DIFFERENTIAL TRANSFORMATION METHOD FOR SOLVING MALARIA –HYGIENE MATHEMATICAL MODEL

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ABSTRACT

In this study, we proposed a malaria-hygiene mathematical model using non-linear differential equation. The model equations are divided into seven compartments consisting of five human compartments (Hygienic Susceptible, Unhygienic Susceptible, Hygienic Infected, Unhygienic Infected, and Recovered) and two vector compartments (Non-Disease Carrier vector and Disease carrier vector). Differential Transformation Method (DTM) is applied to solve the mathematical model. The solutions obtained by DTM are compared with Runge-Kutta order 4th method (RK4). The graphical solutions illustrate similarity between DTM and RK4. It therefore imply that DTM can be consider a reliable alternative solution method.

KEYWORDS

Malaria, Hygiene, Mathematical model, Runge-Kutta order 4th, DTM.

1. INTRODUCTION

Malaria is a parasitic disease which spread by the female Anopheles mosquitoes. The prevalence of malaria has continued to be a global health challenge with spontaneous infection rate in the last five decades, and resulting into huge medical restive economic hardship (Jasminka et al., 2019; Yibeltal et al., 2018). It is transmitted from person to person through an infected female Anopheles mosquito bite (Azeb et al., 2018). Worldwide, 229 million cases of malaria were recorded with 409,000 deaths in 2019. The sub-Saharan African regions are the endemic ancient home of malaria with 94% reported cases and death (World Health Organization, 2021).

According to a study, the number of mosquitoes continues to increase due to the following environmental and human population related factors (Singh et al., 2003):

(i) Discharge of household wastes such as garbage and trash in residential areas,
(ii) Open drainage of sewage in residential areas,
(iii) Plantations of vegetation and hedges in residential areas and in parks,
(iv) Industries and transport systems producing wastes in residential areas,
(v) Open water storage tanks and ponds.

Also the presence of overgrown vegetation and stagnant water with “bola” activities such as unhealthy hygienic practices of sanitary in and around residential environment increases exponential growth of Anopheles mosquitoes (Mauti et al., 2015; Enebeli et al., 2019; Musoke et al., 2018).

Thus, unhygienic environmental conditions in the habitat caused by human populations become responsible for the fast-growing numbers of mosquitoes. DTM is one of the methods used to solve different kinds of linear and nonlinear differential equations. It was first introduced by Zhou in a study about electrical circuits (Zhou, 1986). DTM constructs a semi-analytic numerical technique that uses Taylor series for the solution of differential equations in the form of polynomials.

It is possible to solve integro-differential equations, linear Fredholm integro-differential equation, differential equations, difference equations, differential difference equations, fractional differential equations, pantograph equations, fourth-order parabolic partial differential equations, volterra integral equations and quadratic riccati differential equations by this method (Azuaba and Akinwande, 2018; Arigoglo and Ozkol, 2008; Maleknejad and Kajani, 2004; Maleknejad et al., 2004; Ozdemir and Kaya, 2006; Arigoglo and Ozkol, 2006a; Arigoglo and Ozkol, 2006b; Arigoglo and Ozkol, 2007; Keskin et al., 2007; Ibis, 2014; Jothika and Savitha, 2018; Abiodun et al., 2015). The main advantage of this method is that it can be applied directly to linear and nonlinear ordinary differential equations without linearization, discretization or perturbation. Some works has been done to solve model equation with differential transformation method (DTM); applied DTM to solve a deterministic model of infectious disease, the result shows a positive correlation between DTM and Runge-Kutta solution (Adebisi et al., 2019).

In a study, DTM was applied to determine the approximate solution to a sterile insect technology model for controlling zika virus vector (Atokebola et al., 2021).
The method as applied provides fast convergence rate and was considered a variable alternative tool for solving non-linear and linear problem in science and engineering. A group researcher carried out a study to solve a mathematical model for dengue fever using DTM (Eguda et al., 2019). The work shows that DTM is a very effective tool for solving ordinary differential equation problems. Some researchers solved a mathematical model of yellow fever dynamics incorporating secondary host applying DTM, from the work it showed that DTM was a valid alternative solution method (Somma et al., 2019). With all the study carried out none has solved to the best of the author’s knowledge a malaria hygiene mathematical method using DTM. Hence this work is aimed to solve a malaria hygiene mathematical model. The paper is organized as follows: Section 2 is the model formulation, section 3 presents DTM, section 4 includes model solution, section 5 is presents graphical solution of the model, in section 6 we conclude the study.

2. Model Formulation

In this model, the total human population denoted by \( N_u \) is subdivided into Unhygienic susceptible human population \( S_u \). Hygienic Susceptible Human population \( S_h \), Unhygienic infected human population \( I_u \), hygienic infected human population \( I_h \), and the Recovered Human population \( R_u \). The mosquito population denoted by \( N_m \) is subdivided into susceptible mosquitoes \( S_m \) and infected mosquitoes \( I_m \). Therefore, we have the following sub-populations:

\[
N_u = S_u + S_h + I_u + I_h + R_u.
\]

\[
N_m = S_m + I_m.
\]

Let \( \Lambda_u \) be the recruitment rate of the human population. A fraction \((1 - \alpha)\Lambda_u\) enters unhygienic susceptible human class while the remaining fraction \(\alpha\Lambda_u\) enters the hygienic susceptible human class. The unhygienic susceptible class is increased by the rate at which unhygienic human class lose immunity after recovery given as \(\omega_u\) and reduced by the rate of progression to hygienic class \(\tau_1\), the force of infection for the unhygienic class \(\lambda_u\) and natural human death rate \(\mu_u\). The hygienic susceptible human compartment is increased by the \(\tau_2\), the rate at which hygienic human class lose immunity after recovery at \(\omega_u\), while the compartment is reduced by natural human death rate \(\mu_u\) and the force of infection for the hygienic class \((1 - \zeta)\lambda_u\). The uninfected human class \(I_u\) is increased by \(\lambda_u\) and reduced by natural human death rate \(\mu_u\), rate of progression from \(I_u\) to \(I_h\) given as \(\tau_2\), malaria induced death for unhygienic human class \(\delta_u\) and recovery for unhygienic human class \(\theta_u\). The hygienic infected class \(I_h\) is increased by \((1 - \zeta)\lambda_u\) and \(\tau_1\), then reduced by the recovery rate for a hygienic human class \(\delta_h\), malaria induced death for hygienic human class \(\theta_h\) and natural death rate \(\mu_h\). The recovered human class \(R_u\) is increased by \(\theta_u\) and \(\delta_h\), then reduced by \(\mu_u\), \(\omega_u\) and \(\omega_h\). The susceptible mosquito class \(S_m\) is increased by the Mosquito recruitment rate given as \(\Lambda_m\), reduced by the mosquito’s death rate \(\mu_v\), and force of infection for mosquito given as \(\lambda_v\). The infected mosquito class \(I_m\) is increased by \(\lambda_v\) and \(\mu_v\).

Given the above description and definitions of variables and parameters in Table 1 and 2, the following are the model equations:

\[
\frac{dS_u}{dt} = (1 - \alpha)\Lambda_u - (\tau_1 + \lambda_u + \mu_u)S_u + \omega_u R_u
\]

\[
\frac{dS_h}{dt} = \alpha\Lambda_u + \omega_u R + \tau_1S_u - ((1 - \zeta)\lambda_u + \mu_u)S_h
\]

\[
\frac{dI_u}{dt} = \lambda_uS_u - (\tau_2 + \delta_u + \theta_u + \mu_u)I_u
\]

\[
\frac{dI_h}{dt} = (1 - \zeta)\lambda_uS_h + \tau_2I_u - (\delta_h + \theta_h + \mu_u)I_h
\]

\[
\frac{dR_u}{dt} = \theta_uI_u + \theta_hI_h - (\omega_u + \omega_h + \mu_u)R_u
\]

\[
\frac{dS_m}{dt} = \Lambda_u - \lambda_uS_m - \mu_uS_m
\]

\[
\frac{dI_m}{dt} = \lambda_uS_m - \mu_vI_m
\]

where

\[
\lambda_u = \frac{b_u\beta_uI_u}{N_u}, \lambda_h = \frac{b_u\beta_hI_u}{N_u}, b_1 > b_2, \alpha = \frac{b_2\beta_uI_u}{N_u}, \beta_u > \theta_u, \theta_h > \theta_u
\]

| Table 1. Variables |
|--------------------|
| Symbol | Description |
| \(S_u\) | Unhygienic Susceptible Human |
| \(S_h\) | Hygienic Susceptible Human |
| \(I_u\) | Unhygienic Infected Human |
| \(I_h\) | Hygienic Infected Human |
| \(R\) | Recovered Human |
| \(S_m\) | Non – disease carrier Mosquitoes |
| \(I_m\) | Disease carrier Mosquitoes |

| Table 2. Model Parameters |
|----------------------------|
| Parameter | Definitions |
| \(\Lambda_u\) | Recruitment rate of Human Population |
| \(\Lambda_v\) | Recruitment rate of mosquitoes |
| \(\tau_1\) | progression from \(S_u\) to \(S_h\) |
| \(\tau_2\) | progression from \(I_u\) to \(I_h\) |
| \(\delta_u\) | disease – induced death for the unhygienic human class |
| \(\delta_h\) | disease – induced death for hygienic human class |
| \(b_1\) | biting rate of mosquito for unhygienic human class |
| \(b_2\) | biting rate of mosquito for hygienic human class |
| \(\beta_{uv}\) | transmission probability of infection from mosquito to human |
| \(\beta_{vh}\) | transmission probability of infection from human to mosquito |
| \(\lambda_u\) | the force of infection for unhygienic human class |
| \(\lambda_h\) | the force of infection for hygienic human class |
| \(\lambda_v\) | force of infection for mosquitoes |
| \(\beta_u\) | biting rate of mosquitoes |
| \(\zeta\) | rate of reduction of infection for hygienic class |
| \(\rho\) | Modification Parameter |
| \(\theta_u\) | rate of recovery for unhygienic human class |
| \(\theta_h\) | rate of recovery for hygienic human class |
| \(\omega\) | rate at which recovered human become susceptible |
| \(\alpha\) | hygienic rate |
| \(\mu_u\) | Natural human death rate |
| \(\mu_v\) | natural death rate of mosquitoes |
| \(N_u\) | Total Human Population |

3. Differential Transformation Method (DTM)

In this section, the basic principle of DTM is being utilized as follows. Given
an arbitrary function \( f(t) \) that can be expanded in Taylor series at the point \( t = 0 \) as

\[
f(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} f^{(n)}(0)
\]

We define the differential transformation as

\[
F(t) = \sum_{n=0}^{\infty} \frac{t^n}{n!} f^{(n)}(0)
\]

The inverse differential transform is

\[
f(t) = \sum_{n=0}^{\infty} t^n F(t)
\]

The table below consists of some properties of the DTM. Given that \( c(x) \) and \( f(x) \) are arbitrary functions with \( C(k) \) and \( F(k) \) as the respective transformation functions.

| S/N | Original Functions | Transformed Functions |
|-----|--------------------|-----------------------|
| 1   | \( y(x) = c(x) \pm f(x) \) | \( Y(k) = C(k) \pm F(k) \) |
| 2   | \( y(x) = \rho c(x) \) | \( Y(k) = \rho C(k) \) |
| 3   | \( y(x) = \frac{dc(x)}{dx} \) | \( Y(k) = (k+1)C(k+1) \) |
| 4   | \( y(x) = \frac{d^2c(x)}{dx^2} \) | \( Y(k) = (k+1)(k+2)C(k+2) \) |
| 5   | \( y(x) = \frac{d^n c(x)}{dx^n} \) | \( Y(k) = (k+1)(k+2)...(k+n)C(k+n) \) |
| 6   | \( y(x) = 1 \) | \( Y(k) = \delta(k) \) |
| 7   | \( y(x) = x \) | \( Y(k) = \delta(k-1) \) |
| 8   | \( y(x) = e^{ax} \) | \( Y(k) = \frac{a^k}{k!} \) |
| 9   | \( y(x) = (x+c) f(x) \) | \( Y(k) = \sum_{n=m}^{\infty} F(m) C(k-m) \) |
| 10  | \( y(x) = (1+x)^n \) | \( Y(k) = \frac{(n+1)(n+2)...(n+k+1)}{k!} \) |

4. Model Solution

Here, DTM is applied to solve the model equations (3):

Using the properties in Table 3, we obtain the following system of transformed equations:

\[
S_u(k+1) = \frac{1}{k+1} \left[ (1-\alpha)A_u - (\tau_1 + \mu_y)S_u(k) - \frac{\nu_y}{\nu_y} \sum_{m=0}^{\infty} S_v(m) \iota_y(k-m) \right] + \omega \mu \tau R(k)
\]

5. Graphical Presentation of Model Solution

In this section, the graphical illustration obtained from the analytical solution is presented using Maple software. By using the initial conditions and parameter values from table 4, figures (2) to (8) show the solutions plots by DTM and RK4.

| Table 4: Parameter values of Model |
|----------------------------------|
| Symbols | Values | Source |
|--------|--------|--------|
| \( A_u \) | 100 | (Oluwataya, 2019) |
| \( \Delta_u \) | 1000 | (Bakare and Nwozo, 2017) |
| \( \tau_1 \) | 0.25 | (Assumed) |
| \( \tau_2 \) | 0.5 | (Assumed) |
| \( \alpha \) | 0.13 | (Assumed) |
| \( \rho \) | 0.05 | (Assumed) |
| \( \mu_y \) | 0.17 | (Assumed) |
| \( \mu_\psi \) | 0.1 | (Assumed) |
| \( \beta_{\psi \mu} \) | 0.03 | (Olaniyit et al., 2018) |
| \( \beta_{\psi \nu} \) | 0.09 | (Olaniyit et al., 2018) |
| \( \beta_u \) | 0.12 | (Olaniyit and Obahiy, 2013) |
| \( \gamma \) | 0.08 | (Assumed) |
| \( \rho \) | 0.5 | (Assumed) |
| \( \omega \) | 0.7902 | (Bakare and Nwozo, 2017) |
| \( \sigma \) | 0.46 | (Assumed) |
| \( \mu_\psi \) | 0.000004 | (Oluwataya, 2019) |
| \( \rho_y \) | 0.000569 | (Bakare and Nwozo, 2017) |
In this study, we present a mathematical model to assess the effect of hygiene in malaria transmission and solved the system of equations using the DTM. Graphical illustrations were presented and compared with DTM result and classical Runge-Kutta order 4th method (RK4). It is shown that DTM is a piecewise efficient convergent, cost effective method for solving nonlinear differential equations in the bounded domains. The method gives rapidly converging series solutions and the solutions can be improved by expanding the series. It is observed that the series solutions obtained with DTM can be written in exact closed form.

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