1. Introduction

For a multiple QSO, the propagation time from the source to the observer varies from the image \(i\) to the image \(j\), and this difference \((\Delta \tau_{ij})\) can be measured when the source is variable. In general, assuming a flat universe without cosmological constant, the parameter \(\Delta \tau_{ij} \times H_0\) (\(H_0\) is the Hubble constant) depends on the redshifts of the lens and the source, as well as the positions of the individual images and the source \((\vec{\theta}_i, \vec{\theta}_j, \vec{\beta})\), and the scaled surface potential \(\Psi_\alpha\) at \(\vec{\theta}_i\) and \(\vec{\theta}_j\) (see, e.g., Blandford and Kundic 1996, Williams and Schechter 1997). The observations of multiple images of the same source are used to infer \(\vec{\beta}\) and the adjustable parameters \(\alpha \equiv (\alpha_1, ..., \alpha_p)\) that appear in the picture of the deflector, i.e., a lens model. From the lens model corresponding to the lens picture, \(\Delta \tau_{ij}, \vec{\theta}_i, \vec{\theta}_j\) and the redshifts, one obtains \(H_0\).

A golden system (which is suitable for determining \(H_0\)) must be a multiple QSO verifying some properties. The deflector cannot be dark or very faint (non-observable), and in order to model the gravitational potential, the lensing structure must be simple (e.g., an isolated galaxy). To infer \(\alpha\) and \(\vec{\beta}\), a relatively large number of constraints is also desirable. On the other hand, for obtaining \(\Delta \tau_{ij}\), some typical scales of intrinsic variability of the source should be less than \(\Delta \tau_{ij}\), and the absence of strong short
timescale microlensing is needed. The microlensing events mask the intrinsic variability.

The best studied gravitational lens is the double QSO Q0957+561 A,B. As a result of recent advances in observations and modeling, a ten percent measurement of $H_0$ is attainable. We discuss in detail this system (Sect. 2) and review the perspectives in a near future (Sect. 3).

2. Twin QSO (0957+561 A,B)

For Q0957+561 A,B, at optical wavelengths, there are epochs of a rapid intrinsic variability (a basic condition for measuring the time delay accurately) and epochs of calmness. They also appear in evidence in favor of an important short timescale microlensing (Schild 1996), which masks the intrinsic variability. So, due to the double behavior activity/calmness, the microlensing events, the gaps in the monitoring (three months per year), etc., it is difficult an accurate time delay determination based on a large dataset containing several years of observations.

The rough estimation $\Delta \tau_{BA} \approx T = 420$ days (Pelt et al. 1996; see also the paper by Pijpers in this volume) can be used for making nice datasets. A nice dataset contains an active light curve $A$ during a period $[t_i, t_f]$ and the light curve $B$ in the interval $[t_i + T, t_f + T]$. The microlensing should be weak and, on the other hand, the absence of gaps (each summer) is needed. Very recently, using image $A$ data in the band $r$ (1994 Dec.-1995 May, which were kindly provided us by T. Kundic), and our image $B$ data in the band $R$ (1996 Feb.-July), we concluded that $\Delta \tau_{BA} = 424 \pm 3$ days (Oscoz et al. 1997). The $A$ component was active (Kundic et al. 1995), and also, we have not found evidences of strong microlensing in the $r - R$ comparison (Oscoz et al. 1997, Goicoechea et al. 1998). A similar result (417±3 days; 2σ confidence level) has been derived by Kundic et al. (1997). They use light curves in the $g$ and $r$ bands. However, a reanalysis based on discrete correlation functions and two nice datasets, $A_r + B_r$ and $A_g + B_g$ (we basically exclude the photometric monitoring of image $B$ in the interval 1995 Dec.-1996 Jan.), shows that 424 days is the most probable delay. For every fixed value $\theta$ (days), we construct the function

$$\delta^2(\theta) = \frac{1}{N} \sum_{i=1}^{N} S_i [DCC(\tau_i) - DAC(\tau_i - \theta)]^2,$$

where $DCC$ is the discrete $A - B$ cross-correlation function, $DAC$ is the discrete $A - A$ autocorrelation function and $S_i = 1$ only when both $DCC$ and $DAC$ are defined at $\tau_i$ and $\tau_i - \theta$, respectively, and $S_i = 0$ otherwise. Then, we search for a minimum $\theta_0$, such that $\theta_0 = \Delta \tau_{BA}$. In the two ($g - g$ and $r - r$) comparisons, it is inferred $\Delta \tau_{BA} = 424$ days (see Fig. 1).
Figure 1. DCC (filled circles) vs. DAC (shifted by 424 days; open circles) in the $g-g$ comparison. The agreement is excellent (for details, see main text).

The main deflector is a cD galaxy at the center of a cluster. A good picture of the galaxy could be a King-type surface density profile with velocity dispersion $\sigma_v$ and core angular radius $\theta_c$, plus a point-mass ($M_{bh}$) at the centre of the King profile (Falco et al. 1991). Moreover, the gravitational effect of the cluster can be represented by means of a quadratic lens with a convergence $\kappa$ and a shear $\gamma$ with position angle $\phi$. Grogin and Narayan (1996) used $\alpha \equiv (\sigma_v, \theta_c, M_{bh}, \gamma, \phi)$, $\beta_1 \equiv (\beta_{1x}, \beta_{1y})$, $\beta_5 \equiv (\beta_{5x}, \beta_{5y})$ as free parameters (Garrett et al. 1994, have fitted the A and B radio images with six Gaussian components, being $A_1$ and $B_1$ the respective core components and $A_5$-$B_5$ another jet components), since, in spite of the large number of constraints, there is a degenerancy in the convergence due to the cluster. The whole lens model must include the nine parameters inferred from measurements of the lensed images as well as an estimation of $\kappa$ derived from a direct measurement of the mass either in the cD galaxy or the cluster. A measurement of $\sigma_v(\text{light})$, the 1D velocity dispersion of the luminous stars in the galaxy, allows to eliminate the cluster degenerancy.

Falco et al. (1997) derived $\sigma_v(\text{light}) = 279\pm13$ km/s, and so, with the whole lens model and the time delay above mentioned, one obtains $H_0 = 66^{+15}_{-14}$ km/s/Mpc (2$\sigma$). This estimate is 10% accurate at 1$\sigma$ (Oscoz et al. 1997). We however remark that a new lens model satisfying the constraints from radio mapping as well as new optical constraints deduced from HST observations is now in progress (Bernstein et al. 1997). Also, the approximation $\sigma_v = \sigma_v(\text{light})$ must be reconsidered (Mediavilla et al. 1998).
3. Perspectives in a near future

The combined effort by several groups of astronomers will lead to the detailed analysis of a large number of gravitational mirages. For each system, if the deflectors are not dark, we must be able to obtain a promising set of constraints and some information on suitable pictures of the global lens (primary and secondary deflectors). Through a good picture and the constraints, one can infer a good lens model. Moreover, the system must be extensively monitored to get the light curves of the different images and deduce at least one time delay. This work could be difficult due to probable microlensing events, a weak source variability, an unsuitable sampling of the image light curves, etc. The technique for determining a time delay plays also a role. Even in the old Twin QSO there is an uncertainty (due to the methodology) of about one week, which is irrelevant in order to obtain $H_0$ from this system, but it could be dramatic in another multiply imaged QSO (e.g., the Triple QSO 1115+080$A_1 - A_2, B, C$).

Nowadays, from two individual systems (Twin QSO and Triple QSO), it is inferred a mean value of $H_0 \approx 60$ km/s/Mpc, in good agreement with other methods (see the paper by C. Frenk in this volume). However, new measurements of $H_0$ from gravitational lenses may surprise us. For example, when one measures $H_0$ via gravitational lensing, the influence of the large-scale structure is not taken into account. The effect introduced in the measurement of $H_0$ caused by large clusters and/or large voids, is a very interesting topic, which will be soon studied by our group.

4. References

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