Testing CP-Violation in the Scalar Sector at Future $e^+e^-$ Colliders

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We propose a model-independent method to test CP-violation in the scalar sector through measuring the inclusive cross sections of $e^+e^- \rightarrow Zh_1, Zh_2, h_1h_2$ processes with the recoil mass technique, where $h_1, h_2$ stand for the 125 GeV standard model (SM) like Higgs boson and a new lighter scalar respectively. This method effectively measures a quantity $K$ proportional to the product of the three couplings of $h_1ZZ, h_2ZZ, h_1h_2Z$ vertices. The value of $K$ encodes a part of information about CP-violation in the scalar sector. We simulate the signal and backgrounds for the processes mentioned above with $m_2 = 40$GeV at the Circular Electron-Positron Collider (CEPC) with the integrated luminosity $5ab^{-1}$. We find that the discovery of both $Zh_2$ and $h_1h_2$ processes at $5\sigma$ level will indicate an $O(10^{-2})$ $K$ value which can be measured to 16% precision. The method is applied to the weakly-coupled Lee model in which CP-violation can be tested either before or after utilizing a “$p_T$ balance” cut (see section II B for the definition). Lastly we point out that $K \neq 0$ is a sufficient but not a necessary condition for the existence of CP-violation in the scalar sector, namely $K = 0$ does not imply CP conservation in the scalar sector.

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I. INTRODUCTION

CP-violation was first observed through $K_L^0 \rightarrow \pi\pi$ decay in 1964 [1]. More CP-violation effects have been discovered in K- and B- meson sectors since then [2]. In 1973, Kobayashi and Maskawa propose [3] that if there exist three or more generations of fermions, one or more nontrivial phase(s) will be left in the quark mixing matrix, namely the Cabibbo-Kobayashi-Maskawa (CKM) matrix [3, 4]. In the standard model (SM), only a single nontrivial phase is left which turns out to explain all the measured CP-violation effects successfully [2]. However, it is still necessary and attractive to study additional sources of CP-violation, which may help to understand the matter-antimatter asymmetry in the universe [2, 5].

In the SM, there is no CP-violation in the scalar sector. In models with additional scalars, extra CP-violation may be introduced in the scalar sector [6]. For example, in a minimal extension of SM [7], some kinds of two-Higgs-doublet models (2HDM) like Lee model [8] or Georgi model [9], and Weinberg model which contains three Higgs doublets [10], etc., there exists CP-violation in the scalar sector. In such models, a Higgs boson can be a CP-mixing state. As an example, two of the authors have studied the phenomenology of Lee model which contains spontaneous CP-violation in the scalar sector in detail [11–13]. These papers revealed the possible correlation between the lightness of Higgs boson and the smallness of CP-violation based on spontaneous CP-violation mechanism which provides another important motivation to study CP-violation further in the scalar sector.

In 2012, a SM-like Higgs boson was discovered by the ATLAS and CMS collaborations [14, 15] with its mass around 125 GeV [16]. Its spin and CP properties have also been studied through the final state distributions of $h \rightarrow ZZ^* \rightarrow 4\ell$ decay process with the conclusion that a pure $0^+$ state is favored and a pure $0^-$ state is excluded at over $3\sigma$ level [17–19]. However, a CP-mixing state is still allowed [17, 20] because the contribution from pseudoscalar component is loop induced and thus highly suppressed.

CP-violation beyond the SM may show several kinds of indirect effects [1]. For example, it may contribute to the electric dipole moments (EDM) of electron or neutron [22] which

\footnote{Here “indirect” means these phenomena will show evidence for CP-violation, but we cannot extract the CP-violation vertex through these processes; while in the “direct” effects discussed below, we can obtain the CP-violation vertex through these measurements directly. Besides the effects discussed below, the Higgs cubic self coupling could also be modified [21] though the modification does not imply CP-violation.}
are stringently constrained experimentally \cite{23,24}; it may contribute to meson mixing matrix element and thus a modification from SM prediction could occur \cite{25}; or it may also contribute to the anomalous $ZZZ$ coupling vertex \cite{26,27} which could lead to a nontrivial CP-sensitive asymmetry in $e^+e^- \rightarrow ZZ$ process \cite{28}.

However, to study the exact sources of extra CP-violation, we need their direct effects. For example, a CP-mixing Higgs boson could couple to a fermion through the effective interaction

$$L_{hf} = -h \bar{f} (g_S + ig_P \gamma^5) f,$$  \hspace{1cm} (1)

where $g_S$ and $g_P$ may be of the same order. For $f = \tau$, it is possible to test CP-violation effects in $h \tau^+ \tau^-$ vertex at future $pp$ or $e^+e^-$ colliders \cite{29,30,31} using the final state distribution of $h \rightarrow \tau^+ \tau^- \rightarrow \nu \bar{\nu} + X$ decay process. Similarly, for $f = t$, the top polarization asymmetry in $e^+e^- \rightarrow t \bar{t}h$ process is useful to test CP-violation effects in $ht\bar{t}$ vertex \cite{32}.

In this paper, we will focus on the scalar sector itself and propose a model-independent method to test CP-violation effects in the scalar sector through the interaction between scalars and massive gauge bosons. The paper is organized as follows. In section II we describe our method and perform a simulation study at the CEPC. In section III we apply this method to the weakly-coupled Lee model. And in section IV we give our conclusions and discussions.

II. MODEL-INDEPENDENT METHOD TO TEST CP-VIOLATION IN THE SCALAR SECTOR AT FUTURE $e^+e^-$ COLLIDERS

If more than one neutral scalars are discovered in the future, the tree level interaction between neutral scalars and massive gauge bosons could be written as

$$L_{tree} = \sum_i c_i h_i v \left( \frac{g^2}{2} W^\mu_\alpha W^-_\mu + \frac{g^2}{4c_W^2} Z^\mu Z_\mu \right) + \sum_{i<j} \frac{c_{ij}g}{2c_W} Z_\mu (h_i \partial^\mu h_j - h_j \partial^\mu h_i). \hspace{1cm} (2)$$

Here $g$ is the SU(2)$_L$ coupling constant, $c_W$ denotes the cosine of electro-weak angle $\theta_W$ \footnote{In this paper, we denote $s_\alpha = \sin \alpha$, $c_\alpha = \cos \alpha$, and $t_\alpha = \tan \alpha$ for any angle $\alpha$.}, $v$ is the vacuum expected value for SM scalar field, and $h_i$ represents the $i$th scalar. For the first two terms, a nonzero tree-level $h_i VV$ vertex requires that $h_i$ must contain CP-even component; while for the last term, a nonzero tree-level $h_i h_j Z$ vertex requires that $h_i$ and

$$\text{...}$$
\( h_j \) must contain components with different CP-properties. If CP is a good symmetry, there must be some terms vanishing in (2); on the other hand, if all \( c_i \) and \( c_{ij} \) are nonzero, there must be CP-violation in the scalar sector.

### A. Method for the Minimal Case

For the minimal case, two neutral scalars with non-degenerate masses are required to be discovered. CP-violation can be confirmed with \( c_1, c_2, \) and \( c_{12} \) all measured to be nonzero. It is natural to define

\[
K \equiv c_1 c_2 c_{12}
\]  

which is a useful quantity to measure the CP-violation effect since \( K \neq 0 \) is a sufficient condition for the existence of CP-violation in the scalar sector. As an example, in 2HDMs, there are three neutral Higgs bosons. We can use this idea to search for direct CP-violation effect once two of them are discovered. A straightforward calculation shows \( c_{12} = c_3 \), and \( K \) is just the product for all \( c_i \) in 2HDM. That is an important quantity to measure CP-violation in the scalar sector [26–28, 33].

At the LHC, the 125 GeV Higgs boson \( h_1 \) has already been discovered and the direct \( h_1VV \) vertices have been confirmed [17, 34]. If another Higgs boson \( h_2 \) is discovered and it has tree level decay channels \( h_2 \rightarrow WW, ZZ, Zh_1 \), it would strongly suggest CP-violation in the scalar sector which has already been discussed in [11, 13, 36]. However, the \( \sigma \cdot Br \) measurements at LHC depend on not only \( c_{1,2} \) and \( c_{12} \), but also a lot of other parameters which would affect on the production cross section or branching ratios. Thus it is difficult to extract or constrain the value of \( K \) from these measurements without model-dependent assumptions.

At future \( e^+e^- \) colliders, we can use three associated production processes, \( e^+e^- \rightarrow Z^* \rightarrow Zh_1, Zh_2, h_1h_2 \), to search for CP-violation in the scalar sector. The Feynman diagrams are shown in Figure 1. The cross sections at tree-level are given as [37, 38].

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3 One should aware that \( K \neq 0 \) is not a necessary condition for the existence of CP-violation in the scalar sector which means in some models, there may be CP-violation in the scalar sector with \( K = 0 \), see the discussions in the last section.

4 In some special models, for example, the loop-philic model [35], a loop-induced decay channel can also have a large branching ratio even it is weakly-coupled.
FIG. 1: Feynman diagrams for associated production processes $e^+e^- \rightarrow Zh_1, Zh_2, h_1h_2$.

\[
\begin{align*}
\sigma_{zh_i} &= \frac{\pi \alpha^2 s \cdot c_i^2}{96(s - m_Z^2)^2} \left( \frac{8s_W^4 - 4s_W^2 + 1}{s_W^4c_i^4} \right) \left( f^3 \left( \frac{m_i^2}{s}, \frac{m_Z^2}{s} \right) + \frac{12m_Z^2}{s} f \left( \frac{m_i^2}{s}, \frac{m_Z^2}{s} \right) \right); \\
\sigma_{h_ih_j} &= \frac{\pi \alpha^2 s \cdot c_{ij}^2}{96(s - m_Z^2)^2} \left( \frac{8s_W^4 - 4s_W^2 + 1}{s_W^4c_{ij}^4} \right) f^3 \left( \frac{m_i^2}{s}, \frac{m_j^2}{s} \right).
\end{align*}
\]

Here $s$ is the square of total energy in the center-of-mass frame, $s(c)_W$ denotes the (co)sine of electro-weak angle $\theta_W$, and the function

\[
f(x, y) \equiv \sqrt{1 + x^2 + y^2 - 2x - 2y - 2xy}.
\]

The cross sections are sensitive to $c_i$ or $c_{ij}$, but besides these, they don’t depend on more details of the model.

The recoil mass technique \[39\] would be very effective for precision measurements on these inclusive cross sections. For $e^+e^- \rightarrow Z(f\bar{f})h_i$ process, the recoil mass is defined as \[39, 40\]

\[
m_{\text{rec}} \equiv \sqrt{s + m_{ff}^2 - 2\sqrt{s}(E_f + E_{\bar{f}})}
\]

whose distribution would show a narrow peak around $m_i$ where $m_{ff}^2 = m_Z^2$ is the invariant mass of the fermion pair. With this method, the sensitivity to $Zh_1$ inclusive cross section would reach better than 1% at future Higgs factories \[41\] with $\sqrt{s} = 250\text{GeV}$ and $\mathcal{O}(ab^{-1})$ luminosity. The result doesn’t depend on the decay channels of Higgs boson which means this is a model-independent technique to measure $h_iZZ$ couplings $c_i$. Generalizing this technique to $e^+e^- \rightarrow h_1(b\bar{b})h_2$ process, with $h_1$ the 125 GeV Higgs boson and $m_{bb}^2 = m_{11}^2$, the distribution of $m_{\text{rec}}$ would show a narrow peak around $m_2$ and thus we can measure the $e^+e^- \rightarrow h_1h_2$ inclusive cross section to extract the $h_1h_2Z$ coupling $c_{12}$ in a model-independent way \[5\]. Thus through measuring the three inclusive associated production cross sections, $\sigma_{zh_i}$, $\sigma_{zh_j}$, and $\sigma_{h_ih_j}$, the recoils $m_{\text{rec}_1}$, $m_{\text{rec}_2}$, and $m_{\text{rec}_3}$ can be extracted and the coupling $c_i$ or $c_{ij}$ can be measured.

\[5\] In order to measure $\sigma_{h_ih_2}$ using this method, Br($h_1 \rightarrow b\bar{b}$) is needed as a model-dependent quantity, which can be accurately measured through $e^+e^- \rightarrow Zh_1$ process.
sections, we can extract all the three couplings $c_1, c_2, c_{12}$ and subsequently obtain $K$ in a model-independent way.

B. Model-Independent Simulation Study

Here we perform a simulation study of the signal and backgrounds for the case $m_2 = 40\text{GeV}$ at Circular Electron-Positron Collider (CEPC) [11] which would be a $e^+e^-$ collider with $\sqrt{s} = 250\text{GeV}$ 6. Such a light scalar can occur in many models, such as 2HDMs [6, 12, 13, 45, 46].

Assuming $h_1$ is SM-like, $c_1 \sim 1$ which is consistent with the recent 125 GeV Higgs measurements [47]. In the following we focus on the inclusive measurements on $Zh_2$ and $h_1h_2$ associated production processes. The strictest direct constraints on $c_2$ and $c_{12}$ came from LEP results [48, 49] which give

$$|c_2| < 0.18, \quad |c_{12}| < 0.54$$

for $m_2 = 40\text{GeV}$ at 95\% C.L. assuming all scalars decay only to $b\bar{b}$ final states.

In our simulation analysis, we use WHIZARD-2.3.1 [50] to generate signal and background events with initial state radiation (ISR) and beamstrahlung effects. For beamstrahlung effects, we use the built-in spectra CIRCE2 for the CEPC project [51]. For both processes, we adopt the recoil mass method in which we do not reconstruct $h_2$ directly using its decay final states thus the results do not depend on the properties of $h_2$ except its mass.

For $Zh_2$ process, we choose the $Z \rightarrow \mu^+\mu^-$ decay channel. The corresponding backgrounds are $e^+e^- \rightarrow \mu^+\mu^- X$ where $X = e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, b\bar{b}, \nu\bar{\nu}$, or $\gamma\gamma$ [41, 52–54]. We impose the basic cuts as [41, 52]

$$|\cos \theta_{\mu^+\mu^-}| < 0.98, \quad m_{\mu^+\mu^-} > 15\text{GeV}, \quad m_{\text{rec}} > 15\text{GeV},$$

$$|\cos \theta_{e^+\gamma}| < 0.995, \quad E_\gamma > 0.1\text{GeV}, \quad \Delta R_{ij} > 0.4.$$  \hspace{1cm} (9)

where $m_{\text{rec}}$ is defined in (7) with $f = \mu$ and $\Delta R_{ij} \equiv \sqrt{(\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2}$ with $i$ and $j$ running over all partons in the final state 7. The transverse momentum of muon is smeared

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6 If the extra scalar is a heavier one, we can utilize this method at $e^+e^-$ colliders with larger $\sqrt{s}$, like the International Linear Collier (ILC) [44].

7 The cuts in the second line are useful to avoid the infrared and collinear divergences in background
by a Gaussian distribution with the standard deviation of $[41]$

$$\sigma_{1/p_T} = 2 \times 10^{-5} \oplus 1 \times 10^{-3}/(p_T \sin \theta)[\text{GeV}^{-1}].$$

(10)

**FIG. 2:** Normalized kinematical distributions of the signal and backgrounds in the $e^+e^- \rightarrow Zh_2$ channel after the basic cuts are applied. The first three figures show the $\cos \theta_{\mu^-}$, $p_T(\mu^+\mu^-)$, and $m_{\text{rec}}$ distributions respectively in which we reconstructed only $\mu^+$ and $\mu^-$. The last figure shows the “$p_T$ balance” distribution (see the text below for details) in which we must tag at least one photon that breaks the inclusiveness a little bit.

We show some kinematical distributions in Figure 2. Based on the kinematical differences shown in the first three figures in Figure 2, we impose the selection cuts as

$$|\cos \theta_{\mu^\pm}| < 0.8, \quad p_T(\mu^+\mu^-) > 35\text{GeV}, \quad |m_{\mu^+\mu^-} - m_Z| < 10\text{GeV},$$

and

$$30\text{GeV} < m_{\text{rec}} < 60\text{GeV}.$$  

(11)

The cuts on $\cos \theta_{\mu^-}$ and $p_T(\mu^+\mu^-)$ are helpful to reduce large $\mu^+\mu^-\nu\bar{\nu}$ and $\mu^+\mu^-\gamma\gamma$ backgrounds. The $m_{\mu^+\mu^-}$ cut is imposed to extract the signal events around $Z$ peak in $m_{\mu^+\mu^-}$ distribution, and the recoil mass cut is imposed to extract the signal events around $h_2$ peak in $m_{\text{rec}}$ distribution. After all the selection cuts, the cross sections of the signal and backgrounds are

$$\sigma_{\text{sig}} = c_2^2 \times 7.438\text{fb}, \quad \sigma_{\text{bkg}} = 5.916\text{fb},$$

(12)

in which $e^+e^- \rightarrow \mu^+\mu^-\gamma\gamma$ is the dominant background process with the cross section $\sigma_{\mu^+\mu^-\gamma\gamma} = 4.659\text{fb}$. Moreover, we can take advantage of the “$p_T$ balance” cut $[54, 55]$ to suppress the $\mu^+\mu^-\gamma\gamma$ background further. The observable $p_{T,\text{bal}}$ is defined as

$$p_{T,\text{bal}} \equiv p_T(\mu^+\mu^-) - p_T(\gamma)$$

(13)

processes. We do not consider the decays of $\tau$ leptons in our analysis. The final state with single photon can be totally rejected by the requirement of a large recoil mass $m_{\text{rec}}$ at the parton level.
where $p_T(\gamma)$ is the transverse momentum of the most energetic photon tagged \(^8\). Based on the last figure in Figure 2 if we choose the cut $p_{T,\text{bal}} > 20\text{GeV}$ as \(^5\), we have

$$\begin{align*}
\sigma'_{\mu^+\mu^-\gamma\gamma} &= 0.211\text{fb} \quad \text{thus} \quad \sigma'_{\text{bkg}} = 1.468\text{fb}
\end{align*}$$

(14)

with cross sections of other processes unchanged. Using these results, we summarize the $3\sigma$, $5\sigma$ discovery potential and expected 95% C.L. upper limit (corresponding to $1.64\sigma$) on $|c_2|$ with $5\text{ab}^{-1}$ luminosity at CEPC before and after “$p_T$ balance” cut separately in Table I.

### TABLE I: Expected 95% C.L. upper limit, $3\sigma$, and $5\sigma$ discovery potential for $|c_2|$ with $5\text{ab}^{-1}$ luminosity at CEPC.

|                      | 95% C.L. limit | $3\sigma$ discovery | $5\sigma$ discovery |
|----------------------|----------------|----------------------|----------------------|
| before “$p_T$ balance” cut | $< 0.087$   | $> 0.118$            | $> 0.152$            |
| after “$p_T$ balance” cut       | $< 0.061$   | $> 0.083$            | $> 0.107$            |

For $h_1h_2$ process, we use the $h_1 \rightarrow b\bar{b}$ decay channel. The backgrounds include $e^+e^- \rightarrow b\bar{b}X$ and $e^+e^- \rightarrow Zh_1(b\bar{b})$ where $X = e^+e^-, \mu^+\mu^-, \tau^+\tau^-, q\bar{q}, b\bar{b}, \nu\bar{\nu}, \gamma\gamma, g\gamma$, and $gg$ \(^9\). We impose the basic cuts as

$$\begin{align*}
m_{bb} &> 15\text{GeV}, \quad m_{\text{rec}} > 15\text{GeV}, \\
|\cos\theta_{e^+\gamma}| &< 0.995, \quad E_{\gamma} > 0.1\text{GeV}, \quad E_g > 1\text{GeV}, \quad \Delta R_{ij} > 0.4
\end{align*}$$

(15)

where $m_{\text{rec}}$ is defined in (7) with $f = b$ and $\Delta R_{ij}$ run over all partons in the final state \(^10\). The jet energy is smeared by a Gaussian distribution with the standard deviation of \(^\Pi\)

$$\begin{align*}
\frac{\sigma_E}{E} &= \frac{0.3}{\sqrt{E(\text{GeV})}}
\end{align*}$$

(16)

for the jet energy less than 100GeV. The $b$-tagging efficiency and $c$-faking rate are \(^\Pi\)

$$\begin{align*}
\epsilon_b &= 0.9, \quad P_{c\rightarrow b} = 0.1
\end{align*}$$

(17)

\(^8\) With this method, we must tag at least one photon which breaks the inclusiveness of the measurement. But for most cases, we can assume $\text{Br}(h \rightarrow \gamma\gamma) \ll 1$ so that tagging a photon would make only a little difference on the measurement.

\(^9\) We also considered other background processes like $e^+e^- \rightarrow b\bar{b}h_{1,2}$ and $e^+e^- \rightarrow Z(b\bar{b})h_2$. However, numerically they are all negligible except for a very strong $h_{1,2}b\bar{b}$ coupling, thus we don’t list them here. Again the SM backgrounds $b\bar{b}g$ and $bb\gamma$ can be completely removed at the parton level.

\(^10\) The cuts in the second line are useful to avoid the infrared and collinear divergences in background processes as discussed above.
separately. In an event, at least two b jets should be tagged. The candidates of b jets from $h_1$ decays are selected with the minimal $|m_{bb'} - m_1|$ and then sorted by the transverse momenta. The leading and sub-leading $p_T$ of the selected b jet pairs are denoted as $p_T(b)$ and $p_T^{\text{sub}}(b)$.

FIG. 3: Normalized kinematical distributions of the signal and backgrounds in the $e^+e^- \rightarrow h_1h_2$ channel after the basic cuts are applied and $\geq 2b$ jets are tagged. In the first five figures, only b jets are reconstructed; while in the last figure, at least one photon should be tagged which breaks the inclusiveness a little bit.

Based on the kinematic differences between the signal and backgrounds as shown in the first five figures in Figure 3, we impose the selection cuts as

$$70\text{GeV} < p_T(bb) < 100\text{GeV}, \quad 70\text{GeV} < p_T(b) < 110\text{GeV}, \quad 30\text{GeV} < p_T^{\text{sub}}(b) < 70\text{GeV},$$

$$|m_{bb'} - m_1| < 25\text{GeV}, \quad \text{and} \quad 20\text{GeV} < m_{\text{rec}} < 70\text{GeV}. \quad (18)$$

The cuts in the first line use the differences in b jets $p_T$ distributions to distinguish events from signal and backgrounds. The $m_{bb'}$ cut is imposed to extract the signal events around $Z$ peak in $m_{bb'}$ distribution, and the recoil mass cut is imposed to extract the signal events around $h_2$ peak in $m_{\text{rec}}$ distribution. After these selection cuts, the cross sections of signal
\[ \sigma_{\text{sig}} = \frac{c_{12}^2 \text{Br}(h_1 \to b\bar{b})}{\text{Br}_{\text{SM}}(h_1 \to b\bar{b})} \times 12.5 \text{fb}, \quad \sigma_{\text{bkg}} = \left( 20.54 + 0.577 \left( \frac{\text{Br}(h_1 \to b\bar{b})}{\text{Br}_{\text{SM}}(h_1 \to b\bar{b})} \right) \right) \text{fb} \]  

(19)

where \( \text{Br}_{\text{SM}}(h_1 \to b\bar{b}) = 0.5824 \) for \( m_1 = 125\text{GeV} \). The dominant background is \( b\bar{b}gg \) production with its cross section \( \sigma_{b\bar{b}gg} = 13.2 \text{fb} \). The backgrounds with photon have the cross section \( \sigma_{b\bar{b}g\gamma + b\bar{b}\gamma\gamma} = 4.981 \text{fb} \) which can be suppressed to \( \sigma'_{b\bar{b}g\gamma + b\bar{b}\gamma\gamma} = 1.107 \text{fb} \) by using the “\( p_T \) balance” cut based on the last figure in Figure 3. The “\( p_T \) balance” cut does not affect the signal and other background processes thus the total background can be reduced to

\[ \sigma'_{\text{bkg}} = \left( 16.66 + 0.577 \left( \frac{\text{Br}(h_1 \to b\bar{b})}{\text{Br}_{\text{SM}}(h_1 \to b\bar{b})} \right) \right) \text{fb}. \]  

(20)

As a benchmark point, take \( \text{Br}(h_1 \to b\bar{b}) = \text{Br}_{\text{SM}}(h_1 \to b\bar{b}) \). We use the results above to summarize the 3\( \sigma \), 5\( \sigma \) discovery potential and expected 95% C.L. upper limit on \( |c_{12}| \) with 5ab\(^{-1}\) luminosity at CEPC before and after “\( p_T \) balance” cut separately in Table II.

**TABLE II: Expected 95% C.L. upper limit, 3\( \sigma \), and 5\( \sigma \) discovery potential for \( |c_{12}| \) with 5ab\(^{-1}\) luminosity at CEPC.**

|                        | 95% C.L. limit | 3\( \sigma \) discovery | 5\( \sigma \) discovery |
|------------------------|----------------|--------------------------|--------------------------|
| before “\( p_T \) balance” cut | < 0.092        | > 0.125                  | > 0.161                  |
| after “\( p_T \) balance” cut  | < 0.088        | > 0.119                  | > 0.153                  |

For \( m_2 < 125\text{GeV} \), the three processes \( e^+e^- \to Zh_1, Zh_2, h_1h_2 \) are possible at CEPC. However, the method discussed in this paper is not always effective for the whole mass region. If \( m_2 \lesssim 34\text{GeV} \) when rare decay \( h_1 \to Zh_2 \) process opens, it will set a stricter constraint \( |c_{12}| \lesssim 0.07 \) which make this method invalid. For a larger \( m_2 \), both cross sections \( \sigma_{Zh_2,h_1h_2} \) decrease when \( m_2 \) grows. But when \( m_2 \) is not close to \( Z \) peak, for example, \( m_2 \lesssim 70\text{GeV} \), the cross sections of signal and backgrounds change slowly thus the method will still be effective. For example, when \( m_2 = 70\text{GeV} \), our simulations show that the 5\( \sigma \) discovery bound can reach \( |c_2| > 0.13(0.11) \) and \( |c_{12}| > 0.21(0.20) \) respectively before (after) “\( p_T \) balance” cut. For \( m_2 \sim (70 - 110)\text{GeV} \) which is around the \( Z \) peak, large \( Z \) backgrounds will be difficult to reduce for both \( e^+e^- \to Zh_2, h_1h_2 \) which means the analysis we used above is not enough and we may need more careful analysis. For larger \( m_2 \), the \( h_1h_2 \) production cross section will decrease quickly when \( m_2 \) grows. Thus at CEPC, this method is effective for \( m_2 \sim (35 - 70)\text{GeV} \).
III. IMPLICATION FOR WEAKLY-COUPLED LEE MODEL

In this paper, we choose weakly-coupled Lee model [12, 13] which naturally contains a light scalar in small CP-violation limit as a benchmark model to study the implications of our simulation results.

Lee model was proposed by Lee in 1973 [8] as a 2HDM which is CP-conserved at Lagrangian level but the CP-violation comes from the vacuum. The scalar potential can be written as

\[ V(\phi_1, \phi_2) = \mu_1^2 R_{11} + \mu_2^2 R_{22} + \lambda_1 R_{11}^2 + \lambda_2 R_{11} R_{12} \]
\[ + \lambda_3 R_{11} R_{22} + \lambda_4 R_{12}^2 + \lambda_5 R_{12} R_{22} + \lambda_6 R_{22}^2 + \lambda_7 t_{12}^2 \] (21)

where \( R(I)_{ij} \) is the real (imaginary) part of \( \phi_i^\dagger \phi_j \). Both \( \phi_i \) are scalar doublets which can be written as \( \phi_1 = (\phi_1^+, v_1 + R_1 + i I_1)/\sqrt{2} \) and \( \phi_2 = (\phi_2^+, v_2 \exp(i\xi) + R_2 + i I_2)/\sqrt{2} \). Here \( I_{1,2} \) and \( R_{1,2} \) are scalar degrees of freedom and \( v = \sqrt{v_1^2 + v_2^2} = 246\text{GeV} \). According to the vacuum stability condition, if

\[ |\lambda_2 v_1^2 + \lambda_5 v_2^2| < 2|\lambda_4 - \lambda_7| v_1 v_2, \] (22)

a nontrivial phase difference \( \xi \) between the vacuum expected values (VEV) of the two Higgs doublets would arise thus CP symmetry is spontaneously broken. As a consequence all the three neutral Higgs bosons must be CP-mixing states.

Defining \( t_\beta \equiv v_2/v_1 \), for weakly-coupled scalar sector (\( \lambda_i \lesssim \mathcal{O}(1) \)), in the limit of small \( t_\beta s_\xi \), a new light scalar is predicted with the mass \( m_2 \sim \mathcal{O}(vt_\beta s_\xi) \) [11–13]. We treat it as the 40GeV new scalar. Its couplings to massive vector bosons are also suppressed by \( c_2 \sim \mathcal{O}(t_\beta s_\xi) \sim \mathcal{O}(0.1) \). If the heavy Higgs boson has its mass \( m_3 \sim \mathcal{O}(v) \), there is also additional constraint on \( c_{12} \) from LHC results [17]. If 200GeV < \( m_3 < 300\text{GeV} \), \( c_{12} \equiv c_3 \lesssim (0.3 - 0.4) \) [12, 13, 57] which is stricter than the LEP result. In this scenario, the 125GeV Higgs boson \( h_1 \) has SM-like couplings. The \( h_1 \rightarrow 2h_2 \) decay channel measurements impose a strict constraint on \( h_1 h_2 h_2 \) coupling to \( \mathcal{O}(10^{-2}) \) [12, 58], but this measurement does not give tighter constraints on the \( c_1, c_2 \) and \( c_{12} \) couplings. The electro-weak precision measurements [59] require that the charged Higgs boson mass must be close the the heavy Higgs mass \( m_3 \) [12]. For \( m_3 \sim v \), there is no further constraints from \( t \rightarrow H^+ b \) rare decay [12]. The study in [12] and its update results in [57] showed this scenario is still viable facing all experimental constraints.
The results we obtained above showed we can set stricter constraint or discovery potential on this scenario. For $h_1 h_2$ production channel, we use $\text{Br}(h_1 \to b \bar{b}) = \text{Br}_{\text{SM}}(h_1 \to b \bar{b})$ as a benchmark point. Assuming all $c_{1,2,12} > 0$, we have

$$K = c_{212} \sqrt{1 - c_2^2 - c_{12}^2}. \quad (23)$$

In Figure 4 we show the expected limit or significance for different $(c_2, c_{12})$ points before (see the left figure) or after (see the right figure) “$p_T$ balance” cut discussed above. The four curves are $K = 0.01, 0.02, 0.03, 0.04$ separately from left to right.

If there is no hint for either processes before “$p_T$ balance” cut, it is expected to set an upper limit $K < 7.9 \times 10^{-3}$; while the upper limit is expected to be $K < 5.3 \times 10^{-3}$ after “$p_T$ balance” cut. If both processes are discovered at over $3(5)\sigma$ level before “$p_T$ balance” cut, we have $K > 1.5(2.4) \times 10^{-2}$; while the number should be $1.0(1.6) \times 10^{-2}$ after “$p_T$ balance” cut. In this case, we can confirm CP-violation in the scalar sector and measure $K$ to the accuracy $\delta K/K \lesssim 24(16)\%$. For the case with the largest $K$, both couplings are set to the recent allowed upper limit, $c_2 = 0.18$ and $c_{12} = 0.3$, we will have $K = 5.4 \times 10^{-2}$ and $\delta K/K = 7.9(4.7)\%$ before (after) “$p_T$ balance” cut.
In the discussions above, we just use the inclusive measurements to determine the couplings and hence $K$ in a model-independent way. For the discovery potential of a specific model, it would be better to use exclusive decay channels such as $h_2 \rightarrow b \bar{b}$ which is expected to be dominant in most models. The sensitivity would also increase if we combine the results from more decay channels of $Z$ and $h_1$.

**IV. CONCLUSIONS AND DISCUSSIONS**

Once two scalars are discovered, we can test the CP-violation in the scalar sector through searching for nonzero tree-level $h_1ZZ, h_2ZZ, h_1h_2Z$ vertices according to the CP-properties analysis. Based on this idea, we proposed a model-independent method to confirm CP-violation in the scalar sector at future $e^+e^-$ colliders through measuring the inclusive $e^+e^- \rightarrow Zh_1, Zh_2, h_1h_2$ cross sections with recoil mass technique. We can use a quantity $K = c_1c_2c_{12}$ which is defined in (3) to measure CP-violation in the scalar sector.

We have performed simulation studies for $m_2 = 40$GeV at CEPC assuming the 125GeV Higgs boson $h_1$ is SM-like and the results are shown in Table I and Table II. We have adopted the recoil mass technique to ensure the measurements are inclusive. The $5\sigma$ discovery limit for both $c_2$ and $c_{12}$ are below the recent 95% C.L. upper limits. For $Zh_2$ associated production, the “$p_T$ balance” cut is efficient to drop the photon background but it also lose the inclusiveness a little. We choose the weakly-coupled Lee model which contains CP-violation and allows an extra light scalar as a benchmark model. In the weakly coupled Lee model, both processes, $Zh_2$ and $h_1h_2$, are possible to be discovered at $5\sigma$ level before or after “$p_T$ balance” cut. If both processes are discovered at $3(5)\sigma$ level, $K$ must reach $O(10^{-2})$ and the sensitivity of $\delta K/K$ measurement can reach $24(16)\%$. This method is also applicable for other $e^+e^-$ colliders if all the three processes can be discovered. For example, if the extra scalar is heavier, we can use this method at a $e^+e^-$ collider with larger $\sqrt{s}$, such as ILC.

We should note that $K \neq 0$ is a sufficient but not necessary condition for the existence of CP-violation in the scalar sector. Precisely speaking, we can use this method to confirm

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11 After “$p_T$ balance” cut, it is quasi-model-independent as discussed above.
12 For most perturbative models, this method is useful to extract tree level information instead of loop level since loop-induced processes have small enough cross sections.
the existence of CP-violation in the scalar sector according to the nonzero $K$, but can’t constrain or exclude the CP-violation in the scalar sector if $K$ is unmeasurable small. For example, in a minimal extension of SM mentioned above [7], there is only an additional complex singlet in the extension of the scalar sector. For some parameter choices, the three scalars would become CP-mixing states, but there are still no tree-level $h_i h_j Z$ vertices thus the measurement on $e^+ e^- \rightarrow h_i h_j$ cannot give a positive result.

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