Research on the Operational Modal Prediction of Dry Gas Seal System Based on Response Surface Method

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Abstract. The cross power spectrum function is used to realize the operational modal analysis and identification of the dry gas seal device system through the multi-reference point least squares complex frequency domain method. The steady state diagram and mathematical indicators MAC, MPD, MPC, MOV and MIF are used to verify the modal results. At the same time, based on the response surface method, with two different operating conditions of medium pressure and rotating speed, modal direction and modal order as the response surface variables, a time-varying modal recognition model is established. Through the Full Factorial experiment design, Box-Behnken experiment design and Central Composite experiment design, the suitable variable sample points are formed. A complete quadratic polynomial response surface model of the system operational modal parameters is established. The complex correlation coefficient, the modified complex correlation coefficient and the root mean square error are used to verify the effectiveness of the response surface model. It provides new method and technical support for realizing time-varying modal identification in this paper.

Keywords. Cross Power Spectra function; operational modal; modal verification; response surface model; experiment design; time-varying modal.

1. Introduction

Experimental Modal Analysis (EMA) is a modal analysis under artificial excitation. The excitation and response are known. However, for some complex mechanical structures (offshore platforms, bridges, large buildings, etc.), it is difficult to make them vibrate by artificial excitation [1, 2]. For some special devices, the working conditions are quite different from the experimental conditions. The complete system characteristics can only be formed in the operating state, which requires the response to be measured under the excitation of the natural operating environment to perform modal analysis to complete modal recognition, that is, Operational Modal Analysis (OMA) [3, 4]. Compared with the modal analysis of the experimental state, the modal analysis of the operating state does not require specific experimental conditions (such as simulated free support), and the excitation equipment (such as a hammer or vibration exciter) is not needed. The tested structure can be used normally to obtain the characteristics of its operating conditions, which greatly reduce the cost of the test, improve the efficiency of the test and the practicality of the test [5-7].
In this paper, the cross-power spectrum function of the output response is used to approximate the frequency response function, and the modal identification of the dry gas seal system is realized based on the multi-input and multi-output and multi-reference point least squares complex frequency domain method and the mathematical index is used for modal verification [8]. Using the mathematical indicators, the modal is verified. Based on a variety of experiment designs and response surface method, with different operating conditions (medium pressure \( p \) and rotating speed \( r \)), modal directions \( D \) and modal orders \( N \) as response surface variables, a complete quadratic polynomial operating modal response surface model of the modal parameters of the dry gas seal system is established under multiple operating conditions. The validity of the model is verified, which provides a new method for realizing time-varying modal identification.

2. Derivation Using Cross Power Spectrum Function Instead of Frequency Response Function

In practical engineering, when the modal analysis and parameter identification of the structural system is performed, it can be assumed that the structure has \( N \) order modes and \( L \) excitations meet the white noise stability condition, then the cross power spectrum \( G_{mn}(jw) \) of the point \( m \) and the point \( n \) on the structure can be expressed as

\[
G_{mn}(jw)=\sum_{p=1}^{L} \sum_{q=1}^{L} H_{mp}^* G_{pq}(jw) H_{nq}(jw) \tag{1}
\]

Among them, \( G_{pq}(jw) \) has nothing to do with the frequency and can be expressed by a constant \( C_{pq} \), then equation (1) can be written as

\[
G_{mn}(jw)=\sum_{p=1}^{L} \sum_{q=1}^{L} C_{pq} H_{mp}^* H_{nq}(jw) \tag{2}
\]

For the frequency response function is

\[
H_{mp}(jw)=\sum_{r=1}^{N} \left( a_{mp} \frac{a_{mp}^*}{jw-\lambda_r} + a_{mp}^* \frac{a_{mp}}{jw-\lambda_r^*} \right) \tag{3}
\]

in the equation \( a_{mp}^*Q_r \Phi_r^* \Phi_r^T \), \( \Phi_r^T \) is the \( r \) th element of the mode shape \( \Phi_r \). \( Q_r \) is the \( r \) th order normalization factor. \( \lambda_r \) and \( \lambda_r^* \) is a pair of conjugate eigenvalues of the structure.

Let \( G_{mn}^{pq}(jw)=C_{pq} H_{mp}^* H_{nq}(jw) \) available

\[
G_{mn}^{pq}(jw)=C_{pq} \sum_{r=1}^{N} \left( a_{mp}^* \frac{a_{mp}^*}{jw-\lambda_r} + a_{mp}^* \frac{a_{mp}}{jw-\lambda_r^*} \right) \tag{4}
\]

The above equation can be decomposed into

\[
G_{mn}^{pq}(jw)=C_{pq} \sum_{r=1}^{N} \sum_{s=1}^{N} \left( \begin{array}{cc}
\frac{a_{mp}^*}{jw-\lambda_s} & \frac{a_{mp}^*}{jw-\lambda_s} \\
\frac{a_{mp}^*}{jw-\lambda_s} & \frac{a_{mp}^*}{jw-\lambda_s} \\
\frac{a_{mp}^*}{jw-\lambda_s} & \frac{a_{mp}^*}{jw-\lambda_s}
\end{array} \right) \tag{5}
\]
Each product term\(\frac{a}{-jw-b}\frac{c}{jw-d}\) can be broken down into \(\frac{ac}{-(d-b)}\left[\frac{1}{jw-b}\frac{1}{jw-d}\right]\). Therefore, equation (5) is

\[
G_{mn}^o(jw) = \sum_{r=1}^{N} \left( \frac{A_{pq,mn}^o}{jw-\lambda_r} + \frac{A_{pq,mm}^o}{jw-\lambda_r} \right) + \sum_{r=1}^{N} \left( \frac{B_{pq,mm}^o}{jw-\lambda_r} + \frac{B_{pq,mn}^o}{jw-\lambda_r} \right)
\]

(6)

Considering that all excitation points are available

\[
G_{mn}(jw) = \sum_{r=1}^{N} \left( \frac{A_{pq,mn}}{jw-\lambda_r} + \frac{A_{pq,mm}}{jw-\lambda_r} \right) + \sum_{r=1}^{N} \left( \frac{B_{pq,mm}}{jw-\lambda_r} + \frac{B_{pq,mn}}{jw-\lambda_r} \right)
\]

(7)

\[
G_{mm}^*(jw) = \sum_{r=1}^{N} \left( \frac{B_{pq,mn}}{jw-\lambda_r} + \frac{B_{pq,mm}}{jw-\lambda_r} \right)
\]

(8)

In the equation

\[
A_{pq,mn}^o = Q_o \Phi_o \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} C_{pq} \sum_{r=1}^{N} \left( \frac{a_{pq,r}^{mn}}{\lambda_r - \lambda_q} + \frac{a_{pq,r}^{mm}}{\lambda_r - \lambda_q} \right) = \Phi_o D^o + B_{pq,mm}^o = Q_o \Phi_o \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} C_{pq} \sum_{r=1}^{N} \left( \frac{a_{pq,r}^{nn}}{\lambda_r - \lambda_q} + \frac{a_{pq,r}^{mm}}{\lambda_r - \lambda_q} \right) = \Phi_o D^m
\]

(9)

It can be seen that equation (7) is similar to equation (3). The poles contain frequency and damping information, where \(A_{pq,mn}^o\) is proportional to \(\Phi_o\) and \(B_{pq,mm}^o\) is proportional to \(\Phi_o^*\). Therefore, the response function in the experimental modal analysis can be replaced by the cross power spectrum function between the response reference points, and the modal parameters are obtained by using the response results alone.

3. Dry Gas Seal Vibration Experiment under Ambient Excitation

3.1. Test Condition

The dry gas seal device is a multi-component combined system device. During runtime, the rotating ring is nested in the shaft sleeve and rotates with the shaft. The high-pressure airflow is introduced into the seal groove to push the floating ring away, forming a high-pressure seal gas film. When the external environment or the device itself generates excitation, the excitation frequency will approach to the natural frequency of the system, and the system is likely to resonate, which will affect the reliability and stability of the seal. Due to the special working structure of the dry gas seal device, only the modal analysis of the environmental excitation operating state can be used to obtain the true modals of the dry gas seal [9].

There are 20 measuring points designed. 20 ICP acceleration sensors (model: 333B30, sensitivity: 100mV/g, frequency range: 0.5Hz-3kHz, range: 50g, weight: 3g) are evenly arranged on the seal machine in axial, radial and circumferential directions. The specific distribution is shown in figure 1. A simplified Geometry model of the whole machine is established to obtain the vibration mode.
Figure 1. Teat points layout.

The modal parameters of the dry gas seal system under different operating conditions (medium pressure \( p \) and rotating speed \( r \)) are tested. The axial, radial and circumferential test design is that when the rated medium pressure is 1 MPa, 2 MPa and 3 MPa, the vibration response of 20 test points is obtained under the test speed of 4000r/min, 6000r/min and 8000r/min operating conditions. The M+P Smart Office data acquisition system is used. Each sampling time is 300s, sampling frequency is 2048Hz, and sampling point is 4096 [10, 11].

3.2. Test Results

From the requirements and failure experience of dry gas seal conditions, the operating modal below 1000 Hz is studied. The cross power spectrum function between test points and multi-reference point least squares complex frequency domain method are used to identify each order modal [12]. Taking 1MPa-4000r/min as an example, the results of the steady-state diagram are shown in figure 2, figure 3, and figure 4, respectively.

Figure 2. 1 MPa-4000 r/min axial Cross Power Spectrum function steady state diagram.
Figure 3. 1 MPa-4000 r/min radial Cross Power Spectrum function steady state diagram.

![Radial Cross Power Spectrum Function](image1)

Figure 4. 1 MPa-4000 r/min circumferential Cross Power Spectrum function steady state diagram

![Circumferential Cross Power Spectrum Function](image2)

3.3. Test Verification

After completing the modal identification, modal screening is carried out according to whether the frequency value and mode shape is reasonable and correct, and then the modal assurance criterion (MAC), mean phase deviation (MPD), modal phase linearity (MPC), and modal complexity (MOV) and modal indicator function (MIF) are used for modal verification.

Taking the axial test result as an example, the test result removes the value of the excitation frequency related to the speed. In figure 2, 66.7 Hz, 133.6 Hz, 395.9 Hz and 600 Hz are the excitation frequencies and should be eliminated. The modal of 351.7 Hz(a) is to swing left and right along the axis, the modal of 547.8 Hz(b) is to swing back and forth along the axis, and the modal of 697 Hz(c) is to move up and down along the axis and swing left and right. As shown in Figure 5, the frequency value and the modal shape conform to the modal characteristics.

![Axial Vibration Mode](image3)

(a) (b) (c)

Figure 5. The axial vibration mode of dry gas seal under 1MPa-4000r/min working condition.
MAC correlation analysis of different order modals can be used to verify the modal results. The MAC correlation analysis result of three-order independence mode shape is within the acceptable range, as shown in figure 6.

![Figure 6](image)

**Figure 6.** The natural frequency MAC matrix from the Cross Power Spectra function.

Finally, MPD, MPC, MOV and MIF are used for final modal verification. As shown in table 1, MPD is less than 20, and results of MPC, MOV and MIF above 80% are acceptable results of engineering tests.

| $p-r$            | $D$ | $N$ | $f$/Hz | MP Dcp | MPC cp | MOV cp | MIF cp |
|------------------|-----|-----|--------|--------|--------|--------|--------|
| 1MPa-40 00r/min  | Axial |       | 351.7  | 21.6%  | 81.14% | 100%   | 84.44% |
|                  |       | First Order |          |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  | Radial |       | 547.8  | 6.31%  | 99.29% | 100%   | 96.99% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  | Circumferential | | 697    | 13.3%  | 93.99% | 100%   | 97.7%  |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       |           | 414.4  | 11.0%  | 94.35% | 100%   | 91.71% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       | Radial | 578.4  | 11.1%  | 96.25% | 100%   | 94.50% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       | Circumferential | | 766    | 17.1%  | 89.99% | 100%   | 86.88% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       |       | 81.3   | 17.6%  | 96.44% | 100%   | 94.94% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       | Circumferential | | 227.7  | 14.5%  | 91.87% | 100%   | 92.09% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       |       | 421    | 15.2%  | 93.47% | 100%   | 91.61% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       |       | 421    | 15.2%  | 93.47% | 100%   | 91.61% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 1MPa-40 00r/min  |       |       | 421    | 15.2%  | 93.47% | 100%   | 91.61% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 2MPa-40 00r/min  | Axial |       | 347.3  | 19.3%  | 85.05% | 100%   | 87.13% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 2MPa-40 00r/min  |       |       | 554.2  | 16.9%  | 83.37% | 100%   | 80.73% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
| 2MPa-40 00r/min  |       |       | 692    | 19.8%  | 95.1%  | 100%   | 85.13% |
|                  |       | First Order  |       |        |        |        |        |
|                  |       | Second Order |       |        |        |        |        |
|                  |       | Third Order  |       |        |        |        |        |
4. Establishment of Operational Modal Response Surface Model

4.1. Response Surface Model
According to the engineering experience, the response surface model is in the form of a complete quadratic polynomial [13, 14]. For s variables, the complete quadratic polynomial response surface model is

$$\tilde{y} = \alpha_0 + \sum_{j=1}^{s} \alpha_j x_j + \sum_{l=1}^{s} \sum_{j=l}^{s} \alpha_{lj} x_l x_j$$  \hspace{1cm} (10)

Among
\begin{itemize}
    \item $\alpha_0$ — Undetermined coefficient of constant term
    \item $\alpha_j$ — Undetermined coefficient of primary term
    \item $\alpha_{lj}$ — Undetermined coefficient of quadratic term
\end{itemize}
\[ x_0 = 1 \\
\begin{aligned}
  x_1 &= x_1, x_2 = x_2, \ldots, x_s = x_s \\
  x_{s+1} &= x_1^2, x_{s+2} = x_2^2, \ldots, x_{2s} = x_s^2 \\
  x_{2s+1} &= x_1 x_2, x_{2s+2} = x_1 x_3, \ldots, x_{(s+1)s/2} = x_{s-1} x_s
\end{aligned} \quad (11) \\
\beta_0 &= \alpha_0 \\
\beta_1 &= \alpha_1, \beta_2 = \alpha_2, \ldots, \beta_s = \alpha_s \\
\beta_{s+1} &= \alpha_{s+1}, \beta_{s+2} = \alpha_{s+2}, \ldots, \beta_{2s} = \alpha_{2s} \\
\beta_{2s+1} &= \alpha_{2s+1}, \beta_{2s+2} = \alpha_{2s+2}, \ldots, \beta_{(s+1)^2/2} = \alpha_{(s+1)^2/2} \quad (12) \\
\]

In the equation \( X_v = (x_1, x_2, \ldots, x_s) \), \( x_i \) \((i = 1, 2, \ldots, s)\) is the design variable, \( \beta_k \) is an unknown coefficient, the number of which \( k = (s + 1)(s + 2)/2 \), Therefore \( \beta = (\beta_0, \beta_1, \ldots, \beta_{k-1})^T \). Using the least squares to determine the unknown coefficient \( \beta_k \), the number of independent tests \( t \) is not less than \( k \), \( t \geq k \).

4.2. The Design of Experiments

The design of experiments (DOE) is to select the sample points according to certain criteria, so that only a small number of points can make the approximate response function reach a higher accuracy. Different operating conditions (medium pressure \( p \) and rotating speed \( r \), modal direction and modal order) are used as response surface variables to carry out experiment design.

The Box-Behnken experiment design can be used for experiments with the number of variables between 3-5, which can provide more complete information about the experimental variables with fewer experiment cycles. The Box-Behnken experimental design does not have axial points. Therefore, in actual design, its level setting will not exceed the operating range, as shown in table 2.

Full Factorial Design is to combine the different levels of each factor to form different experimental conditions. This experiment design can get a lot of information and estimate the size of the main effect of each experimental factor, as well as the size of the interaction effect between the factors at all levels, as shown in table 3.

Central Composite Design is an experimental design that combines interpolation node distribution with full factorial or partial factorial. It is also more suitable for the experimental design of quadratic response surface models. It not only has full factor or partial factor experimental points, but also has designed experimental points located in the center of the design space. This experimental design is a straight line that passes through the center point and is parallel to each coordinate axis. This experimental design is that the design point is taken at the distance of \( \pm \delta \) on a straight line that passes through the center point and is parallel to each coordinate axis. The value of \( \delta \) is generally 1 or \( \sqrt{F} \). \( F \) is the number of sample points, as shown in table 4.

4.3. Regression Analysis and Prediction Analysis

After completing the approximate model, the predictive ability of the response surface should be evaluated, that is, regression analysis.

Suppose \( X = \begin{bmatrix} 1 & x_1^{(0)} & x_2^{(0)} & \cdots & x_{k-1}^{(0)} \\ 1 & x_1^{(1)} & x_2^{(1)} & \cdots & x_{k-1}^{(1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_1^{(t-1)} & x_2^{(t-1)} & \cdots & x_{k-1}^{(t-1)} \end{bmatrix} \) is \( t \) sample points.
\[
\begin{align*}
\mathbf{y} = \begin{bmatrix}
y^{(0)} \\
y^{(1)} \\
\vdots \\
y^{(t-1)} 
\end{bmatrix}
\text{ is the corresponding response value.}
\end{align*}
\]

\[
\begin{align*}
\hat{y}^{(0)} &= \sum_{i=0}^{k-1} \beta_i x_i^{(0)} \\
\hat{y}^{(1)} &= \sum_{i=0}^{k-1} \beta_i x_i^{(1)} \\
\vdots \\
\hat{y}^{(t-1)} &= \sum_{i=0}^{k-1} \beta_i x_i^{(t-1)}
\end{align*}
\]

\[
\text{is the response surface function value.}
\]

Let \( S(\beta) = \sum_{j=0}^{t-1} \left( e^{(j)} \right)^2 = \sum_{j=0}^{t-1} \left( \sum_{i=0}^{k-1} \beta_i x_i^{(j)} - y^{(j)} \right)^2 \rightarrow \min 
\]

The necessary conditions for taking a minimum is

\[
\frac{\partial S}{\partial \beta_i} = 2 \sum_{j=0}^{t-1} x_i^{(j)} \left( \sum_{i=0}^{k-1} \beta_i x_i^{(j)} - y^{(j)} \right) = 0 \quad (i = 0, \ldots, k-1)
\]

which is:

\[
(X \beta - \bar{y})^T X = 0
\]

So,

\[
\beta = \left( X^T X \right)^{-1} X^T \bar{y}
\]

The common evaluation indicators which can be used to evaluate the quality of the response surface regression are

\[
R^2 \quad \text{— multiple correlation coefficient}
\]

\[
R^2_{adj} \quad \text{— Modified multiple correlation coefficient}
\]

\[
\sigma \quad \text{— Root mean square difference}
\]

\[
\text{SSE — Sum of squares of the difference between the response value and the response estimate}
\]

\[
\text{SSY — Sum of squares of the difference between the response value and the response mean}
\]

\[
R^2 = 1 - \frac{\text{SSE}}{\text{SSY}}
\]

\[
\text{SSE} = \sum_{i=1}^{t} \left( y_i - \hat{y}_i \right)^2
\]

\[
\text{SSY} = \sum_{i=1}^{t} \left( y_i - \bar{y} \right)^2
\]

\[
R^2_{adj} = 1 - \left( \frac{t-1}{t-k} \right) \frac{\text{SSE}}{\text{SSY}}
\]
\[ \sigma = \frac{1}{ty} \sqrt{n \sum_{i=1}^{n} (y_i - \bar{y})^2} \]  

(20)

The closer the value of \( R^2 \) is to 1, it means that the regression equation can accurately describe the change of \( y \). \( R_{adj}^2 \) considers the influence of the number of parameters \( k \). If there is a big difference between \( R^2 \) and \( R_{adj}^2 \), it indicates that there are unimportant parameters in the response surface approximation model. The closer the value of \( \sigma \) is to 0, the smaller the error between the response value and the response estimate.

| Number | \( P \) | \( D \) | \( r \) | \( N \) |
|--------|--------|--------|--------|--------|
| 1      | 0      | 0      | 0      | 0      |
| 2      | -1     | -1     | -1     | -1     |
| 3      | 1      | -1     | 0      | 0      |
| 4      | -1     | 1      | 0      | 0      |
| 5      | 1      | 1      | 0      | 0      |
| 6      | 0      | 0      | -1     | -1     |
| 7      | 0      | 0      | 1      | -1     |
| 8      | 0      | 0      | -1     | 1      |
| 9      | 0      | 0      | 1      | 1      |
| 10     | -1     | 1      | -1     | 1      |
| 11     | 1      | 0      | 0      | -1     |
| 12     | -1     | 0      | 0      | 1      |
| 13     | 1      | 0      | 0      | 0      |
| 14     | 0      | -1     | -1     | 0      |
| 15     | 0      | 1      | -1     | 0      |
| 16     | 0      | -1     | 1      | 0      |
| 17     | 0      | 1      | 1      | 0      |
| 18     | -1     | -1     | -1     | 1      |
| 19     | 1      | 0      | -1     | 0      |
| 20     | -1     | 0      | -1     | 1      |
| 21     | 1      | 0      | 1      | 0      |
| 22     | 0      | -1     | 0      | -1     |
| 23     | 0      | 1      | 0      | -1     |
| 24     | 0      | -1     | 0      | 1      |
| 25     | 0      | 1      | 0      | 1      |
| 26     | -1     | 1      | -1     | -1     |
| 27     | 1      | 1      | 1      | -1     |

From the experiment design results of table 2, table 3 and table 4, it can be seen that the Box-Behnken experiment design and the Full Factorial experiment design can meet the distribution of 4 response variables of medium pressure, rotating speed, modal direction and modal order. Due to the existence of axial points, there are designed points in table 4 that exceed the original level requirements in the Central Composite experiment design, so the Central Composite experiment
design is not suitable for the establishment of a complete quadratic polynomial response surface model of the system operational modal parameters.

Table 5 shows the complete quadratic response surface model of the Box-Behnken experiment design and the Full Factorial experiment design and its corresponding evaluation indicators. The overall accuracy of the response surface model meets the requirements, but the complete quadratic response surface model obtained by the Box-Behnken experiment design the response surface model evaluation index is better than the response surface model evaluation index of the Full Factorial experiment design. 9 sets of modal prediction results are randomly selected. The comparison between the natural frequency experimental value \( f_e \) and the predicted value \( f_\beta \) is shown in table 6 [15].

| \( f_\text{Box-Behnken} \) | \( f_\text{FullFactorial} \) |
|-------------------------|--------------------------|
| \(-39.1007 + 58.7259x_1 + 45.8831x_2 - 18.7948x_3 - 5.0769x_4 \) | \(-23.8292 + 14.8953x_1 + 55.6513x_2 - 12.1738x_3 - 2.5425x_4 \) |
| \(-32.6723x_1^2 - 23.3768x_2^2 - 1.6758x_3^2 - 3.8130x_4^2 \) | \(-6.8001x_1^2 - 30.3007x_2^2 + 5.3249x_3^2 + 1.1125x_4^2 \) |
| \(-10.4554x_1x_2 + 16.5489x_1x_3 + 2.6257x_1x_4 \) | \(+ 0.35x_1x_2 - 0.1x_1x_3 + 0.55x_1x_4 \) |
| \(+ 4.3029x_2x_3 + 1.5638x_2x_4 + 4.1721x_3x_4 \) | \(-2.5875x_3x_4 + 0.9x_2x_4 + 1.4875x_3x_4 \) |

| \( \rho \) | \( R^2 \) | \( R^2_{\text{adj}} \) | \( \sigma \) |
|---------|-------|----------|------|
| \( f_\text{Box-Behnken} \) | 0.98  | 0.99     | 0.001|
| \( f_\text{FullFactorial} \) | 0.96  | 0.97     | 0.005|

Table 6. Prediction results.

| \( p - D - r - N \) | \( f_e / \text{Hz} \) | \( f_\beta / \text{Hz} \) | Error |
|-------------------|------------------|------------------|------|
| 1-Axial-4 000-2   | 547.8            | 547              | 0.2% |
| 2-Axial-4 000-1   | 347.3            | 364              | 4.8% |
| 3-Radial-8 000-3  | 962              | 971              | 1%   |
| 1-Radial-6 000-2  | 682.4            | 678              | 0.7% |
| 2-Radial-6 000-3  | 880              | 876              | 0.5% |
| 1-Circumferential-6 000-1 | 82.8   | 83.3            | 0.6% |
| 2-Axial-6 000-2   | 586              | 594              | 1.5% |
| 3-Radial-4 000-1  | 416              | 410              | 1.4% |
| 3-Circumferential-8 000-1 | 81     | 88              | 8.6% |

5. Conclusion

Based on the multi-reference point least squares complex frequency domain method, the cross power spectrum function instead of the frequency response function can be used to realize the analysis and identification of the operational modal of the dry gas sealing device system. The accuracy of the verification result is verified through mathematical indicators. Based on a variety of experimental designs and response surface method, a time-varying modal parameter identification model can be established. Through the comparison of model evaluation indicators, the Box-Behnken experimental design is most suitable to establish a complete quadratic polynomial of the system operational modal parameters. The response surface model realizes the prediction of the natural frequency of the operational modal, and provides a new technical solution for the identification of the time-varying mode.

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