A method of two-dimensional filtering of modulated matrix data sequences

V G Getmanov¹, I I Astapov², N S Barbashina², A D Gvishiani¹, A N Dmitrieva², M N Dobrovolsky¹, D V Peregou dov¹, R V Sidorov¹, A A Soloviev¹, V E Chinkin¹, V V Shutenko² and I I Yashin²

¹Geophysical Center of the Russian Academy of Sciences (GC RAS), Molodezhnaya St., 3, Moscow, 119296, Russia
²National Research Nuclear University MEPhI ((Moscow Engineering Physics Institute)), Kashirskoe shosse 31, Moscow, 115409, Russia

r.sidorov@gcras.ru

Abstract. The method of two-dimensional filtering of modulated matrix data sequences is proposed. The structure of a two-dimensional filter based on operations of element-by-element matrix multiplications is developed. An application of the two-dimensional filtering of the URAGAN muon hodoscope matrix data sequence for the elimination of muon flux diurnal modulations is discussed.

1. Introduction and posing of the problem

A method for two-dimensional filtering of sequences of modulated matrix data, consisting of sums of initial and modulated components, is proposed. The need for such filtering – the elimination of component modulations in the matrix data sequences – arises in many problems of experimental physics and various applications associated with the requirements for the separation of time-varying spatially-temporal oscillatory processes: for example, in the analysis of periodic secular and seasonal processes of ice formation in the polar regions; in the studies related to the influence of cyclic solar activity on the dynamics of slow climatic changes of given regions of the Earth's surface; in sea disturbance studies using satellite data etc. Here an application associated with filtering of diurnal modulations in muon fluxes (MF) in the URAGAN [1] muon hodoscope (MH) matrix data is discussed.

MF reaching the MH aperture-type detector are subject to temporal and spatial modulations, which can be divided into periodic: solar 11- and 22-year and 27-day modulations [2], diurnal modulations due to the Earth rotation [3] and aperiodic ones – due to possible Forbush decreases [4] and atmospheric phenomena influences [5], including temperature and pressure influences [6]. In some cases, modulations can distort the useful information about the processes in the heliosphere contained in the MF.

In the MH, two-dimensional MF intensity distribution functions are measured based on the count of registered muons for a set of discrete solid angles for a given time. The measured values of the distribution function are placed into the MH matrices and denoted as \( Y(i,j,Tn) \), where \( i = 1,N_1 \), ...
j = 1, N_j, T is a time step, n = 0, N_j − 1. The indices i and j correspond to the azimuth and zenith angles with given discreteness and ranges.

Due to the rotation of the Earth, the contents of the MH matrices are subject to diurnal modulations with a frequency Ω_0 = 2π / T_0, T_0 is a day duration. As an example, hourly MH matrices recorded in June-July 2015 were analyzed, T_0 = 3600 sec, N_j = 1451. For the data in the MH matrices, corrections were made for temperature and pressure [7]. On figure 1, index 1 is the graph of the component of the function Y(i_0, j_0, T_n) for i_0 = 30, j_0 = 45, π_1 ≤ n ≤ π_2, π_1 = 200, π_2 = 1451; this function of measurements contains noise and modulation diurnal components. Filtering, eliminating the noise components, was performed using a one-dimensional digital low pass Butterworth filter with an order N_s = 8 and a cutoff frequency ω_1 = 1.2Ω_0; the software module from this filter was obtained from [9]. On figure 1, index 2 is the result of noise component filtering, and the remaining modulation diurnal components can be seen.

Figure 1. Y(i_0, j_0, T_n) component function plot (1), the result of Butterworth filtering (2).

To clarify the spectral composition of a set of signals in the MH matrices Y(i, j, T_n), where i = 1, N_i, j = 1, N_j, the total MF intensity function S = S(T_n), muon·sec^{-1}, was introduced. We calculate its discrete Fourier transform (DFT) C(k) for a given N

\[
S(T_n) = \frac{1}{N_i N_j} \sum_{i=1}^{N_i} \sum_{j=1}^{N_j} Y(i, j, T_n), \quad C(k) = (1 / N) \sum_{n=0}^{N-1} S(T_n) \exp(-2\pi in / N), \quad k = 0, N-1. \quad (1)
\]

The physical meaning of a function S(T_n) is obvious. The logarithmic DFT spectrum for the introduced function, denoted as LC(ΔF_k) = 20 log_{10}(C(k)C(k)), allows to estimate the spectral component amplitudes for S(T_n). On figure 2, the LC plot (dB) is displayed against the frequency ΔF_k Hz, where ΔF = 1 / NT is the DFT resolution, N = 1024, ΔF = 2.7126 Hz, k = 0, 1, ..., 260. Two
spectral peaks can be seen, indicating the 1-day and 0.5-day periods, with the frequencies of 1.1574 \times 10^5 Hz and 2.3118 \times 10^5 Hz, respectively. The presence of at least two spectral components allows us to conclude that daily modulations in \(S(Tn)\) are not sinusoidal, which is quite consistent with their physics.

Figure 2. DFT spectrum of the function of total intensity of muon fluxes

In this paper, the task is to develop a method for two-dimensional filtering of sequences of modulated matrix data and the implementation of an application of filtering the MF daily modulations in MH matrix sequences – the formation of corresponding matrix sequences in which these variations are absent.

2. Method of two-stage two-dimensional low-pass filtering

To eliminate modulations, we use one-dimensional digital low-pass filters. To the input functions \(Y(i, j, Tn)\) we apply the standard procedures for digital one-dimensional low-pass filtering with weights \(b_r(i, j), r = 1, \ldots, r_0, a_s(i, j), s = 0, \ldots, s_0\), the parameters \(r_0, s_0\) define the orders of the filters. The filter cutoff frequencies \(\omega_j(i, j)\) must satisfy the condition \(\omega_j(i, j) < \Omega_0\). We obtain the weight values from [9]; let us formulate the difference equation, in which \(Y_f(i, j, Tn)\) is a digital filter output

\[
Y_f(i, j, Tn) = -\sum_{r=1}^{r_0} b_r(i, j) Y_f(i, j, T(n - r)) + \sum_{s=0}^{s_0} a_s(i, j) Y(i, j, T(n - s)).
\]  (2)

The use of digital one-dimensional low-frequency filters (2) for the task posed involves two problems:

1) the need to provide small time costs for performing one-dimensional filtering operations by (2);

2) the need to eliminate the arising phase shifts that occur in \(Y_f(i, j, Tn)\) due to the recurrence relations (2). We realize the filtering of the sequence of matrices from MH based on the two-step method.

At the first stage we form the matrices \(B_1, \ldots, B_{r_0}, A_0, \ldots, A_{s_0}\) consisting of the weight elements \(b_r(i, j), a_s(i, j)\) for (2). With their help we form the structure of a two-dimensional filter on the basis of the difference equation in the matrix form:
\[ Y_{1p}(Tn) = -\sum_{r=1}^{N_B} B_r \circ Y_{1p}(T(n-r)) + \sum_{s=0}^{N_B} A_s \circ Y(T(n-s)) \]  

(3)

where “\( \circ \)” denotes the operation of element-wise multiplication of matrices. Using [9], a superfast matrix element-wise multiplication is realized. The matrix sequence \( Y_{1p}(Tn) \) is output for the filter (3).

At the second stage we perform the elimination of phase shifts. Based on (1), we introduce the total intensity function \( S_{1f}(Tn) \) for \( Y_{1f}(i, j, Tn) \). Let us construct a functional \( F(S, S_{1f}, n_d) \) and find an optimal phase shift \( n_d^* \):

\[ S_{1f}(Tn) = \frac{1}{N_1 N_2} \sum_{i=1}^{N_1} \sum_{j=1}^{N_2} Y_{1f}(i, j, Tn), F(S, S_{1f}, n_d) = \sum_{n=1}^{N_B} (S(Tn) - S_{1f}(T(n-n_d)))^2, \]

\[ n_d^* = \arg\min_{10^9, 50^9} F(S, S_{1f}, n_d) \]  

(4)

The result of the second stage of filtering is defined as the sequence of matrices in which the phase shift correction \( n_d^* \) is performed: \( Y_{1f}(i, j, Tn) = Y_{1f}(i, j, T(n-n_d^*)) \).

3. Filtering of daily modulations in muon fluxes for observation of the hydroscope of URAGAN hodoscope

The examination of the sequence of MH matrices obtained during June-July 2015 was continued.

1. The MH matrices \( Y_{1f}(Tn) \) were calculated using the structure (3) based on Butterworth filters of order \( N_B = 8 \) and \( \omega = 1.2 \Omega_0 \), the \( S_{1f}(Tn) \) function with phase shifts was formed. On figure 3, curve 1 depicts the total intensity function \( S = S(Tn) \) for MH against the time \( Tn \), sec, for the interval \( \bar{n}_1 \leq n \leq \bar{n}_2, \bar{n}_1 = 200, \bar{n}_2 = 1451 \); diurnal modulations with an amplitude of \( \approx 0.0015 \) are seen; the mean intensity value is \( \approx 0.2 \); curve 2 is the plot for the filtered function \( S_{1f}(Tn) \) with a phase shift. On curve 2, the diurnal modulations are absent. With the application of optimization \( F(S, S_{1f}, n_d) \), the phase shift estimate \( n_d^* = 39 \) was obtained, and the shift correction was realized.

![Figure 3](image-url)  

Figure 3. The result of the work of a two-dimensional matrix filter for eliminating diurnal variations of muon fluxes
2. The computational experiments were performed, which allowed to determine that: time costs of the proposed method of two-dimensional filtering, on average, an order of magnitude less than the time costs that are realized for filtering based on one-dimensional filters: phase shift estimates are $\approx 1\text{--}2\text{ grad}$, approximately, and the model sinusoid modulation amplitude decrease $\approx 10\text{--}20$ times.

The method of two-dimensional filtering of sequences of modulated matrix data developed in the article is more effective in comparison with traditional approaches \[7, 8\] in terms of time costs and possibilities for eliminating phase shifts.

4. Conclusions
1. The proposed method of two-dimensional filtering of sequences of modulated matrix data proved to be workable.
2. The filtering of the diurnal modulations of muon fluxes for the matrix observations of the URA-GAN MH was successfully implemented.
3. On the basis of computational experiments, it was established that: the time costs of the proposed two-dimensional filtering method are, on average, one order of magnitude less than the time costs that are realized for filtering based on one-dimensional filters; the errors of phase shift estimates are $\approx 1\text{--}2\text{ grad}$; the amplitudes of the model sinusoidal modulations decrease $\approx 10\text{--}20$ times.
4. The proposed two-dimensional filtering method can be applied to many problems of experimental physics and other applications related to the elimination in the sequences of matrix data of modulation components.

5. References
[1] Yashin I I et al 2015 J. Phys.: Conf. Ser. 632 012086.
[2] Okazaki Y et al 2008 Astrophys. J. 681 1.
[3] Poirier J, Catanach T 2011 Proc. 32-nd Int. Cosmic Ray Conf. (Beijing) (Under the Auspices of the International Union of Pure and Applied Physics (IUPAP)) vol 11 p 173.
[4] Braun I, Engler J, Horandel J R and Milke J 2009 Adv. Space. Res. 43(4) 480–488.
[5] Karapetyan G G 2014 Phys. Rev. D89 093005.
[6] Dmitrieva A N et al 2011 Astropart. Phys. 34(6) 401–411.
[7] Yashin I I et al 2015 Adv. Space. Res. 56(12) 2693–2705.
[8] Dmitrieva A et al. 2015 J. Phys.: Conf. Ser 632 (2015) 012054.
[9] Garello R (ed) 2013 Two-dimensional Signal Analysis (London: Wiley-ISTE).
[10] Dyakonov V P, Abramenko V V 2002 Matlab. Image and Signal processing (in Russian, Saint Petersburg: Piter)
[11] Image Processing Toolbox. http://matlab.exponenta.ru/imageprocess (last accessed: 10.09.2018).