Absorption Mechanism and Optimization of a Subwavelength Acoustic Absorber

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Abstract. A subwavelength acoustic absorber based on parallel coupled Helmholtz resonators is designed to extend the absorption band, where the lateral space is used and verified to reduce the overall thickness. Both theoretical analysis and finite element method are used to demonstrate the low frequency absorption performance. The ratio of absorber thickness to resonant wavelength acquires only 1.87%. It has found that the side location of perforation can move the absorption peak to lower frequency range. The absorption mechanism is investigated by analysing reflection coefficient in the complex frequency plane. The energy dissipation modes underlying absorption performance are further revealed by the viscous energy dissipation patterns. Finally, the absorber is optimized by a DE algorithm to extend the absorption band below 500Hz. The present design offers us an acoustic absorber with excellent stiffness and strength for low-frequency absorption.

1. Introduction

Thin layer material used for low-frequency noise dissipation has attracted lots of interests recently. Acoustic porous materials are widely used to dissipate sound energy but has limits at low frequency if small thickness is to be targeted. Recently, various acoustic metamaterials[1-3], metasurface[4-7] and metaporous[8-9] have been proposed to enhance the low-frequency acoustic absorption under thin thickness constraint. Coupling a decorated membrane resonator (DMR) with a thin air cavity[4] can achieve total absorption in extremely low frequency, the overall thickness is only 1/133 of the working wavelength. The coiling up structures[10-13] can shrink bulky structures into deep subwavelength scale, which has been introduced to the acoustic absorber to reduce the overall thickness. The fewest ratio of the overall absorber thickness to resonant wavelength is about 1/223[5].

The present work tries to design a subwavelength absorber composed of a perforated plate and partitioned cavities (i.e, Helmholtz resonator (HR) array) under thin thickness constraint. A theoretical method and a finite element method (FEM) are used to calculate the absorption coefficient. Then the underlying mechanism of the absorption is revealed by various techniques. Finally, the absorber is optimized by a differential evolution (DE) algorithm to enhance the absorption within frequency range below 500Hz under different thickness.

2. Model and Theory

2.1. Model

Inspired by the coiling up structures, and with some simplification for easy fabrication, the basic idea to design present subwavelength absorber is using the lateral dimension of a straight cavity to reduce
the overall thickness. In order to broaden the sound absorption bandwidth, it overlaps elaborately the absorption peaks of the individual resonator by choosing the proper parameters. The proposed absorber consists of a perforated plate (facial plate) and backing cavity, the absorber is loaded by a rigid backing, as shown in Figure 1(a). The perforated plate has two mixed size holes, which periodically distribute in plate with periods $L$ and $w$ along $x$ and $y$ axes. The thicknesses of the facial plate and back cavity are $t$ and $h$ respectively. The thickness of all the partition panels is $b_0$. $P$ denotes the incident plane wave. Figure 1(b) shows the cross-section of one unit cell. The perforation diameter is $d_i$ ($i=1,2$). The distance between both holes is $w/2$, the position along $x$ axis is marked with $T_i$. The backing cavity is divided into two sections denoted by $S_i$. The width of the cavity is $w_i$, and the same length of both cavities is $l$.

![Figure 1. (a) Schematic structure of the sub-wavelength absorber composed of a perforated panel and partitioned cavities. (b) Cross-section of one unit cell](image)

2.2. Theoretical Method

In general, sound absorption coefficient could be acquired by deriving the relative acoustic impedance. The acoustic impedance of the $i$th resonator in the proposed structure is given by

$$Z_i = Z_{pi} + Z_{Ci}$$

where the subscript $i$ represents the serial number of resonator. $Z_{pi}$ and $Z_{Ci}$ are the acoustic impedance of the perforated panel and cavity respectively. Based on the solution of Crandall and the acoustic impedance of perforation panel only containing $i$th element can be

$$Z_{pi} = \frac{j\omega \rho_i t_i}{p_i} \left[1 - \frac{2J_1(\gamma_i \sqrt{j})}{(\gamma_i \sqrt{j})J_0(\gamma_i \sqrt{j})}\right]^{-1} + \frac{\sqrt{2\eta_i}}{p_i d_i} + j \frac{0.85 \omega \rho_i d_i}{p_i}$$

where $t_i$, $d_i$, $p_i$ are the thickness, hole diameter and porosity of the $i$th perforated panel, respectively. $\rho_0$ denotes air density, $\gamma_i = d_i \sqrt{\rho_0 \omega / 4\eta_i}$, $\eta_i$ is dynamic viscosity coefficient of air and $\omega$ is angular frequency. $J_1$ and $J_0$ are Bessel functions of the first and zeroth order of the first kind. The last two terms in equation (2) are the resistance and reactance from the end correction induced by the $i$th hole.

The rigid back cavity can be regarded as a pure reactance:

$$Z_{Ci} = - j \rho_0 c_0 \frac{S_{eff}}{S_m} \cot(kl_{eff})$$

![Diagram of the sub-wavelength absorber](image)
where $S_{oi} = (w_i + b_i) \times (l + b_i)$ is the outside area of the perforated panel of the $i$th element. $S_{wi} = w_i \times l$ refers to the corresponding cross section of the back cavity. $l_{eff}$ is the effective length of cavity and $k$ denotes wave number.

For two coupled HRs, the parallel connection rule \cite{1} is utilized to calculate the total acoustic impedance $Z_t$

$$\frac{A_t}{Z_t} = \sum_{i=1}^{2} \frac{S_{wi}}{Z_{ri}}$$

(4)

here $A_t = S_{o1} + S_{o2}$ is the total outside area of one unit cell. Then the absorption coefficient $\alpha$ can be expressed as

$$\alpha = 1 - \frac{|Z_t - 1|}{|Z_t + 1|}$$

(5)

where $Z_t = Z_i / \rho_0 c_0$ denotes the normalized specific acoustic impedance.

To verify the correctness of the theoretical method, we use a commercial finite element method (FEM) via COMSOL Multiphysics. The thermo-viscous losses are accounted for using the Acoustic-Thermoacoustic Interaction module. Due to a huge impedance mismatch between air and solid panels, we assume all the walls are hard boundaries. The Bloch theorem is applied on both sides of the unit cell owing to the periodicity.

3. Results and Discussions

3.1. Absorption Performance

Table 1 gives the detailed parameters of the present subwavelength absorber. The air has sound speed $c_0=343 m/s$, density $\rho_0=1.21 kg/m^3$, and dynamic viscosity $\eta=1.814 \times 10^{-5} Pa \cdot s$. We found that the resonant frequency can move to lower frequency while the perforations are placed close to one side (side location) of the cavity (the details in the parametric analysis in the following part). For side placed holes, the effective length $l_{eff}$ can be approximated as $l_{eff} = \sqrt{l^2 + h^2}$. Figure 2 compares the absorption coefficients obtained by the theoretical method and FEM. The both methods show a good agreement. There are two resonant absorption peaks at 420 Hz and 451Hz, and over 98% and 100% of energy absorption are acquired at the both resonant frequencies respectively. The total thickness of the absorber, $t + h = 15.3 \text{ mm}$, is only 1.87% of the wavelength (819mm) at the first resonant frequency (420Hz), which shows the absorber has a deep subwavelength thickness. The bandwidth of the absorption over 80% is 61Hz, i.e., from 405Hz to 466Hz.

![Figure 2](image1.png)

**Figure 2.** The absorption coefficients calculated by the theoretical method (red solid line) and FEM (black circles).

![Figure 3](image2.png)

**Figure 3.** The map of $log |\rho|_2^2$ for the absorber.
Table 1. Parameters of the subwavelength absorber (Units: mm)

|   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|
| d_1 | d_2 | w | T_1 | T_2 | w_1 | w_2 | t | h |
| 1.2  | 1.4  | 30 | 1   | 1   | 14  | 14  | 0.3| 15 |
| 60   | 1    |    |     |     |    |    |   |

3.2. Absorption Mechanism

In order to understand the above absorption performance, a graphic method of the scattering function in the complex frequency plane is utilized. By substituting \( \omega = \omega_R + io\omega_I \) in the wave number \( k \), we analyze the reflection coefficient \( r \) in the complex frequency plane. Generally, in lossless case, pairs of poles and zeros of \( r \) are symmetrically distributed on both sides of the real frequency axis\(^8\). If the loss is introduced into the absorber, the zeros and poles should be down-shifted. When the zero of \(|r|\) is on the real frequency axis in loss case, the critical coupling condition is fulfilled and perfect absorption is achieved. Figure 3 shows the distribution of \( \log |r|^2 \) in the complex plane. One can see that the zero of \( r \) at 451Hz is on the real frequency axis and the critical coupling condition is fulfilled, the absorber acquires perfect absorption. Instead, the zero of \( r \) at 420Hz is located below the real frequency axis and therefore the perfect absorption can not be achieved.

Figure 4 further reveals the energy dissipation modes underlying the sound absorption. At the first resonant frequency (420Hz), the energy is mainly dissipated by the friction among the hole of S1, as shown in the zoom around the hole in Figure 4(a). One can also see that the energy is mainly dissipated near the border of the perforation. Further, there also exists cooperative damping in the HR S2 (see Figure 4(b)), because the resonant frequencies of the both HRs are close. Similarly, at 451Hz, the perforation of S2 dissipates mainly acoustic energy into heat, the energy dissipation pattern is omitted for the paper length.

![Figure 4](image)

**Figure 4.** Contour of viscous energy dissipation (W/m³) at central cross-sectional plane of perforations at 420Hz of (a) S1 and (b) S2

3.3. Effect of Perforations Position

In the design phase, it finds that the resonance frequency of the absorber can be affected obviously by the perforation position \( T_i \), the influence of \( T_i \) on the absorption is shown in Figure 5(a). The low-frequency absorption performance is enhanced by decreasing \( T_i \). To explain the variation of the absorption with \( T_i \), Figure 5(b) depicts the variation of acoustic reactance of the absorber with different \( T_i \). The acoustic reactance becomes larger when \( T_i \) is decreased, hence the frequency of the impedance matching and then the resonant frequency moves to lower frequency range. This can offer us a new technique to design low frequency absorber while keeping the volume of cavity or perforation diameter unchanged.
Figure 5. Variation of (a) the absorption coefficient and (b) the acoustic reactance with $T_i$

4. Absorption Optimization

In order to extend absorption bandwidth of the absorber and get an optimized result, a differential evolution (DE) algorithm together with the FEM is utilized for absorption optimization. In the DE algorithm, the goal is to find a global minimum for all parameters in the search-space. In order to achieve a good sound absorption performance with wide absorption bandwidth over 0.8 under the condition of the rigid backing, the objective function can be expressed as

$$ F = -\exp[\alpha(f, \Omega) - 0.8], \Omega \in k\ell $$

(6)

where $\alpha$ represents the absorption coefficient, $\Omega$ is the set of design parameters, the target frequency range in present optimization is 400-500Hz, and $k$ denotes the range of design variables. On the premise of the cavity volume $V$ remaining a constant, we present two different optimizations: (1) keep the cavity thickness 15mm as above (Opt1), (2) the cavity thickness is optimized under the range from 15mm to 25mm (Opt2). The rest design parameters and the constraints are summarized in Table 2.

Table 2. Parameter range for absorber optimization (Units: mm)

| $d_1$ | $d_2$ | $w$ | $T_1$ | $T_2$ | $w_1$ | $t$ |
|-------|-------|-----|-------|-------|-------|-----|
| [0.5,1.8] | [0.5,1.8] | [20,40] | [1,25] | [1,25] | [8,20] | [0.1,2] |

The optimized parameters obtained by DE are listed in Table 3. Figure 6 compares the absorption coefficients of the subwavelength absorbers with and without optimization. While the thickness is kept 15mm, one can readily see that the absorption (blue dash dotted line) bandwidth in the target frequency range is extended obviously after optimization, the absorption coefficient above 0.8 spans from 411Hz to 486Hz. While the cavity thickness is further optimized, see the red solid line, the absorption bandwidth is further broadened, the absorption coefficient above 0.8 extends from 399Hz to 500Hz. Here the ratio of the thickness to the first resonant wavelength (at 420Hz) is increased to 3.0% with the thickness increasing. One can use different objective function or target frequency ranges for optimization to improve the absorption performance according to the practical requirement.

Table 3. Geometric parameters of optimized absorber (Units: mm)

| $d_1$ | $d_2$ | $w$ | $T_1$ | $T_2$ | $w_1$ | $t$ | $h$ |
|-------|-------|-----|-------|-------|-------|-----|-----|
| Opt 1 | 1.33 | 1.42 | 35.1 | 15.4 | 15.2 | 17.6 | 0.49 | 15 |
| Opt 2 | 1.36 | 1.32 | 27.9 | 18.2 | 19.5 | 11.2 | 0.48 | 24.0 |
Finally, we further place the perforations at $T_1=T_2=1$mm while the rest parameters remain the same as the optimum proposal 2. As shown in Figure 7, we can readily see again that the absorber obtains lower frequency absorption peak, the absorption coefficient over 0.8 extends from 382Hz to 475Hz when the perforations take side position.

$$\text{Figure 6. Comparison of the absorption coefficients of the absorbers with and without optimization}$$

$$\text{Figure 7. Absorption coefficients of absorbers in opt 2 with and without side position of perforations}$$

5. Conclusions
We have designed a subwavelength acoustic absorber based on parallel coupled HRs is designed to extend the absorption band, where the lateral space is used to reduce the overall thickness. The low frequency absorption performance is demonstrated by the theoretical analysis and finite element method. The ratio of the absorber thickness to resonant wavelength is only 1.87%. The graphic method using complex plane is utilized to demonstrate that perfect absorption can be achieved by fulfilling the critical coupling condition. The energy dissipation modes underlying the sound absorption are further revealed by velocity and viscous energy dissipation patterns. While keeping a constant volume of cavity, the parameters of the absorber have been optimized by a DE algorithm to enhance the absorption performance under different thickness constraint. The results show that the absorption bandwidth can be further broadened in target frequency range below 500 Hz. The present design offers us an absorber with simple subwavelength structure, which also possesses excellent stiffness and strength for practical applications.

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7. References
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