The holographic screen at low temperatures

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A permissible spectrum of transverse vibrations for the holographic screen modifies both a distribution of thermal energy over bits at low temperatures and the law of gravitation at small accelerations of free fall in agreement with observations of flat rotation curves in spiral galaxies. This modification relates holographic screen parameters in de Sitter space-time with the Milgrom acceleration in MOND.

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I. INTRODUCTION

The holographic approach\[1-3\] within the thermodynamical formulation of gravity\[4-14\] became logically accomplished in papers by E.Verlinde\[15\] and T.Padmanabhan\[16, 17\] as well as in further developments, modifications and applications of their ideas in various aspects\[18-55\].

In\[56\] we have connected the entropic force acting on a probe particle near a holographic screen with its surface tension\(p_A\). Therefore, we have calculated the surface density of holographic entropy \(s_A = dS/dA\) as follows from the equipartition rule, so that

\[
s_A = \frac{1}{4G},
\]

whereas the variation of screen energy \(E\) is given by the following relation:

\[
dE = TdS - p_A dA,
\]

with \(A\) denoting the screen area, while the surface tension is equal to

\[
p_A = -T s_A.
\]

However, according to the Nernst heat theorem one should expect that the surface entropy density tends to zero at zero temperature: \(s_A \to 0\) at \(T \to 0\), because dynamical degrees of freedom are frozen out at absolute zero. Furthermore, according to\[57\] the derivative of pressure with respect to the temperature at a constant volume should tend to zero, so that in the case under consideration one gets

\[
\left(\frac{\partial p_A}{\partial T}\right)_A \to 0, \quad \text{at} \quad T \to 0.
\]

From\[11\] and\[3\] it follows that

\[
\left(\frac{\partial p_A}{\partial T}\right)_A = -\frac{1}{4G} = \text{const.}
\]

That is in conflict with the Nernst heat theorem, and the same concerns for expression\[11\] determining the entropy density, which is independent of the temperature.

Obviously, one could satisfy the Nernst heat theorem if the degrees of freedom on the holographic screen are frozen out when approaching the absolute zero. Therefore, the equipartition distribution of energy is modified as

\[
E = \frac{1}{2} TN \quad \rightarrow \quad E = \frac{1}{2} TNB(T),
\]

where \(E = M\) is the gravitating mass inside the screen, and the number of “bits” \(N\) is equal to the number of Planck cells\(^1\) on the screen area

\[
N = \frac{A}{G}.
\]

\(^1\) The Newton gravitation constant \(G = \ell_P^2\), where \(\ell_P = 1/m_P\) is the Planck length inverse to the Planck mass. Here we put \(c = 1\), \(\hbar = 1\) and \(k_B = 1\).
while

\[ B(T) \to 0, \quad \text{at} \quad T \to 0. \]

In [58] the following expression has been heuristically suggested

\[ B(T) = D(x), \quad (8) \]

where \( D(x) \) is the one-dimensional Debye-function

\[ D(x) = \frac{1}{x} \int_{0}^{x} \frac{z \, dz}{e^z - 1}, \quad (9) \]

at \( x = T_D/T \). The new empirical parameter \( T_D \) occurs to be comparable with the current value of Hubble constant \( H_0 \). At high temperatures \( T \gg T_D \) the equipartition rule is valid since \( D(0) = 1 \), but at low temperatures \( T \ll T_D \) the Debye function gets the linear behavior, namely

\[ D(x) \xrightarrow{x \to \infty} \frac{T}{T_0}, \quad T_0 = \frac{6}{\pi^2} T_D. \quad (10) \]

It gives the modification of Newton gravitation law for a body of mass \( M \), so that in the framework of holographic formulation the acceleration of free fall \( a = 2\pi T \) becomes equal to

\[ a^2 = \frac{GM a_0}{r^2}, \quad (11) \]

where \( a_0 = 2\pi T_0 \) is the Milgrom acceleration phenomenologically introduced in the MOND paradigm\(^2\) (modified Newtonian dynamics)\(^5\) to explain the flat rotation curves of spiral galaxies, when the velocity of peripheral stars is asymptotically expressed in terms of the mass of visible matter \( M \) in the galaxy according to the formula

\[ v_0^4 = GM a_0, \quad (12) \]

empirically known as the Tully-Fisher law.

Note that the authors of [58] have not physically validated both the Debye factor and the connection between the Debye temperature and Hubble constant, though they have noted that the notion of Debye correction is meaningful for solid states, but not for holographic screens, since the factor describes the wide interval of temperatures from sound vibrations to independent oscillations of lattice notes in the solid state bodies, when the spectrum of vibrations is well approximated by the Debye formula. In addition, the introduction of one-dimensional Debye function in [58] is the guess beyond any reasonable speculations, and it raises a question on the dimension of holographic screen.

Here we consider collective vibrations of holographic screen on the basis of its surface tension. We find the impossibility of propagation of 2-dimensional sound waves with both polarization and wave vector lying in the plane of holographic screen. So, the only permissible collective motion of bits on the holographic screen is given by the one-dimensional scaling of a holographic screen as whole in the transverse direction to the screen plane\(^3\). We analyze the frequency spectrum of such motion and validate an appropriate thermal correction to the energy distribution law using the Debye method, i.e., a cutoff in the frequency spectrum. We investigate a physical interpretation for a number of vibrational modes of the holographic screen and its connection to the scale hierarchy: the Planck scale, the Hubble rate and a sub-Planckian scale which is of the order of the grand unification scale in the particle physics.

II. COLLECTIVE MOTIONS OF HOLOGRAPHIC SCREEN “BITS”

The sound speed squared \( c_s^2 \) in the medium is determined by the adiabatic derivative of pressure with respect to the energy density. In the case of holographic screen we get

\[ c_s^2 = \left( \frac{\partial p_A}{\partial \rho_A} \right)_S, \quad (13) \]

\(^2\) In [60] Y. Bekenstein has constructed a relativistic Lagrangian formalism consistent with MOND.

\(^3\) Surface oscillations of the holographic screen without gradients of density, pressure or temperature are possible with wave vectors tangent to the screen plane (see Appendix A). But because of holographic principle, these oscillations correspond to specific vibrations of gravitating matter inside the screen, that breaks its initial spatial symmetry, for example, the spherical symmetry of field for a massive point-like source.
where the surface energy density is denoted by $\rho_A = \frac{dE}{dA}$ at high temperatures, when the equipartition rule is valid, and it takes the form

$$\rho_A = \frac{1}{2G} T = 2T s_A.$$  \hfill (14)

Taking into account the expression for the surface tension in (3) we find that the speed squared for sound on the holographic screen is negative

$$c_s^2 = -\frac{1}{2},$$

whereas this expression is valid at low temperatures too because thermal correction does not change the relation between the energy density and surface tension $p_A = \frac{1}{2} \rho_A$.

Thus, the sound wave propagation along the screen with the polarization in the screen plane is absolutely impossible. This fact can be predicted from the definition of the holographic screen as the surface with a constant acceleration of free fall and, hence, with a constant temperature. Therefore, a temperature gradient is forbidden, hence, gradients of the energy density and pressure do not propagate. The relation between the surface tension and surface energy density confirms the validity of such the consideration in respect to the sound waves.

The only permissible collective motion is the uniform extension or contraction of holographic screen as a whole in the transverse direction to the screen, i.e. the scaling transformation depending on the time: $r \rightarrow a(t)r$, where the scale factor $a(t)$ completely determines the dynamics of such motion with single degree of freedom. In Appendix A we consider the transverse oscillations of the screen propagating along the screen. However, such oscillation with a wave vector lying in the screen plane are unavoidably accompanied by a motion of matter inside the holographic surface. In the case of spherically symmetric matter the vibrational wave vector should be directed orthogonally to the screen, only.

It is easy to calculate the spectrum of such transverse motion because the scale factor is determined by the Universe evolution. Since we are interested of the present evolution, we can use the parametrization for the Hubble constant $H$ in the form of

$$H^2(t) = H_0^2 \left( \frac{\Omega_M}{a^3(t)} + \Omega_\Lambda \right), \quad a(0) = 1,$$

where the energy balance in the flat Universe for fractions of the matter $\Omega_M$ and the cosmological constant $\Omega_\Lambda$ gives $\Omega_M + \Omega_\Lambda = 1$, and numerically $\Omega_\Lambda \approx 0.76$. In the epoch of cosmological constant the Hubble constant is equal to

$$H_\Lambda = H_0 \sqrt{\Omega_\Lambda},$$

and the scale factor behaves like

$$a(t) = e^{H_\Lambda t}, \quad t \in (-\infty, 0).$$

The spectrum density

$$\nu(\omega) = \int_{-\infty}^{0} dt \ e^{-i\omega t} a(t) = \frac{1}{H_\Lambda - i\omega}$$

gives the number of the scale-vibrational modes for the holographic screen $N_G(\omega)$ in the frequency range $(0, \omega)$

$$N_G(\omega) = \int_{-\omega}^{\omega} \frac{d\omega}{2\pi} |\nu(\omega)| = \int_{0}^{\omega} \frac{d\omega}{\pi} \frac{1}{\sqrt{H_\Lambda^2 + \omega^2}}$$

Notice that at small frequencies $\omega \ll H_\Lambda$ the frequency density becomes constant, and the number of modes is equal to

$$N_G(\omega) \approx \frac{\omega}{\pi H_\Lambda} \ll 1.$$  \hfill (21)

The contribution of such vibrations to the free energy $\mathcal{F}$ can be written as

$$\mathcal{F} = -T \sum_{\text{bits, } \omega} \ln Z(\omega),$$

\hfill (22)
where the sum is taken over both “bits” on the screen and low frequency modes of oscillations, if $N_G \ll 1$, while

$$Z(\omega) = \left(1 - e^{-\omega/T}\right)^{-1}.$$  \hfill (23)

At high temperatures $\omega \ll T$ the equipartition rule is reproduced because

$$\ln Z \approx \ln T - \ln \omega,$$

and

$$\sum_{\text{bits,}\omega} \ln Z(\omega) \approx -\frac{1}{2} \tilde{N}_{\text{bits}} N_G (\langle \ln \omega \rangle - \ln T)$$

where the average value is defined as

$$\sum \omega \ln \omega = N_G \langle \ln \omega \rangle,$$

so that the energy $E = F - \partial F / \partial \ln T$ is equal to

$$E = \frac{1}{2} T N,$$  \hfill (24)

where $N = \tilde{N}_{\text{bits}} N_G$ is the total number of “bits” equal to the number of Planckian cells of the screen area, and $\tilde{N}_{\text{bits}}$ is the number of “bits” per the scale-vibrational mode of the screen.

At low temperatures $T \ll \omega$ by introducing the frequency cutoff $\omega < T_D$, formula (22) gives

$$F = \frac{1}{2} T \tilde{N}_{\text{bits}} \int_0^{T_D} \frac{d\omega}{\pi H^2} \ln \left(1 - e^{-\omega/T}\right),$$

hence, the energy equals

$$E = \frac{1}{2} T N \mathcal{D} \left(\frac{T_D}{T}\right),$$  \hfill (26)

where again $N = \tilde{N}_{\text{bits}} N_G$ and $N_G = N_G(T_D)$, so that

$$E \approx \frac{1}{2} N T^2 \frac{T}{T_0}, \quad T \ll T_0.$$

Obviously, the formulas (25) and (26) can be applicable not only when $T \ll T_D$, but in the full temperature range, if $N_G \ll 1$ (otherwise, the integration over the frequency is essentially different from the one-dimensional oscillator approximation because $|\nu(\omega)|$ depends on the frequency).

We find the relation

$$T_0 = \frac{6}{\pi} H \Lambda N_G = \frac{6}{\pi} \sqrt{\Omega \Lambda} H_0 N_G,$$  \hfill (27)

wherefrom it follows that $T_0 \ll H_0$ in consistency with the empirical data setting $a_0 \approx H_0/2\pi$, so that

$$N_G \approx \frac{1}{24\pi} \frac{1}{\sqrt{\Omega \Lambda}} \sim 10^{-2}.$$  \hfill (28)

One can see that the quantity $N_G$ is the new fundamental parameter in the framework of thermodynamical and holographical gravity.
III. DISCUSSION AND CONCLUSION

In our approach the thermodynamical “bit” on the holographic screen occupies the Planckian area $A_{Pl} = \ell_{Pl} \times \ell_{Pl}$, defining the quantum of area. Note that a binding energy of the screen “bits” is perhaps of the order of Planck scale itself. To speculate on the notion of surface, one should require that energy fluctuations in the transverse direction to the screen should be much less than the Planck energy.

On the other hand, the “bits” on the screen experience the collective transverse motion as a whole. Then, the quantity defined by

$$\Lambda_G = \frac{N_G}{\ell_{Pl}},$$

represents the density of vibrational modes per the Planckian length. By dimension, it is the energy of transverse motion for the area quantum or “bit”. Therefore, the surface structure is rigid or stable, if $\Lambda_G \ll m_{Pl}$.

Thus, we draw the conclusion that the stability of holographic screen as the surface demands $N_G \ll 1$, while by the order of magnitude “the transverse energy of the Planckian quantum area” is $\Lambda_G \approx N_G m_{Pl} \sim 10^{16-17}$ GeV, i.e. it is close to the scale of the grand unification theory (GUT).

For the Milgrom acceleration we obtain

$$a_0 \approx 12 \sqrt{\Omega_{\Lambda} H_0 \Lambda_G / m_{Pl}}.$$  \hspace{1cm} (29)

Thus, the basic parameter of the MOND involves the scale of vacuum energy (the cosmological constant), Planck mass and the scale closed to the GUT energy.

In this paper we have given the justification for the modification of the equipartition rule for the “bits” on the holographic screen at low temperatures due to the proper description for the low-frequency collective transverse oscillations of screen at the presence of cosmological constant. The requirement on the stability of holographic screen with respect to such oscillations is reduced to the introduction of energy scale which should be substantially less than the Planck energy. Then, the corrected low-temperature distribution of thermal energy on the holographic screen is reduced to the modified gravitational law like MOND at accelerations of free fall less than the critical Milgrom acceleration. The Milgrom acceleration has been empirically introduced in the MOND paradigm in order to describe the flat rotation curves in spiral galaxies and to explain the Tully-Fisher law connecting the visible mass of galaxy to the star velocity in the region dominating by the dark matter halo. Phenomenologically, observational data give the scale of transverse oscillating energy for the screen close to the scale of grand unification theory.

Another treatment of Milgrom acceleration in terms of entropic force has been recently presented in [61].

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Appendix A: Transverse oscillations running along the screen

Let us derive the speed of propagation for the transverse oscillations on the holographic screen with wave vector in the screen plane. The screen can be considered as a membrane with the constant surface tension $\sigma = -p_A = T s_A$ and the surface density of energy $\rho_A = E/A = 2T s_A$ in the approximation of high temperatures. Let us regard the sinusoidal wave running in the direction of axis $X$ (Fig.1).

At the initial time the wave equation reads off

$$y(x) = \mathcal{A} \sin \left(\frac{2\pi x}{\lambda}\right),$$

where $\mathcal{A}$ is a small wave amplitude, $\lambda$ is a wave length. Let us select the element of the wave between $x$ and $x'$ coordinates. Forces $F$ and $F'$ are equal to each other by magnitude $\sigma L$, where $L$ is a wave width in the direction perpendicular to the wave vector. But the directions of those forces are tangent to the screen and, hence, different because the points of action slightly differ. Denote the angles between axes $X$ and forces $F, F'$ by $\alpha$ and $\alpha'$, respectively,
FIG. 1: Lateral oscillations of the screen.

and put $\Delta x = x' - x$. Then the mass of selected element equals $m = \rho_A \Delta x L$, where we ignore the slope of the wave to axis $X$, because the wave amplitude is small. Similarly, we neglect the net force along axis $X$. The net force along axis $Y$ equals $\sigma L (\sin \alpha - \sin \alpha') = \sigma L \left( \frac{\Delta x}{\lambda} \right)^2 y$. Immediately, we obtain the equation of oscillations

$$\rho_A \Delta x L \ddot{y} = -\sigma L \Delta x \left( \frac{2\pi}{\lambda} \right)^2 y \Rightarrow \ddot{y} + c_T^2 y = 0.$$ 

So, we straightforwardly get the square of the wave speed

$$c_T^2 = \frac{\sigma}{\rho_A} = \frac{1}{2}.$$ 

Notice, that considered oscillations occur at zero gradients of the energy density and pressure, i.e. it is not a sound. Moreover, since the holographic screen is the surface with a constant acceleration of free fall, these oscillations take place in connection with proper oscillations of the *matter* inside the screen.

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