Simultaneous Multicolor Observations of Starlink’s Darksat by the Murikabushi Telescope with MITSuME

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Abstract

In this paper, we present the SDSS g'-, the Cousins R_c- and I_c-band magnitudes and associated colors of Starlink’s STARLINK-1113 (one of the standard Starlink satellites) and 1130 (Darksat) with a darkening treatment to its surface. Using the 105 cm Murikabushi telescope/MITSuME, simultaneous multicolor observations for the above satellites were conducted four times: on 2020 April 10 and May 18 (for Darksat), and 2020 June 11 (for Darksat and STARLINK-1113). We found that (1) the SDSS g'-band apparent magnitudes of Darksat (6.95 ± 0.11–7.65 ± 0.11 mag) are comparable to or brighter than that of STARLINK-1113 (7.69 ± 0.16 mag), (2) the shorter the observed wavelength is, the fainter the satellite magnitudes tend to become, (3) the reflected flux by STARLINK-1113 is extremely (>1.0 mag) redder than that of Darksat, (4) there is no clear correlation between the solar phase angle and orbital altitude-scaled magnitude, and (5) by flux model fitting of the satellite trails with the blackbody radiation, it is found that the albedo of Darksat is about half that of STARLINK-1113. In particular, result (1) is inconsistent with previous studies. However, considering both solar and observer phase angles and atmospheric extinction, the brightness of STARLINK-1113 can be drastically reduced in the SDSS g' and the Cousins R_c band. Simultaneous multicolor–multispot observations of more than three colors would give us more detailed information regarding the impact of low-Earth-orbit satellite constellations.

Unified Astronomy Thesaurus concepts: Astronomical site protection (94); Astronomical research (91); Observatories (1147)

1. Introduction

SpaceX launched 60 Starlink satellites to low Earth orbit (LEO) on 2019 May 24, as the first batch of a large constellation. Furthermore, SpaceX plans to launch 42,000 Starlink LEO communication satellites in total until the mid-2020s. On 2019 June 3, the International Astronomical Union (IAU) expressed its concern for the fact that the extremely bright magnitude of these communication satellites would affect astronomical observations and the pristine appearance of the night sky. In response to the concerns of the IAU, SpaceX has tried to reduce satellite brightness, and has also asked astronomers to measure the satellite brightness. Since the launch of the Starlink satellites, their apparent magnitude and impact on astronomical observations have been reported in previous studies, using ground-based telescopes. Hainaut & Williams (2020) investigated the impact of mega-constellations of LEO satellites in the optical and IR wavelength regions on the ESO telescopes. They concluded that very wide-field imaging surveys will be ruined due to saturation and/or ghosting by satellites. McDowell (2020) also concluded that certain types of observation, such as long-exposure and twilight observations with wide fields of view (FoV), will be significantly affected by the Starlink satellites.

SpaceX launched the third batch of 60 LEO satellites on 2020 January 7. One of the 60 satellites is the prototype satellite STARLINK-1130 (nicknamed Darksat), whose communication antenna is coated with paint to reduce reflected sunlight to the Earth (see Figure 7 of Tyson et al. 2020). In their initial observation, Tregloan-Reed et al. (2020, hereafter Tr20) estimated the Sloan g' magnitude of Darksat and STARLINK-1113, and suggested that Darksat is 0.77 ± 0.05 magnitude fainter than STARLINK-1113. However, simultaneous multicolor observations for these satellites have not yet been reported, despite the importance of this kind of observation. The simultaneity of the observations ensures the same conditions in multiple bands: same coordinate, airmass, and exposure time, etc. Moreover, it is able to examine the darkening effects of the satellite surface via color estimation and/or radiation model fitting for these LEO communication satellites under simultaneous multicolor observations.

In this paper, we report the multi-band (the SDSS g', the Cousins R_c- and I_c; hereafter g', R_c, and I_c for simplicity) magnitudes and colors of Darksat and STARLINK-1113 with simultaneous multicolor observations, using the 1.05 m Murikabushi telescope/MITSuME system. In Section 2, observations and data analysis for Darksat and STARLINK-1113 are presented. We show the apparent and orbital altitude-scaled magnitudes of the satellites in Section 3. In Section 4, we discuss (1) the effects of atmospheric extinction to the satellites, and (2) modeling the AB flux of the satellites.

2. Observations and Data Analysis

2.1. Observations

The Ishigakijima Astronomical Observatory (IAO) of the National Astronomical Observatory of Japan operates the 105 cm Murikabushi telescope (F12) with MITSuME. The Murikabushi telescope, equipped with the MITSuME system,
Table 1

Observation Log of Darksat and STARLINK-1113 by the Murikabushi Telescope/MITSuME $g'$, $R_c$, and $I_c$ Bands

|                      | Darksat | Darksat | Darksat | STARLINK-1113 |
|----------------------|---------|---------|---------|---------------|
| Observation Date     | 2020 April 10 | 2020 May 18 | 2020 June 11 | 2020 June 11 |
| Start Time of Observation (UTC) | 10:53:50 | 11:13:55 | 12:31:56 | 12:17:56 |
| Central Time of Observation (UTC) | 10:54:00 | 11:14:00 | 12:31:58.5 | 12:17:58.5 |
| End Time of Observation (UTC) | 10:54:10 | 11:14:05 | 12:32:01 | 12:18:01 |
| Exposure Time (s)    | 20.0    | 10.0    | 5.0     | 5.0          |
| R.A.$^a$             | 05:52:35.05 | 09:04:46.3 | 10:42:36.7 | 12:23:17.5 |
| Decl.$^b$            | −05:18:43.8 | −13:58:57.0 | −13:34:37.2 | −25:03:20.0 |
| Azimuth$^a$ (deg)    | 236.67  | 223.29  | 238.57  | 205.66       |
| Elevation$^a$ (deg)  | 42.49   | 40.00   | 26.96   | 35.80        |
| Airmass$^a$          | 1.48    | 1.55    | 2.20    | 1.71         |
| Altitude$^a$ (km)    | 550.16  | 549.88  | 549.72  | 549.56       |
| Distance between Satellite and Observer (km) | 781.54 ± 3.12 | 812.03 ± 25.62 | 1068.69 ± 0.91 | 873.41 ± 6.36 |
| Solar Phase Angle$^a$ [Sun–Target–Observer] (deg) | 109.9 | 93.7 | 93.5 | 67.4 |
| Phase Angle$^a$ [Sun–Observer–Target] (deg) | 70.1 | 86.3 | 86.5 | 112.6 |
| Observer Phase Angle$^a$ (deg) | 42.9 | 44.8 | 55.1 | 48.3 |
| Angular Velocity (arcsec s$^{-1}$) | 1934.10 ± 10.88 | 1322.99 ± 76.20 | 1415.33 ± 2.35 | 1626 ± 22.03 |

Notes.

$^a$ Values at the central time of observations.

$^b$ The north and the east are 0° and 90°, respectively.

has three 1024 × 1024 pixel CCD cameras (Apogee Alta U-6), facilitating simultaneous $g'$ (477 nm), $R_c$ (649.2 nm) and $I_c$ (802 nm)-band observations, where the values in parenthesis indicate the effective wavelengths in each band. Of the three bands, the $R_c$ band has the highest sensitivity. The F-value of the Murikabushi telescope is adjusted from F12 to F6.5 via the conversion lenses in the MITSuME system; it yields a FoV of 12.3 × 12.3 arcmin$^2$ and a pixel scale of 0.72 arcsec pixel$^{-1}$.

Based on the two-line element (TLE) data from the satellite catalog of the Celestrak website$^6$ and the forecast time and equatorial coordinates extracted from Heavensat,$^7$ we carried out simultaneous multicolor observations for the light trail of Darksat and STARLINK-1113 between 2020 April and June, using the Murikabushi telescope/MITSuME. Table 1 lists the observation log for this study. Since the satellites move so fast, we were not able to track them. Instead we pointed the telescope to the calculated position of a satellite, and waited for the satellite to pass through the field of view.

The observed flux of satellites, $f_{\text{sat}}$, is inversely proportional to the angular velocity of the satellite along the celestial sphere, $V_{\text{sat}}$; therefore, the apparent magnitude of satellites, $m_{\text{sat}}$, is written as follows:

$$m_{\text{sat}} = m_{\text{star}} - 2.5 \log \left( \frac{V_{\text{sat}} f_{\text{sat}}}{V_{\text{star}} f_{\text{star}}} \right),$$

where $V_{\text{sat}}$, $f_{\text{sat}}$, and $m_{\text{sat}}$ are the angular velocity along the celestial sphere, the flux, and the magnitude of reference stars (i.e., $V_{\text{sat}} = 15 \times \cos \delta$ arcsec s$^{-1}$, $\delta$: decl.), respectively. Here, we observed the satellites with star tracking, after which the reference stars were observed by stopping the star tracking immediately. The velocity of the reference stars during fixed observation is $V_{\text{star}}$. It is possible to estimate the angular velocity, $V_{\text{sat}}$, by calculating a traverse speed on the great-circular distance, $\lambda$, which is written as follows:

$$\lambda = \arccos(\sin \delta \sin D + \cos \delta \cos D \cos(A - \alpha)), \quad (2)$$

where ($\alpha$, $\delta$) and ($A$, $D$) are the R.A., decl. of satellites at the initial time of observation and those at a certain time, respectively. In order to measure the flux of reference stars, $f_{\text{star}}$, in the same way, so as to evaluate the flux of satellite trails, we elongated the CCD images of reference stars so that the images become the same shape as satellite light trails. Figures 1–4 show the FITS images of the satellites and elongated reference stars around the satellite trail. In Figures 1 and 4, we do not include the bright stars on the right-side edge of the images for reference stars, due to a count bias on the CCD image of the $I_c$ band. Since there are no bright reference stars without the count bias around the STARLINK-1113 trail (Figure 4), we added and observed a bright reference star near the target field (i.e., three reference stars for the STARLINK-1113 trail, see Table 2).

2.2. Data Analysis

We performed dark subtraction, dome flat corrections, and sky subtraction, using the Image Reduction and Analysis Facility. For the actual data analysis, we subtracted bad pixels and stars from the original satellite trail images (see middle panels of Figures 1–4). In this study we adopted the star catalog UCAC4, which is, for instance, able to refer to Johnson’s $B_r$, $V_r$, SDSS $r$, and $i$-band magnitudes. It is possible to derive $g'$-,$R_c$- and $I_c$-band magnitudes using the equations in Table 3 of Jordi et al. (2006):

$$g = V + 0.630(B - V) - 0.124, \quad (3)$$

$$R = r - 0.252(r - i) - 0.152, \quad (4)$$

$$I = r - 1.245(r - i) - 0.387. \quad (5)$$

We measured the count profile perpendicular to the streak for each object, and compared the corresponding profile from the elongated star image. We note again that the flux per unit
length of the streak is inversely proportional to the angular velocity of an object (see Equation (1)). We used Projection in SAOImage DS9\(^8\) to measure the flux (averaged counts of cross sections) in a rectangular region for the integration along the Darksat and STARLINK-1113 trails, and the reference stars around these satellite trails. The rectangular region to measure the averaged satellite flux, \(f_{\text{sat}}\), is 10 times thicker than that of the reference stars, but avoids the edge of the CCDs (i.e., the trail length was limited in measuring its flux), and we then integrated the area under the profile. Figure 5 exhibits averaged CCD counts along the Darksat and STARLINK-1113 trails.

The statistical magnitude errors, \(\sigma_m\), of Darksat and STARLINK-1113 were estimated by applying the law of error propagation to Equation (1):

\[
\sigma_m = \frac{2.5}{\ln 10} \sqrt{\left( \frac{\delta f_{\text{sat}}}{f_{\text{sat}}} \right)^2 + \left( \frac{\delta f_{\text{star}}}{f_{\text{star}}} \right)^2 + \left( \frac{\delta V_{\text{sat}}}{V_{\text{sat}}} \right)^2},
\]

where \(\delta f_{\text{sat}}\), \(\delta f_{\text{star}}\), and \(\delta V_{\text{sat}}\) are the flux errors for the light trails of satellites and reference stars, and the velocity error of satellites, respectively. In this study, these errors take the following ranges: \(0.005 \lesssim \delta f_{\text{sat}} / f_{\text{sat}} \lesssim 0.07\), \(0.004 \lesssim \delta f_{\text{star}} / f_{\text{star}} \lesssim 0.21\), and \(0.002 \lesssim \delta V_{\text{sat}} / V_{\text{sat}} \lesssim 0.06\). Since the angular velocity of the reference stars, \(V_{\text{star}}\), is constant, in Equation (6) we ignored the term relating to the angular velocity of stars. We estimated the standard deviation of sky flux, \(\sigma_{\text{sky}}\), around the satellite trails to evaluate the flux errors of the trails, \(\delta f_{\text{sat}}\), and those of reference stars, \(\delta f_{\text{star}}\). Here, the sky region used to estimate \(\sigma_{\text{sky}}\) has the same rectangular shape as that used to estimate the flux of trails by satellites or reference stars (i.e., \(\delta f_{\text{sat}}\) or \(\delta f_{\text{star}} = \sigma_{\text{sky}} \times \text{pixel width of trails}\)). The velocity error of satellites, \(\delta V_{\text{sat}}\), is an averaged value of the velocity difference between the central to start exposure time, \(|V_{\text{sat}} - V_{\text{start}}|\), and that of the central to end time, \(|V_{\text{sat}} - V_{\text{end}}|\).

3. Results

In this section, we provide the multiband magnitudes and colors of the Darksat and STARLINK-1113 trails.
3.1. Apparent and Normalized Magnitude

First, we measured the apparent magnitudes of Darksat and STARLINK-1113, as described in the previous section. In addition to correcting the difference of the distance between an observer and a satellite, we normalized these magnitudes at a satellite orbital altitude of \( \sim 550 \) km (hereafter, normalized magnitude) by adding a factor of \(+5\log(r/550)\), where \( r \) is the distance between the satellite and observer (see also Tr20 and Tyson et al. 2020). We considered the errors of \( r \) for the estimation of the normalized magnitudes. Table 2 summarizes the reference stars, apparent, and normalized \( g' \), \( R_c \), and \( I_c \)-band magnitudes for each observation epoch. The apparent and normalized \( g' \)-band magnitudes of Darksat (7.37 \( \pm \) 0.10 and 6.35 \( \pm \) 0.10 mag on average) are slightly brighter than those of STARLINK-1113 (7.69 \( \pm \) 0.16 and 6.68 \( \pm \) 0.17 mag) in our measurements, while those of Darksat are fainter than STARLINK-1113 in the \( R_c \) and \( I_c \) bands. We found a tendency whereby the longer the observed wavelength is, the brighter the two satellite magnitudes become. In particular, STARLINK-1113 showed an extremely bright \( I_c \)-band magnitude of 5.25 \( \pm \) 0.07 (apparent magnitude) and 4.25 \( \pm \) 0.07 mag (normalized magnitude).

Next, we estimated the colors of the Darksat and STARLINK-1113 trails. Under simultaneous multicolor observations, the largest advantage of color estimations is that the effects on each band can be tested without considering their dependence on parameters such as a satellite orbital altitude (or the distance \( r \)), angular velocity, and solar (or observer) phase angle. Table 3 lists the colors \( g' - R_c \), \( g' - I_c \), and \( R_c - I_c \) of Darksat and STARLINK-1113 for each observation epoch. In the case of Darksat, while there is no significant color difference between May 18 and June 11, that of April 10 is different from the data of the other two epochs due to the weather conditions. The colors \( g' - I_c \) and \( R_c - I_c \) of STARLINK-1113 are significantly (>1 mag) redder than those of Darksat for any observation epoch.

3.2. Phase Angle Effect

We used orbital information from the HORIZONS Web-Interface in order to examine the solar phase angle dependence of the normalized magnitudes (see also Table 1). In Figure 6 we have plotted our data point on the solar phase angles versus normalized \( g' \), \( R_c \), and \( I_c \)-band magnitude planes, together with the results of Tr20. The \( g' \) and \( R_c \)-band magnitudes of Darksat do not show any clear relation with the solar phase angle. Those of the \( g' \) band are consistent with that of Tr20.
despite a solar phase angle difference of $\sim 40^\circ$–$55^\circ$. On the other hand, the $g'$-band magnitude of STARLINK-1113 is $\sim 1.5$ mag darker than that of Tr20 in spite of a smaller difference of $\sim 6^\circ$. Consequently, the multiband magnitudes do not exhibit a clear correlation with solar phase angles. The phase angle effect alone cannot explain the magnitude difference between Darksat and STARLINK-1113 (see Appendix B in regard to both solar and observer phase angle effects). This may be due to the small number of observation points; we would need to conduct more frequent observations to reveal whether or not there is a relation between satellite brightness and solar phase angle.

### 4. Discussion

#### 4.1. The Effect of Atmospheric Extinction

The STARLINK-1113 trail showed significantly redder color than the Darksat trail (Table 3). Below, we discuss the effect of atmospheric extinction. The STARLINK-1113 trail was observed on 2020 June 11 at 12:18:00 UTC in this study. Shortly after 12:18:10 UTC, this satellite plunged into the Earth’s shadow. As shown in Figure 7, we define the following angles: (1) angle $\theta_1$, between the line from the geocenter to ground surface (i.e., the Earth’s radius $R_\oplus = 6371$ km) and the line from the geocenter to the satellite’s orbital altitude at the beginning of the Earth’s shadow, $l$, of 6921 km (point B), (2) angle, $\theta_2$, between the line from the geocenter to the upper edge of the atmosphere (i.e., $R_\oplus + \Delta R$ of 6471 km) and the line from the geocenter to the intersecting point with the tangent line of the upper atmosphere (point A; the same length as $l$). A range of angle, $\theta_1 - \theta_2$, where atmospheric extinction becomes noticeable, is written as follows:

$$\theta_1 - \theta_2 = \arctan \left( \frac{\sqrt{l^2 - R_\oplus^2}}{R_\oplus} \right) - \arctan \left( \frac{\sqrt{(R_\oplus + \Delta R)^2}}{R_\oplus + \Delta R} \right) \sim 2.22 \text{[deg]}.$$  

(7)

Since the orbital period of STARLINK-1113 was 95.64 min at the observation time, the angular velocity of STARLINK-1113 was $\sim 0.06$ deg s$^{-1}$; the time required to cross the range,
\( \theta_1 - \theta_2 \), was \( \sim 35.4 \) s. In other words, STARLINK-1113 was orbiting in the vicinity of the edge of the Earth’s shadow at our observation time, and was strongly affected by atmospheric scattering in the \( g' \), and \( R_c \) bands. The Earthshine and terrestrial radiation would also contribute to the magnitudes of the satellite’s trails. However, the contribution of these types of radiation can be reduced in the Earth’s shadow. Other than the altitudes and the phase angles of the satellite, the effect of atmospheric extinction should also be taken into account as another factor. In other words, the usual brightness of Starlink’s LEO satellites can be reduced to a level comparable to that of Darksat by taking into account the above effects.

4.2. Modeling the Flux of Darksat and STARLINK-1113

4.2.1. Effective Radiation Temperature of Darksat and STARLINK-1113

We evaluate the surface temperature of Darksat and STARLINK-1113 from the apparent magnitudes, \( m_{\text{Sat}} \), given in Table 2. Although these LEO satellites have a flat panel shape, if we assume the satellites are spherical, the surface temperature, \( T_{\text{sat}} \), is written as follows (Spencer et al. 1989):

\[
T_{\text{sat}} = \left( \frac{I(1 - a_{\text{obs}})}{4 \sigma \epsilon} \right)^{\frac{1}{4}},
\]

where \( I (= 1.37 \times 10^3 \text{ W m}^{-2}) \), \( a_{\text{obs}} \), \( \epsilon (=0.9) \), and \( \sigma (=5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}) \) are the solar constant, measured surface albedo, the infrared emissivity (Lebofsky et al. 1986), and the Stefan–Boltzmann constant, respectively. For an estimation of the surface temperature, \( T_{\text{sat}} \), we need to derive the absolute magnitude of these satellites, \( H \):

\[
H = m_{\text{Sun}} - 2.5 \log \left( \frac{a_{\text{obs}} r_{\text{sat}}^2}{(1 \text{ au})^2} \right),
\]

where \( m_{\text{Sun}} \) and \( r_{\text{sat}} \) are the apparent magnitude of the Sun (Willmer 2018) and the radius of Darksat or STARLINK-1113, respectively. We employed a radius, \( r_{\text{sat}} \), of 1.5 m for the Starlink satellites (i.e., a flat panel with a diameter of 3 m; McDowell 2020). The \( H \) for the LEO satellites is also approximately described by the phase integral, \( p(\theta) \), as a first-approximation function of the solar phase angle, \( \theta \).
Notes.

\(^a\) Converted UCAC4 magnitudes using Equations (3), (4), and (5).

\(^b\) Averaged magnitude with the above reference stars.

(Whitmell 1907):

\[ m_{\text{sat}} \sim H - 2.5 \log \left( \frac{(1 \text{ au})^2 p(\theta)}{r^2} \right), \]  

\[ p(\theta) = \frac{2}{3} \left( 1 - \frac{\theta}{\pi} \right) \cos \theta + \frac{1}{\pi} \sin \theta. \]

where \( r \) is the distance between an observer and a satellite. It is then possible to derive the surface albedo, \( \alpha_{\text{obs}} \), from Equations (9) through (11). Next, the surface temperature, \( T_{\text{sat}} \), is immediately evaluated. Table 4 summarizes the surface temperatures of Darksat and STARLINK-1113 in this study. From Table 4, we adopt a temperature, \( T_{\text{sat}} \), of 280 K to be used in the following discussion. We note that the satellite radius of 1.5 m is an approximate value, and therefore include the uncertainty, \( \delta r_{\text{sat}} \). Based on Equations (8) through (10), the surface temperature is inversely proportional to the square root of the satellite radius \( (T_{\text{sat}} \propto r_{\text{sat}}^{1/2}) \). When assuming an uncertainty \( \delta r_{\text{sat}} \) of 0.5 m, the corresponding uncertainty of the surface temperature will be at least 2 ~ 3°, which is comparable to the standard deviation of the surface temperature listed in Table 4 (\( \sigma_{T_{\text{sat}}} \sim 2 \text{ K} \)).

4.2.2. Flux Model Incorporating Blackbody Radiation

In this section, we model the normalized \( g' \), \( R_c \), and \( I_c \)-band flux of the Darksat and STARLINK-1113 trails in each epoch, together with their blackbody radiation. We derived these flux values from the normalized and AB magnitudes in each band (Blanton & Roweis 2007). When modeling the flux of the satellite trails with blackbody radiation, the following four
components were considered: the thermal radiation of the satellite, $F_{TS}$, the reflection of sunlight, $F_{RS}$, the earthshine, $F_{REs}$, and the reflection of Earth’s thermal radiation, $F_{TE}$. The four components are written as follows (see also Sekiguchi et al. 2003):

\[
F_{TS} = \pi c \left( \frac{r_{sat}}{h_T} \right)^2 B(\lambda, T_{sat}) \frac{\lambda^2}{c},
\]

\[
F_{RS} = \pi \left( \frac{R_\oplus}{1 \text{ au}} \right)^2 B(\lambda, T_\oplus) a_{mod} \frac{p(\theta)}{p(\theta)} \left( \frac{r_{sat}}{h_T} \right)^2 \frac{\lambda^2}{c},
\]

\[
F_{REs} = a_E \left( \frac{R_\oplus}{R_\oplus + h_T} \right)^2 \left[ 1 - \left( \frac{R_\oplus}{R_\oplus + h_T} \right)^2 \right] \frac{p(\chi)}{p(\theta)} F_{RS},
\]

\[
F_{TE} = \pi c \left( \frac{R_\oplus}{R_\oplus + h_T} \right)^2 B(\lambda, T_E) a_{mod} \left( \frac{r_{sat}}{h_T} \right)^2 \frac{\lambda^2}{c},
\]

where $T_\odot$ (=5772 K), $T_E$ (=290 K), $R_\oplus$ (=7.0 $\times$ 10^5 km), $R_\oplus$, $h_T$ (=550 km), $a_{mod}$, $a_E$ (=0.3), and $p(\chi)$ are the temperature of the Sun and the Earth, the radius of the Sun and the Earth, the orbital height of the LEO satellites, the modeled albedo including the blackbody radiation, the albedo of the Earth, and the phase integral as a function of the Sun-observer-target phase angle $\chi$, respectively. The blackbody radiation, $B(\lambda, T)$, is expressed as:

\[
B(\lambda, T) = \frac{2hc^2}{\lambda^5} \exp \left( \frac{hc}{\lambda kT} \right) - 1.
\]

where $c$, $h$, and $k$ are the speed of light, the wavelength, the Planck constant, and the Boltzmann constant, respectively. We note that all of the four components (i) are monochromatic flux density per unit frequency, and (ii) are multiplied by a factor, $(r/h_T)^2$, for normalization with the orbital height, $h_T$. For the derivation of Equations (14) and (15), see Appendix B. The effect of moonlight can be ignored for all three observation epochs, since the Moon was below the horizon at the observation times (see Appendix C about the Moon effects).
In this flux model, we adjusted the albedo of satellite \( a_{\text{mod}} \) with the fixed values of the satellite temperature, \( T_{\text{sat}} = 280 \) K, and radius, \( r_{\text{sat}} = 1.5 \) m, as explained in the above discussion.

Figure 8 shows the blackbody curves with the flux of the Darksat and STARLINK-1113 trails in Jansky. The flux of the Darksat trail is relatively bright in the \( g' \) band (2020 April 10) or comparable in each band (2020 May 18 and June 11). Meanwhile, the \( g' \)- and \( R_c \)-band flux of STARLINK-1113 are relatively dimmer than those of the \( I_c \) band, probably due to the strong atmospheric scattering. It is clear that the reflected radiation is dominant in the UV to optical region, whereas the thermal radiation is dominant in the mid-infrared region (see also Hainaut & Williams 2020). By fitting the model with the blackbody radiation, it is confirmed that the modeled albedo of Darksat (\( a_{\text{mod}} = 0.04 \)) is about half that of STARLINK-1113 (\( a_{\text{mod}} = 0.075 \)); these values are consistent with the measured albedo, \( a_{\text{obs}} \), in Table 4. Our analysis has demonstrated the effectiveness of the darkening treatment for Darksat. This difference in satellite surface albedo corresponds to Darksat being \( \sim 0.75 \) mag darker than STARLINK-1113; the magnitude difference between the two satellites is consistent with the results of Tr20. Since it was difficult to fit the \( g' \)-band flux with
Table 4
The Phase Integral, Absolute Magnitude, Measured Albedo, and Surface Temperature of Darksat and STARLINK-1113

| Target Satellite (Filter, Date) | Phase Integral $p(\theta)$ | Absolute Magnitude $H$ | Albedo $a_{\text{mod}, \text{obs}}$ | Temperature $T_{\text{sat}}$ (K) |
|---------------------------------|----------------------------|-------------------------|---------------------------------|--------------------------------|
| Darksat ($g'$-band, 2020 April 10) | 0.11 | 30.97 | 0.12 | 277.38 |
| Darksat ($g'$-band, 2020 May 18) | 0.19 | 32.05 | 0.04 | 283.04 |
| Darksat ($g'$-band, 2020 June 11) | 0.19 | 31.59 | 0.07 | 281.31 |
| Darksat ($R_c$-band, 2020 April 10) | 0.11 | 30.94 | 0.06 | 282.01 |
| Darksat ($R_c$-band, 2020 May 18) | 0.19 | 31.52 | 0.03 | 283.77 |
| Darksat ($R_c$-band, 2020 June 11) | 0.19 | 31.07 | 0.05 | 282.49 |
| Darksat ($I_c$-band, 2020 April 10) | 0.11 | 30.66 | 0.06 | 282.17 |
| Darksat ($I_c$-band, 2020 May 18) | 0.19 | 31.12 | 0.04 | 283.55 |
| Darksat ($I_c$-band, 2020 June 11) | 0.19 | 31.71 | 0.05 | 282.34 |
| Averaged Temperature | ... | ... | ... | 282.01 |
| STARLINK-1113 ($g'$-band, 2020 June 11) | 0.36 | 32.73 | 0.02 | 284.55 |
| STARLINK-1113 ($R_c$-band, 2020 June 11) | 0.36 | 31.66 | 0.03 | 284.06 |
| STARLINK-1113 ($I_c$-band, 2020 June 11) | 0.36 | 30.30 | 0.08 | 280.49 |
| Averaged Temperature | ... | ... | ... | 283.07 |

Note.
* The radius of the satellites, $r_{\text{sat}}$, is fixed to 1.5 m.
the single-albedo blackbody model, the difference between the \( g' \)-band flux and that of the other bands probably reflects the physical properties of these satellites.

5. Conclusions

Using the 105 cm Murikabushi telescope with the MITSuME system, we conducted simultaneous multicolor (\( g' \), \( R_c \), and \( I_c \) bands) observations for the Starlink’s LEO satellites, Darksat and STARLINK-1113, on 2020 April 10, May 18, and June 11. Our observational results are summarized as follows:

1. The apparent \( g' \)-band magnitudes of Darksat are comparable to or brighter than those of STARLINK-1113.
2. The shorter the observed wavelength, the fainter the satellite magnitudes tend to become.
3. At the vicinity of the Earth’s shadow, the reflected flux for STARLINK-1113 is significantly (>1.0 mag) redder than that of Darksat, excluding the difference of \( g' - R_c \).
4. There is no correlation between solar phase angle and orbital altitude-scaled magnitude.
5. The flux model fitting to the satellite trails with black-body radiation revealed that the albedo of Darksat is about half that of STARLINK-1113.

Result (1) is contrary to that of Tr20. However, our results could be explained by qualitatively taking into account both the solar and observer phase angles and the atmospheric extinction.

On 2020 June 3, SpaceX launched its eighth batch of Starlink’s LEO satellites\(^\text{10}\). These satellites include STARLINK-1436 (nicknamed “Visorsat”), equipped with a deployable Sun visor to prevent sunlight and reduce reflected flux. Henceforth, it is important to compare the astronomical impact of the Starlink’s normal LEO satellites, Darksat, and Visorsat. For a precise verification of the solar and observer phase angle dependence of satellite magnitudes, it will be necessary to carry out multicolor–multispot observation for Starlink’s LEO satellites, including Darksat and Visorsat.

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Appendix A

Approximate BRDF Model

Contrary to the results of Tr20, the apparent and normalized magnitudes of STARLINK-1113 are fainter than or comparable to those of Darksat, except for the \( I_c \)-band magnitude (Figure 6). Here, we examine both the solar and observer phase angle effects of Darksat and STARLINK-1113. The definition of the observer phase angle is the angle between the straight line from the geocenter to the satellite, and the straight line from the observer to the satellite (see the angle \( \phi \) in Figure 1 of Mallama 2020). A previous study, Tr20, estimated a parameterized bidirectional reflectance distribution function (BRDF) model, after Minnaert (1941) to investigate the two phase angle effects approximately. They evaluated the ratio, \( R \), which is the index of solar phase attenuation between the two satellites:

\[
R = \left( \frac{\cos \theta_{1113} \cos \phi_{1113}}{\cos \theta_{DS} \cos \phi_{DS}} \right)^{1-k},
\]

where \( \theta_{DS} \) (or \( \phi_{DS} \)) and \( \phi_{1113} \) are the solar and observer phase angle of Darksat (or STARLINK-1113), respectively. The Minnaert exponent \( k \) ranges from 0 to 1. If the ratio \( R > 1 \) (or \( <1 \), a phase angle-dependent reflectance of Darksat (or STARLINK-1113) is dominant. Figure A1 shows the ratio \( R(k) \) as a function of the Minnaert exponent, \( k \), on 2020 June 11, with the \( R(k) \) in Tr20. In this study, the phase angle component of the reflected flux of Darksat is dominant for any \( k \). Tr20 assumed a dark surface (\( k = 0.5 \); Stamnes et al. 1999) for Darksat and STARLINK-1113. When \( k = 0.5 \), \( R(k) \) in this study will be \( \sim 0.68 \), corresponding to a 0.42 mag darkening of Darksat relative to STARLINK-1113. Since the normalized \( g' \) (or \( R_c \)) band magnitude of Darksat on 2020 June 11 (third data points from the right in Figure 6) is already brighter than (or comparable to) that of STARLINK-1113, the correction by BRDF (i.e., brighten the Darksat magnitude by 0.42 mag) leads to the result that STARLINK-1113 is always darker than Darksat in the \( g' \) and \( R_c \) bands.

\(^{10}\)https://www.spaceflightsinside.com/missions/starlink/first-starlink-visorsat-takes-flight-aboard-spacex-falcon-9/
Appendix B

Reflections of Earthlight and Earth’s Thermal Radiation

This section describes the derivation for the reflection flux of the earthlight, \( F_{\text{REs}} \), and Earth’s thermal radiation, \( F_{\text{TE}} \), from the LEO satellite to an observer. The definition of the following parameters is the same as that defined in Section 4.

B.1. Reflection of Earthlight

The thermal radiation density of the Sun received by the Earth, \( F_{\text{SE}} \), is expressed by:

\[
F_{\text{SE}} = 4\pi B(\lambda, T_\odot) \frac{\pi R_\odot^2}{4\pi(1\text{ au})^2} \frac{\lambda^2}{c}
\]

\[
= \pi \left( \frac{R_\odot}{1\text{ au}} \right)^2 B(\lambda, T_\odot) \frac{\lambda^2}{c},
\]

\[
B(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{\exp \left( \frac{hc}{\lambda kT} \right) - 1}.
\]

As shown in Figure B1, considering the distance from the Earth’s surface and the cross-sectional area of the Earth with respect to the cross-sectional area limited by the sphere with radius \( l \) centered at the LEO satellite, the flux of the earthlight received by the LEO satellite, \( F_{\text{ER,sat}} \), is written as follows:

\[
F_{\text{ER,sat}} = a_E F_{\text{SE}} \frac{(R_\odot \sin \theta_\odot)^2}{\pi R_\odot^2} \times \frac{R_\odot^2}{(R_\odot + h_T)^2}
\]

\[
= a_E F_{\text{SE}} \left( \frac{R_\odot}{R_\odot + h_T} \right)^2 \left( 1 - \left( \frac{R_\odot}{R_\odot + h_T} \right)^2 \right),
\]

where \( \theta_\odot \) is the angle between the line from the geocenter to the LEO satellite and the line from the geocenter to the interacting point on the ground surface with the tangent line between the LEO satellite and ground surface. As a result, the earthlight flux from the LEO satellite received by an observer, \( F_{\text{REs}} \), is described by:

\[
F_{\text{REs}} = F_{\text{ER,sat}} a_{\text{mod}} \left( \frac{r}{d} \right)^2 p(\chi) \left( \frac{r}{h_T} \right)^2
\]

\[
= \pi \left( \frac{R_\odot}{1\text{ au}} \right)^2 B(\lambda, T_\odot) a_E \left( \frac{R_\odot}{R_\odot + h_T} \right)^2
\]

\[
\times \left( 1 - \left( \frac{R_\odot}{R_\odot + h_T} \right)^2 \right)^2 a_{\text{mod}} \left( \frac{r}{h_T} \right)^2 p(\chi) \frac{\lambda^2}{c}
\]

\[
= a_E \left( \frac{R_\odot}{R_\odot + h_T} \right)^2 \left( 1 - \left( \frac{R_\odot}{R_\odot + h_T} \right)^2 \right) \frac{p(\chi)}{p(\theta)} F_{\text{RS}},
\]

where this flux is normalized with the orbital height, \( h_T \), by the factor \( (r/h_T)^2 \).

B.2. Reflection of Earth’s Thermal Radiation

The thermal radiation density of the Earth received by the LEO satellite, \( F_{\text{ET,sat}} \), is evaluated as follows:

\[
F_{\text{ET,sat}} = 4\pi \varepsilon B(\lambda, T_E) \frac{\pi (R_\odot \sin \theta_\odot)^2}{4\pi l^2} \frac{\lambda^2}{c}
\]

\[
= \pi \varepsilon \left( \frac{R_\odot}{R_\odot + h_T} \right)^2 B(\lambda, T_E) \frac{\lambda^2}{c}.
\]
Specifically, the Earth’s thermal radiation from the LEO satellite received by an observer, $F_{TE}$, normalized with the orbital height is expressed by:

$$F_{TE} = F_{ET, sat} \ a_{mod} \left( \frac{r}{hT} \right)^2 = \pi \epsilon \left( \frac{R}{R_{\oplus} + hT} \right)^2 B(\lambda, T_E) a_{mod} \left( \frac{r_{sat}}{hT} \right)^2 \frac{\lambda^2}{c}. \quad (B6)$$

## Appendix C

**Thermal and Reflected Satellite Flux from The Moon**

In Section 4, we discussed the normalized thermal and reflected flux of Darksat and STARLINK-1113 without referencing the effect of the Moon. The normalized thermal, $F_{TM}$, and reflected flux, $F_{RM}$, of the satellites in relation to the Moon are described as:

$$F_{TM} = \pi \epsilon \left( \frac{R_{Moon}}{D_{Moon}} \right)^2 B(\lambda, T_{Moon}) a_{Moon} p(\theta_{Moon}) \left( \frac{r_{sat}}{hT} \right)^2 \frac{\lambda^2}{c}, \quad (C1)$$

$$F_{RM} = \pi \left( \frac{R_{Moon}}{1 \ au} \right)^2 B(\lambda, T_{Moon}) a_{Moon} p(\theta_{Moon}) \times \left( \frac{R_{Moon}}{D_{Moon}} \right)^2 a_{p}(\theta) \left( \frac{r_{sat}}{hT} \right)^2 \frac{\lambda^2}{c}. \quad (C2)$$

where $R_{Moon} (=1.73 \times 10^3 \ km)$, $T_{Moon}$, $a_{Moon} (=0.12)$, $\theta_{Moon}$ (or $\theta$), $p(\theta_{Moon})$ (or $p(\theta)$), and $D_{Moon} (=3.83 \times 10^5 \ km)$ are the Moon’s radius, the irradiated Moon’s surface temperature, the albedo of the Moon, the Sun–Moon–satellite (or Moon–satellite–observer) phase angle, the phase integral, and the distance between an observer and the Moon, respectively. The definitions of the other parameters in Equations (C1) and (C2) are the same as those defined in Section 4.

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![Diagram of the Earth and LEO satellite](image-url)