1. The model

The neutrino masses are assumed to be derived through the seesaw mechanism, \( m_\nu(M_X) = m_D^T D_R^{-1} m_D \), where \( M_X \) is the GUT mass scale, \( m_D \) is a Dirac mass matrix which is related to the neutrino-Yukawa coupling matrix \( Y_\nu \) by \( m_D = Y_\nu v \sin \beta / \sqrt{2} \). We assume that the right-handed neutrino mass matrix is a diagonal form as \( D_R = \text{diag}(M_1, M_2, M_3) \).

We assume that the neutrino mass matrix is diagonalized by the Bi-maximal mixing in the diagonal mass basis of charged leptons, i.e., \( m_\nu(M_X) = O_B D_\nu O_B^T \), where \( O_B \) is the Bi-maximal mixing matrix and \( D_\nu = \text{diag}(m_1, m_2 e^{i\alpha_0}, m_3 e^{i\beta_0}) \) with \( \alpha_0 \) and \( \beta_0 \) being Majorana phases[1].

As for the neutrino mass spectrum, there are three cases, the hierarchical (H) masses, \( m_1 << m_2 << m_3 \), the inverse hierarchical (IH) masses, \( m_1 \simeq m_2 >> m_3 \), and the quasi-degenerate (QD) masses, \( m_1 \simeq m_2 \sim m_3 = m_\). The renormalization group does not give any effect for the H case and this case is ruled out. Therefore, only the IH and QD cases may become consistent with the low energy data.

The \( \tau \)-Yukawa coupling is known to contribute to rotate the solar angle from the Bi-maximal to the dark side[2]. Therefore, the other ingredient is required.

We consider the effect of the neutrino-Yukawa coupling, \( Y_\nu \), which enters in the renormalization group equation as \( Y_\nu^T Y_\nu \). LFV processes, \( l_i \rightarrow l_i + \gamma \), are induced by \( |(Y_\nu^T Y_\nu)_{ij}|^2 \), so that its off-diagonal terms must be suppressed from the null observation. Therefore, we assume that \( Y_\nu^T Y_\nu = \text{diag}(y_1^2, y_2^2, y_3^2) \). It was found that the solar angle rotates to the normal side, only when \( y_1^2 - y_2^2 >> |y_3^2 - y_2^2| \)[3][4]. This fact is shown later. With this assumption, \( m_D \) is written by

\[
m_D = V_R D_D P_{ex} , \quad P_{ex} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} ,
\]

where \( D_D = \text{diag}(m_{D_1}, m_{D_2}, m_{D_3}) \) with \( m_{D_3}^2 - m_{D_2}^2 >> |m_{D_1}^2 - m_{D_2}^2| \) and \( P_{ex} \) is a matrix to exchange the eigenvalues.

In our previous paper[5], we considered the QD neutrino mass case, both for the hierarchical Dirac mass, \( m_{D_3} >> m_{D_2} >> m_{D_1} \). In this note, we consider both the IH and QD neutrino mass cases.

By using the form of \( m_D \), we find

\[
M_R^{-1} = D_D^{-1} (P_{ex} O_B) D_\nu (P_{ex} O_B)^T D_D^{-1} ,
\]

where \( M_R \equiv V_R D_R V_R^T \). The diagonalization is easily made. We observe that \( V_R \) and \( M_i \) are determined by three neutrino masses, two Majorana phases, and three Dirac masses \( m_{D_1} \), so that 6 real positive masses and two Majorana phases. In addition to the masses, we use the data from the neutrino oscillations, which fix two neutrino masses. For simplicity, we assume the ratios of \( m_{D_i} \).

Therefore, one mass scale \( m_{D_3} \) and two Majorana phases, and one mass scale for the QD case are free, so that all data can be computed with these parameters.

2. The input data

We take \( \tan^2 \theta_{\alpha} \simeq 1 \),
\[ \tan^2 \theta_{\odot} \simeq 0.40, \quad \Delta m^2_{\odot} = |m_3^2 - m_2^2| \sim 2.5 \times 10^{-3} (\text{eV}^2), \quad \Delta m^2_{\odot} = m_3^2 - m_1^2 \sim 8.3 \times 10^{-5} (\text{eV}^2). \]

For the IH case, \( m_3 \) is not determined, and for the QD case, the overall mass is unknown. For the sake of argument, we take

\[ m_1 \simeq m_2 >> m_3 = 5.0 \times 10^{-3} \text{eV} \quad \text{for IH}, \]
\[ m_1 \simeq m_2 \sim m_3 = 6.5 \times 10^{-2} \text{eV} \quad \text{for QD}. \quad (3) \]

For Dirac masses, we assume \( m_{D1}/m_{D2} = m_{D2}/m_{D3} = 1/5 \) for simplicity.

Now the remaining unfixed parameters are \( m_{D3} \) and two Majorana phases, aside from the parameters in the renormalization group, which we use \( \tan \beta = 20, M_X = 2 \times 10^{16} \text{GeV} \).

For the solar neutrino angle, we find

\[ \tan^2 \theta_{\odot} = \frac{1 - 2(\epsilon_e - \epsilon_{\mu}) \cos^2(\alpha_0/2)m_{D2}/\Delta m^2_{\odot}}{1 + 2(\epsilon_e - \epsilon_{\mu}) \cos^2(\alpha_0/2)m_{D2}/\Delta m^2_{\odot}}, \quad (4) \]

where \( \epsilon_e = (1/16\pi^2)(y_1^2 - y_2^2) \ln(M_X/M_R), \quad \epsilon_{\mu} = (1/16\pi^2)((y_2^2 - y_3^2) \ln(M_X/M_R) + y_1^2 \ln(M_X/M_Z)). \)

From this, we find \( 2\epsilon_e - \epsilon_{\mu} > 0 \), which implies that \( y_1^2 >> y_2^2 + y_3^2 + y_4^2 \), for this we introduced the matrix \( P_{\nu e} \). In addition, we can derive \( |\cos(\alpha_0/2)| \geq \cos 2\theta_{\odot} \sim 0.43 \).

3. Predictions

\( M_1 \) and \( m_{D3} \): The requirement that the low energy solar neutrino angle is reproduced determines the heavy neutrino mass \( M_3 \) and the Dirac mass \( m_{D3} \) as a function of \( \alpha_0 \). The result is almost independent of \( \alpha_0 \) and \( M_3 \sim 1.4 \times 10^{14} \text{GeV} \) for the IH case, and \( \sim 6 \times 10^{13} \text{GeV} \) for the QD case.

The Dirac mass \( m_{D3} \) is determined by \( m_{D3} = \sqrt{m_2 M_3/|\cos(\alpha_0/2)|} \), which is of order 100GeV. The other Dirac masses are given by our assumption for Dirac masses. Also \( M_1 \) and \( M_2 \) are predicted once \( \alpha_0 \) is given.

\[ |V_{13}| \] and \( \delta \) : Here the Bi-maximal mixing, \( V_{13} = 0 \) so that no \( \delta \). In our case, these are induced by the renormalization group. \( |V_{13}| \) is very small for the IH case and of order 0.01 for the QD case.

\( \delta + \beta_0 \) is roughly between \( -\pi/2 \) and \( -2\pi/3 \).

\( < m_{\nu_c} > \): The effective neutrino mass for the neutrinoless double beta decay is about \( < m_{\nu_c} > \approx m_{23}/|\cos(\alpha_0/2)| \geq 0.025 \text{eV}. \)

Lepton Flavor Violation: The LFV processes take place through the slepton mixing, which is proportional to \( (Y_1^T \mathcal{L} Y_\nu)_{ij} \), where \( \mathcal{L} = \text{diag}(\ln(M_X/M_1), \ln(M_X/M_2), \ln(M_X/M_3)). \)

Since \( m_{\nu}^2 \) is diagonal, the contribution occurs only through \( (Y_1^T \mathcal{L} - < \mathcal{L} > Y_\nu)_{ij} \).

The branching ratios are given by the standard form and the result is \( \text{Br}(\mu \rightarrow e\gamma) >> \text{Br}(\tau \rightarrow e\gamma) >> \text{Br}(\tau \rightarrow \mu\gamma) \). We show the branching ratio for the IH case in Fig.1. The \( \beta_0 \) dependence of branching ratios comes in through the ratios of heavy neutrino masses. The branching ratios have the sizable dependence on Majorana phases, \( \alpha_0 \) and \( \beta_0 \). A similar behaviors are obtained for the QD case.

Leptogenesis: We have

\[ \epsilon_1 = \frac{3 m_1 M_1}{4 \pi v^2} \frac{\cos(\alpha_0/2)}{\sqrt{2(1 + \cos \alpha_0)}} \left[ \sin \left( \frac{\alpha_0}{2} - \beta_0 \right) \right], \quad (5) \]

for the IH case. For the QD case, we find

\[ \epsilon_1 = \frac{3 \Delta m_{12}^2 M_1}{16 \pi v^2 m} \frac{\cos(\alpha_0/2)}{R} \sin \left( \frac{\alpha_0}{2} - \beta_0 \right), \quad (6) \]
where $\Delta m^2_{13} = m^2_1 - m^2_3$ and

$$R = \sqrt{1 + \cos^2 \frac{\alpha_0}{2} + 2 \cos \frac{\alpha_0}{2} \cos \left( \frac{\alpha_0}{2} - \beta_0 \right)}.$$  \hspace{1cm} (7)

For both cases, the asymmetric parameter depends on Majorana phases as $\cos(\alpha_0/2) \cos(\alpha_0/2 - \beta_0)$ and small mass factor $m_3$ for the IH case, and $\Delta m^2_{13} = m^2_1 - m^2_3$ for the QD case. The obtained values of $\eta_B$ is given in Fig.2 for IH and Fig.3 for QD. It is possible to reproduce the experimental result, $\eta_B \simeq 6 \times 10^{-10}$.

**4. Summary** We discuss the Bi-maximal mixing which is realized at the GUT scale. Our question is whether the Bi-maximal mixing can cope with low energy data and the leptogenesis. We showed that this is possible in the case where (11) element of $Y^\dagger \nu Y_\nu$ is much larger than other elements. From this and the LFV must be suppressed, we assume that $Y^\dagger \nu Y_\nu$ is diagonal. Then, CP phases including the Dirac phase are induced by two Majorana phases. This model enables to relate the CP phase which appears in the neutrino oscillation to the one in the neutrinoless double beta decay and also the CP phase in the leptogenesis. We showed the model is consistent with all experimental data.

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