Standard Coupling Unification in SO(10), Hybrid Seesaw Neutrino Mass and Leptogenesis, Dark Matter, and Proton Lifetime Predictions

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Abstract: We discuss gauge coupling unification of $SU(3)_C \times SU(2)_L \times U(1)_Y$ descending directly from non-supersymmetric SO(10) while providing solutions to the three outstanding problems of the standard model: neutrino masses, dark matter, and the baryon asymmetry of the universe. Conservation of matter parity as gauged discrete symmetry for the stability and identification of dark matter in the model calls for high-scale spontaneous symmetry breaking through $126_H$ Higgs representation. This naturally leads to the hybrid seesaw formula for neutrino masses mediated by heavy scalar triplet and right-handed neutrinos. Being quadratic in the Majorana coupling, the seesaw formula predicts two distinct patterns of right-handed neutrino masses, one hierarchical and another not so hierarchical (or compact), when fitted with the neutrino oscillation data. Predictions of the baryon asymmetry via leptogenesis are investigated through the decays of both the patterns of RHν masses. A complete flavor analysis has been carried out to compute CP-asymmetries including washouts and solutions to Boltzmann equations have been utilised to predict the baryon asymmetry. The additional contribution to vertex correction mediated by the heavy left-handed triplet scalar is noted to contribute as dominantly as other Feynman diagrams. We have found successful predictions of the baryon asymmetry for both the patterns of right-handed neutrino masses. The $SU(2)_L$ triplet fermionic dark matter at the TeV scale carrying even matter parity is naturally embedded into the non-standard fermionic representation $45_F$ of SO(10). In addition to the triplet scalar and the triplet fermion, the model needs a nonstandard color octet fermion of mass $\sim 5 \times 10^7$ GeV to achieve precision gauge coupling unification at the GUT mass scale $M^0_U = 10^{15.56}$ GeV. Threshold corrections due to superheavy components of $126_H$ and other representations are estimated and found to be substantial. It is noted that the proton life time predicted by the model is accessible to the ongoing and planned experiments over a wide range of parameter space.

Keywords: Standard Model, Grand Unification, Hybrid Seesaw, Neutrino Masses, Dark Matter, Leptogenesis, Baryon Asymmetry, Proton decay
1 Introduction

The standard model (SM) of particle interactions based upon the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ has been tested by numerous experiments. Also the last piece of evidence in favour of the SM has been vindicated with the discovery of the Higgs boson at the
CERN Large Hadron Collider [1]. Yet the model fails to explain the three glaring physical phenomena: neutrino oscillation [2], baryon asymmetry of the universe (BAU) [3, 4], and dark matter (DM) [5]. Although the electroweak part of the SM provides excellent description of weak interaction phenomenology manifesting in $V-A$ structure of neutral and charged currents, it fails to answer why parity violation is exhibited by weak interaction alone. On the fundamental side, the SM itself cannot explain the disparate values of its gauge couplings. The minimal gauge theory which has the potential to unify the three gauge couplings [6, 7] and explain the origin of parity violation is $SO(10)$ grand unified theory (GUT) [8] that contains the Pati-Salam [9] and left-right gauge theories [10] as its subgroups. However, it is well known that direct breaking of all non-supersymmetric (non-SUSY) GUTs [7, 8] to the SM gauge theory under the assumption of minimal fine tuning hypothesis [11, 12] fails to unify the gauge couplings of the SM whereas supersymmetric GUTs like $SU(5)$ [7] and $SO(10)$ [8] achieve this objective in a profound manner. In fact the prediction of coupling unification in the minimal supersymmetric standard model (MSSM) [13, 14] evidenced through the CERN-LEP data [15–17] led to the belief that a SUSY GUT [18] with its underlying mechanism for solutions to the gauge hierarchy problem [19–21] could be the realistic model for high energy physics. SUSY GUTs also predict wino or neutralino as popular candidates of cold dark matter (CDM). Compared to SUSY $SU(5)$ [13], SUSY $SO(10)$ has a number of advantages. Whereas parity violation in $SO(10)$ has its spontaneous breaking origin, for $SU(5)$ it is explicit and intrinsic. The right-handed neutrino (RH$\nu$) as a member of spinorial representation 16 of $SO(10)$ mediates the well known canonical seesaw mechanism [22, 23] that accounts for small neutrino masses evidenced by the neutrino oscillation data. Further the Dirac neutrino mass matrix that occurs as an important ingredient of type-I seesaw [22, 23] is predicted in this model due to its underlying quark-lepton symmetry [9]. In addition, the presence of the left-handed (LH) triplet scalar, $\Delta_L(1, 3, -1) \subset 126_H \subset SO(10)$, naturally leads to the possibility of Type-II seesaw formula for neutrino masses [23, 24]. Both the heavy RH neutrinos and the LH triplet scalar have the high potential to account for BAU via leptogenesis [25–28].

With R-Parity as its gauged discrete symmetry [29–32], the model also guarantees stability of dark matter.

Another attractive aspect of SUSY $SO(10)$ [33] has been its capability to make a reasonably good representation of all fermion masses and mixings at the GUT scale [34, 35]. Such a data set exhibiting $b - \tau$ Yukawa unification and very approximately satisfying Georgi-Jarlskog [36] type relation is obtained using RG extrapolated values of the masses and mixings at the electroweak scale following the bottom-up approach [37]. In particular $\chi^2$ estimation has been carried out to examine goodness of fit to all fermion masses in SUSY $SO(10)$ [35]. Other interesting aspects of the SUSY GUT such as Yukawa unification with large $\mu$ and a heavier gluino [38], viability of GUT-scale tribimaximal mixing [39], and unified description of fermion masses with quasi-degenerate (QD) neutrinos [40] have been explored. A comparison of quality of different models has been also discussed [41]. Recently existence of flavour symmetries [42] and emergence of ordered anarchy from 5.dim. theory [43], and Sparticle spectroscopy [44] have been also investigated with numerical analyses on fermion masses. However, there exists a large class of SUSY $SO(10)$ models where
a qualitative or at most a semi-quantitative representation of fermion masses have been considered adequate without \(\chi^2\) estimation. Examples from a very small part of a huge list are [18, 45–60]. Even while confronting other challenging problems through SUSY SO(10), explanation of neutrino data only has been considered adequate; some examples out of many such works in this direction include derivation of new seesaw mechanism with TeV scale \(Z'\) [51], prediction of Axions [54], low-mass \(Z'\) induced by flavor symmetry [56], realization of SUSY SO(10) from \(M - \) theory [55, 57], predictions of inflaton mass [58], and Starobinsky type inflation [59], or quartic inflation [60] from SUSY SO(10). Generalised hidden flavour symmetries have been explored without confining to any particular type of fermion mass fits [61].

Despite many attractive qualities of SUSY GUTs including the resolution of the gauge hierarchy problem, no experimental evidence of supersymmetry has been found so far. This has led to search for gauge coupling unification of the standard gauge theory in non-supersymmetric (non-SUSY) GUTs while sacrificing the elegant solution to the gauge hierarchy problem in favour of fine-tuning to every loop order [62, 63]. As stated above, single step breakings of all popular non-SUSY GUTs including SU(5) [7] and SO(10) [8] under the constraint of the minimal fine-tuning hypothesis [11, 12] fail to unify gauge couplings.

Introducing gravity induced corrections through higher dimensional operators [64] or additional fine-tuning of parameters with lighter scalars or fermions, gauge coupling unification in non-SUSY SU(5) GUT has been implemented [65, 66] including RH neutrino as DM [67]. Such unification has been also achieved including triplet fermionic DM [68]. A color octet fermion with mass \(> 10^8\) GeV which is also needed for unification has been suggested as a source of non-thermal DM via non-renormalizable interactions [68]. As the model does not use matter parity [69–74], the stabilising discrete symmetry for DM has to be imposed externally and appended to the GUT framework. Further, issues like neutrino masses and mixings and the baryon asymmetry of the universe have not been addressed in this model. Naturally the non-SUSY SU(5) models [64, 65, 67, 68] have no explanation for the monopoly of parity violation in weak interaction alone [9, 10].

However, with or without broken D-Parity at the GUT scale [75, 76], non-SUSY SO(10) has been shown to unify gauge couplings having one or more intermediate symmetries [75–79]. Extensive investigations in such models have been reported with high intermediate scales [35, 75–84] and also with TeV scale \(W_R, Z_R\) bosons and verifiable seesaw mechanisms [85–93]. Out of a large number of possible models that are predicted from non-SUSY SO(10) [76] fermion mass fit has been investigated only in one class of models with Pati-Salam intermediate symmetry [35, 81, 83] and also including additional vector-like fermions [82]. The issue of DM has been also addressed with different types of high scale intermediate symmetries and by introducing additional fermions or scalars beyond those needed by extended survival hypothesis [94] but without addressing fermion mass fits. The problem of TeV scale \(W_R\) boson prediction along with DM have been also addressed in non-SUSY SO(10) by invoking external \(Z_2\) symmetry [90] without fitting charged fermion masses as also in a number of other models [76, 77, 79, 80, 84–89, 95–97]. As there has been no experimental evidence of supersymmetry so far, likewise there has been also no definite
evidence of any new gauge boson beyond those of the SM. This in turn has prompted authors to implement gauge coupling unification with the SM gauge symmetry below the GUT scale [65–70, 72–74] by the introduction of additional particle degrees of freedom with lighter masses. A natural question in this context is how much of the advantages of the SUSY GUT paradigm is maintained in the case of non-SUSY gauge coupling unification models. While SUSY SO(10) is well known for its intrinsic R-Parity [29, 31] as gauged discrete symmetry [30] for the stability of dark matter, as an encouraging factor in favour of the non-SUSY GUT it has been shown recently [69–71, 73, 74] that matter parity defined as \( P_M = (-1)^{3(B-L)} \) could be the corresponding discrete symmetry intrinsic to non-SUSY SO(10) where \( B(L) \) stands for baryon (lepton) number. Whereas neutralino or wino are predicted as dark matter candidates in SUSY GUTs, in non-SUSY SO(10) the DM candidates could be non-standard fermions (scalars) carrying even (odd) matter parity. In fact all SO(10) representations have been identified to carry definite values of matter parity which makes the identification of a dark matter candidate transparent from among the non-standard scalar(fermion) representations. Thus there is enough scope within non-SUSY SO(10) to implement the DM paradigm along with an intrinsic stabilising symmetry.

Compared to SUSY GUTs, the non-SUSY GUTs do not have the problems associated with the Higgsino mediated proton decay [53, 98] while the canonical proton decay mode \( p \rightarrow e^+ \pi^0 \) has been accepted as the hallmark of predictions of non-SUSY GUTs since more than four decades. Further, the non-SUSY GUT also does not suffer from the well known gravitino problem [99, 100].

Coupling unification in the single step breaking of non-SUSY SO(10) has been addressed in an interesting paper by Frigerio and Hambye (FH) [72] by exploiting the intrinsic matter parity of SO(10) leading to triplet fermion in 45\(_F\) as dark matter candidate. The presence of a color octet fermion of mass \( \geq 10^{10} \) GeV has been also noted for unification. The proton lifetime has been predicted in this model at two-loop level of gauge coupling unification. However details of fitting the neutrino oscillation data including derivation of Dirac neutrino mass matrix and the \( RH\nu \) mass spectrum have not been addressed. Likewise related details of derivation of the baryon asymmetry of the universe via leptogenesis has been left out from the purview of discussion. An added attractive aspect of the model is the discussion of various methods, both renormalizable and non-renormalizable, by which the triplet fermionic DM can have TeV scale mass. Although proton lifetime has been predicted from the two-loop determination of the GUT scale, important modification due to threshold effects that could arise from the superheavy components of various representations [101–105] need further investigation.

The contents of the present paper are substantially different from earlier works in many respects. We have discussed the matching with the neutrino oscillation data in detail where, instead of type-I seesaw, we have used hybrid seesaw which is a combination of both type-I and type-II [106]. Both of the seesaw mechanisms are naturally predicted in matter parity based SO(10) model having their origins rooted in the Higgs representation 126\(_H\) and the latter’s coupling to the fermions in the spinorial representation 16 through
Unlike a number of neutrino mass models adopted earlier, in this work we have not assumed dominance of any one of the two seesaw mechanisms over the other. For the purpose of the present work we have determined the Dirac neutrino mass matrix at the GUT scale from the extrapolated values of charged fermion masses [37] and exploiting the exact quark lepton symmetry [9] at that scale. With a view to investigating basis dependence of leptogenesis, the Dirac neutrino mass estimation has been carried out in two ways: by using the $u$-quark diagonal basis as well as the $d$-quark diagonal basis. Using these in the hybrid seesaw formula which is quadratic in the Majorana coupling $f$ gives two distinct patterns of mass eigen values for the heavy RH $\nu$ masses: (i) Compact scenario where all masses are heavier than the Davidson-Ibarra (DI) bound, and (ii) The hierarchical scenario where only the lightest $N_1$ mass is below the DI bound. Thus each of these sets of RH neutrino masses corresponds to two types of Dirac neutrino mass matrices or Yukawa couplings which play crucial roles in the determination of CP-asymmetry resulting from RH$\nu$ decays. We have carried out a complete flavour analysis in determining the CP asymmetries. We have also exploited solutions of Boltzmann equations in every case to arrive at the predicted results on baryon asymmetry. Successful ansatz for baryogenesis via leptogenesis is shown to emerge for each pattern of RH$\nu$ masses. With the compact pattern of RH$\nu$ mass spectrum, this occurs when the Dirac neutrino masses are determined in the $u$-quark or the $d$-quark diagonal basis. However, in the hierarchical scenario of RH$\nu$ masses, the dominant CP asymmetry that survives the washout due to $N_1$-decay and contributes to the desired baryon asymmetry is generated by the decay of the second generation RH$\nu$ where the Dirac neutrino mass corresponds to the $u$-quark diagonal basis. Because of the heavier mass of the LH triplet scalar, although its direct decay to two leptons [107] gives negligible contribution to the generated CP-asymmetry, the additional vertex correction generated by its mediation to the RH$\nu$ decay is found to lead to a CP-asymmetry component comparable to other dominant contributions. Thus the same heavy triplet scalar $\Delta_L$ and the RH$\nu$s which drive the hybrid seesaw formula for neutrino masses and mixings are shown to generate the leptonic CP asymmetry leading to the experimentally observed value for the baryon asymmetry of the universe over a wide range of the parameter space in the model.

For the embedding of the suggested triplet fermionic DM [108] in SO(10) [72], we assume it to originate from the non-standard fermionic representation $45_F \subset SO(10)$ carrying even matter parity. Having exploited the triplet fermionic DM $\Sigma_F(1, 3, 0)$ and the LH triplet Higgs scalar $\Delta_L(1, 3, -1)$ mediating the hybrid seesaw for neutrino masses and leptogenesis, we justify the presence of these light degrees of freedom as ingredients for coupling unification through their non-trivial contribution to the $SU(2)_L \times U(1)_Y$ gauge coupling evolutions. In addition, we need lighter scalar or fermionic octets with mass $\sim 5 \times 10^7$ GeV under $SU(3)_C$ to complete the precision gauge coupling unification.

The degrees of freedom used in this model having their origins from SO(10) representations $126_H, 10_H, 45_H,$ and $45_F$ are expected to contribute substantially to GUT threshold effects on the unification scale through their superheavy components even without resorting to make the superheavy gauge boson masses non-degenerate as has been adopted in a
number of earlier works for proton stability. It is important to note that if we accept the stabilising symmetry for DM to be matter parity, then the participation of $126_H \subset SO(10)$ in its spontaneous symmetry breaking is inevitable. This in turn dictates a dominant contribution to threshold effects on proton lifetime which has been ignored earlier but estimated in this direct breaking chain for the first time. In addition the superheavy fermions in $45_F$ have been noted to contribute substantially. A possibility of partial cancellation of scalar and fermionic threshold effects is also pointed out. Although it is challenging to rule out the present model by proton decay experiments, the predicted proton lifetime in this model for the $p \rightarrow e^+\pi^0$ is found to be within the accessible range of the ongoing search limits [109, 110] for a wider range of the parameter space.

Unlike the case of direct breaking of SUSY SO(10) to MSSM [35] or non-SUSY SO(10) through Pati-Salam intermediate symmetry [35], but like very large number of cases of model building in non-SUSY GUTs, it is not our present goal to address charged fermion mass fit. But we discuss in Appendix C how all fermion masses may be fitted at least approximately in future without substantially affecting this model predictions.

This paper is planned in the following manner. In Sec. 2 we discuss successful fit to the neutrino oscillation data where we estimate the LH Higgs triplet and the RH $\nu$ masses. In Sec. 3.1 we present the estimations of CP-asymmetry for different flavor states. In Sec. 3.2 we discuss Boltzmann equations for flavour based analysis. In Sec. 3.3 and Sec. 3.4 we present the results of final baryon asymmetry. In Sec. 4 we discuss why the neutral component of fermionic triplet is a suitable dark matter candidate. In Sec. 5 we discuss unification of gauge couplings and determine the unification scale. In Sec. 6 we discuss proton lifetime prediction including GUT-threshold uncertainties. In Sec. 7 we summarize and state conclusions. In Appendix A and Appendix B we provide renormalization group coefficients for gauge coupling evolution and estimation of threshold effects. In Appendix C we discuss the possibility of parameterization of fermion masses.

## 2 Hybrid Seesaw Fit to Neutrino Oscillation Data

In this section we address the issue of fitting the neutrino masses and mixings as determined from the neutrino oscillation data by the hybrid seesaw formula. We then infer on the masses of heavy left-handed triplet and RH neutrinos necessary for leptogenesis.

After SO(10) breaking, the relevant part of the Lagrangian under SM symmetry is

$$-L_{\text{Yuk}} \ni Y^{ij}_{\nu} N_{Ri} L_j h^\dagger + \frac{1}{2} f^{ij} v_R N_{Ri} C N_{Rj} + \frac{1}{2} f_{ij} L_i^T C i\tau_2 \Delta_L L_j$$

$$-\mu H^T i\tau_2 \Delta_L H + M_3^2 Tr(\Delta_L^\dagger \Delta_L) + h.c.$$  \hspace{2cm} (2.1)

The first term on the right-hand side (RHS) of eq.(2.1) is from the SO(10) symmetric Yukawa term $Y^{(10)}_{(16.16.10)_H}$ whereas the second and the third terms are from $f_{16.16.126}^\dagger$ [33]. Also we have defined $v_R \equiv \langle \Delta_R \rangle \sim M_R$ and $\mu = \lambda v_R$. Although the associated RH scalar field $\Delta_R(1,3,-2,1) \subset 126_H$ has the respective quantum number under the LR
gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_{C}(\equiv G_{2213})$, it is the singlet component $\Delta_R(1,1,0)$ under the SM that acquires the vacuum expectation value (VEV) $= v_R$. Similarly the LH triplet scalar field $\subset 126_H$ has the transformation property $\Delta_L(3,1,-2,1)$ under $G_{2213}$ but the quantum numbers under the SM ($= G_{321}$) are $\Delta_L(1,3,-1)$. Here $\lambda$ is the quartic coupling of the SO(10) invariant Lagrangian resulting from the combination of $10_H$ and $126_H$: $\lambda 10_H^2 126_H^T 126_H \supset \mu H^T i \tau_2 \Delta_L H$. The Higgs triplet mass-squared term has its origin from $M^2_{\Delta 126_H^T 126_H}$.

Other notations are self explanatory. The hybrid formula for the light neutrino mass matrix is the sum of type-I and type-II seesaw contributions \[ [12] \]
\[
m_\nu = f v_L - M_D \frac{1}{f v_R} M_D^T,
\]
where $v_L = \lambda v_R^2 v_{ew} / M^2_\Delta$ is the induced VEV of triplet scalar $\Delta_L$, and $M_D \equiv Y_\nu v_{ew}$.

There is the well known standard ansatz to fit fermion masses in SO(10) along the line of \[ [33] \]. To estimate the Dirac mass matrix in this work we have carried out one-loop renormalization group evolution of Yukawa couplings in the bottom-up approach using PDG values of all charged fermion masses. At the electroweak scale $\mu = M_Z$ using experimental data on charged fermion masses we choose up-quark or down-quark mass diagonal bases in two different scenarios. We then evolve them up to the GUT scale $\mu = M_{\text{GUT}}$ using bottom-up approach \[ [37] \]. At this scale we assume equality of the up-quark and the Dirac neutrino mass matrices, $M_D \simeq M_u$, which holds up to a very good approximation in SO(10) due to its underlying quark-lepton symmetry \[ [9] \].

As pointed out in Sec.1, $\chi^2$ fit to all fermion masses and mixings in SUSY SO(10) or in non-SUSY SO(10) with $G_{224}$ intermediate symmetry requires a small departure from this assumption \[ [35, 81, 83] \]. On the other hand a very recent derivation of neutrino mass and mixing sum-rules has been found to require $M_D$ close to $M_u$ \[ [84] \] as in our case. Although in the present case of non-SUSY SO(10) breaking directly to the SM gauge theory, fermion mass fit is not our goal in this paper, we have discussed the issue in Appendix C.

We further assumed that $M_D(M_{\text{GUT}}) \sim M_D(\mu)$ for all lower mass scales $\mu < M_{\text{GUT}}$. We could have done better to estimate the Dirac mass matrix at the electroweak scale by following the top-down approach but since it does not get appreciable correction due to the absence of the strong gauge coupling $\alpha_{3C}$ \[ [37] \] contribution, this approximation does not influence our final result substantially. Another reason is that for leptogenesis we need Dirac neutrino Yukawa couplings at intermediate scales, $\mu \sim (10^6 - 10^{12})$ GeV where the renormalisation group (RG) running effects are expected to be smaller in the top-down approach.

Thus in the down quark diagonal basis under the assumption of negligible RG effects we have at $\mu = M_Z$
\[
M_D^{(d)}\text{(GeV)} = \begin{pmatrix}
0.01832 + 0.00441i & 0.08458 + 0.01114i & 0.65882 + 0.27319i \\
0.08458 + 0.01114i & 0.38538 + 1.56 \times 10^{-5}i & 3.32785 + 0.00019i \\
0.65882 + 0.27319i & 3.32785 + 0.00019i & 81.8543 - 1.64 \times 10^{-5}i
\end{pmatrix}
\] (2.3)

We repeat the above procedure in the up-quark diagonal basis at \( \mu = M_Z \) instead of the down quark diagonal basis leading to

\[
M_D^{(u)}\text{(GeV)} = \begin{pmatrix}
0.00054 & (1.5027 + 0.0038i)10^{-9} & (7.51 + 3.19i)10^{-6} \\
(1.5027 + 0.0038i)10^{-9} & 0.26302 & 9.63 \times 10^{-5} \\
(7.51 + 3.19i)10^{-6} & 9.63 \times 10^{-5} & 81.9963
\end{pmatrix}
\] (2.4)

For the sake of clarity it might be necessary to explain how the mass matrix structure given in eq.(2.4) emerges with very small non-diagonal elements. In the bottom-up approach for the RG evolution of Yukawa matrices, we have assumed the up-quark mass matrix \( M_u(M_Z) \) to be diagonal in one case at the electroweak scale which we designate as up-quark diagonal basis. In this case naturally all elements of the down quark mass matrix \( M_d(M_Z) \) are non-vanishing. In the alternative case, called the d-quark diagonal basis, we have chosen \( M_d(M_Z) \) diagonal for which all nine elements of \( M_u(M_Z) \) are non-vanishing.

In the case of up-quark diagonal basis, however, the non-diagonal elements of \( M_u(M_{\text{GUT}}) \) acquire non-vanishingly small corrections due to RG effects in the bottom-up approach and this is approximated as the Dirac-neutino mass matrix \( M_D^{(u)}(M_{\text{GUT}}) \). This explains the appearance of non-diagonal elements appearing in eq.(2.4). It may be noted further that the RG-corrections in the Dirac neutrino mass matrix \( M_D^{(u)} \) for evolutions from \( \mu = M_{\text{GUT}} \) down to relevant lower scales have been ignored as they are expected to be much smaller.

The Dirac neutrino mass matrices given in eq. (2.3) and eq.(2.4) are used in the second term of the right-hand side (RHS) of eq.(2.2) where in the left-hand side (LHS) we use the value of light neutrino mass matrix for the normally ordered case with \( m_\nu_1 = 0.00127 \text{ eV} \) and the best fit values for other parameters [112]. We have also assumed that Majorana phases are zero at all mass scales.

We then search for solutions for the Majorana coupling \( f \) or, equivalently, the values of RH neutrino masses. Due to strongly hierarchical structure of \( M_D \) matrix, it is impractical to assume the dominance of the type-I or the type-II term in the hybrid seesaw formula of eq.(2.2). Since eq.(2.2) is quadratic in \( f \), it has two solutions for every eigenvalue and thus giving a total of \( 2^3 = 8 \) plausible solutions [111]. But for a given \( M_D \) and \( m_\nu \), there should be only two distinct positive definite solutions. We estimated these solutions for \( f \) using the neutrino oscillation data of ref.[112] as input and numerical iteration. A robust iterative numerical estimation of \( f \) matrix is performed to match the oscillation data. Thus by fixing the lightest neutrino mass and the VEV \( v_L \) in a chosen hierarchy of light neutrino masses, the precise forms of the two solutions with positive definite \( f \) are evaluated upto the desired precision. These solutions are presented in Fig. 1 for two sets of values of quartic coupling, \( \lambda = 0.1 \) and \( \lambda = 0.001 \).
Figure 1. Prediction of heavy RH neutrino masses as a function of the lightest neutrino mass and the quartic coupling $\lambda$ in the case when the three neutrino masses are normally ordered. The top row represents a hierarchical spectrum solution of RH neutrinos and the bottom row represents a not so hierarchical scenario which we call as compact spectrum solution. The values of $M_{\Delta L} = 10^{12}$ GeV and $v_R = 10^{15.5}$ GeV have been kept fixed. The value of the quartic coupling used here has been taken to be $\lambda = 0.1(0.001)$ for the left panel (right panel).

In Fig. 1 we have presented these solutions for the normally ordered values of active light neutrino masses. Solutions in the top row of the figure have strongly hierarchical heavy RH neutrino masses, lightest of them being $M_{N_1} \sim \mathcal{O}(10^3-5)$ GeV, testable in future collider experiments, and the heaviest $M_{N_3} \sim \mathcal{O}(10^{12})$ GeV. We call such solutions of RH neutrino masses to represent a hierarchical spectrum scenario. Solutions in the bottom row of the figure are not so hierarchical and the RH neutrinos only span three orders of magnitude of mass range. We call the solutions of this type given in the bottom row to represent a compact spectrum scenario. Lightest of RH neutrino in this scenario is $\sim \mathcal{O}(10^9-11)$ GeV which is far away from direct detection limit of any collider experiment. In arriving at these solutions we assumed the LH triplet scalar mass $M_{\Delta L} = 10^{12}$ GeV, GUT symmetry breaking VEV $v_R = 10^{15.5}$ GeV, and the value of the quartic coupling $\lambda = 0.1$ (left panel) and 0.001 (right panel). We note that the RH$\nu$ masses increase with decrease in $\lambda$ for the compact spectrum scenario while it almost stays unaffected in the hierarchical spectrum scenario. Also the theory should continue to remain perturbative on
acquiring $N_1$-dominated leptogenesis because increasing $\lambda (\sim 1)$ for the above value of $M_\Delta$ will make $M_{N_1} < 10^9$ GeV and $N_1$-dominated leptogenesis will not be possible.

In the compact spectrum scenario we estimate the $f$ matrix in the $d$-diagonal basis using eq.(2.3), $m_{\nu_1} = 0.00127$ eV, $M_{\Delta L} = 10^{12}$ GeV, and $v_R = 10^{15.5}$ GeV

$$M_D = M_D^{(d)}$$

$$f = \begin{pmatrix}
0.385 + 0.1291i & 0.4617 - 0.4922i & 3.509 + 1.080i \\
0.4617 - 0.4922i & 4.626 + 0.1567i & 22.80 + 0.3317i \\
3.509 + 1.080i & 22.80 + 0.3317i & 511.6 + 0.47i
\end{pmatrix} \times 10^{-6}. \quad (2.5)$$

For the same parameters in the compact spectrum scenario but with $M_D^{(u)}$ in $u$-diagonal basis given in eq.(2.4), we derive

$$M_D = M_D^{(u)}$$

$$f = \begin{pmatrix}
0.3175 + 0.0904i & 0.1232 - 0.6089i & -0.4869 - 0.6918i \\
0.1232 - 0.6089i & 3.610 - 0.0724i & 1.587 + 0.2599i \\
-0.4869 - 0.6918i & 1.587 + 0.2599i & 511.8 + 0.6524i
\end{pmatrix} \times 10^{-6}. \quad (2.6)$$

In the hierarchical spectrum scenario, similarly, we have the two matrices for $f$

$$M_D = M_D^{(d)}$$

$$f = \begin{pmatrix}
-0.0690 + 0.0147i & -0.341 + 0.0164i & -4.0194 + 1.5733i \\
-0.341 + 0.0164i & -1.5745 - 0.2133i & -20.2464 - 0.3306i \\
-4.0194 + 1.5733i & -20.2464 - 0.3306i & -507.895 - 0.4034i
\end{pmatrix} \times 10^{-6}, \quad (2.7)$$

$$M_D = M_D^{(u)}$$

$$f = \begin{pmatrix}
-0.000025 + 0.000008i & -0.00019 - 0.000215i & -0.00538 - 0.00177i \\
-0.00019 - 0.000215i & -0.56091 + 0.0092i & 0.95702 - 0.27084i \\
-0.00538 - 0.00177i & 0.95702 - 0.27084i & -508.16 - 0.60957i
\end{pmatrix} \times 10^{-6}. \quad (2.8)$$

Despite widely varying magnitudes of different elements in the matrix, the mass eigenvalues in the $u$- quark and $d$- quark diagonal bases are not very different in both the compact spectrum and the hierarchical spectrum scenarios. Therefore, we have presented only one set of solutions for the RH$\nu$ masses in Fig. 1. It is quite encouraging to note that despite the GUT scale value of $v_R$, the type-II term does not upset the type-I seesaw term in the hybrid formula, rather both of them contribute significantly to the light neutrino mass matrix. We will explore the plausibility of sufficient leptogenesis using the hybrid seesaw mechanism of this model to explain BAU.

3 Baryon Asymmetry of the Universe

In this section at first we estimate the leptonic CP- asymmetry generated in decays of both RH$\nu$ and $\Delta L$. The dynamically generated lepton asymmetry gets converted into baryon asymmetry due to sphaleron interaction [113]. Leptogenesis is discussed in various
papers [114]. The flavour independent calculation of asymmetry is applicable at high temperatures when all the charged lepton mediated interactions are out of equilibrium i.e. $T \gtrsim 10^{12}$ GeV. Flavour dependent analysis [115] becomes necessary for leptogenesis at lower temperatures. In hierarchical spectrum scenarios we have $M_{N_1} \sim 10^{3-5}$ GeV which violates the Davidson-Ibarra bound [116] badly, therefore it can not produce required amount of flavour independent lepton asymmetry. Instead it washes out the asymmetry produced at the early stage in $N_{2,3}$ decays. In the recent studies [115, 117–120] it has been shown that under such circumstances the next heavy neutrino $N_2$ can produce the required asymmetry, if $M_{N_2} \gtrsim 10^{10}$ GeV and there exists a heavier $N_3$. If the asymmetry produced by $N_2$ is not completely washed out by lightest neutrino $N_1$, it survives and gets converted to baryon asymmetry. On the other hand, in the compact spectrum scenario, the lightest RH neutrino is well within the Davidson-Ibarra bound, therefore the asymmetry can be produced in the lightest RH$\nu$ decay. Since for a large region of the parameter space we have shown that $M_{N_1} << 10^{12}$ GeV, the asymmetry will depend on flavour dynamics.

### 3.1 CP-Asymmetry

The flavoured CP-asymmetry in the decay of $N_i$ to a lepton $l_\alpha$ is generated in the lepton flavor generation $\alpha$, and is defined as [123, 124, 136]

$$
\varepsilon_{i\alpha} = \frac{\Gamma(N_i \to l_\alpha + H^*) - \Gamma(N_i \to \bar{l}_\alpha + H)}{\sum_\beta \left[ \Gamma(N_i \to l_\beta + H^*) + \Gamma(N_i \to \bar{l}_\beta + H) \right]}.
$$

(3.1)

One loop decay contributions of $N_i$ are mediated by either $N_{k\neq i}$ or $\Delta_L$ [107] as shown in Fig. 2. The total asymmetry is sum of the two contributions

$$
\varepsilon_{i\alpha} = \varepsilon_{i\alpha}^N + \varepsilon_{i\alpha}^\Delta.
$$

(3.2)

The asymmetry produced in the $N_i$ decay due to $N_{k\neq i}$ appearing in the loop is [123, 124]

$$
\varepsilon_{i\alpha}^N = \frac{1}{8\pi} \sum_{k \neq i} \text{Im} \left[ \frac{\left( \bar{Y}_\nu^\dagger \nu \right)_{ia} \left( \bar{Y}_\nu \nu \right)_{ak} \left( \bar{Y}_\nu^\dagger \nu \right)_{ik}}{\left( \bar{Y}_\nu \nu \right)_{ii}} \right] \cdot \left( \frac{M^2_{N_k}}{M^2_{N_i}} \right) + \frac{1}{8\pi} \sum_{k \neq i} \text{Im} \left[ \frac{\left( \bar{Y}_\nu^\dagger \nu \right)_{ia} \left( \bar{Y}_\nu \nu \right)_{ak} \left( \bar{Y}_\nu^\dagger \nu \right)_{ki}}{\left( \bar{Y}_\nu \nu \right)_{ii}} \right] \cdot \left( \frac{M^2_{N_k}}{M^2_{N_i}} \right)
$$

(3.3)
The first line of this expression contains lepton number violating terms while the second line is the lepton number conserving but violates lepton flavour. Here, $\hat{Y}_\nu = Y_\nu U_f^*$ is the Dirac Yukawa coupling in the right-handed neutrino diagonal mass basis and $U_f$ is the unitary matrix diagonalizing $f$. The loop functions in the asymmetry expression are \[ g(x) = \frac{1 - x}{(1 - x)^2 + \left( \frac{\Gamma_i}{M_i} - x \frac{\Gamma_k}{M_k} \right)^2} \] \[ h(x) = \sqrt{x} \left[ g(x) + 1 - (1 + x) \log \left( \frac{1 + x}{x} \right) \right]. \]

Here by retaining the Wigner-Eckart term in the loop function we can handle degenerate RH$\nu$ mass scenario without hitting singularity, which is possible in compact spectrum scenario in our model (see Fig. 1). Note that in the degenerate regime CP asymmetry gets largest contribution from self-energy term and may reach to a value of $O(1)$. The
Figure 4. The CP-asymmetry vs. the lightest neutrino mass for hierarchical spectrum scenario of RH$\nu$ masses. The top left (right)-panel correspond to d(u)-quark diagonal basis for $\lambda = 0.1$. The bottom left (right) panel correspond to d(u)-quark diagonal basis but for $\lambda = 0.01$.

CP-asymmetry produced in $N_i$ decay from the $\Delta_L$ mediated diagram is [107]

$$
\varepsilon^{\Delta}_{i\alpha} = -\frac{1}{4\pi} \sum_{\beta} \text{Im} \left[ \left( \hat{Y}_\nu \right)_{i\beta} f^{*}_{\beta\alpha} \left( \hat{Y}_\nu \right)_{i\alpha} \mu \right] \frac{1 - \frac{M^2_{\Delta}}{M^2_{N_i}} \log \left( 1 + \frac{M^2_{N_i}}{M^2_{\Delta}} \right)}{M^2_{N_i}},
$$

(3.5)

which gets contribution proportional to the trilinear coupling mass term $\mu$. Its loop function is larger for smaller $M_{\Delta_L}$. But $M_{\Delta_L}$ can not be made arbitrarily small without decreasing $\mu$ or increasing $v_L$ which is constrained to be below GeV from electroweak (EW) precision constraints. Decreasing $\mu$ would decrease CP asymmetry linearly.

Keeping the GUT scale value of $v_R = 10^{15.5}$ GeV and $M_{\Delta_L} = 10^{12}$ GeV we have estimated the flavored CP-asymmetry for different values of the lightest neutrino mass in the normally ordered hierarchal case of light neutrino masses. Change in the mass of $m_{\nu_1}$ alters $f$ and thus changes the masses and mixings of RH$\nu$s. Flavour asymmetries for $N_i$ decay into $\alpha$ flavour are shown in Fig. 3 for compact spectrum case and in Fig. 4 for the hierarchical spectrum case of RH$\nu$s. We note that variation in quartic coupling changes
CP-asymmetry significantly, particularly in the hierarchical spectrum scenario. The tree level decay widths are unaffected by the presence of the scalar triplet $\Delta_L$ in the scheme.

The presence of the heavy scalar triplet $\Delta_L$ in our theory adds another source of CP-asymmetry ($\epsilon_\Delta$) which is produced by the decay of the triplet scalar itself into two like-sign or neutral leptons [107]. Though one triplet scalar is enough to generate the active neutrino masses and mixings through type-II seesaw, the asymmetry production in $\Delta_L$ decay needs either more than one triplet scalars [125–128] or combination of triplet scalar and right-handed neutrinos [107] as shown in Fig. 5 for our model. The CP-asymmetry generated due to $\Delta_L$ decay and mediated by RH$\nu$ is written as [107]

$$
\epsilon_\Delta = \frac{2}{4\pi} \frac{\Gamma(\Delta_L^* \to l + \bar{l}) - \Gamma(\Delta_L \to \bar{l} + l)}{\Gamma(\Delta_L^* \to l + \bar{l}) + \Gamma(\Delta_L \to \bar{l} + l)} \sum_k M_{N_k} \frac{\sum_{ij} |f_{ij}|^2 Y_{\nu}^* Y_{\nu} |f_{kl}| |f_{ll}|}{\sum_{ij} |f_{ij}|^2 M_\Delta^2 + 4|\mu|^2} \log(1 + M_\Delta^2/M_{N_k}^2).
$$

We note that, since $v_R \approx 10^{15.5}$ GeV and $M_{\Delta_L} \approx 10^{12}$ GeV, either of the two terms in the denominator of $\epsilon_\Delta$ is large enough to keep the $CP$-asymmetry fairly small for the parameters under consideration. For example, if three right-handed neutrino masses are $M_{N_k} = (6.6990, 13.869, 1431) \times 10^9$ GeV, the three CP-asymmetries due to $N_k$ decays from the first two diagrams of Fig. 2 are $|\epsilon_{N_k}| = (4.7 \times 10^{-5}, 5.1 \times 10^{-8}, 1.7 \times 10^{-8})$. Likewise the CP-asymmetries from the third diagram are: $|\epsilon_{N_k}^3| = (5.2 \times 10^{-5}, 4.5 \times 10^{-8}, 2.4 \times 10^{-6})$. Compared to these numbers, the CP-asymmetry due to $\Delta_L$ decay of Fig. 5 is $|\epsilon_\Delta| = 2.1 \times 10^{-12}$. Also, since $M_{\Delta_L} >> M_{1,2}$, the asymmetry generated at the early stage will be washed out at the production phase of lighter RH$\nu$s. Henceforth, we will ignore the $\Delta_L$ asymmetry in our numerical estimations [126]. In the next subsection we will estimate the lepton asymmetry using Boltzmann equations for the system.

### 3.2 Boltzmann Equations

The evolution of number density is obtained by solving the set of Boltzmann equations. The co-moving number density is $Y_X \equiv n_X/s$. The Boltzmann equations for heavy neutrinos

---
number density are \[111\]
\[
\frac{dY_{N_i}(z)}{dz} = -K_i(D_i(z) + S_i(z))(Y_{N_i}(z) - Y_{N_i}^{eq}(z))
\]
\[
\frac{dY_{\Delta \alpha}(z)}{dz} = -\sum_{i=1,2} \varepsilon_{i\alpha}K_i(D_i(z) + S_i(z))(Y_{N_i}(z) - Y_{N_i}^{eq}(z))
+ \sum_{i=1,2} K_{i\alpha} \sum_{\beta} W_i(z)(A_{\alpha\beta}Y_{\Delta \beta}(z) + C_{\beta}Y_{\Delta \beta}). \tag{3.7}
\]
where \(\Delta \alpha \equiv B/3 - l_\alpha\), and \(Y_{\Delta \alpha}\) stands for the total \(\Delta \alpha\) asymmetry stored in the fermionic flavours, and \(z = M_1/T\). The washout parameter for various flavors is
\[
K_{i\alpha} = \frac{\Gamma(N_i \to l_\alpha H^*) + \Gamma(N_i \to \bar{l}_\alpha H)}{H(M_{N_i})} \tag{3.8}
\]
such that \(K_{i} = \sum_{\alpha} K_{i\alpha}\). In eq.(3.7) the equilibrium number density \[111, 122\] is defined as
\[
Y_{N_i}^{eq} = \frac{135\zeta(3)}{8\pi^4 g_*} R_i^2 z^2 K_2(R_i z) \xrightarrow{T \gg M_1} \frac{135\zeta(3)}{4\pi^4 g_*}, \tag{3.9}
\]
where \(R_i = M_i/M_1\). The out-of-equilibrium condition for \(N_i\) decay, \(\Gamma_{N_i} < H(T = M_{N_i})\), requires the lightest right-handed neutrino to acquire mass \(M_{N_i} \gtrsim 4 \times 10^8\) GeV \[106\] where \(H \simeq 1.66g_* M_{N_i}^2/(M_D z_k^2)\) is the Hubble expansion rate. The thermally averaged decay rates are \(D_i(z) = R_i^2 z K_1(R_i z)/K_2(R_i z)\) where \(K_1\) and \(K_2\) are the first and the second order modified Bessel functions \[122, 129\], respectively. The scattering terms \(S_i(z)\) account for Higgs-mediated \(\Delta L = 1\) scatterings involving top quark and anti-quark as \(S_i(z) = 2S_i^t(z) + 4S_i^\bar{t}(z)\) \[129\]. The washout term is \(W_i(z) = W_i^{ID}(z) + W_i^{S}(z)\) where the inverse decay contribution is
\[
W_i^{ID}(z) = \frac{1}{4} R_i^4 z^3 K_1(R_i z). \tag{3.10}
\]
The unit lepton number changing \(\Delta L = 1\) scattering contributing to washout is
\[
W_i^{S}(z) = \frac{W_i^{ID}(z)}{D_i(z)} \left(2S_i^t(z)Y_{N_i}(z)/Y_{N_i}^{eq} + 8S_i^\bar{t}(z)\right). \tag{3.11}
\]
The \(\Delta L = 1\) scattering and related washout from Higgs and lepton mediated inelastic scattering involving top quark are included in the evolution of asymmetry \[129\].

We have ignored the off-shell part of \(\Delta L = 2\) process in the washout term which is a good approximation as long as \(M_{N_i}/10^{13} << K_i[130]\). We have also omitted the \(\Delta L = 0\) scattering such as \(N_iN_j \rightarrow \bar{t}l, N_iN_j \rightarrow HH^*, N_i\bar{l} \rightarrow N_jl, N_i\bar{l} \rightarrow N_j\bar{l}\) which do not contribute to the washout but can affect the abundance of heavy neutrinos. When flavor effects are taken into account, they also tend to redistribute the lepton asymmetry among flavors. These effects are of higher order in the neutrino Yukawa couplings and are expected to have little impact on the final baryon asymmetry. We further neglected the scalar triplet related washout processes, gauge scatterings, spectator processes, and the higher order processes like \(1 \rightarrow 3\) and \(2 \rightarrow 3\). The heavy gauge bosons processes such as

\[\text{--- 15 ---}\]
\( N_i e_R \rightarrow \bar{q}_R q'_R \) and \( N_i N_i \rightarrow f f \) tend to keep the heavy neutrinos in thermal equilibrium, thus reducing the generated lepton asymmetry. This effect is practically negligible because RH es are much lighter than the RH gauge bosons. We also ignore such flavour effects [131] which are relevant for resonant leptogenesis.

### 3.3 Baryon Asymmetry in the Compact Scenario

In this scenario the tau lepton flavour state decouples while the electron and muon states are still coupled. Thus, a flavour dependent analysis is necessary. In the two flavour case \( Y_{\Delta_{e+\mu}} = Y_{\Delta_e} + Y_{\Delta_{\mu}}, \varepsilon_{i,e+\mu} = \varepsilon_{ie} + \varepsilon_{i\mu}, K_{i,e+\mu} = K_{ie} + K_{i\mu} \), and the flavour coupling matrices are [120]

\[
A = \begin{pmatrix}
-417/589 & 120/589 \\
30/589 & -390/589
\end{pmatrix}, \quad C = \begin{pmatrix}
-164/589, & -224/589
\end{pmatrix}.
\tag{3.12}
\]

In this case the baryon asymmetry is expressed as

\[
Y_{\Delta B} = \frac{12}{37} \sum_a Y_{\Delta_a}, \quad \text{(SM)}
\tag{3.13}
\]

where the factor \( 12/37 \) is due to partial conversion of \( \Delta_a \) asymmetry in to baryon asymmetry by non-perturbative sphaleron process [132, 133]. The results of BBN [134] and PLANCK [4] experiments are

\[
Y_{\Delta B}^{BBN} = (8.10 \pm 0.85) \times 10^{-11},
\]

\[
Y_{\Delta B}^{Planck} = (8.58 \pm 0.22) \times 10^{-11}.
\tag{3.14}
\]

Compared to these somewhat higher value of BAU obtained from WMAP 7 years’ data has been reported in ref.[135].

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**Figure 6.** Washout factor vs. quartic coupling in the compact spectrum scenario. Left(right) panel corresponds to \( d(u) \)-quark diagonal basis. The lightest neutrino mass is kept at \( m_{\nu_1} = 0.00127 \) eV. Other parameters are kept fixed as described in the text.
Figure 7. CP-asymmetry vs. the quartic coupling in compact spectrum scenario. Left (right) panel corresponds to \(d(u)\)-quark diagonal basis. The lightest neutrino mass is kept at \(m_{\nu_1} = 0.00127\) eV. Other parameters are kept fixed as described in the text.

The washout coefficients \(K_{i\alpha}\) in the compact spectrum scenario of RH neutrino masses for the lightest neutrino mass \(m_{\nu_1} = 0.00127\) eV and \(\lambda \in [0.0001, 0.5]\) are plotted in Fig. 6. We see that there are two to four orders of variation in the washout for the above allowed range of \(\lambda\) in both the d-diagonal (left panel) and the u-diagonal (right panel) cases. We list the washout parameters for \(\lambda = 0.1\) in the case of the d-quark diagonal basis

\[
K = \begin{pmatrix}
1.27 \times 10^{-1} & 2.28 & 3.81 \times 10^{2} \\
2.77 \times 10^{-1} & 5.16 & 1.03 \times 10^{3} \\
1.34 \times 10^{-2} & 8.04 \times 10^{-2} & 4.44 \times 10^{3}
\end{pmatrix}.
\] (3.15)

In the u-quark diagonal basis the washout parameters are

\[
K = \begin{pmatrix}
2.27 \times 10^{-4} & 5.37 \times 10^{-6} & 5.14 \times 10^{-8} \\
1.46 \times 10^{-1} & 6.19 & 7.88 \times 10^{-5} \\
9.23 \times 10^{-3} & 4.75 \times 10^{-2} & 4.45 \times 10^{3}
\end{pmatrix}.
\] (3.16)

Our observations in the two cases are summarized below.

(a) The d-quark diagonal basis:

We note that \(K_i = \sum_{\alpha} K_{i\alpha} \sim (300 - 4000)\). Therefore the system is in strong washout regime for most of the parameter space. The asymmetry is determined by a balance between production and destruction. The final asymmetry freeze occurs at the decoupling of washout with \(z_f \sim (7 - 10)\). In the single flavour analysis the lepton asymmetry is approximated as \([136]\)

\[
Y_{\Delta L}(\infty) \simeq \frac{\pi^2}{6 z_f K_1} \varepsilon_1 Y_{N_1}^{eq}(0).
\] (3.17)

Using the values of \(K_i\) from Fig. 6 and \(\varepsilon_1 = \sum_{\alpha} \varepsilon_{1\alpha}\) from Fig. 7 we can easily achieve the required lepton asymmetry. In fact it may lead to a constraint on quartic coupling \(\lambda\).
(b) The \(u\)-diagonal basis:

We note that, since \(K_1 = \sum_\alpha K_{1\alpha} \ll 1\), this is a very weak washout regime. Ignoring thermal effect on CP-asymmetry and assuming zero initial abundance in the weak washout regime with initial thermal abundance \(Y_{N_1}(z = 0) = Y_{N_1}^{\text{eq}}(z = 0)\) \[136\] gives

\[Y_\Delta(\infty) \simeq \varepsilon_1 Y_{N_1}^{\text{eq}}(0).\] \(3.18\)

If there is already an initial amount of asymmetry left over, say through \(N_2\) decay, it will not be washed out because the system is in weak washout regime. But with zero initial abundance, \(Y_{N_1}(z = 0) = 0\) \[136\]

\[Y_{\Delta L}(\infty) \simeq \frac{27}{16} \varepsilon_1 K_1^2 Y_{N_1}^{\text{eq}}(0).\] \(3.19\)

We note that even if we assume initial thermal abundance \(Y_{N_1}^{\text{eq}}(0) \sim 0.0039\), the CP-asymmetry \(\varepsilon_1 \sim 10^{-4} - 3 \times 10^{-6}\) (Fig. 7) and \(K \sim 10^{-7} - 10^{-3}\) (Fig. 6). Therefore the generated asymmetry would be determined by initial abundance and, in the zero initial abundance scenario, the required lepton asymmetry can not be produced for any parameter value. Therefore the flavour independent analysis in the \(u\)-quark diagonal scenario with zero initial abundance of \(Y_{N_1}\) fails to give the required asymmetry.

![Figure 8](image_url)

**Figure 8.** The baryon asymmetry in \(e+\mu\) flavours (double-dot-dashed blue curve) and \(\tau\) flavor (dotted-dashed curve) for the \(u\)-quark diagonal basis and compact spectrum RH\(\nu\) mass scenario. Left (right) panel correspond to non-zero (zero) initial thermal abundance. The quartic coupling \(\lambda = 0.05\).

On the other hand a flavor dependent analysis can enhance the asymmetry. The flavour dependent lepton asymmetry is analyzed using Boltzmann equations \(3.7\) and is shown in Fig. 8 for \(u\)-quark diagonal basis. Thus in flavoured analysis we find that final lepton asymmetry is independent of initial abundance and is close to the experimental value for \(\lambda < 0.05\). This explicitly shows that \(N_2\) decay contributes to lepton asymmetry which is not completely washed out in the \(N_1\) decay.

The reason for doing flavoured analysis is that there are enhancements in the final asymmetry compared to the unflavoured case. Using \(d\)-quark diagonal basis Fig. 9 shows the variation of total asymmetry with respect to quartic coupling for a fixed value of the...
scalar triplet mass $M_\Delta = 10^{12}$ GeV, $v_R = 10^{15.5}$ GeV, and the lightest neutrino mass $m_{\nu_1} = 0.00127$ eV in normally ordered case. Similar is the effect in the $u$-quark diagonal basis.

### 3.4 Baryon Asymmetry in the Hierarchical Scenario

The Davidson-Ibarra bound is not respected in the hierarchical spectrum scenario of RH$\nu$ (see Fig. 1). In such a case there is the possibility of leptogenesis if asymmetry is produced by the decay of $N_2$. Lower bound on the lightest RH$\nu$ is passed to $M_{N_2} \gtrsim 10^{10}$ GeV. The $N_2$-dominated leptogenesis can be successful if there is a heavy neutrino, or triplet scalar with $M_{N_3}, M_{\Delta L} > M_{N_2}$, and the washout from the lightest RH$\nu$ ($N_1$) is circumvented. Since $M_{N_1} \ll 10^9$ GeV the lepton flavour states become incoherent and the washout acts separately on each flavour asymmetry. We need to solve Boltzmann equations at the production phase with $z_2 = M_2/T$, and at the washout phase with $z_1 = M_1/T$ [120]. We note from the Fig. 4 that the CP-asymmetry due to $N_1$ decay $\varepsilon_i = \sum_\alpha \varepsilon_{i\alpha}$ is very
small compared to CP-asymmetry due to $N_{2,3}$ decays. The decay and washout are also suppressed by a factor $M_{1}^2/M_{2}^2(\sim 10^{-14} - 10^{-15})$ and $M_{1}^2/M_{3}^2(\sim 10^{-9} - 10^{-10})$. Also we note that in the scenario $M_3 \gtrsim 10^{12}$ GeV $> M_2 > 10^{9}$ GeV $> M_1$, the role of $N_3$ becomes indistinct by the time asymmetry is produced due to $N_2$ decay and when washout is active. Thus $N_{1,3}$ do not contribute to asymmetry generation at the $N_2$ decay phase and we can write

$$\frac{dY_{N_2}(z_2)}{dz_2} = -K_2(D_2(z_2) + S_2(z_2)) \left( Y_{N_2}(z_2) - Y_{eq}^{N_2}(z_2) \right)$$

(3.20)

$$\frac{dY_{\Delta_\nu}(z_2)}{dz_2} = -\varepsilon_{2\alpha} K_2(D_2(z_2) + S_2(z_2)) \left( Y_{N_2}(z_2) - Y_{eq}^{N_2}(z_2) \right)$$

$$+ K_{2\alpha} \sum_{\beta} W_2(z_2) \left( A_{\alpha \beta} Y_{\Delta_\beta}(z_2) + C_{\beta} Y_{\Delta_\beta}(z_2) \right).$$

(3.21)

Figure 10. Washout factor vs. quartic coupling in the hierarchical spectrum scenario of RH$\nu$. The left (right) panel corresponds to the $d$($u$)-quark diagonal basis. The lightest neutrino mass is kept at $m_{\nu_1} = 0.00127$ eV. Other parameters are kept fixed as described in the text.

The flavour coupling matrices in the production phase are the same as given in eq. (3.12). For $T \lesssim 10^9$ GeV, the muon Yukawa interaction also gets equilibrated. Then the flavour coupling matrices are $[120, 137]$

$$A = \begin{pmatrix}
-151/179 & 20/179 & 20/179 \\
25/358 & -344/537 & 14/537 \\
25/358 & 14/537 & -344/537
\end{pmatrix},
C = -(37/179, 52/179, 52/179).$$

(3.22)

The washout parameters in the $d$-quark diagonal basis for $m_{\nu_1} = 0.00127$ eV and $\lambda = 0.1$ are

$$K = \begin{pmatrix}
2.157 & 58072 & 8.19 \times 10^6 \\
0.00021 & 21.80 & 3545.8 \\
1.1 \times 10^{-7} & 0.00154 & 450.1
\end{pmatrix}.$$
In the $u$-quark diagonal basis they are

\[
K = \begin{pmatrix}
2.899 & 4.42 \times 10^{-5} & 6.64 \times 10^{-4} \\
5.57 \times 10^{-4} & 37.11 & 2.346 \times 10^{-4} \\
1.297 \times 10^{-7} & 0.0037 & 451.343
\end{pmatrix},
\]

(3.24)

The washout factors and the CP-asymmetries for different flavours as a function of quartic coupling are shown in Fig.10 and in Fig.11, respectively, for the $d$-quark diagonal (left panel) and the $u$-quark diagonal (right panel) bases in each case. Notice that in the $d$-quark diagonal basis $K_{1\alpha} \gg 1$ for $\alpha = \mu, \tau$. Therefore any such type of flavoured asymmetry produced during $N_2$ decay will be washed out during the $N_1$ decay. But since $K_{1e} \approx 2$ the corresponding flavoured asymmetry will be washed out only partially. However, in the $u$-quark diagonal basis, $K_{1\alpha}(\alpha \neq e) \ll 1$. Therefore the corresponding flavour asymmetries produced during $N_2$ decay would survive. Also noting that in this basis $K_{1e} \approx 2.8$, the $e$-asymmetry generated by the $N_2$ decay will be only partially washed out by the $N_1$ decay. Also, noting from Fig.11 that $\varepsilon_{2\tau}$ is significantly large, it may produce the required amount of asymmetry. The complete flavoured analysis scenario is discussed below.

With the washout caused due to the $N_1$ decay, the solutions to Boltzmann equations can be achieved by the substitution $2 \to 1$ everywhere. Since $N_{2,3}$ abundance has vanished below $10^9$ GeV, the corresponding equations are redundant. We also note from Fig. 4 that the CP-asymmetries $\varepsilon_{\alpha \beta}$ are negligibly small, therefore the first term in the RHS of corresponding equation in eq. (3.21) in the $N_1$ decay can be ignored when $K_1$ is not very large. This results in the redundancy of the equation for $N_1$ in eq. (3.20) and we need to solve only

\[
\frac{dY_{\Delta\alpha}(z_1)}{dz_1} = K_{1\alpha} \sum_{\beta} W_1(z_1) \left( A_{\alpha\beta} Y_{\Delta\beta}(z_1) + C_{\beta} Y_{\Delta\beta}(z_1) \right).
\]

(3.25)
Figure 12. The asymmetry with $e + \mu$ flavors (double-dot-dashed blue curve) and the $\tau$ flavor (dot-dashed curve) due to $N_2$ decay. The left (right) panel represents our estimations for quartic coupling $\lambda = 0.01 (0.1)$.

The washout from the lightest RH$\nu$ is more efficient which acts on the whole of the

Figure 13. The flavor asymmetries in $e + \mu$ and $\tau$ flavors (left panel) and separately for $e, \mu$ and $\tau$ flavors (right panel). The quartic coupling has been fixed at $\lambda = 0.0001$.

generated asymmetry. We found that in the $d$-quark diagonal basis, the asymmetry $Y_{\Delta_{\alpha}}$ produced by the $N_2$ decay as shown in Fig.12 is itself much smaller than the experimentally observed asymmetry. There is no way to enhance it at the stage of $N_1$ decay in the case of $d$-quark diagonal basis leading to insufficient asymmetry. We also note from Fig. 10 and Fig. 11 that variation in quartic coupling is not going to help in enhancing the depleted asymmetry.

On the other hand in the $u$-quark diagonal basis $K_{1e} \sim 2$ and $K_{1(\mu, \tau)} \ll 1$, the asymmetries may survive the washout during the $N_1$ decay. In Fig.13 we have shown
solutions to Boltzmann equations where the flavour asymmetries are found to reach the experimental value. The left-panel of the figure corresponds to asymmetry produced during $N_2$ decay and the right-panel corresponds to the asymmetries surviving the $N_1$ decay washout. The results have been computed for $\lambda = 0.0001$ i.e. for the parameters where CP-asymmetry is the smallest as indicated in the Fig.11. As a matter of fact the behaviours of all the three individual asymmetries in the right-panel clearly follow analytically as solutions to eq. (3.20) for which the coupling parameters are given in eq.(3.22). Noting that $|A_{ee}| \sim 1$ but $|A_{e\mu}| \sim |A_{e\tau}| << 1$, and $C_e \sim C_\mu \sim C_\tau << 1$ gives the rising behaviour of $|Y_{\Delta_e}|$ from eq. (3.20) as $K_{1e} \sim 2$. But because of the negligible values of $K_{1\mu}$ and $K_{1\tau}$, eq. (3.20) gives constant behaviours for $|Y_{\Delta_\mu}|$ and $|Y_{\Delta_\tau}|$ as shown in the right-panel of Fig.13.

Using type-I seesaw and $N_2$ dominated flavoured leptogenesis it has been shown that parts of $e$ and $\mu$ asymmetries, designated as phantom terms [120], can completely escape washouts due to the lightest RH$\nu$ $N_1$ decay. Such phantom terms can give large contribution to the asymmetry resulting in a large $B - L$ asymmetry generation by the $N_1$ wash outs. The $N_2$ dominated leptogenesis generated due to such terms has been termed as “phantom” leptogenesis. In this work [120] each of the phantom terms being proportional to the $N_2$ abundance, the phantom terms vanish in the case of zero initial number density of the heavier RH$\nu$ i.e $N_2$.

However in a subsequent investigation [121] phantom terms have been shown to emerge as a generic feature of flavoured leptogenesis. They have to be taken into account even for initially vanishing RH$\nu$ abundances. In the strong washout regime the phantom terms have been also shown to give a contribution independent of initial conditions.

In the present case with hybrid seesaw as the origin of neutrino masses and leptogenesis, we find that even though we have ignored any such phantom term in the three flavour analysis, the $N_1$ decay does not wash out the produced asymmetry at all. Also since $K_{1e} \sim 1$ it helps increasing $Y_{\Delta_e}$ during the second phase of decay. Thus the conclusion of this analysis is that, in the hierarchical spectrum of RH$\nu$s, the production of the observed baryon asymmetry of the universe in heavy neutrino decays is favoured when Dirac mass matrix is such that it is derived from a GUT in the flavour basis satisfying $Y_u(M_Z) = Y_u^{\text{diag}}(M_Z)$.

To summarize this section, we have attempted to generate the right value of BAU through lepton asymmetry produced by the hybrid seesaw mechanism where the three heavy RH$\nu$s and a LH triplet scalar decay directly or act as mediators in the one-loop Feynman diagrams. Two classes of heavy RH$\nu$ spectra are found to be predicted by the neutrino oscillation data: compact and hierarchical. We have carried out complete flavor dependent analysis in both these cases. We have also examined the possibility of basis dependence that determines the Dirac neutrino mass matrix at the GUT scale by choosing either the $u$-quark diagonal basis, or the $d$-quark diagonal basis. Rigorous solutions to the Boltzmann equations are exploited in every case. In the compact spectrum case, the
decay of the lightest RHν which is heavier than the Davidson-Ibarra bound, produces the desired BAU in both the choices of the Dirac neutrino mass matrix. This is shown in Fig.8 and Fig.9. In the hierarchical spectrum scenario the lightest RHν is much lighter than the Davidson-Ibarra bound. The right value of CP-asymmetry is generated predominantly by the decay of heavier RHν N2 that also survives the wash out caused by the lightest N1. Successful generation of BAU shown in Fig.13 is possible with the Dirac neutrino mass matrix determined in the u-quark diagonal basis. Although direct decay of the LH scalar triplet itself does not produce the lepton asymmetry to produce the required BAU, its one loop mediation to the RHν decay vertex correction generates the desired asymmetry which is comparable to other contributions. Thus the role of the LH triplet predicted by the matter parity based SO(10) model is emphasized in the generation of BAU.

4 Fermionic Triplet as Dark Matter Candidate

4.1 General Considerations with Matter Parity

Usually the prospective DM candidates are accommodated in model extensions by imposing additional discrete symmetries for their stability. But as noted in Sec.1 an encouraging aspect of non-SUSY SO(10) is that [69–71] matter parity is available as an intrinsic gauged discrete symmetry if the neutral component of the RH higgs triplet ΔR(1, 3, −2, 1) ⊂ 126H ⊂ SO(10) is assigned GUT scale VEV to break the gauge symmetry leading to the SM Lagrangian. As the Higgs particle possesses even value of |B−L|, the vacuum with SM gauge symmetry conserves matter parity PM = (−1)3(B−L). This enables to identify the SO(10) representations to be identified with odd value of PM for 16, 144,560,....but with even PM for 10, 45, 54, 120, 126, 210, 210′,660 ..... Then it turns out that the would-be DM fermions must be in the non-standard fermionic representations 10F, 45F, 54F, 120F, 126F, 210F.....Thus the smallest representation to provide a doublet fermion with hypercharge Y = ±1 is 10F and the hyperchargeless triplet needed for this model building is in the next larger representation 45F ⊂ SO(10).

Similarly if it is desired to construct models with scalars as DM candidates, they must belong to the odd PM scalar representations 16H, 144H..... Whereas the phenomenology of scalar DM has been emphasized in [69, 70], the triplet fermionic DM has been found suitable in model construction in [72, 138]. In addition, the color octet fermions have been found to be essential at high scale Mcs ≥ 1010 GeV [72]. The importance of various other types of DM along with the triplet fermions of both types of chiralities has been also discussed in high intermediate scale models [94].

An important advantage of using triplet or doublet fermions over scalars as DM is that in the limit of zero chiral fermion masses, a U(1) global lepton symmetry of the SM is restored. Thus a value of the fermion mass substantially lighter than the GUT scale is naturally protected by this global symmetry in the ’t Hooft sense. [139]. On the other hand if a scalar component is used as DM, its mass lighter than the GUT scale has to be obtained by additional fine-tuning in the Lagrangian. Also matter parity conservation forbids it from acquiring any VEV.
4.2 Light Non-Standard Fermion Masses from SO(10)

In this model with the SM gauge symmetry below the GUT scale, a triplet fermionic DM candidate with zero hypercharge appears to be more appropriate with its mass of the order of TeV scale for gauge coupling unification as would be shown below in Sec.5. The neutral component of fermionic triplet $\Sigma_F(1,3,0) \subset 45_F \subset SO(10)$ would act as a cold dark matter candidate. For accurate coupling unification we also need a Majorana-Weyl type color octet fermion $C_8(8,1,0)$ at lower scale. Using Yukawa interaction via higher dimensional non-renormalizable operators, the light triplet fermion mass $\subset 45_F$ has been obtained in ref. [72]. But both the lighter values of masses of the triplet fermion and the octet fermion can be obtained easily from the renormalizable SO(10) Yukawa Lagrangian at the GUT scale. In the notation $45_F = A_F$, $54_H = E$, and $210_H = \Phi$, the relevant GUT scale Lagrangian is

$$-\mathcal{L}_{Yuk} = A_F (m_A + h_p \Phi + h_e E) A_F,$$

where $m_A \simeq M_U$ and $h_i (i = p, e)$ are Yukawa couplings. Using GUT scale vacuum expectation values for the singlet in $E$ and three singlets in $\Phi$ [140], the mass formulas for different components of $45_F$ are

$$
m(3, 1, 2/3) = m_A + \sqrt{2} h_p \frac{\Phi_2}{3} - 2h_e \sqrt{\frac{E}{\sqrt{15}}},$$

$$
m(3, 2, 1/6) = m_A - h_p \frac{\Phi_3}{3} + h_e \sqrt{\frac{E}{2\sqrt{15}}},$$

$$
m(3, 2, -5/6) = m_A - h_p \frac{\Phi_3}{3} + h_e \sqrt{\frac{E}{2\sqrt{15}}},$$

$$
m(1, 1, 1) = m_A + \sqrt{2} h_p \frac{\Phi_1}{\sqrt{3}} + \sqrt{3} h_e \frac{E}{\sqrt{5}},$$

$$
m(1, 1, 0) = m_A + 2\sqrt{2} h_p \frac{\Phi_2}{3} + \sqrt{3} h_e \frac{E}{\sqrt{15}},$$

$$
m'(1, 1, 0) = m_A + 2\sqrt{2} h_p \frac{\Phi_2}{3} - 2h_e \frac{E}{\sqrt{15}},$$

$$
m_{ps}(8, 1, 0) = m_A + \sqrt{2} h_p \frac{\Phi_2}{3} - 2h_e \frac{E}{\sqrt{15}},$$

$$
m_{s}(1, 3, 0) = m_A + \sqrt{2} h_p \frac{\Phi_1}{3} + \sqrt{\frac{3}{5}} h_e \frac{E}{\sqrt{15}}.$$

Fixing the mass $m_A$, these formulas have the options of finetuning two Yukawa couplings and four VEVs. If we get rid of $210_H$ we find that both the triplet mass $m_{s}(1, 3, 0)$ and the singlet mass $m_{s}(1, 1, 1)$ can be made light by a single fine tuning. On the other-hand if we use only $210_H$, only $m_{s}(1, 3, 0)$ can be made light by a single fine-tuning. By the use of both $54_H$ and $210_H$ several options are available with a rich structure of lighter fermion masses. In order to get both the triplet and the octet fermion masses light, two finetunings are needed. A missing partner mechanism with two sets of fermion representations $45_F^{1,2}$ and a Higgs representation $45_H^Y$ has been used to make the triplet fermionic DM light [72].
4.3 Triplet Fermion Dark Matter Phenomenology

The phenomenology of a hyperchargeless triplet fermionic DM in the non-SUSY model is similar to that of the wino DM in MSSM and SUSY GUTs. This has been extensively investigated recently [141] and also continues to be a subject of current importance [142]. It is worthwhile to mention here different constraints on their masses derived from direct and indirect searches because of their relevance to the present model building. The even matter parity of fermion triplet DM $\Sigma_F^{(1,3,0)}$, compared to odd (even) matter parity of standard fermion (Higgs scalar), guarantees stability of the DM by ruling out Yukawa interactions with SM particles. This may make it difficult for the detection of the triplet fermionic DM at the LHC and other hadron colliders.

(i). Triplet Fermion Mass from Relic Density:

The only interaction of the DM fermion with standard model particles is through gauge interaction that leads to the well known mass difference $m_{\Sigma^+} - m_{\Sigma^0} = 166$ MeV [141]. where we have denoted the mass of the charged (neutral) component of $\Sigma_F^{(1,3,0)}$ as $m_{\Sigma^+}(m_{\Sigma^0})$. Each of its two charged components has been estimated to be heavier by $\sim 166$ MeV [141]. Within the 3σ uncertainty, the observed DM relic abundance is $0.095 < \Omega_{DM}h^2 < 0.125$ where $h$ = Hubble parameter. For the triplet mass $m_\Sigma$ much larger than the W-boson mass, the Sommerfeld resonance enhancement plays a crucial role in the annihilations of components of the $\Sigma_F$ leading to the observed DM relic abundance. Neglecting mass difference between the charged and neutral components, the relevant cross section taking into account the annihilation and co-annihilation of all triplet components has been derived [72],

$$<\sigma v> = \frac{37g_\Sigma^4}{96\pi m_\Sigma}, \quad (4.3)$$

where $v$ = relative velocity of DM particles. The Sommerfeld enhancement enters into the annihilation process because of the fact that the triplet components are non-relativistic at the freezeout temperature. Matching the theoretical prediction within the 3σ uncertainty of the observed value of the relic density $\Omega_{DM}$ [143] results in the triplet mass $m_\Sigma = 2.75 \pm 0.15$ TeV [108, 141, 144] whereas a value of $m_\Sigma = 3.0 - 3.2$ TeV has been also estimated [142]. A non-thermal production of $\Sigma^0$ relic density due to the decay of color octet fermion, $C^8(8,1,0)_F$, has been recently discussed in [145]. Quite recently only the neutral components of DM candidates at the TeV scale originating from RH fermionic triplets, rather than the LH triplets, have been suggested to be produced at high temperature through non-equilibrium thermal production process in non-SUSY SO(10) where the charged components acquire larger intermediate scale masses [94]. The direct detection, indirect detection, and collider search for triplet fermion DM at $p - p$ collider have been analysed in [146]. Phenomenology of wino DM in the mass range 500 – 2000 GeV which has much similarity with this non-SUSY triplet fermionic DM, $\rho_3$, has been also discussed recently [147].

(ii). Direct Detection and Collider Signatures

In general, for elastic scattering of a DM particle (which is electrically neutral) off nucleons
either a standard Higgs or a $Z$-boson exchange is needed in the t-channel of the dominant tree diagrams. In the absence of such couplings of $\Sigma^0$, a sub-dominant process occurs by the exchange of two virtual $W^\pm$ bosons in a box diagram [141]. This process leads to suppression of spin independent cross section by $2-3$ orders below the experimentally detectable value. However, such predicted cross sections are measurable with improvement of detector sensitivities [148]. The inelastic scattering with a charged component ($\Sigma^+$ or $\Sigma^-$) is prevented because of kinematic constraints since the mass difference, $m_{\Sigma^+} - m_{\Sigma^0} = 166$ MeV, is about three orders of magnitude above the kinetic energy of $\Sigma$ and also much above the proton-neutron mass difference, $m_n - m_p \sim 2$ MeV. If the triplet fermion has mass $\sim 400$ GeV, its contribution to the spin independent cross section is found to suffer from deviation from the LUX direct bound [149].

Prospects of observing signatures of the triplet fermion DM at colliders have been investigated in [146, 150–152]. For $m_\Sigma \sim 2.7$ TeV and integrated luminosity of $100fb^{-1}$, the DM pair production cross section at LHC in the channel $pp \rightarrow \Sigma\Sigma X$ has been shown to result in only one event [150, 151]. For better detection capabilities upgradation of LHC with twice energy and more luminosity has been suggested [152].

For detection at $e^+e^-$ collider that requires a collision energy of at least twice the DM mass, observation of $\Sigma^+\Sigma^-$ pair production is predicted via $Z$ boson exchange[141, 150]. The neutral pair $\Sigma^0\Sigma^0*$ can be also produced, although at a suppressed rate, through one-loop box diagram mediated by two virtual $W$ bosons. After production such charged components would provide a clean signal as they would manifest in long lived charged tracks due to their decays via standard gauge boson interactions, $\Sigma^\pm \rightarrow W^\pm \rightarrow \Sigma^0\pi^\pm$, or $\Sigma^\pm \rightarrow W^\pm \rightarrow \Sigma^0t^\pm l^-(l = e, \mu)$. The production of $e^\pm$ and $\mu^\pm$ charged leptons but the absence of $\tau^\pm$ due to kinematical constraint may be another distinguishing experimental signature of the triplet fermionic DM. The decay length of such displaced vertices is clearly predicted [141, 150] to be $L_{\Sigma^\pm} \simeq 5.5$ cm.

A contrasting feature regarding the fate of the produced neutral component of the triplet fermion DM, $\Sigma^0 \subset SO(10)$, different from the prediction of [150, 151], has been observed in ref.[72]. In the case of ref.[150, 151] it has been suggested that the corresponding $\Sigma^0$ can decay into leptons. But it has been noted in the context of the matter parity conserved SO(10) model [72] that the decay product $\Sigma^0$ is stable because of its matter parity. As such the production of this neutral component of the triplet fermion DM originating from SO(10) will be signalled through missing energy [72]. This stability feature of $\Sigma^0$ with its TeV scale mass has negligible impact on electroweak precision variables. These interesting features are applicable also in the present model under investigation.

4.3.1 Prospects from Indirect Searches

PAMELA [153] and FERMI/LAT [154] experiments concluded the positron excess in case of the WIMP as DM candidate which is again confirmed by recent AMS-02 [155] data [156]. The electron and positron flux is still significant in the measurement of FERMI/LAT. There are various constraints on the wino dark matter from different search channels such as antiprotons, leptons, dark matter halo from diffuse galactic gamma rays, high latitude
gamma-ray spectra, galaxy clusters, dwarf spheroids, gamma-ray line feature, neutrinos from the galactic halo, CMB constraints, and antideuterons [142]. In the case of the antiproton search channel the wino dark matter having mass close to the resonance i.e, 2.4 TeV, and thin zone of diffusion is consistent with the antiproton measurement. The wino dark matter having mass near the resonance produces very small amount of leptons and large amount of positrons at very low energy scale. This DM can not solve cosmic ray (CR) lepton puzzle because the lepton data can rule out the very proximity of resonance. The galactic $\gamma$ rays impose a stringent limit on the wino DM model. With the inclusion of the $\gamma$ ray constraint, the limit on the wino DM changes. If the mass of DM is 2.5 TeV and it is in a thin diffusion zone, then it is excluded by the $\gamma$ ray data for a wide variation of galactic CR propagation. There is also a very significant limit on the wino dark matter from high latitude $\gamma$ ray spectra. For a 2.5 TeV wino DM the expected 10 year cross section is $1.5 \times 10^{-25} \text{cm}^3\text{s}^{-1}$ including DM substructures [142]. Possible signatures of DM annihilations are given from $\gamma$ ray observations [157, 158] towards nearby galaxy clusters but observations in ref. [159–164] have not seen any significant limits from $\gamma$ ray excess. The wino dark matter having mass 2.4 TeV can be ruled out in this search channel whereas all the other masses are allowed in the dwarf spheroids channel [142]. The winos with masses heavier than 2 TeV are excluded by the HESS [159] data at 95% CL. A new method to search for the indirect signals of DM annihilation is obtained due to the motion of high energy neutrons towards the galactic center. Wino models having the mass 2.4 TeV can be observed in this search channel [142]. There is also a constraint on the wino dark matter due to the CMB temperature and polarization power spectra. Taking WMAP-5 [165] data and with 98% CL, the DM masses in the region 2.3 TeV to 2.4 TeV have been excluded. With WMAP-9 [168] the excluded limit is 2.25 – 2.46 TeV. But the combined search of WMAP-9 with ACT [166, 167] excludes the mass range of 2.18 – 2.5 TeV . To search for the dark matter, the most effective channel is through antideuterons. Due to the smaller signal to back ground ratio at mass 2.5 TeV, the resultant signal is very low with high uncertainty. With the theoretical and experimental progress, there may be stringent limit on the wino dark matter [142].

In our model the triplet fermionic thermal DM resulting from any one of the nonstandard fermionic representations $45_F$, $54_F$, or $210_F$ would be adequate although we have preferred to choose the minimal of these three representations in order to minimise the impact on GUT threshold uncertainties as discussed in Sec.6.

5 Gauge Coupling Unification

In this section we discuss gauge coupling unification at the two-loop level using lighter scalar and fermionic degrees of freedom motivated by solutions to the neutrino masses by hybrid seesaw, dark matter and leptogenesis. At first exact unification of the three gauge couplings is realized using a triplet scalar $\Delta_L(1,3,0)$ at $M_\Delta = 10^{12}$ GeV, a triplet fermion $\Sigma_F(1,3,-1)$ at $M_F \sim 500 – 1000$ GeV, and, in addition, a color octet fermion of Majorana-Weyl type at $M_{C_8} \sim 5 \times 10^7$ GeV. We then estimate threshold effects on the GUT scale
due to various superheavy components in the theory. We discuss proton life prediction in the model including these threshold uncertainties.

5.1 Unification with Lighter Fermions and Scalars

We use the standard renormalization group equations (RGEs) for the evolution of the three gauge couplings [6] and their integral forms are

$$\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i(M_U)} + \frac{a_i}{2\pi} \ln \left( \frac{M_{\Sigma}}{M_Z} \right) + \frac{a'_i}{2\pi} \ln \left( \frac{M_{C_8}}{M_{\Sigma}} \right) + \frac{a''_i}{2\pi} \ln \left( \frac{M_{\Delta}}{M_{C_8}} \right)$$

$$+ \frac{a'''_i}{2\pi} \ln \left( \frac{M_U}{M_{\Delta}} \right) + \Theta'_i + \Theta''_i + \Theta'''_i - \frac{\lambda_i}{12\pi}, \quad (5.1)$$

where $M_{\Sigma} = \text{triplet fermionic DM mass scale}$, $M_{\Delta} = \text{LH triplet mass mediating type-II seesaw}$, and $M_{C_8} = \text{additional fermion octet mass scale found to be necessary to achieve exact unification of the three gauge couplings at two-loop level}$. The one-loop coefficients $a'_i, a''_i, a'''_i$ in their respective ranges of mass scales are shown in Table 3 in the Appendix. The terms $\Theta'_i, \Theta''_i$, and $\Theta'''_i$ are the two-loop contributions in the three different ranges of the mass scales with the respective coefficients $B'_{ij}, B''_{ij}, B'''_{ij}$ given in Table 3.

$$\Theta_i = \frac{1}{4\pi} \sum_j B_{ij} \ln \frac{\alpha_j(M_{\Sigma})}{\alpha_j(M_Z)}, \quad \Theta'_i = \frac{1}{4\pi} \sum_j B'_{ij} \ln \frac{\alpha_j(M_{C_8})}{\alpha_j(M_{\Sigma})},$$

$$\Theta''_i = \frac{1}{4\pi} \sum_j B''_{ij} \ln \frac{\alpha_j(M_{\Delta})}{\alpha_j(M_{C_8})}, \quad \Theta'''_i = \frac{1}{4\pi} \sum_j B'''_{ij} \ln \frac{\alpha_j(M_U)}{\alpha_j(M_{\Delta})}. \quad (5.2)$$

The term $\frac{\lambda_i}{12\pi}$ represents GUT threshold effects on the respective gauge coupling due to super-heavy particles existing around $\mu = M_U$. These may be superheavy Higgs scalars, fermions, or gauge bosons [77, 101–105].

In terms of the experimentally determined parameters at the electroweak scale [170]: $\sin^2 \theta_W(M_Z) = 0.23126 \pm 0.00005, \alpha(M_Z) = 1/127.9$, and $\alpha_S(M_Z) = 0.1187 \pm 0.0017$, we define

$$P_S = \frac{2\pi}{\alpha(M_Z)} \left( 1 - \frac{8}{3} \frac{\alpha(M_Z)}{\alpha_S(M_Z)} \right),$$

$$P_\Theta = \frac{2\pi}{\alpha(M_Z)} \left( 1 - \frac{8}{3} \sin^2 \theta_W(M_Z) \right). \quad (5.3)$$

From the RGEs of eq.(5.1), the corresponding RGEs for $P_S$ and $P_\Theta$ are obtained. These two are then solved to yield formulas for the two mass scales $M_U$ and $M_{\Delta}$

$$\ln \left( \frac{M_U}{M_Z} \right) = \frac{P_S B_\Theta - P_\Theta B_S}{D} + \frac{C_\Theta B_S - C_S B_\Theta}{D} + \frac{B_S T_\Theta - B_\Theta T_S}{D},$$

$$\ln \left( \frac{M_{\Delta}}{M_Z} \right) = \frac{A_S P_\Theta - A_\Theta P_S}{D} + \frac{C_S A_\Theta - C_\Theta A_S}{D} + \frac{A_\Theta T_S - A_S T_\Theta}{D}. \quad (5.4)$$
In eq.(5.4)
\[A_S = (5/3)a_1'' + a_2'' - (8/3)a_3'',\]
\[A_\Theta = (5/3)\left(a_1'' - a_2''\right),\]
\[B_S = (5/3)a_1'' + a_2'' - (8/3)a_3'' - A_S,\]
\[B_\Theta = (5/3)\left(a_1'' - a_2''\right) - A_\Theta,\]
\[T_S = \frac{1}{6}[(8/3)\lambda_3 - \lambda_2 - (5/3)\lambda_1],\]
\[T_\Theta = \frac{5}{18}[(\lambda_2 - \lambda_1)],\]
\[D = A_S B_\Theta - A_\Theta B_S.\] (5.5)

Apart from depending upon the RG coefficients, the quantities \(C_S\) ans \(C_\Theta\) in eq.(5.4) depend upon the lighter mass scales \(M_\Sigma\) and \(M_C^8\)
\[C_S = \left[(5/3)(a_1' - a_1'') + a_2' - a_2'' - (8/3)(a_3' - a_3'')\right] \ln\left(\frac{M_C^8}{M_Z}\right),\]
\[C_\Theta = \left[(5/3)(a_1' - a_2' - a_1'' + a_2'')\right] \ln\left(\frac{M_C^8}{M_Z}\right),\]
\[C_\Theta = \left[(5/3)(a_1 - a_2 - a_1' + a_2')\right] \ln\left(\frac{M_\Sigma}{M_Z}\right).\] (5.6)

In deriving the analytic formulas in eq.(5.4) we have ignored the two-loop terms for the sake of simplicity although they have been included in numerical estimations of mass scales involved. It is clear that in eq.(5.4) the first two terms in the RHS for the two mass scales \(M_\Sigma\) and \(M_C^8\) represent the one-loop contributions but the third term in each case represents the corresponding threshold correction.

At first retaining only one-loop and the two-loop contributions we find excellent unification of the three gauge couplings for \(M_\Sigma = 500 - 1000\) GeV, \(M_C^8 \sim 5 \times 10^7\) GeV and \(M_\Delta = 10^{12}\) GeV. This is shown in Fig.14.

In this model we have found the necessity of either two color octet scalars \(S_8(8,1,0)\) or a single octet fermion \(C_8(8,1,0)\) at mass \(M_C^8 \sim 5 \times 10^7\) GeV, in addition to the triplet fermionic DM candidate \(\Sigma_F(1,3,0)\) and the LH triplet scalar \(\Delta_L(1,3,-1)\). This color octet fermion is thus safely above the cosmologically allowed limit [169]. The two-loop prediction of the GUT scale and the gauge coupling are
\[M_U^0 = 10^{15.56}\text{GeV},\]
\[g_G(M_U) = 0.573\] (5.7)

6 Threshold Corrections and Proton Lifetime Prediction

6.1 Threshold Effects on the GUT Scale

As pointed out in Sec.1, the superheavy components of the representation 126\(H\) is expected to contribute substantially to the GUT threshold effects on the GUT scale \(M_U\) and hence
Figure 14. Unification of couplings of the SM gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ in the presence of LH triplet scalar $\Delta_L$, the triplet fermionic dark matter $\Sigma_F$, and the color octet fermion $C_8$ as described in the text. The ordinates corresponding to these masses $M_\Sigma, M_{C_8}, M_\Delta$ and the GUT scale $M_U$ are indicated along the X-axis. The ordinates corresponding to these masses $M_\Sigma, M_{C_8}, M_\Delta$ and the GUT scale $M_U$ are indicated along the X-axis. 

The proton lifetime predictions. In this estimation at first we assume all the superheavy DM components in $45_F$ to be exactly degenerate with the GUT scale leading to their vanishing threshold effects. In the next step we estimate the fermionic contribution by following the same procedure [101–105].

From the last term in eq.(5.4), the analytic formula for GUT threshold effects on the unification scale is

$$\Delta \ln(M_U/M_Z) = (54/1829) \left( (40/81)\lambda_1 - (4/27)\lambda_2 - (28/81)\lambda_3 \right)$$

where for the $i$th super-heavy scalar component $\lambda_i = tr(t^2_i) \ln(M_{S_i}/M_U)$. But for Weyl (Dirac) fermions near the GUT scale there is multiplicative factor $4(8)$. The numerical values for $tr(t^2_i)$ for each submultiplet has been given in the corresponding Tables in Appendix B.

We next evaluate the functions $\lambda_i(M_U)$ involving small logs caused due to super-heavy scalar components in the loop. These are contained in the SO(10) Higgs representations $10_H, 45_H$, and $126_H$. We further introduce the “partially degenerate” assumption on the super-heavy component masses of Higgs scalars which has been found to be useful in handling large representations especially in $SO(10)$ [77]. Under this assumption all super-heavy scalar masses belonging to a given representation have a common degenerate mass.
Then using decompositions of representations shown in the Appendix we find

\[
\begin{align*}
\lambda_1 &= 17/5 + 4\eta_{(10)} + (0)\eta_{(45)} + 136\eta_{(126)}, \\
\lambda_2 &= 6 + 4\eta_{(10)} + 2\eta_{(45)} + 140\eta_{(126)}, \\
\lambda_3 &= 8 + 4\eta_{(10)} + 3\eta_{(45)} + 140\eta_{126},
\end{align*}
\]

where \( \eta_X = \ln(M_X/M_U) \). The constant terms on the RHS of eq.(6.2) represent the contributions of 33 super-heavy gauge bosons assumed to be degenerate at the GUT scale \( M_U \). The dominant contributions to the threshold factors \( \lambda_i \) in eq.(6.2) arising out of the super-heavy scalar components of \( 126^H \) are quite explicit.

Using eq.(6.2) in eq.(6.1) and maximizing the uncertainty [77] gives

\[
\left[ \frac{M_U}{M_0^U} \right]_S = 10^{\pm 0.928\eta_S},
\]

\[
\eta_S = |\log_{10} \left[ \frac{M_{SH}}{M_U} \right]|, \tag{6.3}
\]

where \( M_{SH} \) is the super-heavy Higgs mass scale and \( M_0^U \) represents the two-loop solution of eq.(5.7) without threshold effects. Similarly excluding the light triplet DM component \( \Sigma_F(1, 3, 0) \), the rest of the fermionic component of the representation \( 45_F \) contribute to the threshold effects

\[
\left[ \frac{M_U}{M_0^U} \right]_F = 10^{\pm 0.253\eta_F},
\]

\[
\eta_F = |\log_{10} \left[ \frac{M_F}{M_U} \right]|, \tag{6.4}
\]

We also note that the degenerate super-heavy gauge bosons contribute a very small correction with a positive sign

\[
\left[ \frac{M_U}{M_0^U} \right]_V = 10^{0.0227}. \tag{6.5}
\]

In general following Coleman-Weinberg [171] idea, \( M_{SH} \) could vary quite naturally within the range \( M_U/10 \) to \( 10M_U \). As the the super-heavy fermionic components are unaffected by such corrections it may be natural to treat their masses to be degenerate at the GUT scale or at a degenerate mass \( M_F \) around \( M_U \). In the first case they do not contribute to threshold corrections to the corrected unification scale. We have considered the general case with degenerate mass \( M_F = (1/10 \rightarrow 10)M_F \). Adding all corrections together we get

\[
M_U = 10^{15.56 + 0.0227\pm 0.928\eta_S \pm 0.253\eta_F} \text{GeV} \tag{6.6}
\]

Treating this as the mass of super-heavy gauge bosons mediating proton decay, we next estimate proton lifetime prediction in the model.

### 6.2 Proton Lifetime Prediction

As the unification scale predicted by this model has an uncertainty naturally dictated by the matter parity motivated SO(10) model, it would be interesting to examine its impact
on proton life time predictions for $p \rightarrow e^+ \pi^0$ for which there are ongoing dedicated experimental searches [53, 175–177] with measured value of the lower limit on the life time [110, 178]

$$\tau_p^{\text{exp.}} \geq 1.4 \times 10^{34} \text{ yrs.} \quad (6.7)$$

Including strong and electroweak renormalization effects on the $d = 6$ operator and taking into account quark mixing, chiral symmetry breaking effects, and lattice gauge theory estimations, the decay rates for the two models are [53, 173, 174],

$$\Gamma(p \rightarrow e^+ \pi^0) = \frac{m_p}{64\pi f_\pi^2} g^4 G \left| A_L \right|^2 (1 + D' + F)^2 \times R, \quad (6.8)$$

where $R = \left[ A_{SR}^2 + A_{SL}^2 (1 + |V_{ud}|^2) \right]$ for $SU(5)$, but $R = \left[ (A_{SR}^2 + A_{SL}^2) (1 + |V_{ud}|^2) \right]$ for $SO(10)$, $V_{ud} = 0.974 = \{1,1\}$ element of $V_{\text{CKM}}$ for quark mixings, and $A_{SL}(A_{SR})$ is the short-distance renormalization factor in the left (right) sectors. In eq.(6.8) $A_L = 1.25 = \text{long distance renormalization factor}$ but $A_{SL} \approx A_{SR} = 2.542$. These are numerically estimated by evolving the dim.6 operator for proton decay by using the anomalous dimensions of ref.[173] and the beta function coefficients for gauge couplings of this model. In eq.(6.8) $M_U = \text{degenerate mass of super-heavy gauge bosons}$, $\tilde{\alpha}_H = \text{hadronic matrix elements}$, $m_p = \text{proton mass} = 938.3 \text{ MeV}$, $f_\pi = \text{pion decay constant} = 139 \text{ MeV}$, and the chiral Lagrangian parameters are $D = 0.81$ and $F = 0.47$. With $\alpha_H = \tilde{\alpha}_H (1 + D' + F) = 0.012 \text{ GeV}^3$ estimated from lattice gauge theory computations [172], we obtain $A_R \approx A_L A_{SL} \approx A_L A_{SR} \approx 3.18$ and the expression for the inverse decay rate is,

$$\Gamma^{-1}(p \rightarrow e^+ \pi^0) = \frac{4 f_\pi^2 M_U^4}{m_p} \frac{1}{\alpha_G^2} \frac{1}{\alpha_H^2 A_R^2 F_q}, \quad (6.9)$$

where the GUT-fine structure constant $\alpha_G = 0.0263$ and the factor $F_q = 2(1 + |V_{ud}|^2)^2 \approx 7.6$ for $SO(10)$. This formula reduces to the form given in [53, 138] and sets the lower limit for the non-SUSY GUT scale to be $M_U \geq 10^{15.5} \text{ GeV}$ from the lower limit of eq.(6.7).

Now using the estimated values of the model parameters eq.(6.9) gives,

$$\tau_p^{SO(10)} \simeq 1.8 \times 10^{34 \pm 3.712 \eta_S \pm 1.012 \eta_F} \text{ yrs.} \quad (6.10)$$

As an example, a super-heavy scalar mass splitting by a factor $2(1/2)$ from the GUT scale gives $\eta_S = 0.3(-0.3)$ leading to $\tau_p \sim 1.8 \times 10^{34 \pm 1.11}$ yrs even if all fermion masses are at $M_U^0$. Similarly if all super-heavy scalar masses are degenerate at the unification scale $M_U^0$, the super-heavy fermions with their mass splitting factor $2(1/2)$ lead to $\tau_p \sim 1.8 \times 10^{34 \pm 0.3}$ yrs. These lifetimes are clearly above the current experimental limit but accessible to ongoing
Figure 15. Proton lifetime prediction for the decay mode $p \to e^+ \pi^0$ shown by slanting solid lines as a function of $\eta = \eta_S(\eta_F) = |\log_{10}(M_{SH}/M_U)|(|\log_{10}(M_F/M_U)|)$ for super-heavy scalar(fermion) components. The shaded green colored region is ruled out by the current experimental bound. The point at $\eta_S = \eta_F = 0$ represents the model prediction at two-loop level without threshold effects with $\tau_0^p = 1.8 \times 10^{34}$ yrs.

searches. The proton lifetime predictions as a function of $\eta = \eta_S$ or $\eta = \eta_F$ are shown in Fig.15 for the $p \to e^+ \pi^0$ decay mode.

It is clear that most of the uncertainties arise out of the GUT threshold corrections due to the larger Higgs representation $126_H$ which plays the central role in determining the contents of dark matter and their stability in the non-SUSY SO(10) by preserving matter parity as gauged discrete symmetry. We note that such uncertainties which are crucial for proton decay searches have been estimated here for the first time. The DM motivated SO(10) also predicts additional threshold corrections to proton lifetime predictions especially due to fermions. Although this may enhance the uncertainty further, in one class of solutions the model also offers an interesting new possibility compared to GUTs without fermionic dark matter. The fermionic threshold corrections may contribute to cancel out a substantial part of the scalar threshold effects in another class of solutions which are shown by the blue curve marked $S - F$ in Fig.15. With this cancellation, the proton decay has somewhat more probability for detection by the ongoing searches.
7 Summary and Conclusion

In this work we have attempted unification of gauge couplings of the non-SUSY standard
gauge theory by addressing solutions to three of its outstanding problems: neutrino masses,
dark matter, and baryon asymmetry of the Universe (BAU). To achieve these objectives
we have exploited an interesting breaking pattern of non-SUSY SO(10) by assigning GUT
scale VEV to the representation $126_H$ where matter parity is conserved as a natural gauged
discrete symmetry of the SM that guarantees dark matter stability. As the origin of dark
matter candidates, the model classifies non-standard fermionic or scalar representations of
non-SUSY SO(10) carrying even or odd matter parity containing suitable components of
dark matter. It predicts the type-I $\oplus$ type-II as the hybrid seesaw formula for neutrino
masses driven by LH scalar triplet $\Delta_L(1, 3, -1)$ and heavy RH neutrinos. This formula
has been used here to fit the neutrino oscillation data that predicts the heavy masses of
the scalar triplet and the RH$\nu$ masses. We have carried out this fitting procedure using
values of the Dirac neutrino mass matrix derived in two ways by assuming $u$-quark diagonal
or the $d$-quark diagonal bases. For a given intermediate mass value of the scalar triplet,
induced VEV, and Dirac neutrino mass matrix, this seesaw formula being quadratic in Ma-
jorana neutrino Yukawa coupling $f$, predicts two distinct sets of RH$\nu$ masses: (i) Compact
spectrum where all three masses are heavier than the Davidson-Ibarra (DI) bound, and
(ii) Hierarchical spectrum where only $N_1$ is lighter than the DI bound. These solutions
provide a variety of results on the surviving lepton asymmetries after washout factors are
adequately taken into account. We have carried out a complete flavor analysis of the RH$\nu$
decays and exploited solutions to Boltzmann equations in every case to arrive at the model
predictions on the baryon asymmetry. Although the decay of the LH scalar triplet in this
model is found to yield negligible CP-asymmetry, it contributes quite significantly through
the new Feynman diagram it generates for the vertex correction of RH$\nu$ decays. In fact
this contribution to the CP-asymmetry is found to be as dominant as other contributions
without triplet mediation. The decay of the lightest RH$\nu$ in the compact spectrum scenario
predicts the values of BAU in agreement with the existing data when the Dirac neutrino
mass determination is associated with either the $u$-quark diagonal basis or the $d$-quark
diagonal basis. In the case of hierarchical spectrum of RH neutrinos, the right value of
BAU is predicted by the $N_2$ decay where the Dirac neutrino mass is associated with the
$u$-quark diagonal basis. This has been found possible even if the initial condition satisfies
vanishing $N_2$ abundance.

With the matter parity available as the stabilising discrete symmetry for dark matter,
the neutral component of hyperchargeless triplet fermion $\Sigma_F(1, 3, 0) \subset 45_F \subset SO(10)$
having even matter parity is well accommodated as a candidate for thermal dark matter
at TeV scale whose phenomenology has been discussed extensively in the literature and
summarized here. Having thus addressed solutions to the three outstanding problems of
the SM as stated above, we implemented unification of the three gauge couplings which
needed a fermionic color octet of mass $M_C \sim 5 \times 10^7$ GeV, in addition to the heavy
Higgs scalar triplet, and the fermionic triplet dark matter. The two-loop solutions yielded
excellent unification with the predicted GUT scale value $M_U = 10^{15.56\pm0.0288}$ GeV where the small positive fraction in the exponent is due to degenerate masses of all superheavy gauge bosons at $M_U^0$ that causes nearly 30\% increase in the proton lifetime prediction over its two-loop prediction. Noting the compelling requirement of the scalar representation $126_H$ to drive the symmetry breaking in this SO(10) model, its superheavy components predict substantial GUT threshold effects on the unification scale and proton lifetime. We have also estimated threshold corrections on the predicted proton lifetime due to superheavy fermions in $45_F$. An interesting possibility of cancelling out a substantial part of threshold corrections due to scalars by fermions has been pointed out. We find that a large region of the parameter space can be explored by the ongoing searches on proton decay $p \rightarrow e^+\pi^0$.

In conclusion we find that in the non-supersymmetric standard gauge theory, the predictions for neutrino masses, dark matter, baryon asymmetry of the universe, unification of gauge couplings, and proton lifetime accessible to ongoing searches can be successfully implemented through direct breaking of non-SUSY SO(10) with particle content inherent to matter parity conservation. The only additional particle needed beyond these requirements for coupling unification is a color octet Weyl fermion (or a pair of complex color octet scalars) which also belong to the SO(10) GUT representation. The introduction of the scalar triplet $\Delta_L$ at the intermediate scale brought in naturally by matter parity conservation in SO(10) causes remarkable changes in the model predictions over its conventional values. The very fact of successful implementation of the current programme in SO(10) resolves the issue of parity violation as a monopoly of weak interaction.

8 Appendix: Renormalization Group Coefficients for Unification of Gauge Couplings and Threshold Uncertainties

In the Appendix A below we provide various decompositions of SO(10) representations under different subgroups relevant for the present work. In Appendix B we give different beta function coefficients along with particle content for different mass scales.

8.1 Appendix A: Decomposition of Representations and Beta Function Coefficients

In this Appendix we present decompositions of non-SUSY SO(10) representations under SU(5) as shown in Table 1.

8.1.1 Particle Content and Beta Function Coefficients

In this subsection we present the particle content used in various ranges of mass scales as shown in Table 2 and the corresponding beta-function coefficients which have contributed for the gauge coupling unification, leptogenesis, and dark matter as shown in Table 3.
$$\text{SO}(10) \supset \text{SU}(5)$$

- \(10 \supset 5 + 5^\dagger\)
- \(16 \supset 10 + 5^\dagger + 1\)
- \(45 \supset 10 + 10^\dagger + 1 + 24\)
- \(54 \supset 24 + 15 + 15^\dagger\)
- \(120 \supset 5 + 5^\dagger + 45^\dagger + 10 + 45\)
- \(126 \supset 5^\dagger + 45 + 15^\dagger + 50^\dagger + 10 + 1\)
- \(210 \supset 1 + 24 + 10^\dagger + 10 + 40 + 40^\dagger\)
- \(54 \supset 24 + 10 + 10^\dagger + 1 + 24\)
- \(120 \supset 5 + 5^\dagger + 10 + 45^\dagger + 10 + 45\)
- \(126 \supset 5^\dagger + 45 + 15^\dagger + 50^\dagger + 10 + 1\)
- \(210 \supset 1 + 24 + 10^\dagger + 10 + 40 + 40^\dagger\)

Table 1. Decomposition of SO(10) representations into SU(5) representations [18].

| Energy Scale     | Particle content                      |
|------------------|---------------------------------------|
| \(M_Z - M_T\)    | SM Particles                          |
| \(M_T - M_O\)    | SM+(1,3,0)\_F                        |
| \(M_O - M_\Delta\)| SM +(1,3,0)\_F +(8,1,0)\_F          |
| \(M_\Delta - M_U\)| SM +(1,3,0)\_F +(8,1,0)\_F +(1,3,1)\_H |

Table 2. Particle content of the model in different ranges of mass scales.

| \(\mu\)         | \(a_i\)       | \(a_{ij}\)  |
|------------------|---------------|-------------|
| \(M_Z - M_T\)    | \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} | \begin{pmatrix} 199/50 \\ 27/10 \\ 44/5 \end{pmatrix} |
| \(M_T - M_O\)    | \begin{pmatrix} 41/10 \\ -11/6 \\ -7 \end{pmatrix} | \begin{pmatrix} 199/50 \\ 27/10 \\ 44/5 \end{pmatrix} |
| \(M_O - M_\Delta\)| \begin{pmatrix} 41/10 \\ -11/6 \\ -5 \end{pmatrix} | \begin{pmatrix} 199/50 \\ 27/10 \\ 44/5 \end{pmatrix} |
| \(M_\Delta - M_U\)| \begin{pmatrix} 43/10 \\ -7/6 \\ -5 \end{pmatrix} | \begin{pmatrix} 83/10 \\ 171/10 \\ 44/5 \end{pmatrix} |

Table 3. One-loop and two-loop beta function coefficients in the respective ranges of mass scales.

8.2 Appendix B: Super-heavy Particles and Coefficients for Threshold Effects

In this subsection we identify the super-heavy particle contents of various SO(10) representations with their quantum numbers and beta function coefficients under the SM gauge group. These coefficients shown in Table 4, Table 5, and Table 6 have been used for the estimation of threshold effects on proton lifetime predictions.
Table 4. Decomposition of the complex 10 representation under SU(5) and one-loop coefficients.

| SU(5) | (3C, 2L, 1Y) | tr(t^2) |
|-------|---------------|--------|
| 5     | (3, 1; -\frac{1}{3}) | (1, 0, 2/5) |
|       | (1, 2; -\frac{1}{2}) | (0, 1, 3/5) |

| SU(5) | (3C, 2L, 1Y) | tr(t^2) |
|-------|---------------|--------|
| 5     | (\bar{3}, 1; \frac{2}{3}) | (1, 0, 2/5) |
|       | (1, 2; \frac{1}{3}) | (0, 1, 3/5) |

Table 5. Decomposition of the real 45 representation under SU(5) and one-loop coefficients. For the sake of convenience, the would-be goldstone modes of all super-heavy gauge bosons have been provided from the scalar representation 45_H.

| SU(5) | (3C, 2L, 1Y) | tr(t^2) |
|-------|---------------|--------|
| (10)  | (1, 1; -1) | (0, 0, 3/5) |
|       | (3, 2; -\frac{5}{6}) | (1, 3/2, 5/2) |
|       | (\bar{3}, 1; -\frac{2}{3}) | (1/2, 0, 1/5) |

| (10)  | (1, 1; 1) | (0, 0, 3/5) |
|       | (\bar{3}, 2; \frac{5}{6}) | (1, 3/2, 5/2) |
|       | (3, 1; \frac{1}{3}) | (1/2, 0, 1/5) |

| (24)  | (1, 1; 0) | (0, 0, 0) |
|       | (1, 3; 0) | (0, 2, 0) |
|       | (8, 1; 0) | (3, 0, 0) |
|       | (3, 2; \frac{1}{3}) | (1, 3/2, 1/10) |
|       | (\bar{3}, 2; -\frac{1}{6}) | (1, 3/2, 1/10) |

8.3 Appendix C: A discussion on charged fermion mass parametrization

While all single step descents of SUSY GUTs leading to MSSM exhibit almost profound gauge coupling unification, there has been several attempts in SUSY SO(10) to explain fermion masses of three generations of quarks and leptons along with the attractive phenomena like b - \tau or t - b - \tau Yukawa unification. In certain other cases approximate validity of some of the Georgi-Jarlskog [36] type mass relations

\[ m_\mu^0 \approx 3m_e^0, \]
\[ m_\tau^0 \approx m_b^0, \]
\[ m_d^0 \approx 3m_e^0. \]  \hspace{1cm} (8.1)

have been found to hold at the GUT scale. While some recent works have presented very attractive details of data analysis with \chi^2-fit [35] as pointed out in Sec.1, a much larger number of other research papers have confined to partially quantitative or qualitative representations of the charged fermion masses as these latter types of investigations focus on other challenging issues of particle physics. Compared to such interesting results on fermion mass fits in the direct breaking model of SUSY SO(10) [35], non-SUSY models
Table 6. Decomposition of the representation $\mathbf{126}$ under SU(5) and one-loop coefficients

| $SU(5)$ | $(3C, 2L, 1Y)$ | $\text{tr}(t^2_\tau)$ |
|---------|----------------|---------------------|
| $(\mathbf{5})$ | $(3, 1; -\frac{1}{4})$ | $(1, 0, 2/5)$ |
| $(1, 2; -\frac{1}{2})$ | $(0, 1, 3/5)$ |
| $(\mathbf{15})$ | $(6, 1; \frac{2}{3})$ | $(5, 0, 16/5)$ |
| $(3, 2; \frac{1}{3})$ | $(2, 3, 1/5)$ |
| $(1, 3; 1)$ | $(0, 4, 18/5)$ |
| $(\mathbf{10})$ | $(1, 1; -1)$ | $(0, 0, 6/5)$ |
| $(3, 1; -\frac{2}{3})$ | $(1, 0, 8/5)$ |
| $(3, 2; -\frac{1}{6})$ | $(2, 3, 1/5)$ |
| $(\mathbf{50})$ | $(6, 3; -\frac{1}{4})$ | $(15, 24, 12/5)$ |
| $(1, 1; 0)$ | $(0, 0, 0)$ |
| $(3, 1; -\frac{1}{3})$ | $(1, 0, 2/5)$ |
| $(6, 1; -\frac{2}{3})$ | $(5, 0, 16/5)$ |
| $(3, 2; -\frac{1}{8})$ | $(2, 3, 1/5)$ |
| $(8, 2; -\frac{7}{4})$ | $(12, 8, 24/5)$ |
| $(\mathbf{45})$ | $(3, 1; \frac{1}{3})$ | $(1, 0, 2/5)$ |
| $(3, 3; \frac{2}{3})$ | $(3, 12, 6/5)$ |
| $(3, 1; \frac{2}{3})$ | $(1, 0, 8/5)$ |
| $(1, 2; \frac{1}{2})$ | $(0, 1, 3/5)$ |
| $(6, 1; \frac{1}{4})$ | $(5, 0, 4/5)$ |
| $(3, 2; \frac{1}{4})$ | $(2, 3, 1/5)$ |
| $(8, 2; \frac{1}{7})$ | $(12, 8, 24/5)$ |

need at least one intermediate gauge symmetry to ensure gauge coupling unification within the constraint of extended survival hypothesis [11, 12]. Also unlike the MSSM or SUSY SO(10), the RG extrapolated values of charged fermion masses through either SM or two-Higgs doublet model in the bottom-up approach [37] do not exhibit a precise $b - \tau$ Yukawa unification at the scale $\mu \sim 10^{16}$ GeV. Unlike the attempts to present all fermion masses in SUSY SO(10) through $\chi^2$ fit and non-SUSY case with $SU(4)_C \times SU(2)_L \times U(1)_R$ intermediate symmetry [35], to our knowledge no such analysis appears to have been done so far in the direct breaking of non-SUSY SO(10) where gauge coupling unification itself under the minimal fine-tuning constraint [11, 12] is highly challenging. In attempts to confront more challenging problems in SUSY or non-SUSY SO(10), a number of recent works have ignored the question of fitting the charged fermion masses while confining mainly to only neutrino masses and mixings, or at most a qualitative presentation of charged fermion masses [43–60]. However, even though a $\chi^2$ fit [35] is not our present goal, we point out how the charged fermion masses may be parameterized within this direct breaking model of non-SUSY SO(10) while successfully encompassing standard model paradigm at lower scales, neutrino masses, baryon asymmetry, dark matter, gauge coupling unification, and
The Higgs representations $10_H, 126_H$, and $120_H$ are known to contribute to fermion masses through the corresponding renormalizable Yukawa interactions. We include two copies of $10_H$ fields in the corresponding renormalizable part of the Yukawa Lagrangian

$$-\mathcal{L}^{(10)} = \sum_{p=u,d} Y_{ij}^{(p)} 16_i 16_j 10_{H_p},$$  \hspace{1cm} (8.2)$$

The Yukawa term $f_{16.16.126_H}$ has been found to be specifically suitable in approximately satisfying the GJ type relations in the down quark and charged lepton sectors. Conventionally, the same matrix $f$ also contributes to the RH neutrino mass matrix $M_N = f v_R$ which plays a crucial role in the type-I and type-II seesaw components of the hybrid seesaw formula used in this work. Therefore, the prime concern for charged fermion mass fit in the present model may be the smallness of the value of the matrix elements $f_{ij} \sim O(10^{-6}) (i,j = 1, 2)$ as shown in eq.(2.5), eq.(2.6), eq.(2.7), and eq.(2.8) needed for successful predictions of baryon asymmetry in this model. We provide below how this difficulty can be circumvented in two different ways : (i) Non-renormalizable, and (ii) Renormalizable; any one of these can be added to $\mathcal{L}^{(10)}$ for charged fermion mass parametrization.

(i). Non-Renormalizable Yukawa Correction:

There have been attempts to represent fermion masses in SUSY SO(10) via renormalizable interactions with additional flavor symmetries and flavon fields [180]. Without introducing any such additional fields or symmetries, our attempt here is confined to the non-SUSY SO(10) gauge symmetry and the Higgs representations of the model. We note that the following non-renormalizable Yukawa (NRY) interactions are allowed

$$\mathcal{L}_{NR}^{(1)} = \frac{F_{ij}^{(1)}}{M_G} 16_i 16_j 10_{H45_H},$$

$$\mathcal{L}_{NR}^{(2)} = \frac{F_{ij}^{(2)}}{M_G} 16_i 16_j 10_{H45_H45_H}. \hspace{1cm} (8.3)$$

where $M_G = \text{Planck scale } M_{\text{Planck}}, \text{ or the String scale } M_{\text{String}}$. The first Yukawa contribution is suppressed by a factor $\frac{M_{GUT}}{M_G} \sim 10^{-2} - 10^{-3}$. Noting that $10_H \times 45_H \supset 120_H \supset \xi(2,2,15)$, it contributes to non-diagonal elements of all Dirac type mass matrices antisymmetrically which we ignore in this qualitative explanation, but can be included if a $\chi^2$ fit is desired in future works. The second Yukawa interaction in eq.(8.3) containing $10_H \times 45_H \times 45_H$ has an effective $(2,2,15)_H$ component that is contained in $126$ and its contribution is symmetric. It is important to note that at the GUT scale $\mathcal{L}_{NR}^{(2)}$ gives a suppressed factor that adequately qualifies it to parameterize the needed additional corrections with $m_{ij}^0 \sim F_{ij}^{(2)} \frac{M_{GUT}}{M_G} v_{ew} \sim F_{ij}^{(2)} (10^{-4} - 10^{-5}) v_{ew}$. Thus, at the GUT scale the quark and lepton mass matrices can be parameterized as:

$$M_u = G_u + F_u, \hspace{0.5cm} M_D = G_u - 3 F_u,$$

$$M_d = G_d + F_d, \hspace{0.5cm} M_l = G_d - 3 F_d, \hspace{1cm} (8.4)$$
where $G_u = Y^{(u)} < 10H_u >$, $G_d = Y^{(d)} < 10H_d >$, $F_p \sim F_{(2)}10^{-4}$. $< 10H_p >$, $p = u,d$. Details of fermion mass parametrization goes in a manner similar to those discussed in [52, 85, 87–89].

(ii). Renormalizable Correction:

Through renormalizable interaction, the improvement of fermion mass parametrization is also suggested by the introduction of a second $126_H$ representation [87, 89]. We denote this and its corresponding components under $G_{224}$ as $126'_H \supset \Delta'_L(3, 1, 10)$, $\Delta'_R(1, 3, 10)$, $\xi'_L(2, 2, 15), \ldots$. In contrast to the $\Delta L \subset 126_H$ whose mass has been fine tuned to be at $M_{\Delta L} \sim 10^{12}$ GeV for the implementation of the type-II seesaw component of the hybrid seesaw formula, leptogenesis, and coupling unification, all the components of $126'_H$ are naturally assigned masses near the GUT scale consistent with extended survival hypothesis [11, 12]. Also no VEV is needed to be assigned to $\Delta'_R$ either i.e. we fix $< \Delta'_R >= 0$, since the corresponding role of gauge symmetry breaking has been taken over by $< \Delta_R(1, 3, 10)> = v_R \sim M_{GUT} \subset 126_H$.

Thus the presence of the second Higgs representation $126'_H$ does not affect the type-II seesaw and the RH neutrino mass parameters of type-I in the hybrid seesaw formula of eq.(2.2). Even upto the two-loop level it does not affect the gauge coupling unification of the present model. Denoting the corresponding SO(10) invariant Yukawa term as $f'16.16,(126)'$, we have renormalizable corrections to eq.(8.4) where $F_u \rightarrow F_u' = f' < \xi'_u >$, $F_d \rightarrow F_d' = f' < \xi'_d >$. It is well known that such corrections provide reasonable parameterization of the fermion masses of the first and second generations. With degeneracy of all superheavy components of $126'_H$, its threshold corrections to unification scale and proton lifetime are vanishingly small [179]. Similarly, if the renormalizable antisymmetric contributions to fermion mass matrices due to Yukawa interaction of a $120_H \subset SO(10)$ are included, its threshold effects on unification scale and proton lifetime would be also vanishingly small due to degeneracy of the components.

Alternatively the fermion mass parametrization may be improved further by including both the renormalizable and non-renormalizable contributions in eq. (8.4). In addition, the antisymmetric contribution through the first nonrenormalizable term in $\mathcal{L}^{(1)}_{NR}$ may be also included for still further improvement. Further, the antisymmetric NRY due to $\mathcal{L}^{(1)}_{NR}$ can be very well replaced by renormalizable Yukawa contribution $h^{(120)}16.16.120_H$.

The next question is whether this parametrization significantly affects the predicted results of this work where we have used the boundary condition $M_D(M_{GUT}) = M_u(M_{GUT})$. In SO(10) there are two maximal subgroups of rank 5: the Pati-Salam group $G_{224}$ and the flipped $SU(5) \times \tilde{U}(1)(\equiv G_R)$. When $SU(4)_C \subset G_{224}$ is unbroken, the assumed boundary condition is exact. Similarly it is well known that in the presence of $G_R$ symmetry $M_D(M_{GUT}) = M_u(M_{GUT})$. But in the process of SO(10) breaking to the SM, both these gauge symmetries are also broken and the boundary condition is approximate to the extent that $M_u - M_D = 4F_u$. This suggests that $\sigma_u \equiv 4F_u/m_{t_{top}}$ should be a small number in case fermion mass fit is also included as a required ingredient in this model. For a very preliminary estimation of $\sigma_u$, we note the interesting point that the GJ relation $m^0_\mu = 3m^0_e$ is almost exactly satisfied near the GUT scale $\sim 10^{15.56}$ GeV by values obtained in the
bottom-up approach within the SM paradigm [37]:
\[
    m_\mu^0 \sim 93.14 \pm 0.01 \text{ MeV}, \\
    m_\tau^0 \sim 34.59 \pm 5.0 \text{ MeV}.
\] (8.5)

With the dominance of the element \( (F_d)_{22} \) in the (22) elements of down-quark and charged lepton mass matrices, \(|(F_d)_{22}| \gg |(G_d)_{22}| \), gives \( (F_d)_{22} \sim 30 \text{ MeV} \) and a fractional change \( \left( \frac{\Delta M_D^{22}}{M_D^{22}} \right) \sim 0.3 \) compared to the uncorrected value of \( (M_D^p)_{22} = 262 \text{ MeV} \) shown in Sec.2.

We have checked that even after applying these corrections satisfying the first of GJ relation in eq.(8.1), our solutions and predictions on baryon asymmetry made in this work are not significantly affected. Also they remain largely unaffected as long as the corrections to the elements of the Dirac neutrino mass matrix \( M_D \) are either less or at most of the same order as those given in Sec.2. After the GUT symmetry breaking to the SM gauge theory we have assumed only one linear combination of different up type and down type doublets to remain massless to form the standard Higgs doublet.

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