Quantum random walk with Rb atoms

Kia Manouchehri and Jingbo Wang
School of Physics, The University of Western Australia, Australia
E-mail: wang@physics.uwa.edu.au

Abstract. Quantum random walks are known to have non-intuitive dynamics, which makes them an attractive area of study for devising quantum algorithms for long-standing classical problems as well as those arising in the field of quantum computing. In this paper we describe a new implementation scheme for quantum random walks, using ultra cold rubidium atoms trapped in optical lattices.

1. Introduction

Quantum random walks represent a generalised version of the well-known classical random walk, which can be elegantly described using quantum information processing terminology [1]. Despite their apparent connection however, dynamics of quantum random walks are often non-intuitive and far deviate from those of their classical counterparts [2,3]. Among the differences, the superior propagation properties of quantum random walks are particularly noteworthy, making them an attractive area of study for devising efficient quantum algorithms. Areas of study using quantum random walks have included connectivity and graph theory [3,4], as well as quantum search algorithms [5,6].

If quantum walks are to be taken seriously as a viable platform for quantum information processing, it is essential to find an implementation that is experimentally feasible, readily scalable and not limited to problems with specific connectivity criteria. In our previous work [7], we have provided a comprehensive list of theoretical as well as experimental efforts to construct quantum walks using a broad range of physical systems including nuclear magnetic resonance, cavity QED, ion traps, classical and quantum optics, as well as quantum dots. In this paper, we present a new implementation scheme for performing quantum random walks using ultra cold Rubidium atoms trapped inside optical lattices, which satisfies the above-mentioned criteria and is based on existing laboratory techniques reported in recent literature [8, 9, 10].

2. Theoretical model

Quantum random walks follow a similar idea to classical random walks, but incorporate a more complex definition of nodes, coins and steps. In a one-dimensional discrete-time quantum random
walk for instance, each node $n$ is made up of two sublevels, conventionally denoted by $\uparrow$ and $\downarrow$, and the probability of being present at a node $P(n) = |\psi(\uparrow,n)|^2 + |\psi(\downarrow,n)|^2$, where $\psi(\uparrow,n)$ and $\psi(\downarrow,n)$ are complex amplitudes associated with each sublevel at site $n$. A quantum coin $\hat{C}$ is defined as a $2 \times 2$ unitary matrix which in some way mixes the sublevel amplitudes within each node. A commonly used unbiased quantum coin, for instance, is defined as:

$$\hat{C} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix},$$

which mixes the two sublevels in the form of

$$\begin{pmatrix} \psi(\uparrow,n) \\ \psi(\downarrow,n) \end{pmatrix}_{\text{new}} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \begin{pmatrix} \psi(\uparrow,n) \\ \psi(\downarrow,n) \end{pmatrix}_{\text{old}}.$$

As shown in figure 1, each step in such a quantum random walk consists of

1. Mixing the sublevels using a quantum coin $\hat{C}$;
2. Shifting the amplitudes of $\psi(\uparrow,n)$ and $\psi(\downarrow,n)$ from node $n$ to nodes $n-1$ and $n+1$, using the translation operator $\hat{T}_\uparrow(1)$ and $\hat{T}_\downarrow(-1)$ respectively.

![Figure 1: Illustration of a single step in a discrete-time quantum random walk. Green arrows represent sublevel mixing, which is followed by sublevel shifting as indicated by the red and blue arrows.](image)

Repeating the above procedure over a number of steps leads to a characteristic probability distribution for quantum random walks, which is starkly different from that of a classical random walk, as illustrated in figure 2.

![Figure 2. Upper panel: A classical walker making her way through a decision tree; middle panel: a quantum walker making his way through a decision tree; lower panel: probability distribution for a classical (dotted) vs. quantum (solid) random walk in one dimension.](image)
3. Physical implementation using cold atoms

Trapping of neutral atoms in optical lattices have opened fascinating new experimental possibilities in quantum information processing. An optical lattice is formed by interacting a pair of counter-propagating lasers to produce a standing laser wave. When neutral atoms such as $^{87}\text{Rb}$ are placed in the resulting beam line, they will effectively see a spatially varying dipole potential. What we propose here is to implement the quantum walks as described above using cold $^{87}\text{Rb}$ atoms trapped inside an optical lattice. The position states are encoded using the individual trapping sites and the atom acts as the quantum walker with some initial distribution throughout the lattice sites. A series of specially tailored control laser operations are then introduced to address, manipulate and interact the atomic wave packets in individual sites, in a way that corresponds exactly to the action of the coin and translation operators along a row of lattice sites.

The $^{87}\text{Rb}$ atoms trapped inside the optical lattice experience a natural splitting of the $5\text{S}$ level into two hyperfine sub-levels, which are used to define the $|0\rangle$ and $|1\rangle$ basis state. These states can be manipulated using Microwave techniques to drive corresponding Rabi-Oscillations [8]. With wavelengths that are in the order of a few centimeters however, microwaves can not resolve any locations within the lattice. Instead we propose performing arbitrary unitary transformations on states $|0\rangle$ and $|1\rangle$ with the aid of a pair of three-photon Stimulated Raman Adiabatic Passage (STIRAP) operations [9], as illustrated in figure 3 (left panel). Each STIRAP requires the use of three control lasers applied in a counter intuitive order to transfer the atomic population in states $|0\rangle$ and $|1\rangle$, to and from an auxiliary state $|\gamma\rangle$, via an intermediate upper state $|\mu\rangle$ that does not get populated during the transfer. The two-photon $\Lambda$ STIRAP $|1\rangle \leftrightarrow |\mu\rangle \leftrightarrow |\gamma\rangle$ transition has already been experimentally demonstrated using circularly polarized laser [10]. Our proposal extends this implementation through the addition of a third linearly polarized laser to facilitate the additional $|0\rangle \leftrightarrow |\mu\rangle$ transition. This operation allows for the implementation of a generalized quantum coin operator, controlled by the phases, amplitudes and timing of these laser pulses.

![Figure 3. Left panel: schematic diagram of a three-photon STIRAP operation on the trapped $^{87}\text{Rb}$ atom; right panel: state dependent optical potentials moving in opposite directions.](image)
To perform the translation operator \( \hat{T}_r(1) \) and \( \hat{T}_l(-1) \), we utilize a state-dependent transport scheme originally proposed in [11]. By setting the lattice wavelength at \( \lambda_{\text{lat}} = 785\text{nm} \), states \( |0\rangle \) and \( |1\rangle \) experience different optical potentials \( V_{\text{off}}(x, \theta) = V_0(x, \theta)/4 + 3V_0(x, \theta)/4 \) and \( V_{\text{on}}(x, \theta) = V_0(x, \theta) \), respectively as illustrated in figure 3 (right panel). Here \( V_0(x, \theta) = V_{\text{max}} \cos^2(kx \pm \theta/2) \), \( k = 2\pi/\lambda_{\text{lat}} \), and \( \theta \) is the relative polarization angle between the pair of counter-propagating lasers. Hence for an atom in the superposition state \( \alpha|0\rangle + \beta|1\rangle \), increasing the polarization angle \( \theta \) will lead to a split in the spatial wave packet of the atom, with the components \( \alpha \) and \( \beta \) moving in opposite directions.

Figure 4 depicts the control operations involved in performing a single quantum walk step. In the proposed scheme adjacent lattice sites represent the two \( \uparrow \) and \( \downarrow \) sublevels within a node. The atom is initially in its internal ground state \( |0\rangle \). All sites denoted as \( \uparrow \) are then excited to the internal state \( |1\rangle \) using a \( \pi \) control pulse. We utilize the state dependant transport mechanism to shift \( \psi(\uparrow, n) \) to their adjacent lattice sites, still in the excited state \( |1\rangle \), where they overlap with \( \psi(\downarrow, n) \). A series of STIRAP operations (\( R_1, R_2, R_3, \ldots \)) are then performed on the overlapping sites, effectively mixing the two states in accordance with the coin operator \( \hat{C} \). Another state dependant transport is performed displacing \( \psi(\uparrow, n) \) by an additional lattice wavelength such that \( \psi(\uparrow, n) \) is adjacent to \( \psi(\downarrow, n+1) \). Another \( \pi \) pulse brings the excited sites back to their ground state, completing one quantum walk step.

![Figure 4. Basic procedure in a quantum walking step.](image)

4. Conclusions

In this paper, we introduced a proposal for the physical implementation of discrete-time quantum random walks using ultra cold rubidium atoms trapped in an optical lattice. We demonstrate that, by tuning the polarization angles of a pair of counter-propagating lasers and utilizing the 3-photon STIRAP process, one can implement the action of the coin and translation operators and thus the specified quantum random walks.

References

[1] Y. Aharonov, L. Davidovich, and N. Zagury, Phys. Rev. A 48, 1687 (1993).
[2] P. L. Knight, E. Roldán, and J. E. Sipe, Phys. Rev. A 68, 020301 (2003).
[3] J. Kempe, Contemporary Physics 44, 307 (2003).
[4] B. L. Douglas and J. Wang, J. Phys. A: Math. Theor. 41, 075303 (2008).
[5] N. Shenvi, J. Kempe, K. B. Whaley, Phys. Rev. A 67, 052307 (2003).
[6] A. Childs and J. Goldstone, Phys. Rev. A 70, 022314 (2004).
[7] K. Manouchehri and J. Wang, J. Phys. A: Math. Theor. 41, 065304 (2008).
[8] O. Mandel, M. Greiner, A. Widera, T. Rom, T. W. Hansch, I. Bloch, Phys. Rev. Lett. 91, 010407 (2003).
[9] Z. Kis and F. Renzon, Phys. Rev. A 65, 032318 (2002).
[10] K. C. Wright, L. S. Leslie, N. P. Bigelow, Phys. Rev. A 77, 041601 (2008).
[11] D. Jaksch, H.-J. Briegel, J. I. Cirac, C. W. Gardiner, P. Zoller, Phys. Rev. Lett. 82, 1975 (1999).