Transonic flow hysteresis in divergent bent channels

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Abstract. This paper presents a numerical study of the 2D and 3D turbulent airflows in divergent 9°-bent channels. The incoming flow is supersonic, whereas the exit flow may be either supersonic or subsonic. Solutions of the Reynolds-averaged Navier-Stokes equations are obtained with a finite-volume solver ANSYS CFX using the Spalart-Allmaras and Shear Stress Transport \( k-\omega \) turbulence models. The solutions reveal a significant hysteresis of the flow field under variations of the free-stream Mach number or exit pressure. At the ends of hysteresis bands, the flow pattern changes crucially due to instability of a shock wave formed near the bend of channel. The instability is caused by the shock foot interaction with an expansion flow developed over the convex wall of channel. Boundary conditions, in which the flow admits a double hysteresis, are figured out. The occurrence of non-unique flow regimes must be taken into account in the advanced intake design, as different losses of the total pressure may cause different trusts of an air breathing engine.

1. Introduction

Supersonic air intakes are of major importance for the efficient operation of aircraft engines. At on-design conditions, a train of oblique shocks forms in a conventional convergent-divergent intake [1]. However, this shock system is very sensitive to small perturbations. A small reduction of the free-stream Mach number \( M_\infty \) triggers an expulsion of the oblique shocks from the channel and formation of a nearly normal shock in front of it. This results in losses of the total pressure and reduction of the engine efficiency. Numerical studies of the supersonic flow in convergent-divergent channels demonstrated a significant hysteresis under gradual variations of \( M_\infty \) [2, 3].

Supersonic flow in bent channels can be sensitive to small perturbations as well [4]. In this case, instability of the shock wave arising ahead of the concave wall is caused by its interaction with a flow acceleration region over the opposite (convex) wall. Numerical simulations of the flow in channels with a short upper wall (cowl) demonstrated a significant hysteresis under variations of \( M_\infty \) or angle of attack [4-6].

Guo et al. [7] performed a numerical modelling of supersonic flow in a curved channel of nearly constant cross-section. The existence of a flow hysteresis under variations of the pressure given at the outlet was demonstrated.

In practice, supersonic flow in bent channels is utilized, e.g., in Y-shaped intakes of battle-plane and fighter aircraft engines. Kotteda and Mittal [8] studied the effect of sideslip angle on the performance of a slightly divergent Y-intake. Different flow regimes under variations of the outlet pressure were identified, and an effect of the initial condition was examined.

Feng et al. [9, 10] performed a numerical and experimental investigation of supersonic flow in a bent channel which models a variable geometry dual mode combustor. The study indicated that the static
pressure distribution on a wall had an obvious hysteresis phenomenon under continuous variations of the outlet cross section area.

In [11] we studied shock wave hysteresis in a bent divergent channel with a longer upper wall than that considered in [4-6]. It was shown that flow features depend essentially on the thickness \( \delta \) of leading edges of walls, \( 0.1 \leq \delta \leq 1.0 \) mm. In the present paper, we focus on the case \( \delta = 0.1 \) mm and study in detail the behaviour of shock waves. Sections 2 and 3 are concerned with the problem formulation for 2D flow and description of a numerical method. Section 4 addresses locations of the shock wave (SW) generated by the concave upper wall; the SW position as a function of the free-stream Mach number \( M_\infty \) is investigated at zero angle of attack and supersonic exit condition. In Section 5 we study the behaviour of SW under variations of the pressure given at the subsonic exit. In Section 6, preliminary simulations of the 3D flow are discussed.

2. 2D problem formulation

Let the inner part of the lower wall be a convex corner with a vertex at \( x = x_{\text{corner}}, y = 0 \):

\[
y = 0 \text{ at } -60 \leq x \leq x_{\text{corner}};
\]
\[
y = (x - x_{\text{corner}}) \tan(-9^\circ) \text{ at } x_{\text{corner}} < x \leq x_{\text{corner}} + 27,
\]

whereas the inner part of the upper wall be a concave corner with a vertex at \( x = 0, y = 30 \):

\[
y = 30 + x \tan(1^\circ) \text{ at } -60 \leq x \leq 0;
\]
\[
y = 30 + x \tan(-8^\circ) \text{ at } 0 \leq x \leq 55.
\]

Evidently, the walls are 1° divergent before and after the bends. In expressions (1), (2) and further in the paper, the Cartesian coordinates \((x, y)\) are dimensional and given in millimetres. The outer parts of the lower and upper walls are a segment and corner, respectively, see figure 1. The thickness of the leading edges is set to 0.1 in order to attenuate the bow shocks and reduce perturbations they generate.

The above specific geometry is chosen to provide a clear demonstration of the hysteresis phenomenon in bent channels. From a practical point of view, this geometry can be of interest, e.g., for the Y-type intake design [8]. The small dimensions of the channel should be convenient for future experimental verification in a midsize wind tunnel.

The inflow boundary \( \Gamma_{\text{in}} \) of the computational domain is constituted by a vertical segment \( x = -90, -30 \leq y \leq 60 \) and two inclined segments with endpoints \( x = 0, y = \pm 200 \), which are remote enough to eliminate an interaction of \( \Gamma_{\text{in}} \) with the bow shocks produced by the channel. We prescribe the \( x-, y- \) and \( z- \) components of the flow velocity on \( \Gamma_{\text{in}} \) as follows:

\[
U_\infty = M_\infty a_\infty, \quad V_\infty = 0, \quad W_\infty = 0.
\]

In addition, we impose on \( \Gamma_{\text{in}} \) the static pressure \( p_\infty \), the turbulence level of 1%, and static temperature \( T_\infty = 250 \) K which determines the sound speed \( a_\infty = 317.02 \) m/s. The pressure \( p_\infty \) is set to \( 10^5 \) N/m\(^2\) throughout the paper, except for figure 6 in which we compare results obtained for different values of \( p_\infty \).

The outflow boundary \( \Gamma_{\text{out}} \) of the computational domain is constituted by two segments which begin at \( x = 0, y = \pm 200 \) and end at \( x = 20 \) on the outer surfaces of walls. We prescribe the supersonic outflow condition \( M > 1 \) on \( \Gamma_{\text{out}} \). In the exit section \( \Gamma_{\text{exit}} \) of the channel we impose either the supersonic condition \( M > 1 \) or subsonic one and the static pressure \( p_{\text{exit}} \).

The vanishing heat flux and no-slip condition are prescribed on the walls. The air is treated as a perfect gas whose specific heat at constant pressure is 1004.4 J/(kg K) and the ratio of specific heats is 1.4. We use the Sutherland formula for the molecular dynamic viscosity and adopt the value of 28.96 kg/kmol for the molar mass. The Reynolds number based on \( p_\infty = 10^5 \) N/m\(^2\), \( M_\infty = 1.35 \), and height of the channel is \( 1.1 \times 10^6 \). Initial data are either parameters of the free stream or a flow field calculated for other values of \( M_\infty, p_{\text{exit}} \).
3. Numerical method
Solutions of the unsteady Reynolds-averaged Navier-Stokes equations were obtained with a commercial ANSYS-18.2 CFX finite-volume solver of second-order accuracy. Computations showed a convergence of the mean parameters of turbulent flow to steady states in less than 0.1 s of physical time. We used the Spalart-Allmaras [12] and Shear Stress Transport $k$-$\omega$ [13] turbulence models, which are known to reasonably predict aerodynamic flows with boundary layer separations. Hybrid unstructured meshes were constituted by quadrangles in 38 layers on the walls and by triangles in the remaining region, see figure 2. The non-dimensional thickness $y^+$ of the first mesh layer on the walls was less than 1. The sizes of triangles were essentially decreased in the channel for an accurate resolution of shock waves. Test solutions obtained on uniformly refined meshes of approximately $2.5 \times 10^5$, $5 \times 10^5$, and $10^6$ cells showed that a discrepancy between shock wave coordinates calculated on the second and third meshes did not exceed 1%. Global time steps of $5 \times 10^{-7}$ s and $10^{-6}$ s yielded indistinguishable solutions. For this reason, the time step of $10^{-6}$ s and mesh of $5 \times 10^5$ cells were employed for the study of flow behavior at various $M_\infty$ and $p_{\text{exit}}$.

The solver was validated by computation of several benchmark supersonic and transonic flow problems. For instance, we performed numerical simulations for a channel with a bump [4], a supersonic intake [14] and Busemann biplane [11]. The obtained solutions showed good agreement with numerical and experimental data available in the literature.

4. Shock location versus $M_\infty$ at $p_{\text{exit}} = 10^5$ N/m$^2$ and supersonic exit condition
First, we solved the problem at $M_\infty = 1.35$, $x_{\text{corner}} = 7.5$ using the free stream (3) for initialization of the solution. The obtained flow field exhibits a train of weak shocks which propagate from the entrance downstream, being multiply reflected from the walls. A stronger shock wave (SW) begins in front of the upper wall bend and hits the lower wall at a coordinate $x > x_{\text{corner}}$. Such a flow pattern is called hereafter the flow regime with a swallowed SW.

A reduction of $M_\infty$ step-by-step (at each step, initial conditions are the steady flow parameters obtained for the previous value of $M_\infty$) from 1.35 to 1.27 produces a decrease of Mach numbers in the channel and formation of a small subsonic region downstream of SW, see figure 3(a). To analyse streamwise positions of SW, we will trace its coordinate $x_{\text{sh}}$ at the height $y = 10$. Computations showed that, when $M_\infty$ decreases from 1.35 to 1.27, the flow pattern persists while $x_{\text{sh}}$ slightly decreases, see the upper branches of curves 1 and 2 in figure 4(a). Further reduction of $M_\infty$ to 1.268 triggers a jump of SW upstream (and, hence, drop of $x_{\text{sh}}$) due to instability of the shock foot interaction with the flow acceleration region over the lower wall bend. The drop of $x_{\text{sh}}$ means a transition from the upper branches of curves 1 and 2 to lower ones. The oblique SW becomes a nearly normal shock located at $x_{\text{sh}} = -41.5$. Such a flow pattern is called hereafter the flow regime with a partially expelled SW.
Further decrease of \( M_\infty \) to 1.24 yields a gradual shift of SW in the channel to the entrance. A comparison of curves 1 and 2 in figure 4(a) shows a negligible distinction between SW coordinates obtained using SST \( k-\omega \) and Spalart-Allmaras turbulence models.

Inversely, if \( M_\infty \) increases step-by-step from 1.24 to 1.318, then the expelled SW gradually shifts downstream and \( x_{sh} \) rises, as illustrated by the lower branches of curves 1 and 2. In particular, isoMachlines obtained at \( M_\infty = 1.27 \) are depicted in figure 3(b). An increase of \( M_\infty \) beyond 1.318 triggers a jump of SW downstream and relaxation to a flow pattern with a swallowed SW. This corresponds to a transition from the lower branches of curves 1 and 2 to upper ones. As seen, the coordinate \( x_{sh} \) exhibits a hysteresis in the band \( 1.27 \leq M_\infty \leq 1.318 \) in which the realization of a certain flow field depends on the history of free-stream Mach number variation.

Figure 3. Mach number contours at \( M_\infty = 1.27, M_{exit} > 1, p_\infty = 10^5 \text{ N/m}^2, x_{corner} = 7.5, \) SST \( k-\omega \) turbulence model. The flow field is obtained by: (a) decreasing \( M_\infty \) from 1.35 to 1.27, (b) increasing \( M_\infty \) from 1.24 to 1.27.

Figure 4. Coordinate \( x_{sh} \) of SW versus \( M_\infty \) at \( p_\infty = 10^5 \text{ N/m}^2, M_{exit} > 1; \) turbulence models: 1 – SST \( k-\omega \), 2 – Spalart-Allmaras; (a) \( x_{corner} = 7.5 \), (b) \( x_{corner} = 11.5 \).

In a similar way we obtained a plot \( x_{sh}(M_\infty) \) for \( x_{corner} = 11.5 \), see figure 4(b). In this case there exist two lower branches of the curves, which illustrate an extra hysteresis caused by the separated boundary layer like a hysteresis in straight nozzles [15]. The dashed (solid) curves are obtained at decreasing (increasing) free-stream Mach number, as indicated by arrows next to the curves.

Figure 5 illustrates the relative total pressure \( P_{total} = p_{total}/p_{total,\infty} \) (where \( p_{total,\infty} = 296,761 \text{ N/m}^2 \)) in the channel and exit section at \( x_{corner} = 11.5, M_\infty = 1.35 \). As seen, pressure losses are different in the flow regimes which correspond to the upper and lower branches of curves in figure 4(b).

We notice that the thickness \( \delta \) of the leading edges of walls crucially influences the location of SW. When \( \delta \) increases, the bow shocks become stronger, and flow velocities in the channel decrease. Therefore the swallowing and expulsion of SW occur at larger Mach numbers \( M_\infty \). Computations showed that the hysteresis band shifts from \( 1.35 < M_\infty < 1.40 \) to \( 1.41 < M_\infty < 1.473 \) when \( \delta \) increases from 0.1 to 0.5 in the case \( M_{exit} > 1, x_{corner} = 11.5 \).
Figure 5. The relative total pressure $P_{\text{total}}$ at $M_\infty = 1.35$, $x_{\text{corner}} = 11.5$, $M_{\text{exit}} > 1$, SST $k$-$\omega$ turbulence model: (a) contours of $P_{\text{total}}$ in the regime that corresponds to the lower branch of curve 1 in figure 4(b); (b) plots of $P_{\text{total}}$ in the exit section vs the non-dimensional height $\bar{y}$, $0 < \bar{y} < 1$, where curve 1 (curve 2) corresponds to the lower (upper) branch of curve 1 in figure 4(b).

5. Variations of the pressure given on $\Gamma_{\infty}$ and $\Gamma_{\text{exit}}$

A doubling of the free-stream pressure $p_\infty$ (and, consequently, the Reynolds number) produces an insignificant effect on the lower branches of plot $x_{\text{sh}}(M_\infty)$, see figure 6. Meanwhile, the upper branch essentially shifts to smaller values of $M_\infty$; therefore, the hysteresis band expands. Further increase of $p_\infty$ to $4 \times 10^5$ N/m$^2$ virtually does not influence the shock location.

The calculated pressure $p_{\text{exit}}$ is less than $1.28 p_\infty$ when the supersonic condition $M_{\text{exit}} > 1$ is used on $\Gamma_{\text{exit}}$. If, instead of $M_{\text{exit}} > 1$, we impose the subsonic condition $M_{\text{exit}} < 1$ along with $p_{\text{exit}} = 1.65 p_\infty$, then an extra shock appears and terminates the supersonic region near the exit. This shock gradually moves upstream as $p_{\text{exit}}$ increases. Figure 7 illustrates the flow field obtained at $p_{\text{exit}}/p_\infty = 1.88$, $M_{\infty} = 1.40$. Further increase of $p_{\text{exit}}/p_\infty$ triggers a jump of the coordinate $x_{\text{sh}}$ from 2 to −30 and a transition from the upper branch of curve 2 in figure 8 to lower one. Inversely, a decrease of $p_{\text{exit}}/p_\infty$ to 1.83 produces a gradual increase of $x_{\text{sh}}$ to −18. After that a decrease of $p_{\text{exit}}/p_\infty$ to 1.825 triggers a jump of $x_{\text{sh}}$ to 9 and a transition to the upper branch of curve 2.

Figure 6. Coordinate $x_{\text{sh}}$ of SW versus $M_\infty$ at $x_{\text{corner}} = 11.5$, $M_{\text{exit}} > 1$ and SST $k$-$\omega$ turbulence model: 1 – $p_\infty = 10^5$ N/m$^2$, 2 – $p_\infty = 2 \times 10^5$ N/m$^2$.

Figure 7. Mach number contours in the flow at $x_{\text{corner}} = 11.5$, $M_\infty = 1.40$, $M_{\text{exit}} < 1$, $p_{\text{exit}} = 188000$ N/m$^2$, SST $k$-$\omega$ turbulence model.
At the larger free-stream Mach number $M_\infty = 1.41$, computations demonstrated a similar dependence of $x_{sh}$ on $p_{\text{exit}}/p_\infty$, see curve 3 in figure 8. Meanwhile, at the smaller Mach number $M_\infty = 1.39$, a transition from the upper to lower branch of curve 1 is irreversible, since any decrease of $p_{\text{exit}}/p_\infty$ fails to produce a return to the upper branch, i.e., to the flow regime with a swallowed SW.

6. 3D flow simulation
A three-dimensional channel was created by an extrusion of the profile (1), (2) with $\delta = 0.5$, $x_{\text{corner}} = 11.5$ from the plane $z = 0$ to $z = \pm 45$. The sidewall thickness was set to 0.5, so that the outer surfaces of sidewalls were located at $z = \pm 45.5$. The boundaries $\Gamma_{\text{in}}$ and $\Gamma_{\text{out}}$ of the 2D computational domain were extruded from $z = 0$ to $z = \pm 120$. For CPU savings, we assumed the flow to be symmetric about the plane $z = 0$ and solved the problem in a half of the domain, at $-120 < z < 0$. On the side boundary $z = -120$ we imposed the free-slip condition. A hybrid 3D mesh was constituted by about $1.13 \times 10^6$ tetrahedrons, $1.11 \times 10^6$ prisms, and by $2.1 \times 10^6$ hexahedrons in 38 layers on the walls.

Computations showed that, due to a sidewall effect, flow velocities in the 3D channel are noticeably smaller than those in 2D one at the same $M_\infty$. As a consequence, the expulsion of SW takes place when $M_\infty$ becomes smaller than 1.50 instead of 1.41 in 2D flow. The 3D flow hysteresis is realized in the band $1.50 \leq M_\infty \leq 1.52$ instead of $1.41 \leq M_\infty \leq 1.473$ pointed out in the end of Section 4.

Figure 9 displays the bow shocks and partially expelled SW at $M_\infty = 1.49$. Figure 10 exposes Mach number contours in the plane $x = -30$. As seen, a dependence of flow parameters on the spanwise coordinate $z$ is insignificant, except for a vicinity of the sidewall. The boundary layer separation from the walls is larger than that in 2D flow because of the larger $M_\infty$ and, as a consequence, stronger shocks.

Thus, 3D flow simulations confirm the occurrence of flow hysteresis in the channel at hand. With increasing span or Reynolds number, the sidewall effect is expected to decrease.

Figure 9. Isosurfaces $M(x,y,z) = 1.34$ and $M(x,y,z) = 1$ in the 3D flow at $M_\infty = 1.49$, $M_{\text{exit}} > 1$, $x_{\text{corner}} = 11.5$.

Figure 10. Mach number contours in the cross section $x = -30$ of 3D flow at $M_\infty = 1.49$, $M_{\text{exit}} > 1$, $x_{\text{corner}} = 11.5$. 
7. Conclusion
Transonic flow in divergent bent channels at zero angle of attack is studied numerically. Simulations of the 2D flow revealed a shock wave in front of the bend of channel and significant hysteresis in the shock location under variations of the free-stream Mach number or exit pressure. At the ends of hysteresis bands, there are abrupt changes of the flow patterns, whereas within the bands there exist up to three flow regimes. This must be taken into account in the design of air breathing engines, as different losses of the total pressure in different regimes may essentially influence the engine thrust. The 3D flow simulations confirmed the findings, though demonstrated a noticeable sidewall effect.

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