Geons found include non-susy CDM particle and non-singular “Kerr-Newman” models

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Abstract

The antisoliton-soliton $G = S_- \lor S_+$ neutral state is a proper geon in a family of stable squashed-$S^3 \times \mathbb{R}$ pp-wave electrovacua along a primordial $Q/r^2$ field. With $S_-$ propagating backwards in time, the dominant EM field is that of an effective electric-dipole moment $p$. If disjointed, the $S_\pm$ carry $\pm Q$ charge (on a round-$S^2$ physical singularity of radius $r_\circ$) as non-singular alternatives to the Kerr-Newman solution. $G$ has three scales (gravitational $\kappa$, metric scale, NUT-charge $\kappa Q = 2r_\circ$) in a full 4-scale hierarchy without supersymmetry. A particular $G$ with effective mass and a near-zero $p$ is proposed as dark-matter particle. A gas of such $G$s would ‘freeze-out’ before the electroweak era as CDM, whose present mean density is predicted by this model (via Casimir-effect data on earth) as, roughly, 100Mev/cm$^3$.

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1 Introduction

Following Schwarzschild’s solution in 1915, Einstein’s deep interest on ‘point’ singularities relating to particle aspects and geodesic motion was expanded and carried all the way into the 50s with singular solutions (one is the Papapetrou-Majumdar), but also with point-like particles of finite radius $r_o$ and the geon concept [1]. The latter had not been realized up to now with any proper example, namely any exact non-singular electrovacuum which is asymptotically flat, stable, etc [2]. In quite lesser adversity, the Taub-NUT vacuum was formulated within the span of two decades [3], treated as a cosmological model at the time. Remarkably, it still is the only explicitly available solution in the entire class of (even locally) $S^3 \times \mathbb{R}$ non-singular Ricci-flat manifolds [4]. Meanwhile, the introduction of topological geons and concern for stability re-focused interest back to singular models and toward the quantum-mechanical properties of geon black holes and Reissner-Nordström types of Taub-NUT solutions [5]. These developments, combined with the outlook for supersymmetry under recent LHC results [6], have motivated us in uncovering $G$ as the first example of a proper geon model (GM), actually a 2-parameter family of GMs. A specific member therein, the $G_{dmp}$, is proposed as dark-matter particle (DMP) for a new type of cold dark matter (CDM) models, alternative to the current plethora of supersymmetric ones [7].

As a solitonic pp-wave, $G$ propagates along a sourceless primordial electromagnetic (EM) field $F$ of an Einstein-Maxwell electrovacuum. As a manifold, $G$ is a non-singular left-$SU(2)$ invariant Bianchi-type IX, with an extra Killing vector for axial rotations as the only survivor of right-$SU(2)$ invariance. $G$ has a Taub-NUT form of metric and line element with basic scale $L_o$ [4], expressible in terms of the left-$SU(2)$ invariant 1-forms $\ell^i$ (with $\ell^3 = \cos \theta d\phi + d\psi$, $(\ell^1)^2 + (\ell^2)^2 = d\Omega^2$ etc, as parametrized by the $\theta, \phi, \psi$ angles on $S^3$) and duals $L_i$, in

$$\partial_u v^i \cdot \ell^i = -\frac{1}{2} \epsilon_{jkl} \ell^j \wedge \ell^k, \quad [L_j, L_k] = \epsilon_{jkl} L_l, \quad (1.1)$$

$$ds^2 = -L_o^2 \left( g \ell^3 + 2d\omega \right) \ell^3 + r^2 d\Omega^2, \quad (d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2). \quad (1.2)$$

$r = r(u)$, $g = g(u)$ are functions of the scaleless null $u \in \mathbb{R}$, used (for now) in lieu of a $t$-time coordinate. For the $L_i, \partial_u$ vectors we read off [1,2] that $L_3 \cdot \partial_u = -L_o^2$; $\partial_u^2 = 0$, etc, and we’ll also find $(L_3)^2 = -L_o^2/P$ for any $P > 0$ constant. In this geometry, our Lagrangian

$$\mathcal{L} = \frac{1}{\kappa^2} R - \frac{1}{4} F^2 \quad (1.3)$$

complies with our premise that any point-like sources of $(m_s, Q)$ mass, electric charge, etc, can only emerge a posteriori in $G$, effectively or otherwise, if at all [2]. [4].

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1 The Taub and NUT spaces have boundaries (‘null squashed-$S^3$ Misner bridges’ in Taub-NUT) which are not mathematical singularities (as in a black-hole), but physical ones (with non-singular Riemann, etc).

2 This geometric (‘pure-marble’) approach will also avert the formation of a mathematical singularity.
2 \( \mathcal{G} \) compared and contrasted to Taub-NUT

\( \mathcal{G} \) and Taub-NUT share the generic line element \([1.2]\) and its isometries. They also share invariance under \( u \to -u \) reflections or \( u \to u + u_o \) translations, and, as it will turn out, the entire \( r = r(u) \) function itself, including its minimum \( r = r_o \) value as a NUT charge\(^3\). However, \( g(u) \) in \([1.2]\) turns out to have the mentioned \( g = 1/P \) constant value everywhere in \( \mathcal{G} \), whereas the same \( 1/P \) constant is approached only asymptotically by \( g(u) \) in Taub-NUT. This result is at the basis of several (and some very profound) differences in \( \mathcal{G} \) vs Taub-NUT. In addition to the \( P > 0 \) parameter, \( \mathcal{G} \) also has the electric charge \( Q \neq 0 \) as a second free parameter (with no counterpart in the also 2-parameter Taub-NUT solution). And in addition to the metric scale \( L_o \) and the NUT charge \( r_o \) as a pair of two arbitrary lengths, \( \mathcal{G} \) also carries the gravitational coupling \( \kappa^2 = 8\pi G_N \) as a third scale. Expectedly, the \( L_o, r_o \) pair can be expressed in terms of \( \kappa \) via the \( P,Q \) pair, actually as \( L_o = \kappa \sqrt{P} Q \) and \( 2r_o = \kappa Q \). In the case of \( \mathcal{G}_{\text{dmp}} \) in particular, the range of the \( r \geq r_o \) radius of the spacelike dimensions of the \( S^3 \) in \( \mathcal{G} \) covers the realistic and fully diversified hierarchy (without supersymmetry!) of four fundamental scales as

\[
     r_o << r_{ew} \sim L_o << r_{cl} << r_H = H_o^{-1}, \quad r_{cl}^3 = \frac{3m_{\text{dmp}}}{4\pi \rho_{\text{dm}}}. \tag{2.1}
\]

Here, \( r_o \) is Planck-scale, \( r_{ew} \sim L_o \) scales the effective mass in \((m_s, Q)\), \( r_{cl} \) is the classical mean-free-path of DMPs (of mass \( m_{\text{dmp}} \) each) in a fluid of mean density \( \rho_{\text{dm}} \), and \( r_H = H_o^{-1} \) is the Hubble radius in a cosmological model filled with that fluid. Other profound differences in \( \mathcal{G} \) vs Taub-NUT involve \( L_3 \) as an always timelike vector. Thus, with no Taub sector or Misner bridges, \( \mathcal{G} \) must be the \( C^\infty \) union (at \( u = 0 \)) of two NUT-like pieces \( S_\pm \) (in \( u \to -u \) symmetry to each other), as a \( \mathcal{G} = S_- \lor S_+ \). The \( S_\pm \) are solitonic pp-waves along the null \( \partial_u \) wave-vector, which satisfies the \( D\partial_u = 0 \) condition, with \( S_- \) propagating backwards in time (towards \( u = 0 \)) as an antisoliton. Crucial is the presence in \( \mathcal{G} \) (along a spacelike \( L_1 \times L_2 \) radial direction) of the \( E_C = Q/r^2 \) electric field, a primordial one, because no actual charge \( Q \) can exist anywhere in \( \mathcal{G} \), but also a non-singular one, because there exists no \('r = 0'\) origin either. The \( r \) coordinate cannot cover \( \mathcal{G} \) globally. This can be seen in terms of the double-valued \( u = \pm|u(r)| \), obtained from the single-valued \( r = r(u) \) in the upcoming \([3.6]\). However, the \( u = \pm|u(r)| \) branches (defined over the same \( r \geq r_o \) range with a spurious singularity at \( r = r_o \)) cover quite elegantly the \( S_\pm \) submanifolds in \( \mathcal{G} = S_- \lor S_+ \) as single-valued functions over \( u \geq 0, u \leq 0 \), respectively. Although practically identical to a Taub-NUT already at \( r \sim r_{ew} \) and beyond, \( \mathcal{G} \) cannot reduce to Taub-NUT or to any Ricci-flat manifold, or to a \( Q = 0 \) limit. The \( r_o = 0 \) limit is also forbidden, so \( \mathcal{G} \) cannot reduce to a singular limit (e.g., Kerr-newmann or Reissner-Nordström) either.

\(^3\)This concurrence allows the first direct identification of a NUT charge: via the upcoming \([3.6]\) we find \( 2r_o = \kappa Q \) as the geometric mean of the fundamental couplings, thus a Planck-scale length if \( Q^2 = 1/137 \).
The solution for $\mathcal{G}$

Non-holonomic Cartan frames $\theta^\alpha$ (with dual $\Theta_\alpha$) oriented in the line element (1.2) of $\mathcal{G}$ as

$$
\theta^0 = L_o \left( \sqrt{P} du + \frac{1}{\sqrt{P}} v^3 \right), \quad \theta^1 = r e^1, \quad \theta^2 = r e^2, \quad \theta^3 = L_o \sqrt{P} du,
$$

$$
\Theta_0 = \frac{\sqrt{P}}{L_o} L_3, \quad \Theta_1 = \frac{1}{r} L_1, \quad \Theta_2 = \frac{1}{r} L_2, \quad \Theta_3 = \frac{\sqrt{P}}{L_o} \left( \frac{1}{P} \partial_u - L_3 \right),
$$

give a manifest locally Minkowski $ds^2 = \eta_{\alpha\beta} \theta^\alpha \theta^\beta$, with $\eta_{\alpha\beta} = \text{diag}[1, 1, 1, 1]$. The $D\partial_u = 0$ condition is then verified via $D\theta^\alpha := d\theta^\alpha + \Gamma^\alpha_{\beta\gamma} \theta^\beta \wedge \theta^\gamma = 0$, which also supplies the Christoffel $\Gamma^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma} = \Gamma^\alpha_{\beta\gamma}$. The curvature $\mathcal{R}^\alpha_{\beta\gamma\delta} = 1/2 R^\alpha_{\beta\gamma\delta} \theta^\gamma \wedge \theta^\delta = d\Gamma^\alpha_{\beta\gamma} + \Gamma^\alpha_{\beta\gamma} \wedge \Gamma^\gamma_{\beta\delta}$ is calculable from

$$
-\Gamma_{12}^0 = \Gamma_{21}^0 = \frac{L_o}{2\sqrt{P} r^2}, \quad \Gamma_{12}^0 = -\frac{\sqrt{P}}{L_o} - \frac{L_o}{2\sqrt{P} r^2}, \quad \Gamma_{12}^3 = \frac{\sqrt{P}}{L_o}, \quad \Gamma_{13}^1 = \Gamma_{23}^1 = \Gamma_{31}^2 = \frac{\sqrt{P}}{L_o} - \frac{r'}{P} r',
$$

with a prime for $d/du$. Ricci’s $R_{\alpha\beta} = R^\gamma_{\alpha\gamma\beta}$ follows from the contractible Riemann’s

$$
R^0_{101} = R^2_{202} = -\frac{L_o^2}{4Pr^4}, \quad R^1_{212} = -\frac{1}{L_o^2 P} \left( \frac{r'}{r} \right)^2 + \frac{4Pr^2 + 3L_o^2}{4Pr^4}, \quad R^1_{313} = R^2_{323} = -\frac{1}{L_o^2 P} \frac{r''}{r'},
$$

while Weyl’s $W^\alpha_{\beta\gamma\delta}$ (with just one independent non-identically-zero component) vanishes as $O(r^{-3})$. For compatibility with the geometry, $F = -E\theta^\alpha \wedge \theta^\beta + B\theta^1 \wedge \theta^2$ is the general EM field in $\mathcal{G}$. Then, via the sourceless Maxwell equations $dF = d*F = 0$ which give

$$
E = -\frac{Q}{r^2} \cos \rho, \quad B = \frac{Q}{r^2} \sin \rho, \quad r^2 dr = L_o^2 du, \quad E^2 + B^2 = \frac{Q^2}{r^4} =: E_C^2,
$$

Einstein’s $\kappa^{-2} R_{\alpha\beta} = T_{\alpha\beta}^{(\text{em})} = \frac{1}{2} E_C^2 \text{diag}[1, 1, 1, -1]$ give the non-singular general solution

$$
r^2 = r_o^2 + L_o^2 Pu^2, \quad g = \frac{1}{P} > 0, \quad r_o^2 = \frac{L_o^2}{4Pr^4} = \left( \frac{\kappa Q}{2} \right)^2,
$$

$$
E = E_C - \frac{2r_o^2 Q}{r^4}, \quad B = -\frac{2r_o Q \sqrt{r^2 - r_o^2}}{r^4} \approx -\frac{m_s}{r^3}.
$$

Thus, $F$ includes the Coulomb-like $E_C = Q/r^2$ field and analogues of an electric quadrapole and a magnetic-dipole moment $m_s = \kappa Q^2$, with duality rotations allowed. To first clarify the crucial presence of $E_C$ in $\mathcal{G}$ (in spite of $d*F = 0$), we apply the divergence theorem over a 3D volume $\mathcal{V}$, which includes the $u = 0$ locus within its $\partial \mathcal{V}$ boundary in $\mathcal{S}_- \vee \mathcal{S}_+$, as

$$
0 = \int_\mathcal{V} d*F = \int_{\partial \mathcal{V}} *F = \int_{\partial \mathcal{V}_+} *F + \int_{\partial \mathcal{V}_-} *F = [4\pi Q]_{\mathcal{S}_+} + [-4\pi Q]_{\mathcal{S}_-}.
$$

As depicted in Fig[1] the $\partial \mathcal{V}_\pm$ parts of $\partial \mathcal{V}$ are round-$S^2$ sections which can be viewed as
Figure 1: A pair of $S^2[r]$ as (i) $\partial \mathcal{V}_\pm$ boundaries of a volume $\mathcal{V}$ in $S_- \vee S_+$ by (3.8), (ii) sections of $S^3$ (with its TL radius suppressed) in $S_-$, $S_+$ disjointed, hence with $\mp Q$ charge on the respective $S^2[r_o]$ boundary (physical singularity) at $u = 0$. $|E_C| = |Q|/r^2$ everywhere in $\mathcal{G}$.

evolving from $S^2[r_o]$ at $u = 0$ to large absolute values of $\pm |u|$. The minus sign in the second square bracket in (3.8) is due to the backwards-in-time propagation of $S_-$ as an antisoliton. The overall null result holds for any $\mathcal{V}$, so there is no $Q$ to be trapped anywhere in $S_- \vee S_+$, and the electric flux is not interrupted through any $S^2$ section, notably through $S^2[r_o]$. Thus, $E_C = Q/r^2$ must indeed be identified as a sourceless primordial field in $\mathcal{G} = S_- \vee S_+$, which strongly resembles a particle-antiparticle $\bar{p} p$ neutral bound state (e.g., a positronium). Such states are typically unstable, in contrast to $\mathcal{G}$ which inherits its basic aspects (including its stability) as an exact solution via symmetry and dynamics directly from the Lagrangian (1.3). To reconcile the Coulomb-like $E_C = Q/r^2$ in $\mathcal{G}$ with the electric-dipole field of a $\bar{p} p$ state, one must resort to the backwards-in-time propagation of $S_-$ as a submanifold\footnote{This is a geometric analogue for antisolitons, after the QFT standard for the propagation of antiparticles.}. In terms of global $t$-time as in (4.2), for every $(\pm |t|, \theta, \phi, \psi)$ point (where $E_C = Q/r^2$) in $S_+$, there is the equally-present $(-|t|, \theta, \phi, \psi)$ point (where $E_C = -Q/r^2$) in $S_-$, so the $\pm Q/r^2$ contributions cancel-out exactly at $(t, \theta, \phi, \psi)$ as a single event in this $\mathcal{G}$-with-pointwise-identifications-manifold, now renamed $\mathcal{G}$. However, as we’ll see, the same $t$ can also involve differing values of $r$ in $\pm Q/r^2$, hence cancellations which are almost exact. The ‘corrections’ are insignificant for sufficiently large $r$, but they do leave as dominant overall contribution an effective electric-dipole moment $p$. This, for $\mathcal{G}_{dmp}$, is a minuscule $p_{dmp} \sim 4\pi\kappa Q^2 = 4\pi m_s$.

The $S_\pm$ can be separated as independent geodesically-incomplete manifolds, coverable globally by $r$, as seen. The boundaries at $r = r_o$, now exposed as spurious loci of the formerly common $S^2[r_o]$ section (junction) at $u = 0$, become physical $S^2$ singularities of radius $r = r_o$. These, under the new initial-value setting (case ii in Fig.1), must now carry actual $\pm Q$ electric charge, distributed homogeneously on the respective $S^2[r_o]$ of $S_\pm$.\footnote{This is a geometric analogue for antisolitons, after the QFT standard for the propagation of antiparticles.}
4 Asymptotic aspects and stability of $G, S_{\pm}$

With $R_{\nu \rho \sigma}^\mu$ vanishing by (3.4) as $O(r^{-2})$ or faster, the $G, S_{\pm}$ are asymptotically locally flat manifolds. They cannot radiate, due to $W_{\rho \gamma \delta}^\alpha \sim O(r^{-3})$ and to stationarity with vorticity

$$\omega = *(v \wedge dv) = \frac{2r_o}{r^2} \theta^3,$$

as measured by an observer with 4-velocity $V$, dual of $v = \theta^0$ or $v = dt + L_o/\sqrt{P} \cos \theta d\phi$ from the upcoming (4.3). $G, S_{\pm}$ also carry calculable effective mass ($\sim m_s$), spin $S$, dipole moments, etc, well-defined and measurable by our inertial observer. With

$$x^\mu = (x^0, x^i), \quad x^0 = t = L_o \left( \frac{1}{\sqrt{P}} \psi + \sqrt{P} u \right) \mod 8\pi r_o,$$

namely with holonomic time $t \in \mathbb{R}$ and Cartesian $x^i$ coordinates in (1.2), manifest general covariance can be traded for what is usually taken as a perturbation over Minkowski’s $M^4$ to produce $M^4$ with its metric split as $\eta_{\mu \nu} + h_{\mu \nu}$. Here, however, we can have $M^4 = G$ exactly (but at a price, as we’ll see shortly), if (1.2) is re-written (with $L_o \sqrt{P} u = \pm \sqrt{r^2 - r_o^2}$) as

$$ds^2 = - \left( dt + \frac{L_o}{\sqrt{P}} \cos \theta d\phi \right)^2 + \frac{r^2}{r^2 - r_o^2} (dr)^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2)$$

$$= \eta_{\mu \nu} dx^\mu dx^\nu + \frac{r_o^2}{r^2 - r_o^2} (dr)^2 - 4r_o \cos \theta d\phi (dt + r_o \cos \theta d\phi).$$

This general result is particularly important for $G_{\text{dmp}}$, whose Planck-scale $r_o$ elevates the $r \rightarrow r_{\text{ew}}$ limit to asymptotic infinity, with $h_{\mu \nu} \rightarrow 0$ being already well-established there. The result is fundamental because it actually shows that such $G$s could have been created abundantly before the EW era, as essentially non-interacting DMPs. It is also crucial for stability, because those $G_{\text{dmp}}$ would be unable to break-up after the EW era, for lack of sufficient excitation energy. The price for these deeper findings has been the loss of manifest left-$SU(2)$ invariance in (4.3), due to the absorption of the $\psi$ angle in the definition of $t$ in (4.2). In particular, the surviving $\theta, \phi$ angles in (4.4) could (and here they do) hinder the calculation of the mass and spin ($m_s, S_s$) parameters (p.165 ff). Nevertheless, an estimate for $m_s$, with a less reliable one for $S_s$, is possible via $T_{00}^{(\text{em})}$ and $\omega$ from (4.1) as

$$m_s \approx \frac{\sqrt{2} \pi^2 Q^2}{L_o} \sim m_{\text{dmp}}, \quad S_s \sim \frac{\sqrt{2} \pi^2 Q^2}{\sqrt{P}}, \quad m_s = \kappa Q^2, \quad p_{\text{dmp}} \sim 4\pi \kappa Q^2, \quad \rho_{\text{dm}} = \frac{3m_{\text{dmp}}}{4\pi r^3_{\text{cl}}}. \quad \text{(4.5)}$$

These results, which also include the mean density $\rho_{\text{dmp}}$ of the $G$-fluid (a CDM candidate) from (2.1), etc, hold for $S_{\pm}, G, \text{etc, as the case may be.}$

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5The $\psi \in [0, 4\pi)$ angle in (4.2) allows non-trivial homotopy and Planck-scale loops in the $t$ coordinate.

6Stability would be enhanced by spontaneous symmetry breaking for a fixed $\psi = \psi_s$ angle, which would also lift the homotopy in (4.2). Time loops at Planck scale would still exist, but no-longer as geodesic curves.
5 Conclusions

By our last results in (4.5), the disjointed \( S_\pm \) carry mass \( m_s \) and charge \( \pm Q \). \( G_{\text{dmp}} \) as a neutral bound state has mass \( m_{\text{dmp}} \) and no electric charge, but it does carry the effective dipole moment \( p_{\text{dmp}} \). The process of uncovering the latter attribute is actually administering a new type of pp-wave superpositions (via the point-wise-identifications in section 3, etc), by which the generic \( G = S_- \lor S_+ \) manifold has been formed out of its solitonic constituents. Analogous superpositions could be made possibly even in Taub-NUT, but certainly in a class of generalized \( S_- \lor S_+ \) GMs. In fact, there exists a major class of Einstein-Maxwell-Yang-Mills GMs, whose 4D parameter-space includes \( G \) as a 2D subclass therein [9].

\( G \) is a long-overdue proper GM and, as a first application thereof, \( G_{\text{dmp}} \) founds a CDM model based on those results. As a 2-parameter family, \( G \) can involve drastically different physics from the particular \( G_{\text{dmp}} \). This is closely related to the range of the hierarchy in \( (2.1) \) and depends crucially on the virtually free-to-choose NUT charge \( 2r_o = \kappa Q \). For \( G_{\text{dmp}} \), the NUT charge is a Planck-scale \( 2r_o \). It is conceivable, however, that \( r_o \) could be of a very different scale, e.g., electroweak, or all the way up to scales of astrophysical interest. \( Q \) is involved in \( G_{\text{dmp}} \) as the \( Q^2 = 1/137 \) coupling (there is no actual electric charge in any \( G \) but, in general, there is no restriction on its value. This is also important for \( S_\pm \) as independent manifolds, which have emerged as a hardly-expected but nonetheless fundamental result. As such, the \( S_\pm \) offer mathematically non-singular alternatives to the Kerr-Newman solution (Reissner-Nordström is excluded because there is no \( \omega = 0 \) limit in our case). Thus, the \( S_\pm \) upgrade the primitive notion of a singular point-charge \( Q \) to a rigorous mathematically non-singular solitonic model. Also upgraded in the present case is the formerly-singular formation of the \( Q/r^2 \) field-lines and geodesics, in the sense that the geodesic incompletensses of \( S_\pm \) is essentially curable, as actually realized in \( G \). In any \( G \) or the \( S_\pm \) manifolds, the local ‘03’ Minkowski planes spanned by \( \partial_u \) and \( L_3 \) carry \( F, \omega, S, \) etc. They also carry a small subset of 4-velocity vectors \( V \), which, expectedly as in Taub-NUT, generate splitting geodesics and closed timelike loops. This, however, is a better-accepted reality in the case of \( G_{\text{dmp}} \), because it takes place at Planck scale, as we’ll briefly discuss later on.

Another fundamental result for the \( G, S_\pm \) configurations is their mentioned stability under perturbations, EM or gravitational, provided no point-like sources are introduced ‘by hand’ (c.f. footnote2). In the presence of any spacetime singularity, in particular of a mathematical one (e.g., a Kerr-Newman), with which the \( G, S_\pm \) would certainly interact, they would fully comply with the generally established dynamics and conservation laws near and within the respective horizons. We can now review how \( G_{\text{dmp}} \) could indeed provide an alternative to supersymmetric candidates as a geon (solitonic) DMP. For such an identification, and having established neutrality and stability, we must also confirm the virtual absence of interactions.
This is obviously true for the gravitational (with asymptotic infinity already established at the EW scale, as seen via (4.4)), as well as for the strong interaction (which would be totally absent). The stability of $G$ as an exact solution and the actual value of the effective $p_{\text{dmp}}$ in (4.5) will shape any exchange of photons between $G_{\text{dmp}}$ themselves and between a $G_{\text{dmp}}$ and baryonic matter. As we’ll see in an order-of-magnitude calculation, the dominant actual EM-field content in a $G_{\text{dmp}}$, essentially from $m_s$ and the effective $p_{\text{dmp}}$ dipole moments, involves much weaker values than standard-model estimates for, e.g., a neutrino. Moreover, the values for $m_s, p_{\text{dmp}}$ are smaller by a factor of at least $10^{-10}$ compared to the astrophysically-tolerable limits for such EM moments, so the interaction between $G_{\text{dmp}}$ and baryonic matter must be accordingly weaker. The conclusion here is that, in addition to its importance as the first explicit example of a proper GM, $G$ deserves attention because it may have indeed been realized in nature as a $G_{\text{dmp}}$ configuration.

If so, a fluid of these DMPs at asymptotic infinity to each-other would ‘freeze-out’ before the EW era as a viable CDM configuration [7]. Such DMPs could even be trapped today (via induced polarization) as a mono-layer between equipotential plates (as local ‘12’ planes) less than $r_{\text{cl}}$ apart in a Casimir-effect setting. The trapping would grow stronger as the gap between the plates decreases to a sufficiently-smaller-than-$r_{\text{cl}}$ value and as the trapped $G_{\text{dmp}}$ start orienting themselves and contributing to force along the ‘03’ plane. To fix the $(P, Q)$ set of independent parameters for order-of-magnitude estimates, we try as $(m_s, Q)$ values similar to those of an electron. Then, (3.6), (4.5) give $r_o \sim (5 \times 10^{19}\text{Gev})^{-1} \sim 10^{-33}\text{cm}$ (a Planck-scale radius), $r_{\text{ew}} \sim L_o \sim 10^{-14}\text{cm}$ (which decreases if $m_s$ increases toward typical EW values), while $\sqrt{P} \sim 10^{19}$. Finally, for the expectedly tiny dipole moments we find $m_s \sim 10^{-23}\text{µB}$ with a likewise tiny $p_{\text{dmp}}$ in (4.5). This result is $10^{-11}$ times smaller than what is set by the best astrophysical limits for a neutrino magnetic dipole moment, namely $m_\nu < \sim 10^{-12}\text{µB}$. More predictions could come via Casimir-effect data on earth. Thus, from the $r_{\text{cl}} \sim 1\text{mm gap as a lower limit}$ for practically zero-force (from $G_{\text{dmp}}$) between the plates, the model predicts all observables of a homogeneous dark-matter universe, including the (roughly) $\rho_{\text{dm}} \sim 100\text{MeV}/\text{cm}^3$ density, the $r_H = H_o^{-1} \sim 10^{28}\text{cm}$ Hubble radius, etc.

For the $p_{\text{dmp}}$ estimate in (4.5) we can use the (now explicitly available) global t-time as function of the $(\psi, u)$ coordinates in (4.2) for $t \in \mathbb{R}$. This range accommodates the soliton-antisoliton symmetry under $t \to -t$ and it does not reduce to $[0, \infty)$ in $G$ under the point-wise identifications. By these, the $\pm Q/r^2$ contributions from the $(\pm |t|, \theta, \phi, \psi)$ points in $S_{\pm}$, respectively, produced the exact null result at $(t, \theta, \phi, \psi)$ as one single event in $G$. However, we can change the $(\psi, u)$ pair to $(\psi + \delta \psi, u + \delta u)$ and yet leave the given $(t, \theta, \phi, \psi)$ intact, provided we take $P\delta u = -\delta \psi$ (to secure the same $t$ via (4.2)) and $\delta \psi = 4\pi n, n \in \mathbb{Z}$, (to secure the same $\psi$ as an angle on $S^3$). A $u + \delta u$ value would involve by (3.6) the $r + \delta r$ value with $\delta r = -8\pi nr_o$ in the $Q/(r + \delta r)^2$ contribution from $S_{\pm}$, hence possible non-exact
cancellations at the \((t, \theta, \phi, \psi)\) event. Thus, an overall contribution may survive in spite of the \(u \rightarrow -u\) symmetry, because of the presumably random involvement of closed loops near Planck scale and a thereabout concentration of their effectiveness. The latter comes from the enormous difference between the magnitude of the \(\psi\)-term vs that of the \(u\)-term in (4.2) at normal values of \(r\) (that is, not very close to Planck scale). A precisely null result for \(p_{dmp}\) might simplify our DMP, but it is highly unlikely and unnatural to uphold the \(u \rightarrow -u\) symmetry in a random process at Planck scale. The order-of-magnitude estimate in (4.5) as \(p_{dmp} \sim 8\pi r_o Q\) has thus been used in lieu of a rigorous result from a (not-yet available) study in \(\mathcal{G}\) of such Planck-scale dynamics, which here seems to feign (or hint) attributes from an anticipated quantum-gravity environment.

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