Recalculation of the Wolf Series from 1877 to 1893

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Abstract Homogeneity is considered as the most important property of the Wolf series of sunspot relative numbers, or Wolf numbers, since without a stable scale no valid conclusions about variations in the long-term progress of solar activity can be drawn. However, the homogeneity testing of the Wolf series is a difficult task, since the raw data entering the series and the methods of data-reduction and interpolation used to compile the series are largely unknown. In this article we reconstruct the data-reduction algorithms based on hitherto unpublished original sources from the archives of the former Swiss Federal Observatory in Zürich and discuss their impact on the homogeneity of the Wolf series. Based on Alfred Wolfer as reference, we recalculate the progress of the Wolf series from 1877 to 1893, correcting for the widely disregarded diminishing of Wolf’s eyesight, for the change of Wolf’s main instrument from the 40/700 mm Parisian refractor to the 42/800 mm Fraunhofer refractor, and for the inhomogeneities in the data-reduction procedure during the same time period. The maxima of Cycle 12 in 1884 and of Cycle 13 in 1893 are roughly 10% higher in the recalculated and corrected Wolf series than in the original Wolf series as provided by WDC-SILSO version 1.0. From 1877 to 1883 the smoothed monthly means of the recalculated and corrected Wolf series are up to a factor of 0.76 lower than the original values.

Keywords Solar cycle, observations · Sunspots, statistics

1. Introduction

The sunspot relative number, or Wolf number, invented in 1850 by the Swiss astronomer Johann Rudolf Wolf (Figure 1), is the generally used index to measure variations in the long-term progress of solar activity (Hathaway, 2015). Its value \( R \) is determined on a
Figure 1  Johann Rudolf Wolf (1816 – 1893) was Switzerland’s most renowned astronomer and historian of science of the second half of the 19th century. He also made important contributions in geodesy, meteorology, mathematics, and statistics. In 1852 Wolf discovered, together with others, the parallelism of the solar activity and the variations of the Earth’s magnetic field. In 1864, he founded the Swiss Federal Observatory in Zürich and served as its first director from 1864 to 1893 (Friedli and Keller, 1993).

daily basis as

\[ R = k (10g + f) \]  (1)

where \( g \) is the total number of sunspot groups as seen on the solar disk, \( f \) is the number of individual spots within the groups, and \( k \) is a personal reduction factor transforming the observed Wolf numbers from its raw instrumental system to a common standard system.

Originally, the latter was defined by the observations that Rudolf Wolf made during the years from 1849 to 1863 first in Berne and later in Zürich, using an 83/1320 mm Fraunhofer refractor with magnification 64 and an absorbing glass filter (Friedli, 2016). To provide a complete time series, all raw Wolf numbers as observed up to 1893 and as reconstructed back to 1700 were reduced before compilation to this conventional reference scale. In 1894 Wolf’s successor Alfred Wolfer calculated a \( k \)-factor of 0.6 as appropriate to reduce his own raw Wolf numbers to the reference scale of Rudolf Wolf. This value of the \( k \)-factor was preserved along the following generations of observers at the reference stations in Zürich and Locarno. In 2015 the Wolf-number series was rescaled by omitting the \( k \)-factor of 0.6 (Clette et al., 2015; Clette and Lefèvre, 2016). Thus, in the newly published version 2.0 of the Wolf-number series the observed raw Wolf numbers are reduced to the standard system defined by the observations that Alfred Wolfer made in Zürich during the years from 1876 to 1928 using the same 83/1320 mm Fraunhofer refractor with magnification 64 as Rudolf Wolf, but
Figure 2  The 83/1320 mm Fraunhofer refractor, the standard instrument of the Wolf series, on the southern observation terrace of the Swiss Federal Observatory in Zürich around 1940. The instrument is equipped with a polarization helioscope made by G. and S. Merz in Munich and operated at a magnification of 64 (Friedli, 2016). In 1962 the instrument was put on top of the roof of the observatory – and in 1996 it was moved to the Bernese outskirts, where it is still operated by the author for the daily determination of the Wolf number.

equipped with a polarising helioscope made by G. and S. Merz in Munich (Figure 2). Since 1981, this standard system is realized by observations of the Wolf number made at the pilot station in Locarno.

Daily values of the Wolf number, first published by Waldmeier (1961), are available from the World Data Center for Solar Index and Long-term Solar Observations (WDC-SILSO) at the Royal Observatory in Belgium from 1818 on, monthly means from 1749 on, and yearly means from 1700 on. The series of monthly means as shown in Figure 3 is provided in two versions as a series of observed values and as a series of smoothed values. According to Wolf (1873a), the smoothed value \( R_{13,m} \) for month \( m \) is calculated from the observed monthly means \( R_{m-j} \) as

\[
R_{13,m} = \frac{1}{2} \left( \frac{1}{12} \sum_{j=-6}^{5} R_{m-j} + \frac{1}{12} \sum_{j=-5}^{6} R_{m-j} \right)
\]  

(2)

where the summation is taken over 13 consecutive months. As suggested by Wolf (1877a, 1877c, 1890b), we call this time series of Wolf numbers the Wolf series.

Homogeneity is considered as the most important property of the Wolf series, since without a stable scale no valid conclusions about the long-term variation of solar activity can be drawn. Ideally, a thorough homogeneity testing of the Wolf series should be based on a fully transparent reconstruction of the daily Wolf numbers as provided by WDC-SILSO.
However, since the observed raw data entering the series and the methods of data-reduction and interpolation used to compile the series were never published in full detail, the Wolf series remains to date not reproducible, and a thorough homogeneity testing or an appropriate correction – especially of the parts before 1894 – seemed nearly impossible until recently (Clette et al., 2014; see Section 3.3).

The Rudolf Wolf Society (RWG) in Switzerland, founded in 1992, aims to promote the homogeneous continuation of the Wolf series based on the original instruments used by Rudolf Wolf and his successors and to explore the archives of the former Swiss Federal Observatory at Zürich.

Some years ago, members of the Rudolf Wolf Society located, in the archives of the former Swiss Federal Observatory, a manuscript containing the daily raw numbers of sunspot groups and individual spots as well as the implemented data-reduction and interpolation methods of the entire Wolf series from 1610 to 1876 (Wolf, 1878b). The heritage group of the Rudolf Wolf Society digitized parts of this source book covering the years from 1849 to 1876 and placed it on its site: www.wolfinstitute.ch.

A first inspection of the source book by Friedli (2016) revealed that Wolf changed during his observation period from 1849 to 1893 two times his main instrument and that the scale transfer from the 83/1320 mm Fraunhofer refractor as used by Rudolf Wolf in the years from 1849 to 1863 to the 40/700 mm Parisian refractor as used by Rudolf Wolf mainly in the years from 1861 to 1889 was based on a rather limited number of comparison observations during the years from 1859 to 1861. The research by Friedli (2016) revealed, also, that Rudolf Wolf suffered from some sort of eyesight degradation in his later years, which might have affected the scale transfer to Alfred Wolfer in 1894.

In this article we focus on the homogenization of the Wolf series in the period from 1877 to 1893 where we recalculate, based on results from Frenkel (1913) and on the analysis of the newly digitized original manuscript of Wolfer (1912), the progression of the Wolf series for the years from 1877 to 1893 correcting for the hitherto disregarded effect of Wolf’s eyesight diminishment, for the change of Wolf’s main instrument from the 40/700 mm Parisian refractor to the 42/800 mm Fraunhofer refractor, and for the inhomogeneities in the data-reduction procedure during the same time period.

In Section 2 we review the basic architecture of the Wolf series. In Section 3 we give some background information on the surviving original sources in the archives of the former Swiss Federal Observatory in Zürich and on the significance of the source book for the reconstruction of the Wolf series up to 1876. In Section 4 we reconstruct the calibration and data-reduction algorithms used for compiling the Wolf series and discuss the impact of the implemented $k$-factor estimation methods on the homogeneity of the Wolf series. In Section 5 we discuss the scale transfers within the Wolf series and the construction of the daily and monthly Wolf-number series. In Section 6 we recalculate the Wolf series from 1877 to
Figure 4  Number of days per year with a Wolf number. Beginning with 1818 the daily Wolf numbers are available from WDC-SILSO. Up to 1876 only one observation per day was considered. Note that although the daily record of the Wolf series is apparently complete after 1848, some daily values were numerically interpolated. Furthermore, Wolf had to interpolate graphically during some periods before 1817 to provide a complete record of monthly means.

1893 correcting for Wolf’s eyesight diminishment, for the change of his main instrument, and for the inhomogeneities in the data-reduction procedure during the same time period. In Section 7 the conclusions are summarized.

2. Basic Architecture of the Wolf Series

The Wolf series as shown in Figure 3 and provided by WDC-SILSO may be divided in two main parts: i) the older, incomplete, part up to 1848 where the daily Wolf numbers were reconstructed mainly from historical drawings and other sources and ii) the newer, complete, part from 1849 onwards where the daily numbers of sunspot groups and individual spots were directly recorded by visual inspection of the Sun’s disc at a telescope (Figure 4).

According to Wolf (1873a), the Wolf series is constructed as a daily sequence of observations made solely by a single main observer, called the standard observer, where its gaps are filled with observations from other observers and reduced by $k$-factors depending on observer and instrument to a common scale.

This general principle was observed throughout the Wolf series up to 1980 except for the years from 1877 to 1879 when the mean of three observers (Rudolf Wolf, Robert Billwiller, and Alfred Wolfer) and the years from 1880 to 1893 when the mean of two observers (Rudolf Wolf and Alfred Wolfer) served as standard series, using from 1877 to 1889 the 40/700 mm Parisian refractor and from 1890 to 1893 the 42/800 mm Fraunhofer refractor of Rudolf Wolf as a reference for a common scale (Wolf, 1878a, 1881, 1889).

Since 1981 there is no longer a standard observer. The series is calculated by WDC-SILSO at the Royal Observatory of Belgium as an outlier-cleaned daily average of all...
Figure 5  Number of days per year with observations provided solely by the standard observers of the Wolf series. Starting with 1864, this number includes all days with valid observations from the Swiss Federal Observatory in Zürich and its branch stations in Arosa and in Locarno.

contributing observers, which are reduced to a common scale using the Locarno station in Switzerland as reference (Clette et al., 2007, 2014).

As the standard observers provided the bulk of the observing days, the Wolf series may be further divided into some well-defined periods where the scale of the series is implicitly defined by the instrumental system of the respective standard observer (Figure 5). From 1749 to 1980 the standard observers were Johann Caspar Staudacher (1749 – 1793), Honoré Flaugergues (1794 – 1825), Heinrich Samuel Schwabe (1826 – 1848), Rudolf Wolf (1849 – 1893), Alfred Wolfer (1894 – 1927), William Brunner (1928 – 1945), Max Waldmeier, (1946 – 1979), and Antoine Zelenka (1980). As reported by Friedli (2016), three subperiods have to be distinguished for Rudolf Wolf, since he twice changed his main instrument: in 1861 from the 83/1320 mm Fraunhofer refractor to the 40/700 mm Parisian refractor and in 1890 from the Parisian refractor to another 42/800 mm Fraunhofer refractor.

Thus, the homogeneity of the Wolf series is primarily determined by the long-term stability of the instrumental systems of the standard observers and by the validity of the scale transfers from one standard observer to the next. The reliability of the data-reduction procedures filling the few remaining gaps in the series of standard observations with calibrated observations from other observers is of secondary importance for the long-term homogeneity of the series.

3. Surviving Sources of the Wolf Series

The Wolf series as provided by WDC-SILSO is not well-documented, since most of the raw daily numbers of sunspot groups and individual spots entering the series and some of
the necessary details of the methods of data-reduction and interpolation used to compile the series, including the $k$-factor values, were never published and remained unknown.

### 3.1. Historical Development

From January 1849 to June 1855 Wolf published solely his own observations in the *Notices* of the Bernese Society of Natural Sciences (Friedli, 2016). After his move to Zürich in 1855 he began to fill its gaps with observations made by Schwabe, which he considered as equivalent to his own observations made with the standard 83/1320 mm Fraunhofer refractor (Wolf, 1852, 1856, 1862). In 1859 Wolf realized that observations from different observers and instruments are not necessarily on the same scale (Wolf, 1859). To correct for this effect, Wolf introduced a reduction factor $[k]$ in his formula of the Wolf number (Wolf, 1860, 1861).

For the years from 1861 to 1870, Wolf considered these $k$-factors for a given combination of observer and instrument as constants (Wolf, 1864). But in 1872 he recognized that the $k$-factors varied with the level of solar activity and he changed the algorithm of compiling the combined series (Wolf, 1872). Since this correction attempt failed, he formulated in 1873 an improved data-reduction algorithm as described in the next section of this article, which he later applied to all historic sunspot observations back to 1749 and which remained in use up to 1980 (Wolf, 1873a).

Therefore, the Wolf numbers published by Wolf (1873a) for the year 1872 are the first ones that are identical to those given by Waldmeier (1961) and provided by WDC-SILSO version 1.0. In 1877 Wolf published his final version of the reconstructed series of smoothed monthly means, reaching back to 1749 (Wolf, 1877a,b,c). The corresponding series of observed monthly means followed in 1880 (Wolf, 1880). The daily Wolf numbers for the years from 1818 to 1871 were published only by Waldmeier (1961). However, although a brief description of the underlying sources and data-reduction methods of the Wolf series was published by Wolf (1877c), the daily sunspot group and individual spot numbers as well as the $k$-factors used to reduce the raw observations to the scale of the 83/1320 mm Fraunhofer refractor of Rudolf Wolf remained unknown.

### 3.2. The Significance of the Source Book

Among the treasures preserved in the archives of the Swiss Federal Observatory, a manuscript in the combined handwritings of Rudolf Wolf, Alfred Wolfer, and Max Waldmeier was found containing the daily raw data and $k$-factors for the whole Wolf series from 1610 to 1876, including those parts published by Waldmeier (1961) and provided by WDC-SILSO version 1.0 (Wolf, 1878b).

The heritage group of the Rudolf Wolf Society digitized the parts of this source book concerning observations from 1849 to 1876 and placed it on its site [www.wolfinstitute.ch](http://www.wolfinstitute.ch) (Friedli, 2016). As shown in Figure 6 for the first semester of the year 1861, the Wolf numbers for the years from 1749 to 1876 as given by Waldmeier (1961) and provided by WDC-SILSO version 1.0 may be reconstructed in every detail, since the source book contains all of the raw sunspot group and individual spot numbers as well as the details of the implemented methods of data-reduction and interpolation.

Therefore, the source book acts as missing link between the many raw data series as collected and published by Rudolf Wolf and the final Wolf numbers as published by WDC-SILSO version 1.0. Except for the observations made by Schmidt in the years from 1841 to 1867 and by Schwabe, Wolf, Weilenmann, Fretz, and Meyer in the years from 1849 to 1869, the complete raw data series of the observers considered in the final release of the Wolf
Figure 6  Facsimile of a single page from the source book (Wolf, 1878b) containing data from the first semester of the year 1861. The observers are given in the first column of each month, the daily counts of sunspot groups \( g \) (first number) and of individual spots \( f \) (second number) are given in the second column of each month. The daily Wolf numbers are indicated in the third column of each month, calculated as \( R = k(10g + f) \). The \( k \)-factors may be found in the bottom part of the table. Note that the daily mean Wolf numbers are identical to those of Waldmeier (1961) and WDC-SILSO version 1.0.

series were published in the *Mittheilungen über die Sonnenflecken* and in the *Astronomische Mittheilungen*. Some observations of the published series were not used in the final release of the Wolf series, however. The archives of the Swiss Federal Observatory include a manuscript in the combined handwritings of Rudolf Wolf and Alfred Wolfer with copies of all of these series in chronological order up to the year 1908 (Wolfer, 1909a). The heritage group of the Rudolf Wolf Society digitized also the parts of this document covering
observations from 1863 to 1899. In 1902 Alfred Wolfer incorporated the observations from Kremsmünster covering the years from 1802 to 1830 into the source book (Wolfer, 1902a). The corresponding manuscript in the handwriting of Alfred Wolfer containing the calibration and data-reduction calculations is still available in the archives of the former Swiss Federal Observatory (Wolfer, 1902b).

Thus, the three sources Wolf (1878b), Wolfer (1902b), and Wolfer (1909a) form a complete and fully transparent documentation of the daily, monthly, and yearly mean Wolf numbers from 1749 to 1876 as published by Waldmeier (1961) and provided by WDC-SILSO version 1.0.

3.3. Later Sources

The source book (Wolf, 1878b) ends in 1877, since Wolf changed the data-reduction algorithm that year. From 1877 onwards, the Wolf series no longer consisted of one single observation per day, but they contained for days without an observation from the standard observer the average Wolf number from observations of secondary observers, which could no longer be handled in the form of a diary.

Thus, beginning with 1870, all raw data series were published in extenso in the Astronomische Mitteilungen, although it is not known explicitly which of the raw observations actually are entering the published Wolf series as provided by WDC-SILSO version 1.0.

Starting with 1919, Wolfer (1923) published only a selection of the most important observation series including those from the Swiss Federal Observatory, and starting with 1926 Brunner published only the observations of the Swiss Federal Observatory (Brunner, 1927). Waldmeier published no raw data at all. However, it was repeatedly said that all of the original raw data series would remain in the archives of the Swiss Federal Observatory (Wolfer, 1902a, 1923; Brunner, 1927; Waldmeier, 1958), but up to now, only parts of the original registers covering the years from 1944 to 1980 were found in the archives of Zürich, Locarno, and Uccle.

Thus, considering the surviving known sources, a thorough homogenization of the Wolf series based on a fully transparent reconstruction and correction of the published daily Wolf numbers is possible per se only up to 1918 and from 1944 onwards where all relevant information is available. Fortunately, for the remaining years from 1919 to 1943, the most significant original raw data series have recently been found in the archives of the Swiss Federal Observatory in Zürich (F. Clette, personal communication, 2019), including the observations from the Swiss Federal Observatory covering roughly 90% of the days.

4. Reduction of the Raw Data

4.1. The Overall Model

The determination of the daily number of sunspot groups \([g]\) and individual spots \([f]\) depends on various instrumental, personal, and environmental effects, primarily the magnification of the instrument, the education, experience, and visual acuity of the observer, and the local seeing conditions. Note that the visual acuity plays a significant role only in the case of observations made in projection, since in the case of direct observation through an eyepiece, most of the acuity defects will be corrected by the optics. Only effects stemming from resolution degradation or from astigmatism defects will remain, but an observer with the latter will probably not last an observer for very long and the former is a typical aging
effect present in any long-term record. However, it is certainly correct that, for observers counting the groups and spots from a projection screen, the acuity is one of the mayor factors degrading the quality of the provided group and spot numbers. But note also that from 1826 to 1980 the Wolf series is based mainly on direct countings through an eyepiece, not on countings from a projection screen. Thus, the observed raw Wolf numbers have to be calibrated from their instrumental system to a common standard system. Since for $g$ and $f$ no calibration standards are readily available, some instrumental system has to be declared as the conventional standard system. For the Wolf series as provided by WDC-SILSO version 1.0, this standard system is defined by the observations that Rudolf Wolf made during the years from 1849 to 1863 first in Berne and later in Zürich using a 83/1320 mm Fraunhofer refractor with magnification 64 and an absorbing glass filter (Friedli, 2016).

According to Wolf’s definition as given in Equation 1, the Wolf numbers [$R_B$] of each observer [$B$] are reduced to the Wolf numbers [$R_S$] of the standard observer by calculating

$$R_S = k_B R_B.$$  \hspace{2cm} (3)

This simple model was applied throughout the Wolf series and is still in use today (Clette et al., 2007). But it is well known that this model produces valid results only for instruments similar to the standard 83/1320 mm Fraunhofer refractor of the Wolf series and for observers following conventions for determining the number [$g$] of sunspot groups and the number [$f$] of individual spots similar to those of the reference observer. For instruments much weaker than the 83/1320 mm Fraunhofer refractor operated at a magnification of 64, like the 40/700 mm Parisian refractor of Rudolf Wolf operated at a magnification of 20, the reduced Wolf numbers would be too high for small Wolf number values and too low for high Wolf-number values. For instruments more powerful than the 83/1320 mm Fraunhofer refractor the effect would be the inverse.

In Figure 7 the half-year frequency of small Wolf-number values over time is shown. In this article, we will focus on the homogenization of the Wolf series for the years from 1877 to 1893.

4.2. Reconstruction of the Implemented Algorithms

The recipe given by Wolf (1860) to estimate the $k$-factors as given in Equation 3 was quite vague: $k_B$ should be calculated from corresponding observations of the standard observer [$S$] with some individual observer [$B$] (Wolf, 1859, 1860, 1861, 1872, 1877c). But details or retraceable calculation examples were never published.

We identified in the archives of the Swiss Federal Observatory a manuscript authored by William Brunner containing the complete calculation sheets – the so-called registers – of the Wolf numbers for the year 1944 (Brunner, 1945a). The calculations may be completely retraced and crosschecked with the published Wolf numbers given by Brunner (1945b). Another manuscript written by Alfred Wolfer containing notes on the calculation of the Wolf numbers for February and April 1908 provides further details to clarify the calculation algorithm (Wolfer, 1908, 1909b). Furthermore, the archives of the Swiss Federal Observatory contain the originals of the registers for the years 1975 to 1980 where the construction of the Wolf number can be retraced for the years from 1975 to 1976 and from 1978 to 1979 in full detail (Swiss Federal Observatory, 1984). Recently, the original registers for the remaining years from 1945 to 1974 and for the years 1976 and 1980 were found in the archives of the observatories in Locarno and Uccle.
Figure 7  Frequency plot similar to Clette et al. (2014) of small Wolf-number values ranging from 0 to 24 for the year from 1818 to 2014 for the Wolf series as provided by WDC-SILSO version 1.0. The color indicates on how many days during a semester the particular Wolf number could be observed. The smallest positive Wolf number is 11 for an observer with \( k = 1 \). Smaller values originate in \( k \)-factors below 1 or in averaging of positive and zero values. As is easily verified from the plot, the smallest value of the Wolf number during 1894 to 2014 was 7 resulting from the application of a \( k \)-factor of 0.6 for the reference observer. In this article, we will focus on the homogenization of the Wolf series for the years from 1877 to 1893.

Table 1  \( k_S \)-factors of the Standard Observers from 1749 to 1893.

| Standard observer                  | \( k_S \) | Period of observation | Source            |
|------------------------------------|----------|-----------------------|-------------------|
| Staudacher                         | 2.5      | 1749 – 1799           | Wolf (1878b)      |
| Flaugergues                        | 1.92     | 1788 – 1830           | Wolf (1878b)      |
| Schwabe                            | 1.25     | 1826 – 1868           | Wolf (1878b)      |
| Wolf 83/1320 mm Fraunhofer         | 1.0      | 1849 – 1863           | Wolf (1878b)      |
| Wolf 40/700 mm Parisian            | 1.5      | 1849 – 1889           | Wolf (1878b)      |
| Wolf 42/800 mm Fraunhofer          | 1.5      | 1890 – 1893           | Wolf (1891)       |

According to these documents, the raw observations of the standard observer \( S \) were first reduced to standard Wolf numbers \( R_S \) by calculating

\[
R_S = k_S (10g_S + f_S)
\]

where the calibration factors \( [k_S] \) of the standard observers Staudacher, Flaugergues, Schwabe, and Wolf as given in Table 1 were fixed by Wolf (1878b) and resulted from a carefully conducted scale transfer, as is discussed in more detail in Section 5.1. For Wolf’s successors Wolfer, Brunner, Waldmeier, and Zelenka the \( k_S \)-factors were estimated to 0.6 from many years of parallel observations (Wolfer, 1895; Brunner, 1929; Waldmeier, 1961). The reduced \( R_S \) were rounded to integer numbers. Interestingly, Wolf rounded a fraction of 0.5 always to the next lower integer – contrary to the contemporary practice.
For all non-standard observers, instrumental Wolf numbers $[R_B]$ were calculated according to

$$R_B = 10g_B + f_B.$$  \hspace{1cm} (5)

Then, all days with a complete observation of the standard observer $S$ and of the observer $B$ were selected, forming matched pairs of so-called corresponding observations. From these corresponding observations, all days where at least one of the two Wolf numbers was zero were discarded. The remaining days were called comparison days. The $k$-factor of the observer $B$ was then calculated as

$$k_B = \frac{\sum_n R_S}{\sum_n R_B}$$  \hspace{1cm} (6)

where the $k_B$ were rounded to the second digit after the decimal point. For the recalculation of the $k$-factors of the secondary observers, Wolf used for the years from 1849 to 1876 for most of the secondary observers a summation period of one year. Due to the lack of suitable comparison days the $k$-factors for some of the secondary observers, including Main and Tomaschek, were never recalculated, however. For the years from 1877 to 1888, Wolf recalculated the $k$-factors for each semester. For the years from 1889 to 1893, the $k$-factor of Alfred Wolfer was recalculated every quarter based on all comparison observations from the current and the preceding quarter (Wolf, 1890a). The $k$-factors of the secondary observers were recalculated each semester, a practice that was also continued by Wolfer in the years after 1893 (Wolfer, 1895). In 1928 the Swiss Federal Observatory was assigned responsibility for the publication of the *Quarterly Bulletin on Solar Activity* of the IAU (Brunner, 1929). From then on, the $k$-factors were recalculated for every quarter. In 1945 Waldmeier switched back to a yearly evaluation of the $k$-factors (Waldmeier, 1946). Furthermore, he changed the algorithm for the calculation of the $k$-factors according to the formula

$$k_B = \frac{1}{n} \sum_n R_S/R_B.$$  \hspace{1cm} (7)

Thus, the $k$-factor was calculated as the yearly mean of the daily $k$-factors for the comparison days. This new algorithm remained in use until the end of 1980. According to Clette et al. (2007), it is still part of the present data-reduction algorithm at WDC-SILSO.

### 4.3. Impact on the Homogeneity of the Wolf Series

To quantify the impact of the different formulations of the $k_B$-factors identified in Section 4.2 on the homogeneity of the Wolf series, we applied the algorithms to the sunspot observations recorded in the database of the Rudolf Wolf Society (www.wolfinstitute.ch). This database contains more than 60,000 sunspot observations of more than 100 observers since 1986 and is used to calculate the Swiss Wolf Numbers $[R_W]$ as provided by the Rudolf Wolf Society (Friedli, 2012). We used this data set as a testbed to check the validity of the different data-reduction methods (Friedli, 2014).

A simple validity check is to test if the considered data-reduction method is able to transform the individual instrumental systems of the secondary observers correctly to some standard system and to recover the same series of the smoothed monthly means of the Wolf number as the standard series.

Thus, in a first analysis we calculated for the years 1986 to 1995 for each observer $B$ and for each semester a $k$-factor named $[k_{\text{SUM}}]$ according to Equation 6 using the Wolf
numbers \([R_Z]\) provided by the Swiss Federal Department of Defence as reference (Keller and Friedli, 1995). Then, all raw observations were reduced according to Equation 3 and the Wolf numbers falling on the same day were averaged. A few missing days were imputed by standard Wolf numbers \([R_Z]\), which were reduced with an overall mean \(k\)-factor to the scale of the Swiss Wolf Numbers. As shown in Figure 8, the smoothed monthly means of this combined series were nearly identical to the smoothed monthly means of the standard Wolf numbers \([R_Z]\). Thus, Wolfer’s data-reduction algorithm is able to recover the original standard series (Figure 9). Further analyses showed that this result is not affected if we recalculate the \(k\)-factors instead for each semester for each quarter or year.

A second analysis with a \(k\)-factor named \(k_{\text{Day}}\) recalculated yearly according to Equation 7 revealed that the resulting smoothed monthly means were significantly higher than the original standard series. A linear regression of the reduced to the expected standard values shows that the reduced Wolf numbers calculated with a \(k\)-factor according to Equation 7 are about 5% higher than those calculated with a \(k\)-factor according to Equation 6 (Figure 10).

Finally, we conducted a third analysis using a \(k\)-factor \(k_1\) according to Equation 3, which was interpreted as the slope in a linear regression model without intercept. Thus, \(k_1\) was estimated yearly by least squares as

\[
k_1 = \frac{\sum_n R_S R_B}{\sum_n R_B^2}. \tag{8}
\]

This approach led to a consistent underestimation of the level of the smoothed monthly means.
Figure 9  Open circles represent corresponding values of the observed monthly means of the Zürich Wolf numbers $[R_Z]$ and the observed monthly means of the Swiss Wolf numbers reduced with the classical data-reduction method of Wolfer (RwkSUM semesterly) using a $k$-factor definition according to Equation 6. The slope value of the plotted regression line reveals an almost perfect recovery of the Zürich Wolf numbers by the reduced Swiss Wolf numbers.

Theoretically, an unbiased result may be expected only if the $k$-factor will transform the mean $\bar{R}_B$ of the observed Wolf numbers to the mean $\bar{R}_S$ of the standard Wolf numbers, i.e. if the point $(\bar{R}_B, \bar{R}_S)$ lays on the straight line defined by the $k$-factor equation (Draper and Smith, 1998). For the three $k$-factor formulations contained in our evaluation study this is true only for the $k$-factor formulation according to Equation 6.

Thus, Waldmeier’s approach using a $k$-factor named $k_{\text{Day}}$ according to Equation 7 may have introduced an inhomogeneity in the progress of the Wolf series, which should be corrected. The effective impact is not exactly known, since those observation days with a Wolf number provided by Waldmeier and Zelenka alone will not be affected. According to the yearly reports in the Astronomische Mitteilungen for the years 1945 to 1980 this was the case in about 180 days a year, on average. Thus, the systematic overestimation of the Wolf series may be effectively lower than the 5% resulting from our evaluation study. Since the original raw observations of the numbers of sunspot groups and individual spot entering the Wolf series for the years from 1945 to 1980 were recently rediscovered, this inhomogeneity of the Wolf series may be corrected in the near future.

This possible inhomogeneity is also of interest because it coincides in time and direction with two other implementations attributed to Waldmeier: spot weighting (Clette et al., 2014; Clette and Lefèvre, 2016) and a revised group-splitting technique (Svalgaard and Schatten, 2016).

5. Scale Transfer and Construction of the Wolf Series

The Wolf number is a statistical index calculated as the weighted sum of two correlated components: the sunspot group number and the number of individual spots. The scale of an
instrumental system of Wolf numbers is defined as the mapping of the visual perception of sunspots to the number of sunspot groups and the number of individual spots.

For a given instrumental system of Wolf numbers, its scale is not explicitly known. It is rather implicitly realized by the given combination of instrument, observer, and environmental conditions. Over the whole career of a sunspot observer, various aging effects may degrade the long-term stability of the scale of their instrumental system. Especially the training and the experience of the observer play a major role in the long-term consistency and comparability of the resulting Wolf numbers. Originally, Wolf assumed in 1861 that the homogeneity of the scale of an instrumental system is reflected by the constancy of the individual $k$-factors, but in 1872 he learned that at least for observers with significantly different magnification or resolving power of their instruments the $k$-factors showed systematic variations with the progress of solar activity, and so the $k$-factors of the secondary observers had to be recalculated at least every year. Since then, the long-term homogeneity of the Wolf series relies solely on the assumed long-term stability of the instrumental systems of the standard observers and on the assumed reliability of the scale transfers from one generation of standard observers to the next.

5.1. Scale Transfer in the Wolf Series

To construct a more extended time series of Wolf numbers, Rudolf Wolf transferred his scale backwards to 1749 to a small group of fiducial observers who took over his role as a standard observer, thus forming the backbone of the Wolf series. As reconstructed by Friedli (2016), the $k$-factors for Heinrich Schwabe and for Wolf’s 40/700 mm Parisian refractor, which determines the scale for the years 1826 to 1848 and for 1861 to 1889, respectively,
relied on a rather limited number of corresponding observations during the maximum phase of solar activity. For the scale transfers from Schwabe to Flaugergues and from Flaugergues to Staudacher the situation was even worse. Due to the lack of suitable comparison days, Wolf combined according to the source book (Wolf, 1878b) the reduced Wolf numbers of all available standard and secondary observers to one common Wolf-number series and calculated the $k$-factors of new observers to this combined reference series. Therefore, Wolf calculated a calibration factor for Flaugergues of $k_S = 1.92$ relying on 38 observations using Schwabe, Tevel, Heinrich, Adams, and Arago as bridging observers. For the estimation of the calibration factor of Staudacher of $k_S = 2.5$ only two comparison observations with Bode were available using Tevel, Flaugergues, and Schwabe as bridging observers.

The scale transfer to Wolf’s successor Alfred Wolfer, resulting in a calibration factor of $k_S = 0.6$, was established by Wolfer (1895) using parallel observations from 1877 to 1893. Unfortunately, Wolfer did not correct for a scale drift during the years from 1877 to 1883 in the observations of Rudolf Wolf (Friedli, 2016). In the subsequent years Wolfer established an internal quality-management system using the observations of his assistants at the 83/1320 mm Fraunhofer refractor and his own observations at the 40/700 mm Parisian refractor as controls. After his retirement, Wolfer continued his daily observations at the 83/1320 mm Fraunhofer refractor providing comparison observations with his successor William Brunner. In 1928 the analysis of 31 months of parallel observations during the years from 1926 to 1928 revealed an unchanged calibration factor of $k_S = 0.6$ for William Brunner (Brunner, 1929). The same result was obtained for the scale transfers from William Brunner to Max Waldmeier during the years from 1936 to 1939 and from Max Waldmeier to Antoine Zelenka and to Sergio Cortesi during the years from 1966 to 1979 and from 1957 to 1979, respectively. Thus, Wolfer’s scale could be transferred successfully to the next generations of standard observers (Waldmeier, 1957, 1958, 1959).

### 5.2. Construction of the Daily Series

As discussed in Section 2, the Wolf series is constructed from 1749 to 1980 as a daily sequence of Wolf numbers provided by the standard observers and completed by reduced values from secondary observers. This meant that on days where a valid observation of the standard observer was available, the official Wolf number of this day was the Wolf number of the standard observer. No additional observation was considered. Up to Waldmeier, all complete observations from the standard observer providing a sunspot group number and a number of individual spots were considered as valid. Waldmeier and Zelenka considered only those days where the image quality was good enough. All other days had to be filled with appropriate substitutes. Before 1877, all gaps were filled only by one single observation, usually by the most experienced secondary observer. The source book provides the necessary details of which observer was considered for a specific day (Wolf, 1878b). Starting with 1877, the gaps were filled by daily averages of all available reduced secondary observers. From 1906 on, the gaps in the daily record of the standard observer were filled first by daily averages from all assistants at Zürich, including also the observers at the branch stations in Arosa and Locarno. The few remaining gaps were filled by daily averages of all available secondary observers. This two-stage approach led to the high number of the combined Zürich observations as shown in Figure 5.

Before 1877 the number of secondary observers was small. Thus, Wolf used every series that he could find and reduced the observations in quite an innovative way. Since Schmidt, Leppig, and Secchi provided for the years from 1870 to 1875 group numbers only, Wolf calculated individual transformation tables, which allowed the reduced Wolf number to be...
calculated directly from the provided group numbers (Wolf, 1873b, 1875, 1876). Others, like Carrington, de la Rue, and Secchi provided for the years from 1854 to 1860 and for 1864, 1865, and 1875 sunspot areas instead of individual spot numbers, and Wolf constructed similar translation tables (Wolf, 1873b, 1875). Thus, some gaps in the daily records from 1854 to 1875 were filled by such estimates. For the years 1878 and 1879, Wolf incorporated the areas directly in the equation of the Wolf number by adding a second $k$-factor for the transformation of the areas in individual spot numbers, where the additional $k$-factors were estimated by least squares (Wolf, 1879, 1880).

Starting with 1849, all remaining gaps were filled throughout by linear interpolation, as may be checked by the dynamic graph at www.wolfinstitute.ch.

5.3. Construction of the Monthly Series

As reported by Wolf (1877c) and Waldmeier (1961) the monthly mean Wolf numbers were calculated from the daily values. Before 1849 the record of daily values is not complete, however. For the years from 1749 to 1769, in the year 1772, for the years from 1776 to 1809, and for the years from 1811 to 1818 some periods with extremely sparse observations had to be bridged graphically (Wolf, 1877a,c, 1878b, 1890b).

The procedure of graphical interpolation was described in some detail by Wolf (1877c). Therefore, the better known monthly mean values were plotted for a period of roughly ten years, together with the lesser known monthly mean values, which were plotted in a different style. Then, an approximating curve was drawn by hand giving more weight to the better known monthly mean values than the lesser known. In the table of monthly means, the better known values were transferred without any modification, while for the unknown values the values of the interpolating curve were substituted. The lesser known monthly values were substituted by the averages between the interpolated values and the observed monthly means. From the source book (Wolf, 1878a) we learn that Wolf calculated up to three, occasionally up to four, interpolation points per month. The interpolated monthly mean was then calculated as shown in Figure 11 from the original data points, if available, and from the interpolated values of that month.

6. Recalculation of the Wolf Series from 1877 to 1893

As already discussed in Sections 2, 4.2, and 5.2 of this article, Wolf changed in 1877 the data algorithm for the yearly production of the Wolf-number series. He then used the daily average of the reduced series of daily Wolf numbers as observed by Alfred Wolfer and by Robert Billwiller with the 83/1320 mm Fraunhofer refractor and the reduced series of daily Wolf numbers as observed by himself with the 40/700 mm Parisian refractor as reference for the calculation of the $k$-factors of the remaining observers and replaced gaps within this averaged reference series by daily averages of the reduced Wolf numbers of all contributing observation stations of that day. In 1879 Robert Billwiller was named director of the newly founded Swiss Meteorological Institute in Zürich and left the Swiss Federal Observatory. Thus, from 1880 to 1893 only the daily averages of the reduced series of daily Wolf numbers as observed by Rudolf Wolf and Alfred Wolfer served as reference series for the calculation of the $k$-factors of the secondary observers.

After some years of application of this new procedure, Wolf had the impression that the $k$-factors of the contributing stations were changing in a systematic way. In a short study (Wolf, 1885) he found that the semestral $k$-factors of the observing stations in Athens (Würlisch),
Figure 11  Facsimile of a page from the source book (Wolf, 1878b) containing data from the first semester of the year 1759. The observers are flagged in the first column of each month, the daily counts of sunspot groups \([g]\) (first number) and of individual spots \([f]\) (second number) are given in the second column of each month. The daily Wolf numbers are indicated in the third column of each month, calculated as \(R = k (10g + f)\). The \(k\)-factors of the contributing observers Staudacher and Messier may be found in the bottom part of the table. Note that the resulting daily mean Wolf numbers are not contained in Waldmeier (1961) and in WDC-SILSO version 1.0. In the first semester of the year 1759 Staudacher provided six observations, one in February and five in March. The remaining 14 observations are interpolated values according to the procedure described in Section 5.3 of this article.

Madrid (Ventosa), Palermo (Tacchini and Riccó), and Zürich (Wolfer), as shown in the left panel of Figure 12, had dropped linearly with the rise of the Wolf number from the solar minimum in 1878 to the solar maximum in 1884 (Figure 13). Wolf concluded that although the results from this study were significant, the observed negative correlation between the Wolf-number values and the semestral \(k\)-factors of the four observing stations should be monitored during a longer period and more phases of the solar cycle before a substantial revision of the Wolf-number production procedure should take place.
6.1. The Scale Transfer from Wolf to Wolfer According to Wolfer

After the death of Rudolf Wolf in 1893, Wolfer (1895) extended this study up to the year 1893, covering roughly two minima and maxima of solar activity. Since the series of Athens as included by Wolf (1885) was discontinued in 1886, Wolfer replaced it by the series of Tacchini in Rome. As shown in the right panel of Figure 12, the progress of the $k$-factors of the four stations showed no dependency on solar activity after 1884 (Figure 13). Thus, Wolfer considered the results from Wolf (1885) as spurious. Furthermore, he concluded from the close similarity of the series of $k$-factors from Ventosa in Madrid and Riccó in Palermo with his own that his instrumental system was stable and homogeneous and that an overall $k$-factor of 0.6, calculated as the arithmetic mean of the 34 semestral values from 1877 to 1893, would be appropriate to reduce his Wolf numbers to Wolf’s scale (Wolfer, 1895). The decline of his $k$-factor during the years from 1877 to 1884 as drawn in Figure 13 he explained by a continuous degradation of Wolf’s eyesight since the same decline showed up also in the series of $k$-factors from Ventosa in Madrid and Riccó in Palermo as shown in the right panel of Figure 12. According to Wolfer (1895), the degradation of Wolf’s eyesight was also affecting Wolf’s daily life and caused him, as reported in Wolf (1889), to replace in 1890 his beloved 40/700 mm Parisian refractor with magnification 20 by a more powerful 42/800 mm Fraunhofer refractor with magnification 29. Wolfer (1895) concluded that the whole series should be recalculated using the original raw observations to properly correct these flaws, a task that was postponed. Thus, the effects of the diminishment of Wolf’s eyesight and the replacement of Wolf’s main telescope remained uncorrected.

6.2. The Recalculated Series of Frenkel (1913)

Almost forgotten was the recalculation undertaken by Elsa Frenkel, more than a century ago, as part of her PhD thesis on hidden periodicities in the progress of solar activity carried out at the Swiss Federal Institute of Technology in Zürich under the supervision of Alfred Wolfer.
and corefered by Albert Einstein (Frenkel, 1913), although it was never implemented. As stated in her thesis, Frenkel applied exactly the same data-reduction algorithm to recalculate the corrected Wolf series as Wolfer did for the years after 1893 and that is described in Section 4.2 of this article.

Thus, in a first step, Wolfer’s observed daily raw Wolf numbers, calculated from the observed daily group numbers and the observed daily numbers of individual spots as published in the *Astronomische Mittheilungen*, were reduced with a $k$-factor of 0.6 to Wolf’s scale and rounded to the nearest integer. The resulting series of reduced Wolf numbers became the backbone of the corrected Wolf series.

Then, in a second step, Wolfer’s series of reduced Wolf numbers served as the reference for the calculation of semestral $k$-factors according to Equation 6 for all contributing stations from 1877 to 1893 except for Wolf’s and Billwiller’s observations, which were omitted completely (Figure 14). The elimination of Billwiller’s observations is no surprise, since he provided in 1877 only one and in 1878 only four observations, which were not in common with Wolfer, all with a value of zero. In 1879 Billwiller observed with a small spy refractor belonging to himself (Wolf, 1880).

Finally, in a third step, the gaps in Wolfer’s series of reduced Wolf numbers were filled with mean values calculated from the reduced values of all contributing stations on that day. While the recalculated daily values remained unpublished, the resulting series of monthly means was tabulated by Frenkel (1913).

6.3. Correction of the Daily Values of the Recalculated Series

Luckily, the author found in the archives of the former Swiss Federal Observatory a manuscript in the handwriting of Alfred Wolfer containing tables with the hitherto unpublished daily values of Frenkel’s recalculated Wolf-number series (Wolfer, 1912). A facsimile

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**Figure 13** Facsimile from Wolfer (1895) containing the official yearly mean Wolf numbers (right-hand scale) as provided by WDC-SILSO version 1.0 and Wolfer’s yearly $k$-factor to the reduced Wolf numbers of Rudolf Wolf with the 40/700 mm Parisian refractor (left-hand scale).
Figure 14  Semestral k-factors for the recculated Wolf series as published by Frenkel (1913). For 1876 the values of $k_1$ for the stations in Moncalieri, Palermo, Athens, and Peckeloh are calculated using Rudolf Wolf’s observations with the 40/700 mm Parisian refractor as a reference. Therefore, we used only the recalculated and corrected values for the years from 1877 to 1893 as a substitution for the Wolf series as provided by WDC-SILSO version 1.0 in this article.

of a sample page containing the table with the recculated daily Wolf numbers for the year 1882 is shown in Figure 15. The heritage group of the Rudolf Wolf Society digitized the entire data set.

A closer inspection of Wolfer (1912) revealed that years after the original compilation and publication, some calculation errors in the published values were corrected by William Brunner. As indicated by Wolfer (1912), the corrections concerned a total of 52 daily values in the years 1891, 1892, and 1893 where observations of Alfred Wolfer made with the
Figure 15 Facsimile of a table in the handwriting of Alfred Wolfer from Wolfer (1912) containing the hitherto unknown daily Wolf numbers for the year 1882 as recalculated by Elsa Frenkel. The last three rows beneath the table contain the observed monthly means as published by Frenkel (1913), the yearly mean, the official values of the monthly means as published in the Astronomische Mitteilungen, and the number of spotless days. To the right of the seal of the Swiss Federal Observatory at Zürich are the two semestrial means of the recalculated monthly means. Note that for all calculations in this manuscript a fraction of 0.5 was always rounded to the next lower integer. Small crosses to the left of the daily values in the table mark the end of solar rotations. The digitized and corrected values of the recalculated daily Wolf numbers were placed on the site www.wolfinstitute.ch of the Rudolf Wolf Society.
40/700 mm Parisian refractor of Rudolf Wolf were erroneously reduced with a \( k \)-factor of 0.6 like the observations made with the 83/1320 mm Fraunhofer refractor, although all of these observations are properly flagged by Wolf (1892, 1893) and Wolfer (1894) as observations made with the 40/700 mm Parisian refractor. As William Brunner annotated by Wolfer (1912), the original observations made by Alfred Wolfer with the 40/700 mm Parisian refractor were replaced with mean values calculated from reduced observations of Jena, Kalocza, Catania, Philadelphia, Rome, Haverford, and Kremsmünster. Analogously, the author corrected a similar case on 31 December 1892 overlooked by William Brunner using observations from Catania and Philadelphia.

Furthermore, the author corrected a total of 165 observations of Alfred Wolfer during the years from 1877 to 1893 where the reported Wolf numbers in the manuscript showed calculation errors compared to the original observations of Alfred Wolfer as published in the *Astronomische Mitteilungen* and as compiled separately by Wolfer (1921). Additionally, two missing values on 20 November 1877 and on 28 February 1892, respectively, were filled with reduced observations of Rudolf Wolf made with the 40/700 mm Parisian refractor. Although published by Frenkel (1913) and included by Wolfer (1912), the recalculated values for the year 1876 were entirely omitted in the present article, since Wolfer started his observation series only in August 1876 and the \( k \)-factors \([k_1]\) for the first semester of the remaining four stations in Moncalieri, Palermo, Athens, and Peckeloh as given in Figure 14 were based on the originally reduced observations of Rudolf Wolf made with the 40/700 mm Parisian refractor as reference.

The recalculated and corrected daily values of the Wolf series for the years from 1877 to 1893 were placed on the site [www.wolfinstitute.ch](http://www.wolfinstitute.ch) of the Rudolf Wolf Society. The monthly mean values of the Wolf series for the years from 1877 to 1893 as calculated by the author from the recalculated and corrected daily values are given in Table 2. The smoothed monthly means of the recalculated and corrected Wolf series for the years from 1877 to 1893 are shown in Figure 16 together with the smoothed monthly means of the original Wolf series as provided by WDC-SILSO version 1.0.

### 6.4. Discussion

As expected from Wolfer’s \( k \)-factor as given in Figure 13, the smoothed monthly means of the recalculated and corrected Wolf series are all higher than the original ones for the years from 1883 to 1893, except for the years 1889 and 1892. Thus, the maxima of Cycles 12 and 13 in 1884 and 1893, respectively, are roughly 10% higher in the recalculated and corrected Wolf series than in the original Wolf series as provided by WDC-SILSO version 1.0. This result is confirmed on other grounds by Cliver and Ling (2016).

From 1877 to 1883 the recalculated and corrected Wolf series is lower than the original one. Therefore, the \( k \)-factor of the old against the recalculated and corrected Wolf series grows from 1894 back to 1888 and then declines down to a value of 0.76 in 1877, as shown in the lower part of Figure 16.

Thus, the original Wolf series incorporates a hitherto uncorrected scale drift in the standard observations of Rudolf Wolf made with the 40/700 mm Parisian refractor during the years from 1877 to 1893.

To explain the scale drift as displayed in Figure 16 with the effect of Wolf’s eyesight diminishment, we calculated, as is displayed in the right panel of Figure 17, the progress of the yearly \( k \)-factors of the reduced observations of Rudolf Wolf made with the 40/700 mm Parisian refractor to the reduced observations of Alfred Wolfer made with the 83/1320 mm Fraunhofer refractor. The right panel of Figure 17 displays also the progress of the \( k \)-factors.
Table 2  Observed monthly means of the recalculated and corrected Wolf series.

| Year | Jan | Feb | Mar | Apr | May | Jun | Jul | Aug | Sep | Oct | Nov | Dec |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 1877 | 17.8| 5.9 | 9.0 | 11.3| 13.9| 11.2| 5.3 | 4.5 | 12.3| 5.3 | 10.0| 1.6 |
| 1878 | 2.4 | 5.8 | 6.5 | 0.2 | 5.6 | 6.5 | 0.1 | 0.0 | 2.8 | 0.8 | 1.8 | 0.1 |
| 1879 | 1.3 | 0.6 | 0.0 | 5.5 | 2.0 | 4.7 | 6.7 | 9.9 | 5.9 | 11.5| 11.8| 6.0 |
| 1880 | 19.9| 20.9| 13.8| 16.7| 19.6| 27.2| 18.4| 37.3| 55.3| 35.2| 25.0| 21.1|
| 1881 | 30.0| 41.9| 47.1| 48.0| 41.6| 55.8| 68.0| 52.8| 47.3| 52.3| 44.4| 41.9|
| 1882 | 45.5| 69.3| 65.1| 90.0| 60.7| 46.5| 47.9| 40.5| 51.0| 53.6| 72.8| 38.1|
| 1883 | 50.5| 43.2| 38.7| 77.6| 29.5| 76.1| 82.4| 50.8| 57.8| 88.8| 93.9| 79.9|
| 1884 | 99.2| 91.6| 95.4| 86.4| 75.8| 58.9| 63.1| 64.3| 71.3| 60.7| 41.5| 53.5|
| 1885 | 49.0| 79.3| 57.3| 58.3| 76.5| 94.9| 77.0| 56.8| 38.6| 38.4| 33.6| 25.3|
| 1886 | 32.5| 26.7| 54.5| 42.2| 33.5| 28.9| 30.9| 16.3| 27.2| 9.6 | 18.0| 16.2|
| 1887 | 12.4| 13.2| 7.7 | 7.6 | 24.9| 18.3| 24.5| 25.2| 12.0| 10.3| 7.7 | 23.9|
| 1888 | 13.7| 9.2 | 9.3 | 8.3 | 7.6 | 9.2 | 6.0 | 8.2 | 13.6| 3.1 | 12.2| 7.5 |
| 1889 | 0.7 | 7.9 | 5.9 | 3.3 | 2.0 | 5.7 | 11.5| 18.1| 5.9 | 2.8 | 0.7 | 7.1 |
| 1890 | 5.4 | 0.8 | 4.9 | 3.4 | 7.2 | 2.6 | 13.3| 12.4| 23.3| 14.5| 9.6 | 10.7|
| 1891 | 13.3| 27.2| 10.5| 27.3| 47.9| 55.5| 63.2| 40.6| 58.4| 52.5| 42.2| 31.5|
| 1892 | 62.2| 72.9| 48.0| 65.3| 77.1| 79.5| 75.4| 88.4| 61.6| 63.8| 54.4| 65.2|
| 1893 | 72.9| 76.1| 72.3| 98.6| 96.7| 112.0| 103.3| 129.0| 89.0| 84.9| 79.5| 94.5|

Figure 16  Recalculation of the Wolf series from 1877 to 1893. The original smoothed monthly means on Wolf’s scale are labeled R13 Ri V10 (left-hand scale). The smoothed monthly means as recalculated by Frenkel (1913) and corrected by the author are labeled R13 Ri Frenkel (left-hand scale). In the lower part of the panel the monthly values of the k-factor of the smoothed original to the smoothed corrected monthly means of the Wolf series are drawn (right-hand scale).
**Figure 17** Left panel: yearly $k$-factors of Riccó, Tacchini, Ventosa, and Wolfer as given by Wolfer (1895). Right panel: yearly $k$-factors of Riccó and Ventosa as given by Frenkel (1913). Additionally, the course of the yearly $k$-factor of the reduced observations of Rudolf Wolf made with the 40/700 mm Parisian refractor to the reduced observations of Alfred Wolfer made with the 83/1320 mm Fraunhofer refractor as calculated by the author is shown in the right panel indicating a continuous degradation of Wolf’s eyesight from 1877 to 1893.

of Ventosa in Madrid and Riccó in Palermo, also reduced to Wolfer. The trace of the $k$-factors of Tacchini as shown in the right panel of Figure 12 was omitted in the right panel of Figure 17, since a closer inspection of the original values as provided in the Astronomische Mitteilungen revealed that this series was in fact a mixture of the observations from Tacchini and his assistants made in Palermo and Rome. Thus, the progress of its $k$-factors not only reflects inhomogeneities in the series of Wolfer but also by the changing percentages in the number of observations as provided by Tacchini and his assistants. While the $k$-factors of Ventosa and Riccó show no significant trend, the $k$-factor of Wolf shows an increase from 1877 to 1893, which appears to be modulated by solar activity. In 1890 Wolf changed from the 40/700 mm Parisian refractor with magnification 20 to the more powerful 42/800 mm Fraunhofer refractor with magnification 29 (Friedli, 2016), which may have partly compensated for the degradation effect.

A more detailed analysis using a linear regression model for the yearly $k$-factors revealed, as is shown in Figure 18, that especially the estimated intercept coefficients varied with the phase of the solar cycle, indicating that Rudolf Wolf did not see the smallest spots with his 40/700 mm Parisian refractor. Over the years, the intercept values are systematically and significantly growing, indicating a continuous degradation of Wolf’s eyesight. The estimated slope coefficients show considerably less variation with solar activity and no degradation effect. As expected, the slopes of the two last years, 1892 and 1893, were significantly lower due to the change from the 40/700 mm Parisian refractor to the more powerful 42/800 mm Fraunhofer refractor.

Thus, Wolfer (1895) was right in denying a correlation between the Wolf-number values and the $k$-factors of the secondary observers to the 40/700 mm Parisian refractor as suggested by Wolf (1885). Our analysis confirmed that the principal cause for the scale drift as displayed in the right panel of Figure 17 was a cyclic and systematic drift of the intercept in a linear-regression model for the $k$-factor. Therefore, Frenkel (1913) was right in omitting the
Figure 18  Yearly $k$-factors of the reduced daily Wolf numbers of Rudolf Wolf as observed with the 40/700 mm Parisian refractor to the reduced daily Wolf numbers of Alfred Wolfer as observed with the 83/1320 mm Fraunhofer refractor. In the upper-left corner of each panel the yearly $k$-factor according to Equation 6 and as shown in the right panel of Figure 17 is given. The line represents a least-squares fit. The numerical values of the intercepts and the slopes with their standard errors are given for each year at the bottom of each panel. The light-gray shading around the fitted line represents the 95% confidence band for the line. The intercept values vary with solar activity. In 1884 and in 1890 during solar maximum the intercepts are significantly higher than in 1878 and in 1890 during solar minimum indicating that Rudolf Wolf could not see the smallest spots. This effect is more significant in 1893 than in 1884, indicating that there was some sort of degeneration in Wolf’s visual acuity although Wolf was observing in 1893 with a more powerful instrument. The slope values show less variation with solar activity. During the last two years the slopes were smaller, since Wolf observed with a more powerful instrument.
Figure 19  Frequency of small Wolf numbers ranging from 0 to 24 for the years from 1877 to 1893 for the Wolf series as provided by WDC-SILSO version 1.0 (upper panel) and as recalculated by Frenkel (1913) and corrected in this article (lower panel). Due to the averaging of Wolf’s and Wolfer’s observations in the original Wolf series as provided by WDC-SILSO version 1.0, Wolf numbers with values of 2 and 3 were most frequent in the years from 1886 to 1890. The recalculation of Frenkel (1913) corrected this inhomogeneity. Wolf numbers in the range from 1 to 6 remain sparsely populated in the recalculated and corrected series as shown in the lower panel since Alfred Wolfer was now the only observer in Zürich and some of the secondary observers had more powerful instruments with $k$-factors below 0.6 (Figure 14) resulting in average Wolf numbers below 7 on days with very low sunspot activity.

observations of Rudolf Wolf with the 40/700 mm Parisian refractor from the recalculation of the Wolf series from 1877 to 1893, since yearly $k$-factors estimated according to Equation 6 would have introduced systematic biases in the reduced Wolf numbers as observed with the 40/700 mm Parisian refractor.

The effect of the recalculation of Frenkel (1913) on the frequency of small Wolf numbers in the range from 0 to 24 as discussed in Section 4.1 is shown in Figure 19 and in Figure 20. Due to the averaging of Wolf’s and Wolfer’s observations in the original Wolf series as provided by WDC-SILSO version 1.0, Wolf numbers with values of 2 and 3 were most frequent in the years from 1886 to 1890 as is shown in Figure 7 and in the upper panel of Figure 19. After the recalculation of Frenkel (1913), the most frequent positive small Wolf number in the years from 1877 to 1891 is 7. Wolf numbers in the range from 1 to 6 remain sparsely
Figure 20  Frequency of small Wolf numbers ranging from 0 to 24 for the years from 1818 to 2014 for the Wolf series as recalculated by Frenkel (1913) and corrected in this article. The recalculation and correction successfully extended Wolfer’s series from 1894 back to 1877 and homogenized his entire observation period ranging from 1877 to 1928. Before 1877 the original Wolf series has to be homogenized further, correcting in particular for the scale transfer from the 83/1320 mm Fraunhofer refractor as used by Alfred Wolfer to the 40/700 mm Parisian refractor as used by Rudolf Wolf.

populated in the recalculated and corrected series as shown in the lower panel of Figure 19 and in Figure 20, since on days without a standard observation of Alfred Wolfer, the Wolf-number values have to be provided by secondary observers. Compared to the Wolf series as provided by WDC-SILSO version 1.0, these days are more frequent in the recalculated and corrected series as provided by this article. As shown in Figure 14 the power of the instruments of the secondary observers, measured by their $k$-factors, were quite different; some of the stations, including Madrid, Haverford, Paris, and Palermo, even had $k$-factors below 0.6, indicating that their instruments were more powerful than the 83/1320 mm Fraunhofer refractor with magnification 64 as used by Alfred Wolfer. Others, including Moncalieri, Ogyalla, Rome, and Athens, used significantly less powerful instruments, more similar to the 40/700 mm Parisian refractor as used by Rudolf Wolf. Thus, observers with more powerful instruments will see small sunspot groups that remain invisible for observers with less powerful instruments, resulting in average Wolf numbers below 7 on days with very low sunspot activity.

7. Summary and Conclusions

Homogeneity is considered as the most important property of the Wolf series, since without a stable scale no valid conclusions about variations in the long-term progress of solar activity can be drawn. However, the homogeneity testing of the Wolf series is a difficult task, since the raw data entering the series and the methods of data-reduction and interpolation used to compile the series are largely unknown. For the period from 1749 to 1876 we identified in Section 3.2 of this article, from the archives of the Swiss Federal Observatory in Zürich, three hitherto unpublished manuscripts by Wolf (1878a) and Wolfer (1902b, 1909a), which form a complete and fully transparent documentation of the daily, monthly, and yearly mean Wolf numbers as published by Waldmeier (1961) and provided by WDC-SILSO version 1.0.
Based on the hitherto unpublished manuscripts of Brunner (1945a) and Wolfer (1908, 1909b) from the archives of the Swiss Federal Observatory, we reconstructed in Section 4.2 the data-reduction algorithms and discussed in Section 4.3 their impact on the homogeneity of the Wolf series.

Reconsidering in Section 6.1 the scale transfer in 1893/1894 from the 40/700 mm Parisian refractor as used by Rudolf Wolf to the 83/1320 mm Fraunhofer refractor as used by Alfred Wolfer, we analyzed in Section 6.2 an almost forgotten recalculation of the Wolf series from 1877 to 1893 as published by Frenkel (1913). Based solely on Alfred Wolfer as reference and omitting the observations of Rudolf Wolf and of Robert Billwiller from the recalculation, Frenkel (1913) extended Wolfer’s series from 1894 back to 1877, correcting for Wolf’s eyesight diminishment, for the change of his main instrument in 1890, and for the inhomogeneities in the data-reduction procedure as discussed in Sections 2, 4.2, and 5.2.

Luckily, we found in the archives of the Swiss Federal Observatory a manuscript by Wolfer (1912) containing the hitherto unpublished daily values of this recalculated Wolf-number series that we used in Section 6.3 to examine and to correct the published values by Frenkel (1913). We consider for the years from 1877 to 1893 the recalculated and corrected Wolf-number series as provided by this article as a valid substitution of the original Wolf series as provided by WDC-SILSO version 1.0.

The maxima of Cycle 12 in 1884 and of Cycle 13 in 1893 are roughly 10% higher in the recalculated and corrected Wolf series than in the original Wolf series as provided by WDC-SILSO version 1.0. From 1877 to 1883 the recalculated and corrected Wolf series is up to a factor of 0.76 lower than the original one.

Before 1877 the scale transfer from the 40/700 mm Parisian refractor as used by Rudolf Wolf to the 83/1320 mm Fraunhofer refractor as used by Alfred Wolfer will need to be analyzed further.

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