Masses and magnetic moments of triple heavy flavour baryons in hypercentral model

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Abstract. Triple heavy flavour baryons are studied using the hypercentral description of the three-body system. The confinement potential is assumed as hypercentral Coulomb plus power potential with power index $p$. The ground state ($J^P = \frac{1}{2}^+$ and $\frac{3}{2}^+$) masses of heavy flavour baryons are computed for different power index, $p$ starting from 0.5 to 2.0. The predicted masses are found to attain a saturated value with respect to variation in $p$ beyond the power index $p > 1.0$. Using the spin-flavour structure of the constituting quarks and by defining effective mass of the confined quarks within the baryons, the magnetic moments are computed with no additional free parameters.

Keywords. Hypercentral constituent quark model; charmed and beauty baryons; hyper-Coulomb plus power potential; magnetic moments.

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1. Introduction

The investigation of properties of hadrons containing heavy quarks is not only of great interest in understanding the dynamics of QCD at the hadronic scale but also interesting due to the rapid progress in the discovery of hadronic resonances by different experimental groups like BaBar, CLEO, SELEX and other B factories the world over. The last generation of baryons within the Standard Model are the triple heavy baryons and they are the heaviest composite states, predicted by the constituent quark model. Essentially they are the $\Omega_{ccc}$, $\Omega_{cqb}$, $\Omega_{qbb}$ and $\Omega_{bbb}$ baryons \[1\]. After the discovery of the doubly charmed baryon by the SELEX group \[2\], it is expected that the triple heavy flavour baryonic state may be in the offing very soon. The vital properties of these heaviest baryons in nature are their masses and magnetic moments. This has generated much interest in the theoretical predictions of their properties \[3–8\].

Though considerable amount of data on the properties of the single heavy baryons \[6,9,10\], and on the light heavy flavour baryons \[11–13\] are available in literature,
only sparse attention has been paid to the spectroscopy of double and triple heavy flavour baryons, perhaps mainly due to the lack of experimental incentives [5].

The experimental observation of triple heavy baryons also crucially depends on the production environment. It has been shown by Baranov and Slad that the production cross-sections for triply charmed baryons at $e^+e^-$ collider are too small to be observed [14]. Gomshi-Nobary and Sepahvand have recently calculated the fragmentation functions of $c$ and $b$ evolving into various triple heavy baryons, and estimated that the corresponding fragmentation probabilities vary in the range $10^{-7}$–$10^{-4}$ [15]. They consequently estimated cross-sections, which are associated with the production of $bcc$ and $ccc$, to be about 2 and 0.3 nb in the forthcoming Large Hadron Collider (LHC) experiment with cuts of $p_T > 10$ GeV and $|y| < 1$.

For an integrated luminosity of 300 fb$^{-1}$ (about one year of running at the LHC design luminosity $L = 10^{34}$ cm$^2$ s$^{-1}$), the amount of $bcc$ and $ccc$ yield can reach about 6108 and 1108 [5]. It is expected to establish few of the triple heavy flavour baryons in such a large data sample from LHC.

Stimulated by these possibilities of triple heavy baryons production and detection in near future, it is of recent interest to study their properties like mass spectra and magnetic moments. As no predictions to the masses of triple heavy baryons from lattice QCD simulations have emerged yet, one has to resort to other theoretical means at the moment.

Theoretically, baryons are not only the interesting systems to study the quark dynamics and their properties but are also interesting from the point of view of simple systems to study three-body problems. Out of the many approaches and methods available for the three-body systems [10,12,16–26], we employ here the hypercentral approach due to its simplicity to study the triple heavy flavour baryons. As in our earlier study of single and double heavy flavour baryons [9,27], for the present study of triple heavy flavour baryons the confinement potential is assumed in the hypercentral coordinates of the Coulomb plus power potential form.

The paper is organized as follows: Section 2 introduces the hypercentral scheme and a brief discussion about the hyper-Coulomb plus power potential for triple heavy flavour baryons. In §3, we compute the effective quark mass and magnetic moments of triple heavy baryons. Finally, in §4, we discuss the results of masses and magnetic moments of triple heavy baryons and compare our results with other theoretical predictions.

2. Hypercentral scheme for baryons

Quark model description of baryons is a simple three-body system of interest. Generally, the phenomenological interactions among the three quarks are studied using the two-body quark potentials such as the Isgur–Karl model [28], the Capstick and Isgur relativistic model [18,29], the chiral quark model [12], the harmonic oscillator model [25,26] etc. The three-body effects are incorporated in such models through two-body and three-body spin-orbit terms [9,30]. The Jacobi coordinates to describe baryon as a bound state of three constituent quarks is given by [31]

$$\vec{\rho} = \frac{1}{\sqrt{2}} (\vec{r}_1 - \vec{r}_2); \quad \vec{\lambda} = \frac{1}{\sqrt{6}} (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)$$ (1)
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such that

\[ m_\rho = \frac{2m_1m_2}{m_1 + m_2}; \quad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)}. \]  

(2)

Here \( m_1, m_2 \) and \( m_3 \) are the constituent quark mass parameters.

In the hypercentral model, we introduce the hyperspherical coordinates which are given by the angles

\[ \Omega_\rho = (\theta_\rho, \phi_\rho); \quad \Omega_\lambda = (\theta_\lambda, \phi_\lambda) \]  

(3)

together with the hyper-radius \( x \) and hyperangle \( \xi \) respectively as

\[ x = \sqrt{\rho^2 + \lambda^2}; \quad \xi = \arctan \left( \frac{\rho}{\lambda} \right) \]  

(4)

the model Hamiltonian for baryons can be written as

\[ H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{P_\rho^2}{2m} + V(x). \]  

(5)

Here the potential \( V(x) \) is not purely a two-body interaction but it contains three-body effects also. The three-body effects are desirable in the study of hadrons since the non-Abelian nature of QCD leads to gluon–gluon couplings which produce three-body forces [32]. Using hyperspherical coordinates, the kinetic energy operator \( P_x^2/2m \) of the three-body system can be written as

\[ \frac{P_x^2}{2m} = -\frac{1}{2m} \left( \frac{\partial^2}{\partial x^2} + \frac{5}{x} \frac{\partial}{\partial x} - \frac{L^2(\Omega_\rho, \Omega_\lambda, \xi)}{x^2} \right), \]  

(6)

where \( L^2(\Omega_\rho, \Omega_\lambda, \xi) \) is the quadratic Casimir operator of the six-dimensional rotational group \( O(6) \), and its eigenfunctions are the hyperspherical harmonics, \( Y_{\gamma\nu\ell_\rho\ell_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) \) satisfying the eigenvalue relation

\[ L^2 Y_{\gamma\nu\ell_\rho\ell_\lambda}(\Omega_\rho, \Omega_\lambda, \xi) = \gamma(\gamma + 4) Y_{\gamma\nu\ell_\rho\ell_\lambda}(\Omega_\rho, \Omega_\lambda, \xi). \]  

(7)

Here \( \gamma \) is the grand angular quantum number and it is given by \( \gamma = 2\nu + \ell_\rho + \ell_\lambda \), and \( \nu = 0, 1, \ldots \) and \( \ell_\rho \) and \( \ell_\lambda \) are the angular momenta associated with the \( \rho \) and \( \lambda \) variables.

If the interaction potential is hypercentral symmetric such that the potential depends on the hyper-radius \( x \) only, then the hyper-radial Schrödinger equation corresponds to the Hamiltonian given by eq. (5) can be written as

\[ \left[ \frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \gamma(\gamma + 4) \right] \phi_\gamma(x) = -2m[E - V(x)]\phi_\gamma(x), \]  

(8)

where \( \gamma \) is the grand angular quantum number, \( m \) is the reduced mass defined by

\[ m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}. \]  

(9)
For the present study we consider the hypercentral potential $V(x)$ as [9,27]

$$V(x) = -\frac{\tau}{x} + \beta x^{p} + \kappa + V_{\text{spin}}(x). \quad (10)$$

In the above equation the first three terms correspond to the confinement potential in the hyperspherical coordinates. It belongs to a general potential of the form $-Ar^{-\alpha} + kx^{p} + V_{0}$ where $A,k,\alpha$ and $\epsilon$ are non-negative constants whereas $V_{0}$ can have either sign. There are many potentials of this generality with varying values of the parameters which have been proposed for the study of hadrons [34]. For example, Cornell potential has $\alpha = \epsilon = 1$, Lichtenberg potential has $\alpha = \epsilon = 0.75$, Song-Lin potential has $\alpha = \epsilon = 0.5$ and the logarithmic potential of Quigg and Rosner corresponds to $\alpha = \epsilon \to 0$ etc. have already been employed for the study of hadron properties [35–40]. Martin potential corresponds to $\alpha = 0$, $\epsilon = 0.1$ [35–37] while Grant-Rosner and Rynes potential corresponds to $\alpha = 0$, $\epsilon = 0.045$, $\epsilon = 0$, Heikkilä, Törnquisit and Ono potential corresponds to $\alpha = 1$, $\epsilon = 2/3$ [41]. It has also been explored in the region $0 \leq \alpha \leq 1.2$, $0 \leq \epsilon \leq 1.1$ of $\alpha - \epsilon$ values [42]. So, it is important to study the behaviour of different potential schemes with different choices of $\alpha$ and $\epsilon$ to know the dependence of their parameters to the hadron properties. The potential defined by eq. (10) corresponds to $\alpha = 1$ and $\epsilon = p$. Here $\tau$ of the hyper-Coulomb, $\beta$ of the confining term and $\kappa$ are the model parameters.

The parameter $\tau$ is related to the strong running coupling constant $\alpha_{s}$ as [9,27]

$$\tau = \frac{2}{3}b\alpha_{s}, \quad (11)$$

where $b$ is the model parameter and $\frac{2}{3}$ is the colour factor for the baryon and $\beta \approx m_{\tau}$ numerically in terms of (MeV)$^{p+1}$ has been used. The strong running coupling constant is computed using the relation

$$\alpha_{s}(\mu_{0}) = \frac{\alpha_{s}(\mu_{0})}{1 + \frac{33-2n_{f}}{12\pi} \alpha_{s}(\mu_{0}) \ln \left( \frac{\mu_{0}}{\mu} \right)}, \quad (12)$$

where $\alpha_{s}(\mu_{0}) = 1$ GeV $\approx 0.6$ is considered in our study. The fourth term of eq. (10) represents the spin-dependent part of the interaction. Unlike in our earlier study [27], where the hypercentral spin-hyperfine potential is parametrized without the explicit mass dependence of the interacting quarks, here, we consider the explicit mass dependence as given by [9,30] and is taken as

$$V_{\text{spin}}(x) = -\frac{1}{4}\alpha_{s} e^{-x^{2}/x_{0}} \sum_{i<j} \frac{\vec{\sigma}_{i} \cdot \vec{\sigma}_{j}}{6m_{i}m_{j}} \vec{L}_{i} \cdot \vec{L}_{j}, \quad (13)$$

The energy eigenvalue corresponding to eq. (8) is obtained using virial theorem for different choices of the potential index $p$. The trial wave function is taken as the hyper-Coulomb radial wave function given by [32]

$$\psi_{\omega \gamma} = \left[ \frac{(\omega - \gamma)!}{(2\omega + 5)(\omega + \gamma + 4)!} \right]^{1/2} (2gx)^{\gamma} e^{-gx} L_{\omega-\gamma}^{2\gamma+4}(2gx). \quad (14)$$
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The baryon masses are determined by the sum of the quark masses plus kinetic energy, potential energy and the spin-dependent hyperfine interaction as

\[ M_B = \sum_i m_i + \langle H \rangle + \kappa_{cm}. \]  

(15)

Here \( \kappa_{cm} \) corresponds to the centre of mass correction. The hyperfine interaction energy is treated here perturbatively. The computations are repeated for different choices of the flavour combinations for \( QQ \) (\( Q \in b, c \)) systems. The computed mass variations of these baryons without hyperfine interaction (spin average mass) with respect to the potential index \( p \) shown in figures 1a–d, show the mass variations with \( p \) in \( ccc, ccb, bbc \) and \( bbb \) systems respectively. Large variations in the computed masses are seen for the choices of \( p < 1 \), while in the cases of \( p > 1 \), the mass found to attain a saturated value in all the cases of the baryonic systems studied here. Here we combine the potential parameter \( \kappa \) with the centre of mass correction term \( \kappa_{cm} \) as a single model parameter, \( E_0 \) and is related to the total mass of the constituting quarks. It is found that \( E_0 \) is linearly related to the total mass of the system \( \langle \sum_i m_i \rangle \), as

\[ E_0 = X \left( \sum_i m_i \right) + Y, \]  

(16)

where \( X = 0.211 \) and \( Y = -693.34 \) for all \( QQ \) combinations. The mass parameter and hyperfine model parameter \( x_{eqQQ} \) are taken from our previous calculations \([9]\). We relate the double heavy \( x_{eqQQ} \) values to the triple heavy \( x_{eqQQQ} \) values through an ansatz,

\[ \frac{x_{eqQQ}}{x_{eqQQQ}} = A_{eqQQ} \left( \frac{\langle \sum_i m_i \rangle_{QQQ}}{\langle \sum_i m_i \rangle_{eqQQ}} \right)^{0.5} \]  

(17)

with coefficient \( A_{eqQQ} = 0.5 \) (i.e., \( q = u, d, Q = c, b \) only) for the non-strange double heavy flavour baryons. Mass dependence on this parameter of the hyperfine interaction has already been discussed in \([30]\) and \([9]\). The computed masses of the \( J = \frac{1}{2}^+ \) and \( \frac{3}{2}^+ \) states of the triple heavy flavour baryons are listed in table 1 in the range of potential index \( 0.5 \leq p \leq 2.0 \) along with other theoretical model predictions.

3. Effective quark mass and magnetic moments of heavy baryons

Generally, the meaning of the constituent quark mass corresponds to the energy that the quarks carry inside the colour singlet hadrons, and we call it the effective masses of the quarks \([43]\). Accordingly, the effective mass varies from system to system of hadronic states. The effective mass of the quarks would be different from the ad-hoc choices of the model mass parameters. For example, within the baryons the mass of the quarks may get modified due to its binding interactions with other two quarks. Thus, the effective mass of the \( c \) and \( b \) quarks will be different when it
Figure 1. Variation of spin average masses (MeV) with potential index $p$ for triple heavy baryons. (a) ccc, (b) ccb, (c) bbc and (d) bbb.

is in ccb combinations or in bbc combinations due to the residual strong interaction effects of the bound systems.

Accordingly, we define

$$m_{i}^{\text{eff}} = m_{i} \left( 1 + \frac{\langle H \rangle + \kappa_{\text{cm}}}{\sum_{i} m_{i}} \right)$$

such that mass of the baryon,

$$M_{B} = \sum_{i} m_{i}^{\text{eff}}.$$  \hspace{1cm} (19)

Also, the magnetic moment of baryons are obtained in terms of its constituent quarks as

$$\mu_{B} = \sum_{i} \langle \phi_{st} | \mu_{i} \vec{\sigma}_{i} | \phi_{st} \rangle,$$  \hspace{1cm} (20)

where

$$\mu_{i} = \frac{e_{i}}{2m_{i}^{\text{eff}}}. \hspace{1cm} (21)$$
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Table 1. Triple heavy baryon masses (masses are in MeV).

| Baryon | Model | $J^P = \frac{1}{2}^+$ | Others | $J^P = \frac{3}{2}^+$ | Others |
|--------|-------|----------------------|--------|----------------------|--------|
| $\Omega^{++}_{ccc}$ | $p = 0.5$ | – | – | 4897 | 4803 [3] |
| | $0.7$ | – | – | 4777 | 4790 [4] |
| | $1.0$ | – | – | 4736 | 4760 [5] |
| | $1.5$ | – | – | 4728 | 4773 [7] |
| | $2.0$ | – | – | 4728 | 4777 [33] |
| | | | | | 4965 [26] |
| $\Omega^{+}_{ccb}$ | $p = 0.5$ | 8262 | 8018 [3] | 8273 | 8025 [3] |
| | $0.7$ | 8132 | – | 8142 | 8200 [4] |
| | $1.0$ | 8089 | – | 8099 | 7980 [5] |
| | $1.5$ | 8082 | 7984 [33] | 8092 | 8005 [33] |
| | $2.0$ | 8082 | 8245 [26] | 8092 | 8265 [26] |
| $\Omega^0_{bbc}$ | $p = 0.5$ | 11546 | 11280 [3] | 11589 | 11287 [3] |
| | $0.7$ | 11400 | – | 11440 | 11480 [4] |
| | $1.0$ | 11354 | – | 11394 | 11190 [5] |
| | $1.5$ | 11347 | 11139 [33] | 11386 | 11163 [33] |
| | $2.0$ | 11347 | 11535 [26] | 11386 | 11554 [26] |
| $\Omega^{-}_{bbh}$ | $p = 0.5$ | – | – | 14688 | 14569 [3] |
| | $0.7$ | – | – | 14504 | 14760 [4] |
| | $1.0$ | – | – | 14451 | 14370 [5] |
| | $1.5$ | – | – | 14444 | 14276 [33] |
| | $2.0$ | – | – | 14444 | 14834 [26] |

Here $e_i$ and $\sigma_i$ represents the charge and the spin of the quark ($s_i = \frac{\sigma_i}{2}$) constituting the baryonic state and $|\phi_{sa}\rangle$ represents the spin-flavour wave function of the respective baryonic state [12,44]. The details of the spin-flavour combinations and their wave functions corresponding to spin-$\frac{1}{2}$ and spin-$\frac{3}{2}$ baryons are given in table 2. The computed magnetic moments of the triple heavy flavour baryons are listed in table 3 for three choices of the potential indices, 0.5, 1.0 and 1.5. Other existing model predictions are also tabulated for comparison.

4. Results and discussion

We have employed the hypercentral model with hyperspherical potential of the Coulomb plus power potential form to study the masses and magnetic moments of triple heavy flavour baryons ($QQQ$, $Q \in c, b$). It is important to see that the baryon mass does not change appreciably beyond the potential power index $p > 1.0$ (see figures 1a–d). For the present calculation, we have employed the same mass parameters for $m_c$ and $m_b$ which were used to study single heavy and double heavy baryons [9]. It is interesting to note that our predictions of the mass of $QQQ$ baryons ($Q \in c, b$) are in good agreement with the existing predictions based on other theoretical models like RQM [3], NRQM [5,26] and bag model [33].
Table 2. Spin-flavour wave functions and magnetic moments of heavy flavour baryons with \( J^P = \frac{1}{2}^+ \).

| Baryon | Spin-flavour wave function | Magnetic moment |
|--------|-----------------------------|-----------------|
| \( \Omega_{ccc}^{++} \) | \( c_+ c_+ c_+ \) | \( 3 \mu_c \) |
| \( \Omega_{ccb}^{++} \) | \( \frac{2}{\sqrt{3}} (2 b_+ c_+ c_+ - c_+ b_+ c_+ - b_+ c_+ c_+ + 2 c_+ b_- c_+ - c_+ c_+ b_-) \) | \( \frac{4}{3} \mu_c - \frac{1}{3} \mu_b \) |
| \( \Omega_{cccb}^{++} \) | \( \frac{1}{\sqrt{6}} (c_+ c_+ b_+ + c_+ b_+ c_+ + b_+ c_+ c_+) \) | \( 2 \mu_c + \mu_b \) |
| \( \Omega_{bcb}^{0} \) | \( \frac{2}{\sqrt{3}} (2 c_+ b_+ c_+ b_+ - b_+ c_+ b_+ c_+ - c_+ b_+ c_+ b_- + 2 b_+ c_+ b_- - b_+ b_+ c_-) \) | \( \frac{4}{3} \mu_b - \frac{1}{3} \mu_c \) |
| \( \Omega_{bbbc}^{0} \) | \( \frac{1}{\sqrt{6}} (b_+ b_+ c_+ + b_+ c_+ b_+ + c_+ b_+ b_+) \) | \( 2 \mu_b + \mu_c \) |
| \( \Omega_{bbb}^{0} \) | \( b_+ b_+ b_+ \) | \( 3 \mu_b \) |

*indicates \( J^P = \frac{3}{2}^+ \) state.

Table 3. Magnetic moments of triple heavy baryons in terms of nuclear magneton \( \mu_N \).

| Potential index \( p \) | 0.5 | 1.0 | 1.5 | NRQM [8] | NRQM [6] |
|------------------------|-----|-----|-----|---------|---------|
| \( \Omega_{ccc}^{++} \) | 1.149 | 1.189 | 1.190 | – | 1.023 |
| \( \Omega_{ccb}^{++} \) | 0.492 | 0.502 | 0.503 | 0.510 | 0.475 |
| \( \Omega_{cccb}^{++} \) | 0.637 | 0.651 | 0.652 | – | – |
| \( \Omega_{bcb}^{0} \) | –0.199 | –0.203 | –0.203 | –0.200 | –0.193 |
| \( \Omega_{bbbc}^{0} \) | 0.212 | 0.216 | 0.216 | – | – |
| \( \Omega_{bbb}^{0} \) | –0.192 | –0.195 | –0.195 | – | –0.180 |

From table 1, it can be seen that our charge average value for \( ccb \) and \( bbc \) states for \( J^P = \frac{1}{2}^+ \) and \( J^P = \frac{3}{2}^+ \) are closer to the predictions based on NRQM using linear plus Coulomb potential [26], while for \( ccc \) and \( bbb \) states the values are closer to the RQM predictions of Martynenko [3]. Since there are larger disagreement among different model predictions, only the future experiments on these triple heavy flavour baryons would be able to guide the theoretical models.

The predictions of the magnetic moment of triple heavy flavour baryons studied here are with no additional free parameters. Our results for magnetic moments of triple heavy flavour baryons are listed in table 3 and are compared with other model predictions of [6,8]. The inter-quark interactions within the baryons are considered in the calculation of magnetic moments through the definition of effective mass of the constituent quarks within the baryon [eq. (18)]. It is interesting to note that the magnetic moment predicted in our model does not vary appreciably with different choices of \( p \) running from 0.5 to 1.5 as seen from table 3. The predictions of \( J = \frac{1}{2} \) baryons are also in accordance with NRQM results [6,8].

Experimental measurement of the heavy flavour baryonic magnetic moments are sparse and a few experimental groups (BTeV and SELEX Collaborations) are expected to do measurements in near future [45].
We conclude that the three-body description based on hypercentral coordinates and confinement potential assumed in this coordinate has played a significant role in bringing out a possible saturation property of the basic interactions within the heavy baryons as seen from the mass saturation of the baryons for the potential power index $p > 1$. We also observed that the model quark mass parameters contributes significantly to the splitting of the $J = \frac{3}{2}$ and $\frac{1}{2}$ states. The magnetic moment predictions were found to be less sensitive to the potential power index $p$. The disparity observed in our mass predictions with other theoretical predictions can only be resolved with the experimental confirmation of these states. We look forward to the experimental support to our predictions, from different future heavy flavour high luminosity experiments.

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