Reconstruction of the Higgs mass in $H \rightarrow \tau\tau$ Events by Dynamical Likelihood techniques

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Abstract. An algorithm for reconstruction of the Higgs mass in $H \rightarrow \tau\tau$ decays is presented. The algorithm computes for each event a likelihood function $P(M_{\tau\tau})$ which quantifies the level of compatibility of a Higgs mass hypothesis $M_{\tau\tau}$ with measured momenta of the visible tau decay products plus the missing transverse energy reconstructed in the event. The algorithm is used in the CMS $H \rightarrow \tau\tau$ analysis, where it is found to improve the sensitivity to discover the Standard Model Higgs boson in this decay channel by about 30%.

1. Introduction

Decays of neutral particles into pairs of tau leptons are an important experimental signature at the LHC. The main focus is certainly on the search for the Standard Model (SM) Higgs boson [1, 2] at present. The analysis of Higgs decays into tau pairs will provide information whether the boson found around 125 GeV is indeed the SM Higgs boson, or another particle, e.g. one of the neutral Higgs bosons predicted by supersymmetric extensions of the SM [3, 5, 4, 6]. The search for Higgs bosons is not the only motivation to study events containing tau pairs, however: Several scenarios for physics beyond the SM predict an enhanced production of taus. Examples for such scenarios are decays of heavy neutral gauge bosons $Z' \rightarrow \tau\tau$ [7] and decays into leptons of doubly–charged Higgs bosons [8].

A common feature of these searches is that a possible signal would manifest itself as an excess of events containing pairs of oppositely charged taus over the SM expectation. Compared to performing simple counting experiments, the sensitivity of analyses searching for such signals is enhanced significantly by analyzing the distribution of the tau pair mass, $M_{\tau\tau}$, in selected events.

Different methods have been discussed in the literature for reconstructing $M_{\tau\tau}$ [9, 10, 11, 12, 13]. In the following we present the algorithm, $SVfit$, that has been developed for the CMS $H \rightarrow \tau\tau$ search [14]. The algorithm has been improved since its first use in CMS analyses. In this paper we describe the version of the algorithm that is being used for the ongoing analysis of the full proton–proton collision dataset recorded by CMS in 2011 and 2012.
2. Kinematics of Tau lepton decays

With a mass of 1.777 GeV the tau lepton is heavy enough to decay into hadrons. The dominant decay modes are [15]:

- \( \tau \rightarrow e\nu_e\bar{\nu}_e \) (BR = 17.8%)
- \( \tau \rightarrow \mu\bar{\nu}_\mu \nu_\mu \) (BR = 17.4%)
- \( \tau \rightarrow 1 \) charged hadron + ≥ 0 neutrals \( \nu_\tau \) (BR = 49.5%)
- \( \tau \rightarrow 3 \) charged hadrons + ≥ 0 neutrals \( \nu_\tau \) (BR = 15.2%)

The branching fractions for decays into five or more charged hadrons are negligible. The lifetime of the tau lepton amounts to 290 fs, corresponding to \( c\tau = 87 \mu \text{m} \). Most tau leptons decay before reaching the first layer of the tracking detector. Tau leptons are hence detectable only via their decay products.

One (two) neutrinos are produced in decays of tau leptons into hadrons (in decays into electrons or muons). Consequently, the system of invisible decay products is massless in case of hadronic tau decays, while the invisible momentum system produced in leptonic tau decays is in general of non–zero mass. The total momentum of all visible plus invisible decay products is constrained by the condition that it matches \( m_\tau \), the tau lepton mass.

The kinematics of hadronic (leptonic) tau decays is parametrized by two (three) variables:

- \( X \), the fraction of tau lepton energy (in the laboratory frame) carried by the visible decay products.
- \( \phi \), angle specifying the orientation of the tau lepton momentum vector with respect to the momentum vector of the visible decay products.
- \( m_{\nu\nu} \), the mass of the neutrino system produced in leptonic tau decays.

The variable \( X \) is related to the angle (in the laboratory frame) between the visible decay products and the tau lepton momentum vector, \( \theta_{GJ} \):

\[
\begin{align*}
\theta_{GJ} &= \frac{m^2 - m^2_{\nu\nu} + 2E_\tau E_{\text{vis}}}{2|\vec{p}_\tau^2||\vec{p}_{\text{vis}}^2|} \\
&= \frac{m^2 - m^2_{\nu\nu} + 2E_\tau E_{\text{vis}}}{2|\vec{p}_\tau^2||\vec{p}_{\text{vis}}^2|}.
\end{align*}
\]

\( \theta_{GJ} \) is known as Gottfried–Jackson angle in the literature. Solving for \( \theta_{GJ} \) as function of \( X \) and \( m_{\nu\nu} \) yields:

\[
\cos \theta_{GJ} = -\frac{m^2 + m^2_{\text{vis}} - m^2_{\nu\nu} \pm 2E_\tau E_{\text{vis}}}{2|\vec{p}_\tau^2||\vec{p}_{\text{vis}}^2|},
\]

with

\[
E_\tau = \frac{E_{\text{vis}}}{X} \quad \text{and} \quad |\vec{p}_\tau^2| = \sqrt{E_\tau^2 - m^2_\tau}.
\]

Only one of the two signs in equation 1 yields a valid solution in the interval \( \cos \theta_{GJ} \in [-1, +1] \).

The two variables \( X \) and \( m_{\nu\nu} \) constrain the tau lepton momentum vector to lie on the surface of a cone, at a distance \( |\vec{p}_\tau| = \sqrt{(E_{\text{vis}}/X)^2 - m^2_\tau} \) from the apex. The axis of the cone is given by the momentum vector of the visible decay products. \( \theta_{GJ} \) represents the opening angle of the cone. The variable \( \phi \) specifies the angular coordinate of the tau lepton momentum vector around the cone axis.

Given \( X \), \( \phi \) and \( m_{\nu\nu} \), the energy and momentum of the tau lepton in the laboratory system is fully determined.
2.1. Likelihood formalism

The observables measured in events containing tau lepton pairs do not provide sufficient information to solve for the tau pair mass \( M_{\tau \tau} \) analytically. Depending on whether the two taus decay hadronically or leptonically, the kinematics of the tau pair decay depends upon 4–6 parameters:

- 4 if both taus decay hadronically
- 5 if one tau decays to hadrons and the other tau into an electron or muon
- 6 if both taus decay leptonically.

This contrasts with only 2 observables which constrain the momenta of the neutrinos produced in the tau decays: the two components \( E_x \) and \( E_y \) of the reconstructed missing transverse momentum, \( E_T \).

In the SVfit algorithm the fact that the tau decay kinematics is underconstrained by measured observables is handled via a likelihood approach. In the literature, the term Dynamical Likelihood Methods (DLM) has been introduced for the application of likelihood methods to problems of reconstructing kinematic quantities on an event–by–event basis [16, 17]. This is the approach taken by SVfit.

\( M_{\tau \tau} \) values are reconstructed by combining the measured observables \( E_x \) and \( E_y \) with a probability model, which in the present version of SVfit includes terms for tau decay kinematics and for the \( E_T \) resolution. The model makes a prediction for the probability density \( p(\vec{x}|\vec{y}, \vec{a}) \) to observe the values \( \vec{x} = (E_x, E_y) \) measured in an event, given that the unknown parameters specifying the kinematics of the tau pair decay have values \( \vec{a} = (X_1, \phi_1, m_{\nu_e}, X_2, \phi_2, m_{\nu_\mu}) \) and that the four–momenta of the visible decay products are equal to the observed \( \vec{y} = (p_1^{\nu_e}, p_2^{\nu_\mu}) \).

Probabilities \( P(M_{\tau \tau}) \) are computed for a series of mass hypotheses \( M^i_{\tau \tau} \) by marginalization of the unknown parameters \( \vec{a} \):

\[
P(M_{\tau \tau}) = \int \delta (M_{\tau \tau} - M_{\tau \tau}(\vec{y}, \vec{a})) \ p(\vec{x}|\vec{y}, \vec{a}) \ d\vec{a}. \tag{2}
\]

The integral on the right hand side corresponds to taking a weighted average over all hypothetic configurations which are compatible with the measured values \( \vec{y} \). The probabilities \( P(M_{\tau \tau}) \) are computed in steps of \( \delta M_{\tau \tau} \). The steps are defined by a recursive relation: \( \delta M_{\tau \tau}^{i+1} = 1.025 \cdot \delta M_{\tau \tau}^i \), where \( \delta M_{\tau \tau} = 2.5 \text{ GeV} \), within the range \( M_{\tau \tau} \in [5, 2000] \text{ GeV} \). The step size is chosen such that it is small compared to the expected \( M_{\tau \tau} \) resolution. The integration is performed numerically using the VEGAS [18] algorithm.

The best estimate \( \hat{M}_{\tau \tau} \) for the tau pair mass is taken to be the value \( M^i_{\tau \tau} \) which maximizes \( P(M^i_{\tau \tau}) \). Lower (upper) limits on the reconstructed mass \( \hat{M}_{\tau \tau} \) are determined for every event by the 0.16 (0.84) quantiles of the series of mass hypotheses \( M^i_{\tau \tau} \) and associated probability values \( P(M^i_{\tau \tau}) \).

2.2. Leptonic Tau decays

Matrix elements (MEs) for leptonic and hadronic tau decays are given in the literature [19, 20]. We use MEs to model the decays of tau leptons into electrons and muons. Assuming the taus

1 We foresee to include tau decay vertex information provided by the tracking detector in future versions of the SVfit algorithm. The idea of using decay vertex information for the purpose of reconstructing \( M_{\tau \tau} \) is actually what SVfit owes its name to.
2 In case of hadronic tau decays the variable \( m_{\nu_\mu} \) is a constant of value zero.
3 Evaluation of the integral for the series of mass hypotheses \( M_{\tau \tau} \) takes about 1 s per event on a 2.27 GHz Intel® Xeon® L5520 processor.
to be unpolarized, the probability density functions for the decays $\tau \to e\bar{\nu}_e\nu_\tau$ and $\tau \to \mu\bar{\nu}_\mu\nu_\tau$ are:

$$\frac{d\Gamma}{dX dm_{\nu\nu} d\phi} \propto \frac{m_{\nu\nu}}{4m_\tau^2} (m_\tau^2 + 2m_{\nu\nu}^2)(m_\tau^2 - m_{\nu\nu}^2).$$

(3)

The region allowed by tau decay kinematics is restricted to $0 \leq X \leq 1$ and $0 \leq m_{\nu\nu} \leq m_\tau \sqrt{1 - X}$. Outside of this region the integrand in equation 2 is taken to be zero.

### 2.3. Hadronic tau decays

The matrix elements for decays of tau leptons into hadrons depend on the polarization of the tau lepton and on its decay mode. We find that the sum of all hadronic decays of unpolarized tau leptons is well described by an effective model motivated by two–body phase–space considerations, assuming the system of all hadrons produced in the tau decay to constitute one “particle” (of mass $m_{vis}$):

$$\frac{d\Gamma}{dX d\phi} = \frac{1}{2\pi} \left( \frac{1}{1 - \frac{m_{mvis}^2}{m_\tau^2}} \right).$$

(4)

The region allowed by tau decay kinematics is given by $m_{mvis}^2 \leq X \leq 1$.

### 2.4. Missing Transverse Energy

The $E_T$ likelihood quantifies the compatibility of a tau decay hypothesis with the missing transverse momentum reconstructed in an event. Assuming that the neutrinos produced in tau decays are the only source of $E_T$, momentum conservation in the plane perpendicular to the beam axis implies that the vectorial sum of neutrino momenta matches the reconstructed $E_T$:

$$\sum p'_\nu_x \equiv E_x, \quad E_x = -\sum p'^{PF}_x$$

$$\sum p'_\nu_y \equiv E_y, \quad E_y = -\sum p'^{PF}_y.$$

The sums on the right extend over all particles reconstructed by the particle–flow (PF) algorithm [21].

Differences between $\sum p'_\nu_x$ and $E_x$ ($\sum p'_\nu_y$ and $E_y$) may arise due to resolution effects and are accounted for in the probability model, assuming a Gaussian resolution. The $E_T$ likelihood is computed based on the residual between the sum of neutrino momenta for a given tau decay hypothesis and the value of $E_T$ reconstructed in the event:

$$L_{MET} = \frac{1}{2\pi \sqrt{|V|}} \cdot \exp \left( -\frac{1}{2} \left( \frac{E_x - \sum p'_\nu_x}{E_y - \sum p'_\nu_y} \right)^T \cdot V^{-1} \cdot \left( \frac{E_x - \sum p'_\nu_x}{E_y - \sum p'_\nu_y} \right) \right).$$

(5)

$|V|$ denotes the determinant of $V$. The expected resolution of the $E_T$ reconstruction is represented by the covariance matrix $V$ and is estimated on an event–by–event basis using the $E_T$–significance algorithm [22].

### 3. Performance

Distributions of the tau pair mass $M_{\tau\tau}$ reconstructed in $H \to \tau\tau$ signal events of mass $m_H = 125$ GeV and in $Z/\gamma^* \to \tau\tau$ events, the dominant irreducible SM background to analyses of Higgs production in the tau decay channel, are shown in figure 1.

Compared to $M_{vis}$, the mass of all visible tau decay products, the SVfit algorithm improves the separation between the Higgs signal and the dominant irreducible $Z/\gamma^* \to \tau\tau$ background.
Overall, the SVfit algorithm is found to improve the sensitivity of the SM $H \to \tau\tau$ analysis performed by CMS by about 30%, compared to performing the same analysis using $M_{\text{vis}}$.

Alternative mass reconstruction algorithms [9, 11, 12] have been evaluated in the context of the CMS $H \to \tau\tau$ analysis and found to provide less gain in sensitivity compared to SVfit.

4. Summary

An algorithm for reconstruction of the pair mass in the context of the CMS $H \to \tau\tau$ analysis has been presented. The algorithm is found to improve the sensitivity of the SM $H \to \tau\tau$ analysis performed by CMS by about 30%, corresponding to an increase of about 70% in integrated luminosity.

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