Design of Digital Filter Based on Artificial Bee Population Algorithm

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Abstract. With the development of digital signal processing technology, higher requirements are put forward for the accuracy of digital filters. The accuracy of the digital filter has an important influence on the digital signal processing effect, which shows that the optimal design of the filter has practical significance. This paper studies the design of digital FIR filter based on artificial bee colony algorithm. The mathematical optimization model of the digital filter is constructed, in which the unit impulse response amplitude of the filter is used as the optimization variable, and the mean square error of the designed low-pass filter and the ideal low-pass filter is the optimization goal. The artificial bee colony algorithm is used to solve the optimization objective function of the filter, the flow chart and implementation steps are given, and the influence of the artificial bee colony algorithm parameters and the filter parameters on the design results is compared and analysed. The experimental results show that the artificial bee colony algorithm can effectively design the digital filter, and the optimized filter basically agrees with the amplitude-frequency response curve of the ideal filter.

1. Introduction

Digital signal processing theory is widely used in image processing, pattern recognition and other research fields, and higher requirements are put forward for the accuracy of filter functions [1]. The digital filter consists of a multiplier, an adder and a delay. The same filter system has different transformation structures, and the different system structures transformed will directly affect its filtering function, showing different results, indicating that the optimal design of the filter has practical significance [2].

In the circuits used in classic analog filters, if multiple technical indicators are to be met at the same time or to achieve higher accuracy, the design is often complicated, the structure is huge, and the number of components is large. Compared with analog filters, digital filters have many significant advantages such as strong stability, high precision, and flexibility, and do not require impedance matching and large-scale integration [3].

There are a variety of design methods for digital filters, but none of them can achieve the amplitude-frequency characteristics of the ideal filter. Therefore, a variety of studies using optimization algorithms to achieve filter design have been produced [4] - [6]. The optimization algorithm is actually to find the objective function of the problem and solve the optimal solution of the objective function, that is, to find
the maximum or minimum value. This paper establishes an optimal design model of FIR digital filter, uses artificial bee colony algorithm to solve this model, and realizes the optimal design of digital filter.

2. Mathematical Model

Digital filters include finite impulse response filters (FIR) and infinite impulse response filters (IIR) [7]. FIR digital filters have strict linear phase and arbitrary amplitude characteristics. Therefore, if a set of coefficient vectors that meet specific filtering requirements can be found, then a digital filter that meets the requirements can be designed. When the FIR digital filter system is stable, there is only a pole at \( z = 0 \) and converges at \( |z| > 0 \).

The digital FIR filter can be described by a difference equation,

\[
y(n) = \sum_{k=0}^{N-1} h(k)x(n-k)
\]

(1)

Among them, \( x(n) \) is the input of the system, \( y(n) \) is the output of the system, \( h(n) \) is the unit impulse response of the filter, and \( N \) represents the order of the filter.

The system function [8] is expressed as,

\[
H(z) = \sum_{n=0}^{N-1} h(n)z^{-n}
\]

(2)

The frequency response of the system is,

\[
H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)e^{-j\omega n}
\]

(3)

Assuming that the unit impulse response of an ideal digital filter is \( h_\delta(n) \), its frequency response \( H_\delta(e^{j\omega}) \) is,

\[
H_\delta(e^{j\omega}) = \sum_{n=0}^{N-1} h_\delta(n)e^{-j\omega n}
\]

(4)

The frequency response error between the ideal filter and the designed filter is expressed as,

\[
E(e^{j\omega}) = H_\delta(e^{j\omega}) - H(e^{j\omega})
\]

(5)

The mean square error is expressed as \( E_F^2 \), and its calculation formula is expressed as follows,

\[
E_F^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |E(e^{j\omega})|^2 d\omega
\]

(6)

According to the Paseval formula

\[
\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(e^{j\omega})|^2 d\omega
\]

, the mean square error is expressed as,

\[
E_F^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |E(e^{j\omega})|^2 d\omega = \sum_{n=0}^{N-1} |h_\delta(n) - h(n)|^2 + \sum_{other \ n} |h_\delta(n)|^2
\]

(7)

When the mean square error \( E_F^2 \) reaches the minimum value, the unit impulse response of the designed filter and the ideal filter is the closest. In the actual filter design process, only the value of \( \sum_{n=0}^{N-1} |h_\delta(n) - h(n)|^2 \) is considered. In a given frequency range \([0, \pi]\), the number of frequency sampling
points is $M$, then at all discrete frequency points $\omega_k = \frac{k\pi}{M}$ ($k = 0, 1, 2, \cdots, M$), the mean square error of the frequency response is,

$$E^2_F(k) = \sum_{n=0}^{N-1} |h(n) e^{j\omega_k n} - h_d e^{j\omega_k n}|^2$$  \hfill (8)

Based on the above analysis, the design goal of the digital filter is to find a set of filter coefficients $[h(0), h(1), \cdots, h(N-1)]$ to minimize $\sum_{k=0}^{M} E^2_F(k)$.

3. Artificial Bee Population Algorithm

3.1. Principles of Bionics

Artificial bee colony (ABC) [9] algorithm is a cluster intelligent optimization algorithm proposed by Turkish scholar Karaboga after carefully observing the behavior of bee colony collecting nectar in nature. Its advantage is that it only needs to compare the results, and select the pros and cons of the results. Bees are a kind of creatures that like to live in groups. There are two necessary conditions for their swarm intelligence: self-organization and division of labor. Although the behavior of a single bee is very simple, the bee colony exhibits extremely complex behavior. They can collect nectar from flowers with high efficiency in any complex environment, and they can also quickly adapt to the environment.

When the bee colony completes the process of collecting nectar, firstly the leading bees search for the nectar source randomly. After finding the nectar source, the leading bee turns into an observation bee. In this process, the most critical link is how to exchange and transmit information. After observation, it was discovered that the area where the bees danced was the place where they delivered the "real-time news" of the food source.

If the leading bee finds a nectar source suitable for mining, it will dance a "swing dance" in the dancing area, and the observation bee received the available nectar information based on the dance danced by the leading bee. The probability that a bee decides to go to a nectar source is proportional to the rate of return that can be achieved at that nectar source.

In the initial stage of collecting nectar, the search for the leading bee is random. After finding the nectar source, it becomes a scout bee to actively search for nectar sources near the hive. After determining the nectar source, the leading bee remembers the location of the nectar source and starts collecting nectar. At this time, it will become a "collecting bee" and continue to complete the nectar collecting work.

3.2 Mathematical Description

The bee colony contains $NP$ bees, among which the number of leading bees (collecting bees) is $NP/2$, the number of observing bees is $NP/2$, and the remaining bees can be transformed into scout bees under certain conditions [10].

Nectar source: $X = \{X_1, X_2, \cdots, X_{NP/2}\}$, $(i = 1, 2, \cdots, NP/2)$, where $X_i$ is one nectar source, corresponds to one feasible solution for the optimization problem.

Fitness: $y = fit(x)$, represents the nectar quality of the nectar source, that is, the value of the objective function of the optimization problem.

(1) Leading bee behavior

Leading bee searches the nectar source $X_i$ randomly and find a new nectar source location $V_i$,

$$V_i = X_i + \text{Rand} \cdot (X_i - X_k)$$  \hfill (9)

Among them, $k$ is the sequence number generated by random search, $k \in \{i = 1, 2, \cdots, NP\}$ and $k \neq i$. $\text{Rand}$ is one random number in the interval $[-1, 1]$. If the quality of the new nectar source is
better than the original nectar source, leading bee remembers the new nectar source, otherwise keep the old nectar source.

(2) Observation bee behavior
The probability that the observing bee chooses a certain nectar source is proportional to the rate of return to the nectar source. The probability calculation formula is,

$$p_i = y_i / \sum_{j=1}^{NP} y_j$$

(10)

Where, $y_i$ represents the fitness of nectar $X_i$. One number is randomly generated in the interval $[-1, 1]$; if the selection probability of the nectar source is greater than the random number, the observing bee randomly searches for a new nectar source,

$$V_j = X_i + \text{Rand} \cdot (X_j - X_k)$$

(11)

If the quality of the new nectar source is better than the original nectar source of the leading bee, then the observation bee is transformed into the leading bee, otherwise the role remains unchanged.

(3) Scout bee behavior
If the location of the nectar source $X_i$ is searched for limit times and no better nectar source information is obtained, the nectar source will be abandoned, and the leading bee corresponding to the nectar source will be transformed into a scout bee. The scout bee randomly searches for a new source of nectar,

$$X_j = X_i + \text{rand} \cdot (\max(X_j) - \min(X_j))$$

(12)

Where, rand represents a random number in the interval $[0, 1]$.

When all the leading bees have completed the work of collecting nectar, the optimal nectar source found is the optimal solution to the problem.

4. Artificial Bee Population Algorithm for Digital FIR Filter Optimization
Realizing the optimal design of the filter through the artificial bee colony algorithm is actually taking the search for the best filter coefficient as the process by which the bees find the best nectar source. The nectar source in the artificial bee colony algorithm corresponds to a set of filter coefficients $[h(0), h(1), \ldots, h(N-1)]$, and the fitness of the nectar source corresponds to $y = \sum_{k=0}^{M} E_F^2(k)$. Through constant search, the designed filter amplitude-frequency response curve is close to the ideal filter. After the algorithm is executed, the nectar source with the best quality corresponds to the best filter coefficient $[h(0), h(1), \ldots, h(N-1)]$. The flow chart of filter design based on artificial bee colony algorithm is shown in Figure 1. The specific steps are as follows.

Step1: Initialization: Set the population size of bees $NP$, the maximum number of iterations $\text{maxCycle}$ and conversion condition $\text{limit}$, the technical indicators of the filter, the number of discrete sampling points, etc.

Step2: Randomly generate $NP/2$ nectar sources $X = \{X_1, X_2, \ldots, X_{NP/2}\}$ and calculate the fitness of each nectar source.

Step3: Leading bee searches randomly, finds a new nectar source, and calculates the fitness of the new nectar source. If the new nectar source is better than the old nectar source, record the new nectar source, otherwise it remains unchanged.

Step4: Leading bee recruits observation bees using roulette. If the observation bee can be assigned to the nectar source, it will change to the leading bee; otherwise, it will randomly search for the alternate nectar source and update the nectar source.

Step5: It is judged whether a scout bee is generated. If a scout bee is generated, the scout bee continues to search for new nectar sources, otherwise the role remains unchanged.
Step 6: Judge whether the termination condition of the algorithm is satisfied, if it is satisfied, stop the loop and output the optimal filter coefficient \([h(0), h(1), \ldots, h(N-1)]\), otherwise continue to execute Step 3–Step 5.

Figure 1. Flow chart of filter optimization design.
5. Results Analysis
The ideal low-pass filter is optimized and designed, and MATLAB software is used for simulation experiments to analyse the influence of artificial bee colony algorithm parameters and filter parameters on the optimization results.

Given an ideal low-pass filter, its technical indicators are:

$$H_d(e^{j\omega}) = \begin{cases} 1, & 0 \leq \omega \leq \omega_p \\ 0, & \omega_p < \omega \leq \pi \end{cases}$$

The parameters of the artificial bee colony algorithm are set as: the size of the bee colony $NP = 100$, the conversion condition for leading bee to scout bee $limit = 100$, and the number of iterations $maxCycle = 200$. The discrete frequency points of the FIR digital filter are uniformly sampled from 0 to $\pi$, the number of sampling points $M = 51$, the passband cut-off frequency of the filter $\omega_p = 0.5\pi$, and the filter order $N = 10$.

The change of the mean square error value with the number of iterations is shown in Figure 2. When the number of iterations reaches 50, the mean square error value is basically stable, and the mean square error value is 0.7825. The amplitude-frequency response curve comparison is shown in Figure 3. The designed filter still has a certain error compared with the ideal filter.
5.1 The Influence of Filter Parameters on Optimization Design Results

(1) Change the filter order from $N = 10$ to $N = 20$, and other parameters remain unchanged.

The result is shown in Figure 4. The mean square error value is basically stable and approaches 0 at $\text{maxCycle} = 100$, and the optimal mean square error value is 0.4109. The comparison diagram of the amplitude-frequency response curve of the filter is shown in Figure 5. The optimization effect is obviously better than that in Figure 3. It shows that appropriately increasing the filter order can reduce the error between the designed digital filter and the ideal digital filter, so as to get a better optimization effect.
Figure 5. Comparison of amplitude-frequency response curves ($n = 20$).

(2) Change the number of sampling points from $M = 51$ to $M = 81$, other parameters remain unchanged.

The change of the mean square error value with the number of iterations is shown in Figure 6, and the mean square error value eventually stabilizes. The comparison diagram of the amplitude-frequency response curve of the filter is shown in Figure 7. The optimization effect of the front section of the passband and the end section of the stopband is better than that of Figure 5. It shows that by increasing the number of sampling points, the optimization effect will be better.

Figure 6. Mean square error changes with the number of iterations ($M = 81$).
Based on the above analysis, after adjusting the filter parameters for many times, it is found that the optimization effect is better when the filter parameters are adjusted to: filter order $N = 30$ and frequency sampling points $M = 101$. The result is shown in Figure 8.

5.2 The Influence of ABC Algorithm Parameters on Optimal Design Results

Keep the filter parameters as $\omega_p = 0.5\pi$, $N = 30$, $M = 101$ unchanged, and observe the influence of the artificial bee colony algorithm parameters on the optimization design effect.

(1) Change the size of the bee colony from $NP = 100$ to $NP = 300$, other parameters remain unchanged.
The change of the mean square error value with the number of iterations is shown in Figure 9, and the optimal mean square error value obtained is 0.6427. The comparison diagram of the amplitude-frequency response curve of the filter is shown in Figure 10, and the optimization effect is basically the same as that in Figure 8, indicating that increasing the colony size does not significantly improve the optimization effect.

Figure 9. Mean square error changes with the number of iterations ($NP = 300$).

Figure 10. Comparison of amplitude-frequency response curves ($NP = 300$).

(2) Change the conversion conditions from $limit = 100$ to $limit = 80$, and other parameters remain unchanged.

The result is shown in Figure 11, the mean square error value at this time is 0.6149, and the amplitude-frequency response curve of the filter is shown in Figure 12. The optimization effect is significantly better than Figure 10, indicating that the increased value can achieve better optimized effect.
Combining the analysis of the influence of the filter parameters and the artificial bee colony algorithm parameters on the optimization results, it can be found that when the parameters are $N = 30$, $M = 101$, $NP = 300$, $limit = 80$, $maxCycle = 500$, the best design effect is obtained, and the best optimization effect is shown in Figure 13.

Figure 11. Mean square error changes with the number of iterations ($limit = 80$).

Figure 12. Comparison of amplitude-frequency response curves of filters ($limit = 80$).
6. Conclusion

According to the characteristics of the FIR digital filter, the optimal design model of the digital filter is constructed, in which the optimization goal is to minimize the frequency response error between the design filter and the ideal filter, and the unit impulse response function of the design filter is used as the optimization variable. The artificial bee colony algorithm is used to optimize the design of the FIR digital filter, the algorithm flow chart and specific implementation steps are designed, and the performance of the algorithm is verified by MATLAB software. The influence of artificial bee colony algorithm parameters and filter parameters on the optimization effect is discussed, and the effectiveness of the artificial bee colony algorithm for designing digital filters is verified.

The artificial bee colony algorithm can effectively improve the design quality of the filter, which is very necessary to improve the function of the digital signal processing system. Future research may focus on adopting other design criteria to obtain design methods that are closer to ideal filters.

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