Non-standard primordial clocks from induced mass in alternative to inflation scenarios

Yi Wang, Zun Wang and Yuhang Zhu

Department of Physics, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, P.R. China

Jockey Club Institute for Advanced Study, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, P.R. China

E-mail: phyw@ust.hk, zwangdq@connect.ust.hk, yzhucc@connect.ust.hk

Received July 29, 2020
Revised September 11, 2020
Accepted September 22, 2020
Published November 13, 2020

Abstract. In the primordial universe, oscillations of heavy fields can be considered as standard clocks to measure the expansion or contraction history of the universe. Those standard clocks provide a model-independent way of distinguishing inflation and alternative scenarios. However, the mass of the heavy fields may not be a constant mass, but rather mass induced by non-minimal coupling to the Ricci scalar, or self-interactions. In the case of induced mass, the mass of the heavy field is generically of order Hubble, and thus is time-dependent in alternative to inflation scenarios. We show that such generated mass terms can be considered as non-standard primordial clocks for alternative to inflation, providing similar oscillatory frequencies as standard clocks of inflation. Additional information on scale dependence can distinguish such non-standard clocks from standard clocks.

Keywords: alternatives to inflation, cosmological perturbation theory, inflation, non-gaussianity

ArXiv ePrint: 2007.09677
1 Introduction

For the past decades, inflation has become the most popular scenario to describe the very early universe. It can solve the horizon problem and flatness puzzles in the hot big bang [1–4]. Inflation predicts the primordial density fluctuations, which shows excellent agreement with experimental data from Large Scale Structure (LSS) and Cosmic Microwave Background (CMB).

However, many alternative to inflation scenarios were also proposed in the literature. For example, slow contraction scenarios such as the Ekpyrosis [5], fast contraction scenarios such as matter bounce [6, 7], and slow expansion scenarios such as [8], string gas cosmology [9, 10], or Galilean genesis [11, 12].

For both inflation and alternative to inflation scenarios, a large number of models can be built, with a wide spectrum of predictions. To figure out the expansion (or contraction) history of the universe, and distinguish inflation from alternatives, it may be more efficient to start not from the model details, but rather model-independent features purely from the time-dependence of the scale factor, which is the defining features of inflation or an alternative scenario. One example is the primordial gravitational waves [13] which are the tensor part of the primordial fluctuation, and it may be observed in the CMB experiment from B-mode polarization. This is because, in the simplest setup, primordial gravitational waves can tell us the energy scale of the primordial universe scenarios (which gives $H$), and if we measure it at
different scales (different \( k \)) that crossed the horizon at different time, we can reconstruct the scale factor \( a(t) \). Another possible way is to use massive fields that existed in the primordial universe \([14–17]\). The classical \([18, 19]\) or quantum \([20–22]\) oscillations of massive fields work as primordial standard clocks (PSC) which can directly record the time evolution of the scale factor \( a(t) \).

In primordial standard clocks, the mass parameter is regarded as time-independent. However, non-standard clocks with time dependent mass are also discussed \([18, 23, 24]\). For the case of inflation, though it is natural to consider time-independent mass due to the approximate scale invariance of inflation, exceptions arise from mass terms arising from self-interactions \([23]\), or a dilatonic coupling to the inflaton \([24]\). For alternative scenarios, in addition to a fixed mass parameter, a mass of order Hubble parameter may be physically motivated by quantum corrections in curved spacetime. The Hubble parameter is strongly time-dependent in alternative to inflation, and thus provides significant time dependence to the mass of the field.

In this work, we study those non-standard clocks with time-dependent mass in alternative scenarios. Generally, the time-dependent mass can be generated from two different mechanisms. Firstly, the massive fields have the non-minimal coupling with Ricci curvature \( R \) such as \( \xi R \sigma^2 \). In the inflation case, the background is approximately de-sitter geometry and thus Ricci scalar contributes to the effective mass time-independently (proportional to \( \xi H^2 \)). However, in alternative to inflation scenarios, this term can generate a time-dependent mass. Another mechanism is from quantum loop correction. As we know in de-sitter case, with a self-interaction \( \lambda \sigma^4 \), due to the strange IR behaviors it can generate an effective mass \( m_{\text{eff}}^2 \propto H^2 \) for scalar fields \([25–34]\). By introducing this simple self-interaction \( \lambda \sigma^4 \) into alteration to inflation scenarios, it may also generate a time-dependent effective mass. Following the standard procedure to calculate correlation functions, we show that with a time-dependent mass, the clock signal is different from previous works.

The paper is organized as follows: we briefly review the concept of standard clock signals in section 2. Then in section 3 and 4 we discuss the quantum non-standard clock signal with time-dependent mass generated from Ricci scalar and self-interaction term respectively. We summarize in section 5.

2 Standard clock signals

2.1 Classical standard clocks

In this section, we briefly review the idea about the standard clock signal which can directly encode the information about scale factor during the primordial universe. First of all, due to different resonance mechanisms, we can classify the clock signals as classical or quantum ones. In the primordial universe, there are many massive fields which may originate from IR uplifting and UV completion. As for classical PSC, it requires sharp features (such as model with a special potential) to excite the oscillation of the massive field. Once excited, the massive field \( \sigma \) oscillates as

\[
\sigma \propto e^{imt}.
\]  

At sub-horizon regime, the density fluctuation \( \xi \) behaves like,

\[
\xi_k \propto e^{-ik\tau},
\]
where \( k \) refers to the comoving momentum and \( \tau \) is the conformal time. As a result, the power spectrum will contain these two ingredients,

\[
\langle \xi^2 \rangle \supset \int e^{i(mt-2k\tau)} d\tau .
\]

This integral gets the most important contribution when the resonance happens which requires,

\[
\frac{d}{dt}(mt - 2k\tau) = 0 ,
\]

and we label \( t_* \) satisfying the above equation thus

\[
a(t_*) = 2k/m .
\]

Thus, the evolution of scale factor is directly encoded into the phase of the correlation function as a function of the comoving momentum \( k \).

### 2.2 Quantum standard clocks

As the standard clock, the massive field oscillates to record the evolution of scenarios. The curvature mode resonates with the massive clock field when their wavenumbers match with each other. In an arbitrary time-dependent background, when the mass of a massive scalar field is larger than scale of horizon, the excitations of the field would oscillate as \( \propto e^{\pm imt} \) with approximated constant frequency. The same resonance can also happen when the oscillation of massive field has a quantum origin. Quantum PSC is completely general and can record the evolution of scale factor in a model independent way. Here we express the cosmological scale factors as power law functions with different indices \( p \) to describe different possible scenarios.

\[
a(t) = a_0 \left( \frac{t}{t_0} \right)^p = a_0 \left( \frac{\tau}{\tau_0} \right)^{\frac{p}{1-p}} ,
\]

where \( a_0, t_0 \) and \( \tau_0 \) are constants. Different choices of \( p \) represent different possible primordial scenarios. For example, \( 0 < p < 1 \) corresponds to contraction scenarios; \( |p| > 1 \) corresponds to the fast expansion case and \( -1 < p < 0 \) is for the slow-expansion. The conformal time \( \tau \) runs from \(-\infty\) to 0 in all cases. The coordinate time \( t \) runs from 0 to \(+\infty\) when \( p > 1 \) and from \(-\infty\) to 0 for all other cases.

In arbitrary FRW time-dependent background, the equation of motion of a massive scalar field is

\[
\delta\ddot{\sigma} + 3H\dot{\delta}\sigma - \frac{1}{a^2} \partial_i^2 \delta\sigma + m^2 \delta\sigma = 0 ,
\]

or expressed by the conformal time \( \tau \), which is related to \( t \) by \( a\tau = \frac{t}{(1-p)} \),

\[
\delta\sigma'' + 2Ha\delta\sigma' - \partial_i^2 \delta\sigma + m^2 a^2 \delta\sigma = 0 .
\]

Quantum PSC due to a resonance gets the greatest of contribution when those massive fields step into the classical regime. This means that the physical wavelength of the massive mode is large (\( k/a \ll m \)) and the equation of motion (EoM) becomes approximately

\[
\ddot{v}_k + 3H\dot{v}_k + m^2 v_k = 0 .
\]
The solution of this EoM contains an $e^{imt}$ factor when $|t|$ is large,
\begin{equation}
    v_k \propto (c_+ e^{-imt} + c_- e^{imt}).
\end{equation}
When two modes resonate, massive fields satisfy the classical regime conditions and the curvature mode is still sub-horizon. By choosing the Bunch-Davies (BD) vacuum, the mode function of the curvature mode can be approximately written as
\begin{equation}
    u_k \propto \frac{1}{(-\tau)^{3/2}} e^{-i k \tau}.
\end{equation}
The rest of the discussion is similar to the classical PSC case. The integral will pick up the most important contribution around resonance time ($t_*$ or $\tau_*$), so that the integration is proportional to
\begin{equation}
    \exp(im t_* - i k \tau_*) = \exp \left[ -i \frac{p}{1 - p} mt(k/m) \right],
\end{equation}
where $t(a)$ is the inverse function of $a(t)$. However, unlike the classical PSC which can be directly seen from the power spectrum as a source of scale-invariance breaking, the quantum PSC signal hides in the shape of non-Gaussianities. This is because we require two different scales, one is the massive mode satisfying the classical regime condition, the other one is that the curvature mode is still sub-horizon. Thus, in the next section we will calculate the three-point correlation function under the squeezed limit. In order to clarify the properties of it, we consider the two coupling terms between the perturbation of curvature field $\zeta$ and clock field $\delta \sigma$. The bilinear coupling is used as $L_2 \propto c'_2 a^3 \delta \sigma \dot{\zeta}$ and the cubic coupling is $L_3 \propto c'_3 a^3 \delta \sigma \dot{\zeta}^2$, where $c'_2$ and $c'_3$ represent coupling constants.

By using the in-in formalism [35–37], the three-point function of the curvature perturbation can be written as [20]
\begin{equation}
    \langle \zeta^3 \rangle' \supset \int_{t_0}^t dt_1 \int_{t_0}^t dt_2 \langle H_1(t_1) \zeta_3^3 H_1(t_2)|0\rangle'
    \left( -2 \text{Re} \left( \int_{t_0}^t dt_1 \int_{t_0}^t dt_2 \langle H_1(t_1) H_1(t_2)|0\rangle' \right) \right)
    = 2 u_{k_3}^* u_{k_1} u_{k_2} |_{\tau=0} \left( \int_{-\infty}^0 d\tau c'_2 a^3 v_{k_3} u_{k_1}^* u_{k_2}^* \right) \left( \int_{-\infty}^0 d\tau c'_2 a^3 v_{k_3} u_{k_1} u_{k_2}^* \right)
    - 2 u_{k_3} u_{k_1} u_{k_2} |_{\tau=0} \left( \int_{-\infty}^0 d\tau c'_2 a^3 v_{k_3} u_{k_1}^* u_{k_2}^* \right) \left( \int_{-\infty}^0 d\tau c'_2 a^3 v_{k_3} u_{k_1} u_{k_2}^* \right)
    + \left( \int_{-\infty}^0 d\tau c'_3 a^2 v_{k_3} u_{k_1}^* u_{k_2}^* \right) \left( \int_{-\infty}^0 d\tau c'_3 a^2 v_{k_3} u_{k_1} u_{k_2}^* \right) + \text{c.c.} + 2 \text{ perms}.
\end{equation}
Here, $u_k$ and $v_k$ denote the mode functions of $\zeta$ and $\delta \sigma$. The prime refers to the expression without the common factor $(2\pi)^3 \delta(\Sigma;p)$. The term “2 perms” represents the permutation $k_1 \leftrightarrow k_3$ and $k_2 \leftrightarrow k_3$.

3 Time-dependent effective mass from coupling to Ricci curvature

3.1 The effective mass
The non-minimal coupling between the Ricci curvature $R$ and the scalar field $\sigma$ can be considered as a time-dependent contribution to the effective mass. In our paper, we will not
include the case that the field corresponding to curvature perturbation is also non-minimally coupled to gravity. Because the mass of the inflaton is usually either fine-tuned to make slow roll inflation happen or protected by symmetries. If we consider these two fields both non-minimally coupled to gravity, the background evolution would become more complicated, which is not the key point of our this paper. Thus, in our case, the total effective mass $\mu$ of the massive field is

$$\mu^2 = m^2 + \xi R,$$

(3.1)

where $\xi$ is the coupling constant of the non-minimal coupling, and the original mass $m$ is a constant. The total effective mass should be a combination of these two contributions. The Ricci scalar of the FRW metric is

$$R = 6 \left[ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right].$$

(3.2)

We consider the scale factor $a$ as a time-dependent monomial function $a(t) = a_0 \left( \frac{t}{t_0} \right)^p$ so

$$R = 6 \frac{p(2p-1)}{t^2},$$

$$\mu^2 = m^2 + 6 \frac{p(2p-1)}{t^2}.$$  (3.3)

For alternatives scenarios with time-dependent mass, we consider the differential equation of scalar perturbation for the classical regime. We will discuss the general form in A. For simplification, we here focus on the parameters such that the bare mass $m$ is small, though the transition between a constant mass and a induced mass may be interesting, similar to the discussions in [38]. Thus, in this case, we are interested in the time-dependent mass dominate regime which is $|k\tau| < \sqrt{\frac{6\xi p(2p-1)}{(1-p)^2}}$. Neglecting $m$, and using the relation $\tau = \frac{1}{1-p} a_0^{-1-p} t$, we rewrite the time variable of the differential equation into conformal time,$^1$

$$\frac{d^2 v_k}{d\tau^2} + \frac{2p}{(1-p)} \frac{dv_k}{d\tau} + \left[ \frac{6 \xi p(2p-1)}{(1-p)^2 \tau^2} \right] v_k = 0.$$  (3.5)

The general solution is

$$v_k(\tau) = C_1 \left( \frac{\tau}{\tau_k} \right)^{\frac{\sqrt{2\xi p(2p-1)\sqrt{(-A+B)}}}{1+p}} + C_2 \left( \frac{\tau}{\tau_k} \right)^{\frac{\sqrt{2\xi p(2p-1)\sqrt{(-A+B)}}}{1+p}},$$

(3.6)

where $A$ and $B$ are

$$A = \frac{1}{6} \sqrt{-144 - \frac{36}{(2p-1)\xi} + \frac{6}{p(2p-1)\xi} + \frac{54p}{\xi(2p-1)}},$$

$$B = -\sqrt{6(2p-1)q + 3\sqrt{6p\sqrt{(2p-1)p}}}.$$  (3.7)

$$\frac{6p(2p-1)\sqrt{\xi}}{6p(2p-1)\sqrt{\xi}}.$$  (3.8)

Here, $t_k$ refers to the time at $a(t_k) = k/m$ for one boundary of the resonance.

$^1$Actually, if we adopt canonically normalized fields, it is simpler to get the allowed range of $\xi$. Define the variable $f(\tau) = a(\tau)\dot{v}_k \propto \tau^n$, where power index $n$ satisfies $n(n-1) + (6\xi - 1) \frac{p(2p-1)}{(1-p)^2} = 0$. To make sure there is the oscillation part in the mode function (which requires an imaginary part of $n$), the allowed value range of $\xi$ can be more obviously seen from $\xi > \frac{1}{16p^2(1-3p)^2}$. We thank the anonymous referee for pointing this out.
3.2 Appropriate range of $p$

An oscillating massive field is required for generating the clock signal, which restricts the range of $p$ in (3.6). In this case, not all alternative scenarios could generate the clock signal. With $\xi > 0$, the power index

$$\sqrt{\frac{3}{2}\sqrt{p(2p-1)}\sqrt{\xi (\pm A + B)}}$$

(3.9)

must contain an imaginary part to make sure the oscillation.

From (3.9), the index $B(p)$ cannot make any contributions to the imaginary part in any scenarios since

$$\text{Im} \left[ \sqrt{6(2p-1)p} + 3\sqrt{6p(2p-1)} \xi \right] = 0 \ .$$

(3.10)

The index $A$ will only remain the imaginary part when it satisfies

$$-144 - \frac{36}{(2p-1)\xi} + 6 \bigg( \frac{p}{2p-1}\xi + \frac{54p}{\xi(2p-1)} \bigg) < 0 \ .$$

(3.11)

We should make sure the existence of an imaginary part in $\sqrt{p(2p-1)}A$ to generate the oscillatory field. The allowed interval of $p$ depends on the value of coupling coefficient $\xi$. For $\frac{1}{6} < \xi < \frac{3}{16}$, $p$ satisfies

$$\frac{4\xi - 1}{16\xi - 3} - 2\sqrt{\frac{2}{3}} \sqrt{\frac{6\xi^2 - \xi}{(16\xi - 3)^2}} < p < 2\sqrt{\frac{2}{3}} \sqrt{\frac{6\xi^2 - \xi}{(16\xi - 3)^2}} + \frac{4\xi - 1}{16\xi - 3} \ .$$

(3.12)

For the case $\xi = \frac{3}{16}$, $p$ should be larger than 2/3. And for the cases $\xi > \frac{3}{16}$,

$$p < \frac{4\xi - 1}{16\xi - 3} - 2\sqrt{\frac{2}{3}} \sqrt{\frac{6\xi^2 - \xi}{(16\xi - 3)^2}} \quad \text{or} \quad p > 2\sqrt{\frac{2}{3}} \sqrt{\frac{6\xi^2 - \xi}{(16\xi - 3)^2}} + \frac{4\xi - 1}{16\xi - 3} \ .$$

(3.13)

Notice that the cases for $\xi \leq \frac{1}{6}$ is not allowed. When $\xi = \frac{1}{6}$, it leads to the conformally invariance of the Klein-Gordon equation containing the effective mass with $m = 0$, which has no oscillatory clock signals. This is expected since a conformally coupled field does not "know" the FRW expansion of the universe (which is conformally Minkowski).

We show the allowed ranges of $p$ related to different choices of $\xi$ in figure 1. Here we only consider the scenarios with $-1 < p < 1$. In this figure, we redefine the r.h.s. of (3.12) as the function $F(\xi)$ and the l.h.s. as $G(\xi)$, which is related to the dark yellow and blue lines in the figure 1. For $p > 0$, We can figure out that the allowed $p$ should be always larger than 1/2, which is the limit for the $\xi \to \infty$ case. Thus, slow contraction scenarios will not generate such non-standard clock signals from non-minimal couplings. But the slow-expansion scenarios are allowed for some ranges of $\xi$.

---

\footnote{We focus on $\xi > 0$ case here. If $\xi < 0$, the constraint equation with $p$ and $\xi$ should be an opposite one $\xi < \frac{1}{2p} \frac{(1-3p)p}{4p^2-1}$ with allowed $p$ within $(0, \frac{4}{3})$.)}
3.3 The non-time-ordered integral

In this section, we calculate the non-time-ordered integral eq. (2.13) which is part of bispectrum. According to eq. (2.11) and (3.6), the mode functions of the massive field mode and the curvature mode are respectively

\[ v_k(\tau) \rightarrow \left( \frac{\tau}{\tau_k} \right)^{\frac{1-3p}{2p}} \left[ e^{iQ\log(-\tau)}C_1 + e^{-iQ\log(-\tau)}C_2 \right], \]

\[ u_k(\tau) \rightarrow \frac{1}{\sqrt{k(-\tau)^{1-p}}} e^{-ik\tau}, \]  

(3.14)  

(3.15)

where

\[ Q = \sqrt{(48\xi - 9)p^2 + 6(1 - 4\xi)p - 1} \frac{2}{2(p-1)}. \]  

(3.16)

For non-time-ordered integral eq. (2.13), the resonance among these fields can only occur in the second term inside the second parenthesis. To describe the property of this term qualitatively, we use the method of saddle point approximation which we left more details in appendix B. For the curvature field mode \( u_k \),

\[ u_k \propto \frac{1}{\sqrt{k(-\tau)^{1-p}}} e^{-ik\tau} \Rightarrow \quad u'_k \propto \frac{1}{\sqrt{k}} (-\tau)^{-\frac{p}{1-p}} \left[ -\frac{p}{(1-p)\tau} - ik \right] e^{-ik\tau}. \]

(3.17)

The most important resonance term is

\[ \int_{-\infty}^{0} d\tau e^{i\tau} u_k^* u_k u_k^* u_k^* \].

(3.18)
We notice that only the second term of eq. (3.6) with coefficient $C_2$ can form the resonance. So the above integral should be proportional to

$$\sim C_2 \int_{-\infty}^{0} d\tau \left( \frac{\tau}{\tau_{k_3}} \right)^{\frac{p-1}{2-2p}} \left[ \frac{-p \tau + i k_1}{k_1} \right]^2 e^{i \left[ Q \log(-\tau) + 2k_1 \tau \right]},$$

where the parameter $\tau_{k_3}$ is assumed to satisfy

$$a(\tau_{k_3}) = \frac{k}{\mu_{\text{eff}}} \Rightarrow \tau_{k_3} = -\frac{\sqrt{6} \xi p (2 - p - 1)}{1 - p} \frac{1}{k_3}.$$ (3.20)

Under the squeezed limit $k_3/k_{1,2} \to 0$, resonance happens when the conformal time $\tau_s$ satisfies

$$\frac{d}{d\tau} [Q \log(-\tau) + 2k_1 \tau] = 0 \bigg|_{\tau = \tau_s} \Rightarrow \tau_s = -\frac{Q}{2} \frac{1}{k_1}.$$ (3.21)

According to the method of saddle point approximation, the integral describing the resonance should be proportional to

$$\sim C_2 \Omega \exp \left\{ iQ \log \left[ \left( \frac{2k_1}{k_3} \right)^{-1} \right] - iQ \pm i \frac{\pi}{4} \right\} \left( \frac{2k_1}{k_3} \right)^{\frac{3p-1}{2-2p}}.$$ (3.22)

The parameters $\Omega$ and $\omega$ denotes

$$\Omega(p, \xi) = \omega^\frac{3p-1}{2-2p} \sqrt{\frac{\pm \pi Q}{2}} \left[ i + \frac{2p}{(1 - p)Q} \right]^2,$$ (3.23)

$$\omega(p, \xi) = \sqrt{6 \xi p (2p - 1)} \frac{1}{(1 - p)Q}.$$ (3.24)

We collect the $k_1$-independent parameters into $f(p, \xi, k_3)$ and $\phi(p, \xi, k_3)$, which depends on the details of alternative to inflation models. In general, the coefficient $C_1$ and $C_2$ may depend on $k_3$. This introduces parts of $k_3$ dependence in the pre-factor $f(k_3)$ and the phase $\phi(k_3)$. We also assume $u_{k_1,\tau=0}$ to be proportional to $1/\sqrt{k_3^3}$. So the non-time-ordered integral is

$$\propto f(p, \xi, k_3) \left( \frac{2k_1}{k_3} \right)^{\frac{3p-1}{2-2p}} \exp \left\{ iQ \log \left[ \left( \frac{2k_1}{k_3} \right)^{-1} \right] + i\phi(p, \xi, k_3) \right\}.$$ (3.25)

Under the squeezed limit $k_3/k_{1,2} \to 0$, the clock signal is considered as the oscillatory shape function in the three function in the previous work. It is defined as

$$\langle \zeta^3 \rangle \equiv S(k_1, k_2, k_3) \frac{1}{(k_1 k_2 k_3)^2} (2\pi)^7 \tilde{P}_\zeta^2 \delta^3 \left( \sum_{i=1}^{3} k_i \right).$$ (3.26)

where the fiducial power spectrum is $\hat{P}_\zeta \equiv 2.1 \times 10^{-9}$. So the clock signal should be proportional to

$$S^{\text{clock}} \propto f(p, \xi, k_3) \left( \frac{2k_1}{k_3} \right)^{\frac{1+p}{2-2p}} \sin \left\{ Q \log \left[ \left( \frac{2k_1}{k_3} \right)^{-1} \right] + \phi(p, \xi, k_3) \right\}.$$ (3.27)
We obtain the oscillation part of the shape function

\[
S_{\text{oscillation}} = \sin \left\{ Q \log \left( \frac{2k_1}{k_3} \right) + \phi(p, \xi, k_3) \right\},
\]

(3.28)

which is the essential pattern for the clock signal. We find that this oscillation pattern is the same as the inflation case and we will discuss this point later in detail at the end of section 3. It is also important to note the amplitude of the signal as a function of \( k_1 \), when \( k_3 \) is fixed.

From (3.27), the \((k_1/k_3)^{1+p}\) terms represent how the amplitude changes with the amount of squeezeness, and the function \( f(k_3) \) manifests the overall scale dependence. Generally, \( f(k_3) \) cannot be determined before we specify a model. From the perspective of \( k_3 \), these two dependencies are entangled here and we are unable to distinguish them. However, the power of momentum ratio \( k_1/k_3 \) is determined and can be used to differentiate these scenarios. In practice, we can keep \( k_3 \) unchanged and increase \( k_1 \) to study the squeeze shape dependence.

3.4 The time-ordered integral

The outer and inner integrands of the time-ordered integral take similar forms. We rewrite eq. (2.14) as

\[
\propto \left[ \int_{-\infty}^{0} d\tau_1 c_1^2 a_1^2 v_{k_3} u_{k_3} \int_{-\infty}^{\tau_1} d\tau_2 c_2^2 a_2^2 v_{k_2} u_{k_2} \right. \\
\left. + \int_{-\infty}^{0} d\tau_1 c_1^2 a_1^2 v_{k_3} u_{k_3} u_{k_2} \int_{-\infty}^{\tau_1} d\tau_2 c_2^2 a_2^2 v_{k_2} u_{k_2} \right] + \text{c.c.} + 2 \text{ perms}.
\]

(3.29)

To use the saddle point approximation, it is necessary to make sure that both of the extreme value time \( \tau_0 \) are in the interval of \((-\infty, \tau_1)\). In this case, the time-ordered integrals will share the similar oscillating pattern like (3.29). It describes the property of this clock signal, where we can identify the type of alternative scenarios.

For simplicity, we consider \( \tau_0 = -1 \). The modes are denoted as

\[
u_k \propto \frac{1}{\frac{1}{1-p} v_{k_3} u_{k_3}} \left[ C_1 e^{iQ\log(-\tau)} + C_2 e^{-iQ\log(-\tau)} \right].
\]

(3.30)

\[
u_k \propto \frac{1}{\frac{1}{1-p} v_{k_2} u_{k_2}} \left[ C_1 e^{iQ\log(-\tau)} + C_2 e^{-iQ\log(-\tau)} \right].
\]

(3.31)

We denote the variable \( x_1 = -\tau_1 \) and \( x_2 = -\tau_2 \) and the integral (3.30) and (3.31) should be

\[
\propto k_1 \int_{0}^{\infty} dx_1 x_1 \left[ C_1^{*} e^{-iQ\log(x_1)} + C_2^{*} e^{iQ\log(x_1)} \right] e^{-2ik_1 x_1} \\
\times \int_{0}^{\infty} dx_2 x_2 \left[ C_1 e^{iQ\log(x_2)} + C_2 e^{-iQ\log(x_2)} \right] e^{-ik_2 x_2} \\
+ k_1 \int_{0}^{\infty} dx_1 x_1 \left[ C_1^{*} e^{-iQ\log(x_1)} + C_2^{*} e^{iQ\log(x_1)} \right] e^{-2ik_1 x_1} \\
\times \int_{0}^{\infty} dx_2 x_2 \left[ C_1 e^{iQ\log(x_2)} + C_2 e^{-iQ\log(x_2)} \right] e^{-ik_2 x_2} .
\]

(3.32)

(3.33)
Following the condition of the squeezed limit, we consider $k_3 = 0$ approximately in (3.34) and (3.35). Firstly we deal with the integral (3.34). Compared with the constant mass case, this integral over $x_2$ can be done directly. The second integral term is

$$
\int_0^{x_1} \frac{3-p}{2(1-p)} \left[ C_1 \frac{1}{iQ + \frac{3-p}{2(1-p)} x_2^{-iQ}} + C_2 \frac{1}{-iQ + \frac{3-p}{2(1-p)} x_2^{-iQ}} \right] dx_2
$$

(3.34)

where

$$\langle x \rangle
$$

which also contains a similar $x_1$-dependent term like (3.36).

$$
\int_{x_1}^\infty \frac{3-p}{2(1-p)} \left[ C_1^* \frac{1}{-iQ + \frac{3-p}{2(1-p)} x_1^{-iQ}} + C_2^* \frac{1}{iQ + \frac{3-p}{2(1-p)} x_1^{-iQ}} \right] dx_1
$$

(3.35)

which also contains a similar $x_1$-dependent term like (3.36).

$$
\rightarrow -\frac{3-p}{2(1-p)} \left[ C_1^* \frac{1}{-iQ + \frac{3-p}{2(1-p)} x_1^{-iQ}} + C_2^* \frac{1}{iQ + \frac{3-p}{2(1-p)} x_1^{-iQ}} \right] + \ldots ,
$$

(3.36)

where the $\langle \ldots \rangle$ refers to the $x_1$-independent term from the upper infinity case. After combining with the first integral of each term, for (3.34)

$$
\rightarrow k_1 \int_0^\infty dx_1 x_1^2 \left[ C_1 \frac{1}{iQ + \frac{3-p}{2(1-p)} x_1^{iQ}} + C_2 \frac{1}{-iQ + \frac{3-p}{2(1-p)} x_1^{-iQ}} \right] \left[ C_1^* x_1^{-iQ} + C_2^* x_1^{iQ} \right] e^{-2ik_1 x_1} .
$$

(3.37)

Only the terms with resonance remains for the contribution to the clock signal, which is proportional to

$$
\rightarrow \int_0^\infty dx_1 x_1^2 C_1 C_2^* \left( \frac{1}{iQ + \frac{3-p}{2(1-p)}} \right) \exp [2iQ \log(x_1) - 2ik_1 x_1] .
$$

(3.38)

In the same way, from (3.35)

$$
\rightarrow -\int_0^\infty dx_1 x_1^2 \left[ C_1 C_2^* \left( \frac{1}{iQ + \frac{3-p}{2(1-p)}} \right) \exp [2iQ \log(x_1) - 2ik_1 x_1] \\
+ G(p, \xi) \left\{ k_1 \int_0^\infty dx_1 x_1^2 \left[ C_1 e^{iQ \log(x_1)} + C_2 e^{-iQ \log(x_1)} \right] e^{-2ik_1 x_1} \right\} .
$$

(3.39)

We can figure out that the oscillatory terms and coefficients of (3.39) and the first term of (3.40) are consistent with each other. So they can offset each other and only the second of (3.40) remains, which shares the same form with the non-time-ordered integral. To summarize, we can express the time-ordered integral by saddle point approximation as

$$
S^{\text{clock}} \propto (2k_1)^{\frac{1+p}{2(1-p)}} \sin \left\{ Q \log \left[ (2k_1)^{-1} \right] + \text{phase} \right\} ,
$$

(3.40)

which is the same as eq. (3.27), and $S^{\text{clock}}$ is the shape function we defined before.

\[ -10 - \]
Interestingly, this oscillation pattern is the same as the standard clock signal of inflation with constant mass. There are two intuitive ways to understand this similarity. First of all, the clock signal is generated when the massive field oscillates as $e^{i \int m \, dt}$ which transformed into conformal time is $e^{i m H \log(-\tau)}$ in the inflation case. This logarithmic function dependence cause the unique clock signal of inflation. However, the mass generated from Ricci scalar has the property $m \propto t^{-1}$, this would also introduce a logarithmic function on the exponent. Another way to understand it is from equation of motion (2.8). The mass term combines with the scale factor as $(am)^2$, in the inflation case, $am \sim \tau^{-1}$ and in our case $am \sim aH \sim \tau^{-1}$. So that, this two different cases share similar equations of motion. As a result, the oscillation pattern of quantum clock signal becomes similar to the standard clocks for inflation.

Thus, if one uses shape information only to distinguish inflation and alternative scenarios, there is a possibility that alternative to inflation scenarios with dynamically generated mass would be confused with inflation. To break this degeneracy, we note that, although the oscillation patterns are quite similar, the amplitude of different shape functions differs greatly. As we can see from (3.27), in addition to the oscillation part, the amplitude of this signal also shows a momentum ratio $k_1/k_3$ dependence. This $(2k_1/k_3)^{\frac{1+p}{2(1-p)}}$ term represents how the amplitude changes, when $k_3$ is fixed and one varies $k_1$. For inflation, the corresponding factor is $\sim (k_1/k_3)^{-\frac{1}{2}}$, corresponding to the $p \to \infty$ limit which differs\footnote{For comparison, for alternative to inflation with constant mass, the corresponding factor is $\sim (k_1/k_3)^{-\frac{1}{2}+\frac{p}{2}}$, which is also different.} from the case of a finite $p$. We illustrate this difference in figure 2. In this non-standard case, one notes that for inflation (or super-inflation where the universe is quasi-de Sitter but with increasing energy density), $|p| > 1$ and the oscillation is decreasing while $k_1$ increases. But for alternatives, $-1 < p < 1$ and the signal is increasing while $k_1$ increases.\footnote{In standard case, for slow expansion scenarios ($-1 < p < 0$), the signal is increasing while $k_1$ decreases.}

![Figure 2. Squeeze shape dependence of amplitude in three different scenarios: fast contraction ($p=2/3$) with constant mass, fast contraction ($p=2/3$) with mass $\sim H$ and exponential inflation.](image-url)
4 Time-dependent effective mass from nonlinear corrections

4.1 The effective mass

An effective mass of $\sigma$ may also be generated from interactions in alternative scenarios. We consider the simplest case: $V = \frac{1}{4} \lambda \sigma^4$.

Under comparison, it can be separated as $V = \frac{1}{2} \lambda (3 \langle \sigma^2 \rangle) \sigma^2$. We can figure out that the effective mass term should be proportional to $\langle \sigma^2 \rangle$, which is related to the two-point correlation function of this scalar field,

$$\mu^2 = m_{\text{eff}}^2 = 3 \lambda \langle \sigma^2 \rangle, \tag{4.1}$$

where we can estimate the time-dependent contribution of this two-point correlation in two methods [25, 39, 40]. The first method is to figure out the expression of quantum fluctuation in alternative scenarios, and do the integration in one cut-off process. In the second method, we separate the scalar in long and short wavelength part and solve them form an nonlinear stochastic different equation (SDE).

The general EOM of scalar field $\sigma$ is

$$\ddot{\sigma} + 3H \dot{\sigma} + \frac{k^2}{a^2} \sigma + m^2 \sigma = 0. \tag{4.2}$$

The last term should vanishes in this massless case. We combine the scale factor and field as $v = a \sigma$ for simplification. Shown under the conformal time $\tau$, the EOM is

$$v''_k + \left( k^2 - \frac{a''}{a} \right) v_k = 0, \tag{4.3}$$

which is knowns as the Mukhanov-Sasaki (MS) equation. Here, $v''_k$ and $a''$ denote the second derivative with respect to the conformal time $\tau$.

The scale factor $a$ is still expressed as power-law function, under these relations above,

$$a = a_0 \left( \frac{\tau}{\tau_0} \right)^{\frac{p}{1-p}} \Rightarrow \frac{a''}{a} = \left( \frac{p}{1-p} - 1 \right) \frac{\tau^p}{\tau^2}. \tag{4.4}$$

So, the MS equation is

$$v''_k + \left( k^2 - \frac{\nu^2 - 1}{\tau^2} \right) v_k = 0, \tag{4.5}$$

where $\nu(p) = \frac{3}{2} + \frac{1}{p-1}$ is a model-dependent index. The general solution of this MS equation is a linear combination of Hankel functions,

$$v_k = D_1 \sqrt{-k \tau} H^{(1)}_{|\nu|}(-k \tau) + D_2 \sqrt{-k \tau} H^{(2)}_{|\nu|}(-k \tau). \tag{4.6}$$

This solution should be normalised on the sub-Hubble scales to constraint constants $D_1$ and $D_2$. Under the limit: $-k \tau \to 0$, the quantum fluctuation $\sigma_k$ is [41]

$$\sigma_k = i \frac{1}{\sqrt{4\pi ka^2 (-k \tau)^{|\nu|-\frac{1}{2}}} \Gamma(|\nu|) \Gamma(|\nu| - \frac{1}{2})}. \tag{4.7}$$
The time-dependent contribution of quantity $\langle \sigma^2 \rangle$ is proportional to
\[ \int d^3k \frac{1}{a^2} k^{-2|\nu|} (-\tau)^{1-2|\nu|}. \] (4.8)

We use the time dependent physical momentum $p = \frac{k}{a}$ to emphasize the physical meaning,
\[ \langle \sigma^2 \rangle \propto \int_0^H dp \ p^{2-2|\nu|} (-\tau)^{1-2|\nu|} a^{1-2|\nu|}. \] (4.9)

Since $(-a\tau) \propto H^{-1}$ and the time dependent Hubble constant $H = \frac{\dot{a}}{a} = \frac{p}{t}$,
\[ \langle \sigma^2 \rangle \propto H^{3-2|\nu|} H^{2|\nu|-1} = H^2 \propto \frac{1}{t^2}. \] (4.10)

There is a more accurate method to obtain the time-dependent contribution. We still consider the alternative models with the scalar field $\sigma(x,t)$. The idea is to separate this scalar field using the stochastic approach. We split the $\sigma$ field into the long wavelength part $\sigma_L$ and short wavelength part $\sigma_S$ with the expression
\[ \sigma(x,t) = \sigma_L + \sigma_S = \sigma_L + \int \frac{d^3k}{(2\pi)^{3/2}} \Theta[k - \epsilon a(t)] H \left[ a_k \sigma_{S,k}(t) e^{-ikx} + a^*_k \sigma^*_{S,k} e^{ikx} \right], \] (4.11)

where $\epsilon$ is a small constant and $a_k$ and $a^*_k$ are annihilation and creation operator. $\Theta(x)$ is the Heaviside step function. The short wavelength part satisfies the massless EOM of scalar equation (4.2) with $m^2 = 0$ so $\sigma_{S,k} = \sigma_k$ in (4.7).

The long wavelength part $\sigma_L$ satisfies the classical equation of motion $\ddot{\sigma}_L + 3H \dot{\sigma}_L + \partial_i V = 0$. For the alternative scenarios with index $p$,
\[ \ddot{\sigma}_L + 3P_t \dot{\sigma}_L + \lambda \sigma_L^3 = 0. \] (4.12)

We are interested in the damping dominated regime. In this special period, the damping term is much larger than the accelerating term and then we can ignore the second derivative term $\ddot{\sigma}_L$, i.e.
\[ \ddot{\sigma}_L \ll 3H(t) \dot{\sigma}_L = 3P_t \dot{\sigma}_L, \] (4.13)

which requires
\[ \lambda \sigma_L \ll (1 - p) \frac{\sqrt{3p}}{\sigma_L}. \] (4.14)

Using the Starobinsky’s stochastic method, the short wavelength contribution can be added to the original classical equation as a perturbation term $f(x,t)$. The quantum-corrected equation of motion is [42]
\[ 3P_t \dot{\sigma} = -\lambda \sigma^3 + 3P_t f(x,t). \] (4.15)

The stochastic random force term $f(x,t)$ is
\[ f(x,t) = (p - 1) D_1 t^{p-2} \int \frac{d^3k}{(2\pi)^{3/2}} \delta(k - D_1 t^{p-1}) \left[ a_k \sigma_{S,k}(t) e^{-ikx} + a^*_k \sigma^*_{S,k} e^{ikx} \right], \] (4.16)
where $D_1 = \epsilon a_0 t_0^{-p} \nu$ is a combined constant. Here, the integration is constrained only for specific $k$ because of the delta function. $f(x, t)$ is a Gaussian random field and the two-point correlation function is

$$
\langle f(x_1, t_2), f(x_2, t_2) \rangle = \Lambda t^{-3} \delta(t_1 - t_2) j_0 (\epsilon a(t) H |x_1 - x_2|),
$$

where $j_0(x) = \frac{\sin(x)}{x}$ and $\Lambda$ is a time-independent combined coefficient [25].

The quantum-corrected equation of motion then becomes one nonlinear stochastic differential equation (SDE)

$$
\dot{\sigma} = -\frac{\lambda}{3p} t \sigma^3 + \frac{\sqrt{\Lambda}}{3p} t^{-3/2} \eta(x, t),
$$

where $\eta(x, t)$ is the simplified function with $\langle \eta(x, t) \rangle = 0$ and $\langle \eta(x_1, t_1), \eta(x_2, t_2) \rangle = \delta(t_1 - t_2) \delta(|x_1 - x_2|)$. Because of the time and field-dependent drift coefficient $-\frac{\lambda}{3p} t \sigma^3$ and time-dependent diffusion coefficient $\frac{\sqrt{\Lambda}}{3p} t^{-3/2}$, we cannot figure out the analytic solution of this nonlinear SDE. The numerical solution shows that the proportion to $t^{-2}$ is still an appropriate approximation for the two-correlation function $\langle \sigma^2 \rangle$.

In conclusion, the effective mass in this case is expressed as

$$
\mu^2 = m_{\text{eff}}^2 \propto \frac{1}{t^2}.
$$

And the effective mass and the potential term become

$$
\mu^2 = 2C(p) t^2, \quad V = \frac{C(p)}{t^2} \sigma^2,
$$

where $C(p)$ is a time-independent coefficient which is model-dependent. Approximately, by the first method,

$$
C(p) = 3\lambda \Gamma^2(|\nu|) \frac{2^{2|\nu|}}{64\pi^4},
$$

for the interacting coefficient $\lambda > 0$, $C(p)$ and the potential are always non-negative, which is different from the Ricci-dominated case. Note that there is an undefined scenario for $p = \frac{3}{4}$ ($\nu = 0$). This is the only case leading to an infinite Gamma function so this contracting scenario cannot be dealt with in this way.

### 4.2 The clock signal

Since the effective mass is proportional to $t^{-2}$, the clock signal dominated by the quantum fluctuation should be similar to the Ricci-dominated case. Like (3.6), the conformal time basis EOM of scalar field mode $v_k$ is

$$
\frac{d^2v_k}{d\tau^2} + 2\frac{p}{(1-p)\tau} \frac{dv_k}{d\tau} + \frac{2C(p)}{(1-p)\tau^2} = 0.
$$

The general solution is

$$
v_k = (-\tau)^{1-2p} \left[ e^{iB(p)\log(-\tau)} C_1 + e^{-iB(p)\log(-\tau)} C_2 \right],
$$

$$
B(p) = \sqrt{8C(p) - (1 - 3p)^2} \frac{2(p-1)}{2(p-1)},
$$

where $C(p)$ is a time-independent coefficient which is model-dependent. Approximately, by the first method,

$$
C(p) = 3\lambda \Gamma^2(|\nu|) \frac{2^{2|\nu|}}{64\pi^4},
$$

for the interacting coefficient $\lambda > 0$, $C(p)$ and the potential are always non-negative, which is different from the Ricci-dominated case. Note that there is an undefined scenario for $p = \frac{3}{4}$ ($\nu = 0$). This is the only case leading to an infinite Gamma function so this contracting scenario cannot be dealt with in this way.
which has the same form as eq. (3.6). There is also an restriction on $p$, like the discussion in the last section since $B(p)$ must be real. In the same method like (3.28), we can figure out the clock signal in this case should be proportional to

$$S_{\text{clock}} \propto (2k_1)^{\frac{1+p}{2(1-p)}} \sin \left\{ B(p) \log \left[ (2k_1)^{-1} \right] + \text{phase} \right\}, \quad (4.25)$$

where $C(p)$ follows (4.20) expression with $\nu = \frac{3}{2} + \frac{1}{p-1}$.

5 Conclusion

In this paper, we extend the study of quantum primordial non-standard clocks to alternative scenarios with dynamically generated mass. Such time-dependent mass can naturally be generated where the background significantly breaks scale invariance. Two mechanisms are considered:

The first mechanism is through non-minimal coupling with Ricci scalar $R$ which should be a constant ($\propto H^2$) in de-sitter space-time, but it can contribute a time-dependent effective mass in different alternatives to inflation scenarios with $\tau^{-2}$ time dependence.

We note that not all alternatives to inflation scenarios can produce non-trivial quantum primordial non-standard clocks signal from coupling to $R$. For generating oscillatory signals, the mode function of massive fields should have oscillation behaviors, this condition places a constrain on the power-law index $p$ as a function of $\xi$, as plot in figure 1. In a nutshell, the cases for $\xi \leq \frac{1}{6}$ is not allowed. For $p > 0$, the allowed $p$ should be always large than 1/2.

The second mechanism of mass generation is from quantum loop correction, we give a simple approximation of this method’s contribution. For the massless field with $\lambda \sigma^4$ interaction, we show that the time dependence of effective mass is also proportional to $H^2$.

Finally, the oscillation behavior of this situation is quite similar to inflation scenarios with the time-independent mass case, because massive fields in these two different scenarios share the similar equation of motion. However, we can still distinguish them from the shape dependence of the amplitude. The amplitude of shape function in our case as $(k_1/k_3)^{\frac{1+p}{2(1-p)}}$ when $k_1$ varies, which is a decreasing function for inflation and an increasing function for alternative scenarios. It is interesting to see if similar observations can be used to distinguish classical standard and non-standard primordial clocks.

Acknowledgments

We thank Xingang Chen and Siyi Zhou for fruitful discussions and comments. This work was supported in part by ECS Grant 26300316 and GRF Grant 16301917 and 16304418 from the Research Grants Council of Hong Kong.

A General expression of the clock field

In the above discussion, we consider the small $m$ limit, where the effective mass is dominated by the Ricci curvature. Here we show a general case for the equation of motion of the massive scalar field under the conformal time,

$$\frac{d^2 v_k}{d\tau^2} + \frac{2p}{(1-p)\tau} \frac{dv_k}{d\tau} + \left[ k^2 + 6\xi \frac{p(2p-1)}{(1-p)^2} \tau^2 + m^2 a_0^2 \tau^{1-\eta} \right] v_k = 0 . \quad (A.1)$$
For the massless or relativistic case with $k \gg ma$, the EOM is
\[
\frac{d^2v_k}{d\tau^2} + \frac{2p}{(1-p)\tau} \frac{dv_k}{d\tau} + \left[ k^2 + 6\xi \frac{p(2p-1)}{(1-p)^2\tau^2} \right] v_k = 0 \tag{A.2}
\]
The analytical solution is:
\[
v(\tau) \to (2p - 2)^{1\pm 3p\over 2} - (\tau)^{1\pm 3p\over 2} [C_1 J_P(k\tau) + C_2 Y_P(k\tau)] \tag{A.3}
\]
where
\[
P = \sqrt{1 - 6p + 9p^2 + 24p\xi - 48p^2\xi^2}. \tag{A.4}
\]

B Saddle point approximation

We use the saddle point approximation to deal with the complicated integrals in (2.5) and (2.6). If the integrated term contains fast-oscillating part $e^{ig(t)}$ and weak time-dependent function $f(t)$, the integral can be approximated by the saddle point approximation as
\[
\int_a^b dt \ e^{i\gamma(t)} f(t) \approx e^{i\gamma(t_0) \pm i{\pi\over 4}} f(t_0) \sqrt{\pm 2\pi \gamma''(t_0)}. \tag{B.1}
\]
Here, $t_0$ is the time leading to the extreme value of oscillating function $g(t)$, i.e. $g'(t_0) = 0$. The choices of plus or minus signs are for computing convenience which both lead to the same results.

C Single variable Langevin equation

The Langevin equation with a stochastic variable $\phi$ takes the form
\[
\dot{\phi} = f(\phi, t) + g(\phi, t)\Gamma (x, t), \tag{C.1}
\]
where the Langevin force term $\Gamma(t)$ satisfies Gaussian random distribution with
\[
\langle \Gamma(x, t) \rangle = 0, \quad \langle \Gamma(x, t)\Gamma(x', t') \rangle = \delta(x - x')\delta(t - t'). \tag{C.2}
\]
For nonlinear Langevin equations, generically we cannot obtain analytic solutions. However, for special cases like $g(\phi, t) = g(\phi)$, there could be tricks to transfer the form to a solvable one. We can also solve this by the Fokker-Planck equation to get the probability density $W(\phi, t)$ \cite{43}. After that, the correlation function is
\[
\langle h(\phi) \rangle = \int_{-\infty}^{\infty} h(\phi) W(\phi, t) d\phi. \tag{C.3}
\]
References

[1] A.H. Guth, *The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems*, Phys. Rev. D 23 (1981) 347.

[2] A.D. Linde, *A New Inflationary Universe Scenario: A Possible Solution of the Horizon, Flatness, Homogeneity, Isotropy and Primordial Monopole Problems*, Adv. Ser. Astrophys. Cosmol. 3 (1987) 149 [SPIRE].

[3] A. Albrecht and P.J. Steinhardt, *Cosmology for Grand Unified Theories with Radiatively Induced Symmetry Breaking*, Phys. Rev. Lett. 48 (1982) 1220 [SPIRE].

[4] A.A. Starobinsky, *A New Type of Isotropic Cosmological Models Without Singularity*, Adv. Ser. Astrophys. Cosmol. 3 (1987) 130 [SPIRE].

[5] J. Khoury, B.A. Ovrut, P.J. Steinhardt and N. Turok, *The Ekpyrotic universe: Colliding branes and the origin of the hot big bang*, Phys. Rev. D 64 (2001) 123522 [hep-th/0103239] [SPIRE].

[6] D. Wands, *Duality invariance of cosmological perturbation spectra*, Phys. Rev. D 60 (1999) 023507 [gr-qc/9809062] [SPIRE].

[7] F. Finelli and R. Brandenberger, *On the generation of a scale invariant spectrum of adiabatic fluctuations in cosmological models with a contracting phase*, Phys. Rev. D 65 (2002) 103522 [hep-th/0112249] [SPIRE].

[8] Y.-S. Piao and E. Zhou, *Nearly scale invariant spectrum of adiabatic fluctuations may be from a very slowly expanding phase of the universe*, Phys. Rev. D 68 (2003) 083515 [hep-th/0308080] [SPIRE].

[9] R.H. Brandenberger and C. Vafa, *Superstrings in the Early Universe*, Nucl. Phys. B 316 (1989) 391.

[10] A. Nayeri, R.H. Brandenberger and C. Vafa, *Producing a scale-invariant spectrum of perturbations in a Hagedorn phase of string cosmology*, Phys. Rev. Lett. 97 (2006) 021302 [hep-th/0511140] [SPIRE].

[11] P. Creminelli, A. Nicolis and E. Trincherini, *Galilean Genesis: An Alternative to inflation*, JCAP 11 (2010) 021 [arXiv:1007.0027] [SPIRE].

[12] Y. Wang and R. Brandenberger, *Scale-Invariant Fluctuations from Galilean Genesis*, JCAP 10 (2012) 021 [arXiv:1206.4309] [SPIRE].

[13] A.A. Starobinsky, *Spectrum of relict gravitational radiation and the early state of the universe*, JETP Lett. 30 (1979) 682.

[14] X. Chen and Y. Wang, *Large non-Gaussianities with Intermediate Shapes from Quasi-Single Field Inflation*, Phys. Rev. D 81 (2010) 063511 [arXiv:0909.0496] [SPIRE].

[15] X. Chen and Y. Wang, *Quasi-Single Field Inflation and Non-Gaussianities*, JCAP 04 (2010) 027 [arXiv:0911.3380] [SPIRE].

[16] D. Baumann and D. Green, *Signatures of Supersymmetry from the Early Universe*, Phys. Rev. D 85 (2012) 103520 [arXiv:1109.0292] [SPIRE].

[17] N. Arkani-Hamed and J. Maldacena, *Cosmological Collider Physics*, arXiv:1503.08043 [SPIRE].

[18] X. Chen, *Primordial Features as Evidence for Inflation*, JCAP 01 (2012) 038 [arXiv:1104.1323] [SPIRE].

[19] X. Chen, *Fingerprints of Primordial Universe Paradigms as Features in Density Perturbations*, Phys. Lett. B 706 (2011) 111 [arXiv:1106.1635] [SPIRE].

[20] X. Chen, M.H. Namjoo and Y. Wang, *Quantum Primordial Standard Clocks*, JCAP 02 (2016) 013 [arXiv:1509.03930] [SPIRE].

[21] X. Chen, M.H. Namjoo and Y. Wang, *Probing the Primordial Universe using Massive Fields*, arXiv:1601.06228 [SPIRE].
[22] X. Chen, M.H. Namjoo and Y. Wang, A Direct Probe of the Evolutionary History of the Primordial Universe, Sci. China Phys. Mech. Astron. 59 (2016) 101021 [arXiv:1608.01299] [SPIRE].

[23] Q.-G. Huang and S. Pi, Power-law modulation of the scalar power spectrum from a heavy field with a monomial potential, JCAP 04 (2018) 001 [arXiv:1610.00115] [SPIRE].

[24] G. Domènech, J. Rubio and J. Wons, Mimicking features in alternatives to inflation with interacting spectator fields, Phys. Lett. B 790 (2019) 263 [arXiv:1811.08224] [SPIRE].

[25] A.A. Starobinsky and J. Yokoyama, Equilibrium state of a selfinteracting scalar field in the de Sitter background, Phys. Rev. D 50 (1994) 6357 [astro-ph/9407016] [SPIRE].

[26] F. Finelli, G. Marozzi, A.A. Starobinsky, G.P. Vacca and G. Venturi, Generation of fluctuations during inflation: Comparison of stochastic and field-theoretic approaches, Phys. Rev. D 79 (2009) 044007 [arXiv:0808.1786] [SPIRE].

[27] C.P. Burgess, L. Leblond, R. Holman and S. Shandera, Super-Hubble de Sitter Fluctuations and the Dynamical RG, JCAP 03 (2010) 033 [arXiv:0912.1608] [SPIRE].

[28] C.P. Burgess, R. Holman, L. Leblond and S. Shandera, Breakdown of Semiclassical Methods in de Sitter Space, JCAP 10 (2010) 017 [arXiv:1005.3551] [SPIRE].

[29] A. Rajaraman, On the proper treatment of massless fields in Euclidean de Sitter space, Phys. Rev. D 82 (2010) 123522 [arXiv:1008.1271] [SPIRE].

[30] M. Beneke and P. Moch, On “dynamical mass” generation in Euclidean de Sitter space, Phys. Rev. D 87 (2013) 064018 [arXiv:1212.3058] [SPIRE].

[31] D. Marolf and I.A. Morrison, The IR stability of de Sitter: Loop corrections to scalar propagators, Phys. Rev. D 82 (2010) 105032 [arXiv:1006.0035] [SPIRE].

[32] X. Chen, Y. Wang and Z.-Z. Xianyu, Loop Corrections to Standard Model Fields in Inflation, JHEP 08 (2016) 051 [arXiv:1604.07841] [SPIRE].

[33] X. Chen, Y. Wang and Z.-Z. Xianyu, Standard Model Background of the Cosmological Collider, Phys. Rev. Lett. 118 (2017) 261302 [arXiv:1610.06597] [SPIRE].

[34] X. Chen, Y. Wang and Z.-Z. Xianyu, Standard Model Mass Spectrum in Inflationary Universe, JHEP 04 (2017) 058 [arXiv:1612.08122] [SPIRE].

[35] S. Weinberg, Quantum contributions to cosmological correlations, Phys. Rev. D 72 (2005) 043514 [hep-th/0506236] [SPIRE].

[36] X. Chen, Primordial Non-Gaussianities from Inflation Models, Adv. Astron. 2010 (2010) 638979 [arXiv:1002.1416] [SPIRE].

[37] Y. Wang, Inflation, Cosmic Perturbations and Non-Gaussianities, Commun. Theor. Phys. 62 (2014) 109 [arXiv:1303.1523] [SPIRE].

[38] X. Chen, A. Loeb and Z.-Z. Xianyu, Unique Fingerprints of Alternatives to Inflation in the Primordial Power Spectrum, Phys. Rev. Lett. 122 (2019) 121301 [arXiv:1809.02603] [SPIRE].

[39] K.-i. Nakao, Y. Nambu and M. Sasaki, Stochastic Dynamics of New Inflation, Prog. Theor. Phys. 80 (1988) 1041 [SPIRE].

[40] A.D. Linde, Particle physics and inflationary cosmology, vol. 5 (1990) [hep-th/0503203] [SPIRE].

[41] T. Miranda, E. Frion and D. Wands, Stochastic collapse, JCAP 01 (2020) 026 [arXiv:1910.10000] [SPIRE].

[42] J. Liu, Y. Wang and S. Zhou, Nonuniqueness of classical inflationary trajectories on a high-dimensional landscape, Phys. Rev. D 91 (2015) 103525 [arXiv:1501.06785] [SPIRE].

[43] H. Risken, The Fokker-Planck Equation. Methods of Solution and Applications, second edition, Springer-Verlag, Berlin Germany (1989).