Wideband Antireflection Coating Using Metamaterials

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1. INTRODUCTION

Metamaterials are subwavelength metal-dielectric structures designed to provide extraordinary electromagnetic properties in different frequency ranges. Metamaterials properties are mainly determined by their structures not from the properties of their base compositions [1, 2]. Metamaterials can be engineered to have the negative refractive index [3], the zero refractive index [4], tunable negative permeability [5], high index of refraction [6], anomalous dispersion [7], optical magnetism [8] and other electromagnetic properties that cannot be found in the nature. These properties have been widely used in a variety of applications over wide frequency ranges, for instance, THz switches and modulators [9, 10], high-resolution magnetic resonance radio-frequency imaging [11], super-directive and superconductive antennas. Moreover, it has been shown that metamaterials can be tailored to have a desired impedance and this can be used for designing antireflection coatings [12–14].

To date, different approaches have been followed for designing antireflection coatings [15–18]. In the classical approach, single or multilayer dielectric films have been used as an antireflection coating [19, 20]. The certain drawback associated with this approach is the difficulty of finding appropriate coating materials that satisfy the index-matching requirement. Metamaterials solve this problem due to their designable effective permittivity. One of the recent demonstrations of a metamaterial antireflection in the infrared regime has been proposed by Kim et al. in 2015 [21]. It is a metamaterial structure composed of one-dimensional (1D) metallic slit array. This structure operates as an effective dielectric with designable dielectric permittivity for the polarization perpendicular to the metal grating lines solving the problem of index-matching requirement. However, this antireflection is polarization-sensitive which is not desirable in many applications. Moreover, the bandwidth of this structure is limited to 58% (with a criterion of 10% reflectance) for a substrate with refractive index of 4 [21].

In this paper, we design a broadband and polarization-independent metamaterial-based antireflection coating using two-dimensional (2D) array of metallic pillars in the infrared regime. Our proposed structure is symmetric in both directions. So, it is not sensitive to the polarization of the incident wave at normal incidence. Furthermore, using the transmission line theory, we show that the structure of multilayer Chebyshev matching transformer increases the bandwidth of the antireflection up to 107% at the expense of pass-band ripple. This structure can be used from very low frequencies up to infrared regime by appropriate scaling. © 2018 Optical Society of America

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In this paper, we propose a new approach for realizing antireflection coating using metamaterials. In this approach, a subwavelength array of metallic pillars (with square cross-section) is used for implementing antireflection coating. The effective impedance of the array can be duly adjusted by the size and distance of pillars. Therefore, we design the effective impedance of the antireflection coating to be the geometrical mean of the upper and lower mediums impedance and we choose its height to be a quarter of operating wavelength. Consequently, the reflection vanishes at the desired frequency and fractional bandwidth of 56% is achieved with a criterion of 10% reflectance (the refractive index of the substrate is assumed to be 4). The proposed structure is symmetric in both directions. So, it is not sensitive to the polarization of the incident wave at normal incidence. Furthermore, we show that using the multilayer Chebyshev matching transformer of transmission line theory increases the bandwidth of the antireflection up to 107% at the expense of pass-band ripples. This structure can be used from very low frequencies up to infrared regime by appropriate scaling.
tecture is increased using the presented Chebyshev transformer of transmission line theory. The effect of incident angle on the performance of these structures is investigated in Sec. 4. The paper will be concluded in Sec. 5.

2. 1D ANTIREFLECTION COATING

We start this section with the antireflection coating proposed in [21] which is a metamaterial structure consisting of a one-dimensional metal/air-gap subwavelength silver grating. The substrate, whose refractive index is denoted by \( n_s \), is assumed germanium throughout this paper. The schematic of the metamaterial structure is shown in Fig. 1. The metallic grating has a subwavelength period of \( d \) and the width and the height of the metallic strips are \( w \) and \( h \), respectively.

First, we calculate the transmittance and reflectance of the 1D one-layer silver grating structure on a germanium substrate with the dimensions given in [21] at 100 THz. Here, the transmittance and reflectance of the structure are calculated by finite-difference-time-domain (FDTD) method using a commercial electromagnetic solver (Lumerical FDTD Solutions). The incident wave is polarized in the x-direction while propagating in the z-direction. The optical constant of silver has been extracted from [22] and the refractive index of the germanium substrate has been assumed to be \( n_s = 4 \). Fig. 2 shows the reflectance and transmittance of this structure versus frequency. The bandwidth of this structure (with criterion of reflectance under 10%) is 58%.

This structure is equivalent to a transmission line with a quarter-wave transformer \( (h = \lambda/4) \) which is a simple and useful circuit for matching a real load impedance to a transmission line. An important feature of the quarter-wave transformer is that it can be extended to multilayer designs in a methodical manner to achieve broader bandwidth by providing smooth transition of characteristic impedances and thus reducing the reflectance. In order to minimize the mismatch, the impedance of each layer must be arranged duly using a multilayer transformer. The most well-known multilayer transformers are Chebyshev and binomial transformers. The Chebyshev multilayer transformer suggests optimum bandwidth at the expense of pass-band ripples and helps to obtain a smooth impedance transition. The Chebyshev transformer is designed by making total reflection coefficient of the transformer \( \Gamma(\theta) \) proportional to a Chebyshev polynomial \( T_N(\sec \theta_m \cos \theta) \), where \( N \) is the number of sections, \( \theta \) is the electrical length of each section and \( \theta_m \) is the lower edge of the passband in the transformer. Thus, we have [23]:

\[
\Gamma(\theta) = 2e^{-jN\theta}\sum_{n=0}^{(N-1)/2} \Gamma_n \cos(N - 2n)\theta + \begin{cases} \frac{1}{2} \Gamma_{N/2} & N \text{ even} \\ 0 & N \text{ odd} \end{cases}
\]

(1)

where \( \Gamma_n \) is partial reflection coefficient and \( |A| \) is the maximum allowable reflection coefficient magnitude in the passband. The value of \( \theta_m \) can be calculated as follows [23]:

\[
\sec \theta_m = \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \frac{1}{|A|} \frac{Z_L - Z_0}{Z_L + Z_0} \right) \right]
\]

(2)

where \( Z_L \) is the load impedance which should be matched to \( Z_0 \) through the transformer. For the desired \( |A| \) and specified values of \( Z_L \) and \( Z_0 \), \( \sec \theta_m \) can be determined using Eq. 2 for each selection of \( N \). Then, \( \Gamma_n \) can be specified from Eq. 1. Finally, the impedance of other sections can be calculated using [23]:

\[
\Gamma_n \simeq \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}
\]

(3)

In this paper, in order to avoid increasing the size of the structure and fabrication difficulties we choose a two-layer structure \((N = 2)\) to improve the bandwidth of the antireflection coating as shown in Fig. 3. Now, \( Z_L \) and \( Z_0 \) are the free space and the substrate impedances given by:

\[
Z_L = \eta_0 \\
Z_0 = \frac{\eta_0}{n_s}
\]

(4)

where \( \eta_0 \) is the characteristic impedance of the free space. The substrate is germanium with \( n_s = 4 \) and \( A = \sqrt{0.1} \) is assumed to have reflected power under 10%. It should be noted that \( \theta \) is equal to \( \pi/2 \) in the Chebyshev transformer which means that the height of each layer is equal to a quarter wavelength. The calculated impedances of each layer normalized to the free-space impedance are presented in Table. 1.
Fig. 3. Schematic of the two-layer Chebyshev transformer used to design of antireflection coating.

Table 1. Calculated normalized impedances of each layer in a two-layer antireflection coating

| two-layer antireflection | First-layer | Second-layer |
|--------------------------|------------|-------------|
| Normalized-impedance     | 0.4141     | 0.6037      |

The first and second layer of the structure shown in Fig. 3 can be implemented by subwavelength one-dimensional metallic gratings as shown in Fig. 4. These gratings have identical periods but the width of metals in each layer is determined based on the desirable impedance given in Table 1. It is worth mentioning that the impedance of each layer (for x-polarized incident wave) in 1D subwavelength metallic grating is related to its \( \frac{w_i}{d} \) ratio by the following relation [24]:

\[
Z_i = \eta_0 (1 - \frac{w_i}{d}) \quad i = 1, 2
\]  

(5)

where \( d \) is the period of the gratings and \( w_i \) is the width of the metals in each layer specified by \( i = 1, 2 \). In Eq. 5, it is assumed that the medium filling the slits is air. Given the impedance of each layer as listed in Table 1, the dimensions of metallic strips are specified by Eq. 5 as follows: \( w_1 = 0.6037d \), \( w_2 = 0.4141d \) and \( h_1 = h_2 \) are equal to quarter of the wavelength. Let us assume that the center frequency for our design is 100 THz. We set the subwavelength period of the metallic gratings to \( d = \frac{\lambda}{10} = 300 \text{ nm} \). Therefore, the height of both layers will be 750 nm.

Reflectance and transmittance for an \( x \)-polarized incident wave are plotted in Fig. 5(a). It can be seen that the reflectance of the two-layer structure has a peak higher than 10% in the desired bandwidth and this is due to neglecting the effect of the higher diffraction orders in the Eq. 5. This issue can be resolved by fine-tuning the dimensions of the structure. Another problem is matching the obtained center frequency the desired center frequency (100 THz). In order to shift the center frequency of the reflectance and control the reflectance level, the height and width of the pillars should be changed, respectively. So, the reflectance and transmittance of the redesigned two-layer structure are shown in Fig. 5(b) for the following dimensions which slightly changed: \( h = 650 \text{ nm} \), \( d = 260 \text{ nm} \), \( w_1 = 145 \text{ nm} \), and \( w_2 = 80 \text{ nm} \). In this figure the central frequency is 100 THz, and the reflectance is totally lower than 10% in the bandwidth.

Fig. 4. Schematic of 1D two-layer metallic grating on a germanium substrate. The height of both layers are equal but the width of metals in each layer is chosen in accordance with the desired impedance.

Fig. 5. Reflectance and transmittance of the structure shown in Fig. 4 versus frequency with (a) initial dimensions (b) slightly changed dimensions.
It is clear that the bandwidth of the two-layer structure is substantially greater than the one-layer one. Although the bandwidth of two-layer structure is increased up to 100%, this structure is polarization sensitive. In the next section, we present a simple solution to this problem.

3. ANTIREFLECTION COATING USING AN ARRAY OF METALLIC PILLARS

The schematic of the proposed polarization-independent antireflection coating is shown in Fig. 6. This periodic structure is made of metallic pillars with square cross-section on a substrate with refractive index \( n_s \). The two-dimensional (2D) square pillar array has a subwavelength period of \( d \) in both directions. The side length and height of the pillars are \( w \) and \( h \) respectively. The square pillar array is arranged in a square lattice and the structure is symmetric. As a result, this structure is polarization-independent at normal incidence.

The array of metallic pillars can be modeled by an uniaxial anisotropic effective medium by attributing a surface conductivity at the interface of the structure with the surrounding media. The surface conductivity is due to higher diffracted orders and can be ignored when the period of the structure is much smaller than the wavelength [25]. Accordingly, the effective electric permittivity and permeability of the antireflection coating can be obtained from the following equations [25]:

\[
\begin{align*}
\mu_x &= \mu_y = \frac{S_0^2}{\eta_0} \\
\varepsilon_x &= \varepsilon_y = \frac{1}{\varepsilon_0}
\end{align*}
\]  

where \( S_0 \) is the overlap integral between the zeroth-order plane wave and the component of the electric field of the TEM mode across the whole unit cell and it is calculated as follows [25]:

\[
S_0^2 = \left[ \int_{\text{unit \ cell}} p(x,y) dx \right]^2 \int_{\text{unit \ cell}} |p(x,y)|^2 dx
\]  

An analytical approximate expression for \( p(x,y) \) has been given in [25] when the pillars are perfect electric conductors (PECs).

According to Eq. 6 the effective impedance of the structure is equal to \( Z_{\text{eff}} = \eta_0 S_0^2 \), where \( \eta_0 \) is characteristic impedance of the free space. It is worth mentioning that \( S_0^2 \) is only dependent on the \( w/d \) ratio. Consequently, by appropriate designing of the \( w/d \) ratio, the required impedance of the antireflection coating can be acquired. Values of normalized impedances for different \( w/d \) have been presented in Table. 2.

For a one-layer antireflection coating the impedance of the coating should be the geometric mean of the characteristic impedance of the substrate and free space. Hence, we need \( Z_{\text{eff}} = \eta_0 / \sqrt{\eta_s} \). Assuming \( n_s = 4 \) and according to Table. 2 we obtain \( w = 0.56d \). As already pointed out, the height of antireflection coating is set to \( h = \lambda/4 \). It should be noted that the square pillars should have a sufficiently subwavelength period. Thus, the period of the structure is fixed at \( d = \lambda/10 \).

First, we consider the design of antireflection coating at low frequencies where the metals can be approximated by PEC. Our simulations show that PEC assumption is accurate up to the terahertz regime. This is demonstrated in Fig. 7 where the reflectance of the antireflection coating designed at frequency of 5 THz is plotted by assuming metals to be PEC (blue solid line) and silver (red circles).

The results of Fig. 7 show that zero reflectance is observed around the designed center frequency 5 THz. In this case, the bandwidth is 54%. The slight deviation in the minimum reflectance frequency from what we expect from the circuit model is due to neglecting the effect of higher diffraction orders. This negligible discrepancy can be easily resolved by 3% increase in the height of the pillars.

At infrared frequencies, PEC approximation is not very accurate. As an example, consider an antireflection coating designed at 100 THz (\( \lambda = 3\mu m \)) with \( d = \lambda/10 = 300nm \), \( w = 0.56d = 168nm \) and \( h = \lambda/4 = 750nm \). Fig. 8 shows the transmittance and reflectance of this structure versus frequency assuming metals as PEC (blue solid line) and silver (red circles). It can be seen that when the metals are assumed as silver, the minimum reflectance appears at 90 THz which is fairly close to the desired central frequency 100 THz. On the other hand, the normalized bandwidth of this structure is slightly increased to 56% due to the loss of the silver which slightly reduces the amount of reflectance (and also transmittance).

The difference between the desired center frequency and the minimum level of reflectance can be resolved with a slight adjustment of structure dimensions. By reducing the height of the pillars (\( h = 700nm \)) and increasing the width of the pillars (\( w = 180nm \)), the minimum reflectance frequency is shifted to
Table 2. Calculated \( w/d \) ratios to obtain various effective characteristic impedance for square pillar array normalized to the characteristic impedance of free space

| Normalized Impedance | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
|----------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( w/d \)           | 0.8 | 0.78| 0.73| 0.66| 0.56| 0.45| 0.34| 0.22| 0.104|

100 THz. Also, the minimum reflectance is almost zero. The simulation results of this structure have been shown in Fig. 9. In this case, the bandwidth of almost 59% is obtained.

Now, as shown in Fig. 10, we design a two-layer antireflection coating based on Chebyshev transformer to increase the bandwidth. The impedance of each layer is given in Table 1. According to Table 2, these impedances can be realized by \( w_1 = 0.63d \) and \( w_2 = 0.46d \) where \( d = \lambda/10 \) is chosen. Furthermore, the height of each layer for a Chebyshev transformer is \( h = \lambda/4 \).

Reflectance and transmittance of two-layer structure are plotted in Fig. 11(a). Same as the 1D structure the center frequency of the result do not match to the desired center frequency 100 THz. In order to increase the center frequency of the reflectance curve, the height of the pillars should be reduced. The dimensions of the two-layer structure are tuned as \( h = \lambda/4 = 640 nm, w_1 = 0.63d \) and \( w_2 = 0.46d \) with \( d = \lambda/10 \). Using these fine-tuned dimensions, the structure shown in Fig. 10 is simulated and the transmittance and reflectance of this structure versus frequency are shown in Fig. 11(b). In this figure, the pillars are assumed silver and the maximum bandwidth is equal to 103%. In the next section, we investigate the sensitivity of this structure to the angle of the incident wave.

4. OBLIQUE INCIDENCE

We also study the dependence of the metamaterial-based antireflection coating performance to the angle of the incident wave for the structure shown in Fig. 10 for both TE and TM-polarizations. Fig. 12(a) and Fig. 12(b) show the transmittance and reflectance versus frequency and angle of incidence in the y-z plane for TM-polarization. It is found that the performance of the proposed antireflection remains good within the incident angle range from 0 to 30 degree for TE polarization and from 0 to 40 degree for TM polarization.

5. CONCLUSION

We have demonstrated that a broadband metamaterial can be designed by using the multilayer Chebyshev matching transformer as an antireflection coating in wide frequency ranges. Firstly, we increased the bandwidth of the 1D metamaterial-based antireflection coating which has been proposed by Kim et al. up to 100% in the infrared regime using this idea. Also, we proposed a metamaterial-based structure consisting of a sub-
wavelength array of metallic pillars with a square cross-section for implementing a polarization-sensitive antireflection coating. By utilizing analytical relations, the dimensions of the structure were determined such that the effective impedance of the metallic pillar array to be the geometric mean of the air and the substrate impedance to eliminate the reflectance at the desired frequency. The thickness of the subwavelength metallic pillars is chosen a quarter of the free space wavelength. The fractional bandwidth of the proposed one-layer antireflection coating for germanium substrate is equal to 56%. Furthermore, the bandwidth of the structure is increased to 107% by using the two-layer Chebyshev matching transformer. At normal incidence, this structure is polarization independent due to the symmetry of the structure. Also, we have investigated the sensitivity of the antireflection coating performance to the angle of incidence and an acceptable performance at oblique incidence was observed.

Fig. 11. The Reflectance and the transmittance of two-layer 2D antireflection coating versus frequency (a) before fine-tuning (b) after fine-tuning.

Fig. 12. (a) Transmittance and (b) Reflectance of the proposed 2D two-layer antireflection coating versus frequency and angle of incidence for TE polarization

REFERENCES
1. D. Smith and D. Schurig, “Electromagnetic wave propagation in media with indefinite permittivity and permeability tensors,” Phys. Rev. Lett. 90, 077405 (2003).
2. D. Smith, D. Vier, T. Koschny, and C. Soukoulis, “Electromagnetic parameter retrieval from inhomogeneous metamaterials,” Phys. Review E 71, 036617 (2005).
3. D. R. Smith, W. J. Padilla, D. Vier, S. C. Nemat-Nasser, and S. Schultz, “Composite medium with simultaneously negative permeability and permittivity,” Phys. review letters 84, 4184 (2000).
4. M. Silveirinha and N. Engheta, “Design of matched zero-index metamaterials using nonmagnetic inclusions in epsilon-near-zero media,” Phys. Rev. B 75, 075119 (2007).
5. C. Kurter, T. Lan, L. Sarytchev, and S. M. Anlage, “Tunable negative permeability in a three-dimensional superconducting metamaterial,” Phys. Rev. Appl. 3, 054010 (2015).
6. M. Choi, S. H. Lee, Y. Kim, S. B. Kang, J. Shin, M. H. Kwak, K.-Y. Kang, Y.-H. Lee, N. Park, and B. Min, “A terahertz metamaterial with unnaturally high refractive index,” Nature. 470, 369 (2011).
7. X. Yang, J. Yao, J. Rho, X. Yin, and X. Zhang, “Experimental realization of three-dimensional indefinite cavities at the nanoscale with anomalous scaling laws,” Nat. Photonics 6, 450 (2012).
Fig. 13. (a) Transmittance and (b) Reflectance of the proposed 2D two-layer antireflection coating versus frequency and angle of incidence for TM polarization

8. Y. A. Urzhumov and G. Shvets, “Optical magnetism and negative refraction in plasmonic metamaterials,” Solid State Commun. 146, 208–220 (2008).
9. H.-T. Chen, W. J. Padilla, J. M. Zide, S. R. Bank, A. C. Gossard, A. J. Taylor, and R. D. Averitt, “Ultrafast optical switching of terahertz metamaterials fabricated on eras/gaas nanolaminate superlattices,” Opt. letters 32, 1620–1622 (2007).
10. W. L. Chan, H.-T. Chen, A. J. Taylor, I. Brener, M. J. Cich, and D. M. Mittleman, “A spatial light modulator for terahertz beams,” Appl. Phys. Lett. 94, 215311 (2009).
11. S. B. Yeap, X. Qing, Z. N. Chen, and M. Sun, “Metamaterial lens for magnetic resonance imaging,” in Antennas Technol. (IWAT), 2015 International Workshop on, (IEEE, 2015), pp. 202–205.
12. H. Chen, J. Zhou, J. O’Hara, F. Chen, A. Azad, and A. Taylor, “Antireflection coating using planar metamaterials,” arXiv preprint arXiv:1002.2305 (2010).
13. H.-T. Chen, J. Zhou, J. F. O’Hara, F. F. Chen, A. K. Azad, and A. J. Taylor, “A novel approach of antireflection coating using planar metamaterials,” in Photonic Metamaterials and Plasmonics, (Optical Society of America, 2010), p. MMC4.
14. H.-T. Chen, J. Zhou, J. F. O’Hara, F. Chen, A. K. Azad, and A. J. Taylor, “Antireflection coating using metamaterials and identification of its mechanism,” Phys. review letters 105, 073901 (2010).
15. H. Yao, Z. Chen, and T. Chang, “Broadband multilayer antireflection coating in the region,” in Infrared, Millimeter, and Terahertz waves (IRMMW-THz), 2015 40th International Conference on, (IEEE, 2015), pp. 1–2.
16. M. K. Hedayati, S. Fahr, C. Etrich, F. Faupel, C. Rockstuhl, and M. Elbahri, “The hybrid concept for realization of an ultra-thin plasmonic metamaterial antireflection coating and plasmonic rainbow,” Nanoscale 6, 6037–6045 (2014).
17. U. Schulz, “Wideband antireflection coatings by combining interference multilayers with structured top layers,” Opt. express 17, 8704–8708 (2009).
18. B. Zhang, J. Hendrickson, N. Nader, H.-T. Chen, and J. Guo, “Metasurface optical antireflection coating,” Appl. Phys. Lett. 105, 241113 (2014).
19. L. Young, “Synthesis of multiple antireflection films over a prescribed frequency band,” JOSA. 51, 967–974 (1961).
20. K. Chopra, S. Manchanda, and R. Hradaynath, “Triple-layer broadband antireflection coatings using homogeneously mixed dielectric coatings,” Thin Solid Films 55, 49–53 (1978).
21. W. Kim, J. Guo, and J. Hendrickson, “Subwavelength metal grating metamaterial for polarization-selective optical antireflection coating,” JOSA B 32, 1392–1398 (2015).
22. D. Smith, E. Shiles, M. Inokuti, and E. Palik, “Handbook of optical constants of solids,” Handb. Opt. Constants Solids 1, 369–406 (1985).
23. D. M. Pozar, Microwave engineering (John Wiley & Sons, 2009).
24. A. Khavasi and K. Mehrany, “Circuit model for lamellar metallic gratings in the sub-wavelength regime,” IEEE J. Quantum Electron. 47, 1330–1335 (2011).
25. M. Edalatipour, A. Khavasi, and K. Mehrany, “Physics of broadband brezowski transmission through square array of rectangular metallic pillars,” JOSA B 32, 1202–1207 (2015).