A Mass Sensor Based on Two Complete Synchronized Optomechanical Oscillators

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Abstract. In this work, we study the realization of stable complete synchronization in two coupled optomechanical systems with a master-slave configuration. By taking the open-plus-close-loop method as coupling scheme, it is verified that the mechanical and optical mode in the coupled systems with parameters mismatched can be simultaneously synchronized both in linear and nonlinear regime, and even in chaotic state. Based on the achieved complete synchronization, the coupled systems are then explored in mass sensing applications. By taking one of the coupled as sensing part, we develop the coupled system setting in complete synchronization as a mass sensor. It is revealed that a tiny mass added on the sensing part will lead to desynchronization, and the quantities of added mass can be determined by calculating a designed similarity measure.

1. Introduction

Rapid developments in the field of optomechanics have allowed researchers to move on from the investigation of a single optomechanical system to the coupled even complex networks and/or hybrid of optomechanical systems [1-5]. In relevant theoretical and experimental works, a few valuable findings, such as injection locking [6], Anderson localization [7], pattern formation [3] and etc., have been extensively reported. These works not only shed light on new fundamental tests of quantum mechanics [8, 9], but also for exploring the future optomechanics based applications with multi-physical interfaces [10-12]. Synchronization, which finds practical applications in a wide variety of fields [13-15], has particularly attracted much attention in the field of optomechanics. However, because of unavoidable device nonuniformity and weak mutual couplings, to realize reproducible synchronization in practical optomechanical systems remains challengeable, much less for achieving complete synchronization (CS) in which both involved optical and mechanical states are synchronized.

In a typical optomechanical system, the dynamics of mechanical and optical states are of discrepancy that the optical fields evolves with a much faster rhythm, exhibiting a totally different dynamics with mechanical part[16, 17], and this phenomenon has been already reported by experimental works [18, 19]. Moreover, optical field in optomechanical systems can bifurcate into the nonlinear dynamical regime, i.e. chaos, under certain parameter settings [20]. In such a context, to realize synchronization, in particular the CS, new coupling methods that can establish mutual coupling effect both for mechanical and optical parts are highly of necessity.

Here in this work, we report the first CS in two on-chip optomechanical oscillators (OMOs) with master-slave configuration. By taking an open-plus-closed-loop (OPCL) method as coupling scheme [21], the CS in two on-chip OMOs with practical device nonuniformities is for the first time realized. We have considered both the linear and nonlinear working regime of the OMOs, corresponding to
small and large laser driving, respectively, from which it is revealed that under OPCL coupling broad scope of synchronization, even the chaotic synchronization, can be achieved. The work, to some extent, paves the way for employing chaotic OMOs in future secure communication applications. In particular, given the slave OMOs is seen as a sensing part, we study on how the mass change on slave OMOs exert influence on existing CS, which shows the proposed coupled OMOs has the potential in developing mass sensors. In addition, due to the coupling scheme in our work essentially are controlled by electronic signal, it is more compatible with current on-chip integration technology, compared to the pure optical coupling methods.

Theoretical analysis is given in section 2. Numerical simulations including the realization of CS in linear and nonlinear regime of the OMOs, are put in section 3. Section 4 presents the application of mass sensor based on the CS in the proposed coupled OMOs, followed by a brief conclusion in section 5.

2. Theoretical analysis

Our design mainly consists of two OMOs with master-slave configuration, namely OMO-m and OMO-s, as shown in Fig.1. Each OMO is a typical cavity optomechanical system that includes a high-Q on-chip cavity with laser driving underneath.

Figure 1. Schematic diagram of two coupled OMOs in a master-slave configuration. One is called OMO-m and the other one is called OMO-s. There is a circular capacitor (yellow) etched inside of the OMO-m and OMO-s aiming to extract the motion signal of the OMOs. The extracted signal from OMO-m can be processed via a series units including C-V convertor and amplifier unit, signal process unit (SPU) and driving unit before feeding into OMO-s. Laser light is coupled into one end of an optical fiber, which feeds into a fiber polarization controller (FPC). High-speed photodetectors (PD) are employed to collect the optical signal.

When the optical driving increases to a certain level, the radiation pressure arisen in cavity can lead the OMO into a well-known self-oscillation regime, in which the mechanical and optical states are interacted, and depending on the intensity of the laser driving applied the optical fields in the cavity can bifurcate from periodic state to chaotic state. The phenomenon has been confirmed by previous experiments [19, 20]. To synchronize such two OMOs both for mechanical and optical parts, i.e., CS, we have employed the coupling scheme based on OPCL method [21]. In order to apply OPCL method, it is of priority to extract the instant system dynamics of both the optical and mechanical states of the master OMO, so that we can transfer the dynamics into electrical signal, and then use the signal to control the slave OMO via a series of processing units. Specifically, a circular slot is etched through the OMO-m and OMO-s aiming to extract the displacements of the cavity. Two circular capacitor electrodes are patterned on either side of the slot. The design was first reported by work [6]. A C-V convertor and amplifier unit is then connected to the two electrodes in OMO-m. Thus, the instant dynamics of mechanical state of the OMO-M, i.e. the displacement of OMO-m, can be transferred into electrical signal by detecting the instant capacitance from circular capacitor. Meanwhile, as to the optical state in OMO-m, we have used the phonic detector (PD in Fig. 1) to realize the photoelectric
transformation. Next, the two signals representing the mechanical and optical states are combined and flowed into the signal process unit (SPU) in which the OPCL method works. Likewise, the OMO-s is with the same procedure to extract mechanical and optical states. The signals after processed by SPU will further flow into the driving unit, by which the OMO-s receive the driving signal and modulate its own driving source to synchronize with OMO-m. First, the dynamical model of each OMO can be described by:

\[
\begin{align*}
\ddot{x}_1 &= x_2 \\
\ddot{x}_2 &= -ax_2 - bx_1 + c(x_3^2 + x_4^2) \\
\dot{x}_3 &= (e - fx_1)x_4 - dx_3 \\
\dot{x}_4 &= gE_p - (e - fx_1)x_3 - dx_4
\end{align*}
\]  (1)

Here, the first two equations describe the mechanical mode of the OMO, with \(x_1\) and \(x_2\) representing the displacement and velocity of the mechanical cavity, respectively. The third and fourth equations describe the dynamics of the optical mode in the OMO, with \(x_3\) and \(x_4\) denotes the real and imaginary part of the optical field, respectively. It should be noted that the relation of optical field, \(E = E_e(t) + iE_i(t)\), has been used in deriving the Eq. 1, which aims to avoid appearing imaginary numbers and derive a final equation with autonomous form. In addition, parameters including \(a, b, c, d, e, f, g\) appeared in Eq. 1 are derived based on the following relations:

\[
\begin{align*}
a &= \frac{\alpha}{\Omega}, \\
b &= \frac{\Omega}{\omega_0}, \\
c &= n / (c_0\rho\pi^2R), \\
d &= \omega_0^2n / (c_0N), \\
e &= \omega_0 - \omega_0, \\
g &= \sqrt{\rho ac_0 / \pi \alpha},
\end{align*}
\]

where \(\Omega\) represents the frequency for mechanical, \(\omega_0\) represents the frequency for optical, \(c_0\) is vacuum velocity of light, \(n\) is the refractive index, \(\alpha\) is the absorption coefficient in cavity, \(\rho\) is the material density, and \(Q\) and \(R(r)\) are the quality factor and outer (inner)diameter of the mechanical cavity, respectively. \(\omega_0\) represents the optical resonance at mechanical equilibrium. The Eq. 1 has been employed by Tal Carmon et al. in their work \[19, 20\], based on which the reliability of the model is verified by comparing with experimental results. Hereafter, we treat Eq.1 as the dynamics model of OMO-s. For the system of OMO-m, the model for describing its dynamics has the similar expression with OMO-s (Eq.1). However, concerning the practical environment, there is no two exact same OMOs existing, we therefore add mismatched parameters in the model to present the practical nonuniformities. Its model is given by:

\[
\begin{align*}
\dot{y}_1 &= y_2 \\
\dot{y}_2 &= -ay_2 - by_1 + c(y_3^2 + y_4^2) - Ay_2 - By_1 + c(y_3^2 + y_4^2) \\
\dot{y}_3 &= (e - fy_1)y_4 - dy_3 + (E - Fy_1)y_4 - Dy_3 \\
\dot{y}_4 &= gE_p - (e - fy_1)y_3 - dy_4 + GE_p - (E - Fy_1)y_3 - Dy_4
\end{align*}
\]  (2)

, where \(A, B, C, D, E, F,\) and \(G\) are the mismatched parameters and other parameters are taken as the same as Eq.1. Here, we consider the OPCL coupling method. According to the previous articles \[22\], the OPCL acts like a bridge that can couple two dynamical systems and make them synchronized via a series of mathematical procedures. Specifically, its mathematical process can be expressed as:

\[
\dot{X} = F(x) + D(X, aY)
\]  (3)

, Where \(X, Y\) and \(F\) are Vector forms of the dynamical model of OMO-m(s). The coupling \(D(X, aY)\) is defined by: \(D(X, aY) = aY - F(X)(aY) + [H - \partial F(aY)](X - aY)\) where \(J = \partial / \partial aY\) is the Jacobian. \(H\) is an arbitrary constant Hurwitz matrix whose eigenvalues all have negative real parts, i.e., Routh Hurwitz (RH) condition\[23\]. Here in this work, we specifically take the \(H\) as:

\[
H = [0, 1, 0, 0; -b, -a, p_1, p_2; p_3, 0, -d, (e - p_4); p_5, 0, -e, -p_4, -d]^T,
\]

where \(p_i = -1;\)
\[ p_2 = 1; \quad p_3 = 5; \quad p_4 = 6; \quad p_5 = -2. \]

The specific form of the function \( F(\alpha y) \) can be expressed as:

\[
F(\alpha y) = [\alpha y_2, -\alpha y_2 - aby_1 + a^2(y_3^2 + y_4^2); (e - afy_1)ay_4 - ady_3, gEp - (e - afy_1)y_3 - ady_4]^T,
\]

where \( \alpha \) is taken to be 1 with the aim to realize CS. After implementing the OPCL method (Eq.3), the OMO-s is driven into the following equation:

\[
\dot{x}_1 = x_2
\]

\[
\dot{x}_2 = -ax_2 - bx_1 + c(x_3^2 + x_4^2) + (1 - a)ac(y_3^2 + y_4^2) - aAy_2

- aby_1 + ac(y_3^2 + y_4^2) + (p_1 - 2acy_3)(x_3 - ay_3) + (p_2 - 2acy_4)(x_4 - ay_4)
\]

\[
\dot{x}_3 = (e - E\alpha y) x_4 - dx_3 + (\alpha - 1)afy_4 + \alpha(E - Fy_1)y_4 - aDy_3 +

(p_3 + afy_4)(x_1 - y_1)a + (-p_4 + afy_1)(x_4 - ay_4)
\]

\[
\dot{x}_4 = aE\rho_p - (e - E\alpha y)x_3 - dx_4 + (1 - a)afy_3 + \alpha(E - Fy_1)y_3 -

-ady_4 + (p_5 - afy_3)(x_1 - y_1) + (p_4 - afy_1)(x_4 - ay_4)
\]

Based on the Eqs 2, 3 and 4 above, the realization of CS will be numerically verified in next section.

3. Numerical result analysis

Combining Eqs.2 and 4, we first calculate the case when the laser driving for OMO-m(s) are relatively small. The parameters are taken as: \( a = 1.4 \times 10^{-6}, b = 1.2 \times 10^{17}, c = 9779, d = 1.2 \times 10^8, e = 1.3 \times 10^8, f = 1.1 \times 10^{20}, g = 2.2 \times 10^{10}. \) The outer and inner toroid radii are taken as 14.5 \( \mu m \) and 3 \( \mu m \), respectively. The mismatched parameters in OMO-m are taken as: \( A = 1, B = 4, C = -2, D = 0, E = 10, F = 1, G = 1. \) The mechanical \( Q \) factor and optical \( Q \) of the OMO-m(s) are taken to be 250 and 107, respectively. These parameters are all experimentally accessed and have been used in experimental work [19, 20]. In the following part, these parameters will keep unchanged during the numerical simulation unless there are specified indications.

3.1 Small drive

We set laser driving \( E_p = 0.2 \) at a relative small value for both the OMO-m and OMO-s when mechanical mode and optical field are all oscillating in periodic state. To verify whether the OMO-m and OMO-s
are synchronized, we calculate the time series of mechanical and optical modes in OMO-m(s) based on the parameters setting above. The results are presented in Fig. 2, in which $x_1$ and $y_1$ vs time $t$, representing mechanical mode, $x_2$ and $y_2$ vs time $t$, representing optical mode, are plotted. It is seen that both the mechanical mode and optical mode, from OMO-m and OMO-s, respectively, are synchronized, i.e., the realization of CS. Therefore, the OPCL coupling scheme works effectively for mismatched optomechanical systems in linear regime, despite the dynamics of optical field and mechanical mode in such kind of system are quite different in terms of oscillating frequency. Furthermore, we have calculated the trajectory differences of the mechanical and optical mode in OMO-m(s) over time, and the results are shown in Fig. 3, in which the reaching time for synchronization of mechanical and optical state appear to be different that the optical mode of OMO-m(s) reach synchronization faster. This is due to that the optical field in OMOs is oscillating faster with an ultra-high Q factor than mechanical mode so that the optical fields in these two OPCL coupled OMOs are interacted much more faster.

\subsection{Large drive}

When the laser driving ($E_p$) for both OMO-m and OMO-s are increased to a certain level, the optical mode can evolve into chaotic state while the mechanical mode keeps in periodic state, and there are experimental work that have already confirmed this phenomenon [20]. To confirm our method can also realize CS in nonlinear regime, we set $E_p = 2$ with keeping other parameters unchanged, and then the time series of mechanical and optical mode from OMO-m(s) are calculated and plotted correspondingly, see Fig. 4, where it is seen the CS has been achieved with optical mode oscillating in chaotic state. Likewise, we also calculate the $x_1-y_1$, $x_2-y_2$, and $x_3-y_3$ and $x_4-y_4$ over time in Fig. 5, from which the same phenomenon is observed that the synchronization of mechanical state are reached with a time delay.
Based on the above numerical simulation in both linear and nonlinear regime, we have proved OPCL coupling regime works effectively in OMOs with master-slave configuration. Compared to the traditional weak couplings, our design is more robust in achieve synchronization in OMOs, with a wider working range.

4. Mass sensor application

When the two coupled opt-mechanical oscillators are setting in CS, it is believed that the CS can be broken, i.e. desynchronization, if there are any perturbations applied. We restrict this kind of perturbations as external added mass in this work. In order to quantify the desynchronization brought by external perturbation, we first introduce the so called similarity measure here. The similarity measure is an equation that can be used for calculating the differences between two trajectories of dynamical states, which is given by:

\[
Q = \frac{< [Y_j(t) - X_j(t)]^2 >}{[< Y_j(t)^2 > < X_j(t)^2 >]^{1/2}} \tag{5}
\]

where \(X_j(t), Y_j(t)\) represent two corresponding sub-dimensions from dynamical systems of OMO-m and OMO-s, for example, \(y_1(t)\) and \(x_1(t)\), and \(< \cdot >\) means the process of taking average. The value of \(Q\) indicate whether the synchronization between two corresponding sub-dimensions are achieved, i.e., \(Q = 0\), synchronized; \(Q = C_0\), a nonzero value meaning unsynchronized, and the bigger of \(C_0\), the more degree of desynchronization.
Figure 6. Mass sensor application. (a) $Q$ vs $\Delta m'$ with $\Delta m' \in [0 0.008]$. (b) $Q$ vs $\Delta m'$ with $\Delta m' \in [0 0.03]$.

Now, we begin to numerically investigate how the proposed oscillators setting in CS can be potentially developed as a mass sensor. Specifically, if the OMO-s is served to be a mass sensing part at which small mass might be loaded, the added mass would lead to desynchronization occurring between OMO-m and OMO-s. Therefore, it is possible by detecting the signal of desynchronization from the time series of mechanical or optical mode to sense the added mass. To approve the idea, we, in this section, numerically study the case when OPCL coupled OMOs working in linear CS. We first insert a parameter $k_1$ into mass related parameters including $a (a = \Omega / Q_m)$ and $b (b = \Omega^2)$, where $\Omega = \sqrt{2k / m_0}$ with $k$ is spring constant and $m_0$ is effective mass of OMO-s. If there is added mass $m'$ added to the sensing part, the parameters $a$ and $b$, will be varied accordingly. We take $\Delta m'$ as the ratio between added mass $m$ and $m_0$. Then, two working ranges of the proposed mass sensor are investigated. First, when $\Delta m'$ is varying in the range of $[0 0.008]$, the similarity measure (Eq. 5) is calculated, and the result is shown in Fig. 6 (a), from which it is seen that the value of $Q$ is slowly increased at the beginning part followed by an approximately linear changing trend, both for mechanical and optical mode. It is clearly illustrated that tiny mass loaded on the sensing part can be sensed. Furthermore, we set the $\Delta m'$ varying in a wider varying range $[0 0.03]$, and based on the similar calculation it is revealed that our mass sensor has a wide working range (see Fig. 6. (b)), where the value of $Q$ on mechanical mode are keeping with a linear changing trend. However, the $Q$ value of optical mode goes through a plateau after the linear varying region, which is due to the detuning $(\omega_c - \omega_h)$ brought by added mass.

5. Conclusion
To conclude, the stable complete synchronization is achieved in two master-slave configured optomechanical systems with parameters mismatched by using OPCL coupling method. The OMOs based synchronization can even be realized in chaotic region. Particularly, if taking one of the coupled optomechanical systems as mass sensing part, the synchronized OMOs can be served as a mass sensor which can work both in linear and nonlinear regime. In the coupled optomechanical system, the optical information processing between the OMO-m and OMO-s can be achieved both in mechanical and optical way. This is different with traditional designs that only take optical mode as coupling mode, and this merit can benefit the proposal to be integrated with the current photonic systems.
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