New QCD Sum Rules for Nucleon Axial Vector Coupling Constants

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We study the predictive ability of the QCD Sum Rule method for \( g_A \), using a comprehensive Monte Carlo based uncertainty analysis. This is the first application of such analysis to a three-point function.

In this approach, the complete QCD parameter space is explored, allowing a quantitative study of how the uncertainties in the QCD input parameters propagate to the phenomenological parameters. The Borel window over which the phenomenological and QCD sides of the sum rules are matched is determined by the following criteria: a) OPE convergence | the last term of the truncated OPE series is less than 10% of the total OPE side, b) ground-state dominance | all excited state contributions are no more than 50% of the phenomenological side. Those sum rules which do not meet these criteria are considered invalid and are discarded.

Two sets of new QCD sum rules for \( g_A \) are derived in the external field method, using generalized nucleon interpolating fields. Three sum rules are derived from the correlator \( \langle 1_{-2} 1_{+2} \rangle \) of spin-1/2 interpolating fields: \( 1_{-2} = 1 + \frac{\alpha}{2} \) where \( \alpha \) is a real parameter, \( 1 = abc \, u^a \, T \, C \, u^b \, \bar{u} \), and \( 2 = abc \, u^a \, T \, C \, \bar{d} \, \bar{u} \). The Io e current may be recovered by setting \( \alpha = 1 \). However, Io e’s choice is not optimal. Another set of 8 sum rules is derived from the mixed correlator \( \langle 1_{+2} 1_{-2} \rangle \) of spin-1/2 and spin-3/2 fields:

\[
1_{+2} = 5 \, 1_{-2} \quad \text{and} \quad 2_{+2} = 5 \, 2_{-2} = \, \text{abc} \, \{u^a \, T \, C \, d^b \} \, \bar{u} \, \{u^a \, T \, C \, u^b \} \, \bar{d} \,
\]

The latter has both spin-1/2 and spin-3/2 components.

Our analysis reveals that the sum rule from the spin-1/2 interpolators which was chosen in previous works does not have a valid Borel window. Therefore the previous predictions for \( g_A \) are unreliable. On the other hand, the sum rule at the structure \( I_{-2} = 1 + \frac{\alpha}{2} \) has a valid Borel window of 0.91 GeV to 1.16 GeV with the optimal \( \alpha = 12 \). A combined analysis of this sum rule together with the nucleon mass sum rules yields \( g_A = 1.48 \pm 0.05 \). The relative error of approximately 60% is large compared to a 10% error for the nucleon mass obtained from the same input parameters. Fig. 1 shows distributions for selected \( T \) parameters drawn from 1000 sets of QCD input parameters. Fig. 2 shows two of the parameters that have significant correlations with \( g_A \). The origin of the large error in \( g_A \) is mainly due to the poorly determined pole residue, as \( g_A \) is extracted from the form \( \frac{2}{5} g_A \, e^{M^2} \). The
vacuum susceptibility ν also contributes to this large uncertainty. Preliminary analysis of the sum rules from the mixed correlator shows no improvement over this conclusion. Analysis of other axial couplings is underway.

**Figure 1:** Histograms of selected parameters drawn from 1000 sets of QCD parameters. We find: nucleon mass $M_N = 0.94 \pm 0.04$ GeV, pole residue $\gamma_1 = 0.2 \pm 0.8$ GeV$^{-1}$, transition strength $A_3 = 1.08 \pm 0.06$ GeV$^{-1}$, and $g_A = 1.08 \pm 0.04$.

**Figure 2:** Scatter plots of correlations with $q_A$.

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1. D. B. Leinweber, "QCD Sum Rules for Skeptics," UW -17-95, nucl-th/9510051, Phys Rev. D 51, 6383 (1995).
2. V M. Beltyaev and Ya I. Kogan, Phys. Lett. 136B, 273 (1984); C. Chiu, J. Pasupathy, and S. Wilson, Phys. Rev. D 32, 1786 (1985); E M. Henley et al., Phys. Rev. D 46, 431 (1992).