Modified Newtonian Dynamics as an extra dimensional effect

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Modified Newtonian dynamics can be considered as an effect derived from a squeezable extra dimension space. The third law of Newtonian dynamics can be managed to remain valid in the 5-space. The critical acceleration parameter $a_0$ appears naturally as the bulk acceleration that has to do with the expanding universe in this setup. A simple toy model is presented in this Letter to show that consistent theory can be built with the help of the extra dimensional space.

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I. INTRODUCTION

Modified Newtonian dynamics (MOND) was proposed by Milgrom [1]-[11] asserting that gravitational field requires modifications when the gravitational field strength $N$ is weaker than a critical value $a_0$. This has been shown to be a good candidate as an alternative to cosmic dark matter (Sanders 2001). The phenomenological foundations for MOND are based on two observational facts: (1) flat asymptotic rotation curve, (2) the successful Tully-Fisher [12] law, $M \sim V^\alpha$ for the relation between rotation velocity and luminosity observed in many spiral galaxies. Here $\alpha$ is close to 4.

It was pointed out [1]-[11] that there exists a critical acceleration parameter $a_0 = 1.2 \times 10^{-8}$cms$^{-2}$ characterizing the turning point of the effective power law associated with the gravitational field in MOND. Gravitational field of the following form was suggested

$$g \cdot \mu \left( \frac{g}{a_0} \right) = N$$

with a function $\mu$ considered as a modified inertial. Here $N$ is the Newtonian gravitational field produced by certain mass distribution. Milgrom suggests that

$$\mu(g) = \frac{g}{\sqrt{1 + g^2}}$$

and shows that it provides a best fit with many existing observation data including the rotational curve of many spiral galaxies. Recently it was shown that a simpler inertial function of the following form [13, 14]

$$\mu(g) = \frac{g}{1 + g}$$

fits better with RC data of Milky Way and NGC3198.

Evidences accumulated [1]-[11] show that the theory of MOND is telling us a very important message. Either the Newtonian force laws do require modification in the weak field limit or the theory of MOND may just represent some collective effect of the cosmic dark matter. In both cases, the theory of MOND deserves more attention in order to reveal the complete physics underlying these successful fitting results.

There is, however, a known problem with the momentum conservation law associated with the theory of MOND. One finds that the conservation of momentum, a result of the action-reaction principle, can be resolved with the existence of an extra dimensional space. One will also be able to show that the proposed critical acceleration $a_0 \sim H_0 c$ might be closely related to the existence of an extra dimensional space. Here $H_0$ is the current Hubble constant. In fact, one will show that a spring-like extra dimensional space with squeezable thickness can not only absorb the missing piece of momentum but also provide a natural accommodation of the expanding universe in a consistent way.

II. PROPERTIES OF THE EXTRA DIMENSION

Assuming that there exists an extra dimension space with a squeezable thickness $Z$ related to the induced field strength $g$ and the inertial mass $m$ of the associated test particle. 5-space and 4-space will denote the five-dimensional space time and four-dimensional space time respectively. In addition, 3-space, the hyper surface of the four dimensional spatial space (or the 4D bulk space), will denote the conventional three spatial geometry where Newtonian dynamics holds when $N \gg a_0$.

Moreover, $z$-space will also be denoted as the extra fifth dimensional space and $b$-space will denote the 4D bulk space without its 3-space hyper surface.

For simplicity, one will write the four dimensional spatial geometry as a two-dimensional $xz$ strip with the $x$-coordinate as an abbreviate for the conventional $xyz'$ 3-space. [14, 16] For example, each point on the $x$-line represents a two-dimensional surface. $z$-membrane will also denote the 4D bulk space without its 3-space hyper surface.

Three assumptions on the properties of the space will be proposed: (1) One has assumed that the $z$-space has a squeezable thickness $Z(x)$ related to the interaction with the gravitational field $g$. In the presence of a strong field $N$, it is assumed to have a standard thickness $Z(x) = z_0$. (2) One will further assume that the thickness $Z$ also has to do with the existence of inertial mass $m$ of the test particle. In particular, one will assume that there is also
a $z$-space mass $M(x)$ at the same point $x$ associated with the existence of any particle with $3$-space mass $m$. One assumes that the $z$-space mass $M$ only acts in the $b$-space and is also proportional to $3$-space mass $m$ in the strong field limit. Without losing any generality, one sets $M \rightarrow m$ in the strong field limit. (3) Finally, one assumes that the $z$-space mass density $M(x)/Z(x)$ remains constant for all $x$ throughout the $b$-space.

Here is how the idea works: Once the field strength $N$ goes weak, when MOND effect is apparent, the thickness of the extra dimension will be squeezed appreciably in response to the decreasing of $N$ acting on the $3$-space world. As a result, part of its tensional force (or $M$) in the $b$-space is released to the $3$-space world such that the third law of Newtonian dynamics is preserved in $5$-space. To be more specific, one will show how this could be working closely with the real world in a consistent way by studying the simple model proposed by Famaey and Binney [13, 14]. Any different models can also be shown to give similar results with some necessary modification in the intermediate field strength region where the strength of $N$ is close to the critical value $a_0$. Indeed, one can show that the MOND field strength $g$ is

$$g = \frac{\sqrt{N^2 + 4a_0N} + N}{2}$$

(4)

when the Newtonian field strength produced by a source $m_0$ with a $3$-space mass $m_0$. Let us assume that $m_0$ is a point mass such that $N = Gm_0/r^2$ for simplicity for the moment. Generalization to any mass distribution is straightforward. The force $F$ acting on the particle $m$ at a distance $r$ to $m_0$ is $F = mg$. On the other hand, the test particle $m$ will also produce a Newtonian field strength $N_0$ at the location of the particle $m_0$. It is known that $m$ will produce a force $F_0 = mg_0(N \rightarrow N_0)$ on $m_0$. Note that one has $N = Gm_0/r^2$ and $N_0 = Gm_1/r^2$ acting in the $3$-space world. It is apparent that the Newton’s third law of dynamics $m_0g_0 + mg = 0$ fails to be observed if the theory of MOND is the whole story. Here $g$ and $g_0$ represent the MOND field strength acting at $m$ and $m_0$ respectively.

There have been many stimulating research activities trying to find a covariant field theory that is capable of resolving this problem. Instead of these approaches, we will try a simpler approach. Indeed, one possible resolution to this dilemma is to assume that the third law of the Newtonian dynamics is still obeyed in the $5$-dimensional space by assuming that part of the $3$-space force $F$ goes into the extra $z$-dimension. This can be done by assuming

$$F_5 = F_z + F = mN$$

(5)

throughout the $5$-space. If this is true, the total $5$-dimensional force $F_5 = mN$ will automatically follow the Newton’s third law. Assume that the $z$-force $F_z$ is a repelling force directing outward from the source $m_0$ and $F$ is pointing inward representing an attracting force. Note that these directions are all perpendicular to the normal direction of the $3$-space hyper surface, i.e. outward and inward directions do not have any projection in the $z$-direction. One can show that the magnitude of $F_z$ is that

$$F_z = m\frac{\sqrt{N^2 + 4a_0N} - N}{2}.$$  

(6)

Note that we have assumed that : 1) $F$ only acts on the $3$-space world responsible for the dynamical motion of the particle with inertial mass $m$ moving in the $3$-space world. 2) Similarly, the force $F_z$ only acts on the $b$-space with inertial mass $M$ responsible for the expansion or contraction of the $xz$-membrane. $F_z$ is supposed to do nothing directly on the particle’s inertial mass $m$ on the $3$-space world.

Writing $F = mg_z$, with $g_z$ as a parameter of acceleration, one can show that

$$g_z = \frac{\sqrt{N^2 + 4a_0N} - N}{2}.$$  

(7)

It is straightforward to show that

$$g_z \rightarrow a_0, \text{ if } N \gg a_0$$

$$\rightarrow \sqrt{NA_0}, \text{ if } N \ll a_0$$

(8)

Therefore, one finds that the critical parameter $a_0$ shows up naturally as a physical parameter of the bulk acceleration acting on the $xz$-membrane.

Assumption (4): In addition to the first three properties of the bulk geometry assumed earlier in this Letter, one will also assume that the bulk $xz$-space is acting as a spring-like membrane with a small thickness $Z(x) = z_0$ when the field strength $N \gg a_0$. The thickness $Z(x)$ of the $z$-space will be squeezed in response to the change of the field strength $g$ such that $F_z = M a_0$ is observed for all $x$ in the $3$-space world. Note that one has assumed that $M(x)$ is the total bulk mass of the membrane segment associated with the presence of the point mass $m$ at $x$. One has also assumed that $M(x) = mZ(x)/z_0$ is proportional to the thickness of the $z$-space such that $M(x) = m$, up to some proportional constant set as $1$, in the strong $N$ limit. Therefore, one can show that

$$Z(x) = \frac{\sqrt{N^2 + 4a_0N} - N}{2a_0}z_0.$$  

(9)

such that $F_z = M a_0$ is strictly obeyed. In another word, the whole $b$-space membrane is experiencing a constance acceleration $a_0$ everywhere outward from the source $m_0$ at $x_0$. Similarly, point mass $m_0$ will also see a constant acceleration $a_0$ outward from the source $m$ at $x$. Precisely, each particle of the system sees a $b$-space acceleration $a_0$ repelling each other.

In Fig. 1, thin line represents the function $g(N)$ with $g$ and $N$ both written in unit of $a_0$. In addition, thick line represents the function $Z(N)$ with $Z$ and $N$ written.
One has made totally four assumptions on the properties of the bulk z-space: (1) it has a finite thickness $Z(x)$, (2) it has an inertial mass $M(x)$ in response to the force $F_z$ acting on the bulk space, (3) bulk mass density $M/Z = \text{constant throughout the bulk space}$, (4) the force acting on the b-space $F_z = Ma_0$ tends to be acting on an “incompressible” fluid such that particle far apart tends to accelerate outward against each other with a constant acceleration $a_0$.

As a result, one derives the dependence of $Z(x)$ shown in Eq. (9). It indicates that part of the effect of the mass $M$ in the b-space is released to the 3-space world in compensation as a deformation of the inertial $\mu(g/a_0)$ acting on the 3-space world. All forces adding together are therefore managed to obey the third law of Newtonian dynamics.

The first three assumptions are quite reasonable assumptions one can imagine for a well behaved membrane. The forth assumption asserts that the membrane tends to distribute forces acting on it equally such that constant acceleration is achieved everywhere on the bulk space. For example, two distinct galaxies tend to expand or accelerate outward from each other. This property is similar to the equal pressure acting on an ideal fluid enclosed in a closed container. Therefore, the forth assumption is also a reasonable one.

The result shown here with a simple toy model indicates that the theory of MOND might have to do with the input from the extra dimensional space. The expansion of the bulk space (along the 3-space direction $x$) is achieved as a bonus of the toy model. Assumptions made on the properties of the extra dimensional space are considered to be reasonable ones.

Therefore, it appears that the somewhat successful theory of MOND does have close relation to the effect from the extra dimension. One cannot rule out the possibility that the theory of MOND is merely a collective effect of the cosmic dark matter working closely following the models proposed by Milgrom, Famaey & Binney, and many others. In any case, the theory of MOND deserves more attention for a more clear physical picture of the...
galaxy dynamics.

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