Observational verification of CPT invariance with binary black hole gravitational waves in the LIGO-Virgo catalog GWTC-1

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Abstract

A discovery of gravitational waves from binary black hole coalescences, reported by the LIGO Scientific and Virgo Collaborations\cite{1}, raises a possibility that measurements of them can provide strict verification of the fundamental laws of physics\cite{2–10}. We perform such an observational verification of CPT invariance in gravitational waves in this study. When CPT violation exists, if any, gravitational waves with different circular polarizations could gain a slight difference in propagating speeds\cite{10, 11}. Hence the birefringence of gravitational waves is induced and there should be a rotation of plus and cross polarizations\cite{12, 13}. For a CPT-violating dispersion relation $\omega^2 = k^2 \pm 2\zeta k^3$, where a sign $\pm$ denotes two circular polarizations, we find no substantial deviations from CPT invariance by performing Bayesian parameter inference\cite{14} over a set of ten signals of binary black hole coalescences in the LIGO-Virgo catalog GWTC-1\cite{1}. Therefore, we obtain strict constraints on a CPT-violating parameter $\zeta$, namely, $|\zeta| \lesssim \text{few} \times 10^{-15}$m at 90\% confidence level. In particular, we report the strictest limit as $\zeta = -0.09^{+0.34}_{-0.43} \times 10^{-14}$m, which is given by GW151226\cite{15}. It is around five times better than the only existing limit\cite{10}. Hence our systematic study represents the up-to-date strictest verification of CPT invariance in gravitational waves.

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CPT invariance\cite{16}, as a simultaneous reversal of charge, parity and time, is well known as one of the fundamental laws of physics. Since it was proposed, it has been verified with high precision by a variety of observations in laboratories and astronomy\cite{17, 18}. Nevertheless, quantum gravity (QG) at Planck scale $\sim 10^{19}\text{GeV}$ is expected to leave some low-energy relic effects\cite{19–22}, wherein CPT violation\cite{23–25} is one famous example, although a consistent theory is still missing at the present time. Due to importance of QG, as an ultimate theory of physics, and CPT invariance, as a fundamental law in nature, it is well justified to be unstinting in one’s efforts to explore possible deviations from CPT invariance under various circumstances.

The discovery of binary black hole (BBH) coalescences by the Laser Interferometer Gravitational-Wave Observatory (LIGO)\cite{26} opens a clean observational window to verify CPT invariance in gravitational waves (GWs). If CPT invariance was deviated, if any, GWs with left-handed and right-handed circular polarizations could gain a slight difference in propagating speeds\cite{10, 11}. As a consequence, the birefringence is induced and there is a rotation of plus (+) and cross (\times) modes\cite{12, 13}. Depending on the GW frequency, the birefringent effect can be accumulated along the trajectory of GWs, which are emitted from compact binary coalescences at cosmological distances. Therefore, we can perform measurements of the polarizations of GWs to verify CPT invariance or detect possible deviations from it.

This study aims at verifying the CPT invariance in GWs and finding strict limits on the leading-order CPT violation in GWs. Higher-order CPT violations are ignored since they are expected to lead smaller effects in the spirits of effective field theory (EFT)\cite{27}. In this study the CPT-violating dispersion relation is manifested as

$$\omega^2 = k^2 \pm 2\zeta k^3,$$

(1)

where a sign $\pm$ takes $+/-$ for the left-/right-handed circular polarization and $\zeta$ is a length-dimensional parameter characterizing the size of CPT violating effect. We suppose a convention $\hbar = 1$ here. So $\omega$ and $k$ denote the energy and momentum of gravitons, respectively. Eq. (1) can be corresponded to a dimension-5 CPT-odd Lorentz-violating operator $\bar{k}^{(5)}_{(V)}\cite{10}$, which is the leading-order operator that causes the birefringence of GWs, and we note $\zeta \simeq \bar{k}^{(5)}_{(V)}$. Laboratory experiments in the non-relativistic limit are insensitive to such dimension-5 CPT-violating operators, since Newton’s law remains unchanged by these
operators[28]. However, GWs from cosmologically distant BBHs provide a potential approach to measure these operators[10–13].

Let us remind several known aspects of CPT violation in the pure photon sector[23, 25]. The polarization vector of a linearly polarized photon rotates along its propagating direction if CPT violation exists. A net polarization of gamma-ray burst (GRB) can be measured to constrain CPT violation, since the net polarization diminishes significantly if the rotation angle is different by more than \( \pi/2 \) over an interval of energy. In this way, the strictest limit is given as \( |\zeta| \lesssim O(10^{-50}) \mathrm{m} \) for the same dispersion relation as Eq. (1).

In the pure gravity sector, the only existing limit on the dimension-5 CPT-violating operator was reported to be less than \( 2 \times 10^{-14} \mathrm{m} \), which was obtained by measuring the width of the peak at the maximal amplitude of GW150914[26]. However, we adopt one different approach based on Bayesian parameter estimation[14] and obtain stricter limits in this study. As mentioned above, due to CPT violation, the plus and cross modes of GWs rotate along the propagating direction. Instead of the net polarization, the knowledge of gravitational waveform is essential to our data analysis, since a method named as matched filtering[30] is used for extracting the signals. Fortunately, in general relativity (GR), one can predict exactly the coalescing process of a compact binary system as well as the waveform generated during the process[31]. We assume that the CPT-violating effect is minimal and it can be ignored at the source[3]. To detect possible deviations from CPT invariance, we still need knowledge of the CPT-violating propagation effect on gravitational waveform, which is done in Appendix A.

To be specific, the CPT-violating waveform \( h_{+\times} \) can be expressed as a rotation of the CPT-invariant waveform, namely,[12, 13]

\[
\begin{pmatrix}
    h_+ \\
    h_\times
\end{pmatrix}
= \begin{pmatrix}
    \cos(\delta \Psi) & -\sin(\delta \Psi) \\
    \sin(\delta \Psi) & \cos(\delta \Psi)
\end{pmatrix}
\begin{pmatrix}
    h_{+\mathrm{GR}} \\
    h_{\times\mathrm{GR}}
\end{pmatrix},
\]

(2)

where \( h_{+\times\mathrm{GR}} \) is the waveform predicted by GR and \( \delta \Psi(f; z, \zeta) \) is twice the rotation angle due to CPT violation. Such a rotation angle depends on the GW frequency \( f \) and a redshift of the source, i.e.,

\[
\delta \Psi = 4\pi^2 \zeta f^2 \int_0^z (1 + z')/H(z')dz',
\]

(3)

where \( z \) is the redshift of the source and \( H(z') \) is Hubble parameter at a redshift \( z' \). The integration over the redshift reveals that the CPT-violating effect accumulates with an increase
FIG. 1: Observational constraints on CPT-violating parameter $\zeta$ from the ten signals of BBHs in GWTC-1[1]. A capital H/L/V denotes a LIGO-Hanford/LIGO-Livingston/Virgo detector. A dot/square denotes HL/HLV. Blue/orange color denotes O1/O2. The error bars denote 90% confidence intervals.

of cosmological distance. The GR waveform is recovered when we take $\zeta = 0$.

For the first time, we perform Bayesian analysis software named as Bilby[32] to estimate the posterior probability distribution functions (PDFs) of $\zeta$ and fifteen binary parameters. In our data analysis, we consider the ten signals of BBH coalescences in GWTC-1[1]. Since the CPT violating effect is expected small, a uniform prior PDF of $\zeta$ is set as $[-4, 4] \times 10^{-14}$ m, which is proved to be wide enough for our purpose. Other independent parameters have prior PDFs matched with the reference[3]. Our parameter inference is illustrated in Appendix B.

The results of this study are demonstrated as follows. Figure 1 and Table I show the strict observational constraints on the CPT-violating parameter $\zeta$ at 90% confidence level from the ten signals of BBHs in GWTC-1. Since the constraints are well compatible with $\zeta = 0$, we find no substantial deviations from CPT invariance, indicating stringent upper limits on $|\zeta|$. Typically, we obtain $|\zeta| \lesssim \text{few} \times 10^{-15}$ m. The most stringent constraint is revealed as
TABLE I: Same caption as Figure 1. Typically, we report the stringent constraints as $|\zeta| \lesssim \text{few} \times 10^{-15} \text{m}$. The up-to-date strictest limit is given by GW151226[15]. The uncertainties denote 90% confidence intervals.

$\zeta = -0.09^{+0.34}_{-0.43} \times 10^{-14} \text{m}$, which is given by GW151226[15]. It is around five times better than the only existing limit $\lesssim 2 \times 10^{-14} \text{m}[10]$, which is given by GW150914[26]. Even for GW150914 itself, our constraint is still about three times better. Therefore, we obtain the up-to-date strictest limit on CPT violation in GWs. In addition, our results represent the first self-consistent Bayesian verification of CPT invariance in GWs.

It is interesting to qualitatively explore the sources of the observational uncertainties of $\zeta$. Naively speaking, we expect a better limit or smaller uncertainty from a more distant BBH, since the CPT-violating effect accumulates with the increase of cosmological distance according to Eq. 3. Indeed, it is roughly true for BBHs with the same chirp mass. However, a full story should consider the chirp mass, which determines a cutoff frequency of the signal. A BBH system with smaller chirp mass could generate on the detectors a temporally longer signal, which is important for an efficient extraction of the CPT-violating effect. The side effect is a smaller signal-to-noise (SNR)[30], leading larger uncertainties of other parameters. Therefore, the total uncertainty of $\zeta$ mainly originates from a balance between the chirp mass and cosmological distance, as well as the uncertainties of them.

In conclusion, we showed a systematic verification of CPT invariance in GWs which are
emitted from BBHs at cosmological distance. CPT violation induces the rotation of plus and cross modes of GWs. The measurements of GWs from BBHs provide a clean observational window to verify CPT invariance, since only different circular polarizations are involved. We performed the Bayesian parameter inference over the ten signals of BBHs in GWTC-1, but reported no substantial deviations from CPT invariance in GWs. The strictest limit on the CPT-violating parameter was obtained as $\zeta = 0.09^{+0.34}_{-0.43} \times 10^{-14}$m, which is around five times better than the only existing limit. Therefore, it stands for the up-to-date strictest test of CPT invariance in GWs. In addition, our study represents the first self-consistent Bayesian constraints on CPT violation in GWs, even though similar approaches have been used to study other modifications to GR[3]. In principle, we could also study higher-order CPT-violating effects on GWs in the same way. However, we expect significantly weaker limits on them[13], which are left to future works. Furthermore, we should note that a multi-band observation may improve our results significantly, since the CPT-violating effect is proportional to the square of GW frequency. We also expect CPT invariance to be further verified in the near future[13], since more and more BBHs will be detected by upcoming observing runs of LIGO and Virgo and other GW observatories under construction[33, 34].

Appendix A: Waveform.

Following[12, 13], we can evaluate the CPT-violating contribution to the gravitational waveform. The eigenstates are consist of two circular polarizations[12]. Based on Eq. 1, the phase speed of the left-handed circular polarization is given as $v_L \simeq 1 - \zeta \omega$. Up to first order, we obtain the gravitational strain as $h_L(t) \sim e^{-i\omega(t-l/v_L)} \simeq e^{i\zeta \omega^2 l} e^{-i\omega(t-l)}$, where $t$ and $l$ denote the time and distance to the source, respectively. Hence the phase is shifted by $\zeta \omega^2 l$. Due to expansion of the universe, we should take the redshift of the energy into account by considering an infinitesimal change in the phase, namely, $d\Psi_{\text{tot}} = d\Psi_{\text{GR}} + \zeta \omega^2 dl$. Upon an integration from the source to the detector, the former part gives the phase predicted by GR, while the later one gives the correction term due to modified dispersion, i.e., $\delta \Psi = \int \zeta \omega^2 dl$. We can replace $dl$ with $dt = -dz/[(1 + z)H(z)]$ at zeroth order and multiply $\omega$ by a factor $(1 + z)$. Here $H(z)$ denotes Hubble parameter at the redshift $z$. Therefore, we obtain a finite change in the phase, i.e., $\delta \Psi = 4\pi^2 \zeta f^2 \int_0^z (1 + z')/H(z')dz'$[12, 13], where $f = \omega/2\pi$ denotes the frequency in the observer frame. This is exactly Eq. 3. In a similar way, for the
right-handed circular polarization, the phase speed is given as \( v_R \approx 1 + \zeta \omega \) and the finite change in the phase becomes \(-\delta \Psi\).

Due to CPT violation, the gravitational waveform for circular polarizations is given as 

\[
h_{L,R} = h_{L,R}^{GR} e^{\pm i \delta \Psi},
\]

where \( h^{GR} \) denotes the waveform predicted by GR. The circular polarizations are usually decomposed into the plus and cross modes, namely, 

\[
h_{L,R} = h_{+} \pm i h_{\times}
\]

and 

\[
h_{L,R}^{GR} = h_{+}^{GR} \pm i h_{\times}^{GR}.
\]

Therefore, we can explicitly obtain the formula in Eq. 2 through a little algebraic operation\[12, 13\]. Furthermore, the gravitational strain on a given detector is 

\[
h = F_+ h_+ + F_\times h_\times[30],
\]

where \( F_+, \times \) denote a set of response pattern functions for the detector such as LIGO-Hanford, LIGO-Livingston and Virgo\[35\]. For \( h^{GR} \), we adopt the IMRPhenomPv2 waveform\[36, 37\] in this study.

**Appendix B: Parameter inference.**

The Bayesian inference tool, namely **Bilby**\[32\], is used to evaluate the posterior PDFs in parameter space. For a Gaussian noise, the log-likelihood is defined as\[14\]

\[
\log \mathcal{L} = \langle s, h(\theta) \rangle - \frac{1}{2} \langle h(\theta), h(\theta) \rangle,
\]

where \( s \) is a GW signal and \( h(\theta) \) is a template with a parameter space \( \theta \). An inner product is defined as 

\[
\langle a, b \rangle = 4 \Re [ \int_0^{\infty} a(f) b^*(f) / S_n(f) df ],
\]

where \( S_n(f) \) denotes a single-sided power spectral density (PSD) of detector noise and \( \Re \) means the real part. The template modified by CPT violation is given by Eq. 2 and Eq. 3. For each signal in GWTC-1\[1\], we analyze the data collected by detectors which responded to the signal. In addition, we use the noise PSDs of corresponding detectors\[35\]. Multiple detectors are assumed to have uncorrelated noise and their likelihoods should be multiplied together. The parameter space is consist of both binary and CPT-violating parameters, which are sampled over in our parameter inference.
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