Analysis of Phonetic Soliton Propagation in Neutral Weyl Fermion-sea

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We propose application of Machine Learning (ML) and Neural Network (NN) technique for the analysis of ultrasonic Time Reversal based Nonlinear Elastic Wave Spectroscopy (TR-NEWS).

In order to acquire topological reveal based Nonlinear Elastic Wave Spectroscopy (TR-NEWS).

We consider propagation of bosonic phonons in Fermi-sea of neutral Weyl spinors which are described by Clifford algebra. Configurations in momentum space is transformed to real position space via Clifford Fourier Transform.

We consider A-type without hysteresis effects and B-type with hysteresis effects, and via ML or NN technique search optimal weight of 7 A-type FP actions and 13 B-type FP actions using the Monte-Carlo method.

I. INTRODUCTION

In non-destructive-testing (NDT) the time reversal (TR) based nonlinear elastic wave spectroscopy (TR-NEWS) was successful. Use of time reversal mirrors (TRM) for enhancing signal to noise ratios of ultrasonic phonetic waves scattered in materials is recently reviewed in [5].

Since the media that a phonon propagates is not symmetrical, the breaking of Z₂ symmetry and the intrinsic parameters N₁, N₂ and N₃ are defined by the relation

\[
\begin{pmatrix}
N_1 x(t) \\
N_2 x^2(t) \\
N_3 x^3(t)
\end{pmatrix} = \frac{1}{3} \begin{pmatrix}
-1 & -8 & 2 & 2 \\
0 & 0 & 2 & 2 \\
4 & 8 & -4 & -4
\end{pmatrix} \begin{pmatrix}
y_E \\
y_A \\
y_B \end{pmatrix}.
\]

For \( x_E(t) = x(t) \) and \( x_A(t) = -\frac{1}{2} x(t) \), the simultaneous equation

\[
x(y_E, y_{B1}, y_{B2}) = (3y_E + 2y_{B1} + 2y_{B2})/(3N_1)
\]

\[
x(y_E, y_{B1}, y_{B2})^2 = (2y_{B1} + 2y_{B2})/(3N_2)
\]

\[
x(y_E, y_{B1}, y_{B2})^3 = (-4y_{B1} - 4y_{B2})/(3N_3)
\]
The normalizing flow proposed by Papamakarios et al. [19] uses the convolutional neural network (CNN) to distinguish 20 loops. We let $F\{\cdot\} \in L^1(R^2;G_2)$ be a $(2+1)$-D inverse Clifford transform (ICFT) of $\mathcal{F}\{f\}$ with

$$
\mathcal{F}\{f\}(\omega) = \frac{1}{(2\pi)^2} \int_{R^2} e^{ixu_1f(x)} e^{-jxu_2d^2x}
$$

where $\omega = u_1e_1 + u_2e_2 + \omega_1e_{12}$, $x = x_1e_1 + x_2e_2$, $F = \{f_1(x,u), f_2(x,u)\}$, $f_1(x,u) = 2\pi e_1x_1u_1$, $f_2(x,u) = 2\pi e_2x_2u_2$, and

$$
\frac{\partial \sigma}{\partial u_1}(u_1,h_1) \cdots 0
$$

where $\Sigma = \sum_k p_{KL}(\mathbf{u})$ into random number distribution $(0,1)^E$, where $E$ is the number of epochs. The distribution at the epoch $k$ is related to that at the epoch $k-1$ by $u_k = T_k(u_{k-1})$. The inverse is $u_{k-1} = T_k^{-1}(u_k)$. To every distribution $u_i$, we consider the Clifford Fourier transform (CFT) $x_i = \tau(u_i, h_i)$ where $h_i = c_i(u_{k-1})$, and $\tau$ is the transformer and $c_i$ is the conditioner at the $i$th epoch [22].

The position space transformation from $u$ to $x$, which is the inverse CFT is defined as

$$
h(x) = F^{-1}\{x\}(u) = \frac{1}{(2\pi)^2} \int_{R^2} e^{ixu_1f(h)} e^{-jxu_2d^2x} d^2u
$$

where $d^2u = du_1du_2$. For $g \in H \sim CL(0,2)$, $f^2 = g^2 = -1$, or $f, g \in CL(2,0)$.

In the case of $A$ type loops, we consider simple 2D QFT to obtain $f(x)$, from $\omega = \omega_0x_1 + \omega_2x_2$ as in [22]. However, in the case of $B$ type loops, we include the scalar term and consider the energy-time transformation.

Given $x_i$, we can compute $u_i = \tau^{-1}(x_i, h_i)$. Since $x_i$ depends only on $x_j(j < i)$, the Jacobian of the transformation $\tau$ is

$$
J_r(u) = \begin{pmatrix}
\frac{\partial \sigma}{\partial u_1}(u_1,h_1) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
L(u) & \cdots & \frac{\partial \sigma}{\partial u_E}(u_E,h_E)
\end{pmatrix}
$$

Corresponding to $p_L(u)$ the flow

$$
p_F(x) = p_L(T^{-1}(x)|\det J_{T^{-1}}(x)|,
$$

The $T_k$ transforms $z_{k-1}$ to $z_k$, and multiple of $T_1T_2\cdots T_E$ transforms $z_0 = u$ to $z_E = x$. The inverse flow $T^{-1} = T_1^{-1} \circ T_2^{-1} \circ \cdots \circ T_E^{-1}$ takes a collection of samples from $p_F(x)$ and transform them (in a sense ‘normalizes’ them) to a collection of samples from the density $p_L(u)$. Suppose that $p_F(x) > 0$ and assume the conditional probability $Pr(x_i < x_j|x_{<i})$ with $x_i$ being the random variable.

The probability $p_F(x)$ can be decomposed into a product

$$
p_F(x) = \prod_{i=1}^E p_F(x_i|x_{<i})
$$

The transformation from $x$ to random number $z \in [0,1]^E$ is defined as $F$

$$
z_i = F_i(x_i, x_{<i}) = \int_{x_i}^{x_{i+1}} p_F(x_i'|x_{<i}) dx_i' = Pr(x_i' \leq x_i|x_{<i}).
$$

Papamakarios et al. [13] showed that the inversion can be done element-by-element as

$$
x_i = (F_i(\cdot, x_{<i})^{-1}(z_i) \quad \text{for} \quad i = 1, 2, \cdots, E.
$$

Since $\frac{\partial F_i}{\partial x_i} = 0$ for $i < j$ the determinant of Jacobian $J_F$ is a product of diagonal components

$$
det J_F(x) = \prod_{i=1}^E \frac{\partial F_i}{\partial x_i} = \prod_{i=1}^E p_F(x_i|x_{<i}) = p_F(x).
$$
The variable \( z \) is distributed uniformly in the open unit cube \((0,1)^E\).

The transformation from \( u \) to \( z \) can be defined as
\[
z_i = G_i(u_i, u_{<i}) = \int_{u_i}^{u_i} p_i(u_i' | u_{<i})du_i' = P_r(u_i' \leq u_i | u_{<i}).
\]
\[
(17)
\]

We regard phonons as shock waves produced on Fermi surfaces and calculate plaquette actions derived from the fixed point action in \( 4D \) \( (2+1)D \) lattice points with the asymptotic high momentum action subtracted \[12-14\].

In Fig 1.2 the plaquette action of \( A \) type loops as function of \( u_2/\Delta \), where \( \Delta \) is the lattice unit in momentum space at fixed \( u_2/\Delta \) is even or odd are compared. Averages of Monte-Carlo (MC) simulations show that the slope of actions for \( u_2/\Delta \) is odd is about twice of those for \( u_2/\Delta \).

Link actions of \( B \) type loops for large momenta are asymptotically zero, but they depend on the clockwise rotating (f-link) or counterclockwise rotating (e-link). The relative strength of actions of e-link and f-link type depends on the random number sets used in MC. The average of e-link action and f-link action which are negative in the infrared region, plus plaquette action which are similar to that of A type, obtained in a MC is shown in Fig 3.4. In the MC simulation, 50 orderings of 13 random numbers were considered as in the traveling salesman problem \[13\].

The maximum and minimum of the total action of \( u_2/\Delta = 0 \) is about twice of those of \( u_2/\Delta = 1 \). It reflects the difference of edge spin class of Kitaev \[8\], the former is real and the latter is quaternionic. This property reflects properties of CFT for \( d = 2, 3 \) (mod 4) remarked by Hitzer (2022). For a multivector \( A_r \in \mathcal{G}_d \) of odd grade \( r = 2s + 1 \) or even grade \( r = 2s \)

\[
A_{2s+1}i_d = -i_dA_{2s+1}, \quad A_{2s}i_d = +i_dA_{2s}.
\]

We expect an oscillating pattern in the spectrum.

IV. PARAMETER SEARCH BY MACHINE LEARNINGS

Recently, effectiveness of solving gauge theories using machine learnings or neural networks developed for Markov Chain Monte Carlo (MCMC) was realized.

Obtaining optimal weight function in normalizing flow of lattice MC using Python and PyTorch \[17\] was presented by several authors \[12\].

The Markov process consists of movement of \( X_t, (t \geq 0) = T \) and probability \( P_{x_1}(x \in S) \) such that \( P_{t+1}(x,y) = \sum_{z} P_{t}(x,z)P_{z}(z,y) (t \in T, x,y \in S) \) is satisfied. To characterize the flow one defines a Hamiltonian \( H(\eta, \nu) = -\log[p(\eta, \nu)], \) where \( p(\eta, \nu) \) is a joint distribution of the variable of interest \( \eta \) and \( \nu \). The hamiltonian flow is defined by
\[
\frac{d(\eta, \nu)}{dt} = (\frac{\partial H}{\partial \nu}, -\frac{\partial H}{\partial \eta}).
\]

In a perceptron, which is a prototype of artificial neuron, one defines a decision function \( \sigma(z) \) where \( z = w_1x_1 + w_2x_2 + \cdots + w_mx_m \) is given by a weight vector \( w \) and input vector \( x \) as \( z = w^Tx + b \).

The decision function is variant of unit step function
\[
\sigma(x) = \begin{cases} 
1 & \text{if } \quad z \geq 0 \\
0 & \text{otherwise} 
\end{cases}
\]

The weight function \( w_j \) and bias \( b \) are defined sequentially and
\[
w_j = w_j + \Delta w_j \\
b = b + \Delta b
\]
where the output value \( \hat{y}^{(i)} \) for an input \( x^{(i)} \) is related by
\[
\Delta w_j = \eta(y^{(i)} - \hat{y}^{(i)})x_j^{(i)} \\
\Delta b = \eta(y^{(i)} - \hat{y}^{(i)}).
\]

One defines mean squared error (MSE) as
\[
L(w, b) = \frac{1}{2n} \sum_{i=1}^{n} (y^{(i)} - \sigma(z^{(i)}))^2,
\]
which is to be minimized.

In multi-layer neural network (NN), applied to recognitions of a handwritten number shows reduction of errors after training \[17\].

The algorithm consists of setting prior random number sets, forward setting and backward setting and calculation of Kullback-Leibler (KL) divergences. We want to extend the algorithm such that the information of f-link action is gained, learn optimum values of e-link weight parameters. The difference of distributions are measured by Rényi entropy \[16\].

V. PERSPECTIVE OF MACHINE LEARNINGS

In order to simulate propagation of phonetic waves in hysteretic media, we adopted a model that phonetic waves propagate in the sea of Weyl fermions. To compare with experiments, we considered topological effects that emerge from fixed point actions on \( (2+1)D \) lattices. We compared the Shannon entropy of several random number sets. We observed that a sample \( \gamma \) among 4 samples, whose magnitude of the shift of e-link action from the average of e-link and f-link is close to that of f-link, the Shannon entropy becomes small.

The KL divergence does not show significant difference among random number sets.

In NDT, the random number set similar to \( \gamma \) shows a strong signal, however Rényi entropy search will be helpful for more general situations.

For optimization of the weight function in ML, CNN and RNN using stochastic optimization which is called adaptive moment estimation (ADAM) \[17\] would be appropriate. In this method \(|\text{det}J_r(u; \phi)|\) is evaluated by
FIG. 1. The plaquette action of $A$ type as a function of $u_1/\Delta$, $u_2/\Delta = N_{\text{even}}$ ($0, 2, \cdots, 16$).

FIG. 2. The plaquette action of $A$ type as a function of $u_1/\Delta$, $u_2/\Delta = N_{\text{odd}}$ ($1, 3, \cdots, 15$).

FIG. 3. Sum of the plaquette action and link action of $B$ type as a function of $u_1/\Delta$, $u_2/\Delta = 0$. The suffices $21, \cdots, 211$ distinguish an order in the class 2 of random number sets used in the MC simulations.

FIG. 4. Sum of the plaquette action and link action of $B$ type as a function of $u_1/\Delta$, $u_2/\Delta = 1$. The suffices $21, \cdots, 211$ distinguish an order in the class 2 of random number sets used in the MC simulations.

Introducing a scaling parameter $\alpha$, velocity parameters $\beta_1, \beta_2$ and gradients $g_1, \cdots, g_E$ at each epoch.

At each epoch, CFT from $u$ space to $x$ space and inverse CFT from $x$ space to $u$ space is necessary. In order to analyze the sum of plaquette actions, e-link actions and f-link actions, one-to-many RNN can be used. One-to-many reflects existence of many different hysteresis paths, which can be treated by Lie groupoids\cite{14}.

The convolution of a Khokhlov-Zabolotskaya solitonic wave obtained by Lapidus and Rudenko\cite{23}, and its TR solitonic wave show presence of anomalous zero mode\cite{10}, and detailed analysis of TR-NEWS convolution data will present hints on the gravitational anomaly suggested in\cite{9}. Absence of zero modes may indicate mutual cancellations with the gravitational anomaly.

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