Factorial, cumulant and $H_q$-moments in dependence on their rank $q$ for the instanton-induced deep inelastic scattering (DIS) in the frameworks of QCD are calculated and analysed. The obtained correlation moments behaviour has specific form, which can be considered as a new criterion of the QCD-instantons identification on experiment at HERA.

1 Introduction

As it is known, such gauge theories as SM of electroweak interactions and QCD have degenerated vacuum structure on the classical level: potential energy is periodic with respect to the Chern-Simons number

$$N_{cs} = \frac{g^2}{16\pi^2} \int d^3 x \varepsilon_{ijk} \left( A_i^a \partial_j A_k^a + \frac{g}{3} \varepsilon^{abc} A_i^a A_j^b A_k^c \right). \quad (1)$$

Minimal energy (classical vacua) corresponds to integer $N_{cs}$. Neighbouring vacua are separated by a potential barrier of height $E_{sp}$ (Fig.1).

Fig.1. Schematic dependence of potential energy of gauge fields on Chern-Simons number $N_{cs}$. Gauge condition $A_0 = 0$ is used. $E_{sp}$ is so-called sphaleron mass.

Usual perturbative theory (Feynman rules) describes phenomena with $N_{cs} = 0$ only. Quantum tunnelling transitions between neighbouring vacua can be described by means of instantons, which are classical solutions of the Euclidean field equations with finite action. Taking into account such tunnelling transitions leads to the baryon number violation in SM, which is connected with the problem of matter and antimatter asymmetry in the Universe. In QCD instantons lead to the chirality violation, allow to solve U(1)-problem, give contribution to the confinement.
Therefore, the experimental discovery of instantons would be of fundamental significance for particle physics.

It was suggested probability of the instanton transitions can increase in high energy collisions [6]. There is a possibility of the instanton-induced events identification in the electron-proton DIS at HERA (DESY) [7]. Instanton induced DIS final states can be distinguished from ordinary (perturbative) ones through some features:

- high multiplicity (the average number of partons $\sim 10^7$);
- isotropic distribution of partons in the instanton rest system and presence, practically, of all light quarks (u, d, s) in each events [3];
- specific behaviour of gluon structure functions [8] and gluon correlation characteristics [9].

In our report additional "footprints" of QCD-instantons (factorial, cumulant and $H_q$-moments) are studied.

2 Multiplicity distribution of the instanton-induced final states

In DIS instantons can appear in the quark-gluon subprocesses (Fig.2). The following usual designations are used:

$$Q^2 = -q^2, \quad x = \frac{Q^2}{2Pq}, \quad Q'^2 = -q'^2, \quad x' = \frac{Q'^2}{2pq'},$$

where transferred momentum square $Q^2$ and Bjorken variable $x$ describe total DIS process; $Q'^2$ and $x'$ characterise the instanton subprocess.

Fig.2. Instanton induced DIS (figure was taken from [10]).

As it was mentioned above high parton multiplicity is one of the main characteristics of the instanton-induced events. The distribution on numbers of gluons in the instanton-induced events is given by the expression:

$$P^{(g)}_n = \frac{1}{\sigma_{tot}} \frac{1}{n!} \int d^4k_1...d^4k_n |T(k_1,...,k_n)|^2,$$

where $\sigma_{tot}$ – total cross-section, $T(k_1,...,k_n)$ is the amplitude of the production of gluons with the energy-momentum 4-vectors $k_1,...,k_n$. It is calculated by means of LSZ-technique applying to the Euclidean n-points Green function, which is given by the following Feynman path integral (in the quasiclassical approximation):

$$\int DA e^{-S[A]} A^I_{\mu_1} (x_1)...A^I_{\mu_n} (x_n).$$
where $S^c[A]$ is QCD Euclidean action, $A^{\mu}_{\mu}(x)$ – instanton configuration [2]. In quasiclassical approximation Gauss integral (4) is known calculable expression. The integration is carried out on the gluon fields, which connect neighbour classical vacua. Factorisation in (4) leads to the Poisson distribution on the final gluon number [8, 9]:

$$P_{n}^{(g)} = e^{-<n_g>} <n_g>^n / n!,$$
$$<n_g> = \frac{16\pi^2}{g^2} \left( \frac{1-x'}{x'} \right)^2, \quad 0.5 < x' < 1.$$  (5)

The quarks production in the instanton processes is described by the well-known fixed multiplicity distribution (if we take into account zero modes only [3]):

$$P_{n}^{(q)} = \delta_{2n_f,n},$$  (6)

where $n_f$ is a number of massless quark flavours. We suggest that masses of u, d, s are equal to zero.

Thus, if we take into account both gluons and quarks, then the following distribution is obtained:

$$P_{n} = e^{-<n_g>} <n_g>^{n-2n_f} / (n-2n_f)! \Theta(n-2n_f).$$  (7)

### 3 Calculation of the correlation moments for the instanton DIS processes

Study of the correlation moments is more useful sometimes than study of the multiplicity distribution [11]. Let us remind the well-known definition of the normalised factorial moments:

$$F_q = \left. \frac{1}{<n>^q} \frac{d^q Q(z)}{dz^q} \right|_{z=1}, \quad Q(z) = \sum_{n=1}^{\infty} P_n z^n, \quad 0 \leq z \leq 1.$$  (8)

where $<n>$ is the average multiplicity, $Q(z)$ – generating function.

In the case of the instanton-induced multiparticle production processes $Q(z)$ and $<n>$ have the following forms:

$$Q(z) = \sum_{n=2n_f}^{\infty} e^{-<n_g>} <n_g>^{n-2n_f} z^n / (n-2n_f)! = z^{2n_f} e^{<n_g>[z-1]}, \quad <n> = <n_g> + 2n_f.$$  (9)

The corresponding normalised factorial moments dependence on $q$ is shown on the Fig.3. It is well-known, that normalised factorial moments for ordinary perturbative processes of the particle production increase with increasing $q$. Therefore the behaviour of the moments for the instanton-induced processes can be used as a new instanton identification criterion.

Also we can consider the normalised cumulant moments:

$$K_q = \left. \frac{1}{<n>^q} \frac{d^q \ln Q(z)}{dz^q} \right|_{z=1}.$$  (10)

It is not difficult to calculate $K_q$ for the instanton distribution (11):

$$K_q = 2n_f (-1)^{q-1} q! + <n_g> <n_g>^{[q]}.$$  (11)

It is more interesting to consider the instanton contribution to the ratio of the cumulant and factorial moments:

$$H_q = \frac{K_q}{F_q}.$$  (12)
These moments have the following properties for the perturbative QCD: decreasing oscillations, presence of the negative correlations, there is the first minimum at $q = 5$ \[1\].

Unlike this, for the instanton distribution \[5\] $H_q$-moments have the first minimum at $q = 2$ (Fig.4), oscillations, which magnitude increases at large $q$ numbers (Fig.5).

4 Conclusion

The obtained dependences of the factorial, cumulant and $H_q$-moments on their rank have specific forms. Therefore, the behaviour of the correlation moments can be used as a new signal of the QCD nonperturbative vacuum phenomenon in addition to the well-known "footprints". Of course, we need to take into account hadronization stage. Local parton-hadron duality \[12\] allows to apply the obtained results for the experimental QCD-instantons search.

We propose the following procedure for the experimental QCD instantons search at HERA:
1) selection of the events with high multiplicity. For our approximations $n_{\text{hadrons}} \geq 12$ (local parton-hadron duality gives number of the final hadrons $n_{\text{hadrons}} = 2n_{\text{partons}} \geq 2n_{\text{quarks}} = 4n_f = 12$);
2) analysis of the correlation moments for the selected events and comparison with theoretical predictions.

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Fig. 3. The dependence of the normalised factorial moments on their rank $q$ for the different average gluon numbers: $<n_g>=2$ (solid line), $<n_g>=8$ (dash line); $n_f=3$.

Fig. 4. $H_q$ as the function of $q$, $<n_g>=2$, $n_f=3$: upper curve – full plot, lower – first part of the plot.

Fig. 5. Absolute values of $H_q$ as the function of $q$. Upper curve corresponds to the full plot ($<n_g>=3$ for solid line, $<n_g>=2$ for dash line), lower – first part of the plot ($<n_g>=6$ for solid line, $<n_g>=2$ for dash line).