The extragalactic background light (EBL) is the cumulative radiation from the stars and active galactic nuclei (AGNs) through the universe’s history. It is tightly related to the starlight radiation from the stars and active galactic nuclei (AGNs) and generates the electron–positron pairs (Nikishov1962; Gould1966). This effect can cause an absorption feature in the observed gamma-ray spectrum due to the electron–positron pair production against the extragalactic background light (EBL). The attenuation of high-energy gamma-ray spectrum due to the electron–positron pair production against the extragalactic background light (EBL) provides an indirect method to measure the EBL of the universe. From the measurements of the absorption features of the gamma-rays from blazars as seen by the Fermi Gamma-ray Space Telescope to explore the EBL flux density and constrain the EBL spectrum, star formation rate density (SFRD), and photon escape fraction from galaxies out to $z = 6$. Our results are basically consistent with the existing determinations of the quantities. We find a larger photon escape fraction at high redshifts, especially at $z = 3$, compared to the result from recent Ly$\alpha$ measurements. Our SFRD result is consistent with the data from both gamma-ray burst and ultraviolet (UV) observations in the 1σ level. However, the average SFRD we obtain at $z \geq 3$ matches the gamma-ray data better than the UV data. Thus our SFRD result at $z \geq 6$ favors the fact that star formation alone is sufficiently high enough to reionize the universe.

Key words: galaxies: evolution – gamma rays: galaxies – stars: formation

Online-only material: color figures

1. INTRODUCTION

The extragalactic background light (EBL) is the cumulative radiation from the stars and active galactic nuclei (AGNs) through the universe’s history. It is tightly related to the starlight from direct emission and dust re-radiation which are dominant in the ultraviolet (UV) to near-infrared (near-IR) and mid-IR to submillimeter bands, respectively (Baldry & Glazebrook2003; Fukugita & Peebles2004; Franceschini et al.2008; Finke et al.2010; Stecker et al.2012; Inoue et al.2013). Hence the measurements of EBL at different redshifts are important and fundamental for the understanding of the star formation history and galaxy formation and evolution. However, direct EBL observations are limited by large uncertainties due to the foreground contamination that needs to be accounted for (Hauser & Dwek2001). This makes the measurements of the EBL very difficult especially at high redshifts.

The high-energy gamma-rays interact with the EBL photons and generate the electron–positron pairs (Nikishov1962; Gould & Schreder1966; Fazio & Stecker1970; Stecker et al.1992). This effect can cause an absorption feature in the observed gamma-ray spectrum above a critical energy relative to its intrinsic spectrum. The evolution of the EBL therefore can be derived by the observations of the attenuated spectrum of the high-energy gamma-rays at different redshifts, which provides an indirect but feasible measurement of the EBL evolution. The difficulty of this method is how to determine the intrinsic spectrum of the gamma-ray sources and distinguish the absorption feature caused by the EBL from the intrinsic variation in the gamma-ray spectrum.

Ackermann et al. (2012) reported the gamma-ray absorption feature in a sample of gamma-ray blazars in the redshift range $0.03 < z < 1.6$ using the Large Area Telescope (LAT) of the Fermi Gamma-ray Space Telescope.1 This sample contains 150 blazars of BL Lacertae (BL Lac) type in the 3–500 GeV band, providing the constraints on the EBL spectrum from the UV to the near-IR band. To determine the absorbed spectrum, they analyzed the sample in three independent redshift bins, $z < 0.2$, $0.2 \leq z < 0.5$, and $0.5 \leq z < 1.6$, and excluded the possibility that the attenuation is caused by the intrinsic properties of the gamma-ray sources.

We make use of this data set to constrain the EBL spectrum from the UV to the near-IR band, and to extract the information of the photon escape fraction and the star formation history. Our EBL model is based on the work of Finke et al. (2010), which is dependent on a model for the stellar evolution, initial mass function (IMF), photon escape fraction, and the star formation rate density (SFRD). The Markov Chain Monte Carlo (MCMC) method is adopted in our constraint process, and we derive the average values and standard deviations for the EBL, photon escape fraction, and SFRD from the obtained MCMC chains. We assume the flat $\Lambda$CDM with $\Omega_M = 0.27$, $\Omega_b = 0.046$, and $h = 0.71$ for the calculation throughout the Letter (Hinshaw et al.2012).

2. EBL MODEL

We calculate the EBL spectrum based on the model proposed by Finke et al. (2010), where the EBL spectrum is evaluated by the IMF $\xi(M_*)$, the comoving SFRD $\rho_*(z)$, and the photon escape fraction $f_{\text{esc}}(\lambda, z)$, which denotes the fraction of photons that can escape a galaxy without absorption by interstellar dust. It gives an analytic relation between the EBL spectrum and the IMF, SFRD, and $f_{\text{esc}}$, and hence provides a good way to constrain these quantities using the EBL observational data.

We adopt the IMF model from Chabrier (2003) where it is expressed in two parts,

$$\xi(M_*) = \begin{cases} A \exp\left[-\frac{\log_{10}M_* - \log_{10}M_*^0}{2\sigma}\right], & M_* \leq 1 M_\odot \\ B M_*^{-3}, & M_* > 1 M_\odot \end{cases},$$

where $M_*$ is the stellar mass, $A = 0.158$, $M_*^0 = 0.079 M_\odot$, $\sigma = 0.69$, $B = 4.43 \times 10^{-2}$, and $x = 1.3$. As this IMF model is consistent with the other models (e.g., Baldry &
Glazebrook 2003; Razzak et al. 2009), we fix the values of these parameters when performing our MCMC fits.

We make use of the SFRD model proposed by Cole et al. (2001), which takes the form

\[ \dot{\rho}_*(z) = \frac{a + bz}{1 + (z/c)^d}, \]

where \( \dot{\rho}_*(z) \) is in \( hM_\odot \, yr^{-1} \, Mpc^{-3} \), and \( a, b, c, \) and \( d \) are free parameters. At low redshifts \( z \lesssim 2 \), the SFRD can be constrained by the current observational data, and we take \( a_{z<2} = 0.0118, \ b_{<z}=0.08, \ c_{<z}=3.3, \) and \( d_{<z}=5.2 \) (Hopkins & Beacom 2006, hereafter HB06).\(^2\) At \( z > 2 \), the uncertainty of the SFRD becomes very large because of uncertain dust extinction at the high redshifts. Hence we treat these four parameters as free and constrain them in our MCMC fits at \( z > 2 \). Note that we do not consider the contribution from quiescent galaxies and AGNs in our model since they are not the main components of the EBL which can contribute about 10\%–13\% emission to the total intensity (Domínguez et al. 2011).

The photon escape fraction \( f_{esc} \) should be a function of both the photon energy and redshift, and it is still not well determined by current observations. For simplicity, we assume \( f_{esc} \) is linearly increasing with the wavelength \( \lambda \), which agree well with results from the observations of nearby galaxies (Driver et al. 2008). If the slope is independent of the redshift, then the photon escape fraction can be written as

\[ f_{esc}(\lambda, z) = m(1 + z)^n + p \log(\lambda/\mu m). \]  

Here \( m, n, \) and \( p \) are free parameters and are needed to fit in the MCMC process. When the photon energy is greater than 13.6 eV, we set \( f_{esc} = 0 \), since most of the photons in this energy range would be absorbed by the neutral hydrogen in the galaxies.

Next we can estimate the comoving luminosity density \( \epsilon j_\epsilon \) (in units of \( W \, Mpc^{-3} \)), where \( \epsilon = h\nu/m_e c^2 \) is the dimensionless photon energy. In our model, we consider both the emission from stars and the re-radiation from interstellar dust, i.e., we have \( \epsilon_j = \epsilon j_{star} + \epsilon j_{dust} \). The comoving luminosity density from starlight at redshift \( z \) is given by

\[ \epsilon j_{star}(\epsilon, z) = m_e c^2 \epsilon^2 \int_{M_{min}}^{M_{max}} \frac{dM_\odot}{dM_\odot} \xi(M_\odot) \int_{z_{min}}^{z_{max}} d z' \frac{d \tilde{t}}{d z} \left| f_{esc}(\epsilon, z') \dot{\rho}_*(z') N_\epsilon(\epsilon, M_\odot, t_s). \right. \]

\[ \times \int_{z_{min}}^{z_{max}} d z' \frac{d t}{d z'} f_{esc}(\epsilon, z') \dot{\rho}_*(z') N_\epsilon(\epsilon, M_\odot, t_s). \]

(3)

Here we take \( M_{min} = 0.1 M_\odot, M_{max} = 100 M_\odot, z_{max} = 6, \) and \( N_\epsilon(\epsilon, M_\odot, t_s) = \pi R_s^2 c n_\epsilon(T_s) \) is the number of photons emitted per unit time per unit energy from a star with radius \( R_s \) at lifetime \( t_s, n_\epsilon(T_s) \) is the stellar photon emission spectrum where \( T_s \) is the stellar temperature. To estimate these quantities, i.e., \( R_s, T_s, \) and \( n_\epsilon \), we assume the Planck spectrum for the starlight. This approximation is in good agreement with the results given by Bruzual & Charlot (2003) for stars with solar metallicity between 1 and 10 Gyr of age, which dominate the emission between 0.1 and 10 \( \mu m \) (Finke et al. 2010). We use a model of the Hertzsprung–Russell diagram to take into account different stellar evolution phases from the main-sequence to the horizontal branch, the asymptotic giant branch, and the white dwarf which take a stellar mass \( 0.1 M_\odot \leq M_\star \leq 100 M_\odot \) (see Eggleton et al. 1989 for details). We also assume all stars have solar metallicity, and that it is constant over the cosmic history and stellar mass (Finke et al. 2010).

The radiation of dust which dominates the mid- and far-IR bands is not important in our analysis here, since the gamma-ray sample we use is in the 3–500 GeV band. The process of photon–photon interaction between these gamma-rays and EBL photons would mainly occur in the near-IR or higher energy EBL bands where the EBL photons are emitted directly from stars. Here we take the same dust emission model with the three dust components used in Equation (11) of Finke et al. (2010). The proper EBL spectrum (or EBL intensity, in units of \( nW \, m^{-2} \, sr^{-1} \)) at energy \( \epsilon_p \) and redshift \( z \) can be derived using

\[ \epsilon_p I(\epsilon_p, z) = (1 + z)^4 \frac{c}{4\pi} \int_{z}^{z_{max}} d z' \frac{d t}{d z'} \frac{\epsilon' j_{\epsilon'}(\epsilon', z')}{1 + z'}. \]

(4)

where \( \epsilon_p = (1 + z)/(1 + z') \) \( \epsilon' \) is the proper dimensionless photon energy at \( z \). Also, it is easy to obtain the proper EBL density energy if we note \( \rho_b = (4\pi/c) \epsilon_p I, \) which is in units of \( erg \, cm^{-3} \).

After obtaining the EBL spectrum, we can further estimate the optical depth for gamma-ray absorption with observed energy \( E_\gamma \) emitted at redshift \( z_0 \)

\[ \tau_{\gamma\gamma}(E_\gamma, z_0) = \int_{0}^{z_0} dz' \frac{d l}{d z} \int_{-1}^{1} d u \frac{1 - u}{2} \times \int_{E_{min}}^{\infty} d E_b n_b(E_b, z') \sigma_{\gamma\gamma}(E_\gamma(1 + z'), E_b, u), \]

(5)

where \( d l/d z \) is the cosmological line element, \( E_b = \epsilon_p \times m_e c^2 \) is the proper photon energy of the EBL background at \( z, u = \cos(\theta) \) where \( \theta \) is the angle of incidence, and \( \sigma_{\gamma\gamma} \) is the cross-section of the pair production. \( n_b(E_b, z) = \rho_b/E_b \) is the proper number density of EBL photons at \( z \). The \( E_{min} \) is the minimum threshold energy of EBL photons that can interact with a gamma-ray of observed energy \( E_\gamma \)

\[ E_{min} = \frac{2m_e^2 c^4}{E_\gamma(1 + z)(1 - u)}, \]

(6)

where \( m_e \) is the mass of electron. Then the intrinsic gamma-ray spectrum is modified to be

\[ F_{\gamma}^{obs}(E_\gamma) = F_{\gamma}^{int}(E_\gamma) e^{-\tau_{\gamma\gamma}}. \]

(7)

Therefore, we can use the observations of attenuation of gamma-ray spectrum to compare with our theoretical model and constrain the free parameters that describe the SFRD and \( f_{esc} \) in the model.

3. MCMC FITTING

We employ the MCMC method to perform the constraints. There are several advantages to this method, and the most important one is that the time cost of the computations linearly increases with the number of the free parameters. Thus this method is suitable to fit a large set of parameters in an acceptable computation time. The Metropolis–Hastings algorithm is used

\(^2\) These values are obtained using the IMF from Baldry & Glazebrook (2003) which is well consistent with the IMF we use in Chabrier (2003).
in the MCMC process to decide the possibility of accepting a new chain point (Metropolis et al. 1953; Hastings 1970). We use a Gaussian sampler with adaptive step size to estimate the proposal density matrix (Doran & Muller 2004), and assume uniform prior probability distribution for the parameters.

The $\chi^2$ distribution is employed to calculate the likelihood function which is given by

$$
\chi^2 = \sum_{i=1}^{N} \frac{\left[ \exp(-\tau^{\text{obs}}_{\gamma\gamma}) - \exp(-\tau^{\text{th}}_{\gamma\gamma}) \right]^2}{\sigma_i^2},
$$

where $N$ is the number of the data, $\tau^{\text{th}}_{\gamma\gamma}$ is the theoretical optical depth given by Equation (5), and $\tau^{\text{obs}}_{\gamma\gamma}$ and $\sigma_i$ are the optical depth and errors from the observations.

We use the observational data in Ackermann et al. (2012), where they provide the measurements of the absorption feature derived from 150 blazars in the 3–500 GeV band of the Fermi-LAT survey (see the data points in the left panel of Figure 1). This sample covers the redshift range from 0.03 to 1.6, and gives the absorption feature in three redshift bins with central redshifts $z_c \simeq 0.1, 0.35$ and 1, respectively. We finally have 18 data points (6 in each redshift range), and we fit them all in three redshift bins with our model.

We have seven free parameters in our model: $a, b, c, a$ and $d$ in the SFRD given in Equation (1), and $m, n, p$ in the $f_{\text{esc}}$ shown in Equation (2). As we assume uniform prior for the parameters in the MCMC process, their ranges are set as $a \in (0, 0.1), b \in (0, 1), c \in (0, 6), d \in (0, 10), m \in (-4, 4), n \in (-2, 2), p \in (0, 3)$. These ranges are chosen according to the relevant models (Hopkins & Beacom 2006; Driver et al. 2008) and empirical experience. We perform some pre-runs to check these ranges and make sure that they have the correct physical meaning and there is no other maximum out of these ranges. Note that we fix $a, b, c, a$ and $d$ to be the values in the HB06 model when $z \leq 2$, since the SFRD is relatively well-determined in this redshift range by the current observations as we discuss in the last section.

We run eight parallel chains and obtain about $10^5$ points sampled in each chain after the convergence determined by the criterion of Gelman & Rubin (1992). After the burn-in process and thinning the chains, we merge them into one chain and finally collect about 10,000 points that are used to investigate the probability distributions of the parameters and statistical quantities of the components in the model. More details of our MCMC method can be found in Gong & Chen (2007).

4. RESULTS

In the left panel of Figure 1, we show the data of the attenuation of the gamma-ray spectrum by the EBL background in the three redshift bins, and the best fits of our model are denoted by red dashed lines. We fit 18 data points in the three redshift bins simultaneously and perform a global fitting for all seven free parameters in our model. The best fits and 1$\sigma$ errors (68.3% confidence level) of the parameters derived from the one-dimensional probability distribution function are $a = 0.055^{+0.041}_{-0.050}, b = 0.57^{+0.45}_{-0.33}, c = 3.9^{+2.0}_{-3.3}, d = 4.0^{+3.8}_{-5.0}, m = 0.32^{+0.08}_{-0.11}, n = 1.4^{+0.4}_{-0.5}$, and $p = 2.2^{+0.8}_{-1.4}$. The data are measured in three redshift bins $z < 0.2, 0.2 < z < 0.5$, and $0.5 < z < 1.6$. We take central redshifts of $z = 0.1, z = 0.35$, and $z = 1$ to perform our theoretical calculation. Note that our fitting results are only from the UV to near-IR bands of the EBL out to $1\mu m$ at $z = 0$.

The EBL spectrum at $z = 0$ derived from our MCMC chains are shown in the right panel of Figure 1. We calculate the EBL flux density for each chain point (which is a seven-dimensional point and contains the values of seven free parameters in our model) at different wavelengths, and estimate the average values (blue solid line) and standard deviations (1$\sigma$, blue region).
The EBL data evaluated from the galaxy counts (red triangles) and direct measurements (green circles) are also shown for comparison (Gilmore et al. 2012; Abramowski et al. 2012). The purple region gives the 1σ statistical contour derived from different energy bands of High Energy Stereoscopic System (H.E.S.S.; Abramowski et al. 2012). We find that the EBL spectrum from our MCMC chains are consistent with these observational results at $\lambda \gtrsim 0.4 \mu m$ (vertical dashed line). For $\lambda \lesssim 0.4 \mu m$, the energy of the gamma-ray which can interact with the EBL photons is less than 200 Gev (see Equation (6)), and the data points of optical depth $\tau_{\gamma\gamma}$ are close to 0 as shown in the left panel, which cannot give stringent constraints on the EBL at $z = 0$ in this regime.

In Figure 2, we show $f_{\text{esc}}(\lambda, z)$ at three redshifts $z = 0$, $z = 3$, and $z = 6$. The $f_{\text{esc}}$ are calculated by each MCMC chain point at different wavelengths and redshifts, and the average values and standard deviations (1σ) are shown by the blue solid line and shaded region, respectively. The vertical black dotted line denotes the hydrogen ionization energy at 13.6 eV ($\sim 912 \text{Å}$), and note that the $f_{\text{esc}}$ is set to be 0 when the photon energy is greater than 13.6 eV in our calculation since most of these photons would be absorbed by the neutral hydrogen gas in galaxies. The red circles with error bars are the Ly$\alpha$ escape fraction derived from the Ly$\alpha$ luminosity function around these three redshifts given by Hayes et al. (2011) and Blanc et al. (2011, and references therein). The green points and lines in the upper panel give the results and errors from nearby galaxies (Driver et al. 2008).

The purple region gives the 1σ statistical contour derived from different energy bands of High Energy Stereoscopic System (H.E.S.S.; Abramowski et al. 2012). We find that the EBL spectrum from our MCMC chains are consistent with these observational results at $\lambda \gtrsim 0.4 \mu m$ (vertical dashed line). For $\lambda \lesssim 0.4 \mu m$, the energy of the gamma-ray which can interact with the EBL photons is less than 200 Gev (see Equation (6)), and the data points of optical depth $\tau_{\gamma\gamma}$ are close to 0 as shown in the left panel, which cannot give stringent constraints on the EBL at $z = 0$ in this regime.

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In Figure 3, we show the SFRD at different redshifts. The blue dashed line and the shaded region are the average SFRD values and standard deviations (1σ) estimated by the MCMC chains, respectively. The black solid line denotes the model from Hopkins & Beacom (2006) with the IMF of Baldry & Glazebrook (2003). As addressed in Section 2, we fix the SFRD to be the HB06 model at $z \leq 2$ in our MCMC process, since it can be well constrained by the current observations. We find that the average SFRD (blue dashed line) from our MCMC chains is higher than the HB06 model at $z \gtrsim 3$ and the data from the UV observations (Bouwens et al. 2012) with flatter slope, but it agrees with the data from the gamma-ray burst (GRB) measurements shown by red triangles (Kistler et al. 2009) and green circles (Robertson & Ellis 2012; “low-Z SFR” model). Also, we should note that our result is consistent with both of the gamma-ray and UV data in the 1σ level given the large uncertainty. This implies that our SFRD result favors the fact that star formation alone is sufficient to reionize the universe (Madau et al. 1999).

5. DISCUSSION AND CONCLUSION

We explore the EBL spectrum at the near-IR to UV bands by fitting the gamma-ray attenuation data detected by Fermi-LAT measurements shown in Ackermann et al. (2012). Our EBL model is based on the earlier work of Finke et al. (2010). This model can fit well the gamma-ray attenuation data in all three redshift bins within $0.03 \leq z \leq 1.6$. After obtaining the MCMC chains, we derive the average values and standard deviations of the EBL spectrum $\nu I_{\nu}(\lambda, z = 0)$, $f_{\text{esc}}(\lambda, z)$, and $\rho_d(z)$, respectively from chain points. Also, we compare our results...
with the corresponding observational data, and find they are basically consistent with these observations in the regime of the gamma-ray attenuation data used in the constraints.

The $f_{\text{esc}}$ we obtain agrees with the data of Ly$\alpha$ escape fraction measurements at $z = 0$ and 6, but a bit larger at $z = 3$. Also, it is smaller than the result of Driver et al. (2008) around $\lambda = 0.5 \mu m$ at $z = 0$. For the star formation history, we obtain a higher average SFRD (blue dashed line in Figure 3) at $z \gtrsim 3$ with slope flatter than the result from the HB06 model and UV data. But note that our results in fact are still consistent with theirs in the 1$\sigma$ level given the large constraint uncertainty. On the other hand, our average SFRD matches the results given by GRB measurements very well, and this indicates that our SFRD has a trend that favors that star formation alone at high redshifts could reionize the universe.

Recently, Orr et al. (2011) and Yuan et al. (2012) also imposed constraints on the EBL spectrum using the gamma-ray observations from Fermi and ground-based air Cerenkov telescopes with 12 and 7 blazars, respectively. Their results around near-IR band are consistent with ours in the 1$\sigma$ level but are higher than ours in the optical band. However, our EBL spectrum in the optical band agrees well with that from Dominguez et al. (2011), in which they used the observed galaxy luminosity function and galaxy spectral energy distribution type fractions to derive the EBL spectrum.

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REFERENCES

Abramowski, A., Acero, F., Abaronian, F., et al. 2012, arXiv:1212.3409

Ackermann, M., Ajello, M., Allafort, A., et al. 2012, Sci, 338, 1190

Baldry, I. K., & Glazebrook, K. 2003, ApJ, 593, 258

Blanc, G. A., Adams, J. J., Gebhardt, K., et al. 2011, ApJ, 736, 31

Bouwens, R. J., Illingworth, G. D., Oesch, P. A., et al. 2012, ApJ, 754, 83

Bruzual, G., & Charlot, S. 2003, MNRAS, 344, 1000

Chabrier, G. 2003, PASP, 115, 763

Cole, S., Norberg, P., Baugh, C. M., et al. 2001, MNRAS, 326, 255

Dominguez, A., Primack, J. R., Rosario, D. J., et al. 2011, MNRAS, 410, 2556

Doran, M., & Muller, C. M. 2004, JCAP, 09, 003

Driver, S. P., Popescu, C. C., Tuffs, R. J., et al. 2008, ApJL, 678, L101

Eggleton, P. P., Tout, C. A., & Fitchett, M. J. 1989, ApJ, 347, 998

Fazio, G. G., & Stecker, F. W. 1970, Natur, 226, 135

Finke, J. D., Razzouque, S., & Dermer, C. D. 2010, ApJ, 712, 238

Franceschini, A., Rodighiero, G., & Vaccari, M. 2008, A&A, 487, 837

Fukugita, M., & Peebles, P. J. E. 2004, ApJ, 616, 643

Gelman, A., & Rubin, D. 1992, StaSc, 7, 457

Gilmore, R. C., Somerville, R. S., Primack, J. R., & Domiguz, A. 2012, MNRAS, 422, 3189

Gong, Y., & Chen, X. 2007, PhRvD, 76, 123007

Gould, R. J., & Schreder, G. 1966, PhRvL, 16, 252

Hastings, W. K. 1970, Biometrika, 57, 97

Hauser, M. G., & Dwek, E. 2001, ARA&A, 39, 249

Hayes, M., Schrerer, D., Goran, O., et al. 2011, ApJ, 730, 8

Hinshaw, G., Larson, D., Komatsu, E., et al. 2012, arXiv:1212.5226

Hopkins, A. M., & Beacom, J. F. 2006, ApJ, 651, 142

Inoue, Y., Inoue, S., Kobayashi, M. A. R., et al. 2013, ApJ, 768, 197

Kistler, M., Yssel, H., Beacom, J. F., Hopkins, A. M., & Wyithe, J. S. B. 2009, ApJL, 705, L104

Madani, P., Haardt, F., & Rees, M. J. 1999, ApJ, 514, 648

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N., Teller, A. H., & Teller, E. 1953, JCP, 21, 1087

Nikishov, A. I. 1962, Sov. Phys. JETP, 14, 393

Orr, M. R., Krennrich, F., & Dwek. E. 2011, ApJ, 733, 77

Razzaque, S., Dermer, C. D., & Finke, J. D. 2009, ApJ, 697, 483

Robertson, B. E., & Ellis, R. S. 2012, ApJ, 744, 95

Stecker, F. W., de Jager, O. C., & Salamon, M. H. 1992, ApJL, 390, L49

Stecker, F. W., Malkan, M. A., & Scully, S. T. 2012, ApJ, 761, 128

Yuan, Q., Huang, H., Bi, X., & Zhang, H. 2012, arXiv:1212.5866