Rank Distributions in Semiotics

V.P.Maslov, T.V.Maslova∗

Abstract

The notions of real and user cardinality of a sign are introduced. Rank distributions can be extended to arbitrary sign objects, i.e., semiotic systems. The dynamics of the distribution of consumer durables, such as automobiles, is studied.

Usually, in semiotics only relatively short strings of signs of signs (discourses) are considered, while long strings with large parameters have not really been studied. We shall introduce notions generalizing the notion of participant in communicative and non-communicative sign systems: instead of the terms "narrateur" and "narrataire," or "interlocuteur" and "interlocutaire" (see [1], p. 508], we shall use the pair of terms generator and user.

Semiotic objects, i.e., signs, can be of different types [1, 2].

A word of a natural language is a sign. The collection of words is the dictionary of signs. We use the term dictionary of signs rather than "alphabet of signs" to stress that the number of signs can be very large. The activity index of a sign is the number of its occurrences. We shall call this index the real cardinality of the sign. The real cardinality of the dictionary of signs is the total number of occurrences of all the signs from the dictionary (collection of signs). In language systems, the cardinality of a dictionary (collection of words) corresponds to the number of occurrences of the words in the corpus of texts used to compile the dictionary.

Let us now consider books in a bookstore and let us consider the entire collection of books sold. Assume that each book has an inventory number. It is each copy of the book sold which is a sign, and its value (price) is the cardinality ω of this sign. The additional money involved in the value of the book (storage expenses, overhead, etc.) must be included to get the user cardinality ˜ω of the book. Here the generator is the group of accountants who determined the prices of books.

Now consider the catalog of books for sale. Each opus (be it a novel, a collection of poems, or a textbook) is a sign, and its price is the cardinality ω of the given sign. Valuation by user of this sign is the user cardinality ˜ω. In this situation, all the sold copies of the same opus, as opposed to the previous example, are grouped together under one sign, which is specified by the title listed in the catalog (this notion is similar to that of descriptor in linguistics).

Each article of law in a book of statutes is a sign. The entire list of laws is the dictionary of signs. Let us note that, in specific examples, one can incorrectly interpret the notion of sign and its cardinality. For instance, in the given example, the number of people who were arrested under the given article of the law is not, as one might think, the real cardinality of this sign (article), and the number of people who actually broke this article of law (whether they were arrested or not) is not its user cardinality. The person

∗Moscow Institute of Electronics and Mathematics, Moscow Institute of Economics; pm@miem.edu.ru:
(or persons) who created the book of statutes is not the generator of the signs. Similarly, in linguistics, the word forms that constitute a lexeme is not its cardinality. Word forms are actually signs in a lower hierarchy.

The real cardinality $\omega$ of articles of law, regarded as signs, is the corresponding fine or the length of the prison term. The user cardinality $\tilde{\omega}$ also includes all the unpleasantness related to the punishment (the quality of one's CV, separation from relatives, etc.). Here the generators are the lawmakers who specified the punishment for breaking the law.

People sent to prison for breaking the law may also be regarded as signs (the inmates are even given serial numbers). The cardinality $\omega$ is the length of the prison term. The generators in this case are the lawmakers, the judges, the prosecuting attorneys, etc.

The sale of various goods will be considered below. Each type of goods will be a sign, its price is its cardinality. The generator is the person who fixed the prices. The set of all purchased types of goods is the dictionary of signs, the prices are the cardinalities, the customer is the user.

In the last two examples, it is easy to confuse the sign with its cardinality. The user cardinality in these examples is quite realistic for the customers, say for those who are buying cars. Thus to obtain the user cardinality (price) of an automobile, one must add to its list price the actual expenses related to its upkeep, storage, insurance, spare parts, etc.

Similarly, in the case of judicial punishment, the cardinality related to the actual losses for the prisoner (a spoiled CV, the alienation by the family, etc.) becomes quite real for the other people involved: for some the prisoner becomes an outcast, in some cases becomes a hero for others.

In such cases, the user cardinality is not a monotone function of the real cardinality. For cheap cars, it increases with the decrease of the real cardinality, for instance, the insecurity of the car becomes greater when its price decreases. Similarly, with the decrease of prison terms, beginning at some level, the related negative consequences do not decrease, and in fact increase relatively to the real cardinality.

The generator should take into consideration the priorities, the tastes, the possibilities, and so on of the user. If the generator does not do this to a sufficient extent, then the experimental curve will not approximate the theoretical curve as well as it does in the automobile example shown bellow on Figures 1 and 2.

For instance, if the generator (the lawmakers) does not take into consideration the mentality of the given "user" and compiles a set of laws under which practically any citizen constantly breaks the laws, and, since it is impossible to imprison everyone, the system starts putting in jail only those citizens which are in power dislike for some reason, this will lead to a totalitarian state where everyone lives in fear. In this case, the experimental curves will not fit the theoretical ones, because the absence of the preference principle (see [3]) on which the theoretical curves are based no longer applies.

Let us pass to the description of our main approach to the general class of semiotic objets.

The most important and difficult question is how the generator works out the cardinality of the dictionary of signs. These cardinalities are worked out via a system of "agreements" between the generator and the user. The generator "produces a fictional action which places him at a higher level as compared to" the user.

If the generator, having recently passed the bar exam, begins to impose an ideal system of laws to the user, it will be rejected because it does not satisfy the social "rules of the

---

1See A. Grames, J. Courtier, Semiotics, An Explanatory Dictionary, in [1].

2
game,” and the user will start reimplementing the lynching laws, or using the laws of the maffia.

Let us look at another, even more spontaneous, generator, which must include a huge number of people: it is impossible to specify how and by whom the cities and towns of a country were founded. What was the role of the interaction with neighbors, the greater security in numbers, the role of commerce, all these factors must be included in a very complicated and long algorithm.

Our considerations are based on Kolmogorov's approach to randomness as maximal complexity (now known as Kolmogorov complexity, see [4]). This means that the longer the algorithm used by the generator to construct the collection of signs and their cardinalities, the nearer will the result be to the general position of the majority of all possible versions of these collections. This is similar to the fact that in playing "heads or tails," the longer the number of trials, the nearer will the sequence of heads and tails be to the "generic" version, in which in half the trials we get heads, and tails in the other half. And for the most part of the possible strings, we can apply the theorem from [3].

Indeed, let $N_i$ be the number of signs of the same real cardinality $\omega$, while $\bar{\omega}$ is the user cardinality. We denote the whole user energia\footnote{We use here the terminology of Humbolt-Prieto} by

$$E = \sum_{i=1}^{s} N_i \bar{\omega}_i.$$ 

We can assume that the number of signs $N_i$ corresponding to the given user cardinality $\bar{\omega}$ of the sign $s_i$ is a random variable with equiprobable distribution for any collection of $\{N_i\}$ satisfying

1) $\sum_{i=1}^{s} N_i \bar{\omega}_i \leq E$, if $E < \frac{\sum_{i=1}^{s} \bar{\omega}_i}{s} N$;

and 2) $\sum_{i=1}^{s} N_i \bar{\omega}_i \geq E$, if $E = \frac{\sum_{i=1}^{s} \bar{\omega}_i}{s} N \leq \bar{\omega}_{\text{max}} N$.

Obviously, $E \leq \bar{\omega}_{\text{max}} N$, where $N$ is the length of the dictionary of signs.

This axiom should be understood in the sense that the given string of signs of the energia $E$ is one of many such strings with energia not greater than $E$, possessing the same dictionary of signs; here we assume that, at least for the most part of the signs, the energia is in general position with respect to all possible versions of the collection $\{N_i\}$, provided the latter satisfies conditions 1) or 2).

The case 1) has been proofed in [5]. We present bellow the proof of the case 2).

As in [6], the values of the random variable $\bar{\omega}_1, \ldots, \bar{\omega}_s$ are ordered in absolute value. In our consideration, both the number of trials $N$ and $s$ tend to infinity.
Let \( N_i \) be the number of ”appearances” of the value \( \tilde{\omega}_i : \tilde{\omega}_i \leq \tilde{\omega}_{i+1} \), then
\[
\sum_{i=1}^{s} \frac{N_i}{N} \tilde{\omega}_i = M, \tag{1}
\]
where \( M \) is the mathematical expectation.

The cumulative probability \( P_k \) is the sum of the first \( k \) probabilities in the sequence \( \tilde{\omega}_i : P_k = \frac{1}{N} \sum_{i=1}^{k} N_i \), where \( k < s \). We denote \( NP_k = B_k \).

If all the variants for which
\[
\sum_{i=1}^{s} N_i = N \tag{2}
\]
and
\[
\sum_{i=1}^{s} N_i \tilde{\omega}_i \geq \mathcal{E}, \quad \mathcal{E} = MN > N\overline{\omega}, \tag{3}
\]
where \( \overline{\omega} = \frac{\sum_{i=1}^{s} \tilde{\omega}_i}{s} \), are equivalent (equiprobable), then [3] the majority of the variants will accumulate near the following dependence of the ”cumulative probability” \( B_l\{N_i\} = \sum_{i=1}^{l} N_i \),
\[
\sum_{i=1}^{l} N_i = \sum_{i=1}^{l} \frac{1}{e^{\beta' \tilde{\omega}_i - \nu'} - 1}, \tag{4}
\]
where \( \beta' \) and \( \nu' \) are determined by the conditions
\[
B_s = N, \tag{5}
\]
\[
\sum_{i=1}^{s} \frac{\tilde{\omega}_i}{e^{\beta' \tilde{\omega}_i - \nu'} - 1} = \mathcal{E}, \tag{6}
\]
as \( N \to \infty \) and \( s \sim N \). By the condition \([3] \) \( \beta' < 0 \).

We introduce the notation: \( \mathcal{M} \) is the set of all sets \( \{N_i\} \) satisfying conditions \([2]\) and \([3]\); \( \mathcal{N}\{\mathcal{M}\} \) is the number of elements of the set \( \mathcal{M} \).

**Theorem 1.** Suppose that all the variants of sets \( \{N_i\} \) satisfying the conditions \([2]\) and \([3]\) are equiprobable. Then the number of variants \( \mathcal{N} \) of sets \( \{N_i\} \) satisfying conditions \([2]\) and \([3]\) and the additional relation
\[
| \sum_{i=1}^{l} N_i - \sum_{i=1}^{l} \frac{1}{e^{\beta' \tilde{\omega}_i - \nu'} - 1} | \geq N^{(3/4+\varepsilon)} \tag{7}
\]
is less than \( \frac{c_1 \mathcal{N}\{\mathcal{M}\}}{N^m} \) (where \( c_1 \) and \( m \) are any arbitrary numbers, \( l \geq \varepsilon N \), and \( \varepsilon \) is arbitrarily small).

**Proof of Theorem 1.**
Let \( \mathcal{A} \) be a subset of \( \mathcal{M} \) satisfying the condition
\[
| \sum_{i=l+1}^{s} N_i - \sum_{i=l+1}^{s} \frac{1}{e^{\beta' \tilde{\omega}_i - \nu'} - 1} | \leq \Delta;
\]
\[
| \sum_{i=1}^{l} N_i - \sum_{i=1}^{l} \frac{1}{e^{\beta' \tilde{\omega}_i - \nu'} - 1} | \leq \Delta,
\]
where $\Delta$, $\beta$, $\nu$ are some real numbers independent of $l$.

We denote

$$\left| \sum_{i=l+1}^{s} N_i - \sum_{i=l+1}^{s} \frac{1}{e^{\beta \omega_i - \nu} - 1} \right| = S_{s-l};$$

$$\left| \sum_{i=1}^{l} N_i - \frac{1}{e^{\beta \omega_i - \nu} - 1} \right| = S_l.$$

Obviously, if $\{N_i\}$ is the set of all sets of integers on the whole, then

$$N \{ M \setminus A \} = \sum_{\{N_i\}} \left( \Theta \left( \sum_{i=1}^{s} N_i \omega_i - E \right) \delta_{(\sum_{i=1}^{s} N_i) \omega - \nu} \Theta(S_l - \Delta) \Theta(S_{s-l} - \Delta) \right),$$

where $\sum N_i = N$.

Here the sum is taken over all integers $N_i$, $\Theta(\tilde{\omega})$ is the Heaviside function, and $\delta_{k_1,k_2}$ is the Kronecker symbol.

We use the integral representations

$$\delta_{NN'} = \frac{e^{-\nu N}}{2\pi} \int_{-\pi}^{\pi} d\varphi e^{-iN\varphi} e^{\nu N'} e^{iN'\varphi},$$

$$\Theta(y) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} d\tilde{\omega} \frac{1}{\tilde{\omega} - i} e^{\beta y(1+i\tilde{\omega})}.$$ (10)

Now we perform the standard regularization. We replace the first Heaviside function $\Theta$ in (8) by the continuous function

$$\Theta_\alpha(y) = \begin{cases} 0 & \text{for } \alpha > 1, y < 0 \\ 1 - e^{\beta y(1-\alpha)} & \text{for } \alpha > 1, y \geq 0, \end{cases}$$

$$\Theta_\alpha(y) = \begin{cases} e^{\beta y(1-\alpha)} & \text{for } \alpha < 0, y < 0 \\ 1 & \text{for } \alpha < 0, y \geq 0, \end{cases}$$

where $\alpha \in (-\infty, 0) \cup (1, \infty)$ is a parameter, and obtain

$$\Theta_\alpha(y) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} e^{\beta y(1+ix)} \left( \frac{1}{x - i} - \frac{1}{x - \alpha i} \right) dx.$$ (11)

If $\alpha > 1$, then $\Theta(y) \leq \Theta_\alpha(y)$.

Let $\nu < 0$. We substitute (9) and (10) into (8), interchange the integration and summation, then pass to the limit as $\alpha \to \infty$ and obtain the estimate

$$N \{ M \setminus A \} \leq \left| \frac{e^{-\nu N + \beta E}}{i(2\pi)^2} \int_{-\pi}^{\pi} \left[ \exp(-iN\varphi) \sum_{\{N_i\}} \exp \left\{ -\beta \sum_{j=1}^{s} N_j \omega_j + (i\varphi + \nu) \sum_{j=1}^{s} N_j \right\} \right] d\varphi \times \right.$$

$$\times \Theta(S_l - \Delta) \Theta(S_{s-l} - \Delta) \left|,$$ (12)

where $\beta$ and $\nu$ are real parameters such that the series converges for them.

To estimate the expression in the right-hand side, we bring the absolute value sign inside the integral sign and then inside the sum sign, integrate over $\varphi$, and obtain

$$N \{ M \setminus A \} \leq \frac{e^{-\nu N + \beta E}}{2\pi} \sum_{\{N_i\}} \exp \left\{ -\beta \sum_{i=1}^{s} N_i \omega_i + \nu \sum_{i=1}^{s} N_i \right\} \times \Theta(S_l - \Delta) \Theta(S_{s-l} - \Delta).$$ (13)
We denote
\[ Z(\beta, N) = \sum_{\{N_i\}} e^{-\beta \sum_{i=1}^{l} N_i \omega_i}, \] (14)
where the sum is taken over all \( N_i \) such that \( \sum_{i=1}^{s} N_i = N \),
\[ \zeta_l(\nu, \beta) = \prod_{i=1}^{l} \xi_i(\nu, \beta); \]
\[ \zeta_{s-l}(\nu, \beta) = \prod_{i=l+1}^{s} \xi_i(\nu, \beta); \]
\[ \xi_i(\nu, \beta) = \frac{1}{(1 - e^{-\beta \omega_i})}. \]
\( i = 1, \dotsc, l \).

It follows from the inequality for the hyperbolic cosine \( \cosh(x) = (e^x + e^{-x})/2 \) for \(|x_1| \geq \delta; |x_2| \geq \delta\):
\[ \cosh(x_1) \cosh(x_2) > \frac{e^\delta}{2} \] (15)
that the inequality
\[ \Theta(S_{s-l} - \Delta) \Theta(S_l - \Delta) \leq e^{-c\Delta} \cosh(c \sum_{i=1}^{l} N_i - c\phi_l) \cosh(c \sum_{i=l+1}^{s} N_i - c\phi_{s-l}), \] (16)
where
\[ \phi_l = \sum_{i=1}^{l} \frac{1}{e^{\beta \omega_i - \nu} - 1}; \quad \phi_{s-l} = \sum_{i=l+1}^{s} \frac{1}{e^{\beta \omega_i - \nu} - 1}, \]
holds for all positive \( c \) and \( \Delta \).

We obtain
\[ N\{\mathcal{M} \setminus \mathcal{A}\} \leq e^{-c\Delta} \exp(\beta E - \nu N) \times \]
\[ \times \sum_{\{N_i\}} \exp\{-\beta \sum_{i=1}^{l} N_i \omega_i + \nu \sum_{i=1}^{l} N_i \} \cosh\left(\sum_{i=1}^{l} cN_i - c\phi_l\right) \times \]
\[ \times \exp\{-\beta \sum_{i=l+1}^{s} N_i \omega_i + \nu \sum_{i=l+1}^{s} N_i \} \cosh\left(\sum_{i=l+1}^{s} cN_i - c\phi_{s-l}\right) = \]
\[ = e^{\beta E} e^{-c\Delta} \times \]
\[ \times \left(\zeta_l(\nu - c, \beta) \exp(-c\phi_l) + \zeta_l(\nu + c, \beta) \exp(c\phi_l)\right) \times \]
\[ \times \left(\zeta_{s-l}(\nu - c, \beta) \exp(-c\phi_{s-l}) + \zeta_{s-l}(\nu + c, \beta) \exp(c\phi_{s-l})\right). \] (17)

Now we use the relations
\[ \frac{\partial}{\partial \nu} \ln \zeta_l|_{\beta = \beta', \nu = \nu'} \equiv \phi_l; \quad \frac{\partial}{\partial \nu} \ln \zeta_{s-l}|_{\beta = \beta', \nu = \nu'} \equiv \phi_{s-l} \] (18)
and the expansion \( \zeta_l(\nu \pm c, \beta) \) by the Taylor formula. There exists a \( \gamma < 1 \) such that
\[ \ln(\zeta_l(\nu \pm c, \beta)) = \ln(\zeta_l(\nu, \beta)) \pm c(\ln(\zeta_l)'(\nu, \beta) + \frac{c^2}{2}(\ln(\zeta_l)''(\nu \pm c, \beta). \] (18)

We substitute this expansion, use formula (18), and see that \( \phi_{\nu, \beta} \) is cancelled.
Another representation of the Taylor formula implies
\[
\ln (\zeta_l(\nu + c, \beta)) = \ln (\zeta_l(\beta, \nu)) + \frac{c}{\beta} \frac{\partial}{\partial \nu} \ln (\zeta_l(\beta, \nu)) + \int_{\nu}^{\nu+c/\beta} d\nu' (\nu + c/\beta - \nu') \frac{\partial^2}{\partial \nu'^2} \ln (\zeta_l(\beta, \nu')).
\] (19)

A similar expression holds for \(\zeta_{s-l}\).

From the explicit form of the function \(\zeta_l(\beta, \nu)\), we obtain
\[
\frac{\partial^2}{\partial \nu^2} \ln (\zeta_l(\beta, \nu)) = \beta^2 \sum_{i=1}^{t} \frac{\exp(-\beta(\tilde{\omega}_i + \nu))}{\exp(-\beta(\tilde{\omega}_i + \nu)) - 1} \leq \beta^2 s d,
\] (20)

where \(d\) is given by the formula
\[
d = \frac{\exp(-\beta(\tilde{\omega}_s + \nu))}{\exp(-\beta(\tilde{\omega}_s + \nu)) - 1}.
\]
The same estimate holds for \(\zeta_{s-l}\).

Taking into account the fact that \(\zeta_l \zeta_{s-l} = \zeta_s\), we obtain the following estimate for \(\beta = \beta'\) and \(\nu = \nu'\):
\[
\mathcal{N} \{ M \setminus A \} \leq \zeta_s(\beta', \nu') \exp(-c\Delta + \frac{c^2}{2} \beta^2 s d) \exp(\mathcal{E} \beta' - \nu' N).
\] (21)

Now we express \(\zeta_s(\nu', \beta')\) in terms of \(Z(\beta, N)\). To do this, we prove the following lemma.

**Lemma 1** Under the above assumptions, the asymptotics of the integral
\[
Z(\beta, N) = \frac{e^{-\nu N}}{2\pi} \int_{-\pi}^{\pi} d\alpha e^{-iN\alpha} \zeta_s(\beta, \nu + i\alpha)
\] (22)
has the form
\[
Z(\beta, N) = C e^{-\nu N} \frac{\zeta_s(\beta, \nu)}{[(\partial^2 \ln \zeta_s(\beta, \nu))/(\partial^2 \nu)]} (1 + O(\frac{1}{N})),
\] (23)
where \(C\) is a constant.

We have
\[
Z(\beta, N) = \frac{e^{-\nu N}}{2\pi} \int_{-\pi}^{\pi} e^{-iN\alpha} \zeta_s(\beta, \nu + i\alpha) d\alpha = \frac{e^{-\nu N}}{2\pi} \int_{-\pi}^{\pi} e^{NS(\alpha, N)} d\alpha,
\] (24)
where
\[
S(\alpha, N) = -i\alpha + \ln \zeta_s(\beta, \nu + i\alpha) = -i\alpha - \sum_{i=1}^{s} \ln[1 - e^{\nu+i\alpha-\beta\tilde{\omega}_i}].
\] (25)

Here \(S\) depends on \(N\), because \(s, \tilde{\omega}_i\), and \(\nu\) also depend on \(N\); the latter is chosen so that the point \(\alpha = 0\) be a stationary point of the phase \(S\), i.e., from the condition
\[
N = \sum_{i=1}^{s} \frac{1}{e^{\beta\tilde{\omega}_i-\nu} - 1}.
\] (26)

We assume that \(a_1 N \leq s \leq a_2 N\), \(a_1, a_2 = \text{const}\), and, in addition, \(0 \leq \tilde{\omega}_i \leq B\) and \(B = \text{const}\), \(i = 1, \ldots, s\). If these conditions are satisfied in some interval \(\beta \in [0, \beta_0]\) of the
values of the inverse temperature, then all the derivatives of the phase are bounded, the stationary point is nondegenerate, and the real part of the phase outside a neighborhood of zero is strictly less than its value at zero minus some positive number. Therefore, calculating the asymptotics of the integral, we can replace the interval of integration $[-\pi, \pi]$ by the interval $[-\varepsilon, \varepsilon]$. In this integral, we perform the change of variable

$$z = \sqrt{S(0, N) - S(\alpha, N)}. \quad (27)$$

This function is holomorphic in the disk $|\alpha| \leq \varepsilon$ in the complex $\alpha$-plane and has a holomorphic inverse for a sufficiently small $\varepsilon$. As a result, we obtain

$$\int_{-\varepsilon}^{\varepsilon} e^{NS(\alpha, N)} d\alpha = e^{NS(0, N)} \int_{\gamma} e^{-Nz^2} f(z) \, dz, \quad (28)$$

where the path $\gamma$ in the complex $z$-plane is obtained from the interval $[-\varepsilon, \varepsilon]$ by the change (27) and

$$f(z) = \left( \frac{\partial \sqrt{S(0, N) - S(\alpha, N)}}{\partial \alpha} \right)^{-1} \bigg|_{\alpha = \alpha(z)}. \quad (29)$$

For a small $\varepsilon$ the path $\gamma$ lies completely inside the double sector $\text{re}(z^2) > c(\text{re}z)^2$ for some $c > 0$; hence it can be “shifted” to the real axis so that the integral does not change up to terms that are exponentially small in $N$. Thus, with the above accuracy, we have

$$Z(\beta, N) = \frac{e^{-\nu N}}{2\pi} \int_{-\varepsilon}^{\varepsilon} e^{-Nz^2} f(z) \, dz. \quad (30)$$

Since the variable $z$ is now real, we can assume that the function $f(z)$ is finite (changing it outside the interval of integration), extend the integral to the entire axis (which again gives an exponentially small error), and then calculate the asymptotic expansion of the integral expanding the integrand in the Taylor series in $z$ with a remainder. This justifies that the saddle-point method can be applied to the above integral in our case.

**Lemma 2** The quantity

$$\frac{1}{N(M)} \sum_{\{N_i\}} e^{-\beta \sum_{i=1}^s N_i \bar{\omega}_i}, \quad (31)$$

where $\sum N_i = N$ and $\bar{\omega}_i N_i > \mathcal{E} + N^{1/2+\varepsilon}$, tends to zero faster than $N^{-k}$ for any $k$, $\varepsilon > 0$.

We consider the point of minimum in $\beta$ of the right-hand side of (17) with $\nu(\beta, N)$ satisfying the condition

$$\sum e^{\beta \bar{\omega}_i - \nu(\beta, N)} - 1 = N.$$

It is easy to see that it satisfies condition (5). Now we assume that the assumption of the lemma is not satisfied.

Then for $\sum N_i = N$, $\sum \bar{\omega}_i N_i \geq \mathcal{E} + N^{1/2+\varepsilon}$, we have

$$e^{\beta \mathcal{E}} \sum_{\{N_i\}} e^{-\beta \sum_{i=1}^s N_i \bar{\omega}_i} \geq e^{(N^{1/2+\varepsilon}) \beta}.$$

Obviously, $\beta \ll \frac{1}{\sqrt{N}}$ provides a minimum of (17) if the assumptions of Lemma 1 are satisfied, which contradicts the assumption that the minimum in $\beta$ of the right-hand side of (17) is equal to $\beta'$. 

8
We set \( c = \frac{\Delta}{N^{1+\alpha}} \) in formula (21) after the substitution (23); then it is easy to see that the ratio
\[
\frac{\mathcal{N}(\mathcal{M} \setminus \mathcal{A})}{\mathcal{N}(\mathcal{M})} \approx \frac{1}{N^m},
\]
where \( m \) is an arbitrary integer, holds for \( \Delta = N^{3/4+\varepsilon} \). The proof of the theorem is complete.

We prove a cumulative formula in which the densities coincide in shape with the Bose–Einstein distribution with negative temperature. The difference consists also in that, instead of the set \( \tilde{\omega}_n \) of random variables or eigenvalues of the Hamiltonian operator, the formula contains some of their averages over the cells. In view of our theorem, the \( \varepsilon_i \), which are averages of the energy \( \tilde{\omega}_k \) at the \( i \)th cell, are nonlinear averages in the sense of Kolmogorov [7].

Let us number the signs constituting the dictionary in the order of increase of their cardinality, beginning with the minimal cardinality \( \omega_{\min} \). The signs that have the same cardinality are ordered arbitrarily. The number of each sign in this ordering will be called its rank and denoted by \( r \). If \( l \) is the number of the signs of cardinality \( \omega_l \) (beginning from \( \omega_{\min} \)), then by \( r_l \) we shall denote the number of all signs with cardinality less than or equal to \( \omega_l \). By \( r_{-l} \) we shall denote the number of all signs with cardinality greater than \( \omega_l \), so that \( r_l + r_{-l} \) is the total number of all signs.

Exactly as in the article [8], we see that the rank \( r_l \) of the signs, ordered by increasing cardinality, satisfies relations (3), (5), and (8) appearing in [8].

Let us set
\[
\tilde{\omega}_i = \omega_i(1 + \alpha \omega_i^{\gamma} + \alpha^{-1} \omega_i^{-\gamma}),
\]
then as \( \beta \ll 1 \)
\[
r_l = \frac{c_1}{1 + \alpha \omega_l} + c_2 \quad \omega_l = \left( \frac{1}{\alpha} \frac{r_l}{r_{-l}} \right)^{1/\gamma}.
\]

Figures 1 and 2 show how well the generators of the prices of American automobiles estimate the demand. The first plot shows the dependence of the number of cars sold at a price equal to or less than UJ on the price, the second one, the dependence of the number of the car in the "dictionary of cars" (this number can be regarded as the detailed make of the car) in the increasing order of the car prices. The generators here are the people who determined the price. The point of inflection of the graph corresponds to the price level where the additional expenses are minimal. We see that this point is practically the same on both plots. This means that the "agreement" between the generator and the user in this case reaches the high level.

References

[1] Semiotics. The Semiotics of Language and Literature, Ed. by Yu.S. Stepanov (Raduga, Moscow, 1983).

[2] V. V. Ivanov, Essays in the History of Semiotics in the USSR (Nauka, Moscow, 1976).

[3] V. P. Maslov. The lack-of-preference law and the corresponding distributions in frequency probability theory. // Mat. Zametki [Math. Notes]. 2006, 80, No. 2, 220–230.

[4] A. N. Kolmogorov, Theory of Information Transmission, Proceedings of the USSR Acad. of Sc. Session Devoted to Questions of the Automation of Production (Moscow, 1957).
Figure 1: Number of cars with price $< \omega$. The thin line represents the theoretical curve $r(\omega)$. The mean quadratic error is $\sigma = 0.0188674$. 

[5] V. P. Maslov. On a distribution in frequency probability theory corresponding to the Bose-Einstein distribution. [ArXiv:math.PR/0612394]

[6] V. P. Maslov. Negative asymptotic topological dimension, a new condensate, and their connection with the quantized Zipf law. // Mat. Zametki [Math. Notes]. 2006, 80, No. 6, 856–863.

[7] V. P. Maslov. The nonlinear average in economics. // Mat. Zametki [Math. Notes]. 2005, 78, No. 3, 377–395.

[8] V.P.Maslov. Quantum Linguistic Statistics. // RJMP, 2006, v.13, n.3. p. 315-325.

[9] V. P. Maslov, M. V. Fedoryuk. *Semi-Classical Approximation in Quantum Mechanics*. 1981, D.Riedel Publ.Company. Dordrecht, Holland.
Figure 2: Rank. The thin line represents the theoretical curve $r(\omega)$. The mean quadratic error is $\sigma = 0.0184448$. 

