Measurement uncertainty of the fall cone (FC) test

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Abstract. The most common method to determine liquid limit, sensitivity and undrained shear strength of clay in Nordic countries is to use fall cone test (FC). However, all these properties include some uncertainty arising from different sources e.g. the geological conditions, test procedure and sample disturbance. In this study, the measurement uncertainty of the fall cone test was analysed. The parameters of interest were the penetration depth of the cone and the interpreted intact undrained shear strength. The test set consisted of high-quality block samples (d=132mm) collected from six different depths at soft clay site in Finland. The diameter of the samples made it possible to perform four parallel fall cone tests per depth, so total of 24 fall cone tests were done. The calculated COV-values for each parameter at each depth were between 0.02 and 0.08, thus indicating very low measurement uncertainty. These values also included the inherent variability of the soil. Overall, the results showed that the fall cone test is reliable method for determining the intact undrained shear strength, if the test is done with care.

1. Introduction

Tampere University (TAU) has implemented an extensive research program for soil testing in Finland. The aim has been to develop a database from in-situ and laboratory tests, using high-quality test methods and sampling equipment. The database has then been used for example calibrating new calculation and transformation models, developing better soil testing and sampling methods and studying the variation of investigated soil properties e.g. [5][6][7][8][9][10][16][17]. The database has also been beneficial for the ongoing project concerning reliability based design and calibration of partial factors.

In this paper, the measurement uncertainty of fall cone test (FC) is studied. The fall cone test (FC) is a common method to determine liquid limit, sensitivity and undrained shear strength of clay in Nordic countries. The focus in this paper is to investigate the variation of undrained shear strength and the penetration depth of the cone for Finnish soft clay. High-quality soil samples were collected from six different depths at the site, and then the samples were tested in Tampere University soil laboratory. The measured cone penetration depth and the interpreted undrained shear strength are both evaluated statistically and the bias factors and COVs (coefficient of variation) are calculated using simple approach presented by Ching and Phoon (2014) [4] and discussed in chapter 4. This study focuses mainly on the evaluation of measurement uncertainty of the fall cone test. However, as the uncertainty in a single soil property can arise from multiple sources, also the other possible sources of uncertainties are briefly discussed.

2. Uncertainty in geotechnical property

2.1. Sources of uncertainty

Estimation of design soil properties always includes some uncertainties arising from different sources. The types of uncertainty can be divided in to two main categories: aleatory uncertainty and epistemic
uncertainty. Aleatory uncertainty is randomness of a parameter which cannot be reduced. Epistemic uncertainty, however, is due to lack of knowledge of a certain soil property. It can be reduced by using more precise testing methods or increasing the number of the tests.

The uncertainties are usually represented as the values of the coefficient of variation (COV). The COV is a dimensionless ratio between the standard deviation $\sigma$ and mean value $\mu$ of the parameter (COV=$\sigma/\mu$). The magnitude of the COV-value represents the amount of uncertainty a certain parameter has, but these are not entirely comparable. The uncertainties arising from different sources can be combined as the sum of different uncertainty components to a single value using Eq. (1) (Baecher and Ladd 1997; [11]). The equation gives an approximative result for the total uncertainty related to certain parameter.

$$COV^2_X = COV^2_{spat,X} + COV^2_{err,X} + COV^2_{trans,X} + COV^2_{stat,X} + COV^2_{mod}$$  \hspace{1cm} (1)

where, COV$_{spat,X}$ is spatial variability (aleatory), COV$_{err,X}$ is measurement uncertainty, COV$_{trans,X}$ is transformation uncertainty, COV$_{stat,X}$ is statistical uncertainty, COV$_{mod}$ is model uncertainty and COV$_X$ is the total uncertainty related to parameter X.

In everyday design, taking all the possible uncertainties exactly into account is impossible. The resources are very limited. Therefore, it is beneficial to try to estimate the values of uncertainties by searching literature values or by using empirical information. Some literature values can be found from [13][14][15] Baecher and Christian (2003) and [18]. However, it is common for the literature values that those can be a combination of multiple different sites and databases with different accuracies, and thus, those values should be used with extra caution.

2.2. Measurement uncertainty

Measurement error or measurement uncertainty (studied later on for fall cone test) can be divided into three subcategories [12] imperfections with the used instruments (COV$_{err,equip}$), sample disturbance and operator based error when conducting certain tests (COV$_{err,oper}$) and a random error (COV$_{err,rand}$). Imperfections in the used instruments are caused by lack of proper calibration, usage of damaged instruments or the instruments may brake during the tests. However, these sources of imperfections are usually excluded from measurement uncertainty, because the operator responsible for the testing should make sure that the instruments are in good condition and they are used in proper manner. More operator-based uncertainties come from operators’ different approaches of conducting tests, which might be due to lack of guidance and/or standards or these are open to interpretation. Also, the operator’s experience plays a big role as they may have developed their own procedure of doing the test, which has evolved through multiple repetitions. One example is the fall cone test where releasing the cone and locking it for reading of the penetration depth varies between operators, leading to different results between operators.

With respect to soil sampling, sample disturbance during uplift, packing, transportation and storing are also examples of operator-based uncertainties.

The measurement uncertainty includes both a bias and a random error. The bias is a systematic error e.g. due to sample disturbance or operator working procedure, and as already said, it can be reduced by taking the samples and doing the tests with more care. The random error, however, is caused by the measurement tolerances of the equipment and if the testing equipment works efficiently at times during the test. This cannot be reduced. The calculation of total measurement uncertainty is given in Eq. (2) [12].

$$COV^2_{err,X} = COV^2_{err,equip} + COV^2_{err,oper} + COV^2_{err,rand}$$  \hspace{1cm} (2)

The measurement uncertainty calculated in this paper also includes the inherent variability of the soil, which must be subtracted in order to evaluate the actual measurement error (Eq.1). This is because when the soil is tested with some test method (in-situ or laboratory), the test results capture also the inherent variability of the soil. However, the evaluation of inherent soil variability can be challenging, and it is not done in the context of this paper. The presented measurement uncertainties are therefore a lumped number including both, the inherent soil variability and the actual measurement error.
3. Fall cone tests (FC)

3.1. Tested soil samples
The soil samples used for testing were collected from a soft clay site on the southern coast of Finland. The samples were taken from six depths (1.0 m, 2.0 m, 3.0 m, 4.0 m, 5.0 m and 7.0 m) with a new 132 mm open-drive block sampler designed at Tampere University [6]. This sampler enables high quality samples for laboratory testing.

The subsoil at the site consisted of (from top to bottom): 0.8 m thick dry crust layer, 0.7 m thick transitional layer, thick and inclined (NE to SW) soft clay layer, coarser silt and sand layers and finally bedrock at depth of 14 m. The soft clay layer is normally consolidated, medium sensitive and highly plastic. The index properties for the clay layer are water content $w=60-130\%$, plasticity $Ip=16-50$ and sensitivity $St=20-50$.

3.2. Testing procedure
The fall cone tests were conducted by following the CEN ISO/TS 17892-6 [3] standard and supportive laboratory specific instructions. At first, a 30 mm thick sample was extracted from the tube. The sample diameter of 132 mm made it possible to conduct four parallel fall cone tests per depth by using the same sample, which makes the results between separate fall cone tests fully comparable. The sample was divided into four zones and five drops per zone was made (one FC test = 5 individual drops). Sketch is shown in Fig. (1).

![Figure 1. Sketch of the testing plan. Four (4) individual fall cone tests, each containing 5 drops, are done to the same sample.](image)

After conducting the four FC tests for the undisturbed sample, a quarter of the specimen (one zone) was cut off and the fall cone test was repeated for remoulded sample. Also, the index properties were determined from the sample.

The undrained shear strength was interpreted from the fall cone test by first excluding the minimum and maximum values from the five drops and then multiplying the mean value of the remaining three values with an empirical cone depended factor. Sensitivity was determined as the ratio between intact and remoulded undrained shear strength.

In order to avoid possible equipment or operator specific errors, the tests in this study were conducted by the same operator with the same equipment.

4. Soil statistics
Sample sizes are usually very limited in the field of geotechnical engineering, which makes it quite impossible to calculate the statistical parameters for the whole population. In this case, it is possible to estimate the actual population parameters with sample statistics, which are biased estimates of the actual population statistics. The sample statistics are enough to examine the variability of the property and are easier to use in everyday design than complicated probability distributions.

Let $X$ be a soil variable with existing observations $x_1 \ldots x_n$. If observations are assumed independent (e.g. no trend), the mean $\mu_X$ and standard deviation $\sigma_X$ of variable $X$ can be estimated from sample statistics.
The sample mean is estimated from Eq. (3) where \( n \) is the number of observations:

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]  

(3)

The sample standard deviation is estimated from Eq. (4):

\[
s = \left( \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2 \right)^{\frac{1}{2}}
\]  

(4)

The measurement uncertainty is calculated with the method discussed by Ching and Phoon (2014) [4]:

\[
e = \frac{\text{actual target value}}{b \cdot \text{predicted target value}}
\]  

(5)

The \( b \) in Eq. (5) is the bias factor between predicted and actual value. The actual value is the true value of the soil property in the ground, which in practice, is never known precisely. However, this can be chosen to be the best guess value. The best guess value could be for example the parameter value determined with the most precise laboratory method, which uncertainty is known to be very small. For example, in the case of undrained shear strength, the actual value could correspond the triaxial test value, to which the measured value, like field vane \( s_v \)-value, is compared to. The actual value is our benchmark value in the Eq. (5) and can be chosen rather without restrictions. The predicted value is the estimated (measured) soil property value from in-situ or laboratory tests, which is under investigation. In the last example, it would be the field vane \( s_v \)-value. The product of constant \( b \) and the predicted (measured) value leads to unbiased estimation on the average. The bias tells if the measured values tend to underpredict (\( b>1.0 \)) or overpredict (\( b<1.0 \)) the true value. The random variable \( \varepsilon \) has a mean of 1 (by definition). The measurement uncertainty is the COV of the random variable \( \varepsilon \).

5. Evaluation of the measurement uncertainty

The evaluation of the fall cone test results is done in two phases. In the first phase, the variation of the penetration depth of the cone is determined for each depth and test separately. The penetration depth is the value read from the measurement gauge after dropping the cone. The second phase is to investigate the variation of undrained shear strength, as the undrained shear strength is interpreted from the fall cone test results.

5.1. Variation of the penetration depth of the cone

The fall cone tests were performed to soil samples from six different depths. For each of these six samples, four parallel fall cone tests were done, each test containing five separate points. This sums up to twenty points (\( n_2=20 \), 4 tests each including 5 drops) per depth. This can be considered as a moderate number of data points in order to make depth wise comparisons.

For each depth, the COV\(_e\) and bias \( b \) were calculated with Eq. (5). The actual value in Eq. (5) is chosen to correspond the mean value of all 20 measurements per depth, whereas the predicted value is the individual test result from the fall cone test. The bias and COV\(_e\) were also calculated for each test (\( n_1=5 \), 5 drops per test) and the combination of all depths (\( n_3=120 \), 4 tests form 6 depths each including 5 drops). From the latter, the general value can be derived for variation of the penetration depth. The results are shown in Table 1.
Table 1. Bias and coefficient of variation (COV) of the error term related to penetration depth of the cone. For each individual test the sample size \(n_1=5\) (5 drops / test) and for the last column “All tests” the sample size is \(n_2=20\).

| Depth [m] | number of drops per test | Test 1 | Test 2 | Test 3 | Test 4 | All tests (1-4) |
|-----------|--------------------------|--------|--------|--------|--------|----------------|
| 1.36-1.39\(^a\) | 5                        | 1.05   | 0.05   | 0.96   | 0.06   | 0.95 0.03     |
| 2.34-2.37\(^a\) | 5                        | 1.02   | 0.06   | 1.01   | 0.02   | 1.00 0.02     |
| 3.25-3.28\(^a\) | 5                        | 1.04   | 0.03   | 1.01   | 0.03   | 0.99 0.04     |
| 4.35-4.38\(^a\) | 5                        | 0.99   | 0.05   | 0.99   | 0.03   | 1.02 0.03     |
| 5.38-5.41\(^b\) | 5                        | 0.97   | 0.05   | 1.06   | 0.07   | 1.00 0.10     |
| 7.32-7.35\(^b\) | 5                        | 0.98   | 0.07   | 1.03   | 0.05   | 1.00 0.12     |

**Whole site** 120\(^*\) 1.00 0.07

\(^a\) 60g cone used for samples from depths <5.0m
\(^b\) 100g cone used for samples from depths >5.0m
\(^*\) number of drops for the whole site: 5 drops times 4 tests times 6 depths = 120 drops

Generally, the results (Table 1.) show that the variation of penetration depth of the cone is very low. For all results, the COV-values are ranging between 0.02 and 0.12, which indicate very low measurement uncertainty. Also, the bias factors indicate that the measured values are close to each other. The range of COVs is even smaller (0.03-0.08) when considering only the values derived for each depth by considering all four tests as a one.

A bit higher variation (COV=0.07-0.08) can be seen at results from depths of 5m and 7m, where it was necessary to use heavier cone (60g cone to 100g cone). However, finding the explanation to the difference between results obtained with different cones can be challenging. The difference could be due to fact that heavier cone has not fully stopped its movement when the penetration depth was read from the gauge. The other explanation could be that the operator had different kind of touch when using different cones. The procedure was not the same with different cones. Also, the soil properties could have affected even though the same sample was used. Some horizontal variation could have been within the sample.

5.2. Variation of undrained shear strength

The sample size for this evaluation is much smaller, because each fall cone test results just one value for undrained shear strength. This means that only four values per depth are available, which is a very low number to make any reliable depth wise comparisons. However, as having the four values from six different depths, summing up to 24 data points \((n_v=24, 4\ s_v-values\ from\ 6\ depths)\), the general variability of the undrained shear strength can be studied. The results are shown in Table 2.

As can be seen from the results, the variation of the undrained shear strength is low and the COV-values are between 0.02 and 0.07. The lower COV-values here make sense when compared to the COV-values of penetration depths. As only the three mean penetration depth values per test (minimum and maximum values excluded) are used for interpretation of undrained shear strength, it is logical that the variation is smaller. On the other hand, this may cause some statistical inaccuracy as the sample size decreases. However, in overall the COV-values for both investigated parameters are rather low.
Table 2. Bias and coefficient of variation (COVv) of the error term related to undrained shear strength. From each depth, one value for undrained shear strength was calculated for each test, summing up to four values of sv per depth.

| Depth [m] | number of data points | sv bias COVv |
|-----------|------------------------|--------------|
| 1.36-1.39a | 4 | 1.00 0.07 |
| 2.34-2.37a | 4 | 1.00 0.04 |
| 3.25-3.28a | 4 | 1.00 0.06 |
| 4.35-4.38a | 4 | 1.00 0.02 |
| 5.38-5.41b | 4 | 1.00 0.06 |
| 7.32-7.35b | 4 | 1.00 0.04 |
| Whole site | 24 | 1.00 0.05 |

a) 60g cone used for samples from depths <5.0m
b) 100g cone used for samples from depths >5.0m

6. Discussion and conclusion
The results in Table 1 and Table 2 showed that the variation of both, the penetration depth of the cone and the undrained shear strength derived from FC, were low for the tested soft soil. The COV- values for both parameters were between 0.02 and 0.08. Some differences were observed between different depths and individual tests, but generally the repeatability of the fall cone test was good. Moreover, these values included the inherent variability and by taking that into account, the actual measurement uncertainty could be even lower.

The tests were done in Tampere University’s soil laboratory and by its staff. The same operator did the tests with the same fully calibrated equipment in order to reduce the uncertainties. Moreover, the tests were done with high-quality samples collected with block sampler. All four parallel fall cone tests were done for the same soil sample, making the results fully comparable. This study shows that by doing everything as well as possible, we can obtain very accurate test results with the fall cone test.

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