Use of "virtual" antenna array methods for reduction of DOA estimation systematic error caused by the scattering of waves on mobile carrier body

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Abstract. Article shows that "virtual" antenna arrays can be used to effectively improve the DOA estimation accuracy of mobile direction finders. "Virtual" antenna arrays are the samples of the spatial field distribution calculated by approximation of the field near a three-dimensional scattering body that supports physical ("real") antenna system. Signals from the outputs of the "real" array are used as input data for the approximation process. Information on the geometry and material properties of the medium is not required. This procedure can be based on the main provisions of electrodynamics (Kirchhoff-Huygens principle, the Lorenz lemma, method of equivalent currents and fields) or on the use of quasi-static approximation of the field based on the apparatus of the theory of analytic functions of a complex variable (the Cauchy integral and Laurent series). "Virtual" arrays can reduce the error caused by the carrier body without costly and labour-intensive calibration procedures.

1. Introduction
The calibration of receiving antenna in order to compensate the field diffraction distortions introduced by the carrier body is a complex and expensive procedure. In addition, during equipment operation, calibration effect will be lost very quickly, due to vibration, mechanical damage of the carrier body or oxidation of its parts. Also, there is possibility of uncontrolled change in the scatterer geometry due to opening of doors, hatches, etc. Therefore, development of method for calculation the field near antenna carrier body, which does not require information about its geometry, is an actual problem [1-9].

In this article, we researched an effective way for forming the "virtual" antenna array (VAA) based on analytic continuation of the field measured by the antenna using methods of theory of analytic functions of complex variable \( z = x + iy \), in particular, the Cauchy integral or the Laurent series. This approach is applicable only in case of sufficiently small electrical size of the VAA location region.

2. Method description
On the contour of an equidistant circular antenna array with radius \( R \), the \( E_z \) – component can be described, based on the measured field values \( U_1, U_2, U_3, \ldots, U_n \), using following polynomial:

\[ U_{AA}(z = R \cdot \exp[i\phi]) = \sum_{n=1}^{N+1} B_n \cdot \exp[i(n-1)\phi/(N + 1)], \]

which coefficients are found by the least squares method.
Using the Poisson integral (a special case of the Cauchy integral for a contour in form of circle), we can deduce an integral equation of the first kind, which connects fields on the contours of "real" and "virtual" arrays:

\[
U_{AA}(z) = R \cdot \exp[i\phi] = \frac{1}{2\pi} \int_{0}^{2\pi} U_{VAR}(r \cdot \exp[i\phi]) \frac{r^2 - R^2}{r^2 - 2rR \cos(\psi - \phi) + R^2} d\psi =
\]

\[
= \frac{1}{2\pi} \int_{0}^{2\pi} \left( \sum_{k=1}^{K+1} X_{k-1} \cdot \exp \left[ \frac{i(k-1)\psi}{R+1} \right] \right) \frac{r^2 - R^2}{r^2 - 2rR \cos(\psi - \phi) + R^2} d\psi
\]

which can be solved by the collocation method, after the Tikhonov regularization procedure performed (or – after use of the limited damage method).

Another way for field approximation is based on the expansion of function \( U_{AA}(z) \) in circle \( R - \delta \leq r \leq R + \delta \) (where \( \delta \ll R \)) in the Laurent series:

\[
U_{AA}(z) = \sum_{k=-\infty}^{\infty} c_k z^k , \text{ where } c_k = \frac{1}{2\pi i} \int_{C} \frac{U(\zeta)}{\zeta^{k+1}} d\zeta
\]

Next, with use of (3) the derivatives of functions \( U_{AA}(z) \) in the radial directions \( \partial U(\varphi, r)/\partial r, \partial^2 U(\varphi, r)/\partial r^2 \), etc. are evaluated, and the values of function \( U_{VAA}(\varphi, R + L) \) are estimated by use of segment of the Taylor series:

\[
U(\varphi, R + L) \approx U(\varphi, R) + L \cdot \partial U(\varphi, R)/\partial r + 0.5L^2 \cdot \partial^2 U(\varphi, r)/\partial r^2 + \cdots
\]

The possibility of VAA formation follows from the Lorentz lemma, which allows, according to the known tangential field components on a closed surface \( S \), calculate all field components in a volume \( V \) bounded by it; in the derivation of the Lorentz lemma, data on properties of the medium are not used. The scalar analogue of the Lorentz lemma is the Kirchhoff integral (the 3d integral Green's formula), which makes it possible to calculate field at any point of the volume \( V \) bounded by closed surface \( S \) on which values of the field and its normal derivative are known. We can move from the original three-dimensional problem to equivalent two-dimensional problem of estimation field values on the contour \( \zeta \) from its known values \( u(P_n) \) at several points \( N \) of contour \( L \), Fig. 1.

![Figure 1](https://example.com/figure1.png)

**Figure 1.** Antenna array located on the roof of a car.

On outer contour \( \gamma \) there are auxiliary field sources \( \varphi_m(Q_i, q_m) = -(\pi \cdot i/2) \cdot H_3^1(k \varphi Q_{i, q_m}) \), which linear combination (with unknown complex amplitudes \( U_{q_m} \)) simulates the field inside contour \( \gamma \).
The VAA forming procedure is reduced to solving the system of linear algebraic equations with respect to \( U_{q_m} \):

\[
\sum_{m=1}^{N} U_{q_m} \cdot \sum_{i=1}^{l} \left\{ G(P_n, Q_i) \cdot \frac{\partial \varphi_m(Q_i, q_m)}{\partial n} - \varphi_m(Q_i, q_m) \cdot \frac{\partial G(P_n, Q_i)}{\partial n} \right\} \Delta \zeta_i = 
\]

\[
\quad = u(P_n), \quad n = 1, 2, \ldots, N, \quad (5)
\]

where: \( G(P_n, Q_i) = -(\pi \cdot i/2) \cdot H_0^1(k_0 r_{p_n}, q_m); \quad Q_i \in \zeta; \quad P_n \in L; \quad q_m \in \gamma; \quad I \) – number of contour \( \gamma \) partitions; \( \Delta \zeta_i \) – length of element of circle \( \zeta \).

Since the \( \varphi_m \) functions are solutions of the Helmholtz equation, expression (5) is equivalent to the following system of linear algebraic equations:

\[
\|A\| \cdot \overline{u_p} = \overline{u_p}; \quad (\|A\|)_{n,m} = H_0^1(k_0 r_{p_n}, q_m); \quad (\overline{u_p})_m = U_{q_m}; 
\]

\[
\quad (\overline{u_p})_n = U_{p_n}; \quad m, n \in 1, \ldots, N. \quad (6)
\]

Fields on planar contours \( L, \zeta \) and \( \gamma \) (\( u(P) \), \( U_Q(Q) \) and \( U_q(q) \) respectively, defined by Fourier series with coefficients \( a_n, b_n \) and \( c_n \)) are also related by the following formulas:

\[
\quad u(P) = \sum_{n=-\infty}^{\infty} a_n \exp(in \varphi_L) = \oint_{\gamma} \sum_{n=-\infty}^{\infty} c_n \exp(in \varphi_L) H_0^1(k_0 r_{p,q}) dy_q = 
\]

\[
\quad = \sum_{n=-\infty}^{\infty} c_n \oint_{\gamma} \exp(in \varphi_L) H_0^1(k_0 r_{p,q}) dy_q , 
\]

\[
U_Q(Q) = \sum_{n=-\infty}^{\infty} b_n \exp(in \varphi_\zeta) = \oint_{\gamma} \sum_{n=-\infty}^{\infty} c_n \exp(in \varphi_\zeta) H_0^1(k_0 r_{q,q}) dy_q = 
\]

\[
\quad = \sum_{n=-\infty}^{\infty} c_n \oint_{\gamma} \exp(in \varphi_\zeta) H_0^1(k_0 r_{q,q}) dy_q , 
\]

Coefficients \( a_n, b_n \) and \( c_n \) are describe functions that satisfy the Helmholtz equation, i.e. at least twice continuously differentiable. Taking into account the Riemann-Lebesgue theorem, the following relations hold:

\[
|a_n| < \frac{c_1}{n^2}, 
\]

\[
|b_n| < \frac{c_2}{n^2}, 
\]

\[
|c_n| < \frac{c_3}{n^2}, \quad (7)
\]

where \( c_1, c_2 \) and \( c_3 \) – arbitrary constants.

It was found that, when fields on the contours \( L, \zeta \) and \( \gamma \) are calculated with help of a finite number of Fourier harmonics, the following restrictions are valid:
Expressions for the errors of estimation of the field on contours \( L, \zeta \) and \( \gamma \):

\[
\begin{align*}
\delta u_a &< 2 C_1 \left( \frac{\pi^2}{6} - \sum_{n=1}^{N} \frac{1}{n^2} \right), \\
\delta U_c &< 2 C_3 \alpha_2 \left( L i_2 \left( \frac{\exp(-\beta_2)}{n^2} \right) - \sum_{n=1}^{N} \frac{\exp(-\beta_2 n)}{n^2} \right), \\
\delta u_c &< 2 C_3 \alpha_1 \left( L i_2 \left( \frac{\exp(-\beta_1)}{n^2} \right) - \sum_{n=1}^{N} \frac{\exp(-\beta_1 n)}{n^2} \right)
\end{align*}
\]  

(9)

At \( \frac{r_c}{r_l} = 3 \) \( (r_c = 0.5 \text{ m}, r_l = 1.5 \text{ m}) \) and \( \frac{r_c}{r_l} = 10 \) \( (r_c = 5 \text{ m}) \) to reduce the error of field extrapolation on the contour of VAA to the error level of estimation of the field on the antenna array contour, it is necessary to increase 1.8 times the number of used auxiliary sources.

Fig. 2 illustrates increase in DOA estimation accuracy achieved by use of developed method of the VAA.

To calculate bearings, we use the field strength values \( \vec{U} = [U_1, U_2, U_3, \ldots, U_N]^T \), known value of wavelength \( \lambda_0 \), and known values of coordinates of phase centers of array elements \( x_n \) and \( y_n \); bearing value corresponded to maximum modulus of correlation function between measured amplitude-phase distribution \( \vec{U} = [U_1, U_2, U_3, \ldots, U_N]^T \) and calculated under assumption of absence of incident wave diffraction distortions:

\[
D(\varphi) = \left| \sum_{n=1}^{N} U_n \cdot \exp[-i \cdot \frac{2 \cdot \pi \cdot (x_n \cdot \cos(\varphi) + y_n \cdot \sin(\varphi))}{\lambda_0}] \right|
\]

where angle \( \varphi \) varies from 0 to \( 2 \cdot \pi \) radians.
Figure 2. Dependence of bearings from frequency at $\phi_{Source} = 45^\circ$: 1) “real” antenna array, $N = 18$, $r_l = 0.5$ m; 2) VAA, $N = 36$, $r_\gamma = 1.5$ m, formed by 9 field samples measured by “real” antenna array (1, 3, ..., 17 antenna elements); 3) VAA, $N = 36$, $r_\gamma = 0.5$ m, formed by 18 field samples measured by “real” antenna array.

3. Experiment validation of the proposed method

Studies of the proposed method of DOA estimation using field experiment data obtained by the mobile radio direction finder based on chassis of the “Gazel” minibus equipped with the seven-element antenna array with diameter of 1.08 m (Fig. 3) has shown a significant reduction (up to 2-4 times) in maximum and RMS components of DOA estimation systematic error caused by the electromagnetic waves scattering on the antenna array carrier body. Amplitude-phase field structure at the antenna array plane was approximated at points on a circle with diameter of 2 m.

Figure 3. The ARK-MK1M radio monitoring complex with additional ARK-MK4 antenna system on the telescopic mast (JSC IRCOS (Moscow)).

Zero azimuth was directed along the longitudinal axis of the car; frequency dependencies of measured bearings and absolute errors of direction finding for the incidence azimuthal angles equal to 30° and 60° are shown at Fig. 4. Bold line show direction-finding characteristics of “real” seven-element antenna array with diameter of 1.08 m (the correlation phase method of DOA estimation was used); lines with circles correspond to the VAA with diameter of 2 m (same correlation phase method of DOA estimation was used); lines with diamonds show the results of DOA estimation using the correlation amplitude-phase method of direction finding and the VAA with diameter of 2 m. Maximum errors of direction finding at $\phi_{Source} = 30^\circ$ obtained during experiment are: $\delta\phi_{real,AA}^{max} = 16^\circ$, $\delta\phi_{VAA(ampl.-phase)}^{max} = 7^\circ$, $\delta\phi_{VAA(phase)}^{max} = 9^\circ$. Mathematical expectation of the bearings: $M\phi_{real,AA} = 32.6^\circ$, $M\phi_{VAA(ampl.-phase)} = 28.6^\circ$, $M\phi_{VAA(phase)} = 28.5^\circ$. Standard deviation of the bearings: $\sigma\phi_{real,AA} = 6.2^\circ$, $\sigma\phi_{VAA(ampl.-phase)} = 3.2^\circ$, $\sigma\phi_{VAA(phase)} = 3.5^\circ$. 
Figure 4. Dependence of bearings (a) and bearing errors (b) from frequency of "real" array ($R = 0.54$ m; $N = 7$) and the VAA ($r = 1$ m; $N = 56$) at $\phi_{\text{source}} = 30^\circ$. Rejection of fourth harmonic was used.

4. Conclusion
It was found that use of increased number of the VAA elements (compared to the "real" array), as well as the procedure of rejecting the higher harmonics of the FFT spectrum of measured amplitude-phase distribution, makes it possible to get mathematical expectation of the reconstructed bearing closer to its true value.

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