Metal-rich globular clusters in the galactic disk: new age determinations and the relation to halo clusters

M. Salaris and A. Weiss

Max-Planck-Institut für Astrophysik, Karl-Schwarzschild-Str. 1, 85748 Garching, Federal Republic of Germany

Received; accepted

Abstract. New age determinations of the galactic disk globular clusters 47 Tuc, M71 and NGC6352 have been performed with our up-to-date α-enhanced stellar models. We find that all three clusters are about 9.2 Gyr old and therefore coeval with the oldest disk white dwarfs. Several arguments are presented which indicate that the initial helium content of the stars populating these clusters is close to the solar one. We also revisit a total of 28 halo clusters, for which we use an updated [Fe/H] scale. This new metallicity scale leads on average to an age reduction of around 0.8 Gyr relative to our previous results. We compare the predicted cluster distances, which result from our dating method, with the most recent distances based on HIPPARCOS parallaxes of local subdwarfs. We further demonstrate that for the most metal-rich clusters scaled-solar isochrones no longer can be used to replace α-enhanced ones at the same total metallicity. The implications of the presented age determinations are discussed in the context of the formation history of the Galaxy.

Key words: Galaxy: formation – Galaxy: evolution – globular clusters: general – stars: evolution

1. Introduction

The galactic globular clusters (GC) constitute the fossil record of the Galaxy formation epoch; their ages therefore provide fundamental informations about the timescale of the formation process, and put strong constraints on the age of the universe.

In two recent papers (Salaris, Degl’Innocenti & Weiss 1997; Salaris & Weiss 1997; Papers I & II) we have redetermined ages for a sample of GCs by means of new stellar models employing the latest improvements in stellar input physics, noticeably in equation of state and opacities. In Paper I we demonstrated that the age of three extremely metal-poor clusters is around 12 Gyr due to the model improvements. In Paper II the same models were applied to a sample of 25 halo clusters. We put special emphasis on the careful application of two distance and reddening-independent methods for determining absolute and relative ages within groups of clusters of comparable metallicity. Not only did we confirm the new, low value for the age of the oldest clusters, but also found a clear correlation between age and metallicity of the halo clusters. The lowest age in our sample amounted to only 6.5 Gyr for Ter 7, which, however, might not be one of the typical halo clusters (see the discussion in Paper II), for which the lowest age was 8 Gyr. We emphasize that our predicted distances were in agreement with the first HIPPARCOS-based distance determinations. Several aspects of our results confirmed those of other papers (Chaboyer & Kim 1995; Mazzitelli, D’Antona & Caloi 1995) or have been confirmed in the following (Sarajedini, Chaboyer & Demarque 1997).

Since the lowest ages for typical halo clusters were comparable to those of disk white dwarfs (Salaris et al. 1997), it is interesting to ask how disk GC (see, e.g., Armandroff 1989 for a study about the kinematical properties of this GC system) fit into the emerging picture of the history of Galaxy formation. To determine their ages we have used the same rigorous approach we have been using successfully for the halo clusters. We selected three disk clusters, namely 47 Tuc, M71 and NGC6352, with V and B photometry extending well below the turn-off (TO) point, and with almost the same metallicity ([Fe/H]=-0.70 for 47 Tuc and M71, [Fe/H]=-0.64 for NGC6352 according to Carretta & Gratton 1997). In the case of 47 Tuc and NGC6352 the absolute age is directly determined by means of the difference between the Zero Age Horizontal Branch (ZAHB) level and the TO, the so-called ΔV age indicator (or vertical method), while for M71, whose photometry shows a poorly populated HB, the relative age with respect to 47 Tuc is determined by means of the colour difference between the TO and the base of the Red Giant Branch (RGB), the so-called Δ(B−V) age indicator (or horizontal method).

Due to the high metallicity of the disk clusters, additional stellar evolution calculations became necessary. The computational details as well as the age determina-
tion method will be presented briefly in Sect. 2. The application of our isochrones to the three clusters will then be discussed in detail in Sect. 3. After that, we will revisit our halo cluster sample, which we extended by another three objects. Due to new results about the metallicity scale of globular clusters (Carretta & Gratton 1997), a re-evaluation of their ages became necessary. The updated results are contained in Sect. 4. In the final section we will discuss all results.

2. Computations and age determination method

For the details of the stellar evolution computations we refer the reader to Papers I and II. Suffice to repeat here the most important aspects of the physical input for the evolutionary tracks and isochrones.

2.1. Equation of state (EOS)

We have used the OPAL EOS (Rogers, Swenson & Iglesias 1996) supplemented for the regions where the OPAL EOS is not available, with the EOS described in Paper I.

2.2. Heavy elements mixture

For the relative abundances of the elements heavier than helium we use an $\alpha$-enhanced heavy elements mixture (Table 1) selected according to the results presented by Ryan et al. (1991) about chemical abundances in metal-poor field stars. These abundance ratios are also consistent with the results summarized in Wheeler et al. (1989) about the chemical composition of GC stars and with those by Gratton et al. (1986) and Gratton (1987a, b) – as compiled in Carney (1996) – about the $\alpha$-element distribution in the three metal-rich disk clusters we will study in the next section. Very recently, McWilliam (1997) has reported $\alpha$-element enhancements for metal-poor field stars derived from different authors; these overabundances are again in broad agreement with the heavy elements mixture we adopted. Note, however, that the error-bars in these determinations are so large that a constant enhancement value for all $\alpha$-elements cannot be excluded definitely.

Ti is the only element for which our selected enhancement (in agreement with the Ryan et al. (1991) results for the lowest metallicities) does not agree well with the results of other investigations. Nevertheless, this is only of minor concern, since Ti affects appreciably only the opacities through the TiO molecule, but in a temperature region that does not affect the evolution of the stellar models computed for producing our isochrones (see, e.g., the discussion on Alexander & Ferguson 1994).

2.3. Opacities

We have used appropriate and up-to-date opacity tables (taking into account the effect of molecules at low temperatures) for the $\alpha$-element enhanced mixtures discussed above for all temperatures and densities of interest (Rogers & Iglesias 1992; Alexander & Ferguson 1994). Equivalent tables for mixtures with solar metal ratios were used for scaled-solar isochrones.

2.4. Mixing length

A value of 1.80 for the mixing length parameter has been adopted; this value satisfies the solar constraint and at the same time reproduces well the RGB of a selected sample of GC in the $(M_{bol}, T_{eff})$ plane, as derived by Frogel et al. (1983), once their data are adjusted to the distance scale given by our ZAHB models.

2.5. Bolometric corrections and colour transformations

The conversion of $L/L_\odot$ and $T_{eff}$ into $(U, B, V)$-magnitudes and colours is performed by adopting a combination of Buser & Kurucz (1978 - BK78) and Buser & Kurucz (1992 - BK92) transformations (see Paper I). Since in this paper we want also to compare the GC distance moduli obtained from our ZAHB models with the distance scale set by the HIPPARCOS subdwarfs, we have devoted particular care to the calibration of the zero point of the bolometric correction scale.

The BK78 and BK92 bolometric corrections (BC) are normalized such that the maximum value of the BC is zero (see BK78), all others being negative. This BC scale has to be calibrated later in such a way as to reproduce observed $V$ magnitudes of selected stars (as discussed in BK78 and BK92). To this purpose we have used the recent empirical BC determinations by Alonso et al. (1995, 1996) for main sequence (MS) stars of low to solar metallicity. With a constant shift applied, our BC can reproduce these empirical results for all the metallicities considered in this paper within $\approx \pm 0.02$ mag. The Alonso et al. (1995, 1996) BC are on a scale where $BC_\odot=-0.12$; we therefore adopted $M_{bol,\odot}=4.70$ in order to be consistent with the observed

| Element | (1) | (2) | (3) | (4) |
|---------|-----|-----|-----|-----|
| O       | 0.50| 0.35-0.50 | 0.30-0.50 | 0.31-0.48 |
| Ne      | 0.29| ---     | ---     | ---   |
| Mg      | 0.40| 0.20-0.30| 0.36    | ---   |
| Si      | 0.30| 0.25-0.30| 0.38    | 0.29-0.37 |
| S       | 0.33| ---     | ---     | ---   |
| Ca      | 0.50| 0.10-0.45| 0.18-0.47| 0.09-0.29 |
| Ti      | 0.63| 0.30-0.35| 0.29    | 0.23-0.40 |

(1): this paper; cf. Ryan et al. (1991); (2): Wheeler et al. (1989); (3) McWilliam (1997); (4) Carney 1996
solar V brightness \(M_{V,\odot} = 4.82 \pm 0.02\) according to Hayes (1985).

With respect to Paper I and II, where the zero point of our BC was calibrated according to the old empirical BC scale for hot MS stars (\(T_{\text{eff}} > 8000\) K) of nearly solar metallicity by Code et al. (1976), our isochrones in the \(V-(B-V)\) plane are now 0.06 mag brighter. We think that the present recalculation is more reliable since we are using new empirical BC for stars with the same metallicity and the same range of \(T_{\text{eff}}\) as for GC stars, a fact that minimizes possible differential errors in the theoretical BC varying with metallicity and \(T_{\text{eff}}\).

Of course, the zero point of the adopted BC scale does not influence at all the absolute and relative ages obtained by means of the vertical and horizontal methods used here and in Paper I and II. However, it is important for the a posteriori calculations of the distances.

### 2.6. Additional evolutionary calculations

In addition to the metallicities specified in Paper II we added consistent calculations for \(Z = 0.008\) and \(Z = 0.01\), corresponding to [Fe/H] = −0.7 and −0.6 dex. The helium content at these metallicities was either \(Y = 0.242\), which is identical to that for the mixture with \(Z = 0.004\) (the most metal-rich one used in Paper II), or respectively \(Y = 0.254\) and \(Y = 0.260\) obtained when using \(\Delta Y/\Delta Z = 3\) as in Paper II, or \(Y = 0.273\), which is close to the initial solar helium abundance as derived from the computation of solar models (see e.g. Schlattl et al. 1997). For some \((Y, Z)\)-combinations evolutionary calculations for scaled-solar metal abundances were added for comparison (see Sect. 3.4). With these new calculations our set of \(\alpha\)-enhanced isochrones spans the [Fe/H] range −2.3 ≤ [Fe/H] ≤ −0.6.

### 2.7. Age determination method

As already mentioned in the introduction, to determine directly the absolute age of the clusters we use the vertical method (see Stetson et al. 1996 for a review and Paper I), which makes use of the age-dependent brightness difference between TO and ZAHB. This method is reliable when the cluster photometry shows a well-populated HB; in this case the ZAHB level corresponds to the lower envelope of the stellar distribution along the HB, and it is determined by means of a statistical method discussed in Paper II. When it is not possible to apply the vertical method, relative ages between clusters of comparable metallicity are determined by the horizontal method, which uses the age-dependent colour difference between TO and the base of the RGB (see VandenBerg et al. 1990). After the age has been determined, we check our results with various tests: (i) we verify that the corresponding isochrone fits the cluster CMD sufficiently well; (ii) we compare our predicted reddening (obtained from fitting the unevolved isochrone main sequence to the observed one) with literature values; (iii) we use the theoretical ZAHB brightness to derive the distance modulus of the cluster, and compare this with independent determinations.

### 3. Metal-rich disk clusters

#### 3.1. NGC104 (47 Tuc)

The most prominent disk cluster is certainly 47 Tuc on the southern hemisphere. We used the data from the paper by Hesser et al. (1987). The metallicity of 47 Tuc has been determined as \([\text{Fe}/\text{H}] = -0.70 \pm 0.10\) by Carretta & Gratton (1997) and is confirmed by Cannon et al. (1997). Using the vertical method for deriving the cluster age, Hesser et al. (1987) determined \(t = 13.5 \pm 2.0\) Gyr for 47 Tuc from oxygen-enhanced isochrones, while Chaboyer & Kim (1995) obtained, by applying their isochrones without diffusion (computed by using the OPAL equation of state) to the data by Hesser et al. (1987), \(t = 13.23 \pm 1.7\) Gyr; Mazzitelli et al. (1995) derived ages of 12–14 Gyr from the same observational data. Very recently Gratton et al. (1997) determined the age of 47 Tuc matching the observed TO position (from Hesser et al. 1987) with several sets of theoretical isochrones, after deriving the distance to the cluster by means of the main sequence (MS) fitting using HIPPARCOS subdwarfs; they got ages in the range 9.6–10.8 Gyr, with a random error of 12% on the single age values.

There have been several suggestions that the helium content of 47 Tuc stars is close to the solar value. Alonso et al. (1997) find that \(\Delta V_{\text{TO}}^{\text{HB}}\) is almost the same in NGC6366 (a metal-rich halo cluster, with a metallicity close to that of 47 Tuc, see Table 2) and 47 Tuc, implying the same age for both clusters; however, \(\Delta(B-V)\) is definitely larger for 47 Tuc, thus indicating that this cluster has to be younger than NGC6366. This means that two age indicators give contradicting results. Since \(\Delta(B-V)\) is largely independent of the helium content while \(\Delta V_{\text{TO}}^{\text{HB}}\) is dependent (as shown by Salaris et al. 1994 and confirmed by our computations), they take this as evidence for an enriched helium content in 47 Tuc with respect to NGC6366, concluding that a value \(Y = 0.27-0.28\) for 47 Tuc could explain this discrepancy. Buzzoni et al. (1983) and Hesser et al. (1987) found that the \(Y\)-dependent R-parameter, relating the number of stars on the HB and on the RGB at magnitudes brighter than the HB, is \(1.75 \pm 0.21\), respectively \(1.86 \pm 0.36\), corresponding to a helium content of \(0.27 \pm 0.02\) and \(0.28 \pm 0.04\) according to the calibration by Buzzoni et al. (1983).

We investigated independently another helium abundance indicator: the brightness difference between a point on the lower, unevolved MS, that is, for \(M_V \geq 6.0\), and the ZAHB at the same colour, \(\Delta V_{\text{MS}}^{\text{ZAHB}}\) (a slightly different definition of this He indicator was already given by
Caputo et al. 1983). Since the MS is shifted at lower luminosities and the ZAHB at higher ones for an increase of the initial helium abundance in stellar models, the difference \( \Delta V_{ZAHB}^{MS} \) results to be a good indicator of the stellar initial helium content, that complements the R-parameter.

From our definition of \( \Delta V_{ZAHB}^{MS} \) it turns out that this indicator can only be used in clusters with deep MS photometry, an HB redder than the TO and a well-defined reddening to allow for a precise identification of the corresponding point on the theoretical isochrone; all three conditions are fulfilled for 47 Tuc \((E(B-V) = 0.04 \pm 0.02; \text{Zinn 1980})\). Note that this brightness difference is to very good approximation age-independent and neither influenced by the mixing-length parameter as we have verified from our isochrones. At \((B-V)_0 = 0.76\) our models provide, for [Fe/H] values around [Fe/H] = −0.7: \( Y = -0.319 + 0.13 \cdot \Delta V_{ZAHB}^{MS} + 0.21 \cdot [\text{Fe/H}] \). Taking the reddening (and the associated error) from Zinn (1980), we derived \( \Delta V_{ZAHB}^{MS} = 5.66 \pm 0.12\) and obtained, considering also the error of ±0.10 dex in [Fe/H], \( Y = 0.270 \pm 0.026\), in excellent agreement with the quoted values from the R-parameter. Consequently, we have used isochrones with solar helium content to determine the age of 47 Tuc.

Due to the very well-populated red HB, the identification of the ZAHB level is very easy and unambiguous; applying the method devised in Paper II we derive \( V_{ZAHB} = 14.12 \pm 0.04\). The TO-brightness is more difficult to determine, since (as already noted by Grundahl 1996) not many stars populate this region of the colour-magnitude diagram provided by Hesser et al. (1987). Hesser et al. (1987) give a value of \( V_{TO} = 17.70\), although the central V-value of the bluest brightness bin of their main line is at 17.65. Renzini & Fusi Pecci (1987) quote \( V_{TO} = 17.65\), while Buonanno et al. (1997), using the same raw data, determine it to be at 17.50. We have redetermined the TO from the published photometric data of Hesser et al. (1987). Fitting a parabola to the stars in the TO region, we obtain \( V_{TO} = 17.65 \pm 0.10\), in good agreement with the results by Renzini & Fusi Pecci (1987) and Hesser et al. (1987). We also used two alternative points along the main line for the absolute age determination, as recently suggested in the literature: both points are 0.05 mag redder than the (well-determined) colour of the TO, but either on the MS (Buonanno et al. 1997) or on the subgiant branch (Chaboyer et al. 1996). It turns out that the ages as determined from the visual brightness difference between ZAHB and these two points are in mutual agreement (respectively 8.8 and 9.0 Gyr) and agree well with the value of 9.2 ± 1.0 obtained from \( V_{TO} = 17.65 \pm 0.10\). This age is considerably lower than the values quoted at the beginning of this section, but comparable to that based on HIPPARCOS-parallaxes (Gratton et al. 1997).

In Fig. 1 we show the fit of representative isochrones for the mixture \((Y, Z) = (0.273, 0.008)\) (α-enhanced). The agreement with the observations is very good; the reddening we get is \( E(B-V) = 0.05\), consistent with the value quoted by Zinn (1980), which we have used for the He determination from \( \Delta V_{ZAHB}^{MS} \). The derived apparent distance modulus is \((m - M)_V = 13.50\).

For comparison, Fig. 2 shows the fit of representative isochrones for the mixture \((Y, Z) = (0.254, 0.008)\) (α-enhanced) corresponding to our standard case \( \Delta Y/\Delta Z = 3\). Note that the agreement between theoretical and observed RGB is slightly worse; from the vertical method one gets an age higher by 1.1 Gyr with respect to the age obtained adopting \( Y = 0.273\). We consider this age as a possible upper limit, since \( Y = 0.254\) is close to the lower limit of the possible range of the original He content as derived by us.

### 3.2. NGC6838 (M71)

M71 is a disk cluster about 3.5 kpc from the Sun. Its metallicity is, according to Carretta & Gratton (1997), \([\text{Fe/H}] = -0.70 \pm 0.10\). We use the photometric data and mean line from Hodder et al. (1992). Due to the sparsely populated HB, the application of the vertical method is not straightforward. We therefore determined the age of M71 by means of the horizontal method. If the mean line is registered to that of 47 Tuc in the same way as in Paper II (VandenBerg et al. 1990), this gives the same age as for 47 Tuc (as already noted by Richer et al. 1996, Alonso et al. 1997), 9.2 ± 1.2 Gyr (in the error we considered also the contribution due to the formal uncertainty in determining the relative position of the two RGB as discussed in Paper II). Since the horizontal method is very weakly affected...
Fig. 2. Isochrones of 10-11 Gyr for the composition $(Y, Z) = (0.254, 0.008) \, (\alpha\text{-enhanced})$ applied to 47 Tuc. The observational data are the same as in Fig. 1.

by differences in the helium content among the clusters (see previous discussion about 47 Tuc), the relative age between 47 Tuc and M71 that we get is therefore independent of the possible difference in helium abundance between these two clusters. Buonanno & Iannicola (1995) have determined the helium content to be $Y = 0.29 \pm 0.03$ from the R-parameter, consistent with the helium content of 47 Tuc.

In Fig. 3 we show our ZAHB and isochrone $(Z=0.008$ and $Y=0.273)$ for $t=9.2$ Gyr superimposed to the colour-magnitude diagram of M71 in such a way that the theoretical TO matches the position of the observed one. The predicted ZAHB-level from the isochrone agrees with the lower envelope of the sparsely populated HB. The vertical method – if applicable – therefore would result in a similar age as the relative age determination by the horizontal method. The reddening as obtained from the isochrone is $0.28$ and compares well with the value $0.27 \pm 0.03$ derived by Zinn (1980).

3.3. NGC6352

NGC6352 has been observed lately by Buonanno et al. (in preparation); TO- and ZAHB-brightness of its red HB $V_{\text{TO}}$ and $V_{\text{ZAHB}}$ are, however, already given in Buonanno et al. (1997); they are, respectively, $18.73 \pm 0.06$ and $15.20 \pm 0.06$. The value of the horizontal age parameter is also given, $\triangle(B-V) = 0.285 \pm 0.010$.

The iron content of NGC6352 given by Carretta & Gratton (1997) is $[\text{Fe/H}] = -0.64 \pm 0.10$, consistent within the error bar with the metallicity of 47 Tuc and M71. Using the same isochrones applied to 47 Tuc ($Z=0.008$ and $Y=0.273$) we obtain from the vertical method an age of $t=9.4 \pm 0.6$ Gyr. For verifying the validity of our assumption that the helium content of NGC6352 is the same as that of 47 Tuc, we can use the horizontal method for determining the NGC6352 age relative to 47 Tuc; we get an age difference by 0.4 Gyr, NGC6352 being older, completely consistent with the age from the vertical method.

3.4. Scaled-solar vs. $\alpha$-enhanced isochrones

Since Salaris et al. (1993) it has been known that at least for the lowest metallicities the effect of $\alpha$-element enhancement on the evolutionary tracks and isochrones can be simulated by using a scaled-solar mixture of total metallicity equal to the actual one, provided that $[\text{C+N+O+Ne}/\text{Mg+Si+S+Ca+Fe}] \approx 0$ in the $\alpha$-enhanced mixture (this condition is automatically fulfilled, adopting current determinations of the solar heavy elements mixture, if all $\alpha$-elements are enhanced by the same factor). Although this result was based on calculations without low-temperature opacity tables with $\alpha$-enhancement, it was later confirmed by us in Paper I, and it was again verified by Salaris & Cassisi (1996) employing low-temperature $\alpha$-enhanced molecular opacities. On the other hand, Weiss et al. (1994) found that for solar or super-solar total metallicity, the element abundance ratios within the metals do influence the
evolution. In these calculations, again only high-temperature ($kT > 1$ eV) $\alpha$-enhanced opacity tables were available. Obviously, there exists a value for the metallicity, above which $\alpha$-element enhancement – if present – has to be taken into account, not just because it raises the total metallicity, but also because the distribution of elements influences the evolution significantly. Because the disk clusters have a metallicity of $Z \approx Z_\odot/2$, this is of interest for the age determination. (Recall that for the results of this section, $\alpha$-enhancement, also in the opacities for all temperatures, is taken into account.)

In Fig. 4 (panel a) we compare isochrones (for $t=10$ Gyr) and ZAHB with a total metallicity $Z=0.0004$. Solid lines are for the case that all metals are in scaled-solar abundance ratios, the dotted ones for the case of $\alpha$-enhancement. Even at this low metallicity, the RGB colours differ slightly, a fact that seems to contradict the claim made by Salaris et al. (1993). However, one has to recall that their conclusions applied to the case when $[\text{C+N+O+Ne/Mg+Si+S+Ca+Fe}] \approx 0$, while in our heavy elements mixture (see Sect. 2.2) it is $\approx 0.12$. The relatively larger enhancement of oxygen with respect to the average enhancement of all other $\alpha$-elements explains the small colour differences along the RGB. The ZAHB and TO location of the $\alpha$-enhanced isochrone agree perfectly with the scaled-solar one, therefore the same ages would be derived at this total metallicity when using scaled-solar or $\alpha$-enhanced models. Also the relative ages obtained from the horizontal method are in close agreement, because the small difference in the absolute colour of the RGB does not affect the $\Delta(B - V)$ scaling with age. At $Z=0.002$ (panel b) small colour differences along the MS appear; TO and ZAHB luminosities are slightly lower for the scaled-solar isochrones, but the brightness difference between TO and ZAHB is virtually unchanged. The colour differences along the RGB are larger than in the previous case.

The case of $Z=0.01$ (corresponding to $[\text{Fe/H}]=-0.6$ for the $\alpha$-enhanced isochrones), a metallicity close to the one of the disk GC considered in our work, is shown in panel c of the same figure. At this metallicity the two sets of isochrones appear to be very different: the scaled-solar ZAHB is dimmer than the corresponding $\alpha$-enhanced one and the scaled-solar isochrone is redder and its TO brightness lower. The latter behaviour is in agreement with the result of Weiss et al. (1994) obtained from a comparison of evolutionary tracks. At this metallicity, the use of scaled-solar isochrones for deriving the age of GC using the vertical method leads to ages higher by $\approx 1.0$ Gyr with respect to ages determined by means of the appropriate $\alpha$-enhanced isochrones.

An further point has to be mentioned before concluding this section: when using the MS fitting technique for deriving the distance of a GC, one has to correct the observed local subdwarf colours to the colours they would have at the metallicity of the cluster. To do so one needs the derivative $\Delta(B - V)/\Delta Z$ (at fixed $V$ magnitudes), which is taken from theoretical isochrones. At metallicities $Z \geq 0.002$, the use of scaled-solar isochrones can introduce a systematic error in the derived distances. Considering for example $M_V=6.0$, the $(B - V)$ colours at $Z=0.01$ and $Z=0.002$ differ by 0.104 mag, while this difference is equal to 0.116 mag in the case of scaled-solar isochrones. Since the slope of the MS $\Delta M_V/\Delta(B - V)$ is $\approx 5$ at this magnitude, the use of scaled-solar isochrones in place of $\alpha$-enhanced ones would give a distance modulus too high by $\approx 0.06$ mag, if the subdwarfs have the lower, but the cluster stars the higher metallicity.

4. An update of the halo cluster sample

4.1. Influence of a new metallicity scale

Carretta & Gratton (1997) recently have redetermined the metallicity of a number of globular clusters and on this basis recalibrated the Zinn & West (1984) metallicity scale. As a consequence, the metallicities of the 25 clusters investigated in Paper II increased by on average $\approx 0.20$ dex. This effect has not been taken into account in Paper II, but it was estimated that ages should be affected by less than the typical errors in our ages ($\approx \pm 1.1$ Gyr as $1 - 2 \sigma$ error). For completeness, however, we repeated the analysis of Paper II for the present paper. The new results

1 The line types for these tracks have been exchanged erroneously in the caption of their Fig. 4, as is obvious from comparing the tracks with those in their Fig. 3.
Table 2. Age determinations for a sample of 28 halo clusters and the three disk clusters of the present paper. Column 2 contains the iron abundance as given in Carretta & Gratton (1997), while column 3 gives the total metallicity (including α-element enhancement). Next, the age in Gyr as obtained from our analysis follows in column 4. Distance modulus and reddening are listed in the remaining columns. Note that only for the reference clusters these are obtained directly. For all other clusters the redenning are determined indirectly (see text). In this case, the numbers are in brackets.

| Cluster      | [Fe/H] | Z     | Age  | (m − M)_V | E(B − V) |
|--------------|--------|-------|------|------------|----------|
| **Halo Clusters** |        |       |      |            |          |
| NGC4590 (M68) | -1.99  | 4.2 · 10^{-4} | 11.4±1.0 | 15.26  | 0.05     |
| NGC7078 (M15) | -2.12  | 3.0 · 10^{-4} | 11.4±1.1  | (0.11) |          |
| NGC6341 (M92) | -2.16  | 2.9 · 10^{-4} | 11.0±1.1  | (0.04) |          |
| NGC7099 (M30) | -1.91  | 5.4 · 10^{-4} | 11.9±1.1  | (0.04) |          |
| NGC6397      | -1.82  | 6.2 · 10^{-4} | 11.4±1.1  | (0.18) |          |
| NGC2298      | -1.74  | 7.4 · 10^{-4} | 11.7±1.1  | (0.22) |          |
| Arp2         | -1.76  | 7.1 · 10^{-4} | 9.7±1.1   | (0.13) |          |
| Rup106       | -1.78  | 6.8 · 10^{-4} | 9.1±1.1   | (0.22) |          |
| NGC6584      | -1.30  | 2.0 · 10^{-3} | 10.1±1.0  | 16.05  | 0.10     |
| NGC5272 (M3) | -1.34  | 1.9 · 10^{-3} | 10.1±1.1  | (0.01) |          |
| NGC1904 (M79) | -1.37  | 1.7 · 10^{-3} | 10.1±1.1  | (0.03) |          |
| NGC6752      | -1.43  | 1.5 · 10^{-3} | 9.6±1.1   | (0.03) |          |
| NGC6254 (M10) | -1.41  | 1.8 · 10^{-3} | 10.1±1.1  | (0.28) |          |
| NGC7492      | -1.61  | 1.0 · 10^{-3} | 10.1±1.1  | (0.03) |          |
| NGC6101      | -1.60  | 1.0 · 10^{-3} | 10.9±1.1  | (0.09) |          |
| NGC5897      | -1.59  | 1.0 · 10^{-3} | 10.1±1.1  | (0.10) |          |
| NGC5904 (M5) | -1.11  | 3.2 · 10^{-3} | 9.9±0.7   | 14.53  | 0.02     |
| NGC3201      | -1.23  | 2.4 · 10^{-3} | 9.9±0.8   | (0.24) |          |
| Pal5         | -1.24  | 2.4 · 10^{-3} | 8.3±0.8   | (0.05) |          |
| NGC288       | -1.07  | 3.6 · 10^{-3} | 8.8±0.8   | (0.02) |          |
| NGC362       | -1.15  | 3.1 · 10^{-3} | 8.7±0.8   | (0.04) |          |
| NGC1851      | -1.14  | 3.0 · 10^{-3} | 7.9±0.8   | (0.08) |          |
| Pal12        | -1.00  | 4.1 · 10^{-3} | 6.6±0.8   | (0.02) |          |
| NGC1261      | -1.09  | 3.3 · 10^{-3} | 10.9±0.8  | (0.00) |          |
| NGC1671 (M107) | -0.87  | 5.5 · 10^{-3} | 10.4±1.0  | 15.04  | 0.36     |
| NGC6652      | -0.81  | 6.3 · 10^{-3} | 8.0±1.1   | 15.26  | 0.22     |
| Ter7         | -0.87  | 5.5 · 10^{-3} | 7.1±1.1   | (0.07) |          |
| NGC6366      | -0.87  | 5.5 · 10^{-3} | 12.2±1.1  | (0.72) |          |
| **Disk Clusters** |        |       |      |            |          |
| NGC104 (47 Tuc) | -0.70  | 8.1 · 10^{-3} | 9.2±1.0   | 13.50  | 0.05     |
| NGC6838 (M71) | -0.70  | 8.1 · 10^{-3} | 9.2±1.1   | (0.28) |          |
| NGC6352      | -0.64  | 9.3 · 10^{-3} | 9.4±0.6   | 14.56  | —        |

Scaled-solar metallicity clusters:

| Rup106   | -1.78  | 3.0 · 10^{-4} | 9.9±1.1   | (0.24) |          |
| Pal12    | -1.00  | 1.8 · 10^{-3} | 7.7±0.8   | (0.05) |          |

In addition to the 25 clusters of Paper II, we have added three other halo clusters: NGC5897 (Sarajedini 1992) and NGC6101 (Sarajedini & Da Costa 1991), both intermediate metal-poor clusters, and NGC1261 (Bolte & Marleau 1989), an intermediate metal-rich cluster. Note that the cluster NGC3201 moved one metallicity group upwards. Interestingly, since this cluster allows the application of the vertical method, we used it for checking the consistency of ages in the intermediate metal-poor group in Paper II, while we can do the same test now for the intermediate metal-rich group. In both cases, the relative...
age of NGC3201 with respect to the template cluster in the same metallicity group as derived from the horizontal method is consistent with its absolute age as derived from the vertical method. On average, absolute cluster ages are, compared to Paper I, reduced by ≈ 0.8 Gyr due to the increased metallicity.

In Table 3, last two columns, we have listed direct determinations of distance modulus and reddening for the reference clusters (see Sect. 2.7), and indicative reddenings for all others. The latter have been estimated by means of the following procedure: once the age has been determined by the horizontal method, the absolute colour of the TO is known from the theoretical isochrones. Comparison with the observed TO then gives the reddening (the errors associated with this procedure, due to the error on the cluster ages, are of the order of ±0.02 mag). From the theoretical TO one could also obtain a distance by comparing it with the observed apparent TO-magnitude. Since this, however, is very uncertain in some cases (see the discussion about M15 in Paper I), we did not list them. The reader may derive estimates for the distances by employing our Eq. 1 (see below).

There has been some concern that a few clusters – Rup106 and Pal12 in our sample – do not show α-element enrichment (Brown et al. 1997). Although these are preliminary results based on only two stars in each cluster, and still have to be confirmed, we have investigated by how much the ages would be changed, mainly due to the reduced total metallicity. In Table 3 we have therefore added ages for both clusters based on this assumption. To obtain them, we determined the absolute age of the reference cluster in their group (namely, M68 and M5) from the vertical method by means of appropriately scaled-solar isochrones (thus assuming for the reference clusters [α/Fe]=0, too) and then we derived the differential age of Rup106 and Pal12 with respect to M68 and M5 by means of the horizontal method (using the same set of scaled-solar isochrones). The net effect amounts to an age increase by, respectively, 0.8 Gyr and 1.1 Gyr. We note that the scaled-solar isochrone used for Pal12 fits better to the observed RGB than the one including α-element enhancement. This might be taken as independent, theoretical support for the claim that this cluster does not show enriched α-elements.

4.2. Predicted vs. HIPPARCOS-based distances

As emphasized already in Papers I and II, our method to determine cluster ages is independent of any distance determination. Rather, the vertical method, which we use for the absolute age of the reference clusters, together with the theoretical models, allows to predict a cluster distance. This predicted value can then be compared with other distance determinations to check for the reliability of our models.

Table 3. Predicted and HIPPARCOS distances for reference clusters (Table 2), given as the apparent distance modulus \((m - M)_V\). See text for more explanations

| Name   | this paper | Gratton et al. 1997 | other ref. |
|--------|------------|---------------------|------------|
| M68    | 15.26 ± 0.05 | 15.31 ± 0.08        | 15.35 ± 0.10^1 |
| M5     | 14.53 ± 0.05 | 14.60 ± 0.07        | 14.54 ± 0.10^1 |
| 47 Tuc | 13.50 ± 0.05 | 13.62 ± 0.08        | 13.68 ± 0.15^2 |

^1 Reid 1997; ^2 Chaboyer et al. 1997; ^3 Reid 1998

In Paper I, we have shown that we are in agreement with some of the relations relating absolute RR Lyrae brightness with metallicity. In Paper II, this point was further emphasized, but since the very first HIPPARCOS-parallaxes had become available, we also compared with results relevant for cluster distances. Again, we found that we were consistent with these results, though some of them were in conflict with each other.

Since then, a number of papers have appeared, which are devoted to the question of GC distances obtained from the fitting of their MS to subdwarfs, whose distances can be determined by HIPPARCOS-parallaxes (Reid 1997, 1998; Gratton et al. 1997; Chaboyer et al. 1997; Pont et al. 1998). In these papers, different samples of subdwarfs (sometimes including also objects with only ground-based parallaxes), different bias corrections, different metallicities for the same subdwarfs or globular clusters have been used, and the results do not agree in all cases (e.g. the case of M92: Reid 1997, Pont et al. 1998 and Gratton et al. 1997).

For our reference clusters (see Table 3) we found HIPPARCOS-parallaxes based distance determinations (in the following for simplicity called “HIPPARCOS-distances”) for M68 (Reid 1997; Gratton et al. 1997), M5 (Reid 1997; Gratton et al. 1997; Chaboyer et al. 1997), and 47 Tuc (Gratton et al. 1997, Reid 1998). Moreover, we can also compare our theoretical \(M_V\) (ZAHB)-[Fe/H] relation (taken at the average temperature of the RR Lyrae instability strip, \(log T_{\text{eff}}\approx3.85\)) with the observational one derived by Gratton et al. (1997) for their sample of clusters by means of MS fitting.

In Table 4 we show the possible comparisons. The error bars for our ZAHB distances take into account the error on the observational ZAHB location (as given in Paper I) and the small contribution due to an uncertainty by ±0.10 dex in the cluster metallicity. From Gratton et al. (1997) we adopted the binary corrected distance moduli reported in column 8 of their Table 3, while from Reid (1997) we have taken the distance moduli reported in column 7 of his Table 3 (derived by selecting the subsample of subdwarfs with the most accurate parallax) for M5. In the case of M68, due to the paucity of metal poor subdwarfs considered by Reid (1997), we report his result given in
column 9 of the same table (all subdwarfs shifted to a monometallicity sequence corresponding to the metallicity of M68). Since Reid (1997) gives results for different reddening estimates, we could select distances for those reddenings equal (or as close as possible) to the one obtained from our isochrones.

For M68 and M5 our distances are in very good agreement with Gratton et al. (1997), Reid (1997) and Chaboyer et al. (1997). The agreement in the case of M5 (that is the cluster with the best available photometry) is impressive. One should note that the reddening and cluster metallicity in these papers differ slightly from ours in some cases. Both affect the distance determination by MS fitting, while our distance modulus is independent of the reddening, as long as the uncertainty is of the order of some hundredths of magnitude. However, the differences in reddening are always ≤ 0.01 mag, and those in metallicity < 0.10 dex, such that the results of the comparison are not modified appreciably.

A slightly different result is found in the case of 47 Tuc, for which one notes less consistency between our distance modulus and that by Gratton et al. (1997) and Reid (1998), although the distances are still compatible within the respective errors bars. Part of the difference can be explained by the fact that in the quoted papers solar-scaled isochrones were used (see also the discussion at the end of Sect. 3.4).

Gratton et al. (1997) also provide the following empirical relation between ZAHB brightness at the RR Lyrae instability strip and metallicity, based on the distances they obtained by means of the MS fitting technique applied to their sample of 9 clusters,

\[
M_V(ZAHB) = (0.22±0.09)·(\text{[Fe/H]}-1.5)+(0.49±0.04) \quad (1)
\]

From our theoretical models (considering the case with \(\Delta Y/\Delta Z = 3\)) we derive, at \(T_{\text{eff}}=3.85\):

\[
M_V(ZAHB) = 0.17 \cdot (\text{[Fe/H]} - 1.5) + 0.52 \quad (2)
\]

in very good agreement with the empirical relation (Eq. 1).

We can therefore conclude (as we did in Paper II) that our models predict distances completely confirmed by independent, HIPPARCOS-based methods. For completeness we add that our distance moduli as compared to Paper II changed due to two reasons: firstly, we use higher metallicities, which lower the distance modulus; secondly, the new calibration of the bolometric correction scale increases it again. The net result is only a very small change.

Before closing this section we briefly want to show the result of yet another comparison between HIPPARCOS-based results and our theoretical models. In this case we compare the \((B-V)\) colours of our isochrones at \(M_V=6.0\) with the observed subdwarfs colour at the same absolute magnitude, as derived by Gratton et al. (1997), for a sample of 13 single and unevolved MS stars. The result of this comparison is shown in Fig. 5, where empirical data (symbols) are compared with the corresponding quantity from our theoretical isochrones (solid line). The agreement between the colours of our isochrones and the subdwarfs is very good at all metallicities.

5. Discussion

The original motivation for Paper I had been to resolve the apparent “age discrepancy” between globular clusters and the expanding universe. As we have demonstrated in all our papers, the oldest globular clusters appear to be younger than 12 Gyr, possibly only as old as 11 Gyr. Note that the neglect of diffusion even leads to a slight over-estimation of the ages by approximately 0.8 Gyr (Cassisi et al. 1998) for the vertical method (but the relative ages derived by means of the horizontal method are unchanged). Such a cluster age is completely consistent with the general range of recent \(H_0\)-determinations (see Freedman 1997 for a review), which extends from below 50 to 80 km/s/Mpc. Salaris & Cassisi (1998) used theoretical stellar models calculated with the same stellar evolution program to predict the brightness of the tip of the RGB (their tip bolometric luminosities are in complete agreement with our models), which can be used as a primary distance indicator. Applying this to the galaxy NGC3379 in the Leo I group, the distance to the Coma cluster (obtained from the relative distance Coma-Leo I as derived from different secondary distance indicators) then gives...
Fig. 6. The relation between age and metallicity for all clusters dated in Papers I, II and the present one (Table 2). Filled circles mark generic halo clusters, open circles clusters probably associated with satellite galaxies or affected by tidal interaction with the Magellanic Clouds. Crosses are the three disk clusters. For Rup106 and Pal12 ages obtained under the assumptions of $\alpha$-element enhanced or scaled-solar metals are connected by thin lines.

a Hubble constant $H_0 = 60 \pm 11$ km/s/Mpc. Depending on the cosmological model, the age of the universe is marginally or completely consistent with our oldest clusters. Since an open universe with $\Omega = 0.3$ and $\Lambda = 0$ appears to be preferred (the probably most important single evidence coming from the statistics of giant arcs produced by cluster of galaxies; see Bartelmann et al. 1998), $H_0 = 60$ implies an age of $\approx 13$ Gyr. Therefore distance indicator and cluster age result in a completely consistent picture.

The second purpose of our age determinations is to establish a reliable age-metallicity relation for galactic globular clusters. From Fig. 6 the general scenario of Paper II is confirmed and extended: it appears that the more metal-poor clusters formed between 11 and 12 Gyr ago within a timespan of less than 1 Gyr. After that halo clusters continued to be formed at higher metallicity for about 4 Gyr, with a tendency for more metal-rich clusters to form later (with the exception of NGC6366, but see the discussion in Paper II about this cluster). The halo cluster creation continued even after the disk had already been formed, as it is evident from the ages of the oldest disk white dwarfs (Salaris et al. 1997; Leggett et al. 1997), which are around 9 Gyr old. In the present paper, we found that the three disk clusters investigated are coeval with these disk white dwarfs. This is an important result, because previous age estimates (e.g. Chaboyer & Kim) placed disk clusters at the same age as our oldest halo clusters and let them appear to be considerably older than the disk white dwarfs.

In addition, for all metallicities there are clusters (e.g. Rup106 & Arp2) that appear to have been created at a later time, probably due to tidal interactions with dwarf spheroidal galaxies (namely the Sagittarius dSph) or the Magellanic Clouds. Such events are consistent with the finding that cluster formation in the LMC apparently has been triggered by close encounters with the Galaxy (Girardi et al. 1995; Fujimoto & Kumai 1997).

With respect to the age-galactocentric distance relation (Fig. 7), our conclusion of Paper II is supported: within the innermost 10 kpc an age spread of only $\approx 2.5$ Gyr exists, if one neglects the exceptional clusters NGC6652 and NGC6366; the mean being at $\approx 11$ Gyr. The three disk clusters fit into this range at the lower boundary. At larger distances, the age differences approach $\approx 5$ Gyr, with the mean age being somewhat smaller. We agree with the conclusion of Sarajedini et al. (1997) in that there appears to be a tendency that the innermost parts of the galactic halo formed within a timespan shorter than that for the outermost regions. However, we find the age spreads to differ only by a factor of two, and given the small number of clusters, it is not clear whether this is significant enough. In particular, the larger spread for the outer halo clusters again depends on a few clusters. One of these is Pal12, for which there are indications (Brown et al. 1997) that it is not $\alpha$-enhanced. If true, its age would be raised by $\approx 1.0$ Gyr (Sect. 4.1), making both the in-
termediate metal-rich group and the outer halo clusters more homogeneous in age. Another cluster with uncertain α-enhancement is Rup106.

The disk clusters are different from the halo clusters with respect to their composition. We have found strong evidence that they have a solar-like helium content, while there is no indication of this in the halo clusters of comparable metallicity. Recalling that their metallicity is only half the solar one, this implies a different chemical enrichment history for them than for the solar neighbourhood.

We also showed that α-element enhancement has to be taken into account properly in all aspects of the theoretical calculations for total metallicities $Z > Z_\odot/10$, in order to obtain reliable isochrones. If the α-elements are enhanced in such a way that the condition $[C+N+O+Ne/Mg+Si+S+Ca+Fe]/Z > 0$ is violated, a fact that is compatible with current available observations, and the isochrone colours have to be determined with high accuracy for all evolutionary stages, then the computation of α-enhanced isochrones is necessary even for the lowest metallicities.

Finally, distances predicted from our age determinations and ZAHB models are in very good agreement with HIPPARCOS-based data, demonstrating once more the reliability of our results.

Acknowledgements. We acknowledge stimulating and illuminating discussions with R. Peterson and R.D. Cannon about observational details in general and 47 Tuc in particular. Jørgen Christensen-Dalsgaard helped to recover Grundahl’s thesis work. L. Pulone generously made available a copy of his paper prior to publication. The work of one of us (M.S.) was carried out as part of the TMR programme (Marie Curie Research Training Grants) financed by the EC.

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