Scattering in structured two-layered medium

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Abstract. Multiple scattering in two-layered medium was modeled using Monte Carlo numerical simulation. Special case of group formation in such layers was considered. To estimate the impact of structure factor on the overall results, the rigorous calculation of single scattering phase function was used. Comparison with the Monte Carlo simulations for the respective model phase function shows that there were some differences in the scattered radiance. These differences had no significant energy redistribution in the angular dependence of the scattered radiance, but different elongation in the zero-direction occurred. So it was shown that the group formation and special structure still leaded to some changes in the scattering process, even if the medium had several layers with different parameters and had rather large width.

1. Introduction
One of the most difficult cases in the scattering problem is the wave propagation through a medium with densely packed particles, like colloids, powders, photonic crystals, etc. [1], [2], [3], [4], [5], [6], [7], [8], [9]. Most approximations and strong solutions do not work for densely package, because particles are no more independent and thus multiple scattering should be taken into account [10], [11], [12]. The order of particles in such medium can be random or periodic. Also particles can be joined in groups distributed randomly in the scattering volume. This group formation characterizes such media as clouds with ice crystals, liquids at an initial stage of crystallization, disperse systems during the coagulation process. It is evident, that the dense package leads to interference effects for scattering on one group of particles [8], [13]. But the random character of groups distribution may remove such effects. And respectively, the radiation transfer will have no sensitivity to them. As the result, the random approximation for scattering medium is often used.

Monte Carlo (MC) numerical modeling [14], [15], [16], [17], [18] is an effective way to solve radiation transfer equation (RTE) [19]:
\[
L(x) = L_0(x) + \int k(x, x')L(x')dx',
\]
where the integral kernel is:
\[
k(x, x') = \frac{\Lambda \varepsilon}{4\pi} \chi(\hat{\mathbf{r}}, \hat{\mathbf{r}}')\delta \left( \frac{\mathbf{r} - \mathbf{r}'}{||\mathbf{r} - \mathbf{r}'||} \right) \exp(-\varepsilon||\mathbf{r} - \mathbf{r}'||) \left( \frac{\mathbf{r} - \mathbf{r}'}{||\mathbf{r} - \mathbf{r}'||^2} \right).
\]
where \(L_0(r, \hat{\mathbf{r}})\) is radiance of incident field at the point \(x = (r, \hat{\mathbf{r}})\), \(\Lambda\) is single scattering albedo, \(\varepsilon\) is extinction, \(\chi(\hat{\mathbf{r}}, \hat{\mathbf{r}}')\) is single scattering phase function.
Phase function $\chi(\hat{I}, \hat{I}')$ characterizes the interaction between the incident radiation and a single scattering group. In order to account the wave character of scattering process, it should be determined as precise as possible. It is possible to use the numerical solution of Maxwell equations for this purpose [20], [21], [22], [23]. In order to reduce computational time, we will deal only with 2D model of particles and consider that they have cylindrical form. As it was shown in [24], for different particle groups single scattering phase function significantly depends on the group structure type. Also in [24] it was shown, that structure factor has an essential influence on the angular distribution of scattered radiance for one scattering layer.

Here in this work we will discuss the scattering in the two-layered medium. Computational results will be obtained using Monte Carlo simulations in two cases - first, for the rigorously calculated phase functions, second, for the averaged phase function. Since the last one is often used for random medium, it will be possible to find out, whether two-layered medium may be approximated by the random layers.

2. Two-layered structured medium

The considered medium consists of two layers with different properties of scattering groups (figure 1). Scattering group from the first layer is formed by 3 identical particles oriented along $OX$ axis with radius $R/\lambda = 0.2$, period $T/\lambda = 3$ and dielectric permittivity $\epsilon = 2$; groups from the second layer have 4 particles with the same properties, where $\lambda$ is the wavelength of TEM polarized radiation, incident in $OZ$ direction (figure 2).

![Figure 1. Scattering medium model formed by two layers.](image)

Using FDTD numerical algorithm [20], [21], [22] for finding scattered field distribution and the Kirchhoff diffraction integral for complex field amplitudes we can calculate the single scattering phase function $\chi(\theta)$, $\theta$ is the scattering angle. It is shown in the figure 3.

Phase function is characterized by the asymmetry parameter [25]:

$$g = \int_0^\pi \chi(\theta) \sin \theta \cos \theta d\theta,$$

which was calculated for the obtained $\chi(\theta)$ from figure 3. Numerical values are, respectively, $g_1 = 0.2598$ for the first layer and $g_2 = 0.0061$ for the second layer.
Figure 2. Form and orientation of particles in the groups of a) 3 cylinders and b) 4 cylinders (2D model).

Figure 3. Scattering phase functions for groups of a) 3 particles and b) 4 particles.

Parameter $g$, which is the mean cosine $< \cos \theta >$, shows the degree of energy redistribution from the zero-direction to other angles during a single scattering event. It is also used for a random medium as the main parameter for a Henyey-Greenstein phase function model \[26\]:

$$\chi_{HG}(\theta, g) = \frac{1}{4\pi} \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{3/2}}. \quad (4)$$

We will use this average phase function for Monte Carlo simulations of multiple scattering in the considered two-layered medium.

3. Monte Carlo simulation

For the numerical simulation of the multiple scattering, MC computational scheme with single isotropic source was used. The scattered radiance in the single point in the zero direction can be determined applying double local estimation in MC simulation \[17\]. In order to do this, the
scattered radiance should be divided into zero, first, and higher orders:

\[ L(x) = L_0(x) + L_1(x) + \int k_I(x, x')L(x')dx', \]  
(5)

\[ k_I(x, x') = \left( \frac{\Lambda \varepsilon}{4\pi} \right)^2 \chi(\hat{I}, \hat{I}')\chi(\hat{I}, \hat{I}'') \frac{\exp(-\varepsilon|\mathbf{r}'' - \mathbf{r}'|)}{(\mathbf{r}'' - \mathbf{r}')^2}, \]  
(6)

the first and higher orders of scattered radiance should be found separately [24]. For the first order scattering there is the following equation [27]:

\[ L_1(\theta) = \frac{\Lambda \varepsilon \exp(-\varepsilon r)}{4\pi r \sin \theta} \int_0^\pi \chi(\eta) \exp \left[ -\varepsilon r \sin \theta \left( \tan \frac{\eta}{2} - \tan \frac{\theta}{2} \right) \right] d\eta. \]  
(7)

**Figure 4.** Scattered radiance numerically obtained for two cases: using Henyey-Greenstein phase function and using calculated (“real”) phase function.

Higher order scattered radiation will be determined from the MC simulation. For both first and higher orders the probability of crossing the border between layers was equal to unity, because the layers have no real interface. According to the double local estimation in MC simulation, integration of higher orders in the RTE equation (1) over the angular space is applied in order to find the radiance at a given point \( \mathbf{r} \) as a mean value of a random variable:

\[ L_2(x) = M\xi(x), \]  
(8)

\[ \xi(x) = \sum_{i=1}^N Q_i h_1(x_i), \]  
(9)

\[ h_1(x_i) = \chi_1(\theta_1)\chi_2(\theta_2) \frac{\exp(-\varepsilon|\mathbf{r} - \mathbf{r}'|)}{(\mathbf{r} - \mathbf{r}')^2}. \]  
(10)
where $Q_i$ is a “weight” of a photon after the $i$-th collision. If the medium single phase function is the same for different scattering events, $\chi_1(\theta_1)$ and $\chi_2(\theta_2)$ can be substituted in (10) with $\chi(\theta)$.

The number of simulations was $N = 5000$, layer length was $z_0 = 2m$. In the figure 4 the scattered radiance is shown for both computational cases (with Henyey-Greenstein and numerically obtained phase functions). As it can be noticed, differences in the obtained curves are presented, since the structure group formation still impact on the scattered radiance, comparable with the MC results for the averaged phase function, where this impact vanished. On the other hand, special redistribution of energy from zero-direction to other angles is no more present, which can be explained by the large length of layers and by the mismatch of secondary maxima for single layer scattering.

### 4. Conclusion

As we can see from the obtained results, there are no obvious impact of the group formation on the scattered radiation. But also there are no mismatch with the results of numerical simulations using Henyey-Greenstein phase function. All these facts show that such rather large two-layers medium behaves like a random one, but can not be approximated by random model with the single-parameter Henyey-Greenstein scattering phase function.

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