Semileptonic transition of P wave bottomonium $\chi_{b0}(1P)$ to $B_c$ meson

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Taking into account the two-gluon condensate contributions, the transition form factors enrolled to the low energy effective Hamiltonian describing the semileptonic $\chi_{b0} \to B_c \ell \bar{\nu}, (\ell = (e, \mu, \tau))$ decay channel are calculated within three-point QCD sum rules. The fit function of the form factors then are used to estimate the decay width of the decay mode under consideration.

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I. INTRODUCTION

The quarkonia, especially the bottomonium $b\bar{b}$ states, are approximately non-relativistic systems since they do not contain intrinsically relativistic light quarks. Hence, these states are the best candidates to examine the hadronic dynamics and investigate both perturbative and non-perturbative characteristic of QCD. In the past, mainly theoretical calculations on the properties of these states had been made using potential model or its extensions like the Coulomb gauge model (see for instance [1–6] and references therein). In [7], both potential model and QCD sum rule approach have been applied to extract the ground-state decay constant of mesons containing heavy $b$ quark. It is stated that the QCD sum rule technique gives more reliable and accurate determination of bound-state characteristics compared to the potential models by tuning the continuum threshold parameter. The QCD sum rule approach [8] is one of the most powerful and applicable tools to hadron physics. This model has been widely applied to investigate the spectroscopy of hadrons and their electromagnetic, weak and strong decays. The obtained results have very good consistencies with the experimental data to date within the typical (10-20)% error bars of the technique.

The present work is dedicated to investigation of the semileptonic transition of scalar $P$ wave bottomonium $\chi_{b0}(1P)$ meson with quantum numbers $I^G(J^{PC}) = 0^+(0^{++})$ into the pseudoscalar $B_c$ meson. The $\chi_{b0}(1P)$ state has been observed first in radiative decay of the $\Upsilon(2S)$ [9] and recently has been confirmed by ATLAS Collaboration [10] together with the higher $\chi_b(2P)$ and $\chi_b(3P)$ states. In the latter, these quarkonia states have been produced in proton-proton collisions at the Large Hadron Collider (LHC) at $\sqrt{s} = 7$ TeV and through their radiative decays to $\Upsilon(1S,2S)$ with $\Upsilon \rightarrow \mu^+\mu^-$. Our previous theoretical results [11] on the mass of these states done both in vacuum and finite temperature QCD are in good agreement with the experimental results [10]. Note that, we also have applied the QCD sum rules approach both in vacuum and finite temperature to investigate the spectroscopy of the pseudoscalar, vector and tensor quarkonia in [12–14]. As we know the masses and decay constants of the quarkonia, it is possible to investigate their electromagnetic, weak (leptonic-semileptonic) and strong decays. Considering such decay channels can help us obtain more information about the nature of the scalar $\chi_{b0}(1P)$ meson as well as perturbative and non-perturbative aspects of QCD.

The layout of this article is as follows. In the next section, we derive the QCD sum rules for the form factors appearing in the amplitude of the semileptonic decay channel under consideration. To do so, we take into account the two-gluon condensates as the non-perturbative contributions to the correlation function. Section III is devoted to our numerical analysis of the obtained form factors and their behavior in terms of the transferred momentum squared. In this section, we also numerically estimate the decay width of the
semileptonic $\chi_{b0} \rightarrow B_c l \bar{\nu}$ decay mode. The last section encompasses our concluding remarks.

II. QCD SUM RULES FOR TRANSITION FORM FACTORS OF $\chi_{b0} \rightarrow B_c l \bar{\nu}$

The hadronic event under consideration can be described in terms of quark degrees of freedom by the process $b \rightarrow c l \bar{\nu}$ at tree-level, whose effective Hamiltonian can be written as:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\nu} \gamma_\mu (1 - \gamma_5) l \bar{c} \gamma_\mu (1 - \gamma_5) b,$$

where $G_F$ is the Fermi weak coupling constant and $V_{cb}$ is an element of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. The transition amplitude is obtained via

$$M = \langle B_c(p') | H_{\text{eff}} | \chi_{b0}(p) \rangle,$$

or

$$M = \frac{G_F}{\sqrt{2}} V_{cb} \bar{\nu} \gamma_\mu (1 - \gamma_5) l \bar{c} \gamma_\mu (1 - \gamma_5) b | \chi_{b0}(p) \rangle.$$

To proceed, we need to know the transition matrix element $\langle B_c(p') | \bar{c} \gamma_\mu b | \chi_{b0}(p) \rangle$ whose vector part do not contribute due to parity considerations, i.e.,

$$\langle B_c(p') | \bar{c} \gamma_\mu b | \chi_{b0}(p) \rangle = 0.$$

The axial-vector part of transition matrix element can be parameterized in terms of form factors as

$$\langle B_c(p') | \bar{c} \gamma_\mu \gamma_5 b | \chi_{b0}(p) \rangle = f_1(q^2) P_\mu + f_2(q^2) q_\mu,$$

where $f_1(q^2)$ and $f_2(q^2)$ are transition form factors; and $P_\mu = (p + p')_\mu$ and $q_\mu = (p - p')_\mu$.

Our main goal in the present section is to calculate the transition form factors applying the QCD sum rules technique. The starting point is to consider the following tree-point correlation function as the main ingredient of the model:

$$\Pi_\mu = i^2 \int d^4 x \int d^4 y e^{-ipx} e^{ip'y} \langle 0 | \mathcal{T} \left\{ J_{B_c}(y) J_{\chi_{b0}}^\dagger(0) \right\} | 0 \rangle,$$

where $\mathcal{T}$ is the time ordering product, $J_{B_c}(y) = \bar{c} \gamma_\mu b$ and $J_{\chi_{b0}}(x) = \bar{U} b$ are the interpolating currents of the $B_c$ and $\chi_{b0}$ mesons, respectively; and $J_{V}^\dagger(0) = \bar{c} \gamma_\mu b$ and $J_{A}^\dagger(0) = \bar{c} \gamma_\mu \gamma_5 b$ are the vector and axial-vector parts of the transition current. Following the general idea in the QCD sum rules technique, we calculate this correlation function once in terms of hadronic degrees of freedom called physical or phenomenological side and the second in terms of QCD degrees of freedom (quarks and gluons and their interaction with QCD vacuum) called the QCD side. The latter is done in the deep Euclidean region by the help of operator product expansion (OPE). These two representations are then matched together, using the
quark-hadron duality assumption, through a double dispersion relation to obtain the QCD sum rules for the form factors. As we deal with the ground states in this approach, we shall separate the ground state from the higher states and continuum. This is done by two mathematical operations called Borel transformation and continuum subtraction. Such transformations bring some auxiliary parameters namely two Borel mass parameters and two continuum thresholds for which we will find their working regions in the next section.

The phenomenological side of the correlation function is obtained inserting two complete sets of intermediate states with the same quantum numbers as the interpolating currents $J_{Bc}$ and $J_{\chi_b}$. As a result, we obtain

$$\Pi_{\mu}^{PHYS} = \left\langle 0 \mid J_{Bc}(0) \mid B_c(p') \right\rangle \left\langle B_c(p') \mid J_{\chi_b}^A(0) \mid \chi_b(p) \right\rangle \left\langle \chi_b(p) \mid J_{\chi_b}^\dagger(0) \mid 0 \right\rangle \left(\frac{p'^2 - m_{Bc}^2}{(p'^2 - m_{Bc}^2)(p^2 - m_{\chi_b}^2)}\right) + \cdots,$$

(7)

where $\cdots$ represents the contributions coming from higher states and continuum. Besides the transition matrix elements defined previously, the matrix elements of interpolating current between the vacuum and hadronic states are parameterized in terms of the leptonic decay constants, i.e.,

$$\left\langle 0 \mid J_{Bc}(0) \mid B_c(p') \right\rangle = i \frac{f_{Bc} m_{Bc}^2}{m_b + m_c}, \quad \left\langle \chi_b(p) \mid J_{\chi_b}^\dagger(0) \mid 0 \right\rangle = -i m_{\chi_b} f_{\chi_b}.$$

(8)

Putting all expressions together, the final version of the phenomenological side of the correlation function is obtained as

$$\Pi_{\mu}^{PHYS}(p^2, p'^2) = \frac{f_{\chi_b} m_{\chi_b}}{(p'^2 - m_{Bc}^2)(p^2 - m_{\chi_b}^2)} \frac{f_{Bc} m_{Bc}^2}{m_b + m_c} \left[ f_1(q^2) P_\mu + f_2(q^2) q_\mu \right] + \cdots,$$

(9)

where we will choose the structures $P_\mu$ and $q_\mu$, to evaluate the form factors $f_1$ and $f_2$, respectively.

At QCD side, the correlation function is calculated in deep Euclidean region by the help of the OPE. For this aim, we write the coefficient of each structure in correlation function as a sum of a perturbative (diagram a in figure 1) and a non-perturbative (diagrams b, c, d, e, f and g in figure 1) parts as follows:

$$\Pi_{\mu}^{QCD} = (\Pi_1^{pert} + \Pi_1^{nonpert}) P_\mu + (\Pi_2^{pert} + \Pi_2^{nonpert}) q_\mu,$$

(10)

where, the $\Pi_i^{pert}$ functions are written in terms of double dispersion integrals in the following way:

$$\Pi_i^{pert} = -\frac{1}{(2\pi)^2} \int ds \int ds' \frac{\rho_i(s, s', q^2)}{(s - p^2)(s' - p'^2)} + \text{subtraction terms},$$

(11)
where, $\rho_i(s, s', q^2)$ are the spectral densities with $i = 1$ or 2. Applying the usual Feynman integral technique to the bare loop diagram, the spectral densities are calculated via Cutkosky rules, i.e., by replacing the quark propagators with Dirac delta functions:

$$\frac{1}{p^2 - m^2} \rightarrow -2\pi \delta(p^2 - m^2)$$ implying that all quarks are real. After some calculations, the spectral densities are obtained as follows:

$$\rho_1(s, s', q^2) = 2N_c I_0(s, s', q^2)[m_b(m_c - 3m_b) - A(h + s) - B(h + s')],$$

$$\rho_2(s, s', q^2) = 2N_c I_0(s, s', q^2)[m_b(m_b + m_c) - A(h - s) + B(h - s')],$$

(12)

where

$$I_0(s, s', q^2) = \frac{1}{4\lambda^{1/2}(s, s', q^2)},$$

$$A = \frac{1}{\lambda(s, s', q^2)}[(m_b^2 - m_c^2)u'' + us'],$$

$$B = \frac{1}{\lambda(s, s', q^2)}[2(m_b^2 - m_c^2)s + su'],$$

$$h = 2m_b(m_b - m_c),$$

(13)

here also $\lambda(s, s', q^2) = s^2 + s'^2 + q^4 - 2ss' - 2sq^2 - 2s'q^2$, $u = q^2 + s - s'$, $u' = q^2 - s + s'$, $u'' = q^2 - s - s'$ and $N_c = 3$ is the number of colors. The integration region for the perturbative contribution in Eq. (11) (bare loop diagram) is determined requiring that the arguments of the three $\delta$ functions vanish, simultaneously. Therefore, the physical region in the $s$ and $s'$ plane is described by the following inequality:

$$-1 \leq f(s, s') = \frac{2s[m_b^2 - m_c^2 + s' + \frac{u''}{2}]}{\lambda^{1/2}(m_b^2, s, m_c^2)\lambda^{1/2}(s, s', q^2)} \leq +1.$$ 

(14)
For the non-perturbative part, we take into account the two-gluon condensate diagrams (b, c, d) in figure 1. Here we should mention that we deal with the heavy quarks in the present work and the heavy quarks’ condensates are suppressed by inverse of their masses, so we can ignore them safely. After lengthy calculations for the two-gluon condensates diagrams (b, c, d) correspond to the diagrams with two gluon lines coming out from different quark lines, we get

$$\Pi_{i,nonpert}^i = \int_0^1 dx \int_0^{1-x} dy \langle 0 | \frac{1}{\pi} \alpha_s G^2 | 0 \rangle \left\{ \frac{\Theta_1^i}{D^5} + \frac{\Theta_2^i}{D^4} + \frac{\Theta_3^i}{D^3} + \frac{\Theta_4^i}{D^2} \right\}, \quad (15)$$

where

$$D = m_c^2 x - m_b^2 r + p^2 x y + p^2 y f + xy(p^2 + p^2 - q^2), \quad (16)$$

and

$$\Theta_1^i = \frac{1}{24} \left\{ 9m_b^6 m_c y^2 v^2 (-1 + 2y) - 18m_b^6 y^2 v^3 + 3m_b^4 x v \left[ 3m_c^2 \left( r^2 x + 2r x y + 2w y^2 \right) + q^2 y \left( 3r^3 x + 3r^2 t y + r y^2 (-14 + 15x) + y^3 (-22 + 18x + 9y) \right) \right] \right. \right.$$  

$$\left. + 3m_b^3 m_c x v \left[ m_c^2 x \left( 6 - 19x + 13x^2 - 15y + 22xy + 12y^2 \right) + q^2 y \left( 13r^2 x^2 - 6r^2 x + 3ry - 15ry x + 22ry x^2 + y^2 (15 - 22y - 26x + 12x^2 + 12xy + 12y^2) \right) \right] \right. \right.$$  

$$\left. + m_b m_c q^2 x^2 y v \left[ m_c^2 x \left( 22 - 49x + 39x^2 - 62y + 66xy + 36y^2 \right) + q^2 y \left( 39x^4 - 88x^3 + 66x^3 y + 3f y (3 - 7y + 6y^2) + x^2 (71 - 128y + 36y^2) + x \left( 11 - 33y + 18y^2 \right) \right) \right] \right.$$  

$$\left. + 3q^4 x^3 y^2 \left[ m_c^2 \left( f^2 + 6f^3 y + x^3 (-11 + 3x + 9y) + f x \left( 10 - 17x + 18xy - 26y + 18y^2 \right) \right) + q^2 y \left( x^3 (31 + 3x^2 - 14x + 9xy - 50y + 21y^2) + f x \left( -20 + 54y - 51y^2 + 15y^3 + 36x - 59xy + 27xy^2 \right) + f^2 (-4 + 3yz^2) \right) \right] \right. \right.$$  

$$\left. + 3m_b^2 q^2 x^2 y \left[ q^2 y \left( 4r^3 - 2r^3 x + 16r^3 y - 8r^2 xy + 3ry^2 (8 - 7x + 2x^2) + y^3 (17 - 5y - 27x + 12x^2 + 6xy) \right) \left. \right. \right.$$  

$$\left. \right. + m_c^2 \left( 6xy - 4x + 5x^2 - 2x^3 - 5x^2 y + 6x^2 y^2 + f (8y - 8xy + 12xy^2 + 6y^2 z - 1) \right) \right) \right\}, \quad \Theta_2^i = \frac{1}{96} \left\{ 9m_b^6 m_c v \left[ r x^2 (-7 + 13x) + 2x^2 (-8 + 11x) y - 2y^2 (3f + 5x - 6x^2) + 12y^3 (f + x) \right] \right.$$  

$$\left. - 9m_b^4 v \left[ 3x^2 x^2 + 6r x^2 y + y^2 (7 - 14x + 15x^2 - 16y + 18xy + 9y^2) \right] \right.$$  

$$+ 3m_b^2 x \right.$$
\[\times \left[3m_c^2(6y^4 - xr^2 + 6y^2r^2 - 2xry + 12y^3) + q^2y\left(8r^2x - 11r^2x^2 - 12ry + 54rxy - 38ryx^2 - y^2(43 + 153x^2 - 48x^3 + 54ty + 23y^2 - 158x - 48xyt - 48xy^2)\right)\right]
\]
\[+ m_bm_cv\left[9m_c^2x(4 + 13rx - 13y + 22xy + 12y^2) + q^2y\left(312x^4 + x^3(-535 + 528y) + x^2(313 - 683y + 288y^2) + 3y\left(-9y + 53y - 88y^2 + 48y^3\right) + 6x\left(-9 + 40y - 58y^2 + 24y^3\right)\right)\right] + 3q^2x^2y\left[m_c^2\left(r^2(3 - 29x + 24x^2) - y\left(21 - x(151 - 206x + 72x^2)\right)\right) + 78y + 2xy(-127 + 72x) + 36y^2(-3 + 4x) + 48y^3\right] + q^2y\left(45x^5 + x^4(-182 + 135y) + x^3(306 - 584y + 315y^2) + x^2(-253 + 849y - 1004y^2 + 405y^3) + 3f(4 - 30y + 69y^2 - 59y^3 + 15y^4) + x\left(96 - 500y + 965y^2 - 792y^3 + 225y^4\right)\right] \right]\right],
\[\Theta_1^3 = \frac{1}{96}\left[9m_c^2\left[-r^2x^2 - 2rx^2y + r\left(1 - 7xy^2 + 8x^2y^2\right) + 2y^3(1 + 8rx) + y^4(-1 + 8x)\right]
\]
\[+ 9m_c^2x\left[r^2x(-3 + 4x) + 2rxy(-5 + 6x) + 2y^2(2 - 13x + 12x^2) + 12ty^3 + 8y^4\right]
\]
\[+ 4q^2xy\left[r^2x(17 - 64x + 45x^2) + y\left(21 + x(-154 + 413x - 418x^2 + 135x^3)\right) - 120y^2
\]
\[+ y^2x(497 - 718x + 315x^2) + 3y^3\left(76 - 198x + 135x^2\right) + 3y^4(-58 + 75x) + 45y^5
\]
\[+ 3m_bm_cv\left[52x^4 + 24xy^2 + 12fy^2(-1 + 2y) + x^3(-61 + 88y) + 3x^2\left(7 - 19y + 16y^2\right)\right] \right],
\[\Theta_1^4 = \frac{3}{16}\left[5x^5 + y^2f^2(-4 + 5y) + x^4(-14 + 15y) + fxy^2(-19 + 25y) + x^3\left(13 - 28y + 35y^2\right)
\]
\[+ x^2\left(4 - 13y + 48y^2 - 45y^3\right)\right].\]
\[ f_{1,2}(q^2) = \frac{(m_b + m_c)}{f_B m_{Bc}^2} \frac{1}{f_{\chi_0 m_{\chi_0}}} e^{m_{\chi_0}^2/M^2} e^{m_{Bc}^2/M^2} \left\{ -\frac{1}{(2\pi)^2} \int_{m_b^2}^{s_0} ds \int_{m_c^2}^{s_0'} ds' \times \rho_{1,2}(s, s', q^2) \theta[1 - f^2(s, s')] e^{-s/M^2} e^{-s'/M^2} + \hat{B} \Pi_{1,2}^{\text{nonpert}} \right\}, \]  

where the operator \( \hat{B} \) denotes double Borel transformation. Note that to subtract contributions of the higher states and continuum, we also apply the quark-hadron duality assumption, i.e.,

\[ \rho_{\text{higher states}}(s, s') = \rho^{\text{OPE}}(s, s') \theta(s - s_0) \theta(s' - s_0'). \]  

We also perform the double Borel transformation as follows:

- for the perturbative part, we use

\[ \hat{B} \frac{1}{|p^2 - m_1^2|^m |p'{}^2 - m_2^2|^n} \to (-1)^{m+n} \frac{1}{\Gamma[m]} \frac{1}{\Gamma[n]} e^{-m_1^2/M^2} e^{-m_2^2/M^2} \frac{1}{(M^2)^{m-1}(M'^2)^{n-1}}. \]  

- For the non-perturbative part, first we make the transformation

\[ \hat{B} \frac{1}{|p^2 - f(p'{}^2)|^n} = (-1)^n e^{-f(p'{}^2)/M^2} \frac{1}{\Gamma[n](M)^{n-1}}, \]  

to write the terms containing \( p^2 \) in exponential form. Then we apply the following rule to transform the \( (p'{}^2 \to M'^2) \):

\[ \hat{B} e^{-\alpha p'{}^2} = \delta(\frac{1}{M'^2} - \alpha), \]  

where \( \alpha \) is a function of quarks’ masses as well as the parameters used in Feynman parametrization.

III. NUMERICAL RESULTS

In this section, we numerically analyze the related form factors, obtain their fit function in terms of \( q^2 \) and estimate the decay width of the channel under consideration. In calculations, we use the input parameters as presented in table I.

The sum rules for the form factors denote that they also depend on four auxiliary parameters, namely continuum thresholds \( s_0 \) and \( s'_0 \) and Borel mass parameters \( M^2 \) and \( M'^2 \). The continuum thresholds are not completely arbitrary but they are in correlation with the energy of the excited state in initial and final channels. Considering this point and the
TABLE I. Input parameters used in our calculations \cite{9, 11, 15, 16}.

| Parameters | Values                        |
|------------|-------------------------------|
| $m_c$      | $(1.275 \pm 0.015) \text{ GeV}$ |
| $m_b$      | $(4.7 \pm 0.1) \text{ GeV}$    |
| $m_e$      | 0.00051 GeV                   |
| $m_\mu$    | 0.1056 GeV                    |
| $m_\tau$   | 1.776 GeV                     |
| $m_{\chi_0}$ | $(9859.44 \pm 0.42 \pm 0.31) \text{ MeV}$ |
| $m_{B_c}$  | $(6.277 \pm 0.006) \text{ GeV}$ |
| $f_{B_c}$  | $(400 \pm 40) \text{ MeV}$    |
| $f_{\chi_0}$ | $(175 \pm 55) \text{ MeV}$   |
| $G_F$      | $1.17 \times 10^{-5} \text{ GeV}^{-2}$ |
| $V_{cb}$   | $(41.2 \pm 1.1) \times 10^{-3}$ |
| $\langle 0|\frac{1}{2}\alpha_s G^2|0\rangle$ | $(0.012 \pm 0.004) \text{ GeV}^4$ |

fact that the result of the physical quantities (form factors) should weakly depend on these parameters, we choose the intervals $s_0 = (97.7 - 99.2) \text{ GeV}^2$ and $s_0' = (40 - 41) \text{ GeV}^2$ slightly higher than the mass of pole squared of the initial and final mesonic channels for the continuum thresholds. The Borel parameters $M^2$ and $M'^2$ also are not physical quantities, hence the form factors should be independent of them. The reliable regions for the Borel parameters $M^2$ and $M'^2$ can be determined by requiring that not only the contributions of the higher states and continuum are effectively suppressed, but contribution of the operators with the higher dimensions are small, i.e., the sum rules for form factors converge. As a result of these requirements, the working regions for these parameters are determined to be $15 \text{ GeV}^2 \leq M^2 \leq 30 \text{ GeV}^2$ and $10 \text{ GeV}^2 \leq M'^2 \leq 20 \text{ GeV}^2$. The dependence of form factors $f_1$ and $f_2$ on Borel masses at $q^2 = 1 \text{ GeV}^2$ are plotted in figure 2. From this figure, we see good stability of the form factors with respect to the variations of the Borel mass parameters at their working regions. To see the convergence of the OPE, we compare both perturbative and non-perturbative contributions to the form factors in figure 3 at $q^2 = 1 \text{ GeV}^2$ and the presented Borel windows. From this figure and our numerical calculations, it is found that the ratio of non-perturbative contribution to that of perturbative is 0.08 and 0.05 for $f_1$ and $f_2$, respectively. Hence, the non-perturbative contribution constitutes only 7.5% and 4.8% of the total results respectively for the form factors $f_1$ and $f_2$. This means that the series of sum rules for the form factors are convergent. In the presented Borel windows, the contributions of the excited and continuum states are exponentially suppressed. This
FIG. 2. Dependence of the form factors $f_1$ and $f_2$ on Borel mass parameters $M^2$ and $M'^2$ at $q^2 = 1 \text{ GeV}^2$.

FIG. 3. Comparison between perturbative and non-perturbative contributions to the form factors at $q^2 = 1 \text{ GeV}^2$ and chosen Borel windows. The upper (lower) plane belongs to the perturbative contribution in $f_1$ ($f_2$).

guarantees the reliability of the sum rules and isolation of the ground state from the excited states and continuum.

Our calculations show that the form factors are truncated at $q^2 \simeq 9 \text{ GeV}^2$ (see figure 4). After this point up to the higher limit of the $q^2$, the sum rules predictions are not reliable (for details see for instance [17, 18]). However, we need their fit functions in the whole physical region, $m_t^2 \leq q^2 \leq (m_{\chi b_0} - m_{B_c})^2$ to estimate the decay width of the $\chi_{b0} \rightarrow B_c\ell\nu$ transition. To extend our results to the full physical region, we search for parameterization of the form factors in such a way that in the region $0 \leq q^2 \leq 9 \text{ GeV}^2$, predictions of this
parameterization coincide with the sum rules results. The following parametrization adjust well the $q^2$ dependence of the form factors:

$$f_i(q^2) = \frac{a}{(1 - \frac{q^2}{m_{fit}^2})} + \frac{b}{(1 - \frac{q^2}{m_{fit}^2})^2},$$

(23)

where, the values of the parameters $a$, $b$ and $m_{fit}$ obtained using $M^2 = 25 \text{ GeV}^2$ and $M'^2 = 15 \text{ GeV}^2$ for the $\chi_{b0} \rightarrow B_c \ell \nu$ channel are given in table II.

| $f_1(\chi_{b0} \rightarrow B_c \ell \nu)$ | a  | b   | $m_{fit}^2$ (GeV$^2$) |
|----------------------------------------|----|-----|----------------------|
| $f_1(\chi_{b0} \rightarrow B_c \ell \nu)$ | -0.055 | 0.062 | 21.86 |
| $f_2(\chi_{b0} \rightarrow B_c \ell \nu)$ | 0.225 | -0.254 | 19.79 |

TABLE II. Parameters appearing in the fit function of the form factors.

We depict the dependence of form factors $f_1$ and $f_2$ on $q^2$ obtained directly from the sum rules as well as the fit parametrization at whole physical region in figure 4. In the case of sum rules predictions, we present the perturbative, non-perturbative and total contributions in this figure.

FIG. 4. Dependence of the form factors $f_1$ and $f_2$ on $q^2$ at $M^2 = 25 \text{ GeV}^2$ and $M'^2 = 15 \text{ GeV}^2$.

Having obtained the behavior of the form factors in terms of $q^2$ at whole physical region, we would like to calculate the decay width of the process under consideration. Using the amplitude previously discussed, the differential decay width for $\chi_{b0} \rightarrow B_c \ell \nu$ is obtained in terms of form factors as:

$$\frac{d\Gamma}{dq^2} = \frac{G_F^2}{192\pi^3 m_{B_c}^3} |V_{cb}|^2 \lambda^{1/2}(m_{B_c}^2, m_S^2, q^2) \left( \frac{q^2 - m_{fit}^2}{q^2} \right)^2 \left( \frac{q^2}{2} \right) \left[ |f_1(q^2)|^2 (2m_{B_c}^2 + 2m_S^2 - q^2) \right]$$
Performing integration over $q^2$ in Eq. (24) in the interval $m_\ell^2 \leq q^2 \leq (m_{\chi_{b0}} - m_{B_c})^2$, we obtain the expression for the total decay width. The numerical values of the decay width at different lepton channels are presented in Table III. The errors in the values of the decay rates in table III are due to uncertainties in determination of the working regions for continuum thresholds and Borel mass parameters as well as errors of the other input parameters.

|                  | $\Gamma$(GeV)   |
|------------------|-----------------|
| $\chi_{b0} \to B_c e \bar{\nu}_e$ | $1.46 \times 10^{-14}$ |
| $\chi_{b0} \to B_c \mu \bar{\nu}_\mu$ | $1.45 \times 10^{-14}$ |
| $\chi_{b0} \to B_c \tau \bar{\nu}_\tau$ | $0.91 \times 10^{-14}$ |

TABLE III. Numerical results for decay rate at different lepton channels.

IV. CONCLUSION

In the present work, we studied the semileptonic $\chi_{b0} \to B_c \ell \bar{\nu}_\ell$, ($\ell = (e, \mu, \tau)$) decay channel within the framework of the three-point QCD sum rules. In particular, taking into account the two-gluon diagrams as non-perturbative contributions, we obtained the QCD sum rules for the form factors entered the transition matrix elements. After obtaining the working regions for the auxiliary parameters, we found the behavior of the form factors in terms of $q^2$ in whole physical region. The fit function of the form factors were then used to estimate the decay rates at different lepton channels. Any measurement on the form factors as well as decay rate of the channel under consideration and comparison of the obtained results with theoretical predictions in the present study can give valuable information about the internal structures of the participating mesons specially nature of the scalar $\chi_{b0}(1P)$ state.

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