NGC 2419 does not challenge MOND

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ABSTRACT

I show that, in the context of MOND, non-isothermal models, approximated by high order polytropic spheres, are consistent with the observations of the radial distribution of the line-of-sight velocity dispersion in the distant globular cluster, NGC 2419. This calls into question the claim by Ibata et al. that the object constitutes a severe challenge for MOND. In general, the existence and properties of globular clusters are more problematic for LCDM than for MOND.

1 INTRODUCTION

In a recent preprint, Ibata et al. (2011) have claimed that in the distant globular cluster, NGC 2419, the observed radial profile of line-of-sight stellar velocity dispersion, combined with the surface density distribution of starlight, is consistent with Newtonian gravity in the context of a broad class of globular cluster models and inconsistent with modified Newtonian dynamics (MOND). They go further and assert that this object is a “crucible” for acceleration-based theories of gravity primarily because its large galacto-centric distance means that the galactic external field is less than 20% of the MOND critical acceleration, $a_0$. Thus, the complications due to the MONDian external field effect should be negligible; it is a relatively isolated system and can be modeled taking only internal dynamics into account.

I argue here that the elevation of this object to the role of crucible is overstated primarily due to the limitations of the models considered and that a class of MONDian models which deviate slightly from an isothermal state are consistent with the observations. More generally, I claim that the very existence and overall properties of globular clusters are more problematic for LCDM than for MOND.

2 BACKGROUND

The primary models considered by Ibata et al. are Michie models which are an anisotropic extension of the well known King models. These models are based upon the Jeans theorem which states that, in steady state, the phase space distribution of stars, $f(r,v)$ in any stellar system is a function of the integrals of motion, in this case energy and angular momentum. The Jeans theorem does not say what the function should be; this requires an educated guess. In spherical systems, a reasonable supposition, and one justified by observations, is that this function is an exponential of energy $e^{-E/a^2}$ because this yields a gaussian velocity distribution with constant velocity dispersion – an isothermal sphere. The problem is that, with Newtonian dynamics, the isothermal sphere is infinite in extent and mass, which led Michie (1963, see also King 1966) to impose of a cutoff radius: $f \to 0$ at some finite radius corresponding to the tidal limit of the cluster in the gravitational field of the parent galaxy. This is encoded in the distribution function by taking $f \propto e^{-E/a^2} - 1$ so that the phase space density falls to zero where $E = 0$, defined to be the tidal radius.

Long ago, Milgrom (1984) demonstrated that with MOND, isothermal spheres have finite mass but infinite extent; the density falls asymptotically as $r^{-\alpha}$ where $\alpha \approx 4 - 5$. However, one may show that if the system deviates from being isothermal (Sanders 2000), even slightly, then it is finite in both mass and extent; the density falls to zero at a finite radius. This is true when deviations from an isothermal state are represented by high order polytropes, defined by $\sigma_r \propto \rho^{1/n}$ where $\sigma_r$ is the radial velocity dispersion, $\rho$ is the density and $n$ is the polytropic index taken to be greater than 10 (Newtonian polytropes are finite in extent only if $n \leq 5$). Such objects naturally contain a truncation radius which may or may not be identified with a tidal radius. High-order polytropes ($12 < n < 16$) with a radial velocity anisotropy in the outer regions can reproduce the general properties of elliptical galaxies (Sanders 2000) and, in particular, the scaling relations such as the observed fundamental plane and the Faber-Jackson relation; globular clusters lie on the low mass extension of these relations. Therefore, while the polytropic assumption is certainly an idealization, it would seem reasonable to apply such models to globular clusters, particularly given the existence of an intrinsic truncation radius.

3 THE NON-ISOTHERMAL MODEL

The equation solved is the spherically symmetric Jeans equation

$$\frac{dP}{dr} + \frac{2\beta P}{\tau} = -\rho g$$

where $P$ is the pressure given by

$$P = \rho \sigma_r^2$$
\[ \beta = 1 - \sigma^2 / \sigma_r^2 \]  \hspace{1cm} (3)

with \( \sigma \) being the velocity dispersion in the tangential direction. As is usual I assume a radial dependence of \( \beta \) given by

\[ \beta = [1 + (r/a_r)^2]^{-1}; \]  \hspace{1cm} (4)

that is, for \( r < a_r \) the velocity field is isotropic and for \( r > a_r \) the stellar orbits become primarily radial. Here \( g \) is the gravitational acceleration given in this case by the MOND formula

\[ g\mu(g/a_0) = g_N \]  \hspace{1cm} (4)

where \( g_N \) is the standard Newtonian gravitational acceleration and \( \mu \) is the function interpolating between the Newtonian regime \( (\mu(x) \approx 1 \) where \( x \gg 1 \) \) and the MOND regime \( (\mu(x) \approx x \) where \( x \ll 1 \) \). In this case, the equations are closed by taking the polytropic relation

\[ \sigma_r^2 = c_0^2 \left( \frac{\rho}{\rho_0} \right)^{\frac{1}{n}} \]  \hspace{1cm} (5)

where \( c_0 \) is the central velocity dispersion (with MOND this sets the mass of the system), and \( \rho_0 \) is the central density.

Therefore, the parameters of any such model are \( c_0 \), the central velocity dispersion which is essentially determined by the observed central velocity dispersion; \( n \), the polytropic index which is typically between 10 and 20; \( a_r \), the anisotropy radius which must be on the order of the half-light radius to avoid radial orbit instability; \( \rho_0 \), the central density which must be chosen to match the core radius.

The properties of one such model are shown in the two figures (taking \( \mu(x) = x/\sqrt{(1 + x^2)} \) and \( a_0 = 10^{-8} \text{ cm/s}^2 \)). Fig. 1 is the projected surface brightness distribution compared to that observed for NGC 2419. The second is the radial distribution of line-of-sight velocity dispersion again compared to the observations. For this model \( c_0 = 7.5 \text{ km/s}, n = 10, a_r = 18 \text{ pc}, \) and \( \rho_0 = 35 \text{ M}_\odot \text{ pc}^{-3}. \) This yields a total mass of \( 7.7 \times 10^5 \text{ M}_\odot \) and an effective radius of 18 pc. The corresponding mass to light ratio is 1.6. It is evident that this MONDian model provides a reasonable description of the observations. Overall, a comparable match to the observations can be achieved for \( 7 \text{ km/s} \leq c_0 \leq 7.5 \text{ km/s}, 10 \leq n \leq 12, \) and \( 18 \text{ pc} \leq a_r \leq 22 \text{ pc}. \)

Ibata et al. have also applied the Jeans equation to consider a range of non-isothermal, anisotropic models; in these models the density distribution was frozen to be that of the best-fit Newtonian model of the cluster because this provides an accurate description of the surface brightness distribution. On the basis of the maximum likelihood ratio test no MOND fit to the data that is comparable to the best Newtonian model is achieved. But here it is evident that the anisotropic polytropes can provide a reasonable description of both the surface brightness distribution and the velocity dispersion radial profile, reasonable to the eye if not to maximum likelihood. This is striking in that the polytropic assumption is a particularly idealized and rigid method of encoding deviations from an isothermal state. Given this rigidity it is not surprising that the model shown in Figs. 1 and 2 is not a precise match to the data; in particular, the predicted surface brightness distribution is an imperfect fit. But it is perhaps unwise to rely too heavily on formal statistical tests which assume random errors in data that may be
plagued by systematic effects, observational and/or physical (a common problem in the interpretation astronomical data).

3.1 Discussion

Observationally, there are essentially two classes of halo objects: globular clusters and dwarf spheroidal galaxies; these overlap in mass but not in surface brightness or in age and uniformity of the stellar populations. The globular clusters are generally comprised of ancient, though not necessarily coeval, stellar populations and they are numerous (several hundred observed in and inferred in the Galaxy). They are high surface brightness objects and show no dynamical evidence for dark matter within the visible object (dynamical mass-to-light ratios are typical of the observed stellar populations). The dwarf spheroidal galaxies have low surface brightness and a large conventional dynamical M/L (M/L exceeding 100 in some cases). They are few in number (about 20 directly observed in the the Galaxy) and contain generally younger stellar populations covering a range of ages.

In the context the LCDM paradigm, the explanation of the general properties of these halo objects, specifically the presence or absence of “dark matter”, resides in murky creation mythology. LCDM simulations predict that galaxies, assembled over cosmic time via mergers of smaller halos, should contain a large number of dark matter sub-halos (in the Galaxy more than 200 with velocity dispersion greater than a few km/s); this substructure is intrinsic to the theory and a fundamental constituent of galaxy scale halos. It might seem more natural to identify these dark matter subhalos with globular clusters, the more numerous and primordial objects in the Galaxy. However, these are baryon rather than dark matter dominated, so the identification is made with the dwarf spheroidals. Their embarrassing scarcity is then due the fact that most of the predicted dark matter sub-halos have remained dark because they never captured sufficient baryons to initiate star formation or the captured baryons have been blown away by early stellar processes. Then a separate formation scenario must be invoked for globular clusters; e.g., globular clusters are formed in primordial disk-bound supermassive molecular clouds with high baryon to dark matter ratio and later attain a more spheroidal shape due to subsequent mergers (Kratzov & Gueden 2005). These scenarios, while imaginative, are, to the least, difficult to falsify.

In the context of MOND there is no need to speculate about formation processes in order to account for the perceived dark matter content in these two classes of objects. MOND predicts that high surface brightness systems (systems with high internal acceleration) should exhibit no evidence for a mass discrepancy within the visible object (conventionally, no dark matter). Conversely, MOND predicts that low-surface-brightness systems, such as the dwarf spheroidal satellites of the Galaxy, should exhibit a large discrepancy. These observed properties of globular clusters and dwarf spheroidals find natural explanation in the context of MOND based on existent physical law, not on formation scenarios. That is not to say differing formation histories are unimportant in defining the overall observed properties of these two distinct classes of halo objects (such as the stellar populations). With MOND, globular clusters could well be among the first objects formed, prior to or simultaneous with galaxies, as suggested by the old stellar populations; whereas the dwarf spheroidals may have formed subsequently as tidal objects. But the magnitude of the apparent “dark matter content” is directly related to the internal acceleration or observed surface density and not to different formation histories.

It is of interest that with MOND, non-isothermal systems, such as the high-order polytropes shown here, have a cutoff radius (an edge) which is unrelated to the tidal radius. Given the baryonic mass of NGC 2419, the tidal radius should be in excess of 1 kpc, and yet, the observed truncation radius is on the order of 200 pc. In general the cutoff radii of dwarf spheroidals, which have comparable baryonic masses, are larger than those of the globular clusters (Zhao 2005a,b). Perhaps it is the case that the globular clusters do not fill their Roche lobe – that the density cutoff is due to non-isothermal state. On the other hand, the dwarf spheroidals may well extend to their tidal radii because of the different formation history.

With respect to the specific example of NGC 2419 it has been claimed that simultaneously matching the radial distribution of starlight and line-of-sight velocity dispersion is not possible in the context of MOND. This claim is made in the context of a class of isothermal models in which the phase space distribution of stars as a function of the integrals of motion is chosen to be of a quite specific form (the Michie model). This class may be appropriate for Newtonian isothermal spheres with a constructed radial cutoff (identified with the tidal radius) but it is not clearly applicable to MONDian objects which are intrinsically finite. I have presented a counter-example which demonstrates no such inconsistency with the observations: a non-isothermal models, approximated by high order polytropes. I attach no particular significance to the polytropic relation between velocity dispersion and density; it is an idealized assumption. But it does demonstrate that, given the uncertainties of anisotropy or isothermality, it is perhaps rash to claim that this one particular object is problematic for MOND. Each single well-measured rotation curve for a nearby disk galaxy – missing these ambiguities – is far more of a crucible for gravity theories.

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