Research Article

A Computational Model for the Radiated Kinetic Molecular Postulate of Fluid-Originated Nanomaterial Liquid Flow in the Induced Magnetic Flux Regime

Azad Hussain,1 Aysha Rehman,1 Sohail Nadeem,2 M. Riaz Khan,3 and Alibek Issakhov4,5

1Department of Mathematics, University of Gujrat, Gujrat 50700, Pakistan
2Department of Mathematics, Quaid-I-Azam University, Islamabad 44000, Pakistan
3LSEC and ICMSEC, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, School of Mathematical Science, University of Chinese Academy of Sciences, Beijing 100190, China
4Department of Mathematical and Computer Modeling, Al-Farabi Kazakh National University, Almaty, Kazakhstan
5Department of Mathematical and Computer Modeling, Kazakh British-Technical University, Almaty, Kazakhstan

Correspondence should be addressed to Aysha Rehman; aysharehman1986@gmail.com

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The performance of mass transfer rate, friction drag, and heat transfer rate is illustrated in the boundary layer flow region via induced magnetic flux. In this recent analysis, the Buongiorno model is introduced to inspect the induced magnetic flux and radiative and convective kinetic molecular theory of liquid-initiated nanoliquid flow near the stagnant point. The energy equation is modified by radiation efficacy using the application of the Rosseland approximation. Through similarity variables, the available formulated partial differential equations are promoted into the nondimensional structure. The variation of the induced magnetic field near the wall goes up, and very far away, it decays when the size of the radiation characteristic ascends. The velocity amplitude expands by enlargement in the amount of the magnetic parameter, mixed convection, thermophoresis parameter, and fluid characteristic. The nanoparticle concentration reduces if the reciprocal of the magnetic Prandtl number expands. The temperature spectrum declines by enhancing the amount of the magnetic parameter. Drag friction decreases by the increment in the values of radiation and thermophoresis parameters. Heat transport rate increases when there is an increase in the values of Brownian and magnetic parameters. Mass transfer rate increases when there is incline in the values of the magnetic Prandtl and fluid parameter.

1. Introduction

Improving the thermal efficiency of fluid flows under different conditions and applications has always been a famous research area. Besides, the significance of this issue because of the very wide range of applications in industries has made this area attractive to scientists and companies working in this field. To improve effectiveness, any solution proposed for this in any application can have different technical aspects that should be considered and investigated adequately [1]. For example, Ellahi et al. [2] explored the slip effect in the Newtonian fluid two-phase flow. Particles of the nanosized Hafnium are utilized in the base fluid. Two cases are discussed for fluid under consideration, namely, (i) phase of particles and (ii) fluid phase. Three forms of geometries are investigated in both cases. Relevant studies of nano fluids are discussed in recent articles [3–19].

There are many engineering applications of the mixed convective boundary layer flow such as food processing, solidification system, and nuclear reactors. Convection also plays an important role in managing the production cycle such as medications and cosmetics. The transverse magnetic field that merged with the boundary layer-mixed convection flow towards an inclined plane with a wave is examined. The retardation inflow far from the magnetic field and leading-edge yield acceleration in the leading-edge close flow of the wavy sheet is observed by Wang and Chi-Chang [20]. Many
researchers analyzed the regime of mixed convection flows in their articles [21–26].

There are several uses of free convection in the presence of Lorentz forces, such as fire engineering and geophysics. Newly proposed nanotechnology is a new passive way to enhance heat transfer [27]. The induced magnetic field’s influence on temperature curves is represented by Ghosh et al. [28]. Vanita and Kumar [29] have examined transient flow towards a cone with the inclined magnetic field. Akbar et al. [30] investigated nanoparticle interaction for peristaltic flow in an asymmetric channel towards the magnetic field induced. Hayat et al. [32] observed second-grade nanoliquid stenosis arteries in the existence of the induced magnetic effect. Hayat et al. [36] portrayed the Ag-CuO/H2O rotating thermal radiation impacts for the flow of Falkner-Skan.

The radiation may be sunlight, infrared, or visible and the nature of the material emitted by such radiation depends on its exposure. Depending on how solar heat is collected and distributed or converted into solar electricity, a heat source and its systems are also categorized as either passive solar or active solar. Thermal radiations are defined as electromagnetic emissions from a sheet with a temperature greater than zero [34]. Viskant and Grosh [35] noted that when considering the power plants, hypersonic flights, cooling systems, and combustion chambers, radiations became an important part. Using Rosseland’s approximations, they addressed thermal radiation impacts for the flow of Falkner-Skan. Hayat et al. [36] portrayed the Ag-CuO/H2O rotating hybrid nanofluid flow in the existence of partial slip radiation impacts. Hussain et al. [37] noted the non-Newtonian fluid flow with radiation efficacy and time-dependent viscosity. Li et al. [38] examined the radiation impacts in the heat storage system by adding nanoparticles. Zeeshan et al. [39] investigated, due to entropy generation, the impacts of electromagnetohydrodynamics radiative diminishing internal energy of the pressure-driven dioxide-water titanium nanofluid flow.

The non-Newtonian fluid is more naturalistic to consider because of the rheological characteristics of physiological and industrial fluids. There is no extensive model that can describe the moving structure of all fluids due to the complex behavior of non-Newtonian fluids. Thus, to study non-Newtonian fluid flow characteristics, numerous models have been developed. The Eyring–Powell model was obtained from a liquid molecular hypothesis. And the inclusion of additional analytical constants was further improved. It accurately reflects the essence of Newtonian for low and high shear values. For example, rubber melts, condensed liquids, toiletries, cosmetics, and vegetable products are included in these fluids [40]. Appropriate studies of Eyring–Powell fluid may be mentioned in these articles [41–45].

This report is to narrate the specifications of radiative mass and heat transfer enhancement and flow analysis of the molecular kinetic theory of liquid-initiated boundary layer stagnation point nanofluid towards a vertical stretched surface. The non-Newtonian nanofluid model is manifested with the induced magnetic field, radiation efficacy, combined convection, Brownian, and thermodiffusion diffusion. The flow field describes that, in the form of partial differential equations, the laws of conservation of momentum are considered. By reducing the number of independent variables by using the technique of similarity transformation, these coupled equations are then purified into the system of ordinary differential equations. The results are interpreted by the MATLAB bvp4c technique. Induced magnetic pattern near the wall decreases, and far away, it increases with an increase in the values of the reciprocal of the magnetic Prandtl number. The concentration curve enhances when the number of magnetic, stretching, and Prandtl characteristics incline. This study of nanofluid is mainly applied in heat transfer devices such as electrical cooling systems, radiators, and heat exchangers.

2. Mathematical Formulations

Consider the incompressible, steady, two-dimensional (2D) stagnant point flow under the assumption of the induced magnetic field in the molecular kinetic theory of liquid-initiated nanofluid and heat transport enhancement in the existence of combined convection and radiation towards a vertical stretched sheet as shown in Figure 1. The surface stretching with velocity $u^\prime(x) = dx$ and ambient velocity is $u^\prime\infty(x) = bx$ while the origin is fixed at $O$; see Figure 1.

Taken the Cartesian coordinate structure, the velocity of liquid flow will change through $x$- and $y$-axes in a way that $x$–axis is assumed vertically and $y$–axis is supposed horizontally. The fundamental form of the kinetic molecular postulate of liquid-originated nanofluid is [45]

$$\tau = \left[ \mu + \frac{1}{\delta y} \sinh^{-1}\left( \frac{1}{c_1} \dot{y} \right) \right] A_1, \quad (1)$$

where

$$\dot{y} = \sqrt{\text{tr}(A_1^2)} \frac{1}{2}, \quad (2)$$

where $\delta$, $c_1$, $\mu$, $A_1$, and $\text{tr}$ are fluid parameters, dynamic viscosity, first Rivlin-Ericksen tensor, and trace, respectively. Here, $\tau$ is the extra stress tensor and $A_1 = [ (\text{gradv})^2 + \text{gradv} ]$. We consider the second-order approximation for sinh$^{-1}$ function as

$$\sinh^{-1}\left( \frac{1}{c_1} \dot{y} \right) \approx \frac{\dot{y}}{c_1} - \frac{\dot{y}^2}{6c_1}, \quad \text{where} \quad \left| \frac{1}{c_1} \dot{y} \right| \ll 1. \quad (3)$$

Then, equation (1) becomes

$$\tau = \left[ \mu + \frac{1}{\delta c_1} - \frac{\dot{y}^2}{6\delta c_1^2} \right] A_1. \quad (4)$$

Under these premises, the governing equations of this particular investigation are as follows:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \\
\frac{\partial H_1}{\partial x} + \frac{\partial H_2}{\partial y} = 0, \\
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} - \frac{\mu_{\infty}}{4\pi \rho_f} \left( H_1 \frac{\partial H_1}{\partial x} + H_2 \frac{\partial H_1}{\partial y} \right) = \left( u_{\infty} \frac{d u_{\infty}}{dx} - \frac{\mu_{\infty} H_{\infty}}{4\pi \rho_f} \frac{d H_{\infty}}{dx} \right) + \left[ v + \frac{1}{\delta \zeta \rho f} \right] \frac{\partial^2 u}{\partial y^2} \\
- \frac{1}{2\delta \zeta \rho f} \left( \frac{\partial u}{\partial y} \right)^2 + g \left( 1 - C \right) \beta \rho_{\infty} \left( T - T_{\infty} \right) \\
- \frac{g}{\rho_f} \left( \frac{\rho_p - \rho_{\infty}}{\rho_f} \right) \left( C - C_{\infty} \right), \\
\frac{\partial H_1}{\partial x} + \frac{\partial H_1}{\partial y} = H_1 \frac{\partial u}{\partial x} - H_2 \frac{\partial u}{\partial y} + \frac{\partial^2 H_1}{\partial y^2}, \\
\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \frac{\partial^2 T}{\partial y^2} + \frac{D_p}{\rho f} \frac{\partial C}{\partial y} \left( \frac{\partial T}{\partial y} \right)^2 - \frac{1}{(\rho \zeta)^2} \frac{\partial^2 q_r}{\partial y^2}, \\
\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = D_p \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_{\infty}} \frac{\partial^2 T}{\partial y^2},
\]

The above equations narrate the viscosity coefficient \( \nu \), where \((u, v)\) and \((H_1, H_2)\) describe the velocity and magnetic field components along the \( x \) and \( y \) directions, respectively, whereas \( u_{\infty}(x) = dx \) and \( H_{\infty}(x) = x \).
velocity and $y$ magnetic field at the edge of the boundary layer and $H_0$ is the uniform value of the vertical magnetic field at infinity.

The radiation heat flux is given by using Rosseland approximation:

$$q_r = \frac{4\sigma^*}{3k^*} \left( \frac{\partial T^4}{\partial y} \right), \quad (11)$$

where $\sigma^*$, $k^*$ are the Stefan–Boltzmann and the mean absorption coefficient, respectively, whereas via extending $T^4$ about $T_{\infty}$ in Taylor’s series and ignoring the larger terms,

$$T^4 \equiv 4T_{\infty}^4 - 3T_{\infty}^4. \quad (12)$$

Substituting equations (11) and (12) into (9), the heat equation takes the form

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k^*}{(\rho c_p)^*} \frac{\partial^2 T}{\partial y^2} + \frac{16\sigma^* T_{\infty}^3}{3\rho c_p L} \left( \frac{\partial T}{\partial y} \right)^2. \quad (13)$$

The relevant boundary conditions are

$$u = u_w(x) = dx,$$

$$v = 0, \quad \frac{\partial H_1}{\partial y} = 0,$$  

$$H_2 = 0, \quad T \rightarrow T_w, \text{ at } y \rightarrow 0,$$

$$u = u_{\infty}(x) = bx,$$

$$H_1 = H_{\infty},$$

$$T \rightarrow T_{\infty}, \quad C \rightarrow C_w, \text{ at } y \rightarrow \infty. \quad (15)$$

Invoking similarity transformations are defined by

$$\psi = \sqrt{\nu x} F(y),$$

$$y = \sqrt{\frac{d}{\nu}},$$

$$u = \frac{\partial \psi}{\partial y} = dx F(y),$$

$$v = -\frac{\partial \psi}{\partial x} = -\sqrt{\nu} \frac{dF(y)}{dy},$$

$$H_1 = \left( \frac{H_w x}{L} \right) h'(y),$$

$$H_2 = -\left( \sqrt{\frac{d}{\nu}} \left( \frac{H_w}{L} \right) \right) h(y),$$

$$u = u_w(x) = dx,$$

$$H_{\infty} = H_c \frac{x}{L},$$

$$T = T_{\infty} - \frac{T_w - T_{\infty}}{C_w - C_{\infty}},$$

$$\phi(\gamma) = \frac{C - C_{\infty}}{C_w - C_{\infty}}.$$

The magnetized pressure is described as

$$p = \rho + \frac{\mu |H|^2}{8\pi}. \quad (17)$$

Equations (5) and (6) are satisfied identically. Equations (7), (8), (10), and (13)–(15) reduce to

$$[1 + \varepsilon - \varepsilon m_1] f'' + F f'' - (F')^2 + \eta^2 + \beta \left( (h')^2 - hh'' - 1 \right) + \lambda \phi - N_e \phi = 0, \quad (18)$$

$$\alpha h'''' + Fh'' - hF'' = 0, \quad (19)$$

$$\frac{1}{Pr} \left( 1 + \frac{4}{3} R_d \right) \phi'' + \theta' F + N_b \phi' \theta' + N_t (\theta')^2 = 0, \quad (20)$$

$$\phi'' + Le Pr F \phi' + \frac{N_t}{N_b} \theta'' = 0, \quad (21)$$

with boundary conditions.
\[ F = 0, \]
\[ F' = 1, \]
\[ \theta = 1, \]
\[ \varphi = 0, \]
\[ h = 0, \]
\[ h'' = 0, \quad \text{at } y \to 0, \]
\[ F' = \eta, \]
\[ \theta = 0, \]
\[ \varphi = 1, \]
\[ h = 1, \quad \text{at } y \to \infty. \] (22)

Here, prime denotes derivative for \( y \) and other dimensionless parameters are described as
\[ \lambda_i = \frac{(1 - C_j)\beta q_m(T_w - T_\infty) x}{d^3 x}, \]
\[ N_t = \frac{(\rho_p - \rho_l)(C_w - C_\infty) x}{d^2 x}, \]
\[ \eta = \frac{b}{d}R_d = \frac{4T_w^2\alpha^s}{k^2 k_j}, \quad N_t = \frac{(\rho c_p)_j D_T(T_w - T_\infty)}{(pc_p)^2}, \quad \Pr = \frac{\nu}{\alpha} \]
\[ N_b = \frac{(\rho c_p)_{j} D_B(C_w - C_\infty)}{(pc_p)_{j}^2}, \quad \Le = \frac{\alpha}{D_B}, \quad \alpha = \frac{K}{(pc_p)_{j}}, \]
\[ \varepsilon = \frac{1}{\delta c_1 \mu}, \quad m_i = \frac{d^3 x^2}{2c_1^3}, \quad \gamma = \frac{\mu}{\rho_f}, \quad \alpha_1 = \frac{\alpha^s}{\nu}, \quad \Re^{(1/2)}_x = \sqrt{\frac{u_w x}{\nu}}, \quad \beta = \frac{H^2_{n_1} H_{n_0}}{4d^2 \rho p f} \] (24)

Physical quantities are very valuable from an engineering point of view. These quantities reported the flow behavior which is defined by local Nusselt number \( \text{Nu}_x \), skin friction \( C_f \), and local Sherwood number \( \text{Sh}_x \) definitions as follows:
\[ C_f = \frac{\tau_w}{\rho u_w}, \]
\[ \text{Nu}_x = \frac{x q_w}{k(T_w - T_\infty)}, \] (25)
\[ \text{Sh}_x = \frac{x q_m}{D_B(C_w - C_\infty)}. \]

where \( \tau_w \) represents the surface shear stress, \( q_w \) denotes the surface heat flux, and \( q_m \) presents the surface mass flux for fluid:
\[ \tau_w = \left[ \mu + \frac{1}{\delta c_1} - \frac{1}{6\delta c_1^2} \left( \frac{\partial u}{\partial y} \right)^2 \right] \frac{\partial u}{\partial y} \]
\[ q_w = \left[ -k \frac{\partial T}{\partial y} - \frac{16\sigma^s T^3}{3k^2} \left( \frac{\partial T}{\partial y} \right) \right] \]
\[ q_m = -D_B \frac{\partial C}{\partial y}. \] (26)

Using invoking transformation equation (16), the dimensionless local Nusselt number, skin friction, and the local Sherwood number become
\[ C_f \Re_x^{(1/2)} = \left[ (1 + \epsilon) f''(0) - \frac{\epsilon m_1}{3} (f''(0))^3 \right], \]
\[ \text{Nu}_x \Re_x^{(1/2)} = \left[ 1 + \frac{4}{3} R_d \right] \theta (0), \]
\[ \text{Sh}_x \Re_x^{(1/2)} = \phi' (0). \] (27)

3. Results and Discussion

Coupled nonlinear differential equations (18)–(21) and their boundary conditions (22) and (23) are numerically worked out by employing the MATLAB scheme. This portion illustrates the impact of nondimensional sundry characteristics on induced magnetic, temperature, velocity, and concentration flow characteristics numerically and graphically. Figure 2 portrays the impact of \( \eta \) versus \( h'(\gamma) \). It is noticeable that the induced magnetic spectrum falls when increasing in \( \eta \). Figure 3 designates the effects of \( \alpha_1 \) on the induced magnetic pattern. Dual behavior has been seen for \( \alpha_1 \), near the wall, it is getting down, and far away, it moves upward. The field of \( h'(\gamma) \) rises by enhancing the amount of \( \beta \) in Figure 4. With larger values of mixed convection characteristic, the flow of induced magnetic expands as shown in Figure 5. Figures 6 and 7 demonstrate the effect of Brownian and thermophoresis diffusion on the induced magnetic field, respectively. When there is increase in the amount of Brownian diffusion, the induced magnetic field decreases, and the induced magnetic field increases by growing quantity of thermophoresis. Figure 8 shows the consequence of the Prandtl number on the induced magnetic curve, and profile falls by a bigger amount of Prandtl. The impacts of thermal radiation flux on the magnetic field are described in Figure 9. The variation of \( h'(\gamma) \) near the wall moves upward and very far away it moves downward by increasing the amount of thermal radiation. Figure 10 exhibits the outcome of the fluid parameter on \( h'(\gamma) \) expands in the values of \( \epsilon \) which cause rise in \( h'(\gamma) \). Figure 11 exhibits the deviation of \( \beta \) on velocity curve. When there is expansion in the amount of \( \beta \), then there is increase in
velocity flow. Figure 12 portrays the effect of $\lambda_1$ on the flow of velocity. The diagram shows that the fluctuation shoots up by increment in the mixed convection parameter. Nanofluid behaviour is shown by Brownian and thermophoretic characteristics in Figures 13 and 14. It is depicted that the contrary attitude showed variation in velocity decline for $N_b$ and incline for $N_t$ by enhancing the quantity of these parameters. Figure 15 investigates that the velocity graph expands if the fluid parameter $\varepsilon$ rises. Figure 16 demonstrates that the field of velocity shrinks by rising the amount of the reciprocal of the magnetic Prandtl number.

Figure 26 shows the influence of buoyancy ratio parameter on temperature distribution field increases by increasing the amount of buoyancy ratio parameter. The nanoparticle concentration field decreases if the reciprocal of the magnetic Prandtl number rises in Figure 27. Figure 28 exhibits the outcome of buoyancy ratio characteristics on nanoparticle concentration. It is easily observed that the field of concentration reduces if the number of $N_r$ increases. Figures 30 and 31 define the matching outcomes on the concentration profile. When the size of $Pr$ and $\eta$ expands, the nanoparticle concentration grows. Streamlines graphs against the distinct amount $\alpha_1$, and are shown in Figures 32–35. Table 1 shows the impact of different characteristics on drag friction $C_f \Re^{(1/2)}$. The Nusselt number for noticeable amounts of $R_h$, $N_t$, $N_b$, $\varepsilon$, $\lambda_1$, $m_1$, $\beta$, and $\alpha_1$ is analyzed and characterized in Table 2. Table 3 portrays the deviation of different amounts of parameters on the local Sherwood number.

4. Concluding Remarks

We studied the molecular theory of liquid-originated non-Newtonian nanofluids which are commonly used in heat transfer devices such as heat exchangers, engine oils, electrical cooling systems (such as flat plates), nuclear
Figure 4: Upshot of $\beta$ on $h'(\gamma)$.

Figure 5: Results of $\lambda_1$ on $h'(\gamma)$.

Figure 6: Effects of $Nb$ on $h'(\gamma)$.
Figure 7: Consequences of $Nt$ on $h'(γ)$.

Figure 8: Upshot of $Pr$ on $h'(γ)$.

Figure 9: Consequence of $Rd$ on $h'(γ)$.

Figure 10: Upshot of $ε$ on $h'(γ)$.

Figure 11: Result of $β$ on $F'(γ)$.

Figure 12: Outcome of $λ₁$ on $F'(γ)$.
Figure 13: Result of Nb on $F'(\gamma)$.

Figure 14: Impact of Nt on $F'(\gamma)$.

Figure 15: Upshot of $\epsilon$ on $F'(\gamma)$.

Figure 16: Influence of $\alpha_1$ on $F'(\gamma)$.

Figure 17: Outcome of $\eta$ on $F'(\gamma)$.

Figure 18: Upshot of $\alpha_1$ on $\theta'(\gamma)$.
Figure 19: Result of $\beta$ on $\theta(\gamma)$.

Figure 20: Outcome of $Le$ on $\theta(\gamma)$.

Figure 21: Influence of $Nb$ on $\theta(\gamma)$.

Figure 22: Deviation of $Nt$ on $\theta(\gamma)$.

Figure 23: Consequence of $Pr$ on $\theta(\gamma)$.

Figure 24: Influence of $\eta$ on $\theta(\gamma)$.
Figure 25: Conclusion of $R_d$ on $\theta_\gamma$.

Figure 26: Result of $N_r$ on $\theta_\gamma$.

Figure 27: Outcome of $\alpha_1$ on $\theta_\gamma$.

Figure 28: Upshot of $\beta$ on $\theta_\gamma$.

Figure 29: Effect of $N_r$ on $\phi_\gamma$.

Figure 30: Impact of $Pr$ on $\phi_\gamma$. 
Figure 31: Result of Pr on $\phi(y)$.

Figure 32: Flowlines for $\alpha_1 = 0.5$.

Figure 33: Flowlines for $\alpha_1 = 1.5$. 
Figure 34: Flowlines for $\epsilon = 0.1$.

Figure 35: Flowlines for $\epsilon = 1$.

Table 1: Variation of $C_f ((Re_x)^{1/2})$ for distinct amounts of nondimensional parameters.

| $R_d$ | $N_t$ | $N_b$ | $\epsilon$ | $Pr$ | $\lambda_1$ | $m_1$ | $\eta$ | $N_t$ | $Le$ | $\beta$ | $\alpha_1$ | $C_f ((Re_x)^{1/2})$ |
|-------|------|------|-----------|-----|------------|------|------|------|-----|------|------|-----------------|
| 0.1   | 0.1  | 0.1  | 1         | 7   | 0.1        | 0.1  | 0.5  | 0.1  | 0.1 | 0.1  | 0.1  | -0.85974        |
| 0.3   |      |      |           |     |            |      |      |      |     |      |      | -0.87093        |
| 0.5   |      |      |           |     |            |      |      |      |     |      |      | -0.88148        |
| 0.1   | 0.2  | 0.1  | 1         | 7   | 0.1        | 0.1  | 0.5  | 0.1  | 0.1 | 0.1  | 0.1  | -0.93303        |
| 0.3   |      |      |           |     |            |      |      |      |     |      |      | -1.00570        |
| 0.5   |      |      |           |     |            |      |      |      |     |      |      | -1.14835        |
| 0.1   | 0.1  | 0.2  | 1         | 7   | 0.1        | 0.1  | 0.5  | 0.1  | 0.1 | 0.1  | 0.1  | -0.82344        |
| 0.3   |      |      |           |     |            |      |      |      |     |      |      | -0.81182        |
| 0.4   |      |      |           |     |            |      |      |      |     |      |      | -0.80635        |
| 0.1   | 0.1  | 0.1  | 1.2       | 7   | 0.1        | 0.1  | 0.5  | 0.1  | 0.1 | 0.1  | 0.1  | -1.43201        |
| 1.3   |      |      |           |     |            |      |      |      |     |      |      | -1.40042        |
| 1.4   |      |      |           |     |            |      |      |      |     |      |      | -1.37091        |
| 0.1   | 0.1  | 0.1  | 1         | 7   | 0.2        | 0.1  | 0.5  | 0.1  | 0.1 | 0.1  | 0.1  | -0.83200        |
| 0.3   |      |      |           |     |            |      |      |      |     |      |      | -0.80432        |
| 0.4   |      |      |           |     |            |      |      |      |     |      |      | -0.77670        |
Table 1: Continued.

| $R_d$ | Nt | Nb | $\epsilon$ | Pr | $\lambda_1$ | $m_1$ | $\eta$ | $N_r$ | Le | $\beta$ | $\alpha_1$ | $C_j(Re_x)^{1/2}$ |
|-------|----|----|-------------|----|-------------|------|------|------|-----|------|----------|-----------------|
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.2  | 0.4 | 0.1  | 0.1      | −0.84285        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.84659        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.85046        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −0.66689        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.09788        |
|       |    |    |             |    |             |      |      |      |     |      |          | 0.68368         |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −0.84019        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.85400        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.86503        |

Table 2: Variation of $Nu_x(Re_x)^{1/2}$ for distinct amounts of nondimensional parameters.

| $R_d$ | Nt | Nb | $\epsilon$ | Pr | $\lambda_1$ | $m_1$ | $\eta$ | $N_r$ | Le | $\beta$ | $\alpha_1$ | $Nu_x(Re_x)^{1/2}$ |
|-------|----|----|-------------|----|-------------|------|------|------|-----|------|----------|-----------------|
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.2  | 0.4 | 0.1  | 0.1      | 3.68235         |
|       |    |    |             |    |             |      |      |      |     |      |          | 3.46053         |
|       |    |    |             |    |             |      |      |      |     |      |          | 3.25753         |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | 2.10453         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.04608         |
|       |    |    |             |    |             |      |      |      |     |      |          | 1.91548         |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | 2.21832         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.27728         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.33814         |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | 2.46411         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.86748         |
|       |    |    |             |    |             |      |      |      |     |      |          | 3.22622         |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | 2.15845         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.15809         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.15774         |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | 2.18123         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.24457         |
|       |    |    |             |    |             |      |      |      |     |      |          | 2.32438         |

Table 3: Variation of $Sh_x(Re_x)^{1/2}$ for distinct amounts of nondimensional parameters.

| $R_d$ | Nt | Nb | $\epsilon$ | Pr | $\lambda_1$ | $m_1$ | $\eta$ | $N_r$ | Le | $\beta$ | $\alpha_1$ | $Sh_x(Re_x)^{1/2}$ |
|-------|----|----|-------------|----|-------------|------|------|------|-----|------|----------|-----------------|
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.2  | 0.4 | 0.1  | 0.1      | −1.93921        |
|       |    |    |             |    |             |      |      |      |     |      |          | −2.20711        |
|       |    |    |             |    |             |      |      |      |     |      |          | −2.43144        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −3.70463        |
|       |    |    |             |    |             |      |      |      |     |      |          | −5.34866        |
|       |    |    |             |    |             |      |      |      |     |      |          | −8.21711        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −1.03388        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.73383        |
|       |    |    |             |    |             |      |      |      |     |      |          | −0.58575        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −2.21493        |
|       |    |    |             |    |             |      |      |      |     |      |          | −2.57936        |
|       |    |    |             |    |             |      |      |      |     |      |          | −2.90386        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −1.93906        |
|       |    |    |             |    |             |      |      |      |     |      |          | −1.93674        |
|       |    |    |             |    |             |      |      |      |     |      |          | −1.93842        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −1.95958        |
|       |    |    |             |    |             |      |      |      |     |      |          | −2.01667        |
|       |    |    |             |    |             |      |      |      |     |      |          | −2.08858        |
| 0.1   | 0.1| 0.1| 1           | 7  | 0.1         | 0.5  | 0.1  | 0.1  | 0.2  | 0.4  | 0.1      | −1.94083        |
|       |    |    |             |    |             |      |      |      |     |      |          | −1.93972        |
|       |    |    |             |    |             |      |      |      |     |      |          | −1.93872        |
reactors, biomedicine lubricants, and radiators. The key points of observation in the recent analysis are as follows:

(i) Induced magnetic pattern near the wall declines, and far away, it inclines when \((a_1)\) intensifies. The variation of \(h'(y)\) field near the wall goes up and very far away it decays when the size \(R_d\) ascends.

(ii) \(h'(y)\) (nondimensional induced magnetic function) falls, whereas \((\eta)\), Brownian diffusion \((Nb)\), and Prandtl number \((Pr)\) rise. The field \(h'(y)\) expands by enhancing the amount of magnetic parameter \((\beta)\), mixed convection \((\lambda_1)\), thermophoresis parameter \((Nt)\), and fluid parameter \((\epsilon)\). The variation of \(h'(y)\) profile near the wall moves upward and very far away it moves down when the size \(R_d\) ascends.

(iii) The velocity amplitude expands by enlargement in the amount of magnetic parameter \((\beta)\), mixed convection \((\lambda_1)\), thermophoresis parameter \((Nt)\), fluid characteristic \((\epsilon)\), and stretching parameter \((\eta)\). \(F'(y)\) collapses by Brownian motion \((Nb)\) and \((a_1)\).

(iv) The temperature spectrum increases when the values of \((a_1)\), radiation parameter \((R_d)\), and buoyancy ratio \((N_r)\) increases and decreases by Prandtl number, magnetic parameter \((\beta)\), Brownian motion diffusion \((Nb)\), and stretching parameter \((\eta)\). \(\theta'(y)(\text{nondimensional temperature function})\) rises near the wall, and far away, it diminishes when there is increase in the values of \(Le\).

(v) The nanoparticle concentration portrait reduces if the reciprocal of the magnetic Prandtl number \((a_1)\) and \((N_r)\) rises. Concentration enlarges when the number of \((\beta)\), \((Pr)\), and \((\eta)\) grows.

(vi) Drag friction decays by the inclination in the values of \((R_d)\), \((Nt)\), \((m_1)\), and \((a_1)\). When inclining the amount of \((Nb)\), \((\epsilon)\), \((\lambda_1)\), and \((\beta)\), drag force expands.

(vii) Heat transfer rates are increased when there is an increase in the values of \((Nb)\), \((Pr)\), and \((\beta)\) and decrease when there is an increase in the values of \((R_d)\), \((Nt)\), \((m_1)\), and \((a_1)\).

(viii) Mass transfer rates diminish, for \((R_d)\), \((Nt)\), \((Pr)\), and \((\beta)\), but increases by \((Nb)\), \((m_1)\), and \((a_1)\).

### Abbreviations

- \(a_1\): Reciprocal of the magnetic Prandtl number (-)
- \(\mu\): Dynamic viscosity \(\text{(Nsm}^{-2}\text{)}\)
- \(k^*\): Mean absorption coefficient (-)
- \(b\): Body forces \(\text{(Nm}^{-3}\text{)}\)
- \(C\): Nanoparticles concentration \(\text{(kgm}^{-3}\text{)}\)
- \(F\): Dimensionless velocity function (-)
- \(T\): Temperature (-)
- \(Nb\): Brownian motion parameter (-)
- \(N_r\): Local Nusselt number (-)
- \(C_{\infty}\): Ambient fluid concentration \(\text{(kgm}^{-3}\text{)}\)
- \(T_w\): Hot fluid temperature (K)
- \(u, v\): Velocity components \(\text{(ms}^{-1}\text{)}\)
- \(m_1\): Fluid characteristic (-)
- \(r\): Extra stress tensor (-)
- \(\varepsilon\): Fluid characteristics (-)
- \(S_b\): Local Sherwood number (-)
- \(R_d\): Radiation parameter (-)
- \(\lambda_1\): Mixed convection parameter (-)
- \(\rho_f\): Density of the base fluid \(\text{(kgm}^{-3}\text{)}\)
- \(d_w\): Surface heat flux (-)
- \(np\): Nanoparticle (-)
- \(\alpha\): Thermal diffusivity \(\text{(m}^2\text{ms}^{-1}\text{)}\)
- \(\alpha_1\): Magnetic diffusivity (-)
- \(\beta\): Magnetic parameter (-)
- \(\sigma\): Magnetic parameter (-)
- \(\lambda\): Radiation parameter (-)
- \(\rho\): Density \(\text{(kgm}^{-3}\text{)}\)
- \(\tau_r\): Buoyancy ratio characteristics (-)
- \(\rho_f\): Dimensionless magnetic function (-)
- \(\theta\): Dimensionless heat transfer function (-)
- \(\rho_f\): Density of the nanoparticles \(\text{(kgm}^{-3}\text{)}\).

### Data Availability

The data that support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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