ASTROMETRY WITH THE HUBBLE SPACE TELESCOPE: A PARALLAX OF THE FUNDAMENTAL DISTANCE CALIBRATOR RR LYRAE\textsuperscript{1}

G. Fritz Benedict,\textsuperscript{2} B. E. McArthur,\textsuperscript{2} L. W. Fredrick,\textsuperscript{3} T. E. Harrison,\textsuperscript{4} J. Lee,\textsuperscript{5} C. L. Slesnick,\textsuperscript{3} J. Rhee,\textsuperscript{3} R. J. Patterson,\textsuperscript{3} E. Nelan,\textsuperscript{6} W. H. Jefferys,\textsuperscript{7} W. van Altena,\textsuperscript{5} P. J. Shelus,\textsuperscript{2} O. G. Franz,\textsuperscript{8} L. H. Wasserman,\textsuperscript{8} P. D. Hemenway,\textsuperscript{9} R. L. Duncombe,\textsuperscript{10} D. Story,\textsuperscript{11} A. L. Whipple,\textsuperscript{11} and A. J. Bradley\textsuperscript{12}

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ABSTRACT

We present an absolute parallax and relative proper motion for the fundamental distance scale calibrator, RR Lyrae. We obtain these with astrometric data from FGS 3, a white-light interferometer on the Hubble Space Telescope. We find $\pi_{\text{abs}} = 3.82 \pm 0.2$ mas. Spectral classifications and $VRIJHK_{\text{s}}M$ and DDO 51 photometry of the astrometric reference frame surrounding RR Lyr indicate that field extinction is low along this line of sight. We estimate $\langle A_V \rangle = 0.07 \pm 0.03$ for these reference stars. The extinction suffered by RR Lyr becomes one of the dominant contributors to the uncertainty in its absolute magnitude. Adopting the average field absorption $\langle A_V \rangle = 0.07 \pm 0.03$, we obtain $M^R_{\text{RR}} = 0.61^{+0.11}_{-0.10}$. This provides a distance modulus for the Large Magellanic Cloud (LMC) of $m - M = 18.38 - 18.53 = 0.11^{+0.10}_{-0.10}$, with the average extinction-corrected magnitude of RR Lyrae variables in the LMC, $(V(\text{RR}))$, remaining a significant uncertainty. We compare this result with more than 80 other determinations of the distance modulus of the LMC.

Key words: astrometry — distance scale — stars: distances — stars: individual (RR Lyrae) — techniques: interferometric

On-line material: color figure

1. INTRODUCTION

The various methods used to determine the distances to remote galaxies, and ultimately the size, age, and shape of the universe itself all depend on our knowledge of the distances to local objects. Among the most important of these are the RR Lyrae variable stars. Considerable effort has gone into determining the absolute magnitudes, $M_V$, of these objects through statistical methods (see, e.g., Popowski & Gould 1998, 1999; Tsujimoto, Miyamoto, & Yoshii 1998; Fernley et al. 1998; Layden et al. 1996). For RR Lyrae variables, this determination is complicated by dependence on metallicity, rendering the calibration uncertain (compare Fernley et al. 1998; McNamara 1997; Udalski 2000a; Popowski 2001). Only recently has a relatively high-precision trigonometric parallax been available for RR Lyrae from the Hipparcos (RR Lyr = HIP 95497; Perryman et al. 1997).

We have redetermined the parallax of RR Lyrae with FGS 3 on Hubble Space Telescope (HST) with higher precision. We hope to reduce zero-point errors due to the spatially correlated errors in the Hipparcos Catalogue, discussed by Narayanan & Gould (1999). Additionally, our extensive investigation of the astrometric reference stars provides an independent estimation of the line-of-sight extinction to RR Lyr, a significant contributor to the uncertainty in its absolute magnitude, $M^R_{\text{RR}}$.

In this paper, we describe the calibration, allowing us to use a neutral density filter to relate astrometry of very bright targets to faint reference stars; present the results of extensive spectrophotometry of the astrometric reference stars, required to correct our relative parallax to absolute; briefly discuss data acquisition and analysis; and derive an absolute parallax for RR Lyrae. Finally, we calculate an absolute magnitude for RR Lyrae and apply it to derive a distance modulus for the Large Magellanic Cloud (LMC). We briefly review the present status of LMC distance moduli.

Bradley et al. (1991) provide an overview of the FGS 3 instrument, and Benedict et al. (1999) describe the fringe-tracking (position or POS) mode astrometric capabilities of FGS 3, along with the data acquisition and reduction strategies used in the present study. We time tag our data with a Modified Julian Date, $\text{MJD} = JD - 2,444,000.5$.

2. CROSS FILTER CALIBRATION

The filter wheel in each FGS contains a neutral density filter with a 1% transmission (Nelan 2001). These filters, designated FND5, provide 5 mag of attenuation. This reduction of signal is required to obtain astrometry for stars that are brighter than $V = 8.5$, for which the count rate for the FGS photomultiplier tubes would exceed the electronics capacity (Bradley et al. 1991). No filter has perfectly plane-parallel

faces, an effect called filter wedge. Filter wedge introduces a slight shift in position when comparing an observation with the standard astrometry filter, F583W, with the FND5 filter. We require this latter filter to perform astrometry on our primary science target, RR Lyr, $V \sim 7.2$. To obtain milliarcsecond astrometry requires knowledge of the filter wedge effect to that precision or better. Hemenway et al. (1997) describe an early version of this calibration but provide no explicit numbers nor an estimate of the precision with which the calibration can be determined. Motivated by these difficulties, we obtained a second calibration in 1998.

2.1. Cross Filter Calibration Observations

Conceptually, the calibration observations are simple. Observe in POS (fringe-tracking) mode the same star with and without the FND5 filter and compare the positions. The shift so determined is then applied when comparing faint reference stars with bright science targets. The standard astrometry filter is F583W. As a consequence, we will actually measure differential filter wedge because F583W is also a filter with nonparallel faces. Note that each filter is a refractive element. Thus, a star position will depend on the color of the star. This is an element of the lateral color effect discussed in Benedict et al. (1999).

We carried out this calibration with two different stars at two different epochs. Our target in 1995 was HD 41940 in M35. Our target in 1998 was Upgren 69 in the cluster NGC 188, a star used often as a template for fringe scanning with FGS 3 (Franz et al. 1998). We obtained eight to 10 observations with F583W and seven observations with FND5 over 30 minutes on 1995 March 14 and 1998 January 1. The 1998 measurements along each axis, $X$ and $Y$, with $1 \sigma$ errors are plotted in Figure 1. The errors associated with FND5 are larger because the signal from Upgren 69 is reduced by 99%. These observations clearly show the offset due to differential filter wedge and a typical amount of intraorbit positional drift.

2.2. Cross Filter Calibration Results

We remove the effect of drift by fitting each set of measures with a line (see Fig. 1) and adopt the offset between the lines at the midpoint of the sequence as the amount of differential filter wedge for $X$ and $Y$, $\Delta X_F$ and $\Delta Y_F$. These corrections with associated error estimates, the cross filter calibration, are collected in Table 1, along with the actual attenuation in signal, $\Delta n$, due to the FND5 filter. Note that the differential filter wedge is different, comparing 1995 with 1998. This is due the lateral color effect discussed in Benedict et al. (1999). As can be seen in Table 1, the two calibration stars differ in color.

3. OBSERVATIONS AND DATA REDUCTION

Figure 2 shows the distribution in R.A. and decl. of the five reference stars and RR Lyr. Nine sets of data were acquired, spanning 2.09 yr for a total of 120 measurements of RR Lyr and reference stars. Each data set required approximately 40 minutes of spacecraft time. The data were reduced and calibrated as detailed in Benedict et al. (1999) and McArthur et al. (2001). At each epoch, we measured reference stars and the target RR Lyr multiple times to correct for intraorbit drift of the type seen in the cross filter calibration data (Fig. 1).

4. SPECTROPHOTOMETRIC ABSOLUTE PARALLAXES OF THE ASTROMETRIC REFERENCE STARS

Because the parallax determined for RR Lyr will be measured with respect to reference-frame stars that have their own parallaxes, we must either apply a statistically derived correction from relative to absolute parallax (Van Altena, Lee, & Hofleit 1997, hereafter YPC95) or estimate the absolute parallaxes of the reference-frame stars listed in Table 2. In principle, the colors, spectral type, and luminosity class of a star can be used to estimate the absolute magnitude, $M_V$, and $V$-band absorption, $A_V$. The absolute parallax is then simply,

$$\pi_{\text{abs}} = 10^{(V-M_V+5-A_V)/5}. \quad (1)$$

The luminosity class is generally more difficult to estimate than the spectral type (temperature class). However, the derived absolute magnitudes are critically dependent on the luminosity class. As a consequence, we obtained additional photometry in an attempt to confirm the luminosity classes. Specifically, we employ the technique used by Majewski et
al. (2000) to discriminate between giants and dwarfs for stars later than \( \approx \) G5, an approach also discussed by Paltoglou \& Bell (1994).

4.1. Photometry

Our bandpasses for reference-star photometry include: \( BVRI, JHK \) (from the second incremental release of the Two Micron All Sky Survey [2MASS]),\(^{13}\) and Washington-DDO filters \( M, S, \) and \( T_2 \) (obtained at McDonald Observatory with the 0.8 m prime-focus camera). The 2MASS \( JHK \) have been transformed to the Bessell \& Brett (1988) system using the transformations provided in Carpenter (2001).

\(^{14}\) The WIYN Observatory is a joint facility of the University of Wisconsin–Madison, Indiana University, Yale University, and the National Optical Astronomy Observatory.

\(^{15}\) The Apache Point Observatory 3.5 m telescope is owned and operated by the Astrophysical Research Consortium.

4.2. Spectroscopy

The spectra from which we estimated spectral type and luminosity class come from WIYN\(^{14}\) and New Mexico State University (NMSU) Apache Peak Observatory.\(^{15}\) Classifications used a combination of template matching and line ratios. For this field, we have two sets of spectral types and luminosity class for four out of five stars. Table 6 lists the spectral types and luminosity classes for our reference stars. The differences between the WIYN and NMSU spectral types provide an estimate of \( \sigma_M \). In those instances where the spectral types differ, we adopt the classification closest to that suggested by a \( J–H \) versus \( H–K \) color-color diagram, the spectral type–color mapping least affected by reddening. These colors are listed in Table 4.

The Washington-DDO photometry provides a possible confirmation of the estimated luminosity class. In Figure 3,

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
\text{Year} & \text{MJD} & \text{Star} & \text{\( V \)} & \text{\( B–V \)} & \text{\( \Delta X_f \)} & \text{\( \Delta X_y \)} & \text{\( \Delta m \)} \\
\hline
1995 & 49,790.762 & HD 41940 & 8.14 & 0.05 & \( -3.5 \pm 0.7 \) & \( -6.9 \pm 0.8 \) & 5.29 \\
1998 & 50,814.813 & Upgren 69 & 9.58 & 0.50 & \( -4.5 \pm 0.3 \) & \( -7.2 \pm 0.5 \) & 5.21 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|}
\hline
\text{ID} & \xi^\text{a} & \eta^\text{a} \\
\hline
\text{RR Lyr} & 48.5557 \pm 0.0002 & -36.1748 \pm 0.0003 \\
\text{RR-2} & 12.9877 \pm 0.0001 & -122.0915 \pm 0.0002 \\
\text{RR-4} & 0.0000 \pm 0.0002 & 0.0000 \pm 0.0002 \\
\text{RR-5} & 70.6893 \pm 0.0002 & -7.2290 \pm 0.0003 \\
\text{RR-6} & 72.4101 \pm 0.0002 & 60.4006 \pm 0.0004 \\
\text{RR-8} & 83.8194 \pm 0.0002 & -88.7945 \pm 0.0002 \\
\hline
\end{array}
\]

\(^{a}\) \( \xi \) and \( \eta \) are relative positions in arcseconds.

\(^{b}\) \( \mu_x \) and \( \mu_y \) are relative motions in arcseconds per year.

\(^{c}\) R.A. = 19\textdegree 25\textquoteleft 25\textsecond 56, decl. = 42\textdegree 47\textquoteleft 07 (J2000.0), and epoch = MJD 50,201.05711.

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
\text{Year} & \text{MJD} & \text{Star} & \text{\( V \)} & \text{\( V–R \)} & \text{\( V–I \)} & \text{\( V–K \)} \\
\hline
2 & 12.68 \pm 0.02 & 0.51 \pm 0.03 & 0.68 \pm 0.03 & 1.429 \pm 0.06 \\
4 & 13.47 \pm 0.02 & 0.48 \pm 0.04 & 0.61 \pm 0.04 & 1.325 \pm 0.06 \\
5 & 14.50 \pm 0.02 & 0.67 \pm 0.05 & 0.94 \pm 0.05 & 2.199 \pm 0.06 \\
6 & 13.15 \pm 0.02 & 0.49 \pm 0.03 & 0.70 \pm 0.03 & 1.467 \pm 0.06 \\
8 & 14.94 \pm 0.02 & 0.56 \pm 0.06 & 0.76 \pm 0.06 & 1.633 \pm 0.08 \\
\hline
\end{array}
\]
we plot the Washington-DDO photometry, along with a dividing line between dwarfs and giants (Paltoglou & Bell 1994). The boundary between giants and dwarfs is actually far “fuzzier” than suggested by the solid line in Figure 3 and complicated by the photometric transition from dwarfs to subgiants. This soft boundary is readily apparent in Figure 14 of Majewski et al. (2000). Objects just above the heavy line are statistically more likely to be giants than objects just below the line. All but one of our reference stars lies on the dividing line to the left, where giant/dwarf discrimination is poorest. The remaining star, RR-5, moves closer to the other stars on the dividing line by correcting for the $A_V = 0.14$ indicated in Table 5. Except for this one measurement, all the photometry is consistent with a dwarf classification for each reference star.

### 4.3. Interstellar Extinction

To determine interstellar extinction, we first plot these stars on several color-color diagrams. A comparison of the relationships between spectral type and intrinsic color against those we measured provides an estimate of reddening. Figure 4 contains $V-R$ versus $V-K$ and $V-I$ versus $V-K$ color-color diagrams and total Galactic reddening vectors determined by Schlegel, Finkbeiner, & Davis (1998) along this line of sight. Also plotted are mappings between spectral type and luminosity class V and III from Bessell & Brett (1988) and Cox (2000, hereafter AQ00). Again with maximum reddening vectors and the loci of luminosity class V and III stars. Figure 4, along with the estimated spectral

![Figure 3](image1.png)

**Fig. 3.**—$M$–DDO 51 ($M–51$) vs. $M–T_2$ color-color diagram. The solid line is the division between luminosity class V and luminosity class III stars. Giants are above the line, dwarfs below. The reddening vector is for $A_V = 1.0$. Dereddening star RR-5 by the $A_V = 0.14$ value from Table 5 would move it nearer to the dividing line between giants and dwarfs.

![Figure 4](image2.png)

**Fig. 4.**—$V–R$ vs. $V–K$ and $V–I$ vs. $V–K$ color-color diagrams. The dashed line is the locus of dwarf (luminosity class V) stars of various spectral types; the dot-dashed line is for giants (luminosity class III). The reddening vector is the total Galactic reddening determined by Schlegel et al. (1998).
5. Absolute Parallax of RR Lyr

5.1. Astronomic Model

With the positions measured by FGS 3, we determine the scale, rotation, and offset “plate constants” relative to an arbitrarily adopted constraint epoch (the so-called master plate) for each observation set (the data acquired at each epoch). The MJD of each observation set is listed in Table 7, along with a measured magnitude, a phase (based on \( P = 0.5668 \) days (Kukarin et al. 1971, rephased using the more recent photometry of Schoeneich & Lange 1979), and a \( B-V \) estimated by comparison with the \( UBV \) photometry of Hardie (1955). The RR Lyr reference frame contains five stars. We employ the six-parameter model discussed in McArthur et al. (2001) for those observations. For the RR Lyr field, all the reference stars are redder than the science target. Hence, we apply the corrections for lateral color discussed in Benedict et al. (1999).

As for all our previous astrometric analyses, we employ GaussFit (Jefferys, Fitzpatrick, & McArthur 1987) to minimize \( \chi^2 \). The solved equations of condition for RR Lyr are

\[
\begin{align*}
    x' &= x + \xi_x(B - V) - \Delta X F_x, \\
    y' &= y + \xi_y(B - V) - \Delta X F_y, \\
    \zeta &= \alpha x' + \beta y' + \gamma + R_x(x^2 + y^2) - \mu_x \Delta t - P_{\alpha} \pi_x, \\
    \eta &= -B x' + A y' + F + R_y(x^2 + y^2) - \mu_y \Delta t - P_{\beta} \pi_y,
\end{align*}
\]

where \( x \) and \( y \) are the measured coordinates from \( HST; \) \( \xi_x \) and \( \xi_y \) are the lateral color corrections from Benedict et al. 1999; and \( B-V \) are the \( B-V \) colors of each star, including the variable \( B-V \) of RR Lyr (Table 7). Here \( \Delta X F_x \), and \( \Delta X F_y \) are the cross filter corrections in \( X \) and \( Y \), applied only to the observations of RR Lyr. RR Lyr has a full range of 0.2 < \( B-V \) < 0.6. For this analysis we linearly interpolate between the 1995 and 1998 cross filter calibrations (Table 1) as a function of RR Lyr color. \( A \) and \( B \) are scale and rotation plate constants, \( C \) and \( F \) are offsets, \( R_x \) and \( R_y \) are radial terms, \( \mu_x \) and \( \mu_y \) are proper motions, \( \Delta t \) is the epoch difference from the mean epoch, \( P_{\alpha} \) and \( P_{\beta} \) are parallax factors, and \( \pi_x \) and \( \pi_y \) are the parallaxes in \( x \) and \( y \). We obtain the parallax factors from a JPL Earth orbit predictor (Standish 1990), upgraded to version DE405. Orientation to the sky is obtained from ground-based astrometry (USNO-A2.0 catalog, Monet 1998) with uncertainties in the field orientation of ±0.05.

Solutions carried out constraining four reference stars to have no proper motion, but allowing proper motion for the remaining reference star, indicate a statistically significant proper motion for reference star RR-5. Estimating the

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**Table 5**  
Reference Star \( A_V \) from Spectrophotometry

| RR  | \( A_V(V-I) \) | \( A_V(V-R) \) | \( A_V(V-K) \) | \( \langle A_V \rangle \) |
|-----|----------------|----------------|----------------|----------------|
| 2... | 0.07           | 0.05           | -0.01          | 0.04 ± 0.03    |
| 4... | 0.00           | 0.10           | 0.04           | 0.05 ± 0.03    |
| 5... | 0.19           | 0.15           | 0.06           | 0.14 ± 0.04    |
| 6... | 0.12           | -0.10          | 0.03           | 0.02 ± 0.06    |
| 8... | 0.17           | 0.26           | 0.09           | 0.17 ± 0.05    |
| \( \langle A_V \rangle \) | 0.09           | 0.08           | 0.04           | 0.07 ± 0.03    |

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**Table 6**  
Astrometric Reference-Star Spectral Classifications and Spectrophotometric Parallaxes

| RR  | WIYN | NMSU | Adopted | \( V \) | \( M_V \) | \( A_V \) | \( \pi_{obs} \) | \( \sigma_{\pi/\pi} \) (%) |
|-----|------|------|---------|-------|--------|--------|---------------|------------------|
| 2... | G1 V | G3 V | G1 V    | 12.68 | 4.6 ± 0.4 | 0.07   | 2.4 ± 0.4     | 18               |
| 4... | G0 V | F8 V | F8 V    | 13.47 | 4.6 ± 0.4 | 0.07   | 1.7 ± 0.3     | 28               |
| 5... | K1 V | K1 V | K1 V    | 14.50 | 5.5 ± 0.6 | 0.07   | 1.9 ± 0.6     | 18               |
| 6... | G1 V | G0 V | G1 V    | 13.15 | 4.6 ± 0.4 | 0.07   | 1.9 ± 0.4     | 18               |
| 8... | G2 V | G5 V | G5 V    | 14.94 | 5.1 ± 0.4 | 0.07   | 1.1 ± 0.2     | 25               |
proportional motion of that star imposed an 8% decrease in number of degrees of freedom but resulted in a 15% decrease in \( \chi^2 \).

5.2. Assessing Reference-Frame Residuals

The optical field angle distortion calibration (McArthur et al. 1997) reduces as-built HST and FGS 3 distortions with magnitude \( \sim 1'' \) to below 2 mas over much of the FGS 3 field of regard. From histograms of the astrometric residuals (Fig. 5), we conclude that we have obtained correction at the \( \sim 1 \) mas level in the region available at all HST rolls (an inscribed circle centered on the pickle-shaped FGS field of regard). The resulting reference-frame "catalog" in \( \xi \) and \( \eta \) standard coordinates (Table 2) was determined with \( \langle \sigma_\xi \rangle = 0.2 \) and \( \langle \sigma_\eta \rangle = 0.3 \) mas.

To determine if there might be unmodeled—but possibly correctable—systematic effects at the 1 mas level, we plotted the RR Lyr reference-frame \( X \) and \( Y \) residuals against a number of spacecraft, instrumental, and astronomical parameters. These included \( (X, Y) \)-position within the pickle, radial distance from the pickle center, reference-star \( V \) magnitude and \( B-V \) color, and epoch of observation. We saw no obvious trends other than an expected increase in positional uncertainty with reference-star magnitude.

5.3. Absolute Parallax of RR Lyr

In a quasi-Bayesian approach, the calibration values were entered into the model as observations with associated errors. The reference-star spectrophotometric absolute parallaxes also were input as observations with associated errors not as hardwired quantities known to infinite precision.

We obtain for RR Lyr an absolute parallax of \( \pi_{\text{abs}} = 3.82 \pm 0.11 \) mas thus \( \sigma_\pi/\pi = 3\% \). This parallax differs by \( \sim \sigma_{\text{HIP}} \) and \( \sim 4 \sigma_{\text{HIP}} \) from that measured by Hipparcos, \( \pi_{\text{Hip}} = 4.38 \pm 0.59 \) mas. Comparing our various solutions with and without reference-star proper motion, we find somewhat sensitivity in the resulting parallax, with a full range of variation of 0.2 mas. We feel this range represents a more likely error in our determination and adopt as the HST absolute parallax of RR Lyr \( \pi_{\text{abs}} = 3.82 \pm 0.2 \) mas \( (\sigma_\pi/\pi = 4.6\%) \), an error one-third that of Hipparcos. Figure 6 compares the HST, Hipparcos, and YPC95 (a weighted average of past ground-based results) determinations. The horizontal line is a weighted average of all three sources, \( \langle \pi_{\text{abs}} \rangle = 3.87 \pm 0.19 \) mas. Parallax and proper-motion results from HST, Hipparcos, and YPC95 are collected in Table 8.

6. DISCUSSION AND SUMMARY

6.1. HST Parallax Accuracy

Our parallax precision, an indication of our internal, random error, is often less than 0.5 mas. To assess our accuracy, or external error, we must compare our parallaxes with results from independent measurements. Following Gatewood, Kiewiet de Jonge, & Persinger (1998), we plot all parallaxes obtained by the HST Astrometry Science Team with FGS 3 against those obtained by Hipparcos. These data are collected in Table 9 and are shown in Figure 7. We have not considered four Hyades stars whose parallaxes are considered preliminary (van Altena et al. 1997). The dashed

| Set | MJD     | Phase | \( V \) | \( B-V \) |
|-----|---------|-------|--------|---------|
| 1... | 49,984.76525 | 0.36  | 7.99   | 0.3     |
| 2... | 50,047.5617  | 0.24  | 7.70   | 0.3     |
| 3... | 50,173.4915  | 0.34  | 7.96   | 0.3     |
| 4... | 50,201.5778  | 0.65  | 8.19   | 0.44    |
| 5... | 50,229.3208  | 0.62  | 8.24   | 0.44    |
| 6... | 50,563.3976  | 0.29  | 7.86   | 0.3     |
| 7... | 50,568.1053  | 0.46  | 8.08   | 0.4     |
| 8... | 50,745.7069  | 0.65  | 8.26   | 0.44    |
| 9... | 50,749.7438  | 0.16  | 7.39   | 0.2     |

\( a \) Phase based on \( P = 0.5668 \) days (Kukarkin et al. 1971).

\( b \) From FGS photometry. See Benedict et al. 1998 for transformation details.

\( c \) Estimated from phase using \( UBV \) photometry from Hardie 1955.
line is a weighted regression that takes into account errors in both input data sets. The regression demonstrates the lack of scale and zero-point differences between Hipparcos and HST FGS results. The rms Hipparcos residual to the regression line is 1.02 mas.

6.2. Lutz-Kelker Bias

When using a trigonometric parallax to estimate the absolute magnitude of a star, a correction should be made for the Lutz-Kelker (LK) bias (Lutz & Kelker 1973). Because of the galactic latitude and distance of RR Lyr and the scale height of the stellar population of which it is a member, we use a uniform space density for calculating the LK bias. An LK algorithm modified by Hanson (1979) (H) that includes a uniform space density for calculating the LK bias. An correction should be made for the absolute magnitude of a star, a correction should be made for the Lutz-Kelker bias (Lutz & Kelker 1973). Because of the galactic latitude and distance of RR Lyr and the scale height of the stellar population of which it is a member, we use a uniform space density for calculating the LK bias. An LK algorithm modified by Hanson (1979) (H) that includes a uniform space density for calculating the LK bias. An LK algorithm modified by Hanson (1979) (H) that includes a uniform space density for calculating the LK bias. An LK algorithm modified by Hanson (1979) (H) that includes a uniform space density for calculating the LK bias. An LK algorithm modified by Hanson (1979) (H) that includes a uniform space density for calculating the LK bias.

6.3. Absolute Magnitudes of RR Lyr

Adopting for RR Lyr an intensity weighted average of \( (V) = 7.76 \) (Fernley et al. 1998) and the absolute parallax weighted average from Table 8, we determine that in the absence of reddening \( M_V^{\text{RR}} = 0.68 \pm 0.10 \), including the LKH correction and uncertainty. We derived (\( \S 4.3 \)) an average \( \langle A_V \rangle = 0.07 \pm 0.03 \) from the astrometric reference stars that surround RR Lyr. Fernley et al. (1998) obtain for RR Lyr \( A_V = 0.06 \pm 0.03 \) from a log \( P -(V-K) \) relation. If there is no patchy extinction with angular scale less than \( 1^\circ \), it seems reasonable to conclude that RR Lyr, less distant than any reference star, has \( A_V \leq 0.07 \) and, hence, \( M_V^{\text{RR}} \leq 0.61 \). Including this 0.03 mag uncertainty in \( \langle A_V \rangle \) in quadrature, we obtain \( M_V^{\text{RR}} = 0.61 \pm 0.10 \). Alternatively, we could accept the \( A_V \) variations seen in Figure 2 as real and correct for a

![Fig. 6.—Absolute parallax determinations for RR Lyr. We compare HST, Hipparcos, and YPC95. The horizontal dashed line gives the weighted average absolute parallax, \( \langle \pi_{\text{abs}} \rangle \).](image)

![Fig. 7.—Bottom: HST absolute parallax determinations compared with Hipparcos for the five targets listed in Table 9. Top: The Hipparcos residuals to the error-weighted regression line. The error bars on the residuals are Hipparcos 1 \( \sigma \) errors.](image)

### TABLE 8

**RR Lyr Parallax and Proper Motion**

| Parameter                                      | Value    |
|-----------------------------------------------|----------|
| HST study duration (yr)                       | 2.09     |
| Number of observation sets                    | 10       |
| Reference stars \( (V) \)                    | 13.75    |
| Reference stars \( (B-V) \)                   | 0.71     |
| HST absolute parallax (mas)                   | 3.82 ± 0.20 |
| Hipparcos absolute parallax (mas)             | 4.38 ± 0.59 |
| YPC95 absolute parallax (mas)                  | 3.0 ± 1.9 |
| Weighted-average absolute parallax (mas)       | 3.87 ± 0.19 |
| HST proper motion (mas yr\(^{-1}\))            | 224.0 ± 0.5 |
| In position angle (deg)                       | 208.3 ± 0.5 |
| Hipparcos proper motion (mas yr\(^{-1}\))      | 224.2 ± 1.4 |
| In position angle (deg)                       | 209.3 ± 1.3 |
| YPC95 proper motion (mas yr\(^{-1}\))          | 207.4    |
| In position angle (deg)                       | 210.7    |

### TABLE 9

**HST and Hipparcos Absolute Parallaxes**

| ID                  | HST   | Hipparcos | HST Reference |
|---------------------|-------|-----------|---------------|
| Prox Cen            | 769.7 ± 0.3 | 772.33 ± 2.42 | Benedict et al. 1999 |
| Barnard's star      | 545.5 ± 0.3 | 549.3 ± 1.58 | Benedict et al. 1999 |
| Feige 24           | 14.6 ± 0.4 | 13.44 ± 3.62 | Benedict et al. 2000 |
| Gl 748 AB          | 98.0 ± 0.4 | 98.56 ± 2.66 | Benedict et al. 2001 |
| RR Lyr             | 3.82 ± 0.20 | 4.38 ± 0.59 | This paper |

A correction and uncertainty. We derived (\( \S 4.3 \)) an average \( \langle A_V \rangle = 0.07 \pm 0.03 \) from the astrometric reference stars that surround RR Lyr. Fernley et al. (1998) obtain for RR Lyr \( A_V = 0.06 \pm 0.03 \) from a log \( P -(V-K) \) relation. If there is no patchy extinction with angular scale less than \( 1^\circ \), it seems reasonable to conclude that RR Lyr, less distant than any reference star, has \( A_V \leq 0.07 \) and, hence, \( M_V^{\text{RR}} \leq 0.61 \). Including this 0.03 mag uncertainty in \( \langle A_V \rangle \) in quadrature, we obtain \( M_V^{\text{RR}} = 0.61 \pm 0.10 \). Alternatively, we could accept the \( A_V \) variations seen in Figure 2 as real and correct for a
linearly interpolated $A_V = 0.11 \pm 0.10$, local to RR Lyr. Including that uncertainty in quadrature, we obtain $M_{V}^{RR} = 0.57_{-0.14}^{+0.15}$. Our range of values for $M_{V}^{RR}$ is remarkably close to that determined by Tsujimoto et al. (1998) from the Hipparcos parallax. We ascribe this similarity to differing LKH bias corrections and different $A_V$ corrections.

Beers et al. (2000) cite an $[\text{Fe/H}] - M_V$ relation from Chaboyer (1999),

$$M_V^{RR} = 0.23([\text{Fe/H}] + 1.6) + 0.56. \quad (6)$$

An $[\text{Fe/H}] = -1.39$ value for RR Lyr (also from Beers et al.
| No. | Method                        | Object              | Author                          | $m - M$ |
|-----|-------------------------------|---------------------|---------------------------------|---------|
|     |                               |                     |                                 |         |
| 1...| Baade-Wesselink               | Cepheids            | Gieren et al. 2000              | 18.42 ± 0.10 |
| 2...| Cepheids                      | Carretta et al. 2000a |                                | 18.55 ± 0.10 |
| 3...| Cepheid                       | Gieren, Fouqué, & Gómez 1998 |                            | 18.46 ± 0.02 |
| 4...| Cepheid                       | Di Benedetto 1997   |                                 | 18.58 ± 0.024 |
| 5...| RR Lyraes                     | Carretta et al. 2000b |                                | 18.52 ± 0.20 |
| 6...| RR Lyraes                     | Feast 1997          |                                 | 18.53 ± 0.04 |
| 7...| RR Lyraes                     | Cacciari, Clementini, & Fernley 1992, |                  | 18.40 ± 0.20 |
| 8...| RR Lyraes                     | McNamara 1997       |                                 | 18.54 ± 0.10 |
| 9...| Double mode                   | RR Lyraes           | Alcock et al. 1997              | 18.48 ± 0.19 |
| 10...| RR Lyraes                     | Kovač 2000          |                                 | 18.52 ± 0.21 |
| 11...| Eclipsing binaries           | EROS 1044           | Maloney et al. 2001             | 18.25 ± 0.25 |
| 12...| Cepheids                      | HV 2274             | Nelson et al. 2000              | 18.40 ± 0.07 |
| 13...| Cepheids                      | HV 982              | Fitzpatrick et al. 2001         | 18.31 ± 0.09 |
| 14...| Cepheids                      | HV 2274             | Guinan et al. 1998a             | 18.42 ± 0.07 |
| 15...| Cepheids                      | HV 2274             | Guinan et al. 1998b             | 18.30 ± 0.07 |
| 16...| RR Lyraes                     | Udalski 1998a       |                                 | 18.22 ± 0.13 |
| 17...| RR Lyraes                     | Guinan et al. 1997  |                                 | 18.54 ± 0.08 |
| 18...| Globular cluster dyn. mods    | Mira                | Schmidt-Kaler & Oestreicher 1998 | 18.34 ± 0.09 |
| 19...| High amplitude δ Scuti        | δ Scuti             | McNamara 2001                   | 18.66 ± 0.08 |
| 20...| Long-period variables         | Mira                | Whitelock & Feast 2000          | 18.64 ± 0.17 |
| 21...| M stars luminosity            | Ca Stars            | Bergeat, Knapik, & Rutly 1998  | 18.50 ± 0.17 |
| 22...| M stars luminosity            | Mira                | Van Leeuwen et al. 1997         | 18.54 ± 0.18 |
| 23...| M stars luminosity            | Schmidt-Kaler & Oestreicher 1998 | 18.34 ± 0.09 |
| 24...| Main-sequence fitting        | NGC 1866            | Walker et al. 2001              | 18.33 ± 0.05 |
| 25...| Cepheids                      | Carretta et al. 2000a |                                | 18.55 ± 0.04 ± 0.04 |
| 26...| Cepheids                      | Laney & Stobie 1994 |                                 | 18.49 ± 0.04 ± 0.04 |
| 27...| Maser                         | NGC 4258            | Newman et al. 2001              | 18.31 ± 0.17 ± 0.17 |
| 28...| LMC RR Lyraes                 | McNamara 2001       |                                 | 18.61 ± 0.04 |
| 29...| Modeling Li-rich Ca stars     | M31                 | Ventura, D’Antona, & Mazzitelli 1999 | 18.70 ± 0.25 |
| 30...| Linear-pulsation modeling    | Cepheids            | Wood 1998                       | 18.54 ± 0.08 |
| 31...| Planetary nebulae luminosity | M31                 | Walker 1999                     | 18.50 ± 0.18 |
| 32...| Red clump                     | Popowski 2001       |                                 | 18.33 ± 0.07$^a$ |
| 33...| ...                            | Girardi & Salaris 2001 |                                | 18.55 ± 0.05 |
| 34...| ...                            | Sakai, Zaitksy, & Kennicutt 2000 |                      | 18.29 ± 0.03 |
| 35...| ...                            | Popowski 2000       |                                 | 18.27 ± 0.06$^b$ |
| 36...| ...                            | Stanek et al. 2000  |                                 | 18.24 ± 0.08 |
| 37...| ...                            | Udalski 2000a       |                                 | 18.24 ± 0.08 |
| 38...| ...                            | Twarog, Anthony-Twarog, & Bricker 1999 |                  | 18.42 ± 0.16 |
| 39...| ...                            | Stanek, Zaitksy, & Harris 1998 | 18.065 ± 0.12 |
| 40...| ...                            | Udalski et al. 1998b |                                 | 18.08 ± 0.15 |
| 41...| ...                            | Udalski et al. 1998a |                                 | 18.09 ± 0.16 |
| 42...| ...                            | Udalski 1998b       |                                 | 18.18 ± 0.06 |
| 43...| ...                            | Romaniello et al. 2000 |                                 | 18.59 ± 0.04 ± 0.08 |
| 44...| ...                            | Cole 1998           |                                 | 18.36 ± 0.17 |
| 45...| ...                            | Girardi et al. 1998 |                                 | 18.28 ± 0.14 |
| 46...| ...                            | Beaulieu & Sackett 1998 |                                | 18.3 |
| 47...| Red clump and RR Lyraes       | Popowski 2001       |                                 | 18.24 ± 0.08 to 18.44 ± 0.07 |
| 48...| SN 1987A                      | Carretta et al. 2000 |                                | 18.58 ± 0.05 |
| 49...| ...                            | Romaniello et al. 2000 |                                | 18.55 ± 0.05 |
| 50...| ...                            | Walker 1999         |                                 | 18.55 ± 0.07 ± 0.16 |
| 51...| ...                            | Gould & Uza 1998    |                                 | 18.37 ± 0.04 |
| 52...| ...                            | Panagia, Gilmozzi, & Kirchner 1998 |                  | 18.58 ± 0.08 |
| 53...| ...                            | Lundqvist & Sonneborn 1998 |                                | 18.67 ± 0.05 |
| 54...| Statistical parallaxes        | RR Lyraes           | Carretta et al. 2000a           | 18.38 ± 0.12 |
| 55...| ...                            | Popowski & Gould 1999 |                                | 18.33 ± 0.08$^a$ |
| 56...| ...                            | Popowski & Gould 1999 |                                | 18.23 ± 0.08$^b$ |
| 57...| ...                            | Popowski & Gould 1998 |                                | 18.07 ± 0.15$^a$ |
| 58...| ...                            | Popowski & Gould 1998 |                                | 18.31 ± 0.14$^b$ |
| 59...| ...                            | Gould & Popowski 1998 |                                | 18.24 ± 0.14 |
| 60...| ...                            | Fernley et al. 1998 |                                 | 18.26 ± 0.15 |
| 61...| ...                            | Layden et. al 1996  |                                 | 18.28 ± 0.03 |
| 62...| Subdwarf fitting              | Carretta et al. 2000a |                                | 18.64 ± 0.12 |
| 63...| ...                            | Reid 1998           |                                 | 18.79 ± 0.17 |
| 64...| ...                            | Reid 1997           |                                 | 18.65 ± 0.12 |
2000) implies $M_{V}^{RR} = 0.61$, in agreement with either our $M_{V}^{RR}$ value derived from a variable $A_V$ or the $M_{V}^{RR}$ value derived from the average $\langle A_V \rangle$.

### 6.4. Distance Modulus of the LMC

The distance to the LMC is a critical link in determining the scale of the universe. This distance uncertainty contributes a substantial fraction of the uncertainty in the Hubble constant (Mould et al. 2000). The HST Key Project on the extragalactic distance scale (Freedman et al. 2001; Mould et al. 2000) and the Type Ia Supernovae Calibration Team (Saha et al. 1999) have adopted the distance modulus value $m-M = 18.5$. Values from 18.1 to 18.8 are reported in the current literature, with those less than 18.5 supporting the short distance scale, and those greater than 18.5, the long distance scale. Comprehensive reviews of the methods can be found in Carretta et al. (2000a), Gibson (1999), and Cole (1998). A representative sample of opinion about the “best method” can be found in Paczynski (2001), Popowski (2001), Udalski (2000b), and Gould (2000).

Let us proceed with the constant $\langle A_V \rangle$ result $M_{V}^{RR} = 0.61^{+0.11}_{-0.10}$. Because of our two results for absolute magnitude, it has smaller formal errors. We adopt (from Carretta et al. 2000a) $\langle V \rangle = 19.11$ for the RR Lyr, variables in the bar of the LMC and their assumed $\langle [\text{Fe}/\text{H}] \rangle = -1.5$. Correcting for the differential variation of $M_V$ with $\langle [\text{Fe}/\text{H}] \rangle$ (using the slope from Chaboyer 1999), we compute $\langle V \rangle = 19.14$, correcting from $\langle [\text{Fe}/\text{H}] \rangle = -1.50$ to $\langle [\text{Fe}/\text{H}] \rangle = -1.39$. We obtain an LMC distance modulus of $m-M = 18.53 \pm 0.12$. Carretta, Gratton, & Clementini (2000b) address possible luminosity differences between horizontal-branch stars in globular clusters and in the field and conclude that there is little difference, unless the masses are very different. There is another source of uncertainty in the LMC distance modulus, the measured apparent magnitude of the RR Lyr population in the LMC corrected for extinction local to the LMC. For example, adopting (Udalski et al. 1999) $\langle V \rangle = 18.94$ and $\langle [\text{Fe}/\text{H}] \rangle = -1.6$ for the RR Lyr in the LMC, we obtain an LMC distance modulus of $m-M = 18.38 \pm 0.12$, once again correcting $\langle V \rangle$ to $\langle [\text{Fe}/\text{H}] \rangle = -1.39$.

These two estimates, which agree within their respective errors, are included in Figure 8, a summary of the current LMC distance modulus situation, displaying over 80 determinations, based on 21 independent methods. These are listed in Table 10. The weighted average of these is $\langle m-M \rangle = 18.47 \pm 0.04$, where the error is derived as the standard deviation of the mean with $N = 21$.

### 6.5. Summary

HST astrometry yields an absolute trigonometric parallax for RR Lyr, $\pi_{abs} = 3.82 \pm 0.2$ mas. A weighted average of HST, Hipparcos, and YPC95 absolute parallaxes is $\langle \pi_{abs} \rangle = 3.87 \pm 0.19$ mas. This high-precision result requires an extremely small LK bias correction of $-0.02 \pm 0.01$ mas. Spectrophotometry of the astrometric reference stars local to RR Lyr suggest a low extinction, $\langle A_V \rangle = 0.07 \pm 0.03$. The dominant error terms in the resulting absolute magnitude, $M_{V}^{RR} = 0.61^{+0.11}_{-0.10}$, $\langle [\text{Fe}/\text{H}] \rangle = -1.39$, are the parallax and the uncertainty in the amount of extinction for RR Lyr itself. Depending on metallicity determinations and extinction corrections for RR Lyr variables in the LMC, we find a distance modulus range on the low end of $m-M = (18.38-18.53) \pm 0.12$, marginally supporting the “short scale” and an $H_0$ at the higher end of the present range.
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