The Renormalization-Group Improved Higgs Sector of the Minimal Supersymmetric Model

HOWARD E. HABER AND RALF HEMPFLING

Santa Cruz Institute for Particle Physics
University of California, Santa Cruz, CA 95064

Abstract

In the minimal supersymmetric model (MSSM) all Higgs self-coupling parameters are related to gauge couplings at tree-level. Leading-logarithmic radiative corrections to these quantities can be summed using renormalization group techniques. By this procedure we obtain complete leading-log radiative corrections to the Higgs masses, the CP-even Higgs mixing angle, and trilinear Higgs couplings. Additional corrections due to squark mixing can be explicitly incorporated into this formalism. These results incorporate nearly all potentially large corrections. Mass shifts to the neutral CP-even Higgs bosons grow with the fourth power of the top-quark mass and can be significant. The phenomenological consequences of these results are examined.

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1. Introduction

Supersymmetry is one of the most promising theoretical ideas that attempts to explain the origin of the scale of electroweak interactions. The minimal supersymmetric extension of the Standard Model (MSSM) is the most economical among models of this type [1], and deserves close examination as a candidate for a model of physics beyond the Standard Model (SM). In the MSSM, one simply adds a supersymmetric partner to every quark, lepton and gauge boson. In addition, the MSSM must possess two Higgs doublets in order to give masses to up and down type fermions in a manner consistent with supersymmetry (and to avoid gauge anomalies introduced by the fermionic superpartners of the Higgs bosons). The Higgs sector of the MSSM is greatly constrained by supersymmetry [2]. All quartic Higgs coupling constants are related to electroweak gauge coupling constants, which imposes various restrictions on the tree-level Higgs masses and couplings. In particular, all tree-level Higgs parameters can be expressed in terms of one physical Higgs mass and the ratio of vacuum expectation values, \( \tan \beta \equiv v_2/v_1 \).

Any realistic supersymmetric model must incorporate supersymmetry breaking in the low-energy theory. This breaking is parametrized by adding soft supersymmetry breaking mass terms for the squarks, sleptons and gauginos, and trilinear Higgs-squark-squark and Higgs-slepton-slepton interactions which are proportional to the so-called \( A \) parameters [3]. Due to these supersymmetry breaking terms, one finds that the tree-level relations among Higgs masses and couplings acquire radiative corrections. It has been shown that these corrections can indeed become very substantial if the top quark mass is much larger than \( m_Z \) [4-20]. For example, the tree-level bound, \( m_{h^0} \leq m_Z \) receives a radiative correction of order \( g_2^2 m_t^4/m_Z^2 \ln(M_{\tilde{t}}^2/m_t^2) \) which raises the upper limit of \( m_{h^0} \) by as much as 20 (50) GeV for a top-quark mass of \( m_t = 150 \) (200) GeV [5]. The logarithmic enhancement factor is a remnant of the cancellation of divergences generated by virtual particles (in this case the top quark and its supersymmetric partners). Similar logarithmic corrections also arise when contributions from other sectors of the theory are incorporated.

While the exact one-loop radiative corrections to mass sum rules and Higgs mass bounds can be obtained in a straightforward manner, the radiative corrections to individual CP-even Higgs masses and Higgs interactions are far more complex. For instance, computations of the latter type require a careful definition of the parameter \( \tan \beta \). However, significant simplification can be achieved if we include only the leading logarithmic radiative corrections. In this approximation, the definitions of \( \tan \beta \) and the CP-even Higgs mixing angle \( \alpha \) are unambiguous and can easily be related to physical observables. The goal of this paper is to
construct a low-energy effective Lagrangian from which one can directly obtain the leading contributions to the radiatively corrected Higgs masses and couplings.

One possible procedure for achieving this goal is to compute the terms of the full one-loop effective action that depend on the scalar Higgs fields. This requires the computation of the effective potential, $V_{\text{eff}}$, the coefficient of the scalar kinetic energy term, $Z_{\text{eff}}$, and higher derivative terms. Let us assume that the supersymmetry breaking scale ($M_{\text{SUSY}}$) is somewhat higher than the electroweak breaking scale. Here, we use $M_{\text{SUSY}}$ to denote the mass scale that characterizes supersymmetry breaking. (For now, we ignore the possibility of multiple supersymmetric thresholds, which will be addressed later in this paper.) If one expands the effective action about the Higgs vacuum expectation value and discards all terms of order $m_Z^2/M_{\text{SUSY}}^2$, then it suffices to keep only those terms of dimension 4 or less. Note that at this stage, it is not strictly correct to use the effective potential to compute the one-loop Higgs masses and couplings. One must first rescale the scalar fields (i.e., wave function renormalization) in order that the scalar kinetic energy terms are canonical. The end result is a scalar potential that is polynomial in the scalar fields (with terms of dimension 4 or less), whose coefficients reflect the one-loop radiative corrections. The leading one-loop correction terms will depend logarithmically on $M_{\text{SUSY}}$, which suggests that one can use renormalization group methods to explicitly identify the leading logarithmic terms [7,8,13,19]. In this paper, we shall employ the renormalization group method for two reasons: (i) simplicity, and (ii) the integration of the one-loop renormalization group equations (RGEs) effectively sums the leading logarithmic contributions to all orders in perturbation theory. This method can be extended to include the effects of supersymmetric thresholds. In this paper, we derive RGEs that incorporate terms which are logarithmic in the ratio of threshold masses to one-loop. However, to sum such effects to all orders significantly complicates the analysis and is beyond the scope of this paper.

In addition to neatly summarizing the most important radiative corrections, the direct identification of the terms logarithmic in $M_{\text{SUSY}}$ provides an important check for a more precise (and hence more complicated) explicit one-loop computation. Moreover, the full RGE analysis yields leading log terms to all orders in perturbation theory, and hence provides some information on the contributions that lie beyond the one-loop approximation [10]. This provides a check on the reliability of the one-loop results. However, it is important to realize that the

* If $M_{\text{SUSY}}$ is roughly equal to the scale of electroweak symmetry breaking, then the size of the radiative corrections discussed in this paper will be rather small. In this case, one will need exact one-loop computations to determine reliably the effects of the radiative corrections.
renormalization group technique may not detect all significant terms of the one-loop radiative corrections. For example, if the $A$-terms that control squark mixing are large, then radiative correction terms of order $A^2/M_{\text{SUSY}}^2$ may be enhanced by powers of the top-quark mass \cite{9,12,17,18,20}. Such terms can then compete with (and may be more important than) the leading log terms identified above. Thus, an important goal of this paper is to demonstrate how to include such terms within the renormalization group approach in a consistent manner. The end result of our work is a simple and powerful technique that includes in a transparent fashion the most important radiative corrections to the MSSM Higgs sector.

In section 2, we first examine the general non-supersymmetric two Higgs doublet model at tree-level. The most general two-Higgs-doublet model potential depends on three mass parameters and seven dimensionless couplings. The Higgs mass matrices and three-point Higgs vertices are obtained in terms of these parameters. In section 3, we specialize to the MSSM. Radiative corrections to the Higgs sector parameters are obtained by renormalization group evolution. Supersymmetry implies definite relations among the Higgs self-couplings in terms of the gauge couplings. These relations are used as boundary conditions for the renormalization group equations at a scale $M_{\text{SUSY}}$ which characterizes the scale of supersymmetric particle masses. The Higgs couplings are then evolved according to renormalization group equations (RGEs) and the radiatively corrected Higgs masses and couplings are computed in terms of parameters at the electroweak scale. In the evolution of couplings, we first assume that the effective low-energy theory at the electroweak scale contains two light Higgs doublets. Another possibility is to assume that the second Higgs doublet is significantly heavier than the electroweak scale. In this case the low-energy effective theory is identical to the SM with one physical Higgs boson. This case is discussed section 4. In our analysis, an important parameter is $\tan \beta$, the ratio of vacuum expectation values. The relation of $\tan \beta$ to physically measurable observables is elucidated in section 5.

The analysis described above is equivalent to summing the leading log radiative corrections to all orders in perturbation theory. This approximation summarizes almost all potentially large radiative corrections. However, if squark mixing effects are substantial, an additional set of non-logarithmic corrections may be important. We incorporate these terms into our analysis in section 6. Numerical results and their phenomenological implications are given in section 7. The importance of using RGE-improved results for the Higgs masses is subject of section 8. Final conclusions are given in section 9. Details of the RGE analysis and the incorporation of important non-leading log terms are relegated to four appendices.

Some results of this paper were first described in ref. \cite{13} and used in the phenomenological analyses presented in refs. \cite{21–23}. Additional details can be
found in ref. [24].

2. The General Two-Higgs-Doublet Model

We begin with a brief review of the general (non-supersymmetric) two-Higgs doublet extension of the Standard Model [2]. Let $\Phi_1$ and $\Phi_2$ denote two complex $Y = 1$, SU(2)$_L$ doublet scalar fields. We introduce the notation

$$\Phi_n = \left( \begin{array}{c} H_n^+ \\ (H_n^0 + iA_n^0)/\sqrt{2} \end{array} \right).$$

(2.1)

The most general gauge invariant scalar potential is given by

$$V = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}]$$
$$+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$
$$+ \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + [\lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2)] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}. \quad (2.2)$$

In most discussions of two-Higgs-doublet models, the terms proportional to $\lambda_6$ and $\lambda_7$ are absent. This can be achieved by imposing a discrete symmetry $\Phi_1 \rightarrow -\Phi_1$ on the model. Such a symmetry would also require $m_{12} = 0$ unless we allow a soft violation of this discrete symmetry by dimension-two terms. For the moment, we will refrain from setting any of the coefficients in eq. (2.2) to zero. In principle, $m_{12}^2$, $\lambda_5$, $\lambda_6$ and $\lambda_7$ can be complex. In this paper, we shall ignore the possibility of CP-violating effects in the Higgs sector by choosing all coefficients in eq. (2.2) to be real. The fields will develop non-zero vacuum expectation values (VEVs) if the mass matrix $m_{ij}^2$ has at least one negative eigenvalue. Imposing CP invariance and U(1)$_{\text{EM}}$ gauge symmetry, the minimum of the potential is

$$\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_1 \end{array} \right), \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2 \end{array} \right),$$

(2.3)

where the $v_i$ can be chosen to be real. The VEVs have been normalized so that $m_W^2 = \frac{1}{4} g_2^2 (v_1^2 + v_2^2)$. It is convenient to introduce the following notation:

$$v^2 \equiv v_1^2 + v_2^2, \quad t_\beta \equiv \tan \beta \equiv v_2/v_1. \quad (2.4)$$

The gauge symmetry will then be broken spontaneously. As a result three of the eight degrees of freedom of the original Higgs doublets are eaten by the $W^\pm$ and $Z$.

* This latter requirement is sufficient to guarantee the absence of Higgs-mediated tree-level flavor changing neutral currents.
The remaining five physical Higgs particles are: two CP-even scalars ($H^0$ and $h^0$, with $m_{h^0} \leq m_{H^0}$), one CP-odd scalar ($A^0$) and a charged Higgs pair ($H^\pm$). The mass parameters $m_{11}$ and $m_{22}$ can be eliminated by imposing the minimization conditions

\begin{align}
  m_{11}^2 - t_\beta m_{12}^2 &+ \frac{1}{2} v^2 c_\beta^2 \left( \lambda_1 + 3 \lambda_6 t_\beta + \tilde{\lambda}_3 t_\beta^2 + \lambda_7 t_\beta^3 \right) = 0, \\
m_{22}^2 - t_\beta^{-1} m_{12}^2 &+ \frac{1}{2} v^2 s_\beta^2 \left( \lambda_2 + 3 \lambda_7 t_\beta^{-1} + \tilde{\lambda}_3 t_\beta^{-2} + \lambda_6 t_\beta^{-3} \right) = 0,
\end{align}

where we have introduced the following abbreviations: $s_\beta \equiv \sin \beta$ and $c_\beta \equiv \cos \beta$ and

$$
\tilde{\lambda}_3 \equiv \lambda_3 + \lambda_4 + \lambda_5.
$$

It then follows that the mass matrices of the CP-even, CP-odd and the charged scalars are given by

\begin{align}
M_{A^0}^2 &= \begin{bmatrix} m_{12}^2 & \frac{1}{2} v^2 (2 \lambda_5 s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2) \end{bmatrix} \begin{pmatrix} t_\beta & -1 \\ -1 & t_\beta^{-1} \end{pmatrix}, \\
M_{H^\pm}^2 &= \begin{bmatrix} m_{12}^2 & \frac{1}{2} v^2 (\lambda_4 s_\beta c_\beta + \lambda_5 s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2) \end{bmatrix} \begin{pmatrix} t_\beta & -1 \\ -1 & t_\beta^{-1} \end{pmatrix}, \\
M_{H^0}^2 &= m_{12}^2 \begin{pmatrix} t_\beta & -1 \\ -1 & t_\beta^{-1} \end{pmatrix} + \frac{1}{2} v^2 s_\beta c_\beta \begin{pmatrix} 2 \lambda_1 t_\beta^{-1} + 3 \lambda_6 - \lambda_7 t_\beta^2 & -2 \tilde{\lambda}_3 + 3(\lambda_6 t_\beta^{-1} + \lambda_7 t_\beta) \\ 2\tilde{\lambda}_3 + 3(\lambda_6 t_\beta^{-1} + \lambda_7 t_\beta) & 2 \lambda_2 t_\beta + 3 \lambda_7 - \lambda_6 t_\beta^{-2} \end{pmatrix}.
\end{align}

The first two mass matrices possess a zero eigenvalue corresponding to the Goldstone bosons ($G^0, G^\pm$). The masses of the physical Higgs particles are given by

\begin{align}
m_{A^0}^2 &= \text{tr}(M_{A^0}^2) = m_{12}^2 - \frac{1}{2} v^2 (2 \lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta), \\
m_{H^\pm}^2 &= \text{tr}(M_{H^\pm}^2) = m_{12}^2 - \frac{1}{2} v^2 (\lambda_4 + \lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta).
\end{align}

If we now substitute the remaining mass parameter $m_{12}^2$ in favor of $m_{A^0}^2$ we find the following expressions for the charged Higgs mass and the neutral CP-even Higgs boson mass matrix

\begin{equation}
m_{H^\pm}^2 = m_{A^0}^2 + \frac{1}{2} v^2 (\lambda_5 - \lambda_4),
\end{equation}

\begin{equation}
m_{H^0}^2 = \text{tr}(M_{H^0}^2) = m_{12}^2 - \frac{1}{2} v^2 (\lambda_4 + \lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta).
\end{equation}
\[ M_{H^0}^2 = m_{A^0}^2 \left( \begin{array}{cc} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{array} \right) \]
\[ + v^2 \left( \begin{array}{cc} \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ (\lambda_3 + \lambda_4) s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \end{array} \right). \] (2.10)

The CP-even Higgs mass eigenvalues are given by

\[ m_{H^0, h^0}^2 = \frac{1}{2} \left\{ \text{tr} M^2 \pm \sqrt{[\text{tr} M^2]^2 - 4 \text{det} M^2} \right\}, \] (2.11)

where \( M^2 \equiv M_{H^0}^2 \) and the mixing angle \( \alpha \) is obtained from

\[ \sin 2\alpha = \frac{2M_{12}^2}{\sqrt{[\text{tr} M^2]^2 - 4 \text{det} M^2}}, \]
\[ \cos 2\alpha = \frac{M_{11}^2 - M_{22}^2}{\sqrt{[\text{tr} M^2]^2 - 4 \text{det} M^2}}. \] (2.12)

The phenomenology of the two-Higgs doublet model depends in detail on the various couplings of the Higgs bosons to gauge bosons, Higgs bosons and fermions. The Higgs couplings to gauge bosons follow from gauge invariance and are thus model independent. Most of these couplings are proportional to either \( \sin(\beta - \alpha) \) or \( \cos(\beta - \alpha) \). In contrast, the Higgs couplings to fermions are model dependent, although their form is often constrained by discrete symmetries that are imposed in order to avoid tree-level flavor changing neutral currents mediated by Higgs exchange [25]. An example of a model that respects this constraint is one in which one Higgs doublet (before symmetry breaking) couples exclusively to down-type fermions and the other Higgs doublet couples exclusively to up-type fermions. This is the pattern of couplings found in the minimal supersymmetric model (MSSM). Detailed Feynman rules can be found in ref. [2]. Finally, the 3-point and 4-point Higgs self-couplings depend on the two-Higgs-doublet potential [eq. (2.2)]. The Feynman rules for the most important trilinear Higgs vertices are listed below:
\[ g_{h^0 A^0}^{A^0} = \frac{2m_W}{g_2} \left[ \lambda_1 s_\beta^2 c_\beta s_\alpha - \lambda_2 c_\beta^2 s_\beta c_\alpha - \lambda_3 (s_\beta^3 c_\alpha - c_\beta^3 s_\alpha) + 2\lambda_5 s_\beta - \alpha \right. \\
\left. - \lambda_6 s_\beta (c_\beta s_\alpha + s_\alpha c_\beta) + \lambda_7 (c_\alpha c_2 + s_\alpha c_2) \right], \]

\[ g_{H^+A^-}^{A^0} = -\frac{2m_W}{g_2} \left[ \lambda_1 s_\beta^2 c_\beta c_\alpha + \lambda_2 c_\beta^2 s_\beta s_\alpha + \lambda_3 s_\beta^3 s_\alpha - \lambda_4 s_\alpha^3 c_\beta - \frac{2}{3} c_\beta - \alpha \right. \\
\left. - \lambda_6 s_\alpha (c_\beta s_2 + c_\alpha c_2) + \lambda_7 c_\alpha (s_\beta c_2 + s_\alpha c_2) \right], \]

\[ g_{H^0 H^0} = \frac{-6m_W}{g_2} \left[ \lambda_4 c_\beta c_\alpha + \lambda_2 c_\beta^2 s_\beta s_\alpha + \lambda_3 (s_\alpha^3 c_\beta + c_\alpha c_\beta^3 - \frac{2}{3} c_\beta - \alpha) \right. \\
\left. - \lambda_6 s_\alpha (c_\beta c_2 + c_\alpha c_2 + s_\alpha c_2) + \lambda_7 c_\alpha (s_\beta c_2 + s_\alpha c_2) \right], \]

\[ g_{H^0 H^+ H^-} = \frac{2m_W}{g_2} (\lambda_5 - \lambda_4) c_\beta - \alpha, \]

\[ g_{H^0 H^+ H^-} = \frac{2m_W}{g_2} (\lambda_5 - \lambda_4) s_\beta - \alpha. \]

(In our notation, \( g_1 \equiv g' \) and \( g_2 \equiv g \).) It is interesting to note that couplings of the charged Higgs bosons satisfy relations analogous to that of \( m_{H^\pm} \) given in eq. (2.9).

At present, some experimental constraints on the parameters of the two-Higgs doublet model have been obtained at LEP. Here we briefly summarize the results for the Higgs search as compiled by the Particle Data Group [26]. For the charged Higgs boson, \( m_{H^\pm} > 41.7 \) GeV. This is the most model independent bound and assumes only that the \( H^\pm \) decays dominantly into \( \tau^+ \nu_\tau, c\bar{s} \) and \( c\bar{b} \). The LEP limits on the masses of \( h^0 \) and \( A^0 \) are obtained by searching simultaneously for \( Z \to h^0 f \bar{f} \) and \( Z \to h^0 A^0 \) [27,28]. The \( Z Z h^0 \) and \( Z h^0 A^0 \) couplings which govern these two decay rates are proportional to \( \sin(\beta - \alpha) \) and \( \cos(\beta - \alpha) \), respectively. Thus, one can use the LEP data to deduce limits on \( m_{h^0} \) and \( m_{A^0} \) as a function of \( \sin(\beta - \alpha) \) [28]. Stronger limits can be obtained in the MSSM where \( \sin(\beta - \alpha) \) is fixed by other model parameters. The present limits as summarized by the Particle Data Group [26] are \( m_{h^0} > 29 \) GeV and \( m_{A^0} > 12 \) GeV based on supersymmetric tree-level relations among Higgs parameters, but with no assumption for the value of \( \tan \beta \). If leading log radiative corrections are incorporated and \( \tan \beta > 1 \) is assumed, then recent results of the ALEPH Collaboration yield \( m_{h^0} > 41 \) GeV and \( m_{A^0} > 20 \) GeV (at 95% CL). However, as was shown in ref. [29] (and will be discussed briefly in section 7), the limit on \( m_{h^0} \) may be substantially weaker if large squark mixing is permitted.

The experimental information on the parameter \( \tan \beta \) is quite meager. For definiteness, we shall assume that the Higgs-fermion couplings are specified as in the MSSM. In the Standard Model, the Higgs coupling to top quarks is proportional
to $g_2m_t/2m_W$, and is therefore the strongest of all Higgs-fermion couplings. For $\tan \beta < 1$, the Higgs couplings to top-quarks in the two-Higgs-doublet model discussed above are further enhanced by a factor of $1/\tan \beta$. As a result, some weak experimental limits on $\tan \beta$ exist based on the non-observation of virtual effects involving the $H^-t\bar{b}$ coupling. Clearly, such limits depend both on $m_{H^\pm}$ and $\tan \beta$. For example, for $m_{H^\pm} \approx m_W$, limits from the analysis of $B^0\bar{B}^0$ mixing imply that $\tan \beta \gtrsim 0.5$ [30]. No comparable limits exist based on top-quark couplings to neutral Higgs bosons.

Theoretical constraints on $\tan \beta$ are also useful. If $\tan \beta$ becomes too small, then the Higgs coupling to top quarks becomes strong. In this case, the tree-unitarity of processes involving the Higgs-top quark Yukawa coupling is violated. Perhaps this should not be regarded as a theoretical defect, although it does render any perturbative analysis unreliable. A rough lower bound advocated by ref. [30], $\tan \beta \gtrsim m_t/600 \text{ GeV}$, corresponds to a Higgs-top quark coupling in the perturbative region. A similar argument involving the Higgs-bottom quark coupling would yield $\tan \beta \lesssim 120$. A more solid theoretical constraint is based on the requirement that Higgs–fermion couplings remain finite when running from the electroweak scale to some large energy scale $\Lambda$ [31,32]. Beyond $\Lambda$, one assumes that new physics enters. The limits on $\tan \beta$ depend on $m_t$ and the choice of the high energy scale $\Lambda$ [13,31,32]. For example, if there is no new physics (other than perhaps minimal supersymmetry) below the grand unification scale of $10^{16}$ GeV, then based on the CDF limit [33] of $m_t > 91 \text{ GeV}$, one would conclude that $0.5 \lesssim \tan \beta \lesssim 50$. Finally, it is interesting to note that these limits on $\tan \beta$ are not very different from those that emerge from models of low-energy supersymmetry based on supergravity which strongly favor $\tan \beta > 1$ [34].

3. The Radiatively Corrected Higgs Sector of the MSSM

In a general two-Higgs-doublet model none of the relations derived in section 2 are very predictive due to the large number of unknown parameters. However, in the MSSM, supersymmetry (SUSY) implies constraints among these parameters, thereby leading to numerous predictions for Higgs masses and coupling constants in terms of a few basic model parameters.

Consider first the case of unbroken SUSY. Here all the Higgs self-coupling
constants are related to the gauge coupling constants

\[
\begin{align*}
\lambda_1 &= \lambda_2 = \frac{1}{4}(g_2^2 + g_1^2), \\
\lambda_3 &= \frac{1}{4}(g_2^2 - g_1^2), \\
\lambda_4 &= -\frac{1}{2}g_2^2, \\
\lambda_5 &= \lambda_6 = \lambda_7 = 0.
\end{align*}
\]  

(3.1)

As a result the Higgs sector of the MSSM is completely determined by two new measurable quantities, which can be conveniently chosen to be \(m_{A^0}\) and \(\tan \beta\) [2].

However, the parameters of any theory will in general depend on the energy scale \((\sqrt{s})\) at which they are evaluated. This dependence is described by the renormalization group equations (RGEs)

\[
dp_i/dt = \beta_i(p_1, p_2, \ldots), \quad \text{where } t \equiv \ln(s).
\]  

(3.2)

Here the parameters \(p_i\) stand for the Yukawa couplings \(h_f\) \((f = u, d, \ell)\), the gauge couplings of the SU(3)\(_c\) \(\times\) SU(2)\(_L\) \(\times\) U(1)\(_Y\) gauge group \(g_i\) \((i = 1, 2, 3)\), the Higgs self-coupling constants \(\lambda_j\) \((j = 1, \ldots, 7)\), and the mass parameters of the Higgs bosons \(m_{ij}^2\) \((i, j = 1, 2)\). Since eq. (3.1) is valid at an arbitrary energy scale \(\sqrt{s}\) we find analogous relations for the corresponding \(\beta\)-functions \((\beta_{p_i} \equiv \beta_i)\) by taking the derivatives with respect to \(t\)

\[
\begin{align*}
\beta_{\lambda_1} &= \beta_{\lambda_2} = \frac{1}{4}[\beta_{g_2^2} + \beta_{g_1^2}], \\
\beta_{\lambda_3} &= \frac{1}{4}[\beta_{g_2^2} - \beta_{g_1^2}], \\
\beta_{\lambda_4} &= -\frac{1}{2}\beta_{g_2^2}.
\end{align*}
\]  

(3.3)

Note that eq. (3.3) is valid only if the theory is supersymmetric at the scale \(\sqrt{s}\). However, if SUSY-breaking terms are included, the mass degeneracy between the particles and their supersymmetric partners is violated, and the masses of the superparticles can become heavy. For simplicity we will typically assume that the supersymmetric particle masses are roughly of the same order. That is, the scale of SUSY-breaking is characterized by one single parameter, \(M_{\text{SUSY}}\). Then the \(\beta\)-functions at an intermediate scale \(\sqrt{s}\) \((for \ m_Z < \sqrt{s} < M_{\text{SUSY}})\) will no longer satisfy eq. (3.3) and the gauge coupling constants and the self-coupling constants will evolve differently. In the case of multiple SUSY-breaking scales, as \(\sqrt{s}\) decreases one would have to modify the \(\beta\)-functions every time a multiplet of supersymmetric particles decouples. In this section we shall assume that the mass parameters of the Higgs potential are of the order of \(m_Z\). This guarantees that both Higgs doublets are present in the low effective theory which is equivalent to
the (non-supersymmetric) SM with an extended two-doublet Higgs sector. The required \( \beta \)-functions are presented in Appendix A.\

We begin by running the gauge coupling constants from the electroweak scale (where they are measured) up to \( M_{\text{SUSY}} \). At this scale, the SUSY boundary conditions given by eq. (3.1) can be imposed, and we obtain the values of the Higgs self coupling constants \( \lambda_i \) at \( M_{\text{SUSY}} \). Next we determine \( \lambda_i(M_{\text{weak}}) \) by integrating the corresponding RGEs from \( M_{\text{SUSY}} \) down to \( M_{\text{weak}} \). Finally, using \( \lambda_i \) in eqs. (2.9)–(2.13), we find the RGE-improved Higgs masses and trilinear interactions. RGE-improved Higgs masses based on the two-Higgs doublet RGEs have also recently appeared in ref. [19]. Analytic approximations to the trilinear Higgs interactions can also be found in the literature. In ref. [12], the \( h^0 A^0 A^0 \) coupling was computed using the effective potential method (in which only terms explicitly proportional to \( m_t^4 \) were kept). All other three-Higgs interactions were obtained using the same approximation scheme in ref. [20]. In ref. [23], the \( h^0 A^0 A^0 \) coupling was obtained by the method outlined above, in which the RGE-derived expressions for \( \lambda_i(M_{\text{weak}}) \) were inserted into eq. (2.13), and is contrasted with the results of the effective potential technique. Since this work was completed, a number of more complete computations based on one-loop vertex corrections to the Higgs self-couplings have appeared. The correction to the \( hhh \) coupling can be found in refs. [35] and [36]; see also ref. [37].

By using the running parameters of the theory evaluated at the electroweak scale \( (M_{\text{weak}}) \), one has incorporated the leading logarithmic radiative corrections to the Higgs parameters, summed to all orders in perturbation theory. The RGEs can be solved by numerical analysis using the computer. But it is instructive to solve the RGEs iteratively. To first approximation we can take the right hand side of eq. (3.2) to be independent of \( \ln(s) \). That is, we evaluate the \( \beta_i \) by imposing tree-level relations among the parameters \( p_i \) \([i.e., \text{eq. (3.1)}]\) and evaluating the results at the scale \( \sqrt{s} = M_{\text{weak}} \). Then, integration of the RGEs is trivial, and we obtain

\[
p_i(s_1) = p_i(s_2) - \beta_i \ln \left( \frac{s_2}{s_1} \right),
\]

under the assumption that the particle content of the effective low-energy theory

---

* In refs. [7–10], it was assumed that \( m_{A^0} \simeq M_{\text{SUSY}} \), in which case the effective low-energy theory consists of the non-supersymmetric SM with one physical Higgs boson. Our approach extends the results of these authors by allowing for the possibility that all five physical Higgs states \( (h^0, H^0, A^0 \text{ and } H^\pm) \) may have masses substantially below \( M_{\text{SUSY}} \).

† In the leading logarithmic approximation, the masses obtained in this manner are physical masses (corresponding to the pole of the Higgs propagator). Similarly, all one-loop definitions of the three and four-point couplings differ only in their non-leading logarithmic terms.
does not change within the range of integration.

The lower limit of integration is the electroweak scale, $M_{\text{weak}}$. One possible choice for this scale is $M_{\text{weak}} = m_Z$ (which is the appropriate scale for diagrams involving neutral gauge and Higgs bosons inside the loops). A second equally reasonable choice would be $M_{\text{weak}} = m_t$ (which is the appropriate scale for diagrams involving the top-quark). This is a somewhat arbitrary decision, since a different choice would yield results that differ formally from eq. (3.4) by a non-leading logarithmic term (i.e., a term that does not grow as $\ln(s)$ where $\sqrt{s}$ is the large scale). On the basis of more precise one-loop calculations, we have adopted the following strategy. We integrate the RGEs from $M_{\text{SUSY}}$ down to $m_t$. At that point, we formally integrate out the top-quark from the low-energy theory, and finally integrate the appropriate RGEs of the new low-energy effective theory down to $m_Z$. The first order solution of eq. (3.2) becomes

$$p_i(m_Z^2) = p_i(M_{\text{SUSY}}^2) - \beta_i \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) - \beta_0^i \ln \left( \frac{m_t^2}{m_Z^2} \right), \quad (3.5)$$

where the $\beta_i$ are the $\beta$-functions of the SM with two Higgs doublets presented in Appendix A. The $\beta_0^i$ are the $\beta$-functions in the same model with the top-quark decoupled as we now explain.

We obtain $\beta_0^i$ from $\beta_i$ by setting the top-quark Yukawa coupling to zero. For $\beta_0^2$, the situation is more subtle. The reason is that below the top quark-threshold, the SU(2)$_L \times$ U(1)$_Y$ gauge symmetry is broken. As a result, the coupling constants for vertices involving the gauge bosons are no longer constrained by the SU(2)$_L \times$ U(1)$_Y$ gauge symmetry and can evolve independently. Thus, to determine the evolution of the $g_i$ we have to define these couplings more precisely. Since our physical input parameters are the masses of the $Z$ and $W$ bosons, it is appropriate to define $g_1$ and $g_2$ through the interaction

$$\mathcal{L} = \frac{1}{4} \left[ (H_1^0)^2 + (H_2^0)^2 \right] \left( G^{\pm} W^{\mu}_+ W^{\mu}_- + \frac{1}{2} G^{ij} V^{\mu}_{ij} V_j^{\mu} \right)$$

(3.6)

where $V_i = (W^3, B)$ are the neutral SU(2)$_L$ and U(1)$_Y$ gauge fields. The boundary conditions for the coupling constants $G^\pm$ and $G^{ij}$ above the SU(2)$_L \times$ U(1)$_Y$ breaking scale $m_t$ are

$$G^\pm = g_2^2, \quad G^{ij} = \bar{g}_i \bar{g}_j \quad \text{with} \quad \bar{g}_i = (g_1, -g_2). \quad (3.7)$$

After SU(2)$_L \times$ U(1)$_Y$ symmetry breaking, the set of $\beta$-functions becomes much larger and more complicated. However, if we only work to first order in perturba-
tion theory below \( m_t \), it is not necessary to derive a full set of \( \beta \)-functions. The only ones needed, \( \beta_{G^\pm} \) and \( \beta_{G^{ij}} \), are given in Appendix B. The gauge boson masses are related to the coupling constants by

\[
\begin{align*}
    m_W^2 &= \frac{1}{4} v^2 G^\pm(m_Z), \\
    m_Z^2 &= \frac{1}{4} v^2 \text{tr} G^{ij}(m_Z).
\end{align*}
\]

(3.8)

One can solve the RGEs of Appendix B for \( G^{ij} \) with the ansatz \( G^{ij}(t) = \bar{g}_i(t)\bar{g}_j(t) \) and therefore \( G^{ij}(t) \) is a rank one matrix for arbitrary \( t \). Thus the neutral gauge boson mass matrix maintains its zero mass eigenvalue corresponding to the massless photon even below the top quark-threshold. Therefore, it is convenient to define \( g_1 \) and \( g_2 \) in terms of \( G^\pm(m_Z) \) and \( G^{ij}(m_W) \)

\[
\begin{align*}
    g_2^2 &\equiv G^\pm(m_Z), \\
    g_1^2 + g_2^2 &\equiv \text{tr} G^{ij}(m_Z).
\end{align*}
\]

(3.9)

We now have to choose a value for \( M_{\text{SUSY}} \). Of course the simplest possibility is one in which all supersymmetric particle masses are of order \( M_{\text{SUSY}} \). For completeness we briefly discuss the case of multiple mass scales in the supersymmetric particle sector. In the higgsino/gaugino sector we have two free mass parameters \( \mu \) and \( M_2 \) (it is common to fix \( M_1 \) by the grand unification relation \( M_1 = \frac{5}{3} \tan^2 \theta_W M_2 \)). We have computed the contributions of the gauginos and the higgsinos to \( \beta_{\lambda_i} \) (\( i = 1, \ldots, 4 \)) and \( \beta_{g_i^2} \) (\( i = 1, 2 \)). It is clear that the contributions of \( \tilde{W}, \tilde{B} \) and \( \tilde{H} \) to the \( \beta_{g_2^2} \) can be computed separately and have to be included at a scale \( \sqrt{s} > M_2, M_1 \) and \( |\mu| \), respectively.\(^\S\) In the case of the \( \beta_{\lambda_i} \), both gauginos and higgsinos have to be present at the scale \( \sqrt{s} \) to yield a contribution. As a result, the supersymmetric contributions to \( \beta_{\lambda_i} \) that are proportional to \( g_2^2, g_1^2 \) or the product \( g_1 g_2 \) have to be included if the scale \( \sqrt{s} > \mu_2 \equiv \max\{|\mu|, M_2\} \), \( \sqrt{s} > \mu_1 \equiv \max\{|\mu|, M_1\} \) or \( \sqrt{s} > \mu_{12} \equiv \max\{\mu_1, \mu_2\} \), respectively. The gluino contributes only to \( \beta_{g_3^2} \) at one-loop. The gluino mass \( (M_3) \) is typically fixed by the grand unification relation \( M_3 = (g_3^2/g_2^2) M_2 \). Squark mass parameters are defined explicitly in section 6. For simplicity we neglect the possibility of generation mixing in the squark mass matrices. The result for \( \beta_{\lambda_i} \) and \( \beta_{g_i^2} \) for the MSSM are presented in Appendix A.

We can now compute the effective low-energy coupling constants \( \lambda_i(m_Z) \) of the Higgs potential by evolving the coupling constants in eq. (3.1) from \( M_{\text{SUSY}} \) to

\(^\dagger\) That is, we do not solve the full set of RGEs below \( m_t \). For numerical purposes, it is certainly sufficient to isolate the one-loop leading \( \ln(m_t^2/m_Z^2) \) terms.

\(^\S\) We choose a convention in which \( M_2 \) is positive.
\(m_Z\) by means of eq. (3.4). Plugging those \(\lambda_i\) into eq. (2.10) we find the elements of the CP-even Higgs mass matrix. We focus our attention on the simplest case where all supersymmetric mass parameters are roughly degenerate. With the first order leading log results for the \(\lambda_i\) presented in Appendix C these matrix elements become in this case

\[
\mathcal{M}_{11}^2 = m_A^2 s_\beta^2 + m_Z^2 c_\beta^2 + \frac{g_5^2 m_Z^2 c_\beta^2}{96 \pi^2 c_W^2} P_t \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) \\
+ \left( 12 N_c \frac{m_b^2}{m_Z^2 c_\beta^2} - 6 N_c \frac{m_t^2}{m_Z^2 c_\beta^2} + P_f + P_g + P_{2H} \right) \ln \left( \frac{M_{\text{SUSY}}^2}{m_Z^2} \right)
\]

\[
\mathcal{M}_{22}^2 = m_A^2 c_\beta^2 + m_Z^2 s_\beta^2 + \frac{g_5^2 m_Z^2 s_\beta^2}{96 \pi^2 c_W^2} \left( P_f + P_g + P_{2H} \right) \ln \left( \frac{M_{\text{SUSY}}^2}{m_Z^2} \right) \\
+ \left( 12 N_c \frac{m_t^2}{m_Z^2 s_\beta^2} - 6 N_c \frac{m_b^2}{m_Z^2 s_\beta^2} + P_t \right) \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right)
\]

\[
\mathcal{M}_{12}^2 = -s_\beta c_\beta \left( m_A^2 + m_Z^2 + \frac{g_5^2 m_Z^2 s_\beta^2}{96 \pi^2 c_W^2} \left( P_t - 3 N_c \frac{m_t^2}{m_Z^2 s_\beta^2} \right) \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) \\
+ \left( -3 N_c \frac{m_b^2}{m_Z^2 s_\beta^2} + P_f + P_g + P_{2H} \right) \ln \left( \frac{M_{\text{SUSY}}^2}{m_Z^2} \right) \right)
\]

where the constants \(P_i\) are

\[
P_t \equiv N_c (1 - 4 e_w s_W^2 + 8 e_w^2 s_W^4), \\
P_f \equiv N_g \{N_c (2 - 4 s_W^2 + 8 (e_d^2 + e_u^2) s_W^4) + [2 - 4 s_W^2 + 8 s_W^4] \} - P_t, \\
P_g \equiv -44 + 106 s_W^2 - 62 s_W^4, \\
P_g' \equiv 10 + 34 s_W^2 - 26 s_W^4, \\
P_{2H} \equiv -10 + 2 s_W^2 - 2 s_W^4, \\
P_{2H}' \equiv 8 - 22 s_W^2 + 10 s_W^4.
\]

Here the subscripts \(t, f, g\) and \(2H\) correspond to the contributions from the top-quark, the fermions (excluding the top-quark), the gauge bosons and the two Higgs doublets. By diagonalizing the CP-even mass matrix given by eq. (3.10) one obtains the individual neutral CP-even Higgs masses and mixing angle \(\alpha\) [eq. (2.12)]. With the angle \(\alpha\) in hand, one then obtains the couplings of the Higgs bosons to gauge bosons which are proportional to either \(\sin(\beta - \alpha)\) or \(\cos(\beta - \alpha)\). Finally, the \(\lambda_i(m_Z)\) and the angle \(\alpha\) determine the Higgs self couplings [eq. (2.13)]. These results were previously presented in ref. [13]. Similar results have also recently been obtained in ref. [17] using an effective potential computation. However, our results differ from
those in ref. [17] by terms of order $g^2 m_Z^2 s_W^2 \ln(m_t^2/m_Z^2)$ due to our more precise treatment of the theory below the top-threshold [as explained Appendix B].

Of course the CP-even Higgs mass matrix may be computed numerically by employing the $\lambda_i(m_Z)$ obtained through numerical solution of the RGEs. The resulting Higgs mass matrix will then be the RGE-improved version of eq. (3.10), incorporating leading logarithmic effects beyond one-loop order. In section 8 we will compare these two results to see the numerical implications of RGE-improvement.

Radiative corrections to the charged Higgs mass sum rule have been obtained in refs. [38–40]. For completeness, we also give the one-loop leading logarithmic expression for the charged Higgs mass. From eq. (2.9) and (C.6), we obtain

$$m_{H^\pm}^2 = m_A^2 + m_W^2 + \frac{N_c g_Y^2}{32\pi^2 m_W^2} \left[ \frac{2m_t^2 m_b^2}{s^2 c^2 \beta} - m_W^2 \left( \frac{m_t^2}{s^2 \beta} + \frac{m_b^2}{c^2 \beta} \right) + \frac{g^2}{3} m_W^4 \right] \ln \left( \frac{M_{SUSY}^2}{m_t^2} \right)$$

$$+ \frac{g_2 m_W^2}{48\pi^2} \left[ N_c (N_g - 1) + N_g + \frac{1}{2} N_H - 10 + 15 \tan^2 \theta_W \right] \ln \left( \frac{M_{SUSY}^2}{m_W^2} \right),$$

(3.12)

where the number of Higgs doublets is $N_H = 2$. The coefficient of $\ln(M_{SUSY}^2/m_t^2)$ in eq. (3.12) is consistent with the exact one-loop calculations of refs. [39] and [40]. Note that this result differs slightly from the one quoted in ref. [12], which is based on an effective potential computation. This is not surprising in light of the remarks made in the Introduction. As discussed in refs. [15] and [39], the effective potential is determined from Green functions evaluated at zero external momenta, while physical masses are determined by the pole of the (radiatively corrected) propagator. This implies that for a particle whose tree-level mass is nonzero, the effective potential does not yield the complete leading logarithmic contribution to the particle mass.

The one-loop leading log results [eqs. (3.10)–(3.12)] are useful approximations (in the absence of large squark mixing—see section 6) to the fully integrated RGE results. This will be discussed more fully in section 8. One clarification is necessary. Consider the fully integrated RGE result for some parameter $p_i$. Technically, we have only determined $p_i$ down to $\sqrt{s} = m_t$. For $\sqrt{s} < m_t$, we have only used the one-loop approximation [as in eq. (3.5)] to evolve $p_i(s)$ all the way down to $\sqrt{s} = M_{weak}$. Since $m_t$ cannot be very much larger than $m_Z$, this procedure is certainly sufficient for our purposes.
4. The Large $m_{A^0}$ Limit

Until now we have assumed that $m_{A^0} \simeq \mathcal{O}(m_Z)$. In particular, the low-energy effective theory (at the electroweak scale) contains two Higgs doublets. Consequently, when we refer to the VEVs $v_i$ in section 3, we mean $v_i(m_Z)$. In this section we shall generalize our analysis to arbitrary $m_{A^0}$. First we consider what happens when $m_{A^0} \gg m_Z$. If we expand eq. (2.11) in powers of $m_Z^2/m_{A^0}^2$ we find

$$m_{h^0}^2 = c_\beta^2 M_{11}^2 + s_\beta^2 M_{22}^2 + 2s_\beta c_\beta M_{12} + \mathcal{O}\left(\frac{m_Z^4}{m_{A^0}^2}\right)$$

$$= m_Z^2 c_\beta^2 + \frac{g^2 m_Z^2}{96\pi^2 c_W^2} \left\{\left[12N_c \frac{m_t^4}{m_Z^4} - 6N_c c_\beta \frac{m_b^2}{m_Z^2} + c_\beta^2 P_f\right.ight.$$  

$$+ (P_g + P_{2H})(s_\beta^4 + c_\beta^4) - 2s_\beta^2 c_\beta^2 (P_g' + P_{2H}')\right\} \ln\left(\frac{M_{\text{SUSY}}^2}{m_Z^2}\right)$$

$$(4.1)$$

$$+ \left[12N_c \frac{m_t^4}{m_Z^4} + 6N_c c_\beta \frac{m_t^2}{m_Z^2} + c_\beta^2 P_t\right] \ln\left(\frac{M_{\text{SUSY}}^2}{m_t^2}\right)\right\}$$

$$+ \mathcal{O}\left(\frac{m_Z^4}{m_{A^0}^2}\right) + \mathcal{O}\left[g_2^4 m_Z^2 \ln\left(\frac{m_{A^0}}{m_Z^2}\right)\right],$$

after using the results of eqs. (3.10) and (3.11). Since eqs. (3.10) and (3.11) were derived under the assumption that $m_{A^0} \lesssim \mathcal{O}(m_Z)$ it follows that the terms proportional to $P_{2H}$ and $P_{2H}'$ will be modified when $m_{A^0} \gg m_Z$ by terms of $\mathcal{O}[g_2^2 m_Z^2 \ln(m_{A^0}/m_Z^2)]$ as indicated above.

Alternatively, we may investigate the case of large $m_{A^0}$ by integrating out the heavy Higgs doublet. In this scenario one of the mass eigenvalues of $M_{ij}$ is much larger than the weak scale. Then, in order to obtain the effective Lagrangian at $M_{\text{weak}}$, we first have to run the various coupling constants to the threshold $m_{A^0}$. Then we diagonalize the Higgs mass matrix and express the Lagrangian in terms of the mass eigenstates. Notice that in this case the mass eigenstate $h^0$ is directly related to the field with the non-zero VEV [i.e., $\beta(m_{A^0}) = \alpha(m_{A^0}) + \pi/2 + \mathcal{O}(m_Z^2/m_{A^0}^2)$]. Below $m_{A^0}$ there remains only the SM Higgs doublet $\phi \equiv c_\beta \Phi_1 + s_\beta \Phi_2$. The potential is

$$\mathcal{V} = m_{\phi}^2 (\phi^\dagger \phi) + \frac{1}{2} \lambda (\phi^\dagger \phi)^2,$$  

$$(4.2)$$
where the boundary condition for $\lambda$ at $m_{A^0}$ is

$$
\lambda(m_{A^0}) = \left[ c^4_{1\beta} \lambda_1 + s^4_{1\beta} \lambda_2 + 2 s^2_{1\beta} c^2_{1\beta} \lambda_3 + 4 c^3_{1\beta} s_{1\beta} \lambda_6 + 4 c_{1\beta} s^3_{1\beta} \lambda_7 \right] (m_{A^0}) 
$$

$$
= \left[ \frac{1}{4} (g_1^2 + g_2^2) c_{2\beta} \right] (m_{A^0}) + \frac{g_2^4}{384 \pi^2 c_W^4} \ln \left( \frac{M^2_{\text{SUSY}}}{m_{A^0}^2} \right) 
$$

$$
\times \left[ 12 N_c \left( \frac{m_t^4}{m_Z^4} + \frac{m_b^4}{m_Z^4} \right) + 6 N_c c_{2\beta} \left( \frac{m_t^2}{m_Z^2} - \frac{m_b^2}{m_Z^2} \right) 
$$

$$
+ c^2_{2\beta} (P_t + P_f) + (s^4_{1\beta} + c^4_{1\beta}) (P_g + P_{2H}) - 2 s^2_{1\beta} c^2_{2\beta} (P'_g + P'_{2H}) \right], 
$$

where $(g_1^2 + g_2^2) c_{2\beta}$ is to be evaluated at the scale $m_{A^0}$ as indicated. Similarly, the mass parameter $m_\phi$ that appears in eq. (4.2) can be expressed in terms of the soft SUSY mass parameters although we will not need this expression to compute $m_{h^0}$. The RGE in the SM for $\lambda$ is [41,42]

$$
16 \pi^2 \beta_\lambda = 6 \lambda^2 + \frac{3}{8} \left[ 2 g_2^4 + (g_2^2 + g_1^2)^2 \right] - 2 \sum_i N_c c_{h_i^4} - \lambda \left( \frac{9}{2} g_2^2 + \frac{3}{2} g_1^2 - 2 \sum_i N_c c_{h_i^2} \right),
$$

where the summation is over all fermions with $h_i = g m_{f_i} / (\sqrt{2} m_W)$. The RGEs for the gauge couplings are obtained from the $\beta_{g_i^2}$ given in Appendix A by putting $N_H = 1$. By solving the RGEs for scales $m_Z < \sqrt{s} < m_{A^0}$ iteratively to first order we obtain the light CP-even Higgs mass

$$
m_{h^0}^2 = \lambda(m_Z) v^2 = m^2_{2\beta} c_{2\beta} (m_{A^0}) 
$$

$$
+ \frac{g_2^2 m_Z^2}{96 \pi^2 c_W^4} \left\{ 12 N_c \left( \frac{m_t^4}{m_Z^4} - 6 N_c c_{2\beta} \frac{m_t^2}{m_Z^2} + c^2_{2\beta} P_t \right) \ln \left( \frac{m_{A^0}^2}{m_t^2} \right) 
$$

$$
+ \left[ 12 N_c \frac{m_b^4}{m_Z^4} - 6 N_c c_{2\beta} \frac{m_b^2}{m_Z^2} + c^2_{2\beta} P_f + P_{g1} + P_{1H} \right] \ln \left( \frac{m_{A^0}^2}{m_Z^2} \right) 
$$

$$
+ \left[ 12 N_c \left( \frac{m_t^4}{m_Z^4} + \frac{m_b^4}{m_Z^4} \right) + 6 N_c c_{2\beta} \left( \frac{m_t^2}{m_Z^2} - \frac{m_b^2}{m_Z^2} \right) + c^2_{2\beta} (P_t + P_f) 
$$

$$
+ (s^4_{1\beta} + c^4_{1\beta}) (P_g + P_{2H}) - 2 s^2_{1\beta} c^2_{2\beta} (P'_g + P'_{2H}) \right] \ln \left( \frac{M^2_{\text{SUSY}}}{m_{A^0}^2} \right) \right\} + \mathcal{O} \left( \frac{m_Z^4}{m_{A^0}^4} \right).
$$

(4.5)
Here we have defined

\[ P_{1H} \equiv -9c_4^2 + (1 - 2s_W^2 + 2s_W^4) c_2^2 \beta, \]

\[ P_{1g} \equiv c_2^2 \beta \left(-17 + 70s_W^2 - 44s_W^4\right) - (27 - 36s_W^2 + 18s_W^4) \]

\[ = (c_4^2 + s_4^2) P_g - 2s_\beta^2 c_2^2 P'_g. \]  

(4.6)

where the subscripts 1\( H \) and 1\( g \) indicate that these are the Higgs and gauge boson contributions in the one-Higgs-doublet model. However, we still must deal with implicit scale dependence of \( c_2^2 \beta \). Since the fields \( \Phi_i (i = 1, 2) \) change with the scale \([16,18]\), it follows that \( \tan \beta \) scales like the ratio of the two Higgs doublet fields, i.e.,

\[ \frac{1}{\tan^2 \beta} \frac{d \tan^2 \beta}{dt} = \Phi_1^2 \Phi_2^2 \frac{d \Phi_2^2}{d \Phi_1^2} = \gamma_2 - \gamma_1. \]  

(4.7)

Thus we arrive at the RGE for \( \cos 2\beta \) in terms of the anomalous dimensions \( \gamma_i \) given in eq. (A.10)

\[ c_β^2(m_{A^0}) = c_β^2(m_Z) + 4c_β^2 c_β^2 s_β^2 (\gamma_1 - \gamma_2) \ln \left( \frac{m_{A^0}^2}{m_Z^2} \right). \]  

(4.8)

Inserting eq. (4.8) in eq. (4.5), we end up with

\[ m_{h^0}^2 = m_Z^2 c_β^2(m_Z) + \frac{g_2^2 m_Z^2}{96\pi^2 c_W^2} \left\{ -12N_c \left( \frac{m_4^4}{m_Z^4} - 6N_c c_β^2 \frac{m_2^2}{m_Z^2} \right) \right. 

\[ + \left. (P_g + P_{2H})(s_β^4 + c_β^4) - 2s_β^2 c_β^2 \left( P'_g + P'_{2H} \right) \right] \ln \left( \frac{M_{\text{SUSY}}^2}{m_Z^2} \right) \]

\[ + \left[ 12N_c \frac{m_4^4}{m_Z^4} + 6N_c c_β^2 \frac{m_2^2}{m_Z^2} + c_β^2 P_f \right] \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) \]

\[ - \left( c_β^4 + s_β^4 \right) P_{2H} - 2c_β^2 s_β^2 P'_{2H} - P_{1H} \right] \ln \left( \frac{m_{A^0}^2}{m_Z^2} \right) \}

\[ + \mathcal{O} \left( \frac{m_Z^4}{m_{A^0}^4} \right), \]  

(4.9)

which agrees with eq. (4.1). In particular, we have now obtained explicitly the term proportional to \( \ln(m_{A^0}^2) \) which accounts for the fact that there are two Higgs doublets present at a scale above \( m_{A^0} \) but only one Higgs doublet below \( m_{A^0} \). Finally, by taking the limit of large \( \tan \beta \), we have checked that eq. (4.9) reduces to the one-loop leading log result of ref. [5].
5. Physical Definition of \( \tan \beta \)

In eq. (2.4) we implicitly defined \( \tan \beta \) in terms of VEVs evaluated at the electroweak scale. However, this definition is not appropriate in the case where \( m_{A^0} \gg M_{\text{weak}} \), since the definition of \( \tan \beta \) requires the presence of both Higgs doublets in our low-energy effective theory. Consider for example the tree-level relation for the partial hadronic width of \( A^0 \)

\[
\Gamma(A^0 \to b\bar{b}) = \frac{N_c g_s^2 m_b^2}{32\pi m_W^2} (m_{A^0}^2 - 4m_b^2)^{1/2} \tan^2 \beta .
\]  

(5.1)

This tree-level relation is true for an arbitrary two-Higgs-doublet model under the assumption that down-type (up-type) fermions couple exclusively to \( H_1 \) (\( H_2 \)). Using running parameters evaluated at \( m_{A^0} \), eq. (5.1) continues to hold even after leading log corrections are included. From eq. (5.1) we can obtain a practical definition of \( \tan \beta(m_{A^0}) \). By using the RGE for \( \tan \beta \) given in eq. (4.7) we can obtain \( \tan \beta(m_Z) \) which at the leading log level matches the definition of \( \tan \beta \) in terms of VEVs given in eq. (2.4).

It is instructive to show that our results do not depend on the physical definition of \( \tan \beta \). (See ref. [16] for a detailed discussion of the relation between different \( \tan \beta \) definitions.) Consider an alternative definition of \( \tan \beta \) advocated in ref. [11] based on the supersymmetric tree-level relation

\[
\Delta m_L^2 \equiv m_e^2 - m_{\nu}^2 = m_W^2 \cos 2\beta + O \left( \frac{m_e^2}{m_Z^2} \right).  
\]

(5.2)

We can easily obtain the radiative corrections due to the quark/squark sector, for arbitrary \( m_{A^0} \). The relevant terms of the Lagrangian are

\[
\mathcal{L}_{\nu} = \left[ M_L^2 + \Lambda_{ij} \left( \Phi_i^+ \Phi_j \right) \right] \tilde{L}^+ \tilde{L} + \tilde{\Lambda}_{ij} \left( \Phi_i^+ \tilde{L} \right) \left( \tilde{L}^+ \Phi_j \right),
\]

(5.3)

where \( \tilde{L} \equiv (\tilde{\nu}, \tilde{\ell}_L) \) and the indices \( i, j = 1, 2 \) run over the two Higgs doublet fields.

In the MSSM the tree-level quartic coupling constants \( \Lambda \) and \( \tilde{\Lambda} \) can be expressed in terms of the gauge coupling constants. Clearly, only the terms in eq. (5.3) proportional to \( \tilde{\Lambda}_{ij} \) cause a selectron-sneutrino mass splitting. The supersymmetric

\* However, there are non-leading log corrections that generate Yukawa couplings of up-type (down-type) fermions to \( H_1 \) (\( H_2 \)).
boundary conditions for $\bar{\Lambda}_{ij}^L$ are

$$\bar{\Lambda}_{ij}^L = \frac{1}{2} g_2^2 \text{diag} (-1, 1) .$$  \hfill (5.4)

First, we note that there are no vertex corrections due to the fermions (since we can neglect all Yukawa couplings except $h_b$ and $h_t$). Thus the contributions of the fermions to the $\beta$-functions are

$$\beta_f^L = -\bar{\Lambda}_{ij}^L \gamma_i^f ,$$

where

$$\left\{ \begin{array}{l}
16\pi^2 \gamma_1^f = -N_c h_b^2 , \\
16\pi^2 \gamma_2^f = -N_c h_t^2 .
\end{array} \right.$$  \hfill (5.5)

Note that $\bar{\Lambda}_{ij}^L$ will remain diagonal at scales below $M_{\text{SUSY}}$. After removing the heavy Higgs doublet via $\Phi_1 \rightarrow c_\beta \phi$, $\Phi_2 \rightarrow s_\beta \phi$, \hfill (5.6)

we obtain $\Delta m_L^2 = \frac{1}{2} v^2 \bar{\Lambda}$, where $\bar{\Lambda}$ is the coefficient of $|\bar{L}^\dagger \phi|^2$ in the effective scalar potential at a scale $\sqrt{s} < m_{A^0}$. The $\beta$-function and the boundary condition for $\bar{\Lambda}$ at the scale $m_{A^0}$ are

$$\bar{\Lambda}(m_{A^0}) = \left[ c_\beta^2 \bar{\Lambda}_{11}^L + s_\beta^2 \bar{\Lambda}_{22}^L \right] (m_{A^0}) ,$$

$$\beta_{\bar{\Lambda}} = -\bar{\Lambda} \gamma_f = -\bar{\Lambda} \left( c_\beta^2 \gamma_1^f + s_\beta^2 \gamma_2^f \right) .$$  \hfill (5.7)

Thus the resulting slepton squared mass difference is obtained immediately

$$\Delta m_L^2 = \frac{1}{2} v^2 \bar{\Lambda}(M_{\text{weak}}) = m_W^2 \left[ c_2 \beta(m_{A^0}) + \frac{\beta g_2^2}{g_2^2} c_2 \beta \ln \left( \frac{M_{\text{SUSY}}^2}{M_{\text{weak}}^2} \right) \right. \left. - \left( c_\beta^2 \gamma_1 - s_\beta^2 \gamma_2 \right) \ln \left( \frac{M_{\text{SUSY}}^2}{m_{A^0}^2} \right) - c_2 \beta \left( c_\beta^2 \gamma_1 + s_\beta^2 \gamma_2 \right) \ln \left( \frac{m_{A^0}^2}{M_{\text{weak}}^2} \right) \right] .$$  \hfill (5.8)

This equation, which represents the one-loop leading log radiative corrections to the slepton mass difference due to the quark/squark sector, is sufficient for our purposes. Note that the physical quantities in eq. (5.8) \textit{i.e.}, $m_W$ and $\Delta m_L^2$ cannot depend on the arbitrary scale $m_{A^0}$. It is a simple exercise to check that by taking the derivative of eq. (5.8) with respect to $\ln(m_{A^0}^2)$ we recover the RGE for $\cos 2\beta$ [eq. (4.7)].
6. Radiative Corrections due to Soft Squark Interactions

In the last two sections we have studied the leading log corrections to the Higgs self coupling constants $\lambda_i$. By inspecting the Lagrangian we find that all dimension-four operators of the MSSM respect two U(1) symmetries. Under these symmetries, the Higgs fields transform as $\Phi_i \to e^{i\alpha_i} \Phi_i$, $i = 1, 2$. One combination of these symmetries is gauged [namely U(1)$_Y$] whereas the other one is a global symmetry [which we call U(1)$_g$] which imposes constraints on the parameters of the theory. One result of the U(1)$_g$ symmetry is that the $\lambda_i$ ($i = 5, 6, 7$) remain zero even after one-loop leading log corrections are included. However, the U(1)$_g$ symmetry is broken by dimension-two and dimension-three terms of the MSSM. The dominant corrections derive from the squark mixing effects in the top and bottom sector. These effects would lead for example to finite (non-logarithmic) renormalizations of $\lambda_5$, $\lambda_6$ and $\lambda_7$. In this section, we show how to obtain such corrections (see also ref. [8]).

The most general scalar potential (including Higgs fields and one generation of squarks*) takes the following form:

$$V^0 = V_M + V_\Gamma + V_\Lambda + V_{\tilde{Q}}, \quad (6.1)$$

where we have defined

$$V_M = (-1)^{i+j}m_{ij}^2 \Phi_i^\dagger \Phi_j + M_Q^2 \left( \tilde{Q}^\dagger \tilde{Q} \right) + M_U^2 \tilde{U}^* \tilde{U} + M_D^2 \tilde{D}^* \tilde{D},$$

$$V_\Gamma = \Gamma_D^i \left( \Phi_i^\dagger \tilde{Q} \right) \tilde{D} + \Gamma_U^i \left( i\Phi_i^T \sigma_2 \tilde{Q} \right) \tilde{U},$$

$$V_\Lambda = A_{ik}^{ij} \left( \Phi_i^\dagger \Phi_j \right) \left( \Phi_k^\dagger \Phi_l \right) + \left( \Phi_i^\dagger \Phi_j \right) \left( A_{ij}^{Q} \left( \tilde{Q}^\dagger \tilde{Q} \right) + A_{ij}^{U} \tilde{U}^* \tilde{U} + A_{ij}^{D} \tilde{D}^* \tilde{D} \right) + \Lambda_{ij}^Q \left( \tilde{Q}^\dagger \tilde{Q} \right) \left( \tilde{Q}^\dagger \tilde{Q} \right) + \frac{1}{2} \left[ \Lambda_{ij} \left( i\Phi_i^T \sigma_2 \Phi_j \right) \tilde{D}^* \tilde{U} + h.c. \right], \quad (6.2)$$

and $V_{\tilde{Q}}$ contains quartic squark interaction terms. Henceforth, we discard $V_{\tilde{Q}}$ since the contributions of these terms to the finite renormalizations of the $\lambda_i$ enter only at two-loop order. In eq. (6.2), $i, j, k, l = 1, 2$ run over the two Higgs doublet fields, the third generation squark fields are defined below eq. (A.8), and the various $\Gamma$’s

---

* Contributions from the sleptons and the other two squark generations are omitted in the formulae presented in this section.
and Λ’s are determined by the tree-level SUSY relations

\[
\begin{align*}
\Lambda^Q &= \text{diag} \left[ \frac{1}{4} (g_2^2 - g_1^2 Y_Q), h_U^2 - \frac{1}{4} (g_2^2 - g_1^2 Y_Q) \right], \\
\bar{\Lambda}^Q &= \text{diag} \left[ h_D^2 - \frac{1}{4} g_2^2, \frac{1}{4} g_2^2 - h_U^2 \right], \\
\Lambda^U &= \text{diag} \left( -\frac{1}{4} g_1^2 Y_U, h_U^2 + \frac{1}{4} g_1^2 Y_U \right), \\
\Lambda^D &= \text{diag} \left( h_D^2 - \frac{1}{4} g_1^2 Y_D, \frac{1}{4} g_1^2 Y_D \right), \\
\Lambda &= -h_U h_D,
\end{align*}
\]

where \( h_U, h_D \), and the squark hypercharges are given in eq. (A.9) and subsequent text, and

\[
\begin{align*}
\Gamma^U &= h_U (-\mu, A_U), \\
\Gamma^D &= h_D (A_D, -\mu),
\end{align*}
\]

which define the \( A \)-parameters: \( A_U \) and \( A_D \). (The parameter \( \mu \) also appears in the chargino/neutralino sector [43].) The \( \Lambda^j_{ik} \) can be expressed easily in terms of the \( \lambda_i \) (\( i = 1, 2, \ldots \)) of eq. (2.2). If we introduce the collection of fields \( \Psi \equiv (\tilde{Q}, \tilde{U}^*, \tilde{D}^*) \) and use the fact that

\[
\frac{\partial^2 \mathcal{V}^0}{\partial \Psi_a \partial \Psi_b} = \frac{\partial^2 \mathcal{V}^0}{\partial \Psi^*_a \partial \Psi^*_b} = 0
\]

we obtain the squark mass matrix

\[
\mathcal{M}_a^2 = \frac{\partial^2 \mathcal{V}^0}{\partial \Psi_a \partial \Psi^*_b}.
\]

The RGEs due to the squark sector presented in Appendix A can now be computed by demanding that the one-loop renormalized potential (in the Landau gauge using dimensional reduction [44] and the \( \overline{\text{MS}} \)-scheme)

\[
\mathcal{V} = \mathcal{V}^0 + \frac{N_c}{32 \pi^2} \text{tr} \mathcal{M}^4 \left[ \ln \left( \frac{\mathcal{M}^2}{\sigma^2} \right) - \frac{3}{2} \right]
\]

is independent of the arbitrary renormalization scale \( \sigma \). Here, \( N_c = 3 \) colors, and we have included a factor of 2 since the squark fields are complex. The RGEs for the quartic Higgs couplings are

\[
32 \pi^2 \frac{d \Lambda^j_{ik}}{dt} = N_c \left[ 2 \Lambda^Q_{ij} \Lambda^Q_{lk} + \bar{\Lambda}^Q_{ij} \Lambda^Q_{lk} + \Lambda^Q_{ij} \bar{\Lambda}^Q_{lk} + \Lambda^Q_{ij} \bar{\Lambda}^Q_{lk} \right]
\]

\[
+ \Lambda^U_{ij} \Lambda^U_{lk} + \Lambda^D_{ij} \Lambda^D_{lk} + \Lambda^2 \left( \delta_{ij} \delta_{lk} - \delta_{il} \delta_{jk} \right).
\]

The \( \beta \)-functions at scales below the mass of one or more squark fields are obtained from eq. (6.8) by removing the contributions corresponding to these fields.
and assuming that the coupling constants are continuous. It is clear that the dimensionful coupling constants in eq. (6.4) cannot contribute to the $\beta$-functions of dimensionless couplings at scales larger than all the mass parameters in $V^0$. However, in the presence of dimension-three terms the decoupling of heavy squarks becomes non-trivial. To understand what is happening, consider the path integral derivation of eq. (6.7), in the case of $\tilde{M}_D \gg \tilde{M}_U, \tilde{M}_Q$ (i.e., we integrate out $\tilde{D}$; other cases are completely analogous). The generating functional is

$$W \propto \int \left[ d\tilde{D}^* d\tilde{D} \right] \exp \left\{ i \int d^4x \left[ \tilde{D}^*(-i\Delta)^{-1}\tilde{D} + \Gamma^D_i(\Phi^*j\tilde{Q})\tilde{D} + \text{h.c.} + L_\Phi \right] \right\},$$

where all other terms of the Lagrangian are included in $L_\Phi$. The inverse propagator in the presence of non-zero Higgs fields is

$$(-i\Delta)^{-1} = \Box - \left[ M_D^2 + h_D^2\Phi_1^2 + \frac{1}{4} Y_Qg_1^2(\Phi_1^2 - \Phi_2^2) \right].$$

The path integral over $\tilde{D}$ and $\tilde{D}^*$ is straightforward with the result

$$W \propto \exp \left\{ -i \int d^4x \Gamma^D_i(\Phi^*j\tilde{Q})(-i\Delta) \left[ \Gamma^D_j(\Phi^*j\tilde{Q}) \right]^* + L_\Phi \right\}.$$ 

When external momenta are much smaller than $M_D$, the interaction term in eq. (6.11) becomes local and can be absorbed into the scalar potential of the low-energy effective theory by redefining

$$\Lambda^Q_{ij} \rightarrow \Lambda'^Q_{ij} \equiv \Lambda^Q_{ij} - \frac{1}{M_D^2} \Gamma^D_i \Gamma^D_j.$$ 

These are inserted into the $\beta$-function [eq. (6.8)] which is used to run the Higgs self-couplings at scales below $M_D$. The logs arising from such a calculation are only of $O[\ln(M_U^2/M_D^2)]$. However, notice that $\Lambda^Q_{ij}$ is no longer diagonal and as a result the $\lambda_i$ ($i = 5, 6, 7$) can become non-zero. This can generate phenomena that are absent at the leading log level and may be phenomenologically important in some circumstances.

The dimension-three terms also lead to corrections which have no logarithmic dependence on the mass parameters of the model. Consider the case of $M_D = M_U = M_Q \equiv M_{\text{SUSY}}$. In this case the corrections described above are not present. Nevertheless, important non-logarithmic corrections to the $\Lambda^l_{ik}$ can arise which lead to finite shifts in the $\lambda_i$ (denoted by $\Delta\lambda_i$ below). These can be computed by
expanding the effective potential [eq. (6.7)] to fourth order in $\Phi$ as described in Appendix D. There are two types of corrections corresponding to triangle diagrams and box diagrams. The results for the triangle diagrams (with two powers of trilinear coupling constants $A_U$, $A_D$ or $\mu$) denoted by a superscript (3) are

$$
\Delta \lambda_1^{(3)} = \frac{N_c}{16\pi^2 M_{\text{SUSY}}^2} \left\{ A_D^2 h_D^2 \left( 2h_D^2 - \frac{g_2^2 + g_1^2}{4} \right) + \mu^2 h_U^2 g_2^2 + g_1^2 \right\}
$$

$$
\Delta \lambda_2^{(3)} = \frac{N_c}{16\pi^2 M_{\text{SUSY}}^2} \left\{ A_U^2 h_U^2 \left( 2h_U^2 - \frac{g_2^2 + g_1^2}{4} \right) + \mu^2 h_D^2 g_2^2 + g_1^2 \right\}
$$

$$
\Delta \lambda_3^{(3)} = \frac{N_c}{32\pi^2 M_{\text{SUSY}}^2} \left\{ \mu^2 (h_U^2 - h_D^2)^2 + h_U^2 h_D^2 (A_U + A_D)^2 + \frac{g_1^2 - g_2^2}{4} \left[ (A_D^2 - \mu^2) h_D^2 + (A_U^2 - \mu^2) h_U^2 \right] \right\}
$$

$$
\Delta \lambda_4^{(3)} = \frac{N_c}{32\pi^2 M_{\text{SUSY}}^2} \left\{ \mu^2 (h_U^2 + h_D^2)^2 - h_U^2 h_D^2 (A_U + A_D)^2 + \frac{g_2^2}{2} \left[ (A_D^2 - \mu^2) h_D^2 + (A_U^2 - \mu^2) h_U^2 \right] \right\}
$$

$$
\Delta \lambda_5^{(3)} = 0
$$

$$
\Delta \lambda_6^{(3)} = \frac{N_c \mu}{32\pi^2 M_{\text{SUSY}}^2} \left\{ A_D h_D^2 \left( \frac{g_2^2 + g_1^2}{4} - 2h_D^2 \right) - A_U h_U^2 g_2^2 + g_1^2 \right\}
$$

$$
\Delta \lambda_7^{(3)} = \frac{N_c \mu}{32\pi^2 M_{\text{SUSY}}^2} \left\{ A_U h_U^2 \left( \frac{g_2^2 + g_1^2}{4} - 2h_U^2 \right) - A_D h_D^2 g_2^2 + g_1^2 \right\}
$$

and the results for the box diagrams (with four powers of $A_U$, $A_D$ or $\mu$) denoted with a superscript (4) are
\[ \Delta \lambda_{1}^{(4)} = -\frac{N_{c}}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ A_{D}^{4}h_{D}^{4} + \mu^{4}h_{U}^{4} \right\} \]

\[ \Delta \lambda_{2}^{(4)} = -\frac{N_{c}}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ A_{U}^{4}h_{U}^{4} + \mu^{4}h_{D}^{4} \right\} \]

\[ \Delta \lambda_{3}^{(4)} = -\frac{N_{c}}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ \mu^{2}A_{U}^{2}h_{U}^{4} + \mu^{2}A_{D}^{2}h_{D}^{4} + h_{U}^{2}h_{D}^{2}(\mu^{2} - A_{U}A_{D})^{2} \right\} \]

\[ \Delta \lambda_{4}^{(4)} = -\frac{N_{c}}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ \mu^{2}A_{U}^{2}h_{U}^{4} + \mu^{2}A_{D}^{2}h_{D}^{4} - h_{U}^{2}h_{D}^{2}(\mu^{2} - A_{U}A_{D})^{2} \right\} \] (6.14)

\[ \Delta \lambda_{5}^{(4)} = -\frac{N_{c}\mu^{2}}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ A_{D}^{2}h_{D}^{4} + A_{U}^{2}h_{U}^{4} \right\} \]

\[ \Delta \lambda_{6}^{(4)} = \frac{N_{c}\mu}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ \mu^{2}A_{U}^{2}h_{U}^{4} + A_{D}^{3}h_{D}^{4} \right\} \]

\[ \Delta \lambda_{7}^{(4)} = \frac{N_{c}\mu}{96\pi^{2}M_{\text{SUSY}}^{4}} \left\{ \mu^{2}A_{D}h_{D}^{4} + A_{U}^{3}h_{U}^{4} \right\} . \]

Finally, the self energy diagrams yield corrections to the kinetic term of the Higgs fields which have to be absorbed by redefining the Higgs fields

\[ \Phi_{i} \rightarrow \hat{\Phi}_{i} \equiv (\delta_{ij} - \frac{1}{2}A_{ij}^{'}) \Phi_{j} . \] (6.15)

For example, the contributions to the \( A_{ij}^{'} \) coming from the trilinear scalar interactions are given by

\[ A_{ij}^{'} = -\frac{N_{c}}{96\pi^{2}M_{\text{SUSY}}^{2}} \left[ h_{U}^{2} \left( \begin{array}{cc} \mu^{2} & -\mu A_{U} \\ -\mu A_{U} & A_{U}^{2} \end{array} \right) + h_{D}^{2} \left( \begin{array}{cc} A_{D}^{2} & -\mu A_{D} \\ -\mu A_{D} & \mu^{2} \end{array} \right) \right] . \] (6.16)

If we then express the quartic terms of the potential in terms of the new fields \( \hat{\Phi} \) we obtain
\[
\begin{align*}
\Delta \lambda_1^{(2)} &= \frac{1}{2}(g_1^2 + g_2^2)A'_{11}, \\
\Delta \lambda_2^{(2)} &= \frac{1}{2}(g_1^2 + g_2^2)A'_{22}, \\
\Delta \lambda_3^{(2)} &= -\frac{1}{4}(g_1^2 - g_2^2)(A'_{11} + A'_{22}), \\
\Delta \lambda_4^{(2)} &= -\frac{1}{2}g_2^2(A'_{11} + A'_{22}), \\
\Delta \lambda_5^{(2)} &= 0, \\
\Delta \lambda_6^{(2)} &= \frac{1}{2}(g_1^2 + g_2^2)A'_{12}, \\
\Delta \lambda_7^{(2)} &= \frac{1}{2}(g_1^2 + g_2^2)A'_{12},
\end{align*}
\]  
\tag{6.17}

where we have used the (supersymmetric) tree-level results for the \( \lambda_i \) that appeared on the right hand side of eqs. (6.17) evaluated at the scale \( M_{\text{SUSY}} \).

Combining all the results obtained in this section, we conclude that the appropriate boundary conditions for the \( \lambda_i \) at \( M_{\text{SUSY}} \) are obtained by setting

\[
\lambda_i(M_{\text{SUSY}}) = \lambda_i(\text{SUSY}) + \Delta \lambda_i^{(2)} + \Delta \lambda_i^{(3)} + \Delta \lambda_i^{(4)},
\]
\tag{6.18}

where \( \lambda_i(\text{SUSY}) \) are the scalar self-couplings of the unbroken supersymmetric theory given in eq. (3.1) evaluated at \( M_{\text{SUSY}} \). The low-energy effective Higgs potential is given by \( V \) in eq. (2.2) where the \( \lambda_i \) appearing there are \( \lambda_i(M_{\text{weak}}) \) obtained by solving the RGEs [given in Appendix A] subject to the boundary condition given in eq. (6.18). Using the coupling constants so obtained, one may compute the mass eigenvalues, mixing angles and coupling constants in eq. (2.9)–(2.13) numerically. The effective potential formalism (see, e.g., refs. [6,12,17, and 18]) correctly reproduces the terms that arise from \( \Delta \lambda_i^{(3)} \) and \( \Delta \lambda_i^{(4)} \). However, the effective potential method does not pick up the terms arising from \( \Delta \lambda_i^{(2)} \), which derive from wave function renormalization.

In the next section we shall present numerical results of this procedure. However, it is instructive to first look at a few analytic results. In the one-loop logarithmic approximation, the RGEs are solved analytically as shown in section 3. The effect of the new boundary conditions [eq. (6.18)] at the one-loop level is simply additive. We shall include only the effects of the third generation squark mixing. If we denote the one-loop leading log squared mass shifts obtained from eq. (3.10) by \( (\Delta m^2)_{\text{LL}} \), then we obtain the following expressions for the neutral Higgs masses
in the limit of large $\tan \beta$:

\[
(m^{2}_{h^0} - m^{2}_{Z})_{\beta=\pi/2} = (\Delta m^{2}_{h^0})_{1\text{LL}} - \frac{N_{c}g^{2}_{\text{occ}} m_{W}^{2}}{96\pi^{2} M_{\text{SUSY}}^{4}} \left[ \left( \frac{A_{t} m_{t}}{s_{\beta} m_{W}} \right)^{4} + \left( \frac{\mu m_{b}}{c_{\beta} m_{W}} \right)^{4} \right] \\
+ \frac{N_{c}g^{2}_{\text{occ}} m_{W}^{2}}{96\pi^{2} M_{\text{SUSY}}^{4}} \left[ 12 \frac{A_{t}^{2} m_{t}^{4}}{s_{\beta}^{4} m_{W}^{4}} - 4 \frac{A_{t}^{2} m_{t}^{2}}{s_{\beta}^{2} m_{W}^{2} c_{\beta}^{2}} + 2 \frac{\mu^{2} m_{b}^{2}}{c_{\beta}^{2} m_{W}^{2} c_{\beta}^{2}} \right],
\]

and

\[
(m^{2}_{H^0} - m^{2}_{A^0})_{\beta=\pi/2} = - \frac{N_{c}g^{2}_{\text{occ}} \mu^{2}}{96\pi^{2} M_{\text{SUSY}}^{4}} \left( \frac{A_{t}^{2} m_{t}^{4}}{s_{\beta}^{4} m_{W}^{4}} + \frac{A_{b}^{2} m_{b}^{4}}{c_{\beta}^{4} m_{W}^{4}} \right),
\]

assuming that $m_{A^0} > m_{Z}$ [if $m_{A^0} < m_{Z}$ then interchange $m_{h^0}$ and $m_{H^0}$]. Taking the limit $\beta \to \pi/2$ on the right hand side of eqs. (6.19) and (6.20) is subtle because of factors of $c_{\beta}$ in the denominator. The appropriate limit is one where the Yukawa coupling $h_{b} \equiv g_{2} m_{b}/(\sqrt{2} m_{W} c_{\beta})$ is fixed. Thus, in the limit $\beta \to \pi/2$, it follows that $m_{b} \to 0$ such that $m_{b}/c_{\beta}$ is fixed. We also refrain from setting $s_{\beta} = 1$ to preserve the symmetry of the formulae above.

As a second example consider the radiative corrections to charged Higgs mass. Define

\[
\Delta m^{2}_{H^{\pm}} \equiv m^{2}_{H^{\pm}} - m^{2}_{A^0} - m^{2}_{W},
\]

so that $\Delta m^{2}_{H^{\pm}} = 0$ at tree-level. We then find

\[
\Delta m^{2}_{H^{\pm}} = (\Delta m^{2}_{H^{\pm}})_{1\text{LL}} + \frac{N_{c}g^{2}_{\text{occ}} m_{W}^{2}}{96\pi^{2}} \left[ \frac{m_{t}^{2}}{m_{W} m_{h}^{2}} \left( \frac{\mu^{2} - 2 A_{t}^{2}}{M_{\text{SUSY}}^{2}} \right) + \frac{m_{b}^{2}}{m_{W} c_{\beta}^{2}} \left( \frac{\mu^{2} - 2 A_{b}^{2}}{M_{\text{SUSY}}^{2}} \right) \right] \\
+ \frac{N_{c}g^{2}_{\text{occ}} m_{W}^{2}}{64\pi^{2}} \left[ \frac{m_{t}^{2} m_{b}^{2}}{m_{W} m_{h}^{4} c_{\beta}^{2}} \left( A_{t} + A_{b} \right) - \frac{\mu^{2}}{M_{\text{SUSY}}^{2}} \left( \frac{m_{t}^{2}}{m_{W} m_{h}^{2}} + \frac{m_{b}^{2}}{m_{W} c_{\beta}^{2}} \right) \right]^{2} \\
- \frac{N_{c}g^{2}_{\text{occ}} m_{W}^{2} m_{h}^{2}}{192\pi^{2} m_{W} m_{h}^{4} c_{\beta}^{2}} \left( \frac{A_{t} A_{b} - \mu^{2}}{M_{\text{SUSY}}^{2}} \right)^{2},
\]

where $(\Delta m^{2}_{H^{\pm}})_{1\text{LL}}$ is the value of $\Delta m^{2}_{H^{\pm}}$ obtained from eq. (3.12). This result is consistent with the one-loop calculations of refs. [39,40] in the limit of large $M_{\text{SUSY}}$. [In the same limit, the effective potential computations of refs. [12] and [18] differ slightly from the above results for the reasons mentioned below eq. (3.12).] Under the assumption that all the soft-supersymmetry-breaking parameters are of the
same order and $\tan \beta \ll m_t/m_b$, the dominant contribution to $\Delta m_{H^\pm}^2$ is

$$\Delta m_{H^\pm}^2 = -\frac{N_c g_2^2 m_W^2}{64\pi^2} \left( \frac{\mu}{M_{\text{SUSY}}} \right)^2 \left( \frac{m_t}{s_\beta m_W} \right)^4 + \mathcal{O}(g_2^2 m_t^2).$$

(6.23)

For sufficiently large $\mu$, this correction dominates the leading log contributions which grow only as $m_t^2$.

7. Numerical Results

In this section we evaluate the radiative corrections to the Higgs masses and couplings. These have been computed by numerically solving the RGEs for the Higgs self-coupling parameters to determine the $\lambda_i(M_{\text{weak}})$ as described in section 3. These results are then inserted into eq. (2.10)–(2.13) to obtain the radiatively corrected Higgs masses and couplings. In all numerical results presented in this paper, we shall take the squark mass parameters to be equal to a common soft-supersymmetry breaking mass $M_Q = M_D = M_U = M_{\text{SUSY}}$.

In fig. 1(a) and (b) we plot the light CP-even Higgs mass as a function of $\tan \beta$ for $m_t = 150$ and 200 GeV for various choices of $m_{A^0}$. All $A$-parameters and $\mu$ are set equal to zero. The Higgs mass saturates at a maximum value, $m_{h_0}^\text{max}$, when $\tan \beta$ and $m_{A^0}$ become large. Furthermore, $m_{h_0}^\text{max}$ converges to $m_{A^0}$ in the limit $\tan \beta \to \infty$, as long as $m_{A^0} \leq m_{h_0}^{\text{max}}$. The reason for the $h_0^0$–$A^0$ mass degeneracy in this limit is easily understood. The $\tan \beta \to \infty$ limit can be implemented by setting $m_{12}^2 = 0$. In this case, the model possesses an unbroken global $U(1)_g$ symmetry which guarantees that $m_{h_0} = m_{A^0}$ to all orders in perturbation theory. That is, the radiative corrections to $m_{h_0}^\text{max}$ in this particular limit vanish exactly. In the opposite case where $m_{A^0} \geq m_{h_0}^{\text{max}}$, the dominant $m_t^4$-contribution to $m_{h_0}^2$ is independent of $\tan \beta$ [see eq. (4.9)]. As a result, at fixed $m_{A^0} > m_{h_0}^{\text{max}}$, the radiatively corrected $m_{h_0}$ will reach a maximum (minimum) at $\tan \beta \simeq \infty$ ($\tan \beta \simeq 1$), due to the tree-level behavior of $m_{h_0}^2$ on $\tan \beta$.

For fixed $\tan \beta$, $m_{h_0}$ reaches its minimum value, $m_{h_0}^{\text{min}}$, when $m_{A^0} \to 0$. Note that in contrast to the tree-level behavior (where $m_{h_0} < m_{A^0}$), the Higgs mass does not vanish as $m_{A^0} \to 0$. Moreover, $m_{h_0}^{\text{min}}$ increases as $\tan \beta$ decreases but exhibits only a moderate dependence on $m_t$ and $M_{\text{SUSY}}$. This behavior can be understood

* If both $A \neq 0$ and $\mu \neq 0$, then the $U(1)_g$ symmetry is not exact, and non-leading log corrections to $m_{h_0}$ can be generated.
as follows. For \( m_{A^0} \ll m_Z \) and for values of \( m_t \) and \( M_{\text{SUSY}} \) sufficiently large (say, \( m_t \gtrsim 2m_Z s_\beta \) and \( M_{\text{SUSY}} \gtrsim 500 \text{ GeV} \)), the CP-even squared mass matrix [eq. (2.10)] is dominated by the matrix element \( M_{22}^2 \) due to the \( m_t^4 \) dependence of \( \lambda_2 \). This yields

\[
(m_{h^0})_{\text{min}}^2 \approx M_{11}^2 - \frac{(M_{12}^2)^2}{M_{22}^2} \approx m_Z^2 c_\beta^2, \tag{7.1}
\]

which is in good agreement with the results of fig. 1. One interesting phenomenological consequence is that \( m_{h^0} \) can be larger than \( 2m_{A^0} \). This permits a new decay-mode \( h^0 \to A^0 A^0 \) which is kinematically forbidden at tree-level. For a detailed analysis of the \( h^0 \to A^0 A^0 \) decay-mode and its implications see refs. [12] and [23].

In the limit \( m_{A^0} \to \infty \) the couplings of \( h^0 \) to gauge bosons and matter fields are identical to the Higgs couplings of the SM so that the Higgs sector of the two models cannot be phenomenologically distinguished. However, SUSY does impose constraints on the quartic Higgs self-coupling at the scale \( M_{\text{SUSY}} \), and this does influence the possible values of \( m_{h^0} \). To illustrate this point, we plot in fig. 2 the range of allowed \( m_{h^0} \) in the case of large \( m_{A^0} \) (in our plots we take \( m_{A^0} = 300 \text{ GeV} \)). As we have shown above, the lower limit for \( m_{h^0} \) is attained if \( \tan \beta \approx 1 \) and the upper limit is attained in the limit of large \( \tan \beta \) (in our graphs we take \( \tan \beta = 20 \))\footnote{A second maximum for \( m_{h^0} \) would arise for very small \( \tan \beta \); however, this regime is ruled out because the top-Yukawa coupling develops a Landau-pole at energy scales below \( M_{\text{SUSY}} \).}

Suppose the top quark mass is known and that \( h^0 \) is discovered with SM couplings. If \( m_{h^0} \) does not lie in the allowed mass regime displayed in fig. 2, we could conclude that the MSSM is ruled out. Note that one can also derive upper and lower Higgs mass bounds in the SM (at fixed \( m_t \)) as a function of \( \Lambda \). Here, \( \Lambda \) is some high energy scale, below which all Yukawa and Higgs self-coupling \( \lambda \) are finite (and \( \lambda > 0 \) for stability of the electroweak vacuum). The lower SM Higgs mass bound is about the same as the corresponding bound exhibited in fig. 2 for \( \Lambda = M_{\text{SUSY}} \), while the upper SM Higgs mass bound can be substantially larger than the ones exhibited in fig. 2 (if \( \Lambda \) is significantly smaller than the Planck scale). The reason behind this result is the fact that in the MSSM, \( \lambda \) is very small (of order \( g_2^2 \)) at \( \Lambda = M_{\text{SUSY}} \).

In fig. 3(a)–(d) we plot the RGE-improved CP-even Higgs masses \( m_{h^0} \) and \( m_{H^0} \) as functions of \( m_{A^0} \) for \( m_t = 150 \text{ GeV} \), \( M_{\text{SUSY}} = 1 \text{ TeV} \) and four choices of \( \tan \beta \). In (a) and (b) \( m_{h^0} \) exhibits a very weak dependence on \( m_{A^0} \) as long as \( \tan \beta \lesssim 1 \). In (d) with large \( \tan \beta \) we see the near mass degeneracy of \( h^0 \) (\( H^0 \)) with \( A^0 \) for \( m_{A^0} \lesssim m_Z \) (\( m_{A^0} \gtrsim m_Z \)) while \( m_{H^0} (m_{h^0}) \) stays constant. This behavior is due to the global U(1)\(_g\) symmetry in the limit \( m_{T_2}^2 \to 0 \) as pointed out earlier.
In fig. 4 we plot contours corresponding to the radiatively corrected values of \( m_{h^0} \) from 20 to 140 GeV in the \( m_t-m_{A^0} \) plane. Results are presented for \( M_{\text{SUSY}} = 1 \) TeV and \( A_t = A_b = \mu = 0 \) in the case of \( \tan \beta = 1 \) and 5. In fig. 5, contours of the radiatively corrected \( m_{h^0} \) are shown in the \( \tan \beta-m_{A^0} \) plane. Results are presented for \( M_{\text{SUSY}} = 0.5 \) and 1 TeV and \( m_t = 100, 150 \) and 200 GeV.

The Higgs production cross-section in a two-Higgs-doublet model via the process \( e^+e^- \rightarrow Z^* \rightarrow ZH^0(Zh^0) \) is suppressed by a factor \( \cos^2(\beta-\alpha) \left[ \sin^2(\beta-\alpha) \right] \) as compared to the corresponding cross-sections in the SM. In fig. 6 we plot \( \cos \) radiatively corrected \( m \) as a function of \( \tan \beta \) for \( \tan \beta = 0.5, 1, 2 \) and 20, for \( A_t = A_b = \mu^2 = 0 \), \( M_{\text{SUSY}} = 1 \) TeV and two choices of \( m_t \). The behavior is similar to that of the tree-level result in that \( \cos^2(\beta-\alpha) \rightarrow 0 \) as \( m_{A^0} \) becomes large. This is expected since for large \( m_{A^0} \), all heavy Higgs states decouple, while the \( h^0 ZZ \) coupling [which is proportional to \( \sin(\beta-\alpha) \)] approaches its SM value. Nevertheless, it is interesting to note that \( \cos^2(\beta-\alpha) \) approaches 0 more slowly as \( m_t \) increases (i.e., as the radiative corrections become more significant).

Up until now, we have ignored the effects of squark mixing by setting the \( A \)-parameters and \( \mu \) equal to zero. We now examine the implications of squark mixing on the results obtained up to this point. In order to do this, we must go beyond the leading logarithmic approximation and include the effects of non-zero \( A \) and \( \mu \) as explained in section 6. In fig. 7 we plot the Higgs mass \( m_{h^0} \) as a function of \( \tan \beta \) for \( m_t = 150 \) GeV and for two choices of \( m_{A^0} \). All \( A \)-parameters are taken to be equal; the four curves shown correspond to \( \mu = A = 0, 1, 2 \) and 3 TeV, respectively. The behavior of \( m_{h^0} \) at large values of \( \tan \beta \) is noteworthy: for \( m_{A^0} \lesssim m_Z \), we see that \( m_{h^0} \) decreases monotonically with \( A \). In contrast, in the case \( m_{A^0} \gtrsim m_Z \) and large \( \tan \beta \), \( m_{h^0} \) initially increases, reaches a maximum at \( A \approx \sqrt{6}M_{\text{SUSY}} \), and then falls off rapidly [8]. These behaviors can be obtained immediately from eqs. (6.19) and (6.20).

The results just illustrated have significant implications for Higgs phenomenology at LEP [29]. In fig. 8 we plot the Higgs mass \( m_{h^0} \) and the factor \( \sin^2(\beta-\alpha) \) as functions of \( \tan \beta \) for \( m_t = 150 \) GeV and \( M_{\text{SUSY}} = 1 \) TeV. We fix the sum \( m_{A^0} + m_{h^0} = m_Z \) in both plots in order to bound \( m_{h^0} \) while keeping \( Z \rightarrow A^0 h^0 \) kinematically forbidden. Fig. 8(a) displays contours of fixed \( m_{A^0} + m_{h^0} = m_Z \). To the left (right) of these contours, \( Z \rightarrow A^0 h^0 \) is kinematically allowed (forbidden). In fig. 8(b) we see that the decay \( Z \rightarrow Z^* h^0 \), with a rate proportional to \( \sin^2(\beta-\alpha) \), can be sufficiently suppressed in the large \( \tan \beta \) regime to escape detection. On the other hand, the rate for \( Z \rightarrow A^0 h^0 \) is proportional to \( \cos^2(\beta-\alpha) \) which is near unity for large \( \tan \beta \) and \( m_{A^0} \lesssim m_Z \). Thus, \( Z \rightarrow A^0 h^0 \) would be observed in this regime unless it is kinematically forbidden. In the absence of the Higgs discovery at LEP, we can therefore conclude that the parameter regime to the left of the re-
spective curves (for various choices of \( \mu = A \)) shown in fig. 8(a) are excluded. On the other hand, at large \( \tan \beta \), the parameter regime to the right of the respective curves cannot be ruled out based on current LEP data. In particular, for large \( \mu = A \) (and for large \( \tan \beta \)), the true experimental lower limit on \( m_{h^0} \) [i.e., the dotted curve of fig. 8(a)] can be significantly lower than the quoted Higgs mass limits of the LEP detector collaborations [26–28].

In the search for \( A^0 \) at LEP via \( Z \to A^0 h^0 \), the phenomenology depends in detail on the decay branching ratios of \( A^0 \) and \( h^0 \). The main impact of the one-loop radiative corrections is a shift in the Higgs masses such that the new (and typically dominant) decay mode \( h^0 \to A^0 A^0 \) is now permitted. We have already noted that the tree-level limit \( m_{h^0} \leq m_{A^0} \) can be substantially violated when radiative corrections are incorporated. In figs. 9 and 10, we depict the region of parameter space where \( m_{h^0} \geq 2m_{A^0} \). As before, all \( A \)-parameters are taken to be equal; we exhibit curves corresponding to various choices of \( A \) and \( \mu \), for \( M_{\tilde{Q}} = M_{\text{SUSY}} = 1 \) TeV. The parameter space where \( h^0 \to A^0 A^0 \) is kinematically allowed lies to the left of [or within] the displayed curves.

In the analysis presented in this section, predictions for the (radiatively corrected) Higgs masses are obtained as a function of the soft SUSY breaking parameters. However, not all choices of these parameters lead to physically (or phenomenologically) sensible results. For example, consider the cases of \( m_t = 150 \) GeV, \( M_{\text{SUSY}} = 1 \) TeV and \( m_{A^0} = 0, 40, 100 \) or 300 GeV. Then the condition \( m_{h^0} > 0 \) (or \( m_{h^0} > 40 \) GeV if we take recent LEP limits at face values) rules out the parameter region above the solid, dashed or dot-dashed curves in fig. 11.

Finally we consider the corrections to the charged Higgs mass sum rule. In figs. 12–14 we plot the shift in the charged Higgs squared mass due to radiative corrections, \( \Delta m_{H^\pm}^2 \) [see eq. (6.21)], as a function of \( \tan \beta \) for various choices of \( A \) and \( \mu \). We choose \( m_t = 2m_W \) and \( M_{\tilde{Q}} = M_{\text{SUSY}} = 1 \) TeV. Note that in our RGE analysis \( \Delta m_{H^\pm}^2 \) is independent of \( m_{A^0} \). For large values of \( \tan \beta \) (say, \( m_t \tan \beta \geq m_W \)) the leading log term proportional to \( m_t^2 m_b^2 / (m_W^2 s_\beta^2 c_\beta^2) \) [see eq. (3.12)] dominates and causes \( m_{H^\pm} \) to increase above its tree-level value (except in certain cases where the effects of squark mixing become very significant). For more moderate values of \( \tan \beta \), a different term in the leading log radiative corrections, proportional to \( m_t^4 / s_\beta^2 \), lowers \( m_{H^\pm} \) below its tree-level value. In addition, there is a negative contribution to \( \Delta m_{H^\pm}^2 \) proportional to \( m_t^4 \) due to squark-mixing [see eq. (6.23)]. As a result, there exists a regime in the SUSY parameter space where \( \Delta m_{H^\pm}^2 < -m_W^2 \) [see figs. 12 and 14] which would imply that \( m_{H^\pm} < m_{A^0} \). For example, for \( m_t = 2m_W \), \( \tan \beta = 1 \) and \( \mu = 2.5M_{\text{SUSY}} \), we find \( m_{H^\pm} \approx m_{A^0} \). Note that the radiative corrections in the neutral Higgs sector can impose constraints on the parameter space that depend on \( m_{A^0} \) (see fig. 11). These constraints rule out the
area below and to the right of the solid curves in figs. 12 and 14.

8. One-Loop Leading Log vs. RGE-Improved Results

In this section we compare the RGE-improved results (equivalent to summing leading logs to all orders in perturbation theory) to the one-loop leading log result. The one-loop leading log result is obtained as described in section 3 by solving the RGEs iteratively to first order [see eq. (3.5)]. Such terms correspond precisely to a complete one-loop perturbative calculation (with no RGE-improvement) where only the leading log terms are retained. However, there is an ambiguity to this procedure when it comes to diagonalizing the CP-even mass matrix [eq. (3.10)]. If one is performing a full one-loop computation with no RGE-improvement, then one should diagonalize this matrix perturbatively and obtain expressions for the masses that only involve terms to first order in the leading logs. On the other hand, a better approximation is to diagonalize the one-loop leading log CP-even mass matrix exactly. The resulting mass and mixing angle will differ from those obtained by perturbative diagonalization of the mass matrix by terms that are formally of higher order in perturbation theory. However, this result will be a better approximation to the full RGE-improved result, as we shall demonstrate below. Similar considerations have been touched upon briefly in ref. [16].

We introduce the following notation. The one-loop leading log CP-even mass matrix [eq. (3.10)] can be diagonalized exactly to obtain the neutral Higgs masses and mixing angle. These results will be denoted by the abbreviation 1LL. On the other hand, if we diagonalize eq. (3.10) perturbatively and only retain the leading logs to first order, we will use the abbreviation 1LLP. For example, from eq. (3.10), we find

\[
(m_h^2)_{1LP} = (m_h^2)_{\text{tree}} + \frac{g_2^2 m_Z^2}{96 \pi^2 c_w^2} \left\{ 12 N_c \frac{m_h^2 s_\alpha^2}{m_Z^2 c_\beta^2} - 6 N_c c_\alpha c_\beta \frac{m_Z^2 c_\alpha}{m_Z^2 c_\beta} \right. \\
+ (P_f + P_g + P_{2H})(s_\beta c_\alpha^2 + s_\alpha c_\beta^2) - 2 s_\beta c_\beta s_\alpha c_\alpha (P_f + P_g + P_{2H}^\prime) \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) \\
+ \left[ 12 N_c \frac{m_4^2 c_\alpha^2}{m_Z^2 s_\beta^2} - 6 N_c s_\alpha c_\alpha + \frac{m_t^2 c_\alpha}{m_Z^2 s_\beta} + s_\alpha c_\beta P_t \right] \ln \left( \frac{M_{\text{SUSY}}^2}{m_t^2} \right) \right\},
\]

where \((m_h^2)_{\text{tree}}\) is the tree-level Higgs mass with \(\tan \beta\) and \(m_{A\phi}\) as the physical input parameters.\(^\star\) The CP-even mixing angle \(\alpha\) and \((m_h^2)_{\text{tree}}\) are the tree-level

\(^\star\) Of course, one has to use a definition of \(\tan \beta\) that is consistent with \(\tan \beta = v_2/v_1\) at the leading log level. Eq. (5.1) is an example of such a definition.
values defined as:

\[(\tan 2\alpha)_{\text{tree}} = \tan 2\beta \left( \frac{m_{A^0}^2 + m_Z^2}{m_{A^0}^2 - m_Z^2} \right),\]

\[(m^2_{h^0})_{\text{tree}} = \frac{1}{2} (m_Z^2 + m_{A^0}^2) \left[ 1 + \frac{\sin 2\beta}{(\sin 2\alpha)_{\text{tree}}} \right].\] (8.2)

Eq. (8.1) plus non-leading log terms is the result one would obtain by performing a full one-loop calculation.

In fig. 2 we see that the 1LL results are somewhat larger than the fully integrated RGE-improved results. This can be understood by examining the dominant contributions to \(\beta_{\lambda_2}\). Here it is important to note that the self-coupling constant \(\lambda_2\) becomes large at low energies and cannot be neglected. From eq. (A.2) we approximately have

\[16\pi^2 (d\lambda_2)/dt \approx 6\lambda_2^2 - 6h_t^4 + 6h_t^2\lambda_2.\] (8.3)

This equation has an infrared fixed point when the right-hand side vanishes \(i.e., h_t^2 \approx \frac{1}{2} (1 + \sqrt{5})\lambda_2\) or \(m_{h^0} \approx 1.11m_{s\beta}\). Thus the logarithmic growth of the RGE result for \(m_{h^0}\) flattens out for large \(M_{\text{SUSY}}\) as \(m_{h^0}\) approaches its fixed point value [see fig. 2(c),(d)] [45].

In fig. 15 we plot \(m_{h^0}\) as a function of \(\tan \beta\) for various values of \(m_{A^0}\), \(m_t\) and \(M_{\text{SUSY}}\). We contrast the various methods for obtaining the radiatively corrected value for \(m_{h^0}\). The solid curve corresponds to the RGE-improved result obtained by numerically solving the renormalization group equations for the \(\lambda_i\) as described in section 3. The dashed curves (1LL) correspond to the radiatively corrected \(m_{h^0}\) obtained from exactly diagonalizing the one-loop leading log mass matrix [eq. (3.10)] and the dotted curve (1LLP) corresponds to the one-loop perturbative result given in eq. (8.1). These results provide one of the main motivations for this paper. The comparison of the RGE and 1LL masses shows only a small difference in the predicted mass in the case of \(M_{\text{SUSY}} = 1\) TeV. Note that the one-loop perturbative formula for \(m_{h^0}\) [eq. (8.1)] begins to differ substantially from the RGE and 1LL results for low and moderate values of \(\tan \beta\). The comparison of 1LL and 1LLP shows agreement in the large \(\tan \beta\) limit as expected. Furthermore, in the large \(m_{A^0}\) limit (in fig. 15 we take \(m_{A^0} = 300\) GeV) and arbitrary \(\tan \beta\), the difference between the 1LL and 1LLP masses is suppressed by a factor of \(O(m_{h^0}^2/m_{A^0}^2)\) and is only significant for large \(m_t\) and \(M_{\text{SUSY}}\). However, notice the extremely large difference between \((m_{h^0})_{\text{1LL}}\) and \((m_{h^0})_{\text{1LLP}}\) in the case of small \(m_{A^0}\) and small \(\tan \beta\). In particular, in contrast to the RGE and 1LL results, \((m_{h^0})_{\text{1LLP}}\) is independent of \(m_{A^0}\) when \(\tan \beta = 1\).
For values of $M_{\text{SUSY}}$ much above 1 TeV [e.g., $M_{\text{SUSY}} = 10$ TeV in fig. 15(c) and (d)], we begin to see an appreciable deviation between the RGE and the 1LL results when $m_{A^0} > m_Z$. In contrast, for $m_{A^0} \ll m_Z^2$, the RGE and 1LL results are roughly the same. This is easily understood—in this case, the lightest Higgs boson is dominantly $H_1$ and its squared mass is $m_{h^0}^2 \sim M_{11}^2$. Since there are no large $m_t$ corrections to $M_{11}^2$, the radiative corrections to $M_{11}^2$ are modest and thus exhibit very weak dependence on $m_t$ and $M_{\text{SUSY}}$. As a result, the RGE and 1LL results are roughly the same.

To summarize, we have compared three methods for obtaining the radiatively corrected $m_{h^0}$. Clearly, the RGE-improved result should be the most complete of the three methods examined. The 1LL results obtained by exactly diagonalizing eq. (3.10) yields results rather close to the RGE-improved results unless $M_{\text{SUSY}} \gg 1$ TeV.

Of the three methods for computing the Higgs mass discussed in this section, the fully integrated RGE analysis provides the best approximation. How accurate will such an approximation be? To fully answer this question requires a detailed investigation of the non-leading logarithmic terms that have been neglected in our analysis. In this paper, we have identified the most important terms of this type only in the limit of large squark mixing. To make further progress requires a complete one-loop computation [5,14,15]. The most accurate values for the radiatively corrected Higgs mass would then be obtained by identifying the complete set of non-leading logarithmic terms (say by subtracting the results of eqs. (3.10)–(3.12) from the corresponding exact one-loop calculation) and adding these to our fully-integrated RGE results. By comparing our results to those of a full one-loop calculation, we conclude that the effect of the non-leading corrections on the neutral CP-even Higgs masses is typically no more than a few GeV. Clearly, the relative importance of such terms becomes less significant as $M_{\text{SUSY}}$ increases.

We can improve our results slightly by identifying the largest of the non-leading logarithmic corrections not yet included. In the case of the CP-even Higgs mass-squared matrix, when $m_t \gg m_W$, the largest of such terms are ones of $\mathcal{O}(g_2^2 m_t^2)$. From a full one-loop perturbative computation,

$$\mathcal{M}^2 = \mathcal{M}_{\text{RGE}}^2 + \frac{N_c g_2^2 m_t^2}{48\pi^2 s_\beta^2 c_\beta^2 c_W} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$  (8.4)

where $\mathcal{M}_{\text{RGE}}^2$ is the RGE-improved result [i.e., eq. (2.10)] with the $\lambda_i$ obtained by

* For $M_{\text{SUSY}} = 1$ TeV the largest discrepancy between these two methods occurs for large $m_t$ and $m_{A^0} \gg m_Z$. For example, for $\tan \beta = 1$, $m_{A^0} = 300$ GeV and $m_t = 200$ GeV, we find $(m_{h^0})_{\text{RGE}} = 96.8$ GeV while $(m_{h^0})_{\text{1LL}} = 104.4$ GeV. In contrast, for the same set of parameters we find $(m_{h^0})_{\text{1LLP}} = 111.5$ GeV.
numerically solving the RGEs). The shift of the light Higgs mass due to these non-leading log corrections is of the order of 1 GeV. The largest non-leading log corrections to $m_{H^\pm}^2$ are given in ref. [40]. With this final improvement, our formulae for radiatively corrected Higgs masses are quite accurate in nearly all regions of SUSY parameter space for $M_{\text{SUSY}} \gtrsim 400$ GeV.

9. Conclusions

We have calculated the dominant radiative corrections to the Higgs sector parameters in the MSSM. We have obtained analytic formulae for the one-loop leading logarithmic corrections as a function of the various supersymmetric parameters. Our analysis also includes the most important non-logarithmic effects due to squark mixing. In our numerical analysis, we have focused on the case where all the supersymmetric partners are approximately mass degenerate. Summation of the leading logarithms to all orders in perturbation theory is achieved by solving the RGEs numerically. Non-leading logarithmic corrections due to squark mixing effects are included by computing the Higgs 4-point functions at the scale $M_{\text{SUSY}}$ and modifying the supersymmetric boundary condition.

The most significant contribution to the radiative corrections to $m_{h^0}$ grows with $m_t^4$ and logarithmically with $M_{\text{SUSY}}$. A number of the important phenomenological implications of the Higgs radiative corrections have already been obtained elsewhere (see e.g., refs. [9,12, and 46]). The MSSM cannot be ruled out if LEP-200 fails to discover the Higgs boson with a mass $m_{h^0} \lesssim m_Z$. In particular, if $m_t \gtrsim 150$ GeV, then $m_{h^0} \gtrsim m_Z$ for large $m_{A^0}$ and tan $\beta$ if $M_{\text{SUSY}}$ is sufficiently above $m_t$. For small values of $m_{A^0}$ ($\lesssim 30$ GeV) and small values of tan $\beta$ ($\lesssim 1$) the light Higgs mass can be larger than $2m_{A^0}$. In this case the decay $h^0 \rightarrow A^0A^0$ is kinematically allowed and provides the dominant decay mode over a significant region of the parameter space. The leading log corrections to $m_{H^\pm}$ only grow with $m_t^2$. However, there are non-logarithmic contributions due to squark mixing effects that grow as $m_t^4$. These effects could (in extreme cases) yield $m_{H^\pm} \lesssim m_{A^0}$. Large squark mixing effects can also lower the experimental lower bounds on $m_{h^0}$ in the large tan $\beta$ region.

Numerical comparison of the first order leading log corrections obtained in this paper with the full one-loop radiative corrections in the limit of large tan $\beta$ [5,14,15] shows agreement within a few percent or better for $M_{\text{SUSY}} \gtrsim 400$ GeV. The non-leading log terms not included in our analysis are thus of the same order as higher order leading log terms which are summed by renormalization group improvement and both should be included in any full one-loop calculation. The one-loop leading log expressions also serve as a useful check of any complete one-loop computation.
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APPENDIX A

Renormalization Group Equations

The $\beta$-functions in the non-supersymmetric two-Higgs-doublet model with the discrete symmetry $\Phi_1 \to -\Phi_1$ and in the MSSM are well known [41,31,47,48]. Here, we generalize these results in two respects. First, we take the most general (CP-conserving) two-Higgs-doublet potential [i.e., with no discrete symmetry] given in eq. (2.2). Second, we explicitly treat supersymmetric particle contributions to the $\beta$-functions in the step-approximation. That is, a particular SUSY contribution is omitted for scales below the corresponding SUSY particle mass [49]. We then find

$$16\pi^2\beta_{\lambda_1} = \left\{ 6\lambda_1^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \lambda_5^2 + 12\lambda_6^2 + \frac{3}{8}[2g_4^4 + (g_2^2 + g_4^2)^2] \right\} \theta_Z$$

$$+ \sum_i N_c \left\{ -2h_{D_i}^4 \theta_Z + \left( h_{D_i}^2 - \frac{1}{4}g_1^2 Y_{D_i} \right)^2 \theta_{D_i} + \left( \frac{1}{4}g_1^2 Y_{U_i} \right)^2 \theta_{U_i} \right. $$

$$+ \left[ h_{D_i}^4 - \frac{1}{2}h_{D_i}^2 (g_1^2 Y_{Q_i} + g_2^2) + \frac{1}{8}(g_1^4 + g_1^4 Y_{Q_i}^2) \right] \theta_{Q_i} \right\}$$

$$- \frac{5}{2}g_2^2 \theta_{H} \theta_{W} - g_1^2 g_2^2 \theta_{H} \theta_{B} - \frac{1}{2}g_1^4 \theta_{H} \theta_{B} - 32\pi^2 \lambda_1 \gamma_1. $$

(A.1)

$$16\pi^2\beta_{\lambda_2} = \left\{ 6\lambda_2^2 + \lambda_3^2 + (\lambda_3 + \lambda_4)^2 + \lambda_5^2 + 12\lambda_6^2 + \frac{3}{8}[2g_4^4 + (g_2^2 + g_4^2)^2] \right\} \theta_Z$$

$$+ \sum_i N_c \left\{ -2h_{U_i}^4 \theta_U \theta_Z + \left( \frac{1}{4}g_1^2 Y_{D_i} \right)^2 \theta_{D_i} + \left( h_{U_i}^2 + \frac{1}{4}g_1^2 Y_{U_i} \right)^2 \theta_{U_i} \right. $$

$$+ \left[ h_{U_i}^4 + \frac{1}{2}h_{U_i}^2 (g_1^2 Y_{Q_i} - g_2^2) + \frac{1}{8}(g_1^4 + g_1^4 Y_{Q_i}^2) \right] \theta_{Q_i} \right\}$$

$$- \frac{5}{2}g_2^2 \theta_{H} \theta_{W} - g_1^2 g_2^2 \theta_{H} \theta_{B} - \frac{1}{2}g_1^4 \theta_{H} \theta_{B} - 32\pi^2 \lambda_2 \gamma_2, $$

(A.2)

$$16\pi^2\beta_{\lambda_3} = \left\{ (\lambda_1 + \lambda_2)(3\lambda_3 + \lambda_4) + 2\lambda_3^2 + \lambda_3^2 + \lambda_5^2 + 2\lambda_6^2 + 2\lambda_7^2 + 8\lambda_6 \lambda_7 + \frac{3}{8}[2g_4^4 + (g_2^2 - g_1^2)^2] \right\} \theta_Z$$

$$+ \sum_i N_c \left\{ -2h_{U_i}^4 \theta_U \theta_Z + \left( \frac{1}{4}g_1^2 Y_{D_i} \right)^2 \theta_{D_i} + \left( h_{U_i}^2 + \frac{1}{4}g_1^2 Y_{U_i} \right)^2 \theta_{U_i} \right. $$

$$- \left[ h_{U_i}^4 + \frac{1}{4}h_{U_i}^2 (g_1^2 Y_{Q_i} - g_2^2) + \frac{1}{8}(g_1^4 + g_1^4 Y_{Q_i}^2) \right] \theta_{Q_i} \right\}$$

$$- \frac{5}{2}g_2^2 \theta_{H} \theta_{W} + g_1^2 g_2^2 \theta_{H} \theta_{B} - \frac{1}{2}g_1^4 \theta_{H} \theta_{B} - 16\pi^2 \lambda_3 (\gamma_1 + \gamma_2), $$

(A.3)
\[16\pi^2 \beta_{\lambda_4} = [\lambda_4 (\lambda_1 + \lambda_2 + 4\lambda_3 + 2\lambda_4) + 4\lambda_3^2 + 5\lambda_6^2 + 5\lambda_7^2 + 2\lambda_6\lambda_7 + \frac{3}{2}g_2^2g_1^2] \theta_Z + \sum_i N_c \left\{ 2h_{U_i}^2, h_{D_i}^2, \theta_{U_i} \theta_Z - h_{U_i}^2, h_{D_i}^2, \theta_{D_i} - (h_{U_i}^2 - \frac{1}{2}g_2^2)(h_{D_i}^2 - \frac{1}{2}g_2^2) \theta_{Q_i} \right\} + 2g_2^2\theta_{W_+} - 2g_2^2\theta_{B_+} - 16\pi^2\lambda_4(\gamma_1 + \gamma_2), \]  
(A.4)

\[16\pi^2 \beta_{\lambda_5} = [\lambda_5 (\lambda_1 + \lambda_2 + 4\lambda_3 + 6\lambda_4) + 5(\lambda_6^2 + \lambda_7^2) + 2\lambda_6\lambda_7] \theta_Z - 16\pi^2\lambda_5(\gamma_1 + \gamma_2), \]  
(A.5)

\[16\pi^2 \beta_{\lambda_6} = [\lambda_6 (6\lambda_1 + 3\lambda_3 + 4\lambda_4 + 5\lambda_5) + \lambda_7(3\lambda_3 + 2\lambda_4 + \lambda_5)] \theta_Z - 8\pi^2\lambda_6(3\gamma_1 + \gamma_2), \]  
(A.6)

\[16\pi^2 \beta_{\lambda_7} = [\lambda_7 (6\lambda_2 + 3\lambda_3 + 4\lambda_4 + 5\lambda_5) + \lambda_6(3\lambda_3 + 2\lambda_4 + \lambda_5)] \theta_Z - 8\pi^2\lambda_7(\gamma_1 + 3\gamma_2); \]  
(A.7)

where

\[\theta_X \equiv \theta \left( \ln \frac{s}{M_X^2} \right) = \begin{cases} 1 & \text{for } s \geq M_X^2, \\ 0 & \text{for } s < M_X^2, \end{cases} \]  
(A.8)

and \(M_X\) is the mass of the field \(X\). To first order in \(M_{\text{weak}}^2/M_{\text{SUSY}}^2\), we can ignore mixing in the supersymmetric mass matrices. In this case, the supersymmetric particle spectrum consists of the gluino \((\tilde{g})\), neutralinos \((\tilde{B}, \tilde{W}^\pm, \tilde{H}_1^0, \tilde{H}_2^0)\), charginos \((\tilde{W}^\pm, \tilde{H}^\pm)\), L-type squarks and sleptons \((\tilde{Q}_i^l, \tilde{Q}_i^s)\), and R-type squarks and sleptons \((\tilde{U}_i, \tilde{D}_i)\). The corresponding supersymmetric particle masses are given by: \(M_{\tilde{g}} = |M_3|, M_{\tilde{B}} = |M_1|, M_{\tilde{W}} = |M_2|, M_{\tilde{H}} = |\mu|\), the L-type squarks [sleptons] are degenerate with mass \(M_{\tilde{Q}_i}\), and the R-type squark [slepton] masses are \(M_{\tilde{U}_i}\) and \(M_{\tilde{D}_i}\). The label \(i\) runs over three generations and counts both squarks and sleptons; the corresponding fermion partners (i.e., quarks and leptons) are denoted by \(U_i\) and \(D_i\). The up- and down-type Yukawa couplings are proportional to the corresponding quark [lepton] masses

\[h_{U_i} = \frac{g_{U_i} m_{U_i}}{\sqrt{2m_W \sin \beta}}, \quad h_{D_i} = \frac{g_{D_i} m_{D_i}}{\sqrt{2m_W \cos \beta}}. \]  
(A.9)

The color factor \(N_c = 3 [1]\) when the label \(i\) refers to (s)quarks [(s)leptons], and the hypercharges of the squarks [sleptons] are \(Y_{Q_i} = \frac{1}{3} [-1], Y_{D_i} = \frac{2}{3} [2]\) and \(Y_{U_i} = -\frac{4}{3}\) [there is no \(\tilde{\nu}_R\)]. Finally, the anomalous dimensions \(\gamma_j (j = 1, 2)\) in the Landau
The $\beta$-functions for the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ gauge couplings $g_1 \equiv g'$, $g_2 \equiv g$ and $g_3 \equiv g_s$ are

$$
48\pi^2 \beta_{g_1} = \left( \frac{1}{4} \sum_i N_{c_i} \left[ 2Y_Q^2 \left( 2\theta_t + \theta_{Q_i} \right) + Y_U^2 \left( 2\theta_t + \theta_{U_i} \right) + Y_D^2 \left( 2\theta_t + \theta_{D_i} \right) \right] + \frac{1}{2} N_H \theta_t + N_{\tilde{H}} \theta_{\tilde{H}} \right) g_1^4,
$$

$$
48\pi^2 \beta_{g_2} = \left( \frac{1}{2} \sum_i N_{c_i} \left( 2\theta_t + \theta_{Q_i} \right) + \frac{1}{2} N_H \theta_t + N_{\tilde{H}} \theta_{\tilde{H}} + 4N_{\tilde{q}} \theta_{\tilde{W}} - 22\theta_t \right) g_2^4,
$$

$$
48\pi^2 \beta_{g_3} = \left( \sum_i c_i \left( 2\theta_{U_i} + 2\theta_{D_i} + \theta_{Q_i} + \frac{1}{2} \theta_{U_i} + \frac{1}{2} \theta_{D_i} \right) + 6N_{\tilde{q}} \theta_{\tilde{g}} - 33 \right) g_3^4,
$$

where $c_i = 1$ [0] for color triplet [singlet] fermions and their supersymmetric partners. The number of Higgs doublets is $N_H = 2$, the number of higgsino doublets is $N_{\tilde{H}} = 2$, the number of fermionic left-handed triplets is $N_{\tilde{q}} = 1$ (the gluino $\tilde{g}$) and the number of fermionic color octets is $N_{\tilde{g}} = 1$ (the gluino $\tilde{g}$). The $\beta$-functions for $g_1$ and $g_2$ are valid for scales $\sqrt{s} > m_t$ where the electroweak gauge theory is unbroken; for the $\beta$-functions below top quark-threshold see Appendix B. The boundary conditions are $g_k^2(m_Z^2) = 4\pi/(128c_k^2)$, $4\pi/(128s_k^2)$ and $1.38$ for $k = 1, 2, 3$, respectively. We take $s_{\tilde{W}}^2 = 0.23$ in our numerical work. One can explicitly check that eq. (3.3) is satisfied for scales larger than the largest SUSY mass parameter.

It is important to clarify the validity of the above formulae with explicit $\theta$-function treatment of thresholds. If all supersymmetric particle masses are roughly degenerate and $m_t \simeq m_Z$, then integration of the above formulae will correctly sum all leading logarithmic contributions to all orders in perturbation theory. However,
if there are non-degenerate thresholds this is no longer the case. For example, the
treatment of the top quark threshold is discussed in Appendix B. Below this thresh-
old the electroweak gauge group is broken and thus the effective low-energy theory
now has many new terms which require the introduction of new couplings. One
could work out the complete set of RGEs, including those for all couplings (with
appropriate boundary conditions at $\sqrt{s} = m_t$ corresponding to the requirement of
electroweak gauge symmetry). In practice, such a procedure would be overkill; after
all, $m_t$ is not much larger than $m_Z$. In Appendix B we show how to correctly ob-
tain the one-loop leading logarithmic terms proportional to $g_2^2 \ln(m_t^2/m_Z^2)$. These
are certainly sufficient for our purposes.

A similar set of remarks hold for the supersymmetric particle thresh olds.* Let
$M_{\text{SUSY}}^\text{max}$ be the largest supersymmetry-breaking mass parameter in the MSSM. Then
for $\sqrt{s} < M_{\text{SUSY}}^\text{max}$, we should in principle consider a new low-energy effective theory
with vertices that are no longer constrained by SUSY. This would introduce many
new couplings, and the RGEs for these couplings would be required in order to
sum logarithmic terms such as $\ln(M_i^2/M_j^2)$ to all orders. ($M_i$ and $M_j$ are masses
of different supersymmetric particles.) Again, this is much more than we need.
In deriving the RGEs above we have implicitly assumed supersymmetric relations
among the various couplings of the theory for all $M_{\text{SUSY}}^\text{min} < \sqrt{s} < M_{\text{SUSY}}^\text{max}$ (where
we have assumed that the mass of the lightest supersymmetric particle, $M_{\text{SUSY}}^\text{min}$ lies above $m_Z$). Integrating the RGEs listed above will then yield terms of order
$g_2^2 \ln(M_i^2/M_j^2)$, although these terms will not be correctly summed to all orders.†
On the other hand, the set of RGEs corresponding to the (non-supersymmetric)
two-Higgs-doublet model (i.e., for $\sqrt{s} \leq M_{\text{SUSY}}^\text{min}$) is sufficient to sum to all orders
logarithmic terms proportional to $\ln(M_{\text{SUSY}}^2/M_{\text{weak}}^2)$, where $M_{\text{SUSY}}$ is some aver-
age supersymmetry breaking mass. Results obtained in this manner are certainly
accurate enough for our purposes.

Thus, to complete the set of RGEs required for the computation of the $\lambda_i(m_Z)$,
we need RGEs for the Yukawa couplings corresponding to the (non-supersymmetric)
two-Higgs-doublet model [41,31,47]. These are given by

\begin{align}
16\pi^2 \beta_{h_i^2} &= \left(\frac{9}{2} h_i^2 + \frac{1}{2} h_0^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{17}{12} g_1^2\right) h_i^2, \\
16\pi^2 \beta_{h_0^2} &= \left(\frac{9}{2} h_0^2 + \frac{1}{2} h_i^2 + h_\tau^2 - 8 g_3^2 - \frac{9}{4} g_2^2 - \frac{5}{12} g_1^2\right) h_0^2, \\
16\pi^2 \beta_{h_\tau^2} &= \left(\frac{5}{2} h_\tau^2 + 3 h_0^2 - \frac{9}{4} g_2^2 - \frac{15}{4} g_1^2\right) h_\tau^2.
\end{align}

\[(A.12)\]

* In the numerical work presented in this paper, all supersymmetric masses are taken to be
degenerate. Thus the considerations below do not affect these results.
† Such terms are typically not as significant as the terms proportional to $g_2^2 \ln(m_t^2/m_Z^2)$, which
appear multiplied by powers of $m_t/m_Z$ in the formulae for Higgs masses and couplings.
All other Yukawa couplings can be neglected. Note that eq. (A.12) assumes that the Higgs–fermion interactions have the same structure as the ones in the MSSM. For completeness we present the \( \beta \)-functions for the mass parameters (although these are not required for any of the applications presented in this paper)

\[
16\pi^2 \beta_{m_{11}^2} = [3m_{11}^2 \lambda_1 + m_{22}^2 (2\lambda_3 + \lambda_4) - 6m_{12}^2 \lambda_6] - \gamma_1 m_{11}^2 \\
16\pi^2 \beta_{m_{22}^2} = [3m_{22}^2 \lambda_2 + m_{11}^2 (2\lambda_3 + \lambda_4) - 6m_{12}^2 \lambda_7] - \gamma_2 m_{22}^2 \\
16\pi^2 \beta_{m_{12}^2} = [(\lambda_3 + 2\lambda_4 + 3\lambda_5)m_{12}^2 - 3\lambda_6 m_{11}^2 - 3\lambda_7 m_{22}^2] - \frac{1}{2}(\gamma_1 + \gamma_2)m_{12}^2. \quad \text{cr}
\]

(A.13)

**APPENDIX B**

**Coupling Constant Evolution Below the Top Quark Threshold**

In this appendix we present the \( \beta \)-functions for the relevant gauge and Higgs self-interaction coupling constants below top quark-threshold. Note that by formally integrating out the top-quark, we include logarithmic contributions to the Higgs mass matrix of \( \mathcal{O}(g_2^2 m_Z^2 \ln(m_t^2/m_Z^2)) \). However, it is important to emphasize that such terms are formally smaller or of similar size to some non-leading log terms, e.g., terms of \( \mathcal{O}(g_2^2 m_t^2) \). [The latter can be included easily according to eq. (8.4).] Nevertheless, isolating such logarithmic terms serves as a useful check of more complete one-loop computations.

First consider the \( \beta \)-functions for the Higgs self-couplings. These determine the running of the \( \lambda_i \); the values of \( \lambda_i(M_{\text{weak}}) \) enter the calculations of the neutral and charged Higgs parameters. In calculations involving the neutral Higgs sector the contributions from the top quark can be removed by setting \( h_t = 0 \). In the charged sector we note that some (but not all) of the pieces proportional to \( h_b^2 \) arise from \( t-b \) loops and should also be removed. For the purpose of the calculations presented here, the simpler procedure of setting \( h_t = 0 \) for the neutral Higgs processes and \( h_t = h_b = 0 \) for charged Higgs processes suffices.‡

Next we look at the \( \beta \)-functions of the gauge coupling constants. First we note that below the top quark-threshold the SU(2)\(_L\) \( \times \) U(1)\(_Y\) gauge symmetry is broken down to U(1)\(_{\text{EM}}\). The coupling constants of the photon to charged particles are

‡ In a more precise procedure one would have to write down the most general potential for four neutral fields and four charged fields invariant under U(1)\(_{\text{EM}}\) and determine the bottom quark contributions to the \( \beta \)-function for each coupling constant separately. The boundary conditions for these coupling constants are obtained by requiring that the potential reduces to eq. (2.2) at a scale \( \sqrt{s} = m_t \).
constrained by U(1)\textsubscript{EM} gauge invariance. In contrast, the couplings of the $W$ and $Z$ are now independent parameters which thus can evolve differently from $m_t$ to $m_Z$. To determine the evolution of the gauge couplings, we fix as experimental inputs the masses of the $W$ and $Z$ bosons $m_Z$ and $m_W$ which arise through the interaction

$$\mathcal{L} = \frac{1}{4} \left( \frac{1}{2} g^{ij}_k V^i_\mu V^j_\mu + G^\pm_{kl} W^+_- W^\mu_- \right) H^0_k H^0_l,$$

where $V_i = (W^3, B)$ are the neutral SU(2)$_L$ and U(1)$_Y$ gauge fields. At the scale $m_t$ where the SU(2)$_L \times$ U(1)$_Y$ gauge symmetry is restored we impose the boundary conditions

$$G^\pm_{kl} = \delta_{kl} g_2^2,$$
$$G^i_{kl} = \delta_{kl} \bar{g}_i \bar{g}_j \quad \text{with} \quad \bar{g}_i = (g_1, -g_2).$$

In general the $\beta$-functions corresponding to the four-point interactions, eq. (B.1), have two types of contributions: the box diagrams and the wave function renormalizations. In the following we consider only diagrams involving the top-quark. It is clear that the quark loop contributions to the wave function renormalization of the Higgs boson and the box diagrams to $G^i_{kl}$ cancel since the $\beta_{g_2}$ in eq. (A.11) do not contain pieces proportional to $h_t^2$ or $h_b^2$. Below $m_t$ we remove the $H^-tb$ interaction from the theory so that there are no vertex corrections to $G^\pm_{kl}$ from the bottom quark. However, there are self energy diagrams to both the gauge bosons and the Higgs bosons due to the bottom quark. In the regime where $\tan \beta \ll m_W/m_b$ (i.e., $h_b \ll g_2$) we can neglect the Higgs boson self energy diagram. Thus the only other diagrams one has to consider are the self energy diagrams for the gauge bosons. We assert that

$$G^\pm_{kl} = G^\pm \delta_{kl},$$
$$G^i_{kl} = G^i \delta_{kl}. \quad \text{(B.3)}$$

Then the RGEs of $G^{ij}$ and $G^\pm$ differ from the RGEs of the gauge coupling only by the top-quark contributions

$$\beta_{G^\pm} = \beta_{g_2} + G^\pm \gamma_t^\pm,$$
$$\beta_{G^{ij}} = \beta_{g_2} \delta_{ij} + \frac{1}{2} \left( G^{ik} \gamma^t_{kj} + G^{jk} \gamma^t_{ki} \right), \quad \text{(B.4)}$$

(no summation over $i$). The anomalous dimensions $\gamma_{ij}$ and $\gamma^\pm$ are defined as

$$\gamma^\pm \equiv \frac{d \ln (W^+^- W^\mu_-)}{dt} = \frac{d A'_WW(W^2)}{dt},$$
$$\gamma_{ij} \equiv \frac{d \ln (V^i_\mu V^j_\mu)}{dt} = \frac{d A'_{V_iV_j}(p^2)}{dt}, \quad \text{(B.5)}$$

where $A_{VV}$ ($V = V_i, W$) are proportional to the $g_{\mu\nu}$ term of the self-energies of
the gauge bosons and $A'_{VV}(p^2) \equiv dA_{VV}/dp^2$. The contributions involving the top-quark are

\begin{align}
\gamma_t^t &= -\frac{g_2^2}{16\pi^2}, \\
\gamma_{ij}^t &= -\frac{1}{32\pi^2}(a_i^L a_j^L + a_i^R a_j^R),
\end{align}

where $a_i^L = (g_2, \frac{1}{3}g_1)$ and $a_i^R = (0, \frac{4}{3}g_1)$.

\section*{APPENDIX C}

\subsection*{One-Loop Leading Logarithmic Higgs Self-Couplings}

In the analysis presented in this paper, the RGEs of Appendix A are solved numerically for $m_t < \sqrt{s} < M_{\text{SUSY}}$ and iteratively to one-loop order for $M_{\text{weak}} < \sqrt{s} < m_t$ using the RGEs described in Appendix B. However, it is instructive to solve the RGEs completely iteratively to one-loop order [see eq. (3.5)]. In obtaining the results below, we have carefully removed the top-quark from the low-energy effective theory at energy scales below $m_t$ as explained in section 3. We find

\begin{align}
\lambda_1(m_Z) &= \frac{1}{4}[g_1^2 + g_2^2](m_Z) + \frac{g_4^4}{384\pi^2 c_w^4} \left\{ 6s_w^2 \left( 1 - 2s_w^2 \right) t_{\tilde{H}}^2 - 24s_w^2 \left( 1 - s_w^2 \right) t_{\tilde{Z}}^2 \\
&- (42 - 102s_w^2 + 60s_w^4) t_{\tilde{Z}}^2 - 4 \left( 1 - 2s_w^2 + 2s_w^4 \right) t_{\tilde{H}}^2 - 8 \left( 1 - 2s_w^2 + s_w^4 \right) t_{\tilde{W}}^2 \\
+ \sum_i N_{c_i} \left[ \left( \frac{6m_D^4}{m_Z^4 c_\beta} + 12 \frac{m_D^2}{m_Z^2 c_\beta} \right) \frac{e_{D_i} s_w^2 + 4e_{D_i}^2 s_w^4}{4} \right] t_{\tilde{D}_i}^2 \\
+ \left( \frac{6m_D^4}{m_Z^4 c_\beta} - 6 \frac{m_D^2}{m_Z^2 c_\beta} \right) \left[ c_w^2 + s_w^2 (e_{U_i} + e_{D_i}) \right] + 1 + 4e_{D_i} s_w^2 + 4e_{D_i}^2 s_w^4 \right) t_{\tilde{Q}_i}^2 \\
+ 4e_{U_i}^2 s_w^4 t_{\tilde{U}_i} + (1 - 4e_{U_i} s_w^2 + 4e_{U_i}^2 s_w^4) t_{\tilde{Q}_i}^2 \right\}, \quad \text{(C.1)}
\end{align}
\[ \lambda_2(m_Z) = \frac{1}{4}(g_1^2 + g_2^2)(m_Z) + \frac{g_2^4}{384\pi^2 c_W^4} \left\{ 6s_{W}^2 (1 - 2s_{W}^2) t_{Z}^{\lambda_1} - 24s_{W}^2 (1 - s_{W}^2) t_{Z}^{\lambda_{12}} \\ - (42 - 102s_{W}^2 + 60s_{W}^4) t_{Z}^{\lambda_2} - 4 (1 - 2s_{W}^2 + 2s_{W}^4) t_{Z}^H - 8 (1 - 2s_{W}^2 + s_{W}^4) t_{Z}^W \\ + \sum_i N_c \left[ \left( 6 \frac{m_{U_i}^4}{m_{Z}^4 s_{Z}^4} - 12 \frac{m_{U_i}^2}{m_{Z}^2 s_{Z}^2} e_{U_i} s_{W}^2 + 4e_{U_i}^2 s_{W}^4 \right) t_{U_i} \right] \\ + \left( 6 \frac{m_{D_i}^4}{m_{Z}^4 s_{Z}^4} - 6 \frac{m_{D_i}^2}{m_{Z}^2 s_{Z}^2} c_{D_i} \right) \left[ s_{W}^2 - s_{W}^2 (e_{U_i} + e_{D_i}) \right] + 1 - 4e_{U_i} s_{W}^2 + 4e_{U_i} s_{W}^4 \right) t_{U_i} \right] \\ + 4e_{D_i} s_{W}^4 t_{Z}^\tilde{D_i} + (1 + 4e_{D_i} s_{W}^2 + 4e_{D_i} s_{W}^4) t_{Z}^{\tilde{Q}_i} \right) \} , \]

(C.2)

\[ \bar{\lambda}_3(m_Z) = -\frac{1}{4}(g_1^2 + g_2^2)(m_Z) - \frac{g_2^4}{384\pi^2 c_W^4} \left\{ 6s_{W}^2 (1 + 2s_{W}^2) t_{Z}^{\lambda_1} + 24s_{W}^2 (1 - s_{W}^2) t_{Z}^{\lambda_{12}} \\ + (30 - 42s_{W}^2 + 12s_{W}^4) t_{Z}^\bar{X} - 4 (1 - 2s_{W}^2 + 2s_{W}^4) t_{Z}^\bar{H} - 8 (1 - 2s_{W}^2 + s_{W}^4) t_{Z}^\bar{W} \\ + \sum_i N_c \left[ \left( -6 \frac{m_{U_i}^2}{m_{Z}^2 s_{Z}^2} + 4s_{W}^2 e_{U_i} \right) s_{W}^2 e_{U_i} t_{U_i}^\tilde{U_i} + \left( 6 \frac{m_{D_i}^2}{m_{Z}^2 c_{D_i}^2} + 4s_{W}^2 e_{D_i} \right) s_{W} e_{D_i} t_{Z}^\tilde{D_i} \right] \\ + \left( -3 \frac{m_{U_i}^2}{m_{Z}^2 s_{Z}^2} \right) \left[ c_{W}^2 - s_{W}^2 (e_{U_i} + e_{D_i}) \right] + 1 - 4e_{U_i} s_{W}^2 + 4e_{U_i} s_{W}^4 \right) t_{U_i}^{\tilde{Q}_i} \\ + \left( -3 \frac{m_{D_i}^2}{m_{Z}^2 c_{D_i}^2} \right) \left[ c_{W}^2 + s_{W}^2 (e_{U_i} + e_{D_i}) \right] + 1 + 4e_{D_i} s_{W}^2 + 4e_{D_i} s_{W}^4 \right) t_{Z}^{\tilde{Q}_i} \right] \} \]

(C.3)

We remind the reader that \( \bar{\lambda}_3 \equiv \lambda_3 + \lambda_4 + \lambda_5 \) (in the leading log approximation \( \lambda_5 = 0 \)), the number of colors \( N_c = 3 \) [1] and the electric charges of the quarks [leptons] are \( e_{U_i} = 2/3 \) [0] and \( e_{D_i} = -1/3 \) [-1]. In addition, we have introduced the following notation

\[ t_{Z}^X \equiv \ln \left( \frac{M_{Z}^2}{m_{X}^2} \right), \quad M_{X} \geq m_{Z} , \quad \]

(C.4)

\[ t_{U_i}^{X} \equiv \begin{cases} 
\ln \left( \frac{M_{U_i}^2}{m_{X}^2} \right), & i = 1, 2 \\
\ln \left( \frac{M_{D_i}^2}{m_{X}^2} \right), & i = 3 
\end{cases} \]
where $M_X = |M_1|, |M_2|, |\mu|, M_{\tilde{Q}_i}, M_{\tilde{U}_i}, M_{\tilde{D}_i}$ for $X = \tilde{B}, \tilde{W}, \tilde{H}, \tilde{Q}_i, \tilde{U}_i, \tilde{D}_i$. Furthermore, we have defined $\tilde{\chi}_1, \tilde{\chi}_2, \tilde{\chi}_{12}$ such that

$M_{\tilde{\chi}_1} = \mu_1 \equiv \max\{|\mu|, |M_1|\}$

$M_{\tilde{\chi}_2} = \mu_2 \equiv \max\{|\mu|, |M_2|\}$

$M_{\tilde{\chi}_{12}} = \mu_{12} \equiv \max\{|\mu|, |M_1|, |M_2|\}$

(C.5)

If $M_X < m_Z$ for some field $X$, simply set the corresponding $t_X^{X_Z} = 0$.

The couplings $\lambda_1, \lambda_2$, and $\tilde{\lambda}_3$ evaluated at the scale $m_Z$ appear in the CP-even neutral Higgs mass matrix and couplings. For the charged Higgs mass and couplings we require $\lambda_4$ evaluated at $m_W$. We find

$$
\lambda_4(m_W) = -\frac{1}{2} g_2^2(m_W) - \frac{g_2^4}{192 \pi^2} \left\{ -6t_W^2 + 6 \tan^2 \theta_W t_W^X + 24 \tan^2 \theta_W t_{W_{\tilde{X}_1}^{X}}^W 
\right. $$

$$
-4 t_W^H - 8 t_W^W + \sum_i N_c_i \left[ \frac{6 m_{U_i}^2 m_{D_i}^2}{m_W^2 s_W^2 c_W^2} - 3 \frac{m_{U_i}^2}{m_W^2 s_W^2} - 3 \frac{m_{D_i}^2}{m_W^2 c_W^2} + 2 \right] t_{\tilde{Q}_i} \right\}
$$

(C.6)

where the definitions of $t_W^X$ and $t_{U_i}^X$ are obtained from eq. (C.4) by replacing $m_Z$ with $m_W$. Finally, in the leading-log approximation, $\lambda_i(s) = 0$ for $i = 5, 6$ and 7 at all scales $\sqrt{s}$.

APPENDIX D

One-Loop Squark Contributions to the Scalar Potential

In this appendix we present the derivation of the Higgs four-point functions from the one-loop effective potential. The (one generation) squark mass matrix is given by

$$
\mathcal{M}^2 = \mathcal{M}_M^2 + \mathcal{M}_\Gamma^2 + \mathcal{M}_\Lambda^2,
$$

where

$$
(M_X^2)_{ab} \equiv \frac{\partial^2 \mathcal{V}_X}{\partial \Psi_a \partial \Psi_b}.
$$

(D.1)

for $X = M, \Gamma$ and $\Lambda$ and $\mathcal{V}_X$ given in eq. (6.2). In the special case $M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{Q}} \equiv M_{\text{SUSY}}$ [i.e., $M_{\tilde{M}}^2 = M^2_{\text{SUSY}}$] we can expand the effective potential
\[ V = V^0 + \frac{1}{32\pi^2} \text{tr} \left\{ M_{\text{SUSY}}^4 \left[ \ln \left( \frac{M_{\text{SUSY}}^2}{\mu^2} \right) - \frac{3}{2} \right] \text{tr} 1 \right\} 
\]
\[ + 2M_{\text{SUSY}}^2 \left[ \ln \left( \frac{M_{\text{SUSY}}^2}{\mu^2} \right) - 1 \right] \text{tr} \left( M_{\Lambda}^2 + M_{\Gamma}^2 \right) \]
\[ + \ln \left( \frac{M_{\text{SUSY}}^2}{\mu^2} \right) \text{tr} \left( M_{\Lambda}^2 + M_{\Gamma}^2 \right)^2 + \frac{1}{3M_{\text{SUSY}}^2} \text{tr} \left( M_{\Gamma}^2 + M_{\Lambda}^2 \right)^3 \]
\[ - \frac{1}{12M_{\text{SUSY}}^4} \text{tr} \left( M_{\Gamma}^2 + M_{\Lambda}^2 \right)^4 + \mathcal{O}(\Phi^6) \} \].

If we keep in mind that \( M_{\Gamma} \) (\( M_{\Lambda} \)) contains one (two) power(s) of \( \Phi \) we find the quartic terms of the potential

\[ V_{\text{quartic}} = \Gamma^{ij}_{ik} \left( \Phi_i^\dagger \Phi_j \right) \left( \Phi_k^\dagger \Phi_l \right) + \frac{1}{32\pi^2} \left\{ \ln \left( \frac{M_{\text{SUSY}}^2}{\mu^2} \right) \text{tr} \left( M_{\Lambda}^2 \right)^2 \right\} \]
\[ + \frac{1}{M_{\text{SUSY}}^2} \text{tr} \left( M_{\Gamma}^2 \right)^2 M_{\Lambda}^2 - \frac{1}{12M_{\text{SUSY}}^4} \text{tr} \left( M_{\Gamma}^2 \right)^4 \}. \]

The traces can now be computed without diagonalization of the mass matrix. It is now straightforward to absorb the one-loop corrections into the tree-level terms by redefining the Higgs tree-level coupling constants as indicated in eq. (6.13) and (6.14).

In the more general case of unequal diagonal squark mass parameters, the computation of \( V_{\text{quartic}} \) is more complicated. Here, one must compute the eigenvalues of the squark masses (perturbatively in \( M_{\text{weak}}^2/M_{\text{SUSY}}^2 \)) before taking the traces. Details of the more general computation and the resulting shifts in the Higgs tree-level couplings can be found in ref. [24].
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FIGURE CAPTIONS

1) RGE-improved Higgs mass $m_{h^0}$ as a function of $\tan \beta$ for (a) $m_t = 150$ GeV and (b) $m_t = 200$ GeV. Various curves correspond to $m_{A^0} = 0, 20, 50, 100$ and 300 GeV as labeled in the figure. All $A$-parameters and $\mu$ are set equal to zero. The light CP-even Higgs mass varies very weakly with $m_{A^0}$ for $m_{A^0} > 300$ GeV.

2) The range of allowed Higgs masses in the large $m_{A^0}$ limit (in our plots we take $m_{A^0} = 300$ GeV). The lower limit corresponds to $\tan \beta = 1$. The upper limit corresponds to the limit of large $\tan \beta$ (we take $\tan \beta = 20$). In Fig. 2(a) and (b) we vary $m_t$ and keep $M_{\text{SUSY}}$ fixed at 1 and 0.5 TeV, respectively. In Fig. 2(c) and (d) we vary $M_{\text{SUSY}}$ and keep $m_t$ fixed at 150 and 200 GeV, respectively. The solid (dashed) curves in (c) and (d) correspond to the computation in which the RGEs are solved numerically (iteratively to one-loop order).

3) RGE-improved Higgs masses $m_{h^0}$ and $m_{H^0}$ as a function of $m_{A^0}$, for $M_{\text{SUSY}} = 1$ TeV, $m_t = 150$ GeV and for various choices of $\tan \beta$.

4) Contours corresponding to (the radiatively corrected) Higgs mass $m_{h^0} = 20, 40, 60, 80, 100, 120$ and $140$ GeV as a function of $m_t$ and $m_{A^0}$. Results are presented for $M_{\text{SUSY}} = 1$ TeV and $A_t = A_b = \mu = 0$. We exhibit the cases of (a) $\tan \beta = 1$ and (b) $\tan \beta = 5$.

5) Contours for constant $m_{h^0}$ the $\tan \beta$–$m_{A^0}$ plane in the case $A_t = A_b = \mu = 0$. Two adjacent graphs corresponding to $M_{\text{SUSY}} = 0.5$ and 1 TeV, and three choices of $m_t$ are displayed in both cases. The contour labels are exhibited explicitly in the $M_{\text{SUSY}} = 1$ TeV graphs.

6) The factor $\cos^2(\beta - \alpha)$ as a function of $m_{A^0}$ for $\tan \beta = 0.5, 1, 2$ and 20 (dotted, dashed, dot-dashed and solid curves, respectively). Results are presented for $M_{\text{SUSY}} = 1$ TeV. We consider the case of (a) $m_t = 150$ GeV and (b) $m_t = 200$ GeV.

7) The neutral Higgs mass $m_{h^0}$ as a function of $\tan \beta$ for (a) $m_{A^0} = 50$ GeV and (b) $m_{A^0} = 300$ GeV, for $m_t = 150$ GeV and $M_{\tilde{Q}} = M_{\text{SUSY}} = 1$ TeV. All $A$-parameters are taken to be equal ($A = A_t = A_b$); the four contours shown correspond to $\mu = A = 0, 1, 2$ and 3 TeV, respectively.

8) As a function of $\tan \beta$, we plot (a) the neutral Higgs mass $m_{h^0}$ and (b) the factor $\sin^2(\beta - \alpha)$ for $m_t = 150$ GeV and $M_{\tilde{Q}} = M_{\text{SUSY}} = 1$ TeV. All $A$-parameters are taken to be equal ($A = A_t = A_b$); the four curves shown correspond to $\mu = A = 0, 1, 2$ and 3 TeV, respectively. The sum $m_{A^0} + m_{h^0} = m_Z$ is kept fixed in both plots.
9) The range of parameters in the \( \tan \beta - m_t \) plane where \( m_{h^0} = 2m_{A^0} = 40 \) GeV for various choices of \( \mu \) and the \( A \)-parameters, and \( M_{\text{SUSY}} = M_{\tilde{Q}} = 1 \) TeV.

10) The range of parameters in the \( \tan \beta - m_{A^0} \) plane where \( m_{h^0} = 2m_{A^0} \) and \( m_t = 150 \) GeV for various choices of \( \mu \) and the \( A \)-parameters, and \( M_{\text{SUSY}} = M_{\tilde{Q}} = 1 \) TeV.

11) Contours of constant \( m_{A^0} = 0, 40, 100 \) and \( 300 \) GeV in the \( \mu - \tan \beta \) plane for \( m_t = 150 \) GeV, \( M_{\tilde{Q}} = M_{\text{SUSY}} = 1 \) TeV. In (a) and (b) \( m_{h^0} = 0 \) for two choices of the \( A \) parameters, respectively. In (c) and (d) \( m_{h^0} = 40 \) GeV.

12) The shift in the charged Higgs squared mass, \( \Delta m_{H^\pm}^2 \), normalized to \( m_W^2 \) as a function of \( \mu \) and \( \tan \beta \) for \( m_t = 2m_W \) and \( M_{\tilde{Q}} = M_{\text{SUSY}} = 1 \) TeV. The \( A \)-parameters are set to zero. In our RGE analysis \( \Delta m_{H^\pm}^2 \) is independent of \( m_{A^0} \). In addition, we depict in (a) and (b), solid curves corresponding to \( m_{h^0} = 0 \), which depend on \( m_{A^0} \) as indicated in (a).

13) The shift in the charged Higgs squared mass, \( \Delta m_{H^\pm}^2 \), normalized to \( m_W^2 \) as a function of (a) the \( A \)-parameter and (b) \( \tan \beta \) for \( m_t = 2m_W \) and \( M_{\tilde{Q}} = M_{\text{SUSY}} = 1 \) TeV. The parameter \( \mu \) is set to zero.

14) The shift in the charged Higgs squared mass, \( \Delta m_{H^\pm}^2 \), normalized to \( m_W^2 \) as a function of (a) \( A = A_t = A_b = \mu \) and (b) \( \tan \beta \) for \( m_t = 2m_W \) and \( M_{\tilde{Q}} = M_{\text{SUSY}} = 1 \) TeV. In our RGE analysis \( \Delta m_{H^\pm}^2 \) is independent of \( m_{A^0} \). In addition, we depict in (a) and (b), solid curves corresponding to \( m_{h^0} = 0 \), which depend on \( m_{A^0} \) as indicated in (a).

15) One-loop corrected Higgs mass \( m_{h^0} \) as a function of \( \tan \beta \) for \( m_t = 200 \) GeV and 150 GeV [(c),(d)] and \( M_{\text{SUSY}} = 1 \) TeV and 10 TeV. The solid curves correspond to the fully integrated RGE-improved Higgs mass. The dashed curves correspond to the Higgs mass \( (m_{h^0})_{\text{1LL}} \) derived by exactly diagonalizing the one-loop leading log mass matrix [eq. (3.10)]. The dotted curves correspond to \( (m_{h^0})_{\text{1LLP}} \) given in eq. (8.1). In each case, the lower (upper) curves correspond to \( m_{A^0} = 20 \) (300) GeV.