Abstract—A study on the analysis of the power pattern of phased-array antennas (PAs) based on the quantum Fourier transform (QFT) is presented. The computation of the power pattern given the set of complex excitations of the PA elements is addressed within the quantum computing (QC) framework by means of a customized procedure that exploits the quantum mechanics principles and theory. A representative set of numerical results, yielded with a quantum computer emulator, is reported and discussed to validate the proposed method also pointing out its features in comparison with the classical approach based on the discrete Fourier transform (DFT).

Index Terms—Antenna analysis, array antenna, power pattern, quantum computing (QC), quantum Fourier transform (QFT).

NOWADAYS, phased-array antennas (PAs) are a widely adopted technology with countless applications [1], [2], such as multiple-input multiple-output (MIMO) mobile communications (e.g., 5G) [3], [4], [5], space communications [6], [7], [8], automotive [9] and airborne radar systems [10], and weather forecasting [11] as well as biomedical microwave imaging [12], [13] or, more in general, nondestructive evaluation and testing (NDE/NDT) [14]. Indeed, PAs stand out for the robustness, the adaptability to conformal surfaces, the easy reconfiguration capabilities, and the careful control of the radiation features [15]. Moreover, it is worth mentioning that unconventional (i.e., clustered [16], [17] or thinned/sparse [18], [19] architectures) PAs are gaining momentum, since they are interesting solutions as suitable trade-offs between costs and radiation performance [20].

Typically, the design of PAs has been addressed by formulating the synthesis problem at hand as an optimization one and then solved with deterministic [21], [22], [23] or stochastic [24], [25] methods recurring to machine learning techniques [26], as well. However, a common bottleneck for many design methods is the need of iterating the evaluation of a cost function, which is based on the knowledge/computation of the power pattern, that quantifies the mismatch between synthesized pattern features and the ideal one or against user-defined requirements. Therefore, the computational cost becomes prohibitive in case of large arrays [27] because of the wide dimension of the solution space as well as the huge number of pattern samples to be computed for properly accounting for its angular variations.

The goal of this work is to propose an innovative method, based on quantum computing (QC), for the computation of the power pattern starting from the array excitations (also indicated as “PAs analysis” problem) and to analyze its features. Indeed, QC has the potential for offering a huge computational power, unreachable even by current supercomputers. In the recent years, QC has drawn significant attention, since it has proved the ability to solve computationally intensive problems by taking advantage of phenomena exclusively related to the quantum realm [28], [29]. For instance, the superposition and the entanglement have been used in many quantum algorithms to enable a relevant speedup versus classical algorithms [30]. On the other hand, a widespread use of quantum computers is still prevented by different technical factors, such as the high errors intrinsic to the quantum gates [31], which are used to perform quantum operations, and the instability when dealing with large numbers of quantum bits (qubits) [32]. Despite
these technological issues, QC has been already exploited in different research branches starting from physics and computational biology [33] up to, more recently, computational electromagnetics [34], [35] for the simulation and optimization of complex radiating systems. Undoubtedly, the most well-known applications of QC are in the field of information technology as, for instance, quantum machine learning [36] and quantum cryptography [37]. In this latter framework, a very popular QC algorithm is the Shor’s algorithm [38] that has rendered ineffective the Rivest–Shamir–Adleman (RSA) cryptography technique [39] by solving the underlying factoring problem in polynomial time. Moreover, a part of the Shor’s algorithm, namely, the quantum Fourier transform (QFT), has given an exponential advantage in terms of computational efficiency over its classical counterpart [i.e., the discrete Fourier transform (DFT)].

Since the relation between the excitations of a regular PA and its radiation pattern can be numerically solved with a DFT [1], [2], in this article, for the first time to the best of the authors’ knowledge, the prediction of the power pattern of PAs is carried out by benefiting of the QFT and of the QC framework. Toward this end, the PAs analysis problem is adapted, as preliminary introduced in [40], to the quantum domain since, while quantum computers are able to perform all the operations available on classical computers [41], it is mandatory to reformulate the problem at hand to reach the QC speedup. In more detail, it means first to encode the classical information into the quantum one for taking advantage of the set of QC exclusive operations. Moreover, a user of a quantum computer needs to know that the meaning of output data radically differs from the classical one. Indeed, each execution of a quantum program results in the measurement of a quantum state, and it is possible to derive the probability of such a state, which is here strictly related to the result of the Fourier transform operation, only by iterating the execution of the program for a number of runs (indicated as “shots”). The impact of the number of shots on the representability of the power pattern and on the computational complexity in dealing with the PAs analysis problem is also studied.

The rest of this article is organized as follows. In Section II, the PA analysis problem is mathematically formulated in the QC framework by presenting the QFT-based pattern prediction method. Section III is devoted to the numerical validation and assessment of the proposed computational approach. Representative numerical results are also provided to give some insights on the behavior of the QFT algorithm and the dependence of its performance on the control parameters. Eventually, some conclusions and final remarks are drawn (Section IV).

II. MATHEMATICAL FORMULATION

Let us consider an $N$ element PA where the array elements are equally spaced by $d$ along the $z$-axis (Fig. 1). Each $n$th $(n = 0, \ldots, N-1)$ element is connected to a transmit–receive module (TRM) that generates a complex excitation $w_n$, and $w_n \triangleq \alpha_n \exp(j\beta_n)$ $(n = 0, \ldots, N-1)$, where $\alpha_n$ and $\beta_n$ being the corresponding amplitude and phase, respectively, while $j = \sqrt{-1}$ is the imaginary unit. Let $g_n(u)$ be the radiation pattern of the $n$th $(n = 0, \ldots, N-1)$ array element; then, the EM field radiated in far-field (FF) by the PA is given by

$$F(u) = \sum_{n=0}^{N-1} w_n g_n(u) \exp[j(kd)n]$$

where $k = 2\pi/\lambda$ is the free-space wavenumber, $\lambda$ being the corresponding wavelength, and $u$ is the direction cosine $(u \triangleq \cos \theta, \theta (0 \leq \theta \leq \pi)$ being the angle measured from the $z$-axis. If the array elements are equal (i.e., $g_n(u) \equiv g(u); n = 0, \ldots, N-1$), $F(u)$ can be written as the product between the element pattern, $g(u)$, and the array factor, $A(u)$

$$A(u) \triangleq \sum_{n=0}^{N-1} w_n \exp[j(kd)n]$$

[i.e., $F(u) = g(u)A(u)$], and the corresponding power pattern, $P(u)$ ($P(u) \triangleq |F(u)|^2$), which mathematically describes the angular distribution of the power either radiated or received by the PA, turns out to be

$$P(u) = \left| g(u) \right|^2 |A(u)|^2.$$  

As it can be observed (3), $P(u)$ directly depends on the absolute square of the array factor, $|A(u)|^2$, and the samples of this latter (2), $A = \{A_m; m = 0, \ldots, M - 1\}$, are related to the set of the array excitations, $w = \{w_n; n = 0, \ldots, N-1\}$, through the DFT, $A = DFT(w)$

$$A_m = \sum_{n=0}^{N-1} w_n \exp\left[-j\left(\frac{2\pi}{N}\right)m\right], \quad m = 0, \ldots, M - 1$$

where $A_m = A(u_m)$ is the $m$th $(m = 0, \ldots, M - 1)$ sample of the array factor (2) at the angular direction $u_m = -m(\lambda/Nd)$.

Once the discrete sample vector $A$ has been computed, the corresponding continuous function, $A(u); -1 \leq u \leq 1$, is obtained by means of a periodic interpolation

$$A(u) = \sum_{m=0}^{M-1} A_m S\left(\pi du + \frac{m\pi}{N}\right)$$

where $S$ is the sinc function $[S(x) \triangleq \sin(Nx)/N\sin(x)]$.

Under the hypothesis of ideal elements (i.e., $g(u) = 1$) through (3), it turns out that $P(u) = |A(u)|^2$; thus, the relation

$$P_m = |A_m|^2$$
holds true, where $P_m$ being the sample of the power patter at $n = u_m$ (i.e., $P_m = P(u_m)$).

The power pattern of the $N$ elements array antenna is then computed in the QC framework with the quantum counterpart of the DFT, namely, the QFT. Toward this end, the first step consists in allocating a register of qubits, $\Psi$, where the input–output values of the Fourier transform are encoded in a set of $Q$ quantum states. More specifically, $\Psi$ is the concatenation of $L$ single qubits, each $l$th ($l = 0, \ldots, L - 1$) one assuming two possible states, $|\psi_l\rangle \in \{|0\}, |1\rangle\}$, is given by the tensor product of each single qubit state $|\psi_q\rangle \triangleq \bigotimes_{l=0}^{L-1} |\psi_{q,l}\rangle = |\psi_{q,0}\rangle \cdots |\psi_{q,L-1}\rangle$ (Fig. 2). Since an $L$ qubit register, $\Psi$, can encode at most $2^L$ different (input–output) states, then a register with $L = \lceil\log_2 Q\rceil$ qubits must be chosen to yield, through QFT, $M$ samples of the power pattern starting from $N$ array excitations.

The input state vector of the QFT is then initialized by assigning an $n$th $(n = 0, \ldots, N - 1)$ excitation, $w_n$, to each $q$th $(q = 0, \ldots, Q - 1)$ quantum state, $|\psi_q\rangle$. In more detail, the first $N$ states of $\Psi$ are assigned to the $N$-size set of normalized complex excitations $\hat{w}$, $\hat{w} \triangleq |\hat{w}_n\rangle \triangleq \langle w_n | w|\rangle$; $n = 0, \ldots, N - 1$,\(^1\) while the remaining $Q - N$ ones are set to zero [Fig. 3(a)]

$$|w\rangle = \sum_{q=0}^{N-1} \hat{w}_q |\psi_q\rangle + \sum_{q=N}^{Q-1} 0 |\psi_q\rangle. \quad (7)$$

Analogously to the classical theory, the output state vector $|A\rangle$

$$|A\rangle = \sum_{m=0}^{M-1} \hat{A}_m |\Psi_m\rangle \quad (8)$$

is yielded by applying the QFT to the input quantum state vector $|w\rangle$, so that the weight of the state $|\Psi_m\rangle$ ($m = 0, \ldots, M - 1$) is given by [Fig. 3(b)]

$$\hat{A}_m = \frac{1}{\sqrt{M}} \sum_{q=0}^{M-1} \hat{w}_q \exp\left[-j \left(\frac{2\pi}{N}q\right) m\right] |\psi_q\rangle. \quad (9)$$

Unfortunately, the measurable output of a QC operation, as the QFT one, is not a variable (e.g., here, the complex value

\(^1\)The normalization implies that the sum of the squared modulus of all the weights is unitary (i.e., $\sum_{n=0}^{N-1} |\hat{w}_n|_2^2 = 1$).
In all simulations, ideal isotropic antennas have been assumed to faithfully mimic the behavior of a real quantum computer. With the open Python library Qiskit [42] from IBM [43] that set as a reference. The QC computations have been emulated with the classical DFT-based approach have been computed with the arising power-pattern computation time and the accuracy of the arising power-pattern samples.

### III. NUMERICAL ASSESSMENT

In this section, the proposed QFT-based method for the computation of the power patterns of antenna arrays is assessed by considering various arrays affording different FF beam patterns. For comparison purposes, the power patterns computed with the classical DFT-based approach have been set as a reference. The QC computations have been emulated with the open Python library Qiskit [42] from IBM [43] that faithfully mimics the behavior of a real quantum computer. In all simulations, ideal isotropic antennas have been assumed [i.e., \( g_r(u) = g(u) = \frac{1}{2} \) \( n = 0, \ldots, N - 1 \)] to avoid any bias related to the type of the radiating elements of the array.

#### A. Validation and Resolution Threshold

The first example deals with a linear array of \( N = 16 \) elements, spaced by \( d = (\lambda/2) \), having real-valued excitations drawn from a Dolph–Chebychev (DC) distribution and affording a power pattern with sidelobe level (SLL) equal to \( S.L.L. = -15 \) [dB]. The power pattern has been computed in \( M = 1024 \) angular samples; Thus, \( L = 10 \) qubits have been allocated for the QC process. According to the guidelines in Section II, the complex “amplitudes” of the first \( N \) quantum states of \( |w\rangle \) have been initialized with the DC excitations in Table I, while the others \( Q - N = 1008 \) entries have been set to zero as indicated by (7). The QFT algorithm has been then executed \( T = M \times 10^3 \) times, and the measured quantum state probabilities, \( \{\varphi_m; m = 0, \ldots, M - 1\} \) being \( \varphi_{MAX} = 1.418 \times 10^{-2} \), are shown in Fig. 4 along with the reference pattern yielded by interpolating the pattern samples from the classical DFT. One can observe that the QFT samples almost perfectly fit the DFT curve by assessing the reliability of the QC pattern-prediction tool.

Fig. 4 also shows that the samples of the QFT are always above a minimum “resolution” threshold \( \delta \) defined as follows:

\[
\delta = \frac{1}{V_{MAX}}
\]

and corresponding to the minimum normalized power value representable at the output QFT process, where \( V_{MAX} \) is the maximum number of times the most recurring output quantum state has been measured (i.e., \( V_{MAX} \equiv \max\{\varphi_m(V_m)\} \)). In this case, \( \delta = -41.6 \) [dB] being \( V_{MAX} = 1.4518 \times 10^2 \).

Since \( V_{MAX} \) is expected to statistically grow with \( T \), the value of the resolution threshold \( \delta \) should get smaller and smaller (i.e., samples with lower amplitudes can be observed) when more shots are used for the quantum computation. To assess such a relation between \( T \) and \( \delta \), the QFT pattern prediction has been performed by varying \( T \), while keeping the same DC input state vector, \( |w\rangle \), of the previous example. In particular, the \( M = 1024 \) pattern samples in Fig. 5 have been computed using \( T = M \times 8 \) [Fig. 5(a)], \( T = M \times 20 \) [Fig. 5(b)], \( T = M \times 40 \) [Fig. 5(c)], and \( T = M \times 80 \) [Fig. 5(d)] shots and the smallest representable values of the power patterns turn out to be equal to \( \delta = -21.5 \) [dB], \( \delta = -25.0 \) [dB], \( \delta = -27.8 \) [dB], and \( \delta = -30.9 \) [dB], respectively. Moreover, to provide a statistically reliable assessment, each simulation has been repeated \( R = 20 \) times, and Fig. 6 shows the behavior of the average (solid line) value of \( \delta \) along with its minimum and maximum bounds (shaded region). As expected, the plot confirms the monotonic decreasing dependence of \( \delta \) on \( T \).

#### Table I

**NUMERICAL ASSESSMENT \([N = 16 \text{ and } d = (\lambda/2)\]—NORMALIZED EXCITATIONS**

| Element Index, \( n \) | Normalized Excitation Module, \( |\bar{w}_n|\) |
|------------------------|---------------------------------|
| \( S.L.L. = -15 \) [dB] | \( |\bar{w}_n| \) | \( |\bar{w}_n| \) | \( |\bar{w}_n| \) |
| \( 0 \) | 0.4129 | 0.2588 | 0.2794 |
| \( 1 \) | 0.3574 | 0.1535 | 0.2453 |
| \( 2 \) | 0.1814 | 0.1892 | 0.1786 |
| \( 3 \) | 0.2033 | 0.2232 | 0.2226 |
| \( 4 \) | 0.2221 | 0.2534 | 0.2633 |
| \( 5 \) | 0.2370 | 0.2780 | 0.2974 |
| \( 6 \) | 0.2473 | 0.2953 | 0.3219 |
| \( 7 \) | 0.2526 | 0.3043 | 0.3347 |
Fig. 6. Numerical assessment ($N = 16$, $d = (\lambda/2)$, DC excitations, $SLL = -15$ [dB], and $M = 1024$)—behavior of the QFT resolution threshold statistics versus the number of shots $T$ [average value (solid line) and interval values within the upper/lower bounds (shaded region)].

Fig. 7. Numerical assessment ($N = 16$, $d = (\lambda/2)$, DC excitations, $M = 1024$, and $T = M \times 80$)—power patterns when (a) $SLL = -20$ [dB] and (b) $SLL = -25$ [dB].

B. Analysis Versus SLL

In the third numerical experiment, the dependence of the accuracy of the QFT-based analysis method on the SLL of the reference pattern has been evaluated still considering the $N = 16$, $d = (\lambda/2)$-spaced array, but with the excitations in Table I affording the three DC patterns having $SLL = \{-15, -20, -25\}$ [dB]. Fig. 7 summarizes the outcomes of the QFT prediction process ($M = 1024$ and $L = 10$) when applied to the $SLL = -20$ [dB] and the $SLL = -25$ [dB] cases by setting $T = M \times 80$ shots as in Fig. 5(d) ($SLL = -15$ [dB]). From the comparison, it turns out that there is a growing number of misplaced samples around the SLL peaks when lowering the SLL from $SLL = -15$ [dB] [Fig. 5(d)] down to $SLL = -25$ [dB] [Fig. 7(b)], since the prediction of the samples with smaller power-pattern values needs more and more shots ($T \uparrow$) as pointed out by the analysis in Fig. 5.

To quantify the mismatch between the reference DFT pattern and the QFT one, the following (average) pattern matching metric:

$$\Gamma \triangleq \frac{1}{R} \sum_{r=1}^{R} \frac{\sum_{m=0}^{M-1} |\hat{P}_m - \hat{\varphi}_m(r)|}{\sum_{m=0}^{M-1} \hat{P}_m}$$

has been used, $\hat{\varphi}_m(r)$ being the normalized probability of measuring the $m$th ($m = 0, \ldots, M-1$) output quantum states at the $r$th ($1 \leq r \leq R$) run of the QC process. While one should expect a more faithful pattern matching for higher SLL (Fig. 7), the behavior of the plots in Fig. 8 proves the opposite, since the $\Gamma$ value gets worse as SLL increases. Such a (apparently) contradictory outcome can be explained by separately analyzing the mainlobe (ML)

$$\Gamma_{ML} = \frac{1}{R} \sum_{r=1}^{R} \frac{\sum_{m=0}^{\chi_1-1} |\hat{P}_m - \hat{\varphi}_m(r)|}{\sum_{m=0}^{M-1} \hat{P}_m}$$

and the SL

$$\Gamma_{SL} = \frac{1}{R} \sum_{r=1}^{R} \frac{\sum_{m=\chi_1}^{\chi_2-1} |\hat{P}_m - \hat{\varphi}_m(r)| + \sum_{m=\chi_2}^{M-1} |\hat{P}_m - \hat{\varphi}_m(r)|}{\sum_{m=0}^{M-1} \hat{P}_m}$$

contributions to the pattern matching metric (14), $\chi_1$ and $\chi_2$ ($\chi_1, \chi_2 \in [0, M-1]$) being the indexes of the pattern samples in the angular positions closer to the nulls on the left and right.
Fig. 10. Numerical assessment \((N = 16, d = (\lambda/2), \text{DC excitations, and } M = 1024)—power patterns when (a) \(SLL = -20 \text{~[dB]} \) and \(T = 1.8 \times 10^3 \times M \) and (b) \(SLL = -25 \text{~[dB]} \) and \(T = 2.4 \times 10^3 \times M \).

Fig. 11. Numerical assessment \((N = 16, d = (\lambda/2), \text{Taylor excitations, and } M = 1024)—plots of (a) pattern matching metrics versus the number of shots \(T \) and (b) power pattern when \(T = M \times 10^3 \).

Fig. 12. Numerical assessment \((N = 16, d = (\lambda/2))—normalized excitations of the array elements.

of the ML, respectively, while the same normalization factor of \((14)\) has been kept to fulfill the condition \(\Gamma = \Gamma_{ML} + \Gamma_{SL} \).

The dashed-line plots of \(\Gamma_{ML} \) and \(\Gamma_{SL} \) in Fig. 8 indicate that the mismatch is mainly in the SL region (i.e., \(\Gamma_{SL} > \Gamma_{ML} \)), and the \(\Gamma_{SL} \) value is greater when the SL of the reference pattern reduces (i.e., \(\Gamma_{SL} \mid SLL = -25 \text{~[dB]} > \Gamma_{SL} \mid SLL = -20 \text{~[dB]} > \Gamma_{SL} \mid SLL = -15 \text{~[dB]} \)) according to the conclusions drawn from Figs. 5(b) and 7(a) and (b). Otherwise, the ML index, \(\Gamma_{ML} \), decreases (i.e., \(\Gamma_{ML} \mid SLL = -25 \text{~[dB]} < \Gamma_{ML} \mid SLL = -20 \text{~[dB]} < \Gamma_{ML} \mid SLL = -15 \text{~[dB]} \)), and it is almost constant for smaller SLL values independently on \(T \) (e.g., see \(\Gamma_{ML} \) versus \(T \) when \(SLL = -25 \text{~[dB]} \)—Fig. 8). Such a behavior strictly depends on the fact that a fixed number of pattern samples, \(M \), are used for the QFT computation, and the number of samples entering in the ML, which is proportional to the so-called first-null beamwidth (FNBW), grows when lowering the SLL. Therefore, patterns with wider FNBW (i.e., lower SLL) involve more “high-probability” quantum states over the total number \(M \), and, since such states are more carefully predicted (even with few shots) being the most probable, it turns out that the QFT performance (i.e., the total pattern matching error, \(\Gamma \)) improves.

However, one is usually more interested in characterizing the pattern as a whole, thus having sufficient details of the least probable states (i.e., the low pattern values), as well. To infer the optimal trade-off \(T \), so that an accurate matching of the reference power pattern regardless of its SLL is yielded, the \(\Gamma_{SL} \) value of the pattern in Fig. 4 (i.e., \(\Gamma_{SL}^h = 5.8 \times 10^{-2} \)) has been set as quality-target threshold, and the QFT simulations have been run by progressively increasing the number of shots \(T \) until the condition \(\Gamma_{SL}(T) \leq \Gamma_{SL}^h \) has hold true. The accuracy threshold \(\Gamma_{SL}^h \) has been reached after \(T \mid SLL=-20 \text{~[dB]} = 1.8 \times M \times 10^3 \) and \(T \mid SLL=-25 \text{~[dB]} = 2.4 \times M \times 10^3 \) shots, respectively (Fig. 9). Fig. 10 shows the corresponding QFT results when predicting the reference DC pattern with \(SLL = -20 \text{~[dB]} \) [Fig. 10(a)] and \(SLL = -25 \text{~[dB]} \) [Fig. 10(b)]. As expected, there is a more faithful fitting of the reference pattern in the whole angular range and, in particular, within the SL regions [e.g., Figs. 7(a) versus 10(a) and Figs. 7(b) versus 10(b)].

As for the computational complexity of the proposed method and limiting the analysis to that of the QFT, it amounts to \(\Delta_\text{QFT} = \mathcal{O}(\log M^3)\), while the pattern prediction with the classical DFT method needs \(\mathcal{O}(M \times \log M)\) and \(\mathcal{O}(M)\) operations for the fast Fourier transform (FFT) and the square of the amplitudes, respectively. Accordingly, the computational
QC-based method has been assessed next for a \( d = (\lambda/2) \)-spaced array with the \( N = 16 \) Taylor excitations in Table I affording a power pattern with decreasing SL peaks when moving far from the ML (i.e., \( \text{SLL} = -15 \text{[dB]} \) and \( n = 4 \)). The QFT process has been applied by varying the number of shots from \( T = M \times 2 \) up to \( T = M \times 10^3 \) and repeating each test \( R = 20 \) times. The behavior of the pattern matching indexes is shown in Fig. 11(a), and, as a representative example, Fig. 11(b) gives the QFT power pattern characterized by \( \Gamma_{\text{SL}} = \Gamma^{\text{th}}_{\text{SL}} \) (\( \Rightarrow T = M \times 10^3 \) shots) to prove that it is possible to compute a generic pattern with the required degree of accuracy subject to a proper choice of the number \( T \) of QC shots.

The last example is concerned with the shaped beam patterns generated by the complex sets of excitations in Fig. 12. For both the flat-top and the cosecant-square patterns, the results in Fig. 13 indicate that the main deviations from the reference pattern occur in the SL region (i.e., \( \Gamma_{\text{SL}} > \Gamma_{\text{ML}} \)) whatever the \( T \) value, despite the ML occupies a nonnegligible part of the whole visible range \(-1 \leq u \leq 1\). Once again, the reason is that the quantum states associated with the samples in the ML region (i.e., the samples of the power pattern with higher magnitudes) have higher probability of being observed in the measurements also for low values of \( T \) (e.g., \( T = M \times 10 \) [Fig. 14(a) and (b)], while the SL region is better predicted when \( T \) grows to \( T = M \times 10^2 \) [Fig. 14(c) and (d)] and \( T = M \times 10^3 \) [Fig. 14(e) and (f)].

IV. CONCLUSION

An innovative QC-based method for the analysis of the power pattern of array antennas has been proposed. It exploits the QFT algorithm and the QC measurement principles to yield a suitable accuracy in the pattern prediction. A representative set of numerical examples, also in comparison with the classical DFT-based analysis technique, has been reported and discussed to provide in depth observations on the behavior and the performance of the proposed method.

To the best of the authors’ knowledge, the main innovative contributions of this article with respect to the state-of-the-art lie in the following:

1) the formulation of the PA analysis problem in the QC framework;
2) the adaptation of the QFT algorithm for the computation of the PA power pattern and the exploitation of the relationship between the probability values of the output quantum states and the samples of the power pattern;
3) the exploitation of the quantum parallelism to yield an exponential acceleration in the computation of the Fourier transform with respect to the classical DFT method and the comprehension that the computational advantage of the QFT is lost if one wants to measure the whole output state vector;
4) the study of the dependence of the pattern prediction accuracy of the proposed QC-based method on the number of shots/observations to give to the interested readers some useful guidelines to yield a reliable and effective prediction of the power pattern on the basis of the user needs.
Future works, beyond the scope of this article, will be devoted to extend the proposed approach to the analysis of planar arrays and to implement hybrid algorithms where only computationally intensive tasks are delegated to a quantum computer, while leaving other data processing operations to a classical computer. Moreover, novel array antenna analysis and synthesis QFT-based algorithms will be investigated where only few samples of the output state vector will be required, thus fully exploiting the QFT computational advantage.

Acknowledgment

Andrea Massa would like to thank E. Vico for her never-ending inspiration, support, guidance, and help.

References

[1] R. J. Mailloux, Phased Array Antenna Handbook, 3rd ed. Boston, MA, USA: Artech House, 2018.
[2] R. L. Haupt, Antenna Arrays—A Computation Approach. Hoboken, NJ, USA: Wiley, 2010.
[3] G. Yang and S. Zhang, “Dual-polarized wide-angle scanning phased array antenna for 5G communication systems,” IEEE Trans. Antennas Propag., vol. 70, no. 9, pp. 7427–7438, Sep. 2022. doi:10.1109/TAP.2022.3141188.
[4] J. Park, H. Seong, Y. N. Whang, and W. Hong, “Energy-efficient 5G phased arrays incorporating vertically polarized endfire planar folded slot antenna for mmWave mobile terminals,” IEEE Trans. Antennas Propag., vol. 68, no. 1, pp. 230–241, Jan. 2020.
[5] A. Puglielli et al., “Design of energy- and cost-efficient massive MIMO arrays,” Proc. IEEE, vol. 104, no. 3, pp. 586–606, Mar. 2016.
[6] F. Davarian, “Uplink arrays for the deep space network,” Proc. IEEE, vol. 95, no. 10, pp. 1923–1930, Oct. 2007.
[7] R. Schulze, R. E. Wallis, R. K. Stilwell, and W. Cheng, “Enabling antenna systems for extreme deep-space mission applications,” Proc. IEEE, vol. 95, no. 10, pp. 1976–1985, Oct. 2007.
[8] S.-M. Moon, S. Yun, I.-B. Yom, and H. L. Lee, “Phased array shaped-beam satellite antenna with boosted-beam control,” IEEE Trans. Antennas Propag., vol. 67, no. 12, pp. 7633–7636, Dec. 2019.
[9] T. Yu and G. M. Rebeiz, “A 22–24 GHz 4-element CMOS phased array with on-chip coupling characterization,” IEEE J. Solid-State Circuits, vol. 43, no. 9, pp. 2134–2143, Sep. 2008.
[10] G. Gottardi, L. Poli, P. Rocca, A. Montanari, A. Aprile, and A. Massa, “Optimal monopulse beamforming for side-looking airborne radars,” IEEE Antennas Wireless Propag. Lett., vol. 16, pp. 1221–1224, 2016.
[11] Z. Li et al., “Polarimetric phased array weather radar data quality evaluation through combined analysis, simulation, and measurements,” IEEE Geosci. Remote Sens. Lett., vol. 18, no. 6, pp. 1029–1033, Jun. 2021.
[12] O. M. Bucci, L. Crocco, R. Scapaticci, and G. Bellizzi, “On the design of phased arrays for medical applications,” Proc. IEEE, vol. 104, no. 3, pp. 633–648, Mar. 2016.
[13] F. Tofghi, J. Nourinia, M. Azarmanesh, and K. M. Khazaee, “Near-field focused array microstrip planar antenna for medical applications,” IEEE Antennas Wireless Propag. Lett., vol. 13, pp. 951–954, 2014.
[14] P.-F. Li, S.-W. Qu, and S. Yang, “Two-dimensional imaging based on near-field focused array antenna,” IEEE Antennas Wireless Propag. Lett., vol. 18, no. 2, pp. 274–278, Feb. 2019.
[15] J. S. Herd and M. D. Conway, “The evolution to modern phased array architectures,” Proc. IEEE, vol. 104, no. 3, pp. 519–529, Mar. 2016.
[16] A. Benoni, P. Rocca, N. Anselmi, and A. Massa, “Hilbert-ordering based clustering of complex-excitations linear arrays,” IEEE Trans. Antennas Propag., vol. 70, no. 8, pp. 6751–6762, Aug. 2022. doi:10.1109/TAP.2022.3164161.
[17] P. Rocca, L. Poli, N. Anselmi, and A. Massa, “Nested optimization for the synthesis of asymmetric shaped beam patterns in subarrayed linear antenna arrays,” IEEE Trans. Antennas Propag., vol. 70, no. 5, pp. 3385–3397, May 2022.
[18] G. Oliveri, M. Donelli, and A. Massa, “Linear array thinning exploiting almost difference sets,” IEEE Trans. Antennas Propag., vol. 57, no. 12, pp. 3800–3812, Dec. 2009.
Luca Tosi (Member, IEEE) received the bachelor’s and master’s degrees in information and communication engineering from the University of Trento, Trento, Italy, in 2020 and 2022, respectively, where he is currently pursuing the Ph.D. degree with the Department of Civil, Environmental, and Mechanical Engineering (DICAM). His research interests include antenna array design and synthesis and quantum computing for electromagnetics. Prof. Tosi is a member of the ELEDIA Research Center. In 2022, he received the Quantum Technologies Initiative Scholarship in the context of the IEEE Antennas and Propagation Society Graduate Fellowship Program.

Paolo Rocca (Fellow, IEEE) received the M.S. degree (summa cum laude) in telecommunications engineering and the Ph.D. degree in information and communication technologies from the University of Trento, Trento, Italy, in 2005 and 2008, respectively. He was a Visiting Ph.D. Student with Pennsylvania State University, State College, PA, USA, and the University Mediterranea of Reggio Calabria, Reggio Calabria, Italy, in 2012. He was a Visiting Researcher with the Laboratoire des Signaux et Systèmes (L2S@ Supélec), Gif-sur-Yvette, France, in 2013. He was an Invited Professor with the University of Paris Sud, Bures-sur-Yvette, France, in 2015, and the University of Rennes 1, Rennes, France, in 2017. He is currently an Associate Professor with the Department of Civil, Environmental, and Mechanical Engineering, University of Trento, and a Huashan Scholar Chair Professor with Xi'an University, Xi'an, China. He has authored/coauthored two book chapters, 165 journal papers, and more than 290 conference papers. His main research interests include the framework of artificial intelligence techniques as applied to electromagnetics, antenna array synthesis and analysis, electromagnetic inverse scattering, and quantum computing for electromagnetic engineering.

Prof. Rocca is a member of the ELEDIA Research Center, and the Big Data and AI Working Group for the Committee on Engineering for Innovative Technologies (CEIT) of the World Federation of Engineering Organizations (WFEO). He received the National Scientific Qualification for the position of Full Professor in Italy and France in 2017 and 2020, respectively. He also received the IEEE Geoscience and Remote Sensing and the Italy Section with the Best Ph.D. Thesis Award IEEE-GRS Central Italy Chapter. He served as an Associate Editor for the IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS from 2011 to 2016 and the Microwave and Optical Technology Letters from 2019 to 2020. He has been serving as an Associate Editor for IEEE Antennas and Propagation Magazine and Engineering since 2020.

Nicola Anselmi (Senior Member, IEEE) received the master’s degree in telecommunication engineering from the University of Trento, Trento, Italy, in 2012, and the Ph.D. degree from the International Doctoral School in Information and Communication Technology, Trento, in 2018. He is currently an Assistant Professor with the Department of Civil, Environmental, and Mechanical Engineering (DICAM), University of Trento, where he is also a Research Fellow with the ELEDIA Research Center. His research interests include synthesis methods for unconventional antenna array architectures, tolerance analysis of antenna systems, and electromagnetic inverse scattering techniques, with interest on compressive sensing methodologies for microwave imaging applications.

Dr. Anselmi is a member of the IEEE Antennas and Propagation Society. In 2019, he received the Italian National Scientific Qualification for the position of Associate Professor, and the Qualification aux fonctions de maître de conférences. He was a recipient of the “Giorgio Barzilai” Award for Young Researchers by the Italian Electromagnetic Society (SIEM) in 2016, the “Young Scientist” Prize by the Applied Computational Electromagnetics Society (ACES) in 2018, and the “Mini-Circuits Harvey Kaylilie Best Paper Prize” by the IEEE International Conference on Microwaves, Communications, Antennas and Electronic Systems (COMCAS) in 2019. He serves as an Associate Editor for the IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS and the IEEE OPEN JOURNAL OF ANTENNAS AND PROPAGATION. He serves as a reviewer for different international journals, including the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, the IEEE ANTENNAS AND WIRELESS PROPAGATION LETTERS, and the IET ON Microwaves, Antennas and Propagation.

Andrea Massa (Fellow, IEEE) received the Laurea (M.S.) degree in electronic engineering and the Ph. D. degree from the University of Genoa, Genoa, Italy, in 1992 and 1996, respectively. He is currently a Full Professor of electromagnetic fields with the University of Trento, Trento, Italy, where he currently teaches electromagnetic fields, inverse scattering techniques, antennas and wireless communications, wireless devices and devices, and optimization techniques. He is the Director of the network of federated laboratories “ELEDIA Research Center” (www.eledia.org) located in Brunei, China, Czech, France, Greece, Italy, Japan, Peru, and Tunisia with more than 150 researchers. He is a Chang-Jiang Chair Professor with the University of Electronic Science and Technology of China (UESTC), Chengdu, China, a Professor with Centrale-Supélec, Paris, France, and a Visiting Professor with Tsinghua University, Beijing, China. He has been an Adjunct Professor with Penn State University, State College, PA, USA, a Guest Professor with UESTC, and a Visiting Professor with the Missouri University of Science and Technology, Rolla, MO, USA, Nagasaki University, Nagasaki, Japan, the University of Paris Sud, Orsay, France, Keio University, Japan, and the National University of Singapore, Singapore. He has authored or coauthored more than 900 scientific publications among which more than 350 on international journals (more than 15,000 citations H-index = 65 [Scopus], more than 12,000 citations H-index = 59 [ISI-Web], and more than 23,000 citations H-index = 89 [Google Scholar]) and more than 550 in international conferences where he has presented more than 200 invited contributions (more than 40 invited keynote speaker) (www.eledia.org/publications). He has organized more than 100 scientific sessions in international conferences and has participated to several technological projects in the national and international framework with both national agencies and companies (18 international projects, >5 M€; eight national projects, >5 M€; ten local projects, >2 M€; 63 industrial projects, >10 M€; and six university projects, >300 K€). His research interests include inverse problems, analysis/synthesis of antenna systems and large arrays, radar systems synthesis and signal processing, cross-layer optimization and planning of wireless/RF systems, semantic wireless technologies, system-by-design and material-by-design (metamaterials and reconfigurable materials), and theory/applications of optimization techniques to engineering problems (telecommunications, medicine, and biology).

Prof. Massa is a member of the Editorial Board of the Journal of Electromagnetic Waves and Applications, a Permanent Member of the “PIERS Technical Committee” and the “EuMW Technical Committee,” and an ESOA member. He has been appointed in 2011 by the National Agency for the Evaluation of the University System and National Research (ANVUR) as a member of the Recognized Expert Evaluation Group (Area 09, “Industrial and Information Engineering”) for the evaluation of the studies at the Italian University and Research Center from 2004 to 2010. He has been elected as an Italian Member of the Management Committee of the COST Action TU1208 “Civil Engineering Applications of Ground Penetrating Radar.” He is a fellow of the IET and the Electromagnetic Academy. He has been the Senior DIGITEO Chair at the Laboratoire des Signaux et Systèmes (L2S)-CentraleSupélec and CEA LIST, Saclay, France, and the UGC-TU1208 Chair of Excellence at the Universidad Carlos III de Madrid, Spain. He has been appointed in the Scientific Board of the “Società Italiana di Elettromagnetismo (SIEm)” and elected in the Scientific Board of the Interuniversity National Center for Telecommunications (CNIT). He was an IEEE AP-S Distinguished Lecturer from 2016 to 2018. He served as an Associate Editor for the IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION from 2011 to 2014. He serves as an Associate Editor for the International Journal of Microwave and Wireless Technologies.