Abstract

The aim of this paper was to examine preservice teachers’ knowledge of area formulae. Clinical interview technique was employed to collect the data. This paper reports the analysis of the responses of eight preservice teachers related to a particular task, Task 8. In Task 8, preservice teachers were asked to show a Form One student the way to develop area formulae of a rectangle, parallelogram, triangle, and trapezium. Findings of the study suggest that all of them lack conceptual knowledge underpinning the formula for the area of a rectangle. The implication of the finding was also discussed.

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Keywords: Preservice teachers; Knowledge of area formulae; Case study; Clinical interview

1. Introduction

When students engage in developing formulae, they acquire conceptual understanding of the ideas and relationships involved. Thus, there is less possibility that they will confuse perimeter and area or choose the incorrect formula (Van de Walle, 2007). Moreover, “areas of triangles, parallelograms and trapeziums are related to the area of a rectangle. These relationships form the reasoning for the formulae for the areas.” (Lim-Teo & Ng, 2008, p. 106). Specifically, formulae for calculating the area of other shapes are developed from the formula for the area of a rectangle (O’Daffer & Clemens, 1992).

Van de Walle (2001) pointed out that:

Children should never use formulas without participating in the development of those formulas. Formulas for area and volume should all be developed by children. Developing the formulas and seeing how they are connected and interrelated is significantly more important than blindly plugging numbers into formulas, which is primarily computational tedium. (p. 296)

When students develop formulae, they are engaging in one of the real processes of doing mathematics (Van de Walle, 2007). By doing so, students can realize how all area formulae are related to one unifying idea, namely base
times height. Moreover, “students who understand where formulas come from do not see them as mysterious, tend to remember them, and are reinforced in the idea that mathematics makes sense” (Van de Walle, 2007, p. 399).

Sgroi (2001) suggests that:

The formula for the area of a rectangle can be developed by having children form many rectangles on a geoboard or dot paper, count up the squares inside, and eventually generalize that the area can be found by multiplying the length of the rectangle by the width. (p. 183)

Similar strategy (i.e., inductive method) was recommended by other mathematics educators or researchers (Billstein, Liberskind, & Lott, 2006; Cathcart, Pothier, Vance, & Bezuk, 2006; Cavanagh, 2008; Chua, Teh, & Ooi, 2002; NCTM, 2000; O’Daffer, Charles, Cooney, Dossey, & Schielack, 2005; van de Walle, 2007) to develop or derive the formula for the area of a rectangle.

Bennett and Nelson (2001) stated that “one of the basic principles in finding area is that a region can be cut into parts and reassembled without changing its area” (p. 659). This principle (area conservation) is beneficial in developing the formula for the area of a parallelogram. The formula for the area of a parallelogram can be developed from the formula for the area of a rectangle using the strategy of cut and paste (decompose and rearrange, i.e., decompose a parallelogram into a triangle and a trapezium and then rearrange these shapes to form a rectangle) (Beaumont, Curtis, & Smart, 1986; Billstein et al., 2006; Cathcart et al., 2006; Cavanagh, 2008; Cheang, 2002; Chua et al., 2002; Lim-Teo & Ng, 2008; NCTM, 2000; O’Daffer et al., 2005; van de Walle, 2007). Similarly, the formula for the area of a triangle can be developed from the formula for the area of a rectangle (Billstein et al., 2006; Cathcart et al., 2006; Cavanagh, 2008; Cheang, 2002; Chua et al., 2002; Lim-Teo & Ng, 2008; O’Daffer et al., 2005) or a parallelogram (Beaumont et al., 1986; Cavanagh, 2008; Lim-Teo & Ng, 2008; NCTM, 2000; van de Walle, 2007) using the strategy of partition (i.e., partition a rectangle or parallelogram along its diagonal into two triangles). The area of a triangle is half of the area of the rectangle or parallelogram that encloses it.

Van de Walle (2007) suggests that there are at least ten different methods of developing the formula for the area of a trapezium. Similarly, the formula for the area of a trapezium can be developed from the formula for the area of a rectangle (Cheang, 2002; NCTM, 2000; van de Walle, 2007), a parallelogram (Billstein et al., 2006; Chua et al., 2002; Lim-Teo & Ng, 2008; NCTM, 2000; O’Daffer et al., 2005; van de Walle, 2007), or a triangle (Beaumont et al., 1986; Cathcart et al., 2006; van de Walle, 2007). The formula for the area of a trapezium can be developed using the strategies of cut and paste (decompose and rearrange, e.g., decompose an isosceles trapezium into a rectangle and two triangles and then rearrange these shapes to form a rectangle), duplicate (e.g., duplicate the trapezium and arrange the two trapeziums to form a parallelogram), or algebraic method.

However, the findings of previous studies (Baturo & Nason, 1996; Cavanagh, 2008) revealed that high school students as well as preservice teachers demonstrated a limited understanding of the relationship between the areas of triangle and rectangle. Baturo and Nason, (1996) found that only two of the 13 preservice primary school teachers in their study understand the relationship between the formulae for the area of a triangle and rectangle that encloses it. The area of a triangle is half of the area of the rectangle that encloses it. Similarly, Cavanagh (2008) revealed that high school students in his study demonstrated a limited understanding of the relationship between the areas of triangle and rectangle. The following question remain to be answered: Do the Malaysian preservice teachers understand the relationship among area formulae of rectangle, parallelogram, triangle and trapezium? Thus, the purpose of this paper was to examine preservice teachers’ knowledge of area formulae.

2. Methodology

2.1 Research Design

In this study, the researchers employed case study research design to examine, in-depth, preservice teachers' knowledge of area formulae. “A case study design is used to gain an in-depth understanding of the situation and
meaning for those involved” (Merriam, 1998, p. 19). Several researchers (Aida Suraya, 1996; Chew, 2007; Lim, 2007; Rokiah, 1998; Seow, 1989;Sharifah Norul Akmar, 1997; Sutriyono, 1997) employed case study research design to study Malaysian students, preservice teachers, and lecturers.

2.2 The Subjects

Eight subjects were selected for the purpose of this study. They were preservice teachers from a public university in Peninsula Malaysia enrolled in a 4-year Bachelor of Science with Education (B.Sc.Ed.) program, majored or minored in mathematics. Each preservice teacher was given a pseudonym, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha, in order to protect the anonymity of all interviewees. The brief background information about the subjects is shown in Table 1.

| Subject | Ethnicity  | Gender | Age     | Major   | Minor   | CGPA |
|---------|-----------|--------|---------|---------|---------|------|
| Usha    | Indian    | Female | (21, 9) | Mathematics | Biology | 2.92 |
| Mazlan  | Malay     | Male   | (21, 8) | Mathematics | Chemistry | 2.70 |
| Patrick | Bidayuh   | Male   | (21, 7) | Mathematics | Chemistry | 3.04 |
| Beng    | Chinese   | Female | (22, 9) | Mathematics | Physics | 3.82 |
| Roslina | Malay     | Female | (21, 8) | Biology   | Mathematics | 3.15 |
| Liana   | Malay     | Female | (21, 5) | Chemistry | Mathematics | 2.77 |
| Tan     | Chinese   | Male   | (22, 7) | Chemistry | Mathematics | 3.69 |
| Suhana  | Malay     | Female | (20, 10)| Physics   | Mathematics | 2.52 |

2.3 The Task

This paper reports only the responses of the subjects on Task 8 (see Appendix A). In Task 8, subjects were asked to show a Form One student the way to develop area formulae of a rectangle, parallelogram, triangle, and trapezium.

2.4 Data Collection

Data for this study was collected using clinical interview technique. Materials collected for analysis consisted of audiotapes and videotapes of clinical interviews, subject’s notes and drawings, and researcher’s notes during the interviews.

2.5 Data Analysis

The data analysis process consisted of four levels. At level one, the audio and video recording of the clinical interview were verbatim transcribed into written form. The transcription included the interaction between the researcher and the subject during the interviews as well as the subject’s nonverbal behaviours. At level two, raw data in the forms of transcription were coded, categorized, and analyzed according to specific themes to produce protocol related to the description of subjects’ knowledge of area formulae. At level three, case study for each subject was constructed based on information from the written protocol. At this level, analysis was carried out to describe each subject’s behaviours in solving the task. At level four, cross-case analysis was conducted. The analysis aimed to
identify pattern of responses of knowledge of area formulae held by the subjects. Based on this pattern of responses, preservice teachers' knowledge of area formulae were summarized.

3. Findings

In this section, findings of preservice teachers’ conceptual knowledge of the relationships among area formulae were presented in terms of: (a) rectangle, (b) parallelogram, (c) triangle, and (d) trapezium.

3.1 Rectangle

All the preservice teachers in this study, namely Beng, Liana, Mazlan, Patrick, Roslina, Suhana, Tan, and Usha (pseudonyms), could recall the formula for the area of a rectangle. Nevertheless, none of the eight preservice teachers were able to develop it. They just memorized the formula. None of the eight preservice teachers attempted to develop the formula, except Tan. Tan had attempted to develop the formula but unsuccessful, as shown in Excerpt 1(Tan/L1835-1863). It indicated that all of them have no idea how the formula can be developed or derived. They might have rote-learnt the formula. It was apparent that all of them lack of conceptual knowledge underpinning the formula for the area of a rectangle.

Excerpt 1

R: (Puts a card written the following scenario in front of Tan). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following figures:
(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.
How would you show him the way to develop (derive) the formula for calculating the area of these figures? Let’s start with rectangle.

S: Derive the formula…(silent for a while) ok in our school time, the only thing that teacher teach us is memorizing. If I tell him to memorizing, it is one of the ways to memorize the formula. Have to think quite a hard way to derive this formula because all these formula is a very basic formula already. If you really want to derive for example the rectangle (draws a rectangle with the dimension of 3 cm by 2 cm, as shown in Figure 1). Let’s say this is 2 cm, this is 3 cm. I can cut it into three parts where each part is 1 cm.

R: Could you tell me more about it?
S: Ok the way just now I don’t think it is quite work. But the only way that I can think is I can tell him to remember, memorize the formula of rectangle and then let them understand what is area is all about. That is the surface, the total surface of a particular object. So, the rectangle is the horizontal side (refers to the length of the rectangle) times the vertical side (refers to the width of the rectangle).

Figure 1: Tan draws a rectangle with the dimension of 3 cm by 2 cm

3.2 Parallelogram

Five of the preservice teachers, namely Beng, Mazlan, Patrick, Suhana, and Tan, could recall the formula for the area of a parallelogram. They were able to develop the formula for the area of a parallelogram. Beng, Mazlan, Patrick, and Tan mentally transformed the parallelogram to a rectangle by cutting out a right-angled triangle from
one end of the parallelogram and moved it to the other end of the parallelogram to form a rectangle. Suhana mentally transformed the parallelogram to a rectangle by cutting the parallelogram into two triangles along its diagonal. Suhana mentally moved a triangle from one end of the parallelogram to the other end of the parallelogram to form a rectangle. Excerpt 2 shows how Beng develops the formula (Beng/L1315-1332).

Excerpt 2

R: How would you develop the formula for the area of a parallelogram?
S: (Draws a parallelogram and then writes its area formula, as shown in Figure 2).
(Draws the diagrams, as shown in Figure 3, to show how she develops the formula).
R: Could you explain your solution?
S: If I assume that the student knew the formula for rectangle, so the parallelogram is roughly like this (points to the parallelogram that she has drawn). So, I move this triangle over here to form rectangle. So, it will become a rectangle and this one is still "a". Em this is "b". So, it is just a formula for rectangle.
R: What does the "a" mean?
S: "a" is the length here and "b" is the line here perpendicular to "a".

Figure 2: Beng draws a parallelogram and writes its area formula

Figure 3: Beng develops the formula for the area of a parallelogram

It indicated that Beng, Mazlan, Patrick, Suhana, and Tan understand the relationship between the formulae for the area of a parallelogram and rectangle. A parallelogram can always be transformed into a rectangle with the same base, same height, and the same area. Thus, the formula for the area of a parallelogram is exactly the same as the formula for the area of a rectangle, namely ‘base times height’. Three of the preservice teachers were unable to develop the formula for the area of a parallelogram. It was apparent that they did not know the relationship between the area of a parallelogram and the area of a rectangle. Had they been known of this relationship, they would know how to develop the formula for the area of a parallelogram.

3.3 Triangle

All the preservice teachers could recall the formula for the area of a triangle, except Usha. Only three of them, namely Liana, Suhana, and Tan, attempted to develop the formula. Two of the preservice teachers, namely Liana and Tan, were able to develop the formula for the area of a triangle. Suhana attempted to develop the formula but unsuccessful. Liana developed the formula for the area of a triangle based on the formula for the area of a square. A square is a special case of a rectangle. Excerpt 3 depicts how Liana develops the formula (Liana/ L1317-1329).

Excerpt 3

R: How would you show your student the way to develop (derive) the formula for calculating the area of a triangle?
S: (Develops the formula for the area of a triangle, as shown in Figure 4). Let’s say this is base and this is height. If we have a square here and we divide here, it becomes two triangles here. So, that’s how we get the half. Then, if the formula for the square, the height will be "a" and the base will be "b". So, we simply times "a and b" right. This is the formula for the square. But in the square, that’s lies two triangles. That’s why it’s a half here.

Figure 4: Liana develops the formula for the area of a triangle

Tan developed the formula for the area of a triangle based on the formula for the area of a rectangle. It indicated that they knew the relationship between the formulae for the area of a triangle and rectangle that encloses it. Liana and Tan understand the relationship that the area of a triangle is half of the area of the rectangle that encloses it. Six of the preservice teachers were unable to develop the formula for the area of a triangle. It was quite clear that most of the preservice teachers did not know the relationship between the area of a triangle and the area of the rectangle that encloses it. Had they been known of this relationship, they would know how to develop the formula for the area of a triangle.

3.4 Trapezium

Six of the preservice teachers, namely Beng, Mazlan, Patrick, Suhana, Tan, and Usha, could recall the formula for the area of a trapezium. Of the six preservice teachers who could recall the formula for the area of a trapezium, five of them, namely Beng, Mazlan, Patrick, Suhana, and Tan, attempted to develop the formula. Of the two preservice teachers who could not recall the formula for the area of a trapezium, one of them, namely Liana, attempted to develop the formula. Of the six preservice teachers who attempted to develop the formula, three of them, namely Beng, Suhana, and Tan, were able to develop the formula for the area of a trapezium. All of them developed the formula using algebraic method.

Beng viewed the area of the trapezium as the different between the area of the large rectangle formed and the area of the triangle formed. Thus, the area of the trapezium equals to ‘b x t – \frac{1}{2} (b - a) x t’. She simplified it algebraically to become \( \frac{1}{2} (a + b) \times t \). Suhana developed the formula for the area of a trapezium from the combination of the formulae for the area of a rectangle and a triangle, namely \( (a \times \text{tinggi [height]}) + [(b - a) \times \text{tinggi [height]} \times \frac{1}{2}] \) using algebraic method. She correctly simplified it as \( \frac{1}{2} \times \text{tinggi [height]} \times (a + b) \), which is the formula for the area of a trapezium. Excerpt 4 demonstrates how Suhana develops the formula (Suhana/L1619-1661).

**Excerpt 4**

R: How would you develop (derive) the formula for calculating the area of a trapezium?
S: Trapezium. (Draws a trapezium and then writes its area formula, as shown in Figure 5). This is "a", "b", and \text{tinggi [height]}. Ok half times \text{tinggi [height]} times (a plus b). Or divide into two: (a times \text{tinggi [height]}) plus (b – a) times \text{tinggi [height]}. "times half" missing.
R: Could you show me how it is?
S: Ok this is the original formula (points to first formula, \( \frac{1}{2} \times \text{tinggi [height]} \times (a + b) \), as shown in Figure 5).
R: Could you show me how it is?
S: It is half times tinggi [height], this one (points to "a") and this one (points to "b") we have to add on first, "a", "b" like this one. And then for the second part, we identify it into two parts: (a times tinggi [height]) plus (b – a) times tinggi [height] ("times half" missing).
R: Could you show me how it is?
S: So, eh (b – a) times tinggi [height] times half (realizes the missing part of "times half").
R: Could you show me how do you get this formula (points to the first formula, as shown in Figure 5) from here (points to the second formula, as shown in Figure 5)?
S: This one (points to the first formula, as shown in Figure 5) derive from this one (points to the second formula, as shown in Figure 5).
R: How?
S: Factorize strategy. (Develops the formula for the area of a trapezium, as shown in Figure 6). (First attempt to develop the formula): And then got "a plus b minus a times \( \frac{1}{2} \)…(silent for a while) (realizes a mistake and then cancels it, as shown in Figure 6). (Second attempt to develop the formula): Tinggi [height], "a plus b minus a" and then equal to tinggi [height], "a minus \( \frac{1}{2} a \) equal to \( \frac{1}{2} a \) plus \( \frac{1}{2} b \). Therefore, factorize \( \frac{1}{2} \) times tinggi [height] times (a plus b).

Figure 5: Suhana draws a trapezium and then writes its area formula

Figure 6: Suhana develops the formula for the area of a trapezium

Tan developed the formula for the area of a trapezium using the combination of the formula for the area of a triangle and a rectangle or a square. Tan wrote the formula for the total area of a rectangle or a square, and a triangle as \( (AB \times AC) + (\frac{1}{2} \times BE \times ED) \). He then used the algebraic method to simplified it as \( \frac{1}{2} AC \ (AB + CD) \) which is the formula for the area of a trapezium.

It indicated that Beng, Suhana, and Tan knew that the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle. The formula for the area of a trapezium is also related to the formula for the area of a parallelogram. Nevertheless, five of the preservice teachers were unable to develop the formula for the area of a trapezium. It was quite clear that they did not know the relationship between the area formulae of a rectangle, parallelogram, triangle, and trapezium. Had they been known of this relationship, they would know how to develop the formula for the area of a trapezium.

4. Conclusion

None of the preservice teachers in this study were able to develop the formula for the area of a rectangle. They might have rote-learnt the formula. It was apparent that all of them lack conceptual knowledge underpinning the formula for the area of a rectangle. However, five preservice teachers were able to develop the formula for the area
of a parallelogram. They mentally transformed the parallelogram to a rectangle. It indicated that they understand the relationship between the formulae for the area of a parallelogram and rectangle.

Only two preservice teachers were able to develop the formula for the area of a triangle. They developed the formula for the area of a triangle based on the formula for the area of a rectangle or a square. It indicated that they knew the relationship between the formulae for the area of a triangle and rectangle or square that encloses it. This finding is in concurrence with the findings of previous studies (Batur & Nason, 1996; Cavanagh, 2008). Three preservice teachers were able to develop the formula for the area of a trapezium. They developed the formula for the area of a trapezium from the formulae for the area of a rectangle and a triangle. It indicated that they knew the formula for the area of a trapezium is related to the formulae for the area of a rectangle and triangle.

The implication of this finding is that mathematics teacher educators need to organize teaching and learning activities that provide opportunity for their preservice teachers to investigate and develop the formulae for the area of rectangle, triangle, parallelogram, and trapezium in a logical progression and meaningful way. This is in line with the recommendations in the Form One Mathematics Curriculum Specification (Ministry of Education Malaysia, 2003) which suggested that teaching and learning activities in the classroom to provide opportunity for the students to investigate and develop the formulae for rectangle, parallelogram, triangle, and trapezium.

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Appendix A

Task 8: Developing area formulae

(Puts a handout written the following scenario in front of the subject). Suppose that a Form One student comes to you and says that he does not know how to develop (derive) the formula for calculating the area of the following shapes:

(a) Rectangle,
(b) Parallelogram,
(c) Triangle, and
(d) Trapezium.

How would you show him the way to develop (derive) the formula for calculating the area of these shapes?

Probes:
What do you mean by ____?
Could you tell me more about it?

Note: Area formulae for rectangle, parallelogram, triangle, or trapezium will be given to the subjects if they can not recall it.