Energy definition and dark energy: a thermodynamic analysis

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Accepted the Komar mass definition of a source with energy-momentum tensor $T_{\mu\nu}$, and using the thermodynamic pressure definition, we find a relaxed energy-momentum conservation law. Therefore, we study some cosmological consequences of the obtained energy-momentum conservation law. It has been found that the dark sectors of cosmos are unifiable into one cosmic fluid in our setup. While this cosmic fluid impels the universe to enter an accelerated expansion phase, it may even show a baryonic behavior by itself during the cosmos evolution. Indeed, in this manner, while $T_{\mu\nu}$ behaves baryonically, some parts of it, namely $T_{\mu\nu}(e)$ which is satisfying the ordinary energy-momentum conservation law, are responsible for the current accelerated expansion.

I. INTRODUCTION

Friedmann equations, the ordinary energy-momentum conservation law (OCL) (or the continuity equation) and its compatibility with the Bianchi Identity (BI) are the backbone of the standard cosmology which forms the foundation of our understanding of cosmos \cite{1}. Since on scales larger than about 100 Megaparsecs, cosmos is homogeneous and isotropic \cite{1}, the FRW metric is a suitable metric to study the cosmic evolution \cite{1}. Relations between thermodynamics and Friedmann equations have been studied in various setups which help us in getting more close to the thermodynamic origin of spacetime, gravity and related topics \cite{2, 20}.

A thermodynamic analysis can also lead to a better understanding of the origin of dark energy, responsible for the current accelerated universe \cite{15, 33}. In fact, there are thermodynamic and holographic approaches claiming that the cosmos expansion can be explained as an emergent phenomenon \cite{15, 33, 40}. Two key points in these approaches are the definition of energy and the form of the energy-momentum conservation law \cite{14, 19, 34–38}, and indeed, their results are so sensitive to the energy definitions that have been employed \cite{19}.

In order to make the discussion clearer, consider a flat FRW universe with scale factor $a(t)$ \cite{1}

\begin{equation}
\frac{ds^2}{c^2} = -dt^2 + a^2(t)\left[d\mathbf{r}^2 + r^2d\Omega^2\right],
\end{equation}

filled by a prefect fluid source with energy density $\rho$ and pressure $p$. Einstein field equations and energy-momentum conservation law (or equally the Bianchi identity) lead to

\begin{equation}
H^2 = \frac{8\pi}{3} \rho,
\end{equation}

\begin{equation}
3H^2 + 2\dot{H} = -8\pi p,
\end{equation}

\begin{equation}
\dot{\rho} + 3H(\rho + p) = 0,
\end{equation}

forming the cornerstone of standard cosmology. Although, the third equation (OCL) is in full agreement with our observations on the cosmic fluid in matter and radiation dominated eras \cite{1, 47}, its validity for the current accelerated cosmos is questionable \cite{48} which may encourage us to consider the relaxed types of OCL to describe the current universe \cite{15, 49}. Combining the Friedmann equations with each other, we get

\begin{equation}
\dot{H} = -4\pi(\rho + p),
\end{equation}

which can finally be added to the time derivative of the first Friedmann equation to reach at OCL. Therefore, although the third equation of \cite{2} is generally obtained from the conservation law ($T_{\mu\nu} = 0$), one can easily find it by only using the Friedmann equations, a result which is the direct consequence of the Einstein field equations satisfying the Bianchi identity. It means that two equations of the above set of equations \cite{2} are indeed enough to study the cosmos, and the third equation will be automatically valid and inevitable in this situation.

From the viewpoint of thermodynamics, if OCL is valid, then by applying the thermodynamics laws to the apparent horizon, as the proper causal boundary located at \cite{2, 14}

\begin{equation}
\tilde{r}_A = a(t)r_A = \frac{1}{H},
\end{equation}

where $r_A$ is the co-moving radius of apparent horizon, one can reach the Friedmann equations \cite{2} \cite{3} \cite{18}. Indeed, the validity of OCL is crucial, and its relaxation leads to modification of the Friedmann equations \cite{14–16, 18}. These results propose that the cosmos is so close

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to its thermodynamic equilibrium state \[19, 28\], motivating us to be more confident on thermodynamical quantities, such as pressure, to study the cosmos \[19\].

For a FRW background filled by a fluid whose energy density \(\rho\) is at most a function of time, we may reach

\[
E = \int T_{\alpha\beta} u^\alpha u^\beta dV = \int \rho dV = \rho V, \quad (5)
\]
as the total energy confined to the volume \(V\) and felt by a co-moving observer with four velocity \(u^\alpha = \delta_\alpha^\beta\). Eq. (5) is fully compatible with the combination of the Misner-Sharp mass, thermodynamics laws and standard cosmological evolution, which led to a model for the cosmic evolution, which led to appropriate models for dark sectors of cosmos, if one uses Eq. (7) to model the cosmos \[19\]. On the other hand, Eq. (9) claims that if we use the Komar mass, then OCL and thus Eq. (7) are not valid and another modified Friedmann equations should be employed to describe the cosmos.

In the general relativity (GR) framework, where \(G_{\mu\nu} = 8\pi T_{\mu\nu}\), the satisfaction of BI by the Einstein tensor \((G^\mu_\nu)\) is equivalent to the satisfaction of OCL by \(T^\mu_\nu\). We saw that the form of conservation law depends on the energy definition. The above arguments also motivated the authors of Ref. \[19\] to use various energy definitions such as the Komar mass \[54, 51\] and

\[
H^2 = \frac{8\pi}{3} \rho, \quad (7)
\]

\[
3H^2 + 2\dot{H} = 8\pi \left(\frac{\partial E}{\partial \rho}\right),
\]

\[
\dot{\rho} + 3H(\rho + p) = 0, \quad (9)
\]

instead of Eqs. \[2\], in order to study the relations between energy definitions, thermodynamics laws and the cosmic evolution, which led to a model for the cosmic fluid consistent with observations \[19\].

The Einstein field equations also admit another mass definition, namely the Komar mass \[44, 46\] and

\[
\mathcal{M} = 2 \int (T_{\mu\nu} - \frac{1}{2} T g_{\mu\nu}) u^\mu u^\nu dV = (\rho + 3p)V, \quad (8)
\]

which clearly differs from \[3\] unless we have a pressureless fluid \[11, 17\]. Although this energy definition has originally been obtained by using the Einstein field equations \[17\], it is in accordance with various thermodynamic and holographic approaches \[34, 40\]. In fact, if we accept this energy definition as a basic equation and not a secondary equation \[34, 37\], then there are various thermodynamic and holographic approaches employed to get the Friedmann and gravitational field equations in various theories by using the Komar mass definition \[34, 40\].

Now, inserting the Komar energy definition into Eq. (6) and using \(V = V_0 a^3\), one can easily reach a new conservation law as

\[
\dot{\rho} + 3(\dot{\rho} + 3Hp) + 3H(\rho + p) = 0,
\]

met by Komar mass, and we call it the Komar conservation law (KCL). For the pressureless systems, everything is consistent due to the fact that the above addressed energy definitions are equal. Similar types of this energy-momentum conservation law have also been obtained in theories which include a non-minimal coupling between the geometry and matter fields \[14, 49, 52\].

One basic consequence of the Komar mass is that not only \(\rho\), but both of \(\rho\) and \(p\) participate in building the system energy which leads to appropriate models for dark sectors of cosmos, if one uses Eq. (7) to model the cosmos \[19\].

In the general relativity (GR) framework, where \(G_{\mu\nu} = 8\pi T_{\mu\nu}\), the satisfaction of BI by the Einstein tensor \((G^\mu_\nu)\) is equivalent to the satisfaction of OCL by \(T^\mu_\nu\). We saw that the form of conservation law depends on the energy definition \[4\], and indeed, if the Komar mass notion is used, then one can easily find that \(T^\nu_\mu\) should satisfy KCL instead of OCL. Thus, in the GR framework, there is an inconsistency together with the results of recent observations \[48\], admitting the break-down of OCL in the current accelerated era, motivate us to modify GR, and thus, the Friedmann equations by modifying the matter side of the field equations in the manner in which the modified matter part meets OCL, in agreement with the satisfaction of BI by the geometrical part. In summary, we think that OCL is very restrictive constraint on \(T^\nu_\mu\). Probably, OCL should not be applied to whole of \(T^\nu_\mu\), and it should only be applied to some parts of \(T^\nu_\mu\), the parts which may modify GR in a compatible way with the current accelerated cosmos \[18, 48, 49\].

There is also an elegant consistency between Eqs. \[5\] and \[2\], i.e. \(\rho\) is the energy density in Eq. \[5\], and it also appears in the first Friedmann equation. On the other hand, Eq. \[5\] implies that the quantity \((\rho + 3p)\) plays the role of energy density, while it is not present in the first Friedmann equation. In fact, if we define \(p_c \equiv \rho + 3p\), then Eq. \[9\] can be rewritten as

\[
\dot{\rho_c} + 3H(\rho_c + p) = 0, \quad (10)
\]
equivalent to a hypothetical fluid with an effective energy-momentum tensor of \(T^\mu_\nu(c) = diag(-\rho_c, p, p, p)\), meeting OCL (see \[53\] for a debate on effective energy-momentum tensor). In this manner, Eq. \[5\] for \(T^\mu_\nu(c)\) will be equal to Eq. \[8\] for \(T^\mu_\nu\). Indeed, the above arguments (specially Eq. \[10\]) claim that, even in the Einstein framework, OCL \[2\] may not be valid, if Komar mass is
employed. In this situation, because OCL is the backbone of the Friedmann equations, we may conclude that the Friedmann equations [2] should be modified.

In summary, i) The energy definition and the form of energy-momentum conservation law play crucial roles in getting the Friedmann equations in various theories [3–7]. ii) We also showed the form of energy-momentum conservation law depends on the energy definition, and if one uses the Komar mass, then KCL is obtained instead of OCL. iii) The Komar mass is the backbone of some holographic and thermodynamic attempts to obtain field equations of various gravitational theory [8, 14–17]. It also motivates us to use KCL instead of OCL. iv) Observational data admit the break-down of OCL in the current accelerated era [23, 24]. It also motivates us to use the relaxed forms of OCL in order to describe the current universe. On the other hand, there are also various models for the current accelerated universe satisfying OCL [1]. Therefore, it seems that some inconsistency exists between the various definitions of energy, continuity and Friedmann equations. One may conclude that the fluid obtained from the observation may not be the real cosmic fluid, and it indeed represents some less well-known or even unknown parts of the energy source which become important, effective and tangible in the current cosmos. We want to say that the real cosmic fluid may satisfy conservation laws such as KCL instead of OCL, while some of its parts, responsible for the current accelerated expansion, may satisfy OCL.

As we mentioned, based on Eq. [8], one may conclude that pressure has some contribution to the energy density of system. The question arisen here is what if this contribution can describe the current accelerated universe? In other words, are there combinations of \( \rho \) and \( p \) of a baryonic source which can play the role of dark energy? Here, we are going to find out the answers of the mentioned questions by studying some consequences of accepting Eq. [8] and the Komar mass definition, and thus Eq. [9] in cosmological setups. In the next section, using thermodynamics laws, Bekenstein entropy and Eq. [9], we address some models for dark energy. The third section is devoted to a summary and concluding remarks. Throughout this work, the unit of \( c = \hbar = G = k_B = 1 \), where \( k_B \) denotes the Boltzmann constant, has been employed.

II. KOMAR DEFINITION OF ENERGY AND THE FRIEDMANN EQUATIONS

For an energy-momentum source with \( T^b_a = \text{diag}(-\rho, p, p, p) \), the amount of energy crossing the apparent horizon is evaluated as [8]

\[
\delta Q^m = A(T^b_a \partial_t \delta r_A + W \partial_a \delta r_A) dx^a, \tag{11}
\]

where \( A = 4\pi r_A^2 \) and \( W = \frac{\rho - p}{2} \) denote the horizon area and the work density, respectively. \( A_0 = 4\pi r_A^2 \) is also the co-moving area. After some calculations, one obtains [8, 14, 17]

\[
\delta Q^m = -3V H (\rho + p) dt, \tag{12}
\]

combined with the Clausius relation \( T dS_A = -\delta Q^m \), to get [8, 15]

\[
3V H (\rho + p) dt = \frac{H}{2\pi} dS_A, \tag{13}
\]

where the additional minus sign in the Clausius relation is due to the direction of energy flux [8, 15]. Here, \( S_A \) is the horizon entropy and the Cai-Kim temperature \( T = \frac{H}{2\pi} \) is used to reach this equation [8, 15]. The role of energy-momentum conservation law in deriving the Friedmann equations is now completely clear. The effects of considering various approximations and temperatures have been studied in Ref. [12].

Modified Friedmann equations

Here, using some simple examples, we are going to show the effects of considering KCL in modifying the standard cosmology.

Case i) If OCL is valid, then Eq. (13) is reduced to

\[
d\rho = -\frac{3H^2}{8\pi^2} dS_A, \tag{14}
\]

covering the first Friedmann equation whenever \( S = \frac{4}{3} \).

In this situation, Eq. (9) implies that

\[
\dot{\rho} + 3H p = 0 \rightarrow p(a) = p_0 a^{-3}, \tag{15}
\]

where \( p_0 \) is the integration constant and we considered \( a_0 = 1 \) (\( a_0 \) is the current value of the scale factor). In this manner, for \( S_A = \frac{4}{3} \), we reach

\[
H^2 = \frac{8\pi}{3} \rho, \tag{16}
\]

\[
3H^2 + 2 \dot{H} = -8\pi p_0 a^{-3},
\]

\[
\dot{\rho} + 3H (\rho + p_0 a^{-3}) = 0,
\]

instead of Eqs. (2). It is easy to check that for power law regimes in which \( \rho \propto a^\beta \), conservation law leads to \( p_0 = 0 \) and \( \beta = -3 \), which is nothing but the dust source, in full agreement with our previous results indicating that for a pressureless source everything is consistent. The geometrical parts of the two first equations of (16) address us to the Einstein tensor. It is in fact the direct result of attributing the Bekenstein entropy to the horizon [8]. Therefore, mathematically, since the Einstein tensor obeys BI, the appeared \( \rho \) and \( p \) should also satisfy OCL,
an expectation in full agreement with Eq. (15) and thus the third line of Eqs. (10).

However, the general solution of the obtained conservation law (16) is

\[ \rho(a) = a^{-3}[c - 3p_0 \ln(a)], \]

where \( c \) is the integration constant. It clearly covers the ordinary matter era whenever \( p_0 = 0 \). From Eqs. (16) and (17), we reach

\[ p = p_0, \quad \rho = c, \]

for the current era. Thus, if we have \( p_0 = w_0 c \) and \( c = \Lambda > 0 \), where \( \Lambda \) and \( w_0 \) denote the current values of the energy density and the equation of state (EoS) of dark energy, respectively, then the obtained fluid may theoretically generate the current accelerating universe whenever \( w_0 \leq -\frac{1}{3} \). Now, we can write

\[ p(a) = w_0 \Lambda a^{-3}, \quad \rho(a) = \Lambda a^{-3}[1 - 3w_0 \ln(a)]. \]

In this manner, EoS \( (w) \) and the deceleration parameter \( (q) \) are evaluated as

\[ w = \frac{p}{\rho} = \frac{w_0}{1 - 3w_0 \ln(a)}, \]

\[ q = -1 - \frac{\dot{H}}{H^2} = \frac{1}{2}(1 + \frac{3w_0}{1 - 3w_0 \ln(a)}). \]

At the \( a \to 0 \) limit, independent of the value of \( w_0 \), we have \( w \to 0 \) and \( q \to \frac{1}{2} \). Moreover, one can easily see that, at the \( a \to 1 \) limit, the consistent values with observations for \( w \) and \( q \) are also obtainable, depending on the value of \( w_0 \). Despite this compatibility, there is a singularity at the behavior of \( q \) and \( w \) for \( w_0 < 0 \), located at \( a = e^{\frac{1}{3w_0}} \), meaning that this solution may be at most useable to study the \( a > e^{\frac{1}{3w_0}} \) era. But, as it is obvious from Fig. (1), it does not lead to suitable \( q \) and \( w \)

even for \( a > e^{\frac{1}{3w_0}} \). Therefore, as we previously saw, only the \( w_0 = 0 \) case can meet all of the desired expectations, i.e. it is in agreement with the Friedmann equations, the both addressed energy definitions, and thus the matter dominated era.

Case ii) If \( T_\mu^\nu(e) \), satisfying Eq. (10), is used instead of \( T_\mu^\nu \) in order to evaluate \( S_{Qm} \) and we follow the Cai-Kim approach \[8, 14–18\], then some calculations lead to

\[ H^2 = \frac{8\pi}{3} \rho_c, \]

\[ 3H^2 + 2\dot{H} = -8\pi p, \]

\[ \dot{\rho}_c + 3H(\rho_c + p) = 0, \]

whenever \( S_A = \frac{4}{\beta} \). These equations would also be obtainable by modifying the Einstein field equations as \( G_{\mu\nu} = 8\pi T_{\mu\nu}(e) \), in agreement with attributing the Bekenstein entropy to the apparent horizon (see the paragraph after Eqs. (10)). Now, considering the simple case \( w_e \equiv \frac{\dot{w}_e}{\rho_c} \) and using the above equations, we can easily find \( w_e = \frac{w}{1 + 3w} \). This result indicates that a baryonic source, with \( w > 0 \), cannot lead to a hypothetical fluid with \( w_c < 0 \) in cosmos, meaning that the obtained modified Friedmann equations cannot describe the current accelerated universe.

Case iii) Now, bearing Eq. (9) in mind, and defining an effective pressure as \( p_c = \beta \rho \), one reaches

\[ \dot{\rho} + \frac{3}{\beta} [\dot{p}_c + (4 - \beta)Hp_e] + 3H(\rho + p_c) = 0, \]

reduced to

\[ \dot{\rho} + 3H(\rho + p_c) = 0, \]

whenever we have

\[ \dot{p}_c + (4 - \beta)Hp_e = 0 \Rightarrow p_c = p_c^0 a^{\beta - 4}, \]

where \( p_c^0 \) is the integration constant. Inserting this result into Eq. (28), it is easy to obtain

\[ \rho = \frac{c}{a^3} - \frac{3p_c^0}{\beta - 1} a^{\beta - 4}. \]

Here, \( c \) is an integration constant, and it is worthwhile to mention that the ordinary matter and radiation dominated eras are recovered at the limits of \( p_c^0 = 0 \) and \( \beta = 0 \), respectively, meaning that the obtained fluid can unify these eras into one model. In this manner, we can argue that although \( T_{\mu\nu} \) is the true energy-momentum tensor, an effective tensor of \( T_{\mu\nu}(e) = \text{diag}(\rho, p_c, p_c, p_c) \) should be considered, which may address us to a modified gravitational equation of the form \( G_{\mu\nu} = 8\pi T_{\mu\nu}(e) \). Now, combining Eqs. (24) and (25) with each other, one finds

FIG. 1: Here, \( w_0 = -1 \), dot and solid lines represent \( w \) and \( q \), respectively.
\[ w_e \equiv \frac{p_e}{\rho} = \beta w = \frac{(\beta - 1)p_0^0 a^3}{(\beta - 1)\alpha^3 - 3p_0^0 a^3} \]  

(26)

For \( \beta > 1 \) and \( \beta < 1 \), we have \( w_e \rightarrow 0 \) and \( w_e \rightarrow (1 - \beta)/3 \), respectively, at the \( a \rightarrow 0 \) limit. Moreover, independent of the value of \( \beta \), one can see that \( w_e \rightarrow \frac{(\beta - 1)p_0^0}{(\beta - 1)\rho_0 a^3} \equiv w_0 \), leading to \( c = \frac{p_0^0(\beta - 1 + 3w_0)}{(\beta - 1)\rho_0 a^3} \), whenever \( a \rightarrow 1 \). Using the latest result and the \( 1 + z = a^{-1} \) relation, where \( z \) is the redshift, we can finally rewrite Eq. (26) as

\[ w_e = \beta w = \frac{w_0(\beta - 1)}{(\beta - 1 + 3w_0)(1 + z)^{3-1} - 3w_0}. \]  

(27)

FIG. 2: \( w_e \), \( w \) and \( q \) versus \( z \) when \( w_0 = -1 \). Here, solid lines: \( \beta = 5 \) and dot lines: \( \beta = 4.7 \).

If \( w_0 < 0 \) and \( c > 0 \), then for \( \beta > 1 - 3w_0 \) and \( p_0^0 < 0 \) (or equally \( p_e < 0 \)) we have \( \rho > 0 \), \( p < 0 \), and \( w_e(z \gg 1) \rightarrow 0 \) meaning that this fluid may describe the universe history from the matter dominated era to the current accelerating phase. In this manner, if we either modify Einstein equations as \( G_{\mu\nu} = 8\pi T_{\mu\nu}(e) \) or follow the Cai-Kim recipe by using \( T_{\mu\nu}(e) = diag(-\rho, p_e, p_e, p_e) \), then we obtain

\[ H^2 = \frac{8\pi}{3\rho}, \]  

(28)

\[ 3H^2 + 2\dot{H} = -8\pi p_e, \]

\[ \dot{\rho} + 3H(\rho + p_e) = 0. \]

As before, two first equations imply that \( T_{\mu\nu}(e) \) should satisfy OCL, in full agreement with Eq. (28), and thus the third line of (28). Calculations for the deceleration parameter lead to

\[ q \equiv 1 - \frac{\dot{H}}{H^2} = \frac{1}{2}(1 + 3w_e). \]  

(29)

Suitable values of \( \beta \) should be found by considering the transition point at which \( q_t = 0 \) and \( w_e = -\frac{1}{3} \equiv w_e^t \). In addition, simple calculations for the transition redshift lead to

\[ z_t = -1 + (\frac{\beta - 1 + 3w_0}{3w_0(2 - \beta)})^{1/3}, \]  

(30)

indicating that \( \beta \) should also meet the \( \beta > 2 \) condition to reach positive meaningful solutions for \( z_t \). Because \( w_0 < w_e^t \), we have the relation \( 1 - 3w_0 > 2 \) meaning that the \( \beta > 2 \) condition is in agreement with the previously obtained condition \( (\beta > 1 - 3w_0) \). Besides, although we can obtain suitable values of \( z_t \) for \( 1 < \beta < 1 - 3w_0 \), in this manner, negative amounts will be attained by \( \rho \) which finally reject this case. For other values of \( \beta \), energy density will grow much faster than the matter density, proportional to \( (1 + z)^3 \), by increasing \( z \). Thus, only the \( \beta > 1 - 3w_0 \) case can be meaningful. In Fig. 2, deceleration parameter, \( w_e \) and \( w \) have been plotted versus \( z \) for some values of \( \beta \).

Hence, if we accept both the Komar mass definition and the thermodynamic pressure notion, we can modify the standard cosmology as (28) in which the solutions of the field equations can theoretically unify both the matter dominated era and the current accelerated universe. But, in this manner, since \( \beta > 0 \), the original source has a negative pressure \( (p = \frac{\rho}{c^2} < 0) \). Therefore, it is useful to emphasize that, despite the proper obtained results, since \( p < 0 \), this approach does not lead to a baryonic model for the energy source during the cosmos evolution, i.e. a model with \( 0 \leq p \leq \frac{1}{3}\rho \).

FIG. 3: \( w \) has been depicted versus \( z \) for some values of \( \beta \).

Case iv) As another case, bearing Eq. (29) in mind, defining \( p_e = \beta p \), \( \rho_e = \rho + \alpha p \), where \( \alpha \) and \( \beta \) are unknown constants that should meet the \( \alpha + \beta = 4 \) condition, and considering the \( \dot{p} = \dot{\rho}_e = 0 \) case, one can follow the above recipes to obtain
\[ H^2 = \frac{8\pi}{3} \rho_c, \]
\[ 3H^2 + 2 \dot{H} = -8\pi \rho_c, \]
\[ q = \frac{1}{2} \left( 1 + \frac{3\rho_c}{\rho_e} \right), \]
\[ \rho_c + 3H(\rho_c + \rho_e) = 0 \rightarrow \rho_c = (\rho_c^0 + \beta \rho)(1 + z)^3 - \beta \rho, \]
where \( \rho_c^0 \) is the integration constant. As the previous cases, since the rhs of the two first equations are nothing but the Einstein tensor meeting BI, \( \rho_c \) and \( p_e \) should satisfy OCL (or equally the last line). Indeed, since we have \( \rho_c = 0 \), the \( \alpha + \beta = 4 \) constraint is unavoidable only if we want KCL to be reduced to OCL (the last line). We finally reach at

\[ \rho = (\rho_c^0 + \beta \rho)(1 + z)^3 - 4\rho, \]
\[ w \equiv \frac{p}{\rho} = \frac{\rho}{(\rho_c^0 + \beta \rho)(1 + z)^3 - 4\rho}, \]
\[ w_e \equiv \frac{p_e}{\rho_e} = \frac{\rho_e}{(\rho_c^0 + \beta \rho)(1 + z)^3 - \beta \rho}. \]

As we know, the current accelerated universe (\( z = 0 \)) implies \( \rho_c < 0 \) and \( w_e(z = 0) \equiv w_0 \leq -2/3 \) (or equally \( \rho_e(z = 0) > \frac{2}{3}(\rho_{c0}) \)) \[1\]. Thus, the above results admit a fluid with positive pressure whenever \( \beta < 0 \). In this manner, for \( z \geq 0 \), if \( \rho_c^0 > \alpha \rho \), satisfied when \( \beta < \alpha \rho_0 \) leading to \( \beta < \frac{\alpha \rho_0}{1 + 3w_0} \) (or equally \( 4 < \alpha(1 + w_0) \)), then we have \( \rho_c, \rho > 0 \). This result indicates that we should have \( w_0 > -1 \), in agreement with some observations \[1\], to meet the \( \beta < 0 \) condition. For this case, while, unlike \( w, w_e \) is always negative, both \( w \) and \( w_e \) approach zero for \( z \gg 1 \). Using the above results, we finally get leading to \( z_e = -1 + \left( \frac{2w_0}{1 + 3w_0} \right)^{1/3} \) for the transition redshift. The parameters \( w, w_e \) and \( q \) have been plotted in Figs. \[3\] and \[4\] for \( w_0 = -0.73 \) \[1\]. It is also worthwhile reminding that the current observational data on dark energy density and pressure in fact give us the corresponding values of \( \rho_c \) and \( p_e \). As it is apparent, there are some values of \( \beta \) (and thus \( \alpha \)) for which the maximum value of \( w \) is at most equal to \( -\frac{1}{3} \), signalling us to a baryonic source (since \( \beta < 0 \) and \( w > 0 \), we have \( p > 0 \) and \( p > 0 \)). Therefore, \( w, w_e \) and \( q \) display proper behavior, meaning that it is mathematically enough to only consider some parts of \( T_{\mu\nu} \), represented by \( T_{\mu\nu}(e) \), to modify the standard cosmology as \[31\] (or equally the Einstein field equations as \( G_{\mu\nu} = 8\pi T_{\mu\nu} \)) which gives us a suitable description for the current phase of the universe.

### III. CONCLUSION

Mach principle states that geometry inherits its properties from inside it, i.e. the geometrical information is related to the energy of the source. Hence, it does not limit us to a certain information for the energy source. This principle along with OCL and BI are the backbone of Einstein theory, and thus, standard cosmology, in full agreement with the thermodynamics laws. Based on this theory, the Einstein \( (G_{\mu\nu}) \) and energy-momentum \( (T_{\mu\nu}) \) tensors are in a direct relation as \( G_{\mu\nu} = 8\pi T_{\mu\nu} \). Hence, by having the whole information of the matter source \( (T_{\mu\nu}) \), one can find \( G_{\mu\nu} \) and then the spacetime metric which finally gives us the whole information of geometry. It is also worthwhile mentioning that the recent observation admits the break-down of OCL in the current accelerated era \[18\].

Besides, there are various approaches to gravity and cosmology in which the Komar definition of energy is their foundation. Here, by accepting this energy definition and using the thermodynamic pressure definition, a relaxed energy-momentum conservation law (KCL) has been obtained, helping us in getting some modified energy-momentum tensors \( (T_{\mu\nu}(e)) \) satisfying OCL. In continue, applying the thermodynamics laws to the apparent horizon and attributing the Cai-Kim temperature to it, we could find out some simple modified Friedmann equations. Our results are in fact equal to modify the Einstein field equations as \( G_{\mu\nu} = 8\pi T_{\mu\nu}(e) \). Therefore, in agreement with the satisfaction of BI by the Einstein tensor, \( T_{\mu\nu}(e) \) is also meeting OCL. The study shows that an appropriate choice of \( T_{\mu\nu}(e) \) allows us to unify the dark sectors of cosmos into one model. In fact, the analysis of the last case \((iv)\) confirms that if suitable \( T_{\mu\nu}(e) \) is used instead of \( T_{\mu\nu} \) in order to modify the standard cosmology \[31\], then \( T_{\mu\nu}(e) \) can mathematically describe the current accelerated cosmos, while \( T_{\mu\nu} \) can display a baryonic source.

In summary, i) \( T_{\mu\nu}(e) \), made of \( T_{\mu\nu} \), satisfies OCL (or equally \( \nabla^{\mu}T_{\mu\nu}(e) = 0 \)), and can describe the current accelerated universe. ii) In this manner, \( T_{\mu\nu} \), satisfying
KCL, can even show a baryonic behavior by itself. These results may be translated as that the less well-known or even unknown aspects of $T_{\mu\nu}$, represented by $T_{\mu\nu}(e)$, are responsible for the current accelerated phase of the universe. The origin and the emergence conditions of $T_{\mu\nu}(e)$ may be found out by studying this part in the framework of other physical theories such as quantum field theory.

**Conflict of Interests**

The authors declare that there is no conflict of interest regarding the publication of this paper.

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