Path Integral Quantization of Quantum Gauge General Relativity

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Abstract

Path integral quantization of quantum gauge general relativity is discussed in this paper. First, we deduce the generating functional of green function with external fields. Based on this generating functional, the propagators of gravitational gauge field and related ghost field are deduced. Then, we calculate Feynman rules of various interaction vertices of three or four gravitational gauge fields and vertex between ghost field and gravitational gauge field. Results in this paper are the bases of calculating vacuum polarization of gravitational gauge field and vertex correction of gravitational couplings in one loop diagram level. As we have pointed out in previous paper, quantum gauge general relativity is perturbative renormalizable, and a formal proof on its renormalizability is also given in the previous paper. Next step, we will calculate one-loop and two-loop renormalization constant, and to prove that

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the theory is renormalizable in one-loop and two-loop level by direct calculations.

1 Introduction

Quantum gravity is proposed to unify general relativity and quantum theory. One of the biggest troubles for quantum gravity is the problems of perturbative renormalization. Gauge gravity is studied for a long time, and there are many versions of gauge gravity[1, 2, 3, 4]. It is expected that gauge gravity could solve the problem of renormalization of quantum gravity.

Quantum gauge general relativity is proposed to solve this problem[5, 6, 7, 8, 9, 3, 10]. It is a quantum theory of gravity proposed in the framework of quantum gauge field theory. In 2003, Quantum Gauge General Relativity(QGGR) is proposed in the framework of QGTG. Unlike Einstein’s general theory of relativity, the cornerstone of QGGR is the gauge principle, not the principle of equivalence, which will cause far-reaching influence to the theory of gravity. In QGGR, the field equation of gravitational gauge field is just the Einstein’s field equation, so in classical level, we can set up its geometrical formulation[11], and QGGR returns to Einstein’s general relativity in classical level. The field equation of gravitational gauge field in QGGR is the same as Einstein’s field equation in general relativity, so two equations have the same solutions, though mathematical expressions of the two equations are completely different. For classical tests of gravity, QGGR gives out the same theoretical predictions as those of GR[12], and for non-relativistic problems, QGGR can return to Newton’s classical theory of gravity[13]. Based on the coupling between the spin of a particle and gravitoelectromagnetic field, the equation of motion of spin can be obtained in QGGR. In post Newtonian approximations, this equation of motion gives out the same results as those of GR[14]. The equation of motion of a spinning test particle in gravitational field can also obtained[15]. It’s found that this motion deviates from traditional geodesic curve, and the deviation effects is detectable[16], which can be regarded as a new classical tests of gravity theory. QGGR is a perturbatively renormalizable quantum theory, and based on it, quantum effects of gravity[17, 18, 19, 20] and gravitational interactions of some basic quantum fields [21, 22] can be explored. Unification of fundamental interactions including gravity can be fulfilled in a semi-direct product gauge group[23, 24, 25, 26]. If we use the mass generation mechanism which is proposed in literature [27, 28], we can propose a new theory on gravity which contains massive graviton and the introduction of massive graviton does not affect the strict local gravitational gauge symmetry of the action and does not affect the traditional long-range gravitational
The existence of massive graviton will help us to understand the possible origin of dark matter.

In literature [3], a formal proof on the renormalizability of quantum gauge general relativity is given. The proof is not based on the calculation of loop diagrams, but based on generalized BRST symmetry and generalized Ward-Takahashi identities. This case is similar to that of traditional gauge field theory. We know that traditional gauge field theory is a renormalizable quantum theory [30, 31, 32, 33, 34, 35]. In gauge field theory, though there are many divergences in loop diagram calculations, the constraints from gauge symmetry will make all divergences cancel each other.

Now, we want to ask that the divergence cancellation mechanism in quantum gauge general relativity is really work in one- or two-loop level, as what we expected in the literature [3]? In order to prove that quantum gauge general relativity is perturbatively renormalizable in one-loop and two-loop level, we need first to calculate propagators of gravitational gauge field and ghost field, to determine the Feynman rules of various interaction vertices, and to calculate all divergent one-loop and two-loop Feynman diagrams. As a first step, we discuss quantization of quantum gauge general relativity, and determine Feynman rules of various vertices, which is the main goal of this paper. Next step, we will calculate all divergent one-loop Feynman diagram and discuss the renormalization problem of quantum gauge general relativity in one-loop level. Finally, we discuss the renormalization problem in two-loop level. So this paper is the first one of a serial of papers on the renormalization of quantum gauge general relativity. All these calculations are extremely complicated and time consuming. In order to avoid possible mistakes in analytical deductions, all important results are calculated at least two times, and two calculations are completely independent. Some important results are also checked by using Mathematica. How to use Mathematica to perform these calculations will be discussed in another paper.

2 Quantum Gauge General Relativity

In quantum gauge general relativity, the most fundamental quantity is gravitational gauge field $C_\mu(x)$, which is a vector in the corresponding Lie algebra. $C_\mu(x)$ can be expanded as

$$C_\mu(x) = C_\mu^\alpha(x) \hat{P}_\alpha, \quad (\mu, \alpha = 0, 1, 2, 3) \quad (2.1)$$

where $C_\mu^\alpha(x)$ is the component field and $\hat{P}_\alpha = -i \frac{\partial}{\partial x^\alpha}$ is the generator of global gravitational gauge group. The gravitational gauge covariant derivative is given by

$$D_\mu = \partial_\mu - igC_\mu(x) = G_\mu^\alpha \partial_\alpha, \quad (2.2)$$
where $g$ is the gravitational coupling constant and matrix $G$ is given by

$$G = (G^\mu_\alpha) = (\delta^\alpha_\mu - gC^\alpha_\mu). \quad (2.3)$$

Its inverse matrix is

$$G^{-1} = \frac{1}{I - gC} = (G^{-1}_\alpha^\mu). \quad (2.4)$$

Using matrix $G$ and $G^{-1}$, we can define two important composite operators

$$g^{\alpha\beta} = \eta^{\mu\nu}G^\alpha_\mu G^\beta_\nu, \quad (2.5)$$

$$g_{\alpha\beta} = \eta_{\mu\nu}G^{-1}_\mu^\alpha G^{-1}_\nu^\beta. \quad (2.6)$$

In quantum gauge general relativity, space-time is always flat and space-time metric is always Minkowski metric, so $g^{\alpha\beta}$ and $g_{\alpha\beta}$ are no longer space-time metric. They are only two composite operators which consist of gravitational gauge field.

The field strength of gravitational gauge field is defined by

$$F_{\mu\nu}(x) \overset{\Delta}{=} \frac{1}{-ig}[D_\mu, D_\nu] = F^\alpha_{\mu\nu}(x) \cdot \hat{P}_\alpha \quad (2.7)$$

where

$$F^\alpha_{\mu\nu} = G^\beta_\mu \partial_\beta C^\alpha_\nu - G^\beta_\nu \partial_\beta C^\alpha_\mu. \quad (2.8)$$

The Lagrangian of the quantum gauge general relativity is selected to be

$$\mathcal{L} = (\det G^{-1})\mathcal{L}_0, \quad (2.9)$$

where

$$\mathcal{L}_0 = -\frac{1}{16}\eta^{\rho\sigma}\eta^{\nu\sigma}g_{\alpha\beta}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma} - \frac{1}{8}\eta^{\mu\rho}G^{-1}_{\beta}G^{-1}_{\alpha}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma} + \frac{1}{4}\eta^{\mu\rho}G^{-1}_{\alpha}G^{-1}_{\beta}F^\alpha_{\mu\nu}F^\beta_{\rho\sigma}. \quad (2.10)$$

Its space-time integration gives out the action of the system

$$S = \int d^4x \mathcal{L}. \quad (2.11)$$
3 Path Integral Quantization of Gravitational Gauge Fields

Gravitational gauge field $C_\mu^\alpha$ has $4 \times 4 = 16$ degrees of freedom. But, if gravitons are massless, the system has only $2 \times 4 = 8$ degrees of freedom. There are gauge degrees of freedom in the theory. Because only physical degrees of freedom can be quantized, in order to quantize the system, we have to introduce gauge conditions to eliminate un-physical degrees of freedom. For the sake of convenience, we take temporal gauge conditions

$$C_0^\alpha = 0, \quad (\alpha = 0, 1, 2, 3). \quad (3.1)$$

In temporal gauge, the generating functional $W[J]$ is given by

$$W[J] = N \int [DC] \left( \prod_{\alpha, \mu} \delta(C_\mu^\alpha(x)) \right) \exp \left\{ i \int d^4x (\mathcal{L} + J_\mu^\alpha C_\mu^\alpha) \right\} \quad (3.2)$$

where $N$ is the normalization constant, $J_\mu^\alpha$ is a fixed external source and $[DC]$ is the integration measure,

$$[DC] = \prod_{\mu=0}^{3} \prod_{\alpha=0}^{3} \prod_{j} \left( \varepsilon dC_\mu^\alpha(\tau_j) / \sqrt{2\pi i\hbar} \right). \quad (3.3)$$

We use this generating functional as our starting point of the path integral quantization of gravitational gauge field.

Generally speaking, the action of the system has local gravitational gauge symmetry, but the gauge condition has no local gravitational gauge symmetry. If we make a local gravitational gauge transformations, the action of the system is kept unchanged while gauge condition will be changed. Therefore, through local gravitational gauge transformation, we can change one gauge condition into another gauge condition. The most general gauge condition is

$$f^\alpha(C(x)) - \varphi^\alpha(x) = 0, \quad (3.4)$$

where $\varphi^\alpha(x)$ is an arbitrary space-time function. The Fadeev-Popov determinant $\Delta_f(C)$ is defined by

$$\Delta_f^{-1}(C) \equiv \int [Dg] \prod_{x,\alpha} \delta \left( f^\alpha(\partial C(x)) - \varphi^\alpha(x) \right), \quad (3.5)$$
where $g$ is an element of gravitational gauge group, $^gC$ is the gravitational gauge field after gauge transformation $g$ and $[\mathcal{D}g]$ is the integration measure on gravitational gauge group

$$[\mathcal{D}g] = \prod_x \text{d}^4\epsilon(x),$$  \hspace{1cm} (3.6) $$

where $\epsilon(x)$ is the transformation parameter of $\hat{U}_\epsilon$. Both $[\mathcal{D}g]$ and $[\mathcal{D}C]$ are not invariant under gravitational gauge transformation. Suppose that,

$$[\mathcal{D}(gg')] = J_1(g')[\mathcal{D}g], \quad (3.7)$$

$$[\mathcal{D}^gC] = J_2(g)[\mathcal{D}C]. \quad (3.8)$$

$J_1(g)$ and $J_2(g)$ satisfy the following relations

$$J_1(g) \cdot J_1(g^{-1}) = 1, \quad (3.9)$$
$$J_2(g) \cdot J_2(g^{-1}) = 1. \quad (3.10)$$

It can be proved that, under gravitational gauge transformations, the Fadeev-Popov determinant transforms as

$$\Delta^{-1}_f(^gC) = J_1^{-1}(g')\Delta^{-1}_f(C). \quad (3.11)$$

Insert eq.(3.5) into eq.(3.2), we get

$$W[J] = N \int [\mathcal{D}g] \int [\mathcal{D}C] \left[ \prod_{\alpha,y} \delta(g^{-1}C_\alpha^\alpha(y)) \right]\cdot \Delta_f(C)$$

$$\cdot \left[ \prod_{\beta,z} \delta(f^\beta(\tilde{g}C(z)) - \varphi^\beta(z)) \right]\cdot \exp \left\{ i \int \text{d}^4x (\mathcal{L} + J^\mu_{\alpha} \cdot g^{-1}C^\alpha_{\mu}) \right\}. \quad (3.12)$$

Make a gravitational gauge transformation,

$$C(x) \rightarrow ^gC(x), \quad (3.13)$$
then,

$$^gC(x) \rightarrow ^{gg^{-1}}C(x). \quad (3.14)$$

After this transformation, the generating functional is changed into

$$W[J] = N \int [\mathcal{D}g] \int [\mathcal{D}C] \cdot \left[ \prod_{\alpha,y} \delta(g^{-1}C_\alpha^\alpha(y)) \right]\cdot \Delta_f(C)$$

$$\cdot \left[ \prod_{\beta,z} \delta(f^\beta(C(z)) - \varphi^\beta(z)) \right]\cdot \exp \left\{ i \int \text{d}^4x (\mathcal{L} + J^\mu_{\alpha} \cdot g^{-1}C^\alpha_{\mu}) \right\}. \quad (3.15)$$
Suppose that the gauge transformation $g_0(C)$ transforms general gauge condition $f^\beta(C) - \varphi^\beta = 0$ to temporal gauge condition $C^\alpha_0 = 0$, and suppose that this transformation $g_0(C)$ is unique. Then two $\delta$-functions in eq.(3.15) require that the integration on gravitational gauge group must be in the neighborhood of $g_0^{-1}(C)$. Therefore eq.(3.15) is changed into

$$W[J] = N \int [DC] \Delta_f(C) \cdot \left[ \prod_{\beta,z} \delta(f^\beta(C(z)) - \varphi^\beta(z)) \right]$$

$$\cdot \exp \left\{ i \int d^4x (\mathcal{L} + J^\mu_\alpha \cdot g_0 C^\alpha_\mu) \right\}$$

$$\cdot J_1(g_0^{-1}) J_2(g_0) \cdot \int [Dg] \left[ \prod_{\alpha,y} \delta(g_0^{-1} C^\alpha_0(y)) \right].$$

(3.16)

The last line in eq.(3.16) will cause no trouble in renormalization, and if we consider the contribution from ghost fields which will be introduced below, it will become a quantity which is independent of gravitational gauge field. So, we put it into normalization constant $N$ and still denote the new normalization constant as $N$.

We also change $J^\mu_\alpha \cdot g_0 C^\alpha_\mu$ into $J^\mu_\alpha C^\alpha_\mu$, this will cause no trouble in renormalization. Then we get

$$W[J] = N \int [DC] \Delta_f(C) \cdot \left[ \prod_{\beta,z} \delta(f^\beta(C(z)) - \varphi^\beta(z)) \right]$$

$$\cdot \exp \left\{ i \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta + J^\mu_\alpha C^\alpha_\mu) \right\}.$$  

(3.17)

In fact, we can use this formula as our start-point of path integral quantization of gravitational gauge field, so we need not worried about the influences of the third line in eq.(3.16).

Use another functional

$$\exp \left\{ -\frac{i}{2\alpha} \int d^4x \eta_{\alpha\beta} \varphi^\alpha(x) \varphi^\beta(x) \right\},$$

(3.18)

times both sides of eq.(3.17) and then make functional integration $\int [D\varphi]$, we get

$$W[J] = N \int [DC] \Delta_f(C) \cdot \exp \left\{ i \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta + J^\mu_\alpha C^\alpha_\mu) \right\}.$$  

(3.19)

Now, let’s discuss the contribution from $\Delta_f(C)$ which is related to the ghost fields. Suppose that $g = \hat{U}_\epsilon$ is an infinitesimal gravitational gauge transformation.
gravitational gauge transformation of gravitational gauge field $C^\alpha_\mu(x)$ is \cite{5, 6, 7, 8, 9, 3, 10}

$$
C^\alpha_\mu(x) \rightarrow gC^\alpha_\mu(x) = \Lambda^\alpha_\beta(\hat{U}_eC^\beta_\mu(x)) - \frac{1}{g}(\hat{U}_e\partial_\mu\epsilon^\alpha(y)), \quad (3.20)
$$

Then we have

$$
gC^\alpha_\mu(x) = C^\alpha_\mu(x) - \frac{1}{g}D^\alpha_\mu \sigma \epsilon^\sigma, \quad (3.21)
$$

where

$$
D^\alpha_\mu \sigma = \delta^\alpha_\sigma \partial_\mu - g\delta^\alpha_\sigma C^\beta_\mu \partial_\beta + g\partial_\sigma C^\alpha_\mu. \quad (3.22)
$$

In order to deduce eq.(3.21), the following relation is used

$$
\Lambda^\alpha_\beta = \delta^\alpha_\beta + \partial_\beta \epsilon^\alpha + o(\epsilon^2). \quad (3.23)
$$

$D_\mu$ can be regarded as the covariant derivative in adjoint representation, for

$$
(D_\mu \epsilon)^\alpha = D^\alpha_\mu \sigma \epsilon^\sigma. \quad (3.25)
$$

Using all these relations, we have,

$$
f^\alpha(gC(x)) = f^\alpha(C) - \frac{1}{g} \int d^4y \frac{\delta f^\alpha(C(x))}{\delta C^\beta_\mu(y)} D^\beta_\mu \sigma(y) \epsilon^\sigma(y) + o(\epsilon^2). \quad (3.26)
$$

Therefore, according to eq.(3.5) and eq.(3.4), we get

$$
\Delta^{-1}_f(C) = \int [D\epsilon] \prod_{x,\alpha} \delta \left( -\frac{1}{g} \int d^4y \frac{\delta f^\alpha(C(x))}{\delta C^\beta_\mu(y)} D^\beta_\mu \sigma(y) \epsilon^\sigma(y) \right). \quad (3.27)
$$

Define

$$
M^\alpha_\sigma(x, y) = -g \frac{\delta}{\delta \epsilon^\sigma(y)} f^\alpha(gC(x)) \quad (3.28)
$$

$$
= \int d^4z \frac{\delta f^\alpha(C(x))}{\delta C^\beta_\mu(z)} D^\beta_\mu \sigma(z) \delta(z - y).
$$

Then eq.(3.27) is changed into

$$
\Delta^{-1}_f(C) = \int [D\epsilon] \prod_{x,\alpha} \delta \left( -\frac{1}{g} \int d^4y M^\alpha_\sigma(x, y) \epsilon^\sigma(y) \right) \quad (3.29)
$$

$$
= \text{const.} \times (\text{det}\, M)^{-1}.
$$

Therefore,

$$
\Delta_f(C) = \text{const.} \times \text{det}M. \quad (3.30)
$$
Put the above constant into normalization constant, then generating functional eq.(3.19) is changed into

\[
W[J] = N \int [DC] \det M \cdot \exp \left\{ \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta + J^\mu_{\alpha} C^\alpha_{\mu}) \right\}. \tag{3.31}
\]

In order to evaluate the contribution from \(\det M\), we introduce ghost fields \(\eta^\alpha(x)\) and \(\bar{\eta}_\alpha(x)\). Using the following relation

\[
\int [D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x d^4y \ \bar{\eta}_\alpha(x) M^\alpha_\beta(x, y) \eta^\beta(y) \right\} = \text{const.} \times \det M \tag{3.32}
\]

and put the constant into the normalization constant, we can get

\[
W[J] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta + \bar{\eta} M \eta + J^\mu_{\alpha} C^\alpha_{\mu}) \right\}, \tag{3.33}
\]

where \(\int d^4x \bar{\eta} M \eta\) is a simplified notation, whose explicit expression is

\[
\int d^4x \bar{\eta} M \eta = \int d^4x d^4y \ \bar{\eta}_\alpha(x) M^\alpha_\beta(x, y) \eta^\beta(y). \tag{3.34}
\]

The appearance of the non-trivial ghost fields is an inevitable result of the non-Able nature of the gravitational gauge group.

Set external source \(J^\mu_{\alpha}\) to zero, we get,

\[
W[0] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta + \bar{\eta} M \eta) \right\}, \tag{3.35}
\]

Now, let’s take Lorentz covariant gauge condition,

\[
f^\alpha(C) = \partial^\mu C^\alpha_{\mu}. \tag{3.36}
\]

Then

\[
\int d^4x \bar{\eta} M \eta = - \int d^4x (\partial^\mu \bar{\eta}_\alpha(x)) D^\alpha_{\mu} \beta(x) \eta^\beta(x). \tag{3.37}
\]

And eq.(3.35) is changed into

\[
W[0] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta - (\partial^\mu \bar{\eta}_\alpha) D^\alpha_{\mu} \sigma \eta^\sigma) \right\}. \tag{3.38}
\]
For quantum gauge general relativity, the external source of gravitational gauge field should be introduced in a special way. Define the generating functional with external sources as

\[
W[J, \beta, \bar{\beta}] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x (\mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta - (\partial^\mu \bar{\eta}_\alpha) D_\mu \sigma \eta^\sigma + C_\mu^\alpha \delta_{\alpha\nu} (x) J^\nu_\beta + \bar{\eta}_\alpha \beta^\alpha + \bar{\beta}_\alpha \eta^\alpha) \right\}
\]

(3.39)

where \( \sim^{\mu\gamma} \delta_{\alpha\rho} (x) \) is defined by

\[
\sim^{\mu\gamma} \delta_{\alpha\rho} (x) \triangleq \frac{1}{2} \left( \sim^{\mu} \delta_\rho^\alpha (x) \delta_\alpha^\gamma (x) + \sim^{\gamma} \eta_\alpha^\beta (x) \right),
\]

(3.40)

and

\[
J^\mu_\alpha \triangleq \delta_{\alpha\nu} (x) J^\nu_\beta.
\]

(3.41)

In the above definition, \( \sim^{\mu} \delta_\rho^\alpha (x) \), \( \sim^{\gamma} \eta_\alpha^\beta (x) \) and \( \sim^{\mu\gamma} \eta_\mu^\gamma (x) \) are defined by

\[
\sim^{\mu} \delta_\rho^\alpha (x) = \delta^\mu_\rho = \frac{\partial^\mu \partial_\rho}{\Box + i\epsilon},
\]

(3.42)

\[
\sim^{\gamma} \eta_\alpha^\beta (x) = \eta^{\alpha\gamma} - \frac{\partial^\mu \partial_\gamma}{\Box + i\epsilon},
\]

(3.43)

\[
\sim^{\mu\gamma} \eta_\mu^\gamma (x) = \eta_{\mu\gamma} - \frac{\partial_\mu \partial_\gamma}{\Box + i\epsilon},
\]

(3.44)

where

\[
\Box \triangleq \partial^2 = \partial^\mu \partial_\mu = \eta^{\mu\nu} \partial_\mu \partial_\nu.
\]

(3.45)

Using these relations, we can prove that

\[
J^\mu_\alpha = \sim^{\mu\beta} \delta_{\alpha\nu} J^\nu_\beta.
\]

(3.46)

The effective Lagrangian \( \mathcal{L}_{\text{eff}} \) is defined by

\[
\mathcal{L}_{\text{eff}} \equiv \mathcal{L} - \frac{1}{2\alpha} \eta_{\alpha\beta} f^\alpha f^\beta - (\partial^\mu \bar{\eta}_\alpha) D_\mu \sigma \eta^\sigma.
\]

(3.47)
\( \mathcal{L}_{\text{eff}} \) can be separate into free Lagrangian \( \mathcal{L}_F \) and interaction Lagrangian \( \mathcal{L}_I \),

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_F + \mathcal{L}_I,
\]

where

\[
\mathcal{L}_F = -\frac{1}{16} \eta^{\mu\rho} \eta^{\sigma} F_{0\alpha\beta} F_{\rho\sigma} - \frac{1}{8} \eta^{\mu\rho} F_{0\alpha\beta} F_{\rho\sigma}^{\beta} + \frac{1}{4} \eta^{\mu\rho} F_{0\alpha\beta} F_{\rho\sigma}^{\beta} 
- \frac{1}{2\alpha} \eta_{\alpha\beta} (\partial^\mu C_{\mu}^{\alpha})(\partial^\nu C_{\nu}^{\beta}) - (\partial^\mu \bar{\eta}_{\alpha})(\partial^\mu \eta_{\alpha}),
\]

\[
\mathcal{L}_I = +g (\partial^\mu \bar{\eta}_{\alpha}) C_{\mu}^{\alpha}(\partial^\beta \eta_{\alpha}) - g (\partial^\mu \eta_{\alpha})(\partial^\sigma C_{\mu}^{\alpha}) \eta_{\alpha}
+ \text{self interaction terms of Gravitational gauge field}.
\]

From the interaction Lagrangian, we can see that ghost fields do not couple to \( J(C) \).
This is the reflection of the fact that ghost fields are not physical fields, they are virtual fields. Besides, the gauge fixing term does not couple to \( J(C) \) either. Using effective Lagrangian \( \mathcal{L}_{\text{eff}} \), the generating functional \( W[J, \beta, \bar{\beta}] \) can be simplified to

\[
W[J, \beta, \bar{\beta}] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x (\mathcal{L}_{\text{eff}} + J_{\alpha} C_{\mu}^{\alpha} + \bar{\eta}_{\alpha} \beta^{\alpha} + \bar{\beta}_{\alpha} \eta^{\alpha}) \right\},
\]

(3.51)

## 4 Propagators

Using eq.(3.49), we can deduce propagator of gravitational gauge fields and ghost fields. First, after a partial integration, we change the form of eq. (3.49) into

\[
\int d^4x \mathcal{L}_F = \int d^4x \left\{ \frac{1}{2} C_{\mu}^{\alpha} M_{\alpha\beta}^{\mu\nu}(x) C_{\nu}^{\beta} + \bar{\eta}_{\alpha} \partial^2 \eta^{\alpha} \right\},
\]

(4.1)

where the operator \( M_{\alpha\beta}^{\mu\nu}(x) \) is defined by

\[
M_{\alpha\beta}^{\mu\nu}(x) = \frac{1}{7} \eta^{\mu\nu} \eta_{\alpha\beta} \partial^\rho \partial_\rho - \frac{1}{7} \eta_{\alpha\beta} (1 - \frac{4}{\alpha}) \partial^\mu \partial^{\nu} - \frac{1}{4} \delta^{\mu}_{\beta} \delta^{\nu}_{\alpha} \partial^\rho \partial_\rho 
+ \frac{3}{4} \delta^{\mu}_{\beta} \delta^{\nu}_{\alpha} \partial^\rho \partial_\rho - \frac{1}{4} \eta^{\mu\nu} \partial^\rho \partial_\rho + \frac{1}{2} \delta^{\mu}_{\beta} \delta^{\nu}_{\alpha} \partial^\rho \partial_\rho
\]

(4.2)
Denote the propagator of gravitational gauge field as

\[-i \Delta_{F_{\mu\nu}}^{\alpha\beta}(x), \quad (4.3)\]

and denote the propagator of ghost field as

\[-i \Delta_{F_{\beta}}^{\alpha}(x). \quad (4.4)\]

They satisfy the following equation,

\[\begin{align*}
-\mathcal{M}_{\alpha\beta}^{\mu\nu}(x) \Delta_{F_{\nu\rho}}^{\beta\gamma}(x-y) &= \delta_{\alpha\rho}(x) \delta(x-y), \\
-\partial^2 \Delta_{F_{\beta}}^{\alpha}(x-y) &= \delta_{\beta}(x-y),
\end{align*} \quad (4.5)\]

where \(\delta_{\alpha\rho}(x)\) is defined by (3.40).

Make Fourier transformations to momentum space

\[\begin{align*}
-i \Delta_{F_{\mu\nu}}^{\alpha\beta}(x) &= \int \frac{d^4k}{(2\pi)^4} (-i)^{\sim} \Delta_{F_{\mu\nu}}^{\alpha\beta}(k) \cdot e^{ikx}, \\
-i \Delta_{F_{\beta}}^{\alpha}(x) &= \int \frac{d^4k}{(2\pi)^4} (-i)^{\sim} \Delta_{F_{\beta}}^{\alpha}(k) \cdot e^{ikx},
\end{align*} \quad (4.7, 4.8)\]

where \(-i \sim \Delta_{F_{\mu\nu}}^{\alpha\beta}(k)\) and \(-i \sim \Delta_{F_{\beta}}^{\alpha}(k)\) are corresponding propagators in momentum space. They satisfy the following equations,

\[\begin{align*}
-\mathcal{M}_{\alpha\beta}^{\mu\nu}(k) \Delta_{F_{\nu\rho}}^{\beta\gamma}(k) &= \delta_{\alpha\rho}(k), \\
k^2 \sim \Delta_{F_{\beta}}^{\alpha}(k) &= \delta_{\beta}(k),
\end{align*} \quad (4.9, 4.10)\]

where the operator \(\mathcal{M}_{\alpha\beta}^{\mu\nu}(k)\) is defined by

\[\mathcal{M}_{\alpha\beta}^{\mu\nu}(k) \overset{\Delta}{=} -\frac{1}{4} \eta^{\mu\nu} \eta_{\alpha\beta} k^2 + \frac{1}{4} \eta_{\alpha\beta} (1 - \frac{1}{2} \delta_{\alpha\beta}) k^\mu k^\nu + \frac{1}{4} \delta_{\alpha\beta} k^\mu k^\nu k_\alpha - \frac{1}{2} \delta_{\alpha\beta} k^\mu k_\alpha k_\beta - \frac{1}{2} \delta_{\alpha\beta} k^\mu k_\beta k_\alpha \quad (4.11)\]

and \(\sim \delta_{\alpha\rho}(k)\) is defined by

\[\delta_{\alpha\rho}(k) = \frac{1}{2} \left( \delta_{\rho}(k) \sim \delta_{\alpha}(k) + \eta_{\rho}(k) \eta_{\alpha}(k) \right). \quad (4.12)\]
The operator $M^\mu_{\alpha\beta}$ has the following symmetric property

$$M^\mu_{\alpha\beta} = M^\mu_{\beta\alpha}. \quad (4.13)$$

In the above relation, $\delta_\rho^\mu (k)$, $\eta_{\mu\gamma} (k)$ and $\eta_{\mu\gamma} (k)$ are defined by

$$\delta_\rho^\mu (k) = \delta^\mu_\rho - \frac{k^\mu k_\rho}{k^2 - i\epsilon}, \quad (4.14)$$

$$\eta_{\mu\gamma} (k) = \eta^\mu_{\gamma} - \frac{k^\mu k_\gamma}{k^2 - i\epsilon}, \quad (4.15)$$

$$\eta_{\mu\gamma} (k) = \eta_{\mu\gamma} - \frac{k^\mu k_\gamma}{k^2 - i\epsilon}. \quad (4.16)$$

It can be easily proved that $\delta_\rho^\mu (k)$, $\eta_{\mu\gamma} (k)$, $\eta_{\mu\gamma} (k)$, $\delta_\rho^\mu$, $\eta_{\mu\gamma}$ and $\eta_{\mu\gamma}$ satisfy the following relations:

$$\eta_{\mu\gamma} \cdot \eta_{\gamma\nu} (k) = \eta^\mu_{\gamma\nu} (k) = \eta^\mu_{\gamma\nu} (k) \cdot \eta_{\gamma\nu} = \delta^\mu_\nu (k), \quad (4.17)$$

$$\delta_\gamma^\mu \cdot \delta_\rho^\gamma (k) = \delta^\mu_\gamma \cdot \delta_\nu^\gamma (k) = \delta^\gamma_\mu \cdot \delta_\nu^\gamma (k) = \delta^\gamma_\mu \cdot \delta_\nu^\gamma = \delta^\mu_\nu (k), \quad (4.18)$$

$$\delta_\mu^\mu \cdot \delta_\nu^\gamma (k) = \delta_\mu^\gamma \cdot \delta_\nu^\gamma (k) = \delta_\mu^\gamma \cdot \delta_\nu^\gamma = \delta^\mu_\nu (k), \quad (4.19)$$

$$\delta_\mu^\mu \cdot \eta_{\gamma\nu} (k) = \delta_\mu^\gamma \cdot \eta_{\gamma\nu} (k) = \delta_\mu^\gamma \cdot \eta_{\gamma\nu} = \delta^\mu_\nu \cdot \eta_{\mu\gamma} (k), \quad (4.20)$$

$$k^\mu \eta_{\mu\nu} (k) = k^\mu \eta_{\mu\nu} (k) = k^\mu \delta_\mu^\gamma (k) = k^\mu \delta_\mu^\gamma = 0. \quad (4.21)$$

Using all these relations, we can prove that $\delta_{\alpha\rho}^\mu (k)$ satisfies the following relation

$$\delta_{\alpha\rho}^\mu (k) \cdot \delta_{\gamma\nu}^\gamma (k) = \delta_{\alpha\nu}^\mu (k). \quad (4.22)$$

For the propagator of gravitational gauge field, we require that it should satisfy the following gauge conditions

$$\delta_{\beta\rho}^\gamma (x) \cdot \Delta_{\alpha\beta}^\gamma (x) = \Delta_{\alpha\gamma}^\beta (x), \quad (4.23)$$

$$\delta_{\alpha\rho}^\gamma (x) \cdot \Delta_{\alpha\beta}^\gamma (x) = \Delta_{\alpha\beta}^\gamma (x). \quad (4.24)$$

In momentum space, these two gauge conditions become

$$\Delta_{\alpha\beta}^\gamma (k) \cdot \delta_{\beta\rho}^\gamma (k) = \Delta_{\alpha\rho}^\gamma (k), \quad (4.25)$$
\[
\delta_{\alpha\nu}(k) \sim \Delta_{F_{\mu\nu}}(k) = \Delta_{F_{\mu\nu}}(k).
\] (4.26)

These two gauge conditions are related to the zero mass of graviton. The solutions to the two propagator equations (4.9) and (4.10) and gauge conditions (4.25) and (4.26) give out the propagators in momentum space,

\[
-i \sim \Delta_{F_{\mu\nu}}(k) = \frac{-i}{k^2 - i\epsilon} \eta_{\mu\nu}(k) + \delta_{\mu}(k) \delta_{\nu}(k) - \delta_{\mu}(k) \delta_{\nu}(k),
\] (4.27)

\[
-i \sim \Delta_{F_{\beta}}(k) = \frac{-i}{k^2 - i\epsilon} \delta_{\beta}.
\] (4.28)

The forms of these propagators are quite beautiful and symmetric. It can be easily proved that

\[
k^{\mu} \sim \Delta_{F_{\mu\nu}}(k) = k^{\alpha} \sim \Delta_{F_{\alpha\beta}}(k) = \Delta_{F_{\mu\nu}}(k)k_{\nu} = \Delta_{F_{\mu\nu}}(k)k_{\beta} = 0.
\] (4.29)

5 Feynman Rules of Interaction Vertices

The interaction Lagrangian \( L_I \) is a function of gravitational gauge field \( C_\mu^\alpha \) and ghost fields \( \eta^\alpha \) and \( \bar{\eta}_\alpha \),

\[
L_I = L_I (C, \eta, \bar{\eta}).
\] (5.1)

Then eq.(3.51) is changed into,

\[
W[J, \beta, \bar{\beta}] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x L_I(C, \eta, \bar{\eta}) \right\}
\cdot \exp \left\{ i \int d^4x (L_F + J_\mu^\alpha C_\mu^\alpha + \bar{\eta}_\alpha \beta^\alpha + \bar{\beta}_\alpha \eta^\alpha) \right\}
= \exp \left\{ i \int d^4x L_I \left( \frac{1}{1 + i\delta}, \frac{1}{i\delta}, \frac{1}{i\delta} \right) \right\} \cdot W_0[J, \beta, \bar{\beta}],
\] (5.2)

where

\[
W_0[J, \beta, \bar{\beta}] = N \int [DC][D\eta][D\bar{\eta}] \exp \left\{ i \int d^4x \left( L_F + J_\mu^\alpha C_\mu^\alpha + \bar{\eta}_\alpha \beta^\alpha + \bar{\beta}_\alpha \eta^\alpha \right) \right\}
\cdot \exp \left\{ i \int d^4x \left( \frac{1}{1 + i\delta} J_\mu^\alpha(x) \Gamma_{\alpha\beta}(x) J_\beta^\nu(y) \right) \right\}
= \exp \left\{ i \int d^4x d^4y \left[ \frac{1}{2} J_\mu^\alpha(x) \Gamma_{\alpha\beta}(x - y) J_\beta^\nu(y) \right. \right.
+ \left. \bar{\beta}_\alpha(x) \Delta_{F_{\alpha\beta}}(x - y) \beta^\beta(y) \right\}.
\] (5.3)
In order to obtain the above relation, eq. (3.46) is used.

The interaction Feynman rules for interaction vertices can be obtained from the interaction Lagrangian $L_I$. For example, the interaction Lagrangian between gravitational gauge field and ghost field is

$$+ g (\partial^\mu \bar{\eta}_\alpha) C^\beta_\mu (\partial^\beta \eta^\alpha) - g (\partial^\mu \bar{\eta}_\alpha) (\partial^\sigma C^\alpha_\mu) \eta^\sigma.$$  \hspace{1cm} (5.4)

This vertex belongs to $C^\alpha_\mu (k) \bar{\eta}_\beta (-q) \eta^\delta (p)$ three body interactions, its Feynman rule is

$$i g \delta^\beta_\beta q^\alpha p^\alpha - i g \delta^\beta_\alpha q^\mu k^\delta.$$  \hspace{1cm} (5.5)

To calculate the interaction lagrangian of three gravitational gauge field, four gravitational gauge field and higher gravitational gauge field are extremely complicated. Here I only explain how to calculate them and list related results. First, we can expand $\det G^{-1}$, $G^{-1\nu}_\alpha$ and $g_{\alpha\beta}$ in terms of gravitational gauge field

$$\det G^{-1} = 1 + g C^\alpha_\alpha + \frac{g^2}{2} [C^\alpha_\mu C^\mu_\alpha + C^\mu_\mu C^\alpha_\alpha] + \cdots,$$  \hspace{1cm} (5.6)

$$G^{-1\nu}_\alpha = \delta^\nu_\alpha + g C^\nu_\alpha + g^2 C^\nu_\mu C^\mu_\alpha + \cdots,$$  \hspace{1cm} (5.7)

$$g_{\alpha\beta} = \eta_{\alpha\beta} + g [\eta_{\mu\beta} C^\mu_\alpha + \eta_{\mu\alpha} C^\mu_\beta] + g^2 [\eta_{\mu\beta} C^\alpha_\alpha + \eta_{\mu\alpha} C^\alpha_\beta + \eta_{\mu\nu} C^\mu_\alpha C^\nu_\beta] + \cdots.$$  \hspace{1cm} (5.8)

Next, we need to expand the lagrangian $L_0$ in terms of gravitational gauge field. We will make the following expanding

$$L_0 = 2L_0 + 3L_0 + 4L_0 + \cdots,$$  \hspace{1cm} (5.9)

where $L$ contains all $n$-th order interaction terms of gravitational gauge field. Substitute equations (5.7) and (5.8) into (2.10), we can get

$$\bar{L}_0 = V^\mu\nu^\rho\sigma_{\alpha\beta} (\partial_\rho C^\alpha_\mu) (\partial_\sigma C^\beta_\nu),$$  \hspace{1cm} (5.10)

where

$$V^\mu\nu^\rho\sigma_{\alpha\beta} = - \frac{1}{16} \bar{\eta}^\mu\nu^\rho\sigma_{\alpha\beta}.$$  \hspace{1cm} (5.11)

In the above relation, $\bar{\eta}^\mu\nu^\rho\sigma_{\alpha\beta}$ is defined by

$$\bar{\eta}^\mu\nu^\rho\sigma_{\alpha\beta} = \eta^\mu\nu^\rho^\sigma_{\alpha\beta} + \eta^\mu^\rho\sigma_{\alpha\beta} - \eta^\mu^\sigma\rho^\alpha_{\beta} - \eta^\rho^\mu\sigma^\alpha_{\beta},$$  \hspace{1cm} (5.12)
where
\[ \tilde{\eta}_{\alpha\beta}^{\mu\nu} = \eta_{\alpha\beta}^{\mu\nu} + 2 \delta_{\beta}^{\mu} \delta_{\alpha}^{\nu} - 4 \delta_{\alpha}^{\mu} \delta_{\beta}^{\nu}. \] (5.13)

The interaction term of three gravitational gauge field in the \( \mathcal{L}_0 \) is
\[ 3 \mathcal{L}_0 = V_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} C_{\lambda}^{\gamma} \left( \partial_{\mu} C_{\sigma}^{\alpha} \right) \left( \partial_{\nu} C_{\rho}^{\beta} \right), \] (5.14)
where
\[ V_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} = \frac{g}{16} \left( \tilde{\eta}_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} + \tilde{\eta}_{\beta\alpha\gamma}^{\nu\mu\lambda\rho\sigma} \right), \] (5.15)
\[ \tilde{\eta}_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} = \delta_{\beta}^{\mu} \delta_{\alpha}^{\nu} - 32 \delta_{\alpha}^{\mu} \delta_{\beta}^{\nu} \eta_{\alpha\beta\gamma}. \] (5.16)

The interaction term of four gravitational gauge field in the \( \mathcal{L}_0 \) is
\[ 4 \mathcal{L}_0 = V_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} C_{\lambda}^{\gamma} C_{\delta}^{\kappa} \left( \partial_{\mu} C_{\sigma}^{\alpha} \right) \left( \partial_{\nu} C_{\rho}^{\beta} \right), \] (5.17)
where
\[ V_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} = \frac{g^2}{64} \left[ \tilde{\eta}_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} + \tilde{\eta}_{\beta\gamma\delta\alpha}^{\nu\mu\lambda\rho\sigma} + \tilde{\eta}_{\delta\alpha\gamma\beta}^{\mu\nu\lambda\rho\sigma} + \tilde{\eta}_{\gamma\beta\delta\alpha}^{\nu\mu\lambda\rho\sigma} \right]. \] (5.18)
\[ \tilde{\eta}_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} = \delta_{\beta}^{\mu} \delta_{\alpha}^{\nu} + \delta_{\delta}^{\nu} \delta_{\gamma}^{\mu} \eta_{\alpha\beta\gamma} + \delta_{\gamma}^{\mu} \delta_{\beta}^{\nu} \eta_{\alpha\beta\delta} + \delta_{\delta}^{\mu} \delta_{\gamma}^{\nu} \eta_{\alpha\beta\gamma}. \] (5.19)

Substitute above results and (5.6) into (2.9), we get
\[ 3 \mathcal{L} = \bar{V}_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} C_{\lambda}^{\gamma} \left( \partial_{\mu} C_{\sigma}^{\alpha} \right) \left( \partial_{\nu} C_{\rho}^{\beta} \right), \] (5.20)
where
\[ \bar{V}_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} = V_{\alpha\beta\gamma}^{\mu\nu\lambda\rho\sigma} - \frac{g^2}{16} \delta_{\gamma}^{\mu} \tilde{\eta}_{\alpha\beta}^{\nu\rho\sigma}. \] (5.21)

And
\[ 4 \mathcal{L} = \bar{V}_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} C_{\lambda}^{\gamma} C_{\delta}^{\kappa} \left( \partial_{\mu} C_{\sigma}^{\alpha} \right) \left( \partial_{\nu} C_{\rho}^{\beta} \right), \] (5.22)
where
\[ \bar{V}_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} = V_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} - \frac{g^2}{32} \left( \delta_{\delta}^{\mu} \delta_{\gamma}^{\nu} \eta_{\alpha\beta}^{\lambda\rho\sigma} + \delta_{\nu}^{\mu} \delta_{\gamma}^{\lambda} \eta_{\alpha\beta}^{\rho\sigma} \right) \] (5.23)
\[ + \frac{g^2}{32} \left( \delta_{\gamma}^{\mu} \delta_{\delta}^{\nu} \eta_{\alpha\beta}^{\lambda\rho\sigma} + \delta_{\nu}^{\mu} \delta_{\delta}^{\lambda} \eta_{\alpha\beta}^{\rho\sigma} \right) \] (5.23)
\[ - \frac{g^2}{32} \left( \delta_{\gamma}^{\mu} \delta_{\beta}^{\nu} \eta_{\alpha\delta}^{\lambda\rho\sigma} + \delta_{\nu}^{\mu} \delta_{\beta}^{\lambda} \eta_{\alpha\delta}^{\rho\sigma} \right). \] (5.23)
Feynman rules for the vertex of three gravitational gauge field $C^\alpha_{\mu}(p_1)C^\beta_{\nu}(p_2)C^\gamma_{\lambda}(p_3)$ is

$$-2i \left[ V_{\alpha\beta\gamma\delta}^{\mu\nu\lambda\rho\sigma} p_1 p_2 p_3 + V_{\alpha\gamma\beta\delta}^{\mu\lambda\nu\rho\sigma} p_1 p_2 p_3 + V_{\gamma\alpha\beta\delta}^{\lambda\mu\nu\rho\sigma} p_1 p_2 p_3 \right].$$  \hspace{1cm} (5.24)

The Feynman rule for the vertex of four gravitational gauge field $C^\alpha_{\mu}(p_1)C^\beta_{\nu}(p_2)C^\gamma_{\lambda}(p_3)C^\delta_{\kappa}(p_4)$ is

$$-4i \left[ V_{\alpha\beta\gamma\delta\epsilon}^{\mu\nu\lambda\rho\sigma\tau} p_1 p_2 p_3 p_4 + V_{\alpha\epsilon\gamma\delta\beta}^{\mu\lambda\nu\rho\sigma\tau} p_1 p_2 p_3 p_4 + V_{\gamma\alpha\beta\delta\epsilon\tau}^{\lambda\mu\nu\rho\sigma\tau} p_1 p_2 p_3 p_4 \right] + V_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + V_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ V_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + V_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

$$+ \bar{V}_{\beta\gamma\delta\alpha\tau}^{\nu\lambda\mu\rho\sigma\tau} p_2 p_3 p_4 p_5 + \bar{V}_{\gamma\delta\alpha\beta\epsilon\tau}^{\lambda\nu\mu\rho\sigma\tau} p_3 p_4 p_5.$$

6 Discussions

In this paper, path integral quantization of quantum gauge general relativity is discussed, and Feynman rules of various interaction vertices are calculated. These results are needed in the loop diagram calculation.

In the literature [3], we have formally proved that quantum gauge general relativity is a perturbatively renormalizable quantum theory. In that proof, detailed calculations of loop diagrams are not performed. In the next step, we will calculate all divergent one-loop diagrams, discuss renormalization of quantum gauge general relativity in one-loop level, and determine the renormalization constant in one-loop level. These results will be summarize in the further paper.

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