Active Shape Control of Spinning Membrane Space Structures
and Its Application to Solar Sailing*

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Spinning-type membrane space structures easily deform because they have no supporting structure. This may lead to an unexpected change in the effect of solar radiation pressure (SRP) on the membranes. Since SRP is a dominant factor of the dynamics of membrane space structures, especially for solar sails, knowledge of deformation is vital. However, it is almost impossible to precisely predict and design the actual deformation of membranes. This study provides a method to actively control the deformation of spinning membrane space structures. A completely fuel-free solar sailing technique is also shown as one application of the shape-control method developed.

Key Words: Membrane Structures, Solar Sail, Shape Control, Vibration Mode

Nomenclature

\[ \begin{aligned}
  t & : \text{time} \\
  x, y, z & : \text{Cartesian coordinate} \\
  r, \theta, z & : \text{cylindrical coordinate} \\
  r & : \text{position vector} \\
  M & : \text{mass} \\
  w & : \text{out-of-plane deformation} \\
  \Omega & : \text{spin rate (angular velocity)} \\
  r_a, r_b & : \text{inner and outer radii} \\
  h & : \text{thickness} \\
  \rho & : \text{density} \\
  \nu' & : \text{Poisson’s ratio} \\
  \sigma & : \text{tensile stress} \\
  R, \Theta & : \text{modal function} \\
  q & : \text{modal coordinate} \\
  \omega & : \text{natural frequency} \\
  \nu, \mu & : \text{circular/frequency order of vibration} \\
  n, m & : \text{radial order of vibration} \\
  L & : \text{standard length} \\
  l & : \text{offset between the center of mass and center of SRP} \\
  I & : \text{moment of inertia} \\
  n & : \text{normal vector} \\
  s & : \text{Sun vector} \\
  \psi, \phi & : \text{in-plane and out-of-plane Sun angles} \\
  F, f & : \text{force} \\
  T, \tau & : \text{torque} \\
  P & : \text{momentum flow rate} \\
  S & : \text{area} \\
  C_{\text{spec}} & : \text{specular reflectivity} \\
  C_{d, f} & : \text{diffuse reflectivity} \\
  C_{\text{abs}} & : \text{absorptivity} \\
  B_f & : \text{Lambertian coefficient} \\
  A, B, a, b & : \text{constants of integration} \\
  \rho, \lambda, N, Q, \gamma & : \text{other constants} \\
  \Sigma & : \text{coordinate system} \\
  \delta_{\text{Dirac}} & : \text{Dirac’s delta} \\
  \text{Superscripts} \\
  ^* & : \text{dimensionless value} \\
  ^\ast & : \text{complex conjugate} \\
  ^\prime & : \text{forced response} \\
  ^I & : \text{inertial frame} \\
  ^O & : \text{orbit-fixed frame} \\
  ^{SF} & : \text{spin-free-fixed frame} \\
  \text{Subscripts} \\
  r, \theta, z & : \text{direction in cylindrical coordinate} \\
  \nu, \mu & : \text{circumferential order of vibration} \\
  n, m & : \text{radial order of vibration} \\
  0 & : \text{input by forced vibration} \\
  \text{SRP} & : \text{solar radiation pressure} \\
  M & : \text{membrane} \\
  B & : \text{main body} \\
\end{aligned} \]

1. Introduction

Recently, the use of deployable membrane structures is gathering attention in the space field. The major advantages are that they are lightweight, have a large area, and can be stored compactly. These allow large, lightweight structures with high transportation efficiency to be launched. Examples of membrane space structures are membrane antennas, gossamer aeroshells, and solar sails. A solar sail is a spacecraft driven by solar radiation pressure (SRP), so that it can be propelled basically without any propellant. Figure 1 shows the small solar sail demonstrator IKAROS launched in 2010 by the Japan Aerospace Exploration Agency.1)

Membrane space structures are classified roughly into two
types depending on the deployment method of the membranes. One is the mast-extension type, which deploys its membrane using deployable mast structures. The other is the spinning type, which deploys using centrifugal force. Since the spinning type does not require any supporting structure, it is lighter than the mast-extension type. In addition, the spinning type is still more advantageous in terms of solar sails because propulsion performance depends on area-to-mass ratio. However, spinning-type membrane space structures easily deform due to the lack of supporting structures; natural and local deformations such as deflections or wrinkles still remain even though the global surface of the membrane is kept flat. This may cause an unexpected change of SRP exerted on the membranes. Since membrane space structures receive much sunlight, their attitude motion is strongly disturbed by SRP. Hence, this unexpected disturbance makes it difficult to predict and design the attitude motion of the spacecraft.

A reflectivity control device (RCD) is one solution to deal with the problem. It can change the optical property of the surface electrically to control SRP on it. Therefore, switching multiple RCDs distributed on the membrane can cause the imbalance in SRP to generate attitude control torque. However, several problems remain unresolved even with RCDs. One example is that the performance of the RCDs is also affected by local deformations. This makes accurate control using the RCDs difficult. Another example is that solar radiation disturbance caused by unexpected deformation may exceed the controllability of the RCDs. Actually, the RCDs mounted on IKAROS could not manage the strong attitude disturbance, resulting in much more fuel consumption than expected. Additionally, the unexpected deformation state must be estimated after deployment in order to cancel the disturbance.

Thus, knowledge of membrane deformation is vital to realizing the optimal use of spinning membrane space structures. Nevertheless, current studies are limited to estimating the deformation state after launch, mainly using numerical calculations, and few have dealt with on-orbit active shape control.

The use of some actuators such as a shape memory alloy or artificial muscle has been reported for the shape control of rigid bodies. Flexible membrane structures, however, known as infinite-dimensional dynamical systems. The controller in this case must also be infinite-dimensional to realize active shape control. This requires numerous actuators to be placed over the entire surface of the membrane, resulting in weight increase and a complex control law.

This paper proposes an advanced shape control method for spinning membrane space structures utilizing vibration. The method excites out-of-plane vibration by boundary input. This allows controlling the entire surface of the membrane using fewer actuators compared to conventional ways. The control law is also quite simple: only one-dimensional motion of the actuators controls the infinite-dimensional motion of the membrane. Since the active shape control method deforms the membrane dynamically, it eliminates the effect of natural and local deformations. Thus, the effect of SRP can actively be controlled without estimating the deformation state after launch. This realizes completely fuel-free solar sailing. Not only that, the method also broadens the applicability of membrane space structures by extending them to three-dimensional structures. Thin-film devices attached on the membrane, such as thin-film solar cells, planer antennas, or dust counters, are used for three-dimensional configuration. The accurate behavior of RCDs also becomes possible.

In the following sections, the actual method of shape control is described first. Next, the vibration mode of spinning membrane space structures is derived analytically assuming a simple model. The effectiveness of the analytical solution against general membrane space structures is then confirmed numerically. The validity of the shape control method proposed is also shown using numerical simulation. Finally, a technique for fuel-free solar sailing using active shape control is introduced as one application of the method developed.

2. Shape Control Method

Generally speaking, active shape control of a spinning object requires periodic deformation. This is because the deformation state of the object in the inertial frame changes periodically as the object spins. This kind of deformation can be given, for example, by driving actuators attached on the membrane in synchronization with the spin. However, this causes weight to increase and leads to the complex control law, as described in Section 1.

This paper proposes a method to realize spin-synchronized deformation using vibration. Since membrane space structures are a flexible continuum, the entire shape changes with out-of-plane vibration. Therefore, they form a corresponding vibration mode when vibrated at a certain frequency. The waveform of the vibration becomes completely static in the inertial frame when its frequency is synchronized with the spin. Various shapes can also be effected by superposing multiple vibration modes. The static waveform is hereinafter referred to as “static wave.” Figure 2 shows examples of
static waves.

The shape control input can be, for example, given by internal mechanical actuators. In the case of IKAROS, the sail membrane and main body are connected via tethers, as shown in Fig. 3. Driving the tethers periodically effects forced displacement at the inner boundary of the sail membrane. Figure 4 shows two examples for actuation. Example 1 is to shift the connection parts between the tethers and the main body. Example 2 is to alternately pull on the tethers, which are attached at the top and bottom sides of the main body. This study mainly focuses on the method of Example 2 for the following reasons: it does not include sliding mechanisms that may cause friction and wear, and it gives the forced displacement at the exact boundary of the membrane.

The shape control method proposed has the advantage that it controls the deformation of the entire membrane using only boundary input. Furthermore, the method controls deformation of the membrane, which is an infinite-dimensional system, using only one-dimensional motion: sliding or pulling the tethers.

3. Analytical Study of Vibration Mode

Vibration modes of spinning membrane space structures are derived analytically in this section. Here, a uniform and circular membrane is assumed for simplicity. Vibrations of a membrane consist of in-plane and out-of-plane, and they can be treated independently in a linear region.\(^{(9)}\) In the following, out-of-plane vibration, which is the target of shape control, is first analyzed based on the theory of elasticity. The linearized equation of motion and its general solution are derived under the stress field of plane stress state. The validity of these assumptions is evaluated numerically in Section 4. Next, response to the control input is analyzed using the solution derived. Finally, the conditions to generate a static wave are discussed.

3.1. Vibration mode analysis

3.1.1. Derivation of general solution

A spinning, uniform and circular membrane is assumed here, as shown in Fig. 5. The deformation of the membrane is described in the rotating cylindrical coordinate system \(r-\theta-z\), the origin of which is at the center of the membrane. The spin rate of the system \(\Omega\) is assumed to be constant. Additionally, the inner radius of the membrane \(r_a\) is assumed to be zero. This assumption is valid in the actual cases when \(r_a\) is small enough. The force in the \(z\) direction exerted on a microelement \(dM = \rho hr dr d\theta\) is described as follows:

\[
F_z = \left\{ \frac{\partial}{\partial r} \left( \sigma_r \frac{\partial w}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \sigma_\theta \frac{\partial w}{\partial \theta} \right) \right\} h r dr d\theta
\]

Hence, the equation of motion is written as:

\[
\ddot{w} dM = F_z
\]

\[
\Leftrightarrow \rho r \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial r} \left( \sigma_r \frac{\partial w}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \sigma_\theta \frac{\partial w}{\partial \theta} \right)
\]

Considering the plane stress state of a spinning circular membrane, tensile stress in the radial and circumferential directions is, respectively, derived as follows.\(^{(10)}\)
where the following solution is assumed considering the separation of variables.

\[ w(r, \theta, t) = R(r)\Theta(\theta)q(t) \]  

(4)

Substituting Eq. (4) into Eq. (2), the following equations are obtained.

\[ \frac{d^2 \Theta}{d\theta^2} + \omega^2 \Theta = 0 \]  

(5)

\[ \frac{d^2 \Theta}{d\theta^2} + \frac{v^2}{R} \Theta = 0 \]  

(6)

\[ \frac{d}{dr} \left( \frac{dR}{dr} \right) + (\rho \omega^2 r^2 - \nu^2 \sigma_0)R = 0 \]  

(7)

where \( \omega \) and \( \nu \) appear as the constants of the separation of variables. The boundary conditions are given as:

\[ \Theta(\theta) = \Theta(\theta + 2\pi) \]  

(8)

\[ R(r_b) < \infty \]  

(9)

Equations (8) and (9), respectively, are the continuity for circumferential direction and the free-edge boundary condition at the outer boundary of the membrane. Since the solution of \( \Theta(\theta) \) can be described sinusoidally, Eq. (8) gives the following condition.

\[ \nu = 0, 1, 2, 3, \ldots \]  

(10)

Substituting Eq. (3) into Eq. (7), the differential equation of \( R(r) \) is described as:

\[ \frac{\dot{r}}{r} \frac{d}{dr} \left( \frac{dR}{dr} \right) + (\lambda^2 \dot{r}^2 - \nu^2)R = 0 \]  

(11)

where

\[ \dot{r} = \frac{r}{r_b} \]  

(12)

\[ \lambda^2 = \frac{8}{3 + \nu'} \left( \frac{\omega}{\Omega} \right)^2 + 1 + 3\nu' \geq 0 \]  

(13)

The following infinite series, which starts from the \( p \)-th order, is assumed as the solution of Eq. (11).

\[ R(\dot{r}) = \dot{r}^p \sum_{m=0}^{\infty} a_m \dot{r}^m \]  

(14)

Substituting Eq. (14) into Eq. (11), and evaluating the coefficient of \( \dot{r}^p \), \( p \) is derived as

\[ p = \pm \nu \]  

(15)

Since the solution of \( R(\dot{r}) \) diverges at \( \dot{r} = 0 \) when \( p = -\nu \), which conflicts with Eq. (9), only the case of \( p = \nu \) is considered. Therefore, the following recurrence formula of \( a_m \) is derived by substituting Eq. (14) into Eq. (11) under \( p = \nu \).

\[ a_m = \frac{(m + \nu - 2)(m + \nu) - \lambda^2}{m(m + 2\nu)} a_{m-2} \]  

(16)

\[ a_0 \neq 0 \]

\[ a_{-1} = 0 \]

The following condition is required to satisfy Eq. (9).

\[ \exists n \in O, \quad \lambda^2 = (n + \nu - 1)(n + \nu + 1), \quad n \geq 1 \]

\[ O = \{ x \in \mathbb{Z} | x \text{ is odd} \} \]  

(17)

This is because \( a_m \) is zero at the \( (n + 1) \)-th or higher order, and the infinite series, Eq. (14), becomes a polynomial of degree \( n - 1 \) when Eq. (17) holds. Otherwise, the solution diverges at \( \dot{r} = 1 \). Let \( n \) denote the order of vibration for radial direction. Natural frequency is then expressed as the following equation by substituting Eq. (17) into Eq. (13).

\[ \omega_{n,\nu} = \frac{\sqrt{3 + \nu'} \nu (v + n - 1)(v + n + 1) - 1 + 3\nu'}{8} \]  

(18)

The subscript means that the natural frequency depends both on circumferential and radial orders of vibration. Equation (16) can be rewritten as follows when substituting Eq. (17).

\[ a_m = \frac{(m - n - 1)(m + n + 2\nu - 1)}{m(m + 2\nu)} a_{m-2} \]  

(19)

Therefore, the general term of \( a_m \) can be expressed as:

\[ a_{2k} = (-1)^{k+1} \frac{n - 1}{2} \left( \frac{n - 1}{2} - k \right) \left( \frac{k + \nu + n - 1}{2} \right) \]  

(20)

\[ a_{2k+1} = 0 \]

Thus, the general solution of the radial mode function is described as:

\[ R_{n,\nu}(\dot{r}) = \sum_{k=0}^{\infty} (-1)^{k+1} \frac{n - 1}{2} \left( \frac{n - 1}{2} - k \right) \left( \frac{k + \nu + n - 1}{2} \right) \dot{r}^{2k} \]  

(21)

Finally, the general solution for the vibration of a spinning circular membrane is expressed as:

\[ \sigma_r = \frac{3 + \nu'}{8} \rho \Omega^2 \frac{r_b^2}{r^2} \left( 1 - \frac{r^2}{r_b^2} \right) \]

\[ \sigma_\theta = \frac{3 + \nu'}{8} \rho \Omega^2 \frac{r_b^2}{r^2} \left( 1 - \frac{1 + 3\nu' r^2}{3 + \nu' r_b^2} \right) \]  

(3)
Since \( R_{m,n} \) derived.

In the same manner, the following equation is obtained for Eq. (22).

\[
\begin{align*}
\int_0^1 \hat{r} R_{m,n} R_{v,n} d\hat{r} &= N_{v,n} \delta_{m,n} \\
\int_0^{2\pi} \hat{\theta} \hat{\theta} d\theta &= 2\pi \delta_{m,v}
\end{align*}
\]

### 3.1.2 Orthogonality of the mode function

Equation (11) in the \((v,n)\)-th order can be written as

\[
\left( \chi_{v,n}^2 - \frac{\nu^2}{\beta^2} \right) R_{v,n} = -\frac{d}{d\hat{r}} \left\{ \hat{r}(1 - \hat{r}^2) \frac{dR_{v,n}}{d\hat{r}} \right\}
\]

where

\[
\chi_{v,n}^2 = (v + n - 1)(v + n + 1)
\]

from Eq. (17). By integrating the multiple of Eq. (23) and \( R_{v,n} \), the following equation is obtained.

\[
\int_0^1 \left( \chi_{v,n}^2 - \frac{\nu^2}{\beta^2} \right) R_{v,m} R_{v,n} d\hat{r} = \int_0^1 \hat{r}(1 - \hat{r}^2) \frac{dR_{v,m}}{d\hat{r}} \frac{dR_{v,n}}{d\hat{r}} d\hat{r}
\]

In the same manner, the following equation is obtained for the \((v,m)\)-th order.

\[
\int_0^1 \left( \chi_{v,m}^2 - \frac{\nu^2}{\beta^2} \right) R_{v,m} R_{v,n} d\hat{r} = \int_0^1 \hat{r}(1 - \hat{r}^2) \frac{dR_{v,m}}{d\hat{r}} \frac{dR_{v,n}}{d\hat{r}} d\hat{r}
\]

Calculating Eq. \((25) - (26)\), the following equation is derived.

\[
\left( \chi_{v,n}^2 - \chi_{v,m}^2 \right) \int_0^1 \hat{r} R_{v,m} R_{v,n} d\hat{r} = 0
\]

Since \( \chi_{v,n}^2 \neq \chi_{v,m}^2 \) when \( m \neq n \), the following equation holds.

\[
\int_0^1 \hat{r} R_{v,m} R_{v,n} d\hat{r} = 0 \quad (m \neq n)
\]

Therefore, the orthogonality of the radial mode function about its radial order \( n \) is confirmed. In addition to the case of circumferential mode function, the orthogonality is described using the following equations.

\[
\begin{align*}
\int_0^1 \hat{r} R_{v,m} R_{v,n} d\hat{r} &= N_{v,n} \delta_{m,n} \\
\int_0^{2\pi} \hat{\theta} \hat{\theta} d\theta &= 2\pi \delta_{m,v}
\end{align*}
\]

### 3.2 Forced response analysis

When a periodic input is given to the membrane by tethers and actuators, the forced displacement at \( r = r_0 \) can be expressed as

\[
w(r_0, \theta, t) = w_0(\theta, t) = A_0 e^{i(\theta + \alpha_1 t + \alpha_2)} + B_0 e^{i(\theta - \alpha_1 t + \alpha_2)}
\]

Describing the deformation of the membrane as \( w(r, \theta, t) = \hat{w}(r, \theta, t) + w_0(\theta, t) \), the equation of motion, Eq. (2), is rewritten as:

\[
\rho R \ddot{\hat{w}} = \frac{\partial}{\partial r} \left( \sigma_r \frac{\partial \hat{w}}{\partial r} \right) + \frac{\partial}{\partial \theta} \left( \sigma_\theta \frac{\partial \hat{w}}{\partial \theta} \right) + F_0
\]

\[
F_0 = -\rho R \ddot{w}_0 - \frac{\partial}{\partial \theta} \left( \sigma_\theta \frac{\partial w_0}{\partial \theta} \right)
\]

\( F_0 \) denotes a kind of inertial force that appears when considering deformation via \( \hat{w} \). From Eq. (30), \( F_0 \) can be rewritten as:

\[
\begin{align*}
F_0 &= \rho R \ddot{w}_0 - \frac{\partial}{\partial \theta} \left( \sigma_\theta \frac{\partial w_0}{\partial \theta} \right)
= F_{0,\theta} + F_{0,\theta\theta}\frac{\partial^2 w_0}{\partial \theta^2}
\end{align*}
\]

Assuming that the vibration mode here is also described by the same modal function as in Eq. (22), \( \hat{w} \) can be described as:

\[
\hat{w}(r, \theta, t) = \sum_v \sum_n R_{v,n} \left( \frac{\hat{r}}{\beta} \right)^{v/2} \Theta_{v,n}(\theta) \tilde{q}_{v,n}(t)
\]

From Eqs. (6), (7), (31) and (34), the following equation is obtained.

\[
\sum_v \sum_n R_{v,n} \Theta_{v,n} \left( \frac{d^2 \tilde{q}_{v,n}}{dt^2} + \omega^2_{v,n} \tilde{q}_{v,n} \right) = F_{0,\theta}
\]

Applying the orthogonality of the mode functions described in Eq. (29) to Eq. (35), the following equation is derived.

\[
\frac{d^2 \tilde{q}_{v,n}}{dt^2} + \omega^2_{v,n} \tilde{q}_{v,n} = \frac{1}{2\pi N_{v,n}} \int_0^{2\pi} \hat{r} R_{v,n} \Theta_{v,n} F_{0,\theta} d\hat{r} d\theta
\]

Substituting Eq. (33) into Eq. (36), the differential equation of the modal coordinate \( \tilde{q}_{v,n} \) is derived as:

\[
\begin{align*}
\frac{d^2 \tilde{q}_{v,n}}{dt^2} + \omega^2_{v,n} \tilde{q}_{v,n} &= \sum_{m,v} \left( A_{0,1} e^{i(\alpha_1 t + \alpha_2)} + B_{0,1} e^{i(-\alpha_1 t + \alpha_2)} \right) Q_{v,m} \delta_{v,m} \left( A_{0,1} e^{i(\alpha_1 t + \alpha_2)} + B_{0,1} e^{i(-\alpha_1 t + \alpha_2)} \right)
\end{align*}
\]

where
The particular solution for Eq. (37) is

\[
\tilde{q}_{v,n} = \frac{Q_{v,n}}{\omega^2_{v,n} - \omega_0^2} \left\{ A_0 e^{i(\omega t + \omega_0 t + \alpha_0)} + B_0 e^{i(-\omega t + \omega_0 t + \alpha_0)} \right\}
\]  

(39)

Therefore, the response to periodic input is expressed using the following equation.

\[
\tilde{w}(r, \theta, t) = \sum_{n} \frac{Q_{v,n}}{\omega^2_{v,n} - \omega_0^2} R_{v,n} \left( \frac{r}{r_b} \right) \left\{ A_0 e^{i(\omega t + \omega_0 t + \alpha_0)} + B_0 e^{i(-\omega t + \omega_0 t + \alpha_0)} \right\}
\]  

(40)

As can be seen from Eq. (40), only the \(v_0\)-th order vibration mode appears in the circumferential direction, while the resonance magnification of vibration modes in the radial direction depends on the input frequency \(\omega_0\) and natural frequency \(\omega_{v,n}\).

### 3.3 How to generate static waves

From Eq. (40), membrane vibration is expressed by the superposition of the progressive wave and regressive wave that proceed in the counter-clockwise and the clockwise directions, respectively.

progressive wave: \(B_0 e^{i\left(\omega_0 t + \omega t + \alpha_0 + \alpha\right)}\)

regressive wave: \(A_0 e^{i\left(\omega_0 t + \omega t + \alpha_0 - \alpha\right)}\)

Their velocities in the \(\theta\) direction are, respectively, \(\omega_0/v_0\) and \(-\omega_0/v_0\) in the rotating frame. Considering that the angular velocity of the rotating frame is \(\Omega\), the velocity of each wave in the inertial frame is expressed as \(\Omega + \omega_0/v_0\) and \(\Omega - \omega_0/v_0\), respectively. The conditions for the vibration to be a static wave are given by the following equations.

\[
B_0 = 0
\]

\[
\Omega - \omega_0/v_0 = 0 \Leftrightarrow \omega_0 = v_0\Omega
\]

(41)

Equation (41) means that the amplitude of the progressive wave is zero and the circulating velocity of the regressive wave in the inertial frame is zero. When these conditions are satisfied, the progression of the wave stops and the wave surface becomes static in the inertial frame. Therefore, the “spin-synchronized input” should be given so that the progressive wave does not happen and the frequency is an integral multiple of the spin rate. The shape control input to generate a static wave is, from the conditions above, expressed as:

\[
w_0(\theta, t) = A_0 e^{i(\omega_0 t + \omega_0 t + \alpha_0)}
\]

(42)

The corresponding static waveform excited in the inertial frame is described as:

\[
\tilde{w}_s(r, \theta) = \sum_{n} A_0 Q_{v,n} \left( \frac{r}{r_b} \right) e^{i(\omega_0 t + \alpha_0)}
\]

(43)

As can be seen from Eq. (43), only the vibrations whose circumferential orders coincide with those of the input appear. Regarding the radial order of vibrations, the vibration mode having the natural frequency closest to the input frequency appears. The following equations represent the shapes of the static waves in the low-frequency region from \(v_0 = 1\) to 4.

\[
w_s^1(r, \theta, \alpha_1) = A_1 \cos(\theta + \alpha_1) + \tilde{A}_1 R_{1,1} \left( \frac{r}{r_b} \right) \cos(\theta + \alpha_1)
\]

\[
w_s^2(r, \theta, \alpha_2) = A_2 \cos(2\theta + \alpha_2) + \tilde{A}_2 R_{2,1} \left( \frac{r}{r_b} \right) \cos(2\theta + \alpha_2)
\]

\[
w_s^3(r, \theta, \alpha_3) = A_3 \cos(3\theta + \alpha_3) + \tilde{A}_3 R_{3,3} \left( \frac{r}{r_b} \right) \cos(3\theta + \alpha_3)
\]

\[
w_s^4(r, \theta, \alpha_4) = A_4 \cos(4\theta + \alpha_4) + \tilde{A}_4 R_{4,3} \left( \frac{r}{r_b} \right) \cos(4\theta + \alpha_4)
\]

(44)

They are hereinafter called “\(v_0\)-th order static waves.” The number of subscripts corresponds to the order of vibration.

In the case of exciting the static waves by driving tethers, there is a requirement for the number of tethers. The control input by tethers is considered as a digital signal. On the other hand, the required input at the boundary of the membrane consists of two sinusoidal waves: progressive wave and regressive wave. Based on the sampling theorem, the number of tethers required to reproduce a wave is twice the order of the wave: \(2v_0\) tethers for the \(v_0\)-th order wave. Therefore, \(4v_0\) tethers are required in total to reproduce the control input for the \(v_0\)-th order static wave.

### 4. Feasibility Evaluation

In Section 2, the vibration mode and forced response of a spinning membrane space structure are derived analytically assuming a circular membrane model in a linear region. On the other hand, this assumption does not necessarily hold against actual membrane space structures. For example, IKAROS has a square-shaped sail membrane due to the folding and packing method used. Additionally, the membrane is usually hollow (i.e. \(r_a \neq 0\)) because the spacecraft has the main body. However, the principal characteristics of vibration of the actual membrane are expected to be similar to those of the simple analysis model.

In this section, the effectiveness of the analysis model against general-shaped membrane space structures is evaluated using a numerical simulation. Actually, a circular membrane and a square membrane are analyzed. The results obtained for the circular membrane validate the analytical solution against a hollow membrane in a nonlinear region. The results obtained for the square membrane show the effectiveness of the analytical solution to polygonal membranes in general, because membranes for which the number of vertices is larger than that of a square membrane is more similar to a circular membrane: vibrations of polygonal membranes can be well described by the analytical solution if the vibrations of a square membrane correspond to those of the analytical solution.

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In the following, a circular membrane and square membrane are first discretized numerically using a method called the “multi-particle model (MPM).” Next, vibration mode analysis based on the MPM is conducted for both membranes. Finally, the feasibility of the shape control method proposed is evaluated using numerical simulations with the MPM.

### 4.1. Multi-particle model

MPM\(^{(11)}\) is used for numerical analysis. The MPM models the membranes using spring-mass systems. A number of particles are distributed across the entire surface of the membrane, and the particles are connected via springs and dampers. The mass of each particle is given equally from the surface elements to which the particle belongs. Spring constants of the MPM are determined based on the principle of virtual work so that the elastic energy of the spring-mass system is identical to the strain energy of the continuum. MPM advantages include the computational cost is smaller and the implementation is easier compared to those of the finite element method. Furthermore, the MPM is suitable for analyzing the global behavior of the membranes in a short time. The validity of the MPM has been confirmed through the flight results of IKAROS.

Figure 7 shows the configurations of the MPM used in this study. The circular model has a uniform and hollow membrane. The square model has the same configuration as that of the IKAROS sail; the square sail consists of four trapezoid membranes connected via bridges (see Fig. 3). Tip mass is attached at each vertex of the square membrane for the purpose of supporting the centrifugal force deployment. The effect of thin-film devices such as solar cells is eliminated for simplicity. Table 1 shows the properties of the spacecraft. The main body is considered to be a cylindrical rigid body. Tethers and bridges are also modeled as springs and dampers based on their elastic properties. Parameters of the membrane are derived from those of the IKAROS sail membrane. The inner and outer radii of the circular membrane are determined so that its area is identical to that of the IKAROS sail membrane. The structural damping coefficient is determined so that the numerical behavior of the membrane best matches the flight results of IKAROS.

### 4.2. Mode analysis

The equation of motion for the MPM is written by restoring force on each particle. The equation of motion for all particles can be expressed in a matrix form by linearizing the force around the equilibrium point. Hence, eigenvalue analysis of the matrices derives the vibration mode of the membrane.\(^{(2)}\)

Figure 8 shows the plots of the natural frequencies. The figure confirms that the natural frequencies of the MPMs show a similar trend with those of the analytical solution. Since the error between them becomes larger in proportion to the order of vibration, the analytical solution can properly be applied in the low-frequency region. Figures 9 through 12 show the vibration modes obtained as the MPMs and their corresponding analytical solutions. As can be seen from the figures, the results of the numerical analysis and analytical solution are also in good agreement in every order.

The significant reason for the errors between the numerical results and analytical solution is considered to be whether the membrane is solid or hollow. While the analytical model assumes a solid membrane, the numerical model assumes a hollow membrane with a main body. In case of the numerical model, stress distribution and boundary conditions are different from those of the analytical model. Additionally, in the case of the square membrane, the numerical results differ from the analytical solution due to the difference in membrane configuration. However, the results confirm that the global behaviors of vibration mode and natural frequency are well described using the analytical solution.

### 4.3. Evaluation of shape control

The accuracy of the shape control method proposed is evaluated by numerical simulation using the MPM. The control input, which is based on Eq. (42), is given via tethers to

---

**Figure 7. Configuration of the MPM.**

**Figure 8. Natural frequency of each vibration mode obtained by the MPM. Normalized by the spin rate.**

**Table 1. Properties of the spacecraft.**

| Component       | Value                  |
|-----------------|------------------------|
| Total system    | Spin rate 1.0 rpm      |
| Main body       | Mass 295 kg            |
|                 | Diameter 1.58 m        |
|                 | Height 0.845 m         |
|                 | Moment of inertia x, y| 47.71 kg-m²        |
|                 | z axis                |
| Membrane        | Inner radius 1.8 m     |
|                 | Outer radius 7.6 m    |
|                 | Young’s modulus 3.2 GPa|
|                 | Poisson’s ratio 0.33  |
|                 | Structural damping    |
|                 | Thickness 3.9 x 10⁻⁵  |
|                 | Density 7.5 μm        |
|                 | 1420 kg/m³            |
generate a static wave. The natural length of the tethers is changed over time to simulate the control input stated in Section 2. The membrane is completely flat at the initial state \((t = 0)\), and the motion is calculated by numerically integrating the equations of motion of the particles. Simulations are conducted both for the circular and square membranes, and the results are compared.

The target shapes to generate in this simulation are 1st- and 2nd-order static waves. The expected shapes to be generated on the circular membrane are written as follows using Eq. (44).

\[
w_1(r, \theta) = A_1 \cos(\theta + a_1) + \bar{A}_1 \left(\frac{r}{r_0}\right) \cos(\theta + a_1)
\]

\[
w_2(r, \theta) = A_2 \cos(2\theta + a_2) + \bar{A}_2 \left(\frac{r}{r_0}\right)^2 \cos(2\theta + a_2)
\] (45)

Figures 13 and 14 show the simulation results of the 1st- and 2nd-order static waves, respectively, for the circular membrane. Deformation states in the inertial frame at \(t = 2000\) s and \(t = 3000\) s are picked up as the examples in each case. The figures show that the waveform of the membrane is almost the same regardless of time: the tops and bottoms of the waves are in the same locations. In order to evaluate the shape control accuracy against the analytical solution quantitatively, an error analysis is conducted by means of the least squares method. Specifically, the mean error between the displacement of the particles and the expected shapes written in Eq. (45) is calculated. Figure 15 shows the history of the mean error in circular membrane. The figure confirms that the control error is within approximately 20% over time. The possible reasons for the error are:

- discretization error of the membrane using the MPM.
- transient response because of the small damping effect.
linearization error against the large deformation. The error in 2nd-order shape control is larger than that of the 1st-order shape control because the shape of the 2nd-order vibration mode is more complex than that of the 1st-order mode. Though some small fluctuation still remains around the center values, the time-averaged values of the error are steady in the first several tens of seconds. This confirms that the static waves are effected with a short time response compared to the time scale of attitude or orbital motion.

The error tends to be large in the center region. This is because of the difference in boundary conditions. While the analytical expression in Eq. (22) assumes that the membrane is continuous at \( r = 0 \), the actual model, which has a hollow membrane, does not have to satisfy this continuity.

Regarding evaluation of the square membrane, the target shapes are assumed to be expressed by the same solution as that of the circular membrane, Eq. (22). Since the analytical solution of the radial mode function \( R_{v,p}(\tilde{r}) \) is expressed as a function of the normalized variable \( \tilde{r} \), the behavior of the square membrane is also expected to be described using the normalized variable in the radial direction:

\[
\tilde{r} = \sqrt{x^2 + y^2} / L.
\]

\( L \) is set to be the length from the center to one vertex of the membrane. This assumption is considered to be effective in the low-frequency region based on the results of Section 4.2 (Figs. 9–12). Thus, the target shapes of the square membrane are:

\[
w_1^s(r, \theta) = A_1 \cos(\theta + a_1) + A_1 \left( \frac{\sqrt{x^2 + y^2}}{L} \right) \cos(\theta + a_1)
\]

\[
w_2^s(r, \theta) = A_2 \cos(2\theta + a_2) + A_2 \left( \frac{\sqrt{x^2 + y^2}}{L} \right)^2 \cos(2\theta + a_2)
\]

(46)

Figures 16 and 17 show the simulation results of the square membrane. The results show that the waveforms of the square membrane are almost static, similar to the case of the circular membrane. Figure 18 shows the history of the mean error of the shapes generated. The figure confirms that the history shows a similar trend to that of the circular membrane, even though the error for the square membrane is relatively larger. These results prove that the shape control method works properly for both circular and square membranes.

5. Application to Solar Sailing

In this section, an attitude and orbit control strategy is discussed utilizing the shape control method developed. The thrust vector of a solar sail is determined basically by the ori-
entation of the sail membrane to the Sun. Therefore, the attitude of the spacecraft plays a dominant role in its orbital motion. The thrust in most studies is controlled by changing the attitude of the spacecraft. Attitude control is performed, for example, using thrusters or RCDs. This study proposes a new technique for solar sailing based on shape control.

### 5.1 Approach to solar sailing using active shape control

As described in the previous sections, the 1st-order static wave (Figs. 13 and 16) keeps deformation flat and inclined in the inertial frame. This corresponds to changing the orientation of the sail membrane without changing the attitude (i.e., the total angular momentum of the whole system is conserved). Therefore, the 1st-order static wave can be applied to control thrust. As shown in Section 4.3, time constant of shape control is quite small compared to that of the attitude/orbital motion. Hence, this thrust control method can not only be performed without any fuel, but also achieves faster attitude maneuver of the sail than RCDs.

However, the sail cannot keep its orientation toward a certain point forever because of the SRP disturbance torque; the total angular momentum change interrupts the performance of the sail (i.e., the total angular momentum of the whole system is conserved). Therefore, the 1st-order static wave can be applied to control thrust. As shown in Section 4.3, time constant of shape control is quite small compared to that of the attitude/orbital motion. Hence, this thrust control method can not only be performed without any fuel, but also achieves faster attitude maneuver of the sail than RCDs.

The effect of SRP on the sail is expressed by the sum of specular reflection, diffuse reflection, and absorption. The force exerted on a microelement of the sail by each effect is written as

\[
\begin{align*}
    dF_{\text{spe}} &= -2PC_{\text{spe}}(s \cdot n)^2 n dS \\
    dF_{\text{dif}} &= -PC_{\text{dif}}(s \cdot n) s + B_2(s \cdot n)n dS \\
    dF_{\text{abs}} &= -PC_{\text{abs}}(s \cdot n)s dS
\end{align*}
\]

Hence, the thrust and torque exerted on the spacecraft can be derived using the following equations.

\[
\begin{align*}
    F_{\text{SRP}} &= \int (dF_{\text{spe}} + dF_{\text{dif}} + dF_{\text{abs}}) \\
    T_{\text{SRP}} &= \int \hat{r}_M \times (dF_{\text{spe}} + dF_{\text{dif}} + dF_{\text{abs}})
\end{align*}
\]

In order to treat the attitude and orbital motion properly, the thrust is described in \(\Sigma^O\) and the torque is described in \(\Sigma^SF\). Replacing the superscript \(I\) in Eq. (45) with \(SF\), the shape of the sail membrane in \(\Sigma^SF\) is expressed as:

\[
w_{SF}(r, \theta) = w_{1}^{SF}(r, \theta) + w_{2}^{SF}(r, \theta)
\]

The normal vector of the sail microelement can be derived using the following equation.

\[
n = \frac{\partial r_M}{\partial \theta} \times \frac{\partial r_M}{\partial \theta}
\]

From Eqs. (48), (49) and (50), the thrust and torque are respectively expressed as follows.

\[
\begin{align*}
    F_{\text{SRP}}^{O} &= \left[ f_1(\psi, \phi) + f_2(\psi, \phi) \frac{\gamma A_1}{r_b} \cos a_1 + f_3(\psi, \phi) \frac{\gamma A_1}{r_b} \sin a_1 \\
                    &+ f_4(\psi, \phi) + f_5(\psi, \phi) \frac{\gamma A_1}{r_b} \cos a_1 + f_6(\psi, \phi) \frac{\gamma A_1}{r_b} \sin a_1 \\
                    &+ f_7(\psi, \phi) + f_8(\psi, \phi) \frac{\gamma A_1}{r_b} \cos a_1 + f_9(\psi, \phi) \frac{\gamma A_1}{r_b} \sin a_1 \\
                    &- \tau_1 \phi - \tau_2 \sin a_1 - \tau_3 \frac{A_1 A_2}{I r_b} \sin(a_2 - a_1) \\
                    &+ \tau_1 \psi + \tau_2 \frac{A_1}{r_b} \cos a_1 + \tau_3 \frac{A_1 A_2}{I r_b} \cos(a_2 - a_1) \\
                    &- \tau_4 \frac{A_1}{r_b} (\psi \sin a_1 + \phi \cos a_1)
\right] \\
    F_{\text{SRP}}^{SF} &= \left[ f_1(\psi, \phi) + f_2(\psi, \phi) \frac{\gamma A_1}{r_b} \cos a_1 + f_3(\psi, \phi) \frac{\gamma A_1}{r_b} \sin a_1 \\
                    &+ f_4(\psi, \phi) + f_5(\psi, \phi) \frac{\gamma A_1}{r_b} \cos a_1 + f_6(\psi, \phi) \frac{\gamma A_1}{r_b} \sin a_1 \\
                    &- \tau_1 \phi - \tau_2 \sin a_1 - \tau_3 \frac{A_1 A_2}{I r_b} \sin(a_2 - a_1) \\
                    &+ \tau_1 \psi + \tau_2 \frac{A_1}{r_b} \cos a_1 + \tau_3 \frac{A_1 A_2}{I r_b} \cos(a_2 - a_1) \\
                    &- \tau_4 \frac{A_1}{r_b} (\psi \sin a_1 + \phi \cos a_1)
\right]
\end{align*}
\]
where

\[ f_1(\psi, \phi) = -PS(2C_{spe} \sin \psi \cos^2 \psi \cos^3 \phi + B_f \cot \chi \sin \psi \cos \psi \cos^2 \phi) \]
\[ f_2(\psi, \phi) = PS\{2C_{spe} (\sin \psi \cos^2 \phi - 2 \sin^2 \psi \cos \psi \cos^2 \phi) + B_f \cot \chi (\sin^2 \psi - \cos^2 \psi \cos \phi) \}\]
\[ f_3(\psi, \phi) = -PS\{6C_{spe} \sin \psi \cos^2 \psi \sin \phi \cos^2 \phi + 2B_f \cot \chi \sin \psi \cos \psi \sin \phi \cos \phi \}\]
\[ f_4(\psi, \phi) = -PS\{2C_{spe} \sin \psi \cos^2 \psi \sin \phi \cos^2 \phi + \cot \chi \sin \psi \cos \phi \}\]
\[ f_5(\psi, \phi) = -PS\{4C_{spe} \sin \psi \cos \psi \sin \phi \cos \phi + \cot \chi \sin \psi \cos \phi \}\]

\[ \tau_1 = (\cot \chi + C_{abs})PSl \]
\[ \tau_2 = \{\gamma_M(\cot \chi + C_{abs}) + (B_f \cot \chi + C_{spe})\}PSl \]
\[ \tau_3 = \frac{1}{2} \gamma (B_f \cot \chi + C_{spe})PSl \]
\[ \tau_4 = \gamma_M(\cot \chi + C_{abs})PSl \]
\[ \gamma = \frac{I_a}{I_a + I_M}, \quad \gamma_M = \frac{I_M}{I_a + I_M} \]

Thus, the thrust and torque can be controlled via \( A_1, A_2, a_1, \) and \( a_2. \)

Numerical simulation is conducted using the MPM to verify the analytical derivation of the thrust and torque. A circular MPM is used in the simulation. The effect of SRP shown in Eq. (47) is applied to each surface element of the MPM, and the motion of the membrane is numerically integrated in the same manner as described in Section 4.3. Table 2 shows the optical parameters and the shape control input used in the simulation. The attitude motion of the main body includes a damping term proportional to the angular velocity to simulate energy dissipation.

Figures 20 and 21 show, respectively, the simulation results of the force and torque caused by SRP. The blue lines are the actual histories of the force and torque, and red lines are the analytical values expected using Eqs. (51) and (52). Figure 20 shows that the thrust is properly described using the analytical formulation. Though the torque histories in Fig. 21 are vibrating, their center values correspond to those expected. The cause of fluctuation originates in the shape

**Table 2. Optical parameters and shape control conditions for the simulation.**

| Parameter   | Value     |
|-------------|-----------|
| \( \phi \)  | 0 deg     |
| \( \phi \)  | 0 deg     |
| \( C_{spe} \) | 0.882     |
| \( C_{dif} \) | 0.065     |
| \( C_{abs} \) | 0.053     |
| Nutation damping coef. | \( 2.8 \times 10^{-3} \) s |
| \( A_1 \)  | 0.85 m    |
| \( a_1 \)  | 0 deg     |
| \( A_2 \)  | 0.25 m    |
| \( a_2 \)  | 90 deg    |

**Fig. 20.** History of SRP thrust in \( \Sigma^O \) obtained through MPM simulation.

**Fig. 21.** History of SRP torque in \( \Sigma^F \) obtained through MPM simulation.
control error explained in Section 4.3. Since the error fluctuation of the 1st-order static wave in Fig. 15 is small, the fluctuation in thrust is also small. On the other hand, the fluctuation in torque is relatively large because the error fluctuation of the 2nd-order static wave is large. However, the simulation results validate the analytical expression of the force and torque because their time-averaged values are steady and are well-described by Eqs. (51) and (52).

5.3. Trajectory optimization problem

As the summary of this study, an optimization problem for solar sailing is solved. The objective of the problem is to decelerate the spacecraft to minimize its orbital radius. The orbital motion is limited within the ecliptic plane to simplify the problem. Since the x component of $F_{SRP}^O$ describes the deceleration value of the spacecraft, the objective function is given as:

$$\min_{x(t), u(t)} \left( -\int F_{SRP}^O(x(t), u(t)) dt \right)$$  \hspace{1cm} (56)

where

$$x(t) = [r(t), \theta(t), \dot{r}(t), \psi(t), \phi(t)]^T$$

$$u(t) = [A_1(t), a_1(t), A_2(t), a_2(t)]^T$$  \hspace{1cm} (57)

and $r(t)$ and $\theta(t)$ are the position of the spacecraft in the Sun-centered polar coordinate system. The initial position and velocity of the spacecraft are identical to those of Earth, and the initial attitude is in the direction of the Sun. The optimization problem is solved using a method called direct collocation with non-linear programming (DCNLP). This method derives the optimal trajectory, attitude history, and control input, which minimizes the objective function under the equations of motion. Note that the propulsion performance (area/mass) of the solar sail used in this analysis is set to be better than that of IKAROS because the propulsion performance of IKAROS is not sufficient for trajectory maneuver in a realistic time span.

Figure 22 shows the optimization result of the deceleration maneuver. Figure 22(a) confirms that the spacecraft is continuously decelerated and its orbital radius gets smaller. Figure 22(b) shows that the sail keeps orienting toward a certain point. This point is the optimal direction to decelerate the spacecraft in terms of photon propulsion. A noteworthy fact is that the orientation of the sail moves from the initial point to the optimal point almost linearly, despite the presence of disturbance. This is the result of small time constant for generating the 1st-order static wave. The attitude control by the 2nd-order static wave also works properly based on the fact that the orientation of the sail does not deviate largely from the optimal point.

6. Conclusion

An active shape control method for spinning membrane space structures is proposed. This study achieves the active shape control of a spinning membrane by combining two dynamic phenomena: spinning and vibrations. Since the control input is to pull the tethers using internal actuators, it controls infinite-dimensional dynamical systems using only one-dimensional input. Additionally, this method has better energy efficiency compared to the other shape control methods because it utilizes oscillations; a small control input can excite a large deformation.

A solar sailing method using the shape control method developed is also presented as one application of it. Most of the conventional solar sailing methods are based on changing attitude motion by means of reaction control thrusters or reflectivity control devices. This study provides a new technique for solar sailing that is realized by manipulating the shape of the membrane.

This idea of solar sailing is just one application of active shape control. When a number of thin-film solar cells are attached on the membrane, the shape control will act to arrange the amount of power generation. Attaching phased array antennas on the membrane can form a three-dimensional membrane antenna. The shape control method proposed extends conventional flat membrane space structures to three-dimensional structures. As mentioned above, the method has various applications to future space missions.

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