Comparison of PI Controller, Model Reference Adaptive Controller and Fuzzy Logic Controller for Coupled Tank System

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Abstract

Objective: The Coupled Tank System is modeled and its level should be controlled. Methods: The conventional Proportional Integral (PI) controller, Model Reference Adaptive Controller (MRAC) and Fuzzy Logic Controller are used, to control the level of the second tank. The MRAC can alter the controller parameters in response to changes in plant and the reference model indicates properties of the desired control system. The Fuzzy Logic Controller uses a set of rules to control the plant. Findings: The simulation is done, which demonstrates that the MRAC controller delivers better response compared to the PI controller and Fuzzy Logic Controller. Applications: The Model Reference Adaptive controller can be implemented in various level control applications such as conical tank system, spherical tank system, etc.

Keywords: Coupled Tank System, Fuzzy Logic Controller, Model Reference Adaptive Controller (MRAC), PI Controller

1. Introduction

These days, the procedure commercial ventures, for example, petro concoction businesses, paper making and water treatment commercial ventures obligate fluids to be pumped, put away in tanks, and afterward pumped to another tank. The control of fluid in tanks is an essential issue in the process businesses. The aforementioned commercial enterprises are the indispensable businesses where fluid level control is vital. Ordinarily the fluids will be handled by compound or blending treatment in the tanks, yet continuously the liquid level in the tanks must be controlled.

Coupled Tank system utilized for liquid level control is a model of plant that has normally been utilized as a part of commercial ventures particularly chemical process industries. The coupled tank system is nonlinear. The non-linearity happens on the grounds that the transfer function changes with the change in height of the liquid in a tank and the controller should be adaptive for these changes. The issues of level control in coupled tank process are system dynamics and interacting nature. The level will remain unchanged as long the inflow rate, i.e. the inlet flow rate and outlet flow rate remain unaltered. On the other hand, if any disturbances happen which bring about the change in either the inflow rate or the outflow rate, or any other changes, then the liquid level in the tank would change and settle at various levels. In the event that the outflow rate is more than the inflow rate, the liquid level will settle at a lower level. Let us assume that the tank level is maintained in steady state condition before these changes. If the inflow rate is more than the outflow rate, the liquid level will be higher than before. The control target is that the inflow rate should be adjusted to keep up the level in the same condition as before.

The fundamental requirement of the coupled tank system is to keep up the level of liquid in the tank consistent when there is inflow of water into the tank and outflow of water out of the tank. Process variable is the variable which evaluates execution; is really the water level in the
coupled tank control system. To control the water level at a set point, the inlet flow is adjusted by pump voltage. The inlet flow rate is the manipulated variable, i.e., the variable that is utilized to bring the process variable to its set point.

Different control systems were utilized to control liquid level. Simulated annealing\(^1\) was utilized to tune the PI controller. A PID controller, a time optimal controller and switching method was utilized as a hybrid method\(^2\) for control. The Genetic Algorithm (GA)\(^3\) and Particle Swarm Optimization (PSO)\(^4\) was utilized to get PID controller parameters. Set pointer is used to reduce peak overshoot\(^5\)\(^6\). Decentralized PI controller for MIMO system is discussed\(^7\)\(^8\).

2. Coupled Tank System

The coupled tank system comprises of two tower sort tanks with an inner baffle in the middle. It can be raised to control the flow of liquid between the two tanks, which makes it different from the standard two tank system. Every tank is fitted with an outlet. Fluid is pumped into the highest point of every tank by a pump. To gauge the level of the fluid in every tank, a capacitive sensor can be utilized, where the capacitance estimation is then changed over to an electrical signal which is a function of the fluid level.

The coupled tank system is depicted in the Figure 1. The system is designed as a SISO control issue by raising the baffle somewhat, where the plant dynamics can be approximated and linearized. The control target is to control the fluid level in Tank 2 by controlling the voltage of Pump 1.

The nonlinear equations of coupled tank system are\(^9\) shown by equations (1) and (2)

\[ A_1 \frac{dH_1}{dt} = Qi_1 - a_1 \sqrt{H_1} - a_3 \sqrt{H_1 - H_2} \quad (1) \]

\[ A_2 \frac{dH_2}{dt} = Qi_2 - a_2 \sqrt{H_1} - a_3 \sqrt{H_1 - H_2} \quad (2) \]

\[ H_1, \text{Liquid level in tank 1} \]

\[ H_2, \text{Liquid level in tank 2} \]

\[ A_1, \text{Area of cross section tank 1} \]

\[ A_2, \text{Area of cross section tank 2} \]

\[ Qi_1, \text{inlet flow rate into tank 1} \]

\[ Qi_2, \text{inlet flow rate into tank 2} \]

\[ Qo_1, \text{outlet flow rate of out of tank 1} \]

\[ Qo_2, \text{outlet flow rate of out of tank 2} \]

\[ Qo_3, \text{flow rate of between tanks} \]

\[ a_1, a_2, \text{and } a_3 \text{ are proportionality constants} \]

After approximation and linearization of equations (1) and (2), we get (3) and (4) respectively

\[ A_1 \frac{dh_1}{dt} = q_1 - a_1 \frac{h_1}{2\sqrt{H_1}} + a_3 \frac{h_1 - h_2}{2\sqrt{H_1 - H_2}} \quad (3) \]

\[ A_2 \frac{dh_2}{dt} = q_2 - a_2 \frac{h_2}{2\sqrt{H_2}} + a_3 \frac{h_1 - h_2}{2\sqrt{H_1 - H_2}} \quad (4) \]

The above two equations (3) and (4) are used to form the model of coupled tank system.

3. Controllers

The most recent four decades have seen the improvement of a various modeling and controlling methodologies for a coupled tank system. A perfect model and also a good control procedure are exceedingly vital to achieve the desired performance.

3.1 PI Controller

Almost all control systems for industrial applications utilizes some type of Proportional Integral (PI) control. The universal control calculation can be connected to about any system sort and when tuned appropriately will give stable control. The PI controller\(^10\) is the most widely used feedback controller. PI controllers are today found in all regions where control is utilized. It is an essential part in each control designer’s tool kit. It computes the error and endeavors to minimize it. The PI controller includes two separate steady parameters. It is a two term controller. The P (Proportional part) represents present...
estimation of the error and I (Integral part) represents past estimation of the error. The proportional part will cause the output to be more if the error is more. It can cause the rise time to reduce but cannot remove steady state error. When the gain is increased, the error will decrease but oscillations will increase. The integral part will cause the error to accumulate if the output of the controller is not able to reduce the error. It can remove the steady state error but it makes the transient response slow. It increases oscillations with increasing integral time. The total of the two activities is utilized to change the process by means of control parameters. The performance of the controller can be depicted as far as responsiveness of the controller to an error, the extent to which the controller overshoots the set point and the oscillations produced. Controller acts so that the output is so near to the set point.

### 3.2 Model Reference Adaptive Controller (MRAC)

The Model Reference Adaptive Controller (MRAC) is an important controller. It might be viewed as an adaptive servo system in which the reference model has the desired performance specifications. This is a helpful approach to give specifications for a system. This technique has the adaptability, robustness and flexibility. It is able to automatically adjust the controller parameters. The reference model can be computed based on the performance specifications such as overshoot, rise time, etc. In this manner, the controlling performance could be enhanced by altering the reference model parameters which makes the system output to follow the desired response.

Although different methods are used, model reference adaptive control is better for the below reasons:

- It gives a measure of system performance.
- It is not hard to compute a reference model that has the desired specifications.
- It has good stability analysis.

It has an ordinary loop with feedback having process and controller. It is called inner loop. The parameters are changed on the basis of error. Error is difference between system output and reference model output. It is called outer loop.

Lyapunov’s stability theory is utilized to build algorithms for adjusting parameters in adaptive systems. To do this, find a differential equation for error. Then create a Lyapunov function $V$ such that the error will go to zero. $dV/dt$ should be negative semi definite. It is then used to show boundedness and that the error goes to zero.

Lyapunov’s theory can be utilized to form stable MRAC. It can be described by following steps:

1. Find a controller structure.
2. Derive the error equation.
3. Find a Lyapunov function.

The coupled tank system model is given by equation

$$\frac{h_2(s)}{q_1(s)} = \frac{k_0k_2}{\tau_1\tau_2 s^3 + (\tau_1 + \tau_2)s + (1-k_{12}k_{21})} u(s)$$

Where

- $h_2(s)$ is water level in tank 2
- $u(s)$ is control signal
- $k_1, k_2, k_{12}, k_{21}, \tau_1, \tau_2$ are constants

The reference model is given by equation

$$h_m(s) = \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} r(s)$$

The reference model is designed with 1% overshoot and 15% settling time

$$h_m(s) = \frac{0.1041}{s^2 + 0.5332s + 0.1041} r(s)$$

$h_m(s)$ is output of reference model

$r(s)$ is input reference signal

Controller output is given by equation

$$u(s) = (K_p + K_i/s)(r(s) - h_2(s))$$

The error is given by equation

$$\ddot{e} = h_2 - \dot{h}_m$$

Lyapunov function is given by equation

$$V = \dot{\lambda}_1 \dot{e}^2 + \dot{\lambda}_2 e^2 + \dot{\lambda}_3 K_p^2 + \dot{\lambda}_4 K_i^2$$

Where $\dot{\lambda}_1, \dot{\lambda}_2, \dot{\lambda}_3, \dot{\lambda}_4 > 0$ are constants, $V$ is positive definite. For $V<0$,

$$K_p = \frac{1}{\lambda_1} \dot{e}(h_2 - r)$$
\[ K_i = \frac{1}{2} s (h_2 - r) \]  
(11)

The adaptation mechanism is given by equations (10) and (11).

### 3.3 Fuzzy Logic Controller (FLC)

A fuzzy control system is a mathematical system that analyzes analog input values in terms of variables that take values between 0 and 1. The fuzzy logic controller can be used in control of liquid level. Fuzzy logic control systems normally comprise of four parts: Fuzzifier, rule base, Fuzzy inference engine and Defuzzifier.

The input variables are mapped by membership functions, known as fuzzy sets. The procedure of changing a crisp value into fuzzy value is called fuzzification. Some shapes of membership functions are triangular, trapezoidal, bell curves, etc.

The rule base describes the control objectives by means of linguistic control rules. The IF-THEN rules are called fuzzy conditional statements. The fuzzy sets are inputs to fuzzy rule base.

The fuzzy inference engine has the capacity of reproducing human choice making based on fuzzy concepts and infers control actions using the rules of inference in fuzzy logic. The procedure utilized by Fuzzy Inference Engine consists of finding the firing level of each rule, obtaining the output of each rule and aggregating the outputs of rules to determine the overall output. The most commonly utilized fuzzy inference method is the Mamdani method. In Mamdani’s model the conjunction operator is min, the t-norm taken from compositional rule is min and max operator is used for aggregation of the rules.

The conversion of fuzzy set to crisp set is called defuzzification. The most well known defuzzification method is the centroid method. The centroid technique uses the center of mass of the result to calculate the crisp value.

### 4. Results and Discussion

The output response of PI controller is shown in Figure 2. The output response of Model Reference Adaptive Controller is shown in Figure 3. The output response of Fuzzy logic controller is shown in Figure 4. The comparisons of the output response of all the three controllers are shown in Figure 5. The performance comparison of all the three controllers is tabulated in Table 1.
5. Conclusion

The mathematical model of the coupled tank system is designed and simulated. The MRAC generates a rather smooth control signal to drive the liquid level to its set point. It can be concluded that MRAC yields better result than PI controller and fuzzy logic controller.

6. References

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