Longitudinal Response Functions of $^3$H and $^3$He

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Abstract

Trinucleon longitudinal response functions $R_L(q, \omega)$ are calculated for $q$ values up to 500 MeV/c. These are the first calculations beyond the threshold region in which both three-nucleon (3N) and Coulomb forces are fully included. We employ two realistic NN potentials (configuration space BonnA, AV18) and two 3N potentials (UrbanaIX, Tucson-Melbourne). Complete final state interactions are taken into account via the Lorentz integral transform technique. We study relativistic corrections arising from first order corrections to the nuclear charge operator. In addition the reference frame dependence due to our non-relativistic framework is investigated. For $q \leq 350$ MeV/c we find a 3N force effect between 5 and 15 %, while the dependence on other theoretical ingredients is small. At $q \geq 400$ MeV/c relativistic corrections to the charge operator and effects of frame dependence, especially for large $\omega$, become more important. In comparison with experimental data there is generally a rather good agreement. Exceptions are the responses at excitation energies close to threshold, where there exists a large discrepancy with experiment at higher $q$. Concerning the effect of 3N forces there are a few cases, in particular for the $R_L$ of $^3$He, where one finds a much improved agreement with experiment if 3N forces are included.

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I. INTRODUCTION

Inclusive electron scattering can provide detailed information on the transition charge and current densities in nuclei. In the one photon exchange approximation the cross section for this process is given by [1]

\[
\frac{d^2\sigma}{d\Omega\,d\omega} = \sigma_M \left[ \frac{q^4}{q^4} R_L(q, \omega) + \left( \frac{q^2_\mu}{2q^2} + \frac{1}{2} \tan^2 \frac{\theta}{2} \right) R_T(q, \omega) \right]
\] (1)

where \( R_L \) and \( R_T \) are the longitudinal and transverse response functions respectively, \( \omega \) is the electron energy loss, \( q \) is the magnitude of the electron momentum transfer, \( \theta \) is the electron scattering angle and \( q^2_\mu = q^2 - \omega^2 \). Experimental data for both \( R_L \) and \( R_T \) are available for a variety of energy and momentum transfers. However because of our non-relativistic treatment of the nuclear dynamics we restrict our attention to momentum transfers \( q \leq 500 \text{ MeV}/c \) and energy transfers \( \omega \leq 300 \text{ MeV} \). Data covering various regions in this range are given for both \(^3\text{H} \) and \(^3\text{He} \) by Retzlaff et al [2], Dow et al [3], Marchand et al [4], and Morgenstern [5].

The theoretical treatment of these response functions requires the ability to accurately include transitions to the continuum. Techniques for doing this with realistic NN potentials have only been developed and implemented during the past ten years. These include both Faddeev and Lorentz integral transform (LIT) methods [6, 9, 10, 11, 12]. For the 3N photodisintegration total cross sections results obtained with the LIT [13] and Faddeev techniques are compared in [14]. In the work of Viviani et al [15] expansion techniques were applied to solving the ground state and continuum wave equations, but the calculation was restricted to a \(^3\text{He} \) near threshold region where only the two-body breakup occurs. Previous to the above references Faddeev calculations of trinucleon response functions were published by Meijgaard and Tjon [16] in 1992 using the s-wave Malfliet–Tjon potential MT–I/III [17].

Apart from how the quantum mechanics is done there are major differences in physics input between the longitudinal and transverse responses. Whereas the non-relativistic longitudinal response requires only a charge operator and nucleon form factors, the transverse response requires exchange currents in addition to single nucleon currents and nucleon form factors. It is clear that if a given nuclear interaction cannot describe the longitudinal response then it would be pointless to attempt a calculation of the transverse response. In particular if one inquires into the effect of three-body forces in nuclei it would appear natural to first
investigate their impact on the longitudinal response. Otherwise, through a calculation of $R_T$, it would be difficult to disentangle the effects of three–body forces from exchange current effects. Further as shown in [10] the longitudinal response appears in general insensitive to the realistic NN force model thus removing a possible source of ambiguity when comparing the effects of different three body force models on $R_L(q, \omega)$.

II. NUCLEAR FORCES AND CHARGE OPERATOR

The function $R_L$ represents the response of the nucleus through the nuclear charge operator $\rho$ and is given by

$$R_L(q, \omega) = \sum_{M_0} \int \mathcal{d}f \langle \Psi_0 | \rho^\dagger(q, \omega) | \Psi_f \rangle \langle \Psi_f | \rho(q, \omega) | \Psi_0 \rangle \delta(E_f - E_0 + q^2/(2M_T) - \omega).$$

Here $\Psi_0$ and $\Psi_f$ denote the ground and final states, respectively, while $E_0$ and $E_f$ are energies pertaining to them,

$$(H - E_0)\Psi_0 = 0, \quad (H - E_f)\Psi_f = 0,$$

where $H$ is the nuclear non–relativistic Hamiltonian. The above quantities $\Psi_{0,f}$ and $H$ are internal quantities in the hadronic c.m. frame. The integration (summation) goes over all final states belonging to the same energy $E_f$, and $M_0$ is the projection of the ground state angular momentum.

The Hamiltonian includes the kinetic energy terms, the NN and 3N force terms, and the proton Coulomb interaction term in the $^3$He case. The ground state $\Psi_0$ is calculated via an expansion in basis functions which are correlated sums of products of hyperradial functions, hyperspherical harmonics and spin–isospin functions. In the present work three models of the NN force are used, the realistic AV18 [18] and configuration space BonnA (herein referred to as BonnRA) [19] models, and the s-wave MT–I/III potential. We consider two 3N force models, the UrbIX [20] and the TM’ [21], in the combinations AV18+UrbIX, AV18+TM’ ($\Lambda=3.358$ fm$^{-1}$), and BonnRA+TM’ ($\Lambda=2.835$ fm$^{-1}$). As indicated the TM’ cut-off parameter $\Lambda$ is different in the AV18 and BonnRA combinations in order to properly fix the $^3$H binding energy in each case. Table I lists our results for ground state properties for the above potential combinations containing the 3N force.
As nuclear charge operator we take the following one–body operator

\[ \rho(q, \omega) = \sum_{j=1}^{A} \rho_j^{nr}(q, \omega) + \rho_j^{rc}(q, \omega), \]  

where

\[ \rho_j^{nr}(q, \omega) = \hat{e}_j e^{iq \cdot r_j}, \]

\[ \rho_j^{rc}(q, \omega) = -\frac{q^2}{8M^2} \hat{e}_j e^{iq \cdot r_j} + \frac{i}{4M^2} \vec{\sigma}_j \cdot (q \times p_j) e^{iq \cdot r_j}, \]

\[ \hat{e}_j = G_p^E(q_\mu^2) \frac{1 + \tau_{z,j}}{2} + G_n^E(q_\mu^2) \frac{1 - \tau_{z,j}}{2} \equiv \frac{1}{2} [G_E^S(q_\mu^2) + G_E^V(q_\mu^2) \tau_{z,j}], \]

\[ \hat{\mu}_j = G_p^M(q_\mu^2) \frac{1 + \tau_{z,j}}{2} + G_n^M(q_\mu^2) \frac{1 - \tau_{z,j}}{2}. \]

Here \( r, p, \vec{\sigma}, \) and \( \vec{\tau} \) are the nucleon position, momentum, spin and isospin operators, \( M \) is the nucleon mass, and \( G_{E,M}^{p,n} \) are the nucleon Sachs form factors. The two terms in \( \rho \) proportional to \( M^{-2} \) are the Darwin–Foldy (DF) and spin–orbit (SO) relativistic corrections to the main operator \( \rho_{nr} \), see e.g. \([1, 22]\). We refer to the main operator \( \rho_{nr} \) as the non–relativistic one although the dependence of nucleon form factors on \( q_\mu^2 \) does not allow a non–relativistic interpretation.

In this work we mainly use the well known dipole fit for proton electric form factor, while the neutron electric form factor is taken from \([23]\), but we also check the \( R_L \) dependence with a different parametrization, namely the best fit from \([24]\) to these form factors. In case of the SO term we adopt the usual although recently controversial \([29]\) approximation \( G_M^{p,n}(q_\mu^2) = \mu_{p,n} G_E^p(q_\mu^2) \) in \([8]\), \( \mu_{p,n} \) being proton and neutron magnetic momenta. For the calculation of \( R_L \) it is convenient to rewrite the operator \( \rho \) in terms of the isoscalar and isovector charge nucleon form factors from \([4]\),

\[ \rho(q, \omega) = G_E^S(q_\mu^2) \rho_s(q) + G_E^V(q_\mu^2) \rho_v(q). \]

The inclusion of relativistic corrections for the one–body charge operator only is not completely consistent. In fact there exist additional relativistic effects: a wave function boost (as done in \([27, 28]\) for the \( d(e, e') \) reaction) and additional two-body terms in the charge operator (as done in \([15]\) for the low-energy two-body break-up channel of the \( ^3\text{He}(e, e') \)
reaction). In our case there are two reasons why we include the relativistic corrections to
the one-body charge operator:

(i) At higher \( q \) they lead to an important reduction of the \( R_L \) quasi-elastic peak height. As
illustrated in \cite{28} such a reduction is confirmed if boost corrections are included. Moreover,
the frame dependence of the response functions is studied in \cite{27} where it is shown that in the
Breit frame boost corrections are negligibly small for the quasi-elastic peak region (different
kinematics are not shown). We believe that one has a similar frame dependence of boost
corrections also for the electromagnetic response of the three-nucleon systems. Thus we will
make the comparison with experimental data taking \( R_L \) from a Breit frame calculation with
a subsequent transformation into the \( R_L \) LAB frame result (see discussion of Fig. 5).

(ii) They enable us to make a direct comparison of our results with those of \cite{15}. Since
realistic few–body calculations are rather complicated it is of great importance to have
these kind of checks.

III. CALCULATION OF RESPONSE

We calculate \( R_L(q, \omega) \) by the LIT method as described in \cite{6, 7}. The technique is, however,
directly applicable only when the transition operator does not depend on \( \omega \). To separate
out the \( \omega \) dependence of the transition operator we use Eq. \( \text{(9)} \) to represent the response
function \( \text{(2)} \) as

\[
R_L(q, \omega) = [G^S_{E_0}(q^2_\mu)]^2 R_s + G^S_{E_0}(q^2_\mu)G^V_{E_0}(q^2_\mu)R_{sv} + [G^V_{E_0}(q^2_\mu)]^2 R_v,
\]

(10)

where \( R_s \) and \( R_v \) are the responses which emerge if \( \rho_s(q) \) and \( \rho_v(q) \) are taken as transition
operators, and the quantity \( R_{sv} \) is the mixed response,

\[
R_{sv}(q, \omega) = \sum_{M_0} \int df \left[ \langle \Psi_0 | \rho_s^\dagger(q) | f \rangle \langle f | \rho_v(q) | \Psi_0 \rangle + \langle \Psi_0 | \rho_v^\dagger(q) | f \rangle \langle f | \rho_s(q) | \Psi_0 \rangle \right] \times \delta(E_f - E_0 + q^2/(2AM) - \omega).
\]

(11)

To calculate the subsidiary responses entering \( \text{(10)} \) with the LIT method one can solve the
inhomogeneous equations

\[
(H - E_0 - \sigma) \Psi_s(\sigma) = \rho_s \Psi_0, \quad (H - E_0 - \sigma) \Psi_v(\sigma) = \rho_v \Psi_0 \tag{12}
\]

for a set of complex \( \sigma \) values and then form the scalar products \( \langle \Psi_s(\sigma) | \Psi_s(\sigma) \rangle \),
\( \langle \Psi_v(\sigma) | \Psi_v(\sigma) \rangle \), and \( \langle \Psi_s(\sigma) | \Psi_v(\sigma) \rangle \). These scalar products represent integral transform
with Lorentzian kernels, e.g., one has

$$\langle \tilde{\Psi}_s(\sigma) | \tilde{\Psi}_s'(\sigma) \rangle = \frac{\mathcal{R}^{el}_s(q, \omega_{el})}{|\sigma|^2} + \int_{\omega_{th}}^{\infty} d\omega \frac{\mathcal{R}_s(q, \omega)}{(\omega - \omega_{el} - \sigma)(\omega - \omega_{el} - \sigma^*)}. \quad (13)$$

Here $\mathcal{R}^{el}_s$ is the elastic scattering form factor, $\omega_{el} = q^2/(2M_T)$, and $\omega_{th}$ is the threshold for the inelastic energy transfer. From the inversion of such integral transforms one then obtains the response functions $\mathcal{R}_s$, $\mathcal{R}_v$ and $\mathcal{R}_{sv}$.

In previous work [7] equations similar to those in (12) were solved numerically in order to calculate the above mentioned scalar products $\langle \tilde{\Psi}_i(\sigma) | \tilde{\Psi}_j'(\sigma) \rangle$. An alternative and computationally more efficient way of the calculation of the transform is a direct evaluation of the scalar products via the Lanczos technique [26]. Thus we use this method in our calculation.

As for $\Psi_0$ we perform expansions in terms of basis functions $|\mu\rangle$ which are correlated sums of products of hyperradial functions, hyperspherical harmonics and spin–isospin functions. The first Lanczos vector is given by

$$|\varphi_0\rangle = \frac{|\Psi\rangle}{\sqrt{\langle\Psi|\Psi\rangle}} \quad (14)$$

with

$$|\Psi\rangle = g^{-1}_{\mu\nu} \hat{\rho} |\Psi_0\rangle, \quad (15)$$

where $g^{-1}_{\mu\nu}$ denotes the inverse of the norm matrix $g_{\mu\nu} = \langle\mu|\nu\rangle$. One then applies recursively the following relations

$$b_{n+1}|\varphi_{n+1}\rangle = g^{-1}_{\mu\nu} H|\varphi_n\rangle + a_n|\varphi_n\rangle - b_n|\varphi_{n-1}\rangle, \quad (16)$$

$$a_n = \langle\varphi_n|H|\varphi_n\rangle, \quad b_n \equiv ||b_n|\varphi_n||, \quad (17)$$

where $a_n$ and $b_n$ are the Lanczos coefficients. The transform can then be written as a continuous fraction

$$\langle \tilde{\Psi} | \tilde{\Psi} \rangle = \frac{2i}{\sigma - \sigma^*} Im \frac{\langle \Psi|\Psi\rangle}{(z - a_0) - \frac{b_1^2}{(z - a_1) - \frac{b_2^2}{(z - a_2) - \frac{b_3^2}{\ldots}}}} \quad (18)$$

with $z = \sigma + E_0$.

Basis functions possess definite values of parity $P$, angular momenta $J$ and magnetic quantum numbers $M_J$. Various Lanczos sets are separated with respect to these quantum
numbers. Multipole expansions of the operators $\rho_s$ and $\rho_v$ are performed, which allows elimination of dependencies on $M_J$ and on the ground-state angular momentum projection $M_0$. In our case there exists only one multipole $\ell$ compatible with a given $J$ and $P$ value. Indeed, one has $J = \ell \pm 1/2$, and parity equal to $(-1)^\ell$.

Hyperspherical harmonics (HH) belonging to given permutational symmetry types are obtained via application of the corresponding symmetrization operators to HH of the type $Y^{l_1l_2}_{KLM_L}$ where $K$ is the grand–angular momentum, $L$ and $M_L$ are the total orbital momentum and its projection, and $l_1$ and $l_2$ are the orbital momenta associated with a relative motion of a given pair of particles and a relative motion of the third particle with respect to the pair. These basis HH are coupled to the spin–isospin functions of conjugated permutational symmetry types to get basis functions antisymmetric with respect to permutations of nucleons. These spin–isospin functions posess given spin $S$ and isospin $T$. Thus our basis functions have given $J$, $M_J$, $K$, $L$, $S$, $T$ values, given parity equal to $(-1)^K$, and given type of symmetry with respect to permutations of spatial, or spin–isospin, variables. In order to accelerate the convergence of the HH expansion a spin and isospin dependent correlation operator is applied to the basis functions (see [13] for details). Matrix elements are calculated analytically with respect to three Euler angles determining the orientation of the system as a whole, and the remaining three–dimensional integrations are done numerically.

Rather many basis functions are retained to achieve convergence, and a selection of basis HH has been done to reduce their net numbers in the calculation. The selection is based upon the property [25] that the uncorrelated symmetrized basis HH obtained in the above mentioned way from the subset of HH $Y^{l_1l_2}_{KLM_L}$ with only small $l_1$ and small $l_2$ suffice to provide a predominant contribution to bound state wave functions. We have found that in practice this property is also valid for our correlated HH and for the case of our inhomogeneous equations. At the same time, the selection depended on $L$, $K$, $J$ values and symmetry types of HH as well.

The mixed response (11) is calculated as
\[
R_{sv} = R_{s+v} - R_s - R_v , \tag{19}
\]
where $R_{s+v}$ is the response, which emerges from the scalar product $\langle \tilde{\Psi}_{s+v}(\sigma) \mid \tilde{\Psi}_{s+v}(\sigma) \rangle$, and $\tilde{\Psi}_{s+v}(\sigma)$ is obtained if $\rho_s + \rho_v$, instead of $\rho_s$ or $\rho_v$, is taken as the transition operator in (12).
IV. RESULTS AND DISCUSSION

Before showing detailed results of our calculations it is necessary to address the question of convergence with respect to the maximum angular momentum $J_{\text{max}}$ retained in our calculation. This requires some measure of convergence. In this connection we consider here the $^3$H Coulomb sum rule results computed for the case $G_S^S = G_V^V = 1$. The sum rule reads

$$\int_{\omega_{\text{th}}}^{\infty} R_L(q, \omega) \, d\omega + R_{\text{el}}^L(q, \omega_{\text{el}}) = 1.$$  \hspace{1cm} (20)

Since in the case considered a single proton interacts with the electromagnetic field, Eq. (20) does not contain the nucleon charge correlation contribution and is valid for any $q$. It is clear that larger $q$ values require the expansion to include larger values of $J$. Table II shows the results of using $J_{\text{max}} = 15/2$ for the $q = 250$, and 300 MeV/c cases and $J_{\text{max}} = 21/2$ for the $q = 350 - 500$ MeV/c cases. One notes that the lower $q$ sum rules are nearly fully converged while the 500 MeV/c case still requires about 2% more strength. Although this could be improved by increasing $J_{\text{max}}$ we consider the convergence tolerable for the present investigation. Table II also demonstrates that the convergence is faster for the simple MT-I/III potential as compared to the realistic potential models.

In Fig. 1 we illustrate the dependence of $R_L$ on the NN potential. The results with the two realistic potentials, Bonn and AV18, are very similar at $q = 500$ MeV/c, but exhibit somewhat stronger differences for the quasi-elastic peak height at $q = 250$ MeV/c. With the semi-realistic MT-I/III potential one observes a rather similar picture for $q = 250$ MeV/c as with the realistic potentials, whereas at $q = 500$ MeV/c a greater peak height and considerably less high-energy strength than for the realistic potentials is found.

In Fig. 2 we show the 3N force effect. It is seen that it decreases the peak height and enhances the high-energy tail. At lower momentum transfer the reduction of the peak height is more pronounced. Comparing the three cases, where a 3N force is included, one finds only rather small differences among them except for the low-energy range at $q = 500$ MeV/c as will be seen next in Fig. 3.

In order to study the low-energy behavior better, in Fig. 3 we illustrate the nuclear force model dependence of the triton $R_L$ close to threshold at three momentum transfers covered also by the data of [2]. In this figure $R_L$ is shown as a function of $E_x$, the relative kinetic
energy of the outgoing three nucleons. At $q = 174$ MeV/c there is a rather strong decrease of $R_L$ due to the 3N force. The reduction becomes considerably smaller at $q = 324$ MeV/c and at $q = 487$ MeV/c the 3N force leads to an opposite effect, namely a moderate increase. From the comparison of the cases AV18+UrbIX and AV18+TM' it becomes clear that the 3N force model dependence is for all the three momentum transfers very small. The only evident potential model dependence is found at the highest $q$, where the case BonnRA+TM' exhibits considerably more strength than the other cases with inclusion of 3N force.

In Fig. 4 we show the effect of the relativistic corrections on $R_L$. One sees that the SO term leads only to rather small contributions, while the DF term is more important. It occurs that separate contributions from the SO term are not so small, only their net sum proves to be very small. This probably means that in the inclusive case we have an effect of averaging out due to the spin dependence of the SO operator. Because of the smallness of the SO contribution we have neglected it in most of the following cases. In Fig. 4 we also show $R_L$ results, where a different nucleon form factor parametrization \[^{24}\] is taken. At $q = 250$ MeV/c the different form factors lead to very similar results, but at $q = 500$ MeV/c there is a 3\% reduction of $R_L$ with the parametrization of \[^{24}\]. In the results which follow we will always use the dipole nucleon form factors. However as seen here there will be uncertainties in $R_L$ at higher $q$ values due to uncertainties in the nucleon form factors.

Next we would like to check the frame dependence of our calculation. To this end we calculate $R_L$ also in the Breit (B) frame and the so-called anti–lab (AL) frame. In the AL frame the virtual photon and initial target nucleus have momenta $q_{AL}$ and $-q_{AL}$, respectively, whereas the total momentum of the final three–nucleon state is equal to zero. Note that in the LAB frame one has the opposite case: the target nucleus in the initial state is at rest and the total momentum of the final three-nucleon state is equal to $q$. Finally, in the Breit frame one has total momenta of initial and final hadron states equal to $-q_B/2$ and $q_B/2$, respectively, while the photon four-momentum is $(\omega_B, q_B)$. Formally there are no differences between the calculations in the various frames. One obtains a response function which has the arguments $\omega$ and $q$ of the given frame, i.e. $R_L^{LAB}(q_{LAB}, \omega_{LAB})$, $R_L^{AL}(q_{AL}, \omega_{AL})$ and $R_L^{B}(q_B, \omega_B)$. For a comparison of the results we transform $R_L^{AL}(q_{AL}, \omega_{AL})$ and $R_L^{B}(q_B, \omega_B)$ into $R_L^{LAB(AL)}(q_{LAB}, \omega_{LAB})$ and $R_L^{LAB(B)}(q_{LAB}, \omega_{LAB})$, respectively. To this end we use that the various reference frames are connected via Lorentz boosts and thus $\omega_{AL}$, $q_{AL}$, $\omega_B$ and $q_B$ can be expressed through $\omega_{LAB}$ and $q_{LAB}$. However in order to obtain an $R_L$ in the
LAB frame from $R_L$’s in AL and Breit frames it is not sufficient to transform the relative arguments of $\omega$ and $q$ into the corresponding LAB frame arguments. In addition one has

$$R_{LAB(frame)}^L = \frac{q_{LAB}^2}{q_{frame}^2} R_{frame}^L,$$

(21)

where ”frame” stands for AL or Breit. The origin of the additional factor is the following. The cross section of (1) contains three separate pieces, namely $\sigma_M$, a part regarding the electron (e.g., $q_\mu^4/q^4 \equiv V_{LAB}^L$) and a hadronic part (e.g., $R_L$). The latter two originate from a reduction of a product of leptonic and hadronic Lorentz tensors [1]. The product of these two tensors forms a Lorentz scalar and thus is frame independent. One can show that for the longitudinal part of the cross section of (1) one has [27]

$$V_{LAB}^L = \frac{q_{frame}^2}{q_{LAB}^2} V_{frame}^L,$$

(22)

and thus Lorentz invariance requires the additional factor in (21).

In Fig. 5 we compare the longitudinal response functions of the various frames. At $q=250$ MeV/c differences are rather small, in particular between Breit and AL frame results. Except for the threshold region there is not such a similarly good agreement at $q=500$ MeV/c. In the quasi-elastic peak there are rather pronounced differences: $R_{LAB(AL)}^L$ is about 7 % and $R_{LAB(B)}^L$ about 4 % higher than $R_{LAB}^L$, their peak positions are shifted by about 6 (AL) and 5 MeV (B) towards lower energies. In a consistent relativistic theory one would of course have identical results and thus the obtained differences point to a relativistic inconsistency in the calculation.

As mentioned before Beck et al [27] studied the electromagnetic response functions in deuteron electrodisintegration in the quasi-elastic region. They have shown that an inclusion of boost effects on the hadron wave functions leads essentially to the same results for the various reference frames discussed here. In addition they have found that boost corrections are almost vanishing in the Breit frame. We believe that also in the three-nucleon electrodisintegration one probably has a similar picture with a strong cancellation of boost effects in the Breit frame. Therefore we will take the $R_{LAB(B)}^L$ results in comparison with quasi–elastic experimental data.

A comparison of the $^3$H and $^3$He theoretical longitudinal response functions with experimental data of [3, 4, 5] is shown in Fig. 6 at $q = 250, 300, \text{ and } 350 \text{ MeV/c}$. In the peak region one does not find a clear picture, since there is a better agreement once with the 3N
force ($^3$He) and once without the 3N force ($^3$H). Except for the triton case at $q = 250$ MeV/c one observes rather similar theoretical and experimental results for the high–energy tail. At higher energies the size of the experimental errors is larger than the effect of the 3N force, thus nothing can be said there about an improvement of the theoretical result with the 3N force.

In Fig. 7 we show equivalent results as in Fig. 6 but at the higher momentum transfers of 400, 450, and 500 MeV/c. Also here one finds a better agreement with experimental data without the 3N force in case of $^3$H and with the 3N force in case of $^3$He. It is worthwhile to note that for all six cases of Fig. 7 one has a good agreement of theoretical and experimental peak positions. Concerning the low– and high–energy tails one has a rather good agreement between theory and experiment.

Next we turn to a comparison of the triton low–energy longitudinal response functions with the experimental data of [2]. In Fig. 8 we show the $R_L$ of $^3$H at various $q$. Since the $R_L$ frame dependence is very small close to threshold we illustrate directly the results from a LAB frame calculation. For the lower two momentum transfers there is a rather good agreement of experiment and theory, but the size of the experimental error is too large to draw definite conclusions about possible improvements due to the 3N force. At $q = 487$ MeV/c the picture is different, the theoretical response functions are larger than the experimental one, in particular very close to threshold. It is also evident that the effect of the 3N force moves the calculated $R_L$ even further away from the data.

In Fig. 9 we show a similar comparison with experimental data as in Fig. 8 but for the $R_L$ of $^3$He. Again one finds a rather good agreement between theory and experiment for the two lower $q$’s, but contrary to the triton case here the 3N force is important for this agreement at $q = 174$ MeV/c. Also for the highest momentum transfer one finds a similar picture as for the triton case, namely a large overestimation of the experimental data by the theoretical response functions and also an increase of $R_L$ due to the 3N force.

In Fig. 9 we also illustrate theoretical results from [15]. It is an approach to calculating responses which is entirely different from ours. The calculation [15] has been carried out with the AV18+UrbIX potentials, relativistic DF- and SO-terms have been included and the same nucleon form factors as by us have been used (dipole fit, neutron electric form factor from [23]). In order to have a clean comparison of the two different calculations, we also take into account the SO term for our result with AV18 and UrbIX, though its effect is also
here very small. For the two higher momentum transfers there is a rather good agreement between both calculations. Some differences are visible at \( q = 174 \) MeV/c, but the difference between the two calculations is still considerably smaller than the experimental error bars.

The rather large discrepancy between theory and experiment of the low–energy \( R_L \) at \( q = 487 \) MeV/c requires further theoretical and experimental investigations. We should mention that in the calculation of \[15\] relativistic two–body charge operators were also considered. Although they were not sufficient to give agreement with experiment, they did diminish the discrepancy by about a factor of two. Concerning the nucleon form factors one could only obtain a small reduction (about 3 \%) using the parametrization of \[24\]. In addition the potential model dependence should be further studied. In the discussion of Fig. 3 we have already mentioned a rather strong potential model dependence of the low–energy \( R_L \) at \( q = 487 \) MeV/c. Therefore it would be interesting to consider other modern realistic NN potentials in addition to the AV18 and BonnRA models used here.

V. CONCLUSIONS

In the following we give a brief summary of our work. The trinucleon longitudinal response function \( R_L(q, \omega) \) is calculated with realistic NN interactions, 3N, and Coulomb forces for a variety of kinematical settings that include momentum transfers \( q \) between 174 and 500 MeV/c and wide ranges of energy transfers \( \omega \). The results are fully convergent. The calculations are performed via the Lorentz Integral Transform method.

As NN interaction we use a modern realistic (AV18), a realistic (BonnRA), and also a semi–realistic (MT–I/III) potential model. Two models (UrbIX, TM’\) of the 3N force are employed. The treatment of the trinucleon dynamics is completely non–relativistic. Nonetheless we apply a minimal check on the uncertainties related to this. For this purpose we evaluate \( R_L \) in three different reference frames, namely in lab, anti–lab, and Breit frames. For the charge operator we take the leading relativistic corrections into account (Darwin–Foldy and spin–orbit term).

In general we find a rather small NN potential model dependence, but in some cases there are also larger effects. These include the height of the quasi–elastic peak at lower \( q \) and the threshold behavior at higher \( q \). The effect of the 3N force is typically between 5 and 10 \%, but reaches up to 15 \% for the low–energy response at low \( q \). The dependence on the 3N
force model is very small for all considered cases.

Concerning the relativistic contributions to the charge operator, our inclusive case shows negligible effects due to the spin–orbit term, while the DF term leads to non–negligible effects at higher $q$. With respect to the $R_L$ calculation in the various reference frames, we observe a non–negligible frame dependence at higher $q$, except for the threshold region. In order to restore a more consistent relativistic behavior one would need to consider additional relativistic effects. Similar results have been found in $d(e, e')$ and it is shown that additional boost corrections lead to a much better agreement among the various frame results. In the same work it is also shown that boost effects are negligible in the Breit frame. We assume a similar behavior also in trinucleon electrodisintegration. Thus we compare the $R_L$ calculated in the Breit frame with experimental data.

The comparison of our results with experimental data is generally rather satisfying for all considered momentum transfers, in particular for the $R_L$ of $^3\text{He}$. The experimental data, however, are in most cases not precise enough to draw definite conclusions about the 3N force effect. A nice exception is the $^3\text{He}$ low–energy response, where a 3N force proves to be necessary to obtain agreement with experiment. In addition for the $^3\text{He}$ quasi–elastic peak heights at $q \leq 400$ MeV/c three-nucleon forces considerably improve the agreement with experiment. At higher $q$ and low $\omega$ values one finds a considerably higher $R_L$ response in theory than in experiment.

Last but not least we would like to mention that at very low energies, i.e. up to the three–body breakup threshold, we can compare our results with those of [15]. We find quite a good agreement. The differences which do show up a very low $q$ are still smaller than the experimental error bars.

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### TABLE I: $^3$H ground state properties with AV18+UrbIX, AV18+TM$^\prime$ and BonnRA+TM$^\prime$ Potentials for binding energy (EB), point charge radius (r) and probabilities of total orbital angular momentum components in %

|        | AV18 +UrbIX | AV18 + TM$^\prime$ | BonnRA+TM$^\prime$ |
|--------|-------------|---------------------|---------------------|
| EB [MeV]| 8.47        | 8.47                | 8.47                |
| r [fm] | 1.588       | 1.589               | 1.587               |
| S-wave | 90.60       | 90.63               | 92.69               |
| P-wave | 0.13        | 0.13                | 0.08                |
| D-wave | 9.27        | 9.23                | 7.23                |

### TABLE II: $^3$H Coulomb Sum Rule for AV18, AV18+UrbIX and MT-I/III Potentials

| q [MeV/c] | $J_{\text{max}}$ | AV18 | AV18 + UrbIX | MT-I/III |
|-----------|-------------------|------|--------------|----------|
| 250       | $\frac{15}{2}$    | 0.998| 0.999        | 1.000    |
| 300       | $\frac{15}{2}$    | 0.993| 0.994        |          |
| 350       | $\frac{21}{2}$    | 0.992| 0.993        |          |
| 400       | $\frac{21}{2}$    | 1.003| 0.998        |          |
| 450       | $\frac{21}{2}$    | 0.998| 0.999        |          |
| 500       | $\frac{21}{2}$    | 0.977| 0.977        | .994     |
FIG. 1: NN potential model dependence of triton $R_{LAB}^{3H}(q_{LAB}, \omega_{LAB})$ at $q_{LAB}$=250 (a) and 500 (b) MeV/c (charge operator: non-relativistic plus DF term): AV18 (solid), BonnRA (dotted), and MT-I/III (dashed).
FIG. 2: Effect of 3N force on triton $R_L^{LAB}(q_{LAB}, \omega_{LAB})$ at $q_{LAB}=250$ (a) and 500 (b) MeV/c (charge operator: non-relativistic plus DF term): AV18 (solid), AV18+UrbIX (dotted), AV18+TM' (dashed), and BonnRA+TM' (dash-dotted).
FIG. 3: Effect of 3N force on low-energy triton $R_{LAB}^{L}(q_{LAB}, E_x,)$ at $q_{LAB}=174$ (a), 324 (b), and 487 (c) MeV/c (charge operator: non-relativistic plus DF term): AV18 (dashed), AV18+UrbIX (solid), AV18+TM' (dotted), and BonnRA+TM' (dash-dotted).
FIG. 4: Effect of relativistic contributions and nucleon form factor dependence for triton $R_{LAB}^{3H}(q_{LAB}, \omega_{LAB})$ at $q_{LAB}=250$ (a) and 500 (b) MeV/c (potential model: AV18+UrbIX): non-relativistic charge operator (solid), additional inclusion of SO term (dotted), and total result with further inclusion of DW term (dashed); all three cases with neutron electric form factor from [23] and dipole fit for the other three nucleon form factors. Total result also with nucleon form factor parametrization of [24] (dash-dotted).
FIG. 5: Frame dependence of triton $R_L(q_{LAB}, \omega_{LAB})$ at $q_{LAB} = 250$ (a) and 500 (b) MeV/c (potential model: AV18+UrbIX, charge operator: non-relativistic plus DF term): $R_L^{LAB}$ (dashed), $R_L^{LAB(AL)}$ (dotted) and $R_L^{LAB(B)}$ (solid).
FIG. 6: Comparison of theoretical and experimental $R_L^{LB}(q_{LAB}, \omega_{LAB})$ at $q_{LAB}$ as indicated in figure for $^3$H (left) and $^3$He (right) (charge operator: non-relativistic plus DF term): AV18+UrbIX potentials (solid) and AV18 potential (dotted); experimental data from [3] (circles) and [4, 5] (triangles).
Fig. 7: As Fig. 6 but for different momentum transfers $q_{LAB}$ as indicated in figure.
FIG. 8: Comparison of theoretical and experimental $R_{L}^{LAB}(q_{LAB}, E_{x})$ for $^3$H at $q_{LAB}$ as indicated in figure (charge operator: non-relativistic plus DF term): AV18+UrbIX potentials (solid) and AV18 potential (dashed); experimental data from [2].
FIG. 9: Comparison of theoretical and experimental $R_{L}^{LAB}(q_{LAB}, E_{x})$ for $^3\text{He}$ at $q_{LAB}$ as indicated in figure (charge operator: non-relativistic plus DF and SO terms): AV18+UrbIX potentials (solid) and AV18 potential, but without inclusion of SO term (dashed); experimental data from [2]. Theoretical result from [15] (dotted) with AV18+UrbIX potentials and same charge operators as in our AV18+UrbIX case.