Spatially resolved model for studying decarbonisation pathways for heat supply and infrastructure trade-offs

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HIGHLIGHTS

• A new optimisation model to study heat decarbonisation approaches is presented.
• Heat electrification found to be more cost-effective via district level heat pumps.
• Heat network penetration (HNP) dependent on linear heat density and zone topology.
• High temperature HNP over 50/60% for linear heat density over 1500/2500 kWh/m.
• Mid-temperature HNP over 20/30/40% for linear heat density over 1500/2500/3000 kWh/m.

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ABSTRACT

Heat decarbonisation is one of the main challenges of energy system decarbonisation. However, existing energy planning models struggle to compare heat decarbonisation approaches because they rarely capture trade-offs between heat supply, end-use technologies and network infrastructure at sufficient spatial resolution. A new optimisation model is presented that addresses this by including trade-offs between gas, electricity, and heat infrastructure, together with related supply and end-use technologies, with high spatial granularity. The model is applied in case studies for the UK. For the case modelled it is shown that electrification of heat is most cost-effective via district level heat pumps that supply heat networks, instead of individual building heat pumps. This is because the cost of reinforcing the electricity grid for installing individual heat pumps does not sufficiently offset heat infrastructure costs. This demonstrates the importance of considering infrastructure trade-offs. When modelling the utilisation of a decarbonised gas, the penetration of heat networks and location of district level heat supply technologies was shown to be dependent on linear heat density and on zone topology. This shows the importance of spatial aspects. Scenario-specific linear heat density thresholds for heat network penetration were identified. For the base case, penetration of high temperature heat networks was over 50% and 60% by 2050 for linear heat densities over 1500 and 2500 kWh/m. For the case when medium heat temperature networks were additionally available, a mix of both networks was observed. Medium temperature heat network penetration was over 20%, 30%, and 40% for linear heat densities of over 1500, 2500, and 3000 kWh/m, while high temperature heat network penetration was over 20% and 30% for linear heat densities of under 2000 and 1500 kWh/m respectively.

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1. Introduction

Climate change is one of the grand challenges of the 21st century [1], and heat provision has emerged as one of the most difficult energy services to decarbonise [2]. This is because heat provision has historically relied heavily on fossil fuels, and incumbent low cost end-use technologies have been deeply established.

Additionally, there are high costs and complexities associated with the infrastructure transitions that are required for decarbonising heat, such as the installation of heat networks or the reinforcement of electricity distribution networks needed to support the potential electrification of heat [3].

At the time of writing most heat decarbonisation modelling either focuses on specific technologies [4,5], uses coarse temporal/spatial resolution [6,7], or considers only part of the system [8,9]. No systematic framework exists that trades off individual building and district heat supply technologies and associated...
infrastructures at high spatial resolution. This paper formulates and applies such a framework. It presents a mixed integer linear optimisation model that selects gas, electricity, and heat network infrastructure investments, together with heat supply and end-use technology investment and operation, with the objective of minimising overall cost. The model is formulated as a long-term multi-period energy planning approach within a spatially disaggregated region, considering distances between and within sub-regional zones for infrastructure decisions. This model is then applied to study the case of the City of Bristol in the UK for two different scenarios of gas combustion emissions, reflecting the possibility of use of natural gas or a lower carbon gas such as bio-derived or synthetic methane, and for two scenarios of heat network circulation temperature.

The article is organised as follows: the following section sets out the background on heat decarbonisation, and a review of existing analytical approaches for assessing heat decarbonisation pathways. A methodology section then presents the model formulation and states the assumptions made. Results and discussion for a set of case studies are then presented. Finally a conclusion sets out the insights gained and directions for future research.

2. Background

2.1. Approaches to heat decarbonisation

It is clear from most studies that the continued use of natural gas as a core heating fuel is not likely to be consistent with long-term climate change mitigation targets [10]. The literature sets out a range of approaches to heat decarbonisation, which can broadly be categorised into variants that rely on one or a combination of: (a) decarbonised electricity, (b) low or zero carbon gases such as hydrogen or bio-derived/synthetic methane, (c) heat networks supplied with heat from low carbon sources, and (d) efficiency and behavioural change related approaches.

Lund et al. [11], based on the Danish case, conclude that the best heat decarbonisation solution is a gradual expansion of district heating supplied by combined heat and power (CHP) plants, together with individual heat pumps. Connolly et al. [12] propose a heat strategy for the European Union based on district heating and individual heat pumps, which can potentially reduce primary energy consumption and carbon emissions. When identifying challenges for a future non-fossil heat supply, Lund et al. [13] also propose a “4th generation” of district heating that includes lower network temperatures, efficiency improvements, and higher integration and synergies with the rest of the energy system. Also with regard to heat networks, Troup [14] argues that they are a good investment for heat decarbonisation today, with a view to decarbonising the supply of heat to these networks via biofuels or other low carbon alternative towards 2030. Troup [14] also argues that the main capital investments for heat networks are in network infrastructure, which outlines the heat supply equipment, and is therefore an important enabling measure for future low carbon heat. Dominčič et al. [15] show that interconnecting geographically distributed heat networks can potentially reduce primary energy consumption and CO₂ emissions. This could suggest a possibility of gradually installing heat networks when economically feasible and interconnecting them in a posterior stage.

Heat pumps are also commonly cited as a useful alternative to decarbonise heat in less dense areas [14,16], or by using central and booster heat pumps to supply heat networks. One potential advantage of heat pumps is that they perform well with low heat supply temperatures, and therefore could complement lower network temperatures [4]. Heat pumps have been shown to generate natural gas savings with the subsequent reduction in associated carbon emissions [5]. They are also attractive in the sense that at least part of their heat output can be classified as renewable, and further interest is motivated by the fact that the share of low carbon electricity generation has increased consistently in Europe and the UK in the years up until the time of writing [17]. Heat pumps do face some challenges though, including the requirement to reinforcement in the electricity network to cope with augmented electricity demand, high upfront equipment capital costs, and the potential need to replace internal building heat emitters with more expensive low temperature variants.

In [18], Dodds et al. present fuel cell technologies and hydrogen as alternatives for low carbon heat. Hydrogen is proposed as replacement for gas in countries with an extensive gas infrastructure, or for supplying heat networks through hydrogen CHPs. They also suggest the possibility of scaling up and expanding current hydrogen infrastructure, or building new hydrogen networks. Further interest in this possibility has been motivated by the H21 Leeds City Gate project [19], which proposed that the cost of transforming the Leeds city gas network to hydrogen from a low carbon source was both plausible and potentially cost-effective.

Overall, while a broad range of approaches have been presented, the general conclusion of studies is unequivocal; overall decarbonisation targets cannot be met without addressing heat decarbonisation [3,20,21]. The range of options and trade-offs is vast, and while initial studies have recognised this problem, no systematic and comprehensive study has emerged that considers this range with sufficient detail. Moreover, the studies cited above either implicitly or explicitly show that analytical methods that consider not only the different supply and end-use options, but also infrastructure trade-offs, are needed to inform decision makers about the best pathways towards decarbonisation of heat.

2.2. Models for the assessment of heat decarbonisation

Table 1 shows a categorisation of available energy system modelling approaches used to analyse heat decarbonisation or heat techno-economics, based on some of the categories proposed by [22] and other relevant categories for this study. When analysing the models that include heat supply presented in this Table, it can be seen that there is a gap in modelling infrastructure trade-offs with a fine spatial resolution. The models reviewed either include different distribution networks at national or regional levels without considering spatial aspects [23,24], or model heat networks at a higher resolution without considering trade-offs against other distribution networks such as gas or electricity [25,26], or do not include the lifetimes of the technologies and infrastructure when comparing different supply alternatives [27,28]. According to [29], spatial resolution is a key challenge going forwards in energy systems modelling, particularly for heat where demand density and infrastructure costs can potentially lead to very different decisions.

2.3. Research objective

Given the lack of existing analytical tools that (a) include a sufficiently granular spatial characterisation, (b) include supply, infrastructure and end-use technologies, and (c) include a bottom-up techno-economic depiction of the technical options, and (d) the importance of such an approach, the aims of this paper are to:
| Model | Purpose of the model | Geographical coverage/spatial resolution | Sectoral coverage | Time horizon | Time step | Technology inclusion | Demand characteristic inclusion | Cost inclusion | Analytical approach/methodology/Mathematical approach | Distribution networks inclusion | Heat network modelling | Other relevant comments for this research |
|-------|----------------------|-----------------------------------------|------------------|-------------|----------|--------------------|-------------------------------|--------------|-----------------------------------------------|-----------------------------|----------------|---------------------------------------------|
| A model for structural and operational optimisation of distributed energy systems [25] | General: exploring Specific: heat supply. | Demand: Exogenous. Supply: Modelled. | Urban area divided into zones | Year divided into two month intervals, with day/night period for each. | Heat and electricity technologies | Different user-defined heat and electricity demand profiles. | Heat network cost, technology costs, operation and maintenance (O&M) costs, fuel and electricity costs. | Bottom-up/ Optimisation/ Mixed integer linear programming (MILP). | Heat networks included | Heat networks modelled in detail with pipe lengths, diameters, and losses. Binary variables for existence of heat networks modelled. No lifetime of assets considered. |
| EnerGis [26] | General: exploring Specific: geographical information system for a more detailed assessment of heating options. | Demand: heat demand modelled for different building types based on measured and simulated data. Supply: Modelled. | Urban area divided into zones. | Medium term | User-defined, 4 periods in this research. | Heat supply technologies | Demand for heat in different building types (residential, public/commercial, industrial). | Investments, fuel costs, heat network infrastructure costs, fuel and electricity costs. | Bottom-up/ Simulation, optimisation/ Analytical methods MILP. | Heat networks included | Heat networks modelled as MILP. Integer variables for existence of heat networks between zones and within zones. Other distribution infrastructure not modelled. No lifetime of assets considered. |
| EnergyPLAN [30] | General: forecasting, exploring Specific: energy supply and demand | Technologically detailed. Demand: Exogenous. Supply: Modelled. | National, regional Electricity, heat, transport. | 1 year and medium term by aggregating years | Hourly Renewables for electricity supply, heat supply technologies in buildings | Demand for electricity and heat in buildings. | Fuel costs, investments, O&M, carbon. | Bottom-up/ Simulation, operation optimisation/ Analytical programming | Electricity, gas, and heat networks | Optimises operation of user-defined networks and technologies. The model optimises the operation of a user-defined system. No published studies include fine spatial resolution for heat. |
| ESME [24] | General: Exploring Specific: demand, supply, environmental impacts | Models major energy flows. Demand: Exogenous. Supply: Modelled. | UK divided into 12 regions. Whole energy sector, including heat in buildings. | Short and long term Seasons and intraday divisions | Renewables, recoverable heat, national and regional level Residential heat and transport supply not included. | Annualised investment, O&M, fuel costs, energy import costs. | Bottom-up/ Optimisation, Monte-Carlo/Linear programming | Not included | District heating supply options modelled as cost per KW |
| MARKAL [23] | General: exploring Specific: energy supply | Technologically detailed. Demand: Exogenous. Supply: Modelled. | National, local Energy sector | Medium-long term User-defined. | Renewables for electricity generation. Detailed transport modes. | Detailed electricity and heat demand in buildings. | Annualised investment, O&M, fuel costs, energy import costs, costs and revenues of | Bottom-up/ Optimisation/ Linear programming, dynamic programming | Gas, electricity, heat, and other commodities distribution included | Distribution is modelled as exchange processes. MARKAL is not spatially explicit in UK MARKAL or US EPA MARKAL. | (continued on next page)
| Model | Purpose of the model | Model structure | Geographical coverage/spatial resolution | Sectoral coverage | Time horizon | Time step | Technology inclusion | Demand characteristic inclusion | Cost inclusion | Analytical approach/methodology/Mathematical approach | Distribution networks inclusion | Heat network modelling | Other relevant comments for this research |
|-------|----------------------|----------------|---------------------------------------|-------------------|--------------|-----------|---------------------|-------------------------------|---------------|------------------------------------------------|-----------------------------|----------------|-----------------------------------------------|
| Multi-objective design optimisation of distributed energy systems through cost and energy assessments | General: Exploring Specific: Distributed energy systems | Demand: Exogenous. Supply: Existence, types, sizes, and numbers of devices modelled, with corresponding daily operation strategy. | No spatial representation | Electricity, cooling and heating | One year, hourly demands for four representative season days. | Distributed heat, electricity, and cooling supply technologies, and thermal energy storage. | Given user demand includes electricity, domestic hot water, space heating, and space cooling | exogenous energy and material imports/exports, energy taxes and subsidies, carbon costs, Capital costs, O&M. | Bottom-up/ Optimisation/ Multi-objective linear programme | Networks not modelled | No networks modelled | No networks modelled, no spatial representation |
| Multi-objective optimisation of energy systems and building envelope retrofit in a residential community | General: Exploring Specific: Energy systems and building retrofitting | Demand: Modelled in a first step simulation Supply: Modelled technologies sizes, and operation strategies. Retrofit modelled. | No spatial representation | Electricity and heat | One year, hourly | Heat and electricity supply technologies in buildings. Retrofitting measures | Electricity and heat demand for different building categories. | Annualised investment costs, fuel costs | Bottom-up/ Simulation, optimisation/multi-objective optimisation/ Multi-objective MILP | Heating distribution networks modelled as a binary variable associated to cost | Heating distribution networks modelled as a binary variable associated to cost | Networks not modelled in detail and no spatial representation |
| Sifre | General: Exploring Specific: Optimising energy systems operation | Demand: User specified/User specified energy system, Modelled operation | One zone or region, All energy system | User specified, one year in this research | Hourly | Energy conversion, transmission and end-use technologies, User-specified energy service demands. | Total operating costs | Bottom-up/ Optimisation/MILP | Gas, electricity, heat, and other commodities distribution included | Gas, electricity, heat, and other commodities distribution included | Networks are not spatially represented. Capital expenses not included in the model Networks are not spatially explicit in UK TIMES |
| TIMES | General: exploring Specific: decarbonisation pathways, least cost technology assessment | Technologically detailed. Demand: Exogenous. Supply: Modelled. | Global, national, regional, local | Medium-term or long-term Energy sector | User defined Seasonal, weekly, daily variation time slices. | As MARKAL | As MARKAL | As MARKAL | As MARKAL | As MARKAL | As MARKAL | As MARKAL | As MARKAL |

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Formulate and apply a spatially-resolved model of energy supply, infrastructure, end-use technology and demand to study cost effective heat decarbonisation pathways. Include within this model the three key different distribution networks (electricity, gas and heat), different temperature heat networks, lifetime of technologies and of network infrastructure, explicit infrastructure trade-offs, and different demand profiles. Apply this model to study least-cost heat decarbonisation pathways for a case study area in the UK in order to demonstrate the types of insights that it can generate.

The presented research addresses the gap in existing energy planning models, which rarely capture the before mentioned trade-offs at a sufficient spatial resolution for evaluating heat decarbonisation approaches. It is relevant for this special issue as it covers topics such as grid infrastructure trade-offs, energy systems modelling, and distributed versus centralised heat supply technologies. In a broader perspective, this research concerns the optimal use of energy resources, and how to develop plan and develop more sustainable and optimised energy systems.

It is emphasised that improving heat decarbonisation modelling in terms of temporal resolution is beyond the scope of this article. As discussed below, the framework is intended to capture seasonal and diurnal variations of flows, but not to account for second-to-second or minute-to-minute variations. It is noted that the relative flexibility of the supply-demand balance in heat provision may offer value to the overall energy system, and these finer time resolutions will be considered in future research.

3. Methodology

3.1. General description of the model

The model formulated in this research, named HIT (Heat Infrastructure and Technology) model, is a mixed-integer linear program optimisation that minimises the cost of supplying heat and electricity to a spatially-disaggregated region through to 2050. The planning time horizon is divided into time periods, which are further subdivided into time slices, in order to reflect diurnal and seasonal demand variations. Additionally, different demand profiles for different consumer types are modelled.

The region is subdivided into smaller zones, taking electricity and heat demand per zone and time slice as inputs. The outputs are the optimal investments in heat supply technologies in each zone and time period; the investments in infrastructure for heat, gas, and electricity networks in each zone and time period; the optimal operation of heat supply technologies in each zone and time slice; and the consumption and generation of gas, electricity, and heat across all networks in each zone and time slice. Further networks can be added in the framework developed, as has been done in this article for a second lower temperature heat network. The total system cost (investments, operations and carbon cost) are minimised for the whole planning time horizon. Lifetimes of infrastructure and equipment are considered to inform commissioning and decommissioning decisions. Table 2 describes the input data used.

For calculating network costs, two complementary approaches were adopted; an element of cost for connections between zones, and another element for costs within zones. Costs between zones are average costs per unit length and power (£/kW km), in order to account for different pipe or cable sizes connecting two zones and the length of these connections. The general methodology for calculating this was the following: First, cost data for different cable or pipe diameters per unit length was gathered for gas, heat,
Table 2 Description of input data.

| Input                           | Description                                                                 | Comments                                                                                           |
|---------------------------------|-----------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------|
| Spatial resolution              | City of Bristol                                                             | Bristol was spatially disaggregated into the Office for National Statistics [35] into MSOAs.        |
| Modelled area                   | 55 zones corresponding to Bristol Middle Layer Super Output Areas (MSOAs). See Fig. 1. |                                                                                                     |
| Modelled time horizon           | 2015 to 2051, plus 2013 calibrated base year.                               |                                                                                                     |
| Time periods                    | Investment/decommissioning decisions from 2015 to 2050, every 5 year periods. |                                                                                                     |
| Time slices                     | 16 time slices to reflect seasonal and diurnal variation.                   |                                                                                                     |
| Demand types modelled           | Domestic and commercial.                                                    | A simplifying assumption was made that annual demand was constant throughout the modelled time horizon. |
| Heat and electricity demand     | Commercial and domestic electricity and gas average annual demand data obtained from [39,40] for each MSOA. | Gas consumption data was converted into heat demand assuming an efficiency of 81% [27]. Unallocated consumption for a given area for domestic and commercial was distributed proportionally over all MSOAs for each sector. Demand weighted into time slices as explained previously. |
| Gas and electricity prices      | Prices for gas and electricity [41], central scenario.                      | Retail prices for commercial and domestic properties for gas and electricity obtained from [41] where multiplied by 0.84 to discount the price of distribution [42], as the model endogenously includes the cost of distribution networks. Wholesale fuel prices used for district technologies. |
| Emission factors                | Emission factors for gas and electricity [43] by sector.                   |                                                                                                     |
| Carbon prices                   | Non-traded carbon price projections [43] central scenario for the UK.       |                                                                                                     |

Table 3 Calculation assumptions for network costs between zones.

| Network | Cost data sources and assumptions | Sources and assumptions for maximum power through pipes/cables | R² |
|---------|-----------------------------------|---------------------------------------------------------------|----|
| Electricity | Unit costs for overhead and underground lines, for different distribution voltage levels [44]. The cost for different distribution voltage levels was weighted by the split of overhead and underground lines for each voltage level in the UK obtained from [45]. | Maximum current for different cable diameters obtained from catalogues [46] for different voltage distribution levels. With distribution voltage and maximum current, the maximum power was calculated for each cable diameter. | 0.995 |
| Gas     | Wales & West Utilities mains replacement matrix [47]. | Low pressure (LP) lines were assumed, as 83% length of distribution lines are LP [48]. Maximum power calculated for different diameters by assuming gas calorific value at standard conditions as 35.4 [MJ/m³] [49], a line pressure of 50 [mbar], line temperature of 5 °C, and maximum gas velocity of 20 [m/s] [50]. | 0.973 |
| Heat    | Cost data obtained from [51] for 2015, 2030, and 2050. | Maximum flow velocity for different diameters obtained from [52] for maximum pressure drop of 200 Pa/m. Assumed temperature difference of 30 ºC between supply and return. | 0.881 |

and electricity networks. Then, the maximum power transporting capacity for each cable and pipe size was calculated for the three networks [kW]. Finally, a linear regression was calculated between cost data and maximum power for different diameters, with the slope being the cost per unit length and unit power needed. Table 3 shows the data sources, assumptions and coefficient of determination (R²) for the linear regressions for the three networks.

Costs within zones are average costs per length for a typical network. Costs for the three networks were obtained from the cost sources in Table 3 for different pipe and cable sizes. The percentage of the distance per each voltage distribution level with respect to the total electricity distribution distance in the South West Region was obtained from [53]. This was then further weighted by the split between overhead and underground lines for each voltage level from [45], obtaining an average cost per km considering the split between different voltage distribution levels and the split between overhead and underground lines. For gas networks, mains lengths by pipe diameters were obtained from [47], obtaining the share of each pipe size per unit length which was weighted by the costs per pipe size. Finally for heat networks, the pipe size split per unit distance of network was taken from a real heat network case [54], obtaining an average network cost similar to the maximum capital cost per length of non-bulk schemes shown in [55].

Table 4 shows the parameters used for modelling networks.

Table 5 shows technology parameters used herein for individual and district heat and electricity supply technologies. Capital and fixed maintenance costs for individual heat and electricity supply technologies were modelled as zone dependant in order to account for properties needing to purchase equipment in standard sizes rather than the exact power to supply peak demand. To do this, the capital (or maintenance) cost for a given size in Table 5 was divided by peak heat demand, obtaining a cost per kW of after diversity peak demand for each technology and zone. This method ensures that the cost reflects the actual size of technologies, and implies that the heat and electricity supply capacity decision variable reflects the actual after diversity peak demand served by each technology, rather than containing a fraction of spare capacity within it.

3.2. Model formulation

The following sections give a general description of the model and state the main constraints and objective function. The
complete nomenclature and model formulation can be found in Appendix A and B respectively.

### 3.2.1. Main constraints

#### 3.2.1.1. Installed and working heat and electricity supply capacities

In this model there are three groups of supply technologies: individual or end-user heat supply technologies, district level heat supply technologies, and individual or end user electricity supply technologies. General constraints on installation and lifetime of technologies can be found in Appendix B. Eq. (1) states that the capacity of a given individual heat supply technology working on a given time slice can be at most the total installed capacity of that technology. Equivalent constraints are valid for district level heat and electricity supply technologies (Appendix B).

\[ \text{OCH}_i^{\text{bhi indjy}} \leq \text{TCH}_i^{\text{bhi indjy}} \quad \forall i \text{bhi indjy} \]  \hspace{1cm} (1)

Variables for new capacities of installed technologies are expressed in [kW] of a new purchased heat or electricity supply technology. Eqs. (2) and (3) enforce that only whole units of district heat supply technologies can be installed or decommissioned.

\[ N_i^{\text{Ddist jTy}} / C_1^{\text{CapD}} \text{dist T} = N_i^{\text{CHDidist jTy}} \quad \forall i \text{bhi indjy} \]  \hspace{1cm} (2)

\[ N_i^{\text{Ddist jTy}} / C_1^{\text{CapD}} \text{dist T} = N_i^{\text{CHDidist jTy}} \quad \forall i \text{bhi indjy} \]  \hspace{1cm} (3)

### Table 4

Network parameters.

| Name     | Description                  | Capital cost within zones [£2015/km] | Capital cost between zones [£2015/kW km] | Distribution losses | Lifetime [years] |
|----------|------------------------------|-------------------------------------|-----------------------------------------|---------------------|------------------|
|          |                              | 2015 | 2030 | 2050 | 2015 | 2030 | 2050 | Accounted for in electricity prices | 40 |
|          |                              | 2015 | 2030 | 2050 | 2015 | 2030 | 2050 | Accounted for in gas prices | 50 |
|          |                              | 2015 | 2030 | 2050 | 2015 | 2030 | 2050 | 13% of generation | 50 |
|          |                              | 2015 | 2030 | 2050 | 2015 | 2030 | 2050 | 5% of generation | 50 |

### Table 5

Heat and electricity supply techno-economic parameters.

| Supply technology code | Description                     | Thermal output capacity [kW] | Efficiency (Thermal/Electric) | Lifetime [Years] | Capital cost 2015 [£2015] | Capital cost 2030 [£2015] | Capital cost 2050 [£2015] | Operation cost all years [£2015] | Reference source |
|------------------------|---------------------------------|-----------------------------|-------------------------------|------------------|--------------------------|--------------------------|--------------------------|-------------------------------|-----------------|
| ASHPsmall              | Air source heat pump           | 12                          | 2.54                          | 20               | 11,100                   | 9290                     | 9290                     | 54                            | [59,60] Dom/Comm |
| ASHPbig                | Air source heat pump           | 100                         | 2.54                          | 20               | 45,260                   | 39,060                   | 39,060                   | 1300                          | [60] Comm |
| GSHPsmall              | Ground source heat pump        | 12                          | 3.27                          | 20               | 24,900                   | 20,840                   | 20,840                   | 65                             | [60,61] Comm |
| GSHPbig               | Ground source heat pump        | 100                         | 3.27                          | 20               | 200,100                  | 167,380                  | 167,380                  | 430                            | [60,61] Comm |
| Boilerbig              | Gas boiler                     | 24                          | 0.89                          | 15               | 1737                     | 1737                     | 1737                     | 87                             | [51,62] Comm |
| Boilerbig              | Electric resistance radiator   | 100                         | 0.89                          | 15               | 2182                     | 2182                     | 2182                     | 0                              | [59] Comm |
| CHPdistsmall           | Gas CHP unit                   | 12                          | 0.52/0.28                     | 15               | 12,052                   | 12,052                   | 12,052                   | 1085                           | [51] Comm |
| CHPdistbig             | Gas CHP unit                   | 100                         | 0.52/0.28                     | 15               | 100,434                  | 83,870                   | 83,870                   | 6775                           | [51] Comm |
| HXT1/HXT2big           | Heat exchanger unit (HIU) and heat meter high/medium temperature network | 12                          | 0.52/0.28                     | 20               | 2276                     | 2032                     | 1707                     | 68                             | [51] Comm |
| PVdom                  | Solar photovoltaics            | 2.5                         | 5030                          | 3420             | 3420                     | Dom/Comm                 | Dom/Comm                 | [51] Comm |
| PVcomm                 | Solar photovoltaics            | 10                          | 15,640                        | 10,650           | 10,650                   | Comm                     | Comm                     | [51] Comm |
| CHPdist1               | Gas CHP unit                   | 10000                       | 0.52                          | 30               | 8,390,000                | 8,390,000                | 8,390,000                | 677,456                         | [51] District |
| CHPdist2               | Gas CHP unit                   | 20000                       | 0.52                          | 30               | 14,300,000               | 14,300,000               | 14,300,000               | 1,011,368                      | [51] District |
| CHPdist3               | Gas CHP unit                   | 30000                       | 0.52                          | 30               | 19,560,000               | 19,560,000               | 19,560,000               | 1,184,745                      | [51] District |
| Boilerdist             | Gas boiler                     | 10000                       | 0.9                           | 30               | 900,000                  | 900,000                  | 900,000                  | 36,180                         | [51] District |
| ASHPdist               | Air source heat pump           | 10000                       | 3.27/5.5                     | 20               | 37,500,000               | 31,314,400               | 31,314,400               | 10,765                         | [51] District |
| GSHPdist               | Ground source heat pump        | 10000                       | 3.27/5.5                     | 20               | 37,500,000               | 31,314,400               | 31,314,400               | 10,765                         | [51] District |
3.2.1.2. Networks

The model considers two variables for network capacities: networks within zones, and networks between zones. Networks between zones are built along the linear distance between two zones, and the decision variable is expressed in power units [kW], reflecting that the decision is the pipe/cable diameter connecting two zones. Networks within zones are assumed to be built following the roads within each zone. The decision variable is expressed in length units [km], and the built network length within each zone is proportional to the percentage of peak heat demand served by a given network. Section 3.1 explains how the costs associated to both formulations were calculated. Initial networks for electricity and gas are assumed as described in Section 3.1. These initial networks are decommissioned linearly throughout the modelling time horizon.

The following three equations impose that heat, electricity, and gas networks need to be able to supply at least technologies installed in each zone. Because of the way new capacity and capital costs are modelled (explained in Section 3.1), the installed capacity of individual heat supply technologies is the capacity that serves after diversity peak heat demand. This means that the ratio of installed heat supply technologies consuming gas, electricity, or heat, to the total peak heat demand is the percentage of demand served by each network within a zone. Multiplying this by the total road length per zone gives the necessary length of each network within zones. Eqs. (4) to (6) show these constraints for heat, electricity, and gas networks respectively. Note that Eq. (4) should be for each temperature level of heat networks modelled.

Electricity networks need to be able to at least supply electricity consuming heat supply technologies plus electricity demand. Note that for electricity networks Eq. (5) assumes all electricity demand is still supplied by the electricity grid, plus the additional demand generated by the installation of new electricity consuming heat supply technologies.

\[X^b; i \cap \text{ind} = \frac{\sum H \times \text{TCI}_b \cap \text{ind}_j \times y}{\sum \text{Dem}_b \cap \text{peak}_j \times \text{Num}_b \times y} \times r_{lj} \times \text{TLN}_{\text{heat}_n \cap \text{T}} \times y \quad \forall j \cap T \cap y \] (4)

\[X^b; i \cap \text{ind} = \frac{\sum b_{\text{heat} \cap \text{gas}, \text{heat} \cap \text{electricity}} \times \text{TCI}_b \cap \text{ind}_j \times y}{\sum \text{Dem}_b \cap \text{peak}_j \times \text{Num}_b \times y + 1} \times r_{lj} \times \text{TLN}_{\text{elec}_n \cap y} \quad \forall j \cap y \] (5)
Gas network needs to be able to supply at least all gas consuming heat supply technologies:

\[
\sum_{h=\text{boilers\,CHPs}} \frac{\text{TCHI}_{h=\text{boilers\,CHPs}}}{\text{Dem}_{h=\text{boilers\,CHPs}} \cdot \text{Num}_{h=\text{boilers\,CHPs}}} \cdot R_l \leq TL_{\text{gas\,peak}} \quad \forall jy
\]  

(6)

Regarding networks between zones, Eq. (7) ensures that there can only be flow between zones if a network is in place and also determines the size of the connection. Additional constraints for networks include only allowing energy flows between neighbour zones and others listed in Appendix B.

\[
F_{hj/ny} \leq TC_{hj/ny} \quad \forall hj/ny
\]  

(7)

3.2.1.3. Energy balance in each zone

Eq. (8) constrains the energy balances in each zone for different networks. In power units, this equations states that the energy in heat, gas or electricity that is entering a zone minus the energy leaving the zone, needs to equate the energy being consumed in the zone minus the energy being generated in it. More details on energy flows can be found in Appendix B.

\[
F_{hj/ny}^{\text{IN}} - F_{hj/ny}^{\text{OUT}} + \sum_j F_{hj/ny}^{\text{in}} - \sum_j F_{hj/ny}^{\text{out}} - F_{hj/ny}^{\text{CONS}} + F_{hj/ny}^{\text{GEN}} = 0 \quad \forall hj/ny
\]  

(8)

Eq. (8) for electricity ensures electricity demand is met. Eq. (9) ensures individual heat demand is supplied by individual heat supply technologies:

\[
\sum_{b=\text{boilers}} OCHI_{b=\text{boilers}} \geq \text{Dem}_{h=\text{boilers}} \cdot \text{Num}_{h=\text{boilers}} \quad \forall bh/ijy
\]  

(9)

3.2.2. Objective function

Finally, Eq. (10) shows the objective function to be minimised which corresponds to the present value of the total system’s costs throughout the modelling time horizon. The total system’s costs include the annual operation and maintenance of technologies, capital costs for technologies and networks, fuel and electricity annual costs, decommissioning costs, income for selling electricity to the grid, and the salvage value of assets at the end of the modelling period. The definitions of all these individual costs are detailed in Appendix B.

\[
\min_{\text{COSTS}} = MNT + OP + FE + CRB + CPT + DEC - SLV - ES
\]  

(10)

3.3. Calibration, constraints, and scenarios

A base year calibration (chosen as 2013) was adopted for determining network capacities and technologies in place at the beginning of the modelling time horizon. A minimum cost optimisation was run for one year of operation in order to determine base year network topology. The assumptions and constraints for the base year were that heat demand is supplied exclusively by gas boilers, electricity demand is supplied exclusively by the electricity grid, and no heat networks are in place. Only three city boundary zones were available as entry points for gas and electricity. These are zones 14, 31 and 50 shown in Fig. 1. Domestic properties were not allowed to purchase district level heat supply technologies (Table 5). In zones

\[1\] This is a necessary assumption due to lack of available data on existing network topology. It is noted that if pre-existing network topology data were available, this could be provided to the model as an input, negating the need for this step.
where commercial after diversity peak heat demand was higher than 12 kW, commercial properties were not allowed to purchase domestic heat supply technologies apart from small gas boilers, and when their peak demand was higher than 24 kW they were only allowed to purchase larger technologies.

The base year results were used as initial conditions for networks and installed boilers in each zone when running the case from 2015 to 2051. Additional constraints were imposed for decommissioning initial capacity. Networks were enforced to decommission linearly over the modelling time horizon. One half of initially installed gas boilers were enforced to be decommissioned in 2015, and half in 2020, reflecting the average life of gas boilers in the market. The same constraints as for the base case were applied for entry points of gas and electricity and for domestic and commercial property purchases. Fig. 1 shows the modelled zones and their labels, Fig. 2 shows their heat densities, and Fig. 3 shows their linear heat densities.

Finally, two scenarios were defined for study in this research: Scenario 1 used all parameters as explained in Section 3.1. In contrast, Scenario 2 assumes that gas combustion emissions are half of emissions associated to natural gas for district level technologies. This is intended to depict a scenario where bio-derived or synthetic methane, mixed with natural gas, is utilised by these technologies. For Scenario 1, two cases were run: Case 1 where only high temperature heat networks were available, and Case 2 where medium temperature heat networks were also available.

Fig. 3. Linear heat density in the City of Bristol based on MSOA statistics and road lengths [kWh/m].

Fig. 4. Percentage of total individual heat supply installed capacity, Scenario 1, Case 1.
The commercial software GAMS v28.8.3 was used for implementing and solving the problem. For Scenario 1-Case 1, Scenario 1-Case 2, and Scenario 2, the problem has 4,047,186, 5,688,606, and 4,047,186 variables respectively, and takes approximately 6 h, 2.5 days and 11 h to solve with a relative termination tolerance of 0.001.

4. Results and discussion

Figs. 4–6 show the total share of individual heat and electricity supply technologies for both scenarios. Results show that individual-building electricity generation systems (e.g. PV) are not cost effective in the case studies, as all electricity is purchased from the grid. Scenario 1 shows that the most cost-effective pathway for heat supply in the domestic and commercial sectors is the adoption of heat networks, together with the use of individual dwelling gas boilers. In Scenario 1, the optimal heat supply for Case 2 is given by a mix of high and medium temperature heat networks, with an overall lower heat demand supplied by gas boilers. For Scenario 2, when lower emissions of gas for district heating is assumed, there is a faster penetration of heat networks for both the commercial and domestic sectors.

Figs. 7–9 show the district heat supply technologies over time for both scenarios and cases. For Scenario 1, heat networks are mainly supplied by air-source heat pumps, while Scenario 2 is additionally supplied by district level boilers, with a strong adoption of district level air-source heat pumps from 2030 onwards. These results together with the results shown in Figs. 4 and 5 show that for the topology studied heat electrification is more effectively reached through introducing heat grids and supplying them via heat pumps, rather than installing individual heat pumps for single buildings. This highlights the importance of considering the associated infrastructure costs when comparing different end use technology options, not only stand-alone technology life cycle costs (capital, operation and maintenance, carbon, decommission, etc.). In essence, there are trade-offs between infrastructure costs and end-use technology costs which can lead to less obvious solutions. In this case this is demonstrated through the fact that costs of rein-

Fig. 4.

Fig. 5.

Fig. 6.

Fig. 7.

Fig. 8.

Fig. 9.
forcing the electricity grid makes it more effective to electrify heat via district technologies than through individual level technologies.

For Scenario 1 results for Case 1 and Case 2 show that when medium temperature heat networks are also available the optimisation model selects these (Fig. 9) to the detriment of high temperature heat networks and gas boilers. Also, there is a higher participation of air-source heat pumps supplying both networks, with lower participation of district level gas boilers. This means that the higher network cost of the medium temperature network is counterbalanced by the higher coefficient of performance of heat pumps operating at medium temperatures, and by the lower network losses. This again shows the importance of trade-offs between infrastructure and end-use technologies.

Fig. 10 shows in which zones district heat supply technologies are installed over time for both scenarios. Comparing these with Fig. 2, it can be seen that district heat technologies are installed in the most heat dense zones. Comparing these results with Fig. 3 however shows that there is a better correlation with linear heat density and location of district heat technologies. For example, zone 4 has a low heat density, but a higher linear heat density, and particularly in Scenario 2 there is a higher installation of district technologies in it. However, heat density and linear heat densities are not the only factors to be considered in the decision of where to install district technologies. Connectivity with other heat dense zones is also a determining factor. For example, zones 22 and 25 are both heat dense and linear heat dense. They are neighbours, and both are also neighbours with zone 30. For both scenarios neither zone 22 nor zone 25 present a high installation of district heat technologies. Instead, a large number of district heat technologies are installed in zone 30. Therefore the location of district heat technologies is dependent on linear heat density and on zone-to-zone connectivity, which reflects the ability to aggregate demands and build larger heat networks. On the other hand, when comparing high and low temperature network cases in Scenario 1, results show that when two temperature networks are available, district heat supply technologies are more distributed among zones. For example, the total capacity of district heat technologies installed in zone 30 in Case 2 is lower than for Case 1, while other zones such as 1, 13, 17, and 51 have higher overall installed capacities. This shows that when medium heat networks are available there is a wider spread of supply technologies, and smaller and more distributed heat grids are built.

Fig. 11 shows the penetration of heat networks over time for both scenarios. Comparing these figures with Figs. 2 and 3, there is a correlation between heat network penetration and heat density, although linear heat density is a better indicator. For example, zone 51 has a relatively low heat density with a relatively high linear heat density, and for both scenarios there is a relatively high penetration of heat networks within it. The comparison between both scenarios shows that determining linear heat density thresholds for heat network penetration is case specific. The thresholds vary for emission levels of gas considered in both scenarios for district heating technologies. When comparing Case 1 and Case 2 for Scenario 1, a higher overall penetration of heat networks is achieved when two temperature networks are available. Also, as in for the case of district heat technology locations, heat networks penetration also depends on zone connectivity. This makes it even more challenging to determine a universal linear heat density threshold for heat network adoption.

Despite these caveats, in general terms it can be said that for Scenario 1-Case 1, for linear heat densities of over 1500 kWh/m there is a heat network penetration of more than 50% by 2050, and for linear heat densities of over 2500 kWh/m there is a heat network penetration of over 60% by 2050. For Scenario 2, for linear heat densities of over 1500 kWh/m there is a heat network penetration of more than 50% by 2050, and for linear heat densities over 2000 kWh/m there is over 60% of penetration of heat networks. These thresholds however are general guidelines, as they do not take into account the aspect of connectivity between zones, which these results show can materially influence cost effectiveness.

When two heat network technologies are available, there is a less direct heuristic threshold. For Scenario 1-Case 2, the temperature networks complement each other. For the medium temperature heat network, when linear heat density is greater than 1500 kWh/m, 2500 kWh/m, and 3000 kWh/m, penetration is over 20%, 30%, and 40% respectively. Although high temperature heat networks are also present at these higher linear heat densities, no clear thresholds of adoption were identified. However, when linear heat densities are lower than 2000 kWh/m, heat network penetration of high temperature networks is over 20%, and when linear heat density is lower than 1500 kWh/m, more than 30% of high temperature heat network penetration can be observed. Overall, when two heat network temperatures are available a higher total heat network penetration was observed. Clearly zone-specific heat density becomes less relevant, with demand topology and spatial topology playing a greater role.

Finally, a sensitivity analysis was conducted for low and high price scenarios for gas and electricity [41] and for carbon price scenarios [43]. Each parameter was changed to the low and high price scenarios while leaving the other two constant at central price scenarios. Figs. 12–14 show the total heat network penetration over
time for the three scenarios. Fig. 15 shows the total system’s cost, total cost of carbon, and total system CO₂ equivalent emissions, normalised to the highest value for each. These results show that the final heat network penetration by 2050 does not vary considerably when changing gas and electricity prices. By 2050 the total heat network penetration is around 60% for all these scenarios, as well as for central and high carbon price scenarios. The only scenario that shows to considerably affect total heat network penetration by 2050 is the low carbon price scenario, in which total heat network penetration is around 40% instead of 60%. In this scenario a larger share of individual level gas boilers is observed. Although the total heat network penetration by 2050 is not largely affected by gas, electricity and medium and high carbon prices, Figs. 12–14 show that the pathway for heat network penetration does vary.
for the scenarios studied. The uptake of heat networks is faster for high gas prices, for low electricity prices, and for high carbon prices. Fig. 15 shows that for low carbon prices the cost of carbon and total system’s cost are the lowest while total carbon emissions are the highest, as more individual gas boilers are operating. For electricity prices on the other hand, the low price scenario generates lower carbon emissions, carbon costs, and total cost than the central and high price scenarios, as more district level air-source heat pumps are used. Finally, regarding gas prices, the lowest carbon emissions are reached in the high gas price scenario, as this generates the lowest participation of gas boilers.

As set out by [20], action is required now in the UK for lowering carbon emissions from heat. Opportunities include installing individual building heat pumps, low carbon heat networks in cities, or increasing biomethane injection into the gas grid. This model provides a tool for studying the most cost effective pathways for

![Fig. 11. Heat network penetration.](image-url)
implementing these measures. For example, for the three cases and scenarios detailed previously results show a cost-effective timeline for installing high and medium temperature networks in different zones, and the total cost-effective heat network penetration per zone according to its linear heat density and demand topology. For the particular case of Bristol, according to [63], the Bristol City Council is planning to install heat networks across the City for supplying low carbon heat and power, in order to achieve their CO2 emissions reduction targets of 80% by 2050 (and intermediate milestones as well). Among others, one example of how this model can be applied for energy policy planning would be implementing a cap on total carbon emissions for different time periods in order to achieve specific reduction targets, and finding cost-effective strategies to achieve these targets.

5. Conclusions

This paper has presented a spatially resolved model for the analysis of heat decarbonisation pathways. The model includes economic and technical characterisation of the whole heat supply chain, including primary energy costs, end use technologies, and infrastructure. The novelty of this model is that it captures trade-offs between heat supply, end use technologies, and gas, electricity and heat network infrastructure at a high spatial resolution. This paper has applied the model to demonstrate the trade-offs between technology and infrastructure options.

Results show firstly that the inclusion of resolved demand and infrastructure topology in heat decarbonisation models can certainly influence the characteristics of the heat decarbonisation pathways created. For example, for the scenarios considered, heat electrification is more cost-effectively reached through installing heat networks supplied by air-source heat pumps, rather than by installing individual building heat pumps. This is due to the additional costs these individual technologies require related to the reinforcement of the electricity grid.

Secondly, heat network penetration and the location of district heat technologies are dependant not only on linear heat density, but also on zone-to-zone connectivity and topology, in addition to specific techno-economic assumptions. This further demonstrates the importance of considering spatial aspects in the planning of energy transitions. For the cases modelled, it is observed that linear heat density is a good indicator of district heating adoption where a high temperature heat network is available, with adoption in the range of 50% to 60% for linear heat densities from 1500kWh/m to 2500kWh/m. Yet in the scenarios where a medium temperature network is also available, linear heat density is less indicative of optimal heat network adoption, with cases of high adoption with low linear heat density, driven by favourable demand and network topology.

Finally, also when the option of installing a medium temperature heat network was available, a mix between high temperature heat networks and medium temperature networks was selected by
the model, with a higher penetration of the latter, and a higher overall penetration. This means that the higher coefficient of performance for district level heat pumps when delivering lower temperature heat, and the lower network losses in medium temperature networks, were able to offset the higher network costs. Medium temperature heat network penetration was over 20%, 30%, and 40% for linear heat densities of over 1500, 2500, and 3000 kWh/m², while high temperature heat network penetration was over 20% and 30% for linear heat densities of under 2000 and 1500 kWh/m² respectively. All zones had both temperature networks, but no clear threshold was identified outside of these ranges.

Based on the literature review, future work includes using the new model to study more extensive heat decarbonisation options. For example, by including a low temperature heat network together with booster heat pumps and efficiency improvements. Another example is the inclusion of a hydrogen network for individual and district heat supply technologies, or enabling the possibility of switching the current gas network to hydrogen. By including a cap on emissions from heat, this model can be used to effectively study the trade-offs between energy supply options, infrastructure and equipment costs, and thereby provide new insight on possible heat decarbonisation pathways.

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Appendix A

A.1. Nomenclature

This section describes the nomenclature used for the model formulation. The sets are described in Table A1, while Table A2 presents the decision variables for technology capacities. Table A3 shows the decision variables for network capacities. Table A4 shows the decision variables for fuel and electricity consumption, and Table A5 presents the cost decision variables. Table A6 shows the scalars, and Tables A7–A11 show the parameters used in this work.

Appendix B. Model formulation

B.1. Constraints

The following section describes the constraints used in the model.

B.1.1. Installed and decommissioned heat and electricity supply capacities

Eqs. (B.1) to (B.3) show the relationship between vintage capacity, new capacity, and decommissioned capacity for individual heat supply, district heat supply, and electricity supply technologies.

| Variable | Description | Units |
|----------|-------------|-------|
| NCHI | New installed capacity of individual heat supply technology | kW |
| VCHI | Vintage capacity of individual heat supply technology | kW |
| DCHI | Capacity of individual heat supply technology | kW |
| TCHI | Total capacity of individual heat supply technology | kW |
| NCHD | New installed capacity of district heat supply technology | kW |
| VCHD | Vintage capacity of district heat supply technology | kW |
| DCHD | Capacity of district heat supply technology | kW |
| TCHD | Total capacity of district heat supply technology | kW |
| NCC | Capacity of individual heat supply technology | kW |
| VCC | Vintage capacity of individual heat supply technology | kW |
| DCC | Capacity of individual heat supply technology | kW |
| TCC | Total capacity of individual heat supply technology | kW |
| NDC | Capacity of district heat supply technology | kW |
| VDC | Vintage capacity of district heat supply technology | kW |
| DDC | Capacity of district heat supply technology | kW |
| TDC | Total capacity of district heat supply technology | kW |
| NCF | New installed capacity of electricity supply technology | kW |
| VCF | Vintage capacity of electricity supply technology | kW |
| DCF | Capacity of electricity supply technology | kW |
| TCF | Total capacity of electricity supply technology | kW |

Table A1

Sets.

| Index | Description  | Index | Description |
|-------|--------------|-------|-------------|
| b     | Individual heat supply technologies | y     | Time period [Years] |
| j     | District heat supply technologies | h     | Time slice [Hours] |
| e     | Electricity supply technologies | a     | Fuel type |
| j     | Zones | b     | Demand type |
| T     | Temperature levels [K] | n     | Network type |

Table A2

Decision variables – Technology capacities.

| Variable | Description | Units |
|----------|-------------|-------|
| ICN | Capacity of network type n that connects zones j and j′ in year y | kW |
| CCN | Capacity of network type n that connects zones j and j′ in year y | kW |
| DCN | Capacity of network type n that connects zones j and j′ in year y | kW |
| TCN | Total capacity of network type n that connects zones j and j′ in year y | kW |
| NNL | New network length of network type n installed within zone j km in year y | km |
| VLN | Vintage network length of network type n installed within zone j km in year y | km |
| DLN | Network length of network type n installed within zone j km in year y | km |
| TN | Total network length of network type n within zone j km in year y | km |
| F | Flow in network n from zone j to zone j′ in timeslice h in year y | kW |
| g | Flow in network n from zone j to outside the boundaries of the city into km in year y | km |
| f | Flow in network n from zone j to outside the boundaries of the city into km in year y | kW |
| m | Water mass flow from zone j to zone j′ in timeslice h and year kg/s y at temperature level T | kW |
### Table A4
Decision variables and derived quantities – fuel and electricity consumption.

| Variable          | Description                                                                 | Units  |
|-------------------|-----------------------------------------------------------------------------|--------|
| ELECTTotY         | Total electricity consumed from the grid by demand type in year y           | kWh    |
| FUELTotY          | Total electricity consumed from the grid by district technologies in year y | kWh    |
| FUELDshby         | Fuel a consumed by demand type b in zone j in timeslice h in year y         | kWh    |
| FUELtotshby       | Total fuel a consumed by demand type b in year y                            | kWh    |
| CO2               | Total CO₂ equivalent emissions                                             | Ton    |

### Table A5
Costs elements of objective function.

| Variable          | Description                                                                 | Units  |
|-------------------|-----------------------------------------------------------------------------|--------|
| OP                | NPV of total variable system’s operation costs                              | £      |
| MNT               | NPV of total annual system’s maintenance costs                              | £      |
| FE                | NPV of total system’s fuel and electricity costs                            | £      |
| CRB               | NPV of total system’s carbon costs                                          | £      |
| CPT               | NPV of total system’s capital costs                                         | £      |
| EQ                | NPV of total equipment’s capital costs                                      | £      |
| NTW               | NPV of total networks’ capital costs                                        | £      |
| DEC               | NPV of total system’s decommission costs                                    | £      |
| SLV               | NPV of total system’s salvage value at the end of the modelling period      | £      |
| ES                | NPV of total system’s incomes from selling electricity to the grid          | £      |
| COSTS             | NPV of total system’s costs                                                | £      |

### Table A6
Scalars.

| Scalar | Description       | Value          |
|--------|-------------------|----------------|
| r      | Discount rate     | 7%             |
| c      | Specific heat capacity of water | 4.1813 kJ/kg K |
| y0     | Initial modelling year | 2013         |
| yfinal | Final modelling year | 2051         |
| M; M1  | Big numbers       |                |

These equations establish that vintage capacity in a given year is all capacity of a given technology that has been installed previously and hasn’t been decommissioned until that year.

\[ \text{VCHI}_{\text{biindjy}} = \text{NCHI}_{\text{biindjy}} - \sum_{y=0}^{y_{\text{final}}} \text{DCHI}_{\text{biindjy}} \vee \text{biindj} \vee y \leq y_{\text{final}} \]  \hfill (B.1)

\[ \text{VCHD}_{\text{idistjTy}} = \text{NCHD}_{\text{idistjTy}} - \sum_{y=0}^{y_{\text{final}}} \text{DCHD}_{\text{idistjTy}} \vee \text{idistTy} \vee y \leq y_{\text{final}} \]  \hfill (B.2)

\[ \text{VCE}_{\text{e}jy} = \text{NCE}_{\text{e}jy} - \sum_{y=0}^{y_{\text{final}}} \text{DCE}_{\text{e}jy} \vee \text{e}j \vee y \leq y_{\text{final}} \]  \hfill (B.3)

The total installed capacity of individual heat supply, district heat supply, and electricity supply technologies in a given year are defined by Eqs. (B.4),(B.5) and (B.6), respectively. These equations show that the total installed capacity of technologies equals the sum of all remaining vintage capacity in a given year, independent of when technologies were installed.

\[ \text{TCHI}_{\text{biindjy}} = \sum_{y=0}^{y_{\text{final}}} \text{VCHI}_{\text{biindjy}} \vee \text{biindj} \]  \hfill (B.4)

\[ \text{TCHD}_{\text{idistjTy}} = \sum_{y=0}^{y_{\text{final}}} \text{VCHD}_{\text{idistjTy}} \vee \text{idistTy} \]  \hfill (B.5)

\[ \text{TCF}_{\text{e}jy} = \sum_{y=0}^{y_{\text{final}}} \text{VCE}_{\text{e}jy} \vee \text{e}j \]  \hfill (B.6)
Table A10
Resource parameters.

| Parameter Description | Unit |
|-----------------------|------|
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |
| $Cost_{\text{fo}}^{\text{y}}$ | €/kWh |

Table A11
Demand parameters.

| Parameter Description | Unit |
|-----------------------|------|
| $Num_{\text{f},\text{y}}^{\text{y}}$ | – |
| $Dem_{\text{f},\text{y}}^{\text{y}}$ | kW |
| $Dem_{\text{f},\text{y}}^{\text{y}}$ | kW |
| $Dur_{\text{f},\text{y}}^{\text{y}}$ | hours |

Eqs. (B.7) to (B.9), define decommissioned heat supply capacity. The installed capacity needs to be decommissioned at the latest when it has met its lifetime, or though the option remains to decommission it earlier.

$$NCHI_{\text{bi,indj}^{\text{y}}} = \sum_{y' = 0}^{y-1} DCHI_{\text{bi,indj}^{y'}} \land \text{bi,indj}^{y'} (B.7)$$

$$NCHD_{\text{indj}^{\text{y}}} = \sum_{y' = 0}^{y-1} DCHD_{\text{indj}^{y'}} \land \text{indj}^{y'} (B.8)$$

$$NCE_{\text{y}} = \sum_{y' = 0}^{y-1} DCE_{\text{y}} \land \text{y} (B.9)$$

Eqs. (B.10) to (B.15) show border conditions for vintage capacity and decommissioned capacity. These equations establish that there cannot be any remaining capacity if it has not been previously installed, and technologies cannot be decommissioned before being installed.

$$VCHI_{\text{bi,indj}^{y'}} = 0 \land bi,indj,y > y'$$

$$VCHD_{\text{indj}^{y'}} = 0 \land \text{indj}^{T},y > y'$$

$$VCE_{\text{y}'} = 0 \land \text{y},y > y'$$

$$DCHI_{\text{bi,indj}^{y'}} = 0 \land bi,indj,y \geq y'$$

$$DCHD_{\text{indj}^{y'}} = 0 \land \text{indj}^{T},y \geq y'$$

$$DCE_{\text{y}'} = 0 \land \text{y},y \geq y'$$

B.1.2. Installed and working heat and electricity supply capacities

Eqs. (B.16) to (B.18) state that capacity of a given technology that is supplying heat and electricity operating on a given time slice can be at most the total installed capacity of that technology.

$$OCHI_{\text{bi,indj}^{y}} \leq TCHI_{\text{bi,indj}^{y}} \land \text{bi,indj}^{y}$$

$$OCHD_{\text{indj}^{y}} \leq TCHD_{\text{indj}^{y}} \land \text{indj}^{y}$$

$$OCE_{\text{y}} \leq TC_{\text{y}} \land \text{y}$$

Variables for new capacity are expressed in [kW] of a new purchased heat or electricity supply technology. Eqs. (B.19) to (B.21) enforce that only whole units of district heat supply technologies can be installed or decommissioned.

$$ND_{\text{indj}^{y}} \cdot Cap_{\text{indj}^{y}}^{p} = NCHD_{\text{indj}^{y}} \land \text{indj}^{y} (B.19)$$

$$NDD_{\text{indj}^{y}} \cdot Cap_{\text{indj}^{y}}^{p} = \sum_{y' = y}^{y+\infty} DCHD_{\text{indj}^{y'}} \land \text{indj}^{y'} (B.20)$$

$$ND_{\text{indj}^{y}} \cdot NDD_{\text{indj}^{y}} \in \mathbb{Z}^{+} (B.21)$$

Eq. (B.22) shows constrains all capacity variables to be continuous and not negative.

$$NCHI_{\text{bi,indj}^{y}}, \text{VCHI}_{\text{bi,indj}^{y}}, \text{DCHI}_{\text{bi,indj}^{y}}^{y}, \text{TCHI}_{\text{bi,indj}^{y}}^{y}, \text{OCHI}_{\text{bi,indj}^{y}}^{y}, \text{NCHD}_{\text{indj}^{y}}^{y}, \text{VCHD}_{\text{indj}^{y}}^{y}, \text{DCHD}_{\text{indj}^{y}}^{y}, \text{TCHD}_{\text{indj}^{y}}^{y}, \text{OCHD}_{\text{indj}^{y}}^{y}, \text{NCE}_{\text{y}}, \text{VCE}_{\text{y}}^{y}, \text{DCE}_{\text{y}}$$

$$TCE_{\text{y}}, \text{OCE}_{\text{y}} \geq 0 \land \text{behi}^{\text{dist}^{\text{indj}^{y}}} (B.22)$$

B.1.3. Networks

The model considers two variables for network capacities: networks within zones, and networks between zones. Networks between zones are built along the linear distance between two zones, and the decision variable is expressed in power units [kW], reflecting that the decision is the pipe/cable diameter connecting two networks. Networks within zones are assumed to be built following the roads within each zone. The decision variable is expressed in length units [km], and the built network length within each zone is proportional to the percentage of peak heat demand served by a given network. Section 3.1 explains how the costs associated to both formulations were calculated.

B.1.3.1. Infrastructure between zones

Eq. (B.23) states that a network between zones is connected when it has been installed and not decommissioned.

$$CCN_{\text{j,ny}}^{y} = ICN_{\text{j,ny}}^{y} - \sum_{y' = 0}^{y} DCN_{\text{j,ny}}^{y'} \land jy, n y$$

Initial networks for electricity and gas are assumed as described in Section 3.1. These initial networks are decommissioned linearly throughout the modelling time horizon as described by Eq. (B.24), when the time periods are 8.

$$DCN_{\text{j,ny}}^{y} = \frac{ICN_{\text{j,ny}}^{y}}{8} \land jy, y < n \neq \text{heatn}$$

Eq. (B.25) imposes installed networks to be decommissioned at most when they have met their lifetimes.

$$ICN_{\text{j,ny}}^{y} = \sum_{y' = 0}^{y} DCN_{\text{j,ny}}^{y'} \land jy^{y}$$

Eq. (B.26) defines the total connected network capacity between zones in a given year, independent of when it was installed.
\[ TCN_{ijy} = \sum_{y' < y} CCN_{ijy'y'} \quad \forall ij, ny \]  
(B.26)

Eq. (B.27) constrains all network variables to be non-negative.

\[ ICN_{ijy} \geq 0 \quad \forall ij, ny \]  
(B.27)

Eqs. (B.28) to (B.30) establish that connections between zones must be symmetric (which leads to dividing the cost by two in Eq. (B.72)).

\[ ICN_{ijy} = ICN_{jiny} \quad \forall ij, ny \]  
(B.28)

\[ CCN_{ijy'} = CCN_{jiny'} \quad \forall ij, nyy' \]  
(B.29)

\[ DCN_{ijy'} = DCN_{jiny'} \quad \forall ij, nyy' \]  
(B.30)

Border conditions for networks between zones are shown in Eqs. (B.31) and (B.32). These equations say that there can only be one equation per zone when they have met their lifetimes:

\[ CCN_{ijy} = 0 \quad \forall ij, n, y > y' \]  
(B.31)

\[ DCN_{ijy} = 0 \quad \forall ij, n, y > y' \]  
(B.32)

B.1.3.2. Infrastructure within zones

As for networks between zones, Eq. (B.33) defines a network when it has been installed and not decommissioned.

\[ VLN_{jyny} = NLN_{jyny} - \sum_{y' = 0}^{y} DLN_{jyny'} \quad \forall jn, \forall y \leq y' \]  
(B.33)

As described in Section 3.3, initial networks for gas and electricity are in place at the beginning of the modelled time horizon. Eq. (B.34) imposes initial networks to be decommissioned linearly throughout the 8 modelled time periods.

\[ DLN_{jyny} = \frac{NLN_{jyny}}{8} \quad \forall jn, y < n, n \neq \text{heat} \]  
(B.34)

Eq. (B.35) imposes installed networks to be decommissioned at most when they have met their lifetimes:

\[ NLN_{jyny} = \sum_{y' = 0}^{y} DLN_{jyny'} \quad \forall jn, ny \]  
(B.35)

And Eq. (B.36) defines the total installed network capacity within a zone at a given time period.

\[ TLN_{jyny} = \sum_{y' < y} VLN_{jyny'} \quad \forall jn, ny \]  
(B.36)

All network capacities must be non-negative, as shown in Eq. (B.37).

\[ NLN_{jyny}, VLN_{jyny'}, DLN_{jyny'} \geq 0 \quad \forall jnny' \]  
(B.37)

As in Eqs. (B.31) and (B.32), Eqs. (B.38) and (B.39) state that there can only be remaining networks and decommissioned networks if they have been previously installed.

\[ VLN_{jyny'} = 0 \quad \forall jn, \forall y > y' \]  
(B.38)

\[ DLN_{jyny'} = 0 \quad \forall jn, \forall y > y' \]  
(B.39)

B.1.3.3. Network capacity

The following three equations impose that heat, electricity, and gas networks need to be able to supply at least technologies installed in each zone. Because of the way new capacity and capital costs are modelled (explained in Section 3.1) the installed capacity of individual heat supply technologies is the capacity that serves after diversity peak heat demand. This means that the ratio of installed heat supply technologies' capacities is the modelled capacity divided by the demand served by each network. Multiplying this by the total road length per zone gives the necessary length of each network. Eqs. (B.40) to (B.42) impose the network capacity constraint. Note that Eq. (B.40) should be for each temperature level of heat networks modelled.

\[ \sum_{b_{jy}} TCHI_{b_{jy}}^{\text{heat},y} \cdot R_{ijy} \leq TLN_{\text{heat},1}^{y} \quad \forall ij, \forall y \]  
(B.40)

Electricity networks need to be able to supply at least all electricity using heat supply technologies plus electricity demand.

\[ \sum_{b_{jy}} TCHI_{b_{jy}}^{\text{elec},y} \cdot R_{ijy} \leq TLN_{\text{elec},1}^{y} \quad \forall ij, \forall y \]  
(B.41)

Energy balance within zones

As for networks between zones, Eq. (B.41) assumes that demand is still supplied by the electricity grid, plus the additional demand generated by the installation of new electricity-consuming heat supply technologies.

\[ \left( \sum_{b_{jy}} TCHI_{b_{jy}}^{\text{heat},y} \right) \cdot R_{ijy} \leq TLN_{\text{elec},1}^{y} \quad \forall ij, \forall y \]  
(B.42)

B.1.3.4. Energy balance in each zone. Eq. (B.43) constrains the energy balances in each zone for different networks. In power units, this equation states that the energy in heat, gas or electricity that is entering a zone is still supplied by the electricity grid, plus the additional demand generated by the installation of new energy-consuming technologies.

\[ F_{\text{het},1}^{\text{in}} + F_{\text{out}}^{\text{in}} + \sum_{F_{jyn}} + \sum_{F_{jyn}} - \sum_{F_{jyn}} - F_{\text{cons},1}^{\text{in}} + F_{\text{cons},1}^{\text{in}} = 0 \quad \forall hjny \]  
(B.43)

Consumption from heat networks equates heat delivered to buildings by heat exchangers, as shown by Eq. (B.44). Note that one equation would be needed for each heat network of different temperature levels. Flow generated into heat networks in each zone is the heat generated by district heat supply technologies. On the other hand, electricity generated in each zone is the generation from electricity supply technologies plus electricity generated by CHP units, as shown in Eq. (B.47).

\[ F_{\text{cons},1}^{\text{in}} = \sum_{b_{jy}} OCHD_{b_{jy}}^{\text{heat},1} \quad \forall hjT, y \]  
(B.44)

\[ F_{\text{cons},1}^{\text{in}} = \sum_{b_{jy}} (1 - \text{Loss}) \cdot OCHD_{b_{jy}}^{\text{heat},1} \quad \forall hjT, y \]  
(B.45)

For electricity, Eq. (B.46) shows that electricity consumption in each zone is the initial electricity demand for appliances, plus consumption of electricity-powered heat supply technologies. On the other hand, electricity generated in each zone is the generation from electricity supply technologies plus electricity generated by CHP units, as shown in Eq. (B.47).
Gas consumption in each zone is given by the energy that gas consuming heat supply technologies are using to transform into heat, as shown in Eq. (B.48). On the other hand there is no gas generation in the different zones, as shown in Eq. (B.49).

\[
F_{\text{CONS}}^{\text{Hij}} = \sum_b \left( \frac{OCHI_{\text{hij}}}{\eta_{\text{Hij}}^{\text{B}} \cdot \text{Num}_{\text{hij}}} + \sum_{\text{boilers}, \text{CHPs}} \frac{OCHI_{\text{hij}}}{\eta_{\text{Hij}}^{\text{CHPs}} \cdot \text{Dist}_{\text{hij}}^\text{T}} \right) \quad \forall \text{hijy} 
\]

(Eq. (B.48))

\[
F_{\text{CONS}}^{\text{Hij}} = 0 \quad \forall \text{hijy} 
\]

(Eq. (B.49))

\[
\sum_{\text{v}} OCHI_{\text{hij}}^{\text{v}} \geq \text{Dem}_{\text{hij}}^\text{H} \cdot \text{Num}_{\text{hij}} \quad \forall \text{hijy} 
\]

Additionally, the flow between zones for heat networks is given by Eq. (B.51). This equation should be repeated for each heat network temperature level.

\[
\dot{m}_{\text{hij}}^{\text{Ty}} \cdot (T - T_{y}^\text{c}) \cdot c = F_{\text{hij}}^{\text{heat}} \cdot \dot{m}_{\text{hij}}^{\text{Ty}} 
\]

(Eq. (B.51))

\[
F_{\text{hij}}^{\text{Ty}} \leq TCN_{\text{hij}}^{\text{Ty}} \quad \forall \text{hijy} 
\]

(Eq. (B.52))

Eqs. (B.53) and (B.54) ensure that a network can only be installed between two neighbour zones:

\[
ICN_{\text{hij}}^{\text{ny}} \leq \text{Nb}_{\text{hij}} \cdot M \quad \forall \text{hijy} 
\]

(Eq. (B.53))

\[
CCN_{\text{hij}}^{\text{nyy}} \leq \text{Nb}_{\text{hij}} \cdot M \quad \forall \text{hijy} 
\]

(Eq. (B.54))

And Eq. (B.55) states that there cannot be energy flow between a zone and itself:

\[
F_{\text{hij}}^{\text{ny}} = 0 \quad \forall \text{hijy} \quad j = \bar{j} 
\]

(Eq. (B.55))

For heat networks, Eq. (B.56) imposes no mass flow between a zone and itself:

\[
\dot{m}_{\text{hij}}^{\text{Ty}} = 0 \quad \forall \text{hijy} \quad j = \bar{j} 
\]

(Eq. (B.56))

Eq. (B.57) establishes that there can only be flow from outside the city boundary into a zone which is a city boundary, while Eq. (B.58) establishes that there can only be flow to outside the city boundary from a zone which is a city boundary

\[
0 \leq F_{\text{in}}^{\text{hijy}} \leq \dot{C}_{\text{hijy}} \cdot M \quad \forall \text{hijy} 
\]

(Eq. (B.57))

\[
0 \leq F_{\text{out}}^{\text{hijy}} \leq \dot{C}_{\text{hijy}} \cdot M \quad \forall \text{hijy} 
\]

(Eq. (B.58))

Eq. (B.59) defines all energy and mass flow variables as non-negative.

\[
F_{\text{in}}^{\text{hijy}} \cdot F_{\text{out}}^{\text{hijy}} \cdot F_{\text{CONS}}^{\text{hijy}} \cdot F_{\text{CONS}}^{\text{hijy}} \cdot F_{\text{CONS}}^{\text{hijy}} \cdot F_{\text{CONS}}^{\text{hijy}} \cdot m_{\text{hijy}}^{\text{Ty}} \geq 0 \quad \forall \text{hijy} 
\]

(Eq. (B.59))

B.1.4. Fuel and electricity consumption and CO2 emissions

Eqs. (B.60) and (B.61) show hourly fuel consumption by demand type in each zone, for individual heat supply and district heat supply technologies respectively. In these equations the sums are over individual heat supply and district heat supply technologies that consume fuel a. Eq. (B.62) and (B.63) show the total annual fuel consumption for individual heat supply and district heat supply technologies.

\[
F_{\text{Fuel}}^{\text{aby}} = \sum_{\text{aby}} OCHI_{\text{aby}}^{\text{vh}} \cdot \eta_{\text{Hij}}^{\text{V}} \cdot \text{Dur}_{\text{a}} \quad \forall \text{aby} 
\]

(Eq. (B.60))

\[
F_{\text{Fuel}}^{\text{aby}} = \sum_{\text{aby}} OCHI_{\text{aby}}^{\text{vh}} \cdot \eta_{\text{Hij}}^{\text{V}} \cdot \text{Dur}_{\text{a}} \quad \forall \text{aby} 
\]

(Eq. (B.61))

\[
F_{\text{Fuel}}^{\text{aby}} = \sum_{\text{aby}} F_{\text{Fuel}}^{\text{aby}} \quad \forall \text{aby} 
\]

(Eq. (B.62))

\[
F_{\text{Fuel}}^{\text{aby}} = \sum_{\text{aby}} F_{\text{Fuel}}^{\text{aby}} \quad \forall \text{aby} 
\]

(Eq. (B.63))

Eq. (B.64) shows the yearly consumption of electricity from the electricity grid by demand type, while Eq. (B.65) shows total yearly electricity consumption.

\[
E_{\text{elec}}^{\text{TOT}} = \sum_{\text{by}} \left( \frac{\text{Dem}_{\text{by}}^{\text{em}}}{\eta_{\text{Hij}}^{\text{B}} \cdot \text{Dur}_{\text{a}}} + \frac{\sum_{\text{aby}} OCHI_{\text{aby}}^{\text{vh}}}{\eta_{\text{Hij}}^{\text{V}} \cdot \text{Dur}_{\text{a}}} \right) \quad \forall \text{by} 
\]

(Eq. (B.64))

\[
E_{\text{elec}}^{\text{TOT}} = \sum_{\text{by}} \left( \frac{\text{Dem}_{\text{by}}^{\text{em}}}{\eta_{\text{Hij}}^{\text{B}} \cdot \text{Dur}_{\text{a}}} + \frac{\sum_{\text{aby}} OCHI_{\text{aby}}^{\text{vh}}}{\eta_{\text{Hij}}^{\text{V}} \cdot \text{Dur}_{\text{a}}} \right) \quad \forall \text{by} 
\]

(Eq. (B.65))

Eq. (B.66) shows the total CO2e emissions of the system throughout the modelling period.

\[
\text{CO}_{2} = \sum_{\text{aby}} F_{\text{Fuel}}^{\text{aby}} \cdot \text{Em}_{\text{a}}^{\text{em}} + \sum_{\text{aby}} F_{\text{Fuel}}^{\text{aby}} \cdot \text{Em}_{\text{a}}^{\text{em}} + \sum_{\text{by}} E_{\text{elec}}^{\text{TOT}} \cdot \text{Em}_{\text{a}}^{\text{em}} + E_{\text{elec}}^{\text{TOT}} \cdot \text{Em}_{\text{a}}^{\text{em}} 
\]

(Eq. (B.66))

B.1.5. Costs

Finally, all the systems costs are added and minimised. Eq. (B.67) defines the total fixed annual operation and maintenance costs, while Eq. (B.68) shows the total variable annual operation and maintenance costs. Eq. (B.69) quantifies total fuel and electricity costs, and Eq. (B.70) defines total carbon costs.

\[
\text{MNT} = \sum_{\text{by}} TCHI_{\text{hij}}^{\text{vy}} \cdot \text{Cost}_{\text{hij}}^{\text{M}} \cdot \frac{1}{(1 + r)^{B}} + \sum_{\text{by}} TCHD_{\text{hij}}^{\text{vy}} \cdot \text{Cost}_{\text{hij}}^{\text{M}} \cdot \frac{1}{(1 + r)^{B}} 
\]

(Eq. (B.67))
\[ OP = \sum_\text{hbulkry \text{fuel}} \cdot \text{Dur}_\text{hbulkry} \cdot \text{Cost}^{\text{hbulkry}} \cdot \frac{1}{(1 + r)^T} + \sum_\text{hbulkry \text{fuel}} \cdot \text{Dur}_\text{hbulkry} \cdot \text{Cost}^{\text{hbulkry}} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{hry}} \cdot \text{Dur}_\text{hry} \cdot \text{Cost}^{\text{hry}} \cdot \frac{1}{(1 + r)^T} \]

\[ FE = \sum_{\text{aby}} \cdot \text{FUEL}_\text{aby} \cdot \text{Cost}_\text{aby} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{aby}} \cdot \text{FUEL}_\text{aby} \cdot \text{Cost}_\text{aby} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{by}} \cdot \text{ELEC}_\text{by} \cdot \text{Cost}_\text{by} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{by}} \cdot \text{ELEC}_\text{by} \cdot \text{Cost}_\text{by} \cdot \frac{1}{(1 + r)^T} \]

\[ CRB = \sum_{\text{aby}} \cdot \text{FUEL}_\text{aby} \cdot \text{Em}_\text{aby} \cdot \text{Cost}_\text{CO}_2 \cdot \frac{1}{(1 + r)^T} + \sum_{\text{aby}} \cdot \text{FUEL}_\text{aby} \cdot \text{Em}_\text{aby} \cdot \text{Cost}_\text{CO}_2 \cdot \frac{1}{(1 + r)^T} + \sum_{\text{by}} \cdot \text{ELEC}_\text{by} \cdot \text{Em}_\text{by} \cdot \text{Cost}_\text{CO}_2 \cdot \frac{1}{(1 + r)^T} + \sum_{\text{by}} \cdot \text{ELEC}_\text{by} \cdot \text{Em}_\text{by} \cdot \text{Cost}_\text{CO}_2 \cdot \frac{1}{(1 + r)^T} \]

\[ NTW = \sum_{\text{jmy}} \cdot \text{INC}_\text{jmy} \cdot \frac{1}{(1 + r)^T} \]

\[ CPT = EQ + NTW \]

\[ DEC = \sum_{\text{hby}} \cdot \text{DCH}_\text{hby} \cdot \text{Cost}_\text{hby} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{hby}} \cdot \text{DCH}_\text{hby} \cdot \text{Cost}_\text{hby} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{hby}} \cdot \text{DCE}_\text{hby} \cdot \text{Cost}_\text{hby} \cdot \frac{1}{(1 + r)^T} \]

\[ \text{ES} = \sum_{\text{hy}} \cdot \text{F_\text{hy} \text{by}} \cdot \text{Dur}_\text{hy} \cdot \text{Sel}_\text{hy} \cdot \frac{1}{(1 + r)^T} \]

\[ \text{SLV} = \sum_{\text{hbulkry \text{fuel}}} \cdot \text{Cost}_\text{hbulkry} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{hbulkry \text{fuel}}} \cdot \text{Cost}_\text{hbulkry} \cdot \frac{1}{(1 + r)^T} + \sum_{\text{hry}} \cdot \text{Dur}_\text{hry} \cdot \text{Cost}_\text{hry} \cdot \frac{1}{(1 + r)^T} \]

\[ \text{B.2. Objective function} \]

Finally, Eq. (B.77) shows the objective function to be minimised which corresponds to the total system’s costs throughout the modelling time horizon.

\[ \text{minCOSTS} = MNT + OP + FE + CRB + CPT + DEC - SLV - ES \]

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