Modelling and calculating the stiffness of a cross-shaped element of a microelectromechanical gyroscope-accelerometer

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Abstract. This article investigates the design of a linear acceleration and angular velocity sensor with three axes of sensitivity. The modelling of the parametrized geometry of the gyroscope-accelerometer and its cruciform stiffener is carried out based on the ANSYS software package. The dependence of the movement of proof mass on the change in the thickness of the elastic beams of the cruciform element of the gyroscope-accelerometer is obtained.

1. Introduction
The use of microelectromechanical systems (MEMS) in consumer electronics is growing every year, with increasing demand from the smartphone market, which dominates the development of this new technology. MEMS sensors are becoming key elements in product development for consumer and mobile markets such as game consoles, tablets. MEMS gives the user a new way to interact with their smart device. At present, the improvement of microelectromechanical sensors is one of the urgent problems. [1, 2].

In recent years, low acceleration sensors have changed the face of a variety of devices, from game consoles to mobile phones, and from laptops to home appliances, making it is possible to implement motion-activated user interfaces and advanced security features. Now it is the turn of MEMS gyroscopes and geomagnetic sensors, the use of which can open new possibilities for applications and make them more attractive.

Thus, the article solves the problem of design a three-axis micromechanical sensor of linear accelerations and angular velocities with two inertial masses. Work in this direction will help improve the parameters of devices based on Microsystems and increase the competitiveness of Russian devices in the world market.

2. Design of MEMS Gyroscope-Accelerometer
In this work, the investigated design of the linear acceleration and angular velocity sensor with three axes of sensitivity is shown in Figure 1. The developed micromechanical gyroscope-accelerometer contains two proof mass (1, 2), which, together with fixed electrodes located under them on the substrate, form capacitive sensors, also capacitive sensors (3-8), electrostatic drives (9, 10).

In the proposed design of the linear acceleration and angular velocity sensor there is a cruciform stiffening element, consisting of series-parallel connections of elastic beams. It is in the centre of the structure for connecting electrostatic drives with proof mass. The frequency and amplitude of the movement of proof mass depends on many parameters, including the width and thickness of the elastic beams in the cruciform suspension element of the presented micromechanical gyroscope-accelerometer. The coefficient of stiffness of the cruciform element is obtain and the change in the amplitude of
displacement of proof masses in accordance with the change in the width of the elastic beams in the cruciform element of suspension stiffness is considered.

![Figure 1](image1.png)

**Figure 1.** Design of MEMS gyroscope-accelerometer: 1, 2 - inertial masses; 3-8 - capacitive sensors; 9, 10 - electrostatic drives.

3. **Deducing the stiffness coefficient of the cruciform element of the suspension of the MEMS gyroscope-accelerometer**

In this work, the task is to deduce the stiffness of the element shown in Figure 2, to build the dependence of the displacement of the connecting elements on the thickness of the beams.

![Figure 2](image2.png)

**Figure 2.** Cruciform stiffener: 1-9 elastic beams; 10-13 anchor areas; 14, 15 - perforated stiffening plates.
Due to the force $F$ applied to the cruciform stiffener, its elastic beams begin to deform:

- 3, 9, 7, 8 – take an S-shaped bend, since their ends are rigidly fixed in the anchor areas 10, 12, and 13.
- 1, 2 – bend like cantilever beams.
- 4, 5 – are pinched by the applied force $F$ and the elastic force of the beams 1 and 2, deviating towards the plates 14, 15, respectively.

The total stiffness of the cruciform element along the $X$ axis takes the form:

$$k_x = \frac{E(I_{31} \cdot l_1^3 + I_{32} \cdot l_2^3 + 4I_{37} \cdot l_7^3 + 4I_{38} \cdot l_8^3)}{l_1^3 \cdot l_2^3},$$

where $I_{31}, I_{32}, I_{37}, I_{38}$ – the moment of inertia of the cross-section relative to the neutral $Y$-axis of the beams 1, 2, 7, 8, respectively; $E$ – Young's modulus; $l_1, l_2$ – the length of the elastic beams 1, 7.

Based on the equation for the stiffnesses of elastic beams experiencing S-shaped bending, taking into account the identity of beams 3, 9 and their parallel connection, as well as taking into account the identity of the right and left sides of the element in question, the expression of stiffness will take the form:

$$k_z = 16 \frac{E \cdot I_{y3}^3}{l_3^3},$$

The displacement of stiffening plates 14, 15 is determined by the differential equation for clamped rods:

$$\delta_z = B \cos kx + C \sin kx,$$

where $k = (F / E^* I)^{1/2}; B$ and $C$ are arbitrary constants determined from the boundary conditions: by $z = 0 \ \delta = 0$; by $z = l \ \delta = 0$.

The element deformation along the $Z$ axis ($\delta_z$) for beam 4 depends on the bending radius of this beam under the elastic force of beam 1.

The bending radius of the beam has the form:

$$R = \frac{EI}{M(x)},$$

where $M(x)$ – bending moment for a given beam section relative to the $Y$ axis. $I$ is expressed [3]:

$$I = \frac{wh^3}{12},$$

where $w$ – the width of the cross-section of the beam; $h$ – the height of the cross-section of the beam.

Based on the equations above, the displacement of plate 14 in $Z$ will increase in proportion to the ratio of the thickness of beams 4 and 3, which is confirmed by modeling the structure in the ANSYS program in Figure 3.

Figure 4 shows a graph of the dependence of the element deformation along the $Z$ axis ($\delta_z$) on the ratio of the thicknesses of the internal beams to the external ones ($w4 / w3$).

The graph shows that when the thickness of beams 1 and 4 is 5 times greater than beams 3 and 7, $\delta_z$ for plate 14 approaches the maximum value, since the bending radius of beam 4 is limited by its length. To achieve a larger $\delta_z$ for plate 14 within the framework of this design, it is necessary to increase the length of the elastic beams.
Figure 3. Modeling a stiffener with a ratio of 1/1 and 5/1 beam thicknesses.

Figure 4. Dependence of $\delta z$ on the $w_4/w_3$ ratio.

4. Modelling a MEMS gyro-accelerometer
   For a more accurate understanding of how the width of the beams of the cross-shaped suspension element affects the movement of proof masses, a parameterizable model of the MEMS gyroscope-accelerometer was created in the ANSYS software environment. Figure 5 shows the model of the MEMS gyro-accelerometer. A force was applied to the platforms that connect the electrostatic drives to the cruciform suspension element, which displaces them towards each other by 20 µm. Figure 6 shows the simulation results. It can be seen from the simulation results that at a given force there is no displacement of proof masses due to insufficient rigidity of the beams of the cruciform suspension element. In this model of the gyro-accelerometer, the width of the beams of the cruciform element was doubled and the simulation was repeated. Figure 7 shows results of the simulation. It can be seen from the simulation results that under the same load conditions, but with an increase in the thickness of the beams of the cruciform suspension element, the inertial masses of the MEMS gyroscope-accelerometer shift. The displacement was 4.5 µm.
5. Conclusion
This article presents a design of a linear acceleration and angular velocity sensor with three axes of sensitivity. The coefficient of stiffness of the cruciform element of the suspension of the linear acceleration and angular velocity sensor is obtained. Using the ANSYS software, parametric 3D models of the cruciform element and MEMS gyroscope-accelerometer are developed. The results of static modelling showed that an increase in the width of elastic beams from 7.5 µm to 15 µm in a cruciform suspension element leads to a significant change in the displacement of inertial masses in the developed
model of a MEMS gyroscope-accelerometer. Based on the results obtained, it can be seen that to obtain large displacement values from the effect of electrostatic drives on inertial masses, it is necessary to improve the MMGA model by increasing the rigidity of the cruciform element. This is confirmed by simulations in the ANSYS program.

![Simulation of a micromechanical gyroscope – accelerometer with an increased width of elastic beams.](image)

**Figure 7.** Simulation of a micromechanical gyroscope – accelerometer with an increased width of elastic beams.

The legends in Figures 6, 7 show the movement (m) of the suspension under load.

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