Lagrangian dynamics and heat transfer in porous media convection

Shuang Liu\(^1\), Linfeng Jiang\(^1\), Cheng Wang\(^1\), Chao Sun\(^{1,2,3}\)†

\(^1\)Center for Combustion Energy, Key Laboratory for Thermal Science and Power Engineering of Ministry of Education, Department of Energy and Power Engineering, Tsinghua University, Beijing 100084, China

\(^2\)Department of Engineering Mechanics, School of Aerospace Engineering, Tsinghua University, Beijing 100084, China

\(^3\)Physics of Fluids Group, Max PlanckUniversity of Twente Centre for Complex Fluid Dynamics, University of Twente, 7500 AE Enschede, The Netherlands

(Received xx; revised xx; accepted xx)

We report a numerical study of Rayleigh–Bénard convection through random porous media using pore-scale modelling, focusing on the Lagrangian dynamics of fluid particles and heat transfer for varied porosities \(\phi\). Due to the interaction between the porous medium and the coherent flow structures, the flow is found to be highly heterogeneous, consisting of convection channels with strong flow strength and stagnant regions with low velocities. The modifications of flow field due to porous structure have a significant influence on the dynamics of fluid particles. Evaluation of the particle displacement along the trajectory reveals the emergence of anomalous transport for long times as \(\phi\) is decreased, which is associated with the long-time correlation of Lagrangian velocity of the fluid. As porosity is decreased, the cross-correlation between the vertical velocity and temperature fluctuation is enhanced, which reveals a mechanism to enhance the heat transfer in porous media convection.

Key words: Turbulent convection, convection in porous media

1. Introduction

Transport and mixing processes in porous media flows have attracted much attention over the years, owing to their importance in a wide range of natural and industrial settings, such as the contaminant transport in the subsurface, the kinetics of chemical reactions, and the transport in biological systems (Manke et al. 2007; Seymour et al. 2004; Cushman & Tartakovsky 2016). Understanding the dynamics of fluid particles in complex flows is also important from the theoretical perspective, since the features of the fluid flow advecting the particles can be inferred from the particle dynamics (Falkovich et al. 2001; Biferale et al. 2004; Toschi & Bodenschatz 2009; Calzavarini et al. 2020; Mathai et al. 2020).

Significant progress has been achieved for the transport and mixing processes in the pressure-driven porous media flows in the Darcy regime. The transport dynamics are governed by the probability density function (PDF) of velocity in the pores, particularly the distribution in the low-velocity range, which plays a critical role in the anomalous

† Email address for correspondence: chaosun@tsinghua.edu.cn
or non-Fickian transport behavior in porous media (Berkowitz et al. 2006). A large probability of low velocities can result in persistent anomalous transport. The anomalous transport behaviors in heterogeneous flow field have been modeled using the continuous-time random-walk approaches, accounting for the impact of broad distributions of advective times in the pores (Berkowitz et al. 2006; Bijeljic & Blunt 2006; Bijeljic et al. 2011, 2013; De Anna et al. 2013; Kang et al. 2014; Lester et al. 2014; Holzner et al. 2015; Dentz et al. 2016; Morales et al. 2017; Nissan & Berkowitz 2018; Dentz et al. 2018; Souzy et al. 2020). Dentz et al. (2018) identified and quantified the role of advection and molecular diffusion on the preasymptotic non-Fickian transport, and found that the non-Fickian transport features can persist on the scale of representative elementary volume. In the experimental study of dispersion of tracer particles in a 3D porous media flow, Souzy et al. (2020) identified a transition from a ballistic regime to an intermediate, anomalous regime, and found that the transition to the asymptotic Fickian regime is determined by the minimal velocity. There are also studies devoted to investigating the transport and mixing processes in the presence of additional, complex effects, such as the effect of flow inertia (Nissan & Berkowitz 2018) among others.

The flow pattern and flux in buoyancy-driven porous-media flow have received much attention, for its relevance to various processes in nature and industry, such as geothermal energy recovery and geological sequestration of carbon dioxide (Huppert & Neufeld 2014; Hewitt 2020). Recently, it has been found that hydrodynamic dispersion has a significant effect on the flow properties of porous-media convection (Hidalgo & Carrera 2009; Emami-Meybodi et al. 2015; Wen et al. 2018; De Paoli et al. 2020). In the related numerical studies, the Fickian dispersion model is commonly used (Bear 1972), which may only be valid at asymptotically large scales. It is interesting to study when and how the non-Fickian dispersion affects the macroscopic properties of porous-media convection. The construction and evaluation of macroscopic transport models require a good understanding of the pore-scale transport process. Thus, it is important to investigate the pore-scale transport behavior, which vary often exhibits anomalous, non-Fickian features. The fact that hitherto few studies exist for particle transport in the porous-media convection provides a motivation for the present work. Here we report a numerical study on the transport of fluid particles and heat transfer in random porous media based on pore-scale modelling. In pore-scale models, the detailed flow features in the pores are resolved, which are useful for constructing appropriate macroscopic models and understanding the microscopic mechanisms underlying the macroscopic flow properties (Wood et al. 2020; Gasow et al. 2020).

The paper is organized as follows. In §2 the model and numerical approaches are described. The main results are presented in §3, focusing on the flow field, fluid particle transport and heat transfer properties. Finally, summaries of this study are given in §4.

2. Numerical model

We consider two-dimensional Rayleigh–Bénard convection in a square cell. Fixed, circular obstacles of diameter $D$ are distributed randomly in the cell, with the pore length $\ell \geq \ell_{\text{min}}$, where $\ell_{\text{min}}$ is the minimum pore length. The bottom and top plates are heated and cooled, respectively, with a temperature difference $\Delta$. The fluid flow in the pores is governed by the Oberbeck-Boussinesq equations

\[ \nabla \cdot \mathbf{v} = 0, \quad \frac{\partial T}{\partial t} + \nabla \cdot (\mathbf{v} T) = \frac{1}{\sqrt{Pr Ra_f}} \nabla^2 T, \]

\[ \frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \sqrt{\frac{Pr}{Ra_f}} \nabla^2 \mathbf{v} + T \mathbf{e}_z, \]
Lagrangian dynamics and heat transfer in porous media convection

where $v = (u, w)$ is the velocity vector in the $(x, z)$ plane, $T$ is the temperature, and $p$ the pressure. The unit vector $e_z$ denotes the direction of the buoyancy force. The dimensionless control parameters are the fluid Rayleigh number $Ra_f = g \beta \Delta L^3 / (\nu \kappa)$ and the Prandtl number $Pr = \nu / \kappa$, where $g$, $\beta$, $\nu$ and $\kappa$ denote the gravitational acceleration, thermal expansion coefficient, kinematic viscosity and thermal diffusivity, respectively. The cell height $L$, temperature difference $\Delta$, and free-fall velocity $U = \sqrt{g \beta \Delta L}$ are used to non-dimensionalize the governing equations. Yet another dimensionless parameter is the porosity $\phi$, quantifying the volume fraction of the fluid phase. In the traditional RB convection without porous structure, we have $\phi = 1$. Besides porosity, an additional key non-dimensional number for the porous medium is the Darcy number, $Da = K / L^2$, where $K$ is the permeability, measuring the ability for the fluid to flow through the medium. For porous media convection, the appropriate Rayleigh number would be the Darcy Rayleigh number $Ra_D = Da Ra_f$. For a fixed $Ra_f$, $Ra_D$ is dependent on the medium properties. The heat transfer efficiency of the system is measured by the Nusselt number, $Nu = \sqrt{Ra_f Pr \langle w T \rangle_{W,t}} - \langle \partial_z T \rangle_{W,t}$, where $\langle \cdot \rangle_{W,t}$ denotes taking average over the horizontal wall and over time.

We impose no-slip and no-penetration boundary conditions on the cell boundaries and fluid-obstacle interfaces. For the thermal boundary conditions, the horizontal top and bottom plates are kept at fixed temperatures, and the sidewalls are thermally insulated. The fluid and obstacles are assumed to have the same thermal properties.

The simulation is based on a second-order finite difference method (Verzicco & Orlandi 1996; van der Poel et al. 2015), and the immersed boundary approach is employed to account for the obstacles (Uhlmann 2005; Breugem 2012). The heat transfer between the fluid and the obstacles is considered by solving the temperature equations in both the fluid phase and obstacles (Ardekani et al. 2018). We refer the reader to Liu et al. (2020) for more details of the numerical approaches. Lagrangian tracking algorithm is employed in the direct numerical simulation. In total, 200,000 fluid particles are tracked.

For the simulations a uniform Eularian grid is used with sufficient resolution to resolve the boundary layers and bulk flows, satisfying the classical criterion for the direct numerical simulations of turbulent convection (Shishkina et al. 2010). A grid of $1080 \times 1080$ is used for most cases. For the smallest porosity with the smallest characteristic pore scale, a grid of $2160 \times 2160$ is employed. The circular grains of porous medium are resolved with at least 22 grid nodes.

In this study, the transport properties of fluid particles and heat transfer are investigated for $\phi \in [0.654, 1]$ and the fluid Rayleigh number $Ra_f = 10^8$, $10^9$ with Prandtl number $Pr = 4.3$ and obstacle diameter $D = 0.02$. A reasonable estimate of $Da$ can be obtained based on the Kozeny’s equation $Da = \phi^3 D^2 / [\beta (1 - \phi)^2 L^2]$ (Nield & Bejan 2006; Gasow et al. 2020). With the empirical model coefficient $\beta = 150$, the minimum Darcy number reached in this study is $Da = 6.2 \times 10^{-6}$.

3. Results

3.1. Flow field

Figure 1 shows the instantaneous fields of the temperature $T$ and velocity magnitude $v \equiv |v|$, and the typical trajectories of fluid particles at various $\phi$. In the traditional RB convection with $\phi = 1$, the flow consists of a well-organized large-scale circulation and two corner rolls, around which the fluid particles take a quite periodic motion, as depicted in figure 1(c). We find that, in the presence of obstacles, the flow organization and particle movement are strongly modified. With the decrease of $\phi$, the convection strength is
Figure 1. Snapshots of (a, d, g, j) temperature $T$ and (b, e, h, k) velocity magnitude $v \equiv |v|$, and (c, f, i, l) the typical trajectories of fluid particles for various $\phi$ at the same fluid Rayleigh number $Ra_f = 10^9$: (a – c) $\phi = 1$, (d – f) $\phi = 0.984$, (g – i) $\phi = 0.812$, and (j – l) $\phi = 0.654$. In the snapshots of $v$, the obstacles are indicated by circles. In (c, f, i, l), the trajectories of five fluid particles are depicted, indicated by different colors. Particle positions are shown with fixed time intervals in each plot. The magnitude of particle velocity is quantified by the marker size, with larger marker for larger velocity.

reduced due to the enhanced drag of the porous matrix, and the temperature mixing in the bulk is less efficient, as revealed by the snapshots of the temperature and velocity magnitude in figure 1. In the presence of a small number of obstacles, the flow structure is less organized, and consequently the fluid trajectories become more irregular, as shown in figure 1(f). The flow field in porous media is highly heterogeneous due to the interaction between the porous medium and coherent flow structures, and it fluctuates both spatially and temporally. Interestingly, for small $\phi$, the flow is dominated by large-scale plumes, and convection channels with strong flow strength emerge in the pores, which are closely related to the plume dynamics, and the patches with low velocities appear due to the impedance of the obstacles (see figures 1(h, k)). Along the convection channels, long-range transport of fluid particles is observed, while the particles can stay ‘trapped’ for relatively long duration in the low-velocity regions. For small $\phi$, the flow exhibits columnar structures, similar to those observed in the severely confined RB convection (Chong & Xia 2016).

The particle movement is highly irregular in the heterogeneous flow field of thermal convection in porous media. In order to quantify the chaotic particle dynamics, we plot
the PDFs $P(v/\sigma_v)$ of the normalized velocity magnitude $v/\sigma_v$ for various $\phi$, where $\sigma_v$ denotes the standard deviation of $v$. As $\phi$ is decreased, the probability density for low velocity increases significantly. The time series of the normalized velocity magnitude $v/\sigma_v$ of a fluid particle are shown in the inset of figure 2(a). We find that for small $\phi$, due to the emergence of the convection channels and low-velocity regions, the time variation of $v_0$ exhibits strong intermittency, with the existence of high-velocity bursts, interrupted by long-time trapping events with low velocities.

We plot in figure 2(b) the temporal autocorrelation function $C_v(t) = \langle [v_0 - \langle v_0 \rangle] [v_t - \langle v_t \rangle] \rangle / \sigma_v^2$ of the particle velocity $v$ to further quantify the particle dynamics, where $v_0$ and $v_t$ indicate the velocity magnitudes at times 0 and $t$, respectively. The symbol $\langle \cdot \rangle$ denotes taking averaging over an ensemble of fluid particles. $C_v$ decreases with $t$, demonstrating the loss of memory of the particle dynamics. Compared with the large-$\phi$ case, $C_v$ decays slower for smaller $\phi$, and the autocorrelation is enhanced with decreasing $\phi$ at fixed $t$.

We note that, as $\phi$ is decreased, the initial part of the decay process takes an exponential form, as demonstrated by the linear behavior, $\ln(C_v) \sim t$, in the log-linear plot. For the two smallest $\phi$, the decaying behaviors of $C_v(t)$ are similar and approximately follow a stretched-exponential form with $\ln(C_v) \sim \sqrt{t}$, which suggests that the change of the velocity autocorrelation at small enough $\phi$ is attributed to the characteristic time scale of Lagrangian fluid transport. The relaxation behavior in a stretched-exponential form at small $\phi$ is attributed to the existence of very large relaxation times. To show this, the stretched exponential may be expressed as an integral of exponential functions with a spectrum of decaying rates (Bouchaud 2008; Johnston 2006):

$$
\exp \left[ -\left( \frac{t}{\tau} \right)^{1/2} \right] = \int_0^\infty P(\lambda) \exp \left( -\lambda \frac{t}{\tau} \right) d\lambda,
$$

where $\tau$ denotes the characteristic relaxation time, $\lambda$ is the ratio of relaxation time, and $P(\lambda)$ is the PDF of $\lambda$. Smaller $\lambda$ indicates slower relaxation. Since the right-hand side of expression (3.1) is the Laplace transform of $P(\lambda)$, $P(\lambda)$ is obtained via taking the inverse
Laplace transform. For small $\lambda$, one obtains (Johnston 2006)

$$P(\lambda) \approx \exp(-BL^\beta)$$  \hspace{1cm} (3.2)

up to subleading power-law corrections, where $\beta = -1$, and $B$ is a positive constant. Expression (3.2) suggests that a very limited probability of super-slow relaxation can result into a global relaxation in a stretched exponential form (Bouchaud 2008). The appearance of large relaxation times is attributed to the trapping events of fluid particles at low-velocity regions. The purely exponential decay corresponds to $\beta = -\infty$, i.e., no long-time relaxation (Bouchaud 2008).

3.2. Fluid particle transport

Now we study the transport properties of fluid particles by quantifying the particle displacement $s(t) \equiv \int_0^t v(\tau) d\tau$ along the trajectory. The transport behaviors for the ensemble of particles exhibit strong fluctuations, and the ensemble-averaged displacement $\langle s \rangle$ grows with the time-independent mean velocity $\langle v \rangle$, namely, $\langle s \rangle = \langle v \rangle \cdot t$. Figure 3 shows the time evolution of the displacement variance $\sigma_s^2 = \langle (s - \langle s \rangle)^2 \rangle$ of fluid particles for different $\phi$ at $Ra_f = 10^8$ and $10^9$. $\sigma_s^2$ measures the width of the displacement distribution. It is found that $\sigma_s^2$ grows ballistically with $\sigma_s^2 \sim t^2$ at small $t$. This ballistic regime is robust for different $\phi$, while the proportionality constant decreases with $\phi$. The ballistic regime is a universal phenomenon for general transport processes (Batchelor 1950; Bourgoin 2015; Mathai et al. 2018), and is associated with the strong Lagrangian velocity autocorrelations observed at small times.

Figure 3 also shows the deviation from the ballistic regime at large $t$, and $\sigma_s^2$ exhibits a sub-ballistic scaling, $\sigma_s^2 \sim t^\gamma$, with an effective scaling exponent $\gamma < 2$. The transition to a different transport behavior at large $t$ is expected and indicates the loss of memory of fluid particles to the initial conditions (Bourgoin 2015). In the presence of a small number of obstacles, due to the impact of strong velocity fluctuations, the fluid particles can efficiently explore the irregular flow field and lose the memory about the initial conditions. Consequently, the ballistic regime terminates at an early time, and beyond that the fluid particles approximately exhibit a Fickian transport behavior with $\sigma_s^2 \sim t$, as in the case of $\phi = 0.984$ in figure 3. When $\phi$ is further decreased, $\sigma_s^2$ at relatively large time displays a clear deviation from the Fickian behavior, with an effective scaling exponent $1 < \gamma < 2$. This anomalous non-Fickian behavior of particles in porous media is associated with the increased probability density of low velocity (Berkowitz et al. 2006; Souzy et al. 2020). Similar phenomena are observed for both values of $Ra_f$ considered, demonstrating the robustness for the emergence of non-Fickian behavior with the decrease of $\phi$ in porous media convection. Non-Fickian behavior of particle transport is an omnipresent phenomenon and has been reported in many different settings, both with and without porous media (Richardson 1926; Grossmann 1990; Berkowitz et al. 2006; Salazar & Collins 2009; Bijeljic et al. 2011; Bourgoin 2015; Dentz et al. 2018; Souzy et al. 2020; Taghizadeh et al. 2020).

The transport properties of fluid particles can be related to the temporal autocorrelation function $C_v$ of particle velocity $v$, in the spirit of the Green-Kubo relations, which connect a transport coefficient to a correlation function in time (Kubo et al. 2012). Considering that $\langle s \rangle = \langle v \rangle \cdot t$, $\sigma_s^2$ can be directly related to $C_v$ as

$$\sigma_s^2(t) = \langle (s - \langle s \rangle)^2 \rangle = \langle (\int_0^t dt'[v(t') - \langle v \rangle)]^2 \rangle$$

$$= 2\sigma_v^2 \int_0^t dt'[t - t')C_v(t').$$  \hspace{1cm} (3.3)
Figure 3. Displacement variance $\sigma^2_s(t)$ of fluid particles along the trajectory for various $\phi$ at $Ra_f = 10^8$ and $10^9$. The values of parameters $(Ra_f, \phi)$ are given in the figure. The results at $Ra_f = 10^8$ are shifted upward for clarity. The solid lines are the growth curves of $\sigma^2_s(t)$ obtained from expression (3.3) and the idealized velocity autocorrelation functions in exponential (black) and stretched-exponential (cyan and red) forms. For reference, several scaling laws are included as dashed grey lines.

Some derivation details are given in Appendix A. Based on the numerically identified correlation behaviors in figure 2(b), here we consider two empirical, idealized forms of the autocorrelation function: $C_{v,1}(t) = e^{-at}$ and $C_{v,2}(t) = e^{-\sqrt{bt}}$. $C_{v,1}$ and $C_{v,2}$ are in the exponential and stretched-exponential forms, which are similar in form to the correlation properties of fluid particles in porous media with large and small $\phi$, respectively. Based on expression (3.3) and symbolic integration we obtain

$$\begin{align*}
\sigma^2_{s,1}(t) &= \frac{2\sigma_v^2}{a^2} (at - 1 + e^{-at}), \\
\sigma^2_{s,2}(t) &= \frac{4\sigma_v^2}{b^2} [bt - 6 + 2e^{-\sqrt{bt}}(bt + 3\sqrt{bt} + 3)].
\end{align*}$$

(3.4)

Despite that the assumed autocorrelation functions are highly idealized, the global trend of variation of $\sigma^2_s(t)$ can be captured by expressions (3.4), as shown in figure 3. Expressions (3.4) show that, for both forms of the velocity autocorrelation function, the particles will reach the Fickian regime in the long-time limit. The appearance of the initial ballistic regime is evident from the series expansions of (3.4) at $t = 0$. When the particle velocities are exponentially correlated in time, the particles will exhibit Fickian transport behavior for $t \gg 1/a \sim O(1)$; while when the autocorrelation function $C_v$ has a perfect, stretched-exponential form, $C_v = e^{-\sqrt{bt}}$, the term proportional to $e^{-\sqrt{bt}}$ may disrupt the appearance of the Fickian behavior even when $t \gg 1$, resulting in the anomalous transport behavior at relatively large $t$, consistent with the observation in figure 3. Deviations of (3.4) from the simulation results are visible, which are attributed to the deviations of the velocity autocorrelation of fluid particles from the idealized exponential or stretched-exponential forms, as shown in figure 2(b). Since the idealized autocorrelation functions underestimate the autocorrelation of particle velocity in porous media at large $t$ for the parameters considered, the predicted displacement variance (3.4) may fail to capture the behavior of $\sigma^2_s$ for large times.

The above analysis shows that the anomalous transport behavior at small $\phi$ is associated with the qualitatively different time correlation properties of fluid particles. When $t$ is large enough, the term proportional to $t$ will dominate over other terms, and
the Fickian transport behavior will be achieved (Souzy et al. 2020). Considering that the decaying rate of $C(t)$ is expected to decrease for smaller $\phi$, the presymptotic, anomalous transport behavior may last for relatively long time compared to the characteristic time scale of particle transport at small $\phi$, and is expected to be relevant in realistic porous media convection.

### 3.3. Heat transfer

Now we focus on the heat transfer properties of the fluid particles. We plot in figure 4(a) the cross-correlation $C_{w,T}$ between the vertical velocity $w$ and temperature $T$ as a function of $\phi$ for fluid particles in the whole cell at $Ra_f = 10^8$ and $10^9$. The inset shows the typical time series of the convective heat flux $w \cdot \delta T$ in the vertical direction (red lines) and $w$ (blue lines) at $Ra_f = 10^9$ and various $\phi$: from top to bottom $\phi = 1$, 0.984, and 0.812, corresponding to the filled symbols in the $C_{w,T}(\phi)$ plot. (b) $Nu$ as a function of $\phi$ for $Ra_f = 10^8$ and $10^9$.

### Figure 4.

(a) Cross-correlation $C_{w,T}$ between the vertical velocity $w$ and temperature $T$ as a function of $\phi$ for fluid particles in the whole cell at $Ra_f = 10^8$ and $10^9$. The inset shows the typical time series of the convective heat flux $w \cdot \delta T$ in the vertical direction (red lines) and $w$ (blue lines) at $Ra_f = 10^9$ and various $\phi$: from top to bottom $\phi = 1$, 0.984, and 0.812, corresponding to the filled symbols in the $C_{w,T}(\phi)$ plot. (b) $Nu$ as a function of $\phi$ for $Ra_f = 10^8$ and $10^9$. The inset of figure 4(a) shows the time series of the convective heat flux $w \delta T$ in the vertical direction and the vertical velocity $w$ of fluid particles for various $\phi$, where $\delta T = T - T_m$. In the traditional RB convection with $\phi = 1$, the fluid particles with high velocity may not contribute to the vertical heat transfer for relatively long times, which is due to the fact that the temperature in the center core is well mixed with small temperature fluctuation $\delta T$; while for small $\phi$, the low amplitude of the vertical heat transfer is mainly due to the low particle velocity, confirming the strong cross-correlation between the vertical velocity and temperature fluctuation. As a consequence of this enhanced cross-correlation, the probability of negative Lagrangian heat transfer in the bulk region will be decreased compared with that in the traditional RB convection.
4. Conclusion

To conclude, we find that the flow field in porous media is highly heterogeneous, with the presence of convection channels with strong fluid transport and stagnant regions with low velocities. The displacement variance $\sigma_s^2$ of fluid particles shows the existence of distinct transport regimes. For small $t$ a ballistic regime is identified, which transitions to a sub-ballistic regime at large $t$. For small $\phi$ we observe that the fluid particle is able to exhibit anomalous transport with a super-linear growth of $\sigma_s^2$ at relatively long times. The anomalous transport is associated with the long-time correlation of Lagrangian velocity of the fluid in porous media with small $\phi$. Even though the anomaly is expected to cross over into the Fickian regime in the long-time limit, it may last for a significantly long time when compared to the characteristic time scale of particle transport, and can be of serious consequences in realistic porous media convection. Regarding the heat transfer properties, the cross-correlation $C_{w,T}$ between the vertical velocity and temperature fluctuation is significantly enhanced with decreasing porosity, implying the close relation between the plume dynamics and heterogeneous velocity field in porous media convection. The present findings on particle transport have important implications for hydrodynamic dispersion in various processes in nature and industry, such as the geothermal energy recovery and geological sequestration of carbon dioxide, and the results on heat transfer suggest a new approach for enhancing heat transport by controlling the coherence of the bulk flow. In the future it is interesting to extend the work to larger parameter space and to more complex porous media, such as those with broad distributions of pore scales (Bijeljic et al. 2013; Gjetvaj et al. 2015).

Acknowledgments

We thank D. Lohse, V. Mathai, Y. Yang and Z. Wan for fruitful discussions. This work was supported by the Natural Science Foundation of China (Grant Nos. 11988102, 91852202, 1186131005 and 11672156) and Tsinghua University Initiative Scientific Research Program (Grant No. 20193080058). S.L. acknowledges the project funded by the China Postdoctoral Science Foundation.

Declaration of interests

The authors report no conflict of interest.

Appendix A.

In this Appendix, some details for the derivation of expression (3.3) are given. Considering that $\langle s \rangle = \langle v \rangle \cdot t$, the variance $\sigma_s^2(t)$ can be directly related to the autocorrelation function $C_v(t)$ of particle velocity as

$$
\sigma_s^2(t) = \langle (s - \langle s \rangle)^2 \rangle = \langle (\int_0^t dt'[v(t') - \langle v \rangle])^2 \rangle
$$

$$
= \int_0^t dt' \int_0^t dt'' \langle [v(t') - \langle v \rangle][v(t'') - \langle v \rangle] \rangle
$$

$$
= \sigma_v^2 \int_0^t dt' \int_0^t dt'' C_v(t' - t''), \quad (A1)
$$
where the property of time translation invariance is invoked. By introducing a change of variable, $h = t' - t''$, and employing the symmetry property, $C_v(h) = C_v(-h)$, we obtain
\[
\sigma_z^2(t) = 2\sigma_v^2 \int_0^t dh(t - h)C_v(h).
\]

(A2)

REFERENCES

Ardekani, M. N., Abouali, O., Picano, F. & Brandt, L. 2018 Heat transfer in laminar Couett flow laden with rigid spherical particles. J. Fluid Mech. 834, 308–334.

Batchelor, G. K. 1950 The application of the similarity theory of turbulence to atmospheric diffusion. Q. J. Roy. Meteor. Soc. 76 (328), 133–146.

Bear, J. 1972 Dynamics of Fluids in Porous Media. American Elsevier.

Berkowitz, B., Cortis, A., Dentz, M. & Scher, H. 2006 Modeling non-Fickian transport in geological formations as a continuous time random walk. Rev. Geophys. 44 (2).

Biferaile, L., Boffetta, G., Celani, A., Devenish, B. J., Lanotte, A. & Toschi, F. 2004 Multifractal statistics of Lagrangian velocity and acceleration in turbulence. Phys. Rev. Lett. 93 (6), 064502.

Bijeljic, B. & Blunt, M. J. 2006 Pore-scale modeling and continuous time random walk analysis of dispersion in porous media. Water Resour. Res. 42 (1).

Bijeljic, B., Mostaghimi, P. & Blunt, M. J. 2011 Signature of non-Fickian solute transport in complex heterogeneous porous media. Phys. Rev. Lett. 107 (20), 204502.

Bijeljic, B., Raeni, A., Mostaghimi, P. & Blunt, M. J. 2013 Predictions of non-Fickian solute transport in different classes of porous media using direct simulation on pore-scale images. Phys. Rev. E 87 (1), 013011.

Bouchaud, J.-P. 2008 Anomalous relaxation in complex systems: From stretched to compressed exponentials. In Anomalous Transport: Foundations and Applications (ed. R. Klages, G. Radons & I. M. Sokolov), chap. 11, pp. 327–345. John Wiley & Sons.

Bourgin, M. 2015 Turbulent pair dispersion as a ballistic cascade phenomenology. J. Fluid Mech. 772, 678–704.

Breugem, W.-P. 2012 A second-order accurate immersed boundary method for fully resolved simulations of particle-laden flows. J. Comput. Phys. 231 (13), 4469–4498.

Calzavarini, E., Jiang, L.-F. & Sun, C. 2020 Anisotropic particles in two-dimensional convective turbulence. Phys. Fluids 32 (2), 023305.

Chong, K. L. & Xia, K.-Q. 2016 Exploring the severely confined regime in Rayleigh-Bénard convection. J. Fluid Mech. 805.

Cushman, J. H. & Tartakovsky, D. M. 2016 The Handbook of Groundwater Engineering. CRC Press.

De Anna, P., Le Borgne, T., Dentz, M., Tartakovsky, A. M., Bolster, D. & Davy, P. 2013 Flow intermittency, dispersion, and correlated continuous time random walks in porous media. Phys. Rev. Lett. 110 (18), 184502.

De Paoli, M., Alipour, M. & Soldati, A. 2020 How non-Darcy effects influence scaling laws in Hele-Shaw convection experiments. J. Fluid Mech. 892, A41.

Dentz, M., Icardi, M. & Hidalgo, J. J. 2018 Mechanisms of dispersion in a porous medium. J. Fluid Mech. 841, 851–882.

Dentz, M., Kang, P. K., Comolli, A., Le Borgne, T. & Lester, D. R. 2016 Continuous time random walks for the evolution of Lagrangian velocities. Phys. Rev. Fluids 1 (7), 074004.

Emami-Meybodi, H., Hassanzadeh, H. & Ennis-King, J. 2015 CO2 dissolution in the presence of background flow of deep saline aquifers. Water Resour. Res. 51 (4), 2595–2615.

Falkovich, G., Gawedzki, K. & Vergassola, M. 2001 Particles and fields in fluid turbulence. Rev. Mod. Phys. 73 (4), 913.

Gasow, S., Lin, Z., Zhang, H. C., Kuznetsov, A. V., Avila, M. & Jin, Y. 2020 Effects of pore scale on the macroscopic properties of natural convection in porous media. J. Fluid Mech. 891, A25.
Lagrangian dynamics and heat transfer in porous media convection

Gjetvaj, F., Russian, A., Gouze, P. & Dentz, M. 2015 Dual control of flow field heterogeneity and immobile porosity on non-Fickian transport in Berea sandstone. *Water Resour. Res.* **51** (10), 8273–8293.

Grossmann, S. 1990 Diffusion by turbulence. *Ann. der Phys.* **502** (7), 577–582.

Hewitt, D. R. 2020 Vigorous convection in porous media. *Proc. R. Soc. A* **476** (2239), 20200111.

Hidalgo, J. J. & Carrera, J. 2009 Effect of dispersion on the onset of convection during CO2 sequestration. *J. Fluid Mech.* **640**, 441–452.

Kang, P. K., De Anna, P., Nunes, J. P., Bijeljic, B., Blunt, M. J. & Juanes, R. 2014 Pore-scale intermittent velocity structure underpinning anomalous transport through 3-D porous media. *Geophys. Res. Lett.* **41** (17), 6184–6190.

Liu, S., Jiang, L.-F., Chong, K. L., Zhu, X.-J., Wan, Z.-H., Verzicco, R., Stevens, R. J. A. M., Lohse, D. & Sun, C. 2020 From Rayleigh–Bénard convection to porous-media convection: how porosity affects heat transfer and flow structure. *J. Fluid Mech.* **895**, A18.

Nield, D. A. & Bejan, A. 2006 *Convection in Porous Media*. Springer.

Nissen, A. & Berkwitz, B. 2018 Inertial effects on flow and transport in heterogeneous porous media. *Phys. Rev. E* **98** (6), 063012.

Souzy, M., Lhuissier, H., Méheust, Y., LeBorgne, T. & Metzger, B. 2020 Velocity distributions, dispersion and stretching in three-dimensional porous media. *J. Fluid Mech.* **891**, A16.

Taghizadeh, E., Valdés-Parada, F. J. & Wood, B. D. 2020 Preasymptotic Taylor dispersion: evolution from the initial condition. *J. Fluid Mech.* **889**.
Toschi, F. & Bodenschatz, E. 2009 Lagrangian properties of particles in turbulence. *Annu. Rev. Fluid Mech.* **41**, 375–404.

Uhlmann, M. 2005 An immersed boundary method with direct forcing for the simulation of particulate flows. *J. Comput. Phys.* **209**(2), 448–476.

Verzicco, R. & Orlandi, P. 1996 A finite-difference scheme for three-dimensional incompressible flows in cylindrical coordinates. *J. Comput. Phys.* **123**(2), 402–414.

Wen, B.-L., Chang, K. W. & Hesse, M. A. 2018 Rayleigh-Darcy convection with hydrodynamic dispersion. *Phys. Rev. Fluids* **3**(12), 123801.

Wood, B. D., He, X.-L. & Apte, S. V. 2020 Modeling turbulent flows in porous media. *Annu. Rev. Fluid Mech.* **52**, 171–203.