A general scheme for multiparty controlled quantum teleportation of an arbitrary N-particle state

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There is much interest in the multiparty quantum communications where quantum teleportation using high dimensional entangled quantum channel is one of the promising tools. In this paper, we propose a more general scheme for M-party controlled teleportation of an arbitrary N-particle quantum state using N-1 identical Einstein-Podolsky-Rosen pairs and one (M+2)-particle Greenberger-Horne-Zeilinger state together as quantum channel. Based on which a 2-party controlled teleportation of an arbitrary 3-particle state is tested with our scheme as an example.

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I. INTRODUCTION

Quantum teleportation (QT) [1] is considered as one of the most successful inventions in the quantum information research [2], it achieves the destination of quantum state transfer in a different way via prior shared entanglement as the quantum channel as well as the local operation and classical communication (LOCC), letting an arbitrary qubit collapses from the sender and “reborns” at the receiver’s side without distant qubit traveling. It plays significant roles both in the theoretical and experimental fields of quantum information processing. Since the first theoretical protocol for teleporting an unknown qubit in 1993 [1], this technology has been extensively studied [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]: QT of a bipartite Einstein-Podolsky-Rosen (EPR) pair [29] were presented [10, 11, 12, 13, 14]; schemes which teleport a tripartite Greenberger-Horne-Zeilinger (GHZ) state [30] as well as tripartite W state [31] were also investigated [15, 16, 17, 18].

Later, quantum communications were brought into a multiparty based environment and the QT using high dimensional entangled quantum channel is considered as one of the promising tools for this task. This definitely calls for general QT schemes in higher dimensional space. Recently, QT of an arbitrary N-particle state (N ≥ 1) via N EPR states as quantum channel was proposed [19] and a scheme for teleporting an unknown N-particle W state via entanglement swapping [32] was also introduced [20]; based on multiparticle entanglement theory in Ref. [14], a general scheme for teleporting an arbitrary N-particle state via genuine N-particle entangled quantum channel was given in Ref. [21].

More recently, combining with the idea of controlled communication, controlled quantum teleportation (CQT), which allows QT being performed under a third party’s permission, has been paid much attention and developed into the situation where more than one controller are needed [22, 23, 24, 25, 26, 27, 28]: CQT for teleporting a single-particle state have been studied [22, 23, 24, 25] and multiparticle quantum state from Alice to Bob under the control of many networked agents was also provided [26]: A method for symmetric multiparty controlled quantum teleportation (MCQT) of an arbitrary bipartite entanglement using two GHZ states was presented [27] and MCQT of a N-particle W state via N-1 EPR pairs and one (M+2)-particle GHZ state was proposed [28]. In this paper we completely summarize the schemes presented and propose a more general scheme for M-party MCQT of an arbitrary N-particle state via N-1 EPR pairs and an (M+2)-particle GHZ state as quantum channel.

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II. THE MCQT OF AN ARBITRARY N-PARTICLE STATE

A. Assumptions

Suppose the sender Alice wants to teleport an unknown N-particle state to the distant receiver Bob, and she is only able to do so as long as all M agents Charlie_1, Charlie_2, ..., Charlie_M permit, here the original arbitrary unknown state reads

\[ |\phi\rangle_{A_1A_2...A_N} = (x_1|000...00\rangle + x_2|000...01\rangle + ... + x_{2^N}|111...11\rangle)_{A_1A_2...A_N} \]

\[ = \sum_{k=1}^{2^N} x_k \prod_{i=1}^{N} |\delta_{kA_i}\rangle_{A_i}, \]

(1)

where \( \sum_{k=1}^{2^N} |x_k|^2 = 1, x_k \neq 0, \delta_{kA_i} \in \{0,1\}(i = 1,2,...,N) \). To achieve the QT task, Alice and Bob as well as the M agents take use of the quantum channel which is made of \((N-1)\) identical EPR pairs and an \((M+2)\)-particle GHZ state and follow our MCQT protocol. We assume each EPR pair used in the channel are in the state

\[ |\phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \]

(2)

and cases for other EPR pairs: \(|\phi^-\rangle, |\psi^+\rangle\) and \(|\psi^-\rangle\) are studied in the appendix.

B. The scheme

We detail the scheme in the following steps:

1. Alice prepares \((N-1)\) EPR pairs, taking the joint state

\[ |\phi\rangle_{DB} = \prod_{i=1}^{N-1} |\phi^+\rangle_{D_iB_i} \]

\[ = \prod_{i=1}^{N-1} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{D_iB_i} \]

(3)

with one \((M+2)\)-particle GHZ state

\[ |\phi^+\rangle_{A_NB_NC_1...C_M} = \frac{1}{\sqrt{2}}(|000...0\rangle + |111...1\rangle)_{A_NB_NC_1...C_M} \]

(4)

together as quantum channel.

2. Alice sends the \((N-1)\) particles \((B_1, B_2, ..., B_{N-1})\) from the state \(|\phi\rangle_{DB}\) and the GHZ particle \(B_N\) to Bob, she sends the \(M\) GHZ particles \((C_1, C_2, ..., C_M)\) to \(M\) agents \((Charlie_1, Charlie_2, ..., Charlie_M)\) respectively while keeping the other \((N-1)\) EPR particles \((D_1, D_2, ..., D_{N-1})\) and one GHZ particle \(A_N\) to herself. Therefore, including the unknown state \(|\phi\rangle_{A_1A_2...A_N}\), the whole system state reads

\[ |\psi\rangle = |\phi\rangle_{A_1A_2...A_N} \otimes |\phi\rangle_{DB} \otimes |\phi^+\rangle_{D_NB_NC_1...C_M} \]

\[ = |\phi\rangle_{A_1A_2...A_N} \otimes (|\phi^+\rangle_{D_1B_1} \otimes |\phi^+\rangle_{D_2B_2} \otimes ... \otimes |\phi^+\rangle_{D_{N-1}B_{N-1}}) \otimes |\phi^+\rangle_{D_NB_NC_1...C_M} \]

\[ = \sum_{k=1}^{2^N} x_k \prod_{i=1}^{N} |\delta_{kA_i}\rangle_{A_i} \otimes \prod_{i=1}^{N} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{D_iB_i} \otimes \frac{1}{\sqrt{2}}(|000...0\rangle + |111...1\rangle)_{A_NB_NC_1...C_M}. \]

(5)

3. Alice performs the Bell-joint measurements on pairs \((A_i, D_i)\), \((i = 1, 2, ..., N-1)\) separately, obtaining result \(|\psi\rangle_{A_iD_i}\). Then she broadcasts these results via classical channel, according to which Bob correspondingly performs a series of single-particle unitary transformations \(U_i\) on particles \(B_i\), \((i = 1, 2, ..., N-1)\), see Tab. [I].
4. Alice performs Bell-joint measurements on the pairs \((A, B)\) from Eq. 7 and Table I, we have results. According to the results, this time, Bob performs a corresponding unitary transformation \(|\psi\rangle = \sum_{1}^{N} U_{i} D_{i} |N\rangle\). The Bell-joint measurement on pair \((A, B)\) is the \(\psi^{\dagger} M \psi\) unitary \(N_{M} \psi\) unitary \(A_{i} D_{i} |\psi\rangle\). Charlie’s unitary transformation \(U_{C_{i}}\) is the Bell-joint measurement on pair \((A_{i}, D_{i})\).

Now we analyze the whole system in Eq. (8). For simplicity, define

\[
|\beta_{B_{i}}\rangle = U_{i} A_{i} D_{i} (|\delta_{k_{A_{i}}} A_{i} |\phi^{+}\rangle B_{i}),
\]

(7)

\[
|\beta_{B_{i}C_{1}...C_{M}}\rangle = U_{N} U_{C_{i}} \otimes A_{N} D_{N} \langle |\delta_{k_{A_{i}}} A_{i} |\phi^{+}\rangle D_{N} B_{N} C_{1}...C_{M},
\]

(8)

From Eq. (7) and Table II we have

\[
|\beta_{B_{i}}\rangle = \begin{cases} 
I_{B_{i}} (|A, 0|\delta_{k_{A_{i}}} A_{i}, 0)_{B_{i}} + A_{i} (|1|\delta_{k_{A_{i}}} A_{i}, 1)_{B_{i}} & \text{for } |\psi\rangle A_{i} D_{i} = |\phi^{+}\rangle \\
(\sigma_{x})_{B_{i}} (|A, 0|\delta_{k_{A_{i}}} A_{i}, 0)_{B_{i}} - A_{i} (|1|\delta_{k_{A_{i}}} A_{i}, 1)_{B_{i}} & \text{for } |\psi\rangle A_{i} D_{i} = |\phi^{-}\rangle \\
(\delta_{x})_{B_{i}} (|A, 0|\delta_{k_{A_{i}}} A_{i}, 1)_{B_{i}} + A_{i} (|1|\delta_{k_{A_{i}}} A_{i}, 0)_{B_{i}} & \text{for } |\psi\rangle A_{i} D_{i} = |\psi^{+}\rangle \\
(i\delta_{x})_{B_{i}} (|A, 0|\delta_{k_{A_{i}}} A_{i}, 0)_{B_{i}} - A_{i} (|1|\delta_{k_{A_{i}}} A_{i}, 1)_{B_{i}} & \text{for } |\psi\rangle A_{i} D_{i} = |\psi^{-}\rangle 
\end{cases}
\]

(9)
From Eq. 8 and Tab. 11 we have

\[
|\beta_{BN C_1 \ldots C_M}\rangle = \left\{ \begin{array}{ll}
& I_{BN} \left( \prod_{j=1}^{M} I_{C_j} \right) ( A_N \langle 0| \delta_{A_N} \rangle_{k A_N} |000\ldots0\rangle_{B_N C_1 C_2 \ldots C_M} + A_N \langle 1| \delta_{A_N} \rangle_{A_N} |111\ldots1\rangle_{B_N C_1 C_2 \ldots C_M} ) \\
& \text{for } |\psi\rangle_{A_N D_N} = |\phi^+\rangle \\
& (\sigma_x)_{BN} \left( \prod_{j=1}^{M} I_{C_j} \right) ( A_N \langle 0| \delta_{A_N} \rangle_{k A_N} |000\ldots0\rangle_{B_N C_1 C_2 \ldots C_M} - A_N \langle 1| \delta_{A_N} \rangle_{A_N} |111\ldots1\rangle_{B_N C_1 C_2 \ldots C_M} ) \\
& \text{for } |\psi\rangle_{A_N D_N} = |\psi^-\rangle \\
& (i\sigma_y)_{BN} \left( \prod_{j=1}^{M} I_{C_j} \right) ( A_N \langle 0| \delta_{A_N} \rangle_{k A_N} |111\ldots1\rangle_{B_N C_1 C_2 \ldots C_M} + A_N \langle 1| \delta_{A_N} \rangle_{A_N} |000\ldots0\rangle_{B_N C_1 C_2 \ldots C_M} ) \\
& \text{for } |\psi\rangle_{A_N D_N} = |\psi^-\rangle \\
& = A_N \langle 0| \delta_{A_N} \rangle_{A_N} |000\ldots0\rangle_{B_N C_1 C_2 \ldots C_M} + A_N \langle 1| \delta_{A_N} \rangle_{A_N} |111\ldots1\rangle_{B_N C_1 C_2 \ldots C_M} \\
& = \{ |111\ldots1\rangle_{B_N C_1 C_2 \ldots C_M}; |\delta_{A_N} \rangle_{A_N} = 1 \}
& \{ |000\ldots0\rangle_{B_N C_1 C_2 \ldots C_M}; |\delta_{A_N} \rangle_{A_N} = 0 \}
& = |\delta_{A_N} \delta_{A_N} \ldots \delta_{A_N} \rangle_{B_N C_1 C_2 \ldots C_M}.
\end{array} \right.
\]

(10)

Therefore, the whole system in Eq. 6 reads

\[
|\psi_1\rangle_1 = \sum_{k=1}^{2^N} x_k \left( \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |\delta_{A_N} \delta_{A_N} \ldots \delta_{A_N} \rangle_{B_N C_1 C_2 \ldots C_M} \right)
\]

\[
= \sum_{k=1}^{2^N} x_k \left( |\delta_{kA_2 \ldots kA_{N-1}} \rangle_{B_2 \ldots B_{N-1}} \otimes |\delta_{A_N} \delta_{A_N} \ldots \delta_{A_N} \rangle_{B_N C_1 C_2 \ldots C_M} \right)
\]

\[
= \sum_{k=1}^{2^N} \left( \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |\delta_{kA_N} \rangle_{B_N} \otimes |\delta_{A_N} \delta_{A_N} \ldots \delta_{A_N} \rangle_{C_1 C_2 \ldots C_M} \right).
\]

(11)

5. The \( M \) agents (Charlie1, Charlie2, ..., Charlie\( M \)) respectively perform the Hadamard transformations on the particles in their hands. Since \( H(0) = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \) and \( H(1) = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \), the whole system in Eq. 11 becomes

\[
|\psi_2\rangle_2 = (x_1 + x_3 + \ldots + x_{2^{N-1}}) \left[ \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |0\rangle_{B_N} \otimes \prod_{j=1}^{M} (|0\rangle + |1\rangle)_{C_j} \right]
\]

\[
+ (x_2 + x_4 + \ldots + x_{2^N}) \left[ \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |1\rangle_{B_N} \otimes \prod_{j=1}^{M} (|0\rangle - |1\rangle)_{C_j} \right]
\]

\[
= (x_1 + x_3 + \ldots + x_{2^{N-1}}) \left[ \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |0\rangle_{B_N} \otimes \sum_{i=1}^{N-1} (|C_i \rangle_{C_1 \ldots C_M} + |C_0 \rangle_{C_1 \ldots C_M}) \right]
\]

\[
+ (x_2 + x_4 + \ldots + x_{2^N}) \left[ \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |1\rangle_{B_N} \otimes \sum_{i=1}^{N-1} (|C_i \rangle_{C_1 \ldots C_M} - |C_0 \rangle_{C_1 \ldots C_M}) \right]
\]

\[
= \sum_{i=1}^{N-1} |C_i \rangle_{C_1 \ldots C_M} \left[ (x_1 + x_3 + \ldots + x_{2^{N-1}}) \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |0\rangle_{B_N} + (x_2 + x_4 + \ldots + x_{2^N}) \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |1\rangle_{B_N} \right]
\]

\[
+ \sum_{i=1}^{N-1} |C_0 \rangle_{C_1 \ldots C_M} \left[ (x_1 + x_3 + \ldots + x_{2^{N-1}}) \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |0\rangle_{B_N} - (x_2 + x_4 + \ldots + x_{2^N}) \prod_{i=1}^{N-1} |\delta_{kA_i} \rangle_{B_i} \otimes |1\rangle_{B_N} \right].
\]

(12)
6. To permit Alice and Bob’s QT request, each agent measures his particle in basis \{\{0\}, \{1\}\}, and publishes his result in “0” or “1” correspondingly. According to the number of “1” in the \(M\) agents’ published measurement results, Bob choose different operations as below to fully recover the original unknown \(N\)-particle state of Alice:

- If the \(M\) agents’ measurement results contain odd number of “1”, Bob’s \(N\)-particle system must be in the state

\[
|\psi\rangle_B = (x_1 + x_3 + \ldots + x_{2N-1}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |0\rangle_{B_N} + (x_2 + x_4 + \ldots + x_{2N}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |1\rangle_{B_N} \tag{13}
\]

and he has recovered Alice’s original state;

- If the results contain even number of “1”, then Bob’s \(N\)-particle system must be in the state

\[
|\psi\rangle_B = (x_1 + x_3 + \ldots + x_{2N-1}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |0\rangle_{B_N} - (x_2 + x_4 + \ldots + x_{2N}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |1\rangle_{B_N}, \tag{14}
\]

and he only needs to execute the unitary transformation \(\sigma_z\) on his particle \(B_N\), and the state of \(|\phi\rangle_B\) turns into

\[
(\sigma_z)_{B_N} |\phi\rangle_B = (\sigma_z)_{B_N} [(x_1 + x_3 + \ldots + x_{2N-1}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |0\rangle_{B_N} - (x_2 + x_4 + \ldots + x_{2N}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |1\rangle_{B_N}]
\]

\[
= (x_1 + x_3 + \ldots + x_{2N-1}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |0\rangle_{B_N} + (x_2 + x_4 + \ldots + x_{2N}) \prod_{i=1}^{N-1} |\delta_{kA_i}B_i \otimes |1\rangle_{B_N}
\]

\[
= |\phi\rangle_{A_1A_2\ldots A_N}, \tag{15}
\]

and he has also recovered Alice’s unknown state.

In short, by following our scheme above, MCQT of arbitrary \(N\)-particle state can be successfully performed via \((N-1)\) EPR pairs and one \((M+2)\)-particle GHZ state as quantum channel. The correlations among Alice’s measurement results, Charlie’s unitary transformation, the number of “1” in the agents’ measurement results and Bob’s unitary transformations are shown in Tab. III. Note that each EPR pair used in the channel are in the state \(|\phi^+\rangle\), and the

| Alice’s joint measurement result \(|\psi\rangle_{A_iD_i}\) | Alice’s joint measurement result \(|\psi\rangle_{A_ND_N}\) | Charlie’s unitary transformation \(U_{C_j}\) | Bob’s unitary transformation \(U_i\) | Number of “1” \(N\) | Bob’s unitary transformation \(U_N\) |
|---|---|---|---|---|---|
| \(|\phi^+\rangle\) | \(|\phi^+\rangle\) | \(|0\rangle \langle 0| + \mid 1\rangle \langle 1|\) | \(|0\rangle \langle 0| \pm \mid 1\rangle \langle 1|\) | odd | \(|0\rangle \langle 0| + \mid 1\rangle \langle 1|\) |
| \(|\psi^+\rangle\) | \(|\psi^+\rangle\) | \(|0\rangle \langle 1| + \mid 1\rangle \langle 0|\) | \(|0\rangle \langle 0| - \mid 1\rangle \langle 1|\) | even | \(|0\rangle \langle 0| \pm \mid 1\rangle \langle 1|\) |
| \(|\phi^+\rangle\) | \(|\phi^+\rangle\) | \(|0\rangle \langle 0| + \mid 1\rangle \langle 1|\) | \(|0\rangle \langle 0| \pm \mid 1\rangle \langle 0|\) | odd | \(|0\rangle \langle 0| + \mid 1\rangle \langle 1|\) |
| \(|\psi^+\rangle\) | \(|\psi^+\rangle\) | \(|0\rangle \langle 1| \pm \mid 1\rangle \langle 0|\) | \(|0\rangle \langle 0| - \mid 1\rangle \langle 1|\) | even | \(|0\rangle \langle 0| \pm \mid 1\rangle \langle 1|\) |

correlations of the cases for other EPR pairs: \(|\phi^-\rangle, |\psi^+\rangle\) and \(|\psi^-\rangle\) are given in the appendix. In the next section, we give a MCQT example to test the generality of our scheme by specifying \(N\) and \(M\).
III. TWO-PARTY CONTROLLED TELEPORTATION OF ARBITRARY THREE-PARTICLE STATE: AN EXAMPLE

In this section, we test our general scheme by giving an example where \( N = 3 \) and \( M = 2 \), which is a case of MCQT of arbitrary tripartite state

\[
|\phi\rangle_{A_1A_2A_3} = (x_1|000\rangle + x_2|001\rangle + x_3|010\rangle + x_4|011\rangle + x_5|100\rangle + x_6|101\rangle + x_7|110\rangle + x_8|111\rangle).
\]

between Alice and Bob, jointly controlled by Charlie\_1 and Charlie\_2.

Following our scheme in Sec. II, Alice firstly prepares two EPR pairs in the state \( |\phi^\pm\rangle_{D_iB_i} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{D_iB_i}(i = 1, 2) \) and one four-particle GHZ state \( |\phi^+\rangle_{D_3B_3C_1C_2} = \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)_{D_3B_3C_1C_2} \).

Then Alice sends particles \( B_1, B_2, B_3 \) to Bob, and \( C_1 \) and \( C_2 \) to the two agents Charlie\_1 and Charlie\_2 respectively, keeping particles \( (D_1, D_2, D_3) \) to herself. We have the whole system

\[
|\psi\rangle = |\phi_{A_1A_2A_3}\rangle \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{D_1B_1} \otimes \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{D_2B_2} \otimes \frac{1}{\sqrt{2}}(|0000\rangle + |1111\rangle)_{D_3B_3C_1C_2}.
\]

Alice performs Bell\_1-joint measurements on pairs \( (A_1, D_1) \), \( (A_2, D_2) \), \( (A_3, D_3) \) separately. Suppose the measurement results are \( |\phi^+\rangle, |\psi^-\rangle, |\phi^-\rangle \), and the system turns to

\[
|\psi\rangle_1 = |\phi^-\rangle |\psi^-\rangle |\phi^+\rangle |\psi^+\rangle
\]

\[
= (x_1|010\rangle - x_3|000\rangle + x_5|110\rangle - x_7|100\rangle)_{B_1B_2B_3} \otimes |00\rangle_{C_1C_2}
\]

\[
- (x_2|011\rangle - x_4|001\rangle + x_6|111\rangle - x_8|101\rangle)_{B_1B_2B_3} \otimes |11\rangle_{C_1C_2}.
\]

According to Alice’s measurement results, Bob performs single-particle unitary transformations \( I_{B_1}, (i\sigma_y)_{B_2}, (\sigma_z)_{B_3} \) on the particles \( B_1, B_2, B_3 \) respectively, and the system state reads

\[
|\psi\rangle_2 = I_{B_1}(i\sigma_y)_{B_2}(\sigma_z)_{B_3}|\psi\rangle_1
\]

\[
= (x_1|000\rangle + x_3|010\rangle + x_5|100\rangle + x_7|110\rangle)_{B_1B_2B_3} \otimes |00\rangle_{C_1C_2}
\]

\[
+ (x_2|001\rangle + x_4|011\rangle + x_6|101\rangle + x_8|111\rangle)_{B_1B_2B_3} \otimes |11\rangle_{C_1C_2}.
\]

Now the two agents Charlie\_1 and Charlie\_2 perform single-particle unitary operations \( I_z \) on particle \( C_1, C_2 \) respectively according to Alice’s measurement results on pair \( (A_3, D_3) \). Then they respectively perform the Hadamard transformations on particle \( C_1, C_2 \), and the whole system reads

\[
|\psi\rangle_3 = H_{C_1} H_{C_2} |\psi\rangle_2
\]

\[
= \frac{1}{2}(|x_1|000\rangle + x_3|010\rangle + x_5|100\rangle + x_7|110\rangle)_{B_1B_2B_3} \otimes (|0\rangle + |1\rangle)_{C_1} \otimes (|0\rangle + |1\rangle)_{C_2}
\]

\[
+ \frac{1}{2}(|x_2|001\rangle + x_4|011\rangle + x_6|101\rangle + x_8|111\rangle)_{B_1B_2B_3} \otimes (|0\rangle - |1\rangle)_{C_1} \otimes (|0\rangle - |1\rangle)_{C_2}
\]

\[
= (x_1|000\rangle + x_3|010\rangle + x_5|100\rangle + x_7|110\rangle)_{B_1B_2B_3} \otimes (|0\rangle + |1\rangle)_{C_1C_2}
\]

\[
+ (x_1|000\rangle - x_3|010\rangle + x_5|100\rangle - x_7|110\rangle)_{B_1B_2B_3} \otimes (|0\rangle + |1\rangle)_{C_1C_2}
\]

\[
+ (x_1|000\rangle - x_3|010\rangle - x_5|100\rangle + x_7|110\rangle - x_8|111\rangle)_{B_1B_2B_3} \otimes (|0\rangle - |1\rangle)_{C_1C_2}
\]

\[
(20)
\]

Finally, Charlie\_1 and Charlie\_2 make single-particle measurements on particles \( C_1, C_2 \) in basis \( \{|0\rangle, |1\rangle\} \) and publish their measurement results.

If the measurement results contain odd number of “1”, Bob’s N-particle system should be in the state

\[
|\psi\rangle_B = (x_1|000\rangle + x_2|001\rangle + x_3|010\rangle + x_4|011\rangle + x_5|100\rangle + x_6|101\rangle + x_7|110\rangle + x_8|111\rangle)_{B_1B_2B_3},
\]

which is equivalent to the original unknown state of Alice; otherwise, Bob’s N-particle system reads

\[
|\psi\rangle_B = (x_1|000\rangle - x_2|001\rangle + x_3|010\rangle - x_4|011\rangle + x_5|100\rangle - x_6|101\rangle + x_7|110\rangle - x_8|111\rangle)_{B_1B_2B_3}.
\]

Bob then executes the unitary transformation \( \sigma_z \) on particle \( B_N \), the state of \( |\psi\rangle_B \) is thus recovered to the original state of Alice.

In this example, Bob can fully recover the original state of Alice with our general scheme, and it is clear that our scheme can be specified into any other cases for \( M \geq 0, N \geq 1 \). Therefore, we conclude that our theoretical MCQT scheme is highly general and can be deterministically performed.
IV. CONCLUDING REMARK

In this paper, we propose a more general MCQT scheme of an arbitrary N-particle state using \((N - 1)\) EPR pairs and one \((M + 2)\)-particle GHZ state together as quantum channel. We discuss the cases for each kind of EPR pairs used in the quantum channel respectively and also give a specified example to test the generality of our scheme.

We emphasize that, the QT schemes proposed in other literatures such as Ref. [19, 20, 28] can be treated as special cases of our scheme, and our scheme can also be used for quantum secure direct communication (QSDC) and quantum secret sharing (QSS) tasks such as the scheme in Ref. [24, 25, 27]. We hope this work will shed some light for the prospective research on multiparty quantum communications and quantum cryptography [33].

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APPENDIX A: THE CASES OF USING OTHER EPR PAIRS IN THE QUANTUM CHANNEL

In the main context, we assume that the N-1 EPR pairs are in the state \(|\phi^\pm\rangle\). In fact, these EPR pairs can also be either of the other EPR states \(|\psi^-\rangle, |\psi^+\rangle\) or \(|\psi^-\rangle\). The correlations among all the parameters of each case are shown respectively in Tab. IV, V and VI.

TABLE IV: The case where N-1 pairs of \(|\phi^-\rangle\) are used in quantum channel.

| Alice’s joint measurement on \((A_i, D_i)\) | Alice’s joint measurement on \((A_N,D_N)\) | Charlie’s unitary transformation \(U_{C_i}\) | Bob’s unitary transformation \(U_i\) | Number “1” | Bob’s unitary transformation \(U_N\) |
|------------------------------------------|------------------------------------------|------------------------------------------|------------------------------------------|----------------|------------------------------------------|
| \(|\phi^\pm\rangle\)                  | \(|\phi^\pm\rangle\)                    | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) | \(|0\rangle\langle 0| \mp |1\rangle\langle 1|\) | odd          | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) |
| \(|\psi^\pm\rangle\)                  | \(|0\rangle\langle 1| \pm |1\rangle\langle 0|\)       | \(|0\rangle\langle 0| \mp |1\rangle\langle 1|\) | \(|0\rangle\langle 0| \pm |1\rangle\langle 1|\) | even         | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) |
| \(|\psi^\pm\rangle\)                  | \(|0\rangle\langle 1| \pm |1\rangle\langle 0|\)       | \(|0\rangle\langle 0| \mp |1\rangle\langle 1|\) | \(|0\rangle\langle 0| \pm |1\rangle\langle 1|\) | odd          | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) |
|                                          |                                          |                                           |                                           | even         | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) |
|                                          |                                          |                                           |                                           | odd          | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) |
|                                          |                                          |                                           |                                           | even         | \(|0\rangle\langle 0| + |1\rangle\langle 1|\) |
| Alice's joint measurement on $(A_i, D_i)$ | Alice's joint measurement on $(A_N, D_N)$ | Charlie's unitary transformation $U_C_i$ | Bob's unitary transformation $U_i$ | Number of "1" | Bob's unitary transformation $U_N$ |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|----------------|---------------------------------|
| $|\phi^\pm\rangle$ | $|\phi^\pm\rangle$ | $|0\rangle|0\rangle + |1\rangle|1\rangle$ | $|0\rangle|1\rangle \pm |0\rangle|1\rangle$ | odd | $|0\rangle|0\rangle + |1\rangle|1\rangle$ |
| $|\psi^\pm\rangle$ | $|\psi^\pm\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|0\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|1\rangle$ | even | $|0\rangle|0\rangle + |1\rangle|1\rangle$ |
| $|\psi^\pm\rangle$ | $|\psi^\pm\rangle$ | $|0\rangle|0\rangle + |1\rangle|1\rangle$ | $|0\rangle|0\rangle \mp |0\rangle|0\rangle$ | odd | $|0\rangle|0\rangle \mp |0\rangle|0\rangle$ |
| $|\psi^\pm\rangle$ | $|\psi^\pm\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|0\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|1\rangle$ | even | $|0\rangle|0\rangle \mp |0\rangle|0\rangle$ |

**TABLE VI:** The case where N-1 pairs of $|\psi^-\rangle$ are used in quantum channel.

| Alice's joint measurement on $(A_i, D_i)$ | Alice's joint measurement on $(A_N, D_N)$ | Charlie's unitary transformation $U_C_i$ | Bob's unitary transformation $U_i$ | Number of "1" | Bob's unitary transformation $U_N$ |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|----------------|---------------------------------|
| $|\phi^\pm\rangle$ | $|\phi^\pm\rangle$ | $|0\rangle|0\rangle + |1\rangle|1\rangle$ | $|0\rangle|1\rangle \mp |0\rangle|1\rangle$ | odd | $|0\rangle|0\rangle + |1\rangle|1\rangle$ |
| $|\psi^\pm\rangle$ | $|\psi^\pm\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|0\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|1\rangle$ | even | $|0\rangle|0\rangle + |1\rangle|1\rangle$ |
| $|\phi^\pm\rangle$ | $|\phi^\pm\rangle$ | $|0\rangle|0\rangle + |1\rangle|1\rangle$ | $|0\rangle|0\rangle \mp |0\rangle|0\rangle$ | odd | $|0\rangle|0\rangle \mp |0\rangle|0\rangle$ |
| $|\psi^\pm\rangle$ | $|\psi^\pm\rangle$ | $|0\rangle|1\rangle \pm |1\rangle|0\rangle$ | $|0\rangle|1\rangle \mp |0\rangle|0\rangle$ | even | $|0\rangle|0\rangle \mp |0\rangle|0\rangle$ |

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