A Cooling Anomaly of High-mass White Dwarfs

Sihao Cheng1, Jeffrey D. Cummings1, and Brice Ménard1,2
1 Department of Physics and Astronomy, The Johns Hopkins University, 3400 N Charles Street, Baltimore, MD 21218, USA; s.cheng@jhu.edu
2 Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo, Kashiwa 277-8583, Japan

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Abstract

Recently, the power of Gaia data has revealed an enhancement of high-mass white dwarfs (WDs) on the Hertzsprung–Russell diagram, called the Q branch. This branch is located at the high-mass end of the recently identified crystallization branch. Investigating its properties, we find that the number density and velocity distribution on the Q branch cannot be explained by the cooling delay of crystallization alone, suggesting the existence of an extra cooling delay. To quantify this delay, we statistically compare two age indicators—the dynamical age inferred from transverse velocity, and the photometric isochrone age—for more than one thousand high-mass WDs (1.08–1.23 M⊙) selected from Gaia Data Release 2. We show that about 6% of the high-mass WDs must experience an 8 Gyr extra cooling delay on the Q branch, in addition to the crystallization and merger delays. This cooling anomaly is a challenge for WD cooling models. We point out that 22Ne settling in C/O-core WDs could account for this extra cooling delay.

Supporting material: animation

1. Introduction

Until recently, explorations of the white dwarf region in the Hertzsprung–Russell (H–R) diagram were severely limited by the number of objects with available distance estimates. The European Space Agency Gaia mission (Gaia Collaboration et al. 2016) has changed this situation drastically. Gaia is an all-sky survey of astrometry and photometry for stars down to 20.7 magnitude. The H–R diagram of white dwarfs generated by Gaia Data Release 2 (DR2) reveals three branch-like features, called the A, B, and Q branches in Figure 13 of Gaia Collaboration et al. (2018a). The A and B branches have been understood as standard-mass white dwarfs (m_{WD} \sim 0.6 M_\odot) with hydrogen-rich and helium-rich atmospheres, respectively (e.g., Bergeron et al. 2019). However, the Q branch, as an enhancement of high-mass white dwarfs (m_{WD} > 1.0 M_\odot), is still not fully understood. This is a challenge to current white dwarf evolutionary models and an opportunity for studying high-mass white dwarfs.

On the H–R diagram, white dwarves evolve along their cooling tracks. Unlike the A and B branches, the Q branch is not aligned with any cooling track or isochrone, suggesting that it is caused by a delay of cooling instead of a peak in mass or age distribution. This cooling delay makes white dwarves pile up on the Q branch. The Q branch coincides with the high-mass end of the crystallization branch identified by Tremblay et al. (2019). As a liquid-to-solid phase transition in the white dwarf core, crystallization releases energy through latent heat (e.g., van Horn 1968) and phase separation (e.g., Garcia-Berro et al. 1988; Segretain et al. 1994; Isern et al. 1997), which can indeed create a cooling delay. However, the observed pile-up on the Q branch is higher and narrower than expected from the standard crystallization model (Tremblay et al. 2019, Figure 4), suggesting that there exists a cooling anomaly, i.e., an extra cooling delay in addition to crystallization.

2. Data

In this paper, we investigate this cooling anomaly using kinematic information of high-mass white dwarves in Gaia DR2. In Section 2 we describe our white dwarf sample; in Section 3 we show strong evidence for the existence of an extra cooling delay on the Q branch; in Section 4 we build a model for the white dwarf velocity distribution and use our Gaia sample to constrain the properties of this cooling anomaly; in Section 5 we present the best-fitting values of these properties and as a byproduct of our analysis, the fraction of double-WD merger products among high-mass white dwarfs; in Section 6 we show that 22Ne settling in massive C/O-core white dwarves is a promising physical origin of the extra cooling delay; in Section 7 we examine other aspects of the Q branch, which provide evidence that the extra delayed white dwarves are also double-WD merger products; and in Section 8 we conclude on our findings.

2.1. Quality Cuts

Gentile Fusillo et al. (2019) have compiled a catalog of Gaia DR2 white dwarfs based on the G-band absolute magnitude, Gaia color index, and some quality cuts. To select white dwarfs with high-precision measurements, we further apply the following quality cuts:

\begin{align}
\sigma_{G_{BP}-G_{RP}} & < 0.10, \\
\sigma_{\mu/\varpi} & < 2 \text{ km s}^{-1}, \\
\text{parallax\_over\_error} & > 8,
\end{align}

\footnote{Named after the presence of DA, DB, and DQ white dwarfs (Gaia Collaboration et al. 2018a), respectively. DA white dwarfs have hydrogen lines in their spectra, DB and DQ white dwarfs have helium and carbon lines, respectively.}
2.2. Kinematic and Physical Parameters of White Dwarf

Our analysis requires white dwarf absolute magnitude, color index, and the two components of transverse velocity $v_T = (v_L, v_B)$ in Galactic longitude and latitude directions. Except for the color index $G_{BP} - G_{RP}$, which is directly read from the $bp_{rp}$ column in Gaia DR2, we derive the other quantities in the following way:

$$M_G = G + 5 \log(\varpi/{\text{mas}}) - 10,$$
$$v_L = \mu_L - (A \cos 2l + B)\cos b,$$
$$v_B = \mu_B + A \sin 2l \sin b \cos b,$$

where $G$ and $\varpi/{\text{mas}}$ are read from Gaia DR2 columns $phot \_g \_mean \_mag$ and parallax, $\mu_L$ and $\mu_B$ are converted from columns $ra, dec, pmra$, and $pmdec$ with the coordinate conversion function in the ASTROPY package (Astropy Collaboration et al. 2013, 2018), and $A$ and $B$ are the Oort constants taken from Bovy (2017). We do not correct for extinction because within the distance cut, extinction is in general tiny and there is no accurate estimate for it. To avoid the influence of hyper-velocity white dwarfs, we further impose a velocity cut:

$$v_T < 250 \text{ km s}^{-1}.$$
lines in the 845–872 nm wavelength range of Gaia’s spectrometer (Gaia Collaboration et al. 2016).

We then derive white dwarf photometric isochrone ages and masses from the H–R diagram coordinates:

$$(G_{BP} - \Delta G_{BP}, M_G) \rightarrow (\tau_{\text{phot}}, m_{WD})$$

based on a single-star evolution scenario and white dwarf cooling models. We estimate the main-sequence (MS) ages with an initial–final mass relation (Cummings et al. 2018) and the relation between pre-WD time and main-sequence mass from Choi et al. (2016) for non-rotating, solar-metallicity stars. For high-mass white dwarfs, the pre-WD ages are negligible. As for white dwarf cooling, we use a table of synthetic colors for pure-hydrogen atmosphere (Holberg & Bergeron 2006; Kowalski & Saumon 2006; Tremblay et al. 2011) and a grid of cooling tracks for C/O-core white dwarfs with “thick” hydrogen layers (Fontaine et al. 2001). In order to convert any H–R diagram coordinate into $\tau_{\text{phot}}$ and $m_{WD}$, we linearly interpolate between grid points. Stellar models show that in the single-star-evolution scenario, white dwarfs with a mass higher than about 1.05–1.10 $M_\odot$ have oxygen–neon (O/Ne) cores (e.g., Siess 2007; Lauffer et al. 2018). So, we combine the cooling tracks of $m_{WD} \leq 1.05 M_\odot$ C/O white dwarfs with the four cooling tracks of $m_{WD} \geq 1.10 M_\odot$ O/Ne white dwarfs (Camisassa et al. 2019).

The O/Ne white dwarf model only gives slightly lower mass estimates than the C/O white dwarf model (e.g., 1.08–1.23 $M_\odot$ in the combined model corresponds to 1.10–1.28 $M_\odot$ in the C/O-only model), and their estimates of the photometric ages $\tau_{\text{phot}}$ are similar for the white dwarfs we are interested in ($\tau_{\text{phot}} < 3.5$ Gyr). Switching between thick-hydrogen, thin-hydrogen, and helium atmosphere (Bergeron et al. 2011) models does not significantly change the photometric-age estimate of our sample either.

2.3. Mass, Age, and Q-branch Selection

In Figure 1 we show the selected white dwarfs on the H–R diagram. In the top right panel, we use the 150 pc sample to show the density distribution with a higher contrast. The Q branch is a factor-two enhancement at around $-0.4 < G_{BP} - G_{RP} < 0.2$ and $M_G = 13$. In the main panel, we show our main sample within 250 pc, color-coded by their transverse velocities $v_T$ with respect to the local standard of rest. We adopt $(U_v, V_v, W_v) = (11, 12, 7)$ km s$^{-1}$ (Schönrich et al. 2010) to correct for the solar reflex motion. We emphasize the fast white dwarfs ($v_T > 70$ km s$^{-1}$) in Figure 1 with larger dots: they are very likely thick-disk stars. We also plot a grid of photometric age $\tau_{\text{phot}}$ and mass $m_{WD}$ derived from the combined O/Ne-core and C/O-core white dwarf cooling model. Cooling tracks are the curves with constant $m_{WD}$. White dwarfs with different birth times form a “white dwarf cooling flow” on the H–R diagram as they move along their cooling tracks.

We focus on the mass range where the Q branch is most prominent. To maximize sample size and minimize the contamination from standard-mass helium-atmosphere white dwarfs (the B branch), we impose the following photometric age and mass cuts:

$$0.1 \text{ Gyr} < \tau_{\text{phot}} < 3.5 \text{ Gyr},$$  
$$1.08 M_\odot < m_{WD} < 1.23 M_\odot,$$

where $m_{WD}$ is derived from the combined cooling model for O/Ne and C/O white dwarfs. This mass range corresponds to 1.10–1.28 $M_\odot$ in the C/O-only cooling model. In total, 1070 white dwarfs are selected by these criteria. In this region, the Q branch divides the white dwarf cooling flow into three segments: the early, Q-branch, and late segments, as shown in Figure 1. We define the Q-branch segment by

$$|M_G - 1.2 \times G_{BP - RP} - 13.2| < 0.2$$

in addition to the previous photometric-age and mass cuts.

3. An Extra Cooling Delay on the Q Branch

3.1. Evidence from the Photometric-age Distribution

As argued by Tremblay et al. (2019), an enhancement not aligned with mass or age grid, such as the Q branch, should be produced by a slowing down (and therefore a delay) of white dwarf cooling. Such a cooling delay creates a “traffic jam” in the white dwarf flow, and the Q branch is a snapshot of this traffic jam. Is crystallization alone enough to explain the cooling delay on the Q branch? If it is, then the distribution of photometric age $\tau_{\text{phot}}$ derived from a cooling model including crystallization effects should no longer carry signatures of the Q branch. However, observations lead to the antithesis. In Figure 2 we show

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4 We have made this tool a publicly available python 3 module on https://github.com/ShiaCheng/WD_models.
5 http://www.astro.umontreal.ca/~bergeron/CoolingModels/
6 A catalog of all selected white dwarfs is available on VizieR and on the website: https://pages.jhu.edu/~scheng40/Qbranch.
the distribution of $\tau_{\text{phot}}$ in three mass ranges: there is a mass-dependent enhancement tracing the Q branch, which is consistent with the observation by Tremblay et al. (2019) that the pile-up is higher and narrower than what the standard cooling model predicts. Evolutionary delays from binary interactions or a peak in star formation rate cannot explain this mass-dependent enhancement either. Therefore, an extra cooling delay in addition to crystallization effects (latent heat and phase separation) must exist.

3.2. Evidence from the Velocity Distribution

Observations show that the velocity dispersion of disk stars in the Milky Way is related to the stellar age $\tau$: older stars have higher velocity dispersion than younger stars (e.g., Holmberg et al. 2009). So, the transverse velocity $v_T$ derived from Gaia DR2 can be used as a “dynamical” indicator of the true stellar age $\tau$. For the Milky Way thin disk, the dispersion of transverse velocity approximately follows a power law increasing from about 25 km s$^{-1}$ at 1.5 Gyr to 55 km s$^{-1}$ at around 6–8 Gyr (e.g., Holmberg et al. 2009); for the thick disk, the dispersion is about 65 km s$^{-1}$ (e.g., Sharma et al. 2014). Given this age–velocity-dispersion relation (AVR), we observe two anomalous things in the velocity distribution of the Q branch:

1. There is a strong excess of white dwarfs with $v_T > 70$ km s$^{-1}$ in the Q-branch segment, as shown in Figure 1. According to the age–velocity-dispersion relation (AVR) mentioned above, these fast white dwarfs should be old stars. Given that the photometric age on the Q branch is only 0.5–2 Gyr, these white dwarfs must have experienced an extra cooling delay for several billion years. In the left panel of Figure 3 we show that the excess of fast white dwarfs in the Q-branch segment is clear for $m_{\text{WD}} > 1.05 M_\odot$; in the right panel we show that this excess is observed for a variety of velocity cuts.

2. The fraction of fast white dwarfs in the late segment is lower than that in the Q-branch segment. This is anomalous, because white dwarfs in the late segment should be older than those in the Q-branch segment, as long as all white dwarfs follow the same cooling law. The only way to create such a reverse of fraction is to have more than one white dwarf population with distinct cooling behaviors.

3.3. A Two-population Scenario of the Extra Cooling Delay

The simplest scenario that can explain both the number enhancement and velocity anomaly on the Q branch is to have an extra-delayed population of white dwarfs in addition to a normal-cooling population. This scenario requires only two free parameters:

1. The fraction $f_{\text{extra}}$ of the extra-delayed population, and
2. The length $t_{\text{extra}}$ of the extra cooling delay on the branch.

In Figure 4 we illustrate this scenario by showing the normal-cooling and extra-delayed populations on the H–R diagram. Before the branch, the extra-delayed population has no difference from the normal-cooling population. On the branch, the extra-delayed population has a slower cooling rate, which causes two effects: (1) its members pile up there, creating a number-density enhancement, and (2) the photometric ages $\tau_{\text{phot}}$ of its members start to seem younger than their true ages $\tau$, creating an age discrepancy. After the branch, the number-density enhancement disappears, but the age discrepancy remains. A detailed parameterization of this scenario is presented in Appendix A. To create the observed reversal of fast white dwarf fraction in the Q-branch and late segments, the extra-delayed population must have a high number-density contrast between these two regions, which requires that the population fraction $f_{\text{extra}}$ be small and the delay time $t_{\text{extra}}$ be long.

4. Quantitative Analysis

Having shown qualitatively the existence of an extra cooling delay on the Q branch, we now attempt to quantify its properties. We build a statistical model that (i) includes double-WD mergers, (ii) uses an anisotropic AVR, and (iii) makes use of the full constraining power of the observations.
Table 1
Delay Scenarios of the Three Populations

| Population (Abbreviation) | Single-star Evolution | Extra-delayed (Extra) | Double-WD Merger Products with Normal Cooling |
|--------------------------|-----------------------|-----------------------|---------------------------------------------|
| merger delay             | no                    | yes or no (setup 1 or 2) | yes                                         |
| extra cooling delay      | no                    | yes                   | no                                          |

Note. For each population, the delay types are shown in the upper part of the table, and the total delay time $\Delta t = \tau - \tau_{\text{phot}}$ for each segment is shown in the lower part. $\Delta t$, $\Delta t_{\text{merger}}$, and $\Delta t_{\text{extra}}$ are not single numbers but random variables following their distributions. They are used to calculate the distributions of true ages $\tau$ from photometric ages $\tau_{\text{phot}}$.

Figure 4. An illustration of the two-population scenario of the Q branch: a normal-cooling population (blue dots), and a population with the extra cooling delay (red dots). The number density of WDs on the H–R diagram is determined by the cooling rate, because WDs accumulate where the cooling rate is low. Here, we use the best-fitting values ($f_{\text{extra}} = 6\%$ and $t_{\text{extra}} = 8$ Gyr) from our later analysis to generate this mock H–R diagram. An animated version of this figure is available, where blue dots move with the normal cooling rate, while red dots (the extra-delayed population) move slowly on the Q branch. Each second in the animation corresponds to 1 Gyr in physical time, and the duration of this animation is 11 seconds. More animations can be found on the website: https://pages.jhu.edu/~scheng40/Qbranch.

(An animation of this figure is available.)

4.1. Merger Products among High-mass White Dwarfs

Simulations of binary evolution show that double-WD merger products may account for a considerable fraction of high-mass white dwarfs (e.g., Toonen et al. 2017; Temmink et al. 2019). These merger products also have a discrepancy between their true ages and photometric ages due to binary evolution. Therefore, in order to use the velocity distribution to quantitatively constrain the extra cooling delay, the merger population must be modeled simultaneously. Constraining the merger fraction is also of great interest as its value is still a matter of debate (e.g., Giammichele et al. 2012; Wegg & Phinney 2012; Rebassa-Mansergas et al. 2015; Tremblay et al. 2016; Maoz et al. 2018). Therefore, we include the double-WD merger products in our model and set their fraction as a free parameter.

4.2. Description of the Model

In our model, we consider two evolutionary delays: the extra cooling delay, and the merger delay. Accordingly, we consider three populations of white dwarfs with different combinations of the two delays:

1. A generic population of singly evolved white dwarfs that follows normal cooling, denoted by “s”;
2. A double-WD merger population with systematic age offsets due to the merger delay and with a normal cooling, denoted by “m”;
3. A population with the extra cooling delay, denoted by “extra.”

Their delay scenarios are listed in Table 1. For simplicity, we only explore the two extreme situations for the extra-delayed population, where

1. Setup 1: all members of the extra-delayed population also have the merger delay;
2. Setup 2: no members of the extra-delayed population have the merger delay.

The distribution function $p(y)$ of observables $y$ for all white dwarfs can be written as a weighted average of the distribution for each population:

$$p(y) = f_s p_s(y) + f_m p_m(y) + f_{\text{extra}} p_{\text{extra}}(y),$$

where the weight $f$ denotes the fraction of each population, satisfying $f_s + f_m + f_{\text{extra}} = 1$.

Our goal is to use observations to constrain two independent population fractions and the delay time of the extra cooling delay:

$$f_{\text{in}}, f_{\text{extra}}, \text{ and } t_{\text{extra}},$$

the last of which is encoded in the distribution $p_{\text{extra}}(y)$. We have two sets of observables: the transverse velocities $v_f$ and the photometric ages $\tau_{\text{phot}}$. They are connected by the AVR distribution function $p(v|\tau)$ and the delay scenario of each population (listed in Table 1). The delay

$$\Delta t \equiv \tau - \tau_{\text{phot}}$$

includes contributions from the extra-cooling and/or the merger delays. We build a Bayesian model based on Equation (15) to constrain the aforementioned parameters. Our model is similar to

\footnote{We only consider the double-WD mergers because in our mass range, other merger products such as those from MS–RG, MS–MS, and MS–WD mergers usually only have $<0.2$ Gyr delay and therefore are indistinguishable from the singly evolved white dwarfs in terms of kinematics.}
that of Wegg & Phinney (2012), but we include the extra-delayed population and use a much larger sample. In addition, to avoid the need for modeling selection effects, we derive our constraints from the velocity distribution conditioned on observables other than velocity:

\[ p(\nu | \text{other observables}) = p(\nu_T | T_{\text{phot}}, m_{\text{WD}}, l, b). \]  

The details of this statistical technique and the Bayesian framework of our model are shown in Appendix B.

The free parameters in our model include the population fractions \( f_{\text{extra}} \) and \( f_{\text{extra}} \), the extra delay time \( t_{\text{extra}} \), parameters for the AVR, and solar motion. Although constraints on the AVR and solar motion already exist, treating them also as free parameters can avoid potential systematic errors, and the comparison of our best-fitting values with the existing values allows us to check the validity of our method.

Below, we list the main assumptions and simplifications in our model:

1. We assume that upon entering the Q-branch segment, all members of the extra-delayed population suddenly slow down their cooling by a constant factor, and upon leaving the branch, the cooling rates suddenly resume, so that this extra cooling delay can be parameterized by just its length \( t_{\text{extra}} \) and population fraction \( f_{\text{extra}} \) (see Appendix A). The resulting delay-time distribution is described in Section 4.4.3.

2. The velocity distribution of white dwarfs is a superposition of 3D Gaussian distributions as a function of age \( \tau \), i.e.,

\[ p(\nu | \tau) = \mathcal{N}(\mu_0(\tau), \Sigma(\tau)). \]

The details of this Gaussian velocity model are shown in Appendix C.

3. The true-age distribution of high-mass white dwarfs within 250 pc is uniform up to 11 Gyr, i.e., \( \tau \sim U[0, 11 \text{ Gyr}] \).

4. For the double-WD merger products, we follow Wegg & Phinney (2012) and assume that the resulting white dwarf is reheated enough that its cooling age after the merger is almost equal to the photometric cooling age. We also assume a fixed delay-time distribution for double-WD mergers (see Section 4.3) and a parameterization of the AVR (see Section 4.4.4).

4.3. Delay-time Distributions

The three white dwarf populations in our model are defined by their different delay signatures \( \Delta t \), which concern the extra cooling delay \( \Delta t_{\text{extra}} \) and double-WD merger delay \( \Delta t_{\text{merger}} \). The delay scenario of each population in each segment is listed in Table 1.

The extra cooling delay \( \Delta t_{\text{extra}} \) is built up on the Q-branch. We adopt a uniform distribution \( \Delta t_{\text{extra}} \sim U[0, t_{\text{extra}}] \) of this delay for white dwarfs in the Q-branch segment and a constant value in the late segment. Note that \( t_{\text{extra}} \) is a random variable with a probability distribution, whereas \( t_{\text{extra}} \), as a model parameter to be constrained, is the upper limit of \( \Delta t_{\text{extra}} \). In the Q-branch segment, we do not further distinguish if a white dwarf has just started or is about to complete their extra cooling delay, because the uncertainty of H–R diagram coordinate due to different atmosphere types and astrometric/photometric error is comparable to the width of the Q branch on the H–R diagram. In this case, a uniform distribution is a good and efficient approximation for \( \Delta t_{\text{extra}} \).

The double-WD merger delay \( \Delta t_{\text{merger}} \) originates from the binary evolution before the merger. We refer to binary population synthesis results (e.g., Toonen et al. 2014) and approximate the delay by

\[ p(\Delta t_{\text{merger}}) \propto \Delta t_{\text{merger}}^{0.7} \]  

for \( \Delta t_{\text{merger}} > 0.5 \text{ Gyr} \) and zero for smaller \( \Delta t_{\text{merger}} \). Unlike the extra cooling delay, we do not set any free parameter for this merger-delay distribution.

4.4. Parameterization of the AVR

We define the \( U, V, W \) axes as pointing toward \((l = 0^\circ, b = 0^\circ)\), \((l = 90^\circ, b = 0^\circ)\), and \((b = 90^\circ)\), respectively, and assume that the main axes of the Gaussian velocity distribution are aligned with these directions with dispersion \( \sigma_U(\tau), \sigma_V(\tau), \) and \( \sigma_W(\tau) \). Observations show that the AVR in each direction can be fit by a shifted power law. The power index of the in-disk components are around 0.35 and that of the W component is around 0.5 (e.g., Holmberg et al. 2009; Sharma et al. 2014). For old stars including thick-disk members, the AVR is still a matter of debate (e.g., Yu & Liu 2018; Mackereth et al. 2019).

So, in each direction, we use a shifted power law to parameterize the AVR of the younger, thin-disk stars:

\[ \sigma(\tau) = \sigma^{0} + \Delta \sigma^{0-4} \times \left( \frac{\tau}{4} \right) \beta, \quad \tau < 7 \text{ Gyr}, \]

and we use a constant value \( \sigma^{\text{thick}} \) to represent the velocity dispersion of stars older than 10 Gyr (thick-disk stars); in between 7 and 10 Gyr, we linearly interpolate the values from the two ends to reflect the increasing fraction of thick-disk stars. The shape of the AVR with our parameterization is shown in Figure 5. The ratio of the two in-disk components \( \sigma_V \)
and \( \sigma_U \) should be a constant for a local sample (e.g., Binney & Tremaine 2008), so we set \( \sigma_U(\tau) = k \sigma_U(\tau) \). As the assumption for the velocity distribution to be Gaussian gradually breaks down when \( \sigma_U \) increases, we allow the ratio \( k \) to be different for the thin and thick disks. Thus, we use in total 10 parameters to model the anisotropic AVR: two initial velocity dispersions \( \sigma_U, W \); two dispersion increases \( \Delta \sigma_U, \Delta W \) between 0 and 4 Gyr; two power indices \( \beta_U, W \); two thick-disk dispersions \( \sigma_U, W \); and two in-disk dispersion ratios \( k_{\text{thin}}, k_{\text{thick}} \). The best-fitting values of these parameters can be checked against existing estimates presented in the literature.

5. Results

To constrain the extra cooling delay properties and merger fraction, we feed our Bayesian model with the 1070 white dwarfs selected in Section 2. We use the Markov chain Monte Carlo (MCMC) sampler emcee (Foreman-Mackey et al. 2013) to obtain the posterior distribution of the parameters. Details of the settings are described in Appendix D.

5.1. Constraints on the Main Parameters

In Figure 6 we present the constraints we obtain for the parameters of interest: \( f_{\text{extra}}, t_{\text{extra}}, \) and \( f_m \). We find that the extra-delayed population fraction is

\[
f_{\text{extra}} = 6.4^{+2.3}_{-1.5}\% \quad \text{(setup 1)}
= 9.2^{+4.4}_{-1.4}\% \quad \text{(setup 2),}
\]

and the length of the extra cooling delay is

\[
t_{\text{extra}} \geq 8 \text{ Gyr} \quad \text{(setup 1)}
\geq 10 \text{ Gyr} \quad \text{(setup 2).}
\]

These constrains confirm our qualitative conclusion that \( f_{\text{extra}} \) is small and \( t_{\text{extra}} \) is long in Section 3.3. We point out that the difference of \( t_{\text{extra}} \) in the two setups is exactly where the peak of the merger-delay distribution is located (2 Gyr), which is expected. The lower limit for \( t_{\text{extra}} \) slightly depends on the parameterization of the AVR: if we adopt a younger thick-disk age (7–11 Gyr instead of 9–11 Gyr, Mackereith et al. 2019), this lower limit will also decrease by about 1–2 Gyr. \( t_{\text{extra}} \) may be overestimated for the fact that at a high level of dispersion, the velocity distribution is not exactly Gaussian. But it is unlikely that we overestimate \( t_{\text{extra}} \) too much, because we set the thick-disk dispersion as a free parameter. We have also checked that reasonable variations of the input delay-time distribution of the mergers (e.g., Toonen et al. 2012) do not change these two constraints significantly.

The fraction of merger products without the extra cooling delay is found to be \( f_m = 15^{+3}_{-2}\% \) and \( 20^{+3}_{-5}\% \) (setups 1 and 2). Therefore, the total fraction of double-WD merger products is

\[
f_{\text{extra}} + f_m = 22^{+7}_{-5}\% \quad \text{(setup 1)}
\]
\[
f_m = 20^{+6}_{-5}\% \quad \text{(setup 2)}
\]

among 1.08–1.23 \( M_\odot \) white dwarfs. This total fraction is mainly constrained by the fast white dwarfs in the early segment (where the two setups do not differ from each other), so the constraints on this fraction under setups 1 and 2 are similar. A more detailed analysis of the merger products among high-mass white dwarfs is presented in Cheng et al. (2019).

Finally, we calculate the contribution of the extra-delayed population in the Q-branch segment according to the above fractions:

\[
F_{\text{extra}} = \frac{f_{\text{extra}}(t_{\text{extra}}/\Delta t_{\text{branch}} + 1)}{1 + f_{\text{extra}}t_{\text{extra}}/\Delta t_{\text{branch}}}
= (47 \pm 8)\%.
\]

where \( \Delta t_{\text{branch}} \) is the width of the Q branch (see Appendix A). This fraction, derived purely from the velocity information, is consistent with the factor \( \approx 2 \) estimate obtained directly from the number-density enhancement in Figure 2, which confirms that our extra-cooling-delay scenario is a good phenomenological model for the Q branch.

5.2. Validation of the Model

To further validate our model and results, we first check our constraints on the nuisance parameters. For the solar motion we obtain

\[
(U, V, W)_\odot = (10.3 \pm 1.0, 7.3 \pm 1.0, 6.7 \pm 0.5) \text{ km s}^{-1},
\]

which is consistent with the measurement of Rowell & Kilic (2019) based on mainly standard-mass white dwarfs. Our values of \( U_\odot \) and \( W_\odot \) are also consistent with the results of Schönrich et al. (2010). The discrepancy of the \( V_\odot \) measurement comes from the different treatments of the asymmetric drift and is beyond the scope of this paper.
Figure 7. The observed and modeled velocity distributions. \( v_L \) and \( v_B \) are the Galactic longitude and latitude components of the transverse velocity. For the observed distribution, we present the histogram between \(-150 \) and \( 150 \) km s\(^{-1}\) with 23 bins and the Poisson error of each bin. Note that the y-axis is in logarithmic scale. The solid and dashed curves, which are not very different, are the velocity distributions of the best-fit models under setup 1 and 2, respectively. Both models fit the observations quite well. The dotted curves are the velocity distributions when no white dwarf has the extra cooling delay or merger delay. Its discrepancy to observed histograms shows the necessity of the two delays.

Figure 5 shows our constraint on the white-dwarf AVR, which is consistent with the AVR of thin- and thick-disk MS stars (Holmberg et al. 2009; Sharma et al. 2014). Removing either the extra-delayed population or the merger population leads to unreasonably higher AVRs. Before Gaia DR2 came out, Anguiano et al. (2017) reported an unexpectedly high AVR for young white dwarfs (their Figures 21 and 22), without considering the extra cooling delay or merger delay in their age estimate. This unreasonably high AVR is exactly what we see when we remove the extra-delayed and/or merger population from our model. Thus, we verify that both the extra cooling delay and the merger delay are necessary.

To check the goodness of fit, we compare the observed and modeled velocity distributions in Figure 7. Our best-fitting models (in both setups 1 and 2) provide good characterizations of the observed velocity distribution in all the early, Q-branch, and late segments. Adopting a different star formation history introduces no significant changes to our results. We test both a linearly decreasing star formation rate with a 

Additionally, the \( U \) component has a dispersion of \( 0.014 \text{Ne} \) XXWD CNO. This leads to an abundance \( X_{\text{Ne}}^{\text{WD}} \approx X_{\text{CNO}}^{\text{star}} \approx 0.014 \) for solar metallicity.
Due to the additional two neutrons, $^{22}$Ne nuclei feel more downward force from gravity than the upward force from the electron-pressure gradient. So, they gradually settle down to the white dwarf center and release gravitational energy (Bildsten & Hall 2001).

Now, let us check if $^{22}$Ne settling satisfies the three requirements. We first emphasize that the delay effect only depends on the fractional contribution of the extra energy source to the white dwarf luminosity ($L_{\text{extra}}/L_{\text{surf}}$, see Appendix E for details). Therefore, to create a peaked effect, $L_{\text{extra}}$ need not be also peaked.

The luminosity of $^{22}$Ne settling ($L_{\text{extra}}^{^{22}\text{Ne}}$) depends on the $^{22}$Ne abundance, mass, and core composition of the white dwarf, and the inter-to-self-diffusion factor $D/D_s$, which is of order unity but not well determined. As a white dwarf cools down, $L_{\text{extra}}^{^{22}\text{Ne}}$ does not change much, whereas $L_{\text{surf}}$ drops quickly with temperature (e.g., Figure 2 of Bildsten & Hall 2001). So, if no mechanism suppresses the $^{22}$Ne settling, the two luminosities will meet at some temperature. Around this meeting point is the effective zone of $^{22}$Ne settling, where the white dwarf cooling rate is influenced significantly. On the other hand, the meeting temperature is a function of white dwarf mass. We derive this temperature–mass relation in Appendix E and translate it into H–R diagram coordinates. In Figure 8 we show the results for $X^{^{22}\text{Ne}} = 0.014$ and 0.020 ([M/H] = 0 and 0.15 in the progenitor stars), $D/D_s = 3.5$, C/O-core white dwarfs. The effective zone of $^{22}$Ne settling is indeed highly peaked, and it matches the position and shape of the Q branch well.

Crystallization is a mechanism that may suppress the $^{22}$Ne settling by reducing its mobility in the plasma (e.g., Bildsten & Hall 2001; Deloye & Bildsten 2002). Therefore, in order to see a strong effect of $^{22}$Ne settling, $L_{\text{extra}}^{^{22}\text{Ne}}$ must be high enough to let the meeting point precede crystallization (see Appendix E).

Because $^{22}$Ne settling favors C/O-core and previously metal-rich white dwarfs versus O/Ne-core and/or previously metal-poor white dwarfs, and crystallization sets in earlier in O/Ne-core white dwarfs than C/O-core white dwarfs, the delay effect of $^{22}$Ne settling is indeed selective. It is worth noting that high-mass C/O-core white dwarfs are believed not to be singly evolved (e.g., Siess 2007; Lauffer et al. 2018), which means that if the extra cooling delay is really caused by $^{22}$Ne settling, then the extra-delayed white dwarfs should originate from double-WD mergers, i.e., our setup 1 is correct.

The gravitational energy of $^{22}$Ne stored in 1.0 and 1.2 $M_\odot$ white dwarfs ($Z = 0.02$) are $6.8 \times 10^{37}$ and $1.5 \times 10^{38}$ ergs (Bildsten & Hall 2001); the surface luminosity of white dwarfs on the Q branch is $10^{-3.2}$ and $10^{-2.7} L_\odot$ for the two masses. If crystallization sets in later than this luminosity, $^{22}$Ne settling can stop their cooling for around 8.9 and 6.2 Gyr, respectively, close to our observational constraint for the extra cooling delay. Existing numerical simulations (Deloye & Bildsten 2002; Garcia-Berro et al. 2008; Althaus et al. 2010; Camisasca et al. 2016) give shorter delays (0.2–4.1 Gyr) for white dwarfs with even the highest possible $L_{\text{extra}}^{^{22}\text{Ne}}$. However, the delay time is sensitive to the choice of $D/D_s$ and temperature of crystallization, but existing models have only sparsely sampled the parameter space. Moreover, for the two-component C/O plasma, the updated phase diagram (Horowitz et al. 2010; Hughto et al. 2012) suggests a much lower melting temperature than the widely used phase diagram of Segretain & Chabrier (1993) and the naive prescription of using the same condition as in one-component plasma ($\Gamma = 178$). This low melting temperature means a later crystallization, which can lengthen the delay of $^{22}$Ne settling.

In summary, we propose $^{22}$Ne settling as a promising candidate for the physical origin of the extra cooling delay. $^{22}$Ne settling has a more significant effect in C/O-core white dwarfs, which suggests that the extra-delayed white dwarfs are also merger products. To test our idea, detailed cooling models of high-mass C/O white dwarfs are needed.

7. Discussion

In this section, we discuss two other observational features of the Q branch: the concentration of DQ white dwarfs, and the lack of wide-binary systems. Both of them support the idea that the extra-delayed white dwarfs may also be double-WD merger products, which has been suggested from the $^{22}$Ne settling explanation.

7.1. Concentration of DQ White Dwarfs on the Q Branch

The Q branch is named after the presence of DQ-type white dwarfs (Gaia Collaboration et al. 2018a). To explore this dimension, we cross-match our white dwarf sample with the Montreal white dwarf database, MWDD (Dufour et al. 2017).\(^8\)

We note that most high-mass DQs are concentrated on the Q branch (Figure 9) and the fraction of fast DQs on the branch is very high (Table 2). Therefore, all of these Q-branch DQs are likely to belong to the extra-delayed population. However, not all extra-delayed white dwarfs are DQs. We estimate the fraction of DQs in the extra-delayed population to be

$$F_{\text{DQ}} = 19/(76 F_{\text{extra}}) = (53 \pm 16)\%,$$

\(^8\)http://www.montrealwhitedwarfdatabase.org
based on the total number of DQs and DAs in Table 2. Changing the distance limit of the sample does not influence this result much. It remains unclear but is of further interest to investigate the reason why half of the extra-delayed white dwarfs are DQs while the other half are DAs.

The DQs on the Q-branch are abnormal because the convection zone in a normal white dwarf with similar temperature is not deep enough to dredge up carbon convection zone in a normal white dwarf with similar temperature is not deep enough to dredge up carbon. In a similar way, the hot-DQ white dwarfs discovered by Dufour et al. (2007) are also abnormal. In Figure 9 we show the distributions of the Q-branch DQs and hot DQs on the H–R diagram. Below, we argue that although these two groups of DQs are observationally different, they may be related through an evolutionary relation.

The hot DQs and Q-branch DQs appear to be different in some aspects. Hot-DQ white dwarfs are characterized by the high temperature (>18,000 K), highly carbon-dominant atmosphere (Williams et al. 2013), high rate of having a magnetic field (Dufour et al. 2010, 2013), being variable (e.g., Dufour et al. 2009, 2011; Dunlap et al. 2010; Williams et al. 2016), and rarity (e.g., Dufour et al. 2008). In contrast, the Q-branch DQ white dwarfs are concentrated on the Q branch, have helium-dominant atmospheres with 10^{-4}–10^{-3} carbon (Kepler et al. 2015, 2016; Coutu et al. 2019), and have undetectable or no magnetic field (see Figure 9; a caveat for the magnetic field is that most hot DQs have been examined with high-resolution spectroscopy, so their magnetic fields are more likely to be found). As for kinematics, hot DQs are mildly faster than normal white dwarfs, which is an indication of being merger products (Dunlap & Clemens 2015), whereas Q-branch DQs are much faster, which needs the long extra cooling delay to explain. Dunlap & Clemens (2015) discussed one strange hot DQ (SDSS J115305.47+005645.8 or J1153) with a very high proper motion. We note that J1153 has not been reported to have magnetic field or variability and lies on the Q branch (Figure 9), which means that J1153 can be classified as a Q-branch DQ.

We now turn to the similarities between Q-branch DQs and hot DQs: both of them have high masses, and both of them might have a merger origin. These two similarities raise a serious question: are the Q-branch DQs evolved from the hot DQs? We use number counts to explore this possibility. Hot DQs are rare; based on a spectroscopically verified white dwarf sample (as shown in Figure 9), we find that the fraction of hot DQs is 8/203 = 4.0 ± 1.4% in the region earlier than the Q branch (τ_{phot} < 0.5 Gyr, m_{WD} > 0.9 M_{⊙}). As a comparison, our estimate of the extra-delayed population fraction (f_{extra}) in this region is 6.4% (Equation (21)) and about half of them are DQs (Equation (26)). So, these number counts are consistent with the scenario that hot DQs are the evolutionary counterparts of Q-branch DQs.

In summary, based on the velocity distribution, we argue that all Q-branch DQs belong to the extra-delayed population, and these DQs account for 53 ± 16% of this population. In terms of observational properties, Q-branch DQs form a new class of DQ white dwarfs in addition to the well-understood standard-mass DQs and hot DQs. However, number counts show that hot DQs may evolve into Q-branch DQs, and both of them are likely to originate from double-WD mergers.

### 7.2. Lack of Wide Binaries on the Q Branch

One additional way to test if the extra-delayed white dwarfs are also double-WD merger products is to check the wide binary fraction. The kick velocity of a few km s^{-1} (estimated from the results of Dan et al. 2014) from a merger may destroy many wide-separation binaries, making the wide-binary fraction lower. Because the extra-delayed population is significantly enhanced on the Q branch, if the extra-delayed white dwarfs are double-WD merger products, one would expect to see a lower wide-binary fraction on the Q branch.

We cross-match the wide binaries in Gaia DR2 (El-Badry & Rix 2018, 2019) with our high-mass white dwarf sample. In the early, Q-branch, and late segments, we find 5, 4, and 7 white dwarfs, with 19% having a wide binary, as compared to 25% for the sample in our main analysis. In the late segment, only 12% of the extra-delayed white dwarfs are double-WD systems. For a different sample, Williams et al. (2015) found that most hot DQs have been examined with high-resolution spectroscopy, so their magnetic fields are more likely to be found. As for kinematics, hot DQs are mildly faster than normal white dwarfs, which is an indication of being merger products (Dunlap & Clemens 2015), whereas Q-branch DQs are much faster, which needs the long extra cooling delay to explain. Dunlap & Clemens (2015) discussed one strange hot DQ (SDSS J115305.47+005645.8 or J1153) with a very high proper motion. We note that J1153 has not been reported to have magnetic field or variability and lies on the Q branch (Figure 9), which means that J1153 can be classified as a Q-branch DQ.

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In summary, based on the velocity distribution, we argue that all Q-branch DQs belong to the extra-delayed population, and these DQs account for 53 ± 16% of this population. In terms of observational properties, Q-branch DQs form a new class of DQ white dwarfs in addition to the well-understood standard-mass DQs and hot DQs. However, number counts show that hot DQs may evolve into Q-branch DQs, and both of them are likely to originate from double-WD mergers.

#### Table 2

| Velocity (km s^{-1}) | All DQs | Extra-Delayed DQs |
|----------------------|---------|-------------------|
| v_{T} > 50 km s^{-1}  | 23      | 8                 |
|                      | 30 ± 6% | 42 ± 15%          |
|                      | 14      | 26 ± 7%           |
| v_{T} > 60 km s^{-1}  | 16      | 7                 |
|                      | 21 ± 5% | 37 ± 14%          |
|                      | 8       | 15 ± 6%           |
| v_{T} > 70 km s^{-1}  | 9       | 2                 |
|                      | 12 ± 4% | 11 ± 7%           |
|                      | 6       | 11 ± 5%           |

Note. The fraction of fast DQs is consistent with it belonging purely to the extra-delayed population.
dwarfs with wide-separation companions out of 309, 510, and 251 white dwarfs, respectively. So, the wide-binary fraction on the Q branch is 0.8 ± 0.4%, 2σ lower than the value outside the branch (2.2 ± 0.5%). If we assume that the extra-delayed population contributes no wide-binary system, then the wide-binary fraction of the normal-cooling populations on the Q branch can be estimated as 4/[510 × (1 − F_{extra})] = (1.7 ± 0.8)% consistent with the off-branch value 2.2 ± 0.5% within 1σ. Therefore, the fraction of wide binaries provides additional support for the idea that the extra-delayed white dwarfs are double-WD mergers products.

8. Conclusion

The white dwarf H–R diagram derived from Gaia data has revealed a number-density enhancement of high-mass white dwarfs, called the Q branch. This branch coincides with the crystallization branch, but it is more peaked than what crystallization can create. Adding transverse-velocity information to the H–R diagram, we find a clear excess of fast white dwarfs on the Q branch (Figure 1). According to the age–velocity-dispersion relation (AVR) of Milky Way disk stars, these fast white dwarfs are much older than their photometric isochrone ages. Therefore, both the number count and velocity distribution suggest an extra cooling delay on the Q branch.

Motivated by these simple observations, we build a Bayesian model to quantitatively investigate this extra cooling delay. Because double-WD merger products also contribute to high-mass white dwarfs, we consider in our model both the extra cooling delay and the double-WD merger delay. Our model includes three white dwarf populations: one with no evolutionary delay, one with only the merger delay, and one with the extra cooling delay. We explore both situations in which all (setup 1) and none (setup 2) of the extra-delayed white dwarfs also have the merger delay. Our statistical model uses the discrepancy between the dynamical age inferred from transverse velocity and the photometric age to constrain the fraction of each white dwarf population and the length of the extra cooling delay. To eliminate selection effects, we model the conditional probability distribution of the transverse velocity of each white dwarf given its H–R diagram coordinate and spatial position. To avoid systematic errors of the model, we fit the solar motion and the anisotropic AVR together with the main parameters of interest.

We feed the model with 1070 high-mass white dwarfs (1.08–1.23 \( M_\odot \)), 0.1 Gyr < \( t_{phot} < 3.5 \) Gyr, and \( d < 250 \) pc selected from Gaia DR2. Having checked that the AVR and solar motion parameters are all in agreement with standard values from the literature and that our best-fitting model provides a good fit to the observed velocity distribution, we find

1. about 6% of the high-mass white dwarfs experience an extra cooling delay that significantly slows down their cooling and makes them stay on the Q branch for about 8 Gyr;
2. in the Q-branch region, an enhanced fraction (about a half) of white dwarfs are extra delayed due to the pile-up effect;
3. half of the extra-delayed white dwarfs are DQs;
4. as a byproduct of our analysis, the double-WD merger fraction is estimated to be about 20% in our mass range.

The results for the two setups are similar.

This previously unreported extra cooling delay on the Q branch is a challenge for white dwarf cooling models and our understanding of white dwarf physics. We propose that \(^{22}\text{Ne}\) settling (Bildsten & Hall 2001) could be the physical origin of this extra cooling delay. \(^{22}\text{Ne}\) settling favors C/O-core versus O/Ne-core white dwarfs, suggesting that the extra-delayed white dwarfs are also double-WD merger products, i.e., our setup 1 is correct. This idea is also supported by the concentration of DQ white dwarfs and lack of wide-separation binaries on the Q branch. To further investigate the nature of this extra cooling delay, detailed cooling models for \( m_{WD} > 1.1 \) \( M_\odot \) C/O-core white dwarfs with the \(^{22}\text{Ne}\) settling will be needed.

High-mass white dwarfs have been used to explore the AVR, star formation history, and white dwarf mass distribution. Given the existence of the extra cooling delay, the relevant results of those functions in the literature should be reconsidered. In future analyses of these functions, one could reduce the influence of the extra cooling delay by either using only the high-mass white dwarfs above the Q branch or modeling the extra cooling delay.

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Software: astropy package (Astropy Collaboration et al. 2013, 2018), corner.py (Foreman-Mackey 2016), emcee (Foreman-Mackey et al. 2013), numpy (Oliphant 2006), matplotlib (Hunter 2007), SciPy (Virtanen et al. 2019).

Appendix A

Parameterization of the Extra Cooling Delay

Figure 10 shows the parameterization of our extra-cooling-delay scenario on the Q branch. The observed enhancement...
parameters. Among these three steps, the key part is to construct the likelihood.

As each white dwarf provides an independent observation, the likelihood \( \mathcal{L} \) in our model can be written as the product of the likelihoods of each individual white dwarf:

\[
\mathcal{L} = \prod_i p(y_i | \theta). \tag{30}
\]

To avoid a direct dependence on selection effects, we use the conditional likelihood to let the constraining power originate only from velocity distributions: we define the individual likelihood \( p_i \) as the probability density for the \( i \)th white dwarf to have the transverse velocity \( v_T \) given all other observables of this white dwarf:

\[
p_i = p(v_T | \tau_{\text{photon}}, m_{\text{WD}}, l, b, \theta). \tag{31}
\]

We condition on \( \tau_{\text{photon}} \) and \( m_{\text{WD}} \) because their distributions are influenced by the detection completeness, quality cuts, and white dwarf spatial distribution. Moreover, the mass \( m_{\text{WD}} \) in Equation (31) model is only used to identify whether a white dwarf is on the Q branch. In order to decompose the different populations, we derive

\[
p(v_T | \tau_{\text{photon}}, m_{\text{WD}}) = \frac{p(v_T, \tau_{\text{photon}}, m_{\text{WD}})}{p(\tau_{\text{photon}}, m_{\text{WD}})} = \frac{\sum_i f_i p_i(v_T, \tau_{\text{photon}}, m_{\text{WD}})}{\sum_i f_i p_i(\tau_{\text{photon}}, m_{\text{WD}})}. \tag{32}
\]

where the sums are taken over \( x \) with possible values “s,” “m,” and “extra” representing different populations.

To express these observable distributions by the AVR and star formation history, we employ the probability identity:

\[
p(v_T, \tau_{\text{photon}}) = \int p(v_T | \tau_{\text{photon}}) p(\tau_{\text{photon}}, \tau) d\tau
= \int p(v_T | \tau) p(\tau_{\text{photon}}, \tau) d\tau \tag{33}
\]

(where the second step is valid because the velocity is only a function of the true age \( \tau \) and another identity:

\[
p(\tau_{\text{photon}}) = \int p(\tau_{\text{photon}}, \tau) d\tau. \tag{34}
\]

We also assume that the age distribution is uniform, \( \tau \sim U[0, 11 \text{ Gyr}] \). In this way, the likelihood PDF in Equations (30) and (31) can be expressed through the delay distributions \( p(\Delta \tau) \) and a velocity model \( p(v_T | \tau) \).

**Appendix C**

**The Gaussian Velocity Model**

Here, we describe the PDF of transverse velocities \( v_T \) and their true stellar ages \( \tau \) using the AVR. The velocity distribution of disk stars in the solar neighborhood with respect to the local standard of rest can be approximated as a superposition of 3D Gaussian distributions (e.g., Binney \\& Tremaine 2008):

\[
p(v | \tau) = \frac{\exp\left[-\frac{1}{2}(v - v_0)^T \Sigma(\tau)^{-1}(v - v_0)\right]}{\sqrt{8\pi |\Sigma(\tau)|}}, \tag{35}
\]

whose mean and covariance matrix are determined by stellar age \( \tau \). The mean velocity \( v_0(\tau) \) is determined by two effects: the
solar reflex motion ($-U_c, -V_c, -W_c$) with respect to the local standard of rest, and the asymmetric drift in $V$ direction by $-\sigma_U^2/80 \text{ km s}^{-1}$ (e.g., Binney & Tremaine 2008). We set the solar motion as free parameters and use them to check the validity of our model.

To obtain the distribution of the observable transverse velocity $v_T = (v_L, v_B)^T$, we project the 3D Gaussian distribution $p(v|\tau)$ onto the tangential plane for each white dwarf and marginalize the radial component $v_R$. Because the resulting distribution is still a Gaussian distribution for a given age $\tau$, the only task is to find its covariance matrix and mean vector. Let $v_{XYZ} = (U, V, W)$ and $v_{LBR} = (v_L, v_B, v_R)$ be the expressions of the same vector $v$ in the XYZ and LBR coordinate systems, respectively, and matrix $M$ the rotation transformation matrix between the two systems:

$$(v - v_0)_{XYZ} = M \cdot (v - v_0)_{LBR}.$$  

where

\[
M = \begin{bmatrix}
\sin l & -\sin b \cos l & \cos b \cos l \\
\cos l & -\sin b \sin l & \cos b \sin l \\
0 & \cos b & \sin b
\end{bmatrix}. \tag{36}
\]

We ignore the small in-disk rotation between the Cartesian coordinate XYZ and the galactic polar coordinate. Then, we write the 3D Gaussian distribution in both coordinate systems (note that the Jacobian determinant of rotation transform is unity) and obtain the following relation:

$$(v - v_0)_{XYZ}^T \Sigma_{XYZ}^{-1} (v - v_0)_{XYZ} = (v - v_0)_{LBR}^T \Sigma_{LBR}^{-1} (v - v_0)_{LBR}.$$  

Substituting Equation (36), we obtain

$$\Sigma_{LBR} = (M^T \Sigma_{XYZ} M)^{-1} = M^T \Sigma_{XYZ} M,$$ \tag{39}

where we assume

$$\Sigma_{XYZ} = \begin{bmatrix}
\sigma_U^2 & 0 & 0 \\
0 & \sigma_V^2 & 0 \\
0 & 0 & \sigma_W^2
\end{bmatrix}. \tag{40}
\]

The covariance matrix $\Sigma_{LB}$ for the 2D Gaussian distribution marginalized along the $R$ direction is the top left $2 \times 2$ sub-matrix of $\Sigma_{LBR}$. The mean vector $v_{TO} = (v_0)_{LB}$ can be derived directly from vector rotation and projection. Then, the conditional PDF of the two transverse components $v_T = (v_L, v_B)$ of $v$ can be written as

$$p(v_T|\tau, l, b) = \exp\left[\frac{-1}{2} (v_T - v_{TO})^T \Sigma_{LBR}^{-1} (v_T - v_{TO}) \right] \sqrt{4\pi|\Sigma_{LBR}|}, \tag{41}
\]

where the covariance matrix $\Sigma_{LBR}(\tau, l, b)$ and mean $v_{TO}(\tau, l, b)$ depends on ($\tau, l, b$). We use the condition on ($l, b$) for each white dwarf to avoid modeling the spatial selection effects and reduce unnecessary parameters and biases.

**Appendix D**

**MCMC Settings**

In our Bayesian model, we assume uniform distributions for parameter priors. We feed the affine invariant MCMC sampler emcee (Foreman-Mackey et al. 2013) with the natural logarithm of the likelihood function defined in Equation (30). We use 500 walkers to explore the parameter space. After 200 steps of burn-in, the chains are checked to converge by comparing the percentile values of the parameters in each chain. Then, we run another 400 steps and use this sampling to represent the posterior distribution of each parameter. Figure 11 shows the marginal posteriors of the 10 parameters of the AVR and their correlations, under the setup 1. Setup 2 leads to similar constraints of the AVR.

**Appendix E**

**The Peak of $^{22}$Ne-settling Effect**

The delay effect depends on the fractional contribution $L_{extra}/L_{surf}$ of the extra source luminosity to the surface luminosity of the white dwarf because the more this extra energy contributes, the less does the white dwarf need to consume its thermal energy and to cool down. The pile-up factor of this effect can be expressed as

$$A = \frac{\zeta^{-1}}{\zeta_0^{-1}} - 1 = \frac{1}{L_{surf}/L_{extra} - 1}, \tag{42}\]

where $A$ is the same as defined in Equation (27), $\zeta$ and $\zeta_0$ are the cooling rates with and without the extra energy. Assuming no crystallization suppression, $L_{extra}$ will meet $L_{surf}$ at some surface temperature $T_{eff}$. If this occurs, $^{22}$Ne settling will stop the white dwarf cooling at this temperature until $^{22}$Ne is exhausted, creating a peaked delay effect. Here, we estimate the dependence of this meeting temperature as a function of white dwarf mass, which can be translated into a curve on the H–R diagram.

According to Bildsten & Hall (2001), the energy release of $^{22}$Ne settling can be expressed by

$$L_{extra}^c = \int_0^R FVn(\zeta_{\text{Ne}}4\pi r^2 dr,$$  

where $F = 2m_\text{p}g_r$ is the net force felt by each nucleus of $^{22}$Ne, $g_r$ is the gravity at radius $r$, $V = \left(\frac{D}{D_s}\right)18m_\text{p} \zeta \varpi / (Ze^{1/3} / \sqrt{4\pi \rho})$ is the drift velocity of the settling, $\Gamma \equiv (Ze^2 / (akT)) \propto \rho^{1/2} / (TA^{1/2})$ is the Coulomb coupling parameter, $D$ is the inter-diffusion coefficient of $^{22}$Ne, and $D_s$ is the one-component self-diffusion coefficient, which can be used as a reference value. Substituting these quantities in Equation (43), we obtain

$$L_{extra}^c \propto \int_0^R g_r \cdot \frac{D}{D_s} X(\zeta_{\text{Ne}}) c_{\text{eff}}^{0.11} \zeta^{0.67} \rho^{-0.61} \rho^{0.33} \rho^{0.33} r^{0.33} \cdot \rho r^2 dr,$$ \tag{44}

where $X$ is the element abundance in mass, $c_{\text{eff}}$ is the core temperature. For the main composition of a white dwarf, the charge-to-mass ratio $Z/A = 0.5$ is a constant. Assuming the following approximations: $g_r \sim g \sim M/R^2$, $\rho \sim g/R$, \[\int_0^R dr \sim R\], we obtain

$$L_{extra}^c \propto \frac{D}{D_s} X(\zeta_{\text{Ne}})Z^{-1.56} c_{\text{eff}}^{0.33} \zeta^{0.67} \rho^{0.61} \rho^{0.33} r^{0.33} R^2,$$ \tag{45}

Before the convective coupling between core and atmosphere, the core temperature $T_c$ scales with the surface temperature $T_{eff}$ and gravity $g$ as

$$T_c \propto (T_{eff}^4 / g)^{0.41}, \tag{46}$$
which is obtained empirically from existing white dwarf models (Fontaine et al. 2001). This scaling relation is more realistic than the one used by Mestel (1952). Substituting Equation (46) for $T_c$, we obtain

$$\Delta \sigma_{1/4}^{\text{eff}} = 6.07 \pm 0.07$$

We checked the simulation results from Figures 7 and 8 of García-Berro et al. (2008) and found that our scaling relation is accurate to within 10%, which is sufficient for our purpose.

When the surface luminosity $L_{\text{surf}} \propto T_{\text{eff}}^4 \cdot R^2$ is set equal to $L_{\text{extra}}^\text{Ne}$, we obtain

$$T_{\text{eff}} = K^{0.29} g^{0.65} R^{0.18},$$

where $K \propto \frac{D}{D_t} X^{(22\text{Ne})} Z^{-1.56}$ is a constant that has no relevance to the white dwarf mass $M$, and both $g$ and $R$ are determined by $M$. The proportional factor within $K$ can be evaluated from an existing simulation of $^{22}\text{Ne}$ settling.
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