Coincidence problem within dark energy as a coupled self-interacting Bose–Einstein gas

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Abstract
A late accelerated expansion of the Universe is obtained from non-relativistic Bose–Einstein particles with a short-range attractive interaction, with a temperature of the same order as the critical one, and an energy interchange with a dark-matter source. We show that for a sizable range of parameters, these particles evolve as a coupled dark-energy source, solving the coincidence problem, and in agreement with the luminosity distance versus redshift derived from supernova data.

Keywords: dark energy, coincidence problem, Bose–Einstein gas

(Some figures may appear in colour only in the online journal)

1. Introduction

A central tenet in modern cosmology is the existence of an element in its equations that leads to Universe acceleration. Firmer evidence of the latter came with a more accurate measurement of the redshift versus distance, using distant supernovae [1]; additionally, cosmic microwave background (CMB) anisotropy temperature spectrum [2, 3] and baryon acoustic oscillations (BAO) [4] data suggest such a source becomes the more favored explanation. The implication is the existence of a cosmological constant or dark-energy (DE) component within a general relativistic framework.

Indeed, the present model of cosmology has two main components: DE and dark matter, which account for most of the Universe energy content. Although there is no laboratory evidence for their presence, indirect cosmological evidence has mounted. The model of the
Universe most supported by observations is the $\Lambda$CDM model: a flat homogeneous metric with three main sources of energy at present: ordinary observable baryonic non-relativistic matter, cold dark matter (CDM) and DE in the form of a cosmological constant $\Lambda$ with constant energy density [3, 5]. CDM is necessary to explain motion within galaxies [6] and then galaxy formation. DE is also necessary in structure formation [7], within flat-space models, the latter required after evidence from the CMB, and as implied by inflation.

A cosmological constant $\Lambda$ was first proposed in early cosmology as a stabilizing element for the Universe, while more conclusive evidence for its presence was obtained from supernova data [1], playing the role of the element generating the observed accelerated expansion. In the modern interpretation, it is associated to the energy–momentum part of the Einstein equations. The $\Lambda$ term presents several problems, and elucidating its nature or that of DE and dark matter constitutes an important objective in modern cosmology. A relevant clue is derived from the work of Zeldovich [8], who associated the cosmological constant to a vacuum contribution, which produces such an acceleration. However, the associated scale of the vacuum, using either Zeldovich’s strong interaction scale [8] or the fundamental Planck scale, constitutes orders of magnitude beyond the present scale of the Universe energy density (and hence its components). This is known as the fine-tuning problem; a natural way to solve it is to assume an evolving DE component so that the connection between the Planck scale and the value of the DE today can be explained. Bronstein [9] first envisioned this kind of evolution.

The coincidence problem is closely related, and concerns the unlikely similar values of the dark-matter and DE energy densities today; both cosmological constant and evolving DE field models endure it in one way or another.

An alternative to the cosmological constant and the DE field consists of coupled DE models: the DE field interchanges energy with the CDM by means of a coupling term. When the coupling is present, the dynamical system formed by the energy-density evolution equations presents an attractor, their energy-density ratio may have reached its equilibrium value in the past, and there is no coincidence problem (we are no longer privileged observers, as their ratio will remain constant for a long time). Even if the attractor is in the far future, the coincidence problem can be alleviated as soon as the energy-density ratio evolves smoothly, as compared to the characteristic time span of the Universe. A viable direction for the understanding of DE is to consider models that reproduce this behavior, e.g., see references in reviews [10, 11]. These models address the coincidence problem successfully, but they do not explain the nature of the DE field. Several coupling terms can be found in the literature [12, 13], some of them based in thermodynamics [14, 15], dimensional analysis [16, 17], in quantum field theory [18, 19], and particle physics [20]; a degree of plausibility is gained with the latter models. We rely on the idea that these components are linked [21]; then, such a model is also consistent with a unification idea of CDM and DE. To obtain scaling, we use a general decay form present in many physical processes [22].

In this work, we apply the DE model presented in [23], with a definite microscopical nature, and show that it solves (or alleviates) the coincidence problem. The role of the DE field is played by a Bose–Einstein gas of non-relativistic particles that self-interact attractively, which results in a negative pressure. Such a gas highlights low-temperature components, fit for a DE description. However, the generic local interaction, previously proposed, decays faster than either radiation or matter, and is consequently unsuitable for a late-Universe description; we thus assume gas particles are created from the CDM at a defined rate, which enhances the acceleration effect, and results in a coupling between the Bose–Einstein gas and CDM. While this kind of scaling solves the coincidence problem, and is assumed phenomenologically in many works, here we conjecture it stems from the link to the horizon.
of low-momentum modes that naturally appear in condensates, by the uncertainty principle, just as this link is also present in boundary conditions that allow for the stability of the model, as explained in [23]. The dynamics of this model match well the observed data at present, for some parameter choices: the energy densities of both CDM and DE, the accelerated expansion, and the supernovae type Ia data. Additionally, different parameter choices lead to different luminosity distances at high redshift ($z > 1.5$), giving us a tool to discard or validate the gas model in the near future. In some coupled DE models [13], supernova acceleration is not sufficiently constrained by the supernova data, but it is still necessary to check consistency. This we do in our paper.

This article’s plan is as follows: in section 2, we describe the Bose–Einstein gas with self-interaction in a cosmological frame, and introduce a coupling term between the gas and CDM; in section 3, we assume that the rate of gas particle creation takes a feasible physical form, we deduce the coupling term from it, and we solve the dynamics of the Universe in terms of a number of free parameters; we also bind the free parameter space by imposing observational constraints, and solving or nearly solving the coincidence problem; in section 4, we illustrate the dynamics of the model for some choices of the parameters; we also address its future evolution; in section 5, we derive the evolution for a concrete set of parameters for which the coincidence problem is fully solved; in section 6, we illustrate the dynamics of the model for some choices of the parameters; we also address its future evolution; in section 7, we summarize the findings of the previous sections.

From now on, we assume units for which $\hbar = c = k_B = 1$. As usual, a zero subindex refers to a variable’s present value; likewise, we normalize the metric scale factor by setting $a_0 = 1$.

## 2. Interacting Bose–Einstein gas particles and late acceleration

### 2.1. Energy density and pressure of an interacting Bose–Einstein gas

We consider a self-interacting Bose–Einstein gas (IBEG) of non-relativistic particles that experiment a short-range two-particle attractive interaction $V(x)$, thus, modifying free-particle behavior. In this section, we summarize the IBEG description and we refer the reader to [23] for more details.

The average occupation number is

$$\bar{n}_k = \frac{1}{e^{(\epsilon_k - \mu)/T} - 1},$$

where $\epsilon_k$ is the single particle energy, containing the kinetic energy related to its momentum $k$ and mass $m$. The particle number associated to a given energy state reads

$$dN_k = g \frac{V}{(2\pi)^3} \bar{n}_k d^3k,$$

where for spin-zero particles the degeneracy factor is $g = 1$, and $V$ is the volume. The total number of particles is

$$N = \int dN_k = N_c + N_n,$$

where $N_c$ and $N_n$ are the number of condensate and non-condensate particles, respectively.
Near the critical temperature

\[ T_c = \frac{2\pi}{\zeta\left(\frac{3}{2}\right)} \frac{n^{2/3}}{\sqrt{g^{2/3}}} \approx 3.31 \frac{n^{2/3}}{\sqrt{g^{2/3}}} , \]  

(4)

where the total particle number density is \( n = N/V \), \( \zeta \) is the zeta function, and also when \( T < T_c \), its single-particle energy is given to first-order by [24]

\[ \epsilon = m + \frac{k^2}{2m} + 2v_0 n_c , \]  

(5)

where the first term is the rest-mass energy, the second term is the kinetic energy, the third term represents the potential energy of \( N_c \) non-condensate particles interacting with the particle through the potential \( \mathcal{V}(x) \), \( n_c = N_c/V \), and

\[ v_0 = \int d^3x \mathcal{V}(x) . \]  

(6)

For equation (5), the corresponding chemical potential is

\[ \mu = 2n_c v_0 + m . \]  

(7)

The constancy of \( \mu \) remains valid also in this case, as in the non-interactive one.

We can substitute the single-particle distribution into the Bose distribution in equation (2) and integrate over the volume to find the energy

\[ E = mN + \frac{gV}{\sqrt{2} \pi} \int d\epsilon \epsilon^{3/2} \frac{1}{e^{(\epsilon - \mu)/T} - 1} + \frac{1}{2V} \sum_{i \neq j} N_0 \]  

(8)

\[ \simeq mN + \frac{gV}{\sqrt{3} \pi} \left( T^{5/2} \int_0^\infty dz z^{3/2} \frac{3}{e^z - 1} + \frac{v_0}{2V} N^2 , \right. \]  

(9)

where the third term sums over pairs of interactions from equation (6) in the thermodynamic limit, and \( v_0 \) is an interpolated potential term that takes into account the particle exchange potential between the condensate and non-condensate components as

\[ v_0 = v_0 \left[ \left( N_c^2 + 2(N - N_c)^2 + 2N_c(N_c - N_c) \right) \right] / N^2 . \]  

(10)

The gas particles’ total kinetic energy is related to its entropy, as the condensate particles do not contribute to the gas entropy. From the well-known equations for the Bose–Einstein gas [25]

\[ E_c = \frac{3}{5} TS , \]

\[ S = \frac{5}{3} \left[ \frac{3\zeta(5)}{4\sqrt{2} \pi^3 g} \right]^{1/5} m^{3/5} V^{2/5} E_c^{3/5} \approx \frac{5}{3} (1.128 g)^{2/5} m^{3/5} V^{2/5} E_c^{3/5} , \]

\[ E_c = \epsilon T N_c \]
where $\varepsilon = \frac{3(\frac{2}{3})}{2(\frac{2}{3})} \approx 0.77$, it is possible to write the total kinetic energy as

$$E_k = AN^{5/3}V^{-2/3},$$

(11)

where $A \approx \varepsilon^{5/3}(128\gamma)^{-2/3}m^{-1}$. Thus, the kinetic energy density of the IBEG only depends on non-condensate particles and reads

$$\rho_k = An^{5/3}.$$  

(12)

The total IBEG energy density reads from equations (9) and (12)

$$\rho = \rho_\text{m} + \rho_\text{e}^{5/3} + \frac{1}{2}v_0n^2,$$

(13)

and, from the definition $p = -(\partial E/\partial V)$, the gas pressure takes the form

$$p = \frac{2}{3}\rho + \frac{1}{2}v_0n^2.$$  

(14)

In the next section, we consider the IBEG within a cosmological scenario.

### 2.2. IBEG and late acceleration of the Universe

We assume a Lemaitre–Friedman–Robertson–Walker (LFRW) Universe with line element

$$d^2 = -dt^2 + a(t)^2 \left[ dr^2 + r^2d\Omega_\theta^2 \right],$$

where $a(t)$ is the scale factor and $t, r$ and $\Omega_\theta$ are the time, the radius and solid-angle comoving coordinates of the metric, respectively. As stated in [23], the LFRW metric with the IBEG as only energy source and constant particle number (the horizon serving as boundary, and making it stable despite the attractive interaction) cannot produce late acceleration. Indeed, the expansion of the Universe is accelerated at early times, as $\rho_k + 3p_k < 0$ because of the interaction negative term (maintaining $\rho_k > 0$). However, as the Universe expands, this negative term fades away faster than the other terms (it scales as $a^{-6}$, while the kinetic and mass terms scale as $a^{-3}$ and $a^{-1}$, respectively) and $\rho_k + 3p_k$ becomes positive. Consequently, a LFRW Universe containing the IBEG as only energy source can expand with positive acceleration in its early stage but, eventually, will experiment a late decelerated expansion.

We consider now a LFRW Universe that contains baryonic matter, CDM and the IBEG. The baryonic matter and CDM are both non-relativistic, and, thus, with equation of state $p = 0$. Their energy densities ($\rho_\text{b}$ and $\rho_\text{m}$, respectively) evolve with the scale factor as $\rho_i = \rho_{i0}a^{-3}$ in the absence of coupling terms, where $i = b, m$, and $\rho_{i0}$ is the present-day value.

If there is a particle creation process for the IBEG, the particle number is no longer constant with the expansion. As we will see, this creation mechanism could lead to late acceleration of the Universe under certain conditions. In fact, this process produces an energy exchange between the CDM and the IBEG, described by a coupling term $\mathcal{Q}$. The Einstein equations then read

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_T,$$

(15)

$$\dot{\rho}_b + 3H\rho_b = 0,$$

(16)
\[ \dot{\rho}_m + 3H\rho_m = -Q, \quad (17) \]
\[ \dot{\rho}_b + 3H(\dot{\rho}_b + \dot{\rho}_g) = Q, \quad (18) \]

where \( \rho_T = \rho_b + \rho_m + \rho_g \) is the total energy density contained in the LFRW Universe.

3. Creation process of IBEG particles

A low temperature for the gas is chosen to enhance its DE contribution. This condition is maintained as the Universe evolves [23]. For simplicity, we assume that there is no condensation of IBEG particles. We also assume that, at a certain instant in the past with scale factor \( a_{\text{in}} \), a coupling between CDM and the IBEG is activated in such a way that the non-condensate IBEG particle number (with kinetic energy different from zero) evolves with time as a volume power law. Consequently

\[ N_c(t) = cV(t)^x, \quad (19) \]

where \( c > 0 \) and \( 1 \geq x > 0 \) are the parameters that model this creation process, leading to energy exchange between the IBEG and CDM. This process is Markoffian, as it does not depend on previous states; typical dispersion processes and fluid interactions lead to evolution laws of this type in various physical setups (see [22] and references therein).

The IBEG non-condensate and condensate particle-number densities satisfy

\[ \dot{n}_c + 3Hn_c = 3Hcxa^{3(x-1)}, \quad (20) \]
\[ \dot{n}_c + 3Hn_c = 0, \quad (21) \]

implying that they scale as

\[ n_c = n_c a^{-3} + ca^{3(x-1)}, \quad (22) \]
\[ n_c = n_c a^{-3}, \quad (23) \]

where \( n_{ci} \) and \( n_{ci} \) are the particle number density of each kind that were present before the interaction began, as measured today at \( a = 1 \). While \( n_c \) could experience a jump at the scale \( a_{\text{in}} \), it is also possible that it is an always decreasing function of time.

We assume \( x \leq 1 \) in this work, in order to have a decreasing number density \( n_c \) with the expansion of the Universe from \( a_{\text{in}} \) on. Allowing \( x > 1 \), the model evolves to a scenario with both IBEG number particle and number density increasing with time, making the interaction term eventually bigger than the kinetic and mass terms with a total energy of the IBEG particles negative.

We also assume that the IBEG creation is an ongoing process with \( n_{ci} \), \( n_{ci} \ll c \), (i.e., the number density of particles present before the creation process started is much smaller than that of today’s created particles), and, therefore, that equation (7) is valid. The critical temperature \( T_c \) defined in equation (4) will become lower as \( n_c \) decays (with the possibility of experimenting a sudden jump at the instant \( a_{\text{in}} \)), thus always satisfying the non-relativistic assumption \( m \gg T_c \sim T \), once it is satisfied in the past (even in the case that \( T_c \) increases at \( a_{\text{in}} \), it is reasonable to assume that the non-relativistic behavior of the IBEG particles still holds). Then, the total number of IBEG particles at present is \( n_0 = n_c \approx c \). The non-condensate component dominance is a simplifying assumption, and should be clarified, as it has implications on the early evolution of the model right after inflation, and during the radiation era. While the IBEG could be present at the reheating process at the end of the inflationary
stage and later in equilibrium with radiation, the IBEG energy density will eventually
decouple and soon fade away (as its terms evolve with $a^{-5}$ and $a^{-6}$). At the end of radiation
era (around $a = 10^{-4}$), we assume that the IBEG is no longer in equilibrium with radiation, its
temperature decreases until it is close to $T_c$, where the equations used in the previous section
apply, and a condensate forms. If the amount of IBEG particles present at this stage is
residual, the expansion Universe is dominated by the CDM and baryonic sources. It is natural,
then, to assume that $n_{ci} \approx 0$ and $n_{ci} \approx 0$ until the creation process starts. At this point, both $T_c$
and the IBEG temperature $T$ are approximately proportional to $n_{ci}$, the ratio $T/T_c$ remains
constant, and no further condensation of IBEG particles is expected.

With the above assumptions, the IBEG energy density $\rho_g$ reads from equation (22)
$$
\rho_g = mca^{3x-1} + Ac^{5/3}a^{2x-1} + \frac{1}{2}v_0 c^2a^{6x-1},
$$
where we neglect the energy density corresponding to the IBEG particles present before the
interaction begins (of order $m a^{-3}$), which would always be much smaller than baryonic
and CDM energy densities.

Given the definition of the parameter $v_0$ in equation (10), it is clear that $v_0 \approx 2v_0'$ (as
$N_c \approx 0$ and $N \approx N_i$) and remains constant during the creation process, as soon as $v_0'$ is
constant.

By means of the energy conservation equation (18), we can identify the coupling term $Q$ as
$$
Q = 3H x \left( \rho_G a^{5x-1} + \frac{5}{3} \rho_c a^{2x-5} + 2 \rho_i a^{6x-6} \right),
$$
where we defined the parameters
$$
\rho_G = mc, \quad \rho_c = Ac^{5/3}, \quad \rho_i = (v_0c^2)/2.
$$
The interaction term $Q$ above comes naturally from the assumption that the IBEG non-
condensate particles are created at the rate given by equation (19). While in several coupled
DE models the interaction term is chosen on phenomenological grounds, in our work $Q$ is
motivated by the IBEG energy density itself. On the other hand, the choice of the creation-rate
equation (19) is phenomenological to a certain point. We note that the energy density flow, $Q$
can take negative values for certain parameter choices, at an early scale factor $a$; when $Q < 0$,
the energy flows from the IBEG to the CDM. Even as IBEG particles are created, the energy
can flow in the reverse direction, as the IBEG negative energy density interaction term $\rho_i$
may predominate. As in equation (7), a connection between a coupling term $Q$ and the
chemical potential was pointed out before [14, 15].

### 3.1. Energy density evolution

Once we know the form of $Q$, given by equation (25), we can solve the set of equations (15)–
(18) analytically. The CDM energy density $\rho_m$ reads
$$
\rho_m = \rho_{m0}a^{-3} - \rho_G a^{5x-3}
- \rho_c a^{2x-5} \frac{5x}{-2 + 5x} - \rho_i a^{6x-6} \frac{2x}{-1 + 2x},
$$
the baryon energy density evolves as
$$
\rho_b = \rho_{b0}a^{-3},
$$
and, together with \( \rho \) in equation (24), the total energy density is then

\[
\rho_T = \rho_{b0}a^{-3} + \rho_{m0}a^{-3} + \rho_{c0}a^{5x-5} \left( 1 - \frac{5x}{-2 + 5x} \right) + \rho_{i0}a^{4x-6} \left( 1 - \frac{2x}{-1 + 2x} \right),
\]

(29)

Regarding the above equation, it seems likely that some parameter choices lead to a negative total energy density in the future. From the nature of the coupling term \( Q \), it is reasonable to assume that the interaction between CDM and the IBEG disappears when \( \rho_{b0} = 0 \): if no CDM is left, no new IBEG particles should be created. If the coupling \( Q \) is no longer present, the total energy density will be always positive (we refer the reader to section 4.1 for a detailed discussion on the future evolution of the IBEG model).

The free parameters in our model are six: \( \rho_{b0} \), the current baryonic energy density; \( \rho_{G0} \), the mass contribution to the IBEG energy density; \( \rho_{i0} \), the kinetic contribution to the IBEG energy density; \( \rho_{m0} \), the interaction term of the IBEG energy density (negative term due to its attractive nature); \( \rho_{G0} \), the energy density of the CDM particles; and \( x \), the exponent parameter that models the creation rate.

3.2. Bounds to the free parameters of the model

Our purpose is to find general bounds on the free parameters of the LFRW Universe containing the IBEG, assuming that this model reproduces the observed energy densities of its components, and an accelerated expansion. We also have in mind the coincidence problem as, for some choice of the parameters, it is solved or, at least, alleviated. The case with \( x = 1 \) is treated separately in section 5.

(i) Present-day energy densities. We know from different observations of the CMB spectrum, BAO and supernovae, that the energy density of baryonic matter accounts for 4% of the total energy density of the Universe (\( \rho_{b0}/\rho_T^0 = \Omega_{b0} = 0.04 \)), the CDM energy density is 24% (\( \rho_{m0}/\rho_T^0 = \Omega_{m0} = 0.24 \)), and the 72% left corresponds to the DE (\( \rho_{G0}/\rho_T^0 = \Omega_{G0} = 0.72 \) ) [2]. At this point, we fix the energy density scale so that \( \rho_T^0 = 1 \) (i.e. \( 3H_0^2/(8\pi G) = 1 \)).

The role of the DE in our model is played by the IBEG. Evaluating equations (24) and (27) at present (i.e., \( a = 1 \)), and imposing the observed relations between the energy densities, we obtain

\[
\rho_{m0} = \rho_{G0} = \rho_{c0} \frac{5x}{2 + 5x} = \rho_{i0} \frac{2x}{-1 + 2x} = \Omega_{m0},
\]

(30)

\[
\rho_{G0} + \rho_{c0} + \rho_{i0} = \Omega_{G0}.
\]

(31)

As the baryonic matter in our model does not interact with any other sources of energy, \( \rho_{b0} = 0.04 \). Using these relations, it is possible to express the free parameters \( \rho_{c0} \) and \( \rho_{i0} \) in terms of \( \rho_{m0} \), \( \rho_{G0} \) and \( x \) as

\[
\rho_{c0} = \left( -2\rho_{m0} - 3\rho_{G0} + 2\Omega_{m0} \right)x^{-1}
\]

\[
+ 9\rho_{m0} - 9\Omega_{m0} - 4\Omega_{G0} + 10 \left( -\rho_{m0} + \Omega_{m0} + \Omega_{G0} \right)x,
\]

(32)
The definitions in equation (26), \( \rho > 0 \) and \( \rho < 0 \), limit the free parameters \( \rho_G \), \( \rho_m \), and \( x \); figure 1 shows the bounds on the \( \rho_m - \rho_G \) space from this condition for a chosen \( x \), as well as for those obtained in (ii) and (iii).

(ii) Accelerated expansion of the Universe. As many observations suggest, the Universe is undergoing a stage of accelerated expansion. In other words, \( \rho_T + 3p_T < 0 \), and the solid line condition \( |\dot{r}/r| \leq H_0 \). From equations (14) and (29), this relation adds two new bounds to the parameters of our model: \( x > 1/2 \), and

\[
\rho_m < \frac{0.04}{2 - 9x + 10x^2} \left[ 250 \left( \Omega_m + \Omega_\phi \right) x^2 + \left( 250 \Omega_m + 200 \Omega_\phi + 1 \right) x + 50 \rho_G + 50 \Omega_m \right].
\]

(iii) Coincidence problem. It is well known in the literature that a DE model solves the coincidence problem if the ratio \( r = \rho_m/\rho_G \) tends to a constant [26]. In the IBEG model, \( r \) does not evolve to a constant unless \( x = 1 \) (see section 5). However, the coincidence problem is strongly alleviated when \( |\dot{r}/r| \leq H_0 \) [26]. Although this condition seems to imply a great deviation from the \( \Lambda \)CDM model (for which \( |\dot{r}/r| = 3H_0 \)) that would lead coupled DE models to hardly explain observations, it is in fact compatible with them. Indeed, examples abound in the literature on coupled DE models that alleviate the coincidence problem and explain successfully the observational data (e.g., [10, 13, 27]). From the definition of \( r = \rho_m/\rho_G \), it follows that for the IBEG model
The deviation from the $\Lambda$CDM model is due to the dynamical evolution of $\rho_g$ and $\rho_g$ and from the addition of the last term proportional to the coupling $Q$. If we impose condition $|\dot{\rho}/\rho| \lesssim H_0$ to our model, we find an additional bound on the parameters. Using equations (14), (24), (25) and, (27), we find the region of the $\rho_{m0} - \rho_{G0}$ space that fulfills the condition. The region is represented in figure 1 for fixed values of $x$. For $x \leq 0.85$, our Universe model never fulfills the coincidence-problem and the $\rho_{m0} > 0$, $\rho_{G0} < 0$ conditions simultaneously. This represents an additional bound on the parameter $x$ to the previous one ($x > 1/2$ to obtain an accelerated expanding Universe).

The free-parameter space that fulfills all the above conditions for some $x$ values is shown in figure 2. From the plot we conclude that the closer the parameter $x$ is to 1, the wider the $\rho_{m0} - \rho_{G0}$ region, i.e., the conditions imposed are less restrictive for values close to $x = 1$.

For every free-parameter choice made, we evaluate the scale factor $a = a_{in}$ in the past for which the IBEG energy density is null. In fact, we can consider $a_{in}$ as the initial scale factor at which the coupling $Q$ becomes active. If $a_{in}$ has no relation (based on fundamental physics) with the IBEG free parameters, we might conclude that it is fixed by hand to explain the present day observations, and that in order to avoid the coincidence problem, the IBEG model with $x < 1$ introduces a fine-tuning problem: the interaction should start at a concrete instant in the past to explain what we observe nowadays. This is a common problem of dynamic DE models proposed as a solution to the coincidence problem. However, the severity of this objection diminishes as $x$ nears 1, and, as we will see, the coupling term of the IBEG model with $x = 1$ has no need of an initial scale $a_{in}$ and solves the coincidence problem, without the need of a fine tuning of the coupling term.

In the next section, we study the evolution of the different components.
4. Numerical application and luminosity distance

To study the Universe evolution in the IBEG model, we look at their components’ in equations (24), (27) and (28). We plot the relative densities $\Omega_i = \rho_i / \rho_T$ as a function of the scale factor $a$ (with $i = g, b, m$). Next, we plot $\log(r)$ versus $a$ to check the coincidence problem in the model. Finally, we plot the effective magnitude $m_B$ versus the redshift $z = 1/a - 1$ for each choice of the parameters together with the SNIa data from [28]. The effective magnitude is defined as

$$m_B = 5 \log_{10}(d_L) + 25,$$

(35)

where $d_L$ is the luminosity distance. For the plot, we recover the units of the Hubble factor and take $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [2]. The plots for $x = 0.90, 0.95, 0.99$ are given in figures 3–5.
respectively. The allowed region of parameters $\rho_{m0} - \rho_{G0}$ that fulfills the previous section requirements is given in figure 2; we fix the parameter $\rho_{m0}$ and assign different values for $\rho_{G0}$ within that region. The plots for the different sets of $\rho_{G0}$ and $\rho_{m0}$ show similar behavior. The $\Omega_i$ functions have a smooth dependence on $\rho_{G0}$ and $\rho_{m0}$. Some common features are:

- the slope of the $r$ function is larger than that of the $\Lambda$CDM model until the creation process starts; at this point, it is smoothed out and the coincidence problem is alleviated with respect to the $\Lambda$CDM model in all the cases near $a = 1$.
- The parameter $\rho_{G0}$ has a noticeable influence on the $\Omega_{E,m}$ evolution and $r$, but not on the luminosity distance lines at the redshift range considered.
- The IBEG model luminosity distance is similar to that of the $\Lambda$CDM model in the redshift range $z < 0.5$; for $z > 0.5$, it differs, while both models adjust the data well (both models have very similar $\chi^2$ statistics at 1σ confidence level, e.g., a statistical value of 0.5313 for

Figure 4. IBEG model with $x = 0.95$, $\rho_{m0} = 0.1$ and $\rho_{G0} \in [0.4, 1.9]$ in steps of 0.1. (a) Partial energy densities $\Omega_g$, $\Omega_b$ and $\Omega_m$ versus scale factor $a$. (b) Log($r$) versus scale factor. (c) Luminosity distance versus redshift and observational data from supernovae type Ia [28]. In panels (b) and (c), the solid black line represents the $\Lambda$CDM model. Please refer to the text for a detailed description of the plots.
Some particular characteristics of each $x$ are:

- from figure 2, we see that the maximum allowed value for $\rho_{m0}$ is 0.08, 0.165, and 0.219 for $x = 0.90, 0.95, 0.99$, respectively.

Figure 5. IBEG model with $x = 0.99$, $\rho_{m0} = 0.15$ and $\rho_{G0} \in [0.0, 1.9]$ in steps of 0.1. (a) Partial energy densities $\Omega_{i}^{(i)}$, $\Omega_{b}$ and $\Omega_{m}$ versus scale factor $a$. (b) $\log(r)$ versus scale factor. (c) Luminosity distance versus redshift and observational data from supernovae type Ia [28]. In panels (b) and (c), the solid black line represents the $\Lambda$CDM model. Please refer to the text for a detailed description of the plots.

For larger $z$, the luminosity distance for the IBEG model depends on the choice of $x$ and is quite different from that of the $\Lambda$CDM model. Consequently, the IBEG model is distinguishable from the $\Lambda$CDM model and luminosity distance data from far away objects $z > 1.5$ would discard/favor some values of $x$.
From figures 3(a) and 4(a), we conclude that the IBEG creation process starts around $a_{in} = 0.1$ for $x = 0.90, 0.95$ and all choices of the $\rho_{G0}$ parameter, as $\Omega_{g}$ starts to grow from zero. From figure 5(a), we obtain that the creation of IBEG process with $x = 0.99$ starts very early (calculations show that as early as $a_{in} = 10^{-8} - 10^{-9}$) for all parameter choices.

The $x = 0.90$ case is peculiar, as can be seen in figure 3(a): $\Omega_{m}$ grows right after the IBEG creation process is triggered out, and decreases at later times; this fact is related to the interaction term proportional to $\rho_{i0}$ that appears in (27). This term is positive (as $\rho_{i0} < 0$) and grows with the expansion. The $\Omega_{m}$ evolution is dominated by it for a while until the other decreasing terms start to dominate its evolution. This behavior is observed for all choices of the $\rho_{G0}$ and $\rho_{m0}$ parameters. In the $x = 0.95, 0.99$ cases, $\Omega_{m}$ always decreases with $a$.

From figures 3(b), 4(b), and 5(b), we see that the creation process ends in the near future, when the CDM energy density and $r$ tend to 0 drastically. This instant strongly depends on the choice of $\rho_{G0}$, varying from $a = 1.5$ till $a = 1.8$ in the $x = 0.90$ case, up to $a = 2.2$ in the $x = 0.95$ case, and as far as $a = 10$ in the $x = 0.99$ case.

### 4.1. Future evolution of the coupled IBEG model

As can be seen in figures 3–5, the IBEG model predicts that the CDM energy density will be consumed in the IBEG particle creation process at a future time. At this point, we assume the interaction $Q$ is set to 0, if one imposes the condition that negative energy density of the CDM be avoided. The scale factor at this final moment depends on the parameters; in the previous section cases, it ranges from 1.5 up to 10, as can be appreciated in the evolution of $r$ in figures 3(b)–5(b). At this point, the IBEG Universe will be dominated by the IBEG energy density, as the baryonic term will be much lower than it. The number density of IBEG particles will then evolve proportionally to $a^{-3}$, as no new particles will be created. The interaction term in the energy density $\rho_{i}$ is proportional to $a^{-6}$ and will tend to 0 faster than the kinetic ($\approx a^{-5}$) and the mass ($\approx a^{-3}$) terms. The accelerated expansion stage will consequently end. Eventually, the kinetic term of the IBEG will also fade away and the mass term will remain. The Universe at this point will resume a second non-relativistic matter-dominated era of expansion.

### 5. Creation of interacting Bose–Einstein gas particles with $x = 1$

The $x = 1$ case must be resolved apart as it presents very different dynamics. When the IBEG particle number is created at a rate such that $N_{c}(t) = cV(t)$, the number density remains constant with the evolution as

$$n_{c} = c.$$  \hfill (36)

The energy density of the gas then reads

$$\rho_{g} = \rho_{G0} + \rho_{i0} + \rho_{00}.$$  \hfill (37)

Thus, $\rho_{g}$ remains constant with the expansion of the Universe. It plays a similar role to that of a cosmological constant in the $\Lambda$ CDM model, except for the fact that there is a coupling term, and that the pressure is $p = (2/3)\rho_{c0} + \rho_{00} \neq -\rho_{g}$. Then
The CDM density $\rho_m$ from equation (17) is also different from that of the $\Lambda$CDM because of the coupling, and it is

$$\rho_m = \rho_{m0} a^{-3} - \rho_{G0} - \frac{5}{3} \rho_{c0} - 2 \rho_{i0}. \quad (39)$$

The baryonic matter keeps its form as in the previous sections $\rho_b = \rho_{b0} a^{-3}$.

We still have the same free parameters as in the $x \neq 1$ case (except for $x$): $\rho_{m0}, \rho_{G0}, \rho_{c0}, \rho_{i0}$ and $\rho_{b0}$. We can constrain the IBEG model’s free parameters with $x = 1$ as in section 3, imposing the observed present energy densities, acceleration and coincidence-problem bounds.

(i) Present-day energy densities. In this case, imposing $\Omega_{x0} = 0.72$, $\Omega_{m0} = 0.24$ and $\Omega_{b0} = 0.04$, we obtain the relations

$$\rho_{c0} = -3 \rho_{G0} - 3 \rho_{m0} + 3 \Omega_{m0} + 6 \Omega_{x0}, \quad (40)$$

$$\rho_{i0} = 2 \rho_{G0} + 3 \rho_{m0} - 3 \Omega_{m0} - 5 \Omega_{b0}, \quad (41)$$

which allow us to reduce the free parameters space to a bi-dimensional $\rho_{m0} - \rho_{G0}$ space. It is, again, possible to limit the two remaining free parameters by imposing $\rho_{c0} > 0$ and $\rho_{i0} < 0$. The bounds to the $\rho_{m0} - \rho_{G0}$ region are represented in figure 6.

(ii) Present day acceleration. Another bound over the free parameters comes from requiring that the expansion of the Universe at present to be accelerated, i.e. $\rho_{T0} = 3p_{T0} < 0$. This condition reads

$$\rho_{m0} < -0.01 + 0.67 \Omega_{x0} + 0.67 \Omega_{m0} - 3.33 \times 10^{-10} \rho_{G0}. \quad (42)$$

In figure 6, the above condition is represented as the region of $\rho_{m0} - \rho_{G0}$ under the grey line.

Figure 6. Bounds on the free parameters for $x = 1$. The region to the left of the darker (blue) line fulfills condition $\rho_{c0} > 0$. The region to the left of the lighter (green) line fulfills condition $\rho_{i0} < 0$. The region under the grey (red) line represents an accelerated expanding Universe.

$$Q = -3H \left( \rho_{G0} + \frac{5}{3} \rho_{c0} + 2 \rho_{i0} \right). \quad (38)$$
(iii) **Coincidence problem.** In this case, $r$ evolves to the constant value

$$r_m = \frac{\rho_{G0} - \frac{5}{3}\rho_{c0} - 2\rho_{i0}}{\rho_{G0} + \rho_{c0} + \rho_{i0}}. \tag{43}$$

Consequently, the coincidence problem is solved, and no extra bounds can be obtained from it.

Figure 6 shows the region of the $\rho_{m0} - \rho_{G0}$ parameter space that fulfills the above conditions. The allowed region is less restrictive than that of the $x = 0.90$, 0.95, 0.99 cases. Thus, the maximum value of $\rho_{m0}$ allowed by the conditions is 0.626.
In figure 7, we show the evolution of $\Omega_{bgm}$, log($r$) and the luminosity distance for the IBEG model with $x = 1$. In figure 7(a), we see that the energy density of CDM $\Omega_m$ is an always decreasing function of $a$. In this case, the creation process can start at any instant in the past, as the IBEG energy density remains constant. In figure 7(b) we observe that the slope of $r$ is smooth around $a = 1$ and tends to a constant value for $a > 1$. For this choice of parameters, the creation process has no end. On figure 7(c), we see again that this choice of parameters is compatible with supernovae data. The IBEG model is closer to the $\Lambda$CDM luminosity distance than the previous choices of $x$ for $z > 1.5$, but it is still distinguishable.

The future evolution of the model in the $x = 1$ case, under the $\rho > 0$ assumption, also differs from the previous cases. If $\rho_{m0} < 0.24$, the creation process never consumes the CDM present on the Universe and the accelerated expansion remains. In this case, the future evolution of the IBEG Universe is dominated by a mixture of the IBEG (whose energy density acts as a cosmological constant, but with $\rho_b \neq -\rho_g$), and the CDM. From equation (39), it follows that, eventually, the CDM mass term proportional to $a^{-3}$ tends to zero, and the remaining terms are also constants. The future evolution of the IBEG model with $x = 1$ tends to a de Sitter Universe.

On the other hand, if $\rho_{m0} > 0.24$, the creation process will consume the CDM present in the future. The scale factor at this instant depends on the value of $\rho_{m0}$, but is independent of $\rho_{G0}$. When the creation process ends, the IBEG Universe will be dominated by the mass term $a^{-3}$ (as the interaction term will tend to zero faster), and the IBEG Universe will evolve as a non-relativistic matter-dominated Universe.

6. CDM and interacting Bose–Einstein gas linear perturbations

Given the statistic nature of the IBEG, it is reasonable to assume that IBEG density perturbations will be present. Bulk fluctuations in the gas were treated in [23], and associated to primordial perturbations. In this section, we address qualitatively the evolution of IBEG and CDM linear perturbations. A more detailed study involving numerical work will be done in the future.

The background coupling term $Q$ in equation (25) is derived from the assumption that the creation rate for the IBEG particle number evolves as in equation (19). Several generalizations may be envisioned for a covariant form of the IBEG energy–momentum transfer $Q_{\mu}^\nu$. Considering $T_{\Lambda}^{\mu\nu}$ is the energy–momentum tensor of a perfect fluid, $A = g, m$ and $b$ for the IBEG, CDM and baryonic matter respectively, and from the conservation equation of the total energy–momentum tensor, it follows that $Q_{\nu}^\mu = V_\nu T_{\mu}^{\nu} = -V_\nu T_{\mu}^{\nu} = -Q_{\mu}^\nu$. In order to obtain the evolution of linear perturbation and following [29, 30], we make the hypothesis that $Q_{\nu}^\mu = Q^\nu_{\nu}$ where $u_{\nu}^\mu$ is the IBEG volume element four-velocity. Considering the perturbed metric

$$\text{d}s^2 = a^2 \left\{ -(1 + 2\phi)\text{d}r^2 + 2\partial_i B \text{d}r \text{d}x^i + \left[ (1 - 2\psi)\delta_{ij} + 2\partial_i \partial_j E \right] \text{d}x^i \text{d}x^j \right\},$$

where $\eta$ is the conformal time ($a\eta = \text{d}t$) and $\phi$, $B$, $\psi$ and $E$ are scalar functions, the four velocity $u_{\nu}^\mu$ reads

$$u_{\nu}^\mu = a^{-1} \left\{ 1 - \phi, \text{d}v_g \right\},$$

where $v_g$ is the peculiar velocity potential.

Following the same notation as in [29], the equations for the IBEG-fluid linear perturbation $\delta_{\nu}^\rho$ are
\[ \delta \rho_g' + 3H \left( \delta \rho_g + \delta \rho_b \right) - 3 \left( \rho_g + \rho_b \right) \psi - k^2 \left( \rho_g + \rho_b \right) \left( v_g + E' \right) = aQ \phi + a \delta Q, \] (46)

\[ \left[ \left( \rho_g + \rho_b \right) \left( v_g + B \right) \right] + 4H \left( \rho_g + \rho_b \right) \left( v_g + B \right) + \left( \rho_g + \rho_b \right) \phi + \delta \rho_g = \frac{2}{3} k^2 \sigma_g = aQ \left( v_g + B \right) + af_s', \] (47)

where the prime indicates derivative with respect to the conformal time \( \eta' = \frac{d}{d \eta} = a^2 \frac{d}{d a}, H = a' / a, \delta \rho_g \) is the linear perturbations of the pressure of the IBEG, \( \pi_g \) is defined from the anisotropic stress as \( \pi_g^{ij} = (\delta \sigma_{ij} - \frac{1}{3} \delta \sigma \delta \sigma_{ii}) \sigma, \) and \( f_g \) is the momentum transfer potential of the IBEG (note that \( f_g + f_m = 0 \) as the total momentum is conserved).

Assuming the IBEG is a barotropic fluid, \( \delta \rho_g = c^2 \delta p_g \), where \( c_g \) is the propagation speed of the pressure perturbations in the IBEG rest frame. Functions \( B \) and \( E \) can be fixed to null in the longitudinal gauge, and \( \pi_g = 0 \) in our model (no anisotropic pressure term). The perturbation in the coupling term \( \delta Q \) can be obtained from the definition of \( Q \) in terms of \( \delta \rho_g \).

Finally, we can consider adiabatic initial conditions in order to compute the evolution of the linear perturbations.

Regarding CDM and baryonic perturbations, both are affected by the dynamics of the background metric. CDM perturbations are also affected by the coupling term \( Q \). It is possible to obtain the equations for CDM linear perturbations, which would be identical to those of the IBEG perturbations with a subindex \( m \) replacing the subindex \( g \) and with \( -Q \) replacing \( Q \).

Some qualitative comments should be made at this point for the two cases considered previously:

- \( x \neq 1 \). In this case, \( \rho_g, p_g \) and \( Q \) tend to null when \( a \to a_m \), i.e., the early Universe contains a negligible number of IBEG particles and the expansion is fully dominated by uncoupled CDM and baryonic matter. Thus, it seems unlikely that the early evolution of CMD perturbations differs from those of the \( \Lambda \)CDM model, and this applies to the structure formation scenario as well. An interesting open question is the clustering of the IBEG and the CDM perturbation late evolution (from \( a_m \) on).

It has been reported in the literature [29, 30] that some coupling terms lead non-adiabatic CDM perturbations to blow up at early times. A coupling where the energy flows from CDM to DE with constant adiabatic parameter \( w = p/\rho \) experiments those instabilities.

On the other hand, in our model, the energy flow direction depends on the choice of parameters and on the scale factor \( a \), as mentioned in the previous sections. Also, the IBEG adiabatic coefficient is no longer constant as

\[ w(a) = \frac{P_c(a)}{P_f(a)} = \frac{2}{3} \frac{\rho_c a^{2a-1} + \rho_f a^{2a-1}}{\rho_c + \rho_f a^{2a-1} + \rho_f a^{2a-1}}. \] (48)

Additionally, as the coupling term fades away for \( a < a_m \), the term leading the early non-adiabatic perturbations to grow up will fade away as well. As the final word lies in the numerical solution to equations (46) and (47), it is reasonable to affirm that no early instabilities would appear in the IBEG model for \( x \neq 1 \).

- \( x = 1 \). This case should be analyzed more carefully, as the adiabatic coefficient \( w = \frac{P_c(a)}{P_f(a)} = \frac{2}{3} \frac{\rho_C(a) + \rho_f(a)}{\rho_C(a) + \rho_f(a)} \) is, in fact, constant and the interaction term \( Q \) could be effective from early times on. A numerical analysis of this
case is mandatory in order to check whether early unstable non-adiabatic CDM perturbations are present.

As can be seen in figures 3(a), 4(a), 5(a) and 7(a), the IBEG Universe is dominated at early times \(a < 0.1\) by the CDM independently of the choice of parameters made \((x < 1\) or \(x = 1)\). In this sense, it is natural to assume that the structure formation scenario should prevail in the IBEG model. A numerical solution to perturbation equations is required in order to test the IBEG model against BAO and CMB anisotropy data, the two sets of observational data not used in this article. New bounds on the free parameters of the model could be obtained from them, validating or discarding some parameter choices for the IBEG model.

7. Conclusions

In this work, we presented a physical model of DE: a self-IBEG that increases its particle number, which produces an energy exchange with the CDM present in the Universe. A natural assumption is a connection between CDM and DE, and thus, it is plausible to consider such a coupling between them. Indeed, our coupled model solves (or at least alleviates) the coincidence problem.

Assuming that our Universe model contains baryonic matter, CDM and the IBEG, and that the creation mechanism only produces non-condensated IBEG particles at a general decay form \(N = cV^x\) (a Markoff process), the dynamics of the Universe follows and only six free parameters are left: \(\rho_{b0}, \rho_{m0}, \rho_{G0}\) (the present day mass densities from baryonic, CDM and IBEG particles, respectively), \(\rho_{0}\) (the energy density the associated interaction term between IBEG particles), \(x_0\) and \(x\) (both related with the creation mechanism). Additional bounds are imposed over the free parameters. The first bound is obtained by imposing the present day energy-density values, \(\Omega_{b0}, \Omega_{m0}\) and \(\Omega_{g0}\) [3]; a second bound is imposed by assuring the Universe is undergoing an accelerated expansion stage at the present instant, i.e. \(\rho_{T0} + \frac{3}{2}p_{T0} < 0\). Additionally, we can alleviate the coincidence problem in the IBEG model as soon as \(3\langle r(r)\rangle < H_0\). These limits reduce the number of independent free parameters to 3: \(\rho_{m0}, \rho_{G0}\) and \(x > 0.85\). If we additionally fix \(x\), the region of the \(\rho_{G0}-\rho_{m0}\) space is bounded. Giving different values to the free parameters, we observe that the creation process has a beginning at the scale \(a_{in}\) determined by them. The fact that the coupling is not present at earlier instants of the Universe evolution allows CDM linear perturbations to evolve without instabilities (although numerical work will be done in a future work to confirm this conclusion). The IBEG model successfully fits the supernovae type Ia data with similar statistics to the \(\Lambda\)CDM model, and the parameter \(\rho_G\) has a low impact on the luminosity distance. Under the CDM \(\rho_p > 0\) assumption, the IBEG model in the future evolves into a Universe dominated by the IBEG particles acting as non-relativistic matter, once the CDM particles are exhausted and the self-interaction of IBEG particles has faded away.

If \(x = 1\), the dynamics of the IBEG model are quite different. First, the creation process has no starting point, and can go into the past as far as the Big Bang. Secondly, the coincidence problem is solved, as the parameter \(r\) tends to constant in the late expansion of the IBEG Universe. In fact, the IBEG presents a constant energy density and negative pressure (but \(p \neq -\rho\) so in this sense we cannot affirm it acts as a cosmological constant). The luminosity distance in this case, adjusts the supernovae type Ia data with similar statistics to the \(\Lambda\)CDM. Also, when \(\rho_{m0} < 0.24\), the creation process never completely consumes the CDM present in the Universe and the accelerated stage lasts forever similarly to a de Sitter Universe. The CDM linear perturbations instabilities should be not discarded in this case (as the coupling is present at early times).
In any case, the IBEG model presents a different luminosity distance at high redshifts ($z > 0.5$) for every choice of parameters $x$ and $\rho_m$, and also different than the $\Lambda$CDM one. If we obtain observational data from far away objects, we will be able to determine for which set of parameters the IBEG model is more suitable in order to adjust them. Physical plausibility is gained by using microscopic models to predict cosmological variables and, in turn, by using cosmological data to constrain these models.

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