Computational behavioral models in public goods games with migration between groups

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Abstract

In this study we have simulated numerically two models of linear public goods games where players are equally distributed among a given number of groups. Agents play in their group by using two simple sets of rules, called 'blind' and 'rational' model, respectively, that are inspired by the observed behavior of human participants in laboratory experiments. In addition, unsatisfied agents have the option of leaving their group and migrating to a new random one through probabilistic choices. Stochasticity, and the introduction of two types of players in the blind model, help simulate the heterogeneous behavior that is often observed in experimental work. Our numerical simulations of the corresponding dynamical systems show that being able to leave a group when unsatisfied favors contribution and avoids free-riding to a good extent in a range of the enhancement factor where defection would prevail without migration. Our numerical simulation presents results that are qualitatively in line with known experimental data when human agents are given the same kind of information about themselves and the other players in the group. This is usually not the case with customary mathematical models based on replicator dynamics or stochastic approaches. As a consequence, models like the ones described here may be useful for understanding experimental results and also for designing new experiments by first running cheap computational simulations instead of doing costly preliminary laboratory work. The downside is that models and their simulation tend to be less general than standard mathematical approaches.

1. Introduction

Many social and economic situations have the following flavor: it is proposed to all the participants to contribute to a common fund; the sum of the contributions is used to acquire, or create, a desirable good or a facility that will be freely shared by everybody, i.e., the good is non-excludable, and its value for those enjoying it is estimated to be higher than its cost. Examples abound in our societies: national defense and security, street lighting, official statistics, and many others. Unfortunately, public goods provision is easily undermined by 'free-riders' which are individuals or entities that can access the good but are not contributing to it [1]. Other areas in which the problem arises is in all cases in which a resource is in principle available to all but runs the risk of being overexploited by some with the ensuing resource depletion [2]. Well-known examples are fishing in international waters, littering, or non-respect of pollution quota. The dimensions of goods are excludability and rivalry. For example, national defense and street lightning are non-excludable and non-rival while fishing in international waters is non-excludable and rival. In the following we only deal with the first category. In its simplest version this situation can be described as follows. \(N\) individuals are asked to contribute an amount \(c\) to a common fund. Each contribution in the fund is multiplied by an enhancement factor \(r > 1\), contributions are summed, and the sum is equally shared between the \(N\) members. If all people in the group contribute then each of them will receive \((c \cdot r)\) from the common fund. However, the dominant strategy for a member is to not contribute. In fact, if \(m\) members do contribute then each of them gets \((m \cdot c \cdot r) / N - c\) while the \(N - m\) members that do not contribute get \((m \cdot c \cdot r) / N\) each, which is always larger than the former...
for any $m > 0$. Thus, from a rational standpoint it is best for an individual not to contribute, perhaps hoping that others will, which leads to failure in producing the public good. Modern theoretical treatments of public goods games (PGG) under a variety of conditions can be found, for example, in [3–6].

In spite of the gloomy predictions of game theory, which are based on rationality, common knowledge of rationality, and payoff maximization, we observe cooperative behaviors in some cases in the real world in situations where game theory would prescribe not to contribute, such as blood donation, volunteering for the elderly care, and for cleaning beaches and rivers and similar activities. This has been partly confirmed in the last twenty years in many laboratory and online experiments with human subjects. Experimenters have reported a variety of behaviors and, although free-riding is common, it is by no means the only observed behavior. There is a vast literature on the topic, some important milestones and references to further work can be found, for example, in [7–12]. A useful study which performs a meta-analysis of many experiments on linear PGG can be found in [13]. The salient features that emerge from this body of work are the following. When the PGG game is played once it has been observed that the subjects contribute about half of their endowment. However, when the game is repeated contributions tend to decrease steadily down to a residual contribution around 10%–20%, instead of vanishing as prescriptive theory stipulates. Interestingly, if subjects are told about who is free-riding and are allowed to punish them for a cost, then contributions start to rise again [8, 9]. A recent experimental study [14] of the effect of majority-voting punishment on contribution when individuals have homogeneous or heterogeneous marginal per capita returns (MPCRs) on public goods confirms that punishment is effective in promoting cooperation.

In general, the results observed in the laboratory strongly depend on the experimental setting and a key role is played by the amount of information possessed by the subjects [15–21]. If subjects are informed about who is doing what in a given round, then they may choose strategies that are reactive to such decisions by conditioning the amount they contribute in the next round on the degree of cooperativity observed in the group in the previous one. In particular, if the identity of free-riders is known and they can be punished, the punishing behavior shows that agents act as if they took into account the welfare of others when making their decisions [8]. However, when the only feedback to a participant is her own payoff, the steady reduction in contributions can be explained by simple payoff-based learning, as suggested in the experiments performed by Burton-Chellew et al [10–12].

In the last decade PGGs have been modeled and intensively numerically simulated on more complex population structures than plain isolated groups by generalizing them to populations represented as complex networks, following the previous trends for two-person games (see, e.g., [22–27] and [28, 29] for two recent reviews). There have also been purely theoretical models of multi-person games on networks (see, e.g., [30, 31]). In this view, vertices of the network represent players and groups are formed by a central vertex and all its first neighbors. Such studies have opened the way to more realistic interaction structures that resemble real-world ones. Essentially based on replicator dynamics or stochastic processes ideas, these works have confirmed that contributions go to zero in the long term in the average, until a critical value of the enhancement factor is reached. For larger values of the latter full cooperation is the stable state and it is reached at a rate that depends on the network structure and on the particular strategy update rule. In addition, simulations have permitted to show that allowing players to modify their environment by cutting unprofitable links and establishing new ones can be a successful strategy for enhancing the steady-state contributions [32–38].

The above models and similar ones have been invaluable in providing a strong foundation for PGG dynamics on structured populations. However, they contain some unrealistic assumptions that, altogether, explain why they are not as useful for predicting the outcome of actual play in the field or in the laboratory. The main limitation is that, for theoretical reasons and mathematical simplicity, decision rules are homogeneous across the population (but see the exceptions presented in the next paragraph below), and are essentially based on payoff-proportional strategy update. This does not match the more inhomogeneous and information-dependent behavioral rules that agents seem to use in laboratory and online experiments. In addition, and again for mathematical simplicity, in most theoretical models individuals either contribute their full endowment or do not contribute at all. In experiments, on the other hand, participants usually receive a number of tokens and can choose how many to contribute to the common fund, i.e., contributions are a continuous or, more commonly, an integer variable.

In previous recent work on static and dynamic networks researchers tried to bring the models closer to what is observed experimentally by suggesting information-dependent decision rules and fractional contributions [27, 38]. These rules try to mimic the decision processes that people use when confronted with different amounts of information on their environment and they can be inhomogeneous to some extent by simulating probabilistic decisions. Heterogeneities of various kinds and personality-dependent decision have also been studied in a more theoretical setting based on simulation of replicator dynamics. For example, the authors of [39] show that compassionate behavior leading to payoff redistribution leads to enhanced cooperative behavior in spatial evolutionary games. In Wang et al [40] it is shown that if the investment of a player in a group of
agents playing the spatial PGG depends on the fraction of cooperators in the group, then cooperation is promoted as the level of heterogeneity in the contributions increases. In [41], heterogeneity is introduced through a preferential selection mechanism. On scale-free networks, each agent chooses an individual using the degree of the neighbors with a probability proportional to $k^\alpha$, with $\alpha > 0$. The authors find that cooperators are promoted over a large range of $\alpha$.

The results of using our information-dependent decision rules on both static and coevolving networks are encouraging, showing that contributions may attain fairly high levels even for relatively low enhancement factors. Moreover, the possibility of deleting unprofitable links and creating new ones further increases the equilibrium average contribution [38]. In these networks a player in general belongs to several groups at the same time, these groups being determined by the detailed connections between players. This makes it difficult to disentangle the effects of changes in the different groups and, for the same reasons, makes the system difficult to study in a laboratory setting because the environment is too complex to be apprehended by the average participant. A formal improvement is obtained by considering the bipartite graph whose two sets of vertices are, respectively, the players and the groups. Edges can only connect players to the groups to which they belong [23, 26] but, of course, a player can still belong to several groups as before, since the corresponding social network is just the players’ projection of the bipartite graph. In the present contribution we want to simplify still further this model in such a way that players can still change group under some given conditions, but they can only belong to a single group at a time. The corresponding bipartite graph would then have only one edge going from a given player to the group it belongs to.

Our goal in this work is twofold. First of all, we want to study a model of PGG played by groups of individuals that allows participants to possibly quit the group to which they belong if they are unsatisfied in a simpler setting than a full-fledged complex network, in order to disentangle individual dynamics from multiple group structure. As an added value this setting, being simpler and also common in real-life, it should make testing the model in the laboratory a much easier task than managing the network connections. Second, following [27, 38], we use decision rules that appear to be closer to the actual behavior of subjects in the laboratory than the highly abstract replicator dynamics-like update rules commonly used in computational models. From a computational point of view, the advantage of our approach is that it can be tailored to particular decision rules, including learning, and can thus help in understanding and designing interesting experimental settings but it also has the drawback that it can become too specialized thus loosing explanation power in more general situations.

The manuscript is organized as follows. We first describe the structure of the population playing the PGG and its dynamics. This is followed by numerical simulation of the models under various conditions and the discussion of results. A final section with further discussion and conclusions ends the paper.

2. Methods

In this section we present the model for the population and group structures, the strategic decisions of the individuals belonging to the population, and the resulting dynamical process of evolution.

2.1. Population structure

The population of size $N$ is structured initially into $g$ groups of equal size $n = N/g$, $N$ being exactly divisible by $g$. This can be represented by a bipartite graph [23, 26] but it is simpler and more informative to visualize the population as $g$ islands or simply groups in which we shall assume that individuals can leave a group and migrate to a different one under certain given conditions. Groups or islands will thus be considered as synonyms of groups in what follows. Figure 1 provides a schematic illustration of a population divided into communicating groups. This kind of structure has been very common in population biology starting with the seminal work of Wright [42] and its use in evolutionary biology is fully explained in the standard text by Mac Arthur and Wilson [43]. It has also been often used in evolutionary game theory in biological contexts, especially in two-person games (e.g., see [44]) but $n$-person games such as the PGG on group-structured populations have also been studied [4, 45]. Group-structured populations are also relevant outside population biology in socio-economic contexts. Think, for instance, of moving to a nearby country to optimize tax paying, or moving to another town or neighbourhood to benefit from better education, and similar examples. In spite of their importance, group-structured populations, to our knowledge, seem to have been little studied in relation to PGG using models different from replicator dynamics ones and have also been somewhat neglected in experimental work.

2.2. Public goods game

We consider a standard linear PGG in which the sum of the contributions is equally shared between the members of the group independent of their respective contributions, after multiplication by the enhancement factor
Figure 1. Schematic drawing of a population structured into independent groups or islands. Unsatisfied individuals may migrate to another randomly chosen group.

Algorithm 1. Play rounds.

procedure Play Rounds
Form $g$ groups each of size $n = \text{number of players}/g$
Initialize individual contributions randomly; $t \leftarrow 0$
while $t < \text{max number of rounds}$ do
  for $i = 1 \rightarrow \text{number of players}$ do
    $i$ plays with all the other members of its group; $i$ accumulates its payoff $\pi_i$
  end for
  Generate a random permutation of the players
  for $i = 1 \rightarrow \text{number of players}$ do
    Decide next action ($i$)
  end for
  $t \leftarrow t + 1$
end while
end procedure

$r > 1$. The utility, or payoff, $\pi_i$ of individual $i$ after playing a round in its group is:

$$\pi_i = (E - c_i) + \frac{1}{n} \sum_{k=1}^{n} r c_k$$

in which we assume that $i$’s round endowment $E$ is 1 and $i$’s round contribution is $c_i$. The first term is what remains after contributing the amount $0 \leq c_i \leq 1$ and the second term is $i$’s gain after multiplying by $r$ all the members’ contributions and equally sharing their sum among the individuals of the group of size $n$. A key figure for linear PGG is the MPCR. The MPCR governs the payoff of an individual from contributing an extra unit to the public good. $1/n < \text{MPCR} = r/n < 1$ holds for a Prisoner’s Dilemma situation to occur. If MPCR $< 1/n$ non-contributing is the payoff-dominant strategy. Conversely, when MPCR $> 1$, there is no dilemma and contributing becomes the payoff-dominant strategy.

Initial contributions are chosen randomly among the following set of values $\{0.0, 0.25, 0.50, 0.75, 1.0\}$, where 1.0 represents the total endowment that a player is given initially. The above framework is qualitatively similar to many experimental settings for the PGG. The quantitative difference is that participants usually receive a capital of 20–40 tokens that they can spend in integer amounts. Our simulated setting is an abstraction of those more detailed processes. This high-level process is schematized in algorithm 1 (play rounds).

2.3. Blind model

After having played in their respective groups and having got their utilities, agents must decide what to do next. Specifically, if an agent is satisfied, getting a payoff at least equal to the total contribution it made to the group of which it is a member then, with probability 0.5 it will increase its contribution by 0.25 or, with probability 0.5, it will keep its contribution unchanged in the next step. The noisy decision introduces some heterogeneity.
Algorithm 2. Decide next action (blind model).

Require: 
\[ c_i(t): i's \text{ contribution at time step } t; \pi_i: \text{ total gain of } i \text{ in its current group}; p: \text{ probability of migration} \]

procedure BLIND MODEL
  if \( \pi_i \geq c_i(t) \) then \( i \) is satisfied
    \[ r \leftarrow \text{random number} \]
    if \( r < 0.5 \) then \( c_i(t + 1) \leftarrow c_i(t) \)
    else \( c_i(t + 1) \leftarrow c_i(t) + 0.25 \)
  end if
  else \( i \) is unsatisfied
    if \( c_i(t) > 0 \) then
      \[ r \leftarrow \text{random number} \]
      if \( r > p \) then \( c_i(t + 1) \leftarrow c_i(t) - 0.25 \)
      else \( \text{ChangeGroup}(i) \)
    end if
    else \( c_i(t + 1) \leftarrow c_i(t) \)
  end if
end procedure

Algorithm 3. Change group.

procedure CHANGE GROUP \( (i) \)
  \( i \) belongs to group \( G \)
  find a random group \( D \neq G \)
  delete \( i \) from group \( G \) and add it to group \( D \)
end procedure

in the agent’s behavior and tends to avoid mass population behavior, in agreement with what one observes in laboratory results where individual behaviors are usually inhomogeneous. On the other hand, if the agent is unsatisfied because it got less than what it put in the common fund, then it will have two possible choices: either it tries to migrate to another randomly chosen group with probability \( p \), or it decreases its contribution by 0.25 at the next time step with probability \( 1 - p \). Migration allows individuals to leave an unfavorable environment and it can be done freely, without paying a cost. Although the assumption of zero-cost migration does not hold in many real world situations, it is an acceptable working hypothesis as a starting point to keep the model simple. The influence of migration costs will be included in a future extension. The probability \( p \) for joining another group can be set to any value \( 0 \leq p \leq 1 \). For the sake of clarity, the above processes are summarized in pseudo-code form in algorithms 2 (decide next action) and 3 (change group) for the migration to another random group. For ease of reference and because agents cannot access any information other than own contributions and payoffs this model is called ‘blind’.

The simulated process can be implemented with either synchronous or asynchronous update dynamics. In the asynchronous case agents update their contribution as soon as they are chosen according to the order given by a random permutation of the agents’ numbers. In the synchronous case agents are updated one by one in a random permutation order as before but their new contribution is stored in a separate data structure which replaces the old one when all the agents have been given an opportunity to update their state. At this point, that can be called a synchronization barrier, all the agents are updated at once before the next round begins. Asynchronous dynamics is the most natural way to update the agents’ state. However, for technical reasons, synchronous dynamics is usually easier to deal with in an experimental setting.

The above behavioral adaptation rules are extremely simple and certainly do not correspond in detail to the decisions made by human participants in an experiment but they were successful in the static network case, inducing an average population behavior that was qualitatively similar to what was observed in experiments in which people had the same amount of information [27]. We think that the groups with the migration arrangement introduced here is even simpler and more realistic than an entangled network environment and should be particularly suited for an experimental study. In the next section we present the setting and the results of simulating the model numerically.

2.4. Rational model

In the blind model we presented an extremely simple behavioral rule for the agents’ decisions in which the information available to them is minimal, i.e., just their own contribution and their payoff. However, there are many situations, both in laboratory settings as well as in the field in which actors possess more information on what others agents are doing. To stick with our choice of being as simple as possible we made the following
assumptions in our next model. The information set now changes such that players know not only their own contributions and payoffs but also the contribution of the other members of the group. However, the identity of the players remains unknown. This rules out any individual-based reaction rule and would also prevent any individual-directed punishing behavior on the part of the agents. We also assume that individuals do not have sophisticated computational abilities. For example, they could in principle compute average contributions in the group after a round but we assume that they are either unable to do so, or too lazy, or that there is not enough time to implement the computations. In contrast, we do assume that an agent can react to easy-to-spot group behavior such as how many in the group contributed in the previous round.

In more detail, the agents obey the following behavioral decision rules. A generic agent \( i \) in a group of size \( n \) counts the number \( s \) of group members that have contributed \( c > 0 \) in the previous round, not counting his own contribution \( c_i \). If \( s > \lfloor n/2 \rfloor \) then if the agent payoff \( \pi_i \), is larger or equal to its contribution \( c_i \), the agent, with probability 0.5, either increases its contribution or lets it unchanged. Otherwise, if the agent’s payoff \( \pi_i \) is less than its contribution then the agent either migrates to another randomly chosen group maintaining its current contributions with probability \( p \) or, with probability \( 1 - p \), it stays in its group but it decreases its contribution by 0.25. Finally, if less than half of the agents in the group contribute a positive amount, then the current agent \( i \) migrates to a randomly chosen group or stays in the group with the same contribution with probabilities \( p \) and \( 1 - p \), respectively. The decision process is summarized in the algorithm 4. The rest of the dynamics, i.e., playing the game rounds and migrating is the same as in the previous model. To contrast this model with the one presented in section 2.3, we call it the ‘rational model’ because it implies some computational capabilities on the part of the agents. The process is summarized in pseudocode form in algorithm 4.

\section{Results and discussion}

\subsection{Homogeneous blind model}

Here we present and discuss the simulation results for the blind model described in the previous section with a total population of \( N = 300 \) agents. We also run simulations with \( N = 500 \) and \( N = 30 \) agents obtaining similar average results, except that the fluctuation was larger in the system with \( N = 30 \) and simulations take longer in the larger system; thus, we finally set for \( N = 300 \). Let us start with the \( N \) agents subdivided into 10 groups of equal size \( n = 30 \). The left image of figure 2 shows the average contribution after \( t = 30 \) rounds for the whole population averaged over 50 Monte Carlo simulations in the asynchronous case for \( 1.0 \leq r \leq 3.0 \), migration probability \( 0.0 \leq p \leq 1.0 \).

The population behavior is qualitatively similar to that observed in populations structured as social networks [27]. It is also coherent with several experimental results in which subjects have the same information available [10–12]. The beneficial effects of migration are evident from the heat map. Without migration (bottom line of the map) contributions remain below the 0.5 level up to \( r \approx 2.2 \) and only reach full contribution starting at \( r \approx 2.6 \). On the other hand, with a migration frequency of about 0.5 full contribution is reached

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Algorithm 4. Decide next action (rational model).

\begin{verbatim}
Require: 
c_i(t): i’s contribution at time step t; \pi_i: total gain of i in its current group; p: probability of migration

procedure RATIONAL MODEL
  \textbf{if} more than half members of the group contribute then
    if \( \pi_i > c_i(t) \) then
      \( r \leftarrow \text{random number} \)
      if \( r < 0.5 \) then \( c_i(t+1) \leftarrow c_i(t) \)
      else \( c_i(t+1) \leftarrow c_i(t) + 0.25 \)
    end if
  \textbf{else}
    \( r \leftarrow \text{random number} \)
    if \( r \leq p \) then \( \text{ChangeGroup (i)} \)
    else \( c_i(t+1) \leftarrow c_i(t) - 0.25 \)
  end if
\textbf{end if}

\textbf{else} \{more than half members in i’s group do not contribute\}
  \( r \leftarrow \text{random number} \)
  if \( r \leq p \) then \( \text{ChangeGroup (i)} \)
  else \( c_i(t+1) \leftarrow c_i(t) \)
\textbf{end if}
\textbf{end procedure}
\end{verbatim}
Figure 2. Average contribution at round \( t \) as a function of the enhancement factor \( r \) and the migration probability \( p \) for the asynchronous blind model. The number of players is 300 and the number of groups is 10. Averages are taken over 50 independent simulations. Left panel: \( t = 30 \). Right panel: \( t = 300 \).

from \( r \approx 2.3 \). This trend is monotonous in the sense that the more the agents migrate when they are unsatisfied in their group, the larger the final average contribution. In the limit when all unsatisfied agents always migrate (top line of the map) even for low \( r \) values contributions remain above 0.6 and full contribution is quickly obtained for higher \( r \). The reason is simple: unsatisfied agents are those that have contributed to the fund but did not get a positive payoff in exchange. When they migrate to another random group, they carry with them their ‘altruistic’ behavior and thus average contributions increase, as defecting agents are left alone in the process. It thus appears that freedom of migration under adverse condition really helps in mitigating the tendency towards defection that is typical of this game.

The right panel of figure 2 has exactly the same parameters as the left part except for the play length which is 300 rounds per repetition instead of 30. The motivation for doing this is the following. Twenty to thirty is the typical order of magnitude of the number of rounds that are used in experimental work and thus a simulation of this length makes sense for comparison with laboratory work. However, the model is an abstract stochastic dynamical system and it is of interest to investigate its asymptotic or, more exactly, long run properties. When we do this, we see that nothing fundamentally different happens. The only difference is that states that were still only partly cooperative after 30 rounds of play now reach full contribution. This is seen in the extension of the light green/yellow zone to the middle of the investigated space. Simulations with up to 1000 rounds show that there is practically no change from the 300 rounds pattern (not shown to save space). We can thus draw two conclusions. The first is that a relatively limited number of rounds is sufficient to reach a steady-state of the dynamics. And second, the number of runs typically used in laboratory or online work is already sufficient to give a good approximation of the final situation.

Another interesting question is whether the group size has an influence on the level of contribution that can be reached. To study this question we simulated the same asynchronous system but this time with 30 groups of initial size \( n = 10 \). Average results over 50 Monte Carlo simulations of 30 rounds of play each are shown in the left image of figure 3. Comparing with the left panel of figure 2 the effect of the group size is found to be quantitatively small with a slight advantage for small groups for \( r > 2.0 \). When the repetition length is ten times larger, i.e., \( t = 300 \) as in the right image of figure 2, the differences are even smaller and almost negligible. Judging by these results, we conclude that there is little effect of the group size on the total average contribution and thus group size is not a critical parameter.

Now we present in figure 4 below the simulation results for the synchronous update system with \( N = 300 \) and the population initially subdivided into 10 groups of size 30 each. Comparing the left panel with the corresponding image of figure 2 gives the visual impression that results are very similar with migration being slightly more effective in enhancing cooperation in the synchronous case. The same happens with the case \( t = 300 \) depicted in the right images of both figures. Finally, figure 5 confirms the above trend for smaller groups as well, both for repetitions of 30 rounds and 300 rounds. In conclusion, given the small differences, neither the synchronous or asynchronous timing of operations, nor the group size, at least for the two sizes shown here, seem to be critical parameters. In order to save space, only asynchronous results will be shown.
Is there any pattern in the migration process as the enhancement factor $r$ is varied? Intuitively, one would expect that migration should be chosen more often for low $r$ values as those are the ones for which contributing individuals are the least satisfied. This is indeed the case, as shown in figure 6 where we report the average number of migrations as a function of $r$ for 50 Monte Carlo simulations after 30 rounds, with $N = 300$ and 10 groups of size 30. As expected, the number of migrations is larger for higher migration probability. But the remarkable observation is that in all cases there is a clear phase transition in the $r$ region where the mean contribution starts to grow towards full contribution and, at the same time, the number of migrating individuals decreases abruptly. Migration is implemented more often for values of $r < 2.3$; for higher values of $r$ it is no longer needed. Clearly, the threshold is more abrupt and shifted to lower values of $r$ the higher the migration probability is.

The question whether migration patterns show some regularity is more difficult and has no clear answer. In our model we impose a minimal group size of one and thus the number of groups does not change during a run. It would be possible, and perhaps interesting, to let groups disappear but we leave this study for a future work. During a run the size of groups fluctuates in an essentially random manner because migrations are randomly distributed among the groups.
3.2. Heterogeneous blind model

Until now there was a single subject type in the population and individual differences in behavior can only arise through different choices when the choice depends on drawing random numbers. However, it has been observed in experiments that often the behavior of a population of agents can be roughly reduced to a few types (see, e.g., [46–49]). Since there are many potential choices for the number of types and their migration propensities, here we chose to model the simplest case. We have thus modified the blind model in order to accommodate two types. More precisely, half of the agents in the average are born with a probability of migration $p = 0.8$ and the other half has $p = 0.2$. This gives the same average probability of migration as before when all individuals had 0.5 migration probability, i.e., $0.5 \times 0.8 + 0.5 \times 0.2 = 0.5$. Using asynchronous dynamics and the same simulation parameters as before, we obtained the results shown in figure 7.

Individual runs may differ due to statistical fluctuations, but comparing these average results (bottom curves in figure 7) with the corresponding homogeneous population results (top curves), we see that the overall behavior averaged over 30 Monte Carlo simulations is similar but it is slightly better for the heterogeneous case. Thus, either an homogeneous population of stochastic individuals or a population with two types of individuals with respect to their migration attitude, gives similar positive results in terms of contribution when averaged over a sufficient number of runs.

3.3. Rational model

In order to reduce the computational burden, we present results of the rational model only for probability of migration $p = 0.5$ and thus we show only two sets of curves instead of the whole heat maps. The following
Figure 7. Average contribution as a function of round number for the values of the enhancement factor $r$ shown in the inset for the asynchronous models. Averages are taken over 30 simulations of 30 rounds each. The number of players is 300. Top curves: migration probability $p = 0.5$. Bottom curves: half of the population has a migration probability $p = 0.2$ and the other half has $p = 0.8$.

Figure 8 shows contributions averaged over 30 repetitions of a population with 300 agents playing a simulated PGG during 30 rounds for each repetition for ten groups and thirty groups respectively. The population dynamics is asynchronous.

As it happens, there are only small quantitative differences between the average results; the global pattern turns out to be almost the same with two sets of apparently different behavioral decision rules based on different amounts of information. In fact, when at least half of the members of a group contribute, it means that, depending on the particular $r$ factor, it is likely that agents in the group get a positive payoff and the other way around when less than half contribute. Since the repertoire of what an agent does in each case is limited and similar in both cases, the results we got become less surprising. Some recent experimental results do support this point of view. In [10–12] Burton-Chellew et al performed very careful basic linear PGG experiments in which human participants, besides their own contribution and payoff, were given different amounts of information about the behavior of other players. Their conclusion was that the results when only own contribution and payoff were available, were statistically equivalent to those arising when a richer information was provided. Of course we are not claiming that our simulations match these experiments but the analogy is worth mentioning.
4. Conclusions

We have numerically simulated two models of PGG in which players are initially equally distributed among a given number of groups, the ‘blind’ and the ‘rational’ model. In these models, agents play the PGG in their group by using ad hoc decision rules that have been designed taking into account known behavioral patterns observed in experimental laboratory work with human subjects. We suggested two sets of rules depending on the information feedback given to the players. In the first case only own payoff and contribution are provided. In the second model, besides the previous data, the number of those who contributed in the group in the previous run is also given. The decision rules are partly stochastic, in order to simulate some heterogeneity in the players’ behavior. The decision process goes as follows: after evaluating its own payoff and, if available, the information about the other members of the group, the focal agent can do one of the following: if satisfied, the agent can either keep its contribution unchanged in the next round, or increase it. If unsatisfied, the agent will decrease her contribution unless it contributed nothing in the previous round. Besides, when unsatisfied, it is also possible for the agent to leave his group and to migrate to another randomly chosen group.

The numerical simulation of the corresponding dynamical systems show that for low enhancement factors the asymptotic state is one of little average contribution. However, beyond $r \approx 2$, the process tends to the full contribution absorbing state if enough rounds are played. One interesting conclusion from our work is that when unsatisfied players are allowed to migrate, high or even full contribution can be achieved for lower $r$ values. Therefore, as observed in other cases, the possibility of changing one’s environment facilitates cooperation. The information provided to the agents play a less important role. In fact, qualitatively similar results were obtained with either model. Our numerical simulation results are more in line with what one observes in the laboratory than theoretical results or results based on numerical simulation of replicator dynamics. As a consequence, tools like the ones presented here can be useful in understanding experimental results and in predicting interesting experimental settings. However, this flexibility comes at the price of lack of generality. Nevertheless, our conclusion is that modeling approaches based on behavioral observations are a useful tool in quantitative social science.

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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).
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