Reconstructing the interaction term between dark matter and dark energy

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Abstract. We apply a parametric reconstruction method to a homogeneous, isotropic and spatially flat Friedmann-Robertson-Walker (FRW) cosmological model filled of a fluid of dark energy (DE) with constant equation of state parameter interacting with dark matter (DM). The reconstruction method is based on expansions of the general interaction term and the relevant cosmological variables in terms of Chebyshev polynomials which form a complete set orthonormal functions. This interaction term describes an exchange of energy flow between the DE and DM within dark sector. To show how the method works we do the reconstruction of the interaction function expanding it in terms of only the first three Chebyshev polynomials and obtain the best estimation for the coefficients of the expansion as well as for the DE equation of the state constant parameter \( w \) using the type Ia Supernova SCP Union data set (307 SNe-Ia). The preliminary reconstruction shows that in the best scenario there is an energy transfer from DM to DE which worsen the problem of the cosmic coincidence in comparison with the \( \Lambda \)CDM model. We conclude that this fact is an indication of a serious drawback for the existence of such interaction between dark components.

Keywords: dark energy, supernovae, dark matter

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In the last years the accelerated expansion of the universe has now been confirmed by several independent observations including those of high redshift (\( z \leq 1 \)) type Ia Supernovae (SNeIa) data at cosmological distances [1]-[7]. This has been verified by precise measurements of the power spectrum of the cosmic microwave background (CMB) anisotropies [8]-[12] and the galaxy power spectrum [13]. To explain these observations it has been postulated the existence of a new and enigmatic component of the universe so-called dark energy (DE). Recent observations [14]-[16] show that if it is assumed a dark energy (DE) equation of state (EOS) with constant parameter \( w = P_{DE}/\rho_{DE} \), then there remains little room for departure of DE from the cosmological constant, since \( |1 + w| < 0.06 \) at the 1\( \sigma \) confidence level (C.L.). In addition these observations indicate that our universe is flat and it consists of approximately 70% of Dark Energy (DE), 25% of Dark Matter and 5% of barionic matter.

However the cosmological constant model has two serious problems: the first of them is the cosmological constant problem [17]-[19] which consists in why the observed value of the Cosmological Constant \( \rho_{\Lambda}^{obs} \sim (10^{-12} \text{ Gev})^4 \) is so-small compared with the theoretical value \( \rho_{\Lambda}^{Pl} \sim (10^{18} \text{ Gev})^4 \) predicted from local quantum field theory if we are confident in its application to the Planck scale? The second problem is the so named The Cosmic Coincidence problem [20]-[21] consisting in why, in the present, the energy density of DE is comparable with the density of dark matter (DM) while the first one is
subdominant during almost all the past evolution of the universe?.

Recently, in order to solve the *The Cosmic Coincidence problem*, several researchers have considered a possible phenomenological interaction between the DE and DM components [22]-[31]. Some of these studies have claimed that, for reasonable and suitably chosen interaction terms, the coincidence problem can be significantly ameliorated in the sense that the rate of densities $r \equiv \rho_{DM}/\rho_{DE}$ either tends to a constant or varies more slowly than the scale factor, $a(t)$, in late times.

While it is not totally clear if an interaction term can solved the *The Cosmic Coincidence problem*, we can yet put constraints on the size of such assumed general interaction using recent cosmological data. We do this postulating the existence of an general nongravitational interaction between the two dark components. We introduce phenomenologically this interaction term $Q$ into the equations of motion of DE and DM, which describes an energy exchange between these components [22]-[31]. In order to reconstruct the interaction term $Q$ as a function of the redshift we expand it in terms of Chebyshev Polinomials which constitute a complete orthonormal basis on the finite interval [-1, 1] and have the nice property to be the minimax approximating polynomial (this technique has been applied to the reconstruction of the DE potential in [32]-[33]). At the end we do the reconstruction using the observations of type Ia Supernova SCP Union data set (307 SNe-Ia) [7]. In our reconstruction process we assume a DE equation of state parameter $w = \text{constant}$.

The organization of this paper is a follow. In the second section we introduce the equations of motion of the DE model interacting with DM and the reconstruction scheme of the interaction term. In the third section we briefly describe the application of the type Ia Supernova data cosmological test and the priors used in this one. In the last section we present the results of our reconstruction and the best estimated values of the parameters fitting the observations. Finally, we discuss our main results and present our conclusions.

**Equations of motion for dark energy interacting with dark matter.** The background metric is described by the flat Friedmann-Robertson-Walker (FRW) metric as supported by the anisotropies of the cosmic microwave background (CMB) radiation measured by the WMAP experiment

$$ds^2 = -dt^2 + a^2(t) \left( dr^2 + r^2 d\Omega^2 \right),$$

where $a(t)$ is the scale factor and $t$ is the cosmic time.

We assume an universe formed by four components: the barionic matter density $\rho_b(z)$, the radiation density $\rho_r(z)$, the DM density $\rho_{DM}(z)$ and the DE density $\rho_{DE}(z)$, where the variable $z$ represents the redshift. The DE equation of state is assumed as $P_{DE} = w \rho_{DE}$ where $w$ is a constant where as for the dark matter we have $P_{DM} = 0$.

Moreover all these constituents are interacting gravitationally and additionally only the components $\rho_{DE}$ and $\rho_{DM}$ interact nongravitationally through an energy exchange between them mediated by the interaction term $Q(z)$. As we know the solutions for the barionic matter and radiation density are respectively:

$$\hat{\Omega}_b(z) \equiv \frac{\rho_b}{\rho_{crit}} = \Omega_0^b (1 + z)^3,$$

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where $\rho_{crit}$ is the critical density of the universe.
\[ \dot{\Omega}_r(z) \equiv \frac{\rho_r}{\rho^0_{\text{crit}}} = \Omega^0_r(1+z)^4, \]  
\( (3) \)

where \( H_0 \) is the Hubble constant, \( \rho^0_{\text{crit}} \equiv 3H_0^2/8\pi G \) is the critical density today and \( \Omega^0_b \equiv \rho^0_b/\rho^0_{\text{crit}}, \Omega^0_r \equiv \rho^0_r/\rho^0_{\text{crit}} \) are respectively the density parameters of barionic matter and radiation at the present. The energy conservation equations for both dark components are:

\[ \frac{d\rho_{DM}}{dz} - \frac{3}{1+z}\rho_{DM} = -\frac{Q(z)}{(1+z)\cdot H(z)}, \]  
\( (4) \)

\[ \frac{d\rho_{DE}}{dz} - \frac{3(1+w)}{1+z}\rho_{DE} = \frac{Q(z)}{(1+z)\cdot H(z)}, \]  
\( (5) \)

here \( H(z) \) is the Hubble parameter. We complete the equations of motion with the first Friedmann equation,

\[ H^2(z) = \frac{8\pi G}{3} (\rho_b + \rho_r + \rho_{DM} + \rho_{DE}). \]  
\( (6) \)

Using (2) and (3) we write (6) as

\[ H^2(z) = H_0^2 \left[ \Omega^0_b (1+z)^3 + \Omega^0_r (1+z)^4 + \dot{\Omega}_{DM}(z) + \dot{\Omega}_{DE}(z) \right], \]  
\( (7) \)

where we use the definitions \( \dot{\Omega}_{DM}(z) \equiv \rho_{DM}/\rho^0_{\text{crit}}, \dot{\Omega}_{DE}(z) \equiv \rho_{DE}/\rho^0_{\text{crit}} \). We do the parametrization of the \( Q(z) \) coupling in terms of the Chebyshev polynomials, which form a complete set of orthonormal functions on the interval \([-1,1]\). They also have the property to be the minimax approximating polynomial, which means that has the smallest maximum deviation from the true function at any given order. We can then expand \( Q \) in the redshift representation as:

\[ Q(z) = \sum_{n=0}^{N} \lambda_n T_n(z) \cdot H(z) \cdot (1+z)^3, \]  
\( (8) \)

where \( T_n(z) \) denotes the Chebyshev polynomials of order \( n \) with \( n \in [0,N] \) and \( N \) a positive integer. \( H(z) \) represents the Hubble parameter and \( \lambda_n \) are real free parameters representing the coefficients of the linear expansion. We introduce (8) in (4) and (5) and integrate both equations obtaining,

\[ \dot{\Omega}_{DM}(z) = (1+z)^3 \left[ \Omega^0_{DM} - \frac{z_{\text{max}}}{2} \sum_{n=0}^{N} \hat{\lambda}_n \int_{-1}^{x} \frac{T_n(\tilde{x})}{(a+b\tilde{x})} d\tilde{x} \right], \]  
\( (9) \)

\[ \dot{\Omega}_{DE}(z) = (1+z)^3(1+w) \left[ \Omega^0_{DE} + \frac{z_{\text{max}}}{2} \sum_{n=0}^{N} \hat{\lambda}_n \int_{-1}^{x} \frac{T_n(\tilde{x})}{(a+b\tilde{x})^{3w+1}} d\tilde{x} \right], \]  
\( (10) \)

where we have defined the dimensionless coefficients \( \hat{\lambda}_n \equiv \lambda_n/\rho^0_{\text{crit}} \) and the quantities \( a = 1 + z_{\text{max}}/2, b = z_{\text{max}}/2, x = 2z/z_{\text{max}} - 1 \) where \( z_{\text{max}} \) is the maximum redshift at
FIGURE 1. Reconstruction for the density parameters $\Omega_{DM}(z)$, $\Omega_{DE}(z)$ as a function of the redshift for a spatially flat universe with dark energy interacting with dark matter. The reconstruction is derived from the best estimation obtained using the type Ia Supernova SCP Union data set sample. $z_{\text{max}} = 1.551$ corresponding to the farthest Supernova in the sample used. Note that the coincidence cosmic problem persists.

which observations are available so that $x \in [-1, 1]$ and $|T_n(x)| \leq 1$, for all $n \in [0, N]$.

As we can see from (9) and (10), the Hubble parameter (7) depends on the parameters $(H_0, \Omega_b^0, \Omega_r^0, \Omega_{DM}^0, \Omega_{DE}^0, w)$ and the coefficients $\lambda_n$. In general we must take $N \to \infty$ but in the practice we cut to some finite integer. To simplify our analysis and to show how the method works we do the reconstruction taking the first three polynomials $n = 0, 1, 2$ ($N = 2$), which are:

$$T_0(x) = 1, \quad T_1(x) = x, \quad T_2(x) = 2x^2 - 1,$$

using these polynomials we find closed solutions for the expressions (9) and (10), and finally from (7) we can write the dimensionless Hubble parameter $\tilde{H}^2 \equiv H^2 / H_0^2$ as

$$\tilde{H}^2(z) = \Omega_b^0(1+z)^3 + \Omega_r^0(1+z)^4 + \hat{\Omega}_D(z),$$

where $\hat{\Omega}_D$ denotes the sum of the density parameters of both dark components,

$$\hat{\Omega}_D(z) \equiv \hat{\Omega}_{DM}(z, \Omega_{DM}^0, \lambda_0, \lambda_1, \lambda_2) + \hat{\Omega}_{DE}(z, \Omega_{DE}^0, w, \lambda_0, \lambda_1, \lambda_2).$$

**Cosmological test using Ia Supernova data.** In what follows we assume the following priors $H_0 = 72$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_b^0 = 0.233$, $\Omega_r^0 = 4.63 \times 10^{-5}$, $\Omega_{DM}^0 = 4.62 \times 10^{-2}$. This is motivated by the fact that current data are converging around these values [14]-[16]. The density parameter for dark energy at the present is fixed using the first Fried-
FIGURE 2. Reconstruction for the interaction function $Q(z)$ as a function of the redshift for a spatially flat universe with dark energy interacting with dark matter. The reconstruction is derived from the best estimation obtained using the type Ia Supernova SCP Union data set sample. Note that the strength of the interaction is decreasing to the present.

The Friedmann equation evaluated today $\Omega_{DE}^0 = 1 - \Omega_{DM}^0 - \Omega_r^0 - \Omega_b^0$. We constrain the possible values of the remaining parameters $(w, \lambda_0, \lambda_1, \lambda_2)$ using the type Ia Supernova SCP Union data set (307 SNe-Ia) [7].

For the SNe Ia test it is defined the observational luminosity distance in a flat cosmology as $d_L(z, w, \lambda_0, \lambda_1, \lambda_2) = c(1 + z)H_0^{-1} \int_0^z \frac{\tilde{H}(z')^{-1} dz'}{H(z')}$, where $\tilde{H}(z) \equiv H(z)/H_0$ and $c$ the speed of light. The theoretical distance moduli for the $i$-th supernova with redshift $z_i$ is $\mu(z_i) = 5\log_{10}[d_L(z_i)/\text{Mpc}] + 25$. The statistical function $\chi^2_{\text{SNe}}(w, \lambda_0, \lambda_1, \lambda_2) \equiv \sum_{k=1}^{182} \frac{[\mu(z_k) - \mu_k]^2}{\sigma_k^2}$, where $\mu_k$ is the observed distance moduli for the $k$-th supernovae and $\sigma_k^2$ is the variance of the corresponding measurement.

Reconstruction of the parameters. We compute the best estimated values of the parameters $(w, \lambda_0, \lambda_1, \lambda_2)$ to the data through $\chi^2$-minimization, using the SNe Ia test. In this case $z_{\text{max}} = 1.551$ corresponding to the farthest Supernova in the sample used.

We obtain as the best estimation: $w = -1.2755$, $\lambda_0 = -8.5255 \times 10^{-7}$, $\lambda_1 = 7.5755 \times 10^{-9}$ and $\lambda_2 = 5.2755 \times 10^{-10}$, with a $\chi^2_{\text{min}} = 314.811$ ($\chi^2_{\text{d.o.f.}} = 1.038$). For the age of the universe we have 13.83 Gyr. The results of the reconstruction are illustrated in figures 1 and 2. From these figures we conclude that:

- The density parameters $\Omega_{DM}(z), \Omega_{DE}(z)$ as a function of the redshift are shown in the Fig. 1. We can see that the problem of the cosmic coincidence is not solved and in fact it is worse in comparison with the $\Lambda$CDM model due to that $Q(z)$ is negative [23] as it is shown in the Fig. 2. We note that if we extrapolate this curve to early times the DE density parameter becomes negative which is a serious drawback for the existence of an interaction between the dark components.
- Fig. 2 shows that the reconstructed interaction function $Q(z)$ is always negative signifying an energy transfer from DM to DE which worsen the problem of the
cosmic coincidence in comparison with the $\Lambda$CDM model [23].

- The preliminary reconstruction shows that the principal motivation for introducing a recent possible interaction between DE and DM (to solve the problem of the cosmic coincidence) is not supported by recent type Ia Supernova data.

Finally we mention that a extended analysis will be presented elsewhere [34] which includes a reconstruction of the interaction term adding recent Cosmic Microwave Background (CMB) and the Baryon Acoustic oscillations (BAO) data.

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