Evaluating the patterns of air bubble rise in water-air mixtures used in natural and waste water treatment processes

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Abstract. The paper presents the results of theoretical studies into the patterns of air bubble rise in water-air mixtures used in the natural and waste water treatment processes. It is shown that the air bubble size determines the possibility of its rise under various hydrodynamic conditions. Analysis of the balance of the main forces acting on a single free-rising air bubble identified the basic rise types: laminar rise; laminar rise with a sliding surface layer; transient rise; turbulent rise; turbulent rise in the resistance-law self-similarity field. Mathematical dependencies have been obtained to describe the various types of air bubble rise. Using the obtained mathematical dependencies enables an adequate evaluation of the properties of the water-air mixture containing air bubbles as well as using it more efficiently in natural and waste water treatment processes.

1. Introduction

Liquids saturated with gas bubbles are dynamic two-phase systems that only exist if there is a continuous gas supply. Ceasing to aerate such a liquid results in converting the dynamic two-phase system into a single-phase system over the time it takes a gas bubble to surface, since the gas density is several orders of magnitude less than the liquid density, and the buoyant force pushes the bubbles out of the liquid.

Dynamic two-phase water-air systems can be used to intensify many physicochemical processes occurring in a liquid medium. The following phenomena that occur during the interaction of the liquid and gas phases contribute to the intensification of such processes:

- specific high-gradient mixing of local volumes of liquid near the surface of the rising air bubbles;
- formation of a gas-liquid interface having excess surface energy;
- withdrawal (desorption) of volatile compounds from a liquid by rising air bubbles;
- saturation of a liquid with gas contained in the rising bubbles.
2. Statement of Problem
Dynamic two-phase water-air systems are widely used in the processes of pumping, mixing, and treatment of natural and waste waters.

Pumping air-lift (gas-lift) devices have found application in water supply systems; they are also widely used for pumping return sludge from secondary settling tanks at biological waste water treatment plants.

Pneumatic mixing of wastewaters in regulators effectively levels the concentrations of pollutants contained therein and prevents waste waters from rotting. Pre-aeration and biocoagulation processes involving the pneumatic mixing of waste waters can significantly improve the efficiency of waste water settling in primary settling tanks.

The reagent treatment of natural and waste waters by coagulation and flocculation can be boosted by the pneumatic mixing of reagents into the treated liquid. Pneumatic aeration systems support biological waste water treatment in aeration tanks, provide the activated sludge microorganisms with oxygen, and help ensure the sludge mixture in the aeration part of the tank is mixed appropriately. The flotation processes can effectively remove hydrophobic contaminants from waste waters.

Efficient use of a water-air mixture in a specific process requires proper evaluation of its properties. This paper is devoted to searching for mathematical dependencies for evaluating the properties of the water-air mixture containing air bubbles to use it more efficiently in natural and waste water treatment processes.

3. Basic Properties and Characteristics of Water-Air Mixtures
Below are the basic properties of water-air mixtures:

- the lifetime of such a mixture, which depends on the average time it takes an air bubble to surface;
- the gas saturation factor, i.e. the gas-phase volume to total volume ratio;
- the dispersed composition of air bubbles contained in the water-air mixture.

A free-rising air bubble in water can be considered a separate dispersed particle. A dispersed particle features a water-air interface as well as such an important attribute as heterogeneity.

Heterogeneity (multi-phase condition) means the object has an interphase surface (a surface layer). Dispersion (fragmentation), a feature associated with the object size and geometry, is a quantitative assessment of its degree of heterogeneity.

The following properties determine the air bubble dispersion.

3.1. Air bubble linear size, i.e. the diameter of the air bubble cross-section
Since the air bubble cross-section is always circle-shaped, the air bubble linear size \( d_b \) (m) is related to the maximum cross-section area \( f_b \) (m²) as follows

\[
f_b = \frac{\pi d_b^2}{4} \quad \text{(m}^2\text{)}.
\]

3.2. Air bubble dispersed size
The air bubble dispersed size \( \delta_b \) (m) is the ratio of the air bubble volume \( W_b \) (m³) to its surface area \( S_b \) (m²)

\[
\delta_b = \frac{W_b}{S_b} \quad \text{(m)}.
\]

For a spherical bubble
\( \delta_b = \frac{\pi d_b^3}{6} = \frac{d_b}{\delta_b} \text{ (m).} \)  

### 3.3. Dispersion coefficient

The dispersion coefficient is the ratio of the air bubble linear size \( d_b \) to its dispersed size \( \delta_b \) (m)

\[ K_D = \frac{d_b}{\delta_b}, \]  

then

\[ \delta_b = \frac{d_b}{K_D} \text{ (m).} \]

For a spherical bubble, according to (3), \( K_D = 6 \).

#### 3.4. Air bubble shape coefficient

The air bubble shape coefficient is the ratio of the air bubble surface area \( S_b \) to its cross-section area \( f_b \) (m\(^2\))

\[ K_S = \frac{S_b}{f_b}, \]  

then

\[ S_b = K_S f_b, \text{ (m}^2\text{).} \]

For a spherical bubble

\[ K_S = \frac{\pi d_b^2}{4} = 4, \]  

\[ S_b = \frac{4\pi d_b^2}{4} = \pi d_b^2 \text{ (m}^2\text{).} \]

### 3.5. Specific air bubble surface area

The specific air bubble surface area is the ratio of the air bubble surface area \( S_b \) to its volume \( W_b \)

\[ D_s = \frac{1}{\delta_b} = \frac{S_b}{W_b} \text{ (m}^{-1}\text{)}. \]

The specific surface area \( D_s \) (m\(^{-1}\)) is sometimes, too, referred to as dispersion. The dispersion determines the interphase surface area per unit of the dispersed substance volume.

The higher the dispersion, the more important become the surficial phenomena in the system, as higher dispersion is associated with a greater proportion of surface molecules and a greater surface energy, resulting in a more pronounced manifestation of the heterogeneity-associated properties in such a system.

Greater dispersion results in the emergence of new element-specific and system-wide properties.

### 4. Theoretical Studies

Let us analyze the forces acting on a free-rising air bubble to evaluate the basic properties of a dynamic two-phase water-air system.

An air bubble rises in the water column at the constant velocity \( v_b \) (m/s) as a result of the mutual neutralization of the forces acting on it: the buoyant force, which can be found by the formula...
\[ F_{bf} = W_b g (\rho - \rho_a) \] (N), \hspace{1cm} (11)

and the hydraulic resistance to the bubble motion

\[ F_{hf} = f_b \rho \zeta \frac{V_b^2}{2} \] (N), \hspace{1cm} (12)

where \( g \) is the freefall acceleration (m/s^2); \( \rho \) and \( \rho_a \) are the water density and the air density, respectively (kg/m^3); \( \zeta \) is the bubble hydraulic resistance coefficient.

Given (2) and (6), (11) and (12) mean that

\[ \nu_b = \frac{2g W_b \rho - \rho_a}{\zeta f_b} = \sqrt{\frac{2g}{\zeta} \delta_s K_s \rho - \rho_a \rho} \] (m/s). \hspace{1cm} (13)

Given that \( W_b = \pi d_b^3 / 6 \) and \( f_b = \pi d_b^2 / 4 \), for a spherical bubble we get

\[ \nu_b = \sqrt{\frac{2g \pi d_b^3 / 6 \rho - \rho_a}{\zeta / 4 \pi d_b^2}} = \sqrt{\frac{4 g d_b \rho - \rho_a}{3 \zeta \rho}} \] (m/s). \hspace{1cm} (14)

The hydraulic resistance coefficient in the formulas (13) and (14) is variable, and its value is determined by how water flows around the air bubble as it is rising, which in its turn depends on the Reynolds number

\[ \text{Re} = \frac{\nu_b d_b}{\nu} \] , \hspace{1cm} (15)

where \( \nu \) is the kinematic viscosity coefficient of the liquid (m^2/s).

Figure 1 (A) [1] shows the curve of the hydraulic resistance coefficient \( \zeta \) for a solid sphere floating in pure water as a function of the Reynolds number Re. The same figure shows a similar curve for an air bubble (B), which the researchers have plotted on the data presented in [2], see Figure 2. The next section presents the formulas for calculating \( \zeta \) for various air bubble rise types.

Figure 1. Hydraulic resistance coefficient \( \zeta \) as a function of the Reynolds number Re: A for solid spheres [1], with the following measurements: 1 for Schiller-Schmiedel; 2 for Liebster; 3 for Allen; 4 and 5 for Wieselsberger; B for air bubbles: 6 for a plot based on Kutateladze’s data [2].
Figure 2 shows the free-rising velocity in pure water $v_b$ (m/s) as a function of the bubble diameter $d_b$ (m) [2].

\[ v_b (\text{m/s}) \]

\[ d_b (\text{mm}) \]

Figure 2. Free-rising velocity in pure water $v_b$ as a function of the bubble diameter $d_b$ according to [2].

5. Air Bubble Rise Types
The following types of air bubble rise can be distinguished.

5.1. **Laminar rise (creeping flow, viscous flow, Stokes rise)**
This type is observed at $Re \leq 1$ (the curves to the left of the boundary point 1 in Figures 1 and 2). In this domain, the bubble keeps a strictly spherical shape and rises like a solid sphere of identical density. Thus, at $Re \leq 1$ a bubble can be considered a spherical quasi-solid. In this domain, the hydraulic resistance coefficient $\zeta$ is linearly dependent on the Reynolds number and can be found by the formula

\[ \zeta = \frac{24}{Re}. \]  \hspace{1cm} (16)

Substitute (16) in (14) and consider that $Re = \frac{v_b d_b}{\nu}$ to obtain the well-known Stokes formula

\[ v_b = \frac{4}{3} \frac{v_b d_b}{24 \nu} g d_b \frac{\rho - \rho_a}{\rho} = \frac{1}{18} \frac{v_b d_b^2}{\nu} g \frac{\rho - \rho_a}{\rho} \]  \hspace{1cm} (17)

\[ v_b = \frac{1}{18} \frac{g d_b^2}{\nu} \frac{\rho - \rho_a}{\rho} \]  \hspace{1cm} (18)

Substitute the values $g$, $\nu$, $\rho$ for water and $\rho_a$ for air in (18) to obtain

\[ v_b = \frac{1}{18} \frac{9.8 \times 998.2 - 1.2}{998.2} d_b^2 = 5.38 \times 10^4 d_b^2 \]  \hspace{1cm} (19)

At the upper boundary of the laminar rise, $Re = 1$ Given (19), we get
\[ \text{Re} = \frac{\nu_b d_b}{\nu} = \frac{5.38 \cdot 10^5 d_b \nu}{1.01 \cdot 10^{-6}} = 0.533 \cdot 10^{12} d_b^3 = 1, \quad (20) \]

whence

\[ d_b = \sqrt[3]{\frac{1}{0.533 \cdot 10^{12}}} = 0.000123 \text{ (m)}, \quad (21) \]

\[ \nu_b = \frac{\text{Re} \nu}{d_b} = \frac{1 \cdot 1.01 \cdot 10^{-6}}{0.000123} = 0.0082 \text{ (m/s)}. \quad (22) \]

5.2. Laminar rise with a sliding surface layer

In such a rise, the air bubble retains a spherical shape; however, unlike a solid sphere, its laminar rise continues due to the slippage of the liquid film wrapping the bubble (unlike the bubble, a solid sphere would have a stationary surface). Some data suggest that the Stokes rise may continue at Re values of up to 20 [3, 4]. This is why the formula (16) can also be used to calculate the hydraulic resistance coefficient \( \zeta \) in this domain.

5.3. Transient rise

During a transient rise, the bubble retains a spherical shape, but the Stokes law (18) is no longer effective. Similarly to the transient rise of a solid sphere, there rise velocity and the air bubble diameter are in a linear relationship

\[ \nu_b = K d_b \text{ (m/s)}. \quad (23) \]

At the transient boundary point (point 3 in Figures 1 and 2), the bubble switches to turbulent rise (this domain is referred to as the squared resistance law domain). Pursuant to the recommendations stated in [2], the Reynolds number here is assumed to equal \( \text{Re} = 500 \). As shown in Figure 2, the point 3 velocity \( \nu_b = 0.37 \text{ m/s} \). The bubble diameter can then be found as

\[ d_b = \frac{\text{Re} \nu}{\nu_b} = \frac{500 \cdot 1.01 \cdot 10^{-6}}{0.37} = 0.00137 \text{ (m)}. \quad (24) \]

In which case the coefficient \( K \) in the formula (23) is equal to

\[ K = \frac{\nu_b}{d_b} = \frac{0.37}{0.00137} = 270. \quad (25) \]

The hydraulic resistance coefficient \( \zeta \) at point 3 per (14) equals

\[ \zeta = 4 \frac{g d_b}{3 \nu_b^2} \frac{\rho - \rho_a}{\rho} = 4 \frac{9.8 \cdot 0.00137 \cdot 998.2 - 1.2}{3 \cdot 0.37^2} = 0.13. \quad (26) \]

Another transient boundary point (point 2 in Figures 1 and 2) is the point of transition from laminar rise to transient rise, for which both the formula (25) and the Stokes formula (18) hold true. In this case, the following equality holds true for point 2

\[ 270 d_b = \frac{1}{18} \frac{g d_b^2}{\nu} \frac{\rho - \rho_a}{\rho}, \quad (27) \]

whence

\[ d_b \frac{270 \cdot 18 \nu}{g} \frac{\rho - \rho_a}{\rho} = \frac{270 \cdot 18 \cdot 1.01 \cdot 10^{-6}}{9.8} \frac{998.2}{998.2 - 1.2} = 0.0005 \text{ (m)}. \quad (28) \]
Substitute this value in (25) to obtain for point 2
\[ \nu_b = 270d_b = 270 \cdot 0.0005 = 0.135 \text{ (m/s)}. \] (29)

The Reynolds number at point 2 equals
\[ \text{Re} = \frac{\nu_b d_b}{\nu} = \frac{0.135 \cdot 0.0005}{1.01 \cdot 10^{-6}} = 67. \] (30)

The hydraulic resistance coefficient \( \zeta \) at point 2 per (14) equals
\[ \zeta = \frac{4gd_b}{3\nu_b^2} \frac{\rho - \rho_a}{\rho} = \frac{4 \cdot 9.8 \cdot 0.0005 \cdot 998.2 - 1.2}{3 \cdot 0.135^2} = 0.36. \] (31)

The following patterns are observed in a transient rise
\[ \nu_b = 270d_b \text{ (m/s)}, \] (32)
and the empirical formula
\[ \zeta = 2.91 \text{Re}^{0.5} \] (33)

5.4. Turbulent rise
If the air bubble diameter exceeds \( d_b = 1.37 \cdot 10^{-3} \text{ m}, \) the bubble deforms, altering its shape and becoming an oblate spheroid with a major axis normal to the bubble motion direction. This change in shape increases the hydraulic resistance coefficient \( \zeta. \) Now it has the altered-shape resistance instead of the viscous resistance, reducing the rise velocity \( \nu_b. \)

At the turbulent rise boundary point (point 4 in Figures 1 and 2), the hydraulic resistance coefficient peaks at \( \zeta = 0.4 \) (see calculations below).

As can be seen in Figure 2, the point 4 bubble characteristics are: \( \nu_b = 0.21 \text{ m/s}; \ d_b = 0.005 \text{ m}. \)
In this case, the Reynolds number is
\[ \text{Re} = \frac{\nu_b d_b}{\nu} = \frac{0.21 \cdot 0.005}{1.01 \cdot 10^{-6}} = 1040. \] (34)

Based on the data presented in [2], the air bubble shape coefficient at point 4 can be estimated as \( K_s \approx 2.8. \)

The air bubble dispersed size \( \delta_b \) at point 4 can be found by the formula obtained from the equation (13)
\[ \delta_b = \frac{\nu_b^2 \zeta}{2gK_s} \frac{\rho}{\rho - \rho_a} = \frac{0.21^2 \cdot 0.4}{2 \cdot 9.8 \cdot 2.8 \cdot 998.2 - 1.2} = 0.000322 \text{ (m)}. \] (35)

According to (4), the dispersion coefficient of the air bubble at point 4 equals
\[ K_d = \frac{d_b}{\delta_b} = \frac{0.005}{0.000322} = 15.5. \] (36)

The following empirical patterns are observed in a turbulent rise
\[ \nu_b = 0.0206d_b^{-0.438} \text{ (m/s)}, \] (37)
\[ \zeta = 9.35 \cdot 10^{-6} \text{Re}^{1.555}. \] (38)

Substitute \( \text{Re} = 1040 \) for point 4 in the formula (38) to obtain \( \zeta = 0.4. \)
5.5. Turbulent rise in the resistance-law self-similarity field

During such rise, the hydraulic resistance coefficient is constant \( \zeta = \text{const} \) [2]. As shown above, \( \zeta = 0.4 \) at the boundary point 4; this value then remains unaltered.

Based on the data presented in [2], the air bubble shape coefficient decreases as the air bubble diameter \( d_b \) increases; at \( d_b = 0.025 \text{ m} \), \( K_s \) peaks at \( \approx 2.3 \). Further increase in the diameter barely affects the shape coefficient.

The following empirical patterns are observed in such rising

\[
\begin{align*}
\nu_b &= 0.8d_b^{0.25} \text{ (m/s)}, \\
\zeta &= 0.4 = \text{const}.
\end{align*}
\]  

(39)  

(40)

6. Conclusions

The discovered patterns of various air bubble rise types enable an adequate evaluation of the properties of a water-air mixture containing these bubbles for more efficient use in specific processes.

References

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