Nucleon-Antinucleon Interaction from the Skyrme Model

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Abstract

We calculate the nucleon-antinucleon static potential in the Skyrme model using the product ansatz and including some finite $N_C$ (number of colors) corrections. The mid and long range part of the spin-spin and tensor force are correctly given in both iso-spin channels while the central interaction has insufficient mid-range attraction. This is a well known problem of the product ansatz that should be repaired with better Skyrme dynamics.
I. INTRODUCTION

The Skyrme model [1] is an example of what QCD might look like in the classical or large number of colors ($N_C$) limit [2,3]. The dynamics of the $SU(2)$ Skyrme model is carried purely by a classical pion field. Hadrons appear as topological solitons in this non-linear meson field theory. These are the appropriate degrees of freedom for the non-perturbative, long wavelength limit of QCD, and hence for low energy baryon and pion physics. The Skyrme model has been applied to the nucleon static properties [4] and the nucleon-nucleon interactions [5,6,7,8] with reasonable success. In the last few years, nucleon annihilation has been investigated from the Skyrme point of view. Sommermann et al. [9] studied the dynamics of ungroomed Skyrmion-AntiSkyrmion ($SS$) collisions. They found that annihilation proceeds quickly with the creation of a coherent pion pulse. This was confirmed by Shao, Walet and Amado [10]. The notion that annihilation leads to an intense coherent pion field burst gives reason for considering annihilation within the classical Skyrme approach. The idea, that the annihilation products, pion and other mesons, come from a coherent wave of meson fields arising from soliton-antisoliton dynamics, turns out to be very fruitful [11,12]. Experimental data such as annihilation branching ratios among meson types and pion charge types from low energy annihilation are well explained with minimal parameters [12]. Furthermore this picture provides a unified view of annihilation in which all the channels come from a single process.

Previous studies of annihilation in the Skyrme context, with the exception of [9], have concentrated on the final state mesons. A full account of the process requires a description of the initial state nucleon-antinucleon interaction and of the dynamics leading up to annihilation as well. In this paper, as a first step in that directions, we extend the application of the Skyrme model to the interaction of $NN$ in the product ansatz. (We note that the energy of $SS$ in the product ansatz was studied for two configurations by Musakhanov and Musatov [13].) Phenomenologically, the $NN$ potential is not as well established as the $N\bar{N}$ potential. At distances less than one fermi, the interaction is dominated by annihilation.
However, at larger distances, a meaningful potential can be defined and studied either by $G$-transformation on the $NN$ meson exchange potential or phenomenologically. Here we will compare our Skyrme model results to this phenomenology. We will see that at large distance, where the product ansatz makes the best sense, the potentials we find agree qualitatively and, in most cases, quantitatively with phenomenological interactions. At intermediate and short distances, we do less well, but at these distances the product ansatz is not valid. However it is still suggestive. To obtain the interaction at intermediate distances, we would need to study the full Skyrme dynamics at these distances. For the static $S\overline{S}$ this is somewhat more complex than in the corresponding $SS$ case studied by Walhout and Wambach, [14], but is possible and we plan to return to it. The full, time dependent, dynamical $S\overline{S}$ problem is far more difficult than the $SS$ case and is plagued for $S\overline{S}$ by numerical instabilities [9,15]. For all these reasons, and because this paper is a first step, we begin by exploring the interaction in the product ansatz. In order to carry out our study it is necessary to include $\Delta$ and $\overline{\Delta}$ mixing in the potential as was first suggested in the $NN$ case [16,8].

In the context of the product ansatz, we find that the ungroomed $S\overline{S}$ channel studied in [9] is the most attractive channel and that it leads to rapid annihilation. For non-zero grooming, we find that the $S\overline{S}$ interaction can be repulsive. Therefore it seems likely that the dynamics in groomed channels could be very different from that exhibited in [9]. Since the physical nucleon is represented by an average over differently groomed Skyrmions, annihilation in the nucleon-antinucleon system may proceed more slowly than that of ungroomed $S\overline{S}$.

In Sec. 2 we study the interaction energy of $S\overline{S}$ as a function of separation and relative grooming in the product ansatz. We start by studying the very simplified case of Skyrmion and groomed antiSkyrmion on top of each other. This is a physically artificial case, but it permits analytic evaluation and teaches us something, albeit qualitative, about the dependence of the $S\overline{S}$ interaction on grooming. We find that for zero separation and zero grooming, the $S\overline{S}$ system has zero total energy, as we expect. This is the case of complete annihilation. However at relative grooming angle of $\pi$, and still zero separation, the product
ansatz gives a total energy for the $S\bar{S}$ system of four times the single Skyrmion energy, corresponding to a very repulsive interaction. This clearly indicates that the $S\bar{S}$ interaction is a strong function of relative grooming. Next we study, always in the product ansatz, the interaction energy of $S\bar{S}$ at non-zero separation as a function of grooming. We project to the nucleon space by the algebraic methods of [17] which also include finite $N_C$ corrections. In Sec. 3 we consider the effects of rotational excitations by including intermediate states with $\Delta$ and $\bar{\Delta}$. We first evaluate the corrections to the $N\bar{N}$ potential in perturbation theory and then study the effect fully by diagonalizing in the space spanned by $N$, $\Delta$ and corresponding antiparticles. Our results are presented in Sec. 4.

II. THE INTERACTION ENERGY IN THE $S\bar{S}$ SYSTEM AS A FUNCTION OF SEPARATION AND RELATIVE GROOMING

A. The case of an $S$ on top of a groomed $\bar{S}$

We calculate the energy of the Skyrmion and antiSkyrmion system using the Skyrme lagrangian. The density of this lagrangian is given by

$$\mathcal{L} = \frac{f_\pi^2}{4} \text{Tr}(\partial_\mu U \partial^\mu U^+) + \frac{1}{32e^2} \text{Tr}(Q_{\mu\nu}Q^{\mu\nu}) + \frac{f_\pi^2}{2} m_\pi^2 \text{Tr}(U - 1),$$

(1)

where $U$ is a unitary $SU(2)$ valued field and

$$Q_{\mu\nu} = \left[(\partial_\mu U)U^+, (\partial_\nu U)U^+\right].$$

(2)

The first term in the lagrangian comes from the non-linear $\sigma$-model and the second is the Skyrme term. The third term is a pion mass term and we take $m_\pi = 139$ MeV. We take the parameters in the lagrangian to have the values $f_\pi = 93$ MeV and $e = 4.76$ [14]. These values guarantee that the long distance tail of the nucleon-antinucleon interaction will agree with phenomenology, by virtue of the Goldberger-Treiman relation.

We begin by studying the energy in the product ansatz for the case of zero separation. We include the first two terms in the lagrangian. The mass term is neglected since it does
not lead to additional understanding of this simple configuration. Let \( U = \exp(i\tau \cdot \hat{r}F(r)) \) be the ungroomed Skyrme \( SU(2) \) field. The ungroomed \( \mathcal{S} \) would be \( U^\dagger \). The rotation or grooming matrix \( C \) on \( \mathcal{S} \) is
\[
C = \cos(\beta/2) + i\tau \cdot \hat{n} \sin(\beta/2).
\]
(3)
This corresponds to a grooming rotation through angle \( \beta \) about the \( \hat{n} \) axis. The product ansatz with this relative grooming is
\[
U_{PA} = UCU^\dagger C^\dagger.
\]
(4)
Note that the energy is a function of only the relative grooming and should be zero with no grooming (since for \( \beta = 0, U_{PA} = 1 \)).

The energy density is
\[
E = -\frac{1}{4} \text{tr}\mathcal{L}_i\mathcal{L}_i - \frac{1}{32} \text{tr}[\mathcal{L}_i, \mathcal{L}_j]^2
\]
(5)
in Skyrme units (energy in \( ef_\pi \) and length in \( 1/(ef_\pi) \)). The chiral (left handed) derivative is
\[
\mathcal{L}_i = U_{PA}^\dagger \partial_i U_{PA}.
\]
(6)
Suppose for the Skyrme chiral angle \( F(r) \), the mass contribution to the \( B = 1 \) skyrmion from the non-linear \( \sigma \)-model term is \( M_2 \) and from the Skyrme term \( M_4 \). After some algebra, we arrive at the following result for the total energy of the \( S\mathcal{S} \) product ansatz at relative grooming angle \( \beta \) and zero separation.
\[
M_{SS}(\beta) = \frac{8}{3} \sin^2\left(\frac{\beta}{2}\right) M_2 + \frac{16}{3} \sin^4\left(\frac{\beta}{2}\right) M_4.
\]
(7)
For the profile \( F(r) \) which minimizes the \( B = 1 \) Skyrme mass, we have \( M_2 = M_4 = M/2 \) from scaling arguments. Here \( M \) is the mass of the Skyrmion. We then have
\[
M_{SS}(\beta)/M = \frac{4}{3} \sin^2\left(\frac{\beta}{2}\right) \left[ 1 + 2 \sin^2\left(\frac{\beta}{2}\right) \right],
\]
(8)
which is plotted in Fig. [i].
The maximum occurs at $\beta = \pi$, the maximum grooming, with the value $4M$. This indicates that the $S\overline{S}$ is quite repulsive at this setting. Of course for $\beta = 0$ or $2\pi$, the total energy is zero. This shows that, even for the artificial case of zero separation, the $S\overline{S}$ energy is very dependent on grooming.

**B. $SS$ at a separation and a relative grooming**

We now study, in the product ansatz, the energy of the $S\overline{S}$ system as a function of the separation and relative grooming. This can only be done numerically. We now use the full lagrangian including the finite pion mass term. We put the $S$ and $\overline{S}$ on a 3D lattice with the two solitons a distance $R$ apart on the $x$ axis. The product ansatz is

$$U_{PA}(r) = U(r - \frac{R\hat{r}}{2})CU^\dagger(r + \frac{R\hat{r}}{2})C^\dagger,$$

(9)

where $U(r)$ is the $SU(2)$ field for a single Skyrmion and $C$ is the grooming matrix. In Skyrme units for length ($1/ef_\pi = 0.45$ fm), the spatial extension of lattice we use is $20 \times 10 \times 10$. We evaluate the derivatives of the $U$-field by the two-point difference:

$$\nabla_i U = \frac{1}{2h} [U(r + he_i) - U(r - he_i)].$$

(10)

With $64 \times 32 \times 32$ points on the lattice and $h = 0.001$, we find that, for large separations, the total energy is within one percent of twice the single Skyrmion mass. We calculate the energy for three interesting configurations:

1. no grooming ($H\overline{H}$)
2. relative rotation of $\pi$ around $x$-axis ($x-\pi$)
3. relative rotation of $\pi$ around $z$-axis ($z-\pi$)

The results are shown in Fig. [Fig. 2]. Recall that the separation is along the $x$ axis.

It is instructive to compare the energy of the $S\overline{S}$ system with the corresponding result for the SS system [5,6,7]. The $z-\pi$ grooming is the most attractive configuration for $SS,$
while it is the most repulsive for $SS$. The $x$-$\pi$ grooming, while being the most repulsive for $SS$, is mildly attractive for $S\overline{S}$. Finally the ungroomed case is the most attractive for $S\overline{S}$ while it is mildly repulsive for $SS$. This also leads to speculation about the speed the speed of annihilation. In the calculation of [9], the starting configuration is ungroomed and annihilation happens very fast. This may reflect the fact that this is the the most attractive channel. The physical nucleon is a linear superposition of groomed Skyrmions and perhaps in this case annihilation will proceed slower than that seen in [9].

C. Expansion of the $S\overline{S}$ energy in the relative grooming variables

We now turn to an expansion of the energy in the relative grooming variables as a first step in obtaining the projection of the $S\overline{S}$ interaction onto the $NN$ interaction. We follow the methods developed for obtaining the $NN$ interaction from the $SS$. As in the calculation of [6,17] for $SS$, the energy for $S\overline{S}$ can be expanded in the variables $c_4$ and $c \cdot \hat{R}$, with the relative grooming matrix $C = c_4 + i \tau \cdot c$ and $R$ the vector connecting the centers of the two solitons. For the $SS$, the full expansion is

$$V(R, C) = V_1 + V_2 c_4^2 + V_3 (c \cdot \hat{R})^2 + V_4 c_4^4 + V_5 c_4^2(c \cdot \hat{R})^2 + V_6 (c \cdot \hat{R})^4$$

(11)

where $V_i$, $i = 1..6$ are functions of $R$. For $S\overline{S}$, the symmetry of $R \rightarrow -R$ is broken by the product ansatz and we need three additional terms for a consistent expansion

$$V_{S\overline{S}} = V(R, C) + V_7 c_4(c \cdot \hat{R}) + V_8 c_4^3(c \cdot \hat{R}) + V_9 c_4(c \cdot \hat{R})^3.$$  

(12)

These terms odd in $R$ are an artifact of the asymmetry of the product ansatz and should be discarded. One can use the symmetrized energy $(V_{S\overline{S}} + V_{\overline{S}S})/2$ to extract $V_1$ to $V_6$, since the $V_7$ to $V_9$ terms drop out in this combination.

The six terms in (11) can be expressed in terms of operators in the baryon space using the algebraic methods introduced in [17]. One quantizes each Skyrmion with a $u(4)$ algebra and then the relevant operators and baryon states are easily constructed in terms of the
operators of those algebras. The method was developed in [17] for the \(NN\) system, but since each Skyrmion gets its own algebra, the method can be taken over without alteration to the \(NN\) system. The \(SS\) or \(S\bar{S}\) interaction can be expanded in terms of three operators, the identity and the operators \(W\) and \(Z\) given by

\[
W = T^\alpha_{\pi i} T^\beta_{\pi j} / N_C^2, \\
Z = T^\alpha_{\pi i} T^\beta_{\pi j} \left[ 3 \hat{R}_i \hat{R}_j - \delta_{ij} \right] / N_C^2.
\]  

(13)

Here \(\alpha\) and \(\beta\) label the two different set of bosons, used to realize the \(u(4)\) algebras. \(T\) is a one-body operator with spin 1 and isospin 1. The semiclassical (large-\(N_C\)) limit of these operators can be given in terms of \(R\) and \(C\) as [17]

\[
W_{\text{cl}} = 3c^2 - c^2, \\
Z_{\text{cl}} = 6c \cdot \hat{R} - 2c^2.
\]  

(14)

We can therefore expand the interaction in \(W\) and \(Z\), as an alternative to Eq. (11).

\[
V(R, C) = v_1(R) + v_2(R)W_{\text{cl}} + v_3(R)Z_{\text{cl}} + v_4(R)W_{\text{cl}}^2 + v_5(R)W_{\text{cl}}Z_{\text{cl}} + v_6(R)Z_{\text{cl}}^2
\]  

(15)

The relations between \(V_i\) and \(v_i\) can be found in Eq. (24) of [17]. The advantage of the algebraic method is it allows us to study both the large \(N_C\) limit, and to include finite \(N_C\) effects explicitly in a systematic way. It also makes taking baryon matrix elements quite easy. As in the \(SS\) case, we find for \(S\bar{S}\) that the terms quadratic in \(Z\) and \(W\) are quite small, and so we neglect them. Hence we can write

\[
V = v_1 + v_2 W + v_3 Z.
\]  

(16)

We can use the algebraic methods of [17] to take the \(NN\) matrix element of our interaction. Keeping only the leading terms we find

\[
V = V_c + V_s(\sigma_1 \cdot \sigma_2)(\tau_1 \cdot \tau_2) + V_t[3(\sigma_1 \cdot \hat{R})(\sigma_2 \cdot \hat{R}) - \sigma_1 \cdot \sigma_2](\tau_1 \cdot \tau_2)
\]  

(17)

with
Here \( P_N \) is a finite \( N_C \) correction factor, \( P_N = 1 + 2/N_C \). This gives the nucleons only projection of the \( SS \) interaction. To obtain the full phenomenological interaction it is necessary to include the effects of \( \Delta \) and \( \overline{\Delta} \) admixtures that become important as the baryons approach each other. We now turn to those admixtures.

III. ADIABATIC INTERACTION

The \( NN \) potential in Eq. (17) is calculated by projecting Eq. (16) to the nucleon degrees of freedom only. This is certainly the correct procedure for large separation. However as the Skyrmion and anti-Skyrmion approach, they can deform. In terms of the baryon degrees of freedom and \( N_C = 3 \) that means excitation of the \( \Delta \) and \( \overline{\Delta} \) intermediate states. All that is required to define a \( NN \) interaction is that the particles be \( NN \) asymptotically. They may deform or excite as they wish as they interact. In the \( NN \) case we saw that this intermediate excitation plays a significant role in the intermediate range attraction. For neutral atoms, a corresponding virtual excitation process leads to the attractive Van der Waals force at large distance. For Skyrmions, beside this state mixing, there is a dynamical distortion that goes beyond the product ansatz. This is a crucial part of the \( NN \) interaction [14], but the corresponding \( SS \) distortion is beyond the scope of this paper. For the \( NN \) system this distortion, coupled with the state mixing, is crucial for getting the mid-range attraction. For the nucleon-antinucleon system we expect similar enhancements of attraction coming from the distortion. The effects of distortion and state mixing both come in at distances where the product ansatz can be expected to fail. Thus our short and mid-range results with the product ansatz, even including state mixing, should be taken as only indicative and not as the final word.

As in the \( NN \) system, we first study the effect of mixing \( \Delta \) and \( \overline{\Delta} \) on the energy perturbatively and then use the Born-Oppenheimer method to consider the effect exactly in the limited subspace.
A. Perturbation Theory

We first include the effects of the intermediate states, $N\Sigma$, $\Delta N$ and $\Delta \Delta$ on the nucleon antinucleon potential perturbatively. Since we are using separate $u(4)$ algebras for each Skyrmion, the results for $NN$ in [8] can be carried over to the $N\bar{N}$ problem, with Eq. (15) in [8] being the perturbation correction.

$$V_{PT}^{(1)} = -\frac{Q_N^2}{\delta} \left\{ \frac{1}{3} Q_N^2 P_0^T + \frac{16}{27} P_N^2 + \frac{5}{27} Q_N^2 P_1^T \right\} (v_2^2 + 2v_3^2) \right. \left.+ (\sigma^1 \cdot \sigma^2) \left[ -\frac{1}{18} Q_N^2 P_0^T + \frac{16}{81} P_N^2 - \frac{5}{162} Q_N^2 P_1^T \right] (v_2^2 - v_3^2) \right. \left.+ (3\sigma^1 \cdot \hat{R}\sigma^2 \cdot \hat{R} - \sigma^1 \cdot \sigma^2) \left[ -\frac{1}{18} Q_N^2 P_0^T + \frac{16}{81} P_N^2 - \frac{5}{162} Q_N^2 P_1^T \right] (v_2^2 - v_2v_3) \right\}. \quad (19)$$

Here $\delta$ is the $N$-$\Delta$ mass difference, $P_T^T$ is a projection operator onto isospin $T$, and $Q_N$ is another finite $N_C$ correction factor with the value $Q_N = \sqrt{(1 - 1/N_C)(1 + 5/N_C)}$. This expresses the leading order correction from state mixing to the $NN$ interaction of Eq. (17) in terms of the $SS$ terms of Eq. (16). Recall that unlike the work in [8] we are here using the product ansatz to calculate the $S\bar{S}$ interaction rather than a full dynamical scheme.

B. Diagonalization

We now turn to a full diagonalization of the interaction in the $N\bar{N}$, $N\Sigma$, $\Delta N$ and $\Delta \Delta$ space. This is the Born-Oppenheimer approximation, and it is valid for $N_C = 3$. There are three energy scales or time scales in the problem. The fastest or highest energy scale comes in rearrangements of the pion field itself. These we are modeling using the product ansatz and correspond to energies on the scale of baryon masses. The intermediate scale is set by the $N$-$\Delta$ energy difference. This is an order $1/N_C$ effect. Finally the $N\bar{N}$ interaction is the smallest energy scale and it is determined by the matrix diagonalization.

We first need the matrix element of the potential Eq. (19) in the space of $NN$, $N\Sigma$, $\Delta N$ and $\Delta \Delta$ in the angular momentum coupled form. This has been calculated in Eq. (22) of [8] for the baryon-baryon case and the formula remains valid for baryon-antibaryons.
\[ \langle I_1 I_2 LSJT | v | I_1' I_2' L'S'JT \rangle \]
\[ = v_1 \delta_{SS'} \delta_{LL'} \delta_{I_1 I_1'} \delta_{I_2 I_2'} + \frac{v_2}{9} (-1)^{S+T} \delta_{SS'} \delta_{LL'} \left\{ \begin{array}{c} I_1 \ I_2 \ S \\ I_2' \ I_1' \ 1 \end{array} \right\} \left\{ \begin{array}{c} I_1 \ I_2 \ T \\ I_2' \ I_1' \ 1 \end{array} \right\} \langle I_1 || T^{(11)} || I_1' \rangle \langle I_2 || T^{(11)} || I_2' \rangle \\
+ \frac{v_3}{9} \sqrt{30} (-1)^{L+L'+S+S'+J+T+I_2} \hat{L} \hat{S} \hat{T} \left\{ \begin{array}{c} L \ 2 \ L' \\ 0 \ 0 \ 0 \end{array} \right\} \left\{ \begin{array}{c} S \ L \ J \\ L' \ S' \ 2 \end{array} \right\} \\
\times \left\{ \begin{array}{c} I_1 \ I_2 \ S \\ I_2' \ I_1' \ 1 \end{array} \right\} \left\{ \begin{array}{c} I_1 \ I_2 \ T \\ I_2' \ I_1' \ 1 \end{array} \right\} \left\{ \begin{array}{c} I_1 \ I_2 \ S \\ I_1' \ I_2' \ S' \\ 1 \ 1 \ 2 \end{array} \right\} \langle I_1 || T^{(11)} || I_1' \rangle \langle I_2 || T^{(11)} || I_2' \rangle \]
\[ (20) \]

The relevant reduced matrix elements are
\[ \langle N || T^{(11)} || N \rangle = -10, \]
\[ (21) \]
\[ \langle \Delta || T^{(11)} || \Delta \rangle = -20, \]
\[ (22) \]
\[ \langle N || T^{(11)} || \Delta \rangle = -8 \sqrt{2}. \]
\[ (23) \]

The matrix element of the kinetic part are taken to be very simple,
\[ \langle I_1 I_2 LSJT | K | I_1' I_2' L'S'JT \rangle = \delta_{I_1 I_1'} \delta_{I_2 I_2'} \delta_{LL'} \delta_{SS'} \left( \delta [I_1 + I_2 - 1] + \frac{L(L+1)}{2M_{I_1 I_2}} \delta \right). \]
\[ (24) \]

Here \( M_{I_1 I_2} \) is the reduced mass, \( M_{I_1} M_{I_2}/(M_{I_1} + M_{I_2}) \), with \( M_{1/2} = 932 \) MeV and \( M_{3/2} = 1232 \) MeV. The mass difference is \( \delta M = M_{3/2} - M_{1/2} \).

For the purpose of comparison, we parametrize the full \( NN \) interaction by
\[ V_{NN}^T = V_c^T + V_s^T \sigma_1 \cdot \sigma_2 + V_l^T \sigma_1^1 \sigma_2^2 (3 R_i R_j - \delta_{ij}). \]
\[ (25) \]

The potentials have explicit isospin dependence due to the mixing with states of \( \Delta \). To determine the adiabatic potential for \( NN \), we start at large \( R \) where we have nucleons only. As we move to smaller distance, we diagonalize the \( K + V \) matrix and follow continuously the eigenvalue corresponding to the \( NN \) channel. We then subtract the expectation value of \( K \) to obtain the adiabatic interaction energy as a function of \( R \).
We first consider the case $T = 0$. For $J^\pi = 0^-$ (note that nucleon and antinucleon have opposite intrinsic parity), we have three channels $|N\overline{N}L = 0 S = 0\rangle$, $|\Delta\overline{\Delta}L = 0 S = 0\rangle$ and $|\Delta\overline{\Delta}L = 2 S = 2\rangle$. The lowest eigenvalue of the Hamiltonian $K + V$ should be identified with

$$\langle L = 0 S = 0|V_{NN}^{T=0}|L = 0 S = 0\rangle = V_c^0 - 3V_s^0.$$  \hspace{1cm} (26)$$

For $J^\pi = 0^+$, there are three channels $|N\overline{N}L = 1 S = 1\rangle$, $|\Delta\overline{\Delta}L = 1 S = 1\rangle$ and $|\Delta\overline{\Delta}L = 3 S = 3\rangle$. The lowest eigenvalue should be equated to

$$\langle L = 0 S = 0|V_{NN}^{T=0}|L = 0 S = 0\rangle = V_c^0 + V_s^0 - 4V_t^0.$$  \hspace{1cm} (27)$$

We consider one more set of states with $J^\pi = 1^-$ and there are six channels: $|N\overline{N}L = 0 S = 1\rangle$, $|N\overline{N}L = 2 S = 1\rangle$, $|\Delta\overline{\Delta}01\rangle$, $|\Delta\overline{\Delta}21\rangle$, $|\Delta\overline{\Delta}23\rangle$ and $|\Delta\overline{\Delta}43\rangle$. The matrix element to identify the lowest eigenvalue with is

$$\langle L = 0 S = 1|V_{NN}^{T=0}|L = 0 S = 1\rangle = V_c^0 + V_s^0.$$  \hspace{1cm} (28)$$

From these three linear combinations of $V_c$, $V_s$ and $V_t$ in Eq. (26) to Eq. (28), the potentials in Eq. (25) are easily solved for $T = 0$. A similar calculation applies for $T = 1$, except that now $N\overline{\Delta}$ and $\Delta\overline{N}$ channels also appear.

### IV. RESULT AND DISCUSSION

For each total isospin $T = 0, 1$, we calculate $V_c^T$, $V_s^T$ and $V_t^T$ in Eq. (25) as outlined in the last section. We then compare these result with the phenomenological potentials of Brian-Phillips [18] and of the Nijmegen [19] group. These are potentials based on meson exchange at large distances and phenomenology, including an absorptive part to model annihilation, at small distances. The meson exchange part of these potentials for $NN$ is obtained from the corresponding $NN$ potentials by $G$-parity transform – the contribution of a particular meson for $NN$ is equal to its part in $V_{NN}$ multiplied by the meson’s $G$-parity. We only compare with the scalar, tensor and spin-spin parts of the potentials. The
spin-orbit force is of higher order in $1/N$ and we have not calculated it in the Skyrme picture. We should note that various cutoffs are used in the Brian-Phillips, Nijmegen and other similar potentials. As a result, at distances 1 fm or less, the strength of the potentials can be significantly different from their meson exchange value. In addition, at distances less than 1 fm, the interaction is dominated by the absorptive potential of order 1 GeV.

Furthermore, at these short distances the entire static Skyrme approach, to say nothing of the product ansatz, is no longer meaningful. Hence we should not place any faith on comparisons of our results with the phenomenological potentials around 1 fm or less. At intermediate distances, between 1 fm and 2 fm, the results from our Skyrme approach to and intermediate distances, between 1 fm and 2 fm, the results from our Skyrme approach to comparison with phenomenological potentials require a more complete calculation of the Skyrme dynamics. In particular, we expect the product ansatz to underestimate mid-range attraction, as it does for $NN$. This is basically a consequence of the variational theorem.

With these thoughts in mind, let us turn to our results. Figure 3 shows our results for the $T=0$ part of the central potential. We see that the effects of $\Delta$ mixing either in perturbation theory or full Born-Oppenheimer diagonalization is significant, but still does not begin to agree with the strong central, mid-range attraction seen in the phenomenological potentials. This is the fault of the product ansatz we referred to above. It will be important to see if complete Skyrme calculations can repair this fault.

Figure 4 shows the $T=1$ central potential. Here the effects of $\Delta$ mixing are more striking since for $T=1$ single $\Delta$ intermediate states are permitted. Now we do find some central attraction, but not as much as seen phenomenologically. Note that where they differ, the diagonalization result has superior credentials to the perturbation theory result, and it is the diagonalization result that is too weak. Again we must await full Skyrme calculation in this channel. Figure 5 shows the $T=0$ spin-spin part of the interaction. Except at the smallest distances, the results are very satisfactory. Note that this is remarkable, since the spin-spin interaction is very weak (note the scale in Figure 5) and hence arises from cancellation of much larger terms. We believe it is significant that the Skyrme picture of $\Delta$ mixing either in perturbation theory or full Born-Oppenheimer diagonalization is significant, but still does not begin to agree with the strong central, mid-range attraction seen in the phenomenological potentials. This is the fault of the product ansatz we referred to above. It will be important to see if complete Skyrme calculations can repair this fault.
can reproduce this scale and even the correct sign. The effects of cancellations are even more striking in Figure 6 which shows the $T = 1$ spin-spin interaction. Again the order of magnitudes are correct while the sign depends sensitively on how we do the $\Delta$ mixing. Note that the phenomenological potentials are consistent with zero. Finally in Figures 7 and 8 we show the $T = 0$ and $T = 1$ tensor potentials. All calculations of these agree since they are dominated by one-pion exchange. (Recall that the Skyrme picture in the product ansatz gets one pion exchange right.) Hence except for the central attraction, the product ansatz gives a credible account of the nucleon-antinucleon potential, and we understand how the product ansatz fails for the central attraction. Note that there are no free parameters on our calculation.

We have shown that the Skyrme picture with the product ansatz is a reasonable first step to obtaining the real part of the nucleon-antinucleon interaction. We also understand how doing the Skyrme dynamics better can repair the lack of central attraction we find here. Hence the next step is to do that dynamics. Then combing this picture of nucleon-antinucleon interactions in the entrance channel based on the Skyrme model with our previous work on annihilation channels described by this model, we hope to have a unified picture of annihilation based on the large $N_C$, QCD inspired Skyrme picture.

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REFERENCES

[1] T.H.R. Skyrme, Proc. R. Soc. London 262, 237 (1961); Nucl. Phys. 31, 556 (1962).

[2] G. ’t Hooft, Nucl. Phys. B72, 461 (1974); B 75, 461 (1974).

[3] E. Witten, Nucl. Phys. B223, 433 (1983).

[4] G.S. Adkins, C.R. Nappi, and E. Witten, Nucl. Phys. B228, 552 (1983).

[5] A. Jackson, A. D. Jackson, and V. Pasquier, Nucl. Phys. A432, 567 (1985).

[6] R. Vinh Mau, M. Lacombe, B. Loiseau, W. N. Nottingham, and P. Lisboa, Phys. Lett. B 150, 259 (1985).

[7] A. Hosaka, M. Oka and R.D. Amado, Nucl. Phys. A530, 507 (1991).

[8] N. R. Walet and R. D. Amado, Phys. Rev. C 47, 498 (1993).

[9] H.M. Sommermann, R. Seki, S. Larson and S.E. Koonin, Phys. Rev. D 45, 4303 (1992).

[10] B. Shao, N.R. Walet, and R. D. Amado, Phys. Lett. B 303, 1 (1991).

[11] R.D. Amado, R. Cannata, J-P. Dedonder, M.P. Locher, and B. Shao, Phys. Rev. Lett. 72, 970 (1994); Phys. Rev. C 50, 640 (1994).

[12] Yang Lu and R.D. Amado, Phys. Lett. B 357, 446 (1995); Phys. Rev. C 52, 2158 (1995).

[13] M.M. Musakhanov and I.V. Musatov, Phys. Lett. B 273, 309 (1991).

[14] T.S. Walhout and J. Wambach, Phys. Rev. Lett. 67, 314 (1991).

[15] W. Y. Crutchfield and J. B. Bell, J. Comp. Phys. 110, 234 (1994).

[16] A. Depace, H. Mütter, and A. Faessler, Phys. Lett. B 188, 307 (1987); G. Kälbermann and J. M. Eisenberg, J. Phys. G 15, 157 (1989).

[17] M. Oka, R. Bijker, A. Bulgac and R. D. Amado, Phys. Rev. C 36, 1727 (1987).

[18] R.A. Brian and R.J.N. Phillips, Nucl. Phys. B5, 201 (1968).
[19] M.M. Nagels et al., Phys. Rev. D 12, 744 (1975); P.H. Timmers et al., Phys. Rev. D 29, 1928 (1984).
FIG. 1. Mass of the $SS\bar{S}$ at zero separation and with relative grooming angle $\beta$. $M_S$ is the Skyrmion mass.
FIG. 2. Total energy of the S-antiS system as a function of separation for the configuration HH (dashed line), $x$-π (dash-dotted) and $z$-π (solid) in units of the Skyrme mass. Note the horizontal line is twice the Skyrmion mass. The maximum value of the energy at zero-separation is four times the Skyrmion mass, as we derived in the analytical result.

FIG. 3. Central potential $V^T_C$ as a function of $R$ in the region 0.5-2 fm for the $T = 0$ channels. The solid line gives the nucleons only result from the product ansatz. The short dashed line is the result of the state mixing in perturbation theory and the long-dashed line of the full Born-Oppenheimer diagonalization. The meson exchange potentials are shown by the dash-dotted line for Bryan-Phillips potential [18] and by the dotted line for the Nijmegen potential [19].
FIG. 4. Central potential, same as in Fig. 3 but for \( T = 1 \).

FIG. 5. The spin dependent potential \( V_s \) as a function of \( R \) in the region 1-3 fm for \( T = 0 \). Labeling of curves is the same as in Fig. 3.
FIG. 6. Spin-dependent potential, same as Fig. 5 but for $T = 1$.

FIG. 7. Tensor potential $V_t$ as a function of $R$ in the region 1-3 fm for $T = 0$. Labeling of curves is the same as in Fig. 3.
FIG. 8. Tensor potential, same as in Fig. 7 but for $T = 1$. 