I. INTRODUCTION

The recent discovery [1], [2] that the ratio of proton form factors $G_E/G_M$ falls linearly with $Q^2$ and that the ratio $QF_2/F_1$ reaches a constant value for $2 \leq Q^2 \leq 6$ GeV$^2$ has focused attention on understanding nucleon structure. The constant nature of the ratio $QF_2/F_1$ contrasts with the prediction from perturbative QCD [3,4] that $Q^2F_2/F_1$ should be constant. While this latter ratio could be achieved when experiments are pushed to higher values of $Q^2$, it is worthwhile to obtain a deeper understanding of the present results. In particular, the perturbative result is based on the notion that helicity is conserved [5] in high momentum transfer exclusive processes, so that it becomes interesting to understand why this conservation does not seem to be applicable.

The qualitative nature of the experimental results were anticipated or reproduced by several model calculations [6–9], with that of Ref. [6] based on the work of Schlumpf [10–12] being the earliest. The implementation of relativity is an important feature of each of these calculations, so it is natural seek an understanding of the form factors in terms of relativity. Our purpose here is to examine the model of Ref. [6] with the aim of highlighting the essential features which cause the ratio $QF_2/F_1$ to be constant.

We proceed by presenting definitions and kinematics relevant for a light front analysis in Sect. II. The relevant features of our relativistic constituent quark model are displayed in Sect. III. The essential reason for the constant ratio $QF_2/F_1$ is discussed in Sect. IV, and elaborated upon numerically in Sect. V. The paper is concluded with a brief summary.

II. DEFINITIONS AND KINEMATICS

The electromagnetic current matrix element can be written in terms of two form factors taking into account current and parity conservation:
$$\langle N, \lambda' \mid J^\mu \mid N, \lambda \rangle = \bar{u}_\lambda(p') \left[ F_1(Q^2) \gamma^\mu + \frac{\kappa F_2(Q^2)}{2M_N} i\sigma^{\mu\nu}(p' - p)_\nu \right] u_\lambda(p) \quad (1)$$

with momentum transfer $q^\mu = (p' - p)^\mu$, $Q^2 = -q^2$ and $J^\mu$ is taken as the electromagnetic current of free quarks. For $Q^2 = 0$ the form factors $F_1$ and $\kappa F_2$ are respectively equal to the charge and the anomalous magnetic moment $\kappa$ in units $e$ and $e/M_N$, and the magnetic moment is $\mu = F_1(0) + \kappa F_2(0) = 1 + \kappa$. The Sachs form factors are defined as

$$G_E = F_1 - \frac{Q^2}{4M_N^2} F_2, \quad \text{and} \quad G_M = F_1 + \kappa F_2,$$  

(2)

The evaluation of the form factors is simplified by using the so-called Drell Yan reference frame in which $q^+ = 0$ so that $Q^2 = q_1^2$. This means that the plus components of the nucleon momenta (and also those of the struck constituent quark) are not changed by the absorption of the incoming photon.

If light-front spinors for the nucleons are used, the form factors can be expressed simply in terms of the plus-component of the current $[13]$:

$$F_1(Q^2) = \frac{1}{2P^+} \langle N, \uparrow \mid J^+ \mid N, \uparrow \rangle, \quad \text{and} \quad Q\kappa F_2(Q^2) = -\frac{2M_N}{2P^+} \langle N, \uparrow \mid J^+ \mid N, \downarrow \rangle. \quad (3)$$

The form factors are calculated, using the “good” component of the current, $J^+$, to suppress the effects of quark-pair terms.

It is worthwhile to compare the formalism embodied in Eq. (3) with the non-relativistic quark model formalism in which $G_E$ and $G_M$ are the Fourier transforms of the ground state matrix elements of the quark charge ($\sum_{i=1,3} e_i \delta(r - r_i)$) and magnetization ($\sum_{i=1,3} \frac{e_i}{2m_i} \delta(r - r_i)$) density operators. At high momentum transfer one needs to account for the influence of the motion of the proton on its wave function. Since the charged quarks of the initial proton state (or final state, or both initial and final states) are moving, charge and magnetic effects are correlated in a manner consistent with relativity. It is necessary to maintain this relativistic connection, lost in the non-relativistic quark model. The use of light front dynamics, concomitant with Eq. (3), is a particularly convenient way to handle the motion of the initial and final state. This is because the proton wave function, a function of internal relative coordinates, is the same in any reference frame.

### III. RELATIVISTIC CONSTITUENT QUARK MODEL OF THE NUCLEON

We study the form factors using relativistic constituent quark models in general, and starting with the model of Schlumpf $[10,12]$ in particular. Such models have a long history $[14,16]$, and many authors $[17,32]$ have contributed to the necessary developments. Schlumpf’s model is used because his power-law wave functions lead to a reasonably good description of the proton electromagnetic form factors, $G_E$ and $G_M$, at all of the values of $Q^2$ where data were available as of 1992 $[10,12]$. This model uses the Bakamjian-Thomas BT construction, which implies the choice of a very specific model wave function $[23,31]$. The use of a definite model allows us to gain insight, but we also shall discuss the limitations of this approach.
We remind the reader about a few basic features of light front treatments in the BT approach. The light-front formalism is specified by the invariant hypersurface $x^+ = x^0 + x^3 = \text{constant}$. The following notation is used: A four-vector $A^\mu$ is given by $A^\mu = (A^+, A^-, A_\perp)$, where $A^+ \equiv A^0 + A^3$ and $A_\perp = (A^1, A^2)$. Light-front momenta vectors are denoted by $p = (p^+, p_\perp)$, with $p^- = (p_\perp^2 + m^2)/p^+$ for on-shell quarks. The three momenta $p_i$ of the quarks can be transformed to the total and relative momenta to facilitate the separation of the center of mass motion as

$$P = p_1 + p_2 + p_3, \quad \xi = \frac{p_1^+}{p_1^+ + p_2^+}, \quad \eta = \frac{p_1^+ + p_2^+}{p^+},$$

$$k_\perp = (1 - \xi)p_{1\perp} - \xi p_{2\perp}, \quad K_\perp = (1 - \eta)(p_{1\perp} + p_{2\perp}) - \eta p_{3\perp}. \quad (4)$$

It is also useful to consider the mass operator of a non-interacting system of total momentum $P^\mu$:

$$M_0^2 \equiv \sum_{i=1,3} p_i^- P^+ - P_\perp^2 = \frac{K_\perp^2}{\eta(1 - \eta)} + \frac{k_\perp^2 + m^2}{\eta \xi(1 - \xi)} + \frac{m^2}{1 - \eta}, \quad (5)$$

where $m$ is the light quark mass, taken as the same for up and down quarks.

One may express the proton wave function in the center of mass frame in which the individual momenta are given by

$$p_{1\perp} = k_\perp + \xi K_\perp, \quad p_{2\perp} = -k_\perp + (1 - \xi)K_\perp, \quad p_{3\perp} = -K_\perp. \quad (6)$$

The use of light front variables enables one to separate the center of mass motion from the internal motion. The internal wave function $\Psi$ is therefore a function of the relative momenta $p_1, \xi, \eta$. The internal wave function of the proton depends on these relative momenta. One can obtain the wave function in certain special frames via a kinematic boost. In particular, if the proton acquires a transverse momentum (is boosted) by the absorption of a photon of momentum $q = (0, q_\perp)$ by the third quark, the effects of the boost are obtained merely replacing the momenta $k_\perp, K_\perp$ by

$$k'_\perp = k_\perp, \quad K'_\perp = K_\perp - \eta q_\perp. \quad (7)$$

The mass operator, $M_0'$ of the boosted system is obtained by replacing $k_\perp, K_\perp$ by the variables of Eq. (7) so that

$$M_0'^2 = M_0^2 + \frac{-2\eta K_\perp \cdot q_\perp + \eta^2 q_\perp^2}{\eta(1 - \eta)}. \quad (8)$$

We now turn to the construction of the non-perturbative wave function, $\Psi$. This is based on the attempt to construct a state, described in terms of the given light front variables, that also is an eigenstate of angular momentum [20]. We take the proton wave function to be a product of an anti-symmetric color wave function with a symmetric flavor-spin-momentum wave function $\Psi$. To understand the construction [20] of the relativistic wave function, it is worthwhile to start by considering the non-relativistic model. Then

$$\Psi^{NR} = \frac{1}{\sqrt{2}} (\phi_\rho \chi_\rho + \phi_\lambda \chi_\lambda) \Phi \quad (9)$$
where $\phi_\rho$ represents a mixed-antisymmetric and $\phi_\lambda$ a mixed-symmetric flavor wave function and, $\chi_{\rho,\lambda}$ represents mixed symmetric or anti-symmetric spin wave functions (in terms of Pauli spinors). In the non-relativistic model the wave function $\Phi$ depends on spatial variables only, and the computed form factors $G_E$ and $G_M$ will have the same dependence on $Q^2$.

The relativistic generalization of Eq. (9) is

$$
\Psi(p_i) = u(p_1)u(p_2)u(p_3)\phi(p_1, p_2, p_3),
$$

(10)

where $p_i$ represents space, spin and isospin indices: $p_i = p, s_i, \tau_i$ and repeated indices are summed over. The spinors $u$ are canonical Dirac spinors:

$$
\begin{aligned}
&u(p, s) = \frac{\beta + m}{\sqrt{E(p)} + m} \begin{pmatrix}
\chi_{\text{Pauli}}^s \\
0
\end{pmatrix},
\end{aligned}
$$

(11)

with the isospin label suppressed.

The completely symmetric nature of the space-spin-flavor wave function is preserved by using [20]

$$
\begin{aligned}
\psi(p_1, p_2, p_3) &= \Phi \left[ \bar{u}(p_1)\Gamma u^T_2(p_2) \bar{u}_3(p_3) u_N(0) + \bar{u}(p_1)\Gamma^{\mu,\alpha} u^T_2(p_2) \bar{u}_3(p_3) \tilde{\Gamma}^{\nu,\alpha} u_N(0) g_{\mu\nu} \right] \\
\Gamma &\equiv -\frac{1}{\sqrt{2}} \frac{1 + \beta}{2} \gamma_5 C i \tau_2, \quad \Gamma^{\mu,\alpha} \equiv \frac{1}{\sqrt{6}} \gamma^\mu C i \tau_\alpha \tau_2, \quad \tilde{\Gamma}^{\nu,\alpha} \equiv \frac{1 + \beta}{2} \gamma^\nu \gamma_5 \tau_\alpha,
\end{aligned}
$$

(12)

where the charge-conjugation matrix $C \equiv i\gamma^0 \gamma^5 = -i \gamma_5 \sigma_2$. Note that if one takes the non-relativistic limit of (12) by taking $p_i \rightarrow (m, 0_\perp)$ one gets (9) for the spin-isospin dependence.

The momentum wave function can be chosen as a function of $M_0$ to fulfill the requirements of spherical and permutation symmetry. We take the $S$-state orbital function $\Phi(M)$ to be of a power law form:

$$
\Phi(M_0) = \frac{N}{(M_0^2 + \beta^2)^{\gamma}},
$$

(14)

which depends on two free parameters, the constituent quark mass and the confinement scale parameter $\beta$. Schlumpf’s parameters are $\beta=0.607$ GeV, $\gamma = 3.5$, and the constituent quark mass, $m=0.267$ GeV.

The wave function of Eq. (12) now specified. It is an eigenstate of the angular momentum operator (if interactions present in that operator are neglected) and also corresponds to a state of vanishing orbital angular momentum. This state is Poincare invariant according to Ref. [20], but this invariance may be incomplete [32].

**IV. THE ESSENTIAL EFFECT**

The form factors of Eq. (3) are obtained by computing the matrix elements of the current operator $J^+$. Here we take the quarks to be elementary particles, so that the operator $J^+$ is essentially the operator $\gamma^+ \gamma^+$ times the charge of the quarks. The calculations may be simplified by making a unitary transformation which replaces the Dirac spinors of Eq. (10) by light front spinors.
\[ u_L(p, \lambda) = \frac{\not{p} + m}{\sqrt{2p^+}} \gamma^+ \left( \begin{array}{c} \chi_{\lambda \text{Pauli}}^3 \\ 0 \end{array} \right), \]  

(15)

because \[ u_L(p, \lambda) = \frac{\not{p} + m}{\sqrt{2p^+}} \gamma^+ \left( \begin{array}{c} \chi_{\lambda \text{Pauli}}^3 \\ 0 \end{array} \right), \]

because \[ u_L(p^+, \lambda') \gamma^+ u_L(p, \lambda) = 2 \delta_{\lambda \lambda'} p^+. \]

(16)

One then uses the completeness relation, \( 1 = \sum_\lambda u_L(p, \lambda) \bar{u}_L(p, \lambda)/2m \), in Eq. (16), to obtain the light front representation for the wave function:

\[ \Psi(p_i) = u_L(p_1, \lambda_1) u_L(p_2, \lambda_2) u_L(p_3, \lambda_3) \psi_L(p_i, \lambda_i), \]

(17)

\[ \psi_L(p_i, \lambda_i) \equiv \langle \lambda_1 | R_{M}^\dagger(p_1) | s_1 \rangle \langle \lambda_2 | R_{M}^\dagger(p_2) | s_2 \rangle \langle \lambda_3 | R_{M}^\dagger(p_3) | s_3 \rangle \psi(p_1, p_2, p_3), \]

(18)

where \( R_M \) is a Melosh rotation \[ [33] \] acting between Pauli spinors. For example,

\[ \langle \lambda_3 | R_{M}^\dagger(p_3) | s_3 \rangle = \bar{u}_L(p_3, \lambda_3) u(p_3, s_3) = \langle \lambda_3 \left| \frac{m + (1 - \eta) M_0 + i \sigma \cdot (\mathbf{n} \times p_3)}{\sqrt{(m + (1 - \eta) M_0)^2 + p_{3\bot}^2}} \right| s_3. \]  

(19)

The net result of this is that the relativistic spin effect is to replace the Pauli spinors of Eq. (4) with Melosh rotation operators acting on the very same Pauli spinors \[ [21] \]. The spin-wave function of the \( i \)th quark is given by

\[ |\uparrow p_i \rangle \equiv R_{M}^\dagger(p_i) \left( \begin{array}{c} 1 \\ 0 \end{array} \right) \text{ and } |\downarrow p_i \rangle \equiv R_{M}^\dagger(p_i) \left( \begin{array}{c} 0 \\ 1 \end{array} \right). \]

(20)

This means that, for example, the spin wave function \( \chi_\rho \) is replaced by a momentumdependent spin wave function \( \chi_{\rho, p_i} \)

\[ |\chi_{\rho, p_i} \rangle = \frac{1}{\sqrt{2}} |\downarrow p_1 \downarrow p_2 \rangle - |\downarrow p_1 \uparrow p_2 \rangle \equiv |\chi_0^{\text{rel}}(p_1, p_2) \rangle |\uparrow p_3 \rangle \]

(21)

for a spin +1/2 proton. The term \( \chi_0^{\text{rel}} \) represents the relativistic generalization spin-0 wave function of the quark pair labeled by (1,2). Note that the momenta \( p_i \) are to be expressed in terms of the relative variables \( k_\perp, K_\perp, k'_\perp, K'_\perp \) of Eqs. (1) and (7). We shall see that the important relativistic effect is contained in the difference between the spinors of Eq. (20) and Pauli spinors.

The next step is to simplify the calculation by using the symmetry of the wave function under interchange of particle labels to replace the quark current operator by three times that operator acting only on the third quark. The average charge of the third quark of the mixed-symmetric flavor wave function vanishes. This means that the second term of Eq. (13) does not contribute to proton electromagnetic form factors. The first term involves a mixed-antisymmetric wave function, so the third quark carries the spin of the proton, \( s_3 \) of Eq. (19). However, the helicity of the light front spinor \( \lambda_3 \) can either be the same as or different from \( s_3 \). The weighting of the terms is determined by the two terms of the Melosh transformation of Eq. (19). Considering the arguments of the final state wave function allows one to understand that the two terms are comparable. The effect of the boost is incorporated simply by using Eq. (7) in the argument of the final state wave function. Thus the large momentum \( Q \) is involved and both terms of the Melosh rotation (19) are comparable.
There is clearly a substantial amplitude for a spin-up valence quark to carry a negative light front helicity. No suppression of light-front helicity-flip can be expected from the use of such a wave function. This means that helicity conservation does not occur. This conclusion has been obtained by Ralston and collaborators \cite{Ralston1978}, based on the presence of non-zero orbital angular momentum. Here we have no orbital angular momentum, and the mixture of light-front helicity we obtain occurs as a result of imposing Poincare invariance on the constituent quark model. Nonetheless, we certainly support their statements \cite{Ralston1978} that helicity non-conservation is an important effect. We also note that Braun et al. \cite{Braun1990} argue that soft non-factorizable terms in the wave function, with helicity structure similar to ours, are important at intermediate values of $Q^2$. In our model, there is no basis for expecting light-front helicity conservation because the non-perturbative wave function is a mixture of different light-front-helicity states.

If the value of $Q^2$ becomes asymptotically large, the effects of the non-perturbative wave function may disappear and perturbative effects, which do respect helicity conservation, could take over. But for the present, we must take helicity non-conservation as a given and it will be worthwhile to consider the implications of this non-conservation.

**V. PROTON FORM FACTORS**

We obtain the form factors by using the wave function of Eq. (18) in Eq. (3). Our result can be expressed as

$$F_1(Q^2) = \int \frac{d^2q_\perp d\xi}{\xi(1-\xi)} \frac{d^2K_\perp d\eta}{\eta(1-\eta)} \tilde{\Phi}(M'_0) \tilde{\Phi}(M_0) \langle \chi^\text{rel}_0(p_1', p_2') | \chi^\text{rel}_0(p_1, p_2) \rangle \langle \uparrow p_3 | \uparrow (p_3) \rangle$$  \hspace{1cm} (22)

$$Q^2 F_2(Q^2) = 2M_N \int \frac{d^2q_\perp d\xi}{\xi(1-\xi)} \frac{d^2K_\perp d\eta}{\eta(1-\eta)} \tilde{\Phi}(M'_0) \tilde{\Phi}(M_0) \langle \chi^\text{rel}_0(p_1', p_2') | \chi^\text{rel}_0(p_1, p_2) \rangle \langle \uparrow p_3 | \downarrow (p_3) \rangle$$  \hspace{1cm} (23)

The value of $M'_0$ is obtained by using Eq. (8), and

$$\tilde{\Phi}(M_0) \equiv \sqrt{\frac{E_3 E_{12} E_1}{M_0}} \Phi(M_0)$$  \hspace{1cm} (24)

$$E_1 = \sqrt{\frac{k^2_e + m^2}{4\xi(1-\xi)}}, \quad E_{12} = \frac{K^2 + 4E_1^2 + \eta^2 M_0^2}{2\eta M_0}, \quad E_3 = M_0 - E_{12}.$$  \hspace{1cm} (25)

Numerical evaluations of these equations for large values of $Q^2$ were presented in Ref. [6]. Our aim here is to understand the essential features of the different $Q^2$-dependence of $F_1$ and $F_2$. The expressions for the form factors differ only by the presence of the last factor $\langle \uparrow p_3 | \uparrow (p_3) \rangle$, or $\langle \uparrow p_3' | \uparrow (p_3) \rangle$, with $F_1$ depending on the non-spin-flip term, and $F_2$ on the spin-flip term \cite{Braun1990}. To proceed we evaluate these overlaps, using the Melosh transformations with $p_{3\perp} = -K_\perp$, and $p'_{3\perp} = -K_\perp + \eta q_\perp$. One computes the product of the matrices $R_M(p'_3) R_M^\dagger(p_3)$. The upper diagonal element (non-spin-flip term) appears in the expression for $F_1$, and the upper off-diagonal element (spin-flip term proportional to $\sigma$) determines $F_2$. The evaluation is simplified by realizing that integration over $d^2K_\perp$ causes terms linear in the component of $K$ which are perpendicular to $q$ to vanish.

To be definite, we take $q_\perp$ to lie along the $x$-direction, so that we find...
\[
\langle \uparrow p' \mid \uparrow p \rangle = \frac{\sqrt{(m + (1 - \eta)M_0^2 + K_0^2)}}{\sqrt{(m + (1 - \eta)M_0^2 + K_0^2)}} \langle \uparrow p' \rangle \frac{\langle \uparrow p \rangle}{\langle \uparrow p \rangle}
\]

We may understand the qualitative nature of the ratio \( \frac{QF_2(Q^2)}{F_1(Q^2)} \) using the notion that the value of \( Q = \sqrt{q^2} \) can be much larger than the typical momenta, of order \( \beta = 560 \text{ MeV} \), which appear in the wave function. Then for \( Q \gg \beta \), we may approximate Eq. (8) by

\[
M'_0 \approx Q \sqrt{\frac{\eta}{1 - \eta}} \quad \text{(27)}
\]

and take the terms of the bracketed expressions of Eq. (26) which are proportional to \( Q \) as dominant. Then, using Eqs. (26) and (27) in Eqs. (22,23), we see that each of \( F_1 \) and \( QF_2 \) contains an explicit factor of \( Q \); and

\[
\frac{QF_2}{F_1} \approx 2M_N \frac{\langle \eta(m + (1 - \eta)M_0) + \sqrt{\eta(1 - \eta)K_x} \rangle}{\langle -K_x + (m + (1 - \eta)M_0)\rangle}, \quad \text{(28)}
\]

where the expectation value symbols abbreviate the operation of multiplying by the remaining factors of Eqs. (22,23) (without approximation) and performing the necessary six-dimensional integral. The terms \( \eta, M_0, K_x \) can be anticipated to have an expectation value independent of \( Q^2 \), so the ratio is anticipated to be constant.

The exact model calculation and the approximation (28) are compared in Fig. 1. Eq. (28) qualitatively reproduces the constant nature of the ratio and its value. Thus the constant nature of the ratio is understood from the properties of the Melosh transformation, which here embodies the relativistic effects.
Equation (28) represents a simple quick argument which gives a constant ratio. But this is only a rough approximation because each of $F_1, F_2$ is over predicted by about 40%. Numerical work shows that neglecting the terms proportional to $K_x$ in both the numerator and denominator of Eq. (28) leads to a different approximation:

$$\frac{Q\kappa F_2^{As}}{F_1^{As}} \approx \frac{\langle \eta(m + (1 - \eta)M_0) \rangle}{\langle (m + (1 - \eta)M_0) \sqrt{\eta(1 - \eta)} \rangle}. \quad (29)$$

which, as shown in Fig. 2 leads to an even better reproduction of the model results for $F_1$ and $F_2$.

![Graph](image)

FIG. 2. Model calculation of Eqs. (22,23,26) solid, vs. the approximation Eq. (29), dashed.

Equation (29) is a better approximation because the terms involving $K_x$ cancel against terms involving the difference between $M'_0$ and its approximation (27). Thus it seems that values of $Q^2$ going up to 20 GeV$^2$ are not large enough to allow one to completely neglect other terms, and therefore also not large enough to extract the asymptotic behavior.

This feature of not reaching asymptotic values of $Q^2$ may be understood by examining the dependence of the integrands of Eqs. (22,23) on the value of $\eta$. We may write

$$F_{1,2}(Q^2) = \int_0^1 d\eta I_{1,2}(\eta, Q^2), \quad (30)$$

and determine the important regions by examining $I_{1,2}(\eta, Q^2)$. As shown in Figs. (3,4) the important contributions occur for a very narrow band of values close to $1 - \eta = x_3 = 0.145$. The sharp peaking is maintained for all of the values of $Q^2$ considered here, and is a central reason for the qualitative success of the approximations (28,29). The small factor $1 - \eta$ multiplies the large factor $Q$ appearing in Eq. (27), and suppresses the dominance of the terms proportional to $Q$. The integrands peak at $x_3 = 0.85$, a large value (compared to 0.33,
expected if each quark were to carry the same momentum) which indicates the presence of the Feynman mechanism, and a corresponding difficulty of using simple arguments to extract asymptotic properties of form factors.

**FIG. 3.** Important region of integration for $F_1$, Eq. (30). The curves show the derivative of $I_1$ for values of $Q^2 = 2, 4, 6, 8, 10 \text{ GeV}^2$, with the larger values occurring for the smaller values of $Q^2$.

**FIG. 4.** Important region of integration for $F_2$, Eq. (30). The curves show the derivative of $I_1$ for values of $Q^2 = 2, 4, 6, 8, 10 \text{ GeV}^2$, with the larger values occurring for the smaller values of $Q^2$. 
We also find that the computed value of the ratio $Q\kappa F_2/F_1$ is remarkably independent of the parameters of the model. For example, Fig. (5) shows that a 10% increase in the value of $\gamma$, Eq. (14), causes about a 50% decrease in the computed values of $F_2$, but Fig. (6) shows only a 5% change in the ratio.

![Graph](attachment:image.png)

**FIG. 5.** The effect of varying the parameter $\gamma$ which governs the power of the falloff of the wave function of Eq. (14). The curves for $F_2$ are labeled by the value of $\gamma = 3.5$ correct model value (solid) or $\gamma = 3.9$ (dashed).

The solid curve of Fig. (5) represents our final results and predictions for values of $Q^2$ that are not yet measured. The dashed curve of Fig. (5) is closer to the data for the ratio, but it corresponds values of $F_2$ at high $Q^2$ which are much smaller than the data of [37]. Thus the original model of Schlumpf seems to account for both $F_1$ and $F_2$. There is a small disagreement ($\sim 15\%$) with the data for the ratio, which can not be fixed simply by varying the parameters. This small degree of disagreement between the model and the data is remarkable because so many plausible effects, such as configuration mixing involving both quark and gluon degrees of freedom and a non-perturbative $Q^2$ variation of the constituent quark masses [38] are ignored. Pion cloud effects [39] are surely present, but these do not seem to be significant for values of $Q^2$ greater than about 2 GeV$^2$.

We also study the behavior of the ratio $Q\kappa F_2/F_1$ for very large values of $Q^2$; see Fig. 7. The constant nature of the ratio seen in previous figures is actually the result of a broad maximum occurring near $Q^2 \approx 10$ GeV$^2$. The ratio falls for asymptotic values of $Q^2$, the displayed curve can be represented by but not as predicted using perturbative QCD, which would be $Q^2\kappa F_2/F_1 \sim 1/\log(Q^2)$ for $80 < Q^2 < 600$ GeV$^2$. 

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FIG. 6. The effect of varying the parameter $\gamma$ which governs the power of the falloff of the wave function of Eq. (14). The curves for $Q F_2/F_1$ are labeled by the value of $\gamma = 3.5$ (solid) or $\gamma = 3.9$ (dashed). The data for $2 \leq Q^2 \leq 3.5$ GeV$^2$ are from Ref. 1., and that for $3.5 \leq Q^2 \leq 5.5$ GeV$^2$ are from Ref. 2.

FIG. 7. High $Q^2$ behavior of $Q\kappa F_2/F_1$.
VI. SUMMARY AND DISCUSSION

We have seen how a simple relativistic constituent quark model accounts for both \( F_2 \) and \( QF_2/F_1 \) for values of \( Q^2 \) between 2 and 5.5 GeV\(^2\). The most relevant ingredient in the model is its attempt to use a wave function which is Poincare invariant. In such wave functions the helicity conservation rule is not satisfied because the non-perturbative wave function is a mixture of different helicity states. This feature leads to an analytic understanding, embodied in Eqs. (28) and (29), that \( QF_2(Q^2) \) and \( F_1(Q^2) \) have the same variation with \( Q^2 \). The predicted value of the ratio \( QF_2(Q^2)/F_1(Q^2) \approx 0.8 \) for the values of \( Q^2 \) up to 20 GeV\(^2\), and drops slowly for larger values.

The present data set extends to \( Q^2 = 5.5 \) GeV\(^2\), and its measurement of the non-conservation of helicity has implications for other exclusive processes involving protons. Helicity conservation should be not relevant if we consider proton-proton scattering at high momentum transfers, up to \(-t = 5.5 \) GeV\(^2\). This means large values of various analyzing powers can be expected. Perhaps the most interesting mystery in proton-proton scattering is the large value of \( A_{NN} \) observed in 90° proton scattering at \( s \approx 20 \) GeV\(^2\) \[14,15\]. This corresponds to \(-t \approx 7-10 \) GeV\(^2\). If the present measurements are extended to values of \( Q^2 \) such as these, and if the constant nature of the ratio \( QF_2(Q^2)/F_1(Q^2) \) is maintained, one could be able to seek an explanation of the large value of the analyzing power in terms of the non-perturbative proton wave function.

ACKNOWLEDGMENTS

This work was supported by the Department of Energy under Grant No. DE-FG03-97ER41014. We wish to acknowledge helpful conversations with S.J. Brodsky, T. Frederico, D.S. Hwang and H.J. Weber, and to thank O. Gayou for sending us the experimental data.

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