A Monte Carlo algorithm for Multiphoton Beamstrahlung in Monte Carlo event generators

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Abstract

We describe a simple algorithm that calculates the distributions of electrons and positrons under multiphoton beamstrahlung at a future linear collider. The evolution equation as given by Chen is solved by a Monte Carlo algorithm. Explicit multiple beamstrahlung photons are generated. We present first results from an implementation of beamstrahlung into the Monte Carlo event generator WOPPER, that calculates the QED radiative corrections to the process $e^+e^- \rightarrow 4$ fermions through resonating W pairs.

1 Introduction

At future linear $e^+e^-$ colliders with center of mass energies of 0.5 TeV and above, beamstrahlung [1, 2], the synchrotron radiation from the colliding $e^+e^-$ beams, will become of significant physical interest. On one hand, beamstrahlung may carry away a substantial fraction of primary beam energy and lead to a degradation of the effective center of mass energy for $e^+e^-$ collisions, on the other hand, the lower energy $e^+e^-$ and $\gamma$'s contribute to background processes.

In general, the full treatment of beam-beam interactions and beamstrahlung is a complicated many-body problem, and dedicated codes for the calculation of beam-beam effects exist [3] or are being developed [4]. However, from the high energy physics perspective the availability of a simplified analytical or numerical treatment is desired for the study of interesting processes, (e.g., $e^+e^- \rightarrow W^+W^-$), at the level of Monte Carlo event generators.

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As long as the average number of beamstrahlung photons per beam particle is much less than unity, the radiation spectrum is given by the Sokolov-Ternov spectrum [5]. In the case of the proposed linear colliders, the average number of radiated photons per beam particle at design luminosity is typically $O(1)$. Therefore, the effects of multiple radiation of beamstrahlung photons have to be taken into account.

Multiphoton beamstrahlung has already been discussed in [6, 7] (and references quoted therein). In this note we will follow the approach by Chen et al., however, without using any approximation to the radiation spectrum. We shall derive a recursive solution of the evolution equation, which is well suited for a numerical implementation.

The Monte Carlo algorithm for beamstrahlung presented below is considered complementary to a full treatment of all beam-beam effects as performed in [3]. The main purpose of this Monte Carlo approach is to provide a simple package that can be incorporated into or interfaced with Monte Carlo event generators for physics at future $e^+e^-$ colliders. In section 2 we will review the electron energy spectrum under multiphoton beamstrahlung. Section 3 outlines our Monte Carlo approach, while in section 4 we present preliminary results. Further details of the calculation will be presented elsewhere [8].

## 2 Electron Energy Spectrum

In this section we recapitulate the derivation of the electron energy spectrum due to multiphoton beamstrahlung in the spirit of Chen et al. [6, 7]. We use $\hbar = c = 1$.

Let $\psi(x, t)$ be the distribution function of the electron with energy fraction $x = E/E_0$ at time $t$, normalized such that

$$\int_0^1 dx \, \psi(x, t) = 1 .$$

(1)

We shall assume that the emission of the photons takes place in an infinitesimally short time interval. Therefore the interference between successive emissions may be neglected, and one can describe the time evolution of the electron distribution by the rate equation

$$\frac{\partial \psi}{\partial t} = -\nu(x)\psi(x, t) + \int_x^1 dx' \, F(x, x')\psi(x', t) ,$$

(2)

where the first and second term on the r.h.s. correspond to the sink and source for the evolution of $\psi$, respectively. $F$ is the spectral function of radiation, and

$$\nu(x) = \int_0^x dx' \, F(x', x)$$

(3)
represents the average number of photons radiated per unit time by an electron with energy fraction \( x \). Electrons and positrons from pair production in photon-photon processes are not taken into account.

The spectral function of radiation is conventionally characterized by the beamstrahlung parameter \( \Upsilon \), which is defined as
\[
\Upsilon = \frac{E_0 B}{m_e B_c},
\]
where \( B \) is the effective field strength in the beam, and \( B_c = m_e^2/e \) is the Schwinger critical field. The \( e^+e^- \) beams generally have Gaussian charge distributions, and thus the local field strength varies inside the beam volume. However, it has been shown that through integration over the impact parameter and the longitudinal variations, that the overall beamstrahlung effect can be simply described as if all particles experience a uniform mean field \( B_{\text{mean}} \) during an effective collision time \( \tau = l_{\text{eff}}/2 = \sqrt{3} \sigma_z \). In what follows, we will assume the obtained mean beamstrahlung parameter \( \Upsilon_{\text{mean}} \) and effective beam length \( l_{\text{eff}} \) but drop the subscripts.

The spectral function of synchrotron radiation was derived by Sokolov and Ternov and reads
\[
F(x, x') = \frac{\nu_{\text{cl}}}{\Upsilon} \frac{2}{5 \pi x'^2} \left\{ \int_\eta^\infty du \ K_{5/3}(u) + \frac{(\xi \eta)^2}{1 + \xi \eta} K_{2/3}(u) \right\} \theta(x' - x),
\]
where \( \xi = (3 \Upsilon/2)x', \eta = (2/3 \Upsilon)(1/x - 1/x') \), and \( K_\nu \) denotes the modified Bessel function of order \( \nu \).

We define
\[
\nu(x) = \int_0^x dx' \ F(x', x) \equiv \nu_{\text{cl}} \cdot U_0(x \Upsilon),
\]
where \( \nu_{\text{cl}} \) is the number of photons per unit time in the classical limit,
\[
\nu_{\text{cl}} = \frac{5}{2\sqrt{3}} \frac{\alpha m_e^2}{E_0} \Upsilon,
\]
which is independent of the particle energy for a given field strength due to (4).

The function \( U_0(y) \) behaves as
\[
U_0(y) = \begin{cases} 1 & y \to 0 \\ \text{const.} \times y^{-1/3} & y \to \infty \end{cases},
\]
exhibiting the asymptotic approach to the classical limit and the suppression of radiation in the deep quantum regime, respectively.
Finally, the differential $e^+e^-$ luminosity can be expressed as the convolution of the effective electron energy distributions of the colliding beams \[7\],

\[
\frac{dL(s)}{ds} = L_0 \int dx_1 dx_2 \delta(s - x_1 x_2 s_0) \psi(x_1) \psi(x_2) ,
\]

where $L_0$ is the nominal luminosity of the collider, including the enhancement factor due to beam disruption, and the effective energy distribution $\psi(x)$ is obtained by averaging over the longitudinal position within the beam,

\[
\psi(x) = \frac{2}{l} \int_0^{l/2} \psi(x, t) .
\]

3 Multiphoton Beamstrahlung by Monte Carlo

The evolution equation (2) may be solved by a Monte Carlo algorithm. Here we will outline only the basic idea, deferring the full presentation and discussion to a forthcoming publication \[8\].

To determine the electron distribution $\psi(x, t)$, let us decompose it into a suitable set of time independent functions $\phi_n(x)$ using the ansatz

\[
\psi(x, t) = \sum_{n=0}^{\infty} \phi_n(x) \cdot \frac{(\nu t)^n}{n!} e^{-\nu t} ,
\]

where $\nu$ is a parameter that will be determined below. The normalization condition (1) translates into

\[
\int_0^1 dx \phi_n(x) = 1 .
\]

Plugging the ansatz (11) into the evolution equation (2) we obtain a recursion relation for the $\phi_n$,

\[
\phi_n(x) = \int_0^1 dx' \left[ \left(1 - \frac{\nu(x)}{\nu}\right) \delta(x - x') + \frac{F(x, x')}{\nu} \right] \phi_{n-1}(x') \quad \text{for } n \geq 1 ,
\]

with initial condition

\[
\phi_0(x) = \psi(x, 0) .
\]

If we neglect the intrinsic (Gaussian) energy spread of the accelerator, the initial condition of the electron distribution is simply given by

\[
\psi(x, 0) = \delta(1 - x) ,
\]
and thus

$$\phi_0(x) = \delta(1 - x) .$$  \hspace{1cm} (16)

The functions $\phi_n(x), n > 0$, can be determined, e.g., by numerical integration. Here we suggest a different way to exploit the recursion relations. First note that the kernel given by the square brackets in (13) is positive for $x' > x$. Furthermore, since $\nu(0) \geq \nu(x) \geq \nu(1)$, the kernel is positive if we choose $\nu = \nu(0) = \nu_{cl}$. The functions $\phi_n$ will then be positive, and together with the normalization condition (12) they will allow for a probabilistic interpretation.

Using our ansatz (11), the effective electron energy distribution (10) reads

$$\psi(x) = \sum_n \left[ 1 - \frac{\gamma(1 + n; N_{cl})}{n!} \right] \phi_n(x) ,$$  \hspace{1cm} (17)

where $\gamma(1 + n; N_{cl})$ is the incomplete gamma function, and $N_{cl} = \nu_{cl} \cdot l/2$ is the average number of photons per beam particle radiated during the entire collision, in the classical limit. Due to the positivity of the term in square brackets in (17) and the functions $\phi_n$, the electron energy distribution can be determined straightforwardly by a multichannel Monte Carlo method [8].

4 Results

Of the abundance of available parameter sets for future linear $e^+e^-$ colliders, we have taken a hopefully representative selection from [9] that comprises many of the proposed linear collider designs. Table 1 lists some of the parameters that are relevant for our numerical results. $\Upsilon_{\text{mean}}$ was determined similarly to [7].

We have calculated the differential luminosity of the $e^+e^-$ beams under beamstrahlung, eq. (9), using our Monte Carlo algorithm based on (13). The result of the simulation for the normalized luminosity is shown in figure 1. For the sake of brevity we have selected only a subset of four designs which essentially cover the interesting parameter range: the TESLA design with rather large bunch length but small effective beamstrahlung parameter and a correspondingly rather soft beamstrahlung spectrum, the JLC-S design with shorter bunches but large $\Upsilon$ and large mean energy loss due to a hard beamstrahlung spectrum, NLC with short bunches and moderate $\Upsilon$, having a small number of radiated photons $N_{cl}$, and VLEPP with a large multiplicity of beamstrahlung photons. We have also calculated the relative luminosity for collisions with $s > 0.99 s_0$, the result being displayed in the last line of table 1.
In high energy $e^+e^-$ processes, besides beamstrahlung one also has to take into account the radiative corrections to the process under consideration. The single most important universal contribution of the radiative corrections are the leading logarithms $\alpha^n \log^n (s/m_e^2)$, which are due to QED initial state radiation (ISR). These large logarithms can be resummed to all orders in the structure function approach, with the electron structure function [10]:

\[
D(z; \mu^2) = \frac{\beta}{2} (1 - z)^{\frac{\beta}{2} - 1} \left( 1 + \frac{3}{8} \beta \right) - \frac{1}{4} \beta (1 + z) + O(\beta^2),
\]

where

\[
\beta = \frac{2\alpha}{\pi} \left( \log \frac{\mu^2}{m_e^2} - 1 \right),
\]

$\mu^2$ is the factorization scale of order $s$, and $z$ is the electron energy fraction after initial state radiation.

As an application, we have interfaced our Monte Carlo routines for beamstrahlung with the Monte Carlo event generator WOPPER [11]. WOPPER simulates the process $e^+e^- \rightarrow (W^+W^-) \rightarrow 4f$ including QED radiative corrections in the abovementioned leading logarithmic approximation with explicit photons. In figure 2 we show the normalized invariant mass distribution $M_{WW}^2$ of the final state (assuming a 100% reconstruction efficiency for simplicity),

\[
\frac{1}{\sigma_0} \frac{d\sigma}{dM_{WW}^2} = \int dx_1 dx_2 d\epsilon_1 d\epsilon_2 \psi(x_1) \psi(x_2) D(z_1; \mu^2) D(z_2; \mu^2) \cdot \frac{1}{\sigma_0} \sigma_0(x_1 z_1 x_2 z_2),
\]

using the same collider designs as in figure 1. The thin continuous line represents the corresponding distribution from ISR alone. For events with a significantly reduced invariant mass the corrections due to beamstrahlung are numerically much larger than due
to pure ISR, and the shape of the distribution is essentially dominated by the beamstrahlung spectrum. This result clearly shows the importance of beamstrahlung and the need to include beamstrahlung into Monte Carlo event generators for physics at future $e^+e^-$ colliders.

References

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