CHARACTERIZATION OF APPROXIMATELY MONOTONE AND APPROXIMATELY HÖLDER FUNCTIONS

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Abstract. A real valued function $f$ defined on a real open interval $I$ is called $\Phi$-monotone if, for all $x, y \in I$ with $x \leq y$ it satisfies

$$f(x) \leq f(y) + \Phi(y - x),$$

where $\Phi : [0, \ell(I)] \to \mathbb{R}_+$ is a given nonnegative error function, where $\ell(I)$ denotes the length of the interval $I$. If $f$ and $-f$ are simultaneously $\Phi$-monotone, then $f$ is said to be a $\Phi$-Hölder function. In the main results of the paper, using the notions of upper and lower interpolations, we establish a characterization for both classes of functions. This allows one to construct $\Phi$-monotone and $\Phi$-Hölder functions from elementary ones, which could be termed the building blocks for those classes. In the second part, we deduce Ostrowski- and Hermite–Hadamard-type inequalities from the $\Phi$-monotonicity and $\Phi$-Hölder properties, and then we verify the sharpness of these implications. We also establish implications in the reversed direction.

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