Computational analysis of induced magnetohydrodynamic non-Newtonian nanofluid flow over nonlinear stretching sheet

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Abstract
In the current article, induced magnetic field applied on second-grade fluid flow under variable thermal conductivity by an exponentially stretching sheet is taken into account for current analysis. The chemical reaction and viscous dissipation effects under the influence of thermophoresis and Brownian motion are considered on an exponentially stretching sheet. With the above assumptions, a mathematical model was developed in terms of partial differential equations by using the boundary-layer approximations. Similarity transformations in terms of ordinary differential equations considerably simplified this system. The dimensionless system was solved by a numerical procedure, the bvp4c method. The effects of involving physical parameters are presented through graphs and tables. The obtained numerical outcomes of the skin friction coefficient, the Sherwood number, and the Nusselt number are also highlighted in the tabulated form. It is concluded that the velocity and concentration profiles increased due to higher values of material parameter.

Keywords
Second-grade nanofluid, induced magnetic field, variable thermal conductivity, exponentially stretching sheet

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Introduction

The MHD fluid flow has many applications, namely: petroleum, warming systems, electrostatic mechanisms, power generation, modern metallurgy, and pumping having the essential role in usual life. Damesh\(^1\) discussed the thermal boundary layer in the existence of a magnetic field’s impact by virtue of an exponentially stretching surface. Ishak\(^2\) considered the MHD flow of a fluid having a linear radiative effect on an exponentially stretching surface. The unsteady two-dimensional boundary-layer flow of an incompressible Newtonian fluid under the thermal reaction, magnetic field, and absorption/heat generation impacts by virtue of an exponentially stretched surface was discussed by Elbasheshy et al.\(^3\) Nadeem et al.\(^4\) discussed the MHD flow of a Casson fluid by an exponentially shrunked surface. Ferdows et al.\(^5\) presented the numerical solutions for the flow of fluid having nanoparticles due to the effect of MHD across a porous medium by virtue of a vertical exponentially stretching sheet. Impacts of chemical reaction for MHD flow of third-grade liquid by an exponentially stretching surface are discussed by Hayat et al.\(^6\) In the recent period, a few investigators worked on the MHD flow concept under various assumptions (see Refs. 7-10).

Initially, the idea of the flow of fluid by an exponentially stretched sheet was given by Magyari and Keller.\(^11\) They discussed the numerical solutions for the fluid flow toward an exponentially stretched surface, and distributed the fact of mass and heat transfer properties. The idea of the fluid flowing toward an exponentially stretched sheet plays a vital factor in industries and engineering fields such as glass fiber, drilling of sheets and plastic films, paper production, annealing and thinning of the copper wire, etc. It is worth mentioning that the stretched velocity is not linear in all the conditions, so the stretched velocity of the fluid may be nonlinear or exponentially stretching. The flow of Newtonian and non-Newtonian fluid by virtue of an exponentially stretched surface has been discussed by scientists in the recent past. Partha et al.\(^12\) considered an exponentially vertical surface to study the heat transportation with thermal mixed convection flow under the significance of viscous dissipation. Pal\(^13\) provided numerical solutions, and analysis follows to narrate the mixed convection flow by considering a vertical exponential surface with an exponentially dependent stretching velocity. The MHD, velocity, and the thermal slip effects were used by Mukhopadhyay,\(^14\) who analyzed the heat and mass transportation by a porous exponentially stretched surface with suction or blowing. Also, Mukhopadhyay\(^15\) utilized a thermally stratified medium and made numerical solutions for the magnetohydrodynamic flow by virtue of an exponentially stretched surface. Hussain et al.\(^16\) deliberated the two-dimensional MHD flow of second-grade liquid with nanoparticles by virtue of the exponentially stretched surface. Beg et al.\(^17\) discussed the effects of unsteady MHD nanofluid flow over a vertical exponentially stretching sheet. Veeresha et al.\(^18\) surveyed the heat and mass transportation impact in a boundary-layer MHD flow of viscous fluid into a permeable medium by virtue of an exponentially stretched surface. Several authors discuss the different problems of the boundary-layer flow using different models to analyze the behavior of heat transfer and its effects (see Refs. 19-21).

A fluid in which nanoparticles are submerged is called nanofluid. The size of nanoparticles is from 1 nm to 100 nm. The thermal conductivity of the nanofluids as compared to the base fluid is higher. Hence, nanofluids are the best source to raise the thermal conductivity to the base fluid. By virtue of such important features, the nanofluids are used in many fields of life such as heat ex-changer vehicle cooling, nuclear reactors, biomedicine transfer cooling, electronic devices, etc. The magnetic nanofluids are used in MHD power generators, hyperthermia, removal of blockage in the arteries, in cancer therapy, treatment of cancer tumors, etc. Choi\(^22\) was the first scientist who used the term nanofluid. Pop and Khan\(^23\) numerically discussed the flow of the fluid with nanoparticles under the impacts of thermophoresis and Brownian motion over a stretching surface. Makinde and
Aziz\textsuperscript{24} analyzed the nanofluid behavior of the laminar boundary-layer flow with the impact of convective boundary condition on a stretched sheet. Rana and Bhargava\textsuperscript{25} numerically performed and discussed the laminar boundary fluid flow under the effect of nanoparticles through a nonlinear stretching sheet. Hady et al.\textsuperscript{26} considered a viscous nanofluid flow and numerically examined the heat transportation with the significance of nonlinear thermal radiation and variable wall temperature toward a nonlinear stretched sheet. Rashdi et al.\textsuperscript{27} used nanofluid with the thermal and buoyancy radiation’s influence on the magnetohydrodynamic flow toward a stretched surface. Greeshma et al.\textsuperscript{28} characterized the inspiration of heat absorption or generation and nonlinear thermal radiation with a porous medium for the three-dimensional flow of Jeffery nanofluid by a nonlinear porous stretching sheet. Several authors investigated the effects of flow behavior with diffusion with COVID-19 (see Refs.\textsuperscript{29-33}).

In the recent work, we examined the MHD flow of second-grade nanofluid with the existence of induced magnetic field, thermophoretic effect, variable chemical reaction, and variable thermal conductivity. By employing the similarity transformation, the governing boundary-layer PDEs changed into the nonlinear ODEs, and they were numerically solved by the built-in bvp4c technique. The effect of different dimensionless parameters on the induced magnetic field profile, temperature profile, velocity profile, and concentration profile are debated graphically. Further, the skin friction, Sherwood number, and Nusselt number are calculated and discussed. These results are more advanced and may be useful in industrial and engineering problems.

**Mathematical formulation**

Consider an incompressible, steady 2D of MHD second-grade fluid flow with the consequences of viscous dissipation, induced magnetic field, variable chemical reaction, and thermophoretic effect. The flow of the fluid is along the x-axis. The flow of the fluid is considered by virtue of an exponentially stretched sheet with the velocity \( u_w = U_0 e^{x/l} \). The description of the coordinates and the physical chart are given in Figure 1.

By using the upper suppositions, the boundary-layer equations are

\[
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0
\]  

Figure 1. Coordinates and the physical chart.
\[
\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \quad (2)
\]

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \alpha_1 \left[ \frac{\partial}{\partial x} \left( u \frac{\partial^2 u}{\partial y^2} \right) + \frac{\partial^2 u}{\partial x \partial y} + \nu \frac{\partial^3 u}{\partial y^3} \right] + \mu_0 \frac{H_1}{4\pi} \frac{\partial H_1}{\partial x} + H_2 \frac{\partial H_1}{\partial y}, \quad (3)
\]

\[
u \frac{\partial H_1}{\partial x} + v \frac{\partial H_1}{\partial y} = \alpha_2 \frac{\partial^2 H_1}{\partial y^2} + H_1 \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}, \quad (4)
\]

\[
u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{\rho C_p} \frac{\partial}{\partial y} \left[ k(T) \frac{\partial T}{\partial y} \right] + \frac{D_B}{T} \frac{\partial C}{\partial y} + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial y^2} \right)^2 + \frac{\mu}{\rho C_p} \frac{\partial u}{\partial y} + \frac{\alpha_1 \partial u}{\rho C_p} \frac{\partial}{\partial y} \left( \nu \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right), \quad (5)
\]

\[
u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \left( \frac{\partial^2 T}{\partial y^2} \right) - \frac{\partial}{\partial y} (V_T C). \quad (6)
\]

Here \(k(T)\) and \(V_T\) are defined as

\[
k(T) = k_o \left( 1 + \varepsilon \frac{T - T_\infty}{\Delta T} \right), \quad V_T = -\frac{k^*}{T} \frac{\partial T}{\partial y},
\]

Here, \(\rho, \upsilon, k, \mu_0, \alpha_1, \alpha_2, H_1, H_2, C_p, D_B, D_T, T_\infty, \mu, k_\infty, \varepsilon, V_T, \) and \(C\) are the fluid density, kinematic viscosity, thermal conductivity, magnetic permeability, second-grade parameter, magnetic diffusivity of the nanofluid, component of the induced magnetic field along the x-axis, component of the induced magnetic field component along the y-axis, the heat capacity, coefficient of Brownian motion, thermophoresis coefficient, ambient temperature, fluid dynamic viscosity, fluid viscosity, fluid thermal conductivity, small parameter, and chemical reaction coefficient, respectively. The boundary conditions are

\[
u = u_w = U_0 e^y, \quad v = 0, \quad H_1 = 0, \quad \frac{\partial H_1}{\partial y} = H_2 = 0, \quad C = C_w + \lambda_2 \frac{\partial C}{\partial y}, \quad (7)
\]

\[T = T_w + \lambda_1 \frac{\partial T}{\partial y} \text{ at } y \to 0,
\]

\[
u \to 0, \quad H_1 \to H_\infty, \quad T \to T_\infty, \quad C \to C_\infty \text{ at } y \to \infty.
\]

Here, \(H_\infty = H_0 e^{y/l}\).

Following are the similarity transformations.
After using the above similarity transformations (8), the PDEs (3)–(6) are transformed into the following ODEs (9)–(12)

\[ 2f'' - ff'' = f''' + A \left[ 3f'f'' + nf''f''' - \frac{1}{2}ff'' \right] + M \left[ 2h'' - hh'' \right], \]

\[ h'' = Pr_{m} \left[ hh'' - fh'' \right], \]

\[ Pr_{r}(f' \theta - f \theta') = (1 + \epsilon \theta) \theta'' + \epsilon \theta'^{2} + Pr \left( N_{b} \theta \varphi' + N_{i} \theta'' \right) + P_{r,E_{c}} \left[ f'' + Mf'' + Af''(f' f'' - ff'') \right], \]

\[ f' \varphi - f \varphi' = \frac{1}{Sc} \varphi'' + \frac{N_{i}}{Sc N_{b}} \theta'' - \tau_{1} \left( \theta' \varphi - (\theta + C_{1}) \theta' \right), \]

The corresponding boundary conditions are as follows

\[ f = 0, \quad f' = 1, \quad h = 0, \quad h' = 0, \quad \theta = 1 + S_{1} \theta', \quad \varphi = 1 + S_{2} \varphi', \quad \text{at } \eta \to 0 \]

\[ f' = 0, \quad h' = 1, \quad \varphi = 0, \quad \theta = 0, \quad \text{at } \eta \to \infty. \]

\( A = \alpha_{1}/\rho U_{0}/v l e^{v/l} \) (second-grade parameter), \( M = \mu/4 \pi \rho H_{0}^{2}/U_{0}^{2} \) (magnetic field parameter), \( P_{r} = v/\alpha_{1} \) (Prandtl number), \( Pr_{m} = v/\eta \) (magnetic Prandtl number), \( N_{b} = D_{B} C_{0}/v e^{v/l} \) (parameter of Brownian motion), \( N_{i} = D_{r} T_{0}/v T_{e} e^{v/l} \) (thermophoresis effect parameter), \( S_{e} = v/D_{B} \) (Smith number), \( \tau_{1} = -k^{*}(T_{w} - T_{x})/T_{r} \) (thermophoretic effect parameter), and \( C_{1} = (C_{w} - C_{x})/C_{x} \) (concentration difference parameter).

The expressions of the physical quantities of interests like skin friction, Nusselt number, and Sherwood number are defined as

\[ C_{f_{w}} = \frac{1}{2} \mu u_{w}^{2}, \]

\[ \tau_{w} = \left[ \mu \frac{\partial u}{\partial y} + \rho a_{1} \left( 2 \frac{\partial u}{\partial x} \frac{\partial u}{\partial y} + u \frac{\partial^{2} u}{\partial x \partial y} \right) \right]_{v=0}, \]

\[ C_{f_{w|\eta=0}} = \left( \frac{R_{e}}{2} \right)^{1/2} \left[ 1 + 3a f'(0) \right] f''(0), \]
\[ N_{ux} = \frac{xq_w}{k(T - T_s)}, \]
\[ q_w = \left| -k \frac{\partial T}{\partial y} \right|_{y=0} \]
\[ R_{s}^{-\frac{1}{2}} N_{ux} = -\theta'(0), \]
\[ Sh_x = \frac{xh_m}{D\theta(\phi_w - \phi_s)} \]
\[ S_{hx} = -\frac{x}{(C_w - C_s)} \left( \frac{\partial C}{\partial y} \right)_{y=0}, S_{hx} = -\sqrt{\frac{x}{2}} \left( \frac{R_{ex}}{2} \right)^{-\frac{1}{2}} \phi'(0), S_{hx} \sqrt{\frac{x}{2}} (R_{ex})^{-\frac{1}{2}} = -\phi'(0). \]

**Results and discussion**

The second-grade fluid flow analysis of induced magnetic flow over an exponentially stretching sheet is taken into account. The model is developed under flow assumptions, and solved through a numerical technique. The graphical results are observed and effects are highlighted in Figures 2–15. Figure 2 presented the variation of \( A \) and \( f'() \). It is prominent that \( f''() \) declined due to higher values of \( A \). It means that thickness of the boundary layer reduced for higher values of the material parameter because the stress applied resists strain and shear flow linearly. These phenomena reduce the velocity profile because the thickness of the fluid is enhanced. Figure 3 displays the variation of \( f''(\eta) \) with \( M \). The thickness of the velocity function declined due to enhancing the values of \( M \). The velocity function reduced because Lorentz force resists the decline in flow speed at the surface.

**Figure 2.** Variation of \( f''(\eta) \) and \( A \).
Figure 4 presents the variation of the magnetic profile \( h'(\eta) \) and \( A \). The magnetic profile is enhanced due to larger values of \( A \). The thickness of the magnetic profile enhances with augmented values of the material parameter. The relation of \( Pr_m \) and \( h'(\eta) \) is revealed in Figure 5. Raising the values of \( Pr_m \) which resist to increase the magnetic profile because thickness of the magnetic profile rised for higher values of \( Pr_m \). As the induced current density rises due to higher values of \( Pr_m \) due to the effect of these phenomena, the induced magnetic profile thickness is enhanced. The relation of \( M \) and \( h'(\eta) \) is revealed in Figure 6. Raising the values of \( M \) which resist to increase the magnetic profile because thickness of the magnetic profile rised for higher values of \( M \). Figure 7 exposes the impacts of \( N_b \) on the temperature function \( \theta(\eta) \). The thermal thickness is enhanced due to higher values of \( N_b \). The Brownian motion parameter increases which enhances the temperature due to stronger molecular motion. Figure 8 depicted the impact of \( P_r \) on the temperature function \( \theta(\eta) \). It is noted that thermal thickness reduced due to enhanced values of \( P_r \). The thickness of the thermal boundary layer is reduced as the Prandtl number rises. The ratio of momentum diffusivity to heat diffusivity is known as the Prandtl number. \( P_r \) controls the relative thickening of the motion and heat flux zones in heat transfer problems. Figure 9 highlights the effects of \( S_1 \) on the temperature function. The temperature function is enhanced due higher values of \( S_1 \) because the thermal slip is enhanced which enhances the temperature function. The variation of \( N_t \) and \( \phi(\eta) \) is presented in Figure 10. The thickness of \( \phi(\eta) \) is enhanced because \( N_t \) increased. The distribution of concentrations becomes more non-uniform. Likewise, thermophoresis promotes non-uniformity in the concentration distribution, with a stronger effect at significantly greater concentrations. Figure 11 depicts the impacts of \( L_e \) on \( \phi(\eta) \). It is seen that thickness of \( \phi(\eta) \) is reduced due to higher values of \( L_e \). Physically, the Lewis number is ratio of thermal and mass diffusivity. It is used to describe the flow of fluid in which heat and mass transfers occur simultaneously. The effects of \( N_b \) on \( \phi(\eta) \) are revealed in Figure 12. The thickness of concentration profile is enhanced due to enhanced values of \( N_b \) because of the movement of the particles from a higher concentration area to lower concentration area. Figure 13 depicts the effects of \( S_2 \) on \( \phi(\eta) \). It is noted that the higher values of \( S_2 \) resist the

Figure 3. Variation of \( f'(\eta) \) and \( M \).
enhancement of the values of the concentration profile. The concentration profile resisted due to slip of concentration enhances. Figure 14 presents the impacts of $S_c$ on $\varphi(\eta)$. It is noted that the thickness of the concentration function is reduced due to higher values of $S_c$. Actually, $S_c$ is directly proportional to the momentum diffusivity and inversely proportional to the mass diffusivity; consequently, the greater values of $S_c$ conformed to the small mass diffusivity which caused decline in the concentration function. Figure 15 depicts the effects of $\tau_1$ on $\varphi(\eta)$. The findings show that the
concentration distribution gets more non-uniform as the particle size increases which resists the reduction of the values of the concentration profile.

Tables 1–2 presented the effects of physical parameters on $f''(0)$, $-\theta'(\eta)$, and $-\varphi'(0)$. The effects of material parameter $A$ increased which reduces the skin friction. Because the viscosity of the fluid near the surface enhanced due to enhanced the material parameter which resist to reduce the skin friction.
friction. The variation of the skin friction and magnetic field parameter is presented in Table 1. The values of $f^\prime\prime\prime(0)$ are enhanced due to higher values of $M$. These phenomena reduce the velocity profile because the thickness of the fluid is enhanced due to enhanced skin friction. The impacts of $P_{rm}$ on $f^\prime\prime\prime(0)$ are highlighted in Table 1. It is observed that skin friction $f^\prime\prime\prime(0)$ is enhanced for higher values of $P_{rm}$. It is noted that motion of the fluid reduced which reduces the skin friction due to

![Figure 8. Variation of $\theta(\eta)$ and $P_r$](image)

![Figure 9. Variation of $\theta(\eta)$ and $S_1$](image)
higher values of $P_{cm}$. The variation of $E_c$ and $f''(0)$ is presented in Table 2. There is no variation found between $E_c$ and $f''(0)$.

The variation of $N_b$ and $-\theta'(\eta)$ and $-\varphi'(0)$ which reveals in Table 2. It is noted that values of $-\theta'(\eta)$ increase because Brownian motion parameter enhances, which improve the heat transfer rate because the movement of the particles of concentration is from higher area to lower area. The values
of $-\phi'(0)$ increased due to higher values of $N_b$ because the motion of the particle is enhanced which enhances $-\phi'(0)$. The $\phi(\eta)$ thickness enhances due to enhanced values of $N_b$ because the movement of the particles of concentration is from higher area to lower area. The variation of the $N_i$ and $-\theta'(\eta)$ and $-\phi'(0)$ which reveals in Table 2. It is noted that values of $-\theta'(\eta)$ increase with higher
values of thermophoresis parameter which increase the heat transfer rate, while the values of $-\phi'(0)$ decline due to higher values of $N_{t}$ because the effect becomes more pronounced as the average concentration increases. The impacts of $E_{c}$ on $-\theta'(\eta)$ and $-\phi'(0)$ are presented in Table 2. It is noted that the values of $Pr$ are enhanced which enhance the heat transfer rate at the contact point of the
surface and fluid while the values of $-\theta'(0)$ increased. The heat transfer reduced due to higher values of $Pr$. The variation of $\tau_1$ and $-\theta'(\eta)$ and $-\phi'(0)$ which reveals in Table 2. It is noted that values of the $-\theta'(\eta)$ increase, while the values of $-\phi'(0)$ decline due to higher values of $\tau_1$. The

| $A$ | $M$ | $P_m$ | $E_c$ | $f''(0)$ |
|-----|-----|-------|-------|----------|
| 0.1 | 0.3 | 0.3   | 1.2   | 1.27703  |
| 0.2 | —   | —     | —     | 1.46242  |
| 0.3 | —   | —     | —     | 1.63755  |
| 0.2 | 0.1 | —     | —     | 0.25489  |
| —   | 0.2 | —     | —     | 0.24021  |
| —   | 0.3 | —     | —     | 1.22848  |
| —   | —   | 0.1   | —     | 1.24821  |
| —   | —   | 0.2   | —     | 1.23683  |
| —   | —   | —     | —     | 1.21974  |
| —   | —   | —     | —     | 1.24821  |
| —   | —   | —     | —     | 1.24821  |

| $A$ | $N_b$ | $N_c$ | $E_c$ | $P_r$ | $\tau_1$ | $S_c$ | $-\theta'(0)$ | $-\phi'(0)$ |
|-----|-------|-------|-------|-------|----------|-------|---------------|--------------|
| 0.1 | —     | —     | —     | —     | —        | —     | 1.00294       | 0.3483       |
| 0.2 | —     | —     | —     | —     | —        | —     | 1.00145       | 0.3541       |
| 0.3 | —     | —     | —     | —     | —        | —     | 0.999877      | 0.3594       |
| 0.3 | 0.1   | —     | —     | —     | —        | —     | 1.00294       | 0.3483       |
| —   | 0.2   | —     | —     | —     | —        | —     | 1.00172       | 0.3015       |
| —   | 0.3   | —     | —     | —     | —        | —     | 1.0005        | 0.2859       |
| —   | —     | 0.0   | —     | —     | —        | —     | 1.00331       | 0.2556       |
| —   | —     | 0.5   | —     | —     | —        | —     | 0.98525       | 0.4014       |
| —   | —     | 0.6   | —     | —     | —        | —     | 0.981742      | 0.4289       |
| —   | —     | —     | 0.3   | —     | —        | —     | 0.246018      | 0.2891       |
| —   | —     | —     | 0.5   | —     | —        | —     | 0.253782      | 0.2923       |
| —   | —     | —     | 0.7   | —     | —        | —     | 0.261544      | 0.2956       |
| —   | —     | —     | —     | 0.1   | —        | —     | 0.238251      | 0.2859       |
| —   | —     | —     | —     | 0.2   | —        | —     | 0.330193      | 0.3186       |
| —   | —     | —     | —     | 0.3   | —        | —     | 0.41175       | 0.368        |
| —   | —     | —     | —     | 0.1   | —        | —     | 0.238251      | 0.2859       |
| —   | —     | —     | —     | —     | 0.2      | —     | 0.238257      | 0.2834       |
| —   | —     | —     | —     | —     | 0.3      | —     | 0.238263      | 0.2810       |
| —   | —     | —     | —     | —     | 0.1      | —     | 0.238251      | 0.2859       |
| —   | —     | —     | —     | —     | 0.2      | —     | 0.238129      | 0.3872       |
| —   | —     | —     | —     | —     | 0.3      | —     | 0.238033      | 0.4795       |
variation of the $S_c$ and $-\theta'(\eta)$ and $-\varphi'(0)$ which reveals in Table 2. It is noted that values of $-\theta'(\eta)$ increase, while the values of $-\varphi'(0)$ decline due to higher values of $S_c$.

**Final remarks**

We considered the two-dimensional flow of second-grade fluid under the induced magnetic field over an exponential stretching sheet. The variable thermal conductivity and viscous dissipation under the chemical reaction effects at exponentially stretching sheet are discussed in this analysis. The following achievements are highlighted:

- Velocity concentration profile is enhanced due to larger values of $A$ but declines for higher values of $M$.
- Material parameter $A$ and Prandtl number $P_r$ are enhanced which resist the decrease in the temperature and concentration profiles.
- Magnetic profile is raised due to higher values of the material parameter $A$, $P_{rm}$, and $M$.
- $-\theta'(0)$ is enhanced for various values of $A$, but the inverse tendency of the Nusselt number is noticed for larger values of $M$ and $P_{rm}$.
- $-\varphi'(0)$ increases by higher values of $A$, $N_t$, $E_c$, $P_r$, and $\tau_1$ but declines for larger values of $N_h$ and $\tau_1$.

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**Appendix**

**Notation**

- $\rho$ Density
- $\nu$ Kinematic viscosity
- $(u,v)$ Velocity components
- $\mu$ Dynamic viscosity
- $(x,y)$ Space coordinates
- $(H_1,H_2)$ Components of induced magnetic field
- $Pr$ Prandtl number
- $T$ Temperature
- $C_p$ Heat capacity
- $\alpha_1$ Material constant
- $K(T)$ Thermal conductivity depending upon temperature
- $D_B$ Brownian motion
- $C$ Concentration of second-grade fluid
- $D_T$ Thermophoresis
- $T_\infty$ Ambient temperature
- $K_\infty$ Thermal conductivity of the fluid
- $\varepsilon$ Small parameter
- $U_0$ Reference velocity
- $T_0$ Ambient temperature
- $C_\infty$ Ambient concentration
- $C_0$ Ambient estimation for concentration
- $\eta$ Dimensionless variable
- $H_0$ Ambient estimation for magnetic field
- $\alpha$ Second-grade nanofluid parameter
- $M$ Magnetic field parameter
- $Pr_m$ Magnetic Prandtl number
- $N_b$ Brownian motion parameter
\( L_e \)  Lewis number
\( \tau_w \)  Wall shear stress
\( q_{w} \)  Wall heat flux
\( \sqrt{ReC_{f}} \)  Dimensionless expression skin of friction
\( N_u \)  Dimensionless expression of the Nusselt number
\( Sh \)  Dimensionless expression of the Sherwood number
\( N_t \)  Thermophoresis parameter
\( \theta \)  Temperature
\( \phi \)  Concentration