Damping of transversal plasma-electron oscillations and waves in low-collision electron-ion plasmas

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Abstract

Previously developed method [1, 3] for finding asymptotic solutions of Vlasov equations using two-dimensional (in coordinate $x$ and time $t$) Laplace transform is here applied to consider transversal oscillations and waves in low-collision quasi-neutral ($n_i \simeq n_e$) Maxwellian electron-ion plasmas. We obtain two branches of electron waves: the ubiquitous one of high-frequency and high-velocity oscillations and the unusual low-velocity one. Taking into account Coulomb collisions in the limit $m_e \ll m_i$, $\bar{v}_i \ll \bar{v}_e$, and $T_e m_e \ll T_i m_i$ results in expressions for transversal plasma-electron oscillation/wave decrements with a damping of the low-velocity electron branch $\sim n_i^{1/3} / \bar{v}_e^{4/3}$, where $n_i$ is the ion density and $\bar{v}_e$ is the mean electron velocity. It ought to rehabilitate Vlasov principal value prescription for relevant integrals, but to supplement it with representation of an asymptotical solution as a sum of exponents (not a single one!). "Non-damping" kinematical waves in low-collision plasma transform in the damping ones at reasonably chosen iteration process.

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1 Introduction

Propagation of plane transversal electromagnetic waves in plasmas is described by asymptotic in coordinates and time solutions of the coupled system of kinetic equations for electrons and ions and Maxwell equations for the electric field:

$$\frac{\partial f_{1}^{(e)}}{\partial t} + v_x \frac{\partial f_{1}^{(e)}}{\partial x} - \frac{|e|E_z(x, t)}{m_e} \frac{\partial f_{0}^{(e)}}{\partial v_z} = 0, \quad (1)$$

$$\frac{\partial f_{1}^{(i)}}{\partial t} + v_x \frac{\partial f_{1}^{(i)}}{\partial x} + \frac{|e|E_z(x, t)}{m_i} \frac{\partial f_{0}^{(i)}}{\partial v_z} = 0, \quad (2)$$

$$\frac{\partial^2 E_z(x, t)}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 E_z(x, t)}{\partial t^2} + \frac{4\pi |e|}{c^2} \frac{\partial}{\partial t} \int v_z \left( n_e f_{1}^{(e)} - n_i f_{1}^{(i)} \right) d\vec{v} = 0, \quad (3)$$

where

$$\int f_{0}^{(e,i)} d\vec{v} = 1; \quad \int f_{1}^{(e,i)} d\vec{v} \ll \int f_{0}^{(e,i)} d\vec{v}; \quad \int v_z f_{1}^{(e,i)} d\vec{v} \ll \int v_z f_{1}^{(e,i)} d\vec{v}, \quad (4)$$

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the plane wave is moving in $x$-direction, $f_0$ is Maxwell distribution function. Let us note that one can add to the right hand sides of Eqs. (1), (2) some collision integrals.

Here we limit ourselves to a consideration of the particular case of plane electron waves in infinite homogenous fully ionized plasma with the boundary electric field perturbation $E(0, t) = E_0 \exp(i\omega t)$. In close analogy to our previous paper [4], where we have considered the damping of longitudinal waves in the electron-ion low-collision plasmas, the problem is solved using iteration technique with substitution of collisionless distribution functions into the Coulomb collision integrals.

2 The null iteration

Neglecting the ion constituent in Eq. (3) and using Laplace transforms in Eqs. (1)–(3) we arrive at

$$E_x(x, t) = \frac{1}{(2\pi i)^2} \iiint \sigma^{+} \sigma^{-} \int \int E_{p_1 p_2} e^{p_1 t + p_2 x} dp_1 dp_2 ,$$  \hspace{1cm} (5)

$$f^{(e)}_1 (\vec{v}, x, t) = \frac{1}{(2\pi i)^2} \iiint \sigma^{+} \sigma^{-} \int \int f^{(1)}_{p_1 p_2} e^{p_1 t + p_2 x} dp_1 dp_2 ,$$ \hspace{1cm} (6)

$$\frac{\partial f^{(e)}_1 (\vec{v}, x, t)}{\partial x} = \frac{1}{(2\pi i)^2} \iiint \sigma^{+} \sigma^{-} \int \int (p_2 f^{(1)}_{p_1 p_2} - f^{(1)}_{p_1}) e^{p_1 t + p_2 x} dp_1 dp_2 ,$$ \hspace{1cm} (7)

$$\frac{\partial f^{(e)}_1 (\vec{v}, x, t)}{\partial t} = \frac{1}{(2\pi i)^2} \iiint \sigma^{+} \sigma^{-} \int \int (p_1 f^{(1)}_{p_1 p_2} - f^{(1)}_{p_2}) e^{p_1 t + p_2 x} dp_1 dp_2 ,$$ \hspace{1cm} (8)

$$\frac{\partial^2 E_x(x, t)}{\partial x^2} = \frac{1}{(2\pi i)^2} \iiint \sigma^{+} \sigma^{-} \int \int \left(p_2^2 E_{p_1 p_2} - p_2 E_{p_1} - F_{p_1}\right) e^{p_1 t + p_2 x} dp_1 dp_2 ,$$ \hspace{1cm} (9)

$$\frac{\partial^2 E_x(x, t)}{\partial t^2} = \frac{1}{(2\pi i)^2} \iiint \sigma^{+} \sigma^{-} \int \int \left(p_1^2 E_{p_1 p_2} - p_1 E_{p_2} - F_{p_2}\right) e^{p_1 t + p_2 x} dp_1 dp_2 ,$$ \hspace{1cm} (10)

where $\sigma^{+}_{1, 2} \equiv \sigma_{1, 2} \pm i\omega$ and $f^{(1)}_{p_1}, f^{(1)}_{p_2}, F_{p_1}, F_{p_2}, E_{p_1},$ and $E_{p_2}$ are, correspondingly, Laplace transforms of $f^{(e)} (\vec{v}, 0, t)$, $f^{(e)} (\vec{v}, x, 0)$, $\frac{\partial f^{(e)}_1 (\vec{v}, x, t)}{\partial x}_{|x=0'}, \frac{\partial f^{(e)}_1 (\vec{v}, x, t)}{\partial t}_{|t=0'}$, $E_x(0, t)$, and $E_x(x, 0)$.

Neglecting for simplicity initial and boundary values

$$f^{(1)}_{p_1}, f^{(1)}_{p_2}, F_{p_1}, F_{p_2}, E_{p_1}, E_{p_2}$$ \hspace{1cm} (11)

(they do not affect characteristic frequencies $\omega \equiv -ip_1$ and wave numbers $k \equiv -ip_2$) one obtains the following equation for the double poles in $p_1$ and $p_2$:

$$E_{p_1 p_2} \left[p_2^2 - \frac{p_1^2}{c^2} + \frac{\omega_{p_1}^2}{c^2} \int v_z \frac{\partial f^{(e)}_0}{\partial v_z} \frac{d^3 \vec{v}}{p_1 + v_z p_2} \right] = p_2 E_{p_1} .$$ \hspace{1cm} (12)

where $\omega_L \equiv \sqrt{4\pi e^2 n_e/m_e}$ is Langmuir frequency. Using transformation

$$\int_{-\infty}^{\infty} e^{-}\frac{m_e v^2}{2 p_1} \frac{dv_x}{p_1 + v_x p_2} \equiv \int_{0}^{\infty} e^{-}\frac{m_e v^2 \\}{p_1^2 - v_x^2 p_2^2} \approx \int_{0}^{\infty} e^{-}\frac{m_e v^2}{p_1^2}} \frac{dv_x}{p_1 + v_x p_2} \approx \int_{0}^{\infty} e^{-}\frac{m_e v^2}{p_1^2}} \frac{dv_x}{p_1 + v_x p_2}$$
\begin{equation}
\approx \sqrt{\frac{2\pi kT_e}{m_e}} \frac{p_1}{p_1^2 - v_x^2 p_2^2},
\end{equation}

where \( v_x^2 \) can be approximated by the mean square velocity defined by Maxwell exponent

\begin{equation}
v_x^2 \simeq \frac{kT_e}{m_e},
\end{equation}

one obtains

\begin{equation}
E_{p_1 p_2} \simeq \frac{p_2 E_0 / (p_1 - i\omega)}{p_2^2 - (p_1^2 / c^2) \left( 1 + \omega_L^2 / (p_1^2 - p_2^2 v_x^2) \right)}
\end{equation}

and characteristic equation for the poles \( p_1, p_2 \):

\begin{equation}
p_2^2 - \frac{p_1^2}{c^2} \left( 1 + \frac{\omega_L^2}{p_1^2 - p_2^2 v_x^2} \right) = 0,
\end{equation}

where it was assumed

\begin{equation}
E(0, t) = E_0 e^{i\omega t}; \quad E_{p_1} = \frac{E_0}{p_1 - i\omega}.
\end{equation}

This implies the pole in the complex \( p_1 \) plane:

\begin{equation}
p_1 = i\omega
\end{equation}

and the corresponding pole in the complex \( p_2 \) plane defined from Eq. (16).

Taking account for \( v_x^2 \ll c^2 \) one obtains from Eqs. (16) and (17) two solutions:

\begin{equation}
[p_2^{(1)}]^2 = -\frac{\omega^2}{2v_x^2} \left[ 1 + \frac{v_x^2}{c^2} - \sqrt{\left( 1 + \frac{v_x^2}{c^2} \right)^2 - 4 \frac{v_x^2}{c^2} \left( 1 - \frac{\omega_L^2}{\omega^2} \right)} \right]
\end{equation}

\begin{equation}
\simeq -\frac{\omega^2}{c^2} \left( 1 - \frac{\omega_L^2}{\omega^2} \right); \quad \text{(19)}
\end{equation}

\begin{equation}
[p_2^{(2)}]^2 = -\frac{\omega^2}{2v_x^2} \left[ 1 + \frac{v_x^2}{c^2} + \sqrt{\left( 1 + \frac{v_x^2}{c^2} \right)^2 - 4 \frac{v_x^2}{c^2} \left( 1 - \frac{\omega_L^2}{\omega^2} \right)} \right].
\end{equation}

The solution (19) at \( \omega_L < \omega \), that is

\begin{equation}
p_2^{(1)} = ik \simeq \pm i \frac{\omega L}{c} \sqrt{1 - \frac{\omega_L^2}{\omega^2}},
\end{equation}

is the well-known result of non-damping transversal electromagnetic high-frequency waves in fully ionized plasma [2]. Phase velocity of this mode is greater than \( c \), but this result is by no means related to the applicability of Maxwell distribution function \( f_0^{(e)} \) at high velocities \( v \) up to \( c \), but, instead, is due only to the Maxwell field equation Eq. (3).

The solution (20) for non-damping low-velocity waves appears to be more intriguing:

\begin{equation}
p_2^{(2)} = ik \simeq \pm i \frac{\omega}{\sqrt{v_x^2}} \left( 1 + \frac{v_x^2 \omega_L^2}{2c^2 \omega^2} \right) \simeq \pm i \frac{\omega}{\sqrt{v_x^2}}
\end{equation}

with phase and group velocities

\begin{equation}
V_{ph} \simeq V_{gr} \simeq \sqrt{v_x^2},
\end{equation}

which are not dependent on \( \omega_L \).
Since we assume definiteness and convergence of the inverse Laplace transformation we should discuss an appearance of numerous poles \((p_1 + p_2v_x) = 0\) in integrals in \(dv_x\) in Eq.\((12)\) depending on running values \(p_1, p_2\). The results of calculations of poles \(p_1 = i\omega\) and \(p_2(1,2)\), Eqs.\((13)–(24)\), show that at these values of \(p_1\) and \(p_2\) the integral in \(dv_x\) in Eqs.\((12)–(13)\) appears as logarithmically divergent. Strictly speaking, approximation \((13)\) implies that this integral is defined in a principal-value sense. In this case the inverse Laplace transformation from approximate “image” to “original” function is also definite both for \(E_{p_1p_2}\) and \(f^{(1)}_{p_1p_2}\).

The criterium for the validity of some found solutions \(f_1(v_x, x, t)\) and \(E(\omega t)\), Eqs.\((11)–(3)\), is the fulfilment of original Eqs.\((1)-(3)\) on these functions. Such definite solutions can be obtained if one takes the principal value prescription for integrals in \(dv_x\) (using approximation \((13)\) or not). This prescription is not an arbitrary agreement, but is the necessary consequence of Laplace transform existence. So, incorrectness of solving Vlasov equations by Vlasov himself (in case of longitudinal plasma oscillations \([4]\)) was in representing the solution in the form of a single exponent \(\exp(i\omega t - ikx)\) rather than in the form of \(\int dv_x\) in principal value prescription. Asymptotical solution in general case is some complex function which can be expanded in a series of exponents, but it is not necessarily a single exponent.

Substitution of expression \(p_2(2) \simeq \pm i\omega \sqrt{\bar{v}_x^2} \left(1 + \frac{\bar{v}_x^2\omega_L^2}{2c^2\omega^2}\right) = ik\) into Eq.\((13)\) confirms the existence of two opposite in signs poles of the “image” \(E_{p_1p_2}\). Calculation of residua in these poles \(p_2(2)\) and amplitudes of the electric field oscillations for this low-velocity non-damping mode results here in a trivial asymptotic solution in the form of a standing wave

\[
E(x, t)_{\text{asymp}} = E_0e^{i\omega t} \cos(kx) .
\]

However real existence and amplitudes of this mode must be defined at accounting for additive constituents from all other partially coupled\([4]\) or independent boundary and initial conditions \((11)\) which have been omitted till now for simplicity. So, one of such additional conditions might be, for example, even if partly absence of backward wave, etc.

If the boundary and initial conditions are nevertheless such that some oscillatory mode with frequency \(\omega\) is represented only with a single forward wave (and/or backward wave), all contradictions will be removed if the logarithmically divergent integral in the partial dispersion relation of this mode \((24)\) or any other) will be treated in the Vlasov sense of the principal value. In general case including collision plasma and longitudinal waves one can get presence of a single travelling non-damping or damping forward wave only if the latter has non-exponential form \(F(\omega t - kx)\) with a non-exponential boundary condition, for example, \(E = E_0\cos(\omega t)\) (that is at least a sum of two complex-valued exponents).

It might be interesting to note that quite nearly to the poles \((24)\) there are located the values of variables \(p_2\) in the integrand \(E_{p_1p_2}\) of inverse Laplace transformation

\[
p_2^* = \pm i\frac{\omega}{\sqrt{\bar{v}_x^2}}
\]

with zero contribution into Laplace \(dp_2\)-integral in the formula for an asymptotical value of \(E(x, t)\). Such proximity between values of \(p_2\) variable at which \(E_{p_1p_2}\) turns to the infinity and to the zero is intriguing. What will happen in case when \(\bar{v}_x^2 \to 0\)? One can suppose, for example, that this case may be accompanied with a considerable augmentation of the length/time needed to set up the asymptotical regime to really unobservable large values. Be that as it may, strict interpretation of this mode features is possible only at account for relativistic corrections in Eq.\((1)\).

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\(^2\)One of such additional conditions might be, for example, the absence of a backward wave.
3 Coulomb low-collision plasma

Coulomb collision integral is included into Eqs. (11) and (12) in the form

$$f_{p_1p_2}^{(1)} = \frac{1}{p_1 + v_xp_2} \left( \frac{|e|}{m_e} \frac{\partial f_0^{(e)}}{\partial v_x} E_{p_1p_2} + Q_{p_1p_2} \right),$$

where $Q_{p_1p_2}$ is Laplace transform of the usual Coulomb collision integral dominated by the electron-ion collision term $E_{\bar{V}}$:

$$Q(v, x, t) \simeq \frac{2\pi e^4 L_{ni} \omega}{m_e} \int_{-\infty}^{\infty} \frac{dV}{p_1 + v_xp_2 \partial v_i} \left( \frac{v^2 \delta_{ij} - v_i v_j}{v^3} \frac{\partial f_1^{(e)}(\bar{v}, x, t)}{\partial v_j} \right),$$

where $\bar{u} = \bar{V} - \bar{v}$ and $\bar{V}$ is the ion velocity.

Thus, the calculation of the collision contribution into $E_{p_1p_2}$ reduces to the calculation of the term

$$\frac{p_i \omega_L^2}{c^2} \frac{2\pi e^4 L_{ni} \omega}{m_e} E_{p_1p_2} \int_{-\infty}^{\infty} \frac{v_x}{p_1 + v_xp_2 \partial v_i} \left( \frac{v^2 \delta_{ij} - v_i v_j}{v^3} \frac{\partial f_1^{(e)}(\bar{v}, x, t)}{\partial v_j} \frac{\partial f_0^{(e)}}{\partial v_z} \right).$$

After integration by parts and simple transformation of the type (13) one obtains the characteristic equation for determining decrement $\delta$:

$$\left(p_1^2 - p_2^2 \bar{v}_x^2\right)^2 - \left(p_1^2 - p_2^2 \bar{v}_x^2\right)^3 + \frac{4\pi e^4 \omega_L^2 n_i L}{3 \sqrt{3v_x^2 m_e c^2} kT_e} p_1 \left(p_1^6 - 3p_1^4 p_2^2 \bar{v}_x^2 + 7p_1^2 \left[p_2^2 \bar{v}_x^2\right]^2 + 3 \left[p_2^2 \bar{v}_x^2\right]^3 \right) = 0.$$ (30)

Substituting the value $p_1 = i\omega$ and assuming

$$p_2 = p_2^{(1)} + \delta \quad |\delta| \ll |p_2^{(1)}|,$$ (31)

where $p_2^{(1)}$ is defined by Eq.(19), one obtains for the coordinate damping decrement:

$$\delta \equiv \delta_1 = \pm \frac{2\pi e^4 n_i L \omega_L^2}{3 \sqrt{3v_x^2 m_e c^2} kT_e \omega \sqrt{1 - \omega_L^2/\omega^2}}$$ (32)

with $\delta_1 < 0$ for wave with $k < 0$ (21) travelling to the right direction.

But more interesting solution appears for the wave (22). Substituting into Eq.(31) values

$$p_1 = i\omega \quad p_2 = \pm i\frac{\omega}{\sqrt{v_x^2}} + \delta \quad \left( |\delta| \ll \frac{\omega}{\sqrt{v_x^2}} \right),$$ (33)

assuming also

$$|\delta| \gg \frac{\sqrt{v_x^2 \omega_L^2}}{2\omega c^2},$$ (34)
and keeping in Eq. (30) only terms to the order $\delta^3$, accounting for $\bar{v}_x^2 \ll c^2$, one obtains the following striking nonlinear result for the low-velocity electron branch decrement:

$$\delta \equiv \delta_2 = \pm \left( \frac{\pi e^4 n_i L \omega^2}{3\sqrt{3} \left[ \bar{v}_x^2 \right]^2 m_e kT_e} \right)^{1/3}. \quad (35)$$

This intriguing result can be tested experimentally, but one should have in mind that in low-velocity/low-frequency region there might also exist other electron-ion branches defined by the ion current in the Maxwell equation (3), which can complicate interpretation of test results. It is worth also to note that decrements $\delta_{1,2}$, generally speaking, should not be used automatically as simple additive parts of the whole collision damping decrement of partially ionized plasma where collisions of electrons with neutral particles are also present and should be taken into account.

4 Conclusions

Application of our method [1] of 2-dimensional Laplace transformation to plasma transversal oscillation equations with calculating logarithmically divergent integrals with the principal value prescription results in determination of plasma oscillation frequencies (or wave numbers). We have obtained dispersion equations for non-damping oscillatory modes $k(\omega)$ of quasi-neutral Maxwellian collisionless fully ionized plasma and damping modes of low-collision plasma including high-velocity (with phase velocity $> c$) and low-velocity (with phase and group velocities $\approx \bar{v}_e \ll c$) transversal modes.

With the help of the same method we have also obtained the damping decrements of these modes due to Coulomb electron-ion collisions in the low-collision fully ionized plasmas. The most striking thing is a non-linear nature of the decrement for the low-velocity mode

$$\delta_2 \sim \left( \frac{n_i \omega^2}{\bar{v}_x^2} \right)^{1/3},$$

where $\bar{v}_x^2$ is the mean-square velocity of the electrons.

The obtained results on propagation and damping of plasma waves and oscillations can be useful not only in applications to laboratory plasma studies, but mainly in theoretical evaluations of non-thermal energy and damping lengths in the solar atmosphere as well as in interplanetary and interstellar media.

Let us emphasize once more: there is no necessity to appeal to Landau’s rule of passing around poles in calculations of indefinitely (logarithmically) divergent integrals both in the case of transverse as well as of longitudinal plasma waves. The effect of dissipative “Landau damping”, see [1], does not exist in nature and is no more than some abstract great fiction of theorists.

It ought to rehabilitate Vlasov principal value prescription of his relevant logarithmically divergent integrals, however to generalize his solution [4] with using not a single exponent, but some combination of exponents as for an asymptotical solution as well as for functions in boundary and initial conditions (for example, these conditions have to be specially selected in order to avoid unphysical divergent at $x \to \infty$ backward waves, etc.).

Asymptotical solution is some linear combination (a) of exponents (Laplace expansion) which must satisfy linear plasma equations. The linear combination (b) of the exponents from boundary and initial conditions ought to be considered as a selector for the exponents to be included in group (a). It should be noted here that the boundary condition of the type
Then it is natural to expect

\[ E \simeq E_0 \exp(i\omega t) \]

is some mathematical abstraction and can not be realized in a real physical situation.

According to formulas given in this paper for collisionless plasma one has

\[ f_{p_1 p_2}^{(1)}(\vec{v}) = \frac{1}{p_1 + v_x p_2} \left( \frac{|e|}{m_e} \frac{\partial f_0^{(e)}}{\partial v_z} E_{p_1 p_2} + f_{p_2}^{(1)} + v_x f_{p_1}^{(1)} \right), \]  \hspace{1cm} \text{(36)}

\[ E_{p_1 p_2} = \frac{1}{G} \left[ p_2 E_{p_1} + F_{p_1} - p_1 E_{p_2} + F_{p_2} - \lambda \int \frac{(p_1 f_{p_1}^{(1)} - p_2 f_{p_2}^{(1)}) u_x u_z d^3\vec{u}}{p_1 + u_x p_2} \right] \]  \hspace{1cm} \text{(37)}

with \( \lambda = 4\pi |e| n_e / c^2 \) and

\[ G \equiv p_2^2 - \frac{p_1^2}{c^2} + \frac{\omega_L^2 p_1}{c^2} \int u_z \frac{\partial f_0(e)}{\partial u_z} \frac{d^3\vec{u}}{p_1 + u_x p_2}. \]  \hspace{1cm} \text{(38)}

The point \( p_1 + u_x p_2 = 0 \) is not a point of singularity of integrals in Eqs.(37) and (38) because the integrals are defined with the help of the principal value prescription. But the point \( p_1 + v_x p_2 = 0 \) is a pole of \( f_{p_1 p_2}^{(1)} \) in Eq.(36) with functions \( f_{p_1}^{(1)} \), \( f_{p_2}^{(1)} \), \( E_{p_1} \) and others, which, in its turn, can have singularities in \( p_1 \), \( p_2 \).

According to [5, 6] this point corresponds to solutions

\[ C(\vec{v}) e^{i k (x - v_x t)} \quad \left( \text{if } f_{p_2}^{(1)} \propto \frac{1}{p_2 - i k} \right) \]  \hspace{1cm} \text{(39)}

or

\[ C(\vec{v}) e^{i \omega (t - x / v_x)} \quad \left( \text{if } f_{p_1}^{(1)} \propto \frac{1}{p_1 - i \omega} \right) \]  \hspace{1cm} \text{(40)}

of equation

\[ \frac{\partial f_1^{(e)}}{\partial t} + v_x \frac{\partial f_1^{(e)}}{\partial x} = 0 \]  \hspace{1cm} \text{(41)}

with arbitrary \( k \) (or \( \omega \)), \( C(\vec{v}) \), and \( v_x \).

Inverse Laplace transform \( \varphi(\vec{v}, x, t) \) of expression

\[ \varphi_{p_1 p_2} = \frac{\varphi_{p_1}(\vec{v}) + v_x \varphi_{p_2}(\vec{v})}{p_1 + v_x p_2}, \]  \hspace{1cm} \text{(42)}

with some arbitrary initial (\( \varphi_{p_2} \)) and boundary (\( \varphi_{p_1} \)) conditions gives the general complete solution of Eq.(41), including as well the alternative partial solutions (39) and (40). So, one can always select from the set of solutions of Eq.(1)

\[ f_1^{(e)}(\vec{v}, x, t; \varphi) = f_1^{(e)}(\vec{v}, x, t) + \varphi(\vec{v}, x, t) \]  \hspace{1cm} \text{(43)}

with arbitrary initial and boundary conditions a variant with \( f_{p_1}^{(e)} = f_{p_2}^{(e)} = 0 \). This solution, due to Eq.(37), induces appearance of electrical field \( E(x, t) \) and of unphysical backward waves.

Let us assume the boundary electric field to be of the following form

\[ E(0, t) = E_0 \cos(\omega t). \]  \hspace{1cm} \text{(44)}

Then it is natural to expect

\[ f_1^{(e)}(\vec{v}, 0, t) = a_\omega(\vec{v}) \cos(\omega t) + b_\omega(\vec{v}) \sin(\omega t) \]  \hspace{1cm} \text{(45)}

with some arbitrary coefficients \( a_\omega(\vec{v}) \) and \( b_\omega(\vec{v}) \).
In this case Laplace transforms are
\[ E_{p1} = \frac{p_1 E_0}{p_1^2 + \omega^2}, \]
\[ f_{p1}^{(1)} = \frac{p_1 a_{p1}(\vec{v}) + |p_1| b_{p1}(\vec{v})}{p_1^2 + \omega^2}. \]

According to Eq. (37) with account for collision damping (through the additional collision term in \( G \)) the term \( p_2 E_{p1}/G \) leads to asymptotical waves with exponents of \( (i\omega t \pm ikx \pm \delta x) \) and \( (-i\omega t \pm ikx \mp \delta x) \), and the term with \( f_{p1}^{(1)} \) leads to waves with exponents of \( (i\omega t \pm ikx \pm \delta x) \) (with coefficient \( a_\omega \) in Eq. (17)) and \( (-i\omega t \pm ikx \mp \delta x) \) (with coefficient \( b_\omega \) in Eq. (17)). It is evident that it is always possible to eliminate backward waves by the proper selection of \( a_\omega \) and \( b_\omega \). Possible more convenient modification of Eq. (17) can be also, for instance,
\[ f_{p1}^{(1)} = \frac{p_1 a_{p1}(\vec{v}) + p_1^2 b_{p1}(\vec{v})}{p_1^2 + \omega^2}, \]
where \( a_{p1}(\vec{v}), b_{p1}(\vec{v}), \) and \( f_{p1}^{(1)} \) are all proportional to \( E_0 \).

At account for the simultaneous presence of the other mode (22), it is seen that the already defined above \( a \) and \( b \) have to lead to removal of backward waves in this mode too, due to its analogous structure.

The paradox consists in the fact that at damping electrical field, due to collision terms in \( G \), there exist non-damping "ballistic" kinematical oscillations in \( f_{1e}^{(e)}(\vec{v}, x, t) \) (in [5, 6] there was implied, instead, collisionless Landau damping with the same result). These "ballistic" oscillations appear to be unremovable. But there is no mystery. If collision term, even if partly, included some additional term of the form
\[ -C(\vec{v}) f_{1e}^{(e)}(\vec{v}, x, t), \]
then, instead of denominator \( p_1 + v_x p_2 \) and \( p_1 + u_x p_2 \) in Eqs. (36)–(38), one would have denominator \( p_1 + v_x p_2 + C(\vec{v}) \) or \( p_1 + u_x p_2 + C(\vec{v}) \) and the "ballistic" term would be damping too in the same extent as the oscillations of electric field.

It appears that, for instance, as the first collision iteration one could more reasonably use the substitution in Eq. (26)
\[ Q_{p1,p2} \equiv f_{p1p2}^{(1)} \left( \frac{Q_{p1p2}}{f_{p1p2}(1)} \right) \simeq f_{p1p2}^{(1)} \left[ \frac{Q_{p1p2}^o}{(f_{p1p2}^{(1)}(1))_o} \right], \]
where \( (f_{p1p2}^{(1)}(1))_o \) is the collisionless approximation of \( f_{p1p2}^{(1)} \) according to Eq. (39), and \( Q_{p1p2}^o \) is the function of \( p_1, p_2, \) and \( \vec{v} \) defined by the Laplace transformation of expression (28) with substitution \( f_{p1p2}^{(1)} \rightarrow (f_{p1p2}^{(1)}(1))_o \). Here arises also the need for some new correction of coefficients \( a \) and \( b \) in the boundary and initial conditions \( f_{p1}^{(e)} \) and \( f_{p2}^{(e)} \) in order to avoid growing backwards waves.

By this way non-damping of ballistic mode in low-collision plasma can be related with some approximation made in iteration procedures (yet nobody has still investigated convergence of the related iteration processes) and not with intrinsic features of the Coulomb collision integral.

This example of a successful application of our methodology to plasma oscillations supports the hope of further unraveling these and more entangled problems of plasma-echo phenomena. In particular, in the case of boundary conditions with two superimposed frequencies
\[ E(0, t) = E_0 \cos(\omega_1 t) \cos(\omega_2 t) \]
two non-damping waves with frequencies \((\omega_1 - \omega_2)\) and \((\omega_1 + \omega_2)\) arise in collisionless plasma. That is a very simple imitation of plasma echo effect (cf. description in [6]) for both transversal and longitudinal oscillations.

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