Flavour in supersymmetric Grand Unification: 
a democratic approach

Riccardo Barbieri, Gia Dvali*, Alessandro Strumia,

Dipartimento di Fisica, Università di Pisa and
INFN, Sezione di Pisa, I-56126 Pisa, Italy

Zurab Berezhiani*,

INFN, Sezione di Ferrara, I-44100 Ferrara, Italy

and

Lawrence Hall

Department of Physics, University of California at Berkeley, California 94720

Abstract

We consider the flavour problem in a supersymmetric Grand Unified theory with gauged SU(6) group, where the Higgs doublets are understood as pseudo-Goldstone bosons of a larger SU(6) ⊗ SU(6) global symmetry of the Higgs superpotential. A key element of this work is that we never appeal to any flavour symmetry. One main interesting feature emerges: only one of the light fermions, an up-type quark, to be identified with the top, can get a Yukawa coupling at renormalizable level. This fact, together with bottom-tau Yukawa unification, also implied in our scheme, gives rise to a characteristic correlation between the top and the Higgs mass. By including a flavour-blind discrete symmetry and requiring that all higher dimensional operators be mediated by the exchanges of appropriate heavy multiplets, it is possible to give an approximate description of all masses and mixing angles in term of a hierarchy of grand unified scales. A special “texture” arises, implying a relation between the top mass and the third generation mixing angles. Several other possible consequences of this approach are pointed out, concerning the $\mu/s$ mass ratio, the Cabibbo angle and the proton decay.

* Permanent address: Institute of Physics, Georgian Academy of Sciences, 380077 Tbilisi, Georgia.
1 Introduction

It is definitely attractive to speculate that the physics of the elementary particles, as seen nowadays in experiments, is only the extremely low energy debris of a supersymmetric grand-unified world characterized by an energy scale not far from the Planck mass itself. The experimental developments of the last few years have brought some support to this view, more or less directly. Although far from established, the perturbative nature of the physical origin of the Fermi scale is more likely now than it was before the electroweak precision tests performed especially at LEP \cite{1}. On the other hand, these same experiments, with the consequent highly precise determination of the electroweak mixing angle, $\theta_W$, confirm the successful prediction of this quantity in supersymmetric Grand-Unification \cite{2}. Although the uncertainty of this comparison, dominated by theoretical errors, has not been decreased significantly, it has become nevertheless completely clear \cite{3} that, in the non supersymmetric case, it is possible to accommodate the measured $\sin^2 \theta_W$ only at the price of introducing an intermediate scale, namely one extra parameter. It is also clear how these “signals” for supersymmetry should turn into a direct experimental evidence, in a positive or a negative sense: finding, or not finding, a light Higgs and (some of) the superpartners at the Fermi scale.

From a theoretical point of view, a weakness of supersymmetric Grand Unification is the lack of clear progress in the understanding of flavour, where most of the parameters of the current theory remain undetermined\footnote{Bottom-tau Yukawa unification\cite{4} may constitute a case of partial but significant progress.}. The introduction, at the unification scale, of a large number of Yukawa couplings with the same, or similar, unexplained patterns as those ones of the Standard Model is an unsatisfactory feature of current Grand Unified Theories. Even the appeal to their potential understanding in a more fundamental underlying string theory may not satisfy many.

With these general motivations in mind, we describe here an attempt to improve on this situation. After a few considerations of general character, we develop them in the case of an SU(6) Grand Unified theory, where the light Higgs doublets are understood as pseudo-Goldstone bosons of a larger SU(6) $\otimes$ SU(6) global symmetry of the Higgs superpotential\cite{5,6}. The mechanism by which the Higgs doublets are split from the unwanted colour triplets in this theory, (to be briefly recalled later on), makes it appealing enough to be studied further\footnote{The pseudo-Goldstone boson mechanism for the doublet-triplet splitting in supersymmetric SU(5) Grand Unification was first suggested in ref.\cite{7}. Our results, however, are specific of the gauged SU(6) theory.}.

One immediate result of this approach is of great interest. It is likely, in a sense that will be made precise, that only one of the light fermions, an up-type quark, gets a Yukawa coupling at renormalizable level: its identification with the top quark is obvious. Crucial to this result is the fact that the top belongs to a representation of the gauge group not isomorphic to those that contain the other $Q = 2/3$ quarks. The smallness of the ratio between the bottom ($\lambda_b$) and the top ($\lambda_t$) Yukawa coupling, together with bottom-tau Yukawa unification, also implied in our scheme, requires, for consistency with the observed bottom/tau mass ratio, that $\lambda_t$, naturally of order 1, be actually at, or very close to, the infrared fixed point\cite{8}. In turn, this give rise to a correlation between the top and the lightest Higgs mass, illustrated in figure 1.

From a general point of view, our approach to the flavour problem can be characterized as follows. An interesting attempt has been recently made by Anderson et al.\cite{10} in supersymmetric SO(10). They assume that a flavour symmetry will select very few SO(10) operators, perhaps the minimal number that can account for all masses and mixings of quarks and leptons. They find these operators to have a particular flavour dependence and a particular dimensionality. It is an interesting problem to find out which flavour symmetry can explain all these special features. To some extent, the viewpoint adopted here is opposite to the one taken by Anderson et al. We want to find out, at least in the case of the SU(6) model, how far one can go in understanding quark and lepton masses and mixings without ever introducing a flavour symmetry. Is it possible to obtain a realistic theory of fermion masses, with significant predictions, which results from writing the most general Lagrangian involving a set of fields with given transformation under the gauge group.

In section 2 we develop a few considerations relevant for a generic supersymmetric Grand Unified theory with heavy fermions and non-renormalizable interactions scaled by inverse powers of the Planck
mass. In section 3 we focus on the SU(6) model, where we identify the light fields and we study their Yukawa couplings induced by renormalizable interactions. In section 4 we consider the possible effects of the non-renormalizable interactions and show how the ratio of the bottom, tau and charm masses to the top mass can be related to the ratio of the SU(6) breaking scale to the Planck mass. In section 5 we study the possibility of generating the wanted operators by means of appropriate heavy particle exchanges. In section 6 we show how a specific “texture” for the fermion masses arises. Finally, in section 7, we study the couplings of the heavy coloured triplet whose exchange is relevant to the proton decay amplitude $p \rightarrow K\bar{\nu}$ and we identify a possible mechanism for its suppression, intimately related with the origin of the doublet-triplet splitting. Conclusions are given in section 8.

2 General considerations

We consider in this section a generic supersymmetric Grand-Unified theory, based on a semisimple gauge group $G$ broken to SU(3) $\otimes$ SU(2) $\otimes$ U(1) at the scale $M_G \approx 10^{10}$ GeV. In general $G$ may undergo an intermediate step of breaking at a scale between $M_G$ and $M_{\text{Pl}}$. The successful unification of the SU(3), SU(2) and U(1) gauge couplings constrains both the spectrum below $M_G$ and the group at the intermediate stage of breaking. Below $M_G$ we consider the spectrum of the Minimal Supersymmetric Standard Model, and we take the intermediate group containing or coinciding with SU(5).

The vicinity of the Planck scale to the unification mass suggests to consider also the presence of possible non-renormalizable interactions scaled by inverse powers of $M_{\text{Pl}}$. Below the Planck scale, the theory in the global supersymmetric limit will be characterized by the functions $d(\hat{z}_i, \hat{z}_i^*)$ and $W(\hat{z}_i)$ of the chiral superfields $\hat{z}_i$. (A third function, related to the kinetic term of the gauge superfields, is irrelevant to the present discussion). All fields with mass of order $M_{\text{Pl}}$ have been integrated out and their effects included in $d(\hat{z}_i, \hat{z}_i^*)$ and $W(\hat{z}_i)$. We assume that the superfields can be redefined in such a way that

$$d(\hat{z}_i, \hat{z}_i^*) = \sum_i \hat{z}_i \hat{z}_i^* + d^{h.o.}(\hat{z}_i, \hat{z}_i^*)$$

where $d^{h.o.}(\hat{z}_i, \hat{z}_i^*)$ only contains higher order terms in $1/M_{\text{Pl}}$.

The chiral superfields $\hat{z}_i$ include the “matter superfields” $\hat{f}_a$ and the “Higgs superfields” $\hat{H}_n$. The scalar components of the Higgs superfields get vacuum expectation values at the grand scale which respect supersymmetry and break $G$ to SU(3) $\otimes$ SU(2) $\otimes$ U(1) with a possible intermediate step. The $\hat{H}_n$ also contain the light SU(2) doublets $\hat{h}_1, \hat{h}_2$. Under change of sign of the fields $\hat{f}_a$, which contain the standard quarks and leptons, the Lagrangian is taken to be invariant (“matter parity”). We also assume that, in the supersymmetric limit, the Lagrangian only contains the grand scale (with a possible fine structure) and that, in the same limit, the quarks, the leptons and the $\hat{h}_1, \hat{h}_2$ doublet superfields are exactly massless.

After insertion of the vacuum expectation values of the Higgs fields at the grand scale, $\langle H_n \rangle$, the superpotential $W$ acquires the following general form

$$W_{(H)} = \hat{f}_a M_{ab} \hat{f}_b + \hat{f}_a \lambda^{(1)}_{ab} \hat{f}_b \hat{h}_1 + \hat{f}_a \lambda^{(2)}_{ab} \hat{f}_b \hat{h}_2 + \cdots$$

where $M, \lambda^{(1)}, \lambda^{(2)}$ are numerical matrices and all other terms denoted by the dots (quartic terms in the matter fields, terms involving the heavy Higgs fields, etc.) are irrelevant to the present discussion.

By assumption, when reduced to diagonal form

$$M = U^* M_d U^T, \quad U U^\dagger = \mathbb{I}$$

Figure 1: the correlation between the top and the lightest Higgs mass for $\alpha_s(M_Z) = 0.110 \div 0.125$ (region between the dotted lines) for heavy CP odd scalar. The scalar partners of the top are taken unmixed and degenerate with a mass of 1 TeV. Also shown is the general upper bound on the Higgs mass in the MSSM.
the matrix $M$ has vanishing eigenvalues corresponding to the standard quarks and leptons, $\hat{f}_a$, with the remaining eigenvalues, corresponding to heavy matter fields, of order $M_G$. By an obvious change of basis the superpotential in the light fields reduces to

$$W^{\text{light}} = \hat{f}_a (U^T \lambda^{(1)} U)_{\alpha \beta} \hat{f}_\beta \hat{h}_1 + \hat{f}_a (U^T \lambda^{(2)} U)_{\alpha \beta} \hat{f}_\beta \hat{h}_2$$

(4)

In this superpotential, the heavy fields have been “integrated out”. It is at the same time remarkable and non trivial that, if supersymmetry is broken by a hidden supergravity sector \footnote{The possible role of heavy fermion exchanges in the flavour problem has already been explored in the literature \footnote{The assumption of a superpotential which, in absence of matter superfields, consists of a sum of two terms involving different sets of superfields, including possibly some singlets, would be more attractive if it could be shown to follow naturally from some symmetry of the theory. Several possibilities can be envisaged, making use of continuous or discrete invariances of normal or $R$-type nature.}}, this same superpotential with the only possible addition of a “$\mu$-term”, $\mu \hat{h}_1 \hat{h}_2$, (and $\mu$ of order of the effective supersymmetry breaking scale), describes in the usual way also the supersymmetry breaking terms \footnote{The possible role of heavy fermion exchanges in the flavour problem has already been explored in the literature \footnote{The assumption of a superpotential which, in absence of matter superfields, consists of a sum of two terms involving different sets of superfields, including possibly some singlets, would be more attractive if it could be shown to follow naturally from some symmetry of the theory. Several possibilities can be envisaged, making use of continuous or discrete invariances of normal or $R$-type nature.}}.

For later purposes, it is useful to consider the particular case of a fine structure at the grand scale, with an intermediate step of breaking induced by the vacuum expectation values $\langle H_{n1} \rangle \gg \langle H_{n2} \rangle$. Let us also suppose that the mass matrix $M$ of the matter superfields has no other massless eigenvector than the usual quarks and leptons, $\hat{f}_a$, already at the first step of breaking, when $\langle H_{n1} \rangle \neq 0$ but $\langle H_{n2} \rangle = 0$. Among the heavy fields, the only ones relevant to the present discussion are those, $\hat{F}_a$, with the same $SU(3) \otimes SU(2) \otimes U(1)$ quantum numbers of standard quarks and leptons. In the superpotential with $\langle H_{n1} \rangle \neq 0$, one has in general, in the appropriate basis

$$W_{(H_{n1})} = \hat{F}_a \hat{M}_{ab} \hat{F}_b + \hat{F}_a \gamma^{(i)} \hat{f}_a \hat{h}_i + \hat{f}_a \lambda^{(i)} \hat{f}_\beta \hat{h}_1 + \cdots$$

(5)

where $\hat{F}_a$ are superfields transforming under $SU(3) \otimes SU(2) \otimes U(1)$ as the conjugate representation of $\hat{F}_a$. With also the smaller vacuum expectation values $\langle H_{n2} \rangle$ different from zero, one has

$$W_{(H_{n1}), (H_{n2})} = \hat{F}^c (M + \delta M) \hat{F} + \hat{F}^c \mu \hat{f} + \hat{F}^{(i)} \gamma^{(i)} \hat{f} \hat{h}_1 + \hat{f} \lambda^{(i)} \delta \lambda^{(i)} \hat{f} \hat{h}_1 + \cdots$$

(6)

where

$$\frac{\delta M}{M}, \frac{\mu}{M}, \frac{\delta \gamma^c}{\gamma}, \frac{\delta \lambda}{\lambda} \ll O\left(\frac{\langle H_{n2} \rangle}{\langle H_{n1} \rangle}\right)$$

The mass term $\hat{F}^c \mu \hat{f}$ requires a redefinition of the light fields. To leading order in $\langle H_{n2} \rangle / \langle H_{n1} \rangle$ one has

$$W^{\text{light}} = \hat{f} (\lambda^{(i)} + \delta \lambda^{(i)}) - \mu T \frac{1}{M^T} \gamma^{(i)} \hat{f} \hat{h}_i.$$  

(7)

Equations (6) or (7) give the effective Yukawa couplings of the light fields, a part from the need to bring to canonical form the kinetic terms. In fact the higher order $d$-terms in eq. (6) induce, as a consequence of the grand unified vacuum expectation values, a wave function renormalization of the form $Z = 1 + \varepsilon$, where $\varepsilon$ is a hermitian matrix whose elements are at most of order $\langle H \rangle / M_{\text{pl}}$. To leading order in $\varepsilon$, it is easy to show that the needed redefinition of the fields amounts to a shift of the light masses by a factor $1 + O(\varepsilon)$ and to corrections of order $\varepsilon$ of the Cabibbo-Kobayashi-Maskawa matrix.

### 3 An SU(6) theory: renormalizable Yukawa couplings

We want to consider the flavour problem in an SU(6) theory, whose Higgs system has been discussed in \footnote{The possible role of heavy fermion exchanges in the flavour problem has already been explored in the literature \footnote{The assumption of a superpotential which, in absence of matter superfields, consists of a sum of two terms involving different sets of superfields, including possibly some singlets, would be more attractive if it could be shown to follow naturally from some symmetry of the theory. Several possibilities can be envisaged, making use of continuous or discrete invariances of normal or $R$-type nature.}}. The superpotential has the form

$$W = W(\Sigma) + W(H, \bar{H}) + W(\Sigma, H, \bar{H}, f_a)$$

(8)

where $\Sigma, H, \bar{H}$ are the Higgs superfields transforming respectively as $35$, $6$, $\bar{6}$ representations of SU(6). The part of the superpotential which does not depend on the matter superfields has a global SU(6) ×
 SU(6) invariance. Apart from SU(6) ⊗ SU(6) transformations, the scalars in Σ, H, \bar{H} acquire the vacuum expectation values

$$\Sigma = \langle \Sigma \rangle \text{ diag } (1, 1, 1, -2, -2), \quad H = \bar{H} = \langle H \rangle (1, 0, 0, 0, 0). \quad (9)$$

For the relative orientation between the Σ and the \(H, \bar{H}\) vacuum expectation values given in (9), the gauge and the global groups are broken as shown in table 1, where we have taken \(\langle H \rangle > \langle \Sigma \rangle\). (An opposite ordering of these vacuum expectation values is not suggested by the unification of the coupling constants). Of the Goldstone bosons produced by the breaking of the global group, all are eaten but a pair of SU(2) doublets, which play the role, after supersymmetry breaking, of the light Higgs bosons. Their composition is (see also section 7)

$$h_1 = \frac{(H) h_\Sigma - 3(\Sigma) h_H}{\sqrt{(H)^2 + 9(\Sigma)^2}}, \quad h_2 = \frac{(H) \bar{h}_\Sigma - 3(\Sigma) \bar{h}_H}{\sqrt{(H)^2 + 9(\Sigma)^2}}. \quad (10)$$

in term of the doublets (antidoublets) in Σ and \(H, \bar{H}\).

The chiral matter under SU(6) is normally contained in \((15 \oplus \bar{6} \oplus \bar{6}^\prime)i\) representations, where \(i\) is a family index. As it is well known, the representations \(15 \oplus \bar{6} \oplus \bar{6}^\prime\) give the minimal anomaly free set which has the standard 15-plet of chiral fields under SU(3) ⊗ SU(2) ⊗ U(1). We claim, however, that quarks and leptons do not necessarily live in these representations. This is because there are representations which, although self-adjoint, do not contain the identity in their symmetric product and can therefore obtain a heavy invariant mass only if they occur in even number. The 20-dimensional tensor with 3 totally antisymmetric indices is the smallest such representation in SU(6). We assume that they occur in odd number\(^6\). After integrating out all matter multiplets with Planck scale invariant masses, we therefore consider

$$\hat{f}_a = (15 \oplus \bar{6} \oplus \bar{6}^\prime)i \oplus 20 \quad i = 1, 2, 3.$$ 

In a suitable basis, all possible SU(6) invariant Yukawa couplings among these fields, assuming invariance under matter parity, \(\hat{f}_a \rightarrow -\hat{f}_a\), are

$$W^{(3)}(\Sigma, H, \bar{H}, f_a) = \lambda^{(1)}20 \Sigma 20 + \lambda^{(2)}20 H 15_3 + \lambda^{(3)}15_i \bar{H} \bar{6}_j^\prime. \quad (11)$$

For \(\langle H \rangle \gg \langle \Sigma \rangle\), already at the first level of breaking, from SU(6) to SU(5), the light quarks and leptons are identified from this superpotential. Defining the SU(5) decomposition of the various multiplets as

\[
\begin{align*}
20 &= (10 \oplus \bar{10}), & 15_i &= (10 \oplus 5)_i, \\
6_i &= (\bar{5} \oplus 1)_i, & 6_i^\prime &= (\bar{5} \oplus 1)_i^\prime, \\
H &= (5 \oplus 1)_H & \bar{H} &= (\bar{5} \oplus 1)_\bar{H} \\
\Sigma &= (24 \oplus 10 \oplus \bar{5} \oplus 1)_\Sigma
\end{align*}
\]

\[\text{(12)}\]

5Such relative orientation is fixed only after supersymmetry breaking by radiative corrections. In a range of the low energy parameters, it has been shown that such orientation, apart from corrections of order of the supersymmetry breaking scale, corresponds to a local minimum of the effective potential \[\text{[6]}\]. The issue of the global minimum, which depends on the heavy sector of the theory, will be discussed elsewhere.

6We do not include any other self-adjoint SU(6) representation, since the next one with no invariant mass has dimension 540.
the mass term in eq. (2) has the form
\[ \hat{f} \hat{M} \hat{f} = \lambda^{(2)} \langle H \rangle 10_{3} \overline{10} + \lambda^{(3)} \langle H \rangle 5_{i} \overline{5}_{j}, \] (13)
so that, apart from an irrelevant rotation in the heavy sector, \( \hat{M} \) is already diagonal. The light quarks and leptons are in 10, 10_{1}, 10_{2} and 5_{i}. Also light are all the SU(5) singlets \( 1_{i} \) and \( 1'_{j} \). Accordingly, from eqs. (4) and (7), the only Yukawa coupling between light states is contained in \( \lambda^{(1)} 20 \Sigma 20 \rightarrow \lambda^{(1)} 10 5_{\Sigma} 10 \). One has therefore
\[ W^{\text{light}} = \lambda^{(1)} Q^{c} h_{2} \] (14)
where, under SU(3) \( \otimes \) SU(2) \( \otimes \) U(1), \( 10 = Q \oplus u^{c} \oplus e^{c} \). At this stage only one up-type quark gets a mass after SU(2) \( \otimes \) U(1) breaking. This is an encouraging starting point: with a Yukawa coupling of order 1, only the top gets a mass comparable with the \( W \)-mass. This important result depends crucially on two features of the theory. First, the top quark must lie in an SU(6) representation which is not isomorphic to those containing the charm and up quarks. Second, one must take \( \epsilon \rangle \ll \langle \Sigma \rangle \). If \( \langle H \rangle \ll \langle \Sigma \rangle \) then none of the light quarks and leptons have \( O(1) \) Yukawa interactions with the Higgs doublet. Also the case \( \langle H \rangle \ll \langle \Sigma \rangle \) leads to an intermediate symmetry group SU(4) \( \otimes \) SU(2) \( \otimes \) U(1), which does not successfully predict the weak mixing angle. Thus we discover that the requirement of a symmetry breaking pattern which correctly predicts the weak mixing angle leads directly to the prediction that just one of the quark and leptons has a mass comparable to the weak scale, and this particle is a quark of charge 2/3. We take \( \langle H \rangle \gg \langle \Sigma \rangle \) for two reasons. Firstly this leads to a reduction in the GUT scale threshold corrections to the weak mixing angle. Secondly, the two parameters \( \epsilon \rangle \equiv \langle H \rangle /M_{Pl} \) and \( \epsilon \Sigma \equiv \langle \Sigma \rangle /M_{Pl} \) will allow higher dimension operators to account for the observed hierarchy of quark and lepton masses, as will be shown in the next sections.

## 4 Non renormalizable operators

In fact, non renormalizable operators scaled by inverse powers of \( M_{Pl} \) give effects that cannot be ignored. They can either be present in the basic theory, e.g. as an effect of gravitational interactions, or they can arise by integrating out heavy states at the Planck mass. The following is a full list of all independent \( f \)-term contributions involving four superfields
\[ W^{(4)}(\Sigma, H, \tilde{H}, f_{a}) = \frac{\alpha_{1}}{M_{Pl}^{i}} \tilde{H}_{i} H \tilde{6}_{\alpha} + \frac{\gamma_{2}}{M_{Pl}^{i}} (20 \Sigma) H 15_{i} + \frac{\gamma_{3}}{M_{Pl}^{i}} 20 H (\Sigma 15_{i}) + \frac{\gamma_{4}}{M_{Pl}^{i}} 20 H \tilde{H} 20 + \frac{\gamma_{5}}{M_{Pl}^{i}} 15_{i} \tilde{H} (\Sigma \tilde{6}_{j}) + \frac{\gamma_{5'}}{M_{Pl}^{i}} 15_{i} \tilde{H} (\Sigma \tilde{6}_{j}) + \frac{\gamma_{6}}{M_{Pl}^{i}} 15_{i} (\Sigma \tilde{H}) \tilde{6}_{j} + \frac{\gamma_{6'}}{M_{Pl}^{i}} 15_{i} (\Sigma \tilde{H}) \tilde{6}_{j} \] (15)
where \( \alpha, \beta = i, i' \). In the ambiguous cases the parentheses denote, in a self-explanatory notation, how the SU(6) indices are contracted. In next order, the terms giving the dominant contributions to the masses of the light states are
\[ W^{(5)}(\Sigma, H, \tilde{H}, f_{a}) = \frac{\sigma_{ij}^{(1)}}{M_{Pl}^{i}} 15_{i} H \Sigma H 15_{j} + \frac{\sigma_{ij}^{(2)}}{M_{Pl}^{i}} 20 \tilde{H} \Sigma \tilde{H} \tilde{6}_{i} + \frac{\sigma_{ij}^{(3)}}{M_{Pl}^{i}} 15_{i} (\Sigma^{2} \tilde{H}) \tilde{6}_{j} + \frac{\sigma_{ij}^{(4)}}{M_{Pl}^{i}} 15_{i} (\Sigma \tilde{H}) (\Sigma \tilde{6}_{j}) \] (16)
If all these operators are present, as indicated, with dimensionless couplings of order unity and no particular flavour structure, one does not get the correct pattern of masses for two reasons. The operator
\footnote{We consider the Planck scale \( M_{Pl} \) as a natural scale for these operators. One can bear in mind, however, that in certain cases the regulator scale can be different (about \( 10^{18} \) GeV in the context of superstring inspired theories?).}
We therefore obtain, on the basis of equation (18):

\[ 15(\Sigma \bar{H}) \]

and, for all operators depending on 2 indices \( i, j \) would give comparable masses to all generations. We therefore conclude that a completely general operator analysis based on this set of fields lighter than the Planck scale cannot explain the observed fermion masses. The next step is to study whether a restricted set of operators can yield acceptable masses. If so the origin of this restricted set will have to be addressed.

Consider the theory in which \( \gamma^{(5)}, \gamma^{(6)}, \gamma^{(1)}, \gamma^{(3)} \) and \( \gamma^{(4)} \) are absent. In this case only \( \gamma^{(2)} \) and \( \gamma^{(2)} \) lead to important mass terms. Taking into account that the multiplet 153 has already been defined by \( W^{(2)} \), eq. (11), without loss of generality, we can redefine

\[
\begin{align*}
\frac{\gamma^{(2)}}{M_{Pl}^2} (20 \Sigma) H_{15i} & \rightarrow \frac{\gamma^{(2)}}{M_{Pl}^2} (20 \Sigma) H_{153} + \frac{\gamma^{(2)}}{M_{Pl}^2} (20 \Sigma) H_{152} \\
\frac{\sigma^{(2)}}{M_{Pl}^2} 20 \bar{H} \Sigma \bar{h}_i & \rightarrow \frac{\sigma^{(2)}}{M_{Pl}^2} 20 \bar{H} \Sigma \bar{h}_3
\end{align*}
\]

In this way we obtain a massless first generation of quarks and leptons and, using eq. (7), the following Yukawa couplings for the 2\textsuperscript{nd} and 3\textsuperscript{rd} generation:

\[
W^{\text{light}} = \left[ \lambda^{(1)} \frac{1}{2} \cdot 20^{abc} \bar{c}_2 \Sigma_2 H_{4 \delta} + \frac{\gamma^{(2)}}{M_{Pl}^2} 1 \cdot 20^{abc} \bar{c}_2 H_{4 \delta} 15_{6e} + \frac{\sigma^{(2)}}{M_{Pl}^2} \bar{H} \Sigma \bar{h}_3 \right]_{\text{light}}
\]

\[ (\lambda^{(1)} Q c + \gamma^{(2)} \bar{c} \hat{Q} u + \frac{\gamma^{(2)}}{2} \bar{c} \hat{Q} u c + \bar{c} \hat{Q} u c) h_2 + \sigma^{(2)} \bar{c} \hat{Q} (Q d + c L) h_1 \]  

(18)

For the decomposition under SU(3) \( \otimes \) SU(2) \( \otimes \) U(1) we have defined

\[ 10_i = (Q + u^c + e^c)_i \]

\[ 5_i = (d^c + L)_i \]  

(19)

in analogy with (for the 10 in the 20)

\[ 10 = Q + u^c + e^c \]  

(20)

Let us now recall, from standard renormalization group (RG) results \[ \text{[4]} \], the connection between the running masses, \( m_t, m_c, m_b, m_r, \) and the corresponding Yukawa couplings at the unification scale \( (\tan \beta \equiv \langle h_2 \rangle / \langle h_1 \rangle) \):

\[
\begin{align*}
m_t &= \sqrt{m c} \csc \beta y^6 \eta_t \\
m_c &= 1.27 \pm 0.05 \text{ GeV} = \sqrt{m c} \csc \beta y^3 \eta_c \\
m_b &= 4.25 \pm 0.10 \text{ GeV} = \sqrt{m b} \csc \beta y \eta_b \\
m_r &= 1.78 \text{ GeV} = \sqrt{m r} \eta_r \\
V_{cb} &= 0.04 \pm 0.01 = V_{cb}(M_G) / y
\end{align*}
\]

where

\[ y \equiv \exp \left[ - \int_{\ln M_G}^{\ln M_Z} \frac{\lambda^2(\mu)}{16\pi^2} d\ln \mu \right] \]

and, for \( \alpha_s = 0.110 \div 0.125 \),

\[ \eta_t \approx 1, \quad \eta_c = 1.87 \div 2.30, \quad \eta_b = 1.47 \div 1.62, \quad \eta_r = 0.991, \]

\[ \csc \beta = 3.4 \div 3.8, \quad \csc \beta = 3.3 \div 3.7, \quad \csc \beta = 1.5. \]

We therefore obtain, on the basis of equation (18):

\[ \text{[8]} \text{The combinatoric factors in front of the various operators, as well as of the kinetic terms, not shown, are the inverse of the number of all possible irrelevant permutations of fields and SU(6) indices.} \]
Figure 2: diagrams giving rise to the operators (24a,b,c) respectively.

i) from $m_b/m_t$: $\lambda_t = \lambda^{(1)} = 2 \div 5$;

ii) from $m_c/m_b$: $\tan \beta_{c}/\lambda_b = \tan \beta(\gamma_2^{(2)})/(\lambda^{(1)}\sigma^{(2)}) = 0.55 \pm 0.20$;

iii) from $m_c$:

$$\lambda_t \cos \beta = \varepsilon_H^2 \sigma^{(2)} \cos \beta = 0.0070, \quad \varepsilon_H \geq \frac{0.083}{\sqrt{\sigma^{(2)}}}; \quad (21)$$

iv) from $|V_{cb}|$: $\varepsilon_H \gamma_2^{(2)}/\lambda^{(1)} = 0.027 \pm 0.010$.

This gives an elegant understanding of all heavy fermion masses in terms of the ratio $\varepsilon_H = (H)/M_{Pl}$ and of parameters of order unity. As it is well known, bottom-tau Yukawa unification and moderate tan $\beta$, both implied in our scheme, require $\lambda^{(1)}$ not only to be of order 1, but actually close to its infrared fixed point, and, consequently, for the physical top mass

$$M_t = m_t[1 + \frac{4}{3\pi} \alpha_3(m_t)] = \sin \beta (190 \div 205) \text{ GeV} = 140 \div 205 \text{ GeV} \quad (22)$$

Even more interesting is the correlation between the masses of the top and of the lightest Higgs boson. Since the top Yukawa coupling at the low scale is essentially fixed, such correlation is only determined by the value of tan $\beta$ and is shown in figure 1 for $\alpha_s(M_Z) = 0.110 \div 0.125$. As a matter of fact, what is shown in figure 1 is an upper bound on the lightest scalar Higgs mass, which can be lowered by having an equally light pseudoscalar in a well known way. The scalar partners of the top are taken unmixed and degenerate with a mass of 1 TeV.

5 Yukawa couplings generated by heavy particle exchanges

From the previous section, we are left with two problems:

1. the need to suppress the operator $15 (\Sigma \bar{H}) \bar{6}$ which would tend to $V_{cb}$ close to unity;

2. the difficulty of distinguishing the masses of the first two generations.

Here we show how both problems can be solved, still without appealing to any flavour symmetry, by assuming that all the non renormalizable operators be generated by the exchanges of appropriate heavy particles.

At dimension 5, we only need the operator $20 \Sigma H 15_i$ (irrespective of the possible contractions of the group indices). This operator can uniquely be generated, as shown in figure 2b, by the exchange of
a 70. At dimension 6, the operator $20 \bar{H}\Sigma \tilde{H}$ generates the masses for the $b$ and the $\tau$. The simplest possibility, which involves one heavy exchange only, is shown in figure 2b. Notice, however that, by cutting this diagram, one has the operator $(15 \Sigma \Sigma \tilde{H})$, with the heavy 15 denoted by $15_H$. In turn, if it were possible to replace $15_H$ by $15_i$, this would be a disaster. Other than matter parity, we are therefore forced to introduce another $Z_2$ symmetry, hereafter called $D$, under which $15_H$ (and $15_i$) change sign, whereas $15_i$ stay invariant.

Let us now discuss the operators connecting $15_i$ with $\bar{6}_j$. By restricting ourselves to the exchanges of heavy particles with lower or upper indices only, and at the same time consistent with $D$-parity, the only possibility is the one shown in figure 2c, generating the operator

$$15_i(\Sigma \tilde{H})(\Sigma \bar{6}_j) + (15 \tilde{H})\Sigma \Sigma \bar{6}_j$$

(23)

This is what we need, since $D$ parity can be consistently defined on all multiplets entering the diagrams of eq. 22, as prescribed in table 2, always in a flavour blind way. Two SU(6)-singlets are included, to enlarge the Higgs system, which are assumed to get a vacuum expectation value at a scale $M$, with $\langle H \rangle < M \leq M_{Pl}$. The Yukawa superpotential is taken to be the most general set of trilinear interactions among the supermultiplets in table 2 compatible with the gauge symmetry, matter parity and $D$-parity. The heavy supermultiplets acquire their mass at a scale $M$ from the vacuum expectation values of the singlets $S_1, S_2$.

After integrating out the heavy supermultiplets, other than the renormalizable superpotential given in eq. 11, one obtains the required non renormalizable interactions, as described from the diagrams of figures 2. As a matter of fact, from the diagram of fig. 3, one also obtain the dimension-5 operator $(15_i \Sigma \tilde{H})\Sigma \bar{6}_j$. Such an operator, however, is irrelevant for the masses of the light particles, since, in conjunction with the renormalizable term $15_i \tilde{H} \bar{6}_j$, it only slightly redefines the composition of the heavy particles. We shall see in the next section how this construction of the non renormalizable interactions introduces a distinction between the first two generations.

6 A specific “texture”

We now show how the specific model defined in section 5 gives rise to an interesting quasi-realistic texture for fermion masses. To this purpose, as in section 4, it is useful to redefine, without loss of generality, appropriate linear combinations of fields which are not distinguished by any symmetry, gauged or discrete. With reference to the appropriate terms in the superpotential, and without writing explicitly the dimensionless couplings, we do that according to the following progression:

1. $6_H S_2(\bar{6}_H + \bar{6}_i) \rightarrow 6_H S_2 \bar{6}_H$ (defines $\bar{6}_H$);
2. $20 H 15_i \rightarrow 20 H 15_3$ (defines $15_3$);
3. $20 H 15_i \rightarrow 20 H 15_3$ (defines $15_3$);
3. $15_i \Sigma \Sigma_{\eta H} = 15_{3,2} \Sigma \Sigma_{\eta H}$ (defines $15_2$);
4. $6_i \bar{H} 15_{\eta H} = 6_3 \bar{H} 15_{\eta H}$ (defines $6_3$);
5. $6_i \Sigma 6_{\eta H} = 6_{3,2} \Sigma 6_{\eta H}$ (defines $6_2$).

If we know integrate out the heavy vector-like multiplets, we are left with the renormalizable superpotential of eq. (14) and with the following (relevant) non-renormalizable terms

$$\frac{\lambda_i^{(2)}}{M} (20 \Sigma) H \ 15_i \quad i = 1, 2, 3$$

$$\sigma^{(2)} \frac{20 \bar{H} \Sigma H \ 6_3}{M^2}$$

$$\sigma_{ab}^{(4)} \frac{15_a \Sigma \bar{6}_b}{M} \quad a, b = 2, 3; \quad \sigma_{ab}^{(4)} = \delta_{a \tau b}$$

respectively associated with the diagrams shown in figure 2a,b,c. Taking into account that the tree level potential makes heavy the full $15_3$, the leading terms of the $u$, $d$ and $e$ mass matrices are therefore

$$Q_1 \begin{pmatrix} u_1^c & u_2^c & u_3^c \\ 0 & 0 & 0 \\ \gamma_1^{(2)} \varepsilon_H & \gamma_1^{(2)} \varepsilon_H & \varepsilon_H \end{pmatrix} \cdot v \sin \beta$$

$$Q_2 \begin{pmatrix} \gamma_1^{(2)} \varepsilon_H & \gamma_2^{(2)} \varepsilon_H & \varepsilon_H \lambda^{(1)} \\ \gamma_1^{(2)} \varepsilon_H & \gamma_2^{(2)} \varepsilon_H & \varepsilon_H \lambda^{(1)} \\ 0 & 0 & \delta_{2 \tau 2} \varepsilon_H \end{pmatrix}$$

$$Q \begin{pmatrix} \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \\ \lambda & \lambda & \lambda \end{pmatrix} \cdot v \cos \beta$$

$$e_{\tau 2} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -2 \delta_{2 \tau 2} \varepsilon_H & -2 \delta_{2 \tau 2} \varepsilon_H & \varepsilon_H \sigma^{(2)} \varepsilon_H \end{pmatrix} \cdot v \cos \beta$$

where $\varepsilon_H = \langle H \rangle / M$ and $\varepsilon_\Sigma = \langle \Sigma \rangle / M$.

These mass matrices give rise to a picture for the heavy fermion masses $(t, b, c, \tau)$ which is unchanged with respect to section 4. They actually also constitute an interesting “texture”. A simple phase analysis shows, in fact, that the mass matrices (25) have a single irremovable phase and they involve six other real parameters. In term of these seven parameters one gets ten flavour observables and a massless first generation. Of course, it is also especially important that the size of the various mass terms is correctly determined by two small parameters only, $\varepsilon_H \approx 10^{-1}$ and $\varepsilon_\Sigma \approx 10^{-2}$.

In particular, one obtains the $s, \mu$ Yukawa couplings and the CKM matrix at the unification scale as

$$\lambda_s \approx \delta_{2 \tau 2} \varepsilon_\Sigma \varepsilon_H, \quad \frac{\lambda_\mu}{\lambda_s} \approx 2$$

$$V_{\text{CKM}} \approx \begin{pmatrix} 1 & -s & ss \varepsilon_H \\ -s & 1 & ss \varepsilon_H \varepsilon_H \\ -ss \varepsilon_H & ss \varepsilon_H & ss \varepsilon_H \end{pmatrix}$$

where

$$s = \frac{|\gamma_1^{(2)}|}{\sqrt{|\gamma_1^{(2)}|^2 + |\gamma_2^{(2)}|^2}}, \quad ss \varepsilon_H \approx \delta_{2 \tau 2} \varepsilon_H \frac{\varepsilon_\Sigma \varepsilon_H \lambda}{\lambda^{(2)} \lambda^{(1)} \lambda^{(0)}}, \quad \varepsilon_\Sigma = \varepsilon_H \frac{\varepsilon_\Sigma \varepsilon_H \lambda^{(2)} \lambda^{(1)} \lambda^{(0)}}{\lambda^{(2)} \lambda^{(1)} \lambda^{(0)}} = \sqrt{\frac{\lambda_s}{\lambda_t}}$$

After the appropriate rescalings to low energy of the various parameters $[14]$, one finds a remarkable approximation to a realistic description of masses and mixings, obtained without appeal to any flavour
symmetry. Notice in particular the factor of 2 between the muon and the strange quark masses at unification, characteristic of SU(6), as well as the fact that the Cabibbo angle, $\theta_C$, is naturally of order 1 and the only one to be unsuppressed. This is perhaps why $\theta_C$ is the largest among the mixing angles.

The masslessness of the first generation is an exact consequence of the model specified in section 5. In the down-lepton sector, the three $\bar{6}_i$ fields are only coupled, in the superpotential, to two heavy states $6_H$ and $15_H$, whereas, in the up sector, one of the $15_i$ fields and the $21_H$ are only coupled to the $\overline{10}_H$, leading to a massless linear combination. The first generation masses, or at least those of the electron and the down quark, since a massless up quark may be welcome in view of the strong CP problem, will have to come from a suitable extension of the model.

Let us in fact work under the assumption that the dominant contribution to first generation masses will come from the 1, 1 entries in the Yukawa matrices. Although not guaranteed, within the democratic approach to flavour, the best guess is that the first generation masses arise from some higher dimension operator, which is flavour blind and which then puts a comparable entry everywhere in the mass matrices (25). These entries must then be very small and the dominant contribution to first generation masses will come from the 1, 1 entries alone. In this case, no other diagonalization has to be done to obtain the quark physical basis.

Under this assumption, the CKM matrix (27), via eq. (28), leads to an interesting constraint between the top quark mass and the physical mixing parameters. In terms of the RG correction factors $y$ and $\eta_c$, defined in section 4, one obtains

$$A\lambda^2|1 - \rho - i\eta| = \sqrt{\frac{ym_c}{\eta c m_t}}$$

(29)

where $A$, $\lambda$, $\rho$, $\eta$ are the usual Wolfenstein parameters for the CKM matrix. Using $y = 0.70 \pm 0.05$, $\eta_c = 1.87 \div 2.30$, with an uncertainty dominated by the error on $\alpha_3$, and the experimental values

$$\lambda = 0.22, \quad A = 0.86 \pm 0.10 \quad ([V_{cb}] = 0.042 \pm 0.005),$$

(30)

this relation can be viewed as a constraint on the length $|1 - \rho - i\eta|$ of the side of the usual unitarity triangle proportional to $V_{td}$

$$|1 - \rho - i\eta| = (1.2 \pm 0.15) \sqrt{\frac{170 \text{ GeV}}{m_t}}$$

(31)

Such a constraint is perfectly consistent with the present knowledge on the CKM matrix for a top mass value as in eq. (29). It indicates a negative value of $\rho$ ($-0.3 \lesssim \rho \lesssim 0$), sin $2\alpha$ close to 1, sin $2\beta$ close to 0.5 and a relatively small mixing parameter in the $B_s-B_s$ system

$$x_s = (9.8 \pm 2.5) \frac{m_t}{170 \text{ GeV}}.$$

To conclude this section we comment on the neutrino masses. In the multiplets corresponding to one generation, $\bar{6} \oplus \bar{6}' \oplus 15$, there are 5 neutrino states, 3 SU(2)-doublets and 2 SU(2)-singlets. In the mass matrix, such neutrinos mix among each other and with neutrinos in the heavy 6, 15, 21, 70 representations. It is however readily seen from the general superpotential and from the absence of neutrinos in the 20, that the neutrino mass matrix conserves a neutrino number, with opposite charges for the neutrinos in the barred and unbarred representations. As a consequence, one is left with 3 massless neutrinos per generation. Furthermore, by simple inspection one sees that, a part from irrelevant mixings, only one of these neutrinos (per generation) is a SU(2)-doublet, necessarily coupled to the light charged leptons. This situation persists to all order of perturbation theory until supersymmetry is unbroken. After supersymmetry breaking, characterized by the scale $m$, radiative corrections may give rise to Dirac mass terms involving the SU(2)-doublet neutrinos at most of order $mv/M$. Although small, since $M \simeq 10^{17}$ GeV, these masses could be of phenomenological interest for the solar neutrino physics.

7 Proton decay

Quite in general, the dominant diagram for proton decay is shown in figure 4 in superfield notation, where $\hat{T}$, $\hat{T}$ is the heavy triplet with the SU(3) $\otimes$ SU(2) $\otimes$ U(1) quantum numbers of a down quark [10]. The
resulting operator \((\hat{Q}^a \hat{Q}^b)(\hat{Q}^c \hat{L})\delta_{abc}\), where \(a, b, c\) are SU(3) indices, leads, after supersymmetry breaking, to the decay \(p \to K \bar{\nu}\) with a rate that can be very close to, or even exceed, the present experimental bound \([17]\). The actual value of this rate depends, other than on the unification scale, on the masses of some of the superpartners at the Fermi scale and on the couplings of the heavy triplet to quark and lepton supermultiplets, as shown in figure 4.

As it is the case for the SU(2) doublets \(h_H, \bar{h}_H\) and \(h_{\Sigma}, \bar{h}_{\Sigma}\), in the SU(6) model there are two pairs of triplets, \(T_H, \bar{T}_H\) and \(T_{\Sigma}, \bar{T}_\Sigma\). However, at variance with the doublets, the triplets appear as Goldstone bosons only in the breaking of one of the SU(6) factors

\[
\text{SU}(6) \rightarrow \text{SU}(5),
\]

but not in the other

\[
\text{SU}(6) \rightarrow \text{SU}(4) \otimes \text{SU}(2) \otimes U(1).
\]

As a consequence, the triplet \(T_H, \bar{T}_H\) is eaten by the SU(6) gauginos, whereas only \(T_{\Sigma}, \bar{T}_\Sigma\) remains as a heavy state capable of mediating the process of figure 4. Analytically, the difference between \(h's\) and \(T's\) is clearly seen in the relevant mass matrices involving also the doublet \(\lambda_2, \bar{\lambda}_2\) and the triplet \(\lambda_3, \bar{\lambda}_3\) gauginos:

\[
\begin{pmatrix}
\lambda_2 & h_H & h_{\Sigma} \\
\bar{h}_H & 0 & 0 \\
\bar{h}_{\Sigma} & 0 & 0
\end{pmatrix}
\]

The masses coming from the \(d\)-terms are proportional to the gauge coupling \(g\), whereas the mass term \(\lambda(\Sigma)\) comes from the superpotential. While the doublets from \(H, \bar{H}\) and \(\Sigma\) are mixed by the mass matrix, the triplets are not. This property has the remarkable consequence that, whenever the light Higgs doublet that provides the relevant mass term comes from \(H, \bar{H}\), the corresponding triplet coupling is ineffective to the proton decay. An example is provided by the coupling proportional to \(\gamma^{(5)}\) in eq. (15). It is possible to conceive explicit flavour structures of the Yukawa couplings to the light generation which are at the same time realistic and lead to a suppression of the \(p \to K \bar{\nu}\) amplitude by a factor \(\langle \Sigma \rangle / \langle H \rangle \approx 0.1\) or by its square. Such a suppression might be important in view of the existing estimates of the proton decay lifetime in the minimal SU(5) model \([18]\).

8 Conclusions

In this paper we wanted to analyse how far one could go in accounting for the observed pattern of fermion masses and mixings in a grand unified theory without any flavour symmetry. We have studied the problem in an interesting model for the doublet-triplet splitting. To our surprise we have found an elegant understanding of the heaviness of the top quark. Instrumental to this result is the fact that the top comes out belonging to an irreducible representation of the gauge group not isomorphic to those ones containing the other quarks of charge 2/3.
In own framework the splitting of the heavy top from the light fermions results from simply writing down the most general gauge invariant renormalizable Lagrangian. Extending this to non-renormalizable terms does not immediately lead to a satisfactory generation of the lighter fermion masses. However, by including a flavour-blind discrete symmetry and requiring that all higher dimensional operators be mediated by the exchange of appropriate heavy multiplets, it is possible to give an approximate description of all masses and mixings in terms of the hierarchy $M > \langle H \rangle > \langle \Sigma \rangle$. This leads to several interesting features. One finds $\lambda_b/\lambda_t \approx \epsilon^2_H$ and $\lambda_b/\lambda_\tau = 1$ which is successful provided $\lambda_t$ is very close to its infrared fixed point. As a consequence, $m_t$ and $m_h$ are correlated in a specific way. The light quark mass hierarchies are understood as $m_b/m_t \approx m_c/m_t \approx \epsilon^2_H$ and $m_s/m_t \approx \epsilon_H \epsilon_\Sigma$. In addition to the GUT relation $m_b/m_\tau = 1$, the SU(6) theory predicts $m_s/m_\mu = 1/2$. A characteristic “texture” leads to a Kobayashi-Maskawa mixing matrix in which the magnitude of $V_{td}$ can be predicted at the 15% level of accuracy. This leads to a prediction for $x_s$ and the CP violation angles $\sin 2\alpha$ and $\sin 2\beta$ in $\beta$ decays.

Finally, we have pointed out a possible reason for the suppression of the $p \to K \bar{\nu}$ amplitude, intimately related to the mechanism of the doublet-triplet splitting. Needless to say, the next step is to find a source for the first generation masses.

Acknowledgements

Z.B. thanks Alexei Anselm for useful comments.

References

[1] LEP Collaboration results, as summarized by J. Le Francois, Proceedings of the European Conference on High Energy Physics, Marseille, July 1993.

[2] S. Dimopoulos, S. Raby and F. Wilczek, Phys. Rev. D24 (1981) 1681; S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150; L. Ibañez and G. G. Ross, Phys. Lett. 105B (1981) 439.

[3] P. Langacker and M. Luo, Phys. Rev. D44 (1991) 817; U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B260 (1991) 447; J. Ellis, S. Kelley and D. Nanopoulos, Phys. Lett. B260 (1991) 131.

[4] M. S. Chanowitz, J. Ellis and M. K. Gaillard, Nucl. Phys. B128 (1977) 506; A. J. Buras, J. Ellis, M. K. Gaillard and D. V. Nanopoulos, Nucl. Phys. B135 (1978) 66; D. V. Nanopoulos and D. A. Ross, Nucl. Phys. B157 (1979) 273.

[5] Z. Berezhiani and G. Dvali, Sov. Phys. Lebedev Inst. Reports 5 (1989) 55.

[6] R. Barbieri, G. Dvali and M. Moretti, Phys. Lett. B312 (1993) 137.

[7] K. Inoue, A. Kakuto and T. Takano, Progr. Theor. Phys. 75 (1986) 664; A. Anselm and A. Johansen, Phys. Lett. B200 (1988) 331.

[8] J. Giveon, L. Hall and U. Sarid, Phys. Lett. B271 (1991) 138; M. Carena et al., Nucl. Phys. B379 (1992) 33; V. Barger, M. Berger, P. Ohmann and P. Phillips, Phys. Lett. B314 (1993) 351; V. Barger, M. Berger and P. Ohmann, Phys. Rev. D47 (1993) 1093; P. Langacker and N. Polonsky, Phys. Rev. D47 (1993) 4028; M. Carena, S. Pokorsky and C. Wagner, Nucl. Phys. B406 (1993) 140; W. Bardeen, M. Carena, S. Pokorski and C. Wagner, preprint MPI-Ph/93-58.

[9] For a recent numerical analysis, see M. Carena, M. Olechowski, S. Poroski and C. E. Wagner, to be published; P. Langaker and N. Polonsky, University of Pennsylvania preprint UPR-0594T.
[10] G. Anderson, S. Raby, S. Dimopoulos, L. J. Hall and G. D. Starkman, Berkeley preprint LBL-32817 (Sep. 1993).

[11] C. D. Froggatt and H. B. Nielsen, Nucl. Phys. B147 (1979) 277; Z. G. Berezhiani, Phys. Lett. B129 (1983) 99; Phys. Lett. B150 (1985) 177; S. Dimopoulos, Phys. Lett. B129 (1983) 417.

[12] R. Barbieri, S. Ferrara and C. Savoy, Phys. Lett. B110 (1982) 343; P. Nath, R. Arnowitt and A. Chamseddine, Phys. Rev. Lett. 49 (1982) 970.

[13] L. Hall, J. Lykken and S. Weinberg, Phys. Rev. D10 (1983) 2359.

[14] See, e.g., V. Barger, M. S. Berger and P. Ohmann in ref. 8 and references therein.

[15] For a recent review see A. Ali, CERN preprint TH 7123/93 (December 93).

[16] S. Weinberg, Phys. Rev. D26 (1982) 287; N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533.

[17] Particle Data Group, Phys. Rev. D45 (1992).

[18] For a recent analysis of the proton decay in supersymmetric Grand Unified theories see, e. g., R. Arnowitt and P. Nath, Texas A & M University preprint CPT-TAMU-23-93 (1993).