Effective Field Theories, Landau-Migdal Fermi-Liquid Theory and Effective Chiral Lagrangians for Nuclear Matter

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Abstract

We reinterpret Landau-Migdal Fermi-liquid theory of nuclear matter as an effective chiral field theory with a Fermi surface. The effective field theory is formulated in terms of a chiral Lagrangian with its \textit{mass and coupling parameters} scaling a la Brown-Rho and with the Landau-Migdal parameters identified as the fixed points of the field theory. We show how this mapping works out for response functions to the EM vector current and then using the same reasoning, make prediction on nuclear axial current, in particular on the enhanced axial-charge transitions in heavy nuclei. We speculate on how to extrapolate the resulting theory which appears to be sound both theoretically and empirically up to normal nuclear matter density $\rho_0$, to hitherto unexplored higher density regime relevant to relativistic heavy-ion processes and to cold compact (neutron) stars.

* Dedicated to the memory of A.B. Migdal on the 90th anniversary of his birthday
1 Introduction

In a recent beautiful development [1], Landau’s Fermi liquid theory has been re-formulated as a modern effective field theory with the Fermi liquid state identified as a stable fixed point. This theory represents an effective field theory which is as beautiful as chiral Lagrangian field theory for low-energy pionic interactions. It is then most natural that Migdal’s theory of nuclear matter [2] which is based on Landau Fermi liquid theory can also be formulated as an effective field theory. We dedicate this note, which is based on recent work [3, 4], as a tribute to Migdal on the occasion of his 90th anniversary.

2 Effective Field Theory

Effective field theories enter the nuclear physics domain in two different ways. One is to make precise predictions for certain processes involving few-nucleon systems that are connected with fundamental issues of physics. This is often called for to answer questions of fundamental nature in other areas of physics such as astrophysics or particle physics [5]. The other – which is our objective here – is to be able to extrapolate the knowledge available in normal conditions beyond the normal nuclear matter regime into a high temperature or high density regime that will be the focus of experimental efforts in the coming years. In making the extrapolation, the usual quantum mechanical many-body approach lacks the necessary versatility and field theoretic approaches anchored in quantum chromodynamics will be required. Migdal’s formulation of Fermi liquid for nuclear matter has proven powerful at least up to normal nuclear matter density, and has even led to a variety of predictions of phenomena that might take place in extreme conditions [6]. In its original form, however, it is somewhat limited in its scope if one wishes to extrapolate to extreme conditions, where QCD phase changes may be induced. Such densities are expected in upcoming laboratories and probably exist in neutron stars interiors. In this note, we wish to discuss our recent attempt to formulate Landau-Migdal theory of nuclear matter in a modern effective field theory framework. Such a framework, which offers the possibility of extrapolation to extreme conditions, has been quite successful in such different fields as condensed matter and high-energy physics.

2.1 Effective field theory (EFT) for light nuclei

Before going into our main topic of dense matter, we briefly summarize the status of effective field theories in few-nucleon systems. Here the setting for an EFT is straightforward.

The objectives there are essentially two-fold. One is to derive the nucleon-nucleon interactions – which are fairly well understood from phenomenological approaches – from fundamental principles. The basic question is: Can all two-nucleon systems, viz, nucleon-nucleon scattering at low energy and bound states (e.g., the deuteron) be understood in the framework of an effective field theory? This is an old question, which stimulated by the work of Weinberg [7], recently became the focus of intense activities in many theoretical communities. The status of the field is comprehensively summarized in the proceedings of two recent INT-Caltech workshops [8]. The original Weinberg approach had certain apparent inconsistency in the power counting invoked for a systematic calculation but this problem can be readily resolved as shown by the INT-Caltech collaboration [9]. In this work the notion of “power divergence subtraction” was introduced into the dimensional regularization. This enables one to handle the anomalous length scale that ap-
pears when a quasi-bound state is near by in a more straightforward manner. We now know that when done correctly, the two schemes (i.e., Weinberg’s and the INT/Seattle-Caltech scheme), are essentially equivalent in the treatment of low-energy two-nucleon interactions. Although they may differ in specific details, the two schemes reproduce the low-energy observables thus far studied with equal quality.

The other objective is to exploit the power of effective field theories in making **bona-fide predictions** for processes which cannot be accessed by standard nuclear physics methods. Examples that have been discussed recently are the asymmetry observables in the polarized np capture \( n + p \rightarrow d + \gamma \)

and the solar hep process \( p + ^3\text{He} \rightarrow ^4\text{He} + e^+ + \nu_e \).

The first process (1) has been studied theoretically in a variety of different methods \([10, 12]\) and is being measured \([13]\). The second process (2) has been recently measured in the Super-Kamiokande experiment \([14]\) and has generated a lot of excitement among theorists \([15]\). It turns out rather remarkably that these two processes complement each other in providing a theoretical strategy to overcome a non-trivial obstacle on the way to a parameter-free calculation.

Now, in order to increase the predictive power in general and to facilitate accurate calculations of the above processes, a hybrid version of EFT (called MEEFT or “more effective EFT”) was launched by Park, Kubodera, Min and Rho \([16, 17, 11]\). This approach, which combines the sophisticated standard nuclear physics approach with chiral perturbation theory, turns out to be far more powerful and robust than naively expected. Within this scheme one can **actually** make reliable calculations of observables that cannot be obtained by other methods. Of equal importance is the fact that such predictions can be confronted with data. Thus, the validity of this approach will be settled by experiment in the near future. The accuracy with which such predictions can be made is assessed in \([11]\).

2.2 EFT for heavy nuclei and nuclear matter

In both cases mentioned above addressing low-density systems, the effective Lagrangians are defined at zero density and the relevant fluctuations are made on top of the zero-density vacuum which is accessible by various QCD analyses, treating the matter density as an external perturbation. In a dense medium, the situation is expected to be quite different. While in the light systems the parameters that figure in the effective Lagrangian are in principle derivable from QCD (perhaps on a lattice) or more often from experimental data, this is not the case in a dense medium. Deriving an EFT for dense matter from QCD is probably of similar difficulty as deriving the Hubbard model from QED. The best one can do is to start with a Lagrangian defined at zero density and go up in density. Unfortunately this will be limited to low density and cannot be pushed to high enough density to be useful in the regime we would like to explore.

In this note we circumvent the difficulty of deriving such a theory directly. Rather, we construct an effective chiral Lagrangian field theory that maps onto an established many-body theory, specifically Landau-Migdal’s Fermi-liquid theory and then extrapolate that field theory to the regime of higher density. This is certainly in accordance with the original spirit of Landau-Migdal theory though it is not clear that such a scheme will work in all density regimes. We can only say that up to now there is no evidence against the scheme. For a recent review, see \([4]\).
3 Nuclear Matter as a Fermi Liquid Fixed Point

3.1 Chiral liquid

How to obtain a realistic description of nuclear matter from an effective Lagrangian anchored in the fundamentals of QCD is very much an open problem at the moment. There are however several models available. One of them, the skyrmion with an infinite baryon number is yet to be confronted with Nature. The skyrmion is a soliton solution of a Skyrme-type Lagrangian, which is an approximate Lagrangian for QCD at infinite number of colors $N_c = \infty$. Because the mathematical structure of this model is not very well known at the moment, only very little information can be extracted from it.

Another model is the non-topological soliton picture proposed in an embryonic form sometime ago by Lynn [18]. This description has recently been given a more realistic structure by Lutz, Friman and Appel [19]. The idea here is that one writes down an effective potential or energy calculated to the highest order feasible in practice in chiral perturbation theory, suitably taking into account all relevant scales involved and then looks for the minimum of the effective potential to be identified with the nuclear matter ground state. The state so obtained may be identified with Lynn’s chiral liquid state. The connection between the skyrmion with an infinite winding number and the chiral liquid matter – which must exist in large $N_c$ limit – is presently not understood.

The starting point of our consideration is the assumption that we have a chiral-liquid solution of the type described in [19] that represents the ground state (“vacuum”), on top of which fluctuations can be calculated. The discussion of [19] does not specify how these fluctuations are to be made. To proceed, we propose that the parameters of the Lagrangian (such as masses, coupling constants etc) of the fields representing the relevant degrees of freedom are determined at this ground state, not at the zero-density vacuum which gives the starting point of the Lynn strategy and hence run with density $\rho$. The Lagrangian so defined is assumed to satisfy the same symmetry constraints – such as chiral symmetry and scale anomaly – as those intrinsic to QCD at zero density.

3.2 Effective chiral Lagrangian

Let us denote the parameters so defined at a density $\rho$ with a star. The mass of a nucleon in the system will be denoted as $M^*$, the pion decay constant $f_\pi^*$ etc. The simplest chiral Lagrangian for the nuclear system so defined takes the form

$$\mathcal{L} = \bar{N}[i\gamma_\mu(\partial^\mu + iv^\mu + g_A^*\gamma_5a^\mu) - M^*]N - \sum_i C_i^*(\bar{N}\Gamma_iN)^2 + \cdots$$

(3)

where the ellipsis denotes higher dimensional nucleon operators and the $\Gamma_i$’s Dirac and flavor matrices as well as derivatives consistent with chiral symmetry. Furthermore

$$\xi^2 = U = e^{i\pi - \tau/f_\pi^*}$$

$$v_\mu = -\frac{i}{2}(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)$$

$$a_\mu = -\frac{i}{2}(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger).$$

(4)

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1The meaning of density-dependence in the parameters in an effective Lagrangian we shall study will be precisely defined later. There is a subtlety due to the requirement of chiral symmetry that needs to be addressed.
In (3) only the pion ($\pi$) and nucleon ($N$) fields appear explicitly; all other fields have been integrated out. The effect of massive degrees of freedom will be lodged in higher-dimensional and/or higher-derivative interactions. The external electro-weak fields that we will consider below are straightforwardly incorporated by suitable gauging.

If one is considering symmetric nuclear matter and limits oneself to the mean field approximation, one can write, following [20], an equivalent Lagrangian that contains just the degrees of freedom that figure in a linear model of the Walecka-type [21]

$$\mathcal{L} = \bar{N}(i\gamma_{\mu}(\partial^{\mu} + ig^{*}_{\omega} \omega^{\mu}) - M^{*} + h^{*}\phi)N - \frac{1}{4}F_{\mu\nu}^{2} + \frac{1}{2}(\partial_{\mu}\phi)^{2} + \frac{m_{\omega}^{2}}{2}\omega^{2} - \frac{m_{\phi}^{2}}{2}\phi^{2} + \cdots$$

where the ellipsis denotes higher-dimension operators. This Lagrangian is totally equivalent to (3) in the mean field approximation [21, 22]. Unless otherwise noted, we will be using (3).

3.3 Interpreting the density dependence of the parameters

From a field theory point of view, it is unclear what “density dependence” of various constants of the Lagrangian means. This is because the number density $\rho$ is defined as the expectation value of the number density operator $\bar{N}\gamma_{0}N$ with respect to the state vector one is considering. Thus the density $\rho$ is defined only once the state is determined. The only way that such a quantity can be introduced into the Lagrangian is to assume that the parameters of the Lagrangian such as coupling constants and masses are functions of the fields involved. The constraint that the Lagrangian be invariant under chiral symmetry transformation then limits the field dependence. One may choose a chiral singlet scalar field or chiral invariant bilinear in the nucleon fields. In what follows we shall choose the latter.

For this purpose, we define the chirally invariant operator

$$\tilde{\rho}u^{\mu} \equiv \bar{N}\gamma^{\mu}N$$

where

$$u^{\mu} = \frac{1}{\sqrt{1 - v^{2}}}(1, v) = \frac{1}{\sqrt{\rho^{2} - j^{2}}} (\rho, j).$$

is the fluid 4-velocity. Here

$$j = \langle \bar{N}\gamma N \rangle$$

is the baryon current density and

$$\rho = \langle N^\dagger N \rangle = \sum_{i} n_{i}$$

the baryon number density. The expectation value of $\tilde{\rho}$ yields the baryon density in the rest-frame of the fluid. Using $\tilde{\rho}$ it is easy to construct a Lorentz invariant, chirally invariant Lagrangian with density dependent parameters. However, here we shall not use the relativistic formulation.

Now a density dependent mass parameter in the Lagrangian should be interpreted as

$$m^{*} = m^{*}(\tilde{\rho}).$$
This means that the model (5) is no longer linear, but highly non-linear even at the mean field level. We shall illustrate this using the Lagrangian (5) in the mean field approximation and show that our interpretation is thermodynamically consistent.

The Euler-Lagrange equations of motion for the bosonic fields are the usual ones but the nucleon equation of motion is not. This is because of the functional dependence of the masses and coupling constants on the nucleon field:

\[
\delta L \delta \bar{N} = \frac{\partial L}{\partial \bar{N}} + \frac{\partial L}{\partial \bar{\rho}} \frac{\partial \bar{N}}{\partial \bar{\rho}} = [i \gamma^\mu (\partial_\mu + ig^* \omega_\mu - i u_\mu \Sigma) - M^* + h^* \phi] N = 0
\]

with

\[
\Sigma = \frac{\partial L}{\partial \bar{\rho}}
\]

\[
= m^* \omega^2 \frac{\partial m^*}{\partial \bar{\rho}} - m^* \phi^2 \frac{\partial m^*}{\partial \bar{\rho}} - \bar{N} \omega^\mu \gamma_\mu N \frac{\partial m^*}{\partial \bar{\rho}} - \bar{N} \frac{\partial M^*}{\partial \bar{\rho}}.
\]

Here we are assuming that \( (\partial/\partial \bar{\rho}) h^* \approx 0 \). It may be possible to justify this but we shall not attempt it here. The additional term \( \Sigma \), which may be related to what is referred to in many-body theory as "rearrangement terms", is essential in making the theory consistent. This point has been overlooked in the literature.

Here we shall briefly summarize the results. Details can be found in [23, 4]. When one computes the energy-momentum tensor with (5), one finds the canonical term, which is obtained when the parameters are treated as constants, as well as a new term proportional to \( \Sigma \)

\[
T^{\mu \nu} = T^{\mu \nu}_{\text{can}} + \Sigma (\bar{N} u \cdot \gamma N) g^{\mu \nu}.
\]

The pressure is then given by \( \frac{1}{3} \langle T_{ii} \rangle_{u=0} \). The additional term in (13) matches precisely the terms that arise when the derivative with respect to \( \rho \) acts on the density-dependent masses and coupling constants in the formula derived from \( T_{00} \):

\[
p = -\frac{\partial E}{\partial V} = \rho^2 \frac{\partial E}{\partial \rho} = \mu \rho - \mathcal{E}
\]

where

\[
\mathcal{E} = \langle T^{00} \rangle.
\]

This matching assures energy-momentum conservation and thermodynamic consistency.

Once the interpretation of the density dependence is specified, the derivation of the Landau-Migdal parameters, thermodynamic quantities etc. from (5) is completely analogous to the procedure used by Matsui [24] for Walecka’s linear \( \sigma - \omega \) model.

### 3.4 Nuclear matter with BR scaling

We saw above that the masses and coupling constants in (5) (or equivalently (3)) are to be treated as functionals of \( \bar{\rho} \) where

\[
\bar{\rho} \mu = \bar{N} \gamma^\mu N.
\]
When treated at the mean field level, $\hat{\rho}$ is just the number density, so the parameters become density-dependent. The dependence of the parameters in the Lagrangian on the fields rather than on the density is essential for thermodynamic consistency. Note however that these considerations do not require the parameters to satisfy scaling relations. It is the chiral symmetry and scale symmetry that suggest that the masses satisfy BR scaling at the mean field level \[25\]

$$\Phi(\rho) \approx \frac{f_\pi^*(\rho)}{f_\pi} \approx \frac{m_\rho^*(\rho)}{m_\phi} \approx \frac{m_V^*(\rho)}{m_V} \approx \frac{M^*(\rho)}{M}. \quad (17)$$

Here $V$ stands for the light-quark vector mesons $\rho$ and $\omega$. The quantity $\Phi(\rho)$ is the scaling factor that needs to be determined from theory or experiments. For concreteness, we shall assume

$$\Phi(\rho) = (1 + y\rho/\rho_0)^{-1}. \quad (18)$$

The value of $y$ will be determined below by a global fit of the ground state properties of nuclear matter. Now taking the free-space values,

$$M = 938, \quad m_\omega = 783, \quad m_\phi = 700 \text{ MeV} \quad (19)$$

and

$$g_v = 15.8, \quad h = 6.62, \quad (20)$$

with one additional assumption that the vector coupling $g_v^*$ scales like the mass $m_\omega^*$ and $h^*$ is almost constant, one finds the following properties for the ground state of nuclear matter

$$m_N^*/M = 0.62, \quad E/A - M = -16.0 \text{ MeV}, \quad k_F = 257 \text{ MeV}, \quad K = 296 \text{ MeV}. \quad (21)$$

Here $k_F$ is the Fermi momentum at the saturation point and $K$ is the corresponding compression modulus. The best values favored by Nature that are “well determined” and that “can be associated with an equal number of nuclear properties and general features of RMF (relativistic mean field) models” \[26\] are

$$m_N^*/M = 0.61 \pm 0.03, \quad E/A - M = -16.0 \pm 0.1 \text{ MeV}, \quad k_F = 256 \pm 2 \text{ MeV}, \quad K = 250 \pm 50 \text{ MeV}. \quad (22)$$

To arrive at (21), we need $y = 0.28$ which implies that $\Phi(\rho_0) \approx 0.78$. The scaling of $g_v$, which is needed to obtain a good fit, was not incorporated in the original BR scaling \[25\] but it does not invalidate the scaling relation (17) which is a mean field relation. The scaling of the coupling constant is a fluctuation effect on top of the BR scaling ground state, that is, a running as in the renormalization group as discussed in \[27\]. A caveat here is that at this level, the KSRF relation that holds in free space between the vector mass $m_V$ and $f_\pi g_v$ must have a density-dependent correction in order for the scaling of $g_v^*$ to make sense. To date the possible validity of the KSRF relation or some generalization of it in medium is not yet unraveled.

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2It is worth pointing out that the RMF that has been successful so far involves non-linear terms deemed “natural” in the terminology of EFT. These terms can be interpreted as representing high-dimension Fermi operators.
Another observation of interest is the in-medium mass of the scalar $\phi$. In the analysis of [26], the scalar mass does not have a simple scaling since it is a complicated non-linear theory. See [4] for a detailed discussion on this matter. In the present theory, we in fact have the relation

$$m_\phi^*(\rho_0) = m_\phi \Phi(\rho_0)$$

(23)

which for $\Phi(\rho_0) \approx 0.78$ gives the mass of the scalar in nuclear matter to be 546 MeV which should be compared with the value $500 \pm 20$ MeV favored by [26].

It should be stressed that given the simplicity of the model considered here, the agreement between the simple BR scaling model and the sophisticated non-linear mean-field model [26] is most remarkable. Whether there is something deep here or it is just a coincidence is an issue to be resolved.

4 Deriving Migdal’s Formulas from Effective Chiral Lagrangians

Here we sketch Migdal’s derivation of nuclear orbital gyromagnetic ratio and then write an analogous expression for the nuclear axial charge operator following the same steps taken for the vector current. We have no rigorous proof that the axial charge that results is a unique one that follows from the premise of Fermi liquid theory but we are offering it here as a possible one.

4.1 Landau-Migdal formulation

4.1.1 Vector currents

Consider the response of a heavy nucleus to a slowly varying electromagnetic field. We wish to calculate the gyromagnetic ratio $g_I$ of a nucleon sitting on top of the Fermi sea. There are several ways for doing this calculation [29]. Here we shall use the simplest which turns out to be straightforwardly applicable to the axial current, in particular to its time component, i.e., the axial charge.

We are interested in the response of a homogeneous quasiparticle excitation to the convection current. This corresponds to the limit $q/\omega \to 0$ where $(\omega, q)$ is the four-momentum transfer induced by the electromagnetic field. The first step is to compute the total current carried by the wave packet of a localized quasiparticle with group velocity $v_F = k/m_N^*$ where $m_N^*$ is the Landau effective mass of the quasiparticle and $k$ is the momentum carried by the quasiparticle. The convection current for a localized quasiparticle is

$$J_{LQP} = \frac{k}{m_N^*} \left( \frac{1 + \tau_3}{2} \right).$$

(24)

However this is not really what we want. Among other things, it does not conserve the charge. This is because the quasiparticle interacts with the surrounding medium generating what is known as “back-flow.” Consequently we have to incorporate the back-flow to restore gauge invariance. A simple way to account for the back-flow is to compute the particle-hole contributions

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3 The scalar that figures here is an effective degree of freedom which need not be identified with a particle in the Particle Data booklet. From our point of view, it is closer to the “dilaton” discussed by Beane and van Kolck [28].

4 This should be distinguished from the BR scaling effective mass $M^*$ that appears in [3] and [4] and will be defined more precisely later.
of the type given in Fig. 1 with the full particle-hole interaction – represented in the figure by the solid circle given by Eq. (25) – in the limit that $\omega/q \to 0$. (Note that this contribution vanishes in the other limit $q/\omega \to 0$.) The full interaction between two quasiparticles $p_1$ and $p_2$ at the Fermi surface of symmetric nuclear matter written in terms of a few spin and isospin invariants is

$$f_{p_1\sigma_1\tau_1, p_2\sigma_2\tau_2} = \frac{1}{N(0)} \left[ F(\cos \theta_{12}) + F'(\cos \theta_{12}) \tau_1 \cdot \tau_2 + G(\cos \theta_{12}) \sigma_1 \cdot \sigma_2 ight. + F'(\cos \theta_{12}) \sigma_1 \cdot \sigma_2 \tau_1 \cdot \tau_2 + \left. \frac{q^2}{k_F^2} H(\cos \theta_{12}) S_{12}(\hat{q}) \right]$$

(25)

where $\theta_{12}$ is the angle between $p_1$ and $p_2$ and $N(0) = \frac{k_F^2}{2\pi^2} \left( \frac{d\rho}{d\epsilon} \right)_{F}$ is the density of states at the Fermi surface. We use natural units with $\hbar = 1$. The spin and isospin degeneracy factor $\gamma$ equals 4 in symmetric nuclear matter. Furthermore, $q = p_1 - p_2$ and

$$S_{12}(\hat{q}) = 3\sigma_1 \cdot \hat{q} \sigma_2 \cdot \hat{q} - \sigma_1 \cdot \sigma_2,$$

(26)

where $\hat{q} = q/|q|$. The tensor interactions $H$ and $H'$ are important for the axial charge which we will consider later. The functions $F, F', \ldots$ are expanded in Legendre polynomials,

$$F(\cos \theta_{12}) = \sum_{\ell} F_\ell P_\ell(\cos \theta_{12}),$$

(27)

with analogous expansion for the spin- and isospin-dependent interactions.

In terms of (25), the quasiparticle-quasihole graphs of Fig. 1, suitably generalized to the full interaction, yield

$$J_{ph} = -\frac{1}{3\pi^2} k k_F^2 (f_1 + f'_1 \tau_3)$$

$$= -\frac{k}{M} \left( \frac{F_1 + F'_1 \tau_3}{6} \right)$$

(28)
where $M$ denotes the free-space mass of the nucleon and

$$\tilde{F}_i \equiv (M/m_N^*)F_i.$$  \hfill (29)

In order to obtain the desired current, we have to add the backflow term (i.e., $-J_{ph}$) to the localized quasiparticle term (24),

$$J_{migdal} = J_{LQP} - J_{ph} = \frac{k}{M}g_l = \frac{k}{M} \left( \frac{1 + \tau_3}{2} + \frac{1}{6}(\tilde{F}'_1 - \tilde{F}_1)\tau_3 \right),$$  \hfill (30)

where

$$g_l = \frac{1 + \tau_3}{2} + \delta g_l$$  \hfill (31)

is the orbital gyromagnetic ratio and

$$\delta g_l = \frac{1}{6}(\tilde{F}'_1 - \tilde{F}_1)\tau_3.$$  \hfill (32)

In arriving at (30), we have used the relation between the Landau effective mass and the quasi-particle interaction

$$\frac{m_N^*}{M} = 1 + \frac{1}{3}F_1 = (1 - \frac{1}{3}\tilde{F}_1)^{-1}. $$  \hfill (33)

It is important to note that, as a consequence of charge conservation and Galilei invariance the isoscalar term in (30) is not renormalized by the interaction. Thus, the renormalization of $g_l$ is purely isovector. It is also important to note that it is the free-space mass $M$, not the Landau mass $m_N^*$, that appears in (30). This is an analog to Kohn’s theorem for the cyclotron frequency of an electron in an external magnetic field \[31, 32\], and constitutes a strong constraint for a consistent theory to satisfy. The effective Lagrangian theory discussed below does satisfy this condition.

### 4.1.2 Axial currents

Next we turn to the axial charge operator $A^a_0$ (where the superscript $a$ is an isospin index). In deriving the “Migdal formula” for this operator \[4\], we assume that we can follow the exactly the same reasoning as above for the vector current. This assumption needs still to be justified.

In matter-free space, the axial charge operator for a non-relativistic nucleon with mass $M$ is

$$A^a_0 = g_A \frac{\tau^a \cdot k}{2M},$$  \hfill (34)

while in dense matter a localized quasiparticle with a Landau effective mass $m_N^*$ has the axial charge

$$A^a_{0LQP} = g_A \frac{\tau^a \cdot k}{2m_N^*}. $$  \hfill (35)

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5. The cyclotron frequency of an electron with a Landau effective mass $m_e^*$ in an external magnetic field of magnitude $B = 2\pi n_f \phi/e$ where $n_f$ is the electron number density and $\phi$ is the flux integer ($\equiv 2$ for fermions) is not $\omega_0 = 2\pi n_f \phi/m_e^*$ as one would naively expect for a localized quasiparticle but $\omega_0 = 2\pi n_f \phi/m_e$ due to the back-flow effect.

6. We put quotation marks since Migdal did not derive formulas for the axial charge.
Next we calculate the particle-hole contribution – which is minus the back-flow contribution – in the same way as for the vector current (i.e., Fig.3 with the pion exchange replaced by $f$, eq.(25)). The result [29] is

$$A^a_{0ph} = -g_A \frac{\tau^a \cdot k}{m_N} \Delta$$

(36)

with

$$\Delta = \frac{1}{3} G' - \frac{10}{3} H'_0 + \frac{4}{3} H'_1 - \frac{2}{15} H'_2$$

(37)

where $G'$ and $H'$ are the spin-isospin-dependent components of the force given in Eq.(25). Therefore the total is

$$A^a_{0\text{migdal}} = A^a_{0\text{LQP}} - A^a_{0ph} = g_A \frac{\tau^a \cdot k}{m_N} (1 + \Delta).$$

(38)

It will become clearer when we calculate the same quantity based on chiral Lagragian, but at this point it should be noted, that unlike the vector current, here there is no analog of the Kohn theorem. This is because chiral symmetry is realized, not in the Wigner mode, but rather in the Goldstone mode for which the meaning of a conserved charge is different from that of the vector charge. Another point to be noted is that while for the convection current, only $F_1$ and $F'_1$ appear, it is a lot more complicated for the axial charge. It involves spin-isospin dependent interactions as well as tensor forces. These two features will show up non-trivially when we compute the $\delta g_l$ and the $\Delta$ with the effective chiral Lagrangian.

4.2 Calculation with effective chiral Lagrangian

We will now compute $\delta g_l$ and $\Delta$ using a BR scaling chiral Lagrangian. One can use either the Lagrangian (3) or the Lagrangian (5) with BR scaling [25] incorporated. We shall use (3) as we did for the vector current. We need only the two terms of the four-Fermi interactions that correspond to the $\omega$ and $\rho$ channels:

$$- \frac{C^*_{\omega} \cdot \omega}{2} (\bar{N}\gamma_\mu N)^2 - \frac{C^*_{\rho} \cdot \rho}{2} (\bar{N}\gamma_\mu \tau N)^2 + \cdots,$$

(39)

i.e., what remains when the vectors $\omega$ and $\rho$ are integrated out. The subscripts represent not only the vector mesons $\omega$ and $\rho$ nuclear physicists are familiar with but also all vector mesons of the same quantum numbers, so the two “counter terms” subsume the full short-distance physics of the same chiral order.

4.2.1 Landau mass from chiral Lagrangian

We first calculate the single-particle energy with (3). In the nonrelativistic approximation, we have

$$\varepsilon_p = \frac{p^2}{2M^*} + C^*_{\omega} \langle N^\dagger N \rangle + \Sigma_\pi(p),$$

(40)
Figure 2: Feynman diagrams contributing to the EM convection current in effective chiral Lagrangian field theory. Figure (a) is the single-particle term and (b, c) the next-to-leading chiral order pion-exchange current term. Figure (c) does not contribute to the convection current; it renormalizes the spin gyromagnetic ratio.

where $M^* = \Phi M$ is the BR-scaling nucleon mass and $\Sigma_\pi(p)$ is the nucleon self-energy due to the pion Fock term. The Landau effective mass is defined \[33\] by

$$m^*_L = \frac{k_F}{M} \left( \frac{d}{dp} \varepsilon_p \bigg|_{p=k_F} \right)^{-1} = \left( \Phi^{-1} - \frac{1}{3} \tilde{F}_1(\pi) \right)^{-1}$$  \hspace{1cm} (41)

where we have used the fact that the second term of (40) does not contribute and

$$\tilde{F}_1(\pi) = - \frac{3M}{k_F} \frac{d\Sigma_\pi(p)}{dp} \bigg|_{p=k_F} = -\frac{3f^2 M}{8\pi^2 k_F} I_1$$  \hspace{1cm} (42)

where $f = g_A m_\pi/(2f_\pi) \simeq 1$ and

$$I_1 = \int_{-1}^{1} dx \frac{x}{1-x + \frac{m^2}{2k_F^2}} = -2 + (1 + \frac{m^2}{2k_F^2}) \ln(1 + \frac{4k_F^2}{m^2}).$$  \hspace{1cm} (43)

Now using the Landau mass formula (33) and

$$\tilde{F}_1 = \tilde{F}_1(\omega) + \tilde{F}_1(\pi)$$  \hspace{1cm} (44)

we find

$$\tilde{F}_1(\omega) = 3(1 - \Phi^{-1}).$$  \hspace{1cm} (45)

4.2.2 Convection current

In the chiral Lagrangian approach, the isovector magnetic multipole operator to which the convection current belongs is chiral-filter-protected \[34\] which means that the one-soft-pion exchange should dominate in the correction to the leading single-particle term. The single-particle term for a nucleon with the BR scaling mass $M^*$ on the Fermi surface with momentum $k$ corresponding to Figure 2a is

$$J_{1\text{-body}} = \frac{k}{M^*} \frac{1 + \tau_3}{2}.$$  \hspace{1cm} (46)

Note that the nucleon mass appearing in (46) is the BR scaling mass $M^*$ as it appears in the Lagrangian, not the (Landau) effective mass $m^*_L$ that enters in the Fermi-liquid approach for
Figure 3: (a) Feynman diagram contributing to the EM convection current from four-Fermi interactions corresponding to all channels of the $\omega$ and $\rho$ quantum numbers (contact interaction indicated by the blob) in effective chiral Lagrangian field theory. The $\bar{N}$ denotes the anti-nucleon state that is given in the chiral Lagrangian as a $1/M$ correction and the one without arrow is a Pauli-blocked or occupied state. (b) The equivalent graph in heavy-fermion formalism with the anti-nucleon line shrunk to a point. The blob represents a four-Fermi interaction coupled to a photon that enters in (3) as a $1/M$ counter term.

The localized quasiparticle current. To the next-to-leading order, we have two soft-pion terms Fig.2a,b as discussed in [34]. To the convection current, only Fig. 2b contributes

\[ J_{\pi}^{2-body} = \frac{k}{k_F} \frac{f^2}{4\pi^2} I_1 \tau_3 = \frac{k}{M} \frac{1}{6} (\tilde{F}'_1(\pi) - \tilde{F}_1(\pi)) \tau_3. \]  

(47)

In arriving at this formula, it has been assumed that pion properties are unchanged in medium up to nuclear matter density. Since pions are almost Goldstone bosons, this assumption seems reasonable. Indeed it is consistent with what is observed in Nature. Note that there are no unknown parameters associated with the pion contribution (77): the one-pion-exchange contributions to the Landau parameters $\tilde{F}_1(\pi)$ and $\tilde{F}'_1(\pi)$ are entirely fixed by the chiral effective Lagrangian at any density.

The contributions from the four-Fermi interactions (that is, the vector meson degrees of freedom) are subleading to the pion exchange by the chiral filter [34]. They are given by Fig. 3. Both the $\omega$ (isoscalar) and $\rho$ (isovector) channels contribute through the antiparticle intermediate state as shown in Fig. 3a. The antiparticle is explicitly indicated in the figure. However in the heavy-fermion formalism, the backward-going antinucleon line should be shrunk to a point as Fig. 3b, leaving behind an explicit $1/M^*$ dependence folded with a factor of nuclear density indicating a $1/M^*$ correction in the chiral expansion [7]. One can interpret Fig. 3a as saturating the corresponding counter term although this has yet to be verified by writing down the full set of counter terms at the same order. We find

\[ J_{\omega}^{\rho} = \frac{k}{M} \frac{1}{6} \tilde{F}_1(\omega), \]  

(48)

\[ J_{\rho}^{\rho} = \frac{k}{M} \frac{1}{6} \tilde{F}'_1(\rho) \tau_3, \]  

(49)

where

\[ F_1(\omega) = -C^\omega_2 \frac{2k_F^3}{\pi^2 M^*}. \]  

(50)

\footnote{The heavy-baryon formalism must be unreliable once the $M^*$ drops for $\rho \gtrsim \rho_0$. One would then have to resort to a relativistic formulation [35]. We expect, however, that our reasoning will remain qualitatively intact.}
and

\[ F'_1(\rho) = -C'_\rho^2 \frac{2k_F^3}{\pi^2 M^*}. \quad (51) \]

The total current given by the sum of (46), (47), (48) and (49) precisely agrees with the Fermi-liquid theory result (30) when we identify

\[ \tilde{F}_1 = \tilde{F}_1(\omega) + \tilde{F}_1(\pi), \quad (52) \]
\[ \tilde{F}'_1 = \tilde{F}'_1(\rho) + \tilde{F}'_1(\pi). \quad (53) \]

If we further assume the same flavor symmetry as in free space holds in medium, then

\[ \tilde{F}'(\rho) = \tilde{F}'(\omega)/9 \quad (54) \]

which uses the nonet symmetry and

\[ \tilde{F}(\pi) = -\tilde{F}(\pi)/3 \quad (55) \]

which uses the isotopic invariance. The BR scaling chiral Lagrangian prediction reduces to a one-parameter formula

\[ \delta g_i = \frac{1}{6}(\tilde{F'}_1 - \tilde{F}_1)\tau_3 = \frac{4}{9} \left[ \Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1(\pi) \right] \tau_3. \quad (56) \]

Here \( \Phi(\rho) \) is the only parameter in the theory that needs to be determined from theory or experiment. As mentioned, \( \tilde{F}_1(\pi) \) is fixed for an arbitrary density from the (assumed) chiral symmetry. It is important that the result is consistent with charge conservation and Galilei invariance.

### 4.2.3 Axial charge

The axial charge operator in nuclear matter is protected by the chiral filter in the chiral Lagrangian formalism, so all we need is the soft-pion exchange implemented with BR scaling. We shall continue assuming that pions do not scale in medium. It has been shown in [36] that higher order chiral corrections – such as loops, higher derivative and four-Fermi terms – to the soft-pion contribution are small. This means that we can limit ourselves to the tree order in the chiral counting and to the pionic range with shorter-range interactions subsumed in the BR scaling.

The procedure for the case at hand will then be identical to that we used for the convection current. The axial charge for a single particle will be identical to that of a particle in free space except that the nucleon mass \( M \) is to be replaced by the BR scaling mass \( M^* \)

\[ A^a_0 = g_A \frac{\tau^a}{2} \frac{\sigma \cdot k}{M^*}. \quad (57) \]

Now the leading correction to the single-particle term is given by a diagram similar to Fig Bc with the photon replaced by the weak axial charge. There is no term equivalent to Fig 2b due to G-parity invariance. The calculation is straightforward and the result is

\[ A^i_0_{\text{body}} = g_A \frac{\tau^i}{2} \frac{\sigma \cdot k}{M^*} \frac{\Delta}{\Lambda}. \quad (58) \]

Here \( \Phi(\rho) \) is the only parameter in the theory that needs to be determined from theory or experiment. As mentioned, \( \tilde{F}_1(\pi) \) is fixed for an arbitrary density from the (assumed) chiral symmetry. It is important that the result is consistent with charge conservation and Galilei invariance.
with

\[ \tilde{\Delta} = \frac{f^2 k_F M}{2 g_A^2 m_\pi^2 \pi^2} \left( I_0 - I_1 - \frac{m_\pi^2}{2 k_F^2} I_1 \right) \]  

(59)

where \( I_1 \) is as defined in (53) and \( I_0 \) is

\[ I_0 = \int_{-1}^{1} dx \frac{1}{1 - x + \frac{m_\pi^2}{2 k_F^2}} = \ln \left( 1 + \frac{4 k_F^2}{m_\pi^2} \right). \]  

(60)

The factor \( (1/g_A^2) \) in (59) arose from replacing \( \frac{1}{f_\pi} \) by \( \frac{g_A m_N}{g_A M^*} \) using the free-space Goldberger-Treiman relation.

Collecting all terms, the chiral Lagrangian prediction is

\[ A_{0\text{chiral}}^a = g_A \frac{\sigma \cdot k }{M^*} \frac{\tau^a}{2} (1 + \tilde{\Delta}). \]  

(61)

For comparison with the “Migdal formula” \( A_{0\text{migdal}}^i \), we re-express \( 1/M^* \) in terms of \( 1/m_N^* \)

\[ \frac{1}{M^*} = \frac{1}{m_N^*} (1 - \frac{\Phi}{3} \tilde{F}_1(\pi))^{-1}. \]  

(62)

Thus

\[ A_{0\text{chiral}}^a = g_A \frac{\sigma \cdot k }{m_N^*} \frac{\tau^a}{2} (1 + \tilde{\Delta}') \]  

(63)

where

\[ \tilde{\Delta}' = (\tilde{\Delta} + \frac{\Phi}{3} \tilde{F}_1(\pi))(1 - \frac{\Phi}{3} \tilde{F}_1(\pi))^{-1}. \]  

(64)

Comparing with the “Migdal formula” (58), we obtain a formula that expresses a combination of spin-isospin-dependent Landau-Migdal parameters in terms of constants that figure in the chiral Lagrangian with BR scaling:

\[ \frac{1}{3} G_1' - \frac{10}{3} H_0' + \frac{4}{3} H_1' - \frac{2}{15} H_2' = \tilde{\Delta}'. \]  

(65)

Again the result depends on only one parameter \( \Phi \).

There are two points to note here. One is as noted in the Landau-Migdal formulation that there is no equivalent to “Kohn’s theorem” for the axial charge. The other is that the soft-pion contribution combined with BR scaling does not lend itself to a direct term-by-term identification on both sides. These are all different from the case of the convection current. In the axial case, both the Landau-Migdal approach and the chiral Lagrangian approach involve complicated relations: on the right-hand side of (53), the factor \( g_A \) appears in a non-trivial way and exhibits features that are characteristic of the spontaneously broken axial symmetry and on the left-hand side, this complexity is manifested by the fact that, due to the tensor force, the Migdal parameters involved comprise several multipoles \( (l = 0, 1, 2) \) of the quasiparticle interaction.

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5 Comparison with Experiments

In confronting our theory with Nature, we shall assume that data on heavy nuclei represent nuclear matter. This aspect has been extensively discussed elsewhere so we shall be brief.

5.1 Extracting $\Phi(\rho_0)$

If one assumes BR scaling, then there are several ways to determine $\Phi$ at normal nuclear matter density. We shall mention three of them.

1. The first way is that if pions are taken to be non-scaling, then the in-medium Gell-Mann-Oakes-Renner relation

$$m_\pi^2 f_\pi^2 = -(m_u + m_d) \langle \bar{q}q \rangle^*$$

(66)

gives

$$\frac{f_\pi^*}{f_\pi} \approx \left( \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle_0} \right)^{1/2}.$$  

(67)

\[\text{From the value of quark condensate in nuclear matter estimated from the empirical } \pi N \sigma \text{ term and using Feynman-Hellmann theorem in the linear density approximation} \text{, one finds} \]

$$\frac{f_\pi^*}{f_\pi} \approx 0.78.$$  

(68)

2. The second evidence comes from the property of nuclear matter in chiral Lagrangian models with BR scaling. A global fit yields

$$M^*/M \approx 0.78 \pm 0.02.$$  

(69)

3. The third evidence comes from QCD sum rule calculation of the mass of the vector meson $\rho$ in medium [37, 38]. The result is [38]

$$\frac{m_\rho^*}{m_\rho} = 0.78 \pm 0.08.$$  

(70)

All three methods give the same result. We are therefore led to use

$$\Phi(\rho_0) = 0.78.$$  

(71)

As a smooth interpolation which seems reasonable at least up to $\rho \simeq \rho_0$, we take

$$\Phi(\rho) = \left(1 + 0.28\rho/\rho_0\right)^{-1}.$$  

(72)

*The linear density approximation may be suspect already at nuclear matter density, so it is difficult to assess the uncertainty involved with this estimate.
5.2 The orbital gyromagnetic ratio

Given the scaling factor $\Phi(\rho_0) \approx 0.78$ and the pionic contribution (42) which at nuclear matter density yields $\tilde{F}_1(\pi) \approx -0.459$, the anomalous orbital gyromagnetic ratio turns out to be

$$\delta g_l = \frac{4}{9} [\Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1(\pi)] \tau_3 = 0.227\tau_3.$$  \hspace{1cm} (73)

This is to be compared with the experimental value for the proton obtained from the giant dipole resonance in the Pb region [39]

$$\delta g_l^p = 0.23 \pm 0.03.$$ \hspace{1cm} (74)

It is worth commenting at this point which assumptions enter into this calculation and what the possible implication might be. Apart from the BR scaling, we have assumed (1) that pions do not scale, (2) the nonet symmetry for the vector mesons and (3) the isospin symmetry for the pions. The first is based on the observation that the pion is an almost Goldstone boson and a truly Goldstone boson would preserve its symmetry as density is increased beyond normal nuclear matter density. This assumption needs to be verified. The second is hard to check and remains to be verified. The third is most probably solid. The upshot of the result is that the charge is conserved, “Kohn’s theorem” is satisfied and the agreement with experiment essentially confirms in an average sense the BR scaling for the nucleon mass.

5.3 Landau mass for the nucleon

A quantity closely related to $\delta g_l$ is the Landau effective mass $m_N^*$. Given $\Phi$ and $\tilde{F}_1(\pi)$ for $\rho = \rho_0$, we obtain from Eq. (33) that

$$m_N^* = (1/0.78 + 0.153)^{-1} M \approx 0.70 M.$$ \hspace{1cm} (75)

There are two sources of information that can be compared with this prediction. One is theoretical, namely, the QCD sum-rule result [40]

$$\left( \frac{m_N^*(\rho_0)}{M} \right)_{QCD} = 0.69^{+0.14}_{-0.07}.$$ \hspace{1cm} (76)

The other is an indirect semi-empirical indication coming from peripheral heavy ion collisions at the BEVALAC and the SIS [41]

$$m_N^{*HI} \simeq 0.68 M.$$ \hspace{1cm} (77)

The agreement here is essentially a re-confirmation of the gyromagnetic ratio (73).

5.4 Axial-charge transitions in heavy nuclei

Before confronting the chiral Lagrangian prediction (61) (with (59)) with experiments, we compare the left-hand side of (65) (i.e., “Migdal’s axial charge”) with one-pion exchange
only with the right-hand side which is given to next-to-leading order (NLO) chiral perturbation theory with BR scaling chiral Lagrangian.\footnote{The precise agreement is probably coincidental.}

To compute the Migdal charge, it is sufficient to compute the one-pion-exchange graphs of Fig. 4 in the limit $\omega/q \to 0$. The negative of this gives the desired quantity, namely, the “back-flow.” A simple calculation gives

$$\left( \frac{1}{3} G_1' - \frac{10}{3} H_0' + \frac{4}{3} H_1' - \frac{2}{15} H_2' \right)_\pi = \frac{f^2 k_F m_N^*}{4m_N^2 \pi^2} (I_0 - I_1)$$

(78)

where the subscript $\pi$ denotes the pionic contribution. Now since the right-hand side of (65) is valid beyond the leading order in chiral perturbation theory, it contains information that accounts for more than one-pion exchange. In the same vein, (78), although the interaction is evaluated with one-pion exchange, contains a lot more since the mass of the nucleon is given by the Landau mass $m_N^*$. Therefore there is no reason to expect a one-to-one correspondence between the two. Even so, we conjecture that to the extent that the dynamics is governed by the pion exchange corrected by the BR scaling $\Phi$, the two must be approximately the same. That is to say that the combination of the Migdal parameters of (65) should be saturated by the pions modulo what corresponds to higher chiral order terms which are argued to be small. This is required if the chiral filter argument is to hold.

Let us consider how the relation (78) fares with the pion for $\rho = \rho_0/2$ and $\rho_0$. The left-hand side – given by (78) – comes out to be, respectively, 0.42 and 0.50 for $\rho = \rho_0/2$ and $\rho_0$ while the right-hand side – which is the full contribution from the BR Lagrangian – gives 0.37 and 0.55. Thus the pions are seen to saturate $\sim 90\%$ of the total predicted by the chiral field theory with BR scaling.

Although far from direct, there is a beautiful confirmation of the prediction (51) from axial charge transitions in heavy nuclei (denoted by the mass number $A$)

$$A(J^\pm) \to A'(J^\mp) + e^- (e^+) + \bar{\nu} (\nu) \quad \Delta T = 1.$$  

(79)

The quantity we shall look at is Warburton’s $\epsilon_{MEC}$\footnote{Modulo a correction less than 10\%, this is valid to next-next-to-leading order (NNLO) in chiral perturbation theory\cite{36}.}, defined by

$$\epsilon_{MEC} = M_{exp}/M_{sp}$$

(80)
where \( M_{\text{exp}} \) is the measured matrix element for the axial charge transition and \( M_{\text{sp}} \) is the theoretical single-particle matrix element for a nucleon without BR scaling. There are theoretical uncertainties in defining the latter, so the ratio is not an unambiguous object but what is significant is Warburton’s observation that in heavy nuclei, \( \epsilon_{\text{MEC}} \) can be as large as 2:

\[
\epsilon_{\text{MEC}}^{\text{HeavyNuclei}} = 1.9 \sim 2.0. \tag{81}
\]

More recent measurements – and their analyses – in different nuclei \([43, 44]\) quantitatively confirm this result of Warburton.

The theoretical prediction from (61) is

\[
\epsilon_{\text{MEC}}^{\text{chiral}} = \Phi^{-1}(1 + \tilde{\Delta}) \tag{82}
\]

with \( \tilde{\Delta} \) given by (59). For nuclear matter density, we find

\[
\epsilon_{\text{MEC}}^{\text{chiral}} \approx 2.1. \tag{83}
\]

The theory therefore describes correctly the large enhancement of the axial-charge matrix element in nuclei in general and the density-dependent enhancement in particular. There are two elements that account for this enhancement. Pions contribute \( \tilde{\Delta} \sim 1/2 \) with little density dependence and the BR scaling \( \Phi \) accounts for the further enhancement for heavier nuclei. This result represents a strong case for the validity of the theory in the normal density regime.

6 Going to Denser Matter

6.1 Evidence in dense matter?

The real strength in effective field theories is that one may be able to describe quantitatively the state of matter that is formed in high density as one approaches the chiral phase transition. If one assumes that the matter is a Fermi liquid all the way to the phase transition, then one can use the BR scaling chiral Lagrangian in the mean field. But this means that all degrees of freedom, fermionic as well as bosonic, are treated as “quasiparticles.” It is established that nucleons are quasiparticles in nuclear matter as Migdal had argued. The shell model for nuclei is justified by the quasiparticle picture. It is also supposed that at asymptotic density where weak coupling of QCD is operative, quarks can be treated as quasi-quarks \([45]\). The presence of a Fermi sea for nucleons and quarks is one of the ingredients for treating them as quasiparticles.

In the discussions given above, bosons were not required to be “quasiparticles” despite that BR scaling is invoked for both fermions and bosons. In addressing heavy-ion processes, however, properties of bosons in medium might play an important role. For instance, in CERN’s CERES experiments, it is the property (i.e., mass, width etc.) of the \( \rho \) meson in dense and hot medium that seems to play a key role. So the question arises how bosons behave in extreme conditions.

There are some indirect experimental evidences for vector bosons with dropping masses in dense medium. The effect usually manifests in spin-isospin dependent nuclear forces and affect spin-isospin observables \([46, 47]\). These observables probe off-shell properties of the mesons involved up to nuclear matter density and do not in general give direct information on their “physical” properties in medium. There are similar indications from tensor forces in heavy nuclei which also can be explained from the exchange of the \( \rho \) meson with a reduced mass \([48]\). A more direct indication comes from dilepton production in heavy-ion collisions at CERN CERES. There
the quasi-particle picture of the vector mesons with dropping mass in hot and dense matter (at a density greater than that of normal nuclear matter) provides a simple and successful explanation of the observed downward shift of the invariant mass of the lepton pair \cite{14}. The approximation used in \cite{19} consists of taking only tree-order graphs with an effective chiral Lagrangian a la BR scaling discussed above that gives a realistic description of nuclear matter: no loop corrections are taken into account in a proper sense although partial account is of course made in the unitarization of the amplitudes involved. The question as to what happens when loop corrections are properly taken into account in this theory so far remains unanswered. It is also not known whether the tree-order (i.e., quasi-particle) treatment correctly describes the excitation of the vector quantum numbers in such dense matter.

6.2 Perturbing from zero-density vacuum

One might attempt an ambitious program to start from an effective chiral Lagrangian constructed at zero density and do a systematic chiral expansion to arrive at higher density. This is the spirit of \cite{18,19}. Aided by experiments, this program could be made to work up to nuclear matter density but it is a completely different matter if one wants to reach a density where the chiral phase transition can occur. Dense matter probes short distances and chiral perturbation theory (ChPT) cannot access such kinematic regime as is clear from Landau-Migdal Fermi liquid theory. What has been done up to date is a low-order perturbation calculation in a strong-coupling regime. Now if such a calculation is based on an effective Lagrangian satisfying relevant symmetries (e.g., chiral symmetry), leading-order (tree-order) terms are consistent with low-energy theorems and should give reasonable results at low density provided the parameters are picked from experiments. See Rapp and Wambach \cite{50} for review where the relevant references are found. In such low-order calculations, one finds that the mesons, such as the $\rho$ and $a_1$ get “melted” due to increasing width and lose their particle identities. However as density increases away from zero, the tree-order approximations that are essentially all one can work with cannot be trusted. Exactly where this discrepancy will become serious is not known. Being in a strong-coupling regime, anomalous dimensions of certain fields (such as scalar fields) grow too big to be natural, signaling that one is fluctuating around the wrong vacuum. We believe this to be the case already at nuclear matter density. BR scaling corresponds to shifting to and fluctuating around a “vacuum” defined at $\rho \geq \rho_0$ where the effective coupling gets weaker in the sense of quasiparticle interactions. As the density approaches the critical for the chiral phase transition, the picture with quasi-nucleons goes over to the one with quasi-quarks. It seems extremely difficult, if not impossible, to arrive at this picture starting from a strong-coupling hadronic theory effective at zero density. See \cite{51,4} for further discussion on this point.

6.3 Perturbing from BR scaling ground state

Given a Lagrangian \cite{3} or \cite{1} with BR scaling that gives the ground state of nuclear matter correctly, we would like to know how to make fluctuations around the ground state. As an illustration, consider kaon-nucleon interactions in medium \cite{22}. This process is relevant for both laboratory experiments and for the structure of compact stars as we will describe below.

For the problem at hand, it is convenient to generalize \cite{1} to the $SU(3)$ flavor so as to
incorporate kaons in the Lagrangian. The additional term relevant to the process is given by

$$\delta L_{KN} = -\frac{6i}{8f^*} (N \gamma_0 N) K \partial_\tau K + \frac{\Sigma_{KN}}{f^*} (N N) K + \cdots \equiv \mathcal{L}_\omega + \mathcal{L}_\phi + \cdots$$  \hspace{1cm} (84)$$

where \( K^T = (K^+ K^0) \), \( f^* \) is the in-medium Goldstone boson decay constant which within the approximation adopted here, may be taken to be the pion decay constant and the ellipses stand for higher-order terms in the chiral counting. The structure of the first two leading-order terms of the fluctuating Lagrangian is dictated by current algebras, which implies that \( \Sigma_{KN} \) is the usual KN sigma term in free space and also that it may be valid near nuclear matter density.

Within the scheme a la BR, we are to work in the mean field approximation. Assuming that this is valid up to nuclear matter density, one gets from (84) the potential energy for the scalar (\( \phi \)) field \( S_{K^-} \) and the vector (\( \omega \)) field \( V_{K^-} \) that \( K^- \) feels in nuclear matter at \( \rho = \rho_0 \):

$$S_{K^-} + V_{K^-} \approx -192 \text{ MeV}.$$  \hspace{1cm} (85)$$

For this we have used the value for the KN sigma term, \( \Sigma_{KN} \approx 3.2m_\pi \) and \( f^*/f_\pi \approx \Phi \). The exact value is unknown since the sigma term is not fixed precisely. The attraction (85) is consistent with what is observed in kaonic atoms [52] and also with the \( K^-/K^+ \) production ratio in heavy-ion collisions at GSI [53]. When applied to neutron-star matter and extrapolating to higher density, it is more appropriate to adopt the “top-down” approach proposed in [22] in which the kaon field is introduced as a matter field and the relevant fermion field is taken to be the quasi-quark rather than the nucleon. With a suitable modification appropriate for the top-down approach of [22] in the Lagrangian (84), one then predicts \( K^- \) condensation at a matter density \( \rho_c \approx 2 \sim 3\rho_0 \) with the intriguing implication that the maximum stable neutron star mass is 1.5 times the solar mass [54]. These mean field results with BR scaling Lagrangians are in agreement with more refined calculations carried out in high-order chiral perturbation theory [54]. If it turned out that condensation occurs at higher density than the range considered so far (due to some higher order effects that cannot be accessed by the effective Lagrangian method used), then the presently available machinery for handling short-distance physics would not be powerful enough to allow us to pin down the critical density [55]. More work is needed in this area.

### 6.4 “Sobar” model

Among Migdal’s other major contributions to nuclear physics is his work on pion-nuclear interactions, in particular on pion condensation in dense nuclear matter [3]. It is suggested that the Fermi liquid description a la BR scaling chiral Lagrangian can be phrased in a form resembling Migdal’s description of pion condensation. The initial idea is formulated in a series of recent papers by Kim et al [56].

Consider a vector meson, say \( \omega \), which is inserted in a dense medium and look at the excitation of \textit{coherent} modes of the \( \omega \) quantum number. The \( \omega \) meson will be coupled to particle-hole excitations of the same quantum number as depicted in Fig.4. Analogously to the treatment of pion condensation, the lowest-energy collective particle-hole mode is interpreted as an effective vector meson field operating on the ground state of the nucleus, i.e.,

$$\frac{1}{\sqrt{A}} \sum_i \left| N_i^* N_i^{-1} \right|^{-1} \approx \sum_i \left[ \rho(x_i) \text{ or } \omega(x_i) \right] |\Psi_0 >_s ,$$  \hspace{1cm} (86)$$
with the antisymmetrical (symmetrical) sum over neutrons and protons giving the $\rho$-like ($\omega$-like) nuclear excitation. Here the “particle” is taken to be $N^*$ while the “hole” is nucleon-hole. We will ignore the nucleon as particle since in the channel we are concerned with, we expect the nucleon to be more weakly coupled than the $N^*$ to the (near on-shell) vector meson. We call the collective mode (86) “sobar,” i.e., $\rho$-sobar, $\omega$-sobar etc.

The dropping vector meson masses could then be calculated in terms of mixing of the nuclear collective state, Eq. (86), with the elementary vector meson through the mixing matrix elements of Fig. 5. Now building up the collective “nuclear mode,” the latter can be identified as an analog to the state in the degenerate schematic model of Brown for giant dipole resonance [57]. The fields figuring in a BR scaling chiral Lagrangian are then to be identified with interpolating fields for the lowest branch modes that emerge from the mixing. An important development which leads to the assumption Eq. (86) was furnished by Friman, Lutz and Wolf [58]. From empirical values of the amplitudes such as $\pi + N \rightarrow \rho + N$ etc. they constructed the $\rho$-like or $\omega$-like states in consistency with our assumption Eq. (86). Thus the input assumption made for the sobar model receives substantial empirical support.

Since the development is at its initial stage and still quite crude, we briefly summarize what we hope to accomplish in the end.

The property of a vector meson, say, $\omega$, in medium is encoded in the propagator of the meson in interaction with the system. For simplicity of discussion, let us consider a two-level schematic model. In (86), we take only one configuration with $N^* = N^*(1520)$ in the $\omega$ channel. The starting point is the $\omega$-meson propagator in nuclear matter given by

$$D_\omega(q_0, \vec{q}; \rho_N) = \frac{1}{q_0^2 - \vec{q}^2 - m_\omega^2 - \Sigma_{\omega N^* N}(q_0, q; \rho_N)}$$  \hspace{1cm} (87)$$

where we have ignored the $\omega$ decay width, and the density-independent real part of the self-energy has been absorbed into the free (physical) mass $m_\omega$. Here $\rho_N$ is nucleon number density. Note that within the low-order approximation made here the entire density-dependence resides in the in-medium $\omega$ self-energy $\Sigma_{\omega N^* N}$ induced by $N^*(1520)N^{-1}$ excitations. In what follows we will for simplicity concentrate on the limit of vanishing three-momentum where the longitudinal and transverse polarization components become identical. Due to the rather high excitation energy of $\Delta E = M_{N^*} - M_N = 580$ MeV, one can safely neglect nuclear Fermi motion to obtain

$$\Sigma_{\omega N^* N}(q_0) \sim g_{\omega N^* N}^2 \frac{q_0^2 \rho_N}{m_\omega^2} \frac{2(\Delta E)}{(q_0 + i\Gamma_{tot}/2)^2 - (\Delta E)^2}$$  \hspace{1cm} (88)$$
where $\Gamma_{\text{tot}}$ is the sum of the full width of $N^*(1520)$ in free space and density-dependent width due to medium. If the widths of the $\omega$ and $N^*(1520)$ are sufficiently small one can invoke the mean-field approximation and determine the quasiparticle excitation energies from the zeros in the real part of the inverse propagator. In particular, for $\vec{q} = 0$, the in-medium $\omega$ mass is obtained by solving the dispersion relation

$$q_0^2 = m_\omega^2 + \text{Re}\Sigma_{\omega N^*N}(q_0).$$

(89)

The pertinent spectral weights of the solutions are characterized by $Z$-factors defined through

$$Z = (1 - \frac{\partial}{\partial q_0^2} \text{Re}\Sigma_{\omega N^*N})^{-1}.$$ 

(90)

The formulas written above are presumably valid for low density since they can be made consistent (by fiat) with low-energy theorems. However there is no reason to expect that a low-order calculation in strong coupling will be viable at high density. For instance there is no way that the $\omega$ mass will go to zero at any density even in the chiral limit. We are therefore led to make certain assumptions motivated by our objective to model BR scaling. It is clear that with a few-order perturbative calculation in a strong-coupling regime, there is no way to arrive at BR scaling. Lacking a workable scheme to compute systematically, we will simply impose a condition on the model and study the consequence on the model. The simplest condition that we can impose is that $q_0 = 0$ be a solution of (89) at some density $\rho_c$ at which the in-medium pion decay constant $f_\pi$ is to vanish (a la, e.g., in-medium Weinberg sum rule). This is readily achieved if

$$g_{\omega N^*N}^* \frac{q_0^2}{m_\omega^2} \to \text{constant}$$

(91)

independent of density as $\rho \to \rho_c$. Note that we have appended a star on the $\omega N^*N$ coupling constant to indicate that higher order corrections will inject a non-linear density dependence into the vertex (as well as into the width etc.) The limit can be achieved only if the density dependence in $f^*$ cancels the same in $q_0$ as one approaches the critical density. Now the constant cannot be fixed a priori and what one takes for it will determine at what $\rho_c$ the effective $\omega$-sobar mass will vanish. The basic assumption here is that since the vector mass drops while the pion mass does not, the quasiparticle picture gets restored as $\rho$ approaches $\rho_c$ with the width shrinking due to the decreasing phase space. This is consistent with the general premise of BR scaling.

As stressed in [56], nobody has been able to “derive” such a sober description starting from effective field theories defined at zero density. It seems however promising that this is doable in a systematic way in the framework laid down in [54]. How this can come about is sketched in the references [54].

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