A Spectrum Allocation Algorithm Based on Non-cooperative Game

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Abstract. Aiming at the problem of spectrum allocation among cognitive users, this paper proposes a spectrum allocation algorithm under non-cooperative game conditions, and presents a non-cooperative game Nash equilibrium algorithm, which can be iterated faster. Under the model and system settings, the improved fireworks algorithm is used to solve the spectrum allocation problem among cognitive users, and the utility function of cognitive users is improved to ensure the fairness of spectrum allocation.

1. Introduction
In cognitive radio networks, authorized users can obtain economic benefits by selling corresponding idle spectrum while ensuring their own transmission quality. Improving the supply-demand relationship by uncovering system adjustment capabilities from demand is becoming a new trend[1]. Because cognitive users compete with each other, they cannot obtain information from other cognitive users. So the entire cognitive wireless network can be seen as a typical Stackelberg non-cooperative game system model. When the spectrum allocation bandwidth is in Nash equilibrium, the entire cognitive user system maximizes the gain, improving the overall spectrum utilization[2].

This paper mainly constructs the benefit network of cognitive network system; a cognitive wireless network spectrum allocation algorithm is proposed to improve the spectrum utilization of the whole system. When the model is non-convex and discontinuous, the performance of the classical method is degraded and cannot even be solved[3]. Literature[4] proposed a power control algorithm based on normalized utility function; literature[5] introduced a simple linear cost function; literature[6] studied a cooperative mode of authorized users to improve spectrum utilization; [7] Consider the supply and demand of spectrum to adjust the selling price of spectrum resources. Analysis of the above documents does not consider the issue of fairness among cognitive users. Considering that each cognitive user aims to maximize their own interests, this paper analyzes the algorithm by constructing a non-cooperative game model.

2. System model
Fig.1 shows the spectrum trading model of the entire cognitive wireless network system. In this system, one authorized user and N cognitive users are considered. Under the premise of ensuring the communication quality of authorized users, authorized users to sell their own spectrum bandwidth to obtain economic benefits, and cognitive users can obtain transmission information through the obtained spectrum resources.
It is assumed that the authorized user has a continuous spectrum and its transmission power spectral density is evenly distributed. All users use QAM modulation mode, and the bit error rate is $BER$:

$$BER = 0.2 \exp\left(-1.5\gamma/\left(2^k - 1\right)\right)$$

(1)

Where $\gamma$ is the signal to noise ratio and $k$ is the spectral efficiency. Assume that all users have a maximum bit error rate of $BER_{\text{max}}$. Therefore spectrum efficiency $k$ [8]:

$$k_n = \log_2\left[1 + K \gamma_n\right] (n = 1, 2, \ldots, N)$$

$$K = \frac{1.5}{\ln(0.2 / BER_{\text{max}})}$$

(2)

3. Non-cooperative game spectrum allocation among cognitive users

3.1. Cognitive user utility function

The utility function set in this section is different from the literature[7, 9], the utility function proposed in this section contains the price gain factor of the authorized user when selling its spectrum, and a punitive factor is assigned to ensure the fairness of spectrum allocation among cognitive user systems.

In summary, this section presents a fairness-first utility function for spectrum allocation among cognitive users:

$$U(b) = \sum_{i=1}^{N} r_i \eta_i b_i - \sum_{i=1}^{N} u_i p_i b_i \lambda_i$$

(3)

The meaning of each symbol in the formula:

$r_i$: Cognitive user's income factor, $\eta_i$: Cognitive user spectral efficiency, $b_i$: Cognitive user's spectrum allocation, $u_i$: Punitive factor, $p_i$: Spectrum sale unit bandwidth price, $\lambda_i$: Price gain factor.

Where the price gain factor $\lambda_i$:

$$\lambda_i = 1 + \left(b_i / b_{\text{max}}\right)^2$$

(4)

Through the price gain factor, the price of a single cognitive user purchasing spectrum will increase as the number of purchased bandwidth increases.

Introduce the punitive factor $u_i$ in the formula:

$$u_i = \frac{(b_i - \overline{b}) \cdot N}{\sum_{n=1}^{N} (b_n - \overline{b})^2 + \varepsilon}$$

(5)
When the punishment is greater, the cognitive user cost overhead is increased to limit the spectrum of the cognitive user.

The goal of the entire cognitive user system is to maximize its own benefits:

$$\tilde{b}^* = \arg \max \arg U(\tilde{b})$$ \hspace{1cm} (6)

Among them, $$\tilde{b}^* = \{b_1^*, b_2^*, ..., b_N^*\}$$ is the Nash equilibrium solution of N people's limited non-cooperative game. When all followers respond optimally to the leader's strategy and the leader responds to this optimal response, the game reaches the Stackelberg equilibrium[10].

3.2. Cognitive User Spectrum Allocation Algorithm——Based on Simulated Annealing Dynamic Search Fireworks Algorithm (SADFA)

3.2.1. Fireworks algorithm

Fireworks Algorithm (FWA) is a swarm intelligence algorithm proposed by Tan and Zhu in 2010[11], which simulates the sparking phenomenon caused by fireworks explosion. The cooperation mechanism with the ant colony algorithm is different[12]. Based on this natural phenomenon, the solution process for an optimization problem is similar, and the solution of the cooperative game Nash equilibrium is also one of the optimization problems[13].

Fireworks algorithm flow:
In general, assume that the problem solved is an optimization problem:

$$\min \ f(x) \in R, x \in \Omega$$ \hspace{1cm} (7)

Step1: In the entire feasible solution space, randomly select N fireworks to form the initial solution space (population), where the initial solution of the ith fireworks is set to $$x_i (i = 1, 2, 3, ..., N)$$.

Step2: Calculate the explosion radius $$r_i$$ and the number of explosion sparks $$c_i$$ for each fireworks based on the explosion operator:

$$r_i = r * \frac{f(x_i) - y_{\min} + \varepsilon}{\sum_{j=1}^{N} (f(x_j) - y_{\min}) + \varepsilon}$$ \hspace{1cm} (8)

$$c_i = c * \frac{y_{\max} - f(x_i) + \varepsilon}{\sum_{j=1}^{N} (y_{\max} - f(x_j)) + \varepsilon}$$ \hspace{1cm} (9)

In the formula, $$r, c$$ are constants, which are used to adjust the spark radius and the number of sparks produced by fireworks explosion. $$\varepsilon$$ is a machine amount used to avoid dividing 0 operations.

Step3: According to the explosion operator calculated in Step2, where the generated quantity is $$c_i$$ and the explosion radius is $$r_i$$, the explosion spark of each of the fireworks $$x_i$$ is generated.

Step4: In the explosion spark produced in step3, according to the mutation operator, randomly select a certain number of sparks to mutate to generate the variation spark.

Step5: Determine whether the iterative termination condition is satisfied. If it is not satisfied, skip to Step6.

Step6: According to the roulette selection strategy, select N fireworks among the fireworks population as the next generation of fireworks (iterative solution) and enter Step2.

3.2.2. Dynamic Search Fireworks Algorithm Based on Simulated Annealing
The Dynamic Search in Fireworks Algorithm (dynFWA) [14] is an improved algorithm for the original fireworks algorithm. This section proposes a improved fireworks algorithm - Dynamic Search in Fireworks Algorithm based on Simulated Annealing (SADFA). The algorithm proposed in this paper focuses on improving the local search ability of the fireworks algorithm. A simulated annealing algorithm is introduced to make the fitness value of fireworks population within a certain acceptable range. Simulated annealing algorithm can get the local optimal solution better, but it can also receive the solution with poorer than the current solution with a certain probability to enhance the global search ability of the algorithm [15]. Taking into account the search capabilities of the dynamic search fireworks algorithm, this paper improves the simulated annealing algorithm:

Table 1. Improved simulated annealing algorithm

| Improved simulated annealing algorithm |
|---------------------------------------|
| 1. Initialization parameters: The initial temperature \( T \), Initial solution \( S_0 \), Final temperature \( T_f \), Number of iterations \( N \) |
| 2. \( \text{WHILE} (T > T_f \text{ AND } n < N) \text{ DO} : \) |
| 3. Create a new solution \( S^* \) |
| 4. \( \Delta t = f(S^*) - f(S_0) \) |
| 5. \( IF \quad \Delta t < 0 \text{ THEN} \) |
| 6. accept \( S^* \) |
| 7. Update solution |
| 8. \( n+ = 1, T^* = a(0 < a < 1) \) |
| 9. END WHILE |

Through the improved simulated annealing algorithm, thus improving the local search ability of the fireworks algorithm. For dynFWA, its advantage is that it has a strong local search capability, but its global search ability is poor, and the dynFWA algorithm only considers the relationship between the core fireworks of the last iteration and the core fireworks of this iteration. It does not fully consider the influence of iteration times on the core fireworks, and prematurely fall into a local optimum. To solve this problem and ensure that the fireworks population has a certain diversity, this paper proposes a new explosive operator and mutation operator.

Mutation operator: Considering the simulated annealing algorithm is to receive a solution with a certain probability than the current solution to improve the diversity of the population, for this reason, we always select the worst solution in the fireworks population for Gaussian mutation. Based on this, this algorithm fully combines the advantages of simulated annealing and dynFWA algorithm to improve the local search ability of the algorithm and guarantee a certain global search capability.

4. Simulation results and analysis

This section of the simulation mainly considers a spectrum trading model consisting of one authorized user and three cognitive users. Among them, the authorized user obtains benefits by selling certain spectrum resources. The spectrum trading model uses the algorithm proposed in this paper to perform Nash equilibrium to perform spectrum allocation. Without loss of generality, it is assumed that the maximum saleable spectrum bandwidth of the authorized user is 20MHz, the bit error rate \( BER_{\text{aur}} = 10^{-4} \) of the cognitive user, and the cognitive users receiver signal to noise ratio is 30dB.
Fig. 2 shows the Nash equilibrium convergence process of spectrum allocation for three cognitive users in this cognitive system. Assume that three cognitive users get the same spectrum price that authorized users sell. Through simulations, it is shown that three cognitive users perform non-cooperative games during spectrum allocation and eventually obtain authorized users spectrum allocation to maximize the benefits of the cognitive system. It can be seen from the figure that the initial allocated spectrum quantity is different from their respective final spectrum allocation quantity, which indicates the fairness of the cognitive user system utility function designed in this paper. Restrict cognitive users to purchase unlimited spectrum from authorized users. In addition, it can be seen from the simulation graph that SADFA has the ability of rapid iterative solution to the Nash equilibrium solution.

Fig. 3 shows that when authorized users continue to increase the initial spectrum sales price, the total revenue of the cognitive user system will gradually decrease. This is because with the continuous increase in the selling price, the cognitive users will pay higher costs. According to the utility function, it can be concluded that it will reduce its own spectrum requirements to ensure the benefits of its own relevant information transmission.
Figure 4. Spectrum allocation at different SNR

Fig.4 shows the simulation of cognitive user’s SNR and spectrum utilization. As can be seen from the figure, as the cognitive user's signal-to-noise ratio increases, spectrum utilization of cognitive users tends to be stable, and is between 80% and 90%. The simulation diagram shows that using the utility function proposed in this paper, non-cooperative game Nash equilibrium can be used for spectrum allocation to obtain higher spectrum utilization. Fig.4 below shows the change of the SNR and the total return of the entire cognitive system. As the cognitive user's signal-to-noise ratio continues to increase, the total revenue of the cognitive system gradually increases, but the magnitude of change gradually decreases, and the benefits of different spectrum selling prices are different. It can be seen that when the spectrum price is equal, the total income obtained by the cognitive system is higher than the total income at other sale prices.

5. Conclusion
This paper first analyzes and models the information transmission model between cognitive users, constructs a non-cooperative game model based on the non-cooperative characteristics among cognitive users, determines its game optimization utility function, and then solves it through the SADFA algorithm. Through the above simulation experiments, it can be concluded that the SADFA algorithm is an effective algorithm for solving Nash equilibrium problems in non-cooperative games. By applying this algorithm to the game model of cognitive user spectrum allocation, it is used to solve its spectrum allocation Nash equilibrium point to maximize the benefits. The results show that the algorithm can quickly iterate out the Nash Equilibrium Point, which is easy to apply to practical problems. By designing the utility function, it can effectively improve the spectrum utilization rate and ensure the fairness of the spectrum allocation of the entire cognitive system.

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