Branching Fractions and Direct $CP$ Asymmetries of

\[ \overline{B}_s^0 \to K^0 h^+ h^- (h^{(l)} = K, \pi) \] Decays

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Abstract

Motivated by the recent LHCb measurements of charmless three-body decays of $B_s^0$ meson, we calculate the branching fractions of $\overline{B}_s^0 \to K^0 \pi^+ \pi^-$, $\overline{B}_s^0 \to K^0 K^+ K^-$, $\overline{B}_s^0 \to K^0 \pi^+ K^-$ and $\overline{B}_s^0 \to K^0 K^+ \pi^-$ decay modes within the factorization approach. The resonant and nonresonant contributions are studied in detail. For the decays $\overline{B}_s^0 \to K^0 \pi^+ \pi^-$ and $\overline{B}_s^0 \to K^0 K^+ K^-$, our results agree well with experimental data, and the former is dominated by the $K^*$ and $K_0^*(1430)$ poles, while the later one is dominated by the nonresonant contribution. Considering the flavor $SU(3)$ symmetry violation, the sum of branching fractions of $\overline{B}_s^0 \to K^0 \pi^+ K^-$ and $\overline{B}_s^0 \to K^0 K^+ \pi^-$ could accommodate the data well. It should be noted that the branching fractions are very sensitive to the scalar density $\langle K \pi | \bar{q} q | 0 \rangle$. Furthermore, the resonant contributions are dominated by the scalar poles $K_0^*(1430)$. We hope that these branching fractions could be measured separately in the experiments so as to test the factorization approach. Moreover, the direct $CP$ asymmetries of these decays are also explored, which could be measured in the running LHCb experiment and Super-b factory in future.

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1 Introduction

In the recent years, the charmless three-body decays of $B$ mesons have being attached more attentions, because by studying them one can determine the Cabibbo-Kabayashi-Maskawa (CKM) parameters or search for the possible new physics effect beyond the standard model. For example, the Dalitz-plot analysis combined with flavor SU(3) symmetry allows us to extract the angle $\gamma$ cleanly from $B \to K\pi\pi$ and $B \to KKK$ decays\cite{1, 2, 3}. However, the three-body decays of $B$ mesons are more complicated than the two-body cases, because both resonant (vector or scalar) and nonresonant contributions are involving the hadronic matrix elements. The interference between resonant and nonresonant amplitudes makes it rather hard to disentangle these distinct contributions and extract the nonresonant one, it is very difficult to measure the direct three-body decays. Over the recent years, thanks to the two $B$ factories and LHCb experiment, remarkable progress in measuring the branching fractions and direct $CP$ asymmetries of the three-body decays could be made using the Dalitz-plot analysis (for a review see ref. \cite{4}).

In the theoretical side, the charmless three-body decays of heavy mesons have been studied using different approaches, such as the factorization approach (FA)\cite{5, 6, 7, 8, 9, 10, 11, 12, 13}, diagram approach combined with SU(3) symmetry\cite{14, 15, 16, 17}, perturbative QCD approach\cite{18}, and others\cite{19, 20, 21}. FA, based on the phenomenological factorization model has been applied in calculating three-body decays of heavy meson widely, although factorization has not been proved in the three-body decays. Within the FA, the predicted branching fractions and direct $CP$ asymmetries of $B \to PPP$ decays\cite{9, 10, 11, 12, 13} agree with the experimental data well, except for decay $\overline{B}^0 \to K^+K^0\pi^0$.

We will review the FA briefly by taking $B^- \to \pi^+\pi^-\pi^-$ as an example. Under the FA, the amplitude of decay $B^- \to \pi^+\pi^-\pi^-$ is separated into three distinct factorizable terms: (i) the current-induced process with a meson emission, $\langle B^- \to \pi^+\pi^- \rangle \times \langle 0 \to \pi^- \rangle$, (ii) the transition process, $\langle B^- \to \pi^- \rangle \times \langle 0 \to \pi^+\pi^- \rangle$, and (iii) the annihilation process $\langle B^- \to 0 \rangle \times \langle 0 \to \pi^+\pi^-\pi^- \rangle$, where $\langle A \to B \rangle$ stands for a $A \to B$ transition matrix element. One of the nonresonant contribution due to $\langle B^- \to \pi^+\pi^- \rangle$ has been studied on the basis of the heavy meson chiral perturbative theory (HMChPT)\cite{22, 23, 24, 25}. However, it could lead to large branching fraction ($\mathcal{O}(10^{-5})$)\cite{5, 6}, which disagrees with the experimental data ($5.3 \times 10^{-6}$) from BaBar\cite{26}. In fact, this issue can be understood considering the applicability of the HMChPT. When the HMChPT is applied to three-body decays, two of the final-state pseudoscalars should be soft. If the soft meson result is assumed to be the same in the whole Dalitz plot, the decay rate will be greatly overestimated. To overcome this issue, Cheng et al. proposed in refs\cite{10, 11, 12, 13} to parameterize the momentum dependence of nonresonant amplitudes $\langle B \to PP \rangle$ in an exponential form $e^{-\alpha_{NR}(p^+ + p^-)}$ so that the HMChPT results are recovered in the soft pseudoscalar meson limit. The tree-dominated $B^- \to \pi^+\pi^-\pi^-$ decay data is used to fix the unknown parameter $\alpha_{NR}$. Besides from the current-induced process, the matrix elements $\langle \pi^+\pi^- | q\gamma_\mu q | 0 \rangle$ and $\langle \pi^+\pi^- | \bar{d}d | 0 \rangle$ also receive nonresonant contributions. In principle, the weak vector form factor of the former matrix element can be related to the charged pion electromagnetic (e.m.) form factors. However, unlike the kaon case, the time-like e.m. form factors of the pions are not well measured enough allowing
us to determine the nonresonant parts. Therefore, the nonresonant contribution to $\langle \pi^+ \pi^- | \bar{q} \gamma_\mu q | 0 \rangle$ is always ignored. The matrix element $\langle \pi^+ \pi^- | \bar{d}d(0) \rangle$ is related to $\langle K^+ K^- | \bar{s}s(0) \rangle$ via SU(3) flavor symmetry. As for the resonant contributions to three-body decays, vector and scalar resonances contribute to the two-body matrix elements $\langle P_1 P_2 | V_p | 0 \rangle$ and $\langle P_1 P_2 | S_p | 0 \rangle$, respectively. They can also contribute to the three-body matrix element $\langle P_1 P_2 | V_p - A_p | 0 \rangle$. Resonant effects are described in terms of the usual Breit-Wigner formalism. In this manner, the relevant resonances which contribute to the 3-body decays of interest could be figured out.

In conjunction with the nonresonant contribution, the total rates for three-body decays are well calculated. Very recently, corresponding to an integrated luminosity of $1.0 \text{ fb}^{-1}$ recorded at a centre-of-mass energy of 7 TeV, LHCb published their first measurements of the branching fractions of three-body decays of $B^0_s$ meson [27] as following:

$$Br(B^0_s \to K^0 \pi^+ \pi^-) = (14.3 \pm 2.8 \pm 1.8 \pm 0.6) \times 10^{-6},$$

$$Br(B^0_s \to K^0 K^+ \pi^-) = (73.6 \pm 5.7 \pm 6.9 \pm 3.0) \times 10^{-6},$$

$$Br(B^0_s \to K^0 K^+ K^-) \in [0.2; 3.4] \times 10^{-6} \text{ at } 90\% \text{ CL}.$$  

Since these decays have never been explored before, we will calculate the branching fractions in this work using the FA proposed by Cheng et al. so as to test FA in $B_s$ decays. The resonant and nonresonant contributions of these decays will be studied, which are important in determining the branching fractions of $B_s \to KV$ and $B_s \to KS$ experimentally. Furthermore, we will calculate the $CP$ asymmetries of these decays, which may be helpful to extract the CKM angle $\gamma$. All results could be checked in the current LHCb experiment and Super-b factory in future.

In the following work, we will systematically use the FA to calculate the $\bar{B}^0_s \to K^0 h^+ h^-$ and present the formulas in Sec. 2. The numerical results and some discussions are given in Sec. 3. We will summarize this work in Sec. 4 lastly.

## 2 Analytic Formalism

### 2.1 The Effective Hamiltonian

Under the factorization hypothesis, the matrix elements of the decay amplitudes are given by

$$\langle P_1 P_2 P_3 | H_{\text{eff}} | \bar{B}^0_s \rangle = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(r)} \langle P_1 P_2 P_3 | T_p^{(r)} | \bar{B}^0_s \rangle,$$
where $\lambda^{(r)}_p = V_{pb}V_{ps}^*$ with $r = d, s$. For $K\pi\pi$ and $KKK$ modes, $r = d$; and for $KK\pi$ channels, $r = s$. The Hamiltonian $T_p^{(r)}$ has the expression \[28\]

$$T_p^{(r)} = a_1\delta_{pu}\langle \bar{u}b\rangle_{V-A} \otimes (\bar{r}u)_{V-A} + a_2\delta_{pu}(\bar{r}b)_{V-A} \otimes (\bar{u}u)_{V-A} + a_3(\bar{r}b)_{V-A} \otimes \sum_q (\bar{q}q)_{V-A}$$

$$+ a_p^q \sum_q (\bar{q}b)_{V-A} \otimes (\bar{r}q)_{V-A} + a_3(\bar{r}b)_{V-A} \otimes \sum_q (\bar{q}q)_{V+A}$$

$$- 2a_p^q \sum_q (\bar{q}b)_{S-P} \otimes (\bar{r}q)_{S+P} + a_7(\bar{r}b)_{V-A} \otimes \sum_q \frac{3}{2} e_q(\bar{q}q)_{V+A}$$

$$- 2a_p^q \sum_q (\bar{q}b)_{S-P} \otimes \frac{3}{2} e_q(\bar{r}q)_{S+P} + a_9(\bar{r}b)_{V-A} \otimes \sum_q \frac{3}{2} e_q(\bar{q}q)_{V-A}$$

$$+ a_7^p \sum_q (\bar{q}b)_{V-A} \otimes \frac{3}{2} e_q(\bar{r}q)_{V-A},$$

(5)

with $(\bar{q}q')_{V\pm} \equiv \bar{q}\gamma_{\mu}(1 \pm \gamma_5)q'$, $(\bar{q}q')_{S\pm P} \equiv \bar{q}(1 \pm \gamma_5)q'$ and a summation over $q = u, d, s$ being implied. For the effective Wilson coefficients at the renormalization scale $\mu = 2.1$ GeV, we shall follow \[12\] to use

$$a_1 \approx 0.99 + 0.037i, \quad a_2 \approx 0.19 - 0.11i, \quad a_3 \approx -0.002 + 0.004i, \quad a_5 \approx 0.0054 - 0.005i,$$

$$a_4^q \approx -0.03 - 0.02i, \quad a_5^q \approx -0.04 - 0.008i, \quad a_6^q \approx -0.06 - 0.02i, \quad a_6^q \approx -0.06 - 0.006i,$$

$$a_7 \approx 0.54 \times 10^{-4} i, \quad a_7^q \approx (4.5 - 0.5i) \times 10^{-4}, \quad a_8 \approx (4.4 - 0.3i) \times 10^{-4},$$

$$a_9 \approx -0.010 - 0.0002i, \quad a_9^q \approx (-58.3 + 86.1i) \times 10^{-5}, \quad a_9^q \approx (-60.3 + 88.8i) \times 10^{-5},$$

(6)

In the above coefficients, the strong phases are from vertex corrections and penguin contractions, which have been calculated within the QCD factorization approach \[29\].

### 2.2 $B_s^0 \rightarrow K^0\pi^+\pi^-$

With the effective Hamiltonian and the equation of motion, we obtain the $B_s^0 \rightarrow K^0\pi^+\pi^-$ decay amplitude as

$$\langle K^0\pi^+\pi^- | T_p | B_s^0 \rangle = \langle K^0\pi^+ | \langle \bar{u}b\rangle_{V-A} | B_s^0 \rangle \langle \pi^- | (\bar{d}u)_{V-A} | 0 \rangle \left[ a_1\delta_{pu} + \frac{a_p^0}{a_1} + \frac{a_p^0}{a_1} - r_7^5(a_p^0 + a_6^0) \right]$$

$$+ \langle K^0 | (\bar{d}b)_{V-A} | B_s^0 \rangle \langle \pi^+\pi^- | (\bar{u}u)_{V-A} | 0 \rangle \left[ a_2\delta_{pu} + a_3 + a_5 + a_7 + a_9 \right]$$

$$+ \langle K^0 | (\bar{d}b)_{V-A} | B_s^0 \rangle \langle \pi^+\pi^- | (\bar{d}d)_{V-A} | 0 \rangle \left[ a_3 + a_5 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10}) \right]$$

$$+ \langle K^0 | (\bar{d}b)_{V-A} | B_s^0 \rangle \langle \pi^+\pi^- | (\bar{s}s)_{V-A} | 0 \rangle \left[ a_3 + a_5 - \frac{1}{2}(a_7 + a_9) \right]$$

$$+ \langle K^0 | (\bar{d}b)_{V-A} | B_s^0 \rangle \langle \pi^+ | \bar{d}d | 0 \rangle \left[ - 2a_p^0 + a_6^0 \right]$$

$$+ \langle K^0 | (\bar{s}d)_{V-A} | 0 \rangle \langle \bar{s}b | B_s^0 \rangle \left[ a_4^p - \frac{1}{2}a_{10}^p \right]$$

$$+ \langle K^0 | \pi^+\pi^- | \bar{s}(1 + \gamma_5)d | 0 \rangle \langle \bar{s} \gamma_5 b | B_s^0 \rangle \left[ 2a_p^0 - a_6^0 \right],$$

(7)

where $r_7^5(\mu) = \frac{2m_r^2}{m_s(\mu)m_d(\mu)m_u(\mu)}$. It should be noted that $\langle \pi^+\pi^- | (\bar{d}d)_{V-A} | 0 \rangle = -\langle \pi^+\pi^- | (\bar{u}u)_{V-A} | 0 \rangle$ because of isospin symmetry. Besides, the matrix element $\langle \pi^+\pi^- | (\bar{s}s)_{V-A} | 0 \rangle$ is suppressed heavily by the
Okubo-Zweig-Izuka (OZI) rule. Moreover, there exist two weak annihilation contributions, where the \( B_s^0 \) meson is annihilated into vacuum and a final state with three mesons is then created, as shown the last two terms in above equation. However, from the results of \( B \to PPP \) decays, the contributions from annihilations are fairly small, so we will ignore them in the numerical calculations in the current work.

For the current-induced process, the three-body matrix element \( \langle K^0\pi^+|(\bar{u}b)_{V-A}|B_s^0(pB)\rangle \) could be parameterized as \[25\]

\[
\langle K^0(p_1)\pi^+(p_2)|\langle\bar{u}b\rangle_{V-A}|B_s^0(pB)\rangle = i\rho(p_B - p_1 - p_2)_{\mu} + i\omega^+(p_2 + p_1)_{\mu} + i\omega^-(p_2 - p_1)_{\mu} + h\epsilon_{\mu\nu\lambda\beta}\rho_{\mu}^\nu(p_2 + p_1)^\alpha(p_2 - p_1)^\beta.
\] (8)

The form factors \( \omega_{\pm} \) and \( r \) have the expressions as \[25\]

\[
\omega_+ = -\frac{g}{4\pi f_K} \frac{f_B\cdot m_{B^*}\sqrt{m_B\cdot m_{B^*}}}{s_{23} - m_{B^*}^2} \left[ 1 - \frac{(p_B - p_1)_{\cdot}p_1}{m_{B^*}^2} \right] + \frac{f_{B_s}}{4\pi f_K},
\]

\[
\omega_- = \frac{g}{4\pi f_K} \frac{f_B\cdot m_{B^*}\sqrt{m_B\cdot m_{B^*}}}{s_{23} - m_{B^*}^2} \left[ 1 + \frac{(p_B - p_1)_{\cdot}p_1}{m_{B^*}^2} \right],
\]

\[
r = \frac{f_{B_s}}{4\pi f_K} \frac{p_B \cdot (p_2 - p_1)}{m_{B^*}^2} + \frac{2g}{4\pi f_K} \sqrt{m_{B^*}} \frac{(p_B - p_1)_{\cdot}p_1}{s_{23} - m_{B^*}^2}
\]

\[
- \frac{4g^2 f_{B_s}}{4\pi f_K} \frac{m_{B^*} m_{B^*}}{s_{23} - m_{B^*}^2} p_1 \cdot (p_2 - p_1) (p_B - p_1) (p_B - p_{12}) (p_1^2 + m_{B^*}^2 + m_{B^*}^2).
\] (9)

where \( s_{ij} = (p_i + p_j)^2 \). \( g \) is a heavy-flavor independent strong coupling which has been extracted from the CLEO measurement of the \( D^{*+} \) decay width \[30\]. \( |g| = 0.59 \pm 0.01 \pm 0.07 \). In this work, we also follow \[22\] to adopt its sign to be negative. Thus, we drive the current-induced amplitude as:

\[
A_{\text{current-ind}} = \langle \pi^-(p_3)|\langle\bar{u}b\rangle_{V-A}|0\rangle \langle K^0(p_1)\pi^+(p_2)|\langle\bar{u}b\rangle_{V-A}|B^-\rangle
\]

\[
= -\frac{f_{\pi}}{2} \left[ 2m_{B^*}^2 r + (m_{B^*}^2 - s_{12} - m_{B^*}^2)\omega_+ + (s_{23} - s_{13} - m_{B^*}^2 + m_{B^*}^2)\omega_- \right] e^{-\alpha_{N^0PP}(p_1 + p_2)} e^{i\phi_{12}}.
\] (10)

As stated in the Sec \[1\] the exponential form \( e^{-\alpha_{N^0PP}(p_1 + p_2)} \) is introduced so that the HMChPT results are recovered in the soft meson region and

\[
\alpha_{NR} = 0.081^{+0.015}_{-0.009} \text{GeV}^{-2};
\] (11)

which is constrained from the tree dominated decay \( B^- \to \pi^+\pi^-\pi^- \). The unknown strong phase \( \phi_{12} \) is set to be zero for simplicity.

In this decay mode, vector meson \( (K^*) \) and scalar resonances \( (K_0^3(1430)) \) also contribute to the three-body matrix element \( \langle K^0(p_1)\pi^+(p_2)|\langle\bar{u}b\rangle_{V-A}|B_s^0(pB)\rangle \), which effects are described in terms of the usual Breit-Wigner formalism. So, we have the expression as

\[
\langle K^0(p_1)\pi^+(p_2)|\langle\bar{u}b\rangle_{V-A}|B_s^0\rangle_R = \frac{g_{K^0\to K^0\pi^+}}{s_{12} - m_{K^0}^2 + i m_{K^0} \Gamma_{K^0}} \sum_{\text{pol}} \psi \cdot (p_1 - p_2) \langle K^{*+} |\langle\bar{u}b\rangle_{V-A}|B_s^0\rangle
\]

\[
= -\frac{g_{K_0^*\to K^0\pi^+}}{s_{12} - m_{K_0^*}^2 + i m_{K_0^*} \Gamma_{K_0^*}} \langle K_0^{*+} |\langle\bar{u}b\rangle_{V-A}|B_s^0\rangle,
\] (12)
where we have ignored the contribution of $K^*(1410), K^*(1680), \cdots$. Hence,

$$
(K^0(p_1)\pi^+ + p_2)|\bar{u}b\rangle_{V-A}\langle B_s^0|^{R} \langle \pi^- (p_3)|\bar{d}u\rangle_{V-A}|0\rangle = -f_{\pi} \frac{g_{K^0_{\pi} + \pi^0}^{\pi}}{s_{12} - m_{K_0}^2 + i m_{K^*} \Gamma_{K^*}^I} (m_{B_s}^2 - s_{12}) F_0^{B_s K_0^*}(q^2)
$$

$$
- \frac{f_{\pi}}{2\sqrt{2}} \frac{g_{K^* - K^0}^{\pi}}{s_{12} - m_{K^*}^2 + i m_{K^*} \Gamma_{K^*}^I} (s_{13} - s_{23}) \left[ (m_{B_s} + m_{K^*}) A_{1}^{B_s K^*}(q^2)
$$

$$
- \frac{A_{2}^{B_s K^*}(q^2)}{m_{B_s} + m_{K^*}} (m_{B_s}^2 - s_{12}) - 2 m_{K^*} [A_{3}^{B_s K^*}(q^2) - A_{0}^{B_s K^*}(q^2)] \right],
$$

with $q^2 = (p_B - p_1 - p_2)^2 = p_s^2$. In above formulae, the definitions of decay constants and form factors are referred to the Refs. [13, 31, 32].

For the transition process, because the time-like e.m. form factors of two pions have not been measured well, we will ignore the nonresonant contributions and only consider the contributions from the vector and scalar mesons. Hence, the amplitude of the transition process is read as

$$
\langle \pi^+ (p_2)\pi^- (p_3)|\bar{u}u\rangle_{V-A}|0\rangle^{R} \langle K^0(p_1)|\bar{d}d\rangle_{V-A}|B_s^0|^{R} = -F_{1}^{B_s K}(s_{23}) F_{R}^{\pi^+ \pi^-} (s_{23}) (s_{12} - s_{13}),
$$

$$
\langle \pi^+ (p_2)\pi^- (p_3)|\bar{d}d\rangle_{V-A}|0\rangle^{R} \langle K^0(p_1)|\bar{d}d\rangle_{V-A}|B_s^0|^{R} = \frac{-m_{B_s}^2 - m_{K}^2}{m_{B_s} - m_{d}} F_{R}^{B_s K}(s_{23}) \sum \frac{m_{f_0_i} f_{f_0_i} g_{f_0_i \pi^+ \pi^-}}{s_{23} - m_{f_0_i}^2 + i m_{f_0_i} \Gamma_{f_0_i}},
$$

with the definition of the form factor $F_{R}^{\pi^+ \pi^-}$:

$$
F_{R}^{\pi^+ \pi^-} (s) = \frac{1}{\sqrt{2}} \sum \frac{m_{\rho_i} f_{\rho_i} g_{\rho_i \pi^+ \pi^-}}{s - m_{\rho_i}^2 + i m_{\rho_i} \Gamma_{\rho_i}},
$$

where $\rho_i = \rho, \rho(1450), \cdots$ and $f_0 = f_0(980), f_0(1370), f_0(1500), \cdots$. The scalar decay constant $f_{f_0_i}^{q}$ is defined by $\langle \bar{q}q|0\rangle^{R} = m_{f_0_i} f_{f_0_i}^{q}$ - $g_{f_0_i \pi^+ \pi^-}$ is the strong coupling of the $f_0_i \rightarrow \pi^+ \pi^-$ decay.

For the scalar meson $f_0(980)$, we will consider it as the conventional $q\bar{q}$, though the quark structure of the light scalar mesons below or near 1 GeV has been quite controversial. Because some experimental evidences indicate that $f_0(980)$ is not purely an $s\bar{s}$ state [29], we write the flavor wave functions of the $f_0(980)$ as:

$$
|f_0(980)\rangle = |s\bar{s}\rangle \cos \theta + |n\bar{n}\rangle \sin \theta,
$$

with $n\bar{n} \equiv (\bar{u}\bar{u} + \bar{d}\bar{d})/\sqrt{2}$. Experimental implications for the mixing angle have been discussed in detail in [31]. Assuming 2-quark bound state for $f_0(980)$, the observed large rates of $B \rightarrow f_0(980)K$ and $f_0(980)K^*$ modes can be explained in QCDF with the mixing angle $\theta$ in the vicinity of 20° [35]. So, we use $\theta = 20°$ in this work.
2.3 $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$

The factorizable $\overline{B}_s^0 \rightarrow K^0 K^+ K^-$ decay amplitudes given by

$$
\langle K^0 K^+ K^- | T_p | \overline{B}_s^0 \rangle = \langle K^+ K^- | (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle \langle K^0 | (\bar{d}s)_{V-A} | 0 \rangle \left[a_0^p - \frac{1}{2} a_{10}^p - r_K (a_0^p - \frac{1}{2} a_{10}^p)\right] + \langle K^0 | (\bar{d}b)_{V-A} \overline{B}_s^0 | K^+ K^- | (\bar{u}u)_{V-A} | 0 \rangle \left[a_2 \delta_{pa} + a_3 + a_5 + a_7 + a_9\right] + \langle K^0 | (\bar{d}b)_{V-A} \overline{B}_s^0 | K^+ K^- | (\bar{d}d)_{V-A} | 0 \rangle \left[a_3 + a_4 + a_5 - \frac{1}{2}(a_7 + a_9 + a_{10})\right] + \langle K^0 | (\bar{d}b)_{V-A} \overline{B}_s^0 | K^+ K^- | (\bar{s}s)_{V-A} | 0 \rangle \left[a_3 + a_5 - \frac{1}{2}(a_7 + a_9)\right] + \langle K^+ | (\bar{u}b)_{V-A} \overline{B}_s^0 | K^- K^0 | (\bar{d}u)_{V-A} | 0 \rangle \left[a_1 \delta_{pa} + a_4^p + a_{10}\right] + \langle K^+ | (\bar{u}b)_{V-A} \overline{B}_s^0 | K^+ K^- | | 0 \rangle \left[-2 a_6^p + a_8^p\right] + \langle K^+ | (\bar{u}b)_{V-A} \overline{B}_s^0 | K^- K^0 | d\bar{d} | 0 \rangle \left[-2 a_6^p - 2 a_8^p\right] + \langle K^0 | (\bar{u}b)_{V-A} \overline{B}_s^0 | K^+ K^- | | 0 \rangle \left[a_0^p - \frac{1}{2} a_{10}^p\right] + \langle K^0 | (\bar{u}b)_{V-A} \overline{B}_s^0 | K^+ K^- | | 0 \rangle \left[2 a_6^p - a_8^p\right].
$$

(17)

For the current-induced process with a kaon emission, the form factors $r$ and $\omega_{\pm}$ for the three-body matrix element $\langle K^+ K^- (\bar{s}b)_{V-A} | \overline{B}_s^0 \rangle$ evaluated in the framework of HMChPT are similar to that of Eq. (9) except that $f_\pi$ is replaced by $f_K$. This process also receives the contributions of vector ($\phi$) and scalar ($f_0$) resonants by

$$
\langle K^+(p_2) K^- (p_3) | (\bar{d}b)_{V-A} | \overline{B}_s^0 \rangle \langle K^0 (p_1) | (\bar{s}s)_{V-A} | 0 \rangle = \frac{-f_K}{2 \sqrt{2}} \frac{g_{\phi-K^+ K^-}}{s_{23} - m_\phi^2 + i m_\phi \Gamma_\phi} (s_{12} - s_{13}) \left[(m_{B_s} + m_\phi) A_1 B_1 \phi (q^2) - \frac{A_2 B_2 \phi (q^2)}{m_{B_s}^2 + m_\phi^2} (m_{B_s}^2 - s_{23}) - 2 m_\phi (A_3 B_3 \phi (q^2) - A_0 B_0 C (q^2))\right] - f_K \sum_i \frac{g_{f_{1i} \rightarrow K^+ K^-}}{s_{23} - m_{f_{1i}}^2 + i m_{f_{1i}} \Gamma_{f_{1i}}} \frac{1}{4} F_0^{B_1 f_{1i}} (q^2) (m_{B_s}^2 - s_{23}).
$$

(18)

For the transition amplitude, in addition to the $b \rightarrow u$ tree transition, we need to consider the nonresonant contributions to the $b \rightarrow s$ penguin amplitude

$$
A_1 = \langle K^0 (p_1) | (\bar{d}b)_{V-A} | \overline{B}_s^0 \rangle \langle K^+(p_2) K^- (p_3) | (\bar{q}q)_{V-A} | 0 \rangle, \quad A_2 = \langle K^+(p_2) | (\bar{u}b)_{V-A} | \overline{B}_s^0 \rangle \langle K^0 (p_1) K^- (p_3) | (\bar{d}u)_{V-A} | 0 \rangle, \quad A_3 = \langle K^0 (p_1) | db | \overline{B}_s^0 \rangle \langle K^+(p_2) K^- (p_3) | \bar{d}d | 0 \rangle, \quad A_4 = \langle K^+(p_2) | \bar{u}b | \overline{B}_s^0 \rangle \langle K^0 (p_1) K^- (p_3) | \bar{d}u | 0 \rangle.
$$

(19-22)

We firstly calculate the two-kaon creation matrix element $A_1$, which could be expressed in terms of the time-like kaon current form factors as

$$
\langle K^+(p_K^+) K^- (p_K^-) | \bar{q} \gamma_\mu q | 0 \rangle = (p_{K^+} - p_{K^-})_\mu F_q^{K^+ K^-}.
$$

(23)

The weak vector form factor $F_q^{K^+ K^-}$ is related to the kaon electromagnetic (e.m.) form factors $F_{\text{em}}^{K^+ K^-}$. Phenomenologically, the e.m. form factors receive resonant and nonresonant contributions and can be
expressed by
\[ F_{\text{em}}^{K^+ K^-} = F_{\rho}^{KK} + F_{\omega}^{KK} + F_{\phi}^{KK} + F_{NR}. \]  

(24)

It follows from Eqs. (23) and (24) that
\[
\begin{align*}
F_{\mu}^{K^+ K^-} & = F_{\rho}^{KK} + 3F_{\omega}^{KK} + \frac{1}{3}(3F_{NR} - F_{NR}'), \\
F_{d}^{K^+ K^-} & = -F_{\rho}^{KK} + 3F_{\omega}^{KK}, \\
F_{s}^{K^+ K^-} & = -3F_{\phi}^{KK} - \frac{1}{3}(3F_{NR} + 2F_{NR}'),
\end{align*}
\]

(25)

where the isospin symmetry has been used. The resonant and nonresonant terms can be parameterized as
\[ F_{h}(s_{23}) \text{ and } F_{NR}^{(p)}(s_{23}), \]
respectively by using
\[ h(\bar{f}) \]

In A_{3}, although the nonresonant contribution vanishes as both \(K^+\) and \(K^-\) do not contain the valence \(d\) or \(\bar{d}\) quark, this matrix element does receive the contribution from the scalar \(f_0\) pole,
\[
(K^+(p_2)K^-(p_3)|\bar{d}d(0)|R) \equiv f_d^{K^+ K^-}(s_{23}) = \sum_i \frac{m_{f_0}f_{f_0,i}g_{f_0,i}^{K^+ K^-}}{m_{f_0}^2 - s_{23} - im_{f_0}f_{f_0,i}},
\]

(27)

which leads to
\[
A_3 = \frac{m_B^2 - m_K^2}{m_B - m_s} F_{0}^{B,K}(s_{23}) f_{d}^{K^+ K^-}(s_{23}).
\]

(28)

For the equations \(A_2\) and \(A_4\), the contributions from nonresonant could be parameterized as \(F_{NR}\) and \(f_{NR}^d\) respectively by using \(SU(3)\) symmetry. The formulae of \(f_{d}^{NR}\) is expressed and discussed in detail in [13].

After calculation, we obtain
\[
\begin{align*}
A_2 & = (s_{12} - s_{13}) F_{1}^{B,K}(s_{23}) F_{NR}(s_{13}), \\
A_4 & = \frac{m_B^2 - m_K^2}{m_B - m_s} F_{0}^{B,K}(s_{23}) f_{d}^{NR}(s_{13}).
\end{align*}
\]

(29) (30)

2.4 \( \bar{B}_s^0 \rightarrow K^0 K^-\pi^+ \) and \( \bar{B}_s^0 \rightarrow \bar{K}^0 K^+\pi^- \)

The factorizable amplitudes of the \( \bar{B}_s^0 \rightarrow K^0 K^-\pi^+ \) and \( \bar{B}_s^0 \rightarrow \bar{K}^0 K^+\pi^- \) are given as :
\[
\langle K^0 K^-\pi^+ | T_R | \bar{B}_s^0 \rangle = \langle K^0 \pi^+ | (\bar{u}b)_{V-A}| \bar{B}_s^0 \rangle \langle K^- | (\bar{s}u)_{V-A}| 0 \rangle \left[ a_1 \delta_{pu} + a_4^p + a_{10}^p - r^K (a_6^p + a_8^p) \right]
\]
\[
+ \langle K^0 | (\bar{d}b)_{V-A}| \bar{B}_s^0 \rangle \langle K^- \pi^+ | (\bar{s}d)_{V-A}| 0 \rangle \left[ a_4^d - \frac{1}{2} a_{10}^d \right]
\]
\[
+ \langle K^0 | (\bar{d}d)_{V-A}| \bar{B}_s^0 \rangle \langle K^- \pi^+ | \bar{d}d| 0 \rangle \left[ -2a_6^d + a_8^d \right]
\]
\[
+ \langle K^0 |K^- \pi^+ |(\bar{e}u)_{V-A}| 0 \rangle \langle (\bar{s}b)_{V-A}| \bar{B}_s^0 \rangle \left[ a_2 \delta_{pu} + a_3 + a_9 \right]
\]
\[
+ \langle K^0 |K^- \pi^+ |(\bar{d}d)_{V-A}| 0 \rangle \langle (\bar{s}b)_{V-A}| \bar{B}_s^0 \rangle \left[ a_3 - \frac{1}{2} a_9 \right]
\]
\[
+ \langle K^0 |K^- \pi^+ |(\bar{s}b)_{V-A}| 0 \rangle \langle (\bar{s}b)_{V-A}| \bar{B}_s^0 \rangle \left[ a_3 + a_4 - \frac{1}{2} a_9 - \frac{1}{2} a_{10}^d \right]
\]
\[
+ \langle K^0 | \pi^+ \pi^- | \bar{s}(1 + \gamma_5)s(0) | (0) | \bar{B}_s^0 \rangle \left[ 2a_6^p - a_8^p \right].
\]

(31)
For the two-body matrix element

\[
\langle \mathbf{K}^0 | \pi^- | T_p | \mathbf{B}_s^0 \rangle = \langle K^+ \pi^- | (\bar{d}b)_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} (\bar{d}b)_{V-A} | 0 \rangle \left[ a_4^p - \frac{1}{2} a_1^p - r_K^p (a_6^p - \frac{1}{2} a_8^p) \right] \\
+ \langle K^+ \pi^- | (\bar{u}b)_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} (\bar{u}b)_{V-A} | 0 \rangle \left[ a_1 \delta_{pu} + a_7^p + a_1^{10} \right] \\
+ \langle K^+ \pi^- | (\bar{u})_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} (\bar{u})_{V-A} | 0 \rangle \left[ -2a_6^p - 2a_8^p \right] \\
+ \langle K^0 | \pi^- | (\bar{s}u)_{V-A} | 0 \rangle (\bar{s}b)_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} | 0 \rangle \left[ a_2 \delta_{pu} + a_3 + a_9 \right] \\
+ \langle K^0 | \pi^- | (\bar{s}u)_{V-A} | 0 \rangle (\bar{s}b)_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} | 0 \rangle \left[ a_3 - \frac{1}{2} a_9 \right] \\
+ \langle K^0 | \pi^- | (\bar{s}s)_{V-A} | 0 \rangle (\bar{s}b)_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} | 0 \rangle \left[ a_3 + a_4^p - \frac{1}{2} a_9 - \frac{1}{2} a_1^{10} \right] \\
+ \langle K^0 | \pi^- | (\bar{s}s)_{V-A} | 0 \rangle (\bar{s}b)_{V-A} | \mathbf{B}_s^0 \rangle (\mathbf{K}^0)_{V-A} | 0 \rangle \left[ 2a_6^p - a_8^p \right],
\]

(32)

For the current-induced processes, the three-body matrix elements \( \langle K^+ | (\bar{q}b)_{V-A} | \mathbf{B}_s^0 \rangle \) have the similar expressions as Eqs. (9) and (10). Furthermore, these process also receive resonant contributions, which could be written as, for example,

\[
\langle K^0 | (p_3) \pi^+ | (p_2) | (\bar{u}b)_{V-A} | \mathbf{B}_s^0 \rangle R \langle K^- | (p_1) | (\bar{d}u)_{V-A} | 0 \rangle = \frac{f_K}{\sqrt{2}} \frac{g_{K^+ \rightarrow K^0 \pi^+}}{s_{23} - m_{K^+}^2 + im_{K^+} \Gamma_{K^+}} (s_{13} - s_{12}) \left[ (m_{B_s} + m_{K^+}) A_{B_l}^{K^+} (q^2) \right] \\
- \frac{A_{B_s}^{K^+} (q^2)}{m_{B_s} + m_{K^+}} \left[ (m_{B_s} - s_{23}) - 2m_{K^+} [A_{B_s}^{K^+} (q^2) - A_{B_s}^{K^+} (q^2)] \right] \\
- \frac{f_K}{s_{23} - m_{K^+}^2 + im_{K^+} \Gamma_{K^+}} \frac{(m_{B_s} - s_{23}) F_{B_s}^{K^+} (q^2)}{(m_{B_s} - s_{23}) F_{B_s}^{K^+} (q^2)}.
\]

(33)

For the two-body matrix element \( \langle K^- | \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle \), we note that

\[
\langle K^- | (p_1) \pi^+ | (p_2) | (\bar{s}d)_{V-A} | 0 \rangle = (p_1 - p_2) \mu F_{K^+}^{K^+} (s_{12}) + \frac{m_{K^+}^2 - m_{\pi^+}^2}{s_{12}} (p_1 + p_2) \mu \left[ - F_{K^+}^{K^+} (s_{12}) + F_{K^+}^{K^+} (s_{12}) \right].
\]

(34)

The resonant contributions are expressed by:

\[
\langle K^- | (p_1) \pi^+ | (p_2) | (\bar{s}d)_{V-A} | 0 \rangle R = \sum_{s_{12} - m_{K^+}^2 + im_{K^+} \Gamma_{K^+}} c_{s12} \left( p_1 - p_2 \right) \left( K_{s12}^+ | (\bar{s}d)_{V-A} | 0 \right) \\
- \sum_{s_{12} - m_{K^+}^2 + im_{K^+} \Gamma_{K^+}} \langle K_{s12}^+ | (\bar{s}d)_{V-A} | 0 \rangle.
\]

(35)

Hence, form factors \( F_{K^+}^{K^+} \) and \( -F_{K^+}^{K^+} + F_{K^+}^{K^+} \) receive the following resonant contributions

\[
(F_{K^+}^{K^+} (s)) R = \sum_{s_{12} - m_{K^+}^2 + im_{K^+} \Gamma_{K^+}} \frac{m_{K^+} \mu f_{K^+} \mu g_{K^+} (s_{12})}{m_{K^+}^2 - s - im_{K^+} \Gamma_{K^+}}
\]

(36)

As a result, the amplitude \( \langle K^- | \pi^+ | (\bar{s}d)_{V-A} | 0 \rangle \langle K^0 | (\bar{q}b)_{V-A} | \mathbf{B}_s^0 \rangle \) has the expression

\[
\langle K^- | (p_1) \pi^+ | (p_2) | (\bar{s}d)_{V-A} | 0 \rangle \langle K^0 | (\bar{q}b)_{V-A} | \mathbf{B}_s^0 \rangle = F_{K^+}^{K^+} (s_{12}) F_{K^+}^{K^+} (s_{12}) \left[ \frac{m_{K^+}^2 - m_{\pi^+}^2}{s_{12}} \right] + F_{K^+}^{K^+} (s_{12}) F_{K^+}^{K^+} (s_{12}) \left[ \frac{m_{K^+}^2 - m_{\pi^+}^2}{s_{12}} \right],
\]

(37)
where the momentum dependence of the weak form factor $F_{K\pi}(q^2)$ is parameterized as
\begin{equation}
F_{K\pi}(q^2) = \frac{F_{K\pi}(0)}{1 - q^2/\Lambda^2 + i\Gamma_R/\Lambda},
\end{equation}
with $\Gamma_R = 200$ MeV [3] being the width of the relevant resonance and $\Lambda = 0.83$ GeV being a chiral symmetry breaking scale.

For the term $\langle K\pi|\bar{s}d|0\rangle$, it receives contributions of both resonant and nonresonant, the expression of which is shown as
\begin{equation}
\langle K^- (p_1)\pi^+(p_2)|\bar{s}d|0\rangle = \frac{m_{K^0} - m_{\pi^+}}{m_{s} - m_d} F_{NRK} + \langle K^- (p_1)\pi^+(p_2)|\bar{s}d|0\rangle^{NR}.
\end{equation}
In the above equation, the unknown two-body matrix elements of scalar densities $\langle K\pi|\bar{s}d|0\rangle$ is related to $\langle K^+ K^-|\bar{s}s|0\rangle$ via SU(3) symmetry, e.g.
\begin{equation}
\langle K^- (p_1)\pi^+(p_2)|\bar{s}d|0\rangle^{NR} = \langle K^+ (p_1)K^- (p_2)|\bar{s}s|0\rangle^{NR} = f_{s}^{NR}(s_{12}),
\end{equation}
with the expression of $f_{s}^{NR}$ given as
\begin{equation}
f_{s}^{NR} = \langle K^- (p_1)\pi^+(p_2)|\bar{s}d|0\rangle^{NR} = \frac{m_{s}^2 - m_{\pi}^2}{m_{s} - m_d} F_{NR} + \frac{2}{3} F_{NRs} + \sigma_{sn}e^{-\alpha s_{12}},
\end{equation}
where $\sigma_{sn} = e^{3\pi/4}(3.36\pm 1.12)$ GeV is fixed from data of $B^0 \rightarrow K_S K_S K_S$ [4].

3 Numerical Results

To proceed with the numerical calculations, we firstly specify the parameters used in this work. For the CKM matrix elements, we use the updated Wolfenstein parameters $A = 0.823$, $\lambda = 0.22457$, $\bar{\rho} = 0.1289$ and $\bar{\eta} = 0.348$ [36]. The corresponding CKM angles are $\sin 2\beta = 0.689 \pm 0.019$ and $\gamma = (69.7^{+1.3}_{-2.8})^{\circ}$. The form factors used in this work are from refs. [37] [38] [39], which are summarized as follows
\begin{align}
F_{0}^{B_{s}\rightarrow K} = 0.31, V_{B_{s}\rightarrow K} = 0.43, A_{1}^{B_{s}\rightarrow K} = 0.31, A_{0}^{B_{s}\rightarrow K} = 0.36, A_{2}^{B_{s}\rightarrow K} = 0.23, V_{B_{s}\rightarrow K} = 0.31, \\
A_{1}^{B_{s}\rightarrow K} = 0.23, A_{2}^{B_{s}\rightarrow K} = 0.18, F_{0}^{B_{s}\rightarrow K} = 0.42, F_{0}^{B_{s}\rightarrow f_{0}(980)} = 0.44, F_{0}^{B_{s}\rightarrow f_{0}(1500)} = 0.41.
\end{align}

In practical calculation, we shall assign the form factor error to be 0.03. For the strong coupling constants, most of them have been determined from the measured partial width in refs. [12] [13], which are shown as
\begin{align}
g^{(770)\rightarrow K^+K^-} = 6.0, g^{K^+(892)\rightarrow K^+K^-} = 4.59, g^{f_{0}(980)\rightarrow K^+K^-} = 1.18 \text{ GeV}, g^{K^+(1430)\rightarrow K^+K^-} = 3.84 \text{ GeV}, \\
g^{\phi\rightarrow K^+K^-} = -4.54, g^{f_{0}(980)\rightarrow K^+K^-} = 3.7 \text{ GeV}, g^{f_{0}(1500)\rightarrow K^+K^-} = 0.69 \text{ GeV}, g^{f_{0}(1710)\rightarrow K^+K^-} = 1.6 \text{ GeV}.
\end{align}
Table 1: Branching fractions (in units of $10^{-6}$) of resonant and nonresonant (NR) contributions to $\bar{B}_s^0 \to K^0 h^+ h^-$.

| Decay mode | Theory | Decay mode | Theory |
|------------|--------|------------|--------|
| $\bar{B}_s^0 \to K^0 \pi^+ \pi^-$ | | $K^0 f_0(980)$ | $0.01+0.00+0.01+0.00$ |
| $K^+ \pi^-$ | $6.08^{+0.00+1.12+0.02}_{-0.00-1.10-0.04}$ | $K^0 f_0(1370)$ | $0.04+0.00+0.01+0.00$ |
| $K^0_{0}^{*}(1430) \pi^-$ | $7.45^{+0.00+1.19+0.04}_{-0.00-1.05-0.04}$ | NR | $1.18+0.01+0.43+0.00$ |
| $K^0 \rho^0$ | $0.52^{+0.00+0.10+0.01}_{-0.00-0.10-0.01}$ | | |
| Total | $14.46^{+0.32+2.33+0.08}_{-0.34-2.06-0.09}$ |

| $\bar{B}_s^0 \to K^0 K^+ K^-$ | | $f_0(980)K^0$ | $0.10+0.00+0.05+0.00$ |
| $\phi K^0$ | $0.19^{+0.00+0.15+0.01}_{-0.00-0.08-0.01}$ | NR | $0.69+0.01+0.21+0.01$ |
| $f_0(1500)K^0$ | $0.10^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$ | | |
| Total | $0.90^{+0.01+0.38+0.01}_{-0.01-0.25-0.01}$ |

| $\bar{B}_s^0 \to K^0 K^- \pi^+$ | | $K_0^{*0} K^0$ | $3.47^{+0.00+0.76+0.01}_{-0.00-0.67-0.01}$ |
| $K^+ K^-$ | $1.96^{+0.00+2.79+0.04}_{-0.00-1.04-0.05}$ | $K_0^{*0} (1430) K^0$ | $19.94^{+0.00+4.67+0.02}_{-0.00-4.09-0.03}$ |
| $K_0^{*0} (1430) K^-$ | $2.44^{+0.00+2.47+0.09}_{-0.00-1.28-0.09}$ | NR | $11.02^{+0.05+11.19+0.01}_{-0.03-5.54-0.01}$ |
| Total | $40.00^{+0.17+24.63+0.14}_{-0.09-13.43-0.13}$ |

| $\bar{B}_s^0 \to K^0 K^+ K^-$ | | $K^* K^+$ | $3.20^{+0.00+0.73+0.11}_{-0.00-0.64-0.10}$ |
| $K^0 K^0$ | $1.12^{+0.00+2.39+0.00}_{-0.00-0.69-0.05}$ | $K_0^{*0} (1430) K^+$ | $19.70^{+0.00+4.56+0.11}_{-0.00-4.01-0.11}$ |
| $K_0^{*0} (1430) K^0$ | $1.40^{+0.00+2.88+0.00}_{-0.00-0.95-0.00}$ | NR | $10.77^{+0.02+10.94+0.02}_{-0.02-5.42-0.02}$ |
| Total | $37.29^{+0.16+23.27+0.01}_{-0.10-12.59-0.01}$ |

For the running quark masses we shall use $[33, 40]$

$$m_b(m_b) = 4.2 \text{ GeV}, \quad m_b(2.1 \text{ GeV}) = 4.94 \text{ GeV}, \quad m_b(1 \text{ GeV}) = 6.34 \text{ GeV},$$
$$m_c(m_b) = 0.91 \text{ GeV}, \quad m_c(2.1 \text{ GeV}) = 1.06 \text{ GeV}, \quad m_c(1 \text{ GeV}) = 1.32 \text{ GeV},$$
$$m_s(2.1 \text{ GeV}) = 95 \text{ MeV}, \quad m_s(1 \text{ GeV}) = 118 \text{ MeV},$$
$$m_d(2.1 \text{ GeV}) = 5.0 \text{ MeV}, \quad m_u(2.1 \text{ GeV}) = 2.2 \text{ MeV}. \quad (45)$$

With above parameters and formulas in Sec.2, we calculated the branching fractions of resonant and nonresonant contributions to concerned decay modes and presented them in Tab.1. The theoretical errors
are from the uncertainties in (i) the parameter $\alpha_{NR}$ which governs the momentum dependence of the nonresonant amplitude, (ii) the strange quark mass $m_s$, the form factors, the nonresonant parameter $\sigma_{NR}$ and $SU(3)$ asymmetry violation parameter $\beta$, and (iii) the unitarity angle $\gamma$.

From Table. 1 we see that the decay $B_s^0 \to K^0\pi^+\pi^-$ is tree dominated and its main contributions arise from the $K^{*+}$ and $K^{*_0}(1430)$ resonants, while the nonresonant contribution is less important. Compared with experimental data, the calculated branching fraction agrees well with the recent LHCb measurement. As for $B_s^0 \to K^0K^-\pi^+$, although it receives the color-suppressed tree contribution, but it is dominated by transition $b \to d\bar{s}q$. Consequently, it has a small branching fraction $(0.90^{+0.01+0.38+0.01}_{-0.01-0.25-0.01}) \times 10^{-6}$, which is much smaller than that of $B_s^0 \to K^0\pi^+\pi^-$. Noted that this decay is governed by the nonresonant background dominated by $\sigma_{NR}$, hence this decay mode could be an ideal plat for constraining the unknown parameter $\sigma_{NR}$ in turn. Experimentally, however, no significant evidence of this decay mode has been obtained, and its branching fraction is described in $(0.2-3.4) \times 10^{-6}$ at 90% confidence level (CL) based on the CL inferences in Ref. [41]. Obviously, the result we predicted is falling into the experimental range. We hope this decay will be measured precisely in the current LHCb experiment. The results of above two decay modes also confirm the conclusion that nonresonant decays play a prominent role in the penguin-dominated three-body $B$ meson decays in Ref. [12].

For the decay $B_s^0 \to K^0K^-\pi^+$, the current-induced process with a $K^-$ emission is tree dominated, while the transition processes $\langle B_s^0 \to K^0 \rangle \times \langle 0 \to K^-\pi^+ \rangle$ are induced by penguin operators. On the contrary, the its current-induced process of decay $B_s^0 \to K^0K^+\pi^-$ with a neutral kaon emission is induced by penguin, and the transition processes receive the effects not only from tree but from penguin operators. In these two decays, the nonresonant contributions arise dominantly from the transition process via the scalar density $\langle K\pi|\bar{s}q|0 \rangle$, and slightly from the current-induced process. Thus, the nonresonant contributions are sensitive to the matrix elements of scalar densities $f_s^{NR}$, as shown in Table [11]. For the resonant contributions, both of them are dominated by the scalar particles $K^{*}_0(1430)$. Considering parameter $\beta$ standing for effects of the $SU(3)$ symmetry violation and the final states rescattering, the sum of two branching fractions is $(77.29^{+0.32+47.89+0.13}_{-0.19-26.03-0.13}) \times 10^{-6}$, which could accommodate data of the recent LHC measurement well. We hope the future experiment could measure these two decays separately, so as to test our results.

In QCD calculations based on a heavy quark expansion, one faces uncertainties arising from power corrections such as annihilation and hard-scattering contributions. For example, in QCD factorization, there are large theoretical uncertainties related to the modelling of power corrections corresponding to weak annihilation effects and the chirally enhanced power corrections to hard spectator scattering. Even for two-body $B$ decays, power corrections are of order $(10 - 20)\%$ for tree-dominated modes, but they are usually bigger than the central values for penguin-dominated decays. Needless to say, $1/m_b$ power corrections for three-body decays may well be larger. However, in the current work we use the phenomenological factorization model rather than in the established theories based on a heavy quark expansion. Consequently, uncertainties due to power corrections, at this stage, are not included in our calculations, by assumption.
In view of such shortcomings we must emphasize that the additional errors due to such model dependent assumptions may be sizable.

In this work, the $CP$ asymmetries of these four decays are also calculated, and the results are summarized in Table 2. We see from the table that the decay $B_{s}^{0} \rightarrow K^{0}K^{+}K^{-}$ has large $CP$ asymmetries with and without resonant contributions. Noted that the two asymmetries have same sign, because this decay is dominated by the nonresonant background, which can also be read from Table. 1. For other three decays, the resonant contributions are much larger than the nonresonant contribution, they may affect the $CP$ asymmetries through taking large strong phases. In fact, the strong phases could arise from the effective Wilson coefficients, the Breit-Wigner formalism for resonances and the penguin matrix elements of scalar densities. Besides, the final states rescattering may take new phases, which are not calculated directly up to now. Recently, the $CP$ asymmetries of $B \rightarrow KK\pi, KK\pi$ have been measured in LHCb, however the $CP$ asymmetries of three-body of $B_{s}^{0}$ have not been explored till now. The $CP$ asymmetries of these four decays are hoped to be measured in the current LHCb experiment or Super-b in future, so as to test the factorization approach in $B_{s}^{0}$ meson three-body decays.

4 Summary

Recently, LHCb published the measurements of charmless three-body decays of $B_{s}^{0}$ meson, corresponding to an integrated luminosity of 1.0 fb$^{-1}$ recorded at a centre-of-mass energy of 7 TeV. Motivated by this, we calculated the branching fractions of $\overline{B}_{s}^{0} \rightarrow K^{0}\pi^{+}\pi^{-}$, $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}K^{-}$, $\overline{B}_{s}^{0} \rightarrow K^{0}\pi^{+}K^{-}$ and $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}\pi^{-}$ decay modes within the factorization approach, which is generalized by Cheng et al. We calculated the branching fractions with not only from nonresonant contributions but also from resonant contributions in detail. For the decays $\overline{B}_{s}^{0} \rightarrow K^{0}\pi^{+}\pi^{-}$ and $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}K^{-}$, our results agree well with experimental data, and the former is dominated by the $K^{*}$ and $K_{s}^{0}(1430)$ poles, while the later one is dominated by the nonresonant contribution. With flavor $SU(3)$ symmetry violations, the sum of branching fractions of $\overline{B}_{s}^{0} \rightarrow K^{0}\pi^{+}K^{-}$ and $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}\pi^{-}$ could accommodate the data well. It should be emphasized that the branching fractions are very sensitive to the scalar density $\langle K\pi|\bar{s}q|0\rangle$. We hope these branching fractions could be measured separately in the experiments so as to test the factorization approach in three-body

| Final state | Total | Nonresonant |
|-------------|-------|-------------|
| $\overline{B}_{s}^{0} \rightarrow K^{0}\pi^{+}\pi^{-}$ | $1.0^{+0.1+0.5+0.0}_{-0.1-0.3-0.0}$ | $-1.1^{+0.4+0.0+0.0}_{-0.3-0.0-0.0}$ |
| $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}K^{-}$ | $-17.0^{+0.2+0.8+0.0}_{-0.3-1.0-0.0}$ | $-18.1^{+0.1+0.3+0.2}_{-0.1-0.6-0.1}$ |
| $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}\pi^{-}$ | $-3.4^{+0.1+0.6+0.0}_{-0.1-0.4-0.1}$ | $5.9^{+0.8+2.6+0.1}_{-1.1-1.9-0.3}$ |
| $\overline{B}_{s}^{0} \rightarrow K^{0}K^{+}\pi^{-}$ | $0.7^{+0.1+0.4+0.0}_{-0.1-0.4-0.0}$ | $0.8^{+0.1+0.1+0.0}_{-0.0-0.0-0.0}$ |
decays of $B$ mesons. Moreover, the direct $CP$ asymmetries of these decays are also explored, and the sizable results could be measured in the running LHCb experiment and Super-b factory in future.

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Note added

When this paper is being prepared, Hai-Yang Cheng and Chun-Khiang Chua posted their paper to the e-print archiv 42. The same decays have been studied in that work, and most of our results agree with theirs after considering the differences parameters. In 42, more attentions are attached in the $U$-spin asymmetry, while in this work we paid more attentions in disentangling the resonant and nonresonant contribution. Moreover, in dealing with the flavor $SU(3)$ symmetry violation of $\langle K\pi|0\rangle$, different approaches are adopted.

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