DYNAMICAL SPIN II

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ABSTRACT

The possibility of building all particles from spinless constituents is explored. Composite fermions are formed from bosonic carriers of electric and magnetic charge of a composite abelian gauge field. Internal attributes are accounted for by dimensional reduction from a higher-dimensional space-time in which the abelian gauge field is replaced by a composite higher-rank antisymmetric tensor field. The problem of building magnetically neutral fermions is considered.

It is with great sadness that I dedicate this paper to the memory of my friend Wolfgang Kummer. Wolfgang and I met during our student days in Vienna, and as fate would have it, we both went to Geneva for our first post-doctoral appointment. There we collaborated on the paper "The phases of the proton’s electromagnetic form factors in the time-like region", Nuovo Cimento 24, 1160 (1962).

During some work Wolfgang and I did in those early computer-era days, we were asked to use the computing facilities sparingly. For numerical integration we therefore availed ourselves of the services of that marvelous one-of-a-kind CERN employee, Mr. Klein. This Mr. Klein could perform complex arithmetic operations in his head. A Holocaust survivor, he had used this unusual ability to earn his living in a circus in the postwar years. There he was spotted and recruited by CERN. Mr. Klein calculated our integrals to such accuracy, that in the end, the computer’s streamlined task was reduced to not much more than confirming his estimates.

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I cannot resist mentioning here another bond between Wolfgang and me: since we both grew up in Viennas, he in the Austrian capital and I in the Romanian city of Timișoara, the Habsburg Empire’s former “Little Vienna,” we both developed keen musical interests, we both sang, we both were baritones, and we both gave recitals with mezzo-sopranos. Unlike me though, Wolfgang had as his singing partner none other than that wonderful mezzo-soprano Helga Dernesch, who after singing with him was destined to become a major star of the Wiener Staatsoper.

To this memorial volume I decided to contribute a paper I wrote in 1981, which however may still be of some interest today. It has been available on SPIRES (EFI-81/07-CHICAGO, Feb 1981) for over a quarter of a century, but due to all kinds of complications it has never been published before. I chose this paper, because I clearly recall a discussion with Wolfgang about the ideas contained in it. In fact, his interest in this paper prompted me to give him the preprint of the original 1981 version reproduced below without any changes.
1. INTRODUCTION

The search for a simple way of accounting for the observed particle spectrum and interactions has led to ever more remote constituent and subconstituent models [1]. In order to account for the observed fermions it is usually assumed that some or all of the constituents are themselves fermions, and thus carry half-odd-integer spin. Here we wish to explore the opposite case where none of the constituents carry spin so that all angular momentum is of dynamical origin. Spinless bosons can bind into bosonic states of integer angular momentum. If amongst these components there are gauge bosons, then we have the possibility of nontrivial topological objects that carry magnetic charge. Together with electrically charged objects we then have the ingredients to build spinorial fermions [2, 3]. For such a picture to make even remote phenomenological sense, a considerable "attribute" (i.e., flavor, color, etc) proliferation at the level of the electrically and magnetically charged constituents seems to be required. An elegant way to avoid such a proliferation is provided by higher-dimensional Kaluza-type theories [4]. Yet the idea of building fermions from electric and magnetic charges relies heavily on a 4-dimensional space-time. To extend this idea to higher dimensions we propose to replace the abelian vector gauge field of 4 dimensions by gauge fields of higher (totally antisymmetric) tensorial rank [5]. The corresponding carriers of electric and magnetic charge are then not point particles but extended objects, as we shall see. Since abelian structures are natural in this context, the nonabelian gauge fields of electroweak and strong interactions are to be viewed as composites. Both Bose and Fermi composites being possible, dynamical supersymmetry may also arise.

2. MAGNETIC CHARGES IN HIGHER DIMENSIONAL SPACES

Consider a Minkowski space $M_d$ with one time- and $d-1$ space-dimensions. Define over $M_d$ a rank-$n$ antisymmetric tensor potential $A_{\mu_1 \ldots \mu_n}$ ($n \leq d-1$) or, equivalently, the $n$-form $A = A_{\mu_1 \ldots \mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \wedge ... dx^{\mu_n}$. The field strengths are the components of the $n+1$-form $F = dA$. It’s dual $*F$ is a $d-n-1$ form. Introducing the "electric" current $n$-form $J$ and the "magnetic" current $d-n-2$-form $K$, the field equations are

$$d * F = * J, \quad d F = * K.$$
The \(n\)-form \(J\) can be restricted to "live" on a \((d_e + 1)\)-dimensional submanifold of \(M_d\), provided \(d_e + 1 \geq n\). We shall consider here the "minimal" case \(d_e + 1 = n\), and specifically that one of the dimensions of the submanifold is time-like (a proper-time) and \(d_e\) are space-like. At any proper-time the support of the electric charge is then \(d_e\)-dimensional. Similarly, the support of magnetic charge has at least \(d_m = d - n - 3\) dimensions. Notice that

\[
d_e + d_m = d - 4,
\]

so that both pointlike electric and magnetic charges are possible only in 4-dimensions. In general \(d_e \neq d_m\), but in every even dimension there exists an electric-magnetic-dual case in which \(F\) and \(*F\) are both \(\frac{d}{2}\) forms, so that \(n = \frac{d}{2} - 1\) and \(d_e = d_m = \frac{d-4}{2}\). It is this electric-magnetic-dual case that interests us here.

At this point we want to make precise what we mean by an electric or a magnetic field configuration and to find the counterparts of the Coulomb-electric and Dirac-magnetic (monopole) potentials. To this effect we first consider a static configuration such that at all times the support of \(J\) is the \((\frac{d}{2} - 1)\)-hyperplane (our results are obviously generalizable to other \(J\)-supports)

\[
x^1 = x^2 = \ldots = x^{\frac{d}{2}+1} = 0.
\]

Here it is worthwhile to streamline our notation. The last coordinate \(x^d\) is designated as time, the metric signature is thus \((-\ldots-+)\). Indices that range from 1 to \(\frac{d}{2} + 1\) (from \(\frac{d}{2} + 2\) to \(d\)) will be designated by letters from the beginning (middle) of the latin alphabet \(a, b, c, \ldots (m, n, p, \ldots)\). Thus, e.g., the hyperplane equation (2) becomes \(x^a = 0\). A set of totally antisymmetrized indices of either type will be indicated in a generic way by a square bracket containing one of them. Specifically, \([a]\) means \(a_1a_2\ldots a_{\frac{d}{2}+1}\) with all a’s ranging from 1 to \(\frac{d}{2} + 1\), and \([m]\) means \(m_1m_2\ldots m_{\frac{d}{2}-1}\) with all m’s ranging from \(\frac{d}{2} + 2\) to \(d\). Finally, the Levi-Civita symbol for the first \(\frac{d}{2} + 1\) (last \(\frac{d}{2} - 1\)) indices will be written as \(\epsilon_{[a]bc} (\epsilon_{[m]}\)

With this notation the only nonvanishing components of \(J\) in our static situation (2) are given by

\[
J_{[m]} = \epsilon_{[m]} \frac{e}{\Omega_{\frac{d}{2}}} \delta(x^1) \ldots \delta(x^{\frac{d}{2}+1})
\]
where $\Omega_d$ is the $\frac{d^2}{2}$-dimensional total solid angle (area of unit $\frac{d}{2}$-sphere: $\Omega_2 = 4\pi...$). The field equations then yield

$$A_{[m]} = \frac{e}{r^{\frac{d}{2}-1}} \left( \frac{-2}{d-2} \right), \quad (3b)$$

with

$$r^2 = (x^1)^2 + (x^1)^2 + ... (x^{d+1})^2 \quad (3c)$$

The nonvanishing field components are all "electric" and of the form

$$E_a = F_{a[m]} = \frac{e x^a \epsilon_{[m]}}{r^{\frac{d}{2}+1}} \quad (3d)$$

independent of time and of the last $\frac{d-4}{2}$ space coordinates, as expected. The equations (3) define a Coulomb-electric field configuration. A Dirac-magnetic configuration with support in the same hyperplane requires a structure of $K$ of the same type as Eq. (3a) for $J$ but with the "electric charge" $e$ replaced by the "magnetic charge" $g$. For the magnetic field

$$H_a = \frac{1}{\frac{d}{2}!} \epsilon_{abc} F_{b[c]} = \frac{1}{(\frac{d}{2} - 1)!} \epsilon_{abc} \partial_b A_{[c]} \quad (4)$$

we require it to be of the same form as the Coulomb field (3d) but with $e \rightarrow g$:

$$\frac{1}{(\frac{d}{2} - 1)!} \epsilon_{abc} \partial_b A_{[c]} = \frac{g x_a}{r^{\frac{d}{2}+1}}. \quad (5)$$

We now have to solve these equations for $A_{[a]}$. As in the familiar 4-dimensional Dirac case, the Bianchi identities force us to introduce a string of singularities starting in each point of the support of $K$. For convenience we point all these strings along, say, the 3-direction. The proper Ansatz for $A_{[a]}$ is then

$$A_{[a]} = \epsilon_{[a]3b} x^b f(r, \xi), \quad \xi = \frac{x^3}{r}. \quad (6a)$$

Inserting this Ansatz into Eq. (5) we find

$$f(r, \xi) = r^{-\frac{d}{2}} F(\xi) \quad (6b)$$
with $F(\xi)$ obeying the differential equation

$$F'(\xi) - \frac{d}{2} \frac{\xi}{1-\xi^2} F(\xi) + \frac{g}{1-\xi^2} = 0.$$  \hspace{1cm} (7)

Since $|\xi| \equiv |\frac{x^3}{r}| \leq 1$, it is convenient to introduce the variable

$$\theta = \text{Arccos} \xi$$  \hspace{1cm} (6c)

and the function

$$G(\theta) \equiv F(\xi)$$  \hspace{1cm} (6d)

The solution to Eq. (7) is then

$$G(\theta) = g(\sin \theta)^{-\frac{d}{2}} \left[ \int^{\theta} (\sin \psi)^{\frac{d-a}{2}} d\psi + \lambda \right]$$  \hspace{1cm} (6e)

with $\lambda$ an integration constant that goes with the indefinite integral. The equations (6) determine the Dirac potentials. As an example for $d = 4$ we obtain the familiar Dirac result with the string along the positive (negative) 3-axis for $\lambda = -1$ ($\lambda = +1$). From the familiar recursion formula for the indefinite integral in (6e), $G(\theta)$ is periodic in $\theta$ for $d$ an integer multiple of 4. For $d = 2 \mod 4$ the indefinite integral in $G(\theta)$ contains also a linear term in $\theta$ which can be brought to the main determination $0 < \theta < \pi$ by readjusting the integration constant $\lambda$.

At this point we have to consider some global problems. As defined above, the support of both electric and magnetic charges for even $d > 4$ are infinite $\frac{d-4}{2}$-dimensional hyperplanes, which is undesirable. But if the higher dimensions are to be unobservable, then $d - 4$ space-like dimensions must have compact topology (e.g. a torus). But as we saw, $d_e + d_m = d - 4$, so that an electric-magnetic charge pair can always fit into the ”extra” compact space-like dimensions.

From the electric and magnetic charges in $d$ dimensions we can construct spinorial fermions. One way to see that, is to replicate the Tamm-Fierz [2] arguments for our spread-out charges. Heuristically, upon dimensional reduction (i.e., compactification of the $d - 4$ dimensions in which the charges are extended) the $\frac{d}{2} - 1$-tensor field contains ordinary 4-dimensional abelian gauge fields. Spinors can then be constructed from Bose electric and magnetic charges in the usual way [2,3]. But these 4-dimensional spinors must originate in $d$-dimensional spinors; they cannot come from $d$-dimensional tensors.
by dimensional reduction. Both $e$ and $g$ have dimension of $(d\text{-dimensional action})^{\frac{1}{2}}$ so that the ensuing $d$-dimensional Dirac quantization is meaningful.

3. COMPOSITE PICTURE

The way to use the arguments above to construct composite models is as follows. Suppose one starts with a $d$-dimensional space-time $d = \text{even integer larger than 4}$. In this space there exist a set of scalar fields for which one can build a composite $(\frac{d}{2} - 1)$-rank antisymmetric tensor field or, alternatively, this field can be "elementary". There can further appear electric and magnetic extended objects $e$ and $\mu$ and the corresponding anti-objects $\bar{e}$ and $\bar{\mu}$. From $e\mu$, $e\bar{\mu}$, $\bar{e}\mu$, $\bar{e}\bar{\mu}$ one can construct spinor composites, from $e\bar{e}$, $\mu\bar{\mu}$ tensor composites. With suitable dynamics these composites may exhibit a "dynamical" supersymmetry. If $d$ is large this may involve higher rank tensors. A dimensional reduction is precipitated one way or another [4] and in four dimensions we have a proliferation of composites since each spinor and tensor from $d$ dimensions branches into many counterparts in 4 dimensions (just as in extended supergravities). In 4 dimensions the spectrum is very rich, the simplicity is restored in $d$ dimensions. This picture is, of course, very similar to extended supergravity except that the gauged supersymmetry in the original $d$ dimensions is viewed as dynamical, thus allowing higher rank tensors and spin-tensors, or higher spins in 4 dimensions.

As it stands, this picture has a serious flaw: all fermions $e\mu$, $e\bar{\mu}$, $\bar{e}\mu$, $\bar{e}\bar{\mu}$ contain one unit each of electric and magnetic charge. All fermions of the theory must contain an odd number of these basic fermions and, as such, must carry odd, and therefore non-vanishing, electric and magnetic charges. Even though we have not as yet specified the detailed nature of the abelian gauge field in 4 dimensions whose sources these charges are, this is a serious difficulty.

We want to sketch here one possible way out. Consider (in four-dimensional space-time) a spherical shell of uniformly distributed electric charge. Classically this tends to explode and the Casimir effect is known to have the wrong sign [6] and thus does not stabilize the configuration. It has been noted recently by Agostinho Ferreira, Zimerman and Ruggiero [7] that in a distribution of both electric and magnetic charge along a spherical shell the Casimir effect is stabilizing. Specifically, they consider a spherical shell that
is a perfect magnetic conductor at its polar caps, and a perfect electric conductor on the "ring" between these caps: On the ring is uniformly distributed the electric charge while the two polar caps support uniform distributions of magnetic charge $\tilde{g}$ and $-\tilde{g}$ respectively, so that the whole system is magnetically neutral. Here the Casimir effect is stabilizing. We observe that for this system the angular momentum does not vanish as it would, were the magnetic charges at the two polar caps to have the same sign. By adjusting $e$ and $\tilde{g}$, we can fix the total angular momentum at $\frac{\hbar}{2}$, as would befit a spinor (as a model for the electron such a semiclassical argument requires much too large a size). One may object that each polar cap contributes to the total angular momentum, which violates angular momentum quantization (or equivalently, the Dirac quantization). This can be circumvented by postulating that such "polar caps" can never be isolated, but must always come in like- or opposite-charged pairs, as if they were doublets of a confining $SU(2)$ gauge theory. This is similar to what would happen in discussing usual Dirac quantization were one guaranteed that all magnetically charged particles are composites made of an even number of very closely bound inseparable constituents of equal magnetic charge. Obviously then, the Dirac quantization for "monopoles" would translate into a quantization for the constituents.

The challenge is now to construct a detailed model that implements the ideas presented above.

Following the completion of this work I received a Trieste preprint IC/80/180 from J.C. Pati, A. Salam and J. Strathdee, in which similar ideas are explored in a rather different way.

**Note (August 13, 2008):**

The results reported in section 2 of this paper have also been obtained independently by R. Nepomechie [8] and by C. Teitelboim [9].

The paper by Pati, Salam and Strathdee mentioned in the last sentence of the text has since appeared, [10].

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