Chaotic Motion of Double-walled Carbon Nanotubes with Fluid

Tai-Ping Chang*

Department of Construction Engineering, National Kaohsiung University of Science and Technology,
1 University Road, Yanchao, Kaohsiung, 824, Taiwan

(Received March 20, 2017; accepted June 27, 2018)

Keywords: double-walled carbon nanotubes (DWCNTs), chaotic motion, fluid-loaded, nonlinear vibration

In this study, we investigate the chaotic motion of double-walled carbon nanotubes (DWCNTs) with fluid by dealing with the effects of nonlinearities. The governing equations of the system are derived and solved by the variation method with Galerkin’s technique. A chaotic phenomenon occurs when the top Lyapunov exponent of the system becomes positive. Referring to the results of the computations of the largest Lyapunov exponent, we examine and discuss the effects of some parameters, such as flow velocity, driving frequency, and load amplitude, on the chaotic motion of the nonlinear system. These parameters play an important role in determining whether or not chaotic motion will occur on the system. Moreover, these parameters can be used to control the chaos of DWCNTs conveying fluid.

1. Introduction

Carbon nanotubes (CNTs) have been utilized in numerous applications in many fields after the publication of a pioneer paper by Iijima. One of their applications is as nanopipes conveying fluids. Recently, many researchers have been studying the characteristics of nanotube vibration and have investigated, among others, nonlinear vibration problems with nonlocal continuum theories. In the past, much research has been reported on the chaotic behaviors of nonlinear systems, however, investigations about the chaotic behavior of double-walled carbon nanotubes (DWCNTs) with fluid are rare. The chaotic motion is primarily generated thanks to the nonlinear characteristics in the physical system. Here, we examine the chaotic motion of DWCNTs with fluid by studying the nonlinear characteristics. With the use of the variation method and Galerkin’s technique, the governing equations of the system are derived and solved. The chaotic phenomenon occurs when the top Lyapunov exponent of the system becomes positive, as previously described. The goal of the study is to determine whether or not the system of DWCNTs undergoes chaotic motion by computing the largest Lyapunov exponent of the system.
2. Simple Model for DWCNTs with Fluid

In Fig. 1, the DWCNT is modeled as a double-tube pipe composed of an inner tube of radius \( R_1 \) and an outer tube of radius \( R_2 \). The thickness of each tube is \( h \), the length is \( L \), and Young’s modulus of elasticity is \( E \). The internal fluid is assumed to flow steadily through the inner tube at a constant velocity \( U \). In addition, the boundary condition of the DWCNT is assumed to be simply supported at both ends. Following the assumptions and derivations of Chang\(^{(12)}\) after some lengthy formulations, we obtain the governing equations for the system as follows:

\[
\begin{align*}
E_1 \frac{\partial^4 w_1}{\partial x^4} + MU_1^2 \frac{\partial^2 w_1}{\partial x^2} - & \int_0^L \left( \frac{\partial w_1}{\partial x} \right)^2 \left( \frac{E A_1}{2L} + \frac{M U_1^2}{2L} \right) \, dx \frac{\partial^2 w_1}{\partial x^2} \\
+ & \frac{3MU_1^2}{2} \left( \frac{\partial w_1}{\partial x} \right)^2 \frac{\partial^2 w_1}{\partial x^2} + (M + m_1) \frac{\partial^2 w_1}{\partial t^2} + 2MU_1 \frac{\partial^2 w_1}{\partial x \partial t} \\
- & MU_1 \frac{\partial w_1}{\partial t} \frac{\partial w_1}{\partial x} \frac{\partial^2 w_1}{\partial x^2} - c_1(w_2 - w_1) - c_3(w_2 - w_1)^3 = 0,
\end{align*}
\]

where \( x \) is the axial coordinate, \( t \) is the time, \( w_i \) is the transverse displacement of the \( i \)th tube on the neutral axis, and the subscript \( i = 1 \) or 2. Note that tube 1 is the inner tube while tube 2 is the outer tube. \( A_1 \) and \( A_2 \) are the cross-sectional areas of tubes 1 and 2, respectively. \( I_i \) and \( m_i \) are the moment of inertia and the mass of the \( i \)th tube per unit length, respectively; \( M \) denotes the fluid mass. \( c_1 \) and \( c_3 \) are the van der Waals (vdW) force coefficients and are assumed to be \( c_1 = 71.11 \) GPa and \( c_3 = 2.57 \times 10^4 \) GPa nm\(^2\).\(^{(22)}\) \( \xi \) is the damping coefficient, \( F_0 \) is the magnitude of the external load, and \( \Omega \) is the driving frequency of the excitation. On the basis of Galerkin’s approach, for simplicity, one term approximation is used to calculate the displacement for the nonlinear coupled system. Then, the displacement function can be expressed as:

\[
\begin{align*}
E_2 \frac{\partial^4 w_2}{\partial x^4} + & 2 \xi \frac{\partial^2 w_2}{\partial t^2} + m_2 \frac{\partial^2 w_2}{\partial t^2} - \int_0^L \left( \frac{\partial w_2}{\partial x} \right)^2 \left( \frac{E A_2}{2L} \right) \, dx \frac{\partial^2 w_2}{\partial x^2} \\
+ & c_1(w_2 - w_1) + c_3(w_2 - w_1)^3 = F_0 \cos(\Omega t),
\end{align*}
\]

Fig. 1. Simple model for DWCNTs with fluid.
\[ w_i(x, \vec{T}) = \phi(x) \vec{T}_i(\vec{T}). \]  

(3)

where \( i = 1 \) or \( 2 \). \( \phi(x) \) defines the mode shape satisfying the boundary conditions.

Equation (3) is substituted into Eqs. (1) and (2), followed by multiplication with \( \phi(x) \) and integration from 0 to \( L \). For convenience, the following dimensionless parameters are adopted to deal with the tiny values in nanosystems:

\[ t = \frac{\vec{T}}{1/\omega_0} = \vec{t}_0, \quad T_i = \frac{\vec{T}_i}{R_i}, \quad i = 1, 2. \]  

(4)

Here, \( 1/\omega_0 \) is chosen as the characteristic time, \( \omega_0 \) is the fundamental natural frequency, and \( R_i \) is the inner tube radius, which is selected as the characteristic length. Finally, the following dimensionless differential equations can be obtained:

\[ \ddot{\vec{T}}_1 + a_1 \dot{\vec{T}}_1 + b_1 T_1 + \vec{c}_1 T_2 + d_1 T_1^3 + e_1 T_2^2 T_1 + f_1 T_2 T_1^2 + g_1 T_2^3 = 0, \]  

(5)

\[ \ddot{\vec{T}}_2 + a_2 \dot{\vec{T}}_2 + b_2 T_1 + c_2 T_2 + d_2 T_2^3 + e_2 T_2^2 T_1 + f_2 T_2 T_1^2 + g_2 T_1^3 = F \cos(\Omega t), \]  

(6)

where “\( \cdot \)” denotes \( \frac{d}{dt} \). The coefficients are explicitly written in Appendix A.

3. Chaotic Motion of DWCNTs with Fluid

To study the chaotic behavior of the coupled equations, Eqs. (5) and (6), it is convenient to use the following variables: \( y_1 = T_1, y_2 = \vec{T}_1, y_3 = T_2, y_4 = \vec{T}_2 \). Then Eqs. (5) and (6) become

\[ \begin{align*}
\dot{y}_1 &= y_2, \\
\dot{y}_2 &= -a_1 y_2 - b_1 y_1 - \vec{c}_1 y_3 - d_1 y_1^3 - e_1 y_3^2 y_1 - f_1 y_3 y_1^2 - g_1 y_3^3, \\
\dot{y}_3 &= y_4, \\
\dot{y}_4 &= -a_2 y_4 - b_2 y_1 - c_2 y_3 - d_2 y_3^3 - e_2 y_3^2 y_1 - f_2 y_3 y_1^2 - g_2 y_3^3 + F \cos(\Omega t).
\end{align*} \]  

(7)

We must determine whether the above system exhibits chaotic behavior by calculating the top Lyapunov exponent. Lyapunov exponents provide an estimate of the rate of divergence of two initially nearby orbits (or trajectories), as stated in Ref. 20.

Equation (7) can be transformed into a vector form,

\[ \dot{\vec{Y}} = \vec{h}(\vec{Y}; s), \]  

(8)
where $Y = [y_1, y_2, y_3, y_4]^T$ is the state-space vector, and $h = [h_1, h_2, h_3, h_4]^T$ and $s = (a_1, ..., g_2, F, \Omega)$ are parameters. The equations for small deviations $\delta Y$ from the trajectory $Y(t)$ are

$$
\delta \dot{Y} = J_y(Y(t))\delta Y, \quad i, j = 1, 2, 3, 4,
$$

where $J_y = \partial h_i / \partial y_j$ denotes the Jacobian matrix of derivatives.

Equation (9) for the system Eq. (7) can be rewritten as

$$
\begin{bmatrix}
\delta \dot{y}_1 \\
\delta \dot{y}_2 \\
\delta \dot{y}_3 \\
\delta \dot{y}_4
\end{bmatrix} =
\begin{bmatrix}
0 & 1 & 0 & 0 \\
J_{21} - a_1 & J_{23} & 0 & \delta y_2 \\
0 & 0 & 0 & 1 \\
J_{41} & 0 & J_{43} - d_2 & \delta y_4
\end{bmatrix}
= J_{ij}(\delta Y),
$$

(10)

where

$$
J_{21} = -b_1 - 3d_1 y_1^2 - e_1 y_3^2 - 2f_1 y_3 y_1,
J_{23} = -\tilde{C}_1 - 2e_1 y_3 y_1 - f_1 y_1^2 - 3g_1 y_3^2,
J_{41} = -b_2 - 3g_2 y_1^2 - e_2 y_3^2 - 2f_2 y_3 y_1,
J_{43} = -c_2 - 2e_2 y_3 y_1 - f_2 y_1^2 - 3d_1 y_3^2.
$$

From the definition in Ref. 21, the top Lyapunov exponent of the system is denoted as

$$
\hat{\lambda} = \lim_{t \to \infty} \frac{1}{t} \log \left\| \frac{\delta Y(t)}{\|Y(0)\|} \right\|.
$$

(11)

It is noted that Eqs. (7) and (10) must be numerically solved simultaneously to acquire the largest Lyapunov exponent. The Runge–Kutta method of order 4 is adopted to solve Eqs. (7) and (10) simultaneously. In Eq. (7), there are several parameters, such as $a_1, ..., g_2, F, \Omega$, and the flow velocity $U$, which can influence the chaotic motion of the system, except the initial conditions. The goal of the study is to detect whether or not the system of DWCNTs presents chaotic motion due to these parameters by determining the largest Lyapunov exponent of the system.

4. Numerical Results and Discussion

We consider the parameters $E = 0.926$ TPa, $L = 60$ nm, $h = 0.34$ nm, $\rho = 1130$ kg/m$^3$, $\rho_f = 1000$ kg/m$^3$, and $R_1 = 2.5$ nm. The damping coefficient $\zeta$ is considered as 0.1, and the initial conditions for $T_1$ and $T_2$ in Eqs. (5) and (6) are $T_1(0) = T_2(0) = 1$, and $\dot{T}_1(0) = \dot{T}_2(0) = 0$.

In Eqs. (5) and (6), there are several parameters that can influence the chaotic motion of the system, such as $a_1, ..., g_1, a_2, ..., g_2, F, \Omega$, the velocity of the fluid, and the initial conditions.
The causes of the chaotic oscillations are mainly the geometric nonlinearity of the DWCNT, as determined by inspecting Eqs. (5) and (6), particularly those of \( d_1, d_2, g_1, \) and \( g_2 \). The goal of the study is to detect whether or not these parameters cause the system of DWCNTs to present chaotic motion, by computing the top Lyapunov exponent of the system. This can be accomplished by changing one parameter while all the other parameters are fixed. First of all, we deal with the system having the parameters \( F_0 = 1.0 \times 10^{-5} \) N/m and \( \Omega = 1 \). Here, \( \Omega \) is the dimensionless driving frequency in Eq. (6). Then, we compute the top Lyapunov exponent of the DWCNT system by increasing the flow velocity from \( U = 0.0 \) to \( U = 1.0 \times 10^{-3} \) m/s. The top Lyapunov exponent is presented versus the flow velocity \( U \) in Fig. 2. The top Lyapunov exponent is positive when the flow velocity is zero, namely, chaotic motion of the system occurs. However, when the flow velocity \( U \) increases to \( 5.2 \times 10^{-4} \) m/s, the largest Lyapunov exponent changes from positive to negative, which implies that the motion of the DWCNTs has changed from chaotic to periodic. In Fig. 3, the Poincare map of DWCNTs is depicted for \( U = 1.0 \times 10^{-4} \) m/s. As can be seen from the figure, the chaotic motion of the system is quite obvious. Now, for the same system except with the flow velocity \( U \) increased to \( U = 1.0 \times 10^{-3} \) m/s, the largest Lyapunov exponent becomes negative as seen in Fig. 2. The Poincare map of DWCNTs is presented in Fig. 4. It is obvious that the motion of the system is periodic, as deduced from the figure. Therefore, the flow velocity \( U \) is very important in identifying whether or not the motion of the system is chaotic. To examine the effect of the driving frequency \( \Omega \) on the chaotic behavior of the DWCNTs, we set the parameters \( F_0 = 1.0 \times 10^{-5} \) N/m and \( U = 1.0 \times 10^{-3} \) m/s and increase the driving frequency \( \Omega \) from 1 to 30. In Fig. 5, the largest Lyapunov exponent becomes positive when the driving frequency \( \Omega \) is larger than 5, namely, the motion of the system becomes chaotic. In Fig. 6, the Poincare map of DWCNTs is depicted. Obvious chaotic motion of the system is detected. Therefore, it is concluded that the driving frequency has a considerable influence on the chaotic feature of the DWCNTs. The final step is to examine the influence of load intensity on the chaotic behavior of the DWCNTs. We set the parameters \( \Omega = 1 \) and \( U = 5.0 \times 10^{-4} \) m/s and increase the load intensity \( F_0 \) from 0 to \( 1.0 \times 10^{-3} \) N/m. In Fig. 7, the largest Lyapunov exponent is negative when \( F_0 = 0 \), but it becomes positive with a slight

![Fig. 2. (Color online) Largest Lyapunov exponent versus flow velocity \( U \) \((F_0 = 1.0 \times 10^{-5} \) N/m, \( \Omega = 1 \)).](image1)

![Fig. 3. (Color online) Poincare map of DWCNTs \((F_0 = 1.0 \times 10^{-5} \) N/m, \( \Omega = 1 \), \( U = 1.0 \times 10^{-4} \) m/s).](image2)
increase in $F_0$ and remains positive until $F_0 = 1.0 \times 10^{-3}$ N/m. In Fig. 8, the Poincare map of DWCNTs is given for $F_0 = 1.0 \times 10^{-6}$ N/m. Note that the periodic motion of the DWCNTs is observed because the top Lyapunov exponent is negative. For the load intensity $F_0$ increased to $1.0 \times 10^{-3}$ N/m, the Poincare map of DWCNTs is shown in Fig. 9; chaotic motion of the DWCNTs is detected. On the basis of the aforementioned results, it is obvious that the flow velocity, driving frequency, and load intensity play important roles in the chaotic behavior of the system. In particular, it can be concluded that increasing the fluid velocity to a certain range will change the motion of the DWCNTs from chaotic to periodic; on the other hand, increasing the frequency or magnitude of external force to certain ranges will switch the motion of the DWCNTs from periodic to chaotic.
5. Conclusions

The chaotic behavior of DWCNTs with fluid was examined by studying the nonlinear characteristics. The governing equations of the system were derived and solved by the variation technique and Galerkin’s method. Chaotic phenomena occurred when the top Lyapunov exponent of the coupled system changed from negative to positive. The calculations of the top Lyapunov exponent indicated that the chaotic motion of the nonlinear system occurred when the flow velocity $U$ was between 0.0 and $5.2 \times 10^{-4}$ m/s. When $F_0 = 1.0 \times 10^{-5}$ N/m and $\Omega = 1$, the chaotic motion of the nonlinear DWCNT system switched to periodic motion on increasing the flow velocity to above $5.2 \times 10^{-4}$ m/s. Moreover, the periodic motion of the nonlinear system switched back to chaotic motion on increasing the magnitude of the driving frequency $\Omega$ to above 25. In addition, when the amplitude of the applied load increased to $F_0 = 1.0 \times 10^{-3}$ N/m, the periodic motion of the DWCNTs switched back to chaotic motion. It can be concluded that these parameters play an important role in determining whether or not chaotic motion will occur in the system, and these parameters can be utilized to control the chaotic motion of the DWCNTs with fluid.

Acknowledgments

This study was sponsored by MOST in Taiwan under Grant No. MOST-105-2221-E-327-007-MY2. The author is thankful for the financial help.

References

1. S. Iijima: Nature 354 (1991) 56.
2. G. E. Gadd, M. Blackford, S. Moricca, N. Webb, P. J. Evans, A. M. Smith, S. Jacobsen, S. Leung, A. Day, and Q. Hua: Science 77 (1997) 933.
Appendix

\[ a_1 = \frac{1}{(M + m_1)\omega_0} \left[ MU\left(\frac{x}{L}\right)^2 \right], \quad b_1 = \frac{1}{(M + m_1)\omega_0^2} \left[ EI_1\left(\frac{x}{L}\right)^4 - MU^2\left(\frac{x}{L}\right)^2 + c_1 \right], \]

\[ \bar{C}_1 = \frac{-c_1}{(M + m_1)\omega_0^2}, \quad d_i = \frac{R_i^2}{(M + m_i)\omega_0^2} \left[ \frac{E_i}{2L} + \frac{MU_i^2}{2L} \right] \left[ \frac{x}{L} \right]^4 - \frac{3}{8} MU_i^2 \left[ \frac{x}{L} \right]^2 + \frac{3}{4} c_1, \]

\[ e_1 = \frac{9}{4} \frac{c_2 R_i^2}{(M + m_i)\omega_0^2}, \quad f_i = -\frac{9}{4} \frac{c_2 R_i^2}{(M + m_i)\omega_0^2}, \quad g_i = -\frac{3}{4} \frac{c_2 R_i^2}{(M + m_i)\omega_0^2}, \]

\[ a_2 = \frac{2 \bar{\xi}}{m_2 \omega_0}, \quad b_2 = -\frac{c_1}{m_2 \omega_0^2}, \]

\[ c_2 = \frac{1}{m_2 \omega_0^2} \left[ E_2 I_2\left(\frac{\pi}{L}\right)^4 + c_1 \right], \quad d_2 = \frac{R_2^2}{m_2 \omega_0^2} \left[ \frac{E_2 A_2}{2L} \right] \left[ \frac{\pi}{L} \right]^4 + \frac{3}{4} c_3, \]

\[ e_2 = \frac{9}{4} \frac{c_2 R_i^2}{m_2 \omega_0^2}, \quad f_2 = -\frac{9}{4} \frac{c_2 R_i^2}{m_2 \omega_0^2}, \quad g_2 = -\frac{3}{4} \frac{c_2 R_i^2}{m_2 \omega_0^2}, \quad F = \frac{2LF_0}{\pi m_2 R_0 \omega_0^2}, \quad \Omega = \frac{\Omega}{\omega_0}. \]