Given a smooth projective variety $X$, the Hilbert scheme $X^{[n]}$ of $n$ points on $X$ is again a smooth projective variety of dimension $\dim X^{[n]} = n \dim X$. Each vector bundle $E$ on $X$ defines the \textit{vector bundle} $E^{[n]} = p_2_* p_1^* E$. Here $p_k$ is the projection to the $k$-factor from the universal subscheme $\Pi_n \subset X \times X^{[n]}$. The bundle $E^{[n]}$ is called the Fourier-Mukai transform of $E$ (with respect to $\Pi_n$). By work of Lehn and Leh-Sorger, these transforms are important tools to study the topology and geometry of Hilbert schemes. Conversely, they are useful to study bundles on $X$ itself e.g. by work of Voison, Ein-Lazarsfeld and Agostini.

The present article enhances the Fourier-Mukai transform to so-called V-cotwisted Hitchin pairs $(E, \theta)$. Here $E$ and $V$ are vector bundles on $X$ and $\theta : E \otimes V \to E$ is a section. The outcome of the enhanced Fourier-Mukai transform are $V^{[n]}$-cotwisted Hitchin pairs $(E^{[n]}, \theta^{[n]})$ on $X^{[n]}$. Note here that if $V = T_X$, then $V^{[n]} \cong T_{X^{[n]}}(- \log B_n)$ (by a result of Stapleton) where $B_n \subset X^{[n]}$ is the locus of non-reduced subschemes of $X$. In particular, the enhanced Fourier-Mukai transforms of Higgs bundles (i.e. $T_X$-cotwisted Hitchin pairs) are logarithmic Higgs bundles on $X^{[n]}$.

After establishing basic results on the enhanced Fourier-Mukai transform, which are of independent interest, the authors prove various interesting results on the relationship between Hitchin pairs on $X$ and their enhanced Fourier-Mukai transforms (similar results for vector bundles were already obtained by the second author), for example:

- If $(E, \theta)^{[n]} \cong (F, \eta)^{[n]}$ on $X^{[n]}$, then $(E, \theta) \cong (F, \eta)$ on $X$ where $X$ is any smooth projective curve of genus $\geq 1$ or any smooth quasi-projective variety of $\dim X \geq 2$;
- relationship between the stability conditions for $(E, \theta)$ on $X$ and $(E, \theta)^{[n]}$ on $X^{[n]}$ for any smooth projective curve $X$.

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MSC:

- 14D23 Stacks and moduli problems
- 14D20 Algebraic moduli problems, moduli of vector bundles
- 14H30 Coverings of curves, fundamental group
- 14F08 Derived categories of sheaves, dg categories, and related constructions in algebraic geometry
- 14C05 Parametrization (Chow and Hilbert schemes)

Keywords:

- logarithmic Higgs bundle; Hilbert scheme; Fourier-Mukai transformation; stability

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