Tests of QCD using differences between gluon and quark jets†

J. William Gary

Department of Physics, University of California, Riverside, CA, 92521, USA
E-mail: william.gary@ucr.edu

Abstract

I present recent results from LEP which utilize differences between gluon and quark jets to make quantitative tests of QCD. The principal topic is a determination of the ratio of QCD color factors, $C_A/C_F$, using either the multiplicity or fragmentation functions of the jets. In addition, I discuss a recent measurement of the rate of $\eta$ mesons in gluon jets compared to quark jets.

1. Introduction

The physics of differences between gluon and quark jets has enjoyed a golden age at LEP due to the large data samples and good detector capabilities. Starting with samples of symmetric three jet “Y” events, in which the two lowest energy jets both form an angle of about 150° with respect to the highest energy jet [1], the LEP experiments established the basic phenomenology of this field: gluon jets are broader, have a softer fragmentation function and a larger mean multiplicity than quark jets [2]. These discoveries, made in 1991-1996, culminated experimental efforts that had unsuccessfully attempted to establish these differences since the late 1970s. The field has now evolved beyond exposition of gluon-quark jet differences to quantitative tests of QCD. Amongst these are tests of analytic calculations of higher moments of the multiplicity distributions of gluon and quark jets [3, 4] and determinations of the ratio of QCD color factors, $C_A/C_F$. In the following I focus on the latter topic since these results have appeared just this past year. There are two studies I will discuss, presented in OPAL publication CERN-EP/99-028 (in press in Eur. Phys. J C) and in DELPHI paper DELPHI 99-127 Conf. 314. Also note a recent DELPHI publication [5] on a related topic. I will also discuss a new result presented in OPAL Physics Note PN407 on the production rate of $\eta$ mesons in gluon jets compared to quark jets.

2. Gluon and quark jets from a point source

In QCD calculations, quark and gluon jets are produced as virtual $q\bar{q}$ and $gg$ pairs, respectively, from a color singlet point source. The jet properties are defined by an inclusive sum over event hemispheres. The hemispheres are defined by the plane perpendicular to the principal event axis. For jets defined in this manner, referred to as “unbiased,” there is no jet finding algorithm and no ambiguity about which particles to associate with gluon or quark jet production. Experimental access to high energy unbiased quark jets is easy since hadronic events in $e^+e^-$ annihilations result from $q\bar{q}$ production from a color singlet source. In contrast, $gg$ production from a color singlet point source is a process which has been practically unobserved in nature. One channel where the experimental selection of gluon jets matches the theoretical criteria is $e^+e^-$ hadronic annihilation events in which the quark jets $q$ and $\bar{q}$ from the electroweak $Z^0/\gamma$ decay are approximately colinear: the gluon jet hemisphere against which the $q$ and $\bar{q}$ recoil is produced under the same conditions as gluon jets in $gg$ events [6, 7]. OPAL selected events of the type $e^+e^- \rightarrow q_{tag} \bar{q}_{tag} g_{incl}$, in which $g_{incl}$ refers to a gluon jet hemisphere recoiling against two tagged quark jets $q_{tag}$ and $\bar{q}_{tag}$ in the opposite hemisphere. Monte Carlo study shows that these gluon jet hemispheres have almost identical properties to unbiased gluon jets and can be selected with virtually no dependence on a jet finding algorithm, a unique feature of this method. The OPAL results are obtained for jet energies of 40 GeV.

The charged particle multiplicity distributions of the unbiased gluon and quark jets are shown in Fig. 1a and b, respectively [6]. The measured ratio of the mean multiplicity between gluon and quark jets is $1.51 \pm 0.02$ (stat.) $\pm 0.04$ (syst.). This is in excellent agreement with recent QCD calculations of this quantity [5].

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Multiplicities in full phase space are sensitive to higher order corrections and the effects of energy conservation. Multiplicities in limited phase space, such as the multiplicity of soft particles, is much less sensitive to these effects and better satisfies the asymptotic condition of QCD. Thus one can potentially observe the full color factor difference $C_A/C_F=2.25$ in the ratio of soft hadron multiplicities, limited only by possible finite energy and hadronization effects. Indeed it has been predicted that the ratio of soft particles at large transverse momentum $p_T$ between gluon and quark jets should approximately equal 2.25 even at the finite energies of LEP [1]. Fig. 2 shows the $p_T$ spectrum of soft particles in the unbiased gluon and quark jets. $p_T$ is defined relative to the jet axes obtained by summing the particle 3-momenta in the gluon and quark jet hemispheres. Soft particles are defined by momenta $p<4.0$ GeV/c.

Fig. 1. Charged particle multiplicity of unbiased (a) gluon and (b) uds flavored quark jets, defined by inclusive sums over event hemispheres.

Fig. 2. (a) $p_T$ distributions of soft charged particles in unbiased gluon and uds flavored quark jets, (b) Ratio of the gluon to quark jet measurements from (a).

Table 1. Results for the multiplicity ratio between gluon and quark jets for soft particles with large transverse momentum with respect to the jet axes.

|                | OPAL data | HERWIG hadrons, $E_{c.m.}=91$ GeV | HERWIG partons, $E_{c.m.}=91$ GeV | HERWIG hadrons, $E_{c.m.}=10$ TeV | HERWIG partons, $E_{c.m.}=10$ TeV | JETSET partons, $E_{c.m.}=91$ GeV, $C_A=C_F=4/3$ |
|----------------|-----------|----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
|                | OPAL      | 2.29 ± 0.17                      | 2.16                              | 2.09                              | 2.24                              | 2.25                              | 1.00                              |

Table 1 contains next-to-next-to-leading-order QCD plus energy-momentum conservation at each branching and exhibits the correct asymptotic behavior and thus is appropriate for this test. If the experimental analysis is applied to HERWIG events at the parton level at asymptotically large energies (e.g., 10 TeV), the result should equal 2.25 if the analysis truly measures $C_A/C_F$. Similarly one can use parton level Monte Carlo events with $C_A=C_F=4/3$ to verify that the experimental analysis yields unity, again a necessary condition if the analysis truly measures $C_A/C_F$. For the latter test, the JETSET Monte Carlo is used since HERWIG does not allow $C_A=C_F$. The Monte Carlo results for the experimental variable $(r_{ch})_{0.8<p_T<3}$ are given in the bottom portion of Table 1. The results are 2.25 for HERWIG at the parton level with $E_{c.m.}=10$ TeV and 1.00 for JETSET at the parton level with $C_A=C_F=4/3$. This demonstrates that the result [1] is indeed a measurement of $C_A/C_F$: it is in fact one of the most accurate measurements of this ratio yet performed. Note that the Herwig results for $E_{c.m.}=91$ GeV in Table 1 imply a hadronization correction of about 0.97 to the measurement [1].

Beyond the measurement of $C_A/C_F$ discussed above, OPAL uses their $g_{inel}$ jet sample to exclude the AR-2 and AR-3 models of color reconnection implemented in the ARIADNE QCD Monte Carlo program with a significance of about five standard deviations. This is currently the most stringent limit on any realistic model of color reconnection.

3. $C_A/C_F$ from the scale dependence of gluon and quark jet fragmentation functions

The jets discussed in the previous section have a fixed energy of about 40 GeV. It is of interest to obtain information about the internal properties of gluon jets at other scales, using three jet $q_T$ events selected in a standard way with a jet finding algorithm. Higher order terms associated with
the phenomenon of coherence suggest that the characteristics of a jet depend on a transverse momentum-like scale, $\kappa = E_{\text{jet}} \sin(\theta_{\text{min}}/2)$ \cite{10}, where $E_{\text{jet}}$ is the energy of the jet and $\theta_{\text{min}}$ is the smaller of the angles with respect to the other two jets. It has previously been demonstrated that $\kappa$ can be an appropriate variable for comparing jets in different three jet topologies \cite{11}.

DELPHI has examined the scale dependence of gluon and quark jet fragmentation functions in three jet events. Three jet events are selected using either the $k_L$ or Cambridge jet finders. The impact parameters of charged tracks are used to identify gluon jets within those events. The fragmentation functions of the two lower energy jets, one of which is a quark jet and the other a gluon jet with high probability, are measured as a function of $\kappa$. The results for quark jets are shown in Fig. 3. The DELPHI results for quark jets are in good agreement with the unbiased (hemisphere) results from the TASSO and TPC/2$\gamma$ experiments. This agreement supports the appropriateness of $\kappa$ as the scale for this analysis. Fig. 4 shows the corresponding results for gluon jets. The DELPHI results for gluon jets extrapolate well to the OPAL results for the fragmentation function of unbiased gluon jets at $\kappa = 40$ GeV (CERN-EP/99-028). This again supports the appropriateness of $\kappa$ as the scale.

The scale evolution of fragmentation functions in QCD is described by the DGLAP equations. DELPHI parametrizes the fragmentation functions of the gluon and quark jets at a fixed scale, chosen to be $\kappa = 5.5$ GeV. A simultaneous fit is made to the DELPHI data in Figs. 3 and 4, using first order DGLAP evolution, to fix the variables of this parametrization and to determine the color factor $C_A$ and the effective QCD scale parameter $\Lambda$. This strategy is very similar to that employed in deep inelastic scattering to determine the strong coupling strength, $\alpha_s$, from the scale evolution of structure functions. The results for $C_A$ and $\Lambda$ are $2.97 \pm 0.12$ (stat.) and $0.40 \pm 0.11$ GeV (stat.), respectively. Dividing the result for $C_A$ by $C_F = 4/3$ yields $C_A/C_F = 2.23 \pm 0.09$ (stat.) $\pm 0.23$ (syst.)$^\dagger$, where the systematic uncertainty is mostly due to the difference between using the $k_L$ or Cambridge jet finders. This result is in good agreement with the QCD value of 2.25.

4. $\eta$ meson rates in gluon and quark jets

QCD predicts that the ratio of the mean multiplicity in gluon to quark jets, $r_h$, is the same for all hadron species $h$. $r_h$ could differ for different types of hadrons, however, for at least two reasons: (1) because of the decay properties of hadrons, e.g. B hadrons yield many kaons, making $r_h$ smaller for kaons than the corresponding result for other particles since $e^+e^-$ jets with B hadrons are almost always quark jets, or (2) dynamical differences between the hadronization properties of gluons and quarks. Examples of dynamical differences occur in the Lund hadronization model, which predicts that $r_h$ is larger for baryons than for mesons, and in the gluon octet string model \cite{12}, which predicts $r_h$ to be larger for isoscalar mesons than for non-isoscalar ones.

To test for the presence of a dynamical enhancement of isoscalar meson production in gluon jets, L3 \cite{13} measured the $\eta$ meson rate in the lowest energy jet ("jet 3") of three jet events and compared the result to the corresponding Monte Carlo prediction with no isoscalar enhancement mechanism. They found the $\eta$ rate in jet 3 to be $30 \pm 10\%$ larger in the data than in the MC, which was interpreted as evidence for the dynamical enhancement of isoscalars in gluon jets.

$^\dagger$ The systematic uncertainty for this result is based on the full difference between the standard measurement and the measurements with a systematic change in the analysis, and not half the difference as in DELPHI 99-127 Conf. 314, to make it comparable to other results for $C_A/C_F$ such as the one presented in section \cite{13}. 

![Figure 3](https://example.com-figure3.png)

*Figure 3.* Quark jet fragmentation function vs. scale. in good agreement with the unbiased (hemisphere) results from the TASSO and TPC/2$\gamma$ experiments. This agreement supports the appropriateness of $\kappa$ as the scale for this analysis. Fig. 4 shows the corresponding results for gluon jets. The DELPHI results for gluon jets extrapolate well to the OPAL results for the fragmentation function of unbiased gluon jets at $\kappa = 40$ GeV (CERN-EP/99-028). This again supports the appropriateness of $\kappa$ as the scale.

![Figure 4](https://example.com-figure4.png)

*Figure 4.* Gluon jet fragmentation function vs. scale. results for gluon jets extrapolate well to the OPAL results for the fragmentation function of unbiased gluon jets at $\kappa = 40$ GeV (CERN-EP/99-028). This again supports the appropriateness of $\kappa$ as the scale.
OPAL has performed a related study. Three jet events are reconstructed using either the $k_{\perp}$, Luclus or cone algorithms. The charged particle, $\pi^0$ and $\eta$ multiplicities of the jets are compared as a function of the $\kappa$ scale discussed in the previous section. The measurements are unfolded to correspond to pure quark and gluon jets in the regions where the $\kappa$ scales of jets overlap.

The results for all charged particles are shown in Fig. 4 (these are very similar to recently published data from DELPHI [5]). The curves in Fig. 5b, and similarly for Fig. 5c. The scaled parametrizations describe the curves in Fig. 5a, and similarly for Fig. 5b. The scaled parametrizations describe the $\pi^0$ and $\eta$ mesons accurately with the exception of $\eta$ mesons in gluon and quark jets compared to charged particles, the corresponding results in Fig. 5a (these are very similar to recently published data from DELPHI [5]). The curves for charged particle, (b) $\pi^0$, and (c) $\eta$ meson rates in gluon and quark jets vs. scale $Q=\kappa$, using the Luclus jet finder.

in that figure show a polynomial parametrization of the measurements. The corresponding results for $\pi^0$ and $\eta$ mesons are shown in Fig. 4 and 5. To test for dynamical differences between the production rates of $\pi^0$ and $\eta$ mesons in gluon and quark jets compared to charged particles, the parametrizations from Fig. 4 are scaled using the same factor for both gluon and quark jets to obtain the curves in Fig. 5, and similarly for Fig. 5b. The scaled parametrizations describe the $\pi^0$ and $\eta$ measurements accurately with the exception of a slight disagreement in the lowest bin of the $\eta$ distribution (Fig. 5b) which is not statistically significant. Thus the ratios $r_{\pi^0}$ for $\pi^0$ and $\eta$ mesons agree with $r_{\eta}$ for charged particles in all bins of $\kappa$ to within the uncertainties. Therefore OPAL does not obtain evidence for a dynamical enhancement of $\eta$ mesons in gluon jets, in contrast to L3. ALEPH [4] recently reported results for the $\eta$ meson in jet 3 of three jet events, similar to the L3 study. ALEPH observes the measured result to be well described by the Monte Carlo without a mechanism for isoscalar meson enhancement, in contrast to L3. Thus – similar to OPAL – ALEPH does not obtain evidence for isoscalar meson enhancement in gluon jets.

5. Summary

After 20 years of experimental effort, the field of differences between gluon and quark jets has advanced to the level of providing precise, quantitative tests of QCD. The gluon to quark jet multiplicity ratio in full phase space is $1.51 \pm 0.02(\text{stat.}) \pm 0.04(\text{syst.})$, in agreement with QCD calculations which incorporate energy conservation and the correct phase space limits for soft gluon radiation [6]. For soft particles with large transverse momentum with respect to the jet axes, the corresponding result is $2.29 \pm 0.09(\text{stat.}) \pm 0.15(\text{syst.})$, providing one of the most accurate measurements of the ratio of QCD color factors, $C_A/C_F$. $C_A/C_F$ has also been measured using the difference in the scale evolution of gluon and quark jet fragmentation functions to be $2.23 \pm 0.09(\text{stat.}) \pm 0.23(\text{syst.})$. Last, non-perturbative aspects of gluon and quark jet differences have been probed in two recent studies which find no evidence for a enhancement of $\eta$ mesons in gluon jets compared to quark jets, beyond that observed for charged particles. These latter results are in contrast to the conclusions of an earlier L3 study [13].

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