Predictions of Entropic Gravitation

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Abstract

We discuss consequences of an entropic view of gravity, which differs in details from the original proposals of Jacobson and Verlinde. We assume the entropy is localized in the degrees of freedom of the systems interacting gravitationally. We then find there is a density-dependent minimum length for Newtonian gravity. This makes renormalizability issues irrelevant. It also follows that at nanometer scales Newton’s universal law fails, black holes with mass $M_{BH} \lesssim 10^{-13}M_\odot$ do not exist and all Hawking radiation is at unobservably low temperature.
Gauge theories such as quantum electrodynamics [1], constructed by Dyson, Feynman, Schwinger and Tomonaga, quantum chromodynamics [2], due to Fritzsch, Gell-Mann, Gross, Nambu and Wilczek, and the standard model [3], originated by Glashow, 't Hooft, Salam, Veltman and Weinberg, are renormalizable. This despite the divergences which occur because of employing singular products of fields at the same spacetime point.

When the same approach is applied to gravity, one finds non-renormalizability, because the gravitational interaction is too large at very short distances. This is the principal reason why string theory is favored in which the uncontrollable divergences are ameliorated by replacing local fields by extended objects whose interactions are suitably spread out in spacetime and can thereby avoid the previous divergences and lead to a finite theory.

In this note we suggest that the non-renormalizability of the non-string approach may be based on a false presumption.

In the quantum electrodynamics of photons and electrons, there exists a primordial vertex of the photon to the electron with coupling $e$, the electronic charge, and the whole renormalizable theory is built up upon this. Similarly for quantum chromodynamics there is a primordial gluon-quark vertex.

The presumption in quantum gravity is generically that there exists a corresponding primordial graviton-electron vertex (for example) with a non-zero coupling related to Newton’s constant. The non-renormalizability can then be ascribed to the fact that this coupling becomes too large at very short distances, when compared to the corresponding coupling in a renormalizable theory.

If we consider two electrons, at a very short distance apart, conventional wisdom is that the gravitational force is more than forty orders of magnitude smaller than the electromagnetic force.

If we adopt an entropic view [4,5] (for one application, see [6]), the situation is changed.

According to the entropic view, the force of gravity, Newton’s law, is explicable always as an entropy increase. In this case, the gravitational force between two isolated electrons whose entropy cannot increase, is not merely extremely small compared to the electromagnetic force, it is zero.

At least one of the two objects must have sufficiently many subcomponents that its entropy is adjustable. It is known that neutrons are attracted to the Earth which contains $\sim 10^{51}$ nucleons. The question is then how many constituents, $N$, are necessary. Probably it must satisfy $N \gg \sqrt{N}$ in order that fluctuations are sufficiently small, so that e.g. $N \geq 100$.

While it is known that neutrons are attracted to the Earth, from experiment, when both bodies are elementary, like electrons, we expect zero gravitational force.

This then implies that the primordial electron-graviton vertex is not exceptionally large at very short distances, but actually vanishes.
Therefore there is no reason to expect gravity to be renormalizable, or to employ string theory. In the entropic approach, if there were a graviton it would be a collective excitation more like a phonon than a photon, and not an elementary particle. Because the gravitational force between two elementary particles is zero, there is no reason for conventional quantum gravity to be renormalizable.

It is known from the beautiful experiment of Colella, Overhauser and Werner [7] that a single neutron is attracted to the earth. On the other hand, with only two elementary particles, like two electrons (or two neutrons) with fixed positions (and spins) we expect a zero force in the entropic scenario. In principle, the experiment of Colella et al. could be replicated with a small sample of matter (say 1000 particles) as the source of the gravitational field for a split neutron beam. One could then observe that as the number of particles diminishes in the source the fluctuations would destroy the gravitational potential. Of course in practice such an experiment would be impossibly hard to achieve.

It is known that Newton’s law is valid [8] down to a distance of a few tens of microns, or \( \sim 10^{-5} \) m, while we would expect it to break down in matter of normal density before reaching an atomic scale, \( \sim 10^{-10} \) m. If we require one thousand (one million) surrounding atoms it would suggest \( \sim 10^{-9} \) m (\( \sim 10^{-8} \) m) as the shortest scale at which Newton’s law could become approximately valid. We therefore can reasonably expect a deviation from the inverse-square law of Newton to be experimentally detectable in principle, although almost impractically challenging [8] in practice, at such length scales. The assumption is being made of normals densities as is applicable to the material of the massive objects used in the Cavendish- and Adelberger- type experiments [3].

**Black Holes**

In this entropic approach there exists a fundamental length \( L \) providing a cut-off and from our discussions the value is in the range

\[
1nm \leq L \leq 50\mu.
\]

where the upper limit is from experiment [8] and the lower limit derives from theory.

Let us take the most conservative possibility, with the least departure from general relativity, and set \( L = 1 \) nm.

When we consider black hole [4] solutions such as the Schwarzschild solution [9], the cut-off \( L \) means that we must consider only those solutions which exist for larger scales which

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[3] In a system with nuclear density such as a neutron star the corresponding lower limit to the validity of Newton’s universal law would be \( \sim 10^5 \) times smaller.

[4] Since the details of incorporating black holes into entropic gravity are, to say the least, incompletely understood, the present discussion must be regarded only as indicative because we are assuming *faute de mieux* a minimal length \( L \) corresponding to normal matter densities.
means that the Schwarzschild radius $r_S$ should satisfy $r_S \geq L$. With $L = 1$ nm this implies a black hole mass $M_{BH}$ satisfying

$$M_{BH} \geq 10^{18} \text{kg} \sim 10^{-7} M_\oplus \sim 10^{-13} M_\odot$$

where the mass of the Earth and Sun are $M_\oplus \sim 6 \times 10^{24}$ kg and $M_\odot \sim 2 \times 10^{30}$ kg, respectively.

The existence of the high energy cut-off at $\sim 1000$ eV implies that black holes smaller than the lower bound Eq. (2) may not be possible and this is contrary to the standard lore concerning Primordial Black Holes (PBHs). Does this disfavor or favor the cut-off? Since no black holes which violate Eq. (2) have shown up observationally, a neutral researcher could say that our approach is favored.

The radiated photons from a black hole must have wavelength $\lambda > L = 1$ nm. This translates into a maximum energy $E_\gamma \lesssim 1000$ eV. This is insufficient generically to create an $e^+e^-$ pair, and therefore calls into question the standard analysis of Hawking radiation.

Our approach differs in details from those of Jacobson [4] and Verlinde [5]. With respect to [4], that author’s minimal length is the Planck length. With respect to [5], our treatment of entropy is more local so that equality of action and reaction, as well as conservation of momentum, are prima facie compromised.

In summary, while we have not dared or been competent to present a theory, alternative to general relativity, we have speculated on how a putative theory which incorporates the entropic emergence of gravity will likely differ in its predictions. For all large scales, cosmological, galactic, and terrestrial, there will be no measurable difference. At very short scales, however, only a few times the atomic size characterized by the Bohr radius, there are predicted to be very big differences: Newton’s universal gravity is predicted to fail, black holes with such a small radius are predicted not to exist and Hawking radiation will be all at unobservably low temperature.
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References

[1] S.S. Schweber, *QED and the Men Who Made It*, Princeton University Press. (1994).

[2] T. Muta, *Foundations of QCD*, World Scientific. (1998).

[3] C. Burgess and G. Moore, *The Standard Model: A Primer*, Cambridge University Press. (2006).

[4] T. Jacobson, Phys. Rev. Lett. **75**, 1260 (1995). [gr-qc/9504004](https://arxiv.org/abs/gr-qc/9504004).

[5] E. Verlinde, JHEP **1104**:029 (2011). [arXiv:1001.0785[hep-th]](https://arxiv.org/abs/1001.0785).

[6] D. Easson, P.H. Frampton and G.F. Smoot, Phys. Lett. **B696**, 273 (2011). [arXiv:1002.4278[hep-th]](https://arxiv.org/abs/1002.4278); Int. J. Mod. Phys. **A27**, 1250066 (2012). [arXiv:1003.1528[hep-th]](https://arxiv.org/abs/1003.1528).

[7] R. Colella, A.W. Overhauser, and S.A. Werner, Phys. Rev. Lett. **34**, 1472 (1975).

[8] E.G. Adelberger, B.R. Heckel, and A.E. Nelson, Ann. Rev. Nucl. Part. Sci. **53**, 77 (2003).

[9] K. Schwarzschild, Sitzungsberichte der Deutschen Akademie der Wissenschaften zu Berlin, Klasse fur Mathematik, Physik, und Technik (1916). pages 189ff, and 424ff.

[10] B.J. Carr, K. Kohri, Y. Sendouda and J. Yokoyama, Phys. Rev. **81**, 104019 (2010). [arXiv:0912.5297[astro-ph.CO]](https://arxiv.org/abs/0912.5297).

[11] S.W. Hawking, Comm. Math. Phys. **43**, 199 (1975).