Entanglement Between Nanoelectromechanical Systems Mediated by a Transmission Line Resonator

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Entanglement Between Nanoelectromechanical Systems Mediated by a Transmission Line Resonator

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Abstract In this letters, is considered a simple chip composed of three devices, where we have two nanoelectromechanical systems capacitive coupling with a transmission line resonator, in which this interaction is approximated to two non–degenerate parametric oscillator interaction. For Entanglement analysis, we take two identical nanoelectromechanical systems with a thermal initial state.

Keywords Nanoelectromechanical · Systems, Transmission Line Resonator · Bipartite Entanglement

1 Introduction

Quantum processing information require normally the manipulation of entangled states as well as the investigation of the entanglement shared between distant sites [1–4]. In fact, it has been exploited extensively - both experimental and theoretical studies - using trapped atomic ions [5], entangled photons [28] and cavity QED [7] providing suitable scenarios for this propose. On the one hand, such quantum systems inevitably interact with the surrounding environment generating effects of decoherence. Several proposals have been developed for minimizing, or using the environment to control the entangled states theses small quantum system. On the other hand, the manipulation of quantum entangled states can be realized more efficiently using Gaussian operations (operations that keep the Gaussian character) [8]. Gaussian states (GS) have attracted much attention due to facility in prepare and manipulate resources for processing of quantum information [9]-[17]. The investigation of the entanglement features using symplectic and Lorentz groups in a two-mode bipartite GS was explored in Ref.[10]. The tight bounds for the entanglement in two-mode GS employing the properties of Gaussian channels was studied in Ref.[8]. For the case of an arbitrary two-mode GS dissipating in local Gaussian environments has been discussed in other works [11]. Therefore it would be interesting to extend our research for study the dynamics of entanglement between two ions (considering only the vibrational degrees of freedom) using two-mode bipartite GS.

Here we propose a new architecture to model two nondegenerate parametric amplifier through the chip that consists of a Transmission Line Ressonator (TLR) capacitively coupled to two nanoelectromechanical systems (NEMS), we evaluate the entanglement via symplectic [8], and the enhanced of the non-separability of these states in time function through of the radiation intensity of the TLR. This paper is organized as follows: In Sec. 2 we define the notation used for the Gaussian bipartite states and review a form for evaluate the symplectic eigenvalues of the covariance matrix [8]. Sec. 3 contains the theoretical model of the our investigation and we show, a case, how to generate a nondegenerate parametric oscillator type. The main results are show in Sec. 4 and 5, among them we can highlight the improvement of the entanglement time and your measure. In Sec. 6 we summarize all the find out obtained in this work.
2 Gaussian Bipartite States

In the quantum mechanics, gaussian states are ones that are completely characterized by second order moments. Thus, the information on GS can be stored in mean values vectors and covariance matrices [10]. On bipartite systems described by bosonic annihilation (\(\hat{a}_1, \hat{b}_2\)) and creation (\(\hat{b}_1^+, \hat{b}_2^+\)) operators, the Covariance Matrix \(V\) can be represented by [10],

\[
V = \begin{pmatrix} V_1 & C \\ C^T & V_2 \end{pmatrix},
\]

with

\[
V_i = \begin{pmatrix} n_i + \frac{1}{2} \mathbb{1} \\ \mathbb{1} \end{pmatrix}, \quad C = \begin{pmatrix} m_s & m_c \\ m_c^* & m_s^* \end{pmatrix},
\]

where

\[
n_1 = Tr[\hat{b}_1^+ \hat{b}_1 \rho], \quad m_c = -Tr[\hat{b}_1^+ \hat{b}_2 \rho],
\]

\[
n_2 = Tr[\hat{b}_2^+ \hat{b}_2 \rho], \quad m_s = -Tr[\hat{b}_2^+ \hat{b}_1 \rho],
\]

\[
m_s = Tr[\hat{b}_1^+ \hat{b}_2 \rho], \quad m_c = -Tr[\hat{b}_2^+ \hat{b}_2 \rho]
\]

for \(n_1, m_s, m_c \in \mathbb{R}, V_1 \) and \(V_2\) being hermitian matrices containing only local values [10]. We should notice that general covariance matrices for two mode systems can be brought to this form by local unitary transformations. Gaussian operations preserve an input GS character, in other words, Gaussian operations are an entirely positive map acting on the corresponding density operators [14]. The covariance matrix assumes important values for the bipartite systems and these values provide local and global properties. The local symplectic invariants can be defined by:

\[
I_1 = detV_1, \quad I_2 = detV_2,
\]

\[
I_3 = detC, \quad I_4 = 2 |I_3| \sqrt{I_1 I_2}, \tag{3}
\]

stands for the group \(Sp(2, R) \otimes Sp(2, R)\) [8,7]. Finally, with local symplectic invariants demonstrated in (3) we can calculate the symplectic eigenvalues [8] of the covariance matrix (1)

\[
\eta_{\pm}^2 = I_1 + I_2 + I_3 \pm \sqrt{(I_1 - I_2)^2 + (I_1 + I_2)I_4 + I_4}. \tag{4}
\]

Therefore, serving as criterion to define physical states \((\eta_+ > \eta_+)\), entanglement states \((\eta_- < 0, 5)\) and separable states \((\eta_- \geq 0, 5)\). This expression will serve to obtain the results of this work according to our model formulated in the following section.

3 Physical System

Considering the system in chip how figure (1), w have a TLR capacitive coupled with two NEMS can be characterized by the following Hamiltonian

\[
H = H_0 + H_e, \tag{5}
\]

where the \(L, C, \Phi, \) and \(Q\) are the characteristic inductance, the characteristic capacitance, canonical coordinates for TLR representing magnetic flux and charge in a single mode, analogous to an LC circuit. The \(m_j, \omega_j, p_j,\) and \(x_j\) are the masses, the frequencies of oscillations, respectively. The electrostatic interaction energy \((H_e)\) is given by [27]

\[
H_e = \sum_{j=1,2} \frac{Q^2}{2C(x_j)} = \frac{1}{2\epsilon_0 A} Q^2 \sum_{j=1,2} (x_j + d)
\]

where \(C(x_j) = \frac{\epsilon_0 A}{d + x_j}\) is the capacitance between TLR-NEMS in function of the position of the NEMS, with \(d\) is equilibrium and separation TLR-NEMS. Now rewriting the Hamiltonian (5) as follows,

\[
H = \frac{\Phi^2}{2L} + \frac{Q^2}{2C} + \sum_{j=1,2} \left( \frac{p_j^2}{2m_j} + \frac{1}{2} \nu_j^2 \omega_j^2 x_j^2 \right),
\]

for \(i = 1, 2, h\) is the Plank constant, \(\frac{1}{\epsilon} = \frac{1}{\epsilon_0} + \frac{2\phi}{\epsilon_0},\) and \(\omega_c^2 = LC\).

Thus, the Hamiltonian to system is given by

\[
\dot{H} = \dot{H}_0 + \dot{H}_e, \tag{7}
\]
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with

\[ \hat{H}_0 = \hbar \omega \hat{a}^\dagger \hat{a} + h \sum_{j=1,2} \nu_j \hat{b}_j^\dagger \hat{b}_j, \]

\[ \hat{H}_e = \hbar \sum_{j=1,2} \sqrt{\hbar \omega C^2 \over 8 m_j \nu_j} \left( \hat{a} + \hat{a}^\dagger \right)^2 \left( \hat{b}_j + \hat{b}_j^\dagger \right). \]

Applying the first rotation \( U_1(t) = \exp \left[ -i \omega \hat{a}^\dagger \hat{a} t \right] \) in the Hamiltonian (7), we have (with \( m = m_1 = m_2 \)) \( \nu = \nu_1 = \nu_2, \) \( g = \sqrt{\hbar \omega C^2 \over 8 m_\nu} \)

\[ \hat{H}_1 = \hbar \nu \sum_{j=1,2} \hat{b}_j^\dagger \hat{b}_j + h g \sum_{j=1,2} \left( \hat{a} e^{-i \omega t} + \hat{a}^\dagger e^{i \omega t} \right)^2 \left( \hat{b}_j + \hat{b}_j^\dagger \right), \]

we are in the rotating wave approximation (RWA)

\[ \hat{H}_1 = \hbar \omega \hat{a}^\dagger \hat{a} + h \nu \sum_{j=1,2} \hat{b}_j^\dagger \hat{b}_j + h g \sum_{j=1,2} \hat{a} \left( \hat{b}_j + \hat{b}_j^\dagger \right). \]

(8)

Considering a drive intense in the TLR, we can linearize the interaction [18], give \( \hat{a} \approx \sqrt{n} + \hat{a} e^{-i \Omega t}, \) \( \langle \hat{a} \rangle = \sqrt{n}, \) where \( \Omega \) is drive frequency, we have

\[ \hat{H}_1 = \hbar \omega (\sqrt{n} + \hat{a} e^{-i \Omega t}) \left( \sqrt{n} + \hat{a} e^{-i \Omega t} \right) + h \nu \sum_{j=1,2} \hat{b}_j^\dagger \hat{b}_j \]

\[ + h g (\sqrt{n} + \hat{a} e^{-i \Omega t}) \left( \sqrt{n} + \hat{a} e^{-i \Omega t} \right) \sum_{j=1,2} \left( \hat{b}_j + \hat{b}_j^\dagger \right). \]

(9)

Applying the second rotation

\[ U_2(t) = \exp \left[ -i \left( \delta \hat{a}^\dagger \hat{a} + \nu \sum_{j=1,2} \hat{b}_j^\dagger \hat{b}_j \right) t \right] \]

in (9), where \( \delta \) is the detuning between the drive frequency and TLR \( \delta = \Omega - \omega \) (illustrated in figure 1), finally making RWA with \( \delta \approx -\nu, \) we get the effective Hamiltonian

\[ \hat{H}_{eff} = h g \sqrt{n} \sum_{j=1,2} \left( \hat{a} \hat{b}_j + \hat{a}^\dagger \hat{b}_j^\dagger \right), \]

(10)

characterizing two non-degenerate parametric oscillate, an acceptable regime in interactions via radiation pressure [18] and electromechanical circuit [19]. For quantum total domain situations (\( \hbar \omega \gg kT \)), the equations of motion are

\[ \dot{\hat{a}} = -i g \sqrt{n} \hat{b}_1^\dagger - i g \sqrt{n} \hat{b}_2^\dagger, \]

\[ \dot{\hat{b}}_1 = i g \sqrt{n} \hat{a}, \]

\[ \dot{\hat{b}}_2 = i g \sqrt{n} \hat{a}. \]

(11a)

(11b)

(11c)

Therefore, the solutions are

\[ \dot{\hat{a}}(t) = \dot{\hat{a}}(0) W(t) \]

\[ - i {\sqrt{2 \over 2}} \left( \hat{b}_1(0) + \hat{b}_2(0) \right) Z(t), \]

(12a)

\[ \dot{\hat{b}}_1(0) = \frac{i \sqrt{2}}{2} \hat{a}(0) Z(t) + \hat{b}_1(0) \left[ \frac{W(t) + 1}{2} \right], \]

(12b)

\[ \dot{\hat{b}}_2(0) = \frac{i \sqrt{2}}{2} \hat{a}(0) Z(t) + \hat{b}_2(0) \left[ \frac{W(t) - 1}{2} \right], \]

(12c)

where,

\[ W(t) = \cosh \left( \sqrt{2n g} t \right), \]

\[ Z(t) = \sinh \left( \sqrt{2n g} t \right). \]

In the next topic we calculate the symplectic invariants following the entanglement dynamics of the system.

4 Dynamics of the Entanglement Between Two NEMS

Applying the separability criteria on bipartite systems [15,8], in the equation (4), \( \eta_-, \) the state is separable if \( \eta_- \geq \frac{1}{2}. \) The state is entangled if \( 0 < \eta_- < \frac{1}{2}. \) In order to calculate the local symplectic invariants (eq. 3) with the equation (12), assuming the initial thermal state to NEMS, with thermal numbers \( N_1 = N_2 = N, \) given by

\[ I_1 = I_2 = \left[ \left( \hat{n} + N \right) \frac{Z^2}{2} + N + \frac{1}{2} \right]^2, \]

\[ I_3 = \left[ \frac{4 \left( N + 1 \right)^2 + 2 \hat{n} \left( N + 1 \right)}{4} \right] Z^4, \]

\[ I_4 = 2 I_1 |I_3|, \]

(13)

the symplectic eigenvalue (eq. 4) results

\[ \eta_- = \left| N + \frac{1}{2} + \frac{Z^2}{2} \left( \hat{n} + N - \sqrt{4 \left( N + 1 \right)^2 + 2 \hat{n} \left( N + 1 \right)} \right) \right|, \]

(14)

Note that, from figure (2), the entanglement dynamics between the two NEMS, give for \( \hat{n} \approx N \) and on for
to the TLR, with amplitude $g$ entering a controlled entanglement generator. In the sequence, a drive is applied to the TLR acting as the source of the controlled entanglement between the NEMS. The experiment is performed as shown in figure (3). The first scheme we focus on is the behavior of the result is related to entanglement time, when $\eta(0) < \eta_{-}$ we have no entanglement, the radiation field is directed to a microwave beam splitter. The two outputs are then amplified and run through separate IQ MIX [21].

**Fig. 2** The $\eta_{-}(gt)$ for initial states being thermal states. For values of $\bar{n}$ very different from $N$ we have no entanglement as shown in (a). For values close between $\bar{n}$ and $N$, times temporally bipartite entanglement between the NEMS, and may even generate an EPR state.

A bounded time interval $T_{e}$ in:

$$\arcsin \frac{2N}{\sqrt{4(N+1)^2+24(N+1)-8-N}} < T_{e} <$$

$$\arcsin \frac{2N+2}{\sqrt{4(N+1)^2+24(N+1)-8-N}}$$

and may even generate an EPR state ($\eta_{-} = 0$). That is, this model shows us that the TLR can be a means of control for the generation of entanglement between the NEMS.

**5 Proposed of Measure**

The experiment is performed as shown in figure (3). The TLR acts as the source of the controlled entanglement generator. In the sequence, a drive is applied to the TLR, with amplitude $\sqrt{\bar{n}}$. The radiation field there is generated two non-degenerate parametric oscillators. After these interactions, the radiation field can be ejected from the TLR and any correlation can be measured using IQ Mix linear detectors [20]-[21]-[22], giving us the relationship with entanglement quantization

$$\langle \hat{x}_d \rangle = \left\langle \frac{\hat{\alpha}e^{-\imath t} + \hat{\alpha}^{\dagger}e^{\imath t}}{2} \right\rangle = \sqrt{\eta} \left( 1 - Z^2 \right) \cos(\theta), \quad (16)$$

because $Z(t)$ is directly related to the simplectic eigenvalue of equation (14). Other correlations for calculating $\eta_{-}(t)$ can also be made.

**Fig. 3** In the proposed experiment: The output of the TLR is directed to a microwave beam splitter. The two outputs are then amplified and run through separate IQ MIX [21].

**6 Outlook**

In summarized we have demonstrated two central results. The first them, we focus on the behavior of the dynamics of entanglement bipartite in a tripartite circuit using the symplectic eigenvalues ($\eta_{-}$) extracted from covariance matrix as entanglement parameter. We shown the domain of validity for existence of physical separable state ($\eta_{+} \geq \eta_{-} \geq \frac{1}{2}$) and entangled state ($0 < \eta_{-} < \frac{1}{2}$) via relation of $\bar{n}$ and $N$. The second result is related to entanglement time, when $N \neq 0$ and $\bar{n} \approx N$. We believe that this architecture proposal can be used in development of new circuits to quantum computation and temperature estimation [23]-[24]. This scenario represents the first circuit proposal for entanglement between mechanical thermal states. The other proposals for entanglement between device modes already in the literature are between quantum states and nonlinear interactions [25]-[26]-[27]-[28].

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