Competing effects of mass anisotropy and spin Zeeman coupling on the upper critical field of a mixed $d$- and $s$-wave superconductor

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Abstract

Based on the linearized Eilenberger equations, the upper critical field ($H_{c2}$) of mixed $d$- and $s$-wave superconductors has been microscopically studied with an emphasis on the competing effects of mass anisotropy and spin Zeeman coupling. We find the mass anisotropy always enhance $H_{c2}$ while the Zeeman interaction suppresses $H_{c2}$. As required by the thermodynamics, we find $H_{c2}$ is saturated at zero temperature. We compare the theoretical calculations with recent experimental data of YBa$_2$Cu$_3$O$_{7-\delta}$.

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It has been a consensus that a high-$T_c$ cuprate superconductor has a $d$-wave pairing symmetry, and CuO$_2$ plane is responsible for superconductivity. However, a pure $d$-wave symmetry is appropriate only for a tetragonal lattice structure, an orthorhombic material such as YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) is believed to have a subdominant $s$-wave component in the order parameter. The orthorhombicity in YBCO is originated from a mass anisotropy (MA) along $a$- and $b$-directions. In other words, $m_a$ is larger than $m_b$. Such a discrepancy is mainly due to CuO chains in the $b$-direction.

Based on the Ginzburg-Landau theory, Xu et al. have explored effects of the mass anisotropy to show that an $s$-wave component always coexists with a dominant $d$-wave component in the bulk order parameter. Belzig et al. have obtained the phase diagram of a mixed $d$- and $s$-wave superconductor in the quasiclassical theory, and shown that the mass anisotropy gives rise to a non-zero $s$-wave component.

In this paper, we shall investigate effects of the MA on the upper critical field ($H_{c2}$) of a mixed $d$- and $s$-wave superconductor, based on the quasiclassical theory, using a quantum mechanical method we have developed in Ref. and compare a theoretical result with recent experimental data. We also take into account the paramagnetic Zeeman interaction (ZI) because $H_{c2}$ of a high-$T_c$ superconductor is large at low temperature. As in Ref., we neglect any other effects associated with the chains. For example, we assume that lattice constants along $a$- and $b$-directions have the same value.

Since the calculation of $H_{c2}$ is a quantum mechanical problem of a charged particle in a constant magnetic field, first of all we need to check if solvable is a problem of a charged particle with two different effective masses along $x$- and $y$-directions, $m_x$ and $m_y$, respectively, in a constant magnetic field $\mathbf{H} = \nabla \times \mathbf{A} = H\hat{z}$. In the symmetric gauge, the Hamiltonian $\mathcal{H}$ of the particle is given by

$$\mathcal{H} = \frac{1}{2m_x}(p_x + \frac{e}{2}Hy)^2 + \frac{1}{2m_y}(p_y - \frac{e}{2}Hx)^2$$

$$= \frac{1}{2m_x}(p_x^2 + \lambda p_y^2) + \frac{eH}{2m_x}(yp_x - \lambda xp_y) + \frac{e^2H^2}{8m_x}(y^2 + \lambda x^2),$$

where $\lambda = m_x/m_y$. Introducing operators $a_x$ and $a_y$ such that
\[ a_x = \frac{1}{\sqrt{2m_x\omega_0}}(m_x\omega_0\sqrt{\lambda}x + ip_x), \quad (2) \]

and

\[ a_y = \frac{1}{\sqrt{2m_x\omega_0}}(m_x\omega_0 y + i\sqrt{\lambda}p_y), \quad (3) \]

where \( \omega_0 = |e|H/2m_x \), \( \mathcal{H} \) becomes

\[ \mathcal{H} = \omega_0[a_x^+a_x + a_y^+a_y + \sqrt{\lambda} + i(a_x^+a_y - a_y^+a_x)]. \quad (4) \]

Let us introduce a new operator \( b = (a_x + ia_y)/\sqrt{2\sqrt{\lambda}} \) to simplify \( \mathcal{H} \); then, we obtain

\[ \mathcal{H} = \omega_c\left(b^+b + \frac{1}{2}\right), \quad (5) \]

with \([b, b^+] = 1, [b, \mathcal{H}] = \omega_c b, \) and \([b^+, \mathcal{H}] = -\omega_c b^+, \) where \( \omega_c = |e|H/\sqrt{m_xm_y} \). It shows that the problem we are considering reduces to that of a simple harmonic oscillator, and consequently \( H_{c2} \) for an anisotropic mixed \( d \)-and \( s \)-wave superconductor can be exactly calculated.

In the calculation of \( H_{c2} \), we choose \([110]\) and \([-110]\) as \( x \) and \( y \) axis, respectively. An electron spectrum \( \epsilon(k) \) then changes from \( k_x^2/2m_a + k_y^2/2m_b \) to \( k^2/2m + ck_xk_y/m \). Here we have defined \( m_a = m/(1-c) \) and \( m_b = m/(1+c) \), where a small quantity \( c \) is introduced to represent the degree of mass anisotropy. The Fermi surface (FS) now is elliptical, and the density of state \( N(\phi) \) on FS becomes \( N_0(0)/(1 + c\sin(2\phi)) \). By considering the anisotropy effect as a perturbation, we may assume that the order parameter can be still expanded in terms of the set of unperturbed eigenstates \( \{f_N(R)\} \). In Ref.\(^\text{6}\), we have shown that the (singlet) order parameter is written as \( A_0^{(d)}|0\rangle_d + A_2^{(s)}|2\rangle_s + A_4^{(d)}|4\rangle_d \) near \( H_{c2} \), where we denote \( |N\rangle_d(s) \) as eigenstates \( f_N \) of \( d(s) \) channel. However, we expect that some other states such as \( |0\rangle_s, |1\rangle_{s(d)}, \) and \( |2\rangle_d \) may involve in the order parameter due to the perturbation. The linear combination of these states for the order parameter near \( H_{c2} \) will be determined by the symmetry of the system.

The linearized Eilenberger equation is written as\(^\text{5}\)
\[ \hat{L}f(\omega, \mathbf{k}, \mathbf{R}) + \text{sgn}(\omega)\mu_0^* \mathbf{H} \cdot [\mathbf{\sigma} f(\omega, \mathbf{k}, \mathbf{R}) - f(\omega, \mathbf{k}, \mathbf{R})\mathbf{\sigma}^r] = 2\pi\Delta(\mathbf{R}, \mathbf{k}) , \quad (6) \]

with

\[ \Delta(\mathbf{R}, \mathbf{k}) = T \sum_{\omega} \langle V(\mathbf{k}, \mathbf{k}') f(\omega, \mathbf{k}', \mathbf{R}) \rangle_{FS} , \quad (7) \]

and

\[ \hat{L} = 2|\omega| + \text{sgn}(\omega)\mathbf{v}_F \cdot \mathbf{\Pi} . \quad (8) \]

Here \( f(\omega, \mathbf{k}, \mathbf{R}) \) is a quasiclassical Green’s function, \( \mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) are Pauli matrices, \( \omega \) is the Matsubara frequency, \( \mu_0^* \) can be interpreted as an effective magnetic moment of a quasi-electron with a mass anisotropy \( (m_a \neq m_b) \), which will be considered as a phenomenological parameter associated with a coupling strength between an electron spin and a magnetic field. The symbol \( \langle \cdots \rangle_{SF} = \int \frac{d\phi'}{2\pi} N(\phi') \cdots \) represents the angular average over the Fermi surface. Since \( \mathbf{v}_F = v_{F_a}\hat{a} + v_{F_b}\hat{b} \) and \( \mathbf{\Pi} = -i\nabla_{\mathbf{R}} - 2e\mathbf{A} = \Pi_a\hat{a} + \Pi_b\hat{b} \) with \( \hat{a} \) and \( \hat{b} \) being unit vectors along the \( a \) and \( b \) directions,

\[ \mathbf{v}_F \cdot \mathbf{\Pi} = (v_{F_x} + cv_{F_y})\Pi_x + (v_{F_y} + cv_{F_x})\Pi_y , \quad (9) \]

with \( v_{F_{x,y}} = k_{F_{x,y}}/m \) in the \( x-y \) coordinate system. The pairing interaction in this coordinate system can be written as

\[ V(\phi, \phi') = V_s + V_d \sin(2\phi) \sin(2\phi') , \quad (10) \]

where \( \phi = \tan^{-1}(k_y/k_x) \).

For the singlet pairing, \( f = f_0i\sigma_y \) and \( \Delta = \Delta_0i\sigma_y \). Using the inverse of the operator \( \hat{L}_{op} = \hat{L} + \text{sgn}(\omega)\mu_0^*H \), which admits the representation

\[ \hat{L}_{op}^{-1} = \int_0^\infty ds \exp(-s\hat{L}_{op}) , \quad (11) \]

we show

\[ f_0 = 2\pi \int_0^\infty ds e^{-s(\hat{L} + 2\text{sgn}(\omega)\mu_0^*H)}\Delta_0 . \quad (12) \]
Substituting Eq. (12) into Eq. (7), we obtain the linearized gap equation of an anisotropic mixed $d$- and $s$-wave superconductor as follows:

$$\Delta_0(R, \phi) = 2\pi T \sum_\omega \langle V(\phi, \phi') \int d\xi e^{-\xi|2|\omega|+i\text{sgn}\omega v_F} \Pi \rangle \times \cos(2\mu_0 H\xi)\Delta_0(R, \phi') \rangle_{SF}. \quad (13)$$

It is easy to see that $\Delta_0(R, \phi)$ turns out to be $\Delta_s(R) + \Delta_d(R) \sin(2\phi)$ because of the pairing interaction $V(\phi, \phi')$ in Eq. (10).

Let us, first of all, consider equations to determine $T_c$ of such a superconductor. Setting $H = 0$, we obtain

$$\Delta_s = N_0 V_s \left(1 + \frac{c^2}{2}\right) \ln \frac{2e^\gamma \omega_D}{\pi T_c} \Delta_s - N_0 V_s \left(\frac{c^2}{2}\right) \ln \frac{2e^\gamma \omega_D}{\pi T_c} \Delta_d. \quad (14)$$

and

$$\Delta_d = -N_0 V_d \left(\frac{c^2}{2}\right) \ln \frac{2e^\gamma \omega_D}{\pi T_c} \Delta_s + N_0 V_d \left(\frac{1}{2} + \frac{3}{8}c^2\right) \ln \frac{2e^\gamma \omega_D}{\pi T_c} \Delta_d. \quad (15)$$

In order to calculate $T_c$ of an anisotropic mixed $d$- and $s$-wave superconductor, it is convenient to introduce the transition temperature $T_{d(s)}$ of the anisotropic $d(s)$-wave superconductor such that $N_0 V_s \left(1 + \frac{c^2}{2}\right) \ln \frac{2e^\gamma \omega_D}{\pi T_c} = 1$, and $N_0 V_d \left(\frac{1}{2} + \frac{3}{8}c^2\right) \ln \frac{2e^\gamma \omega_D}{\pi T_c} = 1$. Then, $T_c$ can be expressed in terms of $T_{d(s)}$ as follows:

$$\left(1 + \frac{c^2}{2}\right) \left(\frac{1}{2} + \frac{3}{8}c^2\right) \ln \frac{T_s}{T_c} \ln \frac{T_d}{T_c} = \left(\bar{c}\right)^2, \quad (16)$$

where $\bar{c} = c \sum_{n=0}^{n=D}(n + 1/2)^{-1}$ with $\omega_D = (2n_D + 1)\pi T_c$. Here we would like to point out that a phenomenological value $T_c \simeq \frac{1}{2}(T_d + T_s) + \frac{1}{2}\sqrt{(T_d - T_s)^2 + 8\bar{c}^2 T_d T_s}$ can be achieved only if $T_s \simeq T_d \simeq T_C$. If $T_s << T_d$, $T_c$ has to be numerically calculated.

Following Ref.6, we expand $\Delta_{d(s)}(R)$ in terms of $\{f_N(R)\}$, namely, $\Delta_{d,s}(R) = \sum_N A_{N}^{d(s)} f_N(R)$, to obtain

$$\Delta_s(R) = V_s \sum_N A_{N}^{d(s)} \int d\xi \sum_{m=0}^{N} \Phi_{n,m}(\xi, N) C_{n,m}^{d(s)}(\xi, c) f_{N-n+m}(R)$$

$$+ V_s \sum_N A_{N}^{d} \int d\xi \sum_{m=0}^{N} \Phi_{n,m}(\xi, N) C_{n,m}^{d}(\xi, c) f_{N-n+m}(R), \quad (17)$$
and

$$\Delta_d(R) = V_d \sum_N A_d^{(i)} \int d\xi \sum_{n=0}^N \Phi_{n,m}(\xi, N) C_{n,m}^{(sd)}(\xi, c) f_{N-n+m}(R)$$

$$+ V_d \sum_N A_d^{(d)} \int d\xi \sum_{n=0}^N \Phi_{n,m}(\xi, N) C_{n,m}^{(d)}(\xi, c) f_{N-n+m}(R),$$

(18)

where

$$\Phi_{n,m}(\xi, N) = 2\pi N_0 \sum_\omega e^{-2|\omega|\xi} e^{-|e|H\left(\frac{\omega F}{\sqrt{2}}\right)^2} \cos(2\mu_0 H \xi)$$

$$\times \frac{1}{\sqrt{m!n!}} \begin{pmatrix} N \\ n \end{pmatrix}^{1/2} \begin{pmatrix} N-n+m \\ m \end{pmatrix}^{1/2} \left[ -|e| H (v_F \xi)^2 \right]^{(n+m)/2}$$

if \((n+m)\) is even and \(\Phi_{n,m}(\xi, N) = 0\) if \((n+m)\) is odd, and

$$C_{n,m}^{(j)}(\xi, c) = \int \frac{d\phi}{2\pi} \sin(2\phi) \exp \left\{ -|e| H \left(\frac{v_F \xi}{\sqrt{2}}\right)^2 [2c \sin(2\phi) + c^2] \right\}$$

$$\times (e^{-i\phi} - i c e^{i\phi})^m (e^{i\phi} + i c e^{-i\phi})^n$$

with \(C_{n,m}^{(0)} \equiv C_{n,m}^{(s)} \quad C_{n,m}^{(1)} \equiv C_{n,m}^{(sd)} \quad C_{n,m}^{(2)} \equiv C_{n,m}^{(d)}\). (See Appendix A for the detailed derivation.) It is necessary to investigate the symmetry properties of \(C_{n,m}^{(j)}\) to calculate \(H_{e2}\) as we have mentioned early. Since \(c\) is a small quantity, we may expand the integrand of \(C_{n,m}^{(j)}\) up to the \(c^2\) order. After a careful investigation of \(C_{n,m}^{(j)}\), we find

$$\frac{C_{n,m}^{(0)}}{n-m+4(n-m+2)(n-m)(n-m-2)(n-m-4)},$$

(19)

$$C_{n,m}^{(1)} \propto \frac{C_{n,m}^{(0)}}{(n-m+6)(n-m-6)},$$

(20)

and

$$C_{n,m}^{(2)} \propto \frac{C_{n,m}^{(1)}}{(n-m+8)(n-m-8)}.$$  

(21)

As one can easily see, \(C_{n,m}^{(0)} \neq 0\) only if \(|n-m| = 0, 2, 4, 6\), \(C_{n,m}^{(1)} \neq 0\) only if \(|n-m| = 0, 2, 4, 6\), and \(C_{n,m}^{(2)} \neq 0\) only if \(|n-m| = 0, 2, 4, 6, 8\). We also find other symmetry properties of \(C_{n,m}^{(j)}\) such as \(C_{n,m}^{(j)} = -C_{n,m}^{(j)} \) if \(|n-m| = 2, 6\), and \(C_{n,m}^{(j)} = C_{m,n}^{(j)} \) if \(|n-m| = 4, 8\). These properties
are still valid even if we expand \(C^{(j)}_{n,m}\) up to the \(c^4\) order. We, thus, know that \(|2N\rangle_d(s)\) do not couple to \(|2N+1\rangle_d(s)\) but to \(|2N\rangle_d(s)\), and vice versa. Consequently, we expect that the order parameter is represented by \(\sum_N[A^{(d)}_{2N}|2N\rangle_d\sin(2\phi) + A^{(s)}_{2N}|2N\rangle_s]\) near \(H_{c2}\); in other words, \(H_{c2}\) is determined by such coefficients as \(A^{(d)}_{2N}\) and \(A^{(s)}_{2N}\).

Using the orthonormality of \(\{f_N\}\), we obtain the equation for the coefficients \(A^{(s)}_N\) and \(A^{(d)}_N\) from which \(H_{c2}\) will be calculated as follows:

\[
A^{(s)}_m = V_s \sum_N A^{(s)}_N \int d\xi \sum_{n=0}^N \Phi_{n,m+n-N}(\xi, N) C^{(0)}_{n,m+n-N}
+ V_s \sum_N A^{(d)}_N \int d\xi \sum_{n=0}^N \Phi_{n,m+n-N}(\xi, N) C^{(1)}_{n,m+n-N},
\]

(22)

and

\[
A^{(d)}_m = V_d \sum_N A^{(s)}_N \int d\xi \sum_{n=0}^N \Phi_{n,m+n-N}(\xi, N) C^{(1)}_{n,m+n-N}
+ V_d \sum_N A^{(d)}_N \int d\xi \sum_{n=0}^N \Phi_{n,m+n-N}(\xi, N) C^{(2)}_{n,m+n-N},
\]

(23)

with \(m+n-N \geq 0\), and \(|m-N| = 0, 2, 4\) for \(C^{(0)}_{n,m+n-N}\); \(|m-N| = 0, 2, 4, 6\) for \(C^{(1)}_{n,m+n-N}\); and \(|m-N| = 0, 2, 4, 6, 8\) for \(C^{(2)}_{n,m+n-N}\). The equations to determine \(H_{c2}\) are given by \(m = 0, 2, 4, \ldots\), namely,

\[
A^{(s)}_0 = V_s \int d\xi \left[ A^{(s)}_0 \Phi^{0}_{0,0} C^{(s)}_{0,0} + A^{(s)}_2 \Phi^{2}_{2,0} C^{(s)}_{2,0} + A^{(s)}_4 \Phi^{4}_{4,0} C^{(s)}_{4,0} \right]
+ A^{(d)}_0 \Phi^{0}_{0,0} C^{(sd)}_{0,0} + A^{(d)}_2 \Phi^{2}_{2,0} C^{(sd)}_{2,0} + A^{(d)}_4 \Phi^{4}_{4,0} C^{(sd)}_{4,0} \ldots
\]

\[
A^{(s)}_2 = V_s \int d\xi \left[ A^{(s)}_0 \Phi^{0}_{0,2} C^{(s)}_{0,2} + A^{(s)}_2 \sum_{i=0}^2 \Phi^{2}_{i,i} C^{(s)}_{i,i} + A^{(s)}_4 \sum_{i=0}^2 \Phi^{4}_{2+i,i} C^{(s)}_{2+i,i} \ldots \right]
+ A^{(d)}_0 \Phi^{0}_{0,2} C^{(sd)}_{0,2} + A^{(d)}_2 \sum_{i=0}^2 \Phi^{2}_{i,i} C^{(sd)}_{i,i} + A^{(d)}_4 \sum_{i=0}^2 \Phi^{4}_{2+i,i} C^{(sd)}_{2+i,i} \ldots
\]

\[
A^{(s)}_4 = V_s \int d\xi \left[ A^{(s)}_0 \Phi^{0}_{0,4} C^{(s)}_{0,4} + A^{(s)}_2 \sum_{i=0}^2 \Phi^{2}_{i,2+i} C^{(s)}_{i,2+i} + A^{(s)}_4 \sum_{i=0}^2 \Phi^{4}_{i,i} C^{(s)}_{i,i} \ldots \right]
+ A^{(d)}_0 \Phi^{0}_{0,4} C^{(sd)}_{0,4} + A^{(d)}_2 \sum_{i=0}^2 \Phi^{2}_{i,2+i} C^{(sd)}_{i,2+i} + A^{(d)}_4 \sum_{i=0}^2 \Phi^{4}_{i,i} C^{(sd)}_{i,i} \ldots
\]

\[\vdots\]
and similar equations of $A_0^{(d)}, A_2^{(d)}, A_4^{(d)}, \ldots$. $H_c$ is the largest value of solutions which satisfy the condition for a non-trivial solution to exist in these equations of $A_{N}^{(d,s)}$. As we have mentioned, $|2N\rangle_{d(s)}(N = 0, 1, 2, \cdots)$ are involved in the determination of $H_c$; however, it is expectable that the first few states such as $|0\rangle_{d(s)}, |2\rangle_{d(s)}$ and $|4\rangle_{d(s)}$ are important because in the case of $c = 0$, $|0\rangle_d, |2\rangle_s$ and $4\rangle_d$ play the dominant role in determining $H_c$. Inclusion of more states such as $|6\rangle_{d(s)}$ and $|8\rangle_{d(s)}$ gives rise to a difference much less than 1%.

As we did in Ref. 6, we introduce dimensionless unit in the calculation of $H_c$: $t = T/T_c$ and $h = 2|e|H(v_F/2\pi T_c)^2$. In addition to the anisotropy parameter $c$, we also define parameters $\delta = T_s/T_d$ and $\gamma_z = (2\pi\mu_0/ev_F^2)T_c$, which is the strength of spin-magnetic field coupling. For the sake of comparison with experiment measurement, we convert the normalized magnetic field into the dimensional one by using

$$\frac{H(T)}{\frac{dT}{dT}|_{T_c}T_c} = \frac{h(t)}{\frac{dh}{dt}|_1}. \quad (24)$$

By solving the eigen-equations for $A_{N}^{(d,s)}$, we plot $H_c$ in Fig. 1 as a function of temperature for several typical cases. $H_c$ for an anisotropic mixed $d$- and $s$-wave superconductor with ZI taken into account is plotted with the solid line. $H_c$ for an anisotropic mixed $d$- and $s$-wave superconductor without ZI is represented by the dot-dashed line. The involved parameter values are taken to be: $\delta = T_s/T_d = 0.06$, $c = 0.16$, and $\gamma_z = 0.15$. Also plotted are $H_c$ for a two-dimensional $s$-wave superconductor (dotted line) and a pure $d$-wave superconductor (dashed line) without the MA and the ZI. Note that, even though the increase of $\delta$ can enhance $H_c$, for the value of $\delta$ chosen here, the upper critical field for an isotropic mixed $d$- and $s$-wave order parameter (with $\delta = 0.06$) without ZI is more or less same as that of the pure $d$-wave superconductor (dashed line). As shown in Fig. 1, we find that the mass anisotropy enhances $H_c$ while the spin Zeeman coupling suppresses $H_c$. This means that the mass anisotropy supports superconductivity; in other words, it increases $T_c$ (and consequently the gap), which can be easily seen from the equation for $T_c$. In addition, one can also see that $H_c$ is saturated at zero temperature as a reflection of the thermodynamic requirement; namely, on the phase boundary in the $T - H$ plane,

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The differential equation

\[ -\frac{dH_{c2}}{dT} = \frac{\delta S}{\delta M} = 0 \]  

(25)

at \( T = 0 \) near \( H_{c2} \), where \( \delta S \) is the entropy difference between the normal and superconducting states, and \( \delta M \) is the magnetization near \( H_{c2} \).

Recently, the upper critical field \( H_{c2} \) (parallel to the \( c \) axis) of YBCO with \( T_c = 84.3 \) K has been measured down to about 4\( K \). The corresponding data is shown by solid circles in Fig. 1. The slope \( dH/dT|_{T_c} \) has been experimentally observed to be \(-1.9 \) T/K. In the theoretical calculation, we are mainly concerned about the low-temperature data of \( H_{c2} \) most because at the low temperature thermal fluctuation effect is negligible. As shown in Fig. 1, the experimental data can be fit very well, with the above given parameter values, by our calculation for an anisotropic mixed \( d- \) and \( s- \) wave superconductor including the ZI. Here we would like to point out that, since \( \delta \) enhances \( H_{c2} \) more significantly than \( c \), if we take a large value of \( \delta \), we have to choose a physically unacceptable high value of \( \gamma_z \) to fit the low-temperature data. Actually, it is believed that \( T_c/E_F \sim 0.1 \) for a high-\( T_c \) superconductor which yields \( \gamma_z = \pi T_c/2E_F \approx 0.157 \) by assuming \( \mu_0^* = \mu_B \) (Bohr magneton). On the other hand, most of available experiments seem to indicate that \( \delta \) should be very small. All this facts demonstrate that the chosen set of parameter values are physically reasonable. Finally, we would like to mention: (i) Because \( H_p \) for the sample is about 185 T, the experimental data in Ref.\(^7\) are within paramagnetic limit with the critical ratio \( H_p/T_c \) estimated to be 2.2 T/K for a \( d- \) wave superconductor. Therefore, in our consideration, we do not have to include the spin-orbit interaction because it reduces the pair-breaking effect of the Zeeman interaction, and consequently, it allows \( H_{c2} \) to be larger than \( H_p \) In the theoretical point of view, since the strength of the spin-orbit coupling is proportional to \( Z^2 \), where \( Z \) is an atomic number, we may neglect its effect in YBCO as long as no heavy-atomic impurity is taken into account as in this paper. However, it may play an important role in such heavy-fermion superconductors as UBe\(_{13}\) and UPt\(_3\). (ii) Magnetic and non-magnetic impurities are pair breakers so that it is clear the impurities reduce \( H_{c2} \) as well as \( T_c \). However, the impurity concentration in the sample prepared in Ref.\(^7\) seems to be negligible, the corresponding effect
is not considered here. (iii) Recently, O’brien et al.\(^4\) have interpreted the experiment based on a three dimensional s-wave model.\(^5\) However, it is well-known that superconductivity in YBCO is of two dimensional nature.\(^1\) (vi) A small deviation of theoretical results and experimental data occurs in high temperature region because thermodynamic fluctuations of vortices is strong when temperature is high.

In summary, the upper critical field (\(H_{c2}\)) of a mixed \(d\)- and \(s\)-wave superconductor with a mass anisotropy has been microscopically calculated based on the quasiclassical theory. We found the mass anisotropy supports \(H_{c2}\) against Zeeman suppression. \(H_{c2}\) becomes saturated at zero temperature in consistence with a thermodynamic requirement. The theoretical results are compared well with recent experimental data of YBa\(_2\)Cu\(_3\)O\(_{7-\delta}\).

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**APPENDIX A:**

Since \(\Delta_0(\mathbf{R}, \phi) = \Delta_s(\mathbf{R}) + \Delta_d(\mathbf{R}, \phi)\), we have two equations; namely, one is for \(\Delta_s(\mathbf{R})\) and the other for \(\Delta_d(\mathbf{R})\) as follows:

\[
\Delta_s(\mathbf{R}) = V_s \sum_\omega \int \frac{d\phi}{2\pi} \frac{1}{1 + c \sin(2\phi)} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^* H \xi) \Omega(\Pi_+, \Pi_-) \Delta_s(\mathbf{R}) \\
+ V_s \sum_\omega \int \frac{d\phi}{2\pi} \frac{\sin(2\phi)}{1 + c \sin(2\phi)} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^* H \xi) \Omega(\Pi_+, \Pi_-) \Delta_d(\mathbf{R}) \tag{A1}
\]

and

\[
\Delta_d(\mathbf{R}) = V_d \sum_\omega \int \frac{d\phi}{2\pi} \frac{\sin(2\phi)}{1 + c \sin(2\phi)} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^* H \xi) \Omega(\Pi_+, \Pi_-) \Delta_s(\mathbf{R}) \\
+ V_d \sum_\omega \int \frac{d\phi}{2\pi} \frac{\sin(2\phi)^2}{1 + c \sin(2\phi)} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^* H \xi) \Omega(\Pi_+, \Pi_-) \Delta_d(\mathbf{R}), \tag{A2}
\]

where

\[
\Omega(\Pi_+, \Pi_-) = 2\pi T N_0 \exp[-\text{sgn}(\omega) \left(\frac{u_F}{\sqrt{2}}\right)} \xi \left(\xi (e^{-i\phi} - ice^{i\phi})\Pi_+ + (e^{i\phi} + ice^{-i\phi})\Pi_- \right) \tag{A3}
\]
with \( \Pi_{\pm} = (\Pi_x \pm i\Pi_y) / \sqrt{2} \). Note that

\[
-\text{isgn}(\omega) \left( \frac{v_F}{\sqrt{2}} \right) \xi (e^{-i\phi} - ice^{i\phi}) \Pi_+, \quad -\text{isgn}(\omega) \left( \frac{v_F}{\sqrt{2}} \right) \xi (e^{i\phi} + ice^{-i\phi}) \Pi_-
\]

\[
= 2|e|H \left( \frac{v_F \xi}{\sqrt{2}} \right)^2 [1 + 2c \sin(2\phi) + c^2],
\]

(A4)

then we obtain

\[
\Delta_s(R) = V_s \sum_\omega \int \frac{d\phi}{2\pi} \frac{1}{[1 + c \sin(2\phi)]} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^*H\xi) \tilde{\Omega}(\Pi_+, \Pi_-) \Delta_s(R)
\]

\[
+ V_s \sum_\omega \int \frac{d\phi}{2\pi} \frac{\sin(2\phi)}{[1 + c \sin(2\phi)]} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^*H\xi) \tilde{\Omega}(\Pi_+, \Pi_-) \Delta_d(R)
\]

(A5)

and

\[
\Delta_d(R) = V_d \sum_\omega \int \frac{d\phi}{2\pi} \frac{\sin(2\phi)}{[1 + c \sin(2\phi)]} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^*H\xi) \tilde{\Omega}(\Pi_+, \Pi_-) \Delta_d(R)
\]

\[
+ V_d \sum_\omega \int \frac{d\phi}{2\pi} \frac{\sin(2\phi)^2}{[1 + c \sin(2\phi)]} \int d\xi e^{-2|\omega|\xi} \cos(2\mu_0^*H\xi) \tilde{\Omega}(\Pi_+, \Pi_-) \Delta_d(R),
\]

(A6)

where

\[
\tilde{\Omega}(\Pi_+, \Pi_-) = 2\pi T N_0 \exp \left[ -|e|H \left( \frac{v_F \xi}{\sqrt{2}} \right)^2 (1 + 2c \sin(2\phi) + c^2) \right]
\]

\[
\times e^{-\text{isgn}(\omega) \left( \frac{v_F}{\sqrt{2}} \right) \xi (e^{-i\phi} - ice^{i\phi}) \Pi_+} e^{-\text{isgn}(\omega) \left( \frac{v_F}{\sqrt{2}} \right) \xi (e^{i\phi} + ice^{-i\phi}) \Pi_-}.
\]

(A7)

Expanding \( \Delta_{s(d)}(R) \) in terms of \( \{ f_N(R) \} \) and noting

\[
\begin{pmatrix}
\Pi_+ \\
\Pi_-
\end{pmatrix}
\begin{pmatrix}
f_N
\end{pmatrix}
= \sqrt{2|e|H} \begin{pmatrix}
\sqrt{N + 1} \\
\sqrt{N}
\end{pmatrix}
\begin{pmatrix}
f_{N+1} \\
f_{N-1}
\end{pmatrix},
\]

(A8)

we obtain Eqs. (17) and (18).
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FIG. 1. The upper critical field $H_{c2}$ for an anisotropic mixed $d$- and $s$-wave superconductor (solid line) with the spin Zeeman interaction taken into account. Solid circles are experimental data of YBCO. Also plotted are $H_{c2}$ of a two-dimensional isotropic $s$-wave superconductor (dotted line), a pure isotropic $d$-wave superconductor (dashed line), and an anisotropic mixed $d$- and $s$-wave superconductor (dot-dashed line), all obtained without the Zeeman interaction included.