Numerical simulations of nanosecond discharge in gas-dynamic flows

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Abstract. Numerical simulations were performed of the nanosecond high-voltage breakdown in nitrogen at a voltage of 25–35 kV in the sub-centimeter discharge gaps. The streamer configurations were studied at two constant pressures, as well as at the radially varying pressure, simulating the pressure distribution in the axial region of the vortex “columnar” gas-dynamic flow. In each case, the characteristic parameters and propagation velocities of the primary and secondary streamers were calculated. It was ascertained that the law of pressure change in the radial direction strongly affects the spatial structure of the negative streamer.

1. Introduction
The gas-dynamic flows of low-temperature gas discharge plasma are widely studied due to the interest in the technological applications. In plasma aerodynamics, the plasma actuators can be used to control flows near the surface and inside the channel, and to ignite fuel in the engines, due to such their properties as the fast gas heating and the shock wave generation [1–4]. The nature of the gas flow affects the discharge geometry and characteristics. When a pulsed discharge is initiated in the non-uniform medium, it can develop in the form of a localized plasma formation [1–3]. In the presence of vortexes, the discharges become localized in the low-pressure region [2, 3]. The numerical and experimental analysis of the thermal and hydrodynamic effects occurring during the nanosecond breakdown in atmospheric pressure air demonstrated the strong effect of the initial temperature and the fast heating energy on the intensity of the shock-wave generated by the discharge [5, 6]. Thus, the mechanism for the discharge effect on the flow depends on the processes occurring during the breakdown and in the stage of the discharge electrical current. That is why it is important to analyze the streamer developments in inhomogeneous mediums. The goal of the work is to numerically study the mechanism for the streamer formation in the discharges in nitrogen in the sub-centimeter gaps (in the “pin-pin” geometry). We will consider the discharges at atmospheric pressure and at the reduced pressure inside a vortex with allowance for the effects of the gas-dynamic flow parameters on the discharge characteristics.

2. Mathematical model
In the two-dimensional (axisymmetric) case, the plasma dynamics in the discharge gap can be described using the continual drift-diffusion model (the multifluid continuum of the electron and ion fluids) [7, 8].
The drift-diffusion model consists of equations describing the space-time evolution of the electron and positively charged ion densities. Supplemeting these equations with the Poisson equation for the electric field characteristics, we obtain the following set of equations:

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \mathbf{j}_e = \alpha \mu_e n_e |E| - \beta n_e n_p, \quad \mathbf{j}_e = -D_e \nabla n_e - \mu_e E$$

$$\frac{\partial n_p}{\partial t} + \nabla \cdot \mathbf{j}_p = \alpha \mu_p n_e |E| - \beta n_p n_e, \quad \mathbf{j}_p = -D_p \nabla n_p + \mu_p n_e E \quad (1)$$

where $E$ is the electric field strength, $\phi$ is the electric field potential, $n_e, n_p$ are the electron and positively charged ion densities, respectively, and $\mu_e, \mu_p, D_e, D_p$ are the electron and positively charged ion mobilities and diffusion coefficients, respectively. Nitrogen was chosen to be the gaseous medium; then, the transport coefficients are as follows:

$$\mu_e = 4.5 \times 10^3 \frac{\text{Torr} \cdot \text{cm}^2}{\text{V} \cdot \text{s}}, \quad \mu_p = 1.45 \times 10^3 \frac{\text{Torr} \cdot \text{cm}^2}{\text{V} \cdot \text{s}}$$

$$D_e = \mu_e T_e, \quad D_p = \mu_p T_p \frac{\text{cm}^3}{\text{s}}$$

Ionization and recombination rates are as follows:

$$\alpha = 12 \cdot p \cdot e^{342} \frac{\text{cm}^3}{\text{Torr} \cdot \text{cm}^{-1}}, \quad \beta = 2 \times 10^{-7} \frac{\text{cm}^3}{\text{s}}$$

The computational domain and boundary conditions at each boundary are presented in figure 1.

Figure 1. The computational domain of drift-diffusion model with boundary conditions.

Figure 2. The pressure distributions at constant pressure (red line) and in the vortex (blue line).

The origin of the cylindrical coordinate system $(z, r)$ is set on the cathode, and the Oz axis matches with the streamer axis. Near the cathode on the symmetry axis, the seed plasma spot is set with the following initial electron and positively charged ion density distributions; the voltage $V$ is applied to the anode:

$$n_e^0 = n_p^0 = 10^7 \cdot e^{\frac{r^2 + z^2}{\sigma^2}} \text{cm}^{-3}, \quad \sigma = 10^{-2} \text{ cm} \quad (2)$$
2.1. Numerical method
The first two differential equations in the mathematical model are the equations of the convection-diffusion-reaction type. The combined approach is used for their numerical approximation. The time-explicit numerical scheme is used with the approximation of the first-order spatial derivatives in the convective part by the backward finite differences (the upwind scheme) and approximation of the second-order derivatives in the diffusion part by the three-point central-difference formula [9]. The time step was chosen with allowance for the stability conditions for the convective and diffusion parts and the numerical values of the right-hand sides of the equations. For each time step, the electric field potential was calculated by solving the elliptic-type Poisson equation using the Gauss-Seidel upper relaxation method.

3. Numerical simulation results
The adjoint problem of the streamer-spark discharge propagation in nitrogen at a temperature of \( T = 297 \text{ K} \) in the interelectrode gap with a length of \( d = 0.6 \text{ cm} \) and a width of \( R = 0.1 \text{ cm} \) was solved numerically. The applied pulsed voltage was \( V = 25 \text{ kV} \). The effect of the gaseous medium parameters on the streamer space-time structure was studied. Three cases were considered: in the first one, the constant pressure (density) was \( p = 760 \text{ Torr} \), in the second one, \( p = 380 \text{ Torr} \), and in the third one, the pressure varied in the radial direction (figure 2). In the latter case, the conditions were simulated arising in the vortex structures of the real gas-dynamics flows. The uniform 300 × 100 Cartesian computational grid was used in the calculations. The fields of the electron density and the axial electric field strength at times corresponding to the streamer head contact with the anode surface are presented in figures 3 and 4. These images illustrate the streamer spatial structure and time evolution.

![Image](image_url)

**Figure 3.** The electron density (cm⁻³) distributions (axial and radial coordinates – cm × cm):
(a) constant pressure \( p = 760 \text{ Torr} \), \( t = 21 \text{ ns} \); (b) constant pressure \( p = 380 \text{ Torr} \), \( t = 2.1 \text{ ns} \); (c) varying pressure, \( t = 2.6 \text{ ns} \).

It was found that, when the pressure decreases twice, the negative streamer propagation velocity increases by an order of magnitude. The average velocity of the streamer head propagation along the length of the discharge gap is \( 2.7 \cdot 10^7 \), \( 2.5 \cdot 10^8 \) and \( 2.12 \cdot 10^8 \) cm/s in cases (a), (b) and (c), respectively. It can be seen that, in case (c), this parameter is closer to that calculated in case (b). The longitudinal structure of the streamer electric field can be characterized by the distribution along the axis of symmetry shown in Fig. 4. Under the same initial conditions, in case (a), the range of the electric field variation is narrower ((4.0–5.1) \( \cdot 10^4 \) V/cm) than that in cases (b) and (c), ((0.2–1.8) \( \cdot 10^5 \) and \( 0.4-2.2 \) \( \cdot 10^5 \) V/cm, respectively).
The streamer transverse structure is determined by the electron diffusion rate, which is lower in the lower pressure case. This results in the greater streamer head expansion and an increase in the streamer channel radius. In all cases under consideration, the secondary wave (secondary streamer) is observed (in figure 3, the high electron density regions are marked in different shades of red); it propagates through the primary streamer channel and forms the radial field that prevents the channel from the radial expanding [7]. At higher pressures, the secondary wave propagation velocity is higher as compared to that of the streamer head. The pressure in the axial region determines the streamer propagation velocity. In the vortex “columnar” flow, it has considerably different radial structure (the streamer channel radial scale is by an order of magnitude smaller than that in the case of the constant pressure).

4. Conclusions
The plasma dynamics related to the problem of the gas streamer breakdown in nitrogen in the centimeter-sized gap was considered in the framework of the two-dimensional nonstationary mathematical model. The effective numerical scheme was developed and the numerical research was performed of the streamer structure in the regions with the constant pressure and the pressure varying in the radial direction. In the latter case, the conditions were simulated arising in the vortex structures of the real gas-dynamics flows. The effect of the medium parameters in the discharge gap on the spatial and temporal streamer structures is demonstrated.

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