Balitsky’s hierarchy from Mueller’s dipole model and more about target correlations.

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Abstract

High energy scattering formulated as a classical branching process is considered within the framework of the QCD dipole model. Starting from Mueller’s generating functional, we derive the high energy evolution law for the scattering amplitude. The amplitude’s evolution is given by an infinite hierarchy of linear equations equivalent to the Balitsky’s chain reduced to dipole operators. This new derivation of the hierarchy is the central result of the paper. We also comment about target correlations which prevent the hierarchy from being expressed as the Balitsky-Kovchegov equation in closed form.

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1 Introduction

The high energy scattering in QCD can be most efficiently addressed in terms of colour dipole degrees of freedom. In this approach, originally proposed by Mueller [1], one considers a fast moving projectile as a bunch of dipoles created via a classical branching process. High energy evolution of the projectile wavefunction can be then found from the QCD generating functional [1], which obeys a linear functional evolution equation [2]. Though the evolution of the projectile wavefunction is independent of target, the interaction amplitude depends on it.

The energy evolution of the interaction amplitude will be the central object studied in the present paper. It was shown by Kovchegov [3], that if one assumes that all the dipoles produced via evolution interact with the target independently of each other, then the scattering amplitude obeys a nonlinear evolution equation presently referred to as the Balitsky-Kovchegov (BK) equation [4, 3]. The BK equation has been the core of numerous analytical [5] and numerical [6] studies. It has also been applied for phenomenology [7].

It was realised in Ref. [2], that the assumption regarding independent interactions can be incorrect for many targets, and in particular for realistic nuclei which are not very dense. Target correlations are needed in order to account for more realistic structure of targets. It was shown in Ref. [2] that the amplitude obeys a linear functional evolution equation, which is the most appropriate tool for the introduction of target correlations. In general, the linear functional equation for the amplitude with target correlations cannot be represented as an ordinary equation and preserves its functional form. Our goal here is to show that the linear functional evolution equation for the interaction amplitude for a generic target with correlations is equivalent to an infinite chain of linear hierarchy equations. This chain is the hierarchy of Balitsky [4] reduced to dipole operators only \(^1\). Thus we clarify the connection between the Wilson loop approach of Balitsky and the Mueller dipole model. We also demonstrate that the large \(N_c\) limit, which is in the foundation of the dipole model, is insufficient to guarantee a close form equation for the scattering amplitude. This new derivation of the Balitsky’s chain should be view as our main result.

We also demonstrate a possibility to rigorously derive an evolution for the amplitude without restricting ourselves to a single dipole as a specific choice of projectile. Our results are valid for any projectile whose evolution is not affected by high density effects.

At the end of the paper we discuss an ansatz for target correlations which allows one to obtain a certain generalisation of the BK equation. This part is very much along the lines of Ref. [2]. Similar result has been recently reported by Janik [9].

One of our central goals in this letter is to draw more attention to Ref. [2] in which the linear functional formalism was developed as well as many other results appearing in this reference.

\(^1\)The true Balitsky’s hierarchy, as well as its equivalent, the JIMWLK equation [8], also contain non-dipole operators, which account for \(N_c\) corrections. In the present paper we have nothing to say about \(N_c\) corrections which will be systematically ignored.
2 Balitsky’s chain from the QCD generating functional

We first consider a generic fast moving projectile whose wave function can be expanded in a dipole basis. Contrary to many previous studies we do not restrict ourselves to a single dipole as a projectile. We further assume that non-linear effects associated with high dipole densities in the projectile wave function can be ignored. This is a strong assumption which eliminates effects which might be associated with pomeron loops, so called enhanced diagrams.

Let us define a probability density \( P_n \) to find \( n \) dipoles with coordinates \( r_1, b_1, r_2, b_2, \ldots, r_i, b_i, \ldots, r_n, b_n \) and rapidity \( Y \) in the projectile wave function. \( r_i \) and \( b_i \) denote the dipole’s size and impact parameter, both are two dimensional vectors. We define \( P_n \) as a dimensionfull quantity.

The equation for \( P_n \) obeys the classical branching process:

\[
\frac{\partial P_n}{\partial Y} (Y - Y_0; r_1, b_1, r_2, b_2, \ldots, r_n, b_n) = - \sum_{i=1}^{n} \omega(r_i) P_n (Y - Y_0; r_1, b_1, r_2, b_2, \ldots, r_n, b_n)
+ \sum_{i=1}^{n-1} \frac{(r_i + r_n)^2}{(2 \pi)^2 r_i^2 r_n^2} P_{n-1} (Y - Y_0; r_1, b_1, r_2, b_2, \ldots, (r_i + r_n), b_{in}, \ldots, r_{n-1}, b_{n-1})
\]

(2.1)

with \( b_{in} = b_i + r_n/2 = b_n - r_i/2 \) being imposed via \( \delta \)-functions. Two terms of Eq. (2.1) have a very simple meaning: the first one describes the decrease in probability to find \( n \) dipoles due to a decay of one dipole into two of arbitrary sizes. This probability is equal to

\[
\tilde{\alpha}_s \omega(r_i) = \frac{\tilde{\alpha}_s}{2 \pi} \int_\rho \frac{r_i^2}{(r_i - r')^2 r^2} d^2 r' = \tilde{\alpha}_s \ln(r_i^2/\rho^2)
\]

with \( \rho \) being some infrared cutoff, and \( \tilde{\alpha}_s = \alpha_S N_c/\pi \). The second term shows the increase in probability to find \( n \) dipoles due to a creation of a new dipole from \( n - 1 \) dipoles with probability

\[
\frac{\tilde{\alpha}_s}{2 \pi} \frac{(r_1 + r_2)^2}{r_1^2 r_2^2}
\]

Eq. (2.1) has to be supplemented by initial conditions at \( Y = Y_0 \), specifying a dipole distribution in the projectile. In writing the equation (2.1) we explicitly used our assumption that there are no nonlinear effects in the projectile. Otherwise, we would need to include dipole recombination processes.

The hierarchy (2.1) can be resolved by introducing a generating functional \( Z \)

\[
Z (Y - Y_0; [u]) \equiv \sum_{n=1} P_n (Y - Y_0; r_1, b_1, r_2, b_2, \ldots, r_i, b_i, \ldots, r_n, b_n) \prod_{i=1}^{n} u(r_i, b_i) d^2 r_i d^2 b_i
\]

(2.2)

where \( u(r_i, b_i) \equiv u_i \) is an arbitrary function of \( r_i \) and \( b_i \). It follows immediately from (2.1) that the functional (2.2) obeys the condition: at \( u = 1 \)

\[
Z (Y - Y_0; [u = 1]) = 1.
\]

(2.3)
The physical meaning of (2.3) is that the sum over all probabilities is one.

Multiplying Eq. (2.1) by the product \( \prod_{i=1}^{n} u_i \) and integrating over all \( r_i \) and \( b_i \), we obtain the following linear equation for the generating functional:

\[
\frac{\partial Z}{\partial s} Y = - \int d^2r d^2b \ V_{1\rightarrow 1}(r, b, [u]) \ Z + \int d^2r d^2r' d^2b \ V_{1\rightarrow 2}(r, r', b, [u]) \ Z . \tag{2.4}
\]

with the definitions

\[
V_{1\rightarrow 1}(r, b, [u]) = \tilde{\alpha}_s \ \omega(r) \ u(r, b) \ \frac{\delta}{\delta u(r, b)}
\]

and

\[
V_{1\rightarrow 2}(r, r', b, [u]) = \frac{\tilde{\alpha}_s}{2\pi} \ \frac{r^2}{r'^2 (r - r')^2} \ u(r', b + \frac{r - r'}{2}) \ u(r - r', b + \frac{r'}{2}) \ \frac{\delta}{\delta u(r, b)} . \tag{2.6}
\]

The functional derivative with respect to \( u(r, b) \), plays the role of an annihilation operator for a dipole of the size \( r \), at the impact parameter \( b \). The multiplication by \( u(r, b) \) corresponds to a creation operator for this dipole. Eq. (2.1) has an extra \( b \) integration compared to the one first derived in Ref. [2]. Motivated by the fact that evolution kernels do not depend on the impact parameter, we ignored the \( b \)-dependence in Ref. [2] in order to simplify the presentation. This procedure also corresponds to an approximation in which the impact parameter is considered to be much larger than any dipole size. The correct analysis has to preserve the true kinematics and this amounts to tracing the \( b \)-dependence which is done in Eq. (2.4). Eq. (2.4) can be also obtained starting from the JIMWLK equation [9].

The \( n \)-dipole densities in the projectile \( \rho^p_n(r_1, b_1, \ldots, r_n, b_n) \) are defined as follows:

\[
\rho^p_n(r_1, b_1, \ldots, r_n, b_n; Y - Y_0) = \frac{1}{n!} \ \prod_{i=1}^{n} \ \frac{\delta}{\delta u_i} Z (Y - Y_0; [u])|_{u=1}
\]

One can recast the chain of equations (2.1) into hierarchy for \( \rho^p_n \):

\[
\frac{\partial \rho^p_n(r_1, b_1, \ldots, r_n, b_n)}{\partial s} Y = \left( - \sum_{i=1}^{n} \ \omega(r_i) \ \rho^p_n(r_1, b_1, \ldots, r_n, b_n) + \right) \ \\
2 \ \sum_{i=1}^{n} \ \int \frac{d^2r'}{2\pi} \ \frac{r'^2}{r_i^2 (r_i - r')^2} \ \rho^p_n(\ldots r', b_i - r'/2 \ldots) + \ \sum_{i=1}^{n-1} \ \frac{(r_i + r_n)^2}{(2\pi)^4 r^2_i r^2_n} \ \rho^p_{n-1}(\ldots (r_i + r_n), b_{i+1} \ldots) \right) . \tag{2.7}
\]

The evolution for \( \rho^1_1 \) was considered by Mueller in his original work [1]. It obeys the linear BFKL equation [10]. The \( n \)-dipole densities with \( n > 1 \), are necessary generated through the evolution even if the projectile is a single dipole. For this specific choice of the projectile, our linear equation, say, for \( \rho^p_2 \) can be identically reformulated as a nonlinear with respect to \( \rho^p_1 \). This form of the equation for \( \rho^p_2 \) was studied in Refs. [1, 11]. We would like to stress, however, that Eq. (2.7) is more general, as it does not depend on any specific choice of projectile. Our approach cannot be rigorously applied to a hadron or nucleus as a projectile, as the latter need to account for high
dipole densities, in spite of this we view our finding as a first important step towards a symmetric description of target and projectile.

So far we have discussed the evolution of the projectile only. Our goal is to compute the total scattering amplitude. Following Ref. [3] the amplitude $N$ is defined

$$N(Y) = \sum_{n=1}^{\infty} (-1)^n \int \gamma_n(r_1, b_1, \ldots, r_n, b_n; Y_0) \rho_n^t(r_1, b_1, \ldots, r_n, b_n; Y - Y_0) \prod_{i=1}^{n} d^2 r_i d^2 b_i .$$

(2.8)

The amplitude for simultaneous scattering of $n$ dipoles off the target is denoted by $\gamma_n$. It has to be specified at the lowest rapidity $(Y_0)$. $\gamma_n$ can be expressed through $\rho_n^t$, the dipole densities in the target:

$$\gamma_n(r_1, b_1, \ldots, r_n, b_n; Y_0) = \sum_{m=1}^{\infty} \int \sigma^{nm}(r_1, b_1, \ldots, r_n, b_n | \bar{r}_1, \bar{b}_1, \ldots, \bar{r}_m, \bar{b}_m) \rho_n^t(\bar{r}_1, \bar{b}_1, \ldots, \bar{r}_m, \bar{b}_m; Y_0) d^2 \bar{r}_1 d^2 \bar{b}_1 \ldots d^2 \bar{r}_m d^2 \bar{b}_m ,$$

(2.9)

with $\sigma^{nm}$ being the properly normalised amplitude for $n$ dipoles scattering off $m$ dipoles, at the two gluon exchange level for each pair of dipoles.

While the generating functional $Z$ contains information about the projectile only, the target enters the amplitude through the functions $\gamma_n (\rho_n^t)$. Substituting (2.9) into (2.8) we obtain:

$$N(Y) = \sum_{n, m=1}^{\infty} (-1)^n \int \rho_n^t(\bar{r}_1, \bar{b}_1, \ldots, \bar{r}_n, \bar{b}_n; Y_0) \rho_n^p(r_1, b_1, \ldots, r_n, b_n; Y - Y_0) \prod_{i=1}^{n} d^2 r_i d^2 b_i \prod_{j=1}^{m} d^2 \bar{r}_j d^2 \bar{b}_j .$$

(2.10)

Eq. (2.10) appears symmetric with respect to the exchange of the target and projectile. In fact this symmetry is illusive. As soon as we require $\rho_n^p$ to obey the evolution given by Eq. (2.7) the boost invariance would impose a different evolution law (which we will not be able to write explicitly) for $\rho_n^t$. We would like to emphasise the importance of the summation over $n$. It guarantees the unitarity, and boost invariance of the amplitude. It is challenging to include simultaneously high dipole density effects, in both the evolution of target and projectile. Eq. (2.10) would then account for all the effects under discussion in Ref. [13].

If $Y_0 = 0$ Eq. (2.8) yields the expression for the amplitude in which the whole evolution is through the projectile wavefunction. The dipoles produced via the evolution then multiple rescatter off the target resulting in the unitarity respecting amplitude. The unitarisation is achieved without saturation, as no high density effects are accounted by the evolution of the projectile.

Alternatively we can take $Y_0 = Y$. This would bring the whole evolution into the target averaged amplitudes. By requiring the total amplitude $N$ to be independent of our choice of frame we will be able to deduce the evolution law for $\gamma_n(Y)$. As can be expected $\gamma_n$ happens to obey the Balitsky’s chain of hierarchy equations [4] projected onto dipole operators.
We now derive the evolution of $\gamma_n$. We note that

$$\rho_n^p = -(-1)^n \frac{\delta N}{\delta \gamma_n}$$

We obtain the linear functional equation for the amplitude $N$:

$$\frac{\partial N}{\partial Y} = \sum_{n=1}^{\infty} \int \frac{d^2 r_i d^2 b_i}{2 \pi} \frac{r_i^2}{r_i^2 (r_i - r')^2} \frac{\delta N}{\delta \gamma_n} - \frac{n-1}{\sum_{i=1}^{n-1} \frac{(r_i + r_n)^2}{(2 \pi) r_i^2 r_n^2} \frac{\delta N}{\delta \gamma_{n-1}}}.$$  \hspace{1cm} (2.11)

The equation (2.11) can be solved using the ansatz: $N(Y, [\gamma]) = N(\gamma_1(Y), \gamma_2(Y) \ldots)$ such that

$$\frac{\delta N}{\delta Y} = \frac{\delta N}{\delta \gamma_n} \frac{\partial \gamma_n}{\partial Y} \prod_{i=1}^{n} d^2 r_i d^2 b_i.$$  

As all $\gamma_n$ are independent of each other, Eq. (2.11) is equivalent to Balitsky’s hierarchy (restricted to dipole operators):  \hspace{1cm} (2.12)

$$\frac{\partial \gamma_n(r_1, b_1, \ldots, r_n, b_n; Y)}{\partial Y} = -\sum_{i=1}^{n} \omega(r_i) \gamma_n(r_1, b_1, \ldots, r_n, b_n) +$$

$$\sum_{i=1}^{n} \int \frac{d^2 r'}{2 \pi} \frac{r_i^2}{r_i^2 (r_i - r')^2} \left[ 2 \gamma_n(r_1, b_1, \ldots, r', b_i + \frac{(r_i - r')}{2}, \ldots, r_n, b_n) \right]$$

$$- \gamma_{n+1}(r_1, b_1, \ldots, r', b_i + \frac{(r_i - r')}{2}, \ldots, r_i - r', b_i + \frac{r'}{2}) \right].$$

The initial conditions for the evolution chain (2.12) are given by a target model which is supposed to provide $\gamma_n(Y = Y_0)$. All $\gamma_n$ are defined as scattering amplitudes and hence are target averaged quantities. In principal, Eq. (2.12) governs the evolution of the target densities $\rho_n^p$. Though we are not able to write an explicit equation for $\rho_n^p$, it apparently differs from the evolution of $\rho_n^p$ (Eq. (2.11)).

The amplitude $N$ can be rewritten in the following form

$$N(Y) = -\sum_{n=1}^{\infty} (-1)^n \int \gamma_n(r_1, b_1, \ldots, r_n, b_n; Y - Y_0) \rho_n^p(r_1, b_1, \ldots, r_n, b_n; Y_0) \prod_{i=1}^{n} d^2 r_i d^2 b_i.$$  

The $Y$ evolution has been transferred from the evolution in the projectile wavefunction ($\rho_n^p$) to the evolution of the target averaged amplitudes ($\gamma_n$). Note that by construction $N(Y)$ is boost or frame invariant. In other words, Eq. (2.12) can be obtained just by requiring $N(Y)$ to be $Y_0$ independent. The functions $\rho_n^p(Y = Y_0)$ have to be provided by the initial dipole distribution in the projectile. So far, our treatment was general. Note, however, the asymmetry between the target and projectile, as for the latter no high density effects are included. This asymmetry is reflected by the fact that $\rho_n^p$ and $\gamma_n$ obey different hierarchy equations.
We would like to emphasise that so far our discussion was quite general and valid for any projectile. Our results can be trusted until we violate our central assumption, namely the absence of nonlinear effects in the projectile wave function.

We now make a choice of a projectile. Consider the canonical case of a dipole of the size $r$ scattering off a generic target at the impact parameter $b$. Then the initial condition in the projectile wave function at $Y = Y_0$ is $\rho^p_{1} = P_{n=1} = \delta^2(r - r_1) \delta^2(b - b_1)$ while $\rho^p_{n>1} = P_{n>1} = 0$. This choice of the projectile initial conditions implies

$$Z(Y = Y_0, r, b; [u]) = u(r, b). \tag{2.13}$$

For the total amplitude $N$ we immediately obtain

$$N(Y; r, b) = \gamma_1(r, b; Y)$$

For a generic target we cannot write down a closed form equation for $N$. It was shown in Ref. [2] that for this particular choice of the projectile, Eq. (2.4) can be rewritten in the nonfunctional but non-linear form, reproducing the same equation for $Z$ as in Ref. [1]:

$$\frac{\partial Z(Y, r, b; \gamma)}{\bar{\alpha}_s \partial Y} = -\omega(r) Z(Y; r, b; [u]) \tag{2.14}$$

$$+ \int \frac{d^2 r'}{2 \pi} \frac{r'^2}{r^2 (r - r')^2} Z(Y; r', b + \frac{(r - r')}{2}; [u]) Z(Y; (r - r'), b - \frac{r'}{2}; [u]).$$

The linear functional equation (2.14) is more general, however, as it is valid for any projectile.

### 3 Target correlations

Target correlations are defined by having

$$\gamma_n(r_1, b_1, \ldots r_n, b_n; Y = Y_0) = C_n(r_1, b_1 \ldots r_n, b_n) \gamma(r_1, b_1) \ldots \gamma(r_n, b_n)$$

The coefficients $C_n$ are $n$-dipole correlation parameters. They can be also viewed as effective measures of target fluctuations. If we assume $C_n$ be pure numbers independent of coordinates, then Eq. (2.11) is reduced to the following functional equation [2]

$$\frac{\partial N(Y; \gamma)}{\bar{\alpha}_s \partial Y} = -\int d^2 r' V_{1\rightarrow1}(r'; [\gamma(r')]) N(Y; \gamma) \tag{3.15}$$

$$+ \frac{2}{\pi} \int d^2 r' d^2 r^{''} \gamma(r^{''}) \frac{r'^2}{r^2 (r - r')^2} \frac{\delta}{\delta \gamma(r')} N(Y; \gamma)$$

$$+ F \left( \int d^2 \bar{r} \gamma(\bar{r}) \frac{\delta}{\delta \gamma(\bar{r})} \right) \int d^2 r'' d^2 r' V_{1\rightarrow2}(r', r'', [\gamma(r')]) N(Y; \gamma)$$

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Here $F(n) \equiv C_n/C_{n-1}$. If we further simplify the correlations assuming $F$ is constant

$$C_n = F^{n-1} C_1$$

then Eq. (3.15) can be easily reduced to a non-functional non-linear equation (ignoring the $b$ dependence):

$$\frac{\partial N(r, Y)}{\partial Y} = \bar{\alpha}_S \times \int_{\rho_p} \frac{d^2 r'}{2\pi} \frac{r^2}{r'^2 (r - r')^2} \times

\left[ 2 N(r', Y) - N(r, Y) - \frac{F}{C_1} N(r', Y) N(r - r', Y) \right],$$

The initial condition is $N(Y = Y_0) = C_1 \gamma$. Note that unless target properties are specified, $F$ and $C_1$ are arbitrary numbers ($F \geq C_1$).

Our solution first obtained in Ref. [2], proves that under condition (3.16) the hierarchy (2.12) shrinks to a single nonlinear equation (3.17). An identical result has been obtained recently in Ref. [9] by a direct analysis of the hierarchy equations, whereas the result $F/C_1 = 2$ of Ref. [12] is a particular solution.

The new aspect which we find as most important, is the relation between the target correlations and Balitsky's hierarchy, something which we were not aware of when preparing Ref. [2]. Eq. (3.17) which we derived was viewed as a phenomenologically motivated ad hoc modification of the BK equation. By establishing in this letter a relation between the correlations and hierarchy, we attempt to correct the wrong attitude regarding our original work.

If we consider dipole interaction as fully uncorrelated $F = C_1 = 1$ then Eq. (3.17) reduces to the BK equation. Its solution can also be written in the following form

$$N(Y, r, b) = - \sum_{n=1}^{\infty} (-1)^n \rho_n^p(r_1, b_1, \ldots r_n, b_n; Y - Y_0) \prod_{i=1}^{n} N(Y_0, r_i, b_i) \ d^2 r_i \ d^2 b_i.$$  

So far the approximation $F = \text{const}$ looks pure mathematical. In Ref. [2], a realistic model for target correlations was proposed. In that model the target was considered to be a nucleus (not necessary very heavy) but the approach can be viewed more generally. The target correlations were estimated based on pure counting arguments independent of the dipole coordinates. The model of Ref [2] leads to a constant $F$, providing a physically intuitive realisation of a more formal structure outlined above.

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