Perfect fluids with $ω = \text{const}$ as sources of scalar cosmological perturbations

Maxim Eingorn and Ruslan Brilenkov

1 North Carolina Central University, CREST and NASA Research Centers, Fayetteville st. 1801, Durham, North Carolina 27707, U.S.A.
and
2 Department of Theoretical Physics, Odessa National University, Dvoryanskaya st. 2, Odessa 65082, Ukraine

In the given paper we make the suggesting itself generalization of a self-consistent first-order perturbation scheme, being suitable for all (sub-horizon and super-horizon) scales, which has been recently constructed for the concordance cosmological model and discrete presentation of matter sources, to the case of extended models with extra perfect fluids and continuous presentation. Namely, we derive a single equation determining the scalar perturbation and covering the whole space as well as define the corresponding universal Yukawa interaction range and demonstrate explicitly that the structure growth is suppressed at distances exceeding this fundamental range.

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I. INTRODUCTION

Along with the conventional cosmological model, which satisfies the modern observational data and describes the Universe filled with prevailing dark energy (represented by the cosmological constant) and cold dark matter (CDM) as well as standard baryonic matter and radiation (playing secondary roles), there are a number of alternatives, which assume the presence of some additional Universe constituent, e.g., in the form of a perfect fluid with a constant parameter $ω$ in the linear equation of state (EoS). The cosmological constant itself may be interpreted exactly as such a fluid with the parameter $ω = -1$ (or, in other words, with the vacuum-like EoS). If $ω \neq -1$ but $ω$ is still close enough to $-1$, then this fluid may be used instead of the cosmological term in order to ensure the late-time acceleration (see, particularly, [1, 2] for the corresponding experimental constraints imposed on the dark energy EoS of the specified form). At the same time, there can be components with $ω$ being quite far from $-1$. For example, frustrated networks of topological defects (cosmic strings and domain walls) have the form of perfect fluids with the constant parameters $ω = -1/3$ and $ω = -2/3$, respectively [3, 4]. In general, depending on the comparison of the negative parameter $ω$ and the vacuum value $-1$, these components are usually called quintessence [5, 6] and phantom [7–11] for $-1 < ω < 0$ and $ω < -1$, respectively.

In compliance with the cosmological principle, the Universe is considered homogeneous and isotropic and described by the corresponding background Friedmann-Lemaître-Robertson-Walker (FLRW) metric on sufficiently large scales. At the same time, on small enough scales the Universe is apparently highly inhomogeneous (galaxies, groups and clusters of galaxies are observed). In the recent paper [12] the unified first-order perturbation scheme being valid for arbitrary (sub-horizon and super-horizon) distances and incorporating linear and nonlinear effects was developed in the framework of the ΛCDM model in the weak gravitational field limit. It has a number of successes in resolving different challenges and promises to be of importance for the high-precision cosmology and $N$-body simulations covering the whole space. Consequently, it makes sense to generalize this approach, elaborated for the presentation of nonrelativistic matter in the form of discrete gravitating particles (see also the precedent papers [13–15]), to the case of the continuous presentation of sources, namely, the standard ΛCDM components in combination with supplementary inhomogeneous fluids undergoing adiabatic perturbations. We make this necessary generalization in the given paper and thereby provide an ample opportunity to investigate the structure formation and growth within the nonconventional cosmological models at arbitrary scales. This can help to distinguish among them and the concordance paradigm.

The paper is organized in the following way. In Section II we revisit the scalar cosmological perturbations theory for the Universe filled with the above-mentioned ingredients and reduce the system of linearized Einstein equations for the first-order metric corrections with respect to the homogeneous background to the only one basic equation. An illustrative example concerning the structure growth is also given here. Then in Section III...
we remove the reported earlier sham limitations on the parameter \( \omega \) and briefly summarize the main results.

II. SCALAR COSMOLOGICAL PERTURBATIONS THEORY REVISITED

Let us start with the FLRW metric

\[
ds^2 = a^2 \left( d\eta^2 - \delta_{\alpha\beta} dx^\alpha dx^\beta \right), \quad \alpha, \beta = 1, 2, 3,
\]

where \( a(\eta) \) is the scale factor; \( \eta \) is the conformal time; the comoving coordinates are denoted by \( x^\alpha, \alpha = 1, 2, 3 \), and it is supposed for simplicity that the spatial curvature is absent. The corresponding Friedmann equations in the case of the \( \Lambda \)CDM model supplemented with an additional perfect fluid characterized by a constant pressure \( p \) read:

\[
\frac{3H^2}{a^2} = \kappa (\tau_M + \tau_R + \tau_X) + \Lambda
\]

\[
= \kappa \sum_I \tau_I, \quad I = M, R, X, \Lambda,
\]

and

\[
\frac{2H' + H^2}{a^2} = -\kappa \bar{p}_R - \kappa \bar{p}_X + \Lambda
\]

\[
= -\kappa \left( \frac{1}{3} \tau_R + \omega \tau_X \right) + \Lambda = -\kappa \sum_I \omega_I \tau_I,
\]

where \( H = a'/a \equiv (da/d\eta)/a; \) the prime denotes the derivative with respect to \( \eta; \kappa \equiv 8\pi G_N/c^4 \) (\( c \) is the speed of light and \( G_N \) is the Newtonian gravitational constant); \( \tau_M, \tau_R \) and \( \tau_X \) represent the energy densities of the nonrelativistic pressureless matter, radiation and the above-mentioned additional component, respectively. The corresponding pressures \( p_M, p_R \) and \( p_X \) satisfy the following linear equations of state:

\[
p_M = 0, \quad p_R = \frac{1}{3} \bar{p}_R, \quad p_X = \omega \varepsilon_X \Leftrightarrow p_I = \omega_I \varepsilon_I,
\]

where \( \omega_M = 0, \omega_R = 1/3 \) and \( \omega_X = \omega \). Further, the overline indicates the average value, and \( \Lambda \) is the cosmological constant (the corresponding energy density and pressure read: \( \varepsilon_\Lambda = \tau_\Lambda / \kappa \) and \( p_\Lambda = \bar{p}_\Lambda = -\Lambda / \kappa \), so \( \omega_\Lambda = -1 \)). Throughout the paper the extended notation in formulas is combined with the contracted one, which contains the subscript \( I \) and, generally speaking, is valid for the Universe filled with an arbitrary number of perfect fluids with constant parameters in the linear equations of state (or, in other words, with pressureless matter, radiation and an arbitrary number of additional \( X \)-components of the specified form).

It should be noted that we do not strive for replacing the \( \Lambda \)-term by the \( X \)-component, for example, in order to assure the late-time acceleration of the Universe expansion. On the contrary, the full range of values of the parameter \( \omega \) is studied, including those which do not give rise to this acceleration when \( \Lambda = 0 \) (so in this case the nonzero \( \Lambda \)-term is still required in order to be in agreement with the observations). At the same time we do not exclude a possibility \( \omega = -1 \) (in this case there is no need to introduce the \( \Lambda \)-term separately, so one should consider that \( \Lambda = 0 \)).

Following the analysis of scalar cosmological perturbations in [12–19], let us consider the metric

\[
ds^2 \approx a^2 \left[ (1 + 2\Phi) d\eta^2 - (1 - 2\Phi) \delta_{\alpha\beta} dx^\alpha dx^\beta \right],
\]

then the linearized Einstein equations for the function \( \Phi \) read:

\[
\Delta \Phi - 3H(\Phi' + H\Phi) = \frac{1}{2} \kappa a^2 \delta T^0_0,
\]

\[
\frac{\partial}{\partial x^\alpha}(\Phi' + H\Phi) = \frac{1}{2} \kappa a^2 \delta T^0_\alpha,
\]

\[
[\Phi'' + 3H\Phi' + (2H' + H^2) \Phi] \delta_{\alpha\beta} = -\frac{1}{2} \kappa a^2 \delta T^\alpha_\beta.
\]

The average mixed components \( T^i_k, i, k = 0, 1, 2, 3 \), of the total energy-momentum tensor for the investigated multicomponent perfect fluid (pressureless matter + radiation + \( X + \Lambda \)) read:

\[
T^0_0 = \tau_M + \tau_R + \tau_X + \tau_\Lambda \equiv \sum_I \tau_I, \quad T^0_\alpha = 0,
\]

\[
\bar{T}^\alpha_\beta = - (\bar{p}_R + \bar{p}_X + \bar{p}_\Lambda) \delta_{\alpha\beta}
\]

\[
= - \left( \frac{1}{3} \tau_R + \omega \tau_X - \tau_\Lambda \right) \delta_{\alpha\beta} = - \delta_{\alpha\beta} \sum_I \omega_I \tau_I,
\]

while for the corresponding fluctuations we have

\[
\delta T^0_0 = \delta \varepsilon_M + \delta \varepsilon_R + \delta \varepsilon_X \equiv \sum_I \delta \varepsilon_I,
\]

\[
\delta T^0_\alpha = - \sum_I (1 + \omega_I) \frac{\partial \zeta_I}{\partial x^\alpha},
\]

\[
\delta T^\alpha_\beta = - (\delta p_R + \delta p_X) \delta_{\alpha\beta}
\]

\[
= - \left( \frac{1}{3} \delta \varepsilon_R + \omega \delta \varepsilon_X \right) \delta_{\alpha\beta} = - \delta_{\alpha\beta} \sum_I \omega_I \delta \varepsilon_I.
\]

Here, obviously, \( \delta \varepsilon_\Lambda \equiv 0 \), so \( \delta p_\Lambda = - \delta \varepsilon_\Lambda = 0 \). Of course, these equalities do not exclude the case of the inhomogeneous perfect fluid with the vacuum-like EoS, since its role can be played by the additional \( X \)-component with the appropriate parameter \( \omega = -1 \) and nonzero fluctuations \( \delta \varepsilon_X \neq 0 \) and \( \delta p_X = - \delta \varepsilon_X \neq 0 \). As regards the introduced quantities \( \zeta_I \), which are treated as importing the first order of smallness, \( \nabla \zeta_I \) stands for the gradient
part of the spatial vector \( \varepsilon_I \mathbf{v}_I \), where \( \mathbf{v}_I \) is the comoving velocity field of the corresponding \( I \)-th constituent of the Universe.

In addition, in concordance with Reference 12 (see also Refs. therein), we reject the generally accepted assumption of the linear relativistic perturbation theory that the energy density fluctuations \( \delta \varepsilon_I \) are much less than the corresponding average values \( \overline{\varepsilon}_I \), i.e. \( \delta \varepsilon_I \ll \overline{\varepsilon}_I \). In other words, we do not require fulfilment of these inequalities, allowing the fluctuations \( \delta \varepsilon_I \) to be nonlinear. This is an indispensable step in the direction of elaborating a relativistic formalism, which would incorporate nonlinear effects at small distances, while being valid at large distances as well.

Substituting (11) and (12) into (6)-(8), we obtain

\[
\Phi' + \Phi \overline{\varepsilon}_I = -\frac{1}{2} \kappa a^2 \sum_I (1 + \omega_I) \zeta_I, \tag{14}
\]

Substitution of (14) into (13) gives

\[
\Delta \Phi = \frac{1}{2} \kappa a^2 \sum_I \delta \varepsilon_I - \frac{3}{2} \kappa a^2 \overline{\varepsilon}_I \sum_I (1 + \omega_I) \zeta_I. \tag{16}
\]

In order to find \( \Phi \) as a solution of Eq. (10), we need, in particular, to determine the quantities \( \delta \varepsilon_I, I = M, R, X \). For this purpose, let us analyze the well-known background energy conservation equations

\[
\overline{\varepsilon}_M + 3 \frac{a'}{a} \overline{\varepsilon}_M = 0 \quad \Rightarrow \quad \overline{\varepsilon}_M \sim \frac{1}{a^3}, \tag{17}
\]

\[
\overline{\varepsilon}_R + 3 \frac{a'}{a} (\overline{\varepsilon}_R + \overline{\rho}_R) = \overline{\varepsilon}_R + 4 \frac{a'}{a} \overline{\rho}_R = 0 \quad \Rightarrow \quad \overline{\varepsilon}_R \sim \frac{1}{a^4}, \tag{18}
\]

\[
\overline{\varepsilon}_X + 3 \frac{a'}{a} (\overline{\varepsilon}_X + \overline{\rho}_X) = \overline{\varepsilon}_X + 3(1 + \omega) \frac{a'}{a} \overline{\rho}_X = 0 \quad \Rightarrow \quad \overline{\varepsilon}_X \sim \frac{1}{a^{3(1+\omega)}}, \tag{19}
\]

as well as the perturbed energy-momentum conservation equations (see, e.g., Reference 19 for their linearized form):

\[
\delta \varepsilon'_I + 3 \frac{a'}{a} (\delta \varepsilon_I + \delta \rho_I) - 3 (\overline{\varepsilon}_I + \overline{\rho}_I) \Phi' + \nabla \left[ (\varepsilon_I + p_I) \mathbf{v}_I \right] = 0, \tag{20}
\]

where \( \nabla \left[ (\varepsilon_I + p_I) \mathbf{v}_I \right] = (1 + \omega_I) \Delta \zeta_I, \) and

\[
(1 + \omega_I) \zeta'_I + 4 \frac{a'}{a} (1 + \omega_I) \zeta_I + \delta \rho_I + (\overline{\varepsilon}_I + \overline{\rho}_I) \Phi = 0. \tag{21}
\]

Really, we study solely noninteracting pressureless matter, radiation and the \( X \)-component; therefore, Eqs. (20) and (21) are valid for each constituent of the Universe separately. Besides, we have dropped the contributions containing both \( \delta \varepsilon_I \) and \( \Phi \) since these terms would import the second order of smallness in the Einstein equations and therefore lie beyond the accuracy adopted here. From (20) we get

\[
\delta \varepsilon'_M + 3 \frac{a'}{a} \delta \varepsilon_M - 3 \overline{\varepsilon}_M \Phi' + \Delta \zeta_M = 0 \quad \Rightarrow \quad \delta \varepsilon_M = \frac{\delta A_M}{a^3} + 3 \overline{\varepsilon}_M \Phi', \tag{22}
\]

\[
\delta \varepsilon'_R + 4 \frac{a'}{a} \delta \varepsilon_R - 4 \overline{\varepsilon}_R \Phi' + \frac{4}{3} \Delta \zeta_R = 0 \quad \Rightarrow \quad \delta \varepsilon_R = \frac{\delta A_R}{a^3} + 4 \overline{\varepsilon}_R \Phi, \tag{23}
\]

\[
\delta \varepsilon'_X + 3(1 + \omega) \frac{a'}{a} \delta \varepsilon_X - 3(1 + \omega) \overline{\varepsilon}_X \Phi' + (1 + \omega) \Delta \zeta_X = 0 \quad \Rightarrow \quad \delta \varepsilon_X = \frac{\delta A_X}{a^{3(1+\omega)}} + 3(1 + \omega) \overline{\varepsilon}_X \Phi, \tag{24}
\]

\[
\delta A'_X = -(1 + \omega) a^{3(1+\omega)} \Delta \zeta_X. \tag{25}
\]

Thus, the sought-for fluctuations \( \delta \varepsilon_I, I = M, R, X \), are determined. In confirmation of the result (22) for \( \delta \varepsilon_M \), let us mention the important fact that it exactly coincides with what follows directly from the well-known formula \( \varepsilon_M = \rho_M c^2 \left[ \delta \rho_M / (-\bar{g}) \right]^{1/2} \) (20) \( (\overline{\varepsilon}_M = \overline{\rho}_M c^2 / a^3 \) and \( \delta A_M = \delta \rho_M c^2, \) where \( \rho_M \) and \( \overline{\rho}_M \) represent the rest mass density of the pressureless matter in the comoving coordinates and its average value, respectively, while \( \delta \rho_M = \rho_M - \overline{\rho}_M \). The derived expressions (22)-(24) can be certainly united:

\[
\delta \varepsilon_I = \frac{\delta A_I}{a^{3(1+\omega)}} + 3(1 + \omega) \overline{\varepsilon}_I \Phi, \tag{26}
\]

\[
\delta A'_I = -(1 + \omega) a^{3(1+\omega)} \Delta \zeta_I, \tag{27}
\]

and this general expression is also valid for \( I = \Lambda \) since \( \delta A_{\Lambda} \equiv 0 \). Substituting (25) into Eq. (10), we get

\[
\Delta \Phi = -\frac{3}{2} \kappa a^2 \sum_I (1 + \omega_I) \overline{\varepsilon}_I \Phi = \frac{1}{2} \kappa a^2 \sum_I \frac{\delta A_I}{a^{3(1+\omega)}} - \frac{3}{2} \kappa a^2 \overline{\varepsilon}_I \sum_I (1 + \omega_I) \zeta_I. \tag{28}
\]

Thus, we arrive at determination of the scalar perturbation \( \Phi \) by the fluctuations \( \delta A_I, I = M, R, X \), describing the “intrinsic” perturbations of the corresponding energy densities \( \varepsilon_I \) (the first terms in (22)-(24), respectively), and the quantities \( \zeta_I \). The second term on
the left-hand side of Eq. (23) is directly proportional to the sum of the “responses” of the energy densities \( \varepsilon_I \) to the presence of the inhomogeneous gravitational field (the second terms in (22) and (21)). The expression

\[
\frac{3}{2} c \left[ \sum I (1 + \omega_I) \sigma_I \right] = \frac{3}{a^2} \left( \frac{H^2 - \dot{H}}{a^2} \right) \equiv \frac{1}{\chi^2} \tag{27}
\]
determines the Yukawa interaction range \( \lambda \) (in the physical coordinates), which is universal for each component, in complete agreement with the corresponding statement made in (12). Really, the solution of Eq. (20) has the form

\[
\Phi = \Phi_M + \Phi_R + \Phi_X = \sum I \Phi_I, \quad \Phi \equiv 0, \tag{28}
\]
where each contribution \( \Phi_I \) satisfies the equation

\[
\Delta \Phi_I = \frac{a^2}{\chi^2} \Phi_I = \frac{\kappa}{2a^2} \chi \delta A_I - \frac{3}{2} \kappa a^2 H (1 + \omega_I) \zeta_I. \tag{29}
\]
Finding \( \Phi_I \) from (29), one can substitute the result into (28) and, hence, find the total scalar perturbation \( \Phi \), provided that its sources \( \delta A_I \) and \( \zeta_I \) are known. It is interesting that the formulated Yukawa range definition (27) holds true in the case of varying parameters \( \omega_I \) as well (or, in other words, in the case of nonlinear equations of state) if linear perturbations are at the center of attention. Really, this follows directly from the fact that the “response” \( \delta \varepsilon_I \sim 3 (\sigma_I + \sigma_{\parallel}) H \Phi \) still satisfies the equation \( \delta \varepsilon_I + 3 H \delta \varepsilon_I + \delta p_I - 3 (\sigma_I + \sigma_{\parallel}) \Phi' = 0 \) for the EoS \( p_I = f(\varepsilon_I) \) with an arbitrary nonlinear function \( f \), as one can easily prove with the help of the evident equalities \( \sigma_I = f(\varepsilon_I) \), \( \delta p_I = (\partial p_I/\partial \varepsilon_I) \delta \varepsilon_I \), and \( \sigma_{\parallel} + 3H(\sigma_I + \sigma_{\parallel}) = 0 \). The perfect fluid with Chevallier-Polarski-Linder parametrization of the EoS (\( \omega \) as a linear function of \( a \)) and the Chaplygin gas represent concrete popular examples, which belong to the discussed class of supposed Universe components.

It is not difficult to show that the remaining linearized Einstein equations (13) and (15) are satisfied. For example, one can act by the Laplace operator \( \Delta \) on both sides of Eq. (13) and express \( \Delta \Phi \) from (20), \( \delta A_I \) from (25), and \( \zeta_I \) from (21) as well as use the Friedmann equations (12) and (6). Then, in order to prove Eq. (15), one can start with expressing \( \Phi' \) from (14).

Therefore, the initial system of Eqs. (13-15) has been reduced to the only one Eq. (20) supplemented with the perturbed energy-momentum conservation equations. It is noteworthy that, as we emphasized above, fulfilment of the inequalities \( \delta \varepsilon_I \ll \sigma_I \) is not demanded. Eq. (20) for the scalar perturbation \( \Phi \) represents our main novel result. Its derivation became possible due to splitting of \( \delta \varepsilon_I \) into “intrinsic” fluctuations and “responses” to the presence of the inhomogeneous gravitational field (see the first and second terms in (25), respectively). First, this splitting helps to find \( \Phi \) in the whole space for linear as well as nonlinear perturbations of energy densities, without \( 1/e \) series expansion, “dictionaries” or admixtures of second-order quantities. In other words, we have not made any extra assumptions supplementing the weak field limit. Second, separation of intrinsic fluctuations reveals the Yukawa nature of their gravitation, corroborating the ideas reported earlier in (12).

Returning momentarily to the linear perturbation theory, it is very interesting to investigate the structure growth in terms of intrinsic fluctuations. For the illustration purposes, let us restrict ourselves to the matter-dominated stage of the Universe evolution, disregarding all other constituents. Then, being based on (21), (24) and (29), it is not difficult to demonstrate that the Fourier transform \( \delta \rho_M(\eta, k) \) of the rest mass density perturbation \( \delta \rho_M(\eta, \mathbf{k}) \) satisfies the following equation:

\[
\delta \rho_M'' + H \delta \rho_M - \kappa \frac{c}{\chi} \frac{a}{2} \delta \rho_M = 0, \tag{30}
\]
where \( \chi \equiv a^2/ (k^2 \lambda^2) \). Now, if \( a/k \ll \lambda \) (small scales as compared with the Yukawa interaction range), \( \chi \ll 1 \), and then

\[
\delta \rho_M'' + H \delta \rho_M - \kappa \frac{c}{\chi} \frac{a}{2} \delta \rho_M = 0, \tag{31}
\]
admitting the growing mode \( \delta \rho_M \sim a \), as it is generally known. One should remember that the rest mass densities \( \rho_M \) and \( \rho_M^{ph} \) in comoving and physical coordinates, respectively, are interconnected by means of the relationship \( \rho_M^{ph} = \rho_M / a^3 \). The average physical rest mass density \( \bar{\rho}_M^{ph} \) behaves as \( 1/a^3 \), while \( \bar{\rho}_M \) = const, and the density contrast \( \delta_M \equiv \delta \rho_M^{ph} / \bar{\rho}_M^{ph} = \delta \rho_M / \bar{\rho}_M \sim \delta \rho_M \).

However, in the opposite case \( a/k \gg \lambda \) (large scales as compared with the Yukawa range), \( \chi \gg 1 \), and therefore

\[
\delta \rho_M'' + 2 H \delta \rho_M = 0, \tag{32}
\]
admitting the constant mode \( \delta \rho_M = const \). The issued statement remains unchanged if the cosmological constant \( \Lambda \) is also taken into consideration along with the nonrelativistic matter. This is one more novel result obtained in the given paper, which can be formulated as follows: the evolution of intrinsic fluctuations (the rest mass density perturbation in our example) is suppressed at distances exceeding the Yukawa range of gravitational interaction. Of course, this assertion is expected right after the introduction of this range. Really, from the physical point of view, inhomogeneities, which are separated by the distance greater than \( \lambda \), gravitationally almost do not “feel” each other. Consequently, it is hard for them to participate in the formation of the same structure.

One should emphasize that the revealed upper bound set to a spatial domain of the probable structure growth corroborates the hypothesis formulated in (12): the established Yukawa range bears a direct relation to the explanation of existence of the largest known cosmic structures (23-24) (according to (12), \( \lambda \approx 3.7 \) Gpc at present, and their dimensions do not exceed this limiting value).
III. TROUBLESHOOTING AND CONCLUSION

Let us enumerate two important corollaries from the developed description of scalar cosmological perturbations. First, in the framework of the ΛCDM model supplemented with an additional perfect fluid characterized by a constant parameter ω in the linear EoS (or, generally speaking, with an arbitrary number of such fluids) there is no any theoretical limitation imposed on ω at the level of linearized Einstein equations, as distinct from what was reported before in [25]. In that paper only two values ω = −1 (the vacuum-like EoS) and ω = −1/3 survived. Now these severe restrictions are removed irrevocably. Second, there is no need at all in any additional radiation contribution, introduced in [15], and the energy-momentum conservation takes place for each Universe constituent separately.

The following main results, obtained in the present paper, deserve mentioning as well:

— the first-order scalar perturbation Φ is determined at arbitrary (sub-horizon and super-horizon) scales by means of the Helmholtz equation (20) in the weak gravitational field limit;
— owing to [20], the spatial averaging of Φ gives the zero value $\bar{\Phi} = 0$, so there are no first-order backreaction effects, as expected from the very beginning (for demonstration technique see [12]);
— the velocity-independent part of Φ is characterized by the finite time-dependent Yukawa interaction range $\lambda$, defined by the formula [27] and being the same for each supposed component of the inhomogeneous Universe, in full accord with [12]:

— the Yukawa nature of gravitational interaction between inhomogeneities leads to suppression of the structure formation and evolution at distances greater than $\lambda$.

Thus, we have made the promised direct generalization of the approach elaborated in [12] for the concordance cosmological model and discrete presentation of matter sources to the case of unconventional models with extra perfect fluids and continuous (hydrodynamical) presentation. Being based on the obtained results, it is quite possible to construct similarly an appropriate second-order scheme for arbitrary scales and investigate the backreaction effects. Corresponding hydrodynamical simulations of structure growth and related investigations can enable us to distinguish among the ΛCDM paradigm and its various competing alternatives.

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