Calculation of the acoustic efficiency of smart resonant cells

P V Pisarev and K A Maksimova*
Perm National Research Polytechnic University, 614000 Komsomolsky Prospect, 29, Russia

*karina-maksimova0402@yandex.ru

Abstract. In this work, numerical studies of the acoustic efficiency of prismatic resonators with adaptive neck diameters were carried out. In the process of research, the physical and mathematical models for predicting effective acoustic characteristics of a group of sound-absorbing structures cells with different neck diameters were formulated. The concept of the resonant frequency adaptive control and impedance of smart SAS cells by means of piezoelectric elements embedded into the neck is proposed. Verification of the developed numerical models has been carried out. According to the results of the numerical calculations, the dependences of the natural frequencies and sound absorption coefficients of the group of adaptive SAS cells were obtained for different variants of control modes (combinations of resonator neck diameters).

1. Introduction
In the XXI century there is a rapid development of civil aviation and air transport market, which is a catalyst for the economic growth of cities and regions. At the same time, intensified the negative impact of aviation on the environment, especially on the aerodrome sites. Ecology of air transport was the second actual problem released by the International Civil Aviation Organization (ICAO), yielding first place only to flight safety [1]. One of the dominant components of the harmful effects of aviation on the environment is noise on the ground created by the aircraft. ICAO is constantly tightening international standards for aircraft noise in the area, normalizing its level at takeoff and landing, forcing aviation equipment manufacturers to develop new technologies to reduce aircraft noise[2]. One of the major components of aircraft noise is the noise generated by the propulsion - turbofan engines (turbojet).

One of the most effective ways to reduce the noise of aircraft propulsion systems is the inclusion in its structure of sound-absorbing structures (SAS) [3]. Representing a set of Helmholtz resonators formed in a honeycomb structure made of polymeric composites. Such traditional sound absorbing construction are installed on the inner surface of an aircraft engine air intake to reduce noise propagating in the forward hemisphere, and on the walls of the channel of the outer contour of the engine to reduce the noise propagating in the backward hemisphere [4].

To date developed a number of SAS of resonance type: one-, two- and three-layer honeycomb panels of SAS, allowing to provide noise reduction over a wide frequency range, thanks to the work of resonators tuned to different frequencies [1, 5]. Development of SAS performed on the basis of the solutions of direct and inverse problems of determination of the impedance of geometrical parameters of SAS that allows to define the desired geometric parameters of SAS for a given impedance [6, 7]. It should be noted that the approaches currently used in the design of SAS are generally based on semi-
empirical models that were obtained using major simplifications and take into account less factors or do not take into account many factors that affect the attenuation of sound. Experimental research of developed SAS is also limited to test of small samples [4]. As a result, during the creation of full-scale construction of SAS its acoustic characteristics, implemented in practice, differ significantly from those projected in the design. This requires a long and costly design rework already at the stage of implementation it into serial production.

The new requirements to reduce the noise of aircraft propulsion systems cause the formulation of new research tasks in the field of aeroacoustics to develop new methods to maximize the effectiveness of the SAS with a minimal increase in weight and cost of structures [8]. Traditional increasing attenuation coefficient due to the use of multilayer SAS, three- and four-layer leads to an increase in weight and costs and reduce structural rigidity. Existing methods and mathematical models used for the design of SAS, do not allow to improve their acoustic performance. In this regard, it requires the development of new approaches for the design of SAS.

It is of interest to identify new ways to control the impedance of cells and SAS, including the control of the shape and volume of the resonator chamber, structural design of the neck and perforations layout of resonators based on their non-local interactions [5].

In this work, the concept of adaptive control of the resonant frequency and impedance of the SAS cells by means of piezoelectric elements embedded into the neck is proposed. A numerical study of the acoustic efficiency of a group and single resonators with variable neck diameters has being carried out [9].

The presence of a piezoelectric deformable element 3 (Figure 1) in the neck of the resonant cell allows to increase the absorption coefficient in a wide frequency range. That is achieved through the implementation of a controlled resonant mode of the "external acoustic environment/cell" system for different frequencies of acoustic waves in the environment. In addition, minimizing acoustic losses and maximally more complete conversion of the potential energy of air compression inside chamber 2 into the kinetic energy of air movement in the passage of the neck section 1 is achieved. And, in particular, in creating an acoustic screen in the channel cross section, which prevents the propagation of the main acoustic wave via channel [10].

Figure 1. General view of a SMART cell equipped with a piezoelectric element, where: 1 - the neck of the cell, 2 - the free volume of the chamber, 3 - the piezoelectric element.

Moreover, the use of a customizable piezoelectric deformable element reduces the interaction of resonators with the same geometric characteristics [11].

The study of the acoustic efficiency of closely spaced resonators has been research subject by many authors. An experimental study of the mutual influence of closely spaced resonators was carried out in [12, 13]. It was found that when the distance between resonators necks is smaller than the wavelength, the efficiency of resonators pair is less than that of one. In [13], mutual influence of 2 or more cylindrical resonators was studied. In [14], a numerical study of closely spaced cylindrical resonators interaction was performed. The optimal distances between the resonators, as well as the geometric characteristics of the neck were determined.

Usage of 1-DOF resonators group is a special interest.2-DOF [15], as well as the study of their joint work presented in [16, 17]. The authors presented the optimal location system for two different Helmholtz 1DOF resonators as a result of the research. A study on the relative distance, orientation
and geometry of the resonators was conducted. Also, in subsequent work it was revealed that two identical 2-DOF-resonator systems provide a relatively wider attenuation range for a bandwidth of 300 Hz, compared to previously published studies for single 2-DOF, single 1-DOF and two identical 1-DOF.

At the same time, the obtained results and decisions are difficult to implement in the designs of engine nacelles of modern aircraft engines. That also requires serious investments. In this connection, there is a need to carry out computational and experimental studies aimed at developing inexpensive, technological adaptive sound-absorbing structures operating in a wide frequency range.

2. Numerical model

As part of the numerical experiments, the acoustic efficiency of prismatic resonators with adaptive (different) neck diameters was analyzed. The calculation of the sound absorption coefficient of adaptive Helmholtz resonators was carried out when working together in the operating frequency range of 100–3000 Hz based on the numerical solution of the Navier – Stokes equation [18].

The research results will allow to establish the patterns of acoustic pressure fields distribution in the model channel with different configurations of the location and geometric characteristics of the cells and will allow to identify significant factors affecting the SAS acoustic efficiency. The obtained results will allow developing new technological schemes and adaptive sound-absorbing structures for future aircraft engines.

To carry out a series of numerical experiments, geometric models were constructed, which can be divided into four groups: A1–2, B1–6, C1-7, D1-4.

All models consist of a circular channel (free volume of the interferometer) and an attached free volume of a cell or a pair of cells (resonator) of prismatic shape. Figure 2 shows a general view of geometric models where: 1 is the channel of finite length of circular cross section; in the center at one of the ends is the cell (2) of a prismatic shape, connected to the channel by a “narrow” adaptive neck(3) of a cylindrical shape.

![Figure 2](image)

**Figure 2.** Geometric model: a - with a single resonator; b - with a pair of resonators.

Geometrical models of group A contain: A1 - the base resonator with a neck diameter of 2 mm; A2 - two base resonators. For two base resonators, as well as for subsequent groups of resonators, the distance between the axial lines (Δl) passing through the center of the neck of the resonators was assumed to be 170 mm. Geometric models of group B contain a base resonator with a change in the diameter of 1.7 ÷ 2.2 mm (with a step of 0.1 mm) neck diameter. Geometrical models of group C contain a pair of base resonators with the same neck diameters varying in the range of 1.1 ÷ 2.2 mm (with a step of 0.2 mm). All models of group D contain a base resonator with a neck diameter of 2 mm, as well as an auxiliary one with a variable in diameter of 1.8 ÷ 2.2 mm (in 0.1 mm increments) neck diameter.
To calculate the acoustic efficiency of prismatic resonators with adaptive (different) neck diameters, a mathematical model is formulated that describes the normal propagation of acoustic harmonic wave in a cylindrical channel with Helmholtz cellular resonators. The model takes into account viscous friction losses [19] and the thermal conductivity of air and includes a system of Navier – Stokes equations linearized in the frequency domain (1–4).

\[
i \omega \rho + \rho_0 (\nabla \cdot U) = 0
\]

\[
i \omega \rho_0 U = \nabla \cdot \left( -p I + \mu (\nabla U + (\nabla U)^T) - \left( \frac{2\mu}{3} - \mu_B \right) (\nabla \cdot U) I \right)
\]

\[
i \omega \rho_0 C_p T = -\nabla \cdot (-k \nabla T) + i \omega p_0 \alpha_0 + Q
\]

\[
\rho = \rho_0 (\beta_T p - \alpha_0 T)
\]

where \( \rho \) is the density; \( U \) is the vector of oscillatory velocity; \( p \) is the acoustic pressure, \( T \) is the temperature; \( \mu \) - coefficient of dynamic viscosity; \( \mu_B \) is the coefficient of bulk viscosity; \( C_p \) is the specific heat capacity of the medium at constant pressure; \( k \) - coefficient of thermal conductivity; \( \alpha_0 \) is the coefficient of thermal expansion; \( \beta_T \) is the compressibility factor; \( Q \) - external sources of thermal energy.

As the boundary conditions at the entrance to the cylindrical channel, a planar harmonic wave with an amplitude of 10 Pa was set. The walls of the channel were assumed to be isothermal with an ambient temperature of 20°C and a zero velocity vector.

When constructing a grid model, cells were used that have a shape similar to that of an equilateral tetrahedron. The maximum element size was defined as \( N_{\text{max}} = \frac{343 [\text{m/s}] / 6 [\text{kGц}] / 10 = 5.7 \text{ mm} }{ } \), the minimum element size was taken as \( N_{\text{min}} = 0.0167 \text{ mm} \), the total number of elements was 200 thousand elements for the base model. In addition, when grinding the grid, sharp differences in the geometric dimensions of neighboring cells were avoided - the linear dimensions of neighboring cells do not differ by more than 2 times [20-22].

3. Calculation of the acoustic efficiency of adaptive cells of sound-absorbing structures

According to the results of computational experiments for models of the A-C group, the dependences of sound absorption coefficient on frequency were obtained.

When considering the results of numerical experiments obtained by the models of group A, it was revealed that the greatest value of the sound absorption coefficient is observed for model A1 and is 0.994 at a resonant frequency of 1470 Hz. For model A2, the sound absorption coefficient was 0.87 at a resonant frequency of 1470 Hz (Figure 2).

![Figure 3. Sound absorption coefficient dependence on the frequency for group A models.](image-url)
The analysis of the obtained results revealed the interaction of closely located cellular resonators with a normal incidence of the sound wave. It is established that the acoustic efficiency of the considered pair of base resonators when working together is lower than that of a single resonator.

When considering the results obtained by the B1-6 models, it was found that when the diameter decreases from 2.0 mm to 1.7 mm, the sound absorption coefficient decreases by 1.4%, the resonant frequency - by 12.24%. With an increase in the perforation diameter from 2.0 mm to 2.2 mm, a decrease in the sound absorption coefficient by 2.8% and an increase in the resonant frequency by 6.4% are also observed.

![Figure 4. Sound absorption coefficient dependence on the frequency for group B models.](image)

The analysis of the dependences obtained for the C1–7 models revealed that with an increase in the neck diameter of the resonator group, a linear increase in their own frequencies occurs. It was revealed that the maximum value of the sound absorption coefficient is observed at a diameter of 1.4 mm. With a decrease in diameter from 1.4 mm to 1.0 mm, there is a decrease in sound absorption coefficient by 15.2%, and a resonant frequency by 25.2%. With an increase in the perforation diameter from 1.4 mm to 2.2 mm, there is also a decrease in sound absorption coefficient by 19.6% and an increase in the resonant frequency by 30.2%. In addition, analysis of the results showed that a change in the diameter of the neck affects its broadband. It can be concluded that for a group of two closely spaced resonators, it is possible to choose neck diameters at which the effect of interference will be minimal. To increase the bandwidth of the band for nearby resonators, it is necessary to specify different neck diameters.

![Figure 5. Sound absorption coefficient dependence on the frequency for group C models.](image)
The analysis of the dependences obtained for the models D 1-4 revealed that, with diameters differing by 10%, an increase in the absorption coefficient is observed, as well as an increase in the bandwidth of the group. So, for combinations of the diameters of the necks of the base resonator 2 mm with the diameter of the auxiliary resonator 1.8, there is an increase in acoustic efficiency for the base resonator by 13.2%, the auxiliary by 11.7%. At the same time, there is a significant increase in band bandwidth. The operating frequency range for the D1 model was 1340 ÷ 1470 Hz, for the D4 model - 1450 ÷ 1565 Hz with a sound absorption coefficient of 0.9.

Thus, a comparative analysis of the results obtained for resonators of group C and D revealed that the combination of resonators D1 and D4 is most preferable, with the greatest increase in sound absorption coefficient for the base and auxiliary resonators. However, for resonators C3, there is a maximum acoustic efficiency of a pair of resonators with the same geometrical characteristics. This combination can be used in an adaptive arrangement of SAS cells.

4. Verification of the developed numerical models
As part of the verification of the developed numerical models, the results of numerical experiments obtained for models A1, A2 were compared with the results of laboratory experiments conducted on an interferometer with a normal incidence of a sound wave [23].

For carrying out laboratory experiments on the interferometer, two model samples of the SAS were developed and manufactured (Figure 7). The geometric characteristics of the samples correspond to the characteristics of the geometric models A1, A2, the diameter of the samples is 30 mm. Samples are made by 3D printing using FDM prototyping technology.

Testing of manufactured samples A1, A2 was carried out on the interferometer of the laboratory of the mechanisms of noise generation and modal analysis of PNRPU (Figure 8). The experimental setup is a tube of circular cross section, at one end of which a SAS sample is located, on the other - a speaker that irradiates the sample with acoustic waves. Also at some distance from the sample, microphones are installed, which record the acoustic pressure of the incident and reflected waves in time. Next, the recorded pressure is processed by an algorithmic procedure, as a result of which the
impedance of the sample SAS is calculated. Usually two microphones are used for measurements, since the “transfer function method” used to determine the impedance is the easiest to calculate [24].

![Figure 8](image1.png)

**Figure 8.** Interferometer set and ready for operation.

According to the results of laboratory experiments, the dependences of sound absorption coefficient on frequency were determined (Figure 9). Where 1 is the dependence obtained for sample A1, 2 - for sample A2.

![Figure 9](image2.png)

**Figure 9.** Dependence of sound absorption coefficient on the frequency for samples: 1 –A1, 2 –A2.

Comparative analysis of the results of numerical calculations (Figure 3), with the results of laboratory tests (Figure 9) revealed that for models A1, A2 the discrepancy in sound absorption coefficient does not exceed 5%, in frequency - 2%. The resulting error indicates the validity of the developed mathematical and numerical models.

5. Conclusion

Thus, according to the results of the research performed, physical and mathematical models were formulated for predicting the effective acoustic characteristics of a group of cells of sound-absorbing structures with different neck diameters. The concept of adaptive control of the resonant frequency and impedance of smart SAS cells by means of piezoelectric elements embedded into the neck is proposed. Verification of the developed numerical models has been carried out. As part of the verification, the results of numerical calculations were compared with the results of laboratory tests. It is revealed that for models A1, A2 the discrepancy in sound absorption coefficient does not exceed 5%, in frequency - 2%. According to the results of the numerical calculations obtained, the dependences of the natural frequencies and sound absorption coefficients of the group of adaptive SAS cells for different variants of control modes are obtained. It is revealed that the combination of resonators D1 and D4 is the most preferable, at which the greatest increase in sound absorption coefficient is observed for the basic and auxiliary resonator. At the same time, it was found that for resonators C3, the maximum acoustic efficiency of a pair of resonators with the same geometrical characteristics is observed. This combination can be used in an adaptive layout of cells of high-performance broadband SAS.
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