First estimates of the \((\alpha + \beta)^\pi\) from two–photon experiments.

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Abstract.

We carry out the semi–model analysis of existing data on the angular distributions of the \(\gamma\gamma \rightarrow \pi^+\pi^-, \pi^0\pi^0\) reactions with purpose to obtain S-wave cross section and D-wave’s parameters. In the first time we obtain from experiment the sum of electrical and magnetic pion polarizabilities: \((\alpha + \beta)^{\pi^+} = 0.28 \pm 0.07\) (MARK-II [1]), \((\alpha + \beta)^{\pi^0} = 0.38 \pm 0.05\) (CELLO [2]), \((\alpha + \beta)^{\pi^0} = 1.26 \pm 0.06\) (Crystal Ball [3]) in units of \(10^{-42}\, cm^3\, (e^2 = 4\pi\alpha)\).
1. Nowadays there exist an experimental data on the angular distributions of the $\gamma\gamma \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ reactions \[1, 2, 3\]. Probably the most interesting physical question here is related with the S-wave cross section (long–standing problem of scalar resonances spectrum and structure). However the D–wave dominates in these reactions in a wide energy region, so this task needs either a bid statistics or the accurate modelling of the main contribution. Note that the D-wave contains the low–energy parameter, not investigated earlier — the sum of electrical and magnetic pion polarizabilities. The most surprising result of our analysis is the fact that these low–energy parameters may be obtained from existing data with small statistical errors and with minimal model assumptions.

An angular distributions can be analyzed in different ways: from model–independent partial–wave analysis (see \[4\]) up to using of model final state formulae for both helicity amplitudes \[5\]. Here we use some intermediate method of analysis \[2\]: for the main helicity 2 amplitude some model is used with few free parameters, as for S–wave contribution – it is extracted from data independently in every energy bin. The reasons for such a choice are that model–independent analysis gives too ambiguous results \[1\] with present data and just the S-wave contains the biggest theoretical uncertainty. In contrast to \[2\] we use the unitary model \[3\] with final state interaction for helicity 2 amplitude instead of rather rough simple expression. Its using leads \[7\] to other conclusions as compared with \[2\]: a) There is no necessity for additional damping of Born QED contribution (of unclear nature) to describe the data , b) The S-wave below 1 Gev is much less and does not conflict with results of near–threshold analysis \[8\]. Present more detailed analysis confirms the main conclusions of \[7\] and gives the additional arguments in favour of the model \[6\].

In such an analysis the main assumption is the dominantness on helicity 2 state in the decay $f_2(1270) \rightarrow \gamma\gamma$. It has theoretical foundations and does not contradict to previous numerous experiments. The quality of present data does not allow to check this assumption on our analysis.

2. Formulae for helicity 2 amplitude are contained in \[6\], so we shall specify here only the background contribution, interfering with resonance $f_2(1270)$.

\[
W^C = M^{Born}_{+-} + \frac{g^2_{\rho\pi\gamma}}{4} \left( \frac{1}{m^2_\rho - t} + \frac{1}{m^2_\rho - u} \right) + a^C
\]

\[
W^N = \frac{g^2_{\rho\pi\gamma}}{4} \left( \frac{1}{m^2_\rho - t} + \frac{1}{m^2_\rho - u} \right) + (\rho \rightarrow \omega) + a^N
\]

(1)
Here $a^{C,N}$ are some arbitrary constants combining all other possible contributions, $C =$ Charged, $N =$ Neutral. The same idea was used in \cite{8} for helicity 0 amplitude: only the lightest $\rho$- and $\omega$- cross-exchanges are considered as "alive", all other are approximated by an arbitrary constant. It gives the more general formulae with additional degrees of freedom. Note that these amplitudes satisfy the one–channel unitary condition, $I = 0$, $J = 2$ final state interaction we describe in a standard way, $I = 2$, $J = 2$ interaction we don’t take into account, as usual, because of its smallness. All contributions near the Compton–effect threshold contribute to the low–energy structure constants $(\alpha + \beta)/2m_\pi$. We prefer to use them as free parameters instead of $a^{C,N}$. As a result helicity 2 amplitudes $\gamma\gamma \rightarrow \pi^+\pi^-$, $\pi^0\pi^0$ will contain three arbitrary constants: $(\alpha + \beta)^C$, $(\alpha + \beta)^N$, $\Gamma(f_2 \rightarrow \gamma\gamma)$.

3. In analysis we found that the data on $\gamma\gamma \rightarrow \pi^+\pi^-$ are sensitive only to polarizability of $\pi^+$ and $\gamma\gamma \rightarrow \pi^0\pi^0$ only to polarizability of $\pi^0$. So in considering of a single reaction we shall fix this unessential parameters according to theoretical prediction and use the two–parameter expression for helicity 2 amplitude. The fixed parameter may lie in a very broad interval – see below. Let’s recall that predictions of different low–energy models (i.e. \cite{9, 10}) are rather close to each other: $(\alpha + \beta)^{\pi^+} \simeq 0.20$, $(\alpha + \beta)^{\pi^0} \simeq 1.20$ in units of $10^{-42}$ cm$^3$ ($e^2 = 4\pi\alpha$). Slightly the higher values are predicted in the dispersion sum rules \cite{11}: $0.43 \pm 0.06$ and $1.65 \pm 0.13$ correspondingly.

**CELLO data on $\gamma\gamma \rightarrow \pi^+\pi^-$ \cite{2}**. Let’s consider the CELLO angular distributions in 7 energy bins from 0.8 up to 1.4 Gev, they contain 53 points. Let us fix $(\alpha + \beta)^{\pi^0} = 1.20$, one can vary it between 0.6 and 1.8 without any changes. Best fit values are:

$$ (\alpha + \beta)^{\pi^+} = 0.38 \pm 0.05, \quad \Gamma(f_2(1270) \rightarrow \gamma\gamma) = 2.88 \pm 0.12 \text{ KeV} $$

$$ \chi^2 = 54 \quad \text{at} \quad NDF = 53 - 9 = 44 $$

(2)

**MARK-II data on $\gamma\gamma \rightarrow \pi^+\pi^-$ \cite{11}**. Seven energy intervals between 0.8 and 1.4 Gev, 42 experimental points. Again the $(\alpha + \beta)^{\pi^0} = 1.20$ is fixed.

$$ (\alpha + \beta)^{\pi^+} = 0.28 \pm 0.07, \quad \Gamma(f_2(1270) \rightarrow \gamma\gamma) = 2.73 \pm 0.19 \text{ KeV} $$

$$ \chi^2 = 17.0 \quad \text{at} \quad NDF = 42 - 9 = 31 $$

(3)

**Crystal Ball data on $\gamma\gamma \rightarrow \pi^0\pi^0$ \cite{3}**. Consider seven energy intervals between 0.85 and 1.45 Gev, 56 points. $(\alpha + \beta)^{\pi^+} = 0.20$ is fixed, its changing between - 0.5 and 1.0 does not influence on results.

$$ (\alpha + \beta)^{\pi^0} = 1.26 \pm 0.06, \quad \Gamma(f_2(1270) \rightarrow \gamma\gamma) = 3.56 \pm 0.22 \text{ KeV} $$
$\chi^2 = 38 \quad \text{at} \quad NDF = 56 - 9 = 47$ \quad (4)

4. The main observation in such an analysis is that the angular distributions for both reactions in vicinity of the resonance $f_2(1270)$ are very sensitive to value of smooth background, interfering with resonance. Simplest assumption about form of this background allows to relate it with a threshold structure constant $- (\alpha + \beta)^\pi$ and to obtain in the first time some estimates for them. The obtained values are very close to existing theoretical predictions. At first sight there should exist the essential model dependence at an extraction of threshold parameter from analysis in region of $f_2(1270)$. But we found that the $(\alpha + \beta)^\pi$ values are very stable at any attempts "to improve" the model. In particular, one can include the higher mass exchanges to background contribution (1) — it redefines the constants $a^{C,N}$, but does not change polarizability’s estimates.

Another interesting result is the smallness of the S–wave cross section, which is seen in analysis of all existing data [1, 2, 3]. Only in vicinity of 1.3 Gev there arises some resonance–like structure of a rather small amplitude. The results for three considered experiments does not contradict to each other in the first approximation. Such a behaviour of the S-wave must give additional constrains for resonance interpretation of data in scalar sector and it needs a more detailed consideration.

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