Quantum Brownian motion simulation of the control effect for two harmonic oscillators coupling in position and momentum with general environment

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In this paper, we study the dynamical properties of two coupled quantum harmonic oscillators coupled with bosonic non-Markovian environment both in position and momentum. We deduce the exact analytical master equation using Quantum State Diffusion method and give the quantum trajectory description when the control is added to the system by applying interaction between two harmonic oscillators. With numerical simulation, we compare the evolution of entanglement under different controlling effects. At last, we use nonlinear QSD method to strengthen our above results by getting the same evolution.

I. INTRODUCTION

The study of quantum open systems are increasingly important in the field of quantum dynamics because it is impossible to isolate the system from its environment or make a measurement without involving with other systems. Generally, although environment and specific quantum system are initially independent, they will become entangled due to the interaction, as a result, the quantum system will no longer be pure state which means the evolution operator is non-unitary. Most experimental physicists face the quantum open system where a small system of interest coupled to a large system with a large number of freedom, which can be described by heat bath. Traditionally, we describe the open system by a Lindblad master equation which can be derived with Born-Markov approximation which means the flow of energy or information is unidirectional, in other words, the bath is memoryless. However, if the bath memory effects are relevant, for example in the cases of a high-Q cavity, atom laser or complex structured environment, where the Born-Markov approximation does not work, we have to use Non-Markovian process to describe the quantum system.

Quantum Brownian Motion (QBM) is a paradigm of quantum open system motivated by possible observation of macroscopic effects in quantum systems and problem of quantum measurement theory. Also, Quantum Brownian Motion, as a exactly-solvable model, provides us a glance at the relationship between different measure of quantum systems, including entanglement, coherence, purity, during the evolution of quantum system and under the external time-dependent control. To study the quantum-to-classical transition in quantum cosmology, Hu, Paz and Zhang got the exact master equation with nonlocal dissipation and colored noise in a general environment, which beginning the new stage to treat the old problem\cite{1}. Later, Chou, Ting and Hu derived an exact master equation for two coupled quantum harmonic oscillators interacting via bilinear coupling with a common environment at arbitrary temperature made up of many harmonic oscillators with a general spectral density function\cite{2}, which makes it possible to study the decoherence and disentangle in Brownian motion model. Traditionally, we use reduced density matrix to describe the quantum open system when we consider the environment effect to the specific system. Recently, there are tremendous progresses in the development of stochastic Schrodinger equations to describe the quantum open system. We will have the reduced density matrix by tracing over the quantum trajectories of a stochastic Schrodinger equation, which means a possible series of influence of the environment to the system. Quantum state diffusion method provides not only an efficient way in the numerical calculation of quantum open system, but also a way to describe our system, which shed light on the difficulties encountered in environment memory effect.

In this paper, we mainly give the numerical simulation and derivation of the stochastic Schrodinger equation and master equation for open quantum system containing two time-dependent interacting harmonic oscillators, coupled with a thermal bath involving infinite number of bosonic oscillators at zero temperature. The symmetric position-momentum coupling pattern is used, which also can be regarded as a Rotating Wave Approximation of position. We also shows in zero temperature case, the symmetric coupling in position and momentum provides an easy but effective way to have an glance at the quantum system under the influence of environment and external control field, especially for numerical simulations, because ten related differential equations are simplified to one differential equation.

Our paper is organized as follows. We firstly give a brief introduction to Quantum State Diffusion method, in zero temperature in Section II A. The evolution of two harmonic oscillators with the symmetric coupled pattern in position and momentum in Section II B is considered, where the time-local, convolution-less master equation, derived by non-Markovian quantum state diffusion method(NMQSD). We then consider the application of quantum control in Section II C where we control the entanglement and coherence of the specific quantum system by time-dependent interaction. In Section III B we simulate these controlling and non-Markov process, including coherence states, folk states and cat states under the influence of environment and control field.
II. THEORETICAL FRAMEWORK

A. Introduction to QSD in zero temperature

The standard total Hamiltonian in the system-plus-reservoir in quantum open system can always be written as

\[
H_{\text{tot}} = H_{\text{sys}} + H_{\text{int}} + H_{\text{bath}}
\]

\[
= H + \hbar \sum_{\lambda} \left( g_{\lambda} L_{\lambda}^\dagger + g_{\lambda} L_{\lambda}^\dagger b_{\lambda} \right) + \sum_{\lambda} \hbar \omega_{\lambda} b_{\lambda}^\dagger b_{\lambda} \tag{1}
\]

where \( L \) is the system operator providing the coupling between the system and the environment and \( g_{\lambda} \) are coupling constants. We will then set \( \hbar = 1 \).

Using the Schrödinger equation, the non-Markov Quantum State Diffusion (NMQSD) equation is [4]

\[
\partial_t \psi_t = -iH \psi_t + L z_t^* \psi_t - L^\dagger \int_0^t \alpha(t-s) \frac{\delta \psi_t}{\delta z_s^*} ds \tag{2}
\]

where \( \alpha(t-s) \) is the bath correlation function determined by temperature and initial bath states, \( H \) is the system Hamiltonian and \( z_t^* \) are Gaussian random process with correlation functions that mirror the vacuum correlations of the bath operators in the interaction picture.

It is possible to replace the functional derivative by some time-dependent operator \( O \), which depends on the time \( t \), \( s \), and the entire history of the stochastic process \( z_t^* \). By making an ansatz, the evolution equation for operator \( O = O(t,s,z^*) \) is

\[
\partial_t O = \left[ -iH + L z_t^* - L^\dagger \hat{O} (t,z^*) , O \right] - L^\dagger \frac{\delta \hat{O}(t,z^*)}{\delta z_s^*} \tag{3}
\]

where the time-integrated operator \( \hat{O}(t,z^*) \) is the integral of \( O(t,s,z^*) \) from 0 to \( t \) with the weight function \( \alpha(t-s) \)

\[
\hat{O}(t,z^*) = \int_0^t \alpha(t-s) O(t,s,z^*) ds \tag{4}
\]

\( \alpha(t-s) \) is the correlation function to describe the influence of the environment to the system, which depends on the spectral density of the system and temperature.

\[
\alpha(t-s) = \sum_{n=1}^{N} \frac{G_n^2}{2m_n \omega_n} \left[ \coth \left( \frac{\omega_n}{2kB T} \right) \cos \omega_n (t-s) - i \sin \omega_n (t-s) \right] \tag{5}
\]

Our numerical results focus on zero temperature and finite temperature. We can introduce the spectral density of bath oscillators:

\[
J(\omega) = \sum_{n=1}^{N} \frac{G_n^2}{2m_n \omega_n} \delta(\omega - \omega_n) = M \gamma \omega^3 f_c(\frac{\omega}{\Lambda}) \tag{6}
\]

Later we choose super-Ohmic environment in our numerical computation. \( f_c \) is a cutoff function, of which \( \Lambda \) is cutoff frequency, the most important parameters to control our environmental spectrum. In our simulation, cutoff function is chosen to be \( f_c(x) = \exp(-x) \).

The problem of solving stochastic Schrödinger equation is more and less solving \( O \) operator, exactly or approximately. The general QSD equation can be simplified to a form which may be simulated numerically, as long as the ansatz satisfies the consistency condition. In practice, even when the operator \( O(t,s,z^*) \) cannot be determined exactly, perturbation techniques may be used to create a serious of \( O \) operator allowing for an approximate form of general QSD equation. Fortunately, we can find exact \( O \) for two coupled harmonic oscillators. The convolution-less form of NMQSD equation will appear if we replace the functional derivative by the known operator

\[
\partial_t \psi_t = (-iH + L z_t^* - L^\dagger \hat{O}(t,z^*)) \psi_t(z^*) \tag{7}
\]

Once we have the NMQSD equation, the reduced density operator is given by the ensemble mean over the trajectories of the stochastic Schrödinger equation.

\[
\dot{\rho}_t = -i \left[ H, \rho_t \right] + \left[ L, \mathcal{M} \left\{ P_t \hat{O}^\dagger (t,z^*) \right\} \right] + \left[ \mathcal{M} \{ O(t,z^*) P_t \}, L \right] \tag{8}
\]

If the exact operator \( O \) is independent of the noise \( z_t^* \), we can find that

\[
\dot{\rho}_t = -i \left[ H, \rho_t \right] + \left[ L, \rho_t \hat{O}^\dagger (t) \right] + \left[ \hat{O}(t) \rho_t, L \right] \tag{9}
\]

In this paper, we will show that the exact master for two harmonic oscillators in zero temperature is Eq.\ref{eq:rotwave}, while the exact master equation of Rotating Wave Approximation form in zero temperature is Eq.\ref{eq:exactmaster} form.

B. Quantum Brownian motion with coupling symmetric in position and momentum by Quantum State Diffusion

Quantum Brownian motion of a damped harmonic oscillator bi linearly coupled to bath of harmonic oscillators has been studied for decades and the non-Markov exact master equation was provided by path integral techniques [2]. This part will provide another approach to exact master equation by Quantum State Diffusion. In the more generalized forms, the masses and frequencies of the oscillators are different so that the master equation can be used easily in the research of non resonant problems. Also, the couple between two harmonic
oscillators is a function of time to research the quantum control application, and if we are not interested in the control part we can set \( k(t) = k \).

The Hamiltonian of the total system consisting of a system of two mutually coupled harmonic oscillators with different mass and frequency interacting with a bath of harmonic oscillators of masses and frequencies in an equilibrium state at zero temperature:

\[
H_{\text{tot}} = H_{\text{sys}} + H_{\text{int}} + H_{\text{bath}} \tag{10}
\]

where

\[
H_{\text{sys}} = \frac{p_1^2}{2M_1} + \frac{p_2^2}{2M_2} + \frac{1}{2} M_1 \Omega_1^2 q_1^2 + \frac{1}{2} M_2 \Omega_2^2 q_2^2 + k(t) (q_1 - q_2)^2 \tag{11}
\]

is the system Hamiltonian for the two system oscillators of interest, with displacements and momentum and time dependent couple function \( k(t) \).

\[
H_{\text{bath}} = \sum_{n=1}^{N_B} \left( \frac{\pi_n^2}{2m_n} + \frac{1}{2} m_n \omega_n^2 b_n^2 \right) \tag{12}
\]

is the bath Hamiltonian with displacements and conjugate momentum.

One solvable model is that the system and the environment are coupled through different observables:

\[
H_{\text{int}} = (q_1 + q_2) \sum_{n=1}^{N_B} C_n b_n + \frac{p_1 + p_2}{M \Omega} \sum_{n=1}^{N_B} C_n \frac{m_n \omega_n \pi_n}{m_n \omega_n} \tag{13}
\]

For simplification, we assume the masses and frequencies of those harmonic oscillators are same. It is easy to rewrite this interaction Hamiltonian in creation and annihilation operators:\[3\]

\[
H_{\text{int}} = \sum_{n=1}^{N_B} \frac{C_n \sqrt{2}}{\sqrt{m_n \omega_n}} (a b_n^\dagger + a^\dagger b_n) \tag{14}
\]

The same type can also be derived from standard position-position coupling with rotating-wave approximation (RWA). In the expansion of \( (q - q')^2 \), we can rewrite this in terms of creation and annihilation operators as \( (a_1 - a_2 + a_1^\dagger - a_2^\dagger)^2 \). In RWA, high-frequency oscillating terms like \( a^2 \) and \( a^{12} \) are neglected. If we rewrite the system with RWA interaction as:

\[
H_{\text{tot}} = \Omega \left( a_1^\dagger a_1 + a_2^\dagger a_2 \right) + k(t) \left( a_1 - a_2 + a_1^\dagger - a_2^\dagger \right)^2 + \sum_{n=1}^{N_B} \omega_n b_n^\dagger b_n + \sum_{n=1}^{N_B} (g_n a_1 b_n^\dagger + g_n a_2 b_n^\dagger) + \sum_{n=1}^{N_B} (g_n a_1^\dagger b_n + g_n a_2^\dagger b_n) \tag{15}
\]

In this model, the Lindblad operator will be \( a_1 + a_2 \). We find the \( O \) operator without an explicit noise dependence:

\[
O(t, s, z^\gamma) = f(t, s) (a_1 + a_2) \tag{16}
\]

Define the integration of function \( f \) with the correlation function as weight function

\[
F(t) = \int_0^t f(t, s) \alpha(t - s) ds \tag{17}
\]

The complex function \( f \) follows the evolution equation and boundary condition:

\[
\partial_t f = i \Omega f + 2F(t)f \tag{18}
\]

\[
f(t, s = t) = 1 \tag{19}
\]

We can get the analytical solution if the frequency distribution is Lorentz spectrum, where \( \alpha(t, s) = \frac{\Gamma_2}{2} e^{-\gamma |t - s|} \). If we integrate both sides of the evolution equation of \( f \) from 0 to \( t \) with \( \alpha(t, s) \), we will have

\[
\partial_t F = 2F^2 - (\gamma - i \Omega) F + \frac{\Gamma_2}{2} \tag{20}
\]

which is Riccati Equation. Solve the equation with transformation

\[
F(t) = (-\gamma'(t))/(2\gamma(t))
\]

and we will have

\[
y'' + (\gamma - i \Omega) y' + \Gamma \gamma y = 0 \tag{21}
\]

Finally, the solution is

\[
F(t) = \frac{-\lambda_1 e^{\lambda_1 t} + \lambda_1 e^{\lambda_2 t}}{2 \left( e^{\lambda_1 t} - \frac{\Delta}{\lambda_2} e^{\lambda_2 t} \right)} \tag{22}
\]

where the parameters \( \Delta = (\gamma - i \Omega)^2 - 4\Gamma \gamma \) determines whether \( C(t) \) is complex or real. In this case, as long as \( \Omega \neq 0 \) or \( \gamma^2 - 4\Gamma \gamma < 0 \), the system will be non-Markov.

After obtaining the evolution of \( O \), master equation turns to be as follows:
\[
\partial_t \rho = i[\Omega(a_1^+a_1 + a_2^+a_2) + k(t)(a_1^+ - a_2^+ + a_1 - a_2)^2, \rho] + [(a_1 + a_2), \rho F^*(t)(a_1^+ + a_2^+)] + [F(t)(a_1 + a_2), \rho, (a_1^+ + a_2^+)].
\]

\[
(23)
\]

FIG. 1. If \( \Delta > 0 (\Gamma = 1, \gamma = 5, \Omega = 0) \), the system will become Markov when time goes by.

\[
\partial_t \rho = i[\Omega(a_1^+a_1 + a_2^+a_2) + k(t)(a_1^+ - a_2^+ + a_1 - a_2)^2, \rho] + [(a_1 + a_2), \rho F^*(t)(a_1^+ + a_2^+)] + [F(t)(a_1 + a_2), \rho, (a_1^+ + a_2^+)].
\]

\[
(23)
\]

FIG. 2. If \( \Delta < 0 (\Gamma = 1, \gamma = 3, \Omega = 0) \), system has obvious non-Markov properties.

\[
\frac{d|\psi\rangle}{dt} = [-iH_s + \triangle_t(L)|\psi\rangle - \triangle_t(L^\dagger)\bar{O}(t,z^+)|\psi\rangle + \triangle_t(L^\dagger)\bar{O}(t,z^+)|\psi\rangle]_{t=0}.
\]

\[
(24)
\]

Here, we have the notation

\[
\triangle_t(L) \equiv L - \langle L \rangle_t
\]

and the noise term

\[
\bar{z}_t = z_t^* + \int_0^t \alpha^*(t, s) \langle L^\dagger \rangle_s ds.
\]

\[
(25)
\]

\[
(26)
\]

III. APPLICATION IN QUANTUM CONTROL

In this section, we present theoretical analysis and numerical results of the open quantum system.

A. Quantum control for entanglement for gaussian states

The decoherence of two harmonic oscillators has been studied systematically in [2, 3] where coupling constant between two oscillators is constant. If we control the coupling relation as a function of time, we can control the decoherence. With the control function, we can transform the dynamical phase from one to a different one. or simplification, we assume the masses and frequencies of those harmonic oscillators are same (resonant case).

It is convenient to use coordinates \( x_{\pm} = (x_1 \pm x_2)/\sqrt{2} \) since \( x_+ \) couples to the environment while \( x_- \) is controlled:

\[
H_{tot} = \frac{p_+^2}{2m} + \frac{m}{2} \Omega^2 x_+^2 + \sqrt{2} x_+ \sum_{n=1}^{N_B} c_n q_n + \frac{N_B}{2m} \left( \frac{p_n^2}{2m_n} + \frac{m_n \omega_n^2 q_n^2}{2} \right) + \frac{p_+^2}{2m} + \frac{m}{2} \Omega^2 x_-^2 + 2k(t) x_-^2
\]

\[
(27)
\]

The system is simplified to a harmonic oscillator coupled to environment and another harmonic oscillator controlled by external time dependent frequency \( \Omega'(t) = (\Omega^2 + 4k(t)/m)^{\frac{1}{2}} \). The master equation for one harmonic oscillator [4] is
\[ \partial_t \rho = -i [H_R, \rho] - i \gamma(t) [x_+, \{p_+, \rho\}] - D(t) [x_+, [x_+, \rho]] - f(t) [x_+, [p_+, \rho]] \] (28)

where the renormalized Hamiltonian is

\[ H_R = \frac{p_+^2}{2m} + \frac{M}{2} \Omega^2 x_+^2 + \frac{p_-^2}{2m} + \frac{M}{2} \Omega^2 x_-^2 + \frac{M}{2} \delta \omega^2(t)x_+^2 \] (29)

where the coefficients depend on the spectral density of the environment. The second moments of \( x_+ \) and \( p_+ \) according to [3] are

\[ \frac{\partial_t}{2m} \left( \frac{\langle p_+^2 \rangle}{m} \right) + \frac{m}{2} \Omega^2(t) \frac{\partial_t}{m} \langle x_+^2 \rangle = -2 \frac{\gamma(t)}{m} \langle \frac{p_+^2}{m} \rangle + \frac{D(t)}{m} \] (30)

\[ \frac{1}{2} \partial_t^2 \langle x_+^2 \rangle + \gamma(t) \partial_t \langle x_+^2 \rangle + \Omega^2(t) \langle x_+^2 \rangle = \frac{\langle p_+^2 \rangle}{m} - f(t) \frac{\langle \frac{p_+^2}{m} \rangle}{m} \] (31)

Then, the system will become two separable harmonic oscillators: one is coupled with environment and another one is under the control. Similarly, we can have the second moments for \( x_- \) and \( p_- \).

\[ \frac{\partial_t}{2m} \left( \frac{\langle p_-^2 \rangle}{m} \right) + \frac{m}{2} \Omega^2 \left( \frac{4k(t)}{m} \right) \frac{\partial_t}{m} \langle x_-^2 \rangle = 0 \] (32)

\[ \frac{1}{2} \partial_t^2 \langle x_-^2 \rangle + \left( \Omega^2 + \frac{4k(t)}{m} \right) \langle x_-^2 \rangle = \frac{\langle p_-^2 \rangle}{m} \] (33)

We assume that the initial state of system is Gaussian. Because the complete evolution is linear and all the operators are quadratic, the Gaussian nature of the state will be preserved for all times. The entanglement for Gaussian states is entirely determined by the properties of the con-variance matrix. The covariance matrix (CM) \( \sigma \) is a real symmetric and positive \( 4 \times 4 \) matrix.

\[ \sigma(t) = \begin{bmatrix}
\sigma_{x_1x_1}(t) & \sigma_{x_1p_1}(t) & \sigma_{x_1x_2}(t) & \sigma_{x_1p_2}(t) \\
\sigma_{p_1x_1}(t) & \sigma_{p_1p_1}(t) & \sigma_{x_2p_1}(t) & \sigma_{p_1p_2}(t) \\
\sigma_{x_2x_1}(t) & \sigma_{x_2p_1}(t) & \sigma_{x_2x_2}(t) & \sigma_{x_2p_2}(t) \\
\sigma_{x_2p_2}(t) & \sigma_{p_1p_2}(t) & \sigma_{x_2p_2}(t) & \sigma_{p_1p_2}(t)
\end{bmatrix} \] (34)

where the matrix elements are defined as:

\[ \sigma_{ij} = \frac{1}{2} \text{Tr}[(\xi_i \xi_j + \xi_j \xi_i)\rho] - \text{Tr}[(\xi_i \rho) \text{Tr}(\xi_j \rho)] \] (35)

where \( \xi = (x_1, p_1, x_2, p_2) \). CM also has the structure

\[ \sigma(t) = \begin{bmatrix}
A & C \\
CT & B
\end{bmatrix} \] (36)

where \( A, B \) and \( C \) are \( 2 \times 2 \) Hermitian matrices and \( T \) denotes the transposed matrix. \( A \) and \( B \) denote the symmetric covariance matrices for the individual reduced one-mode states, while the matrix \( C \) contains the cross-correlations between modes. The advantage of gaussian state is that a good measure of entanglement for such states is logarithmic negativity \( E_N \), which can be computed as

\[ E_N = \max \{0, -\ln(2 \nu_{min})\} \] (37)

where \( \nu_{min} \) is the smallest symplectic eigenvalue of the partially transposed covariance matrix. Some expressions for \( E_N \) based on previous paper [3] have been known. For two mode squeezed state, obtained from the vacuum by acting with the creation operator \( \exp(-r(a_1^\dagger a_2^\dagger - a_1 a_2)) \), we would have \( E_N = 2 |r| \). For this squeezed state, the dispersions satisfy the minimum uncertainty condition \( \delta x_+ \delta p_+ = \delta x_- \delta p_- = 1/2 \). The squeezing factor determines the ratio between variances since \( m\Omega \delta x_+ / \delta p_+ = \delta p_- / (m\Omega \delta x_-) = \exp(2r) \). If \( r \to \infty \), the state becomes localized in the \( p_+ \) and \( x_- \) variables approaching an ideal Einstein-Podolsky-Rosen state. We will use numerical simulation to see how time-dependent control external field will influence the evolution of entanglement.

### B. Numerical results and discussion

Within the framework of evolution equation, entanglement, system energy and quantum coherence, measured by l1 norm can be investigated numerically. Initially, we consider the simplest case, the symmetric coupling between momentum and position in zero temperature, to show how input control can result in the change of entanglement, energy and coherence. We keep the parameters of environment, which is strong non-Markov case, and the coupling between two harmonic oscillators weak relative to coupling between system and environment. We simulate system by evolve the master equation. The initial states for all simulations are squeezed states created by squeezed operator \( \exp(-r(a_1^\dagger a_2^\dagger - a_1 a_2)) \), we would have \( E_N = 2 |r| \). Without control, the entanglement, coherence and energy will decrease from the initial value with fluctuations, where the non-Markov environment results in fluctuation by information and energy back flow. In long time limit, the entanglement will stay in a stable value.
With the control which frequency is much greater than the response frequency, the entanglement will decrease with oscillations with high input frequency but get stable value more quickly, which means high-frequency input cannot influence the entanglement to reach a new point which does not exist before the control. There are several resonance frequencies as shown in the figure. Keeping the strength of the input signal but changing the frequency, the oscillation of the entanglement, energy and coherence is growing while the frequency is approaching to the resonance frequency, and decaying while the frequency is leaving the resonance frequency. When the input signal frequency is exactly same as the resonance frequency, the energy will accumulate to a high level compared with non-resonance cases and fluctuate, as what happens in classical regime, and the analysis of classical system with comparison between classical system and quantum system will be given later. The coherence will increase with similar trend as energy which can be regarded as the input of information with energy, then the flowing of information and energy to environment is weakened by changing to a specific resonance frequency compared to the incoming flowing information and energy, which results in the accumulation of energy and information. There is an interesting phenomenon for entanglement in the resonance cases, after reaching the highest point, the entanglement drops quickly with fluctuation and reaches zero, which means two harmonic oscillators disentangle just after the maximum of entanglement. The entanglement between two harmonic oscillators cannot reach zero because the environment is non-markov except by resonance control, which is different from classical cases. In experiment, the input signal with specific frequency can protect the system coherence and entanglement from real environment.

In order to make our results more credible, we calculate two above-mentioned models using two different methods, one by quantum state diffusion equation and the other by master equation. If our above results are valid, these two methods should give the same evolutionary picture of entanglement over time. Finally, our
comparision clearly shows when the number of trajectories run by equation of quantum state diffusion is large enough, it gives the highly likely result with master equation, which shows our above numerical discussion is believable enough.

![FIG. 5. Comparision between Quantum State Diffusion method and Master equation method](image)

IV. CONCLUSION

In this paper, we studied the control effect with two interacting oscillators coupled with bosonic thermal bath by the method of quantum brownian motion. Non-Markovian master equation of reduced density matrix is obtained by quantum state diffusion method and one momentum-position coupling patterns are considered. There are several attracting questions remaining in this work. We noticed that after interacting with environment, two quantum oscillators evolve from a pure state to a final mixed state, and correspondingly physical quantities such as energy of system, quantum coherence and purity remain at a non-zero level after fluctuating at the beginning. Environment has the ability to build up coherence and entanglement but cannot totally destroy the connection or squeeze all information contained within system back to surroundings. Classically second law of thermodynamics might help us to understand relative phenomenon in qualitative view but we aim to find out the quantitative interpretation by means of this model. We speculate that there could be some symmetry protected process which makes the system get rid of thorough destruction in long-time limit. Relevant investigation will be presented in future work.

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