Connecting the Galactic and Cosmological Scales: Dark Energy and the Cuspy-Core Problem

A. D. Speliotopoulos

Department of Mathematics, Golden Gate University, San Francisco, CA 94105, and Department of Physics, Ohlone College, Fremont, CA 94539-0390

(Dated: November 30, 2007)

Abstract

We propose a solution to the ‘cuspy-core’ problem by extending the geodesic equations of motion using the Dark Energy length scale $\lambda_{DE} = c/(\Lambda_{DE}G)^{1/2}$. This extension does not affect the motion of photons; gravitational lensing is unchanged. A cosmological check of the theory is made, and $\sigma_8$ is calculated to be $0.68 \pm 0.11$, compared to $0.761^{+0.049}_{-0.048}$ for WMAP. We estimate the fractional density of matter that cannot be determined through gravity at $0.197 \pm 0.017$, compared to $0.196^{+0.025}_{-0.026}$, the fractional density of nonbaryonic matter. The fractional density of matter that can be determined through gravity is estimated at $0.041^{+0.030}_{-0.031}$, compared to $0.0416^{+0.0038}_{-0.0039}$ for $\Omega_B$.  

*Electronic address: achilles@cal.berkeley.edu
I. INTRODUCTION

The recent discovery of Dark Energy [1, 2] has not only broadened our knowledge of the universe, it has brought into sharp relief the degree of our understanding of it. Only a small fraction of the mass-energy density of the universe is made up of matter that we have characterized; the rest consists of Dark Matter and Dark Energy, both of which have not been experimentally detected, and both of whose precise properties are not known. Both are needed to explain what is seen on an extremely wide range of length scales. On the galactic (∼ 100 kpc parsec), galactic cluster (∼ 10 Mpc), and supercluster (∼ 100 Mpc) scales, Dark Matter is used to explain phenomena ranging from the formation of galaxies and rotation curves, to the dynamics of galaxies and the formation of galactic clusters and superclusters. On the cosmological scale, both Dark Matter and Dark Energy are needed to explain the evolution of the universe.

While the need for Dark Matter is ubiquitous on a wide range of length scales, our understanding of how matter determines dynamics on the galactic scale is lacking. Recent measurements by WMAP [3] have validated the ΛCDM model to an unprecedented precision; such is not the case on the galactic scale, however. Current understanding of structure formation is based on [4], and both analytical solutions [5] and numerical simulations [6, 7, 8, 9, 10] of galaxy formation have been done since then. These simulations have consistently found a density profile that has a cusp-like profile [6, 8, 10], instead of the pseudoisothermal profile commonly observed. Indeed, De Blok and coworkers [11] has explicitly shown that the density profile from [6] attained through simulation does not fit the density profile observed for Low Surface Brightness galaxies; the pseudoisothermal profile is the better fit.

This is the cuspy-core problem. There have been a number of attempts to solve it within ΛCDM [9, 10], with varying degrees of success. While the problem does not exist for MOND [12], there are other hurdles MOND must overcome. Our approach to this problem, and to structure formation in general, is more radical; therefore, its consequences are correspondingly broader. It is based on the observation that with the discovery of Dark Energy, $\Lambda_{DE}$, there is a universal length scale, $\lambda_{DE} = c/(\Lambda_{DE}G)^{1/2}$, associated with the universe. Extensions of the geodesic equations of motion (GEOM) can now be made that will satisfy the equivalence principal, while not introducing an observable fifth force. While affecting the motion of massive test particles, photons will still travel along null geodesics,
and gravitational lensing is not changed. For a model galaxy, the extend GEOM results in a nonlinear evolution equation for the density of the galaxy. This equation is the minimum of a functional of the density, which is interpreted as an effective free energy for the system. We conjecture that like Landau-Ginzberg theories in condensed matter physics, the system prefers to be in a state that minimizes this free energy. Showing that the pseudoisothermal profile is preferred over cusp-like profiles reduces to showing that it has a lower free energy.

Here, phenomena on the galactic scale are inexorably connected to phenomena on the cosmological scale, and a cosmological check of our theory is made. The Hubble length scale $\lambda_H = c/hH_0$ naturally appears in our approach, even though a cosmological model is not mentioned either in its construction, or in its analysis. Using the average rotational velocity and core sizes of 1393 galaxies obtained through four different sets of observations [11, 13, 14, 15] spanning 25 years, we calculate $\sigma_8$ to be $0.68_{-0.11}^{+0.049}$, in excellent agreement with $0.761_{-0.017}^{+0.018}$ from [3]. We also calculate $\Omega_{\text{asymp}}$, the fractional density of matter that cannot be determined through gravity, to be $0.197_{-0.017}^{+0.025}$, which is nearly equal to the fractional density of nonbaryonic matter $\Omega_m - \Omega_B = 0.196_{-0.025}^{+0.005}$ [3]. We then find the fractional density of matter in the universe that can be determined through gravity, $\Omega_{\text{Dyn}}$, to be $0.041_{-0.009}^{+0.030}$, which is nearly equal to $\Omega_B = 0.0416_{-0.0026}^{+0.0038}$. Details of our calculations and theory is in [16].

II. EXTENDING THE GEOM AND GALACTIC STRUCTURE

Any extension of the geodesic action requires a dimensionless, scalar function of some property of the spacetime folded in with some physical property of matter. While before no such properties existed, with the discovery of Dark Energy there is now $\lambda_{DE}$ and these extensions can be made. As we work in the nonrelativistic, linearized gravity limit, we consider the simplest extension:

$$\mathcal{L}_{\text{Ext}} = mc \left( 1 + \mathcal{D} \left[ \frac{Rc^2}{\Lambda_{DE} G} \right] \right)^{\frac{1}{2}} \left( g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right)^{\frac{1}{2}} \equiv mc \mathcal{R} \left[ \frac{Rc^2}{\Lambda_{DE} G} \right] \left( g_{\mu\nu} \frac{dx^\mu}{dt} \frac{dx^\nu}{dt} \right)^{\frac{1}{2}}$$

with the constraint $v^2 = c^2$ for massive test particles. Here, $\mathcal{D}(x)$ is a function function given below, and $R$ is the Ricci scalar. For massive test particles, the extended GEOM is $v^\nu \nabla_{\nu} v^\mu = c^2 (g^{\mu\nu} - v^\mu v^\nu / c^2) \nabla_{\nu} \log \mathcal{R}[4 + 8\pi T / \Lambda_{DE} c^2]$, where $v^\mu$ is the four-velocity of a test particle, $T_{\mu\nu}$ is the energy-momentum tensor, $T = T_{\mu}^{\mu}$, and we take $\Lambda_{DE}$ to be the cosmological constant. As the action for gravity+matter is a linear combination of
the Hilbert action and the action for matter, any changes to the equation of motion for test particles can be accounted for in \( T_{\mu\nu} \), and we still have
\[
R = 4\Lambda_{DE}\frac{G}{c^2} + 8\pi\frac{G}{c^4}T
\]
in Eq. (1).

For massless particles, \( v^\nu \nabla_\nu \left( \mathcal{R}[4 + 8\pi T/\Lambda_{DE}c^2]v^\mu \right) = 0 \) instead. With the reparametization \( dt \rightarrow \mathcal{R}dt \), the extended GEOM for massless test particles reduces to the GEOM. Our extended GEOM does not affect the motion of photons.

Because the geodesic Lagrangian is extended covariantly, Eq. (1) explicitly satisfies the strong equivalence principal. For \( T_{\mu\nu} \), we may still take \( T_{\mu\nu} = (\rho + p/c^2)v_\mu v_\nu - pg_{\mu\nu} \) for an inviscid fluid with density \( \rho \) and pressure \( p \) [16]. While for the GEOM \( T_{\mu\nu}^{\mathrm{Geo-Dust}} = \rho v_\mu v_\nu \) for dust, for the extended GEOM the pressure does not vanish [16]; it is a functional of \( \rho \) and \( \mathcal{R} \). Nevertheless, in the nonrelativistic limit \( p \ll \rho c^2 \), and \( T_{\mu\nu}^{\mathrm{Ext-Dust}} \approx \rho v_\mu v_\nu \) still [16]. Moreover, because \( v^\mu v_\mu = c^2 \) for the extended GEOM, the first law of thermodynamics still holds for the fluid, and the standard thermodynamical analysis of the evolution of the universe under the extended GEOM follows much in the same way as before.

All dynamical effects of extension can be interpreted as the rest energy gained or lost by the test particle due to variations in the local curvature. For these effects not to have already been seen, \( \mathcal{D}(4 + 8\pi T/\Lambda_{DE}c^2) \) must change very slowly at current experimental limits. As such, we take \( \mathcal{D}(x) = \chi(\alpha) \int_x^\infty (1 + s^{1+\alpha})^{-1}ds \), where \( \alpha \geq 1 \) and \( \chi(\alpha) \) is set by \( \mathcal{D}(0) = 1 \). This \( \mathcal{D}(x) \) was chosen for three reasons. First, there is only one free parameter, \( \alpha \), to determine. Second, it ensures that the effects of the additional terms in the extended GEOM will not already have been observed; \( \Lambda_{DE} = (7.21^{+0.82}_{-0.84}) \times 10^{-30} \) g/cm\(^3\), and \( \rho \gg \Lambda_{DE}/2\pi \) in all current experimental environments so that \( \mathcal{D} \approx 0 \). A lower experimental bound of 1.35 for \( \alpha \) can be found [16]. Third, \( \mathcal{D}'(x) \) is negative, and will contribute an effective repulsive potential to the extended GEOM that mitigates the Newtonian \( 1/r \) potential.

While definitive, a first principles calculation of the galactic rotation curves using the extended GEOM would be analytically intractable. Instead, we show that given a model, stationary galaxy with a specific rotation velocity curve \( v(r) \), we can derive the mass density profile of the galaxy. We use a spherical model for the galaxy that has three regions. Region I = \( \{ r \mid r \leq r_H, \text{ and } \rho \gg \Lambda_{DE}/2\pi \} \), where \( r_H \) is the galactic core radius. Region II = \( \{ r \mid r > r_H, r \leq r_{II}, \text{ and } \rho \gg \Lambda_{DE}/2\pi \} \) is the region outside the core containing stars undergoing rotations with constant rotational velocity; it extends out to \( r_{II} \), which is determined by the theory. A Region III = \( \{ r \mid r > r_{II}, \text{ and } \rho \ll \Lambda_{DE}/2\pi \} \) also appears in
differentiate between baryonic matter and Dark Matter in $\rho_{\Lambda_{DE}}$ because $\rho_{\alpha}$ where $\kappa \equiv a$ the theory.

As all the stars in the model galaxy undergo circular motion, the acceleration of a star, $a \equiv \ddot{x}$, is a function of is location, $x$, only. Taking the divergence of the extended GEOM,

$$f(x) = \rho - \frac{1}{\kappa^2(\rho)} \left\{ \nabla^2 \rho - \frac{1 + \alpha_\Lambda}{4 + 8\pi \rho / \Lambda_{DE}} \left( \frac{8\pi}{\Lambda_{DE}} \right) |\nabla \rho|^2 \right\},$$

(2)

where $\kappa^2(\rho) \equiv \left\{ 1 + (4 + 8\pi \rho / \Lambda_{DE})^{1+\alpha_\Lambda} \right\} / \lambda_{DE}^2$, and $f(x) \equiv -\nabla \cdot a / 4\pi G$. We do not differentiate between baryonic matter and Dark Matter in $\rho$. Near the galactic core $1/\kappa(\rho) \sim \lambda_{DE}[\Lambda_{DE}/8\pi \rho_H]^{(1+\alpha_\Lambda)/2}$, where $\rho_H$ is the core density. Even though $\lambda_{DE} = 14010^{+800}_{-810}$ Mpc, because $\rho_H \gg \Lambda_{DE}/2\pi$, $\alpha_\Lambda$ can be chosen so that $1/\kappa(r)$ is comparable to typical $r_H$. Doing so sets $\alpha_\Lambda \approx 3/2$.

Given a $v(r)$, $a(r)$ can be found and $f(r)$ determined. We idealize the observed velocity curves as $v^{\text{ideal}}(r) = v_H r/r_H$ for $r \leq r_H$, while $v^{\text{ideal}}(r) = v_H$ for $r > r_H$, where $v_H$ is the observed asymptotic velocity. This $v^{\text{ideal}}(r)$ is more tractable than the pseudoisothermal velocity curve, $v^{\text{p-iso}}(r)$, used in [11]. As it has the same limiting forms in both the $r \ll r_H$ and $r \gg r_H$ limits, $v^{\text{ideal}}(r)$ is also an idealization of $v^{\text{p-iso}}(r)$.

For cusp-like density profiles [10], it is the density profile that is given. While it is possible to integrate the general density profile to find the corresponding curves $v_{\text{cusp}}(r)$, both the maximum value of $v_{\text{cusp}}(r)$ and the size of the core are different depending on the profile. These core sizes would thus have to be scaled appropriately to compare one profile with another. Doing so is possible in principle, but would be analytically intractable in practice. We instead take $f(r) = \rho_H (r_H/r)^{\gamma}$ if $r \leq r_H$, and $f(r) = \rho_H (r_H/r)^{\beta} / 3$ if $r > r_H$ for the density profiles. Here, $\gamma < 2$ and $\beta \geq 2$ agrees with the parameters for the generic cusp-like density profile [2], with the core size set to $r_H$. The $\gamma = 0, \beta = 2$ case corresponds to the idealized pseudoisothermal profile.

Since $\rho \gg \Lambda_{DE}/2\pi$ in Regions I and II, Eq. (2) minimizes

$$\mathcal{F}[\rho] = \frac{\Lambda_{DE} c^2}{8\pi} (\chi^{1/2} \lambda_{DE})^3 \int d^3u \left\{ \frac{1}{2\alpha_\Lambda} \left| \nabla \left( \frac{\Lambda_{DE}}{8\pi \rho} \right)^{\alpha_\Lambda} \right|^2 - \frac{\alpha_\Lambda}{\alpha_\Lambda - 1} \left( \frac{\Lambda_{DE}}{8\pi \rho} \right)^{\alpha_\Lambda - 1} \right. +$$

$$\left. \left( \frac{\Lambda_{DE}}{8\pi \rho} \right)^{\alpha_\Lambda} \frac{8\pi f(u)}{\Lambda_{DE}} \right\},$$

(3)

which we identify as a free energy functional; here, $u = r/\chi^{1/2} \lambda_{DE}$. For $\gamma = 0$, Eq. (2) gives $\rho(r) = \rho_H$ in Region I; the free energy for this solution is $\mathcal{F}_{\gamma=0} = -\Lambda_{DE} r_H^3 (\Lambda_{DE}/8\pi \rho_H)^{\alpha_\Lambda - 1} / 6(\alpha_\Lambda - 1)$. While for $\gamma > 0$ perturbative solutions can be found,
all such solutions have a $^{I}{\mathcal F}_{\gamma}$ greater than $^{I}{\mathcal F}_{\gamma=0}$ \[\boxed{16}\]. This results because $\sim |\nabla \rho|^2 \geq 0$ in Eq. \[3\]; just as in a Landau-Ginzberg theory, $|\nabla \rho|^2$ only vanishes for the constant density solution.

For Region II, the density, $\rho_{II}$, is first found asymptotically in the large $r$ limit. With the anzatz $f(r) \ll \rho(r)$ for large $r$, Eq. \[2\] reduces to a homogeneous equation \[16\] with the solution $\rho_{\text{asymp}}(u) = \Lambda_{DE} \Sigma(\alpha_{\Lambda})/8\pi u^{1+\alpha_{\Lambda}}$, where $\Sigma(\alpha_{\Lambda}) = [2(1+3\alpha_{\Lambda})/(1+\alpha_{\Lambda})]^{1+\alpha_{\Lambda}}$. To include the galaxy’s structural details, we take $\rho_{II}(r) = \rho_{\text{asymp}}(r) + \rho^1_{II}(r)$ and to first order in $\rho^1_{II}$,

$$\rho_{II}(r) = \rho_{\text{asymp}}(r) + \frac{1}{3} A_{\beta} \rho H \left(\frac{r_H}{r}\right)^{\beta} + \left(\frac{r_H}{r}\right)^{5/2} \left(C_{\cos} \cos \left[\nu_0 \log r/r_H\right] + C_{\sin} \sin \left[\nu_0 \log r/r_H\right]\right),$$

(4)

where $\nu_0 = [2(1+3\alpha_{\Lambda})/(1+\alpha_{\Lambda})^2 - 1/4]^{1/2}$, $C_{\cos}$ and $C_{\sin}$ are determined by boundary conditions, and $A_{\beta} = 1$ for $\beta = 2, 3$. The first part, $\rho_{\text{asymp}}(r)$, of $\rho_{II}(r)$ corresponds to a background density. It is universal, and has the same form irrespective of the detailed structure of the galaxy. The second part, $\rho^1_{II}(r)$, gives the structural details.

The free energy, $^{II}{\mathcal F}$, for Region II separates into the sum of three parts. The first part depends only on $\rho_{\text{asymp}}$; it is positive, and is independent of $\beta$. The second part is

$$^{II}{\mathcal F}_{\text{asymp-\beta}} \left(\chi^{1/2}\lambda_{DE}\right)^{3} = c^2 \int_{D_{II}} d^3 u f(u) \left(\frac{\Lambda_{DE}}{8\pi \rho_{\text{asymp}}}\right)^{\alpha_{\Lambda}} + \frac{8\pi \alpha_{\Lambda} c^2}{\Lambda_{DE} \Sigma_f^{2(1+\alpha_{\Lambda})}} \int_{\partial D_{II}} u^{4} \rho^1_{II}(u) \nabla \rho_{\text{asymp}} \cdot d\mathbf{S},$$

(5)

where $D_{II}$ is Region II. It is negative because the minimum $\rho$ must be positive. Indeed, we find that $^{II}{\mathcal F}_{\text{asymp-\beta}} \sim -r_{H}/r_{II}^{\beta}$ for $\beta < 5/2$; $^{II}{\mathcal F}_{\text{asymp-\beta}} \sim -(r_{H}/r_{II})^{5/2}$ for $5/2 \leq \beta < 5 - 2/(1 + \alpha_{\Lambda})$; and $^{II}{\mathcal F}_{\text{asymp-\beta}} \sim \pm (r_{H}/\chi^{1/2}\lambda_{DE})^{5/2/(1+\alpha_{\Lambda})}$ for $5 - 2/(1 + \alpha_{\Lambda}) < \beta$. Clearly, free energy is lowest for $\beta = 2$. The third part depends on $(\rho^1_{II}(r))^2$, and is negligibly small. The total free energy in this region is thus smaller for $\beta = 2$ than for $\beta > 2$. Combined with the calculation for $^I{\mathcal F}$, we conclude that the pseudoisothermal density profile has the lowest free energy, and is the preferred state of the system. We thus take $\gamma = 0$ and $\beta = 2$ in the following.

In Region III, $\rho \ll \Lambda_{DE}/2\pi$, and $\kappa^2(\rho) \approx (1 + 4^{1+\alpha_{\Lambda}})/\chi^{2}\lambda_{DE}^2$; Eq. \[2\] reduces to the undriven, modified Bessel equation. As such, the density vanishes exponentially fast in this region on the scale $1/\kappa(r)$. This sets $r_{II} = [\chi/(1 + 4^{1+\alpha_{\Lambda}})]^{1/2}\lambda_{DE}$.

The extended GEOM can be written as $\ddot{x} = -\nabla \Psi$. The dynamics of test particles is governed by an effective potential $\Psi(x) = \Phi(x) + c^2 \log (\mathcal{R}[4 + 8\pi \rho \Lambda_{DE}])$, and not by the gravitational potential $\Phi(x)$. For $\Phi(r)$ in Region I, we obtain the Newtonian gravity result
\( \Phi(r) = v_H^2 r^2 / 2 r_H^2 + \text{constant} \). In Region II, \( \Phi(r) \) is dominated by four terms. The first is the usual \( 1/r \) term. The second is a \( \log(r/r_H) \) term due to \( f(r) \). This term is long ranged, and in addition to galactic rotation curves, could explain the interaction observed between galaxies and galactic clusters. The third is a \( \rho_{II}^1(r) r^2 \) term, and contains terms \( \sim 1/r^{1/2} \). The fourth term is a \( r^{2\alpha/\lambda(1+\alpha\lambda)} \) term due to \( \rho_{\text{asymp}} \), and is proportional to \( c^2 \).

This last term grows as \( r^{6/5} \) for \( \alpha\lambda = 3/2 \), and would dominate the motion of test particles in the galaxy if the extended GEOM depended on \( \Phi(x) \) instead of \( V(x) \). We instead find that \( \Psi(x) \approx \Phi(x) - \left[ u^2 c^2 (1 + \alpha\lambda)^2 / 4\alpha\lambda (1 + 3\alpha\lambda) \right] (\Lambda_{DE}/8\pi)(\rho_{\text{asymp}} - \alpha\lambda \rho_{II}^1) \). The last two terms in this expression cancel both the \( \rho_{II}^1(r) r^2 \) and the \( r^{2\alpha/\lambda(1+\alpha\lambda)} \) terms in \( \Phi(r) \); the resultant \( \Psi(r) \) increases as \( \log r/r_H \), agreeing with observation.

The \( r^{2\alpha/\lambda(1+\alpha\lambda)} \) term in \( \Phi(x) \) comes from the background density \( \rho_{\text{asymp}} \). Thus, a good fraction of the mass in the observable galaxy does not contribute to the motion of test particles in the galaxy. It is rather the near-core density \( \rho_{II}^1(r) \) that contributes to \( \Psi(x) \). As inferring the mass of structures through observations of the dynamics under gravity of their constituents is one of the main ways of estimating mass, the motion of stars in galaxies can only be used to estimate \( \rho_{II}^1 \); the matter in \( \rho_{\text{asymp}}(r) \) is present, but cannot be “seen” in this way. Moreover, as \( \rho_{\text{asymp}}(r) \gg \rho_{II}^1(r) \) when \( r \gg r_H \), the majority of the mass in the universe cannot be seen using these methods.

### III. A COSMOLOGICAL CHECK

We have extrapolated our results for a single galaxy to the cosmological scale. This is possible because recent measurements from WMAP, the Supernova Legacy Survey, and the HST key project show that the universe is essentially flat; \( h = 0.732^{+0.031}_{-0.032} \) and of the age of the universe \( t_0 = 13.73^{+0.10}_{-0.15} \) Gyr were determined using this assumption. The largest distance between galaxies is thus \( ct_0 = \mathcal{R}(\Omega) \lambda_H \), where \( \mathcal{R}(\Omega) = 1.03^{+0.05}_{-0.05} \).

Next, the density of matter of our model galaxy dies off exponentially fast at \( r_{II} \); the extent of matter in the galaxy is fundamentally limited to \( 2r_{II} \). This size does not depend on the detailed structure of the galaxy; it is inherent to the theory. Given a \( \Omega_\lambda = 0.716^{+0.055}_{-0.055} \), we can express \( r_{II} = \left[ 8\pi \chi / 3\Omega_\lambda (1 + 4^{1+\alpha\lambda}) \right]^{1/2} \lambda_H \) as well \([16]\), and numerically \( r_{II} = 0.52 \lambda_H \) for \( \alpha\lambda = 3/2 \). Although \( \alpha\lambda \) was set to \( 3/2 \) based on analysis at the galactic scale, \( \rho(r) \) naturally cuts off at \( \lambda_H/2 \).
To accomplish the extrapolation, we consider our model galaxy to be the representative galaxy for the observed universe. This representative galaxy could, in principal, be found by sectioning the observed universe into three-dimensional, non-overlapping cells of different sizes centered on each galaxy. By surveying these cells, a representative galaxy, with an average $v_H^*$ and $r_H^*$, can be found, and used as inputs for the model galaxy. Even though such a survey has not yet been done, a large repository of galactic rotation curves and core radii [11, 14, 15] is present in the literature. Taken as a whole, these 1393 galaxies are reasonably random, and are likely representative of the observed universe at large.

While we were able to estimate of $\alpha_\Lambda = 3/2$ by looking at the galactic structure, the accuracy of this estimate is unknown; comparison with experiment is not possible. We instead require that $r_{\text{II}} = R(\Omega)\lambda_H/2$, which in turn gives $\alpha_\Lambda$ as the solution of $R(\Omega)^2(1 + 4^{1+\alpha_\Lambda}) = 32\pi f(\alpha_\Lambda)/3\Omega_\Lambda$; this sets $\alpha_\Lambda = 1.51 \pm 0.11$.

A calculation of $\sigma_8^2$ has been done [16] using Eq. (4). The resultant $\sigma_8^2$ is dominated by two terms. The first is due to the background density $\rho_{\text{asymp}}$. It depends only on $\alpha_\Lambda$, and contributes a set amount of 0.141 to $\sigma_8^2$. The second is the larger one, and is due primarily to the $1/r^2$ term in Eq. (4). It depends explicitly on the rotation curves through the term $(v_H^*/c)^4(8h^{-1}\text{Mpc}/r_H^*)$.

Although there have been a many studies of galactic rotation curves in the literature, both $v_H$ and $r_H$ are needed here. This requires fitting the observed velocity curve to some model. To our knowledge, both values are available from four places in the literature: The de Blok et. al. data set [11]; the CF data set [14]; the Mathewson et. al. data set [15, 17] analysed in [14]; and the Rubin et. al. data set [13]. Except the last set, the observed velocity curves is fitted to either $v_{p\text{-iso}}(r)$, or to a functionally similar velocity curve [14]. The last set gives only the galactic rotation curves, and they have been fitted to $v_{p\text{-iso}}(r)$ in [16]. While the URC of [18] has a constant asymptotic velocity, it has a $r^{0.66}$ behavior for $r$ small. This behavior is different from $v_{\text{ideal}}$, and was not considered here [16].

While $v_H$ is easily identified for all four data sets, determining $r_H$ is more complicated; this is determination is done in [16]. The resultant values are used to obtain $v_H^*$ and $r_H^*$ for each set, which are then used to calculate the $\sigma_8$ and $\Delta\sigma_8$ for it. Results of these calculations are in Table I. Four of the five data sets give a $\sigma_8$ that agrees with the WMAP value at the 95% CL. The Rubin et. al. set does not, but it is known that these galaxies were not randomly selected [13].
TABLE I: The \(v_H^*\) (km/s), \(r_H^*\) (kps), and resultant \(\sigma_8\), \(\Delta \sigma_8\), and t-test comparison with the WMAP value of \(\sigma_8\).

| Data Set                  | \(v_H^*\) | \(\Delta v_H^*\) | \(r_H^*\) | \(\Delta r_H^*\) | \(\sigma_8\) | \(\Delta \sigma_8\) | t-test |
|--------------------------|------------|------------------|-----------|------------------|--------------|------------------|-------|
| deBlok et. al. (53)      | 119.0      | 6.8              | 3.62      | 0.33             | 0.613        | 0.097            | 1.36   |
| CF (348)                 | 179.1      | 2.9              | 7.43      | 0.35             | 0.84         | 0.18             | 0.43   |
| Mathewson et. al. (935)  | 169.5      | 1.9              | 15.19     | 0.42             | 0.625        | 0.089            | 1.34   |
| Rubin et. al. (57)       | 223.3      | 7.6              | 1.24      | 0.14             | 2.79         | 0.82             | 2.46   |
| Combined (1393)          | 172.1      | 1.6              | 11.82     | 0.30             | 0.68         | 0.11             | 0.70   |

We have estimated \(\Omega_{\text{asymp}}\) by averaging \(\rho_{\text{asymp}}(r)\) over a sphere of radius \(r_{II}\), and found \(\Omega_{\text{asymp}} = 0.197 \pm 0.017\). In calculating this average, we assumed that there is only a single galaxy within the sphere, however. While this is a gross under counting of the number of galaxies in the universe, \(\rho_{\text{asymp}}\) is an asymptotic solution, and \(\rho_{II}^1 \to 0\) rapidly with \(r\). Additional galaxies may change the form of \(\rho_{\text{asymp}}\), but these changes are expected to be equally short ranged; we expect that our calculation is an adequate estimate of \(\Omega_{\text{asymp}}\). Such is not the case for \(\Omega_{\text{Dyn}}\), however. Direct calculation of \(\Omega_{\text{Dyn}}\) would require knowing both the detailed structure of galaxies, and the distribution of galaxies in the universe. Instead, we note that \(\Omega_m = \Omega_{\text{asymp}} + \Omega_{\text{Dyn}}\), and using \(\Omega_m = 0.238^{+0.025}_{-0.026}\) from WMAP, find \(\Omega_{\text{Dyn}} = 0.041^{+0.030}_{-0.031}\).

IV. CONCLUDING REMARKS

Given how sensitive \(\sigma_8\) is to \(v_H^*, r_H^*,\) and \(\alpha_\Lambda\), that our predicted values of \(\sigma_8\) is within experimental error of the WMAP value is surprising. Even in the absence of a direct experimental search for \(\alpha_\Lambda\), this agreement provides a compelling argument for the validity of our extension of the GEOM. It also supports our free energy conjecture; our calculation of \(\sigma_8\) would be very different if \(\beta = 3\), say, was used instead of \(\beta = 2\). With \(\alpha_\Lambda = 1.51\) so close to the experimental lower bound for \(\alpha_\Lambda\) of 1.35, direct measurement of \(\alpha_\Lambda\) may also be possible in the near future.

Interestingly, \(\Omega_m - \Omega_B = 0.196^{+0.025}_{-0.026}\) is nearly equal to \(\Omega_{\text{asymp}}\) in value. Correspondingly, \(\Omega_B^{[3]}\) is nearly equal to \(\Omega_{\text{Dyn}}\). It would be tempting to identify \(\Omega_{\text{asymp}}\) with \(\Omega_m - \Omega_B\), especially since matter in \(\rho_{\text{asymp}}(r)\) is not “visible” to inferred-mass measurements. That \(\Omega_{\text{Dyn}}\)
would then be identified with $\Omega_B$ is consistent with the fact that most of the mass inferred through gravitational dynamics are indeed made up of baryons. We did not differentiate between normal and dark matter in our theory, however. Without a specific mechanism funneling nonbaryonic matter into $\rho_{\text{asymp}}$ and baryonic matter into $\rho - \rho_{\text{asymp}}$, we cannot at this point rule out the possibility that $\Omega_m - \Omega_B = \Omega_{\text{asymp}}$ and $\Omega_B \approx \Omega_{\text{Dyn}}$ is a numerical accident.

Acknowledgments

The author would like to thank John Garrison for his numerous suggestions, comments, and generous support during this research. He would also like to thank K.-W. Ng H. T. Cho and Clifford Richardson for their helpful criticisms.

[1] A. G. Riess, A. V. Filippenko, P. Challis, A. Clocchiatti, A. Diercks, P. M. Garnavich, R. L. Gilliland, C. J. Hogan, S. Jha, R. P. Kirshner, B. Leibundgut, M. M. Phillips, D. Riess, B. P. Schmidt, R. A. Schommer, R. C. Smith, J. Spyromilio, C. Stubbs, N. B. Suntzeff, and J. Tonry, Astron. J. 116, 1009 (1998).

[2] S. Perlmutter, G. Aldering, G. Goldhaber, R. A. Knop, P. Nugent, P. G. Castro, S. Deustua, S. Fabbro, A. Goobar, D. E. Groom, I. M. Hook, A. G. Kim, M. Y. Kim, J. C. Lee, N. J. Nunes, R. Pain, C. R. Pennypacker, R. Quimby, C. Lidman, R. Ellis, M. Irwin, R. G McMahon, P. Ruiz-Lapuente, N. Walton, B. Schaefer, B. J. Boyle, A. V. Filippenko, P. Matheson, A. S. Fruchter, N. Panagia, H. J. M. Newberg, and W. J. Couch, Astrophys. J. Suppl. 517, 565 (1999).

[3] D. N. Spergel, R. Bean, O. Doré, M. R. Nolta, C. L. Bennett, J. Dunkley, G. Hinshaw, N. Jarosik, E. Komatsu, L. Page, H. V. Peiris, L. Verde, M. Halpern, R. S. Hill, A. Kogut, M. Limon, S. S. Meyer, N. Odegard, G. S. Tucker, J. L. Weiland, E. Wollack, and E. L. Wright, Astrophys. J. Suppl. 170, 277 (2007).

[4] P. J. E. Peebles, Astrophys. J. 277, 470 (1984).

[5] J. E. Gunn, and J. R. Gott III, Astrophys. J. 176, 1 (1972); J. A. Fillmore, and P. Goldreich, Astrophys. J. 281, 1 (1984); Y. Hoffman, and J. Shaham, Astrophys. J. 297, 16 (1985);
Y. Hoffman, Astrophys. J. 328, 489 (1988).

[6] J. F. Navarro, A. S. Frenk, and S. D. White, Astrophys. J. 462, 563 (1996).

[7] A. V. Kravtsov, A. A. Krispin, J. S. Bullok, and J. Primack, Astrophys. J. 502, 48 (1998).

[8] B. Moore, T. Quinn, F. Governato, J. Stadel, and G. Lake, Mon. Not. R. Astron. Soc. 310, 1147 (1999).

[9] P. J. E. Peebles, and B Ratra, Rev. Mod. Phys. 75, 559 (2003).

[10] G. Bertone, D. Hooper, and J. Silk, Phys. Rep. 405, 279 (2005).

[11] W. J. G. de Blok, S. S. McGaugh, A. Bosma, and V. C. Rubin, Astrophys. J. 552, L23 (2001); W. J. G. de Blok, and A. Bosma, Astro. Astrophys. 385, 816 (2002); S. S. McGaugh, V. C. Rubin, and W. J. G. de Blok, Astron. J. 122, 2381 (2001).

[12] M. Milgrom, Astrophys. J. 270, 265 (1983).

[13] V. C. Rubin, W. K. Ford, Jr., and N. Thonnard, Astrophys. J. 238, 471 (1980); V. C. Rubin, W. K. Ford, Jr., N. Thonnard, and D. Burstein, Astrophys. J. 261, 439 (1982); D. Burstein, V. C. Rubin, N. Thonnard, and W. K. Ford, Jr., Astrophys. J., part 1 253, 70 (1982); V. C. Rubin, D. Burstein, W. K. Ford, Jr., and N. Thonnard, Astrophys. J. 289, 81 (1985).

[14] S. Courteau, Astron. J. 114, 2402 (1997).

[15] D. S Mathewson, V. L. Ford, and M. Buchhorn, Astrophys. J. Suppl. 82, 413 (1992).

[16] A. D. Speliotopoulos, arXiv:0711.3124v1 [astro-ph].

[17] M. Persic, and P. Salucci, Astron. J.Suppl. 99, 501 (1995).

[18] M. Persic, P. Salucci, and F. Stel, Mon. Not. R. Astron. Soc. 281, 21 (1996).