Shadow images of Kerr-like wormholes

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Abstract. The study of shadow continues to be a major source of insight into compact astrophysical objects. Depending on the nature of compact objects and due to the strong gravitational lensing effect that casts a shadow on the bright background. We consider the Kerr-like wormholes spacetime [1] which is a modification of Kerr black holes that turns into the wormholes for nonzero values of deformation parameter $\lambda^2$. The results suggest that the Kerr spacetime can reproduce far away from the throat of the wormhole. We obtain the shapes of the shadow for the Kerr-like wormholes and discuss the effect of spin $a$ and deformation parameter $\lambda^2$ on the size of the shadow. As a consequence, it is discovered that the shadow is distorted due to the rotation and that the radius of the shadow monotonically increases with $\lambda^2$.

Keywords: Kerr-like wormholes, Embedding, Shadow
1 Introduction

In the context of general relativity, wormholes act as tunnel-like structures connecting two different regions in spacetime. This phenomenon necessitates solving Einstein’s equations in a reversed manner to obtain solutions admitting wormholes. Studies on this topic can be divided into two classes: one relating to Euclidean wormholes \[2–5\] and the other concerned with Lorentzian ones. Historically, the notion of wormhole physics may be traced back in 1916 by Flamm \[6\], soon after the discovery of the Schwarzschild solution. Then, Einstein and Rosen \[7\] proposed a mathematical construction in order to eliminate coordinate or curvature singularities, and introduced a bridge-like structure connecting two identical sheets known as Einstein-Rosen bridges. This mathematical representation of physical space was not successful as a model for particles, rather it emerged as the prototype wormhole in gravitational physics. Subsequently, Wheeler \[8, 9\] revived the subject in the 1950s, where Kruskal-Szekeres coordinates were employed to describe the geometry of the Schwarzschild wormhole, although these wormholes were at the quantum scale. Wheeler coined the term wormhole and later his solutions were transformed into Euclidean wormholes by Hawking \[5\] and others.

Interest in traversable Lorentzian wormholes has been on the incline recently following the stimulating work of Morris and Thorne \[10\], possibly through which observers may freely traverse and even permit time travel. They first introduced concepts as tools for teaching general relativity where the Einstein field equations are solved by constructing the spacetime metric first and then deducing the stress-energy tensor components. To obtain such traversable wormhole solutions, one needs to possess a peculiar property, namely exotic matter, whose stress-energy tensor components violate the null, weak and strong energy conditions \[10, 11\], at least in a neighborhood of the wormhole throat. For a more detailed review the reader may consult \[12, 13\] and references therein. However, the introduction of exotic matter sounds unusual, but such matter fields exist in the quantum theory of the gravitational field (namely Casimir effect and Hawking evaporation \[14\]), and in scalar-tensor theories. In fact, wormhole geometries violate the averaged energy conditions \[12\].

Thus various attempts has been made over the decades to avoid the usage of exotic matter, but all were in vain within the context of general relativity. An active part in this discussion was taken to minimize the violation of the energy conditions, namely, the volume...
integral quantifier [15, 16], which quantifies the total amount of matter violating the energy conditions. Nevertheless, Nandi et al. [17] considered an exact integral quantifier for matter violating averaged null energy condition (ANEC). Kuhfittig [18] reported that the region consisting of exotic matter can be made arbitrarily small. Therefore, it is not an easy task to search for promising candidates for exotic matter because all classical forms of matter are believed to obey the standard energy-conditions. Dark energy models with exotic equations of state have been proposed due to the formation and the evolution of the present structures in the Universe. While observational and experimental support is lacking strong theoretical arguments for dark matter have been advanced principally to address the anomalous accelerated expansion of the universe problem. Consequently, much effort has been made to obtain stable wormhole solutions in these new possibilities, such as those supported by the phantom energy [19–21], cosmological constant [22–24], generalized or modified Chaplygin gas [25–27].

The detection of gravitational waves (GWs) by the advanced Laser Interferometer Gravitational Wave Observatory (aLIGO) [28] and Advanced Virgo [29] detectors is exciting opportunities towards the study nature of the black holes. The detection of GWs from five binary black holes (BBHs) demonstrated that stellar-mass black holes really exist in our universe. However, the complete structure of the spacetime inside the light ring and near-horizon is still opaque form the current observations. Thus, the physics in strong gravitational field, e.g., near black holes is still an important topic not only in theoretical but also observational investigations. Analysing the physical nature of black holes requires the actual detection of the event horizon. To confirm the presence of event horizons a number of tests have been performed that help reveal exciting physics and astrophysics possibilities [30–32]. The evidence is compelling however not conclusive [33]. Another important phenomenological feature of black holes is their shadow. The concept of the rotating black hole shadow was suggested by Bardeen [34], with an idea of optical image of black hole appears due to the strong gravitational lensing effect. Black holes are expected to cast shadows on the bright background due to the strong gravitational effect. Current observations suggest that most galaxies contains supermassive black holes at their centers [35, 36], and that galaxies are rotating. This means that the black hole at the center of a galaxy also possesses a spin, and in this environment it is interesting to investigate the nature of the black holes by analyzing the shadow that they cast on the bright background. Motivated by present observational missions [37], the study of the black hole shadow has received significant attention. Various configurations of black hole solutions such as shadow of Schwarzschild black hole [38, 39] have been investigated. The shadow of rotating black holes with gravitomagnetic and electric charge have been found [40]. The existence of shadow for several black holes in the context of modified theories of gravity and higher-dimensional theories has received attention in [40–61].

The main motivation of the paper is to investigate the shadows of Kerr-like wormholes obtained by Bueno et al. [1], where the Kerr black hole turns out to be a wormhole solution that reproduces Kerr’s spacetime away from the throat. The same procedure outlined by Damour and Solodukhin for obtaining the Schwarzschild-like wormhole [62] was followed. The detection of ringdown frequencies from the black holes provides precise tests that astrophysical black holes are verily described by the Kerr spacetime. Therefore, we construct the analysis of shadow for the Kerr-like wormholes. In addition, it is expected that shadow of Kerr-like wormholes can be used to probe the true nature of wormholes by Very Long Baseline Interferometry (VLBI) observations. In this direction a recent paper has been devoted to the study of the shadow boundary associated with the outer spherical orbits [63], and later this work was developed by [64]. Ellis wormholes have been analyzed by use of the images
of wormholes surrounded by optically thin dust in [65]. Shadow of rotating wormholes in a plasma environment was studied in the literature by [66]. Recently, the shadow of a class of charged wormholes solutions in EMDA theory has been studied in [67].

The paper is organized as follows. After a brief introduction in section 1, we review the Kerr-like wormholes solution and construct embedding diagrams to represent a wormhole in section 2. In section 3, we evaluate the geodesic equations and derive photon trajectories around the wormhole in section 4. Section 5 is developed to construct the shadow images of Kerr-like wormholes. A discussion on the results is included in section 6.

2 Kerr-Like wormholes and embedding diagrams

We start with the Kerr-like wormhole spacetime, which was obtained by performing a modification on Kerr metric as similar to that of Damour and Solodukhin did for a Schwarzschild black hole [62]. This spacetime class was introduce by Bueno et al. [1], and the metric written in Boyer-Lindquist coordinates \((t, r, \theta, \phi)\) as

\[
ds^2 = -\left(1 - \frac{2Mr}{\Sigma}\right)dt^2 - \frac{4Mar \sin^2 \theta}{\Sigma}dtd\phi + \frac{\Sigma}{\hat{\Delta}}dr^2 + \Sigma d\theta^2 + \left(r^2 + a^2 + \frac{2Ma^2r \sin^2 \theta}{\Sigma}\right)\sin^2 \theta d\phi^2,
\]

where \(\Sigma\) and \(\hat{\Delta}\) are expressed as

\[
\Sigma = r^2 + a^2 \cos^2 \theta, \quad \hat{\Delta} = r^2 + a^2 - 2M(1 + \lambda^2)r.
\]

It contains a family of parameters where \(a\) and \(M\) correspond to the spin and mass of wormhole, and \(\lambda^2\) is the deformation parameter which provides deviation from the Kerr spacetime. In other words, for any non vanishing \(\lambda^2\) differs this metric from the Kerr metric. Also, one can recover a Kerr metric when \(\lambda^2 = 0\). The throat of the Kerr-like wormhole (2.2) can be evaluated by equating the \(\hat{\Delta}\) to zero, which gives

\[
r_+ = (1 + \lambda^2)M + \sqrt{M^2(1 + \lambda^2)^2 - a^2},
\]

it represents a special surface or region that connects two different asymptotically flat regions. We can construct the embedding diagrams to represent a Kerr-like wormhole and extract some useful information by considering an equatorial slice, \(\theta = \pi/2\) and a fixed moment of time, \(t = \text{const}\), yielding

\[
ds^2 = \frac{dr^2}{1 - \frac{2Ma}{r}} + R^2 d\phi^2,
\]

where

\[
R^2 = r^2 + a^2 + \frac{2Ma^2}{r}, \quad \text{and} \quad b(r) = 2M(1 + \lambda^2) - \frac{a^2}{r}.
\]

We embed the metric (2.4) into three-dimensional Euclidean space to visualize this slice, and the metric can be written in cylindrical coordinates as

\[
ds^2 = dz^2 + dR^2 + R^2 d\phi^2 = \left[\left(\frac{dR}{dr}\right)^2 + \left(\frac{dz}{dr}\right)^2\right]dr^2 + R^2 d\phi^2.
\]
Since the combination of (2.4) and (2.6) reduce the equation for the embedding surface, which is given by

$$\frac{dz}{dr} = \pm \sqrt{\frac{r}{r - b(r)}} - \left( \frac{dR}{dr} \right)^2. \quad (2.7)$$

Now, plugging the value of (2.5) into (2.7), then the above relation turns out to be

$$\frac{dz}{dr} = \pm \sqrt{\frac{M(2r^7(1 + \lambda^2) + 4a^2r^5 - 4Ma^2(1 + \lambda^2)r^4 + 2a^4r^3 - Ma^4r^2 + 2Ma^4(1 + \lambda^2)r - Ma^6)}{r^3(2Ma^2 + a^2r + r^3)(r^2 + a^2 - 2Mr - 2Mr\lambda^2)}}. \quad (2.8)$$

Note that the integral (2.8) cannot be solved analytically. We can evaluate this integral numerically and illustrate the wormhole shape given in Fig. 1 (a). Immediately, if we set the solution when $a \to 0$, it yields

$$z = \pm \sqrt{8M(1 + \lambda^2)(r - 2M - 2M\lambda^2)}. \quad (2.9)$$

In addition, setting the deformation parameter to zero, i.e., when $\lambda^2 = 0$, one can recover the Schwarzschild solution

$$z = \pm \sqrt{8M(r - 2M)}. \quad (2.10)$$

However, we can simplify the problem by considering a slowly rotating Kerr-like wormhole solution, for this purpose we are interested in approximate solution by considering a series expansion, say, around $a$ and $\lambda^2$ in equation (2.8). The solution can be approximated as follows:

$$z = \pm \left\{ \sqrt{8M(r - 2M)} - \frac{\lambda^2 \sqrt{2M} \arctan \left( \frac{2(r - 2M)}{2\sqrt{M}} \right)}{4M^{3/2}r(r - 2M)^{3/2}} \left[ \sqrt{2} \arctan \left( \frac{2(r - 2M)}{2\sqrt{M}} \right) (r - 2M)^{3/2}a^2 + \chi \right] \right\}.$$
\[ + \frac{a^2 \sqrt{2M}}{16r^2M^{3/2}(r - 2M)^{1/2}} \left[ 3 \sqrt{2} \arctan \left( \frac{2(r - 2M)}{2 \sqrt{M}} \right) \right. \]

\[ \left. (r - 2M)^{1/2}r^2 + \zeta \right] \}, \quad (2.11) \]

where

\[ \zeta = 16M^{5/2} - 4M^{3/2}r + 6r^2M^{1/2}, \quad (2.12) \]

and

\[ \chi = 24M^{5/2}r^2 - 32M^{7/2}r + 4M^{5/2}a^2 - 4M^{3/2}r^3 - 8a^2M^{3/2}r + 2r^2a^2\sqrt{M}. \quad (2.13) \]

Finally, introducing a new variable \( \rho \), via

\[ r = 2M(1 + \lambda^2) + \frac{\lambda^2 - 1}{2M}a^2 + \frac{\rho^2}{2M}. \quad (2.14) \]

Fig. 1(b) is a graph of (2.14) with the \( \phi \) direction, which shows the visual image of the slowly rotating Kerr-like wormhole.

### 3 Geodesics in Kerr-like wormholes

The spacetime of rotating Kerr-like wormholes is generally characterized by the four constants of the motion, i.e., Lagrangian \( \mathcal{L} \), energy \( E \), angular momentum \( L_z \) and the Carter constant \( \mathcal{K} \). The geodesic equations of a test particle having the rest mass \( m_0 \), can be derive by using these conserved quantities and the Hamilton-Jacobi equation. Since, the geodesics of the test particle in the background of Kerr-like wormholes spacetime (2.1) satisfy the Hamilton-Jacobi equation [68]

\[ \frac{\partial S}{\partial \sigma} = -\frac{1}{2}g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu}, \quad (3.1) \]

where \( \sigma \) is the affine parameter and \( S \) is the Jacobian action with the following separable ansatz:

\[ S = \frac{1}{2} m_0^2 \sigma - Et + S_r(r) + S_\theta(\theta) + L_z \phi, \quad (3.2) \]

where \( S_r \) and \( S_\theta \) are the functions of \( r \) and \( \theta \), respectively. On substituting equation (3.2) into equation (3.1), and after some straightforward calculation [68], we obtain the following form of geodesic equations

\[ \Sigma \frac{dt}{d\sigma} = -a \left( aE \sin^2 \theta - L_z \right) + \frac{(r^2 + a^2)P}{r^2 + a^2 - 2M(1 + \lambda^2)r}, \]

\[ \Sigma \frac{d\phi}{d\sigma} = - (aE - L_z \csc^2 \theta) + \frac{aP}{r^2 + a^2 - 2M(1 + \lambda^2)r}, \]

\[ \Sigma \frac{dr}{d\sigma} = \pm \sqrt{R}, \]

\[ \Sigma \frac{d\theta}{d\sigma} = \pm \sqrt{\Theta}, \quad (3.3) \]

where the terms \( \mathcal{P}, \mathcal{R} \), and \( \Theta \) are expressed as

\[ \mathcal{P} = (r^2 + a^2)E - aL_z, \]

\[ \mathcal{R} = \mathcal{P}^2 - \left[ r^2 + a^2 - 2M(1 + \lambda^2)r \right] \left[ m_0^2 r^2 + \mathcal{K} + (L_z - aE)^2 \right], \]

\[ \Theta = \mathcal{K} + \cos^2 \theta \left( a^2 E^2 - L_z^2 \csc^2 \theta \right). \quad (3.4) \]

These geodesic equations determine the trajectories of a test particle in the background of Kerr-like wormholes spacetime. Physically, the energy \( E \) is required energy for a distant observer to place the test particle in orbit around the Kerr-like wormholes.
4 Circular photon orbits around Kerr-like wormholes

We consider the photons released from a bright source are moving towards the wormhole that is placed between the observer and the light source. The possible trajectories of the photons around the wormholes are: (i) falling into the wormholes, (ii) scattered away from the wormholes to infinity, and (iii) critical geodesics which separate the first two sets, also known as unstable circular orbits. It turns out that in the observer’s sky, the plunged photon geodesics form dark spots whereas the scattered photon geodesics form bright spots. The critical photon geodesics trajectories form a dark region in the observer’s sky in the presence of bright background, which we called wormhole hole shadow.

Hence, our main task is to evaluate these critical orbits or unstable circular orbits. In order to obtain the boundary of the wormhole shadow, it is required to study the radial motion of photons around the wormholes. Here, we consider photon as a test particle, and hence $m_0 = 0$. The radial geodesic equation can be rewritten as

$$\Sigma^2 \left( \frac{dr}{d\sigma} \right)^2 + V_{\text{eff}} = 0,$$  \hspace{1cm} (4.1)

where $V_{\text{eff}}$ represents the effective potential describing radial motion of photon geodesic. Besides, the radial motion of photons can be expressed by two independent impact parameters such that $\xi = L_z/E$ and $\eta = K/E^2$, hence equation (4.1), yields

$$V_{\text{eff}} = E^2 \left[ r^2 + a^2 - 2M(1 + \lambda^2)r \right] \left[ \eta + (\xi - a)^2 \right] - E^2[r^2 + a^2 - a\xi]^2.$$ \hspace{1cm} (4.2)

We plotted effective potential with radius for a Kerr-like wormhole in figure 2, by varying the angular momentum $L_z/M$ and deformation parameter $\lambda^2$. It turns out that for large value of $L_z/M$, the effective potential is large and its value decreases with large values of $\lambda^2$. Note that the boundary of the shadow is determined by unstable circular photon orbits corresponding to the highest maximum of the effective potential $V_{\text{eff}}$, leading to

$$V_{\text{eff}} = 0, \quad V'_{\text{eff}} = 0,$$ \hspace{1cm} (4.3)
where prime (′) represents derivative with respect to \( r \). By using equations (4.2) and (4.3), we can easily obtain the expressions of the impact parameters,

\[
\xi = \frac{a^2(\lambda^2M + M + r) + r^2[r - 3(\lambda^2 + 1)M]}{a(\lambda^2M + M - r)} ,
\]

\[
\eta = \frac{4Ma^2r^3(\lambda^2 + 1) - Mr^4[r - 3(\lambda^2 + 1)]}{a^2(\lambda^2M + M - r)^2}.
\] (4.4)

Notice that the impact parameters depend on the spin \( a \), mass \( M \), and deformation parameter \( \lambda^2 \). If one substitutes \( \lambda^2 = 0 \), then equation (4.4) reduce to the Kerr case [43]. Furthermore, we solve the equation (4.3) for nonrotating case \( (a = 0) \) of the wormholes, which yields

\[
\eta = 27(1 + \lambda^2)^2M^2 - \xi^2 \quad \text{with} \quad r = 3M(1 + \lambda^2).
\] (4.5)

Moreover, a substitution \( \lambda^2 = 0 \), in equation (4.5) renders it to the Schwarzschild case.

5 Shadow of Kerr-like wormhole

In this section, our main goal is to construct the shadow of Kerr-like wormhole. We are going to turn our attention towards the observer’s sky to detect the optical images cast by the Kerr-like wormholes. In preparation for the investigation of shadow, we consider a plane passing through the center of the wormhole and it’s normal join the center of the wormhole and the line of sight of a observer. For a better visualization, we are going to introduce new coordinates \( (\alpha, \beta) \) [34], widely known as celestial coordinates, where \( \alpha \) and \( \beta \) corresponds to the apparent perpendicular distance of the image as seen from the axis of symmetry and the apparent perpendicular distance of the image from its projection on the equatorial plane, respectively. With this construction, we obtain the projection of wormhole’s throat on the observer’s sky. For a distant observer, these coordinates can be calculated [34] as

\[
\alpha = \lim_{r \to \infty} \left(-r^2 \sin \theta_0 \frac{d\phi}{dr}\right),
\]

\[
\beta = \lim_{r \to \infty} \left(r^2 \frac{d\theta}{dr}\right),
\] (5.1)

where \( \theta_0 \) represents the inclination angle between the rotation axis of the wormhole and the direction to the observer. By using equations (3.3), (3.4), and (5.1), the celestial coordinates transform into

\[
\alpha = -\xi \csc \theta_0,
\]

\[
\beta = \pm \sqrt{\eta + a^2 \cos^2 \theta_0 - \xi^2 \cot^2 \theta_0}.
\] (5.2)

It is clear from equation (5.2) that the shapes of the shadow of Kerr-like wormholes depend on its spin \( a \), inclination angle \( \theta_0 \), and the deformation parameter \( \lambda^2 \). Whilst the observer is located in the equatorial plane \( (\theta_0 = \pi/2) \) of wormholes, in this case, the equation (5.2) reforms as

\[
\alpha = -\xi,
\]

\[
\beta = \pm \sqrt{\eta}.
\] (5.3)
Figure 3: Plot illustrating the shapes of shadows of Kerr-like wormholes by varying spin $a$.

A captured region in the observer’s sky can be obtained when the impact parameters $\eta$ and $\xi$ attain all the possible values. This captured region is the shadow of wormhole. Moreover, a nonrotating wormhole shadow can be construct by the following expression:

$$\alpha^2 + \beta^2 = \frac{2r^2(r^2 - 3(\lambda^2 + 1)^2M^2)}{(\lambda^2M + M - r)^2},$$

(5.4)

which implies that the shadow casts by a nonrotating wormhole ($a = 0$) has a circular shape with a radius $\sqrt{\frac{2r^2(r^2 - 3(\lambda^2 + 1)^2M^2)}{(\lambda^2M + M - r)^2}}$. Moreover, for a rotating wormhole the sum of $\alpha^2 + \beta^2$, takes the complicated form,

$$\alpha^2 + \beta^2 = \frac{a^2(\lambda^2M + M + r)^2 + 2r^2(r^2 - 3(\lambda^2 + 1)^2M^2)}{(\lambda^2M + M - r)^2}.$$

(5.5)

The presence of spin and deformation parameters in the expression (5.5) differs the shape of shadow of the rotating wormhole from the nonrotating one. In this case the shape is oblate instead of circular. This oblateness is due to the rotation of wormholes and it increases with an increase in the value of spin $a$. The wormhole has a more deformed shape of shadow for large value of $a$ as compare to the nonrotating case. We plotted the shadows of Kerr-like wormholes for different values of spin $a$, deformation parameter $\lambda^2$ as well as inclination angle $\theta_0$ (cf. figure 3 and 4). In figure 3, we have shown the variation of spin $a$ whilst the figure 4 depicts the effect of $\lambda^2$ on the shape of shadow. For the comparison, we have included the case of $\lambda^2 = 0$, which corresponds to the Kerr case. We construct the shapes of Kerr-like wormholes shadows for nonzero values of $\lambda^2$. We found that the presence of $a$ and $\lambda^2$ affects the shape and size of shadows. It can be seen from figure 4 that the radius is continuously increasing with a small change in the value of deformation parameter $\lambda^2$. 


Figure 4: Plot illustrating the shapes of shadows of Kerr-like wormholes by varying the deformation parameter $\lambda^2$.

6 Conclusion

Discovery of quasar in 1960s, confirms the existence of supermassive black holes located at least in some galactic binary systems in the centers of most of large galaxies, e.g. Milky Way and Messier 87 have, namely, Sgr A* and M87. Supermassive black holes are considered as the astrophysical black holes and they form from the gravitational collapse of matter. Besides, the detection of ringdown frequencies from the black holes provides precise tests that astrophysical black holes are verily described by the Kerr spacetime. Motivated by these tests, we construct the theoretical analysis of shadow for the Kerr-like wormholes. Therefore, observational investigation of Kerr-like wormhole shadow will be very interesting and a useful tool to demonstrate the true nature of it. The observation of shadow also provides a tentative way to find out the parameters of the wormholes. As we know that one of the approach to detect the supermassive black holes is based on high resolution imaging of them. To resolve the invisibility of the supermassive black holes, there exist an Event Horizon Telescope which is working on the Very Long Baseline Interferometry technique. This technique is able to achieve the angular resolution comparable to sub-millimeter wavelength diapason. This study will be helpful for the current astronomical observations and likely to be observed with the Event Horizon Telescope.

In this paper, we mainly focus on the construction of a shadow by the Kerr-like wormholes. We evaluate the test particle geodesics and determine the trajectories of them around the wormhole. It is seen that the photons approaching the wormhole form the circular orbits around it. We introduce impact parameters that define the boundary of shadow on the bright background. To visualize the optical image or the shadow of Kerr-like wormhole, the celestial coordinates are evaluated. With help of these coordinates, we have discussed the nonrotating and rotating cases. We have obtained different shapes of shadow for Kerr-like wormholes by varying the parameters. The shape of shadow is oblate in case of Kerr-like wormhole. The oblateness occurs due to the rotation of wormhole, it is more oblate for a large value of spin.
It is found that the shapes of shadow is quite sensitive to the deformation parameter $\lambda^2$, a small change in $\lambda^2$ increases the radius of shadow. As a consequence the radius is monotonically increases with $\lambda^2$ as a whole.

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References

[1] P. Bueno, P.A. Cano, F. Goelen, T. Hertog and B. Vercnocke, *Echoes of Kerr-like wormholes*, Phys. Rev. D **97**, 024040 (2018).
[2] S. Ruz, S. Debnath, A. K. Sanyal and B. Modak, *Euclidean wormholes with minimally coupled scalar fields*, Class. Quant. Grav. **30**, 175013 (2013).
[3] N. Arkani-Hamed, J. Orgera and J. Polchinski, *Euclidean wormholes in string theory*, J. High Energy Phys. **0712**, 018 (2007).
[4] S.W. Hawking, *Quantum Coherence Down the Wormhole*, Phys. Lett. B **195**, 337 (1987).
[5] S.W. Hawking, *Wormholes in Space-Time*, Phys. Rev. D **37**, 904 (1988).
[6] L. Flamm, *Beitrag zur Einsteinschen Gravitationstheorie* Phys. Z. **17**, 448 (1916).
[7] A. Einstein and N. Rosen, *The Particle Problem in the General Theory of Relativity*, Phys. Rev. **48**, 73 (1935).
[8] J. A. Wheeler, *Geons*, Phys. Rev. **97**, 511 (1955).
[9] J. A. Wheeler, *Geometrodynamics*, (Academic Press, New York, 1962).
[10] M.S. Morris and K.S. Thorne, *Wormholes in spacetime and their use for interstellar travel: A tool for teaching General Relativity*, Am. J. Phys. **56**, 395 (1988).
[11] M.S. Morris, K.S. Thorne and U. Yurtsever, *Wormholes, Time Machines, and the Weak Energy Condition*, Phys. Rev. Lett. **61**, 1446 (1988).
[12] M. Visser, *Lorentzian Wormholes: From Einstein to Hawking* (American Institute of Physics, New York, 1995).
[13] F.S.N. Lobo, *Exotic solutions in General Relativity: Traversable wormholes and ‘warp drive’ spacetimes*, Classical and Quantum Gravity Research, 1-78, (2008).
[14] G. Klinkhammer, *Averaged energy conditions for free scalar fields in flat spacetime*, Phys. Rev. D **43**, 2542 (1991).
[15] M. Visser, S. Kar and N. Dadhich, *Traversable wormholes with arbitrarily small energy condition violations*, Phys. Rev. Lett. **90**, 201102 (2003).
[16] S. Kar, N. Dadhich and M. Visser, *Quantifying energy condition violations in traversable wormholes*, Pramana **63**, 859 (2004).
[17] K. K. Nandi, Y. Z. Zhang and K. B. Vijaya Kumar, *On volume integral theorem for exotic matter*, Phys. Rev. D **70**, 127503 (2004).
[18] P.K.F. Kuhfittig, *Can a wormhole supported by only small amounts of exotic matter really be traversable?*, Phys. Rev. D **68**, 067502 (2003).
[19] F.S.N. Lobo, *Phantom energy traversable wormholes*, Phys. Rev. D **71**, 084011 (2005).
[20] F.S.N. Lobo, *Stability of phantom wormholes*, Phys. Rev. D **71**, 124022 (2005).
[21] O.B. Zaslavskii, *Exactly solvable model of wormhole supported by phantom energy*, Phys. Rev. D **72**, 061303 (2005).

[22] F. Rahaman, M. Kalam, M. Sarker, A. Ghosh and B. Raychaudhuri, *Wormhole with varying cosmological constant*, Gen. Rel. Grav. **39**, 145 (2007).

[23] J.P.S. Lemos, F.S.N. Lobo and S. Quinet de Oliveira, *Morris-Thorne wormholes with a cosmological constant*, Phys. Rev. D **68**, 064004 (2003).

[24] M. Cataldo, S. del Campo, P. Minning and P. Salgado, *Evolving Lorentzian wormholes supported by phantom matter and cosmological constant*, Phys. Rev. D **79**, 024005 (2009).

[25] S. Chakraborty and T. Bandyopadhyay, *Modified Chaplygin traversable wormholes*, Int. J. Mod. Phys. D **18**, 463 (2009).

[26] M. Jamil, U. Farooq and M.A. Rashid, *Wormholes supported by phantom-like modified Chaplygin gas*, Eur. Phys. J. C **59**, 907 (2009).

[27] P.K.F. Kuhfittig, *A Single model of traversable wormholes supported by generalized phantom energy or Chaplygin gas*, Gen. Rel. Grav. **41**, 1485 (2009).

[28] J. Aasi et al. [LIGO Scientific Collaboration], *Advanced LIGO*, Class. Quant. Grav. **32**, 074001 (2015).

[29] F. Acernese et al. [VIRGO Collaboration], *Advanced Virgo: a second-generation interferometric gravitational wave detector*, Class. Quant. Grav. **32**, 024001 (2015).

[30] A.E. Broderick, A. Loeb and R. Narayan, *The Event Horizon of Sagittarius A*, Astrophys. J. **701**, 1357 (2009).

[31] A.E. Broderick, R. Narayan, J. Kormendy, E. S. Perlman, M.J. Rieke and S.S. Doeleman, *The Event Horizon of M87*, Astrophys. J. **805**, no. 2, 179 (2015).

[32] R. Narayan and J. E. McClintock, *Advection-Dominated Accretion and the Black Hole Event Horizon*, New Astron. Rev. **51**, 733 (2008).

[33] M.A. Abramowicz, W. Kluzniak and J.P. Lasota, *No observational proof of the black hole event-horizon*, Astron. Astrophys. **396**, L31 (2002).

[34] J.M. Bardeen, *Black holes, in Proceeding of the Les Houches Summer School, Session 215239*, edited by C. De Witt and B.S. De Witt and B.S. De Witt (Gordon and Breach, New York, 1973).

[35] M.J. Rees, *Black Hole Models for Active Galactic Nuclei*, Ann. Rev. Astron. Astrophys. **22**, 471 (1984).

[36] J. Kormendy and D. Richstone, *Inward Bound—The Search For Supermassive Black Holes In Galactic Nuclei*, Ann. Rev. Astron. Astrophys. **33**, 581 (1995).

[37] S. Doeleman, E. Agol, D. Backer et al., *Imaging an event horizon: submm-VLBI of a super massive black hole*, Astro2010: The Astronomy and Astrophysics Decadal Survey, Science White Papers, No. 68 (2009).

[38] J.L. Synge, *The escape of photons from gravitationally intense stars*, Mon. Not. R. Astron. Soc. **131**, 463 (1966).

[39] J.P. Luminet *Image of a spherical black hole with thin accretion disk*, Astron. Astrophys. **75**, 228 (1979).

[40] A. Grenzebach, V. Perlick and C. Lammerzahl, *Photon Regions and Shadows of Kerr-Newman-NUT Black Holes with a Cosmological Constant*, Phys. Rev. D **89**, no. 12, 124004 (2014).

[41] R. Takahashi, *Black hole shadows of charged spinning black holes*, Publ. Astron. Soc. Jap. **57**, 273 (2005).
[42] C. Bambi and K. Freese, *Apparent shape of super-spinning black holes*, Phys. Rev. D 79, 043002 (2009).
[43] K. Hioki and K.i. Maeda, *Measurement of the Kerr Spin Parameter by Observation of a Compact Object’s Shadow*, Phys. Rev. D 80, 024042 (2009).
[44] S.W. Wei and Y.X. Liu, *Observing the shadow of Einstein-Maxwell-Dilaton-Axion black hole*, J. Cosmol. Astropart. Phys. 11 (2013) 063.
[45] C. Bambi and N. Yoshida, *Shape and position of the shadow in the \( \delta = 2 \) Tomimatsu-Sato space-time*, Class. Quant. Grav. 27, 205006 (2010).
[46] L. Amarilla, E.F. Eiroa and G. Giribet, *Null geodesics and shadow of a rotating black hole in extended Chern-Simons modified gravity*, Phys. Rev. D 81, 124045 (2010).
[47] L. Amarilla and E. F. Eiroa, *Shadow of a rotating braneworld black hole*, Phys. Rev. D 85, 064019 (2012).
[48] L. Amarilla and E. F. Eiroa, *Shadow of a Kaluza-Klein rotating dilaton black hole*, Phys. Rev. D 87, 044057 (2013).
[49] A. Yumoto, D. Nitta, T. Chiba and N. Sugiyama, *Shadows of Multi-Black Holes: Analytic Exploration*, Phys. Rev. D 86, 103001 (2012).
[50] A. Abdujabbarov, F. Atamurotov, Y. Kucukakca, B. Ahmedov, and U. Camci, *Shadow of Kerr-Taub-NUT black hole*, Astrophys. Space Sci. 344, 429 (2013).
[51] F. Atamurotov, A. Abdujabbarov and B. Ahmedov, *Shadow of rotating non-Kerr black hole*, Phys. Rev. D 88, 064004 (2013).
[52] Z. Li and C. Bambi, *Measuring the Kerr spin parameter of regular black holes from their shadow*, J. Cosmol. Astropart. Phys. 01 (2014) 041.
[53] P.V.P. Cunha, C.A.R. Herdeiro, E. Radu, and H.F. Runarsson, *Shadows of Kerr black holes with scalar hair*, Phys. Rev. Lett. 115, 211102 (2015).
[54] T. Johannsen, *Photon Rings around Kerr and Kerr-like Black Holes*, Astrophys. J. 777, 170 (2013).
[55] A. Abdujabbarov, M. Amir, B. Ahmedov, and S.G. Ghosh, *Shadow of rotating regular black holes*, Phys. Rev. D 93, 104004 (2016).
[56] M. Amir and S.G. Ghosh, *Shapes of rotating nonsingular black hole shadows*, Phys. Rev. D 94, 024054 (2016).
[57] R. Kumar, B.P. Singh, M.S. Ali, and S.G. Ghosh, *Rotating black hole shadow in Rastall theory*, arXiv:1712.09793.
[58] B.P. Singh, *Rotating charge black holes shadow in quintessence*, arXiv:1711.02898.
[59] U. Papnoi, F. Atamurotov, S.G. Ghosh, and B. Ahmedov, *Shadow of five-dimensional rotating Myers-Perry black hole*, Phys. Rev. D 90, 024073 (2014).
[60] M. Amir, B. P. Singh and S. G. Ghosh, *Shadows of rotating five-dimensional charged EMCS black holes*, Eur. Phys. J. C 78, 399 (2018).
[61] B.P. Singh and S.G. Ghosh, *Shadow of Schwarzschild-Tangherlini black holes*, Annals Phys. 395, 127 (2018).
[62] T. Damour and S. N. Solodukhin, *Wormholes as black hole foils*, Phys. Rev. D 76, 024016 (2007).
[63] P.G. Nedkova, V.K. Tinechev, and S.S. Yazadjiev, *Shadow of a rotating traversable wormhole*, Phys. Rev. D 88, 124019 (2013).
[64] R. Shaikh, *Shadows of rotating wormholes*, arXiv:1803.11422.
[65] T. Ohgami and N. Sakai, *Wormhole shadows*, Phys. Rev. D 91, 124020 (2015).

[66] A. Abdujabbarov, B. Juraev, B. Ahmedov and Z. Stuchlík, *Shadow of rotating wormhole in plasma environment*, Astrophys. Space Sci. 361, 226 (2016).

[67] M. Amir, A. Banerjee and S. D. Maharaj, *Shadow of charged wormholes in Einstein-Maxwell-dilaton theory*, arXiv:1805.12435.

[68] S. Chandrasekhar, *The Mathematical Theory of Black Holes* (Oxford University Press, New York, 1992).