Zero-bias anomaly in two-dimensional electron layers and multiwall nanotubes

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The zero-bias anomaly in the dependence of the tunneling density of states \(v(\epsilon)\) on the energy \(\epsilon\) of the tunneling particle for two- and one-dimensional multilayered structures is studied. We show that for a ballistic two-dimensional (2D) system the first order interaction correction to DOS due to the plasmon excitations studied by Khveshchenko and Reizer is partly compensated by the contribution of electron-hole pairs which is twice as small and has the opposite sign. For multilayered systems the total correction to the density of states near the Fermi energy has the form \(\delta v/\nu_0 = \max(|\epsilon|, \epsilon^*)/4\epsilon_F\), where \(\epsilon^*\) is the plasmon energy gap of the multilayered 2D system. In the case of one-dimensional conductors we study multiwall nanotubes with the elastic mean free path exceeding the radius of the nanotube. The dependence of the tunneling density of states energy, temperature and on the number of shells is found.

PACS numbers: 73.63.-b, 73.21.Ac, 73.21.Hb, 73.23.Hk

I. INTRODUCTION

Electron-electron interaction results in a singular suppression of the tunneling (single-particle) density of states at the Fermi surface of low-dimensional metallic systems. The effect, known as the zero-bias anomaly, was first discussed by Altshuler and Aronov for diffusive systems with a short range interaction, and by Altshuler, Aronov and Lee for the Coulomb interaction. In the two-dimensional (2D) case the correction to the density of states (DOS) is double-logarithmic, \(\delta v/\nu_0 \sim \ln(|\epsilon|/\epsilon_F) \ln(\Delta/\epsilon_F)\), where \(\epsilon\) is the energy of a tunneling electron, counting from the Fermi energy, and \(\Delta\) is a high-frequency cutoff related to the Coulomb screening, \(1/\tau \ll \Delta \ll \epsilon_F, \epsilon_F\) being the Fermi energy. Zuzin showed that the second logarithm in this formula is cut off at low energies for the experimental setup of a 2D electron plane screened by a metal shield. Rudin, Aleiner and Glazman generalized the theory of zero-bias anomaly to incorporate the ballistic energies \(\epsilon > 1/\tau\), and argued that the correction actually has the form, \(\delta v/\nu_0 \sim -\ln^2(\epsilon_F/|\epsilon|)\). Khveshchenko and Reizer analyzed the contribution of collective electron excitations, 2D plasmons, to the tunneling DOS and obtained an additional correction \(\delta v/\nu_0 = (|\epsilon| - \epsilon_F)/2\epsilon_F\), less singular near the Fermi surface but dominant in the wide range of energies.

In the present paper we consider the interaction correction to the tunneling DOS of a different system, consisting of a number of identical periodically spaced two-dimensional electron layers. We also neglect the possibility of electron tunneling between the layers. While analyzing such a setup we primarily have in mind high-\(T_c\) materials, which are attracting considerable interest with respect to the properties of electron-electron interactions. The electron transport in these materials is extremely anisotropic, and the inter-layer tunneling amplitude in some crystals can be as weak as \(0.05-2\ K\). Even though the motion of electrons tunneling from the tip of the tunneling electron microscope (TEM) is confined to the outermost layer only, the presence of internal layers is important as they participate in the screening of the Coulomb interaction.

We also consider a similar though different tunneling geometry which is realized in multi-wall carbon nanotubes (MWNT). A typical MWNT consists of a few graphite monolayer sheets rolled concentrically into cylinders with radius \(R \sim 10\ \text{nm}\). At zero doping they can be either metals or semiconductors, depending on the helical arrangement of the carbon hexagons. In the measurements of the tunneling DOS the tunneling current propagates through the outermost shell while the inter-shell tunneling is suppressed. Depending on the degree of disorder the transport around the elastic mean free path can be shorter or longer than the radius of the nanotube corresponding to the diffusive \((l < R)\) or ballistic \((R > l)\) motion along the circumference of the nanotube. For energies \(\epsilon\) exceeding the inverse time of propagation around the circumference the zero-bias anomaly is described by 2D formulas. At lower energies a crossover to the regime of a quasi-one dimensional (quasi-1D) conductor is realized.

The zero-bias anomaly in diffusive quasi-1D conductors was first considered by Altshuler and Aronov. The correction to DOS in the lowest order of perturbation theory was found to be more singular than in the 2D case, \(\delta v/\nu_0 \sim -1/\sqrt{|\epsilon|/\epsilon_F}\). Working beyond the perturbation approximation, Nazarov found that close to the Fermi surface DOS has an exponential behavior, \(\ln(\nu/\nu_0) \sim |\epsilon|^{-1}\) (this result was later obtained by Levitov and Shytov in a different way). Ballistic 1D conductors were studied by several authors and were shown to have a power law behavior \(\nu/\nu_0 \sim |\epsilon|^\alpha\) of the tunneling DOS. The crossover between diffusive and ballistic regimes as well as the temperature behavior of DOS in multi-wall carbon nanotubes were recently studied in...
Ref. 14 under the assumption that electrons reside on the outermost shell only.

Here we study the zero-bias anomaly due to dynamically screened inter- and intra-shell Coulomb interaction in two- and one-dimensional layered systems and its effects on the tunneling DOS. In Section II the system of two-dimensional layers is analyzed, with both regimes of diffusive and ballistic in-plane electron motion considered. In Section II we discuss the zero-bias anomaly in multi-wall nanotubes assuming that the doping electrons are distributed uniformly across the shells.

II. SYSTEM OF TWO-DIMENSIONAL ELECTRON LAYERS

Let us consider a semi-infinite system of identical conducting two-dimensional layers separated by the distance $d$ (see the Figure in Appendix). The tunneling electron from TEM propagates within the upper layer, and the inter-layer tunneling is neglected. The properties of an isotropic two-dimensional electron system are described by the in-plane Fermi velocity $v$ and the electron-impurity scattering rate $1/\tau$. The presence of internal layers is important as they contribute to the screening of Coulomb interaction.

The first order perturbation correction to the tunneling DOS (see Refs. 15,16) of the 2D conductor at zero temperature has the form

$$\frac{\delta\nu(\epsilon)}{\nu_0} = \int_{|r|}^{\epsilon_F} d\omega \mathcal{V}(\omega), \quad (2.1)$$

where $\nu_0$ is the thermodynamic two-dimensional density of states $\nu_0 = m/\pi$, counting both spin directions. The electron-impurity vertex function is given by

$$\Gamma^{-1}(\omega, q) = 1 - \frac{i/\tau}{[(\omega + i/\tau)^2 - q^2v^2]^{1/2}}, \quad (2.2)$$

The function $U(\omega, q)$ stands for the Coulomb interaction of two electrons residing in the outermost plane and screened by the infinite number of conducting layers. To find this function, we consider the Coulomb potential $\phi(\omega, r)$ created by the tunneling electron located in the outermost plane $z = 0$. It satisfies the Poisson equation, that has the following form in the Fourier representation with respect to the in-plane coordinates,

$$\left(\frac{d^2}{dz^2} - q^2 + 4\pi e^2\Pi(q, \omega) \sum_{n=0}^{\infty} \delta(z - nd)\right) \phi(\omega, q, z) = 4\pi e \delta(z), \quad (2.4)$$

where the last term in the brackets describes the polarization charge induced in the system of 2D layers, and the polarization operator of a single 2D electron layer is,

$$\Pi(\omega, q) = i\nu_0 \frac{\omega + i/\tau - [(\omega + i/\tau)^2 - q^2v^2]^{1/2}}{[(\omega + i/\tau)^2 - q^2v^2]^{1/2} - i/\tau}. \quad (2.5)$$

For the solution of Eq. (2.4) we refer the reader to the Appendix. We obtain,

$$U(\omega, q) \equiv -e\phi(\omega, q, 0) = \frac{4\pi e^2 \sinh qd}{q(e^{kd} - e^{-qd})}, \quad (2.6)$$

where $k$ is given by the solution of the equation

$$\cosh kd = \frac{qd - K(\omega, q) \sinh qd}{K(\omega, q)}, \quad (2.7)$$

having a non-negative real part $\Re k \geq 0$. Here $K(\omega, q) = 2\pi e^2\Pi(q, \omega)/q$ is the factor describing the dynamical screening of an external charge by a single metallic layer. When the wavelength decreases $qd \to \infty$, Eq. (2.6) gives

$$U(\omega, q) = \frac{2\pi e^2}{q - 2\pi e^2\Pi(q, \omega)}, \quad (2.8)$$

and the conventional expression for the screened interaction in a single 2D layer is recovered. For static interactions Eq. (2.3) gives $U(\omega \to 0, q) = 2\pi e^2/(q + \kappa)$, where $\kappa = 2\pi e^2\nu_0$ is the inverse static screening length.

In what follows we assume that $kd \gg 1$. This condition ensures that different layers are (at least for low frequencies) weakly coupled. In the opposite limit $kd \ll 1$ the system could be treated effectively as the 3d metal with the cylindrical Fermi surface. This regime is beyond the scope of our paper.

A. Ballistic motion

First we consider the ballistic limit when the tunneling bias exceeds the scattering rate $\epsilon \gg 1/\tau$. The main contribution to the integral in Eq. (2.2) comes from two regions: 1) high-frequency plasmon region, $qv \ll \omega$; 2) particle-hole continuum $qv > \omega$.

1) Using the high frequency ($qv \ll \omega$) approximation for the polarization operator Eq. (2.3), $\Pi(q, \omega) = \nu_0 q^2v^2/2\Omega^2$, where we introduced the notation $\Omega^2 = \omega(\omega + i/\tau)$, and solving equation (2.7) with respect to $e^{kd}$, we get

$$\mathcal{V}_1(\omega) = \frac{1}{4\pi e^2\omega^2} \int_0^{q_0} dq \left( e^{2qd} - 1 \right) \times \left( \Omega^2 - \omega \Omega - \sqrt{\Omega^2 - \omega^2} \right). \quad (2.9)$$

Here $q_0(q)$ represents the upper/lower boundaries of the plasmon continuum (of an infinite system of layers $z = 0, \pm 1, \pm 2, ...$) that can be parametrized by the wavenumber $q_\perp$ in the $z-$direction,
\[ \omega_\pm^2(q) = \frac{\kappa q^2}{2} \left\{ \coth \frac{qd}{2} \right\} \]  
\hspace{1cm} (2.10)

The mode \( \omega_+(q) \) corresponds to the uniform charge distribution across all layers, \( q_\perp = 0 \), while the other mode, \( \omega_-(q) \), describes the alternating charge in adjacent planes (\( q_\perp = \pi/d \)). The former mode has a frequency gap \( \epsilon^* = v(\kappa/d)^{1/2} \) and at \( qd \to 0 \) gives the usual three-dimensional plasmon (in anisotropic metal). At \( qd \to \infty \), both branches tend to the usual plasmon spectrum of a two-dimensional electron gas. This is illustrated by Fig. 1.

![Plasmon spectrum](image)

**FIG. 1.** The plasmon spectrum of an infinite number of 2D metallic layers is shown. Dashed line shows the plasmon spectrum of a single 2D layer. Plasmons of different layers interact with each other thus creating a band (inner area between solid lines). The upper solid line \( \omega = \omega_+(q) \) represents the upper boundary \( \omega = \omega_+(q) \) of the plasmon continuum, with the uniform charge distribution across the layers \( q_\perp = 0 \) while the lower solid line marks the lower boundary corresponding to the alternating charge in adjacent layers \( q_\perp = \pi/d \).

We chose the momentum cutoff \( q_* \) in Eq. (2.9) to be larger than the characteristic momenta of plasmon modes but still less than the momentum of particle-hole excitations, i.e. \( q_* \ll \omega/v \).

When the frequency is high \( \omega > \epsilon^* \), the integral over the momentum is dominated by large values of \( q > 1/d \) where \( \omega_+ \) and \( \omega_- \) converge exponentially, \( \omega_\pm^2 = \kappa q^2 [\frac{1}{2} \mp \exp(-qd)] \). Simple calculations give,

\[ \mathcal{V}_1(\omega) = -\frac{1}{4\epsilon_F} - \frac{1}{2\pi\epsilon_F \omega \tau} \ln \frac{\kappa q \omega}{\omega^2}. \]  
\hspace{1cm} (2.11)

2) High transferred momenta \( q \sim \omega/v \gg q_* \) present the contribution of particle-hole pairs. Under the condition \( \kappa d \gg 1 \), the second term in the right-hand side of Eq. (2.7) always exceeds the first term (overscreened interaction) and the interaction takes the form,

\[ U(\omega, q) = \frac{1}{\nu_0} \sqrt{\left(\omega + i/\tau\right)^2 - q^2 v^2} - \frac{i/\tau}{\sqrt{\left(\omega + i/\tau\right)^2 - q^2 v^2} - \omega - i/\tau} \]  
\hspace{1cm} (2.12)

Substituting Eqs. (2.12) and (2.3) into Eq. (2.2) and integrating from \( q_* \) to infinity we find

\[ \mathcal{V}_2(\omega) = \frac{1}{4\epsilon_F} - \frac{1}{2\pi\epsilon_F \omega \tau} \ln \frac{\kappa q \omega}{\omega^2} \]  
\hspace{1cm} (2.13)

By adding this expression to the plasmon contribution \( \mathcal{V}_1(\omega) \) we obtain the following expression for the total spectral weight function,

\[ \mathcal{V}(\omega) = \mathcal{V}_1(\omega) + \mathcal{V}_2(\omega) = -\frac{1}{4\epsilon_F} - \frac{1}{2\pi\epsilon_F \omega \tau} \ln \frac{\kappa q \omega}{\omega^2} \]  
\hspace{1cm} (2.14)

The results (2.11) and (2.14) hold as long as the frequency exceeds the plasmon gap \( \epsilon^* \). In this large frequency and large wave number \( qd > 1 \) region the plasmons have virtually no dispersion along the z-direction due to weak interaction of electron densities on different layers. At lower frequencies and wave numbers, \( \omega < \epsilon^* \) and \( qd < 1 \), electron densities on different layers interact strongly, and the plasmon spectrum acquires a significant dispersion along the z-direction. In this region we may approximate \( \omega_\pm^2 = \kappa q^2 \pi^2/4, \omega_\pm^2 = \kappa v^2/d, \) \( \exp(2qd) = 1 + 2qd \), and take the inverse interlayer distance as the momentum cutoff \( q_* \sim 2/d \). Taking the integral in Eq. (2.2) explicitly and extracting the imaginary part, we find instead of Eq. (2.11),

\[ \mathcal{V}_1(\omega) = -\frac{1}{4\epsilon_F} - \frac{1}{2\pi\epsilon_F \omega \tau} \ln \frac{\epsilon^*}{\omega^2}. \]  
\hspace{1cm} (2.15)

The high-momentum (particle-hole) contribution does not depend (as long as \( \kappa d \gg 1 \)) on the distance between layers. This can be readily seen from Eq. (2.12) that does not contain \( d \). This is quite natural as the particle-hole pairs do not induce long-range oscillations of electric field. Therefore, we can take the old expression (2.13) but with the new cutoff \( q_* \sim 2/d \), and obtain

\[ \mathcal{V}(\omega) = -\frac{1}{2\pi\epsilon_F \omega \tau} \ln \frac{\kappa v}{\epsilon^*}, \quad \omega < \epsilon^*. \]  
\hspace{1cm} (2.16)

Integrating the expressions (2.14) and (2.16) with the frequency according to Eq. (2.1), we found the perturbation correction to the tunneling DOS in the ballistic regime

\[ \frac{\delta \nu(\epsilon)}{\nu_0} = \max(|\epsilon|, \epsilon^*) - \epsilon^* \]  
\hspace{1cm} (2.17)

\[ \frac{1}{4\epsilon_F} \ln \left[ \frac{\kappa v}{\max(|\epsilon|, \epsilon^*)} \right] \ln \left[ \frac{\kappa v}{\epsilon^*} \right] \]  
\hspace{1cm} (2.18)

We observe that the correction (2.17) is less singular near the Fermi surface than in the case of a single 2D layer.

Let us point out that in the limit \( d \to \infty \) our result differs from the formula of Khveshchenko and Reizer as the leading contribution (the first term of Eq. (2.17)) is twice as small as their result. The difference is due to the contribution from particle-hole excitations, \( \mathcal{V}_2(\omega) \) in Eq. (2.14).
B. Diffusive motion

Let us now address the regime of strong impurity scattering, \( \epsilon \ll 1/\tau \), when the motion of the tunneling electron is diffusive and the polarization operator has the form, \( \Pi(\omega, q) = -\nu_0 D q^2 / (D q^2 - i \omega) \), here \( D = v^2 \tau / 2 \) is the diffusion coefficient.

When \( \omega > \epsilon = D \kappa / d \), the main contribution to the integral of \( \omega^2 q \) comes from the region of transferred momenta \( \omega / D \kappa \sim q < \sqrt{\omega / D} \), while for \( \omega < \epsilon \) the principal contribution is due to \( \omega / D \kappa d < q < \sqrt{\omega / D} \). In this region the interaction takes the form

\[
U(\omega, q) = -\frac{i \omega}{\nu_0 D q^2},
\]

and we obtain

\[
\mathcal{V}(\omega) = -\frac{1}{4\pi^2 \nu_0 \omega} \ln \frac{D \kappa^2}{\max(\omega, \epsilon)}.
\]

For a sufficiently dirty system \( \epsilon \tau < 1 \). In this case integrating over the frequency we obtain for the diffusive DOS correction at \( |\epsilon| > \epsilon \),

\[
\frac{\delta \nu_d(\epsilon)}{\nu_0} = \frac{1}{8\pi^2 \nu_0 D} \ln (D^2 \kappa^4 \tau / |\epsilon|) \ln (|\epsilon| \tau) + \frac{\delta \nu_0(1/\tau)}{\nu_0},
\]

where the first term is the well-known double logarithmic correction of Altshuler, Aronov, and Lee and the second term is the constant contribution of ballistic frequencies \( \omega > 1/\tau \) discussed in the Section 2.1.

For \( |\epsilon| < \epsilon \), we have instead of Eq. (2.18)

\[
\frac{\delta \nu_d(\epsilon)}{\nu_0} = \frac{1}{4\pi^2 \nu_0 D} \left( \ln (\kappa d) \ln (|\epsilon| \tau) + \frac{\ln \epsilon \tau}{2} \right) + \frac{\delta \nu_0(1/\tau)}{\nu_0},
\]

and the first logarithm in Eq. (2.19) is cut off. For a relatively clean system with \( \epsilon \tau > 1 \) (i.e., such that \( l^2 > d / \kappa \)) the second term in the brackets in this equation is absent.

Comparing Eq. (2.20) with the result by Zuzin for the DOS correction for a 2D electron system screened by a bulk material, we can observe that in the diffusive regime at low energies the role of an infinite set of 2D electron layers \( z = d, 2d, 3d \ldots \) is equivalent to a bulk metal screen located at a distance \( d / 2 \) from the outermost plane.

C. Interlayer tunneling

Throughout the analysis performed above we disregarded completely all possible tunneling transitions between different layers. We now consider the corrections to the density of states originating from interlayer tunneling and establish conditions when such processes could be neglected. We treat them perturbatively, in the lowest order in the tunneling Hamiltonian,

\[
\dot{\mathcal{H}}_t = \frac{t}{2} \sum_i \int \frac{d^2 p}{(2\pi)^2} [\hat{\psi}^\dagger_{i+1}(p) + \hat{\psi}^\dagger_{i-1}(q)] \hat{\psi}_i(p),
\]

which conserves the in-plane momentum during tunneling. Having in mind mainly applications to the system with strong in-plane scattering (like high-\( T_c \) materials) we focus here on the diffusive transport regime \( \epsilon \ll 1/\tau \).

The corrections to the density of states to the lowest order in the inter-layer tunneling amplitude \( t \) are shown in Fig. 2. Of the six diagrams drawn here the more important ones are the diagrams e) and f) that contain four diffusons each. They correspond to the tunneling corrections to the screened Coulomb interaction rather than to corrections to Green’s function of the tunneling electron itself, which are given by the diagrams a) – d).

![Diagram](image)

**FIG. 2.** The diagrammatic representation of the correction to the single-particle density of states from the inter-layer tunneling processes to the lowest order of perturbation theory in the tunneling amplitude \( t \). The electron from the microscope probe tunnels into white circles. Solid lines stand for the Green’s functions, wavy lines denote the dynamically screened Coulomb interaction, black dots represent the tunneling matrix elements, and the dashed lines are the impurity ladders (diffusons).

To compute these diagrams one has to know the interaction \( U_{nm}(\omega, q) \) between electrons residing on arbitrary layers \( n \) and \( m \). As finding such a general expression seems to be quite a cumbersome task for a semi-infinite system of layers, we utilize the corresponding expressions for the infinite system to obtain a qualitative estimate of the effect. It has the form (see Appendix),
The diagrams e) and f) are computed to yield for the tunneling correction,
\[ V_I(\omega) = t^2 v_0 \omega \tau \mathbb{R} \int \frac{dq}{2\pi^2} \frac{U_{00}^2(q, \omega) \tanh kd/2}{(Dq^2 - i\omega)^4}. \]  
(2.23)

If the frequency is large \( \omega > \tilde{\epsilon} \), the main contribution to the integral in Eq. (2.23) arrives from the interval \( 1/d < q < \omega/\kappa D \), where we can approximate \( U_{00} \tanh kd/2 \approx k^2/\nu_0^2 q^2 \), to obtain
\[ \frac{\delta \nu(I)}{\nu_0} = \frac{e^4 t^2 v_0 \tau}{2\pi\epsilon^2} \ln |\epsilon|/\epsilon. \]  
(2.24)

For smaller frequencies \( \omega < \tilde{\epsilon} \), the leading contribution comes from the integral \( 0 < q < 1/d \) and reads,
\[ \frac{\delta \nu(I)}{\nu_0} = -\frac{e^4 t^2 v_0 \tau}{2\pi\epsilon} \frac{1}{|\epsilon|}. \]  
(2.25)

It is worth noting that the inter-layer correction changes its sign around the point \( |\epsilon| \sim \tilde{\epsilon} \) and in fact lead to increase in the density of states for \( |\epsilon| > \tilde{\epsilon} \). Comparing the expressions (2.24) and (2.25) with the above formulas (2.19) and (2.21) we observe that inter-layer tunneling correction is small provided that \( t \ll |\epsilon|/(\epsilon_F \tau) \). This much stronger condition than the one that might be naively expected \((t \ll |\epsilon|)\) is due to the enhancement of the electron-electron interaction by the inter-layer tunneling. When this condition is violated the summation of a wider class of diagrams with all possible transitions of diagrams is necessary.

### III. MULTIWALL NANOTUBES

In this section we consider the interaction suppression of tunneling DOS in multiwall carbon nanotubes. A multiwall nanotube is built up of \( M \) concentric carbon tubes (or shells) each of which can be obtained by rolling a graphite sheet into a cylinder.

In tunneling experiments the tunneling current is believed to propagate along the outermost shell only while the tunneling between the shells is largely suppressed. Thus the role of electrons in the inner shells is reduced to merely screening the interaction between the outer shell electrons.

The electron band structure of a single metallic carbon nanotube consists of \( N \) (this number is usually of the order of a few tens and depends on the doping level and gate voltage) conducting sub-bands \( \epsilon_{n}(k) \) characterized by the Fermi velocities along the tube axis \( v_n \) and around the circumference \( v_{n1} \). Each of the \( M \) shells has, in general, its own band structure \( \epsilon_{n}(k) \). The electrons are scattered between different bands (but mostly within the same tube) by impurities, lattice imperfections and incommensurate potential of neighboring tubes. Not all the tubes are necessarily metallic, some of them could well be insulating. Experimental evidence of the internal structure of MWNT is usually not easy to obtain and leaves some room for speculation. In our previous paper we studied the scenario of the dopants residing in the outermost shell only, thus neglecting possible interaction with electrons from inner shells. In the present paper we study a different aspect of the problem concentrating on the effects of the finite number \( M \) of conducting shells on the tunneling DOS of a MWNT. To simplify the problem we consider an approximation in which all \( M \) shells have the same band structure and the same doping level.

Unlike the case of the 2D conductors considered above, in the one-dimensional wires the contribution of plasmon frequencies \( \omega \gg qv \), logarithmically exceeds that of the electron-hole interval \( \omega < qv \). Therefore the weight function can be written in a simple form (compare this with Eqs. (2.22) and (2.23)),
\[ V(\omega) = \frac{1}{\omega^3} \sum_{m=-\infty}^{+\infty} \int dq \frac{2\pi^2 U_{00}(\omega, q, q_m)}{2\pi^2}, \]  
(3.1)

where the sum is taken over the quantized transverse momentum \( q_m = n/R_0, n = 0, \pm 1, \pm 2... \). The function \( U_{00}(\omega, q, q_m) \) should be understood as the 00 element of the matrix \( U_{ij}(\omega, q) \) which represents the dynamically screened Coulomb interaction of an electron in shell \( i \) with an electron in shell \( j \) and satisfies the matrix equation
\[ \hat{U}(\omega, q) = \hat{V}(q) + \hat{\Pi}(\omega, q) \hat{\Pi}(\omega, q) \hat{U}(\omega, q), \]  
(3.2)

where \( \Pi_{ij}(\omega, q, q_m) \) is the polarization operator which, according to our assumptions, is proportional to the unit matrix
\[ \Pi_{ij}(\omega, q, q_m) = \delta_{ij} \Pi = \delta_{ij} \nu_1 \frac{q^2 v_{n1}^2 + q^2 v_m^2}{\omega(\omega + i/\tau)}, \]  
(3.3)

here we introduced the one-dimensional density of states \( \nu_1 = \sum q (\pi v_n)^{-1} \) and the average squares of the longitudinal, \( v_{n1}^2 \) and the transverse, \( v_m^2 \) electron velocities,
\[ v_{n1}^2 = \frac{\sum_n v_n^2}{\sum_n v_n}, \quad v_m^2 = \frac{\sum_n v_n^{-1} v_n^2}{\sum_n v_n^{-1}}. \]

In Eq. (3.2) \( V_{ij}(q, m) \) denotes the bare Coulomb potential,
\[ V_{ij}(q, m) = \frac{2e^2}{\pi} \int_0^{\pi} d\phi K_0(q R_i) \cos m \phi. \]  
(3.4)

In this equation we introduced the notations \( R_{ik}(\phi) = R_k^2 + R_i^2 - 2 R_i R_k \cos \phi \), with \( R_i \) being the radii of concentric shells forming the MWNT \((i = 0 \text{ is the external shell, } i = M - 1 \text{ the innermost shell})\).
To proceed further with Eqs. (3.2, 3.4) we assume that the radius of the \( j \)-th shell is a linear function of its number \( R_j = R_0(1 - j\xi) \). In the long-range limit \( qR_0 \ll 1 \), we obtain from Eq. (3.4) for the bare interaction,

\[
\frac{V_{ij}(q, q_m)}{e^2} = \left\{ \begin{array}{ll}
\frac{1}{m} \left( \frac{1 - \xi \max(i, j)}{1 - \xi \min(i, j)} \right)^{|m|}, & m \neq 0, \\
\ln \left( \frac{\beta}{qR_0} \right) + 2\xi \min(i, j), & m = 0.
\end{array} \right.
\]

Here \( \beta = 4e^{-2C} \approx 1.26 \), with \( C \) being the Euler constant. While the second line in Eq. (3.3) holds provided \( \xi \ll 1 \), the first line utilizes only the approximation \( qR_0 \ll 1 \).

We use the set of eigenvectors \( w_i^{(k)}(q) \) and eigenvalues \( V_k(q, m) \) of the bare interaction matrix (3.4) to write the screened Coulomb interaction in the form

\[
U_{ij}(\omega, q, q_m) = \sum_k \frac{w_i^{(k)}(m)w_j^{(k)}(m)}{V_k^{-1}(q, q_m) - \Pi(\omega, q, q_m)}. \tag{3.6}
\]

The eigenvalues \( V_k(q, m) \) determine the spectrum of collective plasmon excitations \( \omega_i^{(k)}(q) \) through the poles of the interaction \( U_{ij}(\omega, q, q_m) \) in Eq. (3.6), and the eigenvectors \( w_i^{(k)}(m) \) determine the distribution of charge between different shells in these plasmon oscillations. In the limit \( qR_0 \gg 1 \) one can approximate sum with the integral and recover the usual two-dimensional Coulomb potential (2.3).

Typically, MWNTs exhibit ballistic transport around the circumference \( l = v\tau \sim 10 - 100 \text{ nm} > R \). At energies \( \epsilon \) below the Thouless energy \( v/R \) for the transport around the circumference the contribution of the \( m \neq 0 \) terms is non-singular due to the gap in their spectrum and is only weakly dependent on \( \epsilon \). The \( m = 0 \) plasmons, on the other hand, are gapless. Their contribution to Eq. (3.1) continues to depend on \( \epsilon \) and has a singularity at \( \epsilon \rightarrow 0 \). Therefore, concentrating on the low energy \( \epsilon < v/R \) dependence of DOS we can neglect the non-singular contribution of the \( m \neq 0 \) plasmons and retain only the \( m = 0 \) term in Eq. (3.1).

The index \( k \) labels plasmon modes and equals the number of nodes in the charge distribution in the \( k \)-th plasmon across the section of a multi-shell nanotube. The mode \( k = 0 \) is characterized by the uniform distribution of the oscillating charge and corresponds to the logarithmic eigenvalue, \( V_0(q) \simeq e^2M\ln qR_0^2/\pi^2 \), with \( w_0^{(0)} \simeq 1/\sqrt{M} \). All other \( M - 1 \) modes correspond to \( q \)-independent eigenvalues of the bare interaction, \( V_k = 2\xi e^2\alpha_k \) and therefore have sound-like spectrum. The coefficients \( \alpha_k \) are to be computed numerically. They range roughly from a few tenths to a few units, e.g., for \( M = 10 \) we obtain \( \alpha_k = 10.0; 2.6; 1.2; 0.72; 0.50; 0.38; 0.31; 0.28; 0.26 \). The components \( w_0^{(0)} \) of the eigenvectors are also computed numerically. As the eigenvalues of the matrix (3.4) depend (at most) only logarithmically on the momentum \( q \), we can perform the integral in Eq. (3.1) taking these eigenvalues at characteristic plasmon momenta, \( q^2 \sim \omega(\omega + i/\tau)/Nv^2 \),

\[
\mathcal{V}(\omega) = -\frac{q}{N} f_M(\omega) \frac{\sqrt{\omega + i/\tau}}{\omega^{1/2}}, \tag{3.7}
\]

where we defined the average Fermi velocity \( \bar{v} = \sum_n v_n/N \) and the dimensionless coupling strength \( g = e^2/(2\pi\bar{v}) \) in a single channel. The function \( f_M(\epsilon) \) is given by the 00-element of the matrix \( \mathcal{V}^{1/2} \),

\[
f_M(\omega) = \sum_k (\omega_{0}^{(k)})^2 \sqrt{V_k/2e^2} = \frac{1}{M} \ln^{1/2} \frac{\sqrt{NM\bar{v}}}{R\sqrt{[\omega(\omega + i/\tau^{-1})]} + \gamma_M \sqrt{\xi}}, \tag{3.8}
\]

the last term representing the contribution of the \( M - 1 \) sound-like plasmons, with values of \( \gamma_M \) given in the table.

| \( M \) | 1 | 2 | 3 | 5 | 10 |
|-------|---|---|---|---|----|
| \( \gamma_M \) | 0 | 0.35 | 0.60 | 0.84 | 1.38 |

The contribution of the \( k = 0 \) plasmon decreases with the number of tubes \( M \) due to the screening by internal shells, while the contribution of sound-like plasmons increases roughly linearly with \( M \). Since the number of shells may not exceed \( 1/\xi \) the second term in Eq. (3.8) is never greater than the first one, and at most becomes comparable to it at \( M \approx 1/\xi \). The approximation \( \omega > qv \) used above assumes that the case of strong interaction is realized, \( \xi\alpha_k \gg 4e^2\bar{v}_0 \sim 1/4N \). In the opposite limit \( \xi\alpha_k < 1/4N \), the contribution of the \( k - \)th plasmon mode is weak and may be treated perturbatively.

Substituting the expression (3.7) into Eq. (3.1) and evaluating the frequency integral we obtain the following perturbation correction to the tunneling DOS of an \( M \)-shell multi-wall nanotube,

\[
\frac{\delta v_M(\epsilon)}{v_1} = -\frac{q}{N} f_M(\epsilon) \frac{\sqrt{[\omega(\omega + i/\tau^{-1})]} + \gamma_M \sqrt{\xi}}{R} \times \left\{ \begin{array}{ll}
\left( \frac{\sqrt{v}f_M(\epsilon)}{v_1} \right) + f_M(\tau^{-1}) \ln \frac{\sqrt{N\tau^2}}{R}, & |\epsilon| < 1/\tau, \\
\frac{f_M(\epsilon)}{\sqrt{\tau^2}}, & |\epsilon| > 1/\tau. \end{array} \right. \tag{3.9}
\]

We now discuss how the results obtained for multi-wall nanotubes could be extended to the very vicinity of the Fermi surface where the perturbation correction (3.9) diverges and the nonperturbative approach is required. This question was addressed for a single-wall metallic nanotube in Ref. [14] that utilized the phase approximation first proposed by Nazarov [15] and further developed in Ref. [14]. Here we concentrate on the dependence of the DOS on the number of shells \( M \) skipping the derivation that could be found in Ref. [14]. The result for the ballistic regime, \( |\epsilon| > 1/\tau \) mainly utilizes the plain exponentiated perturbative result (3.9).
\[ \nu(\epsilon) \propto \exp \left\{ -\sqrt{\frac{g}{N}} \ln \frac{\bar{\nu}}{R|\epsilon|} \left( \frac{1}{\sqrt{M}} \ln^{1/2} \frac{\bar{\nu}}{R|\epsilon|} + \gamma_M \sqrt{\xi} \right) \right\}. \]

(3.10)

The diffusive regime of lower energies, \(|\epsilon| < 1/\tau\), is more tricky. We obtain,

\[ \nu(\epsilon) \propto \exp \left\{ -\frac{\epsilon_0}{|\epsilon|} \left( \frac{1}{\sqrt{M}} \ln^{1/2} \frac{\bar{\nu}}{R|\epsilon|} + \gamma_M \sqrt{\xi} \right)^2 \right\}, \]

(3.11)

where \(\epsilon_0 = \pi g/2N\tau\). This result holds provided that the argument of an exponent is large and the temperature small enough \(T < \epsilon^2/\epsilon_0\). For higher temperatures \(\epsilon^2/\epsilon_0 < T < \epsilon\) the tunneling DOS acquires the form,

\[ \nu(T) \propto \exp \left\{ -1.07 \sqrt{\frac{\epsilon_0}{T}} \left( \frac{1}{\sqrt{M}} \ln^{1/2} \frac{\bar{\nu}}{R\sqrt{T\epsilon_0}} + \gamma_M \sqrt{\xi} \right)^2 \right\}. \]

(3.12)

The crossover between the low-temperature (3.11) and the high-temperature (3.12) regimes can be shown to depend on the energy through the dimensionless variable \(|\epsilon|/\sqrt{\epsilon_0 T}\) only. We omit the derivation of the corresponding scaling function which is quite straightforward and follows the same route as in the case when logarithms in the exponents are screened off by the presence of a metal shield, see Ref. [4].

IV. DISCUSSION

We considered the zero-bias anomaly in in the tunneling density of states in layered 2D and quasi-1D materials with dynamically screened Coulomb interaction. The theory presented above is applicable to high-T\(_c\) materials and semiconductor heterostructures as well as to the multi-wall carbon nanotubes. We showed that the presence of many conducting shells in 2D systems weakens the singularity in the DOS correction at the Fermi level: for energies below \(\epsilon^* = \kappa D/d\) in diffusive systems or below the plasmon gap \(\nu(\kappa/d)^{1/2}\) in ballistic systems the DOS correction becomes logarithmic \(\propto \ln |\epsilon|\), rather than double-logarithmic \(\propto \ln^2 |\epsilon|\) as in the case of a single layer.

In addition to the logarithmic terms mentioned above that originate from the broad region of the transferred frequency and momentum there are also terms coming from the regions around the plasmon and particle-hole singularities. These contribute correction to DOS \(\max(|\epsilon|, \epsilon^*)/4\epsilon_F\) that is not singular but dominant in the wide range of energies \(|\epsilon| > \ln(\epsilon_F \tau)/\tau\). The contribution of the particle-hole continuum is especially interesting in 2D ballistic systems. It turns out that the contribution of the particle-hole continuum to the DOS correction, which corresponds to the region \(\omega \leq qv\) in the interaction frequency/momentum transfer, of the same order as the contribution originating from the plasmon pole but has a different sign. Namely, the particle-hole continuum increases the density of states. The total effect is still a suppression of DOS for low energies as the plasmon contribution is always larger than that of the particle-hole continuum (twice as large for a single 2D layer).

In derivation of DOS in 2D systems we assumed that the ground state is the free electron model. It is possible, however, to incorporate also the Fermi liquid interaction in the calculations of the tunneling DOS. In particular, the Fermi liquid theory predicts that thermodynamic density of states is renormalized \(\nu_0 \to Z\nu_0\) by a Z-factor (0 \(< Z < 1\)) describing a step in the quasiparticle partition function. We obtain however that the relative correction \(\delta\nu/\nu_0 = |\epsilon|/4\epsilon_F\) due to Coulomb interaction is in fact universal as it is independent of the Fermi liquid interaction strength \(Z\).

The validity condition for neglecting the inter-shell tunneling was found for the diffusive regime. It was shown that instead of the naively expected requirement of the tunneling time \(t^{-1}\) being greater than the characteristic measurement time \(\epsilon^{-1}\), the much more stringent condition \(t^{-1} \gg \epsilon_F T/\epsilon \gg \epsilon^{-1}\) must be satisfied.

With respect to multi-wall carbon nanotubes our analysis was concentrated on the dependence of DOS on the number \(M\) of coaxial shells for both the regimes of perturbative and strong suppression. We found that this dependence can be described by a single function \(\gamma_M\) found numerically. The energy and temperature dependence of DOS found in Ref. [4] for a nanotube with a single conducting shell is preserved for an arbitrary number of shells.

V. ACKNOWLEDGEMENT

The authors are grateful to I. Beloborodov, A. Chubukov, L. Glazman, A. Larkin, M. Pustilnik and M. Turlakov for valuable discussions. We thank K. Efetov for warm hospitality at Bochum University where part of this work was performed. This research was sponsored by the Grants DMR-9984002 and BSF-9800338 and by the A.P. Sloan and the Packard Foundations. E. M. also thanks the Russian Foundation for Basic Research, Grant 01-02-16211.

APPENDIX A: SOLUTION OF THE POISSON EQUATION

To solve the Poisson equation (2.5) we first derive the Green function for the infinite problem,
having a positive real part, \( \text{Re} \, k \) be written in the conventional form

\[
\left( \frac{d^2}{dz^2} - q^2 + 4\pi e^2 \Pi \sum_{n=-\infty}^{\infty} \delta(z - nd) \right) G(z, z') = \delta(z - z'),
\]

(A1)

with the boundary condition \( G(z, z') \to 0 \) when \( |z - z'| \to \infty \). The homogeneous solutions of Eq. (A1) are easily written in terms of quasiperiodic Bloch functions,

\[
\psi_{\pm}(z) = e^{\pm k nd} \left( \sinh[q(z - (n + 1)d)] - e^{\pm kd} \sinh[q(z - nd)] \right), \quad nd < z < (n + 1)d \quad \text{(A2)}
\]

with \( k \) given by the solution of the equation

\[
cosh kd = \cosh qd - \frac{2\pi e^2 \Pi}{q} \sinh qd \quad \text{(A3)}
\]

having a positive real part, \( \text{Re} \, k \geq 0 \). This choice means that the function \( \psi_-(z) \) decreases and the function \( \psi_+(z) \) increases with \( z \) increasing. The Green function can then be written in the conventional form

\[
G(z, z') = -\frac{1}{2q \sinh qd \sinh kd} \begin{cases} 
\psi_-(z)\psi_+(z'), & z > z', \\
\psi_+(z)\psi_-(z'), & z < z'.
\end{cases}
\]

(A4)

To find the solution for the semi-infinite problem, Eq. (2.4), we impose a fictitious charge \( Q \) located at \( z = -d/2 \) (see Fig. 3).

![FIG. 3. The crystal consisting of a semi-infinite system of 2D metallic layers (\( z = 0, d, 2d, \ldots \)) is shown. The thin solid line at \( z = -d/2 \) hosts a fictitious (image) charge \( Q \) that accounts for the absent \( z = -d, -2d, \ldots \) layers.](image)

This charge has to be found from the condition that the total potential of the electronic charge \(-e\) and the fictitious charge \( Q \)

\[
\phi(z) = 4\pi [e \, G(z, 0) - Q \, G(z, -d/2)]
\]

(A5)

contains only the exponent \( e^{qz} \) for the negative values of the coordinate, \(-d/2 < z < 0\).

This condition ensures that the electric field in the outside region \( z < 0 \) decays exponentially with the distance from the outermost plane, with the fictitious charge, therefore, taking care of the absent \( z = -dn, \ n = 1, 2, \ldots \) planes. We obtain the following value of the fictitious charge

\[
Q = e \frac{\psi_-'(0)}{\psi_+(-d/2)} e^{kd} - e^{qd} \quad \text{(A6)}
\]

Substituting Eq. (A6) into Eq. (A3) and making use of Eqs. (A3) and (A4) we obtain the final expression (2.6).

In Section II C the formula for interaction \( U_{nm} \) of two electrons residing at different layers \( z = nd \) and \( z = md \) in an infinite system is used. To obtain it one can write \( U_{nm} = -4\pi e^2 G(nd, md) \), which leads to the expression (2.22).

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