model with $J^z_{ab} = J^z_{ab}$ and $h_a = 0$, where spins have been shown to be delocalized and the spin transport is subdiffusive [13]. For $\phi_{ia\uparrow} \neq \phi_{ia\downarrow}$ the effective model takes the form of an easy-axis XXZ model with random $J^z_{ab} \geq |J^z_{ab}|$ but also with random fields $h_a$. Due to the latter interaction, Eq. (3) resembles the canonical model studied in the context of MBL [10][11]. However, an essential difference in Eq. (3) emerges from random interactions $J^z_{ab}$ with singular distributions, as shown later on. Hence, dynamical properties cannot be simply deduced from previous studies of the standard model.

In order to study numerically the spin dynamics, we first generate random $\epsilon_i$ in Eq. (4) and diagonalize $H_0$ for a chain of $L$ sites. Then, we randomly choose $N$ Anderson states occupied by fermions ($N/2$ for each spin projection). Doubly occupied states $|a\rangle$ are spin singlets and do not contribute to the Hamiltonian (3). Consequently, the squeezed model contains (on average) only $\bar{L} \sim N - N^2/2L$ singly-occupied states $|a\rangle$. Note that the average distance between fermions occupying these states is $L/\bar{L} \geq 2 \gg \lambda$, even for the half-filled Hubbard model, $\bar{n} = N/L = 1$. Moreover, overlaps of the wave-functions in Eqs. (4,6) decay exponentially with the real-space distance between the Anderson states, hence we consider interactions only between nearest neighbors $b = a \pm 1$.

Let us first consider statistical properties of parameters $J^z_{ab}, h_a$ as they occur in the squeezed model [3]. Fig. 1 shows the joint probability density, $p(|J^z|, h)$. One may observe that $J^z$ and $h$ are strongly correlated with each other and Fig. 1 reveals a clear maximum at $J^z \simeq |J^z|$. Moreover, the probability densities, $f^z(J^z)$ and $f^h(|h|)$, are rather insensitive to a modest difference $t_\uparrow - t_\downarrow$. This can be observed from the cumulative distribution functions $F^z(J) = \int_0^J dJ' f^z(J')$ and $F^h(h) = \int_0^h dh' f^h(|h'|)$ shown in Fig 1b for $t_\uparrow = 0.8$. They are quite close to the distribution $F^0(J) = (J/2U)^{\lambda}$ (also plotted in Fig 1b) with $\lambda = \lambda \bar{L}/L$ which describes the distribution of $J$ in the SU(2) symmetric model [13]. The similarity of distributions is important, since they are singular also for the asymmetric model (provided that $\lambda < 1$). We stress that probability for $J^z_a = 0$ vanishes. The latter would induce a trivial spin localization via cutting the chain into disconnected parts. Again, $\lambda$ is the essential parameter which in the SU(2) symmetric case governs the subdiffusive dynamics, $(S^x_a(t) S^x_a(0)) \propto t^{-\lambda/(1+\lambda)}$. In all considered cases we also find $\lambda > 0$, i.e., random $J$ alone is insufficient to cause spin localization.,
At finite temperatures $T > 0$, Fig. 2 shows the strength of random magnetic field $h_a$, i.e. the variance $\langle h_a^2 \rangle$. This quantity shows a power-law decrease with the disorder strength $W$ for arbitrary $t_\uparrow \neq t_\downarrow$. Counterintuitively, strongly disordered Hubbard model maps onto the spin chain with random fields which are too weak to cause an efficient spin localization. The essential physical mechanism behind the onset of spin-localization can be observed in Fig. 1: which shows the joint probability density $p(|J_\perp|, h)$. When compared to Fig. 1a, correlations between $J_\perp$ and $h_a$ seem insignificant. Therefore, there is quite high probability for regions with large ratio $h_a/J_\perp$, which in the following are shown to be essential for the spin localization.

Figs. 2a, b show the central result of this work: the local spin-spin correlation function for the effective spin model

$$C(t) = \langle \psi | S_a^z(t) S_a^z(0) | \psi \rangle_{\text{ave}}.$$ \hspace{1cm} (7)

We have calculated $C(t)$ taking into account parameters from the original Hubbard model in accordance with Eqs. 4-6. Then, numerical results have been averaged over random $\epsilon_i$, as well as over random choice of singly occupied Anderson states $|a\rangle$. Averaging over $|\psi\rangle$ in the corresponding squeezed model has been carried out at high temperature, $T \to \infty$. We stress that results still depend on the filling $\bar{n}$ of the Hubbard chain and on the size of the squeezed chain $L$.

As follows from Fig. 2, there is a quite elevated probability for finding weak links \cite{25,26} with small $J_\perp$, which may result in long-lasting but still transient phenomena. In order to rule out such transient effects we have used two complementary numerical methods and verified their consistency. Namely, data for $L = 20$ and times $t \lesssim 10^3$ are obtained via the time-dependent Lanczos method \cite{29}, whereas longer times but smaller systems $L \lesssim 14$ are studied by exact diagonalization (ED). We have carried out averaging over $10^3$ and $10^4$ realizations of disorder for the former and the latter methods, respectively. For the symmetric case, $t_\downarrow = 1$, $C(t)$ shows unrestricted power-law decay in time corresponding to ergodic but subdiffusive behavior. However, it saturates even for very weak asymmetry $t_\uparrow - t_\downarrow \sim 10^{-2}$ marking the onset of nonergodicity and spin localization. It holds true not only at low--filling ($\bar{n} \ll 1$) shown in Fig. 2, but also for parameters corresponding to the half-filled Hubbard chain ($\bar{n} = 1$) in Fig. 2. Fig. 2 presents a finite size scaling of the spin stiffness $C_0 = C(t \to \infty)$ vs. $1/L$. More precisely, the circles show result obtained from the Lanczos method for $t = 10^3$, whereas
squares show $C_0$ obtained via ED. It appears, that both approaches yield very similar results for the extrapolated stiffness $C_0$, as presented in Figs. 2a and 3 for fillings $\bar{n} = 1$ and $\bar{n} \approx 0.14$, respectively. Finally, extrapolated stiffness shows a power-law dependence on the asymmetry parameter, i.e. $C_0 \propto (t_{\uparrow} - t_{\downarrow})^\gamma$ and $\gamma$ is of the order of $\bar{\lambda}$. Consistently with previous considerations [33], for symmetric case $t_{\downarrow} = 1$ we get ergodic behavior with $C_0 = 0$, but with a subdiffusive dynamics provided that $\bar{\lambda} < 1$.

It follows from Fig. 1 that $f^z(J) \approx f^0(J) = \lambda(J/2U)^{\lambda-1}$ for $J \leq 2U$. The power–law distribution has an integrable singularity at $J = 0$ provided that $0 < \lambda < 1$. Namely, the singularity occurs when the average distance between singly occupied Anderson states $L/L$ is larger than the single–particle localization length $\lambda$. One may further simplify the squeezed model, Eqs. (4)–(6), assuming that $J_{\downarrow}^z = J_{\uparrow}^z = J_a$ are random variables with a distribution function $f^0(J)$ and that random $J_a$ is uncorrelated with field $h_a \sim \Delta \bar{e}_a$. Figs. 2a and 3 show the comparison of $C_0$, obtained for the complete and simplified squeezed model, respectively. One may observe that the simplified version indeed maintains the essential properties of the more general version. Moreover, the simplified version allows to study regimes which cannot be derived from the Hubbard model within our approach, e.g., when $W$ is small or $U$ is too large. In particular, Fig. 3 shows the same results as Fig. 2, but obtained for $\bar{\lambda} = 1$, i.e., for a uniform (nonsingular) distribution of random $J$. Despite the presence of the random fields, $h_a$, spins remain delocalized for non-singular $f(J)$. Absence of spin localization can also be observed in Fig. 3 that shows results for random $h_a$ but with uniform $J_{\downarrow}^z = J_{\uparrow}^z = 1$. Concluding this part, we can therefore stress that the instability of the spin subdiffusion and the onset of spin localization originate in the squeezed model from a coexistence of random fields $h_a$ and the singular distribution of random $J_{\downarrow}^z \sim J_{\uparrow}^z$.

As a final numerical support for our approach, we compare the local spin–spin correlation functions obtained from the squeezed model and directly from the disordered full Hubbard chain, where $C(t) = \langle S_i^z(t)S_j^z(0) \rangle$ is calculated at $T \rightarrow \infty$ via the microcanonical Lanczos method [33]. Since the local correlations in the Hubbard model are defined in terms of the Wannier states $|i\rangle$, one can expect quantitative but not qualitative differences at $t \rightarrow \infty$. A comparison is shown in Figs. 4a, b, c for various parameters. Due to much larger Hilbert space, results for the Hubbard chains are obtained for rather limited system sizes $L \leq 18$, low fillings $\bar{n} \leq 0.6$ and time–windows $t \leq 100$. Both $C(t)$ reveal decay in time for the SU(2) symmetric case (Fig. 4a) and saturation for $t_{\downarrow} \neq t_{\uparrow}$ as shown in Figs. 4b and 4c. The best agreement between the models is expected to show up for modest $U < 4$, large $W > 4$ (small $\lambda$) and low filling $\bar{n} \ll 1$ (large distance between spins). It is still satisfactorily close to quarter–filling, $\bar{n} \approx 1/2$, Fig. 4b, i.e. for the case studied experimentally [35].

While the deviation between both models at $t \lesssim 1$ is not surprising, it is useful to explain its origin. To this end, for the Hubbard chain we have calculated also a (normalized) charge–charge correlation function, $C_n(t) = \langle (n_i(t) - \bar{n})(n_j(t) - \bar{n}) \rangle/\bar{n}^2$, shown in Fig. 4. We note, that in the short–time regime the charge is redistributed over the Anderson states. This feature is missing in the squeezed model and is responsible also for the short–time deviations visible in the spin dynamics of both $C(t)$.

Conclusions. – We have studied how the spin dynamics in the disordered Hubbard chain depends on spin–dependent hopping that breaks the SU(2) symmetry. To this end we have derived an effective spin model assuming that the disorder strength is the largest energy scale (i.e., interaction is weak) and the single–particle localization length is much smaller than the average distance between singly occupied sites. Results obtained for the squeezed model show that the subdiffusive spin dynamics occurs only for strictly SU(2) symmetric system whereas arbitrary $t_{\downarrow} \neq t_{\uparrow}$ localizes spins and restores full MBL in the original Hubbard model. Instability of the subdiffusive dynamics originates from the interplay between two
specific properties of the squeezed model: weak random magnetic field and random $J$ with a distribution function that is singular at $J = 0$. Despite rather obvious numerical limitations, results obtained for the Hubbard chain qualitatively agree with those for the squeezed Hamiltonian. There are nevertheless open questions. It is unclear whether the instability of subdiffusive spin dynamics is restricted to the regime where we can reliably derive the squeezed spin model. Moreover, we have considered only that is singular at $J = 0$. Despite rather obvious numerical limitations, results obtained for the Hubbard chain qualitatively agree with those for the squeezed Hamiltonian. There are nevertheless open questions. It is unclear whether the instability of subdiffusive spin dynamics is restricted to the regime where we can reliably derive the squeezed spin model. Moreover, we have considered only a specific breaking of SU(2) symmetry, and it is a pertinent question whether this instability holds for arbitrary perturbation that breaks the latter symmetry.

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[1] D. M. Basko, I. L. Aleiner, and B. L. Altshuler, “Metal–insulator transition in a weakly interacting many-electron system with localized single-particle states,” Ann. Phys. 321, 1126–1205 (2006).
[2] V. Oganesyan and D. A. Huse, “Localization of interacting fermions at high temperature,” Phys. Rev. B 75, 155111 (2007).
[3] C. Monthus and T. Garel, “Many-body localization transition in a lattice model of interacting fermions: Statistics of renormalized hoppings in configuration space,” Phys. Rev. B 81, 134202 (2010).
[4] D. J. Luitz, N. Laflorencie, and F. Alet, “Many-body localization edge in the random-field Heisenberg chain,” Phys. Rev. B 91, 081103 (2015).
[5] F. Andraschko, T. Enss, and J. Sirker, “Purification and many-body localization in cold atomic gases,” Phys. Rev. Lett. 113, 217201 (2014).
[6] P. Ponte, Z. Papić, F. Huveneers, and D. A. Abanin, “Many-body localization in periodically driven systems,” Phys. Rev. Lett. 114, 140401 (2015).
[7] A. Lazarides, A. Das, and R. Moessner, “Fate of many-body localization under periodic driving,” Phys. Rev. Lett. 115, 030402 (2015).
[8] R. Vasseur, S. A. Parameswaran, and J. E. Moore, “Quantum revivals and many-body localization,” Phys. Rev. B 91, 140202 (2015).
[9] M. Serbyn, Z. Papić, and D. A. Abanin, “Quantum quenches in the many-body localized phase,” Phys. Rev. B 90, 174302 (2014).
[10] D. Pekker, G. Refael, E. Altman, E. Demler, and V. Oganesyan, “Hilbert-glass transition: New universality of temperature-tuned many-body dynamical quantum criticality,” Phys. Rev. X 4, 011052 (2014).
[11] E. J. Torres-Herrera and Lea F. Santos, “Dynamics at the many-body localization transition,” Phys. Rev. B 92, 014205 (2015).
[12] M. Távora, E. J. Torres-Herrera, and L. F. Santos, “Inevitable power-law behavior of isolated many-body quantum systems and how it anticipates thermalization,” Phys. Rev. A 94, 041603 (2016).
[13] C. R. Laumann, A. Pal, and A. Scardicchio, “Many-body mobility edge in a mean-field quantum spin glass,” Phys. Rev. Lett. 113, 200405 (2014).
[14] D. A. Huse, R. Nandkishore, and V. Oganesyan, “Phenomenology of fully many-body-localized systems,” Phys. Rev. B 90, 174202 (2014).
[15] S. Gopalakrishnan, K. R. Islam, and M. Knap, “Noise-induced subdiffusion in strongly localized quantum systems,” Phys. Rev. Lett. 119, 046601 (2017).
[16] J. Hauschild, F. Heidrich-Meisner, and F. Pollmann, “Domain-wall melting as a probe of many-body localization,” Physical Review B 94, 161109 (2016).
[17] J. Herbst, J. Köhler, and P. Prelovšek, “Local spin relaxation within the random Heisenberg chain,” Phys. Rev. Lett. 111, 147203 (2013).
[18] J. Z. Imbrie, “Diagonalization and many-body localization for a disordered quantum spin chain,” Phys. Rev. Lett. 117, 027201 (2016).
[19] R. Steinigeweg, J. Herbst, F. Pollmann, and W. Brenig, “Typicality approach to the optical conductivity in thermal and many-body localized phases,” Phys. Rev. B 94, 180401 (2016).
[20] M. Znidarič, T. Prosen, and P. Prelovšek, “Many-body localization in the Heisenberg XXZ magnet in a random field,” Phys. Rev. B 77, 064426 (2008).
[21] J. H. Bardarson, F. Pollmann, and J. E. Moore, “Unbounded growth of entanglement in models of many-body localization,” Phys. Rev. Lett. 109, 017202 (2012).
[22] J. A. Kjäll, J. H. Bardarson, and F. Pollmann, “Many-body localization in a disordered quantum Ising chain,” 113, 107204 (2014).
[23] M. Serbyn, Z. Papić, and D. A. Abanin, “Criterion for many-body localization-delocalization phase transition,” Phys. Rev. X 5, 041047 (2015).
[24] D. J. Luitz, N. Laflorrence, and F. Alet, “Extended slow dynamical regime prefiguring the many-body localization transition,” Phys. Rev. B 93, 060201 (2016).
[25] M. Serbyn, Z. Papić, and D. A. Abanin, “Universal slow growth of entanglement in interacting strongly disordered systems,” Phys. Rev. Lett. 110, 260601 (2013).
[26] S. Bera, H. Schomerus, F. Heidrich-Meisner, and J. H. Bardarson, “Many-body localization characterized from a one-particle perspective,” Phys. Rev. Lett. 115, 046603 (2015).
[27] E. Altman and R. Vosk, “Universal dynamics and renormalization in many-body-localized systems,” Annu. Rev. Condens. Matter Phys. 6, 383 (2015).
[28] K. Agarwal, S. Gopalakrishnan, M. Knap, M. Müller, and E. Demler, “Anomalous diffusion and Griffiths effects near the many-body localization transition,” Phys. Rev. Lett. 114, 160401 (2015).
[29] S. Gopalakrishnan, M. Müller, V. Khemani, M. Knap, E. Demler, and D. A. Huse, “Low-frequency conductivity in many-body localized systems,” Phys. Rev. B 92, 104202 (2015).
[30] M. Znidarič, A. Scardicchio, and V. K. Varma, “Diffusive and subdiffusive spin transport in the ergodic phase of a many-body localized system,” Phys. Rev. Lett. 117, 046601 (2016).
[31] M. Mierzewski, J. Herbst, and P. Prelovšek, “Universal dynamics of density correlations at the transition to the many-body localized state,” Phys. Rev. B 94, 224207 (2016).
[32] Y. Bar Lev and D. R. Reichman, “Dynamics of many-
body localization,” Phys. Rev. B 89, 220201 (2014).
[33] Y. Bar Lev, G. Cohen, and D. R. Reichman, “Absence of diffusion in an interacting system of spinless fermions on a one-dimensional disordered lattice,” Phys. Rev. Lett. 114, 100601 (2015).
[34] O. S. Barisic, J. Koka, I. Balog, and P. Prelovsek, “Dynamical conductivity and its fluctuations along the crossover to many-body localization,” Phys. Rev. B 94, 045126 (2016).
[35] J. Bonca and M. Mierzejewski, “Delocalized carriers in the t-J model with strong charge disorder,” Phys. Rev. B 95, 214201 (2017).
[36] P. Bordia, H. Luschen, S. Scherg, S. Gopalakrishnan, M. Knap, U. Schneider, and I. Bloch, “Probing slow relaxation and many-body localization in two-dimensional quasiperiodic systems,” Phys. Rev. X 7, 041047 (2017).
[37] P. Sierant, D. Delande, and J. Zakrzewski, “Many-body localization due to random interactions,” Phys. Rev. A 95, 021601 (2017).
[38] I. Protopopov and D. Abanin, “Spin-mediated particle transport in the disordered Hubbard model,” ArXiv e-prints (2018), arXiv:1808.05764 [cond-mat.str-el].
[39] M. Schecter, T. Iadecola, and S. Das Sarma, “Configuration-controlled many-body localization and the mobility emulsion,” Phys. Rev. B 98, 174201 (2018).
[40] J. Zakrzewski and D. Delande, “Spin-charge separation and many-body localization,” Phys. Rev. B 98, 014203 (2018).
[41] D. J. Luitz and Y. Bar Lev, “Anomalous thermalization in ergodic systems,” Phys. Rev. Lett. 117, 170404 (2016).
[42] D. J. Luitz and Y. Bar Lev, “The ergodic side of the manybody localization transition,” Annalen der Physik 529, 1600350 (2016).
[43] M. Kozarzewski, P. Prelovsek, and M. Mierzejewski, “Spin subdiffusion in the disordered Hubbard chain,” Phys. Rev. Lett. 120, 246602 (2018).
[44] P. Prelovsek and J. Herbrych, “Self-consistent approach to many-body localization and subdiffusion,” Phys. Rev. B 96, 035130 (2017).
[45] Y. Bar Lev, D. M. Kennes, C. Klöckner, D. R. Reichman, and C. Karrasch, “Transport in quasiperiodic interacting systems: From superdiffusion to subdiffusion,” EPL (Europhysics Letters) 119, 37003 (2017).
[46] P. Prelovsek, J. Bonca, and M. Mierzejewski, “Transient and persistent particle subdiffusion in a disordered chain coupled to bosons,” Phys. Rev. B 98, 125119 (2018).
[47] S. S. Kondov, W. R. McGehee, W. Xu, and B. DeMarco, “Disorder-induced localization in a strongly correlated atomic Hubbard gas,” Phys. Rev. Lett. 114, 083002 (2015).
[48] M. Schreiber, S. S. Hodgman, P. Bordia, H. P. Luschen, Mark H Fischer, Ronen Vosk, Ehud Altman, Ulrich Schneider, and Immanuel Bloch, “Observation of many-body localization of interacting fermions in a quasirandom optical lattice,” Science 349, 842 (2015).
[49] J.-Y. Choi, S. Hild, J. Zeiher, P. Schauß, A. Rubio-Abadal, T. Yefsah, V. Khemani, D. A. Huse, I. Bloch, and C. Gross, “Exploring the many-body localization transition in two dimensions,” Science 352, 1547 (2016).
[50] P. Bordia, H. P. Luschen, S. S. Hodgman, M. Schreiber, I. Bloch, and U. Schneider, “Coupling Identical 1D Many-Body Localized Systems,” Phys. Rev. Lett. 116, 140401 (2016).
[51] J. Smith, A. Lee, P. Richerme, B. Neyenhuis, P. W. Hess, P. Hauke, M. Heyl, D. A. Huse, and C. Monroe, “Many-body localization in a quantum simulator with programmable random disorder,” Nat. Phys. 12, 907 (2016).
[52] A. Chandran, V. Khemani, C. R. Laumann, and S. L. Sondhi, “Many-body localization and symmetry-protected topological order,” Phys. Rev. B 89, 144201 (2014).
[53] Andrew C. Potter and Romain Vasseur, “Symmetry constraints on many-body localization,” Phys. Rev. B 94, 224206 (2016).
[54] P. Prelovsek, O. S. Barisic, and M. Znidarič, “Absence of full many-body localization in the disordered Hubbard model,” Phys. Rev. B 94, 241104 (2016).
[55] Ivan V. Protopopov, Wen Wei Ho, and Dmitry A. Abanin, “Effect of su(2) symmetry on many-body localization and thermalization,” Phys. Rev. B 96, 041122 (2017).
[56] Aaron J. Friedman, Romain Vasseur, Andrew C. Potter, and S. A. Parameswaran, “Localization-protected order in spin chains with non-abelian discrete symmetries,” Phys. Rev. B 98, 064203 (2018).
[57] M. Mierzejewski, M. Kozarzewski, and P. Prelovsek, “Counting local integrals of motion in disordered spinless-fermion and Hubbard chains,” Phys. Rev. B 97, 064204 (2018).
[58] Ivan Protopopov and Dmitry Abanin, “Spin-mediated particle transport in the disordered Hubbard model,” arXiv e-prints, arXiv:1808.05764 (2018), arXiv:1808.05764 [cond-mat.str-el].
[59] T. J. Park and Light J. C., “Unitary quantum time evolution by iterative Lanczos reduction,” J. Chem. Phys. 85, 5870 (1986).