Multipomeron Cuts and Hard Processes on Nuclei

Wolfgang Schäfer

Institute of Nuclear Physics PAN, ul. Radzikowskiego 152, 31-342 Kraków, Poland

Abstract. With nuclear targets comes a new scale into the pQCD description of hard processes – the saturation scale. In the saturation regime, the familiar linear $k_T$–factorization breaks down and must be replaced by a nonlinear $k_T$–factorization, which accounts for absorptive and multiple scattering corrections to the hard process. Predictions for partial cross sections corresponding to a fixed number of cut Pomerons (the topological cross sections) can be obtained in a surprisingly straightforward manner. We discuss some applications to deep inelastic scattering.

Keywords: QCD, Multiple scattering, Diffraction

PACS: 12.38.Bx, 11.80.La, 13.85.Hd

LINEAR $k_T$ FACTORIZATION IS BROKEN

Heavy nuclei are strongly absorbing targets and bring a new scale into the perturbative QCD (pQCD) description of hard processes [1]. This has severe consequences for the relations between various hard scattering observables. In a regime of small absorption, small–$x$ processes are adequately described by the linear $k_T$–factorization, and the pertinent observables are linear functionals of a universal unintegrated gluon distribution. Not so for the strongly absorbing target, where the linear $k_T$–factorization is broken [2], and has to be replaced by a new, nonlinear $k_T$–factorization [2, 3], where observables are in general nonlinear functionals of a properly defined unintegrated glue. The relevant formalism has been worked out for all interesting processes [3, 4] (see [5] for references on related work), but in this very short contribution we concentrate on deep inelastic scattering (DIS). Here, in the typical inelastic DIS event the nuclear debris will be left in a state with multiple color excited nucleons after the $q\bar{q}$ dipole exchanged many gluons with the target. The partial cross sections for final states with a fixed number of color excited nucleons are the topological cross sections. It is customary to describe them in a language of unitarity cuts through multipomeron exchange diagrams [6]. In our approach [7], color excited nucleons in the final state give a clear–cut definition of a cut pomeron. Topological cross sections carry useful information on the correlation between forward or midrapidity jet/dijet production and multiproduction in the nuclear fragmentation region as well as on the centrality of a collision.
NUCLEAR COLLECTIVE GLUE AND ITS UNITARITY CUT INTERPRETATION

The basic ingredient of the nonlinear $k_T$–factorization is the collective nuclear unintegrated glue, which made its first appearance in our work on the diffractive breakup of pions into jets $\pi A \rightarrow \text{jett} \rightarrow A$ [8]. Indeed, in the high energy limit, the nearly back–to–back jets acquire their large transverse momenta directly from gluons. It is then natural use the diffractive $S$–matrix of a $q\bar{q}$–dipole $S_A(b,\rho,\mathbf{x})$ for defining the nuclear unintegrated glue:

$$Z \frac{d^2 \mathbf{r}}{(2\pi)^2} S_A(b,\rho,\mathbf{x}) \exp \left( i \mathbf{p} \mathbf{r} \right) = S_A(b,\rho,\mathbf{x}) \delta^{(2)}(\mathbf{p}) + \phi(b,\rho,\mathbf{x}) \Phi(b,\rho,\mathbf{x}) : \quad (1)$$

Notice that it resums multiple scatterings of a dipole, so that there is no straightforward relation to the conventional parton distribution which corresponds to just two partons in the $t$–channel. It is still meaningful to call it an unintegrated glue – one reason was given above – another one, besides its role in factorization formulas is its small-$x$ evolution property: The so–defined $\phi(b,\rho,\mathbf{x})$ can be shown [9] to obey the Balitskii–Kovchegov [10] evolution equation.

Close to $x_A \approx (m_NR_A)^{-1}$, for heavy nuclei, the dipole $S$–matrix is the familiar Glauber–Gribov exponential $S_A(b,\rho,\mathbf{x}) = \exp \left( \sigma(x_A,\mathbf{r}) T(b) = 2 \right)$, for large dipole sizes it can be expressed as $S_A(b,\rho,\mathbf{x}) \delta^{(2)}(\mathbf{p}) = \exp \left( \nu_A(x_A,\mathbf{b}) \right)$. Here the nuclear opacity $\nu_A(x_A,\mathbf{b}) = \frac{1}{2} \sigma_0(x_A) T(b)$, is given in terms of the dipole cross section for large dipoles $\sigma_0(x) = \sigma(x,\mathbf{r}) \delta^{(2)}(\mathbf{p})$. In momentum space, a useful expansion is in terms of multiple convolutions of the free–nucleon unintegrated glue (we use a notation $f(x,\mathbf{p}) \propto p^4 \partial G(x,\mathbf{p}^2) = \partial \log (\mathbf{p}^2)$):

$$\phi(b,\rho,\mathbf{x}) = \sum_j w_j \nu_A(x_A,\mathbf{b}) f^{(j)}(x_A,\mathbf{p}) : \quad (2)$$

Here

$$w_j(x_A,\nu_A) = \frac{\nu_A^{(j)}(x_A,\mathbf{b})}{j!} \exp \left( \nu_A(x_A,\mathbf{b}) \right) f^{(j)}(x_A,\mathbf{p}) = \prod_j d^2 \kappa f^{(j)}(x_A,\mathbf{b}) \delta^{(2)}(\mathbf{p}) \sum \kappa : \quad (3)$$

Curiously, the very same collective nuclear glue is proportional to the spectrum of quasielastically scattered quarks:

$$\frac{d \sigma (qA \rightarrow qX)}{d^2 b d^2 \mathbf{p}} \propto \phi(b,\rho,\mathbf{x}) : \quad (4)$$

Now, we can state the first unitarity cutting rule in momentum space: the $k$–th order term in the expansion (2) corresponds to the topological cross section for the quark–

\[1\] Strictly speaking only a few iterations of this equation make good sense.
nucleus scattering with $k$ color excited nucleons in the final state:

$$
\frac{d\sigma^{(k)}(qA \rightarrow qX)}{d^2b d^2p} \propto w_k \ v_A(b) \ f^{(k)}(p) : \tag{5}
$$

This simple substitution rule forms at the heart of the cutting rules applied to the nonlinear quadratures of [4].

**STANDARD AGK VS. QCD**

Given the close relation between the nuclear unintegrated glue and the Glauber–Gribov scattering theory from color dipoles, one may be tempted to play around with various expansions of the exponential. Taking inspiration from 1970’s hadronic models one may then 'derive' expressions for topological cross sections. For example, the inelastic cross section of the $q\bar{q}$-dipole-nucleus interaction is certainly obtained from:

$$
\Gamma^{inel}(b, r) = \exp[\sigma(r)T(b)] = \sum_k \Gamma^{(k)}(b, r); \tag{6}
$$

and $\Gamma^{(k)}(b, r) = \exp[\sigma(r)T(b)][\sigma(r)T(b)]^{k!}$ is then interpreted as the $k$–cut Pomeron topological cross section. This is entirely incorrect, the reason is that this result neglects the color–coupled channel structure of the intranuclear evolution of the color dipole. Interestingly, a simple closed expression can be obtained with full account for color [7]:

$$
\Gamma^{(k)}(b, r) = \sigma(r)T(b) w_k (2v_A(b)) e^{2v_A(b)} \frac{e^{2v_A(b)}}{\lambda^k} \gamma(k, \lambda); \tag{7}
$$

where $\lambda = 2v_A(b)$, $\sigma(r)T(b)$, and $\gamma(k, \lambda)$ is an incomplete Gamma–function. For a more quantitative comparison, consult fig 1. We see that the standard Glauber–AGK predicts a strong hierarchy: $k$ cuts are suppressed by the $k$–th power of the dipole cross section. In the QCD–cutting rules there is an additional dimensionful parameter, the opacity of a nucleus for large dipoles $v_A$, and the distribution over $k$ is substantially broader. This difference will be more dramatic the smaller the dipole and reflects itself in the predicted $Q^2$–dependence of DIS structure functions with fixed multiplicity of cut Pomerons. More figures, as well as another example for the failure of standard AGK, can be found on the conference website.

**CONCLUSIONS**

Topological cross sections can be obtained from nonlinear $k$ factorization formulas by straightforward substitution (cutting) rules. For a correct isolation of topological cross sections a careful treatment of the color coupled channel properties of the color(!) dipole intranuclear evolution is mandatory. Don’t be misguided by simple formulas derived in a single channel context, or by a too literal analogy between color transparency and the Chudakov–Perkins suppression of multiple ionisation by small size $e^+ e^-$ pairs in QED.
FIGURE 1. **Left:** the profile function for $k$ cut Pomerons according to standard Glauber–AGK for a fairly large dipole $r = 0.6\text{ fm}$ at $x = 0.01$ for $A = 208$. **Right:** the same for the QCD cutting rules.

**ACKNOWLEDGMENTS**

It is a pleasure to thank the organizers for the kind invitation. This work was partially supported by the Polish Ministry for Science and Higher Education (MNiSW) under contract 1916/B/H03/2008/34.

**REFERENCES**

1. A. H. Mueller, Nucl. Phys. B 335 (1990) 115; Nucl. Phys. B 558 (1999) 285.
2. N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller, J. Exp. Theor. Phys. 97 (2003) 441.
3. N. N. Nikolaev, W. Schäfer and B. G. Zakharov, Phys. Rev. Lett. 95 (2005) 221803.
4. N. N. Nikolaev and W. Schäfer, Phys. Rev. D 71 (2005) 014023; N. N. Nikolaev, W. Schäfer, and B. G. Zakharov and V. R. Zoller, ibid. D 72 (2005) 114018; N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller, ibid. D 72 (2005) 034033.
5. J. Jalilian-Marian and Y. V. Kovchegov, Prog. Part. Nucl. Phys. 56 (2006) 104; F. Gelis, T. Lappi and R. Venugopalan, Int. J. Mod. Phys. E 16 (2007) 2595.
6. V. A. Abramovsky, V. N. Gribov and O. V. Kancheli, Yad. Fiz. 18 (1973) 595.
7. N. N. Nikolaev and W. Schäfer, Phys. Rev. D 74 (2006) 074021.
8. N. N. Nikolaev, W. Schäfer and G. Schwiete, Phys. Rev. D 63 (2000) 014020.
9. N. N. Nikolaev and W. Schäfer, Phys. Rev. D 74 (2006) 014023.
10. I. Balitsky, Nucl. Phys. B 463 (1996) 99; Y. V. Kovchegov, Phys. Rev. D 60 (1999) 034008.