Non-Decoupling in Two Higgs Doublets Models, Spontaneous CP Violation and $Z_2$ symmetry

Miguel Nebot

Abstract

In Two Higgs Doublet Models shaped by some unbroken symmetry, placing perturbativity requirements on the quartic couplings can imply that the allowed masses of all the fundamental scalars are bounded from above, i.e. there is no decoupling regime for the new scalars. This important property is analysed in detail for the only two viable scenarios, the case with $Z_2$ symmetry and the case with CP symmetry. It is also noticeable that one exception arises in each case: when the vacuum is assumed to respect the imposed symmetry, a decoupling regime can nevertheless appear without violating perturbativity requirements. In both models with no decoupling regime, soft symmetry breaking terms can however lead to such a decoupling regime: the possibility that this regime might be unnatural, since it requires some fine tuning, is also analysed.
1 Introduction

Two Higgs-Doublets Models (2HDMs) were introduced by T.D. Lee in [1,2]. One central and appealing motivation was the possibility that the origin of CP violation is exclusively spontaneous: with CP invariance at the Lagrangian level, CP violation could nevertheless arise from the vacuum configuration. On the other hand, a significant source of concern for 2HDMs is the presence of Scalar Flavour Changing Neutral Couplings (SFCNC): they are already present, a priori, at tree level. Safe strategies to forbid or suppress SFCNC were soon identified, like Glashow and Weinberg’s Natural Flavour Conservation (NFC) [3] (for recent discussions on general flavour conserving 2HDM scenarios, see [4,5]). For 2HDMs shaped by an exact $Z_2$ symmetry (not softly broken), this precludes a spontaneous origin of CP violation: having NFC and spontaneous CP violation (SCPV) requires more than two doublets [6,7]. 2HDMs with spontaneous CP violation have been widely studied in the literature [8–20]. Recently, a 2HDM where all CP violation is originated by the vacuum, which includes SFCNC of controlled intensity, and which is viable, was presented in [21]. One important aspect of that model is the fact that the new scalars are necessarily light: their masses are all below 950 GeV. The absence of a regime in which the new scalars can have arbitrarily large masses, that is a decoupling regime, is a property that has been noticed and explored by different authors in the context of 2HDM [22–35]. On that respect, it is important that the scalar potential respects boundedness from below and that the scattering of scalars at high energies is perturbatively unitary. The introduction of soft symmetry breaking terms opens the possibility of having a decoupling regime. The objective of this work is to explore this non-decoupling property, in particular the bounds on the masses of the new scalars, for the two viable 2HDM with an exact symmetry, the one with CP symmetry and the one with $Z_2$ symmetry (this is the one usually studied in the litterature), and the one with “standard” CP symmetry.

The discussion is organised as follows. Section 2 starts with the SM scalar potential and vacuum, which are briefly revisited, paying special attention to the ingredients that lead to bounds on the Higgs mass à la Lee-Quigg-Thacker [36,37]; the general 2HDM is then discussed. The different symmetric 2HDMs are introduced in section 3. Out of them, the only two viable models, the one with CP symmetry and the one with $Z_2$ symmetry, are discussed in detail. In section 4 numerical analyses of both models are presented, showing in particular that the masses of the new scalars are constrained to be below 1 TeV. Since, as mentioned, the introduction of soft symmetry breaking terms allows the appearance of a non-decoupling regime, that question is addressed in section 5. It is stressed that, from the point of view of the symmetry, obtaining a decoupling regime is related to a rather unnatural or fine-tuned scalar potential. The appendices provide further details on different aspects of the previous sections.
2 Minimization of the potential and non-decoupling

2.1 Standard Model

In the Standard Model (SM), the Higgs potential is
\[ V(\Phi) = \mu^2 \Phi^\dagger \Phi + \lambda (\Phi^\dagger \Phi)^2, \]
where the scalar \( \Phi \) is an \( SU(2)_L \) doublet with hypercharge \( Y = 1/2 \); boundedness from below requires \( \lambda > 0 \). Electroweak symmetry is spontaneously broken (or hidden), \( SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q \) with \( Q = I_3 + Y \), if \( V(\langle \Phi \rangle) \) has a non-trivial minimum for
\[ \langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \]
In order to have an extremum, one needs
\[ \frac{d}{dv} V(\langle \Phi \rangle) = v(\mu^2 + \lambda v^2) = 0, \]
that is, one needs a potential with \( \mu^2 = -\lambda v^2 < 0 \). Then, the mass of the SM Higgs boson, \( m_{h_{\text{SM}}}^2 \), is
\[ m_{h_{\text{SM}}}^2 = \mu^2 + 3\lambda v^2 = 2\lambda v^2 > 0. \]
In order to achieve the desired spontaneous symmetry breaking (that is, the correct Fermi constant \( G_F \)) one chooses \( v \simeq 246 \text{ GeV} \). The crucial aspect is that both \( \mu^2 \) and, most importantly, \( m_{h_{\text{SM}}}^2 \), are fixed in terms of the vacuum expectation value \( v \) and \( \lambda \) (dimensionless) by means of the minimization condition. Before the discovery of 2012 [38, 39], one line of reasoning concerning the previous steps could be simply summarized as: any constraint on \( \lambda \) translates into a constraint on \( m_{h_{\text{SM}}}^2 \).
Different theoretical requirements like the stability (or metastability) of the vacuum, triviality, perturbative unitarity, were considered in order to provide, precisely, that kind of constraint [36, 37, 40–50]. In the SM, among those constraints, the \( 2 \rightarrow 2 \) scattering of longitudinal gauge bosons and scalars at high energies depends quite straightforwardly on the coupling \( \lambda \): requiring perturbative unitarity of those scattering processes gives simple bounds on \( \lambda \), and, as shown by Lee, Quigg and Thacker [36, 37] (see also [40]), this turns into an upper bound on \( m_{h_{\text{SM}}}^2 \). Of course, with the 2012 discovery, the situation is reversed for the SM Higgs: \( m_{h_{\text{SM}}} \) is measured, and \( \lambda \) inferred from it. The idea, however, remains an interesting possibility for extended scalar sectors, in particular 2HDMs.

2.2 General 2HDM

The most general 2HDM scalar potential is
\[ V(\Phi_1, \Phi_2) = \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \left( \mu_{12}^2 \Phi_1^\dagger \Phi_2 + \text{H.c.} \right) \
+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + 2 \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2 \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \
+ \left( \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \text{H.c.} \right) + \left( \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \text{H.c.} \right). \]
\( \mu_{11}^2, \mu_{22}^2 \) and \( \lambda_i, i = 1 \text{ to } 4 \), are real, while \( \mu_{12}^2, \lambda_j, j = 5, 6, 7 \), can be complex.

It is clear that, in analogy with the SM case, for a 2HDM where one can trade the dimensionful \( \mu_{ij}^2 \) for dimensionless \( \lambda_j \)'s and vacuum expectation values through the minimization conditions, an important consequence follows. If the quartic couplings \( \lambda_j \) are bounded, and that is precisely the case when one requires perturbativity or perturbatively unitary high energy scattering, then the masses of all the scalars are necessarily bounded from above, i.e. there is no decoupling regime. If one takes \( \lambda_j < O(10) \) for a very rough estimate, it follows that the new scalars have masses below \( \sim 1 \) TeV. It should be noticed that these bounds on the scalar masses have a somewhat loose nature: the precise values of the largest scalar masses that are allowed will directly depend on the values used in the requirements imposed on the \( \lambda_j \)'s. In any case large \( \lambda_j \)'s signal that a description in which those fundamental scalars are the relevant degrees of freedom would not be valid anymore. Of course, having a strongly interacting scalar sector is not a problem per se, but that is not the approach adopted here: we concentrate on the analysis of the scenarios where the fundamental scalars in 2HDMs are the relevant degrees of freedom.

A candidate vacuum with the desired properties for electroweak symmetry breaking has

\[
\langle \Phi_1 \rangle = e^{i\theta_1} \left( \begin{array}{c} 0 \\ v_1/\sqrt{2} \end{array} \right), \quad \langle \Phi_2 \rangle = e^{i\theta_2} \left( \begin{array}{c} 0 \\ v_2/\sqrt{2} \end{array} \right), \tag{6}
\]

characterized by \( v_1, v_2, \) real and positive, and by \( \theta = \theta_2 - \theta_1 \), the relative phase among \( \langle \Phi_2 \rangle \) and \( \langle \Phi_1 \rangle \), which is a potential source of CP violation\(^2\) \{\( v_1, v_2 \) encode the same information as \( v = \sqrt{v_1^2 + v_2^2} \) (which is of course chosen to be \( v \approx 246 \text{ GeV} \)) and \( t_\beta \equiv \tan \beta, \beta \in [0; \pi/2] \), with \( c_\beta = \cos \beta \equiv v_1/v, s_\beta = \sin \beta \equiv v_2/v \) (in the following, the compact notation \( c_x \equiv \cos x, s_x \equiv \sin x \) is used). Consider now \( V(v_1, v_2, \theta) \equiv \mathcal{V}(\langle \Phi_1 \rangle, \langle \Phi_2 \rangle) \):

\[
V(v_1, v_2, \theta) = \mu_{11}^2 \frac{v_1^2}{2} + \mu_{22}^2 \frac{v_2^2}{2} + \text{Re} \left( \bar{\mu}_{12}^2 \right) v_1 v_2 + \lambda_1 \frac{v_1^4}{4} + \lambda_2 \frac{v_2^4}{4}
+ (\lambda_3 + \lambda_4 + \text{Re} \left( \bar{\lambda}_5 \right)) \frac{v_1^2 v_2^2}{2} + \text{Re} \left( \bar{\lambda}_6 \right) \frac{v_1^2 v_2^2}{2} + \text{Re} \left( \bar{\lambda}_7 \right) \frac{v_1 v_2^3}{2}, \tag{7}
\]

where the \( \theta \) dependence is encoded in

\[
\bar{\mu}_{12}^2 = \mu_{12}^2 e^{i\theta}, \quad \bar{\lambda}_5 = \lambda_5 e^{i2\theta}, \quad \bar{\lambda}_6 = \lambda_6 e^{i\theta}, \quad \bar{\lambda}_7 = \lambda_7 e^{i\theta}. \tag{8}
\]

There are three stationarity conditions

\[
\frac{\partial V}{\partial v_1} = \frac{\partial V}{\partial v_2} = \frac{\partial V}{\partial \theta} = 0, \tag{9}
\]

which involve, linearly, the four\(^3\) dimensionful quantities \{\( \mu_{11}^2, \mu_{22}^2, \text{Re} \left( \bar{\mu}_{12}^2 \right), \text{Im} \left( \bar{\mu}_{12}^2 \right) \}\).

It is then clear that not all of them can be traded for \( \lambda_j \)'s and \{\( v_1, v_2, \theta \)\} and, as a consequence, one may expect that for values of the remaining dimensionful quantity

\[^2\]With no loss of generality, one can set \( \theta_1 = 0 \) in eq. (6).

\[^3\]Although \( \text{Im} \left( \bar{\mu}_{12}^2 \right) \) is absent from eq. (7), \( \frac{\partial}{\partial \theta} \text{Re} \left( \bar{\mu}_{12}^2 \right) = -\text{Im} \left( \bar{\mu}_{12}^2 \right) \).
much larger than \( v \), a decoupling regime can be obtained (without violating bounds on the \( \lambda_j \)'s). Conversely, in 2HDMs where there is less parametric freedom than in the general case in eq. (5), that is in 2HDMs shaped by some symmetry\(^4\) that possibility might be absent, and no decoupling might be expected (this generic property has been mentioned, for example, in \([31]\)). Symmetric 2HDMs are addressed in the next section: for the moment, let us analyse in simple terms what is required to have a decoupling regime in the general 2HDM. Before proceeding with the discussion, we take a small detour (until eq. (16)) to fix notation and introduce the physical fields and the mass terms.

In a Higgs basis \( \{H_1, H_2\} \)\(^{51, 53}\),

\[
\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = R_\beta \begin{pmatrix} e^{-i\theta_1} \Phi_1 \\ e^{-i\theta_2} \Phi_2 \end{pmatrix}, \quad \text{with} \quad R_\beta = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}, \quad R_\beta^T = R_\beta^{-1},
\]

only one combination of \( \Phi_1 \) and \( \Phi_2 \), \( H_1 \), has a non-vanishing vacuum expectation value:

\[
\langle H_1 \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.
\]

The usual expansion of the fields around the candidate vacuum in eq. (6) is

\[
\Phi_j = e^{i\theta_j} \begin{pmatrix} \varphi_j^+ \\ (v_j + p_j + m_j) \end{pmatrix}, \quad H_1 = \begin{pmatrix} G^+ \\ v + H^0 + \sqrt{2} \end{pmatrix}, \quad H_2 = \begin{pmatrix} H^+ \\ \sqrt{2} \end{pmatrix}.
\]

While the would-be Goldstone bosons \( G^0, G^\pm \) and the physical charged scalar \( H^\pm \) are readily identified, the neutral scalars \( \{H^0, R^0, I^0\} \) are not mass eigenstates: their mass terms read

\[
\frac{1}{2} \begin{pmatrix} H^0 & R^0 & I^0 \end{pmatrix} \mathcal{M}_0^2 \begin{pmatrix} H^0 \\ R^0 \\ I^0 \end{pmatrix} \subset \mathcal{V}(\Phi_1, \Phi_2),
\]

with the \( 3 \times 3 \) mass matrix \( \mathcal{M}_0^2 \) real and symmetric. \( \mathcal{M}_0^2 \) is diagonalised with a \( 3 \times 3 \) real orthogonal matrix \( R \),

\[
\mathcal{R}^T \mathcal{M}_0^2 \mathcal{R} = \text{diag}(m_h^2, m_{H}^2, m_A^2), \quad \mathcal{R}^{-1} = \mathcal{R}^T,
\]

which defines the physical neutral scalars \( \{h, H, A\} \):

\[
\begin{pmatrix} h \\ H \\ A \end{pmatrix} = \mathcal{R}^T \begin{pmatrix} H^0 \\ R^0 \\ I^0 \end{pmatrix}.
\]

\( h \) is assumed to be the SM-like Higgs with \( m_h = 125 \text{ GeV} \) (the alignment limit in which its couplings are SM-like corresponds to \( R_{11} \to 1 \)). Coming back to the discussion of the decoupling regime in the general 2HDM, through eqs. (9) one can express \( \{\mu_{11}^2, \mu_{22}^2, \text{Im}(\tilde{\mu}_{12}^2)\} \) in terms of \( \text{Re}(\tilde{\mu}_{12}^2) \), \( \lambda_j \)'s and \( \{v_1, v_2, \theta\} \):

\[
s_{2\beta} \text{Im}(\tilde{\mu}_{12}^2) = -v^2 s_\beta c_\beta \left\{ s_{2\beta} \text{Im}(\tilde{\lambda}_5) + c_\beta^2 \text{Im}(\tilde{\lambda}_6) + s_\beta^2 \text{Im}(\tilde{\lambda}_7) \right\},
\]

\(^4\)Of course, a similar situation is also to be expected in models with more than two scalar doublets.
\[ c_\beta \mu_{12}^2 = -s_\beta \text{Re} (\bar{\mu}_{12}^2) - \frac{v^2 c_\beta}{4} \left\{ \frac{4c_\beta^2 \lambda_1 + 4s_\beta^2 \left[ \lambda_3 + \lambda_4 + \text{Re} (\bar{\lambda}_5) \right]}{4s_\beta^2 \text{Re} (\bar{\lambda}_6) + 2s_\beta^2 t_\beta \text{Re} (\bar{\lambda}_7)} \right\}, \quad (17) \]

\[ s_\beta \mu_{22}^2 = -c_\beta \text{Re} (\bar{\mu}_{12}^2) - \frac{v^2 s_\beta}{4} \left\{ \frac{4s_\beta^2 \lambda_2 + 4c_\beta^2 \left[ \lambda_3 + \lambda_4 + \text{Re} (\bar{\lambda}_5) \right]}{+ 2s_\beta^2 t^{-1}_\beta \text{Re} (\bar{\lambda}_6) + 3s_\beta^2 \text{Re} (\bar{\lambda}_7)} \right\}. \quad (18) \]

Using eqs. (16)–(18), \( \mathcal{M}_0^2 \) is fully expressed in terms of \( \text{Re} (\bar{\mu}_{12}^2) \), \( \lambda_j \)'s and \( \{ v_1, v_2, \theta \} \). For the argument here, it is sufficient to consider \( \text{Tr}[\mathcal{M}_0^2] \) (for further details on \( \mathcal{M}_0^2 \), see Appendix [3]). In the mass eigenstate basis of eq. (15), \( \text{Tr}[\mathcal{M}_0^2] = m_h^2 + m_H^2 + m_A^2 \), while on the other hand

\[ \text{Tr}[\mathcal{M}_0^2] = -2(t_\beta + t_\beta^{-1}) \text{Re} (\bar{\mu}_{12}^2) + 2v^2 \left\{ \frac{2c_\beta^2 \lambda_1 + 2s_\beta^2 \lambda_2 - 2\text{Re} (\bar{\lambda}_5)}{+(s_\beta^2 - t_\beta^{-1}) \text{Re} (\bar{\lambda}_6) + (s_\beta^2 - t_\beta) \text{Re} (\bar{\lambda}_7)} \right\}. \quad (19) \]

Furthermore, the mass of the charged scalar is

\[ m_{H^\pm}^2 = -(t_\beta + t_\beta^{-1}) \text{Re} (\bar{\mu}_{12}^2) - \frac{v^2}{2} \left\{ 2[\lambda_4 + \text{Re} (\bar{\lambda}_5)] + t_\beta^{-1} \text{Re} (\bar{\lambda}_6) + t_\beta \text{Re} (\bar{\lambda}_7) \right\}. \quad (20) \]

The decoupling regime requires masses of the new scalars \( H, A, H^\pm \), much larger than the electroweak scale \( v \). In eqs. (19)–(20), one can roughly identify three scenarios where this could happen without requiring large \( \lambda_j \)'s.

\( (i) \) \( t_\beta^{-1} \gg 1 \) and \( -t_\beta^{-1} [\text{Re} (\bar{\mu}_{12}^2) + 2v^2 \text{Re} (\bar{\lambda}_6)/2] \gg v^2 \), and thus \( \mu_{12}^2 \simeq -t_\beta^{-1} [\text{Re} (\bar{\mu}_{12}^2) + 2v^2 \text{Re} (\bar{\lambda}_6)/2] \gg v^2 \). \( \mu_{22}^2 \gg |\mu_{11}^2| \). \( (21) \)

\( (ii) \) \( t_\beta \gg 1 \) and \( -t_\beta [\text{Re} (\bar{\mu}_{12}^2) + 2v^2 \text{Re} (\bar{\lambda}_7)/2] \gg v^2 \), and thus \( \mu_{12}^2 \simeq -t_\beta [\text{Re} (\bar{\mu}_{12}^2) + 2v^2 \text{Re} (\bar{\lambda}_7)/2] \gg v^2 \). \( \mu_{11}^2 \gg |\mu_{22}^2| \) \( (22) \)

\( (iii) \) \( -\text{Re} (\bar{\mu}_{12}^2) \gg v^2 \) without regard to \( \beta \). \( (23) \)

In the last case, for \( t_\beta \sim t_\beta^{-1} \sim O(1), -\text{Re} (\bar{\mu}_{12}^2) \sim \mu_{11}^2 \sim \mu_{22}^2 \).

The previous analysis can be rephrased in terms of the scalar potential in the Higgs basis of eq. (10):

\[ \mathcal{V}(H_1, H_2) = M^2_{11} H_1^\dagger H_1 + M^2_{22} H_2^\dagger H_2 + \left( M^2_{12} H_1^\dagger H_2 + \text{H.c.} \right) + \Lambda_1 (H_1^\dagger H_1)^2 + \Lambda_2 (H_2^\dagger H_2)^2 + 2\Lambda_3 (H_1^\dagger H_2)(H_2^\dagger H_1) + 2\Lambda_4 (H_1^\dagger H_1)(H_1^\dagger H_2) + \left( \Lambda_5 (H_1^\dagger H_1)^2 + \text{H.c.} \right) + \left( \Lambda_6 (H_1^\dagger H_2)^2 + \text{H.c.} \right) + \left( \Lambda_7 (H_2^\dagger H_2)^2 + \text{H.c.} \right). \quad (24) \]

\( M^2_{11}, M^2_{22} \) and \( \Lambda_i, i = 1 \) to 4, are real, while \( M^2_{12}, \Lambda_j, j = 5, 6, 7, \) can be complex.

Equations (19) and (20), expressed in terms of the parameters in eq. (24), are:

\[ \text{Tr}[\mathcal{M}_0^2] = 2M^2_{22} + 2v^2 [\Lambda_1 + \Lambda_3 + \Lambda_4], \quad (25) \]

\[ m_{H^\pm}^2 = M^2_{22} + v^2 \Lambda_3. \quad (26) \]
One can easily read that $M_{22}^2 \gg v^2$ leads to the decoupling regime: in the Higgs basis, the decoupling regime is simply achieved through a large mass term $M_{22}^2 H_1^\dagger H_2$, and there is no obstacle for that since $M_{22}^2$ does not participate in the minimization conditions. Of course, since $5 M_{22}^2 = s_\beta^2 \mu_{11}^2 + c_\beta^2 \mu_{22}^2 + s_\beta \Re(\bar{\mu}_{12}^2)$, one can substitute the stationarity conditions in eqs. (17)–(18) to obtain

$$M_{22}^2 = -(t_\beta + t_\beta^{-1}) \Re(\bar{\mu}_{12}^2) - v^2 \left\{ c_\beta^2 s_\beta^2 (\lambda_1 + \lambda_2) + (1 - 2 c_\beta^2 s_\beta^2) (\lambda_3 + \lambda_4 + \Re(\bar{\lambda}_5)) \right\}.$$  

Clearly, achieving $M_{22}^2 \gg v^2$ brings us back to eqs. (21)–(23). After this considerations on the general 2HDM, we now turn to 2HDMs with symmetry.

3 2HDM with symmetry

There are two classes of symmetric 2HDMs. In the first class, invariance under “Higgs family symmetries”

$$\Phi_j \mapsto U_{jk} \Phi_k, \quad U \in U(2),$$

leads to three different cases:

- **$Z_2$ symmetry**, with $\Phi_1 \mapsto -\Phi_1$, $\Phi_2 \mapsto \Phi_2$, and

  $$\mu_{12}^2 = 0, \quad \lambda_6 = \lambda_7 = 0,$$

- **$U(1)$ symmetry**, with $\Phi_1 \mapsto e^{i\tau} \Phi_1$, $\Phi_2 \mapsto \Phi_2$ ($\tau \neq 0, \pi$) and

  $$\mu_{12}^2 = 0, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0,$$

- **full $U(2)$ symmetry** with

  $$\mu_{22}^2 = \mu_{11}^2, \quad \mu_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_4 = \lambda_1 - \lambda_3, \quad \lambda_5 = \lambda_6 = \lambda_7 = 0.$$  

Following the discussion of the general 2HDM, it is clear that in all three cases, with $\mu_{12}^2 = 0$, the dimensionful $\mu_{ii}^2$ parameters can be traded for $\lambda_j$’s and vacuum expectation values, and thus non-decoupling is to be expected. In the $U(1)$ and $U(2)$ cases, having global continuous symmetries, spontaneous electroweak symmetry breaking leaves a massless scalar. The appearance of the unwanted massless scalars is avoided introducing soft symmetry breaking terms ($\mu_{11}^2 \neq \mu_{22}^2$ and $\mu_{12}^2 \neq 0$), which could also allow the existence of a decoupling regime. Since the focus in this section is on realistic 2HDMs

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5See [54] for general expressions relating the parameters in the scalar potential under changes of bases $\Phi_i \mapsto U_{ij} \Phi_j, U \in U(2)$.
with an exact symmetry, we do not consider these $U(1)$ and $U(2)$ invariant cases further.

From the point of vue of the scalar sector alone, since there is no unwanted massless scalar in the $\mathbb{Z}_2$ invariant case, we can have a viable model without the need to introduce soft symmetry breaking terms: the 2HDM with $\mathbb{Z}_2$ symmetry is discussed in subsection 3.1 below.

The second class of symmetric 2HDMs is given by symmetry transformations of the generalized CP type [55]

$$\Phi_j \mapsto U_{jk} \Phi_k^*.$$ \hfill (33)

There are, again, three distinct possibilities.

- Symmetry under the usual CP (also referred to as CP1),

$$\Phi_j \mapsto \Phi_j^* \text{ with all } \mu_{ij}^2, \lambda_j \text{ real.}$$ \hfill (34)

- CP2 symmetry with

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \mapsto \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} \Phi_1^* \\ \Phi_2^* \end{pmatrix} \text{ and } \mu_{22}^2 = \mu_{11}^2, \quad \mu_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_7 = -\lambda_6.$$ \hfill (35)

- CP3 symmetry with

$$\begin{pmatrix} \Phi_1 \\ \Phi_2 \end{pmatrix} \mapsto \begin{pmatrix} c_\tau & s_\tau \\ -s_\tau & c_\tau \end{pmatrix} \begin{pmatrix} \Phi_1^* \\ \Phi_2^* \end{pmatrix}, \quad 0 < \tau < \pi/2, \quad \text{and } \mu_{22}^2 = \mu_{11}^2, \quad \mu_{12}^2 = 0, \quad \lambda_2 = \lambda_1, \quad \lambda_5 = \lambda_1 - \lambda_3 - \lambda_4 \in \mathbb{R}, \quad \lambda_6 = \lambda_7 = 0.$$ \hfill (36)

While the usual CP in eq. (34) can be extended to the fermion sector easily (by requiring the Yukawa coupling matrices to be real), extending CP2 and CP3 to the fermion sector is much more involved. As discussed in [56], that is not achievable for the CP2 case, which forces the presence of massless fermions, while in the CP3 case, for $\tau = \pi/3$ in eq. (36), a viable model could, a priori, be constructed. Unfortunately, if the symmetry is exact, there is no mixing in the fermion sector, and one needs CP3 soft breaking terms, $\mu_{22}^2 \neq \mu_{11}^2$ and $\mu_{12}^2 \neq 0$, to overcome that difficulty. This soft breaking can still preserve the usual CP [56], and in that scenario one is led to a particular case of the more general “usual CP symmetry” scenario. Consequently, we focus on the 2HDM with usual CP symmetry, which is discussed in subsection 3.2.

Summarizing the discussion so far, two 2HDMs with an exact symmetry, $\mathbb{Z}_2$ or CP, are a priori viable. We analyse them in more detail in the following two subsections. Although they are not expected to have a decoupling regime, two exceptions arise, one for each symmetry, in which the resulting 2HDM does, nevertheless, have a decoupling regime.
3.1 2HDM with $\mathbb{Z}_2$ symmetry

Imposing symmetry under $\Phi_1 \mapsto -\Phi_1$, $\Phi_2 \mapsto \Phi_2$, the general 2HDM scalar potential in eq. (5) is reduced to

$$
\mathcal{V}(\Phi_1, \Phi_2) = \mu_1^2 \Phi_1^\dagger \Phi_1 + \mu_2^2 \Phi_2^\dagger \Phi_2 \\
+ \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + 2\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
+ \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \lambda_7^2 (\Phi_2^\dagger \Phi_1)^2,
$$

(37)

with $\mu_{jj}^2 \in \mathbb{R}$, $\lambda_k \in \mathbb{R}$ for $k \neq 5$. The stationarity conditions in eqs. (16)–(18) become

$$
0 = v^2 s_{2\beta} \text{Im} \left( \bar{\lambda}_5 \right),
$$

(38)

$$
c_\beta \mu_{11}^2 = -c_\beta v^2 \left\{ c_\beta^2 \lambda_1 + s_\beta^2 \left[ \lambda_3 + \lambda_4 + \text{Re} \left( \bar{\lambda}_5 \right) \right] \right\},
$$

(39)

$$
s_\beta \mu_{22}^2 = -s_\beta v^2 \left\{ s_\beta^2 \lambda_2 + c_\beta^2 \left[ \lambda_3 + \lambda_4 + \text{Re} \left( \bar{\lambda}_5 \right) \right] \right\}.
$$

(40)

One should distinguish between the two cases $s_{2\beta} = 0$ and $s_{2\beta} \neq 0$. Furthermore, a rephasing of the fields only amounts to a rephasing of $\lambda_5$ and thus, without loss of generality, one can set $\text{Im} \left( \lambda_5 \right) = 0$ and $\text{Re} \left( \lambda_5 \right) = \lambda_5$. In that case, eq. (37) can be written in terms of real parameters: as is well known, imposing an exact $\mathbb{Z}_2$ there is no CP violation in the 2HDM.

3.1.1 Inert 2HDM

For $2v_1 v_2 = v^2 s_{2\beta} = 0$, that is either $s_\beta = 0$ or $c_\beta = 0$, the basis $\{ \Phi_1, \Phi_2 \}$ and the Higgs basis $\{ H_1, H_2 \}$ coincide: $s_\beta = 0$ or $c_\beta = 0$ correspond to the two possible identifications $\Phi_1 = H_1$ or $\Phi_1 = H_2$. This 2HDM, together with a fermion sector which only couples to the scalar doublet which acquires a vacuum expectation value (owing to the $\mathbb{Z}_2$ symmetry), is the inert 2HDM \cite{57}, which provides, economically, a dark matter candidate \cite{58, 60}. Then, eq. (38) is trivially satisfied while eqs. (39)–(40) give

$$
\begin{align*}
\text{for } s_\beta = 0, & \quad \left\{ \begin{array}{l}
\text{eq. (39)} \Rightarrow \mu_{11}^2 = -v^2 \lambda_1, \\
\text{eq. (40)} \text{ trivially satisfied, arbitrary } \mu_{22}^2.
\end{array} \right. \\
\text{for } c_\beta = 0, & \quad \left\{ \begin{array}{l}
\text{eq. (39)} \text{ trivially satisfied, arbitrary } \mu_{11}^2, \\
\text{eq. (40)} \Rightarrow \mu_{22}^2 = -v^2 \lambda_2.
\end{array} \right.
\end{align*}
$$

(41) (42)

One can now obtain

$$
\mathcal{M}_0^2 = \text{diag} \left\{ 2v^2, \mu^2 + v^2 \left[ \lambda_3 + \lambda_4 + \text{Re} \left( \bar{\lambda}_5 \right) \right], \mu^2 + v^2 \left[ \lambda_3 + \lambda_4 - \text{Re} \left( \bar{\lambda}_5 \right) \right] \right\},
$$

(43)

with

$$
\lambda = \lambda_1, \quad \mu^2 = \mu_{22}^2 \quad \text{for } s_\beta = 0, \quad \text{and} \quad \lambda = \lambda_2, \quad \mu^2 = \mu_{11}^2 \quad \text{for } c_\beta = 0.
$$

(44)

The charged scalar mass is

$$
m_{H^\pm}^2 = \mu^2 + v^2 \lambda_3.
$$

(45)

The decoupling regime is simply obtained with $\mu^2 \gg v^2$, as anticipated in section 2.2 in the discussion of decoupling in the Higgs basis. The additional ingredient in the inert 2HDM is the requirement of $\mathbb{Z}_2$ invariance in the Higgs basis.
3.1.2 $Z_2$-2HDM

For $2v_1v_2 = v^2s_2 \neq 0$ we have the "$Z_2$-2HDM". Eq. (38) requires $\text{Im} (\lambda_5) = 0$, that is $2\theta = -\text{arg}(\lambda_5) [\pi]$, as mentioned after eq. (40), one can set $\text{Im} (\lambda_5) = 0$ with a simple rephasing, in which case we simply yield eqs. (17)–(18) with $\theta = 0$. Then, eqs. (39)–(40) impose

$$\mu_{11}^2 = -v^2 \{ c_\beta^2 \lambda_1 + s_\beta^2 [\lambda_3 + \lambda_4 + \lambda_5] \},$$

$$\mu_{22}^2 = -v^2 \{ s_\beta^2 \lambda_2 + c_\beta^2 [\lambda_3 + \lambda_4 + \lambda_5] \}.$$  

From the mass matrix of the neutral scalars in Appendix B, one can directly read

$$m_h^2 = 2v^2 \{ \lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 \}, \quad m_\lambda^2 = -2v^2 \lambda_5,$$

while the mass of $H^\pm$ is

$$m_{H^\pm}^2 = -v^2 (\lambda_4 + \lambda_5).$$

It is clear, attending to eqs. (48) and eq. (49), that the $Z_2$-2HDM does not have a decoupling regime. A detailed numerical study of the model is addressed in section 4.

We now turn to the 2HDM with CP symmetry.

3.2 2HDM with CP Symmetry

Following eq. (34), the 2HDM scalar potential

$$V(\Phi_1, \Phi_2) = \mu_{11}^2 \Phi_1^\dagger \Phi_1 + \mu_{22}^2 \Phi_2^\dagger \Phi_2 + \mu_{12}^2 (\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + 2\lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + 2\lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1)$$

$$+ \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_1)^2] + (\lambda_6 \Phi_1^\dagger \Phi_1 + \lambda_7 \Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1),$$

with all $\mu_{ij}^2, \lambda_j$ real, respects CP invariance, eq. (34). The stationarity conditions in eqs. (16)–(18) become

$$\mu_{12}^2 s_\theta = -\frac{v^2}{2} \{ s_{2\beta} \lambda_5 s_{2\theta} + c_\beta^2 \lambda_6 s_\theta + s_\beta^2 \lambda_7 s_\theta \},$$

$$\mu_{11}^2 = -t_\beta \mu_{12}^2 c_\theta - \frac{v^2}{4} \{ 4c_\beta^2 \lambda_1 + 4s_\beta^2 [\lambda_3 + \lambda_4 + \lambda_5 c_{2\theta}] + 3s_{2\beta} \lambda_6 c_\theta + 2s_\beta^2 t_\beta \lambda_7 c_\theta \},$$

$$\mu_{22}^2 = -t_\beta^2 \mu_{12}^2 c_\theta - \frac{v^2}{4} \{ 4s_\beta^2 \lambda_2 + 4c_\beta^2 [\lambda_3 + \lambda_4 + \lambda_5 c_{2\theta}] + 2c_\beta^2 t_\beta \lambda_6 c_\theta + 3s_{2\beta} \lambda_7 c_\theta \}. $$

Attending to eq. (51), one should now distinguish between two cases, $s_\theta = 0$ and $s_\theta \neq 0$, that we address in turn.

3.2.1 Real 2HDM

For $s_\theta = 0$, eq. (51) is fulfilled without regard to $\mu_{12}^2, \lambda_5, \lambda_6$ and $\lambda_7$. Then, eqs. (52)–(53) simply yield eqs. (17)–(18) with

$$\mu_{12}^2 \mapsto \pm \mu_{12}^2, \quad \lambda_5 \mapsto \lambda_5, \quad \lambda_6 \mapsto \pm \lambda_6, \quad \lambda_7 \mapsto \pm \lambda_7.$$
3.2.2 SCPV-2HDM

For $\theta \neq 0$, we have the “SCPV-2HDM”, which incorporates a spontaneous origin for CP Violation. The stationarity conditions, as anticipated, allow us to trade all $\mu_{ij}$ for $\lambda_j$'s, $v$, $\beta$ and $\theta$:

\[
\begin{align*}
\mu_{12}^2 &= -\frac{v^2}{2} \left[ 4 \lambda_5 c_\beta s_\beta c_\theta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \right], \\
\mu_{11}^2 &= -v^2 [\lambda_1 c_\beta^2 + (\lambda_3 + \lambda_4 - \lambda_5) s_\beta^2 + \lambda_6 c_\beta s_\beta c_\theta], \\
\mu_{22}^2 &= -v^2 [\lambda_2 s_\beta^2 + (\lambda_3 + \lambda_4 - \lambda_5) c_\beta^2 + \lambda_7 c_\beta s_\beta c_\theta].
\end{align*}
\]

That is, one can choose a potential in eq. (50) where $\mu_{12}^2$, $\mu_{11}^2$ and $\mu_{22}^2$ are given in eqs. (55)–(57), which depend on $\lambda_j$ ($j = 1$ to 7), $v$, $\beta$ and $\theta$. The mass of the charged scalar $H^\pm$ is

\[m_{H^\pm}^2 = v^2(\lambda_5 - \lambda_4)\]

and, for the neutral scalars, following appendix [3] we have

\[\text{Tr}[M_0^2] = m_h^2 + m_{H^0}^2 + m_A^2 = v^2 \left\{ 2(\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + \lambda_3) + (\lambda_6 + \lambda_7)s_\beta c_\theta \right\}.
\]

Equations eq. (58) and eq. (59) show that the SCPV-2HDM, like the $Z_2$-2HDM, does not have a decoupling regime if perturbativity constraints are respected. The obvious and most relevant consequence is that the masses of the new scalars are forced to be, roughly, below 1 TeV, and thus phenomenologically interesting.

As a closing remark for this section, it is also to be noticed that the two exceptional cases which have a decoupling regime, the inert 2HDM and the real 2HDM, one for each symmetry, appear when the vacuum also respects the imposed symmetry.

4 Analysis

As discussed in the previous section, two 2HDMs with an exact symmetry can be viable while having no decoupling regime. In this section they are analysed in detail to explore their allowed parameter space, with a special emphasis on the masses of the new scalars. One should, of course, impose a number of relevant constraints. They are:

- $m_h = 125$ GeV and $v = 246$ GeV (rather than a constraint, with appropriate parametrisations of the models, this simply amounts to an election of parameter values);

- agreement with electroweak precision observables, in particular the oblique parameters $S$ and $T$ [61];

- $2 \rightarrow 2$ high energy scattering is perturbatively unitary (see appendix [A] for details);

- perturbativity, i.e. $|\lambda_j| < 4\pi$; although the high energy scattering constraint is sufficient, over most parameter space, to ensure that $|\lambda_j| < 4\pi$, the constraint is nevertheless imposed;
the scalar potential is bounded from below and the considered vacuum is the global minimum of the potential. For the $Z_2$-2HDM, this is guaranteed by the following analytic requirements:

\[
\lambda_1 > 0, \quad \lambda_2 > 0, \quad \sqrt{\lambda_1 \lambda_2} > -\lambda_3, \quad \sqrt{\lambda_1 \lambda_2} > |\bar{\lambda}_5| - \lambda_3 - \lambda_4,
\]

and

\[
\left[ \left( \frac{m_{H+}^2}{v^2} + \lambda_4 \right)^2 - |\bar{\lambda}_5|^2 \right] \left[ \frac{m_{H+}^2}{v^2} + \sqrt{\lambda_1 \lambda_2} - \lambda_3 \right] > 0.
\]

For the SCPV-2HDM, there are no simple analytic requirements as the previous, and the complete procedure described in [62] is adopted.

In these analyses, the focus is only in the scalar sector and thus constraints that require the specification of scalar-fermion couplings are not considered, like for example constraints from flavour changing transitions or from LHC production and decay processes. One cannot ignore, however, that the 125 GeV scalar is quite “SM-like” [65]: in order to reflect this, a lower bound is forced on the scalar mixing element $R_{11}$. Furthermore, since no direct limits are imposed on the masses of the new scalars, the results shown in the following separate, for illustration, the sequence of regions in Figure 1 (from left to right, each one includes the next).

For the $Z_2$-2HDM and the SCPV-2HDM, respectively, the allowed regions for the masses of the new scalars are shown in Figures 2 and 3. One can clearly observe that, as anticipated, no decoupling regime is allowed: masses of the new scalars above 1 TeV are not allowed as a result of the perturbativity requirements. Complementing Figures 2 and 3, Figures 4 and 5 show allowed regions for different “spherical slices” of $\bar{m} \equiv \sqrt{m_H^2 + m_A^2 + m_{H^\pm}^2}$: notice the diminishing size of the allowed regions as $\bar{m}$ increases; for $\bar{m} \sim 900\sqrt{3}$ GeV, there are no allowed regions anymore. The allowed regions in both models do not differ substantially, and their shape is mainly determined by the oblique parameters $S$ and $T$.

As expected, Figures 2 and 3 illustrate that the $Z_2$-2HDM and the SCPV-2HDM do not have a decoupling regime when the quartic couplings $\lambda_j$ obey perturbativity requirements. There is, however, a puzzling aspect. As discussed in section 3, the }

\footnote{Although in the popular $Z_2$ symmetric 2HDMs of types I, II (and X,Y, when the lepton sector is also considered) there is flavour conservation and all Yukawa couplings are fixed in terms of the quark masses and $\tan \beta$, that is not the case for other 2HDMs where the $Z_2$ symmetry has a more involved realization in the fermion sector, and which have controlled SFCNC which depend on additional parameters [63,64].}
Figure 2: Allowed regions for the masses of the new scalars in the $Z_2$-2HDM (following conventions in Fig. 1).

Figure 3: Allowed regions for the masses of the new scalars in the SCPV-2HDM (following conventions in Fig. 1).

Figure 4: Allowed regions for the masses of the new scalars in the $Z_2$-2HDM (following conventions in Fig. 1), for different spherical slices in $\bar{m} = \sqrt{m_H^2 + m_A^2 + m_{H\pm}^2}$; the dashed straight line goes from the origin to the point $m_H = m_A = m_{H\pm} = \bar{m}/\sqrt{3}$.

Inert and the real 2HDMs do have a decoupling regime compatible with perturbativity requirements. The inert 2HDM would correspond to $s_{2\beta} \rightarrow 0$ in the $Z_2$-2HDM while the real 2HDM would correspond to $s_\theta \rightarrow 0$ in the SCPV-2HDM. One could have
expected, accordingly, the appearance of a decoupling regime in Figures 2 and 3 due to these limits, \( s_{2\beta} \to 0 \) and \( s_\theta \), respectively. Why is that not the case?

Consider first the \( \mathbb{Z}_2 \)-2HDM. Assuming \( s_{2\beta} \neq 0 \), eqs. (39)–(40) are equivalent to eqs. (46)–(47); however, for \( s_{2\beta} \to 0 \), eq. (40) is trivially satisfied with free \( \mu_{22}^2 \) while eq. (47) gives \( \mu_{22}^2 = -v^2(\lambda_3 + \lambda_4 + \lambda_5) \). It is then clear that the inert 2HDM is not recovered in the limit \( s_\beta \to 0 \): one cannot recover, by construction, a free \( \mu_{22}^2 \) in the \( \mathbb{Z}_2 \)-2HDM. (The argument applies similarly to \( c_\beta \to 0 \) and \( \mu_{11}^2 \)). It is to be noticed, in addition, that the limit \( s_{2\beta} \to 0 \) in the \( \mathbb{Z}_2 \)-2HDM gives \( m_A \to 0 \) (see eq. (97) in appendix B.3).

With \( m_{\text{Min}} \equiv \text{Min}(m_H, m_A, m_{H\pm}) \), Figure 6 illustrates the previous discussion (in Fig. 6(a), \( s_{2\beta} \to 0 \) corresponds to \( t_{1\beta}^\pm \to \infty \)).

### 5 Decoupling and naturalness for softly broken \( \mathbb{Z}_2 \) or CP symmetries

As already mentioned, the introduction of soft symmetry breaking terms, that is symmetry breaking terms with mass dimension smaller than 4, opens the possibility of having a decoupling regime where \( m_A, m_H, m_{H\pm} \gg v \). In general, an important motivation backing the introduction of soft symmetry breaking terms is the following. Since the renormalization group evolution of the soft terms (which are relevant operators), enhances them in the evolution from higher energy scales down to the electroweak scale, one can think of them, at low energies, as arising from a scenario with the symmetry almost exactly realized at high energies. On the issue of decoupling there is, however, a puzzling aspect: while the model with exact symmetry does not have a decoupling regime, a completely different qualitative regime where decoupling is possible does appear when the symmetry is softly broken. In this section it is analysed how, from the point of view of the imposed symmetry, the decoupling can be interpreted.
Figure 6: $m_{\text{Min}}$ vs. $t_\beta$, $s_\theta$ (following conventions in Fig. 1): for $t_\beta^{-1} \to \infty$ in the $Z_2$-2HDM, $m_{\text{Min}} = m_H \to 0$; for $s_\theta \to 0$ in the SCPV-2HDM, $m_{\text{Min}} = m_A \to 0$.

as an unnatural regime, since it involves an implicit fine tuning of all the soft parameters, the symmetry breaking and the symmetry allowed ones. Although fine tuning arguments have been invoked in wider contexts which include a 2HDM sector (e.g. supersymmetric extensions of the SM), there is no pretence, however, that fine tuning in the decoupling regime constitutes a source of concern in the context of the 2HDMs analysed here.

Notice that, attending to the discussion on the Higgs basis and decoupling in section 2.2, one can nevertheless argue that the decoupling regime simply corresponds to the regime in which $H_2$, the scalar doublet which does not acquire a vacuum expectation value, has a mass term much larger than $v$. It would appear that there is no naturalness or fine tuning question in that case. This argument is independent of the presence of the symmetry and, to some extent, misleading: in order to obtain decoupling of $H_2$, the eventual fine tuning is already encoded in the coefficients of the scalar potential rewritten in terms of $H_1$ and $H_2$, which do not have well defined symmetry transformation properties. The exception arises again with the inert and the real 2HDMs, in which the exact symmetry holds in the Higgs basis; one can nevertheless interpret that they require fine tuning to reach a decoupling regime, as commented at the end of this section, after eq. (84).

5.1 $Z_2$-2HDM with soft symmetry breaking

In the $Z_2$-2HDM, the $Z_2$ symmetry is softly broken by adding the term $\mu_2^2 \Phi_1^* \Phi_2 + \text{H.c.}$ to $V(\Phi_1, \Phi_2)$ in eq. (37). Instead of the stationarity conditions in eqs. (38)–(40), we now
have

\[ \text{Im}(\tilde{\mu}_{12}) = -v^2 c_\beta s_\beta \text{Im}(\tilde{\lambda}_5), \]
\[ c_\beta \mu_{11}^2 = -s_\beta \text{Re}(\tilde{\mu}_{12}^2) - c_\beta v^2 \left\{ c_\beta^2 \lambda_1 + s_\beta^2 \left[ \lambda_3 + \lambda_4 + \text{Re}(\tilde{\lambda}_5) \right] \right\}, \]
\[ s_\beta \mu_{22}^2 = -c_\beta \text{Re}(\tilde{\mu}_{12}^2) - s_\beta v^2 \left\{ s_\beta^2 \lambda_2 + c_\beta^2 \left[ \lambda_3 + \lambda_4 + \text{Re}(\tilde{\lambda}_5) \right] \right\}. \]

Then, the mass of the charged scalar \( H^\pm \) is

\[ m_{H^\pm}^2 = -(t_\beta + t_\beta^{-1}) \text{Re}(\tilde{\mu}_{12}^2) - v^2 (\lambda_4 + \text{Re}(\tilde{\lambda}_5)), \]

while, from the mass matrix of the neutral scalars,

\[ \text{Tr}[\mathcal{M}_0^2] = -2(t_\beta + t_\beta^{-1}) \text{Re}(\tilde{\mu}_{12}^2) + 2v^2 \left[ c_\beta^2 \lambda_1 + s_\beta^2 \lambda_2 - \text{Re}(\tilde{\lambda}_5) \right]. \]

The decoupling regime requires \(-(t_\beta + t_\beta^{-1}) \text{Re}(\tilde{\mu}_{12}^2) \gg v^2\); then, eqs. (63)–(64) imply

\[ \text{for } t_\beta \sim \mathcal{O}(1), \quad \mu_{11}^2 \sim \mu_{22}^2 \sim -\text{Re}(\tilde{\mu}_{12}^2), \]
\[ \text{for } t_\beta^{-1} \gg 1, \quad \mu_{11}^2 \sim v^2, \quad \mu_{22}^2 \gg v^2, \]
\[ \text{for } t_\beta \gg 1, \quad \mu_{22}^2 \sim v^2, \quad \mu_{22}^2 \gg v^2. \]

For \( t_\beta \sim \mathcal{O}(1) \), fine tuning manifests in the fact that, while \( \mu_{11}^2 \) and \( \mu_{22}^2 \) respect the symmetry and \( \text{Re}(\tilde{\mu}_{12}^2) \) does not, they all need to be of similar size.

On the other hand, for \( t_\beta \gg 1 \) or \( t_\beta^{-1} \gg 1 \), fine tuning manifests in the strong hierarchy among \( \mu_{11}^2 \) and \( \mu_{22}^2 \) (or equivalently in the strong hierarchy among the vacuum expectation values): such a hierarchy is not motivated by the symmetry. Without entering a discussion on, or invoking, more sophisticated measures of fine tuning (see e.g. [66]), one can illustrate the previous argument within a numerical analysis along the lines of section 4, performed in this case for the \( \mathbb{Z}_2 \)-2HDM with soft symmetry breaking.

Consider first

\[ t_{12}^2 = \frac{\mu_{22}^2 - \mu_{11}^2}{\mu_{22}^2 + \mu_{11}^2}, \quad \text{with} \quad \left\{ \begin{array}{c} \mu_{\text{Max}}^2 \equiv \text{Max}(\mu_{11}^2, |\mu_{22}^2|) \\ \mu_{\text{Min}}^2 \equiv \text{Min}(\mu_{11}^2, |\mu_{22}^2|) \end{array} \right. \]

With no fine tuning, one expects \( \mu_{11}^2 \sim \mu_{22}^2 \), that is \( t_{12}^2 \ll 1 \).

Consider also

\[ t_{22}^2 = \frac{|\text{Re}(\tilde{\mu}_{12}^2)|}{\mu_{\text{Min}}^2}. \]

With no fine tuning one also expects \( t_{22}^2 \ll 1 \) \( (\text{Re}(\tilde{\mu}_{12}^2) \) violates the symmetry while \( \mu_{\text{Min}}^2 \) respects it). Defining

\[ t_{22}^2 \equiv \text{Max}(t_{12}^2, t_{22}^2), \]

Figure 7(a) shows \( t_{22}^2 \) vs. \( m_{\text{Min}} \equiv \text{Min}(m_H, m_A, m_{H^\pm}) \); large values of \( m_{\text{Min}} \) correspond to the decoupling regime. It is clear that \( m_{\text{Min}} \gg v \) cannot be achieved with \( t_{22}^2 \ll 1 \), that is without fine tuning.
5.2 SCPV-2HDM with soft symmetry breaking

In the SCPV-2HDM, the CP symmetry is softly broken for Im(\(\mu_{12}^2\)) \(\neq 0\) in the scalar potential in eq. (50). This model [67] has been extensively explored (see, e.g. [68]). Instead of the stationarity conditions in eqs. (51)–(53), we now have

\[
\text{Re} \left( \mu_{12}^2 \right) = -t_{\theta}^{-1} \text{Im} \left( \mu_{12}^2 \right) - \frac{v^2}{2} \left\{ 2s_2c_\theta \lambda_5 + c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7 \right\},
\]

\[
c_\beta \mu_{11}^2 = \frac{s_\beta}{s_\theta} \text{Im} \left( \mu_{12}^2 \right) - \frac{v^2}{2} c_\beta \left\{ c_\beta^2 \lambda_1 + s_\beta^2 \left[ \lambda_3 + \lambda_4 - \lambda_5 \right] + s_\beta c_\beta c_\theta \lambda_6 \right\},
\]

\[
s_\beta \mu_{22}^2 = \frac{c_\beta}{s_\theta} \text{Im} \left( \mu_{12}^2 \right) - \frac{v^2}{2} s_\beta \left\{ s_\beta^2 \lambda_2 + c_\beta^2 \left[ \lambda_3 + \lambda_4 - \lambda_5 \right] + s_\beta c_\beta c_\theta \lambda_7 \right\}.
\]

Then, the mass of the charged scalar H\(\pm\) is

\[
m_{H\pm}^2 = \frac{t_{\beta} + t_{\beta}^{-1}}{s_\theta} \text{Im} \left( \mu_{12}^2 \right) + v^2 (\lambda_5 - \lambda_4),
\]

while, from the mass matrix of the neutral scalars,

\[
\text{Tr} \left[ \mathcal{M}_\theta^2 \right] = 2 \frac{t_{\beta} + t_{\beta}^{-1}}{s_\theta} \text{Im} \left( \mu_{12}^2 \right) + 2v^2 \left[ c_\beta^2 \lambda_1 + s_\beta^2 \lambda_2 + \lambda_5 + c_\beta s_\beta c_\theta (\lambda_6 + \lambda_7) \right].
\]

The decoupling regime requires \(t_{\beta} + t_{\beta}^{-1} \overset{s_\theta}{\sim} \text{Im} \left( \mu_{12}^2 \right) \gg v^2\). For \(s_\theta \lesssim 1\), the situation is similar to the \(\mathbb{Z}_2\)-2HDM case: from eqs. (73)–(75),

\[
s_\theta \lesssim 1, \text{ for } t_{\beta} \sim O(1), \quad \mu_{11}^2 \sim \mu_{22}^2 \sim \text{Re} \left( \mu_{12}^2 \right) \sim \text{Im} \left( \mu_{12}^2 \right),
\]

\[
s_\theta \lesssim 1, \text{ for } t_{\beta}^{-1} \gg 1, \quad \mu_{11}^2 \sim v^2, \quad \mu_{22}^2 \gg v^2, \quad \text{Re} \left( \mu_{12}^2 \right) \sim \text{Im} \left( \mu_{12}^2 \right),
\]

\[
s_\theta \lesssim 1, \text{ for } t_{\beta} \gg 1, \quad \mu_{22}^2 \sim v^2, \quad \mu_{11}^2 \gg v^2, \quad \text{Re} \left( \mu_{12}^2 \right) \sim \text{Im} \left( \mu_{12}^2 \right).
\]

The reasoning on fine tuning in the \(\mathbb{Z}_2\)-2HDM applies directly to the previous equations (with the only change that \text{Re} \left( \mu_{12}^2 \right) is symmetry preserving while \text{Im} \left( \mu_{12}^2 \right) is symmetry violating); for \(s_\theta \ll 1\), however, such fine tuning seems absent, since we have \(\mu_{11}^2 \sim \mu_{22}^2 \sim \text{Re} \left( \mu_{12}^2 \right) \gg \text{Im} \left( \mu_{12}^2 \right)\). The point is that, rather than in the \(\mu_{12}^2\), fine tuning in this regime directly corresponds to choosing \(s_\theta \ll 1\) in a model in which, by construction, \(s_\theta \neq 0\). As in the \(\mathbb{Z}_2\)-2HDM case, one can illustrate the previous arguments within a numerical analysis of the SCPV-2HDM with soft symmetry breaking. We now have

\[
t_{1}^{SCPV} = \frac{\mu_{\text{Max}}^2 - \mu_{\text{Min}}^2}{\mu_{\text{Max}}^2 + \mu_{\text{Min}}^2}, \quad \text{with} \quad \left\{ \begin{array}{l} 
\mu_{\text{Max}}^2 \equiv \text{Max} \left( |\mu_{11}^2|, |\mu_{22}^2|, |\text{Re} \left( \mu_{12}^2 \right)| \right) \\
\mu_{\text{Min}}^2 \equiv \text{Min} \left( |\mu_{11}^2|, |\mu_{22}^2|, |\text{Re} \left( \mu_{12}^2 \right)| \right)
\end{array} \right.
\]

\[n_{2}^{SCPV} = \frac{\text{Im} \left( \mu_{12}^2 \right)}{\mu_{\text{Min}}^2}, \]

such that with no fine tuning one expects \(n_{1}^{SCPV}, n_{2}^{SCPV} \ll 1\). In addition, to include the eventual fine tuning associated to \(s_\theta \ll 1\) in the analysis, we simply consider

\[n_{3}^{SCPV} \equiv 1 - |s_\theta|, \]

\[17\]
in which case, with no fine tuning, one expects $t^{\text{SCPV}}_3 \ll 1$. Defining

$$t^{\text{SCPV}} \equiv \max(t^{\text{SCPV}}_1, t^{\text{SCPV}}_2, t^{\text{SCPV}}_3),$$

Figure 7(b) shows $t^{\text{SCPV}}$ vs. $m_{\text{Min}} \equiv \min(m_H, m_A, m_{H^\pm})$; again, it is clear that $m_{\text{Min}} \gg v$ cannot be achieved with $t^{\text{SCPV}} \ll 1$, that is without fine tuning.

To close this section, a final comment concerning the models in which the exact symmetry holds in the Higgs basis (the inert and the real 2HDMs) and no soft symmetry breaking terms are required to obtain a decoupling regime. As mentioned in section 2.2, decoupling is achieved with $M_{22}^2 \gg v^2$; in these models both $M_{11}^2$ and $M_{22}^2$ are symmetry preserving. If one subscribes the previous discussions on fine tuning in the $Z_2$-2HDM and the SCPV-2HDM, it is clear that in decoupling regime of the inert and the real 2HDMs, fine tuning is required to have $M_{22}^2 \gg |M_{11}^2|$, since such a hierarchy is alien to the imposed symmetry.

Conclusions

In this work, the possibility that perturbativity requirements on the quartic couplings of a 2HDM could imply that there is no decoupling regime available, that is, all the new scalars cannot have large masses, is analysed. It is discussed how in two models with an exact symmetry, the $Z_2$-2HDM and the SCPV-2HDM, that decoupling regime is absent. Detailed numerical analyses illustrate the point, showing that the new scalars have masses lighter than 1 TeV. Allowed ranges for these masses are fairly similar in both models. For these exact symmetries, it is also shown that a decoupling regime can nevertheless appear for one specific vacuum configuration in each case. Finally, the introduction of soft symmetry breaking terms allows for the appearance of a decoupling
regime: it is argued that this situation might be viewed as unnatural, since it involves a fine tuning of parameters not justified by the symmetry.

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A Perturbative unitarity

At high energies, $2 \rightarrow 2$ scattering processes in the scalar sector are controlled by the quartic couplings $\lambda_j$; the corresponding $2 \rightarrow 2$ tree level scattering matrix $S$ is block diagonal, since the total hypercharge $Y$ and weak isospin $I$ are conserved in that limit $[22–29]$ (for a recent one loop analysis, see $[69]$). The resulting submatrices $S_{[Y,I]}$ are

$$S_{[1,1]} = \frac{1}{8\pi} \begin{pmatrix} \lambda_1 & \lambda_5 & \sqrt{2}\lambda_6 \\ \sqrt{2}\lambda_5^* & \lambda_2 & \sqrt{2}\lambda_7^* \\ \sqrt{2}\lambda_6^* & \sqrt{2}\lambda_7 & \lambda_3 + \lambda_4 \end{pmatrix},$$

$$S_{[1,0]} = \frac{1}{8\pi} (\lambda_3 - \lambda_4),$$

$$S_{[0,1]} = \frac{1}{8\pi} \begin{pmatrix} \lambda_1 & \lambda_4 & \lambda_6 & \lambda_6^* \\ \lambda_4 & \lambda_2 & \lambda_7 & \lambda_7^* \\ \lambda_6^* & \lambda_7^* & \lambda_3 & \lambda_5 \\ \lambda_6 & \lambda_7 & \lambda_5 & \lambda_3 \end{pmatrix},$$

$$S_{[0,0]} = \frac{1}{8\pi} \begin{pmatrix} 3\lambda_1 & 2\lambda_3 + \lambda_4 & 3\lambda_6 & 3\lambda_6^* \\ 2\lambda_3 + \lambda_4 & 3\lambda_2 & 3\lambda_7 & 3\lambda_7^* \\ 3\lambda_6^* & 3\lambda_7^* & \lambda_3 + 2\lambda_4 & 3\lambda_5^* \\ 3\lambda_6 & 3\lambda_7 & 3\lambda_5 & \lambda_3 + 2\lambda_4 \end{pmatrix}.$$  

Requiring that the different $S_{[Y,I]}$ do not yield probabilities larger than 1 is the perturbative unitarity requirement; that is, for values of $\{\lambda_1, \lambda_2, \ldots, \lambda_7\}$ such that some eigenvalue of the above matrices is larger than 1, that point in parameter space is not acceptable.

In the analyses of sections 4 and 5, for the $Z_2$-2HDM one has $\lambda_6 = \lambda_7 = 0$ and the perturbative unitarity requirement can be reformulated easily in terms of analytic conditions. For the SCPV-2HDM that is not the case, and the eigenvalues of $S_{[Y,I]}$ are computed numerically.

B Mass matrices of Neutral Scalars

In this appendix, the elements of the mass matrices of the neutral scalars are shown for the general 2HDM (including expressions in the Higgs basis), for the $Z_2$-2HDM
and for the SCPV-2HDM, for the \( \mathbb{Z}_2 \)-2HDM with soft symmetry breaking, and for the SCPV-2HDM with soft symmetry breaking. In obtaining these mass matrices, the stationarity conditions are, of course, used; for completeness, the mass of the charged scalar \( H^\pm \) is shown again.

## B.1 General 2HDM

For the general 2HDM in section 2.2 the mass matrix of the neutral scalars is given by

\[
[M^2_{0}]_{11} = v^2 \left\{ 2\lambda_1 c_3^2 + 2\lambda_2 s_3^2 + [\lambda_3 + \lambda_4 + \Re (\bar{\lambda}_5)] s_{2\beta}^2 + 2(\Re (\bar{\lambda}_6) c_3^2 + \Re (\bar{\lambda}_7) s_3^2) s_{2\beta} \right\},
\]

\[
[M^2_{0}]_{12} = \frac{v^2}{2} \left\{ 2 \left[ -\lambda_1 c_3^2 + \lambda_2 s_3^2 \right] s_{2\beta} + [\lambda_3 + \lambda_4 + \Re (\bar{\lambda}_5)] s_{4\beta} + \Re (\bar{\lambda}_6) (c_{2\beta} + c_{4\beta}) + 2\Re (\bar{\lambda}_7) s_{\beta} s_{3\beta} \right\},
\]

\[
[M^2_{0}]_{22} = -(t_\beta + t_\beta^{-1}) \Re (\bar{\mu}_{12}^2) + \frac{v^2}{2} \left\{ s_{2\beta}^2 [\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \Re (\bar{\lambda}_5))] \right\},
\]

\[
[M^2_{0}]_{13} = -v^2 \left\{ \Im (\bar{\lambda}_5) s_{2\beta} + c_\beta \Im (\bar{\lambda}_6) + s_\beta \Im (\bar{\lambda}_7) \right\},
\]

\[
[M^2_{0}]_{23} = -\frac{v^2}{2} \left\{ 2\Im (\bar{\lambda}_5) c_{2\beta} + (\Im (\bar{\lambda}_7) - \Im (\bar{\lambda}_6)) s_{2\beta} \right\},
\]

\[
[M^2_{0}]_{33} = -(t_\beta + t_\beta^{-1}) \Re (\bar{\mu}_{12}^2) - \frac{v^2}{2} \left\{ 4\Re (\bar{\lambda}_5) + t_\beta^{-1} \Re (\bar{\lambda}_6) + t_\beta \Re (\bar{\lambda}_7) \right\}.
\]

The mass of the charged scalar is

\[
m_{H^\pm}^2 = -(t_\beta + t_\beta^{-1}) \Re (\bar{\mu}_{12}^2) - \frac{v^2}{2} \left\{ 2[\lambda_4 + \Re (\bar{\lambda}_5)] + t_\beta^{-1} \Re (\bar{\lambda}_6) + t_\beta \Re (\bar{\lambda}_7) \right\}.
\]

In terms of parameters in the Higgs basis

\[
[M^2_{0}]_{11} = 2v^2 \Lambda_1,
\]

\[
[M^2_{0}]_{12} = v^2 \Re (\Lambda_6),
\]

\[
[M^2_{0}]_{22} = M_{22}^2 + v^2 \{ \Lambda_3 + \Lambda_4 + \Re (\Lambda_5) \},
\]

\[
[M^2_{0}]_{13} = -v^2 \Im (\Lambda_6),
\]

\[
[M^2_{0}]_{23} = -v^2 \Im (\Lambda_5),
\]

\[
[M^2_{0}]_{33} = M_{22}^2 + v^2 \{ \Lambda_3 + \Lambda_4 - \Re (\Lambda_5) \},
\]

and

\[
m_{H^\pm}^2 = M_{22}^2 + v^2 \Lambda_3.
\]

The scalar mixing matrix \( \mathcal{R} \) in eq. (15) is a general real \( 3 \times 3 \) orthogonal matrix, which depends on three real parameters.
B.2 \( \mathbb{Z}_2 \)-2HDM

For the \( \mathbb{Z}_2 \)-2HDM, the mass matrix of the neutral scalars is given by

\[
[M^2_0]_{11} = 2v^2 \left\{ \lambda_1 c_\beta^4 + \lambda_2 s_\beta^4 + 2c_\beta^2 s_\beta^2 (\lambda_3 + \lambda_4 + \bar{\lambda}_5) \right\}, \\
[M^2_0]_{12} = v^2 s_{2\beta} \left\{ -\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + c_\beta (\lambda_3 + \lambda_4 + \bar{\lambda}_5) \right\}, \\
[M^2_0]_{22} = 2v^2 c_\beta^2 s_\beta^2 \left\{ \lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \bar{\lambda}_5) \right\}, \\
[M^2_0]_{13} = 0, \\
[M^2_0]_{23} = 0, \\
[M^2_0]_{33} = -2v^2 \bar{\lambda}_5. \\
\tag{93}
\]

The mass of the charged scalar is

\[ m^2_{H^\pm} = -v^2 (\lambda_4 + \bar{\lambda}_5). \tag{94} \]

In this case, \( \mathcal{R} \) is block diagonal: it is customary to introduce \( \alpha \) parametrizing the transformation from \( \{\rho_1, \rho_2\} \) in eq. (12) to \( \{h, H\} \), for example

\[
\begin{pmatrix}
  h \\
  H
\end{pmatrix} = \begin{pmatrix}
  s_\alpha & c_\alpha \\
  -c_\alpha & s_\alpha
\end{pmatrix}
\begin{pmatrix}
  \rho_1 \\
  \rho_2
\end{pmatrix},
\tag{95}
\]

and then

\[
\mathcal{R} = \begin{pmatrix}
  s_{\alpha\beta} & -c_{\alpha\beta} & 0 \\
  c_{\alpha\beta} & s_{\alpha\beta} & 0 \\
  0 & 0 & 1
\end{pmatrix},
\tag{96}
\]

which depends on a single parameter combination \( \alpha + \beta \), with \( s_{\alpha\beta} = \sin(\alpha + \beta) \), \( c_{\alpha\beta} = \cos(\alpha + \beta) \).

B.3 SCPV-2HDM

For the SCPV-2HDM, the mass matrix of the neutral scalars is given by

\[
[M^2_0]_{11} = v^2 \left\{ 2\lambda_1 c_\beta^4 + 2\lambda_2 s_\beta^4 + [\lambda_3 + \lambda_4 + \lambda_5 c_{2\theta}] s_\beta^2 \right\}, \\
[M^2_0]_{12} = v^2 \left\{ -\lambda_1 c_\beta^2 + \lambda_2 s_\beta^2 + [\lambda_3 + \lambda_4 + \lambda_5 c_{2\theta}] c_\beta s_\beta \right\}, \\
[M^2_0]_{22} = v^2 \left\{ \frac{1}{2} s_{2\beta} [\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4)] + \lambda_5 (1 + c_{2\beta}^2) \right\}, \\
[M^2_0]_{13} = -v^2 s_\theta \left\{ 2\lambda_5 s_\beta c_\theta + c_\beta^2 \lambda_6 + s_\beta^2 \lambda_7 \right\}, \\
[M^2_0]_{23} = -v^2 s_\theta \left\{ 2\lambda_5 c_\beta c_\theta + c_\beta s_\beta (\lambda_7 - \lambda_6) \right\}, \\
[M^2_0]_{33} = 2v^2 \lambda_5 s_\theta^2. \\
\tag{97}
\]

The mass of the charged scalar is

\[ m^2_{H^\pm} = v^2 (\lambda_5 - \lambda_4). \tag{98} \]
B.4 \textit{Z}_2-2HDM with soft symmetry breaking

For the \textit{Z}_2-2HDM with soft symmetry breaking term in section 5.2, the mass matrix of the neutral scalars is given by

\[
\begin{align*}
[M^2_{0}]_{11} & = 2v^2 \left\{ \lambda_1 c^4_\beta + 2s^2_\beta + 2c^2_\beta s^3_\beta (\lambda_3 + \lambda_4 + \text{Re}(\tilde{\lambda}_5)) \right\}, \\
[M^2_{0}]_{12} & = v^2 s_{2\beta} \left\{ -\lambda_1 c^4_\beta + 2s^2_\beta + 2s^2_\beta s^2_\beta (\lambda_3 + \lambda_4 + \text{Re}(\tilde{\lambda}_5)) \right\}, \\
[M^2_{0}]_{22} & = - (t_\beta + t^{-1}_\beta) \text{Re}(\bar{\mu}^2_{12}) + 2v^2 c^2_\beta s^2_\beta \left\{ \lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4 + \text{Re}(\tilde{\lambda}_5)) \right\}, \\
[M^2_{0}]_{13} & = -v^2 s_{2\beta} \text{Im}(\tilde{\lambda}_5), \\
[M^2_{0}]_{23} & = -v^2 c_{2\beta} \text{Im}(\tilde{\lambda}_5), \\
[M^2_{0}]_{33} & = -(t_\beta + t^{-1}_\beta) \text{Re}(\bar{\mu}^2_{12}) - 2v^2 \text{Re}(\tilde{\lambda}_5).
\end{align*}
\]

The mass of the charged scalar is

\[
m^2_{H^\pm} = -(t_\beta + t^{-1}_\beta) \text{Re}(\bar{\mu}^2_{12}) - v^2 (\lambda_4 + \text{Re}(\tilde{\lambda}_5)).
\]

B.5 SCPV-2HDM with soft symmetry breaking

For the SCPV-2HDM with soft symmetry breaking terms in section 5.2, the mass matrix of the neutral scalars is given by

\[
\begin{align*}
[M^2_{0}]_{11} & = v^2 \left\{ 2c^4_\beta \lambda_1 + 2s^2_\beta \lambda_2 + s^2_\beta [\lambda_3 + \lambda_4 + c_2\beta \lambda_5] + 2s_{2\beta} c_\theta [c^2_\beta \lambda_6 + s^2_\beta \lambda_7] \right\}, \\
[M^2_{0}]_{12} & = v^2 \left\{ s_{2\beta} [-c^4_\beta \lambda_1 + s^2_\beta \lambda_2] + c_2\beta s_{2\beta} [\lambda_3 + \lambda_4 + c_2\beta \lambda_5] + c_\theta [c_2\beta c_3\beta \lambda_6 + s_3\beta s_{3\beta} \lambda_7] \right\}, \\
[M^2_{0}]_{22} & = \frac{t_\beta + t^{-1}_\beta}{s_\theta} \text{Im}(\mu^2_{12}) + \frac{v^2}{2} \left\{ s^2_{2\beta} [\lambda_1 + \lambda_2 - 2(\lambda_3 + \lambda_4)] + 2(1 + c^2_2 c_\theta) \lambda_5 - s_{4\beta} c_\theta [\lambda_6 - \lambda_7] \right\}, \\
[M^2_{0}]_{13} & = -v^2 s_\theta [2s_2 c_\theta \lambda_5 + c^2_\beta \lambda_6 + s^2_\beta \lambda_7], \\
[M^2_{0}]_{23} & = -v^2 s_\theta [2c_2 c_\theta \lambda_5 - s_\beta c_\beta (\lambda_6 - \lambda_7)], \\
[M^2_{0}]_{33} & = \frac{t_\beta + t^{-1}_\beta}{s_\theta} \text{Im}(\mu^2_{12}) + 2v^2 s^2_\beta \lambda_5.
\end{align*}
\]

The mass of the charged scalar is

\[
m^2_{H^\pm} = \frac{t_\beta + t^{-1}_\beta}{s_\theta} \text{Im}(\mu^2_{12}) + v^2 (\lambda_3 - \lambda_4).
\]

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