Research on the mechanisms of internal gear honing by use of cone-shape honing wheel with tool tilt angle

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Abstract
Internal gear is an important transmission component. However, it is restricted by its ring—shaped structure, which hinders the finishing process of the tooth surfaces after the heat treatment. Honing is the most commonly used finishing technique for external gear, but it is inconvenient for internal gear. In this paper, a new type of cone—shape honing wheel is proposed and the honing mechanism for improving the honing performance of the internal gear is studied. First, the mathematical model of the honing technology that employs a cone—shape honing wheel is built, and the merits of the new honing method are discussed. Then, the contact conditions between the honing wheel and the internal gear are analyzed. It is established that the tool tilt angle of the honing wheel has a good effect on the honing velocity distributions. Finally, the simulations of honing and profile measuring of the internal gear by employing the cone—shape honing wheel are carried out, by which the feasibility of the new honing technique is verified.

Keywords Honing mechanism · Internal gear · Cone—shape honing wheel · Tool tilt angle

Nomenclature

| Symbol | Description |
|--------|-------------|
| $S_1$ | The fixed coordinate system of the work gear |
| $S_w$ | The attached coordinate system of the work gear |
| $S_2$ | The fixed coordinate system of the honing wheel |
| $S_c$ | The attached coordinate system of the honing wheel |
| $S_t$ | Transfer coordinate system |

F Axial feed velocity of the work gear
$F$ Axial feed speed of the work gear
$\Sigma$ Crossed axes angle
$a$ Center distance
$\tau$ Tool title angle of the honing wheel
$c$ The cone angle of honing wheel
$\omega_w$ The angular speed of the work gear
$\omega_c$ The angular speed of the honing wheel
$Z_c$ Number of teeth of the honing wheel
$Z_w$ Number of teeth of the work gear
$P$ Helix parameter of the work gear
$P_z$ Helix lead of the work gear
$\beta_w$ The helix angle of the work gear
$\phi_c$ The helix angle of the honing wheel
$r_w$ Reference radius of the work gear
$r_c$ Reference radius of the honing wheel
$m_t$ The transverse module of the work gear
$m$ The normal module of the work gear
$r_b$ Base radius of the work gear
$u$ Involute parameter
$\theta$ Transverse tooth profile rotation angle
$\phi_w$ The rotation angle of the work gear
$\phi_c$ The rotation angle of the honing wheel
$\omega_w$ Angular velocity of the work gear
$\omega_c$ Angular velocity of the honing wheel
$i$ Transmission ratio

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Due to higher efficiency, lower cost, and better machining quality for hard gears, honing is deemed as an excellent gear finishing technique. It is widely used in the automotive, aerospace, and heavy equipment industries, especially in the areas that require quiet, robust, and reliable gearing [1, 2]. After years of development, theoretical and practical application studies of honing techniques have achieved great progress. The aforementioned mainly includes topological modification for helical gear, high efficiency honing with high accuracy, and electrochemical honing.

Many studies of gear honing are focused on the modification methodology. Tran et al. [3] proposed a new method for longitudinal tooth flank crowning of work gear surfaces during the honing process. Han et al. [4] developed a mathematical model for lead and profile crowning modification of the work gear by setting the movement of the honing wheel as two fourth-order polynomial functions. Wu et al. [5–8] presented methods for topology modification of helical gears using internal—meshing gear honing. Simultaneously, to improve the honing performance, many researchers investigate the contact conditions, cutting force vibration, and teeth elastic deformation of the honing wheel during the honing process. Klocke et al. [9] carried out cutting speed and contact condition analyses of gear honing, thus forming the basis for optimizing the process design. Bagaiskov [10] investigated the sliding parameters and tooth contact geometry in the engagement of gear hone teeth and processed parts with consideration of elastic deformation. Bergs [11] derived a force model of gear honing. Mallipeddi et al. [12] investigated the gear surface characteristics generated by grinding and honing of case hardened steel. The authors concluded that the honed gear has better performance in micro—pitting prevention compared with the ground one. Da Silva et al. [13] applied particle swarm optimization for achieving the minimum profile error in the honing process. In addition, the non—conventional honing technique is also the focus of several studies. Shaikh and Jain [14] investigated the micro—geometry improvement method of straight bevel gears finished by an electrochemical honing process. Moreover, the author demonstrated that ECH has great potential to be developed as a technologically better, more productive, and more economical alternative to the conventional bevel gear finishing processes.

Generally, most of the honing studies are focused on the finish machining of the external gear. Due to the restrictions of the ring—shaped structure of the internal gear, the honing technique is merely used in its finishing processing. Considering the size of the honing wheel is limited to stretch into and hone the entire tooth flank of the work gear, the service life of the honing wheel for the internal gear is diminished compared with the honing wheel for the external gear. Furthermore, to avoid interference between the internal gear and the tool spindle in the honing process, the diameter of the tool spindle must be limited which partially lowers the stiffness of the honing system. The aforementioned decreases the finishing quality of the work gear.

Based on the conducted literature overview, an improved honing method for the internal gear using the new type of cone—shape honing wheel is proposed in this paper. The mathematical model of the new honing technique is first built. Then, the contact conditions between the honing wheel and the work gear are analyzed. Finally, simulations of the honing process for the internal gear are carried out to verify honing feasibility and accuracy.

### 2 Honing principle by use of the cone—shape honing wheel

As shown in Fig. 1, several coordinate systems are defined for describing the relative position between the new type cone—shape honing wheel and the work gear. $S_1(x_1, y_1, z_1)$ is the fixed coordinate system of the work gear that rotates about and moves along the axis of $z_1$, while $S_w(x_w, y_w, z_w)$ is rigidly attached to the work gear. The axial feed velocity of the work gear is $F$, whose direction can either be positive or negative with respect to $z$. $S_2(x_2, y_2, z_2)$ is the fixed coordinate system of the honing wheel that rotates about the axis of $z_2$, while $S_c(x_c, y_c, z_c)$ is rigidly attached to the honing wheel. $S_c(x_c, y_c, z_c)$ is the transfer coordinate system. The crossed axes angle and the center distance between the work gear and the honing wheel are denoted as $\Sigma$ and $a$, respectively. Furthermore, there is an additional tool title angle $\tau$ of the cone—shape honing wheel axis $z_c$ relative to the axis $z$, compared with the original gear honing technology. Due to the tool title angle $\tau$ attached to the honing wheel, the shape of the honing wheel is changed into a cone shape. The new type of honing wheel is composed of a conical segment $b_1$ with a cone angle $\tau_c$ and a cylindrical segment $b_2$, as shown in Fig. 1. The teeth of the
conical segment of the honing wheel are meshing with the teeth of the work gear in the honing process, while the teeth of the cylindrical segment are not, which acts as the spare dressing segment of the honing wheel.

Material removal motion is formed by engagement and differential movement between the honing wheel and the work gear. The angular speed of the cone-shape honing wheel is defined as $\omega_c$, and the axial feed speed of the work gear along $z_1$ is denoted as $F$. The angular speed of the work gear $\omega_w$ can be described as

$$\omega_w = \frac{Z_c}{Z_w} \omega_c + \frac{F}{P}$$

where $Z_c$ and $Z_w$ are the numbers of teeth of the honing wheel and the work gear, respectively, and $P$ is the helix parameter that can be expressed as

$$P = \frac{P_z}{2\pi}$$

where $P_z$ is the helix lead on the reference cylinder of the work gear.

The crossed angle $\Sigma$ between the axes of the honing wheel and the work gear is the sum of the work gear helix angle $\beta_w$ and the honing wheel helix angle $\beta_c$. The center distance $a$ is the difference between the work gear reference radius $r_w$ and the honing wheel reference radius $r_c$:

$$\Sigma = \beta_w + \beta_c$$

$$a = r_w - r_c = (Z_w - Z_c) \cdot m_t$$

where $m_t$ is a transverse module of the work gear.
3 Mathematical model of the cone–shape honing wheel

The tooth flank of the cone–shape honing wheel can be enveloped by the tooth surface of the work gear. If the target tooth surface of the work gear is an involute helicoid, it can be expressed in $S_w$ as

$$
\mathbf{r}_w(u, \theta) = \begin{bmatrix}
x_w \\
y_w \\
z_w \\
1
\end{bmatrix} = \begin{bmatrix}
x_w = r_b \cos(u + \theta) + r_u \cos(u + \theta) + r_u \mu \sin(u + \theta) \\
y_w = r_b \sin(u + \theta) - r_u \mu \cos(u + \theta) \\
z_w = P \cdot \theta \\
1
\end{bmatrix}
$$

(5)

where $r_b$ is the base circle radius of the work gear, $u$ is the involute parameter, and $\theta$ is the rotation angle of the transverse tooth profile of the target tooth surface around $z_w$.

The normal vector of the involute helicoid can be expressed in $S_w$ as

$$
\mathbf{n}_w(u, \theta) = \begin{bmatrix}
r_b \mu \sin(u + \theta) \\
r_b \mu \cos(u + \theta) \\
r_u \mu \\
1
\end{bmatrix} \mathbf{P}.
$$

(6)

If the rotation angle of the work gear about the axis $z_1$ is $\varphi_w$, the position coordinates of a point on the target tooth flank can be expressed in $S_1$ as

$$
\mathbf{r}_w^{(1)}(u, \theta, \varphi_w) = \mathbf{M}_{1w} \cdot \mathbf{r}_w(u, \theta)
$$

(7)

where $\mathbf{M}_{1w}$ is the transfer-matrix from $S_w$ to $S_1$, whose upper-left $3 \times 3$ submatrix is $\mathbf{L}_{1w}$.

$$
\mathbf{M}_{1w}(\varphi_w) = \begin{bmatrix}
\cos \varphi_w & -\sin \varphi_w & 0 & 0 \\
\sin \varphi_w & \cos \varphi_w & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(8)

Normal vector can be expressed in $S_1$ as

$$
\mathbf{n}_w^{(1)}(u, \theta, \varphi_w) = \mathbf{L}_{1w} \cdot \mathbf{n}_w(u, \theta)
$$

(9)

Angular velocity of the honing wheel $\omega_c$ and angular velocity of the work gear $\omega_w$ can be expressed separately in $S_1$ as

$$
\omega_c^{(1)} = \mathbf{L}_{1w}^{-1} \cdot \omega_c = \omega_c \cdot \begin{bmatrix}
-\sin \tau \\
-\cos \tau \sin \Sigma \\
\cos \tau \cos \Sigma
\end{bmatrix}^T
$$

\begin{equation}
\omega_w^{(1)} = \omega_w = i \cdot \omega_c \cdot \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}^T
\end{equation}

(10)

(11)

where $i$ is the transmission ratio, $\mathbf{L}_{1w}$ is the upper-left $3 \times 3$ submatrix of $\mathbf{M}_{1w}$ which is the transfer-matrix from $S_1$ to $S_c$, and $\mathbf{L}_{2w}$ is the upper-left $3 \times 3$ submatrix of $\mathbf{M}_{2w}$ which is the transfer-matrix from $S_1$ to $S_2$.

$$
i = Z_c/Z_w
$$

(12)

$$
\mathbf{M}_{1i} = \begin{bmatrix}
1 & 0 & 0 & -\rho \\
0 & \cos \Sigma & \sin \Sigma & 0 \\
0 & -\sin \Sigma & \cos \Sigma & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(13)

$$
\mathbf{M}_{2i} = \begin{bmatrix}
\cos \tau & 0 & \sin \tau & 0 \\
0 & 1 & 0 & 0 \\
-\sin \tau & 0 & \cos \tau & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(14)

The relative velocity of the point on the target tooth flank about $z_1$ relative to the honing wheel about $z_2$ can be expressed in $S_1$ as

$$
\mathbf{v}_w^{(1)} = \mathbf{v}_w^{(1)} = \mathbf{\omega}_c^{(1)} \times (\mathbf{r}_w^{(1)} - \mathbf{a}) - \mathbf{\omega}_w^{(1)} \times \mathbf{r}_w^{(1)}
$$

(15)

where $r_w$ transforms into an upper $3 \times 1$ submatrix of itself, and $\mathbf{a}$ is the position vector of the honing wheel.

$$
\mathbf{a} = a \cdot \begin{bmatrix}
0 \\
0 \\
1
\end{bmatrix}^T
$$

(16)

The point constraint on the target tooth flank being a meshing point “M” in $S_1$ is that the normal vector $\mathbf{n}_w^{(1)}$ is vertical with the relative velocity $\mathbf{v}_w^{(1)}$.

$$
\mathbf{n}_w^{(1)}(u, \theta, \varphi_w) \cdot \mathbf{v}_w^{(1)}(u, \theta, \varphi_w) = 0
$$

(17)

The meshing point $\mathbf{r}_w^{(1)}(u, \theta)$ in $S_1$ can be obtained by simultaneously solving Eqs. (7) and (17). Therefore, the position vector of the point on the tooth flank of the honing wheel can be expressed in $S_1$ as

$$
\mathbf{r}_w(u, \theta, \varphi_w) = \begin{bmatrix}
x_w \\
y_w \\
z_w
\end{bmatrix} = \mathbf{M}_{1c}(\varphi_w) \cdot \mathbf{M}_{2w} \cdot \mathbf{M}_{1i} \cdot \mathbf{r}_w^{(1)}
$$

(18)

where $\mathbf{M}_{1c}$ is the transfer-matrix from $S_2$ to $S_c$.

$$
\mathbf{M}_{1c}(\varphi_c) = \begin{bmatrix}
\cos \varphi_c & \sin \varphi_c & 0 & 0 \\
-\sin \varphi_c & \cos \varphi_c & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
$$

(19)

By employing the parameters of the tool and the work gear shown in Table 1, the geometric model of the cone–shape honing wheel can be built. By using the new type of cone–shape honing wheel, the honing performance of the internal gear can be improved.

Firstly, the conventional radial direction dressing method that results in the tooth shape change of the honing wheel has a displacement effect on the gears. However, the dressing direction of the cone–shape honing wheel is axial, which will not cause teeth to shape change of the honing wheel during
various dressing procedures. Therefore, the aforementioned is beneficial for improving the consistency of the honing accuracy of the work gears. Moreover, the total available dressing times of the cone-shape honing wheel are mainly decided by the width of the cylindrical part. Therefore, total available dressing times can be increased in accordance with the actual demand by widening the cylindrical segment $b_2$. For example, if the width of the cylindrical segment $b_2$ is increased from 10 to 30 mm and the dressing amount for each time is 1 mm, the total available dressing times can then be increased from 10 to 30.

Secondly, due to the tool tilt angle of the honing wheel, interference between the tool spindle of the honing wheel and the work gear, as shown in Fig. 2 view P, can be avoided more easily. For example, to avoid interference, the tool spindle diameter $d_s$, at a tool spindle length $L_s$ of 40 mm, should be less than 41 mm. On the other hand, if the tool tilt angle is lifted in the “Q” direction and is increased to 10 degrees, the maximum tool spindle diameter $d_s$ can be increased to 50 mm. Consequently, a larger diameter tool spindle can be used, which is beneficial for the stiffness of the honing system.

Thirdly, the honing velocity distribution can be affected by the tool tilt angle of the honing wheel. This may have a good effect on the honing performance for the internal gear. The detailed analysis is carried out in Sect. 4.

### 4 Contact analysis of the honing process

Contact analysis is carried out to investigate the honing velocity distribution and the contact condition between the cone-shape honing wheel and the work gear. The basic parameters of the tool and the internal gear are shown in Table 1. For each rotation angle $\varphi_w$ of the work gear, a contact line exists between the work gear tooth flank and the honing wheel tooth flank. This line consists of meshing points that can be expressed by Eq. (20) in $S_w$. The sliding velocity of the meshing point on the work gear tooth flank relative to the honing wheel can be expressed by Eq. (21) in $S_w$ [15]:

$$r_M^{(w)}(u, \theta, \varphi_w) = M_w^{(1)}(\varphi_w) \cdot r_M^{(1)}$$  \hspace{1cm} (20)

$$v_{cw}^{(w)}(u, \theta, \varphi_w) = L_w^{(1)}(\varphi_w) \cdot v_{cw}^{(1)}$$  \hspace{1cm} (21)

The contact lines between the work gear and the honing wheel at different meshing positions are shown in Fig. 3. Groups of contact lines with different lengths and unanimous directions can be joined up to the meshing zones on the tooth flanks of the work gear and the honing wheel. Due to the restrictions of engagement without undercuts [16], the lengths of the contact lines are different, which may result in parts of the meshing zones being unable to cover the whole gear surface, so the axial feed movement in the honing process is recommended, especially when the tool tilt angle of the honing wheel is large. To distinguish the two sides of the teeth of the work gear, the front flank of the tooth space, referring to the rotation direction, is defined as the approach flank “A.” The other flank of the tooth space is defined as the recess flank “R.”

Honing velocities of the meshing points on the contact lines of the work gear can be calculated via Eq. (21). Theoretical calculation results of the contact patterns and honing velocity distributions with different tool tilt angles on the tooth flanks of the work gear are shown in Fig. 4. When the tool tilt angle is set to zero, honing velocities are

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**Table 1 Parameters of the gear and the honing wheel**

| Parameters of internal gear | Value |
|----------------------------|-------|
| Number of teeth $Z_w$      | 80    |
| Normal module $m$          | 2.25  |
| Pressure angle $\alpha$    | 22.5$^\circ$ |
| Helix angle $\beta_w$      | 12$^\circ$ |
| Revolution speed $\omega_w$| 1200 r/min |

| Parameters of honing wheel | Value |
|----------------------------|-------|
| Number of teeth $Z_c$      | 40    |
| Helix angle $\beta_c$      | 0$^\circ$ |
| Tool tilt angle $\tau$     | 10$^\circ$ |
| Cone angle $\tau_c$        | 10$^\circ$ |

---

**Fig. 2** Interference between the tool spindle of the honing wheel and the work gear
regularly distributed on the tooth flanks, which is identical to the original honing method. Directions of the honing velocities on the addendum are different from the dedendum. They can be separated by the reference lines RL near the pitch circle. Therefore, the honing trajectory on the tooth flanks will be unevenly distributed. Directions of the honing trajectory on the addendum and the dedendum are significantly different from the directions near the pitch circle, which is not good for the tooth contact fatigue life of the work gear [12].

As the tool tilt angle is increased to 5 degrees, the honing velocity distribution on the tooth flanks changes compared to the zero degrees condition. As the tool tilt angle is continuously increased to 10 degrees, the distribution of the honing velocities changes more evidently. The honing

Fig. 3 Contact lines and meshing zones

Fig. 4 Honing velocity distributions with different tool tilt angle }
velocities of the right side of the approach flank and the left side of the recess flank mostly incline to the dedendum, while the opposite flank sides are mainly oriented toward the addendum; the inclination angle of the reference line increases from 0 to 25 degrees for the approach flank and from 0 to 22 degrees for the recess flank. It can be considered that the tool tilt angle of honing wheel has an evident influence on the honing velocity distribution compared with the original honing method, which would have the effect of homogenizing the honing trajectory on the teeth flanks of the work gear if the axial differential motion of the work gear in the honing process is applied [17, 18].

The material removal rate mostly depends on the relative speed of honing movement, so the relative speeds along the contact lines are analyzed. The average honing speed $V_{av}$ is calculated first and then marked on the up-right side of the figures which can be used to estimate the values of the other relative speeds along the contact lines by comparing the lengths of the honing velocities. It can be seen from the figures that the average honing speed rises from 3.0 to 3.5 m/s for the approach flank and from 2.9 to 3.8 m/s for the recess flank as the tool tilt angle increases from 0 to 10 degrees. It also can be seen from the figures that the honing speeds along one contact line are approximate, but change obviously among the different contact lines in the meshing process. For example, the honing speed is smaller near the reference lines, but larger at the meshing in and the meshing out position, which may result in the uneven material removal of the gear teeth flanks. Moreover, due to the restrictions of engagement without undercuts, the height of the meshing zone in the tooth width direction $z_w$ is changed when the tool tilt angle of the honing wheel is increased, as shown in Fig. 4, which may result in parts of the meshing zones being unable to cover the whole gear surface. Therefore, the axial feed movement in the honing process is recommended.

5 Honing simulation and verification

In practice, the machining accuracy of the work gear is inevitably affected by the cutting tool accuracy, the machine tool accuracy, the thermal deformation of the machine tool, and other factors. In order to eliminate the influences of all these factors on the honing precision, verification machining simulations are carried out to verify the feasibility of the internal gear honing by using cone-shape honing wheel. The flow chart of the simulation process is shown in Fig. 5. Based on the theoretical study of honing engagement, the 3D model of the cone-shape honing wheel can be built. After that, the simulations of honing and profile measuring of internal gear by use of cone-shape honing wheel are carried out, which can verify the feasibility of the new honing technique.

5.1 Honing process simulation

The five-axis honing machine tool model is built in the VERICUT software, as shown in Fig. 6. The X, Y, and Z axes are the three orthogonal moving axes whose positive directions are defined as the Cartesian coordinate system. The A axis is the swing axis that rotates about the X-axis. C and the $C_1$ axes are the work gear rotation axis and the honing wheel rotation axis, respectively, which are coupled together by the electronic gearbox.

The machining allowance of the work gear is $d$, as shown in Fig. 6b. The relative position and orientation between the honing wheel and the work gear are set based on the parameters shown in Table 1. The cone-shape honing wheel and the work gear rotate around their axes with respect to their transmission ratio $i$. With the axial differential motion of the work gear, the entire tooth flank of the work gear can be honed. The green surfaces are the honed tooth flanks of the work gear, as shown in Fig. 6c. In the honing process, the machine components can move correctly without motion interference. After the simulation, the honed tooth flanks with the green color and the designed target...
tooth flanks with the blue color are compared, as shown in Fig. 6d. The result shows that the honing allowance is removed completely and the honed tooth flanks are smooth enough which coincides with the designed target tooth flanks very well.

5.2 Simulation of the work gear tooth flanks enveloping and measuring

To test the honing accuracy, tooth flanks enveloping and measuring simulations are carried out in the UG software.

![Three views of honing machine tool model](image_a)
![Machining allowance](image_b)
![Finished tooth flanks](image_c)
![Flanks comparison](image_d)

**Fig. 6 Honing movement simulation**

The model of the cone-shape honing wheel, as shown in Fig. 7a, is built by applying the point set of the tooth flank that can be obtained by solving the aforementioned tooth flank equations. The machining allowance of the work gear is \( d \). The relative position and orientation between the honing wheel and the work gear are set based on the parameters shown in Table 1. The axial motion’s distance per revolution of the work gear is 0.57 mm. During honing simulation, tooth flanks of the work gear are generated by the Boolean calculation between the blank work gear and the cone-shape honing wheel at each meshing position. Therefore, the tooth flanks of the work gear are composed of a series of striped marks left by the teeth of the honing wheel, as shown in Fig. 7b. If the theoretical calculation is correct, the marks should be tangent to the theoretical tooth flanks of the work gear. Moreover, the tangent lines should coincide with the theoretical contact lines. To verify the aforementioned, tooth profile deviation of the work gear should be obtained. Therefore, coordinates of several points on the transverse tooth profiles of the enveloped tooth flanks of the work gear are measured in the UG software, as shown in Fig. 7c. Next, a comparison between tooth profiles and the standard involutes is conducted. In [19, 20], algorithms of the profile deviation calculation are shown.

The calculation results of tooth profile deviation are shown in Fig. 7d, e. The measurement results indicate that transverse tooth profiles of the approach flank and the recess flank of the work gear are composed of a series of small arcs. These arcs are characterized by marks on the tooth flanks of the work gear generated by the teeth of the honing wheel during honing simulation. The current tooth profile deviation mostly reflects the height of the marks and can be decreased by minimizing the simulation steps. Therefore, fluctuation of the lowest points of the marks, approximately one micron, best represents the tooth profile deviation. Considering the modeling accuracy of the UG software is approximately equal to one micron, the tooth profile of the honed work gear can be considered accurate.

Apparently, compared with the original gear honing technique, there are many advantages of the proposed honing method for internal gear by using a new type of cone-shape honing wheel, but the shortcomings obviously exist. Firstly, due to the distribution features of the honing velocities and the meshing zones, the additional axial feed movement in the honing process is recommended. Secondly, there is a one-to-one correspondence between the honing wheel and the work gear, so the universality and the flexibility of the cone-shape honing wheel are weak. Moreover, the dressing of the cone-shape honing wheel is relatively hard compared with the cylindrical honing wheel.
6 Conclusions

An improved honing method using a new type of cone-shape honing wheel with tool tilt angle is proposed in this paper. The total available dressing times of the honing wheel for the internal gear with stable honing accuracy can be increased, and the interference between the tool axis and the work gear can be avoided more easily.

The mathematical model of the new honing technology that employs a cone-shape honing wheel is built. The contact analysis between the honing wheel and the internal gear is carried out, and the results show that the tool tilt angle of the honing wheel has a good influence on the honing velocity distribution of the work gear.

The simulations of honing and profile measuring of the internal gear by using cone-shape honing wheel are carried out, by which the feasibility of the honing movement is verified, and the tooth profile of the honed work gear is proven to be accurate.

Author contribution Zheng Guo: general concept and original draft preparation; writing; and editing. Yan Li: introduction; review; and editing. Wen-Chao Guo: honing simulation by use of the cone-shape honing wheel. Wu-Gang Zhang: further simulation of honing and measuring. Feng Gao: calculation of tooth profile deviation. Dong-Ya Zhang: preparation of conclusions and editing.

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Availability of data and material The datasets used or analyzed during the current study are available from the corresponding author on reasonable request.

Code availability Not applicable.

Declarations

Ethics approval Not applicable.

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Consent for publication Not applicable.

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