Penetration level Permission for DG Considering Transmission-Distribution-Coordination

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Abstract. With the rapid development of distributed power supply such as photovoltaic and fan, the coupling relationship between transmission network and distribution network is greatly enhanced, which also puts forward higher requirements for collaborative management and scheduling of transmission and distribution network. For this reason, this paper proposes a method to calculate the maximum access capacity of distributed generation considering the cooperation of transmission and distribution. First of all, based on the global optimization model of transmission and distribution, considering the characteristics and similarities and differences of the transmission and distribution network, this paper defines the model variables, builds the calculation model of the maximum access capacity of the distributed power supply considering the transmission and distribution coordination, uses the heterogeneous decomposition algorithm to solve the optimization model, and obtains the maximum access capacity of the distributed power supply. The results show that the proposed model has high accuracy and good convergence performance, and it has engineering application value.

1. Introduction

With the rapid development of distributed power supply such as photovoltaic and wind turbine, the coupling relationship between transmission network and distribution network is greatly enhanced. The concept of transmission and distribution collaborative management and scheduling came into being [1]. In the aspect of scientific research, more and more researches focus on the collaborative calculation and optimal scheduling of transmission and distribution, including power flow calculation [2], economic scheduling [3], optimal power flow [4], reactive power optimization [5], unit combination [6], etc... However, there are few related research results on the calculation of the maximum access capacity of the transmission and distribution collaborative distributed generation.

The access of large-scale distributed generation has a great influence on the safe and stable operation of traditional power system. Therefore, in the power grid planning stage, it is necessary to calculate the maximum access capacity of the distributed power supply under the constraints of voltage and power flow, so as to provide basis and reference for the actual access of the distributed power supply. In recent years, many scholars have conducted extensive and in-depth research on this issue from different perspectives. For example, [7-9] carries out specific and detailed model for distributed power sources such as photovoltaic and wind farms, so as to calculate the maximum accessible capacity method. In [10-11], the factors such as load characteristics, load uncertainty, voltage fluctuation constraint, N-1 principle are added to be considered to get a more practical model. However, the background of transmission and distribution coordination has not been fully considered.
The transmission and distribution network is managed by different dispatching departments, the models and data between them are private, and the Centralized Optimal Dispatching calculation is not in line with the engineering practice. Moreover, there are order of magnitude differences in network parameters, voltage levels, power flow and other aspects of the transmission and distribution network. The Centralized Optimal Scheduling calculation may face serious numerical problems. Therefore, the decomposition algorithm is the main calculation method in the existing research of collaborative optimal scheduling of transmission and distribution, including alternative optimal multiplier method (ADMM) [12-13], auxiliary problem principle method (APP) [14-15], target diversion method (ATC) [16-17], heterogeneous decomposition algorithm (HGD) [4, 18], etc. Among them, HGD algorithm is designed for the problem of transmission and distribution collaborative optimization scheduling. Compared with the traditional decomposition algorithm, HGD algorithm has better convergence performance and avoids many complex processes of parameter setting.

Therefore, HGD algorithm is used to calculate the maximum access capacity of the distributed generation. By analysing the accuracy, efficiency and convergence performance of the algorithm, the effectiveness of the proposed optimization model and algorithm is verified.

2. Calculation model of maximum accessing capacity of DG

2.1. Global optimization model of transmission and distribution network

The transmission network and distribution network are connected by boundary nodes. The boundary node generally refers to the root node of the distribution network, which is connected to the transmission network through the transformer. Then the global optimization model of transmission and distribution can be expressed as follows:

\[
\begin{align*}
\min & \ c_T(u^T, u^B, x^T, x^B) + c_D(u^D, x^B, x^D) \\
\text{s.t.} & \qquad f_T(u^T, x^T, x^B) = 0 \\
& \qquad g_T(u^T, x^T, x^B) \geq 0 \\
& \qquad f_B(u^B, x^B, x^D) = 0 \\
& \qquad g_B(u^B, x^B) \geq 0 \\
& \qquad f_D(u^D, x^B, x^D) = 0 \\
& \qquad g_D(u^D, x^B, x^D) \geq 0
\end{align*}
\]

Where, \( x \) represents the vector of the state variable, such as voltage amplitude, voltage phase angle, etc.; \( u \) represent the vector of control variables, such as generator output, reactive power compensation device output, etc.; \( c \) represents the objective function, especially in the calculation of the maximum accessing capacity of the transmission and distribution collaborative distributed power supply, the objective function is the sum of the active power of each distributed power supply; \( f \) and \( g \) represent the equality constraint and inequality respectively In general, inequality constraints include the upper and lower limits of generator output, the upper and lower limits of distributed generation output, the upper and lower limits of reactive power compensation device output, line and transformer capacity constraints, voltage amplitude constraints and so on. The superscripts \( T, B \) and \( D \) represent the transmission network area, boundary area and distribution network area respectively.

In order to model the transmission and distribution network in different regions, the state variables of the transmission and distribution network need to be decoupled. Therefore, the boundary injection power vector \( y_{BD} \) is introduced, and the (4) formula is expressed as follows:

\[
\begin{align*}
\tilde{f}_B(u^B, x^T, x^B) &= y_{BD} \\
\tilde{f}_D(u^D, x^B, x^D) &= y_{BD}
\end{align*}
\]
Furthermore, the generalized global optimization model of transmission and distribution can be simplified as follows:

\[
\min \left( u^T, u^B, x^T, x^B \right) + c_D \left( u^D, x^B, x^D \right) \\
\text{s.t.} \left( u^T, u^B, x^T, x^B, x^D, y^{BD} \right) \in \Omega_T \\
\left( u^D, x^B, x^D, y^{BD} \right) \in \Omega_D
\]

(10)

Where, \( \Omega_T \) represents the feasible region composed of constraints (2), (3), (5) and (8). \( \Omega_D \) represents the feasible region composed of constraints (6), (7) and (9).

The Lagrange function of this model can be expressed as:

\[
L = c_T + c_D + \lambda_T^T f_T + \omega_T^T g_T + \lambda_B^T \left( f_B - y^{BD} \right) + \omega_B^T g_B + \lambda_D^T \left( f_D + y^{BD} \right)
\]

(12)

Where \( \lambda \) is the multiplier vector corresponding to the equality constraint and \( \omega \) is the non negative multiplier vector corresponding to the inequality constraint.

Further, the Karush-Kuhn-Tucker (KKT) condition can be expressed as:

A) the partial differential of \( L \) with respect to each variable is equal to 0;

B) meet the feasibility constraint (11);

C) satisfy the complementary condition constraint (13).

\[
\begin{align*}
\omega_T^T g_T &= 0, \omega_T \geq 0 \\
\omega_B^T g_B &= 0, \omega_B \geq 0 \\
\omega_D^T g_D &= 0, \omega_D \geq 0
\end{align*}
\]

(13)

2.2. Specific definition of model variables

In the process of computing the maximum access capacity of the collaborative distributed power supply, it is necessary to define the state variables, control variables, objective functions and constraints.

2.2.1. State variable and control variable. In this paper, the calculation of the maximum access capacity of the transmission and distribution cooperative distributed power supply is general. The power flow equation is expressed in polar coordinates, so the state variables and control variables can be expressed as follows:

\[
\begin{align*}
\mathbf{u}^T &= \begin{bmatrix} P_G^T \\ Q_G^T \\ P_{DG}^T \\ Q_{DG}^T \\ P_R^T \\ Q_R^T \end{bmatrix} & \mathbf{u}^B &= \begin{bmatrix} P_G^B \\ Q_G^B \\ P_{DG}^B \\ Q_{DG}^B \\ P_D^B \\ Q_D^B \end{bmatrix} & \mathbf{u}^D &= \begin{bmatrix} P_D^D \\ Q_D^D \end{bmatrix} \\
\mathbf{x}^T &= \begin{bmatrix} V_T^T \\ \theta_T^T \end{bmatrix} & \mathbf{x}^B &= \begin{bmatrix} V_B^B \\ \theta_B^B \end{bmatrix} & \mathbf{x}^D &= \begin{bmatrix} V_D^D \\ \theta_D^D \end{bmatrix}
\end{align*}
\]

(14)

Where, \( \mathbf{P} \) and \( \mathbf{Q} \) represent active and reactive power vectors, and \( \mathbf{V} \) and \( \mathbf{\theta} \) represent node voltage amplitude and phase angle vectors. Subscripts \( G, DG \) and \( R \) refer to thermal power unit, distributed power supply and reactive power compensation device respectively. Superscripts \( T, B \) and \( D \) indicate the transmission network area, boundary area and distribution network area respectively.

The boundary injection power vector \( \mathbf{y}^{BD} \) represents the boundary injection active power \( \mathbf{P}^{BD} \) and reactive power \( \mathbf{Q}^{BD} \), namely:

\[
\mathbf{y}^{BD} = \begin{bmatrix} P^{BD} \\ Q^{BD} \end{bmatrix}
\]

(16)
2.2.2. **Objective function.** The optimization model of the maximum access capacity of the distributed generation should aim at the maximum sum of the active power of the distributed generation.

\[ c_f(*) = -\sum P_{DG}^T \quad c_b(*) = -\sum P_{DG}^D \] (17)

2.2.3. **Constraint.**

A) **Equality Constraint -- power flow equation constraint.** For node \( i \) of transmission network area, boundary area and distribution network area, there are equality constraints (18) - (19), (20) - (21), (22) - (23) respectively:

\[ P_i^T - V_i^T \sum_{j \in C^T} V_j^T (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) \]

\[ -V_i^T \sum_{j \in C^T} V_j^B (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \] (18)

\[ -V_i^T \sum_{j \in C^T} V_j^T (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \]

\[ Q_i^T - V_i^T \sum_{j \in C^T} V_j^T (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \] (19)

\[ -V_i^B - V_i^D \sum_{j \in C^D} V_j^T (G_{ij} \sin \theta_{ij} + B_{ij} \sin \theta_{ij}) \]

\[ -V_i^B - V_i^D \sum_{j \in C^D} V_j^B (G_{ij} \sin \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \] (20)

\[ -V_i^D \sum_{j \in C^D} V_j^T (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \]

\[ -V_i^D \sum_{j \in C^D} V_j^D (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \] (21)

\[ P_i^D - V_i^D \sum_{j \in C^D} V_j^D (G_{ij} \sin \theta_{ij} + B_{ij} \sin \theta_{ij}) \]

\[ -V_i^D \sum_{j \in C^D} V_j^D (G_{ij} \sin \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \] (22)

\[ Q_i^D - V_i^D \sum_{j \in C^D} V_j^D (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) \]

\[ -V_i^D \sum_{j \in C^D} V_j^D (G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij}) = 0 \] (23)

Where, \( C \) represents the node set, and \( G_{ij} \) and \( B_{ij} \) represent the elements in row \( i \) and column \( j \) of the conductivity matrix and the susceptance matrix. \( P \) and \( Q \) represent the injected active power and reactive power of node \( i \), equal to the output of thermal power unit, distributed power supply and reactive compensation device connected at node \( i \) and the load at node \( i \). \( \theta_{ij} \) represents the phase angle difference between node \( i \) and node \( j \).

B) **Inequality constraint.** The inequality constraints considered in this paper mainly include: upper and lower limits of generator output (24), upper and lower limits of distributed generation output (25), upper and lower limits of reactive power compensation device output (26), line and transformer capacity constraints (27), voltage amplitude constraints (28).

\[ P_G \leq P_i \leq \overline{P}_G \]

\[ Q_G \leq Q_i \leq \overline{Q}_G \] (24)

\[ P_{DG} \leq P_{DG} \leq \overline{P}_{DG} \]

\[ Q_{DG} \leq Q_{DG} \leq \overline{Q}_{DG} \] (25)

\[ Q_R \leq Q_i \leq \overline{Q}_R \] (26)

\[ A \leq A \leq \overline{A} \] (27)

\[ V \leq V \leq \overline{V} \] (28)

Where, \( A \) represents the capacity vector of the line or transformer, and the upper and lower limits of the variable are indicated by the upper and lower dashes.
3. HGD solving model

3.1. HGD Introduction

There are different scheduling characteristics between transmission and distribution networks, that is, the optimal scheduling of transmission and distribution coordination is different and heterogeneous. HGD algorithm is an effective way to realize this differential and heterogeneous optimal scheduling. Under the HGD algorithm, the global optimization model of transmission and distribution can be decomposed into transmission network optimization model and distribution network optimization model with the aid of auxiliary functions, so that the model and data of transmission network and distribution network can be decoupled, so as to ensure their privacy and autonomy of optimal scheduling. Furthermore, they are optimized alternately in various regions through the voltage and injection power values of boundary nodes, iterating repeatedly until convergence. In reference [6], it is proved that HGD algorithm has the property of local first order convergence.

Specifically, the optimization model of transmission network can be expressed as (29) - (30), and the optimization model of distribution network can be expressed as (31) - (32):

\[
\min c_f(u^T, u^B, x^T, x^B) + c_{auxT} \tag{29}
\]

\[
s.t. (u^T, u^B, x^T, x^B) \in \Omega_T(y_0^{BD}) \tag{30}
\]

\[
\min c_f(u^D, x^D, x^B) = x_0^B + c_{auxD} \tag{31}
\]

\[
s.t. (u^D, x^D, y^{BD}) \in \Omega_D(x_0^B) \tag{32}
\]

Where \( \Omega_T(y_0^{BD}) \) represents the feasible region of reduced operation of transmission network when \( y^{BD} = y_0^{BD} \), and \( \Omega_D(x_0^B) \) represents the feasible region of reduced operation of distribution network when \( x^B = x_0^B \). \( c_{auxT} \) and \( c_{auxD} \) are auxiliary functions, ensuring that the optimization objectives of the two models after decomposition are consistent with those of the original model.

\[
\begin{align*}
\{ c_{auxT}(x^B) &= h_{BD}^T x^B \\
 c_{auxD}(y^{BD}) &= \lambda_{TB}^T y^{BD} \}
\end{align*} \tag{33}
\]

\[
h_{BD} = \frac{\partial c_D}{\partial x^B} + \frac{\partial f_D}{\partial x^B} \lambda_D + \frac{\partial f_D}{\partial x^B} \sigma_D + \frac{\partial f_D}{\partial x^B} \lambda_{BD} \tag{34}
\]

In the specific optimization model of the maximum access capacity of the transmission and distribution cooperative distributed power supply, items 1 and 3 in (34) are zero. The calculation process is as follows:

Step 1: Set the initial value and the convergence precision of \( x^B \) and \( \lambda_{TB} \), set the maximum number of iterations, and the current number of iterations \( k = 0 \).

Step 2: According to the current value of \( x^B \) and \( \lambda_{TB} \), \( y^{BD} \) and \( h_{BD} \) are calculated based on the optimization model (31) - (34).

Step 3: Use the current \( y^{BD} \) and \( h_{BD} \) to calculate and update \( x^B \) and \( \lambda_{TB} \), based on the optimization models (29), (30) and (33).

Step 4: If the \( x^B \) and \( \lambda_{TB} \) meet the convergence accuracy, then the algorithm convergence ends; if they do not meet the convergence accuracy, \( k = k + 1 \), if they reach the maximum number of iterations, then the algorithm does not converge, otherwise repeat step 2 to step 4.

4. Example analysis

In order to test the algorithm proposed in this paper, it is necessary to construct a whole network example of transmission and distribution network integration. Here, the transmission network example case30 of Mat power is spliced with the IEEE distribution network 69 node standard example case69. The specific splicing method is to connect the 30 node of case30 to the root node of case69 through the transformer. The upper and lower limits of voltage amplitude of each node are 1.1pu. U. and 0.9pu. U. At the same time, the generator at nodes 1, 2 and 22 of case30 is regarded as the thermal power generation unit, the generator at nodes 27, 23 and 13 of case14 is regarded as the distributed power supply, and some nodes
of case69 are connected to the distributed power supply (DG) or reactive power compensation device (RPC), as shown in Table 1:

| Access node | Type  | Maximum output active power/MW | Minimum output active power/MW | Maximum output reactive power/MVar | Minimum output reactive power/MVar |
|-------------|-------|--------------------------------|-------------------------------|-----------------------------------|-----------------------------------|
| 15          | DG    | 2.0                            | 0                             | 1.0                               | 0                                 |
| 30          | DG    | 2.0                            | 0                             | 1.0                               | 0                                 |
| 45          | DG    | 0.5                            | 0                             | 1.0                               | 0                                 |
| 61          | DG    | 1.0                            | 0                             | 1.0                               | 0                                 |
| 54          | RPC   | -                              | -                             | 1.2                               | 0                                 |

4.1. Accuracy test

In order to test the accuracy of the model and algorithm proposed in this paper, the calculation results are compared with the maximum access capacity calculation results of the center type and the transmission and distribution network independent distributed generation. The central model refers to the model obtained after the transmission and distribution network model is spliced. Although this model has data privacy, numerical stability and other problems, it is not in line with the engineering practice, but it can be used to calculate the accurate results of the maximum access capacity of the distributed power supply, providing reference and comparison for the calculation results of other methods. The independent model of transmission and distribution network refers to the calculation of the maximum access capacity of the distributed power in the distribution network area assuming that the root node voltage of the distribution network is 1.0 p.u., then the injection power at the root node is substituted into the transmission network to calculate the maximum access capacity of the distributed power in the transmission network area independently, and finally the calculation results of the two are added to get the results of the whole network.

The left and right subgraphs in Figure 1 respectively show the calculation results of each unit in the transmission network area and distribution network area under three models. It is not difficult to find that the output of each unit obtained by using the model and algorithm of transmission and distribution coordination proposed in this paper is basically consistent with the calculation results of the central type, while the independent type calculation results have a large area in the calculation results of the transmission grid area and the central type, such as the output of T1, T2, T22, T23 and other units.

Furthermore, Table 2 shows the access capacity of distributed power in each region, and compares the target function values under different models. It is not difficult to find that the collaborative calculation method proposed in this paper is consistent with the central optimization results, while the independent optimization results deviate.

| Model        | Access capacity of distributed power in transmission network area / MW | Access capacity of distributed power in distribution network area / MW | Total access capacity of distributed generation / MW |
|--------------|------------------------------------------------------------------------|------------------------------------------------------------------------|-----------------------------------------------------|
| Central type | 123.9                                                                   | 5.5                                                                    | 129.4                                               |
| Synergetic   | 123.9                                                                   | 5.5                                                                    | 129.4                                               |
| Independent style | 125.0                                                               | 5.5                                                                    | 130.5                                               |

4.2. Convergence test

In order to test the advantages of HGD algorithm in convergence performance, it is compared with common ADMM and APP decomposition algorithm. It can be seen from Figure 2 that HGD algorithm has strong convergence, and the number of iterations is little affected by the convergence accuracy. Under the accuracy requirements of 1e-7, only six iterations are needed. In contrast, the number of
iterations of ADMM algorithm and app algorithm is greatly affected by the convergence accuracy. When the convergence accuracy is high, the number of iterations is far greater than the HGD algorithm used in this paper.

5. Conclusion
In this paper, the calculation model and solution method of the maximum access capacity of the distributed generation based on the transmission and distribution coordination are proposed.

A) The algorithm used in this paper can get the same calculation accuracy as the central model, and make up for the shortcomings of the data model in the central model, such as poor privacy, potential numerical stability and so on.

B) The algorithm used in this paper has better convergence performance than other common decomposition optimization algorithms, such as ADMM algorithm and app algorithm, especially when the convergence accuracy is high, it can save dozens of iterations.

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