The random first-order transition theory of active glass in the high-activity regime

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Abstract

Dense active matter, in the fluid or amorphous–solid form, has generated intense interest as a model for the dynamics inside living cells and multicellular systems. An extension of the random first-order transition theory (RFOT) to include activity was developed, whereby the activity of the individual particles was added to the free energy of the system in the form of the potential energy of an active particle, trapped by a harmonic potential that describes the effective confinement by the surrounding medium. This active-RFOT model was shown to successfully account for the dependence of the structural relaxation time in the active glass, extracted from simulations, as a function of the activity parameters: the magnitude of the active force \(f_0\) and its persistence time \(\tau_p\). However, significant deviations were found in the limit of large activity (large \(f_0\) and/or \(\tau_p\)). Here we extend the active-RFOT model to high activity using an activity-dependent harmonic confining potential, which we solve self-consistently. The extended model predicts qualitative changes in the high activity regime, which agree with the results of simulations in both three-dimensional and two-dimensional models of active glass.

Introduction

Active glass is a condensed phase of matter with internal sources of active (non-thermal) forces and extremely slow dynamics resembling in many ways the dynamics of passive glass-forming liquids. It has attracted a significant amount of interest as an abstract model for many biological systems [1–9], or synthetic soft active matter systems [10–13], and as a new challenge for non-equilibrium physics [14–19]. Realizations of active glass in simulations are mainly in the form of a dense aggregate of interacting particles that are self-propelled; the self-propulsion appears as random forces applied to each particle, characterized by a force amplitude \(f_0\) and a persistence time \(\tau_p\) [20–23]. An often-used dimensionless quantity \([24]\) that characterizes the strength of activity is the Péclet number \(\text{Pe} \equiv f_0 \tau_p / (\gamma \sigma)\), where \(\gamma\) is a friction coefficient, and \(\sigma\) is a microscopic length related to the particle size. Thus, the strength of activity can be increased by increasing \(f_0\) or \(\tau_p\). Theories of equilibrium glasses have been extended for active systems, and the resulting descriptions provide insights into many aspects of how activity affects glassy properties [25–32]. However, these theories apply in a regime where the activity is weak, i.e., Pe is small. Our aim in this work is to develop a theory for the high activity regime, of large \(f_0\) and \(\tau_p\).

We have recently presented an active random first-order transition (active-RFOT) theory of an active glass [30]. The standard RFOT for passive glasses describes the activated reconfgurations of correlated domains (`mosaics') through a mechanism similar to nucleation. The driving force for the reconfiguration of the mosaics is the configurational entropy density, which is a function of the potential energy density and temperature \(T\). We proposed that the activity manifests as an additional term in the bulk configurational entropy density of the...
mosaics, while leaving their surface energy term unchanged. Furthermore, we assumed that the active contribution to the configurational entropy density is simply proportional to the increase in potential energy density of the particles in the active glass due to activity. This potential energy density was assumed to have the same functional form as analytically derived for a single overdamped particle trapped in a harmonic potential [30,33,34]. This ‘effective medium’ model treats the particles surrounding the test particle as an effective confining harmonic potential with effective spring constant $k$ and friction coefficient $\gamma$. It is a phenomenological model [30], not based on a first-principles microscopic theory of an active glass and the details of the inter-particle potentials. While this model successfully explained simulation data for the relaxation times as a function of the amplitude and persistence time of the active forces, it failed at large values of these activity parameters. Here we present a non-perturbative extension of this model, which captures the relaxation times of the active glass for a much broader range of activity parameters.

In the active-RFOT theory of [30], the effect of activity on the free energy barrier for relaxation is expressed in terms of the quantity

$$\Delta F = \frac{Hf_0^2 \tau_{p}}{1 + k \tau_{p}/\gamma}$$

(1)

which quantifies the activity-induced change in the configurational entropy. Here, $H = \tilde{H}/\gamma$, $\tilde{H} = T_K \kappa_0/\Delta C_p$, $T_K$ is the Kauzmann temperature, defined as the temperature where the configurational entropy of the passive system vanishes [35], $\Delta C_p$ is the jump in specific heat from the liquid to the crystalline state [35], and $\kappa_0$ is an active fragility parameter that quantifies the sensitivity of the configurational entropy to changes in the active force. Using this modification we obtained an expression for the $\alpha$-relaxation time $\tau[30]$,

$$\log(\tau/\tau_0) = \frac{E}{(T - T_K)} + \Delta F$$

(2)

where $\tau_0$ is a microscopic time scale and $E$ represents the surface reconfiguration energy governing the relaxation dynamics of a region, an effective surface energy between regions that are in different amorphous metastable states. In the active-RFOT explored in [30] both $\tau_0$ and $E$ were assumed to be independent of the activity parameters $f_0$, $\tau_{p}$. When comparing data from simulations we use reduced units in which the parameters appearing in the Lennard-Jones interaction potential are used to define the units of mass, length, and time (see SI for more details).

The theoretical expression (equation (2)) was compared to simulation data for a three-dimensional active glass model [30], and found to correctly predict the qualitative dependence of the relaxation time on the activity parameters (see figure 1(a)). In the limit of large $\tau_{p}$, we find that the active term $\Delta F$ approaches a constant, but for large $f_0$ the predicted plateau values deviate significantly from the simulation results (see figure 1(a)).

Understanding the origin of this discrepancy at high activities, and extending the active-RFOT theory beyond the limit of weak forces and short persistence time, is the central subject of this paper.

We assume that the basic RFOT phenomenology remains valid even at high activity and consider what modifications to the active term (equation (1)) can account for this discrepancy. More specifically, at large activity, i.e. large active force $f_0$ and large persistence time $\tau_{p}$, the effective medium parameters ($k$, $\gamma$) may become dependent on the activity. Since in the large $\tau_{p}$ limit the parameter $\gamma$ cancels out, we focus on the effective confinement parameter $k$. Note that the proposed dependence of $k$ on the activity is within the same

Figure 1. (a) Comparison between the measured $\alpha$-relaxation time $\tau$ from simulations of a three-dimensional active glass (at $T = 0.45$) [30] (points) as function of $\tau_{p}$, and for different values of $f_0$ from top to bottom $f_0 = 0.1, 0.25, 0.5, 0.75, 1.0, 2.0$. The dashed lines give the calculated relaxation time according to the active-RFOT (equations (1), (2)), using a constant $k = k_0$ while the solid lines use the mARFOT expression for the effective confinement $k = k_0 + k_\Delta$ (broken lines). Plots of the simulated relaxation data of (a) as a function of $f_0$, plotted as the inverse of equation (2) (red circles). Solid and dashed lines are the mARFOT and active-RFOT results, respectively. (b) $\tau_{p} = 0.05$, so that $f_0 \ll f_t$, i.e., $\tau_{p} = 100$, so that $f_0 \sim 3$, and indeed we see a clear deviation from the quadratic dependence, in agreement with the predicted $f_0^{1/2}$ power law. This is further shown in the log-log version (inset), with the dash-dotted line indicating the exponent of 4/3. The values used in the fit are as in [30]: $\tau_0 = 0.135$, $T_K = 0.29$, $E = 1.55$, $H = 0.042$, $k_0/\gamma = 0.316$, $k_1 = 0.045$, $\gamma = 1$. 

- [30]: A reference to a paper or study.
- [33]: Another reference.
- [34]: Another reference.
- [35]: A reference to a property or concept.
- $H$: Symbol for potential energy, also denoted by $\tilde{H}$.
- $\tau_0$: A microscopic time scale.
- $E$: Surface reconfiguration energy.
- $T_K$: Kauzmann temperature.
- $\Delta C_p$: Jump in specific heat.
- $\kappa_0$: Active fragility parameter.
phenomenological ‘effective medium’ model. The surface reconfiguration energy, $E$, should also depend on activity, but fits of simulation data suggest that the modification of $E$ due to activity remains small even in this regime and we treat $E$ as a constant. The active fragility parameter $\kappa_{\text{am}}$ which relates the change in the potential energy due to activity to the resulting change in configurational entropy, is also treated as a constant. Since the potential energy is directly dependent on the activity, which we describe using the effective confined-particle, it is more natural to further modify this quantity to describe both low and high activity regimes.

**Activity-induced correction to the confinement**

Inside a dense active glass each particle is confined within a potential well formed by its neighbors. The active motion can lead to persistent squeezing of the particles against each other, leading to a stronger effective confinement due to the steep repulsive part of the inter-particle interaction potential, such as the Lennard-Jones potential that is used in the simulations. We keep the treatment of the effective confining potential as harmonic, with the same functional form for the quantity $\Delta F$ (equation (1)). Within this potential we describe how the active fluctuations can stiffen the effective harmonic confinement, for our choice of density and inter-particle potential. As the activity of the confining, neighboring particles, increases it gives rise to an effective stiffening of the harmonic potential. By having a stronger active force and longer persistence, these neighboring particles are able to more strongly squeeze the test particle, for which we calculate the average potential energy. We propose that this squeezing is the origin of the effective stiffening of the confining potential that is manifested in the relaxation times.

For small fluctuations within a toy model consisting a one-dimensional lattice of particles (appendix A), we can obtain the first correction to the effective spring constant (equation (A7)) as of the form

$$k = k_0 + k_1 \delta$$

where $k_0$ is the effective spring constant we used before for low activity [30], $k_1$ is a new parameter of the system, and $\delta$ is some effective measure of the mean-square deflections of the particles, confined within the active glass. This functional form can also be motivated on symmetry grounds, and we propose to use it in a self-consistent calculation to renormalize the effective spring constant of our active-RFOT model. Note that the original use of the potential energy of an active particle confined in a harmonic potential [30] was not meant to give a microscopic description at the single-particle dynamics. Similarly, we do not consider $\delta$ to be a quantitative measure of the single-particle dynamics, and we are able to self-consistently remove it from the final modified version of the active-RFOT (mARFOT).

Note that terms involving higher powers of $\delta$ can be added to equation (3). However, we keep the lowest order term, in order to obtain analytic expressions. The good agreement between the simulation data and the mARFOT model presented below, further supports this choice a posteriori.

**Self-consistent calculation of the renormalized effective harmonic confinement**

The mean-square dispersion of the harmonically-confined active particle, $\langle x^2 \rangle = \delta$, can now be calculated self-consistently. The potential energy of the confined active particle is related to its mean-square displacement [30, 33, 34]

$$\frac{1}{2} k \delta = \frac{1}{2} \frac{f_0^2 \tau_p / \gamma}{1 + k_0 \tau_p / \gamma}$$

where in the active-glass regime the active fluctuations dominate over the thermal component, and we will therefore neglect the thermal contribution. This is a standard method to renormalize elastic constants due to fluctuations (see for example [36]). Substituting equation (3) into equation (4) results in the following implicit equation for $\delta$

$$\delta = \frac{1}{k_0 + k_1 \delta} \frac{f_0^2 \tau_p / \gamma}{1 + (k_0 + k_1 \delta) \tau_p / \gamma}$$

The solutions of equation (5) are the roots of a cubic polynomial in $\delta$

$$a \delta^3 + b \delta^2 + c \delta = d$$

$$a = k_1^2 \tau_p / \gamma$$

$$b = k_1 (1 + 2k_0 \tau_p / \gamma)$$

$$c = k_0 (1 + k_0 \tau_p / \gamma)$$

$$d = 1$$
\[ d = f^2_0 \tau_p / \gamma \] (10)

In the limit of large activity (force and persistence time), the discriminant of equation (6) is negative, and one obtains one real (and positive) solution for \( \delta \), as well as two irrelevant complex solutions. For smaller activities, we have a single positive solution and two negative solutions that are discarded. Explicitly, we find that the self-consistent solution \( \delta_c \) initially increases linearly with \( f^2_0 \) for small \( f_0 \), but for large forces it now increases as \( f^3_0 \). The transition between these limits depends on the value of \( \tau_p \):

\[
\delta_{\infty} \sim \begin{cases} 
\frac{d}{c} = \frac{f^2_0 \tau_p}{\gamma k_0 (1 + k_0 \tau_p / \gamma)} & , \quad f_0 < f_c \\
\left( \frac{d}{a} \right)^{1/3} = \left( \frac{f^2_0}{k_0} \right)^{1/3} & , \quad f_0 > f_c
\end{cases}
\] (11)

where the crossover force is given explicitly by \( f_c = (k_0 (\gamma + k_0 \tau_p) / \tau_p)^{1/3} (\sqrt{k_0 / \gamma}) \).

For \( \tau_p \to \infty \) the critical force has the limiting value \( f_c \to (k_0)^{3/2} / (\sqrt{k_0 / \gamma}) \). In this limit, the mean-square displacement of the particles can be written relatively compactly as

\[
\delta_{\infty} = \frac{k_0 (\Lambda - 1)^2}{3k_0} & , \quad \Lambda = (1 + d' + \sqrt{d'(2 + d')})^{1/3}
\] (12)

where we have defined the dimensionless parameter \( d' = 27 k_0 f^2_0 / (2 k_0) \). In the limit of small \( f_0 \) we thus recover the quadratic dependence of \( \delta_{\infty} \) on \( f_0 \), while at large \( f_0 \) the \( f^3_0 \) power-law of equation (11) is obtained. Note that a modified effective spring constant with higher order terms in equation (3), \( k = k_0 + k_1 \delta + ... \delta^n \), would result in \( \delta_{\infty} \sim f^3_0 / (1 + 2n) \), in this limit.

The self-consistent solution \( \delta_c \) can now be used in the modified confinement spring constant (equation (3)), \( k = k_0 + k_1 \delta_c \), and in the active contribution to the free energy. With this modification we find that \( \Delta F \) increases as \( f^3_0 \) for small \( f_0 \), but at large activity varies more slowly as \( f^2_0 \). We next compare this mARFOT to simulation results.

**Comparison to simulations**

Substituting the self-consistent solution we can compare to the simulation data of a three-dimensional active glassy system (described in appendix C), shown in figure 1 [30]. The simulations shown in this paper are of the ‘model I’ type [30], where the active force correlations have a fixed amplitude \( f_0 \) which is independent of the persistence time. We find that the mARFOT greatly diminishes the discrepancy with the simulation data at large \( \tau_p \) and \( f_0 \) (see figure 1(a)). Note that these calculations use for the parameters \( E, T_{K_i}, \tau_0 \) values that are obtained from fits of \( \tau \) for the passive system as a function of \( T \) to the form of equation (2) with \( \Delta F = 0 \) [22,30] (see appendix D). The values of the active parameters \( k_0 / \gamma, H \) (denoted as \( G, H \), respectively, in [30]) are taken as in [30], determined by the fit at large \( \tau_p \) and small \( f_0 \). This leaves only \( k_1, \gamma \) as free fitting parameters. Without loss of generality, we fixed the friction coefficient to be \( \gamma = 1 \), as was used in the 2D Langevin simulations [37] (described in appendix B). We then fit \( k_1 \) for a single value of the force, and the model then correctly describes the functional dependence on both the force amplitude and persistence time, for all other combinations of these parameters, without any further parameter fitting. The good agreement, in both low and high activity regimes, serves to validate the model.

Another way to expose the qualitative change at large \( \tau_p \) is to plot log \((\tau_0 / \tau)\), i.e. the inverse of equation (2), as a function of \( f_0 \) for different values of \( \tau_p \). This is shown for two values of \( \tau_p \) in figures 1(b)–(c). It is clear that at low \( \tau_p \) this function increases quadratically with \( f_0 \) (see figure 1(b)), as predicted by the original active-RFOT expression (equation (1)). For this value of \( \tau_p = 0.05 \) we expect a crossover force \( f_c \sim 70 \) (equation (1)), so we never enter the regime where the mARFOT is distinguishable. At a larger value of \( \tau_p = 100 \) (see figure 1(c)) we clearly find that the simulation data indicates an increase that is lower than quadratic, and is in good agreement with the predicted \( f^{4/3} \) dependence of the mARFOT (within the limitations of the available simulation data). For this value of \( \tau_p \), the cross-over force is predicted to be \( f_c \sim 3 \), so the range of simulated forces does indeed enter the regime where mARFOT effects are significant.

In figure 2 we compare the model to new simulation data for a two-dimensional active glass (a finite temperature version of the model studied in [37], described in appendix B). As in the three-dimensional case, we fit the \( \alpha \)-relaxation time of the passive system as a function of temperature to extract the parameters \( T_{K_i}, \tau_0 \) and \( E \) (see appendix D, figure 3). The discrepancy between the simulation data and the active-RFOT is less strong compared to the data from the three-dimensional system (see figure 1a), but it is clearly observable. The mARFOT is seen to resolve the major part of the discrepancy (see figure 2(a)). It also captures the transition from
quadratic (figure 2(b)) to non-quadratic dependence of the active term in the free energy on the force amplitude at high $\tau_p$ (figure 2(c)).

**Discussion**

We provide here a self-consistent extension of the active-RFOT model that incorporates the renormalization of the effective confinement due to active fluctuations of the particles. This treatment gives a non-analytic modification of the power-law dependence of the active contribution to the free energy ('effective temperature') on the active force magnitude. The predicted non-quadratic dependence, with power-law $f_0^{4/3}$, is in good agreement with our simulation data, both in three and two dimensions. This result may offer an explanation for similar puzzling observations in configurational entropy calculations [38], where at high active forces a clear lower-than-quadratic dependence of the 'effective temperature' on the active force was found. Future simulations at higher activity may allow for more precise tests of the predicted $4/3$ power-law behavior.

Although our extension of the active-RFOT description improves the agreement with simulation results for high activity, there are regions in the parameter space $(f_0, \tau_p, T)$ where this theory may not provide a complete description of the actual behavior [39,40]. A recent study [37] of an athermal ($T = 0$) active system found a jamming transition as $f_0$ is reduced below a critical value $f_J$ in the $\tau_p \to \infty$ limit. For $f < f_J$ and large but finite values of $\tau_p$, the relaxation time is found to increase with increasing $\tau_p$. It is not clear whether this jamming transition would persist at moderate temperatures. However, our simulations for small values of $T$ and $f < f_J$ show a trend of increasing $\tau$ with $\tau_p$ for very large values ($\sim 1000$) of $\tau_p$. This behavior cannot be reproduced in the mARFOT description, which always predicts a decrease of $\tau$ with increasing $\tau_p$. Also, the active-RFOT and
mARFOT descriptions cannot be used to describe the behavior for $T < T_K$. This is because the factor $T - T_K$ in equation (2), arising from the temperature dependence of the configurational entropy for $T > T_K$, should be replaced by zero for $T < T_K$. Equation (2) would then predict a temperature-independent relaxation time $\tau$ for $T < T_K$, but the (presumably weak) temperature dependence of parameters such as $\kappa$, $\gamma$, and $E$, which we have neglected in our treatment, would lead to values of $\tau$ that depend weakly on $T$. The temperature dependence of these parameters needs to be explored further. Finally, the effect of activity on the surface reconfiguration energy, ignored in our analysis, may become important for values of $f_0$ substantially higher than those considered here. Further examination of this effect would improve our present understanding of the properties of dense active matter in the high-activity regime. Exploration of other effects of activity that are absent within our current framework, such as the one leading to motility induced phase separation [41], as well as the inclusion of external shear forces [42], would also be interesting.

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### Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

### Appendix A. Correction to the effective spring constant for small fluctuations of a 1D lattice

We now estimate the change in the effective confinement $k$ of a test particle in the glass due to the active fluctuations of the neighboring particles, using a density functional theory (DFT) formalism [43,44].

For simplicity we do the calculation in one dimension, where the effective potential in DFT at $x$ has the form

$$u(x) = \int_{-\infty}^{+\infty} dx' v(x - x') [\rho(x') - \rho_l].$$

(A1)

Here, $v(x)$ is the inter-particle potential (note that it is a function of $|x|$ only), $\rho(x)$ is the local number density field and $\rho_l$ is the density of the uniform liquid. In writing this equation, we have approximated the direct pair correlation function of the uniform liquid that appears in the Ramakrishnan-Yussouff form [43] of the free-energy functional by $-v/(k_B T)$. The term involving $\rho_l$ is a constant that is ignored in the rest of the analysis. We consider two particles located at $x = \pm a$ and calculate $u(x)$ near $x = 0$. The density near $x = \pm a$ is assumed to have a Gaussian form with variance $\delta$. The effective potential is given by

$$u(x) = \frac{1}{\sqrt{2\pi\delta}} \int_{-\infty}^{+\infty} dx' v(x - x') \{ \exp[-(x' - a)^2/(2\delta)] + \exp[-(x' + a)^2/(2\delta)] \}.$$  

(A2)

Clearly, $u(x) = u(-x)$, so that all odd derivatives of $u(x)$ at $x = 0$ are zero. We need to calculate the second derivative of $u(x)$ at $x = 0$: this gives the effective ‘spring constant’. Consider the first term in equation (A2):

$$u_1(x) = \frac{1}{\sqrt{2\pi\delta}} \int_{-\infty}^{+\infty} dx' v(x - x') \exp[-(x' - a)^2/(2\delta)]$$  

$$= \frac{1}{\sqrt{2\pi\delta}} \int_{-\infty}^{+\infty} dy \ v(x - y - a) \exp[-y^2/(2\delta)].$$  

(A3)

where we have set $x' - a = y$. The second derivative of $u_1(x)$ at $x = 0$ is given by

$$u_1''(x = 0) = \frac{1}{\sqrt{2\pi\delta}} \int_{-\infty}^{+\infty} dy \ v''(-y - a) \exp[-y^2/(2\delta)].$$  

(A4)

Now we make use of the fact that the exponential function is sharply peaked at $y = 0$ because $\delta$ is small ($\delta \ll a^2$), so that the main contribution to the integral comes from values of $y$ close to zero. This allows us to consider only the first non-vanishing term of the Taylor series expansion of $v''(-y - a) = v''(y + a)$ near $y = 0$, giving...
\[ u''(0) = \frac{1}{\sqrt{2\pi\delta}} \int_{-\infty}^{+\infty} dy \ [v''(a) + \frac{1}{2} v'''(a)y^2] \exp[-y^2/(2\delta)]. \]

Combining this with the contribution of the second term in equation (A2), we get

\[ u''(0) = 2v''(a) + v'''(a)\delta. \]  

(A6)

The effective confinement parameter therefore becomes

\[ k = k_0 + k_1 \delta \]

(A7)

where \( k_0 = 2v''(a) \) and \( k_1 = v'''(a) \).

While the final expression (equation (A6)) depends on the details of the inter-particle potential and lattice spacing, the functional form (equation (A7)) is general, and gives us further motivation for the modification of the effective spring constant that we use in our model (equation (3)).

Appendix B. Model for active glass in 2d

For the 2d active glass simulations, we have used a previously studied active glassy system [37,45]. This model is essentially a 2d binary soft active sphere (65: 35) mixture at high density \( \rho = 1.2 \). To implement the soft interaction between the particles (say between i and j) we use a Lennard-Jones interaction

\[ V_{ij}(r) = 4\epsilon_{ij}\left(\frac{\sigma_{ij}}{r}\right)^{12} - \left(\frac{\sigma_{ij}}{r}\right)^{6}, \]

(B1)

where inter-particle distance is represented by \( r \) and \( \epsilon_{ij} \) and \( \sigma_{ij} \) are the parameters representing the energy scale and the interaction radius, respectively. We have set all the energy scales in units of \( \epsilon_{AA} \) and all the length scales in units of \( \sigma_{AA} \) by choosing \( \epsilon_{AA} = 1.5\sigma_{AA} \), \( \epsilon_{AB} = 0.5\epsilon_{AA}, \sigma_{AB} = 0.8\sigma_{AA} \) and \( \sigma_{BB} = 0.88\sigma_{AA} \). The passive limit of the model is known as Kob-Andersen glass [46,47] and has been extensively used to study glassy dynamics in dense classical particle systems. To simulate the evolution we employ inertial dynamics with the equation of motion,

\[ m\ddot{\mathbf{r}}_i = -\gamma \dot{\mathbf{r}}_i + \mathbf{F}_i - \mathbf{f}_0 \mathbf{n}_i + \xi_i \]

(B2)

where \( m \) is the mass of each particle and \( \gamma \) is the friction coefficient. The position vector of the \( i \)-th particle is \( \mathbf{r}_i \) where \( i = 1, \ldots, N \) and \( \mathbf{F}_i = -\nabla V_i \) is the total interaction force on particle \( i \) derived from the LJ potential as \( V_i = \sum_{j \neq i} V_{ij} \) (see equation (B1) for the definition). The interaction potential \( V_{ij} \) has been truncated at \( R_{ij}^c = 2.5\sigma_{ij} \) and a quadratic smoothing function has been used so that both the energy and forces are continuous at \( R_{ij}^c \). We use \( \epsilon_{AA} = 1, \epsilon_{AA} = 1 \) and \( m = 1 \), so that all results are in reduced LJ units: lengths are in units of \( \sigma_{AA} \), energies in units of \( \epsilon_{AA} \), and all times are in units of \( \sqrt{m\epsilon_{AA}/\epsilon_{AA}} \), \( f_0 \mathbf{n}_i \) is the active force acting on the \( i \)-th particle along the direction \( \mathbf{n}_i \) associated with that particle. The unit vector \( \mathbf{n}_i = (\cos \theta_i, \sin \theta_i) \) where \( \theta_i \) is the angle representing the direction of the active force on the \( i \)-th particle. This angle (for each particle) is assumed to perform rotational Brownian motion following

\[ \dot{\theta}_i = \sqrt{2/\tau_p} \zeta_i \]

with rotational diffusion constant \( 1/\tau_p \). Here \( \zeta_i \) is zero mean uncorrelated Gaussian white noise with correlator

\[ \langle \zeta_i(t)\zeta_i(t') \rangle = \delta(t - t'). \]

(B4)

The thermal noise on the \( i \)-th particle is represented by \( \xi_i \), which has the property

\[ \langle \xi_i(t) \rangle = 0, \quad \langle \xi_i^\alpha(t)\xi_i^\beta(t') \rangle = 2\gamma k_B T\delta_{\alpha\beta}\delta(t - t') \]

(B5)

where \( T \) is the temperature of the heat bath and \( k_B \) is the Boltzmann constant. All the simulations were performed using a square periodic box with \( N = 1000 \) particles. We used modified Langevin dynamics [48] for the MD simulation with \( dt = 0.005 \). All the relevant quantities are averaged over time (after leaving out a transient time \( t_T = 10^3 \) which is 10 times more than the largest \( \tau_p \) studied) and also over 32 independent simulations. To measure the relaxation timescale \( \tau \) we first calculate the two-point overlap correlation function defined as \( Q(t) \),

\[ Q(t) = \frac{1}{N} \left[ \sum_{i} q(|\mathbf{r}_i(t) - \mathbf{r}_i(0)|) \right] \]

(B6)

where \( N \) is the number of particles in the simulation,

\[ q(x) = \begin{cases} 
1 & \text{if } x \leq c \\
0 & \text{otherwise}
\end{cases} \]

(B7)
and we have used $\epsilon = 0.3$ (in units of $\sigma_{AA}$). The relaxation time $\tau$ is defined by $Q(\tau) = 1/e$. For the data presented in the main text, we have kept the temperature at $T = 0.4$ and extracted the time scale $\tau$ for different $f_0$ and $\tau_p$.

**Appendix C. Model for active glass in 3d**

For the three dimensional glass we have used data that has been shown in [30] and has been generated from the model introduced in [22]. The passive limit of the model (described in [22]) is a Kob-Andersen glass in 3d (a 80: 20 mixture of Lennard Jones particles with number density $\rho = 1.2$). The parameters $\epsilon_{AA}$, $\epsilon_{BB}$, $\epsilon_{AB}$, $\sigma_{AA}$, $\sigma_{AB}$, $\sigma_{BB}$ are identical to the 2d Kob-Andersen model, described in the previous section.

Though the passive version of the 3d model is almost identical to the active glass model described in the previous section, in terms of implementation of activity this model is slightly different. For example all the particles in the previous model are active, whereas in this model only B particles were subject to a propulsion force, whilst keeping all the A particles passive. Self-propulsion forces are modelled inspired by an 8-state discrete Clock-model which has the form $f = f_0(k_x i + k_y j + k_z k)$, where $k_x$, $k_y$, $k_z$ are randomly assigned to have values $\pm 1$ (keeping the constraint that the net force is zero). The active particles are driven in the directions of $(k_x i + k_y j + k_z k)$ for a time scale (persistence time $\tau_p$) and then the directions of forcing are randomised by choosing a different set of $k_x$, $k_y$, $k_z$. Simulations are performed for $N = 1000$ particles in a 3d cubic box.

**Appendix D. Extraction of $E$, $\tau_0$ and $T_K$**

For the passive limit, i.e. $\Delta F = 0$ we have used a standard VFT (Vogel-Fulcher-Tammann) form for fitting the relaxation time ($\tau$) versus temperature $T$ data. The VFT form is given by

$$\tau = \tau_0 \exp \left[ \frac{E}{T - T_K} \right]$$

(D1)

where $\tau_0$ is the microscopic time scale, $E$ represents the surface reconfiguration energy governing the relaxation dynamics of a domain and $T_K$ is the Kauzmann temperature. This is the passive limit ($\Delta F = 0$) of equation (2). In figure 3 we have shown a typical fit (dashed line) of the simulation data (red points) of the relaxation time which gives us the fit parameters $E$, $\tau_0$ and $T_K$.

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**References**

[1] Angelini T E, Hannezo E, Trepat X, Marquez M, Fredberg J and Weitz D A 2011 Proc. Natl. Acad. Sci. (USA) 108 4717

[2] Nettu K D, Knorr M, Käs J and Zink M 2012 New J. Phys. 14 115012

[3] Zhou E H, Trepat X, Park C Y, Lenormand G, Oliver M N, Mijailovich S M, Hardin C, Weitz D A, Butler J P and Fredberg J J 2009 Proc. Natl. Acad. Sci. (USA) 106 10632

[4] Parry B, Surovtsev I, Cabeno M, O’Hern C, Dufresne E and Jacobs-Wagner C 2014 Cell 156 183

[5] Garcia S, Hannezo E, Elgeti J, Joanny J F, Silberzan P and Gov N S 2015 Phys. Rev. X 5 062403

[6] Nishizawa K, Fujiwara K, Ikenaga M, Nakajo N, Yanagisawa M and Mizuno D 2017 Proc. Natl. Acad. Sci. (USA) 114 15314

[7] Nandi S K 2018 Phys. Rev. E 97 052404

[8] Sadhukhan S and Nandi S K 2021 Phys. Rev. E 103 062403

[9] Lama H, Yamamoto M, Furuta Y, Shimaya T and Takeuchi K A 2022 arXiv:2205.10436

[10] Klongvessa N, Ginot F, Ybert C, Cottin–Bizonne C and Leocmach M 2019a Phys. Rev. Lett. 123 248004

[11] Klongvessa N, Ginot F, Ybert C, Cottin–Bizonne C and Leocmach M 2019b Phys. Rev. E 100 062603

[12] Geyer D, Martin D, Tailleur J and Bartolo D 2019 Phys. Rev. X 9 031043

[13] Klongvessa N, Ybert C, Cottin–Bizonne C, Kawasaki T and Leocmach M 2022 J. Chem. Phys. 156 154509

[14] Janssen L M 2019 J. Phys. Condens. Matter 31 503002

[15] Berthier L, Flenner E and Sammel G 2019 J. Chem. Phys. 150 200901

[16] Chakri S and Chakraborti R 2020 Soft Matter 16 7103

[17] Debets V, De Wit X M and Janssen L M 2021 Phys. Rev. Lett. 127 278002

[18] Paoluzzi M, Levis D and Pagonabarraga I 2022 Communications Physics 5 1

[19] Kuroda Y, Matsuyama H, Kawasaki T and Miyazaki K 2022 arXiv:2202.04436

[20] Ni R, Stuart M A C and Dijkstra M 2013 Nat. Commun. 4 2704

[21] Berthier L 2014 Phys. Rev. Lett. 112 220602

[22] Mandal R, Bhuyan P J, Rao M and Dasgupta C 2016 Soft Matter 12 6268

[23] Mandal R and Sollich P 2021a J. Phys. Condens. Matter 33 184001
[24] Fily Y, Henkes S and Marchetti M C 2014 Soft Matter 10 2132
[25] Berthier L and Kurchan J 2013 Nat. Phys. 9 310
[26] Szamel G 2016 Phys. Rev. E 93 012603
[27] Liluashvili A, Önody I and Voigtmann T 2017 Phys. Rev. E 96 062608
[28] Feng M and Hou Z 2017 Soft Matter 13 4464
[29] Nandi S K and Gov N S 2017 Soft Matter 13 7609
[30] Nandi S K, Mandal R, Bhuyan P J, Dasgupta C, Rao M and Gov N S 2018 Proc. Natl Acad. Sci. 115 7688
[31] Ghoshal D and Joy A 2020 Phys. Rev. E 102 062605
[32] Caprini I and Marconi U M B 2021 Soft Matter 17 4109
[33] Ben-Isaac E, Fodor É, Visco P, van Wijland F and Gov N S 2015 Phys. Rev. E 92 012716
[34] Wesler D, Gov N, Rasmussen K O and Bel G 2020 Physical Review Research 2 013003
[35] Kauzmann W 1948 Chem. Rev. 43 219
[36] Safran S A 2018 Statistical Thermodynamics of Surfaces, Interfaces, and Membranes (Boca Raton, FL: CRC Press)
[37] Mandal R, Bhuyan P J, Chaudhuri P P, Dasgupta C and Rao M 2020 Nat. Commun. 11 1
[38] Preisler Z and Dijkstra M 2016 Soft Matter 12 6043
[39] Keta Y-E, Jack R I, and Berthier L 2022 Phys. Rev. Lett. 129 048002
[40] Azora P, Sood A and Ganapathy R 2022 Phys. Rev. Lett. 128 178002
[41] Cates M and Tailleur J 2015 Annu. Rev. Condens. Matt. Phys. 6 219
[42] Mandal R and Sollich P 2021b Proc. Natl Acad. Sci. 118 e2101964118
[43] Ramakrishnan T V and Yussouff M 1997 Phys. Rev. B 56 2775
[44] Hoell C, Löwen H and Menzel A M 2019 J. Chem. Phys. 151 064902
[45] Mandal R and Sollich P 2020 Phys. Rev. Lett. 125 218001
[46] Kob W and Andersen H C 1995 Phys. Rev. E 51 4626
[47] Brüning R, St-Onge D A, Patterson S and Kob W 2008 J. Phys. Condens. Matter 21 035117
[48] Beard D A and Schlick T 2000 J. Chem. Phys. 112 7313