SCALAR TENSOR THEORIES AND HADAMARD STATE CONDITION

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Abstract

The Hadamard state condition is used to analyze the local constraints on the two-point function of a quantum field conformally coupled to a background geometry. Using these constraints we develop a scalar tensor theory which controls the coupling of the stress-tensor induced by the two-point function of the quantum field to the conformal class of the background metric. It is then argued that the determination of the state-dependent part of the two-point function is connected with the determination of a conformal frame. We comment on a particular way to relate the theory to a specific conformal frame (different from the background frame) in which the large scale properties are brought into focus.

1 Introduction

The essential feature of scalar tensor theories, such as Brans-Dicke theory, is to generalize general relativity and bring it into accord with Mach’s principle (the origin of physical properties of space is in the matter contained therein [1]). These theories are not completely geometrical since the gravitational effects are described by a scalar field as well as a metric tensor. In fact, the global distribution of matter affects the local gravitational properties through the emergence of a scalar field. The implementation of this interrelation between global and local properties of matter in quantum field theory, as demanded by Mach’s principle, is a complicated problem. Some ideas in this direction can be found in [2, 3].

In a simplified picture one can expect that the role of a scalar tensor theory may be of importance for improving our knowledge on the local properties of a linear quantum field propagating in a gravitational background, in particular the local properties of the quantum stress-tensor induced by the two-point function of the quantum field. The present paper deals with the consideration of this issue. In specific terms, we study a model in which the local properties of

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a linear quantum field conformally coupled to a gravitational background is affected both by the local geometry and a conformal invariant scalar field derived from the state (boundary)-dependent part of the two-point function. To arrive at this model we basically take into account the local constraints imposed on the two-point function by the Hadamard state condition. In this context there is a problem concerning the specification of the state-dependent part of the two-point function. In our presentation we establish a connection between this problem and the problem of the determination of a conformal frame.

To avoid any confusion at the outset, we should note that the scalar tensor theory we wish to consider is meant only to provide an analytical mean to determine the general properties of a quantum stress-tensor that can consistently be coupled to conformally related background metrics, and in this respect its interpretation differs from the standard interpretation of such theories as alternative theories of gravitation.

The organization of this paper is as follows: In section 2 we present the Hadamard prescription and review the derivation of the local constraints on the state-dependent part of two-point function of a linear scalar quantum field conformally coupled to gravity. In section 3, we present a way to use a conformally invariant scalar field for analyzing the state-dependent part of the two-point function. It is shown that the implications of the resulting scalar tensor theory for the stress-tensor are in accord with the standard predictions of the renormalization theory. In section 4, we make some general remarks on the existence of an alternative frame in which the trace of the stress-tensor is determined by a cosmological constant rather than the usual anomalous trace. The existence of this frame indicates that the state-dependent part of the two-point functions may have some large scale characteristics which are basically not present in the conformal frame determined by the local characteristics. Similar arguments were discussed previously in a different context [3].

2 Hadamard state condition

We consider a free scalar quantum field \( \phi(x) \) propagating in a curved background spacetime with the action functional [4] (We use the conventions of Hawking and Ellis [5] for the signature and the sign of curvature)

\[
S[\phi] = -\frac{1}{2} \int d^4x g^{1/2}(g^{\alpha\beta}\nabla_\alpha \phi \nabla_\beta \phi + \xi R \phi^2 + m^2 \phi^2)
\]  

(1)

where \( m \) and \( \xi \) are parameters, and \( R \) is the scalar curvature (In the following the semicolon and \( \nabla \) indicate covariant differentiation). This gives rise to the field equation

\[
(\Box - m^2 - \xi R)\phi(x) = 0
\]  

(2)

The choice of the parameters \( m \) and \( \xi \) depends on the particular type of coupling. For example, the minimal coupling corresponds to \( m=0, \xi = 0 \) and the conformal coupling (in four dimensions) corresponds to \( m=0, \xi = 1/6 \). A state of \( \phi(x) \) is characterized by a hierarchy of Wightman-functions (n-point functions)

\[
\langle \phi(x_1), ..., \phi(x_n) \rangle
\]  

(3)
We are primarily interested in those states which reflect the intuitive notion of a "vacuum". For this aim, we may restrict ourselves basically to quasi-free states, i.e. states for which the truncated n-point functions vanish for $n > 2$ (In a linear theory this property is shared by the vacuum state of Minkowski space). Such states may be characterized by their two-point functions. In a linear theory the antisymmetric part of the two-point function is common to all states in the same representation. It is just the universal commutator function. Thus, in our case all the relevant informations about the state-dependent part of the two-point function are encoded in its symmetric part, denoted in the following by $G^+(x, x')$, which satisfies Eq.(2) in each argument. Equivalence principle suggests that the leading singularity of $G^+(x, x')$ should have a close correspondence to the singularity structure of the two-point function of a free massless field in Minkowski space. In general the entire singularity of $G^+(x, x')$ may have a more complicated structure. Usually one assumes that $G^+(x, x')$ has a singular structure represented by the Hadamard expansions. This means that in a normal neighborhood of a point $x$ the function $G^+(x, x')$ can be written $[3, 4, 5]$ as

$$G^+(x, x') = \frac{1}{8\pi^2} \left\{ \frac{\Delta^{1/2}(x, x')}{\sigma(x, x')} + V(x, x') \ln \sigma(x, x') + W(x, x') \right\}$$

(4)

where $2\sigma(x, x')$ is the square of the distance along the geodesic joining $x$ and $x'$ and $\Delta(x, x')$ is the Van Vleck determinant

$$\Delta(x, x') = -g^{-1/2}(x)\det\{-\sigma_{\mu\nu}\}g^{-1/2}(x')$$

$$g(x) = \det g_{\alpha\beta}$$

(5)

The functions $V(x, x')$ and $W(x, x')$ are regular and have the following representations as power series

$$V(x, x') = \sum_{n=0}^{+\infty} V_n(x, x')\sigma^n$$

(6)

$$W(x, x') = \sum_{n=0}^{+\infty} W_n(x, x')\sigma^n$$

(7)

in which the coefficients are determined by applying Eq.(2) to $G^+(x, x')$, yielding the recursion relations

$$(n+1)(n+2)V_{n+1} + (n+1)V_{n+1;\alpha}\sigma^{\alpha} - (n+1)V_{n+1}\Delta^{-1/2} \Delta^{1/2}_{\alpha} \sigma^{\alpha} + \frac{1}{2}(\Box - m^2 - \xi R)V_n = 0$$

(8)

$$(n+1)(n+2)W_{n+1} + (n+1)W_{n+1;\alpha}\sigma^{\alpha} - (n+1)W_{n+1}\Delta^{-1/2} \Delta^{1/2}_{\alpha} \sigma^{\alpha} + \frac{1}{2}(\Box - m^2 - \xi R)W_n$$

$$(2n+3)V_{n+1} + V_{n+1;\alpha}\sigma^{\alpha} - V_{n+1}\Delta^{-1/2} \Delta^{1/2}_{\alpha} \sigma^{\alpha} = 0$$

(9)

together with the boundary condition

$$V_0 + V_0;\alpha\sigma^{\alpha} - V_0\Delta^{-1/2} \Delta^{1/2}_{\alpha} \sigma^{\alpha} + \frac{1}{2}(\Box - m^2 - \xi R)\Delta^{1/2} = 0$$

(10)

From these relations one can determine the function $V(x, x')$ uniquely in terms of local geometry. Therefore it takes the same universal form for all states. But the biscalar $W_0(x, x')$ 

remains arbitrary. Its specification depends significantly on the choice of a state and may be regarded as the imposition of a boundary condition. However there is a general constraint on $W_0(x, x')$ which can be obtained from the symmetry condition of $G^+(x, x')$ together with the following dynamical equation which can be obtained using (2), (4) and (6).

\[(\Box - m^2 - \xi R)W(x, x') = -6v_1(x) + 2v_{1;\alpha}\sigma^{\alpha x} + 0(\sigma)\]  

(11)

where

\[v_1(x) = \lim_{x' \to x} V_1(x, x') = \frac{1}{720}\{\Box R - R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\delta\gamma}R^{\alpha\beta\delta\gamma}\} \]  

(12)

To get this constraint we first expand the symmetric function $W(x, x')$ into a covariant power series, namely

\[W(x, x') = W(x) - \frac{1}{2}W_{x0}(x)\sigma^{\alpha x} + \frac{1}{2}W_{\alpha\beta}(x)\sigma^{\alpha x}\sigma^{\beta x} + \frac{1}{4}\{\frac{1}{6}W_{\alpha\beta\gamma}(x) - W_{\alpha\beta;\gamma}(x)\}\sigma^{\alpha x}\sigma^{\beta x}\sigma^{\gamma x} + 0(\sigma^2)\]  

(13)

We may insert this into Eq. (11) and compare term by term up to the third order in $\sigma^{\alpha x}$ to obtain

\[W_{\gamma x}(x) = (\xi R + m^2)W(x) - 6v_1(x)\]  

(14)

\[\{W_{\alpha\beta}(x) - \frac{1}{2}g_{\alpha\beta}W_{\gamma x}(x)\}_{\alpha x} = \frac{1}{4}\{\Box W(x)\}_{\alpha\beta} - \frac{1}{2}m^2W_{\alpha\beta}(x) + 2v_1(x)_{\alpha\beta} + \frac{1}{2}R_{\alpha\beta}W^{\alpha x}(x) - \frac{1}{2}\xi RW_{\beta x}(x)\]  

(15)

Then using the covariant expansion of the symmetric function $W_0(x, x')$

\[W_0(x, x') = W_0(x) - \frac{1}{2}W_{0;\alpha}(x)\sigma^{\alpha x} + \frac{1}{2}W_{0\alpha\beta}(x)\sigma^{\alpha x}\sigma^{\beta x} + 0(\sigma^{3/2})\]  

(16)

together with Eqs. (7), (9) and (13), we get

\[W(x) = W_0(x)\]  

(17)

\[W_{\alpha\beta}(x) = (W_{0\alpha\beta}(x) - \frac{1}{4}g_{\alpha\beta}W_{\gamma x}(x)) + \frac{1}{4}\{(m^2 + \xi R)W_0(x) - 6v_1(x)\}g_{\alpha\beta}\]  

(18)

Substituting (17) and (18) into (15) leads to

\[\{W_{0\alpha\beta}(x) - \frac{1}{4}g_{\alpha\beta}W_{0\gamma x}(x)\}_{\alpha x} = \frac{1}{2}v_{1;\alpha\beta}(x) + \frac{1}{4}\{\Box W_0(x)\}_{\alpha\beta} - \frac{1}{2}m^2W_{0;\alpha\beta}(x) + \frac{1}{2}R_{\alpha\beta}W_{0\alpha\beta}(x)\]

\[+ \frac{1}{4}\xi [R_{\alpha\beta}W_{0}(x) - RW_{0;\alpha\beta}(x)]\]  

(19)

This equation is a general constraint imposed on the state-dependent part of the two-point function. The function $W_0(x)$ may be considered as arbitrary, but once a specific assumption has been made on the form of $W_0(x)$, the equation (19) acts as a constraint on $W_{0\alpha\beta}(x)$.

We should note that the constraint (19) is, in principle, the first member of a hierarchy of constraints, because we have used the covariant expansion $W_0(x, x')$ only up to the second order in $\sigma^{\alpha x}$. Thus, in general, there are some additional constraints on the higher order expansion terms. In our analysis we shall neglect these higher order constraints. Such a limitation is suggested by dimensional arguments because the second order expansion terms of $W_0(x, x')$ has already the physical dimension of a stress-tensor.
3 The conformally invariant scalar field

In the case of conformal coupling a local Hilbert space would in general exhibit an essential sensitivity to the pre-existing local causal structure of space-time which in the present case is determined by the conformal class of the background metric. By implication, this causal structure should act as the basic input for the characterization of the local states. Since the conformal transformations leave the causal structure unchanged we expect, in particular, that an essential ambiguity, related to conformal transformations, should enter the dynamical specification of the state-dependent part of the two-point function. Thus, in the case of the conformal coupling, it is suggestive to develop a dynamical model in which the conformal symmetry acts as a fundamental symmetry in the specification of the two-point function, in particular the function $W_0(x)$. In the following we shall use the constraint (19) to develop a dynamical model along this line. We first start with the explicit form of the constraint (19) in the case of conformal coupling, namely

\[
[W_{0\alpha\beta}(x) - \frac{1}{4}g_{\alpha\beta}W_0^\gamma(x) - \frac{1}{2}g_{\alpha\beta}v_1(x) - \frac{1}{4}g_{\alpha\beta}\Box W_0(x)]^{\alpha}
= \frac{1}{2}R_{\alpha\beta}W_0^{\prime\alpha}(x) + \frac{1}{24}(R_{\alpha\beta}W_0(x) - RW_{0;\beta}(x)). \tag{20}
\]

One can use the Bianchi identity

\[
R_{\alpha\beta}^{\prime\alpha} = \frac{1}{2}R_{\alpha\beta} \tag{21}
\]

and the differential identity

\[
\Box(W_{0;\beta}(x)) = (\Box W_0(x))_{;\beta} + R_{\alpha\beta}W_0^{\prime\alpha}(x) \tag{22}
\]

to show that (20) can be written as a total divergence

\[
\Sigma_{\alpha\beta}^{\prime\alpha} = 0 \tag{23}
\]

where

\[
\Sigma_{\alpha\beta} = (W_{0\alpha\beta}(x) - \frac{1}{4}g_{\alpha\beta}W_0^\gamma(x)) - \frac{1}{6}(R_{\alpha\beta} - \frac{1}{4}Rg_{\alpha\beta})W_0(x) - \frac{1}{3}(W_{0;\beta\alpha}(x) - \frac{1}{4}g_{\alpha\beta}\Box W_0(x))
- \frac{1}{2}g_{\alpha\beta}v_1(x) \tag{24}
\]

Now, the basic input is to subject in (23) the choice of $W_0(x)$ to the condition

\[
W_0(x) = \psi^2(x) \tag{25}
\]

where $\psi(x)$ is taken to be a conformally invariant scalar field coupled to the gravitational background, so that its dynamical equation is

\[
(\Box - \frac{1}{6}R)\psi = 0. \tag{26}
\]
For a given Hadamard state the field $\psi$ may be interpreted as measuring the one-point function of the quantum field $\phi$. The conformal invariance of $\psi$ ensures that there exists no pre-assigned dynamical configuration for the one-point function in a local Hilbert space. This is indeed a desirable characteristic of a linear theory.

Technically, the merit of introducing the field $\psi$ is that the tensor $\Sigma_{\alpha\beta} + \frac{1}{2} g_{\alpha\beta} v_1(x)$, which is traceless due to (24), may now be related to the conformal stress-tensor of $\psi$, namely

$$\Sigma_{\alpha\beta} + \frac{1}{2} g_{\alpha\beta} v_1(x) = T_{\alpha\beta}[\psi]$$

(27)

where the conformal stress-tensor $T_{\alpha\beta}[\psi]$ is given by [12]

$$T_{\alpha\beta}[\psi] = \left(\frac{2}{3} \nabla_{\alpha} \nabla_{\beta} \psi - \frac{1}{6} g_{\alpha\beta} \nabla_{\gamma} \psi \nabla^{\gamma} \psi \right) - \frac{1}{3} \left(\psi \nabla_{\alpha} \nabla_{\beta} \psi - g_{\alpha\beta} \psi \Box \psi \right) + \frac{1}{6} \psi^2 G_{\alpha\beta}$$

(28)

in which $G_{\alpha\beta}$ is the Einstein tensor. The tensor $T_{\alpha\beta}$ is traceless due to the dynamical equation (26). The meaning of the relation (27) is that it defines a formal prescription which allows us to relate the tensor $W_{0\alpha\beta}(x)$ in (24) to the function $W_0(x)$ and the metric tensor $g_{\alpha\beta}$, so it characterizes a criterion to select the class of admissible Hadamard states. Taking into account (28) we can write this criterion as

$$G_{\alpha\beta} - 3\psi^{-2} g_{\alpha\beta} v_1(x) = 6\psi^{-2} (\Sigma_{\alpha\beta} + \tau_{\alpha\beta}(\psi)).$$

(29)

Here $\tau_{\alpha\beta}(\psi)$, is equal to $T_{\alpha\beta}[\psi]$ without the $G_{\alpha\beta}$-term, so it coincides up to a sign with the so-called modified energy-momentum (stress-) tensor [13]. Now, the basic strategy is to consider the tensor $\Sigma_{\alpha\beta}$ as the quantum stress-tensor induced by the two-point function. Our criterion can then be interpreted as a rule for relating the latter tensor to the local background geometry, as reflected in (29). The essential point is that this rule is expressed in the form of a scalar tensor theory in which the dynamics of the scalar field $\psi$ makes substantially no distinction between different frames in the conformal class of the background metric. The implication is that at the dynamical level all conformal frames may be considered as equivalent.

This conformal invariance reflects a basic connection between the state-dependent part of the two-point function and the pre-existing causal structure determined by the background metric. In particular, it establishes a basic connection between the properties of a given physical state in a local Hilbert space and those of a corresponding conformal frame. To see this in explicit terms let us consider a conformal transformation

$$\bar{g}_{\alpha\beta} = \Omega^2(x) g_{\alpha\beta}$$

$$\bar{\psi}(x) = \Omega^{-1}(x) \psi(x)$$

(30)

Due to (25), $W_0(x)$ would then transform as

$$\bar{W}_0(x) = \Omega^{-2}(x) W_0(x)$$

(31)

It is now clear from (31) that a given conformal frame may be characterized by the particular configuration of $W_0(x)$ (or alternatively $\psi$) in that frame. Therefore the problem of specification of $W_0(x)$ for a given physical state is basically connected with the problem of
determination of a conformal frame. In particular, different states characterized by conformally related configurations of \( W_0(x) \) should principally be supported on different conformally related metrics. The same conclusion holds for their stress-tensors.

At this point we make a general remark concerning the consistency of our results with the standard prediction of the renormalization theory. Focusing ourselves to the two-point function on the background metric we can take the trace of (27), to obtain

\[
\Sigma_\alpha^\alpha = -2v_1(x)
\]

This together with (23) characterize the general properties of the quantum stress-tensor on the background metric. These properties are consistent with the well-known results of the renormalization theory [14] and \( v_1(x) \) is actually the function that determines what is commonly known as the trace anomaly. In our presentation this quantum anomaly requires a somewhat distinct behavior of the scalar field \( \psi \). In fact, according to (23) and (27) and due to the nonvanishing trace anomaly, the tensor \( T_{\alpha\beta}[\psi] \), which may be considered as the stress-tensor of the field \( \psi \), appears not to be conserved on the background metric, requiring the dynamical properties of \( \psi \) on the background metric not to fit in with the properties of a diffeomorphism invariant action characterizing a C-number (classical) field. But it is necessary to stress that this behavior does not appear to be a physical contradiction in the present case. Actually, the scalar field \( \psi \) which characterizes the local property of the two-point function may in general change its configuration if one varies the two-point function within a local Hilbert space. Therefore, in general it may not act as a C-number field within a local Hilbert space. By implication, the standard results of a diffeomorphism invariant action may not be applied to \( \psi \). We note that a similar process of assigning non-diffeomorphism invariant properties to a local Hilbert space has been previously discussed in the context of generally covariant quantum field theory [2].

4 Λ-frame

The conformal symmetry which was established in the local specification of \( W_0(x) \) would imply that locally the stress-tensor \( \Sigma_{\alpha\beta} \) can be related to different conformal frames. Thus the question arises as to which frame should be considered as a physical frame. To deal with this question it is necessary to emphasize the role of the superselection rules which characterize the boundary conditions imposed on the physically realizable states and the corresponding Hilbert spaces. In general, the identification of a conformal frame as a physical frame depends on the particular superselection rule one wishes to apply. Of direct physical significance, in the present case, is a superselection rule that tells us how a local Hilbert space is linked to the large scale boundary conditions imposed on physical states. If the latter conditions correspond to the presence of large scale distribution of matter whose energy density is measured by a cosmological constant, one may subject the determination of a conformal frame (alternatively a local Hilbert space) to the asymptotic correspondence between the anomalous trace and a nonvanishing cosmological constant at sufficiently large spacelike distances. In general, this condition may not be realized in the underlying background frame, so in this case the physical frame is expected to be different from the background frame.
This observation opens a way to study the transition from the local characteristics of physical states in a local Hilbert space to the large scale characteristics, which is expected to be of particular importance for establishing the large scale gravitational coupling of physical states in a local Hilbert space. Since by such a transition the small distance properties are no more important, we may take the overall correspondence between the anomalous trace and a non-vanishing cosmological constant everywhere as the defining characteristic of a local conformal frame which, by implication, acts as the physical frame if one focuses on large scale characteristics of physical states in the presence of large scale distribution of matter. For the construction of this frame one needs only to apply a conformal transformation to the background frame which establishes the correspondence between the trace anomaly and a nonvanishing cosmological constant. Denoting the cosmological constant by \( \Lambda \), the corresponding conformal factor may be taken to satisfy the equation

\[
-3\Omega^2(x)\bar{\psi}^{-2}v_1(\Omega^2(x)g_{\alpha\beta}) = \Lambda \tag{33}
\]

Under this conformal transformation the equation (27) transforms to

\[
\Sigma_{\alpha\beta} - \frac{1}{6}\Lambda g_{\alpha\beta}\bar{\psi}^2 = T_{\alpha\beta}[\bar{\psi}] \tag{34}
\]

or, equivalently

\[
G_{\alpha\beta}(g_{\alpha\beta}) + \Lambda g_{\alpha\beta} = 6\bar{\psi}^{-2}(\Sigma_{\alpha\beta} + \tau_{\alpha\beta}(\bar{\psi})). \tag{35}
\]

Therefore in the new frame, which we call the \( \Lambda \)-frame, a scalar tensor theory with a cosmological constant is obtained together with Eq.(33) which is a complicated constraint on the conformal factor. In the \( \Lambda \)-frame, contrary to the background frame, the stress-tensor \( \Sigma_{\alpha\beta} \) may not be conserved. However Eq.(34) implies that one can establish a conserved stress-tensor by replacing \( \bar{\psi} \) by a constant average value \( \bar{\psi} = \text{const} \). In this case the usual features of general relativity can be established in the \( \Lambda \)-frame. In particular, the tensor \( T_{\alpha\beta}[\bar{\psi}] \), which was found to be non-conserved in the background frame, becomes a multiple of the Einstein tensor, so a conserved tensor.

For further investigation of the constraint (33), we write its explicit form on the background metric. Using the conformal transformation of the function \( v_1(x) \) \[10\] we find

\[
-e^{2\omega}\bar{\psi}^{-2}\{3v_1(g_{\alpha\beta}) + \frac{1}{240}[2R\Box\omega + 2R_{\alpha\rho\omega}^{\lambda} + 6\Box(\Box\omega) + 8(\Box\omega)^2

-\omega_{\alpha\beta}\omega^{\alpha\beta} - R_{\alpha\beta\omega}^{\gamma\lambda\omega^\gamma\omega^\lambda} - \omega^\gamma\omega_{\gamma\rho\omega - 2\omega_{\alpha\beta}\omega^{\alpha\beta}\omega^{\gamma\lambda}}]\} = \Lambda \tag{36}
\]

where \( \omega = -\ln \Omega \). As an illustration we shall now apply (36) to study an asymptotic relation between the \( \Lambda \)-frame and a specific background metric which we take to be described by a Schwarzschild black hole. In this case the function \( v_1(x) \), which determines the trace anomaly, reduces to

\[
v_1(g_{\alpha\beta}) = \frac{1}{720}R_{\alpha\beta\delta\gamma}R^{\alpha\beta\delta\gamma} = \frac{1}{15} \frac{M^2}{r^6} \tag{37}
\]

where \( M \) is the mass of the black hole. Since the trace anomaly vanishes for \( r \to \infty \), one may generally expect that for a sufficiently small \( \Lambda \) there should be no distinction between
the background and the Λ-frame in a region far from the black hole event horizon. That this behavior is dynamically allowed follows from the equation (36) as we briefly demonstrate: Let us restrict ourselves to the static case and assume that \( \omega \) is only a function of \( r \). For \( r >> 2M \) the equation (36) takes then the form

\[
\omega'''' - 4\omega''\omega'' = -40\Lambda \psi^2 e^{-2\omega} - \frac{8M^2}{r^6}, \quad r >> 2M
\]

where prime indicates differentiation with respect to \( r \). This equation reveals that \( \omega \) as a slowly varying function would be a solution for a large value of \( r \) and a sufficiently small cosmological constant. In particular, for large values of \( r \) an almost constant conformal factor (close to one) can be used to establish the correspondence between the background frame and the Λ-frame.

### 5 Summary and outlook

For a quantum field conformally coupled to a gravitational background we have presented a model in which the role of a scalar tensor theory is emphasized for studying the local constraints imposed on physical states by the Hadamard state condition. The corresponding scalar field is conformally invariant and controls the coupling of the stress-tensor to the conformal class of the background metric. The predictions of this theory are in accord with the standard results of the stress-tensor renormalization if one chooses a conformal frame corresponding to the background metric. We have emphasized that the choice of a specific conformal frame as a physical frame must, in general, be subjected to the superselection rules regulating the coupling of a local Hilbert space to the physical conditions at distant regions. In this context we have discussed the possibility to consider the theory in a distinguished frame, namely the Λ-frame, which may act as the physical frame for the establishing the large scale gravitational coupling of physical states in the presence of large scale distribution of matter. It is suggestive to link this large scale gravitational coupling of physical states, reflected in the Λ-frame, with their cut-off property in the short distance scaling. A dynamical cut-off theory of this type, if properly formulated, would reflect one of the characteristic implication of Mach’s principle in quantum field theory.

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