An anti-outliers SRCKF algorithm for fixed single observer passive location

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Abstract. Outliers often appear in measurement noise of single observer passive location system, which has a negative effect on filtering accuracy and stability, and may even lead to filter divergence. Aiming at the effect of outliers, an anti-outliers square-root cubature Kalman filter algorithm is proposed based on Bayes theorem by using the normalized polluted normal model. In this method, the cubature rule is used to calculate the mean value and variance of the nonlinear function, the normalized contaminated normal model is used to deal with the measurement error, and the variance matrix of the measurement prediction residual is adjusted in real time according to the posterior probability of outliers. The simulation results in the fixed single observer passive location model show that the proposed algorithm is robust and can eliminate the adverse effects of discrete or continuous outliers in the measurement noise.

1. Introduction
Single observer passive location system has many advantages, such as strong concealment, relatively independent equipment and long operating distance, and has gradually become a research hotspot in the field of electronic countermeasures[1-2]. Single observer passive location tracking is a typical nonlinear filtering problem, most of existing studies prior to assume the measurement noise as Gaussian random sequences[3-4], but in practical application, due to the measuring device itself, environmental interference or possible errors in the process of data transmission, make the observation sequence contains some errors observed quantity, engineering field called outliers[5]. With single observer passive location system not like radar system can obtain the target distance information, but through Angle, Angle rate and Doppler frequency rate observed quantity to indirect access to the location of the object information and outliers will more serious impact on the system, the precision and stability of the filter will be markedly reduced. The continuous outliers will even lead to the divergence of the filter, so it is an urgent and significant work to study a robust filtering algorithm against outliers in the single station passive location technology.

Square root cubature Kalman filter is a new type of nonlinear filtering algorithm, which uses a set of equal-weight volume points to calculate the posterior probability density[6-7]. It is simple to implement, has strong adaptability and high numerical accuracy. Therefore, it is applied in the field of single station passive positioning. Based on the Bayes principle and the idea of the normalized polluted normal distribution model[8], an outlier-resistant robust volume Kalman filter algorithm is proposed in this paper. Principle of the algorithm using volume numerical integral calculating mean and variance of nonlinear random function, and the measurement error to establish a normalized contaminated Gaussian model, and then according to the posterior probability of outliers occur to adjust the measurement variance matrix of the new rate enhanced algorithm for possible outliers inhibition ability, enhance the stability of the algorithm.
2. Fixed single observer passive location model

When the target radiation source is cruising at a uniform speed, its motion state in the air can usually be approximated to a uniform linear motion. Fig. 1 is the two-dimensional plane geometric relation diagram of passive positioning of a fixed single station. Suppose a fixed single observation station is at the origin, and the position of the target is denoted as \( T(x, y) \); at \( k \) time, its radial distance relative to the observation station \( O \) is \( r_k \), and its azimuth Angle is \( \beta_k \).

Suppose the state vector of the target radiation source at \( k \) time is \( X_k = (x_k, \dot{x}_k, y_k, \dot{y}_k)^T \). Where, \( x_k \) and \( y_k \) is the position variable of the target radiation source in the two-dimensional Cartesian coordinate system; \( \dot{x}_k \) and \( \dot{y}_k \) is the velocity variable of the radiation source, namely, the derivative of \( \dot{x}_k \) and \( \dot{y}_k \). Then, the state equation of the positioning system is:

\[
X_{k+1} = FX_k + GW_k
\]

(1)

Where, the state transfer matrix of the positioning system is:

\[
F = \begin{bmatrix}
1 & T & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & T \\
0 & 0 & 0 & 1
\end{bmatrix},
G = \begin{bmatrix}
0.5T^2 & 0 \\
T & 0 \\
0 & 0.5T^2 \\
0 & T
\end{bmatrix}
\]

is the state noise transfer matrix, and \( W_k \) is the state noise.

The observation equation can be expressed as:

\[
Z_k = h(X_k) + V_k
\]

(2)

Where, \( h(\cdot) \) is the nonlinear transformation function from the state vector to the observed quantity, \( V_k \) represents the measurement noise, which is the Gaussian white noise with zero mean value, and its covariance matrix is \( R_k \).

The spatial and frequency domain information of the target radiation source can be obtained by the fixed single station positioning model in the two-dimensional rectangular coordinate system. In this paper, the target azimuth Angle, angular velocity and Doppler frequency change rate are taken as the observation quantity, and the observation equation using \( \{\beta_k, \dot{\beta_k}, f_{\alpha_k}\} \) information is as follows:
\[ Z_k = \begin{bmatrix} \beta_k \\ \beta_k' \\ f_{\beta k} \end{bmatrix} = \begin{bmatrix} \arctan \left( \frac{y}{x} \right) \\ \frac{xy - x'y'}{x^2 + y^2} \\ -f_\varepsilon \frac{(xy - x'y')^2}{c(x^2 + y^2)^{3/2}} \end{bmatrix} + \begin{bmatrix} v_{\beta k} \\ v_{\beta k}' \\ v_{f_{\beta k}} \end{bmatrix} \] (3)

3. Anti-outliers square-root cubature Kalman filter algorithm

3.1. Scaled-Contaminated Normal Distribution Model

Since there may be outliers in the measured data, it is not appropriate to assume that the statistical characteristics of the measured noise obey the normal distribution. Considering that the probability of outliers appearing is low relative to the whole measurement sequence, by referring to the idea of SCNM model, outliers can be regarded as an error distribution with larger trailing than the normal distribution. In this case, the error distribution in the measurement equation is:

\[ v(k) \sim N\left(0, R(k)\right) \]

\[ V_\varepsilon(k) \sim p_1N\left(0, R_{1,i}\right) + p_2N\left(0, R_{2,i}\right) \] (4)

Where, \( p_1 + p_2 = 1 \), \( p_2 \in (0, 0.15) \) is usually desirable. \( R_{1,i} = \text{diag} \left[ v_{\beta 1}, v_{\beta 1}', v_{f_{\beta 1}} \right] \), \( R_{2,i} = q^2 R_{1,i} \), \( q^2 > 1 \), that is, in any measurement case, \( R_{2,i} > R_{1,i} > 0 \).

Therefore, the core idea of SCNM model is to use two normalized weighted normal distributions to approximate the error distribution of measurement noise that may have outlandises. Since those \( R_{2,i} \) with outliers are more uncertain than conventional measurements \( R_{1,i} \), they are weighted only with small probabilities \( p_2 \).

3.2. Anti-outliers SRCKF algorithm flow

The calculation process of anti-outliers SRCKF algorithm is summarized as follows:

A. Time update

1. Calculate the Cholesky decomposition of covariance matrix:

\[ P_{kp} = S_{kp} S_{\text{Tria}} \] (5)

2. Calculate the volume point and the propagation of the volume point (\( i = 1, 2, \cdots, m \)):

\[ X_{i,\text{kp}} = S_{kp} \hat{s}_{i,\text{kp}} + \hat{x}_{i,\text{kp}} \] (6)

\[ X'_{i,\text{kp}} = F_i X_{i,\text{kp}} \] (7)

3. One-step prediction for calculating the state vector:

\[ \hat{x}_{i,\text{kp}} = \sum_{i=1}^{m} \omega_i X'_{i,\text{kp}} \] (8)

4. Calculate the square root of the prediction covariance matrix:

\[ S_{\text{Tria}} = \text{Tria} \left[ \left[ X'_{1,\text{kp}} \quad S_{\text{Tria}} \right] \right] \] (9)

In the formula, represents the QR decomposition operation of the matrix.

\[ X'_{i,\text{kp}} = \frac{1}{\sqrt{m}} \left[ X'_{1,\text{kp}}, X'_{2,\text{kp}}, \cdots, X'_{n,\text{kp}}, X'_{i,\text{kp}}, \cdots, X'_{m,\text{kp}} \right] \] (10)

\[ Q_{\text{Tria}} = S_{\text{Tria}} S_{\text{Tria}}^T \] (11)

B. Measurement update

1. Calculate the volume point (\( i = 1, 2, \cdots, m \)):
\[ X_{i+k\mid k} = S_{i+k\mid k} \xi_i + \hat{X}_{i+k\mid k} \]  
(12)

(2) Calculate the propagation volume point of the measurement equation:
\[ Z_{i+k\mid k} = h_{i+k\mid k}(X_{i+k\mid k}) \quad i = 1, 2, \ldots, m \]  
(13)

(3) One-step prediction value of calculation measurement:
\[ \hat{Z}_{k\mid k} = \sum_{i=1}^{m} \rho_i Z_{i+k\mid k} \]  
(14)

(4) Calculate the square root of the new interest covariance matrix:
\[ v_{k+1} = Z_{k+1} - \hat{Z}_{k\mid k} \]  
\[ S_{z_{k+k\mid k}} = \text{Tria} \left( \begin{bmatrix} Z_{k\mid k}^T & S_{R_{k+k\mid k}} \end{bmatrix} \right) \]  
(15)

\[ Z_{k\mid k} = \frac{1}{\sqrt{m}} \left[ Z_{1\mid k} - \hat{Z}_{1\mid k}, Z_{2\mid k} - \hat{Z}_{2\mid k}, \ldots, Z_{n\mid k} - \hat{Z}_{n\mid k} \right] \]  
(16)

\[ R_{k+1} = S_{R_{k+k\mid k}} S_{R_{k+k\mid k}}^T \]  
(17)

(5) Calculate the cross-covariance matrix:
\[ P_{z_{k+k\mid k}} = X_{k\mid k} Z_{k\mid k}^T \]  
(18)

\[ X_{k\mid k} = \frac{1}{\sqrt{m}} \left[ X_{1\mid k} - \hat{X}_{1\mid k}, X_{2\mid k} - \hat{X}_{2\mid k}, \ldots, X_{n\mid k} - \hat{X}_{n\mid k} \right] \]  
(19)

(6) Calculate the posterior weighted probability:
\[ p_{k+1,1} = \left\{ 1 + \frac{P_1}{p_1} \left( \frac{D_{k+1,1}}{D_{k+1,2}} \right)^{1/2} \exp \left( -\frac{1}{2} v_{k+1}^T \left( \frac{D_{k+1,1}^{-1}}{D_{k+1,2}^{-1}} \right) v_{k+1} \right) \right\} \]  
\[ p_{k+1,2} = 1 - p_{k+1,1} \]  
(20)

(7) Calculate the gain matrix:
\[ K_{k+1} = P_{z_{k+k\mid k}} \left( p_{k+1,1} D_{k+1,1} + p_{k+1,2} D_{k+1,2} \right) \]  
(21)

(8) Calculate state vector update:
\[ \hat{x}_{k+1\mid k} = \hat{x}_{k\mid k} + K_{k+1}(Z_{k+1\mid k} - \hat{Z}_{k\mid k}) \]  
(22)

(9) Calculation error covariance matrix square root update:
\[ S_{z_{k+k\mid k}} = \text{Tria} \left( \begin{bmatrix} X_{k\mid k}^T & M_{k\mid k} Z_{k+1\mid k}, M_{k\mid k} S_{R_{k+k\mid k}} \end{bmatrix} \right) \]  
(23)

Among them,
\[ M_{k} = p_{k,1} D_{k,1}^{-1} + p_{k,2} D_{k,2}^{-1} - N_{k} \]  
(24)

\[ N_{k} = p_{k,1} p_{k,2} \left( D_{k,1}^{-1} - D_{k,2}^{-1} \right) v_{k} v_{k}^T \left( D_{k,1}^{-1} - D_{k,2}^{-1} \right)^T \]  
(25)

### 4. Simulation and Analysis

In order to verify the effectiveness of the anti-outliers SRCKF algorithm, the simulation experiment is carried out to verify it. Simulation conditions: in a two-dimensional plane, the fixed observation station is located at the origin of coordinates (0, 0), and the target at \((180, 90) km\) moves uniformly in a straight line at the speed of \((\sim 300,100) m/s\). The observation period lasted for 100s. It is assumed that the frequency of the target emitter remains \(10 GHz\) constant during the observation time. The measurement precision of Angle, Angle change rate and Doppler frequency change rate are respectively \(5 mrad\), \(0.5 mrad/s\), \(1 Hz/s\). Parameters of outlier-resistant robust SRCKF algorithm are set as: \(p_1 = 0.9\), \(p_2 = 0.1\), \(R_{k,2} = 25 R_{k,1}\).

Relative Range Error (RRE) is used to describe algorithm performance, and the definition is:
\[ RRE = \frac{\sqrt{(x_k - \hat{x}_k)^2 + (y_k - \hat{y}_k)^2}}{\sqrt{x_k^2 + y_k^2}} \times 100\% \]  
(28)

Where: \( (x_k, y_k) \) and \( (\hat{x}_k, \hat{y}_k) \) are the true value and filtering estimated value of the location of the target radiation source at \( k \) time.

Two groups of simulation experiments were carried out. In the first group, a discrete measurement field of \( \delta_p = 0.3 \text{ rad}, \delta_\phi = 0.01 \text{ rad/s}, \delta_\beta = 20 \text{ Hz/s} \) was added at the 80s; In the second experiment, six measured outliers of \( \delta_p = 0.3 \text{ rad}, \delta_\phi = 0.01 \text{ rad/s}, \delta_\beta = 20 \text{ Hz/s} \) were added continuously at the 80s. Each group was subjected to 100 Monte-Carlo experiments, and the simulation results were shown in Figure 2 and Figure 3.

According to the positioning curve in the above simulation experiment figure, it can be concluded that:

1. When there are discrete outliers in the observed quantity, the positioning accuracy of SRCKF algorithm is seriously affected. The sudden increase of observation information leads to the increase of filtering gain, which makes the estimated result deviate from the true value greatly. However, the anti-outliers SRCKF algorithm can adjust the posterior weighted probability adaptively to overcome the adverse effects of outliers because of establishing a normalized contaminated normal distribution model for the measurement error.

2. When there are continuous outliers in the observed quantity, the divergence of SRCKF filter is caused. This is because the observation information can no longer be updated in real time at this time, and the single-station passive positioning system cannot directly measure and obtain the target distance information like radar, which leads to the divergence of follow-up tracking results. At this time, the anti-outliers SRCKF algorithm still maintains strong stability, and the algorithm can still converge normally.

3. When there is no measurement outlier, the positioning accuracy of anti-outliers SRCKF algorithm is slightly lower than that of SRCKF algorithm, because the anti-outliers SRCKF algorithm is affected by the small probability weighting of the contaminated observation noise. In this case, positioning accuracy and stability are contradictory to each other, so it is worthwhile to exchange a smaller positioning accuracy for the robustness of the algorithm.

5. Conclusion
In the past researches on passive positioning of single station, the existence of outliers is rarely considered. However, the existence of outliers will affect the accuracy and stability of the filter, and even lead to the divergence of the filter. In view of this, based on the Bayes principle, this paper uses
the idea of normalized polluted normal distribution model to improve the SRCKF algorithm, and proposes a robust SRCKF algorithm against outfield values. Through the simulation example, it can be seen that the anti-outliers SRCKF algorithm has strong stability and high estimation accuracy, and can well suppress the adverse shadow noise generated by observation outliers. It should be pointed out that, when there is no outliers, the estimation accuracy of anti-outliers SRCKF algorithm is slightly lower than that of SRCKF algorithm, but it is worthwhile to exchange a small positioning accuracy for algorithm stability.

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