Generation and detection of vortex rings in superfluid $^4$He at very low temperature

H Yano, A Nishijima, S Yamamoto, T Ogawa, Y Nago, K Obara, O Ishikawa, M Tsubota and T Hata
Graduate School of Science, Osaka City University, Osaka 558-8585, Japan
E-mail: hideo@sci.osaka-cu.ac.jp

Abstract. Motions of vortices are fundamental characteristics of quantum turbulence. These motions are expected to be governed only by quantized circulations in superfluids at the zero temperature limit. In the present paper, we report the motions of vortex rings emitted from a quantum turbulence in superfluid $^4$He, by detecting vortex rings using a vortex-free vibrating wire as a detector. The time of flights of vortex rings are distributed, because vortex rings are emitted in any direction from a turbulent region and the detector can respond only to a reachable vortex ring. By measuring time-of-flights many times, we find an exponential distribution of time-of-flights with a non-detection period, which corresponds to the fastest time of flights of vortex rings. For a larger generation power of vortex rings, a distribution of time-of-flights still shows a single exponential distribution, but a non-detection period becomes shorter. This result implies that sizes of emitted vortex rings are distributed dependently on the generation power of turbulence. The observed exponential distributions are confirmed by numerical simulations of the dynamics of vortex rings.

1. Introduction
Quantum turbulence consists of a disordered tangle of superfluid vortices [1]. Because of quantum mechanical constraints on the rotational motion, these vortices carry the same quantized circulation $\kappa$ and their cores are very thin: core radius $a_0 \sim 0.1$ nm for superfluid $^4$He. The motions of vortices are predicted to be governed only by quantized circulations in superfluids at the zero temperature limit. A vortex ring, for instance, flies in a superfluid sea, propelled by quantized circulation with a velocity [2]

$$v = \frac{\kappa}{4\pi R} \left( \ln \frac{8R}{a_0} - \frac{1}{2} \right),$$

where $R$ is the radius of the vortex ring. Despite of the simple predictions, experimental studies on the motions of vortex rings are quite difficult because of limited methods of vortex observation. Visualization techniques of quantized vortices can trace vortex motions [3, 4], though these techniques have been limited to studies on quantized vortices for high temperatures yet and tracer particles may affect the vortex motions. In the present paper, we report the motions of vortex rings with no tracer particles at a very low temperature, by using vibrating wires as a generator and a detector of vortex rings.

2. Experimental setup
Quantum turbulence is easily generated by oscillating spheres [5], grids [6], forks [7], or wires [8, 9] in superfluid $^4$He, even at very low temperatures, where a normal fluid component is almost absent. Vortex rings emitted from a turbulent region can be detected by using a vortex-free vibrating wire [10].
a vortex ring collides to a vibrating wire, the vibration of the wire is dissipated due to generation of turbulence. After stopping the wire vibration, however, the wire returns to a vortex-free state [11]. To achieve the vortex-free condition, we mounted three vibrating wires (2.4 μm in diameter) parallel to each other in a copper chamber with a pin hole. Superfluid $^4$He was filled in the chamber through the pin hole at temperatures below 100 mK. After that, we found that one of the vibrating wires located near the pin hole cannot generate turbulence by itself, revealing a vortex-free wire. We used this wire as a detector of vortex rings and the next wire as a generator of turbulence. The distance between the generator and the detector is 1.13 mm. The resonance frequencies of the vibrating wires are 3.15 kHz for the generator and 3.50 kHz for the detector measured in vacuum. In this work, the detector was vibrated at a peak velocity of 200 mm/s to detect a vortex ring.

3. Results and discussion

3.1. Time-of-flight measurements

Time-of-flight measurements were performed by the similar methods used in previous studies [12, 13]. We measured a period between the start of turbulence generation by the generator and the detection of a vortex ring by the detector, equal to the creation time of a vortex ring plus the time of its flight from the generator to the detector. We measured the period repeatedly at a temperature of 50 mK for a power of 2.7 pW injected in a turbulence region, and find that observed periods are distributed as shown in Fig. 1a. This is because vortex rings are emitted in any direction from the generator, and the detector can respond only to a reachable vortex ring. To investigate the distribution, we estimated a non-detection probability $1 - P$ in a period of time $t$, and find that $1 - P$ is well fitted to a function $\exp\left(-\left(t - t_0\right)/t_1\right)$ as shown in Fig. 1b. Here $P$ is the probability of the vortex detection within a period of time $t$. The estimated time $t_0 = 8.1 \pm 0.5$ ms corresponds to the flight time of the fastest vortex ring emitted from the generator. If assuming a creation time negligible, the velocity of the fastest ring is estimated to be 140 mm/s. This result indicates that emitted rings are limited in size to diameters larger than 1.2 μm derived from Eq. (1). The limited size is considered to be associated with the wire diameter, the wire vibrating amplitude, or a Kelvin wave length, which are 2.4 μm, 8.7 μm, and 13.3 μm for 3.15 kHz, respectively. The size of the fastest ring is equal to a half of the wire diameter. It is plausible that the sizes of emitted vortex rings are affected by the size of a vibrating structure. The single exponential distribution shown in Fig. 1b indicates a Poisson process: the detections of vortex rings occur continuously and independently.

Figure 1. Distribution of detection times repeatedly measured for a generation power of 2.7 pW at 50 mK (total 100 measurements): (a) Histogram of detection times; (b) Non-detection probability $1 - P$ estimated from the distribution shown in Fig. a. Here $P$ is the detection probability in a period of time $t$. The non-detection probability is well fitted to $\exp\left(-\left(t - t_0\right)/t_1\right)$ shown as the solid line in Fig. b.
at a constant average rate equal to the inverse of the estimated time $t_1 = 13.6$ ms. Actually the time $t_0 + t_1 = 22$ ms is equal to the average of the detection times shown in Fig. 1a.

The number of vortex rings emitted from a turbulence region increases with increasing generation power. At a power of 6.0 pW, the non-detection probability $1 - P$ is also well fitted to $\exp(-t/t_0-t_1)$ as shown in Fig. 2. The estimated times $t_0$ and $t_1$ are 6.0 ms and 12.1 ms, respectively. The both times are shorter than those observed for a power of 2.7 pW, suggesting that smaller sizes of vortex rings are emitted. It is plausible that a distribution of emitted vortex rings becomes wider in size with increasing power as well as those number increases, as reflected in the time $t_1$. Microscopic images showing the creation of vortex rings are necessary to understand the time-of-flights observed here.

3.2. Numerical simulations of time-of-flights

To investigate distributions of the time-of-flights, we simulated the dynamics of vortex rings numerically at the zero temperature, using a vortex filament model with the full Biot-Savart law [14]. The simulations are performed in a cubic box with a 0.5 mm side. Vortex rings with a 4 $\mu$m diameter are injected continuously through a slit with a 6 $\mu$m aperture located in a surface of the box. A vortex ring is detected at a line with a 2 $\mu$m width located in the opposite side of the box parallel to the slit. Therefore, the distance between the slit and the line is 0.5 mm. We measured a period from the start of vortex injections through the slit to the first detection of a vortex ring colliding to the detector line.

When we inject vortex rings with each position and direction given randomly, the time-of-flights of vortex rings are distributed. We plot the non-detection probability $1 - P$ estimated from the distribution of time-of-flights in Fig. 3. Injection rings form a single exponential distribution $1 - P = \exp(-t/t_0-t_1)$ at a rate of 0.5 kHz as shown in Fig. 3a. Vortex rings propagate from the slit and will reach to the opposite side without colliding to each other. In this case, the estimated time $t_0 = 10.9$ ms corresponds to the time of the direct flight of a vortex ring from the slit to the detector line, equal to the flight time calculated by Eq. (1) for an injecting ring with a 4 $\mu$m diameter. The simulated distribution is consistent with the distributions observed in the experiments shown in Figs. 1 and 2. This result suggests that emitted vortex rings propagate without colliding to each other and a non-detection time $t_0$ corresponds to the smallest size of emitted vortex rings.

At a large rate of injections (20 kHz), however, the non-detection probability deviates downwards from a single exponential distribution at low $t$’s as shown in Fig. 3b. In this simulation, vortex rings collide and reconnect themselves with each other, resulting in the creation of smaller or larger vortex rings. Those processes may modify a distribution of time-of-flights; a non-detection period becomes a bit shorter. Collisions and reconnections observed in the simulation might occur in experiments for a large power, though the distributions observed for powers below 6.0 pW show a single exponential distribution.

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**Figure 2.** Non-detection probability $1 - P$ in a period of time $t$ measured for a generation power of 6.0 pW. The probability decreases exponentially, similar to the probability shown in Fig. 1. The estimated times $t_0$ and $t_1$ are shorter than those obtained for the distribution shown in Fig. 1b (see text).
4. Conclusions

We investigate the dynamics of vortex rings in superfluid $^4$He both experimentally and numerically at very low temperature. The time-of-flights of vortex rings for a low injection power shows a single exponential distribution with a non-detection period. By comparing the distribution with numerical simulations of the vortex dynamics of vortex rings, we found that the non-detection period corresponds to the smallest size of injected vortex rings. The smallest size of emitted vortex rings decreases with increasing generation power. These interesting characteristics can be applied to the technique of vortex ring generation for superfluid $^4$He.

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