Capacity and outage analysis of a dual-hop decode-and-forward relay-aided NOMA scheme

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Abstract—Non-orthogonal multiple access (NOMA) is regarded as a candidate radio access technique for the next generation wireless networks because of its manifold spectral gains. A two-phase cooperative relaying strategy (CRS) is proposed in this paper by exploiting the concept of both downlink and uplink NOMA (termed as DU-CNOMA). In the proposed protocol, a transmitter considered as a source transmits a NOMA composite signal consisting of two symbols to the destination and relay during the first phase, following the principle of downlink NOMA. In the second phase, the relay forwards the symbol decoded by successive interference cancellation to the destination, whereas the source transmits a new symbol to the destination in parallel with the relay, following the principle of uplink NOMA. The ergodic sum capacity, outage probability, and outage sum capacity are investigated comprehensively along with analytical derivations, under both perfect and imperfect successive interference cancellation. The performance improvement of the proposed DU-CNOMA over the conventional CRS using NOMA, is proved through analysis and computer simulation. Furthermore, the correctness of the author's analysis is proved through a strong agreement between simulation and analytical results.

Index Terms—Cooperative relaying, Downlink, Ergodic capacity, Non-orthogonal multiple access, Successive interference cancellation, Uplink.

I. INTRODUCTION

To deal with the high data rate requirements of the next generation wireless networks, integration of multiple technologies is anticipated [1, 2]. Cooperative relaying strategy (CRS) is an important technology for wireless networks to improve system capacity, combat fading, and extend service coverage [3]. In addition, non-orthogonal multiple access (NOMA) has garnered substantial attention from the industry and academia, to meet with the large data rate requirements of 5G and beyond [4-6]. In this paper, cooperative diversity and NOMA is integrated, which can be a promising approach to meet the capacity demands for future wireless networks [7-9].

A lot of different varieties of NOMA can be found in the literature [10]. Among them, cooperative NOMA (C-NOMA) is one of the most dynamic areas of research [7-12]. Based on the direction of data transmission C-NOMA can be further classified as uplink C-NOMA [13-14] and downlink C-NOMA [15-17]. In [15], a cooperative relaying scheme (CRS) using NOMA (termed as CRS-NOMA) was proposed to improve the spectral efficiency over independent Rayleigh fading channels, where a source (S) transmits a superposition coded composite signal to the relay (R) and the destination (D), during the first time slot. Then, R decodes own symbol by performing successive interference cancellation (SIC), whereas D decodes own symbol considering other signal as noise. In the subsequent time slot, R retransmits the decoded symbol with full power to D. In [16], the performance of CRS-NOMA [15] was investigated over Rician fading channels. A novel detection scheme for CRS-NOMA [15] was proposed in [17] (termed as CRS-NOMA-ND), where D uses maximal-ratio combining and another SIC to jointly decode transmitted symbols by source. Note that only achievable average rate was analyzed in [15-17]. In [18-19], the authors considered both uplink and downlink NOMA under non-cooperative scenario, whereas [20-21] exploited the concept of both uplink and downlink NOMA under cooperative scenario. In [20], a cooperative relay sharing network was proposed, where multiple sources can communicate with their corresponding destination simultaneously through a common relay. A C-NOMA scheme considering both downlink and uplink transmission systems, was proposed in [21], where a strong user works as a cooperative relay for the weak user.

Unlike the existing works, in this paper, a CRS-NOMA scheme using the concept of downlink and uplink NOMA (termed as DU-CNOMA) is proposed. In the proposed DU-CNOMA, S transmits a superposition coded composite signal consisting of two symbols $s_1$ and $s_2$ to D and R, according to the principle of downlink NOMA as in [15-17], during the first time slot. However, in the subsequent time slot, unlike [15-17], S transmits a new symbol $s_3$ and R transmits decoded symbol $s_2$ to D simultaneously, according to the principle of uplink NOMA [13, 18-19]. Furthermore, unlike [15-17], where only perfect SIC is considered, we consider both perfect and imperfect SIC by taking into account a more realistic scenario. Principal contributions of this paper are outlined as follows:

1) A dual-hop CRS by taking into account NOMA, is proposed and investigated over independent Rayleigh fading channels.

2) The closed-form expressions of the ergodic sum capacity (ESC), outage probability (OP), and outage sum capacity (OSC) of DU-CNOMA are derived under both perfect and imperfect SIC. The analytical results are validated by Monte Carlo simulation.

3) The performance improvement of the proposed DU-CNOMA over CRS-NOMA [15], and CSR-NOMA-ND [17], is manifested through analysis and simulation. Moreover, analytical derivations are validated by computer simulation.

The rest of this paper is organized as follows. The system model with detailed description of the proposed protocol is provided in Section 2. The channel model is also demonstrated in this section. The closed-form expressions of the ESC, OP, and OSC are presented in Section 3, 4, and 5, respectively. The
numerical results that are validated by Monte Carlo simulation, are provided in Section 6, and finally, the conclusion along with future recommendations is drawn in Section 7.

II. NETWORK ARCHITECTURE AND PROTOCOL DESCRIBTIONS

A half-duplex cooperative relaying protocol exploiting the concept of both downlink and uplink NOMA is proposed. The system architecture consists of a source (S), a DF relay (R), and a destination (D), as drawn in Fig. 1. Fig. 1(a) shows the system architecture considered in [15-17], whereas Fig. 1(b) shows the system architecture of the proposed DU-NOMA. All the links (i.e., S-to-R, S-to-D, and R-to-D) are considered available and subjected to independent Rayleigh fading. Channel coefficient with zero mean and variance $\lambda_i = d_i^{-\nu}$ is represented by $h_i \sim CN(0, \lambda_i)$, where $d$ is the distance, $\nu$ is the path loss exponent, and $i \in \{1, 2, 3\}$. Parameters $h_1$, $h_2$, and $h_3$ refer to the respective complex channel coefficient of S-to-D, S-to-R, and R-to-D links. Without loss of generality, it is assumed that $\lambda_1 < \lambda_2$ and $\lambda_1 < \lambda_3$, under statistical channel state information [21]. The data transmission in the proposed protocol is performed by two cooperative phases as follows.

A. Phase-1 ($t_1$)

At the first phase of the transmission, the source S transmits a composite NOMA signal consisting of two symbols $Z_1 = \sqrt{\phi_1} P_1 s_1 + \sqrt{\phi_2} P_2 s_2$ to D and R simultaneously as per law of downlink NOMA. The symbols $s_1$ and $s_2$ are correspond to D and R, respectively. The total power transmit of S, the power allocation factor with $s_1$, and the power allocation factor with $s_2$ are respectively denoted by $P_S$, $\phi_1$, and $\phi_2$ wherein $\phi_1 > \phi_2$ and $\phi_1 + \phi_2 = 1$. Upon receiving the signal, firstly, R extracts $s_1$ by treating $s_2$ as noise. Then, it performs SIC to cancel out the extracted information from the received signal and thus it extracts $s_2$. Hence, the received signal-to-interference plus noise ratios (SINRs) at R for symbols $s_1$ and $s_2$ are respectively represented by

$$\gamma_{s_1 \to s_2}^{t_1} = \frac{\phi_1 P_1 |h_2|^2}{\phi_2 P_2 |h_2|^2 + \sigma^2} = \frac{\phi_1 P_1}{\phi_2 P_2 |h_2|^2 + 1},$$

$$\gamma_{s_2}^{t_1} = \frac{\phi_2 P_2 |h_2|^2}{\phi_1 P_1 |h_2|^2 + 1},$$

where $h_2 \sim CN(0, \kappa_1 \lambda_2)$, $\rho \triangleq P_2 / \sigma^2$ is the transmit SNR of S and $\sigma^2$ is the noise variance. The parameters $\kappa_1$ ($0 \leq \kappa_1 \leq 1$) represents the level of residual interference at R because of SIC imperfection. As a special case, the conditions of perfect SIC and without SIC are represented by $\kappa_1=0$ and $\kappa_1=1$, respectively [13, 20]. On the other hand, D decodes $s_1$ by recking of $s_2$ as noise. So, the received SINR regarding symbol $x_1$ at D is obtained as

$$\gamma_{x_1}^{t_1} = \frac{\phi_1 P_1 |h_1|^2}{\phi_2 P_2 |h_1|^2 + 1},$$

B. Phase-2 ($t_2$)

During the second phase, according to the law of uplink NOMA, R retransmits the decoded symbol $s_2$ and S transmits a new symbol $s_3$ to D at the same instant of time. The respective assigned powers with $s_2$ and $s_3$ are $\sqrt{P_S \rho}$ and $\sqrt{P_S \sigma_3}$, where $P_S$ is the total transmit power of the relay and $\vartheta_2 > \vartheta_3$. As the information related to $s_2$ is dominant over $s_3$ at the destination, D first decodes $s_2$ by considering $s_3$ as noise. After then, by applying SIC procedure, it subtracts the decoded information to get $s_3$. So, the received SINRs concerning $s_2$ and $s_3$ at D are respectively given by

$$\gamma_{s_2}^{t_2} = \frac{\vartheta_2 P_S |h_3|^2}{\vartheta_3 P_S |h_3|^2 + \sigma^2} = \frac{\vartheta_2 P_S}{\vartheta_3 P_S |h_3|^2 + 1},$$

$$\gamma_{s_3}^{t_2} = \frac{\vartheta_3 P_S |h_3|^2}{\vartheta_2 P_S |h_3|^2 + 1},$$

where $h_3 \sim CN(0, \kappa_2 \lambda_3)$, $\rho \triangleq P_S / \sigma^3$ is the transmit SNR by R, and $\kappa_2$ represents the level of residual interference at D. Note that $\kappa_2$ shows similar behavior like $\kappa_1$.

C. Sum capacity

The end-to-end rate of a multi-hop cooperative network is determined by the weakest link. So, the achievable rate related to $s_1$ is depicted by

$$C_1 = \frac{1}{2} \log_2 \left(1 + \min \left(\gamma_{s_1 \to s_2}^{t_1}, \gamma_{s_2}^{t_1}\right)\right),$$

The achievable rate associated with $s_2$ is dependent on (2) and (4), which can be denoted by

$$C_2 = \frac{1}{2} \log_2 \left(1 + \min \left(\gamma_{s_1 \to s_2}^{t_1}, \gamma_{s_2}^{t_1}\right)\right),$$

By using (5), the achievable rate related to $s_3$ is given by

$$C_3 = \frac{1}{2} \log_2 \left(1 + \gamma_{s_3}^{t_2}\right).$$

Therefore, the sum capacity of the proposed DU-CNOMA system can be calculated by summing up (6), (7), and (8) as follows

$$C_{\text{prop}} = C_1 + C_2 + C_3.$$
A. Ergodic capacity related to $s_1$

The achievable rate of (6), can be simplified as (eq. (8)[15])

$$C_1 = \frac{1}{2} \log_2 \left( 1 + \min \left\{ |h_1|^2, |h_2|^2 \right\} \rho \right)$$

$$- \frac{1}{2} \log_2 \left( 1 + \min \left\{ |h_1|^2, |h_2|^2 \right\} \rho \phi_2 \right),$$

(10)

Let $W \triangleq \min \left\{ |h_1|^2, |h_2|^2 \right\}$. Applying PDF $f_{|h_1|^2}(w) = (1/\lambda_i) e^{-w/\lambda_i}$ for $i \in \{1, 2\}$, the CDF of $W$ is derived as $F_W(w) = 1 - e^{-w/(\lambda_1 + \lambda_2)}$. Then, the probability density function of $W$ is derived by taking the derivative of $F_W(w)$ as

$$f_W(w) = \left( \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) e^{-w/(\lambda_1 + \lambda_2)}$$

(11)

Now, using (10) and (11), the EC associated with $s_1$ can be obtained as

$$\bar{C}_1 = \mathbb{E}\{C_1\} = \frac{1}{2} \int_0^\infty \left\{ \log_2 \left( 1 + \rho w \right) - \log_2 \left( 1 + \rho w \phi_2 \right) \right\} f_W(w) \, dw$$

(12)

Using $\log_2(\alpha) = \frac{\ln(\alpha)}{\ln(2)}$, (12) can be written as

$$\bar{C}_1 = \frac{1}{2 \ln 2} \int_0^\infty \left\{ \ln \left( 1 + \rho w \right) - \ln \left( 1 + \rho w \phi_2 \right) \right\} f_W(w) \, dw$$

(13)

By applying

$$\int_0^\infty e^{-mw} \ln(1+mw) \, dw = -\frac{1}{m} e^{-m/n} \text{Ei}(-m/n) \text{ (eq. (4.337.2)[22]),}$$

$$\bar{C}_1 = -\frac{1}{2 \ln 2} e^{\frac{1}{\lambda_1} + \frac{1}{\lambda_2}} \text{Ei} \left( -\frac{1}{\rho} \left( \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right)$$

$$+ \frac{1}{2 \ln 2} e^{\frac{1}{\phi_2 \rho}} \text{Ei} \left( -\frac{1}{\rho} \left( \frac{1}{\lambda_1} + \frac{1}{\lambda_2} \right) \right)$$

(14)

where $\mathbb{E}\{\cdot\}$ and $\text{Ei}\{\cdot\}$ denote the expectation operator and exponential integral function, respectively [22].

B. Ergodic capacity related to $s_2$

Let $U \triangleq \sqrt{\frac{\phi_1}{\phi_2}}, V \triangleq \sqrt{\frac{\lambda_2}{\lambda_1}}$, and $Z \triangleq \min(U, V)$. The CDF of $U$ and $V$ can be respectively written as

$$F_U(u) = 1 - \frac{\phi_2 \lambda_2}{\phi_2 \lambda_2 + \phi_1 \lambda_1 V} e^{-\frac{p}{\rho} \frac{1}{\lambda_1}}$$

$$= 1 - \frac{p}{p + u} e^{-\frac{p}{\rho} \frac{1}{\lambda_1}}, \quad (15)$$

$$F_V(v) = 1 - \frac{\phi_2 \lambda_2}{\phi_2 \lambda_2 + \phi_1 \lambda_1 V} e^{-\frac{g}{\rho} \frac{1}{\lambda_1}}$$

$$= 1 - \frac{g}{g + v} e^{-\frac{g}{\rho} \frac{1}{\lambda_1}}, \quad (16)$$

where $p = \phi_2 A_2 / \phi_1 A_1$ and $g = \phi_2 A_3 / \phi_1 A_1$. Using (15) and (16), the CDF of $Z$ can be obtained as

$$F_Z(z) = 1 - \frac{pg}{(p+z)(g+z)} e^{-qz}, \quad (17)$$

where $q = \frac{1}{\phi_2 \rho A_2} + \frac{1}{\phi_1 \rho A_1}$. So, the exact EC related to $s_2$ is derived as

$$\bar{C}_2^\infty = \mathbb{E}\{C_2\} = \frac{1}{2} \int_0^\infty \log_2 (1+z) f_Z(z) \, dz. \quad (18)$$

Applying $\int_0^\infty \log_2 (1+z) f_Z(z) \, dz = \frac{1}{2 \ln 2} \log_2 \int_0^\infty \frac{1}{1+z} - \frac{2}{1+z} \, dz$, (18) can be represented as

$$\bar{C}_2^\infty = \frac{1}{2 \ln 2} \int_0^\infty \frac{pg}{(1+z)(p+z)(g+z)} e^{-qz} \, dz$$

$$= \frac{p \log_2 e}{2(p-1)} \int_0^\infty \left( 1 + \frac{1}{z} \right) g \left( 1 - \frac{z}{g+z} \right) e^{-qz} \, dz$$

$$= \frac{p \log_2 e}{2(p-1)} \left\{ -e^q \text{Ei}(-q) + e^{qg} \text{Ei}(-qg) \right\}$$

$$- \frac{g}{g-q} \left\{ -e^{qg} \text{Ei}(-pg) + e^{qg} \text{Ei}(-gq) \right\}. \quad (19)$$

Note that (19) is derived by considering imperfect SIC (i.e., $0 < k_2 \leq 1$). Therefore, the exact EC of $s_2$ under perfect SIC is derived as follows.

With perfect SIC, $Z \triangleq \min(U, V)$ can be written as $Z \triangleq \min(\phi_2 \rho |h_2|^2, V)$. The CDF of $Z$ is therefore given by

$$F_Z(z) = 1 - \frac{g}{g+z} e^{-qz}, \quad (20)$$

The exact EC related to $s_2$ under perfect SIC, is derived as

$$\bar{C}_{2p} = \frac{1}{2 \ln 2} \int_0^\infty \log_2 (1+z) f_Z(z) \, dz$$

$$= \frac{p \log_2 e}{2(p-1)} \int_0^\infty \left( 1 + \frac{1}{z} \right) g \left( 1 - \frac{z}{g+z} \right) e^{-qz} \, dz$$

$$= \frac{p \log_2 e}{2(g-1)} \left\{ -e^{qg} \text{Ei}(-q) + e^{qg} \text{Ei}(-gq) \right\}. \quad (21)$$

C. Ergodic capacity related to $s_3$

Let $Y \triangleq \sqrt{\frac{\phi_3}{\phi_1}}$, So, the CDF of $Y$ is derived as

$$F_Y(y) = 1 - \frac{\phi_3 \lambda_1}{\phi_3 \lambda_1 + \phi_2 \lambda_2 \lambda_3 y} e^{-\frac{q}{\rho} \frac{1}{\lambda_2 \lambda_3}}, $$

(22)

So, the exact EC associated with $s_3$ is obtained as

$$\bar{C}_3^\infty = \mathbb{E}\{C_3\} = \frac{1}{2} \int_0^\infty \log_2 (1+y) f_Y(y) \, dy$$

$$= \frac{\log_2 e}{2} \int_0^\infty (1+y)^{-1} \frac{\phi_3 \lambda_1}{\phi_3 \lambda_1 + \phi_2 \lambda_2 \lambda_3 y} e^{-\frac{q}{\rho} \frac{1}{\lambda_2 \lambda_3}} \, dy$$

$$= \frac{\log_2 e}{2} \left\{ \frac{\phi_3 \lambda_1}{\phi_3 \lambda_1 - \phi_2 \lambda_2 \lambda_3} \right\} \times \left\{ \int_0^\infty (1+y)^{-1} \frac{\phi_2 \lambda_2 \lambda_3}{\phi_3 \lambda_1 + \phi_2 \lambda_2 \lambda_3 y} e^{-\frac{q}{\rho} \frac{1}{\lambda_2 \lambda_3}} \, dy \right\}$$

$$= \frac{\log_2 e}{2} \left\{ \frac{\phi_3 \lambda_1}{\phi_3 \lambda_1 - \phi_2 \lambda_2 \lambda_3} \times \left\{ -e^{\frac{q}{\rho} \lambda_2} \text{Ei}(-\frac{1}{\phi_3 \rho} \lambda_1) + e^{\frac{q}{\rho} \frac{1}{\lambda_2 \lambda_3}} \text{Ei}(-\frac{1}{\phi_2 \rho \lambda_3} \lambda_1) \right\} \right\}. \quad (23)$$

Note that (23) is derived by considering imperfect SIC (i.e., $0 < k_2 \leq 1$). The exact EC of $s_3$ under perfect SIC is derived as follows.
With perfect SIC, $Y \triangleq \gamma_i^2$ can be written as $Y \triangleq \phi_{3} \rho | h_1 |^2$. The CDF of $Y$ is therefore given by

$$F_Y(y) = 1 - e^{-\frac{y}{\phi_{3} \rho | h_1 |^2}}.$$  \hfill (24)

Hence, the exact EC associated with $s_3$ is obtained as

$$\bar{C}_{3,p} = \frac{\log_2 e}{2} \int_0^\infty 1 - F_Y(y) \frac{dy}{1+y} = \frac{\log_2 e}{2} e^\gamma (-r),$$ \hfill (25)

where $r = \frac{1}{\phi_{3} \rho | h_1 |^2}$.

### D. Ergodic sum capacity of DU-CNOMA

Using (14), (19), and (23), the exact closed-form expression of ESC of the proposed DU-CNOMA protocol under imperfect SIC can be written by

$$\bar{C}_{\text{sum, ip}} = C_1^{\text{ex}} + C_2^{\text{ex}} + C_3^{\text{ex}}.$$ \hfill (26)

Conversely, using (14), (21), and (25), the exact closed-form expression of ESC of the proposed DU-CNOMA protocol under perfect SIC can be written by

$$\bar{C}_{\text{sum, p}} = C_1^{\text{ex}} + C_2^{\text{ex}} + C_3^{\text{ex}}.$$ \hfill (27)

### IV. OUTAGE PROBABILITY ANALYSIS

According to the required quality of service, $C_1$, $C_2$, and $C_3$ are assumed to be the predetermined target data rate thresholds of the symbols $s_1$, $s_2$, and $s_3$, respectively. The closed-form expressions of outage probabilities related to $s_1$, $s_2$, and $s_3$ are provided over independent Rayleigh fading channel in the following subsections.

#### A. Outage probability of symbol $s_1$

The OP of symbol $s_1$ is given by

$$P_{\text{out},s_1} = P_r \{ y_i^1 < R_{t_1} \} = 1 - e^{-\frac{R_{t_1}}{\phi_1 \rho \gamma_i^1}},$$ \hfill (28)

where $R_{t_1} = 2^{2C_{1}} - 1$ and $\frac{R_{t_1}}{\phi_1 \rho \gamma_i^1} < \phi_1 < 1$.

#### B. Outage probability of symbol $s_2$

The OP of symbol $s_2$ is given by

$$P_{\text{out},s_2} = 1 - P_r \{ \min(y_i^1, y_i^2) > R_{t_2} \} P_r \{ y_i^{1 \rightarrow 2} > R_{t_1} \}$$

$$= 1 - \frac{\phi_2 \lambda_2 \theta_2 \lambda_3}{\phi_2 \lambda_2 + \phi_1 \kappa_1 \lambda_2 R_{t_2}} \frac{R_{t_2}}{\phi_3 \lambda_3 + \phi_1 \kappa_1 \lambda_3 R_{t_2}}$$

$$\times e^{-\frac{R_{t_1}}{\phi_2 \rho \lambda_3} - \frac{R_{t_2}}{\phi_3 \rho \lambda_3} - \frac{R_{t_1}}{\phi_1 \rho \gamma_i^1}},$$ \hfill (29)

where $R_{t_2} = 2^{2C_{2}} - 1$. Now, by putting $\kappa_1 = 0$, the OP of $s_2$ under perfect SIC can be expressed as

$$P_{\text{out,s}_2}^p = 1 - \frac{\phi_2 \lambda_3}{\phi_2 \lambda_3 + \phi_3 \lambda_1 R_{t_2}} e^{-\frac{R_{t_1}}{\phi_2 \rho \lambda_3} - \frac{R_{t_2}}{\phi_3 \rho \lambda_3} - \frac{R_{t_1}}{\phi_1 \rho \gamma_i^1}}.$$ \hfill (30)

#### C. Outage probability of symbol $s_3$

The OP of symbol $s_3$ is given by

$$P_{\text{out},s_3} = 1 - \frac{\phi_3 \lambda_3}{\phi_3 \lambda_3 + \phi_3 \lambda_3 R_{t_3}} e^{-\frac{R_{t_1}}{\phi_3 \rho \lambda_3}},$$ \hfill (31)

where $R_{t_3} = 2^{2C_{3}} - 1$. Now, by putting $\kappa_2 = 0$, the OP of $s_3$ under perfect SIC can be expressed as

$$P_{\text{out},s_3}^p = 1 - e^{-\frac{R_{t_3}}{\phi_3 \rho \gamma_i^3}}.$$ \hfill (32)

### V. OUTAGE CAPACITY ANALYSIS

This section presents analytical derivation for OSC of the proposed DU-CNOMA over independent Rayleigh fading channel.

#### A. Outage capacity of $s_1$

The OC $C_{t_1}$ related to $s_1$, with specified OP $Y_1$ can be computed from (28) as

$$Y_1 = 1 - e^{-\frac{R_{t_1}}{\phi_1 \rho \gamma_i^1}},$$

$$e^{-\frac{R_{t_1}}{\phi_1 \rho \gamma_i^1}} = \ln(1 - Y_1),$$

$$\{(\lambda_1 \phi_1 \rho \ln(1 - Y_1) - 1)R_{t_1} = \lambda_1 \phi_1 \rho \ln(1 - Y_1)\},$$

$$C_{t_1} = \frac{1}{2} \log_2 \left[ 1 + \frac{\lambda_1 \phi_1 \rho \ln(1 - Y_1)}{\lambda_1 \phi_1 \rho \ln(1 - Y_1) - 1} \right].$$ \hfill (33)

#### B. Outage capacity of $s_2$

Using $e^x \approx 1 + x$ at high $\rho$, the OC $C_{t_2}$ related to $s_2$, with specified OP $Y_2$ can be computed from (29) as

$$Y_2 = 1 - \frac{\phi_2 \lambda_2 \theta_2 \lambda_3}{\phi_2 \lambda_2 + \phi_1 \kappa_1 \lambda_2 R_{t_2}} \frac{R_{t_2}}{\phi_3 \lambda_3 + \phi_1 \kappa_1 \lambda_3 R_{t_2}} \times \left( 1 - \frac{R_{t_2}}{\phi_2 \rho \lambda_2} - \frac{R_{t_2}}{\phi_3 \rho \lambda_3} \right),$$

$$Y_2 = 1 - \frac{\phi_2 \lambda_2 \theta_2 \lambda_3}{\phi_2 \lambda_2 + \phi_1 \kappa_1 \lambda_2 R_{t_2}} \frac{R_{t_2}}{\phi_3 \lambda_3 + \phi_1 \kappa_1 \lambda_3 R_{t_2}} \times \left( 1 - \frac{R_{t_2}}{\phi_2 \rho \lambda_2} - \frac{R_{t_2}}{\phi_3 \rho \lambda_3} \right),$$ \hfill (34)

conditioned on $y_i^{1 \rightarrow 2} > R_{t_2}$, where $G = \phi_2 \lambda_2$, $H = \theta_2 \lambda_3$, $I = \phi_1 \kappa_1 \lambda_2$, and $J = \phi_3 \lambda_3$. After some algebraic simplifications, (34) can be rewritten as

$$I \rho \ (1 - Y_2) R_{t_2}^2 + \{(GJ + HI) (1 - Y_2) \rho + H + G \} R_{t_2} + (-G H \rho Y_2) = 0$$

$$K R_{t_2}^2 + L R_{t_2} + M = 0,$$ \hfill (35)
where \( K = 1 \rho (1 - \Upsilon_2) \), \( L = (GJ + H) (1 - \Upsilon_2) \rho + H + G \), \( M = -GH\rho\Upsilon_2 \) are assumed. Solving (35) and considering feasible root, \( C_{t_2} \) can be obtained as
\[
R_{t_2} = \frac{-L + \sqrt{L^2 - 4KM}}{2K} \\
2^{2C_{t_2}} - 1 = \frac{-L + \sqrt{L^2 - 4KM}}{2K} \\
C_{t_2} = \frac{1}{2} \log_2 \left( 1 + \frac{-L + \sqrt{L^2 - 4KM}}{2K} \right)
\]
(36)

On the other hand, the OC \( C_{t_2} \) under perfect SIC can be computed from (30) as
\[
\Upsilon_2 = 1 - \frac{\vartheta_2 \lambda_3}{(\vartheta_2 \lambda_3 + \vartheta_1 \lambda_2 R_{t_2})} \left( 1 - \frac{R_{t_2}}{\vartheta_2 \rho \lambda_2} - \frac{R_{t_2}}{\vartheta_2 \rho \lambda_3} \right) \\
= 1 - \frac{H}{(H + J R_{t_2})} \left( 1 - \frac{R_{t_2}}{G\rho} - \frac{R_{t_2}}{H\rho} \right) \\
R_{t_2} = \frac{GJ \rho \Upsilon_2}{GJ \rho (1 - \Upsilon_2) + G + H} \\
C_{t_2} = \frac{1}{2} \log_2 \left( 1 + \frac{GJ \rho \Upsilon_2}{GJ \rho (1 - \Upsilon_2) + G + H} \right)
\]
(37)

C. Outage capacity of \( s_3 \)

Using \( e^x \approx 1 + x \) at high \( \rho \), the OC \( C_{t_3} \) related to \( s_3 \), with specified OP \( \Upsilon_3 \) can be computed from (31) as
\[
\Upsilon_3 = 1 - \frac{\vartheta_3 \lambda_1}{\vartheta_3 \lambda_1 + \vartheta_2 K_3 \lambda_1 R_{t_3}} \left( 1 - \frac{R_{t_3}}{\vartheta_3 \rho \lambda_1} \right) \\
\Upsilon_3 = 1 - \frac{\vartheta_3 \lambda_1 (\vartheta_3 \rho \lambda_1 - R_{t_3})}{(\vartheta_3 \lambda_1 + \vartheta_2 K_3 \lambda_1 R_{t_3}) \vartheta_3 \rho \lambda_1} \\
\Upsilon_3 = \frac{\vartheta_3 \lambda_2 \lambda_3 R_{t_3} \rho + R_{t_3}}{(\vartheta_3 \lambda_1 + \vartheta_2 K_3 \lambda_1 R_{t_3}) \vartheta_3 \rho \lambda_1} \\
R_{t_3} = \frac{\vartheta_3 \lambda_1 \rho \Upsilon_3}{1 + \vartheta_2 K_3 \lambda_1 \rho - \vartheta_2 K_3 \lambda_3 \rho \Upsilon_3} \\
C_{t_3} = \frac{1}{2} \log_2 \left( 1 + \frac{\vartheta_3 \lambda_1 \rho \Upsilon_3}{1 + \vartheta_2 K_3 \lambda_3 \rho (1 - \Upsilon_3)} \right)
\]
(38)

D. Outage sum capacity

Using (33), (36), and (38), the OSC of the proposed DU-CNOMA under imperfect SIC is given by
\[
C_{\text{out, ip}} = (33) + (36) + (38).
\]
(40)

Conversely, Using (33), (37), and (39), the OSC of the proposed DU-CNOMA under perfect SIC is given by
\[
C_{\text{out, p}} = (33) + (37) + (39).
\]
(41)

VI. NUMERICAL RESULTS

This section presents simulation (Sim.) and analytical (Anl.) results of our proposed DU-CNOMA protocol. In each case, analytical result matches well with simulation result and it confirms the correctness of the author’s analysis presented here. For comparison purpose, the simulation results for CRS-NOMA [15] and CRS-NOMA-ND [17] are also presented. It should be mentioned that analytical derivations for OP and OSC are not provided in [15, 17]. Throughout the simulation, it is assumed that \( v=3 \), \( d_{SD}=1 \), \( d_{SR} = d_{SD}/2 \), \( d_{SD} = 1 - d_{SR} \), \( \varphi_1=0.9 \), \( \varphi_2=0.1 \), \( \varphi_3=0.1 \), \( \varphi_4=0.2 \), \( \varphi_5=1 \), and \( \varphi_6=0.3 \), unless otherwise specified. Note that fixed power allocation method as in [15-17] is assumed for the proposed protocol.
A. Ergodic capacity

ESC versus SNR behavior of DU-CNOMA, CRS-NOMA, and CRS-NOMA-ND is shown in Fig. 2. Performance of the proposed DU-CNOMA is executed under two conditions, i.e., perfect SIC and imperfect SIC. Note that only perfect SIC is considered in CRS-NOMA and CRS-NOMA-ND. For the case of perfect SIC, it is observed from the figure that DU-CNOMA significantly outperforms all other existing protocols. However, with the increasing amount of residual interference the performance of DU-CNOMA starts degrading which causes it to exhibit a saturated value at medium to high SNR values. The performance of the proposed protocol becomes worse for $\kappa_1 = \kappa_2 = 0.04^2$ than $\kappa_1 = \kappa_2 = 0.02^2$. Therefore, at high SNR, the adverse impact of residual interference on DU-CNOMA causes it to achieve less ESC than existing methods. Therefore, it is suggested that an efficient interference cancelation technique can significantly improve the performance of DU-CNOMA, particularly at medium to high SNR. Lastly, strong agreement between simulation and analytical results verifies the appropriateness of the ESC analysis.

ESC behavior for varying relay position between source and destination, $d_{SR}$ is demonstrated in Fig. 3, under perfect SIC. ESC versus $d_{SR}$ performance of DU-CNOMA is compared with CRS-NOMA and CRS-NOMA-ND protocols for two different SNR values, i.e., $\rho = 15$ and $30$ dBs. For both cases, proposed protocol achieves better ESC than existing protocols irrespective of the relay position. In addition, ESC of DU-CNOMA becomes far better than others for the increasing distance between source and relay. However, this behavior is bounded by a maximum $d_{SR}$ value (e.g., around $d_{SR} = 0.5$ for $\rho = 15$ dB and around $d_{SR} = 0.8$ for $\rho = 30$ dB).

ESC with respect to (w.r.t) the power allocation coefficient $\phi_2$ is shown in Fig. 4, where $\phi_2 = 1$ and $\phi_3 = 0.3$ or $0.5$. It is demonstrated that the ESC performance of all protocols degrades with the increase of $\phi_2$. Further, ESC of the proposed DU-CNOMA protocol is higher than existing protocols for all feasible values of $\phi_2$. It is also clear from the figure that ESC of the proposed protocol is higher for $\phi_3 = 0.5$ than $0.3$.

B. Outage probability

OP versus SNR performance of the proposed protocol is demonstrated in Fig. 5 for two different threshold values of target data rate, i.e., $C_t = 1$ and $0.5$ (bps/Hz). Perfect SIC is considered for analyzing all analytical and simulation results. Coincidence of analytical and simulation results for each case refers to the accuracy of OP analysis. OP becomes better with the increase of SNR wherein OP related to symbol $s_1$ is less than $s_2$ and $s_3$ for a specific $C_t$ value. Though OPs related to symbols $s_1$ and $s_3$ decrease linearly with the increase of $\rho$, OP related to symbol $s_2$ takes a saturated value for medium to high $\rho$ range. The reason behind exhibiting this performance by $s_2$ is the interference effect from other symbols on it. The OP related to $s_1$, $s_2$, and $s_3$ for $C_t = 1$ bps/Hz is higher than $C_t = 0.5$ bps/Hz, as expected.
By considering imperfect SIC and target data rate $C_1 = 0.5$ bps/Hz, OP of DU-CNOMA protocol w.r.t SNR $\rho$ is depicted in Fig. 6. Only OP versus SNR analysis related to $s_2$ and $s_3$ are compared as the performance related to $s_1$ is not affected by imperfect SIC condition. OP related to $s_2$ is better than $s_3$ at low to medium $\rho$ ($0 < \rho \leq 15$), whereas OP related to $s_3$ becomes better than $s_2$ at high $\rho$ ($\rho > 15$) for the considered parameters. OP related to any of the symbols is less for small residual interference (i.e., $\kappa_1 = \kappa_2 = 0.01^2$) than comparatively large amount of residual interference (i.e., $\kappa_1 = \kappa_2 = 0.03^2$). Though OP related to $s_3$ shows linear behavior even at high $\rho$ as shown in Fig. 5, it tends to be saturated at high $\rho$ as shown in Fig. 6 due to the impact of residual interference.

C. Outage capacity

10% OSC of the proposed DU-CNOMA protocol w.r.t SNR $\rho$ is plotted under both perfect and imperfect SIC conditions in Fig. 7. Two cases of imperfect SIC is considered, i.e., $\kappa_1 = \kappa_2 = 0.01^2$ and $\kappa_1 = \kappa_2 = 0.03^2$. For perfect SIC condition, OSC of the system linearly increases with the betterment of $\rho$ and maintains it till the high $\rho$. But, for imperfect SIC condition, OSC of the system only increases linearly up to medium value of $\rho$, then it becomes saturated due to the impact of residual interference. If the effect of residual interference increases, OSC of DU-CNOMA decreases and tends to be saturated at a less value of $\rho$ than for comparatively small residual interference impact.

OSC behavior w.r.t specified outage probability $\Upsilon$ for the proposed DU-CNOMA protocol is demonstrated in Fig. 8. Perfect SIC condition is taken into account and the performance behavior is observed for two different values of $\rho$, i.e., $\rho = 15$ dB and $30$ dB. Fig. 8 depicts that OSC of the system increases with the increase of specified outage probability $\Upsilon$. In addition, OSC goes high for higher $\rho$ ($\rho = 30$ dB) than lower $\rho$ ($\rho = 15$ dB).

VII. CONCLUSION AND FUTURE WORKS

A cooperative decode-and-forward relaying strategy using the concept of downlink and uplink NOMA has proposed and analyzed in this paper. Under both perfect and imperfect SIC, the performance of the proposed protocol has studied comprehensively, in terms of ESC, OP, and OSC over independent Rayleigh fading channels. The closed-form expressions of these system parameters have derived and validated by computer simulation. It has shown that the proposed protocol significantly outperforms CRS-NOMA and CRS-NOMA-ND under perfect SIC, whereas under imperfect SIC, performance gains depends on the level of residual interference, particularly at high SNR. Furthermore, hybrid downlink-uplink NOMA for multi-input multi-output systems will be investigated in future works.

DISCLOSURE STATEMENT

The author(s) declare(s) no potential conflict of interest regarding the publication of this paper.

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