On the issue of energy consumption of vibration technological machines

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Abstract. The article provides a comparative analysis of the energy consumption required for the implementation of the resonant and beyond-resonant modes of vibrating technological machines working body’s oscillations. An algorithm for selecting parameters of an electric drive based on an asynchronous electric engine to provide the specified amplitude and frequency of the working body oscillations for both modes of vibration is presented, taking into account frequency regulation, losses arising in the elastic suspension of the machine and in bearings of engine, the occurrence of reactive power and changes in the energy conversion efficiency.

1. Introduction
In modern industry, vibration technological machines with unbalanced vibration exciters (vibrating conveyors, vibrating screens, vibrating crushers, etc.) operating on the beyond-resonant mode are widely used [1-3]. Using the beyond-resonance mode ensures the stability of the vibratory machine in a wide range of load parameters, however, to overcome the resonant frequencies during starting up such machines, electric engines with excess power are installed on them. This leads to the fact that in the operating mode the electric engine is significantly underloaded, as a result of which the energy consumption is increased and the service life of the engine is reduced[3-6].

One of the ways to increase the energy efficiency of vibrating machines is considered to be the usage the resonant oscillation mode of their working bodies. It seems to be obvious that for the implementation of resonant oscillation modes compared with the beyond-resonance mode less disturbing forces are required, which are determined by the value of the imbalance. However, the use of the resonance mode is associated with the need to ensure its stability, which is especially important for nonlinear dynamic systems. At the same time, the problem of ensuring the stable operation of the vibrating machine near resonant frequencies can be solved by using various control systems that configure and maintain the resonant mode of oscillations [6-11]. It is obligatory to note, that in the existing literature there are practically no quantitative estimates illustrating the efficiency of resonance modes usage.

The aim of this work is to substantiate the energy efficiency of the resonance mode based on a comparison of the energy consumption and the required electric drive power of the resonant and beyond-resonant vibrating machines. The presented assessment does not take into account the energy costs necessary for the implementation of the actual technological process, since they are considered the same regardless of the selected operating mode of the vibrating machine. For an adequate comparison of the resonant and beyond-resonant vibration machines operation modes, it is accepted that the engines of the compared vibration machines are fed through frequency converters powered
from an alternating current (AC) network with a voltage amplitude \( U = 380 \text{ V} \), while both machines must implement the same operating mode with a frequency \( \omega^* \) and amplitude \( A^* \). In this case, the nominal shaft rotation speeds \( \omega_N \), corresponding to the passport data of AC engines, in both cases, can significantly differ from the operating frequency \( \omega^* \), which requires taking into account the law of frequency regulation.

2. Calculation scheme

Comparison of the resonance and beyond-resonance modes will be carried out on the basis of a single-mass vibration machine with one unbalanced vibration exciter, which calculation scheme is shown in figure 1. The vibration exciter is a three-phase AC engine with a squirrel-cage rotor, on the shaft of which an unbalanced mass is installed - unbalance. The moment characteristic of the electric engine is described using the Kloss formula [12] and has the form shown in figure 2, where: \( M_S^* \) - starting torque, \( M_N^* \) - nominal torque on nominal frequency \( \omega_N \), \( M_C^* \) - critical torque on critical frequency \( \omega_C \). All of this AC engines parameters are regulated by the manufacturer. The relations between the starting, nominal and critical moments are taken as the average values of the allowed intervals:

\[
M_N = 0.965M_S, \quad M_N = 0.5M_C. \tag{1}
\]

The vibration exciter is installed directly on the working body, which performs unidirectional vibrations along the horizontal axis \( Ox \). The working body of mass \( m \) is mounted on an elastic suspension (spring) with a linear stiffness characteristic \( c \) (it is believed that the mass of the vibration exciter compared to the mass of the working body can be neglected). The energy dissipation in a spring is described by a model of viscous friction with a damping coefficient \( b = 0.06\sqrt{cm} \) equal to 0.03 of the corresponding critical value.

In the following text and formulas, all parameters related to the beyond-resonant machine will contain the index "1", and to the resonant machine - the index "2".

Thus, both of the vibration machines are described by a single calculation scheme. In this case, the masses of the working bodies are considered to be the same, and the springs stiffness differs in \( n^2 \) times, where \( n = p_2 / p_1 \) - the ratio of the natural frequencies of each of the systems \( p_1 = \sqrt{c_1/m} \), \( p_2 = \sqrt{c_2/m} \).

The motion of the oscillatory system (figure 1) with limited excitation is described by the following nonlinear equations [11]:

\[
\begin{align*}
mx + hx + cx &= m_1 r_d (\ddot{\alpha} \sin \alpha + \dot{\alpha}^2 \cos \alpha); \quad J\ddot{\alpha} + H(\dot{\alpha}) - m_d r_d \dot{x} \sin \alpha = M(\dot{\alpha}),
\end{align*}
\tag{2}
\]

where \( y \) - the coordinate the system center mass, counted from the position of its static equilibrium; \( \alpha \) - the unbalance rotation angle, counted from the positive direction of the \( Ox \) axis clockwise; \( m_d, J \) and \( r_d \)
- mass, moment of inertia and eccentricity of unbalance; $M(\dot{\alpha})$ - the torque characteristic of the AC engine.

The resistance to rotation moment of the AC engine rotor will be described using the model of dry friction caused by the reaction in the rotor bearing from the action of centrifugal force:

$$H(\dot{\alpha}) = \mu m_d r_d \dot{\alpha}^2,$$

(3)

where $\mu$ – the friction coefficient, $R$ is the radius of the rotor; points over the variables denote the operation of differentiation with respect to time $t$.

The torque of an AC motor will be described by the Kloss formula [12], which has the form:

$$M(\dot{\alpha}) = \frac{2M_{Cr}}{\left(\omega_s - \dot{\alpha}\right)} \left(\omega_s - \dot{\alpha}\right)^2 + \left(\omega_s - \omega_{Cr}\right)^2,$$

(4)

where $M_{Cr}$ - is the maximum (critical) moment corresponding to the rotational speed $\omega_{Cr}$, $\omega_s = 2\pi f R^{-1}$ - is the idle speed (in the absence of a load on the shaft), $f$ - the frequency of the supply voltage, $K$ - the number of engine poles.

For an adequate comparison of the energy indicators of both modes, it is necessary to establish the mass-geometric parameters of unbalances that provide tuning to a single operating mode, the minimum required AC engines powers and power consumption on the operating and transient conditions (for the beyond-resonance machine), taking into account the frequency regulation law of AC engine.

To implement the required stationary mode with frequency and amplitude, it is necessary, first of all, to determine the imbalance value, the moment of all resistance forces arising from the oscillatory movement of the system, and select the appropriate parameters of the operating characteristic of the engine torque for the resonant and beyond-resonant machines.

Setting the rotational speed of the unbalances of both vibratory machines is carried out using a frequency converter according to the law, $Uf^{-1} = const$ [12]. In figure 3 the graphs of the control characteristics of an AC motor with decreasing power frequency are shown.

With the chosen law of frequency regulation, a decrease in the supply frequency $f > f_1 > ... > f_0 > ... > f_n$ relative to the $f=50$ Hz supply frequency leads to a parallel shift of the rated torque characteristics of the motor along the $\omega$ axis. This offset can be entered into the Kloss formula (4) as a parameter $\Delta\omega = \omega_s - \omega_{s*}$, where $\omega_{s*}$ is the idling frequency of the operating characteristic. Then engine torque characteristics:

$$M_s(\dot{\alpha}) = \frac{2M_{Cr}}{\left(\omega_s - \dot{\alpha} - \Delta\omega\right)} \left(\omega_s - \dot{\alpha} - \Delta\omega\right)^2 + \left(\omega_s - \omega_{Cr}\right)^2.$$

(5)

**Figure 3.** Frequency-controlled AC engine torque characteristics.

**Figure 4.** Cosine of power and engine efficiency depending on the load factor.

Electric power consumption is calculated by the formula [12]:

$$P = \frac{1}{2} \frac{M_s(\dot{\alpha}) \omega_s}{\dot{\alpha}}.$$
\[ P_e = \frac{N}{\eta \eta_f \cos \varphi} \]  

where \( N = \omega M_c(\omega) \) - the mechanical power of the engine necessary to maintain oscillations (net power on the motor shaft), \( \eta \) - the efficiency of the electric motor, \( \eta_f \) - the efficiency of the frequency converter, \( \cos \varphi = PP_e^{-1} \) - the cosine of the power of the electric motor, where \( P = N(\eta_f \eta)^{-1} \) is the active power of the electric motor, is the total (consumed) electric power.

For the effective operation of the engine at the operating frequency \( \omega = \omega_e \), it is necessary that \( \cos \varphi = PP_e^{-1} \) is close to one. The cosine of the load allows you to estimate the amount of reactive power that circulates between the energy source and the consumer. The lowest value of reactive power is achieved when the power on the motor shaft is close to the nominal. Typical graphs of the dependence of the power cosine, as well as the efficiency of the electric motor depending on the load, are presented in figure 4 [12], where the \( \beta = NP_e^{-1} \) - dimensionless parameter of the engine load is plotted along the abscissa, \( P_N = \omega e M_N \) - the rated (passport) power of the electric motor. The efficiency of the electric motor is determined from the graphs shown in figure 4 for a given value \( \beta \).

Efficiency of the frequency converter for both modes \( \eta_f = 0.98 \).

3. Calculation of parameters of vibration exciter when working in the resonant mode

To determine the parameters of the vibration exciter, taking into account the characteristic features of its moment characteristic and the law of frequency control (5), it is necessary to calculate the unbalance value \( D_1 \), the frequency \( \omega_{1c} \), at which the critical moment of the electric motor \( M_{1c} \) is achieved, and the shift of the operating characteristic \( \Delta \omega_t \) from the condition that the operating frequency \( \omega_e \) should be achieved working amplitude \( A^* \).

The value of the imbalance \( D_1 \) necessary for the implementation of a given operating mode is determined from the solution of the system of differential equations (2) describing the stationary vibrations of a vibrating machine with an engine of limited power [11].

In the resonance region, the system of differential equations (2), in the case of stationary oscillations at the operating frequency \( \dot{\alpha} = \omega_e \), is written in the form:

\[
\begin{align*}
\ddot{m}x + b \dot{x} + c x &= m'_d r'_d \omega_e^2 \cos(\omega_e t); \\
\mu m_d r_d^2 R o_x^2 - m_d' r_d^2 \dot{x} \sin \omega_e t &= M_{1t} (\omega_e).
\end{align*}
\]

From the well-known solution of the first equation [2, 4] for the amplitude of the forced oscillations, one can obtain an expression that determines the value of the imbalance at which a given working amplitude \( A^* \) is achieved:

\[ D_1 = A_1 (m^2 (p^2 - \omega_e^2)^2 + b'_d \omega_e^6)^{1/2} \omega_e^{-2}. \]

To obtain expressions that describe the points \( M_{1t}(\omega_e), M_{1t}(p) \) of the required operating characteristics of the engine’s torque, first of all, the equation for determining the required engine torque in the operating mode should be written. To this end, substituting the solution of the first equation of system (7) into the second, and averaging the resulting expression over one unbalance revolution, we obtain the equation:

\[ M_{1t} (\omega_e) = \mu D_1 R o_x^2 + \frac{b A^2 \omega_e}{2}. \]

The value of the moment necessary to overcome the resonance frequency \( p_1 \) can be obtained from the solution of system (2) in the resonance region, provided that \( \dot{\alpha}(t) \) is a slowly changing function per one unbalance revolution [11]:
where $A_{res} = D_i p_i / \beta_i$.

Note that the formula (9) describes the stationary mode of resonant oscillations for a linearized system and gives an overestimated value of the required moment, since in this case the maximum amplitude develops. In fact, to correctly describe the passage of the resonance regime, the original equations (2) should be used. Therefore, the final value of the required moment will be clarified by numerically solving system (2).

In addition, the starting torque of the electric motor must satisfy the condition [1, 4, 5]:

$$M_e(0) > 0.725 Dg.$$  

Equations (1), (8) and (9), taking into account formula (5), form a closed system of algebraic equations with respect to unknowns $\omega_{IC}$, $M_{IC}$, $\Delta \omega$:

$$\frac{2M_{IC}(\omega - \omega_{IC})(\omega - \omega_p - \Delta \omega_p)}{(\omega - \omega_p - \Delta \omega_p)^2 + (\omega - \omega_{IC})^2} = \mu D_i R \omega^2 + \frac{b_i A_{res} \omega_p}{2};$$

$$\frac{2M_{IC}(\omega - \omega_{IC})(\omega_p - \Delta \omega_p)}{(\omega_p - \Delta \omega_p)^2 + (\omega - \omega_{IC})^2} = \mu D_i R \omega^2 + \frac{b_i A_{res} \omega_p}{2};$$

$$\frac{2M_{IC}(\omega - \omega_{IC}) \omega_p}{\omega_p^2 + (\omega - \omega_{IC})^2} = 0.518 M_{IC}.$$  

Solving this system of equations, the required passport characteristic of the electric motor (at $\Delta \omega_1 = 0$) and the operating characteristic of the electric motor are obtained.

Thus, it is possible to obtain engine characteristics that provide stationary modes at all characteristic vibration frequencies. To clarify the moment characteristics, the system of equation (2), in which the obtained values from the solution of system (13) can be used as a first approximation. This refinement is carried out by numerically solving system (2) by iteratively approximating the value of the moment of the operating characteristic $M_{IC}$ to the actual moment of all resistance forces $M_{1R}$ at the resonant frequency $\omega_p$ by varying $M_{IC}$.

As a result, the minimum required values of the required parameters of the engine operating characteristic $M_{1\min}(\omega)$ were calculated, which ensured passage through the resonance and reached the operating mode. However, when using such a moment characteristic, the passage through the resonance is accompanied by significant and dangerous amplitudes of the working body oscillations. Therefore, in a beyond-resonance vibrating machine, the engine power and, accordingly, the torque should exceed by 30 ... 70% the moment of resistance forces at the resonant frequency $M_{1\min}(\omega)$. For this, in this work, the minimum value of the excess power is assumed to be 30%. Then, replacing the second equation of system (13) by the equation

$$\frac{2M_{IC}(\omega - \omega_{IC})(\omega_p - \Delta \omega_p)}{(\omega_p - \Delta \omega_p)^2 + (\omega - \omega_{IC})^2} = 1.3 M_{1\min}(\omega_p),$$

the system of equations (13) is again solved and the final values of the parameters of the torque characteristic of the engine are calculated.

The described algorithm was implemented in the Wolfram Mathematica software package with the following system parameters: $m = 1000 \text{kg}$, $\omega_s = 157 \text{rad} \cdot \text{s}^{-1}$, $K = 2$, $R = 0.04 \text{m}$, $\omega_0 = 100 \text{rad} \cdot \text{s}^{-1}$, $p_i = 33.3 \text{rad} \cdot \text{s}^{-1}$, $A_\omega = 0.004 \text{m}$, $m_d = 150 \text{rad} \cdot \text{s}^{-1}$, $\mu = 0.002$.

As a result, the unbalance values and the characteristics of the motor characteristics were calculated:
\[ D_1 = 3.56 \text{kg} \cdot \text{m}, \quad M_{\text{GC}} = 54.2 \text{Nm}, \quad \omega_{\text{GC}} = 113.0 \text{rad} \cdot \text{s}^{-1}, \quad \Delta \omega_1 = 54.7 \text{rad} \cdot \text{s}^{-1}. \]  

Taking into account (1) and (13), the necessary passport data of the required electric motor are determined: \[ M_{1N} = 27.08 \text{Nm}, \quad \omega_{1N} = 145.4 \text{rad} \cdot \text{s}^{-1}, \quad M_{1S} = 28.05 \text{Nm}. \]

The obtained values of the oscillation amplitude during the passage of the resonance \((A_{\text{res}} = 0.025 \text{m})\) turned out to be 6.25 times greater than the specified working amplitude \((A_{\text{res}} = 0.025 \text{m})\), which corresponds to the data given in [1, 3, 5]. At the same time, the system reaches the operating mode in about 3 s.

According to the obtained mechanical characteristics, the rated engine power is equal \[ P_{1N} = M_{1N} \omega_{1N} = 3.94 \text{kW}. \] In accordance with the existing nomenclature of the series of rated power of electric motors, the calculated value is rounded up to 4 kW.

In operating mode, the power on the motor shaft \(N_s = 0.452 \text{kBt}\). As a result, the electric motor of the resonant vibrating machine is underloaded more than 8 times, which, in accordance with figure 4, leads to a low value of efficiency \((\eta_s = 0.4)\) and load cosine \((\cos \phi_s = 0.4)\). Thus, according to (6), the consumed electric power of the vibrating machine is \[ P_{\text{ps}} = 2.8 \text{kBr}. \]

It should be noted that the power of the engines for the beyond-resonant vibrating machine is determined, first of all, by the power required to overcome the resonant frequency when starting up the machine. There are various technological solutions that make it possible to reduce the value of that power, in particular, the use of unbalances with a varying eccentricity, or separate start-up of electric motors in the case of using several self-synchronizing vibration exciters [5]. It is also possible to increase the cosine of the load \(\cos \phi_1\) by including additional devices compensating reactive power in the electric circuit of the vibrating machine power supply [12].

### 4. Calculation of parameters of vibration exciter for operation in resonance mode.

For a resonant machine, it is necessary to provide an operating mode of oscillations with an amplitude \(A_s\) at a frequency \(\omega_s\) close to the natural frequency of the machine oscillations. Since near the resonant frequency the oscillations of the system are considered stationary, the first equation of system (2) degenerates into equation (7), which describes the motion of the linear system. Then the value of the imbalance \(D_2\) necessary for the implementation of a given operating mode of the resonant vibrating machine is determined from the well-known formula for the resonant amplitude [2,4]:

\[ D_2 = \frac{A \omega_s}{\omega_s}. \]  

To obtain expressions describing the point \(M_{2s}(\omega_s)\) of the required operating characteristic of the engine torque at the operating frequency, we use equation (8) written through the parameters of the resonant vibrating machine

\[ M_{2s}(\omega_s) = \frac{2M_{2C}(\omega_s - \omega_{2C})(\omega_s - \omega_{-\Delta \omega_2})(\omega_s - \omega - \Delta \omega_2)}{(\omega_s - \omega_s - \Delta \omega_2)^2 + (\omega_s - \omega_{2C})^2} = \mu D_2 R \omega_s^2 + \frac{b_2 A^2 \omega_s}{2} \]  

where the operating characteristic of the driving moment \(M_{2s}(\omega_s)\) is determined by the law of frequency regulation of the engine with a rated characteristic \(M_{2s}(\omega_s)\).

The rated torque characteristics of the engine must satisfy conditions (1) and (12).

For the resonant machine to work effectively, it is necessary that the engine load parameter

\[ \beta_s = N_{2s} P_{N2}^{-1} = 1, \] from where \(\omega_s M_{2s}(\omega_s) = M_{2s} \omega_{2N}\)  

Condition (18) corresponds to the maximum efficiency and load cosine \((\cos \phi_s)\) during operation in the operating mode (figure 4).

To determine the frequency, the relationship between the nominal \(M_{2s}\) and critical \(M_{2C}\) moments (1) is used:
Equations (1.17-19), taking into account (5), and (14), form a closed system of algebraic equations with respect to unknowns $\omega_{2G}$, $M_{2G}$, $\Delta \omega_2$, $\omega_{2N}$:

$$
\frac{2}{\omega_0^2 + \left( \omega - \omega_0 - \Delta \omega_2 \right)^2 + \left( \omega - \omega_2 \right)^2} = \frac{M_{2G}}{2} \omega_2 + \frac{b_i \omega_0}{2};
$$

$$
\frac{2}{\omega_0^2 + \left( \omega - \omega_0 - \omega_{2N} \right)^2 + \left( \omega - \omega_2 \right)^2} = \frac{M_{2G}}{2} \omega_{2N};
$$

$$
\frac{2}{\omega_0^2 + \left( \omega - \omega_0 - \omega_{2N} \right)^2 + \left( \omega - \omega_2 \right)^2} = \frac{M_{2G}}{2} \omega_{2N}.
$$

As a result of calculations at $m = 1000 \text{kg}$, $\omega_0 = 157 \text{rad} \cdot \text{s}^{-1}$, $K = 2$, $R = 0.04 \text{m}$, $\omega_k = 100 \text{rad} \cdot \text{s}^{-1}$, $A_s = 0.004 \text{m}$, $m_{d1} = 150 \text{kg}$, $\mu = 0.002$, we obtain the following values of the imbalance and parameters of the characteristics of the electric motor: $D_2 = 0.24 \text{kg} \cdot \text{m}$, $M_{2G} = 7.87 \text{Nm}$, $\omega_{2G} = 113.2 \text{rad} \cdot \text{s}^{-1}$, $\Delta \omega_2 = 43.8 \text{rad} \cdot \text{s}^{-1}$, $\omega_{2N} = 145.3 \text{rad} \cdot \text{s}^{-1}$.

Then from expressions (1) we obtain the passport characteristics of the required electric motor: $M_{2N} = 3.94 \text{Nm}$, $M_S = 4.1 \text{Nm}$.

From the calculation it follows that the resonant vibrating machine reaches the operating mode with the given parameters in about 1.8 s.

According to the obtained mechanical characteristics of the engine, the rated engine power $P_{2N} = M_{2N} \omega_{2N} = 0.57 \text{kJ}$, which, according to a number of electric motor powers, is rounded to 0.75 kW.

In accordance with the graphs shown in figure 4, the efficiency of the resonant machine engine $\eta_{el} = 0.85$, and the cosine of the load $\cos \varphi_{el} = 0.9$. From the expression (6) it follows that the consumed electric power of the vibrating machine $P_{2e} = 0.764 \text{kW}$.

For comparison, table 1 presents the minimum required values of the calculated parameters of the vibrating machines operating in the resonant and beyond-resonance modes and their ratio as an indicator of the efficiency of using the resonant mode.

**Table 1. Comparison of energy indicators of resonant and beyond-resonant vibratory machines.**

| Parameter (Par)            | Operation mode | Ratio Par_2/Par_1 |
|---------------------------|----------------|------------------|
|                          | resonance      | beyond-resonance |
| Imbalance $D$             | 0.24 kg\cdot m | 3.56 kg\cdot m   | 14.8 |
| Rated engine power $P_N$  | 0.75 kW        | 4 kW             | 5.3  |
| Consumed electric power $P_e$ | 0.764 kW     | 2.3 kW           | 3.7  |
| Power on the engine shaft $N*$ | 0.57 kW        | 0.45 kW          | 0.79 |
5. Conclusion
As a result of the analysis, it was shown that the consumed electric power of the resonant machine turned out to be 3.7 times larger than the resonant vibrating machine. It is important that the factor that had the greatest influence on the result turned out to be that the engine in the out-of-resonance mode is significantly (8 times) underloaded, which leads to a decrease in the efficiency of the electric motor and the appearance of large values of reactive power and a decrease in the durability of the engine due to its overheating.

However, it is worth noting that the power required to maintain oscillations in the operating mode of the resonant machine is greater than that of the resonant one, and, first of all, this is due to greater damping in the elastic suspension and, therefore, large values of dissipative forces. This is due to the selected model of energy dissipation, depending on the stiffness of the elastic suspension.

Acknowledgments
The study was funded by a grant from the Russian Science Foundation (project No. 18-19-00708)

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