Simultaneous observation of gravitational and electromagnetic waves

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Assuming that the short gamma-ray burst detected by the Fermi Gamma-Ray Space Telescope about 0.4 seconds after the gravitational waves observed by the LIGO and VIRGO Collaborations originated from the same black hole merger event, we perform a model-independent analysis of different quantum gravity scenarios based on (modified) dispersion relations (typical of quantum gravity models) for the graviton and the photon. We find that only scenarios where at least one of the two particles is luminal (the other being sub- or super-luminal) are allowed, while scenarios where none of the two particles is luminal are ruled out. Moreover, the physical request of having acceptable values for the quantum gravity scale imposes stringent bounds on the difference between the velocities of electromagnetic and gravitational waves, much more stringent than any previously known bound.

I. INTRODUCTION

Quantum gravity effects are usually expected to be relevant at energies around the Planck scale \[^{1\,8}\] \(M_{Pl} \approx 1.2 \times 10^{19} \text{ GeV}\), where a breakdown of classical space-time in favor of a fuzzy/foamy description is usually expected \[^{9\,13}\]. Obviously, quantum gravity theories cannot be tested at laboratory energies, but the propagation for cosmological distances of (ultra-) high energy particles provides an excellent laboratory for testing deviations from classical general relativity. There is great interest for the cases when multiple observations from the same astrophysical source are available and a time-of-flight analysis can be carried on \[^{5\,14\,22}\], in particular on using the observation of gravitational waves \[^{23}\] for this purposes.

Together with the discovery of the Higgs boson \[^{24\,25}\] (and, if confirmed, of the 750 GeV resonance \[^{26\,27}\]), a new great finding provides another breakthrough, crucial for our understanding of fundamental physics and for the ways it opens for future research. This is the recent direct observation by LIGO and VIRGO Collaborations \[^{28}\] of gravitational waves generated by a black hole merger, with a false alarm rate estimated to be less than 1 event per 203,000 years – equivalent to a significance of \(5.1\sigma\), that called for enthusiastic multi-messenger searches from several observatories that tried to identify candidate photons and neutrinos subsequent to this GW150914 event. The gamma-ray observatories INTEGRAL \[^{29}\], SWIFT \[^{30}\] and AGILE \[^{31}\] did not observe significant photon excesses in different energy ranges. The neutrino observatories ANTARES and IceCUBE did not
observe significant candidate neutrinos as well [32]. At variance with other gamma-ray
burst observatories, covering only a fraction of the sky where LIGO detected the GW150914
event, the Fermi Gamma-ray Space Telescope was exposed to a large fraction of the same
region, and reported the detection of a weak gamma-ray burst above 50 keV just 0.4 s
after GW150914, with a positional uncertainty region overlapping with that of the LIGO
observation. The estimated false alarm probability for this observation is 0.0022 [33].

Whether or not the gravitational wave and the gamma-ray burst come from the same
source, i.e whether the theory can accommodate both these signals from the merging of the
two black holes is still a controversial question [34, 35]. According to [34], for instance, the
two signals are unlikely to be related, and from the merging of these two black holes no
burst of photons should be observed. On the other hand, a recent model [35] suggests that
the merging black holes might have been generated by the collapse of a rapidly rotating
massive star, that at the end of its collapse might have produced gamma-ray bursts. Other
studies [36–41] also try to reconcile the two observations, and this possible multi-messenger
signal has even been used to question the concept of cosmic acceleration from a genuine
scalar-tensor modification of gravity [42].

In the following we assume that the gamma-ray burst observed by the Fermi Collaboration
has been caused by the black hole merger that generated the gravitational waves detected by
the LIGO Collaboration. As already noted, there is only a 0.2 percent chance that these two
events originated in the same patch of sky at the same time but were due to two different
high-energy phenomena. Naturally, it is a challenge for present and future theoretical work
to explain the mechanism that produces this (unexpected) electromagnetic signal in a black
holes collision. Here we are interested in investigating the consequences of this joint event,
and we will see that they are very important. This issue has already been partly studied
by a number of authors [37, 43–45] with different purposes, as deriving upper bounds for
the speed of gravitational waves, for the graviton mass, or lower bounds for the quantum
gravity scale. Actually, quantum gravitational effects usually modify the dispersion relation
\( E^2 = p^2c^2 + m^2c^4 \) of a relativistic particle by terms that in an effective field theory approach
depend on inverse powers of the quantum gravity scale \( E_{QG} \), and involve a violation of
Lorentz invariance (LIV) at high energies, with \( E_{QG} \) typically being of the order of the
Planck scale \( M_{Pl}c^2 \sim 10^{19} \text{ GeV} \).

The aim of the present work is twofold. With the help of the modified dispersion relations
for the graviton and the photon, on the one hand we study the upper bounds for the
difference in speed between the gravitational and the electromagnetic wave, so to get in
particular constraints on the gravitons’ speed, on the other hand we perform a general
model-independent analysis to constrain quantum gravity theories. We will find new and
important results that greatly improve our knowledge on these issues.

The rest of the paper is organized as follows. In section II we consider the general set-up
for our analysis, starting by considering the modified dispersion relation for massless par-
ticles and deriving the corresponding difference \( \Delta t \) between the graviton and the photon
propagation times, \( \Delta t_g \) and \( \Delta t_\gamma \), respectively. In section III we will capitalize on the theory
to constrain the speed of gravitons, improving the current estimations by several orders of
magnitude. In section IV we will fully develop the model-independent analysis to con-
strain both astrophysical models and quantum gravity theories, while section V is for our
conclusions.
II. GENERAL SETUP

Quantum gravity theories generically induce LIV, parametrized in terms of a model independent modified dispersion relation, that can be written as [22]

\[ E^2 \approx p^2 c^2 \times \left[ 1 + \sum_{s=1}^{\infty} \xi \left( \frac{E}{E_{QG}} \right)^s \right], \tag{1} \]

where \( E_{QG} \) is the quantum gravity scale, the factor \( \xi \) accounts for sub-luminal (\( \xi = -1 \)) and super-luminal (\( \xi = +1 \)) propagation (while \( \xi = 0 \) is for luminal particles), and \( E (< E_{QG}) \) is the energy of the particle.

These corrections emerge for instance as leading-order terms in nonlinear quantum gravity models [1], and more generally they result from an effective field theory approach. The linear correction accounts for violation of \( \mathcal{CPT} \) symmetry, while when \( \mathcal{CPT} \) symmetry is preserved, the linear term is absent and the dominant term is typically the quadratic correction. Keeping in Eq. (1) only the lowest-order non-vanishing term, say \( s = n \), the particle (group) velocity is:

\[ v = \frac{\partial E}{\partial p} \approx c \times \left[ 1 + \xi \frac{n + 1}{2} \left( \frac{E}{E_{QG}} \right)^n \right], \tag{2} \]

from which we see that when particles with different energies are produced by astrophysical objects at cosmological distances, their observation by terrestrial detectors can be measurably delayed even if they are emitted at the same time.

The idea of this work is to derive model-independent constraints on theories predicting LIV by considering the dispersion relation (1) for both photons and gravitons and applying the time-of-flight analysis to the recent observation of gravitational waves (gravitons) and the subsequent arrival of a gamma-ray burst (high-energy photons), under the assumption that both events have been caused by the same black hole merger event.

To this end, in the following we use Eq. (2) for photons and gravitons, considering all the possible cases obtained by combining \( \xi_\gamma = -1, 0, 1 \) with \( \xi_g = -1, 0, 1 \). In other words, with no reference to any specific model, we allow for both gravitons and photons to be sub-luminal, luminal, or super-luminal, and making use of the experimental results, we analyse each of the corresponding combinations. Our analysis is performed under the very general assumption that that for both of them the dominant LIV contribution occurs for the same value of \( n \).

The difference \( \Delta t = \Delta t_g - \Delta t_\gamma \) between the graviton and the photon propagation times is given by

\[ \Delta t = \Delta t_a - (1 + z_0) \Delta t_e, \tag{3} \]

where \( \Delta t_a \) is the arrival delay observed at Earth and \( \Delta t_e \) the emission delay at the source with redshift \( z_0 \), that in our case is \( z_0 = 0.09 \). While \( \Delta t_a \) is a measured quantity, we do not have informations on \( \Delta t_e \). The typical approach is to assume \( \Delta t_e = 0 \), and to derive then bounds for the physical quantities of interest, for instance for the graviton velocity. In our analysis, we will allow \( \Delta t_e \) to freely vary, and this will give us the possibility to investigate the compatibility of different LIV models with the LIGO and Fermi data.

If \( v_\gamma(z) \) and \( v_g(z) \) are the speeds of the photons and the gravitons respectively at a given
red-shift $z$, $\Delta t$ is given by:

$$\Delta t = cH_0^{-1} \int_0^{z_0} \left( \frac{1}{v_g(z)} - \frac{1}{v_\gamma(z)} \right) \frac{dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}},$$  \hspace{1cm} (4)$$

where $\Omega_m$ is the fractional density of matter, $\Omega_\Lambda$ the fractional density of dark energy and $H_0$ the Hubble constant at present time in the $\Lambda$-CDM cosmological model. In the following, we will consider the values obtained from recent measures of the Planck collaboration [46], namely $\Omega_m \approx 0.31$, $\Omega_\Lambda \approx 0.69$ and $H_0 \approx 67.8$ km/s/Mpc.

Inserting Eq. (2) for the energies $E_\gamma$ and $E_g$ of the photon and the graviton in Eq. (4) we get:

$$\Delta t \simeq \beta_n(z_0) \frac{n+1}{2} \left[ \xi_\gamma \left( \frac{E_\gamma^0}{E_{QG}} \right)^n - \xi_g \left( \frac{E_g^0}{E_{QG}} \right)^n \right],$$  \hspace{1cm} (5)$$

where $E_g^0$ and $E_\gamma^0$ are the energy of the graviton (gravitational wave) and of the photon (gamma-ray burst) measured at Earth, and we have used $E_\gamma = h\nu_\gamma^0 (1+z) = E_\gamma^0 (1+z)$ and $E_g = h\nu_g^0 (1+z) = E_g^0 (1+z)$ (with $\nu_\gamma^0$ and $\nu_g^0$ the frequencies of the electromagnetic and gravitational waves measured at Earth), and $\beta_n(z_0)$ is:

$$\beta_n(z_0) = H_0^{-1} \int_0^{z_0} \frac{(1+z)^n dz}{\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}}.$$  \hspace{1cm} (6)$$

As we will see, Eq. (5) is one of the key ingredients of our analysis. Other important ingredients are the following measured quantities: the time arrival delay $\Delta t_a \sim 0.41$ s between the gamma-ray burst and the gravitational wave (see Eq. (3)) and the energies $E_g^0$ and $E_\gamma^0$ of the graviton and the photon observed at Earth. For the gamma-ray burst detected by the Fermi Gamma-Ray Burst Monitor (GBM), photons with energies between 50 keV and 1 MeV were observed [33]. As for the gravitational wave, the signal sweeps upwards in frequency from 35 to 250 Hz, so that the energy of the gravitons at Earth is in the range $E_g^0 = h\nu_0 \approx 10^{-12} - 10^{-13}$ eV, where $\nu_0$ is the gravitational wave frequency at Earth and $h$ is the Planck constant.

With no reference to any specific model, in the following we will explore different possible cases, namely we will consider that both the photon and the graviton can be super-luminal, luminal or sub-luminal. This means that for both particles we will consider in Eq. (2) the following three different possibilities for $\xi$: $\xi = 0, 1, -1$. This model-independent analysis will allow to use Eq. (5) to read the relation between the Quantum Gravity scale $E_{QG}$ and the difference $\Delta t_e$ (see Eq. (3)) in the emission time between the gravitational and the electromagnetic wave. One of the important results of this analysis will be that some models are ruled out simply by the fact that only too low values of the Quantum Gravity scale would be compatible with the observed values of $\Delta t_a$, $E_g^0$ and $E_\gamma^0$. Before moving to that complete analysis (developed in section IV), in the following section we derive the constraints that Eq. (5) puts on the speed of gravitational waves.

### III. CONSTRAINTING THE SPEED OF GRAVITATIONAL WAVES

When multiple observations from the same astrophysical source are available, which is what we assume for the gravitational waves observed by the LIGO and VIRGO Collaborations and the gamma-ray burst observed by the Gamma-ray Space Telescope of the FERMI
Collaboration, a typical time-of-flight analysis can be applied to derive an upper bound on the speed $v_g$ of gravitational waves. In the following we derive in this manner an upper bound on $v_g$ and show that this bound coincides (as it should be) with the one derived in Ref. [44]. However, we will see that the knowledge of the energies of the gravitational and the electromagnetic waves allows to put much more stringent upper bounds on $v_g$. This will be the first (in our opinion very important) of our new results.

Actually, several bounds have already been derived in the literature. For instance, based on the direct observation of GW150914, an upper bound has been recently given by Blas et al [43] and reads $v_g < 1.87 \, c$. Also, the tightest model-independent lower bound is $1 - v_g/c \leq 2 \times 10^{-15}$, deduced from the absence of gravitational Cherenkov radiation allowing for the unimpeded propagation of high-energy cosmic rays across our galaxy [47]. A stricter (although model-dependent) bound from the same authors is $\approx 10^{-19}$, obtained under the assumption that the highest energy cosmic rays are produced by the so-called Z-burst mechanism. This mechanism predicts that very high energy neutrinos produced at cosmological distances annihilate with relic neutrinos via the Z-boson resonance [48, 49], but it has been almost ruled out by recent limits on cosmic-ray photon fractions at the EeV energy scale [50]. Poorer bounds have been obtained from cosmology [51], whereas $c/v_g - 1 \leq 0.01$ has been deduced from radiation damping in binary systems [52] and observations of the Hulse-Taylor pulsar [53].

Let us see now how from Eq. (2) we can derive an upper bound for the difference between the graviton and the photon speed from the typical time-of-flight analysis. Writing Eq. (2) at Earth, we have:

$$v_0^\gamma - v_0^g = c \left( \frac{E_0^\gamma}{E_{QG}} \right)^n - \frac{n+1}{2} - \frac{E_0^g}{E_{QG}} \right)^n \right) (7)$$

where $v_0^\gamma$, $v_0^g$ are the values of speed of the photon and the graviton measured at Earth ($E_0^\gamma$, $E_0^g$, the energies of the photon and the graviton measured at Earth, have already been introduced before). Inserting then Eq. (5) in Eq. (7) we have:

$$v_0^\gamma - v_0^g = \frac{c}{\beta_n(z_0)} (\Delta t_a - (1 + z_0) \Delta t_e) (8)$$

with $\beta_n(z_0)$ given in (6).

Assuming that the electromagnetic wave is emitted later than the gravitational wave, i.e. assuming that $\Delta t_e \geq 0$, we obtain the typical conservative limit by inserting $\Delta t_e = 0$ in Eq. (8), thus getting the upper bound

$$v_0^\gamma - v_0^g \leq \frac{c}{\beta_n(z_0)} \Delta t_a (9)$$

When the experimental values for $H_0$, $\Omega_M$, $\Omega_\Lambda$, $z_0$ and $\Delta t_a$ are used, we find

$$\frac{v_0^\gamma}{c} - \frac{v_0^g}{c} \leq 9.8 \times 10^{-18} \quad \text{for} \quad n = 1 (10)$$

and

$$\frac{v_0^\gamma}{c} - \frac{v_0^g}{c} \leq 9.4 \times 10^{-18} \quad \text{for} \quad n = 2 (11)$$
that are perfectly consistent with the bound found in Ref. [44].

Up to now we have considered the usual time-of-flight analysis that brings to the upper bound found above for the difference between the photon and the graviton speed. However, the measured energies of the gravitons and photons that reach the terrestrial detectors, $E_0^g \sim 10^{-12} - 10^{-13}$ eV and $E_0^\gamma \sim 50$ keV - 1 MeV, allow to put much more interesting and stringent bounds on the speed of the gravitational waves. Moreover, we will see in the next section that using $\Delta t_e = 0$ to derive bounds on the graviton speed is not always consistent with the measured values of the photons and gravitons energies. In particular, for the case that is typically considered, and that we also investigate below, namely the case of a luminal photon and a sub-luminal graviton, $\Delta t_e$ cannot vanish unless the quantum gravity scale becomes (unacceptably) too low.

Let us consider now the above mentioned case when the photon is luminal and the graviton is sub-luminal (or even super-luminal). From Eq. (7), considering the $n = 1$ case and assuming that the Quantum Gravity scale $E_{QG}$ does not lie below the Planck scale, $E_{QG} \geq M_{Pl} c^2$, we have

$$\left| 1 - \frac{v_0^g}{c} \right| \leq 10^{-40}.$$  \hspace{1cm} (12)

This is a fantastically much more stringent bound than the one obtained in the corresponding “usual” Eq. (10). Moreover, we see that even if we allow for a Quantum Gravity scale $E_{QG}$ down to the (too low) TeV scale, still the upper bound on the speed of the gravitational wave is as high as

$$\left| 1 - \frac{v_0^g}{c} \right| \leq 10^{-24},$$  \hspace{1cm} (13)

that is still much more stringent than the bound in Eq. (10). These results greatly improve the existent upper bounds on the speed of gravitational waves.

We can also consider the opposite case, when the photons are sub- or super-luminal. From Eq. (7) we find that for $E_{QG} \geq M_{Pl} c^2$ the bound is

$$|v_0^\gamma - v_0^g| \leq c \cdot 10^{-22},$$  \hspace{1cm} (14)

that is a very important upper bound for the cases (rarely considered in the literature) of a sub- or super-luminal photon.

**IV. CONSTRAINING QUANTUM GRAVITY MODELS**

Carrying on with our analysis, and with no reference to any specific model, we use now our general, model-independent Eq. (5) to consider all possible scenarios involving sub-, super- and luminal photons and gravitons. Each scenario is governed by a different phenomenological equation, derived from Eq. (5) and reported in Tab. I that provides the expected relationship between the emission time delay $\Delta t_e$ and the quantum gravity scale $E_{QG}$ for different cases ($n = 1, 2, 3$ and 4) satisfying dispersion relations as in Eq. (1).

When both the photons and the gravitons are luminal, that is the case $A$ of Tab. I from Eqs. (3) and (5) we immediately have that the time delay $\Delta t_e$ between the emission of the gravitational wave and the subsequent emission of the gamma-ray burst is $\Delta t_e =$
Figure 1: Theoretical curves relating the time emission delay $\Delta t_e$ (between the gravitational wave and the gamma-ray burst) and the energy ratio $E_g^0/E_{QG}$ (where $E_g^0$ is the measured energy of the gravitons and $E_{QG}$ the quantum gravity scale) for the cases B (left panel) and C (right panel) of Tab. I, and for different values of $n$ ($n = 1, 2, 3$ and 4), are shown. The gravitons and the photons obey to (quantum gravitationally modified) dispersion relations as in Eq. (1) (see also Eq. (2)). The dotted thick vertical line indicates the value of $E_g^0/E_{QG}$ obtained for $E_{QG} = M_{Pl}c^2$.

$\Delta t_e/(1 + z_0) \approx 0.4\, s$. In other words, we find the obvious result that if both particles travel at the speed of light, the emission time and the arrival time are the same, up to the red-shift correction factor. Let us move now to consider the other less trivial cases.

Fig. 1 shows the phenomenological curves, for different values of $n$, relating $\Delta t_e$ and $E_g^0/E_{QG}$ for the cases B and C of Tab. I, where the photon is luminal and the graviton is super-luminal (left panel) or sub-luminal (right panel). For any given value of $E_{QG}$ (and for any given value of $n$), the allowed value of $\Delta t_e$ is the one such that the theoretical curve intersects the corresponding vertical line. For instance, the dotted thick vertical line of the figure indicates the value of $E_g^0/E_{QG}$ obtained for $E_{QG} = M_{Pl}c^2$. Note that $E_g^0$ is the value of the graviton energy measured at Earth, $E_g^0 \sim 10^{-12} - 10^{-13}$ eV, so that the curves actually provide a relation between the emission time delay $\Delta t_e$ and the quantum gravity scale $E_{QG}$.

From this figure we see that the time delay $\Delta t_e$ is practically quenched to the value $\Delta t_e \sim \Delta t_a/(1 + z_0) \approx 0.4\, s$ (that is the value obtained for the case A), for $E_{QG} > 100$ keV in the $n = 1$ case, and $E_{QG} > 100$ µeV for the $n = 2$ case, both values being several orders of magnitudes lower than the most stringent lower bounds currently available [22] ($E_{QG} > 7.6\, M_{Pl}c^2$, for $n = 1$, and $E_{QG} > 10^{-9}\, M_{Pl}c^2$, for $n = 2$). We than see that acceptable values for the quantum gravity scale $E_{QG}$ are obtained only if $\Delta t_e \sim 0.4\, s$.

As a consequence, if the photon is luminal and the graviton is super-luminal, that is the case considered in the left panel of Fig. 1 it is never possible to have $\Delta t_e = 0$, the latter being the condition needed to establish the upper bound in Eq. (10) (or Eq. (11)). Actually, the complete analysis presented above shows that the finding $\Delta t_e \sim \Delta t_a/(1 + z_0) \approx 0.4\, s$, that in turn means $\Delta t \sim 0$, explains why we have obtained such a stringent bound as the one of Eq. (12), by far much more stringent than the bound of Eq. (10).
we can see from the theoretical curves shown in this figure, in this case the value $\Delta t_e$ contemplates the case when the photon is luminal and the graviton is massive [54]. As the graviton is sub-luminal. This is the case mostly studied in the literature, and in particular for magnitudes below the Planck scale, and for greater values of $E_0$ can be reached, but this occurs for a value of $\Delta t_e$ is obtained from the intersection between the curve and the vertical line corresponding $E_g = E_g^0/E_{QG}$, $\Delta t'_a = \Delta t_a/(\beta_n^{a+1})$ and $\Delta t'_e = \Delta t_e/(\beta_n^{n+1})$.

Table I: Phenomenological equations corresponding to models with sub-, super- or luminal photons and gravitons. Note that $E_{\gamma} = E_{\gamma}^0/E_{QG}$, $E_{g} = E_{g}^0/E_{QG}$, $\Delta t'_a = \Delta t_a/(\beta_n^{a+1})$ and $\Delta t'_e = \Delta t_e/(\beta_n^{n+1})$.

| Case $\xi_\gamma, \xi_g$ | Equation          |
|--------------------------|-------------------|
| A 0 0                    | $\Delta t_e = \Delta t_a/(1 + z_0)$ |
| B 0 1                    | $\tilde{E}_{\gamma} = [-\Delta t'_a + (1 + z_0)\Delta t'_e]^{1/n}$ |
| C 0 -1                   | $\tilde{E}_{g} = [\Delta t'_a - (1 + z_0)\Delta t'_e]^{1/n}$ |
| D 1 0                    | $\tilde{E}_{\gamma} = [\Delta t'_a - (1 + z_0)\Delta t'_e]^{1/n}$ |
| E 1 1                    | $\tilde{E}_{\gamma} = [\Delta t'_a - (1 + z_0)\Delta t'_e + \tilde{E}_g]^{1/n}$ |
| F -1 -1                  | $\tilde{E}_{\gamma} = [-\Delta t'_a + (1 + z_0)\Delta t'_e - \tilde{E}_g]^{1/n}$ |
| G -1 0                   | $\tilde{E}_{\gamma} = [-\Delta t'_a + (1 + z_0)\Delta t'_e]^{1/n}$ |
| H -1 1                   | $\tilde{E}_{\gamma} = [-\Delta t'_a + (1 + z_0)\Delta t'_e - \tilde{E}_g]^{1/n}$ |
| I -1 -1                  | $\tilde{E}_{\gamma} = [-\Delta t'_a + (1 + z_0)\Delta t'_e + \tilde{E}_g]^{1/n}$ |

In the right panel of Fig. [1] we consider the case when the photon is luminal and the graviton is sub-luminal. This is the case mostly studied in the literature, and in particular it contemplates the case when the photon is luminal and the graviton is massive [54]. As we can see from the theoretical curves shown in this figure, in this case the value $\Delta t_e \sim 0$ can be reached, but this occurs for a value of $E_{QG}$ that for $n = 1$ is more than 20 orders of magnitudes below the Planck scale, and for greater values of $n$ is even smaller. As for the previous case, we then see that acceptable values for the quantum gravity scale $E_{QG}$ are obtained only if the time emission delay is frozen to the value $\Delta t_e \sim 0.4$ s, and this again provides an explanation for the extraordinarily low value of the upper bound $|1 - \nu^0_0/c| \leq 10^{-40}$ of Eq. (12).

Fig. [2] shows the cases D and G of Tab. [1] where the graviton is luminal and the photon sub- (left panel) or super-luminal (right panel). Similarly to the case of Fig. [1] here we present the curves that relate the time emission delay $\Delta t_e$ between the two waves with the energy ratio $E_{\gamma}^0/E_{QG}$, where $E_{\gamma}$ is the measured energy of the photons, for different values of $n$ ($n = 1, 2, 3$ and 4). As for the previous cases, for any value of $E_{QG}$ the allowed value of $\Delta t_e$ is obtained from the intersection between the curve and the vertical line corresponding to a given value of $E_{\gamma}^0/E_{QG}$. For instance, the two dotted thick vertical lines shown in the figure indicate the values of $E_{\gamma}^0/E_{QG}$ obtained for $E_{QG} = M_{Pl}c^2$ and with the two extremal values of the energies of the observed photons, 50 keV and 1 MeV. Even for these cases we find that $\Delta t_e = \Delta t_a/(1 + z_0) \approx 0.4$ s is the only allowed value for $E_{QG} > 100$ GeV ($n = 1$) and $E_{QG} > 0.1$ eV ($n = 2$), again much below the most stringent lower bounds [22].

From the analysis developed so far we can already draw some important lessons. First of all we note that scenarios where one of the particles (the photon or the graviton) is luminal and the other sub- or super- luminal are allowed, but the requirement of having an acceptable value for the quantum gravity scale imposes that the time emission delay between the gravitational and the electromagnetic waves has to be equal to $\Delta t_a/(1 + z_0) \approx 0.4$ s. This is an important result as it strongly constrains astrophysical models [35–41] that aim to explain the arrival delay of the two signals and the production of short gamma-ray bursts from black hole mergers. Moreover, the fact that $\Delta t_e \sim 0.4$ s, that in turn means $\Delta t \sim 0$, implies very stringent lower bounds on the difference in speed between the photon and the graviton, much more stringent than previous existing bounds, and this is in fact what we.
Similarly to Fig. 1, here we plot the curves corresponding to the cases D (left panel) and G (right panel) of Tab. 1. The dotted thick vertical lines indicate the values of $E_0^g/E_{QG}$ obtained for $E_{QG} = M_{Pl}c^2$ and the extremal energies of the observed photons, 50 keV and 1 MeV.

have found in the previous section.

Up to now we have considered cases where at least one of the two particles (the photon or the graviton) is luminal. Now we move to consider cases when they are both super- or sub- luminal, or one of them is super- and the other is sub- luminal. The corresponding relationships between the physical quantities $E_0^g/E_{QG}$, $E_0^\gamma/E_{QG}$, and $\Delta t_e$ are obtained from Eq. (5) and listed as cases E, F, H, I in Tab. 1.

Results obtained for the case E and F, where the photon is super-luminal and the graviton is sub- (left panels) or super- (right panels) luminal are shown in Fig. 3. In the two top panels we consider the special case when there is no emission delay between the gravitational and the electromagnetic wave, i.e. when $\Delta t_e = 0$. Under this condition, we are left with a relation between the two ratios $E_0^\gamma/E_{QG}$ and $E_0^g/E_{QG}$. As in the previous figures, the thick dotted lines are obtained for the measured values of $E_0^g$ and $E_0^\gamma$ when $E_{QG} = M_{Pl}c^2$. We see that the theoretical curves, even in the $n = 1$ case, do not fit in the experimental allowed range, and only if the quantum gravity scale is lowered of several orders of magnitudes (so that the horizontal thick dotted lines move upwards, while the vertical ones move to the right) the curves become compatible with experiments.

In the two other panels of Fig. 3 we plot again the theoretical curves relating $E_0^\gamma/E_{QG}$ with $E_0^g/E_{QG}$ for several different values of $\Delta t_e$, ranging from $\Delta t_e = 10^{-20}$ s to $\Delta t_e = 10^{20}$ s, considering only the $n = 1$ case. The results are similar to those discussed above for the $\Delta t_e = 0$ case. We see that there are no values of $\Delta t_e$ that are compatible at the same time with the measured value of $E_0^g$ and $E_0^\gamma$ and with a quantum gravity scale that lies at or above the Planck scale. Only if $E_{QG} \ll M_{Pl}c^2$ the theoretical curves would lie in the allowed region. For instance, in the case $E_{QG} \approx 10^{13}$ GeV ($n = 1$) emission delays between -1 s and 0 s would be possible.

Similar conclusions are drawn from the analysis of models H and I of Tab. 1 and the corresponding theoretical curves are shown in Fig. 4. Again, there are no values of $\Delta t_e$
Figure 3: The theoretical curves corresponding to the cases E (left panels) and F (right panels) of Tab. I are plotted. As for the other figures, dotted thick lines correspond to the observed values of $E_g^0$ and $E_g^0$ for $E_{QG} = M_{Pl}c^2$. In top panels, we consider $\Delta t_e = 0$ and vary $n$, whereas in bottom panels we consider $n = 1$ and vary $\Delta t_e$.

compatible with observation and a quantum gravity energy scale above the Planck’s one. Note that there are no curves in the top-left panel because for $\Delta t_e = 0$ the corresponding values of $E_g^0 / E_{QG}$ would be negative. As for the previous cases, only if $E_{QG} \ll M_{Pl}c^2$ the theoretical curves would reach the allowed region. For instance, in the case $E_{QG} \approx 10^{13}$ GeV ($n = 1$) emission delays between 0.4 s and 1 s would be allowed. Once again, these kind of models would be allowed only if the quantum gravity scale is downshifted much below the Planck scale.

Therefore, from the analysis of the cases E F H I of Tab. I we learn the following very important lesson: scenarios where none of the two particles, the photon and the graviton,
Figure 4: As in Fig. 3 here we consider the cases corresponding to models H (left panels) and I (right panels) of Tab. 1. In the top-left panel there are no curves because for $\Delta t_e = 0$ the corresponding values of $E_\gamma/E_{QG}$ would be negative.

is luminal are not allowed, unless the quantum gravity scale lies in an energy range that is several orders of magnitude below the Planck scale.

V. CONCLUSIONS

Let us summarize the results of the present work. Assuming that the gamma-ray burst observed by the Fermi Collaboration was emitted by the black hole merger that produced the gravitational waves detected by the LIGO Collaboration, we have performed a model-independent analysis that allows to obtain bounds for the difference in speed between the
gravitational and the electromagnetic waves and constraints on quantum gravity models.

With the help of dispersion relations typical of many quantum gravity models, we have found that, when the photon is luminal while the graviton is sub- or super-luminal, if (as expected) the quantum gravity scale lies at or above the Planck scale, an extraordinarily stringent constrain on the speed of the gravitational wave emerges, namely \( |1 - v_0^g/c| \leq 10^{-40} \). This is a much more stringent limit than those obtained so far \([37, 44, 54]\). Even if we lower considerably the quantum gravity scale, the upper bound on the speed of gravitational waves stays may orders of magnitudes lower than any previously known bound. Moreover, we have also considered the case where the photons are not luminal. In this case, always considering that \( E_{QG} \geq M_{Pl} c^2 \), we have constrained the difference in speed as \( |v_0^\gamma - v_0^g| \leq c \times 10^{-22} \), that is still five orders of magnitude more stringent than those obtained with other approaches.

Our model-independent analysis also demonstrates that only scenarios with at least one luminal particle – either the photon or the graviton or both – are allowed, regardless of the value of \( n \) in the modified dispersion relation. Other scenarios with no luminal particles are ruled out, unless the energy scale of quantum gravity is downshifted by several orders of magnitude. In all likely scenarios, the allowed values for the time emission delay are strictly constrained to equal \( \Delta t_a/(1 + z_0) \approx 0.4 \text{ s} \), result that strongly constrains the new astrophysical models \([35,40]\) (see Ref. \([36]\) for the description of some possible models and the corresponding emission delay) required to explain the production of short gamma-ray bursts from black hole mergers.
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