Can the Universe escape eternal acceleration?

John D. Barrow\textsuperscript{1}, Rachel Bean\textsuperscript{2}, and João Magueijo\textsuperscript{2}

\textsuperscript{1}DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 0WA.

\textsuperscript{2}Theoretical Physics, The Blackett Laboratory, Imperial College, Prince Consort Rd., London SW7 2BZ

Recent astronomical observations of distant supernovae light-curves, \cite{1}\textsuperscript{-}\cite{4}, suggest that the expansion of the universe has recently begun to accelerate. Acceleration is created by an anti-gravitational repulsive stress, like that produced by a positive cosmological constant, or universal vacuum energy. It creates a rather bleak eschatological picture. An ever-expanding universe’s future appears to be increasingly dominated by its constant vacuum energy. A universe doomed to accelerate forever will produce a state of growing uniformity and cosmic loneliness. Structures participating in the cosmological expansion will ultimately leave each others’ horizons and information-processing must eventually die out\cite{5}. Here, we examine whether this picture is the only interpretation of the observations. We find that in many well-motivated scenarios the observed spell of vacuum domination is only a transient phenomenon. Soon after acceleration starts, the vacuum energy’s anti-gravitational properties are reversed, and a matter-dominated decelerating cosmic expansion resumes. Thus, contrary to general expectations, once an accelerating universe does not mean always an accelerating universe.

The observed cosmic acceleration, if due to vacuum repulsive stresses, is very puzzling. In contrast to conventional forms of matter, like pressureless dust or radiation, the cosmological energy densities contributed by these
repulsive stresses are not diluted by expansion: they remain constant. Consequently, the vacuum energy will eventually exceed the densities of all other forms of matter in an ever-expanding universe, causing its expansion to accelerate. Yet, in order to explain why the vacuum energy comes to dominate other matter only close to the present time, 15 Gyr after the expansion began, we must assume it starts out with a density that is $10^{120}$ times smaller than that of other forms of matter, for no known reason. A possible explanation is the ‘quintessence’ picture, in which a scalar matter field is present in the universe and can display repulsive gravitational behaviour late in the universe’s history, despite remaining innocuous for most of its life. In the most extreme case, its influence can become identical to the presence of a positive cosmological constant with constant energy density but the possible time-variation permitted in the evolution of its density allows acceleration to appear at late times following more natural initial conditions. We shall show that in well-motivated scenarios of this type the observed spell of vacuum domination is only a transient phenomenon.

We consider a homogeneous and isotropic universe with zero spatial curvature that contains two dominant forms of matter: a perfect fluid with pressure $p$ and density $\rho$ linked by an equation of state $p = (\gamma - 1)\rho$, with $\gamma$ constant, together with a hypothetical scalar ‘quintessence’ field $\phi$ defined by its self-interaction potential $V(\phi)$. All variables can depend only on the cosmic co-moving proper time $t$. The cosmological expansion scale factor is $a(t)$ and the Hubble expansion rate is defined by $H \equiv \dot{a}/a$. Separate conservation of mass-energy for the two matter fields as the universe expands requires $\rho \propto a^{-3\gamma}$, while the scalar field, $\phi$, obeys

$$\ddot{\phi} + 3H\dot{\phi} = \frac{\partial V}{\partial \phi},$$

where overdots represent time derivatives. The densities of the perfect fluid, $\rho$, and the $\phi$ field, $\rho_\phi$, drive the expansion, so

$$3H^2 = \rho + \rho_\phi,$$

where the scalar-field’s energy density is the sum of its kinetic and potential parts, $\rho_\phi \equiv \frac{1}{2}\dot{\phi}^2 + V$. We use Planck units defined by $\hbar = c = (8\pi G)^{-1/2} = 1$.

If we choose a monotonic potential of the form $V(\phi) = Ae^{-\lambda\phi}$, with $A$ and $\lambda > \sqrt{2}$ constant parameters, then $V$ need not increasingly dominate the right-hand side of eq. (2) at large $a$. Instead, $\rho_\phi$ evolves in direct proportion to $\rho \propto t^{-2}$ during an era of radiation ($\gamma = 4/3$) or matter ($\gamma = 1$)
domination. Hence \( H = 2/3\gamma t \) and the fraction of the total density in the form of quintessence during this 'scaling' regime depends only on \( \lambda \): 

\[
\Omega_\phi \equiv \frac{\rho_\phi}{\rho_\phi + \rho} = \frac{\rho_\phi}{3H^2} = \frac{3\gamma}{\lambda^2}.
\]

This unusual type of behaviour shows how vacuum domination can be postponed for a long period since the universe’s expansion began. However, eventually, the expansion departs from this special scaling behaviour and the potential of the \( \phi \) field comes to dominate eq. (2), contributing an almost constant value of \( \rho_\phi \) and accelerating expansion. Eventually, the potential energy \( V \) supersedes the kinetic energy \( \frac{1}{2}\dot{\phi}^2 \), making quintessence behave like an effective cosmological constant. In the standard scenarios the early scaling behaviour is a robust feature of quintessence models, but subsequent late-time acceleration can only be achieved with a large degree of fine tuning of the defining parameters - either in the initial conditions or in the parameters of the potential.

Albrecht and Skordis [7] have proposed a particularly attractive model of quintessence. It is driven by a potential which introduces a small minimum to the exponential potential:

\[
V(\phi) = e^{-\lambda \phi} \left( A + (\phi - \phi_0)^2 \right).
\]  

Unlike previous quintessence models, late-time acceleration is achieved without fine tuning of the initial conditions. The authors argue that such potentials arise naturally in the low-energy limit of \( M \)-theory; the constant parameters, \( A \) and \( \phi_0 \), in the potential take values of order 1 in Planck units, so there is also no fine tuning of the potential. They show that, regardless of the initial conditions, \( \rho_\phi \) scales, with \( \rho \propto \rho_\phi \propto t^{-2} \) during the radiation and matter eras, but leads to permanent vacuum domination and accelerated expansion after a time which can be close to the present. Acceleration begins when the field gets trapped in the local minimum of the potential at \( \phi = \phi_0 + (1 \pm \sqrt{1 - \lambda^2 A})/\lambda \), which is created by the quadratic factor in eq. (3) when \( 1 \geq \lambda^2 A \). Once the field gets stuck in the false vacuum its kinetic energy disappears (\( \phi \approx \) constant), and the ensuing dominance of \( \rho + \rho_\phi \) by an almost constant value of the potential value triggers a period of accelerated expansion that never ends. The probability for quantum tunnelling through the barrier is negligible [8].

We have found that this type of behaviour is by no means generic. In Fig. we plot the evolution of \( \Omega_\phi \) and a measure of the effective equation of
Figure 1: Permanent (dashed) and transient (solid) vacuum domination (with $\lambda = 4$, and $A\lambda^2 = 0.5, 1$ respectively). The early scaling behaviour is identical, but in one case the field gets stuck in the local potential minimum, leading to permanent vacuum domination, while in the other case the field resumes rolling down the potential, and a second period of matter-dominated expansion follows the temporary vacuum domination.
state of the total matter content of the universe \( w_{\text{tot}} = p_{\text{tot}}/\rho_{\text{tot}} \), where the total density is \( \rho_{\text{tot}} = \rho + \rho_\phi \) and the total pressure is \( p_{\text{tot}} = (\gamma - 1)\rho + \frac{\dot{\phi}^2}{2} - V \).

We show \( \Omega_\phi \) and \( w_{\text{tot}} \) in two qualitatively distinct cases, created by equally plausible values for potential parameters. Vacuum domination occurs when \( \Omega_\phi > 1/2 \), and accelerated expansion when \( w_{\text{tot}} < -1/3 \). The dashed line traces the case found by Albrecht and Skordis, but the solid line shows the general behaviour. In both cases there is scaling of the densities early in the expansion history: with \( w_\phi = 1/3 \) and \( \Omega_\phi \approx 4/\lambda^2 \) in the radiation era, followed by \( w_\phi = 0 \) and \( \Omega_\phi \approx 3/\lambda^2 \) in the matter era. The expansion decelerates in both scaling eras. But, then, in both cases, as \( \phi \rightarrow \phi_0 \), vacuum domination and accelerated expansion is triggered. Whereas in the first case this phenomenon is permanent, in the latter it is ephemeral. In the latter case, \( \phi \) soon continues rolling down the potential, and the universe resumes scaling evolution with \( \rho \propto \rho_\phi \propto t^{-2} \) and another matter-dominated era ensues. The spectre of never-ending vacuum domination has been lifted.

Transient vacuum domination arises in two ways. When \( A\lambda^2 < 1 \), the \( \phi \) field arrives at the local minimum with enough kinetic energy to roll over the barrier and resume descending the exponential part of the potential where \( \phi >> \phi_0 \). This kinetic energy is determined by the scaling regime, and so by parameters of the potential and not initial conditions. Another instance of transient vacuum domination is the whole region \( A\lambda^2 > 1 \). As \( A \) increases towards \( \lambda^{-2} \), the potential loses its local minimum, and flattens out into a point of inflexion. This is sufficient to trigger accelerated expansion temporarily, but the field never stops rolling down the potential, and matter-dominated scaling evolution with \( a(t) \propto t^{2/3} \) is soon resumed.

Further scrutiny of the theory reveals a critical dividing line separating permanent and transient vacuum domination. If we impose on the parameter space \( \{ A, \phi_0, \lambda \} \) the condition that accelerated expansion be occurring by the recent past, as observations imply, then we require \( \lambda\phi_0 \approx 280 \). This leaves us with two degrees of freedom, which we parametrise by \( \lambda \) and \( A\lambda^2 \). The behaviour of the Universe in this parameter space is shown in Fig. 2. The unshaded area represents the regions of permanent vacuum domination. Outside this area we have plotted contours giving the number of e-foldings of accelerated expansion, defined by \( N = \log(a_f/a_i) \), where \( a_f \) and \( a_i \) are the values of the expansion scale factor at the end and start of acceleration, respectively.

Our investigations have revealed a new type of cosmological evolution. It is possible for the universe to exit from a period of accelerated expansion
Figure 2: The phase space of the Albrecht-Skordis theory. A critical line divides permanent from transient vacuum domination. Above this line we have plotted contours for the number of e-foldings during which the Universe accelerates. The line $\lambda \approx 4$ indicates the nucleosynthesis bound ($\lambda > 4$).

and resume decelerated expansion. Moreover, for the well-motivated family of Albrecht-Skordis potentials this is the most likely form of evolution, rather than a state of continuing acceleration. Models of this type create a different theoretical framework in which to interpret the observation results of [1]-[4] and change their consequences for theories of cosmic structure formation. Other escape routes from acceleration, like introducing a quintessence field that ultimately decays into matter and radiation after producing acceleration, are at present entirely ad hoc. We can also imagine a richer cosmological structure if we admit the possibility of a random variation of $V(\phi)$ around the universe, in the spirit of chaotic inflationary universes. The condition for transient accelerated expansion will arise quasi-randomly around the universe as a set of parameters like $\{A, \phi_0, \lambda\}$ varies stochastically. This will give rise to some environments displaying transient acceleration at different times in their history, in which galaxy formation and clustering can continue to develop for longer than in the rarer, perpetually-accelerating, regions. The ability of expanding regions to recover from the onset of accelerated vacuum-dominated expansion may be an important factor in the global evolution of regions of an infinite universe containing long-lived stars and galaxies.
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