A Modified Deutsch-Anderson Equation for Predicting the Nanoparticle Collection Efficiency of Electrostatic Precipitators

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ABSTRACT

It is of great importance to mitigate nanoparticle exposure in the environment by using effective control devices. For controlling nanoparticles with the particle diameter \(d_p<100\) nm in the air environment, electrostatic precipitators (ESPs) are widely used in many industries because of their low pressure drop and high removal efficiency. However, existing models cannot predict the nanoparticle collection efficiency very well. This is due to the partial charging effect, when some nanoparticles are not charged but penetrate through the ESPs, resulting in a decrease in the collection efficiency. In this study, a modified Deutsch-Anderson model was developed to predict the nanoparticle collection efficiency \(\eta\) (%), as well as that for particles with \(d_p>100\) nm. The present model is

\[
\eta(\%) = \left[1 - \exp(-A(N_{De})^B) + C(N_{De}) - (1 - \alpha)\right] \times 100\%,
\]

where \(A\), \(B\), and \(C\) are regression coefficients, \(N_{De}\) is the modified Deutsch number, and \(\alpha\) is the partial charging factor. Good agreement was obtained between the present predictions and experimental data published in the literature. It is expected that the modified equation can facilitate the design and scale-up of the ESPs for controlling nanoparticle emissions, as well as particle in all size ranges.

Keywords: Deutsch-Anderson equation; Particle control; Nanoparticles; Electrostatic precipitator.

INTRODUCTION

Nanotechnologies are developed and commercialized rapidly in many consumer products and environment-related applications including water, indoor and outdoor air, and soil treatment (Litter et al., 2012). However, in the processes of manufacturing and handling of nano-materials or during the life cycle of nanoproducts, nanoparticles may be released to the environment and pose adverse effects on human health, environment and organisms (Tsai and Pui, 2009; Tsai et al., 2009, 2011). Manufactured nanoparticles such as nano-SiO\(_2\), nano-TiO\(_2\), nano-CuO, nano-Ag, nano-Al\(_2\)O\(_3\), and nano-sized diamond were shown to inhabit the activity, growth and reproduction of aquatic organisms and microorganisms (Xu et al., 2010; Kumar et al., 2011; Li et al. 2011; Mendonça et al., 2011; Zhao et al., 2011). Therefore, it is very important to mitigate nanoparticle exposure in the environment by using effective control devices.

Because of the advantages of low pressure drop and high particle collection efficiency, electrostatic precipitators (ESPs) are widely used in the industry to remove suspended nanoparticles and micro-sized particles in the exhaust gas (Ruttanachot et al., 2011). Fig. 1 shows the schematic diagram of a typical wire-in-plate ESPs, in which aerosol particles are charged by diffusion and field charging mechanisms, drifted by electric field toward grounded collection plates and are removed. To design high efficiency ESPs, several models were derived to predict particle collection efficiency, such as the Deutsch-Anderson equation (Hinds, 1999), the models of Cooperman (1971), Leonard et al. (1980), Zhao and Zhang (1994), Kim et al. (2001), Zhang et al. (2002), and Ortiz et al. (2010).

Previous mathematical models mentioned above are mainly used to predict the collection efficiency for particles with \(d_p>100\) nm. For nanoparticles, the mathematical model is still not available. This study aims at developing a modified Deutsch-Anderson equation applicable for nanoparticles and micro-sized particles. The V-I curve, electric field strength and amount of particle charge calculated theoretically are first compared with the results in previous researches. In the present model, \(N_{De}\) is selected as the dimensionless variable because it contains three important parameters including the particle migration velocity, the area of the collection plates and air flow rate, which all play major roles in...
determining the particle collection efficiency of ESPs (Richards, 1995). The present model is then validated by using the experimental data of for nanoparticles, and the experimental data of Salcedo and Munz (1978) and Riehle and Löffler (1990, 1993) for particles larger than 100 nm in diameter.

**REVIEW OF COLLECTION EFFICIENCY EQUATIONS**

Among the existing mathematical models described in introduction section, the Dutsch-Anderson equation was derived theoretically based on many assumptions, including the constant particle concentration profile at any cross section normal to the gas flow, the constant gas velocity, the constant and uniform distribution of electric field and ion concentration, the constant migration velocity of particle toward the collecting electrodes and the neglect of re-entrainment of collected particles from the collecting electrodes (2004). However, the Deutsch-Anderson equation often underestimates collection efficiencies because actual operation conditions are different from the ideal assumptions (Yoo et al., 1997; Kim and Lee, 1999; Park and Chun, 2002).

In order to predict particle collection efficiency accurately, the Deutsch-Anderson equations was modified by Matts and Öhnfeld (1964) as:

\[
\eta(\%) = [1 - \exp(-N_{De,1})] \times 100\% ,
\]

where \(N_{De,1}\) is the original Deutsch number which is defined as:

\[
N_{De,1} = w_k \frac{A}{Q}
\]

in which \(w_k\) is the effective migration velocity of particles, \(k\) is a constant ranging from 0.4 to 0.6, \(A\) is the area of the collecting plates (m), and \(Q\) is the air flow rate (m³/s).

Cooperman (1971) took into account longitudinal turbulent mixing effects and re-entrainment and proposed the following equation:

\[
\eta(\%) = \left[1 - \exp\left(\frac{u_{ave}L}{2D_x} - \sqrt{\left(\frac{u_{ave}L}{2D_x}\right)^2 + (1 - R)Pe\frac{L}{s_y}}\right)\right] \times 100\%
\]

where \(L\) is the length of collection plate (m), \(u_{ave}\) is the average air velocity (m/s), \(D_x\) is the longitudinal particle diffusion coefficient (m²/s), \(R\) is the re-entrainment effect factor, \(Pe\) is the Peclet number, \(s_y\) is the half of wire to wire spacing (m), and \(s_x\) is the wire to plate spacing (m).

In Eq. (1), \(w_k\) was found to be lower than the experimental data of Wiggers (1982) in Riehle and Löffler (1992), who also suggested that Eq. (1) can’t be used to predict the effective migration velocities. Eq. (3) is not suitable for the design of the ESPs since high longitudinal turbulent mixing only occurs in the extreme condition (Kim et al., 2001) and a general method to estimate the re-entrainment factor and the particle diffusivity is absent (Kim and Lee, 1999).

Several equations for the collection efficiency were derived by solving the particle convection-diffusion equation (Leonard et al., 1980; Zhao and Zhan, 1994; Kim et al., 2001; Zhang et al., 2002). The best agreement was found between the experimental data of Salcedo and Munz (1987) for particles with the geometric mean diameter \(d_g = 13.7\, \mu\text{m}\) and predictions by the following equation of Zhang et al. (2002):

\[
\eta(\%) = [1 - \exp(-(F \times N_{De}))] \times 100\%
\]

where \(N_{De}\) is the modified Deutsch number which is defined as:

\[
N_{De} = \frac{wL}{s_y u_{ave}}
\]

This modified Deutsch number is also used in the present model. In Eq. (5), \(w\) is the particle migration velocity (m/s), and \(F\) is a dimensionless factor, which is a function of \(Pe, u_{ave}\) and \(w\) (m/s). Although applicable for large micro-sized particles, Eq. (4) has not been validated by...
experimental data of submicron particles.

For particles in the size range from 150 nm to 10 μm, the equation by Zhao and Zhang (1994) as shown below was found to predict the experimental collection efficiency data of Kim and Lee (1999) more accurate than the model of Leonard et al. (1980):

\[
\eta(\%) = \left[1 - \frac{\sqrt{\frac{Pe}{2s_y \pi N_{De}}} \times \int_0^s \exp \left(-\frac{Pe}{4N_{De}} \left(\frac{y}{s_y} - N_{De}\right)^2\right) dy \right] \times 100\%
\]  

(6)

Recently, a simple fitted model was developed with the aid of regression analysis for scaling-up, design and improvement of ESPs (Ortiz et al., 2010). With the consideration of several dimensionless parameters, including turbulence factor, electro-hydrodynamics factor, and \( N_{De} \), the following collection efficiency equation was found to have the simplest form and the best fit to the experimental data of Ortiz et al. (2010) for particles with the average diameter of 12 μm:

\[
\eta(\%) = [1 - 1.042 \times \exp(-N_{De}^{0.612})] \times 100\%
\]  

(7)

It is to be noted that the particle charging in this equation is estimated by using Pauthenier’s saturation charging equation, which may not be applicable in the nano-sized range because nanoparticles are partially charged (Yoo et al., 1997; Huang and Chen, 2002). Besides, the electric field is calculated as \( E_{ave} = V_0/s_y \) (\( E_{ave} \): average electric field (V/m), \( V_0 \): applied voltage (V)), which is too simplified as the electric field in the wire-in-plate ESPs is non-uniform. Therefore, the applicability of the model of Ortiz et al. (2010) for nanoparticle collection efficiency requires further investigation.

**PRESENT THEORETICAL MODEL**

**Electric Field and Ion Current**

Dimensions and operation conditions of each ESP in previous researches are shown in Table 1, in which tested particles are MgO (Magnesium Oxide) with \( \varepsilon \) (relative permittivity) of 3.4 (Salcedo and Munz, 1978), limestone with \( \varepsilon \) of 2.7 (Riehle and Löfler, 1990, 1993), sucrose with \( \varepsilon \) of 3.0 (Huang and Chen, 2002) and Ag with \( \varepsilon \) of \( \infty \) (Lin et al., 2010a). The experimental methods were described in detail in Salcedo and Munz (1978) and Riehle and Löfler (1990, 1993) for micro-sized particles, and Huang and Chen (2002) and Lin et al. (2010a) for nanoparticles.

The vertical electric field strength distribution from the wire to the collection electrode, \( E_c \) (V/m) is calculated by Cooperman’s (1981) equation as follows:

\[
E_c(y) = \sqrt{\left(\frac{E_{ave}^2}{2L} \right) + \frac{2J_p}{\varepsilon_0 \varepsilon_r} y} - E_{ave}
\]  

(8)

where \( \varepsilon_0 \) is the permittivity of a vacuum C²/N-m², \( Z_i \) is the ion mobility (m²/V-s), \( J_p \) is the average current density at the plate (A/m²), and \( E_{ave} \) is the average electric field strength at the collection plate (V/m). In Eq. (8), particle space charge effect is neglected since the ion number concentration is much higher than particle number concentration in ESPs in Lin et al. (2010a) and Huang and Chen (2002). The average electric field strength is calculated as:

\[
E_{ave} = \pi r_c E_c \frac{r_c}{2s_y}
\]  

(9)

where \( r_c \) is the radius of the discharge wire, \( s_y \) is the half of wire to wire spacing (m), \( E_c \) is the corona initiating electric field (V/m), which is:

\[
E_c = 3 \times 10^5 f \left(\frac{T_0 P}{TP_0} + 0.03 \frac{TP_0}{TP_r}ight)
\]  

(10)

where \( T \) and \( P \) is the operational temperature (K) and pressure (atm), respectively, \( T_0 = 293 \) (K), \( P_0 = 1 \) (atm), \( f \) is the wire roughness factor. To calculate \( J_p \), the following analytical equation derived by Cooperman (1981) is used:

\[
J_p = \frac{e_0 Z_i}{16s_y} \left(\frac{3}{2} \sqrt{\gamma^2 + 192(V_0 - V_c)(s_y E_f)}\right)
\]  

(11)

where

\[
\gamma = 9(V_0 - V_c + s_y E_f)^2 - 12(s_y E_f)^2
\]  

(12)

\[
E_f = \frac{\pi V_c}{2s_y \ln \frac{r_{eff}}{r_c}}
\]  

(13)

\[
V_c = r_c E_c \ln \frac{r_{eff}}{r_c}
\]  

(14)

**Table 1.** Dimensions and operation conditions of the ESPs in previous researches (Salcedo and Munz, 1978; Riehle and Löfler, 1990, 1993; Huang and Chen, 2002; Lin and Tsai, 2010).

| \( r_w \) (mm) | no. of wires | L (m) | H (m) | \( s_y \) (m) | \( s_y \) (m) | \( V_0 \) (kV) | \( u_{ave} \) (m/s) | References |
|---------------|--------------|-------|-------|---------------|---------------|---------------|-----------------|------------|
| 0.68          | NA           | 0.38  | 2.53  | 0.076         | 0.081         | 30.0, 38.0    | 5.0             | Salcedo and Munz (1978) |
| 0.70          | 5            | 0.28  | NA    | 0.035         | 0.0705        | 20.0, 35.0    | 0.71, 1.41      | Riehle and Löfler (1990) |
| 0.5           | 5            | 0.40  | NA    | 0.05          | 0.1           | 50.0          | 1.0             | Riehle and Löfler (1993) |
| 0.15          | 3            | 0.168 | 0.076 | 0.021         | 0.06          | 16.5–21.5     | 0.183           | Huang and Chen (2002)    |
| 0.05          | 3            | 0.048 | 0.009 | 0.008         | 0.0045        | 3.8–3.9       | 0.123           | Lin and Tsai (2010)      |
In the above equations, $V_c$ is the corona onset voltage (V), $r_{eff}$ is the equivalent cylinder radius (m). After obtaining $J_p$, the total current $I$ (A) per discharge wire is calculated as (McDonald et al., 1977):

$$I = J_p A s d$$

(16)

where $l$ is the wire length (m). The electric field strength at $x$ direction, $E_x$, is assumed to be zero. Therefore, $E_{ave}$ in the ESP is given by:

$$E_{ave} = \frac{\sum_{y=0}^{s_y} E_x (y) \times y}{\sum_{y=0}^{s_y} y}$$

(17)

After $J_p$, $Z_i$ and $E_{ave}$ are obtained, the average ion concentration ($N_i$, #/m$^3$) can then be calculated as:

$$N_i = \frac{J_p}{Z_i E_{ave} e}$$

(18)

**Particle Charging and Collection Efficiency**

The total number of elemental units of particle charges as function of time, $n_p(t)$, is calculated by using Fuch's equation (Fuchs, 1947) for diffusion charging ($n_{diff}(t)$) and Pauthenier and Moreau-Hanot (1932) equation for field charging ($n_{field}(t)$) as:

$$n_p(t) = n_{diff}(t) + n_{field}(t)$$

(19)

where

$$n_{diff}(t) = \frac{d}{2T} \ln \left[ 1 + \frac{\pi K_c d \tilde{c} e^2 N_i}{2kT} \right]$$

(20)

$$n_{field}(t) = \frac{3e}{\varepsilon + 2} \left[ \frac{E_{ave}}{4K_c e} \right] \left( 1 + \pi K_c e Z_i N_i \right)$$

(21)

In the above equations, $k$ is the Boltzmann constant ($1.38 \times 10^{-23}$ J/K), $K_c = 9.0 \times 10^9$ (N-m/C$^2$), $\tau$ is the mean thermal speed of the ions (m/s), $e$ is the elementary charge (C), $N_i$ is the ion concentration (#/m$^3$), $t$ is the residence time of particles in ESPs (s).

However, Eq. (19) underestimates the charge of nanoparticles as compared to that calculated by Lawless model (Lawless, 1996) as shown in Table 2. Therefore, in order to predict nanoparticle charge accurately, the following modified equation for particle charge, $n'_p(t)$, is derived by fitting Eq. (19) to the numerical values of Lawless model (Lawless, 1996):

$$n'_p(t) = n_{diff}(t) \times \exp \left[ a \times n_{diff}(t)^b + cn_{diff}(t) + d \right] + n_{field}(t)$$

(22)

where $a = 1.91588$, $b = -0.1425$, $c = 1.296 \times 10^{-5}$, and $d = -1.2671$. Table 2 shows particle charges predicted by Eq. (22), which are in good agreement with those by Lawless model (Lawless, 1996) with less than 1.0% error. Therefore, Eq. (22) is used to calculate the charge of nanoparticles in the present model.

Considering the important parameter, $N_{De}$, and the partial charging effect, the following semi-empirical equation is

$$r_{eff} = \frac{4s}{\pi}$$

for $s \leq 2.0$

(15)

Table 2. Comparison of different models for particle charging by field, diffusion, and combined charging at $N = 10^{13}$ s/m$^3$, $\varepsilon = 5.1$.

| Particle diameter ($\mu$m) | Number of elemental units of charged acquired | Diffusion charging | Field charging | Combined charging |
|---------------------------|---------------------------------------------|-------------------|---------------|------------------|
|                           | Eq. 15.24 in Hinds (1999)                   | Numerical solution (Lawless, 1996) | The first term of Eq. (22) | Eq. 15.25 in Hinds (1999) | Numerical solution (Lawless, 1996) | Eq. (22) |
| 0.002                     | 0.0006                                      | 0.08              | 0.09          | 0.0007          | 0.08            | 0.09          |
| 0.004                     | 0.002                                       | 0.16              | 0.17          | 0.003           | 0.17            | 0.17          |
| 0.006                     | 0.004                                       | 0.24              | 0.25          | 0.006           | 0.25            | 0.25          |
| 0.008                     | 0.007                                       | 0.33              | 0.32          | 0.01            | 0.34            | 0.34          |
| 0.01                      | 0.10                                        | 0.41              | 0.40          | 0.02            | 0.43            | 0.42          |
| 0.012                     | 0.13                                        | 0.49              | 0.49          | 0.023           | 0.51            | 0.51          |
| 0.014                     | 0.17                                        | 0.57              | 0.57          | 0.032           | 0.61            | 0.60          |
| 0.016                     | 0.21                                        | 0.65              | 0.65          | 0.042           | 0.70            | 0.70          |
| 0.018                     | 0.25                                        | 0.73              | 0.73          | 0.05            | 0.79            | 0.80          |
| 0.02                      | 0.29                                        | 0.82              | 0.82          | 0.07            | 0.88            | 0.87          |
| 0.04                      | 0.79                                        | 1.63              | 1.62          | 0.26            | 1.9             | 1.88          |
| 0.1                       | 2.7                                         | 4.1               | 4.1           | 1.63            | 5.6             | 5.7           |
| 0.4                       | 15.7                                        | 16.3              | 16.1          | 25.9            | 40              | 42            |
| 1                         | 47                                          | 41                | 40.2          | 162             | 162             | 202           |
| 4                         | 237                                         | 163               | 161.4         | 2580            | 2680            | 2741.4        |
| 10                        | 673                                         | 407               | 408.0         | 16200           | 16540           | 16608         |
proposed to calculate the particle collection efficiency of ESPs:

\[
\eta(\%) = \left[1 - \exp\left(-A(N_{De})^B + C(N_{De}) - (1 - \alpha)\right)\right] \times 100\%
\]

\[
\alpha = \text{MIN}\left(1, n'_{p}(t)\right)
\]  

where \(\alpha\) is a partial charging effect factor, which is equal to 1 for particle charge greater than or equal to 1.0 (or when \(n'_{p}(t) \geq 1.0\)), and is equal to \(n'_{p}(t)\) for \(n'_{p}(t) < 1.0\), or when particles are partially charged. A, B and C are regression coefficients obtained by fitting Eq. (23) to the experimental data of Huang and Chen (2002) and Lin et al. (2010a) for nanoparticle, and in Riehle and Löfler (1990, 1993) for particles with \(d_{p} > 100\) nm.

In the above model, only the most important parameter \(N_{De}\) is considered. The parameters, such as particle re-entrainment \(R\), turbulence \(D_{t}\) and electro-hydrodynamics flow factors \(N_{EH}\), are not considered. This is based on the dimensional analysis in Ortiz et al. (2010) which showed the correlation between aerosol penetration and the parameters \(D_{t}\), \(N_{EH}\) and \(R\) was not significant.

**Calculation Procedure**

Fig. 2 shows the calculation procedure for the average electric field and ion concentration, particle charging, and particle collection in the present model. \(J_{p}\) is first calculated by using Eq. (11)–(15). After obtaining \(J_{p}\), \(E(y)\) and \(I\) are calculated by Eq. (8) and (16), respectively. Finally, \(E_{ave}\) and \(N_{i}\) are obtained by Eq. (17) and (18), respectively.

To calculate particle charge \(n'_{p}(t)\), Eq. (22) is used. Eq. (19) is first fitted to the numerical values of Lawless model (Lawless, 1996) shown in Table 2 to obtain regression coefficients of a, b, c and d. Eq. (22) with known a, b, c and d is then used to calculate particle charge in ESPs.

After obtaining the average electric field, ion concentration and particle charge, nanoparticle collection efficiency in ESPs is calculated by using Eq. (23) with regression coefficients of A, B and C, which are obtained by fitting Eq. (23) to the experimental data of Huang and Chen (2002) at the air flow rate of 50 L/min for \(N_{De} < 10\). For \(N_{De} \geq 10\), A, B and C are obtained by fitting Eq. (23) to the experimental data of Lin et al. (2010a) at the applied voltage of 3.9 kV and the air flow rate of 5 L/min. For particles with \(d_{p} > 100\) nm, the regression coefficients are obtained by fitting Eq. (23) to the experimental data in Riehle and Löfler (1990 and 1993) for \(N_{De} < 0.15\) and 0.15 \(\leq N_{De} \leq 2.20\), respectively.

**RESULTS AND DISCUSSION**

**Validation of the V-I Curve and Electric Field Strength**

Fig. 3 shows the comparison of V-I curve between the present theoretical values and the experimental results of Penny and Matick (1960) and Cooperman (1981). Reasonable agreement is obtained when the ion mobility of \(1.5 \times 10^{-4}\) \(\text{m}^{2}/\text{V} \cdot \text{s}\) (Hinds, 1999) is used for negative ions and \(1.4 \times 10^{-4}\) \(\text{m}^{2}/\text{V} \cdot \text{s}\) (Adachi et al., 1985) is used for positive ions. The theoretical values of \(J_{p}\), which are calculated to be

\[
J_{p}(\text{Eq. (1)}-(\text{15}))
\]

\[
E(y)(\text{Eq. (8)})\text{,} J(\text{Eq. (16))}
\]

\[
E_{ave}(\text{Eq. (17)})\text{,} N_{i}(\text{Eq. (18))}
\]

\[
n'_{p}(t)(\text{Eq. (22))}
\]

\[
N_{De}(\text{Eq. (5))}
\]

\[
A\text{, B and C for } N_{De} < 10 \text{ (Eq. (23)) fitted to the experimental data of Huang and Chen (2002) at the air flow rate of 50 L/min)}
\]

\[
A\text{, B and C for } N_{De} \geq 10 \text{ (Eq. (23)) fitted to the experimental data of Lin et al. (2010) at the applied voltage of 3.9 kV and the air flow rate of 5 L/min)}
\]

\[
A\text{, B and C for } N_{De} \geq 10 \text{ (Eq. (23)) fitted to the experimental data of Riehle and Löffler (1990))}
\]

\[
A\text{, B and C for } N_{De} \geq 10 \text{ (Eq. (23)) fitted to the experimental data in Riehle and Löffler (1993))}
\]

\[
\eta(\%)
\]

**Fig. 2. Flow chart for the calculation procedure of particle collection efficiency in ESPs.**
1.05 mA/m² at 38 kV and 0.388 mA/m² at 30 kV, also match with the experimental data of Salcedo and Munz (1987) of 1.28 and 0.387 mA/m², respectively. But there are no experimental data available in Riehle and Löffler (1993) for comparing with the theoretical value of $J_p$ of 7.17 mA/m² at the applied voltage of -50 kV. The characteristics of the V-I curve in Huang and Chen (2002) and Lin et al. (2010a) was described in Lin et al. (2010b).

Fig. 4 shows the comparison of electric field strength distribution between theoretical values and the numerical results in Talaie et al. (2001). When $r_w=1.0$ mm, $s_x = 0.1$ m, $s_y = 0.1$ m, and $V_0 = 80$ kV, analytical solutions calculated by Eq. (17) along a vertical line from the wire to the plate are in good agreement with numerical results with a deviation from 0.05% to 14.6% in the $y$ direction. The $E_{ave}$ predicted by Eq. (17) is $5.79 \times 10^5$ V/m with a deviation of 7.8% as compared to the numerical value of $6.22 \times 10^5$ V/m. For the electric field strength along the perpendicular direction from the wire to the plate, comparison between the numerical $E_{ave}$ of $6.49 \times 10^5$ V/m and theoretical values shows reasonable agreement with a deviation of 10.8%. It is noted that for the $E_{ave}$ of $8 \times 10^5$ V/m, which is calculated by using the simple equation of $E_{ave} = V_0/s_y$ (Kim and Lee, 1999), more deviation of 18.9–22.3% was found as compared to the numerical results calculated by Talaie et al. (2001). Therefore, Eq. (17) is more appropriate for predicting the average electric field strength in non-uniform wire-in-plate ESPs.

**Comparing Particle Collection Efficiencies in Nano-Sized Range**

The present model was first used to predict the collection efficiencies for nanoparticles with the diameter of $d_p \leq 100$ nm. Fig. 5 shows the comparison of particle collection efficiency in the single-stage wire-in-plate dry ESP of Huang and Chen (2002) between the present predictions and the experimental data of Huang and Chen (2002) at the applied voltage of 15.5 kV.

The present model was first used to predict the collection efficiencies for nanoparticles with the diameter of $d_p \leq 100$ nm. Fig. 5 shows the comparison of particle collection efficiency in the single-stage wire-in-plate dry ESP of Huang and Chen (2002) between the present predictions and the experimental data at a fixed applied voltage of –15.5 kV and the air flow rate of 50–150 L/min. The regression coefficients of 1.4018 for $A$, 0.7601 for $B$, and –0.0059 for $C$ are derived by fitting Eq. (23) to the experimental data at the air flow rate of 50 L/min. Eq. (23) was then used to calculate particle collection efficiencies under different operation conditions and then compared with experimental
data. When $N_{N_{\text{th}}}$ are in the range from 0.32 to 13.5 (for particles with $6.79 \leq d_p \leq 88.3$ nm), the predictions agree reasonably with experimental data with a deviation of 0.03–17.8%, as shown in Fig. 5.

Fig. 6 shows the comparison of particle collection efficiency between the present predictions and the experimental data in Huang and Chen (2002) at a fixed air flow rate of 100 L/min and the applied voltage of –15.5 to –21.5 kV. Good agreement is also obtained with deviation of 0.10–16.7% when Deutsch number is in the range of 0.41–9.74 (for particles with $8.83 \leq d_p \leq 97.6$ nm).

When $N_{N_{\text{th}}}$ is in the range of 10.0–17.9 (for particles with $6.89 \leq d_p \leq 9.18$ nm), comparison between the present predictions and experimental data shows a large deviation of over 20% (data not shown). This result suggests that when $N_{N_{\text{th}}}$ is higher than 10.0, the original regression coefficients in the present model are not appropriate for predicting the nanoparticle collection efficiencies. Therefore, for $N_{N_{\text{th}}} \geq 10.0$, new regression coefficients of $3.28 \times 10^{-3}$ for $A$, $7.113$ for $B$ and $-8.51 \times 10^{-4}$ for $C$ are obtained by fitting Eq. (23) to the experimental data of Lin et al. (2010a) at the applied voltage of 3.9 kV and the air flow rate of 5 L/min (Fig. 7). As shown in Fig. 6, better agreement between the predictions and experimental data for the particles in the diameter of $6.86 \leq d_p \leq 9.18$ nm ($10.0 \leq N_{N_{\text{th}}} \leq 17.9$) is obtained with the deviation of 6.84–16.09% at the applied voltage of −18.0 to −21.5 kV.

For particles in the diameter of $d_p \leq 15.0$ nm, partial charging was found to cause the reduction of particle collection efficiencies in the dry ESP of Huang and Chen (2002). The partial charging effect on nanoparticle collection efficiencies can be minimized by increasing particle retention time or applied voltage in ESPs. For example, when the air flow rate is decreased from 150 to 50 L/min, the number of charge of 7 nm particle is increased from 0.72 to 0.81, resulting in an increase of collection efficiency from 68.5 to 799%. By increasing the applied voltage from −15.5 to −16.0 kV, collection efficiency is calculated to increase from 55.9–65.9% with an increasing particle charge from 0.65–0.71.

Fig. 7 shows the comparison of particle collection efficiency between the present predictions and experimental data obtained by using the single-stage wire-in-plate wet ESP (Lin et al., 2010). When the applied voltage and air flow rate is +38.0–+39.0 kV and 5 L/min, respectively, reasonable agreement for the particles with $6.98 \leq d_p \leq 93.1$ nm ($10.0 \leq N_{N_{\text{th}}} \leq 17.9$) is obtained with the deviation of 0.0–23.7%. The partial charging is found to cause reduction of the collection efficiency for silver particles with $d_p \leq 20$ nm. Again, reduction of the partial charging effect on particle collection efficiency can be achieved by increasing the applied voltage in wet ESPs. Taking 10.0 nm particles for example, when the applied voltage is increased from +38.0 to +39.0 kV, collection efficiency increases from 87.4.4 to 91.0% with increasing particle charge from 0.90 to 0.96. That is, the present model can be used to assist the design of ESPs for nanoparticle control with high collection efficiency. In addition to nanoparticle control, ESPs are used frequently as nanoparticle samplers which must have very high collection efficiencies. Efforts were made to enhance the unipolar charging efficiency of nanoparticles so that the collection efficiency can be improved (Tsai et al., 2008; Aliat et al., 2008, 2009; Tsai et al., 2010; Chien et al., 2011).

**Comparing Particle Collection Efficiencies for Particles Larger than 100 nm**

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**Fig. 6.** Comparison of particle collection efficiency in the single-stage wire-in-plate dry ESP between the present predictions and the experimental data in Huang and Chen (2002) at the air flow rate of 100 L/min.

**Fig. 7.** Comparison of particle collection efficiency in the single-stage wire-in-plaque wet ESP (Lin et al., 2010a) between the present predictions and the experimental data at the air flow rate of 5 L/min.
The present model was further used to predict the collection efficiencies for particles with \( d_p > 100 \) nm. However, the predictions disagreed with the experimental data in Riehle and Löffler (1993) when the regression coefficients for nanoparticles in the previous section were used. Therefore, new groups of regression coefficients are derived. By fitting the Eq. (23) to the experimental data in Riehle and Löffler (1990, 1993), 2.273 for \( A \), 0.471 for \( B \), and 0.0168 for \( C \) are obtained for \( 0.15 \leq N_{De} \leq 2.20 \), and 0.0023 for \( A \), –0.5058 for \( B \), and 3.8389 for \( C \) are obtained for \( N_{De} < 0.15 \). The fitted curve is shown in Fig. 8.

Fig. 9 shows the comparison of particle collection efficiency between predictions and the experimental data of Riehle and Löffler (1990). When the applied voltage is 35 kV, air velocities are 0.71 and 1.41 m/s, the present predictions reasonably match with experimental data with the deviation from 3.21–11.6% and 0.7–5.88%, respectively. Good agreement is also obtained at the applied voltage of 20 kV and the air velocity of 1.41 m/s with the deviation from 0.85–6.64%. Fig. 10 shows the comparison of particle collection efficiency between predictions and the experimental data in Salcedo and Munz (1978). Reasonable agreement is obtained with slight over-estimation of the experimental data by 4.65–20.9% and 0.65–15.8% at the applied voltage of 38 kV and 30 kV, respectively.

CONCLUSIONS

This study developed mathematical models to predict the electric field strength the number of particle charge, and a modified Deutsch-Andersen equation for the particle collection efficiency in ESPs. The present model for the particle collection efficiency is

\[
\eta(\%) = [1 - \exp(-A(N_{De})^B) + C(N_{De}) - (1 - \alpha)] \times 100\%,
\]

where \( A, B, \) and \( C \) are regression coefficients, \( N_{De} \) is the modified Deutsch number, and \( \alpha \) is the partial charging factor. For nanoparticles, the regression coefficients in the modified Deutsch-Andersen equation are \( A = 1.4018 \), \( B = 0.7601 \) and \( C = -0.0059 \) for \( N_{De} < 10 \), and \( A = 3.28 \times 10^{-7} \), \( B = 7.113 \), \( C = -8.51 \times 10^{-4} \) for \( N_{De} \geq 10 \). The comparison of predicted collection efficiencies and the experimental data of the dry ESP in previous literature shows reasonable agreement with a deviation smaller than 20% for particles with \( 6 \leq d_p \leq 100 \) nm.

For particles with \( d_p > 100 \) nm, the regression
coefficients are $A = 0.0023$, $B = -0.5058$, and $C = 3.8389$ when $N_{De} < 0.15$, $A = 2.273$, $B = 0.471$ and $C = 0.0168$ when $0.15 \leq N_{De} \leq 2.20$. Good agreement was also obtained with a deviation smaller than 21% for the comparison between the predictions and the experimental data in the literature.

It is expected that the modified equation can facilitate the design and scale-up of the ESPs for controlling emissions of nanoparticles as well as particle in all size ranges. Since the present model provides a simple method to calculate the electric field strength, the number nanoparticle charge and nanoparticle control efficiency, the model also enables the design of efficient electrostatic nanoparticle samplers for sampling and characterization of nanoparticles.

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