Resonant self-pulsations in coupled nonlinear microcavities

Victor Grigoriev and Fabio Biancalana
Max Planck Institute for the Science of Light, Günther-Scharowsky-Str. 1, Bau 26, Erlangen 91058, Germany
(Dated: January 13, 2013)

A novel point of view on the phenomenon of self-pulsations is presented, which shows that they are a balanced state formed by two counteracting processes: beating of modes and bistable switching. A structure based on two coupled nonlinear microcavities provides a generic example of system with enhanced ability to this phenomenon. The specific design of such structure in the form of multilayered media is proposed, and the coupled mode theory is applied to describe its dynamical properties. It is emphasized that the frequency of self-pulsations is related to the frequency splitting between resonant modes and can be adjusted over a broad range.

PACS numbers: 42.65.Pc, 47.20.Ky, 05.45.Xt, 68.65.Ac

I. INTRODUCTION

Optical bistability is a general nonlinear phenomenon, which can be observed in a large variety of systems provided that nonlinearity is complemented by some form of feedback. Apart from the classical Fabry-Perot resonators, the possible configurations of bistable systems include multilayered structures [1, 2], fiber Bragg gratings [3, 4] and microcavities in photonic crystal waveguides [5, 6].

It was noticed however that the description of these systems in the frequency domain does not capture the full complexity of their temporal behavior. After overcoming the first switching threshold, the transient response of these systems may change dramatically, leading to self-pulsations or even chaos [7-9]. These instabilities in the time domain are usually explained from the viewpoint of nonlinear dynamics as a manifestation of the Hopf bifurcations [10-14]. Although such description is appropriate, it is often desirable to have a control over these instabilities, as they might be useful for certain applications. In this work, we present a novel point of view on the phenomenon of self-pulsations which shows that they are a balanced state formed by two counteracting processes: beating of modes and bistable switching. We apply this concept to design a structure where self-pulsations can be excited very efficiently.

The paper is organized as follows. In Section II we consider a typical example of the structure that supports self-pulsations. It is represented by the Bragg grating, and we explain how the beating of modes and bistable switching can interplay in this particular case. In Section III we propose a design of multilayered structure, which is based on two coupled nonlinear microcavities and shows pronounced self-pulsations. The structure has several independent parameters that make the adjustment of its properties easier. In Section IV we derive a set of coupled mode equations which describes the switching dynamics in such system and discuss how the qualitative behavior of the structure agrees with the results obtained by the linear stability analysis. As a possible application, we show that self-pulsations can be used to convert a continuous wave signal into a regular train of ultrashort pulses. The last section summarizes the results and presents the conclusions.

II. BEATING OF MODES

As was mentioned in Introduction, self-pulsations tend to appear after the first switching threshold on the hysteresis curve. However, each loop of the hysteresis curve is related to a resonance in the linear transmission spectrum. For example, a Bragg structure obtained by periodic alternation of two different layers is considered on Fig. 1. It has many resonances near the band edge [Fig. 1(a)] which can be bent into the band gap region by the Kerr nonlinearity producing a multistable hysteresis curve [Fig. 1(b)]. Each resonance creates its own loop when input intensity is gradually increased and decreased. The difference between the first and higher switching thresholds is clearly pronounced in the field profiles of corresponding resonances. It turns out that the first resonance (counting from the upper band edge) has only a single localization center in the field profile, the second has two such centers [Fig. 1(c)], and this number continues to increase by one for the next resonances [15-17]. Therefore, even the Bragg structure without any defects can be considered as a set of coupled microcavities.

It is known however that the beating of modes can exist in coupled systems. Although it is a purely linear effect, it can so strongly redistribute the energy between constituent parts of the system that they will meet conditions for up- and down-switching periodically. The need to overcome the first switching threshold in the case of Bragg structures is thus related to the fact that they can be approximated as a single-mode microcavity near the first resonance, and the beating of modes is not possible in such circumstances. When the input intensity reaches the switching threshold, the higher order resonances shifted by the Kerr nonlinearity will dominate, and the beating of modes can be excited. In principle,
self-pulsations can be observed even without overcoming any threshold, if one chooses the carrier frequency to be initially near a higher order resonance.

There are two requirements for the onset of beating. Firstly, the system should support normal modes with slightly different frequencies. Most often, they can be built as symmetric and antisymmetric combinations of a degenerate mode. The difference of their frequencies determines the period of the beating. Secondly, the system should be excited in a way that creates an initial imbalance of energy between its parts, for example, by rapidly varying input intensity. The latter requirement is particularly important for open systems, because beating and self-pulsations are closely related processes. Each of them can excite the other, and at the same time it needs to be sustained by the other. The interplay between these processes can take a periodic form, resulting in an infinite series of switchings, or self-pulsations.

III. COUPLED MICROCAVITIES

The analysis performed in the previous section leads to conclusion that the simplest system with the ability to self-pulsations should consist of just two microcavities, and each of them can be strictly bistable. We consider a possible design of such system in the form of multilayered structure. It is sufficient to have dielectric layers of two types, which will be denoted as 'L' and 'H'. They correspond to materials with different refractive indices, and it will be assumed that the optical thickness of these layers satisfies the quarter wave condition. In what follows, the specific materials used for the layers 'L' and 'H' are polydiacetylene 9-BCMU with linear (nonlinear) refractive index $n_L = 1.55$ ($n_{2L} = 2.5 \cdot 10^{-5}$ cm$^2$/MW) and rutile with $n_H = 2.3$ ($n_{2H} = 10^{-8}$ cm$^2$/MW), respectively [18, 19]. The quarter wave condition is set to $\lambda_0 = 0.7$ μm, which gives the thicknesses of the layers $d_L = 112$ nm and $d_H = 76$ nm.

The microcavities can be formed by breaking periodicity in the arrangement of layers. We propose the following symbolic formula for the multilayered structure:

$$\text{(HL)}^p\text{(LH)}^m\text{L(HL)}^m\text{(LH)}^{m+p}$$

where the parameters $m$ and $p$ are integer numbers which satisfy the conditions $m \geq 1$ and $p \geq 0$. It is important to consider their meaning in more detail and to emphasize how they are related to various functional parts of the structure [Fig. 2(a)].

The two major parts are represented by the combination $(\text{HL})^m(\text{LH})^m$, which can be viewed as a result of imperfect junction of two periodic sequences. This causes the formation of a point-like defect in the middle, and creates a strong resonance in the transmission spectrum. This resonance is located exactly at the frequency for which the quarter wave condition holds, and the mirror symmetry of the structure ensures that it demonstrates perfect transmission [Fig. 2(b)]. The quality factor $Q$ of this resonance can be adjusted over a broad range by varying the number $m$. When the refractive indices of the layers $n_L$ and $n_H$ have a large contrast, the following formula can be used to estimate the quality factor

$$Q = \frac{\omega_0}{2\gamma} = \frac{\pi n_H n_L}{4(n_H - n_L)} \left(\frac{n_H}{n_L}\right)^{2m}. \quad (2)$$

The purpose of the single layer 'L' placed in the middle of the structure is twofold. Firstly, it smoothly connects
the two microcavities without introducing an additional
defect into the structure. Secondly, it changes the phase
mismatch between microcavities. If it were absent, the
resonance caused by the point defect is
in the middle of the band gap region, where the two spectra
overlap. (c) Transmission spectrum of two coupled microcavities
for $m = 3$ and $p = 0$. Due to the coupling, the degenerate resonance
splits into a doublet of even and odd modes.

IV. SWITCHING AND SELF-PULSING

To describe the switching dynamics in a single micro-
cavity, we apply the coupled mode theory (CMT), which
has been recognized as a useful tool for studying the prop-
erties of linear and nonlinear defects in photonic crystal
waveguides [20,22]. Assuming that the microcavity is
mirror symmetric, the CMT equations [see Fig. 3(a) for
notation] can be written as

$$\frac{dA}{dt} = -\left[ i \left( \omega_0 - \gamma \frac{|A|^2}{I_0} \right) + \gamma \right] A + \gamma (u_+ + v_+),$$

where $A(t)$ is the amplitude of the mode $E_0(x)$ with reso-
nant frequency $\omega_0$ and damping constant $\gamma$, $u_\pm (v_\pm)$ are the amplitudes of in- and out-going plane waves on the
left (right) side of the cavity.

Although the materials are assumed to be lossless,
damping takes place due to the leakage of energy out
of the structure and the decay rate can be estimated as
$\gamma = \left( (1/c) \int_0^L \epsilon(x) dx \right)^{-1}$. The integration in this for-
ma is performed over the full length of the microcavity
(spanning from 0 to $L$), $c$ is the speed of light in the vac-
uum, and $\epsilon(x)$ is the linear permittivity ($\epsilon = n^2$). Since
the electric field inside the microcavity is a product of the
mode amplitude $A(t)$ and the mode profile $E_0(x)$, it is
convenient to consider the mode profile as a dimen-
sionless function which satisfies the boundary conditions
$E_0(0) = E_0(L) = 1$. This ensures that the expression
for the damping constant has a proper dimensionality.

The nonlinearity is responsible for the shift of reso-
nant frequencies and can be taken into account by in-
troducing the characteristic intensity for the microcavity
$I_0 = [((\omega_0/c) \int_0^L n_2 \epsilon |E_0|^2 dx) \gamma I_0]^{-1}$. This formula follows from
the orthogonal projection of the perturbation term which
contains the nonlinear index of refraction $n_2(x)$ on the
microcavity mode.

For the structure described by $m = 3$, the damping
constant and characteristic intensity take the following values
$\gamma^{-1} = 27.6$ fs, $I_0 = 154$ MW/cm$^2$. The question
how accurate the CMT model is can be answered by com-
paring the linear transmission spectrum computed with
a rigorous transfer matrix method and with the following
formula derived from Eqs. (3)–(4)

$$T = \frac{I_{\text{out}}}{I_{\text{in}}} = \left[ 1 + \left( \frac{\omega - \omega_0}{\gamma} + \frac{I_{\text{out}}}{I_0} \right)^2 \right]^{-1},$$

where $I_{\text{in}} = |u_+|^2$ ($I_{\text{out}} = |v_+|^2$) is the intensity of
incident (transmitted) wave. Despite of the fact that the
quality factor of the considered resonance is rather low
($Q = 37.2$), the two spectra overlap in a wide range of
frequencies around $\omega_0$ [Fig. 2(b)].

For a fixed value of frequency detuning, Eq. (5) de-
scribes the hysteresis curve of the single microcavity
[Fig. 3(b)]. It is strictly bistable and the switching re-
region can be determined from the linear stability analy-
sis. The points on hysteresis curve, which have at least
one eigenfrequency with positive imaginary part, cannot be stable, because small perturbations will exponentially grow forcing switching to another branch of hysteresis. It is worth noting that the eigenfrequencies of the linearized system $\omega_p$ should satisfy a quadratic equation, $(\omega_p^{2}+\gamma q+\alpha) q = 0$, where $q$ is a real coefficient. Therefore, the transient response after the switching should have the form of exponentially decaying oscillations $\exp(-i\omega_p t)$, where $\omega_p/\gamma = \pm \sqrt{|q|} - i$.

A qualitatively different behavior is possible in presence of two coupled microcavities. The corresponding CMT equations can be written as

$$\frac{dA_1}{dt} = -\left[i\left(\omega_0 - \frac{\gamma}{2}\right) + \frac{\gamma}{2\kappa^2}\right] A_1 + \frac{n_L \gamma}{2} A_2 + \frac{\gamma}{\kappa} u_1^+, \quad (6)$$

$$\frac{dA_2}{dt} = -\left[i\left(\omega_0 - \frac{\gamma}{2}\right) + \frac{\gamma}{2\kappa^2}\right] A_2 + \frac{n_L \gamma}{2} A_1 + \frac{\gamma}{\kappa} u_2^+, \quad (7)$$

where $n_L$ is the refractive index of the phase-matching layer ‘L’, and the parameter $\kappa = (\alpha + \gamma/\kappa)^2$ takes into account the influence of the Bragg mirrors. The linear normal modes of the system (6)–(7) are given by combinations $A_1 \pm A_2$, and their resonant frequencies are $\omega = \omega_0 \mp n_L \gamma/2$. It is important that the frequency splitting depends only on the number $m$ and can be estimated using Eq. (2), while the number $p$ determines the external quality factor $Q_{\text{ext}} = \kappa^2 Q$ and can lower the effective characteristic intensity $I_{\text{eff}} = I_0/\kappa^4$. The latter property can be useful if materials have a weak Kerr non-linearity, but for simplicity it will be assumed that $p = 0$ and consequently $\kappa = 1$.

The hysteresis curve of this system shows that there is a range of input intensities where only unstable solutions are possible [vertical gray region in Fig. 3(c)]. This range falls exactly between the resonances of the doublet, and the transient dynamics in this region is different from the switching [see Fig. 4 and caption for details]. In the time domain simulation, the input intensity of a continuous wave signal was slowly varied to examine stability of several key points on the hysteresis curve. After overcoming the second threshold the system goes into a state of infinite switching. The period of these pulses extracted from the simulation data was $252 \text{ fs}$, which is of the same order of magnitude as the beating period of the two resonant modes $4\pi/(n_L \gamma) = 224 \text{ fs}$. Therefore, the frequency of self-pulsations $\omega_{\text{sp}} = 4 \text{ THz}$ can be estimated by the following formula

$$\omega_{\text{sp}} = (n_L \omega_0)/(4Q). \quad (8)$$

It shows explicitly that smaller internal quality factors can produce train of pulses with higher repetition rate,
since photons tend to spend less time in each cavity and they exchange energy at a faster rate. This property can be useful in optical communications, where such pulses can transfer bits of information [23]. Another possible application is optical clocks for photonic circuits, where train of pulses can synchronize the operation of different components [24].

There are two reasons why the frequency of beating and self-pulsations do not coincide precisely. Firstly, the structure considered in this paper belongs to the open systems, and thus the resonant frequencies are not real quantities. Depending on the value of the quality factor, this can limit accuracy to about 1%. Secondly, the Kerr nonlinearity changes the refractive indices of the layers and consequently shifts all resonant frequencies. This can increase the discrepancy even further, but since the period of the self-pulsations is determined by the frequency splitting of the doublet rather than its absolute position, the accuracy of about 10% can be considered as acceptable to prove the concept.

The quantitative agreement can be improved if self-pulsations are not very intense. As an example we can apply our interpretation of self-pulsations to a paper of other authors [5], where this phenomenon was studied for microcavities embedded in photonic crystal waveguides. We derived an explicit formula for the frequency of beating in their case and found that it differs from the expected state formed by switching and beating of modes in coupled nonlinear microcavities. We proposed the description of multilayered structure where self-pulsations can be excited very efficiently. Since the phenomenological CMT equations were applied to describe the switching dynamics, the specific choice of configuration is not critical, and we expect qualitatively similar results in other configurations and physical systems.

It is also possible to observe self-pulsations in systems with continuous spectrum such as waveguide couplers or dual-core fibers [25–29]. The coupling between waveguides leads to the beating of modes in space, and the overall dynamics can be described by equations which are similar to Eqs. (6)–(7). However, it is not easy to excite selectively odd or even modes in that case [27], and therefore the usage of coupled microcavities offers some advantage.

V. CONCLUSIONS

In conclusion, self-pulsations can be viewed as a balanced state formed by switching and beating of modes in coupled nonlinear microcavities. We proposed the design of multilayered structure where self-pulsations can be excited very efficiently. Since the phenomenological CMT equations were applied to describe the switching dynamics, the specific choice of configuration is not critical, and we expect qualitatively similar results in other configurations and physical systems.

This work was supported by the German Max Planck Society for the Advancement of Science (MPG).

[1] V. Grigoriev and F. Biancalana, New J. Phys. 12, 053041 (2010).
[2] E. Lidorikis, M. M. Sigalas, E. N. Economou, and C. M. Soukoulis, Phys. Rev. B 61, 13458 (2000).
[3] A. Parini, G. Bellanca, S. Trillo, M. Conforti, A. Locatelli, and C. De Angelis, J. Opt. Soc. Am. B 24, 2229 (2007).
[4] C. Martijn de Sterke and J. E. Sipe, Phys. Rev. A 42, 2858 (1990).
[5] B. Maes, M. Fiers, and P. Bienstman, Phys. Rev. A 80, 033805 (2009).
[6] M. Soljacic, M. Ibanescu, S. G. Johnson, Y. Fink, and J. D. Joannopoulos, Phys. Rev. E 66, 055601 (2002).
[7] K. Ikeda and O. Akimoto, Phys. Rev. Lett. 48, 617 (1982).
[8] K. Ikeda, K. Kondo, and O. Akimoto, Phys. Rev. Lett. 49, 1467 (1982).
[9] H. G. Winful and G. D. Cooperman, Appl. Phys. Lett. 40, 298 (1982).
[10] P. V. Paulau and N. A. Loiko, Phys. Rev. A 72, 013819 (2005).
[11] H. Hashemi, A. W. Rodriguez, J. D. Joannopoulos, M. Soljacic, and S. G. Johnson, Phys. Rev. A 79, 013812 (2009).
[12] M. Tabor, Chaos and Integrability in Nonlinear Dynamics (Wiley, New York, 1989).
[13] S. Longhi, Opt. Commun. 204, 339 (2002).
[14] S. Trillo and M. Haelterman, Opt. Lett. 21, 1114 (1996).
[15] E. Lidorikis, Q. M. Li, and C. M. Soukoulis, Phys. Rev. B 54, 10249 (1996).
[16] V. Grigoriev and F. Biancalana, Photonics Nanostruct. Fundam. Appl. 8, 285 (2010).
[17] F. Eilenberger, C. Martijn de Sterke, and B. J. Eggleton, Opt. Express 18, 12708 (2010).
[18] M. D. Tocci, M. J. Bloemer, M. Scalora, J. P. Dowling, and C. M. Bowden, Appl. Phys. Lett. 66, 2324 (1995).
[19] F. Biancalana, J. Appl. Phys. 104, 093113 (2008).
[20] J. Bravo-Abad, S. Fan, S. G. Johnson, J. D. Joannopoulos, and M. Soljacic, J. Lightwave Tech. 25, 2539 (2007).
[21] S. Fan, W. Suh, and J. D. Joannopoulos, J. Opt. Soc. Am. A 20, 569 (2003).
[22] H. A. Haus, Waves and Fields in Optoelectronics (Prentice-Hall, New Jersey, 1984).
[23] S. Coen and M. Haelterman, Opt. Lett. 26, 39 (2001).
[24] T. Papakyriakopoulos, K. Vlachos, A. Hatziefremidis, and H. Avramopoulos, Opt. Lett. 24, 717 (1999).
[25] B. Daino, G. Gregori, and S. Wabnitz, J. Appl. Phys. 58, 4512 (1985).
[26] S. Trillo, S. Wabnitz, G. L. Stegeman, and E. M. Wright, J. Opt. Soc. Am. B 6, 899 (1989).
[27] G. P. Agrawal, Applications of Nonlinear Fiber Optics, 2nd ed. (Academic Press, New York, 2008).