Non-thermal Gravitino Production
After Large-Field Inflation

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Supersymmetry, gravitino, and cosmology

- **Supersymmetry (SUSY)** is a well-motivated candidate of physics beyond the Standard Model.

- **Supergravity (SUGRA)** is its local version.

- **Gravitino** is the SUSY partner of graviton, and it is the gauge fermion field of SUSY with spin 3/2.

- Basically, its interaction is gravitational strength, and gravitino is **long-lived**. In general, long-lived particles affect cosmology.
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Cosmological gravitino problem

Stable gravitino

- It may exceed the observed dark matter abundance. (The annihilation cross-section is small.)

Unstable gravitino

- Its decay products alters light element abundance by affecting big-bang nucleosynthesis (BBN).
- They also affect cosmic spectrum including cosmic microwave background (CMB), and $\gamma$-ray background etc.
- The abundance of lightest SUSY particle (LSP) from gravitino decay must also satisfy the dark matter bound.

It’s very important to estimate the gravitino abundance!
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Gravitino problem

BBN bound on gravitino abundance

Figure: BBN constraints on the gravitino yield [Kawasaki et al., 2008].

Fitting formula for the gravitino yield $Y_{3/2}$ in [Kawasaki et al., 2008]:

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} \simeq 2.3 \times 10^{-14} \times \left( \frac{T_R}{10^8 \text{GeV}} \right) + \ldots$$  \hspace{1cm} (1)
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Production mechanisms of gravitino

Thermal production

Production by scattering processes in a thermal bath.

Non-thermal production

Productions other than thermal production. Typically, from decay of heavier particles like inflaton or moduli.

- Reheating: perturbative decay of the inflaton around the minimum of the inflaton potential
- Preheating: non-perturbative production of particles at the early stage of inflaton oscillation
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Gravitino production at preheating [∼ 2000]
[Maroto and Mazumdar, 2000, Kallosh et al., 2000a, Giudice et al., 1999, Kallosh et al., 2000b, Nilles et al., 2001c, Nilles et al., 2001b]

- Inflation sector was a toy model.
- SUSY breaking by Polonyi model → Polonyi problem.
- Does not correspond to the late-time gravitino abundance.
- Lack of analytical understanding.

Gravitino from perturbative heavy scalar decay [∼ 2006]
[Endo et al., 2006a, Nakamura and Yamaguchi, 2006, Kawasaki et al., 2006a, Asaka et al., 2006, Dine et al., 2006] and more.
Not so trivial (mixing between inflaton/moduli $\phi$ and SUSY breaking field $z$ is important).

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Problems of steepness and negativity for inflaton potential

An exponential factor and a negative contribution.

\[ V = e^K \left( g^{i\bar{j}} D_i W \bar{D}_{\bar{j}} \bar{W} - 3|W|^2 \right) + \frac{1}{2} f_{AB} D^A D^B, \]  \hspace{1cm} (2)

where \( D_i W = \partial_i W + \partial_i K W \).

Shift symmetry to obtain a flat potential,

\[ \phi \rightarrow \phi + c, \quad K(\phi, \bar{\phi}) = K(i(\phi - \bar{\phi})), \]  \hspace{1cm} (3)

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Two solutions to the steepness/negativity problem

More fields

A stabilizer field $X$ is introduced which satisfies $\langle X \rangle \simeq 0$.

$$W = X f(\phi), \quad V \simeq |f(\phi)|^2. \quad (4)$$

Sometimes, a stabilization term for $X$ is needed,

$$K = -\frac{1}{2} (\phi - \bar{\phi})^2 + |X|^2 - \frac{1}{\Lambda^2} |X|^4. \quad (5)$$

[Kawasaki et al., 2000, Kallosh and Linde, 2010, Kallosh et al., 2011]

More terms

Non-minimal terms are introduced.

$$K = ic(\phi - \bar{\phi}) - \frac{1}{2} (\phi - \bar{\phi})^2 - \frac{1}{\Lambda^2} (\phi - \bar{\phi})^4 + \ldots, \quad (6)$$

$$W = e^{ic\phi} f(\phi), \quad (7)$$

with $|c| \gtrsim \sqrt{3}$. [Ketov and Terada, 2014b, Ketov and Terada, 2014a]
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Our work

- A shift-symmetric inflaton with or without a stabilizer field → Realistic large-field model.

- Strongly-stabilized Polonyi field → No Polonyi problem.

- Taking into account time-dependent mixing between gravitino and a fermion. → True gravitino abundance.

- Analytic estimation → Parametric dependence is clearer.

- Reproduces results of perturbative decay → Consistent with earlier works.
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1. Start from the supergravity Lagrangian.
2. Rewrite it in terms of physical degrees of freedom. (Transverse and longitudinal modes of the gravitino.)
3. Canonically normalize them.
4. Remove mixing with other fermions. (diagonalization)
5. The form of the Lagrangian reduces to that of Majorana spinors.
   Apply the formula of fermion production by preheating.
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Production of Majorana fermion by preheating

The production rate is estimated by the background field method. Suppose that the fermion mass $m(t)$ oscillates with its amplitude $\tilde{m}$ and frequency $\Omega$. The number density grows as

$$n(t) \simeq \frac{C}{16\pi} \Omega^2 \tilde{m}^2 t. \quad (8)$$

$C = 1$ in the case of $m(t) \propto \cos(\Omega t)$. Suppose the inflaton oscillates sinusoidally, $\phi(t) \propto \cos(m_\phi t)$. $m \propto \phi^n$ leads to $\Omega = nm_\phi, (n - 2)m_\phi, \ldots$. $n = 1, 2$ can be interpreted as decay and annihilation, respectively:

$$\Gamma(n \times \phi \rightarrow \psi\psi) \sim \frac{nn_\psi}{2n_\phi t} \sim \frac{n^3C}{32\pi} \frac{\tilde{m}^2}{\phi^2_{\text{amp}}} m_\phi. \quad (9)$$

[Greene and Kofman, 1999, Peloso and Sorbo, 2000, Asaka and Nagao, 2010] See also App. B of [Ema et al., 2016].
Abstract of analyses

Summary of the section

It is very important to estimate the gravitino abundance!

Given a supergravity model, our tasks are

- To diagonalize the fermion mass matrix.
- To obtain the amplitude $\tilde{m}$ and frequency $\Omega$ of the oscillating mass eigenvalues.

\[ \phi = \phi_{\text{amp}} \cos(m_{\phi}t) \text{ and } m = \frac{m_{\phi}^2}{2\Lambda^2} \]

\[ \Rightarrow \tilde{m} = \frac{m_{\phi}^2}{2\Lambda^2} \text{ and } \Omega = 2m_{\phi}. \]

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- To diagonalize the fermion mass matrix.
- To obtain the amplitude $\tilde{m}$ and frequency $\Omega$ of the oscillating mass eigenvalues.

Example

\[
\phi = \phi_{\text{amp}} \cos(m_\phi t) \quad \text{and} \quad m = \frac{m_\phi \phi^2}{2M_{\text{Pl}}^2}.
\]

\[
\rightarrow \tilde{m} = \frac{m_\phi \phi_{\text{amp}}^2}{2M_{\text{Pl}}^2} \quad \text{and} \quad \Omega = 2m_\phi.
\]

- To apply the formula of fermion production to estimate the gravitino abundance.
Abstract of analyses

Summary of the section

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Given a supergravity model, our tasks are

- To diagonalize the fermion mass matrix.
- To obtain the amplitude $\tilde{m}$ and frequency $\Omega$ of the oscillating mass eigenvalues.

**Example**

$$\phi = \phi_{\text{amp}} \cos(m_\phi t) \text{ and } m = \frac{m_\phi \phi^2}{2M_{\text{Pl}}^2}.$$  

$$\rightarrow \tilde{m} = \frac{m_\phi \phi_{\text{amp}}^2}{2M_{\text{Pl}}^2} \text{ and } \Omega = 2m_\phi.$$  

- To apply the formula of fermion production to estimate the gravitino abundance.
1 Introduction
   - Gravitino problem
   - Context of our work
   - Abstract of analyses

2 Gravitino Lagrangian
   - Lagrangian and physical modes
   - Diagonalization of the gradient term

3 Gravitino production without a stabilizer field
   - Model and its dynamics
   - Gravitino production

4 Gravitino production with a stabilizer field
   - Model and its dynamics
   - Gravitino production

5 Conclusion
   - Summary, conclusion, and prospects
Assumptions

- Kinetic terms can be regarded as minimal, $K_{i\bar{j}} \simeq \delta_{i\bar{j}}$ after inflation.

- Scalar field configuration is real.
  (If all the parameters of the model and initial conditions are real, this assumption is satisfied.)

- $D$-term is not important.
  (Inflaton is a gauge singlet, and inflation is $F$-term-driven.)
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Lagrangian and physical modes

Supergravity Lagrangian

\[ e^{-1} \mathcal{L} = \frac{M_{P1}^2}{2} R - g_{ij} \partial_\mu \phi^i \partial^\mu \phi^*_j - V \]

\[ - \frac{1}{2} g_{ij} \left( \bar{\chi}_L^i \hat{\nabla} \chi_R^j + \bar{\chi}_R^j \hat{\nabla} \chi_L^i \right) - \frac{1}{2} \left( m_{ij} \bar{\chi}_L^i \chi_L^j + m_{ij} \bar{\chi}_R^i \chi_R^j \right) \]

\[ - \frac{1}{2} \bar{\psi}_\mu R^\mu + \frac{1}{2} \bar{\psi}_\mu \left( m_{3/2} P_R + m_{3/2}^* P_L \right) \hat{\gamma}^{\mu \nu} \psi_\nu \]

\[ + \frac{\sqrt{2}}{M_{P1}} g_{ij} \bar{\psi}_\mu \hat{\gamma}^{\nu \mu} \left( \chi_L^i \partial_\nu \phi^*_j + \chi_R^j \partial_\nu \phi^*_i \right) + \frac{1}{\sqrt{2} M_{P1}} \left( \bar{\psi} \cdot \hat{\gamma} \right) v + e^{-1} \mathcal{L}_{4f}, \]

where

\[ R^\mu \equiv \hat{\gamma}^{\mu \rho \sigma} D_\rho \psi_\sigma, \quad D_\mu \psi_\nu = \left( \partial_\mu + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} - i A_\mu \gamma_* \right) \psi_\nu - \Gamma^\rho_{\mu \nu} \psi_\rho, \]

\[ A_\mu \equiv \frac{i}{4 M_{P1}^2} \left( \partial_i K \partial_\mu \phi^i - \partial^i K \partial_\mu \phi^* \right), \quad v_L \equiv e^{K/2 M_{P1}^2} D_i W \chi_L^i + g_{ij} \phi^i \chi_R^j \]

\[ m_{3/2} \equiv e^{K/2 M_{P1}^2} \frac{W}{M_{P1}^2}, \quad m_{ij} \equiv e^{K/2 M_{P1}^2} \left[ \partial_i + \frac{\partial_i K}{M_{P1}^2} \right] D_j W - e^{K/2 M_{P1}^2} \Gamma^k_{ij} D_k W. \]
We take unitary gauge $v = 0$, and assume the FLRW background.

Equation of motion

$$\Sigma^\mu \equiv R^\mu - \tilde{\gamma}^{\mu\nu} \left( m_{3/2} \psi_\nu - \frac{\sqrt{2}}{M_{Pl}} g_{ij} \left( \chi_L \partial_\nu \phi^* \bar{j} + \chi_R \partial_\nu \phi^i \right) \right) = 0. \quad (10)$$

Constraints

$$0 = D_\mu \Sigma^\mu + \frac{m_{3/2}}{2} \tilde{\gamma}_\mu \Sigma^\mu, \quad 0 = \Sigma^0. \quad (11)$$

Decomposition

$$\vec{\psi} = \vec{\psi}^t + \left( \frac{1}{2} \vec{\gamma} - \frac{1}{2k^2} \vec{k} \left( \vec{k} \cdot \vec{\gamma} \right) \right) \psi^\ell + \left( \frac{3}{2k^2} \vec{k} - \frac{1}{2k^2} \vec{\gamma} \left( \vec{k} \cdot \vec{\gamma} \right) \right) \vec{k} \cdot \vec{\psi}, \quad (12)$$

where $\psi^\ell \equiv \vec{\gamma} \cdot \vec{\psi}$ and $\vec{\gamma} \cdot \vec{\psi}^t = \vec{k} \cdot \vec{\psi}^t = 0$.

$$i \vec{k} \cdot \vec{\psi} = \left( i \vec{\gamma} \cdot \vec{k} - a \left( m_{3/2} + \gamma_0 H \right) \right) \psi^\ell, \quad (13)$$

where $H \equiv \dot{a}/a.$
We take unitary gauge $\nu = 0$, and assume the FLRW background.

Equation of motion

$$\Sigma^\mu \equiv R^\mu - \tilde{\gamma}^\mu_\nu \left( m_{3/2} \psi_\nu - \frac{\sqrt{2}}{M_{Pl}} g_{i\bar{j}} \left( \chi^i_L \partial_\nu \phi^{*\bar{j}} + \chi^\bar{j}_R \partial_\nu \phi^i \right) \right) = 0. \quad (10)$$

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Equation of motion, constraints, and decomposition

We take unitary gauge $\nu = 0$, and assume the FLRW background.

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$$

**Constraints**

$$
0 = D_\mu \Sigma^\mu + \frac{m_{3/2}}{2} \hat{\gamma}_\mu \Sigma^\mu, \\
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$$

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$$
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$$
\vec{q} \cdot \vec{\psi} = \left( i\vec{\gamma} \cdot \vec{k} - a \left( m_{3/2} + \gamma_0 H \right) \right) \psi^\ell, \quad (13)
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We take unitary gauge $v = 0$, and assume the FLRW background.

Equation of motion

$$\Sigma^\mu \equiv R^\mu - \hat{\gamma}^{\mu \nu} \left( m_{3/2} \psi_\nu - \frac{\sqrt{2}}{M_{Pl}} g_{i j} \left( \chi^i L \partial_\nu \phi^* - \chi^j R \partial_\nu \phi \right) \right) = 0.$$  \hspace{1cm} (10)

Constraints

$$0 = D_\mu \Sigma^\mu + \frac{m_{3/2}}{2} \hat{\gamma}_\mu \Sigma^\mu, \quad \quad \quad \quad \quad 0 = \Sigma^0.$$  \hspace{1cm} (11)

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$$\Sigma^\mu \equiv R^\mu - \tilde{\gamma}^{\mu\nu}\left(m_{3/2}\psi_\nu - \frac{\sqrt{2}}{M_{Pl}}g_{i\bar{j}}(\chi^i_L\partial_\nu \phi^*\bar{j} + \chi^\bar{j}_R\partial_\nu \phi^i)\right) = 0. \tag{10}$$

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where $H \equiv \dot{a}/a$. 
Lagrangian and physical modes

Lagrangian in terms of physical degrees of freedom

\[ e^{-1} L_{3/2} = e^{-1} L_t + e^{-1} L_\ell + e^{-1} L_{\text{mix}}, \quad (14) \]

\[ e^{-1} L_t = -\frac{1}{2a^3} \bar{\psi}^t \dot{\bar{\psi}}^t + \frac{H}{2a^2} \bar{\psi}^t \gamma^0 \bar{\psi}^t - \frac{1}{2a^2} \bar{\psi}^t m_{3/2} \bar{\psi}^t, \quad (15) \]

\[ e^{-1} L_\ell = -\frac{\rho_{\text{SB}}}{4ak^2 M_{\text{Pl}}^2} \bar{\psi}_\ell \left[ \gamma^0 \partial_0 + (i\vec{\gamma} \cdot \vec{k}) \hat{A} - \frac{3a}{2} (m_{3/2} + H\gamma^0) \hat{A} - \frac{1}{2} am_{3/2} \right] \psi^\ell, \quad (16) \]

\[ e^{-1} L_{\text{mix}} = \frac{\sqrt{2}}{a^2 M_{\text{Pl}}} \bar{\psi}^\ell \gamma^0 g_{ij} \left[ (\partial_0 \phi^i) \chi^j_R + (\partial_0 \phi^* j) \chi^j_L \right], \quad (17) \]

where

\[ \hat{A} \equiv \frac{p_{\text{SB}} - \gamma^0 p_W}{\rho_{\text{SB}}}, \quad (18) \]

\[ \rho_{\text{SB}} \equiv \sum_i |\dot{\phi}_i|^2 + V_{\text{SB}}, \quad V_{\text{SB}} \equiv V + 3m_{3/2}^2 M_{\text{Pl}}^2 = \sum_i |F_i|^2, \quad (19) \]

\[ p_{\text{SB}} \equiv \sum_i |\dot{\phi}_i|^2 - V_{\text{SB}}, \quad p_W \equiv 2m_{3/2} M_{\text{Pl}}^2 = -\sum_i (\dot{\phi}_i^* F_i + \dot{\phi}_i F_i^*). \quad (20) \]
Lagrangian in terms of physical degrees of freedom

\[ e^{-1} \mathcal{L}_{3/2} = e^{-1} \mathcal{L}_t + e^{-1} \mathcal{L}_\ell + e^{-1} \mathcal{L}_{\text{mix}}, \]  

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where

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\rho_{SB} \equiv \sum_i |\dot{\phi}_i|^2 + V_{SB}, \quad V_{SB} \equiv V + 3m_{3/2}^2 M_{Pl}^2 = \sum_i |F_i|^2, \]  

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Lagrangian and physical modes

Canonical normalization

canonical fields

\[ \tilde{\psi}_c^t \equiv \sqrt{a} \tilde{\psi}^t, \quad \psi_c^\ell \equiv -\frac{\sqrt{\rho_{SB}} a^{3/2}}{\sqrt{2k^2 M_{Pl}}} i \left( \tilde{\gamma} \cdot \tilde{k} \right) \psi^\ell, \quad \chi'^i \equiv a^{3/2} \chi^i, \quad (21) \]

\[ \mathcal{L}_t = -\frac{1}{2} \tilde{\psi}_c^t \left[ \gamma^0 \partial_0 + i \left( \tilde{\gamma} \cdot \tilde{k} \right) + am_{3/2} \right] \psi_c^t, \quad (22) \]

\[ \mathcal{L}_\ell = -\frac{1}{2} \tilde{\psi}_c^\ell \left[ \gamma^0 \partial_0 - i \left( \tilde{\gamma} \cdot \tilde{k} \right) \hat{A}^i + a\hat{m}_{3/2} \right] \psi_c^\ell, \quad (23) \]

\[ \mathcal{L}_{\text{mix}} = \frac{2}{\sqrt{\rho_{SB}}} \tilde{\psi}_c^\ell i \left( \tilde{\gamma} \cdot \tilde{k} \right) \gamma^0 g_{ij} \left[ \dot{\phi}^i \psi_j^R + \dot{\phi}^* j \psi_j^L \right], \quad (24) \]

where

\[ \hat{m}_{3/2} = \frac{3Hp_W + m_{3/2} (\rho_{SB} + 3p_{SB})}{2\rho_{SB}}. \quad (25) \]
Canonical normalization

**canonical fields**

\[ \vec{\psi}_c ^t \equiv \sqrt{a} \vec{\psi} ^t, \quad \psi_c ^t \equiv - \frac{\sqrt{\rho_{SB}} a^{3/2}}{\sqrt{2 k^2 M_{Pl}}} i (\vec{\gamma} \cdot \vec{k}) \psi_c ^t, \quad \chi'^i \equiv a^{3/2} \chi ^i, \quad (21) \]

\[ L_t = - \frac{1}{2} \vec{\psi}_c ^t \left[ \gamma^0 \partial_0 + i (\vec{\gamma} \cdot \vec{k}) + a m_{3/2} \right] \vec{\psi}_c ^t, \quad (22) \]

\[ L_\ell = - \frac{1}{2} \psi_c ^\ell \left[ \gamma^0 \partial_0 - i (\vec{\gamma} \cdot \vec{k}) \hat{A}^i + a \hat{m}_{3/2} \right] \psi_c ^\ell, \quad (23) \]

\[ L_{\text{mix}} = \frac{2}{\sqrt{\rho_{SB}}} \vec{\psi}_c ^\ell i (\vec{\gamma} \cdot \vec{k}) \gamma^0 g_{ij} \left[ \dot{\phi}^i \chi'^j_R + \dot{\phi}^j_l \chi'^i_l \right], \quad (24) \]

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Preparation of diagonalization 1

Fermions $\chi_i$ ($i = 1, \ldots, N$) can be decomposed as goldstino $v$ and orthogonal fields $v_I^\perp$ ($I = 1, \ldots, N - 1$). They are packaged in $v_i \equiv (v, v_I^\perp)$.

goldstino

$$v = \sum_i \alpha_i (-F_i + \dot{\phi}_i \gamma^0) \chi_i$$

$$\equiv \sum_i \alpha_i (\cos \theta_i + \sin \theta_i \gamma^0) \chi_i = \sum_i \alpha_i e^{\gamma^0 \theta_i} \chi_i,$$

where $\alpha_i \equiv \sqrt{\rho_{SB}^i} / \rho_{SB}^i$ and $\rho_{SB}^i \equiv |\dot{\phi}_i|^2 + |F_i|^2$.

relation of the two bases

$$v_i = (O^T)^{ij} e^{\gamma^0 \theta_j} \chi_j \leftrightarrow \chi_i = e^{-\gamma^0 \theta_i} O_{ij} v_j, \quad O^T = \begin{pmatrix} \alpha_1, \ldots, \alpha_N \end{pmatrix}.$$
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where $\alpha_i \equiv \sqrt{\rho^i_{SB}/\rho_{SB}}$ and $\rho^i_{SB} \equiv |\dot{\phi}_i|^2 + |F_i|^2$.

relation of the two bases

\[ v_i = \left( O^T \right)_{ij} e^{\gamma^0 \theta_j} \chi_j \iff \chi_i = e^{-\gamma^0 \theta_i} O_{ij} v_j, \quad O^T = \begin{pmatrix} \alpha_1, \ldots, \alpha_N \end{pmatrix}. \]
Fermions \( \chi_i \ (i = 1, \ldots, N) \) can be decomposed as goldstino \( v \) and orthogonal fields \( v^I_\perp \ (I = 1, \ldots, N - 1) \). They are packaged in \( v_i \equiv (v, v^I_\perp) \).

goldstino

\[
v = \sum_i \alpha_i (-F_i + \dot{\phi}_i \gamma^0) \chi_i
\]

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where \( \alpha_i \equiv \sqrt{\rho^i_{SB}/\rho_{SB}} \) and \( \rho^i_{SB} \equiv |\dot{\phi}_i|^2 + |F_i|^2 \).

relation of the two bases

\[
v_i = \left( O^T \right)_{ij} e^{\gamma^0 \theta_j} \chi_j \quad \leftrightarrow \quad \chi_i = e^{-\gamma^0 \theta_i} O_{ij} v_j, \quad O^T = \left( \alpha_1, \ldots, \alpha_N \right).
\]
Diagonalization of the gradient term

Preparation of diagonalization 2

In terms of these variables $v_i$, the matter fermion Lagrangian reads

$$\mathcal{L}_{f, \text{kin}} = -\frac{1}{2} v_\perp^T \left[ \gamma^0 \partial_0 \delta_{IJ} + \tilde{O}_{Ti}^T e^{\gamma^0 \theta_i} \left( \tilde{\gamma} \cdot \vec{k} \right) e^{-\gamma^0 \theta_i} \tilde{O}_{iJ} \right. \left. + \tilde{O}_{Ti}^T \left( \partial_0 \theta_i \right) \tilde{O}_{iJ} + \tilde{O}_{Ti} \left( \gamma^0 \partial_0 \tilde{O}_{iJ} \right) \right] v_\perp^J , \quad (29)$$

$$\mathcal{L}_{f, \text{mass}} = -\frac{a}{2} \chi^T m_{ij} \chi_j = -\frac{a}{2} v_\perp^T \left( \tilde{O}_{Ti}^T e^{\gamma^0 \theta_i} m_{ij} e^{-\gamma^0 \theta_j} \tilde{O}_{jJ} \right) v_\perp^J . \quad (30)$$

The mixing term is now

$$\mathcal{L}_{\text{mix}} = 2 \overline{\psi}^\ell_i \left( \tilde{\gamma} \cdot \vec{k} \right) \gamma^0 \left[ \sum_i \alpha_i \sin \theta_i \chi_i \right] = 2 \overline{\psi}^\ell_i \left( \tilde{\gamma} \cdot \vec{k} \right) \gamma^0 \alpha_i \sin \theta_i e^{-\gamma^0 \theta_i} \tilde{O}_{iI} v_\perp^I . \quad (31)$$
Diagonalization of the gradient term

\[ \mathcal{L}_{\text{grad}} = -\frac{1}{2} \left( \psi_{c}^\ell \quad v_\perp \right) \left( \gamma \bar{\gamma} \cdot \bar{k} \right) \hat{A} \left( \psi_{c}^\ell \quad v_\perp \right), \]  

(32)

where

\[
\hat{A} = \begin{pmatrix}
-\hat{A}^\dagger & \alpha_i e^{-2\gamma^0 \theta_i} \tilde{O}_{ij} \\
\tilde{O}_{ii}^T \alpha_i e^{-2\gamma^0 \theta_i} & \tilde{O}_{ii}^T e^{-2\gamma^0 \theta_i} \tilde{O}_{ij}
\end{pmatrix} = O^T \text{ diag} \left( e^{-2\gamma^0 \theta_i} \right) O \equiv e^{-2\gamma^0 \hat{\theta}}.
\]  

(33)

where we used \( \hat{A} = -\sum_i e^{2\gamma^0 \theta_i} \alpha_i^2 \).

Diagonalization

\[
\begin{pmatrix}
\psi_{c}^\ell' \\
v_\perp'
\end{pmatrix} \equiv e^{-\gamma^0 \hat{\theta}} \begin{pmatrix}
\psi_{c}^\ell \\
v_\perp
\end{pmatrix},
\]  

(34)

where \( e^{\pm \gamma^0 \hat{\theta}} = O^T \text{ diag} \left( e^{\pm \gamma^0 \theta_i} \right) O \).

\[
\mathcal{L} = -\frac{1}{2} \left( \psi_{c}^\ell' \quad v_\perp' \right) \left( \gamma^0 \partial_0 + i\bar{\gamma} \cdot \bar{k} + aM \right) \left( \psi_{c}^\ell \quad v_\perp \right).
\]  

(35)
Diagonalization of the gradient term

\[ \mathcal{L}_{\text{grad}} = -\frac{1}{2} \left( \psi_c^\ell \quad \bar{v}_\perp \right) \left( i \vec{\gamma} \cdot \vec{k} \right) \hat{A} \left( \psi_c^\ell \bar{v}_\perp \right), \quad (32) \]

where

\[ \hat{A} = \begin{pmatrix} -\hat{A}^\dagger & \alpha_i e^{-2\gamma^0 \theta_i} \bar{O}_{ij} \\ \bar{O}_{ij}^T \alpha_i e^{-2\gamma^0 \theta_i} & \bar{O}_{ij}^T e^{-2\gamma^0 \theta_i} \bar{O}_{ij} \end{pmatrix} = O^T \text{ diag} \left( e^{-2\gamma^0 \theta_i} \right) O \equiv e^{-2\gamma^0 \hat{\theta}}. \quad (33) \]

where we used \( \hat{A} = -\sum_i e^{2\gamma^0 \theta_i} \alpha_i^2 \).

Diagonalization

\[ \left( \begin{array}{c} \psi_c^{\ell'} \\ v'_\perp \end{array} \right) \equiv e^{-\gamma^0 \hat{\theta}} \left( \begin{array}{c} \psi_c^\ell \\ v_\perp \end{array} \right), \quad (34) \]

where \( e^{\pm \gamma^0 \hat{\theta}} = O^T \text{ diag} \left( e^{\pm \gamma^0 \theta_i} \right) O. \)

\[ \mathcal{L} = -\frac{1}{2} \left( \psi_c^{\ell'} \quad \bar{v}'_\perp \right) \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a \mathcal{M} \right) \left( \psi_c^\ell \quad \bar{v}_\perp \right). \quad (35) \]
Diagonalization of the gradient term

Master formula of the mass matrix

\[ \mathcal{L} = -\frac{1}{2} \left( \overline{\psi^c} \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a \mathcal{M} \right) \mathcal{M} \mathcal{M}^{\dagger} \left( \psi^c \right) = \frac{1}{2} \left( \overline{\psi^c} \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a \mathcal{M} \right) \mathcal{M} \mathcal{M}^{\dagger} \left( \psi^c \right), \]  

where

\[ \mathcal{M} = e^{-\gamma^0 \hat{\theta}} \left[ \begin{array}{cc} \gamma^0 \frac{\partial}{\partial t} + \left( \hat{m}_{3/2} \right) & 0 \\ 0 & \tilde{\Omega}^T \left( \hat{m}_f + \hat{\theta}_i + \gamma^0 \tilde{\Omega} \tilde{\Omega}^T \right) \tilde{\Omega} \end{array} \right] e^\gamma^0 \hat{\theta}, \]

with

\[ \hat{m}_f \equiv e^{\gamma^0 \hat{\theta}_i} m_{ij} e^{-\gamma^0 \hat{\theta}_j}. \]

\( \gamma^0 \)-dependent mass terms (\( \gamma^0 \mathcal{M}_2 \) in \( \mathcal{M} = \mathcal{M}_1 + \gamma^0 \mathcal{M}_2 \)) can be removed, and do not contribute to the mass eigenvalues by time-dependent field redefinition \( \psi_{\text{old}} = L \psi_{\text{new}} \) with \( L \) satisfying

\[ (\partial_0 + \mathcal{M}_2) L = 0. \]

The final mass matrix is \( L^T \mathcal{M}_1 L \) [Nilles et al., 2001b].

The remaining task is to diagonalize the mass term, once specifying the model.
Diagonalization of the gradient term

Master formula of the mass matrix

\[ \mathcal{L} = -\frac{1}{2} \left( \psi_c^{\ell} \quad \nu'_\perp \right) \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a \mathcal{M} \right) \left( \psi_c^{\ell'} \quad \nu'_{\perp} \right) , \quad (36) \]

where

\[ \mathcal{M} = e^{-\gamma^0 \hat{\theta}} \left[ \gamma^0 \frac{\partial}{\partial t} + \begin{pmatrix} \hat{m}_{3/2} & 0 \\ 0 & \tilde{O}^T \left( \hat{m}_f + \dot{\theta}_i + \gamma^0 \tilde{O} \tilde{O}^T \right) \tilde{O} \end{pmatrix} \right] e^{\gamma^0 \hat{\theta}} , \quad (37) \]

with

\[ \hat{m}_f \equiv e^{\gamma^0 \theta_i} m_{ij} e^{-\gamma^0 \theta_j} . \quad (38) \]

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Master formula of the mass matrix

\[
\mathcal{L} = -\frac{1}{2} \left( \overline{\psi_c'} \mathbf{v}_\perp \right) \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a \mathcal{M} \right) \left( \psi_c' \mathbf{v}_\perp \right),
\]

(36)

where

\[
\mathcal{M} = e^{-\gamma^0 \hat{\theta}} \left[ \gamma^0 \frac{\partial}{\partial t} + \begin{pmatrix} \hat{m}_{3/2} & 0 \\ 0 & \hat{O}^T \left( \hat{m}_f + \dot{\theta}_i + \gamma^0 \hat{O} \hat{O}^T \right) \hat{O} \end{pmatrix} \right] e^{\gamma^0 \hat{\theta}},
\]

(37)

with

\[
\hat{m}_f \equiv e^{\gamma^0 \theta_i} m_{i,j} e^{-\gamma^0 \theta_j}.
\]

(38)

\(\gamma^0\)-dependent mass terms \((\gamma^0 \mathcal{M}_2\text{ in } \mathcal{M} = \mathcal{M}_1 + \gamma^0 \mathcal{M}_2)\) can be removed, and do not contribute to the mass eigenvalues by time-dependent field redefinition \(\psi_{\text{old}} = \mathcal{L} \psi_{\text{new}}\text{ with } \mathcal{L}\) satisfying

\[
(\partial_0 + \mathcal{M}_2) \mathcal{L} = 0.
\]

(39)

The final mass matrix is \(\mathcal{L}^T \mathcal{M}_1 \mathcal{L}\) [Nilles et al., 2001b].

The remaining task is to diagonalize the mass term, once specifying the model.
Summary of this section

Assumptions

Minimal Kähler, real configuration, and irrelevant $D$-terms.

After canonical normalization,

Transverse gravitino

\[ \mathcal{L}_t = - \frac{1}{2} \bar{\psi}_c^t \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a m_{3/2} \right) \psi_c^t. \]  \hspace{1cm} (40)

Longitudinal gravitino and fermion system

\[ \mathcal{L} = - \frac{1}{2} \left( \begin{array}{c} \bar{\psi}_c^\ell' \\ v_\perp' \end{array} \right)_{\text{new}} \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a L^T M_1 L \right) \left( \begin{array}{c} \psi_c^\ell' \\ v_\perp' \end{array} \right)_{\text{new}}, \]  \hspace{1cm} (41)

We should express mass eigenvalues in terms of the oscillating inflaton field.
Summary of this section

Assumptions
Minimal Kähler, real configuration, and irrelevant $D$-terms.

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$$\mathcal{L} = -\frac{1}{2} \begin{pmatrix} \psi_c' \\ v_\perp' \end{pmatrix}_{\text{new}} \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a L^T M_1 L \right) \begin{pmatrix} \psi_c' \\ v_\perp' \end{pmatrix}_{\text{new}},$$  \hspace{1cm} (41)

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Summary of this section

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Minimal Kähler, real configuration, and irrelevant $D$-terms.

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$$\mathcal{L}_t = -\frac{1}{2} \overline{\psi}_t^c \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a m_{3/2} \right) \psi_t^c. \quad (40)$$

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$$\mathcal{L} = -\frac{1}{2} \begin{pmatrix} \overline{\psi}_c^{\ell'} & \overline{v}_\perp' \end{pmatrix}_{\text{new}} \left( \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a L^T M_1 L \right) \begin{pmatrix} \psi_c^{\ell'} \\ v_{\perp}' \end{pmatrix}_{\text{new}}, \quad (41)$$

We should express mass eigenvalues in terms of the oscillating inflaton field.
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   - Gravitino problem
   - Context of our work
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   - Lagrangian and physical modes
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   - Model and its dynamics
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5 Conclusion
   - Summary, conclusion, and prospects
Model and its dynamics

Model

\[ K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (42) \]

\[ W = \frac{1}{2}m_\phi \phi^2 + \mu^2 z + W_0. \quad (43) \]

Gravitino mass at the vacuum

\[ m_{3/2}^0 \simeq \mu^2 / \sqrt{3}M_{P1} \simeq W_0 / M_{P1}^2. \quad (44) \]

Strongly stabilized Polonyi field

\[ \langle z \rangle \simeq 2\sqrt{3}M_{P1}(m_{3/2}^0 / m_z)^2, \quad \text{and} \quad m_z^2 = 12(m_{3/2}^0 M_{P1} / \Lambda)^2. \quad (45) \]

SUSY breaking and (transverse) gravitino mass

\[ |\dot{\phi}| \sim |F_\phi| \sim m_\phi \phi_{\text{amp}} \sim H M_{P1}, \quad |F_z| \sim \mu^2 \sim m_{3/2}^0 M_{P1} \gg |\dot{z}|, \quad (46) \]

\[ m_{3/2} \simeq \frac{m_\phi \phi^2}{2M_{P1}^2} + m_{3/2}^0 \simeq H \frac{\phi_{\text{amp}}}{M_{P1}} + m_{3/2}^0. \quad (47) \]
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\[ K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]  
\[ W = \frac{1}{2}m_\phi \phi^2 + \mu^2 z + W_0. \]  

Gravitino mass at the vacuum

\[ m^{0}_{3/2} \simeq \mu^2 / \sqrt{3}M_{Pl} \simeq W_0 / M_{Pl}^2. \]  

Strongly stabilized Polonyi field

\[ \langle z \rangle \simeq 2\sqrt{3}M_{Pl}(m^{0}_{3/2}/m_z)^2, \quad \text{and} \quad m^2_z = 12(m^{0}_{3/2}M_{Pl}/\Lambda)^2. \]  

SUSY breaking and (transverse) gravitino mass

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\[ m^{3/2}_{3/2} \simeq \frac{m_\phi \phi^2}{2M_{Pl}^2} + m^{0}_{3/2} \sim H \frac{\phi_{\text{amp}}}{M_{Pl}} + m^{0}_{3/2}. \]
Goldstino nature and mass eigenvalues

\[ K = -\frac{1}{2} (\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]
\[ W = \frac{1}{2} m_\phi \phi^2 + \mu^2 z + W_0. \]

Who is the goldstino?

\[ \nu \sim \begin{cases} \tilde{\phi} & \text{for } H \gtrsim m_{3/2}^0, \\ \tilde{z} & \text{for } H \lesssim m_{3/2}^0, \end{cases} \quad \nu_\perp \sim \begin{cases} \tilde{z} & \text{for } H \gtrsim m_{3/2}^0, \\ \tilde{\phi} & \text{for } H \lesssim m_{3/2}^0. \end{cases} \quad (48) \]

Longitudinal gravitino-fermion system

\[ \mathcal{L}_f = -\frac{1}{2} \left( \begin{array}{c} \psi^\ell_{\nu_\perp} \\ v_{\perp} \end{array} \right) \begin{pmatrix} \gamma^0 \partial_0 + i\vec{\gamma} \cdot \vec{k} + a\mathcal{M} \end{pmatrix} \left( \begin{array}{c} \psi^\ell_{\nu_\perp} \\ v_{\perp} \end{array} \right), \quad (49) \]

Mass eigenvalues

\[ (m_{\text{heavy}}, \ m_{\text{light}}) \approx \begin{cases} (m_\phi, \ -m_{3/2}(m_{3/2}/H)^2) & \text{for } H \gtrsim m_{3/2}^0, \\ (m_\phi, \ -m_{3/2}) & \text{for } H \lesssim m_{3/2}^0. \end{cases} \quad (50) \]
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\[ K = -\frac{1}{2} (\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]
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Who is the goldstino?

\[ v \sim \begin{cases} \bar{\phi} & \text{for } H \gg m_0^{3/2}, \\ \bar{z} & \text{for } H \lesssim m_0^{3/2} \end{cases}, \quad
v_\perp \sim \begin{cases} \bar{z} & \text{for } H \gg m_0^{3/2} \\ \bar{\phi} & \text{for } H \lesssim m_0^{3/2} \end{cases}. \tag{48} \]

Longitudinal gravitino-fermion system

\[ \mathcal{L}_f = -\frac{1}{2} \left( \bar{\psi}_c \gamma_\ell' \psi_c' \right) \left[ \gamma_0 \partial_\ell + i \vec{\gamma} \cdot \vec{k} + aM \right] \left( \begin{array}{c} \psi_c' \\ \psi_\perp' \end{array} \right), \tag{49} \]

Mass eigenvalues

\[ (m_{\text{heavy}}, \quad m_{\text{light}}) \approx \begin{cases} (m_\phi, \quad -m_{3/2} (m_{3/2}/H)^2) & \text{for } H \gg m_0^{3/2} \\ (m_\phi, \quad -m_{3/2}) & \text{for } H \lesssim m_0^{3/2} \end{cases}. \tag{50} \]
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Longitudinal gravitino-fermion system

\[ \mathcal{L}_f = -\frac{1}{2} \left( \frac{\bar{\psi}_c^{\ell'}}{v_\perp^i} \right) \left[ \gamma^0 \partial_0 + i\vec{\gamma} \cdot \vec{k} + a\mathcal{M} \right] \left( \frac{\psi_\ell'}{v_\perp} \right), \quad (49) \]

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Time dependence of various quantities

Figure: **Left**: Time dependence of $m_\phi, H$ and $m_{3/2}$ in single superfield inflation model. **Right**: Time evolution of mass eigenvalues of $(\psi^\ell, v_\perp)$ are shown by thick solid lines. The red (blue) segments show that the main composition of the mass eigenstate is $\psi^\ell$ ($v_\perp$). Dashed and dot-dashed lines show $M_{22}$ and $M_{11}$, respectively.
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Transverse gravitino production

The annihilation rate is

$$\Gamma(\phi \phi \rightarrow \psi^t \psi^t) \simeq \frac{C}{4\pi} \frac{\tilde{m}^2}{\phi_{amp}^2} m_\phi \simeq \frac{3C}{16\pi} \frac{H^2 m_\phi}{M_{Pl}^2}, \quad (51)$$

where $\sqrt{3}\tilde{m} \simeq H\phi_{amp}/2M_{Pl}$ is the oscillation amplitude of the mass of the produced fermion.

Transverse gravitino yield

$$\frac{n_{3/2}^{(t)}}{s} \simeq \left( \frac{\Gamma(\phi \phi \rightarrow \psi^t \psi^t)}{H} \right) \bigg|_{H=H_{inf}} \frac{3T_R}{4m_\phi} \simeq \frac{9C}{64\pi} \frac{H_{inf} T_R}{M_{Pl}^2}$$

$$\simeq 8 \times 10^{-16} C \left( \frac{H_{inf}}{10^{13} \text{GeV}} \right) \left( \frac{T_R}{10^{10} \text{GeV}} \right). \quad (52)$$
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Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_\perp$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

\[
\Gamma(\phi\phi \to v_\perp v_\perp) \lesssim \frac{3C}{16\pi} \left( \frac{m_{3/2}^0}{H} \right)^2 \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}.
\] (53)

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

\[
\Gamma(\phi\phi \to \psi^\ell\psi^\ell) \lesssim \frac{3C}{16\pi} \frac{H^2 m_\phi}{M_{Pl}^2}.
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The dominant contribution comes when $H \simeq m_{3/2}^0$,

\[
\Gamma(\phi\phi \to \psi^\ell\psi^\ell) \lesssim \frac{3C}{16\pi} \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}.
\] (55)

Longitudinal gravitino yield

\[
\frac{n_{3/2}^{(\ell)}}{s} \simeq 8 \times 10^{-23} C \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right).
\] (56)
Longitudinal gravitino production

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$$\Gamma(\phi\phi \rightarrow v_\perp v_\perp) \lesssim \frac{3C}{16\pi} \left(\frac{m_{3/2}^0}{H}\right)^2 \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}.$$  \hspace{1cm} (53)

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Two sources of longitudinal gravitino

- $v_\perp$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

$$\Gamma(\phi\phi \to v_\perp v_\perp) \lesssim \frac{3C}{16\pi} \left(\frac{m_{3/2}^0}{H}\right)^2 \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}. \quad (53)$$

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

$$\Gamma(\phi\phi \to \psi^\ell \psi^\ell) \lesssim \frac{3C}{16\pi} \frac{H^2 m_\phi}{M_{Pl}^2}. \quad (54)$$

The dominant contribution comes when $H \simeq m_{3/2}^0$,

$$\Gamma(\phi\phi \to \psi^\ell \psi^\ell) \lesssim \frac{3C}{16\pi} \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}. \quad (55)$$

Longitudinal gravitino yield

$$\frac{n_{3/2}(\ell)}{s} \simeq 8 \times 10^{-23} C \left(\frac{m_{3/2}^0}{10^6 \text{ GeV}}\right) \left(\frac{T_R}{10^{10} \text{ GeV}}\right). \quad (56)$$
Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_\bot$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

\[
\Gamma(\phi\phi \rightarrow v_\bot v_\bot) \lesssim \frac{3C}{16\pi} \left(\frac{m_{3/2}^0}{H}\right)^2 \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}. \tag{53}
\]

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

\[
\Gamma(\phi\phi \rightarrow \psi^\ell \psi^\ell) \lesssim \frac{3C}{16\pi} \frac{H^2 m_\phi}{M_{Pl}^2}. \tag{54}
\]

The dominant contribution comes when $H \simeq m_{3/2}^0$,

\[
\Gamma(\phi\phi \rightarrow \psi^\ell \psi^\ell) \lesssim \frac{3C}{16\pi} \frac{(m_{3/2}^0)^2 m_\phi}{M_{Pl}^2}. \tag{55}
\]

Longitudinal gravitino yield

\[
\frac{n_{3/2}^{(\ell)}}{s} \simeq 8 \times 10^{-23} C \left(\frac{m_{3/2}^0}{10^6 \text{ GeV}}\right) \left(\frac{T_R}{10^{10} \text{ GeV}}\right). \tag{56}
\]
Inflatino production

The mass of the heavier spinor $\nu_\perp$ has an oscillating part,

$$m_{\text{heavy}} \simeq m_\phi + 2\hat{m}_{3/2}.$$  \hfill (57)

This allows production at $H \sim H_{\text{inf}}$ if inflaton amplitude is Planck scale.

Inflatino yield

$$\frac{n_{\tilde{\phi}}}{s} \simeq \frac{27 C}{16\pi} \frac{H_{\text{inf}} T_R}{M_{\text{Pl}}^2} \simeq 9 \times 10^{-15} C \left( \frac{H_{\text{inf}}}{10^{13} \text{GeV}} \right) \left( \frac{T_R}{10^{10} \text{GeV}} \right),$$  \hfill (58)

where we have used $\tilde{m} \simeq 2\hat{m}_{3/2} \simeq 3H_{\text{inf}} \simeq \sqrt{3}m_\phi$.  

\[35/55\]
The mass of the heavier spinor $v_\perp$ has an oscillating part,

$$m_{\text{heavy}} \simeq m_\phi + 2\hat{m}_{3/2}.$$  \hfill (57)

This allows production at $H \sim H_{\text{inf}}$ if inflaton amplitude is Planck scale.

**Inflaton yield**

$$\frac{n_{\tilde{\phi}}}{s} \simeq \frac{27C}{16\pi} \frac{H_{\text{inf}}T_R}{M^2_{\text{Pl}}} \simeq 9 \times 10^{-15} C \left( \frac{H_{\text{inf}}}{10^{13} \text{GeV}} \right) \left( \frac{T_R}{10^{10} \text{GeV}} \right),$$  \hfill (58)

where we have used $\tilde{m} \simeq 2\hat{m}_{3/2} \simeq 3H_{\text{inf}} \simeq \sqrt{3}m_\phi$. 
Gravitino production

Higher power inflaton potential

\[
K = -\frac{1}{2} (\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2},
\]

(59)

\[
W = \frac{1}{n} \lambda \phi^n + \mu^2 z + W_0.
\]

(60)

Background expansion is different from the quadratic case.

For the quartic potential \(n = 3\), it is \(\phi_{\text{amp}} \propto a^{-1}\) and \(H^2 \propto a^{-4}\), and the gravitino yields are

\[
\frac{n^{(t)}}{s} \approx \frac{C}{64\pi} \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \left( \frac{H_{\text{inf}}}{M_{\text{Pl}}} \right)^{3/2} \approx 2 \times 10^{-11} C \left( \frac{H_{\text{inf}}}{10^{13} \text{ GeV}} \right)^{3/2},
\]

(61)

\[
\frac{n^{(t)}}{s} \approx \frac{C}{64\pi} \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \left( \frac{m_{3/2}^0}{M_{\text{Pl}}} \right)^{3/2} \approx 6 \times 10^{-22} C \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right)^{3/2},
\]

(62)
Gravitino production

Higher power inflaton potential

\[ K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad \text{(59)} \]

\[ W = \frac{1}{n} \lambda \phi^n + \mu^2 z + W_0. \quad \text{(60)} \]

Background expansion is different from the quadratic case. For the quartic potential \((n = 3)\), it is \(\phi_{\text{amp}} \propto a^{-1}\) and \(H^2 \propto a^{-4}\), and the gravitino yields are

\[ \frac{n^{(t)}_{3/2}}{s} \simeq \frac{C}{64\pi} \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \left( \frac{H_{\text{inf}}}{M_{\text{Pl}}} \right)^{3/2} \simeq 2 \times 10^{-11} C \left( \frac{H_{\text{inf}}}{10^{13} \text{ GeV}} \right)^{3/2}, \quad \text{(61)} \]

\[ \frac{n^{(\ell)}_{3/2}}{s} \simeq \frac{C}{64\pi} \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \left( \frac{m^0_{3/2}}{M_{\text{Pl}}} \right)^{3/2} \simeq 6 \times 10^{-22} C \left( \frac{m^0_{3/2}}{10^6 \text{ GeV}} \right)^{3/2}, \quad \text{(62)} \]
Summary of gravitino abundance

Figure: Dependence of gravitino abundance $Y_{3/2} = n_{3/2}/s$ on the reheating temperature $T_R$. Parameters are set as $m_{3/2} = 10^6$ GeV and $H_{\text{inf}} = 10^{13}$ GeV.
Gravitino production

Summary of this section

**Quadratic inflaton potential**

- The transverse gravitino is produced dominantly at $H \sim H_{\text{inf}}$.

- The longitudinal gravitino is produced dominantly at $H \sim m_{3/2}^0$, and suppressed compared to the transverse mode by a factor $(m_{3/2}^0/H_{\text{inf}})^2$.

- The inflatino production is similar to the transverse gravitino.

**Quartic inflaton potential**

- Gravitino (mainly transverse) is copiously produced even for low reheating temperature.
Summary of this section

**Quadratic inflaton potential**

- The transverse gravitino is produced dominantly at $H \sim H_{\text{inf}}$.
- The longitudinal gravitino is produced dominantly at $H \sim m_{3/2}^0$, and suppressed compared to the transverse mode by a factor $(m_{3/2}^0/H_{\text{inf}})^2$.
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**Quartic inflaton potential**

- Gravitino (mainly transverse) is copiously produced even for low reheating temperature.
Gravitino production

Summary of this section

Quadratic inflaton potential

- The transverse gravitino is produced dominantly at $H \sim H_{\text{inf}}$.

- The longitudinal gravitino is produced dominantly at $H \sim m_3^0/2$, and suppressed compared to the transverse mode by a factor $(m_3^0/H_{\text{inf}})^2$.

- The inflatino production is similar to the transverse gravitino.

Quartic inflaton potential

- Gravitino (mainly transverse) is copiously produced even for low reheating temperature.
Gravitino production

Summary of this section

Quadratic inflaton potential

- The transverse gravitino is produced dominantly at $H \sim H_{\text{inf}}$.
- The longitudinal gravitino is produced dominantly at $H \sim \frac{m_3^0}{2}$, and suppressed compared to the transverse mode by a factor $(\frac{m_3^0}{H_{\text{inf}}})^2$.
- The inflatino production is similar to the transverse gravitino.

Quartic inflaton potential

- Gravitino (mainly transverse) is copiously produced even for low reheating temperature.
Gravitino production

Summary of this section

**Quadratic inflaton potential**

- The transverse gravitino is produced dominantly at $H \sim H_{\text{inf}}$.
- The longitudinal gravitino is produced dominantly at $H \sim m_{3/2}^0$, and suppressed compared to the transverse mode by a factor $(m_{3/2}^0/H_{\text{inf}})^2$.
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**Quartic inflaton potential**

- Gravitino (mainly transverse) is copiously produced even for low reheating temperature.
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   - Gravitino problem
   - Context of our work
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3. Gravitino production without a stabilizer field
   - Model and its dynamics
   - Gravitino production

4. Gravitino production with a stabilizer field
   - Model and its dynamics
   - Gravitino production

5. Conclusion
   - Summary, conclusion, and prospects
Model and its dynamics

Model

\begin{align}
K &= -\frac{1}{2} (\phi - \phi^\dagger)^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \\
W &= m\phi X\phi + \mu^2 z + W_0.
\end{align}

Induced oscillation amplitude of the stabilizer $X$

\[X_{\text{amp}} \sim \frac{m_{3/2}^0}{H} \phi_{\text{amp}}.\]

Time evolution of the gravitino mass $m_{3/2}$

\[m_{3/2} \sim \begin{cases} 
\frac{m\phi \phi^2}{M_{P1}^2} \frac{m_{3/2}^0}{H} + m_{3/2}^0 & \text{for } H > m_{3/2}^0 \\
\frac{m\phi \phi^2}{M_{P1}^2} + m_{3/2}^0 & \text{for } H < m_{3/2}^0
\end{cases}.\]
Model and its dynamics

Model

\[ K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]  
\[ W = m_\phi X\phi + \mu^2 z + W_0. \]

Induced oscillation amplitude of the stabilizer \( X \)

\[ X_{\text{amp}} \sim \frac{m_0^{3/2}}{H} \phi_{\text{amp}}. \]

Time evolution of the gravitino mass \( m_{3/2} \)

\[ m_{3/2} \sim \begin{cases}  
m_\phi \phi^2 \frac{m_0^{3/2}}{H} + m_{3/2}^0 & \text{for } H > m_{3/2}^0 \\
\frac{m_\phi \phi^2}{M_{\text{Pl}}^2} + m_{3/2}^0 & \text{for } H < m_{3/2}^0 \end{cases}. \]
Model and its dynamics

Model

\[ K = -\frac{1}{2}(\phi - \phi^\dagger)^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (63) \]

\[ W = m_\phi X\phi + \mu^2 z + W_0. \quad (64) \]

Induced oscillation amplitude of the stabilizer \( X \)

\[ X_{\text{amp}} \sim \frac{m_0^{3/2}}{H} \phi_{\text{amp}}. \quad (65) \]

Time evolution of the gravitino mass \( m_{3/2} \)

\[ m_{3/2} \sim \begin{cases} 
\frac{m_\phi \phi^2}{M_{Pl}^2} \frac{m_0^{3/2}}{H} + m_{3/2}^0 & \text{for } H > m_{3/2}^0 \\
\frac{m_\phi \phi^2}{M_{Pl}^2} + m_{3/2}^0 & \text{for } H < m_{3/2}^0 
\end{cases}. \quad (66) \]
Rewrite the model

It is convenient to define $\Phi_\pm \equiv \frac{1}{\sqrt{2}}(\phi \pm X)$ so that

$$K = |\Phi_+|^2 + |\Phi_-|^2 - \frac{1}{4} \left[ (\Phi_+ + \Phi_-)^2 + \text{h.c.} \right] + |z|^2 - \frac{|z|^4}{\Lambda^2},$$

(67)

$$W = \frac{1}{2} m_\phi (\Phi_+^2 - \Phi_-^2) + \mu^2 z + W_0.$$  

(68)

longitudinal gravitino-fermion system

$$\mathcal{L}_f = -\frac{1}{2} \begin{pmatrix} \bar{\psi}_c' & v_1^{(1)'} & v_1^{(2)'} \end{pmatrix} \left[ \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + a \mathcal{M} \right] \begin{pmatrix} \psi_c' \\ v_1^{(1)'} \\ v_1^{(2)'} \end{pmatrix},$$

(69)

Mass eigenvalues

$$\begin{cases} 
(m_\phi, -m_\phi, -m_{3/2}(m_{3/2}/H)^2) & \text{for } H \gtrsim m_{3/2}^0, \\
(m_\phi, -m_\phi, -m_{3/2}) & \text{for } H \lesssim m_{3/2}^0.
\end{cases}$$

(70)
Rewrite the model

It is convenient to define $\Phi_{\pm} \equiv \frac{1}{\sqrt{2}}(\phi \pm X)$ so that

$$
K = |\Phi_+|^2 + |\Phi_-|^2 - \frac{1}{4} \left[ (\Phi_+ + \Phi_-)^2 + \text{h.c.} \right] + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (67)
$$

$$
W = \frac{1}{2} m_{\phi} (\Phi_+^2 - \Phi_-^2) + \mu^2 z + W_0. \quad (68)
$$

longitudinal gravitino-fermion system

$$
\mathcal{L}_f = -\frac{1}{2} \left( \frac{\psi_\ell'^r}{v_\perp^{(1)'}} \quad v_\perp^{(2)'} \right) \left[ \gamma^0 \partial_0 + i \vec{\gamma} \cdot \vec{k} + aM \right] \left( \begin{array}{c} \psi_\ell'^r \\ v_\perp^{(1)'} \\ v_\perp^{(2)'} \end{array} \right), \quad (69)
$$

Mass eigenvalues

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(m_{\phi}, -m_{\phi}, -m_{3/2}(m_{3/2}^0/H)^2) & \text{for } H \gtrsim m_{3/2}^0 \\
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\]

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W = \frac{1}{2} m_\phi (\Phi_+^2 - \Phi_-^2) + \mu^2 z + W_0. \tag{68}
\]

longitudinal gravitino-fermion system

\[
\mathcal{L}_f = -\frac{1}{2} \begin{pmatrix} \bar{\psi}_c^{} & \psi_{\ell}^{(1)'} & \psi_{\ell}^{(2)'} \end{pmatrix} \begin{pmatrix} \gamma^0 \partial_0 + i\vec{\gamma} \cdot \vec{k} + a\mathcal{M} \end{pmatrix} \begin{pmatrix} \psi_c^{} \\ v_{\perp}^{(1)'} \\ v_{\perp}^{(2)'} \end{pmatrix}, \tag{69}
\]

Mass eigenvalues

\[
\begin{cases}
\left( m_\phi, -m_\phi, -m_{3/2}^{} (m_{3/2}^0 / H)^2 \right) & \text{for } H \gtrsim m_{3/2}^0 \\
\left( m_\phi, -m_\phi, -m_{3/2}^{} \right) & \text{for } H \lesssim m_{3/2}^0 .
\end{cases} \tag{70}
\]
Time dependence of various quantities

Figure: Same as Fig. 2 but for multi-superfield inflation models.
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Transverse gravitino production

The production rate is suppressed by the small stabilizer amplitude.

\[ \Gamma(\phi\phi \to \psi^t\psi^t) \simeq \begin{cases} \frac{C}{4\pi} \frac{\phi_{\text{amp}}^2}{M_{P1}^2} \frac{m_\phi^3}{M_{P1}^2} \left( \frac{m_{3/2}^0}{H} \right)^2 & \text{for } H > m_{3/2}^0 \\ \frac{C}{4\pi} \frac{\phi_{\text{amp}}^2}{M_{P1}^2} \frac{m_\phi^3}{M_{P1}^2} \simeq \frac{3C}{4\pi} \frac{H^2 m_\phi}{M_{P1}^2} & \text{for } H < m_{3/2}^0 \end{cases} \]

(71)

Transverse gravitino yield

\[ \frac{n_{3/2}^{(t)}}{s} \simeq \left( \frac{\Gamma(\phi\phi \to \psi^t\psi^t)}{H} \right)_{H=m_{3/2}^0} \frac{3T_R}{4m_\phi} = \frac{9C}{16\pi} \frac{m_{3/2}^0 T_R}{M_{P1}^2} \]

\[ \simeq 3 \times 10^{-22} \mathcal{C} \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right). \]

(72)
Transverse gravitino production

The production rate is suppressed by the small stabilizer amplitude.

\[
\Gamma (\phi \phi \rightarrow \psi^t \psi^t) \simeq \begin{cases} 
\frac{C}{4\pi} \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_\phi^3}{M_{\text{Pl}}^2} \left( \frac{m_{3/2}^0}{H} \right)^2 & \text{for } H > m_{3/2}^0 \\
\frac{C}{4\pi} \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_\phi^3}{M_{\text{Pl}}^2} \simeq \frac{3C}{4\pi} \frac{H^2 m_\phi}{M_{\text{Pl}}^2} & \text{for } H < m_{3/2}^0
\end{cases}
\]

Transverse gravitino yield

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\frac{n_{3/2}^{(t)}}{s} \simeq \left( \frac{\Gamma (\phi \phi \rightarrow \psi^t \psi^t)}{H} \right)_{H=m_{3/2}^0} \frac{3T_R}{4m_\phi} = \frac{9C}{16\pi} \frac{m_{3/2}^0 T_R}{M_{\text{Pl}}^2}
\]

\[
\simeq 3 \times 10^{-22} C \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right) .
\]

(71)
Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_\perp^{(2)}$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

$$\Gamma(\phi\phi \rightarrow v_\perp^{(2)} v_\perp^{(2)}) \lesssim \frac{C}{4\pi} \left( \frac{m_{3/2}^0}{H} \right)^6 \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_\phi^3}{M_{\text{Pl}}^2} \approx \frac{3C}{4\pi} \left( \frac{m_{3/2}^0}{H} \right)^4 \frac{(m_{3/2}^0)^2 m_\phi}{M_{\text{Pl}}^2},$$  \hspace{1cm} (73)

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

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Longitudinal gravitino yield

$$\frac{n_{3/2}^{(\ell)}}{s} \lesssim \left( \frac{\Gamma(\phi\phi \rightarrow \psi^\ell \psi^\ell)}{H} \right) \left( \frac{3T_R}{4m_\phi} \right) \sim \frac{9C}{16\pi} \frac{m_{3/2}^0 T_R}{M_{\text{Pl}}^2} \approx 3 \times 10^{-22} C \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right). \hspace{1cm} (75)$$
Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_{\perp}^{(2)}$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

$$
\Gamma(\phi\phi \rightarrow v_{\perp}^{(2)} v_{\perp}^{(2)}) \lesssim \frac{C}{4\pi} \left( \frac{m_{3/2}^0}{H} \right)^6 \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_{\phi}^3}{M_{\text{Pl}}^2} \approx \frac{3C}{4\pi} \left( \frac{m_{3/2}^0}{H} \right)^4 \frac{(m_{3/2}^0)^2 m_{\phi}}{M_{\text{Pl}}^2},
$$

(73)

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

$$
\Gamma(\phi\phi \rightarrow \psi^\ell \psi^\ell) \lesssim \frac{C}{4\pi} \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_{\phi}^3}{M_{\text{Pl}}^2} \approx \frac{3C}{4\pi} \frac{H^2 m_{\phi}}{M_{\text{Pl}}^2}.
$$

(74)

longitudinal gravitino yield

$$
\frac{n_{3/2}^{(\ell)}}{s} \lesssim \left( \frac{\Gamma(\phi\phi \rightarrow \psi^\ell \psi^\ell)}{H} \right)_{H=m_{3/2}^0} \frac{3T_R}{4m_{\phi}} \approx \frac{9C}{16\pi} \frac{m_{3/2}^0 T_R}{M_{\text{Pl}}^2}
$$

$$
\approx 3 \times 10^{-22} C \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right).
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Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_{\perp}^{(2)}$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

$$\Gamma(\phi\phi \rightarrow v_{\perp}^{(2)} v_{\perp}^{(2)}) \lesssim \frac{C}{4\pi} \left( \frac{m_{3/2}^0}{H} \right)^6 \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_{\phi}^3}{M_{\text{Pl}}^2} \approx \frac{3C}{4\pi} \left( \frac{m_{3/2}^0}{H} \right)^4 \frac{(m_{3/2}^0)^2 m_{\phi}}{M_{\text{Pl}}^2},$$ (73)

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

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Longitudinal gravitino yield

$$\frac{n_{3/2}^{(\ell)}}{s} \lesssim \left( \frac{\Gamma(\phi\phi \rightarrow \psi^\ell \psi^\ell)}{H} \right) \bigg|_{H=m_{3/2}^0} \frac{3T_R}{4m_{\phi}} \approx \frac{9C}{16\pi} \frac{m_{3/2}^0 T_R}{M_{\text{Pl}}^2} \simeq 3 \times 10^{-22} C \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right). \quad (75)$$
Longitudinal gravitino production

Two sources of longitudinal gravitino

- $v_{\bot}^{(2)}$ produced when $H > m_{3/2}^0$ becomes $\psi^\ell$ later.

$$\Gamma(\phi\phi \to v_{\bot}^{(2)} v_{\bot}^{(2)}) \lesssim \frac{C}{4\pi} \left(\frac{m_{3/2}^0}{H}\right)^6 \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_\phi^3}{M_{\text{Pl}}^2} \sim \frac{3C}{4\pi} \left(\frac{m_{3/2}^0}{H}\right)^4 \frac{(m_{3/2}^0)^2 m_\phi}{M_{\text{Pl}}^2},$$

(73)

- $\psi^\ell$ produced when $H < m_{3/2}^0$.

$$\Gamma(\phi\phi \to \psi^\ell \psi^\ell) \lesssim \frac{C}{4\pi} \frac{\phi_{\text{amp}}^2}{M_{\text{Pl}}^2} \frac{m_\phi^3}{M_{\text{Pl}}^2} \sim \frac{3C}{4\pi} \frac{H^2 m_\phi}{M_{\text{Pl}}^2}.$$

(74)

Longitudinal gravitino yield

$$\frac{n_{3/2}^{(\ell)}}{s} \lesssim \left(\frac{\Gamma(\phi\phi \to \psi^\ell \psi^\ell)}{H}\right)_{H=m_{3/2}^0} \frac{3T_R}{4 m_\phi} \sim \frac{9C}{16\pi} \frac{m_{3/2}^0 T_R}{M_{\text{Pl}}^2} \sim 3 \times 10^{-22} C \left(\frac{m_{3/2}^0}{10^6 \text{ GeV}}\right) \left(\frac{T_R}{10^{10} \text{ GeV}}\right).$$

(75)
Inflaton and stabilizino production

Heavy fermion states have oscillating parts in their masses.

\[
m_{\text{heavy}}^\pm \simeq \pm m_\phi + 2\alpha^2 \hat{m}^{3/2}_\pm,
\]

(76)

where \( \hat{m}^{3/2}_\pm \) has an oscillating term of \( \mathcal{O}(H) \).

Inflaton/stabilizino yields

\[
\frac{n_{v_{\perp}}^{(1)}}{s} \simeq \frac{n_{v_{\perp}}^{(2)}}{s} \simeq \frac{27C}{16\pi} \frac{H_{\text{inf}}T_R}{M_{\text{Pl}}^2}
\]

\[
\simeq 9 \times 10^{-15} C \left( \frac{H_{\text{inf}}}{10^{13} \text{ GeV}} \right)^{-1} \left( \frac{T_R}{10^{10} \text{ GeV}} \right).
\]

(77)
Heavy fermion states have oscillating parts in their masses.

\[
m_{\text{heavy}}^\pm \simeq \pm m_\phi + 2\alpha_\pm^2 \hat{m}_{3/2}^\pm, \tag{76}
\]

where \(\hat{m}_{3/2}\) has an oscillating term of \(O(H)\).

**Inflatino/stabilizino yields**

\[
\frac{n_{\nu_{\perp}^{(1)}}}{s} \simeq \frac{n_{\nu_{\perp}^{(2)}}}{s} \simeq \frac{27C H_{\text{inf}} T_R}{16\pi M_{\text{Pl}}^2} \simeq 9 \times 10^{-15} C \left(\frac{H_{\text{inf}}}{10^{13} \text{ GeV}}\right)^{-1} \left(\frac{T_R}{10^{10} \text{ GeV}}\right). \tag{77}
\]
Gravitino production

Higher power inflaton potential

\[ K = -\frac{1}{2} (\phi - \phi^\dagger)^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (78) \]
\[ W = \lambda X \phi^n + \mu^2 z + W_0. \quad (79) \]

Scalar mass matrix

\[ V = \begin{pmatrix} \phi \\ X \end{pmatrix} \begin{pmatrix} (\lambda \phi^{n-1})^2 & -2m_0^0 (\lambda \phi^{n-1}) \\ -2m_{3/2}^0 (\lambda \phi^{n-1}) & n^2 (\lambda \phi^{n-1})^2 \end{pmatrix} \begin{pmatrix} \phi \\ X \end{pmatrix}. \quad (80) \]

For \( n \neq 1 \), the masses are not degenerate.

Induced oscillation amplitude of stabilizer \( X \) \( (n \neq 1) \)

\[ X_{\text{amp}} \sim \frac{m_{3/2}^0}{m_\phi} \phi_{\text{amp}}, \quad \text{with} \quad m_\phi \equiv \lambda \phi_{\text{amp}}^{n-1}. \quad (81) \]

Time evolution of the gravitino mass \( m_{3/2} \) \( (n \neq 1) \)

\[ m_{3/2} \simeq \begin{cases} \frac{m_{3/2}^0 \phi^2}{M_{\text{Pl}}^2} + m_{3/2}^0 & \text{for } m_\phi > m_{3/2}^0 \\ \frac{m_\phi \phi^2}{M_{\text{Pl}}^2} + m_{3/2}^0 & \text{for } m_\phi < m_{3/2}^0 \end{cases}. \quad (82) \]
Gravitino production

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\]

Scalar mass matrix

\[
V = (\phi \ X) \begin{pmatrix}
(\lambda \phi^{n-1})^2 & -2m^{0}_{3/2}(\lambda \phi^{n-1}) \\
-2m^{0}_{3/2}(\lambda \phi^{n-1}) & n^2(\lambda \phi^{n-1})^2
\end{pmatrix} \begin{pmatrix}
\phi \\
X
\end{pmatrix}. \quad (80)
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X_{\text{amp}} \sim \frac{m^{0}_{3/2}}{m_{\phi}} \phi_{\text{amp}}, \quad \text{with} \quad m_{\phi} \equiv \lambda \phi_{\text{amp}}^{n-1}. \quad (81)
\]

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m_{3/2} \simeq \begin{cases}
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\[ X_{\text{amp}} \sim \frac{m_0^{3/2}}{m_\phi} \phi_{\text{amp}}, \quad \text{with} \quad m_\phi \equiv \lambda \phi^{n-1}_{\text{amp}}. \quad (81) \]

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\]
Higher power inflaton potential

The transverse gravitino production is suppressed by \((m_{3/2}^0/m_\phi)^2\) compared to the case without a stabilizer field.

For a quartic potential \((n = 2)\),

Gravitino yields

\[
\frac{n_{3/2}^{(t)}}{s} \sim \frac{9C}{16\pi} \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \left( \frac{(m_{3/2}^0)^2}{H_{\text{inf}}^{1/2} M_{\text{Pl}}^{3/2}} \right) \approx 7 \times 10^{-24} C \left( \frac{H_{\text{inf}}}{10^{13} \text{ GeV}} \right)^{-1/2} \left( \frac{m_{3/2}^0}{10^6 \text{ GeV}} \right)^{3/2}, \tag{83}
\]

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Figure: Dependence of gravitino abundance $Y_{3/2} = n_{3/2}/s$ on the reheating temperature $T_R$. Parameters are set as $m_{3/2} = 10^6$ GeV and $H_{\text{inf}} = 10^{13}$ GeV.
Summary of this section

**quadratic inflaton potential**

- The transverse gravitino is produced dominantly at $H \sim m_3^{1/2}$, and suppressed compared to the case without a stabilizer field.

- The longitudinal gravitino is produced dominantly at $H \sim m_3^{0}$.

- The inflatino/stabilizino production is not suppressed. Depending on its interactions and masses, this can be a dominant source of the gravitino by their decay. See [Nilles et al., 2001a] for cosmological consequences of inflatino.

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- Gravitino production is enhanced from the quadratic case, but still much suppressed compared to the case without a stabilizer field.
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1. Introduction
   - Gravitino problem
   - Context of our work
   - Abstract of analyses

2. Gravitino Lagrangian
   - Lagrangian and physical modes
   - Diagonalization of the gradient term

3. Gravitino production without a stabilizer field
   - Model and its dynamics
   - Gravitino production

4. Gravitino production with a stabilizer field
   - Model and its dynamics
   - Gravitino production

5. Conclusion
   - Summary, conclusion, and prospects
Summary

- We studied gravitino production by preheating with focuses on ($Z_2$-symmetric) large-field models.

- Our setup is way more realistic than the previous works: shift symmetry of the inflaton Kähler potential, with or without a stabilizer field, no Polonyi problem, and gravitino as a time-dependent mass eigenstate.

- Gravitino abundance depends very much on the model: with or without a stabilizer field, and quadratic or quartic (or higher) potentials.
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**Figure:** Dependence of gravitino abundance $Y_{3/2} = n_{3/2}/s$ on the reheating temperature $T_R$. Parameters are set as $m_{3/2} = 10^6$ GeV and $H_{\text{inf}} = 10^{13}$ GeV.
Conclusion

Without a stabilizer field $X$

- Longitudinal gravitino abundance is well suppressed.

- For a quadratic potential, transverse gravitino abundance is less than thermal gravitino abundance.

- For a quartic potential, transverse gravitino abundance is large irrespective of the reheating temperature, and cosmologically problematic.

With a stabilizer field $X$

- Due to the small induced oscillation of $X$, the oscillation of the gravitino masses is suppressed.

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Prospects

We can generalize this work to the following cases:

- **Non-minimal Kähler potential**
  → General treatment becomes technically involved.

- **Complex scalar configurations**
  → We cannot neglect the auxiliary vector field in supergravity.

- **$D$-term inflation**
  → Gaugino plays the role of goldstino.

- **Constrained superfields** such as (orthogonal) nilpotent superfield(s)
  → Sound speed of gravitino can be non-relativistic.

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It will be interesting to study these cases.
Notations and conventions

Metric

The sign is the \((-, +, +, +)\) convention.

Dirac \(\gamma\) matrices

\[ \{\hat{\gamma}^\mu, \hat{\gamma}^\nu\} = 2g^{\mu\nu}. \]

Related to the flat space quantities by \(\hat{\gamma}_\mu = e_\mu^a \gamma_a\).

\((\gamma^0)^\dagger = -\gamma^0\) and \((\vec{\gamma})^\dagger = \vec{\gamma}\).

\(\gamma_* \equiv i\gamma_0\gamma_1\gamma_2\gamma_3\), and \(P_L \equiv \frac{1+\gamma_*}{2}\), \(P_R \equiv \frac{1-\gamma_*}{2}\).

Dirac/Majorana conjugate

\(\bar{\psi} \equiv i\psi^\dagger \gamma^0\).

Charge conjugation matrix \(C\): \(\gamma^{\mu T} = -C\gamma^\mu C^{-1}\).

Majorana fermion \(\lambda\) satisfies \(\lambda = -C^{-1}\lambda^T\).

Curvature

\(\omega^{ab}_\mu = 2e^\nu[a\partial^\mu e_\nu]_b - e^\nu[a e^b]_\sigma e^\mu_c \partial^\nu e^\sigma_c\),

\(R^{ab}_{\mu\nu} = 2\partial[\mu \omega_{\nu}]^{ab} + 2\omega[\mu^{ac} \omega_{\nu}]_c^{b}\), \n(\(R_{\mu\nu} \equiv R_{\mu\rho}^{ab} e_{a\nu} e^b_\rho\), \(R \equiv g_{\mu\nu} R^{\mu\nu}\).
More explanations on the derivation of yield quantities

Suppose that gravitinos are produced dominantly at $H \simeq H_*$. 

**Quadratic potential case**

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} = \left( n_\phi \frac{\Gamma(\phi\phi \to \psi\psi)}{H} \right) \bigg|_{H=H_*} \left( \frac{a_*}{a_R} \right)^3$$

$$= \frac{3T_R}{4\rho_{\text{rad, R}}} \left( \frac{a_*}{a_R} \right)^3 \left( \frac{\Gamma(\phi\phi \to \psi\psi)}{H} \right) \bigg|_{H=H_*}$$

$$= \frac{3T_R}{4m_\phi} \left( \Gamma(\phi\phi \to \psi\psi) \right) \bigg|_{H=H_*}$$

because $\rho_{\phi,*} \left( \frac{a_*}{a_R} \right)^3 = \rho_{\phi,R}$ (just before decay) = $\rho_{\text{rad, R}}$ (just after decay). 

(85)
Suppose that gravitinos are produced dominantly at $H \simeq H_*$. 

**Quartic potential case**

$$Y_{3/2} \equiv \frac{n_{3/2}}{s} = \left( \frac{n_\phi}{H} \frac{\Gamma(\phi\phi \to \psi\psi)}{H} \right) \bigg|_{H=H_*} \left( \frac{a_*}{a_R} \right)^3$$

$$= \frac{3T_R}{4\rho_{\text{rad}, R}} \frac{\rho_{\phi,*} \left( \frac{a_*}{a_R} \right)^3}{m_\phi \left( \frac{\Gamma(\phi\phi \to \psi\psi)}{H} \right) \bigg|_{H=H_*}}$$

$$= \frac{3T_R}{4m_\phi} \left( \frac{\Gamma(\phi\phi \to \psi\psi)}{H} \right) \bigg|_{H=H_*} \left( \frac{a_R}{a_*} \right)$$

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(86)

because $\rho_{\phi,*} \left( \frac{a_*}{a_R} \right)^4 = \rho_{\phi,R}^{(\text{just before decay})} = \rho_{\text{rad}, R}^{(\text{just after decay})}$.

Here, we defined $T_* \equiv T_R \left( \frac{a_R}{a_*} \right) = \left( \frac{90}{\pi^2 g_*} \right)^{1/4} \sqrt{H_* M_{\text{Pl}}}$. 
Comment on the linear term in the Kahler potential

**Linear term in the inflaton Kahler potential**

If the inflaton does not have a $Z_2$ symmetry, and the Kahler potential has the linear sinflaton term,

$$K = ic(\phi - \phi^\dagger) + \ldots,$$

(87)

the gravitino production is significantly enhanced.

For definiteness, consider the quadratic potential case ($n = 2$) without a stabilizer field. There is a mixing between $\phi$ and $z$ induced by the following term,

$$V \supset (D_\phi W)(D_\phi W) \supset \frac{K_\phi W}{M^2_{Pl}} m_\phi \phi^* + \text{h.c.} \simeq \frac{\sqrt{3}icm_3/2m_\phi}{M_{Pl}} z\phi^* + \text{h.c.} \ldots$$  

(88)

**Mixing angle**

$$\theta_{\phi z} \sim \begin{cases} 
\frac{\sqrt{3}cm_3/2m_\phi}{m_z^2M_{Pl}} & \text{for } m_\phi < m_z \\
\frac{\sqrt{3}cm_3/2}{m_\phi M_{Pl}} & \text{for } m_\phi > m_z
\end{cases}.$$  

(89)

**Amplitude of induced oscillation**

$$\tilde{z}_{\text{amp}} \sim \theta_{\phi z}\phi_{\text{amp}}$$

(90)
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$$z_{\text{amp}} \sim \theta_{\phi z} \phi_{\text{amp}}$$
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\end{cases} \ldots (89)$$

**Amplitude of induced oscillation**

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Linear term in the inflaton Kähler potential 2

If the inflaton does not have a $Z_2$ symmetry, and the Kahler potential has the linear sinflaton term,

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(91)

the gravitino production is significantly enhanced.

Gravitino production rate

$$\dot{n}^{(\ell)}_{3/2} \simeq \frac{2\rho z}{m_z} \Gamma(z \rightarrow \psi^\ell \psi^\ell) \simeq \frac{2\rho_\phi}{m_\phi} \frac{m_\phi}{m_z} \theta^2_{\phi z} \Gamma(z \rightarrow \psi^\ell \psi^\ell),$$

(92)

where

$$\Gamma(z \rightarrow \psi^\ell \psi^\ell) \simeq \frac{1}{96\pi} \frac{m_z^5}{(m_{3/2}^0)^2 M_{Pl}^2}.$$  

(93)

Inflaton partial decay rate

$$\Gamma(\phi \rightarrow \psi^\ell \psi^\ell) \simeq \frac{m_\phi}{m_z} \theta^2_{\phi z} \Gamma(z \rightarrow \psi^\ell \psi^\ell) \simeq \begin{cases} \frac{c^2 m_\phi^3}{32\pi M_{Pl}^4} & \text{for } m_\phi < m_z \\ \frac{c^2 m_\phi^3}{32\pi M_{Pl}^4} \left( \frac{m_z}{m_\phi} \right)^4 & \text{for } m_\phi > m_z \end{cases}.$$  

(94)
Linear term in the inflaton Kähler potential 2

If the inflaton does not have a $Z_2$ symmetry, and the Kähler potential has the linear sinflaton term,

$$K = ic(\phi - \phi^\dagger) + \ldots,$$

the gravitino production is significantly enhanced.

**Gravitino production rate**

$$\dot{n}^{(\ell)}_{3/2} \simeq \frac{2\rho_z}{m_z} \Gamma(z \to \psi^\ell \psi^\ell) \simeq \frac{2\rho_\phi m_\phi}{m_\psi m_z} \theta_{\phi z}^2 \Gamma(z \to \psi^\ell \psi^\ell),$$

where

$$\Gamma(z \to \psi^\ell \psi^\ell) \simeq \frac{1}{96\pi} \frac{m_z^5}{(m_{3/2}^0)^2 M_{Pl}^2}.$$  

**Inflaton partial decay rate**

$$\Gamma(\phi \to \psi^\ell \psi^\ell) \simeq \frac{m_\phi}{m_z} \theta_{\phi z}^2 \Gamma(z \to \psi^\ell \psi^\ell) \simeq \begin{cases} 
\frac{c^2 m_\phi^3}{32\pi M_{Pl}^4} & \text{for } m_\phi < m_z \\
\frac{c^2 m_\phi^3}{32\pi M_{Pl}^4} \left(\frac{m_z}{m_\phi}\right)^4 & \text{for } m_\phi > m_z
\end{cases}.$$
If the inflaton does not have a $Z_2$ symmetry, and the Kähler potential has the linear sinflaton term,

$$K = ic(\phi - \phi^\dagger) + \ldots,$$

the gravitino production is significantly enhanced.

**Gravitino production rate**

$$\dot{n}_{3/2}^{(\ell)} \sim \frac{2\rho_z}{m_z} \Gamma(z \rightarrow \psi^\ell \psi^\ell) \sim \frac{2\rho_\phi}{m_\phi} \frac{m_\phi}{m_z} \theta_{\phi z}^2 \Gamma(z \rightarrow \psi^\ell \psi^\ell),$$

(92)

where

$$\Gamma(z \rightarrow \psi^\ell \psi^\ell) \approx \frac{1}{96\pi} \frac{m_z^5}{(m_{3/2}^0)^2 M_{Pl}^2}.$$  

(93)

**Inflaton partial decay rate**

$$\Gamma(\phi \rightarrow \psi^\ell \psi^\ell) \sim \frac{m_\phi}{m_z} \theta_{\phi z}^2 \Gamma(z \rightarrow \psi^\ell \psi^\ell) \sim \begin{cases} 
\frac{c^2 m_\phi^3}{32\pi M_{Pl}^4} & \text{for } m_\phi < m_z \\
\frac{c^2 m_\phi^3}{32\pi M_{Pl}^4} \left(\frac{m_z}{m_\phi}\right)^4 & \text{for } m_\phi > m_z 
\end{cases}.$$  

(94)
If the inflaton does not have a $Z_2$ symmetry, and the Kähler potential has the linear sinflaton term,

$$K = ic(\phi - \phi^\dagger) + \ldots,$$  \hspace{1cm} (95)

the gravitino production is significantly enhanced.

The result is consistent with [Endo et al., 2007, Nakayama et al., 2012].

(Longitudinal) gravitino yield

$$n_{3/2}^{(\ell)} \approx \left( \frac{2\Gamma(\phi \rightarrow \psi^\ell \psi^\ell)}{H} \right)_{H=\Gamma_{\text{inf}}} \frac{3T_R}{4m_{\phi}} \sim \frac{3c^2m_{\phi}^2}{64\pi M_{\text{Pl}}^3T_R} \left( \frac{90}{\pi^2g_*} \right)^{1/2}$$

$$\simeq 1 \times 10^{-5} \left( \frac{c}{M_{\text{Pl}}} \right)^2 \left( \frac{m_{\phi}}{10^{13}\text{ GeV}} \right)^2 \left( \frac{10^{10}\text{ GeV}}{T_R} \right).$$  \hspace{1cm} (96)

Very violent production occurs unless $c$ is suppressed!
Consider a fermion with an oscillating mass $m(t)$ with its frequency $\Omega$,

$$\mathcal{L} = -\frac{1}{2} \bar{\psi} (\partial_t - m(t)) \psi. \quad (97)$$

**Creation and annihilation operators**

The Fourier mode $\psi_k(t) = \int \frac{d^3x}{(2\pi)^{3/2}} e^{-i \vec{k} \cdot \vec{x}} \psi(t, x)$ is expanded as

$$\psi_k(t) = \sum_s \left[ u_{k,s}(t) \hat{b}_{k,s} + v_{k,s}(t) \hat{b}_{-k,s}^\dagger \right], \quad (98)$$

where $v_{k,s}(t) = -C^{-1} u^T_{-k,s}(t)$, and mode functions are orthonormal, and the creation/annihilation operators satisfy the standard canonical anti-commutation relations.

**Helicity basis**

$$u_{k,h}(t) = \left( \begin{array}{c} u_{k,h}^+(t) \\
 u_{k,h}^-(t) \end{array} \right) \otimes \xi_{k,h}, \quad v_{k,h}(t) = \left( \begin{array}{c} -u_{-k,h}^-(t) \\
 u_{-k,h}^+(t) \end{array} \right)^* \otimes \xi'_{-k,h}. \quad (99)$$
Quantization II

Here $\xi_{\vec{k},h}$ is the normalized eigenvector of helicity $h = \pm 1$, satisfying

$$(\vec{\sigma} \cdot \hat{\vec{k}})\xi_{\vec{k},h} = h\xi_{\vec{k},h}, \quad \hat{\vec{k}} \equiv \vec{k}/k$$

is a unit vector. We have also defined

$$\xi'_{\vec{k},h} \equiv -i\sigma^2\xi^*_{\vec{k},h},$$

which satisfies

$$(\vec{\sigma} \cdot \hat{\vec{k}})\xi'_{\vec{k},h} = -h\xi'_{\vec{k},h}.$$ 

Now, the normalization condition becomes

$$\left| u^+_{\vec{k},h} \right|^2 + \left| u^-_{\vec{k},h} \right|^2 = 1.$$ 

Equation of motion

$$i\partial_0 u^+_{\vec{k},h} + hku^-_{\vec{k},h} = m(t)u^+_{\vec{k},h},$$

$$i\partial_0 u^-_{\vec{k},h} + hku^+_{\vec{k},h} = -m(t)u^-_{\vec{k},h}.$$ 

Combining them, we obtain

$$0 = \ddot{u}^+_{\vec{k},h}(t) + \tilde{\omega}^2_{\vec{k}}(t)u^+_{\vec{k},h}(t),$$

$$0 = \ddot{u}^-_{\vec{k},h}(t) + \tilde{\omega}^2_{\vec{k}}(t)u^-_{\vec{k},h}(t).$$ 

where $\tilde{\omega}^2_{\vec{k}}(t) \equiv \omega^2_{\vec{k}}(t) + i m(t)$ and $\omega^2_{\vec{k}}(t) \equiv m^2(t) + k^2$. 

Vacuum initial conditions

\[ u_{k, h}^{+} (t \to 0) = \sqrt{\frac{\omega_{k}(0) + m(0)}{2\omega_{k}(0)}}, \quad \dot{u}_{k, h}^{+} (t \to 0) = -i\omega_{k}(0)u_{k, h}^{+} (t \to 0), \quad (104) \]

\[ u_{k, h}^{-} (t \to 0) = -\hbar \sqrt{\frac{\omega_{k}(0) - m(0)}{2\omega_{k}(0)}}, \quad \dot{u}_{k, h}^{-} (t \to 0) = -i\omega_{k}(0)u_{k, h}^{-} (t \to 0). \quad (105) \]
Fermion production in the background field method

Particle production I

Hamiltonian density

$$\langle H(t) \rangle = \langle \psi^\dagger i \partial_0 \psi \rangle$$

$$= \int \frac{d^3 k}{(2\pi)^3} \frac{1}{2} \sum_h \left[ m(t) \left( |u_{k,h}^-(t)|^2 - |u_{k,h}^+(t)|^2 \right) + 2hk \Re \left( u_{k,h}^+(t) u_{k,h}^- (t) \right) \right]$$

$$= 2 \times \int \frac{d^3 k}{(2\pi)^3} \omega_k(t) \left( f_\psi(\vec{k}; t) - \frac{1}{2} \right) \quad (106)$$

where the phase space density is given by

$$f_\psi(\vec{k}; t) \equiv \frac{1}{2\omega(t)} \left[ m(t) \left( |u_{k}^-(t)|^2 - |u_k^+(t)|^2 \right) + 2hk \Re \left( u_k^+(t) u_k^- (t) \right) \right] + \frac{1}{2}$$

$$= \frac{1}{2\omega(t)} \left[ m(t) + 2 \Im \left( u_{k}^+ (t) \partial_0 u_k^-(t) \right) \right] + \frac{1}{2}. \quad (107)$$
Fermion production in the background field method

Particle production II

Number density

\[ n_\psi(t) = 2 \times \int \frac{d^3k}{(2\pi)^3} f_\psi(\vec{k}; t), \quad (108) \]

Ansatz of the mode function

\[ u_{k,h}^+(t) = \frac{A_{k,h}(t)}{\sqrt{2\tilde{\omega}_k(t)}} e^{-i \int_0^t d\tau \tilde{\omega}_k(\tau)} + \frac{B_{k,h}(t)}{\sqrt{2\tilde{\omega}_k(t)}} e^{i \int_0^t d\tau \tilde{\omega}_k(\tau)}, \quad (109) \]

where

\[ \dot{A}_k(t) = \frac{\tilde{\omega}_k(t)}{2\tilde{\omega}_k(t)} e^{2i \int_0^t d\tau \tilde{\omega}_k(\tau)} B_k(t), \quad \dot{B}_k(t) = \frac{\tilde{\omega}_k(t)}{2\tilde{\omega}_k(t)} e^{-2i \int_0^t d\tau \tilde{\omega}_k(\tau)} A_k(t). \quad (110) \]

This satisfies the equation of motion. We solve the time evolution of \( A(t) \) and \( B(t) \) perturbatively in time.
Fermion production in the background field method

Particle production III

Initial conditions of $A$ and $B$

$$A_k(t \to 0) = \sqrt{\omega_k(0) + m(0)}, \quad B(t \to 0) = 0. \quad (111)$$

Initially, we expect $A_k \simeq \sqrt{\omega_k + m}$ and $B_k \simeq 0$ at the leading order.

For modes with $k^2 \gg m^2$, we obtain

$$B_k(t) \simeq \int_0^t dt' \frac{m \dot{m} + i \ddot{m}/2}{2 \omega_k^2} A_k(0) e^{-2i \int_0^{t'} d\tau \omega_k(\tau)} \simeq -i A_k(0) \int_0^t dt' m(t') e^{-2i \omega_k t'}. \quad (112)$$

For given time $t$, the integration cancels out due to oscillations of the phase except for $\Omega - \Delta \Omega \lesssim 2 \omega_k \lesssim \Omega + \Delta \Omega$ with $\Delta \Omega \sim 1/t$.

$$B_k(t) \simeq -\frac{i}{2} A_k(0) \tilde{m} t \quad \text{for} \quad \Omega - \frac{1}{t} \lesssim 2 \omega_k \lesssim \Omega + \frac{1}{t}, \quad (113)$$
where $\tilde{m}$ stands for the amplitude of oscillating $m(t)$. Similarly,

$$A_{\vec{k}}(t) \simeq A_{\vec{k}}(0) - iB'_{\vec{k}}(0)\tilde{m}\frac{t^2}{4} \simeq A_{\vec{k}}(0) \left(1 - \frac{\tilde{m}^2 t^2}{8}\right) \quad \text{for} \quad \Omega - \frac{1}{t} \lesssim 2\omega_{\vec{k}} \lesssim \Omega + \frac{1}{t}. \quad (114)$$

**Growth of number density**

$$f_\psi(\vec{k}; t) \simeq \frac{\tilde{m}^2 t^2}{4} \quad \text{for} \quad \Omega - \frac{1}{t} \lesssim 2\omega_{\vec{k}} \lesssim \Omega + \frac{1}{t}. \quad (115)$$

This expression is valid as long as $f_\psi \ll 1$, namely $q\Omega t \ll 1$ with the resonance parameter being $q \equiv \tilde{m}^2/\Omega^2 \ll 1$. Integrating this,

$$n_\psi(t) \simeq \frac{C}{16\pi} \Omega^2 \tilde{m}^2 t. \quad (116)$$

$C$ is an $\mathcal{O}(1)$ parameter depending on the details of the oscillation. For example, $C = 1$ for $m(t) \propto \cos(\Omega t)$. 


Fermion production in the background field method

Particle production V

Oscillation induced by a scalar field
Suppose

\[
\phi(t) \simeq \phi_{\text{amp}} \cos(m_\phi t), \quad m(t) \propto \phi^n(t). \tag{117}
\]

This involves \( \Omega = jm_\phi \) with \( j = n, n - 2, n - 4, \ldots \).

**Example**
For \( n = 1 \), \( \Omega = m_\phi \), and we can interpret the process as decay of \( \phi \),

\[
\Gamma(\phi \rightarrow \psi\psi) \sim \frac{n_\psi}{2n_\phi t} \sim \frac{C}{32\pi} \frac{\tilde{m}^2}{\phi_{\text{amp}}^2} m_\phi \tag{118}
\]

**Example**
For \( n = 2 \), \( \Omega = 2m_\phi \), and we can interpret the process as annihilation of \( \phi \),

\[
\Gamma(\phi\phi \rightarrow \psi\psi) \sim \frac{n_\psi}{n_\phi t} \sim \frac{C}{4\pi} \frac{\tilde{m}^2}{\phi_{\text{amp}}^2} m_\phi \tag{119}
\]
Small-field model without a stabilizer field

Single-field new inflation model [Izawa and Yanagida, 1997]

\[ K = |\phi|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \tag{120} \]

\[ W = \phi \left( M^2 - \frac{\lambda \phi^n}{n+1} \right) + \mu^2 z. \tag{121} \]

Expansion around the vacuum: \( \phi = \langle \phi \rangle + \delta \phi \) with \( \langle \phi \rangle = (M^2/\lambda)^{1/n} \).

\[ K = \langle \phi \rangle (\delta \phi + \delta \phi^\dagger) + |\delta \phi|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \tag{122} \]

\[ W \simeq \frac{1}{2} m_\phi (\delta \phi)^2 + \mu^2 z + W_0 - m_{3/2}^0 \langle \phi \rangle \delta \phi, \tag{123} \]

where \( m_\phi = nM^2/\langle \phi \rangle \) and \( W_0 = \frac{n}{n+1} \langle \phi \rangle M^2 = m_{3/2}^0 M_{Pl}^2 \).

This is similar to the chaotic inflation with a linear term in the Kähler potential with \( c \sim \langle \phi \rangle \). The inflaton decays through the mixing with \( z \), and the rate is consistent with [Endo et al., 2006b, Endo et al., 2007].
Gravitino production in small-field inflation models

Small-field model without a stabilizer field

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Expansion around the vacuum: \( \phi = \langle \phi \rangle + \delta \phi \) with \( \langle \phi \rangle = (M^2/\lambda)^{1/n}. \)

\[ K = \langle \phi \rangle (\delta \phi + \delta \phi^\dagger) + |\delta \phi|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \quad (122) \]
\[ W \simeq \frac{1}{2} m_\phi (\delta \phi)^2 + \mu^2 z + W_0 - m_{3/2}^0 \langle \phi \rangle \delta \phi, \quad (123) \]

where \( m_\phi = n M^2 / \langle \phi \rangle \) and \( W_0 = \frac{n}{n+1} \langle \phi \rangle M^2 = m_{3/2}^0 M_{Pl}^2. \)

This is similar to the chaotic inflation with a linear term in the Kähler potential with \( c \sim \langle \phi \rangle. \) The inflaton decays through the mixing with \( z, \) and the rate is consistent with [Endo et al., 2006b, Endo et al., 2007].
Gravitino production in small-field inflation models

Small-field model with a stabilizer field

Multi-field new inflation model [Asaka et al., 2000, Senoguz and Shafi, 2004]:

\[ K = |\phi|^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]  \hspace{1cm} (124)

\[ W = X (M^2 - \lambda \phi^n) + \mu^2 z + W_0. \]  \hspace{1cm} (125)

Expansion around the vacuum: \( \phi = \langle \phi \rangle + \delta \phi \) with \( \langle \phi \rangle \simeq (M^2/\lambda)^{1/n} \) and \( \langle X \rangle \simeq 0 \).

\[ K = \langle \phi \rangle (\delta \phi + \delta \phi^*) + |\delta \phi|^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]  \hspace{1cm} (126)

\[ W \simeq m_\phi X \delta \phi + \mu^2 z + W_0, \]  \hspace{1cm} (127)

where \( m_\phi = nM^2/\langle \phi \rangle \).

This is similar to the chaotic inflation with a linear term in the Kähler potential with \( c \sim \langle \phi \rangle \). Since \( \phi \) and \( X \) mixes maximally, and there is a mixing term \( \sim \langle \phi \rangle X z^* \), the decay rate is similar to the previous case at least for \( H \lesssim m_{3/2} \). The inflaton decays through the mixing with \( z \), and the rate is consistent with [Endo et al., 2006b, Endo et al., 2007].
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\[ W = X(M^2 - \lambda\phi^n) + \mu^2 z + W_0. \]  

(124)  

(125)

Expansion around the vacuum: \( \phi = \langle \phi \rangle + \delta \phi \) with \( \langle \phi \rangle \simeq (M^2/\lambda)^{1/n} \) and \( \langle X \rangle \simeq 0. \)

\[ K = \langle \phi \rangle (\delta \phi + \delta \phi^\dagger) + |\delta \phi|^2 + |X|^2 + |z|^2 - \frac{|z|^4}{\Lambda^2}, \]

\[ W \simeq m_\phi X \delta \phi + \mu^2 z + W_0, \]  

(126)  

(127)

where \( m_\phi = nM^2/\langle \phi \rangle. \)

This is similar to the chaotic inflation with a linear term in the Kähler potential with \( c \sim \langle \phi \rangle. \) Since \( \phi \) and \( X \) mixes maximally, and there is a mixing term \( \sim \langle \phi \rangle X z^* \), the decay rate is similar to the previous case at least for \( H \lesssim m_{3/2}. \) The inflaton decays through the mixing with \( z \), and the rate is consistent with [Endo et al., 2006b, Endo et al., 2007].
Dynamical mixing of scalar fields

Induction of oscillation through mixing I

Potential of two real scalars $\phi_1$ and $\phi_2$

$$V = \frac{1}{2} (\phi_1 \phi_2) \mathcal{M}^2 \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \mathcal{M}^2 = \begin{pmatrix} m_1^2 & m_{12}^2 \\ m_{12}^2 & m_2^2 \end{pmatrix}. \quad (128)$$

where we assume $|m_1 m_2| > m_{12}^2$ (no tachyon).

Initial condition: $(\phi_1, \phi_2) = (\phi_i, 0)$.

Mass eigenvalues

$$V = \frac{1}{2} (\phi'_1 \phi'_2) \mathcal{M}'^2 \begin{pmatrix} \phi'_1 \\ \phi'_2 \end{pmatrix}, \quad \mathcal{M}'^2 = \begin{pmatrix} m_1'^2 & 0 \\ 0 & m_2'^2 \end{pmatrix}, \quad (129)$$

where

$$m_1'^2 = \frac{1}{2} \left( m_1^2 + m_2^2 + \frac{m_1^2 - m_2^2}{m_1^2 - m_2^2} \sqrt{(m_1^2 - m_2^2)^2 + 4m_{12}^4} \right), \quad (130)$$

$$m_2'^2 = \frac{1}{2} \left( m_1^2 + m_2^2 - \frac{m_1^2 - m_2^2}{m_1^2 - m_2^2} \sqrt{(m_1^2 - m_2^2)^2 + 4m_{12}^4} \right), \quad (131)$$
Dynamical mixing of scalar fields

Induction of oscillation through mixing II

Mass eigenstates

\[
\begin{pmatrix}
\phi_1' \\
\phi_2'
\end{pmatrix} =
\begin{pmatrix}
c_\theta & s_\theta \\
-s_\theta & c_\theta
\end{pmatrix}
\begin{pmatrix}
\phi_1 \\
\phi_2
\end{pmatrix},
\] (132)

where \(c_\theta \equiv \cos \theta\) and \(s_\theta \equiv \sin \theta\) with \(-\pi/4 < \theta \leq \pi/4\).

\[
c^2_\theta = \frac{1}{2} \left( 1 + \sqrt{1 - \frac{4m^4_{12}}{4m^4_{12} + (m^2_1 - m^2_2)^2}} \right),
\] (133)

\[
s^2_\theta = \frac{1}{2} \left( 1 - \sqrt{1 - \frac{4m^4_{12}}{4m^4_{12} + (m^2_1 - m^2_2)^2}} \right).
\] (134)

with \(\theta \geq 0\) for \((m^2_1 - m^2_2)/m^2_{12} > 0\) and \(\theta < 0\) for \((m^2_1 - m^2_2)/m^2_{12} < 0\).
Induction of oscillation through mixing III

Solution of the equation of motion

\[ \phi_1'(t) = c_\theta \phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \cos(m'_1 t), \]  

\[ \phi_2'(t) = -s_\theta \phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \cos(m'_2 t), \]

In the original basis, this becomes

\[ \phi_1(t) = \phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \left[ c_\theta^2 \cos(m'_1 t) + s_\theta^2 \cos(m'_2 t) \right], \]

\[ \phi_2(t) = -\phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \sin(2\theta) \sin \left( \frac{(m'_1 + m'_2) t}{2} \right) \sin \left( \frac{(m'_1 - m'_2) t}{2} \right). \]
Induction of oscillation through mixing IV

\[ \phi_1(t) = \phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \left[ c_\theta^2 \cos(m'_1 t) + s_\theta^2 \cos(m'_2 t) \right], \]

\[ \phi_2(t) = -\phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \sin(2\theta) \sin \left( \frac{(m'_1 + m'_2)t}{2} \right) \sin \left( \frac{(m'_1 - m'_2)t}{2} \right). \]

**Induced oscillation (non-degenerate case)**

\[ \phi_2(t) \sim \sin(2\theta)\phi_i, \quad (139) \]

after a few oscillation.

**Induced oscillation (degenerate case)**

In the degenerate limit \( m_1 = m_2 \), we have \( m'_1 - m'_2 \sim m_{12}^2/m_1 \), and

\[ \phi_2(t) \sim -\phi_i \left( \frac{a_i}{a(t)} \right)^{3/2} \sin(m_1 t) \frac{m_{12}^2}{2m_1} t \quad \text{for} \quad t \lesssim \frac{2m_1}{m_{12}^2}. \quad (140) \]
Some definition for later convenience

Remember the definition of $\theta_i$, $\tan \theta_i = -\dot{\phi}_i/F_i$. We can estimate its time derivative as

$$
\dot{\theta}_i = \frac{1}{F_i} \left( -\ddot{\phi}_i + \frac{\dot{\phi}_i \dot{\rho}_{SB}^i}{2 \rho_{SB}^i} \right) = \frac{\partial_{\phi_i} V}{F_i} - \frac{3m_{3/2} \dot{\phi}_i^2}{\rho_{SB}^i} + \frac{3H \dot{\phi}_i F_i}{\rho_{SB}^i},
$$

which is of the order of $\sim \mathcal{O}(m_{\phi_i}) + \mathcal{O}(m_{3/2}) + \mathcal{O}(H)$. It is also conveniently expressed as

$$
\dot{\theta}_i = \frac{\partial_{\phi_i} V}{F_i} - m_{3/2} - \hat{m}_{3/2}^i,
$$

where we have decomposed

$$
\hat{m}_{3/2} = \sum_i \alpha_i^2 \hat{m}_{3/2}^i, \
\hat{m}_{3/2}^i = \frac{3Hp_i^W + m_{3/2}(\rho_{SB}^i + 3p_{SB}^i)}{2\rho_{SB}^i},
$$

where

$$
p_{SB}^i \equiv |\dot{\phi}_i|^2 - |F_i|^2, \quad p_W^i \equiv -(\dot{\phi}_i^* F_i + \dot{\phi}_i F_i^*).
$$

Note that $(\rho_{SB}^i)^2 = (p_{SB}^i)^2 + |p_W^i|^2$. 

Estimation of the mass eigenvalues

Single-superfield model I

Trace and determinant

\[ \text{Tr } \mathcal{M} = \hat{m}_{3/2} + m_f - \alpha_1^2 \dot{\theta}_1 - \alpha_2^2 \dot{\theta}_2 \]

\[ \simeq m_\phi + 2\alpha_1^2 \hat{m}_{3/2}^1 + (\alpha_1^2 - \alpha_2^2)m_{3/2}, \]

(145)

\[ \text{det } \mathcal{M} = \alpha_1^2 \alpha_2^2 \sin^2(\theta_1 - \theta_2)(\hat{m}_{3/2} - m_f^c)^2 - \alpha_2 \dot{\alpha}_2 (\hat{m}_{3/2} - m_f^c) \sin(2(\theta_1 - \theta_2)) \]

\[ - (\dot{\alpha}_1^2 + \dot{\alpha}_2^2) \sin^2(\theta_1 - \theta_2) + \alpha_2^2 (\hat{m}_{3/2} - m_f^c)(\dot{\theta}_2 - \dot{\theta}_1) \]

\[ + (\hat{m}_{3/2} - \dot{\theta}_1)(m_f^c - \dot{\theta}_2) \]

\[ \simeq - \alpha_2^2 m_\phi m_{3/2} + \alpha_1^2 \alpha_2^2 \sin^2(\theta_1 - \theta_2)(\hat{m}_{3/2}^1)^2 - \alpha_1^2 \alpha_2^2 (\hat{m}_{3/2}^1 + m_{3/2})^2 \]

(146)

where \( \dot{\theta}_\phi \simeq -m_\phi - m_{3/2} - \hat{m}_{3/2}^1, \ \dot{\theta}_z \simeq 0, \)

\( \hat{m}_{3/2}^1 = (m_{3/2} + 3H \sin 2\theta_1 - 3m_{3/2} \cos 2\theta_1)/2, \ \hat{m}_{3/2}^2 \simeq -m_{3/2}, \)

\( m_f^c \equiv m_f + \alpha_2^2 \dot{\theta}_1 + \alpha_1^2 \dot{\theta}_2 \simeq -\alpha_2^2 (m_{3/2} + \hat{m}_{3/2}^1) \) and it satisfies

\( \hat{m}_{3/2} - m_f^c \simeq \hat{m}_{3/2}^1. \ \dot{\alpha}_i \) have been neglected because

\( \alpha_1 \dot{\alpha}_1 = -\alpha_2 \dot{\alpha}_2 = O(\min [(m_{3/2}^0)^2/H, H^2/(m_{3/2}^0)]) \lesssim O(m_{3/2}^0). \)
Estimation of the mass eigenvalues

Single-superfield model II

Mass eigenvalues
Since \((\text{Tr} \, \mathcal{M})^2 \gg \text{det} \, \mathcal{M}\), the mass eigenvalues of \(\mathcal{M}\) are given by

\[
\frac{1}{2} \left( \text{Tr} \, \mathcal{M} \pm \sqrt{(\text{Tr} \, \mathcal{M})^2 - 4 \text{det} \, \mathcal{M}} \right) = \begin{cases} 
\text{Tr} \, \mathcal{M} & \text{for the heavy state}, \\
\text{det} \, \mathcal{M} / \text{Tr} \, \mathcal{M} & \text{for the light state}. 
\end{cases}
\] (147)

Thus, we obtain the eigenvalues

\((m_{\text{heavy}}, m_{\text{light}}) \simeq (m_\phi + 2\alpha_1^2 \tilde{m}_3^{1/2}, -\alpha_2^2 m_3^{1/2})\).
Multi-superfield model I

Matter fermion mass matrix

\[ \hat{m}_f \simeq m_f \simeq \text{diag}(m_\phi, -m_\phi, 0), \]  

in the “light-cone” basis \((\Phi_+, \Phi_-, z)\) with \(\Phi_\pm = (\Phi \pm X)/\sqrt{2}\).

\(\dot{\theta}_i\) and \(\hat{m}_{3/2}^i\)

\[ \dot{\theta}_\pm \simeq \mp m_\phi - m_{3/2} - \hat{m}_{3/2}^\pm \text{ and } \hat{m}_{3/2}^\pm \simeq (m_{3/2} - 3m_{3/2} \cos 2\theta_1 \pm 3H \sin 2\theta_1)/2. \]

Using these, we can straightforwardly calculate the following quantities.

Trace, determinant, and one more combination

\[ \text{Tr}\mathcal{M} = (1 - 2\alpha_2^2)m_{3/2} + 2\left(\alpha_+^2 \hat{m}_{3/2}^+ + \alpha_-^2 \hat{m}_{3/2}^-\right), \]  

\[ \det \mathcal{M} = s_2^2 m_{3/2}^2 m_\phi + \cdots = \alpha_z^2 m_{3/2}^2 m_\phi + \cdots, \]

\[ m_1m_2 + m_2m_3 + m_3m_1 = \mathcal{M}_{11}\mathcal{M}_{22} + \mathcal{M}_{22}\mathcal{M}_{33} + \mathcal{M}_{33}\mathcal{M}_{11} - \mathcal{M}_{12}^2 - \mathcal{M}_{23}^2 - \mathcal{M}_{31}^2 \]

\[ = -m_\phi^2 - 2(\alpha_+^2 \hat{m}_{3/2}^+ - \alpha_-^2 \hat{m}_{3/2}^- + \mathcal{O}(m_{3/2}))m_\phi + \cdots, \]

where \(m_1, m_2,\) and \(m_3\) are mass eigenvalues.
Mass eigenvalues

The three mass eigenvalues are

\[(m_\phi + 2\alpha_+^2 \hat{m}_{3/2}^+, -m_\phi + 2\alpha_-^2 \hat{m}_{3/2}^-, -\alpha_+^2 m_{3/2})\].

Estimation of the mass eigenvalues

Multi-superfield II
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