Multi-Dimensional Wireless Tomography Using Tensor-Based Compressed Sensing

Takahiro Matsuda$^{1,2}$ · Kengo Yokota$^3$ · Kazushi Takemoto$^1$ · Shinsuke Hara$^3$ · Fumie Ono$^2$ · Kenichi Takizawa$^2$ · Ryu Miura$^2$

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Abstract Wireless tomography is a technique for inferring a physical environment within a monitored region by analyzing RF signals traversed across the region. In this paper, we consider wireless tomography in a two and higher dimensionally structured monitored region, and propose a multi-dimensional wireless tomography scheme based on compressed sensing to estimate a spatial distribution of shadowing loss in the monitored region. In order to estimate the spatial distribution, we consider two compressed sensing frameworks: vector-based compressed sensing and tensor-based compressed sensing. When the shadowing loss has a high spatial correlation in the monitored region, the spatial distribution has a sparsity in its frequency domain. Existing wireless tomography schemes are based on the vector-based compressed sensing and estimates the distribution by utilizing the sparsity. On the other hand, the proposed scheme is based on the tensor-based

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Takahiro Matsuda
matsuda@comm.eng.osaka-u.ac.jp

Kengo Yokota
yokota@c.info.eng.osaka-cu.ac.jp

Kazushi Takemoto
k-takemoto@post.comm.eng.osaka-u.ac.jp

Shinsuke Hara
hara@info.eng.osaka-cu.ac.jp

1 Graduate School of Engineering, Osaka University, Osaka 565-0871, Japan

2 National Institute of Information and Communications Technology, Yokosuka, Kanagawa 2390847, Japan

3 Graduate School of Engineering, Osaka City University, Osaka 558-8585, Japan
compressed sensing, which estimates the distribution by utilizing its low-rank property. With simulation experiments, we reveal that the tensor-based compressed sensing has a potential for highly accurate estimation as compared with the vector-based compressed sensing. In order to show the possibility of the wireless tomography schemes in practical environments, we also show an experimental result in an anechoic chamber.

Keywords Wireless tomography · Compressed sensing · Tensor

1 Introduction

Wireless tomography [12, 15, 16, 20], sometimes called RF (Radio Frequency) tomography or Radio Tomographic Imaging, is a technique for inferring a physical environment within a region by analyzing wireless signals transmitted between wireless nodes, and it can be used for inferring the locations of obstructions from the outside of the region. Wireless tomography may lead to developing building monitoring systems for security applications and an emergency use by fire-fighters and police-officers [16]. For this purpose, in [11, 21], wireless tomography is applied to RSS (Received Signal Strength)-based localization schemes.

In general, wireless signal propagating on a wireless link loses its power due to distance, shadowing and multipath fading. Wireless tomography aims at estimating a spatial distribution of the shadowing loss in a monitored region from a measured power of wireless signals. In [1, 16], a spatially correlated shadowing model is presented, and the power attenuation of wireless signals is represented as a system of linear equations of the spatial distribution. In [16, 20], a regularized weighted least-squared (WLS) error estimator is used to estimate the spatial distribution.

Compressed sensing [4, 9], a new paradigm in signal/image processing, is a promising technique for wireless tomography. By means of compressed sensing, we can solve underdetermined linear inverse problems, that is, we can reconstruct an unknown vector from fewer measurements than the length of the unknown vector. Compressed sensing utilizes the sparsity of the unknown vector, where most of its elements are exactly or approximately zero. When the shadowing loss has a high spatial correlation in the monitored region, the spatial distribution has a sparsity in its frequency domain, which enables us to naturally apply compressed sensing. In [12, 15], compressed sensing-based wireless tomography schemes are proposed. In [12], a compressed sensing-based wireless tomography is proposed by extending the basic idea of wireless tomography in [16]. Mostofi [15] proposes compressive cooperative mapping in mobile networks, where mobile nodes collect measurements and estimate a map of spatial variations of the parameters of interest by means of compressed sensing.

Compressed sensing can be categorized into vector-based compressed sensing [2, 4, 9, 23], matrix-based compressed sensing [17, 19], and tensor-based compressed sensing [3, 7]. Wireless tomography schemes explained in the above are based on vector-based compressed sensing and aim at estimating a spatial distribution of the shadowing loss in two-dimensional monitored regions. In this paper, we try to generalize the wireless tomography framework in order to estimate the spatial distribution in higher dimensional regions and propose a multi-dimensional wireless tomography scheme using tensor-based compressed sensing.
Tensors are a higher order generalization of vectors and matrices \([5, 7, 13, 14]\), that is, vectors and matrices correspond to the 1st-order tensors and the 2nd-order tensors, respectively. Tensor-based compressed sensing is a generalization of matrix-based compressed sensing, and tensor-based and matrix-based compressed sensing utilize the low-rank property of unknown tensors and matrices, respectively, while vector-based compressed sensing utilizes the sparsity of unknown vectors. To the best of our knowledge, tensor-based compressed sensing has not been applied to wireless tomography. The most important contribution of this paper is to reveal that the tensor-based compressed sensing has a potential for highly accurate estimation in multi-dimensional wireless tomography. To do so, with simulation experiments, we compare the performance of the proposed scheme with that of wireless tomography based on vector-based compressed sensing. In the simulation experiments, the spatial distribution of the shadowing loss is estimated with ideally-beamformed wireless signals, which have the zero beamwidth. In order to show the possibility of the wireless tomography in the case of the non-zero beamwidth, we evaluate its performance obtained by experiments in an anechoic chamber, where transmitters and receivers have almost omni-directional antenna patterns.

The remainder of this paper is organized as follows. In Sect. 2, we explain vector, matrix, and tensor-based compressed sensing. In Sect. 3, we describe the problem formulation in multi-dimensional wireless tomography. In Sect. 4, we propose the multi-dimensional wireless tomography scheme based on tensor-based compressed sensing. In Sect. 5, we evaluate the proposed scheme with simulation experiments and experiments in the anechoic chamber. Finally, we conclude this paper in Sect. 6.

# 2 Compressed Sensing

As described in the previous section, we consider three frameworks for compressed sensing: vector-based compressed sensing, matrix-based compressed sensing, and tensor-based compressed sensing, and hereafter, we refer to them as vector recovery, matrix recovery, and tensor recovery, respectively.

## 2.1 Vector Recovery

We first describe the vector recovery, which is a basic problem in compressed sensing \([4, 9]\). In this problem, we consider estimating an unknown vector \(x = (x_1, \ldots, x_N)^\top \in \mathcal{R}^{N \times 1}\) in a linear inverse problem:

\[
y = Ax,
\]

where \(\top\) denotes the transpose operator, and \(y \in \mathcal{R}^{M \times 1}\) and \(A \in \mathcal{R}^{M \times N}\) denote a measurement vector and a sensing matrix, respectively, and we assume \(M < N\), that is, an underdetermined linear system. We also assume that \(A\) is known exactly and \(x\) is sparse in some orthonormal basis \(\Phi = (\phi_1, \phi_2, \ldots, \phi_N) \in \mathcal{R}^{N \times N}\) as \(x = \Phi s\), where

\[
s = (s_1, s_2, \ldots, s_N)^\top \in \mathcal{R}^{N \times 1}, \quad \phi_n \in \mathcal{R}^{N \times 1} \quad (n = 1, 2, \ldots, N).\]

\(s_n\) \((n = 1, 2, \ldots, N)\) can be defined over complex number field \(\mathbb{C}\) when \(\Phi\) is defined by a complex number matrix such as an inverse Fourier transform matrix. In this paper, however, we define \(s_n\) as a real number by using an inverse DCT (Discrete Cosine Transform) matrix in order to simplify the description.
A straightforward approach to the vector recovery is \( \ell_0 \) optimization:
\[
\hat{s} = \arg\min_s \|s\|_0 \quad \text{subject to} \quad y = Ax = A\Phi s,
\]
where \( \|s\|_0 \) is the \( \ell_0 \) norm of \( s \) defined as the number of nonzero elements in \( s \). Finally, we have an estimate \( \hat{x} \) by \( \hat{x} = \Phi \hat{s} \).

Because \( \ell_0 \) norm has the discrete and non-convex natures, the above \( \ell_0 \) optimization problem is difficult to solve in general. Therefore, in compressed sensing, a convex relaxation of \( \ell_0 \) optimization is used:
\[
\hat{s} = \arg\min_s \|s\|_1 \quad \text{subject to} \quad y = Ax = A\Phi s,
\]
where \( \|s\|_1 \) is the \( \ell_1 \) norm of \( s \). Here, for \( z = (z_1, z_2, \ldots, z_L)^T \in \mathcal{R}^{L \times 1} \) and \( p \geq 1 \), \( \ell_p \) norm \( \|z\|_p \) of \( z \) is defined as
\[
\|z\|_p = \left( \sum_{i=1}^{L} |z_i|^p \right)^{1/p}.
\]

When the measurements are noisy, we can also consider another optimization problem such as \( \ell_1-\ell_2 \) optimization \([23]\):
\[
\hat{s} = \arg\min_s \left( \frac{1}{2} \|y - Ax = A\Phi s\|_2^2 + \lambda \|s\|_1 \right), \tag{1}
\]
where \( \|y - Ax\|_2 \) is the \( \ell_2 \) norm (Euclidean norm) of \( y - Ax \) and \( \lambda (\lambda > 0) \) is a regularization parameter. Several algorithms to solve the \( \ell_1 - \ell_2 \) optimization have been proposed, e.g., fast iterative shrinkage-thresholding algorithm (FISTA) \([2, 23]\).

### 2.2 Matrix Recovery

In the matrix recovery problem \([17]\), we consider the following linear inverse problem for an unknown matrix \( X \in \mathcal{R}^{N_1 \times N_2} \):
\[
y = A(X),
\]
where \( A(\cdot) \) represents a linear map \( A : \mathcal{R}^{N_1 \times N_2} \to \mathcal{R}^{M \times 1} \) to obtain the measurement vector \( y \in \mathcal{R}^{M \times 1} \). We assume that \( M < N_1 N_2 \), that is, an underdetermined system.

In the matrix recovery, \( X \) is estimated by means of a rank minimization problem:
\[
\hat{X} = \arg\min_X \text{rank}(X) \quad \text{subject to} \quad y = A(X). \tag{2}
\]

Because \( \text{rank}(X) \) also has the discrete and non-convex natures as \( \ell_0 \) norm, the above rank minimization problem is difficult to solve. In the matrix recovery, therefore, a convex relaxation of the rank minimization problem to the nuclear norm minimization is used:
\[
\tilde{X} = \arg \min_X \|X\|_* \quad \text{subject to} \quad y = A(X),
\]

where \(\|X\|_*\) is the nuclear norm of \(X\), which is defined as the sum of its singular values \(\sigma_i (i = 1, 2, \ldots, \text{rank}(X))\):

\[
\|X\|_* = \sum_{i=1}^{\text{rank}(X)} \sigma_i.
\]

Because all the singular values are nonnegative, the nuclear norm is equal to the \(\ell_1\) norm of the vector composed of singular values [17].

When the measurements are noisy, we can estimate an unknown matrix \(X\) as the nuclear norm regularized linear least squares problem [19]:

\[
\tilde{X} = \arg \min_X \left( \frac{1}{2} \| y - A(X) \|_2^2 + \mu \|X\|_* \right),
\]

where \(\mu (\mu > 0)\) is a regularization parameter. Several algorithms to solve this problem have been proposed, e.g., the \textit{accelerated proximal gradient singular value thresholding algorithm} (APG) [19].

### 2.3 Tensor Recovery

#### 2.3.1 Tensor Rank

For an integer \(D \geq 3\), the \(D\)-th order tensor is referred to as a higher-order tensor. The vectorization of the \(D\)-th order tensor \(X \in \mathcal{R}^{N_1 \times N_2 \times \ldots \times N_D}\) is denoted by \(\text{vec}(X) \in \mathcal{R}^{N_1 N_2 \ldots N_D \times 1}\). By the vectorization, the tensor element \((k_1, k_2, \ldots, k_D)\) of \(X\) is mapped to the \(l\)-th entry of \(\text{vec}(X)\), where

\[
 l = \left\{ \sum_{i=1}^{D-1} (k_i - 1) \left( \prod_{j=i+1}^{D} N_j \right) \right\} + k_D.
\]

The mode-\(n\) matricization \((n = 1, 2, \ldots, D)\) of the \(D\)-th-order tensor \(X \in \mathcal{R}^{N_1 \times \ldots \times N_D}\) is denoted by \(X_{(n)} \in \mathcal{R}^{N_n \times I_n}\), where \(I_n = \prod_{i=1}^{D} 1 = 1 \text{ } N_i\). By the mode-\(n\) matricization, the tensor element \((k_1, k_2, \ldots, k_D)\) of \(X\) is mapped to the matrix element \((k_n, l_n)\) of \(X_{(n)}\), where

\[
 l_n = \sum_{i=1}^{D} (k_i - 1) \left( \prod_{j=i+1}^{D} N_j \right), \quad N_{D+1} = 1.
\]

We refer to \(X_{(n)}\) as the mode-\(n\) unfolding.

There are several notions on the tensor rank [13]. In this paper, we consider the \(n\)-rank of the \(D\)-th order tensor \(X\), which is the tuple of the ranks of the mode-\(n\) unfoldings [7, 14]:

\[
n-\text{rank}(X) = (\text{rank}(X_{(1)}), \ldots, \text{rank}(X_{(D)})).
\]
2.3.2 Tensor Recovery

In the tensor recovery problem, we consider the following linear inverse problem for an unknown $D$-th order tensor $X \in \mathcal{R}^{N_1 \times N_2 \times \cdots N_D}$:

$$y = A(X),$$

where $A(\cdot)$ represents a linear map $A: \mathcal{R}^{N_1 \times N_2 \times \cdots N_D} \rightarrow \mathcal{R}^{M \times 1}$ to obtain the measurement vector $y \in \mathcal{R}^{M \times 1}$.

Now, we define $f(n-\text{rank}(X))$ as $f(n-\text{rank}(X)) = \sum_{i=1}^{D} \text{rank}(X_{(i)})$. In the tensor recovery, we consider the minimization of function $f(n-\text{rank}(X))$ [7]:

$$\hat{X} = \arg \min_X f(n - \text{rank}(X)) \quad \text{subject to} \quad y = A(X).$$

Due to the discrete and non-convex nature of the tensor rank, the following convex relaxation is considered:

$$\hat{X} = \arg \min_X \sum_{i=1}^{D} \|X_{(i)}\|_* \quad \text{subject to} \quad y = A(X).$$

This problem corresponds to the recovery of compressed tensor data via the higher-order singular value decomposition (HOSVD), which is the most commonly used generalization of the matrix SVD to higher-order tensors [5, 14]. Therefore, the tensor recovery is a generalization of the matrix recovery.

When the measurements are noisy, we can estimate an unknown tensor $X$ using the following unconstrained optimization:

$$\hat{X} = \arg \min_X \left( \frac{\mu}{2} \|y - A(X)\|_2^2 + \sum_{i=1}^{D} \|X_{(i)}\|_* \right),$$

where $\mu (\mu > 0)$ is a regularization parameter. Several algorithms to solve this problem have also been proposed, e.g., Douglas-Rachford splitting for tensor recovery (DR-TR) [7].

3 Problem Formulation

In wireless tomography, nodes inject wireless signals into a monitored region, and characteristics such as power attenuation due to obstructions are inferred from the received wireless signals. In this paper, we consider the $D$-dimensional wireless tomography ($2 \leq D \leq 4$), where the monitored region is represented by a $D$-dimensional structure on Cartesian coordinates. For the case of $D = 4$, wireless tomography is described with 3 spatial axes and the time axis. Here, we discretize the time axis into intervals with the same time unit and let $t (t = 1, 2, \ldots)$ denote the $t$-th time interval, which is also referred to as time $t$ hereafter.

The path loss of signal propagated on a wireless link consists of the large-scale path loss due to distance, shadowing loss due to obstructions, and non-shadowing loss due to multipath fading [1, 8, 16]. Let $\mathcal{V}$ denote a set of wireless nodes, which are deployed on the border of the monitored region as shown in Fig. 1a. Suppose that a wireless signal is
transmitted from a transmitter $v_i \in \mathcal{V}$ to a receiver $v_j \in \mathcal{V}$ ($i, j = 1, 2, \ldots, |\mathcal{V}|, j \neq i$). We define $\text{dist}(i, j)$ and $\bar{P}(\text{dist}(i, j))$ as the distance and the large-scale path loss in dB between $v_i$ and $v_j$, respectively. In this case, we can model the received signal power $P_{i,j,t}$ [dBm] at $v_j$ observed at time $t$ as

$$P_{i,j,t} = P_{TX} - \bar{P}(\text{dist}(i, j)) - Z_{i,j,t},$$

$$Z_{i,j,t} = Z_{i,j,t}^{(1)} + Z_{i,j,t}^{(2)},$$

where $P_{TX}$, $Z_{i,j,t}^{(1)}$ and $Z_{i,j,t}^{(2)}$ denote the transmitted power at $v_i$ in dBm, the shadowing loss in dB, and the non-shadow fading loss in dB, respectively. Furthermore, $\bar{P}(\text{dist}(i, j))$ is given by

$$\bar{P}(\text{dist}(i, j)) = 10\alpha \log(\text{dist}(i, j)) + \beta,$$

where $\alpha$ and $\beta$ are constants and $\alpha \geq 2$ [8]. Using the line integral over the wireless link $\text{path}(i, j)$ between $v_i$ and $v_j$, we have

$$Z_{i,j,t}^{(1)} = \int_{\text{path}(i,j)} g(r, t) dr,$$

where $r \in \mathbb{R}^3$ denotes a coordinate in the monitored region and $g(r, t)$ [dB/m] denotes the
power attenuation due to the shadowing loss on location $r$ at time $t$ \cite{1,15,16}. Note that $g(r, t) = 0$ if there is no obstruction on $r$. For $Z_{ij}^{(2)}$, we assume a wide-sense stationary Gaussian process with zero mean and variance $\eta^2$.

Let us divide the monitored region into 3-dimensional voxels $(n_1, n_2, n_3)$ ($n_i = 1, 2, \ldots, N_i$, $i = 1, 2, 3$), and represent $\Delta(n_1, n_2, n_3) \subset \mathbb{R}^3$ as a subset of the monitored region within voxel $(n_1, n_2, n_3)$. Here, we assume that $g(r \in \Delta(n_1, n_2, n_3), t = n_4)$ $(n_1 = 1, 2, \ldots, N_i, i = 1, 2, 3, 4)$ has a constant value $X_{n_1,n_2,n_3,n_4}$ within voxel $(n_1, n_2, n_3)$. Figure 1b, c show examples of a monitored region divided into voxels for 2-dimensional wireless tomography and 3-dimensional wireless tomography, respectively. We then have

$$Z_{ij}^{(1)} = \sum_{n_1,n_2,n_3} \delta_{ij}(n_1, n_2, n_3)X_{n_1,n_2,n_3,n_4},$$

where $\delta_{ij}(n_1, n_2, n_3)$ is the overlapped distance between wireless link $\text{path}(i,j)$ and voxel $(n_1, n_2, n_3)$ (See Fig. 2). Note that $\delta_{ij}(n_1, n_2, n_3) = 0$ if $\text{path}(i,j)$ does not cross voxel $(n_1, n_2, n_3)$.

Now, let $Q_{n_4} = \{(v_i, v_j) \mid v_i, v_j \in \mathcal{V}, i, j \in \{1, 2, \ldots, |\mathcal{V}|\} \}$ $(n_4 = 1, 2, \ldots, N_4)$ denote a set of pairs of nodes used for measurements at time $n_4$. In addition, let $(v_{im}^{(n_4)}, v_{jm}^{(n_4)}) \in Q_{n_4}$ ($m = 1, 2, \ldots, M_{n_4}, i_m, j_m \in \{1, 2, \ldots, |\mathcal{V}|\}, i_m \neq j_m$) denote $M_{n_4}$ pairs of wireless nodes, where $M_{n_4} = |Q_{n_4}|$. Given $P_{im,\text{TX},n_4}, P_{\text{TX}}$ and $\hat{P}(\text{dist}(i_m, j_m))$, we can obtain the following linear equation:

$$y_m^{(n_4)} \triangleq P_{\text{TX}} - P_{im,\text{TX},n_4} - \hat{P}(\text{dist}(i_m, j_m))$$

$$= \sum_{n_1,n_2,n_3} \delta_{im,jm}(n_1, n_2, n_3)X_{n_1,n_2,n_3,n_4} + Z_{im,jm}^{(2)},$$

where $y_m^{(n_4)}$ is referred to as the $m$-th measured shadowing loss at time $n_4$. Furthermore, let $y^{(n_4)} = (y_1^{(n_4)} \ y_2^{(n_4)} \ \cdots \ y_{M_{n_4}}^{(n_4)})^\top \in \mathbb{R}^{N_4 \times N_4 \times 1}$ denote the measurement vector at time $n_4$ and

**Fig. 2** An example of overlap distance $\delta_{ij}(n_1, n_2, \ldots, n_D)$ on wireless link $\text{path}(i,j)$ between wireless nodes $v_i \in \mathcal{V}$ and $v_j \in \mathcal{V}$ for $D = 2$
$X^{(n_4)} = \{X_{n_1,n_2,n_3,n_4} \mid n_i = 1, 2, \ldots, N_i, i = 1, 2, 3\}$ denote the loss field tensor at time $n_4$.

Using a linear map $\mathcal{A}^{(n_4)} : \mathcal{R}^{N_1 \times N_2 \times N_3} \to \mathcal{R}^{M_{n_4} \times 1}$, (5) is rewritten as follows:

$$y^{(n_4)} = \mathcal{A}^{(n_4)}(X^{(n_4)}) + w^{(n_4)},$$

where $w^{(n_4)} = (Z_{i_1,j_1,n_4}^{(2)} Z_{i_2,j_2,n_4}^{(2)} \cdots Z_{i_{M_{n_4}},j_{M_{n_4}},n_4}^{(2)}$).

Defining the measurement vector $y$ as

$$y \triangleq \begin{pmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(N_4)} \end{pmatrix} = \begin{pmatrix} \mathcal{A}^{(1)}(X^{(1)}) \\ \mathcal{A}^{(2)}(X^{(2)}) \\ \vdots \\ \mathcal{A}^{(N_4)}(X^{(N_4)}) \end{pmatrix} + \begin{pmatrix} w^{(1)} \\ w^{(2)} \\ \vdots \\ w^{(N_4)} \end{pmatrix},$$

we finally reformulate (6) with a linear map $\mathcal{A} : \mathcal{R}^{N_1 \times N_2 \times N_3 \times N_4} \to \mathcal{R}^{M \times 1}$:

$$y = \mathcal{A}(X) + w,$$

where $X = \{X^{(n_4)} \mid n_4 = 1, 2, \ldots, N_4\} \in \mathcal{R}^{N_1 \times N_2 \times N_3 \times N_4}$ denote the loss field tensor and $w = ((w^{(1)})^T (w^{(2)})^T \cdots (w^{(N_4)})^T)^T \in \mathcal{R}^{N_1 \times N_2 \times N_3 \times N_4 \times 1}$ denote the noise vector.

Wireless tomography is a linear inverse problem to estimate $X$ from the measurement vector $y$. Note that $D$-dimensional wireless tomography for $D = 2, 3$ can be formulated as special cases of the 4-dimensional wireless tomography. Namely, the 2-dimensional wireless tomography corresponds to the 4-dimensional wireless tomography with $N_1 > 1$, $N_2 > 1$, and $N_3 = N_4 = 1$, where the loss field tensor $X$ has two spatial axes (i.e., $x$-axis, $y$-axis). For the case of $D = 3$, we can consider two cases: three spatial axes (i.e., $x$-axis, $y$-axis, and $z$-axis), and two spatial axes and time axis. The former corresponds to the 4-dimensional wireless tomography with $N_1 > 1$, $N_2 > 1$, $N_3 > 1$, and $N_4 = 1$, while the latter corresponds to the 4-dimensional wireless tomography with $N_1 > 1$, $N_2 > 1$, $N_3 = 1$, and $N_4 > 1$.

4 Wireless Tomography with Compressed Sensing

We assume that $X_{n_1,\ldots,n_D}$ has a high spatial correlation, which enables us to estimate $X_{n_1,\ldots,n_D}$ by means of compressed sensing. In the following subsections, we first describe the wireless tomography scheme based on the vector recovery, which has been studied in [12, 15], and then, we explain the wireless tomography scheme based on the tensor recovery.

4.1 Vector Recovery-Based Wireless Tomography

We define a loss field vector $x$ as $x = \text{vec}(X) \in \mathcal{R}^{N_1 N_2 N_3 N_4 \times 1}$, and reformulate (7) as $y = Ax + w$, where $A \in \mathcal{R}^{M \times N_1 N_2 N_3 N_4}$ denotes a sensing matrix. Let $\mathcal{F}$ denote a linear map that transforms $X$ to its frequency domain representation such as the $D$-dimensional discrete Fourier transform (DFT) and the $D$-dimensional discrete cosine transform (DCT). In addition, let $S = \mathcal{F}(X) \in \mathcal{C}^{N_1 \times N_2 \times N_3 \times N_4}$ and $s = \text{vec}(S)$ denote the frequency domain
representation of $X$ and its vectorization, respectively. The matrix that transforms $s$ to $x$ is denoted by $\Phi \in C^{N_1 \times N_2 \times N_3 \times N_4}$, that is, $x = \Phi s$, so we have

$$y = A \Phi s + w.$$  

Then, let $F_N = \{ F_{k,n} \mid 1 \leq k, n \leq N \}$ $(N \geq 1)$ denote an $N \times N$ unitary matrix such as the one-dimensional DFT matrix and the one-dimensional DCT matrix. In the case of DCT, $F_{k,n}$ is given by

$$F_{k,n} = \begin{cases} \frac{1}{\sqrt{N}}, & k = 1, 1 \leq n \leq N, \\ \frac{2}{N} \cos \frac{\pi (2n + 1)k}{2N}, & 2 \leq k \leq N, 1 \leq n \leq N. \end{cases}$$

By using $F_N$, $\Phi$ can be written as

$$\Phi = F_{N_1}^T \otimes F_{N_2}^T \otimes F_{N_3}^T \otimes F_{N_4}^T,$$

where $\otimes$ represent the Kronecker product, and if $B = \{ b_{i,j} \}$ and $C = \{ c_{k,l} \}$ are $L_1 \times L_2$ and $L_3 \times L_4$ matrices, respectively, $B \otimes C$ is defined as

$$B \otimes C = \begin{pmatrix} b_{1,1}C & b_{1,2}C & \cdots & b_{1,L_2}C \\
1 & \frac{2}{N} \cos \frac{\pi (2n + 1)k}{2N}, & 2 \leq k \leq N, 1 \leq n \leq N. \end{cases}$$

Consequently, we can obtain an estimate $\hat{s}$ of the frequency domain representation $s$ by the sparse vector recovery, as explained in Sect. 2.1, and then have the estimate $\hat{x}$ of loss field vector $x$ by $\hat{x} = \Phi \hat{s}$.

4.2 Tensor Recovery-Based Wireless Tomography

The wireless tomography scheme based on the tensor recovery estimates the loss field tensor $X$ from the measurement vector $y$ by means of the tensor recovery explained in Sect. 2.3. Because each element of measurement vector $y$ includes a noise in a practical situation, we estimate $X$ by means of (4). Namely, from (6) and (7), we can rewrite (4) as

$$\hat{X} = \arg \min_X \left( \frac{\mu}{2} \| y - A(X) \|_2^2 + \sum_{i=1}^D \| X(i) \|_s \right)$$

$$= \arg \min_X \left( \frac{\mu}{2} \sum_{n_4=1}^{N_4} \| y^{(n_4)} - A^{(n_4)}(X^{(n_4)}) \|_2^2 + \sum_{i=1}^D \| X(i) \|_s \right).$$

It is worth mentioning that the matrix recovery-based wireless tomography corresponds to the tensor recovery-based wireless tomography for $D = 2$. Therefore, in the following, we use the tensor recovery-based wireless tomography for $D = 2$ and the matrix recovery-based wireless tomography exchangeably.
5 Performance Evaluation

5.1 Simulation Setup

In this section, we demonstrate the performance of the tensor recovery scheme by comparing it with that of the vector recovery scheme for $D = 2, 3, 4$. Figure 3a–c show the monitored regions for $D = 2, D = 3$, and $D = 4$, respectively. In the case of $D = 2$, we set $N_1 = N_2 = 10$ and place a $3 \times 3$ square-shaped obstruction in the monitored region, which is represented by dark pixels in Fig. 3a. The elements of $X \in \mathbb{R}^{N_1 \times N_2}$ are set to 10 within the dark pixels and 0 within the other pixels. 40 wireless nodes are placed on the border of the monitored region as shown in Fig. 4a. Next, in the case of $D = 3$, we set $N_1 = N_2 = 10, N_3 = 5$ and place a $3 \times 3 \times 2$ obstruction, which is represented by dark voxels in Fig. 3b. The elements of $X \in \mathbb{R}^{N_1 \times N_2 \times N_3}$ are set to 10 within the dark voxels and 0 within the other pixels. 200 wireless nodes are placed on the sides of the monitored region as shown in Fig. 4b. Finally, in the case of $D = 4$, we set $N_1 = N_2 = 10, N_3 = 5, N_4 = 3$, where the monitored region is the same environment as the case of $D = 3$ and the obstruction is moving as shown in Fig. 3c.

In each simulation experiment, $M$ pairs of wireless nodes are randomly chosen to establish $M$ wireless links, and in each pair, a randomly chosen node is set to a transmitter

Fig. 3 Monitored regions for simulation experiments. a $D = 2$. b $D = 3$. c $D = 4$
and the other is set to a receiver. We assume that each measurement is contaminated with a Gaussian noise with zero mean and variance $\eta^2$.

In the vector recovery scheme, we use the DCT to transform the loss field tensor $X$ to its frequency representation, and estimate the loss field tensor by solving the optimization problem (1) with FISTA [2, 23]. The regularization parameter $\lambda$ in (1) is set to 1.0. On the other hand, in the tensor recovery scheme, we estimate the loss field tensor $X$ by solving the optimization problem (3) [19] for $D = 2$ and the optimization problem (4) [7] for $D = 3, 4$. The regularization parameter $\mu$ is set to 1.0. Note that in this paper, we do not consider the optimization of the regularization parameters $\lambda$ and $\mu$, which is beyond the scope of the paper.

We evaluate the performance of the vector and tensor recovery schemes in terms of the normalized reconstruction error $\epsilon$ between the true loss field tensors and the corresponding estimated loss field tensors. In more detail, for given $M$ and $\eta$, we conduct $N_{\text{run}}$ independent simulation experiments to calculate $\epsilon$, which is defined as

$$\epsilon = \frac{1}{N_{\text{run}}} \sum_{i=1}^{N_{\text{run}}} \| \hat{X}_{[i]} - X \|_F / \| X \|_F.$$

In (8), $\hat{X}_{[i]}$ ($i = 1, 2, \ldots, N_{\text{run}}$) denote the estimated loss field tensor obtained by the $i$-th simulation experiment and $\| \cdot \|_F$ represents the Frobenius norm. Here, for $Z = \{ Z_{n_1, n_2, \ldots, n_D} \mid n_i = 1, 2, \ldots, N_i, i = 1, 2, \ldots, D \} \in \mathcal{R}^{N_1 \times N_2 \times \cdots \times N_D}$, its Frobenius norm is defined as

$$\| Z \|_F = \sqrt{\sum_{i=1}^{D} \sum_{n_i=1}^{N_i} Z_{n_1, n_2, \ldots, n_D}^2}.$$
\[ \|Z\|_F = \left( \sum_{n_1=1}^{N_1} \sum_{n_2=1}^{N_2} \cdots \sum_{n_D=1}^{N_D} z^2_{n_1,n_2,\ldots,n_D} \right)^{\frac{1}{2}}. \]

5.2 Simulation Results

Figures 5, 6, and 7 show examples of the estimated loss field tensors for \(D = 2, 3,\) and 4, respectively. For each figure, we show the loss field tensors estimated by the vector and tensor recovery schemes in the noiseless environment (i.e., \(\eta = 0\)). Here, the number \(M\) of measurements is set to 60 for the case of \(D = 2,\) and \(M\) is set to 300 for the case of \(D = 3.\) For the case of \(D = 4,\) furthermore, \(M\) is set to 900, and the number \(M_{n_4} (n_4 = 1, 2, 3)\) of measurements at time \(n_4\) is set to 300. From these figures, we observe that the tensor recovery scheme (i.e., Figs. 5b, 6b, 7b) can estimate the loss field tensor more accurately than the vector recovery scheme (i.e., Figs. 5a, 6a, 7a).

For fair comparison of the results obtained in different dimensions, we define the normalized number \(\gamma\) of measurements as

\[ \gamma = \frac{M}{N_1 N_2 N_3 N_4}, \]

where \(N_3 = N_4 = 1\) for \(D = 2\) and \(N_4 = 1\) for \(D = 3.\) Figure 8a–c show the reconstruction error \(\epsilon\) versus \(\gamma\) in the noiseless environment (i.e., \(\eta = 0\)) for \(D = 2, 3,\) and 4, respectively. In these figures, “DCT”, “Matrix”, and “Tensor” represent the performances of the vector, matrix and tensor recovery schemes, respectively, where we set the number \(N_{\text{run}}\) of simulation experiments for each parameter to 50. From these figures, we observe that the tensor recovery scheme can achieve lower reconstruction errors than the vector recovery scheme.

Figures 9a–c show the normalized reconstruction error \(\epsilon\) versus the standard deviation \(\eta\) of noise for \(D = 2, 3,\) and 4, respectively, where we set \(M = 60\) for \(D = 2,\) \(M = 300\) for \(D = 3,\) and \(M = 900\) and \(M_{n_4} = 300 (n_4 = 1, 2, 3)\) for \(D = 4.\) For the cases of \(D = 2\) and 3, we observe that the tensor recovery scheme has lower reconstruction errors even in the noisy environments. For the case of \(D = 4,\) however, we observe that the tensor recovery scheme achieves a better performance than the vector recovery scheme only for smaller \(\eta.\) These figures indicate that the tensor recovery scheme is vulnerable to the measurement.

Fig. 5 Examples of the estimated loss field tensor \((D = 2, M = 60, \eta = 0).\) a Vector recovery. b Matrix recovery.
noise. Therefore, in order to ensure the performance gain by the tensor recovery scheme, measurement techniques with smaller observation noise are required, where we leave it as a future work.
5.3 Discussion: Vector Recovery Versus Tensor Recovery

In this subsection, we discuss the reason why the tensor recovery scheme can achieve more accurate estimation of loss field tensors than the vector recovery scheme, as shown in the previous subsection. In order to simplify the discussion, we focus on the case of $D = 2$.

In the vector recovery scheme, by using the frequency domain representation $S$, the loss field tensor $X$ can be written as $X = F_{N_1}^T S F_{N_2}$.

When $X$ has a high spatial correlation, most of the energy in $X$ is concentrated in a few low-frequency elements of $S$, that is, $S$ is an approximately sparse matrix. Therefore, when applying the vector recovery to the measurement vector $y$, we obtain a sparsified matrix $\hat{S}$ by replacing the elements with a smaller absolute value in $S$ with zeros. We thus obtain the estimated loss field tensor $\hat{X}^{(V)}$ as $\hat{X}^{(V)} = F_{N_1}^T \hat{S} F_{N_2}$.

The reconstruction error $\kappa^{(V)}$ of $\hat{X}^{(V)}$ is then given by $\kappa^{(V)} = \| \hat{X}^{(V)} - X \|_F$.

Figure 10a, b show $|s_i|$ and $|\hat{s}_i|$ ($i = 1, 2, \ldots, N_1 N_2$) sorted in the decreasing order, where $s = \text{vec}(S) = (s_1 \ s_2 \ \cdots \ s_{N_1 N_2})^T$ and $\hat{s} = \text{vec}(\hat{S}) = (\hat{s}_1 \ \hat{s}_2 \ \cdots \ \hat{s}_{N_1 N_2})^T$ are defined as the vectorization of $S$ and $\hat{S}$, respectively. We observe that $\hat{s}$ is obtained by replacing the smaller elements of $s$ with zeros.

On the other hand, in the tensor recovery scheme, the reconstruction error can be explained with SVD. Namely, let us define $\sigma_i$ ($i = 1, 2, \ldots, r$) as singular values of $X$, where $r = \min\{N_1, N_2\}$. With matrices $U \in \mathbb{R}^{N_1 \times r}$ and $V \in \mathbb{R}^{N_2 \times r}$ whose column vectors are orthogonal, the SVD of $X$ can be written as $X = U \Sigma V^T$,

where $\Sigma = \text{diag}(\sigma_1, \sigma_2, \ldots, \sigma_r)$ denotes an $r \times r$ diagonal matrix.

Without loss of generality, we can assume that singular values $\sigma_i$ ($i = 1, 2, \ldots, r$) are arranged in the decreasing order, i.e., $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0$. Applying the tensor recovery...
scheme to the measurement vector $y$, we obtain a diagonal matrix $\hat{\Sigma} = \text{diag}(\hat{\sigma}_1, \hat{\sigma}_2, \ldots, \hat{\sigma}_r)$ by replacing the $r-K$ smaller diagonal elements of $\Sigma$ with zeros, that is, $\hat{\sigma}_i \approx \sigma_i$ ($i = 1, 2, \ldots, K$), $\hat{\sigma}_{K+1} = \hat{\sigma}_{K+2} = \cdots = \hat{\sigma}_r = 0$. We thus obtain estimated loss field tensor $\hat{X}^{(M)}$ as

$$
\hat{X}^{(M)} = U\hat{\Sigma}V^T.
$$

The reconstruction error $\kappa^{(M)}$ of $\hat{X}^{(M)}$ is then given by

$$
\kappa^{(M)} = \| \hat{X}^{(M)} - X \|_F.
$$

Table 1 shows singular values of the true loss field tensor and an estimated loss field tensor. Because the rank of the loss field tensor $X$ in Fig. 3a is rank$X = 1$, $X$ has only one non-zero singular value $\sigma_1$. The estimated loss field tensor $\hat{X}$ highly approximates $X$ by replacing $\sigma_i$ ($i = 3, 4, \ldots, N_1N_2$) with zeros.

Finally, suppose that both $\hat{S}$ and $\hat{\Sigma}$ generally have $K$ non-zero elements. For the case of $D = 2$, it is well-known that SVD provides the smallest reconstruction error, that is, $\kappa^{(M)} \leq \kappa^{(V)}$ [6, 10], where $\kappa^{(M)}$ is given by

$$
\kappa^{(M)} = \sqrt{\sum_{i=K+1}^{r} \sigma_i^2}.
$$

Therefore, for the case of $D = 2$, the tensor recovery scheme is the best way in terms of the reconstruction error. Actually, in [6], the authors show that the SVD-based image compression obtains better reconstruction errors than the DCT-based image compression.
Furthermore, in [14], HOSVD is studied and it is shown that low-rank approximation of the $D$-th tensor provides a good approximation in terms of the reconstruction error.

### 5.4 Experiments in an Anechoic Chamber

Next, we evaluate the performance of the proposed scheme by experiments in an anechoic chamber of National Institute of Information and Communications Technology (NICT) Headquarter. Figure 11a shows the experimental setup, where there is one radio wave absorber as an obstruction in the monitored region and Table 2 summarizes specifications of the experiment. In the experiments, we use a continuous wave (CW) signal in a 8GHz band, and its transmission powers is adjusted to be received with received power of $-60$ [dBm] $\sim -30$ [dBm]. While ideal wireless signals with zero beamwidth are used in the simulation experiments, the transmitters and receivers of wireless nodes have the same type of antenna with an almost omni-directional antenna radiation pattern in the horizontal plane as shown in Fig. 11b.

Figure 12 shows the monitored region with size of $140$ [cm] $\times 140$ [cm] $\times 100$ [cm], and the obstruction with size of $30$ [cm] $\times 30$ [cm] $\times 60$ [cm]. We set $N_1 = 7$, $N_2 = 7$, and $N_3 = 3$, and divide the monitored region into three slices as shown in Fig. 13. On each slice, 28 nodes are placed, and the wireless signal is transmitted between $M$ pairs of nodes, where each pair is chosen randomly from nodes on the same slice. There are at most 270 measurable paths on each slice, therefore, the total number of measurable paths is $M_{\text{max}} = 270 \times 3 = 810$.

We define $N_1 \times N_2$ matrix $X(n_3)$ ($n_3 = 1, 2, 3$) as $X(n_3) = \{X_{n_1',n_2',n_3} \mid n_1' = 1, 2, \ldots, N_1, n_2' = 1, 2, \ldots, N_2, n_3' = n_3\}$ and an estimated matrix $\hat{X}^{(M)}(n_3)$ of $X(n_3)$. We redefine the reconstruction error $\epsilon(M)$ as a function of the number $M$ of measurements:

---

(a)

(b)

Fig. 11 Experimental setup. a Obstruction in the monitored region. b Antenna radiation pattern used in the experiments
\[
\epsilon(M) = \frac{1}{N_3} \sum_{n_3=1}^{N_3} \frac{\| \hat{X}^{(M)}(n_3) - \hat{X}^{(M_{\text{max}})}(n_3) \|_F}{\| \hat{X}^{(M_{\text{max}})}(n_3) \|_F}
\]

Figure 14 shows the normalized reconstruction error versus the number \( M \) of measurements. From the figure, we observe that the reconstruction error decreases almost linearly as the number of measurements increases. Figures 15 and 16 show the estimated loss field tensor for \( M = 114 (= 38 \times 3) \) and \( M = M_{\text{max}} (= 810) \), respectively. We observe that in both cases, the proposed scheme can estimate the loss field tensor clearly. Note that the transmitters and receivers are placed on the same plane, therefore, from Fig. 12, no obstruction should appear in the estimated loss field tensor at \( n_3 = 3 \). As expected, there are no obstructions estimated in both the figures.

In Sect. 5.2, the ideal wireless signal with zero beamwidth was used to measure the received power and we assumed that phenomena of radio waves such as reflection and diffraction do not occur. On the other hand, in this section, we use an almost omni-directional antenna for the transmitters and the receivers. Although these phenomena may affect the performance of the wireless tomography, the experimental results indicate that the proposed scheme is feasible.

### Table 2 Specifications of the experiments

| Location       | NICT Headquarter |
|----------------|------------------|
| Size of Anechoic Chamber | 840 [cm] \( \times \) 720 [cm] \( \times \) 660 [cm] . |
| Size of Monitored Region | 140 [cm] \( \times \) 140 [cm] \( \times \) 100 [cm] . |
| Size of Obstruction | 30 [cm] \( \times \) 30 [cm] \( \times \) 60 [cm] . |
| Signal Emission | 8GHz (CW) |
| RSS            | \(-60 \text{ [dBm]} \sim -30 \text{ [dBm]}\) |
| Antenna        | UWB Omni-Antenna |
| Number of wireless nodes | 28 nodes on each X-Y plane \( \times 3 = 84 \) |
| Maximum number of measurement paths | 810 |

![Fig. 12](image)

**Fig. 12** Monitored region for the experiments. We set \( N_1 = 7 \), \( N_2 = 7 \), and \( N_3 = 3 \). The shaded area represents the obstruction. **a** Monitored region. **b** Monitored region projected onto the X-Y plane.
Fig. 13 Slices of the monitored region. The black points on the border of the monitored region represent wireless nodes.

Fig. 14 Estimation error versus the number of measurements.

Fig. 15 An experimental result of the tensor recovery-based wireless tomography with $M = 114$. (a) $n_3 = 1$ (b) $n_3 = 2$ (c) $n_3 = 3$
Conclusion

In this paper, we proposed a multi-dimensional wireless tomography using tensor-based compressed sensing, which enables us to estimate the locations of internal obstructions by a small number of measurement signal transmissions. While the conventional wireless tomography using the vector recovery-based compressed sensing utilizes the sparsity of the frequency representation of a given loss field tensor, the proposed wireless tomography utilizes its low-rank property. With simulation experiments, we validated the effects of the proposed wireless tomography, and showed that the tensor recovery-based wireless tomography can provide more accurate estimation of the loss field tensor, especially in a less noisy environment.

We still have some remaining issues to be resolved. For example, in order to achieve noiseless measurements, we require a signal processing technique to extract a direct link in multipath fading environments. In this paper, we assumed that many wireless nodes are deployed on the border of the monitored region, and we chose wireless links randomly by using these wireless nodes. In a situation where there are a small number of wireless nodes, however, we have to consider a wireless node selection scheme to achieve an efficient estimation of a loss field tensor. We will try these issues in the future work.

References

1. Agrawal, P., & Patwari, N. (2009). Correlated link shadow fading in multi-hop wireless networks. *IEEE Transactions on Wireless Communications*, 8(8), 4024–4036.
2. Beck, A., & Teboulle, M. (2009). A fast iterative shrinkage-thresholding algorithm for linear inverse problems. *SIAM Journal on Imaging Sciences*, 2(1), 183–202.
3. Caiafa, C. F., & Cichocki, A. (2013). Computing sparse representations of multidimensional signals using Kronecker bases. *Neural Computation*, 25(1), 186–220.
4. Candès, E. J., & Wakin, M. B. (2008). An introduction to compressive sampling. *IEEE Signal Processing Magazine*, 25(2), 21–30.
5. Chen, J., & Saad, Y. (2009). On the tensor SVD and the optimal low rank orthogonal approximation of tensors. *SIAM Journal on Matrix Analysis and Applications*, 30(4), 1709–1734.
6. Dapena, A., & Ahalt, S. (2002). A hybrid DCT-SVD image-coding algorithm. *IEEE Transactions on Circuits and Systems for Video Technology*, 12(2), 114–121.
7. Gandy, S., Recht, B., & Yamada, I. (2011). Tensor completion and low-n-rank tensor recovery via convex optimization. *Inverse Problems*, 27(2), 025010.
8. Hashemi, H. (1993). The indoor radio propagation channel. *Proceedings of the IEEE*, 81(7), 943–968.
9. Hayashi, K., Nagahara, M., & Tanaka, T. (2013). A user’s guide to compressed sensing for communications systems. *IEICE Transactions on Communications, E96–B(3),* 685–712.

10. Jain, A. K. (1989). *Fundamentals of digital image processing.* Upper Saddle River: Prentice Hall.

11. Kaltiokallio, O., Bocca, M., & Patwari, N. (2014). A fade level-based spatial model for radio tomographic imaging. *IEEE Transactions on Mobile Computing, 13*(6), 1159–1172.

12. Kanso, M. A., & Rabbat, M. G. (2009). Compressed RF tomography for wireless sensor networks: Centralized and decentralized approaches. In *Proceedings of the 5th IEEE international conference on distributed computing in sensor systems* (pp. 173–186).

13. Kolda, T. G., & Bader, B. W. (2009). Tensor decompositions and applications. *SIAM Review, 51*(3), 455–500.

14. Lathauwer, L. D., Moor, B. D., & Vandewalle, J. (2000). A multilinear singular value decomposition. *SIAM Journal on Matrix Analysis and Applications, 21*(4), 1253–1278.

15. Mostofi, Y. (2011). Compressive cooperative sensing and mapping in mobile networks. *IEEE Transactions on Mobile Computing, 10*(12), 1769–1784.

16. Patwari, N., & Agrawal, P. (2008). Effects of correlated shadowing: Connectivity, localization, and RF tomography. In *Proceedings of the international conference on information processing in sensor networks* (pp. 82–93).

17. Recht, B., Fazel, M., & Parrilo, P. A. (2010). Guaranteed minimum-rank solutions of linear matrix equations via nuclear norm minimization. *SIAM Review, 52*(3), 471–501.

18. Takemoto, K., Matsuda, T., Hara, S., Takizawa, K., Ono, F., & Miura, R. (2014). Multi-dimensional wireless tomography with tensor-based compressed sensing. *arXiv:1407.2394, http://arxiv.org/abs/1407.2394.*

19. Toh, K. C., & Yun, S. (2010). An accelerated proximal gradient algorithm for nuclear norm regularized linear least squares problems. *Pacific Journal of Optimization, 6*(3), 615–640.

20. Wilson, J., & Patwari, N. (2010). Radio tomographic imaging with wireless networks. *IEEE Transactions on Mobile Computing, 9*(5), 621–632.

21. Yokota, K., Hara, S., Matsuda, T., Takizawa, K., Ono, F., & Miura, R. (2016). Experimental evaluation on a joint attenuation map estimation/indoor localization by means of compressed sensing-based wireless tomography. In *Proceedings of the 3rd international conference on information and communication technologies for disaster management 2016 (ICT-DM 2016)* (accepted).

22. Yokota, K., Hara, S., Matsuda, T., Mukamoto, M., Uemura, Y., Takizawa, K., Ono, F., & Miura, R. (2015). Experimental evaluation on wireless tomography with compressed sensing in a three-dimensional space. In *Proceedings of the 18th international symposium on wireless personal multimedia communications (WPMC 2015).*

23. Zibulevsky, M., & Elad, M. (2010). L1-L2 optimization in signal and image processing. *IEEE Signal Processing Magazine, 27*(3), 76–88.

Takahiro Matsuda received his B.E. with honors, M.E., and Ph.D. in communications engineering from Osaka University in 1996, 1997, 1999, respectively. He joined the Department of Communications Engineering at the Graduate School of Engineering, Osaka University in 1999. He is currently an Associate Professor in the Department of Information and Communications Technology, Graduate School of Engineering, Osaka University. His research interests include performance analysis and the design of communication networks and wireless communications. He received Best Tutorial Paper Award and Best Magazine Paper Award from IEICE ComSoc in 2012, and Best Paper Award from IEICE in 2014. He is a member of IEICE, IPSJ, and IEEE.
Kengo Yokota received the B.E. degree in Information Engineering from Osaka City University, Osaka, Japan, in 2015. He is currently pursuing the M.E. course at Osaka City University, engaging in the research on indoor localization.

Kazushi Takemoto received his B.E. with honors and M.E., in information and communications engineering from Osaka University in 2012 and 2014, respectively. He is currently with FURUNO Electric Co. LTD.

Shinsuke Hara received the B.Eng., M.Eng., and Ph.D. degrees in communications engineering from Osaka University, Osaka, Japan, in 1985, 1987, and 1990, respectively. He was an Assistant Professor from April 1990 to September 1997 and an Associate Professor from October 1997 to September 2005 in Osaka University. Since October 2005, he has been with the Graduate School of Engineering, Osaka City University as a Professor. From April 1995 to March 1996, he was also a visiting scientist as Telecommunications and Traffic Control Systems Group, Delft University of Technology, Delft, The Netherlands. His research interests include mobile and indoor wireless communications and digital signal processing.
Fumie Ono received her B.E., M.E., and Ph.D. degrees in electrical engineering from Ibaraki University, in 1999, 2001, 2004, respectively. She was with the Faculty of Engineering, Tokyo University of Science, from 2004 to 2006 as a research associate. From 2006 to 2011, she was an assistant professor in the Division of Electrical and Computer Engineering, Yokohama National University. She is currently with Communications Research Laboratory, National Institute of Information and Communications Technology (NICT). She is a member of IEEE and IEICE.

Kenichi Takizawa received the B.E., M.E. and Dr. Eng. degrees from Niigata University, Japan, in 1998, 2000, and 2003, respectively. He joined Communications Research Laboratory, now National Institute of Information and Communication Technology (NICT), in 2003. He is a member of IEEE.

Ryu Miura received the B.E., M.E., and Dr. Eng. degrees in Electrical Engineering from Yokohama National University, Yokohama, Japan, in 1982, 1984, and 2000, respectively. He joined Radio Research Lab, Ministry of Posts and Telecom (reorganized to NICT in 2004), Tokyo, Japan, in 1984. Since then, he has been engaged in R&D on communication systems with satellites and high-altitude/long-endurance aerial platforms. During 1991–1992, he was a visiting researcher in AUSSAT, Pty Ltd., Sydney, Australia, where he served for the prototyping of mobile satellite communication systems. During 1993–1996 and 2009–2011, he was with Advanced Telecommunications Research Institute (ATR), Kyoto Japan, where he was involved in R&D on digital beamforming antennas and intelligent transport systems for driving safety support, respectively. Now he is an executive researcher in Wireless Network Research Center, NICT, Yokosuka, Japan, where he is involved in the R&D on wireless networks for unmanned aircraft systems.