The Indeterministic Einstein Equation:

Quantum Jumps, Spacetime Structure, and Dark Pseudomatter

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Abstract

Current physics is faced with the fundamental problem of unifying quantum theory and general relativity, which would have resulted in quantum gravity. The main effort to construct the latter has been bent on quantizing spacetime structure, in particular metric. Meanwhile, taking account of the indeterministic aspect of the quantum description of matter, which manifests itself in quantum jumps, essentially affects classical spacetime structure and the Einstein equation. Quantum jumps give rise to a family of sets of simultaneous events, which implies the existence of universal cosmological time. In view of the jumps, the requirement for metric and its time derivative to be continuous implies that the Einstein equation should involve pseudomatter along with matter. Pseudomatter manifests itself only in gravitational effects, being thereby an absolutely dark “matter”.

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Introduction

Current physics is faced with the fundamental problem of constructing a unified physical theory. The problem boils down to the unification of quantum theory and general relativity, which would have resulted in quantum theory of gravitation, or quantum gravity. The effort to construct quantum gravity has been bent, for the most part, on quantizing spacetime structure, in particular metric. Meanwhile, taking account of the indeterministic aspect of the quantum treatment of matter, which manifests itself in quantum jumps, essentially affects classical spacetime structure and the Einstein equation. The reasoning is as follows.

In the classical description of spacetime and quantum treatment of matter, which is known as semiclassical gravity, the Einstein equation reads

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi \kappa T_{\mu\nu}, \]

where \( G \) is the Einstein tensor, \( \Lambda \) is the cosmological constant, \( g \) is metric, \( \kappa \) is the gravitational constant, \( T \) is the energy-momentum tensor operator, and \( \Psi \) is a state vector. Quantum jumps of \( \Psi \) have as a consequence jumps of \( T_{\mu\nu} \), which implies those of \( \tilde{G}_{\mu\nu} = G_{\mu\nu} - \Lambda g_{\mu\nu} \). The components \( G_{ij} \) \((i, j = 1, 2, 3)\) involve the second time derivatives \( \ddot{g}_{ij} \equiv \ddot{g}_{i00} \) of the metric components \( g_{ij} \)\([1,2]\). Therefore the six equations \( \tilde{G}_{ij} = 8\pi \kappa T_{ij} \) may remain unchanged: Jumps of \( T_{ik} \) will result in those of \( \ddot{g}_{ij} \), which is physically appropriate. A completely different type of situation occurs in the four equations \( \tilde{G}_{0\mu} = 8\pi \kappa T_{0\mu} \). The only time derivatives of metric involved in the components \( G_{0\mu} \) are \( \dot{g}_{ij} \), which should be continuous, let alone \( g_{\mu\nu} \). Therefore the four equations are violated by jumps of \( T_{0\mu} \) and must be extended. An apparent approach to the problem is to write \( \tilde{G}_{0\mu} = 8\pi \kappa (T_{0\mu} + P_{0\mu}) \) where \( P_{0\mu} \) are to compensate for the jumps of \( T_{0\mu} \). We shall treat the quantities \( P_{0\mu} \) as the components of the pseudomatter energy-momentum tensor.

There immediately arises the uniqueness problem: In order that introducing \( P_{0\mu} \) make sense, the separations of tensors into \((0\mu)\) and \((ij)\) components should be unique. This implies the existence of a distinguished universal time. It is the quantum jumps that provide the existence of such a time. A quantum jump gives rise to a set of simultaneous events—jumps of \( \ddot{g} \). These events allow for synchronizing clocks and thereby furnishing universal time. It is natural to identify cosmological time with quantum-jump universal time. Thus cosmological time is defined on the level of fundamental physical laws—in contrast to the phenomenological approach in classical cosmology\([3,4]\).

Universal time gives rise to the product structure of spacetime manifold, \( M = T \times S \) where \( T \) is cosmological time and \( S \) is cosmological space, and a particular synchronous reference frame—cosmological reference frame.

There remains the problem of pseudomatter dynamics, i.e., the time dependence of the quantities \( P_{0\mu} \). The latter is determined by the dynamics of the tensor \( T_{\mu\nu} \).

The foregoing results in the indeterministic Einstein equation: \( G_{\mu\nu} - \Lambda g_{\mu\nu} = S_{\mu\nu}, \quad S_{\mu\nu} = T_{\mu\nu} + P_{\mu\nu} \), \( S \) stands for source, \( T \) relates to matter and \( P \) to pseudomatter, respectively; in the cosmological reference frame \( P_{ij} = 0 \).

Pseudomatter manifests itself only in gravitational effects, representing therefore an absolutely dark "matter".

The application of the indeterministic Einstein equation to the Robertson-Walker spacetime results in that the source density is \( \rho_s = \rho_m + \rho_{ps} \) where \( m \) relates to matter and \( ps \) to pseudomatter, respectively, in accordance with which the parameter \( \Omega = \rho/\rho_{cr} \) equals \( \Omega_s = \Omega_m + \Omega_{ps} \).
1 Quantum jumps and violation of the Einstein equation

We adopt the classical description of spacetime and quantum treatment of matter. The Einstein equation takes the form:

\[ G - \Lambda g = 8\pi \kappa T \]  \hspace{1cm} (1.1)

\[ T = (\Psi, \hat{T}\Psi) \]  \hspace{1cm} (1.2)

where \( G \) is the Einstein tensor, \( g \) is metric, \( \Lambda \) is the cosmological constant, \( \kappa \) is the gravitational constant, \( \hat{T} \) is the energy-momentum tensor operator, and \( \Psi \) is a state vector. An essential aspect of this treatment is quantum indeterminism, which manifests itself in jumps of the state vector:

\[ \Psi_{\text{before jump}} \equiv \Psi^< \rightarrow \Psi^> \equiv \Psi_{\text{after jump}} \]  \hspace{1cm} (1.3)

A jump of \( \Psi \) results in that of \( T \):

\[ \Delta T = (\Psi^>, \hat{T}\Psi^>) - (\Psi^<, \hat{T}\Psi^<) \]  \hspace{1cm} (1.4)

under the assumption that \( \hat{T} \) is continuous. Discontinuity of \( T \) causes a violation of the Einstein equation (1.1). Let us consider the violation in detail. Write down equations (1.1), (1.2), and (1.4) in components:

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi \kappa T_{\mu\nu} \]  \hspace{1cm} (1.5)

\[ \Delta T_{\mu\nu} = (\Psi^>, \hat{T}_{\mu\nu}\Psi^>) - (\Psi^<, \hat{T}_{\mu\nu}\Psi^<) \]  \hspace{1cm} (1.6)

Let a jump of \( \Psi \) occur at \( x^0 = t = t_{\text{jump}} \). We put

\[ \Psi^< = \Psi(t_{\text{jump}}), \quad \Psi^> = \Psi(t_{\text{jump}} + 0) \]  \hspace{1cm} (1.7)

so that

\[ \Delta T_{\mu\nu}(t_{\text{jump}}, \vec{x}) = T_{\mu\nu}(t_{\text{jump}} + 0, \vec{x}) - T_{\mu\nu}(t_{\text{jump}}, \vec{x}), \quad \vec{x} = (x^i), \quad i = 1, 2, 3 \]  \hspace{1cm} (1.8)

The components \( G_{ij} \) \((i, j = 1, 2, 3)\) of the Einstein tensor involve the second time derivatives

\[ \ddot{g}_{ij} \equiv g_{ij,00} \]  \hspace{1cm} (1.9)

of the metric tensor components \( g_{ij} \) \([1,2]\). This makes it possible to retain the six equations

\[ G_{ij} - \Lambda g_{ij} = 8\pi \kappa T_{ij} \]  \hspace{1cm} (1.10)

unchanged. Jumps of the \( T_{ij} \) will result in those of the \( \ddot{g}_{ij} \), which is quite conceivable from the physical point of view: A jump of force results in a jump of acceleration. As to the four equations

\[ G_{0\mu} - \Lambda g_{0\mu} = 8\pi \kappa T_{0\mu} \]  \hspace{1cm} (1.11)

the situation is completely different. The components \( G_{0\mu} \) of the Einstein tensor involve no second time derivatives of metric tensor components; the only time derivatives of metric involved in \( G_{0\mu} \) are \( \dot{g}_{ij} \). The latter should be continuous, not to mention \( g_{\mu\nu} \). Thus the violation of the four equations (1.11) is intolerable and they must be extended.
2 Introducing pseudomatter

An apparent approach to the improvement of equations (1.11) is to write

\[ G_{0\mu} - \Lambda g_{0\mu} = 8\pi \kappa (T_{0\mu} + P_{0\mu}), \quad \mu = 0, 1, 2, 3 \] (2.1)

in which the terms \( P_{0\mu} \) are to compensate for the jumps of \( T_{0\mu} \). Dynamical equations are (1.10), whereas (1.11) play the role of integrals of motion, so that introducing the terms \( P_{0\mu} \) into (2.1) is quite justified. We treat the quantities \( P_{0\mu} \) as the components of the energy-momentum tensor relating to pseudomatter.

3 The problem of uniqueness of pseudomatter

There immediately arises the uniqueness problem. In order that the above procedure of introducing the terms \( P_{0\mu} \) make sense, the separation of the tensors in question into \((0\mu)\) and \((ij)\) parts needs to be unique. This, in its turn, implies the existence of a distinguished \( x^0 \) coordinate, or a universal time. Cosmological time, i.e., universal time of the standard model of cosmology cannot be taken for this purpose, for it is introduced phenomenologically rather than on the basis of fundamental physical laws [3,4].

4 Quantum-jump universal time

It is quantum jumps that appear again, this time to provide a universal time. A quantum jump of the state vector gives rise to a set of events—jumps of \( \dot{\gamma} \). These events are, by definition, simultaneous, which allows for synchronizing clocks and thereby furnishing the universal time. It is natural to identify phenomenological cosmological time with the quantum-jump universal time. Now cosmological time is defined on the level of fundamental physical laws.

In special relativity the concept of simultaneity in connection with quantum jumps makes no operationalistic sense. Taking gravity into account endows the concept with an operationalistic content.

5 Spacetime structure

Universal time gives rise to a family of sets of simultaneous events, thereby endowing spacetime with a fiber structure. The metric compatible with this structure, i.e., admitting synchronization of clocks, is of the form [2]

\[ ds^2 = g_{00}(dx^0)^2 + g_{ij}dx^idx^j \] (5.1)

or, with

\[ dt = g_{00}dx^0, \quad t(x^0, \vec{x}) = \int g_{00}(x^0, \vec{x})dx^0 \] (5.2)

\[ ds^2 = dt^2 + g_{ij}dx^idx^j, \quad g_{ij} = g_{ij}(t, \vec{x}) \] (5.3)
which relates to a synchronous reference frame. The latter, in its turn, implies the product spacetime manifold:

\[ M = M^4 = T \times S, \quad M \ni p = (t, s), \quad t \in T, \quad -\infty \leq a \leq t \leq b \leq \infty, \quad s \in S \quad (5.4) \]

The one-dimensional manifold \( T \) is universal cosmological time, the three-dimensional manifold \( S \) is cosmological space. By (5.4), the tangent space \( M_p \) at a point \( p \in M \) is

\[ M_p = T_t \oplus S_s, \quad p = (t, s) \quad (5.5) \]

and, in view of (5.3),

\[ T_t \perp S_s \quad (5.6) \]

Thus, quantum jumps give rise to the product spacetime (5.4) and a particular synchronous reference frame

\[ p = (t, s) \leftrightarrow (t, \vec{x}) \quad (5.7) \]

The latter may be called cosmological reference frame and considered as a canonical synchronous reference frame.

In the coordinate-free representation, the metric (5.3) reads

\[ g = dt \otimes dt - h_{tt}, \quad h_{tt} \leftrightarrow -g_{ij}(t, \vec{x})dx^idx^j \quad (5.8) \]

in which \( h_{tt} \) is a Riemannian metric tensor on \( S \) depending on \( t \).

6 Pseudomatter dynamics

In order for equations (2.1) not to be merely a definition of the quantities \( P_{0\mu} \):

\[ P_{0\mu} \equiv \frac{1}{8\pi \kappa}\left(G_{0\mu} - \Lambda g_{0\mu}\right) - T_{0\mu} \quad (6.1) \]

pseudomatter dynamics, i.e., time evolution of these quantities should be determined independently of the dynamics for the right-hand side of (6.1).

We have

\[ G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi \kappa S_{\mu\nu}, \quad S_{\mu\nu} = S_{\nu\mu} \quad (6.2) \]

where \( S \) stands for source (and has nothing to do with the \( S \) in (5.4)). In the canonical synchronous reference frame, which will be used henceforth,

\[ S_{ij} = T_{ij}, \quad S_{0\mu} = T_{0\mu} + P_{0\mu}, \quad P_{ij} = 0 \quad (6.3) \]

and

\[ g_{0i} = 0, \quad g_{00} = 1 \quad (6.4) \]

From (6.2) follows

\[ S^{\mu}_{\nu} = 0 \quad (6.5) \]

For any symmetric tensor \( Y^{\mu\nu} = Y^{\nu\mu} \)

\[ Y^{\mu}_{\mu} \sqrt{-g} = (Y^{\mu}_{\mu} \sqrt{-g})_{,\mu} - (1/2)g^{\alpha\beta,\mu}Y^{\alpha\beta} \sqrt{-g}, \quad g = \det(g_{\mu\nu}) \quad (6.6) \]
Thus or, for brevity, so that holds [5]. We find from (6.5), (6.6), (6.4), and (5.3)

\[ (S^\mu_0 \sqrt{-g})_0 = (1/2)g_{kl,\mu}T^{kl} \sqrt{-g} - (S^\mu_j \sqrt{-g})_j \]  

(6.7)

whence

\[ S^\mu_0 \sqrt{-g} = \int_{t_0}^t dt' \{ (1/2)g_{kl,\mu}T^{kl} \sqrt{-g} - (S^\mu_j \sqrt{-g})_j \} + f_\mu(\bar{x}) \]  

(6.8)

Thus

\[ S_{0\mu}(t, \bar{x}) = \frac{1}{\sqrt{-g(t, \bar{x})}} [F_\mu(t, \bar{x}) + f_\mu(\bar{x})] \]  

(6.9)

or, for brevity,

\[ S_{0\mu} = \frac{1}{\sqrt{-g}} [F_\mu + f_\mu] \]  

(6.10)

where

\[ F_\mu = \int_{t_0}^t dt' \{ (1/2)g_{kl,\mu}T^{kl} \sqrt{-g} - (S^\mu_j \sqrt{-g})_j \} \]  

(6.11)

so that, in view of (6.3), (6.4),

\[ F_i = \int_{t_0}^t dt' \{ (1/2)g_{kl,i}T^{kl} \sqrt{-g} - (T^i_j \sqrt{-g})_j \} \]  

(6.12)

\[ F_0 = \int_{t_0}^t dt' \{ (1/2)g_{kl,0}T^{kl} \sqrt{-g} - (g^{ji}S_{0i} \sqrt{-g})_j \} \]  

(6.13)

\[ = \int_{t_0}^t dt' \{ (1/2)g_{kl,0}T^{kl} \sqrt{-g} - (g^{ji}[F_i + f_i])_j \} \]

As to initial conditions, we have

\[ F_\mu(t_0) = 0 \]  

(6.14)

so that

\[ f_\mu = f_\mu(\bar{x}) = \frac{1}{8\pi \sqrt{-g}} [\sqrt{-g}(G_{0\mu} - \Lambda g_{0\mu})](t_0, \bar{x}) \]  

(6.15)

In view of (6.3), the dynamics of \( P_{0\mu} \) reduces to that of \( S_{0\mu} \). The latter is described by equations (6.10), (6.12), (6.13), and (6.15).

Let us explicitly introduce jumps \( \Delta T_{0\mu} \) into consideration. It follows from (6.6) that

\[ (1/2)g_{kl,\mu}T^{kl} \sqrt{-g} - (T^\mu_j \sqrt{-g})_j = (T^\mu_0 \sqrt{-g})_0 - T^{\mu \nu} \sqrt{-g} \]  

(6.16)

Now we find from (6.12)

\[ F_i = \sqrt{-g} T_{0i} - [\sqrt{-g} T_{0i}](t_0) - \sum_{t_0 \leq t_k < t} [\sqrt{-g} \Delta T_{0i}](t_k) - \int_{t_0}^t dt' T^{\nu \mu} \sqrt{-g} \]  

(6.17)

and from (6.13), (6.10), (6.17), and (6.15)

\[ F_0 = \sqrt{-g} T_{00} - [\sqrt{-g} T_{00}](t_0) - \sum_{t_0 \leq t_k < t} [\sqrt{-g} \Delta T_{00}](t_k) - \int_{t_0}^t dt' T^{\nu \mu} \sqrt{-g} \]  

\[ - \int_{t_0}^t dt' \left( g^{ji} \left\{ \sqrt{-g} \left( \frac{1}{8\pi \sqrt{-g}} G_{0i} - T_{0i} \right) \right\} (t_0) - \sum_{t_0 \leq t_k < t} [\sqrt{-g} \Delta T_{0i}](t_k) - \int_{t_0}^{t'} dt'' T_{i \nu}^{\nu} \sqrt{-g} \right) \]  

(6.18)
Thus we obtain from (6.3), (6.10), (6.17), (6.18), and (6.15)

$$P_{0\mu} = \frac{1}{8\pi\sqrt{\mathcal{g}}} \left[ \sqrt{\mathcal{g}} (G_{0\mu} - \Lambda g_{0\mu} - 8\pi \mathcal{X} T_{0\mu}) \right](t_0)$$

$$- \frac{1}{\sqrt{\mathcal{g}}} \left\{ \int_{t_0}^{t} dt' \sqrt{\mathcal{g}} T_{\mu'}^\nu + \sum_{t_0 < t_k < t} \left[ \sqrt{\mathcal{g}} \Delta T_{0\mu}(t_k) \right] \right\}$$

$$- \frac{g_{0\mu}}{8\pi\sqrt{\mathcal{g}}} \int_{t_0}^{t} dt' \left\{ g^{ji} \left[ \sqrt{\mathcal{g}} (G_{0i} - 8\pi \mathcal{X} T_{0i}) \right](t_0) \right\}_j$$

$$+ \frac{g_{0\mu}}{\sqrt{\mathcal{g}}} \int_{t_0}^{t} dt' \left\{ g^{ji} \left( \int_{t_0}^{t'} dt'' \sqrt{\mathcal{g}} T_{i'}^\nu + \sum_{t_0 < t_k < t} \left[ \sqrt{\mathcal{g}} \Delta T_{0i}(t_k) \right] \right) \right\}_j$$

(6.19)

This explicitly describes pseudomatter dynamics.

Note that the components $P_{ij}$ cannot be introduced, for there are only four pseudomatter dynamics equations (6.5) and they determine the components $P_{0\mu}$.

### 7 The indeterministic Einstein equation

We have obtained the indeterministic Einstein equation

$$G - \Lambda g = 8\pi \mathcal{X} S$$

(7.1)

In canonical synchronous reference frame it reads:

$$G_{ij} - \Lambda g_{ij} = 8\pi \mathcal{X} T_{ij}$$

(7.2)

and (2.1) with $P_{0\mu}$ (6.19), i.e.,

$$G_{0\mu} - \Lambda g_{0\mu} - 8\pi \mathcal{X} T_{0\mu}$$

$$+ \frac{8\pi}{\sqrt{-\mathcal{g}}} \left\{ \int_{t_0}^{t} dt' \sqrt{\mathcal{g}} T_{\mu'}^\nu + \sum_{t_0 < t_k < t} \left[ \sqrt{\mathcal{g}} \Delta T_{0\mu}(t_k) \right] \right\}$$

$$+ \frac{g_{0\mu}}{\sqrt{-\mathcal{g}}} \int_{t_0}^{t} dt' \left\{ g^{ji} \left[ \sqrt{\mathcal{g}} (G_{0i} - 8\pi \mathcal{X} T_{0i}) \right](t_0) \right\}_j$$

$$- \frac{8\pi g_{0\mu}}{\sqrt{-\mathcal{g}}} \int_{t_0}^{t} dt' \left\{ g^{ji} \left( \int_{t_0}^{t'} dt'' \sqrt{\mathcal{g}} T_{i'}^\nu + \sum_{t_0 < t_k < t} \left[ \sqrt{\mathcal{g}} \Delta T_{0i}(t_k) \right] \right) \right\}_j$$

$$= \frac{1}{\sqrt{-\mathcal{g}}} \left[ \sqrt{\mathcal{g}} (G_{0\mu} - \Lambda g_{0\mu} - 8\pi \mathcal{X} T_{0\mu}) \right](t_0)$$

(7.3)

Six equations (7.2) are dynamical ones whereas four equations (7.3) play the role of integrals of motion.

A complete system of dynamical equations should include an equation for the state vector $\Psi$.

Initial conditions at $t = t_0$ are given by

$$g_{ij}(t_0, \vec{x}), \quad \dot{g}_{ij}(t_0, \vec{x}), \quad \Psi_{t_0}$$

(7.4)
The classical Einstein equations (1.11) are obtained by putting

\[ \Delta T_{0\mu} = 0, \quad T_{\mu\nu} = 0, \quad [G_{0\mu} - \Lambda g_{0\mu} - 8\pi \kappa T_{0\mu}](t_0) = 0 \]  

(7.5)
in (7.3). In order that the complete system (1.10), (1.11) be fulfilled in all reference frames, the condition

\[ \Delta T_{ij} = 0 \]  

(7.6)
must hold as well.

8 Pseudomatter as an absolutely dark “matter”

There exists the well known problem of dark matter [6,7,8]. There is no interaction between pseudomatter and matter, except that they interact through gravity. Thus pseudomatter manifests itself only in gravitational effects and may therefore be regarded as an absolutely dark “matter”.

9 Pseudomatter in the Robertson-Walker spacetime

Let us consider \( P_{0\nu} \) (6.19) in the case of the Robertson-Walker spacetime. In this case

\[ G_{0i} = 0, \quad T_{0i} = 0 \]  

(9.1)
so that (7.3) for \( \mu = i \) results in

\[ T_{i\nu} = 0 \]  

(9.2)
Thus

\[ P_{0i} = 0 \]  

(9.3)

\[ P_{00} = \frac{1}{8\pi \sqrt{-g}} \left[ \sqrt{-g}(G_{00} - \Lambda - 8\pi \kappa T_{00}) \right](t_0) \]

\[ \quad - \frac{1}{\sqrt{-g}} \left\{ \int_{t_0}^{t} dt' \sqrt{-g} T_{0\nu} + \sum_{t_0 \leq t_k < t} [\sqrt{-g} \Delta T_{00}](t_k) \right\} \]  

(9.4)
Next we assume that matter energy \( E \) is continuous:

\[ \Delta E(t_k) = 0 \]  

(9.5)
then

\[ \Delta T_{00}(t_k) = 0 \]  

(9.6)
In view of (9.2) we put

\[ T_{0\nu} = 0 \]  

(9.7)
so that

\[ T_{\mu\nu} = 0 \]  

(9.8)
Now (9.4) reduces to

\[ P_{00} = \frac{1}{8\pi \kappa} \sqrt{-g(t_0)} \left[ G_{00} - \Lambda - 8\pi \kappa T_{00} \right](t_0) \]  

(9.9)

We have

\[ \frac{\sqrt{-g(t_0)}}{\sqrt{-g}} = \frac{R^3(t_0)}{R^3(t)} \]  

(9.10)

where \( R \) is the radius of the universe.

Thus the pseudomatter density

\[ \rho_{ps} = \rho_{ps}(t) = P_{00} = \frac{[R^3(G_{00} - \Lambda - 8\pi \kappa \rho_m)](t_0)}{8\pi \kappa} \frac{1}{R^3(t)} \propto \frac{1}{R^3(t)} \]  

(9.11)

where

\[ \rho_m = T_{00} \]  

(9.12)

is the matter density. The source density is

\[ \rho_s = \rho_m + \rho_{ps} \]  

(9.13)

in accordance with which, the parameter \( \Omega = \rho/\rho_{cr} \) is equal to

\[ \Omega_s = \Omega_m + \Omega_{ps} \]  

(9.14)

These results match those of [9].

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