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Research Article

Design, Optimization, and Experiment on a Bioinspired Jumping Robot with a Six-Bar Leg Mechanism Based on Jumping Stability

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A jumping leg with one degree of freedom (DOF) is characterized by high rigidity and simple control. However, robots are prone to motion failure because they might tip over during the jumping process due to reduced mechanism flexibility. Mechanism design, configuration optimization, and experimentation were conducted in this study to achieve jumping stability for a bioinspired robot. With locusts as the imitated object, a one-DOF jumping leg mechanism was designed taking Stephenson-type six-bar mechanism as reference, and kinematic and dynamic models were established. The rotation angle of the trunk and the total inertia moment were used as stability criteria, and the sensitivity of different links to the target was analyzed in detail. With high-sensitivity link lengths as the optimization parameters, a configuration optimization method based on the particle swarm optimization algorithm was proposed in consideration of the different constraint conditions of the jumping leg mechanism. Optimization results show that this method can considerably improve optimization efficiency. A prototype of the robot was developed, and the experiments showed that the optimized trunk rotation angle and total inertia moment were within a small range and can thus meet the requirements of jumping stability. This work provides a reference for the design of jumping and legged robots.

1. Introduction

Bioinspired jumping robots have good environmental adaptability and can overcome obstacles that are several or even ten times larger than themselves. They have become a hotspot in research on bionic robots [1–5]. Many researchers have designed various jumping robots that achieve good jumping performance by using creatures with jumping ability, such as kangaroos [6, 7], spiders [8], locusts [9], and froghoppers [10, 11], as research objects [12, 13]. The leg mechanism is the key to the design of bioinspired jumping robots, and it is the main structure through which a robot performs takeoff. Bionic series [14] and flexible [15] mechanisms have been applied to the design of the leg mechanism of jumping robots. The former is driven by springs or motors, and the latter stores and releases energy through elastic deformation of materials. The one-degree-of-freedom (1DOF) mechanism has also been used in the design of the leg mechanism of jumping robots. Compared with bionic series and flexible mechanisms, the 1DOF mechanism has high rigidity and simple control and thus exhibits a good application prospect [16, 17]. Many studies have been conducted on the four-bar jumping mechanism. For example, Zhang et al. [18] developed two 1DOF jumping modes, namely, four-bar jumping leg mode and slider-crank jumping leg mode. The jumping performance of the two jumping modes is analyzed and compared, and the results show that the motion law of the four-bar jumping leg is closer to that of the hind leg of locust. Li et al. [19] designed a bioinspired jumping robot by adding an auxiliary link on the basis of the original series structure (femur and tibia links), and these three links and the trunk of the robot form a four-bar mechanism with one DOF. By imitating an insect, this design reduces the asymmetric thrust forces between two
legs and destabilizing rotation during the flight phase. Zhang et al. [20] of Southeast University also designed a bioinspired jumping robot with a four-bar jumping leg. The average jumping height and jumping distance were 99.1 and 72.7 cm on a smooth surface, respectively. With the development of material science, the integration of the 1DOF mechanism and flexible materials has also been studied [21, 22]. Compared with the four-bar mechanism, the six- and eight-bar mechanisms have more complex structures, but more abundant motion can be achieved [23]. For example, the eight-bar mechanism was used as the jumping leg for the jumping robot “Salto” designed by Haldane et al. [24]. The robot has good agility and high jumping frequency and can achieve the expected jumping trajectory.

For jumping robots, jumping stability is one of the most important performance indicators. If the jumping process is unstable, the robot will tip over in the air or landing phase, resulting in motion failure. However, the stability of most 1DOF jumping robots is difficult to meet the movement requirements, and this inability limits their application. Wings [25, 26] and tails [27, 28] have been added based on bionic inspiration to ensure robot stability. For example, Woodward et al. [29] designed a gliding mechanism for a jumping robot inspired by bats. After takeoff, the bilateral four-bar link mechanisms unfold, and the jumping legs are converted to a gliding configuration with a semiactive mode to achieve stable gliding. Considering that lizards can control the swing of their tails to redirect angular momentum from their bodies to their tails, thereby stabilizing the body posture on the sagittal plane, Libby et al. [30] designed a lizard-sized robot with an active tail, which can swing up and down on a plane. The tail of the robot swings upward as the controller applies torque to stabilize the body, thus keeping the body angle constant with proportion-differential (PD) feedback control. Festo’s BionicKangaroo robot can perform multiple jumps, and its tail also provides the necessary balance when jumping [31]. In comparison with the 1DOF tail mechanism, the multi-DOF tail mechanism can achieve more flexible attitude adjustment [32, 33]. Hybrid structure design is also applied to jumping robots to maintain stability [34–36]. In particular, Kovac et al. [37–39] also studied the continuous jumping ability of the robot on rugged terrain. On the basis of the miniature 7 g jumping robot they developed, they presented a spherical system with a mass of 9.8 g and a diameter of 12 cm that is able to jump, upright itself after landing, and jump again. The jumping performance was analyzed in detail. The robot can overcome obstacles of 76 cm at a takeoff angle of 75°. However, these auxiliary mechanisms increase the complexity of the robot and make it difficult to control.

In summary, if a robot achieves stable jumping without increasing the complexity of the structure and has the advantages of high rigidity and simple control, then optimizing the one-DOF jumping leg mechanism is feasible. The optimization should adopt the six-bar mechanism, which has many variables and acceptable complexity, as the research object. Many studies have been conducted on the optimization of the six-bar mechanism, and mathematical modeling [40, 41] and simulation methods [42] have been used for optimization analyses. For example, in [43, 44], a six-bar mechanism was adopted as a research object, the constraint conditions (including mass balancing and tip trajectory) were set, and the link length was optimized. The robot can jump steadily along the vertical direction. The optimization method is a heuristic approach. Tsuge et al. [45] presented a synthesis method for the Stephenson III six-bar linkage that combines the direct solution of the synthesis equations with an optimization strategy to achieve increased performance for path generation. The methods mentioned above are heuristic approach, and specific tool MATLAB is used in optimization. In addition, some experts have been focusing on the optimization procedure. For example, Rao et al. [46] proposed a Jaya algorithm in order to solve complex constrained design optimization problems, which does not have any algorithm-specific control parameters. Meanwhile, the burden of tuning the control parameters is minimized. Besides, the performance of the proposed Jaya algorithm is tested, and the results show that the Jaya algorithm is superior to or competitive with other optimization algorithms. Renaud et al. [47] optimized a leg mechanism based on multiobjective design optimization method. By taking into account environmental and design constraints, the best architecture and the optimal geometric parameters of a leg are determined. These optimization methods provide a reference for the optimal design of bioinspired jumping robots. However, due to the characteristics of jumping robots, such as floating rack and short takeoff time, these methods are not fully applicable to the optimization of such robots.

In this study, with the Stephenson-type six-bar mechanism as the research object, kinematic and dynamic models were established, and a comprehensive performance optimization method based on mechanism sensitivity was proposed. An experiment was then conducted. This study provides a theoretical basis for the stability design of bioinspired legged robots.

2. Mechanism Design and Performance Analysis

2.1. Mechanism Design. The locust has a strong jumping ability to escape [48, 49]. Thus, it was imitated in this work. Locusts can not only jump forward but also jump laterally [50]. In this paper, only the jumping of locust in a plane is studied. A locust has three pairs of legs. The forelegs and midlegs are used for crawling, and the hind legs are the main physiological structure for jumping. The jumping leg of a locust is shown in Figure 1(a) [48]. The leg is composed of the femur, tibia, and tarsus. During takeoff, the jumping leg can be regarded as a multi-DOF planar series mechanism. A diagram of locust takeoff process can be seen in Figure 1(b), and movement sequence of jumping leg of locust during takeoff process can be seen in Figure 1(c). In the whole takeoff process, the angle between the tibia and femur of hind leg increases rapidly to realize jumping (not less than 100°). So, the equivalent tibia of the jumping leg mechanism of robot should achieve a large swing angle relative to the equivalent femur [51].
According to the observation results of locust takeoff, the leg structure of the robot should satisfy the following design conditions. (a) The leg mechanism of the robot should be similar to that of the hind leg of a locust; that is, it has equal tibia and femur lengths. (b) The equivalent tibia of the jumping leg mechanism can achieve a large swing angle relative to the equivalent femur. Besides, the mechanism should not have any dead point in the motion range. A one-DOF, four-bar mechanism was applied to the design of the jumping leg to achieve high rigidity, good controllability, and simple robot structure. This mechanism is shown in Figure 2(a), where \( ABCD \) denotes a four-bar mechanism. The elongation of one link can be used as the equivalent tibia, and the rest of the links can be used as the equivalent femur. Literature [17] showed that the rotation angle of the trunk of the robot is about 30° during takeoff. The turnover trend of the robot is magnified in the flight phase because of the high acceleration and long flight time, and the robot inevitably tips over. At this time, although the robot has good bionic characteristics and can take off quickly, the stability of the robot is poor, and the performance requirements of the jumping robot cannot be easily met. Therefore, a one-DOF, six-bar mechanism was designed as a jumping leg in this study. The six-bar mechanism includes two basic configurations, namely, Watt and Stephenson types. The atlas of the six-bar link in the Stephenson-type six-bar mechanism is shown in Figure 2(b). The jumping leg shown in Figure 2(c) can be obtained by selecting the three-pair link \( DEF \) and the two-pair link \( FG \) as the initial tibia and femur, respectively, and setting link \( AB \) as the trunk link. By changing the link length, the jumping leg can have the equivalent tibia and femur and demonstrates good bionic characteristics. Given that the six-bar mechanism has more length parameters than the four-bar mechanism, it can be optimized to ensure that the robot has a stable jumping process, that is, the robot does not tip over in the air and landing phases.

On the basis of the Stephenson-type six-bar mechanism, two possible configurations exist, as shown in Figure 3. In Figure 3(a), two- and three-pair links are selected as the initial tibia and femur, respectively. In Figure 3(b), two two-pair links are selected as the initial tibia and femur.
Considering that the mass distribution of the equivalent tibia and femur should be balanced, the configuration shown in Figure 3(a) was selected in this study.

2.2. Kinematic Modeling. The purpose of kinematic analysis for the jumping leg is to obtain the trunk attitude in the takeoff process, which is one of the main parameters that reflect jumping stability. The modeling method and the basis of biological movement can be obtained from literature [16]. The six-bar jumping leg is shown in Figure 4. The robot is driven by a spring. One end of the spring is connected with link $ACD$, and the other end is connected with link $CG$. The coordinate origin of coordinate system $O_Mx_My_M$ coincides with $M$ (contact point between the jumping leg and ground). The direction of the $x_M$ axis is horizontal to the right, and the direction of the $y_M$ axis is vertical upward. The coordinate origin of motion coordinate system $O_tx_ty_t$ coincides with the midpoint of link $AB$. The direction of the $x_t$ axis coincides with the perpendicular line in $AB$, and the direction of the $y_t$ axis is along link $AB$. Link $ABO_C$ refers to the trunk, and $O_C$ is the center of mass. The center of mass $O_C$ in coordinate system $O_Mx_My_M$ can be written as
where

\[ R_{11} = \cos \left( \frac{\pi}{2} + \gamma_1 + \alpha_8 \right), \]
\[ R_{12} = -\cos \left( \frac{\pi}{2} + \gamma_1 + \sum_{i=9}^{9} \alpha_i \right), \]
\[ R_{13} = \cos \left( \frac{\pi}{2} + \sum_{i=1}^{2} \gamma_i + \sum_{k=8}^{10} \alpha_k \right), \]
\[ R_{14} = -\cos \left( \frac{\pi}{2} + \gamma_6 + \sum_{i=1}^{2} \gamma_i + \sum_{k=8}^{10} \alpha_k \right), \]
\[ R_{15} = \cos \left( \frac{\pi}{2} + \sum_{i=1}^{2} \gamma_i + \sum_{p=5}^{6} \gamma_p + \sum_{k=8}^{10} \alpha_k + \alpha_2 \right), \]
\[ R_{21} = \sin \left( \frac{\pi}{2} + \gamma_1 + \alpha_8 \right), \]
\[ R_{22} = -\sin \left( \frac{\pi}{2} + \gamma_1 + \sum_{i=9}^{9} \alpha_i \right), \]
\[ R_{23} = \sin \left( \frac{\pi}{2} + \sum_{i=1}^{2} \gamma_i + \sum_{k=8}^{10} \alpha_k \right), \]
\[ R_{24} = -\sin \left( \frac{\pi}{2} + \gamma_6 + \sum_{i=1}^{2} \gamma_i + \sum_{k=8}^{10} \alpha_k \right), \]
\[ R_{25} = \sin \left( \frac{\pi}{2} + \sum_{i=1}^{2} \gamma_i + \sum_{p=5}^{6} \gamma_p + \sum_{k=8}^{10} \alpha_k + \alpha_2 \right). \]

In the above expressions, \((x_{OC}, y_{OC})\) is the coordinate of the center of mass \(O_C\) in the coordinate system \(x_M, y_M, z_M\), and they are a series of discrete values that can be determined by the initial position of the center of mass and takeoff direction angle (the angle between the direction of the external force acting on the center of mass and the positive direction of \(x_M\)). \(l_2 (l_3, l_4, l_5, l_6, l_7, l_8)\) is the length of link \(ME (EG, GC, CA, and AO)\). \(\alpha_8\) is the angle between ME and MF, \(\alpha_9\) is the angle between ME and EG, and \(\alpha_{10}\) is the angle between EG and GF in the quadrilateral \(MEGF\). \(\alpha_2\) is the angle between \(AB\) and \(AO\) in the triangle \(AO\). When the length of each link of the jumping leg is determined, \(\alpha_i (i = 2, 8, 9, 10)\) can be obtained by the cosine theorem. \(\gamma_1\) is the angle between MG and the positive direction of the \(y_2\) axis, \(\gamma_2\) is the angle between \(CG\) and \(GF\), \(\gamma_3\) is the angle between \(AB\) and \(AC\), and \(\gamma_6\) is the angle between \(CG\) and \(CA\). The coupling relationship between \(\gamma_5, \gamma_6, \) and \(\gamma_2\) can be solved by the quadrilateral \(DEGC\) and the pentagon \(FGCAB\).

In the crossed quadrilateral \(DEGC\), the following vector equation can be obtained:

\[ \overrightarrow{DE} + \overrightarrow{EG} + \overrightarrow{GC} + \overrightarrow{CD} = \overrightarrow{0}. \]  

Equation (3) can be projected to the \(x_M\) and \(y_M\) axis as follows:

\[ \begin{bmatrix} -\cos(\gamma_9 - \gamma_{10}) \cos \gamma_8 & \cos \gamma_{10} \\ -\sin(\gamma_9 - \gamma_{10}) \sin \gamma_8 & -\sin \gamma_{10} \end{bmatrix} \begin{bmatrix} l_5 \\ l_6 \end{bmatrix} = \begin{bmatrix} l_{10} \\ 0 \end{bmatrix}, \]  

where \(l_5\) is the length of link \(DE\) and \(l_{10}\) is the length of link \(CD\). \(\gamma_9\) is the angle between links \(CD\) and \(DE\), \(\gamma_{10}\) is the angle between links \(CG\) and \(GE\), and \(\gamma_{10}\) is the angle between links \(CG\) and \(CD\). According to the geometric relationship shown in Figure 3, the following formula can be obtained:

\[ \begin{align*}
\gamma_9 &= \gamma_2 + \alpha_{10} \\
\gamma_{10} &= \alpha_6 - \gamma_9
\end{align*} \]

In the pentagon \(FGCAB\), the vector equation can be written as

\[ \overrightarrow{FG} + \overrightarrow{GC} + \overrightarrow{CA} + \overrightarrow{AB} + \overrightarrow{BF} = \overrightarrow{0}. \]

Equation (10) can be further written as

\[ \begin{bmatrix} \cos \gamma_2 \cos \gamma_3 - \cos(\gamma_3 + \gamma_4) & -\cos(\gamma_2 + \gamma_6) \\ \sin \gamma_2 - \sin \gamma_3 \sin(\gamma_3 + \gamma_4) & -\sin(\gamma_2 + \gamma_6) \end{bmatrix} \begin{bmatrix} l_6 \\ l_8 \end{bmatrix} = \begin{bmatrix} l_4 \\ 0 \end{bmatrix}, \]  

where \(l_4, l_7, \) and \(l_8\) is the length of links \(FG, BF, \) and \(AB\), respectively, \(\gamma_3\) is the angle between links \(FB\) and \(FG\), and \(\gamma_4\) is the angle between links \(FB\) and \(AB\). Afterward, the relationships between \(\gamma_3, \gamma_4, \gamma_5, \) and \(\gamma_2\) can be obtained, and they can be expressed as
\[ y_3 = \pi - \arcsin \left( \frac{U_f V_f + V_f \sqrt{U_f^2 + V_f^2 - W_f^2}}{U_f^2 + V_f^2} \right) \]  
\[ = f_4(y_2), \]  
\[ y_4 = 2\pi - \arccos \left( \frac{l_5 \cos y_3 + l_6 \cos y_2 - l_{11} \cos (y_2 + y_6)}{l_8} \right) - y_3 = f_5(y_2). \]  
\[ y_5 = 3\pi - (y_2 + y_3 + y_4 + y_6) = f_6(y_2), \]

where

\[ U_f = 2l_c [-l_6 \sin y_2 + l_{11} \sin (y_2 + y_6)], \]
\[ V_f = 2l_c [l_6 \cos y_2 - l_{11} \cos (y_2 + y_6)], \]
\[ W_f = l_7^2 - l_8^2 + l_6 \cos y_2 - l_{11} \cos (y_2 + y_6))^2 \]
\[ + [-l_6 \sin y_2 + l_{11} \sin (y_2 + y_6))^2. \]

When the coordinates of the center of mass of the robot during the takeoff phase are known, equations (6) and (14) can be substituted into equation (1), and the unknowns contained in equation (1) are \( y_1 \) and \( y_2 \). Then, the joint angles of the jumping leg can be obtained by solving equation (1).

In particular, according to the geometric relationships shown in Figure 3, the attitude angle of the trunk \( \theta_\theta \) (that is, the angle between link \( BO_C \) and the positive direction of \( x_M \) axis) can be written as

\[ \theta_\theta = y_1 - y_3 - y_4 - \alpha_1 - \alpha_{11} + \frac{5\pi}{2}, \]

where \( \alpha_1 \) is the angle between \( AB \) and \( BO_C \) in the triangle \( ABO_C \), \( \alpha_{11} \) is the angle between \( FG \) and \( MF \) in the quadrilateral \( MFGE \), \( y_1 \) is the angle between \( FB \) and \( GF \), and \( y_4 \) is the angle between \( AB \) and \( FB \).

Therefore, the change in the attitude of the trunk during the takeoff process can be expressed as

\[ \Delta \theta_\theta = \Delta y_1 - \Delta y_3 - \Delta y_4. \]

2.3. Dynamic Modeling. The takeoff direction angle of the robot should be determined (the initial coordinate of the center of mass is known) to obtain the coordinates of the center of mass for the robot during the entire takeoff phase, and it can be seen in Figure 4.

The expression of the takeoff direction angle is

\[ \Psi = \arctan \left( \frac{F_{69y-0} + F_{66y-0} - mg}{F_{68} + F_{26x-0}} \right), \]

where \( m \) is the mass of the robot and \( (F_{66y-0}, F_{69y-0}) \) and \( (F_{26x-0}, F_{26y-0}) \) are the forces of links \( ACD \) and \( BF \) acting on the trunk in the initial state, respectively. Suppose that the \( i \)-th link \((i = 1-6)\) subjects the forces \( f_i \) of the other links connected with it on it. The link may also suffer from external force \( f_{in} \) and spring driving force \( f_{si} \). The force and moment balance equations can be written as

\[ \sum_{j=1}^{n1} f_j + \sum_{p=1}^{n2} f_{ip} + \sum_{q=1}^{n3} f_{iq} + m_i g = 0, \]
\[ \sum_{j=1}^{n1} \left( r_j \times f_j \right) + \sum_{p=1}^{n2} \left( r_{ip} \times f_{ip} \right) + \sum_{q=1}^{n3} \left( r_{iq} \times f_{iq} \right) + r_i \times m_i g = 0, \]

where \( r \) is the position vector. \( f_{16-0} \) and \( f_{26-0} \) can be solved according to equations (19) and (20), and the takeoff direction angle can be obtained by equation (18). Given that the takeoff time of the robot is very short, the robot is generally considered to move along the takeoff direction line during the takeoff phase without deviation. Therefore, the relationship between the abcissa and ordinate of the center of mass \( O_C \) in the coordinate system \( x_M-x_M'y_M'z_M' \) can be written as

\[ y_{OC} = \left( x_{OC} - x_{O_C-0} \right) \cdot \tan \Psi + y_{O_C-0}, \]

where \( (x_{O_C-0}, y_{O_C-0}) \) is the initial coordinate of the center of mass.

By substituting equation (1) into (21), the relationship between \( y_1 \) and \( y_2 \) can be obtained as

\[ y_1 = \arccos \left( \frac{W_f V_f - U_f \sqrt{U_f^2 + V_f^2 - W_f^2}}{U_f^2 + V_f^2} \right) - \frac{\pi}{2} = f_7(y_2), \]

where

\[ U_f = w - u \cdot \tan \psi, \]
\[ V_f = u - v \cdot \tan \psi, \]
\[ W_f = s \cdot \tan \psi, \]
\[ s = x_{O_C-0} - \frac{y_{O_C-0}}{\tan \Psi}; \]

\[ u = l_2 \cos \alpha_8 - (l_{11} - l_6) \cos \left( \sum_{i=8}^{10} \alpha_i \right) - l_3 \cos \left( \sum_{i=8}^{9} \alpha_i \right) + l_{13} \cos \left( \sum_{i=8}^{10} \alpha_i \right), \]
\[ v = -l_2 \sin \alpha_8 + (l_{11} - l_6) \sin \left( \sum_{i=8}^{10} \alpha_i \right) + l_3 \sin \left( \sum_{i=8}^{9} \alpha_i \right) - l_{13} \sin \left( \sum_{i=8}^{10} \alpha_i \right), \]
\[ w = l_2 \sin \alpha_8 - (l_{11} - l_6) \sin \left( \sum_{i=8}^{10} \alpha_i \right) - l_3 \sin \left( \sum_{i=8}^{9} \alpha_i \right) + l_{13} \sin \left( \sum_{i=8}^{10} \alpha_i \right). \]
According to these analysis results, all of the joint angles of the jumping leg can be expressed as functions of $\gamma_2$. Hence, the angular velocity of each joint angle and the velocity of each link can be expressed as functions of $\gamma_2$ and $\dot{\gamma}_2$, and the formulas are as follows:

$$\dot{\Gamma} = Q \cdot \dot{y}_2,$$

where

$$\dot{\Gamma} = \begin{bmatrix} \dot{\gamma}_3 & \dot{\gamma}_4 & \dot{\gamma}_5 & \dot{\gamma}_6 & \dot{\gamma}_7 \end{bmatrix},$$

$$Q = \begin{bmatrix} \frac{df_1(y_2)}{dy_2} & \frac{df_2(y_2)}{dy_2} & \frac{df_3(y_2)}{dy_2} & \frac{df_4(y_2)}{dy_2} & \frac{df_5(y_2)}{dy_2} & \frac{df_6(y_2)}{dy_2} \end{bmatrix}^T.$$

Supposing that the center of mass of each link of the jumping leg is located in its geometric center, the velocity of the center of mass of each link can be expressed as

$$V = J \cdot \dot{\Theta},$$

where

$$V = \begin{bmatrix} v_{1x} & v_{1y} & v_{2x} & \cdots & v_{5y} & v_{6x} & v_{6y} \end{bmatrix}^T,$$

$$\dot{\Theta} = \begin{bmatrix} \dot{\theta}_1 & \dot{\theta}_2 & \dot{\theta}_3 & \dot{\theta}_4 & \dot{\theta}_5 & \dot{\theta}_6 \end{bmatrix}^T.$$

where $v_{ix}$ and $v_{iy}$ ($i=1-6$) are the velocity of the center of mass of the $i$-th link along $x_M$ and $y_M$ axes, respectively, $\theta_1$ ($\theta_2$, $\theta_3$, $\theta_4$, $\theta_5$, $\theta_6$) is the angle between link $MF$ ($BF$, $CG$, $DE$, $AC$, $BO_c$) and positive direction of $x_M$ axis, which is a function of $\gamma_2$. $J$ is the Jacobian matrix with 12 rows and 6 columns, and each element is a function of $\gamma_2$. The expression of $J$ is as follows:

$$J = \begin{bmatrix}
  -l_{14} \sin(\theta_1 + \gamma_5) & 0 & 0 & 0 & 0 & 0 \\
  l_{14} \cos(\theta_1 + \gamma_5) & 0 & 0 & 0 & 0 & 0 \\
  -l_1 \sin \theta_1 & \frac{l_2}{2} \sin \theta_2 & 0 & 0 & 0 & 0 \\
  l_1 \cos \theta_1 & \frac{l_2}{2} \cos \theta_2 & 0 & 0 & 0 & 0 \\
  -l_{15} \sin(\theta_1 + \gamma_6) & 0 & \frac{l_6}{2} \sin \theta_3 & 0 & 0 & 0 \\
  l_{15} \cos(\theta_1 + \gamma_6) & 0 & \frac{l_6}{2} \cos \theta_3 & 0 & 0 & 0 \\
  -l_2 \sin(\theta_1 + \gamma_1) & 0 & 0 & \frac{l_5}{2} \sin \theta_4 & 0 & 0 \\
  l_2 \cos(\theta_1 + \gamma_1) & 0 & 0 & \frac{l_5}{2} \cos \theta_4 & 0 & 0 \\
  -l_1 \sin \theta_1 & -l_2 \sin \theta_2 & 0 & 0 & -l_{15} \sin(\theta_5 + \beta_4) & -l_8 \sin(\theta_6 + \alpha_1) \\
  l_1 \cos \theta_1 & l_2 \cos \theta_2 & 0 & 0 & l_{15} \cos(\theta_5 + \beta_4) & l_8 \cos(\theta_6 + \alpha_1) \\
  -l_1 \sin \theta_1 & -l_2 \sin \theta_2 & 0 & 0 & 0 & -l_{12} \sin \theta_6 \\
  l_1 \cos \theta_1 & l_2 \cos \theta_2 & 0 & 0 & 0 & l_{12} \cos \theta_6 
\end{bmatrix},$$

where $l_{14}$ is the length from the center of mass $O_1$ of the 1st link to the point $M$, $l_{15}$ is the length of $MG$, $l_{16}$ is the length from the center of mass $O_5$ of the 5th link to the point $A$, and $\beta_4$ is the angle between $O_5A$ and $AC$. 
The acceleration of each link and the angular acceleration of each joint can be expressed as functions of $\gamma_2$, $\dot{\gamma}_2$, and $\ddot{\gamma}_2$ by deriving equations (24) and (26).

The Lagrange dynamic modeling method [52, 53] was used in this study to establish a dynamic model of the jumping leg. The total potential energy of the jumping leg can be expressed as

$$U = \sum_{i=1}^{6} (m_i g h_i) + \frac{1}{2} k (l_{e-o} - l_i)^2 = \sum_{i=1}^{6} (m_i g \cdot f_{hi}(\gamma_2)) + f_{k1}(\gamma_2) = f_U(\gamma_2),$$

(29)

where $m_i$ is the mass of each link, $k$ is the stiffness coefficient of the driving spring, $l_{e-o}$ is the original length of the spring, $l_i$ is the length of the spring in the takeoff process, and $h_i$ is the height of the center of mass of each link in the fixed coordinate system $o_Mx_My_Mz_M$, and expressions of $h_i$ and $l_i$ are as follows:

$$h_1 = l_{i1} \sin (\theta_1 + \alpha_{i1}) = f_{h1}(\gamma_2),$$
$$h_2 = l_1 \sin \theta_1 + \frac{l_2}{2} \sin \theta_2 = f_{h2}(\gamma_2),$$
$$h_3 = l_{i1} \sin (\theta_1 + \alpha_{i3}) + \frac{l_4}{2} \sin \theta_3 = f_{h3}(\gamma_2),$$
$$h_4 = l_2 \sin (\theta_1 + \alpha_4) + \frac{l_5}{2} \sin \theta_4 = f_{h4}(\gamma_2),$$
$$h_5 = l_1 \sin \theta_1 + l_7 \sin \theta_2 + l_6 \sin (\theta_6 + \alpha_1) + l_{i6} \sin (\theta_6 + \alpha_2) = f_{h5}(\gamma_2),$$
$$h_6 = l_1 \sin \theta_1 + l_7 \sin \theta_2 + l_{i12} \sin \theta_6 = f_{h6}(\gamma_2),$$
$$l_s = \sqrt{(l_9 - l_{i1})^2 + a_1^2 - 2 \cdot (l_9 - l_{i1}) \cdot a_1 \cdot \cos \varphi_1} = f_{s1}(\gamma_2),$$

(30)

where

$$a_3 = \sqrt{l_{i2}^2 + l_{i3}^2 - 2 \cdot l_4 \cdot l_{i2} \cdot \cos \gamma_2},$$
$$\varphi_3 = \gamma_3 - \arcsin \left( \frac{l_{i2} \cdot \sin \gamma_2}{a_3} \right),$$
$$a_2 = \sqrt{l_{i4}^2 + a_3^2 - 2 \cdot l_7 \cdot a_3 \cdot \cos \varphi_3},$$
$$\varphi_2 = \gamma_4 - \arcsin \left( \frac{a_3 \cdot \sin \varphi_3}{a_2} \right),$$
$$\varphi_3 = \gamma_5 - \alpha_4 - \arcsin \left( \frac{a_2 \cdot \sin \varphi_2}{a_1} \right),$$

(31)

in which $l_{i1}$ and $l_{i2}$ are the length of $DH_1$ and $GH_2$, respectively, and $\alpha_7 (\alpha_{i2}, \alpha_{i3})$ is the angle between $O_5A (O_7M, GM)$ and $CA (MF, FM)$.

The total kinetic energy of the jumping leg can be expressed as

$$T = \sum_{i=1}^{6} \left( \frac{1}{2} m_i \dot{\theta}_i^2 + \frac{1}{2} m_i (\dot{v}_{x_i}^2 + \dot{v}_{y_i}^2) \right)$$
$$= \sum_{i=1}^{6} \left( \frac{1}{2} m_i \cdot f_{g1}(\gamma_2, \dot{\gamma}_2) + \frac{1}{2} m_i \cdot f_{hi}(\gamma_2, \dot{\gamma}_2) \right) = f_T(\gamma_2, \dot{\gamma}_2),$$

(32)

where $J_{ai}$ ($i = 1–6$) is the moment of inertia of the $i$-th link of the jumping leg.

According to equations (30) and (32), the potential and kinetic energies of the jumping leg are functions of $\gamma_2$ and $\dot{\gamma}_2$. By substituting equations (30) and (32) into the Lagrange dynamic equation, the following formula can be obtained:

$$\frac{d}{dt} \frac{dU}{d\dot{\gamma}_2} + \frac{d}{dt} \frac{dT}{d\dot{\gamma}_2} + \frac{dU}{d\gamma_2} + \frac{dD}{d\gamma_2} = 0,$$

(33)

where $D$ is the dissipative energy of the system, and its expression is

$$D = \frac{1}{2} c (\dot{v}_{x_i}^2 + \dot{v}_{y_i}^2) = f_D (\gamma_2, \dot{\gamma}_2, \ddot{\gamma}_2),$$

(34)

where $c$ is the damping coefficient; $\dot{v}_{x_i}^2 = \sum_{i=1}^{6} \dot{v}_{x_i}^2$; and $\dot{v}_{y_i}^2 = \sum_{i=1}^{6} \dot{v}_{y_i}^2$.

By substituting equation (34) into (33), the latter can be simplified to

$$G_1(\gamma_2)\ddot{\gamma}_2 + G_2(\gamma_2)\dot{\gamma}_2 + G_3(\gamma_2)\dot{\gamma}_2 + G_4(\gamma_2) = 0.$$

(35)

Equation (35) is a second-order quadratic nonlinear differential equation, and the numerical method [54] is used to solve equation (35). The initial value of $\gamma_2$ is set to $\gamma_{2,0}$, and the initial value of $\dot{\gamma}_2$ is set to 0. The solution of equation (35) can be expressed as

$$\gamma_2 = h_1(t),$$
$$\dot{\gamma}_2 = h_2(t).$$

(36)

By substituting equation (36) into equations (26)–(28), the angular velocity (acceleration) of each angle and the velocity (acceleration) of each link can be solved. Then, the force analysis is conducted taking inertial force $f_{ci}$ and moment $m_{ci}$ into account to obtain the joint forces. On the basis of equations (19) and (20), the equations of force and moment for the $i$-th link can be written as

$$\sum_{j=1}^{n1} f_{ij} + \sum_{p=1}^{n2} f_{ip} + \sum_{q=1}^{n3} f_{iq} + f_{ci} + m_{ci} g = 0,$$

(37)

$$\sum_{j=1}^{n1} (r_{ij} \times f_{ij}) + \sum_{p=1}^{n2} (r_{ip} \times f_{ip}) + \sum_{q=1}^{n3} (r_{iq} \times f_{iq}) + r_{ci}$$
$$\times (f_{ci} + m_{ci} g) + m_{ci} = 0,$$

(38)
where $f_i$ and $m_i$ are the inertial force and inertial moment of the $i$-th link, respectively.

In particular, the total inertia moment can reflect the stability of the robot in the flight and landing phases and can be expressed as

$$\mathbf{m}_i = \sum_{j=1}^{6} (\mathbf{r}_{ij} \times f_{ij} + m_{ij}),$$

where $\mathbf{r}_{ij}$ is the position vector of the inertia force.

### 3. Configuration Optimization

#### 3.1. Determination of the Objective Function

The design goal is to ensure good stability of the jumping robot; thus, the absolute value of total inertia moment $M_t$ and the change of trunk attitude angle $\theta_a$ in the jumping process should be as small as possible [1]. The mass $m_i$ of the mechanism component affecting $\theta_a$ and $M_t$ is linearized according to the link length parameter $l_i$. At this time, $l_i$ is the only variable. The linearization of mass is as follows:

$$m_i = \rho l_i,$$

where $\rho$ is the linear density of the component. For the rotation angle of the trunk in the takeoff process, the variance of $\theta_a$ can be expressed as the objective function, which reflects the change in trunk attitude relative to the initial value:

$$\theta_a = \theta_a(l) = \int_{y_2}^{y_2'} \frac{(\theta_{y_2} - \mu_{\theta_a})^2}{\gamma_2' - \gamma_2} dy_2$$

(41)

For the total inertia moment, the sum of absolute values of $M_t$ can be expressed as an objective function:

$$M_t = M_t(l) = \int_{y_2}^{y_2'} |M_t| dy_2,$$

(42)

where $y_2$ and $y_2'$ correspond to the initial and final values in the takeoff phase, respectively. After dimensionless treatment, the optimization objective function can be expressed as

$$H = H(l) = e^{M_{t0}} \int_{y_2}^{y_2'} \frac{(\theta_{y_2} - \mu_{\theta_a})^2}{\gamma_2' - \gamma_2} dy_2$$

(43)

$$+ \left(1 - e^{M_{t0}} \right) \int_{y_2}^{y_2'} |M_{t2}| dy_2,$$

where $\rho$ is a proportional coefficient and $M_{t0}$ and $\theta_{a0}$ are initial values for dimensionless processing.

In particular, the programming software MATLAB is used to optimize the configuration of the jumping leg.

#### 3.2. Reliability Sensitivity Analysis

The six-bar mechanism has many length parameters, which increase the difficulty of optimization. To simplify the optimization process, this study developed a multiobjective optimization method for the six-bar jumping leg on the basis of the sensitivity analysis results. We set the parameters with low sensitivity to a fixed value, and the parameters with sensitivity higher than the allowable value are regarded as optimization parameters.

Since the Monte Carlo method is recognized as an accurate solution approach in the field of reliability sensitivity analysis [55–57], local reliability sensitivity was analyzed with the Monte Carlo reliability sensitivity method. Then, the jumping process was discretized with $t = (t_1, t_2, \ldots, t_l, \ldots, t_l)_{k}$ as a variable, and a global sensitivity analysis was carried out. $q$ is the number of jumping time discrete points, and $k$ is the $k$-th time point among the $q$ jumping time discrete points.

For the locally reliable sensitivity, because the link parameters $l = (l_1, l_2, \ldots, l_l)$ are independent of each other, we suppose that they obey the normal distribution of $l_i \sim N(\mu_i, \sigma_i^2)$. Given that inert moment $M_t$ and motion attitude angle $\theta_a$ are considered in the judging of jumping stability, the robot has two failure modes, and the system is a typical multifailure mode. The two failure modes can be regarded as a series system. The variance is used to express the distribution of the trunk attitude $\theta_a$ and total inertial moment $M_t$.

The attitude objective function and failure domain can be expressed as

$$g_1(l) = \frac{\sum_{k=1}^{q} (\theta_a - \mu_{\theta_a})^2}{q} - \varepsilon_{\theta_a},$$

(44)

$$F_1 = \{l \in g_1(l) > 0\}.$$

The total inertia moment objective function and the failure domain can be expressed as

$$g_2(l) = \frac{\sum_{k=1}^{q} (M_t - \mu_{M_t})^2}{q} - \varepsilon_{M_t},$$

(45)

$$F_2 = \{l \in g_2(l) > 0\},$$

where $\varepsilon_{\theta_a}$ and $\varepsilon_{M_t}$ are the reliability domain values and $M_{t0}$ is the initial inertia moment.

For the $N$-time sampling of the multiple failure mode series system, the failure probability expression is

$$P_f = \int_{p_{l1}}^{p_{l1}} \int_{p_{l1}}^{p_{l1}} \cdots \int_{p_{l1}}^{p_{l1}} f_{L}(l_1, l_2, \ldots, l_l) dl_1 dl_2 \cdots dl_1$$

(46)

$$= \int_{p_{l1}}^{p_{l1}} \int_{p_{l1}}^{p_{l1}} \cdots \int_{p_{l1}}^{p_{l1}} I_{F}(l_1, l_2, \ldots, l_l) dl_1 dl_2 \cdots dl_1$$

The failure domain indication function is

$$I_{F}(l) = \begin{cases} 1, & l \in F, \\ 0, & l \notin F, \end{cases}$$

(47)

where $f_{L}(l_1, l_2, \ldots, l_l)$ is the joint probability density function of the link parameter $l = (l_1, l_2, \ldots, l_l)$, $R^n$ is an $n$-dimensional variable space, $E(I_{F}(l))$ is the mathematical expectation operator, $P_f$ is the failure probability of the $h$-th failure mode, and $P_{h12}$ is the probability of simultaneous failure of two failure modes.
For the analysis of reliability sensitivity, the dimensionless sensitivity method proposed by Wu is adopted [58]. The reliability sensitivity is a partial derivative of the distribution parameter \( \theta^{(k)} \) of variable \( l \), of failure probability \( P_f \), and it can be expressed as

\[
\frac{\partial P_f^{(i)}}{\partial \theta^{(k)}_j} = \left( \cdots \right) \frac{\partial f_L(l)}{\partial \theta^{(k)}_j} \cdot \frac{\partial \mu_f}{\partial l_j}.
\]  

(48)

To eliminate the effect of the dimension on the reliability sensitivity, the dimensionless sensitivity coefficient \( S_{\theta^{(k)}}^{(i)} \) of failure probability to distribution parameter \( \theta^{(k)}_j \) was used. The dimensionless sensitivity coefficients of the failure probability to the mean \( \mu_0 \) and standard deviation \( \sigma_0 \) of the \( i \)-th variable are as follows:

\[
S_{\mu_0} = \frac{\partial P_f^{(i)}}{\partial \mu_0} = \frac{1}{N} \sum_{j=1}^{N} u_{ji}^2,
\]

\[
S_{\sigma_0} = \frac{\partial P_f^{(i)}}{\partial \sigma_0} = \frac{1}{N} \sum_{j=1}^{N} \left( u_{ji}^2 - 1 \right),
\]

(49)

where \( u_{ji} \) is the standard normalized sample corresponding to the \( i \)-th component \( l_j \) of the \( j \)-th sample \( l_j = (l_{j1}, l_{j2}, \ldots, l_{j11}) \) and \( u_{ji} = (l_{ji} - \mu_0)/\sigma_0 \).

### 3.3. Optimization Process

The optimization is conducted based on PSO [59], and the optimization process is shown in Figure 5. First, the sensitivity of each variable was analyzed, and the variable with low sensitivity was set to a constant. The distribution function was set for variables with high sensitivity, and the dimension of the optimization objective function was reduced. Second, the variables were assigned. Independent variable \( l \) is a D-dimensional vector in objective function \( H(l) \). An independent variable \( l \) forms a particle, and \( N \) particles form a community. Particle initialization of \( l \) was carried out, and in order to avoid the concentration of search range for objective function, \( l \) should obey the uniform distribution: \( l \sim U(\bar{\theta}_1, \bar{\theta}_2) \), where \( \bar{\theta}_1 \) is the initial link length parameter and \( \bar{\theta}_2 \) are the proportionality coefficients. In the optimization target search space of \( l \), the \( b \)-th particle \( l_b \) can be expressed as

\[ l_b = (l_{b1}, l_{b2}, \ldots, l_{bD}), \quad b = 1, 2, \ldots, N, d = 1, 2, \ldots, D. \]

(50)

The probability density function of \( l_b \) is

\[
l_b = \begin{cases} \frac{1}{\omega_1 l_{bd} - \omega_2 l_{bd}} - \omega_1 l_{bd} & \text{if } l_{bd} \in (\omega_1 l_{bd}, \omega_2 l_{bd}) \\
0 & \text{if } l_{bd} \notin (\omega_1 l_{bd}, \omega_2 l_{bd}, +\infty). \end{cases}
\]

(51)

Third, the constraint conditions were set. To improve the efficiency of optimization and the accuracy of the results, the following constraint conditions were added to the optimization process. (a) Geometric constraint condition: the proportional relationship among link lengths should make the mechanism have bionic characteristics, which means that the jumping mechanism has distinct equivalent tibia and femur. (b) Parameter constraint condition: all values should be in the real field. If the conditions are not met, then the optimization process should be recycled. (c) Result constraint condition: the condition for the end of the optimization process is that the objective functions are less than the set values.

Fourth, according to the characteristics of PSO, in order to avoid the concentration of search range of the objective functions, the change velocity \( v \) of the link length that obeys the uniform distribution \( v \sim U(\omega_1 l_{lb}, \omega_2 l_{lb}) \) was initialized. Then, the change velocity of the link length of the \( b \)-th particle can be expressed as follows:

\[
v_b = (v_{b1}, v_{b2}, \ldots, v_{bD}), \quad b = 1, 2, \ldots, N, d = 1, 2, \ldots, D.
\]

(52)

The probability density function of \( v_{bd} \) is

\[
v_{bd} = \begin{cases} \frac{1}{\omega_2 l_{bd} - \omega_1 l_{bd}} - \omega_1 l_{bd} & \text{if } v_{bd} \in (\omega_1 l_{bd}, \omega_2 l_{bd}) \\
0 & \text{if } v_{bd} \notin (\omega_1 l_{bd}, \omega_2 l_{bd}, +\infty). \end{cases}
\]

(53)

After the initialization of link length variation velocity \( l_{bd} \), \( v_{bd} \) is updated according to PSO, and \( l_{bd} \) is updated according to the initial link length parameter and \( v_{bd} \). The updated formula is as follows:

\[
l_{bd} = l_{bd} + v_{bd},
\]

(54)

where \( c_1 \) and \( c_2 \) are learning factors, \( r_1 \) and \( r_2 \) are uniform random numbers in the range of [0,1], and \( w \) is the inertial weight. The values of \( c_1 \), \( c_2 \), \( r_1 \), \( r_2 \), and \( w \) are all constants.

After updating the link length parameter \( l_b \), whether or not \( l_b \) satisfies the geometric and parameter constraint conditions is determined. If not, \( v_b \) is reinitialized and \( l_b \) is reupdated. If the constraint condition is satisfied, it is judged whether or not the objective function value \( H \) satisfies the result constraint condition. In particular, after each update of \( l_b \), the sensitivity of parameters needs to be recalculated to determine if the optimization parameters have changed.

### 3.4. Analysis of Optimization Results

#### 3.4.1. Sensitivity Analysis

In order to make a significant difference in the sensitivity analysis results, we suppose that the variation coefficient of the link is \( V = 10^{-4} \) by multiple calculations. The sensitivity analysis results of link length parameters are shown in Table 1.

The mean sensitivity and standard deviation sensitivity of \( M_j \) of each link are shown in Figure 6. The mean sensitivity
and standard deviation sensitivity of attitude $\theta$ are shown in Figure 7. If the sensitivity is negative, then the distribution parameter of the link is negatively correlated with the failure probability. As the distribution parameter increased, the failure probability decreased and reliability increased. Conversely, the distribution parameters of the links were positively correlated with the failure probability. The sensitivity value was logarithmized for an easy expression. As

![Figure 5: Optimization flow based on particle swarm optimization.](image)

### Table 1: Parameters of reliability sensitivity.

| Parameter | $l_1$ | $l_2$ | $l_3$ | $l_4$ | $l_5$ | $l_6$ | $l_7$ | $l_9$ | $l_{10}$ | $l_{11}$ |
|-----------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Mean ($\mu$) | 26.11 mm | 17.44 mm | 9.73 mm | 1.04 mm | 26.59 mm | 25.42 mm | 21.76 mm | 9.68 mm | 8.93 mm | 1.17 mm |
| $S$ ($\sigma$) | $2.61 \times 10^{-2}$ | $1.74 \times 10^{-2}$ | $9.73 \times 10^{-3}$ | $1.04 \times 10^{-3}$ | $2.66 \times 10^{-2}$ | $2.54 \times 10^{-2}$ | $2.18 \times 10^{-2}$ | $9.68 \times 10^{-3}$ | $8.93 \times 10^{-3}$ | $1.17 \times 10^{-3}$ |

![Figure 6: Reliability sensitivity for $M_1$. (a) Mean reliability sensitivity and (b) standard deviation reliability sensitivity.](image)
shown in Figures 6 and 7, links $l_1$, $l_2$, and $l_4$ have low sensitivity. In subsequent calculations, suppose $l_1 = 26$ mm, $l_2 = 17.5$ mm, and $l_4 = 1$ mm. Other variables with high sensitivity $l_d$ ($d = 3, 5, 6, 7, 9, 10, 11$) continue to be used as variables in the subsequent calculation.

After the link sensitivity analysis was completed, the variables were reduced from 10 to 7, and the PSO is used to optimize the objective function $H(l)$. The parameters of PSO are shown in Table 2. $N$ is the particle number, $c_1$ and $c_2$ are learning factors, $w$ is inertia weight, $M$ is iteration times, and $D$ is variable dimension. In PSO, in order to obtain a high-precision solution, the key is the appropriate collocation of parameters [60].

After the link sensitivity analysis was completed, the optimization time was reduced from 20 hours to 5 hours, which seriously improved the optimization efficiency of the algorithm. The difference is shown in Figure 8.

### 3.4.2. Analysis of Constraint Conditions

1. **Geometric Constraint Conditions.** To achieve good jumping performance, the jumping leg mechanism should satisfy the biomimetic characteristics. Therefore, the geometric constraint conditions should be considered, which means that the jumping leg should have a distinct equivalent tibia and femur. The constraint conditions can be written as

\[
\begin{align*}
9.5 \cdot (l_2 + l_3) &\leq l_5 \leq 1.05 \cdot (l_2 + l_3), \\
9.5 \cdot (l_2 + l_3) &\leq l_6 \leq 1.05 \cdot (l_2 + l_3), \\
9.5 \cdot (l_2 + l_3) &\leq l_7 \leq 1.05 \cdot (l_2 + l_3), \\
0.2 \cdot (l_1 + l_4) &\leq l_8 \leq 0.5 \cdot (l_1 + l_4), \\
0.2 \cdot (l_1 + l_4) &\leq l_9 \leq 0.5 \cdot (l_1 + l_4), \\
0.2 \cdot (l_1 + l_4) &\leq l_{10} \leq 0.5 \cdot (l_1 + l_4), \\
0.2 \cdot (l_1 + l_4) &\leq l_{11} \leq 0.5 \cdot (l_1 + l_4).
\end{align*}
\]  

(55)

To reduce the optimization difficulty, we suppose that $l_1$ and $l_4$ are the same link and form a triangle with $l_2$ and $l_3$. The geometric constraint conditions of the quadrilateral $GEMF$ and triangle $ADC$ are

\[
\begin{align*}
l_1 + l_4 &> l_2 + l_3, \\
|l_1 - l_4| &< l_2 + l_3, \\
l_{10} + l_{11} &> l_9, \\
|l_{10} - l_{11}| &< l_9.
\end{align*}
\]  

(56)
Figure 9 indicates that after the geometric constraint conditions are added, the bionic characteristics of the mechanism become better than those without geometric constraints. This improvement allows the robot to jump by imitating the rapid changes in the relative angles of the tibia and femur of a locust’s hind leg. The structure shown in Figure 9(b) cannot easily simulate the movement of the hind leg of a locust without interference.

(2) Resulting Constraint Conditions. For the PSO algorithm, due to the uncertainty of particle search, the final optimization results differ, which directly affects the jumping stability of the robot. Hence, the optimization results are constrained to prevent them from being a local minimum solution that cannot satisfy the requirement. The value $H(l)$ should satisfy the following equation:

$$H < \lambda H_0,$$  \hspace{1cm} (57)

where $H_0$ is the result obtained by incorporating the initial value of link length into equation (37). To obtain a good result, coefficient $\lambda$ is introduced. Suppose that $\lambda = 0.1$, $\omega_1 = 0.9$, $\omega_2 = 1.1$, $l_1 = 32$ mm, $l_2 = 20$ mm, and $l_4 = 1.3$ mm. The initial values of each parameter in the optimization process are shown in Table 3. In the process of optimization, the coordinate of the center of mass in the fixed coordinate system is $(15$ mm, $20$ mm), the spring stiffness coefficient is $1.0 \times 10^{-3}$ N/mm, and the mass is $55$ g. Among them, the mass of the robot and the coordinate of the center of mass in the fixed coordinate system are obtained according to the actual measurement results of the physiological structure of the locust. The spring stiffness coefficient is calculated by taking the locust’s takeoff velocity (about $1.2$ m/s) as the constraint condition. Link length is determined by experience to make its scale similar to that of locusts. However, when the result constraint conditions are not given, the optimization algorithm can generate a set of results after completing a number of iterations. As indicated in Figure 10, when no result constraint conditions exist, the total inertia moment and trunk rotation angle are large although the optimization process has completed the specified iteration times. The maximum value of total inertia moment is $-4.7 \times 10^{-2}$ Nm, and it fluctuates significantly in the entire takeoff process. The length of the link corresponds to the local minimum. When the result constraint conditions are considered, the maximum value of the total inertia moment is $5.54 \times 10^{-4}$ Nm, and the variance is only $5.33 \times 10^{-8}$. Similarly, the rotation angle of the trunk decreases significantly when the result constraint conditions are considered.

### 3.4.3. Optimization Process and Result Analysis

As the number of iterations increases, the value of the objective function corresponding to 40 particles decreases. After completing 10 iterations, the objective function value is significantly reduced. The iteration process is shown in Figure 11.

After the optimization is completed, the optimized link parameters are obtained, as shown in Table 4. The jumping performance before and after optimization is shown in Figure 12. The optimized total inertia moment is significantly reduced, and the attitude is stable. The horizontal and vertical distances are better than those before optimization. Therefore, the jumping performance of the robot is significantly improved, and the optimization effect is obvious, which proves the effectiveness of the algorithm.

### 4. Experiment

A robot prototype with a six-bar jumping leg is shown in Figure 13. The robot consists of the trunk, buffering leg, and two jumping legs. The coordinate origin of coordinate system $o_r-x_ryrz_r$ coincides with the coordinate origin of coordinate system $o_t-x_ty_tz_t$. The direction of the $x_r$ axis is parallel to the upper edge of the trunk. The direction of the $y_r$ axis is perpendicular to the trunk plane and inward. $z_r$ can be determined by the right-hand rule. The robot structural parameters are presented in Table 5. In order to ensure that the position of the center of mass is consistent with the theoretical calculation results, a counterweight is added at the lower end of the head of the robot. The total mass of the robot without counterweight is $229$ g. The weight of the counterweight is $43$ g. At this time, link length is increased six times on the basis of the optimized results. The robot prototype consists of a limiting cam, motor, sensor, spring, and USB port. We control the motor to drive the cam, and the spring deforms to make the robot take off. Acceleration and trunk attitude of the robot are collected by sensors. The sampling frequency of the sensor is $500$ Hz.

The variable parameters of the robot mainly include the initial attitude angle of trunk (the angle between the upper edge of trunk and the ground), the position of the center of mass, and the spring coefficient. The jumping stability of the robot was analyzed by changing these three parameters. During the process of experiment, acceleration and angle of the robot are collected by sensors. Then, by integrating acceleration using the cumulative trapezoidal numerical integration function in MATLAB, velocity and displacement of the robot can be obtained.

#### 4.1. Initial Attitude Angle of Trunk

The experiment shows that the initial attitude angle of trunk is within $[-20, 0]$ when the coordinates of the position of center of mass is $(0$ mm, $0$ mm, $0$ mm) and $k = 1.0$ N/mm, and the rotation angle of the robot in the process of jumping is within acceptable range (the robot does not have a tendency to tip over in landing process). The attitude changes and jumping distance of the robot during takeoff are shown in Figure 14(a), and the change in velocity and acceleration in the takeoff phase of the robot is shown in Figure 14(b). Figure 14(a) indicates that when the initial attitude angle of trunk is $0$, the maximum rotation angle of the trunk is about $33.56$° during the entire jumping process. When the initial angle of takeoff is $-20$, the maximum rotation angle of the trunk is only $20.36$°. This shows the robot can have good jumping stability by reducing the initial angle of takeoff. In addition, the decrease in the initial attitude angle of trunk...
increases the jumping distance of the robot, but the jumping height decreases. The change in velocity and acceleration is consistent with the jumping distance.

4.2. Spring Stiffness Coefficient. Through multiple experiments, we choose three reasonable spring stiffness coefficients, and they are $k = 0.5 \text{ N/mm}$, $k = 1.0 \text{ N/mm}$, and...

---

**Figure 9:** Mechanism configuration with different geometric constraint conditions. (a) Considering geometric constraint conditions. (b) Without considering geometric constraint conditions.

**Table 3:** Initial values of each parameter.

| $e$ | $M_{m0}/(\text{Nm})$ | $\theta_{d0}$ (rad) | $\gamma_{20}$ (rad) | $\gamma_{21}$ (rad) | $l_5$ (mm) | $l_6$ (mm) | $l_7$ (mm) | $l_8$ (mm) | $l_{10}$ (mm) | $l_{11}$ (mm) |
|-----|-----------------------|----------------------|---------------------|---------------------|-------------|-------------|-------------|-------------|----------------|-------------|
| Value | $7.40 \times 10^{-1}$ | $1.34 \times 10^{-1}$ | $9.94 \times 10^{-2}$ | $0$ | $3.64 \times 10^{-2}$ | $10$ | $26$ | $26$ | $22$ | $10$ | $9$ | $4$ |

---

**Figure 10:** Comparison of motion performance with and without result constraint conditions. (a) Total inertia moment of the robot and (b) rotation angle of the trunk.
Table 4: Parameters of the six-bar jumping leg.

|                | \( l_3 \) (mm) | \( l_5 \) (mm) | \( l_6 \) (mm) | \( l_7 \) (mm) | \( l_9 \) (mm) | \( l_{10} \) (mm) | \( l_{11} \) (mm) |
|----------------|-----------------|----------------|----------------|----------------|----------------|-----------------|------------------|
| Before optimization | 12.07           | 32.78          | 30.93          | 27.43          | 12.05          | 11.36           | 1.41             |
| After optimization  | 12.07           | 32.78          | 30.93          | 27.43          | 12.05          | 11.36           | 1.41             |

Figure 11: Optimized iterative process.

Figure 12: Continued.
Figure 12: Comparison of motion performance of the unoptimized and optimized jumping leg. (a) Total inertia moment, (b) trunk attitude, (c) jumping distance, and (d) jumping height.

Figure 13: Robot prototype with a six-bar jumping leg. (a) 3D model of the robot and (b) the prototype of the robot.
$k = 1.5 \text{ N/mm}$. Suppose that the initial attitude angle of trunk is $-20^\circ$ and the position of center of mass is (0 mm, 0 mm, 0 mm). The attitude changes and jumping distance of the robot during takeoff are shown in Figure 15(a), and the change in velocity and acceleration in the takeoff phase of the robot is shown in Figure 15(b). Figure 15 indicates that the increase in spring stiffness coefficient increases the stored energy, jumping height, and jumping distance, but the robot is likely to overturn. Therefore, the selection of the spring stiffness coefficient needs to be considered comprehensively.

4.3. Position of the Center of Mass. Three different positions of the center of mass were taken as examples for analysis. The coordinates of the three different positions are (0 mm, 0 mm, 0 mm), (16 mm, 0 mm, 0 mm), and (37 mm, 0 mm, 0 mm).
The three positions of the center of mass are $O_{c1}$, $O_{c2}$, and $O_{c3}$, respectively, which are shown in Figure 13(b). The change of the position of the center of mass is achieved by changing head counterweight. The mass of the head counterweight corresponding to the three positions of the center of mass is 43 g, 58 g, and 73 g, respectively. The measurement process of the counterweight is shown in Figure 16. Suppose that the initial attitude angle of trunk is $-20^\circ$ and $k = 1.5 \text{ N/mm}$. The attitude change and jumping distance of the robot during takeoff are shown in Figure 17(a). The change in velocity and acceleration in the takeoff phase of the robot is shown in Figure 17(b). Figure 17 shows that when the center of mass is at the origin of the coordinate system, the rotation angle of the trunk is 35.74° during the entire jumping process. When the coordinate is $(37 \text{ mm}, 0 \text{ mm}, 0 \text{ mm})$, the rotation angle of the trunk is 12.19°. The rotation angle decreases sharply. Thus, as the center of mass moves along the positive direction of the $x_r$ axis, the rotation angle of the trunk during the jumping of the robot decreases, and the jumping distance increases. When the robot leaves the ground instantaneously, the total inertia moment of the robot is not zero, and the robot tends to turn backward. Moving the center of mass forward can effectively reduce the rotation angle of the robot and improve the jumping stability of the robot.

When the coordinates of the three different positions are $(0 \text{ mm}, 0 \text{ mm}, 0 \text{ mm})$, $(16 \text{ mm}, 0 \text{ mm}, 0 \text{ mm})$, and $(37 \text{ mm}, 0 \text{ mm}, 0 \text{ mm})$, the corresponding jumping sequences are as shown in Figure 18. The jumping trajectory of the robot at this time is shown in Figure 19. The robot can realize stable
jumping for these three cases, that is, it will not turn over at a large angle during the jumping process.

In summary, the initial attitude angle of trunk should have a small value, the position of the center of mass should move forward along the $x_r$ axis, and the spring stiffness coefficient should be large but within the allowable range (the spring stiffness coefficient has a more obvious effect on jumping distance than on trunk rotation angle).

This research mainly focuses on the jumping stability of pause-and-leap jumping robot. The link length is taken as the optimization parameter, and the feasible ranges of initial attitude angle of trunk, position of the center of mass, and

Figure 18: Jumping sequence of the robot. The coordinates of the center of mass are (a) $(0\,\text{mm}, 0\,\text{mm}, 0\,\text{mm})$, (b) $(16\,\text{mm}, 0\,\text{mm}, 0\,\text{mm})$, and (c) $(37\,\text{mm}, 0\,\text{mm}, 0\,\text{mm})$. 
spring stiffness coefficient are given preliminarily. However, there are still some limitations in this study. For example, the jump height of a robot is relatively low. It can be seen from the simulation and experimental result that the parameters affecting the jumping height of the robot include the spring stiffness coefficient, the position of the mass center, and the initial attitude of the trunk. 12 groups of experiments are carried out, and the parameters are shown in Table 6. At this time, the jumping height of the robot corresponding to different groups is as shown in Figure 20. It can be seen from Figure 20 that if the jumping height needs to be increased, the spring stiffness coefficient can be appropriately increased, and the initial attitude of the trunk and the position of the center of mass can be appropriately changed so that the robot can increase the jumping height and ensure the jumping stability at the same time. If the jumping height needs to be further improved, the total mass of the robot needs to be reduced to reduce the total inertia moment, and the mass distribution of the robot needs to be optimized with the total mass unchanged. Besides, the coupling relationship among link length, spring stiffness coefficient, and position of center of mass needs to be further analyzed in the optimization process with more optimization parameters to make the jumping robot have better jumping performance.

5. Conclusions

The one-DOF six-bar jumping leg has a simple structure, easy control, and high stiffness. The focus of research is how to provide robots with a one-DOF jumping leg good jumping stability. Through theoretical modeling and analysis, a group of lengths of jumping leg is determined in this paper. Then, the experiment is conducted to clarify the effect of design parameters including the initial attitude angle of trunk, position of the center of mass, and spring stiffness coefficient on the jumping stability. The instructive conclusions of this study include the following. (a) The optimization method proposed in this paper can improve the efficiency of optimization, and the optimization result can guarantee the jumping stability of the robot. (b) When the allowable value of trunk rotation angle is given, the smaller the initial attitude angle of trunk is, the smaller the trunk rotation angle is and the better the jumping stability is. Therefore, in the control of a jumping robot, the initial attitude angle of trunk can be reduced appropriately to make the robot have good jumping stability. (c) When the center of mass moves along the positive position of $X_t$ axis, the rotation angle of the robot decreases. Hence, in the manufacture of the jumping robot, the position of the center of mass can be mounted near the head of the robot so that the robot can have good jumping stability. (d) For the stiffness coefficient of the driving spring, the position of the center of mass and the initial attitude of the trunk should be
considered synthetically. This work provides a reference for research on the stability of robots.

Future work could include a comparative study of the jumping stability of robots with a six-bar jumping leg with different configurations. The jumping stability of robots in 3D space also needs to be investigated. In addition, jumping height and distance should also be taken as optimization objectives, and more optimization parameters should be considered.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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