Modeling and comparative analysis of schemes for returning a payload using a space tethered system

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Abstract. The multiple schemes for returning a payload from LEO using a space tethered system are designed and analyzed in this paper. The space tethered system consists of a main spacecraft, a tether, a returning capsule with the payload (or space debris) and an atmospheric probe. The simulation for the motion of the tethered system is based on a mathematical model with distributed parameters, in which the tether is considered as a sequence of point masses with one-sided elastic mechanical coupling. The aerodynamic forces acting on all parts of the tethered system including on the tether are considered in this work. The atmospheric probe is used as an additional braking device for the proposed schemes for returning the payload.

1. Introduction
In recent years, the space tethered systems (STSs) have gained much interest due to their potential applications such as active space debris removal [1], study of the upper atmosphere [2], change of the asteroid trajectory [3] and returning a payload from the orbit [4, 5]. Returning a payload by using the STS is a promising alternative approach compared to the traditional approach by using the braking corrective thruster. A large number of studies were carried out to reduce the cost of orbital transport operations, which were carried out without using the corrective thruster [6]. In this work the STS consists of a main spacecraft, a tether, a returning capsule with the payload (or space debris) and an atmospheric probe (AP). The AP is used as an additional braking device.

The multiple schemes for returning the payload from LEO are considered. These schemes are compared with each other. A discrete model of motion with distributed parameters is used for analyzing these schemes. The discrete model takes into account the mass and stiffness of the tether. The aerodynamic forces acting on all parts of the STS, including on the tether, are taken into account. The similar models were used in much work such as [7]. The aerodynamic force is calculated under the assumption that the STS’s motion occurs in a free molecular flow. In addition, the hypothesis on the diffuse reflection of molecules is made [8]. In the first motion phase the STS deploys the tether into the position near the local vertical. A large number of papers focus on the deployment control problem such as [7]. The AP is used for the considered schemes. The ballistic coefficient of the AP in this work is relatively large.

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2. Mathematical model of motion of STS with distributed parameters

The following coordinate systems (Figure 1) are used for deriving the motion equations of the STS: 1) the geocentric right-hand coordinate system \( OXYZ \), 2) the geocentric orbital moving coordinate system \( OX_0Y_0Z_0 \), 3) the orbital moving coordinate \( cx_0y_0z_0 \). Plane \( OXY \) is associated with the orbital plane of the system center of mass \( C \), where axis \( OX \) is in the direction along the nodal line, and axis \( OZ \) is in the direction of the angular momentum vector of the orbital motion. Axis \( OX_0 \) is in the direction of \( OC \).

The geocentric orbital moving coordinate system \( OX_0Y_0Z_0 \) rotates with respect to the coordinate system \( OXYZ \) with the angular velocity \( \Omega = dy/dt \), where \( \gamma \) is the argument of latitude. The axes of the coordinate systems \( OX_0Y_0Z_0 \) and \( cx_0y_0z_0 \) are parallel, and the only difference between these two coordinate systems is in the coordinates of the origin.

![Figure 1. Coordinate systems.](image)

The STS with distributed parameters is a mechanical system. In this mechanical system the STS is considered as \( n \) point masses with one-sided elastic mechanical coupling (Figure 2).

![Figure 2. Model with distributed parameters for STS.](image)

The motion equations of the STS are presented as [9]:

\[
\frac{dr_k}{dt} = v_k, \quad m_k \frac{dv_k}{dt} = G_i + R_i + T_i - T_{i-1}
\]

(1)

where \( r_k \) \((k = 1, 2, ..., n)\) are the positions of the main spacecraft \((k = 1)\), point masses of the tether \((k = 2, 3, ..., n-1)\) and returning capsule or AP \((k = n)\), \( m_i = m_i(n-2)^{-1} \) \((k = 2, 3, ..., n-1)\) are the tether’s point mass, \( m_i \) is the tether’s mass, \( n \) is the number of the STS’s point masses, \( v_i \) is the absolute velocity, \( G_i \) is the gravitational force, \( R_i \) is the aerodynamic force. The motion model (1) provides more detailed insight into the system dynamics as this model makes it possible to analyze the tether shape and other features of the STS, such as the tether extensibility and the possibility of tether becoming slack.

The tensional force \( T_i \) is calculated according to the Hooke’s law taking into account that the mechanical coupling is one-sided:

\[
T_i = r_k \frac{r_{k+1} - r_k}{|r_{k+1} - r_k|}, \quad (k = 1, 2, ..., n-1)
\]

(2)
In this work, the motion of the STS is divided into the following phases: 1) the deployment phase of the STS into the position close to the local vertical, 2) the free motion phase of the sub-system, which is separated from the main spacecraft, to the surface of the Earth can be divided into the following phases: 1) the deployment phase of the STS into the position close to the local vertical, 2) the free motion phase of the sub-system, which is separated from the main spacecraft, to the surface of the Earth.

The relationship between the absolute and relative velocities is presented as:

\[ \mathbf{V}_{i,t} = \mathbf{V}_i - \Omega_s \times \mathbf{r}_i, \quad (k = 1, 2, ..., n) \]

where \( \Omega_s \) is the angular velocity of the Earth’s self-rotation.

### 3. Schemes for returning the payload from LEO

In this work, it is assumed that the tether is deployed from the main spacecraft and after the deployment phase the tether is in the position close to the local vertical. The mass of the main spacecraft is much larger than the masses of other parts of the STS: the returning capsule, tether and AP. The schemes in this work differ from each other in the ways of using the AP. The structure of the AP is foldable or inflatable. It means that the ballistic coefficient or cross-sectional area of the AP can change during the motion of the STS. The mission of returning the payload from LEO to the surface of the Earth can be divided into the following phases: 1) the deployment phase of the STS into the position close to the local vertical, 2) the free motion phase of the sub-system, which is separated from the main spacecraft, to the surface of the Earth.

The relationship between the absolute and relative velocities is presented as:

\[ \mathbf{V}_{i,t} = \mathbf{V}_i - \Omega_s \times \mathbf{r}_i, \quad (k = 1, 2, ..., n) \]
dense layers of the atmosphere (110 km); 3) the descent motion of the returning capsule with the payload in the dense layers of the atmosphere (below 110 km).

The deployment of the STS into the position close to the local vertical, is analyzed in [7]. A deployment control law is proposed taking into account the effect of the aerodynamic force in [7]. This deployment control law can bring the STS to the given final state. The sub-system is separated from the main spacecraft when the tether passes through the local vertical. In this case, the direction of the relative velocity component of the end-body below is opposite to the orbital motion velocity direction of the system center of mass due to the swing of the tether. This leads to a decrease in the absolute velocity of the end-body below when the sub-system is separated from the main spacecraft. In addition, the descent trajectory of the sub-system is steeper. A similar maneuver was used in the space tethered experiment YES2 [12].

The following schemes are proposed for returning the payload. The sub-systems of these schemes are different.

3.1. Scheme “Capsule above, AP below”
In this scheme the AP, which is connected with the tether, is separated from the main spacecraft first. The control law in [7] is used for deploying the tether. After the deployment phase the capsule with the payload is separated from the main spacecraft. The capsule is at the other end of the tether. The sub-system in this scheme consists of the capsule, tether and AP. The cross-sectional area of the AP increases at the moment of separating the capsule, for example, the folding structure is unfolded. It leads to an increase in the ballistic coefficient of the AP. In this scheme the ballistic coefficient of the AP is larger than the ballistic coefficient of the capsule. The free motion phase of the sub-system finishes when the orbital altitude of the capsule reaches 110 km. Next, the capsule is separated from the tether and descends to the Earth. This scheme is illustrated in Figure 3.

3.2. Scheme “Capsule below, with tether, without AP”
In this scheme the capsule with the payload is separated from the main spacecraft firstly and then is deployed into the position close to the local vertical. The sub-system, which consists of the capsule and tether, is separated from the main spacecraft after the deployment phase. The free motion phase of the sub-system finishes when the orbital altitude of the capsule reaches 110 km. Next, the capsule is separated from the tether and descends to the Earth. This scheme is illustrated in Figure 4.
3.3. Scheme “Capsule below, without tether and AP”
In this scheme the tether is cut off at both ends of the tether after the deployment phase. Next, only the motion of the capsule with the payload is considered. This scheme is used for the space tethered experiment YES2 [12]. It is illustrated in Figure 5.

![Figure 5. Scheme “Capsule below, without tether and AP”](image)

3.4. Scheme “Capsule below, AP above”
In this scheme the capsule is the end-body below during the deployment phase. The AP is separated from the main spacecraft after the deployment phase. The sub-system in this scheme consists of the capsule, tether and AP. The free motion phase of the sub-system finishes when the orbital altitude of the capsule reaches 110 km. Next, the capsule is separated from the tether and descends to the Earth. This scheme is illustrated in Figure 6.

![Figure 6. Scheme “Capsule below, AP above”](image)

3.5. Scheme “Capsule with variable ballistic coefficient, without tether and AP”
This scheme is similar to the scheme “Capsule below, without tether and AP”. The difference between these two schemes is that the ballistic coefficients of the capsule with the payload in the deployment phase and after the deployment phase are different. In the free motion phase of the capsule (the orbital altitude descends to 110 km) the ballistic coefficient of the capsule is larger than the ballistic coefficient of the capsule in the deployment phase in order to act as the additional braking device. When the orbital altitude of the capsule reaches 110 km, the ballistic coefficient of the capsule is restored as in the deployment phase. This scheme is illustrated in Figure 7.

![Figure 7. Scheme “Capsule with variable ballistic coefficient, without tether and AP”](image)
4. Simulation results
The parameters used for numerical simulations of the schemes in this paper are presented in Table 1–4. The ballistic coefficient $\sigma$ is defined as $\sigma = c \cdot S \cdot m^{-1}$, where $c$ is the drag coefficient of the object, $S$ is the cross-sectional area of the object in the direction of the object’s motion, $m$ is the object’s mass.

| Parameter | Value | Unit |
|-----------|-------|------|
| Initial altitude of circular orbit: $H_c$ | 270 | km |
| Total tether length: $L_{end}$ | 30 | km |
| Line density of tether material: $\rho_i$ | 0.2 | kg/km |
| Tether diameter: $D_i$ | 0.5 | mm |
| Drag coefficients of the main spacecraft, capsule and AP: $c_k$ ($k = 1, n$) | 2.4 |
| Drag coefficient of the tether: $c_t$ | 2.2 |
| Mass of the main spacecraft: $m_i$ | 6000 | kg |
| Ballistic coefficient of the main spacecraft: $\sigma_i$ | $3 \cdot 10^3$ | $m^2/kg$ |
| Mass of the capsule: $m_c$ | 6 | kg |
| Diameter of the capsule: $D_c$ | 0.4 | m |
| Ballistic coefficient of the capsule: $\sigma_c$ | $5 \cdot 10^2$ | $m^2/kg$ |

| Parameter | Value | Unit |
|-----------|-------|------|
| Mass of the AP: $m_p$ | 5 | kg |
| Diameter of the AP (for the deployment phase): $D_{s1}$ | 0.4 | m |
| Ballistic coefficient of the AP (for the deployment phase): $\sigma_{s1}$ | 0.06 | $m^2/kg$ |
| Diameter of the AP (after the deployment phase): $D_{s2}$ | 4 | m |
| Ballistic coefficient of the AP (after the deployment phase): $\sigma_{s2}$ | 6.032 | $m^2/kg$ |

| Parameter | Value | Unit |
|-----------|-------|------|
| Mass of the capsule: $m_c$ | 6 | kg |
| Diameter of the capsule (for the free motion phase $H_{cap} \geq 110$ km): $D_{c1}$ | 4.382 | m |
| Ballistic coefficient of the capsule (for the free motion phase $H_{cap} \geq 110$ km): $\sigma_{c1}$ | 6.032 | $m^2/kg$ |
| Diameter of the capsule (for the deployment phase and descending phase $H_{cap} < 110$ km): $D_{c2}$ | 0.4 | m |
| Ballistic coefficient of the capsule (for the deployment phase and descending phase $H_{cap} < 110$ km): $\sigma_{c2}$ | 0.05 | $m^2/kg$ |

Table 1. Parameters of the STS.
Table 2. Parameters of the scheme “Capsule above, AP below”.
Table 3. Parameters of the scheme “Capsule with variable ballistic coefficient, without tether and AP”.

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The following parameters are used for analyzing the multiple schemes for returning the payload from LEO: the re-entry angle of the capsule into the dense layers of the atmosphere $|\theta|$, the longitudinal range of the capsule on the Earth’s surface $s$ and the landing site dispersion $\Delta s$. The schemes are numbered from 1 to 5. The order is the same as the section order in section 3.

The influence of the variation of the atmospheric density on the re-entry angle of the capsule into the dense layers of the atmosphere and longitudinal range of the capsule on the Earth’s surface is evaluated. The following variations of the atmospheric density ($\Delta \rho_s$) with respect to the nominal atmospheric density are used for the simulation: when the orbital altitude of the capsule $H_{cap} \geq 110 \text{ km}$, $\Delta \rho_s = \pm 20\%$; when $H_{cap} < 110 \text{ km}$, $\Delta \rho_s = \pm 10\%$.

Table 4. Parameters of the scheme “Capsule below, AP above”.

| Parameter         | Value | Unit |
|-------------------|-------|------|
| Mass of the AP: $m_p$ | 5     | kg   |
| Diameter of the AP: $D_p$ | 4     | m    |
| Ballistic coefficient of AP: $\sigma_p$ | 6.032 | m$^2$/kg |

Table 5 shows the the re-entry angle of the capsule into the dense layers of the atmosphere $|\theta|$ for the nominal and variational atmospheric density. The results of numerical simulations show that the re-entry angle of Scheme 5 is the largest. In addition, the re-entry angle dispersion ($\max|\theta| - \min|\theta|$) of Scheme 5 is also the largest. The re-entry angles of Schemes 2 and 4 are close.

Table 5. Re-entry angle of the capsule ($|\theta|$, deg) and its variation under the change of atmospheric density.

| Scheme | $\min |\theta|$, $|\theta| + \Delta \rho_s$ | Nominal value, $|\theta|$ | $\max |\theta|$, $|\theta| + \Delta \rho_s$ | $\max |\theta| - \min |\theta|$ |
|--------|---------------------------------|--------------------------|--------------------------|--------------------------|
| 1      | 1.248                           | 1.256                    | 1.259                    | 0.011                    |
| 2      | 0.785                           | 0.872                    | 0.951                    | 0.166                    |
| 3      | 0.531                           | 0.542                    | 0.554                    | 0.020                    |
| 4      | 0.817                           | 0.845                    | 0.922                    | 0.105                    |
| 5      | 1.919                           | 2.078                    | 2.22                     | 0.301                    |

Table 6 shows the longitudinal range of the capsule on the Earth’s surface $s$ for the nominal and variational atmospheric density and the landing site dispersion $\Delta s$. The longitudinal range of Scheme 4 is the largest. The longitudinal range of Scheme 5 is the smallest. The longitudinal ranges of Schemes 2 and 3 are close. The landing site dispersion of Scheme 2 is the smallest.

Table 6. Longitudinal range of the capsule ($s$, km) and landing site dispersion ($\Delta s$).

| Scheme | $\min s$, $s + \Delta s$ | Nominal value, $s$ | $\max s$, $s + \Delta s$ | $\max s - \min s$ |
|--------|-------------------------|--------------------|-------------------------|-------------------|
| 1      | 19655                   | 18652              | 17881                   | 1774              |
| 2      | 17915                   | 17276              | 16782                   | 1133              |
| 3      | 17696                   | 17541              | 17404                   | 292               |
| 4      | 20110                   | 19734              | 18897                   | 1213              |
| 5      | 12255                   | 11820              | 11456                   | 799               |
5. Conclusions
The schemes for returning the payload using the STS are analyzed by using the model of motion with distributed parameters. All the considered schemes in this work can be used for returning the payload from LEO.

The re-entry angle of the capsule into the dense layers of the atmosphere increases by using the AP in the STS, which expands the possibilities of using the STS.

The scheme “Capsule with variable ballistic coefficient, without tether and AP” is more preferable, because the maximum re-entry angle and the minimum longitudinal range are obtained by using this scheme. The landing site dispersion of this scheme is relatively small.

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