A way to break supersymmetry

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ABSTRACT

I study the spontaneous breakdown of supersymmetry when higher-dimensional Yang-Mills or the type-I $SO(32)$ string theory are compactified on magnetized tori. Because of the universal gyromagnetic ratio $g = 2$, the splittings of all multiplets are given by the product of charge times internal helicity operators. As a result such compactifications have two remarkable and robust features: (a) they can reconcile chirality with extended low-energy supersymmetry in the limit of large tori, and (b) they can trigger gauge-symmetry breaking, via Nielsen-Olesen instabilities, at a scale tied classically to $m_{SUSY}$. I exhibit a compactification of the $SO(32)$ superstring, in which magnetic fields break spontaneously $N = 4$ supersymmetry, produce the standard-model gauge group with three chiral families of quarks and leptons, and trigger electroweak symmetry breaking. I discuss supertrace relations and the ensuing ultraviolet softness. As with other known mechanisms of supersymmetry breaking, the one proposed here faces two open problems: the threat to perturbative calculability in the decompactification limit, and the problem of gravitational stability and in particular of the cosmological constant. I explain, however, why a good classical description of the vacuum may require small tadpoles for the dilaton, moduli and metric.

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1. Introduction

Perhaps the main puzzle of superstring unification [1, 2] concerns the breaking of space-time supersymmetry. Two proposals have so far been put forth in the literature: the non-perturbative scenario based on gaugino condensation in a hidden sector [3, 4, 5, 6, 7, 8], and the tree-level Scherk-Schwarz mechanism [9, 10, 11, 12, 13, 15, 16]. Both assume that the correct vacuum of the theory is an exact solution of the classical string equations, which leave undetermined the values of various continuous moduli. These should then hopefully be fixed by radiative or non-perturbative corrections. The mechanism of Scherk and Schwarz allows in particular for a string realization of the no-scale supergravity models [17, 18]. Supersymmetry breaks classically at a scale proportional to the values of one or more moduli, say Re$T$, and the gauge hierarchy is presumably attributed to the logarithmic running of couplings which yields a very shallow deformation of the classically-flat potential of Re$T$. A merit of this scenario is that it is calculable at tree- and one-loop order, at which point one encounters instabilities due among other things to a non-vanishing cosmological constant. Furthermore the embedding in the string, which makes one-loop gravitational corrections finite, raises a novel problem: within the one-loop approximation there is no hope to stabilize the dilaton and avoid among other things conflict with the principle of equivalence [19]. Gaugino condensation or other non-perturbative phenomena are conceptually more promising in this respect, but we lack the technology to study them directly at the string level. Most of the interesting attempts are therefore limited to guessing superpotential modifications, using as a guide field theory as well as the $S$-duality conjecture [7, 8]. Even with such guesswork a realistic vacuum without runaway dilaton and moduli and with vanishing cosmological constant has not yet been found. It is furthermore questionable whether anything short of solving all of the above gravitational instabilities at once, would constitute real progress.

In view of this unsatisfactory situation, it is I believe fair to ask whether we have not taken too seriously the classical string equations of motion. What if the true string vacuum leads to small classical tadpoles which should be cancelled ultimately by higher-loop and non-perturbative corrections? The Coleman-Weinberg mechanism [20] serves to illustrate forcefully this point: one considers a complex scalar field with quartic potential, $V_{\text{tree}}(\phi) = \lambda (\phi^* \phi)^2$, coupled to a $U(1)$ gauge field as well as to massless fermions through Yukawa couplings. The classical field equations tell us that $<\phi> = 0$, so that the photon and all fermions stay massless. One-loop corrections on the other hand can change the shape of the potential leading to a non-zero vacuum expectation value for $\phi$. Expanding around this non-zero vev gives thus a much more accurate description of the spectrum, even though classically there remains an uncanceled tadpole. Likewise, the price for getting a good description of the low-energy world from the string may be to allow for small metric, dilaton and moduli tadpoles in the classical description of the ground state. To see how small is small, suppose that in what concerns the gauge sector, the vacuum of the heterotic string could be approximated well by compactification on a four-torus times a two sphere with a magnetic monopole in its middle [21]. These backgrounds correspond [22, 23] to a $SU(2)_k$ WZW model and free fields, so that apart from the $\beta$-function of the dilaton: $\beta_\phi = (\hat{c} - 10)/\alpha' \simeq 4/ka' \neq 0$, all other classical equations are satisfied [24]. Now we imagine that quantum-gravity effects cancel the tadpoles of the dilaton and metric, but do not affect the gauge sector of the theory where supersymmetry is broken.
at a scale $m_{SUSY} \sim 1/\sqrt{\kappa a'}$. For this scale to be $\sim 1TeV$ we need $k \sim 10^{30}$. The classical prediction of string theory, that the effective space-time dimension $\hat{c} = 10$, would in this hypothetical case be a very good approximation of reality. But given our lack of control over the full quantum dynamics why should we expect it to be more accurate than one part in ten to the thirty?

Following this logic opens up a host of possibilities: any background deviating a little from a supersymmetric, classical solution of string theory could a priori be a good starting point to describe the vacuum. This nearby supersymmetric solution (NSS) must of course contain the right gross ingredients: a gauge group $G \supseteq SU(3) \times SU(2) \times U(1)$ and enough chiral fermions to describe the families of quarks and leptons. The number of non-compact dimensions in the NSS may or may not be equal to four, since the decompactification theorems [25, 13] only apply when one restores supersymmetry along marginal directions. As for all other low-energy phenomena, such as electroweak and supersymmetry breaking and the generation of mass, these are fine structure when viewed from the Planck scale and they therefore depend crucially on what kind of deviations one is willing to consider. To make some progress we need an ansatz for these deviations and I will here make the choice of turning on constant magnetic fields in torroidally-compactified dimensions. Though by no means compelling, this choice has the following attractive features:

(a) It corresponds, as I will explain below, to classically-stable solutions of higher-dimensional gauge theory, as well as of type-I open string theory, if one arranges for Nielsen-Olesen instabilities [29] to be absent. Tadpoles only arise upon coupling the dilaton and metric, which makes it more plausible that after all the quantum-gravity dust has settled, the classical spectrum is still a good approximation of reality.

(b) The pattern of supersymmetry breaking is elegant and simple, with all splittings being proportional to charge times internal-helicity operators. This is related to the fact that consistently-coupled relativistic particles must have a gyromagnetic ratio $g = 2$ [28, 29], and it implies powerful supertrace relations.

(c) Such compactifications can reconcile chirality with extended low-energy supersymmetry in the limit of large tori. This is a consequence of the index theorem [27, 1], or put differently of the fact that the magnetic field shifts the masses of mirror fermions in opposite directions.

(d) The Nielsen-Olesen instability triggers gauge symmetry breaking when the magnetic field is embedded in non-abelian group factors. This provides a new mechanism to break electroweak symmetry and tie up classically $m_Z$ to $m_{SUSY}$.

The use of open strings facilitates the analysis and postpones gravitational tadpoles to the one-loop (annulus or Möbius strip) level [30]. The above features should, however, continue to hold if one compactifies the heterotic string on magnetized spheres rather than tori. Furthermore, I expect the last two features to be robust and to characterize a much wider class of compactifications. Note that in contrast to the above mechanism, the breaking à la Scherk-Schwarz is explicit rather than spontaneous at the global level, it can accomodate chiral fermions only for minimal low-energy supersymmetry ( $N = 1$ in four dimensions) [12], and it cannot trigger electroweak breaking classically. Both mechanisms face, however, the same two serious difficulties: the problem of gravitational stability due in particular to a non-zero vacuum energy $V \sim m_{SUSY}^4$, and the problem of
large internal dimensions \[25, 13\] which, though not in obvious conflict with experiment \[10, 16\], threaten weak-coupling calculability. This latter difficulty is due in our case to the Dirac quantization condition, which ties up the size of magnetic fields to the inverse area of tori. For chiral theories it poses, as we will see, a problem at one-loop order, while in the Scherk-Schwarz scenario this problem can be postponed to two loops \[15\]. Likewise the vacuum energy has in our case a classical contribution, while the Scherk-Schwarz breaking generates it only at one-loop order. The two mechanisms do not differ qualitatively in these respects, and any way out of these difficulties could a priori apply to either one or to both. A couple of points are nevertheless worth stressing: first, reconciling chirality with extended low-energy supersymmetry raises the exciting possibility that the tractable dynamics of \(N = 4\) and \(N = 2\) supersymmetric gauge theories \[31\] could be of help in addressing these issues. Second, the classical energy of the magnetic fields, which is a drastic departure from the no-scale idea, changes the nature of the gauge-hierarchy puzzle \[\dagger\]: rather than explain why \(m_{\text{SUSY}}\) is so much smaller than \(M_{\text{Planck}}\) we only have to explain why it is not \textit{exactly} zero. Such a tree-level energy was in fact added by hand in recent phenomenological attempts to stabilize the mass of the gravitino \[33\].

Finally a note on references: there is of course a huge literature on compactifications of higher-dimensional gauge theories, Kaluza-Klein theories and supergravities in the presence of gauge-field backgrounds \[34\], and in particular on the two-sphere with a magnetic monopole in its middle \[21\]. The role of gauge backgrounds in creating chirality through the index theorem is also a well-established fact of life \[27, 34, 1\], while monopole compactifications of the \(SO(32)\) superstring were considered early on by Witten \[35\]. What was perhaps not appreciated in these earlier studies, was that magnetic fields can provide an elegant mechanism for spontaneous breaking of both supersymmetry and electroweak symmetry. These efforts were in any case silenced by the advent of string theory, as emphasis shifted to zero-energy exact solutions of the classical equations of motion. The possibility of violating these equations was evoked by Rohm and Witten \[22\]. Their study of antisymmetric-tensor-field backgrounds in relation to gaugino condensation in the heterotic string is closest in spirit to the present paper.

The plan of the paper is as follows: in section 2 I review some elementary facts about magnetic monopoles on the torus, and describe the splitting of \(N = 2\) hypermultiplets. In order to generalize the mass formula to vector and higher-spin massive multiplets, it is technically more convenient to pass to the open superstring. This is explained in section 3, which contains some known material for completeness. In section 4 I discuss the Nielsen-Olesen instability, anomaly cancellation, as well as supertrace relations and the ensuing one-loop ultraviolet softness. I explain why despite this soft behaviour, and the absence of renormalization when the tori are demagnetized, the question of perturbative calculability in the decompactification limit remains open. In section 5 I exhibit a compactification of the \(SO(32)\) superstring with a three-chiral-family standard model in the massless spectrum, broken \(N = 4\) supersymmetry and a negative mass square for the higgs doublets. Though this model has too much structure at the supersymmetry threshold to be considered at this point as phenomenologically viable, it is intriguing

\[\dagger\]Whether it can also help with the problem of the runaway dilaton \[32\] is unclear. Although this energy is multiplied by inverse powers of the coupling when one works with \(\sigma\)-model backgrounds, this changes if one rescales fields so as to normalize the Einstein term in the action \[24\]. Note also that in WZW compactifications of the heterotic string the sign of this classical energy changes sign.
how close it comes to describing low-energy physics. Finally in section 6 I give some
concluding remarks.

2. Breaking SUSY with the magnetized torus

The simplest setting of interest is that of a six-dimensional gauge theory \((A_\mu\) with \(\mu = 0, 1, \ldots, 5\)) compactified down to four dimensions on a two-torus. We assume for simplicity that the torus is generated by orthogonal vectors on the plane: \(x_4 = x_4 + R_4\) and \(x_5 = x_5 + R_5\). Vacuum solutions, invariant under the four-dimensional Poincaré group, can be characterized by a constant magnetic field \(F_{45} = H\), and by the Wilson phases around a basis of non-contractible loops on the torus. Choosing a particular gauge we may write:

\[
A_4(x) = a_4 \quad \text{and} \quad A_5(x) = a_5 + H x_4 ,
\]

where \(a_4\) and \(a_5\) are constant. Charged quantum mechanical particles behave very differently according to whether \(H = 0\) or \(H \neq 0\). In the former case their wavefunction is periodic, leading to a lattice of covariant momenta shifted from the origin by the charge \((q)\) times the constant gauge field. Thus the spectrum of the two-dimensional laplacian, which gives the mass shifts of towers of Kaluza-Klein states, reads:

\[
\delta M^2 = p_4^2 + p_5^2 = \left( \frac{2\pi n_4}{R_4} - qa_4 \right)^2 + \left( \frac{2\pi n_5}{R_5} - qa_5 \right)^2 .
\]

Here \(\delta M^2\) stands for the mass of the \((n_4, n_5)\) excitation minus the mass of the parent field in six dimensions. This spectrum changes continuously with the values of the (periodic) moduli \(a_4\) and \(a_5\), which parametrize inequivalent vacua. For \(a_\mu \neq 0\), all charged excitations are massive, including any charged (non-abelian) gauge bosons. This is the well-known mechanism of gauge-symmetry breaking by Wilson lines. It respects all supersymmetries of the parent theory since the above mass formula is spin-blind.

The situation changes drastically for a non-zero value of \(H\): the continuum of inequivalent vacua is replaced by a discrete set of states, and in the presence of charged fields all supersymmetries are spontaneously broken. To see why let us recall some elementary facts [36] about the gauge field, eq. (2.1). This is the field of a monopole, i.e. a non-trivial \(U(1)\) bundle over the torus with a transition function joining the \(x_4 \approx 0\) to the \(x_4 \approx R_4\) regions:

\[
A_\mu|_{x_4+R_4} = A_\mu - ie^{-i\theta} \partial_\mu e^{i\theta} |_{x_4}, \quad \text{with} \quad \theta = H R_4 x_5 .
\]

Demanding that \(e^{i\theta}\) be single-valued on the \(x_5\) circle leads to the Dirac quantization condition:

\[
H = \frac{2\pi K}{R_4 R_5} , \quad \text{with} \quad K \text{ integer}
\]

if the unit of charge is set equal to one. Thus for fixed area of the torus, \(H\) is a discrete modulus rather than a continuous parameter of compactification. Furthermore the spectra of Kaluza-Klein excitations depend only on the commutator of covariant momenta,

\[
[p_4, p_5] = iqH ,
\]

but not on \(a_4\) and \(a_5\). Indeed \(a_5\) can be absorbed by a shift of \(x_4\), and \(a_4\) by a change of gauge [36]. Thus, although from the 4d point of view, \(\delta A_4\) and \(\delta A_5\) are scalar fields
with a flat potential, their expectation values are physically irrelevant and do not label inequivalent vacuum states. This is of course a common phenomenon: the Goldstone boson of a spontaneously-broken global symmetry has also a physically irrelevant expectation value, as does the dilaton in a linear-dilaton vacuum of string theory.

That the magnetic field breaks supersymmetry could be argued for on the basis of its positive contribution to vacuum energy. Strictly speaking this is incorrect, since in the absence of charged fields (and of gravity) all of the $H$-vacua would still be supersymmetric. Let us assume therefore that the parent $N = 1$ supersymmetric 6d theory contains some charged chiral (and hence massless) hypermultiplet. This yields by trivial reduction two complex scalars and two Weyl spinors of opposite chirality in four dimensions. Compactifying in the background of the magnetic field splits this hypermultiplet in an interesting way. First, the Laplace operator on the torus has now the spectrum of a harmonic oscillator, giving the following mass shifts for the scalar components of the multiplet:

$$\delta M^2_{(0)} = p_4^2 + p_5^2 = (2n + 1)|qH| \quad \text{with} \quad n = 0, 1, 2,..$$  \hspace{1cm} (2.6)

Each Landau level in the above spectrum is, furthermore, $(qK)$ times degenerate, with both $q$ and $K$ being integers by virtue of quantization of charge. A set of wavefunctions that span, for example, the lowest Landau level when $a_4 = a_5 = 0$ are the following:

$$\Phi_{(0),j} = N \sum_{m=-\infty}^{\infty} \exp\left[-\frac{1}{2}|qH| \left(x_4 - (m + \frac{j}{qK})R_4\right)^2\right] \exp\left[2\pi i(qKm + j)\frac{x_5}{R_5}\right],$$  \hspace{1cm} (2.7)

where $N$ is a normalization and $j = 1, 2,..,qK$. These have indeed the required periodicities: $\Phi(x_4 + R_4, x_5) = \exp(iq\theta)\Phi(x_4, x_5)$ and $\Phi(x_4, x_5 + R_5) = \Phi(x_4, x_5)$. For higher Landau levels one must replace the first of the two exponentials above with higher excited harmonic-oscillator eigenfunctions.

Consider next what happens to the 6d Weyl spinor. Denoting by $\Gamma_\mu$ the Dirac matrices which obey $\{\Gamma_\mu, \Gamma_\nu\} = 2\eta_{\mu\nu}$, and using the commutator (2.5), one finds:

$$\delta M^2_{(1/2)} = (p_4\Gamma_4 + p_5\Gamma_5)^2$$  
\hspace{2cm} = (2n + 1)|qH| + 2qH\Sigma_{45},$$  \hspace{1.5cm} (2.8)

Each Landau level is again $(qK)$ times degenerate, and $\Sigma_{45} = \frac{1}{2}[\Gamma_4, \Gamma_5]$ is the 6d spin operator projected along the magnetic field. Since the six-dimensional spinor was Weyl, we can identify the internal helicity $i\Gamma_4\Gamma_5$ with the four-dimensional chirality, so that $\Sigma_{45} = \pm\frac{1}{2}$ for chiral or antichiral 4d spinors. At the lowest Landau level we thus obtain $(qK)$ massless chiral fermions, while their antichiral partners are shifted to a mass equal to $2|qH|$. There they are joined by the chiral fermions of the first Landau level so as to form $(qK)$ massive Dirac spinors, and the tower continues like that forever. All this is of course in agreement with the (two-dimensional) index theorem:

$$\text{index}(\partial_A) = \frac{q}{2\pi} \int dx^4 dx^5 F_{45},$$  \hspace{1.5cm} (2.9)

which we could have used to predict the net number of massless chiral fermions surviving compactification on the magnetized torus \cite{27, 34, 10}.\footnote{This is of course possible because 6d Poincaré invariance has been broken.}
It follows easily from the above expressions that for each Landau level separately

\[ \text{Str } \mathcal{M}^2 = 0 \],

\hspace{1cm} (2.10)

in accordance with the fact that the breaking of supersymmetry is spontaneous. Note that, in contrast, the Scherk-Schwarz mechanism breaks global supersymmetry explicitly, by modifying the boundary conditions of fields as in the case of finite temperature, so that the above supertrace may \cite{10} but need not vanish. Note also how chirality can be reconciled, as advertized, with low-energy \( N = 2 \) supersymmetry in four dimensions. In the stringy Scherk-Schwarz scenario by contrast, chiral fermions live in twisted sectors of orbifolds, which are spectators of the symmetry-breaking process \cite{12}. Equations (2.8) and (2.10) will stay valid for vector as well as massive higher-spin multiplets. This can be shown easier in the context of the open superstring to which we now turn our attention.

3. Type-I superstring.

A constant electromagnetic background adds only quadratic boundary terms to the world-sheet action of an open string, so that the corresponding conformal field theory can be solved exactly \cite{37, 38}. This has for instance been exploited to calculate the rate of string-pair creation in a constant electric field \cite{39}, and to show that the gyromagnetic ratio is \( g = 2 \) for all higher-spin string excitations \cite{28}. The latter is the key observation which we want now to adapt in our context. We consider for definiteness some toroidal compactification of the \( SO(32) \) superstring from ten down to four dimensions, and turn on for the time being a magnetic field in only one of the three planes of the hypertorus: 

\[ H = F_{45} \].

For later convenience we also add some Wilson-line breaking of the gauge group, by exploiting all six compact dimensions: \( a_I \) for \( I = 4, \ldots, 9 \). We take all these backgrounds in the Cartan subalgebra of \( SO(32) \), and denote by \( q_{L(R)} \) the left(right) end-point charges of the string in the direction of \( H \), and by \( q^I_{L(R)} \) the corresponding charges in the direction of \( a_I \). The world-sheet action on the strip reads:

\[
S_{\text{world-sheet}} = -\frac{1}{4\pi \alpha'} \int d\tau \int_0^\pi d\sigma \left\{ \partial_\nu X^\mu \partial^\nu X_\mu - i \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu \right\} \\
- \int d\tau \left( q_L H \left\{ X^4 \partial_\tau X^5 - \frac{i}{2} \bar{\psi}^4 \rho^0 \psi^5 \right\} + \sum_{I=4}^{9} q^I_{L(R)} \partial_\tau X^I \right)_{\sigma=0} \\
- \int d\tau \left( q_R H \left\{ X^4 \partial_\tau X^5 - \frac{i}{2} \bar{\psi}^4 \rho^0 \psi^5 \right\} + \sum_{I=4}^{9} q^I_{L(R)} \partial_\tau X^I \right)_{\sigma=\pi}.
\]

(3.1)

Here \( \alpha' \) is the Regge slope which we set equal to \( \frac{1}{2} \), the \( \psi^\mu \) are real Majorana fermions, and \( (\rho^0, \rho^4) \) are the corresponding Dirac matrices in two dimensions with the conventions of ref. \cite{1}.

Since the gauge fields only couple at the boundary, all fermionic and bosonic coordinates satisfy free-wave equations. For \( I = 6, 7, 8, 9 \), the \( X^I \) and \( \psi^I \) have the usual Neumann, and Ramond (R) or Neveu-Schwarz (NS) boundary conditions. The corresponding momenta \( p^I \) lie on a lattice which is shifted by an amount \( (q^I_L + q^I_R) a_I \) from the

\footnote{The magnetized torus was also considered in ref. \cite{38}. This study was restricted to the bosonic string, so that the issue of supersymmetry breaking did not arise.}
origin, so that all charged states are generically massive. This is up to here a conventional torus compactification with Wilson-line breaking of the gauge group and maximal \( N = 2 \) supersymmetry in six dimensions. We may also reduce the 6d supersymmetry to \( N = 1 \), by orbifolding these extra compact dimensions [10, 11], without affecting the discussion that follows.

In contrast to the Wilson-line backgrounds, a non-vanishing magnetic field changes the boundary conditions of the remaining complex compact coordinate \( X \equiv \frac{1}{\sqrt{2}}(X_4 + iX_5) \) and of its superpartner \( \psi \equiv \frac{1}{\sqrt{2}}(\psi_4 + i\psi_5) \). Recall that variations of the latter must be constrained at the boundary as follows [11]: \( \delta \psi_R = \delta \psi_L \) at \( \sigma = 0 \), and \( \delta \psi_R = -(-)^a \delta \psi_L \) at \( \sigma = \pi \), where \( \psi_{L(R)} \) is the left(right)-moving component of the fermion and \( a = 0 \) or \( 1 \) in the Neveu-Schwarz or Ramond sector. Extremizing the action leads therefore to the equations [38, 28, 39]:

\[
\begin{align*}
\partial_\sigma X &= -i \beta_L \partial_\tau X \quad \text{and} \quad \psi_R = \frac{1 + i\beta_L}{1 - i\beta_L} \psi_L \quad \text{at} \quad \sigma = 0 ; \\
\partial_\sigma X &= i \beta_R \partial_\tau X \quad \text{and} \quad \psi_R = -(-)^a \frac{1 - i\beta_R}{1 + i\beta_R} \psi_L \quad \text{at} \quad \sigma = \pi ,
\end{align*}
\]

(3.2)

where we have used the short-hand notation:

\[
\beta_{L(R)} = \pi q_{L(R)} H .
\]

(3.3)

Since the boundary conditions are linear, we can expand the coordinates in orthonormal modes as usual:

\[
X = x + i \sum_{n=1}^{\infty} a_n \phi_n(\sigma, \tau) - i \sum_{n=0}^{\infty} a_n^\dagger \phi_n(\sigma, \tau)
\]

(3.4a)

with

\[
\phi_n(\sigma, \tau) = (n - \epsilon) \frac{1}{2} e^{-i(n-\epsilon)^r} \cos[(n - \epsilon)\sigma + \arctan(\beta_L)] ,
\]

(3.4b)

and

\[
\psi = \begin{cases} 
\sum_{n=1}^{\infty} b_n \psi_n(\sigma, \tau) + \sum_{n=0}^{\infty} \tilde{b}^\dagger_n \psi_n(\sigma, \tau) & (\text{Ramond}) \\
\sum_{n=0}^{\infty} \left[ b_\sigma \psi_\sigma(\sigma, \tau) + \tilde{b}_\sigma^\dagger \psi_\sigma(\sigma, \tau) \right] & (\text{NS})
\end{cases}
\]

(3.5a)

with

\[
\psi_{(L)}(\sigma, \tau) = \frac{1}{\sqrt{2}} \exp \left[ -i(n - \epsilon)(\tau \mp \sigma) \pm i \arctan(\beta_L) \right] .
\]

(3.5b)

The above expressions depend on the magnetic field through the non-linear function [38]

\[
\epsilon = \frac{1}{\pi} [\arctan(\beta_L) + \arctan(\beta_R)] ,
\]

(3.6)

which summarizes the effects of the non-minimal string coupling. Note that in the weak-field limit that interests us in this paper (\( H \sim m_{SUSY}^2 \sim 10^{-30} \)) we have

\[
\epsilon = (q_L + q_R) H + o(H^3) ,
\]

(3.7)

while for a field of the order of the string tension \( \epsilon \) saturates to the values \( \pm 1 \). Canonical quantization leads to the commutation relations:

\[
[a_n, a^\dagger_m] = [\tilde{a}_n, \tilde{a}^\dagger_m] = \delta_{nm} = \{ b_n, b^\dagger_m \} = \{ \tilde{b}_n, \tilde{b}^\dagger_m \} = \delta_{nm}
\]

(3.8)
and
\[ [x, x^\dagger] = \frac{1}{(q_L + q_R)H} . \] (3.9)

All other commutators are zero.

The upshot of this tedious algebra is that the complex supercoordinate \((X, \psi)\) behaves like the coordinate of an orbifold \(^{[41]}\) in a twisted sector with twist angle \(\epsilon\). There are, to be sure, some significant differences between the magnetized torus and an orbifold: the center-of-mass position has in our case a non-trivial commutator, \(\epsilon\) is not related to a discrete symmetry and can be arbitrarily small for a large torus, and we do not sum over twisted sectors. The orbifold analogy is nevertheless useful in deriving the spectrum of masses. To this end we note the following:

(i) The creation operators \(\tilde{a}_n^\dagger\) and \(\tilde{b}_n^\dagger\) (\(n > 0\)) raise the helicity in the \((X_4, X_5)\) plane by one unit and have their world-sheet frequencies shifted by \(+\epsilon\). Similarly \(\tilde{a}_n\) and \(\tilde{b}_n\) lower the helicity by one unit and have their frequencies shifted by \(-\epsilon\).

(ii) The zero modes require special treatment: if \(\epsilon\) is positive, \(\tilde{a}_0\) annihilates the vacuum while \(\tilde{a}_0^\dagger\) creates the successive excited Landau levels. Likewise \(\tilde{b}_0\) annihilates the lowest-lying Ramond state of internal helicity \(-\frac{1}{2}\), while the action of \(\tilde{b}_0^\dagger\) flips the helicity to \(+\frac{1}{2}\) and raises the square mass of the state by \(2\epsilon\). If \(\epsilon\) is negative one must reverse the roles of daggered and undaggered operators in the zero-mode sector.

(iii) By virtue of world-sheet supersymmetry the magnetic field does not shift the position of the vacuum in the Ramond sector, while a straightforward calculation \([41, 38, 39]\) gives a shift of \(|\epsilon|\) for the square mass of the vacuum in the Neveu-Schwarz sector.

Putting these observations together we arrive at the following remarkably simple expression for the mass shift of all string excitations when going down from six to four dimensions:
\[ \delta M_{\text{string}}^2 = (2n + 1)|\epsilon| + 2\epsilon\Sigma_{45} , \] (3.10)

with \(\Sigma_{45}\) the spin operator projected on the plane of the torus. The non-trivial commutator (3.9) ensures furthermore that each Landau level is degenerate \((q_L + q_R)K\) times. The above expression reduces to eqs. (2.6) and (2.8) in the weak-field limit and for 6d scalar or, respectively, spinor excitations, but generalizes these results to vector and higher-spin multiplets.

Several remarks are in order here: first eq. (3.10) is of course only valid for \(q_L + q_R \neq 0\), in which case the Wilson lines \(a_4, a_5\) are irrelevant. For strings neutral with respect to the magnetic \(U(1)\) these Wilson lines shift the lattice of \((p_4, p_5)\) momenta as previously described. Secondly, the above analysis can be extended trivially to the case where all three tori are magnetized. Labeling these tori by a lower-case Latin index, and working from now on in the weak-field limit, eq. (3.7), we may write:
\[ \delta M_{\text{string}}^2 \simeq \sum_{a=(45),(67),(89)} (2n_a + 1)|q_a H_a| + 2q_a H_a \Sigma_a . \] (3.11)

Here \(H_a\) are the magnitudes of the three magnetic fields pointing in some directions inside the Cartan subalgebra of \(SO(32)\), and \(q_a\) are the corresponding total charges. One can consider more general situations, such as magnetic fields not aligned with the planes of

\(^{\dagger}\)With our conventions the square mass is given by twice the zeroth-moment Virasoro generator.
the tori, but we won’t need these in the discussion that follows. When all the $q_a \neq 0$, a 10d Weyl spinor is split by the three magnetic fields in such a way that only one 4d chiral fermion remains massless. Indeed, one must fix all three internal helicities: $\Sigma_a = -\frac{1}{2} sgn(q_a)$, so as to cancel the positive mass shift common to all excitations at the lowest Landau level. This shows how chirality can be reconciled with maximal ($N = 4$) low-energy supersymmetry in the limit of three large tori. Note finally that the tracelessness of $\Sigma_{\mu\nu}$ ensures that $strM^2 = 0$ for any multiplet and every Landau level.

### 4. Nielsen-Olesen instability, anomalies and UV softness

Let us take now a closer look at vector multiplets. A charged 6d gauge boson (and its conjugate) gives by trivial dimensional reduction two complex scalars with internal helicities $\Sigma_{45} = \pm 1$, and a 4d gauge boson with $\Sigma_{45} = 0$. The mass formula, eq. (3.10) implies that the lowest Landau excitation of one of the two scalars has a negative mass shift equal to $-|qH|$. If the gauge boson was originally massless, the vacuum would therefore be unstable, as Nielsen and Olesen were the first to point out [26]. We can of course eliminate the instability by rendering the charged 6d gauge bosons massive. This can be achieved by turning on Wilson lines in the extra compact dimensions, so that the unbroken 6d gauge group is of the form $U(1)_H \times G$. Turning on several magnetic fields will also eliminate some of the instabilities, since only one internal helicity can be non-zero for low-lying 4d scalars. A third option, whose consistency needs however to be checked in string theory, can a priori also be envisaged: since $H$ does not break the reflection symmetry $(X_4, X_5) \rightarrow (-X_4, -X_5)$ of the world-sheet action, eq. (3.1), we could mod it out and convert the torus into a $\mathbb{Z}_2$ orbifold. This projects out of the spectrum the non-zero helicity components of all 6d gauge bosons, thus eliminating the dangerous, potentially tachyonic states. Note that since the area of the orbifold is half that of the torus, the minimal magnetic charge must in this case be doubled. If despite all of the above cures there remain tachyonic scalars in the spectrum, they will acquire non-zero expectation values in the vacuum. In contrast to Wilson-line breaking, this mechanism can reduce the rank of the gauge group as the example in the following section will illustrate.

Setting magnetic instabilities aside for the moment, let me comment briefly on another important issue, i.e. the cancellation of anomalies. Suppose there are no pure gauge anomalies in six dimensions, so that the relevant box diagram is zero. Since the magnetic field gives mass to all charged gauge bosons, the unbroken gauge group after compactification is necessarily of the form $U(1)_H \times G$. Let the 6d chiral fermions transform in the representations $\oplus(q_{(R)}, \mathcal{R})$ of this gauge group. According to the index theorem, eq. (2.9), the net number of (left minus right) 4d chiral fermions in the $(q_{(R)}, \mathcal{R})$ representation is $q_{(R)}K$. The $G - G - G$ triangle anomaly thus reads:

$$(GGG) \text{ triangle } \propto K \sum_{\mathcal{R}} q_{(\mathcal{R})} \text{tr}_{\mathcal{R}}(T^{[\alpha}T^{\beta}T^{\gamma]})$$

$$\propto (GGG - U(1)_H) \text{ box } = 0 ,$$

where the $T^\alpha$ are generators of $G$. Likewise one can show easily that all triangle anoma-

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[25] Since we work with weak magnetic fields we won’t worry about the extra instabilities which can occur when $H$ is of the order of $\alpha’$. 

[42]
lies involving $U(1)_H$ vanish. This is a special case of the argument given for arbitrary compactifications by Witten \cite{Witten88,1981}. The only subtle point is the fact that anomaly cancellation in ten dimensions makes use of the two-index antisymmetric-tensor $B_{\mu\nu}$, with modified field strength

$$H_{\mu\nu\rho} = \partial_{[\mu} B_{\nu\rho]} - \text{tr} \left( A_{[\mu} \partial_{\nu]} A_{\rho]} + \frac{2}{3} A_{[\mu} A_{\nu} A_{\rho]} \right), \tag{4.2}$$

where we neglect here gravitational backgrounds. Since $H_{\mu\nu\rho}$ contributes to the energy it must be globally well-defined, which means that \cite{Witten88,1981}

$$\int_{\mathcal{K}} d \wedge H = \int_{\mathcal{K}} \text{tr} (F \wedge F) = 0 \tag{4.3}$$

for any compact four-manifold $\mathcal{K}$. It follows that consistent compactifications on several magnetized tori must obey \cite{Witten88}

$$\text{tr} (F_a F_b) = 0 \text{ for } a \neq b, \tag{4.4}$$

i.e. the various magnetic fields should point in orthogonal directions in group space. This guarantees that in the 4d theory any residual anomalies can still be cancelled by the mechanism of Green and Schwarz.

The reader may wonder why anomalies are treated differently from other ultraviolet divergences of the annulus or Möbius-strip, which signal non-vanishing gravitational tadpoles. The reason in ten dimensions is that anomalies give rise to tadpoles of unphysical Ramond-Ramond states \cite{Polchinski98}, and cannot therefore, even in principle, be cured by shifting gravitational backgrounds. In four dimensions, on the other hand, they are a signal of an ill-defined compactification as just noted. Another fact concerning eq. (4.3) is also worth pointing out: let the four-manifold $\mathcal{K}$ be the product of a torus with magnetic field $H$ on its surface, and of a sphere at spatial infinity. If all 4d scalars go to constant values at infinity, the integral factorizes into the product of magnetic fluxes. In order to satisfy (4.3) the compactified theory should therefore have no monopoles with magnetic charge under $U(1)_H$. Does this mean that electric-magnetic duality is necessarily broken in this case? The answer is not obvious because the term $H^2$ in the Lagrangian gives a mass of order $H \sim m_{\text{SUSY}}^2 / M_{\text{Planck}} \sim 10^{-3} \text{eV}$ to the $U(1)_H$ gauge boson, with $B_{45}$ furnishing its longitudinal component. This is a tiny mass indeed, since $B_{\mu\nu}$ has gravitational-strength couplings, but electric charges are all the same screened and duality could be possibly saved.

The last thing I would like to discuss in this section, is the issue of radiative corrections. These pose a threat to perturbative calculability, since above the supersymmetry threshold the theory is higher-dimensional and hence non-renormalizable, at least naively. Let me concentrate in particular on maximal $N = 4$ supersymmetry, as this might offer the best hope of alleviating the problem. To keep the expressions simple I suppose that only one of the tori is magnetized, say $F_{45} \neq 0$, but the results will also hold in the more general situation. All particles belong to spin-1 multiplets of $N = 4$, which contain one gauge boson, four Weyl fermions and six scalars. For massive multiplets one of the scalars is eaten by the longitudinal component of the vector. Out of the eight bosonic states two \footnote{We use $F_a$ to denote the Lie-algebra valued magnetic field, and $H_a$ its appropriately normalized magnitude.}
have internal helicity $\Sigma_{45} = \pm 1$ and all the others zero, while the eight fermionic states 
have $\Sigma_{45} = \pm \frac{1}{2}$ in equal numbers. A simple counting then shows that at each Landau 
level separately
\[ \text{str} \mathcal{M}^{2n} = 0 \quad \text{for all } n < 4 \, . \] (4.5)
As a result the one-loop ultraviolet behaviour is indeed much more soft than naively expected. Consider for instance a multiplet of mass $M_0$ in six dimensions. Using the expansion
\[ \text{str} \ e^{-2\Sigma_{45}} = \sum_{l=2}^{\infty} 2(1 - 4^{1-l}) \frac{z^{2l}}{(2l)!} \, , \] (4.6)
and the Schwinger proper-time parametrization, one finds the following result for its contribution to the one-loop vacuum energy:
\[ \Lambda_{1-\text{loop}} = \sum_{n=0}^{\infty} \frac{1}{32\pi^2} \int_0^\infty \frac{dt}{t^3} \ e^{-t\left(M_0^2+(2n+1)|qH|\right)} \text{str}(e^{-2tqHS_{45}}) \]
\[ = \frac{1}{16\pi^2} \sum_{l=2}^{\infty} (2^{2l} - 4) \frac{(2l - 3)!}{(2l)!} |qH|^{2l} \sum_{n=0}^{\infty} \left[M_0^2 + (2n + 1)|qH|\right]^{-2l} . \] (4.7)
Since the double summation is absolutely convergent, the result is ultraviolet finite despite the infinite tower of Landau levels. Furthermore for massive multiplets, $M_0 \gg m_{\text{SUSY}}$; the result vanishes like the sixth power of the supersymmetry scale:
\[ \Lambda_{1-\text{loop}} \simeq \frac{|qH|^3}{64\pi^2M_0^2} \sim \frac{m^6_{\text{SUSY}}}{M_0^2} . \] (4.8)
Likewise one can show that the one-loop contribution of each 6d multiplet to the effective 
gauge coupling is finite. A 6d multiplet in a representation $\mathcal{R}$ of some simple factor $G$ of 
the gauge group contributes indeed the following correction to $\alpha^{-1}_G$:
\[ \Delta^G_{1-\text{loop}} = \frac{c_2(\mathcal{R})}{2\pi} \frac{2}{12\pi} \sum_{l=1}^{\infty} \frac{(2^{2l} + 8)}{2l} |qH|^{2l} \sum_{n=0}^{\infty} \left[M_0^2 + (2n + 1)|qH|\right]^{-2l} , \] (4.9)
where $c_2(\mathcal{R})$ is the quadratic Casimir of the representation $\mathcal{R}$, and $\chi$ is here the helicity 
in four dimensions [4]. The double summation is once more convergent, and the result is finite even though we are dealing with a 6d theory and supersymmetry is broken. Furthermore for particles much above the susy threshold we find
\[ \Delta^G_{1-\text{loop}} \simeq \frac{c_2(\mathcal{R}) |qH|}{8\pi M_0^2} \sim \frac{m^2_{\text{SUSY}}}{M_0^2} , \] (4.10)
i.e. such particles do not even contribute a finite renormalization to the gauge couplings!

Does this mean that the couplings will stay small as we go up in energy towards the Planck scale? Things are, unfortunately, not so simple. First, this is a one-loop}

\[ \text{Note that } \Delta \text{ can be obtained as the coefficient of the } B^2V/8\pi \text{ term in the one-loop vacuum energy, when one turns on a magnetic field } B \text{ in a three-dimensional volume } V. \text{ This can be used to check the relative normalizations of eqs. (4.7) and (4.9).} \]
result, and there is certainly no guarantee that divergences will not show up at two- or higher-loop order. Second, if more than two compact dimensions become large, we must sum the above expressions over the 6d masses \((M_0)\) of new towers of Kaluza-Klein states. The correction will thus grow logarithmically if there are two extra large radii, and it would grow like the square of energy if all six compact dimensions were large. But six large compact dimensions is precisely what we need in order to reconcile chirality with low-energy \(N = 4\) supersymmetry, as previously noted. This example serves in fact to illustrate a crucial point: since for fixed torus area the magnetic field is a discrete modulus, the limit of supersymmetry restoration cannot be taken independently and need not commute with the sum over Kaluza-Klein states. Put differently, although every multiplet makes a vanishingly-small contribution to the running, the fact that there can be as many as \((M_{\text{Planck}}/m_{\text{SUSY}})^6\) of them can lead to a very large cumulative effect in the decompactification limit. The absence of renormalization in the supersymmetric compactification \(H_a = 0\), does not in particular protect us against potential disaster. In chiral Scherk-Schwarz compactifications power-law corrections can be suppressed at one loop \([15]\), but this difficulty should show up at two- and higher-loop order.

5. A standard model with broken \(N=4\)

I will now describe a compactification of the \(SO(32)\) superstring \(^\ddagger\), that exhibits the two main features of magnetized tori: the reconciliation of chirality with extended low-energy supersymmetry, and the triggering of electroweak breaking by the Nielsen-Olesen instability. I label the three magnetized tori by lower-case Latin indices, as in section 3, and consider the minimal fields allowed by the Dirac quantization condition:

\[ F_a = A_a^{-1}Q_a , \quad \text{for } a = (45), (67), (89). \]

(5.1)

Here \(A_a\) is the area of the \(a\)th torus, and \(Q_a\) is the corresponding generator of \(SO(32)\), normalized so that the elementary charge in the adjoint representation equals one. In order to define the \(Q_a\) we will use the following sequence of embeddings:

\[ \text{SO}(32) \supset \text{SO}(10) \times \text{SO}(6)_{\text{hor}} \times \text{SO}(8) \times \text{SO}(8)' \]

\[ \supset \left[ \text{SU}(5) \times \text{U}(1) \right] \times \left[ \text{SU}(3) \times \text{U}(1) \right]_{\text{hor}} \times \left[ \text{SU}(4) \times \text{U}(1) \right] \times \left[ \text{SU}(4) \times \text{U}(1) \right]' \]

I assume a maximal embedding \(\text{SO}(2N) \supset \text{SU}(N) \times \text{U}(1)\), and will denote by \(T_5, T_3, T_4\) and \(T'_4\) the four \(U(1)\) generators in the order in which they appear above from left to right. Note that when these are normalized to unit charge, their trace in the adjoint of \(SO(32)\) reads

\[ \text{tr}_{\text{adj}} \ T_{(N)}^2 = 60N , \]

(5.2)

where the \(N\) refers to the subgroup \(SO(2N)\) in which the generator is embedded. Let us now choose the directions of the three magnetic fields as follows:

\[ Q_{(45)} = \frac{1}{2} \left( 3T_5 - 5T_3 + T_4 - T'_4 \right) \]

\[ Q_{(67)} = \frac{1}{2} \left( T_5 + T_3 - T_4 - T'_4 \right) \]

\[ Q_{(89)} = \frac{1}{2} \left( T_5 + T_3 + T_4 + T'_4 \right) \]

(5.3)

\(^\ddagger\)See also ref. \([35]\) for an earlier effort.
Using eq. (5.2) one can check easily that these are orthogonal generators of $SO(32)$, so that the anomaly condition is satisfied.

Now recall that chiral fermions can only come from multiplets whose charges $q_a$ under all three magnetic $U(1)$’s do not vanish. This is necessary in order to fix all internal helicities of the 10$d$ Weyl spinor, and can be also seen from the expression for the net chirality in four dimensions:

$$n_L - n_R = q_{(45)}q_{(67)}q_{(89)} .$$

As a result there are no chiral fermions transforming under both the ”observable” gauge group $[SU(5) \times U(1)] \times [SU(3) \times U(1)]_{hor}$, and the ”hidden” one $[SU(4) \times U(1)]^2$. Simple inspection shows in fact that there are only three types of chiral fermions in the ”observable sector”, whose charges and multiplicities are listed in the table below:

| $SU(5) \times SU(3)_{hor}$ | $q_{(45)}$ | $q_{(67)}$ | $q_{(89)}$ | # of states |
|----------------------------|-------------|-------------|-------------|-------------|
| (10, 1)                    | 3           | 1           | 1           | 3           |
| (5, 3)                     | 1           | -1          | -1          | 1           |
| (1, 3)                     | 5           | -1          | -1          | 5           |

**Table 1.** Chiral fermions transforming under the ”observable” gauge group, their charges under the three magnetic $U(1)$’s and their multiplicities.
This is precisely the anomaly-free content of an $SU(5)$ grand-unified model with three chiral families of quarks and leptons, and an extra horizontal $SU(3)_{\text{hor}}$ symmetry under which transform the 5’s as well as standard-model singlets. To proceed further we would like to break $SU(5) \to SU(3)_c \times SU(2)_w \times U(1)_Y$, render massive other unwanted massless states and take care of Nielsen-Olesen instabilities. To these ends we still have Wilson lines, and the choice of the radii at our disposal. Rather than being systematic, let me make some choices and see where they lead us. Denoting by $Y$ the hypercharge generator inside $SU(5)$, we set

$$a_6 = a_8 = b T_{(4)} + b'T_{(4)}', \quad a_7 = -c T_{(5)} + c_Y Y \quad \text{and} \quad a_9 \in SU(3)_{\text{hor}},$$

where $b, b', c$ and $c_Y$ are constants. Recall that a Wilson line is relevant only when the charge under the magnetic field on the corresponding torus does not vanish. Chiral fermions and all their $N = 4$ partners are not therefore affected by the above backgrounds. Gauge bosons of unbroken symmetries, on the other hand, have all three $q_a = 0$, so the $a_7$ and $a_9$ Wilson lines will break $SU(5)$ to the standard model group, and the horizontal symmetry to $U(1)$ factors. Furthermore all particles charged under both the hidden and observable gauge groups have either $q_{(67)} = 0$ or $q_{(89)} = 0$, so they will obtain a mass from either the $a_8$ or the $a_6$ Wilson line. By choosing moduli appropriately we can ensure that none of these states is tachyonic.

What about the breaking of electroweak symmetry? Candidate higgs doublets come from two different places: scalar partners of the chiral fermions in the representation $(\bar{5},3)$ of $SU(5) \times SU(3)_{\text{hor}}$, and scalars in the $(\bar{5},3)$ multiplet, which does not contain chiral fermions since it has $q_{(45)} = -4$ and $q_{(67)} = q_{(89)} = 0$. The $(\bar{5},3)$ scalars are preferable for two reasons: (i) they have non-vanishing (renormalizable) Yukawa couplings with the chiral fermions as can be seen from their $SU(3)_{\text{hor}}$ transformation properties, and (ii) the Wilson line $a_7$ splits the colour-triplet from the doublet, and can thus ensure that $SU(3)_c$ remains unbroken. Using the formulae (3.11) and (2.2), and the fact that $Y = -1$ or $\frac{2}{3}$ for the doublet or (anti)triplet in the 5 of $SU(5)$, we find the following masses for the lowest-lying scalars that transform non-trivially under the standard model gauge group:

$$\mathcal{M}^2_{(5,3)} = \mathcal{A}^{-1}_{(45)} + \mathcal{A}^{-1}_{(67)} + \mathcal{A}^{-1}_{(89)} - 2 \max(\mathcal{A}^{-1}_a),$$

$$\mathcal{M}^2_{(10,1)} = 3 \mathcal{A}^{-1}_{(45)} + \mathcal{A}^{-1}_{(67)} + \mathcal{A}^{-1}_{(89)} - 2 \max(3\mathcal{A}^{-1}_{(45)}, \mathcal{A}^{-1}_{(67)}, \mathcal{A}^{-1}_{(89)}),$$

and

$$\mathcal{M}^2_{(5,3)} = \begin{cases} -4\mathcal{A}^{-1}_{(45)} + (c - c_Y)^2, & \text{(doublet)} \\ -4\mathcal{A}^{-1}_{(45)} + (c + \frac{2}{3}c_Y)^2, & \text{(triplet)} \end{cases}$$

Note that in what concerns eq. (5.7), the contribution of the $a_7$ Wilson line cannot exceed $(\pi/R_7)^2$, the contribution of the $a_9$ background was omitted for simplicity, and there are $|q_{(45)}| = 4$ excitations in the lowest Landau level. Now if we want only colour-singlet scalars to be tachyonic, we must demand that both $(\mathcal{A}^{-1}_{(45)}, \mathcal{A}^{-1}_{(67)}, \mathcal{A}^{-1}_{(89)})$ and $(3\mathcal{A}^{-1}_{(45)}, \mathcal{A}^{-1}_{(67)}, \mathcal{A}^{-1}_{(89)})$ satisfy the triangle inequalities, and that

$$|c - c_Y| < 2/\sqrt{\mathcal{A}^{-1}_{(45)}} < |c + \frac{2}{3}c_Y|.$$  

These constraints are mutually compatible, choosing for instance $\mathcal{A}_{(67)} = \mathcal{A}_{(89)} < \frac{2}{3}\mathcal{A}_{(45)}$ and $c = c_Y = R_7^{-1} = R_6^{-1}$ will satisfy them all. In an appropriate region of parameter
space the higgs doublets will thus acquire non-zero vevs, breaking electroweak symmetry and giving mass to the quarks and leptons. Furthermore, unless we fine tune parameters, the scale of electroweak breaking $m_Z$ will be tied classically to $m_{SUSY}$.

Although this model has too much structure at the supersymmetry threshold to be considered at this point as realistic, it is surprising how close we come to a classical description of our low-energy world with relatively little effort. Several features which have proven hard to achieve in previous string model-building, come out rather easily here: adjoint scalars for $SU(5)$ breaking, three chiral families of quarks and leptons, and negative mass square for the higgs doublets. More conservative uses of magnetized tori can also be envisaged: since for example mass splittings are proportional to charge, one can restrict tree-level supersymmetry breaking to a hidden and possibly hypermassive sector. Alternatively, one may insist that there are only two large compact dimensions, so that the low-energy world is $N = 2$ supersymmetric. Finally, it could be desirable to have both magnetic fields and modified, Scherk-Schwarz boundary conditions: the former would create chirality and trigger electroweak breaking, while the latter would give tree-level mass to all the standard-model gauginos.

6. Outlook

No proposal for breaking supersymmetry avoids at present all gravitational tadpoles, for the dilaton, moduli and conformal factor of the metric. Allowing such tadpoles classically is thus a valid alternative, and may be necessary for getting a good approximation of our low-energy world from the string. This logic opens up a host of possibilities of which perhaps the simplest one, compactification of $SO(32)$ string theory on magnetized tori, was studied in this paper in detail. Two remarkable features of such compactifications, namely the reconciliation of chirality with extended low-energy supersymmetry, and the Nielsen-Olesen-triggered electroweak breaking, should survive in more general settings. It should, in particular, be possible to extend the results of this paper to compactifications of the heterotic string on products of magnetized two-spheres. As in the limiting field theory, the spectrum in the full string theory should also be calculable exactly, because string motion on a magnetized sphere can be described by a WZW model. The heterotic embedding can, however, be subtle. Whether the quantum string dynamics can stabilize such a vacuum, without going to negatively-curved supersymmetric space-times is of course the big and open question. The other major difficulty, shared by the Scherk-Schwarz scenario and other marginal deformations of classical string vacua, comes from the fact that $m_{SUSY}$ is tied to the size of compact dimensions. In the compactification of section 5, for example, the entire $10d$ $SO(32)$ string lies just beyond the supersymmetry threshold! Needless to say this would have dramatic consequences, such as infinite towers of mirror fermions shifted relative to each other by $m_{SUSY}$. Unfortunately, it also poses a threat to perturbative calculability, which as I explained above cannot be addressed by studying the exact supersymmetric limit.

Acknowledgements

§By going to the corner of parameter space where the doublets are just barely tachyonic, we can be sure that their non-zero vevs will not have a big effect on the masses of all other scalars.
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