Collisional triggering of fast flavor conversions of supernova neutrinos

Francesco Capozzi,1,a Basudeb Dasgupta,2,b Alessandro Mirizzi,3,4,c Manibrata Sen,2,d and Günter Sigl5,e

1Max Planck Institut für Physik (Werner-Heisenberg-Institut), Föhringer Ring 6, 80805 München, Germany.
2Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India.
3Dipartimento Interateneo di Fisica “Michelangelo Merli”, Via Amendola 173, 70126 Bari, Italy.
4Istituto Nazionale di Fisica Nucleare - Sezione di Bari, Via Amendola 173, 70126 Bari, Italy.
5II. Institute for Theoretical Physics, Hamburg University Luruper Chaussee 149, D-22761 Hamburg, Germany
(Dated: August 22, 2018)

Fast flavor conversions of supernova neutrinos, possible near the neutrinosphere, depends on an interesting interplay of collisions and neutrino oscillations. Contrary to naive expectations, the rate of self-induced neutrino oscillations, due to neutrino-neutrino forward scattering, comfortably exceeds the rate of collisions even deep inside the supernova core. Consistently accounting for collisions and oscillations, we present the first calculations to show that collisions can create the conditions for fast flavor conversions of neutrinos, but become subdominant thereafter, and oscillations can continue without significant damping. This may have interesting consequences for supernova explosions and the nature of its associated neutrino emission.

Introduction. – Neutrinos and antineutrinos emitted from supernovae can undergo significant changes in their flavor composition, especially due to the refractive effect of their large densities in the star. Each ν (or ¯ν) flavor-oscillates as it propagates, and undergoes forward-scattering off the other ν and ¯ν along their path, which in turn affect its oscillations in a correlated manner. These self-induced or collective oscillations [1–4], associated with ν–ν forward-scattering, have been a continuous source of surprises and puzzling results that have been studied with increasing levels of sophistication. See refs. [5–7] for recent reviews. The most puzzling manifestation of these collective effects was pointed out by Raymond Sawyer [8, 9], who argued that the growth rate of flavor conversions can be proportional to the neutrino potential µ ∼ √2G F nν, which scales with the neutrino density nν but curiously does not seem to depend on the neutrino having a mass. Thus, the fast oscillation rate can exceed the ordinary neutrino oscillation frequency ω = Δm 2/(2E) by a factor of µ/ω ∼ 105, and occur closer to the region where neutrinos are emitted.

Fast neutrino conversions can take place only if the electron lepton number (ELN) distribution, i.e., the difference of the νe and ¯νe angular fluxes, changes its sign across some direction of emission at any given point inside the supernova [10–16]. This necessary condition for fast oscillations, that the ELN has a “crossing” through zero, obviously requires that νe and ¯νe have different collision rates. Such a difference is quite likely near the neutrino decoupling region in a supernova: due to neutron richness of stellar matter, the ¯νe decouples earlier. So the ¯νe are more forward peaked than νe, and if number density of νe does not greatly exceed that of ¯νe, i.e., the lepton asymmetry is modest, the ELN could exhibit the required crossing. However, this also immediately raises a red flag – if collisions are important to create the conditions for fast conversions, wouldn’t they damp oscillations too?

In this Letter, we present the first calculations to explain the interplay of collisions and fast oscillations. We note that the collision rates, Γνe and Γ ¯νe, are significantly smaller that the refractive potential µ even inside the supernova core. As a result, the collisions are dominant only initially and can create conditions for fast oscillations when oscillations are not yet operative. However, once fast oscillations have been triggered, the collision rates, being smaller than µ, are not large enough to lead to damping of oscillations. This represents a major change to the existing paradigm, wherein the collisional and free-streaming regimes are believed to be well-separated. Our results suggest that this simplification may not always hold, with potentially important consequences for supernova astrophysics and neutrino physics.

We will start with a simple illustration of the relevant scales in the problem. Fig. 1 shows neutrino potential µ, and the charged-current collisional rate Γ ∼ nB σ, where nB is the nucleon density and σ is the charged-current scattering.

FIG. 1. Properties of an 11 M⊙ supernova derived from the simulation in ref. [17]. Radial profiles of neutrino potential µ, matter potential λ, and scattering rates Γνe and Γ ¯νe, are shown for a snapshot at a post-bounce time t = 0.170 s.
cross section for the $\nu_e$ and $\bar{\nu}_e$, from an $11M_\odot$ spherically symmetric (1D) supernova model, simulated by the Garching group, at a post-bounce time = 0.170 s [17]. These quantities are energy-averaged; see the appendix for details. As apparent, the neutrino potential $\mu$ is always larger than the $\nu_e$ collisional rate, by no less than $\sim 5$ orders of magnitude. The $\bar{\nu}_e$ collisional rate is a factor $\sim 3$ further smaller than the $\nu_e$ collision rate. Thus, even in the deepest regions, at $r \lesssim 10$ km where these quantities become roughly constant, the refractive effects remain stronger. Notice that in the figure we also show the matter term $\lambda = \sqrt{2}G_F n_e$ that is one order of magnitude larger than $\mu$. However, it has been shown that fast flavor conversions that can grow locally are not suppressed by a large matter effect [12, 16]. This situation implies that if such fast conversions are triggered somewhere in the neutrino decoupling region, they may affect the entire region near the neutrinosphere.

Ideally, one would characterize this situation by simulating the formation of neutrino fluxes, including their energy and especially angular distributions, through the interplay of collisional processes and possible flavor changes triggered by the neutrino potential. Unfortunately, the dynamic range covered by collisions and oscillations is too large and it remains impractical to simulate this in earnest. Supernova simulations assume oscillations do not take place deep in the star, while oscillation calculations, even when considering fast conversions, completely ignore collisions. We adopt a reductionist approach and use a one-dimensional model with a simplified implementation of the collision term to study the interplay of oscillations and collisions. In the following, we first set up the equations, define our model, and present the numerical results for the same. We then conclude with a discussion of the relevance of these results for a realistic scenario.

**Equations of motion including collisions.** In the absence of external forces, the dynamics of the $\nu$ occupation numbers $\varrho_{p,x,t}$ for momentum $p$ at position $x$ and time $t$ is governed by the following equations of motion (EoMs) [18–22]

$$ (\partial_t + v_p \cdot \nabla_x) \varrho_{p,x,t} = -i[\Omega_{p,x,t}, \varrho_{p,x,t}] + C[\varrho_{p,x,t}] , \quad (1) $$

where, in the Liouville operator on the left-hand side, the first term accounts for explicit time dependence, while the second term, proportional to the neutrino velocity $v_p$, encodes the spatial dependence due to particle free streaming. The right-hand-side contains the oscillation Hamiltonian $\Omega_{p,x,t}$, which is a sum of the vacuum term depending on the mass-squared matrix of neutrinos, the matter term depending on background density of electrons, and the self-interaction term $\int \frac{d^3q}{(2\pi)^3}(1 - v_p \cdot v_q)(\varrho_{q,x,t} - \varrho_{q,x,t})$ [6]. The last term on the right-hand-side of Eq. (1) accounts for collisions. Antineutrinos represented by $\varrho_{p,x,t}$ obey the same equation, but with an opposite sign for the vacuum oscillation term.

Our goal here is to capture the main features of the interplay between flavor conversions and collisions. Therefore, we simplify the collisional term as described below. We assume that initially our system has only $\nu_e$ and $\bar{\nu}_e$, and neglect neutral-current interactions that both produce the other flavors and affect kinetic decoupling [23, 24]. Instead, we only include the charged-current processes that will create flavor and angular asymmetries between $\nu_e$ and $\bar{\nu}_e$. For the charged-current processes, the collisional term has been derived in [25] and can be mimicked by [26]

$$ C[\varrho_{p,x,t}] = \frac{1}{2} \left\{ \Gamma_{p,v} (\varrho_{p}^{eq} - \varrho_{p}) \right\} \quad , \quad (2) $$

where $\{ , \}$ denotes an anticommutator and $\varrho_{p}^{eq}$ represents the equilibrium value of the matrix of densities for momentum $p$. The matrix $\Gamma = \text{diag}(\Gamma_{\nu_e}, 0)$ in the flavor basis has a nonzero contribution only for the electron flavor and is proportional to the collision rate $\Gamma_{\nu_e}$ for the processes allowed, e.g., $p e^- \rightarrow n \nu_e$ for $\nu_e$. Analogously, only the process $n e^+ \rightarrow p \bar{\nu}_e$ is relevant for $\bar{\nu}_e$. These rates are shown in Fig. 1.

The collisional term in Eq. (2) is analogous to the one used in the context of neutrino flavor conversions in the early Universe [27]. The effect is twofold: It populates the diagonal components of the density matrix; in particular, if $\varrho_{\nu_{ee}}$ is not the same for all modes, these states get differently populated. However, it damps the off-diagonal term of the density matrix, which destroys coherence and, if sufficiently strong, inhibits any kind of flavor oscillation.

**Numerical examples.** We consider time-dependent flavor evolution in one spatial dimension labeled by $z$. Further, we take only two momentum modes of equal energy, counter-propagating in the forward ($p_z > 0$) and backward ($p_z < 0$) directions, labelled by $f$ and $b$, respectively. Their equilibrium abundances without oscillations, $\varrho_{\nu_{ef}}^{eq}$ and $\varrho_{\nu_{bf}}^{eq}$, are chosen to be unequal so that a crossing in the ELN is created, equivalent to assuming different scattering cross sections. We then numer-
ically solve the non-linear EoMs [Eq. (1)] in the length interval \( z \in [0, L] \), always working in the units in which the neutrino potential is \( \mu = 1 \). For simplicity, we set the matter term \( \lambda \) to zero, while the vacuum oscillation frequency, here \( \omega = 9 \times 10^{-5} \), gives the seed for the flavor conversions. The neutrino velocities are taken to be \( v_f = -v_b = 0.2 \). One can roughly picture this model as mimicking the temporal and radial flavor evolution in a supernova.

In the left panel of Fig. 2 we plot the equilibrium value of the occupation numbers for \( \nu_e \) and \( \bar{\nu}_e \) in the forward and backward directions in the box \( z \in [0 : 2500] \). We divide the interval into three regions, i.e., \( z < 700 \) that represents the trapping regime where both \( \nu_e \) and \( \bar{\nu}_e \) have equally populated forward and backward modes, \( 700 < z < 1500 \) representing the decoupling region where \( \bar{\nu}_e \) decouples, while \( \nu_e \) decouples around \( z \approx 1500 \), and \( z > 1500 \) the free-streaming region where both \( \nu_e \) and \( \bar{\nu}_e \) having decoupled can now free-stream. The specific values of \( z \) demarcating the regions are chosen ad hoc and do not carry any special significance. For \( z < 700 \) we assume \( \bar{\nu}_e = \nu_e \), with an excess of \( \nu_e \) over \( \bar{\nu}_e \). In the decoupling region, \( 700 < z < 1500 \), the neutrino sector has no asymmetry between forward and backward modes, whereas to mimic the decoupling of \( \nu_e \), the antineutrinos are assumed to have an excess of forward over backward modes, as expected in their decoupling region. Moreover, we keep the total number of \( \bar{\nu}_e \) in the first and second region constant. With such a definition of \( \bar{\nu}_e \), collisions will eventually generate a crossing in the ELN, i.e., an excess of \( \nu_e \) over \( \bar{\nu}_e \) in the backward mode, and vice versa for the forward mode at a fixed location. Finally, in the free-streaming region at \( z > 1500 \) we enforce there are no backward modes for both neutrinos and antineutrinos.

In the right panel of Fig. 2 we show the collision rates \( \Gamma_{\nu_e \bar{\nu}_e} \) and \( \Gamma_{\bar{\nu}_e \nu_e} \), both normalized to one at their maximum. \( \Gamma_{\nu_e \bar{\nu}_e} \) is nearly constant up to about \( z = 1500 \), whereas \( \Gamma_{\bar{\nu}_e \nu_e} \) starts decreasing around \( z = 700 \). For \( z > 1500 \) both neutrinos and antineutrinos are free-streaming, i.e., \( \Gamma_{\nu_e \bar{\nu}_e} = \Gamma_{\bar{\nu}_e \nu_e} = 0 \). Note that for both \( \bar{\nu}_e \) and \( \nu_e \) we are considering smooth variations between the three zones, since discontinuities may introduce numerical artifacts in the simulations.

In Fig. 3 we plot several time snapshots of the evolution of the density matrix for the two modes, including the collisional term with \( \Gamma_{\nu_e} = 0.1 \), and setting \( \mu = 0 \) (no fast oscillations). We start with no neutrinos in the box at \( t = 0 \), but they get populated through the collisional term. Already at \( t=2 \) the population of both forward and backward modes for \( \nu_e \) and \( \bar{\nu}_e \) start to grow due to the collisional term. From the snapshot at \( t = 20 \) one can see that three of the modes reach their equilibrium value in the decoupling region. At \( t = 200 \) all modes, including the forward \( \bar{\nu}_e \), have reached their equilibrium value and in the following time snapshot one simply observes the free propagation of the forward modes into the free-streaming region where \( \Gamma = 0 \). Note that backward modes are frozen to their equilibrium value, as the repopulation is efficient.

In Fig. 4 we switch on the neutrino-neutrino interaction term, \( \mu = 1 \), keeping \( \Gamma_{\nu_e} = 0.1 \). Due to the presence of a crossing in the ELN in the central region, flavor conversions start to develop (notice the wiggles in the snapshot at \( t = 200 \)). However, due to the large collisional term, the system quickly tends to equilibrium and at larger times the evolution is very similar to the case with \( \mu = 0 \). In this case, fast conversions is only a transient effect, quickly damped by the collisional effects.

Finally, in Fig. 5 we significantly lower \( \Gamma_{\nu_e} \) to \( 10^{-4} \) in order to represent the realistically expected hierarchy between \( \Gamma \) and \( \mu \), as shown in Fig. 1. In this case, since the collisional production rate is significantly slower than in the previous case, to speed-up the calculation we start at \( t = 800 \) with a significant population of neutrinos, but still without any ELN crossing anywhere. We see that due to the smallness of \( \Gamma \) the creation of a crossing in the decoupling region is much slower. Without the presence of a significant crossing, fast conversions cannot develop, as one observes until \( t = 1600 \). At later times, when a crossing is generated, fast conversions develop in the decoupling region (see the wiggles in the snapshot at \( t = 1600 \)). Such conversions are observed only for the forward modes. This is a consequence of the conservation of flavor lepton number [15]. Indeed, the total lepton number is coming only from the backward modes, since the excess of the \( \bar{\nu}_e \) for the forward modes is negligible. Further, since the collisions are weaker than refractive effects, modes are not efficiently repopulated towards the equilibrium value. Then the oscillated forward modes can propagate towards larger \( z \) (see snapshot at \( t = 2400 \)), and the effects of fast conversions can reach the free-streaming zone.

Discussion and conclusions. – Fast neutrino flavor conversions are possible near the SN core, where the angular distributions of the ELN flux, i.e., the difference of the \( \nu_e \) and \( \bar{\nu}_e \) fluxes, might harbor a crossing. This region is the same in which neutrinos decouple from the matter, so that they still feel residual scatterings. We have studied this in a simple one-dimensional model with two momentum modes that allows us to calculate effects of neutrino flavor conversions and collisions in a consistent manner. We find that for collision rates that are significantly smaller than the neutrino potential, collisions create the conditions for fast conversions but do not damp them. Unexpectedly, state-of-art SN simulations seem to suggest that the neutrino potential indeed dominates over the collisional rate in the SN core. Drawing the insights from our model, this dominance implies that once fast conversions are generated in the decoupling region they will propagate everywhere. With the possibility of such fast conversions, the neutrino fluxes found by SN
FIG. 3. Two-beam model ($\Gamma_{\nu_e} = 0.1, \mu = 0$). Evolution of forward and backward going modes for $\nu_e$ and $\bar{\nu}_e$ as a function of $z$ for different representative times. Blue continuous curve indicates forward-going $\nu_e$, blue dashed curve is for backward-going $\nu_e$, red continuous curve is for forward-going $\bar{\nu}_e$, while red dashed curve is for backward-going $\bar{\nu}_e$.

FIG. 4. Two-beam model ($\Gamma_{\nu_e} = 0.1, \mu = 1$). Evolution of forward and backward going modes for $\nu_e$ and $\bar{\nu}_e$ as a function of $z$ for different representative times in the same format as Fig. 3.

FIG. 5. Two-beam model ($\Gamma = 10^{-4}, \mu = 1$). Evolution of forward and backward going modes for $\nu_e$ and $\bar{\nu}_e$ as a function of $z$ for different representative times.

simulations, computed without including flavor oscillations, may not be representative of reality.

Our finding motivates a detailed analysis of current SN simulations to understand if the conditions for fast conversions are indeed generated by collisions. In this context, a dedicated analysis of angle distributions of the neutrino radiation field for several spherically symmetric (1D) supernova simulations has not found any crossing in the ELN near the neutrinosphere [17]. However, this may change in 3D models. For example, in the presence of Lepton Emission Self-sustained Asymmetry [28], which produces a pronounced large-scale dipolar pattern in the ELN emission, one may expect a change of sign in difference of $\nu_e$ and $\bar{\nu}_e$ angular distributions. The ensuing crossing in the ELN angular distributions can then trigger fast conversions, with possibly drastic impact on further evolution of the SN. Indeed, one has to think of new approaches to include the effect of fast conversions into already challenging supernova simulations. This task, while obviously very challenging, may be necessary to obtain an accurate description of the supernova dynamics and neutrino fluxes.

Acknowledgments. – We acknowledge useful discussions with Hans-Thomas Janka. The work of F.C. is supported partially by the Deutsche Forschungsgemeinschaft through Grant No. EXC 153 (Excellence Cluster “Universe”) and Grant No. SFB 1258 (Collaborative Research Center “Neutrinos, Dark Matter, Mes-
Appendix: Calculation of $\Gamma_{\nu}$

Assuming a neutrino energy $E_\nu$ and a distance from the supernova centre $r$, the inverse of the scattering rate for the process $\nu_e + n \rightarrow e^- + p$ ($\bar{\nu}_e + p \rightarrow e^+ n$) is given by [29]

$$\Gamma_{\nu_e}^{-1}(E_\nu, r) = \frac{G^2}{\pi} n_n(p)(g_\nu^2 + 3g_A^2)(E_\nu + Q)^2 \left(1 - \frac{m_e^2}{(E_\nu + Q)^2}\right) ,$$

where $m_e$ is the mass of the electron, $G^2 = 5.18 \times 10^{-44}$ MeV$^{-2}$ cm$^2$, $Q = 1.2935$ MeV, $g_\nu = 1$, $g_A = 1.23$ and $n_n(p)(r)$ is the neutron (proton) density at a distance $r$. In Eq. 3 we are neglecting the decrease in the scattering rate due to nucleon and electron final state blocking. Such an effect is relevant when matter density becomes of the order of the nuclear density. However, since our purpose is a comparison with $\mu$ (see Fig. 1), we show that $\mu$ dominates over $\Gamma$ even with such assumptions.

For each $r$ we calculate an average of the scattering rate using the neutron density as weight function

$$\langle \Gamma_{\nu_e}(r) \rangle = \int_0^\infty dE_\nu \frac{E_\nu \Gamma_{\nu_e}(E_\nu, r)}{\int_0^\infty dE_\nu n_\nu(E_\nu)} .$$

A similar average is performed also in the calculation of the neutrino potential $\mu(r)$. These energy-averaged quantities are shown in the main text in Fig. 1, where we omit the notation $\langle \cdots \rangle$ for typographic clarity.
[13] I. Izaguirre, G. Raffelt, and I. Tamborra, *Fast Pairwise Conversion of Supernova Neutrinos: A Dispersion-Relation Approach*, Phys. Rev. Lett. **118** (2017), no. 2 021101, [1610.01612].

[14] F. Capozzi, B. Dasgupta, E. Lisi, A. Marrone, and A. Mirizzi, *Fast flavor conversions of supernova neutrinos: Classifying instabilities via dispersion relations*, Phys. Rev. **D96** (2017), no. 4 043016, [1706.03360].

[15] B. Dasgupta and M. Sen, *Fast Neutrino Flavor Conversion as Oscillations in a Quartic Potential*, Phys. Rev. **D97** (2018), no. 2 023017, [1709.08671].

[16] S. Abbar and H. Duan, *Fast neutrino flavor conversion: roles of dense matter and spectrum crossing*, 1712.07013.

[17] I. Tamborra, L. Huedepohl, G. Raffelt, and H.-T. Janka, *Flavor-dependent neutrino angular distribution in core-collapse supernovae*, Astrophys. J. **839** (2017) 132, [1702.00060].

[18] G. Sigl and G. Raffelt, *General kinetic description of relativistic mixed neutrinos*, Nucl. Phys. **B406** (1993) 423–451.

[19] P. Strack and A. Burrows, *Generalized Boltzmann formalism for oscillating neutrinos*, Phys. Rev. **D71** (2005) 093004, [hep-ph/0504035].

[20] A. Vlasenko, G. M. Fuller, and V. Cirigliano, *Neutrino Quantum Kinetic Equations: The Collision Term*, Phys. Rev. **D94** (2016), no. 3 033009, [1605.09383].

[21] C. Volpe, D. Vaananen, and C. Espinoza, *Extended evolution equations for neutrino propagation in astrophysical and cosmological environments*, Phys. Rev. **D87** (2013), no. 11 113010, [1302.2374].

[22] G. G. Raffelt, *Muon-neutrino and tau-neutrino spectra formation in supernovae*, Astrophys. J. **561** (2001) 890–914, [astro-ph/0105250].

[23] M. T. Keil, G. G. Raffelt, and H.-T. Janka, *Monte Carlo study of supernova neutrino spectra formation*, Astrophys. J. **590** (2003) 971–991, [astro-ph/0208035].

[24] G. Raffelt and G. Sigl, *Neutrino flavor conversion in a supernova core*, Astropart. Phys. **1** (1993) 165–184, [astro-ph/9209005].

[25] A. D. Dolgov, *Neutrino oscillations in the early universe: Resonant case*, Nucl. Phys. **B610** (2001) 411–429, [hep-ph/0102125].

[26] A. D. Dolgov, S. H. Hansen, S. Pastor, S. T. Petcov, G. G. Raffelt, and D. V. Semikoz, *Cosmological bounds on neutrino degeneracy improved by flavor oscillations*, Nucl. Phys. **B632** (2002) 363–382, [hep-ph/0201287].

[27] I. Tamborra, F. Hanke, H.-T. Janka, B. Müller, G. G. Raffelt, and A. Marek, *Self-sustained asymmetry of lepton-number emission: A new phenomenon during the supernova shock-accretion phase in three dimensions*, Astrophys. J. **792** (2014), no. 2 96, [1402.5418].

[28] S. W. Bruenn, *Stellar core collapse: Numerical model and infall epoch*, Astrophys. J. Suppl. **58** (1985) 771–841.