Asymptotic method for predicting the resonance during the descent of an asymmetric probe in the Martian atmosphere

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Abstract. This work is devoted to the study of the uncontrolled descent of a probe with small asymmetries in the atmosphere of Mars. The aim of the work is to formulate a method for predicting the principal resonance in the problem of the descent of a space probe with small geometric and aerodynamic asymmetries in the atmosphere of Mars. The noted resonance is observed in the case of a long-term coincidence of two characteristic frequencies of a nonlinear system of equations simulating the motion of the probe relative to the center of mass. The applying of the averaging method makes it possible to analyze non-resonant evolutions in the angular velocity and spatial angle of attack caused by the influence of the principal resonance. Presented resonance prediction method can find application in a whole class of problems of modern astronautics.

1. Introduction
The perturbed uncontrolled motion of a spacecraft relative to the center of mass in the atmosphere is analyzed in a significant number of publications, for example [1-2]. It was shown in [3] that disturbing moments from inertial and aerodynamic asymmetries are the cause of the appearance of the principal resonance in the problem of descent of the spacecraft in the atmosphere. It is also known that resonant phenomena can lead to an accident when the braking parachute system of a spacecraft is deployed. In particular, secondary resonant effects contribute to the violation of the specified restrictions on the angular velocity during the descent of an asymmetric spacecraft [4-5]. These evolutionary phenomena are explained by the influence of resonance on non-resonant regions of motion of a dynamical system [6]. The effects were discovered theoretically in the analysis of the perturbed fast rotation of a satellite containing a magnetic damper on board [7]. It should be noted that small resonance-induced moments were also obtained when studying the rotational motion of an asymmetric spacecraft in the atmosphere [8].

The aim of this work is to formulate a method for predicting the principal resonance during the uncontrolled descent of the probe with small geometric and aerodynamic asymmetries in the atmosphere of Mars.

2. Mathematical model
The mathematical model is based on the application of a nonlinear low-frequency system of equations of motion for a probe with small asymmetry with respect to the center of mass. This system was obtained by the method of integral manifolds [9]. The low-frequency system is considered in conjunction with a
system of equations that determine the motion of the center of mass of the probe [5]. To predict the principal resonance, it is necessary to be able to analyze the evolution of the motion of the probe relative to the center of mass. However, a direct analysis of the evolutions of slow variables in a low-frequency system is difficult, since its right-hand sides contain a fast variable \( \theta \). Let's write a low-frequency system as a system with one fast variable and three slow variables

\[
\frac{dz}{dt} = \varepsilon Z(z, \theta, \varepsilon) \quad (1)
\]

\[
\frac{d\theta}{dt} = \Delta \quad (2)
\]

where \( z = (\omega_x, \alpha, \omega) \) is the vector of three slow variables; \( Z(z, \theta) \) is the vector of right-hand sides of the equations (1). The neighborhood of the principal resonance of system (1)-(2) is estimated by the condition \( \frac{d\theta}{dt} = O(\sqrt{\varepsilon}) \). Averaging the system of equations (1)-(2) taking into account the first two approximations in the non-resonant case

\[
\left\langle \frac{d\omega_x}{dt} \right\rangle = \varepsilon A^\omega_1 + \varepsilon^2 A^\omega_2 \quad (3)
\]

\[
\left\langle \frac{d\alpha}{dt} \right\rangle = \varepsilon A^\alpha_1 + \varepsilon^2 A^\alpha_2 \quad (4)
\]

where \( A^\omega_1, A^\omega_2, A^\alpha_1, A^\alpha_2 \) are functions of the first and second approximations, which depend on slow variables \( \omega_x, \alpha, \omega; \varepsilon \) is a small dimensionless parameter, which explains the small value of the geometric and aerodynamic asymmetries. By defining the functions \( A^\omega_1, A^\omega_2, A^\alpha_1, A^\alpha_2 \), the author gets the averaging equations for the angular velocity \( \omega_x^0 \) and angle of attack \( \alpha^0 \), taking into account the first and second approximations of the averaging method:

\[
A^\omega_1 = 0, \quad A^\alpha_1 = 0, \quad A^\omega_2 = \text{sign}(\omega_x) m_x \frac{\omega^2 I_x \omega_h}{\omega} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
+ \text{sign}(\omega_x) m_x \frac{\omega^4 I_x \omega_h}{\omega} \frac{\partial^2}{\partial \omega^2} \text{cost}(\theta_1 - \theta_2) -
\]

\[
- \frac{1}{2 I_x \omega} \frac{\partial^2}{\partial \omega^2} A^\omega_2 = -\text{sign}(\omega_x) m_x \frac{\omega^4 I_x \omega_h}{\omega} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
+ \text{sign}(\omega_x) \frac{1}{I_x F_\alpha (\omega_h - \omega_h)} \omega \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
+ \text{sign}(\omega_x) \left( \frac{m^2 - m^A}{m^2 - m^A} \right) \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) -
\]

\[
+ \text{sign}(\omega_x) \frac{1}{m^2 - m^A} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
+ \text{sign}(\omega_x) \frac{m^2 - m^A}{m^2 - m^A} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
= 1 \frac{\partial m^A}{\partial \omega} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
= \frac{1}{\omega^2} \frac{\partial m^A}{\partial \omega} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]

\[
= \frac{1}{\omega^2} \frac{\partial m^A}{\partial \omega} \frac{\partial}{\partial \omega} \text{cost}(\theta_1 - \theta_2) +
\]
Here OXYZ is a system of frames rigidly connected to the hull by the probe, \( \alpha \) is the spatial angle of attack, \( m_1^m, m_2^A, \theta_1, \theta_2 \) are generalized asymmetry parameters of geometric and aerodynamic asymmetries, \( \omega_x \) is the angular velocity, \( \theta \) is the fast variable, \( \omega_a = \sqrt{\frac{I_x \omega_x^2}{4 + \omega^2}} \); \( -A \), \( m_1^m \) are dimensionless parameters that determine the values of the asymmetries; \( m_2^m \) is the coefficient of aerodynamic restoring torque, \( \omega \) is the frequency of probe precession in the case \( \omega_x = 0 \), \( I_x, I_y, I_z \) are the principal axial moments of inertia of the probe, \( \Delta = \omega_x - \omega_{A_2} \) is the resonant ratio of the frequencies; \( F_0 \) is the function of the slow variables [5].

From equations (3)-(4) it follows that the first approximations equal to zero. Consequently, the non-resonant evolution of the averaged velocity \( \omega_x \) is defined by the second approximation. Therefore, equation (3) contains dependences on dimensionless parameters of geometric asymmetry \( \frac{A}{m_1^m, \theta_1} \) and geometric-aerodynamic asymmetry \( \frac{m_2^m, \theta_2}{\theta_1} \).

The resonant values of angular velocity are determined from solutions of the equalities \( \omega_x^F - \omega_x = 0 \) \((\omega_x > 0)\) and \( \omega_x^F - \omega_x = 0 \) \((\omega_x < 0)\). As a result, you can get

\[
\omega_x^F = \text{sign}(\omega_x) \frac{\omega}{\sqrt{1 - \omega^2}}
\]

where \( \omega = \sqrt{-m_2^m q S L \tan \alpha / I} \), \( q \) is the dynamic pressure, \( S \) is the area of the probe section, \( L \) is the characteristic longitudinal size of the probe.

It should be noted the presence of frequency ratios \( \Delta, \Delta^2 \) in the denominators of the averaged equation (3). If the derivative (3) is zero, the stationary points are formed. Thus, the sign of the derivative (3) defines the evolutionary process of the angular velocity \( \omega_x \).

Let's write the second derivative of the angular velocity \( \omega_x \). After non-resonant averaging, the second derivative has a form:

\[
\frac{d^2 \omega_x}{dt^2} = \frac{\partial \langle \dot{\omega}_x \rangle}{\partial \omega_x} \frac{d \omega_x}{dt} + \frac{\partial \langle \dot{\omega}_x \rangle}{\partial \theta} \frac{d \theta}{dt} + \frac{\partial \langle \dot{\omega}_x \rangle}{\partial \alpha} \frac{d \alpha}{dt}
\]

The sign of second derivative (6) defines the direction of convexity of the function \( \omega_x(t) \). Furthermore, the equality \( \langle \dot{\omega}_x \rangle = 0 \) determines the inflection points of the averaged angular velocity \( \omega_x(t) \).

Applying expressions (3) and (6), it is possible to obtain a new condition that provides a non-resonant tendency of the angular velocity \( \omega_x(t) \) to resonance values \( \omega_x^F(t) \)

\[
\left\{ \frac{d \omega_x}{dt} \right\}^2 \frac{d^2 \omega_x}{dt^2} > 0
\]
The condition (7) characterizes the effect of the principal resonance on the derivatives of the angular in two non-resonant sites adjacent to this resonance. This condition applies when predicting the principal resonance.

3. Resonance prediction method and numerical results

Averaging the low-frequency system in the non-resonant case, the author obtains the averaged equations of the derivatives (3)-(4), (6) that contains the frequency ratios in the denominators of second approximations. It should be noted that when this system falls into the region of external stability \( \frac{1}{\sqrt{\epsilon}} \leq |A| \leq \frac{1}{\sqrt{\epsilon}} \) of the resonance, the averaged equation indicates that dynamic system approaches to the resonance or moves away from it. In this way, the averaged equations (3)-(4), (6) should be considered as containing numerical data on the possible realization of the principal resonance. It can be proved that the change in slow variables with a non-resonant tendency of the system to resonance is externally stable. Moreover, this evolution has realized with invariable sign of a curvature of the angular velocity \( \omega_x(t) \). In this case, the absence of near-resonant stationary points and inflection points is required. Hence, the averaged equations can be applied to predict the occurrence of resonance.

Let us formulate an algorithm for predicting the possible realization of the principal resonance in the low-frequency system. The resonance prediction algorithm includes the following elements, shown in figure 1. From this algorithm it follows that the process of predicting the principal resonance is iterative. Indeed, it involves the use of numerical solutions of equations (3),(4),(6). Note that predicting of the location of the resonant magnitudes (5) relative to the current value of the angular velocity \( \omega_x(t) \), the simultaneous fulfillment of three conditions is required: 1. fulfillment of the predetermined condition providing a limitation in time of descent \( t_{i+1} < T \) (T-preset limitation in time); 2. fulfillment of the condition \( \langle \omega_x \rangle \langle \dot{\omega}_x \rangle > 0 \) for the evolution of the slow component of the angular velocity into the resonance region; 3. fulfillment of the equality \( \langle A \rangle = O(\sqrt{\epsilon}) \).

![Figure 1. Principal resonance prediction algorithm.](image)

If all three conditions are fulfilled simultaneously, the algorithm displays a message about reaching the resonance region. If at least one of these conditions is not fulfilled, then the algorithm concludes that there is no achievement of the resonance domain. It should be noted that the identification characteristic evolutions performed by monitoring modules and signs derivatives \( \langle \dot{\omega}_x \rangle, \langle \ddot{\omega}_x \rangle \). In addition, this analysis allows predicting the relative location of the resonant values and the current values of the angular
velocity. The result can be illustrated by means of figures 2-3. Figures 2-3 show the evolution of angular velocity $\omega_x$ as a function of time. In figure 2, the solid curve describes the non-resonant evolution in angular velocity. The dotted curve shows the evolution of the resonance magnitude of the angular velocity (5). It follows from figure 2 that the first and second derivatives of averaged angular velocity (3) and (6) are negative while striving the resonant value at $\omega_x(0) > \omega_x^r(0) > 0$.

Figure 3 shows that first and second derivatives (3) and (6) are positive in the process of approaching of the resonant value at $\omega_x(0) > \omega_x(0) > 0$. In figure 3, the solid curve also describes a non-resonant evolution of the angular velocity $\omega_x(t)$. The dashed curve presents the evolution of the resonant value of the angular velocity (5). When obtaining of figures 2-3 applied equations (5) and (3). In addition, the probe had a shape close to conical. The mass-geometrical parameters of the probe Mars Polar Lander were used in figures 2-3. The initial value of the angular velocity is $\omega_x(0) = 1 \text{s}^{-1}$. The dimensionless parameters of asymmetries are $\overline{m_A} = 0.02$, $\overline{m} = 0.02$, $\theta_1 - \theta_2 = \pi$. In obtaining figure 3, similar mass-inertial parameters and initial conditions of the probe motion were used. However, the initial angular velocity $\omega_x(0)$ is $0.01 \text{s}^{-1}$. In addition, the asymmetry parameters are $\overline{m_A} = 0.01$, $\overline{m} = 0.001$, $\theta_1 - \theta_2 = \pi$.

**Figure 2.** Evolution of the angular velocity to the resonance value at $\omega_x(0) > \omega_x^r(0) > 0$.

**Figure 3.** Evolution of the angular velocity to the resonance value at $\omega_x^r(0) > \omega_x(0) > 0$.

4. Conclusion

It is shown in the work that the analysis of the derivatives of the averaged angular velocity makes it possible to predict the relative location of the current angular velocity and its resonance value. Note that a new forecasting algorithm was also formulated for the occurrence of the principal resonance during the descent of an asymmetric probe in the atmosphere of Mars. The presented method for predicting the principal resonance is based only on the asymptotic analysis of the non-resonant evolution of the angular velocity.

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