Quark Masses and Flat Directions in String Models

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Abstract

I discuss a method for producing a quasi–realistic inter–generational quark mass hierarchy in string models. This approach involves non–Abelian singlet states developing intermediate scale vacuum expectation values. I summarize recent investigations into string model realization of this.

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I. Quark Mass Hierarchy

The experimental measurement a few years ago of the top quark mass [1] eliminated the last unknown among the physical masses of all three generations of up–, down–, and electron–like particles. The masses of these particles are displayed in Table I, where I have expressed all masses in top–quark mass units. To first order, the inter–generational mass ratio for up–type quarks is $10^{-5} : 10^{-3} : 1$, and for down–type is $10^{-5} : 10^{-3} : 10^{-2}$.

In minimal supersymmetric standard model (MSSM) physics, quarks (and their supersymmetric partners) gain mass through superpotential couplings to Higgs bosons $H_1$ and $H_2$:

$$W_{u_i} \sim \lambda_{u_i} \hat{H}_2 \hat{Q}_i \hat{U}^c_i; \quad W_{d_i} \sim \lambda_{d_i} \hat{H}_1 \hat{Q}_i \hat{D}^c_i; \quad (1)$$

where $i$ is the generation number. Effective mass terms appear when the Higgs acquire a typical soft supersymmetry breaking scale vacuum expectation value (VEV) $\langle H_{1,2} \rangle \sim m_{\text{soft}} = \mathcal{O}(M_Z)$.†

Inter–generational mass ratios can be induced when the associated first and second generation superpotential terms contain effective couplings $\lambda$ that include non–renormalizable suppression factors,

$$\lambda_{(u,d,e)i} \sim \left(\frac{\langle S \rangle}{M_{\text{Pl}}}\right)^{P_i'}, \quad \text{for } i = 1, 2, \quad (2)$$

where $S$ is a non–Abelian singlet, $M_{\text{Pl}}$ is the Planck scale (which is replaced by the $M_{\text{string}}$ for string models), and $P$ is a positive integer. VEVs only slightly below the Planck/string scale (often resulting from a $U(1)$ anomaly cancellation) imply large values of $P_i'$ for $10^{-5}$ and $10^{-3}$ suppression factors. In contrast, intermediate scale VEVs (between $M_Z$ and $M_{\text{string}}$) require far lower values for $P_i'$. In a series of recent papers intermediate scales have been explored [2,8] and their realization in actual models has been investigated [4,8]. This investigation has been the product of a fruitful collaboration with M. Cvetić, J. Espinosa, L. Everett, P. Langacker, & J. Wang at the University of Pennsylvania.‡

In the following section, I show how intermediate scales can occur and how, in theory, they could produce an inter–generational $10^{-5} : 10^{-3} : 1$ up–quark mass ratio, and a corresponding $10^{-5} : 10^{-3} : 10^{-2}$ down–quark mass ratio.

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* $\hat{X}$ denotes a generic superfield and $X$ its bosonic component.

† Similarly, the three generations of electron–type leptons gain mass via superpotential terms $W_{e_i} \sim \lambda_{e_i} \hat{H}_1 \hat{L}_i \hat{E}^c_i$. 

‡ In addition to this mass ratio study from the perturbative string theory perspective, Katsumi Tanaka of the Ohio State University and I have been investigating quark mass ratios from the non–perturbative approach of Seiberg–Witten duality [8].
II. Theoretical Quark Mass Hierarchy from String Models

One method by which an intermediate scale VEV $\langle S \rangle$ can generate non–renormalizable suppression factors involves extending the SM gauge group by an additional non–anomalous $U(1)'$. This approach requires (at least) two SM singlets $S_1$ and $S_2$, carrying respective $U(1)'$ charges $Q_1'$ and $Q_2'$. $D$–flatness for the non–anomalous $U(1)'$,

$$\langle D \rangle_{U(1)'} \equiv Q_1'|\langle S_1 \rangle|^2 + Q_2'|\langle S_2 \rangle|^2 = 0,$$

(3)

necessitates that $Q_1'$ and $Q_2'$ be of opposite sign. Together the VEVs of $S_1$ and $S_2$ form a $D$–flat scalar field direction $S$ defined by,

$$\langle S_1 \rangle = \cos \alpha_Q \langle S \rangle, \quad \langle S_2 \rangle = \sin \alpha_Q \langle S \rangle, \quad \text{where} \quad \tan^2 \alpha_Q \equiv \frac{|Q_1|}{|Q_2|}.$$

(4)

The $F$–flatness constraints

$$\langle F_{S_p} \rangle \equiv \left( \frac{\partial W}{\partial S_p} \right) = 0, \quad p = 1, 2; \quad \text{and} \quad \langle W \rangle = 0,$$

(5)

imply that the $D$–flat direction $S = S_1 \cos \alpha_Q + S_2 \sin \alpha_Q$ is also a renormalizable $F$–flat direction if (as I assume hereon) $\hat{S}_1$ and $\hat{S}_2$ do not couple among themselves in the renormalizable superpotential.

Consider the real component of this flat direction, $s = \sqrt{2}\text{Re}S = s_1 \cos \alpha_Q + s_2 \sin \alpha_Q$. This scalar’s renormalization group equation (RGE) running mass is,

$$m^2 = m_1^2(\mu) \cos^2 \alpha_Q + m_2^2(\mu) \sin^2 \alpha_Q = \left( \frac{m_1^2}{|Q_1|} + \frac{m_2^2}{|Q_2|} \right) \frac{|Q_1 Q_2|}{|Q_1| + |Q_2|},$$

(6)

which generates a potential

$$V(s) = \frac{1}{2} m(\mu = s)^2 s^2.$$  

(7)

I will assume that $m^2$ is positive at the string scale and of order $m^2_{\text{soft}} \sim O(M_Z^2)$ ($m^2_0$ if universality is assumed). However, through RGE running, $m^2$ can be driven negative (with electroweak (EW) scale magnitude) by large Yukawa couplings (i) of $S_1$ to exotic triplets, $W = hD_1 \hat{D}_2 \hat{S}_1$; (ii) of $S_1$ to exotic doublets and of $S_2$ to exotic triplets, $W = h_D \hat{D}_1 \hat{D}_2 \hat{S}_1 + h_1 \hat{L}_1 \hat{L}_2 \hat{S}_2$; or (iii) of $S_1$ to varying numbers of additional SM singlets $W = h \sum_{i=1}^{N_r} \hat{S}_{ai} \hat{S}_{bi} \hat{S}_1$.

$m(\mu = s)^2$ can turn negative anywhere between a scale of $\mu_{\text{rad}} = 10^4$ GeV and $\mu_{\text{rad}} = 10^{17}$ GeV (slightly below the string scale) for various choices of the supersymmetry breaking parameters $A^0$ (the universal Planck scale soft trilinear coupling) and $M_{1/2}$ (the universal Planck scale gaugino mass).

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§The standard universal scalar EW soft mass–squared parameter $m_0$ has a simple normalizing effect, with $A^0/m_0$ and $M_{1/2}/m_0$ being the actual relevant parameters.
When \( m^2 \) runs negative, a minimum of the potential develops along the flat direction and \( S \) gains a non–zero VEV. In the case of only a mass term and no Yukawa contribution to \( V(s) \), minimizing the potential

\[
\frac{dV}{ds} = \left( m^2 + \frac{1}{2}\beta m^2 \right) \bigg|_{s=0},
\]

(where \( \beta = \frac{\mu}{\mu} \)) shows that the VEV \( \langle s \rangle \) is determined by

\[
m^2(\mu = \langle s \rangle) = -\frac{1}{2}\beta m^2.
\]

Eqs. (8,9) are satisfied very close to the scale \( \mu_{RAD} \) at which \( m^2 \) crosses zero. \( \mu_{RAD} \) is fixed by the renormalization group evolution of parameters from \( M_{\text{string}} \) down to the EW scale and will lie at some intermediate scale.

Location of the potential minimum can also be effected by non–renormalizable self–interaction terms,

\[
W_{\text{NR}} = \left( \frac{\alpha K}{M_{\text{Pl}}} \right)^K \tilde{S}^{3+K},
\]

where \( K = 1, 2 \ldots \) and \( \alpha_K \) are coefficients. Such non–renormalizable operators (NRO’s) lift the flat direction (by breaking \( F \)–flatness) for sufficiently large values of \( s \).

The general form of the potential, \( V(X_p) \), for the scalar components \( X_p \) of corresponding supermultiplets \( \tilde{X}_p \) is

\[
V(X_p) = V_{\text{soft susy}} + \sum_p | \frac{\partial W}{\partial \tilde{X}_p} |^2 + \frac{1}{2} g_\alpha^2 \sum_p Q_\alpha^p | X_p |^2
\]

\[
= V_{\text{soft susy}} + \sum_p | F_p |^2 + \frac{1}{2} g_\alpha^2 \sum_\alpha | D_\alpha |^2
\]

Thus, NRO contributions transform \( V(s) \) in eq. (7) into

\[
V(s) = \frac{1}{2} m^2 s^2 + \frac{1}{2(K+2)} \left( \frac{s^{2+K}}{M^K} \right)^2,
\]

where \( M = C_K M_{\text{Pl}}/\alpha_K \), with \( C_K = [2^{K+1}/((K+2)(K+3)^2)]^{1/(2K)} \).

Even when an NRO is present, the running mass effect still dominates in determining \( \langle s \rangle \) if \( \mu_{RAD} \ll 10^{12} \) GeV. However, an NRO is the controlling factor when \( \mu_{RAD} \gg 10^{12} \) GeV. In the latter case, we find that

\[
\langle s \rangle = \left[ \sqrt{(-m^2)M^K} \right]^{\frac{1}{K+1}} = \mu_K \sim \left( m_{\text{soft}} M^K \right)^{\frac{1}{K+1}},
\]

where \( m_{\text{soft}} = O(|m|) = O(M_Z) \) is a typical soft supersymmetry breaking scale. While \( \langle s \rangle \) is an intermediate scale VEV, the mass \( M_S \) of the physical field \( s \) is still on the order of the soft SUSY breaking scale: For running mass domination,
\[ M_S^2 \equiv \left. \frac{d^2 V}{ds^2} \right|_{s=\langle s \rangle} = \left. \left( \beta m^2 + \frac{1}{2} \mu \frac{d}{d\mu} \beta m^2 \right) \right|_{\mu=\langle s \rangle} \simeq \beta m^2 \sim \frac{m_{sof t}^2}{16\pi^2}, \tag{15} \]

while in the NRO–controlled case,
\[ M_S^2 = 2(K+1)(-m^2) \sim m_{sof t}^2. \tag{16} \]

What powers \( P'_i \) in (2) for first and second generation suppression factors in an NRO–dominated model could produce an up–type quark mass ratio of order \( 10^{-5} : 10^{-3} : 1 \)? From eq. (14), we see the suppression factors become
\[ (\frac{m_{sof t}}{M})^{\frac{P'_i}{K+1}}, \tag{17} \]

where the coefficient \( \alpha_K \) has been absorbed into the definition of the mass scale \( M \equiv M/C_K \).

The mass suppression factors for specific \( P' \) (in the range 0 to 5) and \( K \) (in the range of 1 to 7) are given in Table II. From this table we find that the choices \( P'_1 = 2 \) and \( P'_2 = 1 \) in tandem with \( K = 5 \) or \( K = 6 \) (for the self–interaction terms of \( S \)) can, indeed, reproduce the required mass ratio. These values of \( K \) invoke an intermediate scale \( \langle S \rangle \) around \( 8 \times 10^{14} \) GeV to \( 2 \times 10^{15} \) GeV.

The first and second generation down–quark suppression factors can be similarly realized. However, unless \( tan \beta \equiv \frac{\langle H_2 \rangle}{\langle H_1 \rangle} \gg 1 \), the intra–generational mass ratio of \( 10^{-2} : 10^{-2} : 1 \) for \( m_\tau, m_b, \) and \( m_t \) is not realizable from a \( K = 5 \) or 6 NRO singlet term. For \( tan \beta \sim 1 \), \( m_\tau \), \( m_b \), and \( m_t \) might be associated with an Abelian anomaly may actually contain not just one, but several anomalous \( U(1) \) symmetries. However, all anomalies can all be transferred into a single \( U(1) \) through the unique rotation

III. Realization of Quark Mass Hierarchy in String Models

In string models, one problem generally appears at the string scale that must be resolved “before” possible intermediate scale flat direction VEVs can be investigated. That is, most four–dimensional quasi–realistic \( SU(3)_C \times SU(2)_L \times U(1)_Y \) string models contain an anomalous \( U(1)_A \) (meaning \( TrQ_A \neq 0 \)) \[ 7 \]. In fact, in a generic charge basis, a string model with an Abelian anomaly may actually contain not just one, but several anomalous \( U(1) \) symmetries. However, all anomalies can all be transferred into a single \( U(1)_A \) through the unique rotation

**An intermediate scale VEV \( \langle S \rangle \) can also solve the \( \mu \) problem through a superpotential term \( W_\mu \sim \hat{H}_1 \hat{H}_2 \hat{S} \left( \frac{\hat{s}}{M^2} \right)^{P_\mu} \). With NRO–dominated \( \langle S \rangle \sim \langle m_{sof t} M^K \rangle \), the effective Higgs \( \mu \)–term takes the form, \( \mu_{eff} \sim m_{sof t} \left( \frac{\langle m_{sof t} \rangle}{M^2} \right)^{P_\mu \sim K + 1} \). The phenomenologically preferred choice among this class of terms is clearly \( P = K \): this yields a \( K \)–independent \( \mu_{eff} \sim m_{sof t} \).**
with \( c_A \) a normalization factor. The remaining non–anomalous components of the original set of \( \{U(1)_n\} \) may be rotated into a complete orthogonal basis \( \{U(1)_a\} \).

The standard anomaly cancellation mechanism [8,9] breaks \( U(1)_A \) at the string scale, while simultaneously generating a FI \( D \)–term,

\[
\xi \equiv \frac{g_s^2 M_P^2}{192 \pi^2} \text{Tr} Q_A,
\]

where \( g_s \) is the string coupling and \( M_P \) is the reduced Planck mass, \( M_P \equiv M_{\text{Planck}}/\sqrt{8\pi} \approx 2.4 \times 10^{18} \). The FI \( D \)–term breaks spacetime supersymmetry unless it is cancelled by appropriate VEVs \( \langle X_p \rangle \) of scalars \( X_p \) that carry non–zero anomalous charge,

\[
\langle D \rangle_A \equiv \sum_j Q^{(A)}_j |\langle X_p \rangle|^2 + \xi = 0.
\]

Generalizations of \( D \)– and \( F \)–constraints for \( S_{1,2} \) (i.e., of eqs. (3) and (5)) are imposed on possible VEV directions \( \{\langle X_p \rangle\} \): \( \langle D \rangle_a \equiv \sum_p Q^{(a)}_p |\langle X_p \rangle|^2 = 0 \) and \( \langle F \rangle_p \equiv \langle \frac{\partial W}{\partial \hat{X}_p} \rangle = 0; \langle W \rangle = 0 \).

My colleagues and I at Penn have developed methods for systematically determining [10,11] and classifying [3] \( D \)– and \( F \)–flat directions in string models. We have applied this process [4,5] to the free fermionic three generation \( SU(3)_C \times SU(2)_L \times U(1)_Y \) models (all of which contain an anomalous \( U(1)_A \)) introduced in refs. [12], [13] and [14]. For each model, we have determined the anomaly cancelling flat directions that preserve hypercharge, only involve VEVs of non–Abelian singlet fields, and are \( F \)–flat to all orders in the non–renormalizable superpotential.

Flat directions in Model 5 of [14] have particularly received our attention [4]. In Model 5 we investigated the physics implications of various non–Abelian singlet flat directions. After breaking the anomalous \( U(1)_A \), all of these flat directions left one or more additional \( U(1)_a \) unbroken at the string scale. For each flat direction, the complete set of effective mass terms and effective trilinear superpotential terms in the observable sector were computed to all orders in the VEV’s of the fields in the flat direction. The “string selection–rules” disallowed a large number of couplings otherwise allowed by gauge invariance, resulting in a massless spectrum with a large number of exotics \( \dagger \dagger \) which in most cases are excluded by experiment. This signified a generic flaw of these models. Nevertheless, we found the resulting trilinear couplings of the massless spectrum to possess a number of interesting features which we analyzed for two representative flat directions. We investigated the fermion texture; baryon– and lepton–number violating couplings; \( R \)–parity breaking; non–canonical \( \mu \) terms; and

\[\dagger\dagger \text{Recently it was shown [15] that free fermionic construction can actually provide string models wherein all MSSM exotic states gain near–string–scale masses via flat direction VEVs, leaving only a string–generated MSSM in the observable sector below the string scale. The model of ref. [12] was presented as the first known with these properties.}\]
the possibility of electroweak and intermediate scale symmetry breaking scenarios for a $U(1)' \in \{U(1)_a\}$. The gauge coupling predictions were obtained in the electroweak scale case. We found $t - b$ and $\tau - \mu$ fermion mass universality, with the string scale Yukawa couplings $g$ and $g/\sqrt{2}$, respectively. Fermion textures existed for certain flat directions, but only in the down–quark sector. Lastly, we found baryon– and lepton– number violating couplings that could trigger proton–decay, $N - \bar{N}$ oscillations, leptoquark interactions and $R$–parity violation, leading to the absence of a stable LSP.

IV. Comments

I have discussed how intermediate VEVs hold the potential to yield quasi–realistic quark mass ratios. Four–dimensional string models usually require cancellation of the FI $D$–term contribution from an anomalous $U(1)_A$ by near string scale VEVs. Since some non–anomalous $U(1)_a$ are simultaneously broken at the string scale by these VEVs, which $U(1)_a$ might be associated with intermediate scale VEVs strongly depends on the particular set of (near) string–scale VEVs chosen. As our investigations into flat directions have demonstrated, various choices for flat VEV directions can drastically alter low energy phenomenology. The textures of the quark mass matrices can be strongly effected by choice of flat direction since textures are wrought by effective mass terms in the non–renormalizable superpotential.

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TABLE I. Fermion mass ratios with the top quark mass normalized to 1. The values of $u$-, $d$-, and $s$-quark masses used in the ratios (with the $t$-quark mass normalized to 1 from an assumed mass of 170 GeV) are estimates of the $\overline{\text{MS}}$ scheme current-quark masses at a scale $\mu \approx 1$ GeV. The $c$- and $b$-quark masses are pole masses.

| $m_u$ : $m_c$ : $m_t$ | $m_d$ : $m_s$ : $m_b$ | $m_e$ : $m_{\mu}$ : $m_{\tau}$ |
|-----------------|-----------------|-----------------|
| $3 \times 10^{-5}$ : $7 \times 10^{-3}$ : 1 | $6 \times 10^{-5}$ : $1 \times 10^{-3}$ : $3 \times 10^{-2}$ | $0.3 \times 10^{-5}$ : $0.6 \times 10^{-3}$ : $1 \times 10^{-2}$ |

TABLE II. Non-Renormalizable MSSM mass terms via $\langle S \rangle$. For $m_{\text{soft}} \sim 100$ GeV, $M \sim 3 \times 10^{17}$ GeV.
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