Effects of Weakly Interacting Slim Particles in Cavities with a Moving Boundary Condition

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Abstract

We study a Light Shinning Through the wall type setup with microwave cavities, where the regeneration cavity has a moving boundary condition oscillating harmonically. We find a parametric resonance that could enhance the probability conversion between Weakly Interacting Slim Particles and photons by several orders of magnitude.

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1 Introduction

The existence of Weakly Interacting Slim Particles (WISPs) has been motivated long time ago; the Axion as a solution of the strong CP problem [1], Axion-Like Particles (ALPs) as a pseudo-Nambu-Goldstone bosons emerging from the spontaneous symmetry breaking of global symmetries in some extensions of the Standard Model, and generically in all string compactifications [2] and Hidden Photons (HPs), motivated as massive gauge bosons of a U(1) gauge Hidden Sector that couples via kinetic mixing with Standard Model [3].

From the experimental point of view, WISPs have not been found, but there are many experimental proposal that seek to achieve this goal. Such proposals look for, on the one hand the detection of electromagnetic signals produced by cosmological and astrophysical WISPs [4], and on the other hand experiments, which look for the detection of photons generated by WISPs produced in the laboratory. A prime example of the latter are the Light Shinning Through the Wall type experiments [5] in which WISPs are produced in a certain region of space by photon-WISP oscillations, then the created WISPs can pass to a second region where they oscillate to regenerated photons, which could be detected. Resonant Fabry-Perot cavities have been used in order to amplify the probability conversions in LSW type experiments [6] and the use of microwave cavities has also been implemented.

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Under this context, it is important to study, from the theoretical point of view, new mechanisms to help improve these already established experimental setups. Therefore, we study in this paper, the effects of moving harmonically one of the walls of the microwave cavity implemented in a LSW type experiment. The net effect of the oscillating wall is to change the natural frequency of some EM mode periodically in time. Therefore, we expect a parametric resonance of the EM field enclosed in the cavity. As a consequence, it is found a time dependent enhancement of the probability conversions of regenerated photons. This calculation could be important because although high frequency mechanical motion of massive cavity walls could be difficult to achieve, it has been found that an effective motion is possible using a train of laser pulses [8].

The article is organized as follows: in section II we will solve the wave equation with an oscillating boundary condition and set the conditions for parametric instability, in section III we apply the results in a LSW type setup and find the probability conversion of regenerated photons.

2 Electromagnetic field with an oscillating boundary condition cavity

Suppose a conducting rectangular cavity, which one of its ends moving along the x axis by \( x(t) = L_x + l(t) \) and the other ends are fixed at \( x = 0, y = 0, y = L_y, z = 0 \) and \( z = L_z \). An electromagnetic field, inside the cavity, can be described by a potential vector \( A^\mu \), which will be chosen in the Coulomb’s gauge \( \nabla \cdot A = 0 \). If \( A^\mu \) is induced by a current density \( J^\mu = (0, J) \) where \( J \) is pointing in \( \hat{z} \) direction, then \( A^0 = 0 \), \( A = \hat{z}A \) when there is vanishing initial conditions and consequently, from the gauge election, we have \( \nabla \cdot A = \partial_z A = 0 \) or \( A = A(x, y, t) \).

We can find \( A \), solving the partial differential equation

\[
(\partial_t^2 + \gamma \partial_t - \nabla^2)A(x, y, t) = J(x, t),
\]

where \( \gamma \) is a small dissipative parameter. Since the electric field is \( E = -\partial_t A \), equation (1) must be solved with the boundary conditions \( \partial_t A|_{y=0} = \partial_t A|_{y=L_y} = \partial_t A|_{x=0} = \partial_t A|_{x=L_x+l(t)} = 0 \). If \( l(t) \ll L_x \) we can expand the last boundary to the first order in \( l(t) \) in a taylor series, we have

\[
\partial_t A|_{x=L_x} = -l(t) \partial_x \partial_t A|_{x=L_x}. \tag{2}
\]

To solve (1) we expand \( A \) in an appropriate basis that satisfies the boundary conditions, we have

\[
\partial_t A(x, t) = \frac{2}{\sqrt{V}} \sum_{n,m} \dot{A}_{nm}(t) \varphi_n(x, t) \sin \frac{m\pi y}{L_y}, \tag{3}
\]

where \( V = L_x L_y L_z \) and

\[
\varphi_n(x, t) = \sin \frac{n\pi x}{L_x} - \frac{x}{L_x} l(t) \varphi_n'(L_x, t). \tag{4}
\]
We can derive expression (4) to find \( \varphi_n \) in the explicit form \( \varphi_n(x, t) = \sin \frac{n\pi x}{L_x} - \frac{(-1)^n n \pi x}{L_x} \frac{l(t)}{L_x + l(t)} \).

Inserting (3) into (1) and using orthogonality properties, we have

\[
(\partial_t^2 + \gamma \partial_t + \omega_{nm}^2)A_{nm}(t) = -\frac{2(-1)^n}{n} \sum_{n'} n'(-1)^{n'}(\partial_t^2 + \lambda_{nm}^2) \int dt \eta(t)A_{n'm}(t)
+ \int d^3x \psi_{nm}(x)J(x, t),
\]

(5)

where \( \lambda_m = \frac{m \pi}{L_y}, \omega_{nm} = \sqrt{\lambda_m^2 + \frac{n^2 \pi^2}{L_x^2}} \), \( \eta(t) = l(t)/L_x \), \( \psi_{nm}(x) = \frac{2}{\sqrt{V}} \sin \frac{n \pi x}{L_x} \sin \frac{m \pi y}{L_y} \) and very small terms have been neglected.

In order to excite parametrically the fundamental mode \( (n = m = 1) \), we move the wall in a frequency close to twice the fundamental frequency, let \( \eta(t) = h \cos(2\omega_0 + \epsilon)t \), where \( \omega_0 = \omega_{11} \) is the fundamental frequency and \( \epsilon \) is a small variation such that \( \epsilon/\omega_{11} \sim h \ll 1 \).

In this case, we can truncate the series (3) only to the fundamental mode. We obtain, neglecting \( O(h^2) \) terms, the parametric differential equation

\[
(\partial_t^2 + \omega_0^2)A(t) = -2h(\partial_t^2 + \lambda_1^2) \int dt \cos 2\omega_0 \dot{A}(t) - (\gamma \partial_t - \epsilon \omega)A(t) + F(t),
\]

(6)

where \( A(t) = A_{11}(t), \omega = \omega_0 + \epsilon/2, \lambda = \lambda_1 \) and

\[
F(t) = \int_V d^3x \psi(\bar{x})J(\bar{x}, t)
\]

(7)

with \( \psi(\bar{x}) = \psi_{11}(\bar{x}) \). To look for resonant solutions at first order in \( h \), we can write the ansatz

\[
A(t) = a(t)e^{-i\omega_0 t} + c.c.,
\]

(8)

where \( a(t) \) varies slowly in time compared to \( e^{-i\omega_0 t} \), in fact, we suppose that \( \dot{a} \sim h\omega a \).

Consider an harmonic external force \( F(t) = \alpha e^{-i\omega_0 t} \), inserting the ansatz (8) into (6) and neglecting the second derivatives in \( a(t) \) and other \( O(h^2) \) contributions, we have

\[
2i\dot{a} + (i\gamma + \epsilon)a - \kappa \omega_0 a^* + \alpha/\omega_0 = 0,
\]

(9)

where \( \kappa = L_y^2/(L_x^2 + L_y^2) \). Such a system \( (a, a^*) \) has solutions of the form \( a(t) = a_0 + \bar{a}(t) \), where \( \bar{a} \sim e^{st} \) and \( a_0 \) is a constant given by

\[
a_0 = \frac{\kappa h\alpha^* - (i\gamma - \epsilon)\alpha/\omega_0}{\kappa^2 h^2 \omega_0^2 - \gamma^2 - \epsilon^2}.
\]

(10)

This leads to the homogeneous matrix equation

\[
\begin{bmatrix}
\epsilon + i(2s + \gamma) & -\kappa \omega_0 \\
-\kappa \omega_0 & \epsilon - i(2s + \gamma)
\end{bmatrix}
\begin{bmatrix}
\bar{a} \\
a^*
\end{bmatrix} = 0.
\]

(11)
This has no trivial solutions only if its determinant is equal to zero, which becomes in a relation between $s$ and $\epsilon$ given by

$$s(\epsilon) = \pm \sqrt{\kappa^2 h^2 \omega_0^2 - \epsilon^2 - \gamma^2}. \quad (12)$$

In order to achieve a parametric resonance, one of the roots of $s(\epsilon)$ has to be a positive real number, this bounds $\epsilon$ to

$$-\sqrt{\kappa^2 h^2 \omega_0^2 - \gamma^2} < \epsilon < \sqrt{\kappa^2 h^2 \omega_0^2 - \gamma^2}. \quad (13)$$

This condition allows us to choose $\epsilon = 0$. On the other hand we can despise $\gamma$ in (10) if $\gamma \ll \kappa h \omega_0$. This could be true in a cavity with a very high quality factor; since $\gamma$ is related with the quality factor $Q$ and the frequency $\omega_0$ by $\gamma = \omega_0/Q$, such a quality factor must be greater than $(\kappa h)^{-1}$, for instance, if $L_x = L_y = 10(\text{cm})$ and if the wall oscillates with an amplitude of 1(nm), then, we need a quality factor greater than $2 \times 10^8$. Taking this into account ($\epsilon = 0$ and $\gamma \ll \kappa h \omega$) and imposing the initial conditions of zero electric and magnetic field, the final solution for $A$ is

$$A(t) = \frac{1}{\kappa h \omega_0^2} \left[ i \alpha \sinh \left( \frac{\kappa h \omega_0 t}{2} \right) + \alpha^* \left( 1 - \cosh \left( \frac{\kappa h \omega_0 t}{2} \right) \right) \right] e^{-i\omega_0 t} + O(h). \quad (14)$$

The power inside the cavity is $P = \frac{\omega_0}{2Q} \int_V d^3x(|E|^2 + |B|^2)$, where $E$ and $B$ are the electric and magnetic fields given by $E = -\dot{A}$ and $B = \nabla \times A$, respectively. Writing $\alpha = |\alpha|e^{i\phi}$, we obtain

$$P(t) = \frac{2|\alpha|^2 D(t)}{\kappa h \omega_0 Q}, \quad (15)$$

where

$$D(t) = \left[ \cosh \left( \frac{\kappa h \omega_0 t}{2} \right) - 1 \right] \left[ \cosh \left( \frac{\kappa h \omega_0 t}{2} \right) + \sin 2\phi \sinh \left( \frac{\kappa h \omega_0 t}{2} \right) \right]. \quad (16)$$

3 Application to microwave cavity LSW experiments

Suppose an experimental set-up, where we have two cubic cavities as shown in fig [1]. Following reference [7], we put an electromagnetic field in the first cavity that works like a source for an ALP or HP field. These fields, can pass through the walls of cavity and act as a source for a regenerated electromagnetic field inside the second cavity. The idea is to allow the movement of one of the walls of the regeneration cavity such as in above section and thus enhance parametrically the power inside this cavity.

The interaction between ordinary photons and ALPs is given by the lagrangian density

$$\mathcal{L}_{\gamma \gamma \phi} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{2} (\partial_\mu \phi)^2 - \frac{m^2 a}{2} \phi^2 + \frac{g}{4} \phi F_{\mu \nu} F^{\mu \nu}, \quad (17)$$
Figure 1: Schematic representation of our setup. We have a production cavity of length \( L \), where an electromagnetic field \( A_p \) generates ALPs or HPs. These, pass through the shield and work like a source for photons in the regeneration cavity, which has a variable length \( L + l(t) \).

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) is the photon strength tensor, \( \phi \) the ALP field, \( m_\phi \) its mass and \( g \) the coupling constant of ALPs to two photons. In the case of Hidden Photons we have

\[
\mathcal{L}_{\gamma\gamma'} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{m_{\gamma'}^2}{2} X_\mu X_\mu - \frac{\chi}{2} F_{\mu\nu} X^{\mu\nu},
\]

where \( X_\mu \) is the hidden photon field, \( m_{\gamma'} \) its mass, \( X_{\mu\nu} = \partial_\mu X_\nu - \partial_\nu X_\mu \) the HP strength tensor and \( \chi \) the coupling constants to photons.

Let \( A_p(x, t) = \hat{z} a_p e^{-i\omega_0 t} \psi(x) \) the fundamental mode in the production cavity, which was excited by an external force \( f(t) \sim e^{-i\omega_0 t} \). If \( L \) is the the length of the cavity edge, the eigenfunction \( \psi(x) \) is given by

\[
\psi(x) = \frac{2}{\sqrt{V}} \sin \frac{\pi x}{L} \sin \frac{\pi y}{L}
\]

and \( \omega_0 = \sqrt{2\pi/L} \). At first order in \( g \) or \( \chi \), the field \( A_p \) will induce a WISPs field \( \mathcal{W} \) determined by the equation

\[
(\Box + m_{\mathcal{W}}^2)\mathcal{W}(x, t) = \xi \Omega^2 a_p e^{-i\omega_0 t} \psi(x),
\]

where in the case of ALPs, \( \mathcal{W} \) will be the ALP field \( \phi \) and \( \xi = -igB_0/\omega_0 \), where \( B_0 \) is an external strong magnetic field necessary for ALPs-photons oscillations. In the case of Hidden Photons, \( \mathcal{W} \) will be the \( \hat{z} \) component of the sterile field \( X^\mu - \chi \alpha^\mu \) and \( \xi = \chi m_{\gamma'}^2/\omega_0^2 \).

To find \( \mathcal{W} \) in all space, we write \( \mathcal{W}(x, t) = e^{-i\omega_0 t} \mathcal{W}(x) \) where

\[
(\nabla^2 + k^2)\mathcal{W}(x) = -\xi \omega_0^2 a_p \psi(x)
\]

and \( k = \sqrt{\omega_0^2 - m_{\mathcal{W}}^2} \). The last equation can be solved with the Green’s method, where in this case, the Green function is given by \( G(x - x') = -e^{ik|x-x'|}/(4\pi|x-x'|) \). We have

\[
\mathcal{W}(x, t) = \xi \omega_0^2 a_p e^{-i\omega_0 t} \int_{V'} d^3 x' \frac{e^{ik|x-x'|}}{4\pi|x-x'|} \psi(x').
\]
This field can pass through the walls of production cavity and penetrate the regeneration cavity as a source for an electromagnetic field $A_r(x,t)$, which we can calculate with the equation of motion derived from lagrangians $[17]$ and $[18]$. At first order in $g$ or $\chi$ and adding a dissipation term, we obtain a wave equation such as in $[10]$ and due the oscillating boundary condition of this cavity equation $[6]$ works, where the external force $F(t)$ will be given by

$$F(t) = |\xi|^2 \omega_0^2 a_p e^{-i\omega_0 t} \int_V d^3x \int_V d^3x' \frac{e^{ik|x-x'|}}{4\pi|x-x'|} \psi(x') \psi(x).$$

This implies, that the power in this photon regeneration cavity (see $[15]$) is

$$P_r(t) = \frac{2\omega_0^3 |\xi|^4 |G|^2 |a_p|^2 D(t)}{\kappa h Q_r},$$

where $G$ is an adimensional form factor defined by

$$G = \omega_0^2 \int_V d^3x \int_V d^3x' \frac{e^{ik|x-x'|}}{4\pi|x-x'|} \psi(x') \psi(x).$$

In the production cavity the power is $P_p = \omega_0^3 |a_p|^2/Q_p$, therefore, the probability for regenerated photons is given by

$$P_{\gamma \rightarrow W \rightarrow \gamma} = \frac{P_r}{P_p} = |\xi|^4 Q_p Q_r |G|^2 \left( \frac{2D(t)}{\kappa h Q_r^2} \right).$$

In figure $[2]$, we show the sensibility of this theoretical proposal for axion-like particles and hidden photons, respectively with the expected updates for these kind of experiments. We suppose an optimistic experimental scenario, where we stick together the best implements such as $Q_p = Q_r = 10^{10}$, $P_p = 1(W)$, $P_r = 10^{-20}(W)$, $\omega_0 = 8.17(\text{GHz})$, $h = 10^{-8}$ and, for
the case of ALPs, $B_0 = 5(T)$. We use, for simplicity, $|G| = 1$ for $m_W < \omega_0$ and $|G| = 0$ for $m_W > \omega_0$.

The results of this proposal seem very promising, because the projected sensitivity of such setup covers a wide space of parameters not yet explored. However, this calculation does not consider details in a realistic experimental setup. For instance, it is important that the frequency of the oscillating wall is stable and fluctuations in frequency must be small enough, so that we do not depart from the resonance condition. In reference [13], in the context of the Dynamical Casimir Effect, the authors propose an experimental technique, where they achieve to make an effective motion of the wall in the order of GHz and fluctuations less than 100Hz. The latter could be a possibility to apply the idea proposed in this paper.

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