A solution to $B \rightarrow \pi\pi$ puzzle and $B \rightarrow KK$

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Abstract

The large ratio of color-suppressed tree amplitude to color-allowed one in $B \rightarrow \pi\pi$ decays is difficult to understand within the Standard Model, which is known as the “$B \rightarrow \pi\pi$ puzzle”. The two tree diagrams contain the up- and charm-quark component of penguin amplitude, $P_{uc}$, which cannot be separated by measuring $B \rightarrow \pi\pi$ decays alone. We show that the measurements of the branching ratio and direct CP asymmetry of $B^+ \rightarrow K^+K^0$ decay enable one to disentangle the $P_{uc}$ with two-fold ambiguity. One of the two degenerate solutions of the $P_{uc}$ can solve the $B \rightarrow \pi\pi$ puzzle by giving $|C/T| \sim 0.3$ which is consistent with the expectation in the Standard Model. We also show that the two solutions can be discriminated by the measurement of the indirect CP-asymmetry of $B^0 \rightarrow K^0\overline{K}^0$. We point out that if the $B \rightarrow \pi\pi$ puzzle is solved in this way, the corresponding puzzle in $B \rightarrow \pi K$ decays should have a different origin.

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1 Introduction

In the Standard Model (SM), rare charmless nonleptonic $B \to \pi\pi$ decays provide valuable information on the inner angles of the unitarity triangle of Cabbibo-Kobayashi-Maskawa (CKM) matrix [1]. In these decays, dominated by $b \to dq\bar{q}$ ($q = u, d$) at the quark level, one can measure the angle $\alpha$ through the isospin analysis [2]. In addition, new physics (NP) beyond the SM can affect the decay processes [3].

In the diagrammatic approach [4], the $B \to \pi\pi$ decay amplitudes are dominated by color-allowed (suppressed) tree diagram $T(C)$ and QCD-penguin diagram $P$. These diagrammatic amplitudes are expected to be hierarchical in size, i.e., $|C| \approx |P|$ much less than $|T|$. However, the experimental data of the branching ratio (BR) of $B \to \pi^0\pi^0$ decay requires a large ratio of the color-suppressed tree diagram to the color-allowed one, which is in contradiction to the hierarchy. This is known as the "$B \to \pi\pi$-puzzle".

Recently another $B$-decay modes, $B^+ \to K^+K^0$ and $B^0 \to K^0\bar{K}^0$, which have identical $b \to d$ transitions at the quark level have been observed by the BaBar [5] and Belle [6] Collaborations. They have also measured the time-dependent CP asymmetries, $A_f$ and $S_f$ defined by

$$
\frac{\Gamma_{B^0\to f}(\Delta t) - \Gamma_{B^0\to f}(\Delta t)}{\Gamma_{B^0\to f}(\Delta t) + \Gamma_{B^0\to f}(\Delta t)} \equiv A_f \cos(\Delta m\Delta t) + S_f \sin(\Delta m\Delta t).
$$

We study the implications of these newly discovered channels on the extraction of the SM amplitudes. Other studies on $B \to KK$ can be found in [7].

We take i) $SU(3)$ flavor symmetry of strong interactions and ii) smallness of annihilation and exchange topologies as working assumptions. Then $B^+ \to K^+K^0$ and $B^0 \to K^0\bar{K}^0$ decays become pure penguin modes.

The penguin amplitude can be decomposed into two parts depending on the quarks running inside the loop, up- and charm-quark penguin $P_{uc}$ and top- and charm-quark penguin $P_{tc}$. In $B \to \pi\pi$ decays $P_{uc}$ can always be absorbed into $T$ and $C$. As a consequence, it is impossible to extract pure $T$ or $C$ amplitudes from $B \to \pi\pi$ measurements alone.

We show that when combining the $B \to \pi\pi$ and $B \to KK$ decays, we can extract the
tree amplitudes and $P_{uc}$ separately. So we can test the ratio $|C/T|$ without the contamination of $P_{uc}$ penguin in the $b \to q\bar{q}$ transitions.

We will demonstrate that a solution obtained from the $B \to \pi\pi$ data, $\mathcal{B}(B^+ \to K^+K^0)$, and $\mathcal{A}_{CP}(B^+ \to K^+K^0)$ has $|C/T| \sim 0.3$ which is in accord with the SM hierarchy. First, we use the analytic expressions for the BR’s and CP asymmetries to show that all the relevant amplitudes, $T$, $C$, $P_{tc}$, $P_{uc}$, and the angle $\gamma$ of the unitarity triangle can be extracted with two-fold ambiguity.

Then we get the diagrammatic amplitudes numerically by performing a $\chi^2$-fit to the current data. Discarding the indirect CP asymmetry of $B \to K^0\bar{K}^0$, $S_{CP}(B \to K^0\bar{K}^0)$, which has huge errors, we obtain two degenerate solutions which minimize $\chi^2$. One solution, as mentioned above, is acceptable. However, the other solution gives $|C/T| \sim 1.3$ which is unsatisfactory.

From the two solutions we can predict $S_{CP}(B \to K^0\bar{K}^0)$. The favored solution with $|C/T| \sim 0.3$ gives large positive $S_{CP}(B \to K^0\bar{K}^0)$ while the other solution which looks unphysical predicts large negative $S_{CP}(B \to K^0\bar{K}^0)$. Therefore if the $S_{CP}(B \to K^0\bar{K}^0)$ is measured more accurately in the near future, we can discriminate the two solutions and confirm the physical solution.

It is also well known that $|C'/T'| \sim 1$ in $B \to \pi K$ decays, which is again difficult to understand in the SM framework [9]. In $B \to \pi K$ decays, the contamination of $P_{uc}'$ does not play an important role because it is suppressed by CKM factors. Therefore the “$B \to \pi K$ puzzle”, if it remains in the future experimental data, should be solved by other mechanisms such as new physics effects.

This paper is organized as follows. In section 2, using the current experimental $B \to \pi\pi$ data only, we find that the $B \to \pi\pi$ puzzle is still there. In section 3, we confirm the possibility that the $B \to \pi\pi$ puzzle is simply the effect of $P_{uc}$ and there is actually no $B \to \pi\pi$ puzzle in the “bare” ratio of $|C/T|$. We conclude in section 4.
2 The current status of $B \to \pi\pi$ puzzle

The diagrammatic amplitudes provide a useful parametrization for nonleptonic $B$-meson decay processes because it is independent of theoretical models for the calculation of hadronic matrix elements [4]. The decay amplitudes of $B \to \pi\pi$’s are written as

$$\sqrt{2}A(B^+ \to \pi^+\pi^0) = -(T + C + P_{EW} + P_{CEW}^C),$$

$$A(B^0 \to \pi^+\pi^-) = -\left(T + P + \frac{2}{3}P_{EW}^C + E + PA\right),$$

$$\sqrt{2}A(B^0 \to \pi^0\pi^0) = -\left(C - P + P_{EW} + \frac{1}{3}P_{EW}^C - E - PA\right). \quad (2)$$

Here $T$, $C$, $P$, $P_{EW}^{(C)}$, $E$, and $PA$ represent the tree, color-suppressed tree, QCD-penguin, (color-suppressed) electroweak-penguin, exchange, and penguin-annihilation amplitudes, respectively. In general, the diagrammatic amplitudes may have a weak phase as well as a strong phase.

We can further decompose the QCD penguin diagram $P$ which depends on the quarks running inside the loop, as

$$P = V_{ud}V_{ub}^*P_u + V_{cd}V_{cb}^*P_c + V_{td}V_{tb}^*P_t$$

$$= V_{ud}V_{ub}^*(P_u - P_c) + V_{td}V_{tb}^*(P_t - P_c)$$

$$\equiv P_{uc}e^{i\gamma} + P_{tc}e^{-i\beta}, \quad (3)$$

where we use the unitarity property of CKM matrix and explicitly write the weak phase dependence for the amplitudes. The relative sizes of the diagrammatic amplitudes are expected to have the following hierarchical structure:

| $O(1)$ | $O(\bar{\lambda})$ | $O(\bar{\lambda}^2)$ | $O(\bar{\lambda}^3)$ | $O(\bar{\lambda}^4)$ |
|-------|-----------------|-----------------|-----------------|-----------------|
| $|T|$   | $|C|, |P|$       | $|P_{EW}|$      | $|P_{EW}^C|$    | $|E|, |PA|$ |

(4)

where $\bar{\lambda}$ is expected to be of order of $0.2 - 0.3$. We neglect terms of order $\bar{\lambda}^2$ or higher and thus, consider only $T, C, \text{ and } P$. 

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Table 1: The current experimental data for CP averaged branching ratios ($BR$), direct CP-asymmetry ($A_{CP}$) and indirect CP-asymmetry ($S_{CP}$) of $B \to \pi\pi$ and $B \to \pi K$ decays [5, 6, 10].

Taking into account the decay amplitudes containing terms up to $O(\bar{\lambda})$, we find that the amplitudes in $B \to \pi\pi$ decays are given by

$$-\sqrt{2} A(B^+ \to \pi^+\pi^0) = (T + C) e^{i\gamma},$$

$$-A(B^0 \to \pi^+\pi^-) = T e^{i\gamma} + P_{tc} e^{-i\beta} + P_{uc} e^{i\gamma},$$

$$-\sqrt{2} A(B^0 \to \pi^0\pi^0) = C e^{i\gamma} - P_{tc} e^{-i\beta} - P_{uc} e^{i\gamma},$$

(5)

where the weak-phase dependence is explicitly written. From the above expression we notice that the $P_{uc}$ term can always be absorbed into tree diagrams $T$ and $C$ by the redefinition

$$\tilde{T} = T + P_{uc},$$

$$\tilde{C} = C - P_{uc}.\quad (6)$$

Then the $B \to \pi\pi$ decay amplitudes are further simplified:

$$-\sqrt{2} A(B^+ \to \pi^+\pi^0) = (\tilde{T} + \tilde{C}) e^{i\gamma},$$

$$-A(B^0 \to \pi^+\pi^-) = \tilde{T} e^{i\gamma} + P_{tc} e^{-i\beta};$$

$$-\sqrt{2} A(B^0 \to \pi^0\pi^0) = \tilde{C} e^{i\gamma} - P_{tc} e^{-i\beta}.\quad (7)$$

As a consequence, the $P_{uc}$ term cannot be detected if we use the $B \to \pi\pi$ data only.

The current experimental data for the CP-averaged branching ratio ($B$), direct CP-asymmetry ($A_{CP}$), and indirect CP-asymmetry ($S_{CP}$) are shown in Table 1. If we take the
Table 2: The results for the fit to the $B \to \pi \pi$ data alone. We obtained $\chi^2_{\text{min}} / \text{d.o.f} = 0.81 / 1$. The strong phase of $P_{tc}$ is set to zero. The magnitudes and angles are in the unit of $eV$’s and degrees, respectively.

|  $\gamma$  |  $(|\tilde{T}|, \delta_{\tilde{T}})$  |  $(|\tilde{C}|, \delta_{\tilde{C}})$  |  $|P_{tc}|$  |
|----------|-------------------------------|-------------------------------|---------|
|   65.5 ± 14.1 |   (22.4 ± 0.7, 36.0 ± 22.6)  |   (15.8 ± 4.1, −16.5 ± 12.7)  |   8.33 ± 4.34 |

The strong phase $\beta = 21.2^\circ$ measured through the $b \to c\bar{c}s$ transition, we have 6 parameters in (7). We fit 6 parameters in (7) to the 7 measurements. The results are shown in Table 2. The $\chi^2_{\text{min}} / \text{d.o.f} = 0.81 / 1$ is very good. The angle $\gamma = (65.5 \pm 14.1)^\circ$ (all the angles in this paper are in the unit of degrees) is consistent with the global CKM fit value $\gamma = 55.6^{+11}_{−2.5}$ or $68.0^{+2.6}_{−7.0}$ obtained by CKMfitter group [11]. However, we obtain

$$\frac{|\tilde{C}|}{|\tilde{T}|} = 0.71 \pm 0.18,$$

which is larger than the expectation (4) by about factor 3. This large ratio is mainly due to the unexpectedly large $B(B \to \pi^0 \pi^0)$. We confirm the $B \to \pi \pi$ puzzle [12] is still there.

3 Combining $B \to \pi \pi$ with $B \to KK$

Since from the $B \to \pi \pi$ data alone we cannot disentangle the $P_{uc}$, we include the $B \to KK$ data in the analysis. There are three decay modes for these $b \to d(u)s\bar{s}$ transitions:

$$A(B^+ \to K^+\bar{K}^0) = P + A - \frac{1}{3}P_{\text{EW}}^C,$$

$$A(B^0 \to K^0\bar{K}^0) = P + PA - \frac{1}{3}P_{\text{EW}}^C,$$

$$A(B^0 \to K^+\bar{K}^-) = -E - PA.$$  

(9)

When we neglect small contributions from annihilation and electroweak penguins, we simply get

$$A(B^+ \to K^+\bar{K}^0) = A(B^0 \to K^0\bar{K}^0) = P = P_{tc}e^{-i\beta}(1 + re^{\delta_{uc}e^{i(\beta+\gamma)}}),$$

(10)
where \( r \equiv |P_{uc}/P_{tc}| \) and \( \delta_{uc} \) is the strong phase of \( P_{uc} \) (We set \( \delta_{tc} \equiv 0 \)). The \( \mathcal{A}(B^0 \to K^+K^-) \) is negligible. Therefore we expect \( i) \ \mathcal{B}(B^0 \to K^+K^-) \) is very small, which is confirmed by the experimental data [5, 6], \( ii) \ \mathcal{B}(B^+ \to K^+\bar{K}^0) \approx \mathcal{B}(B^0 \to K^0\bar{K}^0) \), which agrees with the data, and \( iii) \ \mathcal{A}_{CP}(B^+ \to K^+\bar{K}^0) \approx \mathcal{A}_{CP}(B^0 \to K^0\bar{K}^0) \). For the last relation, the results of BaBar and Belle are contradictory with each other and we need to wait for more data to confirm or reject it. As one can see in Table 1, the indirect CP asymmetry \( \mathcal{S}_{CP}(B^0 \to K^0\bar{K}^0) \) has still large experimental errors. For these reasons we use only \( \mathcal{B}(B^+ \to K^+\bar{K}^0) \), \( \mathcal{B}(B^0 \to K^0\bar{K}^0) \), and \( \mathcal{A}_{CP}(B^+ \to K^+\bar{K}^0) \) for the numerical analysis which will be presented below.

First, let us demonstrate that the inclusion of just two more data, \( \mathcal{B}(B^+ \to K^+\bar{K}^0) \) and \( \mathcal{A}_{CP}(B^+ \to K^+\bar{K}^0) \), is enough to extract \( P_{uc} \). Given \( \mathcal{B}(B^+ \to K^+\bar{K}^0) \), one can obtain

\[
R \equiv \frac{|A(B^+ \to K^+\bar{K}^0)|^2 + |A(B^- \to K^-\bar{K}^0)|^2}{2|P_{tc}|^2} = 1 + r^2 + 2r \cos(\beta + \gamma) \cos \delta_{uc} \quad (11)
\]

for the ratio of average amplitude squared to \( |P_{tc}|^2 \). The information on \( |P_{tc}| \) is obtained from \( B \to \pi\pi \).

The direct and indirect CP asymmetries for \( B^{+0} \to K^{+0}\bar{K}^0 \) are given by

\[
\mathcal{A}_{CP}(B^{+0} \to K^{+0}\bar{K}^0) = \frac{2r \sin(\beta + \gamma) \sin \delta_{uc}}{1 + r^2 + 2r \cos(\beta + \gamma) \cos \delta_{uc}},
\]

\[
\mathcal{S}_{CP}(B^0 \to K^0\bar{K}^0) = \frac{-r^2 \sin 2(\beta + \gamma) + 2r \sin(\beta + \gamma) \cos \delta_{uc}}{1 + r^2 + 2r \cos(\beta + \gamma) \cos \delta_{uc}}, \quad (12)
\]

respectively. When the angles \( \beta \) and \( \gamma \) are given, \( R \) and the CP asymmetries are functions of \( r \) and \( \delta_{uc} \) only. From (11) and (12), we can determine \( r \) and \( \delta_{uc} \) with \( |P_{tc}| \) and \( \gamma \) determined from \( B \to \pi\pi \) decays.

In Figure 1, we show filled contour plots of \( \mathcal{A}_{CP}(B^{+0} \to K^{+0}\bar{K}^0) \) (Figure 1(a)) and \( \mathcal{S}_{CP}(B^0 \to K^0\bar{K}^0) \) (Figure 1(b)). We also draw contours for various values of \( R \) with thick lines on each plot. For these plots the CP phases are taken to be \( \beta = 21.2^\circ \) and \( \gamma = 65.5^\circ \).

In Figure 1(a), the lines for \( \mathcal{A}_{CP}(B^{+0} \to K^{+0}\bar{K}^0) = 0.12 \) and \( R = 1.79 \) which correspond to the experimental central values cross at two points denoted by the black and white dots. Thereby we get \( |P_{uc}| \) and \( \delta_{uc} \) with two-fold ambiguity once \( |P_{tc}| \) is obtained from the \( B \to \pi\pi \) data.
Figure 1: Contour plots for constant (a) direct and (b) indirect CP asymmetries as a function of \( r = |P_{uc}/P_{tc}| \) and the strong phase \( \delta_{uc} \) (Filled contours). Also contours for constant \( R \) are shown on each plot. The lines for \( A_{CP}(B^{+}(0) \rightarrow K^{+}(0)\overline{K}^{0}) = 0.12 \) and \( R = 1.79 \) which correspond to the experimental central values cross at two points in (a), denoted by black and white dots. The points are also shown in (b).

In Figure 1(b), we can see that the two solutions obtained from Figure 1(a) predict very distinct \( S_{CP}(B^{0} \rightarrow K^{0}\overline{K}^{0}) \). Therefore we can discriminate the two solutions by measuring \( S_{CP}(B^{0} \rightarrow K^{0}\overline{K}^{0}) \) in near future. From the figure we note that the prediction for \( S_{CP}(B^{0} \rightarrow K^{0}\overline{K}^{0}) \), especially its sign, is almost insensitive to the change of \( r \) unless \( r \) is too small.

Now, we proceed to obtain numerical solutions. For this purpose we perform a \( \chi^2 \)-fit to the seven \( B \rightarrow \pi\pi \) data, \( B(B^{+}(0) \rightarrow K^{+}(0)\overline{K}^{0}) \), and \( A_{CP}(B^{+} \rightarrow K^{+}\overline{K}^{0}) \). We do not include the CP asymmetries of \( B^{0} \rightarrow K^{0}\overline{K}^{0} \) to the fit because they have huge errors and/or BaBar and Belle data are not consistent with each other.

The results for the fit are given in Table 3. Again the quality of fit is very good, \( \chi^2_{\text{min}}/\text{d.o.f} = 1.5/2 \). The weak phase \( \gamma = (65.5 \pm 14.1)^\circ \) is consistent with the global fit.
Table 3: The results for the fit to the $B \to \pi\pi$ and $B \to KK$ data. We obtained $\chi^2_{\text{min}} = 0.81$. The strong phase of $P_{tc}$ is set to 0. The magnitudes and angles are in the unit of $eV$'s and degrees, respectively.

| $\gamma$     | $(|T|, \delta_T)$               | $(|C|, \delta_C)$              |
|--------------|--------------------------------|--------------------------------|
| 65.5 ± 14.1  | (28.7 ± 4.8, 25.3 ± 11.6)      | (8.14 ± 3.19, −26.12 ± 30.6)   |
| 65.5 ± 14.1  | (16.7 ± 1.7, 25.3 ± 11.6)      | (22.3 ± 7.1, −9.25 ± 9.50)     |
| $(|P_{tc}|, \delta_{tc})$ | $(|P_{uc}|, \delta_{uc})$ | $|C/T|$ |
| (8.33 ± 4.33, 0) | (7.89 ± 6.82, 174 ± 10)    | 0.28 ± 0.15                   |
| (8.33 ± 4.34, 0) | (6.93 ± 3.58, 7.42 ± 11.2)   | 1.34 ± 0.50                   |

In Table 3, the solutions in the first (second) row correspond to the black (white) solution in Figure 1. For the first solution, the ratio $|C/T|$, given by

$$|C/T| = 0.28 \pm 0.15,$$

is reduced by about factor 3 compared to (8). This agrees with the conventional hierarchy in (4). However, as we discussed in section 2, the ratio $|\tilde{C}/\tilde{T}|$ should be around 0.7. The large difference between the ratios $|C/T|$ and $|\tilde{C}/\tilde{T}|$ can be explained by the constructive interference between $C$ and $P_{uc}$. As we show in the first solution in Table 3, $C$ and $P_{uc}$ are almost same in sizes, while the difference between strong phases $\delta_C$ and $\delta_{uc}$ is around $\pi$. Hence the first solution results in the large enhancement in $\tilde{C} = C - P_{uc}$. The large $\tilde{C}$ can accommodate the $B(B^0 \to \pi^0\pi^0)$. In Figure 1, we see that this solution predicts large positive $S_{\text{CP}}(B^0 \to K^0\overline{K^0})$. Numerically we predict

$$S_{\text{CP}}(B^0 \to K^0\overline{K^0}) = 0.99 \pm 0.02.$$  

Small errors in (14) is because the large errors appearing in $r$ are canceled in the numerator and denominator.

The second solution which gives too large ratio $|C/T| = 1.34 \pm 0.50$, looks unphysical, if we take the central value seriously. This can be tested by measuring the $S_{\text{CP}}(B^0 \to K^0\overline{K^0})$
more precisely. The Figure 1(b) shows that this solution predicts large negative $S_{\text{CP}}(B^0 \rightarrow K^0 \bar{K}^0)$. The numerical prediction is

$$S_{\text{CP}}(B^0 \rightarrow K^0 \bar{K}^0) = -0.97 \pm 0.19. \quad (15)$$

In [8], the authors could extract the diagrammatic amplitudes from $\mathcal{B}(B^0 \rightarrow K^0 \bar{K}^0)$ and a theoretical input which is free of the endpoint infrared divergences in QCD factorization. The prediction

$$\mathcal{S}(B^0 \rightarrow K^0 \bar{K}^0) = 0.97 \pm 0.02 \quad (16)$$

is in full accord with the prediction (14), reinforcing the conventional SM hierarchy (4).

4 Conclusions

We have considered charmless nonleptonic decay modes, $B \rightarrow \pi\pi$ and $B \rightarrow KK$. We find that the large ratio of color-suppressed tree to the color-allowed tree which is known as the “$B \rightarrow \pi\pi$ puzzle” is mainly due to the up- and charm-quark penguin $P_{uc}$ contributions which are always included in the tree diagrams in the form of $\tilde{T} = T + P_{uc}$ and $\tilde{C} = C - P_{uc}$. The $P_{uc}$ can be determined with two-fold ambiguity by using the measured $\mathcal{B}(B^+ \rightarrow K^+ \bar{K}^0)$ and $\mathcal{A}_{\text{CP}}(B^+ \rightarrow K^+ \bar{K}^0)$. The two-fold ambiguity can be lifted when $S_{\text{CP}}(B^0 \rightarrow K^0 \bar{K}^0)$ is measured with more accuracy. Consequently, the bare ratio $|C/T|$ without the contamination of $P_{uc}$ can be determined.

Conversely, we can predict $S_{\text{CP}}(B^0 \rightarrow K^0 \bar{K}^0)$ by imposing the conventional hierarchy to the solutions. For the theoretically preferred value $|C/T| \sim 0.3$, we get

$$S_{\text{CP}}(B^0 \rightarrow K^0 \bar{K}^0) = 0.99 \pm 0.02. \quad (17)$$

If the $B \rightarrow \pi\pi$ puzzle disappears in this way, the $B \rightarrow \pi K$ puzzle which is represented by $|C'/T'| = 1.6 \pm 0.3$ [13] will become more prominent. Its solution will require other mechanisms in the SM or new physics beyond the SM.
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