The mathematical model of the belt conveyor

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Abstract. Belt conveyors provide the greatest technical and economic efficiency in comparison with other types of transport. Researches show that more than 50% of failures occur due to incorrect operation of automatic control systems, leading to malfunctions of the conveyor belt in extended conveyors. Currently, this fact is a limiting factor in the development of the automobile enterprise. To improve the efficiency and for further development of automatic control systems for belt conveyors, a mathematical model of the belt conveyor has been developed.

1. Introduction

In the process of the automobile enterprise operation, there is a need to regulate the production capacity of the flow-line system and create a control system, which could improve the operational productivity of the conveyor – to increase the durability of the equipment, reduce energy consumption, reduce the complexity of maintenance, reduce the number of downtime.

The existing flow-line transport system of PJSC “AVTOVAZ” is currently operating at the limit modes, which becomes a limiting factor in the operation of the enterprise.

This outlines the relevance and necessity for research on the modeling and development of a belt conveyor control system with the choice of appropriate technical facilities.

2. Relevance

In works [1,2,4] for calculation and modeling of the conveyor, the following assumptions are accepted:
- transmission shafts and couplings are absolutely rigid;
- the weight of the drives is reduced to the drive drum rims;
- the mass of the drive is frictional associated with the tape, and slipping of the tape on the reel is missing.
In this case, the speed of the tape on the drive drum is equal to the speed of its escaping from the drive drum and is equal to the speed of the reduced mass of the drive;
- the weight of the load and the rotating parts of the roller supports is evenly distributed;
- the movement of the belt and load is joint, the slipping of the tape with respect to the rollers is missing;
- the resistance force of the conveyor movement does not depend on the speed of the belt;
- longitudinal deformation of the tape occurs according to Hooke's law;
- wave resistance is constant.

Under such assumptions, the constructed models can not depict the real physical processes occurring during the operation of the belt conveyor. Therefore, there is a need to build a model of the conveyor belt that most fully depicts the real physical processes.

3. Problem statement
The general model of the belt conveyor can be built with some degree of approximation to the real object, so the task in this paper is to obtain a model that allows, if necessary, to make changes in its structure to obtain more reliable simulation results without changing the model as a whole. The problem can be solved by constructing a model of the electromechanical system of the belt conveyor in the form of separate blocks, the inputs of which are the control and disturbing influences for this element, and the outputs are the state variables, which are the subject of study or the input signals for other blocks. In this case, all computational operations associated with the processes occurring in the node under consideration will be carried out inside the blocks. The general model of the object, in our case – a belt conveyor, formed from the designed blocks, and its change is to replace one or more blocks by others.

4. Modelling

To create a model of the belt conveyor, we’ll make a design scheme of the conveyor belt with the load on it and the rollers supporting it [3]. The forces of movement resistance, acting on different sites, can be divided into distributed along the length of the conveyor and concentrated (local). The latter include resistance forces to the turning areas, loading areas, intermediate unloading, cleaning and centering devices, etc.

To determine the tractive force, we used the method of bypass along the belt contour. For generality, it is considered that active forces are applied to the drums, as well as reactive forces, always directed against the movement and are not able to independently bring the system out of balance. Reactive forces are generated mainly by friction forces that prevent the belt movement, and the active ones can be caused either by the gravitational component for inclined conveyors or by the influence from the drive side. Further, it is considered that all moments of inertia and moments of resistance forces are distributed along the length of the conveyor belt. The main features of belt conveyors that affect the operating modes of the drive and determine the requirements for it can be established on the basis of the analysis of the conveyor substitution scheme shown in Figure 1 [3,4].

![Figure 1. Equivalent circuit of belt conveyor.](image)

where $J_1, J_2$ – moments of inertia of the drive drums 1 and 2, respectively. $J_1$ includes the moments of inertia of all the associated movable elements of the drive given to the drum 1 shaft. $J_2$ simulates the conveyor belt, the load on it, and all the movable elements connected to the belt, given to the drum 2 shaft.

$C_{ON}, C_{ESC}$ – rigidities in the oncoming and escaping branches of the belt, respectively taking into account the elastic properties of the belt. It is assumed that the wheels $J_1$ and $J_2$ and springs $C_{ON}$ and $C_{ESC}$ are connected by a weightless absolutely rigid flexible string.

$N_{ON}, N_{ESC}$ – damping coefficients in the oncoming and escaping branches, respectively. $\omega_1, \omega_2$ – frequencies of wheels rotation.

$M_{ON}, M_{ESC}$ – moments due to the forces of tension of the oncoming and escaping branches of the conveyor belt.

$M_{R1}, M_{R2}$ – moments of resistance forces to the movement of the drive drum and conveyor belt, respectively. $M$ is the torque of the electric motor.

Considering that the drive has linear mechanical characteristics and there are conditions to eliminate slip of the tape, we write the system of equations [5,6]:

$$M = \beta(\omega_{SET} - \omega_1)$$
\[ \frac{J_1}{d\omega_1}{dt} = M + M_{ESC} - M_{ON} - M_{R1} \]
\[ \frac{J_2}{d\omega_2}{dt} = M_{ON} - M_{ESC} - M_{R2} \]
\[ M_{ON} = \int_{0}^{t} C_{ON}(\omega_1 - \omega_2)dt + n_{ON}C_{ON}(\omega_1 - \omega_2) + M_{PON} \]
\[ M_{ESC} = \int_{0}^{t} C_{ESC}(\omega_1 - \omega_2)dt + n_{ESC}C_{ESC}(\omega_1 - \omega_2) + M_{PESC} \]

where \( \omega_{set} \) – set speed of the drive; \( M_{PON} \) and \( M_{PESC} \) – moments due to the forces of pre-tension of the oncoming and escaping branches of the conveyor belt, respectively.

However, it should be taken into account that the start mode in operation conditions occurs when the static moment of movement resistance is uncertain. In this case, under the action of excessive force developed by the drive, there may be a belt slip on the drum and significant dynamic forces [1].

As is known [2], in the mode of sliding the conveyor belt on the surface of the drive drum, the following condition is fulfilled:

\[ \frac{M_{ON}}{M_{ESC}} = \mu \alpha \]

where \( \mu \) – the coefficient of adhesion, \( \alpha \) – angle of contact.

The expression written as a relation:

\[ \frac{M_{ON}}{M_{ESC}} \leq e^{\mu \alpha} \]

which is a condition of the operation mode of the conveyor without belt slippage. By substituting into this expression the values \( M_{ON} \), \( M_{ESC} \), it can be obtained:

\[ (C_{ON} + C_{ESC}e^{\mu \alpha}) \int_{0}^{t} (\omega_1 - \omega_2)dt + (n_{ON}C_{ON} + n_{ESC}C_{ESC}e^{\mu \alpha})(\omega_1 - \omega_2) \leq M_{PESC}e^{\mu \alpha} - M_{PON} \]

Taking into account the accepted assumptions and considering that both branches of the belt are symmetric, i.e. the equality is performed:

\[ C_{ON} = C_{ESC} = \frac{C}{2}; \ n_{ON} = n_{ESC} = n; \ M_{PESC} = M_{PON} = M_p \]

we can write:

\[ \left\{ C \int_{0}^{t} (\omega_1 - \omega_2) + nC(\omega_1 - \omega_2) \right\} \leq 2M_p \frac{e^{\mu \alpha} - 1}{e^{\mu \alpha} + 1} \]

In the left part of this ratio there is an expression that determines the value of the resulting moment \( M_{12} \) due to the deformation forces of the conveyor belt, and in the right – a constant value determined by the initial conditions \( M_p \) and parameters of the drum drive of the conveyor \( \mu \) and \( \alpha \). With respect to this, the block diagram of the object, allowing one to take into account the possibility of the slipping mode, takes the form shown in Figure 2.
Figure 2. The block diagram of the conveyor with the slipping mode.

For a refined simulation of the conveyor, it is necessary to take into account that during the start and braking of the conveyor, dynamic forces act, caused by the acceleration of the rotating masses of the drive mechanism, drums, as well as the masses of the belt and load. The change in dynamic forces is of a wave nature, which is especially important in extended belt conveyors, in which the movement of tensions along the belt in the upper and lower tension of the belt lasts from a few seconds to several tens of seconds (depending on the length of the conveyor and the smoothness of the start). As a result, the belt does not move uniformly along the entire length, and the associated masses begin to move as the tension wave moves [7, 8].

Let us consider a conveyor with a load tension device located in the head part. The differential equation of drive motion:

\[ M_D \frac{d\theta}{dt} + c\rho \theta = F_0 - W_0 \pm \beta_M \theta = \Delta F \pm \beta_M \theta \]

The mass of progressively moving and rotating parts of the conveyor \( M_G \), reduced to the rim of the drive (leading) drum, is determined by the formula [9]:

\[ M_D = \frac{k(j)^2 \cdot i_G^2}{g D^2} \]

where \( k \) — the coefficient, taking into account the inertia of the gearbox, equal to 1.2-1.3; \( \beta_M \) — coefficient describing the motor characteristics; \( \beta_M = (F_1 - F_2)/(\theta_1 - \theta_2) \); \( \Delta F \) — drive force, excessive relative to the static resistance of the movement, equal to \( F_0 - W_0 \); \( i_G \) — gear ratio; \( g \) — acceleration of gravity; \( D \) — the diameter of the drive drum.

The solution of the differential equation under the initial condition \( \dot{\nu} \approx 0 \) at \( t = 0 \) has the form:

\[ \theta(t) = \frac{\Delta F}{\pm \beta_M - c\rho} \left[ \exp \left( \frac{\pm \beta_M - c\rho}{M_D} t \right) - 1 \right] \]

The dynamic tension of the incoming branch:

\[ S_{DYN,ON}(t) = c\rho \]

on the escaping branch in accordance with the boundary conditions \( S_{ESC} = \text{const} \), \( S_{DYN,ESC} = 0 \).

For determining the acceleration time of the conveyor, a dependency is used:

\[ t_{ACC} = \frac{J^2 \omega}{4g(M_{DYN} - M_{ST})} = \frac{J^2 n}{3750(M_{DYN} - M_{ST})} \]

where \( n \) — rotational speed; \( M_{ST} \) - static moment.

Dynamic brake torque:

\[ M_{DYN} = F_{DYN} R_D = \beta_M (\theta_{SS} - \theta(t)) R_D \]

where \( F_{DYN} \) — dynamic force; \( R_D \) — the radius of the drum; \( \theta_{SS} \) — speed at steady state.
5. Simulation results

A generalized conveyor model was created in the Matlab Simulink (Figure 3), which includes a mathematical description of the processes arising in the individual elements and the mutual connections between them.

The belt conveyor is represented by an electromechanical system consisting of interconnected elements.

The input signal "Tm" simulates the current state of loading and takes into account the random nature of the load flow, and some constant value of the load "Constant". Input 2 determines the reaction of the conveyor to the input signal "U" (control signal from the control system of the flow-line transport system, depending on the load of the conveyor). At the output of the Tm1 model, we show the load coming to the next conveyor, w is the speed of the conveyor belt, Is – the stator current by which the degree of conveyor loading is determined [4, 10].

By dividing the belt conveyor circuit into 20 sections (10 sections of the upper and 10 sections of the lower branch of the conveyor) of the same length, the following simulation results were obtained.

Specified modes of operation of the belt conveyor:
- Start of the unloaded conveyor - from 0 to 3 sec.
- Work conveyor mounted belt speed 3.5 m/s for cargo 1000 t/h – 3 115 sec.
- Stop (Parking) conveyor - from 115 to 145 sec.
- Start the loaded conveyor - from 145 to 148 sec.
- Conveyor operation with a set belt speed of 3.5 m/s at a cargo flow of 1000 t/h – from 148 to 1300 sec.
Figure 4. The curve of drives torque.

Figure 5. The curve of speed distribution in belt sections along the length of the conveyor at full load.

From figure 5 it follows that the speeds of the belt sections are different along the length of the sections, which is explained by the uneven distribution of tension (deformation) of the belt.

Figure 6. Curve of changes in the belt tension depending on sections

Figure 6 shows that:
- maximum tension values are achieved when the belt conveyor is started, and when the loaded conveyor is started, the tension is higher than when the conveyor is started without load;
- when the belt conveyor is completely stopped, the tension values of all sections are practically the same;
- tension on all sections of the belt conveyor increases with the total load of the conveyor;
- after the conveyor is fully loaded in the steady state of the conveyor, the tension values do not change.

6. Conclusion
The results of the belt conveyor model study in different operation modes showed that they influence the specifics of the conveyor with an elastic carrier element. This is depicted in the dependence of the speed of the belt section on the value of the belt tension at this site, the non-simultaneous beginning of the movement of the belt sections.

The above facts confirm that the simulation results are similar to the results of dynamic processes occurring in the operation mode of conveyors. The proposed simulation model of the belt conveyor depicts all the basic properties of the belt conveyor and can be used in the development and adjustment of the automatic control system.
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