Modeling and Estimation of Massive MIMO Channel Non-Reciprocity: Sparsity-aided Approach

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Abstract—In this paper, we study the estimation of channel non-reciprocity in precoded time division duplexing based massive multiple-input multiple-output (MIMO) systems. The considered channel non-reciprocity model covers both the frequency-response and the mutual coupling mismatches between the transmitter and the receiver chains of a massive MIMO base-station (BS). Based on the assumed non-reciprocity model, it is shown that the effective downlink channel can be decomposed as a product of the uplink channel and another sparse matrix, referred to as the BS transceiver non-reciprocity matrix. Stemming from such modeling, we then propose an efficient estimator of the BS transceiver non-reciprocity matrix exploiting its sparse nature, combined with an appropriate formulation of the associated over-the-air pilot-signaling. The mean-squared error performance of the overall corresponding estimation method is finally evaluated using extensive computer simulations which indicate that the non-reciprocity characteristics can be estimated efficiently and accurately, thus potentially facilitating large system-level performance gains in multi-user massive MIMO systems.

Index Terms—Channel non-reciprocity, frequency-response mismatch, massive MIMO, mutual coupling, sparsity, time division duplexing (TDD).

I. INTRODUCTION

One of the key potential technologies for 5G systems is the so called large array or massive multiple-input multiple-output (MIMO) transmission which can facilitate particularly high cell-level and network-level spectral efficiencies [1]. The key element in such technology is to have high number of antenna units in the base stations (BSs) compared to the number of spatially multiplexed users (user equipment, UE) in the system. In this context, massive MIMO systems typically rely on the reciprocity of the physical downlink (DL) and uplink (UL) channels in time-division duplex (TDD) based radio access to acquire DL channel state information (CSI) for precoding purposes using UL pilots, where the overhead is proportional to the number of UEs in the system [2]. The reason is that collecting DL CSI as in the conventional MIMO systems, where BSs send DL pilots to UEs and UEs report DL CSI back towards BSs using UL control channel or feedback signaling, requires system resources proportional to the number of antennas in the BS side which, in turn, is unfeasible in massive MIMO systems [3].

Although within a coherence interval the physical propagation channels can be assumed reciprocal [2], [3], the responses of transmitters and receivers hardware chains are commonly not identical. Therefore, the resulting effective DL and UL channels are not reciprocal, which is known as channel non-reciprocity problem in TDD systems [4], [5]. As shown in [6], [7], channel non-reciprocity has two main sources, one is the frequency-response (FR) mismatches between transmitter and receiver chains of any individual transceiver, and the other one is the difference in the mutual coupling effects between the antenna elements in antenna array based devices in transmitting and receiving modes.

Several contributions exist in the open technical literature that study the achievable system performance under non-reciprocal channels. In this respect, [5] provides a downlink sum-rate analysis for a general multi-user MIMO system with zero-forcing (ZF) or eigen-beamforming based DL precoding under channel non-reciprocity due to FR mismatch. Then, [8], [9] investigate the achievable downlink sum-rates for maximum-ratio transmission (MRT) and ZF precoding schemes in massive MIMO systems, demonstrating significant performance degradation under practical values of channel non-reciprocity parameters. It is evident that the results of such studies clearly show the need to estimate the channel non-reciprocity parameters in order to mitigate the non-reciprocity problem.

The estimation of channel non-reciprocity in TDD based MIMO systems has also been addressed in various works [3], [4], [6], [10], [11]. These studies can be divided into three main categories: The first two refer to such methods which are performed in BS using a reference antenna and are called “self-calibration” methods [3], [4], [6], [10]. These two categories can be further differentiated based on the availability of additional circuitry. The third category refers to over-the-air (OTA) methods in which BS transmits pilot signals to UEs and receives back properly precoded signals from UEs [6], [11] to facilitate non-reciprocity estimation.

In this work, we propose an OTA-based method to estimate the channel non-reciprocity due to BS side imperfections in multi-user massive MIMO systems. As novel contributions, we consider a massive MIMO system model which incorporates both FR mismatch and mutual coupling mismatches unlike many of the works that consider only FR mismatch, such as [3], [4], [6], [10], [11] or works that focus on more classical small scale MIMO systems [6], e.g., 2-4 BS antennas.

The paper is organized as follows: The considered effective channel non-reciprocity model is presented in Section II, whereas Section III first reviews a novel pilot signaling method between the BS and UEs. Then, we propose a novel BS side non-reciprocity matrix estimation method, building on the pilot-
Large Array Base Station

Frequency Mismatch Matrix
Mutual Coupling Matrix
Mult-user MIMO Propagation Channels

(a)

Large Array Base Station

Multi-user MIMO Propagation Channels
Mutual Coupling Matrix
Frequency Mismatch

(b)

Fig. 1. Principal illustration of physical (a) DL and (b) UL transmissions and receptions including propagation channels, transceivers frequency responses and antenna mutual coupling in the BS.

signaling and the associated system model sparsity characteristics. In Section IV, using the results of empirical performance evaluations, we analyze and demonstrate the performance of our proposed channel non-reciprocity estimator in terms of normalized mean squared error (MSE) and compare it to the existing methods. Finally, conclusions are drawn in Section V.

Notations: Throughout the manuscript, we use upper-case (lower-case) bold letters to denote matrices (vectors), e.g., matrix $X$ and vector $y$. The superscripts $(\cdot)^\dagger$, $(\cdot)^\ast$, $(\cdot)^H$, and $(\cdot)^\top$ indicate transposition, complex-conjugation, Hermitian-transpose, and Moore-Penrose pseudo inverse operations, respectively, whereas $I_n$ denotes the $n \times n$ identity matrix. Finally, we use $CN(0, 1)$ to denote a circular-symmetric zero-mean unit-variance complex Gaussian distribution.

II. EFFECT OF TRANSCIEVER AND MUTUAL COUPLING MISMATCHES ON THE CHANNEL NON-RECIPROCITY

We consider a TDD based system and focus on a single cell with one BS and $K$ single-antenna UEs, while we carry out the basic modeling at an arbitrary OFDM(A) subcarrier. For notational simplicity, the subcarrier index is omitted and thus not explicitly shown. It is also assumed that the BS is equipped with large number of antenna units, $N$, where $N \gg K$.

Owing to the assumed reciprocity of the physical channel in DL and UL transmissions in a TDD network, the BS forms DL precoders based on the estimated UL channel. This leads to high beamforming gains and efficient spatial multiplexing capabilities in an ideal case where the effective DL and UL channels are assumed to be totally reciprocal. However, in practice, in addition to the reciprocal physical channel, the effective DL and UL channels also contain the hardware impacts of the involved BS and UEs. As mentioned earlier, in this work we focus only on the effects of BS transceivers and antenna array on the reciprocity of the effective channels. Thus, as shown in Fig. 1, the effective DL and UL channels at an arbitrary OFDM(A) subcarrier can be expressed as

$$\mathbf{H}_{DL} = \mathbf{H}^\top \mathbf{B}_{TX} \mathbf{A}_{TX},$$

$$\mathbf{H}_{UL} = \mathbf{A}_{RX} \mathbf{B}_{RX} \mathbf{H},$$

where $\mathbf{H}$ is the reciprocal physical multiuser MIMO channel, of size $N \times K$, and $\mathbf{H}_{DL}$ and $\mathbf{H}_{UL}$ are the corresponding effective DL and UL channels, respectively. In the above, $\mathbf{A}$ is a diagonal matrix and incorporates the BS frequency-response characteristics, $\mathbf{B}$ is the antenna mutual coupling matrix of the BS, while the subscripts $TX$ and $RX$ denote the transmit and receive modes, respectively.

Based on (1), the relation between the effective DL and UL channels can be readily expressed as

$$\mathbf{H}_{DL} = \mathbf{H}_{UL}^\top \mathbf{C},$$

where the matrix $\mathbf{C}$ incorporates the overall transceiver non-reciprocity at the BS, and reads as follows

$$\mathbf{C} = \mathbf{A}^{-1}_{RX} (\mathbf{B}_{RX}^\top)^{-1} \mathbf{B}_{TX} \mathbf{A}_{TX}.$$

As can be seen in (2) and (3), due to the mismatches in BS hardware responses and mutual coupling between the transmitting and receiving modes, the effective DL and UL channels are not reciprocal. This phenomenon is referred to as channel non-reciprocity in the open technical literature [5], [6]. The fully reciprocal effective DL and UL channels are obtained as a special case in which $\mathbf{A}_{RX} = \mathbf{A}_{TX}$ and $\mathbf{B}_{RX} = \mathbf{B}_{TX}$, and thus $\mathbf{C} = \mathbf{I}_N$.

III. ESTIMATION OF BS TRANSCIEVER NON-RECIPROCITY

In order to retrieve the reciprocity of the effective DL and UL channels, the BS requires the knowledge of the overall non-reciprocity matrix $\mathbf{C}$. In practice, this information is not directly available to the BSs and thus needs to be estimated. In this respect, we present a novel method to estimate the BS transceiver non-reciprocity matrix building on simple pilot-signaling from [12] and the resulting sparsity of the corresponding signal model.

In general, the values of $\mathbf{C}$ can be assumed to remain constant over many channel coherence intervals, since the channel non-reciprocity values vary rather slowly compared to the variations in the propagation channel [11]. Therefore, as mentioned in [3], there is no need to perform the BS transceiver non-reciprocity estimation frequently, which can be realized, e.g. once in every 10 minutes or even once a day. This renders the system-level overhead of channel non-reciprocity estimation negligible, while offering the possibility for substantially improved system performance.

A. Pilot Signaling and Proposed Estimation Method

In order to estimate the BS transceiver non-reciprocity, we are adopting an OTA-based estimation approach which allows us to obtain the information about the effective DL channel in BS. In this approach, as described in [12] in more details, the BS first transmits an $N \times N$ orthonormal pilot matrix called $\mathbf{P}$ which yields

$$\mathbf{Y}_{UE} = \sqrt{\rho_d} \mathbf{H}_{DL} \mathbf{P} + \mathbf{N}_{UE} = \sqrt{\rho_d} \mathbf{H}_{UL}^\top \mathbf{C} \mathbf{P} + \mathbf{N}_{UE}.$$
where \( Y_{UE} \) collects the received pilot signals at UE side, \( \sqrt{P_d} \) is the DL transmission signal to noise ratio (SNR), and \( N_{UE} \) is a matrix of noise samples at UE receivers with \( CN(0, 1) \) i.i.d. elements.

Then, UEs conjugate their received pilot signal samples and send them back to the BS in the UL pilot phase, which results into a received signal model of the form
\[
Y_{BS} = \sqrt{P_d}H_{UL}Y_{UE}^* + N_{BS}
\]
\[
= \sqrt{P_d}\sqrt{\rho_d}H_{UL}H_{UL}^*P^* + Q,
\]
where \( Y_{BS} \) is the received pilot signal matrix at BS, \( \sqrt{\rho_d} \) is the UL transmission SNR, and \( N_{BS} \) is the receiver noise matrix at BS with \( CN(0, 1) \) i.i.d. elements, while
\[
Q = H_{UL}N_{UE}^* + N_{BS}
\]
incorporates the effects of noise sources in both DL and UL directions.

In the next step, the BS processes the received pilot signal samples as
\[
R = Y_{BS}P^H
\]
\[
= \sqrt{\rho_d}\sqrt{\rho_d}H_{UL}H_{UL}^*C + Z,
\]
where \( R \) is the processed overall signal and \( Z = Q^*P^H \) denotes the corresponding processed noise.

Assuming then that the BS has perfect knowledge of UL CSI, it can estimate its own transceiver non-reciprocity matrix as
\[
\hat{C} = \arg \min_C ||R - \sqrt{\rho_d}\sqrt{\rho_d}H_{UL}H_{UL}^*C||_F^2
\]
\[
= \arg \min_C ||R - UC||_F = \arg \min_C \sum_i ||r_i - Uc_i||_F^2,
\]
where
\[
U = \sqrt{\rho_d}\sqrt{\rho_d}H_{UL}H_{UL}^*.
\]
(9)

\( r_i \) and \( c_i \) denote the \( i \)-th columns of \( R \) and \( C \), respectively, while the subscript \( F \) denotes the Frobenius norm. Since the \( i \)-th term in the above summation depends only on the \( i \)-th column of \( C \), the problem can be reformulated as
\[
\hat{c}_i = \arg \min_{c_i} ||r_i - Uc_i||_F^2 \quad \forall i,
\]
which means that estimating BS transceiver non-reciprocity matrix, as shown in (7), can be simplified to estimating each of its column independently.

As shown in (3), \( C \) incorporates all the transceiver and antenna array mismatches, including both FR and antenna mutual coupling. Since the level of mutual coupling and its corresponding mismatch depends on the distance between the antennas, the power of off-diagonal entries decreases as the distance between two corresponding antenna elements grows. Therefore, if two antenna elements are far apart, the power of the corresponding element in \( C \) is rather low and its effect on the system performance is negligible. For this reason, we define a threshold for the distance between two antennas, called \( T \), and attempt to estimate only the off-diagonal elements in \( C \) which are corresponding to the antennas with distance \( T \) or less, assuming that all other elements are zero. It becomes evident that this leads to a sparse structure for \( C \) and \( \hat{C} \) which clearly reduces the complexity of the BS non-reciprocity estimation process.

The index of the non-zero entries of \( C \) can be determined by the BS antenna array geometry and architecture which are known to the BS. Having the information regarding the sparse structure of BS transceiver non-reciprocity matrix, we define \( \hat{c}^p \) which contains only the non-zero elements of \( c_i \). Similarly, we define \( U^p \) which contains only the columns with the same index as non-zero entries of \( c_i \), i.e., the \( j \)-th column of \( U \) is kept only if \( j \)-th row of \( c_i \) is kept while constructing \( c^p \). Therefore, with the involved sparsity assumption, the estimation problem in (10) can be further simplified to
\[
\hat{c}^p_i = \arg \min_{c^p_i} ||r_i - U^p c^p_i||_F^2 \quad \forall i,
\]
where the solution for \( \hat{c}^p \) can be obtained as
\[
\hat{c}^p_i = (U^p_i)^\dagger r_i.
\]
(12)

Based on the above, once the value of \( \hat{c}^p \) is determined, the BS appends zeros to appropriate rows and obtains \( \hat{c} \).

**B. Practical Considerations**

In the proposed BS transceiver non-reciprocity estimation method, we assumed that the channel is fixed for the duration of pilot signaling which is \( 2N \) samples. The coherence time of the physical channel is mostly defined by the mobility of the UEs and is typically in the order of several milliseconds. Therefore, we essentially assume relatively low-mobility scenarios in which the channel coherence time is at least \( 2N \).

As mentioned earlier, all the derivations are for an arbitrary subcarrier of the underlying OFDM(A) radio access waveform. The transceiver responses and thus their mismatches can be modeled by mildly frequency-selective transfer functions [6], and can be assumed to remain unchanged over a set of subcarriers \( M \), where typically \( M \leq 10 \), while depending on the frequency selectivity of the propagation channel. \( H_{UL} \) can change from one subcarrier to another. Owing to that, in order to improve the estimation accuracy of the BS transceiver non-reciprocity characteristics, we take the average of calculated \( C \) matrices over \( M \) subcarriers as
\[
\hat{C} = \frac{1}{M} \sum_{l=1}^{M} \hat{C}_l,
\]
(13)
where \( l \) denotes the subcarrier index inside one block of subcarriers over which the averaging is performed.

**IV. NUMERICAL EVALUATIONS AND ANALYSIS**

In this section, we evaluate the performance of the BS transceiver non-reciprocity estimation method proposed in Section III, using extensive computer simulations. The considered performance metric is the normalized MSE which is defined as
\[
\Delta = \frac{||C - \hat{C}||_F^2}{||C||_F^2}.
\]
(14)

We also compare the performance of the proposed method to two other BS transceiver non-reciprocity estimation methods available in the existing literature, namely Argos [3] and generalized
neighbor least squares [10], where the latter has already been shown to outperform several other BS non-reciprocity estimation methods available also in the literature [10] and will be called GNELS in the rest of the paper for notational simplicity.

A. Basic Simulation Settings

We consider a BS with linear array of infinitely thin $\lambda/2$ dipole antennas where $N = 100$, serving $K = 20$ single-antenna UEs simultaneously through spatial multiplexing. The UL channel $\mathbf{H}_{\text{UL}}$ is assumed to have i.i.d. $CN(0,1)$ elements, for which multiple random realizations are drawn. The estimated BS transceiver non-reciprocity matrices are averaged over $M = 10$ subcarriers in the proposed method, while the values of $\mathbf{C}$ are assumed to remain unchanged over those subcarriers. In the proposed method, the DL and UL SNRs for pilot signaling are assumed to be $\rho_d = 20$ dB and $\rho_u = 0$ dB, while for the other two reference methods, the SNR of the channel between two neighboring antennas is assumed to be 80 dB [10]. The operating frequency is chosen to be $f_c = 3.5$ GHz, based on which the BS input and the mutual impedances are computed as given in [13]. The detailed modeling of BS transceiver frequency responses and mutual couplings between antennas are based on [6], in which $\delta_2$ denotes the variance of diagonal elements in $\mathbf{A}_{TX}$ and $\mathbf{A}_{RX}$ and is fixed to $\delta_2 = -30$ dB, while the power of elements in $\mathbf{B}_{TX}$ and $\mathbf{B}_{RX}$ is controlled by the reflection coefficient denoted here as $\delta_1$ and indicatively set to $\delta_1 = -20$ dB. Throughout the simulations, the matrices $\mathbf{A}_{TX}$ and $\mathbf{A}_{RX}$ are chosen independently as well as the matrices $\mathbf{B}_{TX}$ and $\mathbf{B}_{RX}$. These are the baseline values in all the simulations, but are also varied in the experiments as indicated in the result figures.

B. Obtained Numerical Results

As mentioned in Section III, the number of non-zero entries in each column of the estimated BS transceiver non-reciprocity matrix is depending on the sparsity threshold $T$. We define $T$ as the antenna distance threshold relative to $\lambda/2$, which means that the coupling mismatch due to any two antennas with the distance greater than $T \times \lambda/2$ is assumed to be zero in the proposed BS transceiver non-reciprocity estimation method. In order to find the optimum value of $T$ for different scenarios, in Fig. 2, the effect of $T$ on normalized MSE of BS transceiver non-reciprocity estimation is evaluated against various levels of $\delta_B^2$, which controls the power of mutual coupling between the BS antennas. As can be seen, for low BS antenna mutual coupling power, e.g., $\delta_B^2 \leq -30$ dB, $T = 0$ which corresponds to a purely diagonal estimation of BS transceiver non-reciprocity matrix produces the best result, whereas $T = 1$ has better performance in the cases where the power of antenna mutual coupling is moderate, e.g., $-20$ dB $\geq \delta_B^2 > -30$ dB, and finally $T = 2$ is the best option if mutual coupling level is high, i.e., $\delta_B^2 > -20$ dB.

The effect of the number of scheduled UEs $K$ on the performance of the proposed BS transceiver non-reciprocity estimation method is examined in Fig. 3. The results show that the normalized MSE of the proposed estimation method decreases as $K$ grows. The reason is that the column space of $\mathbf{U}_T^*\mathbf{H}_{\text{UL}}$ in (12) has higher dimensionality for larger values of $K$, since $\mathbf{H}_{\text{UL}}$ in $\mathbf{U}$ is positive semi-definite matrix and of rank $K$ if $\mathbf{H}_{\text{UL}}$ is of rank $K$. Fig. 3 also shows that when the number of scheduled UEs grows, e.g., $K \geq 25$, the optimum value for sparsity distance increases, from $T = 1$ to $T = 2$.

Fig. 4 shows the comparison between the proposed estimation method and the two other methods with respect to the impact of the power of BS antenna mutual coupling on the normalized MSEs. As can be seen, Argos exhibits the worst performance, while for low levels of BS antenna mutual coupling power GNELS method, which only estimates the diagonal elements of $\mathbf{C}$, is the best option. However, as the power of BS antenna mutual coupling grows, the difference between the proposed method and GNELS method gets lower. From moderately low levels of BS antenna mutual coupling, e.g., $\delta_B^2 > -27$ dB, the proposed method outperforms all other methods, since contrary to them, it has the ability to estimate the off-diagonal elements of BS transceiver non-reciprocity matrix.

Fig. 5 compares the effect of the number of scheduled UEs on the normalized MSEs for all the considered BS transceiver non-reciprocity estimation methods. It can be seen that, even for the lowest number of scheduled UEs $K = 10$, the proposed estima-
The difference between the performance of the proposed method and the other two methods increases as \( K \) grows, up to the case of \( K = 70 \) where the accuracy of the proposed method is already around 10 times better than that of the GNELS method. The reason is that while increasing \( K \) does not have any effect on the performance of the other two estimation methods, as mentioned earlier it improves the accuracy of the proposed method as the rank of \( \mathbf{H}_{UL} \mathbf{H}_{UL}^H \) in \( \mathbf{U} \) grows. It is also noted that the estimation accuracy of the proposed method depicted in Fig. 4 and Fig. 5 can be improved when compared to two other methods, by adaptively selecting the optimum value of \( T \) according to the results shown in Fig. 2 and Fig. 3, respectively.

Overall, owing to the sparse nature of the BS transceiver non-reciprocity matrix \( \mathbf{C} \), the proposed BS non-reciprocity estimation method outperforms other methods for moderate to high levels of BS mutual coupling power and/or higher numbers of scheduled UEs \( K \) as it can estimate off-diagonal entries of \( \mathbf{C} \) with high accuracy.

V. CONCLUSION

This paper proposed an efficient channel non-reciprocity estimation framework for fully capitalizing the channel reciprocity benefits in TDD massive MIMO networks with non-reciprocal transceiver and antenna array hardware. Based on the provided channel non-reciprocity model, it was first shown that the effective DL channel is the product of the effective UL channel and a sparse matrix which incorporates the effects of both transceiver FR and antenna array mutual coupling mismatches at BS. Then, exploiting the sparse nature of the BS transceiver non-reciprocity matrix, a novel OTA-based BS non-reciprocity estimation method with reasonable pilot overhead was pursued. The comprehensive computer simulations demonstrated the superiority of the adopted estimation method compared to two other well-known existing methods for practical levels of BS antenna mutual coupling power. It was also shown that the accuracy of the considered channel non-reciprocity estimation method improves as the number of scheduled UEs in the system grows.

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