Acceleration of charged particles from rotating black holes embedded in magnetic fields

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The aim of the present article is to evaluate the motion of charged test particles in the vicinity of a rotating regular black hole in the presence of magnetic fields. Euler-Lagrange motion equations and effective potential methods are used to characterize the motion out of the equatorial plane. Such approach is of peculiar significance if it is considered, e.g., accretion processes onto rotating black holes. In general investigations concerning accretion focus mostly on the simplest case of particles moving in the equatorial plane. Here it will be considered that particles initially moving around some particular orbit may be perturbed by a kick along the \( \theta \) direction, giving rise to other possible orbits. We confirm the possibility that ultra high energy cosmic rays would be produced at the very center of AGNs, for a specific range of magnetic field magnitudes, since it is possible that ultra-high center-of-mass energies can be produced by particles colliding near the horizon of fastly rotating black holes.

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I. INTRODUCTION

Cosmic rays propagate in the universe in different energy ranges. Those with higher energy, frequently called ultra high energy cosmic rays (UHECRs), are detected in some terrestrial experiments like Pierre Auger [1] and Telescope Array [2] (TA) observatories. One of the main interests related to UHECRs is that their energy level is beyond the so called Greisen-Zatsepin-Kuzmin (GZK) limit \( E > 10^{18} \text{ eV} \). Accordingly, some important questions arise and one of the most intricated is related to the origin of these cosmic rays. The main explanations, depending on energy levels, are associated to mechanisms similar to that proposed by Fermi where charged particles could be accelerated by clouds of magnetized gas moving within our galaxy (aka Fermi’s Mechanism). Nevertheless it is well known that Fermi’s original mechanism is too slow to be effective for UHECRs and nowadays it was adapted to a novel diffusive shock acceleration mechanism, as a way to accelerate charged particles efficiently at shock fronts. Other possible mechanisms are the external shock phase [3], unipolar inductors [4], magnetic reconnection acceleration, re-acceleration in sheared jets [4] or even pure radiative transfer and the energy extraction from poloidal magnetic flux in Seyfert galaxies [5]. A detailed sketch about all possible mechanisms and sources can be found in [4] [6, 7].

In any case, it is estimated that the acceleration of charged particles in the vicinity of the event horizon of central black holes (BHs) of active galaxy nuclei (AGNs) is one of the possible viable UHECR explanations. Astrophysical BHs have been observed by quite a few astronomical measurements as optical and X-ray observations and also gravitational wave detection. In this respect, there is a current endeavor among physicists and astronomers to answer the question if the BH theory studied by general relativity (GR) and astrophysical BHs are indeed the same thing. From this perspective, astrophysical BHs have some of the Kerr BH properties predicted by GR. In other words, at least it appears that astrophysical BHs have mass and spin, with angular momentum \( a = J/M (0 < a < 1) \) [8, 9]. In particular, observations show that several stellar or supermassive BHs have high spin \( (a \gtrsim 0.7) \) [10]. Besides, it is also important to consider magnetic fields since the traditional environment of AGNs consists of a plasma accretion disk surrounding the central black hole. Associated with such accretion phenomena there is the creation of highly relativistic jets and, as well, the dynamics of the accretion disk around such a spinning black hole forms an electrodynamic system which produces magnetic fields as explained by several references (see, e.g., [5, 11–14]). Accordingly, the dynamics of the charged particle will be extremely sensitive to the presence of these fields.

If astrophysical BHs and GR BHs are the same thing, it is possible indeed that UHECRs are produced in the center of AGNs, since it is possible that in GR ultra-high center-of-mass energies \( (E_{\text{c.m.}}) \) can be produced by particles colliding near the horizon of fastly rotating black holes. The literature presents the analyzes of such a possible situation in the vicinity of static black holes [15], rotating black holes [15–17], charged black holes [18], weakly magnetized black

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holes [19, 20] and also Kerr naked singularities [21, 22] (for a more complete list of references about the subject see [23]). Ultra high $E_{c.m.}$ particles were firstly proposed by Bañados, Silk and West [15] who noticed that collisions of two neutral classical particles falling freely into extremal Kerr BHs ($a = 1$) may have infinite values of $E_{c.m.}$ close to the event horizon, if one of the particles is tracking marginally bound geodesics. In this respect, [24] and [10] concluded that, in fact, frame dragging effects in Kerr BHs can accelerate particle to high energies, but astrophysical restrictions on the spin (i.e., apparently, real black holes never reach $a = 1$) and restrictions on maximum $E_{c.m.}$ caused by gravitational radiation and back-reaction would solely permit infinite $E_{c.m.}$ only at infinite time and on the horizon of the black hole. Although the statements pointed out by [10, 24] clearly indicates that the effects of self-force might prevent $E_{c.m.}$ from being arbitrarily high, it would also be feasible to undertaking that the center of mass energy is still high enough to be of astrophysical interest if the mass-ratio of the point particle to the BH is negligible [20, 25, 26]. This is particularly important for understanding UHECR production and acceleration process within AGN sources.

In the present article, we study the collision of two charged particles with equal masses moving in the equatorial plane and also in other possible planes (where $0 < \theta < 90^\circ$). The accelerated particles in the central environment of AGNs could mimic protons, atomic nuclei and other charged particles that could be considered as UHECRs. At the moment this work addresses particles as classical charged point particles, coming from infinity towards the horizon of Kerr black holes. Here the black hole is embedded in a magnetic field. Novel aspects are presented as the calculation of motion equations from a Lagrangian containing the electrodynamics characteristics that results in the interaction between the charged particle and the magnetic field, whose analysis is done in the equator position (as usually made) but also for different positions other than the equator. In other words, Euler-Lagrange motion equations and effective potential methods are used to characterize the motion out of the equatorial plane. Here it will be considered that particles initially moving around some particular orbit may be perturbed by a kick along the $\theta$ direction, giving rise to other possible orbits. This means that, in the context of polar coordinates, $\theta$ varies with $r$ and the geodesics $t$ is non null. In this way, the geodesics $t, r, \theta$ and $\phi$ are computed, resulting in the calculation of the effective potential $V_{eff}$ and the center of mass energy $E_{c.m.}$ of two particles that collide close to the horizon. It is possible to assess how much the system is bond or not and if there is a possibility of particles escape quickly from the system. It is concluded from the results that in fact, black holes with spin function as a kind of slingshot propelling the particles out of the system. The bigger the spin, the more propelled the particle is (even if neutral). In addition, charged particles are also propelled if there is the presence of magnetic fields and, in this case, magnetic fields work by mimicking the effect of black hole spin $a$. It is important to point that only a certain range of magnetic fields actually cause the acceleration of the particle in positions increasingly farther from the horizon. Otherwise, magnetic fields of great magnitude can cause the particle to be trapped in the system, prompting the particle to fall into the black hole. In addition, the axial acceleration of the particle is evaluated, showing that for certain fields and spins, in fact the charged particle tends to accelerate out of the system. Analyzes in future works will include the fact that these particles are also fermions, excluding cases applied to small black holes (where Hawking or Unruh effect would be applicable) since we are dealing with astrophysical black holes, of great mass and large area of the horizon.

This article is divided as follows. In Section [II] we explain particle energy from Kerr background. Section [III] is dedicated to discuss electrically charged particles in the presence of magnetic fields and the impact of this in particle geodesics. Section [IV] presents the main results for geodesics and for the energy of charged particle collisions near Kerr BHs. In this section some discussion is presented as well. In Section [V] we present some concluding remarks. Here it will be considered $G = c = 1$ and metric signature $(-+++)$.

II. SPACETIME BACKGROUND AND PARTICLE ENERGY

Some exterior solutions of the Einstein equations in vacuum could be interpreted as black holes with some particular properties. Here it will be used the Kerr solution, i.e., a stationary vacuum black hole solution with rotation. The Kerr line element in Boyer-Lindquist coordinates is

$$ds^2 = g_{tt}dt^2 + 2g_{t\phi}dtd\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2,$$  (1)

with

$$g_{tt} = -\left(1 - \frac{2M}{\Sigma}\right),$$  (2)

$$g_{t\phi} = -\frac{2aMr\sin^2\theta}{\Sigma},$$  (3)
\begin{align}
g_{rr} &= \frac{\Sigma}{\Delta}, \\
g_{\theta\theta} &= \Sigma, \\
g_{\phi\phi} &= \frac{(r^2 + a^2)^2 - a^2 \Delta \sin^2 \theta}{\Sigma} \sin^2 \theta,
\end{align}

and where \( M \) is the gravitational mass of the black hole and \( a \) is its angular momentum normalized by mass, \( \Sigma = r^2 + a^2 \cos^2 \theta \) and \( \Delta = r^2 + a^2 - 2Mr \). The event horizon is located at \( r_H = M + \sqrt{M^2 - a^2} \). Since static holes have no rotation, then \( a \to 0 \) and the Kerr horizon coincides with the Schwarzschild one \( r_H = r_S = 2M \). The so-called ergoregion is described by \( r_H < r < r_E(\theta) = M + \sqrt{M^2 - a^2 \cos^2 \theta} \). From here, we will consider \( M = 1 \). Note that in this situation the horizons are \( r_H = 2 \) for \( a = 0 \) (static BH) and \( r_H = 1 \) for \( a = 1 \) (extremal BH).

![Rotating black hole with magnetic fields near its horizon](image)

\( r = r_+ = 1 + \sqrt{1 - a^2} \)

\(-2(1 + \sqrt{1 + a}) < t < 2(1 + \sqrt{1 - a})\)

FIG. 1: Rotating black hole with magnetic fields near its horizon. Here when a charged particle is orbiting in the innermost stable circular orbit (ISCO) of a rotating black hole, an incoming neutral or charged particle can shot it, expelling it to infinity. Figure adapted from [15].

The last stable circular orbit around Kerr BHs is called ISCO (inner stable circular orbits). The influence of spin in positioning the ISCO follows the idea that the greater the spin \( a \), the more displaced the minimum of the potential is. This displacement occurs in the direction of the horizon, indicating that the ISCO is closer to the horizon as the value of the spin \( a \) approaches 1. For a test particle of mass \( m_0 = 1 \), this potential \( V(r) \) is calculated from radial geodesics as

\[
\frac{1}{2} \dot{r}^2 + V(r) = 0.
\]

The minimum of potential \( V(r) \) describes where the ISCO is located for some possible spin \( a \) (see Fig. 2). The greater the spin is the closer to the horizon the ISCO become.

Considering the motion of neutral or charged particles near rotating BHs in a background described by [1], i.e., a vacuum rotating black hole, the conserved quantities are attached to Killing vectors \( \xi(t) = \xi^\mu(t) \partial_\mu = \frac{\partial}{\partial t} \) and \( \xi(\phi) = \xi^\mu(\phi) \partial_\mu = \frac{\partial}{\partial \phi} \). The first one is related to the free test particle energy conservation

\[
\mathcal{E} = -g_{\mu\nu} p^\mu,
\]
and the other to the free test particle angular momentum conservation

$$\ell = -g_{\phi \mu} p^\mu.$$ \hspace{1cm} (9)

Here the range of $\ell$, the angular momentum per unit rest mass, for geodesics falling in is $-2(1 + \sqrt{1 + a}) < \ell < 2(1 + \sqrt{1 - a})$.

When two neutral or charged particles approach the horizon of a black hole, since the background is curved, it is necessary to define the center-of-mass frame properly. The center-of-mass energy of the two particle system (each with mass $m_0$) is given by [15]

$$E_{\text{c.m.}} = m_0\sqrt{2}\sqrt{1 - g_{\mu \nu}u^{\mu}_{(1)}u^{\nu}_{(2)}},$$ \hspace{1cm} (10)

where $u^{\mu}_{(1)}$ and $u^{\nu}_{(2)}$ are the 4-velocities of each particle, properly normalized by $g_{\mu \nu}u^{\mu}_{(1)}u^{\nu}_{(2)} = -1$. It is awaited that the main conditions for accelerating the particles are regarded from the BH spin. We will show in next sections that magnetic fields also could affect the particle motion.

FIG. 2: Effective potential for marginally bound critical particles (no magnetic field situation). The potential must be non-positive in the allowed region of particle motion. The minimum describes where the ISCO is located for some possible spin $a$. The greater the spin is the closer to the horizon the ISCO become.

III. ELECTRICALLY CHARGED PARTICLES IN THE PRESENCE OF MAGNETIC FIELDS

When a charged particle is orbiting in the innermost stable circular orbit (ISCO) of a rotating black hole, an incoming particle (here we are considering a charged one) can shot it, leading to three possible outcomes: the charged particle is expelled to infinity, or it could be trapped by the BH, or it converges to stability and persists orbiting in the ISCO.

On the other hand, many observations point that a magnetic field is present in the center of AGN environment.Astrophysically, the origin of such a magnetic field is explained from plasma motion connected to accretion disks [29]. It is supposed that magnetic fields at the AGN core do not perturb the geometry of BHs. Otherwise, they actually could affect and alter the motion of charged particles that are orbiting the central black hole [30].

Only a certain range of magnetic fields actually cause the acceleration of the particle in positions increasingly farther from the horizon. Otherwise, magnetic fields of great magnitude can cause the particle to be trapped in the system, prompting the particle to fall into the black hole. In particular, the observation of the twin-jet system of NGC 1052 at 86 GHz with the Global mm-VLBI Array, derived the magnitude of magnetic fields at the very center of AGNs: it would be between $B \sim 200$ G and $8.3 \times 10^4$ G at 1 Schwarzschild radius [31]. See also [32–34] for similar estimations.

For a particle immersed in an uniform magnetic field in a curved space, its Lagrangian is

$$\mathcal{L} = \frac{1}{2}g_{\mu \nu} \dot{x}^\mu \dot{x}^\nu + \frac{qA_\mu \dot{x}^\mu}{m_0},$$ \hspace{1cm} (11)
where $A_{\mu}$ is the electromagnetic 4-potential, $q$ is the particle charge and $m_0$ is its mass. Respecting the Lorentz gauge $A_{\mu,\nu} = 0$ one can choose for example \[ A_{\mu} = \left( \frac{2am_0B}{q}, 0, 0, \frac{m_0B}{q} \right), \] where $B = \frac{qE}{2m_0}$. The 4-potential is invariant under the symmetries which correspond to the Killing vectors, i.e., $A_{\mu,\nu} + A_{\nu,\mu} = 0$. The 4-momentum of the particle in this case is $p_{\mu} = m_0u_{\mu} + qA_{\mu}$ and the conserved quantities in eqs. (8) and (9) are written as

\[ E = -g_{\mu\nu}(m_0u^\mu + qA^\mu), \]

\[ \ell = -g_{\phi\mu}(m_0u^\mu + qA^\mu). \]

**IV. MAIN RESULTS AND DISCUSSION**

The trajectory of particles is very sensitive to the presence of magnetic fields, even for small values of the magnetic fields. The Euler-Lagrange equations from Lagrangian (11), considering the conserved quantities in (13) and (14),
where $\ell$ is the angular momentum of the particle and $E = 1$ the energy, and the normalization condition $u^\mu u_\mu = -1$, lead to the following system of equations where it is possible to calculate the solution for the geodesic equations $\dot{t}$, $\dot{\phi}$, $\dot{r}$ and $\dot{\theta}$:

$$
\dot{t} = -\frac{1}{r^2} [a(aE(r, \theta) - L(r, \theta)) + (r^2 + a^2)T/\Delta],
$$

and

$$
\dot{\phi} = -\frac{1}{r^2} [(aE(r, \theta) - L(r, \theta)) + aT/\Delta],
$$

$$
-\frac{1}{2} \left( \frac{\partial g_{rr}}{\partial \theta} \right) \dot{r}^2 - \frac{1}{2} \left( 2a^2 \cos \theta \sin \theta + \frac{\partial g_{\theta\theta}}{\partial \theta} \right) \dot{\theta}^2 + 2a \dot{r} \dot{\theta} = \frac{1}{2} \left( \frac{\partial g_{tt}}{\partial \theta} \right) \dot{t}^2 + \frac{1}{2} \left( \frac{\partial g_{\phi\phi}}{\partial \theta} \right) \dot{\phi}^2 + \left( \frac{\partial g_{t\phi}}{\partial \theta} \right) \dot{t} \dot{\phi},
$$

$$
g_{rr} \dot{r}^2 + g_{\theta\theta} \dot{\theta}^2 = -1 - g_{tt} \dot{t}^2 - g_{\phi\phi} \dot{\phi}^2 - 2g_{t\phi} \dot{t} \dot{\phi},
$$

where $E(r, \theta) = 1 - 2aB + 2aBr/\Sigma$, $L(r, \theta) = \ell + 2a^2 Br/\Sigma - (r^2 + a^2)B$, $T(r, \theta) = E(r, \theta)(r^2 + a^2) - L(r, \theta)a$. In particular, the system of equations (17) e (18) for geodesics $\dot{r}$ and $\dot{\theta}$ is solved as

$$
\dot{r} = \pm \sqrt{-W \pm \sqrt{W^2 - 4YF^2}}/2Y,
$$

$$
\dot{\theta} = \sqrt{G - \frac{g_{rr} \dot{r}^2}{g_{\theta\theta}}},
$$

where

$$
Y = \frac{\partial g_{rr}}{\partial \theta} + 16r^2 g_{rr},
$$

$$
W = 2 \frac{\partial g_{rr}}{\partial \theta} F - 16r^2 G,
$$

FIG. 5: (a) Radial velocity for $B = 0$ out of the equatorial plane and for some values of $a$. (b) The same for $B = 0.01$. 

\[ F = \left( \frac{\partial g_{t t}}{\partial \theta} \right) \dot{t}^2 + \left( \frac{\partial g_{\phi \phi}}{\partial \theta} \right) \dot{\phi}^2 + 2 \left( \frac{\partial g_{t \phi}}{\partial \theta} \right) \dot{t} \dot{\phi}, \] (23)

\[ G = \frac{-1 - g_{tt}\dot{t}^2 - g_{\phi\phi}\dot{\phi}^2 - 2g_{t\phi}\dot{t}\dot{\phi}}{g_{\theta\theta}}. \] (24)

The dot refers to proper time derivatives. Figs. 3, 4 and 5 show the variation of radial geodesic \( \dot{r} \) for values of \( a \) and \( B \), in the equatorial plane or not. Fig. 6 shows the behavior of \( \dot{\theta} \) geodesics for four different values of magnetic fields. The action of the magnetic field on motion in Figs. 3-(b), 4-(b) and 6 is to accelerate the particle, indicating the possibility that Kerr black holes can act as natural accelerators. On the other hand, the radial motion for particles moving along geodesics out the equatorial plane (\( \theta = \pi/3 \)) show that particles are found to get trapped by the BHs if the value of magnetic fields and BH spin are small (see Fig. 5).

From the calculated geodesic quantities, one can find the energy of center of mass for a collision of two charged particles. Calculation of center of mass energy from particle collision comes from

\[ E_{c.m.} = m_0 \sqrt{2} \sqrt{1 - g_{\mu\nu}u^\mu_{(1)}u^\nu_{(2)}}, \] (25)

and
FIG. 8: Variation of $E_{c.m.}$ for (a) $a = 0.9999$ and (b) $a = 1.0$, for four values of magnetic field $B$. Before the horizon $r = 1$, the particle is accelerated to infinity. Magnetic field acts accelerating particles further and further away from the horizon.

FIG. 9: The presence of magnetic fields changes the position of ISCO. The greater the magnetic field magnitude is the farther the ISCO position become. In this situation, the ISCO particle is accelerated from positions more distant of the horizon than if there is no magnetic field. It is confirmed from results plotted in Fig. 8.

\[ g_{\mu\nu}u_{(1)}^{\mu}u_{(2)}^{\nu} = \left( \frac{2}{r} - \frac{a^2}{r^2} - 1 \right) \frac{T_1 T_2}{\Delta^2} + \frac{(a E_1 - L_1)(a E_2 - L_2)}{r^2} + \frac{r^2}{\Delta} \dot{r}(1) \dot{r}(2), \]  

with

\[ E_1(r, \theta) = E_2(r, \theta) = 1 - 2aB + 2a Br/\Sigma, \]  

\[ L_1(r, \theta) = \ell_1 + 2a^2 Br/\Sigma - (r^2 + a^2)B, \quad L_2 = \ell_2 + 2a^2 Br/\Sigma - (r^2 + a^2)B, \]  

\[ T_i(r, \theta) = E_i(r, \theta)(r^2 + a^2) - L_i(r, \theta)a. \]  

The plots of $E_{c.m.}$ energy are shown in Figs. 7a and 7b and Fig. 8 for some values of $a$ and $B$. 

![Fig. 10: Axial angular acceleration.](image)

V. CONCLUDING REMARKS

In the present paper we have studied the motion of charged particles near magnetized rotating black holes and the related energetic processes in the vicinity of the Kerr BH horizon. It is investigated the collision of two charged particles falling freely from rest at infinity. Theoretically it is possible to extract all the rest energy of a mass by lowering it into a Schwarzschild BH, and even more energy applying a Penrose process lowering such mass into a Kerr BH. This is the same to say that the rotation dynamics is, in principle, a more than sufficient source of energy for energizing powerful relativistic jets and, in consequence, high energy charged particles. In this respect, we have shown under what conditions particles can escape from the vicinity of the black hole to spatial infinity and the viability of magnetic extraction to drive the acceleration of charged particles.

The presented results follows similar conclusions in literature that points to the possibility that Kerr BHs could indeed act as particle accelerators. In this respect, we have discussed the possibility of degeneracy between the BH spin and the local magnetic field. Here it was showed that it appears that certain values of fine-tuned axial magnetic fields also could help in the process of acceleration (in the cases of large and critical $a$’s, Fig. 8). Otherwise, axial magnetic fields could cause a damping effect beyond certain $B$ values. Besides, the investigation of charged particle motion near magnetized BHs can play an important role to test gravity theories in the strong gravitational field regime.
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