Dilaton Brane Cosmology with Second Order String Corrections and the Cosmological Constant

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Abstract

We consider, in five dimensions, the effective action from heterotic string which includes quantum gravity corrections up to \((\alpha')^2\). The expansion, in the string frame, is in terms of \(|\alpha' R|\), where \(R\) is the scalar curvature and uses the third order Euler density, next to the Gauss-Bonnet term. For a positive tension brane and infinite extra dimension, the logarithmic class of solutions is less dependent from fine-tuning than in previous formulations. More importantly, the model suggests that in the full non-perturbative formulation, the string scale can be much lower than the effective Planck mass, without the string coupling to be vanishingly small. Also, a less severe fine-tuning of the brane tension is needed.

KEYWORDS: Branes, Cosmology.

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1 Introduction

Recent developments in string theory suggest that matter and gauge interactions may be confined on a brane, embedded in a higher dimensional space (bulk), while gravity can propagate into the bulk (for reviews see [1], [2]). Within this context several toy models have been constructed to address such issues as the hierarchy and cosmological constant problems [3]. In particular, the large hierarchy between the Standard Model and Planck scales could be explained for an observer on a negative tension flat brane, if the extra dimension was taken to be compact [4]. The possibility of a large non-compact dimension was realized in [5], while it was shown in [6] that warping of five-dimensional space could lead to localization of gravity on the brane, even though the size of the extra dimension was of infinite proper length.

In [7], [8] a simple, interesting alternative model has been considered, where a bulk scalar field $\phi$ is coupled to the brane tension $T_{br}$. This is the all-loop contribution to the vacuum energy density of the brane, from the Standard Model fields. For the 4D cosmological constant problem considered there, solutions of the field equations were found, which localize gravity, but possess naked singularities at finite proper distance. This proper distance is given by $y_c = 1/\kappa_{(5)}^2$, where the five and four-dimensional Planck scales $\kappa_{(5)}^2 = M_{(5)}^{-3}$, $\kappa_{(4)}^2 = M_{(4)}^{-2}$ are related by

$$T_{br} = \frac{\kappa_{(4)}^2}{\kappa_{(5)}^2} = \frac{M_{(5)}^6}{M_{(4)}^2}.$$  \hfill (1)

Then if we momentarily identify the 4D cosmological term with the brane tension $\lambda = T_{br} \sim (1 TeV)^4 = (10^8 GeV)^4 \sim 10^{-64} M_{(4)}^4$ we obtain

$$M_{(5)} \simeq 10^8 GeV, \quad y_c \simeq 1 mm,$$ \hfill (2)

which is acceptable by present day experiments.

It was soon realized that the bulk action should also contain the classical Gauss-Bonnet (GB) term

$$\mathcal{L}_{GB} = R^2 - 4R_{ab}R^{ab} + R_{abcd}R^{abcd},$$ \hfill (3)

which is the leading quantum gravity correction, and the only to provide second order field equations. Some of the early works on the GB gravity include [9], while the corresponding brane cosmology has been studied, among others, in [10]. The corresponding generalized junction conditions appeared in [11].

The GB term is the first in an infinite series of Euler densities that appear in the generalized Lovelock Lagrangian [12]. The next order term is the third order, given by

$$\mathcal{L}_{(3)} = 2R_{abcd}R_{cdef}R_{ab}^{ef} + 8R_{ab}^{ef}R_{cde}^{ef} + 24R_{abcd}R_{cdef}R_{ab}^{ef} + 3RR_{abcd}R_{cdab} + 16R_{ab}R_{bc}R_{ca} - 12RR_{ab}R_{ba} + R^3.$$ \hfill (4)
It is the only combination of three contractions of the Riemann tensor, that is linear in the second derivatives of the metric tensor [13] and linearization about flat Minkowski space introduces no propagator corrections (i.e., no ghosts) [14]. Discussion of cosmological problems with cubic and/or quartic Lagrangians, in the framework of superstring theories, appeared in [15]. Early work on global symmetries of higher order gravity theories appeared in [16] while solutions without bulk cosmological constant appeared in [17].

The effective gravitational equations on the brane have been studied in [18], while the inclusion of the induced gravity model has been studied, among others, in [19]. The effective equations on the brane, in the presence of the GB contribution in the bulk, were studied in [20], whereas astrophysical implications of higher order gravity models on branes have been analyzed in [21]. The cosmological perturbations in brane models have been examined in [22], while the short scale modifications of Newton’s law have been dealt with in [23].

In [24] it was shown that in general self-tuning is generic in theories with at most two branes, or a single brane with orbifold boundary conditions. Also it was shown that localization of gravity (namely finiteness of the effective Planck mass) and infinite extra dimension is only consistent with fine-tuning of the parameters of the theory. A no-go theorem appeared in [25]. The problem of fine-tuning (i.e., the adjustment of Lagrangian parameters as for example the relation of bulk and brane cosmological constants in RS models) was shown not to be present in [26]. Solutions and stability of them which are potentially self-tuning (i.e., need for adjustment of integration constants rather than Lagrangian parameters, as for example the values of the potential $V(\phi_0)$ and its derivative $V'(\phi_0)$ on the brane) where shown to exist. The issue of self-tuning was studied, for example, in [27].

In this paper we consider the tree-level effective action from heterotic string theory, in the string frame (Eq. (5) below). This has been derived in [28], [31], [32] for a generic spacetime dimension $D$ and it preserves the general symmetries of the underlying theory such as general covariance. In the critical dimension $D = 10$ the bulk cosmological constant vanishes. This action has been used by a number of authors ([29], [30], [33], [34]) also for $D = 5$ to study braneworld models. This is in contrast to the usual models, where in the “Einstein frame” the higher order corrections (such as the Gauss-Bonnet term) enter without the exponential dilaton dependence. This form besides the fact that it stems from string theory, offers a new way to examine ”self-tuning” mechanisms ([27], [29]). So we assume that the higher order ($\alpha'$)-corrections enter in this action in powers of $|\alpha' R|$, where $R$ is the scalar curvature [29]. This is reasonable because, in the absence (for simplicity) of gauge field contributions the only length scale of the theory is $\alpha' = l_s^2$. Exploring this possibility we show that

- It is free from naked singularities, the curvature scalar being regular everywhere. Also the effective Planck mass calculated is finite, while the extra dimension can be infinite.
The solutions do not have a smooth limit as the higher order terms are set to zero, so these are genuine \( O(\alpha'^2) \)-classes of solutions, and more importantly,

- The string scale is much lower than the effective Planck mass, the string coupling assumes higher values, as higher order terms are included, and this is true for a continuous range of parameters, namely the need for fine-tuning is much less severe.

Our line of reasoning and conventions follow mainly those of [29]. All calculations are lengthy and have been performed with great care.

## 2 The Action

The action in the string frame, from the string tree level effective action, must have the same dilaton dependence on the Einstein-Hilbert, Gauss-Bonnet and third order terms with respect to the curvature. This is also true due to dimensional arguments because the string expansion parameter \( \alpha' = l_s^2 = M_s^{-2} \) is the only length scale of the theory and quantum gravity corrections appear in this action as powers of \( |\alpha' R| \) [29].

So, in order to explore the results from this point of view and ignoring any gauge field contributions for simplicity, we must have ([31], [32], [34])

\[
S^{(\text{string})}_{\text{bulk}} = \frac{M_s^{D-2}}{2} \int d^D x \sqrt{-g} e^{-2\Phi} \left\{ \bar{R} + 4(\nabla \Phi)^2 + \lambda_0 (\alpha') [\bar{L}_{\text{GB}} + \ldots] + \right.
\]

\[
+ \lambda_1 (\alpha')^2 [\bar{L}_3 + \ldots] - \frac{2(D-10)}{3\alpha'} + O(\alpha'^3) \right\}.
\]

(5)

Here \( \lambda_0 = 1/4, 1/8, 0 \) for the bosonic, heterotic and type II theories [28], [31]. The constant \( \lambda_1 \) would in principle stem from string theory and the full contribution of the third order term is with a value of the order of unity.

One must apply a conformal transformation \( \bar{g}_{\mu\nu} = e^{2\Phi} g_{\mu\nu} \), to bring the action in the Einstein frame, where \( \zeta = 4/(D - 2) \) and the metric \( g_{\mu\nu} \) is

\[
ds^2 = e^{2A(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2,
\]

(6)

which assures four-dimensional Poincare invariance.

For the metric of Eq. (6) we have \( R = -8A'' - 20(A')^2 \) for the scalar curvature, \( L_{\text{GB}} = 24(A')^2 [4A'' + 5(A')^2] \) for the Gauss-Bonnet term and \( L_3 = -24(A')^4 [4A'' + 15(A')^2] \) for the third order term. The corresponding quantities in the string (barred) frame are given by

\[
\bar{R} = e^{-\zeta \Phi} [-8A'' - 4\zeta A'' + 16\zeta A' \Phi' - 3\zeta^2 (\Phi')^2 - 20(A')^2]
\]

\[
\bar{L}_{\text{GB}} = 24 e^{-2\zeta \Phi} (A' + \frac{1}{2} \zeta \Phi')^2 [4A'' + 2\zeta A'' + 5(A')^2 + \frac{1}{4} \zeta^2 (\Phi')^2 + 3\zeta A' \Phi']
\]

\[
\bar{L}_3 = -24 e^{-3\zeta \Phi} (A' + \frac{1}{2} \zeta \Phi')^4 [4A'' + 2\zeta A'' + 15(A')^2 + \frac{11}{4} \zeta^2 (\Phi')^2 + 13\zeta A' \Phi'].
\]

(7)
We will eventually resort to $D = 5$ dimensions, so $\zeta = 4/3$.

The bulk action, in the Einstein frame, now becomes

$$S^{(Einstein)}_{\text{bulk}} = \frac{M_s^3}{2} \int d^Dx \sqrt{-g} \left\{ R - \zeta(\nabla \Phi)^2 + \lambda_0 (\alpha') e^{-\zeta \Phi} [\mathcal{L}_{GB} + c_2 \zeta^2 (\nabla \Phi)^4] + \lambda_1 (\alpha')^2 e^{-2\zeta \Phi} [\mathcal{L}_{(3)} + c_3 \zeta^3 (\nabla \Phi)^6] - \frac{2(D-10)}{3\alpha'} e^{\zeta \Phi} + \mathcal{O}(\alpha'^3) \right\}.$$  \hspace{1cm} (8)

Here $c_2 = (D-4)/(D-2)$ ([30], [32]) is the coefficient of the fourth order dilaton derivative, which is required by string theory arguments. We assume that the same is true for the third order term and so $c_3$ is a $D$-dependent constant. Its precise value would in principle stem from string theory, however the results presented here do not depend essentially on its exact value.

Finally, on the brane one can in general consider both the induced gravity term, a higher order Gauss-Bonnet term and the four-dimensional cosmological term

$$S_{\text{brane}} = M_s^3 \int d^4x \sqrt{-h} \{ (4) R + b_0 (\alpha')^4 R_{GB} - \lambda(\Phi) \}. \hspace{1cm} (9)$$

Here $b_0$ is a dimensionless parameter, as required from dimensional arguments. For the case of the conformally flat metric in Eq. (6) the first two terms cancel. In general however the induced gravity terms for a generic four-dimensional metric can yield interesting consequences for the junction conditions and for the behaviour of the bulk solution very close to the brane. This possibility requires analysis beyond the scope of this work. The $\lambda(\Phi) = (T/M_s^3) e^{\chi \Phi}$ and $\chi = 5/3$ for a Dirichlet brane, while $\chi = 2/3$ for a Neveu-Schwarz brane. Also a gravitational wave on the brane can for example be considered [34], [37].

### 3 The Equations of Motion

Substituting Eq. (7) into Eq. (5) and taking care to include the terms $(\nabla \Phi)^4$, $(\nabla \Phi)^6$ along with Eq. (9), the total action, in the Einstein frame, becomes

$$S^{(Einstein)}_{\text{tot}} = \frac{M_s^3}{2} \int d^4x dy e^{4\Phi} \left\{ -8A'' - \frac{16}{3} \Phi'' - \frac{64}{3} A' \Phi' - \frac{4}{3} (\Phi')^2 - 20(A')^2 + \lambda_0 (\alpha') [e^{\zeta \Phi} \mathcal{L}_{GB} + c_2 \zeta^2 e^{-\zeta \Phi} (\Phi')^4] + \lambda_1 (\alpha')^2 [e^{\zeta \Phi} \mathcal{L}_{(3)} + c_3 \zeta^3 e^{-2\zeta \Phi} (\Phi')^6] - 2\Lambda e^{\zeta \Phi} + \mathcal{O}(\alpha'^3) + 2^{(4)} R \delta(y) - 2\lambda(\Phi) \delta(y) \right\}. \hspace{1cm} (10)$$
This is the obvious generalization of the action used in [29]. Variation is a tedious but straightforward procedure. Varying with respect to $A$ gives

$$6A'' + 12(A')^2 + \zeta(\Phi')^2 + 2\Lambda e^{\xi\Phi} + 2\lambda(\Phi)\delta(y) - \lambda_0(\alpha') e^{-\xi\Phi} \left[ c_2 \zeta^2(\Phi')^4 + \right. $$

$$+ 24A'(A'A'' + (A')^3 - 2\zeta \Phi' A'' - \zeta A' \Phi'' - 3\zeta \Phi'(A')^2 + \zeta^2 A' (\Phi')^2 \bigg] - \lambda_1(\alpha')^2 e^{-2\xi\Phi} \left[ c_3 \zeta^3(\Phi')^6 + G_{(3)} \right] = 0.$$ (11)

When $\lambda_1 = 0$ it reduces to Eq. (14) of [29]. Varying with respect to $\Phi$ we exactly obtain

$$2\zeta \Phi' + 8\zeta \Phi' A' - 2\Lambda \zeta e^{\xi\Phi} - 2\frac{d\lambda(\Phi)}{d\Phi} \delta(y) - $$

$$- \lambda_0(\alpha') e^{-\xi\Phi} \left[ c_2 \zeta^2 (\Phi')^2 (12\Phi'' + 16\Phi' A' - 3\zeta(\Phi')^2) + \right.$$ $$(4A'' + 5(A')^2) \bigg] - \lambda_1(\alpha')^2 e^{-2\xi\Phi} \left[ c_3 \zeta^3(\Phi')^4 (30\Phi'' + 24\Phi' A' - 10\zeta(\Phi')^2) + F_{(3)} \right] = 0.$$ (12)

When $\lambda_1 = 0$ it reduces to Eq. (16) of [29]. The precise form of the functions $G_{(3)} = G_{(3)}(A', \Phi')$ and $F_{(3)} = F_{(3)}(A', \Phi')$ is given in Appendix I.

If we write the generalized Einstein’s equations we also obtain a first order equation, which is nothing more than the $yy$-component and it is the constraint equation. This is given by

$$12(A')^2 - \zeta(\Phi')^2 + 2\Lambda e^{\xi\Phi} + $$

$$+ \lambda_0(\alpha') e^{-\xi\Phi} \left[ 3c_2 \zeta^2(\Phi')^4 - 24(A')^3 (A' - 4\zeta\Phi') \right] + $$

$$+ \lambda_1(\alpha')^2 e^{-2\xi\Phi} \left[ 5c_3 \zeta^3(\Phi')^6 + H_{(3)} \right] = 0,$$ (13)

where the function $H_{(3)}$ is

$$H_{(3)}(A', \Phi') = - 24 \cdot 59(A')^6 - 24 \cdot 183 \zeta(A')^5 \Phi' - 54 \cdot 105 \zeta^2(A')^4(\Phi')^2 - $$

$$- 30 \cdot 130 \zeta^3(A')^3(\Phi')^3 - \frac{15 \cdot 201}{2} \zeta^4(A')^2(\Phi')^4 + \frac{63 \cdot 113}{10} \zeta^5(A')(\Phi')^5 - $$

$$- \frac{71 \cdot 303}{40} \zeta^6(\Phi')^6.$$ (14)

When $\lambda_1 = 0$ this reduces to Eq. (15) of [29].
Taking an appropriate combination of Eqs. (11), (12) and (13) we obtain, after integration, the first order expression

\[ e^{4A} \left( 2\zeta \Phi' + 3\zeta A' - 4\lambda_0(\alpha') e^{-\xi \Phi} [c_2 \zeta^2 (\Phi')^3 - 3\zeta^2 (A')^2 \Phi' + 9\zeta (A')^3] - \lambda_1(\alpha')^2 e^{-2\xi \Phi} (6c_3 \zeta^3 (\Phi')^5 + I_{(3)}) \right) = C, \] (15)

where

\[ I_{(3)}(A', \Phi') = -18 \cdot 37\zeta (A')^5 - 9 \cdot 191\zeta^2 (A')^4 \Phi' - 9 \cdot 197\zeta^3 (A')^3 (\Phi')^2 - \frac{9 \cdot 203}{2} \zeta^4 (A')^2 (\Phi')^3 - \frac{9 \cdot 209}{8} \zeta^5 (A')^4 (\Phi')^4 - \frac{63 \cdot 193}{20} \zeta^6 (\Phi')^5, \] (16)

and \( C \) is an integration constant. When \( \lambda_1 = 0 \) it reduces to Eq. (17) of [29]. Finally we obtain the junction conditions for a \( \mathbb{Z}_2 \)-symmetric brane configuration located at \( y = 0 \). They can be directly deduced from Eqs. (11) and (12), by integrating on an \( \epsilon \)-interval around the brane. The second derivatives in the first brackets of the cubic terms (i.e., terms containing \( A'' \) in \( F_{(3)}, G_{(3)} \)) are written as total derivatives and after partial integration give a term on the brane and terms in the bulk that exactly cancel the terms in the second brackets (i.e., those containing \( \Phi'' \)). Therefore we have

\[ \left[ 3A' + 4\lambda_0(\alpha') e^{-\xi \Phi} (3\zeta \Phi' (A')^2 - (A')^3) + 9\lambda_1(\alpha')^2 e^{-2\xi \Phi} J_{(1)} \right]^+_{-} = -\lambda(\Phi), \] (17)

where

\[ J_{(1)} = -\frac{2 \cdot 59}{5} (A')^5 - 61\zeta (A')^4 \Phi' - 63\zeta^2 (A')^3 (\Phi')^2 - \frac{65}{2} \zeta^3 (A')^2 (\Phi')^3 - \frac{67}{8} \zeta^4 A' (\Phi')^4 - \frac{69}{80} \zeta^5 (\Phi')^5. \] (18)

Also

\[ \left[ \zeta \Phi - 2\lambda_0(\alpha') e^{-\xi \Phi} (c_2 \zeta^2 (\Phi')^3 + 8\zeta (A')^3) - 3\lambda_1(\alpha')^2 e^{-2\xi \Phi} (c_3 \zeta^3 (\Phi')^5 + J_{(2)}) \right]^+_{-} = \frac{d\lambda(\Phi)}{d\Phi}, \] (19)

with

\[ J_{(2)} = \frac{6 \cdot 122}{5} (A')^5 + 6 \cdot 63\zeta^2 (A')^4 (\Phi') + 6 \cdot 65\zeta^3 (A')^3 (\Phi')^2 + 3 \cdot 67\zeta^4 (A')^2 (\Phi')^3 + \frac{3 \cdot 69}{4} \zeta^5 A' (\Phi')^4 + \frac{3 \cdot 71}{40} \zeta^6 (\Phi')^5. \] (20)
4 Logarithmic Classes of Solutions in the Bulk

We consider Eqs. (13) and (15), taking first $C = 0$ and use the ansatz

$$A(y) = A_0 + x\ln\left(1 + \frac{|y|}{y_*}\right), \quad \Phi(y) = \Phi_0 - \frac{2}{\zeta} \ln\left(1 + \frac{|y|}{y_*}\right). \quad (21)$$

If we had taken $C \neq 0$ we would be forced to set $x = 1/4$. The solutions to be presented here are genuine $O(\alpha'^2)$-classical solutions, without smooth Einstein limit. We abbreviate $\alpha := \lambda_0(\alpha')$ and $\tilde{\alpha}^2 := \lambda_1(\alpha')^2$. Then we obtain, from Eq. (15), the constant $y_*$ in terms of the integration constant $\Phi_0$ as

$$4\alpha e^\Phi y_*^2 = \frac{2(4 - 3\zeta x)\zeta^3}{(8c_2\zeta^2 - 9\zeta^4x^3 - 6\zeta^4x^2)} \left[1 \pm \sqrt{\Delta}\right], \quad (22)$$

where

$$\Delta(x) := 1 + \frac{\tilde{\alpha}^2}{4\alpha^2 (8c_2\zeta^2 - 9\zeta^4x^3 - 6\zeta^4x^2)} \left[-\frac{192c_3}{\zeta^2} + I_3(x, -\frac{2}{\zeta})\right]. \quad (23)$$

Since $\Phi_0$ must be real, this will restrict the values of $x$, as follows below. Also the bulk cosmological constant is given, from Eq. (13), by

$$2\alpha \Lambda(x) = \left(\frac{4\alpha}{e^{\Phi_0}y_*^2}\right) \left(1 - \frac{3x^2}{\zeta}\right) + \left(\frac{4\alpha}{e^{\Phi_0}y_*^2}\right)^2 \left[-\frac{3c_2}{\zeta^2} + \frac{3}{2}x^4 + 12x^3\right] + \left(\frac{4\alpha}{e^{\Phi_0}y_*^2}\right)^3 \left(\frac{\tilde{\alpha}}{\alpha}\right)^2 \left[-\frac{5c_3}{\zeta^3} - \frac{1}{64}H_3(x, -\frac{2}{\zeta})\right]. \quad (24)$$

One can obtain the effective Planck mass up to second order string corrections in a standard manner. If we had consider instead of Eq. (6) the metric

$$ds^2 = e^{2A(y)}g^{(4)}_{\mu\nu}dx^\mu dx^\nu + dy^2 \quad (25)$$

and had substituted

$$R_{\mu\nu\rho\sigma} = e^{2A}R_{\mu\nu\rho\sigma}^{(4)} + e^{4A}(A')^2(g^{(4)}_{\mu\nu}g^{(4)}_{\rho\sigma} - g^{(4)}_{\nu\rho}g^{(4)}_{\mu\sigma}), \quad (26)$$

into the Gauss-Bonnet and third order terms, the integrated coefficient of the four-dimensional scalar curvature $R^{(4)}$ (which is linear in the second derivatives of the metric tensor function $A(y)$) would give the higher order contributions to the effective Planck mass, as perceived by a four dimensional observer [26], [33]. Doing this in a careful manner we obtain, up to $O(\alpha'^2)$ order,

$$M_{Pl}^2 = M_s^3 \int_0^{y_c} dy e^{2A(y)} \left[1 - 4\alpha e^{-\Phi}(2A'' + 3(A')^2) + 48\tilde{\alpha}^2 e^{-2\Phi}(A')^2 (12A'' + 11(A')^2)\right]. \quad (27)$$
It follows that

$$M^2_{Pl} = M^3_{P} \frac{y_*}{(2x+1)} \left[ \left( 1 + \frac{y}{y_*} \right)^{2x+1} \right]^{y_c} \cdot \left[ 1 - \frac{4\alpha}{y_*^2 e^{2\Phi_0}} (-2x + 3x^2) + \frac{48\tilde{\alpha}^2}{y_*^4 e^{2\Phi_0}} x^2 (-12x + 11x^2) \right].$$

(28)

We therefore see that when \( x \geq -1/2 \) we can have a finite effective Planck mass if \( y_* < 0 \), so that \( y_c = |y_*| \) and we have a compact proper distance [33]. On the other hand when \( y_* < 0 \) the curvature scalar \( R = -8A^\prime - 20(A^\prime)^2 \propto (|y| + y_*)^{-2} \) diverges as \( y \to y_* \). So we must choose \( y_* > 0 \), for an infinite \( y \)-direction without curvature singularities and \( x \geq -(1/2) \) in order to assure finite Planck mass.

We now choose \( \zeta = (4/3) \), \( c_2 = (1/3) \) and compute the term \( I_3(x,-2/\zeta) \). Then

$$\frac{4\alpha}{y_*^2 e^{2\Phi_0}/3} = \frac{4(1-x)}{(1-6x^3-4x^2)} \left[ \frac{1}{1 \pm \sqrt{\Delta}} \right],$$

(29)

where

$$\Delta = 1 + \frac{\tilde{\alpha}^2}{\alpha^2} \frac{4^5(1-x)}{3^2(1-6x^3-4x^2)} I_3(x),$$

(30)

with

$$I_3(x) = 37x^5 - 191x^4 + 2 \cdot 197x^3 - 2 \cdot 203x^2 + 209x - \frac{28}{5} \cdot 193 + \frac{9}{2} c_3.$$  

(31)

If we momentarily switch off the second order corrections (\( \tilde{\alpha} = 0 \)) the allowed values of \( x \) (ensuring the positivity of the l.h.s. of Eq. (29)) become \( x > 1 \) or \( x \leq 0.4 \), as previously noted [29].

\( (+) \) \textbf{Branch}: Here we observe the interesting fact that

$$I_3'(x) = 185(x-1)^3 \left( x - \frac{209}{185} \right),$$

(32)

whereas

$$I_3(1) = I_3 \left( \frac{209}{185} \right) = \frac{9}{2} c_3 + 43 - \frac{28}{5} \cdot 193$$

$$I_3(0.4) = \frac{9}{2} c_3 + 39.345 - \frac{28}{5} \cdot 193.$$  

(33)

So for example, we see that for \( c_3 \geq 231.3 \) then \( I_3(0.4) > 0 \) and also \( I_3 \left( 209/185 \right) > 0 \). Since \( I_3'(x) > 0 \) for \( x \geq 209/185 \) and for \( x \leq 0.4 \) we obtain that \( I_3(x) > 0 \) and \( \Delta(x) > 0 \). For this range of \( x \) therefore, the \( O(\alpha^\prime) \) solutions derived in [29] are preserved. It is therefore evident that higher order corrections, in general modify the allowed \( x \)-range, due mainly to algebraic constraints.
We move now to the general study. The (+) branch gives positive l.h.s. in Eq. (29) when $\Delta(x) > 0$ and $x \in (-\infty, 0.4) \cup (1, +\infty)$. The roots of $\Delta(x) = 0$ depend also on $\lambda_1$. Since we deal with the tree level, effective action for the heterotic string $\lambda_0 = (1/8)$, so $(\tilde{\alpha}/\alpha) = (\lambda_1/\lambda_0^2) = 64\lambda_1$. The curves $c_3(x) = C(x; \lambda_1)$ (which originate from $\Delta(x) = 0$) on the $(x, c_3)$-plane, separate the allowed $x$-regions (where $\Delta(x) > 0$) from the forbidden $x$-regions (where $\Delta(x) < 0$), for a given $c_3$. We obtain

$$\frac{9}{2} c_3(x) = \frac{3^2}{4^5} \frac{1}{64\lambda_1} (x - 0.4) (6x^2 + 6.4x + 2.56) - 37x^5 + 191x^4 - 394x^3 + 406x^2 - 209x + \frac{28}{5} \cdot 193.$$  \hspace{1cm} (34)

In the limit $\lambda_1 \to 0$ it becomes singular, so what we present here is a genuine-$\mathcal{O}(\alpha'^2)$ classical solution, not having a smooth limit to a lower order solution.

In Fig. 1 we plot the function $c_3(x)$. For the range of $0.01 \leq \lambda_1 \leq 1$ there exists no significant difference of the plotted curves, as occurred from numerical analysis, so we used for convenience $\lambda_1 = 1$.

![Figure 1](image_url)

Figure 1: The dependence $c_3 = c_3(x)$ for $x \leq 0.4$. The allowed $x$-range is to the right of the curve, for a given $c_3$.

As an example for the case of $c_3 \approx 500$ the allowed values of $x$ (those with $\Delta(x) > 0$) from precise numerical computation are $-0.979 \leq x \leq 0.4$. Of course we have to add to this the requirement $-(1/2) \leq x$ which comes from the requirement of finite effective Planck mass.
In the same way for $c_3 \simeq 100$ the allowed values are for $x \geq 2.7719$ from numerical methods, shown in Fig. 2. As it is evident from the figures, as $c_3$ increases the allowed $x$–range increases.

Figure 2: The function $c_3 = c_3(x)$ for $x > 1.0$. The allowed $x$–region is again to the right of the curve, for a given $c_3$.

All the classes of solutions are $\alpha'$–solutions, because we have the additional argument that the cases $\mathcal{C} = 0$ and $\alpha' = 0$ are incompatible, from Eq. (15).

The parameter $x$ is given implicitly, in terms of the bulk cosmological constant as

$$2\alpha \Lambda(x) = \left( \frac{4\alpha}{y_z^2 e^{4\Phi_0/3}} \right) \frac{3}{4} (1 - 4x^2) + \left( \frac{4\alpha}{y_z^2 e^{4\Phi_0/3}} \right)^2 \left[ -\frac{9}{16} + \frac{3}{2}x^4 + 12x^3 \right] +$$

$$+ 64\lambda_1 \left( \frac{4\alpha}{y_z^2 e^{4\Phi_0/3}} \right)^3 \left[ -\frac{135}{64} c_3 - \frac{1}{64} H(3)(x) \right], \quad (35)$$

with

$$H(3)(x) = -24 \cdot 59x^6 + 48 \cdot 183x^5 - 216 \cdot 105x^4 +$$

$$+ 240 \cdot 130x^3 - 120 \cdot 201x^2 + \frac{63 \cdot 113}{5} x -$$

$$- \frac{568 \cdot 303}{5}. \quad (36)$$

In Fig. 3 the bulk cosmological constant for $c_3 \simeq 500$ and $\lambda_1 = 1$ is plotted.
Figure 3: The bulk Cosmological constant $\alpha \Lambda (x)$ for $-0.979 \leq x \leq 0.4$.

We can also obtain the brane tension, from Eq. (17), as

$$
\frac{T}{M_s^3} = -\frac{2}{y_* e^{\zeta_0}} \left[ 3x + \frac{4\alpha}{y_*^2 e^{\phi_0}} (-6x^2 - x^3) + \frac{9\tilde{\alpha}^2}{y_*^4 e^{2\zeta_0}} J_{(1)}(x, -\frac{2}{\zeta}) \right],
$$

while we must have $\chi = \zeta/2 = 2/3$ as required for a Neveu-Schwarz brane.

Now we are ready to make the order of magnitude estimates that follow from the above considerations and constitute our main arguments for this section. For $x \geq -(1/2)$ up to the allowed value $x \leq 0.4$ and for $c_3 \simeq 500$, from Fig. 3 we see that with high precision $2\alpha \Lambda (x) \simeq -3 \cdot 10^4$. Then using Eq. (36) with $x \simeq 0$ we can estimate the term

$$
l := \frac{4\alpha}{y^2_* e^{\phi_0}} \simeq 1,
$$

because we get $-3 \cdot 10^4 \simeq (3/4)l - (9/16)l^2 - 3.308 \cdot 10^4 l^3$. This is solved for $l \simeq 1$. The same would be obtained for $x \simeq -(1/2)$, because now we get $-3 \cdot 10^4 \simeq -2l^2 - 2 \cdot 10^4 l^3$, so again $l \simeq 1$. In both cases we have computed the term of Eq. (37).

Now we use $l$ into Eq. (28) with $x \simeq -(1/2)$. We obtain

$$
y_* \propto \frac{M_P^2}{M_s^3} \left( 1 + \frac{1}{-1 + 48 \cdot 64 \cdot 2.1875 \lambda_1} \right).
$$

The number 48 comes from the geometric properties of the model, namely from the contribution of the third order term $L_{(3)}$ to the effective Planck mass. It is therefore
expected that higher order Euler densities when considered in this $\alpha'$-expansion, due to the increased Riemann contractions, will give large numbers. The number 64 comes from the full contribution of these higher order terms (i.e., when $\lambda_1 = 1$) into the string frame effective action, Eq. (5). Finally the number 2.1875 comes from the increased powers of the $x$ parameter (namely the brane tension) that will appear as progressively higher order terms are included in the action [29]. Also since the allowed $x$-ranges may include $x$-intervals with large negative numbers, as higher order terms enter, this number is expected to increase. This is due to the combination $x^2(-12x + 11x^2)$, in Eq. (28). As one includes progressively higher order terms in the sense described here, and allows for finite $y$-direction (so the requirement $x \geq -(1/2)$ is relaxed), then allowed $x$-region with large negative values are expected to contribute with larger factors, as above.

The first part of our argument is therefore now more clear. As we include higher order terms in Eq. (28) the requirement of $M_s \ll M_{Pl}$ appears to be fulfilled more easily, without severe fine-tuning for $x$, due to the increased contribution of the term in the second pair of brackets.

Now the string coupling, from Eq. (22) is estimated as $e^{\Phi_0} \sim (4\alpha/y_\alpha^2)^{3/4}$, so

$$g_s = e^{\Phi_0} \sim \frac{M_s^3}{M_{Pl}^3}[-1 + 6.72 \cdot 10^3]^{3/2}. \tag{40}$$

In [29] it was correctly observed that when $x \simeq -1/2$ we can have a large string coupling constant $g_s = e^{\Phi_0} \sim 1$ without having to abandon the requirement of $M_s \ll M_{Pl}$. However this requires a severe fine-tuning of the parameters of the theory, namely of the brane tension, through $x$. Here however we see that the inclusion of higher order quantum gravity correction can induce this result without severe fine-tuning. Namely for a whole continuous range of $(-1/2) \leq x \leq 0.4$ we can have $M_s \ll M_{Pl}$ while the string coupling increases as higher order $\alpha'$ corrections are taken into account.

So, as it was defined in [29], we consider that the solutions presented here do not suffer from the strict fine-tuning problem in the following sense: Namely that for a whole range of the $x$-parameter which is connected to the brane tension, through Eq. (38) and the bulk cosmological constant through Eq. (36), one obtains $M_s \ll M_{Pl}$.

In our point of view this is an indirect support to the claim that the presence of these higher order quantum gravity corrections in a sense necessitate the inclusion of higher order quantum loop-corrections, as well, beyond the tree level! An action originating from non-perturbative quantum gravity theory, that is not yet available, may solve the fine-tuning problem in a natural way.

Finally from Eq. (38) we have that

$$T \simeq M_s^4, \tag{41}$$

so if we assume that the brane tension $T \sim (1 TeV)^4 \sim 10^{-60} M_{Pl}^4$ and the string coupling, from Eq. (41) still remains small, however several (four) orders of magnitude
larger than the value $g_s \sim e^{-45}$ obtained in [29]. However as we have just stated if all the quantum gravity and quantum loop corrections could be included, in a non-perturbative fashion, even for this, toy-model logarithmic, class of solutions the need for fine-tuning could be absent in order to have $M_s \ll M_{Pl}$.

5 Discussion

In this paper we have considered the tree-level, effective action of the heterotic string in five dimensions with expansion parameter, in the string frame, the combination $|\alpha' R|$. If one tries to explore, from this point of view, the consequences of incorporating higher order quantum gravity corrections, then some interesting facts occur. They can be summarized as follows: Even for the simple, toy model, class of logarithmic solutions the string scale may become much lower than the effective Planck mass without need for severe fine-tuning of the brane tension. This is due to the increased contribution of the Riemann contractions, in the Euler densities. Thus the hierarchy $M_s \ll M_{Pl}$ does not depend essentially on the brane tension through the parameter $x$. Also the string coupling $g_s = \exp(\Phi_0)$ increases by some orders of magnitude for each contributing term. In [29] it was pointed that close to the brane, the string coupling is expected to increase, so it destabilizes the classical solutions by quantum loop corrections. While this is true, we prefer to view this as an advantage rather than a disadvantage. This may point to the fact that higher quantum gravity corrections inevitably require also quantum loop corrections. Thus in the hypothetical non-perturbative quantum gravity theory the issue of necessity of fine-tuning, for braneworld solutions, may be completely absent. In this context the four-dimensional cosmological constant, through the freedom of its brane tension component, can be null in an more simple way. The class of solutions presented here offers, from our point of view an additional indirect argument for these claims.

The class of solutions presented here is an extension of solutions that appeared in [26], [29]. It shares the same basic qualitative behaviour as those of [29]. From Eq. (28) we have finite effective Planck mass for $x < -1/2$ namely for the brane tension, if $0 < y < y_c := y_*$. Also the spacetime manifold is regular without curvature singularities, since the curvature scalar $R \propto (|y| + y_*)^{-2}$ is finite. However the introduction of the third order term, through $\lambda_1 (\alpha')^2$, in the action of Eq. (10) results in a new class of solutions.

First this is a genuine $O((\alpha')^2)$ class of solutions because the limit $\lambda_1 \to 0$ is singular in Eq. (34). It cannot therefore be mapped to solutions from the lower order approximation.

Second the $x-$interval breaks into allowed regions from Eq. (30) due to the requirement of $\Delta(x) \geq 0$. For all values of the constant $\lambda_1 \in (0.01, 1)$ which controls the contribution of the third order term in the action, Eq. (10), the curve $\Delta(x) = 0$
gives the dependence of the $c_3 = c_3(x)$ on the brane tension $x$ (Eq. (34) and Figs. 1, 2). This is comparable to the constant $c_2 = (D - 4)/(D - 2)$ which was calculated in [32] and controls the contribution of the dilaton derivative terms in the action, for the lower order term. Here we do not have an argument for its exact value. However we found numerically that for the allowed values of $x < -1/2$, from Fig. 1 and Eqs. (36), (39), that the relation $M_s \ll M_{Pl}$ through Eq. (28), holds without the need for an exact fine-tuning for $x$. So we conjecture that higher order corrections through Euler densities can satisfy this hierarchy more naturally without the need to fine-tune the brane tension, as was for example necessary in [29], with $x \simeq -1/2$.

Third the string coupling was calculated to increase about four orders in Eq. (41) through the introduction of the higher order correction. The string coupling enters at the vertices when loop diagrams are included. So when it increases one must consider such diagrams in a quantum corrected action. This may point to the conclusion that one cannot obtain a fully consistent picture, unless the successive quantum gravity corrections are modified by quantum loop-corrections as well.

Fourth and most important although from Eq. (35) it seems that there exists a fine-tuning between the bulk cosmological constant and $x$, inspection of Fig. 3 shows that for the range of $x$ studied here (namely $-0.5 \leq x < 0.4$) and the choice of the other parameters, the bulk cosmological constant assumes an almost constant value, irrespective of $x$. So the introduction of the third order Euler density gives a less severe dependence on fine-tuning.

There exists also a second class of solutions with non-zero constant $C$, where the relevant set of equations for the metric function $A(y)$ and the dilaton field $\Phi(y)$ admits solutions where the $y-$direction breaks into two allowed regions, one compact and the other infinite. In the first case we can have finite effective Planck mass without curvature singularities, while in the second case one cannot in general obtain localization of gravity on the brane. These require a detailed analysis which will appear elsewhere.

Finally it would be interesting to consider higher codimensional models i.e., with action of the form $\int d^5x d^N X$ [34] and in particular of co-dimension two, as for example in [35]. In this context the imprints of the cosmological perturbation spectra are of particular importance in order to decide on the form of the model that can be viable [36]. Work along these lines is in progress [37].


6 Appendix I

The functions appearing in Eqs. (11) and (12) are

\[ G_{(3)} = 36A'' \left\{ 59(A')^4 + 122\zeta \Phi'(A')^3 + \frac{189}{2}\zeta^2(\Phi')^2(A')^2 + \right. \\
\left. + \frac{65}{2}\zeta^3(\Phi')^3 A' + \frac{67}{16}\zeta^4(\Phi')^4 \right\} + \\
+ 18\zeta \Phi'' \left\{ 61(A')^4 + 126\zeta \Phi'(A')^3 + \frac{195}{2}\zeta^2(\Phi^2)(A')^2 + \right. \\
\left. + \frac{67}{2}\zeta^3(\Phi')^3 A' + \frac{69}{16}\zeta^4(\Phi')^4 \right\} + \\
+ 24 \left\{ 59(A')^6 + 111\zeta \Phi'(A')^5 + \frac{201}{4}\zeta^2(\Phi')^2(A')^4 - \\
\left. - \frac{59}{2}\zeta^3(\Phi')^3(A')^3 - \frac{579}{16}\zeta^4(A')^2(\Phi')^4 - \\
- \frac{201}{16}\zeta^5 A'(\Phi')^5 - \frac{97}{64}\zeta^6(\Phi')^6 \right\} \tag{42} \]

and

\[ F_{(3)} = -36\zeta A'' \left\{ 122(A')^4 + 252\zeta \Phi'(A')^3 + 195\zeta^2(\Phi')^2(A')^2 + \right. \\
\left. + \frac{67}{8}\zeta^3(\Phi')^3 A' + \frac{71}{8}\zeta^4(\Phi')^4 \right\} - \\
- 18\zeta^2 \Phi'' \left\{ 126(A')^4 + 260\zeta \Phi'(A')^3 + 201\zeta^2(\Phi^2)(A')^2 + \right. \\
\left. + \frac{69}{8}\zeta^3(\Phi')^3 A' + \frac{71}{8}\zeta^4(\Phi')^4 \right\} - \\
- 24\zeta \left\{ 170(A')^6 + 378\zeta \Phi'(A')^5 + \frac{591}{2}\zeta^2(\Phi')^2(A')^4 + \\
\left. + \frac{71}{8}\zeta^3(\Phi')^3(A')^3 - \frac{189}{8}\zeta^4(A')^2(\Phi')^4 - \\
- \frac{123}{8}\zeta^5 A'(\Phi')^5 - \frac{71}{32}\zeta^6(\Phi')^6 \right\}. \tag{43} \]

In five dimensions \( \zeta = (4/3) \) as before. Equations (11) and (12) could be used also for direct numerical study of the model.

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