Parametric FEM for computational homogenization of heterogeneous materials with random voids

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Computational homogenization is a powerful tool which allows to obtain homogenized properties of materials on the macro-scale from the simulation of the underlying microstructure. The response of the microstructure is, however, strongly affected by variations in the microstructure geometry. The effect of geometry variations is even stronger in cases when the material exhibits plastic deformations. In this work we study a model of a steel alloy with arbitrary distributed elliptic voids. We model one single unit cell of the material containing one single void. The geometry of the void is not precisely known and is modeled as a variable orientation of an ellipse. Large deformations applied to the unit cell necessitate a finite elasto-plastic material model. Since the geometry variation is parameterized, we can utilize the method recently developed for stochastic problems but also applicable to all types of parametric problems — the isoparametric stochastic local FEM (SL-FEM). It is an ideal tool for problems with only a few parameters but strongly nonlinear dependency of the displacement fields on parameters. Simulations demonstrate a strong effect of parameter variation on the plastic strains and, thus, substantiate the use of the parametric computational homogenization approach.

1 Introduction

Computational homogenization is a widely used technique since it can be equally applied without any simplifications to both linear and nonlinear problems, arbitrary complex geometries, and any types of physical problems — elasticity, plasticity, electromagnetism, etc. This is a well developed technique, however there are still some open questions, like, e.g. the design of the representative volume element (RVE) of the microstructure in case that parameter variations (e.g. randomness) are present. In this paper we follow the work started in [1] where an ergodicity assumption and boundary conditions for a parametric problem were analyzed. The purpose of this work is to increase the overall realism of the presented approach, thereby extending it to large deformation plasticity.

Plasticity with variable model parameters was already intensively studied in the context of stochastic plasticity. According to Rosic, e.g. [2], many problems in stochastic plasticity are considered as solved. However geometry variations, which result in variable (or random, if we consider a stochastic setting) boundaries and interfaces, do not allow application of the common polynomial chaos expansion and related methods. Therefore we propose to use a new method recently developed to treat geometry variations — the stochastic local FEM (SL-FEM) [3]. An important advantage of the SL-FEM is that it is not restricted to stochastic problems, but can be utilized also to propagate interval variables, fuzzy numbers, or any types of parameters through the model. Another advantage of the SL-FEM is the fact that plastic or any other complex material response is implemented in the same way as for the common FEM — no modifications are required. A subroutine written for a standard FEM code can directly be used in the SL-FEM.

2 Representative volume element

In this work we study a model of a steel alloy with arbitrary distributed elliptic voids. We model one single unit cell of the material containing only one single void. The geometry of the void is not precisely known and is modeled as a variable orientation of the ellipse.

For the purpose of demonstration we consider a 2D model, the third dimension in fig. 1 represents the parametric dimension, i.e. the variation of the angle defining the ellipse orientation. The orientation of the ellipse varies in the range \(0; \pi/2\).

We apply 10% uniaxial tension which causes significant plastic deformations. Classical strong periodic boundary conditions are applied to the boundaries of the RVE. The considered material model is isotropic. We use additive von-Mises plasticity in the logarithmic strain space with a saturation-type nonlinear isotropic hardening.

3 Simulation results

The solution obtained using the SL-FEM demonstrates the distribution of all quantities of interest simultaneously for all considered parameter values. As examples, fig. 2 demonstrates the von-Mises stress distribution and the hardening variable
distribution (proportional to the plastic strain). The third dimension shows, how these quantities change if we change the RVE geometry.

Note the strong effect of the parameter variation on the distribution of the plastic strains and the von-Mises stress. Not only the stress magnitude but also the location of the stress peak depends on the void orientation. Compared to the purely elastic simulation the homogenized von-Mises stress strongly decreased. In the elastic case the homogenized stress varies in the range \([1.84; 1.99]\) depending on the ellipse orientation. In the elasto-plastic case the homogenized stress varies in the range \([1.14; 1.26]\). This simulation is performed for a relatively simple model and serves the purpose of demonstration. More detailed studies will be presented in subsequent works.

Fig. 2: The distribution of the von-Mises stress (a) and the hardening variable (b) in the parametric model.

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