The effects of viscosity on circumplanetary disks *

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Received 2012 February 14; accepted 2012 July 16

Abstract The effects of viscosity on the circumplanetary disks residing in the vicinity of protoplanets are investigated through two-dimensional hydrodynamical simulations with the shearing sheet model. We find that viscosity can considerably affect properties of the circumplanetary disk when the mass of the protoplanet \( M_p < \sim 33 \, M_{\oplus} \), where \( M_{\oplus} \) is the Earth’s mass. However, effects of viscosity on the circumplanetary disk are negligibly small when the mass of the protoplanet \( M_p \gtrsim 33 \, M_{\oplus} \). We find that when \( M_p \lesssim 33 \, M_{\oplus} \), viscosity can markedly disrupt the spiral structure of the gas around the planet and smoothly distribute the gas, which weakens the torques exerted on the protoplanet. Thus, viscosity can slow the migration speed of a protoplanet. After including viscosity, the size of the circumplanetary disk can be decreased by a factor of \( \gtrsim 20\% \). Viscosity helps to transport gas into the circumplanetary disk from the differentially rotating circumstellar disk. The mass of the circumplanetary disk can be increased by a factor of \( 50\% \) after viscosity is taken into account when \( M_p \lesssim 33 \, M_{\oplus} \). Effects of viscosity on the formation of planets and satellites are briefly discussed.

Key words: accretion, accretion disks — hydrodynamics — planets and satellites: formation — solar system: formation

1 INTRODUCTION

To date, more than 500 exoplanets have been detected. Most of the exoplanets are gas giant planets, as massive planets are preferentially observed by current detection methods. Thus, it is important to understand the formation process of gas giant planets. According to the core accretion model, a solid core with several \( M_{\oplus} \) first forms through coagulation of planetesimals in the circumstellar disk, where \( M_{\oplus} \) is the Earth’s mass. The protoplanet captures a hydrostatic envelope when its mass is less than \( M_p \lesssim 10 \, M_{\oplus} \) (Mizuno 1980; Stevenson 1982; Bodenheimer & Pollack 1986; Pollack et al. 1996; Ikoma et al. 2000, 2001; Hubickyj et al. 2005). Ikoma et al. (2000) showed that run-away gas accretion is triggered when the solid core mass exceeds \( \sim 5 - 20 \, M_{\oplus} \), and the protoplanet quickly gains mass by gas accretion. A gas giant planet acquires almost all of its mass in the run-away gas accretion phase.

Since gas accreting from a differentially rotating circumstellar disk has nonzero angular momentum, a circumplanetary disk can form around the protoplanet (Tanigawa & Watanabe 2002).

*Supported by the National Natural Science Foundation of China.
The circumplanetary disk can influence many aspects of the protoplanet. For example, previously, when calculating the torque on the protoplanet, the contribution of gas inside the whole Roche lobe (or Hill radius) was neglected. This may not be appropriate. The size of the circumplanetary disk may be smaller than the Roche lobe (as shown in Sect. 3.2); therefore, when calculating the torque on the protoplanet, the gas inside the Roche lobe but beyond the outer boundary of the circumplanetary disk should be taken into account. Also, properties of the circumplanetary disk can determine the evolution of a protoplanet and its resulting mass. Finally, satellites form in the vicinity of the protoplanet. Studying the properties of the circumplanetary disk helps to investigate the formation process of satellites.

The gas accretion process onto a protoplanet has been investigated using global simulations by many authors (e.g. Bryden et al. 1999; Kley 1999; Lubow et al. 1999; Kley et al. 2001; D’Angelo et al. 2002, 2003; Bate et al. 2003). However, since the main purpose of this research was to study gap formation and planet migration on a large scale, the region in the vicinity of the protoplanet has not been modeled with sufficient resolution. Thus, in their simulations, the properties of the circumplanetary disk were not thoroughly explored. The fine structure of the circumplanetary disk has been examined with shearing sheet models without viscosity (Tanigawa & Watanabe 2002; Machida et al. 2008, 2010; Machida 2009). A remaining question in their models is: what is the mechanism of angular momentum transport in the circumplanetary disk? Gravitational interaction between the protoplanet and gas can produce spiral shocks inside the Roche lobe (or Hill radius) of a protoplanet. Gas flows into the Hill sphere of a protoplanet through the inner and outer Lagrange points. The gas that flows into the Roche lobe from the inner (outer) Lagrange point will undergo a strong shock on the opposite outer (inner) Lagrange side of the Roche lobe, so angular momentum is lost through the collision between gas and shocks. The gas spirals inward toward the protoplanet as a result of successive shocks.

Obviously, there should be viscosity in the circumstellar disk, which drives the gas flow in the disk toward the central star. The most promising origin of viscosity in the circumstellar disk should be magnetic turbulence generated by the magnetorotational instability (MRI) (Balbus & Hawley 1991, 1998). Previous work found that despite the ionization rate of a circumstellar disk being low, the magnetic field can remain dynamically important, and MRI perturbations can grow under a wide range of fluid conditions and magnetic field strengths (Salmeron & Wardle 2005). Viscosity may play important roles on the properties of the circumplanetary disk. The angular momentum profiles of a circumplanetary disk may be significantly affected by viscosity. Also, viscosity helps to transport gas into the circumplanetary disk from the differentially rotating circumstellar disk; the mass of the circumplanetary disk may be significantly affected by viscosity. Therefore, it is of great importance to study the effects of viscosity on the properties of circumplanetary disks.

In this paper, we study the effects of viscosity on circumplanetary disks by considering the shearing sheet models. We use an anomalous stress tensor to mimic the shear stress originating from magnetohydrodynamic (MHD) turbulence. In Section 2, we describe our models. Results are described in Section 3. We discuss and summarize our conclusions in Section 4.

2 MODEL

2.1 Equations

We confine our models to two-dimensions. We assume that the temperature is constant and that the self-gravity of the disk is negligible. The orbit of the protoplanet is assumed to be circular in the equatorial plane of the circumstellar disk. The protoplanet is not allowed to migrate.

We consider a local region around a protoplanet, using the shearing sheet model (e.g. Goldreich & Lynden-Bell 1965). We take local Cartesian coordinates rotating with the protoplanet with the origin at the protoplanet and the $x$- and $y$- axes in the radial and azimuthal directions of the disk,
respectively. We solve the equations of hydrodynamics without self-gravity

\[
\frac{\partial \Sigma}{\partial t} + \nabla \cdot (\Sigma v) = 0, 
\]

\[
\frac{\partial v}{\partial t} + (v \cdot \nabla)v = -\frac{1}{\Sigma} \nabla P - \nabla \Phi - 2\Omega_p e_z \times v + \frac{1}{\Sigma} \nabla \cdot T, 
\]

where \( \Sigma \) is the surface density, \( v \) is the velocity, \( P \) is the vertically integrated gas pressure, \( \Phi \) is the gravitational potential, \( \Omega_p \) is the Keplerian angular velocity of the protoplanet, \( e_z \) is the unit vector along the rotation axis of the protoplanet, and \( T \) is the vertically integrated anomalous stress tensor.

We adopt an isothermal equation of state,

\[
P = \Sigma c_s^2, 
\]

where \( c_s \) is the sound speed. The angular velocity of the protoplanet is given by

\[
\Omega_p = \left( \frac{GM_c}{a_p^3} \right)^{1/2}, 
\]

where \( G, M_c \) and \( a_p \) are the gravitational constant, the mass of the central star, and the orbital radius of the protoplanet, respectively. The gravitational potential is given by

\[
\Phi = -\frac{3}{2} \Omega_p^2 x^2 - \frac{GM_p}{r}, 
\]

where \( M_p \) and \( r \) are the mass of the protoplanet and the distance from the center of the protoplanet, respectively. The first term is composed of the gravitational potential of the central star and the centrifugal potential. The second term is the gravitational potential of the protoplanet. The Hill radius inside which the protoplanet’s gravity dominates is defined as

\[
R_H = \left( \frac{M_p}{3M_c} \right)^{1/3} a_p. 
\]

Using the Hill radius, we can rewrite Equation (4) as

\[
\Phi = \Omega_p^2 \left( -\frac{3x^2}{2} - \frac{3R_H^3}{r} \right). 
\]

We use the stress tensor \( T \) to mimic the shear stress, which is in reality magnetic stress associated with MHD turbulence driven by MRI. We assume that the only non-zero component of the stress tensor \( T \) is

\[
T_{xy} = \mu \left( \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} \right). 
\]

This is because MRI is driven only by the shear associated with orbital dynamics. In Equation (7), \( \mu = \Sigma \alpha c_s^2 / \Omega_K \), \( \alpha \), \( c_s \), and \( \Omega_K \) are viscosity coefficient, sound speed and Keplerian angular velocity about the protoplanet, respectively. A previous paper (i.e. Papaloizou et al. 2004) studying the interaction between a protoplanet and a magnetized circumstellar disk found that \( \alpha \sim 3 \times 10^{-3} \). The maximum \( \alpha \) used in this paper is \( 3 \times 10^{-3} \).

In this paper, we normalize length by the scale height of the circumstellar disk \( h = c_s / \Omega_p \), time by the inverse of the Keplerian angular velocity of the protoplanet \( \Omega_p^{-1} \), and the surface density by the unperturbed surface density of the standard solar nebula model (Hayashi 1981; Hayashi et al. 1985).
2.2 Numerical Method

The numerical simulations are performed using the ZEUS-2D code (Stone & Norman 1992a,b). Our initial settings are similar to those of Machida et al. (2008). The gas flow has a constant shear in the $x$-direction as

$$v_0 = \left(0, \frac{3}{2}x\right).$$

(9)

Initially, the gas has uniform surface density in the unperturbed disk. In this paper, our standard computational domain is defined by $|x| \leq 6(= x_{\text{max}})$ and $|y| \leq 12(= y_{\text{max}})$. We adopt logarithmically spaced grids with the finest resolution ($\Delta x = 0.003$) around the protoplanet. The total number of grids for our standard computational domain is $500 \times 1000$, so the resolution is high enough to study the circumplanetary disk in the vicinity of the protoplanet. We inject gas with the linearized Keplerian shear on $y = y_{\text{max}}, \ (0 < x < x_{\text{max}})$ and $y = -y_{\text{max}}, \ (-x_{\text{max}} < x < 0)$. For the rest of the boundaries, we adopt outflow boundary conditions.

In order to avoid singularity in the proximity of the protoplanet, the gravitational potential of the protoplanet is smoothed in its neighborhood with

$$\Phi_p = \frac{-GM_p}{(r^2 + r_{\text{sm}}^2)^{1/2}},$$

(10)

where $r_{\text{sm}}$ is the smoothing length of the protoplanet’s potential. In this paper, we choose $r_{\text{sm}} = 0.05$. Tanigawa & Watanabe (2002) showed that $r_{\text{sm}} = 0.05$ is safe to study the properties of the circumplanetary disk in the vicinity of the protoplanet.

In the runaway gas accretion phase, accretion by a planet is non-negligible. In order to best represent the physical processes in this system, we include accretion. Therefore, we should mimic the accretion process by a planet. Because the growth timescale of a planet is much longer than the typical time of our simulations, we should have a constant accretion rate, which is independent of the parameters used to mimic the accretion process. As done by Tanigawa & Watanabe (2002), the gas inside $r_{\text{sink}}$ is removed by a constant rate $[\Sigma^{n+1} = \Sigma^n(1 - \Delta t)], \ \Delta t(\ll 1) \text{ is a time step in the calculations, and superscript } n \text{ is the number of numerical time steps.}$ Tanigawa & Watanabe (2002) have tested the effects of $r_{\text{sm}}$ and $r_{\text{sink}}$ on the accretion rate of the planet. They found that the results do not depend on the values of $r_{\text{sm}}$ and $r_{\text{sink}}$ as long as $r_{\text{sm}} = r_{\text{sink}} < 0.07$. In this paper, we set $r_{\text{sm}} = r_{\text{sink}} = 0.05$.

2.3 Scaling

In the standard solar nebula model (Hayashi 1981; Hayashi et al. 1985), the temperature $T$, sound speed $c_s$, and gas density $\rho_0$ are given by

$$T = 280 \left(\frac{L}{L_\odot}\right)^{1/4} \left(\frac{a_p}{1 \text{ AU}}\right)^{-1/2},$$

(11)

where $L$ and $L_\odot$ are the protostellar and solar luminosities;

$$c_s = \left(\frac{kT}{\mu m_H}\right)^{1/2} = 1.9 \times 10^4 \left(\frac{T}{10^4 \text{ K}}\right)^{1/2} \left(\frac{2.34}{\mu}\right)^{1/2} \text{ cm} \text{ s}^{-1},$$

(12)

where $\mu = 2.34$ is the mean molecular weight of the gas composed mainly of H₂ and He; and

$$\rho_0 = 1.4 \times 10^{-9} \left(\frac{a_p}{1 \text{ AU}}\right)^{-11/4} \text{ g cm}^{-3},$$

(13)
respectively. When the values of \( M_c = 1 M_\odot \) and \( L = 1 L_\odot \) are adopted, using Equations (4), (11) and (12), we can describe the scale height \( h \) as

\[
h = 5.0 \times 10^{11} \left( \frac{a_p}{1 \text{AU}} \right)^{5/4} \text{ cm}.
\] (14)

The mass of the planet in units of Jupiter mass can be described as

\[
\frac{M_p}{M_{\text{Jup}}} = 0.12 \left( \frac{M_c}{1 M_\odot} \right)^{-1/2} \left( \frac{a_p}{1 \text{AU}} \right)^{3/4} \left( \frac{R_H}{h} \right)^3.
\] (15)

In this paper, we assume that \( M_c = 1 M_\odot \) and \( L = 1 L_\odot \). The planet is located at \( a_p = 5.2 \text{AU} \). Therefore, the temperature of the gas is \( T = 123 \text{K} \). Because our shearing box just represents a local region around the planet, we further assume that the temperature is uniform in the whole computational domain.

In the paper below, the Hill radius \( R_H \) is in units of \( h \). In Table 1, \( R_H = 0.4, 0.5 \) and 0.63 correspond to planet mass of 0.026, 0.05 and 0.1 Jupiter mass, respectively.

### Table 1 Parameters for All of Our Models

| Models      | \( R_H \) | \( M_p(M_\oplus) \) | \( \alpha \) |
|-------------|-----------|----------------------|-------------|
| M0026V0     | 0.4       | 8.5                  | 0           |
| M0026V1     | 0.4       | 8.5                  | \( 10^{-3} \) |
| M0026V2     | 0.4       | 8.5                  | \( 3 \times 10^{-3} \) |
| M005V0      | 0.5       | 16.5                 | 0           |
| M005V1      | 0.5       | 16.5                 | \( 10^{-3} \) |
| M005V2      | 0.5       | 16.5                 | \( 3 \times 10^{-3} \) |
| M01V0       | 0.63      | 33                   | 0           |
| M01V1       | 0.63      | 33                   | \( 10^{-3} \) |
| M01V2       | 0.63      | 33                   | \( 3 \times 10^{-3} \) |

### 2.4 Tests of the Effects of Sheet Size on the Results

In this section, we study the effects of sheet size on the structure of the circumplanetary disk inside the Hill radius. We find that the flow structure inside the Hill sphere (or circumplanetary disk) does not depend on the size of the computational box as long as the size is much larger than the Hill radius. Here we just take \( R_H = 0.5 \) and \( \alpha = 0 \) as an example. Table 2 lists the parameters of the \( R_H = 0.5 \) tests.

Figure 1 shows plots of the specific angular momentum (left panel) and surface density (right panel) in the circumplanetary disks for a 16.5 \( M_\oplus \) protoplanet with different sheet sizes. From this figure, we can see that the effects of sheet size on the structure of the circumplanetary disk are negligible as long as the sheet size is much larger than the Hill radius. The results shown in Section 3 are calculated with our standard sheet size of \( 12 \times 24h \).

### Table 2 Parameters for the Test Models

| Model      | Sheet size (\( h \)) | Grids  | \( R_H \) | \( \alpha \) |
|------------|----------------------|--------|-----------|-------------|
| M005V0     | 12 \times 24         | 500 \times 1000 | 0.5       | 0           |
| BM005V0    | 24 \times 48         | 656 \times 1312 | 0.5       | 0           |
| SM005V0    | 6 \times 12          | 358 \times 716  | 0.5       | 0           |
Specific angular momentum (left panel) and surface density (right panel) of the circumplanetary disks for a $16.5 M_\oplus$ protoplanet as a function of distance $r$ from the protoplanet with different sheet sizes. It can be seen that the effects of sheet size on the structure of the circumplanetary disk are negligible as long as the sheet size is much larger than the Hill radius.

3 RESULTS

Table 1 lists all of the models in this paper. We find that all of the models have settled into their steady state configuration before 20 orbits, so the results shown for orbits at $t = 75$ are fully relaxed.

In order to investigate the effects of viscosity on the circumplanetary disk, for each protoplanet with a different mass, we carry out three simulations with different viscosity coefficients $\alpha$. We find that when the protoplanet mass $M_p \gtrsim 33 M_\oplus$, the effects of viscosity are negligible.

3.1 Circumplanetary Disk Structure and Migration of Protoplanets

Figure 2 shows the structure of a circumplanetary disk around protoplanets with mass of 8.5, 16.5 and 33 $M_\oplus$ (top to bottom rows respectively). The panels in the top row show the circumplanetary disks surrounding an $8.5 M_\oplus$ protoplanet. In the top row, the left, middle and right panels correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. The panels in the middle row show the circumplanetary disks surrounding a $16.5 M_\oplus$ protoplanet. In the middle row, the left, middle and right panels correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. The panels in the bottom row show the circumplanetary disks surrounding a $33 M_\oplus$ protoplanet. In the bottom row, the left, middle and right panels correspond to models M01V0 ($\alpha = 0$), M01V1 ($\alpha = 10^{-3}$) and M01V2 ($\alpha = 3 \times 10^{-3}$), respectively. It can be clearly seen that as the mass of the protoplanet increases, the spiral structure of the circumplanetary disk becomes more prominent, which demonstrates that a higher mass protoplanet can excite a higher amplitude spiral shock.

For a protoplanet with mass of $8.5 M_\oplus$, as the viscosity becomes stronger, the spiral structure gets weaker. The spiral structure completely disappears when $\alpha = 3 \times 10^{-3}$ and the circumplanetary disk is nearly axisymmetric about the protoplanet. For an $8.5 M_\oplus$ protoplanet, the maximum surface density of the spiral waves is only two times bigger than the minimum surface density; when the amplitude of the spiral wave is weak, viscosity can easily disrupt the spiral structure and make the gas in the circumplanetary disk become smoothly distributed. For a protoplanet with mass of $16.5 M_\oplus$, viscosity also weakens the spiral structure of the circumplanetary disk. However, the effect is small compared to the case of a protoplanet with smaller mass ($M_p = 8.5 M_\oplus$). This is because with the increase of the mass of the protoplanet, the amplitude of the spiral waves gets higher, so viscosity
Fig. 2 Contours of logarithmic surface density of the circumplanetary disks surrounding protoplanets with mass of 8.5, 16.5 and 33 $M_\oplus$ (top to bottom rows). The panels in the top row show the circumplanetary disks surrounding an 8.5 $M_\oplus$ planet. In the top row, the left, middle and right panels correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. Panels in the middle row show the circumplanetary disks surrounding a 16.5 $M_\oplus$ planet. In the middle row, the left, middle and right panels correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. The panels in the bottom row show the circumplanetary disks surrounding a 33 $M_\oplus$ planet. In the bottom row, the left, middle and right panels correspond to models M01V0 ($\alpha = 0$), M01V1 ($\alpha = 10^{-3}$) and M01V2 ($\alpha = 3 \times 10^{-3}$), respectively.

can hardly affect the spiral structure. It can be seen that when the mass of the protoplanet reaches 33 $M_\oplus$, viscosity plays almost no role in forming structure of the circumplanetary disk.

The protoplanet excites spiral density waves at the Lindblad resonance and the torques exerted on the protoplanet that are the reaction to exciting waves change the orbit of the protoplanet around the central star. Previous works (Lubow et al. 1999; Tanigawa & Watanabe 2002) found that the
Fig. 3 Gravitational force exerted on the 8.5 $M_⊕$ protoplanet from disk gas in $x > 0$ as a function of $r_{\text{mask}}$, within which gas is neglected from force integration. In this figure, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively.

spiral structure around the protoplanet can significantly affect the torques. In the limiting case, if the gas distribution around a protoplanet is perfectly axisymmetric about the protoplanet, the torques exerted on the protoplanet will be zero. From Figure 2, we find that viscosity can considerably disrupt the spiral structure when the mass of protoplanet $M_p < \sim 33 M_⊕$. Thus, we can expect that when the protoplanet is small ($M_p < \sim 33 M_⊕$), viscosity can markedly affect the torques exerted on the protoplanet.

It has always been believed that the gas inside the Hill sphere of a planet migrates with the planet. Therefore, in most previous works, when calculating the torque exerted on a planet, the contribution from the gas inside the Hill sphere is completely neglected. However, Crida et al. (2009) showed that the gas bound to the planet (circumplanetary disk) only exists inside $0.5 R_H$. In the region $0.5 R_H < r < R_H$, the gas is not bound to the planet. Therefore, when calculating the torque, the gas inside the Hill sphere should not be completely neglected.

Now, we quantitatively study the effects of viscosity on the torques exerted on a protoplanet. Since we adopt the local approximation, the net torque in our simulation becomes exactly zero. But an actual net torque is not zero, owing to slight asymmetry of the density distribution, temperature and the curvature of the protoplanet’s orbit. The net torque exerted on the protoplanet is roughly proportional to the one-sided torque exerted by the gas exterior to the orbit of the protoplanet (Tanigawa & Watanabe 2002)\(^1\); therefore, the one-sided torque would be a clue for solving the migration problem. Because we adopt the local approximation, we just discuss the effects of viscosity on the one-sided ‘force’ exerted on the protoplanet. The ‘force’ corresponds to the torque divided by the semimajor axis of the protoplanet. The one-sided force exerted on a protoplanet is defined by

$$F_{y,\text{out}}(r_{\text{mask}}) = \int_{x_0}^{x_{\text{max}}} \int_{y_{\text{min}}}^{y_{\text{max}}} \Sigma \frac{\partial \Phi}{\partial y} \theta(r - r_{\text{mask}}) dy dx,$$

where $r_{\text{mask}}$ is the artificial inner limit of the force integration, and $\theta$ is the step function. If $r > r_{\text{mask}}$, $\theta = 1$, otherwise $\theta = 0$. The ‘force density’ is defined as $dF(x) = \int_{y_{\text{min}}}^{y_{\text{max}}} \Sigma \frac{\partial \Phi}{\partial y} dy$.

\(^1\) Net torque is the difference between the two one-sided torques with opposite signs, and the ratio of the two one-sided torques does not change very much when one of them changes parameters, such as planet mass or temperature in the linear regime. Thus, at least in this regime, we can say that the net torque is proportional to the one-sided torque.
Fig. 4 Left panel shows the gravitational force exerted on the 16.5 $M_\oplus$ protoplanet from disk gas in $x > 0$ as a function of $r_{\text{mask}}$, within which gas is excluded from force integration. In this panel, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. Right panel shows the torque density as a function of distance from the 16.5 $M_\oplus$ protoplanet. In this panel, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively.

We show $F_{y,\text{out}}$ as a function of $r_{\text{mask}}$ for a protoplanet with mass of 8.5 $M_\oplus$ in Figure 3. In Section 3.2, we show that the outer boundary of protoplanets with masses 8.5 $M_\oplus$ and 16.5 $M_\oplus$ is located inside $r = 0.22$, therefore we can focus on the torque exerted by the gas beyond $r = 0.22$. The gas in $x > 0$ exerts a negative torque on the protoplanet, which pushes the protoplanet to migrate towards the central star. As expected, when the spiral structure around the planet gets weaker, the force exerted on the protoplanet significantly decreases.

Figure 4 (left panel) plots $F_{y,\text{out}}$ as a function of $r_{\text{mask}}$ for a protoplanet with mass of 16.5 $M_\oplus$. When $\alpha = 3 \times 10^{-3}$, the spiral structure is partially disrupted by viscosity, which also leads to the reduction of the torque exerted on the planet. In order to clearly show that the ‘torque’ is affected by viscosity, the right panel of Figure 4 plots the ‘torque density’ as a function of distance from the planet. The case for an 8.5 $M_\oplus$ protoplanet is similar. We find when $M_p \geq 33 M_\oplus$, the effect of viscosity on the torque exerted on protoplanets is negligible. We conclude that the viscosity can markedly affect the torques exerted on protoplanets by disrupting the spiral structure of the gas disk when the mass of the protoplanet is small ($M_p \lesssim 33 M_\oplus$).

3.2 Circumplanetary Disk Size

Now, we explore the effects of viscosity on the size of the circumplanetary disks. The radial edge of a circumplanetary disk is taken as the point of turnover in the specific angular momentum of the disk. Quillen & Trilling (1998) make a simple analytic prediction of the approximate circumplanetary disk radii around accreting protoplanets. They assume that the gas flows into the Hill sphere of a protoplanet via the inner and outer Lagrange points. They also assume that when the gas arrives at the Lagrange points, the velocity, relative to the Lagrange points, is negligibly small. The Lagrange points have the same angular velocity around the central star as the protoplanet. The Lagrange points are approximately located at

$$r = a_p \pm R_H.$$  \hfill (17)

Thus, when the gas at the Lagrange points is captured by the protoplanet, its specific angular momentum relative to the protoplanet is

$$j \approx R_H^2 \Omega_p.$$  \hfill (18)
Fig. 5 Specific angular momentum of the circumplanetary disks surrounding an 8.5 $M_\oplus$ protoplanet. In this figure, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. The long-dashed line corresponds to the Keplerian angular momentum with respect to the protoplanet.

Assuming conservation of angular momentum when the accreting gas flows towards the protoplanet, the centrifugal radius, $r_c$, of the circumplanetary disk is

$$\frac{j^2}{r_c^2} \approx \frac{GM_p}{r_c},$$

which yields

$$r_c \approx \frac{R_H}{3}.$$  \hspace{1cm} (20)

Thus, the typical size of a circumplanetary disk should be approximately $R_H/3$.

Figure 5 shows the specific angular momentum of the circumplanetary disks around an 8.5 $M_\oplus$ protoplanet. In this figure, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. We can see the outer boundary of a non-viscous circumplanetary disk (model M0026V0) around an 8.5 $M_\oplus$ protoplanet located at $r = 0.15$. The disk’s size does not differ greatly from the prediction with $R_H/3$. Beyond $r = 0.15$, the gas rotates around the central star, and the angular momentum becomes smaller and smaller with increasing distance from the protoplanet. The angular momentum relative to the protoplanet even becomes negative when the distance from the protoplanet is sufficiently big which is not shown in Figure 5. When viscosity is included, the size of the circumplanetary disk decreases. In model M0026V2, the outer boundary of the circumplanetary disk is located at $r = 0.11$, and the size of the disk is reduced by a factor of 27% compared to the non-viscous disk. As can be seen from Figure 5, the specific angular momentum of the circumplanetary disk becomes lower after including viscosity, which is due to the outward angular momentum transfer by viscosity. The variation of angular momentum at a given radius is determined by the divergence of the viscous stress; at the outer boundary of the circumplanetary disk, the divergence of viscosity is stronger than that at other radii due to the stronger shear of the gas (this can be clearly seen from Fig. 5), which makes the decreasing amplitude of the angular momentum at the outer boundary much larger than that of the gas at other radii. Therefore, the outer boundary of the circumplanetary disk is moved inward when viscosity is taken into account.

Figure 6 shows the specific angular momentum of the circumplanetary disks around a 16.5 $M_\oplus$ protoplanet. In this figure, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha =$...
Fig. 6 Specific angular momentum of the circumplanetary disks surrounding a 16.5 $M_\oplus$ protoplanet. In this figure, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. The long-dashed line corresponds to the Keplerian angular momentum with respect to the protoplanet.

0), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. The outer boundary of the circumplanetary disk is also moved inward when viscosity is included. The outer boundary of a non-viscous circumplanetary disk (model M005V0) around a 16.5 $M_\oplus$ protoplanet is located at $r = 0.22$. In model M005V2, the outer boundary of the circumplanetary disk is located at $r = 0.17$, and the size of the disk is reduced by a factor of 23% compared to the non-viscous disk. We find that when the protoplanet’s mass $M_p \gtrsim 33 M_\oplus$, the effect of viscosity on the size of a circumplanetary disk is negligibly small.

Crida et al. (2009) have found that the size of the circumplanetary disk $\sim 0.5 R_H$, which is slightly larger than that obtained in this paper. This is because Crida et al. use an energy equation to solve the internal energy of the gas. With the energy equation, the collapse of the gas onto the circumplanetary disk is limited by the heating due to adiabatic compression, which gives a wider circumplanetary disk than that in the locally isothermal case.

From Figures 5 and 6, we see that the rotational velocity of the disk around the planets is significantly sub-Keplerian. The reason is as follows. In the radial direction, gravitational force is balanced by centrifugal force and pressure gradient force. If the temperature of the gas is fixed, with the increase of planet mass, the centrifugal force will increase (due to an increase in rotational velocity). In our paper, the mass of the planet is small (around 10 $M_\oplus$), so the pressure gradient force is important, and the gas rotation is sub-Keplerian. Tanigawa et al. (2012) found that the gas rotation around the planet is almost Keplerian. The reason is as follows. In Tanigawa et al. (2012), the temperature is identical to that in this paper, but their planet mass is 1 Jupiter mass, which is much bigger than the mass of the planet in this paper. Therefore, in Tanigawa et al. (2012), the gas rotation is almost Keplerian.

3.3 Circumplanetary Disk Mass

In the standard picture, a protoplanet residing in a circumstellar disk can exert torques on the circumstellar disk through the excitation of spiral density waves (e.g. Goldreich & Tremaine 1979). The angular momentum carried by the density waves will be deposited in the circumstellar disk where the waves are damped. The circumstellar disk gas exterior to the protoplanet’s orbit acquires positive
Fig. 7 The surface density averaged over $y$ against $x$ for the gas around protoplanets. The left and right panels correspond to protoplanets with masses 8.5 and 16.5 $M_{\oplus}$, respectively. In the left panel, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. In the right panel, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively.

Fig. 8 Surface density of the circumplanetary disks as a function of distance $r$ from the protoplanet. The left and right panels correspond to protoplanets with masses 8.5 and 16.5 $M_{\oplus}$, respectively. In the left panel, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. In the right panel, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively.

angular momentum and thus moves outward. The gas in the circumstellar disk that is interior to the protoplanet's orbit receives negative angular momentum and thus moves inward. For a low-mass protoplanet, a partial density gap forms along the orbit of the protoplanet. However, the viscosity inside the circumstellar disk tries to transport gas into the partial gap region with low density, which makes the density gap less prominent. Viscosity can affect the depth of the density gap around the orbit of the protoplanet and thus affects the mass of the circumplanetary disk.
In Figure 7, we plot the surface density averaged over $y$ against $x$ for the gas around protoplanets. The left and right panels correspond to protoplanets with masses $8.5$ and $16.5 \, M_\oplus$, respectively. In the left panel, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. In the right panel, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. It is obviously seen that the protoplanets try to open a density gap along their orbits. As expected, a protoplanet in inviscid gas produces a deeper and wider gap than others.

In Figure 8, we plot the surface density profiles of the circumplanetary disks around protoplanets. The left and right panels correspond to $8.5$ and $16.5 \, M_\oplus$ protoplanets, respectively. In the left panel, the solid, dotted and dashed lines correspond to models M0026V0 ($\alpha = 0$), M0026V1 ($\alpha = 10^{-3}$) and M0026V2 ($\alpha = 3 \times 10^{-3}$), respectively. In the right panel, the solid, dotted and dashed lines correspond to models M005V0 ($\alpha = 0$), M005V1 ($\alpha = 10^{-3}$) and M005V2 ($\alpha = 3 \times 10^{-3}$), respectively. Viscosity makes it easier for gas to flow towards the protoplanet. Therefore, the surface density of the circumplanetary disk in the viscous gas is higher than that of the inviscid gas. We have calculated the mass of the circumplanetary disk. For an $8.5 \, M_\oplus$ protoplanet, when $\alpha = 3 \times 10^{-3}$, the disk’s mass is bigger than that of the inviscid disk by a factor of $56\%$. For a $16.5 \, M_\oplus$ protoplanet, when $\alpha = 3 \times 10^{-3}$, the disk’s mass is bigger than that of the inviscid disk by a factor of $50\%$. We find that when $M_p \gtrsim 33 \, M_\oplus$, the effect of viscosity on mass of the circumplanetary disk is negligibly small.

3.4 Mass Accretion Rate

We discuss the mass accretion rate in this section. We talk about mass accretion rate in real units (Eqs. (11)–(15)). We assume that the planets are located at $a_p = 5.2$ AU. Also, we assume that $M_c = 1 \, M_\odot$ and $L = 1 \, L_\odot$. In this paper, we find that the mass accretion rates for non-viscous models M0026V0 ($8.5 \, M_\oplus$) and M005V0 ($16.5 \, M_\oplus$) are $2.2 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$ and $5.0 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$, respectively. Almost all of the numerical settings of Tanigawa & Watanabe (2002) are the same as that in this paper. The only difference is that there is no viscosity in Tanigawa & Watanabe (2002). In Tanigawa & Watanabe (2002), the smallest mass used is $22 \, M_\oplus$. Therefore, we cannot directly compare our results to theirs. Fortunately, in equation (19) of Tanigawa & Watanabe (2002), they give the dependence of mass accretion rate on the planet’s mass. Using their equation (19), the mass accretion rates for $8.5 \, M_\oplus$ and $16.5 \, M_\oplus$ are $2.02 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$ and $4.8 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$, respectively. Our results for non-viscous gas are consistent with those in Tanigawa & Watanabe (2002).

If viscosity is included, we find that the mass accretion rate becomes higher. The mass accretion rate for the viscous models M0026V2 ($8.5 \, M_\oplus$ and $\alpha = 3 \times 10^{-3}$) and M005V2 ($16.5 \, M_\oplus$ and $\alpha = 3 \times 10^{-3}$) are $4 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$ and $8.0 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$, respectively. D’Angelo et al. (2002) use global simulations to study the gas flow onto planets. For a $6.4 \, M_\oplus$ planet, when $\alpha = 4 \times 10^{-3}$, they find that the mass accretion rate is $1.2 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$ (fig. 25 in their paper). For a $15 \, M_\oplus$ planet, when $\alpha = 4 \times 10^{-3}$, they find that the mass accretion rate is $1.5 \times 10^{-5} \, M_{\text{Jup}} \, \text{yr}^{-1}$ (fig. 25 in their paper). Given that the viscosity and planet mass are comparable, it seems that the accretion rate found in this paper is much higher than that obtained in D’Angelo et al. (2002). The reason may be due to the depletion of the gas inside the orbital radius of the planet in the global simulations of D’Angelo et al. (2002).

The Bondi accretion rates for planets with masses $8.5 \, M_\oplus$ and $16.5 \, M_\oplus$ located at $5.2$ AU are $1 \times 10^{-4} \, M_{\text{Jup}} \, \text{yr}^{-1}$ and $3.8 \times 10^{-4} \, M_{\text{Jup}} \, \text{yr}^{-1}$, respectively. The angular momentum of the gas makes the actual accretion rate much smaller than the Bondi accretion rate of the planets. We must note that the mass accretion rate obtained in this paper may be not accurate. This is because, inside
the Hill radius, the gas may evolve adiabatically. The increased temperature towards the planet may
decrease accretion rates compared to those obtained in this paper.

4 SUMMARY AND DISCUSSION

We consider the effects of viscosity on the circumplanetary disks forming in the vicinity of a pro-
toplanet through two-dimensional hydrodynamical simulations with the shearing sheet model. We
find that viscosity can significantly affect the properties of the circumplanetary disk when the mass
of the protoplanet $M_p \lesssim 33 \, M_\oplus$.

The local shearing-sheet approximation is only an approximation to the global model, and it may
not be appropriate for investigating the global evolution of the disk structure. However, Muto et al.
(2010) have shown that the local shearing-sheet approximation and full global model share many
essential physics in common. Muto et al. (2010) have also shown that the one-dimensional disk
evolution model constructed from the global model and the local model are very similar. Therefore,
we expect the local simulations to capture the main physics of circumplanetary disks.

Physically, we should use three-dimensional simulations to study circumplanetary disks inside
the Hill radius. However, as a first step, we carry out these simulations in order to understand the
basic effects of viscosity on the circumplanetary disks. The two-dimensional simulations can capture
the main physics of the disks. For example, the mass accretion rate can be accurately calculated by
two-dimensional simulations because the accretion rate is determined mainly by the flow where $r > R_H$
(when $r > R_H$, the disk is very thin and two-dimensional simulations are enough) (Tanigawa &
Watanabe 2002). Although the 2D simulations can capture some properties of the circumplanetary
disk, some important features may be lost in the 2D disks. For example, Tanigawa et al. (2012)
found that most gas accretion onto circumplanetary disks occurs, from high altitude, nearly vertically
toward the disk’s surface, which cannot be implemented in 2D simulations. In a subsequent paper,
we will discuss the effects of viscosity on circumplanetary disks in three-dimensions.

In this paper, we use a smoothing length to smooth gravity close to the planet. Müller et al.
(2012) find that for longer distances, the smoothed length is determined solely by the vertical thick-
ness of the disk. For the case of a planet they find that outside $r = H$, the value of $r_{\text{sm}} = 0.7H$
describes the averaged force very well. However, for shorter distances the smoothing needs to be sig-
nificantly reduced. In this paper, in order to study the structure of circumplanetary disks, we adopt
$r_{\text{sm}} = 0.05H$. This value has proved to be safe to study circumplanetary disks modeled by Tanigawa
& Watanabe (2002).

In this paper, we just use an anomalous stress tensor to mimic the shear stress, which is in reality
magnetic stress associated with MHD turbulence driven by MRI. In a real turbulent circumplanetary
disk, the properties of the disk should fluctuate with time, but we expect that the time-averaged
properties of the turbulent disk should be consistent with our results here. The spiral shocks in
the circumplanetary disk can also affect the amplitude of the turbulent stress, but the effect is small when
the mass of the protoplanet is small ($M_p \lesssim 30 \, M_\oplus$) (Papaloizou et al. 2004). Thus, our calculations
have captured the main physics of the circumplanetary disk.

To properly study a viscous circumplanetary disk, it is necessary to include magnetic effects.
However, the gas in the circumstellar disk surrounding a protostar is just weakly ionized. Weakly
ionized plasma is subject to a number of non-ideal MHD effects due to the collisional coupling be-
tween the ionized species and the neutrals (e.g. ambipolar diffusion effects) (Bai & Stone 2011).
The MRI, which is considered as the major mechanism for angular momentum transport via the
MHD turbulence, is strongly affected by non-ideal MHD effects. Thus, it is necessary to study the
non-ideal MHD effects on MRI before further modeling the properties of the magnetized circum-
planetary disks.

We find that when $M_p \lesssim 33 \, M_\oplus$, viscosity can considerably disrupt the spiral structure around
a protoplanet and make the gas in the disk smoothly distributed, which weakens the torques exerted
on the protoplanet. Thus, viscosity can slow the migration speed of a protoplanet. This is helpful for solving the problem that a protoplanet quickly migrates to the vicinity of the central star before becoming a gas giant.

According to the core accretion theory, the formation process of a gas giant planet can be divided into three phases. (1) In phase 1, the solid core forms, which has a mass of several $M_{\oplus}$. (2) In phase 2, a spherical hydrostatic gas envelope around the solid core forms. The gas accretion rate in this phase is very low. (3) After the point when the core mass and the envelope mass become comparable, and gas is accreted in a runaway fashion. The main problem with the core accretion theory is that the formation time of a gas giant planet exceeds the lifetime of the circumstellar disk. Under the assumption of spherical symmetry and gas being in hydrostatic equilibrium, previous works found that the time needed to complete phase 2 is comparable to or exceeds the lifetime of the circumstellar disk (e.g. Pollack et al. 1996). The reason for the long time evolution in phase 2 is that they assume spherical accretion. The gravitational energy released in the accretion process cannot easily escape from the system. The thermal pressure can support the envelope against the gravity of the core, which significantly decreases the accretion rate in phase 2. Lin (2006) proposed that if a circumplanetary disk (instead of a spherical hydrostatic envelope) exists around a protoplanet of several $M_{\oplus}$, the evolution time of phase 2 may be significantly decreased. This is because in the disk accretion case, the gravitational energy released in the accretion process can easily escape from the surface of the circumplanetary disk. However, our simulation found that when the protoplanet is small (several $M_{\oplus}$), the size of the circumplanetary disk is just $\sim 27\%$ of the Hill radius, $R_H$, when viscosity is included (see Fig. 4). Thus, even though the existence of a circumplanetary disk can shorten the formation time of a gas giant, we expect that the effect is not important because the size of the disk is relatively small compared to the Hill radius.

Satellites may form in the circumplanetary disk. Regular satellites around a gas giant planet are believed to form according to a scenario similar to the formation of the Earth-like planets in our solar system. The protosatellite forms by accumulation through mutual collision of the satellitesimals that form after the dust grains sink towards the equatorial plane of the circumplanetary disk (e.g. Stevenson et al. 1986). After including viscosity, the density of the circumplanetary disk increases, which should be helpful for the formation of satellitesimals by the accumulation of dust grains. However, the increase in density will result in an increase of opacity. It is difficult for high opacity circumplanetary disks to radiate the energy generated by gas accretion. Thus, the increase of density may result in an increase in temperature of the circumplanetary disks. A large fraction of ice is found in some of the Galilean moons around Jupiter (Schubert et al. 1986). This means that the temperature of the circumplanetary disk should be low enough for water to condense into ice (Canup & Ward 2002). In this sense, the increase of density may make it difficult for satellite formation to occur. In order to better understand the properties of the circumplanetary disk, we need to study a viscous circumplanetary disk with radiative transfer, which is beyond the scope of this paper.

Acknowledgements We thank Ruobing Dong and T. Tanigawa for useful discussions. We thank Hsiang-Hsu Wang for numerical support. This work was supported in part by the Natural Science Foundation of China (Grant Nos. 10833002, 10825314, 11103059, 11121062 and 11133005), the National Basic Research Program of China (973 Program, 2009CB824800) and the CASSAFEA International Partnership Program for Creative Research Teams.

References
Bai, X.-N., & Stone, J. M. 2011, ApJ, 736, 144
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, Reviews of Modern Physics, 70, 1
Bate, M. R., Lubow, S. H., Ogilvie, G. I., & Miller, K. A. 2003, MNRAS, 341, 213
