Training Behavior of Sparse Neural Network Topologies

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Abstract—Improvements in the performance of deep neural networks have often come through the design of larger and more complex networks. As a result, fast memory is a significant limiting factor in our ability to improve network performance. One approach to overcoming this limit is the design of sparse neural networks, which can be both very large and efficiently trained. In this paper we experiment training on sparse neural network topologies. We test pruning-based topologies, which are derived from an initially dense network whose connections are pruned, as well as RadiX-Nets, a class of network topologies with proven connectivity and sparsity properties. Results show that sparse networks obtain accuracies comparable to dense networks, but extreme levels of sparsity cause instability in training, which merits further study.

Index Terms—neural network, pruning, sparse, training

I. INTRODUCTION

Neural networks have become immensely popular in recent years due to their ability to efficiently learn complex representations of data. In particular, innovations in the design and training of deep convolutional neural networks have led to remarkable performances in the field of computer vision [1]–[4]. Researchers have found that making networks larger, wider, and deeper is a surefire way to increase performance. Many improvements to network performance come from optimizations which tame the training hiccups and computational demands which result from using larger, deeper networks. Because of the explosion in the size of state-of-the-art networks, fast memory has become a key limit in our ability to improve neural network performance.

One strategy to decreasing the memory requirements of training large neural networks is to introduce sparsity into a network’s connections [5]–[8]. This strategy has a biological motivation: the human brain exhibits very high sparsity with its connections, with each neuron connected to approximately 2000 of the 86 billion total neurons on average, a sparsity of $2 \cdot 10^{-8}$ [9]. In addition, research has shown that trained neural networks contain many unnecessary weights [10]. If we can discover sparse topologies which have ‘built-in’ resolution of these redundancies, then we could build larger and better networks. Sparse network structures also lend themselves to being trained in a high-performance manner via sparse matrix libraries, which would lead the way for extremely large, sparse neural networks to be efficiently trained. The MIT/Amazon/IEEE Graph Challenge now features a Sparse Deep Neural Network Graph Challenge for developing such a system [11].

Existing research on sparsity in neural networks is mostly concerned with model pruning, where large networks are pruned to a small fraction of the original size without a loss in accuracy. Pruning was introduced by Lecun [12], who pruned weights via second derivative information. The work of Han et al. [13] introduced the effective technique of iterative pruning, where one alternates training and pruning a network to increasing levels of sparsity. Several other model compression techniques have been used, including low-rank approximation [14], variational dropout [15], and other pruning variations [16]–[19].

While there is a large body of research on model pruning, these methods typically begin by training a large dense network before pruning to obtain a sparse network. Such methods aid in model compression for the purpose of model deployment, but are not applicable to the challenge of designing sparse networks for the purpose of efficient training. There is little research addressing the training of purely sparse networks, or the design of originally sparse network topologies. Even so, sparsity has played an indirect role in several deep learning innovations. For example, convolutional layers in a network can be understood as a structured sparsity of connections, allowing more complex connections to be made without an explosion of computation requirements. One paper which does address the creation and training of sparse network topologies is the work of Prabhu et al. [20]. They replace convolutional and fully-connected layers with sparse approximations to create sparse networks, achieving a net decrease in memory required to train the network without any loss in accuracy.

In this paper we continue the search for viable sparse network topologies for training and large-scale inference. We focus on the trainability of sparse topologies and how they compare to their dense counterparts. We test two types of sparse topologies. The first are sparse topologies which result from pruning a dense network. We prune a pre-trained dense network with both one-time pruning and the iterative pruning technique developed in [19]. Then we train a new network with the pruned structure. The second type of sparse topologies we test are RadiX-Nets. RadiX-Nets, developed by Robinett and Kepner [21], improve off the work in [20] to provide sparse topologies. 

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topologies with theoretical guarantees of sparsity and connectivity properties. We replace fully-connected and convolutional layers with RadiX-Nets and their random counterparts and train them, comparing the accuracy achieved. Our experiments are done on the Lenet-5 and Lenet 300-100 models trained on the MNIST [22] and CIFAR-10 [23] datasets.

We find that both the topology of the sparse network and the sparsity level of the network affect its ability to learn. With pruning-based sparse topologies, the iteratively-pruned sparse structure could be retrained to higher accuracy than the one-time pruned structure, giving evidence that specific connections are crucial to keep when designing sparse topologies. At higher sparsity levels, the sparse networks exhibit convergence difficulties. Better accuracies were obtained on the extremely sparse topologies with a higher learning rate than that used for training dense networks. With RadiX-Net topologies, sparser structures perform generally close to the original dense networks, with 50% sparse networks performing almost equally to their dense counterparts. The results suggest that for this sparse structure, higher sparsity levels may limit performance even when the number of total connections is kept constant.

II. TRAINING SPARSE NEURAL NETWORKS

Compared to dense networks, there are many more possibilities for designing the structure of sparse networks, as we need to decide which subset of a fully-connected layer will be maintained. We consider two approaches to specifying sparse network structure: pruning-based structures and RadiX-Net structures.

A. Pruning-based structures

Our first method uses model pruning to create a sparse network from a densely-trained network. Among several pruning techniques, we found the pruning technique from [19] to be most efficient and effective. The authors made their pruning code open-source, allowing easy validation and repetition of results. After an initial training period, we prune the network every 200 steps such that the network sparsity matches a given sparsity function $s(t)$, a monotonically-increasing function which starts at zero and finishes at the desired sparsity. Whenever connections are pruned, the corresponding weights remain zero for the rest of the procedure. After the desired sparsity level is reached, we train the network for another period with no pruning. This pruned network is then used as a sparse network structure for a new network, which is trained with new weights.

We are able to achieve 99% sparsity with less than 1% reduction in accuracy, and approximately 95% sparsity without any loss in accuracy whatsoever (before retraining on the sparse structure). These results show that the sparse structures learned from the pruning process have the potential to perform just as well as dense structures. We then see if retraining from scratch on the pruned structure can recover that accuracy.

We prune Lenet-5 and Lenet-300-100 on the MNIST dataset to 50%, 75%, 90%, 95%, and 99% sparsity and trained on the sparse structure. To see how the original accuracy of the sparse model affects the sparsely-trained model’s accuracy, we conduct the same experiment using one-time pruning. With one-time pruning, we prune a percentage of the connections with the smallest weights, without any retraining (leading to lower initial accuracy than the iteratively-pruned models).
B. RadiX-Net structures

Our second method improves on the work done in [20]. We replace fully-connected and convolutional layers with sparse equivalents. We use the RadiX-Nets of Robinett and Kepner [21] to create our sparse structure. Their work improves off [20] by providing more rigorous theoretical guarantees of path-connectedness between the input and output. A RadiX-Net is defined with a mixed radix system denoted with a set of $\mathcal{N} = (N_1, N_2, \ldots, N_i)$ and a Kronecker structure denoted with a set of $\mathcal{B} = (B_1, B_2, \ldots, B_{i+1})$.

In addition, we test random sparse networks, where given the dense network to replace, each edge is present with probability $1-s$, where $s$ is the desired sparsity level from zero to one. Such layers asymptotically hold the same theoretical guarantees of path-connectedness as the RadiX-Net while being simpler to implement. As a result, the majority of our experiments use random sparse layers. If one were to implement a neural network training framework using sparse matrix representation, the structured sparsity of RadiX-Nets would yield much greater computation acceleration compared to the random structure.

We experiment with both fixing the number of total connections as sparsity increases, so that the total neurons increases correspondingly, and fixing the number of neurons as sparsity increases, so that the total connections decreases correspondingly. First, we test networks with two, ten, and a hundred times the number of neurons of the original network, with corresponding sparsities so that the total connections remain constant. In addition, we test networks with one-half, one-tenth, one-twentieth, and one-hundredth the number of connections of the original network, with corresponding sparsities so that the total neurons remain constant. We train sparse versions of Lenet-300-100 and Lenet-5 on MNIST as well as Lenet-5 on CIFAR-10. We also test RadiX-Nets on Lenet-300-100 for networks one-tenth and ten times the size of the original to confirm that their performance was equal to that of the random sparse nets. Our large and small Lenet-300-100 RadiX-Nets use $\mathcal{N} = (10, 10), \mathcal{B} = (8, 30, 1)$ and $\mathcal{N} = (10, 10), \mathcal{B} = (8, 3, 1)$ respectively to create networks with the same shape as Lenet-300-100 but with 90% sparsity. To match MNIST input dimension of 784, the top 16 neurons are removed from the network. For more information on the motivation, construction and properties of RadiX-Nets, refer to [21].
III. RESULTS

Figure 7 shows the result of pruning and retraining on MNIST. We first see that for Lenet-300-100 through 95% sparsity, the iterative pruning actually improves the network’s accuracy, with pruning acting as a form of regularization. Training on the pruned structure does not reach as high accuracy as the initial accuracy from iterative pruning. However, up to 90% it achieves the original accuracy. Training on the structure given by one-time pruning, however, performs noticeably worse. This suggests that there are important connections or patterns of connections which one-time pruned structures do not maintain.

The results on Lenet-5 are much different. Unlike Lenet-300-100, this network contains convolutional layers and achieves much better performance on MNIST. Even though the iterative pruning process allows the network to obtain 99% sparsity with less than one percent drop in accuracy, trainability on the pruned sparse structure exhibits a large variance. After some runs the network is able to achieve almost the same accuracy, as exhibited by the one-time and iteratively-pruned models achieving 98.32% and 98.54% accuracy for the 95% sparse model. However, in general the models fare poorly upon retraining. Figure 8 below gives more insight into the training process, showing test accuracy measured throughout the training process for the sparse networks. The networks appear to get stuck at different levels throughout the process. Higher learning rates improved this behavior but did not get rid of it completely. This behavior suggests that training on sparse networks hinders the ability of stochastic gradient descent to converge on a solution. While we know from the pruning
process that a sparse solution with high accuracy exists, the sparsely-trained model may not recover the same accuracy. One can imagine that having fully-connected layers to train over gives the network flexibility when searching the space of parameters to optimize for a best representation. As it is iteratively pruned, the network slowly settles in to a solution. However, when these pruned connections are set in stone, previously found solutions are now obstructed by high-cost, suboptimal representations. In order to train sparse networks, one needs a way to incorporate sparse structure without causing this sort of roughness in the space of stochastic gradient descent.

One factor that may be causing this instability of training is that our pruning process prunes all layers of the Lenet-5, including the first convolutional layer. The first convolutional layer is the most important to the network performance, so pruning it may be limiting the results. However, the behavior is even seen at relatively low levels of sparsity such as 75%. In addition, the accuracy achieved seems uncorrelated with sparsity. More research into the training process is needed to fully understand why the training process is less stable for sparse convolutional neural networks.

Figure 9 shows results training the RadiX-Nets on MNIST for Lenet-300-100 and Lenet-5 and CIFAR-10 for Lenet-5. (Lenet-300-100 is too small a network to achieve a meaningful result on CIFAR-10.) Lenet-300-100 performs best at full density, and accuracy decreases as sparsity is increased, even while keeping total connections constant. We see the same trend for Lenet-5 on MNIST, but the effect of sparsity is much smaller; it loses only one percent accuracy through 95% sparsity. (Training Lenet-5 100 times larger was not feasible due to memory constraints.) In comparison, the curve for Lenet-5 on CIFAR-10, a much more challenging dataset, is similar to that for Lenet-300-100. One explanation for the divergence in behavior for Lenet-5 on MNIST is that Lenet-5 is overparameterized for the relatively simple MNIST dataset, and hence can afford the sparsity without being penalized. Still, sparse versions of Lenet-5 perform very well on both datasets when the total connections are kept constant. This suggests that if the number of connections is the only concern, then sparse representations are at least as good as dense representations.

Note that RadiX-Nets leave the first and last layer of Lenet-5 dense. As a result, any issues Lenet-5 may have had with pruning the first convolutional layer during the pruning-based sparse training are not present here. More experiments are needed to see how pruning or preserving certain layers of a network affects performance.

IV. CONCLUSION

We trained sparse neural network structures and compared their performance to their dense counterparts to learn more about how sparse networks train. Using pruning-based and RadiX-Net sparse network structures led to different insights in training sparse neural networks. In general, we found that while sparse networks are able to perform just as well as dense networks in some cases, increased sparsity can make the training process less stable.

Future work should focus on obtaining the same state-of-the-art accuracies of dense networks through the use of sparse layers and connectivity topologies. After the same performance can be reached with sparse networks, the network size can be increased dramatically to obtain even better results. Another avenue of future work is further investigation into how stochastic gradient descent behaves when using sparse versus dense network structures. Understanding how sparsity affects model convergence could be key in designing sparse structures which train efficiently and effectively.

Lastly, in order to fully realize the potential benefits of sparse neural networks, work needs to be done developing a sparse neural network training framework which efficiently stores and computes sparse matrices. Traditional dense models enjoy the enjoy a large body of work optimizing computation
on GPUs specifically for neural network training, and much work will be needed for sparse matrices to be competitive, even if novel sparse structures showcase exceptional training potential.

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REFERENCES

[1] A. Krizhevsky, I. Sutskever, and G. E. Hinton, “Imagenet classification with deep convolutional neural networks,” in Advances in neural information processing systems, 2012, pp. 1097–1105.
[2] K. Simonyan and A. Zisserman, “Very deep convolutional networks for large-scale image recognition,” arXiv:1409.1556, 2014.
[3] C. Szegedy, W. Liu, Y. Jia, P. Sermanet, S. Reed, D. Anguelov, D. Erhan, V. Vanhoucke, and A. Rabinovich, “Going deeper with convolutions,” in Proceedings of the IEEE conference on computer vision and pattern recognition, 2015, pp. 1–9.
[4] K. He, X. Zhang, S. Ren, and J. Sun, “Deep residual learning for image recognition,” in Proceedings of the IEEE conference on computer vision and pattern recognition, 2016, pp. 770–778.
[5] M. Kumar, W. Horn, J. Kepner, J. Moreira, and P. Pattnaik, “Ibm power9 and cognitive computing,” IBM Journal of R&D, 2018.
[6] J. Kepner and J. Gilbert, Graph Algorithms in the Language of Linear Algebra. SIAM, 2011.
[7] J. Kepner, M. Kumar, J. Moreira, P. Pattnaik, M. Serrano, and H. Tufo, “Enabling massive deep neural networks with the graphblas,” in HPEC. IEEE, 2017.
[8] J. Kepner and H. Jananthan, Mathematics of Big Data: Spreadsheets, Databases, Matrices, and Graphs. MIT Press, 2018.
[9] F. A. Azevedo, L. R. Carvalho, L. T. Grinberg, J. M. Farfel, R. E. Ferretti, R. E. Leite, W. J. Filho, R. Lent, and S. Herculanou-Houzel, “Equal numbers of neuronal and nonneuronal cells make the human brain an isometrically scaled-up primate brain,” Journal of Comparative Neurology, vol. 513, no. 5, pp. 532–541, 2009.
[10] M. Denil, B. Shakibi, L. Dinh, N. De Freitas et al., “Predicting parameters in deep learning,” in Advances in neural information processing systems, 2013, pp. 2148–2156.
[11] S. Samsi, V. Gadepally, M. B. Hurley, M. Jones, E. K. Kao, S. Mohindra, P. Monticciolo, A. Reuther, S. Smith, W. Song, D. Staheli, and J. Kepner, “Graphchallenges.org: raising the bar on graph analytic performance,” CoRR, vol. abs/1805.09675, 2018. [Online]. Available: http://arxiv.org/abs/1805.09675
[12] Y. LeCun, J. S. Denker, and S. A. Solla, “Optimal brain damage,” in Advances in neural information processing systems, 1990, pp. 598–605.
[13] S. Han, J. Pool, J. Tran, and W. Dally, “Learning both weights and connections for efficient neural network,” in Advances in neural information processing systems, 2015, pp. 1135–1143.
[14] T. N. Sainath, B. Kingsbury, V. Sindhwani, E. Arisoy, and B. Ramabhadran, “Low-rank matrix factorization for deep neural network training with high-dimensional output targets,” in ICASSP. IEEE, 2013, pp. 6655–6659.
[15] D. Molchanov, A. Ashukha, and D. Vetrov, “Variational dropout sparsifies deep neural networks,” arXiv:1701.05369, 2017.
[16] M. Babaeizadeh, P. Smaragdis, and R. H. Campbell, “A simple yet effective method to prune dense layers of neural networks,” 2016.
[17] Y. Guo, A. Yao, and Y. Chen, “Dynamic network surgery for efficient dnn,” in Advances In Neural Information Processing Systems, 2016, pp. 1379–1387.
[18] S. Narang, E. Olsen, G. Diamos, and S. Sengupta, “Exploring sparsity in recurrent neural networks,” arXiv:1704.05119, 2017.
[19] M. Zhu and S. Gupta, “To prune, or not to prune: exploring the efficacy of pruning for model compression,” arXiv:1710.01878, 2017.
[20] A. Prabhu, G. Varma, and A. Namboodiri, “Deep expander networks: Efficient deep networks from graph theory,” arXiv:1711.08757, 2017.
[21] R. Robinett and J. Kepner, “Sparse, symmetric neural network topologies for sparse training,” in MIT URTC. IEEE, 2018.