Non Abelian Fields in Very Special Relativity

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Abstract

We study non-Abelian fields in the context of very special relativity (VSR). For this we define the covariant derivative and the gauge field gauge transformations, both of them involving a fixed null vector $n_{\mu}$, related to the VSR breaking of the Lorentz group to the Hom(2) or Sim(2) subgroups. As in the Abelian case the gauge field becomes massive. Moreover we show that the VSR gauge transformations form a closed algebra. We then write actions coupling the gauge field to various matter fields (bosonic and fermionic). Finally we mention how we can use the spontaneous symmetry breaking mechanism to give a flavor dependent VSR mass to the gauge bosons.

1 Introduction

Special relativity (SR) is valid at the largest energies available today [1]. However the possible violation of the underlying Lorentz symmetry presents us with new experimental and theoretical challenges. In particular, its violation has been considered as a possible evidence for Planck scale Physics, as some theories of quantum gravity predicts it [2]. Experiments and astrophysical observations are used to set stringent bounds upon the parameters describing these violations. Broadly speaking, three basic scenarios have been explored: (1) Non-dynamical tensor fields are introduced to determine preferred directions that break the Lorentz symmetry. Some instances of this are the Myers-Pospelov model [3] together with QED in a constant axial vector background [4]. (2) A second scenario is to assume spontaneous symmetry breaking (SSB) of the Lorentz symmetry as in the standard model extension of [5],[6], where such non-dynamical
tensor fields are assumed to arise from vacuum expectation values of some basic fields belonging to a more fundamental theory. (3) A third possibility has been introduced in [7].

There it is proposed that the laws of nature are not invariant under the whole Lorentz group (with 6 parameters) but instead are invariant under subgroups of the Lorentz group which still preserves the basic elements of SR like the constancy of the velocity of light. It was named very special relativity (VSR). In general space isotropy and CP are violated but if CP is incorporated as a symmetry then the whole Lorentz group is recovered. The most interesting of these subgroups are Sim(2) (with 4 parameters) and Hom(2), (with 3 parameters). These subgroups do not have invariant tensor fields besides the ones that are invariant under the whole Lorentz group, implying that the dispersion relations, time delay and all classical tests of SR are valid for these subgroups too. New effects would be generated by parity violating and non-local terms. VSR admits the generation of a neutrino mass without lepton number violation nor sterile neutrinos [8]. The implications of this novel mechanisms could be tested at non-relativistic neutrino energies such as the end point of the electron spectrum in beta decay. VSR has been generalized to include supersymmetry [9], curved spaces [10], noncommutativity [11], cosmological constant [12], dark matter [13], cosmology [14] and Abelian gauge fields [15]. Some doubts about the phenomenology of Hom(2) VSR has been presented in [16], arguing that it is unable to explain Thomas precession, and further extended in [17].

In this paper we want to consider non-Abelian gauge fields in the context of VSR. We review the results of the Abelian case [15] in section 2. In section 3, we define the covariant derivative, the gauge transformations and the action for non-Abelian fields. Section 4 is then devoted to SSB of the VSR gauge symmetry and section 5 contains the conclusions and open problems.

2 Abelian Gauge Fields

In this section, we follow closely [15]. Let us consider a gauge field $A_\mu$ in VSR with gauge transformation

$$\delta A_\mu = \tilde{\partial}_\mu \Lambda,$$

where the wiggle operator is defined by:

$$\tilde{\partial}_\mu = \partial_\mu - \frac{m^2}{2 n \cdot \partial} n_\mu,$$

and $n \cdot \partial = n^\mu \partial_\mu$. The constant vector $n^\mu$ is a null vector and transforms multiplicatively under a VSR transformation so that any term containing ratios involving $n^\mu$ are invariant. In order to have the usual mass dimension for $\tilde{\partial}_\mu$ a constant $m$ has to be introduced and sets the scale of VSR effects.

Consider now a charged scalar field $\phi$ to be coupled to the gauge field and with gauge transformation

$$\delta \phi = i \Lambda \phi.$$
It can be shown that the operator $D_\mu$ [15]:

$$D_\mu \phi = \partial_\mu \phi - i A_\mu \phi - \frac{i}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot A \right) \phi,$$

(4)

satisfies the fundamental property of transforming as $\phi$ does under gauge transformations:

$$\delta(D_\mu \phi) = i \Lambda D_\mu \phi,$$

We will call $D_\mu \phi$ the covariant derivative of $\phi$. Associated to this covariant derivative, we define the wiggle covariant derivative of the field $\phi$ by

$$\tilde{D}_\mu \phi = D_\mu \phi - \frac{1}{2} \frac{m^2}{n \cdot D} n_\mu \phi.$$

(5)

It reduces to $\tilde{\partial}_\mu \phi$ for $A_\mu = 0$. We will see below that using the wiggle covariant derivative in the action gives to $\phi$ a VSR mass.

The field strength related to $D_\mu$ can be computed as

$$[D_\mu, D_\nu] \phi = -i F_{\mu\nu} \phi,$$

and it is given by

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{1}{2} m^2 n_\nu \left( \frac{1}{(n \cdot \partial)^2} \partial_\nu (n \cdot A) \right) - \frac{1}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} \partial_\mu (n \cdot A) \right).$$

(6)

We call $F_{\mu\nu}$ as the $A_\mu$ field strength. It does not coincide with

$$\tilde{\partial}_\mu A_\nu - \tilde{\partial}_\nu A_\mu,$$

(7)

which is also gauge invariant and will be used below to describe massive gauge fields. However, the difference between them must be gauge invariant

$$d F_{\mu\nu} = F_{\mu\nu} - (\tilde{\partial}_\mu A_\nu - \tilde{\partial}_\nu A_\mu) = \frac{1}{2} m^2 \left( n_\nu \left( \frac{1}{(n \cdot \partial)^2} n_\alpha F_{\mu\alpha} - n_\mu \left( \frac{1}{(n \cdot \partial)^2} n_\alpha F_{\nu\alpha} \right) \right).$$

We then define the $A_\mu$ wiggle field strength by

$$\tilde{F}_{\mu\nu} = F_{\mu\nu} - \frac{1}{2} m^2 \left( n_\nu \left( \frac{1}{(n \cdot \partial)^2} n_\alpha F_{\mu\alpha} - n_\mu \left( \frac{1}{(n \cdot \partial)^2} n_\alpha F_{\nu\alpha} \right) \right),$$

(8)

where by construction:

$$\tilde{F}_{\mu\nu} = \tilde{\partial}_\mu A_\nu - \tilde{\partial}_\nu A_\mu.$$

(9)

Using the wiggle covariant derivative (5) we can write a gauge invariant action for the charged scalar coupled to the Abelian gauge field. Since $\tilde{F}_{\mu\nu}$ is gauge invariant the action can have, besides the usual $\tilde{F}^2$ term, contributions involving the square of $n_\mu \tilde{F}^{\mu\nu}$, so that the most general gauge invariant action quadratic in the gauge field is

$$S = \int d^3 x \left( -\frac{1}{4} \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + g \frac{1}{2} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot \partial} n_\alpha \tilde{F}^\mu + |\tilde{D}_\mu \phi|^2 \right),$$

(10)
where \( g \) is a constant. That the action \( S \) describes a massive gauge field can mostly easily be seen in the simpler case where \( g = 0 \) and disregarding the coupling to the scalar field. The free Abelian field equation of motion derived from \( S \) is then

\[
\tilde{\partial}^\mu \tilde{F}_{\mu\nu} = 0.
\]  

Choosing as gauge condition a VSR type Lorentz gauge \( \tilde{\partial}^\mu A_\mu = 0 \), we get

\[
\tilde{\partial}^2 A_\nu = (\Box - m^2)A_\nu = 0.
\]

i.e. \( A_\mu \) has mass \( m \). A similar argument shows that the scalar field also has mass \( m \).

Similarly for fermions coupled to an Abelian gauge field we have the gauge invariant lagrangian

\[
\mathcal{L} = \bar{\psi} \gamma^\mu i \tilde{D}_\mu \psi,
\]

and again it is possible to show that the fermionic field has mass \( m \).

In order to handle the non-local terms we use the definition

\[
\frac{1}{n \cdot \tilde{\partial}} = \int_0^\infty da \, e^{-an \cdot \tilde{\partial}}.
\]

Notice that replacing the wiggle by the raw definitions still preserve the symmetry of the action but now describes massless particles instead of VSR massive particles.

3 Non Abelian Gauge Fields

This is the most important section of the paper. We obtain the generalization of the covariant derivative and the gauge transformations in the presence of a non-Abelian gauge field in VSR.

We consider a scalar field transforming under a non-Abelian gauge transformation with infinitesimal parameter \( \Lambda \)

\[
\delta \phi = i \Lambda \phi.
\]

As before, we define the covariant derivative by

\[
D_\mu \phi = \partial_\mu \phi - i A_\mu \phi - \frac{i}{2} m^2 n_\mu \left( \frac{1}{n \cdot \tilde{\partial}^2} n \cdot A \right) \phi.
\]

To find out the non-Abelian gauge transformation of the gauge field we write

\[
\delta A_\mu = \partial_\mu \Lambda - i [A_\mu, \Lambda] + f_\mu,
\]

and \( f_\mu \) is determined by imposing the proper transformation property for the covariant derivative,

\[
\delta (D_\mu \phi) = i \Lambda D_\mu \phi.
\]
We then get
\[ f_\mu = \frac{i}{2} m^2 n_\mu \Lambda (\frac{1}{(n \cdot \partial)^2} n \cdot A) - \frac{i}{2} m^2 n_\mu (\frac{1}{(n \cdot \partial)^2} \Lambda) + \frac{i}{2} m^2 n_\mu (\frac{1}{(n \cdot \partial)^2} n \cdot [A, \Lambda]) - \frac{i}{2} m^2 n_\mu (\frac{1}{(n \cdot \partial)^2} n \cdot A) \Lambda. \]

Then the gauge transformation for the gauge field is
\[ \delta_\Lambda A_\mu = \partial_\mu \Lambda - i[A_\mu, \Lambda] + \frac{i}{2} m^2 n_\mu \left[ \Lambda, \left(\frac{1}{(n \cdot \partial)^2} n \cdot A\right) \right] - \frac{i}{2} m^2 n_\mu (\frac{1}{(n \cdot \partial)^2} \Lambda) + \frac{i}{2} m^2 n_\mu (\frac{1}{(n \cdot \partial)^2} n \cdot [A, \Lambda]) \quad (17) \]

For an Abelian gauge field \( A_\mu \) we get (1). We have also checked the closure of the algebra
\[ [\delta_\Lambda_1, \delta_\Lambda_2] A_\mu = -i[\Lambda_1, \Lambda_2] A_\mu \]

We also define the wiggle covariant derivative of the field \( \phi \) by:
\[ \tilde{D}_\mu \phi = D_\mu \phi - \frac{1}{2} m^2 n_\mu. \phi \quad (18) \]

It reduces to \( \partial_\mu \phi \) for \( A_\mu = 0. \)

The commutator of two covariant derivatives defines \( F_{\mu \nu} \), the \( A_\mu \) field strength,
\[ [D_\mu, D_\nu] \phi = -iF_{\mu \nu} \phi, \quad (19) \]

so we get
\[ F_{\mu \nu} = A_\nu,\mu - A_\mu,\nu - i[A_\mu, A_\nu] + \frac{1}{2} m^2 n_\nu (\frac{1}{(n \cdot \partial)^2} n \cdot A_\mu) - \frac{1}{2} m^2 n_\mu (\frac{1}{(n \cdot \partial)^2} n \cdot A_\nu) - \frac{i}{2} m^2 \left[ \left(\frac{1}{(n \cdot \partial)^2} n \cdot A_\nu\right), (n_\mu A_\nu - n_\nu A_\mu) \right] \quad (20) \]

It is hermitian if \( A_\mu \) is hermitian and it coincides with (6) for Abelian \( A_\mu \). Since under a gauge transformation
\[ [D'_\mu, D'_\nu] \phi' = i\Lambda [D_\mu, D_\nu] \phi = i\Lambda (-iF_{\mu \nu}) \phi = (-iF'_{\mu \nu}) i\Lambda \phi, \]
we find
\[ F'_{\mu \nu} = \Lambda F_{\mu \nu} \Lambda^{-1}. \quad (21) \]

The non-Abelian generalization of (6) is
\[ \tilde{F}_{\mu \nu} = F_{\mu \nu} - \frac{1}{2} m^2 \left( n_\nu \frac{1}{(n \cdot D)^2} (n_\alpha F_{\mu \alpha}) - n_\mu \frac{1}{(n \cdot D)^2} (n_\alpha F_{\nu \alpha}) \right), \quad (22) \]
where $D_\mu$ is the covariant derivative acting on fields that transform in the adjoint representation (see (28) below). Using the wiggle covariant derivative (18) we can write a gauge invariant action for the scalar field coupled to a non-Abelian gauge field

$$S = \int d^n x \left[ -\frac{1}{4} \text{tr} \left( \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + \frac{g}{2} \frac{1}{n \cdot D} n_\alpha \tilde{F}^{\alpha\mu} \frac{1}{n \cdot D} n_\alpha \tilde{F}_\mu^\alpha \right) + |\tilde{D}_\mu \phi|^2 \right].$$

(23)

Similarly, for fermions coupled to the non-Abelian gauge field we have the gauge invariant Lagrangian

$$L = \bar{\psi} \gamma^\mu \tilde{D}_\mu \psi.$$  

(24)

We again use the definition

$$\frac{1}{n \cdot D} = \int_0^\infty da e^{-a n \cdot D},$$  

(25)

so that

$$\tilde{F}_{\mu\nu} = \tilde{D}_\mu A_\nu - \tilde{D}_\nu A_\mu + O(A^2).$$  

(26)

Notice that $\tilde{F}_{\mu\nu}$ is the right field strength to describe massive bosons (and not $F_{\mu\nu}$). That is (23) describes massive gauge bosons (when $g = 0$) because the term quadratic in $A_\mu$ in (23) is the Abelian one (see (10) with $g = 0$) and as shown earlier it describes massive gauge bosons.

When the scalar field is in the adjoint representation we have

$$\delta \phi = i[\Lambda, \phi].$$  

(27)

The covariant derivative now takes the form

$$D_\mu \phi = \partial_\mu \phi - i [A_\mu, \phi] - \frac{i}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot A \right),$$  

(28)

while the non-Abelian gauge field transforms as

$$\delta A_\mu = \partial_\mu \Lambda = -i [A_\mu, \Lambda] + \frac{i}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot A \right) - \frac{1}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)} \Lambda \right)$$

+ \frac{i}{2} m^2 n_\mu \left( \frac{1}{(n \cdot \partial)^2} n \cdot [A, \Lambda] \right).$$  

(29)

From these we get the proper transformation for the covariant derivative

$$\delta(D_\mu \phi) = i[\Lambda, D_\mu \phi].$$  

(30)

4 Spontaneous Symmetry Breaking

In the context of VSR the non-Abelian gauge bosons all have mass $m$. In order to give different masses to the gauge bosons we need to break the non-Abelian
gauge symmetry. For this we consider a scalar field in the adjoint representation of the gauge group coupled to the gauge field

\[ S = \int d^nx \left[ -\frac{1}{4} \text{Tr}(\bar{F}_{\mu\nu}F^{\mu\nu}) + \text{Tr}(D\mu\phi)^2 + V(\phi) \right], \]  

(31)

where we defined

\[ \bar{F}_{\mu\nu} = F_{\mu\nu} - \frac{1}{2} \mu^2 \phi^2 \left( n_\nu \frac{1}{(n \cdot D)^2} (n_\alpha F_{\mu\alpha}) - n_\mu \frac{1}{(n \cdot D)^2} (n_\alpha F_{\nu\alpha}) \right), \]  

(32)

with \( \mu \) being a dimensionless parameter. If \( \phi \) gets a vacuum expectation value \( v \), then the gauge field will get a VSR mass matrix

\[ M = \mu v \]

in addition to the usual mass matrix coming from:

\[ \text{Tr}([A_\mu, v])^2 \]

In the Standard Model \( \mu \) must be tiny since there is no evidence of violations of SR there.

5 Conclusions

In this paper we have studied non-Abelian fields in VSR. To do this we have to define a covariant derivative and a modified gauge transformation. We have checked that the VSR covariant derivative commutes with the gauge symmetry. Moreover the non-Abelian VSR gauge transformations of the gauge field form a closed algebra. Having these, we can easily built actions for matter fields coupled to the VSR gauge fields. One important point to notice is that the VSR gauge fields are massive, although with a common mass. Since in nature, gauge fields may have different masses, as the Standard Model shows, we implement the spontaneous symmetry breaking in VSR with non-Abelian gauge symmetry. In this way the gauge fields can get the usual mass coming from spontaneous symmetry breaking plus a flavour dependent VSR mass.

There are several open problems. The main one is that VSR quantum field theory models are non local, so standard properties such as unitarity and renormalizability may be hard to prove in this type of models as well as asymptotic freedom.

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