Stochastic exponential stabilization of hybrid systems with time-varying delays via adaptive control

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Abstract

The problem of stochastic stabilization for a class of hybrid dynamical systems with time-varying delays is investigated by adaptive control approach. An adaptive controller is designed to stabilize the hybrid systems, and a sufficient criterion for exponential stabilization is derived based on certain conditions. The time-varying delay observations on the states of noise are made and the least upper bound of time-varying delays is also obtained. Finally, a numerical example is presented to demonstrate the effectiveness of the proposed new techniques.

Introduction: Over recent years there have been increasing interests in the field of hybrid dynamical systems. The asymptotic behaviour of such systems, the stability in particular, has been widely investigated, for example, see [1–3] and the references therein. This is mainly because the application of hybrid systems depends heavily on their stability. Stable hybrid models have been widely used to simulate industrial systems, such as power systems, communication systems, transportation systems and so on. For unstable hybrid systems, effective and reliable controllers need to be designed to make the systems stable in order to achieve desired system performances. Some control methods, such as distributed control, event-triggered control and formation control, have been developed to stabilize and synchronize different types of hybrid systems, such as the work in [4–6]. We consider employing adaptive control to stabilize the hybrid systems. In this case, the system parameter changes with time and operating environment, and the control parameter is adjusted automatically until the controller adapts to the change of parameter. This control method ensures that the tracking is stable and accurate by gradually approaching the system characteristics.

As we all know, circumstance noise exists in nature. One of the nice features of the noise is that it can change the convergence rate of the stable systems and also make the unstable systems stable. In view of practical applications, we investigate the asymptotic behaviour of hybrid systems with time-varying delays by adaptive control with the help of circumstance noise. In this case, the systems are not deterministic but stochastic when being stabilized via circumstance noise. On the other hand, in practical systems, there are always time delays between the observing time and the arrival time of circumstance noise. Time delays of the noise states are considered in [7, 8]. We introduce the time-varying delay observations of the noise state to hybrid systems based on actual situations.

In another research front line, the pth moment stabilization of differential systems via delay feedback controller has been discussed in [9–11] recently. Note that these works are on stochastic stability of small sampling moment for differential systems when 0 < p < 1. Besides, the hybrid differential systems are stabilized only via circumstance noise in this case. For a specific system, it takes more time to achieve stabilization when the system is stabilized only via the noise. Adaptive control was used to synchronize complex networks and exponential synchronization was derived in [12–14]. In comparison, we combine adaptive control with circumstance noise and make use of them to stabilize the hybrid differential systems. The combination of two control methods will make the system performance more desirable. The model structure of the hybrid systems is quite different from that of the complex networks, hence, the method used in complex networks is generally not applicable to hybrid systems. Moreover, we will consider the pth moment exponential stabilization of hybrid systems when p ≥ 2. The large sample moment stabilization differs from the small one when 0 < p < 1. There are circumstance noise and adaptive control in the systems simultaneously, which makes the systems even more complicated. A few results on this issue have been discussed so far because of the presence of adaptive control and circumstance noise at the same time. Above observations motivated us to investigate the stabilization issue for hybrid systems via adaptive control.

The main contributions of this work are made as follows. First, we employ adaptive control and circumstance noise to stabilize hybrid systems and derive the criterion for the pth moment exponential stabilization of the systems. Note that due to the presence of adaptive control together with the circumstance noise for the systems, the whole design process is not an easy task. Second, we make time-varying delay observations on the states of the noise and obtain the supremum of the time-varying delays. Third, we also consider stabilizing the systems only via circumstance noise with time-varying delays.

The formulation and preliminaries are presented in the following section. Exponential stabilization of hybrid systems and the supremum of time-varying delays are discussed in Section 3. A numerical example is presented in Section 4 to demonstrate the effectiveness and availability of the new controller, followed by the conclusion of this paper in Section 5.

Preliminaries: Hybrid systems stem from some engineering and industrial manufacturing. They are usually closely related to physics, electric accessory and machinery systems. Stabilized hybrid systems have been widely applied in manufacturing and engineering [1, 2]. For unstable systems, there is a need for being stabilized by feedback controller before wide applications. Here, we consider the unstable hybrid systems described by

\[ dx(t) = f(x(t))dt + \sigma(t, x(t), x(t - \tau(t)))dW(t), \]

where \( x(t) = (x_1(t), \ldots, x_m(t))^T \in \mathbb{R}^m \) is the state variable at the time \( t \). \( f(x(t)) \) and \( \sigma(t, x(t), x(t - \tau(t))) \) are the activation function. Notice that random error and disturbance exist in the systems, and we simulate them by circumstance noise. \( \sigma(t, x(t), x(t - \tau(t))) \) is the noise intensity function. \( B(t) \in \mathbb{R}^m \) is Brown motion and Wiener process. Note that systems (1) are not applicable in practice if they are not stable. Now we consider designing an adaptive controller for systems (1), together with the circumstance noise to stabilize the system,

\[ dx(t) = \left[ f(x(t)) + u(t) \right] dt + \sigma(t, x(t), x(t - \tau(t)))dW(t), \]

where \( u(t) = d_t g(x(t)) (j = 1, \ldots, m) \) is the control input to be designed, and we set \( D = \text{diag}(d_1, \ldots, d_m) \). We aim to stabilize system (2) to an equilibrium trajectory. To this end, we introduce three assumptions that will be used to develop our main result.

Assumption 2.1. For the activation function \( f(\cdot) \) and the adaptive control function \( g(\cdot) \), there are Lipschitz constants \( k_1 \) and \( k_2 \) such that

\[ |f(x) - f(y)| \leq k_1|x - y|, \quad |g(x) - g(y)| \leq k_2|x - y|, \quad \forall x, y \in \mathbb{R}^m. \]

Assumption 2.2. For the noise intensity function \( \sigma(t, x(t), x(t - \tau(t))) \), the linear growth condition is true. That is, there are positive numbers \( L_1 \) and \( L_2 \) such that

\[ \text{trace}\left\{ \sigma^T(t, x(t), x(t - \tau(t))) \sigma(t, x(t), x(t - \tau(t))) \right\} \leq L_1|x(t)|^2 + L_2|x(t - \tau(t))|^2. \]

Assumption 2.3. For the varying time delays \( \tau(t) \), \( 0 < \max\{\tau(t), \tau(t)\} \leq T < 1 \) hold.

Stochastic exponential stabilization of hybrid systems via adaptive control: The main result of this paper is given in this section. We consider the large sampling moment stabilization of system (2) for \( p \geq 2 \).
Theorem 3.1. Let Assumptions 2.1, 2.2 and 2.3 hold. For any \( \xi(t) \in L^p_{\infty}(\mathbb{R}_+^n) \), system (2) can be almost sure exponentially stabilized via adaptive control and circumstance noise, that is,

\[
\limsup_{t \to \infty} \frac{\log E|\xi(t)|^p}{t} < 0, \quad a.s.
\]  

Therefore, we have that

\[
L'v(t, x(t)) \leq \left( \kappa_1 - \kappa_2 \kappa + \frac{1}{2} (p - 1) L_1 \right) |v(t, x(t))|^p + \frac{1}{2} L_2 (p - 1) \left( 1 - \frac{2}{p} \right) |v(t, x(t))|^p + L_2 (p - 1) |x(t) - \tau(t)|^p = \alpha_1 |v(t, x(t))|^p + \alpha_2 |x(t) - \tau(t)|^p,
\]

where \( \alpha_1 = (\kappa_1 - \kappa_2 \kappa + \frac{1}{2} (p - 1) L_1 + \frac{1}{2} (p - 1) (1 - \frac{1}{p}) L_2) \) and \( \alpha_2 = (p - 1) L_2 \). Then, the parameters in \( D = \text{diag}(d_1(t), \ldots, d_n(t)) \) of the adaptive controller can be chosen as follows,

\[
d_d(t) = -5.8 \kappa_2 u_1 p |v(t)|^{p-1} x(t), \quad p \geq 2.
\]  

Proof. We can write system (2) as the matrix form

\[
dx(t) = \left( f(x(t)) + Dg(x(t)) \right) dt + \sigma(t, x(t), x(t) - \tau(t)) dB(t).
\]  

It follows from Assumption 2.1 that

\[
dx(t) \leq \left( \kappa_1 x(t) + \kappa_2 Dx(t) \right) dt + \sigma(t, x(t), x(t) - \tau(t)) dB(t).
\]  

Take the Lyapunov function candidate,

\[
V(t, x(t)) = |x^T (t) x(t)|^\frac{p}{2} + \sum_{j=1}^n \frac{1}{\mu_j} |d_j(t) + \kappa|^2,
\]  

where \( \kappa > 0 \) is sufficiently large. By Itô’s formula and the Lyapunov function (7), we have

\[
L'v(t, x(t)) = V_x(t, x(t)) + \int_0^t \left[ \sum_{j=1}^n d_j(t) + \kappa \right] \sigma(t, x(t), x(t) - \tau(t)) dB(t)
\]

\[
+ \frac{1}{2} \text{trace} \left[ \sigma^T (t, x(t), x(t) - \tau(t)) \sigma(t, x(t), x(t) - \tau(t)) \right]
\]

\[
\leq \sum_{j=1}^n \frac{1}{\mu_j} |d_j(t) + \kappa| |\sigma(t, x(t), x(t) - \tau(t))| + \frac{1}{2} \text{trace} \left[ \sigma^T (t, x(t), x(t) - \tau(t)) \sigma(t, x(t), x(t) - \tau(t)) \right]
\]

\[
\leq \frac{1}{2} \sum_{j=1}^n |d_j(t) + \kappa| \sigma(t, x(t), x(t) - \tau(t))
\]

\[
+ \frac{1}{2} \text{trace} \left[ \sigma^T (t, x(t), x(t) - \tau(t)) \sigma(t, x(t), x(t) - \tau(t)) \right]
\]

\[
= \frac{1}{2} \text{trace} \left[ \sigma^T (t, x(t)) \sigma(t, x(t)) \right] - \frac{1}{2} \sigma(t, x(t), x(t) - \tau(t))
\]

\[
\left\{ \begin{array}{l}
\left| x(t) \right|^p \left| x(t) \right|^p - |x(t)|^p \left| x(t) \right| ^p \leq \alpha_1 |v(t, x(t))|^p + \alpha_2 |x(t) - \tau(t)|^p
\end{array} \right.
\]

where \( \alpha_1 = (\kappa_1 - \kappa_2 \kappa + \frac{1}{2} (p - 1) L_1 + \frac{1}{2} (p - 1) (1 - \frac{1}{p}) L_2) \) and \( \alpha_2 = (p - 1) L_2 \). Taking \( u = s - \tau(s) \), we have that

\[
E \left| x(t) \right|^p \leq EV(t, x(t)) = V(t_0, \xi(t_0)) + V(t, x(t))
\]

\[
= V(t_0, \xi(t_0)) + \int_{t_0}^t \left[ \alpha_1 |v(s)|^p + \alpha_2 \left| x(s) - \tau(s) \right| ^p \right] ds.
\]

Then, from (9) and (10) we have

\[
E \left| x(t) \right|^p \leq V(t_0, \xi(t_0)) + \int_{t_0}^t \left[ \alpha_1 |v(s)|^p + \alpha_2 \left| x(s) - \tau(s) \right| ^p \right] ds
\]

\[
= V(t_0, \xi(t_0)) + \int_{t_0}^t \left( \int_0^s |v(u)|^p du \right) ds
\]

\[
+ \frac{\alpha_1 \tau}{1 - \tau} \max_{t \leq s \leq t_0} \left| x(s) \right|^p + \frac{\alpha_2 \tau}{1 - \tau} \int_0^t \left| x(s) \right|^p ds.
\]

Note that \( \kappa > 0 \) is sufficiently large, and \( \alpha_1, \alpha_2 > 0 \), thus we have \( \alpha_1 > s \alpha_1 + \alpha_2 > 0 \). For any \( t \geq t_0 \), by (10) we have that

\[
\limsup_{t \to \infty} E \left| x(t) \right|^p \leq \left\{ \begin{array}{l}
V(t_0, \xi(t_0)) + \frac{\alpha_1 \tau}{1 - \tau} \max_{t \leq s \leq t_0} \left| x(s) \right|^p \exp \left( \alpha_1 + \frac{\alpha_2 \tau}{1 - \tau} \right) \right.
\end{array} \right.
\]

\[
\text{where} \quad V(t_0, \xi(t_0)) + \frac{\alpha_1 \tau}{1 - \tau} \max_{t \leq s \leq t_0} \left| x(s) \right|^p > 0 \quad \text{and} \quad \alpha_1 + \frac{\alpha_2 \tau}{1 - \tau} < 0.
\]

Therefore, we have

\[
\limsup_{t \to \infty} \frac{\log E|\xi(t)|^p}{t} \leq \alpha_1 + \frac{\alpha_2 \tau}{1 - \tau} < 0.
\]

The proof is completed. \( \square \)

Remark 1. We have considered the large sampling moment stability for hybrid systems when \( p \geq 2 \), and derived the criterion for stochastic exponential stabilization via adaptive control and circumstance noise. The small sampling moment of differential systems for \( 0 < p < 1 \) has been investigated in [10, 11]. There is a big difference between the convergence of small and large sampling moments, and thus different approach to the problem is adopted compared with the existing works. In addition, we have designed adaptive control and circumstance noise to stabilize hybrid systems. The stability is expedited as much as possible. It is more effective to stabilize the systems by combining these two feedback control methods. The adaptive control is used to synchronize complex networks in [12–14]. In comparison, we have employed adaptive control with circumstance noise to stabilize hybrid systems. The topological structure of hybrid systems is different from that of complex networks.
It should be emphasized that Theorem 3.1 shows that hybrid system (2) can be stabilized via adaptive control and circumstance noise. It is interesting to know that system (2) can be stabilized by circumstance noise without control input when $D = 0$, while system (2) also can be stabilized by the designed adaptive controller without system noise when $\sigma = 0$. Hence we immediately have the following result from Theorem 3.1.

Corollary 3.2. Let Assumptions 2.1 and 2.2 hold, hybrid system \( \dot{x}(t) = f(x(t)) + \sigma(t, x(t), x(t - \tau(t)))d\beta(t) \) is stabilized via circumstance noise with time-varying delays \( \sigma(t, x(t), x(t - \tau(t))) \) if \( \tau(t) \leq \bar{\tau} \), where \( \alpha_1, \alpha_2 \) are defined in Theorem 3.1.

Remark 2. We consider the asymptotic behaviour of hybrid systems in the extreme case in Corollary 3.2. We achieve stochastic stabilization via circumstance noise when $D = 0$, and in this case, the circumstance noise plays a role of feedback controller. Moreover, system (2) can be stabilized via adaptive control when \( \sigma(t) = 0 \), and in this case system (2) is a deterministic one.

Numerical example: Generally it is more difficult to achieve stabilization for oscillator systems than for other systems [15–17]. Consider the following stochastic harmonic oscillators with time-varying delays described by

\[
\dot{x}(t) + (a_0x(t) + u(t)) + 1.5x(t - \tau(t))d\beta(t) = 0, i = 1, \ldots, 4,
\]

where \( x = (x_1, \ldots, x_4) \) with initial value \( x(0) = (0, 1)^T \), and \( u_i = d_i g_i(x(t)), i = 1, \ldots, 4 \), is the adaptive control scheme. \( d\beta(t) \) is Brown motion simulating the circumstance noise. We take the controlled intervals (time/frequency) on \([0, 6\pi]\).

Simple calculations show that the parameters in system (11) meet Assumptions 2.1–2.3. We derive the supremum of time-varying delays \( \bar{\tau} = 0.047 \), and take the value \( \tau = 0.045 < \bar{\tau} \). Simulation results are illustrated in Figures 1–4 corresponding to the original oscillators and three different components of the stabilized system. Figure 1 shows that the original oscillators are finite all the time, and there is no feedback controller in the system, while Figures 2–4 demonstrate that system (11) via adaptive control and circumstance noise is exponential stabilized \( x_i(t) \to 0 \) as \( t \to \infty \) if \( \tau < \bar{\tau} \). Note that the trajectory \( x_3(t) \) rushes to the equilibrium state after a short rise, while the trajectories \( x_2(t), x_4(t) \) go straight to the equilibrium state over a period of 6\( \pi \). The stabilization performance is most closely related to the noise intensity. The circumstance noise and adaptive control expedite the stability of oscillator systems. It can be seen that the running effectiveness of the system is much better because of the combination of adaptive control and circumstance noise compared with the existing works [9–11].

Conclusion: The problem of stabilization has been studied for hybrid systems with time-varying delays by adaptive control method with cir-
currence noise. An adaptive controller is designed to achieve stabilization and the sufficient criterion for exponential stabilization is also obtained. A right upper bound of the time delays has been derived, and it can be enlarged as much as practically possible, which ensures that the systems are almost surely exponentially stable. A numerical example is given to demonstrate the effectiveness of the new approach proposed. In our future work, we will consider extending the results from this paper to the case of stabilization for hybrid systems by both intermittent noise and pinning control. That is, controllers will be designed only for some selected key nodes among the systems and only during some time intervals. Such a design will greatly cut the control cost by reducing the controlled nodes and time.

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Ethical statement: This article is original and has not been published elsewhere. And the study is not split up into several parts to increase the quantity of submissions and submitted to journals over time.

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