Cosmological constraints on unimodular gravity models with diffusion

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A discrete space-time structure lying at about the Planck scale may become manifest in the form of very small violations of the conservation of the matter energy-momentum tensor. In order to include such kind of violations, forbidden within the General Relativity framework, the theory of unimodular gravity seems as the simplest option to describe the gravitational interaction. In the cosmological context, a direct consequence of such violation of energy conservation might be heuristically viewed a “diffusion process of matter (both dark and ordinary)” into an effective dark energy term in Einstein’s equations, which leads under natural assumptions to an adequate estimate for the value of the cosmological constant. Previous works have also indicated that these kind of models might offer a natural scenario to alleviate the Hubble tension. In this work, we consider a simple model for the cosmological history including a late time occurrence of such energy violation and study the modifications of the predictions for the anisotropy and polarization of the Cosmic Microwave Background (CMB). We compare the model’s predictions with recent data from the CMB, Supernovae Type Ia, cosmic chronometers and Baryon Acoustic Oscillations. The results show the potential of this type of model to alleviate the Hubble tension.

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I. INTRODUCTION

Modern cosmology relies on Einstein’s equation, and thus, on the strict conservation of energy momentum tensor. This is a feature that seems essentially untouchable. However a deeper look at the issue reveals that conservation of energy momentum is unlikely to be an exact feature of nature [1]. When taking into account the quantum nature of matter, it becomes rather apparent that our gravitational theory must also undergo some modifications which are likely to include at least small departures from exact conservation laws. However, these changes might be larger in early cosmological times and could have cumulative effects with empirically relevant consequences.

Recently, a group involving some of us [2], used of the fact that in Unimodular Gravity (UG) the strict conservation of the energy momentum tensor is not required for the consistency of the theory, in contrast with what occurs in General Relativity (GR). The theory might be said to reduce the general invariance under diffeomorphisms, which is characteristic of GR to a more restricted invariance under four-volume preserving diffeomorphisms1. Concretely [2] studied within that scheme—i.e. violation of energy momentum conservation together with unimodular gravity—the idea that a violation of the conservation of the energy momentum tensor might have important and observable consequences on the value of the effective cosmological constant. Of course, the idea under consideration contemplates a violation that is too small to be directly detectable, as no direct observation of any departure from strict energy conservation has been observed at this stage. Further studies [3, 5], considered the idea that such violations might also result from defects in the fabric of space-time, and its ultimate origin might lie at the level of the quantum

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1 Strictly speaking the invariance under diffeomorphisms is tautological in any theory that relies on manifold and tensor fields. The point, as discussed in [3] is whether or not all geometrical entities entering the theory are related (or derived from) to the fundamental degrees of freedom, which in theories of gravitation correspond to the space-time metric. When some additional non-dynamical structure is attached to space-time and left untouched by the diffeomorphic transformation is that one finds what are called violations of full diffeomorphism invariance. In the case under consideration such structures could be either the violations intrinsically associated with resolving the conceptual problems of quantum theory as discussed in [1], or the structure characterizing the discreteness of spacetime.
gravitational description thereof. That analysis resulted on an attractive account for the nature and magnitude of the cosmological constant \([4, 5]\), which without the use of any kind of fine tuning, leads to an estimate for the amount of the dark energy which agrees with those emerging from astronomical and cosmological observations. In that proposal an effective cosmological constant represents the accumulated effect of extremely small departures of such conservation with the dominant part of the effect arising around the electro-weak scale (yielding a value \(\Lambda \sim m_{\text{ew}}^2 (E_{\text{ew}}/m_p)^7\) which can easily be seen to be of correct order of magnitude). The perspective provided by this work motivated the investigation of the possible role of the fundamental granularity of Planckian physics as the primordial cause of inhomogeneities ultimately observed at the CMB. This led to an alternative paradigm to the standard inflationary one where inhomogeneities at the CMB are directly linked with Planckian discreteness during a primordial inflationary era \([6]\).

In some recent work \([7, 8]\) the issue of the so-called \(H_0\) tension—i.e. the incompatibility at more than 4\(\sigma\) between the value of \(H_0\) obtained in the context of the standard cosmological model by the use of latest CMB data \([9]\) and what is obtained in studies based on type Ia supernovae (SNIa) explosions together with local distance calibrations (CC) \([10, 11]\)—has been considered from the present perspective. In the proposal for the generation of a cosmological constant in the context of UG gravity, the conditions that resulted in the relatively large violations of the energy-momentum tensor conservation are tied to regimes in which large space-time curvatures are involved. The proposal to deal with the \(H_0\) tension is meant to be an extension of such account, which thus must involve situations where similar high curvature conditions prevail in the post recombination epoch (where the effective cosmological constant would have to change by a nontrivial amount). At late cosmological times the only situations where such strong gravitational fields can be reasonably assumed to arise correspond to the late time dynamics of black holes, particularly highly rotating ones, which might lose up to \(1/3\) of their mass if slowed down via a diffusion mechanism without violating the second law of thermodynamics\(^2\).

Such preliminary study, which assumed a simple expression for the violation of the energy-momentum tensor, concluded that the overall plausibility of the model appears to be supported by observational data \([8]\). That analysis focused on the effects on the value of the acoustic angular scale of the CMB and the comparison with the corresponding recent observational data, and therefore was able to consider just the changes in the global geometry of the universe introduced by the UG model. A similar analysis which considered different assumptions for the variation of the cosmological constant was performed in \([12]\). Also in \([13]\), the authors consider a cosmological model with sign switching cosmological constant and show that this model can alleviate some tensions that are related to the so called Hubble tension. In the present paper we perform a more complete and strict (methodologically speaking) analysis of the viability of the scenario studied in \([8]\). Concretely we study a relatively simple model parameterized by three quantities: one characterizing the magnitude of the violation of energy conservation involved, and the other two characterizing the cosmological period during which the violation is assumed to have taken place. We analyze the model by simultaneously considering, the anisotropies and polarization of the CMB, involving therefore the growth of the structure of the universe, in addition to the other data related to the global geometry of the universe.

The physical source of the relatively large violation of the conservation of energy momentum is thought to be related to the dragging effect on the rotation of black holes, presumably arising from some fundamental granularity of space-time. Thus, in order to to extract a formula for the energy-momentum violation current \(J_a \equiv \nabla^b T_{ab}\) and produce quantitative calculations, one would need to model the cosmological density of black holes as a function of their mass, their angular momentum and cosmic time \(n = f(M, J, t)\) in combination with the proposed form of the effective dissipation rate described in \([7]\). With that expression at hand we could directly insert it into the equations of UG, and study the resulting modifications of the late time cosmology, allowing us to check whether or not the corresponding predictions can account well for all the relevant observations. Unfortunately, at this stage, our knowledge about \(f(M, J, t)\) is just too vague to carry such analysis. There are only some known bounds on the proportion of matter that might be in the form of primordial black holes which unfortunately seem to refer mostly to monochromatic contributions \([14]\) rather than constraints on something like the function \(f(M, J, t)\) or even the momentum integrated function

\[
F(M, t) = \int_0^{M^2} f(M, J, t) \, dJ
\]

(the upper bound resulting from limiting consideration of up to the extremal case). Beyond that, we have recently learned \([15, 23]\) that the range of masses of black hole is very wide, involving in the lower end objects as light as few solar masses and at the other extreme objects as large as \(10^9\) solar masses, with increasing evidence for the existence

\(^2\) Which we will take to continue to hold even in those extreme circumstances.
of many objects in the 60-100 solar mass range. It is fair to say that we have very little understanding regarding the
process by which all but the lighter black holes formed and also very limited knowledge of the form that even $F(M, t)$
might take at intermediate scales (say $10^2 - 10^5$ solar mass range).

Despite our ignorance concerning the details of the current characterizing the violation of energy-momentum con-
servation in the general cosmological context, it is useful to perform a first analysis using recent cosmological data,
such as the ones provided by the CMB, Baryon Acoustic Oscillations (BAO), SnIa and CC, to test the viability of
cosmological models that are motivated by UG to explain the cosmological evolution including the growth of per-
turbations. However, it follows from the previous discussions that some assumptions, related to the violation of
the energy-momentum conservation, will have to be included in order to describe the behavior of the dark energy
component of the universe as well as the baryonic and dark matter ones.

The organization of this manuscript is as follows: In section II we offer a brief discussion of the general theoretical
ideas underpinning this work. Section III is devoted to specify the concrete cosmological model based on these ideas
and to characterize its effective parametrization as well as the ensuing evolution of the various contributions to the
energy density budget as a function of cosmic time. We also discuss in this section the effects of the modified cosmic
evolution introduced by our model on the CMB spectrum. The details of the analysis method used in this work to
perform the statistical analyses (including the observational data sets) are discussed in section IV. Results of the
statistical analysis of the modified cosmological model motivated by unimodular gravity with CMB, BAO, SnIa and
CC data are shown in Section V. We end in section VI with a brief discussion of the results the overall viability of
the model and of the general theoretical scenario.

II. THEORETICAL SCENARIO

A novel and rather successful scheme to account for the nature and magnitude of the cosmological constant, yielding
in a natural way its correct order of magnitude, was proposed in [4, 5]. This was a particularly encouraging result,
particularly when contrasted with standard estimates that are typically wrong by 120 orders of magnitude. The
ingredients underlining the proposal are:

1. The generic idea that quantum gravity implied some sort of spacetime granularity.
2. The result of [24] where it was showed that the granularity cannot select a preferential reference frame without
generating (presently) detectable violations to Lorentz invariance at low energies.
3. The idea that the two considerations above might be reconciled if the effects of granularity become relevant only
in the presence of strong gravitational fields or high curvature.

A basic assumption is that the interactions with granularity need to be of a relational nature in the sense that
the relevant affected degrees of freedom must be capable (via their excitations) of selecting the local preferential
reference frame with respect to which the fundamental scale can have an operational meaning. This suggested the
rather natural assumption that the probes which could be sensitive to such granularity needed to posses both, a
scale (and thus break scale invariance to relate to the curvature scales) and an intrinsic direction to determine the
directionality of the effects under consideration. The natural candidate was thus identified as spinning and massive
degrees of freedom. The effect would have to be modulated by curvature (to avoid the no-go result of item 2 above).

A mean-field perspective suggests taking the scalar curvature $R$ which (linked to the trace of the energy-momentum
tensor via Einstein’s equations) is a natural order parameter of the violations of scale invariance for the gravitational
sources involved. This led us to propose that, at level of the effective characterization of matter in terms of particles,
the effect would take the form of a deviation from the standard geodesic equation given by:

$$u^\mu \nabla_\mu u^\nu = \frac{m}{m_p^2} \text{sign}(s \cdot \xi) R s^\nu,$$

where $\alpha > 0$ is a dimensionless constant, $m$ is the particle’s mass, $m_p$ is Planck’s mass, $s^\mu$ is the particle’s spin,
and $\xi^\mu$ is a preferred time-like vector field characterizing the state of motion of the matter source. The analysis of
this hypothesis in the cosmological context in terms of standard kinetic theory led to an effective violation of energy
momentum conservation given by [4 5]:

$$J^\nu = 8\pi G \nabla^\mu T_{\mu\nu} = 4\pi \alpha \frac{T}{m_p^2} R [8\pi G \Sigma_i |s_i| T^{(i)}] \xi_\nu;$$

(2)
where \( T \) is the temperature of the cosmological plasma, the sum involves contributions of the different species \( i \) in the standard model of particle physics, \( T^{(i)} \) is trace of the energy momentum tensor of the species \( i \), and \( |s_i| \) the spin of that species.

Moreover, given that violations of the energy-momentum conservation are inconsistent in the context of GR, the proposal needs to be formulated in a context of a gravity theory that allows for such violations \cite{5} \cite{23}. In fact, the theory known as Unimodular Gravity permits a specific kind of violations and is therefore ideal to describe those type of effects expected to arise from a fundamental space-time granularity. The relatively recent interest in this theory, which was initially considered by Einstein himself, can be traced back to works such as \cite{26, 27}. At this point it is worthwhile to briefly describe the theory of Unimodular Gravity. The action can be written in a coordinate independent form, using, say, abstract index notation (see for instance \cite{3}):

\[
S = \int [R_{abcd} + \lambda (\varepsilon_{abcd} - \varepsilon_{abcd}) + \mathcal{L}_{\text{matt}} \varepsilon_{abcd}]
\]

where \( \varepsilon_{abcd} \) is a fiduciary 4-volume element and \( \varepsilon_{abcd} \) is the 4-volume element associated to the metric \( g_{ab} \), while \( \lambda(x) \) is a Lagrange multiplier function. Using coordinates \( x^\mu \) adapted to the fiduciary volume element one is led to the simple relation \( \varepsilon_{abcd} = \sqrt{-g} \varepsilon_{abcd} \) with \( g = \det g_{\mu\nu} \). The theory is said to be invariant under a restricted class of diffeomorphisms, i.e. those that are 4-volume preserving\(^3\). The resulting equations of motion for the metric are:

\[
G_{ab}(x) + \lambda(x)g_{ab}(x) = 8\pi GT_{ab}(x)
\]

while the variation with respect to \( \lambda \) leads to the constraint \( \varepsilon_{abcd} = \varepsilon_{abcd} \). The value of the Lagrange multiplier \( \lambda(x) \) can determined by taking the trace of the previous equation. This leads to :

\[
\lambda(x) = \frac{1}{4}(8\pi GT + R)
\]

which upon substitution result in the standard form of the equations of unimodular gravity, namely:

\[
R_{ab} - \frac{1}{4}g_{ab}R = 8\pi G(T_{ab} - \frac{1}{4}g_{ab}T)
\]

As noted a central feature of this theory, which makes it useful for our purposes, is that, in contrast with general relativity, it does not require the conservation law \( \nabla^a T_{ab} = 0 \), for its self-consistency. In fact one might define the ‘energy momentum non-conservation current’ as \( J_a = 8\pi G \nabla^b T_{ab} \neq 0 \) which provided the integrability condition \( dJ = 0 \) is satisfied\(^4\), can be integrated to yield,

\[
R_{ab} - \frac{1}{2}g_{ab}R + g_{ab} \left( \lambda_{-\infty} + \int J \right) = 8\pi GT_{ab}
\]

which coincides with Einstein’s equation with a term that acts like an ‘effective dark energy component’ which need not be a constant. In the cosmological setting, the role of this term—just as it occurs with the usual cosmological constant term for the range of values that are relevant in our universe—is entirely negligible except for the very late times, where every other contribution to the universe’s energy budget has been diluted to an extreme level, and thus such term can become dominant.

Calculating the contribution to dark energy in terms of equation \( \text{(2)} \) using the standard the cosmological history of our universe \cite{5}, it was found that the effective cosmological constant was given approximately by:

\[
\Lambda \approx 16\pi\alpha \sqrt{\frac{5\pi^5}{3} m_t^4 T_{\text{EW}}^3 \varepsilon(T_{\text{EW}})}
\]

where \( m_t \) is the mass of the heaviest particle with non-vanishing spin (here the top quark), \( T_{\text{EW}} \) is the temperature of the electroweak transition, \( g^* \) is the degeneracy factor and \( \varepsilon \) is a factor that takes into account the running of the top

\(^3\) Here, as is common in these discussions it is assumed that only the dynamical elements are subject to the action of the diffeomorphisms.

\(^4\) The integrability condition is a direct consequence of the volume-preserving diffeomorphism invariance of the action principle \cite{2}.

\(^5\) This is justified because it can be shown that the resulting changes in the dynamics of the background are completely negligible during the period in which the effective cosmological constant is generated, and its effects only become relevant at the very late cosmological times when the dilution of all other forms of energies make the dark energy the dominant component.
quark mass. For standard values of the cosmological parameters the previous prediction coincides with the observed value of the cosmological dark energy in order of magnitude when the dimensionless parameter $\alpha$ is assumed to be of order 1.

It became quite clear after subsequent analysis that, apart from the generation of a cosmological constant, the conditions for other effects to be observable where simply to extreme to be accessible to testing in situations to which we have access at present in our universe. For instance, simple dimensional considerations imply violation of Lorentz invariance parametrized in dangerous\(^6\) dimension 5 operators modulated by $R$ such as:

$$\lambda \psi \gamma_\mu \psi \xi^\mu \frac{R}{m_p},$$

for fermion fields $\psi$, a dimensionless $\lambda$. According to Kostelecky \cite{28} the strongest bound on such operators is

$$\frac{\lambda R}{m_p^2} \approx \frac{\rho}{m_p^4} < 10^{-12}.$$  \hspace{1cm} (10)

With neutron star densities estimated to reach scales of about $\rho_n \approx 10^{-80} m_p^4$, it is easy to see that not even under such extreme conditions, empirically significant direct manifestations of the previous fundamental matter-diffusion-type of effect can take place in matter configurations as presently known.

However, similar effects could be important in the context of non-fundamental matter forms. For instance in the case of black holes produced by the gravitational collapse of fundamental matter fields which latter become (for the purpose of outside observers) seemingly empty region but with a (classically) unbounded internal curvature. Of course, black holes are not simple point-like particles and the applicability of equation (2) to such macroscopic objects might not seem directly justified. Nevertheless, the fact that black holes are intimately connected with regimes of arbitrarily high curvatures—expected reach Planck scale curvatures inside where only a full quantum gravity description could be truly relied on—naturally opens the door to the consideration of the phenomenological possibility that they might be affected in a special way by our hypothetical spacetime granularity. This suggests, as discussed in \cite{7} that black holes could be subject to some kind of *effective friction* related to the motion of the “frames they drag” with them and those associated to the matter that was connected to the space-time structure of their respective environment.

This led to two interconnected effects responsible for a translational and a rotational friction respectively. An early analysis carried out in \cite{7} showed clearly that the one that seems more likely to have any important energetic relevance was the second effect, in which (assuming, conservatively, that the second law would not be violated by the type of effects considered) up to 30% of the mass of a highly rotating black hole could be lost as a result of the rotational friction. Moreover, the same analysis indicates that in the context of unimodular gravity the rotational friction could be a significant source of a change in the effective cosmological constant.

Even when the basic theoretic ingredients for the generalization of (2) to the case of black holes are available from the analysis of \cite{7}, an accurate account of the number density of black holes as a function of their mass angular momentum and cosmic time $n = f(M, J, t)$ for quantitative estimates. Such formula presumably would have to incorporate the formation of new black holes as the result of standard gravitational collapse during cosmic evolutions, the effects of black hole mergers, as well as any possible contribution of primordial black holes. With $n = f(M, J, t)$ one would be able to compute, both, the modification of that distribution due to the novel friction hypothesis, as well as (most importantly for this work) its exact contribution to the evolution of the dark energy density during late cosmological dynamics. A project is underway to try to construct reasonable models for $f(M, J, t)$ taking into account hypothetical primordial distribution, simplified models of merger histories, and direct bounds on abundances of black holes at certain mass scales.

Unfortunately that task will not be completed in the very near future to the level where the form of $f$ will be narrow enough so that we could think of simulating the effects with a single $f(M, J, t)$. That means that even in the best of circumstances we will have a multi-parametric characterization of the possible $f(M, J, t)$’s and the studies of their cosmological effects will have to be performed in conjunction with data analysis of many other sources such as the one we will contemplate here.

In view of this, one can consider rather simple models to describe the evolution of the cosmological constant over time \cite{8}. As this study is motivated by the so called “$H_0$-tension” we focus attention on the result of the effect we have described that might have taken place from the time of decoupling until today. We therefore study the overall feasibility of the model in the light of the available cosmological data.

\hspace{1cm} 

\(^6\) These are terms that might appear in the effective matter Lagrangian, which have the potential to generate violations of Lorentz Invariance that are large enough to enter in conflict with empirical bounds.
Some preliminary results were obtained by setting the current value of Hubble parameter, $H_0$, to the SnIa estimation \cite{10} and considering for the range of values of the model parameters that would be roughly compatible with the observational value of $\theta$, the angular acoustic scale of the CMB. This gave encouraging results in the light of the $H_0$ tension, as the fitting of the models parameters did not require a large change in either, the age of the universe, or the present amount of matter (both baryonic and dark components). We note that this preliminary analysis only studied global geometric quantities like $\theta$, however a change in the evolution of cosmic history introduced by this kind of models would also affects the growth of perturbations that are the seeds of the present cosmic structure. In order to include consideration of such aspects, a more complete analysis, like the one presented in the present paper, that also includes the prediction of the anisotropies and polarization of the CMB, as well as geometric quantities such as the luminosity or angular diameter distance, is necessary.

In this work, we consider a some relatively richer model which will be carefully described in section \ref{III}. The friction-like effect will be characterized by three parameters: two of them are used to describe the period in which the effect of ‘dissipation of black hole rotational energy’ takes place, and the last one characterizes the overall magnitude of the corresponding change in the cosmological constant. At the level of analysis that is possible, given our lack of knowledge regarding $f(M,J,t)$, it makes sense to simplify the treatment and model directly the time dependence of the effective energy transfer to dark energy to study its effects in the dynamics of the scale factor. These modifications of the background evolution are then inserted into the standard analysis of structure growth starting from the usual form of the quasi flat scale invariant primordial spectrum usually taken to emerge during the inflationary epoch.

This paper aims at dealing with two crucial aspects. First, the constraining of the parameters of the theory with the current data using a complete and appropriate statistical analysis. Next, the examination and interpretation of the results obtained in light of other relevant constraints. In particular one has the usual nucleo-synthesis constraint setting the overall contribution to the universe’s current energy budget appearing in the form of baryonic matter at 4-5%, and on the other the high precision data emerging from the CMB, providing bounds on the relative abundances of dark matter to baryonic matter at the last scattering surface. On the other hand, as the model calls for anomalous reduction of the corresponding energy densities at late times, we must ensure compatibility with the data from nearby galaxies and galaxy clusters constraining the present abundance of baryonic matter as well as bounds on non luminous matter (that might include some baryonic components). In other words, the proposed mechanism, if it is to be considered as viable, should not result in too large of a reduction in baryonic or dark matter components today so as to generate a conflict with the relevant observations. As we will see the best fits obtained in this work avoid that ‘danger’ by a rather large margin.

III. THE MATTER DENSITIES IN THE UNIMODULAR MODEL

Let us consider a homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) universe, and introduce our modifications at an effective level in the sector of both matter (including baryonic and dark matter) and dark energy. In the context of the UG theory, the modified conservation equation can be expressed as:

$$\dot{\rho}_M + 3\frac{\dot{a}}{a}\rho_M = -\dot{\rho}_\Lambda$$

(11)

Therefore, provided an expression for $\rho_\Lambda(t)$ we can integrate the latter equation from the beginning of the radiation era ($t_{rad}$) up to an arbitrary later time, $t$, to obtain an expression for $\rho_M(t)$. As discussed previously, even with a phenomenological model for the diffusive process in black holes such as that described in the Appendix A (and originally in\cite{7}), the form of the energy-momentum violation current cannot be well described given our present lack of knowledge of the black holes densities in the late universe. However, we can postulate a simple expression for $\rho_\Lambda$ in order to analyze the generic viability of a UG model with diffusion as follows

$$\rho_\Lambda(a) = \rho_\Lambda(t_{rad}) + f(a)\Delta \rho$$

(12)

At $t_{rad}$, the corresponding scale factor is referred as $a_{rad}$. Assuming $f(a_{rad}) = 0$ and $f(a_0) = 1$, the current (i.e today) value of the dark energy density reads:

$$\rho_\Lambda(a_0) = \rho_\Lambda(t_{rad}) + \Delta \rho = \rho_\Lambda^0$$

(13)

where $\rho_\Lambda^0$ is the current value of the dark energy density. The final expression for $\rho_\Lambda(a)$ yields

$$\rho_\Lambda(a) = \rho_\Lambda^0 + \Delta \rho [f(a) - 1]$$

(14)
For the sake of simplicity we assume $f(a)$ to be constant in two time intervals, while shows a linear behaviour in the interval of interest as follows:

$$f(a) = \begin{cases} 
0 & a \in (a_{rad}, a^* - \delta/2), \\
\frac{a-a^*+\delta/2}{\delta} & a \in (a^* - \delta/2, a^* + \delta/2) \\
1 & a \in (a^* + \delta/2, a_0),
\end{cases}$$

(15)

In this way $\rho_\Lambda$ is constant during the universe evolution, except for the time interval for which the value of the scale factor $a$ is $(a^* - \delta/2, a^* + \delta/2)$ where it changes linearly in $a$. Moreover, the value of the cosmological constant is not always the same, namely $\rho_\Lambda = \rho_\Lambda^0 - \Delta \rho$ when $a \in (a_{rad}, a^* - \delta/2)$ and $\rho_\Lambda = \rho_\Lambda^0$ when $a \in (a^* + \delta/2, a_0)$. We should consider of course $a_{rad} < a^* - \delta/2$ and $a^* + \delta/2 < a_0$. In this way, $\Delta \rho$ accounts for the change in the value of the cosmological constant with respect to its present value $\rho_\Lambda^0$, and $(a^* - \delta, a^* + \delta)$ refers to the time interval in which the energy-momentum tensor is not conserved. Consequently, we can obtain the expression for the total matter density $\rho_M(t)$ as a function of the scale factor integrating Eq. (11) and assuming Eqs. (14) and (15)

$$\rho_M(t) = \begin{cases} 
\frac{a_{rad}^3 \rho_M(t_{rad})}{a(t)^3} & a \in (a_{rad}, a^* - \delta/2), \\
\frac{a_{rad}^3 \rho_M(t_{rad})}{a(t)^3} - \frac{\Delta \rho}{4\delta} \left[a - \left(\frac{(a^* - \delta/2)^4}{a(t)^3}\right)\right] & a \in (a^* - \delta/2, a^* + \delta/2), \\
\frac{a_{rad}^3 \rho_M(t_{rad})}{a(t)^3} - \frac{\Delta \rho}{a(t)^3} \left[a^*^3 + \left(\frac{a^* \delta^2}{4}\right)\right] & a \in (a^* + \delta/2, a_0).
\end{cases}$$

(16)

We now proceed to analyse the observational predictions of this model and how the CMB spectra are modified with respect to the standard model as the $a^*, \delta, \Delta \rho$ parameters vary. For this, we recall that baryons and dark matter have different physical interactions with other particles during the formation of neutral hydrogen and the following universe evolution, and therefore in principle they should be described separately. Let us propose the following expression for the baryon, $\rho_B(t)$, and dark matter, $\rho_{DM}(t)$, densities which follow from Eq. (16)

$$\rho_B(a) = \frac{a_{rad}^3 \rho_B(t_{rad})}{a(t)^3} + a F(a, \Delta \rho, a^*, \delta)$$

(17)

$$\rho_{DM}(a) = \frac{a_{rad}^3 \rho_{DM}(t_{rad})}{a(t)^3} + (1 - a) F(a, \Delta \rho, a^*, \delta)$$

(18)

where

$$F(a, \Delta \rho, a^*, \delta) = \begin{cases} 
0 & a \in (a_{rad}, a^* - \delta/2), \\
-\frac{\Delta \rho}{4\delta} \left[a - \left(\frac{(a^* - \delta/2)^4}{a(t)^3}\right)\right] & a \in (a^* - \delta/2, a^* + \delta/2), \\
-\frac{\Delta \rho}{a(t)^3} \left[a^*^3 + \left(\frac{a^* \delta^2}{4}\right)\right] & a \in (a^* + \delta/2, a_0).
\end{cases}$$

(19)

In order to obtain expressions for $a_{rad}$, $\rho_B(t_{rad})$, $\rho_{DM}(t_{rad})$ in terms of the present values of the baryon density parameter $\Omega_B = \frac{\rho_B(t_0)}{\rho_{crit}}$ and the dark matter density parameter $\Omega_{DM} = \frac{\rho_{DM}(t_0)}{\rho_{crit}}$ where $\rho_{crit}$ is the present critical density, we first define the present fraction of baryon to dark matter density as follows

$$\beta = \frac{\rho_B(t_0)}{\rho_{DM}(t_0)}$$

(20)
Next, we assume that the fraction of baryon to dark matter at the beginning of the radiation era is equal to its present value\(^7\)

\[
\beta = \frac{\rho_B(t_0)}{\rho_{DM}(t_0)} = \frac{\rho_B(t_{\text{rad}})}{\rho_{DM}(t_{\text{rad}})}
\]  

(21)

With a bit of algebra we obtain from Eqs. 20 and 21 that

\[
\alpha = \frac{\beta}{1 + \beta} = \frac{\rho_B(t_0)}{\rho_B(t_0) + \rho_{DM}(t_0)} = \frac{\Omega_B}{\Omega_B + \Omega_{DM}}
\]  

(22)

It is important to note that \(\Omega_B\) and \(\Omega_{DM}\) are different from \(\Omega_B^\Lambda\) and \(\Omega_{DM}^\Lambda\) the current baryonic and dark matter densities in units of the critical density defined in the standard cosmological model.

Therefore, recalling that \(\Omega_B = \frac{\rho_B(a_0)}{\rho_{\text{crit}}}\) and \(\Omega_{DM} = \frac{\rho_{DM}(a_0)}{\rho_{\text{crit}}}\) we obtain the expressions for \(\rho_B(a)\) and \(\rho_{DM}(a)\) in terms of \(\Omega_B\) and \(\Omega_{DM}\)

\[
\rho_B(a) = \begin{cases} 
\frac{\Omega_B \rho_{\text{crit}} + \frac{\Omega_B}{\Omega_B + \Omega_{DM}} \Delta \rho (a^3 + \frac{a^* \delta^2}{4})}{a^3} & a \in (a_{\text{rad}}, a^* - \delta/2) \\
\frac{\Omega_B \rho_{\text{crit}} + \frac{\Omega_B}{\Omega_B + \Omega_{DM}} \Delta \rho (a^3 + \frac{a^* \delta^2}{4})}{a^3} - \frac{\Omega_B}{\Omega_B + \Omega_{DM}} \frac{\Delta \rho }{4\delta} \left[ a - \frac{(a^* - \delta/2)^4}{a^3} \right] & a \in (a^* - \delta/2, a^* + \delta/2), \\
\frac{\Omega_B \rho_{\text{crit}}}{a^3} & a \in (a^* + \delta/2, a_0) 
\end{cases}
\]  

(23)

\[
\rho_{DM}(a) = \begin{cases} 
\frac{\Omega_{DM} \rho_{\text{crit}} + \frac{\Omega_{DM}}{\Omega_B + \Omega_{DM}} \Delta \rho (a^3 + \frac{a^* \delta^2}{4})}{a^3} & a \in (a_{\text{rad}}, a^* - \delta/2) \\
\frac{\Omega_{DM} \rho_{\text{crit}} + \frac{\Omega_{DM}}{\Omega_B + \Omega_{DM}} \Delta \rho (a^3 + \frac{a^* \delta^2}{4})}{a^3} - \frac{\Omega_{DM}}{\Omega_B + \Omega_{DM}} \frac{\Delta \rho }{4\delta} \left[ a - \frac{(a^* - \delta/2)^4}{a^3} \right] & a \in (a^* - \delta/2, a^* + \delta/2), \\
\frac{\Omega_{DM} \rho_{\text{crit}}}{a^3} & a \in (a^* + \delta/2, a_0) 
\end{cases}
\]  

(24)

On the other hand, we point out that the baryon and dark matter fractions that must be considered for the nucleosynthesis predictions are

\[
\Omega_{B}^{\text{early}} = \Omega_B + \frac{\Omega_B}{\Omega_B + \Omega_{DM}} \frac{\Delta \rho }{\rho_{\text{crit}}} (a^3 + \frac{a^* \delta^2}{4})
\]  

(25)

\[
\Omega_{DM}^{\text{early}} = \Omega_{DM} + \frac{\Omega_{DM}}{\Omega_B + \Omega_{DM}} \frac{\Delta \rho }{\rho_{\text{crit}}} (a^3 + \frac{a^* \delta^2}{4})
\]  

(26)

\(^7\) As a rather straightforward motivation for this simplifying assumption we might adopt the view that most black holes in the universe (or at least those that play a substantial role in the process at hand) originate ultimately in a population of primordial black holes, which in turn formed well before the spatial distributions of dark and baryonic matter differed substantially. Thus the great majority of matter in the form of black holes can be viewed as traceable to such components which would therefore naturally contribute in proportion to their cosmic abundances. The result is that, when energy in black holes is lost as a result of our hypothetical aforementioned “effective friction”, the proportion of baryonic and dark matter would remain essentially unchanged in the whole process.
Note that, except for the interval \((a^* - \frac{\delta}{2}, a^* + \frac{\delta}{2})\), the baryon and dark matter densities behave in the same way as the ones of the \(\Lambda\)CDM model. Moreover, when \(a < a^* - \frac{\delta}{2}\), the density parameters are equal to \(\Omega^\text{early}_B\) for baryons and \(\Omega^\text{early}_{DM}\) for dark matter, while for \(a > a^* + \frac{\delta}{2}\), \(\Omega_B\) and \(\Omega_{DM}\) are the ones defined above, and correspond to the “late time values”. On the other hand, the evolution of the baryonic and dark matter densities during the interval \((a^* - \frac{\delta}{2}, a^* + \frac{\delta}{2})\) is different from the usual \(\Lambda\)CDM ones as can be inferred from Eqs. 23 and 24, which reflect what should be expected for the time interval during which the energy-momentum conservation is violated.

Let us stress, that we are considering the simplifying assumption that both components of matter (baryonic and dark) are at the end equally affected by the energy diffusion under consideration. The expressions we have obtained for the evolution of the baryon and dark matter and dark energy densities in our unimodular model with diffusion will result in a modification of the the Friedmann equation as follows:

\[
H^2(a) = \frac{8\pi G}{3} \left[ \rho_R(a) + \rho_B(a) + \rho_{DM}(a) + \rho_{\Lambda}(a) \right] - \frac{k}{a^2}
\]

where \(\rho_B(a), \rho_{DM}(a)\) and \(\rho_{\Lambda}(a)\) are described by Eqs. 23, 24 and 12 respectively and \(\rho_R(a)\) is the radiation density which is not modified in our model. The change in the Hubble factor results in changes in other relevant physical quantities for CMB physics such as \(z_{eq}\), i.e. the redshift at which the density of matter and radiation are equal, \(z_{\text{LS}}\), that is the redshift of last scattering, \(\theta(z_{\text{LS}})\), that is the angular diameter distance at last scattering and \(l_D\), the diffusion damping length. Also, for baryon acoustic oscillations (BAO) physics we have to consider the modification in \(z_{\text{drag}}\), i.e the redshift at which baryons decouple from photons. In turn, \(z_{\text{drag}}\) and \(l_D\) are not only affected by the modification in the Hubble factor, but also depend on the speed of sound, which in turn depends on \(\rho_B\). However, we have checked that the variations of the latter quantities are of order 0.07 \% for \(\frac{\Delta \rho_{\text{crit}} h^2}{\rho_{\text{crit}} h^2} = 0.002\) and therefore the most important effects introduced by our model is the change in the Friedmann equation.

A. Considerations for the CMB anisotropy and polarization spectra

We implement the model discussed in the previous section in the public Code for Anisotropies in the Microwave Background (CAMB) 29, changing the expressions of \(\rho_B, \rho_{DM}\) and \(\rho_{\Lambda}\) to the ones described in Eqs. 12, 15, 23 and 24 both in the background evolution and in the calculation of the growth of perturbations. Next, in order to compare the behaviour of the class of models analyzed in this paper, we assume as a reference a fiducial model, namely a \(\Lambda\)CDM one, with the cosmological parameters fixed to the bestfit values of Planck Collaboration (2018) 9. Also, we define

\[
\Delta \rho_{\Lambda} = \frac{8\pi G}{3} \frac{\Delta \rho}{100^2} = \frac{\Delta \rho h^2}{\rho_{\text{crit}}}
\]

that we use instead of \(\Delta \rho\) to analyze the effects of assuming the unimodular models in the CMB anisotropy and polarization spectrum 9.

In Fig. 1 we show the unimodular behaviour as the parameters \(a^*, \delta, \Delta \rho_{\Lambda}\) vary, exploring the impact of one parameter at a time and leaving the other two fixed (indicated at the top of each column), while for the cosmological parameters we set the \(\Lambda\)CDM best-fit model reported by the Planck Collaboration (2018) 9. Our choice of the cosmological parameters for the fiducial and unimodular model results in that the physics of the late universe (\(a > a^* + \delta/2\)) is the same in both models (see Eqs. 23, 24, 12 and 15) 10.

We note that increasing \(\Delta \rho_{\Lambda}\) results in a decrease in the height of the Doppler peaks and a corresponding increase in the valleys. Also, we note that this produces in addition, a very small shift in the locations of the peaks. These effects are very similar to the ones that occur if the value of \(\Omega_{DM} h^2\) is changed, so we can anticipate a degeneration

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8 Since there are far more photons than baryons, after photon decoupling the photons continued to drag baryons with them slightly longer into the Compton drag epoch. The redshift at which the baryon velocity decouples from the photons is called \(z_{\text{drag}}\).

9 In this way, the difference between the values of the cosmological constant at early and late times can be expressed as a density parameter, in a similar way to the baryon and dark matter density parameters.

10 Here we mean that both the behaviour of the energy densities and the corresponding energy parameters are the same in this period (\(a > a^* + \delta/2\)). Note that as a result the unimodular model will not necessarily fit BBN constraint, but the point of this section is to study the behaviour of the CMB spectra as the unimodular parameters are changed in order to understand the results of the statistical analysis of the next section.
between $\Omega_{DM} h^2$ and $\Delta \rho_\Lambda$, in the statistical analysis and parameter constraints. Moreover, it is useful to recall that a decrease in the value of $\Omega_B$ has the effect of decreasing the height of the odd peaks and enhancing the height of the even peaks. The latter is relevant, because we also note that the decrease (due to the increase in $\Delta \rho_\Lambda$) in the Doppler peaks is larger in the even than in the odd ones. The reason for this, is that modifying $\Delta \rho$ affects both $\rho_{DM}(t)$ and $\rho_B(t)$ in a proportional way and therefore the effect that is shown in Fig. 1 is a combination of the change in both densities through the variation in $\Delta \rho$. Therefore, we also expect a degeneration between $\Omega_B$ and $\Delta \rho_\Lambda$, in the statistical analysis. We further note that, as $a^*$ moves away from 1 in the model, the stage of the universe during which its physical parameters are different from the fiducial model is enlarged, resulting in a decrease in the peaks and valleys of the spectrum. Moreover, a change in $\delta$ affects only the lower multipoles of the CMB spectra which is usually attributed to the Integrated Sachs-Wolff (ISW) effect if a standard model of inflation is assumed. Moreover, it is well known that the ISW effect scales with the amount of dark energy [30]. We recall that the dark energy in our model behaves as $\frac{1}{3} \delta$ (see Eqs. 14 and 15) and this explains why for greater values of $\delta$, the departure from the standard behaviour is less than for lower values. Likewise, the low multipoles are also modified by a change in $\Delta \rho_\Lambda$ and $a^*$, and this can be explained since an change in these parameters also affects the amount of dark energy. Finally, due to the lower sensitivity of the spectra to changes in $a^*$ and $\delta$, we expect the degeneration of these parameters with the usual cosmological ones to be small or almost negligible.
IV. METHODOLOGY OF THE ANALYSIS

The observational predictions of the previous section are now compared with cosmological data, so as to obtain the constraints of the free parameters of the unimodular model. To do this, we use a Monte Carlo Markov chain exploration of the parameters space using the available package CosmoMC [31]. We consider an extended dataset comprising Cosmic Microwave Background measurements, through the Planck (2018) likelihoods [32], the CMB lensing reconstruction power spectrum [32, 33], the Baryon Acoustic Oscillation measurements from 6dFGS [34], SDSS-MGS [35], and BOSS DR12 [36] surveys, the type Ia SNe Pantheon compilation [37] and cosmic chronometers measurements of the expansion rate $H(z)$ from the relative ages of passively evolving galaxies [38–43]. We also consider a Gaussian prior for the SNe Ia absolute magnitude $M$, in order to consider the calibration given by the current $H_0$ local measurements [44], $(-19.2435 \pm 0.0373)$ mag, as suggested in [45]. This approach prevents double counting of low redshift supernovae and avoids to assume a value of the deceleration parameter, considering $M$ constrained by the local calibration of SNeI, which is not included otherwise. Let us stress that several approaches are been considered in the literature to address the so called ‘$H_0$ tension’. The initial approaches started by fixing the value of $H_0$, or imposing a prior on it in the statistical analysis [46, 47], but the method proved to be statistically inadequate [48, 51] as it attempts to combine two data sets that are intrinsically incompatible when considered in the context of the ΛCDM model, i.e. CMB and SNeIa, and limits the parametric space of the analysis in regions not determined by the

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11 We use Plik likelihood “TT,TE,EE+lowE” by combination of temperature TT, polarization EE and their cross-correlation TE power spectra over the range $\ell \in [30, 2508]$, the low-$\ell$ temperature Commander likelihood, and the low-$\ell$ SimAll EE likelihood.
CMB data alone. Thus, it has been shown that it is statistically more acceptable to consider a prior on the supernovae calibration parameter instead [45]. Of course, it is crucial to test if this incompatibility of supernovae and CMB data sets still holds in the context of the unimodular model and this will be analyzed in the Results section.

In our analysis, we vary the usual cosmological parameters, namely, the physical baryon density, $\Omega_B h^2$, the physical cold dark matter density, $\Omega_{DM} h^2$, the ratio between the sound horizon and the angular diameter distance at decoupling, $\theta$, the optical depth $\tau$, the primordial amplitude, $A_s$, and the spectral index $n_s$. We also vary the unimodular model parameters $\Delta \rho_\Lambda$, $a^*$ and $\delta$ and the nuisance foreground parameters [52]. We assume large flat priors for the free parameters, varying the unimodular ones between $\Delta \rho_\Lambda \in [-0.2; 0.2]$ , $a^* \in [0.02; 1]$ and $\delta \in [0.02; 1]$. We consider purely adiabatic initial conditions. The sum of neutrino masses is fixed to 0.06 eV, and we limit the analysis to scalar perturbations with $k^* = 0.05$ Mpc.

We choose to perform two statistical analyses that we describe as follows: i) the main analysis, which we simply call “Unimodular parameters free”, in which we let both the cosmological and the parameters of the unimodular model free to vary , ii) a speculative analysis, which we refer to as “Unimodular parameters fixed” where we fix the value of the parameters of the unimodular model to arbitrary values and let the other cosmological parameters vary. This second choice is due to the fact that, as mentioned in the Section III, a degeneration between $\Delta \rho_\Lambda$ and the cosmological parameters $\Omega_{DM}$, $\Omega_B$ and $H_0$ is expected and will be shown next in the results section. Such degeneration can allow this model to predict higher $H_0$ values, and here we want to show the cost of this achievement and the ability of this model to alleviate the Hubble tension whether future data may show better sensitivity to the constraint of its parameters.

FIG. 3: Constraints on model densities parameters ($\Omega_i h^2$) considering the case of cosmological and unimodular theory parameters free to vary (gray line), setting theory unimodular parameters to arbitrary values ($\Delta \rho_\Lambda = 0.09$, $a^* = 0.04$, $\delta = 0.02$) while the cosmological parameters are free to vary (red line), compared with the standard cosmological model (blue line).
We present the results our statistical analyses in Tab. I and Figs. 2, 3 and 4. Note that we also included the unimodular parameters free to vary, we found an agreement within $1\sigma$ than the formation of neutral hydrogen ($\text{H}_2$) can be established. Here we stress that the data clearly indicate that the anomalous behaviour in the energy densities must take place much later than the modifications to the usual theory analyzed here.

V. RESULTS

The $H_0$ value of this analyses is compatible with that of the $\Lambda$CDM model cosmology, although it should be noted that slightly higher values of $H_0$ are allowed. As expected (see discussion in Section III A), $\Delta \rho_{\Lambda}$ shows degeneracy with the matter densities values and a weak correlation with $H_0$, as shown in the right panel of Fig. 4. On the other hand, the $a^*$ and $\delta$ parameters show no degeneracy with the cosmological parameters, which can be explained by the low sensitivity of the CMB spectrum on these parameters (see discussion in section III A). On the other hand, we recall that in our model, the cosmological constant takes two different fixed values over different epochs of standard cosmological evolution (see equations 14 and 15). Our results for $\Delta \rho_{\Lambda}$ show that the difference between them is small, namely of order 0.33%. Finally, it is worth mentioning that the $\chi^2_{\text{min}}$ of this model is comparable to that of the standard model, but has three more parameters in the theory. On the other hand, one of the parameters of the unimodular model is not well constrained and this deserves further investigation with next generation of data that may be more sensitive to the modifications to the usual theory analyzed here.

Given the difficulty in constraining the $a^*$ parameter, and the apparent lack of degeneracy of $\delta$ with cosmological parameters, we now focus on a model where both $a^*$ and $\delta$ parameter are fixed at the smallest values allowed by our first analysis, 0.4 and 0.02 respectively. Besides, the value of $\Delta \rho_{\Lambda}$ is set at a large positive arbitrary value which lies...
We choose to use the best fit as estimator, so that the DIC can be rewritten as

\[ DIC = -2\ln L(\theta) + 2p_D, \]

where the first term is the posterior mean of \( L(\theta) \), i.e., the likelihood of the data given the model parameters \( \theta \), and the second term is the Bayesian complexity \( p_D = -2\ln L(\tilde{\theta}) + 2\ln \tilde{L}(\tilde{\theta}) \) where the tilde refers to the chosen estimator.\(^\text{12}\) We considered following \([60]\) the following scale to determine the performance of our model in that test: \( \Delta DIC = 10/5/1 \) as indicating, respectively, strong/moderate/null preference for the reference model (if the value of \( \Delta DIC \) is negative, it would indicate a preference for the model under consideration). As we obtained \( \Delta DIC = 3.6 \), we conclude that there is no evidence, according to this criteria, to support the analysed UG model over the standard \( \Lambda CDM \) one. This indicates that the current data are not sensitive enough to detect the full complexity of the model. However, we stress that even using this strict measure the model is not discarded in comparison to \( \Lambda CDM \).

VI. SUMMARY AND CONCLUSIONS

In this work, we perform a methodologically proper analysis on a general idea involving late time violation of energy-momentum conservation, resulting from a kind of granularity of space-time whose ultimate origin lies in quantum gravitational features. For this, it is necessary to formulate the cosmological model in the context of Unimodular Gravity, a modification of GR that allows such violations (under certain conditions that hold automatically in cosmology). This idea was applied to the very early universe to offer an account of the nature and magnitude of the cosmological constant, a fact that serves as a strong motivation to seek a resolution of the Hubble tension and therefore, the models that describe the density of black holes, which are, according to our model, the scenario where the violation of the conservation of the tensor-energy moment occurs and therefore, gives rise to the distinctive behavior of the model. Our results show that higher values of \( \Delta \rho \) shift \( H_0 \) to values that further alleviate the Hubble tension and therefore, the models that describe the energy diffusion in black holes (which are at present under construction) should, in order to fully solve the issue, point toward these values. We recall that in the present work, we assumed a simple model for the behaviour of \( \rho_\Lambda \), which is supposed to arise from an energy diffusion from matter (both dark and baryonic) to dark energy catalyzed by black holes. Likewise the predicted values of \( n_s \) are also not in agreement with those of the \( \Lambda CDM \) model, but still consistent with the predictions of standard inflationary models. About the constraints for \( \Omega_b h^2 \) and \( \Omega_{DM} h^2 \), there is also not agreement within 2\( \sigma \) with the \( \Lambda CDM \) model. However, as discussed in section \([11]\) the behavior of these quantities is not the same in both models and therefore, we do not expect to obtain the same estimation. Indeed, for the model to be viable it is necessary that the predicted values of the baryon density in the early universe \( \Omega_b \) are consistent with the nucleosynthesis constraint \( \Omega_b h^2 = (0.021, 0.024) \) and this is verified by the results shown in Tab.\([11]\).

Finally, we use the Deviance Information Criterion (DIC)\([53]\) in order to test whether the complexity of UG model is statistically supported by the data with respect to the vanilla \( \Lambda CDM \) one. The DIC has been proven to be a useful tool to test the average performance of a model with a penalty given by the Bayesian complexity \([54–56]\) (see \([57–59]\) and references inside for some applications of DIC in cosmology). The DIC value is defined, for the selected model \( M \), as:

\[ DIC_M = -2\ln \mathcal{L}(\theta) + 2p_D, \]

12 We choose to use the best fit as estimator, so that the DIC can be rewritten as \( DIC_M = 2\ln \mathcal{L}(\theta) - 4\ln \mathcal{L}(\tilde{\theta}) \). We obtain the mean likelihood from the output chains of the MCMC analysis, and the best fit likelihood via the BOBYQA algorithm implemented in CosmoMC for likelihood maximisation.
to the best of the current ability, the free parameters of the theory. In addition, we note the sensitivity limits of the data on constraining model complexity and discuss in depth the degeneracies between parameters. In summary, we are only able to constrain one unimodular parameter, $\Delta \rho_A$, while we can at most find upper/lower bounds for the other two, $a^*$ and $\delta$. We do not notice significant changes on the constraint of cosmological parameters compared to the standard cosmological model.

It is however a noteworthy fact, that our results show that the model does not spontaneously reduce the tension on $H_0$, producing only a small shift in the value of the parameter. However, it must be emphasised that the potential of the model is not fully expressed as the data proved unable to full constrain the unimodular parameters. In fact, looking at the results of our speculative analysis (in which we fix the values of the three unimodular parameters at arbitrarily chosen values) we see the $H_0$ tension being relaxed, resulting in the value of $H_0 = 71.6$ Km/s/Mpc at 1$\sigma$. This implies very small modification of most other cosmological parameters and without leading to an a problematic depletion of the dark matter and baryonic components (dark and bright) which are of course required at late times to account for the present features of galaxies and galactic clusters. More specifically the present day energy budget resulting from the model is the following ranges (the following values are at 68% confidence level): $\Omega_B = (0.0412, 0.0543)$, $\Omega_{DM} = (0.2438, 0.2594)$, $\Omega_A = (0.69, 0.71)$ when all parameters are free to vary and $\Omega_B = (0.0453, 0.0458)$, $\Omega_{DM} = (0.2429, 0.2467)$, $\Omega_A = (0.73, 0.74)$ when the unimodular parameters are fixed which can be compared with the standard $\Lambda$CDM values given by $\Omega_B = (0.0505, 0.0574)$, $\Omega_{DM} = (0.2513, 0.2613)$, $\Omega_A = (0.69, 0.70)$.

In our view this clearly illustrates the potential of the proposal to fully resolve the $H_0$ tension, once a more clear picture of the detail form of the function $f(M, J, t)$ in known (or realistically characterized in terms of a suitable set of parameters) allowing a repetition on the analysis carried out in this work with better modeling of the relevant black hole abundances.

Finally, since the predictions of the unimodular model for the B modes are different from the ones of the standard model, future CMB polarization data should provide more strict constraints on the unimodular model.

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Appendix A: Effects of the friction driven force in Black Holes

In this appendix we discuss the effects of the force driven by the friction that arises when the effects of granularity of space-times are considered in black holes. In this regard, general considerations led us to postulate a rotational friction term of the form:

\[ u^\nu \nabla_\mu s^\nu = \frac{\alpha_{bh}}{m_p} \frac{M}{M_{\odot}} \text{sign}(s \cdot \xi) \tilde{R} (s \cdot s) u^\nu - \frac{\beta_{bh}}{m_p^2} \tilde{R}_{BH} s^\nu, \]  

(A1)
where $M$ is the black hole’s mass, $\sigma_{bh}$ is connected to the translational friction term analogous to the one affecting particles (as in eq. 1) and the second term takes into account that, in contrast to elementary particles, the spin of the black hole might change not only in orientation but also in magnitude. Thus $\beta_{bh}$ is a new dimensionless parameter characterizing the “intrinsic” spin diffusion term. In [7] two options for the natural order of magnitude of these effective parameters were considered, involving a “high” and “low” suppression levels. In order to assess the magnitude of the effect under consideration, we compare the order of magnitude or the resulting anomalous force, which we denote by $F$, with that of the standard gravitational force between two black holes at the moment of closest approach in a black hole coalescence event which we denote by $F_{BHC}$. The estimate of $F_{BHC}$ is based on the consideration of two equal mass BH’s and the use of the Newtonian expression, evaluated when the separation between both is of the order of the Schwarzschild radius, and the anomalous force is estimated for the case the two black holes are extremal so $s = (M/m_p)^2$ (the magnitude of the black hole’s spin measured in natural units). The order of magnitude estimate for such quantity turns out to be gives $|F_{BHC}| \sim GM^2/(2GM) = m_p^2/4$. It is worth emphasizing that while $F_{BHC}$ is taken to represent the standard gravitational interaction, the hypothetical anomalous force $F$ we are considering would be also, ultimately, of gravitational origin, although, in this case, tied intrinsically with the granular aspects, of space-time which we take to characterize its underlying fundamental quantum gravity origin. As an analogy one might consider comparing an electromagnetic force of between two magnets with the friction force that affects their motion on a surface, which is, of course, also of electromagnetic origin. The results are the following [7]: in the case involving a high suppression level

$$\sigma_{bh} = \sigma_{bh}^{01} \frac{m_p}{M} \implies \frac{|F|}{F_{BHC}} \leq 4\sigma_{bh}^{01} \frac{10^{-6} \left(\frac{M}{M_{\odot}}\right)^3}{M_{\odot}}.$$ (A2)

and for the second possibility involving a low level of suppression

$$\sigma_{bh} = \sigma_{bh}^{02} \frac{m_p}{M} \implies \frac{|F|}{F_{BHC}} \leq 4\sigma_{bh}^{02} \frac{10^{13} \left(\frac{M}{M_{\odot}}\right)^2}{M_{\odot}}.$$ (A3)

These two possibilities might be seen as derived from a $1/\sqrt{N}$ suppression of some stochastic origin with the number of ‘area quanta’ $N \approx M^2/m_p^2$ or the number of ‘energy-quanta’ $N \approx M/m_p$ involved respectively.

The quantity $\bar{R}_{BH}$ represents an appropriate measure of the mean local curvature in the surroundings of the black hole. Such quantity could be for instance something like an “averaged value” of the local Kretschman scalar $\sqrt{R_{abcd}R^{abcd}}$, in the region occupied by the black hole. For simplicity in [8] we explored the simple case in which that quantity is taken to be a simple estimate of curvature $\bar{R}_{BH} = 1/M^2$. Once this choice is made, and taking natural order of magnitude estimates for the parameters $\sigma_{bh}$ and $\beta_{bh}$, the reminder of the analysis would in principle be quite direct but heavily dependent on aspects of astrophysics and cosmology on which our knowledge is still quite incipient.