ON THE $B \to J/\Psi + K^*$ DECAY

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Abstract
Within the QCD factorization approach, we calculate the process- and polarization-dependent non-factorizable terms $\tilde{a}_\lambda$ of the $B \to J/\Psi + K^*$ decay. The longitudinal part $\tilde{a}_0$ is infrared convergent and large enough to agree with recent experimental data, provided that the B-K* form factors $A_1(m_\Psi^2)$ and $A_2(m_\Psi^2)$ satisfy some constraints met by some (but not all) models. The transverse parts $\tilde{a}_\pm$ on the other hand are infrared divergent, the procedure used to handle such divergence is discussed in relation with the $B \to J/\Psi + K$ case in which the same problem arises. Our nonzero phases of the helicity amplitudes are consistent with experimental data recently measured for the first time by the CDF and BaBar groups.

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1 Introduction

Among a hundred hadronic decay modes of the B mesons investigated from both experimental and theoretical sides, the process $B \to J/\Psi + K^*(892)$ is particularly interesting for many reasons:

(i) - It is the first color-suppressed B decay observed in 1994 by the Argus group [2] with the largest branching ratio for its class. Since then, important measurements are intensively explored in great detail by the Cleo [3], CDF [4], BaBar [5] and Belle [6] collaborations.

(ii) - Over both the vector + pseudoscalar $B \to V + P$ and the pseudoscalar + pseudoscalar $B \to P + P$ modes, the advantage of the vector + vector $B \to V + V$ decays ($B \to J/\Psi + K^*$ considered here) stands out in the possibility of detailed analyses of the three helicity amplitudes. The three decay amplitudes (one longitudinal and two transverse) denoted by $H_0$, $H_{-1}$ and $H_{+1}$ could be separately determined both for their magnitudes $|H_\lambda|$ and phases $\delta_\lambda$. These $|H_\lambda|$ and $\delta_\lambda$ analyses provide a powerful tool to test not only the naive factorization method [7] - usually adopted to deal with exclusive two-body hadronic decays - but also the real and imaginary part of the nonfactorizable terms which are calculable in QCD approaches [8, 9] can then be confronted with experiments. We note that such an analysis cannot be done in $B \to V+P$ and $B \to P+P$ decays since only the absolute value of a single amplitude (the equivalent of $|H_0|$) can be measured in these processes.

(iii) - The transverse $H_{-1}$ and $H_{+1}$ amplitudes with both magnitudes and phases provide also a useful way to test robustness of factorization manifested through the $V \equiv A$ property of the effective weak currents in the Wilson operator product expansion (OPE). It would answer the question [10] whether or not the interactions between the hadronic decay products, usually called final-state interactions (FSI), are strong enough to flip the quark spin in color-suppressed B decays. Although intuitively this spin flip unlikely occurs, this possibility could be tested however.

(iv) - Improved by QCD which gives the $\alpha_s$ corrections to the decay amplitudes in OPE, the three “heavy to light” $B$-$K^*$ form-factors in $B \to J/\Psi + K^*$: $A_1(q^2)$, $A_2(q^2)$ and $V(q^2)$ can be determined and compared to models given in the literature. These form factors are useful for other decays, in particular $B \to \rho + K^*$ and $B \to \Phi + K^*$.

Motivated by new experimental data [8, 9] and recent theoretical developments [8, 9], we are trying in this paper to investigate some aspects of the $B \to J/\Psi + K^*$ process.

2 Decay amplitudes

2-1 Generality, Polarizations and Angular Distributions

The most general $B \to V_1 + V_2$ helicity amplitude takes the following form in which we adopt the sign convention of [11]

$$H_\lambda \left( B(P) \to V(p_1, \epsilon_1) + V(p_2, \epsilon_2) \right) = \epsilon_1^{\mu}(\lambda)\epsilon_2^{\nu}(\lambda) \left( g_{\mu\nu} A + \frac{P_\mu P_\nu}{m_1 m_2} B + i\epsilon_{\mu\nu\alpha\beta} \frac{P_1^{\alpha} P_2^{\beta}}{m_1 m_2} C \right),$$

(1)

where $\lambda$ stands for the three helicities 0, $\pm 1$ of the massive vector mesons with polarizations $\epsilon_1^{\mu}(p_1)$, $\epsilon_2^{\nu}(p_2)$. Since the initial B meson is spinless, the two final vector mesons share the same helicity $\lambda$. 


Here \( M, m_1, m_2 \) are masses of the \( B, V_1, V_2 \) mesons with four-momenta \( p, p_1, p_2 \) respectively. The \( A \) and \( B \) associated to the \( S \) and \( D \) waves are CP-even, while \( C \) corresponding to the \( P \) wave is CP-odd. We note that in two-body decays, the Lorentz invariant amplitudes \( H_\lambda \) have a mass dimension, so are the quantities \( A, B, C \). From (1), we get

\[
H_0 = -\left(a A + (a^2 - 1) B \right), \quad H_{\pm 1} = A \pm \sqrt{a^2 - 1} C,
\]

with (7)

\[
a = \frac{p_1 \cdot p_2}{m_1 m_2} = \frac{M^2 - m_1^2 - m_2^2}{2m_1 m_2}.
\]

Also we define \( K_c \) as the common momentum of the outgoing mesons \( V_1 \) (or \( V_2 \)) in the \( B \) rest frame:

\[
K_c^2 = \frac{\lambda(M^2, m_1^2, m_2^2)}{4M^2},
\]

where \( \lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + xz + yz) \). Thus \( \sqrt{a^2 - 1} = MK_c/(m_1 m_2) \).

From (1) and (2), the decay rate in each polarization state is given by:

\[
\Gamma_\lambda = \frac{K_c}{8\pi M^2} |H_\lambda|^2,
\]

with \( \Gamma = \sum_\lambda \Gamma_\lambda \) is the full decay width \( \Gamma(B \rightarrow J/\Psi + K^*) \).

In the following we normalize the partial widths \( \tilde{\Gamma}_\lambda = \Gamma_\lambda/\Gamma \) for the three independent polarization states

\[
\tilde{H}_\lambda = \frac{H_\lambda}{\sqrt{\sum_\lambda |H_\lambda|^2}}
\]

such that \( \sum_\lambda |\tilde{H}_\lambda|^2 = 1 \).

The normalized dimensionless spin amplitudes \( A_0, A_\parallel \) and \( A_\perp \) are related to the helicity ones by

\[
A_0 = \tilde{H}_0, \quad A_\parallel = \frac{\tilde{H}_{+1} + \tilde{H}_{-1}}{\sqrt{2}}, \quad A_\perp = \frac{\tilde{H}_{+1} - \tilde{H}_{-1}}{\sqrt{2}}.
\]

with again \( \sum_\lambda |A_\lambda|^2 = 1 \). To proceed to the \( A_0, A_\parallel, A_\perp \) determinations, angular measurements are necessary. For that, let us define the transversity angles \( \theta_{tr} \) and \( \Phi_{tr} \) as the polar and azimuthal angles of the \( \ell^+ \) descended from \( J/\Psi \rightarrow \ell^+ + \ell^- \) decay in the \( J/\Psi \) rest frame. The \( K^* \) helicity angle \( \theta_{K^*} \) is the angle between the \( K \) meson direction (coming from \( K^* \rightarrow \pi+K \)) and the opposite direction of the \( J/\Psi \) in the \( K^* \) rest frame. The angular distributions (11) given below allow us to determine both the \( |A_0|, |A_\parallel|, |A_\perp| \) magnitudes and phases \( \delta_0, \delta_\parallel, \delta_\perp \) (up to a two-fold ambiguity [10, 12]). Thus

\[
\frac{1}{\Gamma} \frac{d^3 \Gamma}{d \theta_{tr} d \Phi_{tr} d \theta_{K^*} d \Phi_{tr}} = \frac{9}{32\pi} \left( f_1 |A_0|^2 + f_2 |A_\parallel|^2 + f_3 |A_\perp|^2 + f_4 \text{Im}(A_\parallel^* A_\perp) + f_5 \text{Re}(A_0^* A_\parallel) + f_6 \text{Im}(A_0^* A_\perp) \right),
\]

with

\[
f_1 = 2 \cos^2 \theta_{K^*} (1 - \sin^2 \theta_{tr} \cos^2 \Phi_{tr}), \quad f_2 = \sin^2 \theta_{K^*} (1 - \sin^2 \theta_{tr} \sin^2 \Phi_{tr}),
\]

\[
f_3 = \sin^2 \theta_{K^*} \sin^2 \theta_{tr}, \quad f_4 = \pm \sin^2 \theta_{K^*} \sin 2\theta_{tr} \sin \Phi_{tr},
\]

\[
f_5 = -\frac{1}{\sqrt{2}} \sin 2\theta_{K^*} \sin^2 \theta_{tr} \sin 2\Phi_{tr}, \quad f_6 = \pm \frac{1}{\sqrt{2}} \sin 2\theta_{K^*} \sin 2\theta_{tr} \cos \Phi_{tr}.
\]
The plus sign in $f_{4,6}$ refers to the B mesons which are $\overline{B}q$ \ (q = u, d) bound states, and the minus sign to $B$ mesons (b$\overline{q}$ bound states). In the following, for convenience, the amplitudes are implicitly written for the $B$ mesons, since we are dealing with the b quark and not the antiquark $\overline{b}$.

2-2 Effective Hamiltonian

The basis for nonleptonic weak decays of hadrons is the operator product expansion, and the effective Hamiltonian relevant to $B \to J/\psi + K^*$ may be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left( V_{cb} V_{cs}^* [C_1(\mu)O_1(\mu) + C_2(\mu)O_2(\mu)] - V_{tb} V_{ts}^* \sum_{j=3}^{6} C_j(\mu)O_j(\mu) \right), \quad (7)$$

where the Wilson coefficients $C_i(\mu)$ are evaluated at next-to-leading order and at the renormalization scale $\mu$. We neglect the electroweak penguin operators $O_{7-10}$ since their corresponding coefficients $C_7 \cdots C_{10}$ being proportional to $\alpha_{\text{em}} = 1/137$ are numerically negligible compared to the dominant $C_1$ and $C_2$ associated to the tree diagrams, and $C_3 \cdots C_6$ associated to the gluonic penguin loop diagrams. We have:

V-A current $\times$ V-A current

$$O_1 = (\overline{s}_a c_a)_{V-A}(\overline{c}_\beta b_\beta)_{V-A}, \quad O_2 = (\overline{s}_a c_\beta)_{V-A}(\overline{c}_\beta b_a)_{V-A}, \quad (8)$$

QCD-Penguins

$$O_3 = (\overline{s}_a b_\alpha)_{V-A} \sum_q (\overline{q}_\beta q_\beta)_{V-A}, \quad O_5 = (\overline{s}_a b_\alpha)_{V-A} \sum_q (\overline{q}_\beta q_\beta)_{V+A}, \quad (9)$$

$$O_4 = (\overline{s}_a b_\beta)_{V-A} \sum_q (\overline{q}_\beta q_\alpha)_{V-A}, \quad O_6 = (\overline{s}_a b_\beta)_{V-A} \sum_q (\overline{q}_\beta q_\alpha)_{V+A}, \quad (10)$$

where $\alpha, \beta$ are quark color indices and in (9)-(10), the sum $\sum_q$ is done over $q = u, d, s, c, b$ quarks. Also $(\overline{q}_\beta q_\beta)_{V-A}$ denotes $\overline{q}_\beta(1 - \gamma_5)q_\beta \overline{\gamma}^\rho(1 + \gamma_5)q_\rho$. The coefficients $C_i(\mu)$ are given in Table XXII of [13] at next-to-leading order in the naive dimension regularization (NDR) and in the 't Hooft-Veltman (HV) $\gamma_5$ renormalization schema:

$$C_1 = 1.082 \ (1.105), \quad C_2 = -0.185 \ (-0.228), \quad C_3 = 0.014 \ (0.013),$$

$$C_4 = -0.035 \ (-0.029), \quad C_5 = 0.009 \ (0.009), \quad C_6 = -0.041 \ (-0.033),$$

where the first numbers refer to the NDR scheme and those in the parentheses to the HV scheme, both evaluated at the scale $\mu = \overline{m}_b(m_b) = 4.4\text{GeV}$. The dependence of $C_i(\mu)$ on the scale $\mu$ as well as on the regularization-scheme must be cancelled in principle by the matrix-elements $\langle K^*|\Psi|O_i(\mu)|B \rangle$ since physical amplitudes $\sim C_i(\mu) \times \langle K^*|\Psi|O_i(\mu)|B \rangle$ must be scale and regularization-scheme independent. In the "naive" factorization approach, $\langle K^*|\Psi|O_i(\mu)|B \rangle$ is a product of decay constants and form factors, both are real, moreover they are scale and regularization scheme independent, hence the amplitudes via $C_i(\mu)$ suffer from these dependences and turn out to be ambiguous. After a tentative approach[14], the QCD methods finally solve this problem as we will see.

Using the unitarity condition $V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0$ and neglecting $V_{ub}V_{us}^* \approx 10^{-3}$, we note that (7) has only the unique $V_{cb}V_{cs}^*$ factor. This is crucial to ensure that no matter the penguin $\langle O_{3,6} \rangle$ and tree $\langle O_{1,2} \rangle$ matrix-elements are, we always have $|\mathcal{M}| = 1$ (to a very good precision $V_{ub}V_{us}^* \approx 10^{-3}$), where $\mathcal{M}$ and $|\mathcal{M}|$ are respectively the $B \to J/\psi + K(K^*)$ and $B \to J/\psi + \overline{B}(K^*)$ amplitudes. This condition $|\mathcal{M}| = 1$ allows us to extract from experimental data[15, 16] the CP asymmetry $\beta$ angle without any theoretical uncertainties[17] through the "gold-plated" modes $B \to J/\psi + K(K^*)$. 


2.3 QCD-improved factorization approach

To proceed further, we use the QCD-improved factorization approach \[8\] according to which, in the infinite $b$ quark mass limit and for some classes of two-body hadronic $B \rightarrow M_1 + M_2$ decays, the mass-singularities (infrared divergences) factorize, so that the amplitudes may be written as convolutions of *universal* quantities (the light-cone distribution amplitudes (LCDA) of the mesons $B, M_1, M_2$ with their associated semi-leptonic form factors $F^{BM_1}_{\mu}(m_B^2)$) and QCD perturbatively calculable hard-scattering kernels $T^I(u), T^{II}(\xi, \eta, u)$ which are *process-dependent*.

Here $M_1$ denotes the recoiled meson which can be light ($\pi, \rho, K, K^* \cdots$) or heavy (charm $D, D^* \cdots$) but the emitted $M_2$ can only be a light meson[8] or a $Q\bar{Q}$ quarkonium (heavy or light) and not a heavy meson like $D, D^* \cdots$. The main problem with the latters is that they represent an extended soft hadronic object. In our case $M_1$ is the $K^*$ and $M_2$ is the $J/\Psi$. Schematically we write

$$\langle K^* \Psi \mid O_i \mid B \rangle = F^{BK^*}(m_B^2) \int_0^1 du T^I(u) \Phi_{\Psi}(u) + \int_0^1 d\xi d\eta d\xi T^{II}(\xi, \eta, u) \Phi_B(\xi) \Phi_{K^*}(\eta) \Phi_{\Psi}(u), $$

(11)

$$\langle K^* \Psi \mid O_i \mid B \rangle = \langle K^* \mid J_\mu \mid B \rangle \langle \Psi \mid J^\mu \mid 0 \rangle \left[ 1 + \sum r_n \alpha^2_n + O(\Lambda_{QCD}/m_b) \right],$$

(12)

where $O_i$ are products of different currents $J_\mu$ in (8)-(10).

These equations imply that in the $m_b \rightarrow \infty$ limit, “naive” factorization (corresponding to the first term 1 in (12)) is recovered in the absence of QCD for which $T^I(u)$ is independent of $u$ and $T^{II} = 0$ in (11). Since the $b$ quark mass is large but finite, power corrections $O(\Lambda_{QCD}/m_b)$ could be significant, specifically in some particular cases (for instance in $B \rightarrow \pi + K$ for which the scale is not $\Lambda_{QCD}$ but chirally-enhanced like $m_K^2/(m_s + m_d)$). However it must be noted that in the QCD-improved approach[8], these power corrections (generally associated with nonleading higher-twist LCDA) cannot be reliably computed since in many (but not all) cases, mass singularities again show up thus do not factorize. Keeping this fact in mind, we nevertheless calculate to order $\alpha_s$ the QCD corrections to the naive factorization $B \rightarrow J/\Psi + K^*$ amplitude[7], the simpler case $B \rightarrow J/\Psi + K$ of $B \rightarrow V+P$ has been previously studied[13, 14] within the same theoretical framework.

We also mention another approach[9] called PQCD (perturbative QCD) according to which the double logarithms Sudakov suppression effects could regulate the mass singularities, hence power corrections and form factors may be perturbatively calculable but questionned in[11].

The symbolically written form factors $F^{BK^*}(m_B^2)$ in (11) and the more explicit $\langle K^* \mid J_\mu \mid B \rangle$ ones in (12) are in fact defined according to

$$\langle K^*(\epsilon_1, p) \mid \bar{\psi} \gamma_\mu (1 - \gamma_5) b \mid B(P) \rangle \equiv \langle K^*(\epsilon_1, p) \mid V_\mu - A_\mu \mid B(P) \rangle \equiv V_\mu - A_\mu,$$

where[12]

$$V_\mu = i \epsilon_{\mu \nu \alpha \beta} \epsilon_1^{\nu(*)} \rho_\alpha p_\beta \frac{2V(q^2)}{M + m_{K^*}},$$

$$A_\mu = (M + m_{K^*}) \left[ \epsilon_1^{\mu(*)} - \frac{\epsilon_1^* \cdot q}{q^2} q_\mu \right] A_1(q^2) - \frac{\epsilon_1^* \cdot q}{M + m_{K^*}} \left[ (P + p)_\mu - \frac{M^2 - m_{K^*}^2}{q^2} q_\mu \right] A_2(q^2) + 2m_{K^*} A_0(q^2) \frac{\epsilon_1^* \cdot q}{q^2} q_\mu,$$

(13)

and $q = P - p$ is the four momentum of the emitted $J/\Psi$ meson. In the above equation, terms proportional to $q_\mu$ vanish when multiplied to the $J/\Psi$ polarization vector $\epsilon_2^\mu(q)$. With

$$\langle \Psi \mid J^\mu \mid 0 \rangle = \langle \Psi(\epsilon_2, q) \mid \bar{\psi} \gamma_\mu c \mid 0 \rangle = f_\Psi m_\Psi \epsilon_2^\mu(q).$$
where $f_\Psi \approx 405$ MeV is the $J/\Psi$ decay constant extracted from the $J/\Psi \rightarrow \ell^+ + \ell^-$ rate, the first term $\left< K^* \mid J_\mu \mid B \right>$ on the right hand side of (12) is equal to

$$
e_1^\mu (p) e_2^\mu (q) f_\Psi m_\Psi \left[ -(M + m_{K^*}) A_1 (m_\Psi^2) g_{\mu \nu} + \frac{2 P_{\mu} P_{\nu}}{M + m_{K^*}} A_2 (m_\Psi^2) + \frac{2 i c_{\mu \alpha \beta} q_{\alpha \beta}}{M + m_{K^*}} V (m_\Psi^2) \right].$$

When we compare (1) with (14) using (2), (7) and (12), then in terms of an overall common factor

$$\kappa \equiv \frac{G_F}{\sqrt{2}} V_{cb} V^*_{cs} f_\Psi m_\Psi (M + m_s^*) A_1 (m_\Psi^2),$$

and the dimensionless constants $a, b, c$ given in[7]

$$a = \frac{M^2 - m_{K^*}^2 - m_\Psi^2}{2 m_{K^*}^2 m_\Psi}, \quad b = \frac{2 M^2 K^2}{m_{K^*}^2 (M + m_s^*)^2}, \quad c = \frac{2 M K_c}{(M + m_{K^*})^2},$$

together with the form factor ratios $x, y$ defined by[7]

$$x = \frac{A_2 (m_\Psi^2)}{A_1 (m_\Psi^2)}, \quad y = \frac{V (m_\Psi^2)}{A_1 (m_\Psi^2)},$$

we obtain the helicity amplitudes $H_\lambda$:

$$H_0 = \kappa (a - b x) \tilde{a}_0, \quad H_{\pm 1} = \kappa (\pm cy - 1) \tilde{a}_\pm.$$

Numerically we have $a = 3.165, b = 1.308, c = 0.436$ and the dimensionful factor $\kappa$ (in KeV) is equal to $2.48 A_1 (m_\Psi^2)$.

It remains the most involved computations of the polarization-dependent coefficients $\tilde{a}_\lambda$ in (18) which are the nonfactorizable $a_s$ correction terms derived from the QCD-improved factorization framework which the next section will be devoted. In the "naive" factorization method previously considered[7] without the QCD-improved approach, the $a_2 = C_2 + C_1 / 3$, and the penguin contributions $C_{3,6}$ are neglected. As is well known, this "naive" factorization method suffers from a serious problem related to scale $\mu$ and regularization-scheme dependences of the Wilson coefficients $C_i(\mu)$, while the amplitudes $H_\lambda$ are not. The inclusion of the $a_s$ corrections, as will be shown in (32)-(35), cures this problem.

For $B \rightarrow J/\Psi + K^*$ decay, we remark that the original $V-A$ left-handed property of the current $\pi \gamma_\mu (1 - \gamma_5) b$ in $O_4$ is reflected by the expression (18) of the helicity amplitudes $H_{\pm 1} \sim \pm c V (q^2) - A_1 (q^2)$. This may be understood as following: The $-1 / 2$ helicity left-handed $s$ quark, emitted through the $V-A$ current, picks up the spectator antiquark $\bar{q}$ (which has both $\pm 1 / 2$ helicities) to form the $K^*$ meson. Thus the latter can only have $-1$ or $0$ helicity and not $+1$, since the $s$ quark would maintain its $-1 / 2$ helicity unless final state interactions (FSI) are strong enough to flip it into a $+1 / 2$ state. Therefore $|H_{-1}| \sim |cy + 1|$ would largely dominate $|H_{+1}| \sim |cy - 1|$ unless very strong FSI would reverse the order by making $|\tilde{a}_+| \gg |\tilde{a}_-|$. As we will see in section 2-5, this possibility is not supported by our calculations within the QCD-improved factorization approach. However the answer as always must come from the experimental side: whether or not $|H_{-1}| \gg |H_{+1}|$ can only be settled by future measurements of the muon polarization in $B \rightarrow J/\Psi + K^*$ follows by $J/\Psi \rightarrow \mu^+ + \mu^-$.

The decay rate is obtained using (4) and (18) from which we get the normalized longitudinal $A_0$ and transverse $A_\parallel, A_\perp$ fractions measured by Argus[3], CLEO[5], CDF[6] and BaBar[5] collaborations

$$|A_0|^2 = \frac{|\tilde{a}_0|^2 (a - bx)^2}{\Sigma},$$

(19)
\[ |A_\parallel|^2 = \left| \frac{\bar{a}_+ + \bar{a}_-}{2} \right|^2 c^2 y^2 |\bar{a}_+ - \bar{a}_-|^2 - 2cy \left( |\bar{a}_+|^2 - |\bar{a}_-|^2 \right) \] (20)

\[ |A_\perp|^2 = \left| \frac{\bar{a}_+ + \bar{a}_-}{2} \right|^2 c^2 y^2 + \left( |\bar{a}_+|^2 + |\bar{a}_-|^2 \right) (1 + c^2 y^2) - 2cy \left( |\bar{a}_+|^2 - |\bar{a}_-|^2 \right) \] (21)

where

\[ \Sigma = |\bar{a}_0|^2 (a - bx)^2 + \left( |\bar{a}_+|^2 + |\bar{a}_-|^2 \right) (1 + c^2 y^2) - 2cy \left( |\bar{a}_+|^2 - |\bar{a}_-|^2 \right) . \]

The phases \( \delta_\parallel, \delta_\perp, \delta_\perp \) of the \( A_\parallel, A_\parallel, A_\perp \) are given by those of the \( \bar{a}_\lambda \) since \( \kappa, a, b, c, x, y \) are real.

Provided that \( |\bar{a}_-|^2 \geq |\bar{a}_+|^2 \) which is true from the first line of (40), and for nonzero \( A_1(q^2) \), a remarkable upper bound for the longitudinal fraction \( |A_0|^2 \) can be derived using (19), no matter how are the finite form factors \( A_1(q^2), A_2(q^2) \leq (a/b)A_1(q^2) = 2.42A_1(q^2) \) and \( V(q^2) \):

\[ |A_0|^2 \leq \frac{a^2}{a^2 + \rho} \; , \; \rho = \frac{|\bar{a}_+|^2 + |\bar{a}_-|^2}{|\bar{a}_0|^2} , \] (22)

the derivation of this upper bound can be easily obtained by considering the lower bound of the inverse \( 1/|A_0|^2 \). Of course when all of the three \( \bar{a}_\lambda \) are real and identical, we recover our old result[7] of the naive factorization method, as it should be:

\[ |A_0|^2 = \frac{(a - bx)^2}{(a - bx)^2 + 2(1 + c^2 y^2)} \leq \frac{a^2}{a^2 + 2} = 0.83 \; , \] (23)

\[ |A_\parallel|^2 = \frac{2}{(a - bx)^2 + 2(1 + c^2 y^2)} \] (24)

\[ |A_\perp|^2 = \frac{2c^2 y^2}{(a - bx)^2 + 2(1 + c^2 y^2)} \] (25)

The latest experimental data for \( A_0, A_\parallel, A_\perp \) are[4][5]

\[ |A_0|^2 = 0.597 \pm 0.028 \pm 0.008 \; , \; |A_\parallel|^2 = 0.160 \pm 0.032 \pm 0.036 \; , \]

\[ |A_\parallel|^2 = 1 - |A_0|^2 - |A_\perp|^2 = 0.243 \pm 0.034 \pm 0.033 \; . \] (26)

From the experimental value of \( |A_0|^2 = 0.6 \), we derive a constraint on the ratio \( \rho \) using (22),

\[ \frac{|\bar{a}_+|^2 + |\bar{a}_-|^2}{|\bar{a}_0|^2} \leq \frac{2}{3} a^2 = 6.6 \]

which is however far from being saturated (section 2-5 below).

For the phases \( \delta_\parallel, \delta_\perp \), since measurements of the interference terms in the angular distributions are limited to \( \text{Re}(A_\parallel A_0^*), \text{Im}(A_\parallel A_0^*) \) and \( \text{Im}(A_\perp A_0^*) \), there exists a two-fold ambiguity[10][12]

\[ \delta_\parallel \leftrightarrow -\delta_\parallel, \delta_\perp \leftrightarrow \pi - \delta_\perp, \delta_\perp - \delta_\parallel \leftrightarrow \pi - (\delta_\perp - \delta_\parallel) . \]

The phases quoted in radians are[5][10]

\[ \delta_\perp \equiv \text{arg}(A_\perp A_0^*) = -0.17 \pm 0.16 \pm 0.06 \; (-2.97 \pm 0.16 \pm 0.07) \; . \]
\[
\delta_\parallel \equiv \arg(A_\parallel A_\parallel^*) = 2.50 \pm 0.20 \pm 0.07 \left(-2.50 \pm 0.20 \pm 0.07\right), \tag{27}
\]

the numbers in parentheses correspond to the second solution due to the mentioned ambiguity. Consequently, from (26)-(27), one deduces\([13]\) either \(|H_{+1}/H_{-1}| = 0.26 \pm 0.14\) or \(|H_{-1}/H_{+1}| = 0.26 \pm 0.14\), this ambiguity can be solved in the future by the \(J/\Psi \to \ell^+ + \ell^-\) lepton polarization measurements.

The most important information we can draw from the measured nonzero phase of \(\delta_\parallel\) is that nonfactorizable corrections to the “naive” factorization method must be taken into account.

2-4 Nonfactorizable Corrections

In the QCD-improved factorization approach, the light-cone distribution amplitudes play a central role. For vector mesons, the LCDA are given by\([23, 24]\)

\[
\langle V(p, \epsilon) | \bar{\psi}_i(x) q_j(x) | 0 \rangle = \frac{1}{4} \int_0^1 d\eta \ e^{i(p' \cdot y + (1-\eta) p' \cdot x)} \left( f_M V \left[ \phi_\parallel (\eta) + \phi_\perp \phi_\perp (\eta) \right]_{ij} + F_T^V \left( \phi_\perp \phi_\perp (\eta) \right) + \left[ 1 - \frac{2m_q F_T^V}{f_M V} \right] \epsilon_{\mu\nu\alpha\beta} (\gamma^\mu \gamma_5)_{ij} e^{i\epsilon^\nu p^\alpha z^\beta} \phi_\parallel (\eta) \right) \tag{28}
\]

where \(i, j\) denote the Dirac spinor indices, \(z = y - x, \epsilon_\parallel, \epsilon_\perp\) are the longitudinal (transverse) polarizations of the vector meson and \(f_M, F_T^V\) are respectively its vector and tensor decay constants defined by

\[
\langle V(p, \epsilon) | \bar{\psi}_i(x) \gamma_\mu q(0) | 0 \rangle = f_M V \int_0^1 d\eta \ e^{i(p' \cdot y + (1-\eta) p' \cdot x)} \phi_\parallel (\eta), \tag{29}
\]

\[
\langle V(p, \epsilon) | \bar{\psi}_i(x) \gamma_\mu \gamma_\gamma q(0) | 0 \rangle = -i F_T^V (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu) \int_0^1 d\eta \ e^{i(p' \cdot y + (1-\eta) p' \cdot x)} \phi_\parallel (\eta). \tag{30}
\]

Contracting the above equation \(\langle V(p, \epsilon) | \bar{\psi}_i(x) \gamma_\mu \gamma_\gamma q(0) | 0 \rangle\) with \(p_\nu'\) and applying the equation of motion together with the definition \(\langle V(p, \epsilon) | \bar{\psi}_i(x) \gamma_\mu q(0) | 0 \rangle = f_M V \epsilon_\mu,\) a relation is obtained between the \(f_M, F_T^V\) decay constants\([18, 19]\):

\[
F_T^V \frac{M_V}{2 f_M M_q} = 2 f_T^V M_V = 2 \left( \frac{m_q}{M_V} \right)^2 = 2 \eta^2, \tag{31}
\]

where in the last step the on-shell relation \(\eta^2 = p^2 = m_q^2\) has been applied\([18]\).

Finally \(\phi_\parallel (\eta), \phi_\perp (\eta)\) are the leading twist-2 LCDA amplitudes while the nonleading vector-like twist-3 LCDA is denoted by \(\phi_\perp (\eta)\). The axial-like twist-3 LCDA \(\phi_\parallel (\eta)\) contribution is negligible since it is proportional to \(f_M V - 2 F_T^V M_q = M_V^2 - 4m_q^2 \approx 0\). We thus introduce expressions \(\phi_{\parallel, \perp}^K(\eta), \phi_{\parallel, \perp}^{K+\gamma}(\eta)\) and \(\phi_{\parallel, \perp}^{K+\gamma}(\eta)\) for the vector meson \(K^*\).

For the pseudoscalar B meson, its wave function in the heavy b quark limit is

\[
\langle B(P) | \bar{b}_i(x) q_j(x) | 0 \rangle \big|_{x_+ = x_- = 0} = \frac{i F_B}{4} \int_0^1 d\xi \ e^{i P' \cdot \xi - \xi P \cdot M} \left( (P' + M) \gamma_5 \Phi_B (\xi) + \cdots \right)_{ij}, \tag{31}
\]

where \(\cdots\) denote terms that do not contribute to the decay amplitude calculated later in (39)-(40).

For the vector meson \(J/\Psi\) treated as a heavy charmonium, to the leading order in \(1/m_c\), its wave function has a similar expression:

\[
\langle J/\Psi(p, \epsilon) | \bar{\psi}_i(x) c_j(x) | 0 \rangle \big|_{x_+ = x_- = 0} = \frac{f_{\Psi}}{4} \left[ (p_\mu + M_\Psi) \right]_{jk} \int_0^1 d\mu \ e^{i P_\cdot \mu - \mu P_\cdot M} \left[ \Phi_\Psi (\mu) + \cdots \right]_{ij}. \tag{32}
\]
We note that the use of the light-cone wave function for the heavy J/Ψ is problematic, higher twist effects have to be included, however they may not converge fast enough. We adopt in the following \( \Phi_\parallel(u) \) as the distribution amplitude (DA) of the nonlocal vector current of J/Ψ, thus we treat the J/Ψ wave function on the same footing as the B meson. Comparing the above equation with (28), we see that at the leading order in \( 1/m_c \) one has for the heavy charmonium J/Ψ :

\[
\Phi_\parallel(u) = \Phi_\perp(u) = \Phi_\psi(u) \quad \text{and} \quad F_\psi^T = f_\psi .
\]

Equipped with these ingredients, we are ready to compute the nonfactorizable correction terms. Loop integrations of the four vertex corrections diagrams (Fig. 6 in [8]) which gives \( F_\lambda^I \) and the two spectator diagrams (Fig. 8 in [8]) which gives \( F_\lambda^{11} \) are not detailed here. Only we give the results in the NDR scheme:

\[
\bar{a}_\lambda = (a_2^{\lambda} + a_3^{\lambda} + a_5^{\lambda}) ,
\]

\[
a_2^{\lambda} = C_2 + \frac{C_1}{N_c} + \frac{\alpha_s N_c^2 - 1}{4\pi 2 N_c^2} C_1 \left( -18 + 12 \ln \frac{m_b}{\mu} + F_\lambda^I + F_\lambda^{11} \right) ,
\]

\[
a_3^{\lambda} = C_3 + \frac{C_4}{N_c} + \frac{\alpha_s N_c^2 - 1}{4\pi 2 N_c^2} C_4 \left( -18 + 12 \ln \frac{m_b}{\mu} + F_\lambda^I + F_\lambda^{11} \right) ,
\]

\[
a_5^{\lambda} = C_5 + \frac{C_6}{N_c} - \frac{\alpha_s N_c^2 - 1}{4\pi 2 N_c^2} C_6 \left( -6 + 12 \ln \frac{m_b}{\mu} + F_\lambda^I + F_\lambda^{11} \right) ,
\]

with \( N_c = 3 \) is the color number. For completeness, the constants \(-18, -18, -6 \) in (33)-(35) become respectively \(-14, -14, -18 \) in the HV scheme. These constants and the \( \ln(m_b/\mu) \) term inside the parentheses of (33)-(35) reflect the scale and regularization-scheme dependences, they are cancelled by those of the Wilson coefficients \( C_i(\mu) \), therefore the final expressions of \( \bar{a}_\lambda \) are \( \mu \) and regularization-scheme independent. The \( F_\lambda^I \) and \( F_\lambda^{11} \) are calculated to be:

\[
F_\lambda^0 = \int_0^1 \! du \left[ \Phi_\parallel(u) K(r, u) + \Phi_\perp(u) \mathcal{H}(r, u) \right]
\]

where

\[
K(r, u) = 3 \left[ \ln(1 - r) - i\pi \right] + \frac{3(1 - 2u) \ln u}{1 - u} + \frac{2r(1 - u)}{1 - ru}
\]

\[
+ \left[ \frac{1 - u}{(1 - ru)^2} - \frac{u}{[1 - r(1 - u)]^2} \right] r^2 u \ln(ru) + \frac{r^2 u^2 \left[ \ln(1 - r) - i\pi \right]}{[1 - r(1 - u)]^2} ,
\]

and

\[
\mathcal{H}(r, u) = 8r u^2 \left\{ \left[ \frac{1}{1 - r(1 - u)} - \frac{1}{1 - ru} \right] \ln(ru) - \frac{\ln(1 - r) - i\pi}{1 - r(1 - u)} \right\} ,
\]

with \( r = m_\psi^2/M^2 \). Also

\[
F_\lambda^\pm = \int_0^1 \! du \, \varphi_\pm^\lambda (u) K(r, u)
\]

\[
+ 8 \int_0^1 \! du \, \Phi_\perp(u) u^2 \left\{ -\frac{\ln u}{1 - u} + \frac{r \ln(ru)}{1 - r(1 - u)} - r \frac{\ln(1 - r) - i\pi}{1 - r(1 - u)} \right\}
\]
Similarly to the $B \to J/\Psi + K$ case found in \cite{18,19}, the infrared divergences in $F_I^\lambda$ are mutually cancelled among the four vertex diagrams, this cancellation is essentially due to the symmetric $u \leftrightarrow 1 - u$ of the kernels $I(r,u), \ H(r,u)$ and the wave functions. We note that in (38), the $J/\Psi$ nonleading $\varphi^\perp_\lambda(u)$ wave function also contributes to the transverse $F_I^\perp$ on the same footing as the leading $\Phi^\perp_\lambda(u)$.

For the $F^\perp_{II}$ of the spectator-quark effect, following\cite{20} we get

$$F_{II}^0 = \frac{4\pi^2}{N_c} \frac{f_B f_{K^*}}{m_{\Phi} (M + m_{K^*}) A_1 (m_{\Phi}^2)(a - bx)} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^{\infty} d\eta \frac{\Phi^*_\perp_{\lambda}(\eta)}{\eta} \int_0^1 \frac{d\Phi_{\perp}(u)}{u},$$

(39)

$$F_{II}^\perp = \frac{16\pi^2}{N_c} \frac{f_B f_{J/\Psi} (1 + \epsilon)}{M (M + m_{K^*}) A_1 (m_{\Phi}^2)(1 + cy)} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^{\infty} d\eta \frac{\Phi^*_\perp_{\lambda}(\eta)}{\eta^2} \int_0^1 \frac{d\varphi^\perp_\lambda(u)}{u}$$

$$- \frac{8\pi^2}{N_c} \frac{f_B f_{K^*} m_{K^*}}{M^2 (M + m_{K^*}) A_1 (m_{\Phi}^2)(1 + cy)} \int_0^1 d\xi \frac{\Phi_B(\xi)}{\xi} \int_0^{\infty} d\eta d\nu \varphi^\perp_{\lambda}(u) \eta + u$$

$$\pm \frac{1}{4} \varphi^\perp_{\lambda}(u) \eta + u \frac{1}{\eta^2 u^2} + \frac{1}{4} \varphi^\perp_{\perp}(u) \eta + 2u \frac{1}{\eta^2 u^2}.$$  

(40)

We first emphasize that the infrared-finite longitudinal $F^0_{II}$ in (39) is not $1/M$ power-suppressed contrary to its appearance, since the $B$ meson wave function $\Phi_B(\xi)$ is appreciable only for $\xi$ of the order $\Lambda_{QCD}/M$, hence the integral $\int d\xi \Phi_B(\xi)/\xi \sim M/\Lambda_{QCD}$ compensates the $f_{K^*}/(M + m_{K^*})$ in (39).

As for the transverse $F_{II}^\pm$ parts given in (40), they are infrared divergent although $1/M$ and $1/M^2$ power-suppressed. Indeed, the first term in $F_{II}^\pm \sim \int \Phi^\perp_{\lambda}(\eta)/\eta^2$ unexpectedly diverges even with the $K^*$ leading twist-2 LDCA $\Phi^\perp_{\lambda}(\eta)$, the remaining divergent terms come from the twist-3 LDCA of both the $K^*$ and $J/\Psi$ mesons. We have neglected the $r$ dependences in $F_{II}^\pm$ to simplify the computations of complicated loop integrals, since infrared divergences occur no matter the $r$ dependences are kept or not.

In the numerical applications, we use for the leading twist-2 LDCA their asymptotic form $\Phi_{\parallel}(x) = \Phi(1 - x) = 6x(1 - x)$ for both $J/\Psi$ and $K^*$ mesons, although the former is treated as heavy. The twist-3 LDCA of the $K^*$ vector meson are taken to be $\varphi^\perp_\perp(x) = 3[1 + (2x - 1)^2]/4$ and $\varphi^\perp_\perp(x) = 6x(1 - x)$. For the $B$ meson wave function, we take $\Phi_B(\xi) = N_B \xi^2 (1 - \xi^2) \exp[-\xi^2 m^2/2\omega^2]$ with $\omega = 0.25$ GeV, and $N_B$ is the normalization factor such that $\int_0^1 d\xi \Phi_B(\xi) = 1$. With this $\Phi_B(\xi)$, we get $\int_0^1 d\xi \Phi_B(\xi)/\xi = M/(0.3\text{GeV})$, in agreement with our guess $\int d\xi \Phi_B(\xi)/\xi \sim M/\Lambda_{QCD}$ mentioned above.

Remarks

From (36)-(40) we draw some unexpected features of the nonfactorizable terms in $B \to J/\Psi + K^*$ which are distinctive from $B \to J/\Psi + K$:

1- The chirally-enhanced factor $m^2_{K^*}/(m_s + m_d)$ inherent to the pseudoscalar $K$ meson in $B \to J/\Psi + K$ is absent in $B \to J/\Psi + K^*$ with the vector $K^*$ meson. Therefore, neither $F^\lambda_I$ nor $F^\perp_{II}$ are chirally-enhanced in both the longitudinal and transverse parts.
2- Both the $1/m_c$ leading and nonleading wave functions of the charmonium $J/\Psi$ contribute to the transverse $F_{I}^{\pm}$ and $F_{II}^{\pm}$, moreover $F_{I}^{\pm}$ is infrared-finite with the nonleading $\varphi_{\perp}(u)$ as shown in the first line of (38). The longitudinal $F_{I}^{0}$ and $F_{II}^{0}$ are also infrared-finite and not power-suppressed.

3- At the first order $\Lambda_{QCD}/M$ level, $F_{II}^{0}$ vanishes, only survives the infrared-divergent $F_{I}^{0}$. Unexpectedly, the infrared divergence of $F_{II}^{0}$ already comes from the leading twist-2 LCDA of the $K^*$, its twist-3 is unnecessary to render $F_{II}^{0}$ divergent. At the second order $\Lambda_{QCD}/M^2$, the twist-3 $\varphi_{\perp}^K(\eta)$ and $\varphi_{\perp}(u)$ of both $K^*$ and $J/\Psi$ respectively make $F_{II}^{0}$ infrared divergent.

4- Fortunately, the longitudinal part $\bar{a}_0$ as given by (36) and (39) is infrared convergent, therefore it can be unambiguously used in the following section to check whether or not agreement exists between theoretical calculations and experimental data.

### 2.5 Numerical Results

In (39) we need the decay constant $f_{K^*}$ of the vector meson $K^*$, for that we may use the $\tau$ lepton decay $\tau \to \nu_\tau + K^*$ width given by

$$
\Gamma(\tau \to \nu_\tau + K^*) = \frac{G_F^2 |V_{us}|^2 f_{K^*}^2 M_\tau^3}{8\pi} \left(1 + \frac{2m_{K^*}^2}{M_\tau^2}\right) \left(1 - \frac{m_{K^*}^2}{M_\tau^2}\right)^2,
$$

to extract $f_{K^*} \approx 210$ MeV, a value consistent with $m_{K^*} f_{K^*} = m_\rho f_\rho$ of the SU(3) flavor symmetry which relates $f_{K^*}$ to the decay constant $f_\rho \approx 198$ MeV of the charged $\rho(770)$ vector meson. We also take $f_\Psi \approx 405$ MeV and $f_B \approx 180$ MeV.

Numerical values of the infrared-finite quantities $F_{I}^{0}$ in (36)-(37) and $F_{II}^{0}$ in (39) are

$$
F_{I}^{0} = -0.82 - i 6.61, \quad F_{II}^{0} = \frac{4.72}{A_1(m^2_\Psi)(a - bx)}.
$$

These numerical values may vary within 20 per-cent when different LCDA are used instead of their asymptotic forms. Since both $F_{I}^{0}$ and $F_{II}^{0}$ are finite, we first concentrate on the longitudinal part

$$
\Gamma_L(B \to J/\Psi + K^*) = |A_0|^2 \frac{Br(B \to J/\Psi + K^*)}{\tau_B},
$$

where $Br(B \to J/\Psi + K^*)$ is the branching ratio $= (1.35 \pm 0.18) \times 10^{-3}$ for the neutral and $(1.47 \pm 0.27) \times 10^{-3}$ for the charged $B$ meson and $\tau_B$ is their respective lifetime $[1.56(1.65) \pm 0.04] \times 10^{-12}$ s. We obtain on average $\Gamma_L(B \to J/\Psi + K^*) = (3.37 \pm 0.4) \times 10^{-10}$ GeV, using $|A_0|^2 = 0.6$. When this experimental value is compared with the theoretical expression

$$
\Gamma_L(B \to J/\Psi + K^*) = \frac{K_c}{8\pi M^2} \kappa^2 (a - bx)^2 |\bar{a}_0|^2,
$$

we find that the product $(a - bx) A_1(m^2_\Psi) |\bar{a}_0|$ is constraint to equal $0.156 \pm 0.02$, thus

$$
(a - bx) A_1(m^2_\Psi) |\bar{a}_0| = 0.156 \pm 0.02.
$$

Our formulae (33)-(39) with $\alpha_s(m_b) = 0.23$ give $|\bar{a}_0| \approx 0.14$.

This value of $|\bar{a}_0|$ in turn can easily satisfy the constraint (42), and we get a domain for $x, A_1(m^2_\Psi)$ plotted by a hyperbolic curve and translated into the following numerical values:

$$
0 < x \leq 1.1 \quad \text{and} \quad 0.35 < A_1(m^2_\Psi) \leq 0.60.
$$

The smallest $x$ is associated with the smallest $A_1(m^2_\Psi)$, the latter increases with increasing $x$. For the transverse $\bar{a}_\perp$ which cannot be reliably calculable because of their infrared divergences, we reverse the naive factorization procedure previously proposed[1] in which (23) was used to determine $x, y$.

Now we fix $x \approx 1.1$ and $|\bar{a}_0| = 0.14$ then using the theoretical expressions (19)-(21) matched with
the experimental data (26)-(27), we determine in turn $\tilde{a}_\pm$ and $y$. The resulting contour solutions $|\tilde{a}_+| \approx 0.095 \pm 0.02$, $|\tilde{a}_-| \approx 0.125 \pm 0.02$ confirm the polarization-dependence of $\tilde{a}_\lambda$. Also we get $y \approx 1.75$.

We remark that our favoured values $x \equiv A_2(m_\Psi^2)/A_1(m_\Psi^2) \leq 1.1$ and $y \equiv V(m_\Psi^2)/A_1(m_\Psi^2) \approx 1.75$ are generally satisfied by some models of form factors studied in the literature\cite{23,24,27,28,29}. It is amusing to note however that $x \leq 1.1$ is at variance with the B-K$^*$ form-factor ratio derived below from equation of motion for on-shell massless strange quark. Indeed using

$$p^\mu \pi_\gamma \gamma_5 b = m_s \pi_\gamma \gamma_5 b = 0,$$

then we obtain a relation between the two form factors $A_1(q^2)$ and $A_2(q^2)$. Assuming

$$p^\mu \left< \bar{K}^*(e_1, p) \mid \pi_\gamma \gamma_5 b \mid \bar{B}(P) \right> = p^\mu A_\mu = 0,$$

which gives

$$A_1(q^2) = \frac{\lambda(M^2, m_s^2, q^2)}{(M + m)^2(M^2 - m_s^2 - q^2)} A_2(q^2) + \frac{2m}{M + m} A_0(q^2),$$

we get

$$\frac{A_2(q^2)}{A_1(q^2)} = \frac{(M + m)^2(M^2 - m_s^2 - q^2)}{\lambda(M^2, m_s^2, q^2)} \left[ 1 - \frac{2m}{M + m} \frac{A_0(q^2)}{A_1(q^2)} \right].$$

(45)

From (45), we recover the well-known relation at $q^2 = 0$

$$\frac{M + m}{2m} A_1(0) - \frac{M - m}{2m} A_2(0) = A_0(0).$$

Neglecting $m_s^2$ with respect to $M^2$ and $q^2 = m_\Psi^2$, then we get from (45)

$$x \equiv \frac{A_2(m_\Psi^2)}{A_1(m_\Psi^2)} = \frac{M^2}{M^2 - m_\Psi^2} = 1.52,$$

this too large value indicates that $m_s = 0$ in (44) may not be a good approximation.

Although the relation (45) is derived here with the assumption $m_s = 0$, we remark nevertheless that it bears some similarity with the one\cite{28} derived in the very different context of the large recoil energy $q^2$ limit for which $m$ should be neglected:

$$\frac{A_2(q^2)}{A_1(q^2)} = \frac{(M + m)^2}{M^2 - m_s^2 - q^2} \left[ 1 - \frac{2Mm}{M^2 - m_s^2 - q^2} \frac{\xi}{\xi} \right] \Rightarrow \frac{M^2}{M^2 - q^2}.$$

This is only this large $q^2$ recoil energy limit that can justify the above result of\cite{28} and not the $m_s = 0$ assumed here in (44).

3 Conclusion

We have examined within the QCD-improved factorization approach different aspects of the color-suppressed B decay into two vector mesons $B \rightarrow J/\Psi + K^*$ for which important experimental results are recently obtained\cite{3,4}. The nonzero phases of the helicity amplitudes measured for the first time by\cite{3,4} indicate that nonfactorizable terms must be taken into account. We emphasize that the phases can only be determined in $B \rightarrow V + V$ decay, hence its superiority over the $B \rightarrow V + P$ and $B \rightarrow P + P$ in this aspect. Our calculations (36)-(39) give nonzero imaginary part to the process-dependent and polarization-dependent coefficients $\tilde{a}_\lambda(\Psi K^*)$ that substitute the conventional process-independent
In summary, our results show that the spectator effects and final state interactions reflected by theoretical expressions (19)-(21), we determine in turn $0 \leq x \leq 0.074$, thus corrections – mainly due to the spectator-quark effect $F_{II}^0$ in (38) – are large but under control. This $|\tilde{a}_0(\Psi K^*)|$ is also different from the one in $B \to J/\Psi + K$ case for which experimental data indicate that $|\tilde{a}_0(\Psi K^*)| \approx 0.25$, thus confirming their process-dependence.

On the other hand, our calculations show that the transverse part $\tilde{a}_\pm(\Psi K^*)$ is infrared divergent (although power-suppressed), this infrared divergence may be handled by a cutoff procedure. From remarks in 2.4, we note an important difference between $B \to J/\Psi + K$ and $B \to J/\Psi + K^*$ in their nonfactorizable terms. In the former case, the discrepancy by a factor of three between experimental data and theoretical estimates using QCD-improved factorization approach. We may cure the infrared divergence via $X$ is constraint to make the divergent $\tilde{a}_\pm(\Psi K^*)$ be smaller than the convergent $\tilde{a}_0(\Psi K^*)$, which is somewhat disturbing.

Therefore we are inclined to believe that in our case of $B \to J/\Psi + K^*$, the procedure used to handle the infrared divergence via $X$ may be adequate for the treatment of the discrepancy (if any) between experimental data and theoretical estimates using QCD-improved factorization approach. We may seek the remedy outside the $\tilde{a}_\lambda$, probably in the form factor $A_1(m_\Psi^2)$ and in the ratios $x = A_2/A_1$ and $y = V/A_1$, since the overall factor $A_1(m_\Psi^2)$ in (15) is central to the absolute strength of the decay rate $B \to J/\Psi + K^*$, as well the ratio $x$ is central to the longitudinal part $A_0$ and $y$ to the transverse fraction $A_\perp$. To deal with the transverse part $\tilde{a}_\pm(\Psi K^*)$, we adopt a pragmatic method by fixing $x$ and $|\tilde{a}_0(\Psi K^*)|$ previously obtained from $|A_0|$, then using data on $|A_\parallel|$ and $|A_\perp|$ together with their theoretical expressions (19)-(21), we determine in turn $|\tilde{a}_+(\Psi K^*)| \approx 0.095 \pm 0.02$, $|\tilde{a}_-(\Psi K^*)| \approx 0.125 \pm 0.02$. Moreover the ratio $y \equiv V(m_\Psi^2)/A_1(m_\Psi^2)$ is also bounded around 1.75. The constraints $x \leq 1.1$ and $y \approx 1.75$ have implications on models of B-K* form factors commonly used in the literature.

In summary, our results show that the spectator effects and final state interactions reflected by $F_{II}^0$ play an important role in our quantitative understanding of the color-suppressed $B \to J/\Psi + K^*$ decay for the dominant longitudinal mode. However the power $\Delta_{QCD}/m_b$ corrections within the QCD-improved factorization approach has to be better understood.

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