Effects of Varying Viscosity and Mixed Convection on Nanotubes-water Flow With Reactions by a Stretching Cylinder: a Comparative Study

Zakir Hussain (zakir.qamar@yahoo.com)  
University of Baltistan

Tasawar Hayat  
Quaid-I-Azam University

Ahmed Alsaedi  
King Abdulaziz University

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Effects of varying viscosity and mixed convection on nanotubes-water flow with reactions by a stretching cylinder: A comparative study

Zakir Hussain\textsuperscript{a}, Tasawar Hayat\textsuperscript{b,c}, Ahmed Alsaedi\textsuperscript{c}

\textsuperscript{a}Department of Mathematics, University of Baltistan, Skardu, Pakistan
\textsuperscript{b}Department of Mathematics, Quaid-I-Azam University 45320, Islamabad 44000, Pakistan
\textsuperscript{c}Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80257, Jeddah 21589, Saudi Arabia

Corresponding author\textsuperscript{a}; zakir.qamar@yahoo.com (Zakir Hussain)

\textbf{Abstract:} The addressed work explains SWCNT (Single walled carbon nanotubes) and MWCNT (Multi walled carbon nanotubes) nanofluid flow under the influences of temperature dependent viscosity and mixed convection. Comparative study of SWCNT and MWCNT suspended in base liquid is presented. Further heat and mass transfer are addressed for nanofluid effected by radiation, heat generation/absorption and diffusion species. Mathematical development of problem is taken in cylindrical coordinates. System of highly nonlinear differential equations are constructed via appropriate transformations. The system of equations are tackled numerically by bvp4c MATLAB solver. The findings of the study show that volume fraction contributes to decline the fluid flow by cylindrical shaped nanoparticles. In addition, fluid flow decelerates via curvature and magnetic parameters while it boosts by Grashof number and volume fraction. Further more, temperature dependent viscosity variable corresponds to decrease the temperature close to the surface and it develops away from the surface. The temperature advances in MWCNT-liquid than SWCNT-liquid. Volume fraction and magnetic parameters correspond to skin friction coefficient enhancement. Heat transfer rate increases for larger curvature and heat generation parameters and reverse trend holds against radiation param-
Keywords: Temperature dependent viscosity; CNTs(Single and multi walled carbon nanotubes); heat sources; diffusion species.

1 Introduction

The challenges and demands of advanced industries have been attracted the attention of researchers. The solar collectors used in devices are mostly affected by poor conductivity and low heat up capability of ordinary liquids. Choi [1] was the first one to take lead to enhance thermal conductivity in this way by mixing nano-sized particles in ordinary liquids. Nanoliquids are composed by suspension of chemical stable nano-scaled materials namely oxides, metals and carbides etc in base liquids. These novel fluids are termed as nanoliquids which enhance thermal characteristics of base liquids (engine oil, water, glycol etc). Ordinary liquids namely engine oil, water, glycol etc having low conductivity. Suspension of nanoparticles in low thermal conductivity liquids advance the thermal conductivity and hence accelerates the performance of industrial liquids. Further studies of nanofluids are cited therein [2-12]. Khan et al. [13] addressed MHD nanoliquid with entropy generation in rotating frame. Sohail et al. [14] investigated Darcy Forchheimer hybrid nanoliquid in porous medium under the impact of entropy analysis. Khan et al. [15] analyzed non-axisymmetric Homann stagnation point nanofluid flow through multiple solutions. Huda et al. [16] elaborated Cattaneo-Christov model for nanofluid with moving needle. Reactive stretched flow of $Al_2O_3 - water$ in porous space is examined by Lia et al. [17].

Magnetic field application in fluid flow analysis has gained attention of scientists due to its wide range of utilization in many fields namely industries, drug delivery, MHD (Magnetohydrodynamics) generator, mechanical and physiological phenomena and many others. Nadeem et al. [18] studied MHD nanoliquid flow...
numerically. Malvandi et al. [19] studied mixed convection of MHD nanofluid saturate in vertical annulus. Activation energy in MHD squeezed flow with binary chemical reactions was studied by Ahmad et al. [20]. Hayat et al. [21] investigated magnetohydrodynamics third grade nanoliquid convective flow by nonlinear stretched plate. Partial slip in MHD nanoliquid flow with viscous dissipation near stagnation point was investigated by Emad et al. [22].

Radiation does not need any medium to transmit. It depends on shape, temperature and propagates by electromagnetic waves. It is practiced that system in industries having little temperature difference in fluid caused problems. To overcome this difficulty the researchers incorporated a term named as radiation parameter. The variation in temperature of fluid and wall can be novel by this parameter. Cortell [23] summarized influence of heat generation and radiation in convective flow. A summary about this title is cited in the studies[24-27].

Chemical reactions are categorized mainly in two types namely homogeneous and heterogeneous reactions. Reactions which encounter catalyst in same phase (namely gases, liquids, solids) correspond to homogeneous and reactions which happen in two or several different phases (like solid and gas, solid and liquid) as heterogeneous reactions. Some utilization of chemical reactions are found in iron oxidation, polymer and metallurgical industries. Reactions species have composite link for formation and usage of reactant species. Generally reactions rate depend on the magnitude of mass itself. A simple isothermal model proposed by Merkin et al. [28] investigates homogeneous-heterogeneous reactions in flow. Influence of chemical reaction in liquid flow was studied by Bhattacharyya [29]. Chemical reactive fluid flow was reported by Rashidi et al. [30] to explore mixed convection for heat and mass transfer. Convective flow with homogeneous-heterogeneous reactions saturated a porous medium was analyzed by Hayat et al. [31]. Zakir et al. [32] studied CNTs in flow of liquid by stretched cylinder with
Darcy-Forhheimer effect.

The novelty of this research article accounts water as a base fluid and CNTs as nanoparticles. The outcomes of mixed convection, radiation, source of heat and homogeneous-heterogeneous reactions are incorporated. Viscosity of base liquid dependent on temperature and volume fraction of nanoparticles. Simulation to arising nonlinear problem is made. Velocity temperature and concentration are outlined physically. Graphical outcomes for skin friction coefficient and Nusselt number via different variables are displayed. Present analysis may be useful in some engineering and industrial applications. Main results of this article could be used and fruitful in lubrication phenomenon, polymer industry and academic researches.

2 Problem development

In this analysis it is considered that two-dimensional incompressible mixed convective flow of CNTs nanoliquid by stretchable cylinder. The liquid flow is caused by stretching cylinder. The viscosity of base liquid varies with the variation of temperature. Base liquid contains homogeneous combination of CNTs particles. Further more, CNTs particles and base liquid are in thermal equilibrium. Linearly stretching cylinder (i.e. \( U_w = \frac{U_0 z}{l} \)) is along axial direction (\( z-axis \)) while liquid is assumed to deform in radial direction (\( r-axis \)). Fig. 1 addresses the geometric configuration of flow problem. Heat transfer characteristics are explored via heat generation/absorption, viscous dissipation and Joule heating. Diffusion species are accounted in base liquid for reactions.

The fluid viscosity is treated linear function of temperature i.e.,
Figure 1: Schematic representation of problem.

\[
\begin{align*}
\frac{1}{\mu} &= \frac{1}{\mu_\infty} \left(1 + \gamma_0 (T - T_\infty)\right) \quad \text{or} \quad \frac{1}{\mu} = \frac{2\nu}{\mu_\infty} (T - T_r) \\
\text{where} \quad T_r &= T_\infty - \frac{1}{\gamma_0} \quad \text{and} \quad \gamma_0 = \frac{1}{T_\infty - T_r} \\
\text{or} \quad \frac{1}{\mu} &= a^* (T - T_r) \quad \text{where} \quad a^* = \frac{\gamma_0}{\mu_\infty} \\
\text{or} \quad \frac{1}{\mu} &= \frac{\theta_r - \theta}{\mu_\infty \theta_r}, \quad \theta_r = \frac{T_r - T_\infty}{T_\infty - T_\infty} \\
\text{or} \quad \mu(T) &= \frac{\mu_\infty \theta_r}{\theta_r - \theta}.
\end{align*}
\]
In above \( a^* \) and \( T_r \) indicate reference condition of the liquid. Generally, positive values of \( a^* \) stand for liquid and negative values of \( a^* \) show gases.

For heterogeneous-homogeneous reactions, the model of isothermal is defined by

\[
A_\ast + 2B_\ast \rightarrow 3B_\ast, \text{ rate } = k_{p\ast}a_\ast b_{\ast}^2.
\]

(2)

At surface of catalyst the first-order reaction is

\[
A_\ast \rightarrow B_\ast, \text{ rate } = k_{\ast}a_\ast.
\]

(3)

Where \((a_\ast, b_\ast)\) stand for concentrations of species \((A_\ast, B_\ast)\), \((k_p, k_{\ast})\) show the rate constants. The auto-catalyst \((B_\ast)\) is taken inside the boundary layer and outside it is dealt equal concentration \((a_0)\) of reactant \((A_\ast)\). Eq. (3) reveals that there is no reaction rate outside the boundary layer.

The governing problems via boundary layer approximation in cylindrical coordinates are as follows:

\[
\frac{\partial(rw)}{\partial r} + \frac{\partial(rw)}{\partial z} = 0,
\]

(4)

\[
w \frac{\partial w}{\partial z} + u \frac{\partial w}{\partial r} = \frac{1}{\rho_{nf}} \left( \frac{\partial \mu(T) \partial w}{\partial r} \frac{\partial \mu(T) \partial w}{\partial r} + 1 \frac{\partial T}{\partial r} + \mu(T) \frac{\partial^2 w}{\partial r^2} \right) + gB(T - T_\infty) - \frac{\sigma_{nf} B_0^2 w}{\rho_{nf}},
\]

(5)

\[
u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{k_{nf}}{(C_p\rho)_{nf}} \left( \frac{\partial^2 T}{\partial r^2} + 1 \frac{\partial T}{\partial r} \right) + \frac{16 \sigma^* T_0^3}{3k^*(C_p\rho)_{nf}} \left( \frac{\partial^2 T}{\partial r^2} + 1 \frac{\partial T}{\partial r} \right) + \frac{\mu_{nf}(T) \left( \frac{\partial w}{\partial r} \right)^2}{(C_p\rho)_{nf}} + \frac{Q_0(T - T_\infty)}{(C_p\rho)_{nf}},
\]

(6)

\[
u \frac{\partial a_\ast}{\partial r} + w \frac{\partial a_\ast}{\partial z} = DA_\ast \left( \frac{\partial^2 a_\ast}{\partial r^2} + \frac{1}{r} \frac{\partial a_\ast}{\partial r} \right) - k_{p\ast}a_\ast b_{\ast}^2,
\]

(7)

\[
u \frac{\partial b_\ast}{\partial r} + w \frac{\partial b_\ast}{\partial z} = DB_\ast \left( \frac{\partial^2 b_\ast}{\partial r^2} + \frac{1}{r} \frac{\partial b_\ast}{\partial r} \right) + k_{p\ast}a_\ast b_{\ast}^2,
\]
with conditions

\[
\begin{cases}
  w = w_e = \frac{U_0 z}{r}, \quad u = 0, \quad T = T_w, \\
  D_A \frac{\partial a_s}{\partial r} = -a_s k_s, \quad D_B \frac{\partial b_s}{\partial r} = a_s k_s \quad \text{at}, \quad r = R, \\
  w \to 0, \quad T \to T_\infty, \quad a_s \to a_0, \quad b_s \to 0 \quad \text{as} \quad r \to \infty,
\end{cases}
\]

where \((w, u), ((\rho)_{nf}), ((c_p)_{nf}), (\mu_{nf}, \nu_{nf}), (k_{nf}), (w_e), (U_0), (l), (R), (\beta_0), ((\sigma)_{nf}), (g), (B), (Q_0(T - T_\infty)), (T_w, T_\infty), (\sigma^*), (k^*), (T_0), (D_{A*}, D_{B*})\) the velocity components, the density, the specific heat, the dynamic and kinematics viscosity, the thermal conductivity, the stretching velocity, the reference velocity, the characteristics length, the radius of cylinder, the strength of magnetic field, the electric transport, the gravitational acceleration, the thermal coefficient, the heat generate per unit volume, the wall and ambient temperatures, the Stefan Boltzmann constant, the mean absorption coefficient, the reference temperature and the diffusion coefficients respectively.

Following model by Xue [33] one has

\[
\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}, \quad \rho_{nf} = (1 - \phi) \rho_f + \phi (c_p)_{CNT},
\]

\[
(C_p \rho)_{nf} = C_{pnf} \left((1 - \phi) \rho_f + \phi (c_p)_{CNT}\right),
\]

\[
\alpha_{nf} = \frac{k_{nf}}{\rho_{nf} (c_p)_{nf}}, \quad \frac{k_{nf}}{k_f} = \frac{(1 - \phi) + 2\phi k_{CNT} \ln \frac{k_{CNT} + k_f}{2k_f}}{(1 - \phi) + 2\phi k_{CNT} \ln \frac{k_{CNT} + k_f}{2k_f}},
\]

\[
\frac{\sigma_{nf}}{\sigma_f} = \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} + 1, \quad \sigma = \frac{\sigma_{CNT}}{\sigma_f}
\]

in which \(\phi\) denotes the volume fraction of nanosized particle, \(\rho_f\) the base liquid density, \(k_f\) the base liquid conductivity, \(k_{CNT}\) represents nanotubes thermal conductivity, \(\sigma_{CNT}\) and \(\sigma_f\) the electric conductivity of base liquid and nanoparticle respectively.
Table 1: Thermophysical characteristics of CNTs liquid [34].

| Physical Properties | Base fluid | Nanoparticles |
|---------------------|------------|---------------|
| \(\rho\) (kg/m\(^3\)) | 997        | 2600          | 1600  |
| \(c_p\) (J/kgK)   | 4179       | 425           | 796   |
| \(k\) (W/mK)      | 0.613      | 6600          | 3000  |

Letting

\[
\eta = \sqrt{\frac{U_0}{\nu l}} \left( \frac{r^2 - R^2}{2R} \right), \quad \psi = \sqrt{w e z} R f'(\eta) \quad w = \frac{U_0 z}{l} f'(\eta),
\]

\[
u = -\sqrt{\frac{\nu U_0 R}{l}} f'(\eta), \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \quad \zeta(\eta) = \frac{a}{a_0}, \quad h(\eta) = \frac{b}{a_0}. \quad (10)
\]

Eq. (4) is trivially confirmed and Eqs. (5 – 8) reduce to

\[
(1 + 2\gamma \eta) f''' + 2\gamma f'' - \left( \frac{1}{\theta_r - \theta} \right) (1 + 2\gamma \eta) \theta' f''
\]

\[
+ \left( \frac{\theta_r - \theta}{\theta} \right) (1 - \phi) \left( 1 - \phi + \phi \frac{\rho_{CNT}}{\rho_f} \right) \left( f f'' - (f')^2 + Gr \theta \right)
\]

\[
- \left( \frac{\theta_r - \theta}{\theta} \right) \left( \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} + 1 \right) M_f f' = 0, \quad (11)
\]

\[
Pr \left( \frac{k_n f}{k_f} + \frac{4}{3} R_d \right) (1 + 2\gamma \eta) \theta'' + 2\gamma \theta' + \left( \frac{\theta_r}{\theta_r - \theta} \right) Ec (1 + 2\gamma \eta) (f'')^2
\]

\[
+ Q_0 \theta + \left( \frac{(C_p \rho) n_f}{(C_p \rho)_f} \right) f \theta' = 0, \quad (12)
\]

\[
\frac{1}{Sc} \left( (1 + 2\gamma \eta) \Phi'' + \gamma \Phi' \right) + f \Phi' - K \Phi_n h^2 = 0, \quad (13)
\]

\[
\delta \frac{1}{Sc} \left( (1 + 2\gamma \eta) h'' + \gamma h' \right) + f h' + K \zeta_n h^2 = 0, \quad (14)
\]
with
\[ f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad \Phi_*(0) = Ks\Phi_*(0), \quad \delta h'(0) = -Ks\Phi_*(0), \]
\[ f'(\eta) = 0, \quad \theta'(\eta) = 0, \quad \Phi_*(\eta) \to 1, \quad h_*(\eta) \to 0, \quad \text{as} \quad \eta \to \infty, \quad (15) \]
where \( \gamma = \frac{1}{R} \sqrt{\frac{\mu}{U_0}} \) depicts the curvature parameter, \( Gr = \frac{\mu (T_w-T_\infty)}{U_0} \) the dimensionless Grashof number, \( M^2 = \frac{\beta f}{\rho_f U_0} \) the Hartman number, \( Ec = \frac{4\sigma T_0^3}{3k_f} \) the Eckert number, \( Rd = \frac{4k_\star T_0^3}{3k_f} \) the radiation parameter, \( Q_r = \frac{Q_0}{U_0} \) the heat generation/absorption parameter, \( K = \frac{k_\star}{D_{A*}} \) the homogeneous variable, \( K_s = \frac{k_\star D_{A*}}{D_{A*}} \) the heterogeneous variable, \( Sc = \frac{\nu_f}{D_{A*}} \) the Schmidt number and \( \delta = \frac{D_{B*}}{D_{A*}} \) the diffusion ratio coefficient.

One has, for equal diffusion coefficients \( D_{A*} \) and \( D_{B*} \) as \[ \Phi_*(\eta) + h_*(\eta) = 1. \]

Eqs. (13)-(14) imply that
\[ (1 + 2\gamma \eta)\Phi'' + 2\gamma \Phi' + Scf\Phi' - ScK(1-\Phi)^2 = 0. \]
\[ \Phi'(0) = Ks\Phi(0), \quad \Phi(\eta) \to 1 \quad \text{as} \quad \eta \to \infty. \]

Skin friction coefficient and Nusselt number are
\[ C_f = \frac{\tau_w}{\rho_f U_0^2}, \quad Nu_z = \frac{z q_w}{k_f (T_w - T_\infty)}, \]
where \( \tau_w \) and \( q_w \) are defined as;
\[
\begin{cases}
\tau_w = -\mu f \left( \frac{\partial w}{\partial r} \right)_{r=R}, \quad q_w = -\left( k_n f + \frac{16\sigma T_0^3}{3k_f} \right) \left( \frac{\partial T}{\partial r} \right)_{r=R}, \\
C_f Re_z^{1/2} = -\frac{1}{(1-\phi)^{1/2}} f''(0), \quad Nu_z Re_z^{-1/2} = -\left( \frac{k_n f}{k_f} + \frac{4}{3} R \right) \theta'(0),
\end{cases}
\]
(19)
where \( Re_z^{\frac{1}{2}} = \sqrt{\frac{U_0}{\nu}} \) represents the local Reynolds number.

3 Discussion

This section elaborates the graphical discussion for SWCNT and MWCNT nanofluids. SWCNTs is indicated by the solid lines while MWCNT by dashed lines. The values \( \gamma = M_f = Gr = \phi = R_d = \theta_r = Ec = 0.1, Pr = 6.2, K = 0.7, K_s = 0.9 \) and \( Sc = 1.2 \) are taken for outcomes. Further the values of variables are taken constant except the specific variable considered in Figures. The thermophysical characteristics of CNTs nanoliquid is defined in Table 1. The outcomes of curvature, Hartman number, Grashof number, volume fraction and others involved dimensionless variables are elaborated for the distributions \( (f'(\eta), \theta(\eta), \Phi(\eta), C_f Re_z^{\frac{1}{2}}, Nu_z Re_z^{\frac{1}{2}}) \).

**Velocity:** Curvature variable \( \gamma \) declines the velocity distribution. (See Fig. 2). Clearly \( \gamma \) and \( R \) inversely relate to each other and therefore the resistive force enhances for the liquid flow. Therefore the velocity field for SWCNT and MWCNT behaves similarly. Fig. 3 is sketched to discuss the effect of Grashof number \( Gr \) on velocity field. From figure it is observed that velocity enhances for both SWCNTs and MWCNTs. The velocity field increases via larger \( Gr \). Because the Grashof number directly relates to buoyancy forces \( (Gr = \frac{buoyancy force}{viscous force}) \) and inversely relates with viscous force. The resistive force becomes less against more \( Gr \). Fig. 4 shows curves of Hartman number \( M_f \) for velocity flow. The flow field declines for larger \( M_f \). Lorentz force rises for higher \( M_f \) and velocity of liquid decreases. Impact of volume fraction for SWCTs-water and MWCNTs-water is addressed in Fig. 5. Nanoliquid flow boosts when the values of \( \phi \) increase due to the direct relationship with convective flow. Further more, the flow in MWCNTs is observed higher than SWCNTs.
**Temperature:** The lines for temperature via heat variable is addressed in Fig. 6 with SWCNTs and MWCNTs as nanoparticles. Thermal layers and temperature gradient enhance for larger $Q_r$. Heat in liquid increases as the values $Q_r$ higher. Temperature variations is noted similar for both type CNTs. The temperature curves follow free stream condition for larger $\eta$. Fig. 7 addresses the influence of viscous dissipation (i.e. Eckert number $Ec$) on temperature. The temperature rises when $Ec$ is increased. Larger $Ec$ represent high kinetic energy which corresponds to enhance the temperature of liquid. Moreover, same temperature is noted for cylinder shaped SWCNTs and MWCNTs nanofluids. Fig. 8 shows the effect of curvature variable on temperature. The fluid heats up by increasing $\gamma$. Heat in MWCNTs-water is noted higher in comparison to SWCNTs-water. Temperature enhances against more heat generation parameter $Q_r$ (see Fig. 9). Temperature in case of SWCNTs and MWCNTs are noted same for the base fluid. The curves of radiation variable $R_d$ for $\theta(\eta)$ is shown in Fig. 10. Outcomes of radiation variable results to enhancement in $\theta(\eta)$. Higher values of radiation variable results decrease in absorption coefficient. Hence the temperature field increases. Fig. 11 is sketched for the temperature dependent viscosity variable on thermal field. Figure shows that increment in $\theta_r$, contributes to enhancement of temperature. Physically it reflects that larger viscosity mean higher resistance for flow and consequently the kinetic energy enhances. Temperature close to the surface first declines and then advances far away from cylinder. Same temperature is noted for both CNTs.

**Concentration:** Fig. 12 addresses the concentration via curvature variable $\gamma$. Concentration enhances for larger values of $\gamma$. Fig. 13 shows the curves for concentration gradient via homogeneous variable $K$. The concentration decreases for $K$. In fact there is direct relation between chemical reaction and $K$. Concentration for larger heterogeneous variable $K_s$ can be seen in Fig. 14. Same behavior
is noted for $K_s$. The concentration function boosts for $Sc$ (see Fig. 15). There is inverse relationship between Schmidt number and small mass diffusivity. Further same mass transfer rate is noted for both MWCNTs and SWCNT. Fig. 16 shows the behavior of skin friction coefficients via $M_f$ and $\phi$. Skin friction for the fluid flow enhances for larges values of $M_f$ and $\phi$. Fig. 17 addresses the Nusselt number via variables $R_d$ and $Q_r$. Nusselt number increases for larger $Q_r$ and it decreases for $R_d$. Nusselt number for $\gamma$ and $Q_r$ is opposite (see Fig. 18).
Figure 2: Curves via $\gamma$ for $f'(\eta)$.

Figure 3: Curves via $Gr$ for $f'(\eta)$.

Figure 4: Curves via $M_f$ for $f'(\eta)$.
Figure 5: Curves via $\phi$ for $f'(\eta)$.

Figure 6: Curves via $Q_r$ for $\theta(\eta)$.

Figure 7: Curves via $E_c$ for $\theta(\eta)$.
Figure 8: Curves via $\gamma$ for $\theta(\eta)$.

Figure 9: Curves via $Q_r$ for $\theta(\eta)$.

Figure 10: Curves via $R_d$ for $\theta(\eta)$.
Figure 11: Curves via $\theta_r$ for $\theta(\eta)$.

Figure 12: Curves via $\gamma$ for $\zeta(\eta)$.

Figure 13: Curves via $K$ for $\zeta(\eta)$.
Figure 14: Curves via $K_s$ for $\zeta(\eta)$.

Figure 15: Curves via $Sc$ for $\zeta(\eta)$.

Figure 16: Plots for $C_f Re_z^{-1/2}$ via $M_f$ and $\phi$. 
4 Main findings

This article addressed a comprehensive study of SWCNT and MWCNT suspended in base fluid water transport towards a stretching cylinder effected by mixed convection, MHD, viscous dissipation, thermal radiation and homogeneous-heterogeneous reactions. Further more the base fluid is documented variable viscosity. Cylindrical coordinates are adopted for mathematical development. Nonlinear coupled differential equations are obtained through similarity transformations. These developed equations are tackled via bvp4c implicit finite difference MATLAB soft-
ware. The key results are mentioned below:

1. Velocity decreases is found for curvature and Hartmann variables.

2. Velocity enhancement is observed for Grashof number.

3. Temperature is an increasing function of curvature and radiation variables.

4. An opposite trend of temperature is noticed for radiation and Eckert number.

5. Temperature in case of MWCNT is noted higher than SWCNT.

6. Concentration for Schmidt number is opposite when compared with homogeneous-heterogeneous reactions parameters.

7. Skin friction coefficient develops for $\gamma$ and $\phi$ and Nusselt number $N_{u_{z}}Re_{z}^{\frac{1}{2}}$ boosts against $Q_{r}$, $\gamma$ and it decreases via radiation.

References

[1] S.U.S. Choi, Enhancing thermal conductivity of fluids with nanoparticles, Developments and Applications of Non-Newtonian Flows, D.A. Siginer and H.P. Wang, eds, FED-Vol. 231/MD 66, ASME, New York, (1995) 99-105.

[2] Z. Said, R. Saidur, N.A. Rahim and M.A. Alim, Analyses of energy efficiency and pumping power for a conventional flat plate solar collector using SWCNTs based nanofluid, Energy Buildings 78 (2014) 1-9.

[3] M.M. Rashidi, S. Abelman and N.F. Mehr, Entropy generation in steady MHD flow due to a rotating porous disk in a nanofluid, Int. J. Heat Mass Trans. 62 (2013) 515-525.
[4] M. Sheikholeslami, R. Ellahi, H.R. Ashorynejad, G. Domairry and T. Hayat, Effects of heat transfer in flow of nanofluids over a permeable stretching wall in a porous medium, J. Comput. Theor. Nanosci. 11 (2014) 486-496.

[5] K. Bhattacharyya and G.C. Layek, Magnetohydrodynamic boundary layer flow of nanofluid over an exponentially stretching permeable sheet, Phys. Res. Int. (2014) 592536.

[6] P.M. Patil, M. Kulkarni and P.S. Hiremath, Effects of surface roughness on mixed convective nanofluid flow past an exponentially stretching permeable surface, Chinese J. Phys. 64 (2020) 203-218.

[7] M. Sohail and R. Naz, Modified heat and mass transmission models in the magnetohydrodynamic flow of Sutterby nanofluid in stretching cylinder, Physica A: Statistical Mech. Appl. (2020) 124088.

[8] G.S. Sath and M.K. Mishra, Analysis of transient flow of MHD nanofluid past a nonlinear stretching sheet considering Navier’s slip boundary condition, Adv. Powd. Tech. 28(2017) 375-384.

[9] B.S. Rout and S.R. Mishra, Thermal energy transport on MHD nanofluid flow over a stretching surface: A comparative study, Eng. Sci. Tech.: an Int. J. 21 (2018) 60-69.

[10] T. Hayat, Z. Hussain, A. Alsaedi and S. Asghar, Carbon nanotubes effects in the stagnation point flow towards a nonlinear stretching sheet with variable thickness, Advan. Powder Tech. 27 (2016) 1677-1688.

[11] B.J. Gireesha, B. Mahanthesh, G.T. Thammanna and P.B. Sampathkumar, Hall effects on dusty nanofluid two-phase transient flow past a stretching sheet using KVL model, J. Mol. Liq. 256 (2018) 139-147.
[12] Y.S. Daniel, Z.A. Aziz, Z. Ismail and F. Salah, Impact of thermal radiation on electrical MHD flow of nanofluid over nonlinear stretching sheet with variable thickness, Alexandria Eng. J. 57 (2018) 2187-2197.

[13] M.I. Khan, M.U. Hafeez, T. Hayat, M.I. Khan and A. Alsaedi, Magneto rotating flow of hybrid nanofluid with entropy generation, Comput. Meth. Programs Biomedicine 183 (2020) 105093.

[14] A.S. Khan, M.I. Khan, T. Hayat and A. Alsaedi, Darcy-Forchheimer hybrid (MoS2, SiO2) nanofluid flow with entropy generation, Comput. Meth. Programs Biomedicine 185 (2020) 105152.

[15] A.U. Khan, S. Saleem, S. Nadeem and A.A. Alderremy, Analysis of unsteady non-axisymmetric Homann stagnation point flow of nanofluid and possible existence of multiple solutions, Physica A: Statistical Mechanics and its Applications (2019) 123920.

[16] N.U. Huda, A. Hamid and M. Khan, Impact of Cattaneo-Christov model on Darcy-Forchheimer flow of ethylene glycol base fluid over a moving needle, J. Materials Research Tech. (In press).

[17] C. Liu, M. Pan, L. Zheng and P. Lin, Effects of heterogeneous catalysis in porous media on nanofluid-based reactions, Int. Commun. Heat Mass Transfer 110 (2020) 104434.

[18] S. Nadeem, Rizwan Ul Haq, Z.H. Khan, Numerical study of MHD boundary layer ow of a Maxwell uid past a stretching sheet in the presence of nanoparticles, J. Taiwan Inst. Chem. Eng. 45 (2014) 121126.

[19] A. Malvandi, M.R. Safaei, M.H. Kaffash, D.D. Ganji, MHD mixed convection in a vertical annulus filled with Al2O3- water nanouid considering nanoparticle migration, J. Magn. Magn Mater. 382 (2015) 296306.
[20] S. Ahmad, M. Farooq, N. A. Mir, A. Anjum and M. Javed, Magneto-
hydrodynamic flow of squeezed fluid with binary chemical reaction and ac-
tivation energy, J. Central South University 26 (2019) 1362-1373.

[21] T. Hayat, R. Riaz, A. Aziz and Ahmed Alsaeedi, Influence of Arrhenius acti-
vation energy in MHD flow of third grade nanofluid over a nonlinear stretch-
ing surface with convective heat and mass conditions, Physica A: Statistical
Mech. Appl. (2020) 124006.

[22] A.H. Emad and I. Pop, MHD flow and heat transfer near stagnation point
over a stretching/shrinking surface with partial slip and viscous dissipation:
Hybrid nanofluid versus nanofluid, Powder Tech. (In press).

[23] R. Cortell, Internal heat generation and radiation effects on a certain free
convection ow, Int. J. Nonlinear Sci. 9 (2010) 468479.

[24] S. Nadeem and R.U Haq, Effect of thermal radiation for megneto hydrody-
namic boundary layer flow of a nanofluid past a stretching sheet with convec-
tive boundary conditions, J. Comput. Theoretical Nanosci. 11 (2014) 32-40.

[25] S.A. Mohammadein, K. Raslan, M.S. Abdel-Wahed and E.M. Abedel-Aal,
KKL-model of MHD CuO-nanofluid flow over a stagnation point stretching
sheet with nonlinear thermal radiation and suction/injection, Results Phy. 10
(2018) 194-199.

[26] S. Muhammad, G. Ali, Z. Shah, S. Islam and S.A. Hussain, The rotating flow
of magneto hydrodynamic carbon nanotubes over a stretching sheet with the
impact of non-linear thermal radiation and heat generation/absorption, Appl.
Sci. 8 4 (2018) 482.
[27] T. Hayat, S. Ullah, M.I. Khan, A. Alsaedi and Q.M.Z. Zia, Non-Darcy flow of water-based carbon nanotubes with nonlinear radiation and heat generation/absorption, Results phy. 8 (2018) 473-480.

[28] M.A. Chaudhary and J.H. Merkin, A simple isothermal model for homogeneous-heterogeneous reactions in boundary-layer flow, I Equal d-iffusivities, Fluid dynamics research 16 (1995) 311-333.

[29] K. Bhattacharyya, Dual solutions in boundary layer stagnation-point flow and mass transfer with chemical reaction past a stretching/shrinking sheet, Int. Commun. Heat Mass Trans 38 (2011) 917-922.

[30] M.M. Rashidi, N. Rahimzadeh, M. Ferdows, M.J. Uddin and O.A. Beg, Group theory and differential transform analysis of mixed convective heat and mass transfer from a horizontal surface with chemical reaction effects, Chem. Eng. Commun. 199 (2012) 1012-1043.

[31] T. Hayat, Z. Hussain, A. Alsaedi and M. Mustafa, Nanofluid flow through a porous space with convective conditions and heterogeneoushomogeneous reactions, J. Taiwan Institute Chem. Eng. 70 (2017) 119-126.

[32] Z. Hussain, T. Hayat, A. Alsaedi and B. Ahmed, Darcy Forhheimer aspect-s for CNTs nanofluid past a stretching cylinder; using Keller box method, Results Phy. 11 (2018) 801-816.

[33] Q. Xue, Model for thermal conductivity of carbon nanotube based composites, Phys. B Condens Matter 368 (2005) 302307.

[34] P. Sreedevi, P.S. Reddy and A.J. Chamkha, Magneto-hydrodynamics heat and mass transfer analysis of single and multi-wall carbon nanotubes over vertical cone with convective boundary condition, Int. J. Mech. Scie. 135 (2018) 646-655.