Basic Properties of Compressible MHD Turbulence: Implications for Molecular Clouds

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Abstract. Recent advances in understanding of the basic properties of compressible Magnetohydrodynamic (MHD) turbulence call for revisions of some of the generally accepted concepts. First, MHD turbulence is not so messy as it is usually believed. In fact, the notion of strong non-linear coupling of compressible and incompressible motions is not tenable. Alfven, slow and fast modes of MHD turbulence follow their own cascades and exhibit degrees of anisotropy consistent with theoretical expectations. Second, the fast decay of turbulence is not related to the compressibility of fluid. Rates of decay of compressible and incompressible motions are very similar. Third, viscosity by neutrals does not suppress MHD turbulence in a partially ionized gas. Instead, MHD turbulence develops magnetic cascade at scales below the scale at which neutrals damp ordinary hydrodynamic motions. The implications of those changes of MHD turbulence paradigm for molecular clouds require further studies. Those studies can benefit from testing of theoretical predictions using new statistical techniques that utilize spectroscopic data. We briefly discuss advances in development of tools using which the statistics of turbulent velocity can be recovered from observations.

Keywords: turbulence, molecular clouds, MHD

1. What is Compressible MHD Turbulence?

It is well known that molecular clouds are magnetized with compressible magnetic turbulence determining most of their properties (see Elmegreen & Falgarone 1996, Stutzki 2001). Star formation (see McKee & Tan 2002, Elmegreen 2002, Pudritz 2001), cloud chemistry (see Falgarone 1999 and references therein), shattering and coagulation of dust (see Lazarian & Yan 2003 and references therein) are examples of processes for which knowledge of turbulence is absolutely essential. There are many excellent reviews that deal the theory of turbulent molecular clouds and numerical simulations (see Falgarone 1999, Vazquez-Semadeni et al. 2000, Mac Low & Klessen 2003). This short review is focused on the recently uncovered basic properties of MHD turbulence and their implications for understanding of molecular clouds\(^1\). We also briefly deal with recovering of the 3D statistics of

\(^1\) It is not possible to cite all the important papers in the area of MHD turbulence and turbulent molecular clouds. An incomplete list of the references in a recent review on the statistics of MHD turbulence by Cho, Lazarian & Vishniac (2003a;
turbulent velocity from observations, which is a theoretical problem in itself. A more detailed discussion of MHD turbulence can be found in the companion review by Chandran, while observational aspects of turbulence in molecular clouds are discussed in the review by Falgarone (this volume).

Why do we expect molecular clouds to be turbulent? A fluid of viscosity \( \nu \) becomes turbulent when the rate of viscous dissipation, which is \( \sim \nu/L^2 \) at the energy injection scale \( L \), is much smaller than the energy transfer rate \( \sim V_L/L \), where \( V_L \) is the velocity dispersion at the scale \( L \). The ratio of the two rates is the Reynolds number \( Re = V_L L/\nu \). In general, when \( Re \) is larger than \( 10 - 100 \) the system becomes turbulent. Chaotic structures develop gradually as \( Re \) increases, and those with \( Re \sim 10^3 \) are appreciably less chaotic than those with \( Re \sim 10^8 \). Observed features such as star forming clouds are very chaotic for \( Re > 10^8 \). This makes it difficult to simulate realistic turbulence. The currently available 3D simulations containing 512 grid cells along each side can have \( Re \) up to \( \sim O(10^3) \) and are limited by their grid sizes. Therefore, it is essential to find “scaling laws” in order to extrapolate numerical calculations (\( Re \sim O(10^3) \)) to real astrophysical fluids (\( Re > 10^8 \)). We show below that even with its limited resolution, numerics is a great tool for testing scaling laws.

Kolmogorov theory provides a scaling law for incompressible non-magnetized hydrodynamic turbulence (Kolmogorov 1941). This law provides a statistical relation between the relative velocity \( v_l \) of fluid elements and their separation \( l \), namely, \( v_l \sim l^{1/3} \). An equivalent description is to express spectrum \( E(k) \) as a function of wave number \( k \) (\( \sim 1/l \)). The two descriptions are related by \( kE(k) \sim v_l^2 \). The famous Kolmogorov spectrum is \( E(k) \sim k^{-5/3} \). The applications of Kolmogorov theory range from engineering research to meteorology (see Monin & Yaglom 1975) but its astrophysical applications are poorly justified and the application of the Kolmogorov theory can lead to erroneous conclusions (see reviews by Lazarian et al. 2003 and Lazarian & Yan 2003)

Let us consider incompressible MHD turbulence first. There have long been understanding that the MHD turbulence is anisotropic (e.g. Shebalin et al. 1983). Substantial progress has been achieved recently by Goldreich & Sridhar (1995; hereafter GS95), who made an ingenious prediction regarding relative motions parallel and perpendicular to magnetic field \( B \) for incompressible MHD turbulence. An important observation that leads to understanding of the GS95 scaling is that

henceforth CLV03a) includes about two hundred entries. The list of references in the molecular cloud review by Mac Low & Klessen (2003) is even longer.
magnetic field cannot prevent mixing motions of magnetic field lines if the motions are perpendicular to the magnetic field. Those motions will cause, however, waves that will propagate along magnetic field lines. If that is the case, the time scale of the wave-like motions along the field, i.e. $\sim l_||/V_A$, ($l_||$ is the characteristic size of the perturbation along the magnetic field and $V_A = B/\sqrt{4\pi \rho}$ is the local Alfvén speed) will be equal to the hydrodynamic time-scale, $l_\perp/v_\perp$, where $l_\perp$ is the characteristic size of the perturbation perpendicular to the magnetic field. The mixing motions are hydrodynamic-like\(^2\). They obey Kolmogorov scaling, $v_\perp \propto l_\perp^{\frac{1}{3}}$, because incompressible turbulence is assumed. Combining the two relations above we can get the GS95 anisotropy, $l_|| \propto l_\perp^{\frac{2}{3}}$ (or $k_|| \propto k_\perp^{\frac{2}{3}}$ in terms of wave-numbers). If we interpret $l_||$ as the eddy size in the direction of the local magnetic field\(^3\) and $l_\perp$ as that in the perpendicular directions, the relation implies that smaller eddies are more elongated.

GS95 predictions have been confirmed numerically (Cho & Vishniac 2000; Maron & Goldreich 2001; Cho, Lazarian & Vishniac 2002a, hereafter CLV02a; see also CLV03a); they are in good agreement with observed and inferred astrophysical spectra (see CLV03a). However, the GS95 model considered incompressible MHD, but the media in molecular clouds is highly compressible. Does any part of GS95 model survives? Literature on the properties of compressible MHD is very rich (see reviews by Pouquet 1999, Cho & Lazarian 2003 and references therein). Higdon (1984) theoretically studied density fluctuations in the interstellar MHD turbulence. Matthaeus & Brown (1988) studied nearly incompressible MHD at low Mach number and Zank & Matthaeus (1993) extended it. In an important paper Matthaeus et al. (1996) numerically explored anisotropy of compressible MHD turbulence. However, those papers do not provide universal scalings of the GS95 type.

The complexity of the compressible magnetized turbulence with magnetic field made some researchers believe that the phenomenon is too complex to expect any universal scalings for molecular cloud research. High coupling of compressible and incompressible motions is often quoted to justify this point of view.

\(^{2}\) Recent simulations (Cho et al. 2003c) suggest that perpendicular mixing is indeed efficient for mean magnetic fields of up to the equipartition value.

\(^{3}\) Incidentally, if we identify the size of molecular cloud with the longer dimension of an eddy, we shall get the Larson relations $v_\perp \sim l^{\frac{1}{2}}$ (Larson 1981) for velocity and cloud size.
Figure 1. Separation method. We separate Alfven, slow, and fast modes in Fourier space by projecting the velocity Fourier component $v_k$ onto bases $\xi_A$, $\xi_s$, and $\xi_f$, respectively. Note that $\xi_A = -\hat{\phi}$. Slow basis $\xi_s$ and fast basis $\xi_f$ lie in the plane defined by $B_0$ and $k$. Slow basis $\xi_s$ lies between $-\hat{\theta}$ and $k_\parallel$. Fast basis $\xi_f$ lies between $\hat{k}$ and $k_\perp$.

Below we shall provide arguments that are suggestive that the fundamentals of compressible MHD can be understood and successfully applied to molecular clouds.

In molecular clouds the regular magnetic field is comparable with the fluctuating one. Therefore for most part of our discussion, we shall discuss results obtained for $\delta V \sim \delta B / \sqrt{4\pi \rho} \sim B_0 / \sqrt{4\pi \rho}$, where $\delta B$ is the r.m.s. strength of the random magnetic field.

2. Does the Decay of MHD Turbulence Depend on Compressibility?

Turbulent support of molecular clouds (see review by McKee 1999) critically depends on the rate of turbulence decay. For a long time magnetic fields were thought to be the means of decreasing dissipation of turbulence. Numerical calculations by Mac Low et al. (1998) and Stone et al. (1998) indicated that compressible MHD turbulence decays as fast as the hydrodynamic turbulence. This gives rise to a belief that it is the compressibility that is responsible for the rapid decay of MHD turbulence.

This point of view has been recently subjected to scrutiny in Cho & Lazarian (2002, 2003a, henceforth CL02 and CL03, respectively). In these papers a technique of separating different MHD modes was developed and used (see Fig. 1). This allowed us to follow how the energy was redistributed between these modes.

How is this idealized incompressible model related to the actual turbulence in molecular clouds? Compressible MHD turbulence is a highly non-linear phenomenon and it has been thought that different types of perturbations or modes (Alfven, slow and fast) in compressible media...
are strongly coupled. Nevertheless, one may question whether this is true. A remarkable feature of the GS95 model is that Alfven perturbations cascade to small scales over just one wave period, while the other non-linear interactions require more time. Therefore one might expect that the non-linear interactions with other types of waves should affect Alfvenic cascade only marginally. Moreover, since the Alfven waves are incompressible, the properties of the corresponding cascade may not depend on the sonic Mach number.

The generation of compressible motions (i.e. radial components in Fourier space) from Alfvenic turbulence is a measure of mode coupling. How much energy in compressible motions is drained from Alfvenic cascade? According to closure calculations (Bertoglio, Bataille, & Marion 2001; see also Zank & Matthaeus 1993), the energy in compressible modes in hydrodynamic turbulence scales as $\sim M_s^2$ if $M_s < 1$. CL03 conjectured that this relation can be extended to MHD turbulence if, instead of $M_s^2$, we use $\sim (\delta V)^2_A/(a^2 + V_A^2)$. (Hereinafter, we define $V_A \equiv B_0/\sqrt{4\pi \rho}$, where $B_0$ is the mean magnetic field strength.) However, since the Alfven modes are anisotropic, this formula may require an additional factor. The compressible modes are generated inside the so-called Goldreich-Sridhar cone, which takes up $\sim (\delta V)^2_A/V_A$ of the wave vector space. The ratio of compressible to Alfvenic energy inside this cone is the ratio given above. If the generated fast modes become isotropic (see below), the diffusion or, “isotropization” of the fast wave energy in the wave vector space increase their energy by a factor of $\sim V_A/(\delta V)_A$. This results in

$$\frac{(\delta V)^2_{rad}}{(\delta V)^2_A} \sim \left[ V_A^2 + a^2 \frac{(\delta V)_A}{V_A} \right]^{-1}, \tag{1}$$

where $(\delta V)^2_{rad}$ and $(\delta V)^2_A$ are energy of compressible and Alfven modes, respectively. Eq. (1) suggests that the drain of energy from Alfvenic cascade is marginal\footnote{The marginal generation of compressible modes is in agreement with earlier studies by Boldyrev et al. (2002b) and Porter, Pouquet, & Woodward (2002), where the velocity was decomposed into a potential component and a solenoidal component. A recent study by Vestuto, Ostriker & Stone (2003) is also consistent with this conclusion.} when the amplitudes of perturbations are weak, i.e. $(\delta V)_A \ll V_A$. Results of calculations shown in Fig. 2 support the theoretical predictions.

We may summarize this issue in the following way. For the incompressible motions to decay fast, there is no requirement of coupling with
compressible motions. The marginal coupling of the compressible and incompressible modes allows to study modes separately.

3. Is Turbulence in Molecular Clouds Really Messy?

Is it feasible to obtain scaling relations for the compressible MHD turbulence? Some hints about effects of compressibility can be inferred from the GS95 seminal paper. More discussion was presented in Lithwick & Goldreich (2001), which deals with electron density fluctuations in the gas pressure dominated plasma, i.e. in high $\beta$ regime ($\beta \equiv P_{\text{gas}}/P_{\text{mag}} \gg 1$). The incompressible regime corresponds to $\beta \to \infty$, so it is natural to expect that for $\beta \gg 1$ the GS95 picture would persist. Lithwick & Goldreich (2001) also speculated that for low $\beta$ plasmas the GS95 scaling of slow modes may be applicable. A detailed study of compressible mode scalings is given in CL02 and CL03.

The reported (see Mac Low et al. 1998) decay of the total energy of turbulent motions $E_{\text{tot}}$ follows $t^{-1}$ which can be understood if we account for the fact that the energy is being injected at the scale smaller than the scale of the system. Therefore some energy originally diffuses to larger scales through the inverse cascade. Our calculations (Cho & Lazarian, unpublished), stimulated by illuminating discussions with Chris McKee, show that if this energy transfer is artificially prevented by injecting the energy on the scale of the computational box, the scaling of $E_{\text{tot}}$ becomes closer to $t^{-2}$. 

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Figure 2. Mode coupling studies. (a) left: Square of the r.m.s. velocity of the compressible modes. We use $144^3$ grid points. Only Alfvén modes are allowed as the initial condition. “Pluses” are for low $\beta$ cases ($0.02 \leq \beta \leq 0.4$). “Diamonds” are for high $\beta$ cases ($1 \leq \beta \leq 20$). (b) middle: Generation of fast modes. Snapshot is taken at $t=0.06$ from a simulation (with $144^3$ grid points) that started off with Alfvén modes only. Initially, $\beta$ (ratio of gas to magnetic pressure, $P_{\text{gas}}/P_{\text{mag}} = 0.2$ and $M_s$ (sonic Mach number) $\sim 1.6$. (c) right: Comparison of decay rates. Decay of Alfvén modes is not much affected by other (slow and fast) modes. We use $216^3$ grid points. Initially, $\beta = 0.02$ and $M_s \sim 4.5$ for the solid line and $M_s \sim 7$ for the dotted line. Note that initial data are, in some sense, identical for the solid and the dotted lines. The sonic Mach number for the solid line is smaller because we removed fast and slow modes from the initial data before the decay simulation. For the dotted line, we did not remove any modes from the initial data. From CL03.
Our considerations above about the mode coupling can guide us to predict the mode scaling. Indeed, if Alfvén cascade evolves on its own, it is natural to assume that slow modes exhibit the GS95 scaling. Indeed, slow modes in gas pressure dominated environment (high $\beta$ plasmas) are similar to the pseudo-Alfvén modes in incompressible regime (see GS95; Lithwick & Goldreich 2001). The latter modes do follow the GS95 scaling. In magnetic pressure dominated environments (low $\beta$ plasmas), slow modes are density perturbations propagating with the sound speed $a$ parallel to the mean magnetic field. Those perturbations are essentially static for $a \ll V_A$. Therefore Alfvénic turbulence is expected to mix density perturbations as if they were passive scalar. This also induces the GS95 spectrum.

The fast waves in low $\beta$ regime propagate at $V_A$ irrespectively of the magnetic field direction. In high $\beta$ regime, the properties of fast modes are similar, but the propagation speed is the sound speed $a$. Thus the mixing motions induced by Alfvén waves should marginally affect the fast wave cascade. It is expected to be analogous to the acoustic wave cascade and hence be isotropic.

Results of numerical calculations from Cho & Lazarian (CL03) for magnetically dominated media similar to that in molecular clouds are shown in Fig. 3. They support theoretical considerations above.

Why would we care about those scalings? How wrong is it to use Kolmogorov scalings instead? Dynamics, chemistry and physics of molecular clouds (see Falgarone 1999) presents a complex of problems for which the exact scalings may be required to a different degree. If we talk about dynamics of interstellar dust or propagation of cosmic rays, one must account for the actual scalings and couplings of different modes (see reviews by Lazarian & Yan 2003, Lazarian et al. 2003). There are other problems, e.g. turbulent heat transport where the exact scaling of modes seems to be less important (Cho et al. 2003).

4. **What is the effect of partial ionization?**

Our considerations above assume that the gas is fully ionized. Molecular clouds are partially ionized. What is the consequence of this? An obvious effect of neutrals is that they do not follow magnetic field lines and thus produce viscosity.

In hydrodynamic turbulence viscosity sets a minimal scale for motion, with an exponential suppression of motion on smaller scales. Below the viscous cutoff the kinetic energy contained in a wavenumber band is dissipated at that scale, instead of being transferred to smaller scales. This means the end of the hydrodynamic cascade, but in MHD
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Figure 3. $M_s \sim 2.2, M_A \sim 0.7, \beta \sim 0.2$, and $216^3$ grid points. (a) Spectra of Alfvén modes follow a Kolmogorov-like power law. (b) Eddy shapes (contours of same second-order structure function, $SF_2$) for velocity of Alfvén modes shows anisotropy similar to the GS95 ($r_\parallel \propto r_\perp^{2/3}$ or $k_\parallel \propto k_\perp^{2/3}$). The structure functions are measured in directions perpendicular or parallel to the local mean magnetic field in real space. We obtain real-space velocity and magnetic fields by inverse Fourier transform of the projected fields. (c) Spectra of slow modes also follow a Kolmogorov-like power law. (d) Slow mode velocity shows anisotropy similar to the GS95. We obtain contours of equal $SF_2$ directly in real space without going through the projection method, assuming slow mode velocity is nearly parallel to local mean magnetic field in low $\beta$ plasmas. (e) Spectra of fast modes are compatible with the IK spectrum. (f) The magnetic $SF_2$ of fast modes shows isotropy. From CL02

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turbulence this is not the end of magnetic structure evolution. For viscosity much larger than resistivity, there will be a broad range of scales where viscosity is important but resistivity is not. On these scales magnetic field structures will be created by the shear from non-damped turbulent motions, which amounts essentially to the shear from the smallest undamped scales. Indeed, this new regime of turbulence has been discovered (see Fig. 4)!

A theoretical model for this new regime can be found in Lazarian, Vishniac, & Cho (2003; hereafter LVC03). It explains the spectrum $E(k) \sim k^{-1}$ as a cascade of magnetic energy to small scales under the influence of shear at the marginally damped scales. The spectrum is similar to that of the viscous-convective range of passive scalar in hydrodynamic turbulence (see, for example, Batchelor 1959; Lesieur 1990), although the study in Lazarian, Vishniac, & Cho (LVC03) suggests that the physical origin of it is different. A study confirming that the $k^{-1}$ spectrum is not a bottleneck effect is presented in Cho, Lazarian, & Vishniac (2003b). The mechanism is based on the solenoidal motions
and therefore the compressibility should not alter the physics of this regime of turbulence.

For compressible simulations (see Fig. 4) the inertial range is much smaller due to numerical reasons, but it is clear that the viscosity-damped regime of MHD turbulence persists. The magnetic fluctuations, however, compress the gas and thus cause fluctuations in density. These density fluctuations may have important and yet unexplored consequences for the small scale structure of the molecular clouds. There are indications that small-scale structure is indeed present in molecular gas (Marscher, Moore, & Bania 1993) as well as in other partially ionized phases of the ISM (see Heiles 1997). Those structures can be produced by turbulence with a spectrum substantially more shallow than the Kolmogorov one, e.g. with the spectrum $E(k) \propto k^{-1}$ (see Deshpande 2000). Below the viscous scale the fluctuations of magnetic field obey the damped regime shown in Fig. 4b and produce density fluctuations. For typical Cold Neutral Medium gas (see Draine & Lazarian 1998), these fluctuations can be as large as $\sim 70$ AU and less for denser gas.

An important prediction in LVC03 is that the intermittency of magnetic structures increases with the decrease of the scale. This prediction was confirmed by numerical simulations in Cho, Lazarian & Vishniac (2003), which showed that the filling factor of magnetic field was decreasing with the increase of the wavenumber.

Does the effect of neutrals amounts only to emergence of the viscosity-damped regime of MHD turbulence? The answer to this question is
negative. Partial ionization provides the whole range of new interesting effects. Some of them are studied in LVC03. Here we mention those.

First of all, it is clear that whether ions and neutrals act as one fluid in molecular clouds depends on whether the eddy turnover rate $t_{\text{eddy}}^{-1} \sim k v_k$ is longer or shorter than the rate $t_{\text{ni}}^{-1}$ of neutral-ion collisions. If $t_{\text{eddy}}^{-1} > t_{\text{ni}}^{-1}$, neutrals decouple from ions and develop hydrodynamic Kolmogorov-type cascade. Indeed, the damping rate for those hydrodynamic motions $t_{\text{ni}}^{-1}$ and below the decoupling scale the hydrodynamic motions evolve without much hindrance from magnetic field. Magnetic fields with entrained ions develop the viscosity-damped MHD cascade until ion-neutral collisional rate gets longer than the dynamical rate of the intermittent magnetic structures. After that the turbulence reverts to its normal MHD cascade which involves only ions.

If $t_{\text{eddy}}^{-1} < t_{\text{ni}}^{-1}$ up to the scale at which neutral viscosity damps turbulent motions, the viscosity-damped regime emerges at the scale where kinetic energy associated with turbulent eddies is dissipated. Similarly to the earlier case when the when ion-neutral collisions get insufficient to preserve pressure confinement of the small scale magnetic filaments, outbursts of ordinary ionic MHD turbulence will take place. The turbulence will be intermittent both in time and space because of the disparity of time scales at which turbulence evolves in the viscosity-damped and free ionic MHD regimes. We plan to test those predictions with a two fluid MHD code.

5. How Can One Study Turbulence in Molecular Clouds?

Substantial advances in understanding of MHD turbulence in ordinary and viscosity-damped regime make it most important to test the correspondence of theoretical expectations and observational reality (see review by Ostriker 2003). Important advances in obtaining high resolution spectral data (see Falgarone et al. 2000) make the testing feasible.

The turbulence spectrum characterizes the distribution of energy over spatial scales and reflects the processes of dissipation and energy injection. The GS95 model would also gives the Kolmogorov spectrum of Alfvenic motions if averaging over $k$ directions is performed. This stems from the fact that for the inertial range $k_{\perp} \gg k_{||}$ and therefore $k \sim k_{\perp}$.

It is known that studies of stochastic density provide only indirect insight into turbulence. A stationary (not evolving) density distribution is indistinguishable from the result of active turbulence. However, Doppler shifted spectral lines carry information about turbulent
velocity. The problem is that both velocity and density fluctuations contribute to the observed fluctuations. Therefore numerous attempts to study turbulence in diffuse interstellar medium and molecular clouds using emission lines (see Munch 1958, O’Dell 1986, O’Dell & Castaneda 1987, Miesch & Bally 1994, reviews by Scalo 1987, Falgarone 1999) were facing uncertainty of what quantity was actually measured.

Studies of the velocity field have been attempted at different times with velocity centroids (e.g. Munch 1958, Miesch, Scalo, & Bally 1999, Ossenkopf & Mac Low 2002, Miville-Deschenes et al. 2003). However, it has long been realized that the centroids are affected, in general, by both velocity and density fluctuations (see Stenholm 1989). A criterion of when centroids indeed reflect the velocity statistics was obtained in Lazarian & Esquivel (2003, henceforth LE03). Without this testing, it is not clear \textit{a priori} what is actually being measured\textsuperscript{6}.

An important recent development is a quantitative description of spectral data from turbulent media obtained in Lazarian & Pogosyan (2000, 2003 henceforth LP00 and LP03, respectively), where the fluctuations of intensity in channel maps were related to the statistics of velocity and density. LP00 introduced a new technique that was termed Velocity Channel Analysis (henceforth VCA). Within VCA the separation of velocity and density contributions is obtained by changing the thickness of the analyzed slice of the Position-Position- Velocity (PPV) data cube. The VCA was successfully tested numerically in Lazarian et al. (2001) and Esquivel et al. (2003) and applied to the Small Magellanic Cloud (Stanimirovic & Lazarian 2002).

LP03 accounts for the absorption in the turbulent media. Thus it substantially extends the range of spectral lines that can be used for turbulence studies. Note that the importance of LP00 and LP03 studies is not limited to the development of particular toolkit of how to interpret channel maps. These works provide theoretical description that can be used within different techniques. For instance, LP03 establishes limitations on turbulence spectra that can be recovered by another promising technique, namely, Spectral Correlation Function (see Rosolowsky et al. 1999).

Other important tools have also been studied recently. Principal Component Analysis (PCA) of the emission data (Heyer & Schloerb 1997, Brunt & Heyer 2002ab) provide a new promising way to characterise observations. Recent testing showed that it provides statistics

\textsuperscript{6} A more optimistic claim about utility of centroids was obtained in Miville-Deschenes, Levrier & Falgarone (2003) on the basis of experiments with Brownian noise. However, to make the density in their experiments positively defined the authors added additional large mean density. According to the criterion in LE03 this made their centroids bound to be dominated by velocity.
Figure 5. Dealing with Observations: Testing and Applying New Statistical Tools. 

**Upper Left:** the 21 cm image of SMC, exhibiting strong density structure. According to our analysis this density has a spectrum close to that expected from MHD turbulence. **Upper Right:** variations of 2-D 21 cm spectral slope with the velocity slice thickness (from Stanimirovic & Lazarian 2001). The LP00 study predicts that the thick slice reflects the density statistics, while the thin slice is influenced by the velocity. **Lower Left:** the 2D genus (which counts the number of holes versus the number of islands as the threshold contrast changes) of the Gaussian distribution (smooth curve) against the genus for the isothermal compressible MHD simulations with Mach number 2.5 (dotted curve). Interstellar turbulence shows much more intermittency, with consequences for interstellar physics and chemistry. **Lower Right:** the genus of HI distribution in SMC that has the same power spectrum as the dotted curve in the left panel (Lazarian, Pogosyan & Esquivel 2002). The genus (topology) of the distributions are very different!

different from power spectra (Brunt et al. 2003). Wavelet analysis (see Gill & Henriksen 1990), spectral correlation functions (see Rosolowsky et al. 1999, Padoan, Rosolowsky & Goodman 2001), genus analysis (see Lazarian, Pogosyan & Esquivel 2002) are other statistical tools. They can provide statistics and insight complementary to power spectra. Synergy of different techniques should provide the necessary insight into turbulence and enable the comparison of observational statistics with theoretical expectations.
6. What is the future of the field?

There are whole classes of processes for which we are not sure even about the sign of the effect, for instance, whether turbulence supports or compresses molecular clouds. Or consider imbalanced turbulence, i.e. the turbulence where the flow of energy in one direction is larger than the flow of energy in the opposite direction. This situation is typical for interstellar medium with its localized sources of energy. CLV02a speculated that imbalanced turbulence can propagate over larger distances and feed energy to clouds without star formation. To what degree does this process get modified in the presence of compressibility? Alfvénic modes in imbalanced turbulence live longer and the interaction between density fluctuations and the Alfvén mode becomes more important.

In the field of observational studies, the situation looks remarkably promising. With high resolution surveys available (see Falgarone et al. 2000), studies of turbulence statistics should at last become a mainstream research. The prospects of studies of turbulent velocity are most encouraging. Indeed, for the first time, one has understanding of how to get statistics of velocity and density from channel maps (LP00, LP03), and when the velocity centroids reflect the statistics of velocity (LE03). Moreover, a quantitative description of the statistics of the spectral line data cubes (see LP00, LP03) allows us to devise new techniques of turbulence studies. Apart from testing of the particular scaling laws, this research should identify sources and injection scales of the turbulence (see the companion review by Falgarone et al. 1998). Is turbulence in molecular clouds a part of a large scale ISM cascade (see Armstrong et al. 1997)? How does the share of the energy within compressible versus incompressible motions vary within the Galactic disk? There are examples of questions that can be answered in future.

While this review deals mostly with power spectra, higher order statistics will be widely used in the future. Recent numerical research that employed higher order statistics (Muller & Biskamp 2000, CLV02a, Boldyrev et al. 2002, Cho, Lazarian & Vishniac 2003b) showed it to be a promising tool. For instance, the distinction between the old Iroshnikov-Kraichnan and the GS95 model is difficult to catch using power spectra with a limited inertial range, but is quite apparent for fourth order statistics. The difference in physical consequences of whether the turbulence dissipates in shocks or in intermittent vortices may be very substantial. Our discussion in §2 suggests that incompressible motions do dissipate via vortices. The scaling of their intermittency with the Reynolds number is still to be established. In the meantime, obtaining higher order statistics with spectral line observations is a challenging problem. Higher order statistics obtained from observational data were
reported for observed velocity in Falgarone et al. (1994) and for density in Padoan et al. 2003. According to Falgarone & Puget (1995) and Falgarone et al. (1995) (see also review by Falgarone), the intermittency in vorticity distribution can result in the outbursts of localized dissipation that make tiny regions within cold diffuse clouds chemically active. The testing of those ideas is done in Pety & Falgarone (2000), where synthetic maps obtained using hydrodynamic simulations were analyzed. Since then, more results indicating the hydrodynamic simulations may to some degree reflect the physics of MHD turbulence have emerged. First of all, CLV02a found out that the fluid motions perpendicular to \textbf{B} are identical to hydrodynamic motions. Moreover, for low ionization the turbulence in neutrals and ions decouple with turbulence in neutrals forming a hydrodynamic cascade (see §5).

Studies of the statistics of magnetic field in molecular clouds is the next challenging problem (see Crutcher, Heiles & Troland 2003, Ostriker 2003). Better understanding of grain alignment (see review by Lazarian 2003 and references therein) allows to better identify variations of polarization with the variations of magnetic field. However, this important research has not gained sufficient momentum yet.

7. Summary

1. Understanding of molecular clouds requires understanding of the basics of MHD turbulence. MHD turbulence is not a mess. Scaling relations for its modes have been established recently.
2. Fast decay of MHD turbulence is not due to strong coupling of compressible and incompressible motions. The transfer of energy from Alfvén to compressible modes is small. The Alfvén mode develops on its own and decays fast.
3. Doppler shifts imprinted in spectra lines provide an excellent way of testing theoretical expectations. Advances in understanding how this information can be extracted from spectroscopic data allow this.

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