Mutual consistency of the MINOS and MiniBooNE Antineutrino Results and Possible CPT Violation

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Abstract

Discussing the recent MINOS data on $\bar{\nu}_\mu$ disappearance and the MiniBooNE data on $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ oscillation, we show that while the respective best fits are inconsistent with each other, significant overlap of allowed regions does exist. Assuming only three neutrino species, the data indicates a discrepancy of mass levels and mixing angles between the neutrino and the antineutrino sectors. We show that the existing data can be reconciled with a model of explicit CPT violation in the neutrino sector and estimate the magnitude of the required violation.

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Experimental evidence in support of tiny but finite neutrino masses demands an extension of the Standard Model (SM)\textsuperscript{[1]}. Recent results from the MINOS\textsuperscript{[2]} and MiniBOONE\textsuperscript{[3]} collaborations not only re-assert this, but also force us to think about violation of one of the cornerstones of theoretical physics, namely the CPT theorem. The MINOS collaboration has looked for, and found, signals of oscillations in both $\nu_\mu$ and $\bar{\nu}_\mu$ beams. Perhaps their most surprising finding is that parameters governing $\bar{\nu}_\mu$ disappearance are different from those governing $\nu_\mu$. The most straightforward interpretation, namely that the neutrinos have masses different from those of anti-neutrinos, runs counter to the CPT theorem, which, in turn, is a good symmetry for any local field theory defined in a Minkowski space-time. This is probably the first time in the history of physics that we are faced with a situation where to explain a certain experimental result, violation of CPT is asked for.

Yet another longstanding anomalous result is that from the LSND experiment\textsuperscript{[4]} which claimed an evidence for $\bar{\nu}_e$ oscillating into something. The key issue was that the mass scale was quite distinct from either of those deemed to be responsible for the solar neutrinos (ostensibly $\nu_e \leftrightarrow \nu_\mu$) or the atmospheric ones ($\nu_\mu \leftrightarrow \nu_\tau$). While this could be explained by postulating a sterile neutrino, this explanation...
is highly disfavoured by a global fit of all neutrino data \[5\], in particular the SNO neutral current measurements \[6\]. To resolve this anomaly, various experiments have been performed. The latest in this line is MiniBooNE, which has claimed consistency with the LSND results. It should be noted at this point that the LSND result could, in principle, be explained in the framework of CPT violation \[7\].

In view of these two rather startling results, each demanding a drastic step away from the SM, it is worthwhile to consider the possibility of explaining both within a single framework (whether CPT violation or otherwise). However, before delving into this, it would be prudent to examine the (degree of) consistency of the two experiments with each other, and this is what we begin with.

At this point, it would be wise to remember that, matter being CP asymmetric, $\nu$'s and $\bar{\nu}$'s, while propagating through it, have differing effective masses. Assuming that they can have only SM interactions (and keeping in mind that the $1 \leftrightarrow 3$ mixing is small), the matter effect is negligible in $\nu_{\mu} - \nu_{\tau}$ sector. However, the inclusion of a non-standard interaction (NSI) involving $\nu_{\mu}$ and $\nu_{\tau}$ can offset this result, and the consequent matter effect can be substantial even for the relatively small minos baseline. Ref\[8\] seeks to exploit this to explain the minos result. However, two subtle issues need to be considered. In any realistic NSI framework, allowing for a flavour violating coupling involving $\nu_{\mu}$ and $\nu_{\tau}$ necessarily implies a similar interaction involving $\mu$ and $\tau$, thereby, potentially triggering the so far unobserved decay $\tau \rightarrow \mu \gamma$. Furthermore, any attempt to explain the MiniBOONE results within the same NSI framework would, generically, lead to a larger flavour violation involving $e$ and $\mu$ (on account of the larger mass difference observed), resulting in very large $\mu \rightarrow e \gamma$. Note, however, that the MiniBooNE $\bar{\nu}_e$ excess (albeit in a different energy range) has also been ascribed to an underestimation of SM-induced single-photon background events \[9\].

The Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix\[10\] that relates the neutrino mass $\nu_i (i = 1 \cdots 3)$ and flavour $\nu_\alpha (\alpha = e, \mu, \tau)$ eigenstates through $|\nu_\alpha \rangle = \sum_{i=1}^{3} U^{*}_{\alpha i} |\nu_i \rangle$, is usually parametrized as

$$U = U_{23}(\theta_{23}) U_{13}(\theta_{13}) U_{12}(\theta_{12})$$

where $U_{ij}$ represents an orthogonal rotation in the $ij$ plane through the angle $\theta_{ij}$. In the above, (1,2) is the solar pair, with $m_2$ being marginally larger than $m_1$. As for the atmospheric pair, $m_3 > m_2$ denotes the normal hierarchy (NH), while $m_3 < m_2$ implies the inverted one (IH). For our numerical analysis, we use \[11\]

$$\Delta m^2_{21} = (7.59 \pm 0.20) \times 10^{-5} \text{eV}^2 \quad \sin^2(2\theta_{12}) = 0.87 \pm 0.03$$

$$|\Delta m^2_{32}| = (2.43 \pm 0.13) \times 10^{-3} \text{eV}^2 \quad \sin^2(2\theta_{23}) = 1.0$$

where $\Delta m^2_{ij} \equiv m_i^2 - m_j^2$. In eq. (1), we have neglected the Majorana phases (which are irrelevant for the discussion at hand) and have also assumed that there is no CP-violating phase, an assumption that is not a drastic one in view of the fact that $\theta_{13}$ is constrained to be very small ($\theta_{13} \lesssim 7^\circ$).

Henceforth, we shall adopt the convention whereby lowercase letters ($m$ and $\theta$) are used for the $\nu$'s, and uppercase letters ($M$ and $\Theta$) for $\bar{\nu}$'s. Note that eq. (2) applies only to the $\nu$'s. Furthermore, if CPT is violated, there is no way to relate the $\nu u$ hierarchy with the $\bar{\nu}$ one. A constraint that binds the two
sector is the cosmological one on $m_{\text{tot}} \equiv \sum m_\nu$, with the sum ranging over SM-like $\nu$’s and $\bar{\nu}$’s. Current data restricts $m_{\text{tot}} \leq 0.56 \text{eV}$ if the flat $\Lambda$CDM model is assumed and $m_{\text{tot}} \leq 0.94 \text{eV}$ if a generic dark energy source is considered [12].

MINOS looks at $\bar{\nu}_\mu$ disappearance, so technically it can go to either $\bar{\nu}_e$ or $\bar{\nu}_\tau$. Assuming $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$, MINOS obtained results in the antineutrino 2-3 sector which are slightly different from the corresponding data on neutrinos:

$$\Delta M_{23}^2 = (3.36^{+0.45}_{-0.40} \pm 0.06) \times 10^{-3} \text{eV}^2, \quad \sin^2(2\Theta_{23}) = 0.86 \pm 0.11 \pm 0.01,$$

where the first error is statistical and the second is systematic. Thus, there is more than 2$\sigma$ discrepancy between neutrino and antineutrino data.

![Figure 1: Expected survival probability at MINOS for different $\Theta_{13}$ if the MiniBooNE best fit is to be incorporated in a 3-generation scheme.](image)

Considering the appearance of $\bar{\nu}_e$ in a $\bar{\nu}_\mu$ beam and analysing the counts for quasi-elastic scattering for antineutrino energies on the range $475 \text{MeV} \leq E_{\nu}^{QE} \leq 3 \text{GeV}$, MiniBooNE claims evidence for oscillations with parameters distinctly different from the solar ones. Admittedly, the error bars are yet very large, but the best fit, viz.

$$\Delta M_{12}^2 = 0.064 \text{ eV}^2, \quad \sin^2(2\Theta_{12}) = 0.96,$$

also represents virtually the smallest value for $\Delta M_{12}^2$. Together, the two experiments suggest the need for at least 5 neutrino/antineutrino levels. Moreover, the neutrinos should be Dirac particles.\footnote{Majorana fermions can be CPT violating, but here we need different mass values.}

However, it is imperative to check whether MINOS and MiniBooNE findings are mutually consistent. Note that either analysis is contingent on an effective two-generation hypothesis. Whereas this is a fairly good assumption in the neutrino sector (the solar splitting does not materially affect the atmospheric
oscillation), for the antineutrinos it might not be so, as the lowest possible value (at 99% CL) of $\Delta M^2_{12}$ claimed by MiniBooNE is about one order of magnitude larger than the MINOS value of $\Delta M^2_{23}$. The MINOS survival probability $P$ reaches a plateau ($P \sim 1$) for $E_\nu \gtrsim 8$ GeV. If we incorporate MiniBooNE as well, a three-flavour analysis needs to be performed, and consequently, large oscillations in $P$ survive into the high-$E_\nu$ region, and even after bin-averaging, the resultant $P$ is inconsistent with MINOS (see Fig.1). Evidently, non-zero values of $\Theta_{13}$ do not help reconcile the two, and, in addition, the agreement worsens in the small $E_\nu$ region as well.

![Parameter space consistent with MINOS at 90% (left) and 95% C.L. (right) for two values of $\Theta_{13}$. Also shown is the 90% CL contour of MiniBooNE. The point on the right edge is the MiniBooNE best fit and the diamond is our benchmark point BP2.](image)

Figure 2: Parameter space consistent with MINOS at 90% (left) and 95% C.L. (right) for two values of $\Theta_{13}$. Also shown is the 90% CL contour of MiniBooNE. The point on the right edge is the MiniBooNE best fit and the diamond is our benchmark point BP2.

This disagreement is omnipresent over the entire 1$\sigma$ allowed parameter space for MiniBooNE and, at this level, the two experiments are clearly inconsistent with each other. The 90% CL allowed regions do overlap though (Fig.2). That the overlap decreases with increasing $\Theta_{13}$ can be understood by realising that a large $\Delta M^2_{12}$ implies a large $\Delta M^2_{13}$, and coupled with a non-zero $\Theta_{13}$, it sets off additional oscillations in $P$. The diamond in Fig.2 indicates one of our benchmark points; we show, in Fig.3, the corresponding survival probability as well as its bin-averaged value. It is evident that while this (and similar others) choice of parameters is consistent with MINOS as of now, an improvement in the MINOS energy resolution would go a long way in confirming (or ruling out) this solution. Also note that the benchmark point is allowed by the Bugey and Karmen experiments too.

CPT violation in the neutrino sector has been considered earlier [13] and can be classified mainly into two categories. One class of such field theories are non-local. The others are local, and, generically can be represented as an interaction term often expressible in terms of a preferred direction. However, it has been shown [14] that CPT violating theories necessarily do not respect Lorentz symmetry. Possible mechanisms to violate CPT have been discussed by several authors [15]. Consequences of such CPTV interactions on neutrino oscillation have been extensively studied in the literature [16].

Rather than consider a particular model, we adopt a purely phenomenological approach. Given
that the neutrinos and their CPT partners seem to have differing masses, we assume that the only relevant CPT violation appears in the effective (anti-)neutrino mass matrices. Apart from restricting us to Dirac neutrinos, the rest of the analysis is independent of any particular framework proposed in the literature. The mass matrices, in the flavour basis, can then be parametrized as

\[ m_{ij}^{\text{flav}} = \mu_{ij} + \epsilon_{ij}, \quad M_{ij}^{\text{flav}} = \mu_{ij} - \epsilon_{ij} \]  

where \( \mu \) (\( \epsilon \)) are the CPT conserving (violating) parts respectively. These would then be diagonalized by the respective PMNS matrices \( U(\theta_{ij}) \) and \( \bar{U}(\Theta_{ij}) \), assuming the same diagonalizing matrices for left- and right-chiral \( \nu \) and \( \bar{\nu} \) fields.

With too many parameters and too little data, we adopt the constraint that the lightest of neutrino and antineutrino are exactly degenerate. Still, given that the signs of \( \Delta M_{ij}^2 \) are unknown, as many as twelve hierarchy combinations are possible: \( N_{ijk} \) and \( I_{ijk} \) where \( N \) (\( I \)) refer to the normal (inverted) hierarchy in the \( \nu \) sector while \( ijk \) indicate the hierarchy \( M_i < M_j < M_k \). Rather than scan over the parameter space, we choose to discuss the numerical results for two specific benchmark points. For both, \( \Delta M_{23}^2 \) and \( \sin^2 2\Theta_{23} \) are held to the MINOS central value, while \( \theta_{13} = \Theta_{13} = 0 \) (as Fig 2 demonstrates, a non-zero \( \Theta_{13} \) tends to worsen the fit). BP1 ignores the MiniBooNE result altogether, and holds \( \Delta M_{12}^2 = \Delta m_{12}^2 \) and \( \Theta_{12} = \theta_{12} \). BP2, on the other hand, refers to the point illustrated in Figs 2&3. As can be expected, the results drastically depend on this choice. Of particular importance is the cosmological constraint on \( m_{\text{tot}} \), which is rather severe on BP2, and would effectively rule out the allowed parameter points further away from the MiniBooNE best fit.

Not all hierarchies are allowed for a given benchmark point. While some are ruled out by dint of the values of \( \Delta M_{ij}^2 \), others are strongly disfavoured by cosmological constraints. We show the maximum possible mass of the lowest \( \nu \) (and hence \( \bar{\nu} \)) level consistent with the cosmological bound in the last two columns of Table 1. Note that the allowed schemes essentially come in pairs. Within a pair, the quantitative features are almost identical, a consequence of the fact that they are related by the flip
Table 1: Allowed neutrino and antineutrino hierarchies, and the associated measures of CPTV (see text).

| Scheme | $R_1 < 1$ | $R_2 < 1$ | Max($m_{\text{lowest}}$) (meV) |
|--------|-----------|-----------|-----------------------------|
|        |           |           | 0.56 eV | 0.94 eV |
| BP1    | $N_{123}, N_{213}$ | ✓ | ✓ | 32 | 76 |
|        | $N_{312}, N_{321}$ | ✓ | | 28 | 73 |
|        | $I_{123}, I_{213}$ | ✓ | ✓ | 86 | 153 |
|        | $I_{312}, I_{321}$ | ✓ | ✓ | 83 | 151 |
| BP2    | $N_{123}, N_{132}$ | ✓ | | — | 4.8 |
|        | $N_{231}, N_{321}$ | | | 1.5 | 34 |
|        | $I_{123}, I_{132}$ | ✓ | | — | 2.8 |
|        | $I_{231}, I_{321}$ | ✓ | ✓ | $(I_{321})$ | 7.2 | 101 |

Clearly, to achieve some of these hierarchies, the size of the CPTV parameters $\epsilon_{ij}$ need to be relatively large. In the absence of any theory of the same, one cannot formulate a precise definition of this largeness or the naturalness thereof. To this end, we propose two measures of CPT violation, namely,

$$R_1 = \frac{\max(|\epsilon_{ij}|)}{\max(|\mu_{ij}|)}, \quad R_2 = \max\left(\frac{\epsilon_{ij}}{\mu_{ij}}\right),$$

(6)

If it is some underlying symmetry that keeps the $\mu_{ij}$ small, we would, naively, expect it to suppress the corresponding $\epsilon_{ij}$ too and, thus, from a model-builder’s standpoint, those models where both $R_1$ and $R_2$ are less than unity are a bit more favoured. In Table 1, we also display the $R_{1,2}$ properties of the various hierarchies. As expected, rather than the normal-inverted ($\nu, \bar{\nu}$) and the inverted-normal cases, it is the normal-normal and inverted-inverted hierarchies that are associated with relatively smaller $\epsilon_{ij}$, with the $I_{321}$ combination being the best from this standpoint.

Several other features are worth noting. For BP1, the splitting between $\nu_1(\bar{\nu}_1)$ and $\nu_2(\bar{\nu}_2)$ is small compared to $\Delta m^2_{23}(\Delta M^2_{23})$. Consequently, there is an approximate $1 \leftrightarrow 2$ symmetry (compared to level 3) and we expect $\epsilon_{13}/\mu_{13} \approx \epsilon_{23}/\mu_{23}$ which is indeed satisfied to a great accuracy. For BP2 though, the situation is more complicated, and it is only for the $I_{123}$ and $I_{132}$ hierarchies that a similar relation, viz. $\epsilon_{22}/\mu_{22} \approx \epsilon_{33}/\mu_{33}$ can be found. The large extent of the MiniBooNE-allowed parameter space thus points to the difficulty in identifying underlying textures for $\epsilon_{ij}$ parameters. For example, even for $I_{321}$ alone, the point BP1 is consistent with $(\epsilon_{12}, \epsilon_{13} = 0, \epsilon_{22} \approx \epsilon_{11})$, whereas BP2 prefers $(\epsilon_{i3} = 0, \epsilon_{11} \gg \epsilon_{12}, \epsilon_{22})$. To summarise, each of the $\bar{\nu}$ disappearance results of the MINOS far detector and the recent anomalous MiniBooNE results on $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ individually argues strongly for the oscillation parameters in the $\bar{\nu}$ sector to be significantly different from those in the $\nu$ sector. Taken together, though, the two sets of derived
parameters are shown to be inconsistent with each other at the 68% C.L.; this is because the large values of $\Delta \bar{M}_{12}^2$ and $\Theta_{12}$, as preferred by MiniBooNE would induce large oscillations in $P(\bar{\nu}_\mu \rightarrow \nu_\mu)$ for large $E_{\bar{\nu}}$ whereas MINOS sees a saturating behaviour. However, at 90% C.L., the two experiments turn out to be mutually consistent. It is worth noting that an improvement of MINOS energy resolution at large $E_{\bar{\nu}}$ as well the accumulation of more statistics would lead to a much finer probe of this overlap.

Assuming consistency, the two experiments together present a very strong argument for properties of $\bar{\nu}$’s to be radically different from those of the $\nu$’s, to the extent of violating CPT invariance. We explore the possibility that the effective $\nu$– and $\bar{\nu}$–mass matrices differ on account of a CPT violating interaction. If CPT is indeed violated, there is no necessity of having identical hierarchies for $\nu$’s and $\bar{\nu}$’s. Of the twelve possible hierarchies consistent with the $\nu$-sector measurements, only some are found to be consistent with the $\bar{\nu}$ measurements as well as the cosmological constraints. Furthermore, if we demand that the (natural) condition that $\epsilon_{ij}$, the CPTV contribution to the mass matrix, be smaller than the the CPT conserving part, the choices get restricted even further. Finally, various textures for $\epsilon_{ij}$ are possible.

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