Teleportation with two-dimensional electron gas formed at the interface of a GaAs heterostructure

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Abstract

Inspired by the scenario proposed by Bennett et al., a teleportation protocol of qubits formed in a two-dimensional electron gas formed at the interface of a GaAs heterostructure is presented. The teleportation is carried out using three GaAs quantum dots (say ′PP′, ′QQ′, ′RR′) and three electrons. The electron spin on GaAs quantum dots ′PP′ is used to encode the unknown qubit. The GaAs quantum dot ′QQ′ and ′RR′ combine to form an entangled state. Alice (the sender) performs a Bell measurement on pairs (P, Q) and (′′P, ′′Q). Depending on the outcome of the measurement, a suitable Hamiltonian for the quantum gate can be used by Bob (the receiver) to transform the information based on spin to charge-based information. This work offers relevant corrections to the misconception in Weng and Kais (2006 Chem. Phys. Lett. 421 338).

Keywords: two-dimensional electron gas, quantum gate, entanglement, teleportation

(Some figures may appear in colour only in the online journal)

1. Introduction

In quantum teleportation, the whole object positioned at a point, say A, cannot be teleported, but its state (with the aid of classical communication and previously shared entanglement) between points A and another point, say B, can be. This idea was expounded in a paper by Bennett et al [1]. A description of the standard teleportation protocol is as follows: the sender Alice has a source qubit, say |Φ⟩A = |a⟩0 + |b⟩1, which she wants to teleport to Bob. Suppose Alice and Bob share another qubit. Then, the entangled state of the two parties could be represented by |

\[(\text{var})_{AB} = 1/\sqrt{2} (|0⟩A ⊗ |0⟩B + |1⟩A ⊗ |1⟩B)\]

where the A and B denote the quantum state of Alice and Bob, respectively. Since Alice does not know the state of |Φ⟩A, the laws of quantum mechanics do not permit this since she only has a single copy of |Ψ⟩A in her possession, it is impossible for her to precisely measure and specify it. Now, Alice relates the qubit in her possession to half of the EPR pair (i.e. |Φ⟩A = Φ ′′ A Ψ ′′), where |Φ ′′⟩A denotes the state input to the circuit) and sends the qubits through a C-NOT gate to obtain |

\[(\text{var})_{AB} = 1/\sqrt{2} (a|0⟩(00′ + |1⟩1) + b|1⟩(10′ + |0⟩0) + a|0⟩(00′ + |1⟩1) + b|1⟩(10′ + |0⟩0))\]

and then sends the first qubit through a Hadamard gate. The result of this becomes

\[a|0⟩(00′ + |1⟩1) + b|1⟩(10′ + |0⟩0)\]

+ |1⟩0(a|0⟩ + b|1⟩) + |0⟩1(a|1⟩ + b|0⟩)

+ |1⟩0(a|0⟩ - b|1⟩) + |0⟩1(a|1⟩ - b|0⟩).

(1)
From the equation (1), one can deduce that each state of Alice’s qubits corresponds to a state of Bob’s qubit. Once Bob gets the result of this measurement, he can reconstruct the state \( |\Phi^+\rangle_k \) by applying the appropriate quantum gate. As has just been demonstrated, a quantum state can be recovered in a remote location with the aid of a maximally entangled EPR pair and two bits of classical information. On the basis of this novel idea, there has been continuous interest in studying quantum teleportation. The literature is huge, however, and the following [2–8] give some older as well as more recent studies. Moreover, the usefulness of entanglement is not restricted to teleportation alone: it also forms an imperative component in quantum cryptography and quantum information sharing. All these justify indispensability of an entanglement in quantum information processing. It is a manifestation of intrinsic non-locality in the quantum mechanics. A comprehensive understanding of the behavior of entangled systems in their environments has been an investigation which is common to quantum measurement and quantum information. To the best of our knowledge, little has been achieved in relation to teleportation in semiconductors. Nanoscale systems such as quantum dots and superconducting circuits make good candidates within standard semiconductor technology for practical quantum computers. Within this context, a quantum teleportation might be a decisive confirmation of potentiality.

A quantum dot is a semiconductor nanostructure that incarcerates (confinement can be as a result of the presence of electrostatic potentials, semiconductor surface, or an interface between different semiconductor materials) the motion of conduction-band electrons, valence-band holes, or excitons (bound pairs of conduction-band electrons and valence-band holes) in all three spatial directions. A crucial advancement in quantum computation was the realization of a dot in GaAs [9]. In GaAs quantum dots, electron spins are used as qubits. The qubits are formed in a standard two-dimensional electron gas (2DEG) obtained at the interface of a GaAs/AlGaAs heterostructure. For the realization of well-defined spin qubits, the model equation for the on-site (\( U \)) and nearest-neighbor (\( U_{ij} \)) Coulomb repulsion, respectively, in a magnetic field is given by the Hamiltonian

\[
H = H_0 + V_m = \frac{1}{2} \sum_i U N_i(N_i - 1) + U_{ij} N_i N_j - e \sum_i V N_i + \sum_{i,k} \varepsilon_k n_{i,k} - \mu \cdot B,
\]

(2)

where \( H_0 \) is the unperturbed Hamiltonian [10] and \( V_m \) is the correction to \( H_0 \) which results from magnetic fields and spin couplings. \( \mu = g \mu_B \sum_n S_n \) is the magnetic moment with \( \mu_B \) being the Bohr magneton. The \( mth \) electron’s spin-1/2 in the double dot is denoted by \( S_m \). \( g \) is the Landé g-factor and \( N = \sum n_k \) counts the total number of electrons in dot \( i \), with \( n_k = \sum d_{k,i}^\dagger d_{k,i} \) annihilates an electron on dot \( i \), orbital \( k \) with spin \( \sigma \), \( \varepsilon_k \) denotes the energy of a single-particle orbital level in the dot, which yields the typical orbital-level spacing \( \varepsilon_{k+1} - \varepsilon_k \approx \hbar \omega_0 \).

With a sufficiently large magnetic-field intensity, a system of qubits can be initialized. Xu et al [11] initialized a spin state with a singly charged InAs–GaAs quantum dot by optical cooling with near unity efficiency. The experiment requires magnetic field of 0.88 T in Voigt geometry and a temperature of 5–0.06K. The details can be found in [11]. Very recently Mar et al [12] demonstrated that without the need for a magnetic field, the initialization of a single quantum-dot hole spin with high fidelity (lower bound >97%), on picosecond timescales could also be realized. The spin configuration has relevant eigenstates corresponding to \( |1\rangle \) or \( |\uparrow\rangle \).

By occupying each dot with exactly one electron, the spin qubits are realized. Generally speaking, there are many systems that could be employed as qubits in a quantum computation such as the polarization of a single photon (the two states are vertical polarization and horizontal polarization). In this work, we demonstrate a quantum teleportation protocol by utilizing the spins of electrons confined in GaAs quantum dots. The teleportation is carried out using three GaAs quantum dots (say \( P^P, Q^Q, R^R \)) and three electrons. The electron spin on GaAs quantum dot \( P^P \) is used to encode the unknown qubit. The GaAs quantum dots \( Q^Q \) and \( R^R \) combine to form an entangled state. We perform a Bell measurement and, depending on the outcome of the measurement, a suitable Hamiltonian for the quantum gate can be used to transform the spin-based information to charge-based information.

2. Demonstration of quantum teleportation in quantum dots

Now, let there be two participants, spatially separated at different sites in a quantum network, customarily called ‘Alice’ and ‘Bob’. The qubit that Alice, who is located at site \( P \), wishes to teleport to Bob at site \( R \), has been obtained in a standard 2DEG formed at the interface of a GaAs/AlGaAs heterostructure, and it can be written as \( |\psi\rangle = \alpha |\downarrow\rangle + \beta |\uparrow\rangle \), where \( \alpha \) and \( \beta \) are complex and satisfy the relation \( |\alpha|^2 + |\beta|^2 = 1 \). The entangled state of \( Q \) and \( R \) can be written in an occupation number basis \( |n_Q \downarrow n_R \downarrow n_R \downarrow \rangle \)

\[
|\lambda\rangle_{Q^Q R^R} = \frac{1}{\sqrt{2}} \left( |0011\rangle + |0100\rangle + |1011\rangle + |1100\rangle \right).
\]

Qubit pair (\( Q^Q \)) belongs to Alice while \( (R^R) \) belongs to Bob. It can be deduced from equation (3) that there are four possible states in \( Q^Q \) which correspond to each state in \( R^R \), i.e., \( |00\rangle \leftrightarrow |01\rangle, |01\rangle \leftrightarrow |00\rangle, |10\rangle \leftrightarrow |11\rangle \) and \( |11\rangle \leftrightarrow |10\rangle \). The combined state of the qubits becomes

\[
|\psi^{(0)}\rangle_{PPP\overline{P}Q\overline{Q}R\overline{R}} = |\psi\rangle_{PPP} |\lambda\rangle_{Q\overline{Q}R\overline{R}}.
\]

For Alice to achieve her aim, she performs a Bell state measurement (BSM) on her qubit pair \( (P\overline{Q}) \) to obtain the states of other qubits as

\[
|\psi^{(1)}\rangle_{PPPQ\overline{Q}R\overline{R}} = \frac{1}{\sqrt{2}} \left( |01\rangle + |10\rangle \right) = |\psi^{(0)}\rangle_{PPPQ\overline{Q}R\overline{R}}.
\]

(4a)
Thus, the states of qubit pair \((\alpha\beta)\) respectively. The above states become \((\alpha\beta)^2\), where \((\alpha\beta)\) represent the results corresponding to a BSM on the qubit pairs \((P, Q)\) and \((P', Q')\) respectively. The above states represent the 16 possible states which Alice’s system will collapse into after the measurement. One can also observe that these states are pure entangled two-qubit states. Alice then communicates the results of the measurement to Bob, who can choose an apt unitary transformation via the electron spin-up and spin-down as basis in order to completely recover the information. A more explicit expression for equations (5a)–(5d) and the appropriate unitary transformation which Bob could utilize in reconstructing the original state, are shown in table 1. This process is also illustrated in figure 1.

It is worth mentioning that because of the nonlinearity of interactions which is involved in the model, the only obstacle which could stop our Bell measurement from reaching 100% success probability is noise. Noise may set in while Alice communicates the results of the measurement to Bob, who can choose an apt unitary transformation. Then, Bob could utilize in reconstructing the original state, are shown in table 1. This process is also illustrated in figure 1.

| Alice’s result | State obtained by Bob | Unitary transformation |
|---------------|-----------------------|-----------------------|
| \([\Psi^+]\) | \(\alpha\|1\rangle + \beta\|\uparrow\rangle\) | \((\uparrow\downarrow)\) |
| \([\Psi^-]\) | \(-\alpha\|1\rangle + \beta\|\uparrow\rangle\) | \((\downarrow\uparrow)\) |
| \([\Phi^+]\) | \(\alpha\|1\rangle - \beta\|\uparrow\rangle\) | \((\uparrow\downarrow)\) |
| \([\Phi^-]\) | \(-\alpha\|1\rangle - \beta\|\uparrow\rangle\) | \((\downarrow\uparrow)\) |
| \([\Psi^+]\) | \(\alpha\|1\rangle + \beta\|\uparrow\rangle\) | \((\uparrow\downarrow)\) |
| \([\Psi^-]\) | \(-\alpha\|1\rangle + \beta\|\uparrow\rangle\) | \((\downarrow\uparrow)\) |
| \([\Psi^+]\) | \(\alpha\|1\rangle - \beta\|\uparrow\rangle\) | \((\uparrow\downarrow)\) |
| \([\Psi^-]\) | \(-\alpha\|1\rangle - \beta\|\uparrow\rangle\) | \((\downarrow\uparrow)\) |


Table 1. Alice’s results, the corresponding state obtained by Bob and the appropriate unitary transformation which can be utilized by Bob to reconstruct the original state of the qubit. We have ignored the normalization for convenience.

\[
PQ\langle \Psi^+ | \hat{\Phi}^{0(1)} \rangle_{PPQQ RR} = \frac{1}{2\sqrt{2}} \left[ \pm^{(1)} \alpha \|0001\rangle + |0100\rangle + \beta \|1101\rangle + |1110\rangle \right] |
\]

and then on qubit pair \((P', Q')\). Thus, the states of qubit pair \((RR)\) become

\[
PQ'\langle \Psi^+ | \hat{\Phi}^{0(1)} \rangle_{PP'QQ' RR} = \frac{1}{4} \left[ \pm^{(1)} \pm^{(2)} \right] |
\]

where we have denoted the four Bell states as \([\Phi^\pm] = 2^{-1/2}(|\uparrow\rangle \pm |\downarrow\rangle)\) and \([\Phi^\mp] = 2^{-1/2}(|\downarrow\rangle \pm |\uparrow\rangle)\). The \(\pm^{(1)}\) and \(\pm^{(2)}\) represent the results corresponding to a BSM on the qubit pairs \((P, Q)\) and \((P', Q')\) respectively. The above states represent the 16 possible states which Alice’s system will collapse into after the measurement. One can also observe that these states are pure entangled two-qubit states. Alice then communicates the results of the measurement to Bob, who can choose an apt unitary transformation via the electron spin-up and spin-down as basis in order to completely recover \([\Psi^+]\) on site R. Thus, the spin-based information has undergone a transformation to charge-based information via a proper Hamiltonian for the quantum gate. Moreover, the content of the information is unaffected. A more explicit expression for equations (5a)–(5d) and the appropriate unitary transformation which Bob could utilize in reconstructing the original state, are shown in table 1. This process is also illustrated in figure 1.

It is worth mentioning that because of the nonlinearity of interactions which is involved in the model, the only obstacle which could stop our Bell measurement from reaching 100% success probability is noise. Noise may set in while Alice performs the Bell measurement and Bob performs the unitary operation. This might be due to an imperfection of local operation. The properties of teleportation through noisy quantum channels can be measured by the fidelity. The most significant model for noise is the depolarizing channel, which is known to introduce white noise. The state of a quantum system after transmission through a depolarizing channel can be written as \(\hat{\mathcal{E}}_d(\hat{\rho}) = p\hat{I} + (1 - p)\hat{\rho}\), where \(\hat{\rho}\) is a general mixed state representing the initial state of the qubit and \(d\) denotes the dimension of Hilbert space. This expression can be described by saying that, with probability \(p\) an error occurs, while with probability \(1 - p\), the qubit remains intact.
3. Conclusion

In this paper, we have presented a model of quantum teleportation protocol by utilizing the spins of electrons confined in GaAs quantum dots to perform a teleportation. We have utilized three systems of electrons \( e^3 \). However, for \( e^3 > 0 \), where on-site Coulomb repulsion approaches positive infinity, there will be no double occupation and the anti-parallel configuration will be favored by the neighboring spin. Moreover, the current work has corrected the misconception in [2]. The authors performed an incorrect BSM. We will not go into detail on this here. However, we would like to direct the attention of the reader to equation (16) of [2] which is the output of the Hadamard transformation. The Hadamard transform for two qubits is widely known to be

\[
H = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 \\
1 & -1 \\
1 & 1 \\
1 & -1
\end{pmatrix}
\]

(6)

Obviously, its operation onto \( \alpha|10\rangle \) should be

\[
\frac{1}{2} \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 \\
1 & -1 & 1 & 1 \\
1 & 1 & -1 & -1
\end{pmatrix}
\begin{pmatrix}
0 \\
0 \\
\alpha \\
-\alpha
\end{pmatrix} = \frac{1}{2} \begin{pmatrix}
\alpha \\
-\alpha \\
\alpha \\
-\alpha
\end{pmatrix}
\]

(7)

while for \( \beta|01\rangle \) it should be \( \beta/2 (|00\rangle + |01\rangle - |10\rangle - |11\rangle) \).

Supposing that the idea of Alice performing a BSM on qubit pair (AACC) is right, then, and one can clearly see that equation (16) in [2] is wrong and the correct expression should have been

\[
|\Psi_2\rangle = \frac{\alpha}{2\sqrt{2}} (|00\rangle + |01\rangle - |10\rangle - |11\rangle)(|0000\rangle + |1111\rangle)
\]

\[
+ \frac{\beta}{2\sqrt{2}} (|00\rangle - |01\rangle + |10\rangle - |11\rangle)(|1100\rangle + |0011\rangle).
\]

The authors proceeded with the wrong expression (16) to obtain (17) which is another flawed idea. The well-known circuit for BSMs can be found in [13]. Now, this implies that if we are to follow their idea by performing a BSM on qubit pair (AACC), then \( x \) would be AA and \( y \) would be CC which is not meaningful (see equation (1.27) of [13] for meaning of \( x \) and \( y \)). For instance, let us consider the pair \( |1011\rangle \) in [2]. Then, using equation (1.27) of [13], one will obtain \( |\beta_{10,11}\rangle = (|0,11\rangle + (-1)^{10} |1,1\rangle)\sqrt{2} \) which is not meaningful. The same approach was also applied to qubit \( \beta_1 \) as shown in equations (19)–(22) of [2]. The approach of the current study has corrected this misconception.

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