Comments regarding the use of Monte Carlo method in the analysis of maritime ship maintenance works

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Abstract. Elaboration of a mathematical structure, together with a list of correspondences between the mathematical symbols and the objects of the concrete situation considered, led to what is called the mathematical model. The Monte Carlo simulation method is increasingly applied to the analysis of stochastic problems or in conditions of risk, when the same direction of action can have several consequences, the probabilities of which can be estimated. The Monte Carlo simulation can be used to analyze the flow of processes involved in the maintenance works of a ship in a shipyard, because it represents a system characterized by both input and output variables that can interact and therefore the same course of action may have several consequences. Thus, the execution of a certain type of maintenance work on the ship may lead to an increase or decrease in the volume of work initially estimated depending on: class society inspections, limiting the budget for works due to the crisis on the maritime transport market, labor force resources of the shipyard (limited in certain periods with peak load). Consequently, a stochastic model is outlined in which randomness is present, and the states of the variables are not described by unique values, but rather by probability distributions. The paper highlights, through concrete examples, some applications of the Monte Carlo method for the analysis of maintenance works performed for seagoing vessels.

1. Introduction
Models are much simpler representations of reality from which they retain the essential elements. Modeling is a method of studying processes and phenomena that are achieved by substituting the real object of research with a model.

Mathematical models arose from the need to describe and formally study the behavior of a category of real systems, in order to control and direct their future activity. The elaboration of a mathematical structure, together with a list of correspondences between the mathematical symbols and the objects of the concrete situation considered, led to what is called a mathematical model.

Random numbers are used in applications where it replaces the values of the random variable with a set of values that have its statistical properties, numerical simulation realistically reproducing certain elements of the simulated system. Within very long random number strings, some numbers may be repeated, but they meet certain requirements that bring them closer to the random ones, in which case they are called pseudo-random numbers.
There are several processes for producing random numbers, namely: tables with random numbers, physical processes, arithmetic processes [1].

1.1. Aspects regarding mathematical modeling
In general, a model \( M \) of an \( S \) system is another \( S' \) system which, in some respects, is equivalent to \( S \), but which is easier to study. By an \( S' \) system is meant the following set structure [2]:

\[
S = \{T, X, U, V, Y, \varphi, \eta\}
\]  

In equation (1) the meaning of the terms is the following:
- \( T \) is the base time used for timing and ordering events and it is a real number if the system is continuous time or integer if the system is discrete time;
- \( X \) represents the set of entries in the system;
- \( U \) is the set of system input segments (associated with the function \( u: T \rightarrow X \), graph of the function \( u \) over an interval \([t_0, t]\), which means \( u(t_0, t) = \{t, u(t) \mid t_0 \leq t \leq t_1\}\));
- \( V \) is the set of system states (the state is a concept of modeling the internal structure of the system that contains its history and which affects its present and future and, together with the shape of the inputs, uniquely determines the outputs of the system);
- \( Y \) is the set of system outputs;
- \( \varphi \) is the response function of the system \( \varphi: X \times V \rightarrow Y \); if at an input \( u([t_0, t]) \) the system is in the state \( \sigma_{t_0} \in V \), then the system output is \( Y = \varphi(u([t_0, t]), \sigma_{t_0}) \);
- \( \eta \) is the transition function of states, so if the input \( u([t_0, t]) \) finds the system in the state \( \sigma_{t_0} \in V \), then it transforms him into the state \( \sigma_{t_1} = \eta(u([t_0, t]), \sigma_{t_0}) \).

Knowledge of inputs \( u \in U \) and their corresponding answers \( y \in Y \), represents the behavior of the system.

A model of a system must meet the following three conditions [3,4]: the model must reflect as accurately as possible the represented reality; the model must be a simplification of the represented reality; the model is, by its essence, an idealization of the represented reality.

In the process of mathematical modeling, the components of the system are associated with certain variables / parameters, some known (controllable), called input, others unknown (uncontrollable), called output. The connections and interactions between the components of the system or the connections of the system with the outside are transposed in the mathematical model through functional relations (equations and / or identities).

The purpose of the model is to express uncontrollable variables according to controllable variables, so that performance criteria are met.

Sometimes it is not possible to express in the form of equations all the necessary connections, conditions and interdependencies, which is why some of them are described by logical conditions or procedures that can only be handled by computer.

The mathematical model completed with such procedures is a simulation model that, starting from values of controllable variables (generated with special algorithms), will produce values of uncontrollable variables, offering variants from which to choose the best.

Therefore, the simulation model produces experiments on the system it simulates, which allows the choice of those values of the variables and input parameters that lead to the desired performance.

Three types of simulation models are used in many scientific fields [2]:
- imitative models - transpose reality on another scale, larger or smaller, in order to observe the behavior of that reality;
- analogical models - are specific to a process or phenomenon whose behavior is not known and, in order to be studied, a realistic model of a phenomenon or process that presents analogies is used;
• symbolic models - use various symbols to represent the characteristics of a reality, the correlations leading to the writing of appropriate mathematical relationships and thus to the creation of an abstract (mathematical) model.

Simulation models can also be classified as such [5]:

• static models (do not explicitly take into account the time variable; reflects invariant and timeless situations and states; solutions can also be obtained analytically) or dynamic models (take into account the variation and interaction over time of the variables considered; incorporates time as a fundamental quantity, being a state variable; it is solved using the simulation technique);

• deterministic models (all variables are non-random; operative characteristics are equations of a certain form; the solutions of these models are obtained analytically) or stochastic models (contain one or more random input variables and therefore one of the operative characteristics is given by a density function; random inputs lead to random outputs; events do not occur with certainty, but with a certain probability);

• discrete models (changes in the states of variables are made at discrete moments of time) or continuous models (changes in the states of variables occur continuously).

In the process of mathematical modeling [4], the model is representative of the physical system if the causal condition is met, which leads to the classification of the elements into: input elements (cause) that form the input vector \( \mathbf{x} = (x_1, x_2, ..., x_m) \) and output elements (effect) that form the output vector \( \mathbf{y} = (y_1, y_2, ..., y_n) \), both vectors being generally random.

The simulation step is, by definition, a stage in which all input variables take constant values during program execution. Of great importance in building the simulation model is the process of "moving" the system over time – for this it is necessary to introduce a special variable called "simulation clock", which measures the flow of real time in which the system is simulated, in order to maintain the correct order in time of events.

The simulation models also contain: functional relations (identities and / or equations) and operative characteristics (used to express through mathematical relations the interactions of the variables and the behavior of the system).

After constructing the simulation model, the simulation itself, as an experiment, consists in varying the values of the variables and input parameters of the system and deduce based on the model, as a result of the calculations, their effects on the output variables.

There are two types of simulation: discrete, if the model variables can have only certain discrete values and continuous, if the model variables can have any value on certain real intervals.

The construction of simulation models is a broad process that, in general, involves the following steps [3]:

• defining the problem, the stage in which the objectives of the simulation are established;

• primary data collection, analysis, interpretation and processing;

• formulation of the simulation model; estimating the input parameters of the model;

• model performance evaluation and parameter testing;

• description of the simulation algorithm and writing of the calculation program;

• model validation;

• planning simulation experiments;

• analysis of simulated data.

1.2. Aspects regarding the generation of random number

It is called a generator, an arithmetic method of obtaining random numbers based on a recurrence relation of the form \( X_{n+1} = f(X_n, X_{n-1}, ..., X_{n-m}) \), where \( n > m, m \geq 0 \), \( (X_n)_{n \in \mathbb{N}} \) are natural numbers, assuming that the vector of the initial values \( X_0, X_1, ..., X_m \) it is fixed in advance.

In order for an arithmetic process to be called a generator, it must meet the following conditions: be simple and fast; have as long a period as possible (produce strings of numbers of any length, without
repeating the numbers of the string); to produce stochastically independent numbers from each other; to produce numbers whose distribution is uniform.

Building a random number generator [1] is quite difficult, on the one hand due to the high periodicity and on the other hand due to the statistical requirements of stochastic independence and uniformity. The idea of using arithmetic procedures for the algorithmic generation of numbers that have qualities close to random ones belongs to John von Neumann [3] who proposed a particular method known as the "middle part method". Knuth [4], concluded that random number generators should not be constructed by intuitive methods, but should be based on rigorous mathematical theories, generally based on recurrent methods., and Lehmer [5] initiated congruent methods that produced good results.

Let \( \{ \Omega, \mathbb{K}, P \} \) be a field of probability and \( X: \Omega \rightarrow \mathbb{R} \) a random variable.

Generating a random variable with the computer means choosing a number of elementary events \( \omega_1, \omega_2, ..., \omega_n \) and determine values \( X = X(\omega_i) \) of the random variable.

The values \( X_1, X_2, ..., X_n \) represents a selection on the random variable \( X \).

A generation is an algorithm that is able to produce an \( X_i \) and, iterating that algorithm, it should be able to produce \( X_0, X_1, ..., X_n \) so that they are stochastically independent and identically distributed.

The variables \( X_0, X_1, ..., X_n \) are stochastically independent if

\[
F(x_1, x_2, ..., x_n) = \prod_{i=1}^{n} F_i(x_i),
\]

In equation (2) \( F(x_1, x_2, ..., x_n) = P(X_1 < x_1, X_1 < x_i) \).

The variables \( X_0, X_1, ..., X_n \) are identically distributed only if they have the same distribution \( F_i(x) = F_j(x), i \neq j \).

Usually, random numbers are the values of a selection related to a random variable \( U \) that has a uniform distribution. If \( U \) is a uniform discrete random variable, then all its values are equally likely. If \( U \) is a uniform continuous random variable, then it admits a shape distribution density

\[
f(x) = \begin{cases} k, x \in (a, b) \\ 0, x \notin (a, b) \end{cases},
\]

In equation (3) \( k \) is determined by the condition (4)

\[
1 = \int_{-\infty}^{\infty} f(x)dx = \int_{a}^{b} kdx = k(b - a) \Rightarrow k = \frac{1}{b-a}
\]

and from a uniform distribution function (5)

\[
F(x) = \begin{cases} 0, x \leq a \\ \frac{x-a}{b-a}, a < x < b \\ 1, x \geq b \end{cases}
\]

In this case it is said that the random variable \( U \) has a uniform distribution on \((a, b)\). Uniform random numbers on \((0,1)\) are most important for numerical simulation.

Considering \( \mathcal{C} \) a family of random variables for which generation algorithms are known, the problem arises of determining an algorithm \( T \) that transforms a set of random variables into a set of random variables \( X = T(C_1, C_2, ..., C_n) \).

In other words, starting from a family of random variables for which efficient computational generation algorithms are known, it is required to determine the algorithm that generates random variables that are more difficult to generate.
For the generation of random variables, in the literature are used multiple methods: the inverse transform method; rejection method; methods specific to the given distribution; approximate methods; method of composition (mixing); composition-rejection method; particular methods for generating random variables; other methods of generating random variables.

2. Presentation of the Monte Carlo method

If a deterministic problem is associated with a random (probabilistic) model and by generating random variables functionally related to the solution, model experiments are performed and information about the solution of the deterministic problem is provided, the Monte Carlo simulation is used.

The expression "Monte Carlo method" [6] it is synonymous with the method of statistical experiments. The method is based on some conclusions resulting from the limit theorems of probability theory. To calculate a random variable we start from a random variable that has a uniform distribution over the range \([0,1]\) from the constructive probability field, field with a shape application \(\beta = f(x), x \in [0,1]\), with the property that \(\int_{0}^{\infty} f(x) dx = 1\).

The method involves estimating the parameters of the distribution of a random variable based on its achievements. Thus, the main problem is solved by the Monte Carlo method [6] consists in estimating the average value \(M(\xi)\) of a random variable \(\xi\) depending on a permissible error and a given probability. If \(\theta\) is the parameter associated with \(\xi\) what is to be represented by the value \(\hat{\theta}\) estimable on the basis of a sample \(\omega = (\omega_1, \omega_2, ..., \omega_n)\), then the estimator \(f(\omega)\) will have to satisfy the condition \(M(f(\omega) - \theta) \leq \varepsilon\), where \(\varepsilon\) is an acceptable deviation of the estimator from the theoretical random variable \(\theta\).

In the Monte Carlo method, the simple or weighted arithmetic mean is often used as an estimator.

For a random variable \(\xi\) with normal distribution, with average \(m\) and dispersion \(\sigma^2\), likelihood function in relation to \(m\) is

\[F(x_1, x_2, ..., x_n; m; \sigma^2) = \frac{1}{(2\pi)^{n/2}\sigma^n} \prod_{i=1}^{n} \exp \left[ -\frac{1}{2} \left( \frac{x_i - m}{\sigma} \right)^2 \right]; x_i, i = 1, n \]  

(6)

In equation (6) \(F(x_1, x_2, ..., x_n; m; \sigma^2)\) is maximum in relation to \(m\) and \(\sigma^2\) for achievements \(x_1, x_2, ..., x_n\) of \(\xi\) if condition (7)

\[
\begin{align*}
\frac{\partial \ln F}{\partial m} &= 0 \\
\frac{\partial \ln F}{\partial \sigma^2} &= 0
\end{align*}
\]

\(\hat{m} = \frac{1}{n} \sum_{i=1}^{n} x_i\)

whence it results (8)

\[\hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i + \hat{m})\]

(8)

For the random variable with normal distribution, the mean \(m\) is the maximum likelihood estimator. Sample quality can be assessed by concordance tests that measure the proximity of the empirical distribution to the theoretical distribution.

In order for the results obtained with the Monte Carlo method to be conclusive, the following aspects must be taken into account [7]:

- if after \(n\) simulations the frequency of occurrence of a certain event is \(f^*\), then, in reality, it will be within limits \(f = f^* \pm \sqrt{\frac{f(1-f)}{n}}\).
so that the maximum permissible error in determining the probability of occurrence of the
considered event does not exceed a given value \( \Delta \), it is necessary that the expression be made a
number of times given by the relation \( n = \frac{4p(1+p)}{\Delta^2} \), where \( p \) being the searched value of the
probability of occurrence of an event;

- if after \( n \) experiments a statistical mean value \( m \) was determined for a random variable, then
the average value will be within the limits \( m \pm \Delta \);

- if \( v_1 \) is a particular value of a variable \( V \) and it is desired to obtain the average value \( m \) with an
error of at most \( \Delta \), then the experiment must be repeated with number of times \( n = \frac{4D_v}{\Delta^2} \), where
\( D_v = \frac{1}{n} \sum_{i=1}^{n} V_i^2 - (m)^2 \) represents the dispersion of the size \( v \), determined from the results of
the first series of experiments \( n \);

- the accuracy of the Monte Carlo method can be estimated statistically with a finite degree of
certainty (is considered sufficient 0.990 – 0.997);

- an increase in the accuracy of the Monte Carlo method by one order of magnitude, increases
the calculation time by two orders of magnitude.

The Monte Carlo method is approximate, it can be successfully adapted to economic models, the
accuracy can be estimated correctly only as the calculations are performed, being determined by the
number of independent tests (inversely proportional to the probability) and their variation.

3. Applications of the Monte Carlo method in the analysis of ship maintenance processes

Based on the results obtained by the Monte Carlo method, in the simulation processes are obtained
various evaluations, hierarchies, which allow the substantiation of decisions in the management and
control of economic processes. Various authors have been interested in the subject and the literature
[1,6-16] highlights the following areas of applicability: complex storage processes (storage area is
limited, there are penalties for lack of stock); waiting processes - events are interconditioned; repair
processes; analysis of the critical path for complex projects - estimating the parameters of the
distribution of the total duration, determining the frequency of the critical character of the activities;
work processes - scheduling activities, loading equipment, launching activities; macro-economic
processes - the study of flows between branches, problems of economic growth.

In examples 1-4 are presented some examples, contribution made by the authors of the presented
paper.

3.1. Example for determined static model

Request: a shipyard that performs repair / maintenance works of seagoing ships can carry out
maintenance works in one month on three types of electric motors in the installations and systems of
ships contracted for maintenance works: 4 pcs / hour from the first type of electric motors, 2 pcs / hour
from the second type and 3 pcs / hour from the third type. The benefit obtained from the execution of
these maintenance works on electric motors is presented as follows: 12 USD for the first type of
electric motors, 25 USD for the second type and 18 USD for the third type. Shipyard capacity is
limited to 250 electric motors of the first type, 120 of the second type and 80 of the third type.
Knowing that they work 160 hours a month, it is required to draw up the monthly work schedule,
which should contain the number of electric motors of each type to be repaired, so that the benefit
obtained is maximum.

Solving: the construction of the model can be done this way: is denoted by \( x_1, x_2, x_3 \) the number of
devices of each type and is obtained the system (9) for a static and deterministic model.
\[
\begin{align*}
4x_1 + 2x_2 + 3x_3 & \leq 160 \\
x_1 & \leq 250 \\
x_2 & \leq 120 \\
x_3 & \leq 80 \\
x_1, x_2, x_3 & \geq 0 \\
\max (12x_1 + 25x_2 + 18x_3)
\end{align*}
\] (9)

3.2. Example for stochastic model
Request: a shipyard for maintenance work on ships must order daily a number of oxygen cylinders necessary to carry out sheet metal replacement work on ship structures so that the gain from their use is maximum.

Every day, the shipyard orders a certain number of oxygen cylinders, using some or all of them. Each one generates a certain gain. Unused oxygen cylinders can be returned, but they generate a certain loss.

The number of oxygen cylinders used varies from day to day and the probability of using a certain number of oxygen cylinders in a day can be estimated based on previous use.

Solving: the construction of the model can be done as follows: \( n \) = the number of oxygen cylinders ordered each day; \( c \) = the gain from the use of an oxygen cylinder; \( l \) = loss due to an unused oxygen cylinder; \( r \) = requested number of oxygen cylinders used in a day; \( p(r) \) = the probability that \( r \) oxygen cylinders will be used on any given day; \( P \) = the gain obtained in one day.

If in a day the number of oxygen cylinders required (demand) is greater than or equal to the number of oxygen cylinders ordered \( (r \geq n) \), the gain will be \( P(r \geq n) = nc \). If in a day the number of oxygen cylinders used (demand) is less than the number of oxygen cylinders ordered \( (r < n) \), the gain will be \( P(r < n) = rc - (n - r)l \). The average daily gain will be as in equation (10)

\[
P = \sum_{r=0}^{n} p(r)[rc - (n - r)l] + \sum_{r=n+1}^{\infty} p(r)nc
\] (10)

This is a stochastic model because: \( n \) = controllable variable; \( r \) = uncontrollable variable; \( c, l \) = constants. By solving this model we must find \( n \) for which \( P \) is maximum.

3.3. Example 1 for the Monte Carlo method
Request: in the electrical works workshop of a shipyard, the main electrical distribution panels are checked for the equipment of ships contracting docking works. The number of electrical contacts checked and cleaned on board of the ships under repair works, for 100 days, is shown in the table 1.

| Number of electrical contacts checked (pcs / day) | Number of cases |
|-----------------------------------------------|----------------|
| 120                                           | 2              |
| 200                                           | 5              |
| 220                                           | 11             |
| 280                                           | 15             |
| 320                                           | 20             |
| 410                                           | 15             |
| 450                                           | 12             |
| 490                                           | 13             |
| 520                                           | 5              |
| 590                                           | 2              |

Table 1. Data recording.
The management of the shipyard wants to know the performances obtained in time at the electrical ship repair workshop to establish a competitive price compared to other shipyards in the area. Solving: performance of the electrical works workshop as regards the verification of the connections to the contact terminals of a main electrical distribution panel on board a ship can be determined with the Monte Carlo simulation method, in which the performance evaluation operation is done starting from the observations made during the 100 days (as in table 2).

| No. | Number of electrical contacts checked (pcs / day) | Probability of achievement | Cumulative probability |
|-----|-------------------------------------------------|-----------------------------|------------------------|
| 1   | 120                                             | 0.02                        | 0.02                   |
| 2   | 200                                             | 0.05                        | 0.07                   |
| 3   | 220                                             | 0.11                        | 0.18                   |
| 4   | 280                                             | 0.15                        | 0.33                   |
| 5   | 320                                             | 0.20                        | 0.53                   |
| 6   | 410                                             | 0.15                        | 0.68                   |
| 7   | 450                                             | 0.12                        | 0.80                   |
| 8   | 490                                             | 0.13                        | 0.93                   |
| 9   | 520                                             | 0.05                        | 0.98                   |
| 10  | 590                                             | 0.02                        | 1.00                   |

The cumulative probability is plotted in figure 1.

![Figure 1. Cumulative probability.](image_url)

It extract 20 numbers from a table 3 with random numbers in the range \([0,1]\) and calculate the average \(\bar{X}\), arithmetic mean deviation \(\sigma\), coefficient of variation \(C_v\) and the confidence interval.
Table 3. Data recording refer to the number of electrical contacts checked (pcs / day).

| No. | No. randomly | Number of electrical contacts checked (pcs / day) | \( (x_i - \bar{X}) \) | \( (x_i - \bar{X})^2 \) |
|-----|--------------|--------------------------------------------------|------------------------|--------------------------|
| 1   | 0.0317       | 200                                              | -174.5                 | 30450.25                 |
| 2   | 0.9369       | 520                                              | 145.5                  | 21170.25                 |
| 3   | 0.3406       | 320                                              | -54.5                  | 2970.25                  |
| 4   | 0.0200       | 120                                              | -254.5                 | 64770.25                 |
| 5   | 0.9650       | 520                                              | 145.5                  | 21170.25                 |
| 6   | 0.6568       | 410                                              | 35.5                   | 1160.25                  |
| 7   | 0.7571       | 450                                              | 75.5                   | 5700.25                  |
| 8   | 0.6174       | 410                                              | 35.5                   | 1260.25                  |
| 9   | 0.1511       | 220                                              | -154.5                 | 23870.25                 |
| 10  | 0.0306       | 200                                              | -174.5                 | 30450.25                 |
| 11  | 0.2333       | 280                                              | -94.5                  | 8930.25                  |
| 12  | 0.6603       | 410                                              | 35.5                   | 1260.25                  |
| 13  | 0.5447       | 410                                              | 35.5                   | 1260.25                  |
| 14  | 0.9134       | 490                                              | 115.5                  | 13340.25                 |
| 15  | 0.7861       | 450                                              | 75.5                   | 5700.25                  |
| 16  | 0.6969       | 450                                              | 75.5                   | 5700.25                  |
| 17  | 0.9277       | 490                                              | 115.5                  | 13340.25                 |
| 18  | 0.2388       | 280                                              | -94.5                  | 8930.25                  |
| 19  | 0.5773       | 410                                              | 35.5                   | 1260.25                  |
| 20  | 0.6971       | 450                                              | 75.5                   | 5700.25                  |
|     | Total        | 7490                                             |                        | 268495.00                |

\[
\bar{X} = \frac{7490}{20} = 374.5; \\
\sigma^2 = \frac{268495}{20} = 13424.75; \\
\sigma = 115.86; \\
G_P = \frac{115.86}{374.5} = 0.309
\]

The t distribution (Romanovski test) is used to verify the hypothesis regarding the average number of electrical contacts verified at a main electrical distribution panel. The interval indicating the number of electrical contacts checked daily is \( \bar{X} \pm t_{\alpha} \frac{\sigma}{\sqrt{N}} \).

Performing the calculations, the number of electrical contacts verified as between 320 contacts / day and 428 contacts / day is obtained.

The number of experiments considered (20) makes this interval too large.

Repeating the calculations (EXCEL software was used) for other numbers of experiments, stabilizes around 360 contacts / day, which represents the performance over time of the electrical workshop (as in table 4).

Table 4. Medium value / number of experiments.

| No. experiments | 400  | 2500 | 4500 | 6000 | 7500 | 9000 |
|----------------|------|------|------|------|------|------|
| Medium value   | 359.4 | 359.3 | 361.7 | 359.3 | 360.7 | 360.8 |
3.4. Example 2 for the Monte Carlo method

Request: the valve repair shop within the ship repair mechanical section uses 500x500x0.5 mm klinger sheet metal to replace the gaskets on the valves dismantled from the ships. For a period of 100 days, the consumption for this assortment was followed, respectively pcs / day. Following the observations, the data on registered consumptions were centralized, according to the data presented in the table 5.

Table 5. Registered data.

| Number of klinger sheet metal 500x500x0.5 mm (pcs / day) | No. cases |
|----------------------------------------------------------|-----------|
| 5                                                        | 2         |
| 10                                                       | 5         |
| 15                                                       | 3         |
| 21                                                       | 7         |
| 25                                                       | 12        |
| 30                                                       | 10        |
| 32                                                       | 11        |
| 35                                                       | 15        |
| 37                                                       | 15        |
| 40                                                       | 20        |

The supply department wants to know the optimal daily volume of klinger sheet metal consumption, to cover the demand and avoid the situation of remaining with the product in stock.

Solving:

It is possible to determine the volume of consumption required using the Monte Carlo simulation method, in which the estimation operation is carried out based on observations on the registered orders obtained during the 100 days.

In table 6 was calculate the probability and the cumulative probability.

Table 6. Calculus of the probability and the cumulative probability.

| No. | Number of klinger sheet metal 500x500x0.5 mm (pcs / day) | Probability of achievement | Cumulative probability |
|-----|---------------------------------------------------------|-----------------------------|------------------------|
| 1   | 5                                                       | 0.02                        | 0.02                   |
| 2   | 10                                                      | 0.05                        | 0.07                   |
| 3   | 15                                                      | 0.03                        | 0.10                   |
| 4   | 21                                                      | 0.07                        | 0.17                   |
| 5   | 25                                                      | 0.12                        | 0.29                   |
| 6   | 30                                                      | 0.10                        | 0.39                   |
| 7   | 32                                                      | 0.11                        | 0.50                   |
| 8   | 35                                                      | 0.15                        | 0.65                   |
| 9   | 37                                                      | 0.15                        | 0.80                   |
| 10  | 40                                                      | 0.2                         | 1.00                   |

The cumulative probability is plotted in figure 2.
It extract 20 numbers from a table 7 with random numbers in the range \([0,1]\) and calculate the average \(\bar{X}\), arithmetic mean deviation \(\sigma^2\), coefficient of variation \(C_v\) and the confidence interval.

### Table 7. Data recording refer to the number of klinger sheet metal.

| No. | No. randomly | Number of klinger sheet metal 500x500x0.5 mm (pcs / day) | \((x_i - \bar{X})\) | \((x_i - \bar{X})^2\) |
|-----|-------------|----------------------------------------------------------|----------------------|------------------------|
| 1   | 0.3217      | 30                                                       | -0.2                 | 0.04                   |
| 2   | 0.6274      | 35                                                       | 4.8                  | 23.04                  |
| 3   | 0.9183      | 40                                                       | 9.8                  | 96.04                  |
| 4   | 0.2741      | 25                                                       | -5.2                 | 27.04                  |
| 5   | 0.4320      | 32                                                       | 1.8                  | 3.24                   |
| 6   | 0.6272      | 35                                                       | 4.8                  | 23.04                  |
| 7   | 0.7100      | 37                                                       | 6.8                  | 46.24                  |
| 8   | 0.2304      | 25                                                       | -5.2                 | 27.04                  |
| 9   | 0.8901      | 40                                                       | 9.8                  | 96.04                  |
| 10  | 0.5327      | 35                                                       | 4.8                  | 23.04                  |
| 11  | 0.3421      | 30                                                       | -0.2                 | 0.04                   |
| 12  | 0.7082      | 37                                                       | 6.8                  | 46.24                  |
| 13  | 0.1973      | 25                                                       | -5.2                 | 27.04                  |
| 14  | 0.6327      | 35                                                       | 4.8                  | 23.04                  |
| 15  | 0.4268      | 32                                                       | 1.8                  | 3.24                   |
The distribution (Romanovski test) is used to verify the hypothesis regarding the klinger sheet metal consumption. The interval indicating the number of klinger sheet metal consumption per day is 

\[ \bar{x} \pm t_{\frac{\alpha}{2}, N-1} \frac{\sigma}{\sqrt{N}} \]

Number of experiments, \( N = 20 \); degrees of freedom, \( g = N-1 = 19 \); permissible error, \( \alpha = 0.05 \); confidence interval, \( (1- \alpha) = 0.95 \).

It is obtained results from equations (11) and (12):

\[ \bar{x} - t_{\frac{\alpha}{2}, N-1} \frac{\sigma}{\sqrt{N}} = 30.2 - 2.093 \frac{8.91}{4.47} = 26.3 \]  \hspace{1cm} (11)

\[ \bar{x} + t_{\frac{\alpha}{2}, N-1} \frac{\sigma}{\sqrt{N}} = 30.2 + 2.093 \frac{8.91}{4.47} = 34.37 \]  \hspace{1cm} (12)

The daily consumption of klinger sheet metal is between 26 pcs / day and 34 pcs / day. The number of experiments considered (20) makes this interval too large.

Repeating the calculations (EXCEL software was used) for other numbers of experiments, stabilizes around e 30 sheets / day, which represents the optimum klinger sheet metal consumption per day (as in table 8).

| No.experiments | 3500 | 1750 | 3500 | 5000 | 6500 | 8500 |
|----------------|------|------|------|------|------|------|
| Medium value   | 29.875 | 29.642 | 31.482 | 29.467 | 30.737 | 30.972 |

4. Conclusions
Monte Carlo simulation can be used to analyze the flow of processes involved in the maintenance of a ship in a shipyard, whereas it is a system characterized by input variables and output variables that can interact and therefore the same direction of action can have several consequences.

Thus, the execution of a certain type of maintenance work on the ship (replacements of body structures, treatment of interior surfaces) may lead to an increase or decrease in the volume of work initially estimated depending on: class inspections, limitation of the budget for works due to the crisis on the maritime transport market, labor resources of the shipyard (limited in certain periods with peak load).
Consequently, a stochastic model is outlined in which the random character is present, and the states of the variables are not described by unique values, but rather by probability distributions.

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