Reconciling the cosmic age problem in the $R_h = ct$ Universe

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Many dark energy models fail to pass the cosmic age test. In this paper, we investigate the cosmic age problem associated with 20 extremely red and massive galaxies at very high redshift from Castro-Rodríguez and Lopez-Corredoira (2012) in the $R_h = ct$ Universe. These old galaxies and the $R_h = ct$ Universe have not been used to study the cosmic age problem in previous literature. By evaluating the age of the $R_h = ct$ Universe with the observational constraints from the Type Ia supernovae, and Hubble parameter, we find that the $R_h = ct$ Universe can accommodate the 20 old galaxies and quasar APM 08279+5255 at redshift $z = 3.91$ at more than $3\sigma$ confidence level. So, unlike other cosmological models, the $R_h = ct$ Universe does not suffer the cosmic age problem.

I. INTRODUCTION

Many astronomical observations, such as Type Ia supernovae (SNe Ia) [1–3], the cosmic microwave background (CMB) [4–6] and large-scale structure (LSS) [7], indicate that the Universe is undergoing an accelerated expansion, which suggests that our universe may have an extra component like dark energy. The nature of dark energy is still unknown, but the simplest and most interesting candidate is the cosmological constant model [8]. This model can consist with most of astronomical observations. The latest observation gives that the present cosmic age is about $t_0 = 13.82$ Gyr in ΛCDM model [4], but it still suffer from the cosmic age problem [9, 10]. The cosmic age problem is that all objects at any redshift $z$ must be younger than the age of the universe at $z$. In previous literatures, many cosmological models have been tested by the old quasar APM 08279+5255 with age $2.1 \pm 0.3$ Gyr at $z = 3.91$ [11, 12], such as the ACDM [11, 13], Λ(t) model [14], the interacting dark energy models [10], Chaplygin gas model [15, 16], holographic dark energy model [17], braneworld models [18–20] and conformal gravity model [21]. But all of these models have a serious age problem. The conformal gravity model can only accommodate this quasar at $3\sigma$ confidence level [21].

In this paper, we will use 20 extremely red and massive galaxies at very high redshift [22] to test the conformal gravity model again, and we will find the conformal gravity model can only accommodate these galaxies at $1\sigma$ confidence level. Then we will focus on using these galaxies to investigate the cosmic age problem in $R_h = ct$ Universe. The 20 extremely red and massive galaxies are selected in the XMM-Large scale structure survey [22]. The age of these galaxies listed in Table 1 are obtained with a synthesis model [23, 24]. We find these galaxies can give stronger constraints on the age of universe than the old quasar APM 08279+5255. These old galaxies have not been used to test the cosmic age problem before. The $R_h = ct$ Universe is a cosmic model which is closely restricted by the cosmological principle and Weyl’s postulate [25]. The gravitational horizon $R_h$ is always equals to $ct$. The $R_h = ct$ Universe can fit the SNe Ia data well [26], explain the growth of high-$z$ quasars [27], account for the apparent absence in CMB angular correlation [28]. As we discuss above, many cosmological models can not pass the age test. Whether the $R_h = ct$ Universe suffers the cosmic age problem in still unknown.

The structure of this paper is as follows. In section 2, we will test the conformal gravity model which Yang et al. have done in 2013 [21] again, but use the 20 extremely red and massive galaxies at very high redshift. And we will give our new result that the conformal gravity model can only accommodate these galaxies at $1\sigma$ confidence level. In section 3, we introduce the $R_h = ct$ Universe. In section 4, we give the constraint on the $R_h = ct$ Universe using SNe Ia, and H(z) data. Then we will test the $R_h = ct$ Universe with the 20 extremely red and massive galaxies. Conclusions will be presented in section 5.

II. CONFORMAL GRAVITY MODEL

Conformal gravity (CG) model is a simplified version of the original Weyl’s theory and now it always is used to explain the disk galaxies rotation curves without dark matter [29–31]. And in CG model, the equation about $H(z)$
is:

\[ H^2 = H_0^2(\Omega_\Lambda + \Omega_k a^{-2} - \Omega_m^{-3} - \Omega_r^{-4}) \]  

(1)

where \( \Omega_\Lambda \) is the present dark energy density, \( \Omega_k \) is the present energy density which the spatial curvature constant represents, \( \Omega_m \) is the present energy density of matter, \( \Omega_r \) is the present fractional radiation density. All of these parameter are positive and they have \( \Omega_\Lambda + \Omega_k - \Omega_m - \Omega_r = 1 \) \[30, 31\]. So we can find the difference between Eq. (1) and the standard Friedmann equation (\( H^2 = H_0^2(\Omega_\Lambda + \Omega_k a^{-2} + \Omega_m^{-3} + \Omega_r^{-4}) \)) is that the negative sign in front of the matter parameter \( \Omega_m \) and \( \Omega_r \). the deceleration parameter \( q = -\frac{a\ddot{a}}{a^2} = -\frac{\Omega_m}{2} - \Omega_r - \Omega_\Lambda \) is always positive, so the expansion of universe always accelerate.

We use the same method as Yang et al. in \[21\], and find the conformal gravity model can only accommodate these galaxies, especially the galaxy named 14598 with age is 2.744 Gyr and \( z = 3.287 \), at 1σ confidence level if we only use SNe Ia data (see fig. 1).

### III. THE \( R_h = ct \) UNIVERSE

The \( R_h = ct \) Universe is a cosmic model which is closely restricted by the cosmological principle and Weyl’s postulate \[25, 32\]. For a certain age of universe \( t \), there is a limiting observable distance \( R_h(t) \), which is called cosmic horizon. And any signal beyond cosmic horizon can not be observed by us. It is defined as:

\[ R_h = \frac{2GM(R_h)}{c^2}, \]

(2)

where \( M(R_h) \) is the total mass enclosed within \( R_h \) \[23, 33\]. From Eq. (2), we can find that cosmic horizon is Schwarzschild radius. If we set the matter density is \( \rho \), then \( M(R_h) = 4\pi R_h^3 \rho / 3 \), so it yields

\[ R_h = \frac{3c^4}{8\pi G \rho}. \]

(3)

The expansion of the universe is calculated from Friedmann equation

\[ H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3c^2} - \frac{k c^2}{a^2}, \]

(4)

where \( H = \dot{a}/a \) is Hubble parameter, \( a \) is scale factor, \( k \) is the spatial curvature constant and for \( k = -1, 0 \) and +1 corresponds an open, flat and closed universe, respectively. If we assume the universe is flat, from Eq. (3) and Eq. (4), we have \( H = c/R_h \). For the \( R_h = ct \) universe, we have \( R_h = ct \). We obtain

\[ H = \frac{\dot{a}}{a} = \frac{1}{t}, \]

(5)

where \( t \) is the age of universe. Solving Eq. (5) with \( a = \frac{1}{1+z} \) and initial condition \( H = H_0 \) when \( z = 0 \), where \( H_0 \) is Hubble constant, we can get

\[ H = H_0(1+z). \]

(6)

The luminosity distance in the \( R_h = ct \) Universe is \[33\]

\[ d_L = (1+z)R_h(t_0) \ln(1+z) = \frac{c(1+z)}{H_0} \ln(1+z), \]

(7)

where \( t_0 \) is the age of the local universe.

### IV. OBSERVATIONAL CONSTRAINTS ON THE \( R_h = ct \) UNIVERSE

In this section, we constrain the \( R_h = ct \) Universe using the Union 2.1 SNe Ia data \[34\] and the observed Hubble parameter data \( H(z) \). Then we test the model with the 20 extremely red and massive galaxies at very high redshift and the old quasar APM 08279 + 5255 based on the principle that all objects are younger than its local universe.
A. Constrain the $R_h = ct$ universe with SNe Ia and $H(z)$ data

SNe Ia are considered as the best standard candles to measure distance and investigate the expansion of the universe. The Hubble parameter $H(z)$ reveals the expansion of the universe directly. So we use the SNe Ia and $H(z)$ data to constrain the $R_h = ct$ Universe. The Union 2.1 sample contains 580 SNe Ia at redshift less than 1.5 \[34, 36\]. For each SN Ia, the catalog gives its redshift $z_i$, distance modulus $\mu_{obs}(z_i)$ and its corresponding error $\sigma_i$. The theoretical distance modulus is defined as

$$\mu_{th}(z_i) = 5 \log_{10} d_L(z_i) + 25.$$ (8)

We can get theoretical distance modulus $\mu_{th}(z_i)$ for each SN Ia from Eq. (7). The $\chi^2$ for SNe Ia is

$$\chi^2_{SN}(H_0) = \sum_{i=1}^{580} \frac{(\mu_{th}(z_i) - \mu_{obs}(z_i))^2}{\sigma_i^2}.$$ (9)

So $\chi^2_{SN}$ has only one parameter $H_0$. We can get the best-fit $H_0$ by minimize $\chi^2_{SN}$ (see Table 2). \[26\] also found that the $R_h = ct$ Universe can well fit the Union 2.1 sample.

The 28 Hubble parameters we use are obtained from previous published literatures \[37–43\]. These Hubble parameter data is compiled in \[44\]. The $\chi^2$ for $H(z)$ is

$$\chi^2_H(H_0) = \sum_{i=1}^{28} \frac{(H_{th}(z_i) - H_{obs}(z_i))^2}{\sigma_{H_i}^2}.$$ (10)

The total $\chi^2$ is $\chi^2_{tot}(H_0) = \chi^2_{SN}(H_0) + \chi^2_H(H_0)$. Then we minimize the total $\chi^2_{tot}$ to get the best-fit parameter $H_0$ of the $R_h = ct$ Universe.

From Table 2, we can find that the best-fit Hubble constant is $H_0 = 66.62 \pm 0.22$ km s$^{-1}$ Mpc$^{-1}$ at confidence 1σ confidence level with $\chi^2_{min} = 680.09$. After including the 28 Hubble parameter data, the best-fit Hubble parameter is $H_0 = 66.46 \pm 0.22$ km s$^{-1}$ Mpc$^{-1}$ at confidence 1σ confidence level with $\chi^2_{min} = 722.72$. Recently, Planck program gives the Hubble constant $H_0 = 67.3 \pm 1.2$ km s$^{-1}$ Mpc$^{-1}$ in the ΛCDM model, which is consistent with our result.

B. Testing the $R_h = ct$ universe with old objects

The old objects are usually used to test cosmological models, especial the old high redshift objects \[45\] (Lima et al. 2009). In previous literatures, many cosmological models can not pass the cosmic age test. We use the 20 extremely red and massive galaxies at very high redshift and the old quasar APM 08279 + 5255 to test the $R_h = ct$ Universe. We believe the principle that all objects at any redshift $z$ must be younger than the age of the universe at $z$, i.e.,

$$t_{obj}(z) < t_{cos}(z),$$

where $t_{obj}(z)$ is the age of a object at redshift $z$, and $t_{cos}(z)$ is the age of the universe at redshift $z$. The age of a flat universe is given as \[15\]

$$t_{cos}(z) = \int_0^\infty \frac{d\tilde{z}}{(1 + \tilde{z})H(\tilde{z})}.$$ (11)

From Eq. (11), the age of the $R_h = ct$ Universe at redshift $z$ is

$$t_{cos}(z) = \frac{1}{H_0(1 + z)}.$$ (12)

We use the best-fit value of Hubble constant $H_0 = 66.62 \pm 0.22$ km s$^{-1}$ Mpc$^{-1}$ from SNe Ia data to calculate the age of the universe. From Fig. 2 we find that the $R_h = ct$ Universe accommodates the age of all of the 20 galaxies and the old quasar APM 08279 + 5255 at 3σ confidence level. In Fig. 2, the redline shows the best fit, and blue lines are 3σ dispersion. Then we use the best-fit value of Hubble constant $H_0 = 66.46 \pm 0.22$ km s$^{-1}$ Mpc$^{-1}$ from SNe Ia and Hubble parameter data to test the $R_h = ct$ Universe. In Fig. 3 the redline shows the best fit, and blue lines are 3σ dispersion. We can also find the $R_h = ct$ Universe can accommodate the age of all of the 20 galaxies and the old quasar APM 08279 + 5255 at more than 3σ confidence level.
V. CONCLUSIONS

In this paper, we test the cosmic age problem in the \( R_h = ct \) Universe using 20 extremely red and massive galaxies at very high redshift and the old quasar APM 08279 + 5255. The Hubble constant \( H_0 \) is derived from 580 SNe Ia and 28 Hubble parameter data. We find the best-fit value of Hubble constant \( H_0 = 66.62 \pm 0.22 \text{ km s}^{-1} \text{ Mpc}^{-1} \) at 1\( \sigma \) confidence level using SNe Ia data. For this result, the age of local \( R_h = ct \) universe \( t_0 = 14.72 \pm 0.04 \text{ Gyr} \). And if we fit the \( R_h = ct \) Universe with the SNe Ia and H(z) data, we get the Hubble constant \( H_0 = 66.46 \pm 0.22 \text{ km s}^{-1} \text{ Mpc}^{-1} \) at the 1\( \sigma \) confidence level. The age of local universe is \( t_0 = 14.72 \pm 0.04 \text{ Gyr} \). From Fig. 2 and Fig. 3, we find that the \( R_h = ct \) Universe can accommodate the 20 extremely red and massive galaxies at very high redshift and the old quasar APM 08279 + 5255 at more than 3\( \sigma \) confidence level. So unlike other cosmological models, the \( R_h = ct \) Universe has no cosmic age problem. So the \( R_h = ct \) Universe is a promising cosmological model.

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[1] A. G. Riess et al., Astron. J 116, 1009 (1998).
[2] S. Perlmutter et al., Astrophys. J 517, 565 (1999).
[3] B. P. Schmidt, et al., ApJ, 507, 46 (1998).
[4] D. N. Spergel, et al., ApJS, 148, 175 (2003).
[5] E. Komatsu, et al., ApJS, 180, 330 (2008).
[6] Planck Collaboration, et al., arXiv: 1303.5076 (2013).
[7] M. Tegmark, et al., PhRvD, 74, 123507 (2006).
[8] S. M. Carroll, et al., ARA&A, 30, 499 (1992).
[9] R. J. Yang, S. N. Zhang, MNRAS, 407, 1835 (2010).
[10] S. Wang, X. D. Li, M. Li, PhRvD, 82, 103006 (2010).
[11] A. C. S. Friaca, J. S. Alcaniz, & J. A. S. Lima, MNRAS, 362, 1295 (2005).
[12] S. Komossa, G. Hasinger, & N. Schartel, ApJL, 573, L77 (2002).
[13] J. S. Alcaniz, J. A. S. Lima, & J. V. Cunha, MNRAS, 340, L39 (2003).
[14] J. V. Cunha, & R. C. Santos, IJMPD, 13, 1321 (2004).
[15] J. S. Alcaniz, D. Jain, & A. Dev, PhRvD, 67, 043514 (2003).
[16] F. Y. Wang, Z. G. Dai, & S. Qi, RAA, 547, 557 (2009).
[17] H. Wei, & S. N. Zhang, PhRvD, 76, 063003 (2007).
[18] M. S. Movahed, et al., MNRAS, 388, 197 (2008).
[19] U. Alam, & V. Sahni, PhRvD, 73, 084024 (2006).
[20] N. Pires, Z. H. Zhu, & J. S. Alcaniz, PhRvD, 73, 123530 (2006).
[21] R. J. Yang, et al., PhLB, 727, 43 (2013).
[22] N. Castro-Rodríguez, M. Lopez-Corredoira, A&A, 537, A31 (2012).
[23] M. Lopez-Corredoira, AJ, 139, 540 (2010).
[24] A. Vazdekis, E. Casuso, R. F. Peletier, & J. E. Beckman, ApJS, 106, 307 (1996).
[25] F. Melia, MNRAS, 382, 1917 (2007).
[26] F. Melia, AJ, 144, 110 (2012).
[27] F. Melia, PhRvD, 764, 72 (2013).
[28] F. Melia, AJ, 480, 1 (2014).
[29] P. D. Mannheim, AJ, 419, 150 (1993).
[30] P. D. Mannheim, AJ, 471, 659 (1997).
[31] P. D. Mannheim, AJ, 1, 12 (2001).
[32] F. Melia, & A. Shevchuk, MNRAS, 419, 2579 (2012).
[33] F. Melia, Int. J. Mod. Phys. D, 18, 1113 (2009).
[34] N. Suzuki, et al., ApJ, 746, 85 (2012).
[35] M. Kowalski, et al., ApJ, 686, 749 (2008).
[36] R. Amanullah, et al., ApJ, 716, 712 (2010).
[37] R. Jimenez, L. Verde, T. Treu, & D. Stern, ApJ, 593, 622 (2003).
[38] J. Simon, et al., Phys. Rev. D, 71, 123001 (2005).
[39] E. Gaztañaga, & A. Cabré, et al., MNRAS, 399, 1663 (2009).
[40] D. Stern, R. Jimenez, L. Verde, M. Kamionkowski, & S. A. Stanford, J. Cosmol. Astropart. Phys., 02, 008 (2010).
[41] C. Blake, et al., MNRAS, 425, 405 (2012).
[42] M. Moresco, et al., J. Cosmol. Astropart. Phys. 08, 006 (2012).
[43] C. Zhang, et al., [arXiv:1207.4511] (2012).
[44] O. Farooq, & B. Ratra, ApJL, 766, L7 (2013).
[45] J. A. S. Lima, J. F. Jesus, & J. V. Cunha, ApJL, 690, L85 (2009).
| UDS Id | RA(deg) | Dec(deg) | $z$  | $t_{obs}$(Gyr) |
|--------|---------|----------|------|---------------|
| 06165  | 34.43191| -5.48688 | 2.511| 2.553         |
| 79102  | 34.14750| -4.77846 | 2.535| 2.355         |
| 25371  | 34.73086| -5.29274 | 2.536| 2.238         |
| 01559  | 34.40150| -5.53476 | 2.540| 2.511         |
| 40812* | 34.17666| -5.13515 | 2.565| 2.716         |
| 08303* | 34.29894| -5.46366 | 2.581| 2.536         |
| 24756  | 34.73077| -5.29849 | 2.668| 2.665         |
| 65143  | 34.40916| -4.91072 | 2.688| 2.173         |
| 78891  | 34.59163| -4.78038 | 2.705| 2.360         |
| 51547  | 34.90864| -5.03568 | 2.729| 2.684         |
| 39991* | 34.73174| -5.14295 | 2.754| 2.574         |
| 11488  | 34.64674| -5.43199 | 2.813| 2.712         |
| 85772  | 34.38530| -4.71782 | 2.852| 2.487         |
| 28153  | 34.28822| -5.26259 | 2.889| 2.043         |
| 28841* | 34.14779| -5.25500 | 2.896| 2.716         |
| 44569  | 34.42194| -5.10082 | 2.948| 1.758         |
| 10585  | 34.79973| -5.44024 | 3.033| 2.394         |
| 26888  | 34.24049| -5.27663 | 3.112| 2.430         |
| 14598* | 34.68966| -5.40086 | 3.287| 2.744         |
| 93574* | 34.28374| -4.63816 | 3.744| 2.309         |

**TABLE I:** The properties of 20 red and massive galaxies from [22].

| Observations | $H_0/(\text{km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1})$ | $\chi^2_{\text{min}}/\text{dof}$ |
|--------------|----------------------------------|-------------------------------|
| SNe Ia        | 66.62±0.22                      | 1.17                          |
| SNe Ia + H(z) | 66.46±0.22                      | 1.19                          |

**TABLE II:** The best-fit values of the Hubble constant $H_0$ in the $R_h = ct$ Universe.
The best-fitting value: $H_0 = 69.151, q_0 = -0.291$. And it gives the age of the present universe is 15.82 Gyr. The blue line represents the age of the old quasar APM 08279 + 5255 in the $H_0 - q_0$ plane, and the red line represents the age of the old galaxy 14598* in the $H_0 - q_0$ plane.
FIG. 2: The red line shows the evolution of cosmic age in the $R_h = ct$ using the best-fit from SNe Ia, the blue lines are the 3σ deviation (99.7% confidence region). The blue circles are the 20 extremely red and massive galaxies, and the star is the old quasar APM 08279+5255. We can find all the objects are below the lines, which means all the galaxies are younger than the age of the $R_h = ct$ Universe. The insert shows the dispersion and data clearly.
FIG. 3: The same as Fig 2 but using the best-fit $H_0$ from SNe Ia and Hubble parameter $H(z)$. 