Can The Majorana neutrino CP-violating phases be restricted?

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We reanalyze the constraints in neutrino masses and MNS lepton mixing parameters using the new data from the terrestrial (KamLAND) and astrophysical (WMAP) observations together with the HEIDELBERG-MOSCOW double beta decay experiment. It leads us to the almost degenerate or inverse hierarchy neutrino mass scenario. We discuss the possibility of getting the bound for the Majorana CP violating phase.

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Recently the two important experimental results on neutrino physics have been successively released. One comes from the KamLAND [1] and the other does from the WMAP [2]. In this letter, by using these values together with Heiderberg-Moscow result [3], we constrain one of the two Majorana phases in the framework of our treatment [4]. The other Majorana phase cannot be restricted because of the smallness of $U_{e3}$. We use the following experimental values.

1) Heiderberg-Moscow result on the averaged neutrino mass [3]

$$\langle m_\nu \rangle = 0.39 \text{[eV]} \ (\text{best fit})$$  
$$= 0.11 - 0.56 \text{[eV]} \ (95\% \text{CL}). \quad (1)$$
Here Majorana particles, there appear three CP around the best fit. (1σ suppose the experimental data are distributed as a normal (Gaussian) distribution \(3\), Fig.1 of Hannestad \(8\) and Fig.4 of Holand-Smirnov \(5\), sidered to be not so bad from Table 2 in the paper of Klapdor-Kleingrothaus et al.

(2) WMAP result on the neutrino masses \(^2\)

\[
\sum_{i=1}^{3} m_i < 0.70 \text{ [eV]} \text{ (95\%CL).} \tag{2}
\]

(3) Solar neutrino \& KamLAND (l-LMA solution) \(^5\)

\[
sin^2 2\theta_{12} = 0.82 \text{ (best fit),}
\]

\[
= 0.70 - 0.96 \text{ (95\%CL).} \tag{3}
\]

(4) CHOOZ \(^6\)

\[
sin^2 \theta_{13} < 0.03 \text{ (90\%CL).} \tag{4}
\]

The differences of the squared masses \(\Delta m^2_{ij} \equiv |m^2_j - m^2_i|\) measured by neutrino oscillation experiments are not sensitive to our phase analysis. Therefore we only use these best fit values \(^6\)

\[
\Delta m^2_{12} = 7.32 \times 10^{-5} \text{ [eV]}^2 \text{ (l-LMA),}
\]

\[
\text{and} \quad \Delta m^2_{23} = 2.5 \times 10^{-3} \text{ [eV]}^2 \text{ (Atmospheric } \nu \text{ exp.)} \tag{5}
\]

Moreover, we estimate very roughly the errors of \(\langle m_\nu \rangle^2, \sum_{i=1}^{3} m_i, \sin^2 2\theta_{12}\) and suppose the experimental data are distributed as a normal (Gaussian) distribution around the best fit. \((1\sigma = 68.3 \% \text{ CL}, 1.45\sigma = 85.0 \% \text{ CL}, 1.65\sigma = 90.0 \% \text{ CL,} 1.96\sigma = 95.0 \% \text{ CL})\) Namely, we use the following values.

\[
\langle m_\nu \rangle^2 = 0.39^2 \pm (0.39^2 - 0.11^2) \times \frac{1.65}{1.96} = 0.15 \pm 0.12 \text{ (90\%CL),} \tag{6}
\]

\[
\sum_{i=1}^{3} m_i < 0.00 - (0.00 - 0.70) \times \frac{1.65}{1.96} = 0.59 \text{ (90\%CL),} \tag{7}
\]

\[
\sum_{i=1}^{3} m_i < 0.00 - (0.00 - 0.70) \times \frac{1.45}{1.96} = 0.52 \text{ (85\%CL).} \tag{8}
\]

\[
\sin^2 2\theta_{12} = 0.82 \pm (0.82 - 0.70) \times \frac{1.65}{1.96} = 0.82 \pm 0.11 \text{ (90\%CL).} \tag{9}
\]

The assumptions of normal distribution in \(\langle m_\nu \rangle^2, \sum_{i=1}^{3} m_i, \text{ and } \sin^2 2\theta_{12}\) are considered to be not so bad from Table 2 in the paper of Klapdor-Kleingrothaus et al. \(^8\) Fig.1 of Hannestad \(^8\) and Fig.4 of Holand-Smirnov \(^5\).

Maki-Nakagawa-Sakata (MNS) mixing matrix \(U\) takes the following form in the standard representation:

\[
U = \begin{pmatrix}
    c_1 c_3 & s_1 c_3 e^{i\beta} & s_3 e^{i(\rho - \phi)} \\
    c_1 e^{-i\beta} & c_1 c_2 - s_1 s_2 s_3 e^{i\phi} & s_2 s_3 e^{i(\rho - \beta)} \\
    s_2 c_3 & (s_1 s_2 - c_1 s_3 e^{i\phi}) e^{-i\rho} & c_2 c_3
\end{pmatrix}. \tag{10}
\]

Here \(c_j = \cos \theta_j, s_j = \sin \theta_j \ (\theta_1 = \theta_{12}, \theta_2 = \theta_{23}, \theta_3 = \theta_{31})\). Note that, for Majorana particles, there appear three CP violating phases, the Dirac phase \(\phi\) and
the Majorana phases $\beta$, $\rho$. Irrespectively of the $CP$ violating phases we have the inequality\[\sum_{j=1}^{3} U_{ej}^2 m_j \right| \langle m_\nu \rangle \equiv \frac{|U_{e1}^2 m_1 + |U_{e2}|^2 \sqrt{m_1^2 + \Delta m_{12}^2} + |U_{e3}|^2 \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}|}{m_1^2 m_2^2 m_3^2} \right. (11)
\langle m_\nu \rangle < |U_{e1}^2 m_1 + |U_{e2}|^2 \sqrt{m_1^2 + \Delta m_{12}^2} + |U_{e3}|^2 \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}|. \right. (12)

Here we have used the constraint from the oscillation experiments of CHOOZ\[ and SuperKamiokande\[. It is apparent from Eqs.(1), (2), and (3) that the normal hierarchy, $m_1 \lesssim m_2 \ll m_3$, is forbidden. We know that the inverse hierarchy is disfavored by the observation of Supernova 1987A\[ and by the realistic GUT model\[. However we have no way of distinguishing between the almost degenerate and inverse hierarchy neutrino mass scenarios based on Eq.(1) at this stage, because $|U_{e1}^2 m_1 | |U_{e2}|^2 \sqrt{m_1^2 + \Delta m_{12}^2} \gg |U_{e3}|^2 \sqrt{m_1^2 + \Delta m_{12}^2 + \Delta m_{23}^2}$. Keeping these in mind, we adopt that the neutrino masses are almost degenerate and
\[\langle m_\nu \rangle \simeq m_1 |U_{e1}^2 m_1 + |U_{e2}|^2 e^{2i\beta}, \right. (13)

with $m \equiv m_1 \simeq m_2$. Since Eq.(4), $\sin^2 2\theta_{12}$ becomes $4|U_{e2}|^2 (1 - |U_{e2}|^2)$ and Eq.(13) is rewitten as
\[\sin^2 \beta = \frac{1}{\sin^2 2\theta_{12}} \left( 1 - \frac{\langle m_\nu \rangle^2}{m^2} \right). \right. (14)

Eq.(14) gives
\[\sin^2 \beta \leq \frac{1}{\sin^2 2\theta_{12}} \left( 1 - \frac{\langle m_\nu \rangle^2}{m_{\text{max}}^2} \right). \right. (15)

Here we have denoted the experimental upper limits of $m$ obtained from Eq.(2) as $m_{\text{max}}$. Let us superimpose the constraints of the other experimental bounds of Eqs. (1) and (3) on this inequality in Fig.1. When 1-dimensional restriction is translated into 2-dimensional one, the following region approximately coincide with 85% C.L.
\[\chi^2(\sin^2 2\theta_{12}, \langle m_\nu \rangle^2) \equiv \left( \frac{\sin^2 2\theta_{12} - 0.82}{0.70 - 0.82} \right)^2 + \left( \frac{\langle m_\nu \rangle^2 - 0.39^2}{0.11^2 - 0.39^2} \right)^2 < 1, \right. (16)

because we assume these values are distributed as a normal (Gaussian) distribution. In another respect, Eq.(14) gives the upper limit of $\sin^2 \beta$ as
\[\sin^2 \beta \leq \frac{1}{\sin^2 2\theta_{12})_{\text{min}}} \left( 1 - \frac{\langle m_\nu \rangle_{\text{min}}^2}{m^2} \right). \right. (17)

in the confined region $\chi((\sin^2 2\theta_{12})_{\text{min}}, \langle m_\nu \rangle_{\text{min}}^2) < 1$. Then we obtain the allowed region in the $\sin^2 \beta - m$ plane in Fig.2. By combining this with the WMAP experiments, we have the meaningful constraint on the Majorana phase $\beta$ with $\lesssim 85\%$ C.L. for LMA-MSW solution. Namely, we have
\[\sin^2 \beta \lesssim 0.71 \right. (18)
at 85\% C.L. And we obtain the lower limits of neutrino mass.

We must consider the factor of uncertainty in the nuclear matrix elements as well. This uncertainty enlarges the range of $\langle m_{\nu} \rangle$ to
\begin{equation}
\chi^2(\sin^2 2\theta_{12}, \langle m_{\nu} \rangle^2) = \frac{\sin^2 2\theta_{12} - 0.82}{0.70 - 0.82}^2 + \frac{(\langle m_{\nu} \rangle^2 - (0.05^2 + 0.84^2)/2)}{0.05^2 - (0.05^2 + 0.84^2)/2}^2 < 1. (85\% C.L.)
\end{equation}

In this case, $\sin^2 \beta$ is not restricted as shown by Fig.1 and Fig.2. Therefore, the reliability of the HEIDELBERG-MOSCOW $(\beta\beta)^0_{\nu}$ experimental results must be checked more precisely by other near future $(\beta\beta)^0_{\nu}$ experiments, which may enable us to understand one $(\beta)$ of two Majorana phases more definitely. However, it will be difficult to measure another phase $(\rho)$. Finally, we must note the near future $3\text{H}$ beta decay experiments, KATRIN\textsuperscript{13}. After three years of measuring time, this upper limit will be improved to
\begin{equation}
m \lesssim 0.35[eV] (90\% CL).
\end{equation}

It will be very useful to get more detailed information about the Majorana phases and to check the mutual consistencies among many parameters\textsuperscript{14}.

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References

1. K. Eguchi et al., KamLAND collaboration, hep-ex/0212021.
2. WMAP collaboration (NASA), [http://map.gsfc.nasa.gov/mig/outreach_images/Parameters2_m.jpg](http://map.gsfc.nasa.gov/mig/outreach_images/Parameters2_m.jpg)
3. H.V. Klapdor-Kleingrothaus, A. Dietz, H.L. Harney, and I.V. Krivosheina, Mod. Phys. Lett. A16, 2409 (2001).
4. K. Matsuda, T. Kikuchi, T. Fukuyama, and H. Nishiura, Mod. Phys. Lett. A17, 2597 (2002).
5. P.C. de Holanda and A.Yu. Smirnov, hep-ph/0212270.
6. M. Apollonio et al., Phys. Lett. B420, 397 (1998).
7. M. Shiozawa, talk at Neutrino 2002 (http://neutrino.t30.physik.tu-muenchen.de/)
8. S. Hannestad, astro-ph/0303076.
9. S.M. Bilenky, J. Hosek, and S.T. Petcov, Phys. Lett. 94B, 495 (1980); J. Schechter and J.W.F. Valle, Phys. Rev. D22, 2227 (1980); M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett. 102B, 325 (1981); A. Barroso and J. Maalampi, Phys. Lett. 132B, 355 (1983).
10. T. Kajita, talk presented at Neutrino '98, Takayama, Japan, June 1998; H. Sobel, talk presented at Neutrino 2000, Sudbury, Canada, June 2000; Y. Fukuda et al., Phys. Rev. Lett. 81, 1158 (1998); Phys. Rev. Lett. 81, 1562 (1998); Phys. Lett. B433, 9 (1998); Phys. Rev. Lett. 85, 3999 (2000); Phys. Rev. Lett. 86, 5656 (2001); Q.R. Ahmad, et al., Phys. Rev. 87, 071301 (2001).
11. H. Minakata and H. Nunokawa, Phys. Lett. B504, 301 (2001).
12. T. Fukuyama and N. Okada, JHEP 0211, 011 (2002); K. Matsuda, Y. Koide, T. Fukuyama, and H. Nishiura, Phys. Rev. D65, 033008 (2002); Erratum-ibid. D65, 079904 (2002); K. Matsuda, Y. Koide, and T. Fukuyama, Phys. Rev. D64, 053015 (2001).
13. A. Osipowicz et al., hep-ex/0109033.
14. H. Minakata and H. Sugiyama, hep-ph/0202003; K. Matsuda, N. Takeda, T. Fukuyama, and H. Nishiura, Phys. Rev. D64, 013001 (2001).

![Diagram]

Fig. 1. The possible upper bounds for $\sin^2 \beta$ on the $\sin^2 2\theta_{12} - \langle m_\nu \rangle$ plane. Lines (a) and (b) show the cases where the uncertainty of the nuclear matrix elements is neglected and considered, respectively. These contour lines of $\chi^2$ indicate 85% C.L. from Eqs. 4 and 5.
Fig. 2. The allowed region in the \( m \) – \( \sin^2 \beta \) plane. (a) and (b) regions show the cases where the uncertainty of the nuclear matrix elements is neglected and considered, respectively. Thus \( \sin^2 \beta \) has the upper limit with 85\%C.L. if we do not consider the uncertainty.