Magneto-mechanical stability of axially functionally graded supported nanotubes

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Keywords: axially functionally graded material, size-dependent vibration, nanotube conveying fluid, stability map, magnetic field

Abstract

In this paper, size-dependent vibration analysis of axially functionally graded (AFG) supported nanotubes conveying nanoflow under longitudinal magnetic fields are performed, aiming at performance improvement of fluid-interaction nanosystems. Either the density or the elastic modulus of the AFG nanotube varies linearly or exponentially along the axial direction. Based on the nonlocal continuum theory, the higher-order dynamical equation of motion of the system is derived considering no-slip boundary condition. Galerkin discretization technique and eigenvalue analysis are implemented to solve the modeled equation. The validity of the simplified model is justified by comparing the results with findings currently available in the literature. Influence of material gradient, magnetic strength, and nonlocal parameter on the system’s stability is illustrated. The results indicated that the elastic modulus gradient parameter can profoundly displace the instability threshold of the system and the effect of the density profile is negligible. Stability analysis showed that by fine-tuning of material gradation, the nonlocal effect can be significantly alleviated. Furthermore, it was shown that at high or low values of elastic gradient parameters, the stability borders are highly sensitive to magnetic field strength. Results of this paper can be applied as a benchmark in the optimal design of nanofluidic systems.

1. Introduction

During the last two decades, extensive attention is devoted to the dynamics of structures conveying fluid flow at nanoscale levels. Since nanotubes containing fluid flow exhibit unique and promising dynamic characteristics, they are broadly used in nanoscience and nanotechnology fields such as biomass sensors, and fluid storage, among others [1–4]. Nanotubes are comprehensively studied due to their superior characteristics making them as essential parts of bioengineering related topics. Recently they emerge as potent nanocarriers in precisely targeted drug delivery procedures for further functionalization [5, 6]. As a result, it is not surprising that mathematical modeling and stability analysis of such applicable systems becomes a paradigm in the engineering literature. It is well-known that as dimensions of structures miniaturize, utilizing classical continuum mechanics theories may not provide trustworthy results [7–9]. Consequently, modifying classical continuum theories which incorporate nanostructure-dependent size effects is essential to accurately simulate nanoscale problems [10–13]. In order to overcome the deficiency of traditional classical theories, to precisely interpret the mechanical behavior of the nanosized structures, Eringen introduced a higher-order elasticity theory renowned as nonlocal elasticity theory taking into account small-scale effects to serve as a precise theory to the dynamical modeling of nanoscale structures [14]. Influence of Eringen’s nonlocal parameter on the vibrational behavior of nanomechanical fluid-conveying systems is widely elaborated by several researchers [15, 16].

Amongst all nanostructures ubiquitous in various engineering branches, magnetically sensitive nanotubes embedded in external magnetic fields are extensively examined in diverse high-tech areas. Hence, efforts are conducted to scrutinize the mechanics of nanotubes subjected to an externally imposed magnetic field. In this
regard, Murmu et al [17] proposed an analytical model to investigate the influence of a longitudinal magnetic field on the dynamics of magnetically sensitive carbon nanotubes (CNTs) rested on an elastic matrix. They presented an analytical frequency relation to determine the fundamental frequency of the system as a function of nonlocal and magnetic field parameters. Bahadini and Hosseini [18] addressed the nonlocal divergence and flutter instability of an Euler–Bernoulli and Timoshenko CNTs embedded in elastic and Pasternak foundations in the presence of external magnetic fields, respectively. They demonstrated that applying magnetic fields can significantly influence the overall dynamics of the fluid-conveying systems. Arani et al [19] inspected the nonlocal viscoelastic vibration response of viscous fluid-conveying CNTs subjected to a magnetic medium. They discerned that natural frequencies of the system are substantially dependent on the magnetic intensity. In another relevant study, Sadeghi-Goughari et al [20] investigated the dynamics of a nanotube surrounded by elastic media and under external magnetic fields. They specifically detected that magnetic properties of fluid flow have the capability to shift the bifurcation points to higher fluid flow velocities.

Recently, however, modulating material constituents as one of the attractive approaches for stability enhancement of nanofluidic devices are becoming an appealing field of research in ultra-small scale engineering. With the rapid technological advancements in the mechanical metallurgy and fabrication techniques, functionally graded (FG) materials are considered as the state of the art materials in engineering and industry areas [21–23]. Since the material characteristics continuously and smoothly vary along a preferred spatial orientation, FG materials offer valuable advantages with respect to isotropic and conventional laminated composites, such as superior fracture toughness, low-stress concentrations, high thermal and corrosion resistance [24–26]. Applying FG materials in the micro/nanostructures transferring fluid could lead to outstanding outcomes and provide a benchmark for the future design of nanodevices conveying fluid [27, 28]. For instance, Deng et al [29] employed the modified couple stress theory to model the size-dependent vibration of multi-span FG micro-tubes and confirmed that at relatively small values of volume fraction exponents, the critical flow velocity rapidly changes. Setoodeh and Afraim [30] considered the nonlinear vibrational behavior of simply supported micro-tubes manufactured from FG materials. They indicated that regulating FG power index has a significant impact on the fundamental frequency and critical flow velocities. Also, Tang et al [31] performed a comprehensive study on bi-directional FG beams and deduced that any increase in axial FG index decreases the dimensionless nonlinear frequency of the system. In other relevant advanced studies, She et al [4, 32] considered nonlinear size-dependent bending behavior of porous FG nanotubes. They realized that both porosity and power-law index have a deep effect on vibrational behavior of the system. Hosseini et al [33] focused on the forced vibrations of double piezoelectric FG micro-tubes. They asserted that decreasing the dynamical displacements of the system is influential through increasing the material gradient. Filiz and Aydogdu [34] analyzed the free vibrations of embedded FG nanotubes. They evaluated the influence of material properties, flow velocity, and elastic medium stiffness on the fluid-structure dynamics of the system. Deng et al [35] assessed the stability of multi-span viscoelastic FG nanotubes and disclosed that any increment in volume fraction exponent leads to notable variation in critical flow velocity and natural frequencies. It is noteworthy to state that in a majority of available researches conducted on FG nanostructures conveying fluid, it is assumed that the nanotube materials are graded in the radial direction while nanotubes graded in the axial direction are highly applicable in the control of fluid-conveying systems. Despite the importance of the axial grading of material properties, the dynamics of AFG nanotubes are less addressed in the literature so far.

In the present study, the dynamic analysis of AFG nanotubes rested in a magnetically induced medium is developed. In this regard, the stability improvement of AFG nanotubes with both ends supported exposed to external magnetic fields is analyzed. Theoretical modeling for the fluid-conveying nanotube with varying density and elastic modulus in the axial direction is derived. Galerkin discretization scheme is pursued to acquire the reduced-order model equation of the system. Solution approach and stability methodology are briefly discussed. Finally, the influence of material inhomogeneity in the axial direction, nonlocal parameter and magnetic field strength on the dynamics of AFG supported nanotubes is comprehensively elucidated.

2. Theoretical formulation

In this section, a nonlocal fluid-structure interaction formulation is derived utilizing Newton’s second law. A schematic diagram of an AFG nanotube conveying nanofluid between two end supports is shown in figure 1. The nanotube is of length $L$, cross-sectional area $A$, moment of inertia $I$, and conveying fluid of mass per unit length $m$ subjected to an externally imposed magnetic field with the magnetic permeability and strength $\mu_m$ and $H_m$, respectively. In addition, it is assumed that the magnetic permeability of the nanostructure equals the magnetic permeability of the medium around it. The lateral deflection of the nanotube is considered to be $w(x, t)$. 
The density and elastic modulus of the AFG nanotube may alter either exponentially or linearly through the x-axis from \( \rho_0 \) and \( E_0 \) in the upstream to \( \rho_L \) and \( E_L \) in the downstream. They are determined as:

\[
\begin{align*}
\rho(x) &= \rho_0 g(x) \\
E(x) &= E_0 f(x)
\end{align*}
\]  

in which

\[
\begin{align*}
g(x) &= 1 + \frac{x}{L} (\alpha_{\rho} - 1) \quad \text{or} \quad g(x) = e^{\frac{x \ln(\alpha_{\rho})}{L}} \\
f(x) &= 1 + \frac{x}{L} (\alpha_E - 1) \quad \text{or} \quad f(x) = e^{\frac{x \ln(\alpha_E)}{L}}
\end{align*}
\]  

The density and elastic modulus gradient parameters in the equations (3) and (4) can be described as:

\[
\begin{align*}
\alpha_{\rho} &= \frac{\rho_L}{\rho_0} \\
\alpha_E &= \frac{E_L}{E_0}
\end{align*}
\]  

The governing equation for the vibration of the transversally vibrating fluid-conveying system considering magnetic effects can be expressed as [36, 37]:

\[
\frac{\partial^2 M}{\partial x^2} = \rho(x) A \frac{\partial^2 w}{\partial t^2} + F_m + F_f
\]  

in which \( M \), \( F_m \), and \( F_f \) denote the resultant bending moment, lateral forces per unit length induced by the externally imposed magnetic field and the fluid flow movement, respectively, and they are given by [17, 38]:

\[
\begin{align*}
F_m &= \eta_m A H_z \frac{\partial^2 w}{\partial x^2} \\
F_f &= m_f \left( 2U \frac{\partial^3 w}{\partial x^2 \partial t} + U^2 \frac{\partial^2 w}{\partial x^2} + \frac{\partial^3 w}{\partial t^3} \right)
\end{align*}
\]  

where \( U \) denotes the average axial flow velocity under no-slip boundary conditions. It should be mentioned that the influence of the magnetic field is considered through the Lorentz magnetic force acquired from Maxwell’s relations [39].

It is well established that due to the small-size effect, at ultra-small scales the flow behavior is considerably different from those at macro-scales. It is established that the no-slip boundary condition at macro criteria may not hold true at nanoscales and the influence of slip boundary condition might not be negligible for flows at nano criteria [40]. To capture the nanofluid viscosity and slip boundary condition, a velocity correction factor (VCF) modifying nanofluid velocity is suggested by Rashidi et al [41] as follows:

\[
\text{VCF} \triangleq \frac{U}{U_{\text{avg(no-slip)}}} = 1 + 4 \left( \frac{2 - \sigma_s}{\sigma_v} \right) \left( \frac{K_n}{1 + K_n} \right)
\]  

in which \( U_{\text{avg(no-slip)}} \) represents the average flow velocity under the no-slip boundary condition. Also, \( \sigma_v \) is known as the tangential momentum accommodation coefficient and is typically considered to be equal to 0.7 for most practical purposes [41]. Moreover, \( K_n \) is known as the Knudsen number and is used as a discriminate for identifying different flow regimes [19].

The size-dependent characteristics of nanostructures play a crucial role in the mechanical modeling of nanotubes. The nonlocal elasticity theorem is utilized to capture the size-dependency of the mechanical behavior of the system. The nonlocal constitutive relation between the resultant bending moment and the flexural displacement within the framework of Euler–Bernoulli beam theory is expressed in the following special form [14, 42]:

\[ \text{Figure 1. A schematic view of an axially functionally graded nanotube carrying nanofluid.} \]
In this subsection, utilizing Galerkin scheme, the governing equation of motion reduces to a
3.1. Galerkin approach
By some mathematical manipulation, the nonlocal-based governing equation of motion of an AFG nanotube confronted to an axial magnetic medium may be represented as:
\[
E(x)Iw'''' + 2E'(x)Iw'''' + E''(x)Iw'' + m_t(VCF \times U_{avg(no-slip)})w'' \\
+ 2m_t(VCF \times U_{avg(no-slip)})w' - n_mAH^2w'' + \left[m_t + \rho(x)A\right]w - (e_0a)^2 \\
\times \left(m_t(VCF \times U_{avg(no-slip)})^2w'''' + 2m_t(VCF \times U_{avg(no-slip)})w'' - n_mAH^2w''''
\right)
= 0
\]  
where the dots and primes indicate temporal and spatial derivatives, respectively. It follows from equation (13) that in the absence of magnetic field effect (\(H_0 = 0\)), the considered AFG nanotube when \(\mu = 0, \alpha_E = \alpha_p = 1\), and \(\mu = \alpha_E = \alpha_p = 1\) reduces to an AFG tube [43, 44], an isotropic nanotube [36], and an isotropic tube [45], respectively. To acquire the dimensionless form of the equation, the following dimensionless quantities are introduced:
\[
\xi = \frac{x}{L}, \quad \eta = \frac{y}{L}, \quad \tau = \frac{t}{T}
\]  
where \(T\) is defined as:
\[
T = L^4 \left(\frac{M + \rho_0A}{E_0I}\right)^{0.5}
\]  
By substituting the equations (14) into (13), the non-dimensional form of the governing equation of motion will be derived as:
\[
f(\xi)\eta'''' + \gamma(\xi)\eta'''' + \left[\chi(\xi) + u^2 - \psi \eta'' + 2\beta^0.5ui\eta'\right] \\
+ \left[\beta + \theta g(\xi)\right]i\eta'''' - \mu^2(1u^2 - \psi)\eta'''' + 2\beta^0.5ui\eta'''' \\
+ \left[\beta + \theta g(\xi)\right]i\eta'''' + 2\theta g'(\xi)\eta' + \theta g''(\xi)i = 0
\]  
The non-dimensional parameters appeared in equation (16) are described as:
\[
\beta = \frac{m_t}{m_t + \rho_0A}, \quad u = VCF\left(\frac{m_t}{E_0I}\right)^{0.5}LU_{avg(no-slip)}, \quad \psi = \frac{n_mAL^2H^2}{E_0I} \\
\mu = \frac{e_0a}{L}, \quad \gamma(\xi) = 2f'(x)L, \quad \chi(\xi) = f''(x)L^2, \quad \theta = 1 - \beta
\]  
where \(\beta, \psi\), and \(\mu\) are the mass ratio, magnetic field, and nonlocal parameters, respectively.

3. Solution procedure
3.1. Galerkin approach
In this subsection, utilizing Galerkin scheme, the governing equation of motion reduces to a finite model
\[46–50\]. According to this method, the transverse deflection of the nanotube is considered in a series form as:
\[
\eta(\xi, \tau) = \sum_{j=1}^{+\infty} \phi_j(\xi)q_j(\tau)
\]  
where \(n\) is the number of bending modes, \(q_j(\tau)\) denotes the \(j\)th time-dependent modal coordinate, and \(\phi_j(\xi)\) is the \(j\)th undamped natural mode shape of a beam with corresponding boundary conditions [51] (refer to appendix A). Then, by substituting equations (18) into (16), and multiplying the resultant by \(\phi_\xi(\xi)\) and integrating over the interval \([0, 1]\), the discretized form of governing equation of motion is arranged as:
\[
M\ddot{q}(\tau) + C\dot{q}(\tau) + Kq(\tau) = 0
\]
in which \(q\) represents the vector of modal coordinates, \(M, C,\) and \(K\) denote mass, damping and stiffness matrices, respectively with the following elements:
\[ \mathbf{q} = \begin{bmatrix} q_1(\tau), q_2(\tau), \ldots, q_n(\tau) \end{bmatrix}^T \]

\[
\mathbf{M}_{\mathbf{\dot{\mathbf{q}}}} = \theta \int_0^1 \phi_j(\xi)(g(\xi)\dot{\phi}_k(\xi) - \mu^2 g(\xi)\phi''_k(\xi) + 2g'(\xi)\phi'_k(\xi) + g''(\xi)\phi_k(\xi))d\xi
\]

\[
+ \beta \int_0^1 \phi_j(\xi)\dot{\phi}_k(\xi)\phi'_k(\xi) - \mu^2 \phi''_k(\xi)d\xi
\]

\[
\mathbf{C}_{\mathbf{\dot{\mathbf{q}}}} = 2\beta n^5 \int_0^1 \phi_j(\xi)(\phi'_k(\xi) - \mu^2 \phi''_k(\xi))d\xi
\]

\[
\mathbf{K}_{\mathbf{\dot{\mathbf{q}}}} = \int_0^1 f(\xi)\phi_j(\xi)\phi''_k(\xi)d\xi + \int_0^1 \gamma(\xi)\phi_j(\xi)\phi''''_k(\xi)d\xi + \int_0^1 \chi(\xi)\phi_j(\xi)\phi''_k(\xi)d\xi
\]

\[
+ (u^2 - \psi) \int_0^1 \phi_j(\xi)\phi''_k(\xi)d\xi - \mu^2(u^2 - \psi) \int_0^1 \phi_j(\xi)\phi''''_k(\xi)d\xi
\]

\[3.2. \text{Stability analysis} \]

The second-order matrix form of equation (19) can be degenerated to a first-order one as [32]:

\[
\mathbf{BZ}(\tau) + \mathbf{EZ}(\tau) = 0
\]

in which the following elements are defined as:

\[
\mathbf{B} = \begin{bmatrix} 0 & \mathbf{M} \\ \mathbf{M} & \mathbf{C} \end{bmatrix}, \quad \mathbf{E} = \begin{bmatrix} -\mathbf{M} & 0 \\ 0 & \mathbf{K} \end{bmatrix}, \quad \mathbf{Z}(\tau) = \begin{bmatrix} \mathbf{q}(\tau) \\ \mathbf{q}(\tau) \end{bmatrix}
\]

Supposing the solution has a form of \(\mathbf{Z}(\tau) = \mathbf{A}e^{\lambda\tau}\), the first-order equation (25) yields the following eigenvalue problem:

\[
\mathbf{YA} - \lambda \mathbf{I} = 0
\]

in which \(\mathbf{I}\) represents the identity matrix and \(\mathbf{Y} = -\mathbf{B}^{-1}\mathbf{E}\). Finally, the complex-valued eigenvalues (\(\lambda\)) can be obtained by solving equation (26) and the associated complex frequencies (\(\omega = \lambda/i\)) of the system can be acquired as a function of various parameters such as material gradient, flow velocity, magnetic field, nonlocal parameter, and boundary conditions. It worth mentioning that the real and imaginary parts of the complex frequencies of the system are related to the frequency of oscillation and the damping of the AFG restrained nanotubes. Besides, when the real and imaginary parts of one of the complex frequencies become simultaneously zero (\(\text{Real}(\omega) = \text{Image}(\omega) = 0\)), the system undergoes divergence instability via a pitchfork bifurcation. The associated flow velocity at which divergence phenomenon onsets is known as critical divergence velocity \(\nu_d\). At this point, the effective structural stiffness of the system monotonically tends to zero. Furthermore, as two of the branches of frequency diagram whether coalesce or leave on the imaginary (real) axis, Paidoussis (Hamiltonian) type of coupled-mode flutter bifurcation occurs and the corresponding velocity at which the considered bifurcation takes place is known as Paidoussis (Hamiltonian) coupled-mode flutter velocity \(\nu^*\) [53, 54].

\[4. \text{Results and discussion} \]

In this section, natural frequencies of the AFG supported nanotubes with respect to dimensionless flow velocity for various material gradients, nonlocal and magnetic field parameters are calculated and shown. Besides, stability maps, as a tool for optimal designation of the considered structures, are plotted. The real and imaginary parts of the complex-valued natural frequencies of the nanotube represent the oscillation frequency of vibrations and dissipation of the system, respectively.

\[4.1. \text{Model validation} \]

For validation purposes, the numerical results for simplified cases are computed and then compared with those currently available in the literature. The mechanical properties of the nanotube and fluid flow are set, respectively, as: \(\rho_{\text{CNT}} = 2.3 \text{ g.cm}^{-3}, \rho_l = 0.79 \text{ g.cm}^{-3}\) [36]. A nine terms trial mode approximation is considered for the sake of accuracy and validation in Galerkin discretization procedure. The real part of the natural frequency of the first mode of the system versus flow velocity for pinned-pinned (P–P) and clamped-clamped (C–C) boundary conditions, in the absence of the external magnetic field and non-isotropic axial material distribution, are depicted in figure 2. It shows that the results of the present study are in good agreement with those of Mirramazani and Mirdamadi [36] for different nonlocal parameters. It is well shown that considering the nonlocal parameter is vital for evaluating the dynamical behavior of both P–P and C–C nanotubes and has a profound effect on the dynamical behavior of the nanosystem. Moreover, it is clear that the nonlocal parameter has a softening effect on the nanostructure.
In addition, the critical divergence velocities of isotropic end-supported nanotubes for several values of $\mu$ and $Kn$ is obtained and listed in table 1. Comparing the results of the present study with those reported by Mirramezani and Mirdamadi [36], a good agreement is observed. According to table 1, it can be concluded that $\mu$ effect is higher than $Kn$ on the reduction of critical divergence velocity of the system.

### 4.2. Influence of material gradient

In this subsection, the influence of density and elastic modulus gradients on vibrational frequencies and critical flow velocity of the nanotubes are discussed. The parameters $\alpha_E$ and $\alpha_\rho$ play an important role in the stability of the nanotube. In this regard, modulating these parameters, it is possible to remarkably eliminate the unwanted vibrations and improve the performance of the nanosystem. Also, for an AFG nanotube, the elastic modulus (density) decreases from upstream to downstream when $0 < \alpha_E < 1$ ($0 < \alpha_\rho < 1$), while, increases when $\alpha_E > 1$ ($\alpha_\rho > 1$). In addition, an AFG nanotube can be reduced to an isotropic one when $\alpha_\rho = 1$ and $\alpha_E = 1$.

The influence of linear and exponential density gradient index on the vibrational behavior of the system is presented in figures 3(a)–(b), where the real and imaginary parts of the first two vibration modes of the simply supported nanotube against dimensionless flow velocity are depicted, respectively, for $\mu = 0.1$, $\beta = 0.85$ and the indicated density gradient parameters. From figure 3(a), it can be seen that by increasing the fluid flow velocity, the resonant frequency of the first mode decreases monotonously until it finally reaches zero at $u_{cr} \approx 3$. At this point, the system becomes unstable due to divergence and as illustrated in figure 3(b), by raising the fluid flow velocity, the frequency becomes purely imaginary. Moreover, the frequency of the second mode reduces monotonically until it diminishes at $u \approx 5.32$. It can be seen from these figures that, in general, the critical divergence velocity of the system is independent of the density gradient parameter as well as the type of density.

![Figure 2. Comparison of the first natural mode of isotropic restrained nanotubes ($\alpha_\rho = \alpha_E = 1$) with the results of Mirramezani and Mirdamadi [36] for $Kn = 0.001$, $\psi = 0$.](image)

| $\mu$ | $Kn$ | Present study | Mirramezani and mirdamadi [36] | Present study | Mirramezani and mirdamadi [36] |
|-------|------|---------------|-------------------------------|---------------|-------------------------------|
| 0     | 0    | 3.142         | 3.142                         | 6.284         | 6.283                         |
| 0.0005| 3.130| 3.130         | 3.130                         | 6.261         | 6.259                         |
| 0.001 | 3.117| 3.117         | 3.118                         | 6.238         | 6.238                         |
| 0.1   | 0    | 2.997         | 2.998                         | 5.321         | 5.364                         |
| 0.2   | 0    | 2.660         | 2.661                         | 3.913         | 3.93                          |
| 0.001 | 2.6405| 2.64         | 2.64                          | 3.884         | 3.9                           |

In addition, the critical divergence velocities of isotropic end-supported nanotubes for several values of $\mu$ and $Kn$ is obtained and listed in table 1. Comparing the results of the present study with those reported by Mirramezani and Mirdamadi [36], a good agreement is observed. According to table 1, it can be concluded that $\mu$ effect is higher than $Kn$ on the reduction of critical divergence velocity of the system.
distribution. Hence, although having the ability to influence the overall dynamics of the system, one can state that the density gradient parameter does not affect the instability threshold of the system. This feature also is analytically confirmed by the exact approach presented in appendix B. According to figure 3(a), increasing the density gradient parameter results in lower natural frequencies of the system. Moreover, compared with exponentially density distribution, linear density distribution leads to lower values of natural frequency. From figure 3(b), it is clear that the damping ratio of the system is higher when \( \alpha < 1 \). Moreover, the dynamic characteristics of AFG nanotubes have a slight dependency on the type of density distribution. Based on figure 3(b), comparing to linearly density distribution, exponentially density distribution yields higher damping ratio in the system.

Evolution of the system dynamics toward divergence is pursued by plotting the complex frequency when the flow velocity is varied. In figures 4(a)–(b), the influence of linear and exponential variations of elastic modulus

![Figure 3](image_url)
on the real and imaginary parts of the first two modes of an AFG pinned-pinned nanotube is examined for $\beta = 0.15$ and $\mu = 0.1$. It is observed that regardless of the type of material distribution, larger values of $\alpha_E$ remarkably enhance the critical divergence velocity. Furthermore, choosing $\alpha_E$ to be more than unity will dramatically hinder the initiation of the Paidoussis coupled-mode flutter instability. It can be generalized that the linear distribution tends to make the nanostructure more stable than its exponential counterpart. Consequently, one can assert that in contrast to the density gradient parameter, the elastic modulus gradient parameter plays a predominant role in the stability of AFG supported nanotubes. It can be found from figures 3, 4 that the axial variation of elastic modulus in comparison with the axial variation of density leads to a wider range of frequency. Hence, from the viewpoint of designability and operability, the case of elastic modulus grading could contribute to a broad adjustable range of frequencies to avoid resonance phenomenon. Herein, it

Figure 4. (a) Real and (b) imaginary components of the dimensionless complex frequency of a pinned-pinned nanotube against dimensionless flow velocity for $\mu = 0.1, \alpha_\rho = 1, \psi = 0$, and $\beta = 0.15$. 

Figure 4.
should be mentioned that according to figures 3, 4, neither Hamiltonian nor Paidoussis coupled-mode flutter depends on the type of material distribution and are only influenced by the mass ratio parameter.

To investigate the coupling effects of material gradation and size-dependent nonlocal parameter, dimensionless critical divergence velocities of AFG restrained nanotubes with respect to elastic modulus gradient parameter are plotted in figures 5(a)–(b) for several distinct values of the nonlocal parameter. In these figures, the system is stable only for flow velocities that lie under each curve, otherwise, the system tends toward divergence instability. It is clear that the greater the nonlocal parameter, the less stable the system will be. Furthermore, greater \( \mu \) values cause lower vibration frequencies and as a result, divergence instability takes place for lower flow velocity. One can deduce that size-dependent nonlocal parameter greatly reduces the stiffness of the nanostructures. In addition, as shown in the stability map, the stable region of the system gradually enhances by increasing \( \alpha_E \). It signifies that selecting larger \( \alpha_E \) values, is more suitable to reduce the destabilization effect of

**Figure 5.** Dimensionless critical divergence velocity of (a) pinned-pinned and (b) clamped-clamped nanotubes versus elastic modulus gradient parameter; \( \alpha_p = 1 \) and \( \psi = 0 \).
the nonlocal parameter. It should be noted that except at $\alpha_E = 1$ (isotropic nanotube), the linear distribution of elastic modulus leads to a wider stable region, especially at sufficiently high or low values of elastic modulus gradients. Additionally, for both considered boundary conditions, it can be stated that at relatively high values of nonlocal parameter, the stability boundaries of linear and exponential material distributions are virtually close to each other and have a low sensitivity to material gradient index.

To understand the divergence configuration of AFG restrained nanotubes, stability borders in the $u_d-\mu$ plane for both linear and exponential material distributions are represented in figures 6(a)–(b) for three distinct values of $\alpha_E$. In contrary to the $u_d-\alpha_E$ curves, the $u_d-\mu$ curves are generally descending as the nonlocal parameter increases. From the graphical illustration, it is clear that AFG restrained nanotubes would be more stable in comparison with the isotropic restrained nanotubes when $\alpha_E > 1$. Hence, larger $\alpha_E$ values enhance the performance of the restrained nanotubes. It is also evident that the system experiences less stability as the nonlocal parameter increases.

Figure 6. Dimensionless critical divergence velocity of (a) pinned-pinned and (b) clamped-clamped nanotubes versus nonlocal parameter; $\alpha_p = 1$ and $\psi = 0$. 
In other words, incorporating the nonlocal parameter decreases the stiffness of the nanotube. Since the elastic modulus and nonlocal parameters have opposite effects on the stability of the system, they provide additional degree-of-freedoms in the nanofluid devices vibration control. Obviously, the linear distributed elastic modulus leads to a more stable system. As a result, the influence of material distribution on the system stability is highly sensitive. Consequently, the design of AFG nanotubes in comparison with conventional isotropic nanotubes are more flexible to adjust for a certain dynamic characteristic. It is notable that at high values of nonlocal parameter, the stability borders, regardless of linearly or exponentially material distribution, converge to one other. Also, as anticipated, the C–C boundary condition yields a wider stable region. This is due to the fact that the C–C boundary conditions cause to enhance the rigidity and natural frequency of the structure and consequently delay the divergence instability of the system. Surprisingly, at high values of the nonlocal parameter, the critical flow velocity will no longer depend on the type of supported boundary conditions. Furthermore, it worth to mention that the effect of the nonlocal parameter has a considerable impact on the decrement of critical divergence velocity for C–C nanotubes in comparison with P–P counterparts.

4.3. Influence of magnetic fields
In figure 7, the stability map of isotropic magnetically sensitive tubes (\(\alpha_E = \alpha_\rho = 1\)) restrained tubes (\(\mu = 0\)) versus dimensionless magnetic field parameter.

![Figure 7. Dimensionless critical divergence velocity of isotropic (\(\alpha_E = \alpha_\rho = 1\)) restrained tubes (\(\mu = 0\)) versus dimensionless magnetic field parameter.](image)

In other words, incorporating the nonlocal parameter decreases the stiffness of the nanotube. Since the elastic modulus and nonlocal parameters have opposite effects on the stability of the system, they provide additional degree-of-freedoms in the nanofluid devices vibration control. Obviously, the linear distributed elastic modulus leads to a more stable system. As a result, the influence of material distribution on the system stability is highly sensitive. Consequently, the design of AFG nanotubes in comparison with conventional isotropic nanotubes are more flexible to adjust for a certain dynamic characteristic. It is notable that at high values of nonlocal parameter, the stability borders, regardless of linearly or exponentially material distribution, converge to one other. Also, as anticipated, the C–C boundary condition yields a wider stable region. This is due to the fact that the C–C boundary conditions cause to enhance the rigidity and natural frequency of the structure and consequently delay the divergence instability of the system. Surprisingly, at high values of the nonlocal parameter, the critical flow velocity will no longer depend on the type of supported boundary conditions. Furthermore, it worth to mention that the effect of the nonlocal parameter has a considerable impact on the decrement of critical divergence velocity for C–C nanotubes in comparison with P–P counterparts.

4.3. Influence of magnetic fields
In figure 7, the stability map of isotropic magnetically sensitive tubes (\(\alpha_E = \alpha_\rho = 1\), \(\mu = 0\)) in the presence of magnetic fields is plotted for P–P and C–C boundary conditions. In this figure, the average flow velocity is considered based on classical continuum mechanics; i.e., \(Kn\) equals to zero. It can be deduced from this figure that the obtained results of the analytical approach (presented in appendix B) are compatible with Galerkin method. The results presented in this figure illustrate that increasing the magnetic field parameter leads to an increase in critical divergence velocity. It implies that the magnetic field parameter can be utilized for tailoring the divergence capacity of magnetically sensitive systems conveying fluid. This indicates in general, incorporating the magnetic parameter, the structural stiffness hardening is observed.

To inspect the influence of magnetic fields variations on dynamical behavior of AFG simply supported nanotubes, dimensionless critical divergence velocity of the system in terms of elastic modulus gradient, nonlocal and magnetic field parameters are displayed, in figures 8(a)–(c). As expected, these stability maps demonstrate that the stability of the system principally depends on the nonlocal, magnetic field and elastic modulus gradient parameters. As the illustrative curves indicate, stable region drastically expands by ascending magnetic field strength. It means that any increment in the magnetic field parameter leads to a stiffer nanotube. Also, in figure 8(b), at high values of magnetic field parameter, the stability borders approach to each other. By comparing figures 8(a)–(b), it can be discerned that the stiffness softening effect of the nonlocal parameter in the vibration response of the system could be dampened and even eliminated by the presence of the longitudinal magnetic field. Generally, an increase in the magnetic field and elastic modulus gradient parameters could
shrink the unstable divergence region of the nanotube, whereas, any increase in nonlocal parameter weakens the stability of the system. Nevertheless, simultaneous fine-tuning of these parameters is necessary to achieve an improvement in the performance of nanofluidic systems.
5. Conclusions

Size-dependent stability analysis of linearly and exponentially AFG restrained nanotubes subjected to longitudinal magnetic fields is carried out. To capture the size-dependent effect, the nonlocal elastic theory is employed in conjunction with the Euler–Bernoulli beam theory. It is assumed that the effective material properties of the nanotube change continuously in the longitudinal direction. To accurately predict the dynamic characteristics of the system, the no-slip boundary condition is considered in mathematical modeling. Dynamic equation of motion is derived and Galerkin discretization scheme and eigenvalue analysis were adopted to calculate the critical divergence velocity and complex frequencies of the nanotube. The validity of the model is performed by comparing the obtained results for simple cases with those reported in the literature and an excellent agreement is observed. Also, an analytical treatment is employed to confirm the accuracy of the magnetically-influenced behavior of the system. To clarify the cumulative effect of influential system parameters such as density, elastic modulus gradients, nonlocal parameter, and externally imposed magnetic field strength a detailed parametric study is conducted. Results reveal that:

1. The density gradient parameter has virtually no influence on the stability of the system, whereas, the elastic modulus gradient parameter substantially displaces the stability threshold and contribute to an extended range of frequencies.
2. The greater the elastic gradient parameter is, the more stable the system becomes. In addition, the effect of linear material distribution on stability is higher than its exponential counterpart.
3. The small-size effect is more pronounced for axially graded clamped-clamped nanotube in comparison with the simply supported case. Furthermore, at large nonlocal parameters, the stability borders lose their sensitivity to the boundary conditions.
4. Increasing the magnetic field strength results in enlargement of the stable region, especially at high or low values of elastic modulus gradient parameter.

It is demonstrated that in comparison with the conventional isotropic nanotubes, the magnetically sensitive AFG nanotubes have a better performance in stability behavior by designating appropriate system parameters.

Appendix A

The normalized eigenfunctions of nonlocal beams with P–P and C–C boundary conditions are respectively given by [31]:

\[ \varphi_j(\xi) = \sqrt{2} \sin(\sigma_j \xi) \]  

\[ \varphi_j(\xi) = \cos h(\sigma_j \xi) - \cos(\lambda_j \xi) - \frac{\sigma_j h(\sigma_j) + \lambda_j \sin(\lambda_j)}{\sigma_j (\cos h(\sigma_j) - \cos(\lambda_j))} \times \left( \sin h(\sigma_j \xi) - \frac{\sigma_j}{\lambda_j} \sin(\lambda_j \xi) \right) \]  

where

\[ \left\{ \begin{array}{c} \sigma_j \\ \lambda_j \end{array} \right\} = \frac{q_j \sqrt{4 + (\mu q_j)^2 \pm (\mu q_j)^2}}{2} \]  

The characteristic frequency equations of the aforementioned boundary conditions are as follows, respectively [31]:

\[ \sin(\sigma_j) = 0 \]  

\[ \mu \sigma_j \lambda_j \sin h(\sigma_j) \sin(\lambda_j) + 2 \cosh(\sigma_j) \cos(\lambda_j) = 2 \]

Appendix B

The critical divergence velocity of isotropic (\( \alpha_E = \alpha_\nu = 1 \)) supported tubes (\( \mu = 0 \)) including magnetic effect can be derived from the dynamical equation of the system (equation (15)) as follows:
\[ \eta''' + [u^2 - \psi]\eta'' + 20.5u\eta' + \eta = 0 \quad (B1) \]

By considering the solution \( \eta(\xi, \tau) = \text{Y(\xi)}e^{\text{B1}\tau} \) and substituting into equation (B1) yields [55]:

\[ Y''' + [u^2 - \psi]Y'' + 20.5\Omega Y' + \Omega^2 Y = 0 \quad (B2) \]

As the tube becomes unstable via a pitchfork bifurcation at critical divergence velocity (static instability occurs in the system), the first frequency of the tube equals to zero \((\Omega = 0)\) and all time-dependent terms are set equal to zero; thus, one can conclude that density variation along axial direction does not affect the critical divergence velocity in supported tubes. In the following, at the critical divergence velocity, equation (B2) reduces to the following equation:

\[ Y''' + \delta^2 Y'' = 0, \quad \delta = \sqrt{u^2 - \psi} \quad (B3) \]

The general solution of equation (B3) can be readily considered as:

\[ Y = C_1 + C_2\xi + C_3 \sin (\delta\xi) + C_4 \cos (\delta\xi) \quad (B4) \]

where \(C_1-C_4\) are integration constants can be determined from boundary conditions of the system. The boundary conditions of supported tubes, i.e., pinned-pinned and clamped-clamped can be expressed, respectively, as follows:

\[ Y(0) = Y(1) = Y''(0) = Y''(1) = 0 \quad (B5) \]
\[ Y(0) = Y(1) = Y''(0) = Y''(1) = 0 \quad (B6) \]

Substituting equations (B4) into (B3) and then applying the boundary conditions (equations ((B5), (B6))) leads to the following characteristic equations for \(P-P\) and \(C-C\) boundary conditions, respectively.

\[ \sin (\delta) = 0 \quad (B7) \]
\[ 2 \cos (\delta) + \delta \sin (\delta) = 2 \quad (B8) \]

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**References**

[1] Yengejeh S I, Kazemi S A and Oechsner A 2016 Advances in mechanical analysis of structurally and atomically modified carbon nanotubes and degenerated nanostructures: a review Composites Part B: Engineering 86 95–107
[2] Tang Y, Zhen Y and Fang B 2018 Nonlinear vibration analysis of a fractional dynamic model for the viscoelastic pipe conveying fluid Appl. Math. Modell. 56 123–36
[3] Mirtalebi S H, Ahmadian M T and Ebrahimi-Mamaghani A 2019 On the dynamics of micro–tubes conveying fluid on various foundations, SN Applied Sciences 1 547
[4] She G-L, Yuan F-G, Ren Y-R and Xiao W-S 2017 On buckling and postbuckling behavior of nanotubes Int. J. Eng. Sci. 121 130–42
[5] Pardo J, Peng Z and Leblanc R 2018 Cancer targeting and drug delivery using carbon-based quantum dots and nanotubes Molecules 23 378
[6] She G-L, Ren Y-R, Yuan F-G and Xiao W-S 2018 On vibrations of porous nanotubes Int. J. Eng. Sci. 125 23–35
[7] Farajpour A, Ghayesh M H and Farokhi H 2018 A review on the mechanics of nanostructures Int. J. Eng. Sci. 133 231–63
[8] Kuo S-Y 2018 Free vibration of fully functionally graded carbon nanotube reinforced graphite/epoxy laminates Mater. Res. Express 5 055048
[9] Nikpourian A, Ghazavi M R and Azizi S 2019 Size–dependent secondary resonance of a piezoelectrically laminated bistable MEMS arch resonator Composites Part B: Engineering 173 106850
[10] Shafei N and She G-L 2018 On vibration of functionally graded nano–tubes in the thermal environment Int. J. Eng. Sci. 133 84–98
[11] Talebitioni M 2019 A semi-analytical solution for free vibration analysis of rotating carbon nanotube with various boundary conditions based on nonlocal theory Mater. Res. Express (https://doi.org/10.1088/2053-1591/ab2bcb)
[12] Esfahani S, Esmaeilzade Khadem S and Mamaghani A E 2019 Size–dependent nonlinear vibration of an electrostatic nanobeam actuator considering surface effects and inter-molecular interactions Int. J. Mech. Mater. Des. 15 489–505
[13] Rahimian-Koloor SM, Moshefzadeh-Sani H, Shokrieh M M and Hashernianzadeh S M 2018 On the behavior of isolated and embedded carbon nano–tubes in a polymeric matrix Mater. Res. Express 5 025019
[14] Eringen A C 1983 On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves J. Appl. Phys. 54 4703–10
[15] Arash B and Wang Q 2012 A review on the application of nonlocal elastic models in modeling of carbon nanotubes and graphene Compt. Mater. Sci. 51 303–13
[16] Zhen Y X, Wen S L and Wang Y 2019 Free vibration analysis of viscoelastic nanotubes under longitudinal magnetic field based on nonlocal strain gradient Timoshenko beam model Physica E 105 116–24
[17] Murmu T, McCarthy M and Adhikari S 2012 Vibration response of double-walled carbon nanotubes subjected to an externally applied longitudinal magnetic field: a nonlocal elasticity approach J. Sound Vib. 331 5069–86
[18] Bahadini R and Hosseini M 2016 Nonlocal divergence and flutter instability analysis of embedded fluid–conveying carbon nanotube under magnetic field Microfluid. Nanofluid. 20 198
[19] Arami A G, Amir S, Dashi P and Yousefi M 2014 Flow–induced vibration of double bonded visco-CNTs under magnetic fields considering surface effect Compt. Mater. Sci. 86 144–54
[20] Sadeghi-Ghouhari M, León S and Kwon H J 2018 Flutter instability of cantilevered carbon nanotubes caused by magnetic fluid flow subjected to a longitudinal magnetic field Physica E 98 184–90
[21] Udupa G, Rao S S and Gangadharan K 2014 Functionally graded composite materials: an overview Procedia Materials Science 5 1291–9
[22] Esfahani S, Khadem S E and Mamaghani A E 2019 Nonlinear vibration analysis of an electrostatic functionally graded nano-resonator with surface effects based on nonlocal strain gradient theory Int J. Mech. Sci. 151 308–22
[23] Safarpour M, Rahimi A and Alibeglooo A 2019 Static and free vibration analysis of graphene platelets reinforced composite truncated conical shell, cylindrical shell, and annular plate using theory of elasticity and DQM Mech. Based Des. Struct. Mech. 1–29
[24] Tang Y and Yang T 2018 Bi-directional functionally graded nanotubes: fluid conveying dynamics International Journal of Applied Mechanics 10 115201
[25] Tang Y and Ding Q 2019 Nonlinear vibration analysis of a bi-directionally functionally graded beam under hygro-thermal loads Compos. Struct. 190 111076
[26] Shahgholian-Ghahfarokhi D, Safarpour M and Rahimi A 2019 Torsional buckling analyses of functionally graded porous nanocomposite cylindrical shells reinforced with graphene platelets (GPs) Mech. Based Des. Struct. Mech. 1–22
[27] Gao Y, Xiao W and Zhu H 2019 Nonlinear vibration of functionally graded nano-tubes using nonlocal strain gradient theory and a two-steps perturbation method Struct. Eng. Mech. 69 205–19
[28] Gao Y, Xiao W S and Zhu H 2019 Nonlinear vibration analysis of different types of functionally graded beams using nonlocal strain gradient theory and a two-step perturbation method The European Physical Journal Plus 134 23
[29] Deng J, Liu Y and Liu W 2017 Size-dependent vibration analysis of multi-span functionally graded material micropipes conveying fluid using a hybrid method Microfluid. Nanofluid. 21 133
[30] Setoodeh A and Afrahim S 2014 Nonlinear dynamic analysis of FG micro-pipes conveying fluid based on strain gradient theory Compos. Struct. 116 128–35
[31] Tang Y, Lv X and Yang T 2019 Bi-directionally functionally graded beams: asymmetric modes and nonlinear free vibration Composites Part B: Engineering 156 319–31
[32] She G L, Yuan F G, Karami B, Ren Y R and Xiao W S 2019 On nonlinear bending behavior of FG porous curved nanotubes Int. J. Eng. Sci. 135 58–74
[33] Hosseini M, Maryam A Z B and Bahaadini R 2017 Forced vibrations of fluid-conveyed double piezoelectric functionally graded micropipes subjected to moving load Microfluid. Nanofluid. 21 134
[34] Filiz S and Aydogdu M 2015 Wave propagation analysis of embedded (coupled) functionally graded nanotubes conveying fluid Compos. Struct. 132 1260–73
[35] Deng J, Liu Y, Zhang Z and Liu W 2017 Size-dependent vibration and stability of multi-span viscoelastic functionally graded material nanotubes conveying fluid using a hybrid method Compos. Struct. 179 590–600
[36] Mirmamezani M and Mirdamadi H R 2012 Effects of nonlocal elasticity and Knudsen number on fluid–structure interaction in carbon nanotube conveying fluid Physica E 48 2005–15
[37] Bahaadini R, Hosseini M and Jamali B 2018 Flutter and divergence instability of supported piezoelectric nanotubes conveying fluid Physica B 529 57–65
[38] Bahaadini R and Hosseini M 2016 Effects of nonlocal elasticity and slip condition on vibration and stability analysis of viscoelastic cantilever carbon nanotubes conveying fluid Comput. Mater. Sci. 114 151–9
[39] Hosseini M and Sadeghi-Ghouhari M 2016 Vibration and instability analysis of nanotubes conveying fluid subjected to a longitudinal magnetic field Appl. Math. Modell. 40 2560–76
[40] Wang H, Dong K, Men F, Yan Y and Wang X 2010 Influences of longitudinal magnetic field on wave propagation in carbon nanotubes embedded in elastic matrix Appl. Math. Modell. 34 878–89
[41] Rashidi V, Mirdamadi H R and Shirani E 2012 A novel model for vibrations of nanotubes conveying nanofluid Comput. Mater. Sci. 51 347–52
[42] SoltanRezaee M, Bodaghi M, Farrokhhabadi A and Hedayati R 2019 Nonlinear stability analysis of piecewise actuated piezoelectric microstructures Int. J. Mech. Sci. 60 200–8
[43] Zhou X W, Dai H L and Wang L 2018 Dynamics of axially functionally graded cantilevered pipes conveying fluid Compos. Struct. 190 112–8
[44] An C and Su J 2017 Dynamic behavior of axially functionally graded pipes conveying fluid Mathematical Problems in Engineering 2017 1–11
[45] Paidoussis M P 1998 Fluid-Structure Interactions: Slender Structures and Axial Flow (New York: Academic)
[46] Mamaghani A E, Khadem S and Bab S 2016 Vibration control of a pipe conveying fluid under external periodic excitation using a nonlinear energy sink Nonlinear Dyn. 86 1761–95
[47] Mamaghani A E, Khadem S E, Bab S and Pourkiaee S M 2018 Irreversible passive energy transfer of an immersed beam subjected to a sinusoidal flow via local nonlinear attachment Int. J. Mech. Sci. 138 427–47
[48] Hosseini R, Hamedei M, Ebrahimi Mamaghani A, Kim H C, Kim J and Dayou J 2017 Parameter identification of partially covered piezoelectric cantilever power scavenger based on the coupled distributed parameter solution International Journal of Smart and Nano Materials 8 110–24
[49] Sarpasad H and Ebrahimi-Mamaghani A 2019 Vibrations of laminated deep curved beams under moving loads Compos. Struct. 226 111262
[50] Rezaei M, Khadem S E and Firoozy P 2017 Broadband and tunable PZT energy harvesting utilizing local nonlinearity and tip mass effects Int. J. Eng. Sci. 118 1–15
[51] Lu P, Lee H, Lu C and Zhang P 2006 Dynamic properties of flexural beams using a nonlocal elasticity model J. Appl. Phys. 99 073510
[52] Tang Y and Yang T 2018 Post-buckling behavior and nonlinear vibration analysis of a fluid-conveying pipe composed of functionally graded material Compos. Struct. 185 393–400
[53] Zare A, Eghtesad M and Daneshmand F 2017 Numerical investigation and dynamic behavior of pipes conveying fluid based on isogeometric analysis Ocean Eng. 140 388–400
[54] Ebrahimi-Mamaghani A, Sotudeh-Gharebagh R, Zarghami R and Mostoufi N 2019 Dynamics of two-phase flow in vertical pipes J. Fluids Struct. 87 150–73
[55] Dehrouyeh-Semnani A M, Nikkah-Bahrami M and Yazdi M R H 2017 On nonlinear vibrations of micropipes conveying fluid Int. J. Eng. Sci. 117 20–33