One diagonal texture or cofactor zero of the neutrino mass matrix

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Abstract

In view of the recent measurement of nonzero $\theta_{13}$, we carry out a systematic study of a simple class of neutrino models that has one diagonal texture or cofactor zero in the mass matrix. There are seven free parameters in the model and five of them are already measured by neutrino oscillation experiments; some cases for the normal or inverted hierarchy are excluded and for the rest we obtain the preferred values for the lightest neutrino mass and Dirac CP phase. We find that there are strong similarities between one diagonal texture zero models with one mass hierarchy and one diagonal cofactor zero models with the opposite mass hierarchy. We also make predictions for neutrinoless double beta decay for these models. For the one cofactor zero models, we present a simple realization based on a new $U(1)$ gauge symmetry.
1 Introduction

After the recent observation of nonzero $\theta_{13}$ by the Daya Bay [1], RENO [2], and Double Chooz [3] experiments, five parameters in the neutrino sector have been measured by neutrino oscillation experiments. In general, there are nine parameters in the light neutrinos mass matrix. The remaining four unknown parameters may be taken as the lightest mass, the Dirac CP phase and two Majorana phases. The Dirac phase will be measured in future long baseline neutrino experiments, and the lightest mass can be determined from beta decay and cosmological experiments. If neutrinoless double beta decay ($0\nu\beta\beta$) is detected, a combination of the two Majorana phases can also be probed. If there is some structure in the neutrino mass matrix, the four unknown parameters will be related to each other. In this paper, we study the phenomenological consequence of imposing one texture zero or one cofactor zero in the light neutrino mass matrix; for previous work see Refs. [4, 5, 6, 7]. Since one texture or cofactor zero sets two conditions on the parameter space, only seven free parameters in the light neutrino matrix remain. Here we derive analytic formulas that relate the seven free parameters and determine the constraints on these models. By using recent data measured by neutrino oscillation experiments, we exclude some cases for the normal or inverted mass hierarchy, and for the rest we can obtain the allowed regions for the lightest mass and Dirac CP phase, which can be probed in the next generation of neutrino experiments.

In Sec. 2, we discuss the general properties of texture or cofactor zeros in the light neutrino mass matrix. In Sec. 3, we use current experimental data to study the allowed parameter regions for one diagonal texture zero in the mass matrix. In Sec. 4, we study the allowed parameter regions for one diagonal cofactor zero in the mass matrix. In Sec. 5 we discuss the similarities between one texture zero models with one mass hierarchy and one cofactor zero models with the opposite mass hierarchy. We present a simple realization based on a new $U(1)$ gauge symmetry for the one cofactor zero models in Sec. 6 and summarize our results in Sec. 7.
2 General properties of texture or cofactor zeros of the neutrino mass matrix

The light neutrino mass matrix can be written as

\[ M = V^* \text{diag}(m_1, m_2, m_3) V^\dagger, \]

where \( V = U \text{diag}(1, e^{i\phi_2/2}, e^{i\phi_3/2}) \), and

\[
U = \begin{pmatrix}
    c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}.
\]

The results of a recent global three-neutrino fit [8] are shown in Table 1. The masses of light neutrinos can be obtained from the canonical seesaw mechanism [9], in which the mass matrix of the light neutrinos can be written as

\[ M = Y^\dagger_\nu M_{R}^{-1} Y_\nu v^2, \]

where \( v \approx 174 \text{ GeV} \) is the Higgs vacuum expectation value (VEV), \( Y_\nu \) is the \( 3 \times 3 \) Yukawa coupling matrix and \( M_R \) is the \( 3 \times 3 \) heavy right-handed neutrino mass matrix. Here we assume all three light neutrinos are massive, so that the mass matrix of the light neutrinos is invertible (and therefore \( Y_\nu \) must be invertible), and we can write Eq. (3) as

\[ M_R = Y_\nu M_{R}^{-1} Y_\nu v^2. \]

Since \( (M^{-1})_{\alpha\beta} = \frac{1}{\det M} C_{\beta\alpha} \), where \( C_{\beta\alpha} \) is the \((\beta, \alpha)\) cofactor of \( M \), and both the light and heavy neutrino mass matrices are symmetric, any cofactor zeros in the mass matrix are equivalent to texture zeros in the inverse of the mass matrix. Consequently, Eq. (4) implies that if the Yukawa coupling matrix is diagonal, then a cofactor zero in \( M \) implies a texture zero in \( M_R \) [10]. Similarly, a texture zero in \( M \) implies a cofactor zero in \( M_R \) when the Yukawa coupling matrix is diagonal.

An interesting feature of the structure of a texture or cofactor zero is that it is stable against radiative corrections. The one-loop renormalization group equation (RGE) describing the evolution of the light neutrino masses from the lightest right-handed neutrino mass
Table 1: Best-fit values and 2σ ranges of the oscillation parameters \[8\], with \(\delta m^2 \equiv m_2^2 - m_1^2\) and \(\Delta m^2 \equiv m_3^2 - (m_1^2 + m_2^2)/2\).

| Hierarchy | \(\theta_{12}(^\circ)\) | \(\theta_{13}(^\circ)\) | \(\theta_{23}(^\circ)\) | \(\delta m^2(10^{-5}\text{eV}^2)\) | \(|\Delta m^2|(10^{-3}\text{eV}^2)\) |
|-----------|-------------------|-------------------|-------------------|-------------------|-------------------|
| Normal    | 33.6\(\pm\)2.1   | 8.9\(\pm\)0.9    | 38.4\(\pm\)3.6   | 7.54\(\pm\)0.46  | 2.43\(\pm\)0.12  |
| Inverted  | 33.6\(\pm\)2.1   | 9.0\(\pm\)0.8    | 38.8\(\pm\)5.3\(\oplus\) 47.5 – 53.2 | 7.54\(\pm\)0.46  | 2.42\(\pm\)0.11  |

scale \(M_1\) to the electroweak scale \(M_Z\) is \[11\]

\[
16\pi^2 \frac{dM}{dt} = \alpha M + C[(Y_l Y_l^\dagger) M + M(Y_l Y_l^\dagger)^T],
\]

where \(t = \ln(\mu/M_1)\), \(\mu\) is the renormalization scale and \(Y_l = \text{diag}(y_e, y_\mu, y_\tau)\) is the charged lepton Yukawa coupling matrix. In the Standard Model (SM), \(C = -\frac{3}{2}\) and \(\alpha \approx -3g_2^2 + 6y_t^2 + \lambda\), and in the minimal supersymmetric standard model, \(C = 1\) and \(\alpha \approx -\frac{6}{5}g_1^2 - 6g_2^2 + 6y_t^2\), where \(g_1, g_2\) are the gauge couplings, \(y_t\) is the top quark Yukawa coupling, and \(\lambda\) is the Higgs self-coupling. The solution to Eq. (5) can be written as \[12\]

\[
M(M_Z) = I_\alpha \begin{bmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{bmatrix} M(M_1) \begin{bmatrix} I_e & 0 & 0 \\ 0 & I_\mu & 0 \\ 0 & 0 & I_\tau \end{bmatrix},
\]

where

\[
I_\alpha = \exp \left[ -\frac{1}{16\pi^2} \int_0^{\ln(M_1/M_Z)} \alpha(t) dt \right],
\]

and

\[
I_l = \exp \left[ -\frac{C}{16\pi^2} \int_0^{\ln(M_1/M_Z)} y_l^2(t) dt \right],
\]

for \(l = e, \mu, \tau\). Since multiplying diagonal matrices does not affect a texture or cofactor zero, from Eq. (6), we see that texture or cofactor zero models are stable against the RGE running from \(M_1\) to \(M_Z\).

3 One texture zero in the neutrino mass matrix

We discuss the properties of one texture zero in the diagonal entries of the mass matrix; the results for the off-diagonal cases can be found in Ref. [6], which were obtained in models
with four texture zeros in the Yukawa coupling matrix.

Our analysis proceeds as follows. For one texture zero cases, there are 7 independent parameters in the light neutrino mass matrix, which we take to be $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\delta m^2$, $\Delta m^2$, the Dirac CP phase $\delta$, and either $m_1$ (for the normal hierarchy, NH, $m_1 < m_2 < m_3$) or $m_3$ (for the inverted hierarchy, IH, $m_3 < m_1 < m_2$). For each case we find the allowed regions in the $m_1 - (m_3 - \delta)$ plane given the best-fit values of $\theta_{12}$, $\theta_{23}$, $\theta_{13}$, $\delta m^2$ and $\Delta m^2$, and also the 2σ allowed regions using the experimental uncertainties in the measured parameters. We also find iso-$|M_{ee}|$ contours relevant for neutrinoless double beta decay for the best-fit values.

3.1 \(M_{ee} = 0\)

The condition \(M_{ee} = 0\) can be written as

\[ m_1 = -\frac{m_3 e^{i\phi_3} U^2_{e3} + m_2 e^{i\phi_2} U^2_{e2}}{U^2_{e1}}, \tag{9} \]

and is the same for either mass hierarchy. Taking the absolute square gives

\[ m_1^2|U_{e1}|^4 - m_2^2|U_{e2}|^4 - m_3^2|U_{e3}|^4 = 2\text{Re}(m_3 e^{-i\phi_3} U^2_{e3} m_2 e^{i\phi_2} U^2_{e2}), \tag{10} \]

or, defining $\phi = \phi_3 - \phi_2$,

\[ m_1^2|U_{e1}|^4 - m_2^2|U_{e2}|^4 - m_3^2|U_{e3}|^4 = 2m_2 m_3 e^{i\phi} s^2_{13} s^2_{12} \cos(-\phi + 2\delta). \tag{11} \]

Expanding the cosines yields the form

\[ C = A \cos \phi + B \sin \phi, \tag{12} \]

with $A$, $B$ and $C$ as listed in Table 2. Hence the only condition that must be satisfied when \(M_{ee} = 0\) is $C^2 \leq A^2 + B^2$. Since $C^2$ and $A^2 + B^2$ do not depend on $\delta$, it will only yield a constraint on $m_1$ for the normal hierarchy or $m_3$ for the inverted hierarchy. It can be easily seen that $C^2 \leq A^2 + B^2$ cannot be satisfied for the inverted hierarchy, which means that \(M_{ee} = 0\) is not possible for the inverted hierarchy. For the normal hierarchy and best-fit oscillation parameters, the allowed range for $m_1$ is $0.0022 \text{ eV} \leq m_1 \leq 0.0066 \text{ eV}$, while the allowed range at 2σ is $0.0014 \text{ eV} \leq m_1 \leq 0.0085 \text{ eV}$. 

5
\[ C = A \cos \phi + B \sin \phi \]

| Class   | A                                      | B                                      | C                                      |
|---------|----------------------------------------|----------------------------------------|----------------------------------------|
| \( M_{ee} = 0 \) | \( 2m_2m_3c_{23}^2s_{13}^2s_{12}^2 \cos(2\delta) \) | \( 2m_2m_3c_{23}^2s_{13}^2s_{12}^2 \sin(2\delta) \) | \( m_1^2|U_{e1}|^4 - m_2^2|U_{e2}|^4 - m_3^2|U_{e3}|^4 \) |
| \( M_{\mu\mu} = 0 \) | \( 2m_2m_3s_{23}^2c_{13}^2 \times \) \[ c_{12}^2c_{23} + s_{12}^2s_{23}c_{13}^2 \cos(2\delta) \] | \( 2m_2m_3s_{23}^2c_{13}^2 \times \) \[ s_{12}^2s_{23}^2s_{13}^2 \sin(2\delta) \] | \( m_1^2|U_{\mu1}|^4 - m_2^2|U_{\mu2}|^4 - m_3^2|U_{\mu3}|^4 \) |
| \( M_{\tau\tau} = 0 \) | \( 2m_2m_3s_{23}^2c_{13}^2 \times \) \[ c_{12}^2s_{23}^2 + s_{12}^2c_{23}^2c_{13}^2 \cos(2\delta) \] | \( 2m_2m_3s_{23}^2c_{13}^2 \times \) \[ s_{12}^2s_{23}^2s_{13}^2 \sin(2\delta) \] | \( m_1^2|U_{\tau1}|^4 - m_2^2|U_{\tau2}|^4 - m_3^2|U_{\tau3}|^4 \) |
| \( C_{ee} = 0 \) | \( 2m_2^{-1}m_3^{-1}c_{13}^2s_{13}^2s_{12}^2 \cos(2\delta) \) | \( 2m_2^{-1}m_3^{-1}c_{13}^2s_{13}^2s_{12}^2 \sin(2\delta) \) | \( m_1^{-2}|U_{e1}|^4 - m_2^{-2}|U_{e2}|^4 - m_3^{-2}|U_{e3}|^4 \) |
| \( C_{\mu\mu} = 0 \) | \( 2m_2^{-1}m_3^{-1}s_{13}^2s_{23}^2c_{13}^2 \times \) \[ c_{12}^2c_{23}^2 + s_{12}^2s_{23}^2c_{13}^2 \cos(2\delta) \] | \( 2m_2^{-1}m_3^{-1}s_{13}^2s_{23}^2c_{13}^2 \times \) \[ s_{12}^2s_{23}^2s_{13}^2 \sin(2\delta) \] | \( m_1^{-2}|U_{\mu1}|^4 - m_2^{-2}|U_{\mu2}|^4 - m_3^{-2}|U_{\mu3}|^4 \) |
| \( C_{\tau\tau} = 0 \) | \( 2m_2^{-1}m_3^{-1}s_{13}^2s_{23}^2c_{13}^2 \times \) \[ c_{12}^2s_{23}^2 + s_{12}^2c_{23}^2s_{13}^2 \cos(2\delta) \] | \( 2m_2^{-1}m_3^{-1}s_{13}^2s_{23}^2c_{13}^2 \times \) \[ s_{12}^2s_{23}^2s_{13}^2 \sin(2\delta) \] | \( m_1^{-2}|U_{\tau1}|^4 - m_2^{-2}|U_{\tau2}|^4 - m_3^{-2}|U_{\tau3}|^4 \) |

Table 2: The coefficients A, B and C for each class.
3.2 $M_{\mu\mu} = 0$

From $M_{\mu\mu} = 0$, we get

$$m_1 = - \frac{m_3 e^{i\phi_3} U_{\mu_3}^2 + m_2 e^{i\phi_2} U_{\mu_2}^2}{U_{\mu_1}^2},$$

(13)

which is independent of hierarchy. As before this may be put in the form of Eq. (12), with $A$, $B$ and $C$ given in Table 2. From Eq. (12), we can find the solution

$$\phi = 2 \arctan \left( \frac{B \pm \sqrt{A^2 + B^2 - C^2}}{A + C} \right),$$

(14)

and we can write Eq. (13) as

$$m_1 = e^{i\phi_2} - \frac{m_3 e^{i\phi} U_{\mu_3}^2 - m_2 U_{\mu_2}^2}{U_{\mu_1}^2}.$$  

(15)

Since $m_1$ is a non-negative real number in the standard parametrization, we get

$$\phi_2 = - \arg \left[ - \frac{m_3 e^{i\phi} U_{\mu_3}^2 - m_2 U_{\mu_2}^2}{U_{\mu_1}^2} \right],$$

(16)

and

$$\phi_3 = \phi_2 + \phi.$$  

(17)

It is then possible to calculate the magnitude of the $\nu_e - \nu_e$ element of the neutrino mass matrix

$$|M_{ee}| = |m_1 c_{12}^2 c_{13}^2 + m_2 e^{-i\phi_2} s_{12}^2 c_{13}^2 + m_3 e^{-i\phi_3} s_{13}^2 e^{2i\delta}|,$$

(18)

which determines the rate for neutrinoless double-beta decay, a signal of lepton number violation. The allowed regions of the Dirac CP phase $\delta$ and the lightest mass $m_1$ ($m_3$) are defined by the condition $C^2 \leq A^2 + B^2$. We scan over $\delta$ and $m_1$ ($m_3$) to find the allowed regions; see Fig. 1 for the normal hierarchy and Fig. 2 for the inverted hierarchy, where regions corresponding to the best-fit parameters and those allowed at $2\sigma$ are shown. The lightest mass for the normal hierarchy is always larger than 0.027 eV at $2\sigma$, while for the inverted hierarchy, it is strongly dependent on $\delta$. We also plot iso-$|M_{ee}|$ contours using the best-fit oscillation parameters. Here only the contours for the plus sign of $\phi$ in Eq. (14) are shown because changing $\delta$ to $360^\circ - \delta$ yields the same contours for the minus solution.
3.3 $M_{\tau\tau} = 0$

From $M_{\tau\tau} = 0$, we get

$$m_1 = -\frac{m_3 e^{i\phi_3} U_{\tau 3}^2 + m_2 e^{i\phi_2} U_{\tau 2}^2}{U_{\tau 1}^2},$$

(19)

which is independent of hierarchy. This may be put in the form of Eq. (12), with A, B and C as in Table 2. We find that the normal hierarchy is excluded at $2\sigma$. The allowed regions for the inverted hierarchy are shown in Fig. 3 along with iso-$|M_{ee}|$ contours. Note that for the best-fit oscillation parameters, the lightest mass $m_3$ has an upper bound of $0.047$ eV, but there is no upper bound at $2\sigma$.

4 One cofactor zero of the neutrino mass matrix

We now discuss the properties of one cofactor zero in the diagonal entries of the mass matrix; the results for the off-diagonal cases can be found in Ref. [6], which were obtained in models with four texture zeros in the Yukawa coupling matrix. Our analysis follows the same procedure as for the texture zeros in the previous section.

4.1 $C_{ee} = 0$

If $C_{ee} = 0$, then $(M^{-1})_{ee} = 0$. Since $M^{-1} = V \text{diag}(m_1^{-1}, m_2^{-1}, m_3^{-1})V^T$, we can write the condition as

$$m_1^{-1} = -\frac{m_3^{-1} e^{i\phi_3} U_{e 3}^2 + m_2^{-1} e^{i\phi_2} U_{e 2}^2}{U_{e 1}^2},$$

(20)

which is the same for either mass hierarchy. Taking the absolute square, we write this in the form of Eq. (12), with A, B and C as in Table 2. Since $C^2$ and $A^2 + B^2$ do not depend on $\delta$, it will only yield a constraint on $m_1$ ($m_3$) for the normal (inverted) hierarchy. We find that the normal hierarchy is excluded at $2\sigma$. For the inverted hierarchy and best-fit oscillation parameters, the allowed range for $m_3$ is $0.0013 \text{ eV} \leq m_3 \leq 0.0031 \text{ eV}$, while the allowed range at $2\sigma$ is $0.0010 \text{ eV} \leq m_3 \leq 0.0042 \text{ eV}$. 
4.2 $C_{\mu\mu} = 0$

From $C_{\mu\mu} = 0$, which is equivalent to $(M^{-1})_{\mu\mu} = 0$, we get

$$m_1^{-1} = -\frac{m_3^{-1}e^{i\phi_3}U_{\mu 3}^2 + m_2^{-1}e^{i\phi_2}U_{\mu 2}^2}{U_{\mu 1}^2},$$

which is the same for either mass hierarchy, and may be put in the form of Eq. (12), with A, B and C as in Table 2. The allowed regions for the normal hierarchy are shown in Fig. 4 and the allowed regions for the inverted hierarchy are shown in Fig. 5.

4.3 $C_{\tau\tau} = 0$

From $C_{\tau\tau} = 0$, which is equivalent to $(M^{-1})_{\tau\tau} = 0$, we get

$$m_1^{-1} = -\frac{m_3^{-1}e^{i\phi_3}U_{\tau 3}^2 + m_2^{-1}e^{i\phi_2}U_{\tau 2}^2}{U_{\tau 1}^2},$$

This condition is the same for either mass hierarchy, and may be put in the form of Eq. (12), with A, B and C as in Table 2. We find that for the inverted hierarchy, this case is not allowed for the best-fit parameters, but is allowed at 2$\sigma$, with a lower bound on $m_3$ of 0.033 eV. The allowed regions for the normal hierarchy are shown in Fig. 6 along with iso-$|M_{ee}|$ contours. We see that the lightest mass $m_1$ has an upper bound of 0.044 eV for the best-fit oscillation parameters, and 0.071 eV at 2$\sigma$.

5 Similarity of texture-zero and cofactor-zero models

The allowed regions for Class $C_{\mu\mu} = 0$ IH (Fig. 5) are similar to those for Class $M_{\mu\mu} = 0$ NH (Fig. 1). The similarity of a cofactor-zero IH scenario with a texture-zero NH scenario can be understood by looking at the form of the A, B, and C coefficients in Table 2. If we multiply the coefficients for Class $C_{\mu\mu} = 0$ IH by $m_2m_3$, and divide the coefficients for Class $M_{\mu\mu} = 0$ NH by $m_2m_3$, we see that A and B become the same for the two cases. For the C coefficient, the dominant term in each case is the third one, proportional to $|U_{\mu 3}|^4$ times the ratio of a larger mass to a smaller one. Therefore the allowed regions for these two cases are similar. Class $M_{\tau\tau} = 0$ NH and Class $C_{\tau\tau} = 0$ IH have a similar correspondence in the the A, B, and C coefficients, and they are both not allowed for the best-fit parameters.
Class $M_{ee} = 0$ NH and Class $C_{ee} = 0$ IH also have similar constraints from the the $A$, $B$, and $C$ coefficients, but there is no restriction on $\delta$ in those models. Also, the dominant term in the $C$ coefficient is suppressed by a factor $|U_{e3}|^4$, so the other terms become important, and the allowed ranges of the lightest mass are significantly different in the two models.

One can also see a similarity between cofactor-zero models with NH and texture-zero models with IH, although the correspondence occurs only for larger values of the lightest mass. For example, for Class $C_{\mu\mu} = 0$ NH and Class $M_{\mu\mu} = 0$ IH, after multiplying the $A$, $B$, and $C$ coefficients for the NH by $m_2 m_3$ and dividing the coefficients for the IH by $m_2 m_3$, the $A$ and $B$ coefficients are the same. When the lightest mass is not too small (such that $m_1 \approx m_2$ for NH), the same terms in the $C$ coefficient are dominant and proportional to a large mass divided by a small mass. Thus for higher values of the lightest mass, the allowed regions of Classes $C_{\mu\mu} = 0$ NH and $M_{\mu\mu} = 0$ IH must be similar. This can be seen by comparing Figs. 4 and 2. However, for small values of the lightest mass, the first two terms in the $C$ coefficient have similar size for Class $M_{\mu\mu} = 0$ IH, but only the first term is dominant for Class $C_{\mu\mu} = 0$ NH. Thus the allowed regions are quite different when the lightest mass is below 20 meV.

The allowed regions for Class $M_{\tau\tau} = 0$ IH and Class $C_{\tau\tau} = 0$ NH also have a similar correspondence in the the $A$, $B$, and $C$ coefficients, and they have similar allowed regions for higher values of the lightest mass; see Figs. 6 and 3. Likewise Class $M_{ee} = 0$ IH and Class $C_{ee} = 0$ NH have similar $A$, $B$, and $C$ coefficients, and they are both not allowed at 2$\sigma$.

This similarity between texture-zero models with one mass hierarchy and cofactor-zero models with the other mass hierarchy has been noted before in models with a single off-diagonal texture or cofactor zero [6]. Thus it is a generic property for any texture or cofactor zero in the neutrino mass matrix.

### 6 Symmetry realization

All the texture and cofactor zero cases can be realized from discrete $Z_N$ symmetries but it requires many scalar singlets [13]. Here we present a simple realization of the one cofactor zero models using a new $U(1)$ gauge symmetry that only requires two scalar singlets. We
denote the charge of the new $U(1)$ gauge symmetry as $Y'$, and make the following charge assignments:

\[ Y'(q_L) = -Y'(u_R^c) = -Y'(d_R^c) \]

for all families in the quark sector to avoid flavor changing neutral currents; $Y'(l_{Li}) = -Y'(e_{Ri}^c) = -Y'(N_{Ri}^c)$ and $Y'(l_{Li}) \neq Y'(l_{Lj}) \ (i \neq j)$ for each family in the lepton sector; and $Y'(\phi) = 0$ for the SM Higgs. The anomaly-free requirement yields the condition [7]

\[ 9Y'(q_L) + Y'(l_{L1}) + Y'(l_{L2}) + Y'(l_{L3}) = 0. \] (23)

If we consider the case with $Y'(q_L) \neq 0$, then the condition leads to a $B - \sum \alpha x_{\alpha} L_\alpha$ gauge symmetry with the constraint $\sum \alpha x_{\alpha} = 3$, where $B$ and $L$ are the baryon and lepton flavor numbers, respectively. One of the advantages of this model is that both the charged lepton and Dirac neutrino mass matrices are diagonal spontaneously because of the charge assignments of the $U(1)$ gauge symmetry. Hence a cofactor zero in $M$ is equivalent to a cofactor zero in $M_R^{-1}$, which is equivalent to a texture zero in $M_R$. This can be achieved with a suitable $B - \sum \alpha x_{\alpha} L_\alpha$ gauge symmetry and two SM gauge singlet scalars $S_1$ and $S_2$ with appropriate charges. Taking the $C_{ee} = 0$ case for example, if we impose a $B - 3L_e - L_\mu + L_\tau$ symmetry on the model, then the $U(1)$ charge matrix for the right-handed neutrino mass term $Y'(\bar{N}_i^c N_j)$ is:

\[
Y' = \begin{pmatrix}
-6 & -4 & -2 \\
\cdot & -2 & 0 \\
\cdot & \cdot & 2
\end{pmatrix}.
\] (24)

Without any additional singlet scalars, the mass matrix of right-handed neutrinos will only have one non-vanishing entry with the scale $M_{B-3L_e-L_\mu+L_\tau}$. By adding two additional singlet scalars $S_1$ and $S_2$ with $|Y'(S_1)| = 2$ and $|Y'(S_2)| = 4$ respectively, we can make all entries except the $(1,1)$ entry nonzero after $S_1$ and $S_2$ acquire VEVs:

\[
M_R = M_{B-3L_e-L_\mu+L_\tau} \begin{pmatrix}
0 & \cdot & \cdot \\
\cdot & 0 & \cdot \\
\cdot & \cdot & 0
\end{pmatrix} + \langle S_1 \rangle \begin{pmatrix}
0 & \cdot & \cdot \\
\cdot & 0 & \cdot \\
\cdot & \cdot & 0
\end{pmatrix} + \langle S_2 \rangle \begin{pmatrix}
0 & \cdot & \cdot \\
\cdot & 0 & \cdot \\
\cdot & \cdot & 0
\end{pmatrix}
\sim \begin{pmatrix}
0 & \times & \times \\
\cdot & \times & \times \\
\cdot & \cdot & \times
\end{pmatrix},
\] (25)
Table 3: The anomaly-free $U(1)$ gauge symmetry realization for 6 model classes with one cofactor zero in the light neutrino mass matrix. $Y'$ denotes the charge of the $U(1)$ gauge symmetry, and $S_1, S_2$ are two SM singlet scalars with non-vanishing VEVs.

| Class | Symmetry generator | $|Y'(S_1)|$ | $|Y'(S_2)|$ |
|-------|------------------|----------|----------|
| $C_{ee}=0$ | $B - 3L_e - L_\mu + L_\tau$ | 2 | 4 |
| $C_{\mu\mu}=0$ | $B + L_e - 3L_\mu - L_\tau$ | 2 | 4 |
| $C_{\tau\tau}=0$ | $B - L_e + L_\mu - 3L_\tau$ | 2 | 4 |
| $C'_{e\mu}=0$ | $B - 3L_e - L_\mu + L_\tau$ | 2 | 6 |
| $C'_{\mu\tau}=0$ | $B + L_e - 3L_\mu - L_\tau$ | 2 | 6 |
| $C'_{e\tau}=0$ | $B - L_e + L_\mu - 3L_\tau$ | 2 | 6 |

where $\times$ denotes a non-vanishing entry. The other cases can be also realized similarly; a complete list is shown in Table 3.

7 Conclusions

We studied the phenomenology of one diagonal texture or cofactor zero in the low energy neutrino mass matrix. The cofactor-zero condition is equivalent to a texture zero in $M^{-1}$ for three massive neutrinos. In the case that the Yukawa coupling matrix is diagonal, a texture zero in $M$ is equivalent to a cofactor zero in $M_R$, and a cofactor zero in $M$ is equivalent to a texture zero in $M_R$. We imposed one diagonal texture or cofactor zero on the neutrino mass matrix and used the latest experimental data to obtain the allowed regions for the lightest neutrino mass and Dirac CP phase $\delta$. The texture zero cases $M_{\tau\tau} = 0$ NH and $M_{ee} = 0$ IH, and the cofactor zero case $C_{ee} = 0$ NH are not allowed at $2\sigma$, and the case $C_{\tau\tau} = 0$ IH is not allowed for the best-fit parameters.

Once the lightest neutrino mass and Dirac CP phase were determined, we made definite predictions for neutrinoless double beta decay for one texture or cofactor zero models. The effective mass $|M_{ee}|$ is generally proportional to the lightest mass (see the iso-$|M_{ee}|$ contours in Figs. 1-6), which is clearly evident for the quasi-degenerate spectrum. However, $|M_{ee}|$ is
Table 4: The minimum values of $|M_{ee}|$ (in $10^{-3} \text{eV}$) in each class for the best-fit oscillation parameters, and the $2\sigma$ lower bounds. The symbol $\times$ denotes that there is no allowed region for the model.

| Class          | Best-fit | 2\(\sigma\) lower bound |
|----------------|----------|--------------------------|
|                | NH | IH | NH | IH |
| $M_{ee}=0$     | 0.0 | $\times$ | 0.0 | $\times$ |
| $M_{\mu\mu}=0$ | 34.4 | 19.1 | 26.8 | 15.1 |
| $M_{\tau\tau}=0$ | $\times$ | 18.2 | $\times$ | 14.8 |
| $C_{ee}=0$     | $\times$ | 18.1 | $\times$ | 14.8 |
| $C_{\mu\mu}=0$ | 0.0 | 39.7 | 0.0 | 29.6 |
| $C_{\tau\tau}=0$ | 0.0 | $\times$ | 0.0 | 32.3 |

strongly dependent on the Dirac CP phase $\delta$ when the lightest mass is small, of order 20 meV or less.

The minimum value of $|M_{ee}|$ for the best-fit oscillation parameters and the $2\sigma$ lower bounds for the diagonal cases are shown in Table 4. Results for the off-diagonal cases can be found in Ref. [6]. For the diagonal cases that are not excluded, the minimum $|M_{ee}|$’s are all below 40 meV and can only be completely probed by significant improvements in the sensitivity of $0\nu\beta\beta$ experiments. For Classes $M_{ee} = 0$ NH, $C_{\mu\mu} = 0$ NH and $C_{\tau\tau} = 0$ NH, the current lower bound on $|M_{ee}|$ is zero.

However, as we have shown, for larger values of the lightest neutrino mass (especially in the quasi-degenerate region) the similarity of the allowed regions between a texture-zero NH and the corresponding cofactor zero-IH (and a texture-zero IH and the corresponding cofactor-zero NH) makes it difficult to distinguish them simply with measurements of the oscillation parameters and the neutrino mass scale. In order to resolve this ambiguity, future experiments that can determine the mass hierarchy are strongly needed, such as long baseline neutrino experiments (T2K [14], NO$\nu$A [15], and LBNE [16]), atmospheric neutrino experiments (PINGU [17] and INO [18]) and medium baseline reactor experiments (Daya Bay II [19]).
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Figure 1: The allowed regions in the $(m_1, \delta)$ plane for $M_{\mu\mu} = 0$ and the normal hierarchy. The dark shaded regions correspond to the best-fit oscillation parameters, while the light shaded regions are allowed at $2\sigma$. The solid lines are iso-$|M_{ee}|$ contours (in meV).
Figure 2: Same as Fig. 1 except for $M_{\mu\mu} = 0$ and the inverted hierarchy.

Figure 3: Same as Fig. 1 except for $M_{\tau\tau} = 0$ and the inverted hierarchy.
Figure 4: Same as Fig. 1 except for $C_{\mu\mu} = 0$ and the normal hierarchy.

Figure 5: Same as Fig. 1 except for $C_{\mu\mu} = 0$ and the inverted hierarchy.
Figure 6: Same as Fig. [1] except for $C_{\tau\tau} = 0$ and the normal hierarchy.