Abstract

A multiple round quantum dense coding (MRQDC) scheme based on the quantum phase estimation algorithm is proposed. Using an $m + 1$ qubit system, Bob can transmit $2^{m+1}$ messages to Alice, through manipulating only one qubit and exchanging it between Alice and Bob for $m$ rounds. The information capacity is enhanced to $m + 1$ bits. We have implemented the scheme in a three-qubit nuclear magnetic resonance (NMR) quantum computer. The experimental results show a good agreement between theory and experiment.

PACS numbers: 03.67. Hk, 03.67.Lx
I. INTRODUCTION

Dense coding [1] can transmit more than one bit of information by manipulating only one of two particles in an entangled state. Initially, Alice has two particles in an Bell-state [2]. She sends one of the two particles to Bob, who manipulates the particle via one of four unitary operators, namely $I$, $\sigma_x$, $i\sigma_y$ and $\sigma_z$, where $\sigma_x$, $\sigma_y$, $\sigma_z$ and $I$ are the three Pauli operators and the identity operator respectively, so as to change the two-particle system into another of the four Bell-states and then returns the processed particle to Alice. By determining the Bell-state, Alice can read the encoded information. Compared with the classical communication, in which one bit only transmits one bit of information, dense coding doubles the information capacity.

K. Mattle et al realized dense coding transmission with entangled photon pairs [3]. They can transmit one of three messages by manipulating only one of two entangled photons. X. Fang et al demonstrated dense coding using a nuclear magnetic resonance quantum computer [4]. Four messages, i.e., two bits of information, were transmitted in their scheme. X. Liu et al proposed a general scheme for superdense coding among multiparties, where more than two dimension systems were exploited and the information capacity depended on the number of the dimension of the individual system [6]. Grudka et al proposed a symmetric multi-party superdense coding scheme [7] in which the capacity increase is evenly distributed among the participants. K. Shimizu et al proposed a dense coding scheme with enhanced information capacity through an additional degree of freedom [8]. Dense coding for continuous variables was also realized in experiments recently [9, 10]. A. Harrow et al proposed superdense coding of quantum states through generalizing dense coding to quantum states [11]. I. P. Degiovanni et al exploited dense coding to enhance the transmission capacity of quantum key distribution (QKD) [12].

Our scheme concentrates on dense coding using qubits. In previous scheme, the increase of capacity is limited by the dimension of the quantum system. However, in quantum information processing, qubits are more popularly used. Using an entangled qubit pair, the maximum information dense coding can provide is 2 qubits. One common feature of these dense coding schemes is that the qubits are only transmitted one round. If we allow the qubit to travel between the two users more than once, then it is possible to increase the information capacity greatly. In this paper, we propose a multiple round quantum dense
coding (MRQDC) scheme, which can transmit $2^{m+1}$ messages through manipulating only one qubit. The scheme needs only an $m + 1$ qubit system, in which only one qubit is exchanged between Alice and Bob for $m$ rounds to encode $2^{m+1}$ messages. Alice decodes the $2^{m+1}$ messages through the inverse of quantum Fourier transform (QFT). The MRQDC scheme is the results of combining the quantum quantum phase estimation algorithm [13] with the original dense coding. In contrast, the usual dense coding scheme uses only a single round, the maximum information it can transmit is only 2 bits.

An feasible quantum system to demonstrate the proposed quantum communication protocol is the nuclear magnetic resonance (NMR) system, and it has become an important arena for demonstrating quantum algorithms [14, 15, 16, 17, 18, 19]. Some quantum communication protocols, such as the quantum teleportation, quantum dense coding have been demonstrated in NMR quantum systems [20, 21, 22]. We have implemented the proposed MRQDC scheme in a three qubit NMR quantum computer. The effective-pure state is prepared using the spatial averaging method. Using this method, the scheme is implemented through only one experiment, and the output is directly obtained by observing the NMR spectrum.

II. THE MULTIPLE ROUND QUANTUM DENSE CODING SCHEME

The usual dense coding can be pictured as shown in Fig. [Fig]. The horizontal lines denote qubits. $|0\rangle$ denotes the spin up state and $H$ denotes the Hadamard transform. Here the four different coding unitary operations are expressed as $O_b R_z(\varphi_k)$, where $R_z(\varphi_k)$ denotes a phase manipulation, and $O_b$ denotes a bit manipulation. $O_b$ is chosen as $I$, or $\sigma_x$. $R_z(\varphi_k) = e^{i\varphi_k}I_z$, where $\varphi_k = -k\pi$ ($k = 0, 1$). $F^{-1}$ denotes the inverse of quantum Fourier transform. For one qubit system, the Fourier transform is just the Hadamard transformation, namely $F^{-1} = H$. $|x_1\rangle_1|x_0\rangle_0$ is the output state. The four transformation manipulations can be explicitly expressed as $R_z(0) = I$, $R_z(-\pi) = -i\sigma_z$, $\sigma_x$, and $R_z(-\pi)\sigma_x = \sigma_y$, respectively. The four manipulations correspond to the four output basis states $|0\rangle_1|0\rangle_0$, $|1\rangle_1|0\rangle_0$, $|0\rangle_1|1\rangle_0$, and $|1\rangle_1|1\rangle_1$, respectively. Alice can determines the manipulation by making a joint measurement on qubits 1 and 0. During the dense coding process, qubit 1 remains at Alice’s site, and qubit 0 is sent to Bob, then manipulated by Bob, and then sent back to Alice. The Fourier transform is performed by Alice. Expressing the dense coding lends clarity in generalizing
it into a multiple round dense coding scheme.

Decomposing the encoding transformation into a phase rotation $R_z(\varphi_k)$ and a bit manipulation $O_b$ make it easier to generalize it into MRQDC. The phase manipulation can be viewed as quantum manipulations, and the bit manipulations can be viewed as classical manipulations, just like decomposing information into a quantum piece and a classical piece in Refs. [23, 24]. The number of classical manipulations, either the identity $I$ or the spin-flip $\sigma_x$ is fixed. The number of quantum manipulations, however, can be extended to $2^m$, if the system is extended to $m + 1$ qubits. Combining with the two classical manipulations, the MRQDC scheme can transmit $2^{m+1}$ manipulations, as shown in Fig. 2. The system has $m + 1$ qubits, in which only qubit 0 is exchanged between Alice and Bob. For convenience, qubit 0 is called the flying qubit, and the other $m$ qubits are called the stationary qubits, which can be viewed as the quantum memory. The flying qubit is exchanged between Alice and Bob to encode the messages into the stationary qubit. The phase rotation operations are

$$R_z(2^m-j\varphi_k) = e^{i2^m-j\varphi_k I_z} (j = 1, 2, \ldots, m),$$

where $\varphi_k = -2k\pi/2^m$ ($k = 0, 1, 2, \ldots, 2^m - 1$). The output state can be rewritten as $|k\rangle|x_0\rangle_0$. For example, $|0\rangle|x_0\rangle_0 = |0\rangle_1|0\rangle_2\cdot\cdot\cdot|0\rangle_m|x_0\rangle_0$, $|1\rangle|x_0\rangle_0 = |0\rangle_1|0\rangle_2\cdot\cdot\cdot|1\rangle_m|x_0\rangle_0$, $|2^m-1\rangle|x_0\rangle_0 = |1\rangle_1|1\rangle_2\cdot\cdot\cdot|1\rangle_m|x_0\rangle_0$.

One of $2^{m+1}$ messages can be transmitted from Bob to Alice through the following quantum communication scheme:

1. **step 0**: Initially, Alice has $m + 1$ qubits, each of which lies in state $|0\rangle$.
2. **step 1**: Setting $j = 1$. Alice starts MRQDC by manipulating qubit $j$ and qubit 0.
3. **step 2**: Alice applies a Hadamard transform to qubit $j$, and then a controlled-NOT gate (CNOT$_{j0}$) to qubits $j$ and 0 to transform the two qubits into an entangled state. Then she sends qubit 0 to Bob, who applies $R_z(2^m-j\varphi_k)$ to qubit 0, noting $R_z(2^m-j\varphi_k) = R_z^{2^m-j}(\varphi_k)$, i. e., repeating $R_z(\varphi_k)$ for $2^m-j$ times. Then, he returns qubit 0 to Alice. One should note that the two qubits are still entangled. Alice applies $CNOT_{j0}$ to qubits $j$ and 0 for the second time to disentangle qubit 0 from qubit $j$.
4. **step 3**: Check if $j > m - 1$. If it is not, put change $j$ to $j + 1$ and go to step 2. If true, i. e., $j = m$, goto step 4.
5. **step 4**: Alice applies a Hadamard transform to qubit $m$, and then $CNOT_{m0}$ to qubits $m$ and 0 to transform the two qubits into an entangled state. Then she sends qubit 0 to Bob, who applies $O_b R_z(\varphi_k)$ to qubit 0. Then, he returns qubit 0 to Alice. Alice applies $CNOT_{m0}$ to qubits $m$ and 0 for the second time to disentangle qubit 0 from qubit $m$. 

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step 5: Making an inverse Fourier transform to the \( m \) qubits, and then make a joint measurement on qubits 0 to \( m \). Using this result, Alice reads out the information encoded by Bob.

Now let’s analyze the working details of the MRQDC protocol. In each round described in step 2, the effect on qubit \( j (j < m) \) and qubit 0 can be described as

\[
T_j = \text{CNOT}_{j0} R_z 0 (2^{m-j} \varphi_k) \text{CNOT}_{j0} H_j. \tag{1}
\]

\( T_j \) is the elementary operation for encoding in MRQDC. It can be proved that

\[
T_j |0\rangle_j |0\rangle_0 = \frac{1}{\sqrt{2}} (|0\rangle_j + e^{-i2^{m-j} \varphi_k} |1\rangle_j) |0\rangle_0, \tag{2}
\]

where an irrelative overall phase factor has been ignored.

Bob inputs the bit manipulation into the network in the last round described in step 4. After the completion of \( T_{m-1} T_{m-2} \cdots T_1 \), the \( m+1 \) qubits are in a product state. Let

\[
T_m = \text{CNOT}_{m0} O_b R_z 0 (\varphi_k) \text{CNOT}_{m0} H_m, \tag{3}
\]

one obtains

\[
T_m |0\rangle_m |0\rangle_0 = \frac{1}{\sqrt{2}} (|0\rangle_m + e^{-i \varphi_k} |1\rangle_m) O_b |0\rangle_0, \tag{4}
\]

when \( T_m \) is applied to qubit \( m \) and qubit 0. Let \( T = T_m T_{m-1} \cdots T_1 \),

\[
T |0\rangle_m |0\rangle_{m-1} \cdots |0\rangle_1 |0\rangle_0 = \frac{1}{\sqrt{2^m}} \bigotimes_{j=1}^{m} (|0\rangle_j + e^{-i2^{m-j} \varphi_k} |1\rangle_j) O_b |0\rangle_0 \tag{5}
\]

is obtained. Eq. (5) can be rewritten as

\[
T |0\rangle_m |0\rangle_{m-1} \cdots |0\rangle_1 |0\rangle_0 = \frac{1}{\sqrt{2^m}} \sum_{n=0}^{2^m-1} |n\rangle e^{i2n k \pi / 2^m} O_b |0\rangle_0, \tag{6}
\]

using \( \varphi_k = -2k \pi / 2^m \). The decoding process is implemented by the inverse of quantum Fourier transform, which transforms Eq. (6) into \(|k\rangle O_b |0\rangle_0 \). By measuring the \( m+1 \) qubits, Alice obtains the concrete value of \( \varphi_k \), and know which manipulation Bob has made. In this network, only the flying qubit is manipulated by Bob, and \( 2^{m+1} \) messages can be transmitted from Bob to Alice. When \( m = 1 \), MRQDC returns to the original dense coding, where the flying qubit is exchanged for only one round.
III. IMPLEMENTATION OF MRQDC IN A 3-QUBIT NMR QUANTUM SYSTEM

Our experiments use a sample of Carbon-13 labelled trichloroethylene (TCE) dissolved in d-chloroform. Data are taken at room temperature with a Bruker DRX 500 MHz spectrometer. $^1$H is the flying qubit, denoted by H0. The $^{13}$C directly connecting to $^1$H is denoted as qubit 2, and the other $^{13}$C is denoted as qubit 1. The two carbon nuclei are the stationary qubits, denoted as C2 and C1, respectively. By setting $\hbar = 1$, the Hamiltonian of the three-qubit system is

$$H = -2\pi \nu_1 I^1_z - 2\pi \nu_2 I^2_z - 2\pi \nu_0 I^0_z + 2\pi J_{12} I^1_z I^2_z + 2\pi J_{20} I^2_z I^0_z + 2\pi J_{10} I^1_z I^0_z,$$

where $I^j_z (j = 0, 1, 2)$ are the matrices for the z-component of the angular momentum of the spins. $\nu_1, \nu_2, \nu_0$ are the resonance frequencies of C1, C2 and H0 respectively, and $\nu_1 = \nu_2 + 904.4$ Hz. The coupling constants are measured to be $J_{12} = 103.1$ Hz, $J_{20} = 203.8$ Hz, and $J_{10} = 9.16$ Hz. The coupled-spin evolution between two spins is denoted as

$$[\tau]_{jl} = e^{-i2\pi J_{jl} l^j z l^l z},$$

where $l = 0, 1, 2$, and $j \neq l$. $[\tau]_{jl}$ can be realized by averaging the coupling constants other than $J_{jl}$ to zero [26, 27, 28].

The initial effective-pure state $|000\rangle$ is prepared by spatial averaging [29, 30]. The following radio-frequency (rf) pulse and gradient pulse sequence

$$[\pi/4]^1 x - [1/2 J_{12}]_{12} - [-5\pi/6]^1 y - [\alpha]^0 x - [\text{grad}] z - [\pi/4]^0 y - [9/2 J_{23}]_{23} - [1/2 J_{13}]_{13} - [\pi/4]^0 y - [9/4 J_{23}]_{23} - [1/4 J_{13}]_{13} - [\pi/4]^0 x - [\text{grad}] z,$$

transforms the system from the equilibrium state

$$\rho_{eq} = \gamma_C (I^1_z + I^2_z) + \gamma_H I^0_z,$$

to

$$\rho_0 = I^1_z/2 + I^2_z/2 + I^0_z/2 + I^1_z I^2_z + I^2_z I^0_z + I^1_z I^0_z + 2 I^1_z I^2_z I^0_z,$$

where an overall phase factor has been ignored. $[\pi/4]^1_x$ denotes the $\pi/4$ pulse exciting C1 and C2 simultaneously along x-axis. $[\pi/4]^0_y$ denotes the $\pi/4$ spin-selective pulse for $^1$H along y-axis. $\gamma_C$ and $\gamma_H$ denotes the gyromagnetic ratio of $^{13}$C and $^1$H. $\alpha = \arccos(-\gamma_C \sqrt{6}/\gamma_H).$
$\rho_0$ is equivalent to $|000\rangle$ in NMR experiments $^{31,32,33}$. We find that the compound operations

$$\text{CNOT}_{10}R_{z0}(2\varphi_k)\text{CNOT}_{10} = \left[ -\frac{2\varphi_k}{\pi J_{10}} \right]_{10},$$

(12)

$$\text{CNOT}_{20}R_{z0}(\varphi_k)\text{CNOT}_{20} = \left[ -\frac{\varphi_k}{\pi J_{20}} \right]_{20},$$

(13)

make the network easier to realize. The Hadamard transform simultaneously applied to $C_1$ and $C_2$, denoted by $H_{1,2}$, respectively, is realized by pulse sequence $[-\pi/2]_{y1,2} - [\pi]_{z1,2}$. The inverse of QFT $F^{-1} = H_1H_{(11)}^{-\pi/2}H_2\text{SWAP}$. $H^2$ denotes the Hadamard transform applied to $C_2$, and is realized by $[\pi/4]_{y1,2,0}^1 - [\pi]_z^2 - [-\pi/4]_{y1,2,0}^1$, where $[\pi]_z^2$ denotes the z-pulse, which can be realized by rf pulses $^{27}$. $H_1 = H_1^{1,2}H^2$, using $H^2H^2 = I$. $I_{(11)}^{-\pi/2}$ denotes the controlled-phase shift gate, and is realized by $[1/4J_{12}] - [-\pi/2]_{y1,2}^1 - [\pi/4]_{z1,2}^2 - [\pi/2]_{y1,2}^1$ $^{16,34,35}$. The SWAP gate can be counteracted by another one through optimizing the network $^{13}$.

Our experiments results are represented as NMR spectra obtained by spin-selective readout pulses $[\pi/2]_{y1}$, $[\pi/2]_{y2}$ and $[\pi/2]_{y0}$, for the three spins, respectively. Figs. $3$ (a-c) show the NMR spectra of $C_1$, $C_2$, and $H_0$, when the three-spin system lies in effective-pure state $|000\rangle$, via the readout pulses selective for $C_1$, $C_2$, and $H_0$, respectively. It is seen that only one NMR peak appears in each spectrum. The last gradient pulse in preparing the effective-pure state has killed the non-diagonal elements of the density matrix, the three spectra in Fig. $3$ are sufficient to reconstruct the density matrix of $|000\rangle$ using the state tomography technique. The signals in Figs. $3$ (a-b) are chosen as the reference signals to calibrate the phase of the signals in the following carbon spectra in order to make the phases of NMR signals meaningful $^{36}$.

MRQDC starts with $|000\rangle$. Without loss of generality and considering the convenience in experiments, we demonstrate the case of $O_b = I$. The network shown as Fig. $2$ transforms $|00\rangle_{1,2}|0\rangle_0$ to $|00\rangle_{1,2}|0\rangle_0$, $|01\rangle_{1,2}|0\rangle_0$, $|10\rangle_{1,2}|0\rangle_0$, and $|11\rangle_{1,2}|0\rangle_0$, respectively corresponding to the four different manipulations $R_{z}(0) = I$ (identity manipulation or no manipulation), $R_{z}(-\pi/2)$, $R_{z}(-\pi)$, and $R_{z}(-3\pi/2)$. By applying the spin-selective readout pulses, we obtain the spectra of $C_1$ shown in Figs. $4$ (a-d), and spectra of $C_2$ shown as Figs. $4$ (e-h), corresponding to the above four manipulations, respectively $^{30}$. In experiments, we have noted that if no readout pulse is applied, the amplitudes of peaks in each spectrum is so small that they can be ignored. Due to imperfection of the pulse sequence in NMR, the inhomogeneity in the magnetic field and the decoherence time limit, there are errors in the
experiment. The overall experimental errors are less than 20% (barring Fig. 4 (b) and (h)). The accuracies are acceptable in NMR [16, 18].

The case of $O_b = \sigma_x$ can be implemented in a similar way, where $\sigma_x$ can be realized by pulse $[\pi]_x^0$. The output states are $|00\rangle_{1,2}|1\rangle_0$, $|01\rangle_{1,2}|1\rangle_0$, $|10\rangle_{1,2}|1\rangle_0$, and $|11\rangle_{1,2}|1\rangle_0$, corresponding to the four different manipulations $\sigma_x R_z(0) = \sigma_x$, $\sigma_x R_z(-\pi/2)$, $\sigma_x R_z(-\pi)$, and $\sigma_x R_z(-3\pi/2)$, respectively.

IV. CONCLUSION

The multiple round quantum dense coding has been proposed. It enables two parties to transmit $m + 1$ qubits of information by exchanging only a single qubit in multiple rounds. We have also implemented the protocol in a three-qubit NMR quantum computer. The scheme combines the quantum phase estimation algorithm with the original dense coding protocol. The phase estimation algorithm is a very powerful tool in quantum computing and quantum information processing. It has been successfully used in clock synchronization that outperforms classical counterpart exponentially [37]. By applying the phase estimation algorithm to the dense coding, the information capacity can be enhanced to arbitrary amount of bits in a systematic manner. The elementary operations for encoding can be realized directly through the couplings between the flying qubits and the stationary qubit. Consequently, the scheme is easy to carry out in practice. In practical applications, photons are the natural first choice as the flying qubits. The entangled atom-photon system prepared by Blinov et al [38] is a potential system for the realization of MRQDC, where the photon acts as the flying qubit, because it is easy to be transmitted between Alice and Bob. The $m$ atom qubits act as the stationary qubits. $m + 1$ bits of information can be transmitted through manipulating the photon by Bob, and exchanging it between Alice and Bob. Moreover, one can find that different photons can be used in the different rounds. This fact will greatly simplify the process of the implementation of the scheme.

The quantum advantage of MRQDC mainly lies in the fact that MRQDC transmits messages using quantum manipulations. The information is encoded in the phase of a quantum state described as Eq. (2). The measurement for the flying qubit during the process of transmission does not obtain any information of the message, so that the communication is secure. This property of dense coding has been applied to quantum key distribution in
several quantum key distribution protocols \cite{39, 40}.

V. ACKNOWLEDGEMENT

This work is supported by the National Natural Science Foundation of China under Grant No. 10374010, 60073009, 10325521, the National Fundamental Research Program Grant No. 001CB309308, the Hang-Tian Science Fund, the SRFDP program of Education Ministry of China, and China Postdoctoral Science Foundation. We are grateful for Dr. Fuguo Deng for the fruitful discussions.
[1] C. H. Bennett, and S. J. Wiesner, Phys. Rev. Lett, 69 2881(1992)
[2] http://www.theory.caltech.edu/~preskill/ph229 Chapter 4.
[3] K. Mattle, H. Weinfurter, P. G. Kwiat, and A. Zeilinger, Phys. Rev. Lett. 76, 4656 (1996)
[4] X.- M. Fang , X.- W. Zhu, M. Feng, X.-A Mao, and F. Du Phys. Rev. A 61 022307 (2000)
[5] A. Grudka and Wojcik, Phys Rev A66 014301 (2002)
[6] X.- S. Liu, G.- L. Long, D.- M. Tong and F. Li, Phys. Rev. A, 65, 022304 (2002)
[7] A. Grudka, and A. Wójcik, Phys. Rev. A 66, 014301 (2002)
[8] K. Shimizu, N. Imoto, and T. Mukai, Phys. Rev. A, 59 1092(1999)
[9] J.- T. Jing, J. Zhang, Y. Yan, F.-G. Zhao, C.- D. Xie, and K.- C. Peng, Phys. Rev. Lett, 90, 167903 (2003)
[10] X.- Y. Li, Q. Pan, J.- T. Jing, J. Zhang, C.- D. Xie, and K.- C. Peng, Phys. Rev. Lett, 88, 047904 (2002)
[11] A. Harrow, P. Hayden, and D. Leung, Phys. Rev. Lett, 92, 187901 (2004)
[12] I. P. Degiovanni, I. R. Berchera, S. Castelletto, and M. L. Rastello, Phys. Rev. A, 69 032310 (2004)
[13] M. A. Nielsen, and I. L. Chuang, Quantum computation and quantum information(Cambridge University Press,2000)
[14] I. L. Chuang, N. Gershenfeld, and M. Kubinec. Phys. Rev. Lett. 80, 3408 (1998)
[15] L. M. K. Vandersypen, M. Steffen, G. Breyta, C. S. Yannoni, M. H. Sherwood, and I. L. Chuang, Nature, 414, 883(2001)
[16] Y. S. Weinstein, M. A. Pravia, E. M. Fortunato, S. Lloyd, and D. G. Cory, Phys. Rev. Lett, 86, 1889(2001)
[17] G. L. Long, L. Xiao, J. Chem. Phys. 119, 8473-8481 (2003)
[18] J. S. Lee , J. Kim, Y. Cheong, and S. Lee, Phys. Rev. A, 66, 042316 (2002)
[19] X.-H. Peng, X.-W. Zhu, M. Fang, M.-L. Liu, K.-L. Gao, Phys. Rev. A65 042315 (2002)
[20] M.A. Nielsen, E. Knill and R. Laflamme, Nature 396, 52-55 (1998)
[21] X. M. Fang, X.- W. Zhu, M. Feng, X.- A. Mao, and F. Du, Phys. Rev. A, 61, 022307(2000)
[22] D. X. Wei et al, Chinese Science Bulletin, 48, 2501 (2003)
[23] H. K. Lo, Phys. Rev. A, 62, 012313(2000)
[24] H. K. Lo, Phys. Rev. lett, 83, 1459 (1999)

[25] R. R. Ernst, G. Bodenhausen and A. Wokaum, Principles of nuclear magnetic resonance in one and two dimensions, Oxford University Press (1987)

[26] N. Linden, Ľ. Kupčie, and R. Freeman, Chem. Phys. Lett, 311, 321 (1999)

[27] N. Linden, B. Hervè, R. J. Carbajo, and R. Freeman, Chem. Phys. Lett, 305, 28 (1999)

[28] H. Geen, and R. Freeman, J. Magn. Reson. 93, 93 (1991)

[29] D. G. Cory, M. D. Price, and T. F. Havel, Physica D. 120, 82 (1998)

[30] J.-F. Zhang, Z.-H. Lu, L. Shan, and Z.-W. Deng, Phys. Rev. A 65, 034301 (2002)

[31] E. Knill, I. Chuang, and R. Laflamme, Phys. Rev. A, 57, 3348 (1998)

[32] S. Somaroo, C. H. Tseng, T. F. Havel, R. Laflamme, and D. G. Cory Phys. Rev. Lett, 82, 5381 (1999)

[33] J.-F. Zhang, Z.-H. Lu, L. Shan, and Z.-W. Deng, Phys. Rev. A, 66, 044308 (2002).

[34] G. L. Long et al, Phys. Lett. A 286, 121 (2001)

[35] Zhang J F, Lu Z H, Deng Z W and Shan L 2003 Chinese Physics 12 700

[36] J. A. Jones, in The Physics of quantum Information, edited by D. Bouwmeester, A. Ekert, and A. Zeilinger (Springer, Berlin Heidelberg, 2000) pp.177-189.

[37] I. L. Chuang, Phys. Rev. Lett, 85, 2006 (2000)

[38] B. B. Blinov, D. L. Moehring, L.- M. Duan and C. Monroe, Nature, 428, 153 (2004)

[39] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, Rev. Mod. Phys, 74, 145 (2002)

[40] F. -G. Deng, and G. L. Long, Phys. Rev. A, 69, 052319 (2004)
FIG. 1: The usual dense coding scheme, modified slightly for its generalization. The horizontal lines denote qubits. $|0\rangle$ denotes the spin up state. $H$ denotes the Hadamard transform. $R_z(\varphi_k)$ denotes a phase manipulation, and $R_z(\varphi_k) = e^{i\varphi_k I_z}$, where $\varphi_k = -k\pi$ ($k = 0, 1$). $O_b$ denotes a bit manipulation, which can be chosen as $I$, or $\sigma_x$. $F^{-1}$ denotes the inverse of quantum Fourier transform. $F^{-1} = H$ for the case of one qubit. Time goes from left to right. $|x_1\rangle_1|x_0\rangle_0$ is the output state, which can be obtained through measuring qubits 1, and 0.

FIG. 2: The quantum network to implement the multiple round quantum dense coding, obtained through generalizing the scheme shown as Fig. 1. $R_z(2^{m-j}\varphi_k) = e^{i2^{m-j}\varphi_k I_z}$, where $\varphi_k = -2k\pi/2^m$ ($k = 0, 1, 2, \cdots, 2^m - 1$). $|x_1\rangle_1 \cdots |x_j\rangle_j \cdots |x_m\rangle_m|x_0\rangle_0$ is the output state, which can be obtained through measuring qubits 1, 2, $\cdots$, $m$, and qubit 0.
FIG. 3: The NMR spectra of C1, C2, and H0, shown as Figs. (a-c), when the three-spin system lies in effective pure state |000⟩ via readout pulses selective for C1, C2, and H0, respectively. The amplitude has arbitrary units. When the system lies in |000⟩, only one NMR peak appears in each spectrum if a spin selective readout pulse is applied.
FIG. 4: The spectra of C1 (shown by the left column) and C2 (shown by the right column) obtained through $[\pi/2]_y^1$ for C1 and $[\pi/2]_y^2$ for C2 after the completion of multiple round quantum dense coding (MRQDC). Figs. (a-d) and Figs. (e-h) correspond to states $|00\rangle_{1,2}|0\rangle_0$, $|01\rangle_{1,2}|0\rangle_0$, $|10\rangle_{1,2}|0\rangle_0$, and $|11\rangle_{1,2}|0\rangle_0$, respectively.