Efficient Query Answering over Conceptual Schemas of Relational Databases*

Abstract

We develop a query answering system, where at the core of the work there is an idea of query answering by rewriting. For this purpose we extend the DL DL-Lite [5] with the ability to support n-ary relations, obtaining the DL DLR-Lite, which is still polynomial in the size of the data [3,4]. We devise a flexible way of mapping the conceptual level to the relational level, which provides the users an SQL-like query language over the conceptual schema. The rewriting technique adds value to conventional query answering techniques, allowing to formulate more simple queries, with the ability to infer additional information that was not stated explicitly in the user query. The formalization of the conceptual schema and the developed reasoning technique allow checking for consistency between the database and the conceptual schema, thus improving the trustiness of the information system.

1 Introduction

The research we are currently carrying out is aimed at the development of a query answering system that enables users to pose queries over the conceptual schema of a database. Such a system provides added value against conventional DBMSs, where the users are exposed the relational schema only. At the core of our work there is an idea of query answering by rewriting.

In general, query answering by rewriting is divided into two phases. The first one re-expresses a user query posed over the conceptual schema in terms of the relations at the underlying database, and the second evaluates the rewriting over the underlying database (e.g.,[1]).

Our approach uses a formalism based on Description Logics (DLs) [2] to formalize the conceptual schema of the database. Specifically, we have

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*This technical report in anonymous form was accompanying a submission to the student session of ESSLI'06. It was written by Lina Lubytė, Živilė Norkūnaitė, Mantas Šimkus and Daniel Trivellato as part of the Master student project carried out at the Free University of Bolzano (Italy) under the supervision of Diego Calvanese and Sergio Tessaris. Evaldas Taroza has contributed in preparing the submission to ESSLI’06.
extended the DL DL-Lite [5] with the ability to support n-ary relations, obtaining the DL DLR-Lite. Such a formalism is expressive enough to capture basic Entity-Relationship or UML Class diagrams, while allowing query answering that fully takes into account the constraints in the conceptual schema and is still tractable (i.e., polynomial) in the size of the data [3,4].

We have devised a flexible way of mapping the conceptual level to the underlying relational level, which provides the users an SQL-like query language over the conceptual schema. Queries at the conceptual level are first translated into the relational level queries by taking into account the mapping of entities and relationships to the actual database relations. To provide a complete answer to the query, the system then uses the developed query rewriting technique to take into account the constraints expressed in the conceptual schema. The initial user query is thus translated to a set of SQL queries that are evaluated by the DBMS.

This rewriting technique adds value to conventional query answering techniques. Firstly, the user is allowed to formulate more simple queries using terms defined in the conceptual schema only, without taking into account some relational database related details (e.g., join attributes). Moreover, the query rewriting technique allows one to infer additional information that was not stated explicitly in the user query but is implied by the constraints at the conceptual level. Last but not least, the formalization of the conceptual schema and the developed reasoning technique allow checking the consistency of the underlying database against the conceptual schema, therefore, the trustiness of the information system is improved.

2 Formal Framework

DLR-DB system is a triple $S = \langle K, R, M \rangle$, where $K$ is the knowledge base (KB) of $S$, $R$ is a relational schema for $S$ and $M$ is the mapping between the KB $K$ and the relational schema $R$.

2.1 Conceptual Level

We call our description logic language DLR-Lite, that allows to represent the domain of interest in terms of concepts, denoting sets of objects, and relationships, denoting relations between objects. In the language, basic concepts are defined as follows:

$$B ::= A \mid \exists[i]R$$

where $A$ denotes an atomic concept, $R$ an $n$-ary relationship, and $1 \leq i \leq n$. Intuitively, $\exists[i]R$ denotes the projection of $R$ on the $i$-th component. Note, that all concepts denote unary predicates.
For representing intensional knowledge in the KB, we have assertions of the form:

\[ B_1 \sqsubseteq B_2 \]  
\[ B_1 \text{disj} B_2 \]  
\[ \text{funct } \exists[i]R \]

(inclusion)  
(disjointness)  
(functionality)

An inclusion assertion expresses that a basic concept is subsumed by another concept, a disjointness assertion states that the set of objects denoted by a basic concept \( B_1 \) is disjoint from the ones denoted by another concept \( B_2 \), while a functionality assertion expresses the (global) functionality of a certain component of a relationship.

The formal meaning of concept descriptions above is given in terms of interpretations over a fixed infinite countable domain \( \Delta \). We assume, we have one constant for each object, denoting exactly that object.

An interpretation \( I = (\Delta, \cdot^I) \) consists of a first order structure over \( \Delta \) with an interpretation function \( \cdot^I \) such that:

\[ A^I \subseteq \Delta \]
\[ R^I \subseteq \Delta^n \]
\[ (\exists[i]R)^I = \{ \ c \ | \ \exists(c_1,\ldots,c_n) \in R^I, \ c = c_i \}. \]

An interpretation \( I \) satisfies an inclusion assertion \( B_1 \sqsubseteq B_2 \) iff \( B_1^I \subseteq B_2^I \); \( I \) satisfies a disjointness assertion \( B_1 \text{disj} B_2 \) iff \( B_1^I \cap B_2^I = \emptyset \); \( I \) satisfies a functionality assertion \( \text{funct } \exists[i]R \) if \( (c_1,\ldots,c_i,\ldots,c_n) \in R^I \land (c'_1,\ldots,c_i,\ldots,c'_n) \in R^I \supset c_1 = c'_1,\ldots,c_n = c'_n. \)

A model of a KB \( K \) is an interpretation \( I \) that satisfies all the assertions in \( K \). A KB is satisfiable, if it has at least one model. A KB \( K \) logically implies an assertion \( \alpha \) if all the models of \( K \) satisfy \( \alpha \).

All presented assertions allow us to specify the typical constructs used in conceptual modeling. Specifically:

- ISA, using assertions of the form \( B_1 \sqsubseteq B_2 \), stating that the class \( B_1 \) is a subclass of the class \( B_2 \);

- class disjointness, using assertions of the form \( B_1 \text{disj} B_2 \), stating disjointness between the two classes \( B_1 \) and \( B_2 \);

- role-typing, using assertions of the form \( \exists[i]R \sqsubseteq B \), stating that the \( i \)-th component of the relationship \( R \) is of type \( B \);

- participation constraints, using assertions of the form \( B \sqsubseteq \exists[i]R \), stating that instances of class \( B \) participate to the relationship \( R \) as the \( i \)-th component;
- non-participation constraints, using assertions of the form
  \( B \text{ disj } \exists[i]R \), stating that instances of class \( B \) do not participate
  to the relationship \( R \) as the \( i \)-th component;

- functionality restrictions, using assertions of the form \( \text{funct } \exists[i]R \),
  stating that an object can be the \( i \)-th component of the relationship \( R \) at most once.

**Example 1** Consider atomic concepts *Student*, *Professor* and *Course*, the
relationships *Attends* between *Student* and *Course*, *Teaches* between *Professor* and *Course*,
and *HasTutor* between *Student* and *Professor*. We can now define the following
inclusion, disjointness and functionality assertions:

\[
\begin{align*}
(A_1) & \quad \exists[1] \text{Attends} \sqsubseteq \text{Student} & (A_8) & \quad \text{Student} \sqsubseteq \exists[1] \text{Attends} \\
(A_2) & \quad \exists[2] \text{Attends} \sqsubseteq \text{Course} & (A_9) & \quad \text{Student} \sqsubseteq \exists[1] \text{HasTutor} \\
(A_3) & \quad \exists[1] \text{Teaches} \sqsubseteq \text{Course} & (A_{10}) & \quad \text{Course} \sqsubseteq \exists[2] \text{Attends} \\
(A_4) & \quad \exists[2] \text{Teaches} \sqsubseteq \text{Professor} & (A_{11}) & \quad \text{Course} \sqsubseteq \exists[1] \text{Teaches} \\
(A_5) & \quad \text{Professor} \sqsubseteq \exists[2] \text{Teaches} & (A_{12}) & \quad (\text{funct } \exists[1] \text{HasTutor}) \\
(A_6) & \quad \exists[1] \text{HasTutor} \sqsubseteq \text{Student} & (A_{13}) & \quad (\text{funct } \exists[1] \text{Teaches}) \\
(A_7) & \quad \exists[2] \text{HasTutor} \sqsubseteq \text{Professor} \\
\end{align*}
\]

where \( A_1 \) states that everyone attending a course must be a student, while \( A_2 \) states that all attended courses has to be only those that are
offered in general, etc. \( A_{12} \) states that a student can have only one tutor, and \( A_{13} \) states that a course can be taught by only one professor.

We denote by \( \text{Normalize}(\mathcal{K}) \) the DLR-Lite KB obtained by transforming
the KB \( \mathcal{K} \) as follows. The KB \( \mathcal{K} \) is expanded by computing all disjoint
inclusions between basic concepts implied by \( \mathcal{K} \). More precisely, the \( \mathcal{K} \) is
closed with respect to the following inference rule: if \( B_1 \sqsubseteq B_2 \) occurs in \( \mathcal{K} \)
and either \( B_2 \text{ disj } B_3 \) or \( B_3 \text{ disj } B_2 \) occurs in \( \mathcal{K} \), then add \( B_1 \text{ disj } B_3 \) to
\( \mathcal{K} \).

It is immediate to see that, for every DLR-Lite KB \( \mathcal{K} \), \( \text{Normalize}(\mathcal{K}) \) is
equivalent to \( \mathcal{K} \), in the sense that the set of models of \( \mathcal{K} \) coincides with that
of \( \text{Normalize}(\mathcal{K}) \).

Given a DLR-DB system \( \mathcal{S} = \langle \mathcal{K}, \mathcal{R}, \mathcal{M} \rangle \), \( \text{Normalize}(\mathcal{S}) = \langle \mathcal{K}_n, \mathcal{R}, \mathcal{M} \rangle \),
where \( \mathcal{K}_n = \text{Normalize}(\mathcal{K}) \).

**2.2 Relational Level**

At the relational level we consider *relations*, where each relation has an
associated sequence of typed attributes. Each relation may have a sequence
of one or more components, where each component is a sequence of attributes of the relation. Components may not overlap. We call attributes that do not belong to any component, additional attributes of the relation. Note, that the order of components and the order of attributes may not necessarily be related to each other.

### 2.3 Mapping from Conceptual to Relational Level

We can now define the mapping $\mathcal{M}$ between conceptual and logical level as follows:

- to each atomic concept $A$, $\mathcal{M}$ associates a relation $\mathcal{M}(A)$ with a single component;
- to each $n$-ary relationship $R$, $\mathcal{M}$ associates a relation $\mathcal{M}(R)$ with $n$ components.

The mapping induces a signature on basic concepts, and specifically

- for an atomic concept $A$, the signature is the sequence of types of attributes of the component of the relation corresponding to $A$.
- for a concept of the form $\exists[i]R$, the signature is the sequence of types of the $i$-th component of the relation corresponding to $R$.

A mapping $\mathcal{M}$ is consistent with the conceptual level $\mathcal{K}$ and the relational level $\mathcal{R}$ of a system $\mathcal{S} = \langle \mathcal{K}, \mathcal{R}, \mathcal{M} \rangle$, if for each inclusion assertion $B_1 \sqsubseteq B_2$ in $\mathcal{K}$, the signature of $B_1$ is equal to the signature of $B_2$. Note that for disjointness assertions $B_1 \text{ disj } B_2$, we do not require $B_1$ and $B_2$ to have the same signature. Indeed, if $B_1$ and $B_2$ have different signatures, the disjointness assertions will trivially be satisfied at the relational level. In the following, we will always assume that in a system $\mathcal{S} = \langle \mathcal{K}, \mathcal{R}, \mathcal{M} \rangle$, the mapping $\mathcal{M}$ is consistent with $\mathcal{K}$ and $\mathcal{R}$.

**Example 1 (contd.)** In the table below for all atomic concepts and relationships the mapping associates the corresponding relations with components (underlined) and additional attributes.

| Concept/Relationship | Relation                                      |
|----------------------|------------------------------------------------|
| Student              | StudentTable(SName, SSurname, EnrollNumber)   |
| Course               | CourseTable(CourseId, Name, Category)         |
| Professor            | ProfessorTable(PName, PSurname, Degree)       |
| Attends              | AttendsTable(SName, SSurname, CourseId, Year) |
| Teaches              | TeachesTable(PName, PSurname, CourseId, Semester) |
| HasTutor             | HasTutorTable(SName, SSurname, PName, PSurname) |
2.4 Semantics of a System \( \mathcal{S} \)

In order to define the semantics of a system \( \mathcal{S} = \langle \mathcal{K}, \mathcal{R}, \mathcal{M} \rangle \), we first extend the mapping \( \mathcal{M} \) to a mapping \( \mathcal{M}_c \) from basic concepts to components of relations as follows:

- for an atomic concept \( A \), let \( \mathcal{A} \) be the sequence of attributes corresponding to the only component of \( \mathcal{M}(A) \). Then \( \mathcal{M}_c(A) = \pi_{\mathcal{A}}(\mathcal{M}(A)) \);
- for a relationship \( R \), let \( \mathcal{A} \) be the sequence of attributes corresponding to the \( i \)-th component of \( \mathcal{M}(R) \). Then \( \mathcal{M}_c(\exists[i]R) = \pi_{\mathcal{A}}(\mathcal{M}(R)) \).

A database instance (or simply database) \( \mathcal{D} \) over the relational schema \( \mathcal{R} \) is the set of facts of the form \( R(\vec{c}) \), where \( R \) is a relation of arity \( n \) in \( \mathcal{R} \) and \( \vec{c} \) is an \( n \)-tuple of constants of \( \Delta \). A database \( \mathcal{D} \) satisfies w.r.t. \( \mathcal{S} \)

- an inclusion assertion \( B_1 \sqsubseteq B_2 \), if \( (\mathcal{M}_c(B_1))^D \subseteq (\mathcal{M}_c(B_2))^D \);
- a disjointness assertion \( B_1 \mathrm{disj} B_2 \), if \( (\mathcal{M}_c(B_1))^D \cap (\mathcal{M}_c(B_2))^D = \emptyset \);
- a functionality assertion (\( \mathrm{funct} \exists[i]R \)), if the cardinality of \( (\mathcal{M}_c(\exists[i]R))^D \) is equal to the cardinality of \( (\mathcal{M}(R))^D \). In other words, the set of attributes of the \( i \)-th component of \( R \) is a key of \( R^D \).

A database \( \mathcal{D} \) is said to be \emph{consistent} w.r.t. a system \( \mathcal{S} = \langle \mathcal{K}, \mathcal{R}, \mathcal{M} \rangle \), if it satisfies w.r.t. \( \mathcal{S} \) all assertions in \( \mathcal{K} \). A database \( \mathcal{D} \) is said to be \emph{df-consistent} w.r.t. \( \mathcal{S} \), if it satisfies w.r.t. \( \mathcal{S} \) all disjointness and functionality assertions in \( \mathcal{K} \).

3 Queries

3.1 Queries over Conceptual Level

Queries over a DLR-DB system \( \mathcal{S} = \langle \mathcal{K}, \mathcal{R}, \mathcal{M} \rangle \) are specified using an SQL-like syntax corresponding to SPJ queries. More precisely, such a query is written in the form:

\[
\begin{align*}
\text{SELECT} & \ (\text{attribute specifications}) \\
\text{FROM} & \ (\text{relationship specifications}) \\
\text{WHERE} & \ (\text{selection conditions})
\end{align*}
\]

where

- \( \langle \text{relationship specifications} \rangle \) denotes the concepts and relationships involved in the query and the way they join together. It is defined as
follows:

\[
\langle \text{relationship specifications} \rangle := \langle \text{rel.spec} \rangle \mid \langle \text{relationship specifications} \rangle, \langle \text{rel.spec} \rangle
\]

\[
\langle \text{rel.spec} \rangle := \langle \text{join} \rangle \text{ ON } \langle \text{conditions} \rangle
\]

\[
\langle \text{join} \rangle := \langle \text{relationship} \rangle \mid \langle \text{join} \rangle \text{ JOIN } \langle \text{relationship} \rangle
\]

\[
\langle \text{relationship} \rangle := C_i \text{ AS } V_i
\]

\[
\langle \text{conditions} \rangle := \langle \text{equality} \rangle \mid \langle \text{conditions} \rangle \text{ AND } \langle \text{equality} \rangle
\]

\[
\langle \text{equality} \rangle := e_i = e_j
\]

Intuitively, \( \langle \text{relationship specifications} \rangle \) is a sequence of expressions of one of the following forms:

- \( C \text{ AS } V \)
- \( C_1 \text{ AS } V_1 \text{ JOIN } C_2 \text{ AS } V_2 \text{ JOIN } \cdots \text{ JOIN } C_k \text{ AS } V_k \text{ ON } e_1 = e_2 \text{ AND } \cdots \text{ AND } e_{h-1} = e_h \)

where

- each \( C_j \) denotes the name of a relationship or an atomic concept in \( K \);
- each \( V_j \) is a unique variable name, associated to \( C_j \);
- in the equalities \( e_i = e_j \), each \( e_i \) or \( e_j \) is either
  * \( V_i \), if \( V \) is a variable corresponding to an atomic concept;
  * \( V_i .i \), if \( V \) is a variable corresponding to a relationship of arity \( n \geq i \),
- the signatures of the two associated concepts/relationships components must be the same.

- \( \langle \text{attribute specifications} \rangle \) is a sequence of attributes of the form \( V.a \), where \( V \) is a variable in \( \langle \text{relationship specifications} \rangle \), associated to concept or relationship \( C \), and \( a \) is an attribute of relation \( M(C) \);

- \( \langle \text{selection conditions} \rangle \) is a set of equalities, each of one of the following forms:
  - \( V_1.a_1 = V_2.a_2 \),
  - \( V_1.a_1 = c \),

where \( V_i \) is a variables in \( \langle \text{relationship specifications} \rangle \), associated to concept or relationships \( C \); \( a_i \) is an attribute of relation \( M(C) \), and \( c \) is a constant.

\footnote{Note that relationships and atomic concepts may be repeated.}
3.2 Conjunctive Queries over the Relational Level

In this section we first recall the notion of a conjunctive query (CQ). Afterwards we present how a CQ over the relational level can be obtained from a query over the conceptual level.

### 3.2.1 Conjunctive Queries

A *term* is either a variable or a constant. An *atom* is an expression $p(z_1, \ldots, z_n)$, where $p$ is a predicate (relation) of arity $n$ and $z_1, \ldots, z_n$ are terms. A conjunctive query $q$ over a knowledge base $K$ is an expression of the form

$$q(\vec{x}) \leftarrow \exists \vec{y}.\operatorname{conj}(\vec{x}, \vec{y})$$

where $\vec{x}$ are the so-called *distinguished variables*, $\vec{y}$ are existentially quantified variables called *non-distinguished variables*, and $\operatorname{conj}(\vec{x}, \vec{y})$ is a conjunction of atoms of the form $T(z_1, \ldots, z_n)$, where $T$ is a relation of $\mathcal{R}$ with $n$ attributes and $z_1, \ldots, z_n$ are terms. $q(\vec{x})$ is called the *head* of $q$ and $\exists \vec{y}.\operatorname{conj}(\vec{x}, \vec{y})$ the *body* of $q$.

The *answer* of a query $q(\vec{x}) \leftarrow \exists \vec{y}.\operatorname{conj}(\vec{x}, \vec{y})$ over a database $D$ is the set $q^D$ of tuples $\vec{c}$ of constants in a domain $\Delta$ such that when we substitute the variables $\vec{x}$ with the constants $\vec{c}$, the formula $\exists \vec{y}.\operatorname{conj}(\vec{x}, \vec{y})$ evaluates to true in $D$.

A *union of conjunctive queries (UCQ)* is an expression

$$q(\vec{x}) \leftarrow \exists \vec{y}_1.\operatorname{conj}_1(\vec{x}, \vec{y}_1) \lor \cdots \lor \exists \vec{y}_m.\operatorname{conj}_m(\vec{x}, \vec{y}_m)$$

where for each $i \in \{1, \ldots, m\}$ $\operatorname{conj}_i(\vec{x}, \vec{y}_i)$ is a conjunction of atoms.

The answer of a UCQ $q(\vec{x}) \leftarrow \exists \vec{y}_1.\operatorname{conj}_1(\vec{x}, \vec{y}_1) \lor \cdots \lor \exists \vec{y}_m.\operatorname{conj}_m(\vec{x}, \vec{y}_m)$ over a database $D$ is the union of the answers of the conjunctive queries

$$q_1(\vec{x}) \leftarrow \exists \vec{y}_1.\operatorname{conj}_1(\vec{x}, \vec{y}_1)$$

$$\vdots$$

$$q_m(\vec{x}) \leftarrow \exists \vec{y}_m.\operatorname{conj}_m(\vec{x}, \vec{y}_m)$$

### 3.2.2 Converting Conceptual Queries to Conjunctive Queries

Given a *DLR-DB* system $S = (K, \mathcal{R}, \mathcal{M})$, the conversion of a query $q$ over the conceptual level into a conjunctive query is done in two steps:

1. the query $q$ is converted into a standard SQL select-project-join query $q'$ over the relational schema $\mathcal{R}$;

2. $q'$ is converted into a conjunctive queries using the standard translation.
In this conversion, the order of attributes of a relation \( R \), specified at the relational level, is preserved in the atoms for \( R \).

In order to convert our conceptual queries to standard SQL queries, first each relationship \( C_j \) is substituted with \( M(C_j) \). For each equality \( e_1 = e_2 \) in the conceptual query, we substitute it with the conjunction of equalities between the attributes corresponding to the components mentioned in \( e_1 \) and \( e_2 \).

**Example 1 (contd.)** Suppose we want to know the surnames of all students that attend the course with ID "AB23INF". We formulate the conceptual query as follows:

```
SELECT S.Surname
FROM Student AS S JOIN Attends AS A ON S = A.1
WHERE A.Course = "AB23INF"
```

After the rewriting we get the following SQL query:

```
SELECT S.Surname
FROM StudentTable AS S JOIN AttendsTable AS A
  ON S.Name = A.Name AND S.Surname = A.Surname
WHERE A.Course = "AB23INF"
```

Given a query \( q \) over \( S \), we denote with \( CQ(q, S) \) the conjunctive query over \( R \) resulting from the above conversion.

In order to evaluate, using a relational DBMS, the queries we get from the rewriting procedure, we need to convert them back to SQL. In doing so, we again make use of the order of attributes specified at the relational level. We denote the conversion of a CQ \( q \) to SQL with \( SQL(q, R) \).

### 3.3 Reasoning in DLR-DB system \( S \)

Given a DLR-DB system \( S = \langle K, R, M \rangle \), a conceptual query \( q \) over \( S \) and a database \( D \) over \( R \), the certain answers \( ans(q, S, D) \) is the set of tuples \( \vec{c} \) of constants of \( \Delta \), such that \( \vec{c} \in q_{S}^{D} \) for every database \( D' \) that includes \( D \) and is consistent with \( S \).

The basic reasoning services over a DLR-DB system \( S = \langle K, R, M \rangle \) are:

- **KB satisfiability**: verify whether a KB is satisfiable.
- **query answering**: given a DLR-DB system \( S = \langle K, R, M \rangle \), a conceptual query \( q \) over \( S \) and a database \( D \) over \( R \), return the certain answers \( ans(q, S, D) \). 
- **query rewriting**: given a DLR-DB system \( S = \langle K, R, M \rangle \), and a conceptual query \( q \) over \( S \), return a query \( q_{r} \) over \( R \), such that \( q_{r}^{D} = ans(q, S, D) \) for every database \( D \) that is df-consistent with \( Normalize(S) \).
4 Query Rewriting in System S

In this section we present an algorithm that computes the perfect rewriting of a UCQ. Before proceeding, we address some preliminary issues.

**df-consistency of** $\mathcal{D}$ **w.r.t.** $S$ The algorithm Consistent takes as input a normalized KB $\mathcal{K}$ and verifies the following conditions:

- if there exists a disjunction assertion $B_1 \text{ disj } B_2$, such that $(\mathcal{M}_C(B_1))^D \cap (\mathcal{M}_C(B_2))^D \neq \emptyset$

- if there exists a functionality assertion $(\text{funct } \exists[i] R)$, such that the cardinality of $(\mathcal{M}_C(\exists[i] R))^D$ is not equal to the cardinality of $(\mathcal{M}(R))^D$.

Informally, the first condition corresponds to checking whether $\mathcal{D}$ explicitly contradicts some disjunction assertion in $\mathcal{K}$, and the second condition corresponds to check whether $\mathcal{D}$ violates some functionality assertion in $\mathcal{K}$. If at least one of the above conditions holds, then the algorithm returns false, i.e., $\mathcal{D}$ is not fk-consistent w.r.t. $S$. Otherwise, the algorithm returns true.

4.1 Rewriting

The basic idea of the method used is to reformulate the query taking into account the KB $\mathcal{K}$ [4]: in particular, given a query $q$ over the conceptual schema $\mathcal{K}$, we compile the assertions of the KB into the query itself, thus obtaining a new query $q'$. Such a new query is then evaluated over the database instance $\mathcal{D}$.

We say that an argument of an atom in a query is *bound* if it corresponds to either a distinguished variable or a shared variable, i.e., a variable occurring at least twice in the query body, or a constant, while we say that it is *unbound* if it corresponds to a non-distinguished non-shared variable.

**Definition 4.1** We indicate with $gr(g, I)$ the atom obtained from the atom $g$ by applying the inclusion assertion $I$ as follows:

an inclusion assertion $B \subseteq A$ (resp. $B \subseteq \exists[i] R$) is applicable to an atom $T(x_1, \ldots, x_n)$ if

(i) $\mathcal{M}(A) = T$ (resp. $\mathcal{M}(R) = T$)

(ii) all variables among $x_1, \ldots, x_n$ that are in positions of $T$ that are not part of the only (resp. the $i$-th) component of $T$ are unbound.

For $g = T(x_1, \ldots, x_n)$, $gr(g, A_1 \subseteq A_2)$ is the atom $T'(x'_1, \ldots, x'_n)$, where

- $T' = \mathcal{M}(A_1)$, $T = \mathcal{M}(A_2)$;
• the variables in $T'(x'_1, \ldots, x'_n)$ that correspond to the only component of $T'$ are equal to the ones that correspond to the only component of $T$;

• the remaining variables in $T'(x'_1, \ldots, x'_n)$ are fresh.

**Definition 4.2** Given an atom $g_1 = r(X_1, \ldots, X_n)$ and an atom $g_2 = r(Y_1, \ldots, Y_n)$, we say that $g_1$ and $g_2$ unify if there exists a variable substitution $\theta$ such that $\theta(g_1) = \theta(g_2)$. Each such a $\theta$ is called unifier. Moreover, if $g_1$ and $g_2$ unify, we denote as $\text{mgu}(g_1, g_2)$ a most general unifier of $g_1$ and $g_2$.

We are now ready to define the algorithm **Rewrite**.

**Algorithm** At first, SQL query is translated to conjunctive query using standard SQL-to-CQ algorithm. Then the **Rewrite** algorithm is applied. Note, that the order of the variables, which is the one given by the translation from SQL to CQ, must be considered.

```plaintext
algorithm Rewrite(q, S)
input: conjunctive query q, DLR-DB system $S = \langle K, R, M \rangle$
output: union of conjunctive queries $P$

$P := \{q\}$;
repeat
    $P' := P$;
    for each $q \in P'$ do
        (a) for each $g$ in $q$ do
            for each $I$ in $K$ do
                if $I$ is applicable to $g$
                    then $P := P \cup q[g/gr(g, I)]$

        (b) for each $g_1, g_2$ in $q$ do
            if $g_1$ and $g_2$ unify
                then $P := P \cup \{\text{reduce}(q, g_1, g_2)\}$;
    until $P' = P$;
return $P$
```

In the algorithm, $q[g/g']$ denotes the query obtained from $q$ by replacing the atom $g$ with a new atom $g'$.

Informally, the algorithm **Rewrite** first reformulates the atoms of each query $q \in P'$ and produces a new query for each atom reformulation (step (a))\[5\]. More precisely, if there exists an inclusion assertion $I$ and a conjunctive query $q \in P'$ containing an atom $g$, then the algorithm adds to $P'$ the query obtained from $q$ by replacing $g$ with $gr(g, I)$. For the step (b), the algorithm **Rewrite** for each pair of atoms $g_1, g_2$, that unify, computes the query $q' = \text{reduce}(q, g_1, g_2)$, obtained from $q$ by the following algorithm:
algorithm reduce\((q, g_1, g_2)\)

**input:** conjunctive query \(q\), atoms \(g_1, g_2 \in \text{body}(q)\)

**output:** reduced conjunctive query \(q'\)

\[
q' := q;
\]

\[
\sigma := \text{mgu}(g_1, g_2)
\]

\[
\text{body}(q') := \text{body}(q') - \{g_2\}
\]

\[
q' := \sigma(q')
\]

**return** \(q'\)

Informally, the algorithm \texttt{reduce} starts by eliminating \(g_2\) from the query body; then the substitution \(\text{mgu}(g_1, g_2)\) is applied to the whole query (both the head and the body).

In order to compute the answers of \(q\) to \(S\), we need to evaluate the set of conjunctive queries \(P\) produced by the algorithm \texttt{Rewrite}. Every query \(q\) in \(P\) is transformed into an SQL query. The algorithm \texttt{Answer}, given a satisfiable KB \(K\) and a query \(q\), computes the answer to \(q\) over \(K\). \(\text{Eval}(q, D)\) denotes the evaluation of the SQL query \(q\) over the database \(D\).

algorithm \texttt{Answer}(\(q, S, D\))

**input:** conceptual query \(q\), DLR-DB system \(S = \langle K, R, M \rangle\), database \(D\) for \(R\)

**output:** \(\text{ans}(q, S, D)\)

\[
K := \text{Normalize}(K);
\]

**return** \(\text{Eval}(\text{SQL}(\text{Rewrite}(\text{CQ}(q, S), S), R), D)\)

5 Conclusions

In this document we have described \textit{DLR-DB}, a query answering system that enables to pose queries over the conceptual schema of a database, re-expressing a conceptual query in terms of relations at the underlying database and evaluating the rewriting over the underlying database. We have extended the DL \textit{DL-Lite} to the DL \textit{DLR-Lite} which supports n-ary relations, without losing nice computational properties of the developed reasoning techniques.

These results are advantageous in formulating more simple queries, using terms defined in the conceptual schema only, and inferring additional information that was not stated explicitly in the user query but is implied by the constraints at the conceptual level. At the same time, the formalization of the conceptual schema and the reasoning techniques allow for checking the consistency of the underlying database against the conceptual schema.
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