A Novel Practical Robust Control Inheriting PID for SCARA Robot

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ABSTRACT In this paper, we propose a novel practical robust control algorithm for the Selective Compliance Articulated Robot Arm (SCARA) robot and verify the effectiveness through experiments. The dynamic model of the SCARA robot is established considering uncertainties, which include the nonlinear friction, parameter uncertainty, and external disturbance. To restrain the reversal chattering, we apply a modified Strubeck friction model with Gaussian compensation term as the friction description. The algorithm is composed of a proportional-derivative (PD) feedback term based on the model and a robust term. The formation of the robust part comprises the upper bound of the uncertainty. The Lyapunov minimax method is adopted to prove that the system is uniformly bounded and uniformly ultimately bounded, thus guaranteeing the practical stability of the system. Moreover, rapid controller prototyping cSPACE, as the experimental platform, can eliminate the tedious programming work and provide a great convenience for the experiments. The experimental results indicate that the robust control algorithm has good performance, which provides accurate trajectory tracking under the influence of uncertainties.

INDEX TERMS SCARA robot, robust control, uncertainty, nonlinear friction.

I. INTRODUCTION

In recent years, the trend of replacing humans with robots is on the rise with the rapid development of the Computer, Communication, and Consumer Electronic (3C) industry. Selective Compliance Articulated Robot Arm (SCARA) robot plays an increasingly important role in 3C assembly, welding, and handling with its high speed, short cycle, accurate path control, and reliable flexible operation [1], [2]. Due to the characteristics of fast action beat in the 3C industry, we need dynamic control of the SCARA robot to ensure its accuracy. Nevertheless, SCARA robot is difficult to control with its time-varying, strong coupling, and other dynamic characteristics. Therefore, using the traditional control algorithms to deal with the uncertainties of the complex system cannot get satisfactory results.

During the past decades, there have been many typical algorithms to build up the accurate tracking performance of robot system, such as proportional-integral-derivative (PID) control [3]–[5], robust control [6], [7], sliding mode control [8], [9], adaptive control [10]–[12], fuzzy control [13], [14], genetic algorithm and particle swarm optimization [15], [16] and so on. Each algorithm has its advantage and limitation. PID control is widely used due to its low dependence on the dynamic model, whereas it is not satisfactory in the case of nonlinear friction and external interference. Based on PID control, a dual-loop control with active disturbance rejection control (ADRC) was proposed in [17] to achieve stable control of the robot. Sliding mode control is prone to chattering, which affects the tracking accuracy. [18] proposed a robust recursive sliding mode controller combined with an adaptive disturbance observer, which eliminates the system chattering in the reaching control input. Adaptive control is considered to be more suitable for the uncertainty.
of the system but requires high real-time performance. Fuzzy control and neural network control, which are based on the data model rather than the mathematical model, make their precision difficult to meet the high accuracy requirements of 3C industry. Moreover, [19] proposed a tracking control scheme with radial basis function (RBF) neural network, which combines the incremental PID control and sliding mode control. [20] used an adaptive inertia weight particle swarm optimization. [21] proposed a hybrid coordinated control based on port-controlled Hamiltonian and backstepping. In engineering, working environment of the robot is complex, where uncertainties are increasing and hard to predict. In order to solve engineering problems better, a practical and effective control scheme needs to be proposed.

Robust control is widely used for its strong robustness to system uncertainties and simple implementation. [22] proposed an adaptive recursive terminal sliding-mode controller to make the tracking error converge to zero in a finite time. [23] proposed a hierarchical sliding mode control with perturbation estimation technique on a two-wheeled self-balancing vehicle, which achieved good balancing and velocity tracking performance even under external disturbances. It is important to explore a practical and effective robust control algorithm for the SCARA robot in engineering applications. Since robust control has characteristic advantages in dealing with model uncertainties and external disturbance, we aim to come up with a novel practical robust control scheme in this paper via a creative description of uncertainties. An adaptive model-free control (AMC) in [24] can make the tracking error converge to a region in finite time. AMC scheme is more complicated in practical applications, because of the need to design and adjust the adaptive law. Relatively speaking, the advantages of the proposed algorithm are simple structure, few tuning parameters, and simple tuning process. The system is divided into a nominal part and an uncertain part. The uncertainties of SCARA robot system are nonlinear time-varying but bounded. The proposed algorithm is composed of a PD feedback term based on the model and a robust term. The formation of the robust term comprises the upper bound of the uncertainties. The Lyapunov minimax method is employed to prove that the controller is uniformly bounded and uniformly ultimately bounded, thus ensuring the practical stability of the system.

In this paper, the main contributions are three parts. First, we propose a specific robust controller and apply the modified Stribeck friction model with Gaussian compensation term as the friction description to reduce the reversal chattering caused by nonlinear friction in motion. Second, the Lyapunov minimax method is adopted to prove the actual stability of the system. Finally, the numerical simulation and experimental verification, comparing with other algorithms, are carried out based on the SCARA robot. The results indicate that the proposed algorithm has good performance, which provides accurate trajectory tracking under the influence of uncertainties.

II. THE ESTABLISHMENT OF DYNAMIC MODEL OF SCARA ROBOT

We choose a kind of 4-DOF SCARA robot as the research object. The four degrees of freedom are lifting motion, joint 1 rotation, joint 2 rotation, and terminal rotation. Since there is no coupling between the lifting motion and other motions, it can be considered separately. Meanwhile, the robot is mainly focused on end-position control, so the terminal rotational motion has little influence on the research results and can be ignored. The two degrees of freedom of the joint 1 and 2 on the plane are coupled to each other, and the positioning of the robot is mainly guaranteed by the two rotating joints. Therefore, the robot is simplified to a planar two-degree-of-freedom manipulator.

The schematic representation of the simplified SCARA robot model is shown in Fig.1, where \( m_1 \) and \( m_2 \) are masses of link 1 and link 2, \( m_3 \) is the mass of the motor of joint 2, \( m_4 \) is the mass of the end components (including the end motor), \( l_1 \) is the length of the link 1, \( l_2 \) is the length of the link 2, \( c_1 \) is the distance from the center of mass of the link 1 to point \( O \), \( c_2 \) is the distance from the center of mass of the link 2 to point \( A \), \( l_3 \) is the distance from the motor of the joint 2 to point \( O \).

\[
\frac{d}{dt}(\frac{\partial L}{\partial q}) - \frac{\partial L}{\partial \dot{q}} = \tau - \tau_f, \tag{1}
\]

where the Lagrangian \( L = T - V \), \( T \) is the kinetic energy and \( V \) is the potential energy, \( q, \dot{q} \) are generalized coordinates.
and generalized velocities respectively, \( \tau \) is the control output, and \( \tau_f \) is the friction force and external disturbances torque. We have
\[
T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_3 \dot{\theta}_1^2 \dot{\theta}_2 + m_2 \dot{\theta}_2^2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \\
+ 2 \dot{\theta}_1 \dot{\theta}_2 \cos \theta_2 (\dot{\theta}_1 + \dot{\theta}_2) + \frac{1}{2} J_2 (\dot{\theta}_1 + \dot{\theta}_2)^2 \\
+ \frac{1}{2} m_3 (\dot{\theta}_1 \dot{\theta}_2)^2 + \frac{1}{2} m_4 (\dot{\theta}_1^2 \dot{\theta}_2^2 + \dot{\theta}_2^2 (\dot{\theta}_1 + \dot{\theta}_2)).
\]
(2)

Because the SCARA robot is simplified to a planar robot, the potential energy into the Lagrange equation, we have the dynamic equation of SCARA robot system
\[
M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) + F(q, \dot{q}) = \tau,
\]
where \( M(q) \) is the inertia matrix, \( C(q, \dot{q}) \) is the Coriolis/centrifugal force matrix, \( G(q) \) is the Gravity vector, \( F(q, \dot{q}) \) is the friction force and external disturbances.
\[
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{pmatrix}
= \begin{pmatrix}
\theta_1 \\
\theta_2
\end{pmatrix}, \quad
\begin{pmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2
\end{pmatrix}
= \begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2
\end{pmatrix}, \quad \tau = \begin{pmatrix}
t_1 \\
t_2
\end{pmatrix}
\]
(4)

\[
M(q) = \begin{bmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{bmatrix}
\]
(5)

\[
M_{11} = I_1 + I_2 + (m_2 + m_4) l_2^2 + m_3 l_2^2 + m_2 c_2^2 + m_1 l_2^2 + 2(m_2 l_1 c_2 + m_4 l_1 c_2) \cos \theta_2
\]
(6)

\[
M_{12} = M_{21} = I_2 + m_2 c_2^2 + m_4 l_2^2 + (m_2 l_1 c_2 + m_4 l_1 c_2) \cos \theta_2
\]
(7)

\[
M_{22} = I_2 + m_2 c_2^2 + m_4 l_2^2
\]
(8)

\[
C(q, \dot{q}) = \begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}
\]
(9)

\[
C_{11} = -2(m_2 l_1 c_2 + m_4 l_1 c_2) \sin \theta_2 \dot{\theta}_2
\]
(10)

\[
C_{12} = -(m_2 l_1 c_2 + m_4 l_1 c_2) \sin \theta_2 \dot{\theta}_2
\]
(11)

\[
C_{21} = (m_2 l_1 c_2 + m_4 l_1 c_2) \sin \theta_2 \dot{\theta}_1
\]
(12)

\[
C_{22} = 0.
\]
(13)

Since the two axes of the SCARA robot move in the horizontal plane, the gravity does not influence the system. We have \( G(q) = [0, 0] \). For \( F \), we have \( F = F_{fric} + F_d \), where the \( F_{fric} \) denotes the nonlinear friction force and the \( F_d \) denotes the external disturbances.

**Assumption 1:** The inertia matrix \( M(\cdot) \) in the SCARA robot system is uniform positive definite for all \( \theta \), which means that there are scalar constants \( \gamma, \tilde{\gamma} > 0 \) such that
\[
0 < \gamma I \leq M(\cdot) \leq \tilde{\gamma} I,
\]
(14)

**Theorem 1:** The matrix \( \dot{M}(\cdot) - 2C(\cdot) \) is skew symmetric for all \( \theta, \dot{\theta} \) [25]. That is, for any vector \( \xi \), there is
\[
\xi^T (\dot{M}(\cdot) - 2C(\cdot)) \xi = 0.
\]
(15)

In the process of robot movement, due to unreasonable structure design, insufficient lubrication, or too tight assembly, the nonlinear friction of joints has a particularly great impact on the movement of the robot. It affects both the dynamic and static performance of the system, causing problems such as system creep and limit cycle oscillation, especially when the system is moving at a low speed. Therefore, the establishment of a correct and effective friction model can greatly improve joint control performance. In many pieces of research, the friction model is directly treated as Coulomb-Viscous friction model or Stribeck friction model. These models are simple and effective, and the parameters are easy to identify, but they can only describe the static characteristics of friction at a constant speed and cannot deal with the friction when reversing. In this case, we use a modified Stribeck friction model with Gaussian compensation term in commutation [26]. The friction can be described as
\[
F_{fric}(\dot{q}_i(t)) = [f_{si} + (f_{si} - f_{ci}) e^{−(\dot{\theta}_i/\omega_0)^2}] \text{sgn}(\dot{\theta}_i) + f_{ci} \dot{\theta}_i - f_{gi}, \quad i = 1, 2,
\]
(16)

where \( f_{si} \) is the Coulomb Friction coefficient, \( f_{si} \) is the Static Friction coefficient, \( f_{ci} \) is viscous friction coefficient, \( \omega_0 \) is Stribeck velocity and \( f_{gi} \) is the Gaussian function. For \( f_{gi} \), we have
\[
\begin{cases}
\dot{f}_{gi} = s_i e^{−(\dot{\theta}_i/\nu_i)^2} \\
f_i = s_i \text{sgn}(\dot{\theta}_i) f_{si} + f_{ci} l \\
v_i = a_i \omega_0 [d \cdot \text{sgn}(a) + c \cdot \text{sgn}(\dot{\theta}_i)], \quad i = 1, 2
\end{cases}
\]
(17)

where \( a_i \) is the acceleration at the output of the corresponding motor and \( l, d, c \) are all parameters that need to be identified.

Considering the manufacturing assembly, external disturbance, and other factors, there are uncertainties in the system. The dynamic model of the system can be described as
\[
M(\theta, \sigma, \dot{\theta}) \ddot{\theta}(t) + C(\theta, \dot{\theta}, \sigma, \dot{\theta}) \dot{\theta}(t) + F(\dot{\theta}, \sigma, t) = \tau(t)
\]
(18)

where \( \sigma \in \sum \subseteq R^\sigma \) is the uncertain parameter, the set \( \sum \subseteq R^\sigma \), which means the bound of \( \sigma \), is assumed to be known and compact. We assume the uncertainties are time-varying but bounded.

**III. CONTROLLER DESIGN AND STABILITY ANALYSIS**

**A. CONTROLLER DESIGN**

Robot dynamic control should make each joint of the robot have good performance in tracking a given trajectory. The robust controller for the SCARA robot is designed to suppress the influence of various uncertainties and make the tracking error converge to zero gradually.

For the SCARA robot, \( \theta_d(t), t \in [t_0, t_1] \) is the given trajectory, with the desired velocity and acceleration \( \dot{\theta}_d(t), \ddot{\theta}_d(t) \). Assume \( \theta_d : [t_0, \infty) \to R^\theta, \) of class \( C^2 \) and they are uniformly bounded. We define the tracking error as:
\[
e(t) = \theta(t) - \theta_d(t),
\]
(19)

then we can get
\[
\dot{e}(t) = \dot{\theta}(t) - \dot{\theta}_d(t), \ddot{e}(t) = \ddot{\theta}(t) - \ddot{\theta}_d(t).
\]
(20)
The tracking error vector can be written as
\[ \dot{e}(t) = [e(t), \dot{e}(t)]^T. \]  
(21)

Therefore, we are supposed to propose a controller to make the tracking error vector \( \dot{e} \) of the SCARA robot system uniformly bounded and uniformly ultimately bounded.

Moreover, the equation (18) can be rewritten as
\[
M(\theta, \dot{\sigma}, t)(\ddot{\theta}^d(t) + \ddot{\sigma}(t)) + C(\theta, \dot{\sigma}, \sigma, t)(\dot{\theta}^d(t) + \dot{\sigma}(t)) + F(\dot{\theta}, \sigma, t) = \tau(t).
\]
(22)

The functions \( M(\cdot) \), \( C(\cdot) \) and \( F(\cdot) \) consist of two parts
\[
M(\theta, \dot{\sigma}, t) = \tilde{M}(\theta, t) + \Delta M(\theta, \dot{\sigma}, t),
\]
(23)
\[
C(\theta, \dot{\theta}, \sigma, t) = \tilde{C}(\theta, \dot{\theta}, t) + \Delta C(\theta, \dot{\theta}, \sigma, t),
\]
(24)
\[
F(\dot{\theta}, \sigma, t) = \tilde{F}(\dot{\theta}, t) + \Delta F(\dot{\theta}, \sigma, t),
\]
(25)
where \( \tilde{M}(\cdot), \tilde{C}(\cdot) \) and \( \tilde{F}(\cdot) \) are the nominal portions, whereas \( \Delta M(\cdot), \Delta C(\cdot) \) and \( \Delta F(\cdot) \) are the uncertain portions which depend on \( \sigma \). We now define a vector
\[
\Phi(e, \dot{e}, \sigma, t) := -\Delta M(\theta, \dot{\sigma}, t)(\ddot{\theta}^d - \dot{S}e) - \Delta C(\theta, \dot{\theta}, \sigma, t)(\dot{\theta}^d - Se) - \Delta F(\dot{\theta}, \sigma, t),
\]
(26)
where \( S = \text{diag}[s_1, s_2], s_1, s_2 > 0 \) are constants. Obviously \( \Phi \equiv 0 \) if all uncertainties disappear. Then, we choose a scalar \( \rho \) based on the assumed bound of model uncertainty and external disturbances, such that
\[
\| \Phi(e, \dot{e}, \sigma, t) \| \leq \rho (e, \dot{e}, \sigma, t).
\]
(27)

In the absence of special stated, \( \| \cdot \| \) is always treated as the Euclidean norm.

The trajectory following problem of the SCARA robot to be solved is to design a controller \( \tau(t) \). The proposed controller should ensure the \( e(t) \) remains within the predetermined boundary. [27] proposed an approach of guaranteeing prescriptive performance bounds. The deterministic robust control scheme can be expressed as
\[
\tau(t) = \tilde{M}(\ddot{\theta}^d - \dot{S}e) + \tilde{C}(\dot{\theta}^d - Se) + \tilde{F}
\]
\[
- \frac{Pe - \dot{D}e}{\eta + \epsilon},
\]
(28)
where \( P = \text{diag} [kp_i]_{i=1}^2, kp_i > 0, D = \text{diag}[kd_i]_{i=1}^2, kd_i > 0, i = 1, 2 \) are proportional and differential parameters, which play a similar role to nominal PD control. \( \epsilon \) is a positive design parameter, and
\[
\beta = \gamma^2 (e + Se) \eta, \quad \eta = \gamma \| e + Se \|^2.
\]
(29)
(30)
where the scalars \( \gamma \) and \( \rho \) are positive design parameters. The values of \( \rho \) and \( \gamma \) depend on the practical engineering application. The controller (28) ensures \( \dot{e}(t) \) of the system (22) to be uniformly bounded and uniformly ultimately bounded. Moreover, we can make the ultimate boundedness small enough by choosing the suitable design parameters.

**B. PROOF OF STABILITY**

The stability of the system can be proved by the Lyapunov minimax method. First, we choose a Lyapunov function candidate and prove the validity of it [28]. The Lyapunov function candidate is chosen as
\[
V(\dot{e}) = \frac{1}{2} (\dot{e} + Se)^T M(\dot{e} + Se) + \frac{1}{2} e^T (P + Se) e.
\]
(31)

Obviously \( V(\dot{e}) \) is legitimate if we can prove that \( V(\dot{e}) \) is (globally) positive definite and decreasent.

From Eq.(14), \( M \) is bounded, thus
\[
V(\dot{e}) \geq \frac{1}{2} \| \dot{e} + Se \|^2 + \frac{1}{2} e^T (P + Se) e
\]
\[
= \frac{1}{2} \sum_{i=1}^n (\dot{e}_i^2 + 2s_i \dot{e}_i + s_i^2 e_i^2)
\]
\[
+ \frac{1}{2} \sum_{i=1}^n (k_{pi} s_i + k_{di} s_i) e_i^2
\]
\[
= \frac{1}{2} \sum_{i=1}^n [e_i - \dot{e}_i] \Psi_i \left[ \begin{array}{c} e_i \\ \dot{e}_i \end{array} \right],
\]
(32)
where
\[
\Psi_i = \left[ \begin{array}{cc} \gamma s_i^2 + k_{pi} s_i + k_{di} s_i & \gamma s_i \\ \gamma s_i & \gamma \end{array} \right].
\]
(33)
It is easy to prove that \( \Psi_i > 0, \forall i \). Then V is positive definite.

\[
V(\dot{e}) \geq \frac{1}{2} \sum_{i=1}^n \lambda_{\min}(\Psi_i)(e_i^2 + \dot{e}_i^2) \geq \lambda_1 \| \dot{e} \|^2,
\]
(34)
where \( \lambda_1 = \min \left\{ \frac{1}{2} \lambda_{\min}(\Psi_1), \frac{1}{2} \lambda_{\min}(\Psi_2) \right\} > 0 \). By Assumption 1, there is
\[
V(\dot{e}) \leq \| \dot{e} + Se \|^2 \gamma + e^T (P + Se) e.
\]
(35)

For the first term on the right-hand side,
\[
\tilde{\gamma} \| \dot{e} + Se \|^2 = \tilde{\gamma} (\dot{e} + Se)^T (\dot{e} + Se)
\]
\[
= \tilde{\gamma} \left[ \begin{array}{c} e \\ \dot{e} \end{array} \right] \left[ \begin{array}{cc} S^2 & S^2 \\ S^2 & S^2 \end{array} \right] \left[ \begin{array}{c} e \\ \dot{e} \end{array} \right]
\]
\[
\leq \tilde{\gamma} \lambda_{\max} \left( \left[ \begin{array}{cc} S^2 & S^2 \\ S^2 & S^2 \end{array} \right] \right) \| \dot{e} \|^2
\]
\[
= \tilde{\gamma} \| \dot{e} \|^2.
\]
(36)

For the second term, by Rayleigh’s principle,
\[
e^T (P + Se) e \leq \lambda_{\max} (P + Se) \| e \|^2
\]
(37)

With Inequalities (35) and (36) into Inequality (34), we have
\[
V(\dot{e}) \leq \tilde{\gamma} \| \dot{e} \|^2 + \lambda_{\max} (P + Se) \| e \|^2 =: \tilde{\lambda} \| \dot{e} \|^2,
\]
(38)
where \( \tilde{\lambda} = \tilde{\gamma} + \lambda_{\max} (P + Se) \). Note that \( \tilde{\lambda} \) in Inequality (38) is a strictly positive constant, which proves that \( V(\dot{e}) \) is decreasent. Therefore, it can be proved that \( V(\dot{e}) \) is a valid Lyapunov function candidate from (34) and (38).
Then, we prove the stability of the system. The time derivative of $V(\dot{e})$ is given by

$$
\dot{V}(\dot{e}) = (\dot{e} + Se)^T M(\dot{e} + Se) + \frac{1}{2} (\dot{e} + Se)^T \dot{M}(\dot{e} + Se) + \epsilon^T (P + SD) \dot{e},
$$

(39)

for $\tilde{e} = \tilde{q} - \tilde{q}^d$ and Eq.(3), the first two terms become

$$(\dot{e} + Se)^T M(\dot{e} + Se) + \frac{1}{2} (\dot{e} + Se)^T \dot{M}(\dot{e} + Se)$$

$$= (\dot{e} + Se)^T (M\tilde{q} - M\tilde{q}^d + MS\dot{e} + \frac{1}{2} \dot{M}(\dot{e} + Se))$$

$$= (\dot{e} + Se)^T (\tau - C(\tilde{q} - Se) - G - F - \frac{1}{\eta + \epsilon} Pe - D\dot{e})$$

$$= (\dot{e} + Se)^T (-\Delta M(\tilde{q}^d - \tilde{q}) - \Delta C(\tilde{q} - Se) - \Delta G - \Delta F - \frac{\beta}{\eta + \epsilon} Pe - D\dot{e})$$

(41)

By Eqs.(26) and (27), we have

$$(\dot{e} + Se)^T M(\dot{e} + Se) + \frac{1}{2} (\dot{e} + Se)^T \dot{M}(\dot{e} + Se)$$

$$= (\dot{e} + Se)^T [\Phi - \frac{\beta}{\eta + \epsilon} Pe - D\dot{e}]$$

$$\leq \|\dot{e} + Se\|^2 \Phi \|\dot{e} + Se\|^T (Pe + D\dot{e})$$

$$= (\dot{e} + Se)^T \frac{\beta}{\eta + \epsilon}.$$

(42)

Since

$$- (\dot{e} + Se)^T \frac{\beta}{\eta + \epsilon}$$

$$= - \|\dot{e} + Se\|^2 \frac{\beta^2}{\eta + \epsilon}$$

$$\leq - \frac{\eta^2}{\eta + \epsilon} \leq - \frac{\eta^2 - \epsilon^2}{\eta + \epsilon}$$

$$= - \frac{(\eta + \epsilon)(\eta - \epsilon)}{\eta + \epsilon}$$

$$= - \epsilon \|\dot{e} + Se\|^2 + \epsilon,$$

(43)

and

$$- (\dot{e} + Se)^T (Pe + D\dot{e}) = - \epsilon^T PSe - \epsilon^T D\dot{e}$$

(44)

Substitute (43) and (44) into (42) and combine (39)

$$\dot{V}(\dot{e}) \leq \rho \|\dot{e} + Se\| - \gamma \rho^2 \|\dot{e} + Se\|^2 + \epsilon - \epsilon^T PSe$$

$$+ \epsilon^T (P + SD) \dot{e}$$

$$\leq \frac{1}{4\gamma} + \epsilon - \epsilon^T PSe - \epsilon^T D\dot{e}$$

$$\leq \frac{1}{4\gamma} - \lambda \|\dot{e}\|^2 + \epsilon,$$

(45)

where $\lambda = \min(\lambda_{\text{min}}(PS), \lambda_{\text{min}}(D))$. It shows that $\dot{V}$ is negative definite for all $\|\dot{e}\|$ such that

$$\frac{1}{4\gamma} - \lambda \|\dot{e}\|^2 + \epsilon < 0.$$

(46)

Both $\gamma, \lambda, \epsilon$ are crisp. Thus, $\dot{V}(\dot{e})$ is negative for sufficiently large $\|\dot{e}\|$.

Thus, the controller (28) can ensure the uniform boundedness and uniform ultimate boundedness of the SCARA robot system (18). The uniform boundedness is guaranteed with the following performance. That is, for any $y > 0$ with $\|\dot{e}(t_0)\| > 0$, we have

$$d(y) = \begin{cases} 
\sqrt{\frac{\lambda}{2}}, & y > Y, \\
\sqrt{\frac{\lambda}{2}}, & y \leq Y,
\end{cases}$$

(47)

$$Y = \sqrt{\frac{1}{4\gamma}} y,$$

(48)

such that $\|\dot{e}(t)\| \leq d(y)$ for all $t \geq t_0$. Uniform ultimate boundedness also follows. That is, for any $\ddot{d}$ with

$$\ddot{d} > Y \sqrt{\frac{\lambda}{2}},$$

(49)

we have $\|\dot{e}(t)\| \leq \ddot{d}$, $\forall t \geq t_0 + T(\ddot{d}, y)$, with

$$T(\ddot{d}, y) = \begin{cases} 
0, & y \leq Y, \\
\frac{\lambda y^2 - \lambda Y^2}{2}, & \text{otherwise},
\end{cases}$$

(50)

$$\ddot{Y} = \ddot{d} \sqrt{-\frac{\lambda}{2}}.$$ (51)

The stability of the system is guaranteed and tracking error vector $\|\dot{e}\|$ can be made small enough by choosing larger $\lambda$ and $\gamma$.

**IV. SIMULATION AND EXPERIMENTAL ANALYSIS**

**A. PARAMETERS SELECTION AND SIMULATION RESULTS**

In the simulation, we choose step signal and sinusoidal signal respectively as the reference signal to verify the three control algorithms: PID control, Model-Based PD (MPD) control without robust component, and the proposed novel robust control (NRC). To distinguish easier in the figures, we call the Model-Based PD control MPD control and the proposed novel robust control algorithm NRC control. The step signal can be described as

$$q^d = \begin{bmatrix}
\theta^d_1 \\
\theta^d_2 \\
\theta^d_3 \\
\theta^d_4
\end{bmatrix} = \begin{bmatrix}
\pi \\
\frac{18}{18} \\
\frac{18}{18} \\
\frac{18}{18}
\end{bmatrix}.$$ (52)
The sinuosoidal signal can be described as
\[ q^d = \begin{bmatrix} \theta_1^d \\ \theta_2^d \end{bmatrix} = \begin{bmatrix} \frac{\pi}{6} \sin t \\ \frac{\pi}{6} \sin t \end{bmatrix}. \] (53)

The inherent system parameters of the SCARA robot, as the simulation parameters, which are exported from the software CAD and shown in Table 1. Furthermore, the friction force \( F \) of (16) is
\[ F_i(\dot{q}_i(t)) = [f_{ci} + (f_{si} - f_{ci})e^{-(\dot{\theta}_i/\omega_s)^2}] \text{sgn}(\dot{\theta}_i) + f_{si}\dot{\theta}_i - f_{gi}, \quad i = 1, 2, \] (54)
\[ f_{gi} = s_i e^{-\dot{\theta}_i/\omega_s^2}, \]
\[ s_i = \text{sgn}(\frac{\dot{\theta}_i}{\dot{\theta}_i}), \]
\[ v_i = a_i \omega_s \{d \cdot \text{sgn}(\alpha) + c \cdot \text{sgn}(\dot{\theta}_i)\}, \quad i = 1, 2 \] (55)

with \( f_{si} = [0.4, 0.6] \), \( f_{ci} = [0.32, 0.44] \), \( f_{vi} = [0.04, 0.03] \), and \( \omega_s = 0.04 \), \( d = 1.16 \), \( c = 2.08 \), \( \dot{\theta}_i = 0.18 \).

Testing the three control algorithms repeatedly, we select one of the results with a good performance from each algorithm for comparison. The optimal PID parameter are as follows, \( P = [120, 95]^T \), \( I = [0.1, 0.1]^T \), \( D = [20, 22]^T \), whereas the MPD control and the NRC control are finally determined \( S = \text{diag}[1; 1] \), \( P = \text{diag}[260; 195] \) and \( D = \text{diag}[37; 25] \). We choose \( \rho = 2 \), \( \gamma = 1.55 \), \( \epsilon = 1.5 \) as robust term parameters. The uncertainties are assumed as
\[ m_1 = \bar{m}_1 + \Delta m_1, \] (56)
\[ m_2 = \bar{m}_2 + \Delta m_2, \] (57)
\[ m_3 = \bar{m}_3 + \Delta m_3, \] (58)

where
\[ \Delta m_1 = \Delta m_2 = 0.05 \sin t, \] (59)
\[ \Delta m_4 = 0.3 \sin t. \] (60)

The initial time \( t_e = 0 \) and the initial condition \( \hat{e}(0) = [0.1, 0.1, 0, 0]^T \). The simulation results of step response and sinusoidal signal tracking are as follows:

1) STEP RESPONSE

The results of two joints tracking the reference step signals 10° with three control algorithms are shown in Fig.2 and Fig.3. The step response time with NRC is shorter and the steady-state tracking error is smaller than the others of both joints. Specifically, the response time of the NRC is close to 0.6s, whereas the others are more than 0.8s. The steady-state error with the NRC is −0.002 to 0.004°, whereas the MPD’s is −0.008° to 0.009° and the PID’s is −0.01° to 0.02°.

2) SINUSOIDAL SIGNAL TRACKING

Fig.4-Fig.9 show the sinusoidal signal tracking result comparison among three algorithms. From Fig.4 and Fig.5, it can be found that although almost all controllers can quick track the reference signal at the initial time, the steady-state error with the NRC algorithm is smaller than the others.

B. EXPERIMENTAL PLATFORM AND EXPERIMENTAL RESULTS

Just simulation may not fully indicate the superiority of the proposed control algorithm. In order to ultiorily verify
that our proposed algorithm is effective, we complete the corresponding dynamics experiments on the SCARA robot platform. Fig.10 shows the experimental equipment, which consists of the SCARA robot, cSPACE control system, computer and cSPACE upper-computer software, motor driver, and so on. The cSPACE control platform is a kind of rapid control prototype system, which adopts the method of real-time simulation and control the hardware in loop to design. It uses the C2000 Support Package provided by the TI company to directly generate the code on MATLAB/Simulink, from which we can realize the combination of computer simulation and real-time control.

The experimental process can be summarized as the following three steps:

Step1: The absolute value encoder collects the motor position signal and feeds it back to the driver.

Step2: The digital signal processor (DSP TMS320F28335) in the cSPACE receives the drive signal and computes it in combination with the control algorithm.

Step3: The calculated control signal is output through CAN communication and then amplified to drive the motor, to realize the motion control of the robot.

1) TRANSIENT PERFORMANCE

Consistent with the simulation, both joints are required to refer to the step signals of 10°. The step response curves of the system with three algorithms are shown in Fig.11 and Fig.12. It can be found that the step response time of NRC is not significantly different from the other two algorithms, since the proposed algorithm is mainly concerned with the steady-state performance of the uncertain system. Specifically, we can get the steady-state errors of the NRC algorithm $e_1 = -0.005°$ to 0, $e_2 = 0$ to 0.003°, whereas the MBD control $e_1 = -0.02°$ to 0, $e_2 = -0.01°$ to 0 and the PID control $e_1 = -0.06°$ to 0, $e_2 = -0.018°$ to 0.

2) STEADY-STATE PERFORMANCE

Fig.13 to Fig.18 show the experimental comparison results of the three different algorithms with no load. To keep consistent
with the simulation, the amplitude of sinusoidal signals at both joints are 30°. It can be found that the steady-state performance of the NRC algorithm is better than the other two, and one could get more details in the $e_1$ and $e_2$ of Fig.15 and Fig.16. The maximum tracking error with the NRC algorithm is minimal.

From Fig.15 and Fig.16, we notice that the maximum of tracking errors always occur when the $\dot{\theta}$ is around zero with all controllers. The reasons may be the gear backlash and Coulomb friction. Fig.19 and Fig.20 show the comparison of tracking error curves with the NRC algorithm which adds a modified Strubeck friction model with Gaussian compensation term, Coulomb-Viscous friction, and no friction respectively. The error curve with Gaussian compensation is smoother and the maximum tracking error is lower than the other two, which reveals the friction model (17) we select in the research has a certain effect on restraining the reversal chattering.
To quantify the steady-state performance of the three algorithms, Table 2 shows the maximum displacement error (MAXE) and the root mean square of displacement error (RMSE) of the three algorithms, which are defined as:

\[
\text{MAXE} = \max(|e_i|), \quad (61)
\]
\[
\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2}, \quad (62)
\]

where \(e_i\) denotes the \(i\)-th sampled tracking error and \(n\) is the number of samples. The improvement refers to the performance improvement of NRC relative to PID and MPD, respectively. Table 2 clearly shows the performance of the three algorithms. The NRC algorithm possesses a smaller RMSE and MAXE than the others. Table 3 shows the performance comparison of RMSE and MAXE.

To quantify the steady-state performance of the three algorithms, Table 2 shows the maximum displacement error (MAXE) and the root mean square of displacement error (RMSE) of the three algorithms, which are defined as:

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\text{MAXE} = \max(|e_i|), \quad (61)
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\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} e_i^2}, \quad (62)
\]

where \(e_i\) denotes the \(i\)-th sampled tracking error and \(n\) is the number of samples. The improvement refers to the performance improvement of NRC relative to PID and MPD, respectively. Table 2 clearly shows the performance of the three algorithms. The NRC algorithm possesses a smaller RMSE and MAXE than the others. Table 3 shows the performance comparison of RMSE and MAXE.

The improvement refers to the performance improvement of the modified Stribeck friction model relative to the Coulomb-Viscous friction model and without friction model, respectively. The results also prove the effectiveness of the modified Stribeck friction model.

3) ROBUSTNESS OF LOAD CHANGE

Load variation is one of the main uncertainties of the SCARA robot. The performance of each control algorithm varies

![FIGURE 19. The comparison curves of tracking error with different friction models (joint 1).](image1)

![FIGURE 20. The comparison curves of tracking error with different friction models (joint 2).](image2)

![FIGURE 21. The tracking error curves of joint 1 with 0kg payload.](image3)

![FIGURE 22. The tracking error curves of joint 2 with 0kg payload.](image4)

![FIGURE 23. The tracking error curves of joint 1 with 0.5kg payload.](image5)

![FIGURE 24. The tracking error curves of joint 2 with 0.5kg payload.](image6)
with different payloads. Thus, we conduct three comparative experiments with 0kg, 0.5kg, and 1kg payload. Fig. 21 to Fig. 26 show the comparison results of the three control algorithms. The tracking error under the NRC algorithm just increases a little, whereas the increase of payload has a great impact on the other two algorithms, especially joint 2.

Table 4 and Table 5 show the RMSE and MAXE of the three algorithms with different payloads. The improvement refers to the performance improvement of NRC relative to PID and MPD, respectively. The NRC algorithm, compared with the other two, possesses smaller RMSE and MAXE in each case. Moreover, as the load increases, the performance of the NRC algorithm has improved even more, which means that the NRC algorithm shows better robustness when external disturbances increase.

V. CONCLUSION

We propose a novel practical robust control scheme for the SCARA robot and verify the effectiveness through experiments in this paper. The algorithm is composed of a PD feedback term based on the model and a robust term. The formation of the robust part comprises the upper bound of the uncertainty. To restrain the reversal chattering, we apply the modified Strubeck friction model with Gaussian compensation term as the friction description. The algorithm is demonstrated the guaranteed system performance with the Lyapunov minimax method. Simulation and experimental results prove that the proposed algorithm has a good steady-state performance. Moreover, rapid controller prototyping cSPACE, as the experimental platform, can eliminate the tedious programming work and provide a great convenience for the experiments. The experiments mainly compare the trajectory tracking capabilities of the robot under three algorithms. The results indicate that the proposed algorithm has better robustness, which provides accurate trajectory tracking under the influence of uncertainties. The algorithm could solve the control design problem of similar time-varying nonlinear systems, especially appropriate for engineering applications. We will further consider the effect of electromagnetic interference on the SCARA robot system in future work.

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