Abstract

It is experienced that auxiliary information when suitably incorporated yields more efficient and precise estimates. Mishra et al. (2017) have introduced a log type estimator for estimating unknown population mean using ancillary information in simple random sampling. Here we propose an improved log-product type estimator for population variance under double sampling. Properties of the estimators are studied both mathematically and numerically.

Key Words: Bias, Mean Square Error, Auxiliary information, Double Sampling, Unbiased Estimator.

1. Introduction

Estimating population variance in finite sampling is an important issue in survey sampling. Also, it is known and established actuality that appropriate use of supplementary or auxiliary information leads to considerable increase in the efficiency of estimates of the parameters of an estimator. Ratio and regression methods are commonly employed in improving efficiency of estimates. In order to obtain more precise estimates researchers have utilized distinguishing forms of auxiliary information. To measure variability within y values (study variable) the problem of estimation of finite population variance has seized considerable importance in survey sampling. Several authors including Das and Tripathi (1978), Srivastava and Jhajj (1980), Misra (2016), Isakii(1983), Singh et.al (2001), Kadilar and Cingi (2006), Singh and Malik (2014), Singh et al. (2014), Sharma and Singh (2014), Mishra and Singh (2016), Sharma et al. (2018), Adichwal et al. (2016), Bandopadhayya and Singh (2015) and others have suggested improved variance estimator using auxiliary information. Consider a situation where no prior information on auxiliary variable is available, in such situations initially we select a large sample from population for obtaining auxiliary information only and then select a second sample from the selected large sample in which variable of interest(y) is observed in addition to auxiliary information. Above procedure of drawing sample is referred to as double sampling scheme. In double sampling Singh and Singh (2003), Ahmed et al. (2003), Jhajj et al. (2005), Jhajj
et al. (2011), Grover (2010, 2011), Jararha et al. (2002), Singh (1991), Giancarlo et al. (2004) had proposed estimators for population variance. Here, motivated by Mishra et al. (2017), we have suggested an improved estimator for variance and studied its properties.

1.1 Notations

Let $U$ be a finite population of size $N$ from which we draw a sample of size $n$ using SRSWOR. Let $Y_i$ and $X_i$ denote the respective values of variable $y$ and $x$ on the $i$th $(i=1,2,\ldots,N)$ unit of the population. Denoting,

$$S_Y^2 = \frac{1}{N-1} \sum_{i=1}^{N} (Y_i - \bar{Y})^2, \quad S_X^2 = \frac{1}{N-1} \sum_{i=1}^{N} (X_i - \bar{X})^2,$$

$$s_Y^2 = \frac{1}{n-1} \sum_{i=1}^{n} (y_i - \bar{y})^2, \quad s_X^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2, \quad s_Y^2 = \frac{1}{n'-1} \sum_{i=1}^{n'} (x_i - \bar{x})^2,$$

$$\lambda = \left( \frac{1}{n} - \frac{1}{N} \right), \quad \lambda' = \left( \frac{1}{n} - \frac{1}{N} \right)$$

where, $s_Y^2$ is an unbiased estimator of $S_Y^2$ and denotes the sample variance of variable $y$ based on sample of size $n$ and $s_X^2$ and $s_X^2'$ are unbiased estimator of $S_X^2$ and denote the sample variance of variable $x$ based on second phase sample of size $n$ and first phase sample of size $n'$ respectively.

Let, $\mu_{pq} = \frac{1}{N} \sum_{i=1}^{N} (y_i - \bar{Y})^p (x_i - \bar{X})^q$ where, $(\bar{Y}, \bar{X})$ denote the population means of $(y, x)$.

Let $\lambda_{pq} = \frac{\mu_{pq}}{\mu_p^2 \mu_q^2}$ and taking

$$\beta_{2y} = \lambda_{40} \beta_{2x} = \lambda_{04}, \quad \beta_{2y} = \lambda_{40} - 1, \quad \beta_{2x} = \lambda_{04} - 1, \quad \lambda_{22} = \lambda_{22} - 1$$

Let $\rho_{yx}$ be the population correlation coefficient between $y$ and $x$.

Defining,

$$e_0 = \frac{S_Y^2}{S_Y^2} - 1, \quad e_1 = \frac{S_X^2}{S_X^2} - 1, \quad e_2 = \frac{s_X^2}{S_X^2} - 1$$

we assume that, $E(e_0) = E(e_1) = E(e_2) = 0$ and

$$E(e_0^2) = \lambda \beta_{2y}, \quad E(e_1^2) = \lambda \beta_{2x}, \quad E(e_2^2) = \lambda' \beta_{2x}, \quad E(e_0 e_1) = \lambda \lambda_{22}, \quad E(e_0 e_2) = \lambda' \lambda_{22}, \quad E(e_1 e_2) = \lambda' \beta_{2x}.$$
2. Estimators in Literature

| S. No. | Estimators                                                                 | MSE                                                                 |
|--------|---------------------------------------------------------------------------|----------------------------------------------------------------------|
| 1      | $t_0 = s_y^2$                                                             | $\text{Var}(s_y^2) = \lambda \cdot S_y^4 \cdot \beta_{2y}$          |
| 2      | $t_1 = s_y^2 \cdot \frac{s^2}{s_x^2}$, Isaki (1983)                      | $\text{MSI}(t_1) = S_y^4 (\lambda \beta_{2y} + (\lambda - \lambda')(\beta_{2x} - 2\beta_{22}))$ |
| 3      | $t_2 = \frac{s_y^2}{s_x^2} + b$, Isaki (1983)                            | $\min\text{MSE}(t) = S_y^4 \cdot \frac{\beta_{2y}^2}{\lambda - \lambda'} \cdot \frac{2 \cdot \beta_{22}}{}$ |
| 4      | $t_3 = k_1 \cdot s_y^2 + k_2 (s_x^2 - s_y^2)$                           | $\min\text{MSE}(t_3) = \frac{\min\text{MSE}(t_1)}{1 + \frac{\min\text{MSE}(t_2)}{S_y^4}}$ |
| 5      | $t_4 = \left[k_1^2 + k_2 (s_x^2 - s_y^2)\right] \cdot \exp\left(\frac{s_x^2 - s_y^2}{s_x^2 + s_y^2}\right)$, Shabbir and Gupta (2007) | $\min\text{MSI}(t_4) = \frac{\min\text{MSI}(t_1)}{1 + \frac{\min\text{MSI}(t_2)}{S_y^4}} \cdot \frac{(\lambda - \lambda') \beta_{2x}^2}{\min\text{MSI}(t_2)} + \frac{(\lambda - \lambda') S_y^4 \beta_{1x}^2}{16}$ |

Table 1: Estimators considered in this paper along with respective minimum MSE’s

3. Proposed Estimator

The use of auxiliary information in increasing precision of estimates is implied in sampling survey. Different transformations based on auxiliary information are also used like linear transformation, use of exponential transformation by Behl and Tuteja (1991), transformed estimator by Sahai and Ray (1980). Recently, Mishra et al. (2017) introduced estimators using log type transformation which was found to be more efficient than usual mean and ratio estimator. Motivated by Mishra et al. (2017), we propose estimators for estimating population variance under double sampling scheme.

(1) $Pl_1 = s_y^2 + w_0 \log \left(\frac{s_x^2}{s_y^2}\right)$ \hspace{1cm} (3.1)

Rewriting estimator $Pl_1$ in terms of relative error terms, we have

$$Pl_1 = S_y^2 (1 + e_0) + w_0 \left( e_1 - e_2 - \frac{e_1^2}{2} + \frac{e_2^2}{2}\right)$$
For the estimator $P_{1}$, we have

\[
\text{Bias}(P_{1}) = \frac{w_{0}}{2} \{ \lambda^{*} \beta_{2x}^{*} - \lambda \beta_{2x}^{*} \} \tag{3.2}
\]

\[
\text{MSE}(P_{1}) = S_{y}^{2} \lambda \beta_{2y}^{*} + w_{0}^{2} \{ \lambda^{*} \beta_{2x}^{*} + \lambda \beta_{2x}^{*} - 2\lambda^{*} \beta_{2x}^{*} \} + 2w_{0}S_{y}^{2} \{ \lambda \lambda_{22}^{*} - \lambda \lambda_{22}^{*} \} \tag{3.3}
\]

At optimum value of $w_{0}$ in Eq. (3.3), expression for min MSE of $P_{1}$ is given by Eq. (3.4).

\[
w_{0}^{*} = -\frac{S_{y}^{2} \lambda_{22}^{*} (\lambda - \lambda^{*})}{(\lambda - \lambda^{*})^{2} \beta_{2x}^{*}} = -\left( \frac{S_{y}^{2} \lambda_{22}^{*}}{\beta_{2x}^{*}} \right) \tag{3.4}
\]

\[
\text{min MSE}(P_{1}) = S_{y}^{2} \left[ \lambda \beta_{2y}^{*} - (\lambda - \lambda^{*}) \frac{\lambda_{22}^{*}}{\beta_{2x}^{*}} \right] \tag{3.5}
\]

(2) \( P_{2} = s_{y}^{2} (w_{1} + 1) + w_{2} \log \left( \frac{s_{x}^{2}}{s_{y}^{2}} \right) \) \( \tag{3.6} \)

Expressing the estimator $P_{2}$ in terms of $e$'s, we have

\[
P_{2} - S_{y}^{2} = S_{y}^{2} e_{0} + w_{1}S_{x}^{2} (1 + e_{0}) + w_{2} (e_{1} - e_{2}) \]

Expression for Bias and MSE of $P_{2}$ is given by Eq. (3.6) and Eq. (3.7) respectively,

\[
\text{Bias}(P_{2}) = w_{1}S_{x}^{2} \quad \text{and} \quad \text{MSE}(P_{2}) = S_{y}^{2} (1 + \lambda \beta_{2y}^{*}) + w_{1}^{2} (\lambda^{*} \beta_{2x}^{*}) + 2w_{1} (S_{y}^{2} \lambda \beta_{2y}^{*}) + 2w_{2} (S_{y}^{2} (\lambda - \lambda^{*}) \lambda_{22}^{*} + 2w_{1}w_{2} (S_{y}^{2} (\lambda - \lambda^{*}) \lambda_{22}^{*}) \tag{3.7}
\]

Partially differentiating Eq. (3.7) with respect to $w_{1}$ and $w_{2}$, we get

\[
w_{1}^{*} = \frac{(A - C)D}{D^{2} - AB} \]

\[
w_{2}^{*} = \frac{CB - D^{2}}{D^{2} - AB} \]

Using optimum values of $w_{1}$ and $w_{2}$ in equation (3.7), we get

\[
\text{min MSE}(P_{2}) = C + \frac{BC^{2} + (A - 2C)D^{2}}{D^{2} - AB} \tag{3.8}
\]

where, \( A = S_{y}^{2} (1 + \lambda \beta_{2y}^{*}) \), \( B = (\lambda^{*} - \lambda^{*}) \beta_{2x}^{*} \), \( C = S_{y}^{2} \lambda \beta_{2y}^{*} \), \( D = S_{y}^{2} (\lambda - \lambda^{*}) \lambda_{22}^{*} \)

Now, we propose another estimator $P_{3}$ given by Eq. (3.9)

\[
(3) P_{3} = \left[ s_{y}^{2} (1 + w_{3}) + w_{4} \log \left( \frac{s_{x}^{2}}{s_{y}^{2}} \right) \right] \exp \left( \frac{s_{x}^{2} - s_{y}^{2}}{s_{x}^{2} + s_{y}^{2}} \right) \tag{3.9}
\]

Expressing equation (3.9) in terms of $e$'s, we have
The expression for the bias and MSE are given in Eq. (3.10) and Eq. (3.11).

\[
\text{Bias (} P_{l_3} \text{)} = S_1^4 \left( \frac{3}{8} \lambda \beta^*_2 x - \frac{1}{8} \lambda \beta^*_2 x - \frac{1}{2} \lambda^* \lambda^*_2 x + \frac{1}{2} \lambda^* \lambda^*_2 x - \frac{1}{4} \lambda^* \beta^*_2 x \right) + S_1^4 w_3 \left( 1 + \frac{3}{8} \lambda \beta^*_2 x - \frac{1}{8} \lambda \beta^*_2 x - \frac{1}{2} \lambda^* \lambda^*_2 x + \frac{1}{2} \lambda^* \lambda^*_2 x - \frac{1}{4} \lambda^* \beta^*_2 x \right) + w_4 \left( \lambda^* \beta^*_2 x - \frac{1}{2} \lambda^* \beta^*_2 x - \frac{1}{2} \lambda^* \beta^*_2 x \right)
\]

\[(3.10)\]

\[
\text{MSE(} P_{l_3} \text{)} = F + w_3^2 A_1 + w_4^2 B_1 + 2 w_3 C_1 + 2 w_4 D_1 + 2 w_3 w_4 E_1
\]

\[(3.11)\]

\[
\text{Partially differentiating Eq. (3.11) with respect to } w_3 \text{ and } w_4, \text{ we get}
\]

\[
w_3^* = \frac{C_1 B_1 - D_1 E_1}{E_1 - A_1 B_1}
\]

\[
w_4^* = \frac{A_1 D_1 - C_1 E_1}{E_1 - A_1 B_1}
\]

Substituting the optimum values of \( w_3 \) and \( w_4 \) in Eq. (3.11), we obtain expression for min. MSE given by Eq. (3.12):

\[
\begin{align*}
\text{min MSE (} P_{l_3} \text{)} &= F + \frac{B_1 C_1^2 + A_1 D_1^2 - 2 C_1 D_1 E_1}{E_1 - A_1 B_1} \\
\end{align*}
\]

\[(3.12)\]

Where

\[
D_1 = S_1^2 \left( \lambda - \lambda^* \right) \left( \lambda^*_2 - \frac{\beta^*_2 x}{2} \right), \quad E_1 = S_1^2 \left( \lambda - \lambda^* \right) \left( \lambda^*_2 - \beta^*_2 x \right)
\]

\[
F_1 = S_1^4 \left[ \lambda \beta^* \gamma + (\lambda - \lambda^*) \left( \frac{\beta^*_2 x}{4} - \frac{\beta^*_2 x}{2} \right) \right]
\]

We define another estimator \( P_{l_4} \) as follows:

\[
(4) \quad P_{l_4} = s_3^2 (w_2 + 1) + w_6 \log \left( \frac{s_x^2}{s_y^2} \right) \exp \left( \frac{s_x^2 - s_y^2}{s_x^2 + s_y^2} \right)
\]

\[(3.13)\]

Expressing equation (3.13) in terms of \( e \)'s, we have

\[
P_{l_4} = S_2^2 + S_2^2 w_3 (1 + e_0) + w_6 \left( e_1 - e_2 - \frac{e_1^2}{2} - \frac{e_2^2}{2} + e_1 e_2 \right)
\]

Expressions for Bias and MSE of estimator \( P_{l_4} \) is given by Eq. (3.14) and Eq. (3.15) respectively,
Bias \( (P_{I_4}) = S_3^* w_5 + w_6 \left( \lambda - \beta_{2x}, \beta_{2x}, \lambda - \beta_{2x}, \frac{1}{2} \lambda \beta_{2x} - \frac{1}{2} \lambda \beta_{2x}, \frac{1}{2} \lambda \beta_{2x} \right) \) (3.14)

MSE \( (P_{I_4}) = C_3 + w_5^2 A_3 + w_6^2 B_3 + 2 w_5 C_3 + 2 w_6 D_3 + 2 w_5 w_6 E_3 \) (3.15)

partially differentiating Eq. (3.15) w. r. to \( w_5 \) and \( w_6 \), we get optimum values given by:

\[
\begin{align*}
\dot{w}_5 &= \frac{C_3^2 B_3 - D_3 E_3}{E_3^2 - A_3 B_3} \\
\dot{w}_6 &= \frac{A_3 D_3 - C_3 E_3}{E_3^2 - A_3 B_3}
\end{align*}
\]

Using optimum values of \( w_5 \) and \( w_6 \) in equation (3.15), we have:

\[
\begin{align*}
\min \text{MSE} \ (P_{I_4}) &= C_3 + \frac{B_3 C_3^2 + A_3 D_3^2 - 2 C_3 D_3 E_3}{E_3^2 - A_3 B_3} \quad (3.16)
\end{align*}
\]

where,

\[
\begin{align*}
A_3 &= S_3^2 (1 + \lambda \beta_{2y}) \\
B_3 &= \beta_{2x} (\lambda - \lambda^*) \\
C_3 &= S_3^2 \lambda \beta_{2x} \\
D_3 &= S_3^2 (\lambda - \lambda^*) \lambda_{22} \\
E_3 &= S_3^2 \left\{ \lambda^* \lambda_{22} + \lambda \lambda_{22} - \lambda^* \lambda_{22}^* - \frac{\lambda \beta_{2x}^2}{2} - \frac{\lambda \beta_{2x}^*}{2} \right\} \\
&= S_3^2 (\lambda - \lambda^*) \left\{ \lambda_{22}^* - \frac{\beta_{2x}^*}{2} \right\}
\end{align*}
\]

4. Efficiency Comparison

(i) \( \min \text{MSE} \ (P_{I_1}) \leq \text{var} (s_{I_1}) \) Or
\[
\text{var} (s_{I_1}) = \min \text{MSE} \ (P_{I_1}) \geq 0 \text{ if }
\]

\[
S_3^2 (\lambda - \lambda^*) \frac{\lambda_{22}}{\beta_{2x}} \geq 0
\]

(ii) \( \min \text{MSE} \ (P_{I_2}) \leq \text{var} (s_{I_2}) \) Or
\[
\text{var} (s_{I_2}) = \min \text{MSE} \ (P_{I_2}) \geq 0 \text{ if }
\]

\[
BC^2 + (A - 2C)D^2 \leq \frac{D^2 - AB}{D^2 - AB}
\]

(iii) \( \min \text{MSE} \ (P_{I_1}) \leq \text{var} (s_{I_1}) \) Or
\[
\text{var} (s_{I_1}) = \min \text{MSE} \ (P_{I_1}) \geq 0 \text{ if }
\]

\[
\left( \lambda_{22} - \frac{\beta_{2x}}{4} \right) (\lambda - \lambda^*) = \frac{BC^2 + A \lambda_{22}^* - 2 C \lambda_{22} E_3}{E_3^2 - A \lambda_{22} E_3} \geq 0
\]

(iv) \( \min \text{MSE} \ (P_{I_1}) \leq \text{MSE} \ (t_{1}) \) Or
\[
\text{MSE} \ (t_{1}) = \min \text{MSE} \ (P_{I_1}) \geq 0 \text{ if }
\]

\[
S_3^2 (\lambda - \lambda^*) \frac{\beta_{2x}^*}{\beta_{2x}} \geq 0
\]

(vi) \( \min \text{MSE} \ (P_{I_1}) \leq \text{MSE} \ (t_{1}) \) Or
\[
\text{MSE} \ (t_{1}) = \min \text{MSE} \ (P_{I_1}) \geq 0 \text{ if }
\]

\[
S_3^2 (\lambda - \lambda^*)(\beta_{2x}^* - \lambda_{22}^*)^2 = \frac{BC^2 + (A - 2C)D^2}{D^2 - AB} \geq 0
\]
(vii) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
S^4_d (\lambda - \lambda') \left( \frac{3\beta^*_{2x} - \lambda^*_{22}}{4} \right) \geq 0

(viii) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
S^4_d (\lambda - \lambda') \left( \frac{\beta^*_{2x} - 2\lambda^*_{22}}{\beta^*_{2x}} \right) - \frac{B_i C^2_i + A_i D^2_i - 2C_i D_i E_3}{E_i^2 - A_i B_3} \geq 0

(ix) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
S^4_d (\lambda - \lambda') \left( \frac{3\lambda^*_{22}}{\beta^*_{2x}} \right) - \frac{2BC^2 + (A - 2C)D^2}{D^2 - AB} \leq 0

(x) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
S^4_d (\lambda - \lambda') \left( \frac{4\beta^*_{2x} - 2\lambda^*_{22}}{4\beta^*_{2x}} \right) + \frac{B_i C^2_i + A_i D^2_i - 2C_i D_i E_3}{D^2 - AB} \leq 0

(xi) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
S^4_d (\lambda - \lambda') \left( \frac{\lambda^*_{22}}{\beta^*_{2x}} \right) + \frac{B_i C^2_i + A_i D^2_i - 2C_i D_i E_3}{E_i^2 - A_i B_3} \leq 0

(xii) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
S^4_d (\lambda - \lambda') \left( \frac{2\beta^*_{2x} - \lambda^*_{22}}{\beta^*_{2x}} \right) - \frac{2BC^2 + (A - 2C)D^2}{D^2 - AB} \geq 0

(xiii) min MSE (P_i) \leq MSE (t_i) \quad \text{Or} \\
MSE (t_i) - \min MSE (P_i) \geq 0 \quad \text{if} \\
\left[ \frac{\min .MSE (t_i)}{\min .MSE (P_i)} \right] - \frac{(\lambda - \lambda') \beta^*_{2x} (\min .MSE (t_i)) + (\lambda - \lambda') S^4_d \beta^*_{2x}}{4 \left[ 1 + \frac{\min .MSE (t_i)}{S^4_d} \right]} \geq 0

\frac{E_i^2 - A_i B_3}{E_i^2 - A_i B_3} \geq 0

(xiv) min MSE (P_i) \leq min MSE (P_i) \quad \text{Or} \\
min MSE (P_i) - min MSE (P_i) \geq 0 \quad \text{if} \\
\frac{S^4_d (\lambda - \lambda') \beta^*_{2x} - 2\lambda^*_{22}}{4\beta^*_{2x}} + \frac{B_i C^2_i + A_i D^2_i - 2C_i D_i E_3}{E_i^2 - A_i B_3} \leq 0

(xv) min MSE (P_i) \leq min MSE (P_i) \quad \text{Or} \\
min MSE (P_i) - min MSE (P_i) \geq 0 \quad \text{if} \\
\frac{S^4_d (\lambda - \lambda') \beta^*_{2x} - 4\lambda^*_{22}}{\beta^*_{2x}} + \frac{BC^2 + (A - 2C)D^2}{D^2 - AB} \leq 0
(xvi) \( \min \text{MSE} (P_{1i}) \leq \min \text{MSE} (P_{1, j}) \) Or

\[
\frac{S^2_i (\lambda - \lambda')}{\beta_{22}} + \left( \frac{B_i C_{12}^2 + A_i D_{12} - 2 C_i E_{12}}{E_{12} - A_i B_i} \right) \leq 0
\]

(xvii) \( \min \text{MSE} (P_{12}) \leq \min \text{MSE} (P_{1, j}) \) Or

\[
\frac{S^2_i (\lambda - \lambda')}{\beta_{22}} - \left( \frac{B_i C_{12}^2 + A_i D_{12} - 2 C_i E_{12}}{E_{12} - A_i B_i} \right) \geq 0
\]

(xviii) \( \min \text{MSE} (P_{21}) \leq \min \text{MSE} (P_{1, j}) \) Or

\[
\frac{S^2_i (\lambda - \lambda')}{\beta_{22}} - \left( \frac{B_i C_{12}^2 + A_i D_{12} - 2 C_i E_{12}}{E_{12} - A_i B_i} \right) \geq 0
\]

5. Data description and Numerical Calculation

In this section, we consider four Populations for numerical comparison of Percent Relative Efficiency of proposed estimators with relevant existing estimators. Four real data sets are used for numerical illustration:

**Population 1:** Cochran (1977, page 325)

\( N=100, n=10, n'=85, S_{y^2}=214.69, S_{x^2}=56.76, \lambda_{40}=2.2387, \lambda_{04}=2.2523, \lambda_{22}=1.5432. \)

**Population 2:** (Cochran 1977, page 152)

\( N=196, n=49, n'=158, S_{y^2}=1515558.83, S_{x^2}=10900.42, \lambda_{40}=8.5362, \lambda_{04}=7.3617, \lambda_{22}=7.8780. \)

**Population 3:** (Cochran 1977, page 203)

\( N=200, n=29, n'=159, S_{y^2}=99.18, S_{x^2}=85.09, \lambda_{40}=1.9249, \lambda_{04}=2.5932, \lambda_{22}=2.1149. \)

**Population 4:** (Sukhatme and Sukhatme 1970, page 185)

\( N=170, n=10, n'=139, S_{y^2}=26456.89, S_{x^2}=22355.76, \lambda_{40}=3.1842, \lambda_{04}=2.2030, \lambda_{22}=2.5597. \)

| Population | 1     | 2     | 3     | 4     |
|------------|-------|-------|-------|-------|
| Estimators |       |       |       |       |
| \( t_0 \)  | 100.00| 100.00| 100.00| 100.00|
| \( t_1 \)  | 88.394| 1025.763| 292.561| 736.2863|
Table 5.1: Percent Relative Efficiencies of Estimators

|      |        |        |        |        |
|------|--------|--------|--------|--------|
| $t_2$| 122.923| 1082.550 | 517.205 | 1134.776 |
| $t_3$| 134.072| 1094.090 | 519.932 | 1155.333 |
| $t_4$| 138.180| 1132.780 | 529.036 | 1201.728 |
| $Pl_1$| 122.923| 1082.550 | 517.205 | 1134.776 |
| $Pl_2$| 134.072| 1094.090 | 519.932 | 1155.333 |
| $Pl_3$| 138.939| 1084.503 | 521.422 | 1145.068 |
| $Pl_4$| 141.368| 1550.532 | 563.085 | 1883.728 |

From the results of Table 5.1, it is evident that the proposed estimators $Pl_1, Pl_2, Pl_3$ and $Pl_4$ are more efficient than $t_0$ and $t_1$. It can be seen from Table 5.1 that $t_2$ and $t_3$ are equally efficient to $Pl_1$ and $Pl_2$ respectively and $Pl_3$ and $Pl_4$ are more efficient than all the estimators considered in Table 1. Among the proposed estimators, $Pl_3$ and $Pl_4$ are uniformly more efficient than $Pl_1$ and $Pl_2$ respectively.

6. Conclusion

Based on theoretical and numerical results obtained it turns out that percent relative efficiencies of estimators $Pl_1, Pl_2, and Pl_3$ are found to be more than existing estimators in literature (as defined in paper) under certain specified conditions. Estimator $Pl_4$ is found to be uniformly more efficient than other existing as well as proposed estimators. It is therefore, suggested to use proposed estimators for estimating population variance more efficiently when double sampling is used.

7. References

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