Longitudinal wave instability in magnetized high correlation dusty plasma†

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Abstract

Low frequency longitudinal wave instability in magnetized high correlation dusty plasmas is investigated. The dust charging relaxation is taken into account. It is found that the instability of wave is determined significantly by the frequency of wave, the dust charging relaxation, the shear viscosity and viscoelastic relaxation time, the coupling parameter of high correlation of dust as well the strength of magnetic field.

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Low frequency longitudinal volume dust waves have been extensively studied for weakly coupled and unmagnetized dusty plasmas [1–5]. Recently the corresponding research on

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dust waves have been also extended to the strongly coupled [6–9] as well the magnetized dusty plasmas [10–13]. In this letter we study the instability of longitudinal dust wave in the strongly coupled dusty plasmas. The stationary external magnetic field is applied. Some important factors such as the high-correlation on dusts, the wave-number dependence of the elastic relaxation time and effect of dust charging relaxation are included. It is shown that there are some frequency regime for the occurrence of longitudinal dust wave instability because of magnetized effect and the charging collisions between the dusts and the plasma particles. The regime of wavenumber that longitudinal wave is allowed depend also on the competition between shear viscosity and applied magnetic field strength. The behaviors of dust wave instability is also modified significantly in magnetized dusty plasmas by magnetized effect.

The applied external magnetic field is assumed along z-direction as $B = Bz$. Because we consider the longitudinal waves, we just need one of the Maxwell equations, i.e. Poisson equation, as

$$\nabla^2 \varphi = -4\pi e(\delta n_i - \delta n_e - Z_{d0}\delta n_d - n_{d0}\delta Z_d),$$

where $\varphi$ is the electrostatic potential, $n_{\alpha0}$ and $\delta n_\alpha$ are the steady and perturbation density of species $\alpha = e, i, d$ (electrons, ions and dusts respectively), $Z_{d0}$ and $\delta Z_d$ are the steady and perturbation charge numbers of dust negative charge as $q_d = -eZ_{d0} - e\delta Z_d$. Furthermore, the overall charge neutrality condition holds in the steady state, $n_{i0} = n_{e0} + Z_{d0}n_{d0}$. For longitudinal modes, one has to assumes that the wave propagates nearly perpendicular to the magnetic field. For simplicity we shall consider wave along $x$. The perturbations associated with the problem are assumed as in the form of $\sim \exp[i(kx - \omega t)]$. Then the perturbed electrons (ions) number density in magnetized plasmas can get by ordinary fluid equation. On the other hand in terms of a generalized viscoelastic hydrodynamic model [14] the perturbed density for the dusts in magnetized plasmas are also obtained. In addition to the dust charging equation by orbit-motion-limited (OML) theory the problem would be closed by Eq.(1). In Ref. [13] we have obtained the general dispersion relation for low
frequency longitudinal dust waves in strongly coupled magnetized dusty plasmas (please refer to Eq.(11) of Ref. [13])

\[
A \frac{\omega_{pi}^2}{k^2 V_{Ti}^2 + \Omega_i^2} + B \frac{\omega_{pe}^2}{k^2 V_{Te}^2 + \Omega_e^2} + \epsilon_{pd} = 0
\]  

(2)

where \( \omega_{pe,pi} = \sqrt{4\pi n_{e0,i0} e^2/m_{e,i}} \), \( n_{e0,i0} \), \( m_{e,i} \), \( V_{Te,Ti} = \sqrt{T_{e,i}/m_{e,i}} \) and \( \Omega_{e,i} = eB/m_{e,i}c \) are the characteristic oscillation frequencies, unperturbed number densities, masses, thermal velocities and the gyro-frequencies of the electrons and ions, respectively,

\[
A = 1 + \frac{P}{P + 1} \frac{\tau + z}{(1 + \tau + z)}
\]  

(3)

and

\[
B = 1 + \frac{P(\tau + z)}{z(1 + \tau + z)}
\]  

(4)

are two parameters which are associated with the dust charging, and

\[
\epsilon_{pd} = 1 - \frac{\omega_{pd}^2}{\omega \left[ \omega + i n_{d1}(k, \omega) \right] - \gamma_d \mu_d k^2 V_{Td}^2 - \Omega_{d}^2 / [1 + \eta_{d2}(k, \omega)]}
\]  

(5)

is the dielectric function of the dust fluid. The other quantities are \( P = Z_{d0} n_{d0}/n_{e0} \), \( \tau = T_i/T_e \) is the ratio of ion temperature to the electron temperature, and \( z = Z_{d0} e^2/aT_e \) is the normalized dust-dust interaction potential energy, \( \omega_{pd} = \sqrt{4 \pi n_{d0} Z_{d0}^2 e^2/m_d} \), \( n_{d0} \), \( m_d \), \( V_{Td} = \sqrt{T_d/m_d} \) and \( \Omega_d = Z_{d0} eB/m_d c \) are the characteristic oscillation frequency, unperturbed number density, mass, thermal velocity and the gyro-frequency of the dust, especially two quantities

\[
\eta_{d1}(k, \omega) = \frac{(\zeta + 4\eta/3) k^2}{m_d n_{d0} (1 - i\omega \tau_m)},
\]  

(6)

and

\[
\eta_{d2}(k, \omega) = \frac{\eta k^2}{m_d n_{d0} (1 - i\omega \tau_m)},
\]  

(7)

are associated with the bulk viscosity \( \zeta \), the shear viscosity \( \eta \) and the viscoelastic relaxation time \( \tau_m \) for highly correlated dusts. Note that \( \gamma_d \) is the adiabatic index and \( \mu_d = 1 + \)
\( u(\Gamma)/3 + (\Gamma/9)\partial_{\Gamma} u(\Gamma) \) is the compressibility and, \( u(\Gamma) \), the normalized correlation energy, or the excess internal energy, of the system [14]. For example when Coulomb coupling parameter \( \Gamma = (Z_d e)^2/a_d T_d \), \(-Z_d e\) is the dust charge, \( a_d \) the interdust distance and \( T_d \) the dust temperature) becomes high enough, the negative dispersion occurs for dust waves [9].. For weakly coupled dusts \((\Gamma < 1)\), or under certain conditions also for strongly coupled dusts we have \( u(\Gamma) \approx -\sqrt{3}\Gamma^3/2 \) [7,8,14]. In practice \( u(\Gamma) \) is usually obtained by fitting results from experiments and/or Monte Carlo or Molecular Dynamics simulations. Accordingly, fitting the internal-energy results from a liquid simulation one obtains \( u(\Gamma) \approx -0.90\Gamma \) for \( 1 \leq \Gamma \leq 200 \). Other quantities such as the transport coefficients \( \zeta(\Gamma) \), \( \eta(\Gamma) \), and \( \tau_m(\Gamma) \) are usually too complex to be expressed analytically, so that often simplifying models are employed [14].

Introducing the dimensionless quantities \( \omega = \omega/\omega_{pd} \), \( \Omega_d = \Omega_d/\omega_{pd} \), \( k = k\lambda_p \) (where \( \lambda_p = (\lambda_i^{-2} + \lambda_e^{-2})^{-1/2} \) the plasma Debye scale length, \( \lambda_i = \sqrt{T_i/4\pi n_0 e^2} \) is the ion Debye length, \( \lambda_e = \sqrt{T_e/4\pi n_0 e^2} \) is the electron Debye length), \( \tau_m = \tau_m\omega_{pd} \), \( \eta_i^* = (\zeta + 4\eta/3)/m_d n_0 \omega_{pd} a_d^2 \), \( \eta_e^* = \eta/m_d n_0 \omega_{pd} a_d^2 \), and \( \lambda_{e,i,d,p} = \lambda_{e,i,d,p}/\lambda_p \), we can express the general dispersion relation of dust waves Eq. (2) as

\[
\omega^2 - \gamma_d k^2 \lambda_d^2 + \frac{i \omega k^2 \kappa^2 \eta_i^*}{1 - i\omega \tau_m} - \frac{\Omega_d^2}{1 + i k^2 \kappa^2 \eta_e^*/(1 - i\omega \tau_m)\omega} = \frac{1}{1 + C(k)},
\]

where \( \kappa \equiv a_d/\lambda_p \) is the ratio of interdust distance to plasma Debye length and

\[
C(k) = A \frac{1}{k^2 \lambda_i^2 + \mathcal{E}_i} + B \frac{1}{k^2 \lambda_e^2 + \mathcal{E}_e}.
\]

with

\[
\mathcal{E}_i = \frac{\Omega_i^2}{\omega_{pi}^2}, \quad \mathcal{E}_e = \frac{\Omega_e^2}{\omega_{pe}^2}.
\]

For weak magnetized plasmas or unmagnetized plasmas we have \( C(k) \approx A/k^2 \) and \( \Omega_d \ll 1 \) so that Eq.(8) will be reduced to the former extensively studied cases [7,9]. However for relatively strongly magnetized plasmas we have
\[ C(k) \approx \frac{A}{k^2 + \Omega_i^2/\omega_{pi}^2} \]  

(10)

here we ignore the contribution from electrons since usually \( \lambda_e \gg \lambda_i \) and \( \Omega_e \ll \omega_{pe} \). In the following we would study two typical cases, i.e., the kinetic regime of \( \omega \tau_m \ll 1 \) and hydrodynamic regime of \( \omega \tau_m \gg 1 \).

In the kinetic regime of \( \omega \tau_m \ll 1 \) from Eq.(8) we have

\[ \omega^2 + i \omega k^2 \kappa^2 \eta_1^* - \frac{\Omega_d^2}{1 + i k^2 \kappa^2 \eta_2^*/\omega} - \gamma_d \mu_d k^2 \lambda_d^2 - \frac{1}{1 + C(k)} = 0. \]  

(11)

Furthermore we discuss two limiting cases.

(a) For long-wavelength wave, i.e. the wavelength is much longer than the inter-dust distance, \( k \kappa \ll 1 \), we have

\[ \omega^2 + i \omega k^2 \kappa^2 \left( \eta_1^* + \frac{\Omega_d^2}{\omega^2 \eta_2^*} \right) - \Omega_d^2 - \gamma_d \mu_d k^2 \lambda_d^2 - \frac{A}{(V_{Ai}^2 + A)^2} k^2 - \frac{V_{Ai}^2}{V_{Ai}^2 + A} = 0. \]  

(12)

where \( V_{Ai} = \Omega_i/\omega_{pi} \) is the normalized Alfven velocity by light speed \( c \). From this we can always get \( k = k_r + i k_i \) that means there exist wave instability. Now we look when we can get the pure exponent-type unstable wave which needs \( k_r = 0 \). In fact the frequency of wave is in the same order of or less much than the dusty plasma frequency \( \omega_{pd} \) so that the normalized \( \omega \ll O(1) \) or \( \ll 1 \). On the other hand \( \Omega_d \ll O(\omega) \) or \( \ll \omega \) and \( \eta_2^* \sim O(\eta_1^*) \) or \( < \eta_1^* \). Thus if \( \eta_1^* \sim O(1) \) or \( < 1 \) then the second term of left hand in Eq.(12) can be ignored. For this case where the shear effect is not strong we get

\[ k^2 = \frac{\omega^2 - \Omega_d^2 - V_{Ai}^2/(V_{Ai}^2 + A)}{A/(V_{Ai}^2 + A)^2 + \kappa^2(1 - 0.4\Gamma)/3\Gamma}. \]  

(13)

Usually \( \Omega_i \ll \omega_{pi} \) and

\[ \omega > \omega_H \approx \sqrt{\frac{\Omega_d^2 + \Omega_i^2/\omega_{pi}^2}{A}} \]  

(14)

that is a hybrid waves of dust and ions in the value of less than the dust plasma frequency. For \( \tau \ll 1, \ z \sim O(1) \), in general \( A \approx 1 \) to 2 then it leads to that when \( \kappa > \kappa_c = \sqrt{15/2A} \) and \( \Gamma > \Gamma_c = 2.5\kappa^2/(\kappa^2 - \kappa_c^2) \) then \( k^2 < 0 \) that indicates the existence of pure imaginary
part of wavenumber. For example for $A = 2$, $\kappa_c \approx 1.9365$, then for $\kappa = 2, 3, 4, 5$ we have correspondingly $\Gamma_c \approx 40, 4.3, 3.3$ and $3$ respectively.

Now let us discuss the condition of $\eta_1^* \leq O(1)$ and $\omega \tau_m \ll 1$ for the limitation of $\Gamma$. In general it is difficult to obtain exact expressions for $\tau_m$, $\eta_1$ and $\eta_2$ etc. However, approximate models can be constructed from the results of theories and numerical simulations. For example, for the range $10 \leq \Gamma \leq 200$, one can obtain from Table 5 and Fig. 33 of Ref. [14] the dependence $\eta_1^* \approx 0.02 \sqrt{\Gamma}$ from a one component plasma theory. On the other hand in terms of $\eta_1^*/\tau_m = \kappa^{-2} \lambda_d^2 (1 - \gamma_d \mu_d + 4u(\Gamma)/15)$ we have $\tau_m \approx 3 \sqrt{\Gamma}/8$. Obviously in this range $\eta_1^* \leq O(1)$ and $\omega \tau_m \ll 1$ are both hold for $\omega_H < \omega \ll 1$.

Now we have concluded that in this case, long-wavelength wave in the kinetic regime of $\omega \tau_m \ll 1$, the spatial instability occurs for moderate and high strong-coupling, weak viscosity and moderate viscoelasticity of dust, hybrid order of low-frequency of wave and moderate inter-dust distance.

(b) For short-wavelength wave, i.e. the wavelength is in the same order of or less than the inter-dust distance, $k \kappa \leq O(1)$, and $\eta_2^* \leq O(\eta_1^*) \ll 1$, we have

$$\omega^2 - \gamma_d \mu_d k^2 \lambda_d^2 - 1 = 0$$

and therefore

$$k^2 = \frac{\omega^2 - 1}{\kappa^2 (1 - 0.4\Gamma)/3\Gamma}.$$  

That means the wave instability can occur only if $\Gamma < 2.5$ for $\omega < 1$ and $10 > \Gamma > 2.5$ for $\omega > 1$ and $O(1)$. In later case $\Gamma < 10$ is need for meet the conditions of weak shear viscosity $\eta_1^* \approx 0.02 \sqrt{\Gamma} \ll 1$ as well $\tau_m \approx 3 \sqrt{\Gamma}/8 \ll O(1)$. The wave instability occurs independently to the dust changing relaxation and the applied magnetic strength. Certainly $\Omega_d/\omega_{pd} \ll 1$ should not be breaked.

Now we have concluded that in this case, short-wavelength wave in the kinetic regime of $\omega \tau_m \ll 1$, the spatial instability occurs for moderate strong-coupling, very weak viscosity, weak viscoelasticity of dust, dusty-plasma characteristic oscillation order of frequency of wave and has nothing to do with inter-dust distance.
In the hydrodynamic regime of \( \omega \tau_m \gg 1 \) we have

\[
\omega^2 - k^2 \kappa^2 \frac{\eta_1^*}{\tau_m} - \frac{\Omega_d^2}{1 - k^2 \kappa^2 (\eta_2^*/\tau_m \omega^2)} - \gamma_d \mu_d k^2 \lambda_d^2 - \frac{1}{1 + C(k)} = 0. \tag{17}
\]

Similarly we discuss two limiting cases.

(a) For long-wavelength wave, i.e. the wavelength is much longer than the inter-dust distance, \( k \kappa \ll 1 \), Eq.(17) would be reduced as

\[
\omega^2 - k^2 \kappa^2 \frac{\eta_1^*}{\tau_m} - \Omega_d^2 \left(1 + \frac{\eta_2^* k^2}{\tau_m \omega^2} \right) - \gamma_d \mu_d k^2 \lambda_d^2 - \frac{\mathcal{A}}{(V_{\text{Ai}}^2 + \mathcal{A})^2} k^2 - \frac{V_{\text{Ai}}^2}{V_{\text{Ai}}^2 + \mathcal{A}} = 0. \tag{18}
\]

We know that in the long-wavelength limit the \( k \) dependence of \( \tau_m \) may be modeled by [14] \( \eta_1^*/\tau_m = \kappa^{-2} \lambda_d^2 (1 - \gamma_d \mu_d + 4u(\Gamma)/15) \). On the other hand \( (\Omega_d^2/\omega^2)(\eta_2^*/\tau_m) \sim \mathcal{O}(\eta_1^*/\tau_m) \). Therefore one can get the wavenumber of dust wave as

\[
k^2 \approx \frac{\omega^2 - \Omega_d^2 - V_{\text{Ai}}^2/(V_{\text{Ai}}^2 + \mathcal{A})}{\mathcal{A}/(V_{\text{Ai}}^2 + \mathcal{A})^2 + \kappa^2(1-0.3\Gamma)/\Gamma} \tag{19}
\]

which depends on magnetic field and other parameters. Similarly in the case \( \Omega_i \ll \omega_{pi} \) and

\[
\omega > \omega_H \approx \sqrt{\frac{\Omega_i^2/\omega_{pi}^2}{\mathcal{A}}} \tag{20}
\]

we have, therefore, when \( \kappa > \kappa_c \approx \sqrt{10/3\mathcal{A}} \) and \( \Gamma > \Gamma_c \approx 3.3k^2/(\kappa^2 - \kappa_c^2) \), then \( k^2 < 0 \). For example for \( \mathcal{A} = 2 \), \( \kappa_c \approx 1.3 \), then for \( \kappa = 1.4, 2, 3, 4, 5 \) we have correspondingly \( \Gamma_c \approx 22.0, 5.7, 4.0, 3.7 \) and 3.5 respectively.

It is worthy to note what does mean \( \omega \tau_m \gg 1 \). In general \( \omega \ll 1 \) or \( \sim \mathcal{O}(1) \) so that \( \tau_m \gg 1 \) is at least required. That means the highly strong-coupling is needed.

Now we have concluded that in this case, long-wavelength wave in the hydrodynamic regime of \( \omega \tau_m \gg 1 \), the spatial instability occurs for highly strong-coupling, highly strong viscoelasticity of dust, hybrid order or/and dust-oscillation order frequency of wave and moderate inter-dust distance.

(b) For short-wavelength wave, i.e. the wavelength is in the same order of or less than the inter-dust distance, \( k \kappa \leq \mathcal{O}(1) \), and, when \( \Omega_d \ll \sqrt{\eta_2^*/\tau_m} \) for moderate or/and strong shear dusty plasmas, we have
\[
\omega^2 - k^2 \kappa^2 \frac{\eta^*_1}{\tau_m} - \gamma_d \mu_d k^2 \lambda_d^2 - 1 = 0
\]  
(21)

or equivalently

\[
k^2 = \frac{\omega^2 - 1}{\kappa^2 (1 - 0.24 \Gamma)/3 \Gamma}
\]  
(22)

that shows the wavenumber \(k^2 < 0\) for \(\Gamma < 4.16\) when \(\omega < 1\) and otherwise \(\Gamma > 4.16\) when \(\omega > 1\). Obviously the former case is not realized for the requirement of \(\tau_m \gg 1\). Certainly the later case holds for very highly strong coupling dusty system. On the other hand, when \(\Omega_d \gg \sqrt{\eta^*_2/\tau_m}\) for very weak shear dusty plasma, we have not get the existence of pure unstable wave because of its contradictive to the requirement of \(\omega \tau_m \gg 1\).

Now we have concluded that in this case, short-wavelength wave in the hydrodynamic regime of \(\omega \tau_m \gg 1\), the spatial instability occurs for highly strong-coupling, moderate or/and strong viscosity, highly strong viscoelasticity of dust, dusty-plasma oscillation order frequency of wave and has nothing to do with inter-dust distance.

Finally let us discuss briefly that the condition for the occurrence of wave instability in the regime of \(\omega \tau_m \sim O(1)\). In this case the occurrence condition of wave instability is almost same as in the kinetic regime of \(\omega \tau_m \ll 1\) except \(\tau_m \sim O(1)\) add limitation to \(\Gamma\). In fact in the middle regime of \(\omega \tau_m \sim O(1)\) we have

\[
\omega^2 - k^2 \kappa^2 \frac{\eta^*_1}{\tau_m} \frac{1 - i}{2} - \frac{\Omega_d^2}{1 - k^2 \kappa^2 (\eta^*_2/\tau_m \omega^2)(1 - i)/2} - \gamma_d \mu_d k^2 \lambda_d^2 - \frac{1}{1 + C(k)} = 0.
\]  
(23)

For very small \(k \kappa \ll 1\), we have

\[
\omega^2 - \frac{1 - i}{2} k^2 \kappa^2 \left( \frac{\eta^*_1}{\tau_m} + \frac{\Omega_d^2 \eta^*_2}{\omega^2 \tau_m} \right) - \Omega_d^2 - \gamma_d \mu_d k^2 \lambda_d^2 - \frac{\mathcal{A}}{(V_{Ai}^2 + \mathcal{A})^2} k^2 - \frac{V_{Ai}^2}{V_{Ai}^2 + \mathcal{A}} = 0.
\]  
(24)

since \(k \kappa \ll 1\) and \(\eta^*_1/\tau_m \approx 0.05\) so that the second term is ignored in Eq.(24) and it is recover to the Eq.(13). For large \(k \kappa \sim O(1)\), we have

\[
\omega^2 - \frac{1 - i}{2} k^2 \kappa^2 \frac{\eta^*_1}{\tau_m} + (1 + i) \frac{\Omega_d^2 \omega^2}{k^2 \kappa^2 (\eta^*_2/\tau_m)} - \gamma_d \mu_d k^2 \lambda_d^2 - 1 = 0.
\]  
(25)

When \(\eta^*_1/\tau_m \ll 1\) and \(\Omega_d \ll \sqrt{\eta^*_2/\tau_m}\), it leads to
\[ \omega^2 - \gamma_d \mu_d k^2 \lambda_d^2 - 1 = 0 \]  

which is same as Eq.(15). However \( \omega \tau_m \sim O(1) \) indicates that the occurrence condition of wave instability in this case requires that the wave frequency is moderately increased and the strong-coupling is moderately decreased, compared to the case of \( \omega \tau_m \ll 1 \).

In summary, we have investigated the low frequency longitudinal wave instability in strongly coupled dusty plasmas in presence of magnetized field. The effect of dust charging relaxation is taken into account. It is found that the occurrence of instability of wave is determined significantly by the frequency of perturbation wave, the dust charging relaxation, the shear viscosity and viscoelastic relaxation time, the coupling parameter of high-correlated dusts as well the strength of magnetic field.
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