Influence of Excitation Condition on Evaluating Critical Hunting Speed

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In order to verify train bogie running stability, usually, hunting motion tests are carried out on roller rigs. Two types of test are performed: a simple rotation test and an excitation test. It is well known that the critical hunting speed can vary depending on the excitation mode, therefore experiments were conducted to investigate how roller rig excitation waveforms influenced the critical hunting speed. These experiments made it possible to confirm that the occurrence of hunting oscillation depended on the initial lateral amplitude of the free oscillation that was generated by the excitation.

Keywords: hunting motion test, critical hunting speed, excitation method, nonlinearity, bifurcation

1. Introduction

Railway vehicle hunting or oscillation through self-excitation adversely affects running stability. When designing bogies, therefore, it is important to consider a range of factors, such as the stiffness and damping force of the support springs and the wheel tread profile, to try to prevent hunting. One method for evaluating the running stability of a manufactured bogie is to perform hunting motion tests, which are stationary tests conducted on a roller rig. The roller, which has the same cross-sectional profile as an actual rail, is run at high speed to simulate a vehicle running on a track. Broadly speaking, there are two types of hunting motion test that can be performed on a roller rig: simple rotation tests and excitation tests. In a simple rotation test, the roller is run without any obvious intervention on the setup. Excitation tests however, can be conducted in a variety of ways, for example, by directly subjecting the test vehicle to external forces and by exciting the roller. In contrast to simple rotation tests, which simulate a vehicle running on straight rails, whatever excitation method is used is designed to evaluate the running stability of a test vehicle whilst subjected to obvious outside intervention. Experience shows nonetheless that different excitation methods lead to different critical hunting speeds: even when only varying the method used to excite the roller, the critical hunting speed varies with the excitation waveforms.

Accordingly, the hunting motion tests in this study were conducted using an actual bogie to examine how the critical hunting speed is affected by different excitation methods. The results showed that the relationship between the excitation method and the resultant critical hunting speed was determined by the critical motion curve unique to the bogie being tested and the amplitude of oscillation triggered by the excitation [1].

First, this paper discusses the differences in critical hunting speeds caused by different excitation waveforms of the roller. Second, the paper clarifies the overall stability in the speed range between the critical hunting speed in the simple rotation test and the convergence speed based on the results of simple rotation tests conducted to examine hunting to identify its basic characteristics and experiments conducted under initial conditions, and shows that the initial amplitude of free oscillation determines whether the oscillation leads to hunting or converges instead. Then, this paper draws a rational conclusion based on the above results to explain how the different roller excitation methods affect the resulting critical hunting speeds, and presents an excitation method equivalent to the roller excitation method as a running stability disturbance factor. Finally, the overall stability in the speed range between the critical hunting speed and the convergence speed is also discussed from the viewpoint of bifurcation, a phenomenon unique to nonlinear systems.

2. Hunting motion test using a roller rig and different excitation methods

2.1 Half carbody with a load frame

Hunting motion tests using a roller rig can be conducted using either an actual carbody equipped with two bogies, or a load frame equipped with one bogie on which a dead weight is mounted to simulate the mass of half a
carbody. The test reported in this paper was conducted using half a carbody equipped with one bogie. Figure 1 shows the test unit schematically. As seen in Fig. 1, the roller was run counterclockwise. Thus, with this direction of travel, the test setup simulated a car’s rear bogie.

### 2.2 Modes of hunting and bogie hunting

Hunting can be classified into a number of types by the mode of oscillation. There are two types of hunting that occur at relatively high running speeds: wheelset hunting in which individual wheelsets oscillate violently, and bogie hunting in which a bogie and its wheelsets widely oscillate almost as a unit. Depending on the disturbance factors, carbodies can be made to oscillate widely at lower running speeds, and with relatively low frequencies. This is called carbody hunting.

All types of hunting observed in tests conducted as part of this study were bogie hunting. Neither carbody nor wheelset hunting were observed. All hunting discussed hereafter in this paper therefore refers solely to bogie hunting.

The bogie hunting wavelength that is actually produced is often similar to what is called the kinematic bogie hunting wavelength \( S_b \). \( S_b \) is the kinematic wavelength of a two-axle rigid bogie with the wheelsets rigidly mounted onto the bogie frame, and can be calculated based on the wavelength of kinematic hunting of a single wheelset \( S_i \), which can be calculated based on the wheel diameter, wheel tread gradient and gauge, and the wheelbase of the two-axle rigid bogie.

### 2.3 Roller excitation methods

Generally, in hunting motion tests using the roller excitation method, periodic functions and track waveforms are used to simulate actual track irregularity. A range of parameters including frequency and wave number can be considered in various ways for the periodic functions. In the tests discussed in this paper, the following three different periodic functions were used for the excitation:

1. Three consecutive periods of a 1 Hz sine wave with a peak amplitude of 3 mm were used to excite the roller. Hereafter, this is referred to as waveform A.
2. A period of a periodic function, i.e. \( a(1-\cos \omega t) \), was used to excite the roller where \( \omega \) is the kinematic bogie hunting angular frequency, which is obtained by dividing the running speed by the wavelength of the kinematic bogie hunting \( S_b \). \( a \) is 1.5 mm when the peak-to-peak amplitude is 3 mm. Hereafter, this is referred to as waveform B.
3. Three consecutive periods of a sine wave with the same kinematic bogie hunting frequency that was used for a waveform B in (2) above and a peak amplitude of 3 mm were used to excite the roller. Hereafter, this is referred to as waveform C.

Table 1 shows the waveforms of those excitations.

### 3. Speed range and conditions for hunting

#### 3.1 Verification of results of simple rotation tests

The simple rotation tests were conducted to clarify the basic characteristics of the test bogie with respect to hunting stability. The tests can be interpreted as tests on the stability of the bogie to stay in the equilibrium position against minor disturbances caused during the simple rotation of the roller by minor misalignment of the roller rig and other factors.

The roller speed was gradually increased until the bogie started hunting. The roller was then gradually slowed down from that speed until the hunting converged to achieve equilibrium. The results are shown in Fig. 2. The horizontal axis represents the roller speed while the vertical axis represents the peak amplitude of the steady-state oscillation, or the lateral displacement of the wheelsets, during the hunting.

When the rollers reached 282 km/h during the acceleration, the wheelsets, which had stayed in equilibrium, suddenly started hunting with a large amplitude. As the rollers were slowed down when the hunting started, the amplitude of the steady-state oscillation subsided slightly whilst the hunting continued. At 240 km/h, the hunting
Fig. 2  Acceleration to hunting and deceleration to convergence (simple rotation test)

Fig. 3  Acceleration to hunting and deceleration to convergence (excitation test)

Fig. 4  Relationship between roller excitation and initial amplitude

3.2 Bogie behavior in the excitation test

The excitation tests were conducted to examine the behavior of the test bogie including hunting and its convergence. Figure 3 shows the results of the excitation tests, in which the three excitation methods mentioned previously were used, superimposed on the results of Fig. 2. While hunting started at different speeds for the excitation methods, the subsequent behavior of the bogie with respect to the speed for each of the methods closely resembles the behavior observed in the simple rotation test. With the excitation using waveform C, hunting converged to equilibrium immediately after deceleration. In addition, with respect to the steady-state oscillation frequency, the bogie behaved in a similar manner to that shown in Fig. 3.

Through the experiments described above including the simple rotation test, it was found that the different excitation methods led to the same steady state of hunting, although it started at different speeds.

In the simple rotation test discussed previously, two separate steady states were observed in the speed range between hunting and convergence to equilibrium: one with a large amplitude of hunting oscillation and the other in equilibrium. This suggests that there may be one threshold in the initial state of motion related to displacement for example, and one related to speed (referred to hereafter as “the initial condition”) above which hunting occurs and below which convergence to equilibrium occurs. This is discussed in detail in the next section.

3.3 Threshold between convergence to equilibrium and hunting

To find the threshold between convergence to equilibrium and hunting, it is necessary to conduct the experiment while varying the initial state as much as practically possible. One possible method is to subject the test bogie to a static external force, which is removed when the initial target state is reached. This, however, was not possible with the current test facilities. As an alternative, therefore, free oscillation was triggered by exciting the roller with a single input, and the initial amplitude of oscillation, as shown by a circle in Fig. 4(a) and Fig. 4(b), was considered to be the initial state.

By adjusting the amplitude of oscillation of the excited roller and thereby changing the initial amplitude of free oscillation, a threshold was sought for the free oscillation to either converge to equilibrium shown in Fig. 4(a) or start hunting shown in Fig. 4(b).

The results of the experiment are shown in Fig. 5. The circles indicate the initial amplitudes that led to convergence to equilibrium while the crosses indicate the initial amplitudes that led to hunting. As shown in Fig. 5, the experiment revealed a clear threshold in the initial amplitudes between convergence to equilibrium and hunting. This imaginary threshold line is hereafter referred to as
the critical hunting curve. As shown in Fig. 5, the speed at the higher-speed end of the critical hunting curve roughly corresponds with the critical hunting speed observed in the simple rotation tests. Furthermore, the amplitude of the steady-state oscillation immediately before convergence back to equilibrium (the left end of the gray line in Fig. 5) roughly corresponds with the amplitude at the lower-speed end of the critical hunting curve.

To summarize, the experiment revealed that the critical hunting curve runs between the speed at which hunting started in the simple rotation test (Point A in Fig. 5) and the amplitude of the steady-state oscillation immediately before convergence back to equilibrium (Point B in Fig. 5), and that whether hunting occurs or not is determined solely by whether or not the initial amplitude of the free oscillation surpasses the critical hunting curve.

Excitation tests were conducted using roller excitation methods. The roller excitation was simply considered as a trigger to generate free oscillation as it was in Section 3.3. The results were examined with respect to the initial amplitude immediately after the excitation. The tests were conducted using the three excitation methods discussed in Section 2.3. Figure 6 shows the results, which are superimposed on the critical hunting curve from Fig. 5. The triangles indicate the initial amplitudes of free oscillation that led to convergence and to equilibrium. The squares indicate the initial amplitudes of free oscillation that led to hunting.

With all three excitation methods, the free oscillation converged to equilibrium whenever the initial amplitude was located on the left side of the critical hunting curve, and to hunting whenever the initial amplitude was on the right side of the curve. The tests have shown that regardless of which of the three excitation methods was used, whether hunting occurs or not is determined by whether the initial amplitude of oscillation surpasses the critical hunting curve or not.

Fig. 6  Relationship between roller excitation waveforms and critical hunting speed

Based on the above, it appears that the difference in the magnitude of the initial amplitude of excitation-generated free oscillation could be the cause of critical hunting speeds varying in accordance with different excitation waveforms.

4.2 Alternative equivalent excitation methods as a running stability disturbance factor

The focus so far has been on roller excitation, which showed that the relationship between different excitation waveforms and critical hunting speed could be explained using the critical hunting curve. Hunting motion tests can also be conducted by directly subjecting the test body to external forces, or disturbance factors.

To compare the alternative method with the roller excitation method, numerical simulations were conducted on a model half carbody equipped with a bolsterless bogie. In the simulations, a wire attached to the center of the bogie was pulled and the force was removed when a predetermined value was reached (the method is hereafter referred to as the "wire cut-off method").

In the first step, the wire was cut while the running speed was increased in stages. Figure 7 (a) shows the time history of the wheelset lateral displacement during the wire-cutting induced hunting. In the next step, roller excitation sessions were carried out while the oscillation amplitude of the excited roller was increased in stages and the running speed was fixed at the speed at which hunting occurred in the first step. Figure 7 (b) shows the time history of the wheelset lateral displacement during the hunting that occurred after roller excitation. The initial amplitude of the wheelset lateral displacement immediately after the wire was cut in Fig. 7 (a) is roughly equal to that found immediately after roller excitation in Fig. 7 (b).

Accordingly, it can be assumed that if the same bogie is evaluated for running stability using an excitation system different to that used for the known evaluation, the new system can serve as a disturbance factor equivalent to the known system if the initial amplitude of oscillation after excitation is adjusted to the magnitude equivalent to that of the known system. This, however, assumes that only the excitation system of the vehicle tester is different. If
there is a difference in any of the key specifications of the rest of the vehicle tester, such as the roller radius and half-carbody load frame length, appropriate corrections would need to be made.

![Diagram of hunting motion test using different excitation methods](a) Wire cut-off  (b) Roller excitation

Fig. 7  Hunting motion test using different excitation methods (excitation mechanisms)

5. Hunting and bifurcation phenomenon

The experiments discussed previously show that hunting can occur when the initial amplitude of excitation-caused oscillation surpasses a threshold even when the speed at that time is lower than the critical hunting speed of the simple rotation test, and that the threshold changes with the speed (forming the critical hunting curve).

When examining stability against hunting using analytical methods, an eigenvalue analysis is typically employed whereby dynamic models are linearized near the equilibrium position. The speed at which the real part of an eigenvalue turns from negative to positive is generally considered to be the critical hunting speed in eigenvalue analysis. In real phenomena, this corresponds to the critical hunting speed in simple rotation tests. As discussed previously, in actual hunting motion tests, minor misalignment of the roller rig induces minor disturbances even with simple roller rotation. Therefore, simple rotation tests can be interpreted as an evaluation of the local stability against minor disturbances. With linear systems, no type of disturbance will result in a boost in the amplitude of the oscillation equivalent to hunting, unless the critical hunting speed is exceeded. In reality, however, hunting can occur even at speeds below the critical hunting speed in simple rotation tests if the appropriate initial conditions are met. To theoretically reproduce this using a dynamic model, appropriate levels of nonlinearity must be incorporated into the model.

This chapter highlights the similarities between bifurcation, a phenomenon unique to nonlinear systems, and the possibility, observed in the experiment, of hunting occurring even at a speed lower than the critical hunting speed of simple rotation tests due to the presence of the critical hunting curve.

5.1 Bifurcation phenomenon

Bifurcation is a phenomenon whereby even a slight change in parameters can dramatically change related dynamic characteristics. For example, the buckling of a beam is a bifurcation phenomenon. A beam remains straight even when subjected to a minor disturbance, as long as the vertical load acting on it is below the buckling threshold. Once the vertical load exceeds the threshold even by the slightest amount, the beam deforms. In this example, if the vertical load is considered a parameter, the buckling load is called the bifurcation point. In the simple rotation tests discussed above, a slight rise in the running speed, which can be considered a parameter, near the critical hunting speed, caused the wheelsets that had stayed in equilibrium to suddenly start oscillating with a larger amplitude in a steady hunting state. This can also be called a bifurcation phenomenon. In this case, the critical hunting speed is the bifurcation point. To reproduce hunting stability dependent on initial values near the bifurcation point, appropriate levels of nonlinearity must be incorporated in dynamic models. Possible examples of nonlinearity include springs with a nonlinear restoring force and nonlinear characteristics of contact between the wheel tread and the rail surface. Nonlinear elements that dictate hunting stability near the bifurcation point are not discussed in the following sections, which instead look at the relationship between the bifurcation phenomenon and the critical hunting curve identified in the experiment.

5.2 Subcritical Hopf bifurcation [2]

Figure 8 shows a schematic of subcritical Hopf bifurcation, a type of bifurcation phenomenon. The schematic relates to a two-dimensional nonlinear system with state quantities $x_1$ and $x_2$ and a variable parameter, $\mu$, showing the motion of the state quantities and subcritical Hopf bifurcation occurring at $\mu = 0$ in a three-dimensional phase space. Hereafter, a two-dimensional plane showing the motion of state quantities $x_1$ and $x_2$ with a fixed $\mu$ in three-dimensional space is called a phase plane.

With subcritical Hopf bifurcation, when $\mu$ is greater than 0, the equilibrium is unstable, causing a boost in the oscillation amplitude of the state quantities. On the other hand, when $\mu$ is smaller than 0 while the equilibrium position is locally stable, some state quantities on the phase plane sometimes do not converge to the equilibrium position. Stability differs across the dotted closed curve that circles the equilibrium position. State quantities that are inside the closed curve converge to the equilibrium position while those outside the curve diverge infinitely. The closed curve on a phase plane that determines whether state quantities converge to equilibrium or diverge is called an unstable limit cycle. When $\mu$ is smaller than 0, an unstable limit cycle becomes greater as $\mu$ becomes smaller and becomes smaller as $\mu$ becomes greater, ultimately disappearing when $\mu = 0$.

![Fig. 8 Subcritical Hopf bifurcation](image)
5.3 Critical hunting curve and bifurcation chart

The following paragraphs discuss the critical hunting curve from the viewpoint of bifurcation. For simplification, when focusing on the wheelset lateral displacement and considering displacement and speed to be a state quantity, the critical hunting curve is a constellation of points where the speed is zero and around which the state quantities are on the border of convergence and hunting in the phase space.

Figure 9 (a) shows the bifurcation diagram of radius \( r \), which is based on the motion of the state quantities on the phase plane in Figure 8 represented by radius \( r \) and rotation angle \( \theta \) on a polar coordinate system, and Fig. 9 (b) shows the critical hunting curve from the experiment. Figure 9 shows the following similarities between subcritical Hopf bifurcation and hunting. The unstable limit cycle forms a border between convergence to the equilibrium point on the inside and divergence on the outside. Similarly, the initial amplitudes on the inside of the critical hunting curve converge while those outside lead to hunting. The radius of the unstable limit cycle shrinks as \( \mu \) nears the bifurcation point of zero. Similarly, the critical hunting curve nears zero as the speed nears the critical hunting speed of simple rotation. Furthermore, the equilibrium point becomes unstable once the bifurcation point is surpassed, making any initial state quantity diverge. Similarly, once the critical hunting speed is exceeded, any initial amplitude leads to hunting even with no disturbance.

The above shows that the stability near the critical hunting speed can be explained by the dynamic characteristics of subcritical Hopf bifurcation.

Initial displacements on the inside of the critical hunting curve are equivalent to initial states inside the unstable limit cycle on a phase plane. Initial displacements outside the critical hunting curve are equivalent to initial states on the outside of the unstable limit cycle. In accordance with the dynamic characteristics of the unstable limit cycle, subsequent behavior is to either converge or diverge. In reality, however, amplitudes will never develop into infinite hunting and instead will be limited through contact with the root of the flange and turn into steady oscillations.

The conclusion of Chapter 4 regarding what determines whether hunting occurs or not can be interpreted as follows: whether the various waveforms used for roller excitation induce hunting or not in the hunting motion test is determined by whether or not the initial states immediately after roller excitation are outside or inside the unstable limit cycle.

6. Conclusion

Hunting motion tests were conducted using an actual bogie on a roller rig to clarify the relationship between the roller excitation methods and the critical hunting speed. The results were as follows:

1. Hunting can occur when the initial amplitude of the wheelset free oscillation surpasses a threshold even when the speed at that time is lower than the critical hunting speed of the simple rotation test. The thresholds are distributed continuously between the critical hunting and convergence speeds of the simple rotation test. The constellation of these thresholds is defined as the critical hunting curve.

2. The difference in the magnitude of the initial amplitude of excitation-generated free oscillation is essentially the cause of the different critical hunting speeds between the various excitation methods used in the hunting motion test.

3. From the viewpoint of bifurcation in nonlinear dynamics, the stability near the critical hunting speed corresponds with the dynamic characteristics of subcritical Hopf bifurcation and the critical hunting curve is considered equivalent to the unstable limit cycle associated with subcritical Hopf bifurcation.

Further work will aim to develop methods to analyze the critical hunting curve found in these experiments, as discussed in this paper. In addition, the nonlinear characteristics of contact between the wheel tread and the rail surface will be studied to identify any impact they may have on the critical hunting curve.

References

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