Minimal timing jitter in superconducting nanowire single photon detectors

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Using two-temperature model coupled with modified time-dependent Ginzburg-Landau equation and current continuity equation we calculate the delay time $\tau_d$ in appearance of growing normal domain in the current-biased superconducting strip after absorption of the single photon. We demonstrate that $\tau_d$ depends on the place in the strip where photon is absorbed and monotonically decreases with increasing of the current. We argue, that the variation of $\tau_d$ (timing jitter), connected either with position-dependent response or Fano fluctuations could be as small as the lowest relaxation time of the superconducting order parameter $\sim \hbar/k_B T_c$ ($T_c$ is the critical temperature of the superconductor) when the current approaches the depairing current.

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I. INTRODUCTION

In superconducting nanowire single photon detector (SNSPD) absorption of single photon of visible or infrared range with energy $E_\nu$ leads to appearance of voltage pulse at relatively large transport current in superconducting strip. Experiments demonstrate that there is a finite delay time $\tau_d$ in appearance of the voltage response and moreover there is a variance in $\tau_d$ (called as a timing jitter) which depends on the material or bias current $I$. The origin for the timing jitter may come from the electronics, read-out system or finite length of the strip (geometrical jitter) but it also may have an intrinsic origin connected with dynamics of the superconducting order parameter $\Delta$ in response of current-carrying superconducting strip on absorbed photon. Indeed, the photon heats electrons (theoretical estimations show that locally the electron temperature may well exceed critical temperature of the superconductor) but due to finite relaxation time of magnitude of $\Delta$ ($\tau_\Delta$) the superconductivity is not destroyed instantly. This effect is well-known, for example, from the study of time response of superconducting bridge/stripe on the supercritical current pulse (current pulse with an amplitude exceeding critical current) $I_c$. In that works it was found finite delay time which is strongly reduced with increasing of the current pulse amplitude - qualitatively similar result is found in experiments with SNSPD $I_c$.

In SNSPD timing jitter could be connected with position-dependent response, when photon absorbed in different sites across the strip produces the voltage signal at the different detection (critical) currents $I > I_{det}(y)$ ($y$ is a coordinate across the strip). Then in accordance with results of Refs. $I_c$ one may expect the different delay time at fixed current $I$: $\tau_d(I/I_{det}(y))$, depending where in the superconductor the photon is absorbed.

Additional mechanism of timing jitter in SNSPD comes from so-called Fano-fluctuations (lose of the part of energy of the photon due to fluctuated nature of escape of nonequilibrium Debye phonons to the substrate) or local variations of material parameters of the superconducting strip (mean free path, local $T_c$ or thickness of the strip). Because local detection current $I_{det}(y)$ depends on the deposited energy $E$ to the electron/phonon system (it determines how strong electrons and phonons are heated) and on the material parameters, at fixed current the ratio $I/I_{det}$ varies from one absorption event to another one and it produces the variance in the delay time.

In this paper, based on the two-temperature model coupled with modified time-dependent Ginzburg-Landau equation and current continuity equation we calculate the position-dependent delay time in SNSPD both in absence and in presence of Fano fluctuations and study how $\tau_d$ depends on the current and deposited energy. Effect of Fano fluctuations in our model are taken into account via introduction of probability $P(E)$ to deposit energy $E < E_\nu$ to the electron/phonon systems of superconductor. Effect of variations of material parameters may be considered in a similar way and we do not study them explicitly. We define the delay time $\tau_d$ as a time needed for formation of the growing normal domain after absorption of the photon somewhere in the superconducting strip. We find, that $\tau_d$ is drastically reduced as the current approaches to the depairing current and timing jitter may be as small as $\hbar/k_B T_c$ ($\sim 0.8$ ps for superconductor with $T_c = 10K$). We also show that the considered model with position-dependent response predicts stronger deviation of dependence of photon counts on the delay time (in the literature its is called as probability density function (PDF) or instrument response function (IRF) from the Gaussian-like distribution than the hot belt model predicts). We argue that it occurs due to photons absorbed near the edge of the strip which provide the largest delay time.

The structure of the paper is following. In section II we
introduce our theoretical model. In section III we present our results on dependence of position-dependent $\tau_d$ on current in absence of Fano fluctuations and in section IV we include effect of Fano fluctuations and calculate energy dependence of $\tau_d$. In section V we discuss the value of delay time and timing jitter at low currents, when intrinsic detection efficiency of the detector is much smaller than unity and in section VI we relate our results with existing experiments and theoretical works.

II. MODEL

The main assumption of our model implies that in any moment of time distribution function of electrons and phonons could be described by Fermi-Dirac and Bose-Einstein functions with local temperatures of electrons $T_e$ and phonons $T_p$ different from the bath temperature $T$. In Ref. [7] it is shown that this assumption is approximately valid in rather dirty (with diffusion coefficient $D \lesssim 0.5 cm^2/s$) thin superconducting films and energy of photon $E_\nu \gtrsim 1 eV$. In this model temporal and space evolution of $T_e$ and $T_p$ is governed by heat conductance and energy balance equations (Eq. (30) and (31), respectively, in [7]) where heat capacity and heat conductivity (Eq. (31) in [7]) of electrons take into account the presence of the superconducting gap. These equations are coupled to the time-dependent Ginzburg-Landau (TDGL) equation for superconducting order parameter $\Delta$ (Eq. (36) in [7]) which is modified to take into account correct temperature dependence of coherence length, superconducting order parameter and critical (depairing) current at temperatures far below $T_c$. Together with these equations one also has to solve current continuity equation - Eqs. (37) in Ref. [7].

In this model the absorbed photon is modelled by instant local heating of both electrons and phonons up to $T_e = T_p = T_{e,ff}$ in the area $2\xi_c \times 2\xi_c$ - so called initial hot spot [7], where $T_{e,ff}$ should be determined from the energy conservation law (Eq. (15) in [7]). Here $\xi_c = (hD/k_B T_c)^{1/2} \approx \xi_0 = (hD/1.76k_B T_c)^{1/2}$ is convenient in numerical calculations length scale, the initial hot spot is placed at $x = L/2$ and different transverse coordinates $y = 0 - w$ ($L$ is a length of the strip and $w$ is its width). In Ref. [7] it is discussed the eligibility and limitation of this choice of initial condition on the basis of kinetic equations approach.

In numerical calculations we use parameters of typical NbN strip: $w = 20\xi_c \approx 130 nm$ ($\xi_c = 6.4 nm$), thickness $d = 4 nm$, $T_c = 10 K$. Important parameters $\gamma = 10$ and $\tau_0 = 900 ns$ which stay in front of electron-phonon and phonon-electron collision integrals in kinetic equations (see Eqs. (3,4,6,7) and Eqs (30,31) in [7]) control corresponding electron-phonon $\tau_{ep}$ and phonon-electron $\tau_{pe}$ inelastic relaxation times. We also use $L = 4w = 80\xi_c$, $\tau_{exc} = 0.05\tau_0$ (escape time of nonequilibrium phonons to the substrate) and the boundary conditions for $\Delta$, $T_e$ and electrostatic potential $\varphi$ in $x$ and $y$ directions from Ref. [7].

Strictly speaking TDGL equation was derived at temperatures close to $T_c$, and is quantitatively valid when $|\Delta| < k_B T_e$. Note that in the hot spot area the local temperature satisfies this condition (at least at the initial stage of its time evolution) and to the moment of appearance of first vortices does not strongly deviate from $T_c$ (see for example Fig. 8 in [7]). Secondly, we also did our calculations at temperatures close to $T_e$ (at $T/T_c = 0.9$ and $0.95$) and did not find any qualitative difference with results found at lower $T$. Namely, when initial hot spot appears in the central part of the strip the vortex-antivortex nucleate in that place and move in opposite directions, while when it appears near the edge the vortex enters the strip when $I > I_{det}(y)$. The only quantitative difference is that at $T \sim T_c$ delay time becomes much larger (which favors energy leakage of photon’s energy to substrate) and due to lower absolute value of detection current the normal domain grows much slowly or even does not appear in the strip (depending on choice of $\tau_{exc}$ and $T$).

As a result the order parameter relaxes in hot spot area to the equilibrium value after passage of several vortices(antivortices) across the strip without appearance of large voltage signal.

In our work we do not consider fluctuation assisted photon counting at $I < I_{det}$ which is connected with penetration vortices via the energy barrier formed near the hot spot (so we are working strictly in so-called deterministic regime [4]). Therefore delay time is not defined at $I < I_{det}(y)$ and it is finite at $I \geq I_{det}(y)$ (see section II).
III. POSITION-DEPENDENT DELAY TIME

In Fig. 1 we show time dependence of the voltage response of superconducting strip after appearance in the superconductor of the initial hot spot at \( t = 0 \). One can see that depending on the site of the initial hot spot (associated with the site where the photon is absorbed) there is different delay time in appearance of large (growing linearly in time) voltage, connected with appearance of the growing normal domain. From these dependencies it is clear that the variance in \( \tau_d \) does not depend on choice of threshold voltage \( V_{th} \) (if it is large enough) and in the following we choose level \( V_{th} = 20V_c = 20k_BT_c/2e \) for quantitative evaluation of \( \tau_d \).

\[
\begin{align*}
\text{(a)} & \quad \tau_d / \tau_c = 1.1, 1.5, 2.0 \quad \text{for} \quad y = 0.5 \xi_c \\
\text{(b)} & \quad \tau_d / \tau_c = 1.1, 1.5, 2.0 \quad \text{for} \quad \xi_c = 20, 50, 100 \xi.
\end{align*}
\]

FIG. 2: (a) Position-dependent \( \tau_d \) at different currents and deposited energies \((E = 1eV \rightarrow T_{\text{crit}} = 2T_c \text{ and } E = 2eV \rightarrow T_{\text{crit}} = 2.4T_c)\). (b) Dependence of \( \tau_d \) on current for three positions of initial hot spot \( y = 2, 5, 10 \xi_c \) and two deposited energies 1 and 2 eV.

In Fig. 2(a,b) we present position and current dependence of \( \tau_d \). Delay time is minimal in the place where \( I_{\text{det}}(y) \) is minimal (compare Fig. 2a with Fig. 3a) and \( \tau_d \) monotonically decreases with current increase (at fixed position of hot spot - see Fig. 2b). Both these results are not surprising and resemble previous theoretical findings on the time delay in destruction of superconducting state by current pulse [10, 12, 14]. According to these results \( \tau_d \) monotonically decreases with increase of ratio \( I/I_c \), where \( I_c \) is the critical current of the superconducting bridge/strip. In our problem role of \( I_c \) is played by \( I_{\text{det}}(y) \) and situation is more complicated, because we are looking not for the suppression of superconductivity (appearance of the phase slip center/line or moving vortices) but for nucleation of the growing normal domain. From Fig. 1 it is clear that these are not the same. For example at \( y = 2\xi_c \) (photons is absorbed near the edge of the strip) the vortex appears earlier than the vortex/antivortex pair nucleates at \( y = 10\xi_c \) (photons is absorbed in the center of the strip) because \( I_{\text{det}} \) in that place smaller but the normal domain appears earlier in the last case due to shorter time needed to cross the strip by the vortex and antivortex than by single vortex (only after that the normal domain appears and expands along the strip, leading to large voltage response). Moreover in the considered model the appearance of the vortices does not obligatory lead to appearance of the normal domain when the bias current is close to the retrapping current (see discussion in Ref. [7]). Therefore in our 2D case \( \tau_d \) is not only function of ratio \( I/I_{\text{det}}(y) \), but it may also depend on location of initial hot spot. For example at \( y = 2\xi_c \) detection current is the same as at \( y = 7\xi_c \) (see Fig. 3a in case of \( E = 1eV \)) but \( \tau_d \) are different (see Fig. 2a).

\[
\begin{align*}
\tau_d(y) & \quad \text{is minimal (compare Fig. 2a with Fig. 3a) and } \tau_d \text{ monotonically decreases with current increase (at fixed position of hot spot - see Fig. 2b)}.
\end{align*}
\]

In contrast to problem with supercritical current pulse [10, 12, 14] \( \tau_d \) does not diverge as \( I \rightarrow I_{\text{det}}(y) \) (see Fig. 2b). This fact is connected with dynamic nature of the hot spot and its finite life-time. When the hot spot expands the electronic temperatures goes down, but size of hot spot increases and there is an ‘optimal’ hot spot (with ‘optimal’ size and ‘optimal’ \( T_c \)) for given deposited energy \( E \) which provides the ‘minimal’ detection current (do not confuse it with \( I_{\text{det}}^{\text{opt}} \) in Fig. 3a) for fixed photon absorption site. But there is also finite relaxation time of \( |\Delta| \), leading to finite \( \tau_d \) which grows with decreasing of the current. So, roughly speaking the maximal \( \tau_d \) in Fig. 2b (and corresponding current is the ‘minimal’ detection current those coordinate dependence is shown in Fig. 3a) corresponds to the time needed for the growing hot spot to become the ‘optimal’ one.

From Fig. 2a,b one can see that with increasing of the current the variance in delay time decreases and it approaches \( \hbar/k_BT_c \) as the current goes to the depairing current. In the next section we discuss how Fano fluctuations affect this result.
In this section we consider effect of Fano fluctuations on delay time and timing jitter. We follow the Ref. \[9\] and introduce normalized probability of energy deposition \(E\) both to electron and phonon systems after absorption of the photon

\[
P(E) = \frac{1}{\sigma \sqrt{2\pi}} e^{-(E-E_c)^2/2\sigma^2}.
\]  

(1)

In this model it is assumed that the part of energy of photon \(E_\nu - E\) is lost due to fluctuations in escape of nonequilibrium Debye phonons to the substrate and the most probable deposited energy is equal to \(E = E_\nu < E_c\). In Fig. 4a we show position-dependent delay time for different \(E\) and in Fig 4b we demonstrate energy dependence of \(\tau_d\) at different sites where photon is absorbed. Based on these results and Eq. (1) we calculate and plot in Fig. 5 the local probability to observe some \(\tau_d\) in case of absorption of the photon in the center of the strip \(P(\tau_d, y = 10\xi_c)\) (for this purpose we convert dependence \(\tau_d(E)\) to \(E(\tau_d)\) and insert it to Eq. (1)). In calculations we use \(E = 1.5, 2.5 eV\) and \(\sigma = 0.1 E\). One

FIG. 3: (a) Position-dependent detection current for different deposited energies \(E\). (b) Dependence of intrinsic detection efficiency (IDE) on the current for three values of deposited energy 0.4, 1 and 2 eV. Symbols are obtained using results shown in (a) assuming equal probability of photon absorption across the width of the strip and no fluctuations in the deposited energy. Solid curves are obtained in presence of both position-dependent \(I_{\text{det}}(y)\) and Fano fluctuations which provide local fluctuations of \(I_{\text{det}}(y)\). In our model instead of error-function \(\mathcal{E}\) we use simpler expression for local detection efficiency \(LDE(y) = 0.5 \cdot (1 + \tanh((I - I_{\text{det}}(y))/dI))\) with control parameter \(dI = 0.05I_{\text{dep}}\) \((\text{IDE}(I) = \int_0^\infty LDE(y)dy/w)\) to show qualitatively effect of Fano fluctuations. When Fano fluctuations are absent \((dI = 0)\) one comes to curves with symbols.

FIG. 4: (a) Position-dependent \(\tau_d\) for different deposited energies \(E\) and \(I = 0.55I_{\text{dep}}\). At \(E = 0.95 eV\) central part of the strip does not 'detect' photons (absorbed photon does not 'produce' vortices and normal domain does not appear) and formally \(\tau_d = \infty\). (b) Dependence of \(\tau_d\) on deposited energy for three photon's absorption cites \(y = 2, 5, 10\xi_c\) and two values of the current.
can see that with increasing of the current (at fixed $\bar{E}$) or $\bar{E}$ (at fixed current) the function $P(\tau_d, y)$ tends to Gaussian-like form. This result follows from the dependence $\tau_d(E)$ - at large currents and $E$ it is better approximated by linear dependence $\tau_d(E) = a + bE$ in the finite range of energies $\sim 2\pi$ which together with Eq. (1) automatically leads to Gaussian-like dependence. Because nonlinearity is stronger at smaller $\bar{E}$ (large $\tau_d$) dependence $P(\tau_d, y)$ is not Gaussian-like at large $\tau_d$.

![FIG. 5: Local normalized probability to have delay time $\tau_d$ at different currents and different $\bar{E}\sim E_d$ (solid curves - $\bar{E} = 1.5$ eV, dashed curves - $\bar{E} = 2.5$ eV) for the photon absorbed in the center of the strip ($y = 10\xi_c$). In calculations we take $\sigma = 0.1\bar{E}$.](image1)

![FIG. 6: Normalized probability to have delay time $\tau_d$ (parameters are the same as in Fig. 5). At relatively low current ($I = 0.55I_{dep}$) there is a 'shoulder' on dependence $P(\tau_d)$ connected with contribution of photons absorbed in near-edge regions of the strip which provide large $\tau_d$.](image2)

Now we can combine this result with position-dependent $\tau_d$. We calculate $P(\tau_d, y)$ at each discrete point of our numerical grid, integrate it over the $y$ and assume equal probability for photon absorption across the strip. In this way we find $P(\tau_d) = \int P(\tau_d, y)dy$ which is proportional to experimentally found probability density function $\omega$, instrument response function $R$ or dependence of photon counts on delay time - see Fig. 6. Local $P(\tau_d, y)$ at any $y$ has the shape similar to the one shown in Fig. 5 but centered at different $\tau_d$. Contribution from the near edge regions, which give the largest $\tau_d$, provides on dependence $P(\tau_d)$ some kind of 'shoulder' at relatively small current $I = 0.55I_{dep}$ ('oscillations' on the 'shoulder' visible for $\bar{E} = 2.5$ eV have artificial origin and are connected with finite step $dy = 0.5\xi_c$ used in numerical calculations), while at relatively large current the 'shoulder' practically disappears. Visibility of the 'shoulder' depends on parameter $\sigma$ in Eq. 1 and with its increase the 'shoulder' smears out, leading to shape of $P(\tau_d)$ qualitatively similar to the one shown in Fig. 5, while with its decrease the 'shoulder' becomes more pronounced.

V. JITTER AT LOW DETECTION EFFICIENCY

So far we consider delay time and timing jitter at currents exceeding $I_{det}^{\text{max}}$ (see Fig. 3a) when intrinsic detection efficiency reaches unity (or photon count rate (PCR), system detection efficiency (SDE) reaches the plateau or saturate at relatively large current). At $I > I_{det}^{\text{max}}$ both $\tau_d$ and timing jitter decreases with increasing of the current. What one can expect at low currents $I \gtrsim I_{det}^{\text{min}}$ when IDE $\ll 1$?

In the model with position dependent response and no Fano fluctuation the detector stops to operate at $I < I_{det}^{\text{min}}$. At current slightly exceeding $I_{det}^{\text{min}}$ only part of the strip where $I > I_{det}(y)$ detects photons and it is clear that position dependent timing jitter in this case should be small. To illustrate it in Fig. 7 we show $\tau_d$ at different currents just above $I_{det}^{\text{min}}$.

In presence of Fano fluctuations $I_{det}^{\text{min}}$ varies from one act of photon’s absorption to another one because of variation of the deposited energy $E$. But the maximal deposited energy cannot exceed the energy of the photon $E_\nu$ and, hence, there is a minimal $I_{det}^{\text{min}}$ which corresponds to $E = E_\nu$. The same situation is with variation of material parameters - in the ‘weakest’ place of the strip $I_{det}$ reaches the minimal value when $E = E_\nu$. Therefore in framework of the used model we expect that at low currents timing jitter decreases (while delay time increases) with decrease of the current.
I pair current for 100 nm wide strip from [6] gives us E and timing jitter is smaller at fixed current for larger with current is found for wide range of Therefore we make only semi-quantitative comparison of thermalized electrons (compare Figs. 3 and 6 in [14]). It was found different (but qualitatively [14]) and nonthermalized distribution function (nonther-

tal model). It was found different (but qualitatively 

In Ref. [6] minimal experimental timing jitter is about of 3 ps). Sheet resistance for MoSi meanders is not present in Ref. [3] and we cannot estimate depairing current for studied structures. Because Tc in MoSi is smaller than in NbN we expect larger value for minimal timing jitter for this material.

In Refs. [4, 5] nonmonotonous dependence of jitter on current is observed in the range of currents where intrinsic detection efficiency (IDE) is smaller than unity. As we discuss in section V decrease of jitter at relatively small currents could be connected with decreasing of active area of detector and/or contribution to photon counts only absorbed photons with the largest deposited energy Ev. Does this effect exist in Ref. [3] or not is not clear because timing jitter is not presented for the currents where IDE ≪ 1.

The presence of 'shoulder' on dependence of photon counts on τd is a fingerprint of position dependent response. 'Shoulder', qualitatively similar to the one marked in Fig. 6 could be recognized in supplementary Fig. 8 of Ref. [3], while in Refs. [4, 5] it looks absent. We have to stress that the existence of the 'shoulder' depends on probability of photon absorption across the strip, and hence, on wavelength of the photon and its polarization. The 'shoulder' is most visible when photon absorption does not depend on coordinate, as we assume in our calculations. From another side relatively strong Fano fluctuations (σ > 0.2Ev for our parameters) may wash out this feature. But even in this case the position-dependent response could be revealed in the experiment with external magnetic field, where it leads to increasing of the width of dependence P(τd), and, hence, the timing jitter, while no 'shoulder' is seen - see Fig. 3 in Ref. [21].

The main qualitative difference of our results with theoretical results found in [3] for the timing jitter and delay time is the presence of the 'shoulder' on dependence of photon counts on delay time. This difference is not surprising because in Ref. [3] position-dependent response was not studied. Besides, there are also two quantitative differences with the model from Ref. [3]: i) we do not have coefficient in front of time derivative ∂|Δ|/∂t in TDGL equation which is proportional to inelastic τep and/or τee - see Eq. (31) in [3] and ii) in our model maximal delay time is finite which is connected with finite life-time of the hot spot. Coefficient in front of ∂|Δ|/∂t appears due to nonequilibrium effect connected with variation of |Δ| in time and leads to relatively long relaxation time of |Δ| [10, 12, 14, 17]. In the form used in Ref. [3] it is valid at condition that the delay time is much larger than min{τep, τee}. When this is not the case (as in Ref. [4] at large currents) its usage overestimates the delay time as it was first discussed in Ref. [10] (see Fig. 5 there). In the problem with response on supercritical current pulse already at I/Ic ≳ 1.8 the delay time practically does not depend on τep as it could be seen from Figs.

VI. DISCUSSION

We do not compare quantitatively our numerical results with available experiments on dependence of timing jitter on the current and energy of the photon [4, 6] because we believe that used theoretical model may give at most semi-quantitatively correct results. The assumption of the used model (complete thermalization in electron system) is fulfilled only partially due to relatively large inelastic electron-electron relaxation τee for electrons with energy about |Δ| above the Fermi level. As a result the electron distribution function deviates from Fermi-Dirac distribution with effective temperature Te which should affect τd. For example in Ref. [14] two limiting cases were considered: complete thermalization of electrons (quasiequilibrium model in notations of Ref. [14]) and nonthermalized distribution function (nonthermal model). It was found different (but qualitatively similar) dependencies τd(I) and τd was shorter in case of thermalized electrons (compare Figs. 3 and 6 in [14]). Therefore we make only semi-quantitative comparison of our results with available experiments.

In Ref. [6] the monotonous decay of the timing jitter with current is found for wide range of Ev and widths of NbN strips (similar effect is found for MoSi meanders in [3]). According to our result this effect is connected with decreasing of the delay time as current increases - effect comes out from the current dependent relaxation time of |Δ| [10, 14]. Because τd is function of ratio I/Idet and Idet decreases with increasing of Ev the delay time and timing jitter is smaller at fixed current for larger Ev - this effect is observed in [3]. Estimation of the depairing current for 100 nm wide strip from [6] gives us Idep ≃ 45μA (T = 0.9K). It means that the experimental critical current for this strip (Ic ≃ 28μA) is about of 0.62 Idep and therefore the timing jitter does not reach its minimal, from theoretical point of view, possible value ∼ h/kBTc ≃ 1ps for that NbN strip with Tc = 8.65K (in Ref. [6] minimal experimental timing jitter is about of 3 ps). Sheet resistance for MoSi meanders is not present in Ref. [3] and we cannot estimate depairing current for studied structures. Because Tc in MoSi is smaller than in NbN we expect larger value for minimal timing jitter for this material.

FIG. 7: Position-dependent delay time at currents close to Idep min ≃ 0.460Idep. We present results only for left half of the strip, in the right half τd(y) is symmetric.
3.6 in \([14]\). In our model this nonequilibrium effect is already included via term \(\partial (E_0 \mathcal{E}_c(T, \Delta))/\partial t\) (see Eq. (30) in Ref. \([7]\)) which is equivalent to the term \(\sim \partial |\Delta|^2/\partial t\) in Eq. (6) of Ref. \([14]\) as \(T \to T_c\). Moreover, our model automatically takes into account that there is no effect (proportional in this problem to \(I/I_{\text{max}}\)) of vortex nucleation inside the hot spot comes also from the recent experiment \([22]\) where single photon detection with IDE \(\sim 1\) is observed in several micron-wide NbN strips which cannot be explained by vortex penetration via the edge.

VII. CONCLUSION

In the framework of two-temperature model combined with modified time-dependent Ginzburg-Landau equation we find following:

i) delay time and variation of delay time (timing jitter) in SNSPD connected either with position-dependent response or Fano fluctuations monotonically decreases with increasing of the current when \(I > I_{\text{det}}^{\text{max}}\) and timing jitter may be of \(\hbar/k_B T_c\) at the current close to the depairing current. The effect is connected with fast decrease of relaxation time of the superconducting order parameter at large currents. At fixed current the delay time and timing jitter are smaller for photons with larger energy due to larger ratio \(I/I_{\text{det}}\).

ii) Fano fluctuations and nonlinear dependence of \(\tau_{\text{det}}(E)\) provide non-Gaussian dependence of photon counts on delay time, most pronounced at larger \(\tau_{\text{ex}}\). Position-dependent response leads to the appearance of the ‘shoulder’ on this dependence connected with contribution from the photons absorbed in near-edge area of the strip. The ‘shoulder’ decreases with the current and is maximal in case of coordinate-independent photon absorption across the strip.

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