Rescaled Einstein-Hilbert gravity: Inflation and the Swampland Criteria

Based on the work of V. K. Oikonomou, I. Giannakoudi, A. Gitsis and K. Revis
Introduction

• Modified gravity ( f(R) ) contains higher order curvature terms, appealing to describe an inflationary effective Lagrangian.

• The standard inflationary scenario consists of a canonical scalar field φ with equation of motion \( \ddot{\phi} + 3H\dot{\phi} + V' = 0 \) (FRW Metric).

• The rescaling \( 0 \leq \alpha \leq 1 \) may give rise to the compliance of the inflationary theory with the Swampland criteria.
Equations of motion

- The general f(R) gravity is of the form

\[ f(R) = R - \gamma \lambda \Lambda - \lambda R \exp \left( -\frac{\gamma \Lambda}{R} \right) - \frac{\Lambda \left( \frac{R}{m_s^2} \right)^\delta}{\zeta} \]

yielding the eom

\[ 3H^2 f_R = \frac{R f_R - f}{2} - 3H f_R + \kappa^2 \left( \frac{1}{2} \dot{\phi}^2 + V \right), \quad -2 \dot{H} f_R = \kappa^2 \phi^2 + f_R - H \dot{f}_R \]

- In the large curvature limit, we can approximate the f(R) gravity action as

\[ S = \int d^4x \sqrt{-g} \left( \frac{1}{2\kappa^2} \left( \alpha R + \frac{\gamma^3 \lambda \Lambda^3}{6R^2} - \frac{\gamma^2 \lambda \Lambda^2}{2R} - \frac{\Lambda}{\zeta} \left( \frac{R}{m_s^2} \right)^\delta + O(1/R^3) + \ldots \right) - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) \right) \]

yielding at leading order the eom

\[ 3H^2 \alpha \simeq \kappa^2 \left( \frac{1}{2} \dot{\phi}^2 + V \right), \quad -2 \dot{H} \alpha \simeq \kappa^2 \dot{\phi}^2 \quad (\alpha = 1 - \lambda) \]

- From the 2 sets of eom, it appears at leading order as if we have rescaled the E-H action (\alpha R instead of R).
Slow-roll inflation

- The conditions for slow-roll inflation are \( \frac{\dot{H}}{H^2} \ll 1 \) and \( |\ddot{\phi}| \ll 3H|\dot{\phi}| \).

- The spectral index is given by \( n_s = 1 + 2\alpha \eta - 6\alpha \epsilon \) and the tensor-to-scalar ratio by \( r = 16\alpha \epsilon \).

  where \( \epsilon = \frac{1}{2\kappa^2} |\dot{V}|^2 / V^2 \) and \( \eta = \frac{1}{\kappa^2} |V''| / V \).

- The constraints from the observational data of Planck in 2018 are \( n_s = 0.9649 \pm 0.0042 \), \( r < 0.056 \).

(Flat FRW metric assumed)
Swampland criteria

- Conjectures ensuring the concordance of the theory with quantum gravity (expressed in reduced Planck units).

- The distance conjecture: limits the maximum traversable range of a scalar field: \( \Delta \phi \leq f \sim \mathcal{O}(1) \)

- The De Sitter conjecture: sets a limit to the gradient of the scalar potential: \( \frac{V'}{V} \geq g \sim \mathcal{O}(1) \) or, alternatively, \( \frac{V''}{V} \leq -h \sim \mathcal{O}(1) \)

- We are looking for values of \( \alpha \) that satisfy both the inflationary constraints and the Swampland criteria.
D – Brane (p = 2)

\[ V(\phi) = \Lambda^4 \left( 1 - \left( \frac{m}{\kappa \phi} \right)^2 \right) \]
\[ m = [10^{-6}, 10^{0.3}] \]

The constraints for \( \alpha \) are
\[ 0.00598m^2 \leq \alpha \leq 0.03438m^2 \]

\[ n_s \simeq \frac{3m^{2/3}}{8 \sqrt[3]{2} \sqrt[3]{\alpha N^2}} - \frac{3\sqrt{\alpha m^2 N}}{4 \sqrt[3]{2} \alpha N^2} - \frac{3}{2N} + 1 \]
\[ r \simeq \frac{\sqrt{2} \sqrt{\alpha m^2 N}}{\alpha N^2} \]
The constraints for $\alpha$ are

$$0.00313782m^2 \leq \alpha \leq 0.0209707m^2$$
Natural inflation

The constraints for $\alpha$ are

$$\frac{0.02525}{l^2} < \alpha < \frac{0.02553}{l^2}$$

- In this case, we have a very narrow range.
The maximum value of $l$ is narrowed down. Its value is really close to 0.9667 for all values of $\alpha$.

The constraints for $\alpha$ are

\[0.03376l < \alpha \leq 1\]

- The maximum value of $l$ is narrowed down.
$V(\phi) = \Lambda^4 \tanh^4 \left( \frac{\kappa \phi}{\sqrt{6 l}} \right)$

$l = [10^{-2}, 10^4]$  

The constraints for $\alpha$ are

$0.045381 l \leq \alpha \leq 30l$, $l = [10^{-2}, 0.03]$

$0.0453846 l \leq \alpha \leq 1$, $l = [0.033, 22.034]$

- Again, a maximum value for $l$ has been imposed.
Potential with exponential tails model

\[ V(\phi) = \Lambda^4 (1 - e^{-\kappa q \phi}) \]
\[ q = [10^{-3}, 10^3] \]

The constraints for \( \alpha \) are
\[ 0.993/q^2 \leq \alpha \leq 1, \quad q = [0.9967, 1.0059] \]
\[ 0.993/q^2 \leq \alpha \leq 1.012/q^2, \quad q = [1.0059, 10^3] \]

\[ n_s = -\frac{4\alpha q}{\sqrt{2\sqrt{\alpha} + 2\alpha q N}} - \frac{12}{(2\sqrt{\alpha q N} + \sqrt{2})^2} + 1 \]
\[ r = \frac{32}{(2\sqrt{\alpha q N} + \sqrt{2})^2} \]
The range of $w$ is severely restricted. The constraints for $\alpha$ are $0.0949w \leq \alpha \leq 1$, $w = [10^{-2}, 10.538]$.

- The range of $w$ is severely restricted.
E – Model (n = 2)

Again, the range of $w$ is quite limited.

The constraints for $\alpha$ are

$$0.0817w \leq \alpha \leq 1, \ w = [10^{-2}, 12.24]$$

- Again, the range of $w$ is quite limited.
From the plot of $r$, we see that this model is not viable for our theory.

The constraint for $\alpha$ is

$$\frac{\alpha}{q^2} \geq 0.0286389$$
Power – law Potential (I)

\[ V(\phi) = \frac{\lambda \phi^{2/3}}{\kappa^{10/3}} \]

- The observational quantities are now functions only of the e-foldings number N.

\[ n_s \simeq \frac{3}{8N^2} - \frac{3}{2N} + 1 \]

\[ r \simeq \frac{4N - 1}{N^2} \]

(Inflation ends at \( N = [50, 60] \))

- Planck contraints are not satisfied simultaneously so from the plots, the model is a not viable one.
Power – law Potential (II)

\[ V(\phi) = \frac{\lambda \phi}{\kappa^3} \]

- Again, Planck constraints are not satisfied simultaneously.

\[ n_s \approx \frac{3}{8N^2} - \frac{3}{2N} + 1 \]

\[ r \approx \frac{4N - 1}{N^2} \]

- Not a viable model for our theory.
Like before, Planck constraints are not satisfied simultaneously.

Yet another not viable power-law model.

\[ V(\phi) = \frac{\lambda \phi^{4/3}}{\kappa^{8/3}} \]

\[
\begin{align*}
n_s &\approx \frac{5}{9N^2} - \frac{5}{3N} + 1 \\
r &\approx \frac{16(3N - 1)}{9N^2}
\end{align*}
\]
Power – law Potential (IV)

\[ V(\phi) = \frac{\lambda \phi^2}{k^2} \]

- Planck constraints not satisfied simultaneously.
- Another not viable model for our theory.
Power – law Potential (V)

\[ V(\phi) = \frac{\lambda \phi^3}{\kappa} \]

- As in the previous cases, Planck constraints are not satisfied simultaneously.

\[ n_s \approx \frac{4N - 7}{4N + 3} \]

\[ r \approx \frac{3(4N - 3)}{N^2} \]

- Another not viable model.
None of the Power–law models has been proven a viable one.

Just like before, Planck constraints are not satisfied simultaneously.
Swampland for T – Model (m = 1)

The range of $\alpha$ is

$$\alpha = [0, 0.008]$$

- The range is really narrow.

$$\frac{V'(\phi_i)}{V(\phi_i)} = \frac{2\sqrt{\frac{2}{3}} \kappa \text{csch} \left( \kappa \cosh^{-1} \left( l \sqrt{\frac{12\alpha}{l} + 9 + 4\alpha N} \right) \right)}{\sqrt{l}}$$

$$\frac{-V''(\phi_i)}{V(\phi_i)} = -\frac{l \sqrt{\frac{12\alpha}{l} + 9 + 240\alpha - 6l}}{3\alpha \left( 40l \sqrt{\frac{12\alpha}{l} + 9 + 4800\alpha + l} \right)}$$
Swampland for D – Brane ($p = 4$)

The ranges for $\alpha$ and $m$ are

$$\alpha = [0, 0.03]$$

$$m = [0.75, 1.99526]$$

$$512.802\alpha - 0.318 < m$$

$$m < 164.067\alpha + 0.146$$

• We can also evaluate relations between the two parameters.
Swampland for E – Model (n = 1)

The ranges for $\alpha$ and $w$ are

\[
\alpha = [0, 0.05138] \\
w = [0.01, 0.05138] \\
w > 3366.41\alpha^2 + 0.01
\]

- One of the cases we can also find relations between the parameters.

\[
\frac{V'(\phi_i)}{V(\phi_i)} = \frac{\sqrt{6}\kappa\sqrt{w}}{\sqrt{3}\sqrt{\alpha\sqrt{w} + 2\alpha N}}
\]

\[
- \frac{V''(\phi_i)}{V(\phi_i)} = \frac{\kappa^2 (2\sqrt{3}\sqrt{\alpha\sqrt{w} + 4\alpha N} - 3w)}{\alpha (2\sqrt{\alpha N} + \sqrt{3}\sqrt{w})^2}
\]
Swampland for Natural Inflation

- Up till now, none of the Swampland criteria is satisfied.

\[
\frac{V'(\phi_i)}{V(\phi_i)} = -\frac{l \sin \left( 2 \sin^{-1} \left( \frac{\sqrt{2e - 3\alpha l^2}}{\sqrt{\alpha l^2} \sqrt{\alpha l^2 + 1}} \right) \right)}{\cos \left( 2 \sin^{-1} \left( \frac{\sqrt{2e - 3\alpha l^2}}{\sqrt{\alpha l^2} \sqrt{\alpha l^2 + 1}} \right) \right) + 1}
\]

\[
-\frac{V''(\phi_i)}{V(\phi_i)} = \frac{l^2 \cos \left( 2 \sin^{-1} \left( \frac{\sqrt{2e - 3\alpha l^2}}{\sqrt{\alpha l^2} \sqrt{\alpha l^2 + 1}} \right) \right)}{\cos \left( 2 \sin^{-1} \left( \frac{\sqrt{2e - 3\alpha l^2}}{\sqrt{\alpha l^2} \sqrt{\alpha l^2 + 1}} \right) \right) + 1}
\]
Swampland for Natural Inflation

\[ \Delta \phi = \frac{2 \tan^{-1} \left( \frac{\sqrt{2}}{\sqrt{\alpha l}} \right)}{\kappa l} - \frac{2 \sin^{-1} \left( \frac{\sqrt{2e^{-\frac{1}{2} a l^2 N}}}{\sqrt{\alpha l} \sqrt{\frac{2}{\alpha l^2} + 1}} \right)}{\kappa l} \]

- The Swampland criteria are not satisfied for this potential.
Swampland for E – Model (n = 2)

\[ \Delta \phi = \sqrt{\frac{3w}{2}} \ln \frac{1 + \frac{4\sqrt{\alpha}}{\sqrt{3w}}}{1 + \frac{4\sqrt{\alpha}}{\sqrt{3w}} + \frac{8N\alpha}{3w}} \]
Swampland for E – Model (n = 2)

The ranges for $\alpha$ and $w$ are

$\alpha = [0, 0.52]$  
$w = [0.01, 10000]$
Conclusions

- Most models are compatible with the Planck data while some comply with the Swampland criteria, too.

- The Swampland criteria restrict severely the range of $\alpha$. 
Appendix (I)

- From Einstein eqs, we get \[ \frac{3\alpha}{\kappa^2} H^2 = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad \text{and} \quad \frac{2\alpha}{\kappa^2} \dot{H} = -\dot{\phi}^2 \]

- From \[ \left| \frac{\dot{H}}{H^2} \right| \ll 1 \] we define \[ \epsilon_1 = -\frac{\dot{H}}{H^2} = \alpha \epsilon \] and from \[ \left| \ddot{\phi} \right| \ll 3H|\dot{\phi}| \] we define

\[ \epsilon_2 = \frac{\ddot{\phi}}{H \dot{\phi}} = -\alpha \eta + \epsilon_1 \quad \text{where} \quad \epsilon = \frac{1}{2\kappa^2} \frac{V''}{V^2} \quad \text{and} \quad \eta = \frac{1}{\kappa^2} \frac{V''}{V} \]

(The conditions for slow-roll inflation are \( \epsilon_1 \ll 1 \) and \( \epsilon_2 \ll 1 \))

- The spectral index and tensor-to-scalar-ratio are defined as \( n_s - 1 = -4\epsilon_1 - 2\epsilon_2 \) and \( r = 8\kappa^2 \frac{\dot{\phi}^2}{H^2} \)

In our case, \( n_s = 1 + 2\alpha \eta - 6\alpha \epsilon \) and \( r = 16\alpha \epsilon \).
Appendix (II)

- From the de Sitter criterion, \( \left| \frac{V'}{V} \right| = \sqrt{2\epsilon} \geq 1 \)

- For the rescaled case, the slow-roll has to do with \( \epsilon_1 \), not \( \epsilon \).

- We can choose an appropriately small value of \( \alpha \) so that both slow-roll conditions and Swampland criteria hold true.
THANK YOU FOR YOUR TIME!