Whirling waves in interference experiments

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In a double slit interference experiment, the wave function at the screen with both slits open is not exactly the sum of the wave functions with the slits individually open one at a time. The three scenarios represent three different boundary conditions and as such, the superposition principle should not be applicable. However, most well known text books in quantum mechanics implicitly and/or explicitly use this assumption, the wave function hypothesis, which is only approximately true. In our present study, we have used the Feynman path integral formalism to quantify contributions from non-classical paths in interference experiments which provide a measurable deviation from the wave function hypothesis. A direct experimental demonstration for the existence of these non-classical paths is hard. We find that contributions from such paths can be significant and we propose simple three-slit interference experiments to directly confirm their existence.

Quantum mechanics has been one of the most successful theories of the twentieth century, both in describing fundamental aspects of modern science as well as in pivotal applications. However, inspite of these obvious triumphs, there is universal agreement that there are aspects of the theory which are counter-intuitive and perhaps even paradoxical. Furthermore, understanding fundamental problems involving dark matter and dark energy [1,2] in cosmology may need a consistent quantum theory of gravity. Unification of quantum mechanics and general relativity towards a unified theory of quantum gravity [3,4] is the holy grail of modern theoretical physics. Such unification attempts involve modifications of either or both theories. However, all such attempts would rely very strongly on precise knowledge and understanding of the current versions of both theories. This makes precision tests of fundamental aspects of both quantum mechanics and general relativity very important to provide guiding beacons for theoretical development.

The double slit experiment (figure 1) is one of the most beautiful experiments in physics. In addition to its pivotal role in optics, it is frequently used in classic textbooks on quantum mechanics [5,7] to illustrate basic principles. Consider a double slit experiment with incident particles (eg. photons, electrons). The wave function at the detector with slit A open is $\psi_A$. The wave function with the slit B open is $\psi_B$. What is the wavefunction with both slits open? It is usually assumed to be $\psi_A + \psi_B$ [5,7], an assumption we refer to as the “wave function hypothesis.” This is illustrated in figure 1. From the mathematical perspective of solving the Schrödinger equation, this assumption is definitely not true. The three cases described above correspond to three different boundary conditions [8,9] and as such there is no superposition principle. The wavefunction hypothesis is thus only approximately true. How do we quantify this?

An intuitive and simple way of understanding this problem is to appeal to Feynman’s path integral formalism [10]. The path integral formalism involves an integration over all possible paths that can be taken by the particle through the two slits. This not only includes the nearly straight paths from the source to the detector through either slit (the classical paths) like the green paths in figure 2 but also includes paths of the type shown in purple in figure 2 (non-classical paths). These looped paths are expected to make a much smaller contribution to the total intensity at the detector screen as opposed to the contribution from the straight line paths. However, their contribution is finite. This leads to a modification of the wave function hypothesis which now becomes:

$$\psi_{AB} = \psi_A + \psi_B + \psi_L,$$

where $\psi_L$ is the contribution due to the looped i.e., non-classical paths.

In this paper, we will quantify the effect of such non-classical paths in interference experiments, thus quantifying the violation of the wave function hypothesis in different possible experimental conditions. A well-known example of a direct experimental demonstration of such non-classical paths involves the measurement of
the Aharonov-Bohm phase \cite{12}. Berry’s “many-whirls” representation \cite{13} provides insight into simple explanations of the Aharonov Bohm effect in terms of interference between whirling waves passing around the flux tube. However, in most experimental attempts to measure the Aharonov Bohm phase, the detection relies on rather complicated experimental architecture and the results are also open to interpretational issues and further discussion \cite{14,15}. In this work, we propose simple triple slit based interference experiments \cite{16} which can be used as table top demonstrations of non-classical paths in the path integral formalism.

The triple slit experiment provides a simple way to express the failure of the wavefunction hypothesis in terms of directly measurable quantities. The triple slit (path) setup has been used as a test-bed for testing fundamental aspects of quantum mechanics over the last few years \cite{16–21}. Three-state systems are also fast becoming a popular choice for fundamental quantum mechanical tests \cite{22,23}. In order to analyse the effect of non-classical paths in interference experiments, we have considered the effect of such paths on an experimentally measurable quantity \( \kappa \). \( \kappa \) (defined below) has been measured in many experiments over the last few years in order to arrive at an experimental bound on possible higher order interference terms in quantum mechanics \cite{24,25} and in effect the Born rule for probabilities \cite{16,18,19}. Investigations of this quantity may also be relevant to theoretical attempts to derive the Born rule \cite{20}. If Born’s postulate for a square law for probabilities is true and if \( \psi_L = 0 \), then the quantity \( \epsilon \) defined by

\[
\epsilon = p_{ABC} - (p_{AB} + p_{BC} + p_{CA}) + (p_A + p_B + p_C).
\]

is identically zero in quantum mechanics.

Here \( p_{ABC} \) is the probability at the detector when all three slits are open, \( p_{AB} \) is the probability when slits A and B are open and so on.

In the experiments reported in the literature, the normalization factor has been chosen to be the sum of the three double slit interference terms called \( \delta \) given by:

\[
\delta = |I_{AB}| + |I_{BC}| + |I_{CA}|,
\]

where \( I_{AB} = p_{AB} - p_A - p_B \) and so on. This choice of normalization can sometimes lead to false peaks in the \( \kappa \) as a function of detector position due to the denominator becoming very small at certain positions. We use a somewhat different normalization, \( \delta = I_{\text{max}} \), where \( I_{\text{max}} \) is the intensity at the central maximum of the triple slit interference pattern to avoid this problem. Then the normalized quantity \( \kappa \) is given by:

\[
\kappa = \frac{\epsilon}{\delta}.
\]

In discussions which invoke the “zeroness” of \( \kappa \), it is implicitly assumed that only classical paths contribute to the interference \cite{27}. In his seminal work \cite{17}, Sorkin had also assumed that the contribution from non-classical paths was negligible. Now, what is the effect of non-classical paths on \( \kappa \)? If one can derive a non-zero contribution to \( \kappa \) by taking into account all possible paths in the Feynman path integral formalism, that would mean that the wave function hypothesis is not strictly true and experimentalists should not be led to conclude that a measurement of non-zero \( \kappa \) would immediately indicate a falsification of the Born Rule for probabilities in quantum mechanics. A measured non-zero \( \kappa \) could also be explained by taking into account the non-classical paths in the path integral. There is thus a theoretical estimate for a non-zero \( \kappa \). Of course, the immediate expectation would be a clear domination of the classical contribution and perhaps a very negligible contribution from the non-classical paths which would in turn imply that the wave function hypothesis is true in all “experimentally observable conditions.” However, what we go on to discover is that this expectation is not always true. It is possible to have experimental parameter regimes in which \( \kappa \) is measurably large. This in turn leads to a paradigm shift in such precision experiments. Observation of a non-zero \( \kappa \) which is expected from the proposed modification to the wavefunction hypothesis would in fact also serve as an experimental validation of the full scope of the Feynman path integral formalism.

As mentioned before, in calculating \( \kappa \), one inherently assumes contributions only from the classical straight line paths as shown in green in figure 2. In this paper, we have estimated the contribution to \( \kappa \) from non-classical paths, thus providing the first theoretical estimate for \( \kappa \). For simplicity, we will use the free particle propagator in our calculations. For a particle in free space and away from the slits, this is a reasonable approximation. We account for the slits by simply removing from the integral all paths that pass through the opaque metal. An estimate for the error due to this assumption has been worked out in \cite{28}.

The normalized energy space propagator \( K \) \cite{28} for a free particle with wave number \( k \) from a position \( \vec{r} \) to \( \vec{r}' \) is
given by
\[ K(\vec{r}, \vec{r}') = \frac{k}{2\pi i} \frac{1}{|\vec{r} - \vec{r}'|} e^{ik|\vec{r} - \vec{r}'|}. \] (4)

Although in this paper, we will be mainly focusing on analyzing optics based experiments using photons, this propagator equation can be used both for the electron and the photon as argued in [28]. We should point out that there are corrections to the propagator due to closed loops in momentum space from quantum field theory considerations. We have explicitly estimated that the effects of such corrections will be negligibly small [29].

Consider the triple slit configuration shown in figure 2. The entire set of paths from the source to the detector through the slits can be divided into two classes:

1. Paths which cross the slit plane exactly once pertaining to a probability amplitude \( K_c \) as represented by the green line and

2. Paths which cross the slit plane more than once pertaining to a probability amplitude \( K_{nc} \) as represented by the purple line.

\[ \therefore K = K_c + K_{nc}. \] (5)

Here the superscripts \( c \) and \( nc \) stand for classical and non-classical respectively.

We wish to estimate \( K_{nc} \) relative to \( K_c \) in order to test the domain of validity of the wave function hypothesis. An example of a representative \( K_c \) in our problem is the probability amplitude to go from the source \((-L, 0, 0)\) to the detector \((D, y_D, 0)\) following the kind of path shown in figure 2. In this case, the particle goes from the source to the first slit and then loops around the second and third slits before proceeding to the detector. We represent this by \( K_c^A(S, D, k) \). This is approximated by [28]:

\[ K_c^A = -\gamma \left( \frac{k}{2\pi} \right)^2 \int_{d - \frac{w}{2}}^{d + \frac{w}{2}} \! \! \int_{-h}^{h} \! \! dy \, dz \, e^{ik(y_1^2 + z^2) + (y_{D} - y)^2 + z^2}. \] (7)

Here \( \gamma = \frac{1}{DD} e^{ik(L + D)} \). These are Fresnel integrals and have been evaluated using Mathematica.

Let us now proceed to the probability amplitude for multiple slit crossings i.e., \( K_{nc} \). An example of a representative \( K_{nc} \) in our problem is the probability amplitude to go from the source \((-L, 0, 0)\) to the detector \((D, y_D, 0)\) following the kind of path shown in figure 2. In this case, the particle goes from the source to the first slit and then loops around the second and third slits before proceeding to the detector. We represent this by \( K_{nc}^A(S, D, k) \). This is approximated by [28]:

\[ K_{nc}^A = i\left( \frac{k}{2\pi} \right)^3 \int dy_1 \, dy_2 \, dz_1 \, dz_2 \, \frac{e^{ik(l_1 + l_2 + l_3)}}{l_1 l_2 l_3}. \] (8)

Here the \( y_1 \) integral runs over slit \( A \) and \( y_2 \) integral runs over slits \( B \) and \( C \) and where \( l_1^2 = (y_1 - y_8)^2 + L^2 + z_1^2 \), \( l_2^2 = (y_2 - y_1)^2 + (z_2 - z_1)^2 \), and \( l_3^2 = (y_D - y_2)^2 + D^2 + z_2^2 \).

Making approximations appropriate to the Fraunhofer regime, using stationary phase approximation [30] for the oscillatory integrals the integral becomes:

\[ K_{nc}^A = \gamma i^{3/2} \left( \frac{k}{2\pi} \right)^{5/2} \int dy_1 \, dy_2 \, dz_1 \, (y_2 - y_1)^{-1/2} e^{ik \left[ \frac{y_1^2 + y_2^2}{4L} + \frac{(y_2 - y_1)^2 + (y_{D} - y_2)^2}{2D} \right]}, \] (9)

where \( d \) is the inter-slit distance, \( w \) is the slit width, \( h \) is the slit height, \( l_1^2 = y_1^2 + L^2 + z_1^2 \) and \( l_2^2 = (y_{D} - y_2)^2 + D^2 + z_2^2 \) as shown in figure 2. For the source and the detector far apart from one another, i.e., in the Fraunhofer regime, \( D \gg d \) & \( L \gg d \) in the region of integration, therefore, \( l_1 \approx L + \frac{y_1^2 + z_1^2}{2L} \). Similarly \( l_2^2 = (y_D - y_2)^2 + D^2 + z_2^2 \) giving \( l_2 \approx D + \frac{(y_{D} - y_2)^2 + z_2^2}{2D} \). Thus we have

\[ K_{nc}^A = -\gamma \left( \frac{k}{2\pi} \right)^2 \int_{d - \frac{w}{2}}^{d + \frac{w}{2}} \! \! \int_{-h}^{h} \! \! dy \, dz \, e^{ik(y_1^2 + z^2) + (y_{D} - y)^2 + z^2}. \] (7)
\[ K^{AB} = K_c^A + K_c^B + K_{nc}^{AB}. \]  
(11)

\[ K_{nc}^{AB} \] are non-classical terms involving only A and B. Similarly for AC and BC. Thus, in terms of propagators, 
\[ \epsilon = |K^{ABC}|^2 - |K^{AB}|^2 - |K^{AC}|^2 - |K^{BC}|^2 + |K^{A}|^2 + |K^{B}|^2 + |K^{C}|^2, \]  
(12)

and the normalization \( \delta \) is given by \[ \delta = |K^{ABC}(0)|^2, \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Normalised values of \( \kappa \) as a function of detector position. Here \( I_n = |K^{ABC}(y)|^2/|K^{ABC}(0)|^2 \). a) This is for incident photons, slit width = 30µm, inter slit distance = 100µm, distance between source and slits and slits and detector = 18cm and incident wavelength = 810nm [10]. b) This is for incident electrons, slit width = 62nm, inter slit distance = 272nm, distance between source and slits = 30.5cm and slits and detector = 24cm and deBroglie wavelength = 50pm [21].}
\end{figure}

where \( |K^{ABC}(0)|^2 \) is the value of \( |K^{ABC}|^2 \) at the central maximum. By numerical integration, we find \( \kappa \) at the central maximum of the triple slit interference pattern to be of the order of \( 10^{-6} \) for the parameters used in the triple slit experiment reported in reference [10]. What would have been expected to be zero considering only straight line paths now turns out to be measurably non-zero having taken the non-classical ones into account [32]. In figure 3, we show \( \kappa \) as a function of detector position. We also show a plot of the triple slit interference pattern as a function of detector position which gives a clearer understanding of the modulation in the plot for \( \kappa \).

The experiment reported in reference [10] was not sensitive to a theoretically expected non-zeroness in \( \kappa \) due to systematic errors. However, in the absence of such systematic errors, it is definitely possible to use a similar set-up to measure a non-zero \( \kappa \). Simulation results indicate that the set-up could have measured a much lower value of \( \kappa \) but the presence of the systematic error due to one misaligned opening in the blocking mask set the limitation of the experiment making it possible to only measure a value of \( \kappa \) upto \( 10^{-2} \). There is no reason why this systematic error cannot be removed in a future version of the experiment thus making it a perfect table-top experiment to test for the presence of non-classical paths in interference experiments. However, experiments of the kind reported in [18] are not as ideally suited for this purpose. This is because, in our analysis, we have worked in the thin-slit approximation. The effective “slit-thickness” in a diffraction grating based interferometer set-up would be quite big and hence the resulting \( \kappa \) would certainly be smaller.

What we go on to also find in our current analysis is that \( \kappa \) is very strongly dependent on certain experimental parameters and one can definitely find a parameter regime where \( \kappa \) would be even bigger, hence easier to observe. We find that keeping all other experimental parameters fixed, \( \kappa \) increases with an increase in wavelength. Thus, for instance, for an incident beam of wavelength 4cm (microwave regime) and slit width of 120cm and interstitial distance of 400cm, a theoretical estimate for \( \kappa \) would be \( 10^{-3} \). This is an experiment which can be performed for instance in a radio astronomy lab.

Experiments of this kind where the value of \( \kappa \) due to non-classical paths can be estimated would definitely be of great interest as they would serve as a simple experimental demonstration of how the basic assumption that a composite wavefunction is just the sum of component wavefunctions i.e. the wave function hypothesis is not always true. In a sense they would also serve as a direct table-top demonstration of the complete scope of the Feynman path integral formalism where not only the straight line paths are important but also the looped paths can make a sizeable contribution depending on one’s choice of experiment. The effects due to such non-classical paths may also be used to model possible decoherence mechanisms in interferometer based quantum computing applications.

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[11] We have verified that in the absence of non-classical terms, the parameter $\kappa$ evaluates to zero within numerical accuracy.
Supplementary Material

S.1. ASSUMPTIONS

Stationary Experiments

We suppose that we have a monochromatic source of light (monoenergetic source of electrons) and that the detectors integrate over the duration T of the experiment. Assuming that T is much longer than any other time scale in the problem, like the travel time across the apparatus, we can use a steady state description. Both electrons and light are then described (in a scalar approximation) by the Helmholtz equation

\[(\nabla^2 + k^2) \psi_k(\vec{r}) = 0,\]  

(S13)

which is satisfied away from the sources and detectors. \(\psi_k\) is a scalar field representing the wave function of the electron or a component of the electromagnetic vector potential. For light \(k = \omega/c\) and for electrons \(k = \sqrt{2m_\epsilon E}\) (setting \(\hbar = 1\)). Both electron and photon diffraction can be treated on the same footing in the time independent case. Below we will drop the superscript \(k\) on physical quantities to avoid cluttering the formulae with it. We will also suppose throughout that \(\lambda\) is much smaller than any other length scale in the problem, the sizes and separations of the slits and the distance to the source and the detector.

Free Propagation and Huygens principle

To describe free propagation, we use the Kernel (see section 59 of [34])

\[K(\vec{r}_1, \vec{r}_2) = \frac{k}{2\pi i} e^{ik|\vec{r}_1 - \vec{r}_2|},\]  

(S14)

which

(i) satisfies the Helmholtz equation away from \(\vec{r}_1 = \vec{r}_2\) and

(ii) satisfies the Fresnel-Huygens principle:

\[K(\vec{r}_1, \vec{r}_3) = \int d\vec{r}_2 K(\vec{r}_1, \vec{r}_2) K(\vec{r}_2, \vec{r}_3)\]  

(S15)

where \(\vec{r}_2\) is integrated over any plane between \(\vec{r}_1\) and \(\vec{r}_3\) and perpendicular to \(\vec{r}_1 - \vec{r}_3\).

Use of time independent Feynman path integral

By repeated application of equation (S15), we can express the propagator for free space in the form

\[K(\vec{r}_1, \vec{r}_2) = \int D[\vec{x}(s)] \exp [ik \int ds],\]  

(S16)

where \(s\) is the contour length along the path \(\vec{x}(s)\) and the sum is over all paths connecting \(\vec{r}_1\) with \(\vec{r}_2\). In the classical limit of \(k \to \infty\), paths near the straight line path joining \(\vec{r}_1\) to \(\vec{r}_2\) contribute by stationary phase. We refer to these as “classical paths” in the text. All of these would contribute “in phase”. Paths away from the classical path are expected to contribute with rapidly oscillating phase. In describing diffraction by a system of slits, we would have to sum over all paths connecting the source to the detector. This would include paths of the kind shown in purple in Fig. 2, which are far from classical. We would expect (in the limit of small \(\lambda\)) that the contributions of such paths are negligible because of rapid oscillations of the phase. We would like to know just how “small” these contributions are.

S.2. CONTRIBUTION OF CLASSICAL AND NON-CLASSICAL PATHS TO THE KERNEL

In Fresnel’s theory of diffraction by a slit [30] we use equation (S15) to insert a single intermediate state on the slit plane and find the amplitude \(K_1\) to reach \(D\) from \(S\):

\[K_1^\Omega(\vec{r}_S, \vec{r}_D) = \int_{\Omega} d\vec{r} K(\vec{r}_S, \vec{r}) K(\vec{r}, \vec{r}_D),\]  

(S17)

where \(\vec{r}\) is an intermediate point on the slit plane (taken to be the \((y, z)\) plane). The range of integration in equation (S17) is over the two dimensional region \(\Omega\) in the slit plane, where \(\Omega\) is the union of the open slits. This gives a remarkably accurate account of the phenomenon of diffraction. In Fresnel’s theory we find because there is a single integration over \(\Omega\), that the outcomes of the three possible experiments with two slits (A, B and AB open) are related by

\[K_1^{AB} = K_1^A + K_1^B\]  

(S18)

and so the wave function hypothesis is satisfied [31, 7]. Thus the wavefunction hypothesis can be recast in terms of probability amplitudes.

Going beyond Fresnel’s theory, we insert two intermediate points on the slit plane and integrate twice over \(\Omega\), the open parts of the slit plane.

\[K_2^\Omega(\vec{r}_S, \vec{r}_D) = \int_{\Omega} d\vec{r}_1 \int_{\Omega} d\vec{r}_2 K(\vec{r}_S, \vec{r}_1) K(\vec{r}_1, \vec{r}_2) K(\vec{r}_2, \vec{r}_D).\]  

(S19)

A typical path in this integration has two “kinks” (at \(\vec{r}_1\) and \(\vec{r}_2\) and thus the integral is highly oscillatory. These integrals in \(K_2^\Omega\) can be computed numerically and seen to

\footnote{Strictly Speaking, in equation (S19) one should excise the region where \(|\vec{r}_1 - \vec{r}_2| \sim \lambda\) (the diagonal of the \(\Omega \times \Omega\) integration). This integral is already accounted for in \(K_1^\Omega\). However this does not affect any of the terms in the \(\epsilon\) expression (S22).}
be much smaller than $K_1^0$. In this order of approximation the amplitude for detection at $D$ is given by
\[
K^0(r_S, r_D) = K_1^0(r_S, r_D) + K_2^0(r_S, r_D) + \ldots \ldots ,
\]
where $K_2$ is the leading order contribution to the amplitude due to non-classical paths $K_{nc}$. As mentioned before $K_2^0$ is expected to be much smaller than $K_1^0$. Most importantly, because of the two integrations over $\Omega$ in equation S19, the wave function $K_2^0$ results in violations of the wave function hypothesis.

Sorkin suggested that the wave function hypothesis can be tested by performing a three slit experiment\[17\]. By keeping each slit either open or closed, we can perform seven distinct non trivial experiments. Theoretical predictions for the outcomes of these seven experiments are given by choosing $\Omega$ to be one of the seven domains $\{A, B, C, A \cup B, B \cup C, C \cup A, A \cup B \cup C\}$. Subsequently Sorkin derived the quantity
\[
\epsilon = |K_{ABC}|^2 - (|K_{AB}|^2 + |K_{BC}|^2 + |K_{CA}|^2) + |K_A|^2 + |K_B|^2 + |K_C|^2.
\]
A straightforward calculation shows that after cancellations and to linear order in $K_2$,
\[
\epsilon = 2\text{Re}(K_2^C(K_2^{AB} + K_2^{BA}) + K_1^A(K_1^{BC} + K_2^{CB}) + K_2^B(K_2^{AC} + K_2^{CA})).
\]
This final expression shows clearly that it is the $K_2$ terms that violate the wave function hypothesis and make $\epsilon$ non zero. $\kappa = \epsilon/\delta$ has been computed numerically and the resulting graphs are shown in Fig.3.

There are several subtleties associated with Huygens principle, which do not however affect the order of magnitude we get for $\kappa$. Huygens initially gave a construction for evolving the wavefront using secondary wavelets. It was Fresnel\[35\] who applied Huygen’s construction to understand diffraction effects. However, Fresnel had to introduce some “inclination factors”. Subsequently Kirchoff derived Fresnel’s theory from Maxwell’s equations using an integral equation derived from Helmholtz’s equation (see equation 17 on page 422 of\[35\]). He was also able to derive (and correct) Fresnel’s “inclination factors”. In equation S19 these factors result in an additional factor of $1/4$ (because of two right angle kinks) and this leads to a factor of $1/4$ multiplying $\epsilon$.

Classically the path taken by a particle is a path of least action. But quantum mechanics tells us that each physically possible path has a probability amplitude associated to it. The final probability amplitude is summation of probability amplitudes from all paths\[10\].

In solving the problem of scattering due to the presence of slits using Feynman path integral (equation 15) we simply excise all those paths which go through the solid portion of the slits. In Fresnel’s theory we suppose that the amplitude at slit $A$ is the same as it would be in free space. In the next order, we allow for the fact that the amplitude at $A$ could also be influenced by waves arriving through slit $B$ (if $B$ is open). This is why the non-classical effects violate the wave function hypothesis.

### S.3. Detailed Discussion on Errors

**Transmission through metal**

We assume that the penetration of light through the opaque metal is zero. The transmission amplitude can be found heuristically. If $\alpha$ is the attenuation constant and if $\psi_i$ is the incident wave amplitude the transmission amplitude is given by, $\psi_t = e^{-2\pi\alpha\zeta}\psi_i$. $\zeta$ is the thickness of the layer in units of the wavelength. This quantifies the approximation that there is no path passing through the solid metal. Here $\psi_i \approx K_A$, therefore the transition amplitude is, $e^{-2\pi\alpha\zeta}K_A$. $\alpha = 2.61$ as refractive index of steel is $2.29 + 2.61i$. $\zeta = 1\mu$m, therefore the error is $K_A \times 10^{-8}$.

**Error due to stationary phase method**

We assumed that the propagator from the source to the slit and the slit to the detector is a free particle propagator. To quantify this approximation we integrate over an intermediate plane ($x = L/2$) similar to equation S15.

The integral is of the form,
\[
I = \int_{-\infty}^{\infty} f(y) e^{ikg(y)} dy.
\]

Here, $g(y)$ is the total distance from the source to the slit. $y$ is a variable. The stationary point for the above integral is a point lying on the straight line joining the source and the detector.

A Taylor series expansion around a stationary point $y_0$, retaining the first two non-zero terms in series gives,
\[
I = \int f(y_0) e^{ikg(y)} \left[ g(y_0) + \frac{2g'(y_0)}{y_0} (y-y_0) + \frac{g''(y_0)}{2y_0^2} (y-y_0)^2 \right] dy.
\]

The integral is then explicitly written as,
\[
I = f(y_0) e^{ikg(y_0)} \sqrt{\frac{\pi}{kg''(y_0)}} e^{i\pi/4} \left[ 1 + O \left( \frac{g_4(y_0)}{g_2(y_0)^2 k} \right) \right].
\]

For our purpose, $g(y) = \sqrt{(L - L/2)^2 + y^2} + \sqrt{(L/2)^2 + (y - d)^2}$, $y_0 = d/2$, $g_2(y_0) = \frac{4L^2}{(d^2 + L^2)^{3/2}} \approx 4/L$ as $L \gg d$ and $|g_4(y_0)| = \frac{8L^2}{(d^2 + L^2)^{5/2}} \approx 48/L^3$.

Therefore,
\[
\frac{|g_4(y_0)|}{g_2(y_0)^2 k} = \frac{3}{LK}.
\]

For $k = \frac{2\pi}{\lambda}$ and $L = 10^5 \lambda$, the error is $K_A \times 10^{-6}$.
Error due to Fraunhofer approximation

In the Fraunhofer limit $L \gg d$, the errors due to this approximation are of the order $K_A \times d/L \approx K_A \times 10^{-4}$.

These errors result in error in calculating $\kappa$. The final leading order error in $\kappa$ is $\kappa \times 10^{-4}$. 