Passive orientation of the parts in the mechanical disk hopper feeding device with an annular orientator and radial grooves

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Abstract. The paper considers mathematical models describing the process of passive orientation of axisymmetric parts in the form of the bodies of rotation with implicit asymmetry at the ends in the mechanical disk hopper feeding device with an annular orientator and radial grooves made on a rotating disk. The developed mathematical models make it possible to estimate the boundary conditions under which the reliable orientation of the parts and the time of orientation of the parts will be ensured, depending on the geometric parameters of the part, the coefficient of sliding friction of the part against the guide surfaces, the structural and kinematic parameters of the hopper feeding device.

1. Introduction
At present, the enterprises of special engineering industries are developing new technologies for the production of items and the nomenclature of such enterprises is significantly increasing. At the same time, technical equipment for their manufacture should be carried out as soon as possible. Therefore, it is an urgent task to develop reliable systems for automatic feeding of piece parts into the process and assembly equipment. Mechanical hopper feeding devices, being part of such systems, are used in many industries for automatic feeding of parts into the equipment in the desired oriented position and with the given productivity.

Theoretical issues of functioning, design and calculation of most mechanical hopper feeding devices are sufficiently considered in the works of domestic and foreign authors [1 – 4].

There are mechanical disc hopper feeding devices where the rotating disk has gripping elements in the form of radial grooves passing into the profile pockets. Such feeding devices are used for active orientation of axisymmetric parts in the form of the bodies of rotation with explicit asymmetry at the ends. Active orientation of such parts is realized by the installation of the feeding device of the fixed spear in the upper part of the hopper. It places in an oriented position those parts that have incorrectly fused into radial profile pockets.

The size of the gap between the outer surface of the part and the inner surface of the profile pocket has a significant effect on the probability of gripping parts and the feed rate of the device. This is shown in the works of the authors [5, 6] based on developed mathematical models describing the process of gripping parts in disk mechanical hopper feeding devices with radial grooves and pockets. The size of the gap should be minimum for reliable active orientation of parts and maximum for required and sufficient probability of gripping. For parts with explicit asymmetry at the ends, this contradiction is resolved by selecting a rational pocket configuration and gap size. For parts with implicit asymmetry at the ends, this contradiction was practically unsolvable in well-known designs of hopper feeding devices.

This led the authors to the idea of separating the functions of gripping and orienting parts based on the transition from active orientation to passive orientation by installing an annular orientator. Its groove profile corresponds to the profile of the asymmetric end of the part [7]. In this case, the radial pockets on the disc are rectangular and open towards the orientator groove. The gap between the external cylindrical surface of the part and the internal surface of rectangular pocket was selected as
maximum. The gap between the surface of annular orientator groove and the outer surface of asymmetric end face of the part was selected as minimum. This design of the mechanical disc hopper feeding device provided reliable passive orientation of the parts in the form of the bodies of revolution with implicit asymmetry at the ends.

Installation of special combs in the radial grooves of the rotating disk was a further improvement of the found structural solution of the hopper feeding device [8]. It increases the probability of parts falling into the pockets.

This paper presents mathematical models for describing the process of passive orientation of axisymmetric parts in the form of the bodies of revolution with implicit asymmetry at the ends in the improved mechanical disc hopper feeding device with an annular orientator and radial grooves. The developed mathematical models make it possible to evaluate boundary conditions of reliable orientation of the parts and the time of orientation of the parts depending on the geometric parameters of the part, the coefficient of sliding friction of the part against the guide surfaces, the structural and kinematic parameters of the hopper feeding device.

2. Formulation of the problem
The scheme of the improved design of the mechanical hopper feeding device with an annular orientator and grooves in the form of a comb is shown in Fig. 1. The hopper feeding device includes a hopper 1, a fixed base 2, a rotating disk 3 around the circumference of which gripping elements in the form of pockets are uniformly arranged in the radial direction 4. Pockets pass into radial grooves representing a comb 5 along the surface of which protrusions and recesses alternate. The orienting element of the device is the annular orientator 6, the profile of which follows the shape of conical end [8].

During the rotation of the disc the parts, filled into the hopper of the device, move along radial grooves to the gripping elements. If the part moves with a conical end forward, then it freely slides along the radial groove with alternating protrusions and recesses and sinks into the pocket completely. If the part moves the cylindrical end forward, it encounters an obstacle from the side of the alternating protrusions and stops moving towards the gripping element. If the part was able to approach the pocket with a cylindrical end, it is only partially captured by the pocket, since the shape of the annular orientator does not allow the cylindrical end of the part to fully penetrate into the pocket. Such parts in the upper part fall out of the pockets when the disk rotates, end up on the surface of radial grooves in the form of a comb, move along protrusions and recesses, leave the radial groove and enter the total mass of parts. Those parts, that have been completely captured by the pocket, fall into a special window which is located in a fixed base in the feeding area.
Since the depth of the radial grooves increases uniformly from the center of the rotating disk to the pockets, it prevents sinking into the pockets of the parts moving with the cylindrical end and ensures the removal of such parts from the pockets and radial grooves during passive orientation. This increases the probability of properly oriented parts falling into the gripping elements and increases the feed rate of the hopper feeding device.

The difference between the improved design and the previous one is as follows. Firstly, the radial groove is formed by alternating protrusions and recesses. Secondly, it has a variable depth from a value equal to half the diameter of the part to a zero value. Therefore, unlike the previous devices, in the hopper feeding device with radial grooves in the form of a comb the part freely falls out of the groove into the total mass of the parts.

Therefore, the process of passive orientation of the parts with implicit asymmetry in the improved device consists in the complete removal of the part from the pocket until the part is in the position where its cylindrical end is completely on the surface of the radial groove. In this case, the time of the motion of the part \( t_{or} \) during passive orientation should be less than the time \( t_d \) during which the pocket with the part passes the possible orientation zone, determined by the expression

\[
t_d = \frac{\pi \psi_0}{90 \omega},
\]

where \( \omega \) – is the angular velocity of the disk rotation; \( \psi_0 \) – is the angle at which the part begins to fall out of the pocket, measured from the highest point of the hopper.

Thus, to ensure reliable passive orientation of the part, the condition must be met

\[
t_{or} \leq t_d.
\]

Therefore, the main task is to determine the angle \( \psi_0 \) and the time \( t_{or} \) of passive orientation of the part.

3. Theoretical part

To determine the angle \( \psi_0 \), it is necessary to analyze the conditions under which the part begins the passive orientation process, and to draw up the static equilibrium equations of the part in the gripping element of the device (Figure 2).

![Figure 2. Scheme for finding the angle where the process of passive orientation of the part begins: a – the forces acting on the part; b – the part in section and projection of all forces](image)

The boundary conditions for the implementation of the process of passive orientation will be composed in the form of the system of equations. The sum of the moments of the forces acting on the part relative to the supports \( A \) and \( B \), as well as the sum of the projections of these forces on the \( OX \) axis, will be zero. Then we obtain the system of static equilibrium equations of the part in the pocket.
\[
\begin{align*}
\sum M_A^I &= F_2 l_{F2A} + N_2 l_{N2A} - G_1 l_{G1A} - (F_3 - G_2 \cos \psi_0) l_{F3A} = 0; \\
\sum M_B^I &= F_1 l_{F1B} - N_1 l_{N1B} + (G_2 \cos \psi_0 - F_3) l_{F3B} + G_1 l_{G1B} = 0; \\
\sum F_{OX} &= -F_1 \cos \beta - N_1 \sin \beta - (G_1 - N_2) \sin \alpha_1 - (F_2 + F_3 - G_2 \cos \psi_0) \cos \alpha_1 = 0,
\end{align*}
\]

where \(G = mg\) is the gravity force of the part; \(m\) – its mass; \(G_1 = G \cos \alpha_h, \ G_2 = G \sin \alpha_h\) – components of gravity, directed perpendicular and on the surface of the disc along the pocket; \(\alpha_h\) – the inclination angle of the hopper base to the horizon; \(l_{G1A}, \ l_{G1B} = l_1 - l_{G1A}\) – arms of gravity component relative to points A and B, respectively; \(l_1\) – length of cylindrical end of the part; \(F_2 = N_2 h_1\) – friction force of the part against fixed base; \(N_2\) – force of base reaction; \(F_3 = N_3 h_1\) – friction force of the part against side surface of the pocket; \(N_3 = G_2 \sin \psi_0\) – reaction force of side surface of the pocket; \(\mu\) – friction coefficient of the part against structural elements of the device; \(F_1 = N_1 \mu\) – friction force of conical end of the part along protrusion of the pocket; \(N_1\) – reaction force of the pocket protrusion; \(l_{F3B} = l_{F3A} + h_k\) – arm force relative to point B; \(h_k\) – pocket depth; \(l_{F1B}, l_{N1B}\) – arms of forces \(F_1\) and \(N_1\) relative to point B; \(\alpha_1\) – angle formed by the cylindrical surface of the part and the bottom of the pocket (see Figure 2).

The expressions for determining the arms of forces are given in the table 1.

| Designation | Formula for Definition |
|-------------|------------------------|
| \(l_{G1A}\) | \(f \cdot \cos(\alpha_3 - \alpha_1 - \alpha_2)\) |
| \(l_{N1B}\) | \(l_1 \cdot \cos \beta + b\) |
| \(l_{F3A}\) | \(f \cdot \sin(\alpha_3 - \alpha_1 - \alpha_2)\) |

The parameters of the expressions presented in the table are determined by formulae

\[
f = \sqrt{n^2 + b^2 + 2nb \cdot \cos(\alpha_3 + \beta)}, \ \ n = \sqrt{(l_1 - x_c)^2 + \frac{d_1^2}{4}}, \ \ b = \sqrt{(l_1 \cos \beta)^2 + h_k^2 - l_1 \cos \beta};
\]

\[
\begin{align*}
\alpha_0 &= \arccos \left( \frac{b_1^2 + 2l_1^2 - h_k^2}{2l_1 \cdot \sqrt{b_1^2 + l_1^2}} \right); \ \alpha_1 &= \arctan \left( \frac{h_k}{l_1} \right) - \alpha_0; \ \alpha_2 &= \arccos \left( \frac{n^2 + b^2 - h_k^2}{2nf} \right); \\
\alpha_3 &= \arctan \left( \frac{l_1}{2(l_1 - x_c)} \right),
\end{align*}
\]

where \(d_1\) – is the diameter of the cylindrical end of the part; \(\beta\) – half of the angle at the top of the conical end of the part; \(x_c\) – the distance from the cylindrical end of the part to its center of mass.

We obtain the condition of the process of passive orientation of the part after transformations of the system of equations (2)

\[
\begin{bmatrix}
\cos \alpha_i l_{G1A} - l_{F1A} \sin \alpha_i (\cos \psi_0 - \mu \sin \psi_0) - (\sin \beta + \mu \cos \beta) l_{F1B} \sin \alpha_i (\cos \psi_0 - \mu \sin \psi_0) + \sin \alpha_i \cos \alpha_i (\cos \psi_0 - \mu \sin \psi_0) - \\
\sin \alpha_i (\sin \psi_0 + \mu \cos \psi_0) + \sin \alpha_i (\cos \psi_0 - \mu \sin \psi_0) - (\sin \beta + \mu \cos \beta) l_{F1B} \sin \alpha_i (\cos \psi_0 - \mu \sin \psi_0) + \sin \alpha_i \cos \alpha_i (\cos \psi_0 - \mu \sin \psi_0) - \\
\mu l_{F1A} - l_{N1B}\end{bmatrix} > 0. \ (3)
\]
If we equate the left part of the expression (3) to zero, then we find the boundary value of the angle when the process of passive orientation of the part begins

$$\psi_0 = \arcsin \left( -\mu K + \sqrt{\mu^2 + 1 - K^2} \right),$$

(4)

the coefficient $K$ is calculated by the formula

$$K = \frac{\cos \alpha_\text{h} \sin \alpha_1 + (\sin \beta + \mu \cos \beta) \frac{\cos \alpha_\text{h} (l_1 - \ell_G \text{A})}{l_{N\text{B}} - \mu \ell_{F\text{B}}} + \frac{\cos \alpha_\text{h} \ell_G \text{A} (\sin \alpha_1 - \mu \cos \alpha_1)}{l_1 + \mu \ell_h}}{\sin \alpha_\text{h} \cos \alpha_1 - \frac{\ell_{F\text{A}}}{l_1 + \mu \ell_h} \frac{\sin \alpha_\text{h}}{(\sin \alpha_1 - \mu \cos \alpha_1) - \frac{(\sin \beta + \mu \cos \beta) \cdot \ell_{F\text{B}}}{l_{N\text{B}} - \mu \ell_{F\text{B}}}} \sin \alpha_\text{h}}.$$

(5)

Graphs allow to determine the angle $\psi_0$, depending on the coefficient of friction (from 0.3 to 0.6) and the angle of inclination of the hopper ($45^\circ$, $50^\circ$, $55^\circ$) for parts with the conical end at the ratio of the length of the part to the diameter of its cylindrical end $l/d_1$ (Figure 3).

In order for the part to fall out of the socket, it is necessary that the angle was at least $0^\circ$. It is possible to increase the angle by increasing the angle of the hopper base.

The mathematical model of the passive orientation process was developed at all four stages of the part motion (Figure 4): stage 1 (I-II) – motion by the cylindrical end along the base of the pocket and by the conical one along the support $A$ until the cylindrical end contacts the support $A$; stage 2 (II-III) – until the cylindrical base of the part is detached from the base of the pocket; stage 3 (III-IV) – turning on the support $A$; stage 4 (IV-V) – motion along the groove to the distance equal to $c$. The model allows to define the overall orientation time $t_{or}$ of the part.

The process of passive orientation of the part is described by the Lagrange equations of the second kind

$$\frac{d}{dt} \frac{\partial T}{\partial \dot{\psi}} = \frac{\partial T}{\partial \psi} = Q,$$

(6)
where $T$ is the kinetic energy of the system (the part and the rotating disk with constant angular velocity); $\psi, \dot{\psi}, Q$ – generalized coordinate, velocity and force, respectively.

To determine the coordinates of the points of the part as it moves, we use the $XZ$ axis, aligning the origin of the axes with the center of rotation of the disk. Taking into account that the distance $R_1$ is constant from the center of the rotating disk to the beginning of the transition of the radial groove into the pocket, we can postpone the beginning of the concerned coordinate system. We will obtain the point $Oxz$. The angle $\psi$ will be a generalized coordinate. The equations $x_O(\psi)$ and $z_O(\psi)$ will describe the motion of the center of mass of the part. The connection between the coordinates $x_O(\psi)$, $z_O(\psi)$ and the coordinate $\psi$ was obtained with Zinoviev’s method. Then we obtain the equations of closed vector contours $OxzNDOODOxz$ and $OxzOAMFOOxz$ (Figure 5, a), which can be written in the form

$$ (l_1 - x_1) + \frac{d_1}{2} + x_c = r_O; \quad h_k + x_2 + \frac{d_2}{2} + (l - x_c) = r_O. $$

**Figure 5.** Schemes for finding the coordinates of the center of mass of the part and generalized force in stages 1 (a) and 2 (b)

After designing the obtained equations on the axis $OxzX$ and $OxzZ$ and performing transformations, we obtain

$$ x_O(\psi) = -0.5(d_1 - d_2)\cos\psi - h_k + l\sin\psi \operatorname{ctg}(\beta + \psi) + 0.5d_2\sin\psi + (l - x_c)\cos\psi; $$

$$ z_O(\psi) = 0.5d_1\cos\psi + x_c\sin\psi, $$

where $d_2$ – is the diameter of the conical end of the part.

Coordinates of application of forces $F_1$ – $x_1 = 0$, $z_1 = h_k$ and $F_2$ – $z_2 = 0$, $x_2(\psi) = l - 0.5(d_1 + d_2)\sin\psi - l\cos\psi + F(\psi)\operatorname{ctg}(\beta + \psi)$.

The kinetic energy of the part is determined by the expression

$$ T = \frac{1}{2}m v_O^2 + \frac{1}{2}I_O\omega_O^2, $$

where $v_O$ – is the absolute velocity modulus of the center of mass of the part; $I_O$ – the moment of inertia of the part relative to the $O$ axis passing through the center of mass perpendicular to the plane of motion; $\omega_O = \dot{\psi}$ – instantaneous angular velocity,

$$ v_O = \sqrt{x_O^2 + z_O^2} = \psi \sqrt{\left(\frac{dx_O}{d\psi}\right)^2 + \left(\frac{dz_O}{d\psi}\right)^2}. $$
Then the kinetic energy will be written in the form \( T = \frac{\psi^2}{2} J \), where

\[
J = m \left[ \left( \frac{dx_O}{d\psi} \right)^2 + \left( \frac{dz_O}{d\psi} \right)^2 \right] + I_O
\]

is the given moment of inertia. We obtain expressions for finding the partial derivatives of kinetic energy by the generalized velocity \( \psi \), that is

\[
\frac{\partial T}{\partial \psi} = \psi \frac{d}{dt} J.
\]

Then we calculate the derivatives from these partial derivatives over time, considering that \( \psi \) and \( \psi \) are time functions

\[
\psi, \psi = \frac{d}{dt} T.
\]

The partial derivatives of kinetic energy by the generalized coordinate are determined by the expression

\[
\frac{\partial T}{\partial \psi} = \frac{\psi^2}{2} m \frac{d}{d\psi} \left( \frac{dx_O}{d\psi} \right)^2 + \frac{\psi^2}{2} m \frac{d}{d\psi} \left( \frac{dz_O}{d\psi} \right)^2.
\]

Then the left side of the equation of motion of the part in stage 1 can be represented as an expression

\[
\psi J + \frac{d}{dt} \left\{ m \left[ \left( \frac{dx_O}{d\psi} \right)^2 + \left( \frac{dz_O}{d\psi} \right)^2 \right] \right\} - \frac{\psi^2}{2} m \frac{d}{d\psi} \left( \frac{dx_O}{d\psi} \right)^2 - \frac{\psi^2}{2} m \frac{d}{d\psi} \left( \frac{dz_O}{d\psi} \right)^2.
\]

In order to find the expression for the generalized force \( Q \), all the forces acting on the part were identified: the gravity force was decomposed into components \( G_1 \), \( G_2 \); the active forces are the friction forces \( F_1 \), \( F_2 \), \( F_3 \) the parts about the pocket wall, which are determined from the static equilibrium equations of the part in the pocket (active forces); we add the force of inertia \( J_e \) in the transferable motion and the Coriolis force of inertia \( J_c \). The force of inertia \( J_e \) after the transformations will be equal in magnitude to the normal force of inertia \( J_{en} \) and is determined by the expression

\[
J_{en} = m h \omega^2
\]

where \( h = R + x_O \) is the distance to the fixed axis. The modulus of the Coriolis inertia force will be determined by the formula

\[
J_c = 2 m v_r \sin(\omega, \nu_r),
\]

where \( v_r = \dot{x}_O \), \( \sin(\omega, \nu_r) = 1 \). The generalized force \( Q \) will be determined by the expression

\[
Q = \sum_{k=1}^{n} \frac{\delta A}{\delta \psi} \delta \mathbf{r} \cos(\mathbf{F}_k, \delta \mathbf{r}),
\]

where \( \delta A \) is the operation of all \( n \) forces acting on the part on possible motion \( \delta r \). Then the sum of elementary work of all forces is

\[
\sum \delta A = F_3 \delta x - G_2 \cos \psi, \delta x - G_1 \delta z + J_{en} \delta x + F_2 \delta x_2 + F_1 \cos(\beta + \psi) \delta x_1 + F_1 \sin(\beta + \psi) \delta z_1,
\]

where \( \psi, \psi \) is the angle of rotation of the rotating disk, determined from the conditions:
The sum expression of elementary work for determining the generalized force is obtained by dividing the expression (8) into $\delta \psi$ and performing the transformations.

Then the right side of the equation (6) for stage 1 will be

$$
\left[-G_2 q - 2m_0 \left( \frac{\delta x}{\delta \psi} \right)^2 + \frac{\delta x}{\delta \psi} \left(R_1 + x_0(\psi)\right) \right] \frac{\delta x}{\delta \psi} - \frac{G_1}{L_N} \frac{\delta z}{\delta \psi} + \mu \left( \frac{I_{F_{1}} G_{1} G_{1}}{L_{A}} \right) \frac{\delta x}{\delta \psi},
$$

where $q = \mu \sin \psi f - \cos \psi f$.

For the second stage of motion of the part (Figure 5, b) the equations of closed vector contours $OxzDOOxz$ and $OxzOAOxz$ are composed. The we will obtain the expressions

$$
(l_1 - x_1) + \frac{d_1}{2} + x_c = r_0: d_1 + x_1 + \frac{d_1}{2} = (l_1 - x_c) = r_0.
$$

Having projected these equations on the axis $OxzX$ and $OxzZ$, and having transformed them we will obtain

$$
x_O(\psi) = \frac{d_1}{2} \sin \psi - \left(x_c + \frac{d_1}{\sin \psi}\right) \cos \psi; z_O(\psi) = \frac{d_1}{2} \cos \psi + x_c \sin \psi.
$$

The dependence of the coordinate of the application of forces $F_2$ and $F_1$ on $\psi$: $z_2 = 0$, $x_2(\psi) = x(\psi) - 0.5d_1 \sin \psi + x_c \cos \psi$, $x_1 = 0$, $z_1 = d_1$.

At the second stage, the left side of the equation of motion of the part will take the form of equation (7), and the generalized force will be determined by the expression (9).

At the third stage of motion of the part (Figure 6, a), we accept the rotation angle $\sigma$ of the center of mass of the part on the support $A$ as a generalized coordinate.

The coordinates of the center of mass of the part

$$
x_O(\sigma) = \frac{d_1}{2 \cos \alpha} \sin(\alpha - \sigma); z_O(\sigma) = \frac{d_1}{2 \cos \alpha} \cos(\alpha - \sigma) + d_1,
$$

where $\alpha$ is the angle determined by the expression $\alpha = a(\sin(\psi f \sin \sigma))$.

The kinetic energy of the part rotating around a fixed axis is determined by the formula

$$
T = \frac{1}{2} I_O \dot{\sigma}^2
$$

where $\dot{\sigma}$ is the value of the instant angular velocity of the part.

To determine the partial derivatives of the kinetic energy by the generalized velocity $\dot{\sigma}$, it is necessary to calculate their time derivatives. Then we will obtain the expression $\frac{d}{dt} \dot{\sigma} = I_O \dot{\sigma}$. Since the partial derivative of the kinetic energy in the generalized coordinate is equal to zero, the left side of this equation at the third stage will be written in the form $I_O \dot{\sigma}$. 

\[ \psi_i = \begin{cases} \psi_0 - \omega i \tau, & \text{если } \psi_0 - \omega i \tau \geq 0, \\ -\psi_0 + \omega i \tau, & \text{если } \psi_0 - \omega i \tau \leq 0. \end{cases} \]
Figure 6. Schemes for finding the coordinates of the center of mass and the generalized forces at stages 3 (a) and 4 (b)

The generalized force at stage 3 is calculated by the expression
\[ \sum \delta A = F_3 \delta x - J_6 \delta x + J_{en} \delta x - G_2 \cos \psi \delta x - G_1 \delta z. \]

Then at this stage, the motion of the part will be described by the expression
\[ I_0 \ddot{\sigma} = G_2 (\mu \sin \psi - \cos \psi) - 2m \omega^2 \left( \frac{\delta x}{\delta \sigma} \right)^2 + \left( \frac{\delta z}{\delta \sigma} \right)^2 \frac{\delta x}{\delta \sigma} + m \omega^2 (R_1 + x_0 \dot{\sigma}) \frac{\delta x}{\delta \sigma} - G_1 \frac{\delta z}{\delta \sigma}. \] (10)

At stage 4 (Figure 6, b), the generalized coordinate is the distance \( x_2 \) from the support \( A \) to the point \( K \). Having written the equation of the contour \( OxzAKOxz \), we obtain the expression
\[ d_1 + x_2 + 0.5d_1 = r_0, \]
and having projected it on the axis \( OxzX \) and \( OxzZ \), we obtained
\[ z_O(x_2) = 1.5d_1. \]
The point of application of force \( F_1 \) will be described by the coordinates
\[ x_1(x_2) = -x_2, \quad z_1(x_2) = d_1. \]

The kinetic energy of the part moving progressively is
\[ T = \frac{1}{2} m v_O^2. \]
The center of mass speed is
\[ v_O = \sqrt{\dot{x}_O^2 + \dot{z}_O^2} = x_2 \sqrt{\left( \frac{dx_O}{dx_2} \right)^2 + \left( \frac{dz_O}{dx_2} \right)^2}. \]
At the same time
\[ \frac{dx_O}{dx_2} = -1; \quad \frac{dz_O}{dx_2} = 0. \]
Then the kinetic energy of the part is
\[ T = \frac{m}{2} \dot{x}_2^2. \]
Since \( T = T(\dot{x}_2) \), first calculating the partial derivative of the kinetic energy by the generalized velocity \( \dot{x}_2 \), and then the derivative of this derived derivative by time, we find that
\[ \frac{d}{dt} \frac{\partial T}{\partial \dot{x}_2} = m \dot{x}_2. \]
The left side of the equation of motion (6) of the part is written in the form \( m \ddot{x}_2 \), since the partial derivative of the kinetic energy by the generalized coordinate \( \dot{x}_2 \) is equal to zero.

The elementary work of all forces are calculated by analogy with previous stages to determine the generalized force \( Q_{x_2} \). Having divided the desired expression by \( \delta x_2 \) and having converted it, we obtained the equation of motion of the part at stage 4 in the form of Lagrange equations of the second kind.
The mathematical model of the process of motion of the part at all stages of its passive orientation is represented by differential equations (7) - (11). It allows to determine the total time of passive orientation of the part in the improved device by expression \[ t_{or} = t_1 + t_2 + t_3 + t_4 \]. The solution of differential equations, describing the plane motion of the part through stages, can be found by numerical integration methods, for example, with the use of MathCad software package.

4. Discussion of results

Let us consider an example of using the developed mathematical model of the part motion process to determine the conditions that ensure reliability of the orientation of the parts with a conical end with parameters \( \beta = 11.5^\circ, \ d_1 = 0.01 \text{ m}, \ d_2 = 0.007 \text{ m}, \ l = 0.03 \text{ m}, \ m = 5 \text{ g} \) and a coefficient of friction \( \mu = 0.5 \) against the guiding surfaces of the hopper feeding device.

Using the graphs for the parts with a conical end at \( l/d_1 = 3 \) and \( \mu = 0.5 \): at \( \alpha_h = 45^\circ \) we determine the angle \( \psi_0 = 6^\circ \) at \( \alpha_h = 50^\circ - \psi_0 = 18.5^\circ \), at \( \alpha_h = 55^\circ - \psi_0 = 27^\circ \) (see Figure 3, b).

At first, we make the dependence of time \( t_d \) on angular velocity \( \omega \) using the expression (1) for each pair of values \( \alpha_h \) and \( \psi_0 \). Then we solve the differential equations (7) - (11) by numerical methods and obtain the time of motion of the part at each stage and the total time \( t_{or} \) of passive orientation of the part for each pair of values \( \alpha_h \) and \( \psi_0 \). At the same time, the integration boundaries of both \( \psi_0 \) and \( \psi_k \), \( \sigma_0 \), \( x_20 \) and \( x_2k \) for each stage were determined by graphical and graphoanalytical methods using the standard MathCad software package. The graphical representation of the obtained dependences is shown in Figure 7.

![Figure 7](image-url)  
**Figure 7.** Graphs of the dependences of the passive orientation time of the parts \( t_{or} \) and the time of passage of the pocket with the part of orientation zone \( t_d \) on the angular velocity at \( \alpha_h = 45^\circ \ (t_{or} - 1, t_d - 1), \alpha_h = 50^\circ \ (t_{or} - 2, t_d - 2) \) and \( \alpha_h = 55^\circ \ (t_{or} - 3, t_d - 3) \).

The range of permissible values of the angular velocities of the rotating disk with pockets, when passive orientation of the part will be ensured, is determined by the joint solution of inequality \( t_{or} \leq t_d \), equation (1) and differential equations (7) - (11). The intersection of the graphs \( t_{or} (\omega) \) and \( t_d (\omega) \) allow to determine the limit value of the angular velocity of the disk for the parts under consideration for each pair of values. For \( \alpha_h = 45^\circ \) we obtain \( \omega_{lim} \approx 0.3 \text{ rad/s} \), at \( \alpha_h = 50^\circ - \omega_{lim} \approx 1.1 \text{ rad/s} \), at \( \alpha_h = 55^\circ - \omega_{lim} \approx 1.6 \text{ rad/s} \) (see Figure 7). To the left of the limit value of the
Angular velocity of the disc is the area of admissible values of the angular velocity at the specified value of the inclination angle of the base of the improved hopper feeding device.

A boundary value curve is obtained by connecting the intersection points of each pair of graphs \( t_{op}(\omega) \) and \( t_{df}(\omega) \). It separates the boundary of the passive orientation zone from the zone in which passive orientation of the part is impossible. This curve makes it possible to evaluate the conditions under which reliable passive orientation will be provided for the parts under consideration at any values of the inclination angle of the hopper.

5. Conclusions

The developed mathematical model of orientation of the parts in the form of the bodies of revolution with implicit asymmetry of ends, one of which is conical, makes it possible to determine the conditions of reliable orientation of the parts depending on their geometric parameters, structural and kinematic parameters of the hopper feeding device with circular orientator and radial grooves made in the form of a comb.

The methodology and theoretical foundations of constructing the mathematical model of the process of passive orientation of the parts discussed in this article make it possible to determine the conditions for reliable orientation of the parts with their geometric parameters, structural and kinematic parameters of various hopper feeding devices.

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