Dissipative Kerr solitons in a photonic dimer on both sides of an exceptional point

A. Tikan\textsuperscript{1,2,*}, K. N. Komagata\textsuperscript{1,2,3}, A. Tusnin\textsuperscript{1}, J. Riemenberger\textsuperscript{1}, M. Churaev\textsuperscript{1}, H. Guo\textsuperscript{1,4}, T. J. Kippenberg\textsuperscript{1,4}

\textsuperscript{1}Institute of Physics, Swiss Federal Institute of Technology Lausanne (EPFL), CH-1015 Lausanne, Switzerland  
\textsuperscript{2}These authors contributed equally.  
\textsuperscript{3}Present address: Laboratoire Temps-Fréquence, Institut de Physique, Université de Neuchâtel, 2000 Neuchâtel, Switzerland.  
\textsuperscript{4}Present address: Key Laboratory of Specialty Fiber Optics and Optical Access Networks, Shanghai University, 200444 Shanghai, China

*alexey.tikan@epfl.ch, tobias.kippenberg@epfl.ch

Abstract: We study experimentally and theoretically dissipative Kerr soliton formation in a driven photonic dimer on both sides of exceptional points. We describe a diversity of nonlinear coherent states emerging from the additional dimensionality. © 2022 The Author(s)

Exceptional points (EPs) are a ubiquitous concept widely present in driven-dissipative coupled systems described by a non-Hermitian Hamiltonian. They manifest themselves as a particular point on the parameters space where the eigenvalues of the coupling matrix coincide, together with the corresponding eigenvectors. They also mark the transition between the Parity-Time ($\mathcal{PT}$) symmetric and $\mathcal{PT}$-symmetry broken regimes. To date, exceptional points have been extensively examined in the systems supporting only a few optical modes, thereby leaving the observation of collective (multimode) effects outside of the scope of the study [1]. In parallel, another field of study has been rapidly developing in photonics: the generation of dissipative Kerr solitons (DKS) in driven-dissipative nonlinear optical micro-resonators as a way to provide stable and coherent frequency comb [2]. Recently, DKSs have been discovered in a high-Q multimode photonic dimer (a pair of strongly-coupled, almost identical nonlinear resonators) [3]. In this work, we study the generation of DKS in a photonic dimer in the context of $\mathcal{PT}$ symmetry via dissipational dissipation rates. We observe that critical coupling can be achieved in both $\mathcal{PT}$ symmetric and $\mathcal{PT}$-symmetry broken regimes, and both exhibit rich dynamics such as co-existence of solitonic states [4].

In a single microresonator, the complex nonlinear dynamics is governed by the so-called Lugiato-Lefever equation (LLE) [2]. In the case of two linearly coupled microresonators as shown in Fig. 1a, the coupled LLEs (in the frequency domain) can be written as follows:

\begin{equation}
\begin{aligned}
\frac{d}{dt} A_\mu &= -\frac{i}{2} (\kappa_0 + \kappa_{ex,1}) + i(\omega_\mu + \frac{1}{2} \delta - \mu D_1 - \omega_p) A_\mu + ig_K \mathcal{F}[|A|^2]_\mu + iJ_\mu B_\mu + \delta_{\mu,0} \sqrt{\kappa_{ex,1}} s_{in} \\
\frac{d}{dt} B_\mu &= -\frac{i}{2} (\kappa_0 + \kappa_{ex,2}) + i(\omega_\mu - \frac{1}{2} \delta - \mu D_1 - \omega_p) B_\mu + ig_K \mathcal{F}[|B|^2]_\mu + iJ_\mu A_\mu,
\end{aligned}
\end{equation}

here $g_K$ is the Kerr coefficient, $\delta_{\mu,0}$ is the Kronecker delta, $s_{in}$ - the input pump field amplitude, $A_\mu$, $B_\mu$ are the field amplitudes of the modes with index $\mu$ in the first and second resonator, $\mathcal{F}[\ldots]_\mu$ denotes the $\mu^{th}$-component of the Fourier transform. The microresonators have identical intrinsic loss rate $\kappa_0$, and free spectral range. They are evanescently coupled to the through and drop ports with rates $\kappa_{ex,i}$, $i = 1, 2$ and between each other at rate $J_\mu$.

We revisit the definition of the critical coupling conditions and generalize them to the case of the photonic dimer. The result of our investigation is depicted in Fig. 1b. We differentiate two types of critical coupling (zero transmission) conditions: (i) one which depends quadratically on the value of the coupling, (ii) which is independent of the coupling rate. The first type is found in the region of the weak coupling (violet area), while the second one corresponds to the case of the strong coupling (orange area). These two conditions intersect with each other for the coupling rates of 0.5$\kappa_0$. Remarkably, providing the coupling matrix analysis, we discovered that this intersection point (marked by a blue star in Fig. 1b) also corresponds to an exceptional point of the system. More explicitly, the exceptional point condition, depicted in Fig. 1b with the dashed line, represents the separation between two fundamentally different regions: with effective $\mathcal{PT}$-symmetry and broken effective $\mathcal{PT}$-symmetry [5].

We provide additional (to our previous work [3]) experimental investigations in a strongly coupled dimer (effective $\mathcal{PT}$-symmetry). Besides the demonstrated emergent phenomena, we show that the supermode interaction leads to the on-demand single DKS generation. Further, we demonstrate the origin of the emergent phenomena by considering novel four-wave mixing pathways between the dimer supermodes. We also show that despite the complex nonlinear interactions, the supermode soliton can be effectively uncoupled from the dispersive waves.
Dissipative Kerr Solitons on both sides of exceptional points. (a) Schematic of the two resonators. (b) Critical coupling coefficients as a function of $J$ for $\kappa_2 = 0$, in the split dissipation (resonances) regime above (below) the exceptional point line (dotted). The two conditions branch off at the exceptional point of $J = \kappa_0/2$, highlighted by the star. (c) Schematic phase diagram represented as a function of input power and $J$. Four states are identified: soliton in resonator 1 (S1, blue region), soliton crystal in resonator 1 (C1, cyan region), coexistence of soliton in resonator 2 and periodic coherent structure resonator 1 (S2/C1, green region), and soliton in resonator 2 (S2, yellow region). The insets show the intracavity field intensity in the resonators 1 (blue) and 2 (red), at the parameter indicated by the grey dot. White color refers to the absence of stable solitonic state.

belonging to another supermode. Therefore, the supermode representation brings the critical insights for understanding of inherent nonlinear dynamics.

The weakly coupled regime (broken effective $\mathcal{PT}$-symmetry) is investigated numerically by integrating the coupled LLEs 1. We observe the generation of four different stable solitonic states localized in both resonators represented as a diagram in Fig. 1. Among the novel states, we highlight the interlocked bright-dark soliton pairs (in resonators 2 and 1, respectively), the cnoidal wave interaction in resonator 1 with the bright-dark soliton pairs, the deterministic generation of soliton crystal states with high pump conversion efficiency into the comb lines, and bright solitons in resonator 1.

Thereby, we show that even the simplest element of a photonic lattice - a photonic dimer - demonstrates a vast variety of nonlinear effects. The presence of the internal symmetry leads to fundamentally different regions on the parameter space, both exhibiting the DKS formation and unique nonlinear dynamical features not present in the single resonator case. The fundamental aspects of this system can be of interest far beyond the photonics community.

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