Numerical investigation of magnetic field on forced convection heat transfer and entropy generation in a microchannel with trapezoidal ribs

Lixuesong Han\textsuperscript{a}, Chenji Lu\textsuperscript{b}, Alexei Yumashev\textsuperscript{c}, Dariush Bahrami\textsuperscript{d}, Rasool Kalbasi\textsuperscript{e}, Mehdi Jahangiri\textsuperscript{f}, Arash Karimipour\textsuperscript{g}, Shahab S. Band\textsuperscript{h}, Kwok-Wing Chau\textsuperscript{i} and Amir Mosavi\textsuperscript{j,k}

\textsuperscript{a}School of Physical Science and Technology, ShanghaiTech University, Shanghai, People’s Republic of China; \textsuperscript{b}Department of Aeronautical and Astronautical Engineering, University of Southampton, Southampton, UK; \textsuperscript{c}Department of Prosthetic Dentistry, The State Education Institution of Higher Professional Training, The First Sechenov Moscow State Medical University under Ministry of Health of the Russian Federation, Moscow, Russian Federation; \textsuperscript{d}Department of Mechanical Engineering, Shahrekord University, Shahrekord, Iran; \textsuperscript{e}Department of Mechanical Engineering, Najafabad Branch, Islamic Azad University, Najafabad, Iran; \textsuperscript{f}Department of Mechanical Engineering, Shahrekord Branch, Islamic Azad University, Shahrekord, Iran; \textsuperscript{g}Future Technology Research Center, College of Future, National Yunlin University of Science and Technology, Douliou, Taiwan; \textsuperscript{h}Department of Civil and Environmental Engineering, Hong Kong Polytechnic University, Hong Kong, People’s Republic of China; \textsuperscript{i}Faculty of Civil Engineering, Technische Universität Dresden, Dresden, Germany; \textsuperscript{j}John von Neumann Faculty of Informatics, Obuda University, Budapest, Hungary; \textsuperscript{k}Department of Informatics, J. Selye University, Komarno, Slovakia

ABSTRACT

In this study, the effects of adding trapezoidal ribs to microchannel on functionalized multi-walled nano-tubes/water nanofluid heat transfer are examined. The dimensionless slip coefficient (0–0.1), Reynolds number (50–400) and Hartmann number (0–20) are considered as independent variables and the heat transfer along with the entropy generation are considered as the output parameters. The simulation outcomes confirm that the addition of trapezoidal ribs, on the one hand, increases the heat transfer area and, on the other hand, intensifies the possibility of vortex formation. The presence of a vortex decreases the heat transfer potential and thus reduces the performance of the trapezoidal-wall microchannel compared to the base one. With increasing Reynolds number (Re), the probability of vortex formation intensifies, which in turn diminishes the positive effects of using trapezoidal ribs. However, it is found that, with increasing Hartmann number (Ha) and dimensionless slip coefficient (\( \beta^* \)), the vortex strength is weakened, and consequently heat transfer is improved. Based on numerical computations, it is found that at Re = 400, Ha = 0 and \( \beta^* = 0 \) and adding trapezoidal ribs to the base microchannel increases heat transfer by 11.12%.

1. Introduction

Looking at the history of applicable devices, it can be seen that advances in technology have always changed the size of objects (Dinarvand et al., 2017; Sheikholeslami and Rokni, 2017). This resizing has often led to size reduction on mille, micro and even nanoscales. Size reduction in equipment such as electronic devices (Kalbasi, 2021; Muhammad et al., 2020), pumps (Alipour et al., 2019), turbines (Xu et al., 2019) and microfluidics (Bayareh, 2020), are examples of resizing. Size reduction in the flow area leads to microchannels, which provide more heat transfer area per unit volume than conventional ducts (Abdollahi et al., 2020; Bayareh et al., 2020). Thermal and hydraulic behavior in microchannels can be improved by various techniques such as the use of nanofluids (Arjmandfard et al., 2020; K. G. Dehkordi, Karimipour, et al., 2021; Turkyilmazoglu, 2016, 2020b, 2020c, 2021), magnetic fields (MFs) (R. B. Dehkordi, Toghraie, et al., 2021; Dinarvand et al., 2021; Turkyilmazoglu, 2011; Turkyilmazoglu, 2020a; Zhong et al., 2021) and installing ribs (Akermann et al., 2020; Jiang et al., 2020).

Alfellag et al. (2020) examined the effects of installing inclined plate-fins on the hydrothermal performance of a minichannel. Numerical results showed that the Nusselt number (Nu) increased with increasing slotted height up to 1.5. Additionally, their minichannel hydrothermal efficiency and heat transfer are about 1.84 and 1.54 times more than those of a simple one.

CONTACT Amir Mosavi \textsuperscript{a} amir.mosavi@mailbox.tu-dresden.de; Shahab S. Band \textsuperscript{h} shamshirbands@yuntech.edu.tw

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The effects of installing triangular ribs on the walls of a microchannel in which an MF has been exerted were investigated by Nguyen, Bahrami, Kalbasi, and Bach (2020). Internal extended surfaces (ribs) can have two simultaneous effects: on the one hand, they increased heat exchange by increasing the heat transfer area, and on the other hand, a vortex could form behind each rib, which drastically diminishes heat transfer. The presence of an MF weakens the vortex and thus improved heat transfer. Eventually through adding extended surfaces as well as MFs, heat transfer and entropy generation are intensified by 25% and 328%, respectively.

The effect of installing different ribs in a U-shaped channel was numerically simulated by Ravi et al. (2017). They focused on heat transfer and found that the presence of ribs in the channel increases heat transfer. They proved that heat transfer can be enhanced up to 35% by using V-shaped ribs.

Table 1 summarizes the results of studies that were focused on heat transfer improvement using ribs. In another work, Bahrami, Nadooshan, et al. (2020) applied an MF to mix magnetic particles in water numerically. Their result strongly confirmed that there is an optimum value in the strength of an MF to produce a high mixing performance. Nojoomizadeh and Karimipour (2016) investigated heat transfer within a microchannel in the presence of multi-walled cabon nano-tubes (MWCNT)/oil nanofluid applying a slip condition. Their microchannels were considered porous and the results showed that changing the Darcy number as well as the nanofluid volume fraction could lead to an improvement in the amount of heat exchange.

Karimipour et al. (2017), by using computational fluid dynamics techniques, examined a microchannel (with slip condition) in the presence of an MF and revealed that that the MF exerted a resistive force. By comparing two kinds of nanoparticle (Ag and Al₂O₃), they found that Ag nanoparticles performed 20% better than Al₂O₃ in heat transfer.

Forghani-Tehrani et al. (2017) studied a microchannel containing silver/water nanofluid under variable heat flux. In their work, they considered the microchannel in two scenarios. In the first scenario, a slip condition was applied, while in the second one, zero velocity was considered for the wall. They proved that, when the slip coefficient increased, the slip velocity along the wall also grew and an MF could amplify this velocity.

The effects of exerting an MF on heat exchange between a functionalized multi-walled carbon nano-tubes (FMWCNT)/water nanofluid and the wall of a microchannel with triangular ribs were investigated by Shiriny et al. (2019). They applied a slip condition and affirmed that the slip velocity along the wall did not always amplify heat transfer. They reported that, at high volume fraction, an MF boosted heat transfer by 20%.

Mohebbi et al. (2018) used the lattice Boltzmann approach to evaluate nanofluid flow heat transfer from a microchannel equipped with square ribs. They found that applying nanofluid had positive effects on heat transfer. They revealed that increasing nanofluid volume fraction from 0 to 0.05 caused an approximately 40% improvement in heat transfer.

Karimipour et al. (2020) simulated a microchannel equipped with injections by exerting an MF. They proved

| Authors                | Problem          | Effect of                                      | Result                                                                 |
|-----------------------|------------------|-----------------------------------------------|------------------------------------------------------------------------|
| Gravndyan et al. (2017) | Microchannel     | Rib aspect ratios (AR = 10, 15, 20, 25)       | The rib with AR = 10 and AR = 25 led to the highest and lowest PEC. The increment of the rib pitch at higher fluid velocity led to more pressure drop. Up to 200% increase in the pressure drop. |
| Wang et al. (2019)    | Microchannel     | Bidirectional rib                             | Up to 200% increase in the pressure drop.                               |
| Shen et al. (2017)    | Microchannel     | Y-shaped metal foams attached with rib         | 44% decrement in thermal resistances.                                   |
| Wang et al. (2015)    | Microchannel     | Micro-scale ribs                              | Heat transfer in microchannel with ribs was 1.11 ~ 1.55 times better than that in the smooth one. 46.4% boosting in the heat transfer. 234% drop in entropy generation. |
| Bahrami et al. (2020) | Microchannel     | Injectable rib                                 | Maximum heat transfer corrersponded to the triangular rib.             |
| Salman (2019)         | Tube             | Comparing triangular, rectangular and trapezoidal ribs | Maximum heat transfer observed compared smooth microchannel.             |
that the presence of nanofluid, as well as an MF, greatly improved forced convection heat transfer.

Cheng et al. (2021) proposed a wavy microchannel to make vortices in order to boost heat transfer. Their microchannel included several heat sources. They revealed that the rate of heat transfer improves as the inlet velocity increases as a result of higher and stronger vortices. However, in a smooth microchannel the heat transfer was lower due to the lack of vortices.

Samadifar and Toghraie (2018) applied a wide range of vortex generators to improve the efficiency of a heat exchanger. Their observations confirmed that vortex generation with a rectangle shape is much more effective than other forms of geometry.

Bahrami, Abbasian-Naghneh, et al. (2020) proposed a microchannel with a slip condition and ribs. In their work, nanofluid moved through the ribs, which resulted in vortices being generated. They found that the rib height affected heat transfer by creating vortices and irreversibility, and as the inlet velocity increased the vortices became stronger. Based on these results, the former and latter were amplified by 46.4% and 45.3%, respectively, becoming stronger. Based on these results, the former and latter were amplified by 46.4% and 45.3%, respectively, owing to the increase in rib height.

Gallegos and Sharma (2017) used a flexible plate to make a vortex in order to increase convective heat transfer. They proved that, when the plate is rigid, there is no vortex, whereas a flexible plate creates a considerable number of vortices, which increases the thermal efficiency.

In another study, Nguyen, Bahrami, Kalbasi, and Karimipour (2020) changed an inner smooth wall of a double-pipe heat exchanger to a sinusoidal-wall to increase the heat transfer area. They found that the vortex formation in the diverging section did not allow heat transfer to improve. At best, the heat exchanger effectiveness was improved by 1.56%. To reduce the strength of vortices, they exerted an MF and showed that the effectiveness could be improved by 19%. The sinusoidal walls generated a vortex and thus increased irreversibility by 18%, which could be reduced from 18% to 8% by applying an MF.

A literature review revealed that the efficacy of adding trapezoidal ribs on improving heat transfer, as well as entropy generation, had not been investigated. The presence of trapezoidal ribs seems to have two effects. On the one hand, they increase the heat transfer area and therefore have a positive effect on heat exchange between FMWNT/water nanofluid flow and microchannel walls. On the other hand, they might cause vortex formation that negatively affects heat exchange. In this study, an MF is applied to diminish the negative effects of the vortex. However, the MF intensifies the resistive forces and thus amplifies the pressure drop. In this way, by computing the entropy generation, the behavior of a simple microchannel can be compared with the modified microchannel (equipped with trapezoidal ribs and under the influence of an MF) from the second law perspective.

2. Problem description

Figure 1 shows a microchannel with trapezoidal ribs as internal extended surfaces. The microchannel height is 100 μm and the ratio of the length to the height is considered to be 40.

Considering \( a = 120 \mu m \) and \( b = 360 \mu m \), the geometric characteristics of the trapezoidal ribs can be specified. The nanofluid of the FMWNT/water enters the microchannel at \( T_c = 293 K \) and leaves the microchannel after exchanging heat with the trapezoidal ribs at \( T_h = 300 K \). The nanofluid flow is considered to be laminar with a Reynolds number (Re) of 50–400. As shown in Figure 1, a magnetic field MF (at Hartmann numbers Ha up to 20) is exerted. A slip boundary condition is applied on the walls and the results are compared with the non-slip condition. Table 2 presents the thermo-physical properties of FMWNT/water.

3. Mathematical formulation

The main purpose of this study is to compute the rate of heat exchange between nanofluid flow and microchannel walls, which depends on the temperature gradient. The temperature gradient in the solution domain is obtained by solving the energy equation (Afrand et al., 2016):

\[
\frac{\partial T}{\partial x} u + \frac{\partial T}{\partial y} v = \alpha_{nf} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  

In the above equation, \( u \) and \( v \) are the velocity in the \( x \) and \( y \) directions, respectively, \( T \) is temperature and \( \alpha_{nf} = k_{nf}/\rho_{nf}c_{nf} \) denotes the thermal diffusivity. Moreover, \( k_{nf}, \rho_{nf} \) and \( c_{nf} \) are the thermal conductivity, density, and heat capacity of the nanofluid, respectively. The property values are reported in Table 2. The velocity components are determined through solving the momentum/continuity equations in the horizontal \((x)\) and vertical \((y)\) directions:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

\[
\frac{\partial u}{\partial x} u + \frac{\partial u}{\partial y} v = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial x} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
- \frac{\mu_{nf} B_0^2}{\rho_{nf} \mu}
\]  

\[
\frac{\partial v}{\partial x} u + \frac{\partial v}{\partial y} v = -\frac{1}{\rho_{nf}} \frac{\partial p}{\partial y} + \frac{\mu_{nf}}{\rho_{nf}} \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\]
where $\mu_{nf}$ is the viscosity of the nanofluid and $p$ is the pressure. Note that the term $\left(\frac{\sigma_{nf} B_0^2}{\rho_{nf}} u\right)$ denotes the resistive force exerted by the magnetic field (the Lorentz force). Additionally, $B_0$ and $\sigma_{nf}$ are the magnetic intensity and nanofluid electrical conductivity, respectively (Afrand et al., 2016). It should be noted that nanofluid flow is considered under steady, incompressible, laminar and single phase. The boundary conditions are as follows:

$$u = u_c, \quad v = 0, \quad T = T_c \quad \text{inlet} \quad (x = 0, \ 0 \leq y \leq h)$$

$$\frac{\partial u}{\partial x} = 0, \quad v = 0, \quad \frac{\partial T}{\partial x} = 0 \quad \text{outlet} \quad (x = l, \ 0 \leq y \leq h)$$

velocity along the lower wall = $u_s$, \quad at lower wall

$$T = 0.1 T_h \quad 1 \leq x \leq 0.75 l \quad \text{for upper wall}$$

$u = u_s, \quad v = 0$ \quad at upper wall

$T = 0.1 T_h \quad 1 \leq x \leq 0.75 l \quad \text{for upper wall}$

where $u_c$ and $T_c$ are the nanofluid’s incoming velocity and temperature, respectively. Additionally, $u_s$ denotes the slip velocity and is obtained by the following equation:

$$u_s = \pm \beta \left(\frac{\partial u}{\partial n}\right) \quad \text{at upper and lower walls}$$

$$\beta = \frac{\partial u}{\partial n} = \tau$$

where $\beta$ is the slip coefficient.

In present work, the authors consider some separated linear parts along the bottom boundary. This means that the slip velocity is simulated along all these linear boundaries step by step; and finally all of them would be just plotted continuously in the one figure. The positive and negative coefficients correspond to the lower and upper walls of the microchannel, respectively. In this study, the results are presented in terms of dimensionless numbers according to the following equations:

$$\beta^* = \frac{\beta}{h}, \quad U_s = \pm \beta^* \frac{h^2}{u_c} \left(\frac{\partial u}{\partial n}\right) \quad \text{at upper and lower walls}$$

$$\theta = \frac{T - T_c}{T_h - T_c}, \quad \text{Re} = \frac{u_c h}{\mu_{nf}},$$

$$Ha = B_0 h \left(\frac{\sigma_{nf}}{\mu_{nf}}\right)^{0.5}$$

In the above dimensionless equations, $\beta^*$, $U_s$ and $\theta$ are slip coefficient, slip velocity and dimensionless temperature, respectively.
The local and average Nusselt numbers are obtained by the following equations (Nguyen, Bahrami, Kalbasi, & Bach, 2020):

\[ N_u_x = -\frac{k_n h}{k_f} \left( \frac{\partial \theta}{\partial y} \right)_{y=0,h} \]

\[ N_{u_{ave}} = \frac{1}{l} \int_0^l N_u_x \, dx \] (11)

The computations correspond to the study of the second law leading to entropy generation, which includes viscous and thermal terms and is obtained from the following equations (Nguyen, Bahrami, Kalbasi, & Bach, 2020):

\[ \dot{s}_{th} = \int\int \frac{k_n f}{T^2} \left[ \left( \frac{\partial T}{\partial x} \right)^2 + \left( \frac{\partial T}{\partial y} \right)^2 \right] \, dA \] (12)

\[ \dot{s}_f = \int\int \left( \frac{\mu_n f}{T} + 2 \left( \frac{\partial u}{\partial x} \right)^2 + \left( \frac{\partial v}{\partial y} \right)^2 \right) \, dA + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 + \frac{\sigma_n f (\nu/u_c)^2 B_0^2}{T} \, dA \] (13)

where \( \dot{s}_{th} \) and \( \dot{s}_f \) are the thermal and viscous terms, respectively.

4. Validation and grid study

All governing equations and boundary conditions are solved by the finite volume method. For the discretization of the balance equations, a second-order scheme is used. For coupling velocity and pressure, the SIMPLE algorithm is implemented. Additionally, double precision convergence criteria of \( 10^{-6} \) and \( 10^{-16} \) are applied for fluid and energy equations, respectively. For validity, the present numerical study is compared with that of Afrand et al. (2016). They considered a smooth microchannel at \( \beta^* = 0.05 \) and \( Re = 20 \), and exerted Lorentz force through applying magnetic field \( (Ha = 20) \). Numerical results for \( N_u_x \) as well as \( U_x \) along the microchannel for the study conducted by Afrand et al. (2016) are presented in Figure 2 and compared with the present simulations. There is good agreement between the results and it can be claimed that the method of solving the balance equations in this study can well simulate the hydrothermal behavior of microchannels.

The main purpose of this paper is to investigate heat transfer from the microchannel. A schematic of the mesh is illustrated in Figure 1. To examine the mesh independency, Nusselt number variations at 0.125 vol.%, \( Re = 100 \) and \( Ha = 20 \) are inspected. Figure 3 presents the values of \( N_{u_{ave}} \) with respect to the number of computational cells and it is found that a network with 11,600 nodes is appropriate.

5. Results and discussion

Heat transfer, pressure drop and entropy generation depend on the temperature and velocity gradients so that larger gradients will lead to higher values of these parameters. The results are presented within the ranges \( 50 \leq Re \leq 400, 0 \leq Ha \leq 20 \) and \( 0 \leq \beta^* \leq 0.1 \). In the following, the streamlines and isotherms in the microchannel are illustrated to identify regions with high velocity and temperature gradients.

5.1. Streamlines and isotherms

Figure 4 illustrates the streamlines and isotherms at \( Re = 400 \) at \( 0 \leq \beta^* \leq 0.1 \). Because the bottom wall of the microchannel is of trapezoidal shape, flow separation
is expected to occur in the divergent part of the trapezoid, which consequently creates a vortex. Figures 4 and 5 show that trapezoidal ribs create a vortex. In this study, the effect of two parameters, namely $\beta^*$ and $Ha$, on the strength of a vortex is investigated. The main purpose of installing trapezoidal ribs is to boost the heat transfer area, which in turn intensifies the needed heat transfer. But unfortunately, these types of rib produce vortices and therefore affect and reduce the ribs’ efficacy. One way to diminish the strength of a vortex is to apply slip velocities (as shown in Figure 4).
In the case where $\beta^*$ is zero (non-slip), several vortices are observed in Figure 4. As $\beta^*$ rises to 0.05, the zone under the effects of the vortex (the zone affected by the vortex) becomes smaller. With a further increase in $\beta^*$ (from 0.05 to 0.1), it is observed that the vortex has become so weak that the zone affected by the vortex is not easily discernible.

On the other hand, the effect of the slip coefficient on isotherms is also significant. It is observed that, with increasing $\beta^*$, the temperature distribution is limited to the regions next to the wall and hence the temperature gradient increases.

Applying a magnetic field is another technique for diminishing vortex intensity. Streamlines and isotherms are also sensitive to the presence of a magnetic field. Figure 5 shows streamlines as well as isotherms at different Hartmann numbers.

As shown in Figure 5, the trapezoidal wall separates the flow and creates vortices. It is clear that, as $\text{Ha}$ rises, the vortices become weaker or the zone affected by vortices is reduced. Hence, the isotherm density close to the wall amplifies, which means that heat transfer is enhanced by $\text{Ha}$ growth. Finally, it can be claimed that, with increasing $\text{Ha}$ and slip coefficient, the vortex strength is weakened and consequently heat transfer is improved.

5.2. Nusselt number

Figure 6 shows the variations in $\text{Nu}_x$ along the walls of the base and trapezoidal-wall microchannels, respectively, at $\text{Re} = 200$ and $\beta^* = 0.05$.

For the base microchannel, it can be seen that $\text{Nu}_x$ decreases at $\text{Ha} = 0$ as well as at $\text{Ha} = 20$. This is because the thickness of the boundary layer starts to grow from the beginning of the microchannel and increases along the microchannel. Consequently, temperature gradients decrease, and hence $\text{Nu}_x$ is reduced. A remarkable point is a higher $\text{Nu}_x$ at a higher Hartmann number ($\text{Ha} = 20$). It is because the MF exerts a resistive force (Lorentz force) on the nanofluid molecules and thus reduces the velocity. Reducing the velocity of molecules adjacent to the wall weakens the boundary layer thickness growth and therefore increases the temperature gradient as well as the velocity gradient. The same is true for the trapezoidal-wall microchannel. Moreover, the MF greatly weakens the vortices and this results in a smaller thermal boundary thickness; as a result, heat transfer is improved.

Figure 7 illustrates how $\text{Nu}_x$ changes for the base and trapezoidal-wall microchannels. Clearly, in the no-slip condition ($\beta^* = 0$), the nanofluid velocity at the microchannel wall is zero, which implies that it is stuck to the wall. By applying a slip condition, the molecules...
that are in direct contact with the wall do not stick and, by increasing the slip coefficient, the nanofluid molecules are replaced by other molecules at a lower temperature, thus increasing heat transfer.

The average Nusselt number variation (Figure 8) is illustrated at different Re, Ha and $\beta^*$. The sensitivity of the Nusselt number to Re, Ha and $\beta^*$ is such that, as they increase, an improvement in the Nusselt number is
observed. As Re or Ha rises, the boundary layer thickness is reduced, hence temperature gradients, as well as heat transfer, are improved. The effect of an MF on improving heat transfer is affected by Re. The main purpose of using an MF in this study is to reduce the vortex strength. With an increase in Re, the possibility of vortex formation is reinforced. Thereby the effect of an MF in destroying them becomes more pronounced. For $\beta^* = 0.1$, at $Re = 50$, if the magnetic intensity (Ha) is boosted from 0 to 20, the average Nusselt number is intensified by 10.4%, while at $Re = 400$, it is amplified by 18%. Note that at $\beta^* = 0$, these figures are 14.31% (for $Re = 50$) and 20.13% (for $Re = 400$), respectively.

It should be noted that, in the presence of slip velocity, the vortex formation probability is less and therefore the effect of an MF on diminishing the vortex intensity becomes less and less. In conclusion, it can be said that, at higher Re and in the non-slip condition, the possibility of vortex formation is more, and in this case, the use of an MF has more obvious positive effects.

Therefore, the following conclusions can be drawn.

1. With increasing Re number, the probability of vortex formation increases. Therefore, with increasing Re, the positive effect of using trapezoidal-walls diminishes. This is because, at high Re, the heat transfer capability is low in the vortex region. As shown in Figure 9, the value of $Nu/Nu_{\text{base}}$ is reduced by increasing Re.

2. With increasing $\beta^*$, the probability of vortex formation reduces and therefore it is expected that the effect of trapezoidal-walls on heat transfer is greater. As shown in Figure 9, the value of $Nu/Nu_{\text{base}}$ is raised by increasing $\beta^*$.

3. With increasing Ha, the vortex disappears and therefore the positive effect of increasing the heat transfer area (owing to using trapezoidal ribs) is more pronounced. As shown in Figure 9, the value of $Nu/Nu_{\text{base}}$ is amplified by increasing Ha.

### 5.3. Comparison of heat transfer in trapezoidal-wall and base microchannels

In this section, heat transfer in the trapezoidal-wall is compared with that the base case. As shown in Figure 9, for all Re and Ha (with slip/non-slip conditions) the value of $Nu_{\text{trapezoidal-wall}}/Nu_{\text{base}}$ is greater than one, which means that heat transfer from the microchannel with trapezoidal-walls is higher than the corresponding value in the base microchannel. The effect of creating trapezoidal ribs on heat transfer is influenced by the values of Re, Ha and $\beta^*$. Since the problem with trapezoidal-walls is creating vortices, any factor that reduces the vortex formation probability leads to better performance.

### 5.4. Entropy generation

In this section, entropy generation through a trapezoidal-wall microchannel is discussed. According to Equations (12) and (13), it can be observed that the values of gradients (of temperature/velocity) have a significant effect on entropy generation. On the other hand, the temperature gradient near the heating source and the velocity gradient near the wall have the highest values. Therefore, it is expected that the highest viscous entropy generation occurs near the wall and the highest thermal entropy generation is near the heating source. This trend is shown in Figures 10 and 11 for thermal entropy generation and Figures 12 and 13 for viscous entropy generation.
Figure 9. Trapezoidal rib microchannel heat transfer compared to base microchannel.

Figure 10 shows that, as $Ha$ rises, $S_{th}$ increases owing to temperature gradient growth (see Figure 5). Therefore, it should be noted that an MF enhances heat transfer irreversibly. In other words, the increase in heat transfer due to exerting an MF is accompanied by irreversibility.

The values of thermal entropy generation as well as viscous entropy generation are reported in Table 3, the
most important results of which are summarized as follows.

1. The amount of thermal entropy generation is much higher than viscous entropy generation, which means that the total entropy generation variation is similar to the thermal one.
2. It has already been mentioned that the effects of an MF on heat transfer is influenced by Re and $\beta^*$. In general, the use of an MF in conditions with a high
probability of vortex formation has more obvious positive effects. On the other hand, at high Re and low \( \beta^* \), a vortex is more likely to form. In this case, the use of an MF has more effect (see Section 5.2). Therefore, it can be claimed that, in this condition (high Re and low slip coefficient), the effect of an MF on thermal entropy generation is also greater. At \( \beta^* = 0 \) and \( Re = 400 \) (where vortex formation probability is high), an increment in Ha from 0 to 20 amplifies thermal entropy generation by 16.31%. It is expected that, with decreasing Re and increasing \( \beta^* \) (reducing the vortex formation probability), MF efficacy on thermal entropy generation will decrease. The computation reveals that this figure for \( \beta^* = 0.1 \) and \( Re = 50 \) is 6.42%.

3. In the previous section, it was observed that the maximum value of the mean Nusselt number occurs when the temperature gradient reaches its maximum value. All three parameters, Re, Ha and \( \beta^* \), affect the temperature gradient. Growth in Re, Ha and \( \beta^* \) intensify the temperature gradient and thus increase entropy production. The results show that the highest thermal entropy generation \( (0.246 \, \text{W/mK}) \) occurs at the highest values of Re, Ha and \( \beta^* \) (400, 20 and 0.1).

6. Conclusions

In this study, heat transfer in a simple microchannel (the ‘base’ case) and a microchannel with trapezoidal-walls is investigated. At the micro dimension, the slip condition affects the growth of the boundary layer. As such, slip coefficient efficacy on heat transfer is examined. In addition, the effects of MF application on heat transfer and entropy generation are evaluated. The most important results are as follows.

- The performance of a microchannel with trapezoidal walls is highly dependent on vortex formation. The
results show that, with vortex formation, heat transfer is reduced.

- With increasing Re number, the possibility of vortex formation in microchannels with trapezoidal-walls is intensified. Therefore, with increasing Re, although heat transfer rises, the effect of the presence of a trapezoidal-wall on heat transfer improvement is lost. At \( \beta^* = 0 \) and \( Ha = 0 \), if trapezoidal ribs are installed to a simple microchannel, heat transfer increases by 17\% at \( Re = 50 \) and 11.2\% at \( Re = 400 \). This is because, at higher Re, using ribs may lead to vortex formation, which decreases heat transfer.

- As the slip coefficient is increased, the effects of adding trapezoidal ribs on heat transfer are augmented. At \( \beta^* = 0 \), owing to adding ribs, heat transfer can be augmented up to 22.3\%. At \( \beta^* = 0.1 \), this figure is 23.16\%.

- The presence of an MF destroys vortices. Therefore, it is expected that, in the presence of an MF, the addition of ribs will have a greater effect on heat transfer. At \( \beta^* = 0 \) and \( Re = 50 \), adding ribs enhances heat transfer by 11.2\% at \( Ha = 0 \) and 20\% at \( Ha = 20 \), respectively.

- Computations show that the use of trapezoidal ribs in all conditions has a positive effect on heat transfer. The greatest effect of using ribs is a 23.16\% increase in heat transfer, which occurs at the lowest Re and highest Ha and slip coefficient.

- Heat transfer and entropy generation in a microchannel with trapezoidal ribs is intensified with increasing slip coefficient, Ha and Re.

**Nomenclature**

- \( a \) Trapezius short length (\( \mu m \))
- \( b \) Trapezius long length (\( \mu m \))
- \( B_0 \) Magnetic field strength (T)
- \( C_p \) Specific heat (J/kg \cdot K)
- \( E \) Electric field
- \( F \) Lorentz force
- \( h \) Microchannel height (\( \mu m \))
- \( H \) Dimensionless microchannel height
- \( Ha \) Hartmann number
- \( I \) Microchannel length (\( \mu m \))
- \( J \) Current density (A/m\(^2\))
- \( L \) Dimensionless microchannel
- \( MF \) Magnetic field
- \( Nu \) Nusselt number
- \( Pr \) Prandtl number
- \( \bar{p} \) Pressure (Pa)
- \( P \) Dimensionless pressure
- \( \bar{q} \) Heat flux (W/m\(^2\))
- \( Re \) Reynolds number

\[ \dot{S}_f \] Frictional entropy generation \( (W/m^3 \cdot K) \)
\[ \dot{S}_{th} \] Thermal entropy generation \( (W/m^3 \cdot K) \)
\( T \) Temperature (K)
\( u \) Horizontal velocity (m/s)
\( U \) Dimensionless horizontal velocity
\( u_s \) Slip velocity (m/s)
\( U_s \) Dimensionless slip velocity
\( v \) Vertical velocity (m/s)
\( V \) Dimensionless vertical velocity
\( x \) Horizontal axis
\( X \) Dimensionless horizontal axis
\( y \) Vertical axis
\( Y \) Dimensionless vertical axis

**Greek symbols**

- \( \beta \) Slip coefficient (\( \mu m \))
- \( \beta^* \) Dimensionless slip coefficient
- \( \phi \) Volume fraction of nanoparticles (\%)
- \( \mu \) Dynamic viscosity (N \cdot s/m\(^2\))
- \( \tau \) Shear stress (Pa)
- \( \theta \) Dimensionless temperature
- \( \rho \) Density (kg/m\(^3\))
- \( \nu \) Kinematic viscosity (m\(^2\)/s)
- \( \alpha \) Thermal diffusivity m\(^2\)/s
- \( \sigma \) Electrical conductivity (S/cm)

**Super and subscripts**

- ave average
- \( c \) cold
- \( f \) fluid
- \( h \) hot
- nf nanofluid
- \( s \) solid
- th thermal

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**ORCID**

Dariush Bahrami https://orcid.org/0000-0001-6676-4458
Kwok-Wing Chau https://orcid.org/0000-0001-6457-161X
Amir Mosavi https://orcid.org/0000-0003-4842-0613
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