A New Approach to Quintessence and a Solution of Multiple Attractors

Shuang-Yong Zhou

Department of Physics, Shandong University, Jinan, 250100, P. R. China
Email:zhsousy@mail.sdu.edu.cn

We take a new approach to construct Quintessential models. With this approach, we first easily obtain a tracker solution that is different from those discovered before and straightforwardly find a solution of multiple attractors, i.e., a solution with more than one attractor for a given set of parameters. Then we propose a scenario of Quintessence where the field jumps out of the scaling attractor to the de-Sitter-like attractor, by introducing a field whose value changes a certain amount in a short time, leading to the current acceleration. We also calculate the change the field needs for a successful jump and suggest a possible mechanism that involves spontaneous symmetry breaking to realize the sudden change of the field value.

Recent observations and experiments strongly indicate that the universe is spatially flat and currently undergoing accelerated expansion [1, 2, 3]. A negative pressure energy component, termed dark energy, is suggested to be responsible for the acceleration. The simplest candidate for dark energy seems to be a positive cosmological constant, which is conventionally associated with the quantum vacuum energy. However, it is very tiny, compared with typical particle physics scales, which is the so-called fine-tuning problem [4]. It also suffers the so-called coincidence problem [5]. Rather than dealing directly with the dark energy a cosmological constant, various alternative routes have been proposed, which usually invoke dynamical scalar fields, such as Quintessence [6, 7, 8, 9, 10], Phantom [11] and Quintom [12].

Quintessence invokes an evolving canonical scalar field slowly rolling down its potential (to some extent like the inflaton which drives the inflation in the early universe) with equation of state $w = p/\rho < -1$. Motivated from observational data [13, 14], the Phantom invokes a negative kinetic energy with effective equation of state $w_{\phi} < -1$, having led to many interesting phenomena [15].

Among the various Quintessential models, tracker solutions have attracted a lot of attention. The tracker field has an equation of motion with attractor-like solutions in the sense that a very wide range of initial conditions rapidly converge to a common, cosmic evolutionary track of $\rho_{\phi}(t)$ and $w_{\phi}(t)$. The tracking behavior with $w_{\phi} < w_m$ occurs when $\Gamma > 1$ and is nearly constant $(\text{d}(|\Gamma - 1|)/\text{d} \ln \alpha \ll |\Gamma - 1|)$, where $\Gamma$ is defined as $V''/V^2$, with $V$ the potential and $'$ the derivative w.r.t. the field [16]. It has been found that the general inverse power-law ($V(\phi) = \sum c_\alpha/\phi^\alpha$) and exponential ($V(\phi) = V_0 \exp(1/\phi)$) potentials are tracker solutions (we have chosen $\kappa^2 = 8\pi G = 1$).

Another important class of Quintessential models are scaling solutions [17, 18, 19, 20, 21] in which the energy density of the scalar field mimics the background fluid energy density. Namely scaling solutions are characterized by the relation $\rho_{\phi} \propto \rho_m$, whose simplest realization is the exponential potential $V_0 e^{-\mu \phi}$. As long as the scaling solution is the dynamical attractor, for any generic initial conditions, the field would sooner or later enter the scaling regime, being sub-dominant during radiation and matter dominated eras to satisfy the tight constraints from nucleosynthesis and structure formation, thereby opening up a new line of attack on the fine-tuning problem [22]. However, exit from the scaling regime is needed so as to give rise to recent acceleration.

The double exponential potential [8, 23] of the form

$$V(\phi) = V_0 \left( e^{-\mu \phi} + e^{-\nu \phi} \right),$$

provides a simple realization of the exit from the scaling regime. Such potentials can arise as a result of compactifications in superstring models. By properly choosing $\mu$ and $\nu$ and initial conditions, one term in the potential dominated over the other before nucleosynthesis, giving rise to the scaling solution, while the situation has reversed recently, giving rise to a de-Sitter-like acceleration. However, whether it is possible to obtain required values of $\mu$ and $\nu$ remains a problem. In [9], the authors considered the potential

$$V(\phi) = V_0 \left[ \cosh(\mu \phi) - 1 \right]^n,$$

which has two interesting asymptotical regions. One of these with $(|\mu \phi| \gg 1, \phi < 0)$ gives the scaling solution, while the other with $(|\mu \phi| \ll 1)$, according to virial theorem, gives current acceleration with average equation of state $w_{\phi} = (n - 1)/(n + 1)$. As current data favor an equation of state close to $-1$ [2], $n$ should be close to 0, which is mathematically viable, but seems unnatural physically. See [24, 25] for another two popular models.

On the other hand, there is an attempt to search for a solution of two scaling regimes by coupling Quintessence to the matter [26]. Nonetheless, this scenario has faced severe challenges since it has been showed that it cannot be realized for a vast class of scalar field Lagrangians [27].

In this letter, we take a new approach to construct Quintessential models. With this approach, instead of proposing an interesting Quintessential potential directly, we first propose a relation between two quantities, $\Gamma$ and $\lambda$ (defined as $-V'/V$), and then figure out the potential. First, we show that a tracker potential which is different from that discovered before can be easily obtained. Then we find it straightforward to get a solution of multiple attractors, that is, a solution with more than one
attractor for a given set of parameters. In the particular case given in this letter, we have a scaling attractor and a de-Sitter-like attractor. We thus propose a model in which the universe first evolves to the scaling attractor, and then, by introducing a field whose value changes a certain amount in a short time, the universe jumps out to the de-Sitter-like attractor to give the current acceleration. We also calculate the change the field needs for a successful jump and justify the introduction of this kind of field.

To start, we consider the action of Quintessence ($\epsilon = 1$) (or Phantom ($\epsilon = -1$)) minimally coupled to gravity,

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{2} \nabla(\phi)^2 - V(\phi) \right],$$  \hspace{1cm} (3)

where we use the metric signature $(-,+,+,+)$ and $(\nabla(\phi))^2 = g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$. In the flat Friedmann-Robertson-Walker spacetime, the equation of state for the Quintessential field $\phi$ is given by

$$w_\phi = \frac{\rho_\phi}{\rho} = \frac{\epsilon \dot{\phi}^2 - 2V(\phi)}{\epsilon \dot{\phi}^2 + 2V(\phi)}. \hspace{1cm} (4)$$

The variation of the action (3) with respect to $\phi$ gives

$$\dot{\phi} + 3\epsilon H \dot{\phi} + V' = 0. \hspace{1cm} (5)$$

Since we carry out cosmological dynamics of the Quintessential field $\phi$ in the presence of a barotropic fluid whose equation of state is given by $w_m = p_m / \rho_m$ (in this paper, we assume that $w_m$ is constant), Einstein equations reduce to

$$H^2 = \frac{1}{3} \left[ \frac{\epsilon \dot{\phi}^2 + V(\phi) + \rho_m}{\epsilon \dot{\phi}^2} \right], \hspace{1cm} (6)$$

$$\dot{H} = -\frac{1}{2} \left[ \epsilon \dot{\phi}^2 + (1 + w_m)\rho_m \right]. \hspace{1cm} (7)$$

Introducing the following dimensionless variables

$$x \equiv \frac{\dot{\phi}}{\sqrt{6} \dot{H}}, \hspace{0.5cm} y \equiv \sqrt{\frac{V}{3H}},$$

$$\lambda \equiv -\frac{V'}{V}, \hspace{0.5cm} \Gamma \equiv \frac{V''}{V'^2}, \hspace{1cm} (8)$$

Eq. (5), (6), (7) can be recast in the following form [17, 22, 28]:

$$\frac{dx}{dN} = -3x + \frac{\sqrt{6}}{2} \epsilon \lambda y^2$$

$$+ \frac{3}{2} [(1 - w_m)\epsilon x^2 + (1 + w_m)(1 - y^2)], \hspace{1cm} (9)$$

$$\frac{dy}{dN} = -\frac{\sqrt{6}}{2} \lambda xy$$

$$+ \frac{3}{2} y[(1 - w_m)\epsilon x^2 + (1 + w_m)(1 - y^2)], \hspace{1cm} (10)$$

$$\frac{d\lambda}{dN} = -\sqrt{6} \lambda \lambda (\Gamma - 1)x, \hspace{1cm} (11)$$

where $N = \ln a$ ($a$ is the scale factor), together with a constraint equation

$$\epsilon x^2 + y^2 + \frac{\rho_m}{3H^2} = 1. \hspace{1cm} (12)$$

The equation of state $w_\phi$ and the fraction of the energy density $\Omega_\phi$ for the field $\phi$ are, respectively,

$$w_\phi \equiv \frac{\rho_\phi}{\rho} = \frac{\epsilon x^2 - y^2}{\epsilon x^2 + y^2}, \hspace{1cm} (13)$$

$$\Omega_\phi \equiv \frac{\rho_\phi}{3H^2} = \epsilon x^2 + y^2. \hspace{1cm} (14)$$

To warm up, we note that for many Quintessential (or Phantom) potentials $\lambda$ can be written as a function of $\lambda$. Let us take the case of Phantom potential of the form

$$V(\phi) = \frac{V_0}{\cosh(\tau_0 \lambda)}, \hspace{1cm} (15)$$

for example. It is found

$$\Gamma = 1 + \frac{1}{n} - \frac{n \tau^2}{\lambda^2}. \hspace{1cm} (16)$$

Substituting Eq. (15) into Eq. (11), we can perform three-dimension dynamical analysis of the autonomous system. For a barotropic fluid background, there is a unique stable fixed point ($x = 0$, $y = 1$, $\lambda = 0$), which is a de-Sitter-like dominant attractor. For the case $n = 1$, it confirms the results of [29]. Note that we neglect the cases with $y < 0$, as the system is symmetric under the reflection $(x, y) \rightarrow (x, -y)$ and time reversal $t \rightarrow -t$.

The direct way to get a Quintessential (or Phantom) model is to conceive (usually fairly carefully) a potential that meets constraints from observations and experiments. However, encouraged by what has been showed above, let us take another route.

We note that the dynamical system (22, 28) is obviously autonomous except for $\Gamma$. In fact, since the potential $V(\phi)$ is only a function of the field $\phi$, by the definition (3), $\lambda$ and $\Gamma$ can be written as

$$\lambda = P(\phi), \hspace{1cm} \Gamma = Q(\phi). \hspace{1cm} (17)$$

If the inverse function of $P(\phi)$ exists, then we have

$$\Gamma = Q(P^{-1}(\lambda)) \equiv f(\lambda). \hspace{1cm} (18)$$

It is noteworthy that in principle we can figure out the potential as a function of the field $\phi$. Using the definition of $\lambda$ and $\Gamma$, Eq. (15) can be rewritten as

$$V'' = \frac{V^2}{V} f\left(-\frac{V'}{V}\right) \equiv F(V, V'). \hspace{1cm} (19)$$

Let $h = V'$, then we get

$$\frac{dh}{dV} = \frac{1}{h} F(V, h). \hspace{1cm} (20)$$
Having figured out \( h(V) \), we can solve \( V'(\phi) = h(V(\phi)) \) to obtain the potential \( V(\phi) \). Thus we can perform three-dimension dynamical analysis of the system with a fairly large amount of potentials beyond the exponential case where the dynamical system reduces to two-dimension autonomous system.

In the viewpoint of \( \Gamma \) as a function of \( \lambda \) and considering the powerful theorem presented by [16], it is easy to obtain a tracker field. As an example, we write \( \Gamma \) as

\[
\Gamma = 1 + \frac{\alpha}{\lambda},
\]

which can be solved to give the potential

\[
V(\phi) = V_0 e^{\frac{-\alpha}{\lambda} (\phi + \beta)},
\]

where \( V_0 > 0 \) and \( \beta \) are integral constants. We note that it is different from the general inverse power-law \( (V(\phi) = \sum c_{\alpha}/\phi^\alpha) \) or exponential \( (V(\phi) = V_0 \exp(1/\phi)) \) potentials.

Obviously, \( \Gamma > 1 \) if \( \alpha > 0 \). To confirm this is a real tracker solution, we perform the condition \( d(|\Gamma - 1|)/dN \ll |\Gamma - 1| \), and we get

\[
|2\frac{d\lambda}{dN}| \ll 1.
\]

Substituting Eq. (11) for Eq. (23), we obtain

\[
|\alpha x| \ll |\lambda|.
\]

Considering the tracking condition \(|\lambda| \gg |1/x|\) [10], we finally get

\[
\frac{\alpha}{\lambda^2} \ll 1.
\]

Tracking behaviors exist for a wide range parameters and initial conditions, which solves the fine-tuning problem. However, due to the \( w-\Omega \) relation that alleviates the coincidence problem, it is difficult to obtain current equation of state \( w_0 < -0.8 \). A numerical solution of the cosmic dynamical evolution with tracking behavior, where for simplicity we have neglected the matter-dominated era, is given by Fig. 1.

Having witnessed the utility of this approach, we would like to go further. As showed below, we find it straightforward to parameter \( \Gamma \) as function of \( \lambda \) to get a solution of multiple attractors, i.e., a solution with more than one attractor for a given set of parameters. In the particular case given below, we have a scaling attractor and a de-Sitter-like attractor and it is worth noting that two scaling solutions are problematic [27]. Thus we are encouraged to consider a scenario that the initial conditions of the cosmic scalar field are in the basin of a scaling solution and first the field evolves toward the scaling attractor. Then recently, the field jumps out to the basin of a de-Sitter-like dominant attractor, giving rise to the current acceleration. In this scenario, the mechanism of exit from the scaling regime is different from those mentioned in the introduction, which typically invoke fairly carefully conceived potentials with two asymptotical behaviors corresponding to the scaling case and the de-Sitter-like case respectively. Therefore attractors in those models are not exact. On the contrary, the two attractors considered below are exact and we suggest some other physical reason to realize the exit from the scaling regime, rather than connect the two interesting regimes with more or less contrived potentials. The physical reason is formulated as the sudden change of the field value.

Considering Eq. (11), we parameter \( \Gamma \) as

\[
\Gamma = 1 + 1/\beta + \frac{\alpha}{\lambda}.
\]

There are at least the following two fixed points:

- **Point(a):**
  \[
  (x = -\frac{\sqrt{6}\alpha\beta}{2}, \quad y = \frac{1 - \alpha^2 \beta^2/6}{2}, \quad \lambda = -\alpha\beta) \]
  is a de-Sitter-like dominant attractor, in which \( w_\phi = -1 + \alpha^2 \beta^2/3 \). The eigenvalues of the Jacobi matrix of the dynamical system are
  \[
  \mu_1 = -\alpha^2 \beta, \quad \mu_2 = -3 + \frac{\alpha^2 \beta^2}{2}, \quad \mu_3 = -3(1 + w_m) + \alpha^2 \beta^2.
  \]
  It exists if \( \alpha^2 \beta^2 < 6 \) and is stable if \( \alpha^2 \beta^2 < 3(1 + w_m) \) and \( \beta > 0 \).

- **Point(b):**
  \[
  (x = -\frac{\sqrt{3}(1 + w_m)}{2\alpha\beta}, \quad y = \frac{\sqrt{3}(1 - w_m^2)}{2\alpha^2 \beta^2}, \quad \lambda = -\alpha\beta) \]
  is a scaling attractor, in which \( w_\phi = w_m \) and \( \Omega_\phi = 3(1 + w_m)/\alpha^2 \beta^2 \). The eigenvalues of the
FIG. 2: Evolution of $x$ (red solid line), $y$ (green dot-dashed line), $\lambda$ (blue dashed line) of $\Gamma = 1 + 1/\beta + \alpha/\lambda$ with respect to $N = \ln a$ in the background fluid with $w_m = 0$. $\alpha$ is chosen as $-2.6$, $\beta$ chosen as $2$. We choose initial conditions as $x_i = 0.2$, $y_i = 0.36$ and $\lambda_i = -1.3$ for the thin lines, and $x_i = 0$, $y_i = 0.06$ and $\lambda_i = -1.4$ for the thick lines. Note that the attractor is a stable spiral.

The corresponding potential of this case is

$$V(\varphi) = \frac{V_0}{(\eta + e^{-\alpha\varphi})^\beta},$$

where $V_0(>0)$ and $\eta$ are integral constants. For the stability of the potential, that is, the potential should be bounded below, we should choose $\beta$ as $2,4,6,...$ and to obtain interesting cases we choose $\eta < 0$.

Now we come to consider the scenario that the field exits the scaling regime to the de-Sitter-like regime due to a sudden change of the field value. To this end, we conceive $\varphi$ as

$$\varphi = f(t)\varphi,$$

with

$$f(t) = \begin{cases} 1 & t < t_j \\ f_j & t \geq t_j \end{cases},$$

where $t$ is the cosmic time and $f_j$ is a constant with the subscript $j$ representing some recent time when the field jumps. Note that $f(t)$ is not necessarily of the form above, but it should have a certain amount of change of its value in a short time so that the field will not evolve back to the scaling attractor. We will first calculate the change the field needs for a successful jump and then suggest a possible mechanism to realize the sudden change of the field value.

In order to calculate the change the field needs to realize the jump, we choose, without losing generality, $\alpha > 0$ so that the region of $\lambda > 0$ is the basin of the de-Sitter-like dominant attractor ($c$). To meet the constraints from nucleosynthesis and structure formation, we require that the Quintessential field have well scaled with the background before nucleosynthesis. So we choose $\alpha^2\beta^2 > 20$ for $(\Omega_\varphi(T \sim 1\text{ MeV}) < 0.2)$. At some recent time just before the jump the field $\varphi = \phi_j = \varphi_j$ and then $\varphi$ rapidly changes from $\phi_j$ to $\phi_j + \delta \varphi$ (or from $\varphi_j$ to $f_j\varphi_j$). For jumping from the basin of the scaling attractor ($b$) to that of the de-Sitter-like dominant attractor ($c$) happening, we have

$$\delta \varphi > -\frac{1}{\alpha} \ln(-\eta) - \phi_j = \frac{1}{\alpha} \ln \frac{\lambda_j}{\lambda_j + \alpha \beta}$$

where $\lambda_j = -\alpha \beta/(1 + \eta e^{\alpha \varphi_j})$, or

$$f_j\varphi_j > -\frac{1}{\alpha} \ln(-\eta).$$

Now we shall justify the introduction of the field whose value has a sudden change. Below, we suggest a possible mechanism, which resorts to spontaneous symmetry breaking, to realize the sudden change of the field value. We first note that scalar fields are ubiquitous in supersymmetric theory of particle physics. Thus it is reasonable to assume that a few of them are relevant to the cosmic evolution. Considering $\varphi$ as an effective field, we involve three fields with the Lagrangian

$$\mathcal{L} = -\frac{1}{2} f^2(\sigma) g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{V_0}{(\eta + e^{-\alpha f(\sigma)})^\beta}$$

$$-\frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - V(\sigma, \theta),$$

where $\sigma$ and $\psi$ are two other scalars.
so that \( \phi \) is a continuous function, a better choice of \( \theta \) dominates the universe, the coupling through gravity is negligible. Since \( f(\sigma) \) is more reasonable to be a continuous function, a better choice of \( f(\sigma) \) might be

\[
 f(\sigma) = \begin{cases} 1 & \sigma^2 < \sigma^2_0 \\ f_j & \sigma^2 \geq \sigma^2_0, \end{cases}
\]

so that \( \varphi \) decouples from \( \sigma \) and \( \theta \) except at the points \( \sigma = \pm \sigma_0 \) (they surely couple to each other through Friedmann equation; nevertheless, when the radiation or matter dominates the universe, the coupling through gravity is negligible.). Since \( f(\sigma) \) is more reasonable to be a continuous function, a better choice of \( f(\sigma) \) might be

\[
 f(\sigma) = \frac{1 + f_j}{2} - \frac{1 - f_j}{2} \tanh[a(\sigma^2 - \sigma^2_0)], \quad a \gg 1.
\]

We note that \( V(\sigma, \theta) \) is famous for its realization of Hybrid inflation models [31]. In these models, first, the field evolves toward the scaling attractor and first, the field evolves toward the scaling attractor and then, when \( \theta \) rolls down a critical value \( \theta_c \), \( \sigma \) is destabilized and quickly rolls down from 0 to \( \pm M \), ending the inflation. Comparing two ways of writing the potential, we obtain

\[
 m^2_\sigma = \lambda M^2, \quad V_0' = \frac{1}{4} \lambda M^4.
\]

And the critical value of \( \theta \) is

\[
 \theta^2_c = m^2_\sigma / \lambda' = \lambda M^2 / \lambda'.
\]

For this potential to be viable for current purpose, \( \theta \) does not necessarily slow roll. Yet we do need \( \sigma \) quickly roll down from 0 to \( M \) after \( \theta \) rolls down \( \theta_c \), which implies

\[
 m^2_\sigma \gg V_0'.
\]

and we require \( 0 < \sigma^2 < M^2 \) so that when \( \sigma \) rolls down from 0 to \( \pm M \), \( f(\sigma) \) changes from 1 to \( f_j \).

Note that when the field \( \phi \) jumps to the de-Sitter-like regime, \( V(\phi = f(\sigma)) \) will increase and the kinetic term of \( \varphi \) will also change. At the same time, \( V(\sigma, \theta) \) should decrease so as to vanish when Quintessence begins to dominate the universe. For this scenario to be viable, we require that the decrease of \( V(\sigma, \theta) \) be larger than the increase of \( V(\phi = f(\sigma)) \) and the kinetic term (note that when \( |f_j| < 1 \), the kinetic term will decrease; however, it is easy to show that the decline of the kinetic term in this case is small, compared to the increase of the \( V(\phi = f(\sigma)) \)). One may worry that this might spoil the analysis of the dynamics of \( \phi \) above, as this require the energy associated with \( \sigma \) and \( \theta \) to be comparable with that associated with \( \varphi \) around the jump point. However, we argue that it will not, because \( \varphi \) almost decouples to \( \sigma \) and \( \theta \), and the energy associated with \( \sigma \) and \( \theta \) is only comparable with that associated with \( \varphi \) when the radiation or matter dominates the universe and vanishes when the dark energy begins to dominate the universe.

In summary, we suggest a new approach to construct Quintessentail dark energy models, with which we first propose a relation \( \Gamma = f(\lambda) \) between \( \Gamma = VV'/V' \) and \( \lambda = -V'/V \), and then figure out the potential \( V(\phi) \). It is showed that a tracker solution that is different from those discovered before can be easily obtained and a solution of multiple attractors is also found straightforwardly. Then we suggest a scenario that the initial conditions of the cosmic scalar field are in the basin of a scaling attractor and first, the field evolves toward the scaling attractor and then recently, the field jumps out to the basin of a de-Sitter-like dominant attractor, giving rise to current acceleration. For this scenario to be realized, we introduce a field whose value changes a certain amount in a short time. Then we calculate the change the field needs for a successful jump and invoke a mechanism that is similar to the case of Hybrid inflation to justify the introduction of this kind of field.

We thank Yun-Song Piao, Yi Wang, Zuo-Tang Liang, Jian-Hua Gao and Ye Chen for useful discussions.

[1] A. G. Riess et al., Astron. J. 116, 1009 (1998); S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
[2] C. L. Bennett et al., Astrophys. J. Suppl. 148, 1 (2003); D. N. Spergel et al., Astrophys. J. Suppl. 148, 175 (2003); D. N. Spergel et al., arXiv:astro-ph/0603449.
[3] M. Tegmark et al., Phys. Rev. D 69, 103501 (2004); K. Abazajian et al., Astron. J. 128, 502 (2004); E. Hawkins et al., Mon. Not. Roy. Astron. Soc. 346, 78 (2003).
[4] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[5] I. Zlatev, L. -M. Wang, and P. J. Steinhardt, Phys. Rev. Lett. 82, 896 (1999).
[6] C. Wetterich, Nucl. Phys. B 302, 668 (1988).
[7] B. Ratra and J. Peebles, Phys. Rev. D 37, 321 (1988).
[8] T. Barreiro, E. J. Copeland and N. J. Nunes, Phys. Rev. D 61, 127301 (2000).
[9] V. Sahni and L. M. Wang, Phys. Rev. D 62, 103517 (2000).
[10] A. Albrecht and C. Skordis, Phys. Rev. Lett. 84, 2076 (1999).
[11] R. R. Caldwell, Phys. Lett. B 545, 23-29 (2002).
[12] B. Feng, X. L. Wang and X. M. Zhang, Phys. Lett. B 607, 35 (2005); Z. K. Guo, Y. S. Piao, X. M. Zhang and Y. Z. Zhang, Phys. Lett. B 608, 177 (2005); H. Wei, R. G. Cai and D. F. Zeng, Class. Quant. Grav. 22, 3189 (2005); X. F. Zhang, H. Li, Y. S. Piao and X. M. Zhang, Mod. Phys. Lett. A 21, 231 (2006); H. Wei and R. G. Cai, Phys. Lett. B 634, 9 (2006).
[13] P. S. Corasaniti, M. Kunz, D. Parkinson, E. J. Copeland and B. A. Bassett, Phys. Rev. D 70, 083006 (2004).
[14] U. Alam, V. Sahni, T. D. Saini and A. A. Starobinsky, Mon. Not. Roy. Astron. Soc. 354, 275 (2004).
[15] Z. K. Guo, Y. S. Piao and Y. Z. Zhang, Phys. Lett. B 594, 247 (2004); T. Chiba, JCAP 0503, 008 (2005); J. Q. Xia, B. Feng and X. M. Zhang, Mod. Phys. Lett. A 20, 2409 (2005);
[16] P. J. Steinhardt, L. M. Wang and I. Zlatev, Phys. Rev. D 59, 123504 (1999).
[17] E. J. Copeland, A. R. Liddle and D. Wands, Phys. Rev. D 57, 4686 (1998).
[18] R. J. van den Hoogen, A. A. Coley and D. Wands, Class. Quant. Grav. 16, 1843 (1999).
[19] A. de la Macorra and G. Piccinelli, Phys. Rev. D 61, 123503 (2000).
[20] A. Nunes and J. P. Mimoso, Phys. Lett. B 488, 423 (2000).
[21] Y. Gong, A. Wang and Y-Z. Zhang, Phys. Lett. B 636, 286 (2006), arXiv: gr-qc/0603050.
[22] Edmund J. Copeland, M. Sami and Shinji Tsujikawa, Int. J. Mod. Phys. D 15, 1753 (2006), arXiv: hep-th/0603057.
[23] A. A. Sen and S. Sethi, Phys. Lett. B 532, 159 (2002);
I. P. Neupane, Class. Quant. Grav. 21, 4383 (2004);
I. P. Neupane, Mod. Phys. Lett. A 19, 1093 (2004);
L. Jarv, T. Mohaupt and F. Saueressig, JCAP 0408, 016 (2004).
[24] L. A. Urena-Lopez and T. Matos, Phys. Rev. D 62, 081302 (2000).
[25] A. Albrecht and C. Skordis, Phys. Rev. Lett. 84, 2076 (1999).
[26] L. Amendola, Phys. Rev. D 62, 043511 (2000).
[27] L. Amendola, M. Quartin, S. Tsujikawa and I. Waga, Phys. Rev. D 74 (2006) 023525, arXiv:astro-ph/0605488.
[28] S. C. C. Ng, N. J. Nunes and F. Rosati, Phys. Rev. D 64, 083510 (2001).
[29] P. Singh, M. Sami and N. Dadhich, Phys. Rev. D 68, 023522 (2003).
[30] S. Tsujikawa, Phys. Rev. D 73, 103504 (2006).
[31] A. D. Linde, Phys. Lett. B 259, 38 (1991).