Engineering networks simulation and assessment of the mathematical model accuracy

Umankhodja Baxramov*, Uchqon Umarov, and Axror Obidzhonov
Tashkent State Transport University, Tashkent, Uzbekistan

Abstract. The article shows the way to implement a quasilinear mathematical model of flow distribution in pipeline engineering networks that is effective in a wide range of changes in the multidimensional random vector of loads at network nodes and provides reliable determination of the parameters of the probability distribution functions of flows in active and passive network elements. The proposed model consists of determining the matrix of generalized network parameters—the load distribution coefficients along the branches of the circuit, calculated at the point corresponding to the mathematical expectation of the node loads. Based on the obtained model, the convergence of the results obtained with the results of simulation of engineering networks is proved using a numerical experiment on an electronic computer. The effectiveness of the developed model, the corresponding algorithms and a set of programs for an electronic computer is shown—the value of the criterion of reduced costs for parametric optimization of engineering networks can be reduced by 5-7% compared to the methods used in practice. The possibility of obtaining at the design stage the equivalent hydraulic characteristics of engineering networks in the form corresponding to the data of experimental measurements of pressures at the nodes of real complex engineering networks is proved.

1 Introduction

The main task of mathematical modelling of engineering networks in this work is to assess the reliability of the mathematical model of stochastic flow distribution proposed in [1, 2]. For simulation, three calculation schemes of the utility network are used, shown in Figures 1, 2 and 3. It is easy to see that these calculation schemes differ in their dimensionality, the number of nodes and branches, which allows us to objectively reveal the advantages of the mathematical model on networks of varying complexity.

2 Materials and Methods

*Corresponding author: umarxodja@bk.ru
Simulation is reduced to carrying out a large number of calculations of the steady-state flow distribution at various values of the loads in the nodes of the network circuit. With the accumulation of data from such calculations, it becomes possible to estimate the parameters of the following probability distribution functions:

1. Flow values in each passive circuit element (network section) \( q_i \).
2. The values of the head loss in each passive element - \( h_i \).
3. The values of the total feeds of the target product to all nodes of the scheme - \( \Sigma Q_i \).
4. The pressure difference values at the active sources and at the dictating point of the circuit - \( H_{\Delta} \) (these values correspond to the largest values in the matrix [1]).

To simplify the analysis, a normal distribution of all random variables is assumed. Therefore, only two non-random parameters are determined for each of the distributions - the mathematical expectation and variance. Also, for active sources, the covariance values of the dependent random variables are determined – \( h_0 \) and \( \Sigma Q_j \), which, as shown below, is necessary to calculate the total energy consumption for the transportation of the target product.

A general algorithm for simulation (figure - 4) consists of three blocks – A1 in which random load values are generated for all consumption nodes of the target product; A2 providing the calculation of the steady-state flow distribution, and A3, designed for statistical processing of the results obtained.

Fig. 1. The first design scheme of a utility network with 2 rings

Fig. 2. The second design scheme of the utility network with 4 rings

Fig. 3. The third design scheme of the utility network with 20 rings
Simulation is reduced to carrying out a large number of calculations of the steady-state flow distribution at various values of the loads in the nodes of the network circuit. With the accumulation of data from such calculations, it becomes possible to estimate the parameters of the following probability distribution functions:

1. Flow values in each passive circuit element (network section) $q_i$.
2. The values of the head loss in each passive element $h_i$.
3. The values of the total feeds of the target product to all nodes of the scheme $\sum Q_i$.
4. The pressure difference values at the active sources and at the dictating point of the circuit $H_\Delta$ (these values correspond to the largest values in the matrix $[1]$).

To simplify the analysis, a normal distribution of all random variables is assumed. Therefore, only two non-random parameters are determined for each of the distributions — the mathematical expectation and variance. Also, for active sources, the covariance values of the dependent random variables $h_0$ and $\sum Q_j$, which, as shown below, is necessary to calculate the total energy consumption for the transportation of the target product.

A general algorithm for simulation (figure 4) consists of three blocks $A_1$ in which random load values are generated for all consumption nodes of the target product; $A_2$ providing the calculation of the steady-state flow distribution, and $A_3$, designed for statistical processing of the results obtained.

Considering the block $A_2$, it should be noted that almost any of the known algorithms and programs for calculating the steady-state flow distribution can be used here [7, 8]. The only requirement for them from the standpoint of the features of simulation is the need for a fairly convenient software replacement of the values of nodal loads based on the results of the block $A_1$. The block algorithm used in this work $A_2$ detailed in [2] section 3.3

Block $A_3$ the simulation algorithm is quite simple, and its essence boils down to the fact that for all elements of the network design scheme, including active elements, mathematical expectations are calculated, variance, standard deviations and coefficients of variation for each of the distributions of interest of random variables $(P)$ by well-known [6] formulas:

$$
M(P) = \frac{\sum P}{N}; \quad D(P) = \frac{\sum P^2}{N} - [M(P)]^2;
$$

$$
\sigma(P) = \sqrt{D(P)}; \quad \nu_p = \frac{\sigma(P)}{M(P)};
$$

(1)

Initial information used to operate the unit $A_1$ is presented below. The algorithm for simulation modelling of the utility network Figure 3, described in [9], is given in Table 3.1.

When modelling a network, Figure 1 used data for nodes 1, 2, 3 from table 3.1, but for the network Figure 2 received data corresponding to nodes 19 from table 1. In table 3.1, amplitude values harmonics and standard deviation are given in relative units — in shares for each of the nodes of the design scheme.

### 3 Results and Discussion

The results of the simulation modelling of three engineering networks are presented in tables 3.2 ÷ 3.4. Figure 4 shows the relationship between the coefficients of variation of flows in the lines of networks $(\nu_q)$ and head losses $(\nu_h)$, obtained from modelling. Also shown here is the line corresponding to the above [1]) the relationship between these coefficients. Good agreement between the experimental and theoretical data confirms the correctness of the latter and the possibility of calculating the parameters of the distribution functions of the pressure delivery in passive elements from the data on the parameters of the flow distribution functions.
Input of initial data

The subroutine for generating control actions when generating random numbers

- Calculation of the value of a random number with a uniform distribution law in the interval 0 - 1
- Calculating the Amplitude and Phase Shift for Harmonics
- Calculation of linearizing coefficients

I = I + 1

A1. Simulation of random load values for all network nodes

A2. Calculation of the values of the steady-state flow distribution

I = L

NO

YES

A3. Calculation of the values of generalized network parameter

Print results

The end

Data block used to generate random numbers

Solving a system of linear equations

Fig. 4. Block diagram of the simulation algorithm
Comparison of the parameters of the distribution functions of flows in passive elements obtained during simulation and calculation by [1] and [2] (see table 2 ± 4), shows that the proposed mathematical model of stochastic flow distribution in nonlinear pipeline networks provides an accuracy sufficient for practical purposes – calculation error $\delta_i$ less than -8%, and for $\nu_q$ -10%.

When calculating the parameters of the distribution function of the total loads in the network (1) and head losses in the network (1) by formulas [1] and [2], [1] a single value of the correlation coefficient between the process of consumption of the target product in the nodes of the utility network was adopted $r_{ijr}=0.25$. The quantity $r_{ijr}$ obtained from the graph of figure 3.5, where the change in the variance of the total network load is shown depending on the value $r_{ijr}$ in [1]. For all three considered networks, the value $r_{ijr}$ at which the calculated value of the variance of the total load (in simulation modelling) coincides with the value obtained from [1]) approximately equals 0.25. The same meaning $r_{ijr}$ is also used in calculating the dispersion of head losses in the network, which is quite acceptable since the discrepancy between the simulation data and the calculation by the mathematical model of the stochastic flow distribution does not exceed 10% (see tables 2 - 4).

Calculation of the parameters of stochastic flow distribution for networks figures 3, 2 and 3, 3 very cumbersome due to the large dimension of the matrix of load distribution coefficients $C_{ijr}$ and are performed only using an electronic computer.

According to the results of the calculation (network 3.3) on the graph (figure 3.6), the field of possible changes in the pressure losses in the network was constructed by $H_A$, and total load $\sum Q_i$, two points of which (A and B) correspond to the limiting (smallest and largest) values of the head losses in the network at the minimum and maximum values of the total network load.
For the correct selection of pumping equipment and tormented points, it is necessary to find the limits of the possible change in head losses in the network at different values of the total load $\sum Q_i$. This can be done by considering a system of two random variables $H_n$ and $\sum Q_i$, assuming for each of them the normal probability distribution law. Let's consider the correlation coefficient between the values of these random variables as known, for example, take it equally as before 0.25. We can find the so-called conditional distribution $H_n$, that is, the laws of its distribution for various fixed values $\sum Q_i$, known [6, 9, 10], that the density of the conditional distribution of two correlation normally distributed random variables is determined by the expression:

$$
\rho(H_n, \sum Q_i = E) = \frac{1}{2\sigma(H_n)\sigma(\sum Q_i)\sqrt{1-r^2}} \exp\left(\frac{-1}{2\sigma^2(H_n)\sigma^2(\sum Q_i)}\left(\left(\mu(H_n) - \mu(\sum Q_i)\right)^2 - 2\sigma(H_n)\sigma(\sum Q_i)\rho(\sum Q_i)\right)\right)
$$

(2)

Fig. 6. Change in variance of total load network depending on the value of the coefficient correlations between the target consumption process product at network nodes. a is the network in Figure 1; b is the network in Figure 2. $D(\sum Q_j)$ is variance value total loads obtained by imitation modelling.

From (2), the probability of the appearance of different values $H_n$ at $\sum Q_i = E$. In (2), all the necessary quantities are known from the results of the above-described calculation of the stochastic flow distribution. The correlation coefficient between $n$ can be refined based on the simulation results. For the network figure 2, the graph of values $H_n$ for various $\sum Q_i$ shown in Figure 5 - the correlation coefficient here is 0.3, which is quite close to the one used earlier. Results of calculating conditional probability distribution functions $H_n$ at 6 values $\sum Q_i$ shown in figure 3.8, and the parameters of these functions are given in table 3.5, the data of which show that the calculation for 2 quite well converges with simulation modelling and is quite consistent with the data of field experiments in engineering networks shown in figure 7.
**Fig. 7.** Distribution functions of possible changes in head losses in the network: A - the highest value of head loss; B - the smallest value of the head loss

Comparison of simulation results and calculations of the mathematical model of stochastic flow distribution for the network in Figure 1

| Rooms plot networks | Simulation modelling | Math modelling |
|---------------------|----------------------|----------------|
|                     | \( \bar{q}_i \) | \( \bar{v}_i \) | \( \bar{h}_i \) | \( \bar{q}_i \) | \( \bar{v}_i \) | \( \bar{h}_i \) |
| 1                   | 27.05 | 0.217 | 766 | 0.411 | 27.27 | 0.259 | 791.18 | 0.416 |
| 2                   | 36.86 | 0.215 | 755 | 0.403 | 26.64 | 0.265 | 759.15 | 0.499 |
| 3                   | 3.08 | 0.782 | 15.3 | 1.37 | 3.19 | 0.980 | 19.9 | 1.22 |
| 4                   | 15.29 | 0.258 | 249 | 0.484 | 15.12 | 0.262 | 244.4 | 0.498 |
| 5                   | 15.29 | 0.264 | 260 | 0.508 | 15/57 | 0.256 | 258.3 | 0.487 |
| A source (Node 0)   | 513.56 | 0.205 | 1.4 | 0.410 | 513.1 | 0.189 | 1.35 | 0.408 |

Comparison of simulation results and calculations of the mathematical model of stochastic flow distribution for the network in Figure 2

| Rooms plot networks | Simulation modelling | Math modelling |
|---------------------|----------------------|----------------|
|                     | \( \bar{q}_i \) | \( \bar{v}_i \) | \( \bar{h}_i \) | \( \bar{q}_i \) | \( \bar{v}_i \) | \( \bar{h}_i \) |
| 1                   | 27.05 | 0.217 | 766 | 0.411 | 27.27 | 0.259 | 791.18 | 0.416 |
| 2                   | 36.86 | 0.215 | 755 | 0.403 | 26.64 | 0.265 | 759.15 | 0.499 |
| 3                   | 3.08 | 0.782 | 15.3 | 1.37 | 3.19 | 0.980 | 19.9 | 1.22 |
| 4                   | 15.29 | 0.258 | 249 | 0.484 | 15.12 | 0.262 | 244.4 | 0.498 |
| 5                   | 15.29 | 0.264 | 260 | 0.508 | 15/57 | 0.256 | 258.3 | 0.487 |
| A source (Node 0)   | 513.56 | 0.205 | 1.4 | 0.410 | 513.1 | 0.189 | 1.35 | 0.408 |

Comparison of simulation results and calculations of the mathematical model of stochastic flow distribution for the network in Figure 3
| Rooms | Simulation modelling | Math modelling |
|-------|----------------------|----------------|
| plot | $q_i$ | $v_i$ | $h_i$ | $v_{hi}$ | $q_i$ | $v_i$ | $h_i$ | $v_{hi}$ |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 1 | 136.89 | 0.388 | 1.28 | 0.684 | 137.0 | 0.391 | 1.33 | 0.691 |
| 2 | 376.67 | 0.396 | 2.13 | 0.703 | 376.1 | 0.391 | 2.01 | 0.701 |
| 3 | 80.22 | 0.396 | 3.45 | 0.705 | 80 | 0.39 | 3.25 | 0.695 |
| 4 | 50.95 | 0.381 | 1.72 | 0.679 | 51 | 0.389 | 1.8 | 0.683 |
| 5 | 14.51 | 0.389 | 4.16 | 0.711 | 14.5 | 0.394 | 4.12 | 0.699 |
| 6 | 30.74 | 0.385 | 2.34 | 0.707 | 30.8 | 0.388 | 2.41 | 0.715 |
| 7 | 14.99 | 0.399 | 4.44 | 0.71 | 15.1 | 0.403 | 4.51 | 0.78 |
| 8 | 169.23 | 0.397 | 2.9 | 0.689 | 161.3 | 0.399 | 2.95 | 0.702 |
| 9 | 194.6 | 0.391 | 2.59 | 0.694 | 195.1 | 0.402 | 2.71 | 0.71 |
| 10 | 159.8 | 0.393 | 2.79 | 0.695 | 159.1 | 0.389 | 2.67 | 0.691 |
| 11 | 95.5 | 0.384 | 2.43 | 0.721 | 95.4 | 0.381 | 2.41 | 0.72 |
| 12 | 15.07 | 0.401 | 2.63 | 0.711 | 15.3 | 0.411 | 2.72 | 0.719 |
| 13 | 75.15 | 0.409 | 2.7 | 0.771 | 74.8 | 0.4 | 2.63 | 0.769 |
| 14 | 14.39 | 0.448 | 3.64 | 0.745 | 14.5 | 0.451 | 3.72 | 0.75 |
| 15 | 52.1 | 0.444 | 4.68 | 0.691 | 52.9 | 0.449 | 4.76 | 0.688 |
| 16 | 90.41 | 0.388 | 2.49 | 0.699 | 90.1 | 0.381 | 2.41 | 0.698 |
| 17 | 139.7 | 0.399 | 2.33 | 0.72 | 138.3 | 0.388 | 2.21 | 0.69 |
| 18 | 75.67 | 0.428 | 3.14 | 0.714 | 75.2 | 0.417 | 3.1 | 0.702 |
| 19 | 107.3 | 0.409 | 3.56 | 0.722 | 108.3 | 0.415 | 3.72 | 0.735 |
| 20 | 86.41 | 0.419 | 4.04 | 0.807 | 86.2 | 0.403 | 3.91 | 0.798 |
| 21 | 37.99 | 0.446 | 2.49 | 0.701 | 37.7 | 0.425 | 2.33 | 0.692 |
| 22 | 54.15 | 0.399 | 1.54 | 0.734 | 54.0 | 0.391 | 1.52 | 0.733 |
| 23 | 16.54 | 0.428 | 5.39 | 0.655 | 16.1 | 0.421 | 5.23 | 0.651 |
| 24 | 3.31 | 0.345 | 0.94 | 0.755 | 3.44 | 0.355 | 0.99 | 0.761 |
| 25 | 37.92 | 0.448 | 4.62 | 0.709 | 37.7 | 0.432 | 4.24 | 0.697 |
| 26 | 18.1 | 0.399 | 3.24 | 0.727 | 18.2 | 0.405 | 3.41 | 0.731 |
| 27 | 52.61 | 0.424 | 4.99 | 0.866 | 52.2 | 0.421 | 4.91 | 0.683 |
| 28 | 21.33 | 0.37 | 1.07 | 0.726 | 20.6 | 0.362 | 1.08 | 0.701 |
| 29 | 70.52 | 0.426 | 0.89 | 0.759 | 71 | 0.431 | 0.95 | 0.772 |
| 30 | 10.01 | 0.441 | 6.34 | 0.726 | 9.97 | 0.417 | 0.14 | 0.711 |
| 31 | 46.3 | 0.423 | 6.49 | 0.709 | 46.2 | 0.421 | 6.21 | 0.695 |
| 32 | 16.02 | 0.405 | 3.63 | 0.751 | 16.41 | 0.396 | 3.47 | 0.742 |
| 33 | 3.86 | 0.386 | 1.32 | 0.696 | 4.22 | 0.392 | 1.41 | 0.707 |
| 34 | 14.87 | 0.371 | 3.06 | 0.898 | 14.33 | 0.361 | 3 | 0.876 |
| 35 | 8.85 | 0.58 | 0.89 | 0.778 | 8.49 | 0.471 | 0.83 | 0.77 |
| 36 | 48.79 | 0.449 | 0.95 | 0.717 | 49.2 | 0.457 | 0.99 | 0.731 |
| 37 | 8.6 | 0.411 | 6.66 | 0.777 | 8.56 | 0.4 | 6.51 | 0.769 |
| 38 | 32.67 | 0.471 | 3.29 | 0.775 | 32.4 | 0.469 | 0.768 |
| 39 | 21.13 | 0.432 | 3.48 | 3.48 | 0.931 | 21 | 0.43 | 0.927 |
| 40 | 4.9 | 0.632 | 0.402 | 0.807 | 4.88 | 0.622 | 0.389 | 0.8 |
| 41 | 21.18 | 0.474 | 0.44 | 1.39 | 20.3 | 0.465 | 0.37 | 1.2 |
### 4 Conclusions

In this article, we propose ways to implement a quasilinear mathematical model of flow distribution in pipeline engineering networks, which determines the matrices of generalized network parameters – load distribution coefficients along the branches of the scheme, calculated at a point corresponding to the mathematical expectation of node loads. Based on the model obtained in the work, the convergence of the obtained results with the results of simulation modelling of engineering networks is proved by numerical experiment on an electronic computer.

The effectiveness of the developed model, the corresponding algorithms and the software package is proved. The values of the criterion of reduced costs for parametric optimization of engineering networks are given by comparing the results obtained, it is shown that they can be reduced in comparison with the methods currently used in practice. The article indicates the possibility of obtaining equivalent hydraulic characteristics of engineering networks at the design stage in the following cases.

### References

1. Umargadhja Baxramov. Development of a mathematical model of stochastic flow distribution in pipeline engineering networks, TASHIIZhT, Bulletin. Quarterly. 1. (2015)
2. Umargadhja Baxramov. Determination of the stochastic parameters of flow distribution in engineering networks, TASHIIZhT, Bulletin Quarterly. 1. (2016)
3. Buslenko N.P., Golenko D.I., Sobol I.M., Sragovich V.G., Shreider Yu.A. Statistical Test Method (Monte Carlo Method) –M : Fizmatgiz. -332 p. (1962).
4. Umargadhja Baxramov, Irkin Makhamataliev. Modeling of the Random Process of Changing the Structure of an Engineering Network, International Jornal of Scientific & Technology Research, 9 (2), pp. 2676-2678. (2020)
5. Umargadhja Baxramov, G.K.Abduganieva. Determination of the reliability of pipeline engineering networks during their work in various conditions. Bulletin of Karakalpak State University named after Berdakh, 2 (34) 2017.
6. Chetverikov I.N., Belyakovich E.A., Menkov A.V. Computing technology for
7. Umekhodja Baxramov, G.K. Abdiganieva. Methods for describing the loads of a water supply and distribution system for long-term forecasting. “Bulletin of Karakalpak State University named after Berdakh. Nukus. 2019. No. 1 (38) 2018, ISSN 2010-9075

8. Halperin EM. Reliability of water supply and sanitation systems, Modern problems of science and education, 1. 2009

9. Lykin A.V. Mathematical modeling of electrical systems and their elements, 2nd edition, Novosibirsk: Publishing house NSTU.-228 p. (2009)

10. Sveshnikov A.A. Applied methods of the theory of random functions, M., Nauka 232 p. (1968)