**Abstract**

We present a recent calculation of the transverse momentum dependent gluon distributions inside unpolarised protons and show how the ratio of the linearly polarized and the unpolarized gluon distribution in the proton can be probed by looking at \( \cos(2\phi_h) \) asymmetry in \( J/\psi \) production in unpolarized \( ep \) collision. We use NRQCD for estimating \( J/\psi \) production and include contribution both from color singlet and color octet states.

1 Introduction

Transverse momentum dependent (TMD) parton distribution functions (PDFs) play an important role in understanding the spin and spatial structure of the proton. They not only provide the information about the longitudinal momentum fraction but also give us information about the internal transverse momentum \( k_\perp \) carried by partons. Unlike collinear PDFs, which are universal, TMDs are process dependent due to their initial and final state interactions. The two experiments where TMDs can be extracted are SIDIS and DY. In these experiments the observables of most interest are the single-spin asymmetries (SSA) and azimuthal asymmetries. The leading-twist distributions, unpolarised gluon TMD \( f_{1g}(x, k_\perp^2) \) and \( h_{1g}(x, k_\perp^2) \), the so-called Boer–Mulders function, are greatly relevant to these asymmetries. The function \( f_{1g}(x, k_\perp^2) \) represents the probability of finding an unpolarized gluon, within an unpolarized hadron, with a longitudinal momentum fraction \( x \) and transverse momentum \( k_\perp \), while as \( h_{1g}(x, k_\perp^2) \) represents the distribution of linearly polarized gluons within the unpolarized hadron.

Due mainly to the lack of experimental data, information on the \( h_{1g}(x, k_\perp^2) \) is very limited. Heavy quark pair or dijet production in SIDIS [1], diphoton pair [2] and \( \Upsilon(1S) \)+jet [3] production in \( pp \) collision have been suggested for extracting \( h_{1g}(x, k_\perp^2) \). It has been seen in these processes that measuring azimuthal asymmetries, we can probe \( h_{1g}(x, k_\perp^2) \).

Quarkonium production is an important tool to probe the gluon TMDs (see [4] for a recent review). In Ref. [5] the authors probed \( h_{1g} \) in \( \cos(2\phi_h) \) asymmetry in \( J/\psi \) production through the leading order (LO) process \( \gamma^* + g \rightarrow J/\psi \) at the future EIC at \( z = 1 \), where \( z \) measures the fraction of photon energy transferred to \( J/\psi \). This was extended to the region \( z < 1 \) in [6], where only the color singlet contribution within the non-relativistic QCD (NRQCD) was considered. In this article we review a recent calculation of the \( \cos(2\phi_h) \) asymmetry in \( J/\psi \) production in electron-proton collision [7]. The production of \( J/\psi \) is calculated in the NRQCD framework with the inclusion of both color singlet and color octet contributions. We
will be taking $\gamma^* + g \rightarrow J/\psi + g$ partonic subprocess into the consideration, and investigate mainly the small-$x$ region, where the gluon TMDs play a major role.

The rest of this paper is organized as follows. The analytical framework of our calculation is discussed in Section 2. In Section 3, we present our numerical results and finally, in Section 4, we conclude.

2 Azimuthal asymmetry in $J/\psi$ leptoproduction

The $\cos(2\phi_h)$ asymmetry for $e(l) + p(P) \rightarrow e(l') + J/\psi(P_h) + X(P_x)$ process is defined as

$$\langle \cos(2\phi_h) \rangle = \frac{\int d\phi_h \cos(2\phi_h) d\sigma}{\int d\phi_h d\sigma},$$

where $\phi_h$ is the azimuthal angle of $J/\psi$ production plane with the lepton plane and $d\sigma$ is the differential scattering cross section. We consider the frame in which the incoming proton and the virtual photon exchanged in the process move in $+z$ and $-z$ directions. The kinematics here is defined in terms of two light-like vectors with the help of a Sudakov decomposition, here chosen to be the momentum $P(= n_-)$ of the incoming proton, and a second vector $n(= n_+)$, obeying the relations $n \cdot P = 1$ and $n_+^2 = n_-^2 = 0$. Since at small-$x$ the proton is rich in gluons, the dominant partonic sub-process at NLO for the $J/\psi$ production is $\gamma^*(q) + g(k) \rightarrow J/\psi(P_h) + g(p_g)$. The differential scattering cross-section can be written as a convolution of leptonic tensor, a soft parton correlator for the incoming hadron and a hard part:

$$d\sigma = \frac{1}{2s} \frac{d^3l'}{(2\pi)^3 2E_l' (2\pi)^3 2E_{P_h}} \int \frac{d^3p_g}{(2\pi)^3 2E_g} \int dx dk_\perp (2\pi)^4 \delta(q + k - P_h - p_g)$$

$$\times \frac{1}{Q^4} \mathcal{L}^{\mu\nu}(l, q) \Phi^{\nu\nu'}(x, k_\perp) \mathcal{M}_{\mu\nu}(\mathcal{M}_{\mu\nu'})^*$$

The term $\mathcal{M}_{\mu\nu}$ represents the amplitude of $J/\psi$ production in the $\gamma^* + g \rightarrow J/\psi + g$ partonic sub-process and $\mathcal{L}^{\mu\nu}(l, q)$ is the leptonic tensor. At leading twist, the gluon correlator of the unpolarized proton contains two TMD gluon distribution functions

$$\Phi^{\nu\nu'}_{g/g}(x, k_\perp) = -\frac{1}{2x} \left\{ g^{\nu\nu'} f_1^g(x, k_\perp^2) - \left( \frac{k_\perp^2}{M_p^2} + g^{\nu\nu'} k_\perp^2 \right) h_1^{+g}(x, k_\perp^2) \right\}.$$  \hspace{1cm} (3)

Here $g^{\nu\nu'} = g^{\nu\nu'} - P^{\nu} n^{\nu'} / P \cdot n - P^{\nu'} n^{\nu} / P \cdot n$. Following Refs. \cite{7} and references within, to which we refer the reader for more details, we could write the final expression for the azimuthal asymmetry as:

$$\langle \cos(2\phi_h) \rangle \propto \frac{\int k_\perp dk_\perp \left( A_2 f_1^g(x, k_\perp^2) + \frac{k_\perp^2}{M_p^2} B_2 h_1^{+g}(x, k_\perp^2) \right)}{\int k_\perp dk_\perp \left( A_0 f_1^g(x, k_\perp^2) + \frac{k_\perp^2}{M_p^2} B_0 h_1^{+g}(x, k_\perp^2) \right)}.$$  \hspace{1cm} (4)

In the kinematics considered above we found that the unpolarized gluon distribution and the linearly polarized distributions are not disentangled, however the above result can be used for the extraction of their ratio.

3 Results and discussion

In this section, we present numerical estimates of the $\cos(2\phi_h)$ asymmetry in the kinematical region to be accessed at EIC and to avoid the contribution from virtual diagrams. We have
also imposed a lower cutoff $z > 0.1$ to avoid the gluon fragmentation contribution to $J/\psi$. We verified that changing the lower cutoff does not affect the asymmetry much. We took mass of the proton to be $M_p = 1$ GeV.

The contraction in the calculation above for the different states $i.e., \, ^1S^0_0, \, ^3S^1_1$ and $\, ^3P^0_0(8)$ is calculated using the FeynCalc \cite{8, 9}. In all the plots of the asymmetry, the long distance matrix elements (LDMEs) are taken from Ref. \cite{10} except for the right panel of Fig. 1, where we have used different sets of LDMEs. We have used two sets of parameterization for the TMDs to calculate the $\cos(2\phi_h)$ asymmetry: 1) Gaussian-type parameterization \cite{11, 12} and 2) McLerran-Venugopalan model \cite{13–15} at small-$x$ region.

3.0.1 $\cos(2\phi_h)$ asymmetry in the Gaussian parameterization

The results for the Gaussian are obtained for $\sqrt{s} = 100$ GeV and $Q^2 = 15$ GeV. We have incorporated only few of the results here, the reader could find the more details in Ref. \cite{7}. In the left panel of Fig. 1 we present the contribution to the asymmetry from the color singlet(CS) and the color octet(CO) states. From the plot, we see that the color octet states are giving a significant contribution to the asymmetry, whereas the contribution from the CS is almost zero and slightly positive in higher $P_{h\perp}$ region. In the right panel of Fig. 1 we showed the asymmetry for different sets of LDMEs. We see that the magnitude and the sign of the asymmetry depends on the set of LDMEs used. In fact, this is because of different states contributing to the asymmetry depending on the LDMEs. We have a larger asymmetry for LDMEs set BK.

3.0.2 $\cos(2\phi_h)$ asymmetry in the McLerran-Venugopalan (MV) model

For the MV parameterization, the results are obtained in the kinematic region defined by $\sqrt{s} = 150$ GeV, $x = 0.01$ and $z = 0.7$. The integration ranges are $y \in [0.2, 0.9]$ and $x_B \in [0.005, 0.009]$. The value of $Q$ is set according to $y, x_B$ and $s$. In Fig. 2 (Left), we present the contribution to the $\cos(2\phi_h)$ asymmetry for the $J/\psi$ coming from the individual states, as a
Figure 2: $\cos(2\phi_h)$ asymmetry in $e+p \rightarrow e+J/\psi+X$ process as function of $P_{h\perp}$ at $\sqrt{s} = 150$ GeV, $x = 0.01$ and $z = 0.7$. Left: contribution to the $\cos(2\phi)$ asymmetry coming from the individual states, as a function of $P_{h\perp}$ in the NRQCD framework using color octet model. Right: comparison of the Gaussian and MV model (with two different values of $Q_{sg0}$). For both plots, CMSWZ set of LDMEs [10] is used.

function of $P_{h\perp}$. The maximum contribution to the asymmetry comes from the $^1S_0^{(8)}$ state. In the same Fig. 2 (right), we show the comparison of the asymmetry in the Gaussian and MV model, respectively (with two different values of $Q_{sg0}$) within the same kinematical region as discussed above. The asymmetry in MV model depends on the saturation scale. We note that the Gaussian parameterization of the TMDs gives larger $\cos(2\phi_h)$ asymmetry.

4 Conclusion

We investigated the $\cos(2\phi_h)$ asymmetry in $J/\psi$ production in an electron-proton collision in the kinematics of the future EIC. We calculated them in the small-$x$ domain, where the gluon TMD, namely the unpolared and linearly polarized gluon TMD are important in unpolarized scattering. The virtual-photon-gluon fusion process $\gamma^* + g \rightarrow J/\psi + g$ is the dominant sub-process in this kinematical region for $J/\psi$ production. We used the NRQCD-based color octet formalism to calculate the $J/\psi$ production rate. A small but significant $\cos(2\phi_h)$ asymmetry is obtained. We used the Gaussian and MV models for parameterization of TMDs. The magnitude of the asymmetry was found to be larger with Gaussian parameterization. We incorporated contributions both from CO and CS states. The asymmetry depends on the LDMEs used; specifically, contributions from individual states were found to be significantly dependent on the set of LDMEs used. Overall, our results indicate that the $\cos(2\phi_h)$ asymmetry in $J/\psi$ production might be a very useful tool for probing the ratio of the linearly polarized gluon TMD and the unpolarized gluon TMD in the small-$x$ region at the EIC.

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