Neutrino Decays and Neutrino Electron Elastic Scattering in Unparticle Physics

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Abstract

Following Georgi’s unparticle scheme, we examine the effective couplings between neutrinos and unparticle operators. As an immediate consequence, neutrinos become unstable and can decay into the unparticle stuff. Assuming the dimension transmutation scale is around \( \Lambda_u \sim 1 \text{ TeV} \), we implement the cosmological limit on the neutrino lifetime to constrain the neutrino-unparticle couplings for different scaling dimensions \( d \). In addition, provided that the electron-unparticle coupling is restricted due to the precise measurement of the anomalous magnetic moment of electron, we calculate the unparticle contribution to the neutrino-electron elastic scattering. It is more important to jointly deal with the couplings of the unparticle to the standard model particles rather than separately. Taking into account both electron- and neutrino-unparticle couplings, we find that the scaling dimension of the scalar unparticle should lie in the narrow range \( 1 < d < 2 \) by requiring the observables to be physically meaningful. However, there is no consistent range of \( d \) for the vector unparticle operator.

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1 Introduction

It has been shown by Banks and Zaks [1] that the non-Abelian gauge theories with massless fermions can have an infrared-stable fixed point. However, this kind of scale-invariant theory requires the non-integral number of fermion generations and thus is not realized by nature. Recently, Georgi has pointed out that Banks and Zaks (BZ) fields and the standard model (SM) fields may coexist at some high energy scale, where the interaction between these two sets of fields is mediated by the messenger field with the mass scale $M_U$ [2]. At the energy scale lower than $M_U$, physical phenomena are described by the non-renormalizable operators, which are suppressed by the inverse powers of $M_U$ and of the form $\lambda O_{SM}O_{BZ}/M_U^k$, just as in the conventional effective theories. Note that $\lambda$ is the dimensionless coupling constant, $O_{BZ}$ and $O_{SM}$ are respectively the operators composed of BZ and SM fields. It is well known that the radiative corrections in the scale-invariant theory will induce the dimension transmutation [3], which means that an energy scale $\Lambda_U$ appears even if there is only one dimensionless coupling in the generic theory of BZ fields. As argued by Georgi [2], below the scale $\Lambda_U$, the BZ field operator $O_{BZ}$ should match onto the unparticle operator $O_U$ with a non-integral scaling dimension $d$. Therefore, we have the low-energy operators $\lambda' \Lambda_U^{(d_{BZ}-d)}O_{SM}O_{U}/M_U^k$, where $d_{BZ}$ is the scaling dimension of $O_{BZ}$. In such a setup, Georgi has further claimed that the unparticle effects can show up at the colliders as the missing energy and may be promising to be discovered prior to the other new physics beyond the SM [2].

Shortly after Georgi’s proposal, enormous studies have been performed to investigate the unparticle phenomenology [4]-[19]. Since the interaction between the unparticle and SM particles is unclear, one may introduce the operator which can influence the processes well measured in experiments. In this direction of thought, the invisible unparticle $U$ as the final state has been considered in the top quark decay $t \rightarrow u + U$ [2], the electron-positron annihilation $e^+ + e^- \rightarrow \gamma + U$ and the hadronic processes, such as $q + q \rightarrow g + U$ [4, 5]. The importance of the interference between the SM and unparticle-induced contributions to a specific process is highlighted in Ref. [4], where the typical channel $e^+ + e^- \rightarrow \mu^+ + \mu^-$ is analyzed in detail. In some sense, the unparticle sector serves as one kind of new physics beyond the SM. One should take into account the unparticle effects on all the familiar processes. Bearing this in mind, some authors have discussed the possible new origin of CP violation [7], the deep inelastic scattering [8], the anomalous magnetic moment of charged leptons ($g - 2$) [5, 6, 9] and lepton-flavor-violating processes [10, 13] in unparticle physics. On the other hand, since the Lorentz group representations that the unparticle operators belong to are not restricted, they can be of scalar, vector [4, 5] or spinor types [6]. It is also natural to assume that the unparticle operator is invariant under the gauge symmetry of the SM, then one can systematically write down the gauge invariant effective operators as in Ref. [17].

However, the couplings between neutrinos and the unparticle have not yet been touched thus far. Thanks to the elegant neutrino oscillation experiments, we are now convinced that neutrinos are massive and lepton flavors are mixed [20]. Massive neutrinos play an important role in astrophysics and cosmology, for instance, the energy density of active neutrinos may
affect the light element abundance in the big bang nucleosynthesis scenario and the cosmic microwave background. If the unparticle sector couples to neutrinos, heavier neutrinos can decay into the light ones and the invisible unparticle stuff as we will show later. At present, the most stringent limit on the neutrino lifetime comes from the solar neutrino experiment \[21\]: 
\[ \tau/m \geq 10^{-4} \text{ s eV}^{-1} \] 
with \( m \) and \( \tau \) being the mass and lifetime of neutrinos. The detection of the decay of neutrinos from other astrophysical sources, such as a Galactic supernova or a distant Active Galactic Nuclei, may improve the constraint by several orders of magnitude.

It has been recently proposed \[22\] that future cosmological observations can measure the sum of neutrino masses to the accuracy about \( 10^{-2} \) eV, thus they may serve as the best probe of the neutrino lifetime \( \tau/m \geq 10^{16} m_{50}^{-5/2} \text{ s eV}^{-1} \), where \( m_{50} \equiv m/(50 \text{ meV}) \). Obviously, this bound is more serious than that from solar neutrino analysis and can be used to constrain the neutrino-unparticle couplings. Note that we will concentrate on the non-radiative decays of neutrinos, the cosmological limit on the radiative decays can be found in \[23\].

This paper is organized as follows. In Sec. 2, we introduce the interaction between neutrinos and the scalar unparticle operator in addition to the electron-unparticle coupling. The latter is restricted by the precise measurement of the anomalous magnetic moment of electron, while the former is constrained from the bound on neutrino lifetime. Furthermore, we also calculate the cross section of neutrino-electron elastic scattering, in which these two kinds of couplings simultaneously appear. It is found that the scaling dimension of the scalar unparticle operator, which couples with both electrons and neutrinos, should stay in the range \( 1 < d < 2 \), since the observables should be physically meaningful. In comparison, the vector unparticle operator is considered in Sec. 3, where we show that there is no consistent range of the scaling dimension. Therefore, it is more important to jointly deal with the unparticle effects in different physical processes, in which the common unparticle operator is present. Sec. 4 is devoted to the summary of conclusions.

2 Scalar Unparticle Operator

The simplest case is to consider the scalar unparticle operator, and our working effective Lagrangian takes the following form

\[ \mathcal{L}_S = \frac{\lambda_{\alpha\beta}^{\nu}}{\Lambda_{\nu}^{d-1}} \bar{l}_\alpha l_\beta \mathcal{O}_\nu + \frac{\lambda_{\mu}^{\nu}}{\Lambda_{\mu}^{d-1}} \bar{\nu}_\alpha \nu_\beta \mathcal{O}_\mu + h.c., \] 

(1)

where \( \alpha, \beta = e, \mu, \tau \) are the flavor indices, \( \Lambda_{\nu} \) the dimension transmutation scale, \( d \) the scaling dimension of the scalar unparticle operator, \( \lambda_{\alpha\beta}^{\nu} \) and \( \lambda_{\mu}^{\nu} \) the relevant coupling constants. The most important consequence of the first term in Eq. (1) is that the unparticle contributes to the anomalous magnetic moments of charged leptons. On the other hand, because of its flavor-violating feature, the scalar unparticle can mediate the lepton-flavor-changing rare decays, such as \( \mu^- \rightarrow e^-e^+e^- \) \[10\]. In the following we will concentrate on the flavors in the neutrino sector and thus only consider the electron-unparticle coupling \( \lambda_{e}^{ee} \equiv \lambda_{e} \). Note that the unparticle contribution to the anomalous magnetic moment of electron has already been discussed in
Refs. [4, 5, 9]:
\[
\Delta a_e = -\frac{3A_d\lambda_e^2}{16\pi^2\sin(d\pi)} \Gamma(2-d)\Gamma(2d-1) \left( \frac{m^2}{\Lambda_{U^2}} \right)^{d-1},
\]
where \( m_e = 0.51 \) MeV is the electron mass and \( A_d \) is a normalization constant defined in Eq. (4) below. The scaling dimension should be \( d < 2 \) in order that the integral is finite. As argued by Georgi [4], one can choose the theoretically consistent values of \( d \) in the range \( 1 < d < 2 \). The difference between the current experimental data and the SM prediction of \( a_e \) is \( |\Delta a_e| \leq 15 \times 10^{-12} \) [9], which can place a strict constraint on the parameters \( \Lambda_U, d \) and \( \lambda_e \). As shown in Fig. 1, the severest bound on the electron-unparticle coupling is \( \lambda_e < 10^{-4} \) if \( d = 1.1 \), where the dimension transmutation scale is typically taken to be \( \Lambda_U = 1 \) TeV. Note that if \( d \) gets larger values, the constraint on \( \lambda_e \) can be relaxed.

We introduce the lepton-flavor-violating couplings of neutrinos to the unparticle, which is well motivated by neutrino flavor mixing as observed in the neutrino oscillations. From the second term in Eq. (1), one can observe that heavier neutrinos become unstable and can decay into the unparticle stuff and the light ones. Nevertheless, the mass ordering of neutrinos is not uniquely determined. For simplicity, we assume that the lightest one is massless: for the normal mass hierarchy, \( m_1 = 0, m_2 \approx 9.0 \) meV and \( m_3 \approx 50 \) meV; for the inverted mass hierarchy, \( m_3 = 0 \) and \( m_1 \approx m_2 \approx 50 \) meV. It is more convenient to work in the neutrino mass eigenstate basis, which is defined as \( \nu_i = \sum_\alpha V^*_\alpha i \nu_\alpha \) with \( V \) being the neutrino mixing matrix. In this basis, the interaction between neutrinos and the unparticle can be written as \( \lambda^{ij}_\nu \nu_i U_{\alpha j}/\Lambda_U^{d-1} \) with \( \lambda^{ij}_\nu \equiv \sum_\alpha \lambda_\nu^{* \alpha \beta} V^*_\alpha \lambda^\beta \). According to the scale invariance in the unparticle sector, we have [4]
\[
\int d^4x \left< \langle 0 | T[O_{U}(x)O_{U}^\dagger(0)]|0 \rangle \right> e^{ipx} = iA_d \frac{1}{2} \frac{1}{\sin(d\pi)} (-p^2 - i\epsilon)^{d-2},
\]
while there is an additional Lorentz factor \((-g^{\mu\nu} + p^\mu p^\nu/p^2)\) for the vector unparticle operator \( O_{U}^\mu \), which satisfies the transverse condition \( \partial_\mu O_{U}^\mu = 0 \). The normalization constant is defined as
\[
A_d \equiv \frac{16\pi^{5/2}}{(2\pi)^{2d}} \frac{\Gamma(d + 1/2)}{\Gamma(d - 1)\Gamma(2d)}.
\]
It is straightforward to figure out the differential decay rate of neutrinos, namely the process \( \nu_j(p, s_1) \to \nu_i(k, s_2) + U(q) \),
\[
d\Gamma_j = \frac{1}{2m_j} (2\pi)^4 \delta^4(p - k - q) |M|^2 \frac{d^4k}{(2\pi)^4 2k^0} \left[ A_d \theta(q^0)\theta(q^2)(q^2)^{d-2} \frac{d^4q}{(2\pi)^4} \right],
\]
where the invariant matrix element \(|M|^2 = 2|\lambda^{ij}_\nu|^2 k \cdot p/\Lambda_U^{2(d-1)}\) has been summed over the final state spins and averaged over the initial state spin. After integrating over the phase space, we get
\[
\frac{d\Gamma_j}{dE_i} = \frac{4\pi^2 \lambda^{ij}_\nu^2}{4\pi^2 \Lambda_U^{2(d-1)} (m_j^2 - 2m_j E_i)^{2-a}},
\]
where $\nu_i$ is the lightest neutrino and its mass has been set to be vanishing for simplicity, and $E_i$ is the energy of the final state neutrino. The total decay rate is given by

$$\Gamma_j = \int_0^{m_j/2} \left( \frac{d\Gamma_j}{dE_i} \right) dE_i = \frac{A_d |\lambda_{ij}|^2}{16\pi^2(d^2 - 1)} \left( \frac{m_j^2}{A_U^2} \right)^{d-1} m_j.$$  \hspace{1cm} (7)

Combining the expression of $a_e$ and the above equation, we see that the scaling dimension should lie in the range $1 < d < 2$ in order that these physical quantities are well defined. To make clear the dependence of the differential decay rate on $d$, we can define

$$m_j \frac{d\ln\Gamma_j}{dE_i} = 4d(d^2 - 1)(1 - 2y)y^{d-2},$$ \hspace{1cm} (8)

where $y \equiv E_i/m_j < 1/2$. This is the same as the process $t \rightarrow u + U$ considered in [2], however, the scaling dimension is now $1 < d < 2$ as restricted by the anomalous magnetic moment of electron and the neutrino decay rate. It is evident that the behavior of the decay rate with the unparticle stuff in the final state is drastically different from the ordinary two-body decay case. Since it is almost impossible to measure decay products of neutrinos unlike the decay of top quark [2], the most important and relevant quantity is the total decay rate $\Gamma_j$, or equivalently the neutrino lifetime $\tau_U \equiv \Gamma_j^{-1}$. From Eq. (7), we can obtain

$$\frac{\tau_U}{m_j} = \frac{16\pi^2 d(d^2 - 1)}{A_d |\lambda_{ij}|^2} \left( \frac{A_U^2}{m_j^2} \right)^{d-1} \frac{1}{m_j^2},$$ \hspace{1cm} (9)

which should be contrasted with the future cosmological constraint $\tau/m \geq 10^{16} m_{50}^{-5/2} \text{ s eV}^{-1} \approx 1.5 \times 10^{31} m_{50}^{-5/2} \text{ eV}^{-2}$. As is mentioned before, the mass hierarchy of neutrinos is still undetermined. However, the most crucial limit on $\lambda_{ij}$ is the case with $m_j = 50$ meV and $m_i = 0$, for which the numerical analysis is shown in Fig. 2. One can observe that the neutrino-unparticle couplings are restricted to be on the order of $10^{-5}$ for $d = 1.1$ and $A_U = 1 \text{ TeV}$. This bound will be relaxed when $d$ becomes larger, for instance, $\lambda_{\nu} \sim 0.5$ if $d = 1.7$. Note that the constraint on $\lambda_{ij}$ can be directly converted into that on $\lambda_{\nu}^{\alpha\beta}$ by using the neutrino mixing matrix, which is now measured in neutrino oscillation experiments to an acceptable degree of accuracy. Roughly we expect them to be of the same order.

Now we proceed to discuss the physical processes, in which both electron- and neutrino-unparticle couplings are present. It is easy to note that the unparticle will contribute to the neutrino-electron elastic scattering. For the $\nu_\mu e^-$ elastic scattering, the unparticle contribution will interfere with the charged- and neutral-current amplitudes, while for the $\nu_\alpha e^-$ ($\alpha = \mu, \tau$) interference between the unparticle and neutral-current components arises. The relevant neutrino-unparticle couplings $\lambda_{\nu}^{\alpha\alpha}$ in these two cases can be probed by measuring the cross sections of neutrino electron elastic scattering. However, the $\nu_\alpha e^- \rightarrow \nu_\beta e^-$ for $\alpha \neq \beta$ can not occur in the SM, and the couplings are also relevant to the neutrino decays. So we will calculate this flavor-changing process, and point out its implication for the unparticle physics. Note that this case is similar to the non-standard interaction discussed in Refs. [24, 25]. The
invariant matrix element can be computed for $\nu_\alpha(k) + e^-(p) \rightarrow \nu_\beta(k') + e^-(p')$ scattering:

$$\frac{1}{4} \sum_s |M|^2 = \frac{1}{16} \left[ \frac{A_d \lambda_\nu \lambda_{\nu \beta}}{\Lambda_{U d}^{2(d-1)} \sin(d\pi)} \right]^2 (k' \cdot p') (p \cdot p') \left[ -(k - k')^2 - i\epsilon \right]^{2(d-2)}, \quad (10)$$

where we have summed over the final spins and averaged over the initial spins. The total cross section in the center-of-mass reference frame is given by

$$\sigma(s) = \int \frac{1}{4(k \cdot p)} (2\pi)^4 \delta^4(k + p - k' - p') \left( \frac{1}{4} \sum_s |M|^2 \right) \frac{d^3k'}{(2\pi)^3 k'_0} \frac{d^3p'}{(2\pi)^3 2p'_0}, \quad (11)$$

where the lepton masses have been neglected. A straightforward calculation leads to the differential cross section

$$\frac{d\sigma(s)}{d\cos\theta} = \frac{1}{32\pi \cdot 4^d} \left[ \frac{A_d \lambda_\nu \lambda_{\nu \beta}}{\Lambda_{U d}^{2(d-1)} \sin(d\pi)} \right]^2 s^{2d-3} (1 - \cos\theta)^{2(d-1)}, \quad (12)$$

where $s = (k + p)^2$ is the center-of-mass energy square and $\theta$ is the azimuthal angle. Note that the total cross section $\sigma(s) \propto (s/\Lambda_{U d}^{2(d-1)})/[64\pi(2d - 1)s]$ is always regular for $1 < d < 2$. Given the information about $\lambda_\nu$ and $\lambda_{\nu \beta}$, one can predict the total cross section of the flavor-changing neutrino-electron scattering. For example, we take the scaling dimension $d = 1.7$ and $\Lambda_{U d} = 1$ TeV, then $\lambda_\nu \leq 1.0$ and $\lambda_{\nu \beta} \leq 0.5$ as respectively indicated by Fig. 1 and Fig. 2. Finally we get $\sigma \leq 1.4 \times 10^{-36}$ cm$^2$ for $\sqrt{s} = 200$ GeV, which should be compared with the SM prediction of the neutrino-electron elastic scattering. Note that the present limit on neutrino lifetimes is just $\tau/m \geq 10^{-4}$ s eV$^{-1}$, which will hardly constrain $\lambda_\nu$. In this case, we may inversely use the experimental data on extra contributions to neutrino-electron elastic scattering to extract the information about neutrino-unparticle couplings.

3 Vector Unparticle Operator

If the vector unparticle operator couples both to charged-leptons and to neutrinos, the Lagrangian can be written as

$$\mathcal{L}_V = \frac{\lambda_{\nu}}{\Lambda_{U d}^{d-1}} \bar{l}_\alpha \gamma_\mu l_\beta \mathcal{O}_U^\mu + \frac{\lambda_{\nu}}{\Lambda_{U d}^{d-1}} \bar{\nu}_\alpha \gamma_\mu \nu_\beta \mathcal{O}_U^\mu + \text{h.c.}, \quad (13)$$

which will cause some interesting implications for unparticle physics. For simplicity, we still focus on the neutrino flavors and are just concerned about the electron-unparticle coupling. The vector unparticle contribution to the anomalous magnetic moment of electron has also been calculated in [5, 9],

$$\Delta a_e = -\frac{A_d \lambda_\nu^2}{8\pi^2 \sin(d\pi)} \frac{\Gamma(3-d)\Gamma(2d-1)}{\Gamma(d+2)} \left( \frac{m_e^2}{\Lambda_{U d}^2} \right)^{d-1}, \quad (14)$$

where the requirement $d < 2$ should be satisfied. On the other hand, one can analogously discuss the total rate of neutrino decays into the vector unparticle stuff, i.e. $\nu_j(p, s_1) \rightarrow$
\[ \nu_i(k, s_2) + U(q, S). \] The invariant matrix element then is
\[ \frac{1}{2} \sum_{s_1, s_2, S} |\mathcal{M}|^2 = \frac{2 |\lambda_{ij}^\nu|^2}{\Lambda_{2(d-1)}^2} \left[ 2(k \cdot q)(p \cdot q)/q^2 + k \cdot p \right]. \tag{15} \]

After substituting the above equation into Eq. (5) and integrating over the phase space, we get
\[ \frac{d\Gamma_j}{dE_i} = \frac{A_d |\lambda_{ij}^\nu|^2}{4\pi^2 \Lambda_{2(d-1)}^2} \frac{m_j E_i^2 (3m_j - 4E_i)}{(m_j^2 - 2m_\nu E_i)^{3-d}} \theta(m_j - 2E_i), \tag{16} \]
where \( E_i \) is the energy of the final state neutrino. The total decay rate is given by
\[ \Gamma_j = \int_0^{m_j/2} \left( \frac{d\Gamma_j}{dE_i} \right) dE_i = \frac{3A_d |\lambda_{ij}^\nu|^2}{16\pi^2 d(d-2)(d+1)} \left( \frac{m_j^2}{\Lambda_{2(d-1)}^2} \right)^{d-1} m_j. \tag{17} \]

In the above equation, \( d > 2 \) is demanded in order that the total decay rate is finite and positive. Unfortunately, this condition on the scaling dimension conflicts with that from the calculation of \( a_e \). It seems very strange that the vector unparticle interacting with electrons cannot simultaneously interact with neutrinos in the way depicted in Eq. (13).

### 4 Conclusions

In summary, we have introduced effective couplings between neutrinos and the scalar unparticle operator in addition to the electron-unparticle coupling. Because of the neutrino-unparticle interaction, heavier neutrinos become unstable and can decay into the unparticle stuff. The decay rate of neutrinos has been calculated and confronted with the cosmological limit on the neutrino lifetime. Provided that the electron-unparticle coupling is constrained from the precise measurement of the anomalous magnetic moment of electron, and the neutrino-unparticle coupling from the bound on the neutrino lifetime, we also figure out the cross section of the lepton-flavor-changing neutrino-electron scattering. The scaling dimension turns out to be in the range \( 1 < d < 2 \) in order that the anomalous magnetic moment of electron in Eq. (2) and the neutrino decay rate in Eq. (7) are well defined. In comparison, we also consider the effective interactions of electrons and neutrinos with the vector unparticle operator. It is found that there is no consistent region for the scaling dimension in this case. Therefore, we remark that it is necessary to systematically consider the physical processes in which the common unparticle operator couples to the SM particles. This has been proved useful for the determination of the scaling dimension.

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Figure 1: Numerical illustration of $(\Delta a_e, \lambda_e)$ for different scaling dimensions of the scalar unparticle operator: $d = 1.1$ (solid line), $d = 1.3$ (dotted line), $d = 1.5$ (dotted-dashed line), $d = 1.7$ (dashed line), where the horizontal line corresponding to $\Delta a_e = 15 \times 10^{-12}$ is the difference between the SM prediction and the experimental data.
Figure 2: Numerical illustration of \((\tau U/m_j, |\lambda_{ij}|)\) for different scaling dimensions of the scalar unparticle operator: \(d = 1.1\) (solid line), \(d = 1.3\) (dotted line), \(d = 1.5\) (dotted-dashed line), \(d = 1.7\) (dashed line), where the horizontal line corresponds to the future cosmological bound \(\tau/m \geq 1.5 \times 10^{31} \text{ eV}^{-2}\).