The application of mathematical modeling for the development of devices as an example of viscous fluid purification from magnetic impurity

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Abstract. The mathematical model of a coagulation phenomenon and a coagulation process of ferromagnetic particles under the influence of a constant magnetic field are given in the article. The principles that need to be followed during mathematical modeling are stated.

1. Introduction
Methods of technologies and hardware with application of classical modeling techniques are well-known. They are logical, geometrical, physical and mathematical ones, the basis of which is presented by the fundamental laws of natural sciences.

All the above mentioned facts, in conjunction with a qualified use of contemporary computing machinery, allow us to introduce leading technologies and corresponding equipment into various branches of human activity: power engineering, metallurgy, mechanical engineering, chemistry, transport, etc.

Colligating and analyzing the existing modelling techniques, facilities of computing machinery as well as training of specialists and a highly qualified staff in higher education establishments, it is possible to establish the fact that the use of previous experience in the area of equipment modelling, mechanism, and manufacturing processes is still not enough.

2. Statement of the problem
Intuition and the downsized versions of physical and mathematical modeling techniques are used now in most cases for solution of scientific and technical tasks of hardware creation that, in our opinion, significantly affects the quality of the equipment being developed. Complexity of machines, mechanisms, devices is well-known: the significant means and amount of time are spent for their creation, and manufacturing lead time and implementation last for many years. Therefore, another approach to creation of technical means is necessary, namely – engaging the modern complex modeling techniques, based on comprehension of substance processes.

The main component of such methods of hardware development is the mathematical model created on the basis of classical laws of natural sciences and iteratively tested regularities. As a rule, the mathematical models have to be formulated in the form of differential equations that allows investigating various modes of a prototype system, including transient processes.
The objective of work is a demonstration of use of mathematical modeling to coagulation process of particles in magnetic field.

3. Materials and methods
Mathematical modeling of the physical phenomenon, created on basic physical laws of natural sciences was used as a method. The process of the numerical solution was conducted with use of a MathCAD package. Input data and initial data are represented in the article. The pictures of behavior of globular steel particles in magnetic field were taken as experimental verification of coagulation. The particles were placed into the transparent flask filled with glycerin. Field strength changed by means of increasing of current in an electromagnet as follows: $H_1=0.8\cdot10^4$ A/m; $H_2=1.6\cdot10^4$ A/m; $H_3=2.4\cdot10^4$ A/m; $H_4=3.2\cdot10^4$ A/m; $H_5=4.0\cdot10^4$ A/m.

4. Results and discussion
The authors’ experience gained in mathematical modeling of processes and devices of various applications [1-3] is useful if fragments of hardware creation for granular and dispersive media (cleaning of magnetic impurity) are given as an example.

The analysis of theory bases, calculation methods and design of purifiers which have different operating principles, designed for extraction of mechanical impurities from viscous fluids, showed prospects of application of force field of electrical nature which raise a rate of gases purification and liquids purification [4].

On the basis of the above-stated information, during modernization of technologies and devices for purifying medium, it is necessary to observe the following principles:

- a maximum application of the positive characteristics of the existing designs (high performance of work, manufacturability, reliability and simplicity of operation);

- an application of auxiliary energy in the form of force field of electrical nature of the apparatus being designed, should be carried out so that the influence upon particles promoted increase in effectiveness of their extraction from the medium violating the flow patterns insignificantly. Purifiers with the field (electric, magnetic, ultrasonic or combined) can be divided into coagulators, sumps, filters, inertial devices and separators, in which particles of impurity interact with each other under certain conditions, forming flocules, and with the corresponding electrodes (poles or other settling surfaces, which finally leads to an increase of purification efficiency and reliability of mechanisms) [5-6]. These devices are shown in figure 1.

The basic forces influencing ferromagnetic particles are attractive forces (1), the magnetic force (2), Coulomb force for magnetic particles (3), the centrifugal force (4) and the drag force (5):

$$F_G = m \cdot g = (\rho_p - \rho) \cdot V \cdot g$$  \hspace{1cm} (1)

$$F_m = \mu_0 \cdot \chi \cdot V \cdot H \cdot \text{grad} H$$  \hspace{1cm} (2)

$$F_p = \frac{4 \cdot \pi \cdot M_1 \cdot M_2}{\mu_0 \cdot \mu \cdot r^2}$$  \hspace{1cm} (3)

$$F_C = \frac{m \cdot U^2}{R}$$  \hspace{1cm} (4)

$$F_D = -3 \cdot \pi \cdot d \cdot \eta \cdot U$$  \hspace{1cm} (5)

where $m$ – the mass of the body, kg; $g = 9.81$ – free-fall acceleration, m/s²; $\rho_p$ – the density of the body, kg/m³; $\rho$ – the density of the medium kg/m³; $V$ – the volume of the body, m³; $\mu_0$ – the magnetic constant, equal to $4 \cdot \pi \cdot 10^{-7}$, H/m; $\chi$ – the magnetic perception of the body, which is dimensionless; $M_1$, $M_2$ – magnetic forces of the particle, m³kg/s² A; $\mu$ – magnetic permeability, H/m; $H$ – the magnetic field strength, A/m; $r$ – the interparticle distance, m; $U$ – the velocity of the body, m/s; $R$ – the radius along which the particle is moving, m; $d$ – the diameter of the particle, m.
Figure 1. Schemes of electromagnetic cleaning devices:
 a) coagulator, b) sump; c) filter; d) separator; e) hydrocyclone

The coagulation process is presented in figure 2.

Figure 2. The coagulation process in the magnetic field \((H_1<H_2<H_3<H_4<H_5)\).

Effectiveness of extraction of mechanical impurities from the granular medium and viscous fluid depends on:
- properties of particles – density \(\rho\), size \(d\), form \(\psi\), their concentration \(C\), permittivity \(\varepsilon\), magnetic susceptibility \(\chi\), energy absorption rate \(\alpha\), etc.;
- key parameters of the granular medium and viscous fluid – density \(\rho_p\), viscosity \(\eta\) and hydrodynamics of the flow;
- characteristics of the force field – strength \(H\) \((E)\), speed of its change - grad \(H\) \((\text{grad} \ E)\), intensity \(J\) and frequency \(f\) of a source of ultrasonic vibrations.

Mathematical modeling of dynamics of particles in clearing devices with force field of electric nature was carried out with the help of the following algorithm:
- setting up of mathematical models;
- computational investigation;
processing and the analysis of results of computational investigation - physical (experimental) simulation for the purpose of specification of the coefficients connecting parameters of particles, the medium and the field;
- correction of initial mathematical models taking into account the results of physical modeling;
- computational reinvestigation or multiple computational investigations of the corrected mathematical model;
- development of the computational methods suitable for designing the defined (desired) construction;
- experimental-industrial control and commercialization.

Setting up mathematical models was carried out with use of the well-known laws and regularities of solid mechanics, liquid mechanics and gas mechanics, electrodynamic mechanics. Other components of the algorithm, in our opinion, require no explanation. We will consider a problem of coagulation of ferromagnetic particles in the magnetic field as an example. Let us assume that the magnetic field in the area of particles arrangement is homogeneous \((dH/dt=0)\), as shown in figure 3 a. The effect of forces on them coincides with the direction of a vector of field strength \(H\). The force of magnetic concentration is equal to zero. Without gravity and buoyancy force, particles will be affected by forces of coagulation \(F_F\) and resistance of medium \(F_D\). It is possible to write down the equations in a differential form according to the second law of dynamics for each of the two particles having mass \(m\):

\[
\begin{cases}
-m_1 \frac{d^2 y_1}{dt^2} = -3 \cdot \pi \cdot d_1 \cdot \eta \cdot U_1 + \frac{\pi^3 \cdot \mu_0 \cdot d_1^2 \cdot d_2^2}{(y_1 + y_2)^2} \cdot \chi^2 \cdot H^2 \\
-m_2 \frac{d^2 y_2}{dt^2} = -3 \cdot \pi \cdot d_2 \cdot \eta \cdot U_2 - \frac{\pi^3 \cdot \mu_0 \cdot d_1^2 \cdot d_2^2}{(y_1 + y_2)^2} \cdot \chi^2 \cdot H^2
\end{cases}
\]  

(6)

Initial conditions:

\[
t = 0; t(0) = 0; \frac{dy_1}{dt}(0) = U_{10}, \frac{dy_2}{dt}(0) = U_{20}; y_1(0) = y_{10}; y_2(0) = y_{20}
\]

(7)

**Figure 3.** Coagulation of particles in the magnetic field: a) direct problem; b) inverse problem

The system of equations (6) with initial conditions (7) is a mathematical model of the process of magnetic coagulation of two particles, which sizes are close to each other in the viscous fluid. Solution (6) allows finding conditions of formation of flocules, if the values of parameters of particles are known, as well as basic data for computation of the magnetic field.
However, as it appears from equation (3), the effect of force $F_F$ is infinite in space, i.e. particles, being in the field, always experience the influence of each other and whereupon they move having, at each instant of time, defined values of path $y_1$ and $y_2$ and speed of $U_1$ and $U_2$. Therefore, it is impossible to solve the system of equations (6) even by means of a computational method if initial conditions are unknown (7).

For the solution of the system of equations (6), we will consider two ferromagnetic particles which are affected by forces of $F_D$ and $F_F$, as shown in figure 3 b. In this case, the physical meaning of the effect of forces is as follows. Floccules consisting of two particles collapses under the influence of $F_F$. Force $F_D$ prevents this. At the start of a motion path at $t = 0$, $F_F$ has the maximal value, $F_D$ is equal to zero, therefore the motion of particles to the opposite sides is accelerated. $F_F$ decreases with an increase of the distance between particles, while the force of $F_D$ increases at the expense of speed increase $U$. At instant $t = t_\nu$, forces $F_F$ and $F_D$ converge to a maximum. $F_F$ and $F_D$ converge to zero while further motion of particles and forces $F_F$, $F_D$ can be neglected at time $t = t_\nu$ on account of their smallness. Thus, during time $0...t_\nu$, particles will diverge from distance $R$. At the same time, dynamics of particles, characterized by the same differential equations (only symbols in front of members change), will be described. The equations, more suitable for computational investigation, will be of the following kind:

$$\begin{align*}
\frac{d^2y_1}{dt^2} &= \frac{18 \cdot \eta \cdot d_1 \cdot \rho}{d_1 \cdot \rho \cdot dt} - \frac{2 \cdot \pi^2 \cdot \mu_0 \cdot d_2^2}{3 \cdot d_1 \cdot \rho} \left(\frac{y_1}{y_1 + y_2}\right)^2; \\
\frac{d^2y_2}{dt^2} &= \frac{18 \cdot \eta \cdot d_2 \cdot \rho}{d_2 \cdot \rho \cdot dt} + \frac{2 \cdot \pi^2 \cdot \mu_0 \cdot d_1^2}{3 \cdot d_2 \cdot \rho} \left(\frac{y_1}{y_1 + y_2}\right)^2;
\end{align*}$$

(8)

Initial conditions: $t=0$; $t(0)=0$; $y_1(0)=y_2(0)=d/2$; $U_1(0)=U_2(0)=0$.

Equations (8) are described in [7] in more detail.

The system of equations (8) allows one to define the main dynamic characteristics of coagulation of particles: speed $U=f(t,R,d,\eta,\chi,H)$ and the distance at which they interact $R=f(t,U)$.

The results of a numerical integration of the system of equations (8), representing a coagulation of particles in the magnetic field are adduced in figure 4: variation of distance $y_1$ and $y_2$ between them, and their speeds $U_1$ and $U_2$ depending upon time: $d=5 \cdot 10^{-6}$ m; $H=4 \cdot 10^4$ A/m; $\chi=29.25 \cdot 10^{-3}$; $\rho=7.8 \cdot 10^3$ kg/m$^3$; $\eta=1.85 \cdot 10^{-3}$ Pa·s.

Figure 4. Paths (a) and speed (b) of particles.
Varying the parameters of particles the parameters of medium and the parameters of field iteratively from solution to solution, the optimal conditions of coagulation of particles of cleaning units (purifiers) are identified.

The data obtained by the method of computational investigation of the inverse problem of coagulation of particles were checked experimentally with the help of a primal problem. The maximal speed, observed at the time of particle collision, differed by no more than 5% of the maximal speed obtained as a result of the solution of the system of equations (8). The computational solution of a direct and inverse problem of coagulation is given in figure 5. Thus, the possibility of using the system of differential equations (8) is decisive.

![Figure 5. Dependence \( U = f(y) \) of the direct and inverse problems of coagulation.](image)

The differential equations for the electric field or ultrasound are being composed and investigated numerically in a similar manner [8-11]. We hope that the above-mentioned examples and the example in mathematical modeling of the phenomena and processes when imposing fields onto dispersive media will be useful for scientists, graduate students and students.

4. Conclusion

Mathematical modeling plays an important role in development and designing of various devices and mechanisms. The inverse problem of coagulation of particles in the magnetic field was considered as an example of successful utilization of modeling in engineering, owing to which, the most optimal conditions of coagulation of particles in purifiers are identified which is impossible for a direct problem.

For example, so that the particles of 5 microns in diameter could interact with each other at the field strength of \(4 \times 10^4\) A/m, their concentration should not be less than \(0.5\) g/m\(^3\).

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