Effects of Van Hiele's Phase-based Teaching Strategy and Gender on Pre-service Mathematics Teachers’ Geometry Achievement in Niger State, Nigeria

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Citation: Usman, H., Yew, W. T., & Saleh, S. (2020). Effects of Van Hiele's Phase-based Teaching Strategy and Gender on Pre-service Mathematics Teachers’ Geometry Achievement in Niger State, Nigeria. International Journal of Pedagogical Development and Lifelong Learning, 1(1), e02006. https://doi.org/10.30935/ijpdll/8363

ABSTRACT
The study investigated the effects of Van Hiele’s phase-based teaching strategy and gender on pre-service mathematics teachers’ geometry achievement in Niger State, Nigeria. Based on the theoretical framework of the study, three research objectives with corresponding null hypotheses were formulated to guide the study. The study employed a two-by-two (2x2) quasi experimental factorial design. One hundred and forty-nine (149) pre-service mathematics teachers from two colleges of education situated in Niger state, Nigeria were used as research sample. The sample colleges were selected using a purposive sampling technique. The experimental group was exposed to Van Hiele’s phase-based teaching strategy while the control group was taught same topics with conventional teaching strategy. Geometry Achievement Test (GAT) covering topics in Geometry was used to collect data for both pre and post achievement test. A Cronbach alpha was computed to ascertain the internal consistency of the instrument (GAT) and reliability coefficient of 0.79 was obtained. The data was analyzed using Analysis of Covariance (ANCOVA) at 0.05 level of significance. The results of the study revealed that Van Hiele’s phase-based teaching strategy is more effective than conventional teaching strategy in improving pre-service mathematics teachers’ achievement. It was therefore recommended that, since it has been established from the findings of this study that Van Hiele phase-based teaching strategy is more effective than conventional teaching strategy in improving pre-service mathematics teachers’ geometry achievement, both prospective (pre-service teachers) and practicing teachers who seek to enhance their instructional practices and promote their learners' geometric understanding to embrace the Van Hiele phase-based teaching strategy in their classrooms.

Keywords: geometry achievement, gender, pre-service teachers, van Hiele’s phase-based teaching strategy

INTRODUCTION
Ever since mid-1980s quite a number of Geometrical researchers Cho-Koh (2000) Crowley (1987) Halat, (2008a) Serow (2008) Usiskin (1982) are of the believe that the learning process in geometry topics should be based on the geometrical model proposed by van Hiele, (1986). The model therefore has been the focus of recent academic research in the field of geometry due it’s prescriptive nature and growing interest in its application in various fields of geometry studies (Halat, 2008a; Noraini, 2005).

The van Hiele’s geometric model describes how children learn to reason in geometry, it consists of five levels and five phases of instruction which have been applied in many studies (Abdullahi & Zakaria, 2013a; Abu et al., 2012; Alex & Mammen 2016; Atebe, 2008; Chew, 2007; Chew & Lim, 2013; Usiskin, 1982) that are related to teaching and learning of geometry, it was however found to be effective in developing students’ academic achievement. The model was thus developed by two Dutch mathematicians in the 1950s, Pierre van Hiele and his wife Dina van Hiele-Geldof. The five levels according to Van Hiele (1986) are: Recognition, Analysis, Order, Deduction and Rigor.

The levels are attained as a result of experience and instruction rather than age. Consequently, a learner is required to have enough knowledge of (classroom or otherwise) geometric thoughts to move to a higher stage of complexity. That is to say that the feature of the model is hierarchical in nature. Each of the levels (levels 1 - 5) is accompanied by five phased-based instruction strategies. Chew (2009) affirmed that learners must go through the entire five phases to be able to achieve each of van Hiele’s level. The point here is, each level of geometry classroom instruction is attained as a result of sequence of phases (van Hiele, 1986). The five phases of instruction are: Information, Guided orientation, Explicitation, Free-orientation and Integration. Hence, when a teacher is able to move up level of geometry instruction process...
as a result of phase-base instruction, it will help in improving learner’s achievement in geometry.

Geometry as a subject is an essential area in the school mathematics curriculum throughout history (Robert, Primrose, & Christopher, 2018). It is perceived to be the centre or rather the focal point of mathematics (Modinow & Gray, 2001). It is also one area of mathematics that addresses properties and relations of constructible plane figures. In addition, it is a specific mathematical axiomatization of the properties and relations of plane shapes as studied, for instance, under Euclidean Geometry (Atebe, 2008). In other subject areas such as engineering drawing, and geometry drawing, geometry is applied (Abdullah & Zakaria, 2013a). Mak (2016) for instance observed that geometry is prerequisite to fundamental mathematics concepts such as number lines which are frequently employed in studying elementary skill of arithmetic, addition and subtraction, directed numbers and linear inequalities. Furthermore, Chew and Lim (2013) affirmed the position of learning geometry as vital to learning other topics in mathematics such as fractions, decimals, percentage, functions and calculus.

It is on the basis of the above that the National Policy on Education (Federal Government of Nigeria, 2004) states that mathematics and indeed geometry play a central role both as academic discipline and as knowledge that everybody needs in the society. It is however, the worst performed topic in mathematics in all Nigerian internal and external examinations making the overall Mathematics performance very poor (Chief Examiner’s report WAEC, 2014; NECO, 2012). Going by these statements, Atebe (2008) regarded academic achievement in geometry as a good pointer to academic achievement in mathematics specifically. It is on this note that Ogundele (2014) lamented that poor performance of students in mathematics precisely is so pitiful that interested parties keep on doubting reason why this level of students’ achievement is constantly unsuccessful in meeting the desires and aims of the society.

Consequently, Benjamin and Agwagah (2006) came up with helpful findings on the factors leading to learners’ poor academic achievement in mathematics and geometry at all levels of education among which include teachers’ subject matter incompetency. In affirming this, Benjamin and Agwagah (2006), Sanchez and Lopez (2011) were of the opinion that few of the existing difficulties in the teaching and learning of mathematics particularly geometry may be attributed to deficiency of mathematical understanding of learners learning to become teachers. In another related development Ali, Bhagawati, and Sarmah (2014), and Sunzuma, Masocha, and Zezekwa (2013) in their separate submissions attributed cause of poor academic achievement to inappropriate method of teaching and learning mathematics in schools. Ifamuwiya and Ajioloba, (2012) also observed that non-utilization of proper teaching methods and other apparatus, develop activities and experience for the learner so that understanding would grow from within. This is because it describes how to move up the level of instruction processes.

Hence, in accordance with existing research gap on consistent poor academic achievement of students at all level of education in mathematics generally and geometry in particular in Nigeria, the present research aimed at improving the academic achievement of prospective teachers (pre-service teachers) along with preparing them to teach at secondary school level using an effective teaching strategy. Therefore, the general goal of this study is to find out the effectiveness of van Hiele’s phase-based teaching strategy and gender on pre-service mathematics teachers’ geometry achievement in Niger state, Nigeria.

**THEORETICAL FRAMEWORK**

In line with van Hiele’s theory of geometric thinking put forward by Pierre and Dina van Hiele; the theory describes how children learn to reason in geometry, it is made up of three main features, which include: Levels of geometric thinking, properties of the Levels and phases of learning (Phase-based Instruction). Likewise, it a dual in nature; the descriptive and the prescriptive aspect. The descriptive aspect is that which the levels of geometric thinking that the student goes through are specified, while the prescriptive aspect is that which establishes the learning stages the teacher must guide the students
through so that they can acquire a determined level of knowledge (Sánchez-García 2016 p. 1195). The phases of learning provide a platform or model through which teachers could apply in classroom to promote students’ achievement in geometry (Crowley, 1987; Usiskin, 1982; van Hiele, 1986). One of the most important aspect of the theory/model as stressed by van Hiele (1986), and van Hiele (1999), is the sequential features of its operation, students has to pass through numerous levels of reasoning about geometric concepts, each of the levels is characterised by thinking process as the students move from simply identifying geometric figures to further be able to build a formal geometric proof.

**Review of Empirical Studies on van Hiele Theory**

Literature about van Hiele model and academic achievement in mathematics exist with different views and finding. To reinforce the current study, other related works done using van Hiele model were reviewed. Abdullah and Zakaria (2013a) affirmed that the treatment employing the van Hiele phase-based teaching can be employed in classrooms to improve learners’ geometric understanding. This is supported by Chew (2009) that learners must go via entire five phases to realize every aspect of the van Hiele’s level. In the opinion of Crowley (1987), the van Hiele phase-based instruction provides teachers the opportunity of allowing the learners to experience geometry stages associated with the van Hiele model.

Several studies such as Abdullah and Zakaria (2013b) Abdullah, Ibrahim, Surif, and Zakaria (2014) Alex and Mammen (2016) Al-ebous (2016) Atebe and Schafer (2011) Mostafa, Javad, and Reza (2016) were carried out employing van Hiele’s phase-based teaching strategy on academic achievement. In all of these studies, Quasi-experimental design, a pretest posttest control group design was employed. The findings however, revealed a substantial difference between treatment and control group. In contrary, Halat (2008) Parkin and Barkai (2014) Luneta (2015) in their separate findings though not directly link to quasi experimental studies indicates that there is no substantial difference on the focus group. For this reason, it became very necessary to investigate in order to reinforce previous findings.

Gender differences in mathematics achievement have continued to be an issue in the Nigeria educational system and indeed the whole of African countries and the world at large (Farajimakin, 2010). According to Egorova and Cherktkova (2016), despite extensive research on gender differences in mathematics achievement, many controversial issues and contradictions remains. Despite this, gender is one issue that has continued to receive attention in recent times especially in science, technology and mathematics education (Lin, Lin, Potvin, & Tsai 2018). The gap on mathematics achievement based on gender distribution according to Cimpian, Lubinski, Timmer, Makowski, and Miller (2016) deserves unique consideration in schools, since that is where potential mathematicians, computer scientists, and other science, technology, engineering, and mathematics (STEM) professionals tend to reside.

Ajai and Imoko (2015) study on gender revealed a significant difference in the academic achievement while other findings indicated that gender factor had no effect on students’ academic achievements. Adesoji and Fisuyi (2001), Kovas et al. (2015), and Musa et al. (2016), in their separate research reported that male students outperform their female counterparts in mathematics and science related subjects at secondary school level. In contrast, Anagboju and Ezehiora (2007), and Contini at al. (2016) notice that female learners outperformed their male counterparts. On the other hand, the result of this study tends to agree with the report of other researches such as Egorova and Cherktkova (2016), Iwendi and Oyedum (2014), whose results revealed no gender differences in achievement of males and females in mathematics and science subjects. For this reason, there is a necessity to investigate further on gender differences in Nigerian school context on pre-service mathematics teachers’ achievement based on the present trend in the world and attention given by united nations to gender matters in the millennium statement of September 2000.

It is on this note, that this research is meant to cover existing gap discovered on the consistent poor academic achievement of students at all level of education in mathematics generally and geometry in particular in Nigeria, to possibly improve academic achievement of prospective teachers (pre-service teachers). In addition, majority of the studies reviewed, employed van Hiele model in determining geometric thinking level in schools and little research study on the model was carried out in African context. As a result, non-existence or rather dearth of research regarding the van Hiele model is discovered in Nigeria. Again, to the best of researcher’s knowledge from the reviewed work, no study was conducted seeking to address issue of interaction effects of teaching strategy and gender on achievement. Hence, in accordance with existing research gap on consistent poor academic achievement of students at all level of education in mathematics generally and geometry in particular in Nigeria, the present research aimed at improving the academic achievement of prospective teachers (pre-service teachers) along with preparing them to teach at secondary school level using an effective teaching strategy. Therefore, the general goal of this study is to find out the effectiveness of van Hiele’s phase-based teaching strategy and gender on pre-service mathematics teachers’ geometry achievement in Niger state, Nigeria.

**Objectives of the Study**

The aim of this study is to determine the effects of van Hiele’s phase-based teaching strategy and gender on pre-service mathematics teachers’ geometry achievement and attitude towards geometry in Niger state, Nigeria.

Specifically, the research objectives were as follows:

1. Determine the main effect of teaching method on pre-service mathematics teachers’ achievement in geometry.
2. Determine the main effect of gender on pre-service mathematics teachers’ achievement in geometry.
3. Determine the interaction effect of teaching method and gender on pre-service mathematics teachers’ achievement in geometry.

**Null Hypotheses**

The null hypotheses of the study were as follows:

- **H₀₁** There is no significant main effect of teaching method on pre-service mathematics teachers’ achievement in geometry.
- **H₀₂** There is no significant main effect of gender on pre-service mathematics teachers’ achievement in geometry.
- **H₀₃** There is no significant interaction effect of teaching method and gender on pre-service mathematics teachers’ achievement in geometry.
METHODOLOGY

Research Design

The study employed a two-by-two (2x2) quasi experimental factorial design. This research design signifies two levels of treatments (van Hiele’s phase-based teaching strategy and conventional teaching strategy) and two levels of gender (male and female). As noted by Creswell (2013), Gay, Mills, and Airasian (2009) factorial design independently and simultaneously studies the effects of two or more independent treatment variables on an outcome. That is the design allows researcher to look at the individual (main effect) and joint effect (interaction effect) of two independent variables on one dependent variable. Main effect however, is the effect of just one of the independent variables on the dependent variable and an interaction effect occurs when the effect of one independent variable on the dependent variable depends on the level of a second independent variable (Pallant, 2010).

Sample and Sampling Technique

The sample of this study consisted of 149 pre-service mathematics teachers enrolled in MAT 122 from two purposively selected colleges of education in Niger State, Nigeria. Purposive sampling technique was used to obtain the 100 level pre-service mathematics teachers. The reason for choosing 100 level pre-service mathematics teachers is because MAT 122 which is designed to prepare the pre-service mathematics teachers to teach students based on secondary school content constitutes part of the course to be studied at this level. Two course lecturers taken MAT 122 were purposively selected one each from the selected colleges. The sample size of the pre-service mathematics teachers in the experimental and control group are 86 (62 male and 24 female) and 63 (54 male and 9 female) respectively captured from the intact class. This gave the total sample size of 149 pre-service mathematics teachers.

Instrumentation

Atebe (2008) observed that “following the development of the van Hiele theory of the levels of thought in geometry, experts and professional bodies have since developed tests instrument that can be used to measure the attainment of the van Hiele levels and achievement in geometry” (p. 99). Cognitive Development and Achievement in Geometry (CDAG) is one such test instrument. CDAG is a well-known and widely used in the United States (Usiskin, 1982). It is also a well-known geometric test instrument which has since been used in several PhD works (Atebe, 2008; Bafioe & Mereku, 2010; Burger & Shaughnessy, 1986; Knight, 2006; Usiskin, 1982). In studies investigating van Hiele level of understanding and achievement in geometry, CDAG was used and it was effective (Alex & Mammen, 2012; Halat & Sahin, 2008; Yazdani, 2007). For the purpose of this study, which involves determining the first-year pre-service mathematics teachers' geometry achievement, CDAG was adapted and named Geometry Achievement Test (GAT). The reason for adapting CDAG test was as a result of the view of van Hiele (1986) and Senk (1989) who observed that the understanding of the procedures that describe the van Hiele levels should be content specific. This suggests “that as the CDAG test was constructed in accord with the U.S. geometry curriculum” (Atebe, 2008, p. 99). It made sense to adapt the test questions in ways that reflect the Nigerian geometry curricular prescriptions. More so the study is designed for pre-service mathematics teachers based on secondary school content to prepare them to teach same content at secondary school level. The instrument offers a useful model, the contents and style of questioning in the instrument reflects van Hiele's model. The test contains 30 multiple choice items with five options (A-E). The aspect of geometry concepts focused are straight lines and circles.

Experimental Treatment

In this study, lessons plan on van Hiele’s phase-based teaching strategy and conventional teaching strategy was prepared and developed by the researcher for lecturers who handled pre-service mathematics teachers in both the experimental and control group respectively. Specifically, the lesson plan for the experimental group consisted of instructional units in line with “van Hiele’s phase-based instruction” (information, guided orientation, explicitation, free orientation and integration). This lesson plan serves as the treatment and guide to lecturer who handled experimental group. It explains how to implement instruction on geometry lesson based on van Hiele’s phase of teaching strategy. The lesson plan was however designed following van Hiele’s phases. In the control group, the pre-service mathematics teachers in this group were also taught the same content as in the experimental group but with conventional strategy using prepared lesson plan for the control group.

Validity and Reliability of the Instrument

All the 30 items in the instrument were validated by team of expert one each from sample colleges based on the objectives raised. The instrument was rated appropriate, adequate and satisfactory to measure what it intends to measure. Subsequently, the instrument was pilot tested to determine its reliability. In determining the reliability, Cronbach alpha the famous index described and stated in testing and evaluation’s item analysis was used because it measures the degree to which a given items of a test can offer a persistent and well-balance report regarding learners’ mastery of the domain (Pallant, 2010). The reliability coefficient was found to be 0.79 which is considered reliable (Pallant, 2010; Wells & Wollack, 2003).

Procedure for Data Collection

The entire study lasted for 8 weeks. The first week was spent on administering pre-test on GAT to the two colleges involved in the study, the College of Education A and College of Education B. This was followed by application of treatment; the van Hiele's phase-based teaching strategy by the assigned mathematics lecturer to the experimental group and conventional teaching strategy to the control group for six weeks. Both groups were taught the same contents. The posttest, which is identical to pretest, was administered at the eight weeks of the experiment.

Data Analysis

Descriptive statistics such as mean and standard deviation were used to answer all the research questions. Two-way Analysis of Variance (ANOVA) statistics in Statistical Package for Social Sciences (SPSS) version 22.0 was used to test all the hypotheses at 0.05, level of significance. The two-way analysis of variance (ANOVA) according to Pallant (2010) is an extension of the one-way ANOVA. The technique allows the researcher to look at the individual and joint effect of two independent variables on one dependent variable. Pallant (2010) further stressed the benefit of employing a two-way ANOVA was that; the ‘main effect’ for respective independent variable can be tested in
addition to the possibility to explore the 'interaction effect'. An interaction effect takes place immediately the effect of either one independent variable on the dependent variable rely on the level of a second independent variable. Having establish significant different in pre-test, to take care of the covariate that may be responsible for the significant difference, ANCOVA was used to analyse post test scores.

**RESULTS**

**Table 1.** Mean and Standard Deviation of Experimental and Control Groups on GAT

| Treatment            | N   | \( \bar{x} \) | SD  |
|----------------------|-----|----------------|-----|
| Experimental group   | 86  | 57.70          | 10.90 |
| Control group        | 63  | 46.03          | 14.97 |
| **Total**            | 149 | 52.77          | 13.98 |

**Table 2.** Mean and Standard Deviation on GAT by Gender

| Treatment   | N   | \( \bar{x} \) | SD  |
|-------------|-----|----------------|-----|
| Male        | 116 | 51.94          | 13.79 |
| Female      | 33  | 55.67          | 14.48 |
| **Total**   | 149 | 52.77          | 13.99 |

**Table 3.** Posttest Mean Achievement Score on the Basis of Interaction Strategies (Treatment and Gender)

| Treatment            | Gender | Achievement scores Mean (\( \bar{x} \)) |
|----------------------|--------|----------------------------------------|
| Experimental group   | Male   | 61.59                                  |
|                      | Female | 60.98                                  |
| Control group        | Male   | 41.50                                  |
|                      | Female | 37.72                                  |

females achieved higher than males considering their higher mean achievement scores at posttest. As a result of this observed difference in mean achievement scores, hypothesis 2 was tested at 0.05 to determine if the observed difference was significant.

**Table 3** shows mean achievement scores of pre-service mathematics teachers on the basis of the interaction strategies of treatment and gender. An examination of the table revealed that male pre-service mathematics teachers in experimental group (van Hiele's phase-based teaching strategy) outperformed male pre-service mathematics teachers in control group (conventional teaching strategy) in the posttest achievement scores. This is shown by the mean achievement score of 61.59 against 41.50. This trend was also observed for female pre-service mathematics teachers with mean achievement scores of 60.98 against 37.72 respectively. In view of this, male pre-service mathematics teachers achieved better across different interaction strategies confirming the absence of any interaction between gender and instructional strategy in the posttest achievement scores. Nevertheless, to determine if these observed differences were significant, hypotheses 3 was tested at 0.05 levels of significance.

**Table 4. Summary of Analysis of Covariance (ANCOVA) Result of GAT Posttest Achievement Scores of Experimental and the Control Group**

| Source              | Type III Sum of Squares | DF | Mean Square | F    | Sig. | Partial Eta Squared |
|---------------------|-------------------------|----|-------------|------|------|---------------------|
| Corrected Model     | 23706.672a              | 4  | 5926.668    | 163.303 | .000 | .819                |
| Intercept           | 9871.992                | 1  | 9871.992    | 272.013 | .000 | .654                |
| Preachiev           | 18697.348               | 1  | 18697.348   | 515.186 | .000 | .782                |
| Treatment           | 9005.780                | 1  | 9005.780    | 248.145 | .000 | .633                |
| Gender              | 101.188                 | 1  | 101.188     | 2.788  | .097 | .019                |
| Treatment * Gender  | 53.530                  | 1  | 53.530      | 1.475  | .227 | .010                |
| Error               | 5226.107                | 144| 36.292      |       |      |                     |
| **Total**           | 443772.000              | 149|             |       |      |                     |
| **Corrected Total** | 28932.779               | 148|             |       |      |                     |
dependent variable is accounted for by the independent variable. Therefore, hypothesis 3 was failed to be rejected. In other words, the effect of treatment (i.e., teaching method) on achievement in geometry of the pre-service mathematics teachers on posttest scores does not depend on the gender of the pre-service mathematics teachers. This therefore suggests that, irrespective of gender, learning through van Hiele’s phase-based teaching strategy was significantly more effective in improving the geometry achievement of pre-service mathematics teachers in comparison with conventional teaching strategy.

**DISCUSSION**

**Table 4** showed the ANCOVA result of the comparison of posttest achievement scores of pre-service mathematics teachers in Experimental and Control Group using the pretest as a covariate. The result indicated that treatment using van Hiele’s phase-based teaching strategy produce significant difference on pre-service mathematics teachers’ achievement in geometry. Based on the above results, it was inferred that the significant difference observed may be credited to the uniqueness of van Hiele’s phase-based teaching strategy which are descriptive and prescriptive aspect of the model. The descriptive aspect is that in which the level of geometric thinking of learner are specified, and a prescriptive aspect, establishes the learning stages the teacher must guide the learner so that they can acquired determined level of knowledge. The significant difference observed could also be attributed to the view of Mostafa (2016) in which he said that the uniqueness of the teaching model is based on the importance ascribed to learning action between learner and teacher that was stressed within the framework of van Hiele phases of instruction.

The result of this study considering the above therefore, is in conformity with the research findings of Abdullah and Zakaria (2013a), Abdullah, Ibrahim, Surif, and Zakaria (2014), Atebe and Schäfer (2011), Mostafa, Javad, and Reza (2016) in their separate research’s reports. Also, in consistent is the findings of Al-ebous (2016), y Reza (2016), Chew (2009), and Chew and Lim (2013) and Alex and Mammem (2016) whose studies were respectively meant at finding out the effect of the van Hiele model over conventional teaching strategy. This therefore implies the introduction of van Hiele’s phase-based teaching strategy is timely as learners show quite a significant achievement in geometry. In addition, the result has reinforced the need for employing van Hiele’s phase-bases teaching strategy in the classroom instruction with the view of achieving improved teaching and learning. On the other hand, the result of this study tends to disagree with the report of other research such as Egorova and Chertkova (2016)), Iwendi and Oyedum (2014).

**CONCLUSION**

The result obtained based on the aforementioned finding of the study indicated that there was significant difference between pre-service mathematics teachers taught using van Hiele’s phase-based teaching strategy and those taught using conventional teaching strategy. The main effect of gender on pre-service teachers’ achievement in geometry was found to be non-significant, that is, there is no significant main effect of gender on the posttest achievement scores of pre-service mathematics teachers. There was no statistically significant interaction effect of the independent variables (treatment-gender) on the achievement of pre-service mathematics teacher.

Hence, the results of the study indicate that van Hiele phase-based teaching strategy is more effective than conventional instructional strategy in improving pre-service teachers’ geometry achievement. Consequently, the use of van Hiele phase based instructional strategy could be regarded as one of the leverable strategies for enhancing achievement of pre-service mathematics teacher in Niger state and Nigeria in general.

**Recommendation**

Having established from the findings of this study that van Hiele phase-based teaching strategy is more effective than conventional teaching strategy in improving pre-service mathematics teachers’ geometry achievement, the researchers encourages both prospective (pre-service teachers) and practicing teachers who seek to enhance their instructional practices and promote their learners’ geometric understanding to embrace the van Hiele phase-based teaching strategy in their classrooms.

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