A new multi-objective algorithm for underactuated robotic finger during grasping and pinching assignments

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Abstract. This study presents new kinematic and kinetic equations of grasping and pinching assignments, which have been derived to consider the angles for the ternary solid links of a three-phalanx linkage underactuated robotic finger mechanism. Numerical and experimental investigations are conducted. Numerically, eight design criteria are adopted for finding an optimal solution using a new multi-objective function algorithm (grasping stability percentage, forces of grasping, squeezing force, grasping assignment mimic function, transmission angle for grasping assignment, mimic function for pinching assignment, transmission angle for pinching assignment and pinching force). The gradient descent method, comprising three steps, is used to find the optimal geometric parameters. These parameters are modified to fabricate the prototype of the robotic finger. Experimentally, the three-phalanx linkage finger is adjusted via the addition of a DC motor at the second linkage coupler link, having four bars for length adjustment for performing the aspect of human finger through the grasping and pinching assignments. The results show that adding ternary solid links to the robotic finger enhances the grasping stability and assists the complete embrace of objects, particularly objects that are smaller than those for the same finger without ternary solid links. In addition, the results indicate that the pinching force enhancement is 7–25% with reduction of the objects’ sizes, due to the ternary of solid links.

1. INTRODUCTION

In robotics, prosthetic grippers and hands can be important and challenging subsystems in terms of design [1-2]. Generally, fingers that are totally actuated are too intricate mechanically, and require associated control algorithms. Uncomplicated designs, like underactuated fingers, have thus attracted attention for reasons including weight and cost.

Whereas fingers that are totally actuated have the same number of actuators and DOFs, the underactuated forms possess a lower number of actuators than the number of DOFs and, therefore, their kinematic aspects are not completely restricted by such actuators. This feature gives underactuated fingers the capability to be self-adaptive, and thus to mechanically fit their grasping objects. Because the underactuated fingers possess fewer actuators than totally actuated corresponding parts, these fingers mostly grasp with uncomplicated governing algorithms. Also, underactuated fingers need fewer sensors, like tactile pads, encoders or potentiometers [3]. As an example, the Barrett Hand shape has three fingers, and each one possesses two phalanxes, but it only utilizes four actuators; one for each finger, and one to relatively reshape such fingers to the inner surface of the hand [4].

Human fingers can also pinch objects by the fingertips, for example when writing with a pen or holding paper [5]. A robotic finger has begun to appear in research laboratories that has more active fields in design, analysis and control, and this has caught the attention of researchers [6].
With an anthropomorphic underactuated finger, the mechanism has been designed to achieve grasping and pinching assignments. A linkage mechanism for an underactuated finger was presented as a model to achieve such grasping and pinching tasks [7]. This was basically a mechanism of the underactuated finger, having a seven-bar linkage and resembling the human finger in its ability to pinch and grasp. Others have proposed a design whereby the linkage of a finger had three phalanxes with self-adaptation and coupling benefits [8]. Available topology designs, motions and finger grasping patterns were manifested and compared with those for conventional underactuated finger and coupled finger. Furthermore, simulation of the grasp stability was conducted via static analysis. That finger, having three phalanxes with self-adaptation and coupling benefits, was made to adapt to the requirements of either humanoid robot hands or prosthetic hands.

A parametric optimization approach for the mechanism of an anthropomorphic finger during self-adaptive grasping and pinching assignments has also been presented [9]. This mechanism was constructed, depending upon the human finger through pinching and grasping. It had linkages with seven bars, and its top middle phalanx was combined with a lead screw mechanism and a curved guiding slot, to adopt the influential length of link in performing these assignments. The geometrical parameters of fingers were optimized depending on four design criteria: grasping force and kinematics, and anthropomorphic pinching force and kinematics, to improve finger performance. The numerical results gave enhanced group parameters in comparison with the first guess.

A design for of a robotic gripper was introduced by [10] to realize locked and robust grasps. A solution was proposed, to design a smart underactuated mechanism with a self-locking set in parallel to the actuators for triggering automatically where the demanded grasp is performed. The results demonstrated this mechanism may influence the adaptable power spread between the brakes, via the gripper and a differential gear.

Kinematic optimization and design of an underactuated, linkage-based robotic hand exoskeleton, were shown by [11] to perform a grasping operation. The system was designed to stratify just the normal forces to the phalanxes of the finger through the fingers’ flexion/extension, while supplying automatic adaption for various finger sizes. In addition, an analysis of kinematic optimization for the device by grasp stability and statics was performed. This analysis was employed to optimize the lengths of the mechanism link, to ensure a motion of proper range while maximizing the force transmission on the finger joints.

The implemented augmentation of underactuated fingers through supplemental actuators was exhibited by [12]. A small finger was designed and built, its control algorithms being used to change both movement and grasp conductance. It was also proposed to utilize more than a single actuator to drive the underactuated fingers and obtain better, distinctive metrics for use in measuring grasp characteristics, like stability and stiffness. A general kinetostatic analysis was conducted and acclimatized, for converting the underactuated fingers to a state utilizing more than a single actuator. An investigation of geometric optimization for an underactuated linkage three-phalange robotic finger was introduced by [13]. The proposed robotic finger was studied for grasp function only.

However, none of the projects described above has focused on the analysis of kinetic and kinematic of underactuated robotic fingers during the grasping and pinching assignments, by considering the angles of the ternary solid links in the four-bar linkages. Therefore, the main objective of the present study was to derive and introduce new equations for kinetic and kinematic analysis of a robotic finger through grasping and pinching assignments, considering angles of the ternary solid links in the four-bar linkages. In addition, experimental work was conducted for constructing a robotic finger according to the optimal dimensions of robotic fingers and test performance by grasping and pinching objects.
2. MECHANISM OF ROBOTIC FINGER

In the present research, the robot finger is intended to be capable of grasping and pinching assignments. The mechanism of the robotic finger consists of three phalanxes: proximal, middle and distal phalanxes, as shown in Figure 1. The functional upper link length of the middle phalanx of underactuated robotic finger must be comparatively long in the grasping assignment, whereas it is short during the pinching assignment. A DC motor is connected to a screw and coupling system, to shorten and lengthen the coupler link to achieve the required length. The movement of the robotic finger begins from the slider mechanism by giving the input torque to the system provided at link 1. The solid link (link 1) of the first linkage is the input link, having four bars, and will accelerate and move the second link, which is the coupler link (link 2), to transmit the movement to the output link (link 3) to accelerate the first phalanx. Because the first mechanism has four bars, link 3 is the input to the second four-bar mechanism that will move the coupler link (link 4) of the mechanism, to transmit the movement to the output link, which leads to motivation of the second phalanx. Because the second mechanism has four bars, the third solid link input will move the third phalanx.

![Robotic finger mechanism](image)

Figure 1. Robotic finger mechanism

3. KINETIC ANALYSIS OF FINGER

3.1. GRASPING FORCE

In this study, a model of grasping forces was introduced for the finger mechanism shown in Figure 2. Based on the principle of virtual work, the grasping force can be written as: [3]

$$ F^T = t_a^T J_w^{-1} J_v^{-1} $$  \hspace{1cm} (1)

Where, $t_a^T$ : The vector of input torque applied via the actuator and springs mounted among the phalanxes. $J_v$ : The lower triangular matrix that describes the Jacobian matrix for a three phalanx robotic finger, considering that the friction between the grasped object and the contact points can be written as: [14]
The friction coefficient ($\mu$) relies upon the object–finger surface pair material, for the solid–rubber, the numerical values are 1–4, and a conservative magnitude of 2 has been selected [14].

![Figure 2. Static model of three-phalanx robotic finger](image)

### 3.2. CALCULATION OF THE TRANSMISSION MATRIX ($J_{\omega}$)

In this study, $J_{\omega}$ is the matrix of transmission, which characterizes the underactuation between the fingers using four-bar linkage mechanism and considering the angle of ternary solid links (i.e. $\gamma_2$). The transmission matrix ($J_{\omega}$) relates the vector $\omega_\alpha$ of angular velocities in the joints hosting either the actuator or the torsional spring, to the phalanx joints angular velocities, i.e. [15]

$$\dot{\theta} = J_\omega \omega_\alpha$$ (3)
\[
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix} =
\begin{bmatrix}
n_1 & n_2 & n_3 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_a \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix}
\] (4)

When fingers are completely actuated, \(J_a\) should identify the dimension matrix (3). \(n_i\) is a function of linkage geometrical parameters utilized to transmit the torque of actuator to the phalanges.

Where:
\[
\dot{\theta}_1 = n_1\dot{\theta}_a + n_2\dot{\theta}_2 + n_3\dot{\theta}_3 
\] (5)

To obtain \((\dot{\theta}_a)\) as a function of vector \(\dot{\theta}_i\) for present model [16], for the three-phalanx finger, equation (5) can be written as:

\[
\dot{\theta}_1 = \dot{\theta}_a + A\dot{\theta}_2 + AB\dot{\theta}_3 
\] (6)

Referring to Figure 2, the parameters A and B of equation (6) can be found by starting from the second linkage having four bars \((a_2b_2c_2l_2)\) at the known angle \((\psi_2)\) to obtain the third joint velocity \((\dot{\theta}_3)\) as a function of \((\dot{\psi}_2)\). Where, \((\psi_2)\) represents the second four-bar linkage input angle, while \((\theta_3)\) represent, the angle of the third phalanx and \((\dot{\psi}_2)\) is the velocity of the input link, as shown in Figure 2 using cosine law:

\[
r_{p_2}^2 = l_2^2 + a_2^2 - 2l_2a_2\cos(\psi_2) 
\] (7)

\[
r_{p_2}^2 = c_2^2 + b_2^2 - 2c_2b_2\cos(\psi_3 - \theta_2 - \theta_3 + x) 
\] (8)

Equating the equations (7) and (8) and simplifying to get:

\[
l_2^2 + a_2^2 - 2l_2a_2\cos(\psi_2) = 
\]

\[
c_2^2 + b_2^2 - 2c_2b_2(\cos x \cos(\psi_3 - \theta_3 - \theta_2)) + 2c_2b_2 \sin x \sin(\psi_3 - \theta_3 - \theta_2) 
\] (9)

Referring to Figure 3, the value of cosine and sine of the angle \((x)\):

\[
\sin(x) = \frac{a_2\sin(\psi_2 - \theta_2) - c_2\sin(\psi_3 - \theta_3 - \theta_2) + l_2\sin(\theta_2)}{b_2} 
\] (10)
\[
\cos(x) = \frac{l_2 \cos(\theta_2) + c_2 \cos(\psi_3 - \theta_3 - \theta_2) - a_2 \cos(\psi_2 - \theta_2)}{b_2}
\] (11)

Figure 3. The angle (x) at the second four-bar linkage

Angle (\(\dot{\theta}_3\)) can be taken as a function of (\(\dot{\psi}_2\)) by substituting equations (10) and (11) into equation (9):

\[
\dot{\theta}_3 = \frac{2l_2 a_2 \sin(\psi_2) + 2a_2 c_2 \sin(\psi_2 - \psi_3 + \theta_3)}{2c_2 l_2 \sin(\psi_3 - \theta_3) + 2a_2 c_2 \sin(\psi_2 - \psi_3 + \theta_3)} \dot{\psi}_2
\] (12)

Back to the first linkage having four bars (\(a_1, b_1, c_1, l_1\)), the angle (\(\psi_2\)) can be found as a function of \(\theta_2, \theta_1\) and \(\theta_1\) by adding the constant angle (\(\gamma_2\)) to the output angle of the first four-bar linkage. According to cosine law,

\[
r^2_{p_1} = l_1^2 + a_1^2 - 2l_1 a_1 \cos(\psi_1)
\] (13)

\[
r^2_{p_1} = c_1^2 + b_1^2 - 2c_1 b_1 \cos[(\psi_2 - \theta_2 - \theta_1 + \gamma_2) + \psi]
\] (14)

Equating equations (13) and (14) and simplifying to get:
\[ l_1^2 + a_1^2 - 2l_1a_1 \cos(\psi_1) = c_1^2 + b_1^2 - 2c_1b_1[\cos(\psi_2 - \theta_2 - \theta_1 + \gamma_2) \cos(\psi)] + 2c_1b_1[\sin(\psi_2 - \theta_2 - \theta_1 + \gamma_2) \sin(\psi)] \]  

Referring to Figure 4, the value of cosine and sine of the angle (\( \psi \)) can be calculated:

\[
\cos(\psi) = \frac{l_1 \cos\theta_1 - c_1 \cos[\pi - (\psi_2 - \theta_2 - \theta_1 + \gamma_2)] - a_1 \cos(\psi_1 - \theta_1)}{b_1} 
\]  

\[
\sin(\psi) = \frac{l_1 \sin\theta_1 - c_1 \sin[\pi - (\psi_2 - \theta_2 - \theta_1 + \gamma_2)] + a_1 \sin(\psi_1 - \theta_1)}{b_1} 
\]  

![Figure 4](image.png)  

Figure 4. The angle (\( \psi \)) at the first four-bar linkage

Substituting equations (16) and (17) into equation (15) and differentiating with respect to time gives:

\[
\dot{\psi}_2 = \theta_2 - \left[ \frac{l_1a_1 \sin(\psi_1) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))}{c_1l_1 \sin((\psi_2 - \theta_2 + \gamma_2) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))} \right] \theta_1 
\]  

\[
+ \left[ \frac{l_1a_1 \sin(\psi_1) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))}{c_1l_1 \sin((\psi_2 - \theta_2 + \gamma_2) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))} \right] \theta_1 
\]
Substituting equation (18) into equation (12) yields:

\[
\dot{\theta}_3 = - \left[ \frac{2l_2a_2 \sin(\psi_2) + 2a_2c_2 \sin(\psi_2 - \psi_3 + \theta_3)}{2c_2l_2 \sin(\psi_3 - \theta_3) + 2a_2c_2 \sin(\psi_2 - \psi_3 + \theta_3)} \right] \dot{\theta}_2 \\
+ \left[ \frac{l_1a_1 \sin(\psi_1) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))}{c_1l_1 \sin((\psi_2 - \theta_2 + \gamma_2) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))} \right] \dot{\theta}_a \\
+ \left[ \frac{l_1a_1 \sin(\psi_1) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))}{c_1l_1 \sin((\psi_2 - \theta_2 + \gamma_2) + a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))} \right] \dot{\theta}_1
\]  

(19)

Simplifying equation (19) and rearrangement to get the constants A and B, which can be written as:

\[
A = \frac{l_1c_1 \sin(\theta_2 - (\psi_2 - \gamma_2)) - a_1c_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))}{l_1a_1 \sin(\psi_1) + c_1a_1 \sin(\psi_1 - (\psi_2 + \theta_2 - \gamma_2))}
\]

(20)

\[
B = \frac{l_2c_2 \sin(\theta_3 - \psi_3) - a_2c_2 \sin((\psi_2 - \psi_3 + \theta_3)}{l_2a_2 \sin((\psi_2) + a_2c_2 \sin((\psi_2 - \psi_3 + \theta_3)}}
\]

(21)

Considering the four-bar linkages as transmission finger mechanisms, it is obtained that [8]:

\[
\begin{bmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\dot{\theta}_3
\end{bmatrix} = J_\omega \omega_a =
\begin{bmatrix}
1 & A & -AB \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(22)

When the torques exerted by the springs is ignored, the grasping force can be expressed as:
\[ f = \begin{bmatrix} h_2h_3 + AXh_3 + ABXY - ABZ \\ h_1h_2h_3 \\ Ah_3 + ABY \\ - \frac{h_2h_3}{h_3}t_a \\ \frac{ABt_a}{h_3} \end{bmatrix} \] (23)

Where:

\[ X = l_1 (\cos \theta_2 + \mu \sin \theta_2) + h_2 \] (24)

\[ Y = l_2 (\cos \theta_3 + \mu \sin \theta_3) + h_3 \] (25)

\[ Z = l_1 \cos (\theta_2 + \theta_3) + l_1 (\cos \theta_3 + \mu \sin \theta_3) + h_3 \] (26)

It can be noted that the equation (20), contains the terms of angle \( \gamma_2 \) which differs from the previous research, in particular [8]. If the angle \( \gamma_2 \) is equal to zero, the matrix in equation (22) will be the same as the transmission matrix for the four-bar linkage without \( \gamma_2 \). This implies that the newly-derived form of transmission matrix is more general than the previous one.

3.3. PINCHING FORCE

In this research, the static analysis of the free-body diagram for the proposed finger through the assignment of pinching is shown in Figure 5.

Figure 5. Kinetic analysis of finger during pinching assignment
The performed pinching force is calculated. Summation of the forces in the (Y-axis) is zero, then:

\[ F_p = N \]  \hspace{1cm} (27)

Where, \( F_p \) is the pinching force, and \( N \) is the normal force, they respectively act on a distal phalanx. By assuming that the force of pinching is applied at the mid of distal phalanx[9], \( F_p \) can be computed using the moment expression about the point (A) as:

\[ F_p = \frac{t_a}{(l_a - l_b \mu)} \]  \hspace{1cm} (28)

\[ l_a = l_1 \cos(\theta_{11}) + l_2 \cos(\theta_p) + 0.5l_3 \cos(\theta_{31}) \]  \hspace{1cm} (29)

\[ l_b = -l_1 \sin(\theta_{11}) - l_2 \sin(\theta_p) - 0.5l_3 \sin(\theta_{31}) - d \sin(\theta_{31} - \frac{\pi}{2}) \]  \hspace{1cm} (30)

4. KINEMATIC ANALYSIS OF FINGER

4.1. KINEMATIC ANALYSIS FOR GRASPING ASSIGNMENT

The movement of phalanxes is restricted via the part. Two steps are regarded: the first step is when only the proximal phalanx contacts the part, while the second step is when both distal and proximal phalanxes touch the part. At the first stage, the link \( l_1 \) represents the base, while angle \( (\theta_{12}) \) is angle of input of first linkage with four bars, as shown in Figure 6.

![Figure 6. Static model of three-phalanx finger](image-url)
To calculate \((\theta_{12})\), \((\theta_s)\) needs to be defined, which represent the output angle of the slider mechanism that can be written as:

\[
\theta_s = 2\tan^{-1}\left[ \frac{-B_s \pm \sqrt{B_s^2 - C_s^2 + A_s^2}}{C_s - A_s} \right] \quad (31)
\]

\[
A_s = 2x_3x_4 \cos \theta x_4 - 2x_1x_3 \cos \theta x_1 \quad (32)
\]

\[
B_s = 2x_3x_4 \sin \theta x_4 - 2x_1x_3 \sin \theta x_1 \quad (33)
\]

\[
C_s = x_1^2 + x_3^2 + x_4^2 - x_2^2 - 2x_1x_4(\sin \theta x_1 \sin \theta x_4 + \cos \theta x_1 \cos \theta x_4) \quad (34)
\]

In this study, \((\theta_{12} = \theta_s - \gamma_1)\) isangle of input of first linkage with four bars, as shown in Figure 6. \(\theta_{m1}\) represents the output angle of the first four-bar linkage, which can be computed as:

\[
\theta_{m1} = 2\tan^{-1}\left[ \frac{-B_1 \pm \sqrt{B_1^2 - C_1^2 + A_1^2}}{C_1 - A_1} \right] \quad (35)
\]

\[
A_1 = 2l_1 c_1 \cos(\theta_{11}) - 2a_1 c_1 \cos(\theta_{12}) \quad (36)
\]

\[
B_1 = 2l_1 c_1 \sin(\theta_{11}) - 2a_1 c_1 \sin(\theta_{12}) \quad (37)
\]
\[ C_1 = a_1^2 + c_1^2 + l_1^2 - b_1^2 - 2a_1 l_1 [\cos(\theta_{11}) \cos(\theta_{12}) + \sin(\theta_{11}) \sin(\theta_{12})] \]  

(38)

In the second stage, \( (\theta_{22} = \theta_{m1} - \gamma_1) \) is the input angle of the second four-bar linkage. \( \theta_{m2} \) represents the output angle of the second four-bar linkage, which can be calculated as:

\[ \theta_{m2} = 2\tan^{-1}\left[-\frac{B_2 \pm \sqrt{B_2^2 - C_2^2 + A_2^2}}{C_2 - A_2}\right] \]  

(39)

\[ A_2 = 2l_2 c_2 \cos(\theta_{21}) - 2a_2 c_2 \cos(\theta_{m1} - \gamma_2) \]  

(40)

\[ B_2 = 2l_2 c_2 \sin(\theta_{21}) - 2a_2 c_2 \sin(\theta_{m1} - \gamma_2) \]  

(41)

\[ C_2 = a_2^2 + c_2^2 + l_2^2 - b_2^2 \]
\[ - 2a_2 l_2 [\cos(\theta_{21}) \cos(\theta_{m1} - \gamma_2) + \sin(\theta_{21}) \sin(\theta_{m1} - \gamma_2)] \]  

(42)

Referring to Figure 6, the angle \( \theta_{21} \) and angle \( \theta_{31} \) can be calculated as:

\[ \theta_{21} = \theta_{m1} - \pi - \tan^{-1}\left(\frac{a_1}{l_1}\right) - \cos^{-1}\left[\frac{b_1^2 - c_1^2 - (a_1^2 + l_1^2)}{-2c_1 \sqrt{a_1^2 + l_1^2}}\right] \]  

(43)

\[ \theta_{31} = \theta_{m2} - \cos^{-1}\left[\frac{c_3^2 - c_2^2 - l_3^2}{-2c_2 l_3}\right] \]  

(44)
Equations (31–38) were used to find the angle of output of the first linkage with four bars; it is noticed that they are the same equations for the four-bar linkage in reference [7], but the equations in the present study included the angle of the ternary solid link ($\gamma_1$), as shown in Figure 6. Also, the angle ($\gamma_2$) was noticed in equations (39–42) used to find the angle of output of the second linkage with four bars. As was stated before, the new form of equation is more general than the previous one.

4.2. KINEMATIC ANALYSIS FOR PINCHING ASSIGNMENT

Through pinching configuration, the mechanism of the robotic finger is composed of two linkages with four bars connected in series. The first linkage with four bars is $a_1, b_1, c_1, l_1$ and the second linkage with four bars is $a_2, b_2, c_2, l_2$. When the direction of the torque of input is clockwise and is exerted on the link $a_1$, this mechanism is moving as a whole till the distal phalax contacts the part. The triangular link changes the counter to the direction of clockwise if the distal phalange surface contacts the ground. This leads the link having length $l_2$ to move in a vertical direction. Only the links with the length of $a_2, b_2, c_2$ and $l_2$ are regarded in pinching for the kinematics expressing, as shown in Figure 7.

![Figure 7. Kinematic analysis of finger during the pinching assignment](image)

In this case, the linkage ($a_2$) works as a base and the linkage ($c_2$) is a moving link. Thus, $\theta_{22} = (\theta_{m1} - \gamma_2)$ and the angle of output ($\theta_p$) is written as: [7]

$$\theta_p = 2\tan^{-1}\left[ \frac{-B_p \pm \sqrt{B_p^2 - C_p^2 + A_p^2}}{C_p - A_p} \right]$$  \hspace{1cm} (45)

Where:

$$A_p = 2l_2c_2\cos(\theta_{m2}) - 2a_2l_2\cos(\theta_{m1} - \gamma_2)$$  \hspace{1cm} (46)

$$B_p = 2l_2c_2\sin(\theta_{m2}) - 2a_2l_2\sin(\theta_{m1} - \gamma_2)$$  \hspace{1cm} (47)
\[ C_p = a_2^2 + c_2^2 + l_2^2 - b_{2p}^2 - 2a_2c_2[\cos(\theta_{m1} - \gamma_2)\cos(\theta_{m2}) + \sin(\theta_{m1} - \gamma_2)\sin(\theta_{m2})] \] (48)

4.3. KINEMATIC ANALYSIS FOR TRANSMISSION ANGLE

The transmission angle is posture-dependent; it is a function of linkage variable input. The transmission angle of the four-bar linkage mechanism should be between 45° and 135°, so that the efficiency of transmission angle of the four-bar linkage mechanism will be accepted [17]. Referring to Figure 8, the angle of transmission of first linkage with four bars can be written as:

\[ \mu_{t1} = \cos^{-1}\left[\frac{b_1^2 + c_1^2 - O_1^2}{2b_1c_1}\right] \] (49)

Where:

\[ O_1^2 = a_1^2 + l_1^2 - 2a_1l_1\cos(\theta_{12} - \theta_{11}) \]
\[ \theta_{12} = \theta_s - \gamma_1 \] (50)

Similarly, for the mechanism of second linkage with four bars,

\[ \mu_{t2} = \cos^{-1}\left[\frac{b_2^2 + c_2^2 - O_2^2}{2b_2c_2}\right] \] (51)

Where:

\[ O_2^2 = a_2^2 + l_2^2 - 2a_2l_2\cos(\theta_{22} - \theta_{21}) \]
\[ \theta_{22} = \theta_{m1} - \gamma_2 \] (52)
5. MULTI-OBJECTIVE FUNCTION

The gradient descent method was chosen to find the optimal geometric parameters within a specific range of variables. This method is one of the most widely-used approaches in research aiming to solve optimization problem [18]. It starts with creation of all the possible solutions to find the objective function. When all possible solutions have been determined, the optimal design parameters are calculated for each objective function. Once the optimal solutions for each objective function are found, then the best solutions of all the objective functions together will be chosen by collecting the minimum optimization formulation \( G_T \), which is:

\[
G_T = \sum_{i=1}^{n} G_i \times w_i
\]

Where:

- \( G_T \): Optimization formulation.
- \( G_i \): The \( i \)th objective function.
- \( n \) = Number of objective functions; in this study \( n=8 \)
- \( w \) : Weight of the optimization.

The weight of the multi-objects optimization is assumed one for all objective functions in this study. The gradient method consists of three steps of solution to determine the optimal geometric parameters with high precision. When the first step is finished, the second step starts by searching for the optimum solution parameters at the optimal solution of the first step. Making a new range depends on the optimal of the first step to start with the second step of optimization. The new range can be calculated using the following equations:
\[ M = (Max.\ value - Min.\ value) \times \frac{1}{8} \]  

\[ New\ range = (best\ of\ first\ stage - M\ best\ of\ first\ stage + M) \]  

The third step is the determination of whichever is the better of the two stages to achieve the optimal geometric parameters. Some of the parameters were chosen as input parameters as listed in Table 1, which illustrates the optimization input parameters. The parameters \( l_1, l_2 \) and \( l_3 \) were taken as one and a half times of the original length of the human finger [19].

| Parameters  | Unit  | Minimum Values | Maximum Values |
|-------------|-------|----------------|----------------|
| \( a_1 \)    | mm    | 108            | 118            |
| \( b_1 \)    | mm    | 85             | 95             |
| \( c_1 \)    | mm    | 51             | 65             |
| \( a_2 \)    | mm    | 65             | 78             |
| \( b_2 \) (for grasping) | mm    | 70             | 80             |
| \( c_2 \)    | mm    | 28             | 37             |
| \( \gamma_1 \) | Degree | 30             | 60             |
| \( \gamma_2 \) | Degree | 30             | 60             |
| \( x_2 \)    | mm    | 70             | 80             |
| \( x_3 \)    | mm    | 30             | 40             |
| \( b_2 \) (for pinching) | mm    | 50             | 60             |
| \( l_1 \)    | mm    | 64             | 64             |
| \( l_2 \)    | mm    | 37             | 37             |
| \( l_3 \)    | mm    | 34             | 34             |
| \( h_1 \)    | mm    | 32             | 32             |
| \( h_2 \)    | mm    | 18.5           | 18.5           |
| \( h_3 \)    | mm    | 17             | 17             |
| \( \gamma_3 \) | Degree | 60             | 60             |
| \( x_1 \)    | mm    | 40             | 40             |
| \( x_4 \)    | mm    | 100            | 100            |
6. GEOMETRIC OPTIMIZATION

It is very complicated to separate each parameter to find the optimum solution. To fix the problem, a gradient descent method was chosen. The criteria were defined to find the parameters.

6.1 First criterion: the percentage of the grasping stability

In order to characterize the capability for the underactuated finger to create the full grasp which characterized the robustness against ejection for the phalanx [20],

\[ G_{1n} = \frac{\int_{w} \delta(\theta^*, h^*) dh^* d\theta^*}{\int_{w} dh^* d\theta^*} \] (56)

Where, \( \delta(\theta^*, h^*) \) is a Kronecker-like representative for the \((f)\) vector’s positive, which removes the non-whole-phalanx grasps:

\( \delta \) also, \( W \) is the finger workspace in term of \((\theta^*, h^*)\); that means the hyper parallelepiped is clarified via \( l_i > k_i > 0 \) and \( \frac{\pi}{2} > \theta_i > 0 \), \( i > 0 \). Physically, such index denotes the stability percentage that is conductible by the complete grasps of phalanx, like the workspace of the whole-hand grasping. In this study, minimizing of objective function will minimize the term \((G_1)\).

\[ G_1 = |100 - G_{1n}| \] (57)

6.2 Second criterion: the grasping forces

This is defined as the ratio of the total force on the three phalanxes to the largest force as given in the equation below [20]:

\[ G_{2n} = \frac{f_1 + f_2 + f_3}{\max(f_i)} \quad \forall \theta_i, \quad i = 1, 2, 3 \] (58)

In this study, to transform the objective function to a minimum one, there is a need to deduct the above objective function from the maximum number, so:

\[ G_2 = |3 - G_{2n}| \] (59)
6.3. Third criterion: squeezing force

To assure the stability of the grasp, one needs a certain squeezing force. This force should be as high as possible, then [20]:

\[
G_{3n} = \frac{(f_1 + f_2 + f_3)_{max}}{F_a} \quad \forall \theta_i, \quad i = 1, 2, 3 \tag{60}
\]

Taking the maximum squeezing force as (14.82 N), then the minimum objective function of the squeezing force is [21]:

\[
G_3 = |14.82 - G_{3n}| \tag{61}
\]

6.4. Fourth criterion: mimic function for grasping assignment

In this case, the objective function is minimizing the unlikeness between the two angles made via the robotic and human finger through the grasping, which are mathematically expressed as [7]:

\[
G_4 = |\theta_{h21} - \theta_{21}| + |\theta_{h31} - \theta_{31}| \tag{62}
\]

The values of the two angles that made by the human finger are \(\theta_{h21} = 325^\circ\) and \(\theta_{h31} = 280^\circ\), which are for the cylindrical objects grasping with a radius of 3cm [9].

6.5. Fifth criterion: transmission angle for grasping assignment

Generally, the best transmission angle for the four-bar linkages mechanism is \(90^\circ\)[17], therefore to obtain the minimum objective function:

\[
G_5 = |\mu_{t1} - 90^\circ| + |\mu_{t2} - 90^\circ| \tag{63}
\]

6.6. Sixth criterion: mimic function for pinching assignment

The induced angle via a human finger through the pinching \(\theta_{hp}\) is quantified by taking a photo and comparing it with the angle of the robotic finger \(\theta_p\) induced through the pinching. Then, the objective function is minimizing the difference between the angles made by the human finger and the robotic finger[7].

\[
G_6 = |\theta_{hp} - \theta_p| \tag{64}
\]
The angle that is made by the human finger is taken $\theta_{hp} = 290^\circ$[9].

6.7. Seventh criterion: transmission angle for pinching assignment

The best transmission angle for the four-bar linkages mechanism is $90^\circ$, so to get the minimum objective function for the pinching configuration when the coupler link is $b_2$[17],

$$G_7 = |\mu_{tp1} - 90^\circ| + |\mu_{tp2} - 90^\circ|$$  \hspace{1cm} (65)

Where:

$$\mu_{tp1} = \cos^{-1}\left[\frac{b_1^2 + c_1^2 - O_1^2}{2b_1c_1}\right]$$  \hspace{1cm} (66)

$$\mu_{tp2} = \cos^{-1}\left[\frac{b_2^2 + c_2^2 - O_2^2}{2b_2c_2}\right]$$  \hspace{1cm} (67)

6.8. Eighth criterion: pinching force

If the practical pinching force of human per finger is 2.96 N [21], then the minimum objective function of the pinching force is[9]:

$$G_8 = |2.96 - F_p|$$  \hspace{1cm} (68)

The flow chart of the optimization program is shown in Figure 9.
7. RESULTS AND DISCUSSION

A set of initial lengths of robotic finger is selected to cover the minimum and maximum boundaries of each chosen parameter, as listed in Table 1. The grasping forces at three phalanxes are studied in the space bounded by a hyper-parallelepiped. This seems to be a reasonable workspace for this application of the human finger during grasping assignment. Optimization is performed using a gradient descent method with three stages. The optimum solution is given in Table 2.

As has been stated, the goal of this optimization is to find a suitable and appropriate solution. Therefore, the optimal design is modified to facilitate the fabrication of the robotic finger prototype, as shown in Table 3.
The variation in these parameters does not change the value of the optimization formulation $GT$. Figure 10 represents the contact forces and grasping stability of three phalanxes for the robotic finger with adding the angles of ternary solid links.

Table 2. Optimum solution of geometric parameters.

| Parameters | Unit | Final Results |
|------------|------|---------------|
| $a_1$      | mm   | 116.3333      |
| $b_1$      | mm   | 87.5000       |
| $c_1$      | mm   | 63.8333       |
| $a_2$      | mm   | 76.9167       |
| $b_2$      | mm   | 78.3333       |
| $c_2$      | mm   | 36.2500       |
| $\gamma_1$| Degree| 34.9999       |
| $\gamma_2$| Degree| 34.9999       |
| $x_2$      | mm   | 79.1667       |
| $x_3$      | mm   | 34.1667       |
| $b_{2\text{pinch}}$ | mm | 50.8333     |
| $G_T$      |       | 25.97034      |

Table 3. Modified optimum solution for geometric parameters to facilitate the fabrication of the finger prototype.

| Parameters | Unit    | Modified solution |
|------------|---------|-------------------|
| $a_1$      | mm      | 116               |
| $b_1$      | mm      | 88                |
| $c_1$      | mm      | 64                |
| $a_2$      | mm      | 77                |
| $b_2$      | mm      | 78                |
| $c_2$      | mm      | 37                |
| $\gamma_1$| Degree  | 35                |
| $\gamma_2$| Degree  | 34                |
| $x_2$      | mm      | 80                |
| $x_3$      | mm      | 34                |
| $b_{2\text{pinch}}$ | mm | 51               |
| $G_T$      |         | 25.99302          |
It can be seen that the percentage of grasping stability is 55% when all the forces of the three phalanxes are positive. Figure 11 shows that when the angles of the first and second ternary solid links are considered to be zero, the percentage of stability decreases to 53%. So, adding the ternary solid links leads to a slight (2%) increase in the percentage of stability.
The robotic finger with solid links can grasp small objects, whereas the finger without solid links is unable to grasp the small objects, as Figure 12 shows.

![Figure 12](image)

(a) Robotic finger with solid links, (b) robotic finger without solid links

On this basis, the purpose of the entire design is to provide full grasp and containment to the objects. Adding the ternary solid links assists in the complete embrace of objects, especially small ones. Also, appending the ternary solid links to the robotic finger makes the robotic finger more flexible and adaptable, thus increasing the finger portability to grasp a small object. Figure 13 refers to the relationship between the percentage of stability and the coefficient of friction between the finger and the object. It is observed that the increasing in the coefficient of friction leads to a linear increase in the area of grasping stability.

![Figure 13](image)

Figure 13. Relation between stability region and the coefficient of friction
Finally, Figure 14 displays the relationship between the angle of first phalanx ($\theta_{11}$) and the pinching force of the robotic finger with and without ternary solid links. The results revealed that the pinching force enhances by (+7 to +25 %) due to the ternary of the solid links when the angle $\theta_{11}$ is less than 50° (i.e. reducing the size of objects is minimum). However, the effect of ternary solid links is reduced or vanished when the angle increases for more than 50°. Accordingly, utilizing the ternary solid links to the robotic finger is necessary for grasping and pinching of small objects.

Next, after analysis of the problem using analytical techniques, and experimental work to manufacture the robotic to analyze the mechanical behaviour relevant to its problem the results were compared to give the agreement and maximum error for results.

![Figure 14. Relationship between angle of first phalanx ($\theta_{11}$) and pinching force](image)

8. MANUFACTURE OF THE ROBOTIC FINGER

The rapid prototyping of a robotic finger was built from hard plastic (polylactic acid) using a 3D printing machine. The robotic finger mechanism possesses three phalanxes composed of two four-bar linkages with solid links. This robot was fabricated with a DC motor (9 volt) that accelerates the finger in both directions: the finger will accelerate in advance for the object grasping or pinch and come back to the original position after finishing the assignment. This motor is connected to a power screw and coupling system to translate the DC motor’s rotational motion into a linear one. The motor and the power screw are fixed to the finger utilizing a bearing housing, as shown in Figure 15. The coupler link of the second four-bar linkages also contains a DC motor (12 volt) with a screw and coupling system to shorten and lengthen the coupler link in order to obtain the required length to achieve the required configuration, either grasping or pinching. The peak length is utilized in grasping assignments, whereas the minimum length is utilized in pinching assignments.
Two torsional springs are mounted in the second and third joints, to help the finger to achieve good adaption to the shape and size of the objects to be grasped or pinched [22]. Figure 16 illustrates a series of tests during the grasping and pinching assignments that were performed with the prototype of the underactuated robotic finger.

9. CONCLUSIONS

In this paper, new kinematic and kinetic equations for grasping and pinching finger are formulated by considering the angle for the ternary solid links of the four-bar linkages. The multi-objective
functionality of a robotic finger is optimized, based on eight criteria for obtaining an optimal solution that leads to increase the finger performance in grasping and pinching assignments.

The optimization method used comprises three stages to obtain a reasonable accuracy of design parameters. The final solution of the optimization procedure is modified and redesigned to facilitate manufacturing of the robotic finger. This modification does not change the characteristics of forces.

The results show that adding the ternary solid links to the robotic finger leads to a better percentage of grasping stability and pinching forces, and assisted full embrace of the objects, especially those that are small, than the same finger without ternary solid links. In addition, it is shown that the increase in the coefficient of friction to the grasping kinetic analysis leads to a linear increase in the percentage of the grasping stability. It is found that a good performance with a relatively short time can be obtained using a DC motor with a coupler link of a second linkage with four bars with a screw and coupling system, to shorten and lengthen the length of link for grasping and pinching assignments.

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NOTATIONS

| Symbols | Definition                                      | Units |
|---------|------------------------------------------------|-------|
| a₁, a₂, b₁, b₂ | The lengths of the corresponding bars | mm    |
| b₂p | Coupler link length of the 2nd four-bar linkages | mm    |
| d | The linkage’s thickness | mm    |
| f | Grasping Force | N     |
| F_a | Actuator force | N     |
| F_p | Pinching force | N     |
| f_t | Tangential force | N     |
| f_n | Normal force | N     |
| f_T | Transpose Force | N     |
| h₁, h₂, h₃ | Grasping force location for each phalanx | mm    |
| J_w | Transmission matrix | mm    |
| J_v | Jacobian matrix of the finger | mm    |
| l₁, l₂, l₃ | Length of the proximal, Middle and distal phalanx | mm    |
| N | Normal Force | N     |
| t_a | Input torque | N.m   |
| t_s1, t_s2 | Torque of torsional springs | N.m   |
| V | Velocity vector of contact points | m/sec. |
| x₁, x₂, x₃ | The lengths of the corresponding bars of slider mechanism | mm    |
| θ_a | The rotating angle of the driving bar | Deg.  |
\[ \theta_1, \theta_2, \theta_3 \] The rotating angles of each phalax

\[ \theta_{h1}, \theta_{h2}, \theta_{h3} \] Angles of human finger for grasping assignment

\[ \theta_{m1}, \theta_{m2} \] The output angle of the first and second four-bar linkages respectively

\[ \theta_{bp} \] Angles of human finger during pinching assignment

\[ \theta_p \] Angles of robotic finger during pinching assignment

\[ \theta_s \] Output angle of slider mechanism

\[ \theta_{x1}, \theta_{x2}, \theta_{x3} \] Angles of corresponding bars of slider mechanism

\[ \Psi_1, \Psi_2, \Psi_3 \] The angle between the driving bar phalanges

\[ \gamma_1, \gamma_2 \] The angle of solid links for the 1st and 2nd four-bar linkages

\[ \mu \] Coefficient of Friction

\[ \mu_{t1}, \mu_{t2} \] 1st and 2nd transmission angle

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