In this essay I argue that the Hilbert space of states dual to a traversable wormhole is smaller than the tensor product of the independent Hilbert spaces of the boundary field theories. From the point of view of semiclassical physics, the decrease in the number of states is perceived as an emergent, non-local interaction stabilizing the wormhole. This presents new possibilities for models of radiating black holes and raises questions about results established under the spell of the tensor product.

In this essay I will entertain the preposterous idea that – as far as radiating black holes are concerned – such a split does not occur. I will begin by considering wormholes and argue the following:

**Claim 1.** Let \( \mathcal{H} \) denote the Hilbert space of states dual to a two-sided, traversable, holographic wormhole, and let \( \mathcal{H}_L \) and \( \mathcal{H}_R \) be the Hilbert spaces of the decoupled dual theories living on its asymptotic boundaries. In general, \( \mathcal{H}_L \otimes \mathcal{H}_R \neq \mathcal{H} \).

By wormhole we mean a spacetime which has two asymptotically AdS boundaries and allows a signal to pass from one boundary to the other, as shown in Figure 2.

The structure of quantum black holes is a perpetual source of confusion and inspiration. Any quantum gravity proposal must explain how information leaks through the horizon of a radiating black hole. So many ideas have been put forth that the field has become difficult to navigate, see Figure 1. The most persistent assumption, permeating hundreds of proposals, is the idea that the Hilbert space of states of a radiating black hole splits into the tensor product of the modes associated with the outgoing Hawking radiation and their interior partners.

In essence, the split is a consequence of applying the equivalence principle when the Hawking radiation is derived as the Unruh radiation for an observer orbiting a black hole. Despite the fact that the validity of the equivalence principle in the presence of horizons has been repeatedly questioned, e.g. [1], breaking the spell of the tensor product is rarely considered.

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By wormhole we mean a spacetime which has two asymptotically AdS boundaries and allows a signal to pass from one boundary to the other, as shown in Figure 2. At this point, we do not care how the wormhole was opened, we simply assume its existence and traversability for at least a few bits.

Consider a scalar field \( \Phi \) in the bulk of the wormhole with normalizable boundary conditions. We can define its boundary limits as \( f_{R,L} = \lim_{z_{R,L} \to 0} z_{R,L}^2 F \Phi \), where \( z_{R,L} \) are the Fefferman-Graham coordinates associated with the two boundaries at \( z_{R,L} = 0 \) and \( \Delta \) is the dimension of the dual operators. Note that \( f_{R,L} \) represent the state of the theory, not the deformation. Furthermore, by \( \Phi[f_L,f_R] \) let us denote any bulk field with the boundary data \( f_L \times f_R \).

Holography implies that the Hilbert spaces of the quantized bulk and boundary theories match. This is the starting point of proposals such as the HKLL reconstruction [5], where bulk operators are constructed from boundary operators. Here, we will invert this logic. We start by decomposing the bulk field into a complete set of bulk modes, \( \Phi = \sum_F \Phi_F \), where \( \Phi_F = \phi_F a_F + \phi_F^* a_F^\dagger \). Next, the coefficients \( a_F \) are promoted to creation-annihilation operators, and the Fock space \( \mathcal{H} \) is constructed. The vacuum state \( \Omega \) is annihilated by all \( a_F \)'s, and the one-particle states are \( \Phi_F = \Phi_F(\Omega) \). Finally, the boundary operators are defined by taking the boundary limits of \( \Phi_F \).

One can see that the Hilbert space \( \mathcal{H} \) splits into the tensor product \( \mathcal{H}_L \otimes \mathcal{H}_R \) if and only if the boundary conditions on the left and right boundaries are independent, i.e., if the map \( f_L \times f_R \mapsto \Phi[f_L,f_R] \) is well-defined for any boundary data. This is certainly the case for the semiclassical eternal AdS black hole. In particular, by taking the boundary limits of the quantum bulk operators \( \Phi[0,f] \) and \( \Phi[f,0] \) we obtain two boundary operators.
$O^{R,L}_f$. These are the usual operators smeared over each boundary, $O^{R,L}_f = \int d^d x f(x)O^{R,L}(x)$. They act on the separate factors in the tensor product and create factorized one-particle states, $|O^{R,L}_f\rangle$.

However, the situation is different for traversable wormholes. Consider a wave packet $w$ located at the right boundary, as in Figure 2, which travels through the wormhole and emerges on its left boundary as the wave packet $\tilde{w}$. Uniqueness of the bulk solution implies that no bulk field with the boundary data $0 \times w$ nor $\tilde{w} \times 0$ exists. We conclude that the Hilbert space does not split into the tensor product.

One might worry that this set-up suffers from issues such as a lack of coupling to gravity, the wormhole collapsing under the incoming wave, etc. Notice, however, that the essence of the argument relies only on the fact that some field configurations, created at one boundary, hit the opposite boundary. The precise mechanism is irrelevant. Furthermore, the argument does not depend on the choice of modes in the decomposition of the bulk field nor on the choice of the vacuum.

So what are the states that violate the split into the tensor product? Just to keep the discussion simple, consider a single wormhole mode, $\Phi_{\text{wh}} = \Phi[\tilde{w}, w]$, which traverses the wormhole. The states we are looking for, $|O^R|=0$ and $|O^L|=0$, are the would-be one-particle states associated with the boundary data carried by the wormhole mode. They can be defined using the boundary limits of any bulk fields $\Phi[f_L, w]$ and $\Phi[\tilde{w}, f_R]$ for $f_L$ and $f_R$ consistent with $w$ and $\tilde{w}$ on opposite boundaries. However, since we cannot set $f_L = f_R = 0$, we conclude:

**Claim 2.** In the semiclassical approximation, where the split into the tensor product is assumed, we would expect that $|O^R|=0$ belongs to $\mathcal{H}_R$, while $|O^L|=0$ is an element of $\mathcal{H}_L$. But in reality, the two states are linearly dependent, as they are the boundary limits of the single state $\Phi_{\text{wh}}(\Omega) = \phi_{\text{wh}}|\Omega\rangle$.

We can now address the question of how the non-trivial structure of the Hilbert space manifests itself in the semiclassical approximation. Since we would expect that the states $|O^R|=0$ and $|O^L|=0$ are different, their non-vanishing scalar product $\langle O^R|O^L\rangle$ could be interpreted as the presence of an interaction.

**Claim 3.** In the semiclassical approximation, the departure of the Hilbert space $\mathcal{H}$ from the tensor product $\mathcal{H}_L \otimes \mathcal{H}_R$ manifests itself as the emergent, non-local interaction $H_{\text{int}} = O^R_L O^R_L$.

In the seminal work of Gao, Jafferis, and Wall, \cite{6}, a wormhole was opened by introducing a coupling between the QFTs living on the boundaries of the BTZ black hole. As we can see, this non-local interaction between the two QFTs, which sustains the wormhole, is emergent. It is simply the avatar of the non-trivial structure of the Hilbert space of states describing the holographic wormhole.

Using our new-found knowledge of wormholes, let us now speculate about the structure of the Hilbert spaces of radiating black holes. From the early days, we know that in order to unitarize black hole evolution, the interior modes must be somehow encoded in the Hawking radiation. And after decades of research, what is generally accepted today is that, in Susskind’s words, \cite{7}, ‘Entanglement is not enough.’ There exists some mysterious long-range, non-local interaction between the degrees of freedom on both sides of the horizon. Based on the wormhole story, is it possible that what we are actually looking for is a change in the structure of the Hilbert space $\mathcal{H}_L \otimes \mathcal{H}_R$ of the semiclassical black hole rather than a genuine interaction?

As an oversimplified example, consider a pair of entangled modes on both sides of the horizon. The key observation is that in order to introduce a coupling we do not need to take the tensor product of their Hilbert spaces. Instead, the modes are coupled in the way two approximate harmonic oscillators are coupled in the double-well system, see Figure 3. The excitations around the two minima corresponds to semiclassical physics close to the asymptotic boundaries, the tunneling effects are interpreted as Hawking radiation, as in \cite{8}, the dynamics of the inverted harmonic oscillator close to the tip of the potential barrier governs the near-horizon scattering processes, as found in \cite{9, 10}, and the binding energy of the true ground state is interpreted as the negative energy required for sustaining the wormhole.

The double-well construction introduces a coupling, which leads to both perturbative and non-perturbative effects. However, in the case of the black hole, there exist models where the physics in the left and right wedges is modified purely by non-perturbative effects and thus is indistinguishable from the original black hole in the semiclassical approximation. These include models based on various identifications (e.g., the antipodal identification, \cite{11}) or additional boundary conditions (e.g.,
end-of-the-world branes, [12]). Such additional conditions relate bulk field configurations between the left and right wedges of the black hole. While the evolution of the entire system remains causal, a boundary observer perceives non-perturbatively small non-localities, which grow when probing regions closer to the horizon.

To summarize, in this short essay I argued that the Hilbert space of states dual to a wormhole or a radiating black hole does not split into the tensor product, $\mathcal{H}_L \otimes \mathcal{H}_R$, of its boundary theories. Although this possibility was hinted at in e.g., [14], in this essay I have outlined a general proof. The consequences of this observation raise a number of questions regarding the validity of results obtained under the spell of the tensor product. This includes any geometrical methods in the bulk which rely on the evocation of the equivalence principle beyond the horizon. Furthermore, any reasoning based on entanglement, where tracing is carried out over a factor in the tensor product, must be reexamined. In particular, to what extent can methods relying on holographic entanglement entropy, e.g., [15], capture the non-triviality of the Hilbert space of states?

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[1] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, JHEP 02, 062 (2013), arXiv:1207.3123 [hep-th].
[2] K. Papadodimas and S. Raju, Phys. Rev. Lett. 112, 051301 (2014), arXiv:1310.6334 [hep-th].
[3] J. De Boer, R. Van Breukelen, S. F. Lokhande, K. Papadodimas, and E. Verlinde, JHEP 01, 062 (2020), arXiv:1901.08527 [hep-th].
[4] S. Ghosh and S. Raju, Phys. Rev. D 96, 066033 (2017), arXiv:1706.07424 [hep-th].
[5] A. Hamilton, D. N. Kabat, G. Lifschytz, and D. A. Lowe, Phys. Rev. D 74, 066009 (2006), arXiv:hep-th/0606141.
[6] P. Gao, D. L. Jafferis, and A. C. Wall, JHEP 12, 151 (2017), arXiv:1608.05687 [hep-th].
[7] L. Susskind, Fortsch. Phys. 64, 49 (2016), arXiv:1411.0690 [hep-th].
[8] M. K. Parikh and F. Wilczek, Phys. Rev. Lett. 85, 5042 (2000), arXiv:hep-th/9907001 [hep-th].
[9] P. Betzios, N. Gaddam, and O. Papadoulaki, JHEP 11, 131 (2016), arXiv:1607.07885 [hep-th].
[10] K. Hashimoto, K.-B. Huh, K.-Y. Kim, and R. Watanabe, JHEP 11, 068 (2020), arXiv:2007.04746 [hep-th].
[11] G. ’t Hooft, Found. Phys. 47, 1503 (2017), arXiv:1612.08640 [gr-qc].
[12] M. Rozali, J. Sully, M. Van Raamsdonk, C. Waddell, and D. Wakeham, JHEP 05, 004 (2020), arXiv:1910.12836 [hep-th].
[13] A. Bzowski, A. Gnecci, and T. Hertog, JHEP 06, 167 (2018), arXiv:1802.02580 [hep-th].
[14] D. Harlow and D. Jafferis, JHEP 02, 177 (2020), arXiv:1804.01081 [hep-th].
[15] A. Almheiri, T. Hartman, J. Maldacena, E. Shaghoulian, and A. Tajdini, (2020), arXiv:2006.06872 [hep-th].