Anisotropic random resistor networks: a model for piezoresistive response of thick-film resistors.

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A number of evidences suggests that thick-film resistors are close to a metal-insulator transition and that tunneling processes between metallic grains are the main source of resistance. We consider as a minimal model for description of transport properties in thick-film resistors a percolative resistor network, with conducting elements governed by tunneling. For both oriented and randomly oriented networks, we show that the piezoresistive response to an applied strain is model dependent when the system is far away from the percolation threshold, while in the critical region it acquires universal properties. In particular close to the metal-insulator transition, the piezoresistive anisotropy shows a power law behavior. Within this region, there exists a simple and universal relation between the conductance and the piezoresistive anisotropy, which could be experimentally tested by common cantilever bar measurements of thick-film resistors.

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I. INTRODUCTION

Thick-film resistors (TFRs) are a particular class of granular metals made of RuO\textsubscript{2}, Bi\textsubscript{2}Ru\textsubscript{2}O\textsubscript{7}, or other metal-oxide granules embedded in an insulating glassy matrix. The importance of TFRs in applicative problems is due to their large piezoresistive response which, together with high stability and low manufacturing costs, makes the TFRs highly competitive as pressure and force sensors.\textsuperscript{1,2} Hence, the full understanding of the factors governing the piezoresistive response is a very important issue in optimizing the performances of TFRs. The interest in the piezoresistive response however concerns also more fundamental aspects connected, as we discuss in the following, with the role of transport anisotropy in disordered systems.

In practical measurements, the piezoresistive response is obtained by the so-called gauge factor, \( K \), commonly defined as \( K = \delta R/\varepsilon R \), where \( \delta R \) is the change of resistance \( R \) for a given applied strain \( \varepsilon \). When the strain is directed along the voltage drop, the change in resistance determines the longitudinal gauge factor \( K_L \). Usually, in TFRs the gauge factor ranges from \( K_L \sim 2 \) up to \( K_L \sim 35 \) or more,\textsuperscript{3} following roughly the empirical relation:

\[
K_L \sim \ln R. \tag{1}
\]

Both the high values of \( K_L \) and Eq.\textsuperscript{[1]} point towards a scenario in which tunneling processes between metallic grains are the main source of resistance.\textsuperscript{1,2} In fact, if \( d \) is the mean tunneling distance, then transport between adjacent metallic grains is approximatively

\[
R \propto \exp(2d/\xi), \tag{2}
\]

where \( \xi \) is the localization length. Let us assume that under an applied strain \( \varepsilon \) the distance \( d \) changes to \( d \rightarrow d + \delta d = d(1 + \varepsilon) \), where we have set \( \varepsilon = \delta d/d \). The strain-induced change of \( R \) is therefore \( \delta R/R \sim 2\delta d/\xi \), which implies the empirical relation Eq.\textsuperscript{[1]}.\textsuperscript{4}

In obtaining this simple result, we have assumed that the direction of intergrain tunneling is along the direction of the applied strain. In this way we obtained an upper limit of the longitudinal gauge factor \( K_L \). In structurally isotropic systems, however, the direction between two neighbouring grains is oriented randomly and can be even orthogonal to the direction of \( \varepsilon \). This leads to a reduction of \( K_L \) and to a contemporary appearance of a nonzero transverse gauge factor \( K_T \), defined by the change of resistance when the voltage drop is orthogonal to the direction of \( \varepsilon \). In addition, if we consider also that TFRs are disordered conductors, then it is natural to expect that, in flowing from one end to the other of the sample, the microscopic currents are subjected to drastic changes of direction sampling therefore regions where the intergrain distance is \( d(1 + \varepsilon) \) and regions where \( d \) remains unstrained. This effect contributes to the decrease of \( K_L \) and to the enhancement of \( K_T \).\textsuperscript{5}

What is commonly observed in TFRs is that \( K_L \) and \( K_T \) are comparable,\textsuperscript{5} indicating therefore that the current tortuosity induced by disorder and random bond orientations is an important element and needs further investigation. To this end, we have recently addressed the problem of the piezoresistance in a simplified model of TFRs (or more generally of granular metals) consisting of a random resistor network with a fraction \( p \) of bonds having resistances of the form of Eq.\textsuperscript{[2]} and the remaining fraction \( 1 - p \) having zero conductance.\textsuperscript{6} In accord with the qualitative discussion of above, we have found that as \( p \) is reduced from the unity, the longitudinal gauge factor is diminished and the transverse one is enhanced. This trend holds true all the way down to the critical bond concentration \( p_c \) at which the conductance \( G \) vanishes and \( K_L \to K_T \) for an uniaxial strain.\textsuperscript{6}
The observation that $K_L \sim K_T$ in TFRs could therefore be an indication that these systems are close to a metal-insulator transition, an hypothesis sustained by the very low metallic volume fractions $v$ in TFRs and the observation that the conductance $G$ follows a power law behavior $G \sim (v - v_c)^t$, where $v_c$ is the critical volume fraction and $t$ is a critical exponent.

The aim of this paper is to study further the problem of piezoresistive response in random resistor networks by focusing on the equivalence between transport in anisotropic networks and the piezoresistive response of isotropic ones. Close to the percolation threshold, the two systems are equivalent in the sense that transport is governed by the same critical exponents. In addition we consider also the effect of random bond orientations and provide explicit formulas from which it is possible to extract the piezoresistive anisotropy.

II. PIEZORESISTIVE ANISOTROPY OF RANDOM RESISTOR NETWORKS

In this section, we describe in full generality how to extract from a random resistor network model the relevant piezoresistive coefficients. Although real materials like TFRs are quite complex from the point of view of both microscopic electronic processes and microstructure, we believe that the essential physics of the piezoresistive response is already contained in our simplified model. Possible limitations of our approach are however discussed in Sec.

In common piezoresistive measurements, the sample is subjected to a geometrical distortion characterized by a strain field with coefficients $\varepsilon_{ii}$ ($i = x, y, z$) and the piezoresistive response is obtained by recording the change of resistance (conductance) under the effect of $\varepsilon_{ii}$. In the following, we shall consider only situations in which the shearing strains can be neglected, so that $\varepsilon_{ij} = 0$ for $i \neq j$. In the absence of imposed strains, we assume that the sample is isotropic and characterized by a conductance $G$. Hence, under the effect of $\varepsilon_{ii}$, the conductance for a voltage drop in the $i$ direction changes to $G_i = G + \delta G_i$, where the variation $\delta G_i$ can be expressed in terms of the conductivity change $\delta \sigma_i$ and a geometric factor $\xi$:

$$\frac{\delta G_i}{G} = \frac{\delta \sigma_i}{\sigma} - \varepsilon_{ii} + \varepsilon_{jj} + \varepsilon_{kk},$$

where $\sigma$ is the conductivity for the unstrained sample and the indexes $i$, $j$, and $k$ assume the values $x$, $y$, and $z$ with cyclic permutations.

We are interested on the intrinsic conductivity change $\delta \sigma_i$ which is governed by the microscopic electronic processes taking place in the bulk. In agreement with the empirical relation Eq.(2), we assume that bulk transport is mainly governed by tunneling processes between metallic grains and we consider a cubic random-resistor network whose unstrained bond conductances are either proportional to $\exp(2d/\xi)$, where $d$ is the distance between two neighbouring sites, or zero. The probability of having zero bond conductance can be independent of the particular bond considered, as in bond percolation models, or correlated, as in site percolation models. The following considerations apply also for more complicated distributions, like the ones arising in segregated site percolation models where all the sites within spheres of a given radius centered at random are excluded from the network or when the non-zero bond conductances have some distribution of tunneling probability.

To model the effect of the applied strain, let us assume that the resistor network is embedded in a homogeneous elastic medium and that the elastic coefficients of the network and the medium are equal. Moreover we assume also that the directions of the bonds in the network are aligned to the $x$, $y$, and $z$ axes. Therefore, in general, when a strain field $\varepsilon_{ii}$ is applied on the sample, the conducting bonds change their values according to their orientation with respect to $\varepsilon_{ii}$. For example, the tunneling distance for a bond directed along the $i$ axis changes to $d \rightarrow d_i = d(1 + \varepsilon_{ii})$ which, up to the first order in $\varepsilon_{ii}$, implies a bond conductance change

$$\exp\left(\frac{2d}{\xi}\right) \rightarrow \exp\left(\frac{2d_i}{\xi}\right) = \exp\left(\frac{2d}{\xi}\right)\left(1 + \frac{2d}{\xi}\varepsilon_{ii}\right).$$

Now if we consider an uniaxial strain along, for example, the $x$ direction ($\varepsilon_{xx} = \varepsilon, \varepsilon_{yy} = \varepsilon_{zz} = 0$), thus the bond conductances directed along $i = x$ are modified according to Eq.(3), while those directed along the $y$ and $z$ axes remain unchanged. Therefore, up to first order in $\varepsilon$, the resulting conductivities can be expressed as $\sigma_x = \sigma - \sigma \Gamma_{\parallel} \varepsilon$, and $\sigma_y = \sigma_z = \sigma - \sigma \Gamma_{\perp} \varepsilon$, where we have defined

$$\Gamma_{\parallel} = -\frac{\delta \sigma_x}{\varepsilon \sigma},$$

$$\Gamma_{\perp} = -\frac{\delta \sigma_y}{\varepsilon \sigma} = -\frac{\delta \sigma_z}{\varepsilon \sigma},$$

as the longitudinal and transverse piezoresistive coefficients, respectively. The above reasoning holds true also for uniaxial strains along the $y$ and the $z$ axis and, since the problem is linear, for a general strain field $\varepsilon_{ii}$ ($i = x, y, z$) the conductivity variations $\delta \sigma_i$ reduce to:

$$\frac{\delta \sigma_i}{\sigma} = -\Gamma_{\parallel} \varepsilon_{ii} - \Gamma_{\perp} (\varepsilon_{jj} + \varepsilon_{kk}).$$

As already pointed out in the introduction, the commonly measured quantities used to extract the strain sensitivity of the sample are the piezoresistive gauge factors which for a general strain are defined as:

$$K_{ij} = -\frac{\delta G_i}{\varepsilon_{jj} G}.$$
If we assume that the strains $\varepsilon_{ii}$ are known, then Eqs. (3,7) permit to express the different gauge factors $K_{ij}$ in terms only of the two intrinsic piezoresistive coefficients $\Gamma_{\parallel}$ and $\Gamma_{\perp}$. For example, in a typical cantilever bar experiment with the cantilever main axis directed along the $x$ direction, the strains are approximatively $\varepsilon_{xx} = \varepsilon$, $\varepsilon_{yy} = -\nu\varepsilon$, and $\varepsilon_{zz} = -\nu'\varepsilon$, where $\nu$ and $\nu'$ are the Poisson ratios of the cantilever and the resistive sample, respectively. By using Eqs. (3,7), the longitudinal ($K_L$) and transverse ($K_T$) piezoresistive gauge factors are:

$$K_L \equiv K_{xx} = (1 + \Gamma_{\parallel}) + (1 - \Gamma_{\perp})(\nu + \nu'),$$

(9)

$$K_T \equiv K_{yx} = -(1 + \Gamma_{\parallel})\nu - (1 - \Gamma_{\perp})(1 - \nu').$$

(10)

From a measurement of $K_L$ and $K_T$, the above expressions permit to extract $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ which are intrinsic quantities. That is it, they do not depend on the particular strain applied. Note that instead of measuring $K_L$ and $K_T$, the piezoresistive coefficients can also be extracted from Eqs. (3,7) by measuring a gauge factor under two different imposed strains. For example, as in Ref. [14], a cantilever bar measurement provides a first value of $K_L$, Eq. (9), and a successive measurement of the longitudinal gauge factor under hydrostatic pressure provides a second value of $K_L$. These two values are then sufficient to extract $\Gamma_{\parallel}$ and $\Gamma_{\perp}$.

The importance of $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ resides on the fact that they permit to extract useful informations on the percolative nature of transport. To illustrate this point, we show in Fig. 1 the results of numerical Monte Carlo calculations of $\Gamma_{\parallel}$ and $\Gamma_{\perp}$ (filled circles) for a site percolation model in which a concentration of sites $x$ is removed at random. For each missing site the bond conductances connecting the six neighbouring sites are set equal to zero, while the remaining bonds have conductance $\exp(2d/\xi)$ with $2d/\xi = 4$ when unstrained. Note that in this model, the fraction of conducting bonds $p$ is equal to $x^2$, since a bond is present only if the sites at both ends are present. We calculate the total conductance by solving numerically the Kirchhoff equations for all the nodes of the cubic network. In practice, we impose a unit voltage difference between two opposite sides of the network with periodic boundary conditions to the remaining sides. The piezoresistive coefficient $\Gamma_{\parallel}$ ($\Gamma_{\perp}$) is then obtained by calculating, for a fixed configurations of bond resistors, the difference in conductance when $\varepsilon = 0$ and $\varepsilon = 0.01$ for strain directed parallel (orthogonal) to the imposed voltage drop direction.

When there are no missing sites ($x = 1$, $p = 1$), the current flows exclusively along paths directed along the direction of the voltage drop. In this case, the longitudinal piezoresistive coefficient is $\Gamma_{\parallel} = 2d/\xi = 4$ while the transverse one $\Gamma_{\perp}$ is zero [see Eqs. (3,7)]. When sites are removed ($x < 1$), $\Gamma_{\parallel}$ gets reduced and at the same time the transverse coefficient is enhanced in such a way that $\Gamma_{\parallel} > \Gamma_{\perp}$. This behavior is due to the fact that as sites are removed from the network, the missing bonds force the current to flow also along directions perpendicular to the voltage drop. Hence, when the strain is along the voltage drop, the current visits also regions where instead the voltage drop is perpendicular to the direction of $\varepsilon$, the current is influenced also by regions where the bonds are strained (enhancement of $\Gamma_{\parallel}$). This trend gets amplified as $x$ is further reduced and at the percolation threshold ($p_c = x_c^2 \approx 0.098$) $\Gamma_{\parallel} \rightarrow \Gamma_{\perp}$. Qualitatively, this behavior is observed also when the bond conductances have some distribution or when different statistics like that of bond percolation models are considered. Hence, as a general rule, the longitudinal and transverse piezoresistive coefficients become equal as the random resistor network reaches its percolation threshold.

In relation to the gauge factors $K_L$ and $K_T$, it is worth to point out that, from Eqs. (3,7), $K_L - K_T = (\Gamma_{\parallel} - \Gamma_{\perp} + 2)(1 + \nu)$ in accord with Ref. [22]. However, since $\Gamma_{\parallel} > \Gamma_{\perp}$ for $p_c < p \leq 1$, it is not justified to assume $K_L - K_T = 2(1 + \nu)$ unless the system is very close to its percolation threshold. Instead a more general relation is $K_L - K_T \geq 2(1 + \nu)$, which holds true in the whole range of $p \geq p_c$ values.

Summarizing the results of Fig. 1, in the presence of an external uniaxial strain, the network is anisotropic when $p \sim 1$ while is electrically quasi-isotropic when $p \sim p_c$. The metal-to-insulator transition can therefore be viewed also as a piezoresistive anisotropic-to-isotropic transition when $p \rightarrow p_c$. We call the quantity which characterizes this transition the piezoresistive anisotropy factor $\chi$.
defined as:
\[ \chi = \frac{\Gamma_{\|} - \Gamma_{\perp}}{\Gamma_{\|}}. \]  
(11)

As shown in Fig. 2, \( \chi \) (filled circles) is equal to the unity for \( p = 1 \) while it decreases monotonically as the system moves towards its percolation threshold and, as shown in log-log plot of the inset of Fig.2, \( \chi \) goes to zero by following a power law behavior in the critical region \( (p - p_c) \ll 1 \):

\[ \chi \sim (p - p_c)^{\lambda}, \]  
(12)

where \( \lambda \) is a critical exponent. A fit of the numerical data of Fig.2 to Eq.(12) leads to \( \lambda = 0.44 \pm 0.07 \). We have tested that Eq.(12) holds true also for bond percolation models and for conducting bonds having a distribution of tunneling probability.

The power law behavior of Eq.(12) is directly related to the transport properties of anisotropic bond percolation models studied some time ago. In those works, percolating networks with random bond conductances were defined in such a way that the conducting bonds were equal along two directions, for example \( y \) and \( z \), but different from those along the third direction, that is \( x \). Topological considerations and renormalization group analysis showed that the quantity \( \sigma_y/\sigma_x - 1 = \alpha(p) \) has a power law behavior \( \alpha(p) \sim (p - p_c)^{\lambda'} \) close to the percolation threshold, independently of the microscopic bond anisotropy in our model, bond anisotropy is induced by the effect of an applied strain, Eq.(11), so that the two situations are equivalent. It is in fact easy to see from Eqs.(11) that for our piezoresistive model \( \sigma_y/\sigma_x - 1 \approx (\Gamma_{\|} - \Gamma_{\perp}) \sim \chi \), so that the two exponents \( \lambda \) and \( \lambda' \) are equal. Since the exponent \( \lambda \) does not depend on the anisotropy of the microscopic bond conductances, a measurement of the piezoresistive anisotropy factor \( \chi \) is in principle a practical and alternative way to study the role of anisotropy in percolative conductors.

In relation to TFRs, a measurement of the piezoresistive anisotropy factor \( \chi \) as a function of metallic volume concentration would be an useful tool to characterize their transport properties. The advantage of \( \chi \) over other quantities like for example \( K_L - K_T \) is that it measures the anisotropy without being seriously influenced by other factors like the change of tunneling distances with the metallic concentration \( x \). By lowering \( x \) in fact the mean distance between metallic granules should increase leading to an enhancement of the tunneling distance \( d \). This situation can be modeled by assuming that \( d \) is some function of the bond probability \( p \), \( d(p) = df(p) \), where \( f(p) \) increases as \( p \) is reduced and is independent of strain. It is easy to see that, if the unstrained conducting bonds have conductance \( \exp(2d/\xi) \) then the longitudinal and transverse piezoresistive coefficients get multiplied by \( f(p) \). This factor exactly cancels out in the definition of \( \chi \), Eq.(11).

III. AVERAGE OVER BOND ORIENTATIONS

Equation (11) of the last section has been obtained by assuming that the bond directions are aligned parallel to the \( x, y \), and \( z \) axes of an orthogonal frame. In this way, under the effect of an applied strain \( \varepsilon_{ii} \), the conducting bonds change according to the simple relation of Eq.(11). In this section we generalize Eq.(11) in order to describe networks with bonds oriented at random with respect to the reference frame. As already pointed out in the introduction, we expect that in this more realistic situation the longitudinal and transverse piezoresistive responses should be different from those obtained for the oriented network, at least for \( p \approx 1 \).

Instead of studying a network with bond directions changing from site to site, we consider an ensemble of perfectly cubic networks oriented at random. The piezoresistive responses will then be obtained by averaging over
all network orientations. We start by choosing the three bond directions of the cubic network as oriented along the $x'$, $y'$, and $z'$ axes which constitute an orthogonal frame $\hat{A}$ rotated by $\hat{R}$ with respect to the reference frame $A$ specified by the $x$, $y$, and $z$ axes. The rotation matrix $\hat{R}$ is given by the triple product of successive rotations defined by Euler angles. To fix an explicit rotation matrix, let us consider a $z$-$x$-$z$ transformation given by a rotation of angle $\phi$ around the $z$-axis, followed by rotations of $\theta$ and $\psi$ around the the new $x$ and $z$ axes, respectively. With this choice of angles, $\hat{R}$ can take values from $0$ to $\pi$, while $\phi$ and $\psi$ from $0$ to $2\pi$. By using the usual rules of strain transformations, the strains $\varepsilon'_{ij}$ in reference frame $\hat{A}$ are given by:

$$\varepsilon'_{ij} = \sum_l R_{il} R_{jl} \varepsilon_{ll}, \quad (14)$$

where $R_{ij}$ are elements of the matrix $\hat{R}$ and, as before, we have neglected the shearing strains with respect to frame $A$. Now, let us consider in the absence of strains a conducting bond directed along the $x'$ direction having tunneling length $d' = d$. Under the effect of Eq. $(14)$, the bond length changes to

$$\sqrt{(d + \delta d_{x'})^2 + (\delta d_{y'})^2 + (\delta d_{z'})^2} = d \sqrt{(1 + \varepsilon'_{xx})^2 + (\varepsilon'_{xy})^2 + (\varepsilon'_{xz})^2} = d(1 + \varepsilon'_{xx}) + O(\varepsilon'_{xy}) + O(\varepsilon'_{xz}), \quad (15)$$

where $\delta d_{x'}$ is the variation of length along the $i' = x'$, $y'$, $z'$ axis. Compared to $\varepsilon'_{xy}$, the effect of the shearing strains $\varepsilon'_{xy}$ and $\varepsilon'_{xz}$ is therefore of higher order and can be neglected. Hence, for a bond along the $i'$ direction the bond tunneling length changes simply to $d \rightarrow d' = d(1 + \varepsilon'_{ii})$. At this point, with respect to the $\hat{A}$ frame, we can follow the same reasonings of last section so that the conductivity changes are of the same form of Eq. $(1)$:

$$\frac{\delta \sigma'_{ij}}{\sigma} = -\Gamma \| \varepsilon'_{ii} - \Gamma (\varepsilon'_{jj} + \varepsilon'_{kk}) = -\Gamma \| \Delta + (\Gamma - \Gamma \|) \varepsilon'_{ii}, \quad (16)$$

where $\delta \sigma'_{ij}$ is the variation of conductivity $\sigma'_{ij}$ in the $\hat{A}$ frame and we have introduced for later convenience the volume dilatation $\Delta = \sum_j \varepsilon'_{jj}$.

Now, the local Ohm law is $j_{i} = \sigma'_{i} E'_{i}$, where $j'_{i}$ and $E'_{i}$ are the components with respect to $A'$ of the current density and the electric field, respectively. These are related to the corresponding quantities in frame $A$ by $j'_{i} = \sum_j R_{ij} j_{j}$ and $E'_{i} = \sum_j R_{ij} E_{j}$, so that $j_{i} = \sigma_{ij} E_{j}$ where:

$$\sigma_{ij} = \sum_l R^{-1}_{il} \sigma'_{l} R_{lj}. \quad (17)$$

Since the dilatation $\Delta$ is invariant under rotation, equation $(17)$ reduces to:

$$\sigma_{ij} = \delta_{ij} \sigma + R^{-1}_{ij} \delta \sigma'_{ij} = \delta_{ij} \sigma(1 - \Gamma \| \Delta) + \sigma(\Gamma - \Gamma \|) R^{-1}_{ij} \varepsilon''_{ii} R_{ij}, \quad (18)$$

where we have used Eq. $(11)$ and $\delta_{ij}$ is the Kronecker delta symbol. Finally, the average over the ensemble of networks with different bond orientations is achieved by performing the average over the angles $\theta$, $\phi$, and $\psi$ appearing in $\hat{R}$. For a general function $f(\theta, \phi, \psi)$, this is achieved by

$$\langle f \rangle = \frac{\int_{0}^{\pi} \int_{0}^{2\pi} \int_{0}^{2\pi} d\psi J(\hat{R}) f(\theta, \phi, \psi)}{\int_{0}^{\pi} \int_{0}^{2\pi} \int_{0}^{2\pi} d\psi J(\hat{R})} = \frac{1}{8\pi^2} \int_{0}^{\pi} \int_{0}^{2\pi} \int_{0}^{2\pi} d\psi \sin(\theta) f(\theta, \phi, \psi), \quad (19)$$

where $J(\hat{R}) = \sin(\theta)/8$ is the Jacobian of $\hat{R}$. It is easy to see that when $\varepsilon'_{ii}$ in Eq. $(18)$ are replaced by Eq. $(14)$, all the off-diagonal components of $\sigma_{ij}$ vanish under angle integrations, $(\sigma_{ij}) = \delta_{ij} (\sigma_{ii})$, and that the averaged conductance variations $\langle \delta \sigma_{ij} \rangle = (\sigma_{ii}) - \sigma$ reduce to:

$$\frac{\langle \delta \sigma_{ij} \rangle}{\sigma} = -\Gamma \| \Delta + (\Gamma - \Gamma \|) \frac{3 \varepsilon_{ii} + \varepsilon_{jj} + \varepsilon_{kk}}{5}. \quad (20)$$

By using $\Delta = \sum_j \varepsilon_{jj}$, this expression can be recast in a form more similar to Eq. $(3)$:

$$\frac{\langle \delta \sigma_{i} \rangle}{\sigma} = -\Gamma \| \varepsilon_{ii} - \Gamma \| (\varepsilon_{jj} + \varepsilon_{kk}), \quad (21)$$

where we have introduced the effective longitudinal and transverse piezoresistive coefficients

$$\Gamma \| = \frac{3 \Gamma \| + 2 \Gamma \perp}{5}, \quad \Gamma \perp = \frac{\Gamma \| + 4 \Gamma \perp}{5}. \quad (22, 23)$$

If all bonds are present and have all equal conductances, $\exp(2d/\xi)$, then from the above equations we obtain $\Gamma \| = 3 \Gamma \|/5$ and $\Gamma \perp = \Gamma \|/5$, since $\Gamma \perp = 0$ and $\Gamma \| = 2d/\xi$. Therefore, as expected, with respect to a network with bonds oriented along the $x$, $y$, and $z$ axes, the longitudinal response is diminished and the transverse one is enhanced. On the other hand when the network approaches to its percolation threshold, $\Gamma \| \rightarrow \Gamma \perp$, also the effective responses become equal $\Gamma \perp \rightarrow \Gamma \|$. This behavior is summarized in Fig. 1 where the open circles refer to $\Gamma \|$ and $\Gamma \perp$ obtained from the corresponding coefficients for the oriented network via Eqs. $(22, 23)$.

The effect of averaging over all network orientations is therefore substantial away from the critical region and unimportant when $p - p_c \ll 1$. This reflects the universality of the piezoresistive response close the metal-insulator transition which is more evident in the behavior of the effective piezoresistive anisotropy factor.
\[ \tilde{\chi} = \frac{\tilde{\Gamma}_\parallel - \tilde{\Gamma}_\perp}{\Gamma_\parallel} = \frac{2\chi}{3 + 2(1 - \chi)}, \]  

which, according to Eq.(12), shows the power law behavior \( \tilde{\chi} \simeq 2\chi/5 \sim (p - p_c)^\chi \) for \( p - p_c \ll 1 \) (open circles of Fig. 2).

**IV. DISCUSSION AND CONCLUSIONS**

In the previous sections, we have shown how to extract the intrinsic piezoresistive responses \( \Gamma_\parallel \) and \( \Gamma_\perp \) of a granular metal from the knowledge of the gauge factors and the values of applied strains. The so-obtained longitudinal and transverse coefficients permit to define a piezoresistive anisotropy factor \( \chi \), Eq.(11), which follows a power law behavior if the system is sufficiently close to its percolation threshold. Concerning the properties of TFRs, a measurement of \( \chi \) as a function of the volume fraction could be a useful tool to investigate the closeness to a metal-insulator transition. We have also considered the effect of random bond orientations by employing an average procedure over an ensemble of networks with different orientation. This scheme has permitted us to obtain a straightforward and simple generalization of the results obtained for networks with fixed orientations. Unfortunately, we are not aware of published data from which it is possible to extract the piezoresistive coefficients as a function of metallic concentrations, so our predictions cannot yet be tested. We hope that the present study will encourage acquisition of data capable of estimating \( \chi \) as a function of metallic concentrations.

Before concluding, it is useful to remind possible limitations of the theory presented in this work. First of all, all through the previous analysis we have assumed that the granular metal is elastically homogeneous, so that the strain values inside the bulk can be considered as independent of the position. However, TFRs are strongly heterogeneous composites in which quite stiff metallic granules are embedded in an elastically soft glassy matrix. This gives rise to important fluctuations in the local values of the strain. Since the microscopic tunneling processes are affected by the local rather than the macroscopic strain values, such an elastic heterogeneity can influence the piezoresistive response in an important way. However, this effect influences the absolute values of \( \Gamma_\parallel \) and \( \Gamma_\perp \), but should leave the piezoresistive anisotropy factor \( \chi \) relatively unaffected. This is indeed confirmed by the analysis of a simplified model of elastic heterogeneity in TFRs [1], from which actually it results that \( (\Gamma_\parallel - \Gamma_\perp)/\Gamma_\parallel \) is independent of the metal and glass elastic coefficients. This is of course only a preliminary result which needs further investigations.

Another basic assumption of our analysis has been the use of a simple tunneling form for the intergrain hopping. For TFRs, which show a quite weak temperature dependence of resistivity, this should not be a too serious approximation, and simple tunneling captures the essential physics, at least as regards piezoresponse effects. For other granular metals, or for TFRs at very low temperatures, other processes than intergrain tunneling (grain charging effects, Coulomb gap, etc.) play an active role in transport. However, as a function of metallic concentration, we do not expect qualitative deviations from the power law behavior of \( \chi \) in the critical region. Instead, for fixed metallic concentrations, \( \chi \) should develop a temperature dependence proportional to the importance of the other-than-tunneling contributions. For example, in the statistical description of the variable-range-hopping mechanism of Mott [2], the temperature governs the availability of sites energetically favourable for hopping. This is a kind of correlated bond percolation model for which the considerations here presented apply in a straightforward way.

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