Annihilation Contributions in $B \to K_1 \gamma$ decay in next-to-leading order in LEET and CP-asymmetry

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Abstract

The effect of weak annihilation and $u$-quark penguin contribution on the branching ratio $B \to K_1 \gamma$ at next-to-leading order of $\alpha_s$ are calculated using LEET approach. It is shown that the value of LEET form factor remains the same in the range of unitarity triangle phase $\alpha$ favored by the Standard Model. CP-asymmetry for above mentioned decay has been calculated and its suppression due to the hard spectator correction has also been incorporated. In addition, the sensitivity of the CP-asymmetry on the underlying parameters has been discussed.

Exclusive decays involving $b \to s \gamma$ transition are best exemplified by the decay $B \to K^* \gamma$, which provide abundant issue for both theorists and experimentalists. Higher resonances of kaons such as $K_2^*$ (1430) are also measured by CLEO [1] and the B factories [2, 3]. Recently, Belle [4] has announced the first measurement of $B \to K_1^+(1270) \gamma$

$$\mathcal{B}(B^+ \to K_1^+ \gamma) = (4.28 \pm 0.94 \pm 0.43) \times 10^{-5}$$  \hspace{1cm} (1)

There are several reason to focus on higher kaon resonances. First and the most promising is that they share lot of things with $B \to K^* \gamma$, like at quark
level both of them are governed by $b \to s\gamma$. Therefore all the achievements of $b \to s\gamma$ can be used in these decays, e.g. the same operators in the operator product expansion and the same Wilson coefficients that are available. The light cone distribution amplitudes (DA) are same except the overall factor of $\gamma_5$ and this gives few differences in many calculations\cite{5}. Secondly, it was suggested that $B \to K_{\text{res}} (\to K\pi\pi) \gamma$ can provide a direct measurement of the photon polarization\cite{6} and it was shown that large polarization asymmetry $\approx 33\%$ has been produced due to decay of $B$ meson through the kaon resonances. In the presence of anomalous right-handed couplings, the polarization can be severely reduced in the parameter space allowed by current experimental bounds of $B \to X_s \gamma$. It was also argued that the $B$ factories can now make a lot of $B\bar{B}$ pairs, enough to check the anomalous couplings through the measurement of the photon polarization.

The theorists are also facing challenges from the discrepancy between their predictions and experiments. It was pointed out that the form factor obtained using the LEET approach for $B \to K^*\gamma$ is found to be smaller compared to the values obtained by QCD sum rules or light-cone sum rules (LCSR)\cite{7}. At this stage, the source of this mismatch is not well understood.

On $B \to K_1\gamma$ side the situation is more complicated. Based on the QCDF framework combined with the LCSR results, it is predicted that $B(B^0 \to K_1^0(1270)\gamma) = (0.828 \pm 0.335) \times 10^{-5}$ at the NLO of $\alpha_s$ which is very small as compared to the experimental value [cf. Eq. (1)]\cite{5}. The value of the relevant form factor has been extracted from the experimental data and its value is found to be $F_{K_1^0(1270)}^+(0) = 0.32 \pm 0.03$ which is very large as compared to $F_{K_1^0(1270)}^+(0)|_{\text{LCSR}} = 0.14 \pm 0.03$ obtained by the LCSR. This is contrary to the case of $B \to K^*\gamma$ where the form factor obtained from LCSR is larger than the LEET one and the source of discrepancy is not yet known. But for $B \to K_1\gamma$ case the possible candidates to explain this discrepancy, like higher twist effects in DA, non zero mass effects of axial kaon, the framework of QCDF, possible mixing in $K_1(1270)$ and $K_1(1400)$ and annihilation topologies, have also been discussed in detail in the literature\cite{8}. The calculation done in \cite{8} is for the leading twist and it was pointed out that higher twist may have some effect on the form factors because all others are not the suitable candidates. Recently it has been shown that the value of form factor is not sensitive to the higher twists\cite{9}.

In this paper the effect of weak annihilation and also the $u$-quark contribution $A^u$ from the penguin to the branching ratio for $B \to K_1\gamma$ at NLO of $\alpha_s$.
are calculated using the LEET approach \cite{12, 13}. We have followed the same frame work as done by Ali et. al. \cite{7} for \( B \to K^*\gamma \), because \( B \to K_1\gamma \) shares many things with it. As it is pointed out in the literature on \( B \to K^*\gamma \), the effect of annihilation contribution to the charmed quark part of amplitude is numerically small because only the penguin operator with tiny Wilson coefficients can contribute. On the other hand the annihilation contribution to the up-quark part of the amplitude contributes significantly because of large Wilson coefficients but again the CKM suppression \( \left| \frac{\lambda_u^{(s)}}{\lambda_c^{(s)}} \right| \approx 0.02 \) puts this large correction for \( B \to K_1\gamma \) into perspective \cite{14}. Finally, by incorporating these annihilation and up-quark contributions we compute the CP-asymmetry \( A_{CP} \left( K_{\pm}^1\gamma \right) \) involving the decay \( B \to K_1\gamma \). The CP-asymmetry arises due to the interference of the various penguin amplitudes which have clashing weak phases, with the required strong interaction phase provided by the \( \mathcal{O}(\alpha_s) \) corrections entering the penguin amplitudes via the Bander-Silverman-Soni (BSS) mechanism \cite{15}. We find that the hard spectator corrections reduce the CP-asymmetry calculated from the vertex contributions alone. The resulting CP-asymmetry, depend rather sensitively on the ratio of the quark masses \( m_c/m_b \). This parametric dependence, combined with the scale dependence of \( A_{CP} \left( K_{\pm}^1\gamma \right) \) makes the prediction of direct CP-asymmetry rather unreliable and present work will be devoted to this issue.

The effective Hamiltonian for \( b \to s\gamma \) can be written as

\[
\mathcal{H}_{\text{eff}}(b \to s\gamma) = -\frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{8} C_i(\mu) O_i(\mu) ,
\]

where

\[
\begin{align*}
O^{(p)}_1 &= (\bar{s}_i p_j)_{V-A} (\bar{p}_j b_i)_{V-A} , \\
O^{(p)}_2 &= (\bar{s}_i p_j)_{V-A} (\bar{p}_j b_j)_{V-A} , \\
O_3 &= (\bar{s}_i b_i)_{V-A} \sum_{q=u,c,t} (\bar{q}_j q_j)_{V-A} , \\
O_4 &= (\bar{s}_i b_j)_{V-A} \sum_{q=u,c,t} (\bar{q}_j q_i)_{V-A} , \\
O_5 &= (\bar{s}_b b_i)_{V-A} \sum_{q=u,c,t} (\bar{q}_j q_j)_{V+A} , \\
O_6 &= (\bar{s}_b b_j)_{V-A} \sum_{q=u,c,t} (\bar{q}_j q_i)_{V+A} , \\
O_7 &= \frac{e m_b}{8 \pi^2} \bar{s}_i \sigma^{(1 + \gamma_5)} b_i F_{\mu\nu} ,
\end{align*}
\]
\[ O_8 = \frac{g_s m_b}{8 \pi^2} \bar{s}_i \sigma^{\mu \nu} (1 + \gamma_5) T^a_{ij} b_j G_{\mu \nu}^a. \]  

(3)

Here \( i, j \) are color indices and \( p \) stands for \( u \) or \( c \) quark. We neglect the CKM element \( V_{ub} V_{us}^* \) as well as the \( s \)-quark mass. The leading contribution to \( B \to K \gamma \) comes from the electromagnetic operator \( O_7 \) as shown in Fig. 1.

As in the case of the real photon emission \( (q^2 = 0) \), the only form factor appears in the calculation is \( \xi^{(K_1)}_\perp \). Therefore one can write

\[
\langle O_7 \rangle_A \equiv \langle K_1(p', \epsilon) | O_7 | B(p) \rangle = \frac{e m_b}{4 \pi^2} \xi^{(K_1)}_\perp \left[ \epsilon^* \cdot q(p + p') \cdot \epsilon^* - \epsilon^* \cdot \epsilon^* (p^2 - p'^2) + i \epsilon_{\mu \nu \alpha \beta} e^{* \mu} e^{* \nu} q^\alpha (p + p')^\beta \right],
\]

(4)

with \( \epsilon^{* \nu} \) and \( e^\mu \) being the polarization vector for axial kaon and the photon respectively. The decay rate is straightforwardly obtained to be \[ \Gamma(B \to K \gamma) = \frac{G_F^2 \alpha m^2_b m^3_B}{32 \pi^4} |V_{ub} V_{us}^*|^2 \left( 1 - \frac{m^2}{m^2_B} \right)^3 |\xi^{(K_1)}_\perp|^2 |C^{eff(0)}_7|^2, \]

(5)

where \( \alpha \) is the fine-structure constant and \( C^{eff(0)}_7 \) is the effective Wilson coefficient at leading order.

At next to leading order of \( \alpha_s \), one has to consider the contribution from operator \( O_2 \) and \( O_8 \) along with that of the \( O_7 \) in \( B \to K \gamma \) decay. For operator \( O_7 \) all the subleading contributions shown in Fig. 2 are absorbed in the form factor where as the Wilson coefficient contains next to leading order parts

\[ C^{eff}_{7} (\mu) = C^{eff(0)}_{7} (\mu) + \frac{\alpha_s (\mu)}{4 \pi} C^{eff(1)}_{7} (\mu). \]

On the other hand, for operators \( O_2 \) and \( O_8 \) the leading order \( C^{(0)}_2 \) and \( C^{(0)}_8 \) are sufficient for \( C_2 \) and \( C_8 \) because these operators contribute at NLO. Each operator has its vertex contribution and hard spectator contribution terms which are calculated explicitly in [9] and are depicted in Figs. [2-6].

The branching ratio for \( B \to K \gamma \) is given by

\[
\mathcal{B}_{th}(B \to K^* \gamma) = \tau_B \Gamma_{th}(B \to K^* \gamma) = \tau_B \frac{G_F^2 \alpha |V_{ub} V_{us}^*|^2}{32 \pi^4} m^2_{b, pole} M^3 \left[ \xi^{(K_1)}_\perp \right]^2 \left( 1 - \frac{m^2_{K^*}}{M^2} \right)^3 \left| C^{(0)eff}_7 + A^{(1)}(\mu) \right|^2.
\]

(6)
where \( G_F \) is the Fermi coupling constant, \( \alpha = \alpha(0) = 1/137 \) is the fine-structure constant, \( m_{b,\text{pole}} \) is the pole \( b \)-quark mass, \( M \) and \( m_{K_1} \) are the \( B \)- and \( K_1 \)-meson masses, and \( \tau_B \) is the lifetime of the \( B^0 \)- or \( B^+ \)-meson. The value of these constants is used from Refs. [9]. The complex function \( \xi^{(K_1)} \) as a free parameter and we will extract its value from the current experimental data on \( B \to K_1 \gamma \) decays.

The function \( A^{(1)} \) in Eq. (5) can be decomposed into the following three components:

\[
A^{(1)}(\mu) = A^{(1)}_{C_7}(\mu) + A^{(1)}_{\text{ver}}(\mu) + A^{(1)}_{sp}\langle K_1 \rangle (\mu_{sp}) \tag{7}
\]

Here, \( A^{(1)}_{C_7} \) and \( A^{(1)}_{\text{ver}} \) are the \( O(\alpha_s) \) (i.e. NLO) corrections due to the Wilson coefficient \( C_7^{\text{eff}} \) and in the \( b \to s \gamma \) vertex, respectively, and \( A^{(1)}_{sp}\langle K_1 \rangle \) is the \( O(\alpha_s) \) hard-spectator corrections to the \( B \to K_1 \gamma \) amplitude and their explicit expressions are as follows:

\[
A^{(1)}_{C_7}(\mu) = \frac{\alpha_s(\mu)}{4\pi} C^{(1)\text{eff}}_7(\mu), \tag{8}
\]

\[
A^{(1)}_{\text{ver}}(\mu) = \frac{\alpha_s(\mu)}{4\pi} \left\{ \frac{32}{81} \left[ 13C_2^{(0)}(\mu) + 27C_7^{(0)\text{eff}}(\mu) - 9C_8^{(0)\text{eff}}(\mu) \right] \ln \frac{m_b}{\mu} 
- \frac{20}{3} C_7^{(0)\text{eff}}(\mu) + \frac{4}{27} \left[ 33 - 2\pi^2 + 6\pi i \right] C_7^{(0)\text{eff}}(\mu) + r_2(z) C_7^{(0)}(\mu) \right\}, \tag{9}
\]

\[
A^{(1)}_{sp}\langle K_1 \rangle (\mu_{sp}) = \frac{\alpha_s(\mu_{sp})}{4\pi} \frac{2\Delta F^{(K_1)}(\mu_{sp})}{9\xi^{(K_1)}(K_1)} \left\{ 3C_7^{(0)\text{eff}}(\mu_{sp}) 
+ \left[ 1 - \frac{6\alpha_s(K_1)}{\langle \bar{u}^2 \rangle_{(K_1)}(\mu_{sp})} \right] + C_2^{(0)}(\mu_{sp}) \left[ 1 - \frac{h(K_1)z}{\langle \bar{u}^2 \rangle_{(K_1)}(\mu_{sp})} \right] \right\}. \tag{10}
\]

The terms proportional to \( \Delta F^{(\rho)}(\mu_{sp}) \) above are the \( O(\alpha_s) \) hard-spectator corrections which should be evaluated at the typical scale \( \mu_{sp} = \sqrt{\mu\Lambda_H} \) of the gluon virtuality. The complex function \( r_2(z) \) of the parameter \( z = m_c^2/m_b^2 \), and the Wilson coefficients in the above equations can be found in Refs. [10] [11]; the function \( h(\rho)(z, \mu) \) and the dimensionless quantity \( \Delta F^{(\rho)}(\mu) \) are defined through Eqs. (25) and (27), respectively of Ref. [9]. Now \( C_7^{(1)\text{eff}}(\mu) \) and \( A^{(1)}_{\text{ver}}(\mu) \) are process independent and encodes the QCD effects only, whereas \( A^{(1)}_{sp}\langle K_1 \rangle (\mu_{sp}) \) contains the key information about the out going mesons.

The factor \( \frac{6\alpha_s(K_1)}{\langle \bar{u}^2 \rangle_{(K_1)}(\mu_{sp})} \) appearing in the Eq. (10) is arising due to the Gegenbauer moments.
By calculating the numerical value from the above expressions and varying the parameters in the standard range, the value of the form factor is extracted from the experimental measurements and it was found to be
\[ \xi_{K_1}^{(K_1)}(0) = 0.32 \pm 0.03 \]
which is for the leading twist and remains unchanged if one includes the higher twist effects.

It is already pointed out in the literature that it is unlikely if annihilation topology would give considerable contributions, but these are important if one wants to study the CP-asymmetry and this is one of the purpose of this article. Before calculating the CP-asymmetry we will check the effects of annihilation contribution on the branching ratio of \( B \to K_1 \gamma \) decays.

Since weak annihilation is the power correction, we will content ourselves with the lowest order result (\( O(\alpha_s^0) \)) for our estimate and to check its effect on the branching ratio. The reason for including this class of power corrections is that they come with numerical enhancement from the large Wilson coefficients \( C_{1,2} (C_1 \approx 3C_7) \) but are CKM suppressed and thus these contributions are expected to be very small for the decay under consideration.

The amplitude for charged \( B \) meson decay in terms of weak annihilation \( A \), charmed penguin \( P_c \), gluonic penguin \( M \) and short distance amplitude \( P_t \) can be written as [following the notation of]
\[
A \left( B^- \to K^-_i \gamma \right) = \lambda^{(s)}_u a + \lambda^{(s)}_t p \\
A \left( B^0 \to K^0_1 \gamma \right) = \lambda^{(s)}_t \left( P_t + M^{(1)} - P_c^{(1)} \right) + \frac{2}{3} \left( M^{(2)} - P_c^{(2)} \right) 
\]
(11)
where \( \lambda^{(s)}_q = V_{qb} V_{qs}^* \), \( a = A - P_c \) and \( p = P_t + M - P_c \). As it is known \( P_c \approx 0.2A \), \( A \approx 0.3P_t \)
i.e. we can safely neglect charmed penguin \( P_c \) and gluonic penguin \( M \) amplitudes relative to the short-distance amplitude \( P_t \) and weak annihilation amplitude \( A \). Thus Eq. (11) becomes
\[
A \left( B^- \to K^-_i \gamma \right) = \lambda^{(s)}_t p \left( 1 + \frac{\lambda^{(s)}_u d}{\lambda^{(s)}_t p} \right) \\
= \lambda^{(s)}_t p \left( 1 + \epsilon A e^{i \phi A} \frac{\lambda^{(s)}_u}{\lambda^{(s)}_t} \right)
\]
and
\[ A \left( B^0 \to K^0 \gamma \right) = \lambda_i^{(s)} p \]
where \( \epsilon_A e^{i\phi_A} \equiv a/p \), \( \phi_A \) is the strong interaction phase and it disappears in \( \mathcal{O}(\alpha_s) \) in the chiral limit. Hence we will set it equal to zero in the further calculation. Following the same lines as for the charged \( B \) meson the ratio of the branching ratios for charged to neutral \( B \) meson decays can be written as
\[
\frac{\mathcal{B}(B^- \to K^- \gamma)}{\mathcal{B}(B^0 \to K^0 \gamma)} \simeq 1 + \epsilon_A e^{i\phi_A} \frac{V_{ub} V_{us}^*}{V_{tb} V_{ts}^*} F_1 + i F_2
\]
(13)
The estimates in the framework of the light-cone QCD sum rules yields typically [19, 20]: \( \epsilon_A = -0.35 \) and \( \epsilon_A = 0.046 \) for the decays \( B^- \to K^- \gamma \) and \( B^0 \to K^0 \gamma \), respectively. Let’s define
\[
\frac{V_{ub} V_{us}^*}{V_{tb} V_{ts}^*} = e^{i\alpha} = F_1 + i F_2
\]
(14)
where \( \alpha \) is the unitarity triangle phase.

We also recall that the operator basis in \( \mathcal{H}_{eff} \) is larger than what is shown in Eq. (2), in which the operator multiplying the CKM factor \( V_{ub} V_{us}^* \) have been neglected. To calculate CP-asymmetry we have to put them back. Doing this, and using the unitarity relation \( V_{cb} V_{cs}^* = -V_{ub} V_{us}^* - V_{tb} V_{ts}^* \) the effective Hamiltonian reads [21]
\[
\mathcal{H}_{eff} = -\frac{G_F}{\sqrt{2}} \left\{ \frac{V_{tb} V_{ts}^*}{V_{ub} V_{us}^*} \left[ C_7(\mu) O_7(\mu) + C_8(\mu) O_8(\mu) + C_1(\mu) O_1(\mu) + C_2(\mu) O_2(\mu) \right] \right\} .
\]
(15)
In the above equation the ellipses denote the terms proportional to the Wilson coefficients \( C_3 \ldots C_6 \) and we have dropped them because they are very small as compared to \( C_1 \) and \( C_2 \). The operators \( O_{1u} \) and \( O_{2u} \) are defined as
\[
O_{1u}(\mu) = \left( \bar{s}_L \gamma_\mu T^a u_L \right) \left( \bar{u}_L \gamma^\mu T_a b_L \right)
\]
\[
O_{2u}(\mu) = \left( \bar{s}_L \gamma_\mu u_L \right) \left( \bar{u}_L \gamma^\mu b_L \right)
\]
The values of Wilson coefficients in Eq. (15) are same as we have already used in Eqs. (8-10). Thus by including the annihilation contribution and
also the effect of the operator $O_{1u}$ and $O_{2u}$, the branching ratio from Eq. (6) can be written as

$$\mathcal{B}_{th}(B^\pm \to K_1^{\pm} \gamma) = \tau_{B^\pm} \Gamma_{th}(B^\pm \to K_1^{\pm} \gamma)$$

$$= \tau_{B^\pm} \frac{G_F^2 |V_{tb}V_{ts}^*|^2}{32\pi^4} m_b^{2} m_{\text{pole}} M^3 \left(1 - \frac{m_{K_1}^2}{M^2}\right)^3 \left[\xi^{(K_1)}(0)\right]^2$$

$$\times \left\{ (C_7^{(0)\text{eff}} + A_{R}^{(1)})^2 + (F_1^2 + F_2^2) (A_{R}^u + L_{R}^u)^2 \right\} + 2F_1 [C_7^{(0)\text{eff}} (A_{R}^u + L_{R}^u) + A_{R}^{(1)} L_{R}^u] + 2F_2 [C_7^{(0)\text{eff}} A_{I}^u - A_{I}^{(1)} L_{R}^u] \right\} ,$$

(16)

where $L_{R}^u = \epsilon A_{C_7^{(0)\text{eff}}}$ and the subscripts $R$ and $I$ denote the real and imaginary parts of the quantities involved. $A^{(1)}$ is same as defined in Eq. (7) and $A^u$ corresponds to the contribution from $O_{1u}$ and $O_{2u}$ which can be written as

$$A^u(\mu) = \frac{\alpha_s(\mu)}{4\pi} C_2^{(0)}(\mu) [r_2(z) - r_2(0)] - \frac{\alpha_s(\mu_{\text{sp}})}{18\pi} C_2^{(0)}(\mu_{\text{sp}}) \frac{\Delta F_{(K_1)}(\mu_{\text{sp}})}{\xi^{(K_1)}(0)} \frac{h^{(K_1)}(z, \mu_{\text{sp}})}{\langle \bar{u}_L^{-(K_1)}(\mu_{\text{sp}}) \rangle} .$$

(17)

We now proceed to calculate numerically the branching ratios for the decay $B^+ \to K_1^+ \gamma$. Using the value of CKM elements from [22], the values of $A^{(1)}(\mu)$ from [9] and the value of $C_2^{(0)}(\mu)$ from [10, 11], the branching ratio is plotted with unitarity triangle phase $\alpha$ as shown in Fig. [7].

One can easily see that varying the value of $\alpha$ in the range $77^0 \leq \alpha \leq 113^0$ with $\alpha = 93^0$ as the central value, there is a slight change in the value of branching ratio for the decay $B \to K_1(1270) \gamma$ leaving the value of the form factor to be unchanged in this range as shown in the Fig. [8]. We also note that the region of $\alpha$ where the branching ratio is effected is not allowed by the CKM unitarity constraints within the SM which typically yields $77^0 \leq \alpha \leq 113^0$.

We now compute the leading order CP-asymmetry $A_{CP}(K_1^{\pm} \gamma)$ for the decay $B^\pm \to K_1^{\pm} \gamma$. The CP-asymmetry arises from the interference of the penguin operator $O_7$ and the four-quark operator $O_2$[16 17]. The direct CP-asymmetry in the $B^\pm \to K_1^{\pm} \gamma$ is:

$$A_{CP}(K_1^{\pm} \gamma) = \frac{B(B^\pm \to K_1^{\pm} \gamma) - B(B^\pm \to K_1^{\mp} \gamma)}{B(B^\pm \to K_1^{\pm} \gamma) + B(B^\pm \to K_1^{\mp} \gamma)}$$
The dependence of CP-asymmetry on different parameters involved is shown in Fig. [9] and Fig. [10]. In Fig. [9] we have plotted the CP-asymmetry vs the unitarity triangle phase $\alpha$. It is seen that in the SM favored interval of $\alpha$, $77^0 \leq \alpha \leq 113^0$, the CP-asymmetry increases and reaches to its maximum value (on negative side like $K^*$) which is $-0.75$% which reduces to the value $-0.45$% if one includes the hard spectator corrections in addition to the vertex corrections and annihilation contributions.

Fig. [10] shows the plot of $A_{CP}(K^{\pm}\gamma)$ with $\alpha$ at different values of the scale $\mu$. It is very clear that the CP-asymmetry has the marked dependence on the scale $\mu$. The maximum value of CP-asymmetry decreases from $-0.8$% to $-0.3$% in the interval $m_{b,\text{pole}}/2 \leq \mu \leq 2m_{b,\text{pole}}$. A similar discussion for $B \to \rho\gamma$ is given in [7].

In conclusion, we have incorporated the effect of annihilation and $u$-quark penguin contributions on the branching ratio for the decay $B \to (1270)\gamma$. It is shown that the value of LEET form factor remains the same even with inclusion of these annihilation contributions for the value of unitarity triangle phase $\alpha$ favored by Standard Model. Then CP-asymmetry $A_{CP}(K^{\pm}_{1}\gamma)$ for $B \to K_{1}(1270)\gamma$ has also been calculated. The CP-asymmetry received contribution from the hard-spectator corrections which tend to decrease its value estimated from the vertex corrections alone. Unfortunately, the predicted value of CP-asymmetry is sensitive to the choice of scale as well as to the quark mass ratio. The typical value of CP-asymmetry lies around $-0.5$% which is almost same as for $B$ to $K^*$ decays.

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References
[1] T. E. Coan et al. [CLEO Collaboration], Phys. Rev. Lett. 84, 5283 (2000) [arXiv:hep-ex/9912057]
[2] S. Nishida et al. [Belle Collaboration], Phys. Rev. Lett. 89, 231801 (2002) [arXiv:hep-ex/0205025].

[3] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0308021.

[4] Belle Collaboration, K. Abe et al., BELLE-CONF-0411, ICHEP04 11-0656, arXiv:hep-ex/0408138.

[5] J.-P. Lee, Phys. Rev. D 69, 014017 (2004); Proceeding of the 2nd ICFP 03, Seoul, Korea [hep-ph/0312010]; J. P. Lee, Phys. Rev. D69 114007 (2004) [arXiv: hep-ph/0403034].

[6] M. Gronau, Y. Grossman, D. Pirjol, and A. Ryd, Phys. Rev. Lett. 88, 051802 (2002); M. Gronau and D. Pirjol, Phys. Rev. D 66, 054008 (2002).

[7] A. Ali and A.Ya. Parkhomenko, Eur. Phys. J. C 23, 89 (2002).

[8] Y. J. Kwon and J. P. Lee, Phys. Rev. D 71 014009 (2005) arXiv:hep-ph/0409133.

[9] M. Jamil Aslam and Riazuddin, Phys. Rev. D 72 094019 (2005) arXiv:hep-ph/0509082 and reference therein.

[10] C. Greub, T. Hurth, D. Wyler, Phys. Rev. D 54, 3350 (1996) hep-ph/9603404.

[11] K. Chetyrkin, M. Misiak, M. Munz, Phys. Lett. B 400, 206 (1997) [Erratum-ibid. B 425, 414 (1995)] [hep-ph/9612313].

[12] M. J. Dugan and B. Grinstein, Phys. Lett. B 255, 583 (1991).

[13] J. Charles, A. Le Yaouanc, L. Oliver, O. Pene and J. C. Raynal, Phys. Rev. D 60, 014001 (1999) hep-ph/9812358.

[14] S.W. Bosch and G. Buchalla, Nucl. Phys. B621, 459 (2002).

[15] M. Bander, D. Silverman, A. Soni, Phys. Rev. Lett. 44, 7 (1980) [Erratum-ibid. 44, 962 (1980)]

[16] J.M. Soares, Nucl. Phys. B367 (1991) 575; Phys. Rev. D 49 283 (1994).
[17] C. Greub, H. Simma, and D. Wyler, Nucl. Phys. B434 39 (1995); Erratum ibid. 444 447 (1995).

[18] B. Grinstein and D. Pirjol, Phys. Rev. D 62 093002 (2000) arXiv:hep-ph/0002216

[19] A. Ali and V. M. Braun, Phys. Lett. B 359, 223 (1995) arXiv:hep-ph/9506248

[20] A. Khodjamirian, G. Stoll and D. Wyler, Phys. Lett. B 358 129 (1995) arXiv:hep-ph/9506242

[21] A. Ali, H. Asatrian and C. Greub, Phys. Lett. B429 87 (1998) hep-ph/9803314

[22] S. Eideman et al., Phys. Lett. B592 01 (2004)

**Figure Captions**

a): Leading order contribution by operator $O_7$.

1): Feynman diagram contributing to the spectator corrections involving the $O_7$ operator in the decay $B \rightarrow K_1 \gamma$. The curly (dashed) line here and subsequent figures represents a gluon (photon).

2): Feynman diagram contributing to the spectator corrections involving the $O_8$ operator in the decay $B \rightarrow K_1 \gamma$.

Row a: Photon is emitted from the flavor-changing line
Row b: Photon radiation off the spectator quark line.

3): Feynman diagram contributing to the spectator corrections involving the $O_2$ operator in the decay $B \rightarrow K_1 \gamma$.

Row a: Photon is emitted from the flavor-changing line
Row b: Photon radiation off the spectator quark line.

4): Feynman diagram contributing to the spectator corrections involving the $O_2$ operator for the case when both the photon and virtual gluon are emitted from the internal (loop) quark line.

5): Feynman diagram contributing to the spectator corrections involving the $O_2$ operator for the case when only the photon is emitted from the internal (loop) quark line in the $b\gamma$ vertex.
6): Branching ratio for \( B \to K_1\gamma \) decay vs unitarity triangle phase \( \alpha \).

7): Branching ratio for \( B \to K_1\gamma \) decay vs LEET form factor for fixed value of \( \alpha = 93^0 \).

8): CP-asymmetry (\( A_{CP}\% \)) vs the unitarity triangle phase \( \alpha \); dashed line shows the value without hard spectator correction and solid line shows the value with hard spectator correction.

9): CP-asymmetry (\( A_{CP}\% \)) vs the unitarity triangle phase \( \alpha \) for different value of the scale \( \mu \); dashed line shows the value at \( m_{b,\text{pole}}/2 \); solid line shows the value at \( m_{b,\text{pole}} \) and the dotted line shows at \( 2m_{b,\text{pole}} \).
Figure 3
Figure 4
Figure 7
Figure 8
Figure 9
Figure 10