Transformations between the WISE, 2MASS, SDSS and BVRI photometric systems – I. Transformation equations for dwarfs

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ABSTRACT

We present colour transformations for the conversion of the W1 and W2 magnitudes of the Wide-field Infrared Survey Explorer (WISE) photometric system to the Johnson–Cousins BVRI, Sloan Digital Sky Survey gri and Two-Micron All-Sky Survey JHKₜ photometric systems, for dwarfs. The W3 and W4 magnitudes were not considered due to their insufficient signal-to-noise ratio (S/N). The coordinates of 825 dwarfs along with their BVRI, gri and JHKₜ data taken from Bilir et al. were matched with the coordinates of stars in the preliminary data release of the WISE, and a homogeneous dwarf sample with high S/N had been obtained using the following constraints: (1) the data were dereddened; (2) giants were identified and excluded from the sample; (3) sample stars were selected according to data quality; (4) transformations were derived for subsamples of different metallicity ranges; and (5) transformations were twocolour-dependent. These colour transformations, coupled with known absolute magnitudes at shorter wavelengths, can be used in space-density evaluation for the Galactic (thin and thick) discs, at distances larger than the ones evaluated with JHKₜ photometry.

Key words: techniques: photometric – catalogues – surveys.

1 INTRODUCTION

All sky surveys from X-ray to radio regions of the electromagnetic spectrum have great impact on our understanding of the Galactic structure and also changed our view of the Universe dramatically. While surveys in optical wavelengths give detailed information about Galactic haloes, longer wavelength surveys give us useful results for the Galactic disc. Recent photometric surveys such as the Sloan Digital Sky Survey (SDSS; York et al. 2000), Two-Micron All-Sky Survey (2MASS; Skrutskie et al. 2006) and Wide-field Infrared Survey Explorer (WISE; Wright et al. 2010) are used to classify objects in colour spaces, while spectroscopic surveys such as the SDSS and RADial Velocity Experiment (Steinmetz et al. 2006) are used to determine stellar atmospheric parameters. Analysis of astrometric, radial velocity and stellar atmospheric data, especially in the solar neighbourhood, allows us to understand the structure, formation and evolution of the Galaxy. Deep sky surveys are powerful tools in examining the far side of our Galaxy. However, information on nearby space can be obtained from shallow wavelength surveys. Therefore, for a complete understanding of the whole Galaxy, multiple systems are needed. This implies the use of transformation equations between various photometric systems.

The SDSS obtains images almost simultaneously in five broad bands (u, g, r, i and z) centred at 3540, 4760, 6280, 7690 and 9250 Å, respectively (Fukugita et al. 1996; Gunn et al. 1998; Hogg et al. 2001; Smith et al. 2002). The magnitudes derived from fitting a point spread function are currently accurate to about 2 per cent in g, r and i, and 3–5 per cent in u and z for bright (~20 mag) point sources. The data have been made public in a series of yearly data releases where the eighth data release (Aihara et al. 2011) covers 14 555 deg² of the imaging area. The limiting magnitudes are (u, g, r, i, z) = (22, 22.2, 22.2, 21.3, 20.5). The data are saturated at about 14 mag in g, r and i, and at about 12 mag in u and z (see Bilir et al. 2008, for more detail).

The 2MASS provides the most complete data base of near-infrared (near-IR) Galactic point sources available to date. Observations cover 99.998 per cent (Skrutskie et al. 2006) of the sky with simultaneous detections in the J (1.25 μm), H (1.65 μm) and Ks (2.17 μm) bands up to the limiting magnitudes of 15.8, 15.1 and 14.3, respectively. Bright source extractions have 1σ photometric uncertainty of <0.03 mag and astrometric accuracy of the order of 100 mas (see Bilir et al. 2008, for more detail).

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Another IR survey was made by the *Spitzer Space Telescope* which is one of the space-baring observatories (Werner et al. 2004). *Spitzer* has various instruments onboard and one of them is the Infrared Array Camera (IRAC) which was used for the Galactic Legacy Mid-Plane Survey Extraordinaire (GLIMPSE) programme. The GLIMPSE survey was devised to obtain images of IR sources towards the inner Galactic plane at 3.6, 4.5, 5.8 and 8.0 μm with angular resolution between 1.4 and 1.9 arcsec using the IRAC onboard the *Spitzer Space Telescope* (Churchwell et al. 2001; Benjamin et al. 2005).

The *WISE*, an up-to-date IR survey, began surveying the sky on 2010 January 14 and completed its first full coverage of the sky on 2010 July 17 with much higher sensitivity than comparable previous IR survey missions (Wright et al. 2010). The *WISE* has four IR filters, W1, W2, W3 and W4, centred at 3.4, 4.6, 12 and 22 μm, with the angular resolutions 6.1, 6.4, 6.5 and 12.0 arcsec, respectively, and has a 40-cm telescope feeding array with a total of four million pixels. The increased number of detectors leads to a much higher sensitivity: the *WISE* has achieved 5σ point source sensitivities better than 0.08, 0.11, 1 and 6 mJy at 3.4, 4.6, 12 and 22 μm, respectively. These sensitivities correspond to the Vega magnitudes 16.5, 15.5, 11.2 and 7.9. Thus, the *WISE* will go a magnitude deeper than the 2MASS *K* data in W1 for sources with spectra close to that of an A0 star, and even deeper for moderately red sources like K stars or galaxies with old stellar populations. *WISE* filters fill the wavelength gap between the 2MASS (Skrutskie et al. 2006) at 1.2–2.2 μm and AKARI mission (Murakami et al. 2007) which used an ingenious technique to survey the middle-IR sky at 9 and 18 μm with the sensitivities of 50 and 100 mJy (Ishihara et al. 2010), respectively, and with better angular resolution than the InfraRed Astronomical Satellite (Neugebauer et al. 1984; Beichman et al. 1988).

The scientific goals of the survey are the Solar system objects (asteroids, comet trails and zodiac bands), solar neighbourhood objects (dwarf and giant stars, brown dwarfs, young stars and debris discs, interstellar dust) and the majority of the extragalactic objects (the most ultraluminous Seyferts, starburst galaxies and quasars) in the Universe. The preliminary data (released on 2011 April 14) cover 57 per cent of the sky which are from the first 105 days of *WISE* survey observations. A source catalogue contains positional and photometric information for over 257 million objects detected on the *WISE* images, and an Explanatory Supplement that serves as a user’s guide to the *WISE* mission and format, content, characteristics and cautionary notes for the release products.

In Bilir et al. (2008), we presented the transformations between the *BVRI*, SDSS and 2MASS photometric systems. The passband profiles for the *BVRI*, *ugriz*, *JHK*, and W1 W2 W3 W4 photometric systems are given in Fig. 1. Here we add a middle-IR system, *WISE*, to this photometric set and we follow the same procedure with slight modifications: as the transformations were planned for dwarfs, the W3 and W4 magnitudes were not used due to low energy of dwarfs in those bands. The transformations are metallicity-dependent and are a function of two colours, similar to Bilir et al. (2008). However, in the inverse transformations, that is, from the *WISE* system to *BVRI* and *gri*, the *J − H* colour of the 2MASS photometric system is used as a second colour combined linearly with *W1 − W2*. Thus, the *J − H* colour filled the gap of W3 − W4. Such a modification seems reasonable, as both the *J − H* and the W3 − W4 colours correspond to the IR photometric systems.

In Section 2, we present the sources of our star sample and the criteria applied to the chosen stars. The transformation equations are given in Section 3. Finally, in Section 4, we give a summary and conclusion.

2 DATA

The data used for our transformations were taken from Bilir et al. (2008) and the preliminary data release of the *WISE* (Wright et al. 2010). Bilir et al. (2008) used the original *BVRI*, *gri* and *JHK*, magnitudes taken from Stetson (2000), Saha et al. (2005), Adelman-McCarthy et al. (2007) and Cutri et al. (2003), respectively, and formed a catalogue of 825 dwarfs (the identification of dwarfs is explained in Bilir et al. 2008). We matched the coordinates of these stars in 10 fields (NGC 2419 and 2420, Draco, NGC 2683 and 3031, L106, L107, Pal 5, PG1633 and M92) investigated by Stetson (2000) and mapped by the *WISE*, and revealed that 311 out of the 825 stars appeared in the catalogue of Bilir et al. (2008) and preliminary data release of the *WISE*.¹ We confirmed the overlapping

¹ http://irsa.ipac.caltech.edu/cgi-bin/Gator/nph-scan?mission=irsa&submit=Select&projshort=WISE_PRELIM
of 311 stars by comparing their $J$ magnitudes which appeared in two different sources. We reduced the number of stars to 289 whose magnitudes were of best quality (AA) for $W1$ and $W2$, and we had to omit all $W3$ and $W4$ magnitudes due to their insufficient quality. A further reduction has been carried out for 72 stars whose $B - V$ and $R - I$ colours were missing in the catalogue of Bilir et al. (2008). The data of our final catalogue for 289 stars are given in Table 1 in electronic format. The positions of the fields (total 27) investigated by Stetson (2000) and those mapped by the WISE (10 fields) are shown in the equatorial coordinate system in Fig. 2(a). Stetson’s fields with WISE data are also shown in the Galactic coordinate system in Fig. 2(b) and the corresponding reduced $E(B - V)$ colour excesses taken from Schlegel, Finkbeiner & Davis (1998) are shown in Fig. 2(c).

The mean errors, standard deviations and the number of stars available for magnitudes for four photometric systems are given in Table 2. The mean errors are less than 1 and 2 per cent for the magnitudes $BVRI$ and $gri$, respectively, whereas they lie between 3 and 6 per cent for near- and middle-IR photometric systems.

### 2.1 Reddening and metallicity

Dereddening of the magnitudes was carried out through the following procedure. First, we used the $R_{I}/R_{J}$ data of Cardelli, Clayton & Mathis (1989) and obtained a spline function for the IR wavelengths between 2 and 35 μm. Then, we used the colour excess $E(B - V)$ values from Schlegel et al. (1998) at appropriate Galactic latitudes to evaluate the total extinction $R_{I}$ for the wavelengths 3.4, 4.6, 12 and 22 μm corresponding to the bands $W1$, $W2$, $W3$ and $W4$, that is, $0.158$, $0.093$, $0.087$ and $0.056$ mag, respectively (Fig. 3). Thus, the magnitudes $W1$ and $W2$ were dereddened by $0.158$ and $0.093$ mag, respectively. The $BVRI$, $gri$ and $JHK_{s}$ magnitudes had already been dereddened by Bilir et al. (2008). The distribution of errors for 12 magnitudes is shown in Fig. 4 as a function of intrinsic colours.

The transformation formulae are given as a function of metallicity. For this purpose, we used the procedure of Karaali, Bilir & Tuncel (2005) and determined the metallicities of 289 stars (Table 3). It turned out that 177 stars of our sample are metal rich ($-0.4 < [M/H] \leq +0.2$ dex), 43 stars are of intermediate metallicity ($-1.2 < [M/H] \leq -0.4$ dex) and 69 stars are metal poor ($-3 < [M/H] \leq -1.2$ dex).

### Table 1. Johnson–Cousins, SDSS, 2MASS and WISE magnitudes and colours of the sample stars (in total 289 stars). The columns give: (1) star name; (2) Galactic coordinates; (3) $V$ – apparent magnitude; (4) and (5) $(B - V)$ and $(R - I)$ colour indices, respectively; (7) $g$ – apparent magnitude; (8) and (9) $(g - r)$ and $(r - i)$ colour indices, respectively; (10) $J$ – apparent magnitude; (11) and (12) $(J - H)$ and $(H - K_{s})$ colour indices, respectively; (13) $W1$ – apparent magnitude; (14) $(W1 - W2)$ colour index; and (15) reduced $E_{d}(B - V)$ colour excess. The complete table is available in electronic format (see Supporting Information).

| Star     | $l$ (°) | $b$ (°) | $V$  | $(B - V)$ | $(R - I)$ | $g$  | $(g - r)$ | $(r - i)$ | $J$  | $(J - H)$ | $(H - K_{s})$ | $W1$ | $(W1 - W2)$ | $E_{d}(B - V)$ |
|----------|---------|---------|------|-----------|-----------|------|-----------|-----------|------|-----------|----------------|------|--------------|---------------|
| Pal5-S10 | 0.78766 | 45.8528 | 16.742 | 0.936 | 17.209 | 0.765 | 0.337 | 14.782 | 0.447 | 0.059 | 14.162 | -0.023 | 0.058 |
| Pal5-S13 | 0.79079 | 45.8505 | 16.499 | 0.693 | 16.824 | 0.553 | 0.241 | 14.941 | 0.442 | -0.218 | 14.430 | -0.064 | 0.058 |
| Pal5-S14 | 0.79423 | 45.8522 | 17.798 | 1.120 | 18.424 | 1.072 | 0.417 | 15.520 | 0.643 | 0.191 | 14.795 | -0.049 | 0.058 |

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L106-S10 351.06012 51.84716 14.892 0.760 15.227 0.561 0.219 13.444 0.517 0.041 12.886 0.039
L106-S13 351.06807 51.81729 15.321 0.618 15.566 0.426 0.148 14.069 0.297 -0.007 13.628 -0.013
L106-S11 351.07185 51.84046 16.234 0.726 16.550 0.548 0.208 14.771 0.429 0.117 14.237 0.035

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Table 2. Mean errors, standard deviations and number of stars for the filters of the Johnson–Cousins, SDSS, 2MASS and WISE photometries.

| Filter | Mean error (mag) | $s$ | $N$ | Photometry |
|--------|-----------------|-----|-----|------------|
| $B$    | 0.0071          | 0.0045 | 246 | BVRI       |
| $V$    | 0.0042          | 0.0028 | 281 |            |
| $R$    | 0.0082          | 0.0044 | 263 |            |
| $I$    | 0.0069          | 0.0039 | 271 |            |
| $g$    | 0.0145          | 0.0046 | 289 | SDSS       |
| $r$    | 0.0136          | 0.0055 | 289 |            |
| $i$    | 0.0144          | 0.0036 | 289 |            |
| $J$    | 0.0356          | 0.0106 | 289 | 2MASS      |
| $K_s$  | 0.0591          | 0.0227 | 289 |            |
| $W1$   | 0.0322          | 0.0066 | 289 | WISE       |
| $W2$   | 0.0513          | 0.0018 | 289 |            |

3 RESULTS

3.1 Transformations between the WISE and Johnson–Cousins photometries

We used the following general equations and derived four sets of transformations between the $W1$ and $W2$ magnitudes of the WISE and Johnson–Cousins $BVRI$. We did not consider the transformations for $W3$ and $W4$ due to the reason explained in Section 2. The transformation sets are as follows: transformations for the whole sample, metal-rich stars, intermediate-metallicity stars and metal-poor stars. The definitions for the last three subsamples are already given in the previous section. The general equations are

$$ (V - W1)_0 = a_1(B - V)_0 + b_1(B - V)_0(R - I)_0 + c_1(R - I)_0 + d_1, $$

(1)

$$ (V - W2)_0 = a_2(B - V)_0 + b_2(B - V)_0(R - I)_0 + c_2(R - I)_0 + d_2. $$

(2)

The transformation equations consist of two colours but three terms. The term $b_i(B - V)_0(R - I)_0$ ($i = 1, 2$) provides more accurate colours and residuals less than the ones obtained considering only two terms. The numerical values of the coefficients $a_i$, $b_i$, $c_i$ and $d_i$ ($i = 1, 2$) for the four sets are given in columns (1) and (2) of Table 4. The fifth and sixth numbers in each column are the squared correlation coefficient and the standard deviation for the colour indicated at the top of the column, respectively. There are similarities between the values of the coefficients evaluated for different samples, except the ones for intermediate-metallicity stars. The metallicity distribution of these subsamples (−0.4 < [M/H] ≤ +0.2, −1.2 < [M/H] ≤ −0.4 and −3.0 < [M/H] ≤ −1.2 dex) resembles the metallicity ranges for the Galactic (thin and thick) discs and the halo, respectively. Thus, the transformations are luminosity-dependent and metallicity-dependent.

3.2 Transformations between the WISE and SDSS photometries

The transformations between the $W1$ and $W2$ magnitudes of the WISE and SDSS have similar general equations and are as follows:

$$ (g - W1)_0 = a_3(g - r)_0 + b_3(g - r)_0(r - i)_0 + c_3(r - i)_0 + d_3, $$

(3)

$$ (g - W2)_0 = a_4(g - r)_0 + b_4(g - r)_0(r - i)_0 + c_4(r - i)_0 + d_4. $$

(4)

The numerical values of the coefficients $a_i$, $b_i$, $c_i$ and $d_i$ ($i = 3, 4$) for the four sets defined above are given in columns

Figure 3. Typical interstellar extinction curve for the mid-IR normalized to $R_V = 3.1$ and the passbands of the WISE filters. The filled circles correspond to the values taken from Cardelli et al. (1989) and the solid line represents the spline fit to the data. The dashed lines denote the positions of the effective wavelengths of the WISE filters and the open circles correspond to the extinction values of the WISE filters.
Table 3. Metallicity distribution of the sample. Stars with \((g - r)_0 > 0.95\) mag were assumed to have a metallicity of \(-0.4 < [M/H] \leq +0.2\) dex.

| Metallicity (dex) | Number of stars |
|-------------------|-----------------|
| \(-0.4 < [M/H] \leq +0.2\) | 177 |
| \(-1.2 < [M/H] \leq -0.4\) | 43 |
| \(-3.0 < [M/H] \leq -1.2\) | 69 |

3.3 Transformations between the WISE and 2MASS photometries

The transformations between the \(W1\) and \(W2\) magnitudes of the WISE and 2MASS have similar general equations and are as follows:

\[
(J - W1)_0 = a_1(J - H)_0 + b_1(J - H)_0(H - K)_0 + c_1(H - K)_0 + d_1 ,
\]

\[
(J - W2)_0 = a_0(J - H)_0 + b_0(J - H)_0(H - K)_0 + c_0(H - K)_0 + d_0 .
\]

The numerical values of the coefficients \(a_i, b_i, c_i\) and \(d_i\) for the four sets defined above are given in columns (5) and (6) of Table 4. As it was obtained both for Johnson–Cousins and for SDSS photometries, the transformations between the magnitudes \(W1\) and \(W2\) of the WISE and 2MASS are luminosity-dependent, metallicity-dependent and two-colour-dependent.

3.4 Residuals

We compared the observed colours and those evaluated via equations (1)–(6) to analyse the residual distribution. As seen in Table 5, the mean of the residuals is rather small. The residuals are plotted versus the observed \((B - V)_0, (g - r)_0\) or \((J - H)_0\) colour in Fig. 5. Although the number of stars is not the same in each panel, there is no systematic deviation from the zero-point in any panel. However, the ranges of the residuals for different colours are not the same. Those for \(V - W1\), \(g - W1\) and \(J - W1\) are smaller than those for \(V - W2\), \(g - W2\) and \(J - W2\). This indicates that the \(W1\) (absolute) magnitudes evaluated via the transformations given above would be more accurate than those for \(W2\).

3.5 Inverse transformation formulae

As it was explained before, the W3 and W4 magnitudes cannot be used for the star sample. Hence, we adopted the following procedure to get the inverse transformations with two colours: by combining linearly the near- and mid-IR colours, that is, \((J - H)_0\) and \((W1 - W2)_0\), we transformed them to the optical colours, that is, \((B - V)_0, (r - i)_0\) and \((r - i)_0\). Additionally, in Table 6, we separated the stars in the four metallicity ranges defined above into bins and used the locus of each bin for the numerical evaluation of the coefficients in the inverse transformations, given as follows:

\[
(B - V)_0 = a_1(J - H)_0 + b_1(W1 - W2)_0 + c_1 + \gamma_1,
\]

\[
(R - I)_0 = a_2(J - H)_0 + b_2(W1 - W2)_0 + c_2 + \gamma_2,
\]

\[
(g - r)_0 = a_3(J - H)_0 + b_3(W1 - W2)_0 + c_3 + \gamma_3,
\]

\[
(r - i)_0 = a_4(J - H)_0 + b_4(W1 - W2)_0 + c_4 + \gamma_4 .
\]
Table 4. Coefficients $a_i$, $b_i$, $c_i$ and $d_i$ for the transformation equations (1)–(6) in column matrix form for the four different metallicity ranges. The subscript $i = 1, 2, 3, 4, 5$ and 6 corresponds to the column number. The numerical values in the fifth, sixth and seventh lines of each metallicity range are the squared correlation coefficients ($R^2$), the standard deviations ($s$) and number of stars ($N$), respectively.

| [M/H] (dex) | $(V - W1)_b$ | $(V - W2)_b$ | $(g - W1)_b$ | $(g - W2)_b$ | $(J - W1)_b$ | $(J - W2)_b$ |
|------------|---------------|---------------|---------------|---------------|---------------|---------------|
| $[-3, +0.2]$ | $a_1$ 1.754 ± 0.048 | 1.596 ± 0.056 | 1.987 ± 0.031 | 1.916 ± 0.037 | 1.246 ± 0.041 | 1.225 ± 0.064 |
| | $b_1$ −0.665 ± 0.161 | −0.395 ± 0.136 | −1.161 ± 0.107 | −0.971 ± 0.127 | 0.599 ± 0.313 | 1.458 ± 0.491 |
| | $c_1$ 2.742 ± 0.200 | 2.549 ± 0.236 | 3.487 ± 0.176 | 3.418 ± 0.207 | 0.377 ± 0.159 | 0.025 ± 0.249 |
| | $d_1$ −0.214 ± 0.061 | −0.189 ± 0.072 | 0.579 ± 0.022 | 0.519 ± 0.026 | 0.127 ± 0.018 | 0.052 ± 0.029 |
| | $R^2$ 0.991 | 0.989 | 0.994 | 0.999 | 0.906 | 0.823 |
| | $s$ 0.096 | 0.113 | 0.100 | 0.118 | 0.079 | 0.125 |
| | $N$ 217 | 217 | 289 | 289 | 289 | 289 |
| $[-0.4, +0.2]$ | $a_1$ 1.737 ± 0.061 | 1.565 ± 0.070 | 1.975 ± 0.034 | 1.903 ± 0.040 | 1.345 ± 0.048 | 1.358 ± 0.084 |
| | $b_1$ −0.669 ± 0.167 | −0.231 ± 0.192 | −1.136 ± 0.138 | −0.873 ± 0.162 | −0.398 ± 0.407 | 0.356 ± 0.709 |
| | $c_1$ 2.782 ± 0.287 | 2.305 ± 0.330 | 3.454 ± 0.220 | 3.283 ± 0.258 | 0.878 ± 0.224 | 0.720 ± 0.390 |
| | $d_1$ −0.230 ± 0.085 | −0.139 ± 0.098 | 0.594 ± 0.027 | 0.534 ± 0.031 | 0.073 ± 0.023 | −0.020 ± 0.040 |
| | $R^2$ 0.992 | 0.990 | 0.996 | 0.994 | 0.928 | 0.838 |
| | $s$ 0.102 | 0.118 | 0.100 | 0.118 | 0.078 | 0.137 |
| | $N$ 120 | 120 | 177 | 177 | 177 | 177 |
| $[-1.2, −0.4]$ | $a_1$ 2.443 ± 0.502 | 2.225 ± 0.600 | 2.157 ± 0.274 | 2.316 ± 0.329 | 0.735 ± 0.198 | 0.474 ± 0.227 |
| | $b_1$ −1.768 ± 1.086 | −1.482 ± 1.296 | −2.087 ± 1.022 | −2.370 ± 1.229 | 2.242 ± 1.705 | 4.629 ± 1.953 |
| | $c_1$ 2.731 ± 0.731 | 2.651 ± 0.873 | 3.669 ± 0.614 | 3.490 ± 0.738 | −0.371 ± 0.542 | −1.153 ± 0.620 |
| | $d_1$ −0.383 ± 0.287 | −0.366 ± 0.342 | 0.512 ± 0.110 | 0.408 ± 0.133 | 0.301 ± 0.069 | 0.308 ± 0.079 |
| | $R^2$ 0.958 | 0.939 | 0.969 | 0.954 | 0.634 | 0.572 |
| | $s$ 0.075 | 0.089 | 0.079 | 0.095 | 0.071 | 0.081 |
| | $N$ 37 | 37 | 43 | 43 | 43 | 43 |
| $[-3, −1.2]$ | $a_1$ 1.567 ± 0.329 | 1.568 ± 0.410 | 1.847 ± 0.223 | 1.828 ± 0.262 | 1.084 ± 0.072 | 1.019 ± 0.103 |
| | $b_1$ 0.078 ± 0.669 | −0.086 ± 0.835 | −0.841 ± 0.823 | −1.080 ± 0.967 | −0.222 ± 0.730 | 1.051 ± 1.046 |
| | $c_1$ 1.865 ± 0.628 | 2.166 ± 0.783 | 3.451 ± 0.750 | 3.782 ± 0.881 | 0.380 ± 0.331 | −0.165 ± 0.474 |
| | $d_1$ 0.090 ± 0.252 | −0.065 ± 0.314 | 0.639 ± 0.128 | 0.538 ± 0.150 | 0.220 ± 0.033 | 0.169 ± 0.048 |
| | $R^2$ 0.972 | 0.957 | 0.971 | 0.961 | 0.861 | 0.759 |
| | $s$ 0.087 | 0.109 | 0.107 | 0.126 | 0.061 | 0.087 |
| | $N$ 60 | 60 | 69 | 69 | 69 | 69 |

The numerical values of the coefficients $a_i$, $b_i$ and $c_i$ ($i = 1, 2, 3, 4, 5$ and 6) for the four metallicity ranges are given in Table 7.

4 SUMMARY AND CONCLUSION

We presented the colour transformations for the conversion of the $W1$ and $W2$ magnitudes of the *WISE* into three photometric systems: Johnson–Cousins $BVRi$, SDSS $gri$ and 2MASS for dwarfs. The $W3$ and $W4$ magnitudes were not included due to their insufficient photometric quality (however, transformations to convert all magnitudes into the three photometric systems used here will be the second subject for discussion in a forthcoming paper). The following constraints were applied to the sample taken from Bilir et al. (2008) to obtain the most accurate transformations: (1) the data were re-determined; (2) giants had been identified and excluded from the sample; (3) sample stars had been selected according to data quality; (4) transformations had been derived for subsamples of different metallicity ranges; and (5) transformations were two-colour-dependent.

The squared correlation coefficients ($R^2$) for the transformations carried out for the four categories (the whole sample, metal-rich, intermediate-metallicity and metal-poor stars) are rather high with three exceptions. $R^2$ is equal to 0.634 and 0.572 in the transformations of $(J - W1)_b$ and $(J - W2)_b$ for the intermediate-metallicity stars, respectively, while it is as small as 0.759 in the $(J - W2)_b$ transformation for the metal-poor stars.

The transformation equations have been designed with two colours but with three terms (plus the constant), that is, each transformation equation consists of a linear combination of two colours plus a quadratic term corresponding to their multiplication. The coefficients of the linear colour terms in most transformation equations are compatible with each other, indicating that transformations are two-colour-dependent indeed. The coefficient of the quadratic term in the equations (1)–(6) is smaller than the ones...
Figure 5. Colour residuals for different metallicity ranges: (a) for the whole sample, $-3 < [\text{M/H}] \leq 0.2$; (b) for high metallicity, $-0.4 < [\text{M/H}] \leq 0.2$; (c) for intermediate metallicity, $-1.2 < [\text{M/H}] \leq -0.4$; and (d) for low metallicity, $-3 < [\text{M/H}] \leq -1.2$ dex. The notation used is $\Delta (\text{colour}) = (\text{evaluated colour}) - (\text{adopted colour})$.

in the linear terms. However, it provides a better $R^2$ and smaller residuals.

There are similarities between the corresponding coefficients for metal-rich and metal-poor stars, whereas the coefficients for intermediate-metallicity stars are different for other categories. Hence, we can conclude that transformations are metallicity-dependent. Additionally, it is also remarkable that the metallicity ranges for the three subsamples, $-0.4 < [\text{M/H}] \leq 0.2$, $-1.2 < [\text{M/H}] \leq -0.4$ and $-3 < [\text{M/H}] \leq -1.2$ dex, correspond to the metallicity values assumed for the thin disc, thick disc and halo, respectively. Hence, the transformations are also luminosity-dependent.

The WISE will go a magnitude deeper than the 2MASS $K_s$ data in $W1$ for sources with spectra close to that of an A0 star, and even deeper for K and M spectral-type stars. The present transformations can be applied to stars with known absolute $V$, $g$ or $J$ magnitudes, and absolute magnitudes for $W1$ can be provided. We can use these two advantages to investigate the Galactic (thin and thick) discs more accurately and, for example, to find the answers to some of the following questions: do the late-type dwarfs, such as brown dwarfs, obey the two-exponential space-density law for discs that we use today? If so, will the Galactic model parameters that would be estimated with the addition of the brown dwarfs and the ones appearing in the literature be similar? The answers to such questions will be helpful in better understanding of the Galactic disc structure. Moreover, the different metallicity ranges correspond to different Galactic populations, making the relationships a powerful tool in disentangling the Galactic structure. They will be rendered even more powerful when a similar relation is obtained for giant stars, discussed in Paper II of this series (Bilir et al., in preparation).
Table 6. \((B - V)\), \((g - r)\) and \((r - h)\) bins used for the inverse transformation equations for Johnson–Cousins BVRI and SDSS gri, for the metallicity ranges \(-3 < [M/H] < +0.2\), \(-0.4 < [M/H] < 0.2\), \(-1.2 < [M/H] < -0.4\) and \(-3 < [M/H] < -1.2\) dex. \(N\) is the number of stars in each bin. The other columns refer to the colours used in the transformations.

| [M/H] (dex) | \((B - V)\) | \((g - r)\) | \((r - h)\) | \((W1 - W2)\) |
|-------------|-------------|-------------|-------------|-------------|
| \([-3.0, +0.2]\) | 22 | 0.434 | 0.284 | 0.214 | \(-0.056\) | \([-3.0, +0.2]\) | 37 | 0.245 | 0.079 | 0.216 | \(-0.071\) |
| \([-0.4, +0.2]\) | 13 | 0.435 | 0.261 | 0.216 | \(-0.053\) | \([-0.4, +0.2]\) | 35 | 0.232 | 0.079 | 0.216 | \(-0.072\) |
| \([-1.2, +0.4]\) | 6 | 0.447 | 0.292 | 0.180 | \(-0.060\) | \([-1.2, +0.4]\) | 46 | 1.404 | 0.902 | 0.624 | \(-0.078\) |
| \([-3.0, -1.2]\) | 3 | 0.427 | 0.376 | 0.271 | \(-0.066\) | \([-3.0, -1.2]\) | 10 | 0.595 | 0.212 | 0.392 | \(-0.078\) |

Table 7. Numerical values of the coefficients for the inverse transformation equations for the different metallicity ranges. \(a_i\), \(b_i\) and \(\gamma_i, (i = 1, 2, 3\) and 4) correspond to equations (7), (8), (9) and (10), respectively. The numerical values in the fourth and fifth rows are the squared correlation coefficients \(R^2\) and the standard deviations \(s\). \(N\) is the number of stars used in the evaluation of the coefficients.

| [M/H] (dex) | \((B - V)\) | \((g - r)\) | \((r - h)\) |
|-------------|-------------|-------------|-------------|
| \([-3.0, +0.2]\) | \(a_i\) | 2.002 \(\pm\) 0.064 | 1.198 \(\pm\) 0.142 | 2.203 \(\pm\) 0.103 | 1.050 \(\pm\) 0.065 |
| \([-0.4, +0.2]\) | \(a_i\) | 1.914 \(\pm\) 0.132 | 1.122 \(\pm\) 0.191 | 2.137 \(\pm\) 0.083 | 1.028 \(\pm\) 0.051 |
| \([-1.2, +0.4]\) | \(a_i\) | 1.454 \(\pm\) 0.603 | 0.693 \(\pm\) 0.227 | 2.315 \(\pm\) 0.156 | 0.838 \(\pm\) 0.023 |
| \([-3.0, -1.2]\) | \(a_i\) | 2.061 \(\pm\) 0.045 | 0.810 \(\pm\) 0.077 | 1.832 \(\pm\) 0.095 | 0.857 \(\pm\) 0.041 |

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REFERENCES

Adelman-McCarthy J. K. et al., 2007, VizieR On-line Data Catalog: II/276
Aihara H. et al., 2011, ApJS, 193, 29
Beichman C. A., Neugebauer G., Haining H. J., Clegg P. E., Chester T. J., 1988, Infrared Astronomical Satellite (IRAS) Catalogs and Atlases. Volume 1: Explanatory Supplement. NASA, Washington, DC
Benjamin R. A. et al., 2005, ApJ, 630, L149
Bilir S., Ak S., Karaali S., Cabrera-Lavers A., Chonis T. S., Gaskell C. M., 2008, MNRAS, 384, 1178
Cardelli J. A., Clayton G. C., Mathis J. S., 1989, ApJ, 345, 245
Churchwell E. et al., 2001, BAAS, 33, 821
Cutri R. M. et al., 2003, The IRSA 2MASS All-Sky Point Source Catalog, NASA/IPAC Infrared Science Archive, http://irsa.ipac.caltech.edu/applications/Gator/
Fukugita M., Ichikawa T., Gunn J. E., Doi M., Shimasaku K., Schneider D. P., 1996, AJ, 111, 1748
Gunn J. E. et al., 1998, AJ, 116, 3040
Hogg D. W., Finkbeiner D. P., Schlegel D. J., Gunn J. E., 2001, AJ, 122, 2129
Ishihara D. et al., 2010, A&A, 514, A1
Karaali S., Bilir S., Tunçel S., 2005, PASA, 22, 24
Murakami H. et al., 2007, PASJ, 59, 369
Neugebauer G. et al., 1984, ApJ, 278, L1
Saha A., Dolphin A. E., Thim F., Whitmore B., 2005, PASP, 117, 37
Schlegel D. J., Finkbeiner D. P., Davis M., 1998, ApJ, 500, 525
Skrutskie M. F. et al., 2006, AJ, 131, 1163
Smith J. A. et al., 2002, AJ, 123, 2121
Steinmetz M. et al., 2006, AJ, 132, 1645
Stetson P. B., 2000, PASP, 112, 925
Werner M. W. et al., 2004, ApJS, 154, 1
Wright E. L. et al., 2010, AJ, 140, 1868
York D. G. et al., 2000, AJ, 120, 1579

SUPPORTING INFORMATION

Additional Supporting Information may be found in the online version of this article.

Table 1. Johnson–Cousins, SDSS, 2MASS and WISE magnitudes and colours of the sample stars (total 289 stars).

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