Majority Rule for Belief Evolution in Social Networks

Abstract
In this paper, we study how an agent’s belief is affected by her neighbors in a social network. We first introduce a general framework, where every agent has an initial belief on a statement, and updates her belief according to her and her neighbors’ current beliefs under some belief evolution functions, which, arguably, should satisfy some basic properties. Then, we focus on the majority rule belief evolution function, that is, an agent will (dis)believe the statement iff more than half of her neighbors (dis)believe it. We consider some fundamental issues about majority rule belief evolution, for instance, whether the belief evolution process will eventually converge. The answer is no in general. However, for random asynchronous belief evolution, this is indeed the case.

1 Introduction
How agents change their beliefs is a fundamental problem in Artificial Intelligence. Traditionally in the area of Knowledge Representation and Reasoning, this problem is normally formalized in some logical formalism, e.g., propositional logic, and minimal change serves as the first principle [Alchourrón et al., 1985; Katsuno and Mendelzon, 1989].

In this paper, we consider this problem from a social aspect, that is, how an agent’s belief is affected by her neighbors in a social network. For instance, an agent might form a belief that the share price of IBM will increase tomorrow when she discussed this issue with her colleagues. However, she might also change this belief, i.e., to believe that the price will actually decrease, after convinced by her family members later on.

In fact, this aspect, also known as opinion formation and social learning, has been widely studied in other relevant fields such as mathematics and statistics [DeGroot, 1974; Holley and Liggett, 1975; Liggett, 1985; Krause, 2000], economics [Ellison and Fudenberg, 1993; DeMarzo et al., 2003; Sandholm, 2010; Acemoglu et al., 2010; Golub and Jackson, 2010], sociology [Friedkin and Johnsen, 2010; Hgeselmann and Krause, 2002], biology [Clifford and Sudbury, 1973] and so on. Recently, it has also attracted some attentions in theoretical computer science [Bindel et al., 2011]. However, as far as we know, it has been long neglected in the AI community.

Consider a multiagent context, where a social community is formed by some agents. This is usually modeled by a graph as a social network, where each node represents an agent and each edge represents a social tie between two agents. Now consider the agents’ beliefs about a particular statement, e.g., whether the share price of IBM will increase tomorrow. An agent may form a belief about this statement based on some observations and evidences, e.g., the financial situation of IBM recently and the trading volume of the stock today. However, her belief is also heavily affected by other agents, in particular, her friends’ opinions on this statement.

In this paper, we first introduce a general framework to model this phenomenon. We consider the agents’ beliefs about a particular statement in a social network. Initially, each agent has a prior belief, which might be formed based on her own experiences, observations and evidences. Then, the agents start to communicate each other synchronously or asynchronously, and update their beliefs according to their neighbors’ current beliefs in the social network under some belief evolution function. We argue that these belief evolution function, although can be defined in many different ways, should satisfy a list of basic properties.

Next, we focus on the majority rule as the belief evolution function. That is, an agent will (dis)believe the statement iff more than half of her neighbors (dis)believe it. Majority rule is one of the most natural and well studied function in related fields such as voting [Gaertner, 2009]. It does make sense under our context as well. For instance, in the IBM share price scenario, if a majority number of traders believe that the stock price will increase/decrease, this will likely be the case.

We investigate some fundamental properties about this function. Among them, a key issue is the convergence of the belief evolution process, i.e., will all evolution sequences be stable eventually. It can be observed that this is not the case in general. However, we show that, for random asynchronous belief evolution, this is indeed the case.
2 Belief Evolution in Social Networks

We consider a multiagent context. A society is formed by some agents that are connected. Formally, a social network \( N \) is a directed graph \((A,T)\), where \( A \) is a set of actors (also called individuals or agents) in a society, and \( T \subseteq A \times A \) is a set of dyadic ties (also called relationships or connections) among agents. We assume that \((a,a) \in T\) for all \( a \in A \) as an agent must know herself. We say that \( b \) is connected to \( a \) if \((b,a) \in T\), and connected by \( a \) if \((a,b) \in T\). For convenience, we simply use \( a \in N \) to denote \( a \in A \).

Now we consider the agents’ beliefs about a statement \( s \), for instance, whether the share price of IBM will increase tomorrow. The agents’ beliefs might be formed according to many different reasons. We group them into two categories: evidence based influences and communication based influences. The former includes some facts such as the financial situation of IBM in the last three months while the latter includes some facts such as the opinions of the agents’ friends on this statement.

In this paper, we separate the influences of these two categories into two steps. First, all the agents form an initial belief on the statement based on the evidence based influences. Then, the agents start to communicate each other to update their beliefs. We are mainly focused on the latter step, called belief evolution.

Once formed an initial belief, at a certain time point, all the agents’ beliefs on the statement can be viewed as a belief profile.

Definition 1 (Belief profile) Let \( N = (A,T) \) be a network, and \( s \) a statement. A belief profile \( P \) of \( N \) on \( s \) is a mapping from \( A \) to \{0,1\}, i.e., \( P : A \rightarrow \{0,1\} \).

Here, \( P(a), a \in A \) is the opinion of agent \( a \) on the statement \( s \). Particularly, \( P(a) = 1 \) means that the agent \( a \) believes \( s \), while \( P(a) = 0 \) means that \( a \) misbelieves \( s \), i.e., \( a \) believes \( \neg s \).

Given a statement \( s \) and a network \( N \), we say that a belief profile \( P \) of \( N \) on \( s \) is a consensus if all agents have the same belief, i.e., either for all \( a \in N \), \( P(a) = 1 \) or for all \( a \), \( P(a) = 0 \). We say that two profiles are isomorphic if there is a one-to-one correspondence between them. We say that a profile \( P \) is less or equal than another profile \( P' \), denoted by \( P \leq P' \), if for all \( a \in N \), \( P(a) \leq P'(a) \). We use \( \mathcal{P} \) to denote a new profile obtained from \( P \) by flipping over all beliefs, i.e., for all \( a \in N \), \( \mathcal{P}(a) = 1 - P(a) \).

In this paper, we only consider a single statement \( s \). This is because the influences of other statements on \( s \) are categorized as evidence based, and their influences are taken into account in the agents’ initial beliefs. Hence, we omit \( s \) in the belief profile \( P \) if it is clear from the context. Also, we assume that the network structure is fixed throughout the paper. Hence, we sometimes omit \( N \) in the belief profile \( P \) as well.

We can visualize a belief profile \( P \) as a labeled graph based on the network \( N \) with a label on each node, either 1 or 0, indicating this agent’s opinion on the statement.

![N_0](image)

Figure 1: Belief profiles of the network \( N_0 \)

Example 1 Figure 1 depicts four belief profiles of the same network \( N_0 \). Here, \( P_1 \) is a consensus while the rest are not; \( P_2 \) and \( P_3 \) are isomorphic; also \( P_2 = \mathcal{P}_3 \); \( P_4 \) is not comparable with \( P_2 \).

The belief profile defines all agents’ opinions on the statement at a certain time point. At this time point, the agents will communicate with other agents and reconsider their beliefs. The reconsideration is based on their own strategies.

Definition 2 (Belief evolution function) Let \( N = (A,T) \) be a social network, \( a \in A \) an agent and \( s \) a statement. A belief evolution function \( f \) of \( a \) on \( s \), denoted by \( f_a \), is a mapping \( f_a : \mathcal{P} \rightarrow \{0,1\} \), where \( \mathcal{P} \) is the set of all possible belief profiles of \( N \) on \( s \).

Intuitively, after taking into account the overall belief profile \( P \) on \( s \) in the network \( N \), the agent \( a \) decides to revise her belief to a new one \( f_a(P) \). For convenience, we use \( f_N \) to denote the collection of all belief evolution functions of agents in \( N \) and use \( f_N(P) \) to denote the belief profile obtained from \( P \) by applying \( f_N \) on all agents. In this sense, \( f_N \) updates a belief profile to a new one. Again, \( s \) might be omitted in \( f_a \) and \( f_N \).

The belief evolution function in Definition 2 can be defined arbitrarily. For example, a special one is so-called stubborn, that is, the agent never changes her belief. On the contrary, another function is that the agent always flips over her belief at every step. There are other possibilities, for instance, an agent will change her belief as soon as there are at least 3 of her friends taking an opposite opinion.

However, not every belief evolution function is rational, for instance, the one that always flipping over her beliefs. Of course, we are only interested in those rational ones. For this purpose, we propose a list of desirable properties. Let \( N = (A,T) \) be a network and \( a \in A \). We say that a belief evolution function \( f_a \) is

- **bounded** if \( \min \{P(b) \mid b \in N\} \leq f_a(P) \leq \max \{P(b) \mid b \in N\} \) for all profiles \( P \).
- **neutral** if \( f_a(\mathcal{P}) = 1 - f_a(P) \) for all profiles \( P \).
- **congruent** if \( f_a(P) = f_a(P') \) for any two isomorphic profiles \( P \) and \( P' \), where \( a \) and \( a' \) are corresponded.
- **local** if for any two profiles \( P \) and \( P' \) that agree \( s \) for all agents connected by \( a \) (including \( a \) herself), i.e., for all \( b \in N \) such that \((a,b) \in T\), \( P(b) = P'(b) \), we have \( f_a(P) = f_a(P') \).
- **monotonic** if \( P \leq P' \) implies that \( f_a(P) \leq f_a(P') \) for all profiles \( P \) and \( P' \).
non-slavish if there does not exist an agent \( b \) such that for all profiles \( P \), \( f_a(P) = P(b) \).

We say that the collection \( f_N \) of all believe evolution functions is bounded (neutral, congruent, local, monotonic and non-slavish) if for all \( a \in N \), \( f_a \) is bounded (neutral, congruent, local, monotonic and non-slavish).

We argue that a rational belief evolution function should satisfy these properties. Boundedness means that the agent will not change her belief if all agents reach a consensus. Neutrality means that the function will not be affected by how the statement is represented, for instance, from \( s \) to \( \neg s \). Congruency means that the function will not be affected by how the social network is represented. Locality means that the agent only has local information, that is, the agent is not able to get the beliefs of those agents not known by her. Monotonicity means that if an agent changes her belief to a new positive (negative resp.) one under a circumstance, then in another circumstance that is at least as positive (negative as this one, she will do the same thing. Finally, non-slavishness means that the agent cannot be dominated by a single agent in any circumstance. In particular, non-slavishness implies non-stubbornness.

Not every belief function satisfies these properties. For instance, the stubborn function satisfies all but not non-slavishness. The one that always flips over beliefs is neutral, congruent, local, non-slavish but does not satisfy the rest. The one that changes the belief iff 3 of her friends having an opposite opinion is bounded, congruent, local, neutral, non-slavish (if connected to at least 3 other agents) but not monotonic.

By applying the belief evolution functions, the belief profile of a network changes from one to another. We call this a belief evolution step. First, we consider the case that all agents have to reconsider their beliefs at every step, called synchronous belief evolution. Given an initial belief profile \( P^0 \) and the belief evolution function \( f_N \) for all agents in the network \( N \), the synchronous belief evolution will perform iteratively.

**Definition 3 (Synchronous belief evolution)** Let \( N = (A,T) \) be a network, and \( f_N \) a collection of belief evolution functions for all agents in \( N \). Let \( P^0 \) be an initial belief profile of \( N \) on a statement \( s \). The synchronous belief evolution for \( P^0 \) under \( f_N \) is a sequence \( \{P^0, \ldots, P^i, \ldots\} \) of belief profiles, where \( P^{i+1} \) is obtained from \( P^i \) by applying \( f_N \) as follows:

\[
\text{for any } a \in N, P^{i+1}(a) = f_a(P^i).
\]

We simply write \( P^{i+1} = f_N(P^i) \).

Also, we consider asynchronous belief evolution, in which not all agents are forced to reconsider their beliefs at a certain time point. The rationale for asynchronous belief evolution is twofold. First, different agents may communicate with their friends asynchronously due to, e.g., the frequency of contact and/or the strength of their friendship. Second, it is possible that an agent may sometimes stick on her own belief even she knows her friends’ opinions.

Let \( B \) be a subset of agents in a network \( N \) and \( P \) a belief profile of \( N \). The belief profile obtained from \( P \) by applying \( f_N \) on agents in \( B \), denoted by \( f_N^B(P) \), is

- \( f_N^B(P)(a) = f_a(P) \), for \( a \in B \).
- \( f_N^B(P)(a) = P(a) \), for \( a \notin B \).

**Definition 4 (Asynchronous belief evolution)** Let \( N = (A,T) \) be a network, and \( f_N \) a collection of belief evolution functions for all agents in \( N \). Let \( P^0 \) be an initial belief profile of \( N \) on a statement \( s \), and \( \sigma = B_1, \ldots, B_n, \ldots \) be a sequence of groups of agents. The asynchronous belief evolution for \( P^0 \) under \( f_N \) with respect to \( \sigma \) is a sequence \( \{P^0, \ldots, P^i, \ldots\} \) of belief profiles, where \( P^{i+1} = f_N^{B_i+1}(P^i) \).

Here, \( B_i \) means those agents who want to reconsider their beliefs at time point \( i \). In asynchronous belief evolution, \( P^{i+1} \) is obtained from \( P^i, f_N \) together with \( B_{i+1} \). Clearly, synchronous belief evolution can be regarded as a special case of asynchronous belief evolution by setting the sequence of groups of agents as \( A, A, \ldots, A, \ldots \).

We are mainly interested in the dynamics of the agents’ beliefs. In the framework, the agents will update their beliefs according to other agents’ opinions. A question is, will this evolution process stop, if yes, at what kind of belief profiles?

**Definition 5 (Equilibrium)** Let \( N \) be a network and \( f_N \) the belief evolution functions. We say that a belief profile \( P \) is an equilibrium under \( f_N \) if \( P = f_N(P) \).

**Proposition 1** That \( P \) is an equilibrium under \( f_N \) iff for any subset \( B \) of agents, \( P = f_N^B(P) \).

**Proof:** That \( P \) is an equilibrium under \( f_N \) iff for all agents \( a \), \( P(a) = f_a(P) \) iff for any subset \( B \) of agents, \( P = f_N^B(P) \). □

Proposition 1, although simple, shows that equilibrium in terms of synchronous belief evolution is the same as equilibrium in terms of asynchronous belief evolution. Obviously, if \( f_N \) is bounded, then the consensus profile must be an equilibrium under any evolution function \( f_N \). However, if \( f_N \) is not bounded, maybe there exists no equilibrium under \( f_N \) at all. A simple counterexample is that an agent always flips over her belief.

In this paper, our main concern is whether a belief evolution process will eventually terminate on an equilibrium.

**Definition 6 (Convergence)** Let \( N \) be a network, \( f_N \) the belief evolution functions, \( P^0 \) an initial belief profile of \( N \) and \( \sigma \) a sequence of groups of agents. The belief evolution for \( P^0 \) under \( f_N \) with respect to \( \sigma \) converges if there exists a number \( k \) such that \( P^k \) is an equilibrium of \( f_N \). In this case, \( P^k \) is called the convergence of this belief evolution process. For synchronous evolution, we simply say that the evolution process for \( P^0 \) under \( f_N \) converges.

It can be observed that for some \( f_N \), the evolution process will never converge, e.g., the one that an agent always flips over her beliefs. Of course, this is an extreme case as the belief evolution function is not rational.
3 Majority Rule for Belief Evolution

This section dedicates to a natural yet representative belief evolution function of the framework presented in the previous section, namely the majority rule function, originated from the majority rule for voting system [Gaertner, 2009]. For majority rule evolution function, an agent will change her belief iff a majority number of her friends (including herself) have an opposite opinion.

Definition 7 (Majority rule) Let \( N = \langle A, T \rangle \) be a social network and \( a \in A \) an agent. The majority rule belief evolution function \( m_a \) is defined as

\[
m_a(P) = \begin{cases} 
1 & \text{if } N^+(a, P) > N^-(a, P) \\
0 & \text{if } N^+(a, P) < N^-(a, P) \\
P(a) & \text{if } N^+(a, P) = N^-(a, P),
\end{cases}
\]

where \( P \) is a belief profile of \( N \), \( N^+(a, P) = |\{b | (a, b) \in T, P(b) = 1\}| \) and \( N^-(a, P) = |\{b | (a, b) \in T, P(b) = 0\}| \) respectively. We use \( m_N \) to denote the collection of all majority rule functions of agents in \( N \).

Here, \( N^+(a, P) (N^-(a, P)) \) is the number of agents connected by a and (dis)believing in the statement.

The majority rule function satisfies all desirable properties mentioned in the previous section.

Theorem 2 The majority rule belief evolution function is bounded, neutral, congruent, local, monotonic, and is non-slavish if every agent is connected to at least two other agents.

Proof: For space reasons, we only show that the majority rule function is monotonic here. Suppose that \( P \) and \( P' \) are two profiles such that \( P \leq P' \). Then, for agent \( a \) and every agent \( b \) (including \( a \)) connected by \( a \), i.e., \( (a, b) \in T \), we have \( P(b) \leq P'(b) \). There are three cases:

- \( m_a(P) = 0 \). In this case, \( m_a(P) \leq m_a(P') \).
- \( m_a(P) = 1 \) and \( N^-(a, P) = N^+(a, P) \). In this case, \( P(a) = 1 \) and \( N^-(a, P') \leq N^+(a, P') \). Therefore, \( P'(a) = 1 \) and \( N^-(a, P') \leq N^+(a, P') \). Hence, \( m_a(P') = 1 \) so that \( m_a(P) \leq m_a(P') \).
- \( m_a(P) = 1 \) and \( N^-(a, P) < N^+(a, P) \). In this case, \( N^-(a, P') \leq N^-(a, P) < N^+(a, P') \). Hence, \( m_a(P') = 1 \) so that \( m_a(P) \leq m_a(P') \).

We apply the majority rule function for belief evolution. We are mainly interested in some fundamental issues related to equilibrium, convergence and consensus. For instance, given a network, what are the equilibria under the majority rule function? Does every evolution sequence converge for any initial belief profile? Can the consensus be reached eventually?

First of all, let us consider some examples for synchronous belief evolution.

Example 2 Figure 2 depicts the synchronous belief evolution processes under the majority rule function for two different instances, where the agents and their initial beliefs are the same, but \( N_2 \) has an extra edge than \( N_1 \). Both evolution processes converge after several steps.

Example 3 Figure 3 depicts the synchronous belief evolution process for \( P_2 \) in Example 1 under \( m_{N_0} \), which falls into a loop \( P_2 \rightarrow P_3 \rightarrow P_2 \ldots \rightarrow P_3 \rightarrow P_2 \ldots \).

Example 4 Figure 4 depicts three different asynchronous evolution sequences for \( P_2 \) under \( m_{N_0} \). The

Figure 2: An example of synchronous belief evolution

Figure 3: Synchronous belief evolution for \( P_2 \)
agents who evolved their beliefs at the previous round are shadowed. In this example, all evolution processes converge.

Compared to the synchronous belief evolution for $P_2$ in Example 3, there exists an (actually many) asynchronous evolution sequence that converges. However, different asynchronous evolution sequences may lead to exactly opposite results, for instance, the first and the last sequences in Example 4.

The following theorem shows that there always exists a converging sequence for any initial belief profile under the majority rule function.

**Theorem 3** Let $N = (A, T)$ be a finite social network. For any belief profile $P$ of $N$, there exists a sequence $σ$ of groups of agents such that the asynchronous belief evolution for $P$ under $m_N$ w.r.t. $σ$ converges.

**Proof:** We prove a stronger result that this theorem holds under any monotonic belief evolution function $f_N$. We directly construct such a sequence, which is divided into two phrases as follows.

**Increasing phrase** At each round in this phrase, we flip over those negative beliefs to positive ones (i.e. from 0 to 1) whenever possible. Multiple rounds might be needed. Let $P'$ be the final belief profile obtained in this phrase. Clearly, $P \leq P'$.

**Decreasing phrase** On the contrary, at each round in this phrase, we flip over those positive beliefs to negative ones (i.e. from 1 to 0) whenever possible. Similarly, multiple rounds might be needed. Let $P''$ be the final belief profile obtained in this phrase. Clearly, $P'' \leq P'$.

We prove that $P''$ must be an equilibrium by contradiction. Assume that there exists $a \in N$ such that $f_a(P''(a)) \neq P''(a)$. Then, $P''(a) = 0$ and $f_a(P''(a)) = 1$ according to the construction of the decreasing phrase. Otherwise, $a$ will be further selected in the decreasing phrase so that $P''$ is not the final profile obtained, a contradiction. There are two cases:

- $P'(a) = 1$. In this case, $a$ must be selected at some round in the decreasing phrase to flip over from 1 to 0. Therefore, there exists $P^*$ in the decreasing phrase such that $P^*(a) = 1$ but $f_a(P^*) = 0$. Clearly, $P'' \leq P^* \leq P'$. Therefore, $1 = f_a(P'') \leq f_a(P^*) = 0$, a contradiction.

- $P'(a) = 0$. In this case, $f_a(P'') = 1$ since $f_a(P'') = 1$ and $P'' \leq P'$, which contradicts to our construction that $P'$ is the final profile obtained in the increasing phrase.

This completes our proof. □

However, the existence of converging asynchronous evolution does not mean this convergence will eventually be reached. Firstly, the agents themselves do not know which sequence will lead to a convergence because they do not have global information. Secondly, even if they know a converging sequence, perhaps they are not cooperative enough to follow it.

Let us go back to the two major reasons for considering asynchronous belief evolution. First, different agents may have different frequency of communication with their friends. Second, agents might be sometimes over-confident even if they know their friends’ objections. We can use a random variable to simulate both cases.

This motivates us to consider random asynchronous belief evolution, in which the agents randomly evolve their beliefs at each round. More precisely, each agent is associated with a random Boolean variable to decide whether or not she will evolve her belief at a certain time point. The value of the random variable will be determined according to a probability distribution. For instance, at the current stage, agent $a$ might have 0.8 chance of evolving her belief while agent $b$ might only have 0.4. Hence, at a certain time point, a subset of agents will be generated according to the random variables, which is the set of agents who will evolve their beliefs at the current stage. In this sense, random asynchronous belief evolution can be regarded as a statistical process of asynchronous evolution. We argue that random asynchronous belief evolution is more realistic than synchronous belief evolution and asynchronous belief evolution based on intentionally selected sequences.

The following theorem shows that random asynchronous belief evolution will eventually converge.

**Theorem 4** For any finite social network and initial belief profile, random asynchronous belief evolution under the majority rule function always converges.

**Proof:** To prove this, we need to introduce a notion called belief profile transition graph. Let $N$ be a social network and $f_N$ the belief evolution functions for agents in $N$. The belief profile transition graph for $f_N$ is an edge labeled graph $(P, T)$, where $P$ is the set of all belief profiles of $N$, and $T$ is the set of transitions among profiles. Each edge is labeled with a subset of agents in $N$. For two profiles $P, P'$ and a subset $B$ of agents, $(P, P') \in T$ labeled by $B$ iff $P' = f^B_N(P)$, i.e., $P'$ is the belief profile obtained from $P$ by applying $f_N$ on agents in $B$. 
Now we consider the strongly connected components of the belief profile transition graph for the majority rule function. First of all, each equilibrium itself forms a strongly connected component. Consider the condensation of the transition graph, which is a directed acyclic graph. On the one hand, by Theorem 3 a node in the condensation is a leaf (i.e. no outgoing edge) iff it is an equilibrium under the majority rule function. On the other hand, if the network is finite, then for any initial belief profile, random asynchronous belief evolution will eventually lead to a leaf in the condensation because any subset of agents might be generated by randomness for any profile. This shows that random asynchronous belief evolution eventually converges. □

To end up this section, we summarize the main observations and results for belief evolution under the majority rule function.

- The network structure is critical for belief evolution.
- For some networks, there exists non-consensus equilibrium. As a consequence, consensus may never be reached in belief evolution.
- For synchronous belief evolution, the evolution process may never converge for some initial belief profiles.
- The majority opinion of the convergence might not be the same as the one of the initial profile.
- For asynchronous belief evolution, there always exists an evolution sequence leading to a convergence for any initial belief profile. In some cases, there might exist many, and the resulting final equilibria could be quite different.
- Random asynchronous belief evolution processes always converge.

4 Related Work

Belief and opinion dynamics in social network is widely studied in related fields. Several rule of thumbs models are proposed in the literature [DeGroot, 1974; Friedkin and Johnsen, 2010; Krause, 2000; Hegselmann and Krause, 2002; DeMarzo et al., 2003; Ellison and Fudenberg, 1998; Golub and Jackson, 2010; Acemoglu et al., 2010]. In DeGroot’s [1974] seminal work, each agent has an initial opinion (a continuous value), and iteratively evolves her belief by taking a weighted sum of her neighbors’ opinions. Our framework shares some similar ideas but differs from it on the following aspects. First, we intend to propose a framework rather than a model, in which the belief evolution function can be defined arbitrarily under some restrictions. Yet we also consider a representative model of this framework. Second, in our framework, the value is discrete rather than continuous because we are concerned with beliefs rather than opinions. Finally, in DeGroot’s model, the agents evolve their beliefs simultaneously, while we consider both synchronous and asynchronous belief evolutions.

Another highly related work is from the area of statistical mechanics, particularly interacting particle systems [Clifford and Sudbury, 1973; Holley and Liggett, 1975]. In the voter model, each agent has an initial belief (either true or false) and randomly evolves her belief according to her neighbors’ beliefs under a transition function, which should satisfy some properties as well. However, the transition function in the voter model calculates a probability rate, based on which the agent will flip over her beliefs. In contrast, the belief evolution function in our framework directly calculates the value of the belief.

The majority rule evolution function is named from the same well known approach in voting system [Glauber, 1969]. Generally speaking, belief evolution can be considered as voting in a social network for two opposite candidates. However, in belief evolution, the majority rule voting is performed locally, individually, distributively and iteratively, while in voting system, it is performed globally, wholly, centralizedly and only once.

There are other related works, actually from several different disciplines. For instance, an alternative model of opinion formation is to take just a single (but not all) friend’s opinion into account based on their contact frequency [Acemoglu et al., 2010]. Another interesting approach, called replicator dynamics [Sandholm, 2010], takes the historical performance of beliefs into account, and the belief that performs better in the past will be more likely replicated. However, for space reasons, we are not able to discuss all of them in details.

5 Conclusion

In this paper, we considered the problem of belief evolution in social networks. To sum up, the main contributions are as follows.

- We introduced a general framework for belief evolution in social networks. In this framework, the agents form an initial belief profile on a statement based on evidences and observations, and then start to communicate each other in the social network to update their beliefs according to their belief evolution functions. The belief evolution process is performed iteratively, either synchronously or asynchronously.
- We argued that a rational belief evolution function should satisfy some desirable properties such as boundedness and monotonicity.
- We focused on the majority rule belief evolution function, which satisfies all properties mentioned above. The main discoveries for majority rule belief evolution are summarized in the end of Section 3.
- In particular, we focused on the convergence problem. For synchronous belief evolution, the process may never terminate. For asynchronous belief evolution, we show that there always exists a converging evolution sequence for any initial profile. More interestingly, random asynchronous belief evolution, arguably corresponding to belief evolution in the reality, always converges.
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