Collective transport of polar active particles on the surface of a corrugated tube

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Abstract
We study collective transport of polar active particles on the surface of a corrugated tube. Particles can be rectified on the surface of the asymmetric tube. The system shows different motion patterns which are determined by the competition between alignment strength and rotational diffusion. For a given alignment strength, there exist transitions from the circulating band state to the travelling state, and finally to the disordered state when continuously changing rotational diffusion. The circulating band is a purely curvature-driven effect with no equivalent in the planar model. The rectification is greatly improved in the travelling state and greatly suppressed in the circulating band state. There exist optimal parameters (modulation amplitude, alignment strength, rotational diffusion, and self-propulsion speed) at which the rectified efficiency takes its maximal value. Remarkably, in the travelling state, we can observe current reversals by changing translational diffusion.

1. Introduction
Active matter is made of self-driven agents that individually dissipate energy and organise in collective self-sustained motion. The important feature of active matter systems is the energy input on each individual unit, which drives the system completely out of equilibrium [1–6]. The motion of active matter on curved surfaces at the microscopic level is a very common situation in nature. Active systems where curvature plays an important role range from biology to physics. There are a few prominent examples: tissue folding and curvature are crucial during gastrulation [7], epithelial and endothelial cells move on constantly growing, curved crypts and villi in the gut [8], geometric constraints during epithelial jamming [9], and the motion of proteins on the surface of eukaryotic cells [10].

Recently, there has been increasing interest in theoretical and experimental work on active matter on curved surfaces [11–36]. The presence of intrinsic surface curvature frustrates local order giving rise to novel physics. On the one hand, when the manifold is getting curved, defects emerge due to topological constraints (curvature-induced defects) [11–21]. Keber and coworkers [11] studied the spatiotemporal patterns in active nematic vesicles and found that defects are largely static structures and move spontaneously. In the active nematic film, Alaimo et al [12] found a relation between Gaussian curvature and defects. Sknepnek and Henkes [14] studied active swarms on a sphere and frustration due to curvature leads to stable elastic distortions storing energy in the band. Ellis [16] studied the effect of self-propulsion and curvature on the degree of defect unbinding. On the other hand, curvature can induce spontaneous flow and flocking [22–28]. Sanchez and coworkers [22] experimentally studied self-propelled microtubule bundles on curved surfaces and reported the spontaneous generation of a streaming flow. For high curvature, Bruss and coworkers [24] found that particles converge to a common orbit to form symmetry-breaking microswarms. Shankar et al [25] found that curvature and active flow together result in symmetry-protected topological modes that get localized to special geodesics on the surface.

The problem of rectifying motion in random environments is a long-standing issue, which has many theoretical and practical implications [37, 38]. Active matter can be rectified on the asymmetric substrates

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2. Model and methods

We consider \( N \) active spherical particles of radius \( \sigma \) confined to move on the surface of a corrugated tube, shown in figure 1. The dynamics of particle \( i \) is described by the position \( \mathbf{r}_i \equiv (x_i, y_i, z_i) \) of its center and the orientation \( \mathbf{n}_i \). The surface of the tube is algebraically described by \( g(\mathbf{r}) = 0 \). The position \( \mathbf{r}_i \) and the orientation \( \mathbf{n}_i \) are constrained to the tangent plane at every point. \( g(\mathbf{r}) \) can be interpreted as a potential and the constraint trajectories will then lie on the isopotential surface with potential value 0 [12, 14, 46]. For each constraint there exists a constraint force that penalizes any deviations from the isopotential surface. For simplicity, we only focus on overdamped particles and neglect all hydrodynamic effects. In the overdamped limit, the dynamics of particle \( i \) are described by the following Langevin equations with holonomic constraint

\[
\frac{d\mathbf{r}_i}{dt} = v_0 \mathbf{n}_i + \mu \left[ \sum_{j \neq i} \mathbf{F}_{ij} - G(\mathbf{r}_i) \mathbf{J}_i + \sqrt{2D_0} \mathbf{\xi}_i(t) \right],
\]

where \( v_0 \) is the self-propulsion speed, \( \mu \) is the mobility and \( D_0 \) is the translational diffusion coefficient. Here \( G(\mathbf{r}) = \nabla \cdot g(\mathbf{r}) \) is the Jacobian of \( g(\mathbf{r}) \) and \( \lambda_i \) is the Lagrange multiplier [46].

The tube constraint is described as

\[
g(\mathbf{r}_i) = y_i^2 + z_i^2 - \left[ R - A_0 \left( \frac{2\pi x_i}{L_x} + \frac{\Delta}{4} \sin \frac{4\pi x_i}{L_x} \right) \right]^2,
\]

where \( L_x \) is the period of the tube. \( R \) is the average radius of the tube and \( A_0 \) is the amplitude of the modulation. \( \Delta \) is the asymmetry parameter of the periodic tube and the tube is symmetric at \( \Delta = 0 \). In order to avoid the appearance of blocked tube, the condition \( R > |A_0\left(\frac{2\pi x_i}{L_x} + \frac{\Delta}{4} \sin \frac{4\pi x_i}{L_x}\right)| \) must be satisfied.

For the polar alignment, we use the \( XY \)-like angular dynamics described in [14]

\[
\frac{d\mathbf{n}_i}{dt} = \left[ \mathbf{P}_N \left( \hat{N}_i - J \sum_j \mathbf{n}_j \times \mathbf{n}_i \right) + \sqrt{2D_t} \mathbf{\zeta}_i(t) \right] (\mathbf{n}_i \times \mathbf{n}_i),
\]

where \( \mathbf{P}_N \) is the normal projection of a vector on the unit normal to the tangent plane as \( \mathbf{P}_N(\hat{N}_i, \mathbf{a}) = (\mathbf{a} \cdot \hat{N}_i) \hat{N}_i \) and \( \hat{N}_i \) is the local surface normal. The summation in equation (3) is carried over all neighbors within a 4.0\( \sigma \) cutoff radius. \( J \) is the polar alignment strength and we only consider the ferromagnetic case (\( J > 0 \)). \( \mathbf{\zeta}_i(t) \) and \( \mathbf{\zeta}_j(t) \) are uncorrelated Gaussian white noises with zero mean and unit variance. \( D_t \) denotes the rotational diffusion coefficient.

The interactions between particles are taken as short-ranged harmonic repulsive force: \( \mathbf{F}_{ij} = k (2\sigma - r_{ij}) \hat{r}_{ij} \) if particles overlap \( (r_{ij} < 2\sigma) \), and \( \mathbf{F}_{ij} = 0 \) otherwise. \( r_{ij} = |\mathbf{r}_i - \mathbf{r}_j| \) is the distance between particle \( i \) and particle \( j \) and \( \hat{r}_{ij} = (\mathbf{r}_i - \mathbf{r}_j) / r_{ij} \). Here \( k \) describes the interaction strength.

Time and length are in units of the elastic time \( \frac{1}{\mu k} \) and the diameter of particles \( 2\sigma \). The parameters in the dimensionless forms can be rewritten as \( \hat{v}_0 = \frac{v_0}{2\mu k}, \hat{D}_i = \frac{D_i}{\mu k}, \hat{D}_0 = \frac{D_0}{4\mu k^2}, \hat{J} = \frac{J}{\mu}, \hat{R} = \frac{R}{2\sigma}, \hat{L}_x = \frac{L_x}{2\sigma}, \) and \( \hat{A}_0 = \frac{A_0}{2\sigma} \). From now on, we will use only the dimensionless variables and shall omit the hat for all quantities occurring in the above equations.

We measure the rectification of active particles by the average velocity in the \( x \)-direction. In the asymptotic long-time regime, the average velocity can be obtained from the formula \( \mathbf{V}_x = \frac{1}{N} \sum_{i=1}^{N} \lim_{t \to -\infty} \frac{\langle x_i(t) - x_i(0) \rangle}{t} \), where \( \langle \ldots \rangle \) denotes the average over trajectories of the particle with random initial conditions and noise realizations. We define the ratio between the area occupied by particles and the total available area as the packing fraction \( \phi = \frac{N\pi \sigma^2}{2\pi R_u} \).

When particles diffuse on the surface of the tube, the dimensionless cross-sectional circle length \( 2\pi R_u / 2\pi (R - A_0) \) varies along the \( x \)-axis. The effective entropic potential along \( x \)-direction can be obtained after the elimination of the \( y \) and \( z \) coordinates by assuming the equilibrium in the orthogonal direction. The more
detailed approach can be seen in [47]. The corresponding potential is simply \( U_{\text{eff}}(x) = -TS \), where the entropic barrier is \( S = k_B \ln \left( \frac{2 \pi R_x}{2 \pi (R - A_0)} \right) \). Therefore, the effective entropic potential is defined as

\[
U_{\text{eff}}(x) = -T k_B \ln \left( \frac{2 \pi R_x}{2 \pi (R - A_0)} \right),
\]

where \( R_x = R - A_0 \left( \sin \frac{2 \pi x}{L_x} + \frac{\Delta}{4} \sin \frac{4 \pi x}{L_x} \right) \) is the radius of the tube at \( x \), \( k_B \) is the Boltzmann constant and \( T \) is the absolute temperature. The profiles of the effective entropic potential are shown in figure 2 for different \( \Delta \). It is found that the surface with positive Gaussian curvature (the maximum radius of the tube) corresponds to the peak of the effective entropic potential, while the surface with negative Gaussian curvature (the minimum radius of the tube) corresponds to the valley of the effective entropic potential. Therefore, particles have a tendency to move on the surface with positive Gaussian curvature.

3. Results and discussion

Equation (1) can be numerically solved by using RATTLE discretization [46]. From the RATTLE algorithm, we can obtain a set of \( M \) equations to determine \( M \) multipliers \( \lambda_i \) and these equations will be linearly independent and offer a unique set of \( \lambda_i \) [14]. Particles are constrained to the curved surface by projecting the positions and force vectors back onto the local tangent plane after every time step. Equation (3) has been integrated numerically, the torques were projected onto the surface normal at \( r_i \), and finally, \( n_i \) was rotated by a random angle around the same normal [14]. The integration step time was chosen to be smaller than \( 10^{-4} \) and the total...
integration time was more than $10^5$. We have considered 500 realizations to improve accuracy and minimize statistical errors. Unless otherwise noted, our simulations are under the parameter sets: $L_x = 20.0$ and $R = 10.0$.

### 3.1. Motion patterns

In figure 3, we present the typical steady-state snapshots of motion patterns for different values of $J$ and $D_r$ at $D_0 = 0$ and $\phi = 0.8$ (we also include movies in the supplemental materials is available online at stacks.iop.org/NJP/21/093041/mmedia). When alignment strength dominates the transport ($J \gg D_r$), one can observe a circulating band state (shown in figure 3(a)). In this state, particles are compressed toward the maximum radius (positive Gaussian curvature) of the tube and there exist the bald belts at the minimum radius (negative Gaussian curvature) of the tube. The flock rotates around the $x$-axis and cannot move along the $x$-direction. Note that the circulating band is a purely curvature-driven effect with no equivalent in the planar model. When rotation diffusion and alignment strength ($J; D_r$) are comparable, the circulating band state is broken and the travelling state appears (shown in figure 3(b)). In this state, the circular motion disappears and all particles move together along the $x$-direction. When rotation diffusion dominates the transport, polar alignment can be neglected and one can observe the disordered state (shown in figure 3(c)). Therefore, for a given $J$, there exists a transition from the circulating band state to the travelling state and finally to the disordered state when $D_r$ increases continuously from zero.

The phenomena above can be explained as follows. For the sake of discussion, we have plotted the profiles of the effective entropic potential in figure 2 for different $\Delta$. From figure 2, we can find that the bottom (top) of the potential corresponds to the maximum (minimum) radius of the tube. When $J \gg D_r$, rotation diffusion is too small to drive particles out of the potential bottom, and particles tend to stay at the bottom of the potential (the maximum radius of the tube). On the other hand, due to the alignment strength, particles have a tendency to move together. Therefore, all particles rotate around the $x$-axis at the maximum radius of the tube (the

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1 See supplemental material for movies of motion patterns.
circulating band state). When \( J \approx D_r \), active particles can pass across the effective entropic barriers. In this case, particles can be rectified on the surface of the asymmetric tube and move along \( x \)-direction. Owing to alignment interactions between particles, all particles move together along the \( x \)-direction (the travelling state).

### 3.2. Rectification

In this section, we studied the rectification of polar active particles on the surface of the asymmetry corrugated tube. We consider the case of zero translational diffusion in figures 4–7 and the effects of translational diffusion on rectification in figure 8.

Figure 4(a) shows the dependence of the average velocity \( V_x \) on the asymmetry parameter \( \Delta \) of the tube. We find that the average velocity \( V_x \) is positive for \( \Delta < 0 \), zero at \( \Delta = 0 \), and negative for \( \Delta > 0 \). A qualitative explanation of this behavior is presented as follows. When \( \Delta < 0 \), the left side from the minima of the potential is steeper (shown in the top panel of figure 2), and it is easier for particles to move toward the slanted side than toward the steep side, so particles on average move to the right. Similarly, particles on average move to the left \( (V_x < 0) \) when \( \Delta > 0 \). When \( \Delta = 0 \), the tube is completely symmetric (shown in the middle panel of figure 2), rectified transport disappears. In the large \( \Delta \) limits, the effective entropic barrier is too high and particles are trapping in local minima, so current is suppressed. Therefore, there exists an optimal value of \( \Delta \) at which \( |V_x| \) takes its maximal value. In the discussion below, we only consider the case of \( \Delta < 0 \).

The dependence of the average velocity \( V_x \) on the amplitude \( A_0 \) of the modulation is shown in figure 4(b). It is found that the average velocity \( V_x \) is a peaked function of the amplitude \( A_0 \). When \( A_0 \to 0 \), the corrugated tube becomes the straight tube, rectified transport disappears and \( V_x \to 0 \). When \( A_0 > 4.0 \), the effective
entropic barrier is too high for the particles to pass through, rectified transport also disappears. Therefore, there exists an optimal of $A_0$ at which the average velocity takes its maximum value.

Figure 5 (a) describes the average velocity $V_x$ as a function of alignment strength $J$ for different $D_r$. When $J \rightarrow 0$, rotational diffusion dominates the transport, the system shows the disordered state (see figure 3(c)), the rectification is very weak, thus $V_x$ goes to zero. When $J$ is very large, alignment interactions are dominated, all particles rotate around the $x$-axis and the system shows the circulating band state. In this case, particles are trapped on the surface with positive curvature (the maximum radius of the tube), so $V_x$ tends to zero. There exists an optimal value of $J$ at which alignment strength and rotational diffusion are comparable. In this case, all particles move together along the $x$-direction (the travelling state in figure 3(b)), thus the average velocity is large. Therefore, there is a peak in each curve and the peak shifts to larger $J$ with the increase of $D_r$.

The dependence of the average velocity $V_x$ on the rotational diffusion $D_r$ is shown in figure 5(b) for different $J$. Similar to figure 5(a), there is a peak in each curve. When $D_r \rightarrow 0$, the system shows the circulating band state, rectified transport disappears. When $D_r \rightarrow \infty$, alignment strength can be ignored and the motion of polar active particles becomes an effective Brownian motion, so particles cannot be rectified. When $D_r \simeq J$, the system shows the travelling state, the average velocity $V_x$ is maximal.

From figures 5(a) and (b), we can conclude that the rectified efficiency is maximal when the system shows the travelling state, while the rectification will disappear when the system shows the circulating band state or the disordered state. To study in more detail the dependence of rectification on $J$ and $D_r$, we plotted contour plots of

Figure 7. Average velocity $V_x$ versus $\phi$ for different $D_r$ at $J = 0.1$. Other parameters are $\Delta = -1.0, A_0 = 3.0, D_0 = 0.0$, and $v_0 = 1.0$.

Figure 8. (a) Average velocity $V_x$ versus $D_0$ for different $D_r$ at $v_0 = 1.0$. (b) Average velocity $V_x$ versus $D_0$ for different $v_0$ at $D_r = 0.1$. Other parameters are $J = 0.1, \Delta = -1.0, A_0 = 3.0$, and $\phi = 0.25$ ($N = 400$ in one period).
average velocity $V_x$ as functions of $J$ and $D_r$ in figure 5(c). It is found that the rectification is greatly improved in the diagonal region (region B) and is greatly suppressed in the nondiagonal region (region A and region C). Note that regions A, B and C correspond to the circulating band state, the travelling state, and the disordered state, respectively.

The average velocity $V_x$ as a function of $v_0$ is shown in figure 6 for different $D_r$ at $J = 0.1$. It is found that all curves are observed to be bell shaped, and there exists an optimal value of $v_0$ at which $V_x$ takes its maximal value. When $v_0 \to 0$, no nonequilibrium driving appears in the system and the rectification disappears. When $v_0$ is very large, the modulation of tube can be negligible, the rectification also disappears. There exists an optimal self-propulsion speed $v_0$ at which $V_x$ is maximal. As $D_r$ increases, the position of the peak shifts to large $v_0$. Note that for large $D_r$, the average velocity $V_x$ is very small and the peak is very low.

The dependence of the average velocity $V_x$ on the packing fraction $\phi$ is shown in figure 7 for different $D_r$ at $J = 0.1$. An increase of the packing fraction $\phi$ can cause two results: (A) reducing the self-propelled driving, which blocks the ratchet transport and (B) activating motion and breaking the circulating band state, which facilitates the ratchet transport. When $D_r$ is small (e.g. $D_r = 0.05, 0.06$ and $0.07$), alignment strength is dominated, the system shows the circulating band state. In this case, when $\phi$ increases from zero, the factor $B$ first dominates the transport, thus $V_x$ increases with $\phi$. When $\phi \to 1$, the factor A is dominated, so $V_x$ decreases with the increase of $\phi$. There exists an optimal value of $\phi$ at which $V_x$ takes its maximal value. When $D_r$ competes with $J$ (e.g. $D_r = 0.1$), $V_x$ monotonously increases with $\phi$. Because the system shows the travelling state and the factor A will not take effect even for large $\phi$, $V_x$ saturates to a constant when $\phi > 0.4$. When $D_r > J$ (e.g. $D_r = 0.5$), the system shows the disordered state. In this case, it is not easy for particles to pass across the effective entropic barrier and the factor A always dominates the transport, thus $V_x$ monotonously increases with $\phi$. Because particles cannot form the travelling state, $V_x$ is very small when $\phi < 0.6$.

In figures 8(a) and (b), we show how translational diffusion affects the average velocity for different $D_r$, and $v_0$ at $J = 0.1$ and $\phi = 0.25$. When $D_r = 0.0$ (the circulating band state), $V_x$ tends to zero when $D_r \to 0$ or $D_0 \to \infty$ and there is a valley in the curve. When $D_r = 1.0$ (the disordered state), $V_x$ is always zero when changing $D_0$. When $D_r = 0.1$ (the travelling state), on increasing $D_0$ from zero, $V_x$ first changes from a positive maximum to zero, then from zero to a negative maximum, and finally tends to be zero. Therefore, in the travelling state, we can obtain the current reversals by changing translational diffusion. Note that when $D_0 \to \infty$, the modulation of the tube and the self-propulsion can be ignored, the rectification disappears and $V_x \to 0$.

Current reversals can be explained as follows. When $D_0$ is very small, the self-propulsion is the main driving for particles to pass across the effective entropic barrier. In this case, the slope of the effective potential dominates the transport, particles tend to pass across the barrier from the slanted side, thus $V_x$ is positive. When $D_0$ is large, translational diffusion is the main driving for particles pass across the effective entropic barrier. In this case, the diffusion distance is dominated, particles have a tendency to pass across the barrier from the side where the diffusion distance is short. From the top panel of figure 2, we can see that the distance between an entropic minima and its nearest neighbor maximum from the steep side (the left side) is less than that from the slanted side (the right side), thus particles on average move to the left ($V_x < 0$).

4. Concluding remarks

In this work, collective transport of polar active particles is investigated on the surface of a corrugated tube. Nonequilibrium driving comes from the self-propulsion, which can break the thermodynamics equilibrium and induce directed transport on the surface of the asymmetry tube. The system shows different motion patterns which are determined by the competition between alignment strength and rotational diffusion. We observe the travelling state when alignment strength and rotational diffusion are comparable, the circulating band state when alignment strength dominates the transport, and the disordered state when rotational diffusion is dominant. The circulating band is a purely curvature-driven effect with no equivalent in the planar model. The rectification is maximized when alignment strength and rotational diffusion are comparable (the travelling state) and minimized when alignment strength dominates the transport (the circulating band state). When translational diffusion is absent, the direction of transport is completely determined by the asymmetry of the tube, the average velocity $V_x$ is positive for $\Delta < 0$, zero at $\Delta = 0$, and negative for $\Delta > 0$. There exist optimal parameters (modulation amplitude, alignment strength, rotational diffusion, and self-propulsion speed) at which the rectified efficiency takes its maximal value. When translational diffusion is considered, translational diffusion can strongly affect rectified transport of particles. Interestingly, in the travelling state, we can observe current reversals by suitably tailoring translational diffusion.

The present model is too simple to provide a realistic description of real systems. Future work should address the effects of hydrodynamics interactions between particles, the different internal symmetry of the active
particles (e.g. nematic), the shape of the particles, and the inertial effect. We hope this work will be realized by the possible experiments (e.g. self-propelled colloids on the curved surfaces).

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References

[1] Bechinger C, Leonardo R D, Löwen H, Reichhardt C, Volpe G and Volpe G 2016 Rev. Mod. Phys. 88 045006
[2] Menzel A M 2015 Phys. Rep. 554 1
[3] Marchetti M C, Joanny J F, Ramaswamy S, Liverpool T B, Prost J, Raso M and Simha R A 2013 Rev. Mod. Phys. 85 1143
[4] Cates M E 2012 Rep. Prog. Phys. 75 042601
[5] Vicsek T and Zafeiris A 2012 Phys. Rep. 517 71
[6] Speck T 2016 Eur. Phys. J. Spec. Top. 225 2287
[7] Vasse B, Baller A, Chaplain M, Glazier J A and Weijer C J 2010 PLoS One 5 e10571
[8] Ritsma I, Ellenbroek S I, Zomer A, Snipper H J, de Sauvage F J, Simons B D, Clevers H and van Rheenen J 2014 Nature 507 362
[9] Aita L et al 2018 Nat. Phys. 14 613
[10] Batada N, Sheppard L, Siegmund D and Levit M 2006 PLoS Comput. Biol. 3 e24
[11] Keber F C, Loiseau E, Sanchez T, DeCamp S J, Giomi L, Bowick M J, Marchetti M C, Dogic Z and Bausch A R 2018 Soft Matter 14 3451
[12] Alaimo F, Köhler C and Voigt A 2017 Sci. Rep. 7 5211
[13] Praetorius S, Voigt A, Wittkowski R and Löwen H 2018 Phys. Rev. E 97 052615
[14] Sknepnek R and Henkes S 2015 Phys. Rev. E 91 022306
[15] Henkes S, Marchetti M C and Sknepnek R 2018 Phys. Rev. E 97 042605
[16] Ellis P W, Pearce D J G, Chang-Y-W, Goldsztein G, Giomi L and Fernandez-Nieves A 2017 Nat. Phys. 14 85
[17] Ellis P W, Nayan K, McInerney J P, Rocklin D Z, Park J O, Srinivasarao M, Matsumoto E A and Fernandez-Nieves A 2018 Phys. Rev. Lett. 121 247803
[18] Shankar S, Ramaswamy S, Marchetti M C and Bowick M J 2018 Phys. Rev. Lett. 121 108002
[19] Norton M M, Baskaran A, Opatthalage A, Langeslay B, Fraden S, Baskaran A and Hagan M F 2018 Phys. Rev. E 97 012702
[20] Ehrig S, Ferracci I, Weinkamer R and Dunlop J W C 2017 Phys. Rev. E 95 062609
[21] Yao Z 2016 Soft Matter 12 7202
[22] Sanchez T, Chen D T, DeCamp S J, Heymann M and Dogic Z 2012 Nature 491 431
[23] Janssen L M C, Kaiser A and Löwen H 2017 Sci. Rep. 7 5667
[24] Brusas R I and Glotzer S C 2017 Soft Matter 13 5117
[25] Shankar S, Bowick M J and Marchetti M C 2017 Phys. Rev. X 7 031039
[26] Fei W, Driscoll M M, Chaikin P and Bishop K J 2018 Soft Matter 14 4661
[27] Mickelin O, Slomka J, Burns K J, Lecoanet D, Vasil G M, Faria L M and Dunkel J 2018 Phys. Rev. Lett. 120 164503
[28] Fonda P, Rinaldini M, Kraft D I and Giomi L 2018 Phys. Rev. E 98 032801
[29] Pearce D J G, Ellis P W, Fernandez-Nieves A and Giomi L 2019 Phys. Rev. Lett. 122 160802
[30] Fily J, Baskaran A and Hagan M F 2016 arxiv:1601.00324
[31] Apaza L and Sandovol M 2017 Phys. Rev. E 96 022606
[32] Apaza L and Sandovol M 2018 Soft Matter 14 9928
[33] Castro-Villarruel P and Sevilla F J 2018 Phys. Rev. E 97 052605
[34] Giomi L 2015 Phys. Rev. X 5 031003
[35] Yan W and Brady J F 2018 Soft Matter 14 279
[36] Klaus C S, Raghunathan K, DiBenedetto E, Kenworthy A K and Bassereau P 2016 Mol. Biol. Cell. 27 3937
[37] Hänggi P and Marchesoni F 2009 Rev. Mod. Phys. 81 387
[38] Reimann P 2002 Phys. Rep. 361 S7
[39] Olson Reichhardt C J and Reichhardt C 2017 Annu. Rev. Condens. Matter Phys. 8 51
[40] Ghosh P K, Mislov V R, Marchesoni F and Nori F 2013 Phys. Rev. Lett. 110 268301
[41] Ao X, Ghosh P K, Li Y, Schmid G, Hänggi P and Marchesoni F 2014 Eur. Phys. J. Spec. Top. 223 3227
[42] Li Y, Ghosh P K, Marchesoni F and Li B 2014 Phys. Rev. E 90 062301
[43] Ghosh P K, Hänggi P, Marchesoni F and Nori F 2014 Phys. Rev. E 89 062115
[44] Ai R Q, He Y F and Zhong W R 2017 Phys. Rev. E 95 012116
[45] Ai R Q, Chen Q Y, He Y F, Li F G and Zhong W R 2013 Phys. Rev. E 88 062129
[46] Leimkuhler B and Reich S 2004 Simulating Hamiltonian Dynamics (Cambridge: Cambridge University Press)
[47] Reguera D and Rubi J M 2001 Phys. Rev. E 64 061106