High energy neutrino oscillation at the presence of the Lorentz Invariance Violation

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Due to quantum gravity fluctuations at the Planck scale, the space-time manifold is no longer continuous, but discretized. As a result the Lorentz symmetry is broken at very high energies. In this article, we study the neutrino oscillation pattern due to the Lorentz Invariance Violation (LIV), and compare it with the normal neutrino oscillation pattern due to neutrino masses. We find that at very high energies, neutrino oscillation pattern is very different from the normal one. This could provide an possibility to study the Lorentz Invariance Violation by measuring the oscillation pattern of very high energy neutrinos from a cosmological distance.

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I. INTRODUCTION

Nowadays the violation of the Lorentz symmetry has attracted increasing attention, because the Lorentz symmetry is the main concept of special relativity and any relativistic theory which is invariant under continuous Lorentz transformations. A growing number of speculations suggests that Lorentz invariance might be violated or deformed at very high energies \([1,2]\). The local Lorentz symmetry has been examined in many sectors of the standard model (SM) relating to photons, electrons, protons, and neutrons \([3]-[5]\), and none of Lorentz invariance violation (LIV) has been identified so far in these sectors for low-energies. The Lorentz invariance should be violated at very high energy scale or the Planck scale, since the Lorentz group is unbounded at the high boost (or high energy) end, in principle it might subject to modifications in the high boost limit \([6,7]\). The Lorentz symmetry is based on the assumption that space-time is scale-free, namely there is no fundamental length scale associated with the Lorentz group. However, due to violent fluctuations...
of quantum gravity at the Planck scale $M_{\text{pl}} = \hbar c/\lambda_{\text{pl}} \sim 10^{19}\text{GeV}$, $\lambda_{\text{pl}} = \sqrt{\hbar G/c^3} = 10^{-33}\text{cm}$, the space-time manifold is no longer continuous, but discretized, and as a consequence, the Lorentz symmetry is broken. The discretization (foam structure) of space-time manifold with a minimal spacing $\sim \lambda_{\text{pl}}$ was first discussed by Wheeler [8], and have been intensively studied in literatures (see for example [9, 10]). In Ref. [11], by using the universal entropy bound, it has been shown that the space-time has a minimum length scale proportional to the Planck length, leading to a discrete space-time structure.

The possibilities of Lorentz invariance violation have been studied in quantum-gravity models [12], string theory [13, 14], Loop gravity [15, 16], non-commutative geometry [17]-[19], the doubly special relativity (DSR) [20]. In addition, there are some other effective field theories for Lorentz violation, for examples, the Coleman- Glashow model [21], the minimal standard model extension (SME) [22], and the newly proposed standard model supplement (SMS) [23, 24].

In recent years, there has been much interest in testing LIV effects. However, observational tests face a major obstacle of practical nature: LIV effects due to quantum-gravity are expected to be extremely small because of Planck-scale suppression, and low-energy measurements are likely to require very high sensitivities [25]. This leads to the use of high energy astrophysics data to provide constraints on LIV effects. For examples, Gamma-Ray Burst (GRB) data are analyzed to see LIV effects on the arrival time of photons at different energies [26]. However, high energy photons can be annihilated via pair creation with the IR background, and this limits the distances that high energy photons can travel, and the photon number fluxes lower for higher energies limit [27].

Very high energy neutrinos [28] provide an alternative to test LIV effects. Practically all current GRB models [29] predict bursts of very high energy neutrinos, with energy ranging from 100 TeV to $10^4$ TeV (and possibly up to $10^6$ TeV) [30, 31]. In addition, neutrinos with energy up to $\sim 10^{21}$ eV are supposed to be produced by cosmological objects like GRB and Active Galactic Nuclei (AGN) [32]. These high-energy neutrinos from cosmological distances can open a new window on testing LIV effects. It was suggested that neutrinos of energies as high as $10^{22} - 10^{24}$ eV could be produced by topological defects like cosmic strings, necklaces and domain walls [33]. Theoretical framework for Lorentz violation and neutrino oscillation probabilities is proposed in Ref. [34].

In this article, we formulate the discretization of space-time manifold as a hyperbolic lattice with the lattice spacing $a$, and adopt the Lorentz-symmetry-breaking Wilson operator [35] for neutrino fields on the lattice. We show that neutrino oscillations depend not only on their non-vanishing masses, but also on the Lorentz-symmetry-breaking Wilson term in high energies. This provides
the possibility to test LIV effects by studying high-energy cosmic neutrinos oscillations.

II. BOSONS AND FERMIONS ON A DISCRETE SPACE-TIME

We first give a brief review on the energy-momentum relation of free bosons and fermions on a hypercubic lattice of space-time. The Klein-Gordon equation for a free boson field \( \phi(\vec{x}, t) \) in \( 3 + 1 \) dimension space-time,

\[
\ddot{\phi} = \nabla^2 \phi - m^2 \phi, \tag{1}
\]

where \( m \) is the boson mass. Eq. (1) gives the energy-momentum relation of the free boson field

\[
E^2 = k^2 + m^2. \tag{2}
\]

In a hypercubic spatial lattice (time continuum), one can write

\[
a^2 \nabla^2 \phi \rightarrow \phi(\vec{n} + a) + \phi(\vec{n} - a) - 2\phi(\vec{n}) = (d^+ + d^- - 2)\phi(\vec{n}), \tag{3}
\]

where 3-dimension vector \( \vec{n} \equiv a(n_1, n_2, n_3) \), \( a \) is the lattice spacing and the shift operators \( d^\pm \phi(\vec{n}) = \phi(\vec{n} \pm a) \). Then, the Klein-Gordon equation on the lattice is given by

\[
\ddot{\phi} = (d^+ + d^- - 2)\phi(\vec{n}) - m^2 \phi, \tag{4}
\]

and the energy-momentum relation on the lattice is given by

\[
E^2 = m^2 - \frac{2[\cos(k \cdot a) - 1]}{a^2}, \tag{5}
\]

Eqs. (3,5) are not invariant under the Lorentz transformations. For low-energy particles \( k \cdot a \ll 1 \), the energy-momentum relation can approximately be written by

\[
E^2 = m^2 + k^2 - \frac{1}{12}(k^4 a^2) + O(k^6 a^4), \tag{6}
\]

which approaches the energy-momentum relation \( \tag{2} \) and the Lorentz symmetry is restored. Assuming a more complicate discretization of space-time, we parameterize the energy-momentum relation as

\[
E^2 = m^2 + k^2 - \beta k^2(k^2 a^2)^\alpha + O(k^6 a^4), \tag{7}
\]

where the third term breaks the Lorentz symmetry.
FIG. 1: Spectrum of the native lattice Dirac equation.

We turn to consider the energy-momentum relation of Dirac fermions on a lattice. The Dirac equation in the continuum space-time

\[(i \not{\partial} - m) \psi(x) = 0,\]  

and the energy-momentum relation is

\[E = k^2 + m^2, \quad -\infty < k < +\infty,\]  

where \(k\) is the 3-momentum of fermions. On a spatial lattice (time continuum), one uses

\[\not{\partial} \psi(x) = \gamma_{\mu} \partial_{\mu} \psi(x) \Rightarrow \gamma_{\mu} \frac{[\psi(x + a^{\mu}) - \psi(x - a^{\mu})]}{2a}.\]  

where \(a^{\mu} \equiv an^{\mu}\) and

\[\psi(x + a^{\mu}) = \psi(k)e^{ik^{\mu}a^{\mu}}.\]  

As a result, the energy-momentum relation of fermion fields on a lattice is

\[E = \pm \frac{\sin(ka)}{a}.\]  

For \(k \cdot a \ll 1\) the energy-momentum relation becomes

\[E = \pm k + O(k^3a^2),\]  

the usual energy-momentum relation. However, Eq. (12) has a problem of fermion doubling as shown in Fig. 1, \(ka = \pm \pi\) also present fermion spices.

A chiral symmetry breaking term is necessarily added into Hamiltonian \(\mathcal{H}\) so that \(E vs k\) does not have secondary minima at \(ka = \pm \pi\). Considering two-component fermions on a spatial lattice
with Wilson term \[35\], the new Hamiltonian is

\[
H = -\frac{i}{2a} \sum_n \psi^\dagger(n) \alpha \left[ \psi(n + 1) - \psi(n - 1) \right] + m \sum_n \bar{\psi}(n) \psi(n) + \frac{B}{2a} \sum_n \bar{\psi}(n) \left[ 2\psi(n) - \psi(n + 1) - \psi(n - 1) \right],
\]

(14)

The equation of motion for \( \psi \) is

\[
i \dot{\psi}(n) = -\frac{i}{2a} \gamma_5 \left[ \psi(n + 1) - \psi(n - 1) \right] + \frac{B}{2a} \gamma_0 \left[ 2\psi(n) - \psi(n + 1) - \psi(n - 1) \right].
\]

(15)

Substituting a plane wave \( \psi = \exp(iEt - ikna) \) solution into Eq. (15), one obtains the energy-momentum relation

\[
E^2 = m^2 + \frac{\sin^2 ka}{a^2} + 4B^2 \sin^4(ka/2) a^2.
\]

(16)

For low-energy particles \( ka \to 0 \) Eq. (16) reduces to

\[
E^2 \simeq k^2 + m^2 + \frac{1}{4} B^2 k^4 a^2 + O(k^6 a^4),
\]

(17)

where the third term (\( B \)-term) violates the Lorentz symmetry. In principle \( B \) is a free parameter characterizing the deviation from the Lorentz symmetry.

From gravitational theories, for low-energy particles with \( E \ll \xi M_{pl} \), an energy-momentum relation is parametrized as \[37\]

\[
E^2 - p^2 - m^2 \simeq \pm E^2 \left( \frac{E}{\xi_n E_{pl}} \right)^n,
\]

(18)

where \( \xi_2 \gtrsim 10^{-9} \) determined by the flaring AGN \[38\] for photons \( m = 0 \). Comparing Eq. (17) with Eq. (18), one finds that the lattice spacing \( a \lesssim 10^9/M_{pl} \), which indicates the Lorentz symmetry breaking scale. We will adopt this scale to study effects of the Lorentz symmetry breaking on high-energy neutrino oscillations.

III. NEUTRINO OSCILLATIONS DUE TO LORENTZ INVARIANCE VIOLATION

In this section we study neutrino oscillations due to the Lorentz symmetry breaking \( B \)-term in Eq. (16). Flavor neutrinos \( (\nu_e, \nu_\mu, \nu_\tau) \) are always produced and detected via their interacting with intermediate gauge bosons \( W_\mu^{(\pm)} \) and \( Z_\mu^0 \) in the SM. Due to the parity violation, flavor neutrinos are not the eigenstates of the Hamiltonian and in principle they are superpositions of the Hamiltonian eigenstates \( |\nu_i\rangle \)

\[
\mathcal{H}|\nu_i\rangle = E_i|\nu_i\rangle, \quad i = 1, 2, 3,
\]

(19)
where $E_i$ are the energy eigenvalues of the type-$i$ neutrino. Using Eq. (17) ultra-relativistic neutrinos, the energy-momentum relation can be approximately written as

$$E_i \approx k_i + \frac{m_i^2}{2k_i} + \frac{1}{8} B_i^2 k_i^3 a^2 + ..., \quad (20)$$

for the type-$i$ neutrino.

Flavor eigenstates and Hamiltonian eigenstates (mass eigenstates) are related by an unitary transformation represented by a matrix $U$,

$$|\nu_l \rangle = \sum_{i=1}^{3} U_{li} |\nu_i \rangle, \quad (21)$$

where the flavor index $l = e, \mu, \tau$. This shows that flavor eigenstate is a mixing of the Hamiltonian mass eigenstates $|\nu_i \rangle, (i = 1, 2, 3)$ and vice versa. Time evolution of flavor neutrino states is given by

$$|\nu_l(t) \rangle = e^{-iHt} |\nu_l \rangle = \sum_{i=1}^{3} e^{-iE_i t} U_{li} |\nu_i \rangle, \quad (22)$$

indicating, after some time $t$, the evolution of these flavor neutrino states leads to flavor neutrino oscillations. The probability of such neutrino oscillations is given by

$$P_{\nu_l \rightarrow \nu_{l'}} = |\langle \nu_{l'} | \nu_l \rangle|^2 = \sum_{i,j} |U_{li} U_{l'i}^* U_{l'j} U_{lj}| \cos[(E_i - E_j)t + \varphi_{ll'}], \quad (23)$$

where

$$(E_i - E_j) = \frac{(m_i^2 - m_j^2)}{2E} - \frac{(B_i^2 - B_j^2)}{8} k_i^3 a^2, \quad (24)$$

and $\varphi_{ll'} = \arg(U_{li} U_{l'i}^* U_{l'j} U_{lj})$ [39]-[41]. In the right-handed side of Eq. (24), the first term is normal one and the second term is due to the Lorentz symmetry breaking $B$-term in Eq. (20).

For a two-level system of electron and muon neutrinos ($\nu_e, \nu_\mu$). The unitary matrix $U$ is explicitly given by

$$U = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad (25)$$

where $\theta$ is a mixing angle. Eq. (21) becomes

$$|\nu_e \rangle = \cos \theta |\nu_1 \rangle + \sin \theta |\nu_2 \rangle,$$
$$|\nu_\mu \rangle = -\sin \theta |\nu_1 \rangle + \cos \theta |\nu_2 \rangle. \quad (26)$$
The Hamiltonian \((19)\) in the base of the mass eigenstates \(|\nu_i\rangle\) is

\[
\mathcal{H}_{\text{mass}} = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} \simeq E + \begin{pmatrix} \frac{m_1^2}{2E} & 0 \\ 0 & \frac{m_2^2}{2E} \end{pmatrix} + \frac{1}{16} \begin{pmatrix} B_1^2 a^2E^3 & 0 \\ 0 & B_2^2 a^2E^3 \end{pmatrix},
\]

where the leading contribution to the neutrino energy \(E_i\) is obtained by assuming \(p_1 \approx p_2 = E\).

By using Eqs. \((25, 27)\) the Hamiltonian in the base of flavor eigenstates is given by

\[
\hat{\mathcal{H}} = \mathbf{U} \mathcal{H}_{\text{mass}} \mathbf{U}^\dagger = E + \frac{m_1^2 + m_2^2}{4E} + E^3a^2 \left( \frac{B_1^2 + B_2^2}{16} \right) + \left( \frac{\Delta m_{12}^2}{4E} + E^3a^2 \frac{\Delta B_{12}^2}{16} \right) \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix},
\]

where \(\Delta B_{12}^2 = B_2^2 - B_1^2, \Delta B_{12}^2 = B_2^2 - B_1^2, (B_2 > B_1)\) and the mixing angle \(\theta\) is given by

\[
\tan 2\theta = \frac{2\mathcal{H}_{12}}{\mathcal{H}_{22} - \mathcal{H}_{11}}.
\]

**IV. THE CONVERSION PROBABILITY**

Based on Eq. \((23)\) for the system of two neutrino flavors, the conversion and the survival probabilities of a particular flavor of neutrino with the mixing angle \(\theta\), can be written as

\[
P_{\text{conv}}(t, t_i) = \sin^2 2\theta \sin^2 \left( \frac{\Phi}{2} \right),
\]

\[
P_{\text{surv}} = 1 - P_{\text{conv}},
\]

where the oscillation phase \(\Phi\) is given by \([42]\)

\[
\Phi = \int_{t_i}^t \varepsilon(\tau)d\tau,
\]

where \(t_i\) and \(t\) are respectively the initial and final time of the evolution of the system. In the case for vacuum oscillations, \(\varepsilon\) equals to \([39]\)

\[
\varepsilon = \varepsilon_{12} = \frac{\Delta m_{12}^2}{2E},
\]

In the case that the Lorentz violation is present in Eq. \((28)\), \(\varepsilon\) can be written as

\[
\varepsilon = \frac{\Delta m_{12}^2}{2E} + \frac{1}{8} E^3a^2 \Delta B_{12}^2,
\]

which shows \(\Delta B^2\) can also generate neutrino oscillations. The discussions and calculations are also applied for other two-level systems of neutrinos, \((\theta_{23}, \Delta B_{23}^2)\) and \((\theta_{13}, \Delta B_{13}^2)\).
Using the scaling relation
\[ E = E_0 \left( \frac{t_0}{t} \right)^{2/3} = E_0(1 + z), \] (35)

where \( t_0 \sim 10^{18} \text{s} \) is the present epoch, the redshift \( z \equiv \left( \frac{t_0}{t} \right)^{2/3} - 1 \) and \( E_0 \) is the energy at the presence epoch, \( z \equiv 0 \). We separate the oscillation phase (32) into two parts:

\[ \Phi = \Phi_{\text{vac}} + \Phi_{\text{LV}}. \] (36)

then using Eqs. (32,34) and (35), we obtain the vacuum and LV phases

\[ \Phi_{\text{vac}}(x, x_i) = \frac{3}{10} \frac{\Delta m^2 t_0}{E_0} (x^\frac{2}{3} - x_i^\frac{2}{3}), \] (37)

\[ \Phi_{\text{LV}}(x, x_i) = \frac{a^2}{8} \Delta B^2 E_0^3 t_0 \left( \frac{1}{x_i} - \frac{1}{x} \right), \] (38)

where \( x \equiv t/t_0 \) and \( x_i \equiv t_i/t_0 \). From (37) and (38), we find that the neutrino vacuum oscillation does not occur, \( \Delta m^2 \to 0 \), however neutrino oscillations due to the Lorentz symmetry violation take place.

Taking \( x_i = 0.125 \), corresponding to the initial time of neutrino productions at redshift \( z \simeq 3 \), and \( x = 1 \), we obtain

\[ \Phi_{\text{vac}}(1, 0.125) \simeq \frac{3}{10} \frac{\Delta m^2 t_0}{E_0}, \] (39)

\[ \Phi_{\text{LV}}(1, 0.125) \simeq \frac{7}{8} a^2 \Delta B^2 E_0^3 t_0. \] (40)

Eqs. (39) and (40) show that for very high energy neutrinos, the LV oscillation phase becomes more important than the vacuum oscillation phase.

Since neutrino detectors have a finite accuracy in the reconstruction of the neutrino energy, by averaging Eq. (30) over the interval \( \Delta E_0 \simeq E_0 \), one computes the conversion probability [42]

\[ P_{\text{conv}}(E_0) = \frac{1}{\Delta E_0} \int_{E_0/2}^{3E_0/2} dE' P(E'). \] (41)

Considering very high energy neutrinos, which are produce at \( z = 3 \), using Eqs. [31,39,41], we plot in Fig. (2) the survival probability as a function of energy \( E_0 \). We find that for large neutrino energies, the vacuum oscillation phase [39] is suppressed and its contribution to the conversion probability \( P_{\text{conv}}(\nu_{\alpha} \to \nu_{\beta}) \) is almost zero, and the conversion probability \( P_{\text{conv}}(\nu_{\alpha} \to \nu_{\beta}) \) is mainly contributed from the neutrino oscillation phase [40] due to the Lorentz symmetry breaking. This implies that any observation of high-energy neutrino oscillations indicates the Lorentz symmetry breaking. In addition, the neutrino oscillation pattern \( (E_0\text{-dependence}) \) due to the Lorentz symmetry breaking is very different from the neutrino oscillation pattern in vacuum. This might provide
FIG. 2: The survival probability $1 - P_{\text{conv}}(\nu_\alpha \rightarrow \nu_\beta)$ is plotted as a function of neutrino energy $E_0$, for different values of the Lorentz symmetry breaking scale $a$. $\Delta m^2 \approx 10^{-7} \text{eV}^2$ and the mixing angle $\sin^2 2\theta \simeq 1$ \cite{42} and $|\Delta B^2| \approx |\Delta m^2|$.

The possibility that using high-energy cosmic neutrinos, one can study neutrino oscillation pattern to gain some insight into the Lorentz symmetry breaking, in connection with the study of arrival time delay of high-energy cosmic gamma ray due to the Lorentz symmetry breaking \cite{37}. In addition, from the theoretical point view, it would be interesting to see how the Lorentz violation term (15-17) relate to Lorentz violation operators in effective field theories, see for example Ref. \cite{34}. 

[1] H. Sato and T. Tati, Prog. Theor. Phys. 47, 1788 (1972).
[2] G. Amelino-Camelia et al., Nature 393, 763 (1998).
[3] G. Amelino-Camelia, C. Lammerzahl, A. Macias and H. Muller, AIP Conf. Proc. 758, 30 (2005) arXiv:gr-qc/0501053.
[4] V. A. Kostelecky, Phys. Rev. D 69, 105009 (2004) arXiv:hep-ph/0312310.
[5] D. Mattingly, Living Rev. Rel. 8, 5 (2005) arXiv:gr-qc/0502097.
[6] S. R. Coleman and S. L. Glashow, Phys. Rev. D 59, 116008 (1999).
[7] F. W. Stecker and S. L. Glashow, Astropart. Phys 16, 97 (2001) arXiv:astro-ph/1102.2784v1.
[8] J. A. Wheeler, “Geometrodynamics and the Issue of the Final State”, in Relativity, groups and Topology, B. DeWitt and C. DeWitt (eds.) (Gordon and Breach, New York, 1964) 463.
[9] We recall the “Planck lattice”, G. Preparata and S.-S. Xue, Phys. Lett. B264, (1991) 35; S. Cacciatori, G. Preparata, S. Rovelli, I. Spagnolatti, and S.-S. Xue: Phys. Lett. B 427 (1998) 254; G. Preparata, R. Rovelli and S.-S. Xue, Gen. Rel. Grav. 32 (2000) 1859.
[10] S.-S. Xue, Phys. Lett. B682 (2009) 300, Phys. Rev. D82, 064039 (2010) and Phys. Lett. B706 (2011) 213 [arXiv:hep-ph/1110.1317].
[11] Y. Xu and B.-Q. Ma, Mod. Phys. Lett. A 26,2101 (2011) [arXiv:hep-th/1106.1778].
[12] G. Amelino-Camelia, New. J. Phys. 6, 188 (2004) arXiv:gr-qc/0212002.
[13] A. Matusis, L. Susskind and N. Toumbas, JHEP 0012, 002 (2000).
[14] N. R. Douglas and N. A. Nekrasov, Rev. Mod. Phys. 73, 977 (2001).
[15] R. Gambini and J. Pullin, Phys. Rev. D 59, 124021 (1999).
[16] J. Alfaro, H. A. Morales-Tecotl and L. F. Urrutia, Phys. Rev. Lett. 84, 2318 (2000).
[17] S. M. Carroll, J. A. Harvey, V. A. Kostelecky, C. D. Lane and T. Okamoto, Phys. Rev. Lett. 87 141601 (2001) arXiv:hep-th/0105082.
[18] G. Amelino-Camelia and S. Majid, Int. J. Mod. Phys. A 15 4301 (2000) arXiv:hep-th/9907110.
[19] M. Chaichian, P. P. Kulish, K. Nishijima and A. Tureanu, Phys. Lett. B 604 98 (2004) arXiv:hep-th/0408069.
[20] G. Amelino-Camelia, Int. J. Mod. Phys. D 11, 35 (2002) arXiv:gr-qc/0012051; J. Magueijo and L. Smolin, Phys. Rev. Lett. 88, 190403 (2002) arXiv:hep-th/0112090; X. Zhang, L. Shao, B.-Q. Ma, Astropart. Phys. 34, 840 (2011).
[21] S. R. Coleman, S. L. Glashow, Phys. Rev. D 59, 116008 (1999).
[22] D. Colladay and V. A. Kostelecky, Phys. Rev. D 58:116002,(1998), arXiv:hep-ph/9809521.
[23] L. Zhou, B.-Q. Ma, Mod. Phys. Lett. A 25, 2489 (2010) arXiv:1009.1331.
[24] L. Zhou, B.-Q. Ma, Chin. Phys. C 35, 987 (2011) arXiv:1109.6387.
[25] R. Lehner, arXiv:hep-ph/0611177 (2006).
[26] M. R. Martinez, T. Piran and Y. Oren, Jour. Cosm. Astr. Phys 0605 017 (2006),
[27] M. R. Martinez and T. Piran, Jour. Cosm. Astr. Phys 0604 006 (2006), arXiv:astro-ph/0601219.
[28] S. Choubey and S. F. King, Phys. Rev. D 67, 073005 (2003).
[29] T. Piran, Rev. Mod. Phys. 20.429 (2004)
[30] E. Waxman and J. Bahcall, Phys. Rev. Lett. 78 2292 (1997)[arXiv:astro-ph/9701233], Phys. Rev. D 59 023002 (1999) arXiv:hep-ph/9807282.
[31] M. Vietri, Astrophys. J. 507 40 (1998)arXiv:astro-ph/9806110.
[32] see e.g. the reviews: R.J. Protheroe, Nucl. Phys. Proc. Suppl. 77 (1999) 465. astro-ph/9809144, E. Waxman, hep-ph/ 0009152, and references therein.
[33] For a review, see P. Bhattacharjee, in College Park 1997, Observing giant cosmic ray air showers, 168-195; Edited by J.F. Krizmanic, J.R. Ormes, R.E. Streitmatter. Woodbury, AIP, 1998. 536p. (1997), astro-ph/9803029.
[34] S. Yang, B.-Q. Ma, Int. J. Mod. Phys. A24,5876 (2009)[arXiv:hep-ph/0910.0897], references therein.
[35] K. G. Wilson, Phys. Rev. D 10 2445 (1973).
[36] J. B. Kogut, Rev. Mod. Phys. 55, 775 (1983).
[37] U. Jacob and T. Piran, Nature Phys. 3:87-90 (2007) arXiv:hep-ph/0607145.
[38] S. D. Biller et al., Phys. Rev. Lett. 83, 2108 (1999).
[39] R. N. Mohapatra and P. B. Pal, Massive Neutrinos in Physics and Astrophysics, Singapore: World Scientific Publishing Co. Pte. Ltd. (2003).
[40] C. Giunti, C. W. Kim, Fundamentals of Neutrino Physics and Astrophysics, New York: Oxford University Press Inc, (2007).
[41] K. Zuber, Neutrino Physics, New York: Taylor and Francis Group, LLC, (2004).
[42] C. Lunardini, A. Yu. Smirnov, Phys, Rev, D64,073006 (2001), arXiv:hep-ph/0012056.