A study of magneto-transport through quantum dots is presented. The model allows to analyze tunnelling both from bulk-like contacts and from 2D accumulation layers. The fine features in the I-V characteristics due to the quantum dot states are known to be shifted to different voltages depending upon the value of the magnetic field. While this effect is also well reproduced by our calculations, in this work we concentrate on the amplitude of each current resonance as a function of the magnetic field. Such amplitudes show oscillations reflecting the variation of the density of states at the Fermi energy in the emitter. Furthermore the amplitude increases as a function of the magnetic field for certain features while it decreases for others. In particular we demonstrate that the behaviour of the amplitude of the current resonances is linked to the value of the angular momentum of each dot level through which tunnelling occurs. We show that a selection rule on the angular momentum must be satisfied. As a consequence, tunnelling through specific dot states is strongly suppresses and sometimes prohibited altogether by the presence of the magnetic field. This will allow to extract from the experimental curves detailed information on the nature of the quantum dot wavefunctions involved in the electronic transport. Furthermore, when tunnelling occurs from a 2D accumulation layer to the quantum dot, the presence of a magnetic field hugely increases the strength of some resonant features. This effect is predicted by our model and, to the best of our knowledge, has never been observed.

I. INTRODUCTION

The manufacturing of structures where the electronic motion is confined in 1,2,3 directions has been possible for several years and a large amount of experimental evidence has been collected to study the behaviour of electrons in 2,1,0 dimensions. This has resulted in a series of new physical phenomena and has raised hopes of building devices where the behaviour of single electrons is controlled. Quantum dots are structures where the electron motion is confined in all directions and therefore their energy spectrum is discrete. Several spectroscopic techniques such as linear and non-linear transport, capacitance measurements and far-infrared spectroscopy have been used to investigate the energy spectra of quantum dots. The addition of an external magnetic field has proved to be an excellent way to probe the low-dimensional systems since it introduces a structure-independent quantisation that is superimposed to that induced by the fabrication process. Quantum dots obtained from double-barrier resonant tunnelling semiconductor heterostructures are now used by several groups for magneto-tunnelling investigations [1–4], since it is possible to probe the quantum dot spectra in several regimes from the very low to the very high electron number occupancy, tuning the relative strength between confinement and electron-electron interactions. The spectroscopic investigation is still exciting and far from complete, and several issues such as the importance of the electrostatic interaction among the carriers, detailed characteristics of the quantum dot spectra, degeneracy of the states and spin of the different multiplets must still be fully solved.

Practically, in all the previous studies on non-linear transport, the emphasis was placed on the displacement of the I-V features produced in the quantum dot states as a function of the magnetic field in order to probe the quantum dot spectrum [1–4]. On a more fundamental point lies the issue of actually probing the confined wavefunctions inside a quantum dot. This analysis has been carried out by P.H. Beton et al. [5] for the states in a quantum wire. The method consists in measuring the current voltage characteristics of a quantum wire in the presence of a magnetic field as in the previous studies, but to focus the analysis on the magnitude of the current of each peak. Indeed, it is shown that the amplitude of the current can be associated with the Fourier transform of the electron wave function in the direction across the wire. Very recently [6], we reported on a similar analysis of the experimental data collected on quantum dots fabricated from a double-barrier resonant tunnelling structure by an ion implantation and electrode regrowth process [7]. To the best of our knowledge this was the first time such an analysis had been performed for quantum dots.

In this work we lay the theoretical foundations to ob-
tain a detailed understanding of the electronic properties of quantum dots from magneto-transport experiments. In particular we address the problem of non-linear magneto-transport in 3D-0D-3D and 2D-0D-2D structures. This can yield informations not only on the energy position of the dot states, but also on the actual properties of the wavefunctions, i.e. their angular momentum and their spatial extension. Extending the work performed by P.H. Beton et al., we find that also for quantum dots the height of the resonances in the current-voltage characteristics depends strongly on the magnetic field. The height of each resonance shows oscillations as a function of the magnetic field which reflect the oscillations in the density of states at the Fermi energy. Furthermore the behaviour of the amplitude of the fine features changes considerably depending on the particular quantum dot state considered. It is found that a general trend is obeyed depending on the value of the angular momentum of the dot state, and that the angular momentum introduces a selection rule that strongly suppresses tunnelling through states with angular momentum collinear to the magnetic field. Above certain values of the magnetic field the selection rules totally prohibits tunnelling through such states. The last section deals with the 2D-0D magneto-transport occurring in those experimental structures where an accumulation layer is formed in the emitter. Here the effect of the magnetic field is totally different from the 3D-0D-3D case. We show in particular that the amplitude of the current resonances are a direct measure of the overlap between the Landau levels in the emitter and the dot states involved in the tunnelling process. The presence of the magnetic field hugely increases the tunnelling current flowing through the quantum dot ground state and indeed through all the states with principal quantum number \( n = 0 \) and non-positive angular momentum. It must be noted that this effect applies to the quantum dot ground state, i.e. when only a single electron occupies the dot. Therefore it remains valid even in the presence of electron-electron repulsion inside the dot. The theoretical analysis developed here will help to distinguish the nature of the various features in the experimental investigations, to establish unambiguously the dimensionality of the electrodes and to gain further knowledge of the properties of quantum dots.

II. GENERAL REMARKS

The current-voltage characteristics of quantum dot structures are dominated by the concept of resonant tunnelling. The current flows through a specific quantum dot state only if its energy falls within the energy range of the occupied states in the emitter conduction band. The total current flowing through the structure for a given applied voltage bias is the sum of the current flowing through each quantum dot state. In systems such as double-barrier resonant tunnelling heterostructures, at zero voltage no current flows through the structure because the quantum dot states lie above the Fermi energy in the emitter. As the voltage is raised, the quantum dot states fall below the Fermi energy, resonant tunnelling becomes possible and the current increases. It follows that the voltage at which each dot state becomes available for tunnelling is associated with the energy of the dot state itself. Therefore transport measurements provide informations on the quantum dot spectrum.

It has long been argued that transport measurements must be performed in the presence of a magnetic field in order to be a powerful spectroscopic tool. This is because the presence of impurity states in the quantum dot region can produce features in the I-V characteristics that are similar to the ones produced by the quantum dot states. The presence of a magnetic field allows to alter the geometry and the strength of the confinement of the quantum dot in an controllable fashion so that the properties of the quantum dot can be differentiated from those due to impurities. Furthermore the features produced by the quantum dot can be analyzed in much greater details by the application of a magnetic field, hence the importance of understanding in details the physics of magneto-tunnelling in quantum dots.

The typical structure to study vertical transport in quantum dots consists of a double-barrier quantum well which is confined in the lateral directions. The lateral confinement can be introduced by means of several methods such as etching, ion-bombardment, and the presence of gates. For all these methods the resulting confining potential can be very accurately approximated by a parabola. In the growth direction the heterostructure confining potential \( V(z) \) is much stronger than the lateral one and has the shape of a square well. Therefore the Hamiltonian for the dot containing \( N \) particles can be written as:

\[
H = \sum_{i=1}^{N} \left( \frac{\mathbf{p}_i^2}{2m^*} + \mathbf{A}_i \cdot \mathbf{e} \right)^2 + \frac{1}{2} m^* \omega_0^2 \sum_{i=1}^{N} (x_i^2 + y_i^2) + V(z) \quad (1)
\]

In this work we are going to analyze the case where the magnetic field is applied along the vertical direction of the structure, i.e. along the direction of the current. As we are going to deal with small structures where the single electron effects dominate over the electron-electron repulsion, we do not take into account the Coulomb repulsion inside the dot. This approximation considerably simplifies the problem of transport through quantum dots in the non-linear regime and, while it is not theoretically satisfactory, it has proven correct in several circumstances both experimentally and theoretically. Nevertheless, the analysis presented here is valid also in the presence of the electron electron interaction when this can be approximated with the Hubbard-
like hamiltonian. This has been proved a valid approach for small dots of radius 50 nm or less, as those ones considered here. While many theoretical efforts, based on non-equilibrium Green’s functions, have been devoted to the development of a theory of transport in the non-linear regime for correlated systems, a comprehensive theory capable of dealing with a multi-electron system and including partial occupancy and non-equilibrium filling of the dot states in a quantitative fashion does not exist. In the single particle approximation eq.(1) can be solved exactly. Thus the single electron spectrum, neglecting the effects of spin, is given by:

$$E_{n_d,m_d} = \hbar \omega_d (2n_d + |m_d| + 1) + \frac{1}{2} \hbar \omega_c m_d + E_0$$  (2)

where \(\omega_d = \sqrt{\omega_0^2 + \omega^2}\), \(\omega_0\) characterizes the strength of the lateral confinement in the dot, \(\omega_c\) is the cyclotron frequency and \(E_0\) is the energy of the level due to the confinement in the growth direction. The principal quantum number \(n_d\) can take any non-negative integer value. On the contrary, the quantum number \(m_d\), associated with the angular momentum in the direction of the current and the magnetic field, can take any positive or negative integer value. The eigenfunctions related to the eq. (2) are given by:

$$\Phi_d(r, \varphi, z) = \frac{e^{im_d \varphi}}{\sqrt{2\pi}} R_{n_d,m_d}(r) \Psi_d(z)$$  (3)

where \(R_{n_d,m_d}(r)\) is a function that can be expressed in terms of the hypergeometric function:

$$R_{n,m}(r) \propto e^{-\xi r^2/2\hbar} |m|^{n+1,1} F(-n, |m| + 1, \xi)$$  (4)

where \(\xi = \omega_d m^* r^2/\hbar\). \(\Psi_d(z)\) is the wavefunction in the growth direction which consists of the familiar ground state of a square of finite height.

As pioneered by Bardeen and further developed by Payne and by Liu and Aers, the tunnelling current can be calculated in the framework of a transfer matrix approach. The system is divided into three sub-systems, the two contacts and the quantum dot, and the spectrum for each sub-system considered isolated is determined.

The tunnelling process is modeled as a two-step process in which electrons first tunnel from the emitter to the dot and eventually from the dot to the collector. In spite of its sequential nature, this approach is well-known to reproduce the correct physics even in the resonant tunnelling regime. The current for each step can be calculated from the knowledge of the wavefunctions in the three different regions. At equilibrium the current flowing from the emitter to the dot must be equal to the one going from the dot to the collector. Imposing this condition, enables one to calculate the current as well as the occupancy of the dot states. The strength of the tunnelling current flowing through each dot state is determined by the overlap integral between the dot wavefunctions and those of the contacts. Following Bardeen, this is given by:

$$M_{e\rightarrow d} = \left( \frac{\hbar^2}{2m^*} \right) \int_S (\Phi_e, \nabla \Phi_e^* - \Phi_e^* \nabla \Phi_e) dS$$  (5)

for the transition of the electrons from the emitter to the dot, here \(S\) is any surface across the emitter-dot barrier. A similar formula applies for the tunnelling between the dot and the collector.

In order to calculate the current it is necessary to know not only the electron wave functions in the dot, but also those of the contacts. As these depend on the actual shape of the contacts, we are going to discuss two different cases separately.

### III. First Case: 3D Contacts

Let us first suppose that no geometrical confinement is present in the contacts. This is indeed the case in many experimental situations where dots are either obtained by means of deep etching or by ion bombardment. In the presence of an applied magnetic field, the energy spectrum in the contacts takes the form of highly degenerate 1D Landau sub-bands which are occupied up to the Fermi energy. The energy spectrum is given by:

$$E_{n_e,m_e,k_z} = \hbar \omega_c (n_e + m_e + |m_e| + \frac{1}{2} + \frac{\hbar^2 k_z^2}{2m^*})$$  (6)

where \(k_z\) is the momentum in the growth direction. As for the quantum dot, \(m_e\) can take any integer value while \(n_e\) can take all non-negative integer values. Note however the difference between positive and negative values of \(m_e\). While each Landau sub-bands contains all negative values of \(m_e\), the lowest sub-band, of minimum energy \(\frac{1}{2} \hbar \omega_c\), does not contain positive values of \(m_e\), the first excited sub-band contains only \(m_e = 1\), the second excited sub-band \(m_e = 1, 2\) and so on. In general terms the maximum value of the angular momentum in the \(j\)-th sub-band is \(m_e = j - 1\).

The eigenfunctions are plane waves along the vertical growth direction while for the lateral part they are of the same shape as those of the quantum dot:

$$\Phi_e(r, \varphi, z) = \frac{e^{im_e \varphi}}{\sqrt{2\pi}} R_{n_e,m_e}(r) \Psi_e(z)$$  (7)

where \(\Psi_e(z)\) is the wavefunction defined by the one dimensional square potential: \(V(z) = 0\) before the emitter barrier for \(z \leq b\) and \(V(z) = V_0\) for \(z > b\), \(b\) being the abscissa of the left side of the barrier and \(V_0\) the height of the barrier. In this case \(\Psi_e(z)\) is given by:
\[ \Psi_z = \begin{cases} \sqrt{\frac{2\alpha}{\pi E_z}} \sin(k_c(z - b + L_e)) & z \leq b \\ \sqrt{\frac{2\alpha}{\pi E_z}} \sin(k_c L_e) \exp(-K_c(z - b)) & z \geq b \end{cases} \]  

where \( L_e \) is a normalization constant, \( k_c = \sqrt{2m^*E_z/\hbar} \) is the lateral momentum and \( K_c \) is defined as \( K_c = \sqrt{2m^*(V_0 - E)}/\hbar \). \( E \) is the energy in the vertical direction. The spectrum and wavefunctions of the collector are exactly of the same shape as those of the emitter.

The overlap between the quantum dot states and those in the contacts depends on the height and width of the barriers along the direction through which the current flows. In the lateral directions it depends on the particular quantum dot and contact states considered. The overlap integral for a one dimensional problem with barriers of square shape has been studied in details by Payne [16]. In this work we will adopt the same approach to treat the overlap in the growth direction and we will extend the method to the three dimensional case. In order to do so one must calculate the overlap integrals in the lateral directions between the quantum dot and the contact states. These are of the form:

\[
M_{n_d,m_d;n_e,m_e}^\parallel = \int_0^{2\pi} e^{-i(m_d-m_e)} \varphi_d \varphi_e \int_0^\infty r R_{n_d,m_d}(r) R_{n_e,m_e}(r) dr
\]

where the \( e \) and \( d \) refer to states in the contacts and in the dot respectively. The overlap integral [3] can be calculated analytically for all values of \( m_d, n_d, m_e, n_e \). Furthermore it is clear from [3] that only the states with \( m_d = m_e = m \) produce a non zero overlap. Therefore from the highly degenerate Landau sub-bands in the contacts, at most only one particular state will contribute to the tunnelling current flowing through each dot state: the one with the right value of the angular momentum \( m_d \). For example from the lowest emitter sub-band there cannot be any contribution to the tunnelling through those dot states with positive angular momenta.

The overlap integrals [3] can be calculated analytically and are given by [13]:

\[
\delta_{m_d,m_e} \frac{\omega_d\omega_e}{2} \Gamma(\gamma) \lambda^{-n_d-n_e} \gamma(\lambda-k_d)^{n_d}(\lambda-k_e)^{n_e} \times F(-n_d,-n_e,\gamma,\lambda-k_e)/F(-n_d,-n_e,\gamma,\lambda-k_d) = \frac{\omega_d\omega_e}{2} \frac{\Gamma(\gamma) \lambda^{-n_d-n_e} \gamma(\lambda-k_d)^{n_d}(\lambda-k_e)^{n_e} \times F(-n_d,-n_e,\gamma,\lambda-k_e)}{F(-n_d,-n_e,\gamma,\lambda-k_d)}
\]

where \( \alpha_{d,e} = 1/\omega_{d,e} \sqrt{2m_e\gamma_e/\hbar^2} \), \( \Gamma(\gamma) = \frac{\Gamma(\gamma) \lambda^{-n_d-n_e} \gamma(\lambda-k_d)^{n_d}(\lambda-k_e)^{n_e} \times F(-n_d,-n_e,\gamma,\lambda-k_e)}{F(-n_d,-n_e,\gamma,\lambda-k_d)} \), \( \omega_d = \omega_e = \sqrt{2m_e\gamma_e/\hbar^2} \), \( \gamma = |m_{d,e}| + 1 \) and \( \omega = \omega_c/2 \).

Therefore combining formula [10] with the overlap integrals \( M_{n_d,m_d;n_e,m_e}^\perp \) in the direction of the current as outlined by Payne [16] and by Liu and Aers [17], one can calculate the current flowing through the quantum dot in an analytical way. The above technique has been adopted to calculate the I-V characteristics of 3D-0D-3D structures in the presence of an external magnetic field applied in the vertical direction, i.e. parallel to the current. Fig. 4 shows the calculated I-V characteristics at \( T = 4K \) for several values of the magnetic field, calculated with \( \hbar \omega_0 = 15\text{meV} \). This value of the confinement strength together with our choice of \( GaAs_{0.33}Al_{0.67}As \) barriers of widths 8.7nm and of a \( GaAs \) quantum well 5.1nm width, is in accordance with some of our experimental devices and measurements [1]. At low temperature as the voltage bias is raised, electrons tunnel towards unoccupied states in the collector so that tunnelling is dominated by the emitter barrier.

The triangular shape of the peaks at \( B=0T \) is typical of a 3D-0D tunnelling. As expected, the presence of the magnetic field shifts the position of the peaks on the voltage scale. This effect is due to the change in the energy of the quantum dot states as the magnetic field is increased. More important is the current amplitude of the peaks that oscillates as a function of \( B \) and have a different behaviour from peak to peak. To better illustrate this effect in fig. 4 we show the behaviour of the current amplitude for different states. States \( n_d = 0, m_d = -2, -1, 0, 1, 2 \) and \( n_d = 1, m_d = 0 \) are represented.

The amplitude for all the states shows oscillations as a function of the magnetic field. These oscillations have the characteristic \( \frac{1}{\hbar} \) dependence and are due to the oscillations of the density of states at the Fermi energy in the emitter as the magnetic field is changed.

Furthermore, as the magnetic field is increased, the peaks show a rather different behaviour in terms of their amplitude. The peak associated with the ground state shows an amplitude that decreases systematically as a function of the magnetic field, while the first excited state \( n_d = 0, m_d = -1 \) initially increases in amplitude, attains a maximum, and eventually decreases. The state \( n_d = 0, m_d = -2 \) has the same qualitative features as the \( n_d = 0, m_d = -1 \) peak, but the maximum is more pronounced and shifted at higher magnetic field. These effects have indeed been observed recently by B. Jouault et al [3].

The presence of the magnetic field produces two effects. Firstly it reduces the momentum along the direction of the current for the electrons in the contacts by moving the sub-bands’ minima to higher energies. Secondly it confines the electrons towards the center of the dot. The first effect tends to diminish the current, while the second one increases it by increasing the overlap in the lateral direction between the quantum dot wavefunctions and those of the contacts. Thus each Landau sub-band in the emitter gives a current which attains a maximum at a finite magnetic field. The position of this maximum depends on the relative strength of the two effects. Moreover increasing the angular momentum \( m_d \) decreases the overlap integral and shifts this maximum to higher fields. The total current is obtained by summing the contribu-
tion of all the occupied Landau levels in the emitter and it follows that the shift of the maximum current peak from the origin \( B = 0 \)T is a direct measurement of the magnitude of the angular momentum \( m_d \).

Perhaps even more striking is the totally different behaviour of the states \( n_d = 0, m_d = 1 \) and \( n_d = 0, m_d = -1 \). At \( B = 0 \)T these states are degenerate and the radial part of the wave function is identical for both of them for all values of the magnetic field. This is confirmed by the fact that at \( B = 0 \) the amplitude is the same for both of them. However, the lowest Landau sub-band in the contacts does not contain the state \( n_d = 0, m_d = 1 \) so that there is no contribution to the current flowing through such state from this Landau sub-band. Since the radial part of the overlap integrals is at a maximum between states with the same principal quantum number, as soon as well defined Landau levels are formed in the contacts tunnelling through the state \( n_d = 0, m_d = 1 \) is severely limited. Indeed whenever the magnetic field is strong enough so that only the lowest Landau sub-band is occupied in the contacts, tunnelling through the states with angular momentum \( m > 0 \) is altogether forbidden. This is a manifestation of the general selection rule previously mentioned, tunnelling through a state with positive angular momentum \( m \) is forbidden whenever there are only \( m \) occupied Landau sub-bands in the contacts. The consequence of this is clearly observed in fig. 4: all the current peaks which are not related to the first Landau level in the emitter disappear quickly when the magnetic field increases. These peaks related to a positive angular momentum with \( n = 0 \) or to a strictly positive value of \( n \) (with all values of \( m \)) shift to higher voltage bias as the magnetic field increases. As an example the second current peak due to the first excited quantum box state at about 15meV splits into two peaks with \( B (n = 0, m = \pm 1) \), the third one into three \( (n = 0, m = \pm 2 \) and \( n = 1, m = 0) \) and so on. All the current contributions of the peaks shifting to higher voltages \((n = 0, m > 0) \) or \( n > 0 \) wash out very quickly with the magnetic field.

Finally fig. 6 shows that the contribution of the first Landau level for the state \( n_d = 1, m_d = 0 \) is weak relatively to the other dot states \( n_d = 0, m_d = 0, -1, -2 \). This effect reflects the poor overlap of the radial part between the states \( n_e = 0, m_e = 0 \) and \( n_d = 1, m_d = 0 \). Therefore, the current is sensitive to the shape of the confining potential, even if the shift of the maximum is a general feature for every type of quantum dot.

IV. SECOND CASE: THE 2D-0D TRANSPORT

The same calculations can be performed for a one-dimensionally confined emitter, which corresponds to those experimental structures where an accumulation layer is formed in the emitter. In this case a two-dimensional electron gas (2DEG) is formed to the left-side of the emitter-dot barrier and the transport properties of the structures are dominated by the resonant tunnelling between the 2DEG and the quantum dot. The form of the confining potential that produces the 2DEG in the contacts is roughly of triangular shape and only the lowest level \( E_0^0 \) of this accumulation layer is usually filled and participate to the tunnelling current \( 20 \). Therefore we are only going to consider the lowest emitter state \( E_0^e \) in the following analysis. The method adopted in the previous section for the calculation of the I-V characteristics can be readily used also in this case. The only difference is that the energies and eigenfunctions of the emitter are modified and can be written as:

\[
E_{n_e,m_e} = \hbar \omega_e (n_e + \frac{m_e + |m_e|}{2} + \frac{1}{2}) + E_0^e \tag{11}
\]

the wavefunctions are always given by the equation \[\] but here \( \Psi_e(z) \) is the same for all the electrons in the accumulation layer. In other words the presence of a magnetic field produces a fully-discrete spectrum in the accumulation layer. Several different forms of the electron wavefunction can be taken in the triangular groove. However for the scope of our calculations the detail of the wavefunction is not important, the only restriction being that under the barrier it attains an exponentially decaying form. Furthermore as such wavefunction is the same for all electrons, the overlap integral in the growth direction is a constant. Assuming a simple Lorentzian spectral function of characteristic width \( \Gamma \) for each Landau level in the accumulation layer \[21\], the current can be calculated as:

\[
I = 2e |M|^2 \sum_{n_d,m_d,n_e,m_e} \frac{|M_{n_d,m_d,n_e,m_e}|^2 f(E_d - E_f) \Gamma / \pi}{\Delta E_{e,d}^2 + \Gamma^2} \tag{12}
\]

where \( \Delta E_{e,d} \) is the difference in energy between the state in the emitter and the state in the dot at a given voltage bias.

Figure 6 shows the contribution to the I-V characteristics for the ground state in the dot as the magnetic field increases. At \( B = 0 \)T the ground state current increases exponentially with the energy, as expected if there is neither magnetic nor geometric lateral confinement in the contacts. Indeed the overlap integral is greater with the states of the emitter near the conduction band edge because their small lateral momentum is of the same order of magnitude than size of the dot state. Thus the sharp closure at \( V = 20 \)meV arises when the bottom of the conduction band in the emitter is aligned with the state of the dot. The Landau levels in the accumulation layer become well-defined as the ratio \( \Gamma / \omega_e < 1 \), and their formation is observable in the current flowing through the quantum dot ground state by the presence of several peaks. The Landau levels are shifted towards the emitter Fermi level by the magnetic field. This is reflected in
fig. 3 by the fact that the peak associated to each Landau level shifts towards the threshold voltage as the magnetic field increases.

As the overlap integrals increase with B, the oscillations shown by the current as a function of B increase in amplitude with the magnetic field. It is important to notice that for a 2D emitter the increase in the amplitude of the current with B is much more pronounced than the one seen for 3D contacts. This is due to the fact that in 3D the increase in the overlap between the wavefunctions in the lateral directions is counteracted by a decrease in the electron momentum along the direction of the current, as shown in the previous section. The latter effect does not occur in a 2D emitter and the change in the amplitude of the current resonances as a function of the field is only due to the change in the overlap integral. This is a key difference between tunnelling from 2D and 3D contacts. The contribution to the current associated with each Landau level is directly proportional to the overlap integral between the Landau level and the quantum dot wavefunction. As the overlap with the first Landau level is the only one which tends to one when B increases, the contribution of the first Landau level is dominant.

To the best of our knowledge, such an increase has never been observed and only 3D-0D and 1D-0D transport has been reported. Nevertheless our theory clearly predict this effect in quantitative terms and highlights the profound difference in the behaviour of the fine features associated to the quantum dot states as a function of the magnetic field depending on whether an accumulation layer is formed or otherwise. Furthermore, at least for the quantum dot ground state, our prediction is not affected by the presence of the electron-electron interaction in the quantum dot as when the ground state becomes resonant there is only one electron in the quantum dot.

Finally, in fig. 4 we show the current threshold for different dot states as a function of the magnetic field. As in the 3D-0D case (see fig. 3) we clearly observe here the selection rule over the angular momentum which suppresses some peaks for the \( n_d = 0, m_d > 0 \) states, e.g.: peaks at about 23T for the current contribution of the first Landau level through the dot states \( m_d = 1 \) and 2 or peak at about 7T for the current contribution of the second Landau level through the dot state \( m_d = 2 \). Let us point out that the current contribution of the first Landau level at 23T through the \( n_d = 1 \), \( m_d = 0 \) is very weak but not suppressed as expected from the angular momentum selection rule. Furthermore the increase in the current peak with magnetic field can be observed for all those dot states with non-positive angular momentum and \( n = 0 \).

V. CONCLUSION

We have presented a theoretical model that allows us to calculate analytically the current flowing through a quantum dot state as a function of the magnetic field and the voltage bias. Calculations have been performed for 3D-0D, 2D-0D tunneling. The results show that the amplitude of the current is very sensitive to the angular momentum of the quantum dot state. In particular the presence of the magnetic field hinders tunnelling through the states with angular momentum co-linear with the magnetic field. Furthermore in the case of tunnelling from 2D accumulation layers, the current amplitude associated with certain dot states is predicted to increase hugely as an external magnetic field is applied. The analysis presented in this work can be used to interpret transport experiments in quantum dots and directly access details of the wavefunctions of the zero-dimensional states.

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FIG. 1. I-V for several dot states with a 3D emitter at $T = 4 K$; $V=0$ corresponds to the threshold voltage. From the bottom to the top curve the magnetic field varies from 0 to 10.25T with a step of 0.25T. The curves are shifted for clarity.

FIG. 2. Current maximum as a function of B for several dot states in the 3D-0D tunnelling case.
FIG. 3. I-V curves of the ground state for a 2D emitter at $T = 4K$; $V=0$ corresponds to the threshold voltage. From the bottom to the top curve the magnetic field varies from 0 to 12T with a step of 0.5T. The curves are shifted for clarity.

FIG. 4. Current threshold as a function of $B$ for different dot states in the 2D-0D tunnelling case.