The $B_c$ Meson Lifetime in the Light–Front Constituent Quark Model

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Abstract
We present an investigation of the total decay rate of the (ground state) $B_c$ meson within the framework of the relativistic constituent quark model formulated on the light-front (LF). The exclusive semileptonic (SL) and nonleptonic (NL) beauty and charm decays of the $B_c$ meson are described through vector and axial hadronic form factors, which are calculated in terms of quark model LF wave functions. The latter ones are derived via the Hamiltonian LF formalism using as input the update constituent quark models. The inclusive SL and NL partial rates are calculated within a convolution approach inspired by the partonic model and involving the same $B_c$ wave function which is used for evaluation of the exclusive modes. The framework incorporates systematically 84 exclusive and 44 inclusive partial rates corresponding to the underlying $\bar{b} \to \bar{c}$ and $c \to s$ quark decays. We find $\tau_{B_c} = 0.59\pm0.06$ ps where the theoretical uncertainty is dominated by the uncertainty in the choice of LF wave functions and the threshold values for the hadron continuum. For the branching fractions of the $B_c^+ \to J/\psi\mu^+\nu_\mu$ and $B_c^+ \to J/\psi\pi^+$ decays we obtain 1.6% and 0.1%, respectively.

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The theoretical interest in the study of the $B_c$ meson, the bound state of the $\bar{b}c$ system with open charm and beauty, is stimulated by the experimental search at CDF and LHC. Recently the CDF Collaboration reported the observation of $B_c$ in 1.8 TeV $p\bar{p}$ collisions at Fermilab [1]. The CDF results for the $B_c$ mass and lifetime are $M_{B_c} = 6.40\pm0.39\,(stat)\pm0.13\,(syst)\,\text{GeV}/c^2$ and $\tau_{B_c} = 0.46^{+0.18}_{-0.16}\,(stat)\pm0.003\,(syst)\,\text{ps}$. The physics of $B_c$ mesons has stimulated much recent works on their properties, weak decays and production cross section on high energy colliders. For a review see [2].

Similar to $D$ and $B$ mesons the ground $\bar{b}c$ state is stable against strong or electromagnetic decay and disintegrates only via weak interactions. The weak $B_c$ decays occur mainly through the CKM favored $\bar{b} \to cW^+$ transitions with $c$ being a spectator, leading to final states like $J/\psi\ell\nu$, and $c \to sW^+$ transitions with $\bar{b}$ being a spectator, leading to final states like $B_s\pi$, $B_s\ell\nu$. Weak decays of charmed and bottom hadrons are particularly simple in the limit of infinite heavy quark mass, where the decay rate of a hadron $H_Q$ containing a heavy quark $Q$ is completely determined by the decay rate of the heavy quark itself. In this limit, one might expect that $\Gamma(\bar{B}_c) \approx \Gamma(\bar{B}^0) + \Gamma(D^0)$ yielding $\tau_{B_c} \approx 0.3\,\text{ps}$, with $c-$decay dominating over $\bar{b}-$decay [4]. In reality, hadrons are bound states of heavy quarks with light constituents. The account of the soft degrees of freedom generates important pre–asymptotic contributions due to the Fermi motion of a heavy quark inside the hadron. These effects have a significant impact on the lifetime and various branching fractions of $B_c$. The various calculations of $\tau_{B_c}$ have been reported in the literature [3]–[5]. The wide range of predicted lifetimes $\tau_{B_c} = 0.4 – 0.9\,\text{ps}$, reflects the uncertainty due to the various model assumptions on the modification of the free decay rates due to the bound state effects and the limited knowledge of the heavy quark masses.

In this paper, we use an approach in which non–perturbative QCD effects are mocked up by a light–front (LF) wave function of the hadron [3]. The internal motion of a heavy $Q-$quark inside the heavy flavour meson $H_Q$ is described by the distribution function $F(x) = \int d^2p_\perp |\psi(x,p_\perp^2)|^2$, where $|\psi(x,p_\perp^2)|^2$ represents the probability to find a quark $Q$ carrying a LF fraction $x = p_Q^2/H_Q^+$ of the meson momentum and a transverse relative momentum squared $p_\perp^2$. A relevant feature of this approach is that both exclusive and inclusive decays are coherently treated in terms of the same heavy quark wave function. So far the approach has been applied only for the exclusive and inclusive partial widths of $B^0$ [6], where it has been found that the overall picture is quantitatively satisfactory. Here we extend previous calculations started in Ref. 8 to compute the lifetime and various decay branching fractions of $B^+_c$. Our aim is to constrain the model dependence of the calculated $\tau_{B_c}$ related to various choices of $\psi(x,p_\perp^2)$. To this end we will make use of different LF wave functions, constructed via the Hamiltonian LF formalism (see e.g. 8) adopting recently developed relativized and a non–relativistic quark models 3, 10.

To start with, we briefly remind the main points of the procedure used to calculate heavy meson partial widths. For more details see Refs. 6, 7. Consider the SL decay rates first. Instead of considering the exclusive modes individually we will sum over all possible hadronic final states $X$. This sum includes hadronic states with a large range of invariant mass $M_X$. For the heavy–flavour mesons, the energy which flows into hadronic system is typically much larger than the energy scale $\Lambda_{QCD}$ which characterizes the strong interactions. Consequently, for a wide range of the phase space an inclusive description based on quark-hadron duality is appropriate. It will be valid over almost all of the Dalitz plot, failing only in the narrow corner region where the observed mass spectrum is dominated by the two narrow peaks corresponding to the transitions $H_Q \to P$ and $H_Q \to V$, where $P$ and $V$ are the lowest lying pseudoscalar and vector mesons. Accordingly, the total SL rate of the $H_Q$ meson has been represented in
the following hybrid form
\[ \Gamma(H_Q \rightarrow X(\nu_e)) = \Gamma(H_Q \rightarrow P(\nu_e)) + \Gamma(H_Q \rightarrow V(\nu_e)) + \Gamma(H_Q \rightarrow X'(\nu_e)), \]
(1)
where \( X' \) represents the hadroncontinuum including also the resonance states higher than \( P \) and \( V \). The usefulness of such an expansion rests on large energy release in the inclusive decay.

Our calculations of the exclusive rates \( \Gamma(H_Q \rightarrow P(\nu_e)) \) and \( \Gamma(H_Q \rightarrow V(\nu_e)) \) use the hadronic form factors that depends explicitly on dynamics of specific channels. The relevant formulae valid also in the case when the lepton masses are not negligible are collected in [7] and we do not quote them here. Instead, we will concentrate on the calculation of the inclusive rate \( \Gamma(H_Q \rightarrow X'(\nu_e)) \). The modulus squared of the amplitude summed over the final hadronic states is given through the optical theorem by the imaginary part of the quark box diagram describing the decay probability for which no reference to a particular hadronic state is needed equals to one

\[ \Gamma(Q \rightarrow H) = |q|^2 \left( \frac{G_F^2}{2} \right) |V_{Q2}'|^2 L^{\alpha\beta} W_{\alpha\beta}, \]

where \( L^{\alpha\beta} \) is the leptonic tensor and \( W_{\alpha\beta} \) is the hadronic tensor. We use the notation of Ref. [11].

The hadronic tensor can be expressed in terms of five structure functions \( W_i \) to \( W_5 \) that depend on two invariants, \( q^2 \) and \( q_0 \), where \( q \) is the 4–momentum of a lepton pair and \( q_0 \) is related to \( M_X \) by:

\[ q_0 = \frac{(M_{H_Q}^2 + q^2 - M_X^2)}{2M_{H_Q}}. \]

It is convenient to scale all momenta by \( M_{H_Q} \), letting \( M_X^2 = s \cdot M_{H_Q}^2 \) and \( q^2 = t \cdot M_{H_Q}^2 \). Evaluating the contraction \( L^{\alpha\beta} W_{\alpha\beta} \) we arrive at the formula for the SL inclusive width

\[ \Gamma_{SL} = \frac{2}{3} \Gamma_0 |V_{Q2}'|^2 J_{SL} \int_{t_{min}}^{t_{max}} \int_{s_{min}}^{s_{max}} dt^s_{M_{H_Q}} G(t, s), \]

(2)
where the prefactor \( \Gamma_0 = (G_F^2M_{H_Q}^5)/(4\pi^3) \) sets the overall scale of the rate, and \( J_{SL} \approx 0.9 \) represents the effect of the radiative corrections [12]. In Eq. (2) \( \Phi(q^2, m_1^2, m_2^2) = \sqrt{1 - 2\lambda_+ + \lambda_2} \), with \( \lambda_+ = (m_1^2 \pm m_2^2)/q^2 \), \( m_{1,2} \) being the lepton masses, \( \lambda_1 = \lambda_+ - 2\lambda_2 \), \( \lambda_2 = \lambda_+ - \lambda_2 \). Furthermore, \( 2|q|/M_{H_Q} \equiv \alpha(t, s) = \sqrt{(1 + t - s)^2 - 4t} \), and the limits of integrations in the \( t - s \) plane are given by \( s_{min} = (M_X^0/M_{H_Q})^2 \), \( s_{max} = 1 - \sqrt{t} \), \( t_{min} = (m_1 + m_2)^2/m_{Q}^2 \), \( t_{max} = (1 - M_X^0/M_{H_Q})^2 \), where \( M_X^0 \) is the threshold value at which the hadronic continuum starts.

The function \( G(s, t) \) is expressed in terms of the linear combination of \( W_i(t, s) \) [1]:

\[ G(t, s) = 3t(1 + \lambda_1 - 2\lambda_2)W_1 + \left( 1 + \lambda_1 \right) \frac{q^2}{M_{H_Q}^2} + \frac{3}{2} \lambda_2 j \right) W_2 + \frac{3}{2} \lambda_2 t \left( (1 + t - s)W_4 + tW_5 \right). \]

(3)
To calculate the structure functions \( W_i \) we use the parton approach of Refs. [3] (see also [13]) based on the hypothesis of quark-hadron duality. This hypothesis assumes that the inclusive decay probability for which no reference to a particular hadronic state is needed equals to one into the free quarks. The basic ingredient is the expression for the hadronic tensor \( W_{\alpha\beta} \) which is given through the optical theorem by the imaginary part of the quark box diagram describing the forward scattering amplitude:

\[ W_{\alpha\beta} = \int_0^1 dx w_{\alpha\beta}(p_{Q'}, p_Q)\theta(\epsilon_{Q'})\delta[(p_Q - q)^2 - m_{Q'}^2] F(x), \]

(4)
where \( w_{\alpha\beta}(p_{Q'}, p_Q) = \frac{1}{2} \sum_{\text{spins}} \bar{u}_{Q'}O_\alpha u_\beta \cdot \bar{u}_Q O_{\beta}^\dagger u_{Q'} \) is the parton matrix element squared. In what follows we choose the purely longitudinal kinematics with \( q_\perp = 0 \). In Eq. (4) the \( \delta \)-function

\footnote{The function \( W_3 \) does not appear in the total SL width integrated over the lepton energy [13].}
corresponds to the decay of a $Q$–quark with momentum $p_Q = x P_{HQ}$ to a $Q'$–quark with the momentum $p_{Q'} = p_Q - q$ and has two roots in $x$, viz. $\delta((p_Q - q)^2 - m^2_{Q'}) = [\delta(x - x_+) + \delta(x - x_-)]/(M^2_{HQ}|x_+ - x_-|)$, where $x_\pm(t,s) = \frac{1}{2}(1 + t - s \pm \sqrt{(1 + t - s)^2 - 4t + 4m^2_{Q'}/M^2_{HQ}})$ and the quark transverse momenta being neglected. The root $x_-$ is related to the contribution of the $Z$–graph arising from the negative energy components of the $Q'$–quark propagator and is prohibited by the $\theta(\epsilon_{Q'})$ in Eq. (4). Using the explicit expression for $w_{\alpha\beta}$ in Eq. (4) one obtains

$$W_1 = F(x_+), \quad W_2 = 4\frac{x_+ F(x_+)}{|x_+ - x_-|}, \quad W_3 = W_4 = -2\frac{F(x_+)}{|x_+ - x_-|}, \quad W_5 = 0. \quad (5)$$

In this way, the SL inclusive width gets related to the distribution $F(x)$. Substituting $W_i$ from (3) into (6) we obtain

$$\Gamma(H_Q \rightarrow X'l\nu_e) = \frac{2}{3} \Gamma_0 J_{SL}|V_{QQ'}|^2 \int_{t_{\min}}^{t_{\max}} dt \Phi(t) \int_{s_{\min}}^{s_{\max}} ds \alpha(t,s) \left[ (1 + \lambda_1)\alpha^2(t,s) \frac{x_+}{x_+ - x_-} + 3t(1 - \lambda^2) \right] F(x_+). \quad (6)$$

The calculation of the NL decay rate closely follows the SL one. We expect $H_Q$ decays to multimeson states to proceed predominantly via the formation of a quark–antiquark state, followed by the creation of the additional $q\bar{q}$ pairs from the vacuum. The effective weak Lagrangian, e.g. for $\bar{b} \rightarrow \bar{c}u\bar{q}$ processes with $q = d, s$ is given by $L(\mu) = \frac{\alpha_s}{2\pi} V_{cb} V_{uq}(c_1 O_1 + c_2 O_2)$, where $O_1$ and $O_2$ denote current–current operators with the color–non–singlet and color–singlet structure, respectively. The lepton pair is substituted by a quark pair, and the Wilson coefficients $c_1(\mu)$ are the perturbative QCD corrections that describe the physics between the $W$ boson mass and the characteristic hadronic scale $\mu \approx m_Q$ of the process. We shall use the values $c_1(m_b) = 1.132, \quad c_2(m_b) = -0.286; \quad c_1(m_c) = 1.351, \quad c_2(m_c) = -0.631$, obtained at next–to–leading order with the evolution of the running coupling constant being done at two–loop order using the normalization $\alpha_s(m_Z) = 0.118 \pm 0.003$.

When calculating the NL decays we use again the hybrid approach. We write $f dt$ in (2) as $f dt = \int_{t_{\min}}^{t_{\max}} dt \delta(t - \zeta_\mu^2) + \int_{t_{\min}'}^{t_{\max}'} dt \delta(t - \zeta_v^2) + \int_{t_{\min}'}^{t_{\min}'} dt$, where $t_{\min}'$ is now related to the threshold for the hadron continuum produced by the $W$ current, and $\zeta_{P,V} = M_{P,V}/M_{HQ}$, with $M_{P,V}$ being the mass of a pseudoscalar or vector meson. The latter integral is treated in the same way as for SL decays. According to Eq. (3) it is the sum of exclusive rates and inclusive rates. The inclusive NL rate is given by Eq. (4) with the substitution $\Gamma_0 J_{SL} \rightarrow 3\Gamma_0 |V_{uq}|^2 \eta$ where $\eta = c_+^2(N_c + 1)/(2N_c) + c_-^2(N_c - 1)/(2N_c)$, and $c_+ = c_1 \pm c_2$. In the limit of large number of colors $N_c$ which we use below $\eta = c_1^2 + c_2^2$.

Additional contributions arise from the first two terms containing $\delta(s - \zeta_{P,V}^2)$. For the exclusive two–meson decays $H_Q \rightarrow PP, PV, VP, and VV$ we use the factorization approach with the effective QCD coefficients $a_1 = 1, \quad a_2 = -0.3$ and $a_1 = c_1(m_c), \quad a_2 = c_2(m_c)$ for the $\bar{b} \rightarrow \bar{c}$ and $c \rightarrow s$ transitions, respectively. The partial width of the inclusive NL decay $\bar{H}_Q \rightarrow PP, PV, VP, and VV$

$^2$By the quark masses $m_Q$ and $m_{Q'}$ we hereafter understand the “constituent” quark masses taken from a particular constituent quark model.

$^3$The coefficients $a_{1,2}$ correspond to two flavour–flow topologies relevant for our discussion: the so–called "tree topology" (class I amplitudes), dominated by color allowed external $W$–decay and the "colour–suppressed tree topology" (class II amplitudes), dominated by colour-suppressed internal $W$–decays. There is the third class decays in which the $a_1$ and $a_2$ amplitudes interfere. These decays play the important role in the case of $D$ mesons, but are less important for $B_c$ and are absent for $B^0$. 

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\( H_Q \rightarrow P + X_{Q'Q} \) in which the final state contains a charged or neutral pseudoscalar meson directly generated by a color–singlet current are given by

\[
\Gamma_{PX} = \frac{4\pi^2 \eta f_P^2}{M^2_{H_Q} \Gamma_0|V_{Q'Q}|^2} \int_{s_{\text{min}}}^{(1-\xi)^2} ds \frac{[\tilde{x}_+(\tilde{x}_+ + \tilde{x}_-)^2 - (\tilde{x}_- + 3\tilde{x}_+)^2]}{[\tilde{x}_+ - \tilde{x}_-]} a(\xi^2, s) F(\tilde{x}_+). \tag{7}
\]

The analogous inclusive width \( \Gamma_{VX} \) for the production of a vector meson is obtained from (7) by the substitution \( f_P \rightarrow f_V \) and \([\tilde{x}_+(\tilde{x}_+ + \tilde{x}_-)^2 - (\tilde{x}_- + 3\tilde{x}_+)^2] \rightarrow [\tilde{x}_+(\tilde{x}_+ + \tilde{x}_-) - (3\tilde{x}_- + \tilde{x}_+)^2]\). Here \( f_{PV} \) are the pseudoscalar and vector meson coupling constants to the \( W \) current, and \( \tilde{x}_\pm = x_\pm(\xi^2, s) \). The constants \( f_{PV} \) are taken from Ref. [10]. For the \( B_c \) meson mass we use \( M_{B_c} = 6.3 \) GeV, the other meson masses are taken from the recent PDG publication [17].

The non–perturbative ingredient of our calculations is the LF wave function \( \psi(x, p^2_L) \). In what follows, we will adopt for the latter the functions corresponding to the various equal time (ET) quark model wave functions. There is a simple operational connection between ET and LF wave functions [8], which allows to convert the ET wave function \( w(p^2) \) into relativistic LF wave function \( \psi(x, p^2_L) = \frac{w(p^2)}{\sqrt{4\pi}} \frac{\partial}{\partial x} \), where \( p^2 = p^2_t + p^2_z \), \( p_z \) is the longitudinal momentum defined as \( p_z = (x - \frac{1}{2}M) + \frac{m_{sp}^2 - m_Q^2}{2m_0} \), and the free mass \( M_0 \) acquires the familiar form \( M_0 = \sqrt{m_Q^2 + p^2 + \sqrt{m_{sp}^2 + p^2}} \), with \( m_{sp} \) being the mass of the quark–spectator.

For the sake of brevity we shall present the results for the two ET wave functions corresponding to the AL1 [9] and DSR [10] quark models. These functions result from solving either the Schrödinger equation (non-relativistic kinematics \( T = p^2/2\mu_{\text{red}}, \mu_{\text{red}} = m_Q m_{sp}/(m_Q + m_{sp}) \); this is the case AL1) or the Salpeter equation (semi-relativistic kinematics \( T = M_0 \); this is the case DSR). The AL1 potential is the usual Coulomb+linear potential supplemented with a gaussian hyperfine term whose range is mass dependent. The DSR potential is much more sophisticated one since in addition to the previous terms, it contains also effects from instantons (which are necessary to describe light pseudoscalar mesons), and from finite quark size. The DSR potential is then convoluted with a quark density to take into account finite size effects for quarks. The detailed form of this potential is given in Ref. [10]. The parameters of the AL1 and DSR potentials have been determined with help of a fit procedure on many experimental meson resonances, from \( \pi \) to \( B \).

We choose these two different prescriptions for the kinematics to see whether our results are sensitive to some “relativization” at the level of the ET wave function. The main difference between \( w(p^2) \) in the AL1 and DSR models relies in the behaviour at high value of the internal momentum. The AL1 model results in the soft wave function similar to that of the ISGW2 model [18] with \(< p^2 > = 0.235 \) GeV\(^2 \) for \( \bar{B}^0 \) and \(< p^2 > = 1.075 \) GeV\(^2 \) for \( B_c \), while the DSR wave function exhibits high momentum components leading to \(< p^2 > = 0.517 \) GeV\(^2 \) for \( \bar{B}^0 \) and \(< p^2 > = 1.388 \) GeV\(^2 \) for \( B_c \). As a result the DSR distribution function is broader than that for the case AL1. This is illustrated in Fig.1 which shows the distribution functions \( F_B(x) \) for \( \bar{B}^0 \) and \( B_c \) mesons. Note that \( F_B(x) = F_{B'_c}(1 - x) \). In terms of the mean value \( \bar{x} = \int_0^1 x F(x) dx \) and the variance \( \sigma^2 = \int_0^1 [(x - \bar{x})^2 F(x) dx, \) one obtains \( \bar{x}_{\bar{B}^0} = 0.9 \) (AL1), 0.89 (DSR), \( \sigma^2_{\bar{B}^0} = 0.0028 \) (AL1), 0.0065 (DSR), \( \bar{x}_{B_c} = 0.72 \) (AL1 and DSR), and \( \sigma^2_{B_c} = 0.0062 \) (AL1), 0.0081 (DSR). The constituent quark masses in a relativized quark models are systematically lower that those determined using the non–relativistic models. In our case \( m_u = m_d = 0.315, m_s = 0.577, m_c = \)}
1.836, \( m_b = 5.227 \) GeV (case AL1) and \( m_u = m_d = 0.221, m_s = 0.434, m_c = 1.686, m_b = 5.074 \) GeV (case DSR). Note that the constituent quark masses \( m_b \) and \( m_c \) satisfy approximately the relation \( m_b = m_c + 3.4 \) GeV, which is consistent with the well known formula relating the pole masses \( m_{b,pole} \) and \( m_{c,pole} \) in Heavy Quark Effective Theory.

We are now ready to present an overview over different \( B_c \) decays and their relative importance as obtained within the framework we are advocating. To this end we first calculate the partial \( \bar{B}_0 \) decay modes corresponding to various underlying quark subprocesses. In Table 1 we report the \( \bar{B}_0 \) partial widths for AL1 and DSR LF models. For comparison, we show also the results obtained in [7] using the Gaussian–like ansatz of the ISGW2 model which are very similar to the AL1 case.

Our analysis incorporates 54 exclusive SL and NL decays and 29 inclusive decays including two baryon-antibaryon channels. The latter ones were calculated using the Stech approach [9]. We have also included the CKM suppressed \( b \rightarrow u \) contributions with \( |V_{ub}/V_{cb}| \approx 0.1 \). The vector and axial form factors for \( \bar{B}_0 \rightarrow D(D^*)\ell\nu_{\ell} \) and \( \bar{B}_0 \rightarrow \pi(\rho)\ell\nu_{\ell} \) transitions have been calculated using the formalism of Refs. [20]. When calculating the inclusive rates the important question is which value to use for the hadron threshold \( M_X^{(0)} \). This value is very badly defined theoretically. In our calculations we have adopted two “natural” choices: \( M_X^{(0)} = M_P + m_\pi \) and \( M_X^{(0)} = M_V + m_\pi \). In Table 1 we show only the results obtained using \( M_X^{(0)} = M_V + m_\pi \). For \( M_X^{(0)} = M_P + m_\pi \) we obtain \( \approx 10\% \) increasing of the calculated \( \bar{B}_0 \) rate. For each case we then fix the effective value of \( |V_{cb}| \) by the requirement that the measured \( \bar{B}_0 \) meson lifetime \( \tau_{\bar{B}_0} \approx 1.56 \) ps is obtained. Our goal here is not to establish a new value of \( |V_{cb}| \), but rather to illustrate how our approach works. Moreover, imposing the \( |V_{cb}| \) constraint strongly reduces the dependence of the predicted value of \( \tau_{B_c} \) from the uncertainty related to the choice of the continuous \( c\bar{c} \) threshold [1].

We apply the same strategy to calculate different partial rates and the lifetime of the \( B_c \). The critical point with regards to this issue is inclusive charm decay. Here the energy release is not as comfortably large as it is in the case of bottom decay. As a result, our estimations of the inclusive charm decay should be more sensitive to a hadronization model. However, this decay contributes only \( \approx 10\% \) to the total \( c \rightarrow s \) rate. For this reason we did not include any hadronization corrections in our calculations.

The results for the partial \( B_c \) decay modes corresponding to the various underlying quark subprocesses are collected in Table 2 for \( M_X^{(0)} = M_V + m_\pi \). We also include non–spectator contributions from weak annihilation (WA) and Pauli interference (PI). The contribution of the annihilation channel is

\[
\Gamma_a = \sum_{i=r,e} \frac{G_F^2}{8\pi} |V_{cb}|^2 M_{B_c}^3 \left( \frac{f_{B_c}}{M_{B_c}} \right)^2 \left( \frac{m_i}{M_{B_c}} \right)^2 \left( 1 - \left( \frac{m_i}{M_{B_c}} \right)^2 \right)^2 \cdot \tilde{c}_i, \tag{8}
\]

where we take \( f_{B_c} \approx 0.5 \) GeV, \( \tilde{c}_r = 1 \) for the \( \tau^+\nu_\tau \) channel and \( \tilde{c} = (2c_+(2\tilde{\mu}_{red}) + c_-(2\tilde{\mu}_{red})^2)/3 \) for the \( c\bar{s} \) channel, with \( \tilde{\mu}_{red} = m_u m_c/(m_b + m_c) \) being the reduced mass of the \( \bar{b}c \) system. We use \( c_+(\tilde{\mu}_{red}) = 0.8, c_-(\tilde{\mu}_{red}) = 1.5 \). For the \( \Gamma_{PI} \) we use the expression given in [21]. Note that because of the substantial cancellation of weak annihilation rate and the effect of the Pauli interference diagrams, no significant uncertainty on the lifetime arises from the limited knowledge of the decay constant \( f_{B_c} \). We find \( \Gamma_a = 0.268 \text{ ps}^{-1} \), \( \Gamma_{PI} = -0.142 \text{ ps}^{-1} \).

\(^4\)The values of other CKM parameters entering calculations are \( |V_{sc}| = 0.974, |V_{dc}| = |V_{us}| = 0.221.\)
We also show in Table 2 the results obtained in [7] for the ISGW2 LF model [1] and in [8], using Operator Product Expansion (OPE). Viewing the latter comparison with due caution, regarding the model dependence and other uncertainties in the estimation of the decay modes as well as the quark mass uncertainty for the inclusive prediction, it is reassuring that the order of magnitude comes out to be consistent. Our bound state corrections are numerically larger than very small effects found in [4], especially for the $c \to s$ transitions. One possible reason being is that an analysis of $c \to s$ decays using OPE is of limited validity, since due to smaller energy release the convergence of OPE is slower than for $b \to \bar{c}$ decays.

For the sum of $b \to \bar{c}$ spectator contributions we obtain $\Gamma^{(b\to c)} = 0.501 \, \text{ps}^{-1}$ (AL1) and $0.557 \, \text{ps}^{-1}$ (DSR), while the total $c$-decay contribution is $\Gamma^{(c\to s)} = 0.900 \, \text{ps}^{-1}$ (AL1) and $1.079 \, \text{ps}^{-1}$ (DSR). One observes a dominance of the charm decay modes over $b$-quark decays. Various branching fractions can be also deduced from Table 2. For instance the semileptonic branching ratio $BR(B_c \to \ell \nu_X)$ is found to be 9.3%. The absolute rates are $\Gamma(B_c \to \ell \nu_X c^-) = 4.0 \times 10^{13} |V_{cb}|^2 \, \text{sec}^{-1}$ and $\Gamma(B_c \to \ell \nu_X s^-) = 8.0 \times 10^{13} |V_{sc}|^2 \, \text{sec}^{-1}$ for AL1 (DSR) cases, respectively.

Putting everything together one finds $\Gamma(B_c) = \Gamma^{b\to \bar{c}}(B_c) + \Gamma^{c\to s}(B_c) + \Gamma_a + \Gamma_{PI} = (0.65 \, \text{ps})^{-1}$ (AL1), and $(0.57 \, \text{ps})^{-1}$ (DSR). For $M_X^0 = M_p + m_q$ we obtain $\Gamma_{B_c} = (0.61 \, \text{ps})^{-1}$ (AL1) and $(0.53 \, \text{ps})^{-1}$ (DSR). We consider the dispersion in predicted values of $\tau_{B_c}$ as a rough measure of our theoretical uncertainty in calculation of inclusive decay rates arising both from the model dependence of $\psi(x, p^2)$ and the different choices of the continuum threshold $M_X^{0}$. Averaging our predictions we obtain

$$\tau_{B_c} = (0.59 \pm 0.06) \, \text{ps}. \tag{9}$$

This result compares favourably with estimates obtained using OPE [3] and also agrees with the most recent CDF measurement within one standard deviation.

Finally, we note that the experimental extraction of $B_c$ signal in the hadronic background requires the reliable estimation of the branching fraction $BR(B_c \to J/\psi + X)$, because $J/\psi$ can be easily identified by its $\mu^+ \mu^-$ decay mode, while the experimental registration of the final states containing the $\eta_c$ or $B_s^{(*)}$ is impeded by the large hadron background. We obtain $BR(B_c \to J/\psi + \pi) = 0.1\%$ (AL1 and DSR), and $BR(B_c \to J/\psi + X) = 14.8\%$ (AL1), 13.6\% (DSR). For the exclusive $B_c \to J/\psi \mu^+ \nu_\mu$ channel whose signature would be quite clean experimentally when $J/\psi$ decays into a pair of muons, providing a three muon vertex the calculated branching fractions are 1.68\% and 1.57\% for cases AL1 and DSR, respectively.

In conclusion, adopting a LF constituent quark model we have investigated the partial widths and the lifetime of the $B_c$ meson. The hadronic form factors and the distribution function were calculated from meson wave functions derived from an effective $q \bar{q}$ interaction intended originally to describe the meson mass spectra. In this way the link between $B_c$ physics and the “spectroscopic” quark models was explicitly established. For numerical estimates we have employed the LF functions related to the ET eigenfunctions of the AL1 and DSR quark models. In several important aspects our analysis goes beyond the quark model estimations obtained previously. In addition to the SL and the two–meson NL exclusive decay modes we have included a number of the $B_c \to H_c(H_{bs})q_1q_2$ exclusive and $B_c \to X_{bc}(X_{bs}^{(*)})q_1q_2$ inclusive channels. These channels have considerable impact on the predicted overall $b \to c$ rate. To sum up, the LF constituent quark model makes clear predictions on the global pattern: (i) a short $B_c$ lifetime $\approx 0.6 \, \text{ps}$ and (ii) a predominance of charm over beauty decays.

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5The result of Ref. [3] corresponds to $f_{B_c} = 0.42 \, \text{GeV}$. After correcting for the value of $f_{B_c}$ used in this paper, one gets the result quoted in Table 2.
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Table 1. The partial widths of the $\bar{B}^0$ meson (in the units $|V_{cb}/0.039|^2 \text{ps}^{-1}$) calculated adopting the different choices of the LF wave functions. Each SL width is the sum of 2 exclusive and one inclusive widths. For a NL decay the width is the sum of 12 exclusive and 6 inclusive channels corresponding to the external and internal $W$ decays. The CKM matrix element $|V_{cb}|$ is calculated using the experimental value of $\Gamma_{\exp}(B^0) = 0.641 \text{ ps}^{-1}$. Also shown are the SL branching ratios $BR_{SL}$ and the charm counting $n_c$. 

| Model                  | AL1  | DSR  | ISGW2 |
|------------------------|------|------|-------|
| $b \to \bar{c} + e\nu_e$ | 0.076| 0.072| 0.074 |
| $b \to \bar{c} + \mu\nu_\mu$ | 0.075| 0.071| 0.074 |
| $b \to \bar{c} + \tau\nu_\tau$ | 0.016| 0.016| 0.015 |
| $b \to \bar{c} + ud$ | 0.322| 0.311| 0.313 |
| $b \to \bar{c} + cs$ | 0.122| 0.137| 0.123 |
| $b \to \bar{c} + u\bar{s}$ | 0.016| 0.016| 0.016 |
| $b \to \bar{c} + c\bar{d}$ | 0.006| 0.007| 0.006 |
| $b \to \bar{u}$ | 0.007| 0.007| 0.007 |
| $B^0 \to N\Lambda_c$, $\Lambda_c\Xi_c$ | 0.023| 0.031| 0.023 |
| $\Gamma(B^0)$ | 0.662| 0.668| 0.652 |
| $|V_{bc}|$ | 0.0383| 0.0382| 0.0386 |
| $n_c$ | 1.193| 1.205| 1.198 |
| $BR_{SL}$ | 11.42%| 10.78%| 11.35% |
Inclusive partial rates of $B_c$ (in units ps$^{-1}$) for the choice of the continuum threshold $M_X^{(0)} = M_V + m_{\pi}$. The $b \to c$ rates are calculated using the effective value of $|V_{cb}|$ from Table 1. The $c \to s$ rates are calculated using $|V_{cs}| = 0.974$. $\Gamma_a$ and $\Gamma_{PI}$ are calculated using $f_{B_c} = 0.5$ GeV.

**Table 2**

| Model                          | AL1  | DSR  | ISGW2 | OPE  |
|-------------------------------|------|------|-------|------|
| $b \to \bar{c} + e\nu_e$     | 0.058| 0.060| 0.061 | 0.075|
| $b \to \bar{c} + \mu\nu_\mu$ | 0.058| 0.060| 0.061 | 0.075|
| $b \to \bar{c} + \tau\nu_\tau$ | 0.013| 0.014| 0.013 | 0.018|
| $b \to \bar{c} + ud$         | 0.244| 0.261| 0.259 | 0.310|
| $b \to \bar{c} + c\bar{s}$   | 0.093| 0.117| 0.102 | 0.137|
| $b \to \bar{c} + u\bar{d}$   | 0.012| 0.013| 0.013 |      |
| $b \to \bar{c} + c\bar{d}$   | 0.005| 0.006| 0.006 |      |
| $b \to \bar{u}$              | 0.005| 0.007| 0.005 |      |
| $B_c \to \Sigma_c \Sigma_c$  | 0.013| 0.019| 0.011 |      |
| $\Gamma^b$                   | 0.501| 0.557| 0.531 | 0.615|
| $c \to s + e\nu_e$           | 0.078| 0.090| 0.081 | 0.162|
| $c \to s + \mu\nu_\mu$       | 0.073| 0.085| 0.077 | 0.162|
| $c \to s + ud$               | 0.688| 0.814| 0.698 | 0.905|
| $c \to s + u\bar{s}$         | 0.024| 0.029| 0.023 |      |
| $c \to d$                    | 0.037| 0.061| 0.036 |      |
| $\Gamma^c$                   | 0.900| 1.079| 0.915 | 1.229|
| $bc \to \tau\nu_\tau$       | 0.074| 0.074| 0.074 | 0.056|
| $bc \to c\bar{s}$           | 0.194| 0.194| 0.194 | 0.138|
| $\Gamma_{PI}$                | -0.142| -0.142| -0.142 | -0.124|
| $\Gamma_{tot}$               | 1.527| 1.762| 1.571 | 1.914|
| $\tau_{B_c}$                 | 0.65 | 0.57 | 0.64 | 0.52 |
Figure 1: The distribution functions $F_b(x) = \int d^2 p_\perp |\psi_b(x, p_\perp^2)|^2$ for the $B^0$ and $B_c$ mesons calculated using the AL1 and DSR LF wave functions (this work) and the ISGW2 LF wave function (Ref. 7).