HOW TO DISTINGUISH HYDRODYNAMIC MODELS UTILIZING PARTICLE CORRELATIONS AND SPECTRA?

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We demonstrate on examples that a simultaneous study of the Bose-Einstein correlation function and the invariant momentum distribution can be very useful in distinguishing various hydrodynamic models, which describe separately the short-range correlations in high energy hadronic reactions as measured by the NA22 collaboration. We also analyze Bose-Einstein correlation functions, measured by the NA44 experiment at CERN SPS, in the context of the core-halo model. Values for the core radius and the fraction of direct bosons are obtained, and found to be independent of the structure of the correlation function at small relative momenta of $Q \leq 40$ MeV.

1 Introduction

The NA22 Collaboration performed recently a detailed study of Bose-Einstein Correlation Functions (BECF-s) in one- two- and three-dimensions for $\pi^+ / K^+ + p$ reactions at 250 GeV. Their study concluded as follows: “Our data do not confirm the expectation from the string-type model ... . A good description of our data is ... achieved in the framework of the hydrodynamic expanding source models ... . Alternatively, our data are also described in a non-expanding, surface-emitting fireball-like source... .” The above cited hydrodynamic models, successful in describing the NA22 data, belong to the following three different classes: i) Non-expanding, spherically symmetric fireballs, ii) Expanding, spherically symmetric shells, iii) Longitudinally expanding, cylindrically symmetric models with possible transverse flow, transverse and temporal temperature profiles.

2 Particle correlations and spectra in various hydrodynamic models

The experimental evidence reported in ref. indicates that it is rather difficult to distinguish the Fourier-transformed emission functions of the models of type i) – iii). However, the same emission function, which determines the two-particle BECF-s, prescribes also the single-particle spectra. In the forthcoming, we shall summarize the particle spectra and the BECF-s of the models and discuss what are the essential
differences among these. We shall utilize the Wigner-function formalism along the lines of refs. 2, 11, 6, 8, where the particle emission is described by the emission function \( S(x, p) \), \( x = (t, r) \) and \( p = (E, \mathbf{p}) \) with \( E = \sqrt{\mathbf{p}^2 + m^2} \). The two-particle BECF for completely chaotic sources is given by

\[
C(K, \Delta k) = \frac{\langle n \rangle^2}{\langle n(n-1) \rangle} \frac{N_2(\mathbf{p}_1, \mathbf{p}_2)}{N_1(\mathbf{p}_1) N_1(\mathbf{p}_2)} \approx 1 + \frac{|\tilde{S}(\Delta k, K)|^2}{|\tilde{S}(0, K)|^2},
\]

where \( N(\mathbf{p}) = E dN/d\mathbf{p} = \tilde{S}(0, \mathbf{p}) \) stands for the single-particle inclusive invariant momentum distribution, normalized to the mean multiplicity as \( \int N(\mathbf{p}) d\mathbf{p} = \langle n \rangle \) and \( \tilde{S}(\Delta k, K) = \int d^4 x S(x, K) \exp(i\Delta k \cdot x) \) with \( \Delta k = p_1 - p_2 \) and \( K = (p_1 + p_2)/2 \), where \( \Delta k \cdot x \) stands for the inner product of the four-vectors.

### 2.1 Spectra and Correlations for Kopylov and Podgoretskii model

The source considered by Kopylov and Podgoretskii (KP) in refs. 9, 10 was one of the earliest models of particle emission in multiparticle physics: a uniformly illuminated, surface emitting sphere. The emission function itself was not discussed in detail, since the source was assumed to be non-expanding, static. If we assume that the source is thermalized, the KP emission function is given by

\[
S_{KP}(x, p) = \frac{g}{(2\pi \hbar)^3} E_c \delta(R_{KP} - |\mathbf{r}|) \exp \left( -\frac{t}{\tau_{KP}} \right) \frac{1}{\exp(E_c/T) - 1},
\]

which is defined in the CMS of the source. The degeneracy factor is indicated by \( g \), the pre-factor \( E_c \) is due to the invariant normalization of the momentum distribution. The parameter \( R_{KP} \) is the radius of the source, \( \tau_{KP} \) is the decay time of the quanta, or in another interpretation, the thickness of the emitting layer, \( E_c \) is the energy of the particles in the CMS of the source and \( T \) is the temperature of the fireball. The single-particle spectrum can be obtained as

\[
E \frac{dN_{KP}}{d\mathbf{p}} = N_{KP}(\mathbf{p}) = \frac{g}{(2\pi \hbar)^3} E_c V_{KP} \frac{1}{\exp(E_c/T) - 1},
\]

where \( V_{KP} = 4\pi R_{KP}^2 \tau_{KP} \) is the effective volume of the fireball. In Boltzmann approximation, this invariant momentum distribution can be rewritten as

\[
N_{KP}(\mathbf{p}) = \frac{dN}{2\pi m_t dm_t dy} \simeq \frac{g}{(2\pi \hbar)^3} m_t \cosh(y - y_0) V_{KP} \exp \left( -\frac{m_t}{T_{KP}(y)} \right),
\]

\[
T_{KP}(y) = T / \cosh(y - y_0),
\]

where \( m_t \) and \( m_t = \sqrt{p_{x}^2 + p_y^2 + m^2} \) is the transverse mass, \( y = 0.5 \log((E + p_z)/(E - p_z)) \) denotes the rapidity of the particle, and \( y_0 \) stands for the rapidity of the CMS of the fireball. The effective rapidity-dependent temperature distribution, as given by
eq. (5), decreases fast with increasing difference between the rapidity of the particle as compared to the CMS of the fireball. This $1/\cosh(y - y_0)$ decrease is a typical result for non-expanding thermalized fireballs, as discussed in ref. The spectrum of eq. (5) can be re-written approximately as

$$\frac{dN}{2\pi m_t dm_t dy} \simeq \frac{g}{(2\pi \hbar)^3} m_t \cosh(y - y_0) V_{KP} \exp\left(-\frac{(y - y_0)^2}{2\Delta \eta_T^2}\right), \quad (6)$$

where $\Delta \eta_T^2 = T/m_t$. This yields a specific transverse mass dependence for the rapidity-width of the spectrum at a given value of $m_t$, which can be checked easily in experimental data analysis. The BECF for the KP emission function is given as

$$C(q_T, K) = 1 + \lambda \left[ I(R_{KP}q_T) \right]^2 (1 + \tau_{KP}^2 q_0^2)^{-1}, \quad (7)$$

where $I(x) = 2J_1(x)/x$ and $J_1(x)$ is the Bessel function of first order, with $I(0) = 1$ and $I(\infty) = 0$. The parameter $\lambda$ is introduced phenomenologically to account for intercepts $1 + \lambda < 2$, $q_T = \Delta k \cdot K/|K|$ is the relative momentum transverse to $K$ and $q_0 = E_1 - E_2$ is the energy difference in the CMS. This BECF is independent of the mean momentum $K$ due to the non-expanding source described by the KP model.

2.2 Spectra and Correlations for Expanding Shells

The model introduced by S. Pratt (P) and discussed recently by J. Bjorken corresponds to a uniformly illuminated, expanding shell, given by

$$S_P(x, p) = \frac{g}{(2\pi \hbar)^3} E_c \delta(R_P - |p|) \exp(-t^2/\tau_P^2) \frac{1}{\exp(p \cdot u(|p|)/T) - 1}, \quad (8)$$

which is defined in the CMS of the source. The parameter $R_P$ is the radius of the source, $\tau_P$ is the decay time of the quanta or the thickness of the emitting layer in the CMS of the source and $u(|p|) = \gamma(1,v|p|/R_P)$ is a spherically symmetric flow profile characterizing the expansion of the shell, with $\gamma = 1/\sqrt{1 - v^2}$. This model corresponds to a three-dimensional, spherically symmetric expansion in the rest frame of the source, and the KP model is recovered in the $v = 0$ limiting case.

The single-particle spectrum can be evaluated in Boltzmann approximation as

$$\frac{dN}{2\pi m_t dm_t dy} \simeq \frac{g}{(2\pi \hbar)^3} m_t \cosh(y - y_0) V_P \frac{\sinh(a)}{a} \exp(-m_t/T_P(y)) \quad (9)$$

$$T_P(y) = \frac{T}{\gamma \cosh(y - y_0)} [1 + \mathcal{O}(v^2|\gamma|^2)] \quad (10)$$

where $V_P = 4\pi R_P^2(vr_P)^2$ is the volume of the fireball, $a = a(p) = v\gamma|p|/T$ is a momentum-dependent parameter related to $v$, the surface velocity of the shell and $T_P(y)$ is the effective rapidity-dependent slope-parameter distribution. Thus for
small values of the parameter $a(p)$ the effective temperature is decreased as compared to the static fireball (KP) case, however, the rapidity-width of this effective temperature distribution remains unchanged even if a small spherical expansion is included. The BECF for the spherically expanding shells can be written in a direction-dependent, analytic form as given by Eq. (13) of ref. 2. Although this analytic form is too complicated to be repeated here, one arrives at a simpler expression after averaging over the direction of the relative momentum in the CMS of the source.

Here $q = |\Delta k|$ in the CMS and the radius parameter reads

$$R_{s,p} = R_p[(a(p) \tanh(a(p))^{-1} - \sinh(a(p)))^{-2}]^{1/2}.$$ (12)

The momentum dependence of the effective radius parameter $R_{s,p}$ is a direct consequence of the expansion of the source.

2.3 Spectra and Correlations for Longitudinally Expanding Systems

Systems which are dominantly expanding longitudinally appear to be the relevant models to high energy heavy ion reactions at CERN SPS energy region, $\sqrt{s} = 20$ GeV A. The detailed presentation and elaboration of these type of models is outside of the scope of the present contribution, we recommend refs. 4, 8, 20, 6, 7 for further details. However, we summarize here approximate results for the single-particle spectra:

$$\frac{dN}{2\pi m_tdm_tdy} \simeq \frac{g}{(2\pi \hbar)^3} m_t \cosh(y - \eta_s(p)) V_s(p) \exp(-m_t/T_L(y)),$$ (13)

$$\frac{dN}{2\pi m_tdm_tdy} \simeq \frac{g}{(2\pi \hbar)^3} m_t \cosh(y - \eta_s(p)) V_s(p) \exp\left(-\frac{(y - y_0)^2}{2\Delta y(m_t)^2}\right),$$ (14)

$$V_s(p) \approx V_0 (T/m_t)^{\alpha-1},$$ (15)

$$T_L(y) = \frac{T_s}{1 + a_T(y - y_0)^2}, \quad \text{and} \quad \Delta^2 y(m_t) = \Delta^2 \eta + T/m_t.$$ (16)

These relations indicate that the rapidity dependence of the temperature parameter $T_L(y)$ can be described with a new fit parameter $a_T$ (which was shown to be related to $\Delta \eta$, the total longitudinal extension of the particle emitting source, e.g. $a_T = 0$ for infinite systems). A more direct access to the longitudinal size of this expanding system is provided by the transverse mass dependence of the rapidity-width of the invariant momentum distribution, because $\Delta^2 y(m_t) \approx \Delta^2 \eta$ for $m_t >> T$.

The BECF-s for such systems can be written to the following form:

$$C(\Delta k, K) = 1 + \lambda_s(K) \exp\left(-R_{\text{side}}^2(K)Q_{\text{side}}^2 - R_{\text{out}}^2(K)Q_{\text{out}}^2 - R_{\text{long}}^2(K)Q_{\text{long}}^2\right) \times \exp\left(-R_{\text{olong}}^2(K)Q_{\text{out}}Q_{\text{long}}\right),$$ (17)
where the intercept parameter can be interpreted\(^2\) in the core-halo model, the side, out and longitudinal radius parameters as well as the out-long cross-term\(^6\) may in general depend on the mean momentum \(K\) in a complicated manner\(^3\). However, in some specific limiting cases\(^8\) this can be simplified as \(R_{\text{side}} \approx R_{\text{out}} \approx R_{\text{long}} \propto 1/\sqrt{m_t}\).

### 2.4 How to distinguish the three model-classes?

The following tests can be performed experimentally to check the hypothesis of Kopylov and Podgoretskii, Pratt and Bjorken or the model-class of longitudinally expanding, finite systems in a more detailed manner:

1. Measure the \(m_t\) dependence of the effective rapidity-width of the \(N(p)\) distribution at a fixed value of \(m_t\), and try to fit the result with Eq. (16). For static or spherically expanding shells, the parameters \(\Delta y(m_t)\) decrease to 0 as \(1/\sqrt{m_t}\), while for longitudinally expanding finite systems the large \(m_t\) limit is \(\Delta y(\infty) = \Delta \eta > 0\), the longitudinal size of the system in space-time rapidity.

2. Measure the rapidity dependence of the temperature parameter of the \(N(p)\) distribution at a fixed value of \(y\), and try to fit the result with Eq. (16). A slow drop of the effective temperature with increasing values of \(|y - y_0|\) results in the breakdown of the static fireball picture. Longitudinally very extended, expanding systems are predicted to have a Lorentzian effective temperature distribution, \(T_{\text{eff}}(y) = T_*/(1 + a_T(y - y_0)^2)\) where the parameter \(a_T\) carries information about the longitudinal extension of the source\(^8\).

3. Check if the parameters \(R_{KP}, R_P, \tau_P\) and \(\tau_{KP}\) of the Bose-Einstein correlation function of eq. (7) are independent or not of various values of \(K\). Static models predict \(R_{KP}(K) = \text{const}\) and \(\tau_{KP}(K) = \text{const}\), a deviation from this behavior results in the breakdown of the KP or any other static model. Spherically symmetric expanding shells also have a specific transverse momentum dependent \(R_{s_P}(K)\) parameter, decreasing with increasing values of \(|K|\) as given by eq. (12). Longitudinally expanding finite systems have momentum dependent longitudinal radius parameter, the transverse radius parameters may or may not be transverse mass dependent\(^4\),\(^8\).

Let us now focus on the interpretation of the intercept parameter of the BECF-s.

### 3 Core-halo model analysis of NA44 data

Having first performed a Monte Carlo simulation to justify the analysis technique to be used, we then analyze the Bose-Einstein correlation functions from NA44\(^{16,19}\).
3.1 Monte-Carlo test of the method

In a Monte-Carlo study, we simulate the actual and background distribution \(A(Q)\) and \(B(Q)\) of particle pairs. For the CERN experiment NA44, studying S+Pb collisions, the shape of \(B(Q)\) is given approximately by \(B(Q) = Q^3 e^{-(3.6Q^0.3)}\), which form reproduces the experimentally measured background distribution. This background distribution is peaked at 25 MeV similarly to the NA44 data. We have started with a core-halo model correlation function, assuming Gaussian source functions for the core and for the halo, choosing the core radius parameter to be \(R_c = 4\) fm. The radius parameter of the halo was taken to be \(R_h = 40\) fm and the core fraction was \(f_c = 0.7\). The actual distribution \(A(Q)\) was sampled according to the formula

\[
A(Q) = B(Q)[1 + f_c^2 \exp(-R_c^2 Q^2) + 2 f_c (1 - f_c) \exp(-0.5(R_c^2 + R_h^2)Q^2) + (1 - f_c)^2 \exp(-R_h^2 Q^2)]
\]

(18)

which corresponds to the full correlation function including correlations of \((c, c)\), \((c, h)\) and \((h, h)\) type of particle pairs. Here \(c\) refers to the core, which is assumed to be resolvable since \(R_c < h/Q_{bin}\), where the two-particle relative momentum resolution \(Q_{bin}\) is about the size of a bin (cca. 10 MeV for NA44). In the core-halo model, the halo (index \(h\)) is assumed to change on large length-scales, which are un-resolvable by the two-particle correlation measurements, \(R_h > h/Q_{min}\).

Having sampled the actual and background distributions of particle pairs in a Monte-Carlo simulation, the correlation function is now calculated in each bin of size \(Q_{bin}\) as \(C_2(Q) = \frac{A(Q)}{B(Q)}\). This form is fitted by the expression

\[
C_2(Q) = 1 + \lambda_* \exp(-R_*^2 Q^2).
\]

(19)

If the core-halo model is applicable, it predicts that \(\lambda_* = f_c^2\) and \(R_* = R_c\) after the fitting, i.e. the intercept can be utilized to measure the fraction of particles emitted by the core, and \(R_*\) the radius parameter will coincide with the radius parameter characterizing the core. Furthermore, if the assumptions of the core-halo model are satisfied by some data, then the fitted \(\lambda_*\) and \(R_*\) parameters should become independent of the fitted region, obtained by excluding more and more part of the correlation function at the lowest values of \(Q\), as long as the inequalities \(h/R_h < Q_{min} < h/R_c\) are satisfied by \(Q_{min}\). We chose the range from \(Q_{bin} = 10 \leq Q_{min} \leq 60\) MeV.

We have checked in the Monte-Carlo simulation if the effect of the halo on the deviation from a Gaussian form does really cancel or not from the simulation by increasing the value of \(Q_{min}\), the size of the low \(Q\) region excluded from the fit. We have found that indeed \(\lambda_* = f_c^2 \approx 0.5\) and \(R_* = R_c = 4.0\) fm was reproduced by the fitting within the statistical uncertainties even for as large values of the excluded region as \(Q_{min} = 60\) MeV, when already more then one half of the peak was removed from the correlation function, utilizing 300 K pairs. Armed with this experience, we then applied the method to the analysis of NA44 data for pions and kaons in various one-dimensional slices of the correlation functions at given values of \(K\).
3.2 Analysis of NA44 data

In order to extract values for $R$ and $f_c$ from data, we first fitted the side, out and longitudinal slices of the NA44 data on $S+Pb$ reactions at 200 AGeV for pions at low and high transverse mass as well as for kaons. Thus we obtained radius parameters and intercept parameters (with their errors, respectively), as a function of $Q_{min}$. We determined these fit parameters in the $Q_{min} = 0 − 60$ MeV region. The fitted $R_c(Q_{min})$ and $\lambda_*(Q_{min})$ parameters up to $Q_{min} = 40$ MeV/c were (meta)-fitted with a $Q_{min}$ independent constant. Our findings are summarized in Table 1. More details of the study shall be reported elsewhere.

| parameter | high $p_t \pi^+$ | low $p_t \pi^+$ | $K^+$ |
|-----------|------------------|-----------------|-------|
| $R_c$ out (fm) | 2.92 ± .13 | 4.29 ± .13 | 2.54 ± .18 |
| $R_c$ side (fm) | 2.90 ± .18 | 4.24 ± .26 | 2.22 ± .19 |
| $R_c$ long (fm) | 3.31 ± .16 | 5.43 ± .30 | 2.67 ± .22 |
| $f_c$ out | .704 ± .012 | .725 ± .011 | .802 ± .027 |
| $f_c$ side | .735 ± .021 | .647 ± .021 | .736 ± .033 |
| $f_c$ long | .738 ± .014 | .724 ± .015 | .789 ± .029 |

Table 1: Extracted values for $R_c$ and $f_c$ from NA44 S+Pb data.

In ref. the halo has been interpreted as being created essentially by the decay products of the $\omega$, $\eta$ and $\eta'$ resonances. The measured core fractions of Table 1 are indeed in the vicinity of the core fractions of Fritiof and RQMD if $\omega$, $\eta$ and $\eta'$ are taken as unresolved long-lived resonances, as indicated by Table 2. Thus we do not find a complicated, non-Gaussian structure of the measured BECF in $S+Pb$ reactions at CERN SPS, in contrast to some earlier expectations which argued that resonance decays, coupled to a freeze-out hyper-surface obtained from (hydro)dynamic evolution may lead to resolvable deviations from a Gaussian structure. Such a deviation has been predicted by detailed simulations of resonance decay effects by calculations with SPACER, HYLANDER, and recently by H. Heiselberg. Assuming that such non-Gaussian structures are created by the effect of $\eta, \eta'$ and $\omega$ resonance decays to pions, but that they are experimentally not yet resolvable in the NA44 $S+Pb \rightarrow 2\pi + X$ experiment, we obtain a consistent interpretation of the data and a physical interpretation of the measured intercept and radius parameters.
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