DNS of turbulent channel flow at very low Reynolds numbers

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Direct numerical simulations (DNS) of fully-developed turbulent channel flows for very low Reynolds numbers have been performed with a larger computational box sizes than those of existing DNS. The friction Reynolds number was decreased down to 60, where the friction Reynolds number is based on the friction velocity and the channel half width. When the Reynolds number was decreased to 60 with small computational box size, the flow became laminar. Using a large box, we found that a localized turbulence was observed to sustain in the form of periodic oblique band. This type of locally disordered flow is similar to a equilibrium turbulent puff in a transitional pipe flow. Various turbulence statistics such as turbulence intensities, vorticity fluctuations and Reynolds stresses are provided. Especially, their near-wall asymptotic behavior and budget terms of turbulence kinetic energy were discussed with respect to the Reynolds-number dependence and an influence of the computational box size. Other detailed characteristics associated with the turbulence structures were also presented and discussed.

1 Introduction

Low-Reynolds-number turbulent flow in a channel is of practical importance with respect to many engineering applications, such as heat exchange equipment. Laminar flows show much smaller drag, mixing and heat transfer than turbulent flows do. Transition from a turbulent flow to a laminar flow — so-called laminarization — has been studied experimentally by a number of researchers, e.g., Narasimha & Sreenivasan [1], since the process of turbulent-laminar transition is also important in both the fields of industrial applications and fundamental flow physics.

Direct numerical simulation (DNS, hereafter) of a fully developed turbulent channel flow has been increasingly performed for higher Reynolds numbers with an aid of recent development of computers. DNS provides various information, such as velocity and pressure. Special attention has been paid to their near-wall asymptotic behavior and their derivatives at any time and point in an instantaneous field, which are extremely difficult to be measured in experiments.

The first DNS of the fully developed turbulent channel flow was made by Kim et al. [2]. Their Reynolds number was \( Re_\tau = 180 \), which is based on the channel half width \( \delta \), the kinematic viscosity \( \nu \) and the friction velocity \( u_\tau = \sqrt{\tau_w/\rho} \), where \( \tau_w \) is the statistically averaged wall shear stress and \( \rho \) is the density. Kuroda et al. [3] carried out the DNS for a slightly lower Reynolds number of \( Re_\tau = 150 \). Kawamura and co-workers [4,5,6,7] performed the DNS with respect to Reynolds number and Prandtl number dependences for \( Re_\tau = 180–1020 \). As for low Reynolds numbers, several research groups carried out DNS or LES down to \( Re_\tau = 80 \) in order to study turbulence control [8,9,10]. Iwamoto et al. [11] have executed DNS for \( Re_\tau = 110–650 \): their published results of \( Re_\tau = 110 \) and 150 are also included in this paper for comparison. Iida & Nagano [12] performed DNS for \( Re_\tau = 60–100 \) to investigate flow fields with emphasis

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Table 1. Reynolds numbers of the present DNS and friction coefficient $C_f$.
Reynolds numbers: $Re_\tau = u_\tau \delta/\nu$, $Re_m = u_m 2\delta/\nu$ and $Re_c = u_c \delta/\nu$. Computational domain size: MB, medium box size; LB, large box size; XL, extra-large box size, cf., table 2.

| $Re_\tau$ | 180   | 150   | 110   | 80    | 70    | 64    | 60$^a$ |
|-----------|-------|-------|-------|-------|-------|-------|-------|
| $Re_m$    | 5730  | 4620  | 3290  | 2290  | 2320  | 2010  | 1860  | 1580$^b$ |
| $Re_c$    | 3360  | 2720  | 1960  | 1400  | 1430  | 1270  | 1200  | 930$^b$   |
| Box size  | MB    | MB    | MB    | MB    | XL    | LB    | LB    | XL    |
| $u_c/u_m$ | 1.17  | 1.18  | 1.19  | 1.22  | 1.24  | 1.26  | 1.29  |       |
| $C_f \times 10^3$ | 7.90  | 8.42  | 8.95  | 9.60  | 9.52  | 9.65  | 9.40  |       |

$^a$ This case resulted in a laminarization.
$^b$ A value estimated by equations (3) and (4) on the streamwise vortexes. These studies employed rather small computational boxes because of limited calculation resources.

A great deal of effort has also been devoted to experimental studies of the turbulent channel flow. Laufer [13] first obtained the detailed turbulence statistics in the channel flow. Patel & Head [14, 15] measured skin friction and mean velocity profiles in a range of $Re_m = 1,000–10,000$, which includes the transition from the laminar to the turbulent flow. The bulk Reynolds number $Re_m$ is based on the bulk mean velocity $u_m$ and the channel width. Their measurement with a hot-wire showed that turbulent bursts occurred above $Re_m = 1,380$. Later, Kreplin & Eckelmann [16, 17] made their experiments for low Reynolds numbers of $Re_c = 2,800–3,850$, based on the centreline velocity $u_c$ and $\delta$. The measurements of the turbulent channel flow at $Re_c = 2,850$ and 3,220 were executed by Niederschulte et al. [18]. Durst & Kikura [19] carried out their experiments by means of laser-Doppler anemometry (LDA) at Reynolds number range of $Re_m = 2,500–9,800$, which corresponds to $Re_\tau = 87–293$.

It should be emphasized that comparing the existing experimental results [20, 21, 22] showed wide variation in the critical Reynolds numbers from $Re_c = 266$ to 8000. This is because the transition in plane channel flows is sensitive to the background turbulence, e.g., inlet and initial conditions. According to the linear instability analysis using the Orr-Sommerfeld/Squire equations, no exponentially growing solution is found below $Re_c = 5,772.22$ [23]. Later, theoretical results of Orszag’s group [24, 25] inferred a transitional Reynolds number of about 1,000, when finite-amplitude three-dimensional disturbances were considered. Other researchers also indicated that there is a possibility of transient energy growth of disturbance at a Reynolds number as low as $Re_c = 1,000$ [26, 27, 28].

With the aid of developed supercomputing system, DNS of a turbulent channel flow at high Reynolds number has been carried out with a large computational domain, since much attention is paid to a large-scale motions (LSM) in the outer region [29, 30, 6, 7]. For a lower Reynolds number, on the other hand, near-wall streaky structures are so elongated that their lengths exceed the usual computational box sizes. Thus DNS of a low Reynolds number flow also requires a larger box size to capture the near-wall streaky structures and the LSM.

In the present work, DNS of a fully-developed turbulent channel flow has been carried out with larger computational boxes than those of previous works. The purpose of this study is to obtain the turbulence statistics and structures of the turbulent channel flow at very low Reynolds numbers, cf. table 1.

2 Numerical procedure

The mean flow is driven by the uniform pressure gradient: see figure 1. It is assumed to be fully developed in the streamwise ($x$-) and spanwise ($z$-) directions. The coordinates and the flow variables are normalized by $u_\tau$, $\nu$ and $\delta$. Periodic boundary conditions are imposed in the $x$- and $z$-directions and a non-slip condition is applied on the walls. The fundamental equations are the
continuity and the Navier-Stokes equations:

\[ \frac{\partial u_i}{\partial x_i} = 0, \tag{1} \]

\[ \frac{\partial u_i^*}{\partial t^*} + u_j^* \frac{\partial u_i^*}{\partial x_j} = - \frac{\partial p^*}{\partial x_i} + \frac{1}{Re} \frac{\partial^2 u_i^*}{\partial x_j^2}, \tag{2} \]

where \( u_i, t \) and \( p \) are velocity vector, time and pressure, respectively. Note that the superscript * indicates a quantity normalized by \( u_\tau \) and/or \( \delta \).

For the spatial discretization, the finite difference method was adopted. The numerical scheme with the 4th-order accuracy was employed in the \( x \)- and \( z \)-directions, while the one with the 2nd-order is applied in the \( y \)-direction. Time advancement was executed by the semi-implicit scheme: the 2nd-order Crank-Nicolson method for the viscous terms on the \( y \)-direction and the 2nd-order Adams-Bashforth method for the other terms.

In the present work, a series of DNS were made for \( Re_\tau = 60–180 \). Table 1 summarizes the friction Reynolds numbers \( Re_\tau \) and some mean-flow parameters. The computational domain should capture at least a couple of near-wall low-speed streaks, and \( L_i^* \) should be relatively large in a low-Reynolds-number DNS because of existence of an elongated streaky structure. For the lower Reynolds numbers of \( Re_\tau = 60–80 \), the larger boxes than those in the literature were adopted. The computational conditions are shown in Table 2. The stream- and spanwise lengths of the computational domain of XL were chosen as \( L_x^* \times L_z^* = 51.2 \times 22.5 \), which were approximately 4100 and 1800 wall units (at \( Re_\tau = 80 \)), respectively, whereas the characteristic sizes of the near-wall structure are \( \lambda_x^+ \approx 1000 \) and \( \lambda_z^+ \approx 100 \). This was found to be large enough to capture the elongated near-wall streaky structure.

The non-uniform meshes were applied in the wall-normal direction. Abe et al. [5] confirmed, \textit{a posteriori}, that the 2nd-order scheme retains high accuracy on a non-uniform mesh and acceptable results can be obtained when fine enough grids are adopted. The minimum wall-normal grid spacing was approximately equal to 0.3\( \eta \) (\( \eta \) is referred to as a local Kolmogorov scale) at \( Re_\tau = 180 \). In the present study, a coarser mesh \((N_y = 96)\) was used only for \( Re_\tau = 80 \) (XL). In this case, the grid spacings were \( \Delta y^+ = 0.22–3.59 \), which corresponded to 0.13\( \eta \)–1.25\( \eta \). With the finer mesh of \( N_y = 128 \), the maximum spacing (at the centreline of the channel) is also adjusted to keep the resolution, less than 1.6\( \eta \), for the other Reynolds numbers. The grid resolutions
of the present simulations are sufficiently fine to resolve the essential turbulent scales, and the reliable results can be achieved even with the 2nd-order scheme in the wall-normal direction.

In the present DNS, the pressure gradient was decreased stepwise down to an estimated level for an aimed Reynolds number. A fully-developed flow field at a higher Reynolds number was successively used as the initial condition for a one-step lower Reynolds number, e.g., \( Re_\tau = 180 \rightarrow 150, 150 \rightarrow 110, 110 \rightarrow 80 \) and so on. Note that various statistical data were obtained after the flow had reached a statistical-steady state.

3 Results and Discussion

3.1 Mean velocity profile

Figure 2 shows the wall-normal profile of the dimensionless mean velocity in the wall units, where quantities with the superscript of \( + \) indicate those normalized by the wall variables, e.g., \( y^+ = yu_\tau/\nu \). Statistics are denoted by an overbar of ( ), that are the spatial (in the horizontal directions) and temporal averaging. The obtained data from the experiments of Patel & Head [15] and the DNS by Iwamoto et al. [11], who used the spectral method are also shown for comparison. The present results for \( Re_\tau = 110–180 \) are in reasonable agreement with the existing DNS. For the lower Reynolds numbers of \( Re_\tau \leq 110 \), the Reynolds-number dependence in the DNS data is also consistent with the trend of the mean velocity profiles measured by Patel & Head [15]. In these low Reynolds number flows, the mean velocity distributions do not indicate an evident logarithmic region. Thus, the von Kármán constant (not shown here) does not exhibit any constant range at all. In the outer region \( (y^+ > 10) \), a significant Reynolds-number dependence is found when normalized by the inner variables.

The mean flow variables such as the bulk mean velocity \( u_\tau^+ \) and the mean centreline velocity \( u_c^+ \) are given in figure 3 for each Reynolds number. It is interesting to note that the values of \( u_c^+ \) and \( u_m^+ \) increase with decreasing Reynolds number for \( Re_\tau \leq 80 \). Both \( u_c^+ \) and \( u_m^+ \) are expected to approach gradually to laminar values. Dean [31] found that the ratio between \( u_c \) and \( u_m \) was given by

\[
\frac{u_c}{u_m} = 1.28 Re_m^{-0.0116}
\]
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Figure 3. Reynolds number versus centreline velocity $u_c^+$ and bulk mean velocity $u_m^+$. Dotted lines (---, ----) show the laminar flow relations of $u_c^+ = Re/2$ and $u_m^+ = Re/3$, respectively.

Figure 4. Variation with bulk Reynolds number of $C_f$, for a turbulent channel flow. In the range of the present Reynolds numbers, the increasing rate of $u_c/u_m$ (see table 1) with decreasing $Re_m$ is larger than the one indicated by equation (3). The discrepancy between the obtained $u_c/u_m$ and equation (3) reflects the low Reynolds-number effect. If emphasis is placed on the data at $Re_\tau = 80$ (MB and XL), one can see slight increases in $u_c$, $u_m$ and $u_c/u_m$ with extending the box size since the quasi-laminar region locally appeared in the flow field of XL, as will be discussed in section 3.5.

### 3.2 Laminarization

A skin friction coefficient in the transitional region is one of the most fundamental turbulence characteristics. A large number of experimental studies have been devoted to this issue, while it has not been examined yet through DNS owing to the lack of the low Reynolds number
simulations. Figure 4 shows a variation of the skin friction coefficient $C_f = 2\tau_w / (\rho \cdot u_m^2)$ of the channel flow in comparison with the empirical correlations proposed by Blasius for a pipe flow, and by Dean [31] for the channel flow as,

$$C_f = 0.073\left(Re_m\right)^{-0.25}.$$  (4)

Both the measurement result and the numerical data from the previous DNS on the turbulent channel flows are also shown in figure 4.

The present results are in good agreement with the reference data for $Re_m > 3,000$. It is worth noting that, for $Re_m < 3,000$, $C_f$ tends to be smaller than the empirical correlations and decreases for $Re_m$ less than about 2,000. This tendency agrees well with the data from the measurement of Patel & Head [15] and the DNS of Iida & Nagano [12]. The approximate value of 2,000 mentioned above roughly coincides with a critical Reynolds number, below which the flow became intermittently laminar and turbulent in the experiment by Patel & Head [14, 15]. They also showed that the ($C_f$ vs. $Re_m$)-curve, not shown in figure 4, remained unaltered for different entry conditions. This implies that the decrease in $C_f$ would be a universal phenomenon independent of the inlet condition in the case of their experiment. In the present investigation, there may be also inconsiderable differences between the values (of $C_f$ or $u_m$) in the two flows of $Re_\tau = 80$ (MB and XL), although an evidence of the effect of box size appears in the $u_c$ as described above.

The second lowest Reynolds number ($Re_\tau = 64$) of the present study, which corresponds to $Re_m = 1,860$ in the present DNS, still lies in the transitional flow regime. Further decrease in the Reynolds number below $Re_\tau = 60$ resulted in a laminar flow field. On the other hand, the DNS performed by Iida & Nagano [12] revealed that the flow field remains turbulent accompanied with a temporally intermittent quasi-laminar state even at $Re_\tau = 60$ ($Re_m = 1,720$). Their computational box was $5\pi\delta \times 2\delta \times 2.5\pi\delta$. Since it was smaller than that of the present one, a smaller box size might tend to retain the turbulent state down to a lower Reynolds number than a larger box. Although the lower limit for sustaining turbulence cannot be clearly determined by the calculations up to now, the critical Reynolds number is estimated to lay between $Re_\tau = 60$ and 64, namely $Re_m = 1,580$ and 1,860 or $Re_c = 930$ and 1,200 under the present condition.

Transition experiments show that the plane channel flows typically undergo transition to turbulence at Reynolds numbers as low as $Re_c = 1,000$ [15, 32]. These experimental and the present numerical critical Reynolds numbers are both far below the critical Reynolds number of $Re_c = 5,780$ [33] predicted with the two-dimensional linear stability analysis theory. The discrepancy between the present critical Reynolds number ($\sim 1,200$) and the experimental one ($\sim 1,000$) may be attributed to an influence of periodic boundary condition in DNS and/or to that of sidewalls unavoidable in experiments.

### 3.3 Turbulence intensity

The root-mean-square (r.m.s.) of the streamwise velocity fluctuation normalized by $u_\tau$ is given in figure 5. It increases remarkably at the centreline of the channel with decreasing the Reynolds number in the region of $Re_\tau < 80$. The value of the local maximum near the wall decreases remarkably for $Re_\tau < 80$.

With respect to the computational box size, a significant influence of the box size on $u_{rms}^\prime$ can be seen for $Re_\tau = 80$ at the central region. With extending box size, $u_{rms}^\prime$ is enhanced, with a deviation of about 12% at the channel centre. This tendency is also seen for the spanwise fluctuations in the whole channel (figure 6(b)), with a deviation of 7–27%. The influence on the wall-normal component is small (figure 6(a)). Moreover, the influence of the box size on the mean flow variables and other turbulence quantities, such as $\overline{u}$ and vorticity fluctuations (shown later), are also significant at the very low Reynolds number of $Re_\tau = 80$, whereas
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Figure 5. Root-mean-square of the streamwise velocity fluctuation $u'$. 

Figure 6. Root-mean-square of (a) the wall-normal fluctuation $v'$ and (b) the spanwise fluctuation $w'$.

Table 3. Near-wall expansion coefficient

| $Re_{\tau}$ | $b_1$   | $c_2$   | $b_3$   | $b_{1,3} - b_{3,1}$ | $b_{1,c2}$ | $(b_1 d_2 + c_1 c_2)$ |
|-------------|---------|---------|---------|---------------------|-------------|----------------------|
| 180         | 0.360   | 8.70 × 10^{-3} | 0.189   | 2.65 × 10^{-2}     | 7.29 × 10^{-4} | 2.0 × 10^{-5}        |
| 180*        | 0.356   | 8.5 × 10^{-3}  | 0.190   | 2.7 × 10^{-2}      | 7.0 × 10^{-4}  | ——                   |
| 150         | 0.354   | 8.60 × 10^{-3} | 0.172   | 2.60 × 10^{-2}     | 7.08 × 10^{-4} | 1.9 × 10^{-5}        |
| 110         | 0.355   | 6.75 × 10^{-3} | 0.145   | 2.46 × 10^{-2}     | 5.41 × 10^{-4} | 1.5 × 10^{-5}        |
| 80 MB       | 0.302   | 4.55 × 10^{-3} | 0.105   | 2.06 × 10^{-2}     | 3.42 × 10^{-4} | 0.8 × 10^{-5}        |
| 80 XL       | 0.333   | 6.85 × 10^{-3} | 0.144   | 2.28 × 10^{-2}     | 4.45 × 10^{-4} | 2.0 × 10^{-5}        |
| 70          | 0.291   | 4.65 × 10^{-3} | 0.101   | 2.08 × 10^{-2}     | 3.11 × 10^{-4} | 0.9 × 10^{-5}        |
| 64          | 0.268   | 3.90 × 10^{-3} | 0.086   | 1.91 × 10^{-2}     | 2.45 × 10^{-4} | 0.6 × 10^{-5}        |

*Antonia & Kim [35]

Effects of Reynolds number significant in the $u_{rms}^{+}$ of both the spanwise and the wall-normal directions. All the component values decrease as the Reynolds number decreases. Antonia et al. [34] indicated that the Reynolds-number dependence of $w_{rms}^{+}$ is more significant compared to these of $u_{rms}^{+}$ and $v_{rms}^{+}$. In the present work, both $v_{rms}^{+}$ and $w_{rms}^{+}$ decrease remarkably with $Re_{\tau}$. This is because the production term of $v_{rms}^{+}$ and the redistributions for $v_{rms}^{+}$ and $w_{rms}^{+}$ are reduced with the decreasing $Re_{\tau}$ as discussed later.

To analyse the near-wall asymptotic behaviour, the velocity fluctuations can be expanded in Taylor series about-the-wall value as follows:

$$
\begin{align*}
    u^{+} &= b_1 y^{+} + c_1 y^{+2} + d_1 y^{+3} + \cdots \\
    v^{+} &= c_2 y^{+2} + d_2 y^{+3} + \cdots \\
    w^{+} &= b_3 y^{+} + c_3 y^{+2} + d_3 y^{+3} + \cdots
\end{align*}
$$

(5)
The values of the coefficients of the first terms in equation (5) are shown in table 3. The r.m.s. values of the vorticity fluctuations are shown in figure 4. The streamwise and spanwise vorticity fluctuations, namely $\omega_x^+ (= b_3)$ and $\omega_z^+ (= b_1)$, decrease with decreasing Reynolds number. However, the influence of insufficient box size for $Re_\tau = 110$ and 80 (MB) cannot be neglected. The ratio $\omega_y^+/y^+ (= b_{1,3} - b_{1,1} \equiv \partial b_3/\partial x^+ + \partial b_3/\partial x^+)$ tends to become constant in the near-wall region as reported by Antonia & Kim [35].

The use of equation (5) in the expression for the Reynolds shear stress, $-\overline{u'^+v'^+}$, yields

$$-\overline{u'^+v'^+} = -\left[ \overline{b_1 c_2 y^+^3 + (b_1 d_2 + c_1 c_2)y^+^4 + \cdots} \right].$$

The near-wall values of $(b_1 c_2)$ and $(b_1 d_2 + c_1 c_2)$ are extrapolated up to the wall, and are also given in table 3.

The present results at $Re_\tau = 180$ agree well with those of Antonia & Kim [35]. All of the coefficients, shown in table 3, decrease with the decrease of the Reynolds number. This is because the production rate of the turbulent kinetic energy decreases with decreasing the Reynolds number as discussed in section 3.4. The decrease in $b_1$ with $Re_\tau$ is smaller than that in either $c_2$ or $b_3$, which is comparable to the tendency toward more anisotropic turbulence at lower Reynolds numbers, cf. figures 5 and 6. When the Reynolds number is decreased from 180 to 64, the decrease in $b_1$ is only 25%, compared with 55% in the cases of $c_2$ and $b_3$. The decreases of $c_2$, $b_3$, $b_1 c_2$ and $(b_1 d_2 + c_1 c_2)$ are significant when the Reynolds number falls to 70 or 64. The dependence of the ratio $\omega_y^+/y^+ (= b_{1,3} - b_{3,1})$ on the Reynolds number is not large but still appreciable, i.e., drop of 28% with $Re_\tau = 180 \rightarrow 64$.

In the case of $Re_\tau \leq 80$ (MB), however, the obtained coefficients could be further influenced by the box size. For all the values, the increases due to extending the box size are not negligible compared to the variation with the Reynolds number.

### 3.4 Reynolds shear stress

Figure 8 shows the Reynolds shear stress $-\overline{u'^+v'^+}$ and the total shear stress $\tau_{\text{total}}$. In all the cases of the present calculations, the profile of the total shear stress is given as a straight line, indicating that the flow reaches a statistically steady state. As the Reynolds number decreases, the peak value of $-\overline{u'^+v'^+}$ decreases and its position moves close to the wall, if scaled with the wall unit. When $Re_\tau$ is 180, the peak of $-\overline{u'^+v'^+}$ reaches 0.72 at $y^+ = 32$, while it becomes 0.37 at $y^+ = 26$ in the case of $Re_\tau = 64$. In the fully-developed channel flow, the production term of the turbulent kinetic energy is
Figure 8. Reynolds shear stress $-u'v'$ and total shear $\tau_{\text{total}} = -u'v' + \frac{\partial u'}{\partial y}$ distribution. Blue lines, DNS by Iida & Nagano [12] at $Re = 80$ (——) and $Re = 60$ (— — —); symbol (□), experiment by Niederschulte et al. [18] at $Re = 179$.

Figure 9. Budget of turbulent kinetic energy $k$ normalized by $\nu/u_+^4$ expressed as

$$P_k + k = -u'v'\frac{\partial u'}{\partial y} = -u'v'[1 - \frac{y}{Re} - (-u'v')] \quad (7)$$

Thus the peak value of $P_k$ can be calculated as

$$P_k^+ = \frac{1}{4} \left[1 - \frac{y_{\text{max}}}{Re} \right]^2 \text{ at } \left\{ y^+ - u'v' = \frac{\partial u'}{\partial y} \right\} \quad (8)$$

Here, the wall-normal position of the peak of $P_k^+$ is denoted as $y_{\text{max}}^+$, at which the Reynolds shear
stress and the viscous stress are equal to half of the total shear stress.

In the present calculation, \( y_{\text{max}}^+ \) is 12.5 for \( Re_\tau = 180 \). When the Reynolds number is decreased, it moves away from the wall and reaches at \( y_{\text{max}}^+ \approx 19.5 \) for \( Re_\tau = 64 \). Sahay & Sreenivasan \[36\] and Laadhari \[37\] examined the Reynolds-number dependence of \( y_{\text{max}}^+ \), and related it to those positions at which turbulence production and momentum transport attained their respective maxima. They investigated the evolution of \( y_{\text{max}}^+ \) in the form \( y_{\text{max}}^+ = y_{\text{max}}^+ + f(Re_\tau) \) where \( y_{\text{max}}^+ \rightarrow y_{\text{max}}^+ (Re_\tau \rightarrow \infty) \) is an average value of \( y_{\text{max}}^+ = 11 \) from DNS data for channel flows. A power law fitting of the present data gives

\[
y_{\text{max}}^+ \approx 11 + \left( \frac{243}{Re_\tau} \right)^{\frac{2}{3}}.
\]

The peak value of \( P_{k,\text{max}}^+ \) decreases with decreasing \( Re_\tau \) (see figure 9). Figure 10 shows the peak value of \( P_{k,\text{max}}^+ \) and dissipation rate \( \varepsilon_k^+ \) at \( y_{\text{max}}^+ \) for each Reynolds number. The empirical relation between \( P_{k,\text{max}}^+ \) and \( Re_\tau \), derived by Laadhari \[37\], shows a good agreement with the present DNS results. The decrease in \( P_{k,\text{max}}^+ \) with decreasing \( Re_\tau \) is clearly more prominent than that of \( \varepsilon_k^+ \). This is a reason why the turbulent intensities are decreased in the near-wall region with the decreasing \( Re_\tau \) as seen in figures 5, 6 and in table 3.

3.5 Turbulence structures: coherent structure and LSM

3.5.1 Two-point correlation coefficient. The effect of the box size can be most clearly observed in the streamwise and spanwise two-point correlations of the velocity fluctuations \( R_{ii} \), which is shown in figure 11. The data of \( R_{ii} \) were obtained in a near-wall region at \( y^+ \approx 5 \) for \( Re_\tau = 80 \). In the case of the medium box (MB), \( R_{uu} \) does not fall down to zero, indicating that the box size is not large enough: especially, the streamwise box length is too short to contain the streaky structures in the near-wall region (see figure 11(a)). On the other hand, \( R_{uu} \) and \( R_{ww} \) fall down to negative values at the maximum separation (half the domain size) for both stream- and the spanwise directions for the large box (XL), while \( R_{uv} \) falls off to almost zero. It indicates that one long wavelength structure is captured with XL. In addition, a significant decrease in the
magnitude of the negative maximum is found at the spanwise separation distance of $\Delta z^+ \approx 50$, which corresponds to the spanwise spacing of the near-wall streaky structures (figure 11(b)). The negative values of $R_{uu}$ and $R_{uw}$ at the mid box length become the largest in the channel centre region (not shown here). These suggest that an influence of a structure that is larger than a streaky structure exists not only in the channel centre but also in the near-wall region. The spacing of the streaky structures and the large-scale structures are investigated with the use of the pre-multiplied energy spectra, which will be discussed in more detail.

3.5.2 Pre-multiplied energy spectra. The streamwise and spanwise pre-multiplied energy spectra for $Re_\tau = 80$ (XL) are shown in figure 12 with reference to Jiménez [29]. To examine the hypothesis that the higher-Reynolds-number turbulence structure differs significantly from that at low Reynolds numbers, spectra of $u$ and $v$ were examined by the experiment of Wei & Willmarth [38] and DNS of other researchers [6, 29, 34]. Their spectra were presented at several wall-normal positions, generally in the form of $k_i^+ E_{uu}$ vs. $\log \lambda_i^+$, where $k_i$ and $\lambda_i \equiv 2\pi/k_i$ are wavenumber and wavelength, respectively, and the normalization for the wavenumber power spectral $E_{uu}$ is such that

$$\int_0^\infty E_{uu}(k_x)dk_x = \int_0^\infty E_{uu}(k_z)dk_z = \overline{u'u'}^2.$$  

(10)

When the spectrum is plotted in the form $k_i E_{uu}$, the area under the curve are proportional to the contribution to the total energy from the wavenumber bands. Since the pre-multiplied spectrum is proportional to the power in a logarithmic band at $k_i$ or $\lambda_i$ (see, e.g., Perry et al. [39]), the peak position of the pre-multiplied energy spectra gives the most energetic wavelength (MEW).

It is interesting to note that MEW in the wall vicinity stays still at about $\lambda_i^+ \approx 100$ and not much changed from that of the higher Reynolds numbers even though $Re_\tau$ is as low as 80 (figure 12(b)). This corresponds to the well-known streaky structure in a wall vicinity of wall-bounded turbulence. Moving away from the wall, the MEW shifts slightly to the longer wavelengths: $\lambda_x^+ \approx 130$ or $\lambda_z^+ \approx 1.6\delta$ at the mid height. In the central region, a peak of MEW arises at about $\lambda_x^+ = 180$. This MEW corresponds to about $2.3\delta$, which shows a significant deviation from $1.3-1.6\delta$ obtained for the higher Reynolds numbers, cf. Abe et al. [6]. The streamwise MEW arises at $\lambda_x^+ \approx 1,000$ in the near-wall region. With increase in the distance from the wall, the streamwise MEW moves towards the shorter wavelength of $\lambda_x^+ \approx 400$ or $\lambda_x \approx 0.5\delta$ (figure 12(a)).

On the other hand, in the both directions, another peak appears at the longest wavelength in the core region, indicating consistency with the two-point velocity correlation $R_{uu}$, shown in
3.5.3 Instantaneous flow field. Figure 13 shows the high- and low-speed regions and the second invariant of deformation tensor $u'_{ij}u'_{ji}$ which is equivalent to the vortical position. (a) $Re_\tau = 180$, (b) $Re_\tau = 80$ (XL), (c) $Re_\tau = 80$ (MB). All of the box size (a)–(c) are scaled by $\delta$. Direction of the mean flow is from bottom-left to top-right. The visualized volume is the lower half of the computational box, namely (a,c) $12.88 \times 6.4\delta$ or (b), $51.2 \times 6.4\delta$.

The substantial secondary peaks of $w$ (the figure is not shown here) with wavelengths of $\lambda_x = L_x$ and $\lambda_z = L_z$ are also observed, this is an evidence that there exist a large-scale motion in spanwise direction.
on the contrary, the near-wall streaks are intermittently distributed, and a rather calm region can be observed. If we pay attention to the localized cluster of streaks, the spanwise spacing of low-speed streaks is essentially invariant with Reynolds number and the box size, exhibiting consistent value of $\Delta z^+ = 100$. The flow visualization study of Carlson et al. [32], on the transition structure at $Re_c = 1,200$, revealed that natural turbulence spots appeared randomly across the span and the additional oblique waves play an important role in the breakdown to turbulence. Both of their Reynolds number and the shape of the oblique structure are consistent with those we have obtained.

The well-known vortex structures such as quasi-streamwise vortexes are dominant and exist in the buffer region. In addition, these vortices are associated closely with the crowded near-wall streaks. It may be observed that strong turbulence-production regions, ejections and sweeps, in the buffer layer appear in the crowded streaks and vortexes area. The long-wavelength structure of the strong/weak-turbulent regions occurs periodically. Its streamwise and spanwise wavelengths occupy almost the whole box lengths on each direction. Moreover, the streamwise velocity profile changes from a more flat, turbulent-like profile, to a laminar Poiseuille-like profile inside the weak-turbulent region. In other words, the flow field can be separated into two areas: a upstream quasi-laminar state and a downstream turbulent state, which are relatively high- and low-speed regions, respectively. Accordingly, the relatively high-speed region overtake the low-speed one, which produces the strong turbulence and resultant puff-like structure at the interface, as discussed in the following section. This is a reason why the streamwise two-point correlations $R_{uu}$ falls down to a negative value at the middle of the box as seen in figure 11.

It is interesting to note that, with using a computational domain as large as XL, it enables us to capture the large-scale oblique structure — consisting of quasi-laminar and strong turbulent states — which has been unable to emerge in a usual computational box size, i.e., MB and LB. From DNS with a domain similar size to MB, neither the quasi-laminar nor the turbulent state are stable at a very low Reynolds number, and in a relatively short time the flow repeatedly returns to the other state (as discussed by Iida & Nagano [12]).

The flatness factor of the velocity fluctuation is shown in figure 14. For $Re_f = 80$, comparison of the results with different box sizes indicates that the influence of box size on the flatness factor is significant. If the box size is extended large enough to capture the highly disordered turbulent region and weak-turbulent regions, intermittency of fluctuation is enhanced.

### 3.6 Turbulence structures: puff-like structure

Wygnanski and co-workers [40,41] conducted a study of the structures and phenomena associated with transitional and turbulent pipe flow in the range of $1,000 < Re_m < 50,000$. They identified two transitional flow states, the type observed being dependent on $Re_m$. For $2,000 < Re_m <$
2,700, the transition structure was termed as ‘turbulent puff’; the second state was found at $Re_m > 3,500$ and consists of structures termed ‘turbulent slug’. The puff and the slug are characterized by a distinct trailing edge over which the flow changes almost discontinuously from turbulent flow to laminar flow. In the channel flow of the present study, the computational results, shown in figure 15, are strikingly similar to experimental velocity traces obtained by Wygnanski and Champagne \cite{40} for a pipe flow. Figure 15 shows a representative sample of $u^+$ and $v^+$ measured at several wall distances in the case of $Re_\tau = 80$ (XL). It is observed that the sequence of events of the localized turbulence is advected past the measurement point. It can be interpreted as: (i) quasi-laminar flow, (ii) a gradual reduction of $u$, (iii) a highly disordered turbulent region, (iv) a interface with quasi-laminar flow, again return to (i) quasi-laminar flow.

The intensities of both $u^+$ and $v^+$ are enhanced periodically with an interval of $\Delta t u_+ / \delta \approx 3.5$, so the travel speed of the interface is estimated as $(L_x / \Delta t)_+ \approx 14.6$, which is close to the bulk mean velocity $u_+ \approx 14.5$. With respect to the present Reynolds number, i.e., $Re_m=2,320$, the obtained structures of co-existing weak-turbulent region as discussed above are consistent with and remarkably similar to the ‘turbulent puffs’ observed in a pipe flow.

The shape of structure in the wall-normal direction can be clearly illustrated by the two-dimensional correlation for velocity in the $(x, y)$-plane. The fixed point is $y_{ref} = 5$. The correlation coefficient is defined as

$$R_{uu}(\Delta x, y) = \frac{u'(x, y_{ref}, z) u'(x + \Delta x, y, z)}{u_{rms}(y_{ref}) u_{rms}(y)}.$$  \hfill (11)

While a region of negative correlation appears on the side of the other wall for $Re_\tau = 180$ (figure 16(a)), a negative region appears in a wide region of $|\Delta x| > 13\delta$ for $Re_\tau = 80$ (XL). This indicates that a puff-like structure with a large streamwise wavelength of $51.2\delta$ fills the entire channel width in the case of $Re_\tau = 80$ (XL).

A typical well-developed structure at $Re_\tau = 80$ (XL) is shown as a sequence of flow visualizations in figure 17. The puff-like structure, shown in figure 17(a), consists of a quasi-laminar state (region marked A) and a highly disordered turbulence (region B). The wavelength of the near-wall streaks is much shorter than that of the spatial distribution of large-scale fluctuations.
induced by the puff-like structure, since the former is scaled in wall units, i.e., \( \lambda_+ = 100 \), and the latter is almost equal to the box size as shown in section 3.5.2. The actual flow field is a superposition of these two kinds of structures. In consequence, the regions where either high- or low-speed streak is dominant are discriminable, as seen in figures 13(b) and 17.

Figures 17(b)-(h) indicate that the puff-like structure is equilibrium and self-sustained. Moreover, the both regions (A and B) propagate with the streamwise velocity same with the bulk velocity and are inclined at an angle of 24° with respect to the streamwise direction. (This angle is easily determined by the aspect ratio of the horizontal domain size — calculated from \( \tan^{-1}(L_z/L_x) \) — in the case of the present result.) Therefore, the puff-like structures are spatially distributed not only in streamwise but also spanwise direction, while the puff of a pipe flow is homogeneous in azimuthal and intermittent only in the streamwise direction. The angle of the oblique structure can be affected by the aspect ratio and the dimension of the computation domain. As discussed in the previous section, the similar oblique structure has been observed in an experiment of the plane Couette flow [42]. One may regard the inclination of the puff-like structure as essential in the transitional channel flows. Note that the present results, such as visualized flow fields, two-point correlation coefficients (figure 11) and energy spectra (figure 12), show signs of being constrained by the periodicity of the boundary, even when the box size was extended to the XL. The numerical box requires to be enlarged further to allow us to neglect the periodicity of the computational domain.

4 Conclusions

In the present study, we performed DNS of the turbulent channel flow with larger computational boxes and investigated the turbulence statistics with respect to low-Reynolds-number effect. The Reynolds number was decreased down to \( Re_\tau = 60 \) to study the characteristics of the transitional channel flow and to estimate the critical Reynolds number at which a laminarization occurs. Based on the results of the computations we can draw the following conclusions:

(i) Turbulent/transitional state can be sustained in the computational box size of \( 25.6\delta \times 2\delta \times 12.8\delta \) in the range of \( Re_\tau \geq 64 \).
(ii) For \( Re_m < 3,000 \), \( C_f \) tends to be smaller than the empirical correlation and decreases with decreasing Reynolds numbers for \( Re_m < 2,000 \).
Figure 17. Contour of streamwise velocity fluctuation $u'^+\,$ for a series of consecutive times, in an $(x, z)$-plane at $y/\delta \approx 0.5$ for $Re_\tau = 80$ (XL). All of the contours (a)--(f) shows the whole $(x, z)$-plane of $(L_x \times L_z) = (51.2\,\delta \times 22.5\,\delta)$. The direction of the mean flow is from left to right.

(iii) The Reynolds-number dependence of the mean velocity profile is significant in the outer region when scaled with the wall units. The turbulence statistics indicate that the anisotropy of turbulence is enhanced at low Reynolds numbers.

(iv) For $Re_\tau = 80$ ($Re_m = 2,320$), the periodic localized turbulence is observed with using the largest box of $51.2\,\delta \times 2\,\delta \times 22.5\,\delta$ and very similar to a ‘turbulent puff’ observed in a transitional pipe flow.

(v) The significant influence of the captured puff-like structures exist on the turbulence statistics,
such as a mean velocity, turbulence intensities and vorticity fluctuations.

(vi) The equilibrium puff-like structures observed in the channel flow inclines against the stream-wise direction. The propagation velocity of the puff-like structure is approximately equal to the bulk mean velocity.

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