Case report

Joint B₀ and image estimation integrated with model based reconstruction for field map update and distortion correction in prostate diffusion MRI

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ABSTRACT

In prostate Diffusion Weighted MRI, differences in susceptibility values exist at the interface between the prostate and rectal-air. This can result in off-resonance magnetic field leading to geometric distortions including signal stretching and signal pile-up in the reconstructed images. Using a set of EPI data acquired with blip-up and blip-down phase encoding gradient directions, model based reconstruction has recently been proposed that can correct these distortions by using a B₀ field estimated from a separate B₀ scan. However, change in the size of the rectal air region across time can occur that can result in a mismatch of the B₀ field to the EPI scan. Also, the measured B₀ field itself can be erroneous in regions of low Signal to Noise ratio around the prostate rectal-air interface. In this work, using a set of singleshot EPI data acquired with blip-up and blip-down phase encoding gradient directions, a novel joint model based reconstruction is proposed that can account for changes in the off-resonance effects between the B₀ and EPI scans. For ten prostate patients, using a measured B₀ field as an initial B₀ estimate, on a 5-point scale (1–5) image quality scores evaluated by an experienced radiologist, the proposed framework achieved scores of 3.50 ± 0.85 and 3.40 ± 0.51 for b-values of 0 and 500 s/mm², respectively compared to 3.40 ± 0.70 and 3.30 ± 0.67 for model based reconstruction. The proposed framework is also capable of estimating a distortion corrected EPI image even without an initial B₀ field estimate in situations where a separate B₀ scan cannot be obtained due to time constraint.

1. Introduction

Prostate diffusion MRI scans are used for tumour detection within a multi-parametric MRI protocol. However, in some patients the diffusion scans are of limited use due to geometric distortions including signal pile-up and signal drop out at the interface between prostate and rectal-air. These distortions occur due to susceptibility differences at the interface creating an additional off-resonance field (called B₀ field) so that the position-frequency relationship is changed. This results in signal stretching in areas within the image where the gradient of the B₀ field has the same polarity as the phase encoding gradient. Conversely, signal compression or pile-up occurs in regions where the B₀ field gradient direction is opposite to that of the phase encoding gradient.

Several techniques have been proposed to correct for both signal pile-up and stretching distortions in diffusion EPI images. These techniques acquire the same slice twice with opposite phase encoding gradient directions, resulting in two data sets, blip-up and blip-down [1–3]. Image registration based distortion correction methods [1,2] attempt to correct the distortions by finding the B₀ field as a symmetrical displacement field that will result in identical corrected images from blip-up and blip-down data sets. In areas of low SNR such as the region around the prostate rectal-air interface, these techniques may fail due to registration problems [3–5]. Model based reconstruction [6,7] provides an alternative that formulates the distortion correction problem as a model linking the corrupted k-space data to the corrected image. Using a previous estimate of the B₀ field obtained from a separate B₀ scan, this technique solves a full inverse problem using iterative reconstruction and can achieve better reconstruction results in terms of resolution and distortion correction than image registration based methods [7]. However, the model based reconstruction framework assumes the B₀ field to be static except for an offset resulting from scanner frequency drifts. In cases of bowel movements and/or rectal gas changes in the rectal area adjacent to the prostate region between acquisitions of the B₀ and the EPI scans, the estimated B₀ field may be
mismatched to the EPI scans. Also, there can be errors in the estimation of B₀ field itself in the low SNR regions [8]. Thus, the model based distortion correction techniques may benefit from an update of the B₀ field estimation as part of the reconstruction problem.

There has been some works on dynamic B₀ field estimation in functional MRI as part of reconstruction process [9–12]. Sutton et al [10] and Matakos et al. [11] proposed joint image and dynamic B₀ field estimation framework for brain functional MRI that used a set of interleaved gradient echo EPI data sets acquired with different echo times to facilitate the correction of B₀ field errors due to magnetic field drift and respiratory induced phase oscillations. However, as the EPI data sets were acquired with only one phase encoding gradient direction, they did not provide complementary information from opposite phase encoding direction to be able to correct for signal pile-up artifacts [1,13].

In this work, we propose a model based joint image and B₀ field estimation that combines the strength of model based reconstruction with dynamic B₀ field estimation. Using a set of single shot EPI data acquired with blip-up and blip-down phase encoding gradient directions, the proposed framework can account for changes in the off resonance effects between B₀ and EPI scans and can simultaneously compensate for geometric distortions including both signal pile-up and drop-out in the reconstructed EPI images. Results with and without an initial measured B₀ field are presented, as well as using the model based reconstruction alone.

2. Material and methods

The proposed framework acquires two EPI data sets (blip-up and blip-down) with opposite phase encoding gradient directions. For data acquired with a b-value of 0 s/mm², starting from an initial B₀ field, a joint estimation framework is proposed that can estimate both the corrected EPI image and the corrected B₀ field. The corrected B₀ field is then used in subsequent steps of phase correction and model based reconstruction for diffusion weighted data (b-value > 0 s/mm²).

The proposed reconstruction framework has the following three steps:

Step 1) Dynamic B₀ field estimation: Starting from an initial B₀ field, this step estimates the corrected B₀ field by iteratively solving a joint image and B₀ field estimation problem. The initial B₀ field can be estimated from a separate dual echo gradient echo scan. In case, a separate scan is not available, the initial B₀ field is set to 0.

With B₀ field inhomogeneities, the corrupted k-space Yj from the jth coil corresponding to trajectory k(t) at time t can be related to the undistorted image x via the following model [14].

\[
Y(k(t), t) = \sum_{n=0}^{N-1} C(r_n) x \exp(-2\pi i k(t) \cdot r_n) = \sum_{n=0}^{N-1} C(r_n) x \exp(-2\pi i k(t) \cdot r_n) = \sum_{n=0}^{N-1} C(r_n) x \exp(-2\pi i k(t) \cdot r_n)
\]

We can summarize the above equation as:

\[
Y = E(\Delta B_0) x
\]

where Y has dimensions Mx1 and is the acquired distorted k-space from the jth coil, E(ΔB₀) is the measurement matrix with dimensions MxN, x is the undistorted image with dimensions Nx1. The above expression implicitly takes into account any undersampling that is used for accelerated acquisition such as SENSE, Partial Fourier etc.

Let k_up and k_down be the k-space trajectory for blip-up and blip-down scans, the acquired k-space data Y_up and Y_down b-value of 0 s/mm² corresponding to blip-up and blip-down EPI scans for the jth coil are given as:

\[
Y_{up} = E_{up}(\Delta B_0) x
Y_{down} = E_{down}(\Delta B_0) x
\]

where E_{up}(\Delta B_0) = E(\Delta B_0) for k = k_up and E_{down}(\Delta B_0) = E(\Delta B_0) for k = k_down.

By stacking and encoding operators from all J coils, we can write:

\[
Y_{up} = [Y_{up}^{1}, Y_{up}^{2}, ..., Y_{up}^{J}]^T \quad \text{and} \quad Y_{down} = [Y_{down}^{1}, Y_{down}^{2}, ..., Y_{down}^{J}]^T
\]

\[
E_{up}(\Delta B_0) = [E_{up}(\Delta B_0)^{1}, E_{up}(\Delta B_0)^{2}, ..., E_{up}(\Delta B_0)^{J}]^T
\]

\[
E_{down}(\Delta B_0) = [E_{down}(\Delta B_0)^{1}, E_{down}(\Delta B_0)^{2}, ..., E_{down}(\Delta B_0)^{J}]^T
\]

The data from both phase encoding directions can be combined into a single formulation by setting Y = [Y_up Y_down] T and E = [E_up E_down] T in

\[
Y = E(\Delta B_0) x
\]

The proposed joint image and B₀ field estimation framework can be summarized as a minimization problem with optimization over both image and B₀ field [11,15]:

\[
\arg \min_{x, \Delta B_0} \Psi(x, \Delta B_0)
\]

where \(\Psi(x, \Delta B_0)\) = \(\| Y - E(\Delta B_0) x \|^2 + \beta_1 R(x) + \beta_2 R(\Delta B_0)\)

R(x) and R(ΔB₀) are quadratic regularization terms \(\| D x \|^2\) and \(\| D \Delta B_0 \|^2\) respectively, D is the first order finite difference operator that computes derivatives in all in-plane spatial dimensions; \(\beta_1, \beta_2\) are regularization weights that provide a balance between resolution and noise.
in the reconstructed image and B₀ field, respectively; ||.||² denotes the l₂ norm.

The above joint image and B₀ field problem can be solved iteratively
using a two stage alternating minimization scheme [11,15,16] that
splits the joint problem into two sub-problems. In the first stage within
each iteration, the image update is estimated using a previous field
estimate. In the second stage, we minimize over the B₀ field using the
estimated image from the first stage. Mathematically, the image update
xₖ in the kth iteration (k = 1, 2, ..., K) is estimated using a previous field
estimate ΔB₀(k-1) as:

\[ xₖ = \text{argmin}_{x} \| Y - E(\Delta B₀(k-1))x \|^2 + \beta \| R(x) \| \]  

(9)

Using estimate \( xₖ \), the updated B₀ field in the kth iteration ΔB₀ₖ is
estimated as:

\[ \Delta B₀ₖ = \text{argmin}_{\Delta B₀} \| Y - E(\Delta B₀)x \|^2 + \beta \| R(\Delta B₀) \| \]  

(10)

By using the assumption that the dynamic changes between the
current and updated estimates of the B₀ field are small, Eq. (10) can be
linearized using first order Taylor series expansion [11] to formulate it
in a similar form as Eq. (9). The details of linearization procedure are
given in Appendix I. The standard Conjugate Gradient (CG) method can
then be used to solve both sub problems in Eqs. (9) and (10).

The convergence of the CG iterations is achieved when either the
maximum number of iterations is reached or the normalized residual

\[ r = \frac{\| Y - E(\Delta B₀(k-1))x \|}{\| Y \|} \]  

(11)

denotes the l₂ norm) in the current iteration be-
comes smaller than ε (ε being a small number). The B₀ field in the final
Kth iteration (ΔB₀ₖ) is then used in the subsequent phase correction
and model based reconstruction steps for data acquired at b-value > 0.

Step 2) Phase correction: For diffusion weighted data (b-value > 0 s/
mm²), a phase correction has to be performed because even
small physiological motion occurring during diffusion sensitiza-
tion gradients can cause phase changes that may lead to
signal cancellation when data from opposite phase encoding
gradient directions is combined, leading to signal drop out in the
final reconstructed images.

Using the corrected B₀ field (ΔB₀ₖ) found in Step 1 of the pro-
fessed framework, the model based reconstructions xₕ and xₕₖ of the
separate blip-up and blip-down phase encoding gradient direction scans are
given as:

\[ xₕ = \text{argmin}_{x} \| Yₕ - Eₕ(\Delta B₀ₖ)\| x \|^2 + \beta \| R(x) \| \]  

(11)

\[ xₕₖ = \text{argmin}_{x} \| Yₕₖ - Eₕₖ(\Delta B₀ₖ)\| x \|^2 + \beta \| R(x) \| \]  

(12)

With k-space data Yₕ as reference, Yₕₖ is corrected with the phase
correction ΔΦ obtained by taking the Hermitian inner product between
the two reconstructions xₕ and xₕₖ [7].

Step 3) Model based reconstruction

Using the corrected ΔB₀ₖ, the phase corrected Diffusion weighted
data \( Yₕ \) and \( Yₕₖ \) from blip-up and blip-down phase encoding gradient
directions can be combined into a single formulation \( Y = E(ΔB₀ₖ)\) \( x \) by setting

\[ Y = [Yₕ, Yₕₖ]^T \text{ and } E(ΔB₀ₖ) = [Eₕ(ΔB₀ₖ), Eₕₖ(ΔB₀ₖ)]^T \]

Model based reconstruction using the corrected ΔB₀ₖ is per-
formed on k-space data \( Y \) by solving the following minimization pro-
blem [7]:

\[ \text{argmin}_{x} \| Y - E(ΔB₀ₖ)\| x \|^2 + \beta \| R(x) \| \]  

(13)

The above reconstruction from diffusion weighted data is performed
for each b-value and diffusion direction.

2.1. Experiments

Ten male patients (median weight 83 (range: 68–98) kg and age 73
(57–94) years old were recruited from the clinical prostate imaging
pathway and were consented for additional image acquisitions. The
study was approved by the local Ethics committee and written signed
consent was obtained from all patients for the research scans. Patients
were placed feet first into the scanner. No antispasmodic agent was
administered.

Scanning was performed on a 3T scanner (Achieva, Philips
Healthcare) equipped with 16 anterior +16 posterior channel receive
coil array. Single shot EPI data in both blip-up and blip-down phase
encoding directions were acquired with a SENSE factor of 2. The EPI
scans had the following parameters: resolution = 2 × 2 × 4 mm³,
FOV = 180–220 × 180–220 × 55–90 mm³, partial Fourier acquisition
with half scan factor of 0.75, TE/TR = 55 ms/2000 ms, phase encoding
direction = Anterior-Posterior (AP) axis with fat shift in the direction
‘P’ for blip-up and direction ‘A’ for blip-down scans, b-values = 0 and
500 s/mm², number of isotropic diffusion directions (3 for b-value of
500 s/mm²), number of averages (1 and 3 for b-value of 0 and 500 s/
mm², respectively), phase-encode bandwidth per pixel = 20.6–21.5 Hz/
pixel, scan time = 40 s (for both blip-up and blip-down scans). For re-
ference, axial T₂ weighted images were acquired using a turbo spin
echo scan with the following parameters: resolution = 2×2×4 mm³,
FOV = 180–220 × 180–220 × 55–90 mm³, SENSE acceleration
factor = 2, TE/TR = 100 ms/4700 ms, scan time = 40 s. For calcula-
tion of the B₀ field, a separate 3D dual echo gradient echo scan was
acquired at the end of scanning session with the following parameters:
resolution = 2 × 2 × 2 mm³,
FOV = 200–250 × 200–250 × 70–120 mm³, flip angle = 6°, right
to left phase encoding direction, SENSE acceleration factor = 1, echo time
difference ΔTE = 2.3 ms, TE1/TE2/TR = 4.6 ms/6.9 ms/8.7 ms, scan
time = 1 min. Volume shimming was performed in all scans to cover
the whole prostate and surrounding areas. To keep the same shimming
from one scan to the next, SAMEPREP option was selected in the
scanner software that forces to use the same preparation data for all the
scans. In our scans, the patients were asked to lie still and no breathing
management was used.

2.2. Data post processing

To save the raw data together with the relevant information needed
for the reconstruction framework, a software patch was implemented
using ReconFrame software (Gyrotools Zurich, CH). EPI phase cor-
correction was performed using the ReconFrame tool to correct for ghosts
originating from the opposing directions of alternate readouts.
Subsequent post processing was implemented in MATLAB. The coil
sensitivities were calculated using ReconFrame tool by dividing in-
dividual coil images by the body coil images followed by smoothing and
extrapolation as proposed in SENSE paper [17]. The measured B₀ field
was processed using the quantitative susceptibility mapping toolbox
[18] that estimates a B₀ field map by a weighted least squares fit of
temporally unwrapped phases in each voxel over echo time. A robust
spline based smoothing [19] was applied to the B₀ field map in the
image domain to smooth out noisy components. The volumes from B₀
field map and T₂-weighted images were resampled to match the EPI

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scan resolution and the associated field of view. The resampled B₀ field map was further refined by compensating for any frequency offset that may exist due to scanner frequency drift or coordinate shift in scanner software [7].

2.3. Joint image and B₀ field estimation

For joint image and B₀ field estimation, we alternated 30 times between updating the image and then updating the field map. In each update of image or B₀ field, 50 sub iterations of CG method were performed. For the CG method, the matrix vector multiplications used for computing the gradients in CG were performed efficiently with time segmentation and FFT algorithms available in Fessler’s Image Reconstruction Toolbox [20]. To balance between resolution and noise amplification, the regularization parameters β₁ and β₂ in Eqs. (9) and (10) were chosen to achieve a specific target spatial resolution [21,22], such that the Full Width at Half Maximum (FWHM) of Point Spread Function (PSF) was 1.01 pixels for image estimation and 1.2 pixels for B₀ field estimation, respectively. The computation of PSF was done by using quadratically penalized shift invariant least squares method as implemented in function qpwls.psf.m of Fessler’s Image Reconstruction Toolbox [20].

To investigate the dependence of the proposed framework on the initial B₀ field estimate, the proposed joint B₀ and image reconstructions were performed both without an initial estimate (i.e. initial B₀ field as zero) and with an initial estimate set to measured B₀ field. The joint reconstructions were compared against the uncorrected reconstruction without B₀ field (setting ΔB₀ to 0 in Eq. (9) and model based distortion corrected reconstruction using a measured single B₀ field map. The same value for the regularization parameter β₁ was used both for joint and uncoupled reconstructions.

For qualitative assessment, the images reconstructed with different methods were presented in a random order and blinded to the reconstruction method to avoid subjective assessment. A radiologist with 25 years of experience scored each image in terms of overall image quality on a 5-point scale (1: poor, 2: below average, 3: average, 4: above average, 5: excellent) [23].

3. Results

For patient 1, results for the proposed joint reconstruction without an initial B₀ field estimate are shown in Fig. 1 for data acquired at b-values of 0 and 500 s/mm². Without distortion correction, the variation in B₀ at the prostate rectal air interface resulted in significant artifacts.
| T2W | Uncorrected (no B₀ map) | Distortion corrected (measured B₀ map) | Joint (initial B₀ zero) | Joint (initial B₀ from map) |
|-----|--------------------------|--------------------------------------|------------------------|--------------------------|
| P1  |                          |                                      |                        |                          |
| P2  |                          |                                      |                        |                          |
| P3  |                          |                                      |                        |                          |
| P4  |                          |                                      |                        |                          |
| P5  |                          |                                      |                        |                          |
| P6  |                          |                                      |                        |                          |
| P7  |                          |                                      |                        |                          |
| P8  |                          |                                      |                        |                          |
| P9  |                          |                                      |                        |                          |
| P10 |                          |                                      |                        |                          |

(caption on next page)
The proposed framework was able to correct for most of the distortion artifacts in the final reconstructed images even in cases of starting with a zero initial B0 field estimate (Fig. 1c).

For ten patients, the reconstruction results are shown for b-values of 0 and 500 s/mm² in Figs. 2 and 3, respectively. For patient 2 and patient 5, significant errors in the measured B0 field map resulted in imperfect distortion corrections in the model based reconstruction. With erroneous B0 field map as an initial estimate, the proposed joint reconstruction framework was able to correct for those imperfections. There was no obvious bulk motion observed in any of the patient prostate scans.

A comparison of measured B0 field maps and the B0 field maps estimated using the proposed framework is shown in Fig. 4 for selected patients to include both cases a) when measured B0 is accurate and results in reasonable distortion correction (patient 8), and b) when measured B0 has significant errors that result in imperfect distortion correction (patient 2 and patient 5) and joint reconstruction helps to correct these errors.

A summary of image quality scores for different reconstruction methods corresponding to b-value of 0 s/mm² and b-value of 500 s/mm² are given in Fig. 5a and b, respectively. In the uncorrected reconstruction, the image quality scores were 1.90 ± 0.99 and 1.60 ± 0.51. When no B0 field map was available, the proposed joint reconstruction was able to achieve significant improvement in image quality scores (3.00 ± 0.67 and 2.90 ± 0.57). When a B0 field map is available, the joint reconstruction still leads to an improvement in the average image quality score (3.50 ± 0.85 and 3.40 ± 0.51 versus 3.40 ± 0.70 and 3.30 ± 0.67 for model based reconstruction).

4. Discussion

A novel joint reconstruction framework is proposed that combines the strength of model based reconstruction [7] with dynamic B0 field estimation [11]. The power of complementary encoding information available from both blip-up and blip-down directions enables the correction of both signal pile-up and signal stretching artifacts within the diffusion EPI images. The joint B0 field and image estimation method used in our proposed framework allows for compensation of the inaccuracies in the B0 field due to motion or physiological changes near the prostate-rectal interface between the EPI and B0 scans. Furthermore, in the case of not having an initial B0 field estimate, by alternating between the stages of B0 field update and image update, the proposed method is able to correct the distortions in most of the patients. This is the main advantage of the proposed method as it eliminates the need of a separate B0 scan. In cases where a B0 scan is available, starting with the measured B0 field map, the proposed method performs better than all the other reconstructions. Thus, the proposed framework can be beneficial either when a B0 field is performed or in cases where a B0 scan cannot be performed due to time constraints. Our proposed method estimates the changes in the B0 field as a single update that occurs between B0 scan and one set (blip-up and blip-down) of EPI scans. In case of having multiple sets of b-value = 0 s/mm² EPI data acquired in an interleaved manner (for example, in DWI applications requiring long scans due to many b-values, diffusion directions and number of averages, Diffusion Tensor Imaging, functional MRI etc.), the proposed joint reconstruction could be used to estimate corrected B0 fields and corrected EPI images corresponding to each EPI data set.

The proposed method may be further improved by having slightly different echo times for the two EPI scans such that centre of k-space in the two EPI scans is sampled at two different echo times [10,11]. For gradient echo scans, this time shift has been shown to facilitate better initial B0 field estimation that can help for better convergence of joint estimation. For the spin echo EPI sequences used in our framework, a time shift may be achieved by shifting the timing of the refocusing pulse using pulse sequence development tools. In our proposed framework, the joint B0 and image reconstruction was done only for EPI data acquired at b-value of 0 s/mm². Reconstructing images jointly for all b-values and one B0 map may result in better reconstruction performance at the expense of increased computational complexity.

The proposed framework used computationally efficient quadratic regularization with first order differences. This results in slightly blurred reconstructions in both image and B0 field updates. Better regularizers such as l1 norm based minimization [24] may achieve better results. In our framework, standard CG based optimization was used to solve the sub problems in the joint reconstruction. In future, the CG based optimization might be replaced with alternatives such as quasi Newton methods that ensure the global cost function decreases.

Our proposed reconstruction method solves the full inverse problem constrained to the acquired original raw k-space data rather than the scanner reconstructed images. Image based distortion correction methods (such as Topup method [11]) are designed for practical cases where the imaging community do not have access to k-space data and only the scanner reconstructed distorted images are available. The Topup method reconstructs undistorted images based on image registration based optimization. This may result in an erroneous calculation of the B0 field attributed to the lack of a unique solution between the corresponding locations in the blip-up and blip-down images especially in regions with severe signal pileup, leading to artifacts or blurring in the final images [3]. Moreover, the Topup reconstructed images may also contain the bias from noise when magnitude images are combined [25–28]. Our proposed reconstruction avoids this bias due to complex summation/averaging done implicitly via the encoding operators. This is likely to be beneficial for high b-value DWI images with low SNR.

The proposed framework can correct for changes or errors in the B0 field due to any potential origin such as scanner frequency drift or motion etc. However, it does not correct for physiological motion effects that may occur between the EPI blip-up and blip-down scans. In some cases, modification to the B0 field might compensate for the motion effects, for example, a rigid translational shift in the PE direction can be compensated by an offset correction in the B0 field. Integrating the framework with motion field estimation and correction [29,30] may further make the method robust against both distortion and motion corruption effects.

Distortion correction in prostate diffusion MRI is challenging in some patients due to erroneous measured B0 field that is used to compensate for off-resonance effects. The proposed framework improves the overall image quality by combining the strength of model based reconstruction with dynamic B0 field estimation. Validation of proposed framework was performed successfully in ten clinical patients. The proposed framework offers a potential to improve the diagnostic value of prostate images for tumour detection in diffusion weighted imaging - a technique that is now commonly used for detecting prostate cancer.
Fig. 3. In-vivo reconstruction results for 10 patients (P1 to P10) for Diffusion Weighted data acquired at b-value of 500 s/mm². The reference T2-weighted image (left column), uncorrected reconstruction (no $B_0$ map), distortion corrected reconstruction with measured $B_0$ field map (measured $B_0$ map), proposed model based joint reconstruction results without initial $B_0$ estimate (initial $B_0$ zero) and with initial $B_0$ estimate set to measured $B_0$ field map (initial $B_0$ from map) are shown.
Fig. 4. Example $B_0$ fields. The reference T2W image and measured $B_0$ field map are shown in the first and second columns. The $B_0$ fields obtained from the joint reconstruction framework without initial $B_0$ estimate (initial $B_0$ zero) and with initial $B_0$ estimate set to measured $B_0$ field map (initial $B_0$ from map) are shown in third and fourth columns, respectively. In Patients 2 and 5, the jointly estimated $B_0$ fields yield improved reconstructions (see Figs. 2 and 3 for details). In Patient 8, the joint reconstruction without initial $B_0$ estimate (initial $B_0$ zero) was of inferior quality compared to the other reconstructions.

Fig. 5. Image quality assessment of proposed framework: Bar plots showing average expert scores for overall image quality (1: poor to 5: excellent) for different reconstruction methods (uncorrected reconstruction (No $B_0$ map), Model based distortion corrected reconstruction using measured $B_0$ field map (measured $B_0$), joint reconstruction with initial $B_0$ field set to zero (initial $B_0$ zero) and joint reconstruction with initial $B_0$ field set to measured $B_0$ map (initial $B_0$ from map) are shown in (a) and (b) for b-value of 0 s/mm$^2$ and b-value of 500 s/mm$^2$, respectively. The associated standard deviations are also indicated. The proposed joint reconstruction with measured $B_0$ field map as an initial estimate performed better on average than all other reconstructions.
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Appendix A. Linear approximation to dynamic changes in \( B_0 \) field

Let \( \omega = 2 \pi B_0 \), Eq. (1) is rewritten as:

\[
Y'(k(t),t) = \sum_{n=0}^{N-1} C'(t_n)X(t_n)e^{i(2\pi(k(t_n)\omega) + \omega t_n)} \tag{A1}
\]

The above Eq. (A1) is updated to include the dynamic change in \( \omega \) as:

\[
Y'(k(t),t) = \sum_{n=0}^{N-1} C'(t_n)X(t_n)e^{i(2\pi(k(t_n)\omega(t)) + \omega(t) t_n)} \tag{A2}
\]

where \( \hat{\omega} \) is the reference or initial estimate of the \( B_0 \) field.

If the difference between \( \omega \) and \( \hat{\omega} \) is small, we can use first order Taylor approximation:

\[
\omega - i'\left(\omega(t) - \hat{\omega}(t)\right)t \approx 1 - i(\omega(t) - \hat{\omega}(t))t
\]

Eq. (A2) becomes:

\[
Y'(k(t),t) = \sum_{n=0}^{N-1} C'(t_n)X(t_n)e^{i(2\pi(k(t_n)\omega(t)) - i(\omega(t) - \hat{\omega}(t))t) + i(-t)} \sum_{n=0}^{N-1} C'(t_n)X(t_n)e^{i(2\pi(k(t_n)\omega(t)) - i(\omega(t) - \hat{\omega}(t))t)} \hat{\omega}(t) + i(-t) \sum_{n=0}^{N-1} C'(t_n)X(t_n)e^{i(2\pi(k(t_n)\omega(t)) - i(\omega(t) - \hat{\omega}(t))t)} \omega(t) \tag{A3}
\]

For \( M \) number of data points in k-space acquired at sampling time points \( t_1, t_2, \ldots, t_M \) Eq. (A3) can be written in matrix vector form as:

\[
Y = A(\hat{\omega},C)x - B(\hat{\omega},x,C)x + B(\hat{\omega},x,C)x + B(\hat{\omega},x,C)x \tag{A4}
\]

where \( A(\hat{\omega},C) \) and \( B(\hat{\omega},x,C) \) are \( M \times N \) matrices with elements

\[
a_{m,n} = C'(t_n)e^{-i(2\pi(k(t_n)\omega(t) + \omega(t) t_n)} \tag{A5}
\]

and

\[
b_{m,n} = i(-t)C'(t_n)e^{-i(2\pi(k(t_n)\omega(t) + \omega(t) t_n)} \tag{A6}
\]

By stacking data and encoding operators from all \( J \) coils and both blip-up and blip-down phase encoding gradient directions, we can write:

\[
Y = A(\hat{\omega},x) - B(\hat{\omega},x)x + B(\hat{\omega},x)x + B(\hat{\omega},x)x \tag{A7}
\]

Similar to Eq. (8), the equivalent joint minimization problem can be expressed as:

\[
\arg\min_{\omega} ||Y - A(\hat{\omega},x) - B(\hat{\omega},x)x||^2 + \beta_1 |R(x)| + \beta_2 |R(\omega)| \tag{A8}
\]

The above problem can be solved iteratively using two stage alternating minimization scheme. In the first stage, the image estimate \( x^k \) in the \( k^{th} \) iteration (\( k = 1,2,\ldots,K \)) is found by using previous field map estimate \( \omega = \hat{\omega} = \omega^{k-1} \). Thus, Eq. (A6) reduces to:

\[
x^k = \arg\min_{x} ||Y - A(\omega^{k-1})x||^2 + \beta_1 R(x) \tag{A9}
\]

In the second stage, an updated field map estimate \( \omega^k \) in the \( k^{th} \) iteration is found by setting \( x = x^k \) in Eq. (A9):

\[
\omega^k = \arg\min_{\omega} ||Y - A(\omega,x^k) - B(\omega,x^k)x||^2 + \beta_2 R(\omega) \tag{A10}
\]

The above expression is summarized as:

\[
\omega^k = \arg\min_{\omega} ||Y - B(\omega,x^k)||^2 + \beta_2 R(\omega) \tag{A11}
\]

where \( Y = Y - A(\omega,x^k)x + B(\omega,x^k)x \).

Both Eqs. (A7) and (A9) are of similar form and this linear approximation means that both can be solved efficiently using standard Conjugate Gradient methods.

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