Cosmic quantum optical probing of quantum gravity through a gravitational lens

Charles H.-T. Wang, Elliott Mansfield, Teodora Oniga

Department of Physics, University of Aberdeen, King’s College, Aberdeen AB24 3UE, United Kingdom

Abstract

We consider the nonunitary quantum dynamics of neutral massless scalar particles used to model photons around a massive gravitational lens. The gravitational interaction between the lensing mass and asymptotically free particles is described by their second-quantized scattering wavefunctions. Remarkably, the zero-point spacetime fluctuations can induce significant decoherence of the scattered states with spontaneous emission of gravitons, thereby reducing the particles’ coherence as well as energy. This new effect suggests that, when photon polarizations are negligible, such quantum gravity phenomena could lead to measurable anomalous redshift of recently studied astrophysical lasers through a gravitational lens in the range of black holes and galaxy clusters.

Keywords: Astronomical effects of quantum gravity, gravitational lensing, gravitational waves

PACS: 04.60.Bc, 98.62.Sb, 04.30.w

1. Introduction

The effort to find a quantum theory of gravity remains as a key objective of theoretical physics in the 21st century. To better guide theoretical development, there has been substantial interest in searching for possible experimental indications [1, 2]. There exist a number of important studies that could lead to a future detection of quantum gravity effects. Recently, the data from the first detection of gravitational waves by LIGO [3] have been analyzed further to search for echoes that would indicate the Planck-scale structure that some theories predict would exist on the event horizon of a black hole [4]. Gravitational wave astronomy is a promising prospect in the effort to sense quantum gravity and there have been recent developments suggesting coherent Rydberg atoms could be sensitive to the zero-point spacetime fluctuations and stochastic gravitational waves [5]. Attempts to probe quantum gravity can go back as far as 1981, when an argument against semiclassical relativity was given in Ref. [6], claiming indirect evidence of quantum gravity. While this may be an incomplete dichotomy, as there are other theories that attempt to “fix” gravity, such as Modified Newtonian Dynamics [7] and Emergent Gravity [8], it at least reflects the need to quantize gravity as being increasingly recognized.

A few years later, Ref. [9] outlined some geophysical experiments that seem to suggest a repulsive term in the full gravitational equation, but this had a large uncertainty. Recently, there have been considerable developments in testing quantum phenomena in the astronomical domain, noticeably including tests for vacuum birefringence [10], quantum nonlocality [11], and indeed attempts to observe spacetime foam as a quantum gravity effect [12].

Decoherence is known to be a large problematic factor for quantum applications that causes qubits to lose quantum information [13, 14]. Yet, in exploring quantum nature, decoherence can provide useful tools [15, 16]. Decoherence is an essential part of open quantum dynamics, where dissipative interactions with the environment lead to decays from excited states towards the ground state in systems such as atoms in quantum optics [14, 17]. Recently, much work has been carried out in regard to gravitational decoherence with more general systems [18–21].

Any cosmic search for quantum gravitational decoherence are constrained by sources not emitting coherent radiation. While pulsars could be potentially considered, since they do not emit coherently in the optical spectrum, their tests for gravitational decoherence using modern quantum optics techniques may be inviable.

Fortunately, celestial objects known as astrophysical lasers have been studied in more recent times [22, 23]. These sources naturally induce a population inversion by one of two methods: (i) the rapid cooling of ionized plasma, causing electronic recombination, and (ii) selective radiative excitation, where emission lines of similar wavelengths may induce the transition to a higher state [22, 23].

Therefore astrophysical lasers appear to be suitable radiation sources for cosmic probing of gravitational decoherence. However, it has been pointed out that photons and other particles in free space are insensitive to spacetime fluctuations [19]. While radiation can undergo gravitational decoherence in a cavity, the size of the effects are negligible in normal laboratory scales that limit the quadrupole moments of the quantum system, which provide the effective coupling to gravitational fluctuations [20].

In view of these recent developments, here we show that gravitationally lensed coherent optical radiations on the astronomical scale could emerge as a viable test bed for quantum...
gravity through decoherence and the resulting anomalous redshift. We consider astrophysical lasers to provide the coherent optical radiation sources and objects ranging from black holes to galaxy clusters as lensing masses.

We work in the interaction picture with the relativistic units where \( e = 1 \) unless otherwise stated. The framework of quantized linearized gravity is adopted with metric signature \((-1, 1, 1, 1)\) and transverse-traceless (TT) coordinates.

2. Open quantum dynamics of a gravitational lensing system

To focus on the gravitational decoherence and the resulting dissipative phenomena of gravitationally lensed light, here we will neglect self gravity, environmental gravitational temperature, and the spin polarizations of the photons and model them with a neutral massless scalar field \( \phi(r, t) \) subject to an external potential represented by \( u(r) \) described by the Lagrangian density

\[
\mathcal{L} = -\frac{1}{2} \eta^\mu_\nu \phi_{,\mu} \phi^\nu - u \phi^2. \tag{1}
\]

Writing \( \phi = \psi(r) e^{-i\omega t} + \text{H.c.} \), we see that the field equation (1) is satisfied by an effective time independent Schrödinger equation for \( \psi(r) \).

We consider a gravitational potential \( \Phi = -G M e^{-\epsilon r}/r \) where \( M \) is the lensing mass and \( \epsilon \) is a small Yukawa parameter to regularize the infinite range of the Newtonian potential. Here we consider the wave vector \( p \) of the scalar field to have a characteristic norm \( p = |p| \sim p_* \). This allows us to approximate the potential function by

\[
u(r) = -2 \delta^2 G M e^{-\epsilon r}.\]

The effective time independent Schrödinger equation for \( \psi(r) \) can then be solved in terms of the Fourier transform

\[
\psi_q(r) = \int \psi_q(p) e^{ip \cdot r} \, dp
\]

by the approximate expansion

\[
\psi_q(p) = \delta(p - q) + \frac{2 G M p_0^2}{\pi^2 |p^2 - (q + i\epsilon)^2|^2} \epsilon_p \]

analogous to the momentum representation of the Coulomb scattering wavefunctions \([24, 25]\). The first term above represents the plane wave associated with the asymptotically free particle and the second term represents the scattered spherical wave due to gravitational lensing. The full scalar field operator then takes the form

\[
\phi = \int d^3 q \frac{\hbar}{2(2\pi)^3 q} a_q \psi_q(r) e^{-i q \cdot r} + \text{H.c.} \tag{2}
\]

under the Markov approximation which neglects short-term memory effects \([17]\), the general gravitational master equation recently obtained in Ref. [19] yields

\[
\rho(t) = -\frac{8\pi G}{\hbar} \int \frac{d^3 k}{(2\pi)^3} \int_0^{\infty} ds \, e^{-iks} \left[ \tau_{i,j}(k, t) \right] \rho(t) + \text{H.c.} \tag{3}
\]

In the adopted TT coordinates, spacetime fluctuations generally induce a fluctuating correction to the potential and hence \( \tau_{ij} \), which is significant for a harmonic potential \([21]\). However, for the present gravitational potential, such corrections are negligible.

The Fourier-transformed TT stress tensor \( \tau_{ij}(k, t) \) that provides the coupling between the particles and spacetime fluctuations using Eq. (3) follows from Eqs. (3) to be

\[
\tau_{ij}(k, p, p') = \tau_{ij}^{(\text{free})}(k, p, p') + \tau_{ij}^{(\text{scat})}(k, p, p')
\]

in terms of

\[
\tau_{ij}^{(\text{free})}(k, p, p') = \frac{G M p_0^2}{\pi^2} \frac{1}{\sqrt{p p'}} \delta(p - p' - k)
\]

\[
\tau_{ij}^{(\text{scat})}(k, p, p') = \frac{G M p_0^2}{\pi^2} \frac{a_q \tilde{a}_p P_{ij}(k)}{\sqrt{p' p} \left( |p - p' - k|^2 + \epsilon^2 \right)^2} \times \left\{ \frac{p |p|}{|p - k|^2 - (p' - i\epsilon)^2} + \frac{|p'|^2 p}{|p' + k|^2 - (p + i\epsilon)^2} \right\}
\]

Here the first term \( \tau_{ij}^{(\text{free})}(k, p, p') \) is associated with the asymptotically free particle states which does not contribute to the master equation (3) as discussed in Ref. [19], whereas the second term \( \tau_{ij}^{(\text{scat})}(k, p, p') \) contains the scattered spherical waves and contributes to Eq. (3) with decoherence effects.

Considering the gravitational interaction to be weak, we assume the density matrix of the particles to evolve perturbatively over time so that

\[
\rho(t) = \rho_0 + \Delta \rho(t)
\]

with \( |\rho_0| > |\Delta \rho(t)| \) where \( \rho_0 = \rho(t = 0) \). Then by substituting Eq. (4) into Eq. (3), taking the limit \( t \to \infty \), i.e. \( t \gg 1/p_* \), applying the Sokhotski-Plemelj theorem \([18]\) and neglecting the nondissipative Lamb-Stark shift terms, and repetitively using the same argument leading to free particles suffering no Markovian gravitational decoherence \([19]\), we obtain the asymptotic
change of the density matrix of the form

\[
\Delta \rho = -\frac{\zeta}{2} \int d^3k \, \phi^{\dagger}(k) \phi(k) \Delta \rho \frac{d^3q}{d^3p} \frac{d^3q'}{d^3p'} \frac{d^3q''}{d^3p''} \frac{d^3q'''}{d^3p'''} \\
\times \delta(k-p+p') \delta(k+q-q') \frac{p_1 p_1'}{d^{4}q} \frac{p_2 p_2'}{d^{4}q'} \frac{p_3 p_3'}{d^{4}q''} \frac{p_4 p_4'}{d^{4}q'''} \frac{d^{4}k}{d^{4}p} \frac{d^{4}k}{d^{4}p'} \frac{d^{4}k}{d^{4}p''} \frac{d^{4}k}{d^{4}p'''}
\]

with \( \Delta \rho = \Delta \rho(t \to \infty) \) and a dimensionless parameter

\[
\zeta = \frac{32 \pi G^3 M^2 f^4}{c^{10}}.
\]

In the above, the speed of light \( c \) has been restored, \( h \) is the Planck constant, and \( f_p = c p_p /2 \pi \) is the frequency of the particle. As will be clear below, since this frequency is subject to a redshift due to quantum dissipation of the particle when scattered by the lensing mass determined by \( \zeta \), we will refer to it as the “redshift coefficient”.

3. Quantum gravitational redshift through gravitational lensing

To understand the implication of Eq. (5), let us start with a relatively simple one-particle case. Then the corresponding matrix elements read

\[
\langle p_2 | \Delta \rho | p_1 \rangle = -\frac{\zeta}{2} \int d^3p \int d^3q \int d^3q' \int d^3q'' \int d^3p' \int d^3p'' \int d^3p''' \int d^3q''' \frac{d^4k}{d^4p} \frac{d^4k}{d^4p'} \frac{d^4k}{d^4p''} \frac{d^4k}{d^4p'''}
\times \delta(k-p+p') \delta(k+q-q') \frac{p_1 p_1'}{d^{4}q} \frac{p_2 p_2'}{d^{4}q'} \frac{p_3 p_3'}{d^{4}q''} \frac{p_4 p_4'}{d^{4}q'''}
\times \frac{\delta(k-p+p') \delta(k+q-q') \delta(k-q-k^2 + e^2)}{d^{4}q} \frac{\delta(k-p+p') \delta(k+q-q') \delta(k-q-k^2 + e^2)}{d^{4}q'} \frac{\delta(k-p+p') \delta(k+q-q') \delta(k-q-k^2 + e^2)}{d^{4}q''} \frac{\delta(k-p+p') \delta(k+q-q') \delta(k-q-k^2 + e^2)}{d^{4}q'''}
\times \langle q | q' | q'' | q''' \rangle H.c.
\]

By evaluating Eq. (7) with this initial density matrix (8), we find that the scattered states contain broadened frequencies no higher than the initial frequency as \( \langle p_2 | \Delta \rho | p_1 \rangle = 0 \) for \( p_1 > p_0 \) or \( p_2 > p_0 \). It also follows that \( \langle p_2 | \Delta \rho | p_1 \rangle = 0 \) for \( p_1 \neq p_2 \) and hence there is no superposition between different redshifted frequencies in the scattered states.

Therefore the nonunitary dynamics of the particles experience redshift through gravitational lensing as a result of quantum dissipation. Physically, as illustrated in Fig. 1 the loss of energy is induced by the spacetime granularity of scattered states having different gravitational energies with the lensing mass causing spontaneous decay of states accompanied by spontaneous emission of gravitons. This process can be understood as the gravitational analogue of the electromagnetic spontaneous decay of an electron with spontaneous emission of photons in an atom.

The total probability of transition, i.e. decay ratio, into to
redshifted states can be evaluated by tracing $\Delta \rho$ as follows

$$\int_{p < p_c} \langle p | \Delta \rho | p \rangle \, dp \bigg|_{t = 0} \approx \zeta$$

up to an order one numerical factor. This justifies $\zeta$ as the one-particle redshift coefficient through the described quantum gravitational lensing mechanism.

For $N$ particles, following the collective quantum analysis in Refs. [5, 19–21], the gravitational decoherence of $N$ bosonic particles occupying the same state generally scales with $N^2$, by virtue of the quadratic dependence of the gravitational master equation on $\tau_{ij}$ in Eq. (3). We therefore introduce the effective redshift coefficient $\zeta_N = \zeta N^2$ for coherent $N$-particle states. Using Eq. (6), we can evaluate $\zeta_N$ for different frequencies, lensing masses and particles numbers.

As shown in Fig. 2, for optical frequencies at $10^{14}$– $10^{15}$ Hz, the redshift coefficient $\zeta_N$ can come close to order one with a galaxy cluster mass for $N = 1$ to a supermassive black hole mass for $N = 10^{10}$. This could lead to measurable anomalous redshift of gravitationally lensed radiations from astrophysical lasers as observational evidence of quantum gravity, which may further guide theoretical work.

Acknowledgments

The authors are grateful for financial support to the EPSRC and Cruickshank Trust (C.W.), the Scottish Qualifications Authority (E.M.), and the Carnegie Trust for the Universities of Scotland (T.O.).

References

[1] G. Amelino-Camelia, Quantum-Spacetime Phenomenology, Living Rev. Relativ. 16, 5 (2013), and references therein.
[2] E. Berti et al., Testing general relativity with present and future astrophysical observations, Classical Quantum Gravity 32, 243001 (2015), and references therein.
[3] B. P. Abbott et al., (LIGO Scientific and Virgo Collaborations), Observation of Gravitational Waves from a Binary Black Hole Merger, Phys. Rev. Lett. 116, 061102 (2016).
[4] V. Cardoso et al., Echoes of ECOs: gravitational-wave signatures of exotic compact objects and of quantum corrections at the horizon scale, Phys. Rev. D 94, 084031(2016).
[5] D. A. Quillen, T. Oniga, B. T. H. Varcoe, and C. H.-T. Wang, Quantum principle of sensing gravitational waves: From the zero-point fluctuations to the cosmological stochastic background of spacetime, Phys. Rev. D 96, 044018 (2017).
[6] D. N. Page and C. D. Geilker, Indirect Evidence for Quantum Gravity, Phys. Rev. Lett. 47, 979 (1981).
[7] M. Milgrom, A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, Astrophys. J. 270 365 (1983).
[8] E. Verlinde, Emergent Gravity and the Dark Universe, SciPost Phys. 2, 016 (2017).
[9] T. Goldman, R. J. Hughes, and M. M. Nieto, Experimental evidence for quantum gravity? Phys. Lett. B 171, 217 (1986).
[10] R. P. Mignani et al., Evidence for vacuum birefringence from the first optical polarimetry measurement of the isolated neutron star RX J1856.5-3754, Mon. Not. R. Astro. Soc. 465, 492 (2017).
[11] J. Handsteiner et al., Cosmic Bell Test: Measurement Settings from Milky Way Stars, Phys. Rev. Lett. 118, 004161 (2017).
[12] V. Viasklein, J. Granot, T. Piran, and G. Amelino-Camelia, A Planck-scale limit on spacetime fuzziness, Nat. Phys. 11, 344 (2015).
[13] I. Chuang, R. Laflamme, P. Shor, and W. Zurek, Quantum computers, factoring and decoherence, Science 270, 1635 (1995).
[14] M. Schlosshauer, Decoherence and the quantum-to-classical transition (Springer, Berlin, 2014).
[15] I. Pikovski et al., Probing Planck-scale physics with quantum optics, Nat. Phys. 8, 393 (2012).
[16] M. Milgrom, A modification of the Newtonian dynamics as a possible alternative to the hidden mass hypothesis, Astrophys. J. 270, 365 (1983).
[17] H.-P. Breuer and F. Petruccione, The Theory of Open Quantum Systems (Oxford University Press, New York, 2002).
[18] T. Oniga and C. H.-T. Wang, Quantum gravitational decoherence of light and matter, Phys. Rev. D 93, 044027 (2016).
[19] T. Oniga and C. H.-T. Wang, Spacetime foam induced collective bundling of intense fields, Phys. Rev. D 94, 061501(R) (2016).
[20] T. Oniga and C. H.-T. Wang, Quantum dynamics of bound states under spacetime fluctuations, J. Phys. Conf. Ser. 845, 012020 (2017).
[21] T. Oniga and C. H.-T. Wang, Quantum coherence, radiance, and resistance of gravitational systems, Phys. Rev. D 96, 084014 (2017).
[22] D. Dravins, Photonic Astronomy and Quantum Optics, in High Time Resolution Astrophysics, Eds. D. Phelan, O. Ryan, and A. Shearer, Astrophysics and Space Science Library (Springer, 2007).
[23] S. Johansson and V. S. Letokhov, Astrophysical lasers operating in optical Fe II lines in stellar ejecta of Ñ Carinae, A&A 428, 497 (2004).
[24] E. Guth and C. J. Mullin, Momentum Representation of the Coulomb Scattering Wave Functions, Phys. Rev. 83, 667 (1951).
[25] S. S. M. Wong, Introductory Nuclear Physics (WILEY-VCH Verlag, Weinheim, 2004).