Electromagnetic crystal with half-wave vibrators sublattice

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Abstract. Volumetric metamaterials with a regular structure can have elemental structural cells that translation along coordinate axes simulates the entire structure of macroscopic sample. If there are elements in this metamaterial that are responsible for electrical and magnetic properties, then we can say that this is an electromagnetic crystal. Samples that electrical (effective permittivity) properties are determined by long conductors, and magnetic ones - by half-wave vibrators, were investigated. Measurements of transparency spectra and forbidden electromagnetic band gap were carried out in the frequency range of 8 - 12 GHz. Measurements of refractive index in the material were carried out on the angular deviation of microwave radiation passing through a prism. Measurements show that samples with the same structure (unit cell) can significantly change their characteristics even with a small change in one of the internal parameters of the cell.

1. Introduction
Metamaterials with negative refractive index must have negative effective permittivity and permeability [1]. Such materials with a regular structure consist usually of two types of elements that form two sublattices. Long conductors, in particular wires, form a sublattice with negative effective permittivity [2], and sublattice with negative effective permeability often consists of inductive-gap microresonators (split rings resonators - SRR [3] - [5]). The unit cell of such a regular structure must contain both types of elements. The Q-factor of microresonator is usually about hundred units. Their presence in the structure of metamaterial leads to a delay in propagation of signal.

In most of experimental works, split rings SRR [4], [5] are used as magnetic elements. In [6], a scheme for converting a simple ring resonator into a resonator consisting of two parallel conductors is described. Such a resonator of two conductors is exactly two vibrators with electromagnetic coupling known in VHF and microwave antenna technology [7]. A half-wave vibrator with \( \frac{l}{\lambda} = 0.5 \) (\( l \) is the length of conductive rod, \( \lambda \) is the wavelength) in the equatorial plane has a circular radiation pattern. When resonant excitation currents flow in the vibrator that radiate circular magnetic field into the surrounding space. Hence, half-wave vibrators can be used in metamaterials as magnetic elements [6] instead of circular SRR resonators.

Electromagnetic crystals of the simplest structure have been studied in detail [2]. Specific feature of the spectra of such structures is band structure: an alternation of propagation and
Figure 1. Functional diagram of transmission coefficient measurement setup - a; calibration curve - b.

forbidden bands. Forbidden bands are interpreted as frequency zones with negative effective permittivity.

In present paper, the results of research of metamaterial that structure is two sublattices displaced relatively to each other: sublattice of long wires and second sublattice of half-wave vibrators, are presented. The frequency range and structure in which the effect of negative refraction is observed is found.

2. Objects and methods of investigation
Transmission and scattering spectra of metamaterials were measured in the frequency range of $8-12\text{GHz}$ with a scalar reflection and transmission coefficient meter P2M-40. The measurements were carried out in free space using horn antennas. Measuring setup diagram is shown in figure 1.

Measurements to find the optimal value of lattice constant $a$, as well as influence of resonant characteristics of the vibrators on $T(f)$ spectra, were carried out on cubic samples (figure 2 - a and b). Microwave radiation was directed to the sample along normal to $xy$ plane in which wires or rows of vibrators lay. Electric vector $E$ of radiation is parallel to wires. The transmission spectrum of metamaterial composed of long wires consists of several alternating bands: allowed and forbidden [2]. Therefore, the optimal value $a \times a = 21 \times 21\text{mm}$ was found in preliminary experiments on measuring the dependence of the spectrum on the lattice constant $a$ (figure 2 - a). The $T(f)$ spectrum in the frequency range of $8-12\text{GHz}$ is shown in figure 2 - c, where it is indicated by a solid line. In experiments with metamaterial samples from long wires ($l = 150-170\text{mm}$), similar to figure 2 - a, it was revealed that resonance effects in the range of $8-12\text{GHz}$ do not occur in these structures. $T(f)$ Spectra occupies a range of $8-12\text{GHz}$ with optimal cell sizes $a \times a = 21 \times 21\text{mm}$ and is shown in figure 2 - c. In the second sample rows of half-wave vibrators were used instead of wires. A forbidden band appeared in transmission spectra due to scattering and absorption of microwave power by a three-dimensional vibrator array. Electromagnetic coupling between vibrators [7] determines the width of the forbidden band (figure 2 - c, dotted line). The absorption spectra of vibrators array with a different number of layers was investigated and is given in [8].

The two structures shown in figure 2 - a and b were merged into one. The structure fragment of the obtained metamaterial with an assumed negative refractive index had dimensions of
Figure 2. Lattice fragments from wires - a, from rows of half-wave vibrators - b, and transmission spectra in the transparency frequency zone - c.

Figure 3. Electromagnetic crystal structure - a and b. Prism-like sample for refractive index measurements (fragment of the samples structure studied in this work) - c.

160 × 150 × 160mm and consisted of two sublattices of figure 3 - a The sample size along the z axis was 10.5mm larger than that along the y axis. The wire sublattice consisted of wires 160mm long with a diameter of 0.71mm. Half-wave vibrators with a length of 13.6mm ($f_{res} = 11\text{GHz}$) were assembled into chains also 160mm long with a distance between the vibrators of 6.4mm. The square structure of the sublattices had a constant $a \times a = 21 \times 21\text{mm}$. The sublattice of the vibrators was shifted relative to the wires by 10.5mm along the z axis (figure 3 - b). The angular dependence of the spectrum $T(f)$ was measured on a prism sample, shown schematically in figure 3 - c, vertex angle $\Theta = 25^\circ$. All measurements were carried out in the frequency range of 8 − 12GHz.

3. Results and discussion
As it is already noted, the measurements were carried out in free space on sample of 150 × 150 × 160mm size. In figure 4 (red line - 0°), transmission spectrum $T(f)$ is shown under radiation of the investigated crystal along z direction. Immediately it should be noted that the spectrum of the sample under study differs from the spectra of the wire sample (figure 3.
Figure 4. Bandwidth of the first metamaterial sample (0°); and spectrum of radiation refracted by a prism (75°).

- a) and the sample of vibrators (figure 3 - b). The bandwidth of wire sublattice 8.5 – 12GHz is divided by forbidden band 9.2 – 10.5GHz due to absorption of scanning microwave radiation by vibrators sublattice. In the low frequency region of the frequency response of the vibrator array, starting with a frequency of 9.2GHz, the condition l < 1/2 is satisfied. In this frequency range, the equivalent circuit of the vibrator [6] is a parallel oscillating circuit, and the phase characteristic has a positive shift. Scanning radiation propagates in the dipole sublattice with a delay. The wire sublattice can be considered as a passive bandpass filter, and in the low-frequency domain of the frequency response the phase response has a negative offset. Taking these factors into account, a signal appears at a frequency of 8.5GHz with a negative n outside the transparency bands.

The same figure shows the spectrum of 8.5GHz beam (blue line) shifted to the top of prism by 75° (in laboratory coordinate system). This is a small frequency range, in which (at a level of 1/2Pmax), the forbidden band of the wire sublattice ends and the forbidden band of the vibrator sublattice begins. The lateral lines of spectrum at frequencies of 9.7 and 11.2 can be explained by the fact that volume of sample subjected to radiation was 7 × 7 × 7 cells, and scattering small power of scanning radiation occurred.

In the coordinate system tied to refractive edge of the sample, the angle of radiation on the face is 25°. The output beam deviates from normal to the top of prism by 50°. The incident
Figure 5. Spectrum of the second metamaterial sample - a; and spectrum of beam deflected by 22° to the top of prism - b.

and refracted beams lie on one side of normal. The refractive index is:

\[ n = -(\sin(50)/\sin(25)) = -1.81 \]  

(1)

The second metamaterial sample was also investigated. It has dimensions of 150 \times 150 \times 160 \text{mm}, exactly the same structure as the first (see figure 3 - a) with the same dimensions of elementary cells of sublattices. The only difference in comparison with the first sample is that the sublattice of vibrators is shifted relative to wires by 7 mm along \( z \) axis. Transmission spectra were measured on a cubic sample. The angular dependence of spectrum \( T(f) \) was measured on a prism sample, the setup is shown schematically in figure 3 - c, vertex angle \( \theta = 25° \).

In figure 5 - a, transmission spectrum of the second metamaterial sample is shown. Transparency band of wire structure from 8.5 to 11 GHz is divided by the absorption band of 9.2 – 10.3 GHz of vibrator sublattice. The spectrum of 9 GHz beam deflected to the top of prism is shown in figure 5 - b.

The face of prism through which radiation is emitted at angle \( \varphi \) (the case of negative refractive index) has an angle at the apex of prism of 25°. In this case, long wires and chains of vibrators in first approximation (with square sublattices) lie in the plane of this face and alternate. Radiation from sample volume falls on this face at angle of 25° to the normal of face. The measured emission angle from this face is 22° in laboratory coordinate system. Given measurement errors, the output angle between beam and normal to face is approximately zero (25° – 22° = 3° with a measurement error of \( \pm 3° \)). In this case, the refractive index \( n \approx 8.4 \).

Let consider the lattice in plane of face as a diffraction amplitude slit, figure 6.

Main highs condition of lattice is described by expression (2):

\[ \Delta = d(\sin\psi + \sin\varphi_{\max}) = \pm m\lambda \]  

(2)

In the case of sample under study, \( d = 22 \text{mm} \) and \( \psi = 25° \). Let consider that vibrator gives a phase shift of 90°, then angle \( \varphi = 88° \). According to expression (2), we can calculate that \( \Delta = 31.2 \text{mm} \), and \( m\lambda = m \cdot 30 \text{mm} \), i.e. first diffraction maximum is observed. It can be seen that the vibrators are really reradiated in this sublattice with the \( \pi \) phase shift. And one more remark, if in the first sample, when the distance between sublattices \( \Delta z \) along the \( z \) axis is
10.5\text{mm}, a negative refractive index is observed, then in the second sample with $\Delta z = 7\text{mm}$ there is no negative refraction effect.

4. Conclusion
The provided results show that metamaterial with half-wave vibrator elements can have negative refractive index, and this value is very sensitive to structural parameters.

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