New, high statistics measurement of the $K^+ \to \pi^0 e^+\nu$ ($K_{e3}^+$) branching ratio

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E865 at the Brookhaven National Laboratory AGS collected $\approx 70,000 K_{e3}^+$ events to measure the $K_{e3}^+$ branching ratio relative to the $K^+ \to \pi^+\pi^0$, $K^+ \to \pi^0\mu^+\nu$, and $K^+ \to \pi^+\pi^0\pi^0$ decays. The $\pi^0$ was detected using the $e^+e^-\gamma$ pair from $\pi^0 \to e^+e^-\gamma$ decay and no photons were required. Using the Particle Data Group branching ratios [11] for the normalization decays we obtain $BR(K_{e3}^+(\gamma)) = (5.13 \pm 0.02_{stat} \pm 0.09_{sys} \pm 0.04_{norm})\%$, where $K_{e3}^+(\gamma)$ includes the effect of virtual and real photons. This result is $\approx 2.3\sigma$ higher than the current Particle Data Group value. Implications for the $V_{us}$ element of the CKM matrix, and the matrix’s unitarity are discussed.

Although $V_{us}$ can be determined either from hyperon or from $K \to \pi e\nu$ decays, the purely vector $K_{e3}$ decays, with less intrusion of hadronic physics, provide a smaller theoretical uncertainty [11][10]. Theoretical contributions $V_{us}$ were reevaluated recently [7][8][11][11], but since uncertainties of $|V_{ud}|^2$ and $|V_{us}|^2$ are comparable, a high statistics measurement of the $K_{e3}^+$ branching ratio (B.R.) with good control of systematic errors is welcome.

The bare (without QED corrections) $K_{e3}^+$ decay rate [6][7][8][11] can be expressed as

$$d\Gamma(K_{e3}^+)=C(t) |V_{us}|^2 |f_+(0)|^2 [1 + \lambda_+ \frac{t}{M_\pi^2}]^2 dt$$

where $t = (P_K - P_\pi)^2$, $C(t)$ is a known kinematic function, and $f_+(0)$ is the vector form factor value at $t = 0$ which has to be determined theoretically [6][8]. Two recent experiments [12][13] provide $\lambda_+$ (the form factor slope) measurements consistent with each other and with previous measurements. An omitted negligible term in Eq. 2 containing the form factor $-f_+$ is proportional to $M_\pi^2/M_\pi^2$.

E865 [11] searched for the lepton flavor number violating decay $K^+ \to \pi^+\mu^+\nu$. The detector (Figure 1) resided in a 6 GeV/c positive beam. The first dipole

**Figure 1.** The cathedral at Durham.

This contribution follows the thesis of Alexander Sher [2] and the E865 group preprint [3] prepared after the conference. I will comment on some differences in presentation, and emphasize systematic checks. Discussions during all the conference activities (as for example in Figure 1) were valuable in clarifying the connection of our data to other experiments.

The experimentally determined Cabibbo-Kobayashi-Maskawa (CKM) matrix describes mixing between the “intrinsic” and physically observed quarks and is unitary within the Standard Model. One particularly interesting unitarity condition involves the first row elements:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 - \delta$$

where $\delta \neq 0$ indicates a problem with unitarity, and a sum less than one might indicate an additional quark generation. Since LEP results seem to have ruled out more than three neutrinos, such a shortcoming would strain the standard model. Therefore, Eq. 1 has generated substantial interest. The $V_{us}$ element is obtained from nuclear and neutron decays. While the precise shortfall in Eqn. 1 varies with different determinations of $V_{ud}$, the qualitative result is the same. For example, using an alternate, precise, recent value from the nuclear superallowed Fermi beta decays leads to $\delta = (3.2 \pm 1.4) \cdot 10^{-3}$ [8]. $V_{ub}$, too small to affect Eqn. 1, is determined from semileptonic decays of B mesons [11].
magnet separated decay products by charge. The second magnet together with four multiwire proportional chambers (MWPCs: P1-P4) formed the spectrometer. The particle identification system consisted of threshold multichannel Čerenkov counters (C1 and C2, each separated into left and right parts, for four independent volumes) filled with gaseous methane ($after 
\gamma_{\text{eff}} \approx 0.98$, an electromagnetic calorimeter, and a muon range system. The D and A scintillator hodoscopes gave left/right and crude vertical position. The muon system was not used for this analysis.

The decay $\pi^0$ was detected through the $e^+e^-$ pair from the $\pi^0 \to e^+e^-\gamma$ decay mode, with the $\gamma$ detected in some cases. In order to eliminate the uncertainty of the $\pi^0 \to e^+e^-\gamma$ B.R. (2.7%), and to reduce experimental systematic uncertainty we used the other three major decay modes with a $\pi^0$ in the final state ($K^+ \to \pi^+\pi^0(K_{\pi2})$, $K^+ \to \pi^0\mu^+\nu(K_{\mu3})$, $K^+ \to \pi^+\pi^0\pi^0(K_{\pi3})$) for the normalization sample ("Kdal").

The $K_{e3}^+$ data were collected during a one-week dedicated run in 1998, with beam intensity reduced to $\approx 10^7 K^+/2.8$ second pulse, a factor of 10 less than the standard $K^+ \to \pi^+\mu^+e^-$ running intensity, and trigger logic prepared especially for this measurement.

The normalization (Kdal) and $K_{e3}^+$ data were collected by the "ELER" trigger, which identified $e^+/e^-$ pairs (one left, one right) and required time coincidence of the four Čerenkov counters with at least one D-counter scintillator slab on each (left and right) side of the detector. The prescaled Čerenkov efficiency trigger, requiring only three out of four Čerenkov counters (no D-counter requirement), was used to measure the Čerenkov and D counter efficiencies. About 50 million triggers were accumulated, with $\approx 37$ million in the ELER trigger. The ELER events were about 3/4 accidentals, often with a muon from high momentum beam particle decays $K \to \mu\nu$ or $\pi \to \mu\nu$ making part of the Čerenkov portion of the trigger.

Off-line event reconstruction, using the spectrometer only, required a three charged track vertex inside the decay volume. The Čerenkov and D counter efficiencies were obtained from the Čerenkov efficiency triggers. The redundancy of the MWPCs (4 planes/chamber), and track reconstruction was used to extract MWPC efficiencies. The absence of the electromagnetic calorimeter from the trigger allowed its efficiency determination from the data. Each efficiency was measured over its appropriate phase space.

The Monte Carlo simulation used GEANT [15]. All relevant kaon decay modes were simulated, measured efficiencies were applied, and accidental detector hits (from the data) were added to the simulated events. The PDG value $\lambda_+ = 0.0278 \pm 0.0019$ [11] was used for the $K_{e3}^+$ simulation.

The radiative corrections are calculated both outside and inside the 3-body $K_{e3}$ Dalitz plot:

1. Outside: the $K_{e3}^+$ (inner bremsstrahlung and structure dependent) decays which have photons hard enough to move the events outside outside of the 3-body $K_{e3}$ Dalitz plot boundary. These were explicitly simulated [11], and are 0.5% of the $K_{e3}^+$ process. This 0.5% is present in our data, increasing our observed events from the "bare" $K_{e3}^+$ but subtracted when the "bare" $K_{e3}^+$ B.R. is calculated.

2. Inside: The difference between the observed events and the events from the bare $K_{e3}^+$ process. Here we used the procedure of Ref. [11]. The observed events include four terms: a) the bare process ($|a_0|^2$); b) the $K_{e3,\gamma}$ process ($|a_\gamma|^2$; c) the virtual photons $|a_\nu|^2$ correcting the bare process (small compared to the other corrections), and d) the interference of the bare process with the virtual photon corrections ($2Rea_0a_\nu$). This difference is $-1.3\%$, i.e., the virtual photon corrections and their interference with the bare process decrease the number of physically observable events. The overall difference between the "observed, acceptance corrected" events and the events from the "bare" process is $N_{\text{observed, acceptance corrected}} = N_{\text{bare}}(1 + 0.005 - 0.013)$.

An overall correction of 1.0232 (the short distance enhancement) is also applied [11]. Following Cirigliano, et al. [11], we do not apply this to our data in calculating the B.R., but apply it in the calculation of $V_{us}$ from the B.R..
The difference, inside the Dalitz plot, between our detector’s acceptance for the “bare” Ke3 process and for the observed process is an increase of 0.5%. Initially we expected the acceptance to decrease, since radiative corrections shift data to lower electron energies, where we lose events. However, we also have decreased acceptance at high electron energies, where the electron energy distribution itself peaks.

For the $\pi^0 \rightarrow e^+e^-\gamma$ decay, radiative corrections have been taken into account according to Ref. [10].

Table 1 shows how the data were reduced by event selection. Selection criteria, common to Ke3 and Kdal samples, included requirements for a good quality vertex, for the three tracks to cross the active parts of the detector, for the low ($M_{ee} < 0.05 \text{ GeV/c}^2$) mass $e^+e^-$ pair to be identified in the Čerenkov counters, and for the second positive track to have less than 3 GeV/c momentum. The momentum cut avoids any situation in which a mode other than Ke3 can have the second positive track from a normalizer mode properly satisfy the Ke3 criteria. A geometric Čerenkov ambiguity cut rejected events where the Čerenkov counter response could not be unambiguously assigned to separate tracks ($\approx 30\%$ loss of both $K_{e3}$ and Kdal).

The $K_{e3}^+$ sample was then selected by requiring the second positive track to be identified as $e^+$ in 2 of the 3 electron detectors: C1, C2, and the calorimeter. Events entering the normalization sample had no response in at least one of the two Čerenkov counters. These criteria minimized systematic uncertainties [2], but resulted in a small overlap of the $K_{e3}^+$ and Kdal samples, which was taken into account in the final result calculation. Final acceptances for the three charged particles differed by no more than 4% among the three normalization decays [2].

The final signal and normalization samples were 71,204 and 558,186, respectively. Extensive comparisons between quantities in the data and simulation were presented in the thesis [2] and the HEP preprint [3].

Contamination of the $K_{e3}^+$ sample by other $K^+$ decays occurred when $\pi^+$ or $\mu^+$ from normalization decays were misidentified as $e^+$ from $K_{e3}^+$, or as a result of $\pi^0 \rightarrow e^+e^-\gamma$ decay. Care was taken with the PWC simulation so that track chisquares and vertex distributions agree between data and Monte Carlo for well-measured tracks. Decays ($\approx 10\%$ of $\pi^0$’s) are modelled in the simulation. Agreement of the standard vertex quality between data and simulation indicates successful modelling of the decays at the level required for our measurement. Systematic uncertainties were estimated by variation of the vertex quality cut.

Total contamination of the Ke3 sample was estimated to be $(2.49 \pm 0.05_{\text{stat}} \pm 0.32_{\text{sys}})\%$, with the systematic uncertainty caused by the simulation accuracy of the Čerenkov counters’ response to $e^+$ and $\mu^+$. Contamination due to overlapping events was $(0.25 \pm 0.07)\%$ and $(0.12 \pm 0.05)\%$ of the selected normalization and signal samples respectively. Figure 4 shows energy deposited in the calorimeter by the $e^+$ from the selected $K_{e3}^+$ sample. The contamination is manifest in the minimum ionization spike at 250 MeV. The small excess of data in the spike agrees with our contamination uncertainty estimate.

The final $K_{e3}^+$ sample included $\approx 30\%$ events with the fully reconstructed $\pi^0$. We used these events as a consistency check but did not require photon detection in our main analysis. This eliminated an additional systematic uncertainty from photon detection and reconstruction in the calorimeter. However, lack of a $\pi^0$ reconstruction increased vulnerability to contamination from upstream decays and photon conversion. Upstream decays whose photon produced pairs before the decay volume were suppressed by requiring the three track vertex to be more than two meters downstream of the decay volume entrance. In addition, the results obtained from the two independent samples (one with and one without the $\pi^0$ reconstructed) did not show a statistically significant discrepancy. The decay volume was evacuated to about $10^{-8}$ nuclear interaction length, which suppressed beam pion interactions.

The $K_{e3}^+$ sample size gives a statistical precision of
The systematic error was determined from stability of the result under variation of the reconstruction procedure, selection criteria, detector efficiencies applied to the Monte Carlo, and subdivision (in possibly biased distributions) of both signal and normalization samples. No significant correlations between different systematic uncertainties were observed. Systematic uncertainties are summarized in Table 2.

In addition to the check using fully reconstructed $\pi\pi^+$'s, as a second consistency check, we estimated the $K^+ \rightarrow \pi^+\pi^+\pi^-$ decay B.R. relative to the Kdal sample. The result was $(1.01 \pm 0.02) \times R_{PDG}$, where $R_{PDG}$ is the prediction based on the PDG compilation, and the theoretical prediction [17] was used for the $\pi^0 \rightarrow e^+e^−\gamma$ decay rate. The 2% error was determined by combining all relevant uncertainties in quadrature, and was dominated by the uncertainty in the prescale factor of the trigger used to collect $K^+ \rightarrow \pi^+\pi^+\pi^-$ events. This result checks both the treatment of decays and the particle identification.

A third check compared the $K^+_{el}$ B.R. from 1998 and 1997 data. The 1997 $K^+_{el}$ data used a trigger requiring hits in the calorimeter, A and D-counters. That trigger neither allowed measurement of these detector efficiencies, nor of the $K^+_{el}$ trigger efficiency. While we did not use the 1997 data for our final result, the 1997 $K^+_{el}$ B.R. was statistically consistent (within one sigma) with that from the 1998 data. This agreement is important since the momentum comparison in the 1997 data looks qualitatively different from the 1998 data[2]. A preliminary reconstruction version was used for the 1997 data, without the final magnetic field tuning and detector realignment. Our intuition is that the discrepancies in decay product momenta in the momentum (Figure 4) and spatial distributions [2]. The systematic error was determined by dividing $K^+_{el}$ and Kdal events in roughly equal samples using the relevant parameter as a separator and observing the result variation[2]. The sensitivity of the vertical spatial discrepancy to the MWPC alignment and of the momentum discrepancy to the spectrometer parameters [2] indicate possible origins of these discrepancies.

![Figure 3](image_url) Energy deposited in the calorimeter by the second positive track from the selected $K^+_{el}$ sample ($e^+$ which is not from the low mass $e^+e^−$ pair). No calorimeter information was used for the $e^+$ identification. Markers with errors represent data; the histogram is simulation.

![Figure 4](image_url) Reconstructed momentum of the $e^+$ from the low mass $e^+e^−$ pair for the selected $K^+_{el}$ and Kdal samples. Histograms represent Monte Carlo; points with errors are data. Monte Carlo to data ratios are shown on the right.

Table 2. Systematic uncertainty sources and estimates of their respective contributions to the error of the final result. The total error was calculated as the sum of errors taken in quadrature.

| Source of systematic error      | Error estimate |
|---------------------------------|----------------|
| Magnetic field                  | 0.3%           |
| Vertex, quality                 | 0.6%           |
| Vertex position                 | 0.2%           |
| Cerenkov Ambig.                 | 0.3%           |
| $M_{ee}$ cut                    | 0.2%           |
| Aperture                        | 0.2%           |
| $(\pi/\mu)^+$ iden.             | 0.04%          |
| MWPC effic.                     | 0.2%           |
| D ctr. effic.                   | 0.15%          |
| Cerenkov effic.                 | 0.3%           |
| Sample contam.                  | 0.3%           |
| Vertical distrib.               | 0.8%           |
| $e^+/e^−$ momentum distrib.     | 1.3%           |
| $K^+_{el}$ trigger effic.       | 0.1%           |
| $K^+_{el}$ f. f. ($\lambda^+$)  | 0.1%           |
| Total uncer.                    | 1.8%           |

The two largest contributions to the error come from the discrepancies [2] between data and Monte Carlo...
We estimated the form factor slope $\lambda_+$ from both 1998 and 1997 $K_{\ell 3}^+$ data samples. We obtained: 

$$\lambda_+ = 0.0324 \pm 0.0044_{\text{stat}}$$ for the 1998, and 

$$\lambda_+ = 0.0290 \pm 0.0044_{\text{stat}}$$ for the 1997 data, both consistent with the current PDG fit.

After subtraction of contamination in the Ke3 sample [2], our result is 

$$BR(K_{\ell 3}^+)/(BR(K_{\pi 2}^+) + BR(K_{\mu 3}^+) + BR(K_{\tau 3}^+)) = 0.2002 \pm 0.0008_{\text{stat}} \pm 0.0036_{\text{sys}},$$

where $K_{\ell 3}^+$ includes all QED contributions (loops and inner bremsstrahlung). As noted above, for this result the $\pi^0$ was detected using the $e^+e^-$ pair from $\pi^0 \to e^+e^-\gamma$ decay and no photons were required.

Using the PDG fit [1] values for the normalization branching ratios we infer $BR(K_{\ell 3}^+)/(BR(K_{\pi 3}^+)) = (5.13 \pm 0.02_{\text{stat}} \pm 0.09_{\text{sys}} \pm 0.04_{\text{norm}})\%$ where the normalization error is determined by the PDG estimate of the normalization B.R. uncertainties. The PDG fit to the results to the previous $K^+$ decay experiments yields 

$$BR(K^+ \to \pi^0 e^+\nu) = (4.87 \pm 0.06)\%$$ [1], $\approx 2.3\sigma$ lower than our result.

As discussed above, the total radiative correction was 0.8%, yielding $BR(K^+ \to \pi^0 e^+\nu_{\text{bare}}) = (5.17 \pm 0.02_{\text{stat}} \pm 0.09_{\text{sys}} \pm 0.04_{\text{norm}})\%$. This differs slightly from the thesis (5.16%) due to recalculation with the 2002 PDG, while the thesis used the 2001 PDG values.

Using the current PDG value for $G_F$, the short-distance enhancement factor $S_{EW}(M_\mu, M_Z) = 1.0232$[11,12], and our result for the bare $K_{\ell 3}^+$ decay rate we obtain $|V_{us} f^+(0)| = 0.2239 \pm 0.0022_{\text{rate}} \pm 0.0007_{\lambda_+}$, which leads to $|V_{us}| = 0.2272 \pm 0.0023_{\text{rate}} \pm 0.0007_{\lambda_+} \pm 0.0018_{f^+(0)}$ if $f^+(0) = 0.9874 \pm 0.0084$[6,7]. With this value of $V_{us}$ and $V_{ud}$ from superallowed nuclear Fermi beta decays[5], we obtain $\delta = 0.0001 \pm 0.0016$.

This result is consistent with CKM unitarity, but our $K_{\ell 3}^+$ result is 5.3% higher than the PDG 2002 value, with a statistical error 0.4%, systematic error 1.8%, and normalization error 0.7%.

We conclude: a) the Ke3 B.R. may be somewhat higher than that listed in the PDG tables; and b) Ke3 experiments are now precise enough that radiative corrections should be applied consistently to all entries in the PDG average.

Even without our result, the $V_{us}$ extracted from the PDG average $K_{\ell 3}^+$ decay rate is higher than the $V_{us}$ from the $K_{e3}^+$ rate. [13,14]. Our $K_{e3}^+$ result increases this difference. $K_{e3}^+$ decay measurements (both charged and neutral) in progress (CMD2, NA48, KLOE)[14] should clarify the experimental situation.

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