Median Statistics and the Hubble Constant

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ABSTRACT. Following Gott et al., we use Huchra’s final compilation of 553 measurements of the Hubble constant ($H_0$) to estimate median statistical constraints on $H_0$. Our median statistical analysis yields $H_0 = 68 \pm 5.5$ (or ±1) km s$^{-1}$ Mpc$^{-1}$, where the errors are the 95% statistical and systematic (or statistical) errors. These results are close to what Gott et al. found a decade ago, with smaller statistical errors and similar systematic errors. With about two-thirds more measurements, we are also able to clearly illustrate the presence and magnitude of systematic errors for different methods.

Online material: color figure

1. INTRODUCTION

The long and involved history of increasingly more accurate and precise measurements of the Hubble constant has resulted in an extensive list of more than 550 $H_0$ values recorded by Huchra. Rowan-Robinson (2009) notes that most recent (central) estimates of $H_0$ lie in the range of 62 to 72 km s$^{-1}$ Mpc$^{-1}$, although individual estimates can differ among themselves by 2 or 3 standard deviations (for recent reviews see Jackson 2007; Tammann et al. 2008; Freedman & Madore 2010). Here, we explore what a median statistical analysis of Huchra’s extensive compilation yields.

From time to time, the up-to-then status of measuring $H_0$ is reviewed, and normally a combined summary result for $H_0$ is presented. In Table 1 we list a few typical results from some of these reviews. Measuring errors are carefully compared and discussed in these reviews, and the combined summary estimate is very dependent on the error analysis.

Here, we follow Gott et al. (2001, hereafter G01) and use median statistics to determine what Huchra’s $H_0$ central values alone (i.e., ignoring the quoted errors) tell us about the true value and uncertainty of the Hubble constant. With about two-third more data than G01 (553 measurements versus 331), we confirm and strengthen the results of G01 and statistically illustrate the presence and size of systematic errors for different methods and research groups. We also examine how the estimated value of $H_0$ changes as we consider different subsamples of the complete list, and we argue that the estimate from the complete list is a robust estimate of the Hubble constant.

Our article is organized as follows. We first review some basic median statistics concepts from G01 in the following section. In § 3 these are applied in an analysis of Huchra’s $H_0$ list, where we also discuss some consistency tests and systematic errors and give constraints on the Hubble constant. We conclude in § 4.

2. MEDIAN STATISTICS AND ERRORS

Compared with a $\chi^2$ analysis, a median statistics analysis requires fewer hypotheses and is much less sensitive to being biased by outliers. See G01 for a comprehensive introduction to median statistics and its applications. Here, we restate the basic idea from G01 and emphasize some key points that are relevant to our analysis.

The basic idea of median statistics is that the true value of a physical quantity is the median of the set of (error-affected) measurements. This is based on the assumption that the data set meets two statistical requirements: (1) all the measurements are independent, and (2) there is no (overall) systematic error for the whole data set as a group. In other words, as the number of independent measurements goes to infinity, the median will converge to the true value. The median does not depend on the measurement errors (G01). There is no guarantee, however, that Huchra’s compilation of $H_0$ values satisfies these conditions.

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3 See http://cfa-www.harvard.edu/~huchra/.
4 For an analysis of an intermediate version of Huchra’s list with 461 measurements, see the Appendix of Chen et al. (2003). Chen et al. (2003) did not estimate systematic errors bars; instead, they used the earlier systematic error estimate of G01. In this article we estimate systematic error bars for the new list of $H_0$ measurements.
TABLE 1
SOME SUMMARY ESTIMATES OF THE HUBBLE CONSTANT
(in km s\(^{-1}\) Mpc\(^{-1}\))

| Estimate          | Reference                      |
|-------------------|--------------------------------|
| 67 ± 12           | Rowan-Robinson (1985)          |
| 80 ± 11           | Jacoby et al. (1992)           |
| 73 ± 11           | Jacoby et al. (1992)           |
| 76 ± 9            | van den Bergh (1992)           |
| 72 ± 8            | Freedman et al. (2001)         |
| 62 ± 1 ± 4        | Tammann et al. (2008)          |
| 73 ± 2 ± 4        | Freedman & Madore (2010)       |

Consider a data set consisting of \(N\) measurements for a quantity that meets the two preceding requirements. Sort the \(N\) measurements from the lowest value to the highest and label them \(M_i\), respectively, where \(i = 1, \ldots, N\). Then the probability that the true value for the quantity lies between \(M_i\) and \(M_{i+1}\) is

\[
P_i = \frac{2^{-N}N!}{i!(N-i)!},
\]

where we set \(M_0 = -\infty\) and \(M_{N+1} = +\infty\) (G01). The range from \(M_j\) to \(M_{N+1-j}\) (where \(j \leq N/2\)) defines a confidence limit (hereafter c.l.) of \(C_j\) percent, where

\[
C_j = 100 \times (P_j + P_{j+1} + \cdots + P_{N-j}).
\]

The \(C_j\) are a finite number of discrete values, with the number depending on \(N\). So for any confidence limit commonly used, for example, the 95% c.l., we take the c.l. corresponding to the \(C_j\), which is the smallest among those larger than 95 (G01). These confidence limits do not depend on the measurement errors (G01).

Note that the systematic error in the second preceding requirement is different from the individual systematic error quoted as part of the error for an individual measurement. If the systematic errors for the individual measurements are not correlated, we can treat them as random errors when combining individual measurements of a whole data set, as discussed in G01 and subsequently, and the total error can be estimated by studying the histogram of the whole data set, without going into details of the error analysis. But if all measurements are affected by the same systematic shift, i.e., there is a systematic error at the whole data set level, a median statistics analysis will give an incorrect result. This is unlikely to be an issue for the \(H_0\) data (G01). The intermediate case is that a subgroup of data share a similar individual systematic error. Here, we use “subgroup systematic” error to denote the parts of individual systematic errors that are common to all measurements within the subgroup.

Subgroup systematic error is likely the main reason that the first preceding requirement (statistical independence) is not satisfied. One estimate of the error contribution from this effect may be derived by dividing the \(N\) measurements into subgroups that belong to different measurement techniques (measurements in each such subgroup could very likely be affected by similar systematic effects) and then studying the differences between results from each subgroup (G01).

3. APPLICATION TO HUCHRA’S \(H_0\) LIST

3.1. Huchra’s \(H_0\) List

The final version of Huchra’s Hubble constant measurements list, updated on 2010 October 7, contains 553 published estimates (rounded to the nearest 1 km s\(^{-1}\) Mpc\(^{-1}\)), some as recent as 2010 September. All but three of them come with error bars. Most include both statistical and systematical errors, although a few have only statistical errors. In this article we use only the quoted central \(H_0\) value, and not the error, for each measurement. For simplicity, we also use a dimensionless number \(h\) instead of \(H_0\), where \(H_0 = 100h\) km s\(^{-1}\) Mpc\(^{-1}\).

Aside from some that restate previous results, most values on Huchra’s list are measurements that are either from different observations (different raw data), different data processings (including calibration and correction), or different methods (different relation between distance and observable) and may include different biases. Each of these has an associated error, any of which may make the final \(H_0\) value differ significantly. There are examples where the same observations and the same estimation technique results in differences as large as two standard deviations (see, for example, Rowan-Robinson 2009). The complexity of error sources and the difference in systematic errors estimated by different workers in the field make it a worthy goal to use median statistics to derive a summary estimate of \(H_0\) from Huchra’s list.\(^7\)

All but one measurement in Huchra’s list have a primary type label that indicates method used, and less than half of them also have a secondary type label that indicates the research group involved. For four measurements in the list the type labels in Huchra’s list file look ambiguous. We picked their type labels according to our understanding of both the corresponding references as well as Huchra’s definition of types. (The revised list file is available upon request.) The primary type provide a simple, but quite likely typical (G01), criterion for the subgroup study. For conciseness, we focus on the primary type classification in the text, mentioning secondary type results (shown along with the primary type results in Table 2 and Fig. 1) only when necessary. More sophisticated classification schemes require a careful analysis of the systematic effects, which is beyond the purview of this article.

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\(^7\) For the properties of some sources of error, see, for example, G01 and Rowan-Robinson (2009) and references therein.

\(^8\) Other techniques for analyzing heterogeneous collections of measurements, with possibly different systematic errors, can be found in, e.g., Press (1997), Bayesian method; Podariu et al. (2001) and Tammann et al. (2008), error-weighted averaging; and Freedman & Madore (2010), both Bayesian and frequentist methods.
3.2. Analysis of the Complete List

A median statistics analysis of the 553 $H_0$ values results in a formal median $h = 0.68$ and 95% statistical confidence limits of $0.67 < h < 0.69$. However, caution is in order when quoting these, because the two requirements of median statistics are very likely not fully met by this $H_0$ list. There are two main concerns here: systematic errors that are shared by some of the measurements (those in a subgroup) and the restating of prior results (in proceedings and summary articles) that result in restating correlations. Both of these effects make measurements in the list statistically dependent. Since we do not make use of the error information from the list, we will refer to these effects as (subgroup) systematic errors. As mentioned in §2, we can only check the reliability of the preceding results by studying the effects of subgroup systematics, since we choose to ignore the individual errors.

We first group the 553 measurements into 18 subgroups according to the primary type label value in Huchra’s list file. The size of each subgroup and the corresponding median statistics results are shown in Table 2, and Fig. 1 shows the histograms for all but the two smallest primary type subgroups. Note that the subgroup medians are different, and many differences between two subgroups are larger than half of the 95% confidence range of either group. It is fair to say that most subgroups have a subgroup systematic error that is close to their median minus the true value (see the discussion at the end of next subsection), and within each subgroup, statistical errors result in different measurements having different values. Clearly, systematic errors for different subgroups have different signs and different values. Following G01, from the row labeled “Subgroup medians” in the table, we see that $\pm 5.5$ km s$^{-1}$ Mpc$^{-1}$ is a reasonable estimate of the $\pm 95\%$ systematic errors. Furthermore, considering

### Table 2

**Hubble Constant Medians (in km s$^{-1}$ Mpc$^{-1}$) by Type**

| Type of estimate | Subgroup of the type | Subgroup excluding the type |
|------------------|----------------------|-----------------------------|
|                  | Number | Median | 95% c.l. (range) | Newest | Number | Median | 95% c.l. (range) |
| All data         | 553    | 68     | 67–69 (2)       | ...    | ...    | ...    | ...            |
| Global summary   | 111    | 70 (+2)| 68–72 (4)       | 73     | 442    | 67     | 65–69 (4) |
| SNe I            | 92     | 64 (–4)| 60–65 (5)       | 64     | 461    | 69     | 68–70 (2) |
| Other            | 83     | 68 (0) | 60–71 (11)      | 72     | 470    | 68     | 67–69 (2) |
| Grav. lensing    | 75     | 64 (–4)| 62–68 (6)       | 62     | 478    | 69     | 67–70 (3) |
| Sunyaev-Zel’dovich | 46     | 60.5 (–7.5) | 57–66 (9) | 74 | 507    | 69     | 67–70 (3) |
| IR Tully-Fisher  | 23     | 60 (–8) | 56–72 (16) | 71     | 530    | 68     | 67–70 (3) |
| SB fluctuations  | 19     | 82 (+14)| 65–90 (25) | 60     | 534    | 68     | 66–69 (3) |
| Tully-Fisher     | 18     | 75 (7) | 71–82 (11)      | 63     | 535    | 68     | 66–69 (3) |
| CMB fit          | 16     | 69.5 (+1.5) | 58–72 (14) | 71     | 537    | 68     | 67–69 (2) |
| Glob. cluster LF | 14     | 76.5 (+8.5) | 65–82 (17) | 69     | 539    | 68     | 66–69 (3) |
| $D_\odot - \sigma$ | 10     | 75     | ...            | 78     | ...    | ...    | ...            |
| I, R Tully-Fisher | 9      | 74     | ...            | 77     | ...    | ...    | ...            |
| SNe II           | 8      | 59.5   | ...            | 76     | ...    | ...    | ...            |
| Plan. nebulae LF | 6      | 85     | ...            | 77     | ...    | ...    | ...            |
| Novae            | 3      | 69     | ...            | 56     | ...    | ...    | ...            |
| Red giants       | 1      | 74     | ...            | 74     | ...    | ...    | ...            |
| No 1st type      | 2      | 85     | ...            | 85     | ...    | ...    | ...            |
| Subgroup medians | ...    | 71     | 64–75 (11)     | ...    | ...    | ...    | ...            |
| Newest values    | ...    | ...    | 63–76 (13)     | 71.5   | ...    | ...    | ...            |
| No 2nd type      | 315    | 69     | 67–70 (3)      | 76     | 238    | 67     | 63–70 (7) |
| Cosm. depend.    | 75     | 67     | 63–70 (7)      | 62     | 478    | 68     | 67–70 (3) |
| Sandage          | 71     | 55     | 55–57 (2)      | 63     | 482    | 70     | 69–71 (2) |
| Key project      | 62     | 72.5   | 71–74 (3)      | 73     | 491    | 67     | 65–68 (3) |
| de Vaucouleurs   | 21     | 95     | 80–99 (19)     | 80     | 532    | 68     | 66–69 (3) |
| Irvine conf.     | 5      | 65     | ...            | 63     | ...    | ...    | ...            |
| Theory           | 4      | 52.5   | ...            | 72     | ...    | ...    | ...            |

*a* Most of the names are self-explanatory; to avoid confusion, we elaborate on a few of the less obvious ones next. Global summary: combines several different results; Other: cannot be classified as one of Huchra’s named measurement methods; No 1st type: does not have a primary type (method used) label in Huchra’s file; No 2nd type: does not have a secondary type (research group) label in Huchra’s file; Cosm. depend.: cosmological-model-dependent; Irvine conf.: presented at the 1997 Irvine Proceedings of the National Academy of Sciences meeting; Theory: derived from fundamental theory.

*b* The parentheses is the corresponding systematic shift from 68.

*c* We only show the c.l. for subgroups with more than 10 measurements, because the c.l. for a smaller subgroup is not statistically reliable. The range is defined as the difference between the upper and lower limits.
the debates about systematic errors in this field, it is possible that the subgroup systematic errors are complex enough that we can consider them as pseudorandom errors at the level of the whole list (G01). In this case we can use these 18 subgroup medians to estimate the overall uncertainty. The result is $h = 0.68 \pm 0.055$ (95% total error). This may be quoted as a conservative constraint on the Hubble constant, since we are pretty sure that the 18 measurements (i.e., the subgroup medians) are statistically independent.

Now the all-data result has an extremely small uncertainty (95% confidence level) range of $2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, while the subgroup medians result has a relatively large one, $11 \text{ km s}^{-1} \text{ Mpc}^{-1}$. But since the number of measurements ($N$) affects the uncertainty estimate in a manner similar to

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9This is qualitatively different from the procedure of G01, where the subgroup medians are used to estimate the overall systematic uncertainty. Here, we think that the median statistics of the subgroup medians will give the overall uncertainty, including both the systematic uncertainty and the statistical uncertainty. However, since the statistical error here is significantly smaller than that determined in G01 (from a smaller set of measurements), while the systematic error has not changed significantly, resulting in the systematic error becoming even more dominant, the procedure adopted by us does not result in a quantitatively different total error bar compared with what the G01 prescription would give.

10As an alternate estimate, for those concerned about the reliability of early measurements, we can estimate a systematic uncertainty by using the latest measurement of each primary type. These are listed in the Table 2 column labeled “Newest,” and the corresponding results are listed in the row labeled “Newest values,” with a $2\sigma$ total error of $\pm 6.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$.
the $1/\sqrt{N}$ factor in mean statistics (G01), we cannot simply conclude that the larger error estimate includes more uncertainty information (the so-called systematic errors, G01) than the smaller one. To examine the effect of subgroup size on the uncertainty estimate, we perform a simulation. The simulation randomly regroups the 553 measurements into 18 subgroups of the same sizes listed in Table 2 and computes the median statistics of the subgroup medians of these new subgroups. Regrouping 100 times results in 100 sets of median statistics values (each set consists of the median of 18 subgroup medians and corresponding 95% confidence limits). The median of the 100 95% c.l. ranges is 5 km s$^{-1}$ Mpc$^{-1}$, with the largest one equal to 10 km s$^{-1}$ Mpc$^{-1}$. So we see that the uncertainty estimates from the 18 group medians in Table 2, 11 km s$^{-1}$ Mpc$^{-1}$, do indeed include a source of systematic error. This confirms that it is reasonable to use $\pm 0.055$ as the 95% total error.

3.3. Analysis of Subsamples of the List

One concern regarding the preceding results is that the median of all data, $h = 0.68$, may be affected by subgroup systematics. To check this, we perform a median statistics analysis of truncated lists of measurements, truncated by excluding one subgroup of measurements at a time. We only exclude the largest subgroups, since excluding a subgroup with only a few measurements does not result in a discernible change. The results are shown in the right-hand part of Table 2. We see that excluding any single subgroup does not significantly alter the median and c.l. range, at least in comparison with the preceding 95% total error. We ignore the “No 2nd type” results here, because it is not really a type and also has too many measurements included. The only suspicious cases are that excluding the global summary set, where the 95% c.l. range expands most, from 2 to 4 km s$^{-1}$ Mpc$^{-1}$, and that excluding the Sandage set, where the median changes most, from 68 to 70 km s$^{-1}$ Mpc$^{-1}$.

The two subgroups chosen previously have relatively smaller 95% c.l. ranges. Another subgroup that has a similar small 95% c.l. range is the “Key Project” type. We also point out here that the global summary type includes results from many summary articles, and is likely the main contributor to restating correlations. If we look at the histograms of the subgroups (Fig. 1), we see that, except for these three subgroups, the scatter within each subgroup is pretty large compared with 2 km s$^{-1}$ Mpc$^{-1}$, the 95% range for the 553 measurements, even after considering the approximate $1/\sqrt{N}$ factor effect. That may explain why none of these subgroups affect the median of all data significantly. As a further check, we construct a subsample that contains all the measurements except those belonging to either the primary type of global summary or the secondary types of key project or Sandage. There are 362 measurements in this subsample, and the median and 95% confidence limits are 68 and 66–69 km s$^{-1}$ Mpc$^{-1}$.

Another consistency check is a historical analysis (G01). Here, we consider two subsamples. One, the HST era set, only includes the 367 measurements after 1996; the other, the post-G01 set, only includes the 196 measurements added to Huchra’s list after G01. The corresponding results are 67 and 65–69 km s$^{-1}$ Mpc$^{-1}$, and 69 and 67–70 km s$^{-1}$ Mpc$^{-1}$, respectively. Note that there are 13 articles in Huchra’s list added after G01, although they predate G01. These are not included in the post-G01 set. As a reference, we also compute for two more subsamples, pre-HST and pre-G01, that are the complements of the preceding subsamples, respectively. The pre-HST set gives 71 and 67–75 km s$^{-1}$ Mpc$^{-1}$, while the pre-G01 set gives 67 and 65–69 km s$^{-1}$ Mpc$^{-1}$, identical to the G01 result.

As a consequence of these consistency checks, we believe that the all-data median and the 95% c.l. range of subgroup medians are fairly robust and together provide a reasonable summary estimate of $H_0$.

Now that we have a robust median value $H_0 = 68$ km s$^{-1}$ Mpc$^{-1}$, we can go back to look at the systematic errors mentioned in the last subsection. We can now quantify the systematic error of a subgroup as the difference between the median value of the subgroup and $H_0 = 68$ km s$^{-1}$ Mpc$^{-1}$. Using the corresponding values in Table 2, we can see that for most of the subgroups (with size larger than 10), the systematic errors are larger than or close to half of the 95% c.l. range. This is consistent with our conclusion from the simulations discussed in the last subsection and indicates a significant systematic shift from the true median for different estimate methods or different research groups. We point out here that the 95% c.l. range for each subgroup may be underestimated because of the restating correlations inside the subgroup. But the effect is likely small considering that even if half of the values in each group is the duplication of the other half, the 95% c.l. range will only expand by a factor of $\sqrt{2}$.

4. CONCLUSION

We use median statistics to study Huchra’s list of 553 Hubble constant measurements. Ignoring the errors associated with individual measurements, and assuming there is no systematic error at the whole list level, we determine a constraint on $H_0$. We use the median of the complete list and estimate the error by only sampling one value, the median, from every primary type of subgroup, in an attempt to eliminate any possible correlations. This constraint is $H_0 = 68 \pm 5.5$ km s$^{-1}$ Mpc$^{-1}$, where the 95% error bar includes both systematic and statistical errors. By studying various data subsets, we argue that this result is robust and so should be used as a summary estimate of $H_0$. From a purely statistical study, we also illustrate the size of systematic errors of different measurement methods and different research groups.

However, without diving into the detailed systematics of each measurement, the statistical independence required by the median statistics technique cannot be conclusively established.
for the all-data set. Nevertheless, given the complexity of the systematic errors associated with measuring distances (as evidenced by the heated debates about them), we believe that the preceding constraint is a reasonable summary value. It is probably significant that this lies in the middle of the low Tammann et al. (2008) value of $H_0 = 62.3 \pm 1.3$ km s$^{-1}$ Mpc$^{-1}$ and the high Freedman & Madore (2010) value of $H_0 = 73 \pm 4.5$ km s$^{-1}$ Mpc$^{-1}$ (both 1σ errors).

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