Experimental Study on Determining Elastic Modulus of Integrated Bamboo with Deflection Method

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Abstract. This paper mainly studies the experimental method of elastic modulus measurement of integrated bamboo. In this paper, experiments were carried out using rectangular section rods made of integrated bamboo. During the test, the rod was fixed at one end and the other end was cantilevered. A concentrated load is applied to the cantilever end of the rod to measure the projection of the rod end laser pointer on the projection plate. According to the calculation relationship between the projection displacement, the load and the elastic modulus, the elastic modulus of the integrated bamboo is measured.

1. Introduction
In order to study the mechanical properties of the integrated bamboo material with uniform distribution of materials on the cross section, especially the stability performance of the pressure bar, the author compares and analyzes more measurement methods, and finally uses the deflection method to measure the elastic modulus of the material. The key step in the experiment was how to accurately measure the micro-deformation of the bamboo. This paper introduces a simple method for measuring the micro-deformation of bamboo cantilever beam, on the basis of which the elastic modulus of the material is determined.

This paper mainly studies the experimental method of elastic modulus measurement of integrated bamboo. In this paper, experiments were carried out using rectangular section rods made of integrated bamboo. During the test, the rod was fixed at one end and the other end was cantilevered. A concentrated load is applied to the cantilever end of the rod to measure the projection of the rod end laser pointer on the projection plate. According to the calculation relationship between the projection displacement, the load and the elastic modulus, the elastic modulus of the integrated bamboo is measured.

2. Experimental scheme for measuring elastic modulus of bamboo with deflection method
According to Hooke's law, the stress $\sigma$ is linearly proportional to the strain $\varepsilon$ within a certain range of proportional limits. The ratio of stress to strain - $E$ is called elastic modulus or Young's modulus. That is, relationship (1).

$$E = \frac{\sigma}{\varepsilon} \quad (1)$$
Based on the basic theoretical knowledge of mechanics, the author deducts a method based on the cantilever beam deflection method to measure the elastic modulus.

The measurement diagram is shown in Figure 1: the distance between the fixed end and the loading position is l, the distance from the loading position to the projection wall is a, the deflection of the rod is ω, the rotation angle is θ, and the vertical deformation of the rod occurs during loading is b, the applied load is F.

![Figure 1 measurement diagram](image)

3. Related assumptions and solutions for deflection measurement

Assuming that the axis after the beam deformation is a smooth curve, the line displacement of the beam cross-section centroid in the x direction belongs to a high-order trace, which can be neglected compared to the deflection, so the line displacement in the x direction is not counted in the experiment.

The predetermined deflection ω is positive downward, the rotation angle θ is positive clockwise, and the bending moment M is drawn on the tension side, so that the micro-section beam is concave upwards to be positive. I is the moment of inertia, b and h are the outer cross-sectional dimensions of the rod, and b₀ and h₀ are the hollow section dimensions of the rod.

Using the deflection method to measure the elastic modulus of the integrated bamboo, the relationship between the measured and the elastic modulus is ultimately established.

The moment of inertia of a hollow rectangular section is known:

\[ I = 12^{-1} (bh^3 - b_0h_0^3) \]  

(2)

The relationship between bending moment and curvature:

\[ \rho(x)^{-1} = M(x)(EI)^{-1} \]  

(3)

The approximate curve of the deflection curve is:

\[ d^2\omega(dx^2)^{-1} = -M(x)(EI)^{-1} \]  

(4)

The bending moment equation of the cantilever beam:

\[ M(x) = -F(l - x) = F(x - l) \]  

(5)

Seeking Calculus:

\[ EI\omega' = -M(x) = -F(x - l) \]  

(6)

\[ EI\theta = Flx - Fx^22^{-1} + C \]  

(7)

\[ EI\omega = -6^{-1}Fx^3 + 2^{-1}Fxl^2 + Cx + D \]  

(8)

Fixed boundary conditions:

\[ \theta = \theta|_{x=l} = F(2EI)^{-1} \]  

(9)

\[ \omega = \omega|_{x=l} = F(3EI)^{-1} \]  

(10)

The equations of the corner and the curve of the curve obtained by bringing C=0, D=0 into the beam of equations (9) and (10) can be obtained:

\[ \theta = F(lx - x^2 / 2)(EI)^{-1} \]  

(11)

\[ \omega = F(lx^2 / 2 - x^3 / 6)(EI)^{-1} \]  

(12)
Bringing $x=l$ into equations (11) and (12) is calculated:

\[
\theta_b = \theta_{x=l} = Fl^2 (2EI)^{-1}
\]

(13)

\[
\omega_h = \omega_{x=l} = Fl^3 (3EI)^{-1}
\]

(14)

\[
\theta \approx \tan \theta = (b - \omega)a^{-1}
\]

(15)

After finishing and calculating, we can get:

\[
E = F(l^2a + \frac{2l^3}{3})(2lb)^{-1}
\]

(16)

4. Experimental steps and techniques

The rod with the laser is clamped on the fixed end support, and the principle of not crushing the rod and firmly fixing the rod is adopted, and the length of the cantilever end is 40 cm each time.

The loading position is loaded at a certain distance after the laser pointer. The four loading positions in the experiment are $=0.30m$, $=0.31m$, $=0.30m$, $=0.31m$. The laser is projected onto the opposite white paper.

In this experiment, an infrared rangefinder was used to measure the height of the fixed end from the ground and the horizontal distance of the fixed end from the white paper projection surface. The tester used a spirit level to adjust the level of the support, and at the same time, the projection point of the laser on the opposite white paper and the fixed end were approximately on the same horizontal line. During the test, the tester recorded the position of the center of the laser projection point with a steel ruler fixed on a white paper with a pen and a triangular plate.

The 20 g weight is loaded in sequence, and when the projection on the opposite white paper tends to be stable, the position of the center of the projection point is recorded. Until the fourth loading, at this time, a total of 5 positions are recorded on the white paper, and the distance between the centers of the projection points and the distance from the first position is taken as the deflection of the cantilever beam in the vertical direction.

Unload the 20g weight in turn, and observe and record the position of the center of the projection point on the projection surface again until all the weights are removed and recorded.

Remove the rod, perform the loading experiment with the other end of the rod, and repeat the previous steps until the end of the last experiment.

Finally, the experimental equipment is removed, the experimental site is sorted, and the experimental data is processed and calculated accordingly.

5. Calculation method and content

The corresponding a value is calculated by the horizontal distance and each loading position l. The corresponding a, l values and the corresponding relationship between the load F and the vertical displacement b will be recorded, and the relationship between the load and the vertical displacement will be plotted.

Figure 2. F vs. b (1st test)  
Figure 3. F vs. b (2nd test)
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Figure 4. F vs. b (3rd test)  

Figure 5. F vs. b (4th test)

It can be seen from the correlation coefficient calculated from the graph that the correlation between force and vertical displacement is high. The correlation coefficient is almost close to 1, which indirectly verifies the linear relationship between stress and strain in Hooke's law, indicating the effectiveness of the experiment.

In the deflection measurement, after each loading and unloading, the laser projection point can be restored to the projection point of the previous loading, and all the unloaded and restored to the origin, indicating that the material is experimental in the elastic range.

The slope is obtained from the graph to better establish the correlation between the elastic modulus and the slope, so as to better determine the elastic modulus to improve the accuracy of the experiment.

6. Processing of experimental data

According to Hooke's law, stress and strain are roughly proportional to the strain in the elastic range. The slope is obtained from the graph, and is calculated by the formula (16) of the elastic modulus measured by the deflection method, and the average value of the four sets of calculated data is taken as the elastic modulus of the integrated bamboo.

Table 1. b value of each group under load at each level

| F/N  | b₁/m | b₂/m | b₃/m | b₄/m |
|------|------|------|------|------|
| 0    | 0    | 0    | 0    | 0    |
| 0.2  | 0.097| 0.099| 0.099| 0.115|
| 0.4  | 0.197| 0.201| 0.203| 0.232|
| 0.6  | 0.296| 0.309| 0.308| 0.351|
| 0.8  | 0.398| 0.415| 0.412| 0.473|
| l/m  | 0.30 | 0.31 | 0.30 | 0.31 |
| a/m  | 5.577| 5.567| 5.577| 5.567|
| E/GPa| 10.81| 11.04| 10.41| 9.71 |

The slope of the fitted line from the first set of data is k₁=2.0099, which is calculated by the formula (16). It is known that the integrated bamboo section is rectangular section b×h=5mm×6mm, b₀×h₀=4mm×5mm, brought into Formula (2) gets:

\[ I = \frac{48.33 \times 10^{-11} m^4}{4.833 \times 10^{-11} m^4} \]  

(17)

Bring the known data into equation (16) and we get:

\[ E_i = 10.81 \times 10^9 Pa = 10.81 GPa \]  

(18)

Similarly, according to k₂=1.9225, E₂=11.04 GPa can be obtained; according to k₃=1.9359, E₃=10.41 GPa can be obtained; according to k₄=1.6918, E₄=9.71 GPa can be obtained. The average of the above measurements was obtained: Eavg = 10.49 GPa.

7. Analysis of results

In this paper, reasonable and effective tests are carried out according to certain test procedures. The amount of influence is controlled reasonably during the test. The material is always tested in the elastic range, and the obtained test data is accurately processed. Finally, the elastic modulus of the material after multiple tests is obtained. The average value has reached the purpose of this experimental study.
Although the correlation between load and vertical displacement in the test reached $R^2=0.9998$, there was still some error in the test. The bamboo used in the test was carefully selected bamboo with uniform texture, which excluded the influence of the test materials. The production of test rods has a reasonable division of labor. Only one person is responsible for the completion of each task, that is, the bonding process at both ends is exactly the same. The cut bamboo pieces are finely sanded with sandpaper, eliminating the bonding process. Different test errors. At the same time, all the test data are measured and recorded by a special person according to certain standards, so that one person is responsible for supervision and eliminates the influence of accidental error.

During the test, the distance between the fixed end and the projection surface is reasonably controlled. Each time the distance is kept constant, under this reasonable distance control, it is convenient to read the displacement of the material in the vertical direction. The above describes the ease of operation of this test method.

8. Conclusion

After the test is graded and loaded, the projection point position can be returned to the projection point of the previous loading. When all the unloading is completed, the projection point returns to the starting point, indicating that the force applied during the entire loading process is within the proportional limit of the material. Therefore, this test is effective and has certain research value.

The effect of shear force is not included in the test, and there is a certain error. The elastic modulus finally calculated in the test is the elastic modulus of the integrated bamboo material including glue, not the elastic modulus of the bamboo.

Acknowledgements

This paper has been supported by the scientific research foundation project of Yunnan Provincial Department of Education (2014Y386).

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