Modeling the time evolution of geothermal boreholes during peak heating and cooling demands

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Abstract. A geothermal heat exchanger requires special care in its design when it comes to peak heating and cooling demands of the building as the installation may incur in material damages due to the extreme temperatures reached by the heat carrying liquid. The peak demands tend to last a few days at most and the theoretical model used to predict the thermal response of the geothermal heat exchanger has, therefore, to consider the thermal inertia of the heat carrying liquid, the grout, and the ground close to the boreholes. With this in mind, the present work discusses a theoretical model that provides, among other things, the heat injection rates per unit pipe length of the different pipes in the borehole in terms of the bulk temperatures of the heat carrying liquid during those peak heating and cooling demands.

1. Introduction
At present day, it is clear that the global environment is being severely damaged by the massive burning of fossil fuels. Along that line, heating and cooling of buildings represents more than 25% of the world energy consumption [1], so the use of energy-efficient HVAC (heating, ventilation, and air conditioning) systems can be a highly beneficial action. An example of it is the use of geothermal HVAC systems which consist in a water-to-water heat pump connected to a geothermal heat exchanger composed of geothermal boreholes. Each of these boreholes is drilled hundreds of meters into the ground and equipped with one or more U-shaped probes that exchange heat with the surrounding ground with the aid of a circulating liquid.

During the hottest and coldest moments of the year, the heat carrying liquid reaches its maximum and minimum operating temperatures. Due to safety requirements, structural integrity considerations, and environmental concerns, a certain temperature range needs to be enforced during the design of the geothermal HVAC system for which the theoretical model describing the thermal response of the geothermal heat exchanger needs to consider the thermal inertia of the heat carrying liquid, of the grout, and of the ground located close to the boreholes. Many of such models already exist [2, 3, 4] but they have limitations: analytical models require geometrical or physical approximations and detailed numerical simulations need excessive computational resources.

The current work discusses some interesting aspects of the theoretical model recently developed by the authors [5] that takes into account the aforementioned thermal inertias without the need of any simplifications in the geometrical or physical configurations. To accomplish this, the unsteady heat conduction problem in the grout and the ground is solved exactly by
combining the Laplace transform with an expansion of the grout and ground temperatures in terms of conveniently forged multipoles centered at the different pipes.

2. Transient thermal response of slender geothermal boreholes

A geothermal borehole consists in a borehole of radius $r_b$ and depth $H$ drilled vertically into the ground in which U-shaped probes are placed with a heat carrying liquid circulating through them to exchange heat with the surrounding ground. Due to the slenderness of the borehole, $H/r_b \gg 1$, vertical heat conduction is only important close to the borehole endings [6]. Hence, for most of the borehole heat conduction is two dimensional and perpendicular to the borehole’s axial direction. To illustrate this, figure 1 shows a common borehole configuration with two pipes, where $T_1(t)$ and $T_2(t)$ are the time-dependent bulk temperatures of the fluid circulating inside pipes 1 and 2, respectively. The figure also shows the thermal diffusivities ($\alpha_g$ and $\alpha_b$) and thermal conductivities ($k_g$ and $k_b$) of the ground and the grout which in general attain different values.

The borehole is subject to the heat injection rate required by the building which has a characteristic time $t_q$ with a wide spectrum of values [6]. The current work focuses on values of $t_q$ of the order of hours or days which require the thermal inertias of the heat carrying liquid, the grout, and the ground close to the borehole to be taken into account. This means the heat transfer problem to solve consists in the unsteady heat conduction equation in grout and ground, the continuity conditions in temperature and normal heat flux at the borehole wall, the unperturbed ground temperature far from the borehole, and proper boundary conditions at the pipe walls that take into account the thermal conductivity in the pipe walls and the turbulent transport of heat inside the pipes [5].

![Figure 1](image_url)

**Figure 1.** Sketch of a typical geothermal borehole, of depth $H$ and radius $r_b$, equipped with a single U-shaped probe (black) through which a heat carrying liquid (cyan) with temperature $T_i(t)$ flows with a bulk velocity $V$ to exchange heat with the surrounding ground (brown). The empty space between probe and ground is filled up with grout (gray), whose thermal conductivity $k_b$ and thermal diffusivity $\alpha_b$ differ from those of the surrounding ground, $k_g$ and $\alpha_g$. 
3. Heat injection rates per unit pipe length

The solution to the aforementioned heat transfer problem is found by applying the Laplace transform to the unsteady heat conduction equations in grout and ground so that time $t$ is substituted by position $s$ in the complex-valued Laplace plane. By expanding then the grout and ground temperatures in terms of an infinite number of conveniently chosen multipoles, an exact solution to the problem is obtained. All details of the formulated problem and its solution can be found in [5]. The most interesting outcome are the heat injection rates per unit pipe length $\tilde{q}_i$ that appear as source terms in the energy conservation equations that describe the convective transport of heat along the pipes. It is shown in [5] that for a borehole with $N_p$ pipes they can be written as follows in terms of the bulk temperatures of the fluid in the pipes:

$$\tilde{q}_i = \tilde{w}_i \frac{\tilde{T}_m - \tilde{T}_a}{\tilde{R}_b} + \sum_{j=1}^{N_p} \frac{\tilde{T}_i - \tilde{T}_j}{\tilde{R}_{a,ij}},$$  

where the superscript $\sim$ denotes the Laplace-transformed character of the variables. The first term represents the heat exchanged by pipe $i$ with the ground surrounding the borehole while the second term represents the heat exchange between pipe $i$ and the rest of pipes inside the borehole. In this second term, the borehole’s inner thermal resistances $\tilde{R}_{a,ij}$ represent the resistances to the heat exchange between the liquids of different pipes inside the borehole. They account for the turbulent transport of heat in the fluid inside the pipes and for the heat conduction in the pipe’s walls, the grout, and the ground surrounding the borehole. They have the property of being symmetrical, $\tilde{R}_{a,ij} = \tilde{R}_{a,ji}$ [7], which is exploited next to derive an expression for the heat injection rate per unit borehole length $\tilde{q}$ that represents the total amount of heat exchanged by all the heat carrying liquid with the ground:

$$\tilde{q} = \sum_{i=1}^{N_p} \tilde{q}_i = \frac{\tilde{T}_m - \tilde{T}_a}{\tilde{R}_b} \sum_{i=1}^{N_p} \tilde{w}_i + \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} \frac{\tilde{T}_i - \tilde{T}_j}{\tilde{R}_{a,ij}} = \frac{\tilde{T}_m - \tilde{T}_a}{\tilde{R}_b} \sum_{i=1}^{N_p} \tilde{w}_i.$$  

The primary weighting coefficients $\tilde{w}_i$ represent the contribution of each pipe $i$ to the overall heat exchange of the borehole with the surrounding ground. Because of this their sum is unity, $\tilde{w}_1 + \tilde{w}_2 + ... + \tilde{w}_{N_p} = 1$, which simplifies even further the above expression for $\tilde{q}$:

$$\tilde{q} = \frac{\tilde{T}_m - \tilde{T}_a}{\tilde{R}_b}.$$  

It becomes evident from (3) that the heat injection rate per unit borehole length $\tilde{q}$ is proportional to a representative temperature difference between the borehole and the surrounding ground, $\tilde{T}_m - \tilde{T}_a$, and is inversely related to the borehole’s outer thermal resistance $\tilde{R}_b$. This thermal resistance represents the resistance to the heat exchange between the borehole and the ground and accounts for the same phenomena than the borehole’s inner thermal resistances.

Meanwhile, the weighted mean fluid temperature $\tilde{T}_m$ represents the temperature with which the borehole exchanges heat with the surrounding ground. It is defined in terms of the secondary weighting coefficients $\tilde{v}_j$ given in [5], which sum is unity as well, and the fluid temperatures $\tilde{T}_j$ as

$$\tilde{T}_m = \sum_{j=1}^{N_p} \tilde{v}_j \tilde{T}_j.$$  

Finally, the apparent temperature $\tilde{T}_a$ represents the temperature at which the borehole perceives the ground. It is defined as
\[ T_a = \frac{T_\infty}{s} + \sum_{j=1}^{N_p} \tilde{R}_{g,j} \tilde{q}_j, \]  

(5)

with the first term representing the unperturbed ground temperature and the second term expressing the thermal self-influence of the borehole due to its operation. The coefficients \( \tilde{R}_{g,j} \) are obtained as part of the solution to the heat conduction problem in grout and ground [5].

The same temperature interpretations and the same structure for the heat injection rates per unit pipe length \( \tilde{q}_i \), shown in (1), were found by Hermanns and Pérez in 2014 [8] for the quasi-steady thermal response of the borehole and the ground close to it. Hence, as briefly shown in the present work and in greater detail in [5], the use and interest of these concepts are significantly extended by the work presented in [5].

4. Results and conclusions

To illustrate the relevance of the thermal inertias in the response of geothermal boreholes, a time-harmonic test case is presented using a symmetrical borehole configuration with a single U-shaped probe that has the same geometrical and thermal characteristics than the one used in [9]. Figure 2 portrays the moduli and arguments of the weighting coefficients \( \tilde{w}_1 \) and \( \tilde{w}_2 \) and of the borehole’s outer and inner thermal resistances \( \tilde{R}_b \) and \( \tilde{R}_{a,12} \) as functions of the nondimensional parameter \( r_2^2/(\alpha_g/\omega) \), where \( \omega \) is the angular frequency of the time-harmonic variation of the problem. The equality and invariance of the values of \( \tilde{w}_1 \) and \( \tilde{w}_2 \) denote that, due to the symmetry of the borehole, both pipes contribute the same to the heat exchange from the borehole to the ground. This geometric symmetry is also responsible for the equality in values of the primary, \( \tilde{w}_i \), and secondary, \( \tilde{v}_i \), weighting coefficients. The values of \( \tilde{R}_b \) and \( \tilde{R}_{a,12} \) are also constant when \( r_2^2/(\alpha_g/\omega) \) is small compared to unity, but deflect considerably from these values when \( r_2^2/(\alpha_g/\omega) \) is higher than unity. This exemplifies the influence thermal inertias can have in the thermal characteristics of a borehole, and hence the importance of using a theoretical model that includes them when assessing the thermal response of geothermal boreholes during peak heating and cooling demands.

**Figure 2.** (Left) Modulus and (right) argument of the borehole’s inner and outer thermal resistances \( \tilde{R}_{a,12} \) and \( \tilde{R}_b \), respectively, and the primary and secondary weighting coefficients of the borehole, \( \tilde{w}_i \) and \( \tilde{v}_j \), as functions of the modulus of \( r_2^2/(\alpha_g/\omega) \).
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