The black-hole information puzzle, first discussed by Hawking four decades ago, has attracted much attention over the years from both physicists and mathematicians. One of the most intriguing suggestions to resolve the information paradox is due to Bekenstein, who has stressed the fact that the low-energy part of the semi-classical black-hole emission spectrum is partly blocked by the curvature potential that surrounds the black hole. As explicitly shown by Bekenstein, this fact implies that the grey-body emission spectrum of a (3+1)-dimensional black hole is considerably less entropic than the corresponding radiation spectrum of a perfectly thermal black-body emitter. Using standard ideas from quantum information theory, it was shown by Bekenstein that, in principle, the filtered Hawking radiation emitted by a (3+1)-dimensional Schwarzschild black hole may carry with it a substantial amount of information, the information which was suspected to be lost. It is of physical interest to test the general validity of the “information leak” scenario suggested by Bekenstein as a possible resolution to the Hawking information puzzle. To this end, in the present paper we analyze the semi-classical entropy emission properties of higher-dimensional black holes. In particular, we provide evidence that the characteristic Hawking quanta of (D+1)-dimensional Schwarzschild black holes in the large \( D \gg 1 \) regime are almost unaffected by the spacetime curvature outside the black-hole horizon. This fact implies that, in the large-\( D \) regime, the Hawking black-hole radiation spectra are almost purely thermal, thus suggesting that the emitted quanta cannot carry the amount of (non-thermal) information which is required in order to resolve the information paradox. Our analysis therefore suggests that the elegant information leak scenario suggested by Bekenstein, which is based on the effective grey-body (rather than a black-body) nature of the black-hole emission spectra, cannot provide a generic resolution to the intriguing Hawking information paradox.
I. THE HAWKING BLACK-HOLE INFORMATION PUZZLE

The black-hole evaporation phenomenon, first predicted by Hawking [1] more than four decades ago, imposes a great challenge to our understanding of the interplay between gravity and quantum theory. In particular, Hawking’s semi-classical analysis [1] asserts that black holes which were formed from the gravitational collapse of pure quantum states will emit thermally distributed radiation and thus eventually evolve into mixed thermal states. This intriguing physical scenario is in sharp contradiction with the fundamental quantum mechanical principle of unitary temporal evolution, which asserts that pure quantum states should always remain pure as they evolve in time [2].

The incompatibility between gravity and quantum physics, which is realized most dramatically in the Hawking evaporation process of black holes, may also be discussed in terms of the fundamental principles of quantum information theory [3]. In particular, it is well known that perfectly thermal (black-body) radiation cannot convey detailed information about the physical properties of its emitting body. Thus, according to Hawking’s semi-classical analysis [1], the information hidden in the intermediate black-hole state about the initial quantum state of the collapsed matter is lost forever with the complete thermal evaporation of the black hole. This physically intriguing scenario is known as the Hawking black-hole information puzzle.

II. THE ELEGANT RESOLUTION SUGGESTED BY BEKENSTEIN

Several physical scenarios have been suggested in order to resolve the Hawking black-hole information puzzle, see e.g. [3–5] for excellent reviews. In the present paper we would like to analyze a particular intriguing resolution originally proposed by Bekenstein [3]. The possible solution suggested by Bekenstein [3] to the Hawking information paradox [1] belongs to the family of “information leak” scenarios. According to this suggested resolution, the information about the initial quantum state of the collapsed matter, which is supposed to be lost during the semi-classical Hawking evaporation process, is actually encoded into the emitted black-hole radiation quanta [3].

Specifically, Bekenstein [3] has correctly pointed out that, due to the effective curvature potential that surrounds the emitting black hole, the Hawking radiation spectrum of a (3 + 1)-dimensional Schwarzschild black hole departs from the familiar purely thermal radiation spectrum of a perfect black-body emitter. In particular, the characteristic curvature (scattering) potential of the black-hole spacetime [see Eq. (10) below] partly blocks the low-frequency part of the semi-classical Hawking emission spectrum. The departure of the black-hole radiation spectrum from the purely thermal spectrum of a perfect black-body emitter can be quantified by the dimensionless energy-dependent gray-body factors \( \Gamma(\omega) \) [3] of the composed black-hole-radiation-fields system. In particular, the low-frequency part of the semi-classical black-hole radiation spectra is known to be characterized by the simple limiting behavior [4, 7]

\[
\Gamma(\omega_{\text{BH}}) \to 0 \quad \text{for} \quad \omega_{\text{BH}} \to 0 .
\]  

The characteristic relation (1) reflects the physically interesting fact that, due to the effective curvature potential of the black-hole spacetime, the low frequency part of the Hawking black-hole emission spectra is characterized by occupation numbers which are smaller than the corresponding occupation numbers of a purely thermal black-body radiation [3].

In his highly interesting work [3], Bekenstein stressed the fact that, due to the partial backscattering of the emitted quanta by the effective curvature potential that surrounds the (3+1)-dimensional evaporating black hole, the Hawking black-hole (BH) emission spectrum is less entropic than the corresponding purely thermal emission spectrum of a perfect black-body (BB) emitter with the same radiation power \( P \). In particular, (3 + 1)-dimensional evaporating black holes are characterized by the relation [3]

\[
S_{\text{BH-radiation}}^{3D}(P) < S_{\text{BB-radiation}}^{3D}(P) .
\]  

Using standard ideas from quantum information theory, it was pointed out by Bekenstein [3] that the entropic deficiency of the semi-classical black-hole radiation spectrum, as described by the characteristic inequality (2), implies that the emitted Hawking quanta may carry with them a substantial amount of information. In particular, as intriguingly discussed by Bekenstein [3] (see also the pioneering work [8]), the maximum rate at which information can be recovered from the (non-thermal) black-hole emission spectrum is given by the difference [4]

\[
\hat{I}_{\text{max}} = S_{\text{BB-radiation}}^{3D}(P) - S_{\text{BH-radiation}}^{3D}(P)
\]  

between the entropy outflow rate \( S_{\text{BB-radiation}}^{3D} \) from a perfect black-body emitter and the corresponding entropy outflow rate \( S_{\text{BH-radiation}}^{3D} \) which characterizes the (partially filtered [10]) Hawking semi-classical radiation spectrum, both with the same radiation power [3]. Using the relation (3), Bekenstein [3] has provided compelling evidence that,
for evaporating \((3 + 1)\)-dimensional Schwarzschild black holes, the maximum information outflow \(\dot{I}_{\text{max}}^{3D} \) [as defined by (3)] may actually exceed the entropy outflow in the Hawking black-hole radiation spectrum. That is,

\[
\dot{I}_{\text{max}}^{3D} > \dot{S}_{\text{BH-radiation}}^{3D}
\]

for semi-classical \((3 + 1)\)-dimensional Schwarzschild black holes.

Recalling the general relation \(\Delta I = -\Delta S\) between information and entropy \([11, 12]\), Bekenstein \([3]\) has stressed the intriguing fact that the characteristic \((3 + 1)\)-dimensional inequality \((4)\) implies that, given an appropriate quantum mechanism which codes the information in the emitted Hawking quanta, this information may reduce the uncertainty (that is, the lack of information) about the internal quantum state of the Hawking black-hole radiation \([3]\). Thus, the black-hole radiation can, in principle, end up in a pure quantum state, in accord with the known fundamental principles of quantum physics \([2]\). This is the essence of the proposed \((3 + 1)\)-dimensional Bekenstein resolution to the Hawking information puzzle \([3]\).

III. THE BEKENSTEIN RESOLUTION: INSIGHTS FROM THE HAWKING EMISSION SPECTRA OF HIGHER-DIMENSIONAL BLACK HOLES

It is of considerable physical interest to test the general validity of the intriguing \((3 + 1)\)-dimensional conclusion reached by Bekenstein \([3]\) regarding the amount of quantum information that, in principle, can be carried by the emitted Hawking quanta. In particular, one naturally wonders whether the information-entropy inequality \(\dot{I}_{\text{max}}^{3D} > \dot{S}_{\text{BH-radiation}}^{3D}\) [see (4)], which characterizes the semi-classical radiation spectrum of a \((3 + 1)\)-dimensional Schwarzschild black hole \([3]\), is a generic property of the Hawking emission spectra of all \((D + 1)\)-dimensional Schwarzschild black holes.

In order to address this interesting physical question, in the present paper we shall analyze the semi-classical entropy emission properties of higher\(-\)dimensional black holes. In particular, below we shall provide evidence that the characteristic semi-classical Hawking radiation spectra of \((D + 1)\)-dimensional Schwarzschild black holes in the large \(D \gg 1\) regime are characterized by the opposite inequality \(\dot{I}_{\text{max}}^{(D\gg 1)} < \dot{S}_{\text{BH-radiation}}^{(D\gg 1)}\). This characteristic large\(-D\) information-entropy relation implies that the emitted Hawking quanta of higher-dimensional Schwarzschild black holes in the large \(D \gg 1\) regime cannot carry the amount of information which is required in order to solve the intriguing Hawking paradox. Our analysis (to be presented below) therefore suggests that the elegant “information leak” scenario proposed by Bekenstein \([3]\) more than two decades ago cannot provide a generic resolution \([13]\) to the Hawking information puzzle.

We first recall that the energy emission rate per one bosonic degree of freedom out of a \((D+1)\)-dimensional black hole is given by the integral relation \([1, 6, 14, 15]\)

\[
P_{\text{BH}}(D) = \frac{\hbar}{2^{D-1}\pi^{D/2}\Gamma(D/2)} \sum_j \int_0^\infty \frac{\omega^D d\omega}{e^{\omega/T_{\text{BH}}}-1},
\]

where \(j\) stands for the dimensionless angular momentum indices of the emitted Hawking quanta and \(\Gamma = \Gamma(\omega; j, D)\) are the energy-dependent grey-body factors of the composed black-hole-radiation-fields system \([6]\). These dimensionless barrier penetration factors quantify the imprint of passage of the emitted Hawking quanta through the effective curvature potential that surrounds the radiating black hole. The physical parameter

\[
T_{\text{BH}} = \frac{(D - 2)\hbar}{4\pi r_H},
\]

in \([16, 17]\) is the characteristic Bekenstein-Hawking temperature of the evaporating \((D+1)\)-dimensional Schwarzschild black hole \([16, 17]\).

As pointed out in \([18]\), the \((D+1)\)-dimensional thermal distribution \(\omega^D/(e^{\omega/T_{\text{BH}}}-1)\) [see Eq. \((5)\)] has a sharp peak at the characteristic black-hole emission frequency

\[
\omega_{\text{peak}}(D) = \frac{DT}{\hbar}[1 + O(e^{-D})].
\]

Substituting the semi-classical Bekenstein-Hawking temperature \([16]\) of the evaporating \((D + 1)\)-dimensional black holes into \((7)\), one finds the strong inequality

\[
\omega(D) \times r_H = \frac{D^2}{4\pi}[1 + O(D^{-1})] \gg 1
\]
for the characteristic Hawking quanta emitted by the \((D+1)\)-dimensional Schwarzschild black holes in the large \(D \gg 1\) regime.

The strong inequality \([8]\) implies that, in the large \(D \gg 1\) regime, the typical wavelengths in the Hawking black-hole emission spectra are very short on the length-scale set by the spacetime curvature \([8]\). This physically interesting fact suggests that, in the large-\(D\) regime, the emitted Hawking quanta are almost unaffected by the spacetime curvature outside the black-hole horizon.

In particular, it is important to emphasize the fact that the dynamics of the emitted Hawking fields in the curved black-hole spacetime is governed by the Schrödinger-like differential equation \([1, 6, 14, 15]\)

\[
\left(\frac{d^2}{dr_*^2} + \omega^2 - V_{BH}\right)\phi = 0 ,
\]

where \(r_*\) is the familiar ‘tortoise’ radial coordinate \([14, 15]\) and, for a massless perturbation field of dimensionless angular harmonic index \(l\), the effective \((D+1)\)-dimensional black-hole curvature potential in \([8]\) is given by \([15, 19]\)

\[
V(r; D) = \left[ 1 - \left(\frac{r_H}{r}\right)^{D-2} \right] \left[ l(l + D - 2) + \frac{(D-1)(D-3)}{4r^2} \right] + \frac{(1 - p^2)(D-1)^2r_H^{D-2}}{4r^{D-2}}.
\]

Taking cognizance of the fact that the effective curvature potential \([10]\) of the \((D+1)\)-dimensional black-hole spacetime is characterized by the asymptotic large \(D \gg 1\) behavior \([8]\)

\[
\omega^2 \frac{V_{BH}^{\max}}{V_{BH}^{\max}(D \gg 1)} = O\left(\frac{D^2}{r_H^2}\right),
\]

one concludes that the typical emitted field quanta that constitute the Hawking black-hole radiation spectra are characterized by the dimensionless strong inequality [see Eqs. \([8]\) and \([11]\)] \([18]\)

\[
\omega^2 \frac{V_{BH}^{\max}}{V_{BH}^{\max}(D \gg 1)} = O(D^2) \gg 1 \quad \text{for} \quad D \gg 1
\]

in the large \(D \gg 1\) regime.

The characteristic strong inequality \([12]\) reflects the physically intriguing fact that, in the large \(D \gg 1\) regime, the propagation of the emitted Hawking fields in the black-hole exterior region is practically unaffected by the curvature potential [see Eqs. \([9]\) and \([12]\)]. In particular, this characteristic large-\(D\) behavior can be quantified by the compact dimensionless relation

\[
\Gamma(\omega_{peak}^{peak}_{BH}; D \to \infty) \to 1^{-}
\]

for the barrier penetration factors (grey-body factors) of the composed \((D+1)\)-dimensional black-hole-radiation-fields system.

The large-\(D\) asymptotic behavior \([13]\) implies that higher-dimensional evaporating black holes in the large \(D \gg 1\) regime are characterized by almost perfect black-body (thermal) emission spectra. In particular, this physically interesting fact can be quantified by the dimensionless large-\(D\) relation

\[
\hat{S}_{BB}^{\text{radiation}}(P; D \to \infty) = \left(\frac{8\pi}{e}\right)^{1/2} \left(\frac{P}{4\pi}\right)^{(D+1)/2} \times \left(\frac{P}{\hbar}\right)^{1/2}
\]

is the characteristic entropy emission rate out of a \((D+1)\)-dimensional perfect black-body (thermal) emitter with a given radiation power \(P\).

Interestingly, and most importantly for our analysis, the compact dimensionless ratio \([14]\) yields the large-\(D\) asymptotic relation

\[
\hat{I}_{\text{max}}(D \to \infty) = \hat{S}_{BB}^{\text{radiation}}(P) - \hat{S}_{BH}^{\text{radiation}}(P) \to 0
\]

for the maximum rate at which information can be recovered from the Hawking black-hole radiation spectra in the large \(D \gg 1\) limit. As emphasized above (see, in particular, the discussion in Sec. II), a thermally distributed radiation spectrum, which is characterized by the relation \([10]\), cannot carry with it the missing information about the initial quantum state of the collapsed matter fields.
IV. SUMMARY AND DISCUSSION

One of the most promising solutions to the Hawking information puzzle [1] has been raised by Bekenstein more than two decades ago [3]. In his physically intriguing work [3], Bekenstein has stressed the fact that the black-hole emission spectrum is partly blocked by the effective curvature potential [see Eq. (10)] that surrounds the emitting black hole. This simple fact implies that the Hawking emission spectrum of a (3+1)-dimensional black hole is considerably less entropic than the corresponding radiation spectrum of a perfectly thermal black-body emitter [3]. Using standard ideas from quantum information theory, it was shown by Bekenstein [3] that the filtered Hawking radiation emitted by a (3+1)-dimensional Schwarzschild black hole may, in principle, carry with it a substantial amount of information, the information which was suspected to be lost. In particular, a (3 + 1)-dimensional black hole is characterized by the relation [see Eq. (4)]

\[ \dot{I}^{3D}_{\text{max}} > \dot{S}_{\text{BH-radiation}}. \]  

(17)

One naturally wonders whether the information-entropy relation (17), which characterizes the Hawking emission spectrum of a (3 + 1)-dimensional Schwarzschild black hole [3], is a generic property of the radiation spectra of all (D + 1)-dimensional Schwarzschild black holes? In order to address this physically interesting question, in the present paper we have analyzed the entropy emission properties of semi-classical higher-dimensional black holes.

In particular, we have examined the intriguing “information leak” resolution suggested by Bekenstein in the context of higher-dimensional gravitational theories. Taking cognizance of the fact that (D + 1)-dimensional Schwarzschild black holes in the large D \( \gg 1 \) regime behave as almost perfect black-body emitters [see Eqs. (12) and (13)], we have stressed the interesting fact that the Hawking radiation spectra of these higher-dimensional black holes are characterized by the inequality

\[ \dot{I}_{\text{max}}(D \gg 1) < \dot{S}_{\text{BH-radiation}}(D \gg 1). \]  

(18)

The characteristic relation (18) implies, in particular, that the emitted Hawking quanta of the (D + 1)-dimensional Schwarzschild black holes in the large D \( \gg 1 \) regime cannot carry the missing information about the initial quantum state of the collapsed matter fields.

Our compact analysis therefore suggests that, for higher-dimensional evaporating black holes in the large D \( \gg 1 \) regime, the elegant “information leak” scenario proposed by Bekenstein [3] more than two decades ago cannot provide a generic [13] solution to the Hawking information puzzle.

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Here $r_H$ is the black-hole horizon radius, which is given by the $(D + 1)$-dimensional relation $r_H = [16\pi M / (D - 1) A_{D-1}]^{1/(D - 2)}$, where $M$ is the ADM mass of the black-hole spacetime, and $A_{D-1} = 2\pi^{D/2} / \Gamma(D/2)$ is the generalized area of a unit $(D - 1)$-dimensional sphere.

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