OBSERVATIONAL CONSTRAINTS ON PLANET NINE: ASTROMETRY OF PLUTO
AND OTHER TRANS-NEPTUNIAN OBJECTS

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ABSTRACT

We use astrometry of Pluto and other trans-neptunian objects to constrain the sky location, distance, and mass of the possible additional planet (Planet Nine) hypothesized by Batygin & Brown. We find that over broad regions of the sky, the inclusion of a massive, distant planet degrades the fits to the observations. However, in other regions, the fits are significantly improved by the addition of such a planet. Our best fits suggest a planet that is either more massive or closer than argued for by Batygin & Brown based on the orbital distribution of distant trans-neptunian objects (or by Fienga et al. based on range measured to the Cassini spacecraft). The trend to favor larger and closer perturbing planets is driven by the residuals to the astrometry of Pluto, remeasured from photographic plates using modern stellar catalogs, which show a clear trend in decl. over the course of two decades, that drive a preference for large perturbations. Although this trend may be the result of systematic errors of unknown origin in the observations, a possible resolution is that the decl. trend may be due to perturbations from a body, in addition to Planet Nine, that is closer to Pluto but less massive than Planet Nine.

Key words: astrometry – ephemerides – Kuiper Belt: general – Kuiper Belt objects: individual (Pluto)

Supporting material: machine-readable table

1. INTRODUCTION

Uranus had made nearly one full orbit since its discovery when astronomers reported large deviations in its observed sky positions compared with those predicted by available ephemerides (Bouvard 1824). Based on those residuals, Le Verrier & Adams (Adams 1846; Le Verrier 1846a, 1846b) predicted the existence of Neptune, which was discovered shortly thereafter (Galle 1846). That celebrated success ensured that scientists would sporadically revisit the possibility of yet undiscovered planets in the solar system (see Tremaine 1990; Hogg et al. 1991 and Gaudi & Bloom 2005 for reviews).

Although similar anomalies in the positions of Neptune motivated the search that ultimately resulted in the discovery of Pluto (Lowell 1915, p. 1), Pluto was not sufficiently massive to account for them. Improved planetary ephemerides based on more accurate planetary masses and careful vetting of observational data eliminated most of those residuals (Quinlan 1993; Standish 1993).

Nevertheless, the discovery of the Kuiper Belt (KB; Jewitt & Luu 1993), including its intricate dynamical structure (Malhotra 1995; Gladman et al. 2012), detailed size distribution (Gladman et al. 2001; Bernstein et al. 2004; Fraser et al. 2014), numerous large members (Brown et al. 2004, 2005; Schwamb et al. 2010), and distant components (Gladman et al. 2002; Brown et al. 2004; Trujillo & Sheppard 2014), rekindled enthusiasm for the possibility of additional planets orbiting undetected in the far-reaches of the outer Solar System. A number of current solar system formation models feature large-scale planet migration (Malhotra 1993, 1995; Fernández & Ip 1996; Levison & Morbidelli 2003; Morbidelli et al. 2005; Tsiganis et al. 2005; Levison et al. 2008), additional giant planets formed ~5–35 au from the Sun and then scattered within or ejected from the solar system (Chiang et al. 2007, p. 895; Bromley & Kenyon 2014, 2016), or the formation of planets at larger distances (Kenyon & Bromley 2015, 2016).

An additional planet could leave observable signatures in the orbital distribution of trans-neptunian objects (TNOs). Brunini & Melita (2002) investigated the effect of a relatively close (~60 au), Mars-mass object embedded in the KB region, proposing that such an object could naturally explain the observed “edge” in the KB distribution at ~50 au, although later work found that such a body would be inconsistent with other features of the observed orbital distribution of TNOs (Melita et al. 2004). Lykawka & Mukai (2008) also simulated the dynamical evolution of the outer solar system under the influence of a planet with a mass a few tenths that of the Earth. They hypothesized that such a body was scattered out by one of the giant planets, causing it to excite the primordial KB to the levels observed at 40–50 au while also truncating it at ~50 au. Subsequent interactions then pushed the outer planet into a distant (≥100 au) and inclined (20°–40°) orbit.

Gomes et al. (2006) examined the long-term, secular influence of a planet in the inner Oort cloud on the orbits of scattered-disk objects with initial perihelion values near Neptune (32 < q < 38 au). They found that such a planet with a $M_\text{pl}/b_\text{pl}^3 > 0.8 \times 10^{-14} M_\odot$ au$^{-3}$, where $b_\text{pl} = a_\text{pl}(1 - e_\text{pl}^2)$ ($a_\text{pl}$ and $e_\text{pl}$ being the respective semimajor axis and eccentricity of the companion), can lift the perihelia of scattered-disk objects (SDOs) to q > 75 au (values seen for Sedna and 2001 CR105 and other extreme scattered-disk objects) on Gyr timescales.

In their paper announcing the discovery of 2012 VP113, Trujillo & Sheppard (2014) pointed out that the known “extreme scattered-disk objects” (semimajor axes $a > 150$ au and perihelion distances $q > 30$ au) have arguments of perihelion clustered near $\omega \approx 340^\circ \pm 55^\circ$. They emphasized that observational bias cannot account for this clustering. Their simulations showed that a super-Earth-mass body at 250 au can maintain the values of $\omega$ near zero for billions of years. Trujillo & Sheppard (2014) speculated that such a planet is responsible for the observed argument of perihelion clustering and note that
such a planet, with very low albedo, would be fainter than the detection limits of current surveys. de la Fuente Marcos & de la Fuente Marcos (2014) used simulations of observational surveys to confirm that the orbital clustering of extreme scattered-disk objects pointed out by Trujillo & Sheppard (2014) is unlikely to be the result of observational bias.

Gomes et al. (2015) examined the orbital distribution of distant centaurs ($a > 250$ au) and found that the large fraction of such distant objects which are luminous (compared to classical centaurs) can be best explained by the existence of one or more distant planets in the extended scattered disk.

Batygin & Brown (2016) discovered that the orbits of distant KBOs cluster not only in argument of perihelion, but also in physical space. They found that the observed orbital alignment could be maintained by a distant, eccentric planet of mass $\sim 10\, M_\oplus$ orbiting in approximately the same plane as the KBOs, but with a perihelion $180^\circ$ away. In addition, they found that such a planet could explain the orbits of high semimajor axis objects with inclinations between $60^\circ$ and $150^\circ$ whose origin was previously unclear. Despite its otherwise impressive specificity, the long-term dynamical analysis of Batygin & Brown (2016) does not directly constrain the location of Planet Nine within its orbit.

Most of the constraints on the mass and orbit of Planet Nine have come from the observed orbital distribution of TNOs and how Planet Nine could account for peculiar features of that distribution, upper limits from dynamical effects that are not observed (Hogg et al. 1991; Iorio 2009, 2012, 2014), non-detections in optical (Brown et al. 2015) and infrared surveys (Luhman 2014), and models of the physical properties such a planet might have (Ginzburg et al. 2016). Other investigations have used the simulation of orbital distributions and survey results to constrain the sky plane location of Planet Nine (Brown & Batygin 2016; de la Fuente Marcos & de la Fuente Marcos 2016).

However, the analysis of the precision ranging measurements to the Cassini spacecraft (Fienga et al. 2016) or studies of the resonant interactions that distant objects would have with Planet Nine (Malhotra et al. 2016) can also favor or rule out particular sky plane positions for the planet on a physical basis. To our knowledge, the Cassini range measurements are the only direct observations that support the existence, mass, and orbit of Planet Nine.

In the present investigation, we examine the astrometry of Pluto and other TNOs to search for direct evidence of Planet Nine. Precise positional observations of these bodies have the potential to be particularly sensitive to the mass and orbit of Planet Nine for a number of reasons. The astrometry of Pluto spans more than a century (pre-discovery observations date to 1914), and a number of the large TNOs have been observed for several decades. These long time spans allow the effects of weak perturbations to accumulate. The large heliocentric distances of these bodies also make them sensitive probes to tidal perturbations. Finally, the availability of improved stellar catalogs, with reliable proper motions, permit the accurate remasurement of archival data.

We organize the remainder of this paper as follows. In Section 2, we present our model and numerical methods. In Section 3, we describe the observational data to which we apply our model. In Section 4.3, we extend our method to the full sky and, in Section 4, we present those results. In 5, we discuss our conclusions and review the overall constraints on the existence of Planet Nine.

2. NUMERICAL METHOD

Following the approach of Hogg et al. (1991), we test whether fits to the orbits of Pluto and a set of well-observed TNOs are improved by the inclusion of a massive, distant perturber. We use $\chi^2$ to evaluate the fit, where $\chi^2$ is approximately

$$\chi^2 = \sum_i \cos^2(\delta_{o,i}) \left( \frac{\alpha_{o,i} - \alpha_{c,i}}{\sigma_\alpha} \right)^2 + \left( \frac{\delta_{o,i} - \delta_{c,i}}{\sigma_\delta} \right)^2.$$  

The observed R.A. and decl. of the $i$th observation are $\alpha_{o,i}$ and $\delta_{o,i}$, and the corresponding calculated values from the model are $\alpha_{c,i}$ and $\delta_{c,i}$. Their respective uncertainties are $\sigma_\alpha$ and $\sigma_\delta$. (Internally, our code uses a local tangent plane to evaluate each difference between observed and calculated values to avoid dependence on a particular coordinate system.)

First, we determine the minimum chi-squared, $\chi^2_{\text{min}}$, for a reference model that includes the Sun and known planets, as well as the large TNOs as gravitational perturbers. Then, we determine the corresponding value, $\chi^2_{\text{pert}}$, for a model that includes an additional perturbing planet (details of the orbit of the perturbing planet are given below). If the value of $\Delta \chi^2 = \chi^2_{\text{pert}} - \chi^2_{\text{min}}$ is negative and significant, i.e., if the fit is improved, then we take that as evidence supporting the hypothesis. If the $\Delta \chi^2$ is positive and sufficiently large, then we take that as excluding a planet with those parameters.

2.1. Orbit Fitting

We carry out the fits with an extensively modified version of Orbfit (Bernstein & Khushalani 2000), which is software developed to fit the orbits of TNOs to astrometric observations. Orbfit determines the cartesian position and velocity of the fitted TNO in an inertial frame at a given epoch. The trajectory of the TNO is integrated in the gravitational field of the Sun and planets to the time of each observation, with the topocentric position of the observatory and the light travel time properly accounted for. The $\chi^2$ of the fit of the model to the observations is minimized using the Levenberg–Marquardt algorithm (Press et al. 1986). The parameter basis of Orbfit is orthogonal and provides an approximation to the orbital motion that is nearly linear in the parameters. Thus, in this case, the Levenberg–Marquardt algorithm is well-behaved.

Our modifications to Orbfit include the following.

1. We eliminated Orbfit’s dependence on the tangent plane approximation. Although this approximation is well suited to fitting TNO observations that cover a small number of years, Pluto has traversed nearly $180^\circ$ in true anomaly over the span of its observations. Thus, a tangent plane does not accurately capture Pluto’s observed trajectory. Internally, we represent the sky position of the fitted object with a three-dimensional, topocentric unit vector directed from the observatory to the observed location. However, when comparing the calculated sky position with those observed, we resolve the differences along the R.A. and decl. directions using local tangent plane projections (Herget 1965).

2. We replaced Orbfit’s analytic but approximate expressions for the derivatives of the observable properties as a
function of orbital parameters (required by the Levenberg–Marquardt minimization routine) with numerical but fully accurate derivatives.

3. We included the Sun and all of the known planets as perturbers, rather than just the four giant planets. In addition, we include 10 massive TNOs as perturbers: see Section 2.3 for further discussion of the perturbation magnitudes arising from the known TNOs.

4. We upgraded the underlying ephemeris from JPL’s DE405 to DE432.

5. We replaced Orbit’s calculation of the topocentric position of the observatory with routines that use current data for the geocentric observatory positions from the Minor Planet Center (MPC).

6. We reduced the time step in Orbit’s symplectic leap-frog integrator to 10 days from 20 days.

We emphasize that these modifications are only necessary because Pluto has been observed for such a long orbital arc and because we want to ensure that our results are not limited by the accuracy of the model. Orbit results for typical TNOs investigations are still valid.\(^1\)

2.2. Tidal Perturbations from Planet Nine

The principal effect of a distant planet is to accelerate all of the masses in the solar system toward that planet. Any perturbation that leaves the relative positions and velocities of the Sun and planets essentially unchanged, and that does not significantly accelerate the solar system barycenter, would be unobservable (the uniform velocity of the solar system barycenter is removed in the DE432 ephemeris, as is the case for other JPL ephemerides). Only changes in the relative position or velocity between an observing station and some other reference position or velocity can be detected. Thus, in our case, only the effects of tidal acceleration between the observing station and the observed object are important.

As pointed out by Tremaine (1990) and Fienga et al. (2016), the most conservative approach is to vary all of the parameters for all of the bodies when fitting an ephemeris model to the observations. However, to simplify our analysis, we assume that Sun and planets follow the fixed barycentric trajectories given by the DE432 ephemeris and are not affected by Planet Nine, and we vary only the barycentric orbital parameters of Pluto and other TNOs. We have several justifications for this approach. First, the DE432 ephemeris is an accurate fit of a very comprehensive dynamical model to essentially all of the relevant, constraining observations of the solar system bodies. There is little room for the Sun and planets to deviate from the trajectories specified by DE432 while satisfying the observational constraints.

Second, the planets only weakly perturb the orbits of Pluto and other TNOs. Small changes in the trajectories of the solar system planets themselves due to an additional planet will lead to negligible changes to the perturbations of the solar system planets on Pluto and the other TNOs.

Third, changes in the location of the observing sites (primarily through changes in the location of the geocenter) relative to the solar system barycenter would be a factor of \(~30\) smaller than corresponding changes in the location of Pluto or other TNOs due to any additional acceleration (see below).

Fourth, in Holman & Payne (2016) we extended Fienga et al. (2016)’s study of the Cassini range residuals, demonstrating that a perturbative approach is able to rapidly and accurately reproduce the results of a full ephemeris fitting.

Finally, in Appendix A, we explicitly demonstrate the equivalence between full \(n\)-body integrations and our standard model for some select examples.

We include the acceleration of Pluto and TNOs due to an additional planet in two different ways. First, we explicitly incorporate the relative acceleration between Pluto (or the other TNOs) and the barycenter due to Planet Nine into the equations of motion. The additional acceleration of Pluto is given by

\[
a_P = \frac{GM_X}{r_P^3} (r_P - r_X) - \frac{GM_X}{r_X^3} r_X, \tag{2}
\]

where \(r_P\) is the barycentric position of Pluto (or TNO), \(M_X\) and \(r_X\) are the respective mass and barycentric position vector of the distant planet, \(r_P = |r_P|\), \(r_X = |r_X|\), and \(r_PX = |r_P - r_X|\). The primary approximation, as noted above, is that the additional planet accelerates the barycenter of the solar system, rather than the individual solar system bodies. We further assume that Planet Nine follows a keplerian orbit and is itself unperturbed by the other planets. We refer to this approach as the “moving-planet model.”

For our second approach, following Hogg et al. (1991), we assume that the additional planet is stationary and is sufficiently distant that its gravitational potential can be approximated as

\[
\Phi(r_P) = \frac{GM_X}{2r_X^3} [r_P^2 - 3(r_P \cdot \dot{r}_X)^2], \tag{3}
\]

where \(\dot{r}_X = r_X/r_X^3\), and the corresponding tidal acceleration is given by

\[
da_P = -\frac{GM_X}{r_PX^3} [r_P - 3(r_P \cdot \dot{r}_X)\dot{r}_X]. \tag{4}
\]

The strength of the tidal perturbation scales with \(M_X/r_PX^3\). We refer to this approach as the “tidal model.” We note that if Planet Nine has a semimajor axis \(~700\) au (Batygin & Brown 2016), then its period will be \(~18,000\) years; hence over the \(~85\) year span of our observations, Planet Nine is approximately stationary on the sky.

2.3. Large TNOs

There are a number of large TNOs that might also perturb the orbits of Pluto and other TNOs. Although these are all substantially less massive than Planet Nine might be, they are much closer. Thus, in addition to the gravitational perturbations from the Sun and planets, we include the perturbations from a set of large TNOs. The masses of the majority are known from the orbits of their satellites. For the rest, we estimated masses. The large TNOs and their properties are provided in Table 1. Paralleling our assumption that the Sun and planets follow the trajectories given by DE432, we assume that the large TNOs follow fixed orbits.

Over the span of Pluto’s observations, the acceleration of Pluto due to other TNOs at a given time can exceed that from Planet Nine, for the smaller tidal parameters of Planet Nine. For example, the direct acceleration due to Haumea exceeds the tidal acceleration from Planet Nine for several years in our simulations. This does not imply that the long-term effect of

\(^1\) We will describe these modifications in greater detail in a forthcoming publication. The code is available upon request.
those accelerations is larger than that from Planet Nine. It is simply a measure of the effect on shortest timescales. The tidal influence from Planet Nine, with its acceleration oriented in the same direction over long periods, can dominate over other accelerations that tend to average out. Similarly, resonant interactions between Pluto and other bodies, even those with small masses, can dominate.

### 3. OBSERVATIONAL DATA

In order to guide the trajectory of the *New Horizons* spacecraft to its encounter with the Pluto system (Stern et al. 2015), a number of groups concentrated on improving the determination of Pluto’s orbit (Assafin et al. 2010; Benedetti-Rossi et al. 2014; Buie & Folkner 2015; Girdiuk 2015; Pitjeva 2015).

As part of that effort, Buie & Folkner (2015) used the DASCH (Grindlay et al. 2009) and DAMIAN (Robert et al. 2011) scanning systems to remeasure a collection of photographic plates of Pluto taken by Carl Lampland at Lowell Observatory from 1930 through 1951. The primary motivation for the re-analysis was to identify and correct any significant but previously unrecognized systematic errors in the historical astrometry of Pluto that might be affecting its orbit determination. JPL’s DE432 ephemeris incorporates the results of the Buie & Folkner (2015) analysis. The three-dimensional positional uncertainty of the Pluto system just prior to the encounter was ~1000 km. The successful *New Horizons* flyby of the Pluto system demonstrates the accuracy with which Pluto’s location and orbit are currently known.

Initially, we considered using all of the astrometric positions of Pluto and TNOs available from the Minor Planet Center’s (MPC). However, the data set is heterogeneous and lacks reported astrometric uncertainties. Thus, we decided to build upon the carefully selected data of Buie & Folkner (2015) for our analysis of Pluto. We use the MPC astrometry for TNOs only.

For Pluto, we include the astrometry from the remeasured Lampland plates (Buie & Folkner 2015); that from a selection of photographic plates from Pulkovo Observatory that were also remeasured with modern stellar catalogs (Ryklov et al. 1995); Pluto or Charon positions from recent occultation measurements of Pluto and Charon (Assafin et al. 2010; Benedetti-Rossi et al. 2014); and CCD observations from Pico dos Dias Observatory (Benedetti-Rossi et al. 2014), the USNO’s Flagstaff Station (Stone et al. 2003), and JPL’s Table Mountain Observatory (described in Buie & Folkner 2015). Note that, like Buie & Folkner (2015), we exclude the Lowell data from 1930, as those data appear to have a systematic trend that may be due to systematic uncertainties in their observation times.

We adopt the uncertainties of Buie & Folkner (2015) for the Pluto astrometry from Lowell and Pulkovo Observatory. However, we used significantly smaller astrometric uncertainties for the Pico do Dias data. Buie & Folkner (2015) argued that the observations taken within a single night would have correlated errors due to the astrometry being measure by a common set of stars. Accordingly, they increased the reported astrometric uncertainties by a factor $\sqrt{N}$, where $N$ is the number of observation within a night. We find that this overestimates the uncertainties based on the post-fit rms. We find that increasing the astrometric uncertainties of Benedetti-Rossi et al. (2014) by a factor of ~2 results in a match with the post-fit rms.

We also used somewhat smaller astrometric uncertainties for the remaining data sets. For the USNO astrometry, we adopt 0''09 for both R.A. and decl. For the Table Mountain Observatory astrometry, we adopt 0''07 and 0''05 for the R.A. and decl., respectively. For the occultation data we adopt 0''05 and 0''03 for the R.A. and decl., respectively. The astrometric uncertainties we adopted result in a $\chi^2$ per degree of freedom = 1 for the unperturbed Pluto orbit fits.

We included all TNOs, including SDOs, with semimajor axes $a \gtrsim 30$ au for which we could fit reliable orbits. For these objects, we adopt a fixed astrometric uncertainty of 0''27, which results in $\chi^2$ per degree of freedom = 1 for the ensemble of TNOs.

We include a total of 6677 observations for Pluto, plus 35,646 observations of other TNOs.\(^2\)

\(^2\) A machine-readable table that includes all observations is available online.

### Table 1

The TNOs Which Were Assigned as Massive Perturbers within Our Version of Orbit

| Designation | Name           | Separation (au) | Mass ($M_\oplus$) | $M_f/\epsilon^3$ ($M_\oplus$ au$^{-3}$) | Mass Reference |
|-------------|----------------|-----------------|-------------------|----------------------------------------|-----------------|
| (136199)    | Eris           | 102–128         | $8.35 \times 10^{-9}$ | $3.3 \times 10^{-16}$–$6.6 \times 10^{-16}$ | Brown & Schaller (2007) |
| (134340)    | Pluto+Charon   | ...             | $7.33 \times 10^{-9}$ | ...                                    | Stern et al. (2015) |
| (136108)    | Haumea         | 11–58           | $2.01 \times 10^{-9}$ | $3.4 \times 10^{-15}$–$5.3 \times 10^{-13}$ | Ragozzine & Brown (2009) |
| (500000)    | Quaoar         | 13–39           | $7.0 \times 10^{-10}$ | $1.2 \times 10^{-12}$–$2.1 \times 10^{-13}$ | Fraser et al. (2015) |
| (90377)     | Sedna          | 108–130         | $5.0 \times 10^{-10}$ | $3.2 \times 10^{-15}$–$5.6 \times 10^{-16}$ | Assumed         |
| (90482)     | Orcus          | 35–73           | $3.2 \times 10^{-10}$ | $1.8 \times 10^{-15}$–$1.6 \times 10^{-14}$ | Carry et al. (2011) |
| (307261)    | 2002 MS4       | 17–55           | $3.2 \times 10^{-10}$ | $4.2 \times 10^{-15}$–$2.1 \times 10^{-13}$ | Assumed         |
| (120347)    | Salacia        | 47–71           | $2.2 \times 10^{-10}$ | $2.0 \times 10^{-15}$–$6.7 \times 10^{-15}$ | Stansberry et al. (2012) |
| (136472)    | Makemake       | 16–68           | $1.76 \times 10^{-10}$ | $2.2 \times 10^{-15}$–$1.7 \times 10^{-13}$ | Assumed         |
| (225088)    | 2007 OR10      | 68–113          | $1.76 \times 10^{-10}$ | $4.9 \times 10^{-16}$–$2.2 \times 10^{-15}$ | Assumed         |
those observations were serendipitous, rather than part of a systematic observing program. In all cases, there are just a few archival data points that are separated from rest by decades. Although the archival observations tightly constrain the orbital elements of the TNOs, they do not appear to add significantly to the constraints on Planet Nine. That could change if the plates were remeasured, but we are not aware of a program to systematically remeasure those photographic plates, such as the Buie & Folkner (2015) program for the Lampland plates.

4. RESULTS

Using the methods described in Section 2, we perform fiducial orbit fits for Pluto and all other TNOs in an unperturbed Solar System model (i.e., with no Planet Nine). The resulting orbits of Pluto and the other TNOs are in detailed agreement with those available from JPL or the MPC. In Figure 1, we plot the unperturbed orbital fit of Pluto. In Appendix B we provide observatory-specific plots of subsets of this data.

4.1. Level of Significance

After determining the best-fit orbit of Pluto assuming no additional perturber, we now evaluate whether the inclusion of Planet Nine improves or degrades the fits.

As we note in Section 4.3, we evaluate our two models, tidal and moving planet, at a full range of locations on the sky. At each of those locations, the direction of the perturbation, corresponding to either the tidal perturbation vector or the location of Planet Nine at a reference epoch, is fully specified by two position angles. We add a perturbation due to Planet Nine using the moving planet and tidal methods of Equations (2) and (3), respectively.

For the tidal model, we introduce one additional degree of freedom: the magnitude of the perturbation (which scales as $M_X R_X^{-3}$ as per Equation (3)). For the moving-planet model, we introduce two additional degrees of freedom: the separate mass and semimajor axis of the planet.

If, in reality, there is no additional planet in the solar system, then the fits will be slightly improved simply by introducing additional degrees of freedom, $\Delta \chi^2 = 1$ and $\Delta \chi^2 = 2$, for tidal and moving-planet models, respectively. However, with $3 - \sigma$ confidence, we expect $\Delta \chi^2 < 9$ and $\Delta \chi^2 < 11.4$ (Press et al. 1986). More substantial improvements in the fits can be taken as ruling out the null hypothesis that there is no additional planet or unmodeled acceleration.

Likewise, if the inclusion of Planet Nine degrades the fits by more than these thresholds, then we take that as strongly favoring the null hypothesis over an additional planet, i.e., excluding a perturber with those parameters.

We caution that these confidence intervals assume that the errors are normally distributed and uncorrelated, which is not the case. However, our results do not depend sensitively on the threshold value of $\Delta \chi^2$.

4.2. Perturbed Orbital Fits

We begin by illustrating in Figure 2 the results we obtain for a perturber at a few specific locations on the sky.

For each of these locations, we define a unit vector toward that point and then explore a variety of masses and semimajor axes which span a range of $10^{-14}$–$10^{-10} M_{\odot}$ au$^{-3}$ in the tidal perturbation. (For reference, $M_X \sim 10 M_{\oplus}$ at $R_X \sim 600$ au yields a tidal parameter of $M_X R_X^{-3} \sim 1.4 \times 10^{-13}$.) For both the tidal and moving-planet perturbation models, we refit all of the orbits using Orbit and calculate the change in $\Delta \chi^2 = \chi^2_{\text{pert}} - \chi^2_{\text{ref}}$ as described in Section 2.

In Figure 2, we plot $\Delta \chi^2$ as a function of the perturbation magnitude for three different example locations on the sky. We find that the tidal model and the full orbiting planet model are similar in all cases. The small variations that can be seen in the moving-planet models are due to the spacing of the grid points in mass and semimajor axis, rather than being a numerical artifact.

In the top panel, we plot the results for a perturber at R.A., decl. = 180°, −27°. For regions similar to this example, all
perturbation magnitudes lead to a worsening of the fit, and hence we can only provide upper limits on the perturbation scale (dashed lines in the above plots), which are within $\Delta \chi^2_{3\sigma}$ of zero. In this top plot, we rule out perturbations $\gtrsim 2 \times 10^{-13} M_\odot \text{au}^{-3}$. For regions of the sky similar to the middle and bottom plots, not only can we provide upper limits, but we can also find the perturbation scale at the absolute minimum $\Delta \chi^2$. That minimum, and an upper limit on the perturbation. For both of these example regions, we see that the best-fit perturbation is approximately $10^{-11} M_\odot \text{au}^{-3}$.

4.3. Tiling the Sky

The gravitational influence of a perturbing planet depends upon its direction and distance from other bodies in the solar system. We use the HEALPix tiling of the sky (Górski et al. 2005) to test a complete set of perturbing planet locations uniformly distributed on the sky. For the results presented in Section 4, we concentrate on the $N_{\text{side}} = 2^4 = 16$ resolution level, resulting in 3072 tiles. We interpret each HEALPix location as representing an individual possible perturber position in an equatorial frame. We then transform these within OrbitFit to ecliptic coordinates, as well as the body-specific projection coordinates used in OrbitFit (Bernstein & Khushalani 2000). For a given HEALPix tile, i.e., a given R.A. and decl., we study a range of distances and masses for Planet Nine.

When we adopt the tidal potential of a stationary, distant planet, nothing more needs to be specified as the object is stationary.

In the case of a moving planet, we assume that its orbit is circular ($e = 0$) and prograde with semimajor axis $a = r_\text{G}$. For components of the unit vector to Planet Nine in ecliptic coordinates $x, y$, and $z$, we take the inclination to be $f = \sin^{-1}z$. We adopt $\omega = \pi/2$ for $z \geq 0$ and $\omega = -\pi/2$ for $z < 0$ for the argument of perihelion, and $\Omega = \theta - \omega - M_0$ for the longitude of the ascending node, where $\theta = \tan^{-1}(y/x)$ is the ecliptic longitude and $M_0$ is the mean anomaly at a reference date which we take to be zero at JD 2436387.0 (1958 July 2.5). This places Planet Nine at the specified location toward the middle of our data span for Pluto.

To test the dependence of our results on the rate and direction of motion of Planet Nine, we also explored retrograde and polar orbits following a similar approach. As we demonstrate below, our results do not depend sensitively to the details of Planet Nine’s orbit.

We now divide the sky into 3072 regions using the HEALPix tessellation described above, and repeat the fitting procedure illustrated in Figure 2 at each HEALPix location. We plot these all-sky results in Figure 3.

In Figure 3, the plots on the left show the results from the tidal model of Equation (3), while the plots on the right are from the full moving-planet model of Equation (2). The plots at the top illustrate the maximum allowed perturbation (corresponding to the dashed lines in Figure 2). The plots at the bottom illustrate the perturbation scale at the absolute minima.

We draw a few key conclusions from Figure 3.

1. The results from the tidal and moving-planet models are essentially identical. The figures only differ near the margins.
2. There are broad regions of the sky (located on the opposite side of the ecliptic from the orbit of Pluto) for which we can rule out perturbation magnitudes $\gtrsim 3 \times 10^{-13} M_\odot \text{au}^{-3}$.
3. For regions straddling the orbit of Pluto, we can only limit the perturbations to being $\leq 3 \times 10^{-11} M_\odot \text{au}^{-3}$. Much of this same region also permits a significant decrease in $\chi^2$ of the kind illustrated at the middle and
bottom of Figure 2, with a minimum centered around $\sim 10^{-11} M_\odot$ au$^{-3}$.

4. We add to the physical and dynamical alignments already noted in the literature (Trujillo & Sheppard 2014; Batygin & Brown 2016; Malhotra et al. 2016), and highlight an interesting (but possibly coincidental) dynamical alignment between the ascending node of Pluto and the proposed ascending node for Planet Nine. While this may be a coincidence, it may instead indicate that Pluto is now (e.g., via secular effects) or was in the past (e.g., by scattering) directly influenced by Planet Nine.

We elaborate on the moving-planet model results from Figure 3 in Figure 8 of Appendix C, where we demonstrate that in the region broadly straddling the orbit of Pluto, the minima are relatively narrow for an individual healpix, but difference regions of the sky permit perturbations in the range $10^{-12} - 10^{-10} M_\odot$ au$^{-3}$.

5. DISCUSSION

Our results are sensitive to the tidal parameter, $M_X/r_X^3$, and insensitive to the individual mass or radius (see Figures 2 and 3). The upper limits to the tidal parameter seen in Figure 3 span the range $10^{-12} - 10^{-10} M_\odot$ au$^{-3}$, or

$$3 \times \left( \frac{M_X}{1 M_\odot} \right) \left( \frac{r_X}{100 \text{ au}} \right)^{-3} < 300.$$  

This is much closer and/or more massive than the nominal Planet Nine model of Batygin & Brown (2016), which would predict $3 \times 10^{-14} - 10^{-12} M_\odot$ au$^{-3}$ for a $10 M_\odot$ planet in an orbit ranging from $\sim 300$–$1000$ au.
A few caveats accompany our results. Some of these concern the data themselves and some concern our model.

5.1. Data

The most important caveat is that long-term systematic errors in the astrometry could strongly influence the fits. Buie & Folkner (2015) took great care to develop a reliable astrometric data set with which to better determine Pluto’s orbit. The successful New Horizons encounter demonstrates the overall accuracy of the resulting ephemeris. Our work is built upon that strong foundation, but systematic errors are still evident in the residuals.

In particular, it was clear in the unperturbed residuals from Figure 1 that there existed a clear trend in the decl. residuals for the period 1930–1950. In Figure 4, we illustrate the residuals obtained using three different perturbation magnitudes at the specific locations illustrated in Figure 2, plotting on the left-hand side the residuals, and then plotting on the right-hand side the difference between those perturbed residuals and the unperturbed residuals of Figure 1.

For regions with a significant χ² minimum, the inclusion of Planet Nine significantly reduces this trend in decl. residuals, but it does not generally eliminate it. We believe that the large residuals are affecting our fits, and large perturbation magnitudes are required to compensate for those residuals: a particular example of this can be seen in the blue model plotted in the middle rows of Figure 4, where it can be seen that this model gains a highly significant χ² reduction by significantly lowering the decl. residuals around 1930–1950.

The early, photographic observations are the most likely to suffer from systematic errors. The magnitude of the scatter in the astrometric measurements is understood (Buie & Folkner 2015), but the trend in decl. is not. We are not aware of a systematic error that would result in a persistent trend over two decades. It seems unlikely to be the result of zonal errors in the stellar catalogs: Buie & Folkner (2015) measured Pluto’s position against modern stellar catalogs with accurate proper motion estimates.

The relatively few points in the 1950–1990 era, when photographic plates were still used and before CCD cameras were available, have little influence on the fits. However, we...
note that the residuals in the 1960–1980 era tend to be *high* in decl., which is opposite that seen in the earlier data. We recommend that other archival photographic plates of Pluto and other TNOs, particularly from the 1960–1980 era, be remeasured with the same care with which Buie & Folkner (2015) remeasured the Lampland plates.

Timing uncertainties could be substantial for the early observations, as the accuracy of the recorded observation times rely upon the care and attention of the observers, as well as the accuracy of the time-keeping itself (see Buie & Folkner 2015 for an interesting discussion of time-keeping at Lowell Observatory). However, it is not clear how a trend might result from timing uncertainties. We explored allowing for time offsets for the early observations (results not illustrated here), effectively trading one dimension of each astrometric observation for more precise determination of the observation. Preliminarily, we find that this effectively eliminates the R.A. residual, but it leaves the decl. residuals broadly the same.

If the residual trend in decl. were smaller, then it is possible that the favored perturbations would also be smaller. In a future publication, we explore the dependence of our results on these trends.

### 5.2. Model

As discussed at length in Section 2.2, the masses in our model are not fully interacting in a self-consistent manner. However, we validate the assumptions of our model and its underlying numerical integrations in Appendix A (Figure 6) by illustrating that full *n*-body integrations are essentially indistinguishable from our standard model. Thus, the effect of ignoring some planetary interactions must be small.

The acceleration from TNOs, e.g., Haumea, can exceed that from Planet Nine for some brief periods of time in our integrations, but without significantly altering the results, as in almost all stages of our integrations the dominant perturbation is from Planet Nine. We estimated the masses of several of the large TNOs that we included as perturbers, and while the uncertainty in these masses obviously affects our results, any mass corrections will be small, and hence remain sub-dominant to the modeled effects from Planet Nine.

Some features of the KB, such as the edge near 50 au for the cold classical, are not obviously explained by Planet Nine. Some investigations have argued for closer planets, possibly more than one (Brunini & Melita 2002; Lykawka & Mukai 2008; Bromley & Kenyon 2016; Kenyon & Bromley 2016). There are undoubtedly unknown solar system masses that are not included in our model which are yet to be discovered and which may be significant perturbers. There could be more than one additional massive and distant object, or there could be additional smaller and closer planets. Investigating the influence of more than one additional planet is beyond the scope of the present investigation. Closer planets would be more easily detected through means such as astrometric microlensing (Gaudi & Bloom 2005).

In Figure 5, we compare the results of Fienga et al. (2016) with our own results for the specific nominal orbit of Planet Nine from Batygin & Brown (2016). The tidal parameters favored by our models are larger than those suggested by Batygin & Brown (2016) and supported by the *Cassini* range observations (Fienga et al. 2016; Holman & Payne 2016). One possible resolution of this apparent inconsistency is the presence of more than one additional planet. If one planet were at 60–100 au, closer to Pluto, then it would not have to be as massive as Planet Nine to significantly perturb Pluto for a period of time. The *Cassini* range observations would not necessarily be significantly affected by such a planet because those data are only sensitive to the tidal acceleration, rather than the direct acceleration. Furthermore, the *Cassini* data are from a very different range of times.

### 6. CONCLUSIONS

We have used astrometry of Pluto and other TNOs to constrain the sky location, distance, and mass of Planet Nine. We find that over broad regions of the sky, the inclusion of a massive, distant planet degrades the fits to the observations. However, in other regions, the fits are significantly improved by the addition of such a planet. Our best fits suggest a planet that is either more massive or closer than argued for by either Batygin & Brown (2016) or Fienga et al. (2016). The trend to favor larger and closer perturbing planets is driven by the residuals to the astrometry of Pluto, remeasured from photographic plates using modern stellar catalogs (Buie & Folkner 2015), which show a clear trend in decl. over the course of two decades that drives a preference for large perturbations. Although this trend may be the result of systematic errors of unknown origin in the observations, a possible resolution is that the decl. trend may be due to perturbations from a body, in addition to Planet Nine, that is closer to Pluto but less massive than Planet Nine.

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APPENDIX A
COMPARISON BETWEEN FULL N-BODY INTEGRATIONS AND OUR STANDARD MODEL

In Figure 6, we show the results of full n-body integrations from initial conditions of the Sun, planets, Planet Nine, and Pluto. We obtain the initial conditions at a reference time from our Orbit fit modeling. In the figure, we show the difference between the sky position of Pluto when Planet Nine is included and when it is not. This demonstrates that the results of a model in which the positions of the planets are allowed to vary are essentially indistinguishable from our standard model, supporting our simplified model.

APPENDIX B
OBSERVATORY-SPECIFIC PLOTS OF OBSERVATIONAL RESIDUALS

For convenience, in Figure 7, we provide observatory-specific plots to facilitate comparison between our fits and...
Figure 7. Top: zoom-in on the unperturbed model of Figure 1 for the 1930–1950 period, plotting the residuals (observations—model) for different observatories. The Y-axis scale is chosen to facilitate comparison with Buie & Folkner (2015). Bottom: zoom-in on the unperturbed model of Figure 1 for the 1970–2010 period, plotting the residuals (observations—model) for different observatories. The Y-axis scale is chosen to facilitate comparison with Folkner et al. (2014).
those obtained by Folkner et al. (2014) and Buie & Folkner (2015) for these observatories.

APPENDIX C
RANGE OF IMPROVING PERTURBATIONS
To elaborate on the moving-planet model results from Figure 3, in Figure 8, we show the minimum and maximum perturbations which are permissible while retaining \( \Delta \chi^2 > \Delta \chi^2_{\text{min}} \). These plots demonstrate that in the region broadly straddling the orbit of Pluto, the minima are relatively narrow for an individual healpix, but difference regions of the sky permit perturbations in the range \( 10^{-12} - 10^{-10} M_{\odot} \text{ au}^{-3} \).

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Figure 8. All-sky constraints on the perturbation from Planet Nine, derived from simultaneous orbital fits to \( > 500 \) TNOs (including Pluto): all plotted results are from the full moving-planet model. The plots at the top left illustrate the maximum allowed perturbation (corresponding to the dashed lines in Figure 2). The plots at the top right illustrate the perturbation scale at the absolute minima (corresponding to the dotted lines from the bottom plot in Figure 2). The plots at the bottom left illustrate the minimum allowed perturbation within \( \Delta \chi^2_{\text{min}} \) of the absolute minimum. The plots at the bottom right illustrate the maximum allowed perturbation within \( \Delta \chi^2_{\text{min}} \) of the absolute minimum. We note that the plots in the top row are reproductions of the full moving-planet model plots in Figure 3. To guide the eye we plot the ecliptic (gray dashed line), Pluto (green line) and the nominal orbit of Planet Nine (black line) from Batygin & Brown (2016). Note the orbit of Pluto also has the observational data points overplotted in larger green points. It can be seen that over large regions of the sky we can exclude perturbation scales larger than \( \sim 3 \times 10^{-13} M_{\odot} \text{ au}^{-3} \) (darker blue regions, top left plot). Close to the orbit of Pluto and the nominal path of Planet Nine, we can only rule out perturbations larger than \( \sim 3 \times 10^{-13} M_{\odot} \text{ au}^{-3} \). The plots at the top right and in the bottom row illustrate that large perturbations (\( \gtrsim 10^{-11} M_{\odot} \text{ au}^{-3} \): orange and red regions) are able to significantly improve the fits. The relative similarity between the best-fit solutions at the top right and the upper and lower perturbations allowed for good fits in the bottom row, indicate that in general the \( \chi^2 \) minima are relatively narrow, and that (for most HEALPix) only a relatively small range of perturbations can promote a significant improvement to the fit.
