Steady-states of relativistic Pierce diode in a regime with electron reflection from a potential barrier

A B Gerasimenko and V I Kuznetsov

1 Ioffe Institute, Politekhnicheskaya Street 26, Saint-Petersburg 194021, Russia
E-mail: gerasimenko.alexander@mail.ioffe.ru

Abstract. In this work we present the study of steady states of a relativistic Pierce diode. We take into account electron scattering by heavy background ions as well as electron reflection from a potential barrier. Using Lagrangian variables we solve the problem numerically. Steady-state solutions are visualized through the ‘Bursian’ and ‘Non-Bursian’ branches in a parametric plane. Features of these branches with varying in mean collision frequency are analysed.

1. Introduction

The description of a number of astrophysical objects, for example, the vicinity of pulsars [1], as well as the operation of technical devices, such as powerful microwave generators (vircators) [2], are based on plasma diodes. The development and improvement of these devices requires the study of various problems of the physics of relativistic plasma diodes [3]. In recent works [4, 5], the states of a diode with a relativistic electron beam were investigated. However, the authors did not take into account the possible reflection of electrons from the potential barrier existing near the emitter. In this paper, we study the effect of electron reflection on steady states of a diode. However, for the time being we do not take into account the energy losses associated with radiation during electron braking.

We studied steady state solutions for the Pierce diode (a plasma diode with an electron flow whose charge is fully compensated by the charge of the background ions). Dependences for the velocities and potential in the inter-electrode gap on the coordinate are obtained. To represent the solutions, we used the $(\epsilon_0, \delta)$–diagram [6].

2. Statement of the problem

When considering the problem, we limited ourselves to a one-dimensional diode model (Figure 1). As shown in previous work [7], the use of such a model allows a fairly accurate qualitative analysis of the physics of the process under study. In our model, the relativistic electron flow coming out of the emitter is assumed to be monoenergetic. Background ions are considered as immobile. Collisions of electrons with background particles are simplified by introducing friction force into the moment equation proportional to the mean collision frequency and the electron velocity. Earlier, a similar problem was solved in Ref. [5] for the case when there is no potential barrier in the inter-electrode gap reflecting a portion of the electrons. To take into account the reflection of electrons with a $\delta$–shaped energy distribution function, we introduce, as in Ref. [7], the reflection coefficient $r$ equal to the ratio of the number of electrons reflected from the...
potential barrier to the number of electrons entering into the gap. In this paper, we do not take into account the process of radiation by electrons.

To obtain process characteristics, we solve a system consisting of three equations: the continuity equation, the relativistic momentum equation for electrons, and the Poisson equation. We use the following dimensionless quantities for a more convenient presentation of the results: coordinate $\zeta = z/\lambda_D$, velocity $u = v/\sqrt{2W_0/m}$, time $\tau = t\omega_0$, potential $\eta = e\varphi/(2W_0)$ and electric field strength $eE\lambda_D/(2W_0)$, here $\omega_0 = [e^2n_0/(m\varepsilon_0)]^{1/2}$ has the meaning of a characteristic frequency. We use the energy of electrons at the emitter and the Debye length as units of energy and length:

$$W_0 = (\gamma_0 - 1)mc^2, \quad \lambda_D = \left[\frac{2\varepsilon_0 W_0}{e^2 n_0}\right]^{1/2}.$$

Here $c$ is the speed of light in vacuum, $\varepsilon_0$ is the permittivity of free space, $m$ and $e$ are the rest mass and electron charge, relativistic factor

$$\gamma = \left[1 - (v/c)^2\right]^{-1/2},$$

and $\gamma_0$ is the relativistic factor at the emitter. Then the velocity of the electrons on the emitter is expressed via the relativistic factor as

$$u_0 = \frac{\gamma_0}{\sqrt{\gamma_0 + 1/\gamma}}.$$

The basic equations in a dimensionless form take the following form:

$$nu = u_0 H(\zeta; r, \zeta_r),$$

$$u \frac{\partial}{\partial \zeta} (p) = -\epsilon - \nu u, \quad \frac{\partial}{\partial \zeta} \epsilon = \alpha - n.$$

Here $p = \gamma u$, $\tau_r$ and $\zeta_r = \zeta(\tau_r)$ is the moment of reflection and the position of the electron at this moment, and the function $H(\zeta; r, \zeta_r) = 1 + r$ when $\zeta < \zeta_r$, and $1 - r$ when $\zeta > \zeta_r$. Relativistic factor can be expressed in terms of velocity

$$\gamma = 1/\sqrt{1 - 2(\gamma_0 - 1)u^2}. $$
3. Results
When solving the set of equations (3) it is more convenient to proceed from Eulerian set of variables \((\zeta, \tau)\) to the Lagrangian ones \((\tau_0, \tau)\) [8]. In the area located between the emitter and the reflection point this system takes the form:

\[
\begin{align*}
\frac{dp}{d\tau} &= -\epsilon(\tau) - \nu u, \\
\frac{d\epsilon}{d\tau} &= \alpha u - u_0(1 + r)
\end{align*}
\]  

(5)

with boundary conditions

\[
\begin{align*}
u(0) &= u_0 = \sqrt{\gamma_0 + 1}/(\sqrt{2}\gamma_0), \\
\frac{du}{d\tau}(0) &= -(\epsilon_0 + \nu u_0)[1 - 2u_0^2(\gamma_0 - 1)]^{3/2}.
\end{align*}
\]  

(6)

To the right of the reflection point equations (3) are transformed into the equations:

\[
\begin{align*}
\frac{dp}{d\tau} &= -\epsilon(\tau) - \nu u, \\
\frac{d\epsilon}{d\tau} &= \alpha u - u_0(1 - r)
\end{align*}
\]  

(7)

with boundary conditions

\[
\begin{align*}
u(\tau_r) &= 0 \\
\frac{du}{d\tau}(\tau_r) &= 0.
\end{align*}
\]  

(8)

Equations (5), (6), (7) and (8) were solved numerically.

![Dependences of the velocity and the potential on the coordinate](image)

**Figure 2.** Dependences of the velocity (a) and the potential (b) on the coordinate for \(\gamma_0 = 2\), \(\nu = 0.05\). (——) \(r = 0\); (- - - -) \(r = 0.3\); (— · —) \(r = 0.5\).

First, the system (5) is solved with boundary conditions (6) and the dependence of the particle velocity on time \(u(\tau)\) is calculated in the area located between the emitter and the reflection point \(\zeta_r\). The solution depends on the reflection coefficient \(r\). Since it should satisfy the boundary conditions (8), iterations are carried out by varying the parameter \(r\). As a result, we find the moment \(\tau_r\) when the particles are reflected and the electric field strength of the...
emitter $\epsilon_0$. The coordinate of the electron reflection point $\zeta_r$ is found by integration of the velocity

$$\zeta(\tau_r) = \int_0^{\tau_r} u(t; r) dt.$$  \hspace{1cm} (9)

Now, using the known values of $r$, $\epsilon_0$ and $\zeta_r$, the dependence of the potential on the coordinate was found for the area located to the left of the electron reflection point. After that, the calculation was performed for $\tau > \tau_r$, i.e. to the right of the point $\zeta_r$. During these calculations, the coordinates of the points were found where the potential values are equal to the collector potential. There may be several such solutions.

The calculation results are presented in Figure 2 in the form of spatial dependences of the potential and electron velocity. It can be seen that with the raise in the reflection coefficient, the intervals of velocity change turn more narrow as coordinate increase. Similar behaviour is observed in the dependence of the potential on the coordinate. Steady state solutions are presented in Figure 3 as curves on the $\{\epsilon_0, \delta\}$ parametric plane.

![Figure 3](image)

**Figure 3.** Steady state solutions for a diode with $\gamma_0 = 2$ and $\nu = 0(\cdots \cdots)$, $0.02(- - -)$, $0.05(---)$, $0.1(\cdots)$ in the representation on $(\epsilon_0, \delta)$ plane.

As in the case of a non-relativistic electron beam, all solutions are divided into two groups: the first group is Bursian curves which are characterized by one minimum potential distribution only, and the second group is non-Bursian one for which there are several of the minima. It is shown that non-Bursian solutions disappear with an increase in the mean collision frequency $\nu$ at $\nu/\omega_0 \sim 0.1$. 
4. Conclusion
Thus, potential distributions in the Pierce diode with a relativistic electron beam scattered by background particles and without radiation processes, are qualitatively similar to potential distributions in a diode with non-relativistic electrons [7]. However, taking radiation into account for large relativistic factors can lead to the appearance of new features.

References
[1] Shibata S 1997 *Monthly Not. Royal. Astron. Soc.* **287** 262
[2] Carter R G 2018 *Microwave and RF Vacuum Electronic Power Sources* (Cambridge: Cambridge University Press)
[3] Humphries S 2013 *Charged Particle Beams* (New York: Dover Publications)
[4] Greenwood A D, Hammond J F, Zhang P and Lau Y Y 2016 *Phys. Plas.* **25** 072101
[5] Pramanik S and Ghosh S 2019 *Phys. Plas.* **26** 113503
[6] Ender A Ya, Kuznetsov V I and Schamel H 2011 *Phys. Plasmas* **18** 033502
[7] Kuznetsov V I, Gerasimenko A B, Pramanik S and Chakrabarti N 2019 *Phys. Plas.* **26** 123513
[8] Schamel H and Bujarbarua S 1993 *Phys. Fluids* B **5** 2278