Small Signal Stability of Three-phase and Six-phase Synchronous Motors: A Comparative Analysis

G. K. Singh\textsuperscript{1}\textsuperscript{*} and Arif Iqbal\textsuperscript{2}

(1. Department of Electrical Engineering, Indian Institute of Technology, Roorkee 247667, India; 2. Department of Electrical Engineering, Rajkiya Engineering College Ambedkar Nagar, Akbarpur 224122, India)

Abstract: Small signal stability analysis is conducted for an asymmetrical six-phase synchronous motor in comparison with its equivalent three-phase counterpart. For this purpose, a linearized model of the six-phase synchronous motor is developed using the \(dq0\) approach, which is used in eigenvalue criteria to determine absolute stability in comparison with its equivalent three-phase counterpart. The analysis includes a comparison of the variation in evaluated eigenvalues associated with the stator and rotor sides according to changes in both the three and six-phase machine parameters and working conditions. Key analytical results are experimentally investigated and validated on a test rig.

Keywords: Linearized modeling, stability analysis, six-phase synchronous motor, three-phase synchronous motor

Notation Explanation

\begin{itemize}
\item \(V_{d1}, V_{q1}\) Stator voltage of winding set \(abc\) along \(d-q\) axes
\item \(V_{d2}, V_{q2}\) Stator voltage of winding set \(xyz\) along \(d-q\) axes
\item \(I_{d1}, I_{q1}\) Stator current of winding set \(abc\) along \(d-q\) axes
\item \(I_{d2}, I_{q2}\) Stator current of winding set \(xyz\) along \(d-q\) axes
\item \(V_{k}\) Field circuit excitation voltage
\item \(I_{k}\) Field circuit excitation current
\item \(V_{k_d}, V_{k_q}\) Damper windings \(K_d\) and \(K_q\) voltage, respectively
\item \(I_{k_d}, I_{k_q}\) Damper windings \(K_d\) and \(K_q\) current, respectively
\item \(R_1, R_2\) Per-phase resistance of stator winding sets \(abc\) and \(xyz\), respectively
\item \(X_{d1}, X_{q1}\) Per-phase leakage reactance of stator winding sets \(abc\) and \(xyz\), respectively
\item \(X_{d2}, X_{q2}\) Leakage reactance of damper windings \(K_d\) and \(K_q\) respectively
\item \(X_{f}\) Field winding leakage reactance
\item \(X_{a}, X_{m}\) Magnetizing reactance along \(d-q\) axes respectively
\item \(X_{cm}\) Common mutual leakage reactance between winding sets \(abc\) and \(xyz\)
\item \(X_{cd}, X_{cq}\) Cross-mutual coupling reactance between \(d-q\) axes of stator windings
\item \(T_s, T_e\) Load torque and developed electromagnetic torque, respectively
\item \(\omega_s, \omega_r\) Base speed and rotor speed, respectively
\item \(J\) Moment of inertia
\end{itemize}

1 Introduction

The recent trend of research activity has been focused on replacing existing three-phase machines with higher-phase machines (i.e., multiphase, particularly six-phase machines) because of various potential advantages associated with multiphase machines when compared with their equivalent three-phase counterparts. Some important advantages are reduced harmonics (space harmonic and DC link harmonic), reduced torque pulsation, reduced current per phase without change in phase voltage, better power to weight ratio in the same frame with increased reliability, etc.\textsuperscript{[1-3]}. Hence, multiphase machines are preferably used in many applications, particularly in electric vehicle/ship propulsion, aircraft, higher-power drive applications, wind power generation, etc.

Stability is an important factor for successful operation of an electrical machine in a steady state under small signal disturbances. In this regard, the stability analysis of three-phase induction machines has been reported using the root locus technique \textsuperscript{[4]}, boundary layer model bracket \textsuperscript{[5]}, and Lyapunov’s first method \textsuperscript{[6]}. Stability analysis of an induction motor fed by a variable frequency inverter \textsuperscript{[7]} was carried out by using Nyquist stability criteria \textsuperscript{[8]}, whereas a current source inverter was employed and analyzed in Ref. \textsuperscript{[9]}. Some studies investigated the stability of three-phase synchronous machines \textsuperscript{[10]}, where small signal stability analyses were performed using the root locus technique\textsuperscript{[11]} and the Nyquist criteria technique\textsuperscript{[12]}.

Few studies have presented small signal stability analysis of multiphase induction motors, particularly

\* Corresponding Author, Email: gksngfee@gmail.com
Digital Object Identifier: 10.23919/CJEE.2020.000002
five-phase \cite{13} and six-phase motors \cite{14}. Introductory analysis of a six-phase synchronous motor was presented by the present authors \cite{15}, together with the determination of stability limits for various parametric variations and working conditions \cite{16}. However, a comparative analysis of a synchronous motor with a three-phase and six-phase stator winding configuration has not yet been reported from the stability point of view to the best of the authors’ knowledge. Therefore, this paper presents the small signal stability analysis of a six-phase synchronous motor in comparison with its equivalent three-phase counterpart. Initially, a linearized model of an asymmetrical six-phase synchronous machine was developed and used to investigate its small signal stability in comparison with its three-phase equivalent using the dq0 approach. This analysis included the evaluation and comparison of eigenvalues using the developed linearized model of a six-phase synchronous machine under parametric variations and different working conditions (i.e., variations of voltage and speed/frequency). Key analytical results were experimentally investigated and validated on a test rig.

2 Linearized modeling of six-phase synchronous machine

Linearized modeling of a six-phase synchronous motor was carried out by considering some important simplifying assumptions \cite{15-18}: the two three-phase stator winding sets, namely, $abc$ and $xyz$, are identical and symmetrical; the effect of harmonics is neglected; and saturation and hysteresis effects are ignored. Six-phase stator winding of the motor was realized by having two sets of three-phase windings, namely, $abc$ and $xyz$, that were physically displaced by angle $\xi = 30^\circ$ electrical, i.e., an asymmetrical six-phase stator winding. Field winding $f_r$ was used along the $d$-axis, and damper windings $K_d$ and $K_q$ were aligned along the $d$ and $q$ axes, respectively \cite{19-20}. For motor operation, the formula of voltage (stator and rotor) and electromagnetic torque were written in machine variables, resulting in a set of nonlinear differential formulas. Nonlinearity was due to the dependency of motor inductance on rotor position, which is time-dependent. Therefore, for simplification with a constant inductance term, motor formulas were preferably expressed in a rotor reference frame using Park’s variable. Considering the current as a state variable (i.e., an independent variable), motor formulas can be written in matrix form

$$V = ZI$$

(1)

where

$$V = [V_{q1}, V_{d1}, V_{q2}, V_{d2}, V_{Kd}, V_{Kq}]^T$$

(2)

$$I = [I_{q1}, I_{d1}, I_{q2}, I_{d2}, I_{Kq}, I_{Kd}]^T$$

(3)

and $Z$ is the impedance matrix.

The equivalent circuit of a six-phase synchronous motor can be developed by considering the above formulas in a $d-q$ reference frame \cite{21}.

The developed electromagnetic motor torque may be written in terms of state variables, i.e., the current, as

$$T_e = \frac{3 P}{2} \frac{1}{\omega_b} \left[ (I_{q1} + I_{q2}) X_m (I_{d1} + I_{d2} + I_{Kd} + I_{Kq}) - (I_{d1} + I_{d2}) X_m (I_{q1} + I_{q2} + I_{Kq}) \right]$$

(4)

The relationship for rotor speed and rotor angle is given by formulas (5) and (6), respectively.

$$\omega = \frac{1}{p} \left[ 1 - \frac{1}{2 J} \frac{P}{\omega_b} (T_e - T_f) \right]$$

(5)

$$\delta = \frac{1}{p} \left( \frac{\omega - \omega_b}{\omega_b} \right)$$

(6)

where $P$ and $p$ indicate the number of poles and differentiation function with respect to time, respectively.

The concept of Taylor series expansion was used to linearize machine formulas (1), (4), (5) and (6), yielding a set of formulas written in matrix form.

$$\begin{bmatrix} \Delta V_{qds} \\ \Delta V_{qds} \\ \Delta V_{rr} \end{bmatrix} = \begin{bmatrix} W_1 X_1 Y_1 \\ X_2 W_2 Y_2 \\ Q_1 Q_2 S \end{bmatrix} \begin{bmatrix} \Delta I_{qds} \\ \Delta I_{qds} \\ \Delta I_{rr} \end{bmatrix}$$

(7)

The matrix elements are explained later in this section.

An input voltage with constant magnitude and frequency is fed to the synchronous motor (connected to an infinite bus bar) in a synchronously rotating reference frame. Therefore, it is essential to correlate the variables in the synchronously rotating reference frame (i.e., $F^{e}_{qds}$ and $F^{e}_{dqs}$) to those in the rotor reference frame (i.e., $F^{*}_{qs}$ and $F^{*}_{ds}$) using formula (8).

$$\begin{bmatrix} F^{e}_{qds} \\ F^{e}_{dqs} \end{bmatrix} = \begin{bmatrix} \cos \delta_K & -\sin \delta_K \\ \sin \delta_K & \cos \delta_K \end{bmatrix} \begin{bmatrix} F^{*}_{qs} \\ F^{*}_{ds} \end{bmatrix}$$

(8)

where
The term \(\zeta\) is the angular displacement between the two three-phase winding sets \(abc\) and \(xyz\) and \(\gamma\) denotes the phase difference between the terminal voltages of phases \(a\) and \(x\). Numerically, the value of both \(\gamma\) and \(\zeta\) is \(30^\circ\) electrical.

Linearizing the nonlinear formula (8) with suitable approximations (\(\cos\Delta\delta_K = 1\) and \(\sin\Delta\delta_K = \Delta\delta_K\)) results in

\[
\Delta F^e_{kdqs} = T_K \Delta F^e_{kdqs} + F^r \Delta \delta
\]

(9)

Linearization of the inverse transformation yields

\[
\Delta F^e_{kdqs} = (T_K)^{-1} \Delta F^e_{kdqs} + F^r \Delta \delta
\]

(10)

The steady-state \(d-q\) performance indices in the rotor and synchronously rotating reference frames are indicated by \(F^r\) and \(F^e\), respectively.

\[
T_K = \begin{bmatrix}
\cos \delta_K & -\sin \delta_K \\
\sin \delta_K & \cos \delta_K 
\end{bmatrix}
\]

(11)

\[
(T_K)^{-1} = \begin{bmatrix}
\cos \delta_K & \sin \delta_K \\
-\sin \delta_K & \cos \delta_K 
\end{bmatrix}
\]

(12)

Substitution of formulas (9) and (10) into formula (8) results in

\[
\begin{bmatrix}
T_1 \Delta V_{1dqs}^e \\
T_2 \Delta V_{2dqs}^e \\
\Delta V_{r}^e
\end{bmatrix} =
\begin{bmatrix}
W_1 & X_1 & Y_1 & T_1 \Delta I_{1dqs}^e \\
X_2 & W_2 & Y_2 & T_2 \Delta I_{2dqs}^e \\
Q_1 & Q_2 & S & \Delta I_r^e
\end{bmatrix}
\]

(13)

which is arranged as

\[
\begin{align*}
\Delta V_{1dqs}^e &= \begin{bmatrix}
(T_1)^{-1} W_{1} T_1 & (T_1)^{-1} X_{1} T_1 & (T_1)^{-1} Y_{1} \\
(T_2)^{-1} W_{2} T_2 & (T_2)^{-1} X_{2} T_2 & (T_2)^{-1} Y_{2}
\end{bmatrix} \Delta I_{1dqs}^e \\
\Delta V_{2dqs}^e &= \begin{bmatrix}
(T_1)^{-1} W_{1} T_1 & (T_1)^{-1} X_{1} T_1 & (T_1)^{-1} Y_{1} \\
(T_2)^{-1} W_{2} T_2 & (T_2)^{-1} X_{2} T_2 & (T_2)^{-1} Y_{2} \\
Q_1 T & Q_2 T & S
\end{bmatrix} \Delta I_{2dqs}^e \\
\Delta V_{r}^e &= \begin{bmatrix}
Q_1 T & Q_2 T & S
\end{bmatrix} \Delta I_{r}^e
\end{align*}
\]

(14)

Formula (14) can be written in the following simplified form

\[
E p_x = F x + u
\]

(15)

where

\[
x^T = \begin{bmatrix}
(\Delta I_{1dqs}^e)^T \\
(\Delta I_{2dqs}^e)^T \\
(\Delta I_{r}^e)^T
\end{bmatrix}
\]

\[
\begin{align*}
\Delta L^e_{q}, \Delta L^e_{d1}, \Delta L^e_{d2}, \Delta L^e_{kq}, \Delta L^e_{k1}, \Delta L^e_{k2}, \Delta L^e_{k3}, \Delta L^e_{k4}, \Delta L^e_{oq}, \Delta L^e_{ob}
\end{align*}
\]

(16)

\[
u^T = \begin{bmatrix}
(\Delta V^e_{1dqs})^T \\
(\Delta V^e_{2dqs})^T \\
(\Delta V^e_{r})^T
\end{bmatrix}
\]

\[
\begin{align*}
\Delta V^e_{1q}, \Delta V^e_{1d}, \Delta V^e_{1k}, \Delta V^e_{2q}, \Delta V^e_{2d}, \Delta V^e_{2k}, \Delta V^e_{k3}, \Delta V^e_{k4}, \Delta V^e_{oq}, \Delta V^e_{ob}, \Delta T
\end{align*}
\]

(17)

\[
E = \begin{bmatrix}
(T_1)^{-1} W_{1} T_1 & (T_1)^{-1} X_{1} T_1 & (T_1)^{-1} Y_{1} \\
(T_2)^{-1} W_{2} T_2 & (T_2)^{-1} X_{2} T_2 & (T_2)^{-1} Y_{2} \\
Q_1 T & Q_2 T & S
\end{bmatrix}
\]

(18)

\[
F = \begin{bmatrix}
(T_1)^{-1} W_{1} T_1 & (T_1)^{-1} X_{1} T_1 & (T_1)^{-1} Y_{1} \\
(T_2)^{-1} W_{2} T_2 & (T_2)^{-1} X_{2} T_2 & (T_2)^{-1} Y_{2}
\end{bmatrix}
\]

(19)

It is to be noted that variables with the additional subscript “0” (\(I_{d0}, I_{q0}, I_{f0}, I_{q10}, I_{q20}\)) signify the steady-state values. Rewriting formula (15) in fundamental form gives us

\[
p x = A x + B u
\]

(20)

where

\[
A = (E)^{-1} F
\]

(21)

\[
B = (E)^{-1}
\]

(22)

In the above linearized modeling of the motor, the mutual coupling between the two sets of three-phase stator windings \(abc\) and \(xyz\) has been taken into consideration using the respective leakage reactances, \(X_{in} \) and \(X_{idq}\). In the following sections, results are presented for the asymmetrical six-phase synchronous motor (with \(\zeta = 30^\circ\) electrical) in comparison with its equivalent three-phase counterpart.

### 3 Analytical results

Small signal stability analysis is preferably carried out by evaluating the system eigenvalue from its characteristic formula.

\[
\det(A - \lambda I) = 0
\]

(23)

where \(A\), \(I\), and \(\lambda\) are the system matrix, identity matrix, and roots of characteristic formula, respectively. A system is said to be stable only if all the real and/or real components of the eigenvalues are negative \([14-16]\).

State formula (20) and formulas (8.3)-(8.45) of Ref. [21] yield nine and seven state variables for six-phase and three-phase synchronous motors, respectively. Hence, nine and seven eigenvalues are obtained for the six-phase and three-phase synchronous motors, respectively. The three complex (i.e., three complex conjugate pairs) and remaining real eigenvalues are obtained for the six-phase synchronous motor. However, two complex (i.e., two complex conjugate pairs) and the remaining real eigenvalues are obtained for the three-phase synchronous motor. The correlation of eigenvalues...
with machine parameters is computationally complex \[^{[21]}\], but such a correlation is established and explained in Refs. \[^{[14-15]}\]. A 3.2-kW, 6-pole, 36-slot, 50-Hz six-phase synchronous motor, whose parameters are given in Tab. 1, was used during the analysis. The evaluated eigenvalues for both the six-phase and three-phase synchronous motors are presented in Tab. 2 and Tab. 3, respectively. During the evaluation of the eigenvalues, the magnetic flux level in both the three-phase and six-phase motor were assumed to be the same by taking the input phase voltage of the three-phase motor to be twice that of the six-phase motor \[^{[22]}\]. Therefore, the per-phase input voltage was maintained at 120 V and 240 V for the six-phase and three-phase synchronous motors, respectively, operating on the same load torque at 50\% at power factor 0.85 (lagging).

### Tab. 1  Parameters of six-phase synchronous motor

| Parameter    | Value  | Parameter    | Value  |
|--------------|--------|--------------|--------|
| $X_{mq}/\Omega$ | 3.9112 | $R_{f}/\Omega$ | 0.056  |
| $X_{md}/\Omega$ | 6.1732 | $R_{r}/\Omega$ | 0.210  |
| $X_{dq}/\Omega$ | 0.66097 | $R_{eq}/\Omega$ | 2.535  |
| $X_{id}/\Omega$ | 1.550 | $X_{ij}/\Omega$ | 0.1785 |
| $X_{iq}/\Omega$ | 0.1785 | $X_{ij}/\Omega$ | 0.2402 |

### Tab. 2  Calculated eigenvalues of six-phase synchronous motor

| Nomenclature | Eigenvalue |
|--------------|------------|
| Stator eigenvalue I | $-107.8 \pm j104.7$ |
| Stator eigenvalue II | $-17.3 \pm j99.8$ |
| Rotor eigenvalue | $-7.8 \pm j48.4$ |
| Real eigenvalue | $-9136.3, -710.7, -12.3$ |

### Tab. 3  Calculated eigenvalues of three-phase synchronous motor

| Nomenclature | Eigenvalue |
|--------------|------------|
| Stator eigenvalue | $-38.0 \pm j88.8$ |
| Rotor eigenvalue | $-22.3 \pm j61.5$ |
| Real eigenvalue | $-8719.7, -516.6, -9.3$ |
\[
Y_{1k} = \begin{bmatrix}
0 & X_{md} & X_{md} \\
-X_{mq} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
(X_{d1}I_{d10} + X_{md}(I_{d10} + I_{d20} + I_{f0})) \\
-(X_{d1}I_{q10} + X_{mq}(I_{q10} + I_{q20}))
\end{bmatrix}
+ \begin{bmatrix}
(-R_{i} I_{d10} + (X_{d1} + X_{ln} + X_{nd})I_{q10} - X_{ldq} I_{d20}) \\
(X_{md} + X_{ln})I_{q10} + V_{d10}
\end{bmatrix}
\]

\[
Y_{2k} = \begin{bmatrix}
0 & X_{md} & X_{md} \\
-X_{mq} & 0 & 0
\end{bmatrix}
\begin{bmatrix}
(X_{d2}I_{d10} + X_{md}(I_{d10} + I_{d20} + I_{f0})) \\
-(X_{d1}I_{q10} + X_{mq}(I_{q10} + I_{q20}))
\end{bmatrix}
+ \begin{bmatrix}
(-R_{i} I_{d20} + (X_{d2} + X_{ln} + X_{md})I_{q20} - X_{ldq} I_{d10}) \\
(X_{md} + X_{ln})I_{q10} + V_{q10}
\end{bmatrix}
\]

\[
Q_{1k} = Q_{2k} = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
Y_{1p} = \frac{1}{(s_{th})} \begin{bmatrix}
X_{dq} & 0 & 0 \\
0 & X_{md} & X_{md}
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
-(X_{d1} + X_{ln} + X_{mq})I_{d10} - (X_{d1} + X_{ln})I_{d20}
\end{bmatrix}
\]

\[
Y_{2p} = \frac{1}{(s_{th})} \begin{bmatrix}
X_{dq} & 0 & 0 \\
0 & X_{md} & X_{md}
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
-(X_{d2} + X_{ln} + X_{mq})I_{d20} - (X_{d2} + X_{ln})I_{d10}
\end{bmatrix}
\]

\[
S_{p} = \frac{1}{(s_{th})} \begin{bmatrix}
0 & 0 & 0 \\
0 & X_{md} & X_{md}
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix}
+ \begin{bmatrix}
-(X_{dq}I_{d10} + X_{mq}I_{d20}) \\
X_{dq}I_{q10} + X_{mq}I_{q20}
\end{bmatrix}
\]

\[
W_{1k} = \begin{bmatrix}
R_i & (X_{d1} + X_{ln} + X_{md}) \\
-(X_{d1} + X_{ln} + X_{mq}) & R_i
\end{bmatrix}
\]

\[
W_{2k} = \begin{bmatrix}
R_i & (X_{d2} + X_{ln} + X_{md}) \\
-(X_{d2} + X_{ln} + X_{mq}) & R_i
\end{bmatrix}
\]

\[
X_{1k} = \begin{bmatrix}
X_{ldq} & (X_{ln} + X_{md}) \\
-(X_{ln} + X_{mq}) & -X_{ldq}
\end{bmatrix}
\]

\[
X_{2k} = \begin{bmatrix}
X_{ldq} & (X_{ln} + X_{md}) \\
-(X_{ln} + X_{mq}) & -X_{ldq}
\end{bmatrix}
\]

\[
W_{1p} = \frac{1}{(s_{th})} \begin{bmatrix}
(X_{d1} + X_{ln} + X_{mq}) & 0 \\
0 & (X_{d1} + X_{ln} + X_{md})
\end{bmatrix}
\]

\[
W_{2p} = \frac{1}{(s_{th})} \begin{bmatrix}
(X_{d2} + X_{ln} + X_{mq}) & 0 \\
0 & (X_{d2} + X_{ln} + X_{md})
\end{bmatrix}
\]

\[
X_{1p} = \frac{1}{(s_{th})} \begin{bmatrix}
(X_{mq} + X_{ln}) & -X_{ldq} \\
X_{ldq} & (X_{md} + X_{ln})
\end{bmatrix}
\]

\[
X_{2p} = \frac{1}{(s_{th})} \begin{bmatrix}
(X_{mq} + X_{ln}) & -X_{ldq} \\
X_{ldq} & (X_{md} + X_{ln})
\end{bmatrix}
\]

\[
Q_{1p} = Q_{2p} = \frac{1}{(s_{th})} \begin{bmatrix}
0 & 0 \\
0 & X_{md}
\end{bmatrix}
\]

\[
Q_{2p} = \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]
\[
S_k = \begin{bmatrix}
R_{kq} & 0 & X_{md} & 0 & 0 & 0 \\
0 & R_{kd} & \left(\frac{X_{md}}{R_{kd}}\right) & 0 & 0 & 0 \\
-X_m q (I_{d10} + I_{d20}) F_{qs} & X_m d (I_{q10} + I_{q20}) F_{qs} & 0 & 0 & F_{qs} S_{45} \\
0 & 0 & 0 & 0 & \omega_b & 0
\end{bmatrix}
\]

where

\[
S_{45} = -I_{d10}(X_m d (I_{d10} + I_{d20}) - X_m q (I_{d10} + I_{d20})) +
I_{q10}(X_m d (I_{d10} + I_{d20}) - X_m q (I_{d10} + I_{d20})) -
(X_m d (I_{d10} + I_{d20}) + I_{d10} - X_m q (I_{d10} + I_{d20}))+
I_{q20}(X_m d (I_{d10} + I_{d20}) - X_m q (I_{d10} + I_{d20}))
\]

\[
F_{qs} = \frac{3P}{4\omega_b}
\]

In formula (15), the derivative part is shown by coefficient matrix \(E\) with subscript \(p\), and the remaining terms of the linearized motor formulas are indicated by coefficient matrix \(F\) with subscript \(k\). The elements of matrices \(E\) and \(F\) are defined above.

### 3.1 Variation in stator parameters

In the following sections, the six-phase winding configuration is assumed to consist of two balanced and identical three-phase windings, \(abc\) and \(xyz\). Hence, the parameters of the winding sets, which are identical, are the same (i.e., resistance \(R_1 = R_1 = R_2\) and leakage reactance \(X_s = X_{11} = X_{12}\)). Also, in the depicted figures, the solid lines and dash lines show the real and imaginary components, respectively, of the eigenvalues.

Variations in the motor eigenvalues with change in the stator parameters for the three-phase and six-phase stator winding connections are shown in Figs. 1-4. With increasing stator resistance, the motor operation from the stator side was found to be stable, with increased magnitude of the negative real part of the stator eigenvalues (Fig. 1a and Fig. 2a for three-phase and six-phase, respectively). However, rotor operation tended toward instability for both the three-phase and six-phase motors, as shown in Fig. 1a and Fig. 2b, respectively. Small variations in the real eigenvalues were noted for the three-phase (Figs. 1b-1d) and six-phase (Figs. 2c-2e) motors.
Fig. 2  Eigenvalue variations of six-phase synchronous machine with change in stator resistance

Fig. 3  Eigenvalue variations of three-phase synchronous machine with change in stator leakage reactance
A reverse effect was noted with the same variation in stator leakage reactance, $X_{ls}$. With increasing stator leakage reactance, the real component of the stator decreased (with negative magnitude) by 38.9% for the three-phase motor (shown in Fig. 3a) and by 50% and 29.9% for the six-phase synchronous motor (shown in Fig. 4a), corresponding to stator eigenvalues I and II, respectively. However, a slight increase in the negative magnitude of the real component of the rotor eigenvalue of both the three-phase and six-phase synchronous motor was observed, as shown in Fig. 3a and Fig. 4b, respectively. All three real eigenvalues decreased in both the three-phase (shown in Figs. 3b-3d) and the six-phase (shown in Figs. 4c-4e) synchronous motor. However, the value for the six-phase synchronous motor remained higher (with a negative sign), showing its more stable operation in comparison with its equivalent three-phase motor.

### 3.2 Variation in field circuit parameters

In this section, variations in eigenvalues are presented for the change in field circuit parameter ($R_{f}$, $X_{lf}$) for the three-phase and six-phase synchronous motor. The results are presented for the increased field circuit resistance of the synchronous motor. No change in the real components of the stator eigenvalue was noted for the three-phase and six-phase synchronous motors (shown in Fig. 5a and Fig. 6a, respectively). Both motors showed a similar operation from the stator side. For the rotor eigenvalue, variation was not noted for the three-phase motor (shown in Fig. 5a), but the rotor eigenvalue increased linearly (with a negative sign) in the six-phase synchronous motor (shown in Fig. 6b), signifying more stable operation from the rotor side because the time constant of the field circuit had the largest value, giving rise to the smallest eigenvalue $^{[15,21]}$. Hence, the increase in the value of $R_{f}$ was reflected in eigenvalue III, with almost no or only slight variations in real eigenvalues I and II for the three-phase (shown in Figs. 5b-5d) and the six-phase (shown in Figs. 6c-6e) synchronous motors.
Variations in field leakage reactance $X_{lf}$ had an effect on both the stator and the rotor side. With increasing $X_{lf}$, the real component of the stator eigenvalue decreased for both the three-phase and the six-phase synchronous motors, as shown in Fig. 7a and Fig. 8a, respectively. Also, a slight variation in rotor eigenvalue was noted for both motors, as shown in Fig. 7a and Fig. 8b, respectively. A slight linear decrease in real eigenvalue I was noted for both motors, with no or only slight variation in real eigenvalue II, as shown in Figs. 7b-7c for the three-phase and in Figs. 8c-8d for the six-phase synchronous motor. However, a pronounced effect was noted in real eigenvalue III, whose negative magnitude decreased for both the three-phase and six-phase synchronous motors, shown in Fig. 7d and Fig. 8e, respectively.

### 3.3 Variation in damper winding, $K_d$ parameters

The effect of parametric variation of damper winding $K_d$ along the $d$-axis on the eigenvalues of the three-phase and six-phase synchronous motors is shown in Figs. 9-12. Variations in the damper winding resis rotor eigenvalues for the three-phase (shown in Fig. 9a) tance $R_{Kd}$ had no effect on either the stator and
Fig. 7 Eigenvalue variations of three-phase synchronous machine with change in field circuit leakage reactance

(a)

Fig. 8 Eigenvalue variations of six-phase synchronous machine with change in field circuit leakage reactance

(a)

Fig. 9 Eigenvalue variations of three-phase synchronous machine with change in damper winding resistance along $d$-axis

(a)
six-phase (shown in Figs. 10a-10b) synchronous motors. Only real eigenvalue I was noted to decrease with increasing $R_{Kd}$, shown in Fig. 9b and Fig. 10c for the three-phase and six-phase synchronous motors, respectively. No variation was noted for real eigenvalues II and III for either the three-phase (shown in Figs. 9c-9d) or six-phase (shown in Figs. 10d-10e) synchronous motors.

Variations in the leakage reactance $x_{Kd}$ had no effect on either the stator and rotor eigenvalues for the three-phase (shown in Fig. 11a) and six-phase motor (shown in Figs. 12a-12b). Variation was found only in real eigenvalue I, whose negative magnitude decreased for both the three-phase and the six-phase synchronous motor, shown in Fig. 11b and Fig. 12c, respectively. The magnitude of real eigenvalue I remained higher in the six-phase motor, signifying more stable operation than its equivalent three-phase motor. No variation was noted in real eigenvalues II and III for either the three-phase (Figs. 11c-11d) or the six-phase motor (Figs. 12d-12e).
3.4 Variation in damper winding, $Kq$ parameters

Variations in the damper winding resistance $R_{Kq}$ were minimal for the real component of the stator eigenvalue for the three-phase and six-phase synchronous motor, shown in Fig. 13a and Fig. 14a, respectively. With increasing $R_{Kq}$, the rotor eigenvalue tended toward instability by reducing the negative component magnitude by 58.3% and 41.0% for the three-phase and six-phase synchronous motors, shown in Fig. 13a and Fig. 14b, respectively. The only major effect was noted in real eigenvalue II, but there were no or only slight changes in real eigenvalues I and III for the three-phase (Figs. 13c-13d) and six-phase (Figs. 14c-14e) synchronous motors.

Analytically, the variation in damper winding leakage reactance $X_{Kq}$ had little impact on the real component of the motor eigenvalues for both the three-phase and six-phase synchronous motors; hence, those values are not shown. The only variation was in real eigenvalue II. This value (with a negative sign) decreased with increasing $X_{Kq}$, with a higher
negative magnitude for the six-phase synchronous motor, showing its more stable operation when compared with its equivalent three-phase counterpart. This is depicted in Fig. 15a and Fig. 15b for the three-phase and six-phase synchronous motors, respectively.

Fig. 13  Eigenvalue variations of three-phase synchronous machine with change in damper winding resistance along $q$-axis

Fig. 14  Eigenvalue variations of six-phase synchronous machine with change in damper winding resistance along $q$-axis
3.5 Variation in magnetizing reactance

The effect of variations in the magnetizing reactance along the $q$-axis, $x_{mq}$, on the eigenvalues is presented for both the three-phase and six-phase synchronous motor in Fig. 16 and Fig. 17, respectively. By doubling $x_{mq}$, the value of the real component of the stator eigenvalue increased by 14% and 35.2%, shown in Fig. 16a and Fig. 17a for the three-phase and six-phase synchronous motors, respectively. Hence, more stable operation is obtained from the stator side in the six-phase synchronous motor. However, on the rotor side, the real component decreased by 4.1 rad/s and 2.9 rad/s, as shown in Fig. 16a and Fig. 17b for three-phase and six-phase synchronous motor, respectively, thus tending toward operational instability.
We noted that no variation in real eigenvalue I for the six-phase synchronous motor, and hence the values are not shown. A slight variation was noted for real eigenvalue II, shown in Fig. 16b and Fig. 17c for the three-phase and six-phase synchronous motors, respectively. However, a pronounced decrease in the value of real eigenvalue III by 4.6 rad/s is noted in Fig. 16c and by 9.1 rad/s in Fig. 17d for the three-phase and six-phase synchronous motors, respectively.

Variations in the magnetizing reactance along the \( d \)-axis, \( x_{md} \), had no impact on the motor eigenvalues, and hence those values are not shown.

### 3.6 Variation in voltage

The per-phase input voltage of motor is presented in the form of bar graph for both the three-phase and six-phase synchronous motor in Fig. 18. With increasing phase voltage, the real component of the stator eigenvalue of the three-phase motor decreased by 6.5 rad/s (with a negative sign), thus, tending towards operational instability. This effect was not noted in stator eigenvalue I (hence, the values are not shown), but there was a decrease (with a negative sign) by 4.6 rad/s in stator eigenvalue II of the six-phase motor. The decrease in stator eigenvalue II was smaller for the six-phase motor in comparison to the three-phase synchronous motor, shown in Fig. 18a. This signifies more robust operation of the six-phase motor.

On the rotor side, the operation of the three-phase synchronous motor became unstable (shown by positive value of 2.6 rad/s at 100 V), but the value of the real component of the rotor eigenvalue remained negative with a higher magnitude, as shown in Fig. 18b. These results show the more stable operation of the six-phase synchronous motor in comparison with its equivalent three-phase motor. Furthermore, the negative magnitude of real eigenvalues II and III of the six-phase motor was found to be higher, showing its more stable operation in comparison with its equivalent three-phase motor, as shown in Fig. 18c and Fig. 18d, respectively. Variations in real eigenvalue I were not noted, and hence those values are not shown.
3.7 Variation in frequency/rotor speed

The effect of changes in frequency/rotor speed on the motor eigenvalues is presented in Fig. 19. The magnetic flux in both the three-phase and six-phase synchronous motor remained constant by maintaining the V/Hz ratio. With increasing operating frequency, the real component of the stator eigenvalue decreased by 12.6 rad/s (with a negative sign) for the three-phase synchronous motor. However, this variation was only noted in stator eigenvalue II by 11.9 rad/s, showing relatively robust operation by the six-phase synchronous motor, shown in Fig. 19a. On the rotor side, however, the real component of the rotor eigenvalue increased (with negative sign), indicating operational stability for both the three-phase and six-phase synchronous motor. However, the positive value of the real component of the rotor eigenvalue (=1.3 rad/s at 30 Hz) signifies that the six-phase synchronous motor is not suitable for low-frequency operation in comparison with its equivalent three-phase motor, as shown Fig 19b. Furthermore, a decrease in the value of real eigenvalues II and III (with a negative sign) was noted for both the three-phase and six-phase synchronous motor, as shown in Fig. 19c and Fig. 19d, respectively.

3.8 Load variation

The effect of load torque variation on the motor eigenvalues is depicted in Fig. 20. The results are presented for the same magnetic flux level in machines having per-phase voltages of 240 V and 120 V for the three-phase and six-phase synchronous motors, respectively. By increasing the load torque from 0.25 p.u. to its rated value, the real components of the stator eigenvalue of the three-phase synchronous motor increased (with a negative sign). The same pattern was noted in stator eigenvalue II, as shown in Fig. 20a, with no change in stator eigenvalue I (not shown) of the six-phase synchronous motor. A major effect was noted for the real component of the rotor eigenvalue, whose numerical value decreased (with a negative sign) by 11.9 rad/s and 3.1 rad/s for the three-phase and six-phase synchronous motors, respectively, hence tending toward operational instability, as shown in Fig. 20b. Furthermore, no effect on real eigenvalue I was noted (not shown), but there was slight variation in the value of real eigenvalue II for both the three-phase and six-phase synchronous motor, as shown in Fig. 20c. Also, a decrease in the numerical value (with a negative sign) of real eigenvalue III was noted for the three-phase and six-phase synchronous motors, as shown in Fig. 20d.
4 Experimental results

To investigate the motor from a stability point of view, a 3.2-kW, 6-pole, 36-slot synchronous machine was used, where all 72 end terminals were brought out to the terminal box, enabling the connection of stator winding as three-phase or asymmetrical six-phase using the concept of phase belt split [1, 3]. The six-phase AC supply was obtained by using a three- to six-phase three-winding transformer: 10 kVA, 415/415-415, and Y/Y-Δ. The synchronous motor was mechanically coupled with a DC generator (5 kW, 250 V, 21.6 A, 1 300/1 750 r/min) that was previously calibrated separately. Measurement and recording of performance indices were performed using various analog voltmeter, ammeter, together with a power quality analyzer (Hioki 3197). The schematic and general view of the experimental setup [16-18, 23] is shown in Fig. 21a and Fig. 21b, respectively.

The synchronous motor was operated with the maximum permissible load (under the safe limit) with various input phase voltages for both the three-phase and six-phase stator winding configuration. Experimentally, the stability limit of the six-phase synchronous motor was found to be higher than that of its equivalent three-phase motor. This is because of the lower value of the per-phase stator resistance of the six-phase synchronous motor; the variation in the real component of the rotor eigenvalue (which is the dominant eigenvalue [16]) was lower in comparison with its equivalent three-phase, as shown in Fig. 1a and Fig. 2b.

The motor was further operated at different loads with the same magnetic condition in both the three-phase and six-phase winding configuration by maintaining the three-phase motor terminal input voltage at twice that of the six-phase motor. Under this condition, the motor was operated with the maximum permissible load under the safe limit. Experimentally, the power handling capability of the six-phase motor was found to be higher than that of the equivalent three-phase motor with increasing input phase voltage and frequency, as shown in Fig. 22 and Fig. 23, respectively. One such operation of the synchronous motor with a 35-Hz supply is depicted in Fig. 24a and Fig. 24b for the three-phase and six-phase motors, respectively. As shown in Fig. 24c, where the input
phase voltage was approximately 200 V and the power handling capability was 1.42 p.u. (higher than the three-phase rated power).

5 Conclusions

In this study, small signal stability was investigated for an asymmetrical six-phase synchronous motor and its equivalent three-phase through the development of a linearized model using Park’s variable (i.e., dq0 approach). For the purpose of stability analysis, the developed linearized model of motor was used in terms of eigenvalue criteria. The eigenvalues were evaluated and comparatively presented according to parametric variations and changes in working conditions (changes in voltage and frequency with load) for both the six-phase and three-phase synchronous motor. From the stator side, a pronounced effect was noted for the variation of stator resistance, whose effect was similar in both the three-phase and six-phase synchronous motor. The decreased stator resistance made the six-phase motor more stable than its equivalent three-phase motor, which was experimentally validated. On the field side, smallest real eigenvalue III was noted to increase (with a negative value) with increasing circuit resistance. However, the magnitude of real eigenvalue III decreased with increasing field circuit leakage reactance. This magnitude for the six-phase synchronous motor remained higher than that of the three-phase motor, again showing more stable operation by the six-phase synchronous motor. However, the field circuit parameters should be selected for an acceptable time constant of the synchronous motor. A similar effect was noted for the variation in damper winding parameter on both the three-phase and six-phase synchronous motors. The negative magnitudes of real eigenvalues I and II were found to be higher in the six-phase motor than in its equivalent three-phase motor.

It was analytically and experimentally found that the six-phase synchronous motor is more suited for operation at higher voltages with higher stability limits as compared with its equivalent three-phase motor. Also, the motor operation is more stable at higher frequency operation, maintaining a constant ratio of V/Hz.

References

[1] G K Singh. Multi-phase induction machine drive research- A survey. Electric Power Systems Research, 2002, 61(2): 139-147.
[2] E Levi. Multi-phase electric machines for variable-speed applications. *IEEE Transactions on Industrial Electronics*, 2008, 55(5): 1893-1909.

[3] E A Klingshirn. High phase order induction motor-Part-I: description and theoretical consideration. *IEEE Trans. Power App. Sys.*, 1983, 102(1): 47-53.

[4] G J Rogers. Linearized analysis of induction motor transients. *IEEE Proc.*, 1965, 112(10): 1917-1926.

[5] O T Tan, G G Richards. Decoupled boundary layer model of induction machines. *IEEE Proc.*, 1986, 133(4): 255-262.

[6] M M Ahmed, J A Tanfig, C J Goodman, et al. Electrical instability in a voltage source inverter fed induction motor drive. *IEEE Proc.*, 1986, 133(4): 299-307.

[7] F I Fallside, A T Wortley. Steady-state oscillation and stabilization of variable frequency inverter fed induction motor drives. *IEEE Proc.*, 1969, 116(6): 991-999.

[8] T A Lipo, P C Krause. Stability analysis of a rectifier-inverter induction motor drive. *IEEE Trans. Power App. Sys.*, 1969, 88(1): 55-66.

[9] E P Cornell, T A Lipo. Modeling and design of controlled current induction motor drive systems. *IEEE Trans. Power App. Sys.*, 1977, 13(4): 321-330.

[10] T A Lipo, P C Krause. Stability analysis of a reluctance-synchronous machine. *IEEE Trans. Power App. Sys.*, 1967, 86(7): 825-834.

[11] T A Lipo, P C Krause. Stability analysis for variable frequency operation of synchronous machines. *IEEE Trans. Power App. Sys.*, 1968, 87(1): 227-234.

[12] C A Stapleton. Root-locus study of synchronous-machine regulation. *IEEE Proc.*, 1964, 111(4): 761-768.

[13] M J Duran, F Salas, M R Arahal. Bifurcation analysis of five-phase induction motor drives with third harmonic injection. *IEEE Transactions on Industrial Electronics*, 2008, 55(5): 2006-2014.

[14] G K Singh, V Pant, Y P Singh. Stability analysis of a multi-phase (six-phase) induction machine. *Computers and Electrical Engineering*. 2003, 29(7): 727-756.

[15] A Iqbal, G K Singh, V Pant. Stability analysis of an asymmetrical six-phase synchronous motor. *Turk J Elec Eng. & Comp Sci*. 2016, 24(3): 1674-1692.

[16] A Iqbal, G K Singh. Eigenvalue analysis of six-phase synchronous motor for small signal stability. *EPE Journal*, 2018, 28(2): 49-62.

[17] A Iqbal, G K Singh, V Pant. Steady-state modeling and analysis of six-phase synchronous motor. *Sys. Sci. & Cont. Eng.*, 2014, 2(1): 236-249.

[18] G K Singh. Modeling and analysis of six-phase synchronous generator for stand-alone renewable energy generation. *Energy*, 2011, 36(3): 5621-5631.

[19] R F Schiferl, C M Ong. Six phase synchronous machine with AC and DC stator connection, Part-I. *IEEE Trans. Power App. Sys.*, 1983, 102(8): 2685-2693.

[20] G K Singh, A Iqbal. Modeling and analysis of six-phase synchronous motor under fault condition. *Chinese Journal of Electrical Engineering*, 2017, 3(2): 62-75.

[21] P C Krause, O Waszczuk, S D Sudhoff. Analysis of electric machinery and drive systems. *A John Wiley & Sons, Inc. Publication, IEEE Press*, 2nd edition, Piscataway, NJ, 2002.

[22] R Bojoi, M Lazzari, F Profumo, et al. Digital field-oriented control of dual three-phase induction motor drives. *IEEE Trans. Ind. Appl.*, 2003, 39(3): 752-760.

[23] G K Singh. A six-phase synchronous generator for stand-alone renewable energy generation: experimental analysis. *Energy*, 2011, 36(3): 1768-1775.

---

**G. K. Singh** received his B.T. in Electrical Engineering from G B Pant University of Agriculture and Technology, Pantnagar, India, in 1981, and Ph.D. in Electrical Engineering from Banaras Hindu University, Varanasi, India, in 1991. He worked in the industry for nearly five-and-a half years. In 1991, he became a Lecturer at M.N.R. Engineering College, Allahabad, India. In 1996, he moved to the University of Roorkee, Roorkee, India. Currently, he is a Professor in the Electrical Engineering Department, Indian Institute of Technology, Roorkee. He has been involved in the design and analysis of electrical machines in general and high-phase-order AC machines in particular, as well as power system harmonics and power quality. He has coordinated a number of research projects sponsored by the CSIR and UGC, Government of India. He was a Visiting Associate Professor in the Department of Electrical Engineering, Pohang University of Science and Technology (POSTECH), Pohang, Korea, and Visiting Professor in the Department of Electrical and Electronics Engineering, Middle East Technical University, Ankara, Turkey. He also served as a Visiting Professor to University of Malaya, Malaysia for one year. Prof. Singh received the Pt. Madan Mohan Malaviya Memorial Medal and the Certificate of Merit Award 2001–2002 from The Institution of Engineers (India).

---

**Arif Iqbal** received his B.T. and M.T. in Electrical Engineering from Aligarh Muslim University, Aligarh, India, in 2005 and 2007, respectively. He has completed his Ph.D. from Indian Institute of Technology, Roorkee, India, in 2015. He has four years of experience in industry and academia in the field AC drives and power systems. Currently, he is working as an Assistant Professor in the Electrical Engineering Department, Rajkiya Engineering College Ambedkar Nagar, Akbarpur, India. His area of interest is multiphase AC machine and drives, power electronics, and renewable energy systems.