Equal-time temperature correlators of the one-dimensional Heisenberg XY chain

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Abstract
Representations as determinants of $M \times M$ dimensional matrices are obtained for equal-time temperature correlators of the anisotropic Heisenberg XY chain. These representations are simple deformations of the answers for the isotropic XX0 chain. In the thermodynamic limit, the correlators are expressed in terms of the Fredholm determinants of linear integral operators.

Introduction
The description of the correlation functions for the models solved by means of Bethe ansatz is based on the representation for the correlators as the Fredholm determinants of linear integral operators. Such representations were obtained for the first time in [1, 2] for the simplest two point equal-time correlators for the one-dimensional impenetrable bosons model. Later they were generalized for the case of time-dependent correlators for the models being the free-fermion point of models solved by means of Bethe Ansatz (impenetrable bosons [3] and the isotropic XX0 Heisenberg chain [4, 5]) and also for the case of finite interaction ( [6, 7] for the one-dimensional Bose gas and [8] for the XXZ chain). Such representations allow to write classical integrable equations for correlators which can be used, in particular, to calculate the long time and large distance asymptotics for the correlation functions.

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It was understood a time ago \cite{16, 17, 18} that the language of the classical differential equations is natural for the description of the correlation functions of quantum integrable models. The recent progress in this direction is described in detail in the book \cite{19} for the example of impenetrable bosons based on the approach elaborated in \cite{10, 11, 12, 13, 14, 15}. The most important part of this approach is to consider the Fredholm determinant of the linear operator appearing in the representation of the correlators as a $\tau$-function for a new system of classical integrable equations (see also \cite{8}). These linear operators should be of a very special form - so called “integrable” integral operators (see \cite{19, 20}).

In this paper we obtain some determinant representations for equal-time temperature correlators for the anisotropic Heisenberg XY chain both for finite lattice and in thermodynamic limit. This model was introduced and studied for the first time in \cite{21}. Later it was investigated by many authors (see \cite{22, 23, 24} and references therein).

To compute the correlation functions we use a modification of the approach proposed in \cite{25} and based on using the integration over the Grassman variables and the corresponding coherent states. It is an essential point in our paper. We don’t use, however, the functional integral as in \cite{25}. Using the coherent states simplifies the calculations leading automatically to the answers in the necessary form (see for example Appendix B where we reproduce the results \cite{4, 5} for the XX0 chain using our method).

The representations for the equal-time temperature correlators of the anisotropic XY chain as determinants of $M \times M$ matrices obtained in this paper being a direct generalization of the representations for the isotropic case \cite{4, 5, 26} differ from them only by changing “Fermi” (or “Bose”) weight in the kernels of integral operators for a weight depending on the anisotropy parameter. On the other hand these representations generalize the results for the anisotropic chain obtained in \cite{27} for zero temperature. The “integrability” of the integral operators appearing in the thermodynamic limit is evident.

One should note that the isotropic XX0 chain is the “free-fermion point” for the XXZ Heisenberg chain which is a model solved by means of the standard Bethe Ansatz. The anisotropic XY chain is the “free-fermion line” for the XYZ chain which is a model where the usual Bethe Ansatz doesn’t work. Therefore it is very interesting from our point of view that the answers for the equal-time correlators have the form of a simple deformation of the answers for the isotropic case. We hope to give a corresponding description for the time-depending correlation functions in our next publication.

Our paper is organized as follows.

In the first section we describe the model and give the basic facts about the diagonalization of the Hamiltonian using the Bogolyubov transformation following the classical works \cite{21, 22}. In section 2 we introduce the coherent states for isotropic XX0 model and calculate the matrix elements of operators between these states. In section 3 we describe the coherent states for the anisotropic XY chain and consider their relations with the coherent states of the isotropic chain. In section 4 the simplest correlator is calculated. In section 5 we calculate the local spin correlator for the finite anisotropic lattice. In section 6 we obtain the results in the thermodynamic limit. In appendix A the basic facts about Grassmanian coherent states are given. In appendix B the derivation of the results for the isotropic case is presented.

1 The XY Heisenberg chain

The Hamiltonian of the XY spin chain describing the interaction between the nearest neighbors spins $1/2$ placed in the sites of one-dimensional periodical lattice in a constant magnetic
field $h$ is

$$H = H_0 + \gamma H_1 - hS^z, \quad (1.1)$$

where

$$H_0 = -\frac{1}{2} \sum_{m=1}^{M} (\sigma_m^m \sigma_{m+1}^m + \sigma_{m+1}^m \sigma_m^m); \quad (1.2)$$

$$H_1 = -\frac{1}{2} \sum_{m=1}^{M} (\sigma_m^m \sigma_{m+1}^m + \sigma_{m+1}^m \sigma_m^m); \quad (1.3)$$

and the third component of the total spin is

$$S^z = \sum_{m=1}^{M} \sigma_m^m. \quad (1.4)$$

The total number of sites $M$ is supposed to be even. Pauli matrices are defined as usual

$$[\sigma^\alpha_m, \sigma^\beta_n] = 2i\delta_m^n \epsilon^{\alpha\beta\gamma}(\alpha, \beta, \gamma = x, y, z); \quad (1.5)$$

$$\sigma^\pm_m = \frac{1}{2}(\sigma_m^x \pm i\sigma_m^y),$$

with the periodical boundary conditions

$$\sigma^\alpha_{M+1} = \sigma^\alpha_1. \quad (1.6)$$

Due to the symmetries of the Hamiltonian the sign of the magnetic field (as the sign of the Hamiltonian) is not essential. We will assume that $h \geq 0$.

The Jordan-Wigner transformation

$$a_m = \exp \{i\pi Q(m-1)\} \sigma_m^+;$$

$$a_m^+ = \sigma_m^- \exp \{i\pi Q(m-1)\}, \quad (1.7)$$

introduces the canonical fermion fields $a_m, a_m^+$ on the lattice,

$$[a_m, a_n^+] = a_m a_n + a_n a_m = 0;$$

$$[a_m^+, a_n^+] = 0, [a_m, a_n^+] = \delta_{mn}. \quad (1.8)$$

Operator $Q(m)$ is the operator of number of particles on the first $m$ sites of the lattice,

$$Q(m) = \sum_{j=1}^{m} q_j, \quad (1.9)$$

where $q_m$ is the operator of number of particles in the site $m$:

$$q_m = a_m^+ a_m = \sigma_m^- \sigma_m^+ = \frac{1}{2}(1 - \sigma_m^z). \quad (1.10)$$

The operator of the total number of particles,

$$N = Q(M), \quad (1.11)$$

commutes with the operators $H_0$ and $S^z$ but does not commute with the operator $H_1$ and thus the total Hamiltonian $H$ does not conserve the number of ”a-fermions”. At the same
time the operator \((-1)^N = \exp\{\pm i\pi N\}\), anticommuting with the creation and annihilation operators

\[\left[(-1)^N, a_m^+\right] = \left[(-1)^N, a_m\right] = 0, \tag{1.12}\]

commutes with any bilinear in \(a_m, a_m^+\) operators, in particular, with the Hamiltonian

\[\left[(-1)^N, H\right] = 0. \tag{1.13}\]

Periodical boundary conditions (1.6) for the spins lead to the following conditions for the fermions:

\[a_{M+1} = (-1)^Na_1; \quad a_{M+1}^+ = a_1^+(-1)^N. \tag{1.14}\]

Introducing projectors \(P^\pm\)

\[P^\pm = \frac{1}{2}(1 \pm (-1)^N); \quad (P^\pm)^2 = P^\pm; \quad P^+P^- = 1; \quad P^+P^+ = P^-P^- = 0; \tag{1.15}\]

\[P^\pm a_m = a_mP^\pm; \quad [H, P^\pm] = 0, \]

one can rewrite the Hamiltonian in the following form [23]

\[H = H^+P^+ + H^-P^- \tag{1.16}\]

Both operators \(H^\pm\) can be rewritten formally in the same form

\[H^\pm = \frac{1}{2} \sum_{m=1}^M \left[(a_m^+a_{m+1}^+ + a_m^+a_m) + \gamma(a_m^+a_{m+1}^+ + a_m^+a_m)\right] + \]

\[+ \frac{1}{h} \sum_{m=1}^M a_m^+a_m - \frac{hM}{2}; \tag{1.17}\]

the only difference between \(H^+\) and \(H^-\) being the boundary conditions:

\[a_{M+1} = -a_1; \quad a_{M+1}^+ = -a_1^+; \quad \text{for} \quad H^+; \]

\[a_{M+1} = a_1; \quad a_{M+1}^+ = a_1^+; \quad \text{for} \quad H^-\tag{1.18}\]

Hence the Fourier transformations to the momentum representation are different for these Hamiltonians. We denote the sets of permitted quasimomenta \(X^+\) for \(H^+\) and \(X^-\) for \(H^-\):

\[X^+ = \{p : \exp\{ipM\} = \pm 1, \quad p \in (-\pi, \pi]\}, \tag{1.19}\]

or explicitly:

\[X^+ = \left\{p_l = -\pi - \frac{\pi}{M} + \frac{2\pi}{M}l, \quad l = 1, 2, \ldots, M \right\}; \tag{1.20}\]

\[X^- = \left\{p_l = -\pi - \frac{2\pi}{M}l, \quad l = 1, 2, \ldots, M \right\}.

The corresponding formulae for the Fourier transformation can be written in the following form

\[a_m = \frac{\exp\{-i\pi/4\}}{\sqrt{M}} \sum_{p \in X^\pm} a_p \exp\{i(m-1)p\}, \tag{1.21}\]

\[a_m^+ = \frac{\exp\{i\pi/4\}}{\sqrt{M}} \sum_{p \in X^\pm} a_p^+ \exp\{-i(m-1)p\}.

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(the summation is taken over \( p \in X^+ \) for \( H^+ \) and over \( p \in X^- \) for \( H^- \)) and

\[
a_p = \frac{\exp\{i\pi/4\}}{\sqrt{M}} \sum_{m=1}^{M} a_m \exp \{-i(m-1)p\},
\]

\[
a^+_p = \frac{\exp\{-i\pi/4\}}{\sqrt{M}} \sum_{m=1}^{M} a^+_m \exp \{i(m-1)p\}.
\]

The Hamiltonians \( H^\pm \) in the momenta representation are written as

\[
H^\pm = \sum_{p \in X^\pm} \left[ \varepsilon(p) a^+_p a_p + \frac{\Gamma(p)}{2} (a^+_p a^+_p + a^-_p a^-_p) \right] - \frac{Mh^2}{2},
\]

where

\[
\varepsilon(p) = h - \cos p; \quad \Gamma(p) = \gamma \sin p.
\]

Diagonalization of these Hamiltonians can be done using the Bogolyubov transformation (different for \( H^+ \) and \( H^- \)) leading to new canonical fermion operators \( A_p \) and \( A^+_p \)

\[
A_p = \alpha(p)a_p - \beta(p)a^+_p;
A^+_p = \alpha(p)a^+_p + \beta(p)a^-_p,
\]

where

\[
\alpha(p) = \cos \frac{\theta(p)}{2}; \quad \beta(p) = \sin \frac{\theta(p)}{2},
\]

and the angle \( \theta(p) \) is defined by relations:

\[
\cos \theta(p) = \frac{\varepsilon(p)}{E(p)}; \quad \sin \theta(p) = -\frac{\Gamma(p)}{E(p)};
\]

\[
E(p) = \sqrt{\varepsilon^2(p) + \Gamma^2(p)} \geq 0, \quad (p \neq 0, \pi).
\]

The momenta \( p = 0, \pi \) (appearing only in \( H^- \)) should be treated separately. Following [2], we put

\[
A_0 = a_0; \quad A^+_0 = a^+_0;
A_\pi = a_\pi; \quad A^+_\pi = a^+_\pi,
\]

and

\[
E(0) = h - 1 = \varepsilon(0) \quad (E(0) < 0 \quad \text{for} \quad h < 1)
E(\pi) = h + 1 = \varepsilon(\pi) > 0.
\]

The Hamiltonians can be diagonalized as follows

\[
H^\pm = \sum_{p \in X^\pm} E(p)A^+_p A_p + E^\pm_0,
\]

where the "vacuum energy" is

\[
E^\pm_0 = -\frac{1}{2} \sum_{p \in X^\pm} E(p),
\]

(to calculate \( E^-_0 \) one should take into account the definition (1.30)). One should note that for \( h < 1 \) the value \( E^-_0 \) is not the ground state energy \( E^-_g \), in this case \( E^-_g = E^-_0 + \varepsilon(0) \).
2 Coherent states for the XX0 chain

In this section we give some formulae for matrix elements of operators between the coherent states of the isotropic XX0 chain which are necessary for the following calculations. The corresponding Hamiltonians $H_{\pm}^{\pm}$ (see (1.23))

$$H_{\pm}^{\pm} = \sum_{p \in X^{\pm}} \varepsilon(p) a_{p}^+ a_{p} - \frac{hM}{2} \quad (2.1)$$

will be denoted simply $H_{\pm}$ in this section for the simplification. They are diagonal already in terms of the operators $a_{p}, a_{p}^{+}$ (1.21). One should note, however, that the Fock vacuum (which is the same for $H_{\pm}$),

$$a_{m} |0\rangle = 0, \quad \langle 0 | a_{m}^{+} = 0 \quad (m = 1, 2, \ldots, M), \quad (2.2)$$

$$a_{p} |0\rangle = 0, \quad \langle 0 | a_{p}^{+} = 0 \quad (p \in X^{\pm}), \quad \langle 0 | 0 \rangle = 1,$$

is the ground state only for $h > 1$.

One introduces the coherent states (see Appendix A) different for the Hamiltonians $H_{+}$ and $H_{-}$

$$|\phi, \pm\rangle = \exp \left\{ \sum_{q \in X^{\pm}} a_{q}^{+} \phi_{q} \right\} |0\rangle;$$

$$\langle \phi^{*}, \pm | = \langle 0 | \exp \left\{ \sum_{q \in X^{\pm}} \phi_{q}^{*} a_{q} \right\}. \quad (2.3)$$

The parameters $\phi_{q}, \phi_{q}^{*}$ (Grassman algebra elements) anticommute with other parameters and with all the operators $a_{p}, a_{p}^{+}$. The main properties of the coherent states (2.3), (2.4) are described in Appendix A. They are eigenstates for the operators $a_{p}$ and $a_{p}^{+}$:

$$a_{p} |\phi, \pm\rangle = \phi_{p} |\phi, \pm\rangle \quad (p \in X^{\pm}), \quad (2.5)$$

$$\langle \phi^{*}, \pm | a_{p}^{+} = \phi_{p}^{*} \langle \phi^{*}, \pm | \quad (p \in X^{\pm}). \quad (2.6)$$

The scalar product of the coherent states of one type is given by the usual formulae (A.4):

$$\langle \phi^{*}, + | \phi, + \rangle = \exp \left\{ \sum_{p \in X^{+}} \phi_{p}^{*} \phi_{p} \right\};$$

$$\langle \phi^{*}, - | \phi, - \rangle = \exp \left\{ \sum_{q \in X^{-}} \phi_{q}^{*} \phi_{q} \right\}. \quad (2.7)$$

When it cannot cause misunderstandings the sums on the right hand sides of the equations (2.3), (2.4), (2.7), (2.8) will be denoted $a^{+} \phi = \sum_{q} a_{q}^{+} \phi_{q}; \phi^{+} \phi = \sum_{p} \phi_{p}^{+} \phi_{p}$ etc.

The scalar products of the coherent states of different type are given by

$$\langle \phi^{*}, + | \psi, - \rangle = \exp \left\{ \sum_{p, q \in X^{+}, q \in X^{-}} \phi_{p}^{*} L_{pq} \psi_{q} \right\} = \exp \{ \phi^{*} L \psi \}, \quad (2.9)$$

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\[ \langle \psi^*, -| \phi, + \rangle = \exp \left\{ \sum_{p \in X^-, q \in X^+} \psi^*_p L_{pq} \phi_q \right\} = \exp \{ \psi^* L \phi \}, \]

where the matrix element of the \( M \times M \) matrix \( L \) are

\[ L_{pq} = \frac{2}{M} \frac{1}{1 - \exp\{-i(p - q)\}} = \frac{i}{M} \left( \cot \frac{q - p}{2} - i \right). \tag{2.10} \]

It becomes evident if one takes into account that the Fock vacuum is the same for both types of states and rewrites the scalar products in the "coordinate representation" using the formulae (1.21) and (1.22). For example,

\[ \langle \phi^*, +| \psi, - \rangle = \langle 0 | \exp \left\{ M \sum_{m=1}^M \phi^*_m a_m \right\} \exp \left\{ M \sum_{m=1}^M a^+_m \psi_m \right\} | 0 \rangle = \exp \left\{ \sum_{m=1}^M \phi^*_m \psi_m \right\}, \tag{2.11} \]

where

\[ \phi^*_m = \frac{\exp\{i\pi/4\}}{\sqrt{M}} \sum_{p \in X^+} \phi^*_p \exp \{-i(m-1)p\}, \]

\[ \psi_m = \frac{\exp\{-i\pi/4\}}{\sqrt{M}} \sum_{q \in X^-} \psi_q \exp \{i(m-1)q\}. \tag{2.12} \]

Consider now the matrix elements of the operator \( \exp\{\alpha Q(m)\} \) where (see (1.9)) \( Q(m) \) is the number of particles operator at the first \( m \) sites of the lattice:

\[ Q = \sum_{l=1}^m a^+_l a_l = \sum_{p_1, p_2} a^+_{p_1} Q_{p_1, p_2}(m) a_{p_2} \equiv a^+ Q(m) a, \tag{2.13} \]

(the quasimomenta \( p_1 \) and \( p_2 \) here correspond to the states of the same type: \( p_1, p_2 \in X^+ \) or \( p_1, p_2 \in X^- \)). Using the properties of the matrix \( Q(m) \) (evident in the coordinate representation),

\[ Q^2(m) = Q(m); \quad \exp\{\alpha Q(m)\} = I + (e^\alpha - 1)Q(m), \tag{2.14} \]

one has for the matrix elements between two states of the same type

\[ \langle \phi^*, \pm| \exp\{\alpha Q(m)\} | \phi, \pm \rangle = \exp \{ \phi^*[I + (e^\alpha - 1)Q(m)]\phi \} = \]

\[ = \exp \left\{ \sum_{p \in X^+, q \in X^+} \phi^*_p [\delta_{pq} + (e^\alpha - 1)Q_{pq}(m)] \phi_q \right\}, \tag{2.15} \]

the matrix elements of the \( M \times M \) matrix \( Q(m) \) being given as

\[ Q_{pq}(m) = \exp\{-i(m-1)p/2\} Q^{(0)}_{pq}(m) \exp\{i(m-1)q/2\}, \tag{2.16} \]

\[ Q^{(0)}_{pq}(m) = \frac{1}{M} \frac{\sin \frac{m(p-q)}{2}}{\sin \frac{m}{2}}. \tag{2.17} \]
(for the diagonal matrix elements one should use the l’Hôpital rule, \( Q_{pq}(m) = Q_{pq}^{(0)}(m) = m/M \)).

One has for the states of different types, analogously to (2.9),

\[
\langle \phi^*, \pm | \exp{\alpha Q(m)} \rangle | \phi, \mp \rangle = \exp \{ \phi^*[L + (e^\alpha - 1)Q(m)]| \phi \}\]

\[
= \exp \left\{ \sum_{p \in X^+, q \in X^\mp} \phi_p[L_{pq} + (e^\alpha - 1)Q_{pq}(m)]\phi_q \right\},
\]

(2.18)

where the matrix elements \( L_{pq} \) and \( Q_{pq}(m) \) are given by the formulae (2.10), (2.16) (one should note however that now if \( p \in X^+ \) then \( q \in X^- \) and vice versa). In particular, one has for the operator \( \exp{i\pi Q(m)} \) entering the Jordan-Wigner transformation

\[
\langle \phi^*, \pm | \exp{i\pi Q(m)} \rangle | \phi, \mp \rangle = \exp \{ \phi^* L(m)| \phi \}\]

\[
= \exp \left\{ \sum_{p \in X^+, q \in X^\mp} \phi_p[L_{pq}(m)]\phi_q \right\},
\]

(2.19)

where

\[
L_{pq}(m) = \exp{-imp}L_{pq}\exp{imq}.
\]

(2.20)

Turn now to the matrix elements (“form factors”) of the local spins. Since

\[
P^\alpha \sigma_m^\alpha = \sigma_m^\alpha P^\alpha \quad (\alpha = x, y, \text{ or } \alpha = \pm),
\]

(2.21)

we need only the matrix elements of the local operators \( \sigma_m^\alpha \) between the states of different type. A direct calculation using (2.10), (2.11) and (2.13) gives:

\[
\langle \phi^*, \pm | \sigma_m^\alpha | \psi, \mp \rangle = \psi_m(\mp) \exp \{ \phi^* L(m - 1)| \psi \} = 
\]

\[
= \psi_m(\mp) \exp \left\{ \sum_{p \in X^+, q \in X^\mp} \phi_p^* L_{pq}(m - 1)\psi_q \right\},
\]

(2.22)

\[
\langle \phi^*, \pm | \sigma_m^- | \psi, \mp \rangle = \phi_m^*(\pm) \exp \{ \phi^* L(m - 1)| \psi \},
\]

where we use natural notations

\[
\psi_m(\pm) = \frac{\exp{-i\pi/4}}{\sqrt{M}} \sum_{q \in X^\pm} \psi_q \exp{i(m - 1)q},
\]

\[
\phi_m^*(\pm) = \frac{\exp{i\pi/4}}{\sqrt{M}} \sum_{p \in X^\pm} \phi_p \exp{-i(m - 1)p}.
\]

(2.23)

One should note that the matrices \( L \) (2.10), \( L(m) \) (2.20) and \( Q(m) \) (2.16) are related by the following simple formula which can be proved by direct calculation:

\[
\sum_q L_{p1,q}(m)L_{qp2} = \delta_{p1,p2} - 2Q_{p1p2}(m),
\]

(2.24)

\((p_1, p_2 \in X^+, \quad q \in X^- \quad \text{or} \quad p_1, p_2 \in X^-, \quad q \in X^+).\)

Using these formulae one can reproduce the time-dependent temperature correlation functions for the isotropic XX0 chain obtained in [4, 5, 26]. The corresponding calculation is given it in Appendix B. In the next sections we consider the anisotropic XY chain.
3 Coherent states for the anisotropic XY chain

In this section the formulae for the matrix elements of operators between the coherent states of the anisotropic XY chain are given. We consider now two sets of canonical fermion operators, \( a_p, a_p^+ \) and \( A_p, A_p^+ \), related by the Bogolyubov transformation \((1.25)\). For each of these sets we introduce the coherent states ("old" and "new" ones) and calculate the scalar products of old and new states.

Consider first the relations between the two vacuum states \(|0\rangle\) and \(|0\rangle\rangle\) for the old and new sets

\[
\begin{align*}
\langle a_p | 0 \rangle &= 0, & \langle a_p^+ | 0 \rangle &= 0, & \langle 0 | 0 \rangle = 1, \quad \forall p; \\
\langle A_p | 0 \rangle &= 0, & \langle A_p^+ | 0 \rangle &= 0, & \langle 0 | 0 \rangle = 1. \\
\end{align*}
\]

(3.1)

These states are related as follows:

\[
\begin{align*}
|0\rangle\rangle &= \frac{N}{2} / \Omega |0\rangle, & \langle 0|0\rangle\rangle &= 1, \\
A_p |0\rangle\rangle &= 0, & \langle A_p^+ | 0 \rangle &= 0, \langle 0 | 0 \rangle = 1. \\
\end{align*}
\]

(3.2)

where the operators \( \Omega^+ \) and \( \Omega \) are

\[
\begin{align*}
\Omega^+ &= \exp \left\{ \frac{1}{2} \sum_p \tau(p) a_p^+ a_{-p}^+ \right\}; \\
\Omega &= \exp \left\{ \frac{1}{2} \sum_p \tau(p) a_{-p} a_p \right\},
\end{align*}
\]

(3.4)

and

\[
\tau(p) \equiv \tan \frac{\theta(p)}{2}, \quad \frac{1}{2} \sum_p \tau(p) (\xi_{-p}^* \xi_{-p} - \xi_{p} \xi_{p}^*) = -\omega^* T \omega.
\]

(3.5)

The normalization coefficient \( N \) can be represented as a determinant of a diagonal \( M \times M \) matrix,

\[
N = \langle 0 | \Omega^+ \Omega | 0 \rangle = \det(I + T),
\]

(3.6)

where \( I \) is the identity matrix and

\[
T = \text{diag}(i\tau(p)).
\]

(3.7)

Properties \((3.2)\) for the states \((3.3)\) can be easily checked using the commutation relations

\[
[a_p, \Omega^+] = \tau(p) \Omega^+ a_{-p}^+ \quad \text{and} \quad [\Omega, a_p^+] = \tau(p) a_{-p} \Omega.
\]

To calculate the normalization coefficient one makes use of the equation \((3.6)\) and representing

\[
N = \int d\xi d\xi^* \exp \left\{ -\xi^* \xi + \frac{1}{2} \sum_p \tau(p) (\xi_{-p}^* \xi_{-p} + \xi_{p} \xi_{p}^*) \right\}.
\]

Let us make the change of variables

\[
\omega_p = \frac{1}{\sqrt{2}} (\xi_p + i \xi_{-p}^*), \quad \omega_p^* = \frac{1}{\sqrt{2}} (\xi_p^* + i \xi_{-p}).
\]

(3.8)

The Jacobian of this transformation is equal to 1, \( d\xi d\xi^* = d\omega d\omega^* \). One can easily check that

\[
\xi^* \xi = \omega^* \omega; \quad \frac{1}{2} \sum_p \tau(p) (\xi_{-p}^* \xi_{-p} + x \xi_{p} \xi_{p}^*) = -\omega^* T \omega,
\]

(3.9)

where the matrix \( T \) is defined in \((3.7)\). Hence,

\[
N = \int d\omega d\omega^* \exp \{ -\omega^* (I + T) \omega \} = \det(I + T),
\]

which proves the relation \((3.6)\).
Introduce now the coherent states for the new set of operators (compare with (A.2)):

\[ |X\rangle = \exp \left\{ \sum_p A^+_p X_p \right\} |0\rangle; \]

\[ \langle \langle X^\ast | = \langle \langle 0 | \exp \left\{ \sum_p X^*_p A_p \right\}. \]  
(3.10)

The Grassman algebra elements \( X_p, X^*_p \) have the same properties as the old parameters \( \xi_\beta, \xi^*_\beta \). The formulae (1.12)-(A.10) are valid, of course, also for the new states. The direct calculation gives the following representations for the scalar products of old and new states

\[ \langle \langle \xi^\ast | X \rangle = N^{-1/2} \exp \left\{ \frac{1}{2} \sum_p \tau(p)(\xi^*_\beta - X^-\!_p \xi^*_\beta) + \sum_p \alpha^{-1}(p)\xi^*_\beta X_p \right\}, \]

\[ \langle \langle X^\ast | \xi \rangle = N^{-1/2} \exp \left\{ \frac{1}{2} \sum_p \tau(p)(\xi^-\!_\beta - X^\ast_p \xi^-\!_\beta) + \sum_p \alpha^{-1}(p)X^\ast_p \xi^-\!_\beta \right\}. \]  
(3.11)

Considering the XY model it is necessary to introduce different sets of operators \( a_p, a^+_p \) and \( A_p, A^+_p \) corresponding to the sets of momenta \( X_+ \) (for \( H^+ \)) and \( X_- \) (for \( H^- \)), see (1.19), (1.20). Thus the new vacuums \(|\Omega\rangle\) will be also different for \( H^+ \) and \( H^- \) (unlike the old vacuum \(|0\rangle\)). We will not usually mark in our notations this difference but one should have it in mind.

Using the equation (3.11), one can calculate the matrix elements of the operators \( \exp \{-\beta H^\pm\} \) diagonal in the new representations (different for \( H^+ \) and \( H^- \)) between old coherent states (also different for \( H^+ \) and \( H^- \));

\[ \langle \langle \xi^\ast | e^{-\beta H^\pm} | \xi \rangle = N^{-1} e^{-\beta E^\pm_\beta} \det(I - J_\beta T) e^{\omega^\ast T D \omega}, \]  
(3.12)

where the variables \( \omega^p, \omega^-_p \) \( (p \in X^+ \) for \( H^+ \) and \( p \in X^- \) for \( H^- \)) are defined by the equation (3.8), and diagonal \( M \times M \) matrices \( J_\beta \) and \( D \) are

\[ J_\beta = \text{diag}(j_\beta(p)); \quad j_\beta(p) = \exp\{-\beta E(p)\}, \]  
(3.13)

\[ D = \text{diag}(d(p)); \quad d(p) = \frac{j_\beta(p) - i\tau(p)}{1 - i\tau(p)j_\beta(p)}. \]  
(3.14)

(the matrix \( T \) is defined in (3.7)).

To prove it we use the completeness of the new states \( |0\rangle \),

\[ \langle \langle \xi^\ast | e^{-\beta H^\pm} | \xi \rangle = \int dXdX^\ast dY^\ast dY \times \]

\[ \times \langle \langle \xi^\ast | X \rangle \langle Y^\ast | e^{-\beta H^\pm} | Y \rangle \langle \langle X^\ast | \xi \rangle \exp\{-Y^\ast X - X^\ast Y\}, \]

and note that the operators \( H^\pm \) are diagonal in the corresponding new representations (3.11):

\[ \langle \langle Y^\ast | e^{-\beta H^\pm} | Y \rangle = e^{-\beta E^\pm_\beta} \exp \left\{ \sum_{p \in X^\pm} e^{-\beta E(p)} Y^\ast_p Y_p \right\}. \]  
(3.15)
After the integration over $Y,Y^*$ one gets
\[
\langle \xi^* | e^{-\beta H^\pm} | \xi \rangle = \int dX dX^* \langle \xi^* | X \rangle \langle X^* | \xi \rangle \times \\
e^{-\beta E^\pm_0} \exp \left\{ \sum_{p \in X^\pm} e^{-\beta E(p)} X_p^p X_p \right\}.
\]
The integral over $X,X^*$ can be calculated by means of the change of variables as in (3.8). Finally one gets the formula (3.12).

In the following sections the results obtained here will be used to calculate the equal-time correlators.

4 The simplest correlator for the XY chain

In this section the partition function $Z$ and the generating functional of the third components of local spins are calculated for the anisotropic chain. We begin by considering the partition function. The initial representation is the same as in the isotropic case (B.1)
\[
Z = \frac{1}{2} (Z_F^+ + Z_F^- + Z_B^+ - Z_B^-),
\]
where
\[
Z_F^\pm = \text{Tr} \exp \{-\beta H^\pm\}; \\
Z_B^\pm = \text{Tr}(\exp\{-\beta H^\pm\})(-1)^N).
\]
One gets the following representations for the contributions (see for example [25]; the $M \times M$ matrix $J_\beta$ is defined in (3.13)):
\[
Z_F^\pm = e^{-\beta E^\pm_0} \text{det}(I + J_\beta) = \prod_{p \in X^\pm} \left( 2 \cosh \frac{\beta E(p)}{2} \right),
\]
\[
Z_B^\pm = e^{-\beta E^\pm_0} \text{det}(I - J_\beta) = \prod_{p \in X^\pm} \left( 2 \sinh \frac{\beta E(p)}{2} \right).
\]
To obtain, e.g., the fermionic contributions one should use the representation (A.5) for the trace of operators:
\[
Z_F^\pm = \int dY dY^* \langle Y^* | e^{-\beta H^\pm} | Y \rangle \exp\{Y^* Y\},
\]
and the representation (3.13) for the matrix element involved; after that one should calculate the Gaussian integral leading to the equality (4.3). The "bosonic" contributions can be calculated analogously.

It is worth mentioning that it is sometimes convenient to represent the answer differently using the old coherent states, for example,
\[
Z_F^\pm = \int d\xi d\xi^* \langle \xi^* | e^{-\beta H^\pm} | \xi \rangle \exp\{\xi^* \xi\}.
\]
By means of equation (3.12) for the corresponding matrix element one gets
\[
Z_F^\pm = N^{-1} e^{-\beta E^\pm_0} \text{det}(I - J_\beta T) \text{det}(I + D),
\]
\[ Z_B^\pm = N^{-1} e^{-\beta E_B^\pm} \det(I - J_q T) \det(I - D). \] (4.6)

Turn now to the simplest equal-time temperature correlator

\[ G(M) = \frac{1}{Z} \text{Tr} \left( e^{\alpha Q(m)} e^{-\beta \mathbf{H}} \right). \] (4.7)

As for the isotropic chain ((B.8), (B.9)) it can be represented as a sum of four contributions

\[ G(m) = \frac{1}{2Z} \left( Z_F^+ G_F^+ + Z_F^- G_F^- + Z_B^+ G_B^+ - Z_B^- G_B^- \right). \] (4.8)

where

\[ Z_F^\pm G_F^\pm = \text{Tr} \left( e^{\alpha Q(m)} e^{-\beta \mathbf{H}^\pm} \right), \]
\[ Z_B^\pm G_B^\pm = \text{Tr} \left( e^{\alpha Q(m)} e^{-\beta \mathbf{H}^\pm (-1)^N} \right). \] (4.9)

One has for the contributions \( G_{F,B}^\pm \) the following representations as determinants of \( M \times M \) matrices

\[ G_F^\pm = \det \left( I + (e^\alpha - 1) Q^{(0)}(m) \Omega_F \right), \] (4.10)
\[ G_B^\pm = \det \left( I - (e^\alpha - 1) Q^{(0)}(m) \Omega_B \right). \] (4.11)

Here the matrix elements of \( Q^{(0)}(m) \) are given by (2.16) with \( p, q \in X^+ \) for \( G_{F,B}^+ \) and \( p, q \in X^- \) for \( G_{F,B}^- \). Diagonal \( M \times M \) matrices \( \Omega_F \) and \( \Omega_B \) are given by the following formulæ (\( p \in X^\pm \) for \( G_{F,B}^\pm \))

\[ \Omega_F = D(I + D)^{-1} = \text{diag}(\omega_F(p)); \]
\[ \omega_F(p) = \frac{1}{2} \left( 1 - e^{i\theta(p)} \tanh \frac{\beta E(p)}{2} \right), \] (4.12)
\[ \omega_B = D(I - D)^{-1} = \text{diag}(\omega_B(p)); \]
\[ \omega_B(p) = \frac{1}{2} \left( e^{i\theta(p)} \coth \frac{\beta E(p)}{2} - 1 \right). \] (4.13)

(the matrix \( D \) is defined in (8.13)).

Explain now how to calculate the contribution \( G_F^\pm \):

\[ Z_F^\pm G_F^\pm = \int d\xi d\eta d\eta^* d\xi^* e^{\alpha Q(m)} |\eta\rangle \langle \xi^*| e^{-\beta \mathbf{H}^\pm} |\xi\rangle e^{\eta^* \xi - \xi^* \eta}. \] (4.14)

The matrix elements on the right hand side are given by (2.15) and (3.12). We change the variables \( \eta^* \rightarrow -\eta^* \); since \( M \) is even, the integration measure is invariant. After that we change the variables as in (3.8)

\[ \omega_p = \frac{1}{\sqrt{2}} (\xi_p + i\xi^*_p); \quad \omega_p^* = \frac{1}{\sqrt{2}} (\xi^*_p + i\xi_p); \]
\[ \rho_p = \frac{1}{\sqrt{2}} (\eta_p + i\eta^*_p); \quad \rho_p^* = \frac{1}{\sqrt{2}} (\eta^*_p + i\eta_p). \] (4.15)

As a result of this change of variables the measure remains invariant and

\[ \eta^* \xi + \xi^* \eta = \rho^* \omega + \omega^* \rho, \]
\[ \eta^* (I + (e^\alpha - 1) Q(m)) \eta = \rho^* (I + (e^\alpha - 1) Q(m)) \rho. \] (4.16)
The integration over $\omega, \omega^*$ of the factors depending on these variables in (4.14) gives
\[ \det D \exp \{-\rho^* D^{-1} \rho\}. \]
One can calculate the remaining Gauss integral on $\rho, \rho^*$ taking into account the representation (4.5) for the partition functions and equation (4.12):
\[ Z^+ G^+ = Z^+ \det(I + (e^\alpha - 1)Q(m)\Omega) \] (4.17)
One should note that the matrix $Q(m)$ (4.10) differs only by an evident similarity transformation with a diagonal matrix from the matrix $Q(0)(m)$. Thus the representation (4.10) for $G^+$ is proved. The derivation of the contribution $G^-$ is almost the same, one should only use the momenta from the set $X^-$. To calculate the bosonic contributions one should use the property of the operator $(e^\alpha - 1)$,
\[ (e^\alpha - 1)\mid \xi \rangle = \mid -\xi \rangle, \]
(4.18)
in the representation similar to (4.14) which change evidently the calculations leading to the result (4.11).

We should make a remark about the equations (4.10), (4.11). Since the sets $X^+$ and $X^-$ are symmetric under the change of momenta $p_i \rightarrow -p_i$ (there is an exception, it is the momenta 0 and $\pi$ from the set $X^-$; but the result is valid also in this case) one can rewrite the answer as (the sign “+” on the right hand side corresponds to $G^+$ and the sign “−” corresponds to $G^−$)
\[ G^+ = \det(I + (e^\alpha - 1)Q(m)\Omega) \]
(4.19)
The bar means here the complex conjugation and $\omega_{F,B}(p) = \bar{\omega}_{F,B}(p)$

To conclude we discuss some limiting cases.
In the zero temperature limit ($\beta \rightarrow \infty$) the "odd" contributions to the correlator are cancelled and one gets the representation for $G(m)$
\[ G(m) = \det(I + (e^\alpha - 1)Q(0)(m)\Omega_0) \] (T = 0),
(4.20)
where
\[ \Omega_0 = \text{diag}(\omega_0(p)), \quad \omega_0(p) = \frac{1}{2} \left(1 - e^{i\theta(p)}\right), \]
(4.21)
coinciding with the result obtained in [27]. On the other hand for the isotropic case ($\gamma = 0$) taking into account that the angle $\theta(p)$ in the Bogolyubov transformation is
\[ \theta(p) = -\pi \text{sign} p, \quad |p| < k_F; \quad \theta(p) = 0, \quad |p| > k_F \] ($\gamma = 0$),
(4.22)
one gets for the weights
\[ \omega_F(p) = \frac{1}{2} \left(1 - \tanh \frac{\beta \varepsilon(p)}{2}\right), \]
\[ \omega_B(p) = \frac{1}{2} \left(\coth \frac{\beta \varepsilon(p)}{2} - 1\right) \] ($\gamma = 0$).
(4.23)
Here $k_F = \arccos h$ is the Fermi momentum and $\varepsilon(p)$ is the dispersion of the XX0 chain (see (1.24)). Thus one reproduces the answers for the isotropic case [3, 29].
5 Equal-time correlators of the local spins

Here we consider the equal-time correlation functions of the local spin operators ($\beta \equiv 1/T$):

$$G^{(ab)}(m) = \langle \sigma^a_{m+1} \sigma^b_1 \rangle_T = \frac{1}{Z} \text{Tr}(\sigma^a_{m+1} \sigma^b_1 e^{-\beta H}), \quad a, b = +, -, \quad (5.1)$$

These correlators on a finite lattice of length $M$ can be represented (as in (3.8)) as a sum of four contributions

$$G^{(ab)} = \frac{1}{2Z} \left( Z_F G^{(ab),+}_F + Z_F G^{(ab),-}_F + Z_B G^{(ab),+}_B - Z_B G^{(ab),-}_B \right), \quad (5.2)$$

where

$$Z_F^\pm G^{(ab),\pm} = \text{Tr} \left( a_{m+1} \right)^\pm e^{-\beta H^\pm} $$

$$Z_B^\pm G^{(ab),\pm} = \text{Tr} \left( a_{m+1} \right)^\pm e^{-\beta H^\pm} (-1)^N, \quad (5.3)$$

(partition functions $Z^\pm_F, Z^\pm_B$ are given by (4.13)). For the contributions we obtain the following representations as determinants of $M \times M$ matrices:

$$G^{(+),\pm}_F = G^{(-),\pm}_F = \frac{\partial}{\partial q} \det (I + U_F + \alpha C \Omega_F) \bigg|_{\alpha = 0} (m > 0), \quad (5.4)$$

$$G^{(-),\pm}_F = \frac{1}{M} \text{tr} \Omega_F = \frac{1}{M} \sum_p \omega_F(p) \quad (m = 0); \quad (5.5)$$

$$G^{(+),\pm}_F = 1 - \frac{1}{M} \text{tr} \Omega_F \quad (m = 0), \quad (5.6)$$

$$G^{(-),\pm}_B = G^{(+),\pm}_B = \frac{\partial}{\partial q} \det (I - U_F - \alpha C \Omega_F) \bigg|_{\alpha = 0} (m > 0), \quad (5.7)$$

$$G^{(+),\pm}_B = - \frac{1}{M} \text{tr} \Omega_B; \quad G^{(-),\pm}_B = 1 + \frac{1}{M} \text{tr} \Omega_B \quad (m = 0), \quad (5.8)$$

$$G^{(-),\pm}_B = G^{(+),\pm}_B = \frac{\partial}{\partial q} \det (I - U_F - \alpha C \Omega_F) \bigg|_{\alpha = 0} (m > 0). \quad (5.9)$$

The diagonal $M \times M$ weight matrices $\Omega_F$ and $\Omega_B$ are defined in (4.12), (4.13). Matrix elements of $M \times M$ matrices $U, C, S$ are given by the following formulae

$$U_{p_1 p_2} = -\exp \left( \frac{i}{2} (p_1 - p_2) \right) Q^{(0)}_{p_1 p_2} (m), \quad (5.10)$$

(the definition of the matrix $Q^{(0)}(m)$ is in (2.17)),

$$C_{p_1 p_2} = \frac{1}{M} \cos \frac{m}{2} (p_1 + p_2); \quad (5.11)$$

$$S_{p_1 p_2} = \frac{1}{M} \sin \frac{m}{2} (p_1 + p_2). \quad (5.12)$$

It is necessary to emphasize that the momenta numerating the matrix elements $p_1, p_2 \in X^+$ for the contributions $G^{(ab),+}_{F,B}$ and $p_1, p_2 \in X^-$ for the contributions $G^{(ab),-}_{F,B}$; the same thing is true for the weight matrices.
Calculating the functions $G_{F,B}^{(ab),\pm}$ is reduced to calculating Gaussian integrals in the Grassmanian variables. For example, using (A.5) and (A.6) we represent

$$Z_+^{F,B}G_{F,B}^{(ab),+} = \int d\xi d\xi^* d\eta d\eta^* \langle \eta^* | \sigma_{m+1}^a \sigma_1^b | \eta \rangle \langle \xi^* | e^{-\beta H^+} | \xi \rangle e^{\alpha^* \xi - \xi^* \eta}. \quad (5.13)$$

Consider the calculation of the matrix element

$$F_{ab} = \langle \eta^* | \sigma_{m+1}^a \sigma_1^b | \eta \rangle = \int d\zeta d\zeta^* \langle \eta^* | \sigma_{m+1}^a | \zeta \rangle \langle \zeta^* | \sigma_1^b | \eta \rangle e^{-\zeta^* \zeta}. \quad (5.14)$$

Let us use the formulae (2.22) for the matrix elements of spin operators and make the change of variables

$$\tilde{\eta}_p = e^{-\frac{im}{2}p} \eta_p; \quad \tilde{\eta}_\bar{p} = e^{\frac{im}{2}p} \eta_p \quad (p \in X^+),$$
$$\tilde{\zeta}_q = e^{-\frac{im}{2}q} \zeta_q; \quad \tilde{\zeta}_\bar{q} = e^{\frac{im}{2}q} \zeta_q \quad (q \in X^-). \quad (5.14)$$

One gets the representation for the matrix element using the new variables (we omit tildes over the new variables (5.14)):

$$F_{ab} = \frac{\partial}{\partial \alpha} \int d\zeta d\zeta^* \exp \{ \omega + \alpha f_{ab} \} \big|_{\alpha=0}, \quad (5.15)$$

where

$$\omega = \eta^* P \mathcal{L} \zeta + \zeta^* \bar{P} \mathcal{L} P \eta - \zeta^* \zeta, \quad (5.16)$$
$$f_{++} = \eta^* R_+ \eta; \quad (R_+)_{p_1p_2} = \frac{1}{M} e^{-\frac{im}{2}(p_1+p_2)}, \quad (5.17)$$
$$f_{+-} = \zeta^* \bar{R}_+ \zeta; \quad (\bar{R}_+)_{q_1q_2} = \frac{1}{M} e^{\frac{im}{2}(q_1+q_2)}, \quad (5.18)$$
$$f_{++} = i\eta R_- \zeta; \quad (R_-)_{pq} = \frac{1}{M} e^{-\frac{im}{2}(p-q)}, \quad (5.19)$$
$$f_{--} = i\zeta^* \bar{R}_- \eta^*; \quad (\bar{R}_-)_{qp} = \frac{1}{M} e^{\frac{im}{2}(q-p)} = (R_-)_{pq}, \quad (5.20)$$

(the bar means the complex conjugation). The diagonal matrix $P$ has the form

$$P = \text{diag} \left( e^{-\frac{im}{2}p} \right). \quad (5.21)$$

Use now the following identities

$$\sum_q L_{qp} e^{-imq} = (2\delta_{m,0} - 1)e^{im}; \quad (5.22)$$
$$\sum_q e^{imq} L_{qp} = e^{im} \quad (p \in X^+, q \in X^-, \quad m = 0, 1, \ldots, M - 1), \quad (5.22)$$

$$PLPR_+ \mathcal{L} P = -(R_+)_1 \mathcal{L} (1 - 2\delta_{m,0}) \quad (5.23)$$
$$\bar{P} \mathcal{L} P = (R_-)_1 \mathcal{L} \quad (5.24)$$
$$PLPR_- \mathcal{L} P = -(1 - 2\delta_{m,0}) \quad (5.25)$$
Finally we change the variables as in (4.15),
besides (3.8), we changed the variables as in (5.14) and also we changed
$R$ and $M$ get for the contribution

and on the right hand side ($\bar{R}_+|p_1 \bar{p}_2$, $R_-|p_1 \bar{p}_2$). As a result, the matrix elements $F_{ab}$ (5.13) are represented as follows

$$F_{-+} = \frac{\partial}{\partial \alpha} \exp\{\eta^* (I + U) \eta + \alpha \eta^* R_+ \eta\} \bigg|_{\alpha=0},$$

$$F_{+-} = (\delta_{m,0} + \frac{\partial}{\partial \alpha}) \exp\{\eta^* (I + U) \eta + \alpha \eta^* \bar{R}_+ \eta\} \bigg|_{\alpha=0},$$

$$F_{++} = \frac{\partial}{\partial \alpha} \exp\{\eta^* (I + U) \eta + \alpha \eta^* S_- \eta\} \bigg|_{\alpha=0},$$

$$F_{--} = \frac{\partial}{\partial \alpha} \exp\{\eta^* (I + U) \eta - \alpha \eta^* S_- \eta\} \bigg|_{\alpha=0}. $$

(5.26)

Here the matrix elements of the $M \times M$ matrix $U$ are defined as $U_{p_1 \bar{p}_2} = (PLD^2LP)_{p_1 \bar{p}_2} - \delta_{p_1 \bar{p}_2}$ and this representation leads to (5.10) if one takes into account (2.24); the matrices $R_+,$ $\bar{R}_+$ are given by (4.17), (5.18) and (5.12); the matrix $\bar{R}_+$ is defined by the relation

$$(\bar{R}_+)_{p_1 \bar{p}_2} = (\bar{R}_+)_{p_1 \bar{p}_2} (1 - 2\delta_{m,0}).$$

(5.27)

$$(S_-)_{p_1 \bar{p}_2} = \frac{1}{M} \sin \frac{m}{2} (p_1 - p_2).$$

(5.28)

Now we put the expressions (5.12) into (5.3) using also (5.12). Then it is not difficult to get for the contribution $G_{F}^{(-+)\pm}$

$$Z_{L}^{\pm} G_{F}^{(-+)\pm} = N^{-1} e^{-\beta E_{F}^{\pm}} \det(I - J_{3\alpha}T) \frac{\partial}{\partial \alpha} \int d\xi d\eta d\rho d\rho^* \times$$

$$\times \exp\{-\eta^* (I + U + \alpha R_+) \eta + \omega^* D \omega - \eta^* \xi - \xi^* \eta\},$$

(5.29)

(besides (3.8), we changed the variables as in (5.14) and also we changed $\eta^* \rightarrow -\eta^*$). Finally we change the variables as in (4.12); $(\eta, \eta^*) \rightarrow (\rho, \rho^*)$; $(\xi, \xi^*) \rightarrow (\omega, \omega^*)$. Using (5.16) and equalities

$$\eta^* (I + U + \alpha R_+) \eta = \rho^* (I + U + \alpha C) \rho - \frac{\alpha}{2} (\rho^* S_- \rho^* - \rho S_- \rho),$$

we integrate over $\omega, \omega^*$ and then over $\rho, \rho^*$ in (5.29). As a result we get the following representation as a determinant of a $2M \times 2M$ matrix:

$$Z_{L}^{\pm} G_{F}^{(-+)\pm} = N^{-1} e^{-\beta E_{F}^{\pm}} \det(I - J_{3\alpha}T) \det D \times$$

$$\times \frac{\partial}{\partial \alpha} \det^{1/2} \begin{pmatrix} \alpha S_- & -B \\ \cdots & \cdots \\ B^T & -\alpha S_- \end{pmatrix},$$

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where we denoted $B = I + D^{-1} + U + \alpha C$. Using the generalized Gauss algorithm it is not difficult to check that

$$\det \begin{pmatrix} \alpha S_+ & \cdots & -B \\ \cdots & \cdots & \cdots \\ B^T & \cdots & -\alpha S_+ \end{pmatrix} = \det^2 B + O(\alpha^2),$$

(it is valid also for $m=0$). It leads to the representation for $G_F^{(-+)+}$. Analogously one can calculate other contributions.

One should note that the expressions (5.4)-(5.9) for equal-time temperature correlators of the anisotropic XY chain differ from the corresponding expressions for the isotropic XX0 chain only by changing the weights. For example, the corresponding modification for the ”fermionic” weight is as before

$$\frac{1}{2} \left( 1 - \tanh \frac{\beta \epsilon(p)}{2} \right) \rightarrow \frac{1}{2} \left( 1 - e^{i\theta(p)} \tanh \frac{\beta E(p)}{2} \right). \quad (5.30)$$

### 6 The thermodynamic limit

Now we consider the correlators for the anisotropic XY chain in the thermodynamic limit (the length of the chain $M \to \infty$, the magnetic field $h$ is fixed). In this limit the partition functions $Z_{F,B}$ are divergent and using (4.3), (4.4) one can estimate

$$Z_B/Z_F < C_M; \quad C = \tanh \frac{\beta E_{\text{max}}}{2} < 1,$$

where $E_{\text{max}}$ is the maximal value of the quasiparticle energy $E(p)$ (28). So only the ”fermionic” contributions survive in the thermodynamic limit in (4.8) and (5.2). The determinants of the $M \times M$ matrices in the expressions (4.10), (5.4) for these contributions become in the thermodynamic limit the Fredholm determinants of the corresponding integral operators. It is explained explicitly, for example, in [5]. So we get the following answers.

The correlator $G(m)$ (17) is given by

$$G(m) = \lim_{Z} \frac{1}{Z} \text{Tr} (e^{\alpha Q(m)} e^{-\beta \hat{H}}) = \det \left( \hat{I} + (e^{\alpha} - 1) \hat{V} \right). \quad (6.1)$$

At the right hand side, there is the Fredholm determinant of a linear integral operator acting on functions $f(p)$ on the interval $-\pi \leq p \leq \pi$

$$\hat{V} f(p) = \int_{-\pi}^{\pi} V(p,q) f(q) dq, \quad (6.2)$$

$\hat{I}$ means the identity operator on the interval and the kernel $V(p,q)$ is (see (4.11), (2.17), (4.12))

$$V(p,q) = \frac{1}{2\pi} \frac{\sin \frac{\pi(m-q)}{2}}{\sin \frac{\pi}{2}} \omega_F(q), \quad (6.3)$$
The weight $\omega_F(q)$ is equal to

$$\omega_F(q) = \frac{1}{2} \left( 1 - e^{i\theta(q)} \tanh \frac{\beta E(q)}{2} \right) \quad (6.4)$$

(the angle $\theta(p)$ and the quasiparticle energy $E(p)$ are defined in (1.27), (1.28)).

In the thermodynamic limit the correlators (5.1)

$$G^{(ab)}(m) = \lim \frac{1}{Z} \text{Tr}(\sigma^a_{m+1} \sigma^b_1 e^{-\beta H}) \quad (6.5)$$

can be also represented as Fredholm determinants of linear integral operators on the interval $[-\pi, \pi]$. From (5.4)-(5.6) one gets for $m > 0$:

$$G^{(-+)}(m) = G^{(+ -)}(m) = \frac{\partial}{\partial \alpha} \det(\hat{I} - \hat{W} + \alpha \hat{C}) \bigg|_{\alpha = 0} \quad (m > 0), \quad (6.6)$$

$$G^{(++)}(m) = G^{(---)}(m) = \frac{\partial}{\partial \alpha} \det(\hat{I} - \hat{W} - i\alpha \hat{S}) \bigg|_{\alpha = 0} \quad (m > 0). \quad (6.7)$$

The kernels of the operators $\hat{W}$, $\hat{C}$ and $\hat{S}$ are

$$W(p, q) = \frac{1}{\pi} e^{i\frac{(p-q)}{2}} \frac{\sin \frac{m(p-q)}{2}}{\sin \frac{m}{2}} \omega_F(q), \quad (6.8)$$

$$C(p, q) = \frac{1}{2\pi} \cos \frac{m(p+q)}{2} \omega_F(q) \quad (6.9)$$

$$S(p, q) = \frac{1}{2\pi} \sin \frac{m(p+q)}{2} \omega_F(q). \quad (6.10)$$

These answers differ only by changing the weight (5.30) from the answers for the isotropic XX0 chain.

One should note that the formal answers for the time-dependent correlation functions were obtained recently in [29]. In our next paper we hope to give more clear answers for the time-dependent correlators.

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**Appendix A**

Consider a set of canonical fermion operators

$$a_q, a_q^+ \quad ([a_p, a_q] = [a_p^+, a_q^+] = 0, [a_p, a_q^+] = \delta_{p,q}).$$

We denote by $M$ the number of operators $a$ (or $a^+$) in the set. It is supposed that the Fock vacuum exists and has the following properties

$$a_q |0\rangle = 0; \quad |0\rangle a_q^+ = 0, \quad \forall q; \quad \langle 0|0\rangle = 1. \quad (A.1)$$

We introduce the coherent states

$$|\xi\rangle = |\xi_q\rangle = \exp \left\{ \sum_q a_q^+ \xi_q \right\} |0\rangle;$$
where the summation is taken over the whole set. The parameters \( \xi_q, \xi^*_q \) (Grassman algebra elements) anticommute among themselves and with all the operators \( a_q, a^+_q \). One should emphasize that the star in \( \xi^* \) means only that the corresponding parameter is connected with a bra-vector; we don’t consider involutions on the Grassman algebra; parameters \( \xi \) and \( \xi^* \) are entirely independent. The coherent states (A.2) are the eigenstates for the creation and annihilation operators, respectively

\[
|\xi\rangle = |\xi^*\rangle = \langle 0| \exp \left\{ \sum_q \xi^*_q a_q \right\},
\]

(A.2)

One can easily calculate the scalar product of two coherent states using the commutation relations between \( a, a^+, \xi, \xi^* \):

\[
\langle \xi^* | \xi \rangle = \exp \left\{ \sum_q \xi^*_q \xi_q \right\} \equiv \exp \{\xi^* \xi\}. \tag{A.4}
\]

The trace of an operator \( O \) can be represented as an integral over the anticommuting variables of the matrix elements of the operator between the coherent states (see [30]):

\[
\text{Tr}O = \int d\xi d\xi^* \exp\{\xi^* \xi\} \langle \xi^* | O | \xi \rangle, \tag{A.5}
\]

and the expansion for the identity operator is given by

\[
1 = (-1)^M \int d\xi d\xi^* \exp\{-\xi^* \xi\} |\xi\rangle \langle \xi^*|,
\]

(A.6)

(we supposed that the number of sites \( M \) is even and the coefficient \((-1)^M\) in such formulae is usually omitted). We use the following notation

\[
d\xi = \prod_q d\xi_q; \quad d\xi^* = \prod_q d\xi^*_q.
\]

If an operator \( L \) has the following form

\[
L = \sum_{p,q} a^+_p L_{pq} a_q \equiv a^+ L a,
\]

where \( L \) is a \( M \times M \) matrix (with matrix elements \( L_{pq} \)) which can be diagonalized by an unitary matrix \( U \),

\[
L = U^+ D U; \quad U^+ U = U U^+ = I; \quad D = \text{diag}(D_q),
\]

then the matrix elements of the operator \( \exp L \) are given by

\[
\langle \xi^* | \exp\{L\} | \xi \rangle = \exp \left\{ \sum_{p,q} \xi^*_q (\exp\{L\})_{pq} \xi_p \right\} \equiv \exp\{\xi^* \exp L \xi\}, \tag{A.7}
\]

and

\[
\text{Tr} \exp\{L\} = \text{Tr} \exp\{a^+ L a\} = \det[I + \exp\{L\}], \tag{A.8}
\]

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The last equality follows from a well-known formula for the Gaussian integral over the anticommuting variables
\[ \int d\xi d\xi^* \exp\{\xi^* K \xi + \xi \eta + \eta^* \xi\} = \exp\{-\eta^* K^{-1} \eta\} \det K. \] (A.9)

We need also to use a formula valid for an antisymmetric matrix \( A \),
\[ \int d\xi \exp\left\{ \frac{1}{2} \xi A \xi + \eta \xi \right\} = \exp\left\{ \frac{1}{2} \eta A^{-1} \eta \right\} \sqrt{\det A}, \] (A.10)

Appendix B
Here we give the derivation of the formulae for the time-dependent correlators for the isotropic XX0 chain with finite number of sites \( M \) (\( M \) is supposed to be even). We hope that readers will appreciate the simplicity of the derivation of the results using the integration over the Grassmanian variables (compared with paper \[5, 26\]). Below \( H \) denotes the Hamiltonian of the XX0 model We begin with the calculation of the partition function \( Z \). Following \[23, 25\] we represent it in the form
\[ Z = \text{Tr} \exp\{ -\beta H \} = \frac{1}{2} (Z_+ + Z_- + Z_+ - Z_-), \] (B.1)
where \( \beta \equiv 1/T \) is the inverse temperature and the contributions on the right hand side are defined by the formulae
\[ Z_\pm = \text{Tr} \exp\{ -\beta H \pm \} \equiv e^{\beta M \hbar/2} \det (I + J_\beta); \]
\[ Z_\pm^B = \text{Tr}[(-1)^N \exp\{ -\beta H \pm \}] e^{\beta M \hbar/2} \det (I - J_\beta), \] (B.2)
where \( J_\beta \) is a diagonal matrix
\[ J_\beta = \text{diag}(e^{-\beta \varepsilon(p)}), \] (B.3)
(we emphasize that \( p \in X^+ \) for \( Z_\pm^\pm \) and \( p \in X^- \) for \( Z_\pm^B \)). To obtain these representations one should use the following relation
\[ (-1)^N |\phi, \pm \rangle \rightarrow | -\phi, \pm \rangle, \] (B.4)
\[ \exp\{ -\beta H \pm \} |\phi, \pm \rangle \equiv e^{\beta M \hbar/2} [J_\beta \phi, \pm \rangle \equiv \]
\[ \equiv e^{\beta M \hbar/2} \exp \left\{ \sum_{p \in X^\pm} a_p^* e^{-\beta \varepsilon(p)} \phi_p \right\} |0\rangle, \] (B.5)
and rewrite, for example, \( Z_\pm^B \) as a Gaussian integral
\[ Z_\pm^B = e^{\beta M \hbar/2} \int d\phi d\phi^* \exp\{\phi^*[I - J_\beta]\phi\}, \] (B.6)
(see \[A.4, A.5\]).

Consider now the simplest equal-time temperature correlator,
\[ G(m) \equiv \frac{1}{Z} \text{Tr} \left[ e^{\alpha Q(m)} e^{-\beta H} \right], \] (B.7)
which can be represented in the following form \[26\]:

\[
G(m) = \frac{1}{2Z} (Z_F^+ G_F^+ + Z_F^- G_F^- + Z_B^+ G_B^+ - Z_B^- G_B^-),
\]

where

\[
Z_F^\pm G_F^\pm = \text{Tr} \left( e^{\alpha Q(m)} e^{-\beta \mathcal{H}^\pm} \right),
\]

\[
Z_B^\pm G_B^\pm = \text{Tr} \left( e^{\alpha Q(m)} e^{-\beta \mathcal{H}^\pm} (-1)^N \right).
\]

The contributions can be represented as determinants of matrices:

\[
G_F^\pm = \text{det} \left( I + (e^\alpha - 1) Q^{(0)}(m) \Theta_F \right),
\]

\[
G_B^\pm = \text{det} \left( I - (e^\alpha - 1) Q^{(0)}(m) \Theta_B \right)
\]

where diagonal matrices of the Fermi and Bose weights \( \Theta_F \) and \( \Theta_B \) are

\[
\Theta_F = J_\beta [I + J_\beta]^{-1} = \text{diag} \left[ \Theta_F(p) = \frac{1}{1 + e^{2\beta(p)}} \right],
\]

\[
\Theta_B = J_\beta [I - J_\beta]^{-1} = \text{diag} \left[ \Theta_B(p) = \frac{1}{e^{2\beta(p)} - 1} \right],
\]

and the matrix \( Q^{(0)}(m) \) with matrix elements \( Q_{pq}^{(0)}(m) \) was defined in \[21\] \((p,q \in X^+ \text{ for } G_{F,B}^+ \text{ and } p,q \in X^- \text{ for } G_{F,B}^-) \). To obtain these representations one should rewrite, for example, the contribution \( Z_B^- G_B^- \) as

\[
Z_B^- G_B^- = e^{\beta \mathcal{H}^2/2} \int d\phi d\phi^* e^{\sigma^* \cdot \phi - |e^\alpha Q(m)| - J_\beta \phi, -},
\]

use the expression \[2.18\] for the matrix element under the integral, calculate the Gaussian integral, perform the similarity transformation \( Q(m) \to Q^{(0)}(m) \) (see \[2.16\], \[2.17\]) and extract the coefficient \( \text{det}(I - J_\beta) \) from the determinant obtained.

Now we turn to the time-dependent correlation function of the local spins; as usual

\[
\sigma_m^a(t) = e^{it \mathcal{H} \sigma_m^a} e^{-it \mathcal{H}}.
\]

Because of the translation invariance, the local spin correlators \( \sigma_{m_2}^a(t_2), \sigma_{m_1}^a(t_1) \) depend only on differences \( m = m_2 - m_1, t = t_2 - t_1 \):

\[
G^{(ab)}(m,t) = \frac{1}{Z} \text{Tr} (e^{it \mathcal{H} \sigma_m^a} e^{-it \mathcal{H} \sigma_m^b} \sigma_1 e^{-\beta \mathcal{H}}), \quad a,b = \pm.
\]

Due to the property \[2.21\] the correlator can be represented as a sum of four contributions:

\[
G^{(ab)}(m,t) = \frac{1}{2Z} \left( Z_F^+ G_F^{(ab),+} + Z_F^- G_F^{(ab),-} + Z_B^+ G_B^{(ab),+} - Z_B^- G_B^{(ab),-} \right),
\]

where

\[
Z_F^\pm G_F^{(ab),\pm} = \text{Tr} \left( e^{it \mathcal{H} \sigma_m^a} e^{-it \mathcal{H} \sigma_1 e^{-\beta \mathcal{H}}} \right),
\]

\[
Z_B^\pm G_B^{(ab),\pm} = \text{Tr} \left( e^{it \mathcal{H} \sigma_m^a} e^{-it \mathcal{H} \sigma_1 e^{-\beta \mathcal{H}}} (-1)^N \right).
\]
We begin with the correlator $G^{(-)}(m,t)$. One can represent, e.g., the contribution

\[ Z^+_F G^{(-)} = e^{\beta M h/2} \int d\phi d\phi^* d\psi d\psi^* e^{\phi^* \phi - \psi^* \psi} \times \]

\[ \langle \phi^* J(0,t) + |\sigma_{-1}^- m \rangle J(0,-t) \psi, - |\sigma_{-1}^+ m \rangle J(0,+) \rangle \]

where we introduced a diagonal $M \times M$ matrix $J(m,t)$:

\[
J(m,t) = \text{diag} (e^{-ipm/2 + it/2}) \tag{B.15}
\]

Now we use an expression (2.22) for the matrix elements and perform the integration over $\psi, \psi^*$ and then over $\phi, \phi^*$:

\[ Z^+_F G^{(-)} = e^{\beta M h/2} \frac{\partial}{\partial \alpha} \int d\phi d\phi^* d\psi d\psi^* e^{\phi^* \phi - \psi^* \psi} \times \]

\[ e^{\phi^* L(m,t) \psi + \phi^* R J \phi} \bigg|_{\alpha = 0} = e^{\beta M h/2} \frac{\partial}{\partial \alpha} \det |I + L(m,t) + \alpha R J| \bigg|_{\alpha = 0} , \tag{B.16}
\]

where we defined the matrix

\[ L(m,t) = J(0,t) L(0) J(0,-t) \]

(in the matrix elements $L_{p,q}(m,t)$, $p \in X^+$ and $q \in X^-$), and also the matrix $R$ of rank 1 (all the columns of this matrix are equal):

\[ R_{p_1,p_2} = 1 \frac{e^{-ip_1 p_2}}{M} \tag{B.17}
\]

After the similarity transformation with the diagonal matrix $J(-\frac{2\pi}{M}, -\frac{1}{M})$ one gets

\[ Z^+_F G^{(+,+)} = e^{\beta M h/2} \frac{\partial}{\partial \alpha} \det |I + L \beta + \alpha \tilde{R} J\beta| \bigg|_{\alpha = 0} \] \tag{B.16}

where matrix elements of matrices $\tilde{L}$ and $\tilde{R}$ are

\[ \tilde{L}_{p_1 p_2} = e^{\text{imp}_1 / 2 + it c(p_1) / 2} \sum_q L_{p_1 q}(m,t) L_{q p_2}(0) e^{-\text{imp}_2 / 2 + it c(p_2) / 2} \tag{B.17}
\]

\[ \tilde{R}_{p_1 p_2} = 1 \frac{e^{-\text{imp}_1 / 2 + it c(p_1) / 2}}{M} e^{-\text{imp}_2 / 2 + it c(p_2) / 2} \tag{B.18}
\]

(here $p_1, p_2 \in X^+, q \in X^-$).

To represent these matrices in a more convenient way we introduce the following functions (we follow the paper, changing a bit some notations; recall that $\varepsilon(q) = h - \cos q$)

\[ g(m,t) = \frac{1}{M} \sum_q e^{imq + it \varepsilon(q)} ; \tag{B.19}
\]

\[ e(m,t,p) = \frac{1}{M} \sum_q \frac{e^{imq + it \varepsilon(q)}}{\tan \frac{\varepsilon(q)}{2}} ; \tag{B.20}
\]

\[ 22 \]
\begin{equation}
d(m, t, p) = \frac{1}{M} \sum_{q} \frac{e^{imq + it \cos q} - e^{imp + it \cos p}}{\sin^2 \frac{q + p}{2}}; \tag{B.21}
\end{equation}
\begin{equation}
e_-(m, t, p) = e^{-i\frac{m + p}{2} \cos \frac{p}{2}}; \tag{B.22}
\end{equation}
\begin{equation}
e_+(m, t, p) = e_-(m, t, p)e(m, t, p). \tag{B.23}
\end{equation}
All these functions are of order \(O(1)\) as \(M \to \infty\). It is also convenient to introduce four one-dimensional projectors \(\Pi^{++}, \Pi^{+-}, \Pi^{-+}, \Pi^{--}\) which are \(M \times M\) matrices with matrix elements
\begin{equation}
\Pi^{ab}_{p_1,p_2} = e_a(m, t, p_1)e_b(m, t, p_2) \quad ((a, b) = ++, -+). \tag{B.24}
\end{equation}
Using identities
\begin{equation}
\sum_{q} \frac{1}{\sin^2 \frac{q + p}{2}} = M^2 \quad (q \in X^-, p \in X^+); \tag{B.25}
\end{equation}
\begin{equation}
\cot(x - u) \cot(x - v) = \cot(u - v)[\cot(x - u) - \cot(x - v)] - 1, \tag{B.26}
\end{equation}
one can represent the matrix \(\tilde{L}\) in the following form
\begin{equation}
\tilde{L} = I + \frac{1}{M} [S + i\Pi^{+-} - i\Pi^{-+}]. \tag{B.27}
\end{equation}
The diagonal and nondiagonal matrix elements of the matrix \(S\) are given by
\begin{equation}
S_{pp} = d(p)e^{-imp - it \cos p} \tag{B.28}
\end{equation}
\begin{equation}
S_{p_1,p_2} = \frac{e_+(p_1)e_-(p_2) - e_-(p_1)e_+(p_2)}{\tan \frac{m + p_2}{2}}. \tag{B.29}
\end{equation}
The matrix \(\tilde{R}\) is a projector,
\begin{equation}
\tilde{R} = \frac{e^{ith}}{M} \Pi^{-} \tag{B.30}
\end{equation}
(For brevity, the arguments \(m\) and \(t\) are omitted, being the same for all the functions \(d, e, e_{\pm}\)).

Other contributions can be calculated analogously. We give only the results:
\begin{equation}
G^{(-+),\pm}_F = e^{ith} \frac{\partial}{\partial \alpha} \det \left[ I + \frac{1}{M}(S + i\Pi^{-+} - i\Pi^{+-} + \alpha \Pi^{-+})\Theta_F \right] \bigg|_{\alpha = 0}; \tag{B.31}
\end{equation}
\begin{equation}
G^{(-+),\pm}_B = e^{ith} \frac{\partial}{\partial \alpha} \det \left[ I + \frac{1}{M}(S + i\Pi^{+-} - i\Pi^{-+} + \alpha \Pi^{-+})\Theta_B \right] \bigg|_{\alpha = 0}. \tag{B.32}
\end{equation}
Though formally both contributions \(G^{(-+),\pm}_F\) (as \(G^{(-+),\pm}_B\)) are written in the same form, really they are, of course, different. It is necessary to take into account that
\begin{align*}
p, p_1, p_2 \in X^+, \quad q \in X^- \quad &\text{for} \quad G^{(-+),+}_F; \\
p, p_1, p_2 \in X^-, \quad q \in X^+ \quad &\text{for} \quad G^{(-+),-}_F;
\end{align*}
in all the formulae \((B.11+12), (B.28)\) and \((B.29)\) defining the functions \(g, e, d, e_{\pm}\) and the matrix elements of the matrices \(S\) and \(\Pi^{ab}\).

Analogously one can calculate also the correlator \(G^{(-+)}\). The corresponding contributions are
\begin{equation}
G^{(++),\pm}_F = e^{ith} \left[ g(m, t) + \frac{\partial}{\partial \alpha} \right] \det \left[ I + \frac{1}{M}(S + i\Pi^{+-} - i\Pi^{-+} - \right. \tag{B.33}
\end{equation}
\[-\alpha(\Pi^{++} + ig\Pi^{+-} - ig\Pi^{-+} + g^2\Pi^{--})\Theta_F\right]_{\alpha=0}; \quad (B.32)

\begin{align*}
G_{F,B}^{(+,-),\pm} &= e^{ith} \left[ \mathcal{G}(m,t) + \frac{\partial}{\partial \alpha} \right] \det \left[ I + \frac{1}{M}(S + i\Pi^{+-} - i\Pi^{-+} - \\
&\quad - \alpha(\Pi^{++} + ig\Pi^{+-} - ig\Pi^{-+} + g^2\Pi^{--})\Theta_B\right]_{\alpha=0};
\end{align*}

(one should take into account once again the difference between the contributions $G_{F,B}^{(+,-),\pm}$ and $G_{F,B}^{(+,-),-\pm}$ explained above).

One should underline that the results obtained here coincide with the representations obtained using another method (for which rather complicated calculations are required) in [5, 26]. Precisely our expression can be obtained by choosing arbitrary constants $c_1 = i, c_2 = -i$ in the formulae (A.1)-(A.10) of the Appendix A of the paper [28].

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