Generating merger trees for dark matter haloes: a comparison of methods

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ABSTRACT
Halo merger trees describe the hierarchical assembly of dark matter haloes, and are the backbone for modelling galaxy formation and evolution. Merger trees constructed using Monte Carlo algorithms based on the extended Press–Schechter (EPS) formalism are complementary to using N-body simulations and have the advantage that they are not trammelled by limited numerical resolution and uncertainties in identifying and linking (sub)haloes. This paper compares multiple EPS-based merger tree algorithms to simulation results using four diagnostics: progenitor mass function, mass assembly history (MAH), merger rate per descendant halo and the unevolved subhalo mass function. Spherical collapse-based methods typically overpredict major-merger rates, whereas ellipsoidal collapse dramatically overpredicts the minor-merger rate for massive haloes. The only algorithm in our comparison that yields results in good agreement with simulations is that by Parkinson et al. (P08). We emphasize, though, that the simulation results used as benchmarks in testing the merger trees are hampered by significant uncertainties themselves: MAHs and merger rates from different studies easily disagree by 50 per cent, even when based on the same simulation. Given this status quo, the P08 merger trees can be considered as accurate as those extracted from simulations.

Key words: methods: analytical – methods: statistical – galaxies: haloes – dark matter

1 INTRODUCTION
Halo merger trees describe the hierarchical mass assembly of dark matter haloes. They are the backbone for modelling the formation and evolution of galaxies (see Mo, van den Bosch & White 2010), and they are the core ingredient in semi-analytical models that aim to describe the substructure of dark matter haloes (e.g. Oguri & Lee 2004; Taylor & Babul 2005a,b; van den Bosch, Tormen & Giocoli 2005; Zentner et al. 2005; Gan et al. 2010). Two different methods are used to construct halo merger trees: Monte Carlo methods based on the extended Press–Schechter (EPS; Bond et al. 1991) formalism and numerical N-body simulations. Although the rapid advances in computer technology have shifted focus from EPS-based merger trees to extracting merger trees from numerical simulations (e.g. Roukema et al. 1997; Kauffmann et al. 1999a,b; Benson et al. 2000; Helly et al. 2003; Kang et al. 2005; Springel 2005; Han et al. 2012; Behroozi et al. 2013), EPS-based methods remain an important and powerful alternative for a number of reasons.

First of all, EPS methods are typically much faster and more flexible. Although a cosmological simulation typically yields merger trees (after analysis) of thousands of haloes at once, whereas an EPS-based method constructs halo merger trees for each halo at a time, the limited force resolution and mass resolution of N-body simulations introduce serious systematics. In particular, the merger trees of more massive haloes are better resolved, i.e. probe down to progenitor masses that are a smaller fraction of the mass of the final host halo. This complicates a proper analysis of how the (statistical) properties of merger trees scale with halo mass.

Furthermore, to explore the dependence on cosmological parameters typically requires one to run large sets of simulations. The EPS-based method, on the other hand, can typically construct halo merger trees at high mass resolution (i.e. down to progenitors with a mass as small as 10^5 times that of the final host halo mass) in a matter of seconds, and can therefore construct merger trees for large sets of haloes of different masses and/or cosmologies in a fraction of the time required to run and analyse a full blown cosmological simulation.

In addition, it is important to realize that although simulations more reliably capture the physics of gravitational collapse and halo growth in a hierarchical universe than EPS theory, extracting reliable merger trees from simulations is subject to a large number of tricky, systematic issues. In particular, depending on the algorithms used to identify haloes and subhaloes, and to link haloes between different snapshots, one can obtain merger trees that differ substantially (e.g. Harker et al. 2006; Fakhouri & Ma 2008, 2009; Genel et al. 2008, 2009, 2010; Fakhouri, Ma & Boylan-Kolchin 2010). Indeed, a recent comparison of 10 different merger tree construction algorithms applied to the same simulation output has revealed a disconcerting amount of disparity (Srisawat et al. 2013).

In this paper, we compare a number of different methods, available in the literature, that are used to construct EPS-based merger
trees. These cover a variety of strategies: some algorithms make
the implicit assumption that each branching point in the tree
represents a binary merger, while others allow for multiple mergers per
branching point; some algorithms assume that the sum of the progenitor
mass exactly equals the mass of the descendant, while others admit mild violation of mass conservation; some algorithms are
based on the (standard) spherical collapse (SC) model, while others
adopt the more realistic picture of ellipsoidal collapse (EC); and
finally, some algorithms are self-consistent, while others use a progenitor mass function that is inconsistent with EPS. By comparing
with numerical simulations, we test how accurately these methods
can reproduce various statistics of the hierarchical assembly of dark
matter haloes, such as the unevolved subhalo mass function (USMF, i.e. the mass function of subhaloes at accretion), merger rates and
mass assembly histories (MAHs).

This paper is organized as follows. Section 2 discusses the
anatomy of merger trees and the challenges associated with their
construction using either numerical simulations or semi-analytical
methods based on the excursion set formalism. Section 3 describes
the various merger tree algorithms that are tested and compared
in terms of their progenitor mass functions (Section 4.1), MAHs
(Section 4.2), merger rates per descendant halo (Section 4.3) and
their USMFs (Section 4.4). Results are summarized in Section 5.

2 HALO MERGER TREES

2.1 Anatomy of a merger tree

Before describing how to construct halo merger trees using the
EPS formalism, we first define some terminology used throughout
this paper. Fig. 1 shows a schematic representation of a merger
tree illustrating its anatomy. We refer to the halo at the base of
the tree (i.e. the large purple halo at \( z = z_0 \)) as the host halo.
For each individual branching point along the tree (one example
is highlighted in Fig. 1), the end-product of the merger event is
called the descendant halo, while the haloes that merge are called
the progenitors. The main progenitor of a descendant halo is the
progenitor that contributes the most mass. For example, for the
branching point highlighted in Fig. 1, the purple halo at \( z = z_2 \)
is the main progenitor of its descendant at \( z = z_1 \). The main branch of
the merger tree is defined as the branch tracing the main progenitor
of the main progenitor of the main progenitor, etc. (i.e. the branch
connecting the purple haloes). Throughout we shall occasionally
refer to the main progenitor haloes of a given host halo as its zeroth-
order progenitors, while the mass history, \( M(z) \), along this branch
is called the MAH. Note that the main progenitor halo at redshift
\( z \) is not necessarily the most massive progenitor at that redshift. In
numerical simulations, which typically use only a limited number of
snapshots to construct an MAH, this can result in systematic errors
(i.e. the MAH does not trace the actual main branch of the merger
tree). In EPS merger trees, on the other hand, the time resolution is
typically high enough that this problem is avoided.

Haloes that accrete directly on to the main branch are called first-
order progenitors, or, after accretion, first-order subhaloes. Simi-
larly, haloes that accrete directly on to first-order progenitors are
called second-order progenitors, and they end up at \( z = z_0 \) as
second-order subhaloes (or sub-subhaloes) of the host halo. The
same logic is used to define higher order progenitors and subhaloes,
as illustrated in Fig. 1. Note that with our definition, the mass of an
nth-order subhalo includes the masses of its own subhaloes (i.e.
those of order larger than n). Finally, the small shaded boxes present at
each branching reflect the mass accreted by the descendant halo
in the form of smooth accretion (i.e. not part of any halo) or in the
form of progenitor haloes with masses below the mass resolution
of the merger tree. Throughout this paper, we shall refer to this
component as smooth accretion.

2.2 Merger trees based on EPS formalism

The EPS formalism, also known as the excursion set formalism,
developed by Bond et al. (1991) and Bower (1991), uses the statistics
of Gaussian random fields to compute the conditional probability
\( P(M_1, z_1|M_0, z_0) \), that a halo of mass \( M_0 \) at redshift \( z_0 \) has a progenitor with mass in the range \( [M_1, M_1 + dM_1] \) at redshift
\( z_1 > z_0 \). This conditional probability function is the basis from
which one can construct an (EPS-based) halo merger tree.

Following Lacey & Cole (1993), we use the variables \( S \equiv \sigma^2(M) \)
and \( \omega \equiv \delta_s(z) \) to label mass and redshift, respectively. Here \( \sigma^2(M) \)
is the variance of the density field, linearly extrapolated to \( z = 0 \)
and smoothed with a sharp k-space filter of mass \( M \), and \( \delta_s(z) \) is the
critical overdensity for collapse at redshift \( z \). In the case of SC,
\( \delta_s(z) = 1.686/D(z) \) with \( D(z) \) the linear growth rate normalized to
unity at \( z = 0 \). According to the EPS formalism, the conditional
probability function \( P(M_1, z_1|M_0, z_0) \) is given by

\[
P(M_1, z_1|M_0, z_0) = f_{\delta S}(S_1, \omega_1|S_0, \omega_0) \frac{dS_1}{dM_1},
\]  

(1)
where \( S_i = S(M_i), \omega_i = \omega(z_i) \) and

\[
f_{SC}(S_1, \omega_1|S_0, \omega_0) = \frac{1}{\sqrt{2\pi}} \frac{\Delta \omega}{\Delta S^{3/2}} \exp \left( -\frac{\Delta \omega^2}{2 \Delta S} \right),
\]

with \( \Delta S = S_1 - S_0 \) and \( \Delta \omega = \omega_1 - \omega_0 \). The progenitor mass function (hereafter PMF) at \( z = z_1 \) for a host halo of mass \( M_0 \) at \( z_0 \) is simply related to the mass-weighted conditional probability function by

\[
n_{\text{EPS}}(M_1, z_1|M_0, z_0) \, dM_1 = \frac{M_1}{M_0} \, p(M_1, z_1|M_0, z_0) \, dM_1.
\]

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\]

Note that the PMF \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \) is also sometimes denoted as \( dM_1/dM(M_1, z_1|M_0, z_0) \), where \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \) is the ensemble average number \( dM(M_1, z_1|M_0, z_0) \) that a halo of mass \( M_0 \) at redshift \( z_0 \) has a progenitor with mass in \([M_1, M_1 + dM_1]\) at redshift \( z_1 > z_0 \). In the case of EC, the same formalism can be used, but with \( f_{SC} \) replaced by

\[
f_{EC}(S_1, \omega_1|S_0, \omega_0) = \frac{A_0}{\sqrt{2\pi}} \frac{\Delta \omega}{\Delta S^{3/2}} \exp \left( -\frac{\Delta \omega^2}{2 \Delta S} \right) \times \left\{ \exp \left( -A_1 \frac{\Delta \omega^2}{\Delta S} + A_2 \Delta S^{3/2} \right) \right\},
\]

where \( A_0 = 0.8661 \left( 1 - 0.133v_0^{0.15} \right) \), \( A_1 = 0.308v_0^{-0.15} \), \( A_2 = 0.0373v_0^{-0.15} \), \( A_3 = A_0^2 + 2A_0A_1\sqrt{\Delta S/\Delta \omega}, \) \( v_0 = \omega_0^2/S_0 \) and \( \Delta S/S_0 \) (see Zhang, Ma & Fakhouri 2008; Zhang, Fakhouri & Ma 2008b for details).

In order to construct an EPS merger tree, one starts from some target host halo mass, \( M_0 \), at some redshift \( z_0 \), and uses the PMF to draw a set of progenitor masses \( M_1, M_2, \ldots, M_n \) at some earlier time \( z_1 = z_0 + \Delta z \), where \( \sum_1^n M_i = M_0 \) in order to assure mass conservation. The time step \( \Delta z \) used sets the ‘temporal resolution’ of the merger tree. This procedure is then repeated for each progenitor with mass \( M_i > M_0 \), thus advancing ‘upwards’ along the tree. The minimum mass \( M_{\text{min}} \) sets the ‘mass resolution’ of the merger tree and is typically expressed as a fraction of the final host mass \( M_0 \).

There are two problems with this approach. First of all, although EPS provides the PMF, it does not explicitly specify how to split descendants into progenitors. In fact, this can be done using many different ways, resulting in merger trees with different statistics. Secondly, the EPS formalism is at best a crude approximation, and the PMF that it predicts may not be sufficiently accurate to yield reliable merger trees. We now discuss each of these two issues in turn.

2.2.1 The self-consistency constraint

The requirement for mass conservation implies that the probability for the mass of the \( n \)th progenitor of some descendant needs to be conditional on the masses of the \( n - 1 \) progenitor haloes already drawn. Unfortunately, these conditional probability functions are not derivable from the EPS formalism, which results in ambiguity as to how to partition the descendant mass into progenitor masses. This has resulted in the construction of a variety of different Monte Carlo algorithms to construct halo merger trees within the same EPS framework, i.e. relying on the same \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \).

In order to be consistent with EPS, it is crucial that the Monte Carlo algorithm used to construct the merger trees exactly reproduces the EPS conditional mass function \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \) for a single time step \( \Delta z = z_1 - z_0 \). As shown by Zhang et al. (2008a), this is a sufficient condition for the algorithm to also reproduce the \( n_{\text{EPS}}(M, z|M_0, z_0) \) for any \( z \), regardless of the number, or width, of intervening time steps. We shall refer to this as the self-consistency requirement for the Monte Carlo algorithm.

Several Monte Carlo merger tree algorithms rely on the assumption that in the limit of sufficiently small time steps, all mergers are binary in nature (e.g. Cole 1991; Lacey & Cole 1993; Cole et al. 2000; Moreno, Giocoli & Sheth 2008). Under this assumption, it is trivial to assign the progenitor masses using the constraint of mass conservation; after drawing the first progenitor mass, \( M_1 \), from \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \), the mass of the second progenitor is simply \( M_2 = M_0 - M_1 \). An implicit assumption of this binary method is that the PMF is symmetric, such that \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) = n_{\text{EPS}}(M_0 - M_1, z_1|M_0, z_0) \). However, as shown by Sheth & Pitman (1997), this is only correct for Poisson initial conditions \( P(k) = k^n \) with \( n = 0 \). For more relevant cases, such as cold dark matter (CDM), \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \) is (slightly) asymmetric, even in the limit \( \Delta z \to 0 \). Consequently, all binary methods violate the self-consistency constraint. Cole et al. (2000) tried to remedy this by explicitly accounting for accretion of objects below the mass resolution, \( M_{\text{res}} \), of the merger tree. However, as shown in Zhang et al. (2008a), this method still violates the self-consistency constraint, albeit at a much reduced level (see Section 3.1 below).

In order to overcome these problems, several algorithms have been developed that do not make the implicit assumption of binarity (e.g. Kauffmann & White 1993; Sheth & Lemson 1999; Somerville & Kolatt 1999; Neistein & Dekel 2008a; Zhang et al. 2008a). Among these, the methods of Kauffmann & White (1993, hereafter KW93), Zhang et al. (2008a) and Neistein & Dekel (2008a) fulfil the self-consistency requirement. The method of Sheth & Lemson (1999) is only exact for Poisson initial conditions, while the method of Somerville & Kolatt (1999) violates self-consistency, because it discards progenitors drawn from the conditional mass function that overflow the mass budget (see Section 3.1 below).

2.2.2 Beyond spherical collapse

In addition to problems related to the self-consistency constraint, EPS-based merger trees are also hampered by the fact that EPS is an approximate theory at best. This implies that the PMF, \( n_{\text{EPS}}(M_1, z_1|M_0, z_0) \), obtained using EPS theory, may not be sufficiently accurate. Indeed, a comparison with numerical simulations has shown that the EPS conditional mass function based on the assumptions of SC overpredicts (underpredicts) the number of low-mass (massive) progenitors (Cole et al. 2008). Related to this is the well-known problem that EPS predicts halo assembly to occur later than what is found in numerical simulations (e.g. van den Bosch 2002; Lin, Jing & Lin 2003; Neistein, van den Bosch & Dekel 2006).

These problems seem to be related to the assumption that halo collapse is a spherical process. Several studies have shown that assuming EC, rather than SC, conditions results in overall halo mass functions and halo formation times in better agreement with numerical simulations (e.g. Del Popolo & Gambera 1998; Sheth, Mo & Tormen 2001; Hiotelis & Del Popolo 2006; Giocoli et al. 2007). In the excursion set approach, the problem of estimating halo abundances reduces to that of computing the number of time steps a Brownian-motion random walk must take before it crosses an overdensity barrier. In the SC picture, this barrier has a constant height (i.e. the critical overdensity for collapse is independent of
3 MERGER TREE ALGORITHMS

The main goal of this paper is to assess the performances, compared to numerical simulations, of a number of merger tree algorithms regarding a variety of statistics. In this section, we briefly describe the various merger tree algorithms that enter our comparison. These are the ‘N-branch method with accretion’ method developed by Somerville & Kolatt (1999, hereafter SK99), the binary method of Cole et al. (2000, hereafter C00) and its modification by Parkinson et al. (2008, hereafter P08), and several of the algorithms suggested by Zhang et al. (2008a, hereafter Z08). What follows is a description of how these different algorithms select progenitor haloes for a single descendant halo, which constitutes the building block of a merger tree.

3.1 Somerville & Kolatt (1999)

SK99 developed a merger tree algorithm that does not make the assumption of binary mergers. Their ‘N-branch method with accretion’ allows for an arbitrary number of progenitors per time step, and has been widely used, especially in analytical models for the population of dark matter subhaloes (see Jiang & van den Bosch, in preparation). The algorithm is based on drawing progenitor masses from the (mass-weighted) conditional mass function. With each new halo drawn it is checked whether the sum of the progenitor masses exceeds the mass of the descendant. If this is the case, the progenitor is rejected and a new progenitor mass is drawn. Any progenitor with mass \( M < M_{\text{res}} \) is added to the smooth accretion component \( M_{\text{smooth}} \) (i.e. the formation history of these small mass progenitors is not followed further back in time). This procedure is repeated until the total mass left \( (M - M_{\text{smooth}} - \sum M_i) \) is less than \( M_{\text{res}} \). This remaining mass is assigned to \( M_{\text{smooth}} \).

3.2 Cole et al. (2000)

The method of C00 is an improvement over the ‘block model’ developed by Cole et al. (1994) and can be described as a binary method with (fixed) accretion. Similar to SK99, it treats the mass in progenitors below the mass resolution, \( M_{\text{res}} \), as accreted mass. However, unlike SK99, the smooth accretion mass for a given time step is deterministic, calculated by integrating the mass-weighted conditional mass function, i.e.

\[
M_{\text{smooth}}(z_1 \to z_0) = \int_0^{M_{\text{max}}} n_{\text{EPS}}(M_1, z_1 | M_0, z_0) M_1 \, dM_1,
\]

where \( M_i \) is the descendant mass. For each branching point, it is first decided how many progenitors the descendant has by calculating the mean number of progenitor haloes in the mass range \( [M_{\text{res}}, M_0/2] \), given by

\[
P = \int_{M_{\text{res}}}^{M_0/2} n_{\text{EPS}}(M_1, z_1 | M_0, z_0) \, dM_1.
\]

The merger tree time steps, \( \Delta t \), are chosen such that \( P \ll 1 \), to ensure that multiple fragmentation is unlikely. A random number, \( R \), generated in the interval \([0, 1]\) is used to determine whether the descendant has one (\( R > P \)) or two progenitors (\( R \leq P \)). In the case of a single progenitor, its mass is

\[
M_1 = M_0 - M_{\text{res}}.
\]

In the case of two progenitors, one progenitor mass, \( M_1 \), is drawn from the PMF \( n_{\text{EPS}}(M_1, z_1 | M_0, z_0) \) in the range \( [M_{\text{res}}, M_0/2] \), and the second progenitor is assigned a mass

\[
M_2 = M_0 - M_1 - M_{\text{res}}.
\]

3.3 Parkinson et al. (2008)

P08 modified the C00 algorithm by using a PMF tuned to match results from the Millennium Simulation (MS; Springel et al. 2005), rather than the EPS PMF. Specifically, they used the PMF

\[
n(M_1, z_1 | M_0, z_0) = n_{\text{EPS}}(M_1, z_1 | M_0, z_0) G(S_1, S_0, \omega_1^0/\omega_0^0),
\]

where \( G(S_1, S_0, \omega_1^0/\omega_0^0) \) is a perturbing function that is tuned to match simulation results. P08 adopted the functional form

\[
G(S_1, S_0, \omega_1^0/\omega_0^0) = G_0 \left( \frac{S_1}{S_0} \right)^{\gamma_1} \left( \frac{\omega_1^0}{\omega_0^0} \right)^{\gamma_2},
\]

which has the advantage that the two terms \( G_0 \) and \( (\omega_1^0/\omega_0^0)^{\gamma_2} \) only enter the integral equations for \( M_{\text{smooth}} \) and \( P \) [equations (5) and (6), respectively] as multiplicative constants. Using merger trees constructed from the MS by Cole et al. (2008), based on friends-of-friends groups, P08 inferred the following best-fitting values for the free parameters of their perturbing function: \( G_0 = 0.57, \gamma_1 = 0.19 \) and \( \gamma_2 = -0.005 \). We adopt these parameters throughout.

3.4 Zhang et al. (2008a)

Z08 developed three new algorithms (called A, B and C) that, all by construction, satisfy the self-consistency constraint discussed in Section 2.2.1. And each algorithm can use either the PMF for SC, based on equation (2), or that for EC based on equation (4). In what
follows, we only focus on methods A and B, and refer to them as Z08X[YY], where X is either A or B and YY is either SC (for spherical collapse) or EC (for ellipsoidal collapse).

Overall, the Z08 algorithms are considerably more involved than any of the algorithms discussed above. Here we only sketch the rough idea behind them, and we refer the interested reader to Z08 for details. The Z08 algorithms are similar to that of SK99, in that they allow for more than two progenitors per time step. The most massive progenitor for a branching point is called the primary progenitor, while all other progenitors are called secondary. The main difference between methods A and B is the mass range over which the primary progenitor is drawn from the PMF: in the case of method A, this mass range is \( [M_0/2, M_0] \). In the case of method B, this mass range is modified to \([\alpha M_0, M_0] \), where \( \alpha \) is defined by

\[
\int_{\alpha M_0}^{M_0} n_{\text{EPS}}(M_1, z|M_0, z_0) \, dM_1 = 1. 
\]  

(9)

A somewhat unsatisfactory characteristic of the Z08 algorithms is that, if there are multiple secondary progenitors in a given time step, they all have identical masses. In addition, in those cases, neither method A nor method B perfectly conserves mass. However, as shown in Z08, despite these shortcomings both methods A and B accurately satisfy the EPS self-consistency constraints, both for the SC and EC cases.

4 PUTTING THE MERGER TREES TO THE TEST

In our comparison of the various merger tree algorithms described above [namely SK99, C00, Z08A(SC), Z08A(EC), Z08B(SC) and Z08B(EC)], we use the following diagnostics: (i) the PMF for a tiny time step, (ii) the MAH of the main progenitor, (iii) the merger rate per descendant halo and (iv) the USMF. These diagnostics are chosen because they have the potential to reveal subtle differences between the various merger tree algorithms, and because they have been studied using high-resolution cosmological N-body simulations, which provide a benchmark for the comparison. In what follows, we discuss each of these diagnostics in turn.

Throughout what follows, we adopt a flat ΛCDM cosmology with \( \Omega_m,0 = 0.25, \Omega_{\Lambda,0} = 0.75, h = H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1}) = 0.73 \) and with initial density fluctuations described by a Harrison–Zeldovich power spectrum with normalization \( \sigma_8 = 0.9 \). We use the transfer function of Eisenstein & Hu (1998) with a baryonic mass density \( \Omega_b,0 = 0.045 \). This is exactly the same cosmology as that used for the MS (Springel et al. 2005), thus allowing for a direct comparison. We will refer to this cosmology as the ‘Millennium cosmology’. Note that the parameters of this cosmology are close to, but not perfectly consistent with, the most recent cosmic microwave background constraints. Since this paper mainly focuses on a comparison between model and simulations, the exact cosmology is not particularly relevant, though, as long as the comparison is done for the same cosmology. Since the MS provides ideal benchmarks for this comparison project, the Millennium cosmology is the logical choice.

Unless specifically stated otherwise, we always adopt a time step of \( \Delta z = 0.002 \), independent of which merger tree algorithm we use. This easily meets all the time-step criteria described in the original papers, and we have verified that none of our results are sensitive to this choice for \( \Delta z \), as long as it does not get significantly larger than \( \sim 0.01 \). Finally, all merger trees are constructed using a mass resolution of \( M_{\text{reso}} = 10^{-4} M_0 \), unless specifically stated otherwise.

4.1 Progenitor mass functions

As a first test of the various merger tree algorithms, we check how well they perform in terms of the self-consistency test described in Section 2.2.1. The symbols in Fig. 2 indicate the PMFs obtained using \( 10^7 \) realizations of a single time step with \( \Delta z = 0.002, M_0 = 10^{13} h^{-1} M_\odot \) and \( z_0 = 0 \) for the various merger tree algorithms discussed in this paper. The solid line, for comparison, shows the EPS prediction in the case of SC (equations 2 and 3). Clearly, SK99 does not meet the self-consistency criterion, in that the PMF that results from the algorithm does not match the EPS prediction. This is a consequence of the fact that the SK99 algorithm rejects any progenitor drawn from the EPS PMF that overflows the mass budget.

The C00 algorithm clearly improves upon this, but it still fails to meet the self-consistency criterion (at the few per cent level) for \( M \gtrsim M_0/2 \). This is a consequence of the binary assumption inherent to this algorithm. This is evident from the dashed curve in the middle panels of Fig. 2, which shows the symmetrized PMF

\[
n_{\text{EPS}}^\text{sym}(M_1, z|\{M_0, z_0\}) = \begin{cases} 
    n_{\text{EPS}}(M_1, z|\{M_0, z_0\}) & \text{if } M_1/M_0 \leq 0.5 \\
    n_{\text{EPS}}(M_0 - M_1, z_0|\{M_0, z_0\}) & \text{otherwise}
\end{cases}
\]

(10)

with \( n_{\text{EPS}}(M_1, z|\{M_0, z_0\}) \) given by equation (3). Clearly, the binary assumption made in the C00 algorithm results in a PMF that is symmetric with respect to \( M_1/M_0 = 0.5 \), in disagreement with EPS (see also Neistein & Dekel 2008b).

The P08 algorithm results in a PMF that strongly violates the self-consistency criterion. This is a consequence of the fact that the P08 algorithm sidesteps EPS by using a ‘perturbing’ function that has been calibrated such that the resulting PMF is in agreement with that obtained by Cole et al. (2008) using merger trees extracted from the MS. Hence, the blue dots in the middle panel of Fig. 2 may be regarded as representative of the PMF in numerical simulations. Note, though, that as a consequence of the binary nature of the P08 algorithm, this PMF is symmetric, whereas this is not necessarily the case of the true PMF.

The right-hand panel of Fig. 2 shows the results from the Z08B algorithms, both in the case of SC (green dots) and EC (blue dots). Results for the Z08A algorithm are not shown, as they are basically indistinguishable from those of the corresponding Z08B algorithms. Note that the Z08B[SC] algorithm satisfies the EPS self-consistency criterion to high accuracy. In addition, it is interesting to point out that the PMF that results from EC conditions falls below that for SC, similar to the PMF of the P08 method. Since the latter is calibrated against numerical simulations, this suggests that the PMFs in simulations are more reminiscent of EC conditions than of SC conditions. Note, though, that the PMFs of the P08 and Z08B[EC] methods do differ at the 10 per cent level.

4.2 Mass assembly histories

As discussed in Section 2.1, the MAH of a (host) halo is the mass history, \( M(z) \), of the halo’s ‘main’ progenitor (also called the zeroth-order progenitor). The MAHs of dark matter haloes have been studied in a large number of papers, using either the EPS formalism (e.g. Lacey & Cole 1993; Eisenstein & Loeb 1996; Nusser & Sheth 1999; van den Bosch 2002) or N-body simulations (e.g. Wechsler et al. 2002; McBride, Fakhouri & Ma 2009; Zhao et al. 2009; Fakhouri et al. 2010; Yang et al. 2011; Wu et al. 2013). In this section, we compare the median MAHs obtained using the different merger tree algorithms to fitting functions obtained from N-body simulations.
Zhao et al. (2009) used a set of cosmological N-body simulations (for different cosmologies) to study the mass assembly of dark matter haloes, and generalized from their results a universal model that predicts the median MAH for any host halo mass and any cosmology. Yang et al. (2011) subsequently showed that the scatter in $M(z)/M_0$ at fixed $z$ is well described by a log-normal distribution, with median given by the Zhao et al. (2009) model and with a dispersion (in 10-based logarithm) given by

$$\sigma_{\text{MAH}} = 0.12 - 0.15 \log (M(z)/M_0),$$

(11)

where $M(z)$ is the median main progenitor mass at $z$. Because of this log-normal form, it is straightforward to compute the mean MAH. After all, for a log-normal distribution the mean, $\langle M \rangle$, is related to the median according to

$$\langle M \rangle = \exp \left[ \frac{1}{2} \left( \ln(10) \sigma_{\text{MAH}} \right)^2 \right] M.$$  

(12)

Thus, assuming a log-normal distribution, we can convert a median MAH into a mean MAH, and vice versa.

Mcbride et al. (2009) used the MS to study the MAH and mass growth rate of dark matter haloes. They provided a fitting formula for the mean mass growth rate $\langle M \rangle$ as a function of the instantaneous halo mass $M$ and redshift $z$. In a subsequent paper, Fakhouri et al. (2010) used a combination of the MS I and II simulations to (slightly) revise these results, which resulted in a best-fitting mean mass growth rate

$$\langle M \rangle(z) = 46.1 M_\odot \gamma^{-1} \left[ \frac{M(z)}{10^{12} M_\odot} \right]^{1.1} (1 + 1.11 z) \times \sqrt{\Omega_{m,0}(1 + z)^3 + \Omega_{\Lambda,0}}.$$  

(13)

where $\gamma = 0.55$.

In what follows, we compare the simulation results of Zhao et al. (2009), Fakhouri et al. (2010) and Wu et al. (2013) to the MAHs obtained using the various merger tree algorithms. Specifically, for a given cosmology and halo mass, we construct 2000 merger trees from which we compute the median MAH. When computing the median, it is important to take account of the mass resolution of the merger trees (which we take to be $M_{\text{res}} = 10^{-4} M_0$). Throughout, we follow Zhao et al. (2009) and only perform our statistical analysis of the EPS MAHs up to the redshift where the main progenitors of $>90$ per cent of all host haloes in consideration can be traced (i.e. have $M > 10^{-4} M_0$).

Figs 3 and 4 plot the MAHs for haloes of $M_0 = 10^{13} h^{-1} M_\odot$ (at $z=0$) in the Millennium cosmology. The thin grey curves denote the realizations for a random subset of 100 MAHs, while the black solid and dotted curves indicate the median and the 68 percentiles of the distribution of 2000 MAHs. Note that we only plot this median out to the redshift below which less than 10 per cent of the MAHs have dropped below the mass resolution of the merger tree ($10^{-4} M_0$), which can vary substantially from one method to
Generating merger trees

Figure 3. MAHs for present-day host haloes with $M_0 = 10^{13} \, h^{-1} M_\odot$ in the Millennium cosmology. The thin grey lines in the upper panels denote 100 random realizations obtained using the merger tree algorithm indicated in the upper-right corner of each panel. The solid black line indicates the median obtained using 2000 MAHs, while the two dotted curves that enclose the shaded region indicate the corresponding 68 percentiles of the distribution in $\log \left[ \frac{M(z)}{M_0} \right]$ at fixed redshift. Note that we only plot the median and 68 percentiles up to the redshift where the main progenitors of more than 90 per cent of all host haloes in consideration can be traced (i.e. have masses $M > 10^{-4} M_0$), which can vary substantially from one method to the other. For comparison, the red curves represent the model predictions of the median from Fakhouri et al. (2010, dotted curve) and Zhao et al. (2009, dashed curve), both of which are obtained from numerical simulations. The insets show the distributions of $\log \left[ \frac{M(z)}{M_0} \right]$ at $z = 3$ (black histograms), as well as two log-normal distributions with the medians taken from the Fakhouri et al. and Zhao et al. models, and with the scatter given by equation (11). Finally, the lower panels show the ratios of the Monte Carlo results (and the Fakhouri et al. model) with respect to the Zhao et al. model.

Figure 4. Same as Fig. 3 but for the merger tree algorithms Z08A[SC] (left-hand panels), Z08B[SC] (middle panels) and Z08B[EC] (right-hand panels). Note that the histogram in the inset for Z08A[SC] has a different normalization than the model curves; this reflects the fact that 60 per cent of all MAHs constructed using this algorithm have already dropped below the mass resolution of $10^{-4} M_0$ by $z = 3$. 
the other. For comparison, the red curves represent the model predictions, based on numerical simulations, of Fakhouri et al. (2010) and Zhao et al. (2009), as indicated. In the case of Fakhouri et al. (2010), we integrated their model for the mean mass accretion rate (equation 13) to obtain the mean MAH, which we converted to the median using equations (11) and (12). The insets in Figs 3 and 4 show the distributions of log \( \frac{M(z)}{M_0} \) at \( z = 3 \) (black histograms), as well as two log-normal distributions with the medians taken from the Fakhouri et al. and Zhao et al. models, and with the scatter given by equation (11). Finally, the lower panels show the differences in log \( \frac{M(z)}{M_0} \) with respect to the Zhao et al. model.

The Z08A[SC] method differs from all other methods in that it yields a large fraction of MAHs that drop below the mass resolution at very low redshift. In fact, already at \( z \simeq 0.4 \), more than 10 per cent of the MAHs have dropped below \( 10^{-1} M_0 \). At \( z = 3 \), this fraction has increased to 40 per cent; for all other methods, 0 per cent of the MAHs have dropped out from the sample by \( z = 3 \). Similarly large ‘drop-out’ fractions are obtained when using method Z08A[EC]. As discussed in Z08, this arises due to a subtlety in how method A assigns progenitors, and is the main motivation why the authors considered an alternative, method B. Our results show that this subtlety yields MAHs that are seriously flawed, and we therefore no longer consider method Z08A, neither the SC nor the EC version, in what follows.

Comparing the median MAHs obtained using the various EPS algorithms with the simulation results, it is clear that the three SC-based algorithms, SK99, C00 and Z08B[SC], share a common feature: they all predict that halo assembly occurs too recent compared to simulations. As already discussed in Section 2.2.2, this is a well-known problem of SC-based EPS. When comparing the scatter in the MAHs, it is further evident that the SK99 method predicts too much scatter, while the scatter in the C00 MAHs appears to be in good agreement with the simulation results. The Z08B[SC] results are intermediate between those of SK99 and C00. The EC-based method, Z08B[EC], yields a median MAH in excellent agreement with the simulation results, although the method seems to predict slightly too much scatter. Finally, the MAHs obtained using the P08 method also are in good agreement with simulations, both in terms of the median and in terms of the scatter, although there is some indication that it yields MAHs whose early stages of halo assembly occur too early.

Fig. 5 plots the mean MAHs (averaged over 2000 random realizations) obtained using SK99, C00, P08 and Z08B (both SC and EC) for host haloes at \( z = 0 \) with masses of \( M_0 = 10^{11} h^{-1} M_\odot \) (left-hand panel), \( 10^{13} h^{-1} M_\odot \) (middle panel) and \( 10^{14.8} h^{-1} M_\odot \) (right-hand panel). Note that contrary to Figs 3 and 4 we now plot the mean of log \( \frac{M(z)}{M_0} \), which is identical to the median for a log-normal distribution. In the left and middle panels, we have used our fiducial ‘Millennium cosmology’, which allows us to compare the EPS-based MAHs to the simulation results of Fakhouri et al. (2010), shown as thick dashed curves. We also show, for the same cosmology, the predictions based on the Zhao et al. (2009) model.

Figure 5. Mean log \( \frac{M(z)}{M_0} \) as a function of redshift for host haloes at \( z = 0 \) with masses of \( M_0 = 10^{11} h^{-1} M_\odot \) (left-hand panel), \( 10^{13} h^{-1} M_\odot \) (middle panel) and \( 10^{14.8} h^{-1} M_\odot \) (right-hand panel). The solid curves denote the results obtained using the SK99, C00, P08 and Z08B (both SC and EC) merger tree algorithms, as indicated, where, as before, the mean is taken over 2000 random realizations. In the left and middle panels, we have used our fiducial ‘Millennium cosmology’, which allows us to compare the MAHs to the simulation results of Fakhouri et al. (2010, dotted red curve) and Zhao et al. (2009, dashed red curve) model. The grey shaded region in the left-hand panel roughly marks the mass resolution of the simulations used by Fakhouri et al. and Zhao et al., as described in the text. Simulation results in this region are largely based on extrapolation, and have to be considered less reliable. In the right-hand panel, in order to facilitate a comparison with the results of Wu et al. (2013, dotted red curve), we have adopted the ‘Rhapsody cosmology’. The lower panels show the ratios of the Monte Carlo results (and the other model) with respect to the Zhao et al. model.

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model. The grey shaded region in the left-hand panel marks the region where the main progenitor mass $M < 2 \times 10^7 \, h^{-1} \,\, M_\odot$. This roughly marks the mass scale below which the simulations used by Zhao et al. (2009) and Fakhouri et al. (2010) can no longer reliably resolve the MAHs. The region with $z \geq 8$ is also shaded, since this is the redshift range where the Fakhouri et al. (2010) results drop below this mass limit. Hence, in the grey region the simulation results are less reliable and largely based on extrapolation. Ignoring this region, the Fakhouri et al. and Zhao et al. models agree roughly at the 0.1–0.2 dex level. In the right-hand panel, in order to facilitate a comparison with the results of Wu et al. (2013), we adopt the Rhapsody cosmology (which has lower $\sigma_8$ than the Millennium cosmology, and a slightly different Hubble parameter). Again, the agreement between different simulation results, here between Wu et al. and Zhao et al., is (only) at the level of 0.1–0.2 dex.

A comparison with the MAHs obtained using the various merger tree algorithms shows once again that the three SC-based algorithms (SK99, C00 and Z08B[SC]) yield MAHs that systematically fall below the simulation results. In fact, it is interesting how similar the average MAHs obtained with these very different methods are. The P08 and Z08B[EC] algorithms yield MAHs that are in reasonable agreement with the simulation results, whereas P08 seems to overpredict $M(z)/M_0$ at early times and Z08B[EC] seems to slightly underpredict $M(z)/M_0$ for the most massive haloes.

To summarize, EPS merger trees based on SC consistently yield MAHs in which haloes assemble too late compared to simulations. In terms of halo substructure, this implies that the SK99, C00 and Z08B[SC] algorithms will all underpredict the accretion redshifts of subhaloes, and are therefore not well suited to build analytical models for dark matter substructure or to model satellite galaxies. Both the P08 and Z08B[EC] algorithms fare much better in that respect. They both yield MAHs in reasonable agreement with numerical simulations, both in terms of their median and the scatter. The median MAHs predicted by these two methods are in excellent agreement with each other and with the simulation results at low redshifts ($z \lesssim 2$), but start to diverge at larger redshifts. At $z = 7$, they typically differ at the 0.3 dex level. Unfortunately, because of the ∼0.2 dex discrepancy among the different simulation results, we cannot significantly prefer one of these two methods over the other.

### 4.3 Merger rate per descendant halo

The next diagnostic to consider for our EPS merger tree algorithms is the merger rate per descendant halo, $(1/N)dN_{\text{merger}}/d\omega \, dx$, which characterizes the rate at which the population of haloes of mass $M = M_1 + M_2$ is created by mergers between progenitors with a mass ratio $x \equiv M_1/M_2$. Here the notation is such that $M_i \geq M_{i+1}$, which implies that $x \geq 1$. The quantity $dN_{\text{merger}}(M, x, z, \omega, x_\text{w})/d\omega \, dx$ is the number of merger events of mass ratio $x \pm dx/2$ that result in descendant haloes of mass $M \pm dm/2$ per unit time interval $d\omega$ at redshift $z$ and $N$ is the number of descendant haloes of mass $M \pm dm/2$.

Both Fakhouri et al. (2010, hereafter F10) and Genel et al. (2010, hereafter G10) measured this merger rate per descendant halo from the MS. A problem is that the time steps between successive outputs of the MS are relatively large, and a descendant halo often has more than two progenitor haloes at the previous time step. Both F10 and G10 deal with this complication in the same way: whenever a multiple merger event, consisting of $N_p$ progenitors, occurs, they interpret this as a series of $N_p - 1$ binary mergers between $M_1$ and $M_i$ where $i = 2, 3, \ldots, N_p$. Although this is not necessarily a proper description of the true merger history during this time step, this procedure can be repeated using the EPS formalism, thus allowing for a fair comparison.

Using a combination of the MS I and II simulations, F10 and G10 found that the merger rate per descendant halo, for the Millennium cosmology, is well described by

$$\frac{1}{N} \frac{dN_{\text{merger}}}{dx} (M, z, x) = f(z) \left[ \frac{M(z)}{10^2 M_\odot} \right] x^{\omega x} \omega x^{\epsilon x},$$

where

$$f(z) = \begin{cases} 0.065 & (G10) \\ 0.010 \approx (1 + z)^{\epsilon z} & (F10) \end{cases}$$

and the best-fitting parameters are $(a_1, a_2, a_3, a_4) = (0.15, -0.3, 1.58, -0.5)$ and $(a_1, a_2, a_3, a_4, a_5) = (0.133, -0.005, 3.38, -0.263, 0.0993)$ for the G10 and F10 results, respectively. Note that the fitting equation of G10 is only valid over the range $0.5 \lesssim z \lesssim 5$. The dashed and dotted curves in Fig. 6 show the merger rates per descendant halo according to these F10 and G10 fitting formulæ, as indicated. They are in reasonable, mutual agreement for $x \lesssim 10$, but diverge quite strongly for larger values of the merger mass ratio. In particular, F10 predict roughly twice as many minor mergers with $x = 1000$ as G10. This discrepancy mainly arises from the subtle differences in how these authors extract halo merger trees from their simulation outputs (see F10 and G10 for details). As with the MAHs, we are therefore forced to conclude that different authors obtain halo merger rates that differ substantially, even when they base their results on the same simulation. In what follows, we will simply treat this discrepancy between the F10 and G10 results as a rough indicator of the uncertainty on $(1/N)dN_{\text{merger}}/d\omega \, dx$ in simulations.

Using EPS merger trees, it is straightforward to compute the merger rate per descendant halo. For a given host halo mass, $M_0$, at a given redshift, $z_0$, we construct $N = 10^4$ realizations of the population of progenitor haloes a time $\Delta \omega = 0.002$ earlier. These are used to compute the merger rate per descendant halo, strictly following the procedure used by F10 and G10 to treat multiple mergers. Fig. 6 compares the results obtained using our five remaining merger tree algorithms to the fitting functions of F10 and G10. All merger rates in this figure are for a redshift $z_0 = 1$, while different panels correspond to different host halo masses, as indicated. We have verified that the results look almost indistinguishable at any other redshift in the range $0.5 \lesssim z_0 \lesssim 5$. We have also verified that the results are not sensitive to the temporal resolution; as we vary $\Delta \omega$ between 0.001 and 0.1, the merger rates at $z_0 = 1$ change by no more than 10 per cent.

Upon inspection, a number of trends are apparent. First of all, the merger rates obtained with the C00 and Z08B[SC] algorithms are virtually indistinguishable. Both overpredict the rate of major mergers (mergers with $x \lesssim 3$) by a factor of about 2 with respect to the numerical simulation results of F10 and G10. This discrepancy becomes smaller for larger $x$, at least when compared to the F10 fitting function. Interestingly, compared to the G10 fitting function, the C00 and Z08B[SC] merger rates follow almost exactly the same $x$-dependence, but with a normalization that is a factor $\sim 2$ too high. A similar trend was noticed by G10. The third SC algorithm, SK99, dramatically overpredicts the merger rates of dark matter haloes compared to both G10 and F10, for all host halo masses (and at all redshifts). The discrepancy is most pronounced for mergers with a mass ratio $x \sim 20$, for which SK99 predicts a rate that is a factor of 3–4 too high. This failure of the SK99 algorithm is a
direct manifestation of its failure to satisfy the EPS self-consistency constraint.

The EC algorithm Z08B[EC] yields an excellent match to the F10 merger rates for haloes with $M_h = 10^{11} h^{-1} M_\odot$. However, there is a clear trend that the Z08B[EC] algorithm starts to overpredict the merger rates (compared to F10 and G10) for more massive haloes. This problem is more pronounced for mergers with a larger mass ratio. For cluster-size host haloes with $M_h = 10^{15} h^{-1} M_\odot$, and compared to F10, the Z08B[EC] algorithm overpredicts the major-merger rate by a factor of 1.7 and that of minor mergers with mass ratio $x = 1000$ by a factor of 3.7. We believe that this problem arises from the method used to assign halo masses to the secondary progenitors, which are all assigned the same mass (see Section 3.4 and Zhang et al. 2008a for details). Finally, the P08 algorithm yields merger rates that are in excellent agreement with the F10 results. The only exception seems to be for cluster-size haloes with $M_h = 10^{15} h^{-1} M_\odot$, where the P08 merger rates are a factor of ~1.3 too high compared to the F10 fitting function.

To summarize, EPS merger tree algorithms that are based on SC overpredict the rate of major mergers by about a factor of 2. EC seems able to alleviate this tension. However, the Z08B[EC] implementation of EC has a problem in that it vastly overpredicts the number of minor mergers for massive host haloes. Overall, the P08 algorithm yields merger rates in significantly better agreement with the simulation results than any of the other merger tree algorithms considered here. It still overpredicts the merger rates for cluster-size host haloes by about 30 per cent, but we emphasize that the disparity in merger rates obtained by different authors from the same simulation is of a similar magnitude.

### 4.4 The unevolved subhalo mass function

The final diagnostic that we consider for testing the various EPS merger tree algorithms is the USMF. $dN/m(M/M_0)$, where $m$ is the mass of the subhalo at accretion and $M_0$ is the present-day host halo mass.

Using EPS merger trees, van den Bosch, Tormen & Giocoli (2005) noticed that the USMF of first-order subhaloes (i.e. only counting those subhaloes that accrete directly on to the main progenitor) is universal, in that it does not reveal any significant dependence on either host mass, redshift or cosmology. This was later confirmed by Giocoli, Tormen & van den Bosch (2008, hereafter G08) and Li & Mo (2009, hereafter LM09) using numerical simulations. Note, though, that this universality is only approximate. It is adequate for host haloes with masses in the range $10^{10} \lesssim M_0 \lesssim 10^{15} h^{-1} M_\odot$ in a $\Lambda$CDM cosmology, but does not necessarily hold for more extreme halo masses and/or cosmologies. Indeed, using simulations for cosmologies with scale-free power spectra, $P(k) \propto k^n$, Yang et al. (2011) have shown that the USMF depends significantly on the value of the spectral index $n$. The apparent universality noticed by van den Bosch et al. (2005), G08 and LM09 arises because the effective spectral index of the $\Lambda$CDM power spectrum only varies slightly over the mass range $10^{10} \lesssim M_0 \lesssim 10^{15} h^{-1} M_\odot$.

Using the universality of the USMF of first-order subhaloes, it is straightforward to compute the USMF of $n$th-order subhaloes, which is defined as the mass function of $n$th-order subhaloes at their moment of accretion (i.e. when they transit from being host haloes to being subhaloes). After all, since subhaloes can themselves be considered as host haloes at the time of accretion, their subhaloes,
which are of second order, are also expected to obey the universal USMF. As emphasized in LM09, this implies that
\[
n_{\text{un},i}(m|M_0) = \int_{m_0}^{M_i} n_{\text{un},i}(m|M_0) \, dm_0 \, dm, \tag{17}
\]
(for \(i = 2, 3, \ldots \)). Here
\[
n_{\text{un},i}(m|M_0) \equiv \frac{dN}{dm} = \frac{1}{m} \frac{dN}{d \ln(m/M_0)} \tag{18}
\]
is the \(i\)th-order unevolved subhalo mass function, for which we will use the shorthand USMF[\(i\)] in what follows.

Using the high-resolution GIF simulations, G08 found that the USMF[1] is well fitted by
\[
\frac{dN}{d \ln(m/M_0)} = \gamma \left( \frac{m}{M_0} \right)^{\alpha} \exp \left[ -\beta \left( \frac{m}{M_0} \right)^{\zeta} \right], \tag{19}
\]
with best-fitting parameters \((\gamma, \alpha, \beta, \zeta) = (0.18, -0.80, 12.27, 3.00)\). Note that this implies a total normalization
\[
F_{\text{norm}} = \frac{1}{M_0} \int_0^{M_0} \frac{dN}{dm} \, dm = \int_0^1 \frac{dN}{d \ln(m/M_0)} \, dm_0 \simeq 0.735. \tag{20}
\]

The fact that \(F_{\text{norm}}\) is substantially smaller than unity implies that dark matter haloes accrete a significant fraction of their mass ‘smoothly’, either in the form of matter not locked up in any halo or in the form of haloes with masses below the resolution limit of the simulation.\(^2\)

LM09 used the MS and found slightly different best-fitting parameters, given by \((\gamma', \alpha, \beta, \zeta') = (0.2, -0.76, 6.00, 3.20)\), for which \(F_{\text{norm}} \simeq 0.701\). The open circles in Fig. 7 denote the actual data used by LM09 in their fitting procedure, for three different bins in host halo mass, as indicated. The dotted and dashed curves represent the best-fitting functions of the form of equation (19) obtained by G08 and LM09, respectively. As is apparent from the lower panel, showing the residuals, neither is a good fit to the actual data. In particular, the LM09 data reveal a clear ‘shoulder’ around the mass scale where the exponential cut-off kicks in. Since this feature is not captured by the fitting function of the form (19), we adopt an alternative fitting function for the USMF, which simply is a linear combination of two components of the form (19), but with a common exponential-decay part:
\[
\frac{dN}{d \ln(m/M_0)} = \gamma_1 \left( \frac{m}{M_0} \right)^{\alpha_1} + \gamma_2 \left( \frac{m}{M_0} \right)^{\alpha_2} \exp \left[ -\beta \left( \frac{m}{M_0} \right)^{\zeta} \right], \tag{21}
\]
\[\text{Figure 7. The unevolved subhalo mass function of first-order subhaloes (USMF[1]). Symbols denote the results obtained by Li & Mo (2009, LM09) using the MS, for three different bins in host halo mass, as indicated. The dotted and dashed curves represent the fitting functions obtained by Giocoli et al. (2008, G08) and LM09, respectively, while the solid line denotes the best-fitting fitting function of the form given by equation (21). The lower panel shows the ratios of the simulation results (and the other two fitting functions) with respect to the LM09 fitting function. Note that our new fitting function, equation (21), which has more freedom, better describes the ‘shoulder’ around log(m/M_0) \simeq -1.}\]

\[\text{a mass resolution of } \sim 2 \times 10^{10} h^{-1} M_\odot, \text{ and our fitting function therefore has to be considered an extrapolation for any } m \lesssim 2 \times 10^{10} h^{-1} M_\odot.\]

We have used our best-fitting function for the USMF[1] to compute the USMF for subhaloes of higher order. The results are shown in Fig. 8, where different curves correspond to USMFs for different orders, as indicated. Note how USMF[2] is higher than USMF[1] for \(m/M_0 \lesssim 3 \times 10^{-2}\), and dominates the total USMF, defined as the sum of USMF[i] for all \(i\), over the entire range \(-4 \leq m/M_0 \leq -2\). The contribution to the total USMF due to subhaloes of order 5 or higher is negligible for all \(m/M_0 > 10^{-4}\). The data points in Fig. 8 show the simulation results for all order subhaloes obtained by LM09 using the MS. Note that our prediction for this USMF[all], which we compute by summing USMF[i] for \(i = 1 \text{ to } 4\), is in excellent agreement with these data.\(^3\)

\[^{2}\text{Note, though, that the values for } F_{\text{norm}} \text{ quoted here are based on extrapolating fitting functions for the USMF[1] to subhaloes masses well below the mass resolution of the simulations.}\]

\[^{3}\text{As shown in LM09, this USMF[all] is accurately fitted by equation (19) with } (\gamma, \alpha, \beta, \zeta) = (0.22, -0.91, 6.00, 3.00).\]
Figure 8. The USMF for different order subhaloes, as indicated. The first-order USMF is characterized by the fitting function of equation (21) with the best-fitting parameters given in the text, while the higher order USMFs have been computed from this first-order USMF using equation (17). The data points represent the simulation results for all order subhaloes obtained by LM09 using the MS, for three different bins in host halo mass, as indicated. The solid line labelled ‘all orders’ shows the corresponding analytical prediction, which has been computed by summing the USMFs for orders 1 to 4 (the contribution of higher order USMFs is negligible). The fact that this prediction is an excellent match to the data supports the notion that the first-order USMF is universal.

Figure 9. The unevolved subhalo mass function of first-order subhaloes (USMF[1]) for host haloes (at \(z = 0\)) with mass \(M_0 = 10^{11} h^{-1} M_\odot\) (left-hand panel), \(10^{13} h^{-1} M_\odot\) (middle panel) and \(10^{15} h^{-1} M_\odot\) (right-hand panel). The solid curves denote the results obtained using the SK99, C00, P08 and Z08B (both SC and EC) merger tree algorithms, as indicated. For comparison, the red dashed line indicates the best-fitting representation of the simulation results given by equation (21) with the best-fitting parameters given in the text (cf. red solid curve in Fig. 7). The lower panels show the ratios of the Monte Carlo results with respect to these simulation results.
4.4.2 The USMF of higher order

The upper and middle rows of panels in Fig. 10 show the ratios of USMF[2] and USMF[3] obtained with the various merger tree algorithms with respect to the analytical predictions based on equation (17). A few trends are apparent. First of all, the USMFs obtained with SK99, C00 and Z08B[SC] all become progressively worse for higher order subhaloes, overpredicting the USMF by large amounts at the low-mass end. The USMFs obtained using the Z08B[EC] method also become progressively worse for higher order, but in the sense that it starts to underpredict the USMF at the massive end. The higher order USMFs obtained using the P08 method, however, become progressively better with increasing order. The lower panels of Fig. 10 show similar residuals but for the USMF of all subhaloes, here defined as the sum of all USMF[i] for i = 1, 2, ..., 4 (as discussed above, the contribution from higher order USMFs is negligible for m/M0 ≥ 10^{-4}). The P08 predictions are in excellent agreement with the analytical prediction, which in turn is in excellent agreement with the simulations results (cf. Fig. 8). The predictions based on the Z08B[EC] method perform almost equally well for M0 < 10^{13} h^{-1} M_{\odot}, but overpredict the abundance of small subhaloes (m ≤ 10^{-2} M_{\odot}) for more massive host haloes. The three SC-based algorithms SK99, C00 and Z08B[SC] all overpredict the USMF[all] by significant amounts. In the case of C00 and Z08B[SC], the offset is roughly independent of subhalo mass, such that they at least predict the correct power-law slope for the USMF[all]. The SK99 algorithm, on the other hand, predicts a power-law slope that is clearly too steep. As discussed at the end of Section 4.4.1, the sharp drop-offs at the massive end are mere manifestations of the fact that the analytical model used does not adequately account for the fact that there is a hard, upper limit to the masses of nth-order subhaloes equal to M0/2^n.

5 CONCLUSION AND DISCUSSION

In this paper, we have compared and tested a number of different algorithms for constructing halo merger trees. The diagnostics that we have used are the PMFs, the MAHs, the merger rates per descendant halo and the USMFs.

Of all the algorithms tested, the one that fares worst is that of SK99. The main reason is the strong violation of the self-consistency constraint (i.e. the progenitor masses drawn fail to sample the actual PMF), which arises from the fact that the SK99 algorithm discards progenitor masses drawn from the PMF that overflow the mass budget (see Section 3.1 for details). In general, the SK99 algorithm yields (i) haloes that assemble too late, (ii) too much scatter in halo MAHs, (iii) merger rates that are too high by factors of 2–3 and (iv) USMFs that are much too high, especially for subhaloes with a mass at accretion m ~ M0/100. The latter explains why van den Bosch et al. (2005) inferred an average subhalo mass-loss rate that is too high, as previously pointed out by G08. It also implies that other models for dark matter substructure that also used the SK99 algorithm (Zentner & Bullock 2003; Taylor & Babul 2004; 2005a,b; Zentner et al. 2005; Purcell & Zentner 2012) are likely to suffer from similar systematic errors.

The various methods introduced by Z08 have the advantage that, by construction, they accurately satisfy the self-consistency constraint. This is true for both the SC- and EC-based methods. We have tested and compared both methods A and B, and both for SC and EC. Method A, however, is flawed in that it yields MAHs that are unrealistic (see the left-hand panel of Fig. 4), which is why we have not considered this method for any of the other diagnostics. Method B, however, fares much better. The SC implementation yields MAHs that assemble too late compared to simulations and overpredicts the major-merger rate by a factor of 2. These are generic problems for SC-based EPS, and are therefore shared by the SK99 and C00 algorithms. The Z08B[SC] algorithm also overpredicts the USMF at the massive end, by about 50 per cent. This problem is present for each order of the USMF. At the low-mass end (i.e. for m/M0 ~ 10^{-4}), the Z08B[SC] method underpredicts the abundance of first-order subhaloes, but overpredicts that of higher order subhaloes.

The EC implementation of method B, Z08B[EC], solves most of these problems. In particular, it yields MAHs, merger rates and
USMFs that are in much better agreement with simulation results. However, the Z08B[EC] algorithm dramatically overpredicts the minor-merger rate for massive (cluster-size) host haloes. This in turn results in USMFs for such host haloes that are much too high at the low-mass end. We believe that this failure of the Z08B[EC] algorithm has its origin in the fact that it assigns all progenitors of a descendant halo, other than the most massive one, the same mass.

The binary merger method developed by C00 yields halo merger trees that are extremely similar to those constructed with the Z08B[SC] algorithm. In particular, it yields MAHs that assemble too late, overpredicts the major-merger rate by a factor of 2 and underpredicts the USMF of first-order subhaloes at the massive end. The small (largely insignificant) differences with respect to the Z08B[SC] algorithm mainly come from the fact that the C00 method violates the self-consistency constraint, but only for progenitors with a mass more than half the descendant mass, and only by a few per cent.

Of all the algorithms tested, the one that yields results in closest agreement with the simulations is the P08 algorithm. It slightly overpredicts the merger rates for massive descendant haloes, by about 20 per cent, and underpredicts the USMF for first-order subhaloes at the low-mass end, by about 15 per cent, but given the uncertainties in the actual simulation results, these discrepancies are barely significant. The fact that the P08 method yields the best results should not come entirely as a surprise. After all, P08 draws its progenitor masses from a PMF that has been modified with respect to the EPS prediction to match the simulation results presented in Cole et al. (2008). Hence, one ought to expect that the P08 algorithm yields results in better overall agreement with simulations. We emphasize, though, that even if a method is tuned to reproduce the PMF of simulations, there is no guarantee that it reproduces any of the other diagnostics. This requires a merger tree algorithm that successfully partitions the descendant mass over progenitors, which is a non-trivial task (as discussed in Section 2.2.1).

An important (but unavoidable) caveat of the work presented in this paper is that the benchmarks that we have used to test the various merger tree algorithms are all based on numerical simulations. As discussed in Section 2, and as highlighted in this paper, simulation results carry significant uncertainties that mainly arise from issues related to identifying haloes and tracking them across different simulation outputs. The most problematic aspect is how to properly link (sub)haloes between different snapshots in a manner that properly accounts for the fact that some subhaloes are on orbits that take them outside the host halo’s virial radius. As we have shown, the discrepancies in average halo MAHs or merger rates per descendant halo obtained from simulations by different authors can easily exceed 50 per cent, even when they are based on the same simulation. Clearly, this situation has to improve if the goal is to build (semi-)analytical models that are accurate to this level or better. At this point in time, though, taking these uncertainties into account, we conclude that the accuracy of the P08 merger tree algorithm is not significantly worse than that of the simulations themselves.

As a final remark, we point out that there are several other EPS or EPS-based algorithms that can be used to construct merger trees (e.g. Kauffmann & White 1993; Moreno et al. 2008; Neistein & Dekel 2008a,b; method C of Zhang et al. 2008a). Our choice not to include those methods in this study is simply to keep the project manageable and to prevent the paper from becoming overly dense. However, we believe that it would be useful to perform similar tests for these alternative methods as well, and we encourage the community to do so. Based on our experience, we consider the merger rates and the unevolved SHMF to be the most informative diagnostics to test the performance of a merger tree algorithm.

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