The application of Geometric Brownian Motion in stock forecasting during the coronavirus outbreak in Indonesia

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Abstract. Geometric Brownian Motion is a mathematical model that can be used in stock price forecasting. This research aimed to predict the stock prices during the outbreak of coronavirus in Indonesia. There are four important steps of this research, such as calculating the return of the stocks, analyzing the normality test of stock price return, forecasting the stock price by using Geometric Brownian Motion, and calculating the errors of the forecasting. Based on the research, the MAPE for three forecasted stock prices is mostly around 10%.

1. Introduction

According to Indonesian Stock Exchange, the definition of the stock is a sign of the capital participation of a person or entity in a company. There are two advantages that would be gained by investors for buying or owning the stock i.e getting the dividends (profit of sharing) and the capital gains (the difference of buying and selling). The stock investment has a great risk due to fluctuations in the stock price of the capital market on a daily basis. The risks of the stock investment are capital loss and liquidity risk.

There are two ways to minimize the risk in the stock investment decision making, such as collecting the update information related to stock trading and making diversification of investments. Diversification of investments is a spread of investments in several companies by forming a stock portfolio. A portfolio is a combination or a set of assets, both financial assets and real assets owned by investors. Through the establishment of an investment portfolio, investors can minimize the potential of loss that gained from investment in the capital market as the result of stock price fluctuation. There are some studies regarding portfolio optimization in stock prices, such as the works that explained in [1], [2], and [3].

The fluctuations of the stock price in the capital market are reflected through uncertain stock price changes at all the times. Based on the efficient market hypothesis, the stock price movement is influenced by two factors i.e. the present of the stock price that reflects the price of the stock in the past and market response to company information. The stock price is assumed as a random variable and the fluctuation of daily stock price in the capital market is considered a random movement (random walk)[4].

In [4], Dmouj modeled the factors which affected the stock price into Geometric Brownian Motion model. Geometric Brownian Motion is a stochastic model of continuous time that used to forecast the stock price. In the study, the stock price forecasting was developed by using confident level and mean function of lognormal distribution. The other study that implemented Geometric Brownian Motion
model is conducted by Agustini et. al. [5]. The study was forecasted IHSG stock and divided the time period of forecasting. Refers to the works of [4] and [5], the Geometric Brownian Motion provides the high accuracy in forecasting the stock price. Therefore, it is interesting to apply the same method using Geometric Brownian Motion and investigate the performance of the model to forecast the stock price during the coronavirus outbreak in Indonesia.

The outbreak of coronavirus has caused a pandemic of respiratory disease (COVID-19). Many sectors have been poorly affected by this outbreak, including the economic sector. At the same time, there is growing concern about the fluctuations of the stock price in the capital market in many countries around the world as an impact of the outbreak. Based on that issue, this research aimed to predict the stock price of the three prominent issuers in Indonesia i.e PT. Bank Central Asia Tbk., PT. Bank Rakyat Indonesia Tbk., and PT. H.M Sampoerna Tbk. by using Geometric Brownian Motion model. These issuers are chosen by considering the largest stock capitalization in 2019.

2. Research methodology

The methodology in conducting this research as follows:

2.1. Literature study

This is the first methodology of the research. At this stage, reference of theories are collected to support the fundamental theory regarding the stock price and the Geometric Brownian Motion model.

2.1.1. Return

Each investor expects the maximum profit gained from investing in the capital market. The advantage gained by an investor in a stock investment is known as a return. According to [6] return is the result obtained from the investment. Mathematically the stock return is a change in the price of the stock per the price of the past shares that can be formulated as follows [7]:

\[ R_t = \frac{S_t - S_{t-1}}{S_{t-1}} \]  

where \( R_t \) and \( S_t \) denote the return and the stock price at time \( t \) respectively.

2.1.2. Stochastic differential equation

The change in the stock price is one example of the stochastic process, because of its change over time in an uncertain manner. Stock price fluctuations are influenced by drift and volatility. Drift is the expected growth rate on stocks and volatility is the measure of uncertainty about the price movement of future stocks[5]. According to [4], the change of stock price can be observed through two parts:

1. the predictable, calculated and anticipated part is the expectation of the return of the stock price over a certain period of time \( (dt) \) formulated as \( \mu S_t dt \).
2. the stochastic and unpredictable parts that reflect the random change in the stock price over a certain period of time \( (dt) \) in response to market events that are formulated as \( \sigma S_t dB_t \).

Based on the assumptions above, the changes of the stock can be modeled as follows:

\[ dS_t = \mu S_t dt + \sigma S_t dB_t \]  

where:
\[ \mu \] : drift of the stock price,
\[ \sigma \] : volatility of the stock price,
\[ S_t \] : stock price when \( t \),
\[ B_t \] : brownian motion,
\[ dS_t \] : stock price change.

The Equation (2) is a stochastic differential equation. Since Equation (2) is a stochastic differential equations then the equation can be solved by analytical solutions using Ito’s Lemma [4].
2.1.3. Ito’s Lemma
Ito’s Lemma is the most important result of manipulating random variables required in the Ito process. Ito’s Lemma was formed through the manipulation of random variables using a Taylor series expansion with $G(x, t)$ functions so that the following Taylor series expansion was obtained [4]:

$$dG \approx \frac{\partial G}{\partial X} dX + \frac{\partial G}{\partial t} dt + \frac{1}{2} \left( \frac{\partial^2 G}{\partial X^2} (dX)^2 + \frac{\partial^2 G}{\partial t^2} (dt)^2 \right).$$

**Ito’s Lemma:**
If $G(x, t)$ is a function of two variables and $X_t$ is a stochastic process that meets the $dX_t = adt + bdB_t$ for a Brownian motion $B_t$. Then:

$$dG(t, X_t) = \left( \frac{\partial G}{\partial X} a + \frac{\partial G}{\partial t} + \frac{1}{2} b^2 \frac{\partial^2 G}{\partial X^2} \right) dt + \frac{\partial G}{\partial X} b dB_t. \quad (3)$$

2.1.4. Geometric Brownian Motion
Using the Ito’s Lemma in Equation (3) to complete the stochastic differential Equation (2) then the analytic solution obtained from the stochastic differential equation is as follows. Based on Equation (2) It is known that:

$$dS_t = \mu S_t dt + \sigma S_t dB_t.$$  

The two sections are divided by $S_t$ so that:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dB_t,$$

where $\frac{dS_t}{S_t} = d(\ln(S_t))$ therefore can be assumed as $G = \ln(S_t)$.

Then proceed with looking for the derivative $G$ function of each related variable as follows:

$$\frac{\partial G}{\partial S_t} = \frac{1}{S_t},$$

$$\frac{\partial G}{\partial S_t} = 0,$$

$$\frac{\partial^2 G}{\partial S_t^2} = -\frac{1}{S_t^2}.$$

Based on Ito’s Lemma, Equation (2) can be written as follows:

$$d[\ln(S_t)] = \left[ \frac{1}{S_t} \mu S_t + 0 + \frac{1}{2} \left( -\frac{1}{S_t^2} \right) (\sigma S_t)^2 \right] dt + \frac{1}{S_t} \sigma S_t dB_t,$$

$$d[\ln(S_t)] = \left[ \mu - \frac{1}{2} \sigma^2 \right] dt + \sigma dB_t,$$

where $dB_t = \varepsilon \sqrt{dt}$ therefore:

$$d[\ln(S_t)] = \left[ \mu - \frac{1}{2} \sigma^2 \right] dt + \sigma \varepsilon \sqrt{dt}.$$

$d[\ln(S_t)]$ can be written as $\ln(S_t) - \ln(S_{t-1})$ therefore,

$$\ln(S_t) = \ln(S_{t-1}) + \left[ \mu - \frac{1}{2} \sigma^2 \right] dt + \sigma \varepsilon \sqrt{dt},$$

$$e^{\ln(S_t)} = e^{[\ln(S_{t-1}) + \left[ \mu - \frac{1}{2} \sigma^2 \right] dt + \sigma \varepsilon \sqrt{dt}]} \quad (4)$$

Equation (4) is an analytic solution that obtained from the stochastic differential Equation (2) by using Ito’s Lemma. Assumed to be $S_t = F_t$, $dt = \delta t$ and $\varepsilon$ is the smallest positive integers so the Geometric Brownian Motion model is used to predict the following stock price follows [4]:

$$S_t = S_{t-1} e^{[\mu - \frac{1}{2} \sigma^2] dt + \sigma \varepsilon \sqrt{dt}}.$$
\[ F_t = S_{(t-1)} e^{\left[ \mu \frac{t}{\sigma^2} - \frac{1}{2} \sigma^2 t \right] + \sigma \varepsilon \sqrt{\delta t}} \]  

where:
- \( F_t \): stock price forecasting at \( t \),
- \( S_{(t-1)} \): actual stock price at \( t-1 \),
- \( \mu \): drift of the stock price,
- \( \sigma \): volatility of the stock price,
- \( \delta t \): time scale.

### 2.1.5. Drift and volatility

Drift is the mean value of the return price of the stock that is formulated as follows [7]:

\[ \mu = \frac{1}{M \delta t} \sum_{i=1}^{M} R_t \]  

where:
- \( \mu \): drift of the stock price,
- \( M \): amount of stocks return,
- \( R_t \): return of stocks,
- \( \delta t \): time scale.

Volatility is defined as fluctuations of the returns of a securities (stock) or portfolio within a certain period of time formulated as follows [7]:

\[ \sigma = \sqrt{\frac{1}{(M - 1) \delta t} \sum_{i=1}^{M} (R_t - \bar{R})^2} \]  

where:
- \( \sigma \): stock price volatility,
- \( M \): amount of stocks return,
- \( R_t \): stocks return at \( t \),
- \( \delta t \): time scale.

### 2.1.6. MAPE

Mean Absolute Percentage Error (MAPE) or commonly known as MAPE is one measure of precision used to determine the accuracy of forecasting methods. This approach is useful when size or large variable forecasts are important in evaluating predictive accuracy. If the MAPE value resulting from a smaller forecasting method then the forecasting method is better. The MAPE formula is defined as follows [8]:

\[ MAPE = \frac{\sum_{t=1}^{N} |S_t - F_t|}{S_t} \]  

where:
- \( S_t \): actual stock price at \( t \),
- \( N \): amount of stocks price,
- \( F_t \): stock price forecasting at \( t \).

The level of forecasting accuracy of MAPE is presented in Table 1 [8].
Table 1. The forecasting accuracy level of MAPE

| MAPE       | Accuracy           |
|------------|--------------------|
| < 10%      | Highly accurate    |
| 10% - 20%  | Good forecast      |
| 21% - 50%  | Reasonable forecast|
| > 50%      | Inaccurate forecast|

The smaller the value of MAPE, the more accurate the forecasting model is. By using MAPE formula and adjusting the scale on the Table 1, some conclusions of the model can be made.

2.2. Data collection
At this second stage of the methodology, stock price data collection is conducted by using the source of yahoo finance. There are three issuers of stock that have been selected to be forecasted under IHSG which are PT. Bank Central Asia Tbk., PT. Bank Rakyat Indonesia Tbk., and PT. H.M Sampoerna Tbk. based on the largest stock capitalization in 2019. The daily stock price of these three companies was taken from January 2019 to March 2020, then the data is used to generate the Geometric Brownian Motion model for forecasting the next period until June 2020.

2.3. Stock price forecasting
The third methodology is stock price forecasting by using Geometric Brownian Motion. To obtain the results of stock forecasting, there are four steps as follow:
a. Calculating the return of stock price
b. Normality test of stock price return
c. Forecasting stock price by using Geometric Brownian Motion
d. Calculating MAPE

3. The forecasting result
In this research, the return of the stock price is calculating follows the formula in Equation (1). Furthermore, the return of the stock that used in Geometric Brownian Motion must be normally distributed. Hence, normality test is needed by using Kolmogorov-Smirnov test. Based on that test, if \( p\text{-value} \geq 0.05 \) then \( H_0 \) is rejected. It indicates that the return is normally distributed.
The result of normality test for the return of PT Bank Central Asia Tbk., PT Bank Rakyat Indonesia Tbk., and PT H.M Sampoerna Tbk. is presented in Table 2.

Table 2. The result of normality test

| Data                          | \( p\text{-value} \) |
|-------------------------------|---------------------|
| PT Bank Central Asia Tbk.     | 0.112               |
| PT Bank Rakyat Indonesia Tbk. | 0.091               |
| PT H.M Sampoerna              | 0.084               |

Based on the data in Table 2, the \( p\text{-value} \) of all stocks is greater than 0.05. It means that the return of PT Bank Central Asia Tbk., PT Bank Rakyat Indonesia Tbk., and PT H.M Sampoerna Tbk.

The next step is forecasting the stock prices using Geometric Brownian Motion in Equation (5). The drift and the volatility are calculated based on the formula (6) and (7). In this study, model of Geometric Brownian Motion is applied and simulated to forecast the stocks with several test of \( n \) iterations, such as with \( n = 50, n = 100, n = 500, \) and \( n = 1000 \). Then it obtained the smallest
MAPE using $n = 50$ iterations. The Figure 1 (a)-(c) show the results of stock prices forecasting on three issuers during the outbreak of coronavirus in Indonesia. The colorful lines indicate there are 50 possibilities of the stock price would be in 54 days, from 1 April 2020 to 30 June 2020.

From 50 possibilities of the stock price forecasting, there is one possibility of the line that would be a single result of the forecasting. These several results that have been compared with the actual results shown on Figure 2. Figure 2 (a) and Figure 2 (b) show the comparison graph of actual stock price and forecast stock price from PT. Bank Central Asia Tbk. and PT. Bank Rakyat Indonesia Tbk. in 54 days forward from 1 April 2020 to 30 June 2020. Both the greatest gap between the actual price and the forecast price happened from 8 June 2020 to 18 June 2020 in day 38 to 46, it caused by the effect of volatility of the stock. Meanwhile the greatest gap in PT. H.M Sampoerna Tbk., shown in Figure 2 (c), happened in day 52 and 54.

![Figure 1. Forecast graph of (a) PT. Bank Central Asia Tbk., (b) PT. Bank Rakyat Indonesia Tbk., and (c) PT. H.M Sampoerna Tbk.](image)

Overall the large fluctuation of three issuers of stocks from April to June mostly caused by the large-scale social restrictions as an impact of the coronavirus outbreak. The issue about the large-scale social restrictions in Jakarta in April, has encouraged the investors to sell the stocks. It reflected in day 1 to day 20 on the graph, the actual price was decreased. The price began to increase after 30 days mostly seen at the end of June 2020. It indicates the investor confidence to invest again following the end time of large-scale social restrictions in Jakarta. The value of MAPE for the forecasting results is shown in Table 3.
### Table 3. The MAPE of forecasting result

| Issuers                              | MAPE  |
|--------------------------------------|-------|
| PT. Bank Central Asia Tbk.           | 10.2% |
| PT. Bank Rakyat Indonesia Tbk.       | 8.6%  |
| PT. H.M Sampoerna Tbk.               | 10.7% |

![Actual Price and Forecast Price](image1)

(a)

![Actual Price and Forecast Price](image2)

(b)

![Actual Price and Forecast Price](image3)

(c)

**Figure 2.** The graph forecast vs. actual price of (a) PT. Bank Central Asia Tbk., (b) PT. Bank Rakyat Indonesia Tbk., and (c) PT. H.M Sampoerna Tbk.

### 4. Conclusion

Investment in capital market through the stock is one of the way to allocate the capital and generate additional income. Based on references, the Geometric Brownian Motion is highly accurate model to forecast the stock price, in short or long term. It proved both by the results of MAPE value which lower than 10% in normal condition (without pandemic), for example in [5]. However, in this study, Geometric Brownian Motion does not work very accurately to forecast stock prices during the outbreak of coronavirus, it proved by the MAPE that mostly higher than 10%. It is indicated because the historical data that used to calculate the drift and volatility are mostly from normal condition (before pandemic), but it is used to forecast the stock prices during the pandemic. This occurs as an impact of the coronavirus outbreak that influences the global economy including Indonesia, which causes the high volatility of the stock prices. Thus, it cannot be predicted well. Nevertheless, the
results in this study still need to be improved and discussed, especially to investigate and analyze the causes of high errors obtained from the forecasting results.

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References
[1] Dombrovskiy V V, Dombrovskiy D V and Lyashenko E A 2004 Investment portfolio optimization with transaction costs and constraints using model predictive control KORUS \emph{Proc. The 8th Russian-Korean Int. Symp.} 3 202-5
[2] Syaifudin W H 2015 Penerapan Model Predictive Control (MPC) pada optimasi portofolio saham \emph{Master Degree Thesis, Mathematics Department, Institut Teknologi Sepuluh Nopember, Surabaya}
[3] Fitria I, Apriliani E and Putri E R M 2016 Investment management using portfolio optimization with stock price forecasting \emph{J. Applied Mathematical Sciences} 10 2405-13
[4] Dmouj A 2006 Stock price modelling : theory and practice \emph{Masters Degree Thesis, Vrije Univ.}
[5] Agustini W F, Affianti I R and Putri E R M 2018 Stock price prediction using Geometric Brownian Motion \emph{J. Phys. Conf. Ser.} 974 1
[6] Jogianto H 2017 \emph{Teori Portofolio dan Analisis Investasi} 11th ed. (Yogyakarta: Universitas Gadjah Mada)
[7] Wilmott P 2017 \emph{Introduces Quantitative Finance} 2nd Ed. (America: John Wiley and Sons)
[8] Lawrence K, Klimberg R K and Lawrence S M 2009 \emph{Fundamentals of Forecasting Using Excel} (America: Industrial Press Inc.)