Optimum design for permittivity of dielectric absorbing materials

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Abstract. In the design of absorbing materials, investigating the ideal value range of electromagnetic parameters under given conditions and adjusting the electromagnetic parameters by existing materials to achieve the ideal range need are two important issues to be solved. Based on transmission line theory and Debye theory, numerical simulations are used to explore the ideal value range of various parameters of absorbing materials under different conditions as well as their selection laws. The Cole-Cole diagram based on combining theory and experiments was given for optimizing design of absorption performance. When the thickness and reflection attenuation rate of the material are presupposed, the real and imaginary parts of the corresponding ideal permittivity should form an approximately elliptic curve. When the permittivity of the material is in the elliptical region, the reflection attenuation rate is lower than its given value. Ellipses with lower reflection attenuation locate in ellipses with higher reflection attenuation, but the centers of the ellipses do not coincide. As the reflection attenuation rate decreases, the corresponding ellipse area shrinks sharply, and the range of electromagnetic parameters becomes rigorous.

1. Introduction

Research on absorbing functional materials is one of the leading topics in the field of military stealth technology, and its purpose is to minimize or eliminate the target detection capabilities of radar, infrared and other detecting equipments [1, 2]. In the daily treatment of electromagnetic radiation pollution, absorbing materials have also been paid more and more attention [3, 4]. Therefore researches on absorbing materials have far-reaching significance for both military and civilian applications. The absorption performance of a single-layer absorbing composite is mainly determined by its thickness and electromagnetic parameters. And the electromagnetic parameters of composites mainly depend on the electromagnetic parameters of the absorbent and its filling content. Nanoparticles are considered to be excellent absorbents. Although the research on the optimal design of absorbing materials has made some progress [5-10], it is still limited to qualitative description, which cannot meet the actual requirements of absorbing materials, and it is difficult to obtain ideal materials that meet the conditions. Therefore, it is of great research value to explore the quantified expression form of the best electromagnetic parameters of absorbing materials. Here the electromagnetic parameters include permittivity and permeability, the former reflects the polarization degree of dielectric materials under the action of electric field, and the latter reflects the variation rate of the magnetic induction intensity in the magnetic medium with the intensity of external magnetic field.

Two issues need to be solved in the practical application of design results of compound absorbing material. The first one is to make sure the range of the complex permittivity and the complex
permeability when the thickness and absorbing properties of the material are determined. The second one is to choose or adjust the electromagnetic parameters of the material to reach the specified range. In view of these two problems, the spectrum characteristics of dielectric polarization was described in this paper according to the transmission line theory and Debye theory [11-13], and the electromagnetic parameters of pure dielectric absorbing composites were optimized. Besides, the ideal value range of material parameters under different conditions was explored by computer simulation, guiding the optimal design of absorbing materials.

2. Optimum design of permittivity for single layer absorbing material

2.1. Basic absorbing theory

According to the electromagnetic wave transmission theory, when a uniform planar electromagnetic wave at frequency \( f \) vertically injects on the conductor surface coated with a single layer of absorbing material, the power reflectivity of the material to the electromagnetic wave is as follows [6].

\[
R = 20 \lg \left| \frac{Z - Z_0}{Z + Z_0} \right|
\]

where, \( Z \) is the input impedance of the material, \( Z_0 = (\mu_0/\varepsilon_0)^{1/2} = 120\pi \) is the air resistance, \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and magnetic permeability in vacuum, respectively.

\[
Z = Z_r \tanh(\gamma \cdot d) = Z_o (\mu_r/\varepsilon_r)^{1/2} \cdot \tanh j2\pi f (\mu_0\varepsilon_0\mu_r\varepsilon_r)^{1/2} \cdot d
\]

where, \( \mu_r = \mu' - j\mu'' \) is the relative permeability, \( \varepsilon_r = \varepsilon' - j\varepsilon'' \) is the relative permittivity, \( Z_r = Z_o (\mu_r/\varepsilon_r)^{1/2} \) is the characteristic impedance of the material, \( \gamma = j2\pi f (\mu_0\varepsilon_0\mu_r\varepsilon_r)^{1/2} \) is the propagation constant of electromagnetic waves in the material, and \( d \) is the material thickness.

2.2. Optimum design of permittivity for wideband absorbing material

For pure dielectric absorbing materials, interference cancellation is an effective way to achieve the purpose of wide-band absorption. According to the principle of type-absorbing materials, the complex permittivity of the material varies with frequency, which could make that the thickness is an odd multiple of 1/4 wavelength of the electromagnetic wave in the material, so as to obtain the maximum reflection attenuation rate. When an electromagnetic wave propagates in the material, its wavelength \( \lambda \) is as follows [14].

\[
\lambda = \lambda_0 (\mu'_r\varepsilon'_r)^{1/2}
\]

where, \( \lambda_0 \) is the wavelength when electromagnetic waves travel in vacuum. When \( d = (2n+1)\lambda/4 \) is defined, where \( n = 0, 1, 2, 3, \ldots \), the thickness of the material is an odd multiple of 1/4 wavelength in the medium, and the expression is as follows.

\[
\lambda = \frac{4d}{2n+1}
\]

For non-magnetic materials, \( \mu_r = 1 \). Combining formula (3) and (4) gives the following formula.

\[
\varepsilon'_r = \frac{1}{16} \cdot \frac{c^2(2n+1)^2}{d^2 \cdot f^2} = L'_c (d \cdot f)^{-2}
\]
where, $L_n^2 = c^2 (2n+1)^2 / 16$. Here $c$ is the speed of light in vacuum. From above analysis, if a single-layer dielectric absorbing material wants to achieve the purpose of wide-band absorption, the real part of its permittivity must be inversely proportional to $f^2$.

The conditions for interference cancellation of two waves are the same frequency, same vibration direction, opposite phase and equal amplitude. Equation (5) guarantees the opposite phase, the value of the dielectric imaginary part can adjust the amplitudes of the two waves to make them equal, thereby, the maximum reflection attenuation rate is obtained. Given the thickness of the material and the real part of permittivity, if the imaginary part of permittivity is greater than its ideal value, it will cause the primary energy reflection at the interface to be greater than the energy transmitted from the material. Conversely, the primary energy reflection at the interface will be less than the energy transmitted from the material. And the above two values will both weaken the effect of interference cancellation. It is found that $\varepsilon''_r$ needs to be inversely proportional to the product of $f$ and $d$ [5].

$$\varepsilon''_r = L^2_2(d \cdot f)^{-1}$$  \hspace{1cm} (6)

where, $L^2_2$ is related to $L^2_1$. When $n = 0$, $L^2_1 = 5617.3 \text{GHz}^2 \cdot \text{mm}^2$, $L^2_2 = 95.28 \text{GHz} \cdot \text{mm}$.

Therefore, in order to achieve the objective of no reflection, the permittivity of the material needs to be inversely proportional to the frequency of the incident electromagnetic wave, and to possess obvious frequency dispersion characteristics. To achieve the same absorbing effect, the corresponding permittivity in the low frequency band should be higher than the corresponding one in the high frequency band. Permittivity should be inversely proportional to the thickness of the material. Therefore, properly increasing the thickness of the coating could correspondingly reduce the requirements for the permittivity of the material. There are various combinations of the values of electromagnetic parameters for single-layer materials without reflection, which also provides a theoretical basis for selecting materials in a wide range [15].

According to expressions (5) and (6), when $L^2_1 = 5617.3 \text{GHz}^2 \cdot \text{mm}^2$ and $L^2_0 = 95.28 \text{GHz} \cdot \text{mm}$, the values of the ideal electromagnetic parameters were calculated in the frequency range of 2 to 18GHz. The results are shown in Table 1.

| $f$ / GHz | $d$=1mm | | | $d$=2mm | |
|---|---|---|---|---|---|
| $\varepsilon'_r$ | $\varepsilon''_r$ | $R$ / dB | $\varepsilon'_r$ | $\varepsilon''_r$ | $R$ / dB |
| 2 | 1404.3 | 47.6 | -47.4 | 351.08 | 23.82 | -41.4 |
| 6 | 1560 | 15.9 | -37.9 | 39.01 | 7.94 | -31.9 |
| 8 | 878 | 11.9 | -35.4 | 21.94 | 5.95 | -29.4 |
| 10 | 562 | 9.5 | -33.5 | 14.04 | 4.76 | -27.5 |
| 12 | 390 | 7.9 | -31.9 | 9.75 | 3.97 | -25.9 |
| 18 | 173 | 5.3 | -28.4 | 4.33 | 2.65 | -22.4 |

The absorption performance of the materials in the table reaches 99%. As can be seen from table 1, in the 2GHz - 6GHz frequency band, both the real and imaginary parts of the permittivity decline rapidly with the increase of frequency. While in the 6GHz - 18GHz band, the decline trend of the real and imaginary parts of the permittivity is relatively gentle, and the difference between the two parts gradually decreases. Numerical analysis shows that materials with thin thickness have higher requirements on permittivity to achieve the same absorption capability.
For real materials, the electromagnetic parameters can hardly meet the requirements of equations (5) - (7). Some materials may meet the frequency matching at a certain point, so as to obtain a good attenuation effect, while the effects at other frequency points may be difficult to meet the application requirements. Nevertheless, the above research results still have important applications in the frequency point optimization design of absorbing materials. The above research gives the value of the ideal permittivity without reflection, but does not solve a common problem in practical application. The problem is how to get the value range of complex permittivity and permeability of materials under the premise of given thickness and absorbing performance. In the following section, by setting parameters such as material thickness and reflection attenuation rate in advance, the value range of ideal permittivity satisfying the presupposed conditions at a certain incident frequency is solved.

3. Optimum calculation of complex permittivity

3.1. Value range of ideal complex permittivity

For single-layer uniform absorbing materials, the permittivity of high frequency band slightly deviates from the ideal value. In order to completely realize zero reflection, it is necessary to numerically solve equation (2). For pure dielectric loss absorbing materials, formula (2) can be simplified as follows.

\[
\tanh \left[ j2\pi f d \left( \epsilon'_r - j\epsilon''_r \right) \right]^{1/2} \cdot e^{-1} = \left( \epsilon'_r - j\epsilon''_r \right)^{1/2}
\]

(7)

Through the numerical solution for formula (7), the Cole-Cole diagram of complex permittivity \( \epsilon'_r, \epsilon''_r \) and \( d \) at the frequency \( f \) is obtained, as shown in Figure 1. When \( \epsilon'_r, \epsilon''_r \) and \( d \) is corresponding to this curve, the material can achieve zero reflection. It can be seen from Figure 1(a) that when the thickness of the material increases from 1.5mm to 4.0mm, the real part of the complex permittivity decreases from 17.74 to 2.84 and the imaginary part decreases from 5.28 to 1.93, indicating that the real part of the permittivity has a greater impact on the thickness of the material than the imaginary part. Figure 1(b) exhibits that the values of \( \epsilon'_r - \epsilon''_r \) calculated at different frequencies and different thicknesses are on the same curve. According to \( f \) and \( d \), the corresponding electromagnetic parameters can be selected to obtain the absorption effect without reflection.

![Figure 1. Cole-Cole diagram of single-layer non-reflective absorbing material.](image-url)
absorbing material is selected as the research object, and the ideal value range of the complex permittivity is solved to guide the optimization of absorbing materials.

The influence of reflection attenuation rate and material thickness \( d \) on the value of electromagnetic parameters was analyzed. The incident wave frequency was set as 8 GHz, and the material thickness was 2mm and 4mm, respectively. The value range of permittivity corresponding to different reflection attenuation rates were calculated, as shown in figure 2. When the reflection attenuation rate is determined as \( A \), the corresponding complex permittivity forms a curve that approximates an ellipse. When the permittivity of the material is in the ellipse region, the reflection attenuation rate is lower than \( A \). With the decrease of the reflection attenuation rate, the corresponding ellipse area shrinks sharply, and the range of permittivity becomes harsher. Ellipses with lower reflection attenuation locate in ellipses with higher reflection attenuation, but the centers of the ellipses do not coincide. As can be seen from the figure, zero reflection could be achieved when the ellipse is reduced to a point and the permittivity of the material is exactly equal to the value corresponding to the point.

By comparing Figures 2(a) and 2(b), it can be seen that the value range of the ideal permittivity gradually decreases with the thickness of the material increasing. This also explains why some electrical loss materials with thin thickness achieve poor absorption effect in the low-frequency band, because the permittivity of the material is difficult to reach the required permittivity value when the thickness is too thin.

![Figure 2](image.png)

**Figure 2.** Influence of material thickness on the value of complex permittivity.

The influence of the frequency of the incident wave on the value of permittivity is analyzed below. The thickness of the material was set as 2mm, and the frequency of the incident wave was 6GHz, 10GHz, 12GHz and 18GHz, respectively. The values of permittivity corresponding to different reflection attenuation rates were calculated, as shown in figure 3.
As shown in the figure 3, while the frequency of the incident electromagnetic wave increases, the value region of the ideal permittivity gradually decreases. As illustrated, the value of the ideal permittivity is relatively large in the low frequency band, and the permittivity of many dielectric materials cannot meet the requirements in this frequency band. And in the high frequency band, the value of the ideal permittivity is easier to meet the requirements. It could be concluded that for thin layer dielectric loss materials, it is easy to obtain satisfactory absorbing performance in the high frequency band.

3.2. Optimization method for absorbing performance

Optimization of absorbing performance is to enable the material to achieve excellent absorption in wide frequency band at given thickness. For dielectric single-layer absorbing materials, it is generally difficult to meet these two requirements at the same time. The Cole-Cole diagram based on combining theory and experiments was given for optimizing design of absorption performance.

3.2.1. Optimization of frequency point. The complex permittivity of the absorbent nano-silicon carbide (SiC) and nano-carbon black (CB) is measured by using a vector network analyzer in the frequency range of 2GHz to 18GHz, and the clamp is a coaxial transmission/reflection system. The nano-absorbent is mixed with dissolved paraffin wax and then made into a circular coaxial shape with a 7.0mm outer diameter, a 3.0mm inner diameter and a length of 2.0mm ~ 5.0mm. Figure 4 shows the permittivity of SiC/paraffin matrix and CB/paraffin matrix composite with different filling.
concentration. For single-layer dielectric absorbing materials, the difference between the growth rate of the real and imaginary parts of complex permittivity has a decisive influence on the matching thickness of the composite. By utilizing the Cole-Cole diagram calculated theoretically and combining with the measured permittivity, the content and matching thickness of these two nanometer absorbers could be found out to obtain optimal capability.

Figure 5 is the Cole-Cole diagram of the complex permittivity of the above absorbent at 8 GHz and 12 GHz, which exhibits the measured and calculated permittivity. The figures present the values of the ideal permittivity at the frequency of 8 GHz and 12 GHz to achieve non-reflection under different thickness conditions. Then the complex permittivity of the two composites obtained by experiments was also inserted into the figure. The intersection point between the experimental value curve and the theoretical value curve is the optimal matching thickness and concentration of the composite material at 8GHz and 12GHz, and the corresponding real and imaginary parts are the ideal values of complex permittivity. This figure has important guiding significance for the design of dielectric absorbing materials.

3.2.2 Optimization of frequency band. The preconditions for optimization are set as follows. The thickness of material is not greater than the given limit value. The bandwidth of absorbing performance that is better than -10dB should be as wide as possible, and the maximum absorption peak is better than -20dB. The optimization steps are as follows.

The first step is to seek the equivalent permittivity of the material according to the preconditions and figure 2 and figure 3, and set it as the initial value for calculation.

The second step is to calculate the content of the absorbent in the material according to the equivalent permittivity of and appropriate equivalent electromagnetic parameter calculation formula. If the content value is reasonable, the calculation result is retained for comparison with other optimization solutions. Otherwise, the result should be discarded.

The third step is to compare the obtained optimization schemes and select the best one for examination.

4. Conclusions
Based on the transmission line theory and Debye theory, the spectrum characteristics of electromagnetic parameters of dielectric absorbing materials were calculated and analyzed. The optimum value range and adjustment method of electromagnetic parameters were put forward. Besides, the optimal design method of absorbing performance, which is based on the Cole-Cole diagram combining theory and experiment, is proposed. The absorbing principle of a single-layer pure dielectric absorbing coating belongs to interference type. Under the condition of given thickness of absorbing material, when the real part of permittivity is inversely proportional to the square of the
frequency of the incident wave, and the imaginary part of permittivity with an appropriate value is inversely proportional to the frequency, the material could achieve the goal of non-reflection in the entire frequency band. When the thickness and reflection attenuation rate of the material are presupposed, the real and imaginary parts of the corresponding ideal permittivity should form an approximately elliptic curve. When the permittivity of the material is in the elliptical region, the reflection attenuation rate is lower than its given value. The ellipse with lower reflection attenuation locates in the ellipse with higher reflection attenuation, but the centers of the ellipses do not coincide. As the reflection attenuation rate decreases, the corresponding ellipse area shrinks sharply, and the range of electromagnetic parameters becomes harsher.

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