Hadronic Sector in the 4-d Pseudo-Conformal Field Theory

C. N. Ragiadakos
email: ragiadak@gmail.com

ABSTRACT

The pseudo-conformal field theory (PCFT) is a 4-d action, which depends on the lorentzian Cauchy-Riemann (LCR) structure. Like the 2-d Polyakov action, it does not depend on the metric tensor. But the invariance under the pseudo-conformal transformations (in the terminology of E. Cartan and Tanaka) imposes in the action the existence of a gauge field instead of the scalar field of the Polyakov action. The tetrad of the LCR-structure defines a class of metrics and a corresponding class of self dual 2-forms. I prove that the inverse is also valid. Einstein has showed that the equations of motion of the black-hole essential singularities is a consequence of the regularity of the metric tensor. Hence its equivalence with the LCR-structure implies that these equations assure the regularity of the LCR-structures. This permit us to determine the multisolitons of the PCFT. After the expansion of the action around the static LCR-structure soliton, the quadratic part of the Yang-Mills-like term implies a linear partial differential equation (PDE). I solve this PDE using the Teukolsky method of solution of the electromagnetic field in the background of the Kerr black hole. The angular and radial ODEs are different to the corresponding Teukolsky master equations, permitting the possible identification of the quark as a soliton bound state of the static LCR-structure and the gluon. These two results opens up the possibility of the numerical computations of the standard model parameters and the hadron form factors, permitting the experimental check of PCFT.
Contents
1. INTRODUCTION
2. SOLITONS AND MULTISOLITONS
3. THE QUARKS
   3.1 Study of the angular differential equation
   3.2 Study of the radial differential equation
4. PERSPECTIVES
1 INTRODUCTION

A renormalizable generally covariant 4-dimensional quantum field theory\cite{16}, which depends on the lorentzian Cauchy-Riemann (LCR) structure and not the spacetime metric, seems to provide the appropriate framework to describe current phenomenology. Following the terminology of Cartan and Tanaka, I call this lagrangian model pseudo-conformal field theory (PCFT). In my last work\cite{24}, I derived the electromagnetic and weak interactions of the standard model (SM), where the particle-like static LCR-manifold is identified with the electron and its complex conjugate LCR-manifold with the positron. The massless limit of this LCR-manifold has only a left-handed part and it is identified with the neutrino. The SM is the effective action implied after the introduction of quantum fields for the electron and the neutrino solitons, in complete analogy to the condensed matter considerations. The interaction terms are introduced using the Bogoliubov-Medvedev-Polivanov (BMP)\cite{3} S-matrix computational procedure, which in the present case happens to close up to the well known SM action. The spontaneously broken $SU(2) \times U(1)$ group naturally emerges from the BMP closing up procedure. The internal symmetry is purely effective, and implied by the fact that the initial LCR-structure solitons (the massive electron and the massless neutrino) break the symmetry. The renormalizability condition restricts the masses and the coupling constants to take the appropriate relations. The same condition does not permit the incorporation of the linearized soliton-graviton interaction into the SM, because the closing up procedure generates non-renormalizable terms. The gluonic Yang-Mills-like term in the PCFT action does not coincide with conventional quantum chromodynamics (QCD). In the linearized gravity approximation, its Poincaré invariance is not conventional. Therefore it cannot be included in the effective SM quantum field theory\cite{2}. On the other hand ordinary QCD is apparently wrong, because it does not describe confinement. I point out that the PCFT gluon perturbative potential is linear, while QCD implies the $(1/r)$ Coulomb-like potential. In the present work I use solitonic techniques\cite{25}, and I identify the quark as the possible (colored) gluon bound state in the static LCR-structure background.

The original idea\cite{16,21} to study LCR-structure dependent field theories emerged from the observation that the Polyakov (linearized) string action

$$I_S = \frac{1}{2} \int d^2 \xi \sqrt{-g} \gamma^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu \nu}$$

(1.1)
does not essentially depend on the metric $\gamma^{\alpha \beta}$ of the 2-dimensional surface, because in the light-cone coordinates ($\xi_-$, $\xi_+$) the metric and the action take the form

$$ds^2 = 2\gamma d\xi_+ d\xi_-$$

and

$$I_S = \int d^2z \partial_- X^\mu \partial_+ X^\nu \eta_{\mu \nu}$$

(1.2)
The action does not depend on the metric, while it is not topological. This metric independence of the action, without being topological, is the crucial property of the Polyakov action, which should be transferred to four dimensions.
and not the simple Weyl invariance. That is, the four dimensional analogous symmetry must be of pseudo-conformal symmetry (Cauchy-Riemann structure) and not the conventional Weyl symmetry. The pioneers of the CR-structure, E. Cartan and Tanaka, used to call it pseudo-conformal structure, therefore I prefer to call the present lagrangian model pseudo-conformal field theory in order to point out that it is essentially a 4-dimensional analog of the 2-dimensional conformal models.

Four dimensional spacetime metrics cannot generally take the form (1.2). Only metrics which admit two geodetic and shear free null congruences \( \ell^\mu \partial_\mu, \ n^\mu \partial_\mu \) can take the analogous form

\[
ds^2 = 2g_{a\beta}dz^\alpha dz^\beta, \quad \alpha, \beta = 0, 1
\]

where \( z^b = (z^\alpha(x), \bar{z}^\beta(x)) \) are generally complex coordinates. In this case we can write down the following metric independent Yang-Mills-like action

\[
I_G = \int d^4z \sqrt{-g} g^{\alpha\tilde{\beta}} F_{j\alpha\beta} F_{j\tilde{\alpha}\tilde{\beta}} + c.c. = \int d^4z K_{j01} F_{j0\tilde{1}} + c.c.
\]

which depends on the CR-structure coordinates, and it does not depend on the metric.

Notice the similarity of this 4-dimensional action with the 2-dimensional Polyakov action (1.2). In the place of the "field" \( X^\mu \), which is interpreted as the background 26-dimensional Minkowski spacetime in string theory, we now have a gauge field \( A_{j\nu} \), which we have to interpret as the gluon, because the field equations generate a linear potential instead of the Coulomb-like \( \frac{1}{r} \) potential of ordinary Yang-Mills action.

The present action is based on the lorentzian CR-structure, which is determined by two real and one complex independent 1-forms \((\ell, n, m, \bar{m})\). This LCR-structure tetrad satisfy the relations

\[
d\ell = Z_1 \wedge \ell + i\Phi_1 m \wedge \bar{m}
\]

\[
dn = Z_2 \wedge n + i\Phi_2 m \wedge \bar{m}
\]

\[
dm = Z_3 \wedge m + \Phi_3 \ell \wedge n
\]

where the vector fields \( Z_{1\mu}, Z_{2\mu} \) are real, the vector field \( Z_{3\mu} \) is complex, the scalar fields \( \Phi_1, \Phi_2 \) are real and the scalar field \( \Phi_3 \) is complex. This structure essentially replaces the riemannian structure of the spacetime in the Einstein general relativity. The form (1.5) is completely integrable via the (holomorphic) Frobenius theorem, which implies that the lorentzian CR-manifold (LCR-manifold) is defined as a 4-dimensional real submanifold of \( \mathbb{C}^4 \) determined by four special (real) functions,

\[
\rho_{11}(\bar{z}^\alpha, z^\alpha) = 0, \quad \rho_{12}(\bar{z}^\alpha, \bar{z}^\beta) = 0, \quad \rho_{22}(\bar{z}^\alpha, z^\alpha) = 0
\]
where $\rho_{11}$, $\rho_{22}$ are real and $\rho_{12}$ is a complex function and $z^b = (z^\alpha, \bar{z}^\alpha)$, $\alpha = 0,1$ are the local structure coordinates in $\mathbb{C}^4$. Notice the special dependence of the defining functions on the structure coordinates. They are not general functions of $z^b$. The LCR-structure may be viewed as a restricted totally real CR-structure\[1\]. The separation of chiralities in the standard model is caused to this property. The LCR-structure is more general than the riemannian structure of general relativity and permits the invariance of the set of solutions to the pseudo-conformal transformations\[22\].

The action (1.4) takes the following generally covariant form

$$I_G = \int d^4x \sqrt{-g} \{ (\ell^\mu m^\nu F_{j\mu\rho} (m^\sigma F_{j\nu\sigma}) = \ell^\mu m^\nu F_{j\mu\rho} (m^\sigma F_{j\nu\sigma}) \}$$

$$F_{j\mu\nu} = \partial_{\mu} A_{j\nu} - \partial_{\nu} A_{j\mu} - \gamma f_{jkl} A_{j\mu} A_{k\nu}$$

where we have to consider the additional action term with the integrability conditions on the tetrad

$$I_C = -\int d^4x \{ \phi_0 (\ell^\mu m^\nu - \ell^\nu m^\mu) (\partial_{\mu} \ell_{\nu}) +$$

$$+ \phi_1 (\ell^\mu m^\nu - \ell^\nu m^\mu) (\partial_{\mu} m_{\nu}) + \phi_2 (n^\mu m^\nu - n^\nu m^\mu) (\partial_{\mu} n_{\nu}) +$$

$$+ \phi_3 (n^\mu m^\nu - n^\nu m^\mu) (\partial_{\mu} m_{\nu}) + c.conj. \}$$

These Lagrange multipliers introduce the integrability conditions of the tetrad and make the complete action $I = I_G + I_C$ self-consistent and the usual quantization techniques may be used\[18\]. The action is formally renormalizable\[20\], because it is dimensionless and metric independent. Its path-integral quantization is also formulated\[23\] as functional summation of open and closed 4-dimensional lorentzian CR-manifolds in complete analogy to the summation of 2-dimensional surfaces in string theory\[15\]. These transition amplitudes of a quantum theory of LCR-manifolds provides the self-consistent algorithms for the computation of the physical quantities.

The LCR-structure defining tetrad is invariant under the following tetrad-Weyl transformations

$$\ell'_\mu = \Lambda \ell_{\mu} \; , \; \ell'_\mu = N n_{\mu} \; , \; m'_\mu = M m_{\mu}$$

with non-vanishing $\Lambda$, $N$, $M$. I point out that we have not yet introduced a metric. The tetrad with upper and lower indices is simply a basis of tangent and cotangent spaces. But the tetrad does define a class $[g_{\mu\nu}]$ of symmetric tensors

$$g_{\mu\nu} = \ell_{\mu} n_{\nu} + \ell_{\nu} n_{\mu} - m_{\mu} m_{\nu} - m_{\nu} m_{\mu}$$

Every such tensor may be used as a metric to build up the riemannian geometry of general relativity, because its local signature is $(1, -1, -1, -1)$. But this form always admits two geodetic and shear-free null congruences and hence
it does not cover all the metrics of general relativity. The existence of the metric permit us to consider solitonic LCR-manifolds with energy-momentum and angular momentum. The purpose of the present work is to clarify the interacting solitonic LCR-manifolds.

The defining relations (1.6) of quite general class of LCR-manifolds\[22\] take the following form of real surfaces of the grassmannian manifold $G_{4,2}$

$$
\begin{align*}
\rho_{11}(X^{m1},X^{n1}) &= 0 = \rho_{22}(X^{m2},X^{n2}) \\
\rho_{12}(X^{m1},X^{n2}) &= 0 = K(X^{mj}) = 0 \\
\end{align*}
$$

(1.11)

where all the functions are homogeneous relative to the homogeneous coordinates $X^{m1}$ and $X^{n2}$ independently, which must be roots of the homogeneous holomorphic (generally reducible) Kerr polynomial $K(Z^m)$. The charts of its typical nonhomogeneous coordinates are determined by the invertible pairs of rows. If the first two rows constitute an invertible matrix, the chart is determined by $\det \lambda \neq 0$ and the corresponding affine space coordinates $r$ are defined by

$$
X = \begin{pmatrix}
X^{01} & X^{02} \\
X^{11} & X^{12} \\
X^{21} & X^{22} \\
X^{31} & X^{32}
\end{pmatrix} = \begin{pmatrix}
\lambda^{Aj} \\
-ir_{A'A}A^{Aj}
\end{pmatrix}
$$

(1.12)

$$
\sigma_{A'} = \eta^{ab}r^aA_B^{b}A
$$

In this context, we see that the LCR-structures determined by the relations

$$
X^{m1}E_{mn}X^{nj} = 0 , \quad K(X^{mj}) = 0
$$

(1.13)

are flat, i.e. they generate a minkowiskian class of metrics $[\eta_{\mu\nu}]$. Notice that $SU(2,2)$ is the symmetry group of these solutions. Its Poincaré subgroup is identified with the observed Poincaré symmetry in nature.

Using the results of the E. Cartan work on the automorphisms of the 3-dimensional CR-structures, I found\[23\] the automorphisms of the LCR-structure. The case of two commuting generators coincides with the two commuting generators of the Poincaré group. This permits the computation\[21\] of a static massive soliton and its massless stationary limit, as the basic free solitons of the model, which are identified with the electron and the neutrino of the SM. The stability of these solitons is assured by their topological characteristics. The electron LCR-manifold is determined by an irreducible quadratic polynomial (1.11) of $CP^3$, while the neutrino one is determined by the corresponding reducible quadratic polynomial.

In section II, the classical configurations which describe the electron-electron and the positronium bound state are revealed, in complete analogy to sinh-Gordon soliton-soliton scattering and the "breather". It is explicitly proven
that the compatibility of the LCR-structures is achieved if the trajectories of the electron LCR-structure solitons satisfy the equations of motion. In the case of the electron-positron system the positronium is expected.

In section III, the linearized part of the gluon field partial differential equations (PDE) are solved in the massive static soliton (electron) background. Following the Teukolsky technique\cite{4} of the solution of the analogous problem of a photon in the Kerr black hole, I first achieve to bring them to a form, which permits a separation of variables. The implied angular and radial ordinary differential equations (ODE) are completely different to the Teukolsky master equations. They are studied and found that they may admit bound states. The reasonable physical interpretation is to identify these bound states with the colored quarks.

2 SOLITONS AND MULTISOLITONS

Solitons are field configurations with finite energy-momentum and angular-momentum. A typical example of solitons are the finite energy classical solutions of the 2-dimensional sine-Gordon equation\cite{25}. These objects are additional states (besides the ordinary mesons) in the quantum Fock space of the sine-Gordon lagrangian. The general method to study the quantum scattering of solitons is to start from the classical scattering solution and proceed with the higher $\hbar$ terms. The bound states of two quantum solitons is indicated by the existence of a corresponding classical solution, with the typical example the sine-Gordon ”breather”. This procedure is well known\cite{25} and I am not going to review it. Through the present work I will refer to this picture in order to facilitate the reader to understand my solitonic approach in the context of the present 4-dimensional PCFT. I will precisely briefly review\cite{19} the massive static solution\cite{17} of PCFT. The required energy-momentum and the angular momentum of the soliton is defined using the ordinary linearized gravity definition\cite{10} with metric (1.10) defined by the LCR-structure.

Newman has found\cite{11},\cite{12} that the Kerr function condition (for a null congruence to be geodetic and shear-free) may be replaced with a (generally complex) trajectory $\xi^a(\tau)$. In the present case of the LCR-structure formalism, this is done by assuming that the $G_{4,2}$ two homogeneous coordinates $i = 1, 2$ have the form

$$X^i = \begin{pmatrix} \lambda^i \\ -i\xi(\tau_i)\lambda^i \end{pmatrix}$$

$$\xi(\tau_i) = \xi^a(\tau_i)\sigma^b\eta_{ab}$$

(2.1)

where $\sigma^b$ and $\eta_{ab}$ are the Pauli matrices and the Minkowski metric respectively. Here, I have to point out that the consideration of two generally different complex Kerr homogeneous functions is somehow misleading. In conventional algebraic geometry the notion of reducible polynomial is used. The irreducible quadratic Kerr polynomial (1.11) of the electron LCR-structure is equivalent with the complex trajectory $\xi^a = (\tau, 0, 0, ia)$. 

7
The flatprint LCR-structure coordinates are determined by the condition

\[(x - \xi(\tau_i))\lambda^i = 0 \tag{2.2}\]

that admits one non-vanishing solution for every column \(i = 1, 2\) of the homogeneous coordinates of \(G_{4,2}\). This is possible if

\[\det(x - \xi(\tau_i)) = \eta_{ab}(x^a - \xi^a(\tau_i))(x^b - \xi^b(\tau_i)) = 0 \tag{2.3}\]

which gives the two solutions \(z^0 = \tau_1(x)\) and \(z^0 = \tau_2(x)\). The other structure coordinates are \(z^1 = \lambda_{11}^1\lambda_0^1\) and \(z^1 = -\lambda_0^2\lambda_{12}\) where

\[\lambda^{Aj} = \left( \begin{array}{c}
(x^1 - i x^2) - (\xi_1(\tau_j) - i \xi_2(\tau_j)) \\
(x^0 - x^3) - (\xi_0(\tau_j) - \xi_3(\tau_j))
\end{array} \right) \tag{2.4}\]

Notice that the trajectory technique for computation of the structure coordinates incorporates the notion of the classical causality, which is apparently respected by (2.3).

The singularity of the flatprint LCR-structure occurs at \(\det[\lambda^{A1}(x), \lambda^{B2}(x)] = 0\). Recall that the left and right columns of the homogeneous coordinates of \(G_{4,2}\) may be determined ("move") with different trajectories, if the corresponding homogeneous Kerr polynomial is irreducible. In the simple case when both move with the same trajectory \(\xi^a(\tau) = (\tau, \xi_1(\tau), \xi_2(\tau), \xi_3(\tau))\), the singularity occurs at \(\tau_1(x) = \tau_2(x)\), which is

\[(x^i - \xi_1^i(t))(x^j - \xi_2^j(t))\delta_{ij} = 0 \tag{2.5}\]

If \(\xi_1^i\) and \(\xi_2^i\) are the real and imaginary parts of the trajectory, we find that the locus of the solitonic LCR-structure is

\[(x^i - \xi_1^i(t))(x^j - \xi_2^j(t))\delta_{ij} - \xi_1^i(t)(\xi_2^j(t)\delta_{ij} = 0 \tag{2.6}\]

Note that if \(\xi_1^i(t)\) is bounded, the LCR-structure may be interpreted as a soliton with trajectory \(\xi_R^i(t)\) and a locus at the perimeter of the circle of radius \((\xi_1^i(t))^2\) around its trajectory. This locus (a two dimensional surface) is a singularity of the gravitational potential and a source of the corresponding gravitational radiation, but it is not a singularity of the LCR-structure viewed as a surface of the \(G_{4,2}\) grassmannian, because the matrix \(X^{mi}\) has not rank two at this surface.

The class of symmetric tensors \(g_{\mu\nu}\) are identified with the Einstein metric, which defines the curvature and the Einstein tensor \(E^{\mu\nu}\). Because of the singularity of the metric, the Einstein tensor must be singular on the cylinder \(\mathbb{R}^4\) of \(\mathbb{R}^4\) and the circumference of \(\mathbb{R}^3\). Besides the metric tensor, the LCR-structure tetrad defines the class of self-dual 2-forms

\[V = 2\ell \wedge n - 2m \wedge m \tag{2.7}\]
Notice that this 2-form and the metric define the complex tensor
\[ V^\mu_\nu = \ell^\mu n_\nu - n^\nu \ell_\mu - m^\mu \overline{m}_\nu + \overline{m}^\mu m_\nu \]  
(2.8)
which is invariant under the tetrad-Weyl transformation. The LCR-structure tetrad vectors are eigenvectors of this tensor. Hence this tensor completely defines the LCR-structure. It is the complex pseudo-hermitian tensor, introduced by Flaherty [7] [8] to study the metrics with two geodetic and shear free congruences.

Throughout the present work I will use the following form of the static LCR-structure
\[ \ell^\mu dx^\mu = dt - \frac{\omega^2}{\Delta} dr - a \sin^2 \theta d\varphi \\
n^\mu dx^\mu = \frac{\Delta}{\eta^2} (dt + a \sin^2 \theta d\varphi) \\
m^\mu dx^\mu = \frac{1}{\eta^2} (ia \sin \theta dt - \rho^2 d\theta - i(r^2 + a^2) \sin \theta d\varphi) \]
(2.9)
Its contravariant components are
\[ \ell^\mu \partial_\mu = \frac{1}{\Delta} ((r^2 + a^2) \partial_t + \Delta \partial_r + a \partial_\varphi) \\
n^\mu \partial_\mu = \frac{1}{\eta^2} ((r^2 + a^2) \partial_t - \Delta \partial_r + a \partial_\varphi) \\
m^\mu \partial_\mu = \frac{1}{\eta^2} (ia \sin \theta \partial_t + \rho^2 \partial_\theta + \frac{i}{\sin \theta} \partial_\varphi) \]
(2.10)
We will also need its spin coefficients
\[ \varepsilon = 0 \quad , \quad \beta = \frac{\cos \theta}{\sin \theta \eta 2 \sqrt{2}} \quad , \quad \gamma = -\frac{\Delta}{\eta^2} \quad , \quad \alpha = \pi - \beta = \frac{ia \sin \theta}{\eta^2} - \frac{\cos \theta}{sin \theta \eta 2 \sqrt{2}} \]
(2.11)
The tetrad-Weyl factors have been chosen such that to give the Kerr-Newman manifold. They are imposed[19] by the existence of the classical electric charge and Poincaré charges, as follows

1) The 2-form (2.7) of the static soliton with the precise tetradWeyl factors has the characteristic property to admit a complex multiplication function such that
\[ d[(\gamma - ia \cos \theta)(2\ell \wedge n - 2m \wedge \overline{m})] = 0 \]
(2.12)
where \( C \) is an arbitrary complex constant. This the conserved electric charge reducing the general tetrad-Weyl symmetry [19] down to the ordinary Weyl transformation.

2) The precise tetrad-Weyl factors give a metric which coincides with its linearized gravity approximation, and hence define the Poincaré conserved quantities without any “approximation”. This fact fixes the remaining ordinary Weyl transformation.
The SM was derived by identifying the electron as the static LCR-manifold determined by the complex linear trajectory

\[ \xi^b(s) = v^b s + c^b + i a^b , \quad (\xi^b)^2 = (v^b)^2 = 1 \quad (2.13) \]

where \( v^b, c^b, a^b \) are the real constants, which represent the constant velocity, the initial position and the spin of the classical configuration of the electron. The closed self-dual 2-form \((2.12)\) of the static LCR-structure is identified with the self-dual 2-form \( F^+ \) of the electromagnetic field. Hence the intuition suggests to consider the particle-like LCR-manifold determined by a general complex trajectory, which asymptotically at \( t \to \mp \infty \) describes an electron interacting with an "external" field. In fact this is the electron-electron current that was used to derive\[24\] the effective SM action.

This point of view suggests to identify the classical solitonic configuration of the electron-electron elastic scattering with the LCR-manifold, which has two holes determined by non-intersecting general complex trajectories \( \xi^b_1 \) and \( \xi^b_2 \), which become (time) asymptotically linear. This will be used as a multisoliton, in complete analogy to the calculations of the 2-dimensional solitonic models.

The Flaherty pseudo-hermitian structure must be uniquely defined outside the "holes". That is, the class of metrics and the class of 2-forms must have representatives uniquely defined outside the "holes" of the trajectories. In the linearized gravity approximation

\[ g_{\mu \nu} \simeq \eta_{\mu \nu} + h_{\mu \nu} , \quad \hat{h}_{\mu \nu} = h_{\mu \nu} - \frac{1}{2} \eta_{\mu \nu} h^\rho_\rho , \quad \partial_\mu \hat{h}^{\mu \nu} = 0 \quad (2.14) \]

we have to solve the gravitational and electromagnetic laplacian problems

\[ \partial^2 \hat{h}_{\mu \nu} = 0 , \quad dF^+ = 0 \quad (2.15) \]

taking into account the existence of the two "holes" of the trajectories. It is exactly the Einstein framework that implied the equations of motion. The self-consistency imposed the momentum and the spin of every "hole"

\[ p^\mu_\beta = \int T^{\mu \nu_0} d^3x , \quad s^{\mu \nu}_\beta = \int (x^\nu T^{\mu \nu_0} - x^\nu T^{\mu \nu}) d^3x \quad (2.16) \]

to satisfy the equations of motion. The derivations of the equations of motion, using either the Einstein\[6\] or the Fock-Papapetrou\[13\] points of view, have been extensively studied and no review is needed. This procedure has to be accommodated\[5\] to include electromagnetic interaction. But the ring singularity suggests the Papapetrou procedure\[14\], which is extensively used under the name of Mathisson-Papapetrou-Dixon equations for spinning particles.

My conclusion is that the equivalence of the LCR-structure with the pair of class of metrics and 2-forms permit us to consider that the equations of motion are the self-consistency conditions for the regularity of a LCR-structure determined by the two complex trajectories. The precise form of the equations have
to be accommodated to the physical process and the approximative solitonic approach in the context of PCFT.

The conjugate LCR-manifold, determined by the pairs \((\ell, m)\) and \((n, m)\), has opposite charge and corresponds to the positron. The classical attraction of the electron and the positron implies a time periodic classical solution for the trajectories and subsequently for the LCR-structure coordinates and finally the LCR-tetrad. This classical solution corresponds to the sine-Gordon”breather”, which determines the bound state of two sine-Gordon solitons. In complete analogy, an electron-positron (positronium) bound state is expected.

The use of these LCR-structure multisolitons to make the computations of the corresponding quantum solitonic calculations is actually under consideration. The convenient form of the action is that based on the structure coordinates, which permits the noetherian definition of the infinite number of conserved currents. Among the numerical inputs, it is the radius of the Kerr-singular-ring, which can be computed using the spin of the electron

\[
a M_e = \frac{\hbar}{2c}
\]

This is the half of the (reduced) Compton wavelength \(\lambda_e = 386 \text{ fm}\). The Kerr-Newman metric has no horizons. If the Kerr-singular-rings intersect, the corresponding surface of \(CP^3\) is no longer locally quadratic, and it seems that the trajectory representation is no longer valid. I will use the method of infinite number of conserved currents in order to provide a more systematic study of the solitonic picture of PCFT.

3 THE QUARK BOUND STATE

The 2-dimensional kink soliton of the \(\phi^4\) model[25] makes a bound state with the perturbative meson. At the classical level this state appears as a discrete energy level of the stability quadratic part, after the expansion of the lagrangian around the \(\phi_{kink}\) configuration. An analogous phenomenon may appear in the present PCFT model after the expansion of the gluon field \(A_{j\mu}\) field equation around the static LCR-structure. The existence of a discrete energy state is colored and apparently it has to be identified with the quark, explaining the lepton-quark correspondence. In this section I will solve these partial differential equations. The mathematical problem is analogous to Teukolsky procedure for the solution of the photon scattering with the Kerr-Newman black hole[4], but apparently the equations are different.

We assumed the vacuum of the PCFT to be determined by the degenerate LCR-structure and vanishing gluon field \(A_{j\mu} = 0\). The standard model[23] was derived with this simple vacuum. The usual solitonic quantization of the static
soliton gives the following linear field equation of the gluonic field

\[-\frac{1}{\sqrt{\gamma}} \partial_\nu \{ \sqrt{-\gamma} (\ell^\nu m^\tau - \ell^\tau m^\nu) (\phi_{j2}) + (n^\nu m^\tau - n^\tau m^\nu) (-\phi_{j0}) + \\
+ (\ell^\nu m^\tau - n^\nu m^\tau) (\phi_{j1}) + (n^\nu m^\tau - n^\tau m^\nu) (-\phi_{j0}) \} = 0\]  

(3.1)

\[
\phi_{j0} = -\ell^\nu m^\nu (\partial_{\mu} [\mathbf{J}_{j}\mu] - \partial_{\mu} [\mathbf{J}_{j}\mu] ) \\
\phi_{j1} = -\frac{1}{\eta} (\ell^\nu m^\nu + m^\tau m^\nu) (\partial_{\mu} [\mathbf{J}_{j}\mu] - \partial_{\mu} [\mathbf{J}_{j}\mu] ) \\
\phi_{j2} = n^\nu m^\nu (\partial_{\mu} [\mathbf{J}_{j}\mu] - \partial_{\mu} [\mathbf{J}_{j}\mu] )
\]

For convenience, I use the Newman-Penrose (NP) quantities \(\phi_0, \phi_1, \phi_2\) without the color index, because in the present linearized terms the gluon interaction does not appear. The separation of variables is achieved with the tetrad form \(2.10\) and the NP quantities (and not the gluon potential) like in the already solved problem \(3\) of the photon scattering with the Kerr black hole. Applying this tetrad form and its spin coefficients \(2.11\), these partial differential equations (PDE) take the final form

\[
\phi'_{0} = \frac{1}{2} \phi_{0}, \quad \phi'_{2} = \frac{\phi_{2}}{\eta} \\
m^\nu \partial_{\mu} [\sin \theta \phi'_{0}] + \tau m^\nu \partial_{\mu} [\sin \theta \phi'_{2}] = 0 \\
m^\nu \partial_{\mu} [\sin \theta \phi'_{0}] + \mu m^\nu \partial_{\mu} [\sin \theta \phi'_{2}] = 0 \\
-n^\nu \partial_{\mu} [\sin \theta \phi'_{0}] + \ell^\nu \partial_{\mu} [\sin \theta \phi'_{2}] = 0
\]

(3.2)

with the tetrad redefined as follows

\[
\ell^\nu \partial_{\mu} = \ell^\nu \partial_{\mu} = \frac{\sqrt{2} + \alpha^2}{\Delta} \partial_{\tau} + \frac{\Delta}{\Delta} \partial_{\phi} \\
n^\nu \partial_{\mu} = \frac{2 \sqrt{2}}{\Delta} \partial_{\mu} = \frac{\sqrt{2} + \alpha^2}{\Delta} \partial_{\tau} - \partial_{\tau} + \frac{\Delta}{\Delta} \partial_{\phi} \\
m^\nu \partial_{\mu} = \eta \sqrt{2m^\nu \partial_{\mu} = i a \sin \theta \partial_{\xi} + \partial_{\phi} + \frac{i}{\sin \theta} \partial_{\phi}}
\]

(3.3)

Notice that these PDEs do not contain the NP field component \(\phi_1\). This component appears by imposing the integrability of gluon 2-form. Using the following notation

\[
F = \frac{\phi}{2} V - \phi_2 V^0 + \phi_0 V^\tilde{0} + c.c. \\
V^0 = \ell \wedge m \ , \ V^\tilde{0} = n \wedge \tau \ , \ V = 2 \ell \wedge n - 2m \wedge \tau
\]

(3.4)

and the formula

\[
dV^0 = [(2 \pi - \rho) n + (\tau - 2 \beta \tau m] \wedge V^0 \\
dV^0 = [(\pi - 2 \gamma) \ell + (2 \alpha - \pi) m] \wedge V^\tilde{0} \\
dV = [2 \mu \ell - 2 \rho m - 2 \pi m + 2 \tau m] \wedge V
\]

(3.5)

the definition of the gluon field \(F_{\mu}^{\nu\tau}\) via the gluonic potential \(A_{j\mu}\) implies the linearized integrability condition

\[
(m + 2 \tau \phi_0 - (m + \tau - 2 \mu) \phi_0 = (\ell - 2 \mu) \phi_1 - (\ell - 2 \mu) \phi_1 \\
(m + 2 \tau \phi_0 - (m + 2 \tau - 2 \mu) \phi_2 = (n + 2 \tau) \phi_1 - (n + 2 \tau) \phi_1 \\
(\ell + 2 \tau \phi_0 + (n + \mu - 2 \mu) \phi_0 = (m + 2 \tau) \phi_1 + (m + 2 \tau) \phi_1
\]

(3.6)
After the substitution of the electron tetrad, I find
\[
\phi'_0 = \frac{
abla}{
abla\theta} \phi_0 , \quad \phi'_1 = i \frac{1}{\sqrt{2}} \phi_1 , \quad \phi'_2 = \nabla \phi_2 \\
\pi^\mu_\nu \partial_\mu [\sin \theta \phi'_0] - m^\mu_\nu \partial_\mu [\sin \theta \phi'_0] = \Delta \sin \theta (\ell^\mu \partial_\mu + \frac{2}{\eta}) \phi'_1 - (\bar{\ell}^\mu \partial_\mu - \frac{2}{\eta}) \bar{\phi}'_1 \\
m^\mu_\nu \partial_\mu [\sin \theta \phi'_2] - m^\mu_\nu \partial_\mu [\sin \theta \phi'_2] = \Delta \sin \theta (n^\mu \partial_\mu - \frac{2}{\eta}) \phi'_1 - (\bar{n}^\mu \partial_\mu - \frac{2}{\eta}) \bar{\phi}'_1 \\
n^\mu_\nu \partial_\mu [\sin \theta \phi'_0] + \ell^\nu \partial_\mu [\sin \theta \phi'_2] = \sin \theta (m^\mu \partial_\mu + \frac{2\sin \theta}{\eta}) \phi'_1 + (m^\mu \partial_\mu - \frac{2\sin \theta}{\eta}) \bar{\phi}'_1 \\
\tag{3.7}
\]

I will now solve all the equations by separating the real and imaginary parts.

We precisely have for the FE \textbf{3723}:
\[
\phi'_0 = \frac{\Delta}{\sin \theta} \phi_0 , \quad \phi'_1 = i \frac{1}{\sqrt{2}} \phi_1 , \quad \phi'_2 = \frac{\ell^2}{\theta} \phi_2 \\
\phi'^1 + i \phi'^2 = \phi^1 \sin \theta , \quad \phi'^1 + i \phi'^2 = \phi^2 \sin \theta \\
\frac{\partial}{\partial \theta} \phi'^1 + \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} + a \sin \theta \frac{\partial}{\partial \phi} \right) \phi'^2 = 0 , \quad \frac{\partial}{\partial \theta} \phi'^2 + \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} + a \sin \theta \frac{\partial}{\partial \phi} \right) \phi'^1 = 0 \\
\frac{\partial}{\partial \phi} \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 = 2 \sin \theta \phi^1 + 2 \sin \theta \phi^2 \\
\frac{\partial}{\partial \phi} \phi'^2 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 = 2 \sin \theta \phi^2 + 2 \sin \theta \phi^1 \\
\tag{3.8}
\]

and the integrability conditions \textbf{3724}:
\[
\frac{\partial}{\partial \theta} \phi'^1 + \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} + a \sin \theta \frac{\partial}{\partial \phi} \right) \phi'^2 = 0 , \quad \frac{\partial}{\partial \theta} \phi'^2 + \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} + a \sin \theta \frac{\partial}{\partial \phi} \right) \phi'^1 = 0 \\
\frac{\partial}{\partial \phi} \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 - \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 = 2 \sin \theta \phi^1 + 2 \sin \theta \phi^2 \\
\frac{\partial}{\partial \phi} \phi'^2 - \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 = 2 \sin \theta \phi^2 + 2 \sin \theta \phi^1 \\
\tag{3.9}
\]

After summing and subtracting the first two rows, they become
\[
\frac{\partial}{\partial \theta} \phi'^1 + \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} + a \sin \theta \frac{\partial}{\partial \phi} \right) \phi'^2 - \left( \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} + a \sin \theta \frac{\partial}{\partial \phi} \right) \phi'^1 = 2 \Delta \sin \theta \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 \\
\frac{\partial}{\partial \phi} \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 - \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 = -2 \Delta \sin \theta \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 \\
\frac{\partial}{\partial \phi} \phi'^2 - \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 = 2 \sin \theta \phi^1 + 2 \sin \theta \phi^2 \\
\frac{\partial}{\partial \phi} \phi'^2 - \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^2 + a \frac{\partial}{\partial \phi} \right) \phi'^1 + \left( \frac{\ell^2 + \frac{a^2}{\cos \theta}}{\ell^2} \phi^1 + a \frac{\partial}{\partial \phi} \right) \phi'^2 = 2 \sin \theta \phi^2 + 2 \sin \theta \phi^1 \\
\tag{3.10}
\]

Considering the Fourier transform of \( \phi^*_\omega = \sum m_{\omega} \phi^*_\omega (\omega, m_\theta, r, \theta) e^{im_\theta \phi} \) the FE take the form
\[
\frac{\partial}{\partial \theta} \tilde{\phi}'_0 + iQ \tilde{\phi}'_2 = 0 , \quad \frac{\partial}{\partial \theta} \tilde{\phi}'_1 - iQ \tilde{\phi}'_0 = 0 , \quad Q = \omega a \sin \theta + \frac{m}{\sin \theta} \\
\frac{\partial}{\partial \phi} \tilde{\phi}'_0 + iK \tilde{\phi}'_1 - iK \tilde{\phi}'_0 = 0 , \quad \frac{\partial}{\partial \phi} \tilde{\phi}'_2 - \tilde{\phi}'_0 = 0 , \quad \frac{\partial}{\partial \phi} \tilde{\phi}'_2 - \tilde{\phi}'_0 = 0 \\
K = \omega (r^2 + a^2) + am \\
\tag{3.11}
\]
and the integrability conditions become

\[
\frac{\partial}{\partial r}(\phi^2_1 - \phi^2_0) + iQ(\phi^1_2 - \phi^1_0) = 2iK \sin \theta \phi^1_0
\]

\[
\frac{\partial}{\partial r}(\phi^2_0 - \phi^0_0) + iQ(\phi^0_2 + \phi^0_0) = -2\Delta \sin \theta[(\partial_r + \frac{2r}{K})\phi^2_1 + \frac{2\cos \theta}{Q}\phi^1_1]
\]

\[
-\frac{\partial}{\partial r}(\phi^2_0 + \phi^0_0) - iK(\phi^2_0 - \phi^0_0) = 2iQ \sin \theta \phi^1_0
\]

(3.12)

Making the substitutions

\[
\phi^0_1 = \phi^0_2 + \phi^0_0 , \quad \phi^1_0 = \phi^1_2 - \phi^1_0
\]

\[
\phi^2_0 = \phi^2_2 + \phi^2_0 , \quad \phi^2_2 = \phi^2_2 - \phi^2_0
\]

(3.13)

the FE take the simple form

\[
\frac{\partial}{\partial r}(\phi^1_0) - iQ(\phi^2_0) = 0 , \quad \frac{\partial}{\partial r}(\phi^1_1) - iQ(\phi^2_1) = 0
\]

\[
\frac{\partial}{\partial r}(\phi^1_0) + iK(\phi^1_1) = 0 , \quad \frac{\partial}{\partial r}(\phi^2_1) + iK(\phi^2_0) = 0
\]

(3.14)

\[
K = \omega(r^2 + a^2) + am , \quad Q = \omega a \sin \theta + \frac{m}{\sin \theta}
\]

Notice that only the three from the four relations are independent. The general solution of these equations is

\[
\phi^1_0 = \bar{f}(\omega, m; r, \theta) , \quad \phi^1_1 = i\Delta \frac{\partial \bar{f}}{\partial r} , \quad \phi^2_0 = -i\frac{\partial \bar{f}}{\partial \theta} , \quad \phi^2_1 = (\frac{1}{2Q} \frac{\partial}{\partial \theta})(\frac{\Delta}{2Q} \frac{\partial}{\partial r})\bar{f}
\]

(3.15)

Note that the general solution depends on the general function \(\bar{f}(m, \omega, r, \theta)\), which is will be fixed by the integrability conditions (3.12). Replacing these relations into the integrability conditions, I find

\[
\phi^1_0 = \frac{1}{2Q} \frac{\partial}{\partial \sin \theta} \partial_0 (\partial_r \Delta \partial_r + \frac{K}{2})\bar{f} , \quad \phi^1_1 = \frac{1}{2Q} \frac{\partial}{\partial \sin \theta} \partial_0 (\partial_0 \Delta \partial_0 - Q)\bar{f}
\]

(3.16)

Notice that the last two PDEs are equivalent to the following PDE

\[
\frac{\partial}{\partial \sin \theta} (\partial_0 \Delta \partial_0 - Q)(\partial_r \Delta \partial_r + \frac{K}{2})\bar{f} - 2\Delta \frac{\cos \theta}{Q^2} \partial_0 (\partial_r \Delta \partial_r + \frac{K}{2})\bar{f} + \frac{2arQ\Delta}{K^2} \partial_r (\partial_0 \Delta \partial_0 - Q)\bar{f} = 0
\]

(3.17)

Using the relation \(K = aQ \sin \theta = \eta \omega\), it takes the form

\[
\frac{K - aQ \sin \theta}{\sin \theta} (\partial_0 \Delta \partial_0 - Q)(\partial_r \Delta \partial_r + \frac{K}{2})\bar{f} - 2\Delta \frac{\cos \theta}{Q^2} \partial_0 (\partial_r \Delta \partial_r + \frac{K}{2})\bar{f} + \frac{2arQ\Delta \omega}{K} \partial_r (\partial_0 \Delta \partial_0 - Q)\bar{f} = 0
\]

(3.18)
This is the final unique PDE that the unknown function $\tilde{f}$ of (3.15) satisfies. The solutions of this PDE implies solutions for all the linearized NP components of the gluon field. It is apparent that the looking for solutions of the form $\tilde{f} = R(r)S(\theta)$, the ordinary method of separation of variables does not apply. But notice that the following two ratios

$$\left(\frac{Q}{\sin \theta} \frac{\delta}{\delta \theta} - Q^2 - \frac{2a\omega \cos \theta}{Q(1-\lambda Q \sin \theta)} \frac{d}{d\theta}\right)S = 0$$

$$\left(\frac{K}{\Delta} \frac{\delta}{\delta r} + \frac{K^2}{\Delta^2} - \frac{2a\omega}{K(\Delta K \Delta)} \frac{d}{d\theta}\right)R = 0$$

depend on different variables. The first depends on $\theta$ and the second on $r$. Then as usual, the two ratios must be equal to a constant $\lambda$. The derived ODEs are

$$\frac{Q}{\sin \theta} \frac{\delta}{\delta \theta} \sin \theta \frac{d}{d\theta}S - \frac{2a\omega \cos \theta}{Q(1-\lambda Q \sin \theta)} \frac{d}{d\theta}S - Q^2 S = 0$$

$$Q = \omega a \sin \theta + \frac{m}{\sin \theta}$$

(3.21)

Notice the difference with the angular $s = 0$ Teukolsky master equation

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \sin \theta \frac{d}{d\theta} S + (E + a^2 \omega^2 \cos^2 \theta - \frac{m^2}{\sin^2 \theta}) S = 0$$

(3.22)

The essential difference is that (3.22) is a smooth deformation (with parameter $a$) to the ordinary spherical harmonics. For $a = 0$ the ODE (3.21) takes the form

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \sin \theta \frac{d}{d\theta} S - \frac{m^2}{\sin^2 \theta} S = 0$$

(3.23)

which is apparently singular for $m \neq 0$. The above equation (3.22) has the form of differential equation of the spherical harmonics with $l = 0$. This implies that only the constant spherical harmonic $Y_{00}$ can be deformed to a regular solution of the ODE (3.21). This imposes a very strong restriction to this ODE, which we have to solve only for $m = 0$. In this case and for $a\omega \neq 0$, the initial ODE (3.20) takes the form

$$\frac{1}{\sin \theta} \frac{d}{d\theta} \sin \theta \frac{d}{d\theta} S - \frac{2\cos \theta}{\sin^2 \theta(1-\lambda a\omega \sin^2 \theta)} \frac{d}{d\theta} - (a\omega)^2 S = 0$$

(3.24)
Using the variable $c = -\cos \theta$, it becomes

$$[\frac{d^2}{dc^2} + \left(\frac{2\omega}{1-c^2} + \frac{2\lambda \omega}{1-\lambda \omega + \lambda \omega c^2}\right) \frac{dc}{dc} - (a\omega)^2]S = 0$$

$$c = -\cos \theta, \quad [0, \pi] \rightarrow [-1, +1]$$

(3.25)

Hiding the first derivative into $S$, through the transformation

$$S = \Phi \exp \int \kappa dc$$

(3.26)

I find

$$[\frac{d^2}{dc^2} - \frac{dc}{dc} - 2\lambda \omega c - (a\omega)^2]\Phi = 0$$

$$\Phi = \frac{1}{1-\lambda \omega + \lambda \omega c^2}$$

(3.27)

That is

$$[\frac{d^2}{dc^2} - U]\Phi = 0$$

$$U(c) = \frac{1+2c^2}{(1-c^2)^2} + \frac{\lambda \omega (1-\lambda \omega)}{(1-\lambda \omega + \lambda \omega c^2)^2} + \frac{2\lambda \omega c^2}{(1-c^2)(1-\lambda \omega + \lambda \omega c^2)} + (a\omega)^2$$

(3.28)

The function $U(c)$ becomes infinite at the boundaries $c = \pm 1$. It also diverges to $-\infty$ for

$$0 < c^2 = \frac{\lambda \omega - 1}{\lambda \omega} < 1$$

(3.29)

which is possible if $\lambda \omega > 1$. Apparently the search of solutions of $\omega$ and $\lambda$ have to be done in combination with the possible solutions of the radial ODE. Taking into account the complexity of the function $U(c)$, the WKB approximation is indicated to be used, in order to find numerical results.

### 3.2 Study of the radial differential equation

Analyzing the angular ODE, I found the condition $m = 0$, which we have to impose to the radial ODE. I finally find

$$(\frac{d^2}{dr^2} + \omega^2 - 2\sigma \frac{dr}{dr})R = 0$$

$$\sigma = \frac{ar\Delta}{(r^2 + a^2)^2(a - \lambda \omega (r^2 + a^2))}$$

$$r' = r - \frac{2M^2-q^2}{r} \arctan \frac{dr}{rM} + M \ln \Delta + C, \quad d = \sqrt{a^2 - M^2 + q^2}$$

(3.30)

where I have also made a change of variable such that $\frac{dr'}{dr} = \frac{r^2 + a^2}{\Delta}$. After making the transformation

$$R(r') = \Psi(r') \exp \int \sigma(r(r'))dr'$$

$$\sigma(r) = \frac{ar\Delta}{(r^2 + a^2)^2(a - \lambda \omega (r^2 + a^2))}$$

(3.31)

the 1st derivative term is removed and the radial ODE takes the form

$$[\frac{d^2}{dr^2} - V(r(r'))]\Psi(r') = 0$$

$$V(r) = \sigma^2 - \frac{\Delta}{r^2 + a^2} \frac{dr}{dr} - \omega^2$$

$$\sigma(r) = \frac{ar\Delta}{(r^2 + a^2)^2(a - \lambda \omega (r^2 + a^2))}$$

(3.32)
From the analysis of this angular ODE, we find that a bounded state may appear if \( \lambda \omega a > 1 \). This condition implies that \( \sigma(r) \) does not diverge and it is always negative with \( \sigma(0) = 0 \) and for \( r \to \infty \) I find \( \sigma \simeq -\frac{a^2}{\lambda \omega r} \) and \( V(r) \simeq \frac{3a}{\lambda \omega r^3} - \omega^2 \).

For asymptotic values of \( r \), the behavior of the solution is \( \Psi(r) \simeq e^{\pm i \omega r} \). This “free” gluon behavior should not confuse us, because it is a solution of the linear part of the gluonic equation. The confining gluon propagator[22], will appear in its interactions, implying the expected confinement.

The “potentials” \( U(r) \) and \( V(r) \) are too complicated to find formal solutions of the angular and radial ODEs. In order to find the “spectrum” of (3.28) and (3.32) ODEs, i.e. the possible solutions of \( \omega \) and \( \lambda \), we have to make numerical calculations using the ordinary WKB approximation.

4 PERSPECTIVES

The recent experimental results of the LHC experiments at CERN imply that supersymmetric particles do not exist. Hence quantum string theory does not describe nature. Hence, the 4-dimensional PCFT remains the only known model, compatible with quantum theory, which provide the general experimentally observed framework. The still missing ingredient is the numerical computation of standard model parameters (masses and coupling constants) and the hadronic form factors.

The solitonic approach seems to be the computationally convenient procedure. These techniques have been already developed[9] in the skyrmion model for the simulation of hadronic dynamics and the Bogomolny-Prassad-Sommerfield (BPS) model for the experimental search of monopoles.

In the PCFT the proper procedure seems to be the LCR-structure coordinates \((z^\alpha(x), \tilde{z}^\alpha(x))\), \( \alpha = 0, 1 \) implied by the application of the (holomorphic) Frobenius theorem and defined as follows

\[
\begin{align*}
dz^\alpha &= f_\alpha \ell_\mu dx^\mu + h_\alpha m_\mu dx^\mu, \\
dz^\tilde{\alpha} &= f_\tilde{\alpha} n_\mu dx^\mu + h_{\tilde{\alpha}} \overline{m}_\mu dx^\mu \\
dz^0 &\wedge dz^1 \wedge dz^\tilde{0} \wedge dz^\tilde{1} \neq 0 \\
\ell &= \ell_\alpha dz^\alpha, \\
m &= m_\alpha dz^\alpha, \\
n &= n_{\tilde{\alpha}} dz^\tilde{\alpha}, \\
\overline{m} &= \overline{m}_{\tilde{\alpha}} dz^\tilde{\alpha}.
\end{align*}
\]

(4.1)

If the field equations are transcribed into these variables, an infinite number of conserved currents are implied. But it is more convenient to start from the equivalent lagrangian[23]

\[
I_G = \int d^4x \left[ \det(\partial_\alpha z^\alpha) \right] \left\{ (\partial_\alpha x^\mu) (\partial_\alpha x^\nu) F_{\mu \nu}\alpha \right\} \left\{ (\partial_\alpha x^\sigma) (\partial_\alpha x^\tau) F_{\sigma \tau} \right\} + c. c.
\]

\[
I_C = \int d^4x \epsilon^{\mu \nu \rho \sigma} [\phi_0(\partial_\mu z^0)(\partial_\nu z^1)(\partial_\rho \overline{z}^0)(\partial_\sigma \overline{z}^1) + \phi_\tilde{0}(\partial_\mu \overline{z}^0)(\partial_\nu \overline{z}^1)(\partial_\rho z^0)(\partial_\sigma z^1) + \phi(\partial_\mu z^0)(\partial_\nu \overline{z}^1)(\partial_\rho \overline{z}^0)(\partial_\sigma z^1) + \phi(\partial_\mu \overline{z}^0)(\partial_\nu z^1)(\partial_\rho z^0)(\partial_\sigma \overline{z}^1)]
\]

(4.2)
where the $4 \times 4$ matrix $(\partial_\mu x^\mu)$ is the inverse of $(\partial_\mu z^\mu)$.

This action is invariant under the following two infinitesimal pseudo-conformal (LCR-structure preserving) transformations

$$ \delta z^\beta \simeq \varepsilon \psi^\beta (z^\gamma) , \quad \delta z^{\bar{\beta}} \simeq \bar{\varepsilon} \psi^{\bar{\beta}} (z^{\bar{\gamma}}) $$

$$ \delta \phi_0 = -\phi_0 (\partial_\alpha \psi^\alpha) \varepsilon + (\partial_\alpha \psi^\alpha) \bar{\varepsilon} $$

$$ \delta \phi = -\phi (\partial_\alpha \psi^\alpha) \varepsilon + (\partial_\alpha \psi^\alpha) \bar{\varepsilon} $$

(4.3)

Notice that the transformations of the "left" $z^\alpha (x)$ and "right" $z^{\bar{\alpha}} (x)$ structure coordinates are independent, like the conformal transformations in the ordinary 2-dimensional CFT. The following "left" and "right" LCR-currents

$$ J^\lambda \equiv -\det (\partial_\tau z^a) \ F_{j01} \psi^j F_{j\bar{0}1} \tilde{\psi}^\lambda (\partial_\bar{\mu} x^\mu) - 
- \epsilon_{\alpha\beta} \psi^\alpha \epsilon^{\mu
u\rho\sigma} (\partial_\nu z^\beta) [\phi_0 (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}}) + \bar{\phi} (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}})] $$

$$ \tilde{J}^\lambda \equiv -\det (\partial_\tau z^a) \ \tilde{\psi}^\lambda \epsilon^\lambda_{\alpha\beta} F_{j01} \psi^j (\partial_\beta x^\mu) - 
- \epsilon_{\alpha\beta} \tilde{\psi}^\alpha \epsilon^{\mu
u\rho\sigma} (\partial_\nu z^{\bar{\beta}}) [\tilde{\phi}_0 (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}}) + \tilde{\phi} (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}})] $$

(4.4)

are derived.

The independent conserved quantities are implied by the formally independent functions

$$ \psi^\alpha (z^\gamma) = -(z^\alpha)^a (z^1)^{m_0} (z^1)^{m_1}, \quad \Rightarrow \quad \frac{T^{(\alpha)}}{m} = \int d^4x J_0^{(\alpha)} $$

$$ \tilde{\psi}^{\bar{\beta}} (z^{\bar{\gamma}}) = -(z^{\bar{\beta}})^{\bar{\alpha}} (z^{\bar{1}})^{m_0} (z^{\bar{1}})^{m_1}, \quad \Rightarrow \quad \frac{T^{(\bar{\beta})}}{m} = \int d^4x \tilde{J}_0^{(\bar{\beta})} $$

(4.5)

The implied "left" and "right" LCR-currents

$$ J^\mu \equiv -\det (\partial_\tau z^a) \ F_{j01} \psi^j F_{j\bar{0}1} \tilde{\psi}^\lambda (\partial_\bar{\mu} x^\mu) - 
- \epsilon_{\alpha\beta} \psi^\alpha \epsilon^{\mu
u\rho\sigma} (\partial_\nu z^\beta) [\phi_0 (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}}) + \bar{\phi} (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}})] $$

$$ \tilde{J}^\mu \equiv -\det (\partial_\tau z^a) \ \tilde{\psi}^\lambda \epsilon^\lambda_{\alpha\beta} F_{j01} \psi^j (\partial_\beta x^\mu) - 
- \epsilon_{\alpha\beta} \tilde{\psi}^\alpha \epsilon^{\mu
u\rho\sigma} (\partial_\nu z^{\bar{\beta}}) [\tilde{\phi}_0 (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}}) + \tilde{\phi} (\partial_\rho z^\mu) (\partial_\sigma z^{\bar{\tau}})] $$

(4.6)

determine the following independent conserved quantities

$$ \psi^\alpha (z^\gamma) = -(z^\alpha)^a (z^1)^{m_0} (z^1)^{m_1}, \quad \Rightarrow \quad \frac{T^{(\alpha)}}{m} = \int d^4x J_0^{(\alpha)} $$

$$ \tilde{\psi}^{\bar{\beta}} (z^{\bar{\gamma}}) = -(z^{\bar{\beta}})^{\bar{\alpha}} (z^{\bar{1}})^{m_0} (z^{\bar{1}})^{m_1}, \quad \Rightarrow \quad \frac{T^{(\bar{\beta})}}{m} = \int d^4x \tilde{J}_0^{(\bar{\beta})} $$

(4.7)
where $\vec{m} = (m_0, m_1)$ and $m_3$ take integer values.

In order to computationally control these conserved quantities, it is convenient to identify the LCR-structure coordinates with the ratio of the components of the spinors $\lambda^j$ and the parameters $\tau_j$ of (2.1) as follows

$$
z^0 = \tau_1, \quad z^1 = \frac{\lambda^{11}}{\lambda^{01}}, \\
z^0 = \tau_2, \quad z^1 = -\frac{\lambda^{02}}{\lambda^{12}}
$$

(4.8)

This formulation of the solitonic procedure will permit the calculation of all the phenomenological quantities and provide the necessary experimental check of PCFT.
References

[1] M. S. Baouendi, P. Ebenfelt and L. Rothschild, "Real submanifolds in complex space and their mappings", Princeton University Press, Princeton, (1999).

[2] N. N. Bogoliubov, A.A. Logunov and I.T. Todorov, "Introduction to Axiomatic Quantum Field Theory", W.A. Benjamin Publishing Company, Inc. USA (1975).

[3] N. N. Bogoliubov and D. V. Shirkov (1982), "Quantum Fields", Benjamin/Cummings Publishing Company, Inc. USA.

[4] S. Chandrasekhar (1983), “The Mathematical Theory of Black Holes”, Clarendon, Oxford.

[5] D. M. Chase (1954), Phys. Rev. 95, 243.

[6] A. Einstein, L. Infeld and B. Hoffman (1938), Ann. Math. 39, 65.

[7] E. J. Jr Flaherty (1974), Phys. Lett. A46, 391.

[8] E. J. Jr Flaherty (1976), “Hermitian and Kählerian geometry in Relativity”, Lecture Notes in Physics 46, Springer, Berlin.

[9] N. Manton and P. Sutcliffe (2004), “Topological solitons”, Cambridge Univ. Press, Cambridge (2004).

[10] C. N. Misner, K. S. Thorn and J. A. Wheeler, "GRAVITATION", W. H. Freeman and Co (1973).

[11] E. T. Newman (1973), J. Math. Phys. 14, 102.

[12] E. T. Newman (2004), Class. Q. Grav. 21, 3197 (arXiv:gr-qc/0402056).

[13] A. Papapetrou (1948), Proc. Roy. Irish Acad. A52, 69.

[14] A. Papapetrou (1951), Proc. Phys. Soc. (London) 64, 57.

[15] J. Polchinski, "STRING THEORY", vol. I, Cambridge Univ. Press, Cambridge, (2005).

[16] C. N. Ragiadakos (1990), "A Four Dimensional Extended Conformal Model", Phys. Lett. B251, 94.

[17] C. N. Ragiadakos (1991), "Solitons in a Four Dimensional Generally Covariant Conformal Model" Phys. Lett. B260, 325.
[18] C. N. Ragiadakos (1992), "Quantization of a Four Dimensional Generally Covariant Conformal Model", J. Math. Phys. 33, 122.

[19] C. N. Ragiadakos (1999), "Geometrodynamic solitons", Int. J. Math. Phys. A14, 2607.

[20] C. N. Ragiadakos (2008), “Renormalizability of a modified generally covariant Yang-Mills action”, arXiv:hep-th/0802.3966v2.

[21] C. N. Ragiadakos (2008), “A modified Y-M action with three families of fermionic solitons and perturbative confinement”, arXiv:hep-th/0804.3183v1.

[22] C. N. Ragiadakos (2013), "Lorentzian CR stuctures", arXiv:hep-th/1310.7252.

[23] C. N. Ragiadakos (2017), "Pseudo-conformal Field Theory", arXiv:hep-th/1704.00321.

[24] C. N. Ragiadakos (2018), "Standard Model Derivation from a 4-d Pseudo-conformal Field Theory", arXiv:hep-th/1805.11966.

[25] R. Rajaraman (1989), “Solitons and instantons”, Elsevier Science Publishing Company, The Netherlands (1989).