Research Article

Inverse Family of Numerical Methods for Approximating All Simple and Roots with Multiplicity of Nonlinear Polynomial Equations with Engineering Applications

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A new inverse family of the iterative method is interrogated in the present article for simultaneously estimating all distinct and multiple roots of nonlinear polynomial equations. Convergence analysis proves that the order of convergence of the newly constructed family of methods is two. The computer algebra systems CAS-Mathematica is used to determine the lower bound of convergence order, which justifies the local convergence of the newly developed method. Some nonlinear models from physics, chemistry, and engineering sciences are considered to demonstrate the performance and efficiency of the newly constructed family of inverse simultaneous methods in comparison to classical methods in the literature. The computational time in seconds and residual error graph of the inverse simultaneous methods are also presented to elaborate their convergence behavior.

1. Introduction

Considering nonlinear polynomial equation of degree \( n \),

\[
f(r) = r^n + a_{n-1}r^{n-1} + \cdots + a_0 = \prod_{j=1}^{n}(r - \zeta_j)
\]

with arbitrary real or complex coefficient \( a_{n-1}, \ldots, a_0 \). Let \( \zeta_1 \ldots \zeta_n \) denote all the simple or complex roots of (1) with multiplicity \( \sigma_1 \ldots \sigma_n \). Newton’s method [11] is one of the most basic and ancient methods that is used to estimate single roots of (1) at a time as below:

\[
s(t) = r(t) - \frac{f(r(t))}{f'(r(t))} (t = 0, 1, \ldots).
\]

Iterative method (2) has local quadratic convergence. Nedzhibov et al. [13] presented corresponding inverse numerical technique of the same convergence order as

\[
s(t) = r(t) - \frac{f(r(t))}{f'(r(t))} \left(1 - \alpha f(r(t))/1 + f'(r(t)) \right),
\]

Here, we propose the following family of the optimal second-order convergence method for finding simple roots of (1) as

\[
s(t) = r(t) - \frac{f(r(t))}{f'(r(t))} \left(1 - \alpha f(r(t))/1 + f'(r(t)) \right),
\]

where \( \alpha \in \mathbb{R} \). Method (4) is optimal and the convergence order of (4) is 2 if \( \zeta \) is a simple root of (1) and \( \epsilon = r - \zeta \).
\[ \frac{s^{(i)} - \zeta}{(r^{(i)} - \zeta)^2} \rightarrow (-\alpha + C_2); \quad C_k(r) = \frac{f^{(k)}(x)}{k!f'(x)}, \quad (5) \]

or

\[ s^{(i)} - \zeta = O(\varepsilon^2). \quad (6) \]

Corresponding inverse methods of (4) is constructed as

\[ g(r) = \frac{(r^{(i)})^2 \ast f'(r^{(i)})}{r^{(i)}f'(r^{(i)}) + f(r^{(i)})(1/(1 - \alpha f(r^{(i)})/1 + f(r^{(i)})))}, \quad g(\zeta) = \zeta \text{ (fixed point)}, \quad (8) \]

\[ g'(\zeta) = 0. \quad (9) \]

Inverse iterative schemes (7) are second-order convergence as it is easy to prove \( g''(\zeta) \neq 0. \)

Besides simple root finding methods [3–5, 13, 15, 16, 18–20] in literature, there exists another class of numerical methods which estimate all real and complex roots of (1) at a time, known as simultaneous methods. Simultaneous numerical iterative schemes are very prevalent due to their global convergence properties and its parallel execution on computers [1, 6, 8–10, 12, 14].

The most prominent method among simultaneous derivative-free iterative technique is the Weierstrass–Dochive [17] method (abbreviated as MWM1), which is defined as

\[ s^{(i)}_{\text{MWM1}} = r^{(i)} - \omega(r^{(i)}_{\text{MWM1}}), \quad (10) \]

where

\[ \frac{f(r^{(i)}_{\text{MWM1}})}{f'(r^{(i)}_{\text{MWM1}})} = \frac{\omega(r^{(i)}_{\text{MWM1}})}{\prod_{j \neq i}(r^{(i)}_{\text{MWM1}} - r^{(j)}_{\text{MWM1}})}, \quad (i, j = 1, 2, 3, \ldots, n), \quad (11) \]

is Weierstrass' correction. Method (10) has local quadratic convergence. For finding all multiple roots of (1), we use the following correction [17]:

\[ \omega_i(r^{(i)}_{\text{MWM1}}) = \frac{f(r^{(i)}_{\text{MWM1}})}{\prod_{j \neq i}(r^{(i)}_{\text{MWM1}} - r^{(j)}_{\text{MWM1}})^{\sigma_i}}, \quad (i, j = 1, 2, 3, \ldots, n), \quad (12) \]

where \( \sigma \) is the multiplicity of the roots.

2. Construction of the Inverse Simultaneous Method

Using Weierstrass correction \( \omega(r^{(i)}_{\text{MWM1}}) = f(r^{(i)}_{\text{MWM1}})/\prod_{j \neq i}(r^{(i)}_{\text{MWM1}} - r^{(j)}_{\text{MWM1}}) \) in (7), we get a new family of inverse modified Weierstrass method (abbreviated as MWM2):

\[ s^{(i)}_{\text{MWM2}} = \frac{(r^{(i)})^2 \ast f'(r^{(i)})}{r^{(i)}f'(r^{(i)}) + f(r^{(i)})(1/(1 - \alpha f(r^{(i)})/1 + f(r^{(i)})))}, \quad (7) \]

If \( \zeta \) is an exact root of (1), \( f(\zeta) = 0 \), and the following is obtained:

\[ s^{(i)}_{\text{MWM2}} = \frac{(r^{(i)})^2 \ast f'(r^{(i)})}{r^{(i)}f'(r^{(i)}) + f(r^{(i)})(1/(1 - \alpha f(r^{(i)})/1 + f(r^{(i)})))}, \quad (7) \]

Inverse simultaneous iterative method (13) can also be written as

\[ s^{(i)}_{\text{MWM2}} = \frac{(r^{(i)})^2 \ast f'(r^{(i)})}{r^{(i)}f'(r^{(i)}) + f(r^{(i)})(1/(1 - \alpha f(r^{(i)})/1 + f(r^{(i)})))}, \quad (7) \]

Thus, we construct a new derivative-free family of inverse iterative simultaneous scheme (13), abbreviated as MWM2, for estimating all distinct roots of (1). To estimate all multiple roots of (1), we use correction (12) instead of (11) in (7).

2.1. Convergence Framework. In this section, we demonstrate convergence theorem of inverse iterative scheme MWM2.

**Theorem 1.** Let \( \zeta_1, \ldots, \zeta_n \) be single zero of (1) and for necessarily close primary distinct guess \( r^{(0)}_1, \ldots, r^{(0)}_n \) of the zero, respectively; then, MWM2 has local 2\( \alpha \)-order convergence.

**Proof.** Let \( e_i = r^{(i)}_i - \zeta_i \) and \( e^{(i)}_i = s^{(i)}_i - \zeta_i \) be the errors in \( r_i \) and \( s_i \) respectively. For the simplicity of the calculation, we omit the iteration index. Then,

\[ s_i = \frac{(r_i)^2(1 + 1 - \alpha f(r_i))}{r_i(1 + 1 - \alpha f(r_i)) + \omega(r_i)(1 + f(r_i))}, \quad (15) \]

or

\[ s_i = r_i - \frac{r_i\omega(r_i)(1 + f(r_i))}{r_i(1 + 1 - \alpha f(r_i)) + \omega(r_i)(1 + f(r_i))}. \quad (16) \]

Using \( f(r_i)/\prod_{j \neq i}(r_i - r_j) = \omega(r_i) \) in (15), we have

\[ s_i = r_i - \frac{\omega(r_i)(1 + f(r_i))}{(1 + 1 - \alpha f(r_i)) + \omega(r_i)r_i(1 + f(r_i))}. \quad (17) \]
Thus, we obtain

\[ s_i - \zeta_j = r_i - \zeta_j - \frac{w(r_i)(1 + f(r_i))}{(1 + (1 - \alpha)f(r_i)) + w(r_i)r_i(1 + f(r_i))}, \]

(18)

\[ \epsilon_i' = \epsilon_i \left[ 1 - \frac{\prod_{j \neq i}^n (r_i - \zeta_j)/(r_i - r_j) \left( 1 + \epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j) \right)}{(1 + (1 - \alpha)\epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j)) + \left( \prod_{j \neq i}^n (r_i - \zeta_j)/(r_i - r_j) \right) \left( 1 + \epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j) \right)} \right]. \]

(19)

\[ \epsilon_i' = \epsilon_i \left[ 1 - \frac{\left( (1 + (1 - \alpha)\epsilon_i) \prod_{j \neq i}^n (r_i - \zeta_j) \right)}{(1 + (1 - \alpha)\epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j)) + \left( \sum_{k \neq i}^n \epsilon_k/r_k - r_k \prod_{j \neq i}^n (r_i - \zeta_k)/(r_i - r_j) + 1 \right) \left( 1 + \epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j) \right)} \right]. \]

(20)

Using the expression \[ \prod_{j \neq i}^n (r_i - \zeta_j)/(r_i - r_j) - 1 = \sum_{k \neq i}^n \epsilon_k/r_k - r_k \prod_{j \neq i}^n (r_i - \zeta_k)/(r_i - r_j) \]

[9] in (20), we have

\[ \left\{ \epsilon_i' = \epsilon_i \left[ 1 - \frac{(1 + (1 - \alpha)\epsilon_i) \prod_{j \neq i}^n (r_i - \zeta_j)}{(1 + (1 - \alpha)\epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j)) + \left( \sum_{k \neq i}^n \epsilon_k/r_k - r_k \prod_{j \neq i}^n (r_i - \zeta_k)/(r_i - r_j) + 1 \right) \left( 1 + \epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j) \right)} \right] \right\}. \]

(21)

If all the errors are assumed of the same order, i.e., \( |\epsilon_i| = |\epsilon_k| = O(|\epsilon|) \), then

\[ \epsilon_i' = |\epsilon| \left[ 1 - \frac{(1 + (1 - \alpha)\epsilon_i) \prod_{j \neq i}^n (r_i - \zeta_j)}{(1 + (1 - \alpha)\epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j)) + \left( \sum_{k \neq i}^n \epsilon_k/r_k - r_k \prod_{j \neq i}^n (r_i - \zeta_k)/(r_i - r_j) + 1 \right) \left( 1 + \epsilon_i \prod_{j \neq i}^n (r_i - \zeta_j) \right)} \right], \]

(22)

\[ \text{Hence, it is proved.} \]

3. Lower Bound of Convergence of MWM1 and MWM2

Computer algebra system, Mathematica, has been used to find the lower bound of convergence of MWM1 and MWM2.

Consider

\[ f(r) = (r - q_1)(r - q_2)(r - q_3), \]

(23)

where \( q_1, q_2, \text{ and } q_3 \) are exact zeros of (23). The first component of \( N_1(r) \) (where \( r = [r_1, r_2, r_3] \)) of numerical iterative methods is for finding zeros of (23), \( r^{(1)} = N(r^{(1)}) \), and so on.

We obtain the lower bound of convergence order till the first nonzero element of row is found. The Mathematica
notebook codes are used for the following MWM1 and MWM2:

**Weierstrass–Dochive method, MWM1:**

\[
N_1(r_1, r_2, r_3) = r_1 - \frac{f(r_1)}{(r_1 - r_2) \cdot (r_1 - r_3)}
\]

\[
\text{In}[1] = \frac{D[N_1[r_1, r_2, r_3], r_1]}{r_1 \to q_1, r_2 \to q_2, r_3 \to q_3}
\]

\[
\text{Out}[1] = 0,
\]

\[
\text{In}[2] = D[N_1[r_1, r_2, r_3], r_2]
\]

\[
\text{Out}[2] = 0,
\]

\[
\text{In}[3] = D[N_1[r_1, r_2, r_3], r_3]
\]

\[
\text{Out}[3] = 0,
\]

\[
\text{In}[4] = \text{Simplify}\left[\frac{D[N_1[r_1, r_2, r_3], r_1, r_2]}{r_1 \to q_1, r_2 \to q_2, r_3 \to q_3}\right]
\]

\[
\text{Out}[4] = 0,
\]

\[
\text{In}[6] = \text{Simplify}\left[\frac{D[N_1[r_1, r_2, r_3], r_1, r_2]}{r_1 \to q_1, r_2 \to q_2, r_3 \to q_3}\right]
\]

\[
\text{Out}[6] = \frac{1}{(q_1 - q_2)}.
\]

**Modified inverse family of iterative schemes, MWM2:**

\[
N_1(r_1, r_2, r_3) = \frac{(r_1)^2 (1 + (1 - \alpha) f(r_1))}{r_1 (1 + (1 - \alpha) f(r_1)) + (r_1 - r_2) (r_1 - r_3) (1 + f(r_1))}.
\]

\[
\text{In}[1] = D[N_1[r_1, r_2, r_3], r_2]/[r_1 \to q_1, r_2 \to q_2, r_3 \to q_3],
\]

\[
\text{Out}[1] = 0,
\]

\[
\text{In}[2] = D[N_1[r_1, r_2, r_3], r_3]/[r_1 \to q_1, r_2 \to q_2, r_3 \to q_3],
\]

\[
\text{Out}[2] = 0,
\]

\[
\text{In}[3] = D[N_1[r_1, r_2, r_3], r_3]/[r_1 \to q_1, r_2 \to q_2, r_3 \to q_3],
\]

\[
\text{Out}[3] = 0,
\]

\[
\text{In}[4] = \text{Simplify}[D[N_1[r_1, r_2, r_3], r_1, r_1], [r_1 \to q_1, r_2 \to q_2, r_3 \to q_3],
\]

\[
\text{Out}[4] = \frac{4 + 8 \times \alpha \times q_1}{q_3}.
\]

**4. Numerical Results**

Some engineering problems are considered to demonstrate the performance and effectiveness of the simultaneous method, MWM2 and MWM1. For computer calculations, we use CAS-Maple-18, and the following stopping criteria for termination of computer are programmed:

\[
\epsilon_l^{(i)} = \left| r_l^{(i+1)} - r_l^{(i)} \right| < \epsilon = 10^{-30},
\]
where \( e_i^{(1)} \) signifies the absolute error. In Tables 1–5, C-Time represents computational time in second.

4.1. Engineering Applications. Some engineering applications are deliberated in this section in order to show the feasibility of the present work.

Example 1. (see [2]). Considering a physical problem of beam positioning results in the following nonlinear polynomial equation:

\[
 f(r) = r^4 + 4r^3 - 24r^2 + 16r + 1, \\
 = (r - 2)^2(3r^2 + 8r + 4). 
\]  

(30)

The exact root of (30), \( \xi_{1,2} \), is 2 with multiplicity 2 and the remaining other two roots are simple, i.e., \( \xi_3 = -4 - 2\sqrt{3} \) and \( \xi_4 = -4 + 2\sqrt{3} \). We take the following initial estimates:

\[
 r_1^{(0)} = 1.17, \\
 r_2^{(0)} = 1.17, \\
 r_3^{(0)} = -7.4641, \\
 r_4^{(0)} = -0.5354. 
\]  

(31)

Table 1 clearly demonstrates the superiority of MWM2 over MWM1 in terms of predicted absolute error and CPU time for guesstimating all real roots of (30) on the same number of iterations \( n = 3 \).

Example 2. (see [16]). In this engineering application, we consider a reactor of stirred tank. Items \( H_1 \) and \( H_2 \) are fed to the reactor at rates of \( \beta \) and \( q-\beta \), respectively. Composite reaction improves in the apparatus as below:

\[
 H_1 + H_2 \rightarrow H_3, H_3 + H_2 \rightarrow H_4, H_4 + H_2 \rightarrow H_5, H_4 + H_2 \rightarrow H_6. 
\]  

(32)

Douglas et al. [7] first examined this complex control system and obtained the following nonlinear polynomial equation:

\[
 2.98 \times (r + 2.25) - (r + 1.45) \times (r + 2.85)^2 \times (r + 4.35) = \frac{1}{T_c}, 
\]  

(33)

where \( T_c \) is the gain of the proportional controller. By taking \( T_c = 0 \), we have

\[
 f(r) = r^4 + 11.50r^3 + 47.49r^2 + 83.06325r + 51.23266875 = 0. 
\]  

(34)

The exact distinct roots of (34) are calculated as \( \xi_1 = -1.45, \xi_2 = -2.85, \xi_3 = -2.85, \) and \( \xi_4 = -4.45 \), and we take the following initial guessed values:

\[
 r_1^{(0)} = -1.0, r_2^{(0)} = -1.1, r_3^{(0)} = -1.8, r_3^{(0)} = -3.9. 
\]  

(35)

Table 2 evidently illustrates the supremacy behavior of MWM2 over MWM1 in terms of the estimated absolute error and in CPU time on the same number of iterations \( n = 7 \) for guesstimating all real roots of (34).

Example 3. (see [4]). Consider the function

\[
 \frac{8(4 - r)^2r^2}{(6 - 3r)^2(2 - r)} - 0.186 = 0, 
\]  

(36)

\[
 f(r) = 8r^4 - 62.326r^3 + 117.956r^2 + 20.088r - 13.392. 
\]  

(37)

The problem describes the fractional alteration of nitrogen-hydrogen (NH) feed into ammonia at 250 atm pressure and 500°C temperature. Since the (37) is of order four, it has four roots:
\[ \zeta_1 = 0.2777, \]
\[ \zeta_2 = 3.9485 + 0.3161i, \]
\[ \zeta_3 = -0.3840, \]
\[ \zeta_4 = 3.9485 - 0.3161i. \] (38)

The initial approximated value for (27) is taken as
\[ (0)_{r_1} = 0.4, (0)_{r_2} = 3.7 + 0.5i, (0)_{r_3} = -0.4, (0)_{r_4} = 3.7 - 0.5i. \] (39)

Table 3 evidently shows the supremacy behavior of MWM2 over MWM1 in terms of estimated absolute error and in CPU time on the same number of iterations \( n = 8 \) for guesstimating all real and complex roots of (37). Minuscule alteration of nitrogen-hydrogen (NH) feed into ammonia lies between (0,1); therefore, our desire root is \( \zeta_1 \) up to 1900 decimal places:
\[ \zeta_1 = 1.12956568412579833521973452e - 1912. \] (40)

Remaining other approximating roots are \( \zeta_2 = -1.283404526e - 1457 - 8.93219631521e - 1457i, \)
\[ \zeta_3 = -8.21745232237462e - 1091 + 0i, \]
and \( \zeta_4 = -1.834045268801e - 1457 + 8.99321963152e - 1457i. \)

**Example 4.** (see [8]). Consider
\[ f (r) = (r - 0.3 - 0.6i)^{100} (r - 0.1 - 0.7i)^{200} (r - 0.7 - 0.5i)^{300} (r - 0.3 - 0.4i)^{400}, \] (41)

with multiple exact roots:
\[ \zeta_1 = 0.3 + 0.6i, \zeta_2 = 0.1 + 0.7i, \zeta_3 = 0.7 + 0.5i, \zeta_4 = 0.3 + 0.4i, \] (42)

The initial estimations have been taken as
\[ (0)_{r_1} = 0.301 + 0.601i, (0)_{r_2} = 0.100 + 0.702i, (0)_{r_3} = 0.702 + 0.489i, (0)_{r_4} = 0.289 - 0.400i, \] (43)

For distinct roots,
\[ f_4 (r) = (r - 0.3 - 0.6i)(r - 0.1 - 0.7i)(r - 0.7 - 0.5i) (r - 0.3 - 0.4i), \] (44)

Table 4 evidently shows the supremacy behavior of MWM2 over MWM1 in terms of estimated absolute error and in CPU time on the same number of iterations \( n = 5 \) for guesstimating all real and complex roots of (41).

**Example 5.** (see [5]). The sourness of a soaked solution of magnesium-hydroxide (MgOH) in hydroelectric acid (HCl) is given by
\[ \frac{3.64 \times 10^{-11}}{\left[H_3O^+ \right]} = \left[H_3O^+ \right] + 3.6 \times 10^{-4}, \] (45)
Figure 3: Continued.
for the cranium ion concentration \([H_3O^+]\). If we set
\[ r = 10^9[H_3O^+] \]
we obtain the following polynomial:
\[ f(r) = r^3 + 3.6r^2 - 36.4, \]  
(46)

with exact roots of (46), \(\zeta_1 = 2.4\) and \(\zeta_{2,3} = -3.0 \pm 2.3i\) up to one decimal places. The initial estimates have been taken as
\[ r_1^{(0)} = 2.45, r_2^{(0)} = -3.0261 + 2.3834i, r_3^{(0)} \]
\[ = -3.0261 - 2.3834i. \]  
(47)

Table 5 evidently illustrates the supremacy behavior of MWM2 over MWM1 in terms of estimated absolute error and in CPU time on the same number of iterations \(n = 8\) for guesstimating all real and complex roots of (46).

**Example 6.** (see [21]). In general, mechanical engineering, as well as the majority of other scientists, uses thermodynamics extensively in their research work. The following polynomial is used to relate the zero-pressure specific heat of dry air, \(C_p\), to temperature:
\[ C_p = 1.9520 \times 10^{-14}r^4 - 9.5838 \times 10^{-11}r^3 + 9.7215 \times 10^{-8}r^2 + 1.671 \times 10^{-4}r + 0.99403. \]  
(48)

The temperature that corresponds to specific heat of 1.2 \((kJ/kgK)\) needs to be determined. Putting \(C_p = 1.2\) in (48), we have
\[ f(r) = 1.9520 \times 10^{-14}r^4 - 9.5838 \times 10^{-11}r^3 
+ 9.7215 \times 10^{-8}r^2 + 1.671 \times 10^{-4}r + 0.99403. \]  
(49)

with exact roots \(\zeta_1 = 1126.009751, \ zeta_2 = 2536.837119 + 910.5010371i, \) and \(\zeta_3 = -1289.950382, 2536.837119 - 910.5010371i. \) The initial estimations of (49) have been taken as
\[ r_1^{(0)} = 1126, r_2^{(0)} = 2536 + 910i, r_3^{(0)} = -1289, r_4^{(0)} = 2536 - 910i. \]  
(50)

Table 6 clearly illustrates the supremacy behavior of MWM1 over MWM2 in estimated absolute error and in CPU time on the same number of iterations \(n = 4\) for guesstimating all real and complex roots of (49).

**5. Conclusion**

A new derivative-free family of inverse numerical methods of convergence order 2 for simultaneous estimations of all distinct and multiple roots of (1) was introduced and discussed in this paper. Tables 1–5 and Figure 1 clearly show that computational order of convergence of the proposed and existing methods are agreed with the theoretical results. Simulation time, from Figure 2, clearly indicates the supremacy of our newly proposed method MWM2 over...
existing Weierstrass method MWM1. The results of numerical test cases from Tables 1–5, CPU time, and residual error graph from Figure 3 demonstrated the effectiveness and rapid convergence of our proposed iterative method MWM2 as compared to MWM1.

Data Availability
No data were used to support this study.

Disclosure
The statements made and views expressed are solely the responsibility of the authors.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

Authors’ Contributions
All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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