Correlation-enhanced control of wave focusing in disordered media

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A fundamental challenge in physics is controlling the propagation of waves in disordered media despite strong scattering from inhomogeneities. Spatial light modulators enable one to synthesize (shape) the incident wavefront, optimizing the multipath interference to achieve a specific behaviour such as focusing light to a target region. However, the extent of achievable control is not known when the target region is much larger than the wavelength and contains many speckles. Here we show that for targets containing more than $g$ speckles, where $g$ is the dimensionless conductance, the extent of transmission control is substantially enhanced by the long-range mesoscopic correlations among the speckles. Using a filtered random matrix ensemble appropriate for coherent diffusion in open geometries, we predict the full distributions of transmission eigenvalues as well as universal scaling laws for statistical properties, in excellent agreement with our experiment. This work provides a general framework for describing wavefront-shaping experiments in disordered systems.

Waves propagating through a disordered medium undergo multiple scattering from the inhomogeneities. Interference among the multiply scattered fields has important consequences that cannot be described with incoherent diffusion.\textsuperscript{1,2} By controlling the incident wave (‘wavefront shaping’, WFS), one can manipulate this interference and drastically modify the transport of light, microwaves, and acoustic waves.\textsuperscript{3} One early and notable example is focusing light onto a local speckle-sized target through aligning the scattered fields there\textsuperscript{4,7}, which has led to advances in imaging within biological tissue and other scattering materials.\textsuperscript{6} The transport through disordered structures is described by a random field transmission matrix, and the use of WFS over such local properties has treated the matrix elements as having only short-range correlations on the scale of a single speckle\textsuperscript{6-13}. However, it has long been known that diffusive waves also exhibit long-range and infinite-range correlations\textsuperscript{14-17}; this was previously noted in the context of electron transport through mesoscopic structures, where correlations lead to anomalously large conductance fluctuations\textsuperscript{18}. The long-range correlations are related to the existence of near-unity-transmission input states (‘open channels’)\textsuperscript{19-24}, and have measurable effects on other global statistical properties of diffusive waves such as the total transmission variance\textsuperscript{25-28}, the increased background for maximally focused waves\textsuperscript{29-32}, and the singular values of large transmission matrices\textsuperscript{33-36}. With the rapid growth of WFS, an important question, both scientifically and technologically, is how correlations affect the coherent control over targets larger than a single speckle and smaller than the full transmitted pattern—that is, in between local and global. This intermediate regime remains poorly understood but is relevant for many applications, ranging from telecommunications and cryptography to photothermal therapy and the optical or ultrasound imaging of large objects behind a scattering medium.

Here, we demonstrate the effects of correlations by means of optical WFS experiments in this interesting regime. WFS enables dynamic control over how much light is transmitted into a given target, and we find that for large targets, correlations increase the range of control significantly beyond what would be achievable if correlations were negligible (as is typically assumed). Physically this is because in a multiply scattering medium, the transmitted flux is carried by roughly only $g$ open channels\textsuperscript{19-22}; here $g$ is the analogue of the dimensionless conductance or Thouless number for electron transport in a waveguide, and its definition in an open geometry will be discussed below. When the target region is large enough that the number $M_2$ of speckles in it exceeds $g$, the output channels are necessarily correlated. Such positive correlations reduce the effective degrees of freedom $M_1^{(df)}$ that need to be controlled, and lead to the increased control range and other correlation effects.

WFS experiments on strongly scattering media are typically performed in an open geometry: the illumination spot spreads laterally as the wave diffuses into the medium. A rigorous random matrix theory for such a set-up was not known until recently, when the ‘filtered random matrix’ ensemble (FRM) was introduced\textsuperscript{37} and conjectured to apply to diffusion with an open boundary\textsuperscript{38}. There were some partial tests of the FRM eigenvalue distributions\textsuperscript{34-36}, but not with lateral diffusion accounted for rigorously. Here we show precisely how the FRM can be applied to an arbitrary open diffusion experiment, and confirm its predictions for the full distributions of transmission eigenvalues. We prove that the ratio $M_2/g$ determines the presence or absence of significant correlation effects, and derive new scaling laws for measurable statistical quantities that are found to agree very well with our experimental data. This theoretical framework is applicable to the WFS for light, as well as microwaves and acoustic waves.

When we modulate $M_1$ incident channels with a spatial light modulator (SLM) and collect $M_2$ channels transmitted into a target region, the input–output relation can be written as $|\psi_{\text{out}}\rangle = t|\psi_{\text{in}}\rangle$, where $|\psi_{\text{in}}\rangle$ and $|\psi_{\text{out}}\rangle$ are vectors of length $M_1$ and $M_2$ that contain the input and output field amplitudes. Here $t$ is an $M_2$-by-$M_1$ transmission matrix, and we use the tilde to indicate the exclusion of unmodulated inputs and uncollected outputs\textsuperscript{37}. The total flux into the focal target is $T = \langle \psi_{\text{out}}|\psi_{\text{in}}\rangle = \langle \psi_{\text{in}}|t^\dagger t|\psi_{\text{in}}\rangle$; the variational principle guarantees that the maximal and minimal

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transmitted fluxes are the extremal eigenvalues of \( \hat{T} \hat{T} \). The corresponding eigenvectors are the optimal input wavefronts that the SLM should synthesize. Thus, the variance \( \text{Var}(\hat{T}) \), where \( \hat{T} \) denotes the eigenvalues of \( \hat{T} \hat{T} \), is a measure of the range of focused transmission that is achievable by WFS. If the matrix elements of \( \hat{T} \) were uncorrelated random numbers, for sufficiently large \( M_1 \) and \( M_2 \), the eigenvalues would follow the Marcenko–Pastur (MP) distribution\( ^{39} \), which has variance \( \text{Var}(\hat{T}^{\text{MP}}) = \langle \hat{T} \rangle \text{Var}(\hat{T}) \). The ratio between \( \text{Var}(\hat{T}) \) and \( \text{Var}(\hat{T}^{\text{MP}}) \) is a measure of how correlations affect the range of coherent control.

We study the transport through a slab of zinc oxide (ZnO) microparticles (median diameter \( \approx 200 \text{ nm} \)) deposited on a cover slip, with slab thickness \( L \approx 60 \mu \text{m} \) and total transmission \( T \approx 3\% \). The incident wavefront (wavelength \( \lambda = 532 \text{ nm} \)) is modulated with a phase-only SLM and then focused onto the sample with a high-NA objective, and the transmitted light is collected on a charge-coupled device (CCD) camera; see the schematic illustration in Fig. 1a and details in Methods. In our set-up, the SLM/CCD pixels modulate/detect different angles incident onto/transmitted from the sample. The nearby SLM pixels are grouped into macropixels; smaller macropixels correspond to more finely spaced incident angles, with a larger illumination spot and more available input channels. The illumination spot size determines the crucial parameter \( g \) in an open geometry, and we consider three macropixel sizes that correspond to illumination diameters \( D_m \approx 6, 12, 24 \mu \text{m} \); the number of modes we modulate is \( M_1 \approx 128, 512, 2,048 \), respectively. Our output speckle grains are slightly larger than one CCD pixel (the intensity autocorrelation width is 1.5 pixels), and we keep only one pixel out of 2 × 2 pixels in the data recorded on the CCD to remove correlations among neighbour pixels; thus each remaining CCD pixel corresponds to one output channel, free of short-range correlations. With this set-up, we measure the transmission matrix \( \hat{T} \) using a phase-shifting common-path interferometric method (Methods). Once the transmission matrix is measured, we use it to predict the optimal phase-only wavefront for a given target (Methods), and then measure the output on the CCD when such a wavefront is applied on the SLM. Example outputs for enhanced and suppressed transmission into a large target are shown in Fig. 1b–d.

The intensity correlation functions between the output channels are calculated from the measured transmission matrices and shown in Supplementary Fig. 1 (Supplementary Section I). Long-range correlations are readily seen in the data, which explain why, in Fig. 1c–d, the background speckle intensities outside the target increase (or decrease) with those inside the target, as observed previously\( ^{28,32} \).

We obtain the eigenvalues of \( \hat{T} \) from the measured transmission matrices, for circular targets of increasing sizes. As the target size grows, the eigenvalue variance becomes larger than the uncorrelated MP variance, as shown in Fig. 2a. For small targets (\( M_1 \lesssim 10^3 \)), we observe \( \text{Var}(\hat{T}) \approx \text{Var}(\hat{T}^{\text{MP}}) \) with no obvious correlation effects, consistent with prior work involving small transmission matrices\( ^{8,13} \). However, for large targets (\( M_1 \gtrsim 10^3 \)), \( \text{Var}(\hat{T}) \) becomes significantly larger than \( \text{Var}(\hat{T}^{\text{MP}}) \), indicating an enhanced range of control due to correlations. The ratio between the eigenvalue range \( \hat{T}_{\text{max}} - \hat{T}_{\text{min}} \) and that from an uncorrelated matrix \( \hat{T}^{\text{MP}}_{\text{max}} - \hat{T}^{\text{MP}}_{\text{min}} \) is shown in Supplementary Fig. 2 (Supplementary Section II), which follows the same trend as the eigenvalue variance.
The extremal eigenvalues $\tilde{\tau}_{\text{max}}$ and $\tilde{\tau}_{\text{min}}$ are shown in Fig. 3 (green filled symbols) for the case $D_{\text{in}} \approx 12 \, \mu$m. They stretch a wider range than the uncorrelated ones $\tilde{\tau}_{\text{max/min}}^{(\text{MP})} = (\tilde{\tau}) (1 \pm \sqrt{M_i/M_j})^2$ (grey dotted lines), and the difference grows with $M_i$. We can readily achieve this extended range experimentally: when $M_i \gtrsim 3,000$, the largest and the smallest focused transmission reached with our phase-only SLM (red crosses) cover a wider range than the uncorrelated extrema, even though the uncorrelated extrema were calculated assuming both phase and amplitude modulations. Having access to the transmission matrix is important as it helps us find near-optimal wavefronts (Methods); a recent experiment used feedback-based optimization and reported enhancements lower than the uncorrelated value $\tilde{\tau}_{\text{max}}^{(\text{MP})}/\langle\tilde{\tau}\rangle$ because it did not reach a near-optimal wavefront.

The full distributions of eigenvalues are shown in Fig. 4 for two representative target sizes. When $M_i \approx 10^3$, the experimental distribution already differs detectably from the uncorrelated MP law (Fig. 4a). With a much larger target of $M_i \approx 4 \times 10^4$, the experimental distribution spreads five times the width of the corresponding MP distribution (Fig. 4b)—a drastic change due to correlations.

To understand and to describe quantitatively these correlation effects, we make the following ansatz: The $M_j$-by-$M_i$ partial transmission matrix $\tilde{T}$ of an open disordered slab measured with a finite illumination area can be treated as a filtered matrix drawn from a larger $N_j$-by-$N_i$ full transmission matrix $t$ of a disordered coherent conductor in a closed waveguide of non-uniform width.

The parameters of the unknown full matrix $t$ account for the effects of finite illumination area and lateral diffusion in an open geometry, and they remain to be determined. It is known that for closed diffusive waveguides in the absence of absorption, $t' t$ has a bimodal eigenvalue density

$$ P_{t' t}(\tau) = \frac{T}{2 \tau \sqrt{1 - \tau}} \quad (1) $$

that is universal and parametrized only by the average transmission $\tilde{T}$; this allows us, using the FRM, to predict the statistical properties of $\tilde{T}$.

Equation (1) differs drastically from the MP distribution, indicating that the matrix elements of $t$ are necessarily correlated.

In particular, this asymmetric bimodal distribution indicates that the transmitted waves consist mainly of a relatively small number, $g \ll N_i$, of 'open' eigenchannels with order-unity transmission ($\tau \approx 1$), while most of the other eigenchannels have $\tau \approx 0$ and barely contribute to transmission. The available degrees of freedom at the output is not the number of output channels defined by the geometry, $N_j$, but rather it is approximately $g \equiv \langle \text{Tr}(t' t) \rangle \approx N_j \tilde{T}$: more precisely, it can be defined by the participation number $\langle (\sum_{n=1}^{N_j} \tau_n)^2/(\sum_{n=1}^{N_j} \tau_n^2) \rangle$ (refs 30,31,41), which is $3g/2$ for the
bimodal distribution in equation (1). If we collect more output channels than the available degrees of freedom at the output (when $M_2 > 3g/2$), we expect to see strong correlation effects.

The intuitive discussion in the preceding paragraph can be made quantitative by using the analytic FRM formalism\(^1\) to describe the matrix filtering process in our ansatz. In general, this requires knowing the three parameters $N_i, N_t, T$ of the unknown matrix $t$, in addition to the known size $M_1, M_2$ of the measured matrix $t$. But for thick samples (specifically, when $T < 2/3$, which is always the case in the diffusive regime), our derivation (Supplementary Section III) shows that the variance normalized to the uncorrelated MP variance is simply

$$\frac{\text{Var}(\tilde{t})}{\text{Var}(\tilde{t}^{\text{MP}})} = 1 + \frac{2M_1}{3g}$$

which depends on a single parameter, $M_1/g$ (recall that $\text{Var}(\tilde{t}^{\text{MP}}) = (\tilde{T})^2 M_2/M_1$). As expected, when the target region contains more than $3g/2$ channels, the eigenvalues exhibit substantially more variation than the uncorrelated MP behaviour, leading to a wider-than-expected range for coherent control.

To compare the experimental data with equation (2), the only parameter we need is the dimensionless conductance $g$. For a fixed input, the intensity correlation between far-away output speckles equals $2/(3g)$ (refs 14–16), allowing us to determine $g$ from the experimental correlation functions (Supplementary Fig. 1). We also determine $g$ through the measured variance of the normalized total transmission (as in refs 25,27), which also equals $2/(3g)$. The two methods yield almost the same values of $g$, whose average values are $g = 894 \pm 26, 1164 \pm 38, 1642 \pm 82$ for the three illumination sizes $D_{in} \approx 6, 12, 24 \mu m$ considered here (Supplementary Section IV).

Analytic expressions of $g$ for an open geometry\(^{32,27,38,42,43}\) predict similar values, which we describe in Supplementary Section V.

With $g$ determined, we compare equation (2) to the eigenvalue variance without any free parameter and observe excellent quantitative agreement (Fig. 2a). Equation (2) reveals that the MP-normalized variance $\text{Var}(\tilde{t})$ follows a scaling law with respect to a single-parameter $M_1/g$. Furthermore, we show in Supplementary Section III that when $T \ll 4/15$, the third central moment $\text{Skew}(\tilde{t}) \equiv ((\tilde{t} - \langle \tilde{t} \rangle)^3 / \langle (\tilde{t} - \langle \tilde{t} \rangle)^2 \rangle)^{3/2}$ of the eigenvalues also follows a single-parameter scaling law

$$\frac{\text{Skew}(\tilde{t})}{\text{Skew}(\tilde{t}^{\text{MP}})} = 1 + \frac{2M_1}{g} + \frac{8M_1^2}{15g^2}$$

once normalized by the eigenvalue skewness of an uncorrelated matrix, $\text{Skew}(\tilde{t}^{\text{MP}}) = (\tilde{T})^3 (M_2/M_1)^3$. Again $M_1/g$ sets the departure from the uncorrelated behaviour. This scaling is validated with the experimental data in Fig. 2b, again with no free parameter. In Supplementary Section III, we also calculate the MP-normalized fourth central moment (kurtosis) and find that in general it depends on two parameters $M_1/g$ and $M_2/M_1$, with the dependence on $M_1/M_2$, dropping out when it is small; this is validated with experimental data in Supplementary Fig. 3.

We also calculate the full eigenvalue distributions with the analytic FRM formalism. For the parameters of the unknown matrix $t$ in our ansatz, we used the measured average transmission $\tilde{T} = 3\%$, take $N_i = g/T$, and $N_t \approx 6 \times 10^4$ as the number of CCD pixels when output collection is complete; note that $N_i$ is not simply the number of modes in the illumination area because $g$ is enlarged by the lateral diffusion in an open geometry (Supplementary Section V).

The predicted eigenvalue distribution of the filtered matrix $t$ and its extremal values are obtained by solving Supplementary Eqs (S8) and (S13) in Supplementary Section III. These predictions, plotted in Figs 3 and 4 as green solid lines, agree with the experimental data (green circles) and differ from the uncorrelated MP law (grey dotted lines).

We provide a simple heuristic model that approximates the FRM results. The correlations reduce the degrees of freedom at the output, but have no effect on the input channels, which are modulated independently by the $M_1$ SLM macropixels. This suggests modelling the correlated $M_2$-by-$M_1$ matrix $t$ using an uncorrelated $M_1^{\text{eff}}$-by-$M_1^{\text{eff}}$ matrix $t^{\text{eff}}$ with fewer output channels $M_2^{\text{eff}} \leq M_1^{\text{eff}}$. By choosing

$$M_2^{\text{eff}} = M_1^{\text{eff}} \left(1 + \frac{2M_1}{3g}\right)^{-1}$$

we match the eigenvalue variance of $t^{\text{eff}}$ with that of $t$ given in equation (2). As expected, this effective degrees of freedom follows $M_2^{\text{eff}} \approx M_1$ when $M_2 \ll g$, and $M_2^{\text{eff}} \approx 3g/2$ when $M_1 \gg g$. Within this model, all statistical quantities of $t^{\text{eff}}$ follow the simple MP law with the reduced $M_2^{\text{eff}}$, and all correlation effects are encapsulated in this reduction (which only depends on $M_1/g$). For example, the correlation enhancement of control range $\langle (\tilde{t}^{\text{eff}})^2 - \tilde{t}^{\text{eff}} \rangle / \langle (\tilde{t}^{\text{MP}})^2 - \tilde{t}^{\text{MP}} \rangle$ plotted in Supplementary Fig. 2 is simply $\sqrt{1 + 2\lambda g}$ when $M_2^{\text{eff}} \approx M_1$. For large targets ($M_1 \gg g$), the ratio $\langle (\tilde{t}^{\text{eff}})^2 - \tilde{t}^{\text{eff}} \rangle / \langle (\tilde{t}^{\text{MP}})^2 - \tilde{t}^{\text{MP}} \rangle$ approaches $1 \pm \sqrt{2\lambda g}$ and becomes independent of $M_1$, meaning that the achievable total transmission enhancement (as studied in ref. 38) is determined solely by $M_1/g$ within this model. Predictions of this ‘effective MP model’ are reasonably good, and are shown in Figs 3 and 4 for the extremal eigenvalues and the eigenvalue distributions, and in Supplementary Figs 2 and 3 for the eigenvalue range and kurtosis.
The correlation-enhanced control enables more energy delivery into a multi-speckle-sized region, such as a photodetector for optical communications. Meanwhile, the existence of non-local correlations\textsuperscript{11-13} prevents the generation of truly independent random numbers through coherent diffusion\textsuperscript{14}. These correlation effects can be enhanced or suppressed by changing the illumination spot size $D_m$ (through the SLM macropixel size or through focus alignment\textsuperscript{15-17}), which changes $g$. Note that optimizing the target intensity does not necessarily optimize the target-to-background contrast, so imaging and photothermal therapy applications will require objective functions that account for both. The theory here is applicable to microwaves and acoustic waves, for which smaller $g$ can be readily achieved. The correlations are even stronger when approaching localization\textsuperscript{18}, and deep inside the localized regime ($g \ll 1$) the maximal transmission enhancement equals $M$, independent of the target size, since a single eigenchannel dominates the transmission\textsuperscript{21-24}. Future work may extend the present formalism to broadband\textsuperscript{25} and spatial–temporal control\textsuperscript{26,27}, and to the control of reflection and absorption in open disordered systems\textsuperscript{28-30}.

Methods

Methods, including statements of data availability and any associated accession codes and references, are available in the online version of this paper.

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Author contributions
C.W.H. and S.F.L. performed the experiment. C.W.H. analysed the data, C.W.H. developed the theory descriptions. A.G. proposed the effective MP model. H.C. and A.D.S. supervised the project. All authors discussed and interpreted the results. C.W.H. and A.D.S. wrote the manuscript with input from all authors.

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Competing financial interests
The authors declare no competing financial interests.
Methods

Experimental set-up. As illustrated in Fig. 1a, the expanded beam from a continuous-wave Nd:YAG laser (Coherent, Compass 215M-50 SL, wavelength \( \lambda = 532 \) nm) is split into two parallel beams with orthogonal linear polarizations and equal intensity. The two beams are modulated with different areas of a SLM (Hamamatsu, X10468-01) and then recombined. Using a 4-f system (lenses \( L_1 \) and \( L_2 \); focal lengths = 21 cm), the surface of the SLM is imaged onto the entrance pupil of a microscope objective (NA\(_o\) = 0.95, Nikon CF Plan 100\( \times \)) and then focused onto the scattering sample. The transmitted light is collected with an oil-immersion objective (NA\(_o\) = 1.25, Edmund DIN Achromatic 100\( \times \)), and the exit pupil of the objective is imaged onto a CCD camera (Allied Vision, Prosilica GC 660) through another 4-f system (lenses \( L_1 \) and \( L_2 \); focal lengths = 20 cm). In this way, the SLM pixels and the CCD pixels correspond to the different angles incident onto and transmitted from the sample.

The nearby SLM pixels are grouped into square macropixels; smaller macropixels correspond to more finely spaced incident angles, which yields a higher illumination spot and provides more input channels (macropixels) for modulation. For example, when one macropixel consists of 8\( \times \)8 SLM pixels, we have \( M^{\text{tot}} = 978 \) macropixels (489 per polarization) imaged onto the entrance pupil of the input objective and available to use. We measure the transmission matrix for the \( M_1 = 512 \) input channels (macropixels) at the centre (16\( \times \)16 per polarization), using the other \( M^{\text{tot}} = 466 \) available macropixels as the reference. Then we synthesize the desired wavefront at the \( M_1 \) macropixels; at this stage the other macropixels are ‘switched off’ by displaying a high-spatial-frequency phase pattern, making them blocked by the iris placed at the Fourier plane of the lens \( L_1 \).

For the theoretical prediction of the dimensionless conductance \( g \) (Supplementary Section V), we need to know the size and spatial profile of the illumination spot. In our set-up, the SLM macropixels are mapped to the space \((k_x, k_y)\) of transverse wavevectors on the surface of the sample. Let \( q \times q \) denote the size of one macropixel in the \( k \) space. Then the intensity profile of the incident light is

\[
I(x, y) \sim \sin^2(\pi q / 2) \sin^2(\pi q / 2)
\]

The illumination area is \( A_o = \int \int dx \, dy \mid (x, y) \mid = (\pi q)^2 \). The available number of macropixels is the number of \( q \times q \) squares within two circles of radii \((2\pi / \lambda)A_o\) in the \( k \) space, so \( M^{\text{tot}} = 2\pi (2\pi / \lambda)^2 (A_0)^2 / q^2 \). Thus we can determine the area \( A_o \) and the diameter \( D_o = \sqrt{4A_0 / \pi} \) of the illumination spot from \( M^{\text{tot}} \) and \( \lambda \). In this study we use 16\( \times \)16, 8\( \times \)8, and 4\( \times \)4-sized macropixels, corresponding to \( M_1 = 128, 512, 2048 \) and \( D_o \approx 5, 12, 24 \) \( \mu \)m.

Sample preparation and characterization. We dissolve 6 g of ZnO microparticle powder (Alfa Aesar Puratronic) into 3 ml of deionized water and 3 ml of ethanol, and sonicate the mixture in an ultrasonic bath for 30 min for thorough dispersion. The solution is then spin-coated onto a microscope cover slip (22 \( \times \) 22 mm\(^2\), thickness 0.15 mm) and allowed to dry. The resulting sample is shown in the inset of Fig. 1a. Scanning electron microscopy shows that the ZnO particle diameters center around 200 nm. We use the centre of the sample, where the thickness of the ZnO film is measured by a profilometer to be \( L \approx 60 \) \( \mu \)m, and the total transmission is measured with a photodetector to be \( \bar{t} \approx 3\% \).

Transmission matrix measurement. We measure the transmission matrix \( \bar{t} \) using a modified version of the phase-shifting, common-path interferometric method. As in refs 11,12, we control \( M_1 \) macropixels on the SLM and switch on additional \( M^{\text{tot}} \) macropixels with a flat phase as the reference. When the input mod is sent in with a relative phase of \( \phi \), the measured intensity on the \( j \)th pixel of the CCD is

\[
|\bar{t}_j^{\text{ref}} + \bar{s}_j^{\text{ref}}|^2,
\]

where \( s_j \) denotes the transmitted field from the reference pixels. With four measurements at \( \phi = 0, \pi / 2, \pi, 3\pi / 2 \), we obtain \( \bar{u}_j = \bar{t}_j^{\text{ref}} \). We perform the measurements with the input channels \( |u| \) in the Hadamard basis, and then transform back to the basis of SLM macropixels (that is, incident angles).

So far this method yields only \( \bar{u}_j \), which is the transmission matrix \( t_{j}^{\text{ref}} \) multiplied by an unwanted field \( s_j^{\text{ref}} \) from the reference, which obscures the relative phase and amplitude between the output channels \( \{t\} \). To study \( \bar{t} \) and the transmitted intensity, it is necessary to recover the relative amplitude between channels in \( \{t\} \). Thus, for each input channel \( u \), we perform an additional measurement with the \( M^{\text{tot}} \)-reference pixels switched off\(^6\), which provides \( |\bar{t}_j| \) that contains the relative amplitude between the output channels. Specifically, we use \( u_0 |\bar{t}_j|/|u_0| = \bar{t}_j^{\text{ref}} / |u| \) as our transmission matrix; the relative phase between \( \{t\} \) is still unknown, but is irrelevant for us. The whole measurement process takes 2, 8, 32 min, respectively, for \( D_o \approx 6, 12, 24 \) \( \mu \)m.

For each illumination diameter, we measure ten transmission matrices at sufficiently different times that there is no discernible correlation between the measured matrices. The ten sets of data are used to improve the statistics of the eigenvalues and the intensity correlations.

Determination of optimal phase-only wavefront. With the transmission matrix measured, we can instantly determine the optimal wavefront for any given target. When both amplitude and phase can be modulated, the optimal wavefront for maximal (minimal) transmission into the target is simply the eigenvector of \( \bar{t}^\dagger \) with the largest (smallest) eigenvalue; this is not the case for phase-only modulation. For maximal transmission of a phase-only wavefront, we use the phase part of the maximal eigenvector. For minimal transmission, taking the phase of the minimal eigenvector is not optimal (for reasons explained in ref. 24), so we look for the phase profile \( \rho_0, \rho_1, \ldots, \rho_{M-1} \) that minimizes \( \sum_{j=0}^{M-1} |\bar{t}_j^{\text{ref}}|^2 \), using the gradient-based low-storage Bryden–Fletcher–Goldfarb–Shanno (BFGS) algorithm\(^3\) implemented in the free optimization package NLopt\(^4\). The optimization takes only a few seconds.

Data availability. The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.

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