Quantum-enhanced radiometry via approximate quantum error correction

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By exploiting the exotic quantum states of a probe, it is possible to realize efficient sensors that are attractive for practical metrology applications and fundamental studies [1–5]. Similar to other quantum technologies, quantum sensing is suffering from noises and thus the experimental developments are hindered [6–8]. Although theoretical schemes based on quantum error correction (QEC) have been proposed to combat noises [9–15], their demonstrations are prevented by the stringent experimental requirements, such as perfect quantum operations and the orthogonal condition between the sensing interaction Hamiltonian and the noise Lindbladians [15]. Here, we report an experimental demonstration of a quantum enhancement in sensing with a bosonic probe with different encodings, by exploring the large Hilbert space of the bosonic mode and developing both the approximate QEC and the quantum jump tracking approaches. In a practical radiometry scenario, we attain a 5.3 dB enhancement of sensitivity, which reaches $9.1 \times 10^{-4} \text{Hz}^{-1/2}$ when measuring the excitation population of a receiver mode. Our results demonstrate the potential of quantum sensing with near-term quantum technologies, not only shedding new light on the quantum advantage of sensing by revealing its difference from other quantum applications, but also stimulating further efforts on bosonic quantum technologies.

The large Hilbert space of a quantum system and the quantum superposition principle offer the potential advantages of quantum physics in many applications and laid the foundation of quantum information science [16]. Recently, supported by the established quantum state engineering and control techniques, quantum sensing emerges as one of the most promising near-term applications to achieve quantum advantage and has attracted tremendous attentions [1–5]. For a sensor interrogation Hamiltonian $H_{\text{int}}$, the intriguing Greenberger–Horne–Zeilinger entanglement states of a collection of spins [17] or the Schröinger-cat-like states in a large Hilbert space [18–21] could enhance the sensitivity of a quantum probe, because they provide a large variance of energy $\langle H_{\text{int}}^2 \rangle - \langle H_{\text{int}} \rangle^2 \propto N^2$ with $N$ being the number of excitations. However, these exotic quantum states are also more prone to environmental noises, and thus the coherence times are reduced and the ultimate sensing sensitivity is hard to be enhanced [6].

Fortunately, the large Hilbert space also offers the redundancy for realizing quantum error correction (QEC) that protects quantum states from decoherences and imperfections. It has been expected that the Heisenberg limit, i.e. the sensitivity of the sensing scales with the measurement time ($T$) and excitation number as $\propto (NT)^{-1}$, could be achieved by protecting the exotic quantum states via QEC [9–15], in sharp contrast to the standard quantum limit $\propto (NT)^{-1/2}$. However, these QEC-based quantum sensing schemes demand stringent conditions in experiments. On one hand, the non-local QEC operations are challenging for multi-qubit systems [22–24]. Although a pioneering experiment proves the principle of QEC-enhanced sensing by prolonging the coherence time of a single electron spin [25], the extension to a larger Hilbert space is in absence. On the other hand, the theoretical assumptions of perfect ancilla or error-free quantum operations are impractical, and the noises of experimental systems could not meet the orthogonality requirement in general [15]. The unraveled fact that the Heisenberg limit could not be practically attainable [26, 27] discourages further experimental exploration of quantum-enhanced sensing via QEC.

In this work, we demonstrate the enhancement of the sensing sensitivity by approximate QEC with a bosonic probe. Instead of pursuing the Heisenberg limit, our quantum sensing is implemented with optimized experimental strategies in a hardware-efficient superconducting architecture. By using non-exact QEC codes based on two-component Fock states for carrying the coherence, the sensing information could be protected by QEC, and the imperfection due to decoherence could even be further suppressed by mitigating the ambiguity of the quantum evolution trajectories. Benefiting from both the enhanced sensing–information–gain rate and prolonged coherence time of the exotic quantum states, the bosonic probe is applied for practical radiometry and achieves a detection limit of the receiver excitation population of $9.1 \times 10^{-4} \text{Hz}^{-1/2}$, which shows a 5.3 dB enhancement of the sensitivity by QEC compared to the encoding with the two lowest Fock states. Our work develops practical quantum sensing technologies, proves the quantum enhancement, and could also stimulate further experimental efforts in this direction.

Figure 1 illustrates the principle of the quantum-enhanced sensing. When a single two-level system (TLS) is used to probe an external field through coherent coupling, the quantum state in the two-dimensional Hilbert space would acquire a phase proportional to the sensing duration $t_{\text{int}}$. By contrast, when extending the probe to a higher dimension, it is possible to find a subspace in which the phase accumulation rate is higher, while the rotation angle can be preserved even though the states are mapped to disjoint error subspaces by noises. Therefore, the sensitivity could be enhanced by both the larger rate and the QEC protection. A single bosonic mode provides an excellent probe for realizing such an idea in practice because of its large Hilbert space dimension and hardware-efficient quantum control capability [20, 21, 28]. In general, a
The principle of quantum sensing with approximate quantum error correction (QEC). The errors \( \{E_1, E_2, \ldots \} \) map the quantum states in the code space to disjoint subspaces and the recovery operations \( \{R_1, R_2, \ldots \} \) convert the states back to the code space with the acquired phase being preserved. b. One example of practical quantum sensing protocol with a bosonic probe. The Wigner functions illustrate the evolution of the probe quantum state: When the probe state is initialized to \( \langle 1 \rangle \) or \( \langle 3 \rangle \), the phase can be preserved even if there is a single-photon-loss error \( \alpha \), which could be tracked and corrected via the recovery operation \( R_1 \). c. Schematic of the experimental setup for the quantum radiometry implemented with a superconducting architecture. The device is constructed with a bosonic probe that couples to a receiver mode.

A bosonic probe could sense a physical quantity \( \omega \) through the interrogation Hamiltonian

\[
\hat{H}_{\text{int}}/\hbar = \omega \hat{a}^\dagger \hat{a},
\]

where \( \hat{a} \) denotes the annihilation operator of the probe and \( \hbar \) is the Planck constant.

However, exact QEC codes for sensing with such a model are excluded, since the photon loss errors due to damping are not orthogonal with \( \hat{H}_{\text{int}} \) [see Methods]. For the convenience of experiments, we propose the two-component Fock state subspace span \( \{ |m\rangle , |n\rangle \} \) (\( m < n \)) as the QEC code space for sensing. The evolution of the probe state after an interrogation time \( t_{\text{int}} \) becomes

\[
|\psi_{m,n}\rangle = \alpha_{m,n} |m\rangle + \beta_{m,n} e^{-i(n-m)\omega t_{\text{int}}} |\phi_0\rangle |n\rangle,
\]

where \( \alpha_{m,n}, \beta_{m,n} \in \mathbb{R} \) are the amplitudes and \( \phi_0 \) is the initial phase. It is easy to verify that for loss errors up to \( m \) photons, the acquired phase \( \phi = (n-m)\omega t_{\text{int}} \) is preserved with the phase accumulation rate \( (n-m) \omega \) being fixed irrespective to the time when the photon jump occurs. By repetitively implementing the recovery operations \( R_j = |m\rangle \langle m-j| + |n\rangle \langle n-j| + \hat{R}_j \) for the \( j \)-photon-loss error, where \( \hat{R}_j \) being a complementary operator to make \( R_j \) unitary, the phase coherence could be protected in the code space. Therefore, such a code in a high-dimensional Fock space could simultaneously enhance the phase accumulation rate by \( n-m \) times when compared to the encoding with the lowest two levels and prolong the coherence time via QEC. However, the coherence time could only be enhanced by a limited factor, since the code is an approximate QEC code and the amplitudes of the probe state \( \alpha_{n,m} \) and \( \beta_{n,m} \) change after each photon loss, as illustrated by the deformation of the error subspaces in Fig. 1a.

An example of this proposal is schematically shown in Fig. 1b with a probe state \( |\psi_{1,3}\rangle = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\pi/2} |3\rangle) \), i.e. \( m = 1 \) and \( n = 3 \). The single-photon-loss error maps the state to \( \frac{1}{\sqrt{2}} (|0\rangle + e^{i\pi/2} |2\rangle) \) and recovery \( R_1 \) converts the state back to the code space, while the Wigner functions of both states have the same rotation angle and symmetry. However, the phase information is completely corrupted for a two-photon-loss error since \( m < 2 \).

To demonstrate the efficacy of the approximate bosonic QEC scheme for sensing, we first experimentally characterize the performance of the scheme by measuring a virtual phase \( \phi_0 \) introduced in the initial probe state instead of an acquired phase \( \phi \). Our experimental device schematically shown in Fig. 1c consists of a superconducting transmon qubit as an ancilla dispersively coupled to two three-dimensional cavities [29–31]. The cavity (blue) with a long coherence time \( T_1 = 143 \mu s \) serves as the probe, while the ancilla and the other short-lived cavity (green) assist the manipulation and readout of the probe, respectively [see Methods for more details]. In the radiometry experiments studied later, the readout cavity serves as a receiver to collect microwave signals from outside, while the probe can sense the excitation in the receiver through the cross-Kerr interaction.

As shown by the experimental circuits in Figs. 2a and 2b, the ancilla assists the encoding and decoding of the probe by mapping the ground and the excited states \( \{|g\rangle, |e\rangle\} \) to the two Fock states \( \{|m\rangle, |n\rangle\} \). The sensing procedure is reminiscent of the Ramsey interferometer [32] with the output probability of \( |g\rangle \) as \( P_g = A + B \cos(\phi + \phi_0) \) manifesting the interference fringe, where \( A \) and \( B \) are the fitting parameters. Since the decay rate of Fock state \( |m\rangle \) is proportional to the photon number \( m \), we fix \( m = 1 \) and select \( n = 3, 5, 7 \) for a relatively small decay rate. The corresponding error set of the probe is \( \{E_0 = e^{-t_{\text{int}}a^\dagger a/2T_1}, E_1 = \sqrt{1 - e^{-t_{\text{int}}T_1}} e^{-t_{\text{int}}a^\dagger a/2T_1} a\} \) for zero- and single-photon-loss errors with an interrogation time \( t_{\text{int}} \), and we only tackle the dominant error \( E_1 \) by \( R_1 \). The QEC of the probe state is implemented through an autonomous manner, i.e. by applying the correcting pulse after an interrogation time \( t_{\text{int}} \) to act \( R_1 \) on the probe and flip the ancilla if \( E_1 \) occurred [see Methods]. Note that \( \alpha_{m,n}, \beta_{m,n} \) are optimized in order to maximize the visibility of the output fringes for each \( |\psi_{m,n}\rangle \) in the following experiments, therefore the best detection sensitivity can be achieved experimentally [see Methods].

Figures 2c and 2d compare the measured probability \( P_g \) against the virtual phase \( \phi_0 \) and the sensing duration \( t_{\text{int}} \) for the cases without and with the protection by QEC. The results
with QEC indeed show a much slower decaying of the fringe visibility. To evaluate the potentially achievable improvement of the sensing performance, we extract the Ramsey visibility against $\phi_0$ and derive the normalized quantum Fisher information $\mathcal{D}$ with respect to $t_{\text{int}}$, which is the total experimental time for a single-shot measurement including the initialization, encoding, interrogation, decoding, and readout, as well as the time needed for the QEC process and feedback when QEC is performed [see Methods]. $\mathcal{D}$ determines the best achievable sensitivity of $\omega$ in a unit time, i.e.

$$\sigma_\omega \leq 1/\sqrt{\mathcal{D}},$$

which is in the unit of Hz/$\sqrt{\text{Hz}}$ and corresponds to the noise floor of our sensor.

The results of $\mathcal{D}$ with and without QEC are summarized in Figs. 3b-d, and $\mathcal{D}$ of the simple TLS case with the probe being encoded in the lowest two levels is also provided as a reference. All curves of $\mathcal{D}$ first grow up with $t_{\text{int}}$, since a longer interrogation time gives a larger phase acquisition. However, due to more accumulated decoherence, $\mathcal{D}$ saturates at an optimal $t_{\text{opt}}$ and decreases for even longer $t_{\text{int}}$. Benefiting from the larger Hilbert space dimension, the best achievable $\mathcal{D}$ without QEC increases with $n$, while the optimal $t_{\text{opt}}$ decreases with $n$ due to the shorter Fock state lifetime. When performing QEC, $\mathcal{D}$ is clearly improved than that without QEC especially when $t_{\text{int}}$ exceeds the optimal value, confirming the enhanced coherence time by QEC. Comparing the results for different $n$, the improvement induced by QEC reduces with increasing $n$, due to the stronger uncorrectable noise effects for larger $n$ and also higher operation errors when performing QEC.

One main limitation on the performance of the approximation QEC would be the code state deformation, which causes decoherence even when the QEC is successfully implemented. For example, after one round of QEC, the amplitudes of $|\psi_{m,n}\rangle$ evolve as

$$\left\{\alpha_{m,n}, \beta_{m,n} e^{-(n-m)\tau_{\text{QJ}}}\right\}$$

$$\left\{\alpha_{m,n}, \sqrt{\frac{\tau_{\text{QJ}}}{m}} \beta_{m,n} e^{-(n-m)\tau_{\text{QJ}}}\right\}$$

(normalization factors are neglected) for $E_0$ and $E_1$ occurring, respectively, with the relative amplitude of the two Fock components being either amplified or suppressed. As illustrated in Fig. 3a, the final probe state $\rho_{m,n}^{(\text{tot})}$ becomes a mixed state composing of different possible quantum evolution trajectories ($|\psi_{m,n}^{(0)}\rangle, |\psi_{m,n}^{(1)}\rangle, ...$), although the phase is preserved irrespective of the total number of photon jumps during $t_{\text{int}}$. If we can distinguish the number of photon jumps that have occurred, the ambiguity of the possible trajectories could be mitigated, and thus $\mathcal{D}$ could be improved.

Therefore, we propose and demonstrate a quantum jump tracking (QJT) approach: record the output of each autonomous QEC, count the number of single-photon jumps ($j \in \{0, 1, ..., M\}$), and process the data according to $j$ [see Methods]. By doing so, $\mathcal{D}$ is further improved in all cases and even the optimal $t_{\text{opt}}$ is extended, as shown in Figs. 3b-d. These experimental results demonstrate the protection and recovery of $\mathcal{D}$ from decoherence by QEC and QJT, and indicate the benefits of approximate QEC in sensing.

Finally, the scheme is applied in a practical sensing scenario as a quantum radiometer. Based on the device shown in Fig. 1c and using the readout cavity as a receiver to the microwave field under detection, the excitation population of the readout cavity $p = \omega/\chi$ could be derived by the quantum sensor with a calibrated cross-Kerr coefficient $\chi$ between the probe and the receiver. Following the sequence shown in Fig. 4a, the resulting oscillations shift due to the acquired
achieved for $\sigma$ with $\Delta \sigma$ potentially achievable in Methods. Number (component Fock state $|\psi_{m,n}\rangle$), although it could be confined in the code space span $\{ |m\rangle, |n\rangle \}$ via QEC, the amplitudes vary depending on the number ($j$) of the single-photon-loss errors occurring, i.e. $|\psi_{m,n}\rangle \rightarrow |\psi_{m,n}^{(j)}\rangle$. The output would be a mixed state $\rho_{m,n}^{(\text{tot})}$, if different evolution trajectories could not be distinguished.

Quantitative performance of different quantum sensing strategies characterized by $\mathcal{D}$ for the probe states $|\psi_{1,3}\rangle$, $|\psi_{1,5}\rangle$, and $|\psi_{1,7}\rangle$, respectively. TLS: the two-level system encoding with the two lowest Fock states. QEC & No QEC: results with and without QEC, respectively. QEC+QJT: the strategy that combines QEC and QJT. The error bars are obtained through error propagation of the fit parameter uncertainties. Inset: Wigner function of the corresponding probe states, with the same axes and color scale bar as in Fig. 1b.

The Bloch-sphere illustration of the quantum jump tracking (QJT). For a two-component Fock state $|\psi_{m,n}\rangle$, although it could be confined in the code space span $\{ |m\rangle, |n\rangle \}$ via QEC, the amplitudes vary depending on the number ($j$) of the single-photon-loss errors occurring, i.e. $|\psi_{m,n}\rangle \rightarrow |\psi_{m,n}^{(j)}\rangle$. The output would be a mixed state $\rho_{m,n}^{(\text{tot})}$, if different evolution trajectories could not be distinguished.

Quantitative performance of different quantum sensing strategies characterized by $\mathcal{D}$ for the probe states $|\psi_{1,3}\rangle$, $|\psi_{1,5}\rangle$, and $|\psi_{1,7}\rangle$, respectively. TLS: the two-level system encoding with the two lowest Fock states. QEC & No QEC: results with and without QEC, respectively. QEC+QJT: the strategy that combines QEC and QJT. The error bars are obtained through error propagation of the fit parameter uncertainties. Inset: Wigner function of the corresponding probe states, with the same axes and color scale bar as in Fig. 1b.

A demonstration of quantum-enhanced sensing by a bosonic probe is performed with a superconducting circuit. Utilizing the large Hilbert space of the bosonic mode, we realize a radiometer that shows a quantum enhancement of 5.3 dB and opens the door to practical quantum sensing. The gain of quantum Fisher information by approximate QEC and QJT reveals the significant difference between the quantum sensing and other quantum information processing applications: the goal is to acquire the sensing information as much as possible instead of pursuing the perfect protection of an unknown quantum state. The extensions of the scheme to tens of photons by developing sophisticated quantum control method and optimal approximate QEC codes, as well as to multiple bosonic modes, are appealing and worth further investigations. The bosonic radiometry demonstrated here will also excite immediate interests for other quantum sensing applications, such as the force sensing [33], because the scheme is
applicable to all physical quantities that could induce a change of $\omega$. The bosonic probe having the advantages of hardware efficiency and avoiding the non-local interactions is extensible to other bosonic degrees of freedom including phonons coupled with trapped ions [20] and superconducting qubits [34], and also extensible to the collective excitations in spin and atom ensembles to promote the atomic and optical quantum metrology technologies [1–5].

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Methods
Experimental implementation. As described in the main text, our experiments are implemented with a superconducting quantum circuit and the device consists of a superconducting transmon qubit as an ancilla dispersively coupled to two superconducting rectangular microwave cavities, the probe and the receiver. The input and output couplings of the receiver cavity are designed to be asymmetric, $k_{\text{in}} \gg k_{\text{out}}$, offering a decay rate of $k_{\text{in}} = 1/44\,\text{ns}$, which is three orders of magnitude higher than that of the probe cavity. As a consequence, a static coherent state with a mean excitation number $p$ in the receiver cavity can be created within a negligible period of time. Such a design of coupling is fit for the high-fidelity readout of the ancilla. However, for practical applications the receiver mode could be over-coupled to the external fields that are to be detected and an extra readout resonator could be employed to perform the readout. Crucially, the cross-Kerr interaction induced by the nonlinearity of the transmon allows the detection of the excitation in the receiver by utilizing the bosonic quantum states in the probe cavity. The calibrated cross-Kerr coefficient is $\chi/2\pi = 15.3$ kHz.

Quantum Fisher Information (QFI). To characterize the sensitivity that can be achieved in the experiment, the measured data are fitted with sinusoidal curves. For the simplest case with binary outputs $\{ |g\rangle, |e\rangle \}$, the results obey $P_g(\omega) = A + B \cos(\omega(n-m)t_{\text{int}} + \phi_0)$ and $P_e = 1 - P_g$. The final readout outputs follow the binomial distribution, giving a variance of $(\Delta P_g)^2 = P_g(\omega) - P_g^2(\omega)$. Therefore, the attainable resolution (uncertainty $\Delta\omega$) for measuring $\phi$ by one round of the sensing experiment, including the initialization, encoding, interrogation, decoding, and readout, reads

$$\Delta\omega \leq \max_{\phi_0} \frac{\Delta P_g}{\partial P_g(\omega)/\partial \omega} = \frac{\sqrt{A(1-A)}}{(n-m)t_{\text{int}}B}.$$  

Alternatively, the precision could be studied in a more general frame of quantum metrology, where the QFI gain $\mathcal{F}$ could be deduced as

$$\mathcal{F} = \max_{\phi_0} \left\{ \frac{1}{P_g(\omega)} \left[ \frac{\partial P_g(\omega)}{\partial \omega} \right]^2 + \frac{1}{P_e(\omega)} \left[ \frac{\partial P_e(\omega)}{\partial \omega} \right]^2 \right\} \equiv \frac{(n-m)^2t_{\text{int}}^2B^2}{A(1-A)}.$$  

The corresponding achievable measurement uncertainty is

$$\Delta\omega \leq 1/\sqrt{\mathcal{F}}.$$  

Introducing the normalized QFI per unit time $\mathcal{D} = \mathcal{F}/T$, with $T$ being the experimental time, we then could derive the practical sensitivity of the experiment $1/\sqrt{\mathcal{D}}$ that is associated with the measurement bandwidth and corresponds to the noise floor of the sensing. For example, if the duration for a single-shot sensing experiment is $t_{\text{tot}}$, the QFI gain rate could be obtained as

$$\mathcal{D}_{\omega} = \frac{B^2(n-m)^2t_{\text{int}}^2}{A(1-A)t_{\text{tot}}},$$

corresponding to the achievable measurement sensitivity

$$\sigma_{\omega} \leq \frac{1}{\sqrt{2\mathcal{D}_{\omega}}}.$$  

Parameter optimization. For the probe quantum states, the damping channel could be represented by the Kraus operators as $\mathcal{D}(\rho) = \sum_k E_k \rho E_k^\dagger$, where $\rho$ is the density matrix of the probe and

$$E_k = \frac{(1 - e^{-\frac{t_{\text{int}}}{T}})^{k/2}}{\sqrt{k!}} e^{-\frac{t_{\text{int}}}{4}} a^k$$

is the operator for the $k$-photon-loss error during a sensing interrogation time of $t_{\text{int}}$. In the experiments, we only consider the two dominant errors $\{E_0, E_1\}$. Due to the Fock state damping $e^{-\frac{t_{\text{int}}}{4}} a^k$ and the photon-number-dependent photon jump rate, the amplitudes of the two-component Fock states experience unbalanced amplitude change. Therefore, we numerically optimize the coefficients $\alpha_{m,n}$ and $\beta_{m,n}$ of the initial probe quantum states, $\tau_{\text{int}}$, and the repetition number $M$ to maximize $\mathcal{D}$ by considering the full damping channel and the imperfections of QEC and other operations. We further optimize the interrogation time $\tau_{\text{int}}$ experimentally in order to acquire the maximum $\mathcal{D}$. For the purpose of achieving the maximum sensitivity of the radiometry, we also optimize the initial phase $\phi_0$ in the encoding step to maximize the slope $\partial P_g/\partial p$.

Autonomous implementation of QEC. The probe quantum states studied in this work are the approximate QEC codes, and thus the recovery $R_j = |m\rangle \langle m-j | + |n\rangle \langle n-j | + \tilde{R}_j$ for the $j$-photon-loss error are derived according to the transpose channel, which provides a universal approach for constructing the recovery operation with reasonable performance. Such recovery operations are implemented autonomously and repetitively during the sensing. This type of autonomous QEC can map the error state back into the code space span $\{|m\rangle, |n\rangle \}$ whenever the system jumps to an error state, by sending pulses to both the probe cavity and the ancilla qubit simultaneously without the requirement for error detection and real-time adaptive control. Such a protocol can avoid additional fast feedback electronics and also circumvent the delay in the electronics, and thus saves the hardware and suppresses potential decoherence due to the delay. After the autonomous implementation of QEC, the error entropy is transferred from the probe state to the ancilla. Once a single-photon-loss error ($E_1$) occurs, the probe state is recovered and the ancilla is flipped to the excited state $|e\rangle$. However, when there is no error, both the probe and the ancilla remain unaltered during the QEC operation. The subsequent measurement of the ancilla gives a result of $|e\rangle$ or $|g\rangle$ that can be recorded in real time, indicating a single-photon-loss error occurs or not. A conditional $\pi$-pulse is then applied to the ancilla to reset it to $|g\rangle$ for the next repetition of QEC. For more information about the autonomous QEC, see Refs. [35, 36]. The whole correction
process is repeated for \(M\) times followed by a decoding for the final readout to end the sensing experiment.

**Quantum jump tracking (QJT).** The ancilla output of \(|e\rangle\) after the QEC pulse indicates a single-photon-loss error occurs during the interrogation time. In the QJT experiments, the number of \(|e\rangle\) at the output of the QEC pulse is counted and recorded, which allows us to improve the sensitivity. In general, the decoding operation should be adaptively selected according to the number of the quantum jumps \((j)\). The optimal decoding scheme is to map the Fock states \(|m\rangle\) and \(|n\rangle\) to the ancilla state \(|\pm\rangle = |g\rangle \pm e^{\pm i\phi_1} |e\rangle\) for all cases, with \(\phi_1 \approx 0\) being the readout phase, therefore we only selectively process the data after the experiments. We divide the final measurement outputs into groups by \(j\). The conditional probability of output \(|g\rangle\), \(P_{g,j|M}(\omega)\), for \(j\) jumps, corresponding to the case with the single-photon-loss error occurring \(j\) times among \(M\) repetitions of sensing, can be fitted with \(P_{g,j|M}(\omega) = A_{g,j} + B_{g,j} \cos \left[\omega(n-m) t_{\text{int}} + \phi_{g,j}\right]\). Similarly, for output \(|e\rangle\), \(P_{e,j|M}(\omega)\) can be fitted with \(P_{e,j|M}(\omega) = A_{e,j} + B_{e,j} \cos \left[\omega(n-m) t_{\text{int}} + \phi_{e,j}\right]\). Here, \(\phi_{g(e),j}\) includes the initial and the readout phases. The resulting normalized QFI \(\mathcal{Q}\) with QJT can be calculated as

\[
\mathcal{Q}_\omega = \frac{1}{t_{\text{tot}}} \max_{\phi_j} \left\{ \sum_{j=0}^{M} \sum_{l \in \{g,e\}} \frac{1}{P_{l,j|M}(\omega)} \left[ \frac{\partial P_{l,j|M}(\omega)}{\partial \omega} \right]^2 \right\} 
\]

\[
= \frac{1}{t_{\text{tot}}} \sum_{j=0}^{M} \sum_{l \in \{g,e\}} \frac{B_{l,j}^2 (n-m)^2 t_{\text{int}}^2}{A_{l,j}}.
\]

The experimental \(\mathcal{Q}_\omega\) is calculated by the coefficients \(A_j\) and \(B_j\), which are obtained in the experiments of measuring the virtual phase by varying \(\phi_0\). The results indicate a sensitivity of measuring \(p\) as

\[
\sigma_p \leq \frac{1}{\mathcal{Q}_\omega} \frac{\partial \omega}{\partial p} = \frac{1}{\mathcal{Q}_\omega \chi}.
\]

For the direct implementation of the radiometry, the sensitivity of measuring \(p\) is provided as

\[
\sigma_p = \frac{1}{\sqrt{\mathcal{Q}_p}},
\]

with

\[
\mathcal{Q}_p = \frac{1}{t_{\text{tot}}} \sum_{j=0}^{M} \sum_{l \in \{g,e\}} \left[ \frac{\partial P_{l,j|M}}{\partial p} \right]^2,
\]

where the probabilities \(P_{l,j|M}\) and their slopes \(\frac{\partial P_{l,j|M}}{\partial p}\) are obtained directly from experiments with an optimized \(\phi_0\).