The Application of Improved Bayesian Method in Reliability Evaluation

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Abstract. Aiming at the problem that the traditional reliability method with small failure samples cannot accurately calculate the reliability, this paper makes full use of existing information and expert information, etc. Based on the combination of prior information and constrained no prior information, the improved Bayesian method is used to establish reliability. Sexual analysis model. First, a weighted fusion method is used to establish a joint prior distribution that combines prior information with constrained and no prior information according to the credibility of the prior information to the posterior information. At the same time, the robustness of the joint prior distribution is analyzed; The prior distribution is combined with the posterior information to calculate the posterior distribution form of the distribution parameter, and the distribution parameter value and the mean time between failures are calculated.

Keywords: hydraulic system, prior information, improved bayesian, joint prior distribution, MTBF.

1. Introduction
Hydraulic transmission (including oil hydraulic and water hydraulic) has outstanding advantages such as good rigidity, compact structure, high load-bearing capacity, large power-to-weight ratio, and fast response speed. It has a good application prospect in deep-sea equipment. The hydraulic system of the submersible is mainly a manipulator operating tool system, a trim adjustment system, an adjustable ballast system, a ballast tank system, and an emergency load dump system, which provide hydraulic power. The reliability of its work directly determines the submersible's dive operation. The success of the task, or even the safety of the personnel[1-2]. However, the actual application data of hydraulic systems in deep-water submersible vehicles is insufficient. How to carry out the reliability evaluation of the newly developed system against the existing data and expert information is an urgent matter.

The large-scale test of the previous large-depth hydraulic system is unbearable, and the traditional reliability prediction has great defects. The Bayesian method can make full use of a priori information to effectively evaluate the reliability of a product with a small sample. The Bayesian method can use the empirical or existing subjective and objective information that reflects the overall distribution situation, and comprehensively sample the distribution to estimate the overall level, so it has obvious advantages for analyzing small sample problems with insufficient sample information[3-5]. Literature[6-9] used Bayesian method to analyze the reliability of the equipment and achieved good results. Based on the characteristics of the small sample of the test, , the scale parameters and growth
parameters of the AMSAA model were regarded as variables, and the time of the reliability growth test was predicted by Newton's iteration method, but the prediction results were intrusive. Literature [11] used Bayesian method to fuse multiple sources of information to predict the number of failures in the future time period, and discussed the future failure time and reliability of the system.

At this stage, the hydraulic system collects some similar product information, test data during development, and experts' evaluation of this type of submersible. In order to make full use of all the above information, this paper improves the Bayesian method. An improved Bayesian method combined with constrained no prior information is used for reliability analysis. In order to make full use of the prior information (similar to product information) and the product without prior information (expert information) under constraints, this paper builds an exponential distribution parameter estimation model based on the improved Bayesian method. The two cases of constrained no prior information are combined organically to make full use of all possible information of the product and improve the accuracy of the distribution parameter estimation. At the same time, the posterior distribution of the parameters is calculated by combining the posterior information (test data developed at the current stage), the point estimates and the lower limit estimates of the parameters are estimated, and the standard and minimum values of MTBF of the product are calculated. At the same time, the functions of reliability and lower limit of reliability over time were calculated to better understand the performance and usage requirements that the product meets, and to provide a reference for product optimization maintenance strategies and diagnostic work.

2. Estimation model of exponential distribution parameters

Traditional reliability estimation requires a large amount of data, and the failure data collected for new products is not much. At this time, the reliability estimation value has a large error, and even incorrect estimation occurs. In order to improve the accuracy of reliability estimation, this paper adopts an improved Bayesian method to further improve the accuracy of reliability estimation. Fully combine field data with previous test data and constrained no-a priori information. With few field data, you can accurately estimate the reliability of the product.

The distribution function is used to represent the prior information, which can make full use of the test data of different working environments and different working properties. In order to make full use of these multiple sources of prior information in the process of estimating the reliability index and accurately estimate the reliability index, this paper proposes a method of combining reliability-weighted fusion based on prior information and a conjugate distribution method to establish a reliability analysis model.

2.1. The prior and posterior distribution of information parameters in the exponential distribution

When using the conjugate prior distribution, it is assumed that the prior and posterior distributions of unknown parameters have the same distribution type, which is very convenient to calculate. The prior parameters of the prior distribution generally have clear physical meaning, and the exponential distribution is widely used in engineering applications. The following uses the conjugate method to calculate the exponential distribution of the exponential distribution parameter. It is assumed that the information prior distribution of the parameter $\lambda$ is the Gamma distribution. The test data $X$ is collected on site, including the failure data of $n$ products of this type. The total time is $T$. The posterior distribution can be calculated as:

$$
\pi(\lambda | X) = \frac{\pi(\lambda; \alpha_0, \beta_0)f(X | \lambda)}{\int_{\lambda} \pi(\lambda; \alpha_0, \beta_0)f(X | \lambda) d\lambda} = \frac{(\alpha_0 + T)^{\beta_0 + n}}{\Gamma(\beta_0 + n)} \lambda^{\beta_0 + n - 1} e^{-(\alpha_0 + T)\lambda} 
$$

(1)

It can be known from the posterior distribution that the posterior distribution of the parameter $\lambda$ also satisfies the Gamma distribution, that is, the conjugate prior distribution of the parameter is the
Gamma distribution, and thus the prior distribution of the parameter with information is determined as: \( \pi_0(\lambda) \sim G(\lambda; \alpha_0, \beta_0) \). The posterior distribution is: \( \pi(\lambda | X) \sim G(\lambda; \alpha_0 + T, \beta_0 + n) \).

2.2. The exponential distribution is constrained without information prior distribution

There are two main reasons for conducting in-depth research on uninformed prior distributions: first, the use of Bayes' method of sequential law to study multi-stage experimental information will eventually involve uninformed prior distributions; and second, when there is no clear prior information. When no relevant experimental information is available, it is natural to use a priori distribution without information. The parameters in the exponential distribution are scale parameters, and the Jeffrey analysis method is used in the selection of the informationless prior distribution.

Let \( X = (x_1, x_2, \ldots, x_n) \) be a sample from the exponential distribution \( f(x|\lambda) \), then the uninformed prior density of the parameter \( \lambda \) is:

\[
\pi_x(\lambda) = c \exp(-b\lambda) / \lambda^{1/2}
\]

(3)

In this paper, some engineering practice and expert experience can be used in the determination of the prior information-free distribution. At the same time, in order to improve the accuracy of the information, the mean value of the prior failure data can be referred to as a constraint on the prior information distribution without information. The accuracy of the prior distribution of information. According to the principle that the kernels of the prior and posterior distributions in the Bayes distribution are invariant, the restricted and information-free prior distribution of the parameter \( \lambda \) can be determined through the principle of maximum entropy:

\[
\pi_0(\lambda) = c \exp(-b\lambda) / \lambda^{1/2}
\]

(4)

Therefore, the form of the above formula can determine the constrained and information-free prior distribution of the parameter \( \lambda \) to satisfy the Gamma distribution with a shape parameter of \( 1/2 \) and a scale parameter of \( b \). In the presence of the constraint condition \( E(\lambda) = \lambda_0 \), the scale parameter \( b = 1/2\lambda_0 \) can be calculated, so that the informationless prior distribution of the parameter \( \lambda \) can be obtained as \( G(\lambda; 1/2\lambda_0, 1/2) \). The posterior distribution without information is:

\[
\pi_0(\lambda) \sim G(\lambda; 1/2\lambda_0 + T, 1/2 + N).
\]

3. Test for compatibility of prior and posterior information

Suppose that two types of data are collected, one is \( n \) test information of the previous product, and the other is \( m \) test data of the product obtained for the field test of the product. The two types of data are \( X_1 \) and \( X_2 \). Check the compatibility of the two types of information. Credibility is defined as \( P(H_0| \text{Accept } H_0) \). According to the Bayes formula, there are:

\[
P(H_0 \mid \text{accept } H_0) = \frac{1}{1 + \frac{P(H_1)}{P(H_0)} \frac{\beta}{1 - \alpha}}
\]

(4)

From the above hypothesis test, it is known that the probability \( \beta \) of the second type of error is the probability that the mean of the prior distribution and the mean of the posterior distribution are not equal to exceed the allowable value. Take \( \overline{\mu}_{X_1} \) and \( \sigma^2_{\mu X_1} \) as the mean and variance of the average failure time before the test; take \( \overline{\mu}_{X_2} \) and \( \sigma^2_{\mu X_2} \) as the mean and variance of the average failure time after the test; then the distribution form that \( \overline{X}_1 - \overline{X}_2 \) satisfies is:
\[
(X_1 - X_2) \sim N\left(\mu_{X_1} - \mu_{X_2}, \frac{1}{n}\sigma^2_{X_1} + \frac{1}{m}\sigma^2_{X_2}\right)
\]  

(5)

The corresponding OC function of the test is:

\[
\beta(\mu_{X_1} - \mu_{X_2}) = P_{\mu_{X_1} - \mu_{X_2}}(\text{accept } H_0) = \Phi(-\lambda + u_{\alpha/2}) + \Phi(\lambda + u_{\alpha/2}) - 1
\]

\[\alpha\] is the probability of making the first type of error.

\(P(H_0|\text{ accept } H_0)\) is the credibility when it is verified that the prior information meets the requirements.

\(P(H_0)\) is the credibility of the a-priori information, which can be analyzed by analyzing the source of the a-priori information or given by the expert. Generally, \(P(H_0)=0.5\) is selected when the a-priori information is uncertain.

### 4. Weighted fusion method based on credibility to calculate the prior distribution of exponential distribution parameters

Let \(\pi_i(\theta) (i=1,2,\cdots,m)\) are the various prior distributions obtained from the prior data, \(\varepsilon_i\) is the credibility of each information source., and the prior distribution after fusion is recorded as:

\[
\pi(\theta) = \sum_{i=1}^{m} \varepsilon_i \pi_i(\theta)
\]

(7)

For the two types of cases considered in this paper: prior information and no information, the prior distribution of the exponential distribution parameters, that is, when \(m=2\), the corresponding prior distribution. which is

\[
\pi_0(\lambda) = p\pi_1(\lambda) + (1-p)\pi_2(\lambda)
\]

(8)

\(\pi_1(\lambda)\) is the conjugate prior distribution of parameter \(\lambda\), that is \(\pi_1(\lambda) \sim G(\lambda; \alpha_0, \beta_0)\); \(\pi_2(\lambda)\) is the non-information prior distribution of parameter \(\lambda\), that is \(\pi_2(\lambda) \sim G(\lambda; 1/2\lambda_0, 1/2)\) and \(\lambda_0\) are constraint conditions without information. \(P\) is the credibility of the prior information;

### 5. Determination of posterior distribution

The test data of \(n\) products obtained at the site is \(X=(x_1, x_2, \ldots, x_n)\), and the posterior distribution of \(\lambda\) can be determined by using the Bayes formula:

\[
\pi(\lambda \mid X) = \lambda(X)\pi_1(\lambda \mid X) + \left[1 - \lambda(X)\right]\pi_2(\lambda \mid X)
\]

(9)

In the formula:

\[
\lambda(X) = \left(1 - p\right)^{n+1} \left(\alpha_0 + T\right)^{\beta_0+n} \left(\alpha_0 + T\right)^{\beta_0+n} \left(\alpha_0 + T\right)^{0.5+n} \left(\alpha_0 + T\right)^{0.5+n}
\]

\[
\pi_1(\lambda \mid X) = \frac{(\alpha_0 + T)^{\beta_0+n}}{\Gamma(\beta_0+n)} \lambda^{\beta_0+n-1} e^{-(\alpha+T)\lambda} \pi_2(\lambda \mid X) = \frac{(0.5\lambda_0 + T)^{0.5+n}}{\Gamma(0.5+n)} \lambda^{0.5+n-1} e^{-(0.5\lambda_0+T)\lambda}
\]

The point estimate \(\hat{\lambda}\) and the confidence upper limit \(\hat{\lambda}_{U}\) hen the confidence level is \(1-\alpha\) are:
\[
\hat{\lambda} = E(\lambda) = \int_0^{+\infty} \lambda \pi(\lambda \mid X) d\lambda
\]
\[
\int_0^{\frac{1}{\lambda_U}} \lambda \pi(\lambda \mid X) d\lambda = 1 - \alpha
\]

(10)

Thus, the point confidence of MTBF and the lower confidence limit of 1-\(\alpha\) can be obtained:

\[
\begin{align*}
MTBF &= \frac{1}{\hat{\lambda}} \\
MTBF_{\text{min}} &= \frac{1}{\hat{\lambda}_U}
\end{align*}
\]

(11)

6. Reliability Evaluation of Hydraulic System

A large number of researches have been carried out on hydraulic systems, but the relevant test data are insufficient, and the traditional probability and statistical methods cannot be used for reliability assessment. This article is based on the actual product information collected (see Table 1), including similar system failure time, expert information, and actual working time of the hydraulic system. Then the improved Bayesian method was used to fuse the above data to evaluate the reliability of the hydraulic system. The specific calculation steps are as follows:

| Table 1. Hydraulic system failure information. |
|-----------------------------------------------|
| Number | Expiration time (hours) |
|--------|-------------------------|
| Similar product data | 135, 256, 367, 453, 556, 718, 903, 1335, 1709, 2263, 2382, 2664 |
| Expert Information | [1500 20000] |
| Actual failure time | 235, 363, 752, 932, 1342, 1764, 2164, 2872 |

(1) Verify that the prior failure data meets the exponential distribution. According to the characteristics of the exponential distribution, the unbiased estimate of the parameters of the exponential distribution is

\[
\hat{\lambda} = \frac{1}{n} \sum_{i=1}^{n} \frac{1}{t_i} = 0.0019
\]

(2) Estimate the prior distribution parameters of the exponential distribution parameter \(\lambda\).

In the case of prior information, the distribution parameter \(\lambda\) satisfies Gamma distribution. According to the prior information, the mean and variance of the prior information failure rate can be calculated, and then the distribution position parameter \(\alpha_0\) and the shape parameter \(\beta_0\) can be calculated.

\[
\begin{align*}
E(\lambda) &= \frac{\beta_0}{\alpha_0} = 6.7125 \times 10^{-5} \\
\sigma^2(\lambda) &= \frac{\beta_0}{\alpha_0^2} = 2.0080 \times 10^{-9}
\end{align*}
\]

\[
\begin{align*}
\alpha_0 &= 3.3430 \times 10^4 \\
\beta_0 &= 2.2440
\end{align*}
\]

When there is a constraint without prior information, the distribution parameter \(\lambda\) also satisfies the Gamma distribution in the presence of constraints. When the constraint condition is the mean \(\lambda_0\) of the failure rate, the information of the personnel and experts is averaged. The scale parameter of the parameter is 275. Therefore, when there is no prior information, the parameter \(\lambda\) satisfies a Gamma distribution with a shape parameter of 0.5 and a scale parameter of 275.

(3) Establishing the prior distribution form of the parameter \(\lambda\).

According to the weighted fusion method of credibility, the credibility \(P(H_0/\text{accept } H_0)\) of the calculated prior information can be obtained from the prior distribution form:

\[
\pi_0(\lambda) = 0.7316 \times \frac{(3.3430 \times 10^4)^{\lambda_0}}{\Gamma(2.2440)} \lambda^{2.2440-1} e^{-3.3430 \times 10^4 \lambda} + (1-0.7316) \times \frac{(263.1579)^{1/2}}{\Gamma(1/2)} \exp(-263.1579 \lambda) \frac{1}{\lambda^{1/2}}
\]
(4) The posterior distribution and point estimate \( \hat{\lambda} \) and the confidence upper limit \( \hat{\lambda}_U \) when the confidence is 1-0.05 can be obtained from formula (10),

\[
\hat{\lambda} = E(\lambda) = \int_{0}^{\infty} \lambda \pi(\lambda \mid X) d\lambda = 5.8941 \times 10^{-4} \text{ 1/h}
\]

\[
\int_{0}^{8.2196 \times 10^{-4}} \pi(\lambda \mid X) d\lambda = 1 - 0.05 \Rightarrow \hat{\lambda}_U = 8.2196 \times 10^{-4} \text{ 1/h}
\]

Furthermore, the lower confidence limit of the MTBF estimated value and the confidence of the product obtained from formula (11) is 1-0.05, MTBF can be obtained as:

\[
\begin{align*}
\text{MTBF} &= 1/\hat{\lambda} = 1697h \\
\text{MTBF}_{\min} &= 1/\hat{\lambda}_U = 1217h
\end{align*}
\]

(5) According to the estimated values of MTBF and MTBF_{\min}, the product reliability estimation function \( R(t) \) and the reliability estimation upper limit function \( R_d(t) \) can be obtained as

\[
\begin{align*}
R(t) &= \exp(-5.8941 \times 10^{-4} t) \\
R_d(t) &= \exp(-8.2196 \times 10^{-4} t)
\end{align*}
\]

Figure 1. Curve of the reliability change with time.

Figure 1 clearly reflects the change curve of the reliability and lower limit of the product over time. Can calculate the reliability and lower limit of the reliability of the product at any moment. Therefore, the time that the product can work can be calculated according to the product with different reliability requirements, and whether the reliability of the product meets the use requirements can be judged according to the working time required by the product. It makes us more clear about the performance and usage conditions that the product meets, and at the same time provides a reference for product optimization maintenance strategy and diagnostic work.

7. Summary

(1) In order to make full use of the a priori information of the product and the constrained non-a priori information, this paper uses a weighted fusion based on the Bayesian method to establish a reliability analysis model of the submersible hydraulic device. This method makes full use of all the product's information Including a priori information and constrained non-a priori information effectively solve the shortcomings of small product failure samples.

(2) This method has broad application prospects for small samples and expensive testing products, and provides a reference for the safety design and reliability analysis of equipment.

(3) The MTBF of a hydraulic system is estimated to be 1697h, and the lower confidence limits of 0.95 are 1217h.
References

[1] Yang Yi-fan, Zhao Sheng-ya, Qi Hai-bin, Tang Jia-ling, Ding Zhong-jun, Yang Lei. Fault Analysis and Improvement Measures Research of Hydraulic System for Deep-sea Manned Submersible[J]. Chinese Hydraulics & Pneumatics, 2018(04):15-21.

[2] Yang Shenshen, Wang Xuan, Liu Hao, Hu Zhen. Design of Hydraulic System for Deep-sea Submersible[J]. Machine Tool & Hydraulics, 2017, 45(09):93-95.

[3] Li Guodong, Masuda S, Zhou Yuanquan. A new reliability prediction model in manufacturing system [J]. IEEE Transactions on reliability, 2010, 59 (1): 170-178.

[4] Jiang Yingjie, YanZhiqiang, Xie Hongwei. The Method of Bayesian Reliability Evaluation Based on Mixed Gamma Distribution[J]. Computing Technology and Automation, 2010, 29(1): 106-109.

[5] Li Su-fang, Zhu Hui-ming, Li Rong. Bayesian Unit Root Tests in Panel Data by Using MCMC Algorithm[J]. Journal of Hunan University (Natural Sciences), 2015, 42(1):137-140.

[6] Yao Jitao, Wang Xudong. Bayesian Methods for Inferring Representative Values of Variable Actions in Small Sample Situations[J], Journal of Southwest Jiaotong University, 2014, 49(6):995-1001.

[7] Gebraeel N Z, Lawley M A. Residual life distributions from component degradation signals: a Bayesian approach[J]. IEEE Transactions on Automation Science and Engineering, 2005, 37(1): 543-557.

[8] Gebraeel N Z. Sensory-updated residual life distributions for components with exponential degradation patterns [J]. IEEE Transactions on Automation Science and Engineering, 2006, 3(4):382-393.

[9] Gebraeel N Z, Lawley M A. A neural network degradation model for computing and updating residual life distributions[J]. IEEE Transactions on Automation Science and Engineering, 2008, 5(1): 154-163.

[10] Dong Huiru, Huang Yanfeng, Liu jing. An Improvement of AMSAA Model on Reliability Growth[J]. Journal of Air Force Engineering University (Natural Science Edition) , 2002, 3(5):79-82.

[11] Julia A, Steven E. Bayes prediction for the number of failure of a repairable system [J], IEEE Transactions on Reliability.1997, 46 (2): 291-295.