A NOTE ON THE GROWTH FACTOR IN GAUSSIAN ELIMINATION FOR GENERALIZED HIGHAM MATRICES

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Abstract. The Higham matrix is a complex symmetric matrix $A = B + iC$, where both $B$ and $C$ are real, symmetric and positive definite and $i = \sqrt{-1}$ is the imaginary unit. For any Higham matrix $A$, Ikramov et al showed that the growth factor in Gaussian elimination is less than 3. In this paper, based on the previous results, a new bound of the growth factor is obtained by using the maximum of the condition numbers of matrixes $B$ and $C$ for the generalized Higham matrix $A$, which strengthens this bound to 2 and proves the Higham’s conjecture.

1. Introduction

In this paper, we mainly consider the linear system

$$(1.1) \quad Ax = b,$$

where the matrix $A$ is a complex symmetric matrix. This kind of linear systems (1.1) arise from different physical applications, for example, in modeling electromagnetic waves under the assumption of time-harmonic variation in the electromagnetic fields, Helmholtz’s equations with a complex shift (see, e.g., [3, 4, 5]) etc. Moreover, the complex-valued linear system (1.1) can be directly generated in the field of lattice quantum chromo dynamics (QCD), where a model of the interactions of fermions (or quarks) on a lattice is given in terms of a complex-valued gauge field that directly leads to the linear system (1.1) (see, [6, 7]). In addition, this complex symmetric linear system also arises in centered difference discretization of the $R_{22}$-Padé approximations in the time integration of parabolic partial differential equations ([8]) and in direct frequency domain analysis of an $n$-degree-of-freedom ($n$-DOF) linear system ([9]). There are some examples of scientific applications in [10]. Therefore, researches on numerical solutions of the linear system (1.1) are greatly needed.

Next, for convenience, let $M_n(\mathbb{C})$ denote the set of $n \times n$ complex matrices and $A$ be a nonsingular matrix in $M_n(\mathbb{C})$. $\lambda_1$ and $\lambda_n$ are the largest and smallest eigenvalues of $A^*A$, respectively.

Recently, a complex symmetric positive definite (CSPD) matrix arising from the linear system (1.1) in Gaussian elimination without pivoting was firstly studied by Higham in [2], which is called by Higham matrices in [1]. Subsequently, the paper [1] gave a broader class of complex matrices—generalized Higham
matrices (sometimes they are also called accretive-dissipative matrices (see [19])), i.e., for any $A \in M_n(\mathbb{C})$, if its Hermitian decomposition \(^1\) (see, [12])

$$A = B + iC$$

satisfies that

$$B = B^* > 0, \ C = C^* > 0,$$

where $B^*$ is the conjugate transpose of $B$, then the matrix $A$ is said to be a generalized Higham matrix and denoted by $A \in M_n^{++}(\mathbb{C})$. Here, the sign $\geq$ is usually called the Loewner partial order of Hermitian matrices; i.e., we write $B \geq C$ if the matrix $B - C$ is positive semidefinite, similarly, $B > C$ means that $B - C$ is positive definite. In addition, a related class of matrices defined by

$$A = B + iC, \ B = B^* > 0, \ C = C^* < 0,$$

will be denoted by $M_n^{+-}(\mathbb{C})$ as in [1].

This paper is a continuation of [1], and they both originate from the Higham’s paper [2].

As it is well known, for the complex symmetric linear system (1.1), the growth factor $\rho_n(A)$ in Gaussian elimination for $A$ is defined by

$$\rho_n(A) = \frac{\max_{i,j,k} |a_{ij}^{(k)}|}{\max_{i,j} |a_{ij}|},$$

where $A \triangleq (a_{ij}), \ A^{(k)} \triangleq (a_{ij}^{(k)})$ and $A^{(k)}$ is the matrix obtained through the application of the first $k$ steps of Gaussian elimination to $A$ (see, e.g., [11]). In particular, $A^{(n-1)}$ is the upper triangular matrix resulting from the $LU$ factorization of $A$.

Obviously, for the matrix $A$ in the linear system (1.1), if one is able to prove a satisfactory priori bound for $\rho_n(A)$, then it is safe not to pivot in computing the $LU$ factorization of the matrix $A$ (or to choose diagonal pivots based on other considerations such as sparsity preservation) (see, [12]).

For any Higham matrix $A$, the growth factor in Gaussian elimination

$$\rho_n(A) \leq 2$$

was firstly conjectured by Higham (see, [12], P.210). An incorrect proof was given in [2], but Ikramov et al. [1] subsequently showed that

$$\rho_n(A) < 3$$

for any Higham matrix $A$. In addition, if the Higham matrix is extended by allowing $B$ and $C$ to be arbitrary Hermitian positive definite matrices, then

$$\rho_n(A) < 3\sqrt{2}.$$ 

Moreover, Ikramov et al noted that the above bound (1.5) remains true when $B$ or $C$ or both are negative (rather than positive) definite (see, [1]).

For a very restricted subset of Higham matrices, i.e., when $B = I_n$ and $C$ is real, symmetric and positive definite, the authors in [13] proved the better bound

$$\rho_n(A) \leq \frac{1 + \sqrt{17}}{4} \approx 1.28078 \ldots.$$ 

\(^1\)It is also called the Toeplitz decomposition (see, [19]).
In addition, A. George and K. D. Ikramov ([14]) assumed that $B$ and $C$ with being positive definite, satisfy the inequality
\[ C \leq \alpha B, \quad \alpha \geq 0, \]
and they established a bound for the growth factor $\rho_n(A)$ that has the limit 1 as $\alpha \to 0$.

Recently, Lin [20] proved that if $A$ is a generalized Higham matrices, then the growth factor for such $A$ in Gaussian elimination is less than 4. Specially when $A$ is a Higham matrix, then the growth factor is less than $2\sqrt{2}$.

However, as authors in [1] pointed out, in no case have they observed the growth bigger than Higham’s guess of 2 from extensive numerical experiments with Higham matrices. Therefore, they believed that the bound (1.3) is correct and took the proof of this bound for an open problem (see also Problem 10.12 in [12]).

In this work, we continue studying this open problem and then give a new result
\[ 0 \leq \frac{4\kappa}{(1 + \kappa)^2} \leq \rho_n(A) \leq \frac{2(1 + \kappa^2)}{(1 + \kappa)^2} \leq 2, \]
for the generalized Higham matrix $A$, where $\kappa \in [1, +\infty)$ is the maximum of the condition numbers of $B$ and $C$. This directly leads to the Higham’s result (1.3) for any Higham matrix $A$, which proves the open problem. Here, for a nonsingular matrix $A$, its condition number is denoted by $\kappa(A) \triangleq \sqrt{\frac{\lambda_{\max}(A^*A)}{\lambda_{\min}(A^*A)}}$, i.e., the ratio of the largest and smallest singular value of $A$.

This paper is organized as follows. In Section 2, we show some new bounds on the growth factor, based on the condition number. In Section 3, some figures and numerical examples are given to illustrate our results.

2. Main results

In this section, let $A \in M_n(\mathbb{C})$ be partitioned as
\[ A \triangleq \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix} + i \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}, \]
where $A$ is an $n \times n$ nonsingular matrix. If $A_{11}$ is invertible, then the schur complement of $A_{11}$ in $A$ is denoted by $A/A_{11} = A_{22} - A_{21}A_{11}^{-1}A_{12}$ (see, [17]).

**Lemma 2.1.** ([13]). Let $A$ be a CSPD matrix, then $A$ is nonsingular, and any principal submatrix of $A$ and any schur complement in $A$ are also CSPD matrices.

Obviously, Lemma 2.1 shows that, being a CSPD matrix is an hereditary property of active submatrices in Gaussian elimination.

**Lemma 2.2.** ([13]). The largest element of a CSPD matrix $A$ lies on its main diagonal.

Thus, for any CSPD matrix $A$, the definition (1.2) can be replaced by
\[ \rho_n(A) = \frac{\max_{j,k} |a(k)_{jj}|}{\max_j |a_{jj}|}, \]
which greatly simplifies the analysis on bounding the growth factor for a CSPD matrix $A$. 

Lemma 2.3. ([15]). If $B$ is a nonzero $n \times n$ positive definite matrix having eigenvalues $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_n$, then for all orthogonal vectors $x, y \in \mathbb{C}^n$, and $x^*$ denotes the conjugate transpose of $x$, the following equality holds,

\begin{equation}
|x^*By|^2 \leq \left(\frac{\lambda_1 - \lambda_n}{\lambda_1 + \lambda_n}\right)^2 (x^*Bx)(y^*By).
\end{equation}

Lemma 2.4. ([16]). Let $B$ be as in Lemma 2.3, then for any $n \times p$ matrix $X$ satisfying $X^*X = I_p$, where $X^*$ means the conjugate transpose of the matrix $X$, we have that

\begin{equation}
X^*B^{-1}X \leq \frac{(\lambda_1 + \lambda_n)^2}{4\lambda_1\lambda_n}(X^*BX)^{-1}.
\end{equation}

By Lemma 2.4, it is easy to obtain the following lemma.

Lemma 2.5. ([16]). Let $B = \begin{pmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{pmatrix}$ be an $n \times n$ Hermitian positive definite matrix, where $B_{22}$ is any $k \times k$ principal submatrix of $B$ ($k > 0$), then

\begin{equation}
B_{21}B_{11}^{-1}B_{12} \leq \left(\frac{1 - \kappa(B)}{1 + \kappa(B)}\right)^2 B_{22},
\end{equation}

where $\kappa(B)$ is the condition number of $B$.

Theorem 2.6. ([15]). Let $B$ be a Hermitian positive definite matrix, then $\lambda_{n-t+i}(B) \leq \lambda_i(B) \leq \lambda_i(B_k)$, $(t = 1, 2, \cdots, t)$, where $B_k = B(i_1, \cdots, i_t)$ is the $t \times t$ principal submatrix of $B$.

Corollary 2.1. ([18]). Let $B$ be a Hermitian positive definite matrix, and partitioned as in Lemma 2.5, then $\kappa(B) > \kappa(B_{11})$.

Lemma 2.7. ([19]). Let $A = B + iC$, $B = B^*$, $C = C^*$, be partitioned as in (2.7), if $B_{11}$, $C_{11}$ are invertible, then

\begin{equation}
A/A_{11} = B/B_{11} + i(C/C_{11}) + X(B_{11}^{-1} - iC_{11}^{-1})^{-1}X^*,
\end{equation}

where $X = B_{21}B_{11}^{-1} - C_{21}C_{11}^{-1}$.

Corollary 2.2. ([19]). Let $A = B + iC$, $B = B^*$, $C = C^*$ be a generalized Higham matrix and be partitioned as in (2.7), if $A/A_{11} = R + iS$ is its Hermitian decomposition, then $R \geq B/B_{11}$, $S \geq C/C_{11}$.

Next, we give our main result.

Theorem 2.8. Let $A$ be a generalized Higham matrix, then

\begin{equation}
\frac{4\kappa}{(1 + \kappa)^2} \leq \rho_n(A) \leq \frac{2(1 + \kappa^2)}{(1 + \kappa)^2},
\end{equation}

where $\kappa$ is the maximum of the condition numbers of $B$ and $C$.

Proof. Fix the number $k \in \{1, 2, \cdots, n-1\}$ and $j$, where $j \geq k + 1$. Denote $A_k$ by the leading principal submatrix of order $k$ in $A$.

We consider the $(k + 1) \times (k + 1)$ matrix

\[ A_{kj} = \begin{pmatrix} A_k^{T} & \alpha \\ \beta \alpha & a_{jj} \end{pmatrix} = B_{kj} + iC_{kj}, \]
where
\[ a^T = (a_{1j}, a_{2j}, \cdots, a_{kj}) \text{ and } \beta^T = (a_{1j}, a_{2j}, \cdots, a_{kj}) ,\]
\[ B_{kj} = \begin{pmatrix} B_k & b \\ b^* & b_{jj} \end{pmatrix} \text{ and } C_{kj} = \begin{pmatrix} C_k & c \\ c^* & c_{jj} \end{pmatrix} .\]

Note that \( A_{kj}, B_{kj} \) and \( C_{kj} \) are principal order \( k + 1 \) submatrices of \( A, B \) and \( C \), respectively.

It is easy to see that \( a_{kj}^{(k)} \) can be obtained by performing block Gaussian elimination in \( A_{kj} \); i.e.,
\[ a_{kj}^{(k)} = a_{jj} - \beta^T A_k^{-1} \alpha .\]

Setting \( a_{kj}^{(k)} = \beta + i\gamma, \beta, \gamma \in \mathbb{R} \). Since both \( B_{kj} \) and \( C_{kj} \) are Hermitian positive definite, according to the result of the Lemma 2.2, we have
\[ b^* B_k^{-1} b \leq \left( \frac{1 - \kappa(B_{kj})}{1 + \kappa(B_{kj})} \right)^2 b_{jj} \text{ and } c^* C_k^{-1} c \leq \left( \frac{1 - \kappa(C_{kj})}{1 + \kappa(C_{kj})} \right)^2 c_{jj} .\]

Next, by the corollary 2.2 and \( f(x) = \frac{4x}{(1+x)^2} \) is decreasing in \( x \in [1, +\infty) \), we get
\[ |a_{kj}^{(k)}| = |\beta + i\gamma| \geq |B_{kj}/B_k + iC_{kj}/C_k| \geq |(b_{jj} - b^* B_k^{-1} b) + i(c_{jj} - c^* C_k^{-1} c)| \geq \left( \frac{4\kappa(B_{kj})}{(1 + \kappa(B_{kj}))^2} \right) |b_{jj} + i c_{jj}| \geq \left( \frac{4\kappa(B_{kj})}{(1 + \kappa(B_{kj}))^2} \right) |a_{jj}| ,\]
where \( \kappa_{kj} = \max \{ \kappa(B_{kj}), \kappa(C_{kj}) \} \).

Since \( a_{kj}^{(k)} = a_{jj} - \beta^T A_k^{-1} \alpha = b_{jj} + i c_{jj} - \beta^T A_k^{-1} \alpha \), \( \beta/B_{11} = b_{jj} - b^* B_k^{-1} b \) and \( C/C_{11} = c_{jj} - c^* C_k^{-1} c \), by the corollary 2.2, we have
\[ b_{jj} - \text{Re}(\beta^T A_k^{-1} \alpha) \geq b_{jj} - b^* B_k^{-1} b, c_{jj} - \text{Im}(\beta^T A_k^{-1} \alpha) \geq c_{jj} - c^* C_k^{-1} c .\]

Noting that \( g(x) = \left( \frac{x}{1+x} \right)^2 \) is increasing in \( x \in [1, +\infty) \), one has
\[ \text{Re}(\beta^T A_k^{-1} \alpha) \leq b^* B_k^{-1} b \leq \left( \frac{1 - \kappa(B_{kj})}{1 + \kappa(B_{kj})} \right)^2 b_{jj} \]
and
\[ \text{Im}(\beta^T A_k^{-1} \alpha) \leq c^* C_k^{-1} c \leq \left( \frac{1 - \kappa(C_{kj})}{1 + \kappa(C_{kj})} \right)^2 c_{jj} ,\]
equivalently,
\[ |\beta^T A_k^{-1} \alpha| \leq \left( \frac{1 - \kappa_{kj}}{1 + \kappa_{kj}} \right)^2 |a_{jj}| .\]

Thus, we can obtain
\[ |a_{kj}^{(k)}| = |a_{jj} - \beta^T A_k^{-1} \alpha| \leq |a_{jj}| + |\beta^T A_k^{-1} \alpha| \leq |a_{jj}| + \left( \frac{1 - \kappa_{kj}}{1 + \kappa_{kj}} \right)^2 |a_{jj}| \leq \left( \frac{2(1+\kappa_{kj}^2)}{(1+\kappa_{kj})^2} \right) |a_{jj}| .\]
According to the above inequalities (2.7) and (2.8), the following inequalities is obvious that

\[
\frac{4\kappa_{kj}}{(1 + \kappa_{kj})^2} \leq \rho_n(A) \leq \frac{2(1 + \kappa_{kj}^2)}{(1 + \kappa_{kj})^2}.
\]

Note that \( f(x) = \frac{4x}{(1 + x)^2} \) is decreasing in \( x \in [1, +\infty) \), and \( g(x) = \frac{2(1 + x^2)}{(1 + x)^2} \) is increasing in \( x \in [1, +\infty) \) (see, Fig. 1), by Corollary 2.1, we have that

\[
\frac{4\kappa}{(1 + \kappa)^2} \leq \rho_n(A) \leq \frac{2(1 + \kappa^2)}{(1 + \kappa)^2}.
\]

The proof is completed. □

\[
\begin{array}{c}
\text{Figure 1. Left: the variation curve of } f(\kappa) = \frac{4\kappa}{(1 + \kappa)^2} \text{ with } \kappa \text{ decreasing. Right: the variation curve of } g(\kappa) = \frac{2(1 + \kappa^2)}{(1 + \kappa)^2} \text{ with } \kappa \text{ increasing.}
\end{array}
\]

**Corollary 2.3.** If \( A \) is an \( n \times n \) generalized Higham matrix, then

\[
(2.10) \quad 0 \leq \rho_n(A) \leq 2.
\]

**Proof.** Since \( \frac{4\kappa}{(1 + \kappa)^2} \leq \rho_n(A) \leq \frac{2(1 + \kappa^2)}{(1 + \kappa)^2} \), we calculate simultaneously the limit of both sides of the inequality, we have

\[
\lim_{\kappa \to \infty} \frac{4\kappa}{(1 + \kappa)^2} = 0 \text{ and } \lim_{\kappa \to \infty} \frac{2(1 + \kappa^2)}{(1 + \kappa)^2} = 2.
\]

Therefore, the result (2.10) holds. □

**Remark 2.1.** Obviously, the above result (2.10) holds also for any Higham matrix, and hence the Higham’s conjecture (see (1.3)) is correct, which solves this open problem. In addition, since \( \kappa \geq 1 > 0 \), then \((\kappa + 1)^2 = \kappa^2 + 2\kappa + 1 > \kappa^2 + 1\), i.e., \( \frac{2(\kappa^2 + 1)}{(\kappa + 1)^2} < 2 \). So, generally speaking, \( \rho_n(A) < 2 \), see the following numerical experiment.
3. Numerical experiments

In this section, a numerical example will be described. The goal of the experiment is to examine the effectiveness of our result.

We consider the complex symmetric system of linear equation (1.1) arises in the centered difference discretizations of the \( R_{22} \)-Padé approximations in the time integration of parabolic partial differential equations, further details refer to [10].

For convenience, the complex coefficient symmetric matrix (see, [8]) may be written as

\[
A = (K + \frac{3 - \sqrt{3}}{\tau} I) + i(K + \frac{3 + \sqrt{3}}{\tau} I), \quad i = \sqrt{-1},
\]

where \( I \) is the identity matrix, \( \tau \) is the time step-size and \( K \) is the five-point centered difference matrix approximating the negative Laplacian operator \( L = -\Delta \) with homogeneous Dirichlet boundary conditions, on a uniform mesh in the unit square \([0, 1] \times [0, 1]\) with the mesh-size \( h = \frac{1}{m+1} \).

In our tests, we take \( \tau = h \). The matrix \( K \in \mathbb{R}^{n \times n} \) possesses the tensor-product form \( K = I \otimes V_m + V_m \otimes I \), with \( V_m = h^{-2} \text{tridiag}(-1, 2, -1) \in \mathbb{R}^{m \times m} \). Hence, \( K \) is an \( n \times n \) block tridiagonal matrix, with \( n = m^2 \).

Denote \( B = K + \frac{3 - \sqrt{3}}{\tau} I \) and \( C = K + \frac{3 + \sqrt{3}}{\tau} I \),

and apply MATLAB 2011b functions to compute the condition number of \( B \) and \( C \):

\[
t_1 = \text{condest}(B), \quad t_2 = \text{condest}(C).
\]

Let \( \kappa = \max\{\kappa(t_1), \kappa(t_2)\} \), \( L = \frac{2(1+\kappa^2)}{(1+\kappa)} \), respectively. The numerical results as follows (see, Table 1).

| Size (m) | \( t_1 \) | \( t_2 \) | \( \kappa \) | \( L \) |
|---------|----------|----------|---------|-------|
| 700     | 4.4239e+003 | 1.1861e+003 | 4.4239e+003 | 1.9991 |
| 800     | 5.0548e+003 | 1.3552e+003 | 5.0548e+003 | 1.9992 |
| 900     | 5.6858e+003 | 1.5242e+003 | 5.6858e+003 | 1.9993 |
| 1000    | 6.3167e+003 | 1.6933e+003 | 6.3167e+003 | 1.9994 |
| 1100    | 6.9477e+003 | 1.8623e+003 | 6.9477e+003 | 1.9994 |
| 1200    | 7.5786e+003 | 2.0314e+003 | 7.5786e+003 | 1.9995 |
| 1300    | 8.2095e+003 | 2.2005e+003 | 8.2095e+003 | 1.9995 |
| 1400    | 8.8405e+003 | 2.3695e+003 | 8.8405e+003 | 1.9995 |

Obviously, the results on this experiment conform with our theoretical analysis.

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