This study considers seven commonly used surface fitting methods within Golden Software and ArcGIS™ environments. Using grid sizes of 68 rows by 100 columns (6800 grids) and 680 rows by 1000 columns (680,000 grids) and 294,208 elevation points covering the entire landmass of Nigeria, the study evaluates the performance of these methods in terms of execution time and faithfulness in the representation of the spatial elements. Results show marked differences in time taken to execute the fitting and that Inverse Distance, the Natural Neighbor, the Nearest Neighbor, and Triangulation with Linear Interpolation seem to give the highest level of correspondence or faithfulness.

Keywords: image surface fitting; digital elevation modelling; GIS; cartography

1. Introduction

The contour map remains one of the most useful maps to a geographer but also one that has always posed some of the greatest challenges to the cartographer. It is a type of quantitative thematic map, an isarithmic map the origin of which dates back to the mid-sixteenth century, when isobaths were first charted (1, 2). The contour map typifies all isarithm maps such as isobar, isotherm, isohypse (contour), isohyets, isochrones and isodapanes. Isarithmic map involves mapping a real or conceptual three-dimensional geographical volume with quantitative line symbols. Today, two forms of isarithm maps are isometric and isoplethic. Each involves the planimetric mapping of traces of intersections of horizontal planes with a three-dimensional surface. The isarithm is placed by “threading” it through a series of control points (see (1) for the list of Isarithmic Maps).

The increasing use of these types of maps in geography and many of the spatial sciences including geology, mining, and civil engineering, among others is due in part to the use of the computer and mapping software to compose the maps. Hitherto, these types of maps were notorious for the difficulty of compilation and interpretation (1–3). Geographic Information Systems (GIS) in more recent times have developed robust techniques to facilitate not only the compilation of data for, and the computation of these isarithms but also have given the interpretations of the isarithms new meaning and new uses.

Like many things in cartography, automation and digital mapping has affected the production methods for isarithms. In the GIS literature, methods for determining the isolines are called Digital Elevation Models (DEM). However because these methods are based on the transformation of an initial set of points into grids for constructing the isarithms, they are also called gridding methods. In this study, gridding models are described and investigated for the purposes of comparison between one another, and for determining which of them is appropriate for what situation. While many theoretical statements have been made about the potency of the various methods, users are oftentimes left to infer which methods to use. Furthermore, given the nature of the object of investigation (isarithmic map), there are often few avenues for comparing models results with reality thus making empirical investigation difficult. More recently software has been developed to facilitate computation and evaluation processes. We shall therefore utilize some of the existing software and employ real life data obtained from one of our studies on DEM of Nigeria to demonstrate the potency of these models on the one hand and the extent to which they depict the topographical surface of Nigeria on the other hand. It is believed this can lead to a re-examination of some of the assumptions of these models. It should also lead to the formation of simple rules about which of the methods can be relied upon and under what circumstances.

2. Gridding methods

Isarithmic maps are three-dimensional spatial models of surface elements. A surface is a continuous field of
values that may vary over an infinite number of points. Raster surface data represents a surface as a grid of equally sized cells that contain the attribute values for representing the z-value and the x,y location coordinates. Surface interpolation functions create a continuous (or prediction) surface from sampled point values by making predictions from sample measurements for all locations in a raster data set, whether a measurement has been taken at the location or not. Gridding fills in these holes by extrapolating or interpolating z values at those locations where no data exists. The variety of methods to derive a prediction for each location is referred to as a model. For each model, there are different assumptions made of the data so that each model produces predictions using different calculations. In addition, some methods are better than others in preserving data.

Furthermore, some models are referred to as deterministic interpolation methods because they assign values to locations based on the surrounding measured values and on specified mathematical formulas that determine the smoothness of the resulting surface. A second family of interpolation methods consists of geostatistical methods, which are based on statistical models that include autocorrelation (the statistical relationship among the measured points). Because of this factor, not only do geostatistical techniques have the capability to produce a prediction surface, but they can also provide some measure of the certainty or accuracy of these predictions.

Different GIS software includes varying numbers of gridding methods. ArcGIS™ emphasizes four major ones namely Inverse Distance Weighting, Kriging, Natural Neighbor, and Spline (4) as methods for interpolation. Surfer™ from Golden Software describes more methods than any other software on the market (5) and includes Inverse Distance to a Power, Kriging, Minimum Curvature, Modified Shepard’s Method, Natural Neighbor, Nearest Neighbor, Polynomial Regression, Radial Basis Function, Triangulation with Linear Interpolation, Moving Average, Data Metrics, and Local Polynomial.

In this study, we examine the following methods, Inverse Distance to a power, Kriging, Minimum curvature, Modified Shepard’s Method, Natural Neighbor, Nearest Neighbor, Triangulation with Linear interpolation.

2.1. Inverse distance to a power

Inverse Distance to a Power is a quick exact deterministic interpolator. There are no assumptions required of the data and so there are very few decisions to make regarding model parameters. The Inverse Distance to a Power is a weighted average interpolator. It can be either an exact or a smoothing interpolator. With Inverse Distance to a Power, data are weighted during interpolation such that the influence of one point relative to another declines with distance from the grid node (6). Weighting is assigned to data through the use of a weighting power that controls how the weighting factors drop off as distance from a grid node increases. The greater the weighting power, the less effect points far from the grid node has during interpolation. As the power increases, the grid node value approaches the value of the nearest point. For a smaller power, the weights are more evenly distributed among the neighboring data points.

One of the characteristics of Inverse Distance to a Power is the generation of “bull’s-eyes” surrounding the position of observations within the gridded area. The “bull’s-eye” can however be smoothened by using a smoothing parameter during the interpolation.

2.2. Kriging

Kriging is one of the more flexible methods and is useful for gridding almost any type of data set. The flexibility of Kriging requires lots of decision-making in respect of the choice of the parameters (7), (8). Kriging is a geostatistical gridding method that has proven useful and popular in many fields. Kriging assumes that data come from a stationary stochastic process, and some methods assume normally-distributed data. Kriging can be either an exact or a smoothing interpolator depending on the user-specified parameters. It incorporates anisotropy and underlying trends in an efficient and natural manner (9), (10). Kriging produces visually appealing maps from irregularly spaced data. It attempts to express trends suggested in the data, so that, for example, high points might be connected along a ridge rather than isolated by bull’s-eye type contours. It is very flexible and allows the investigation of graphs for spatial autocorrelation. Kriging uses statistical models that allow a variety of map outputs including predictions, prediction standard errors, probability, and so on (11).

2.3. Minimum curvature

Minimum curvature is widely used in the earth sciences. It generates that is analogous to a thin, linearly elastic plate passing through each of the data values with a minimum amount of bending (12). Minimum Curvature generates the smoothest possible surface while attempting to honour the data as closely as possible. Compatibility Minimum Curvature is not an exact interpolator (13).

This method produces a grid by repeatedly applying an equation over the grid in an attempt to smooth the grid. Each pass over the grid is counted as one iteration (14). The grid node values are recalculated until successive changes in the values are less than the Maximum Residuals value, or until the maximum number of iterations is reached.

2.4. Modified Shepard’s method

Modified Shepard’s method uses an inverse distance weighted least squares method (15), (16), and as such, it is similar to the Inverse Distance to a Power interpolator. Designed to eliminate or reduce the “bull’s-eye” appearance of the contours generated by the IDW.

The Modified Shepard’s Method starts by computing a local least squares fit of a quadratic surface around each observation. The Quadratic Neighbors parameter...
specifies the size of the local neighborhood by specifying the number of local neighbors. The local neighborhood is a circle of sufficient radius to include exactly these many neighbors (17). The interpolated values are generated using a distance-weighted average of the previously computed quadratic fits associated with neighboring observations. The Weighting Neighbors parameter specifies the size of the local neighborhood by specifying the number of local neighbors. The Modified Shepard’s Method can be either an exact or a smoothing interpolator.

2.5. Natural neighbor
The Natural Neighbor gridding method is quite popular in many fields. Natural Neighbor interpolation uses a set of Thiessen polygons (the dual of a Delaunay triangulation) to add a new point (target) to a data set, by modifying the Thiessen polygons, some by shrinking in size, while none would increase in size (18). The area associated with the target’s Thiessen polygon that was taken from an existing polygon is called the “borrowed area.” The Natural Neighbor interpolation algorithm uses a weighted average of the neighboring observations, where the weights are proportional to the “borrowed area” (19).

2.6. Nearest neighbor
The Nearest Neighbor gridding method assigns the value of the nearest point to each grid node. This method is useful when data are already evenly spaced, but need to be converted to a grid file. Alternatively, in cases where the data are nearly on a grid with only a few missing values, this method is effective for filling in the holes in the data (20). The method can be particularly useful when some grid values have to be extracted from a data file. In this case, one can set the Search Ellipse to a value so the areas of no data are assigned the blanking value in the grid file. By setting the search ellipse radii to values less than the distance between data values in the file, the blanking value is assigned at all grid nodes where data values do not exist.

2.7. Triangulation with linear interpolation
Triangulation with Linear Interpolation is an exact interpolator method that uses the optimal Delaunay triangulation (21), (22) to create triangles by drawing lines between data points. The original points are connected in such a way that no triangle edges are intersected by other triangles. The result is a patchwork of triangular faces over the extent of the grid.

Each triangle defines a plane over the grid nodes lying within the triangle, with the tilt and elevation of the triangle determined by the three original data points defining the triangle. All grid nodes within a given triangle are defined by the triangular surface. Because the original data are used to define the triangles, the data are honored very closely. Triangulation with Linear Interpolation works best when data sets are evenly distributed over the grid area as data sets that contain sparse areas result in distinct triangular facets on the map.

When small data sets are used Triangulation generates distinct triangular facets between data points (23). One advantage of triangulation is that, with enough data, it can preserve break lines as defined in a data file. For example, if a fault is delimited by enough data points on both sides of the fault line, the grid generated by triangulation will show this discontinuity.

3. Methodology for evaluation of the DEM
The empirical evaluation of the DEM models utilizes three-dimensional \((x, y, z)\) data collected by the authors in a recent project. The data set is a digital elevation database assembled in order to produce a digital terrain map of Nigeria to be used in various forms of analysis such as contouring, hachuring, line of sight studies, viewshed analysis and cut and fill by engineers among others. There could be many ways of assembling such data including land surveys and GPS methods.

The approach used to assemble the three dimensional data set is based on recent developments in remote sensing using facilities provided by high resolution satellite images and GIS software particularly ArcGIS™, KMLer™ and Google Earth™ for extracting heights on such images (4). Google Earth is a virtual globe program that was originally called Earth Viewer, and was created by Keyhole Inc, a company acquired by Google in 2004. It maps the earth by the superimposition of images obtained from satellite imagery, aerial photography and a GIS 3-D globe. Google Earth turns a computer into a window to view high-resolution aerial and satellite imagery, photos, elevation terrain, road and street labels, business listings, and more. Google Earth™ also uses DEM data collected by NASA’s Shuttle Radar Topography Mission (SRTM) to produce 3-D views of features on the earth’s surface.

ESRI’s ArcGIS™ possesses an Extension called KMLer™ that can be used to generate a system of grid-ded coordinates that are immediately useable in ESRI’s ArcGIS™ or Golden Software Surfer™ to produce 3-D surfaces. For this study, the base images were from Quickbird® and Ikonos® high definition satellite images of 2.5 and 0.6 meter resolutions respectively. KMLer™ was used to extract a total of some 294,208 points and their elevations extracted from the images (Figure 1).

The database covers an area of about 1.8 million km\(^2\) (1500 km in the west-east direction and 1200 km in the north-south direction). This is approximately twice the area of Nigeria. The minimum point \(P_{\min}\) is \((2.0529, 3.8389)\) while the maximum \(P_{\max}\) is \((17.2513, 14.1177)\). The spot heights vary from 0.0 m to 2664.19 m (see Table 1). Other statistical properties such as median, mean, standard deviation and coefficients of variation...
and skewness of the x,y,z measurements are given in Table 1.

### Table 1. The characteristics of the area covered for the DEM.

|          | X       | Y       | Z       |
|----------|---------|---------|---------|
| Minimum  | 2.0529  | 3.8389  | 0       |
| Median   | 9.7659  | 9.0476  | 337.6196|
| Maximum  | 17.2513 | 14.1177 | 2664.1895|
| Range    | 15.1984 | 10.2787 | 2664.1895|
| Mean     | 9.7344  | 9.0119  | 398.3967 |
| Standard deviation | 4.2988 | 2.9181  | 282.8001 |
| Coef. of Variation   | –       | –       | 0.7099  |
| Coef. of Skewness    | –       | –       | 1.1902  |

3.1. Comparative analysis of the gridding methods

The strategy for comparing the performance of the various digital elevation models consists of using the Surfer8™ software from Golden Software Inc. to produce the grids for each of the models. Board (3) posited two requirements that are germane when one is examining the qualities of maps as models of reality. These are (i) faithfulness in respect of the gradient between reality and abstraction, and (ii) faithfulness in terms of spatial properties. The real world is so complex that no picture can portray it completely. The map therefore is a model of this complexity and contains only those features that are determined by both the purpose of mapping and the scale of presentation. As abstractions of reality, the more abstract a map is, the farther it is from showing reality. Furthermore, since most isarithmic maps are conceptual, faithfulness in terms of the extent to which the maps represent reality is difficult to test primarily because there is no reality to which comparisons can be made in the first instance. Remember, even the DEM database is an abstraction of reality and not reality in itself. Nevertheless, faithfulness in terms of the representation of the spatial properties remains a major avenue for evaluating the accuracy of the models.

Surfer8™ uses 68 rows by 100 columns (6800 grids) as standard template for generating grids, but it is capable of generating grids at ratios in proportions to this. This research generates grids at the standard template of Surfer8™ to gain insight to what happens both to computation times and accuracy of results.

#### 3.1.1. Speed of computation

Speed of computation was the first variable investigated. The Hardware–Software configuration used for all the analysis was an HP Pavilion 4000 Centrino Laptop running on Windows Vista Ultimate at 1.60 GHz. The same data were run in the different models using the ArcGIS™ 9.2 ArcInfo-ArcMap® software as well as Surfer8™ for the comparative evaluation of the results of the models.

It was found that the fastest result was 0.09 s obtained from the Polynomial Regression for which a quadratic surface was run. The Minimum Curvature was second at 0.11 s. Nearest Neighbor, Triangulation with Linear interpolation and Natural Neighbor came in that order. However, while Nearest Neighbor was only ten times slower than Polynomial Regression, Triangulation with Linear Interpolation was 49.78 times slower and Natural Neighbor 80.44 times slower. Kriging took less time than Modified Shepard’s method. Kriging was 200 times slower than Polynomial Regression. The Modified Shepard’s took 23.2 s and was 236.67 times slower than the fastest method. Other details are shown in Table 2.

The general conclusion is that with small grid sizes, one does not need to overemphasize computational times. Kriging would be slower to compute in about 19.56 times while the Modified Shepard’s Method that took the longest to compute would only be 25.33 times slower than the Nearest Neighbor method.

#### Table 2. Comparative analysis of the speed of computation of the DTM.

| Model                  | Speed (s) | Degree of slowness | Rank |
|------------------------|-----------|--------------------|------|
| 1 Inverse Distance     | 13.2      | 146.67             | 6    |
| 2 Kriging              | 18.0      | 200.00             | 8    |
| 3 Minimum Curvature    | 0.11      | 1.22               | 2    |
| 4 Natural Neighbor     | 7.24      | 80.44              | 5    |
| 5 Nearest Neighbor     | 0.92      | 10.22              | 3    |
| 6 (Modified) Shepard’s Method | 23.3 | 236.67             | 10   |
| 7 Triangulation with Linear Interpolation | 4.48 | 49.78 | 4 |

Notes: Degree of slowness was defined using the fastest as the base (1.00) and the rest as multiples of its speed of 0.09 s.
3.1.2. The use of information

The ability to use all the information supplied as input can be important in determining the goodness of fit of a model. The term used for the information is blanking. It consists of the process of removing grid nodes data from a grid file. Usually along the edges of the surface because the DEM polygon contained within a 7.5-min-quadrangle consists of files that do not always have the same number of data points as a result the variable angle between the UTM coordinate system and true north. The different node blanks differently so that the number of blanked nodes produced by the Surfer™ software is therefore a measure of the ability or otherwise of model to fully utilize all information.

Three of the models namely Triangulation with Linear Interpolation, Natural Neighbor and Modified Shepard’s had 341,417 and 214 blanked nodes respectively. These values represent 5.02, 6.13 and 3.00% respectively. The other nodes freely analyzed the information.

3.1.3. Characteristics of the resulting DEM grids

Certain common attributes of data such as mean, median and range can also be used to investigate the closeness of the grids to reality or to investigate differences of one from the other. For the data set, the statistical properties selected are the Minimum, the Median, the Maximum, the Range, the Mean, the Standard Deviation, the Coefficient of Variation, the Coefficient of Skewness, and the Root Mean Square for the z values. There were no variations in the x,y values of the observed and the models. These results are shown in Table 3.

Figure 2 shows an overlay of the contour surfaces generated by the seven models. By visually inspecting the maps, it is possible to comment on the faithfulness of the models. For instance, the Inverse Distance, the Natural Neighbor, the Nearest Neighbor, and Triangulation with Linear Interpolation would seem to give the highest level of correspondence or faithfulness. Minimum Curvature showed extremities in the characteristics shown by Kriging and those other models in group 2 and so could be in a class of its own. Following these were Kriging, Radial Basis Functions and Modified Shepard’s (the results have been colour-coded for Table 3). Nonetheless, Kriging, Minimum Curvature and Radial Basis Function models behaved erratically south of the Atlantic coast and the spatial elements produced are both difficult to justify or explain.

The degree of faithfulness of the models can also be investigated by the overlay of the maps from the models. Except for the observations made earlier on in terms of the erratic behaviour of some of the models, a generally
similar pattern of contours (land-forms) was observed although there were deviations one from the other. It would be difficult to determine the significance of the variations for many reasons. One good reason is that contour lines are imaginary lines and so one that is in reality does not exist, no matter the level of sophistication that could have gone into making them!

As observed earlier on Inverse Distance, Natural Neighbor, Nearest Neighbor and Triangulation by Linear Interpolation made predictions that are closer to reality and to conceptual expectations of contour surfaces in Nigeria. The extent of this generalization is not known and may need further empirical investigation.

4. Conclusion
In line with conceptual and theoretical framework, this study has shown that it does matter what method is used for DEM in GIS and Cartography. The arguments: First the models operate under varying assumptions some simple, other sophisticated; some depicting our poor state of understanding of operating processes to determine elevations; others indicating what we consider pragmatic in utilizing digital elevation models. Geographers are interested in providing surfaces but little has been done in the evaluation of the closeness of the surfaces to reality, which in any case is difficult to define. Kriging with its observed shortcomings shows that one can make these surfaces probabilistic rather than deterministic, Kriging, nevertheless represents a probabilistic line of research.

This study has come a long way in describing methods for constructing digital elevation models, as well as empirically evaluating results of such models. Some light has been thrown that indicates that the less complex models seem to give better predictions at least in the simplest of situations as those dealt with in this study. Further work may be necessary to investigate the mathematical rigor, assumptions and correctness of the more theoretically induced models such as Minimum Curvature, Modified Shepard’s and Kriging. Kriging has been particularly favored in the literature as providing the best of predictions either in deterministic or probabilistic situations (5). Our experience in producing this paper has not justified such confidence. Of course, in the area of probabilistic modeling of surfaces it is theoretically unsurpassed and has thus constituted the basis of geostatistical modeling in ArcGIS™. We have not been able to prove that these are misplacements of assumptions, but we have been able to show that there is need to match our theoretical preoccupations and predictions with empirical evaluation.

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