Impact of ionization and electron density gradients in X-ray reflection spectroscopy measurements

Gitika Mall,1 Ashutosh Tripathi,1 Askar B. Abdikamalov,1,2,3 and Cosimo Bambi1⋆

1Center for Field Theory and Particle Physics and Department of Physics, Fudan University, 200438 Shanghai, China
2Ulugh Beg Astronomical Institute, Tashkent 100052, Uzbekistan
3Institute of Fundamental and Applied Research, National Research University TIIAME, Tashkent 100000, Uzbekistan

ABSTRACT

The models currently used for the analysis of the reflection spectra of black holes usually assume a disk with constant ionization and electron density. However, there is some debate on the impact of these assumptions on the estimate of the properties of the sources, in particular when the fits suggest very steep emissivity profiles in the inner part of the accretion disk. In this work, we re-analyze a selected set of high-quality NuSTAR and Suzaku data of Galactic black holes and we fit the reflection component with three different models: relxill_nk, in which the ionization parameter and the electron density are constant, relxillion_nk, where the electron density is still constant but the ionization profile is described by a power law, and relxilldgrad_nk, where the electron density profile is described by a power law and the ionization profile is calculated self-consistently from the electron density and the emissivity. While relxillion_nk can fit the data better, we do not find any substantial difference in the estimate of the properties of the sources among the three models. Our conclusion is that models with constant electron density and ionization parameter are probably sufficient, in most cases, to fit the currently available X-ray data of accreting black holes.

Key words: accretion: accretion disks – black hole physics – X-rays: binaries

1 INTRODUCTION

Blurred reflection features are commonly observed in the X-ray spectra of accreting black holes (Fabian et al. 1989; Tanaka et al. 1995; Nandra et al. 1997). They are thought to be produced by illumination of a cold accretion disk by a hot corona (Fabian et al. 1995; Reynolds & Nowak 2003; Risaliti et al. 2013; Bambi et al. 2021). The process can be briefly described as follows. The black hole is accreting from a geocentrically thin and optically thick disk. The thermal spectrum of the accretion disk turns out to be peaked in the soft X-ray band (0.1–10 keV) in the case of stellar-mass black holes and in the UV band (1–100 eV) for supermassive black holes. The corona is some hotter plasma (Tc ~ 100 keV) near the black hole and the inner part of the accretion disk. The corona may be some hot atmosphere above the accretion disk, the hot flow in the plunging region between the inner edge of the disk and the black hole, the base of the jet, etc. Some thermal photons of the accretion disk can inverse Compton scatter off free electrons in the corona. The resulting spectrum can be usually approximated well by a power law with photon index Γ ≈ 1-3 and exponential high-energy cutoff Ecut ≈ 2-3 Tc. A fraction of Comptonized photons can illuminate the disk and interact with the material of the disk: Compton scattering and absorption followed by fluorescent emission generate the reflection spectrum.

In the rest-frame of the gas in the disk, the reflection spectrum presents narrow fluorescent emission lines in the soft X-ray band and a Compton hump peaked at 20–30 keV (Ross & Fabian 2005; García & Kallman 2010). The most prominent emission line is usually the iron Kα complex, which is at 6.4 keV in the case of neutral or weakly ionized iron and can shift up to 6.97 keV in the case of H-like iron ions. Far from the source, the fluorescent emission lines appear broadened and skewed as a result of relativistic effects in the strong gravity region around the black hole: gravitational redshift, Doppler boosting, and light bending (Fabian et al. 1989; Laor 1991; Dauser et al. 2010; Bambi 2017a). The analysis of these broadened reflection features can potentially be a powerful tool for studying the properties of the inner part of the accretion disk, measuring black hole spins, and even testing fundamental physics (Brenneman 2013; Johannsen & Psaltis 2013; Reynolds 2014; Bambi 2017b; Bambi et al. 2021).

Models for the analysis of the reflection spectra of accreting black holes have been significantly improved in the past decade (Dauser et al. 2013; García et al. 2013, 2014); for a review, see Bambi et al. (2021). However, they still rely on a number of simplifications that may introduce unacceptably large systematic uncertainties in the final measurements of the properties of the sources. It is thus crucial to understand well the systematic uncer-
tainties of the theoretical models, as well as to develop more and more sophisticated theoretical models, in order to obtain precise and accurate measurements (Reynolds & Fabian 2008; Zhou et al. 2020; Cárdenas-Avendaño et al. 2020; Riaz et al. 2020, 2021, 2022; Tripathi et al. 2020a, 2021b). Otherwise, with the possibility of analyzing higher and higher quality spectra, there is the risk to get very precise but not very accurate measurements of accreting black holes, which would nullify the efforts to design and launch more powerful X-ray observatories.

Current analyses of the reflection spectra of accreting black holes normally employ reflection models with accretion disks with constant ionization and electron density. The ionization parameter is defined as

$$\xi = \frac{4\pi F_X}{n_e},$$

(1)

where $F_X$ is the X-ray flux from the corona illuminating the disk and $n_e$ is the electron density of the disk. The radial profile of the X-ray flux $F_X$ is determined by the coronal geometry (Wilkins & Fabian 2012; Dauser et al. 2013). Compact coronae very close to their black hole can naturally produce very steep X-ray flux in the inner part of the accretion disk as a result of light bending in the strong gravity region near the compact object (Martocchia & Matt 1996; Dauser et al. 2013; Riaz et al. 2022). The radial profile of the electron density $n_e$ depends on the properties of the accretion disk, but it is normally a function that moderately decreases as the radial coordinate increases.

Both the ionization parameter $\xi$ and the electron density $n_e$ affect the reflection spectrum in the rest-frame of the gas and are not expected to be constant over radii, which is instead the typical assumption in the analysis of reflection spectra. The assumption of constant ionization parameter and the electron density is certainly a simplification of the models, but it is also motivated by the fact that the strong light bending near the black hole can focus the Comptonized photons from the corona on a quite small portion of the inner part of the accretion disk, which we may thus be approximated well with a one-ionization region. On the other hand, very steep radial profiles of the X-ray flux $F_X$ should lead to very steep radial profiles of the ionization parameter $\xi$, which has quite a strong impact on the shape of the reflection spectrum. Svoboda et al. (2012) and Kammoun et al. (2019) found that employing reflection models with constant ionization profile may lead to overestimate the steepness of the emissivity profile of the inner part of the accretion disk and get inaccurate black hole spin measurements (see also Shreeram & Ingram 2020). Wilkins et al. (2022) found that an ionization gradient is required to model the broadband (0.3-50 keV) reflection spectrum of the Seyfert galaxy 1 Zwicky 1.

The aim of our work is to explore whether observations require reflection models with non-constant ionization parameter $\xi$ and electron density $n_e$. To do this, we select three high quality spectra of Galactic black holes and we fit every spectrum with three reflection models: relxill_nk (Bambi et al. 2017; Abdikamalov et al. 2019, 2020), relxillion_nk (Abdikamalov et al. 2021a), in which $n_e$ is constant but $\xi$ has a radial profile described by a power law, and relxillgrad_nk (Abdikamalov et al. 2021b), in which $n_e$ has a radial profile described by a power law and $\xi$ is calculated from Eq. (1) assuming that $F_X$ is proportional to the emissivity profile of the reflection spectrum. We find that relxillion_nk can usually provide a better fit, but we do not find any substantial difference in the estimate of the parameters of the sources among the three different models. Such a conclusion confirms the result found in Abdikamalov et al. (2021b), where we presented relxillgrad_nk and we analyzed a NuSTAR spectrum of the Galactic black hole in EXO 1846–031 with the three reflection models, finding that relxillion_nk fits the data better, but the three models provide consistent measurements of the parameters of the source.

The manuscript is organized as follows. In Section 2, we briefly review the three reflection models used in our work, pointing out their main differences. In Section 3, we select the observations for our study. In Section 4 and in Section 5, we present, respectively, the data reduction and the spectral analysis of the selected observations. Discussion of the results and conclusions are reported in Section 6.

2 REFLECTION MODELS

In our study, we employ three different reflection models: relxill_nk, relxillion_nk, and relxillgrad_nk. In this section we clarify the specific properties of each model.

relxill_nk (Bambi et al. 2017; Abdikamalov et al. 2019, 2020) was developed as an extension of the relxill package (Dauser et al. 2013; García et al. 2013, 2014) to non-Kerr spacetimes. In the present work, we will use relxill_nk with the deformation parameter frozen to zero to impose that the spacetime is described by the Kerr metric. With such a choice, relxill_nk formally reduces to the relxill model, but it reads a different table to include all relativistic effects in the final spectra and has some different subroutines, which are the same as in the other two models used in our work. We thus use relxill_nk instead of relxill in order to be sure that any difference in the fits is only due to the assumptions on the ionization and electron density profiles. The ionization parameter $\xi$ is assumed to have the same value over the whole disk and is left free in the fits. The electron density $n_e$ is frozen to $10^{15}$ cm$^{-3}$ over the whole disk since relxill_nk uses the xillver table (Garcia et al. 2013, 2014) for the reflection spectrum in the rest-frame of the gas.

Even relxillion_nk (Abdikamalov et al. 2021a) employs the xillver table for the reflection spectrum in the rest-frame of the gas and therefore the electron density is assumed to be $10^{15}$ cm$^{-3}$ over the whole disk. The ionization has instead a radial profile described by a power law

$$\xi(r) = \xi_{\infty} \left( \frac{R_{\infty}}{r} \right)^{\alpha_\xi}. \quad (2)$$

The ionization of the disk is thus described by two parameters: the ionization parameter at the inner edge of the accretion disk $\xi_{\infty}$ and the ionization index $\alpha_\xi$. $R_{\infty}$ is the radial coordinate of the inner edge of the accretion disk, which is already a parameter of the model and in our analysis will be set at the innermost stable circular orbit. For $\alpha_\xi = 0$, relxillion_nk exactly reduces to relxill_nk, while for $\alpha_\xi > 0$ the value of the ionization parameter decreases as the radial coordinate $r$ increases. Even relxillion_nk is designed to calculate reflection spectra in non-Kerr spacetimes, but in this work we will not consider such a function and we will freeze the deformation parameter of the model to zero in order to impose the Kerr background. Fig. 1 shows some synthetic reflection spectra calculated by relxillion_nk for different values of the ionization parameter at the inner edge of the accretion disk $\xi_{\infty}$, ionization index $\alpha_\xi$, and inclination angle of the disk, assuming that the spacetime is described by the Kerr metric with spin parameter $a_*$ = 0.998. For $\log \xi_{\infty} = 1$ (left panels,
Ionization and electron density gradients in reflection spectra

Figure 1. Synthetic reflection spectra calculated by relxillion_nk for \(\log \xi_{in} = 1, 3.1, \) and 4, \(\alpha_{\xi} = 0, 1, \) and 2, and a disk’s inclination angle \(\iota = 20^\circ\) and 70°. All spectra are calculated in the Kerr spacetime with spin parameter \(a_*=0.998\) assuming that the inner edge of the disk \(R_{in}\) is at the innermost stable circular orbit, the spectrum illuminating the disk has photon index \(\Gamma = 2\) and high-energy cutoff \(E_{cut} = 300\) keV , the emissivity profile of the disk is described by a power law with emissivity index \(q = 3\), and that the disk has Solar iron abundance, \(A_{Fe} = 1\).

\(\xi_{in}\) (in units of erg cm s\(^{-1}\)), a non-vanishing ionization gradient only affects the spectrum below 2 keV, while for \(\log \xi_{in} = 3.1\) (central panels) and 4 (right panels) we see differences even at the iron line and Compton hump regions.

In relxillgrad_nk (Abdikamalov et al. 2021b), the electron density profile is described by a power law and there are two parameters: the electron density at the inner edge of the accretion disk \(n_{in}\) and the electron density index \(\alpha_n\)

\[
n_n(r) = n_{in} \left( \frac{R_{in}}{r} \right)^{\alpha_n}.
\]

Unlike relxill_nk and relxillion_nk, relxillgrad_nk uses the table of xillverD for the reflection spectrum in the rest-frame of the gas. The electron density \(n_{in}\) is thus allowed to vary in the range \(10^{15}\) cm\(^{-3}\) to \(10^{19}\) cm\(^{-3}\). However, in order to limit the size of the table, xillverD assumes that the high-energy cutoff of the corona \(E_{cut}\) is fixed to 300 keV (in xillver, \(E_{cut}\) can range from 5 keV to 1 MeV). The ionization parameter is calculated assuming that the X-ray flux from the corona \(F_X\) is proportional to the emissivity profile of the reflection spectrum \(\epsilon\) and we have

\[
\xi(r) = \xi_{max} \left[ \frac{4\pi \epsilon(r)}{n(r)} \right]_{\text{norm}},
\]

where \(\xi_{max}\) is the maximum value of the ionization parameter and the expression in square brackets is normalized with respect to such a maximum value. Note that \(\xi_{max}\) may not be at the inner edge of the accretion disk \(R_{in}\) in some systems, depending on the values of \(n_{in}, \alpha_n,\) and \(\epsilon(r)\). Even for relxillgrad_nk, in this work we will set the deformation parameter to zero to work in the Kerr geometry. Fig. 2 shows some synthetic reflection spectra calculated by relxillgrad_nk for different values of the maximum ionization parameter \(\xi_{max}\), electron density index \(\alpha_n\), and inclination angle of the disk, assuming that the spacetime is described by the Kerr metric with spin parameter \(a_*=0.998\) and the electron density at the inner edge of the accretion disk is \(\log n_{in} = 19\) \((n_{in}\) in units of cm\(^{-3}\)). While the value of the electron density gradient mainly affects the spectrum below 3 keV, we have differences even in the iron line and Compton hump regions.

In the case of a point-like corona with a power law spectrum, we have \(F_X \propto g^{-\Gamma}\), where \(g = E_d/E_c\) is the redshift experienced by photons to travel from the corona to the disk and \(\Gamma\) is the photon index of the power law spectrum of the corona. For \(\Gamma = 2, F_X \propto \epsilon\), but this is not the case in general. However, the calculation of the redshift \(g\) requires to know the coronal geometry (we have to know the locations of the emission point in the corona and of the absorption/scattering point on the disk). For an arbitrary coronal geometry, we can only make the approximation \(F_X \propto \epsilon\). See Abdikamalov et al. (2021b) for more details.

MNRAS 000, 1–13 (2022)
Figure 2. Synthetic reflection spectra calculated by relxillgrad_nk for log $\xi_{\text{max}} = 1$, 3.1, and 4, $\alpha_n = 0$, 1, and 2, and a disk’s inclination angle $\iota = 20^\circ$ and $70^\circ$. All spectra are calculated in the Kerr spacetime with spin parameter $a^* = 0.998$ assuming that the inner edge of the disk $R_{\text{in}}$ is at the innermost stable circular orbit, the spectrum illuminating the disk has photon index $\Gamma = 2$ and high-energy cutoff $E_{\text{cut}} = 300$ keV, the emissivity profile of the disk is described by a power law with emissivity index $q = 3$, and that the disk has electron density at the inner edge of the accretion disk log $n_{\text{in}} = 19$ ($n_{\text{in}}$ in units of cm$^{-3}$) and Solar iron abundance, $A_{\text{Fe}} = 1$.

3 SOURCES AND OBSERVATIONS

For our study, we have selected three observations of three bright Galactic black holes. Sources and observations are listed in Tab. 1

3.1 GS 1354–645

GS 1354–645 was discovered with the All Sky Monitor aboard the Ginga satellite during its outburst in 1987 (Kitamoto et al. 1990). The source went in an outburst again in 1997 and was observed with RXTE (Revnivtsev et al. 2000). In June 2015, Swift/BAT detected a new outburst of GS 1354–645: interestingly, the source was only found in the low/hard spectral state (Miller et al. 2015b). NuSTAR observed the source in July. The NuSTAR data show very strong reflection features and the spectral analysis suggests a high inclination angle of the disk, a black hole spin parameter close to 1, and a very steep emissivity profile in the inner part of the accretion disk (El-Batal et al. 2016).

3.2 GRS 1739–278

GRS 1739–278 was discovered with the SIGMA telescope aboard Granat (Paul et al. 1991). The X-ray spectral and timing characteristics strongly suggested that the compact object was a black hole. GRS 1739–278 went into outburst again, after an extended quiescent period, in March 2014 and was detected by Swift/BAT (Krimm et al. 2014). The source was observed even by NuSTAR and the analysis of the data was reported in Miller et al. (2015a). The NuSTAR spectrum presents strong relativistic reflection features. Miller et al. (2015a) find a high black hole spin parameter, a steep inner emissivity profile, and a low inclination angle of the accretion disk.

3.3 GRS 1915+105

GRS 1915+105 is quite a peculiar source. It is a low-mass X-ray binary, as the mass of the companion star is less than 1 $M_\odot$. GRS 1915+105 was discovered by Granat in 1992 (Castro-Tirado et al. 1992), and since then it has never returned to the quiescent state. GRS 1915+105 shows remarkable variability across the whole electromagnetic spectrum (Belloni et al. 2000) and is identified as a micro-quasar because of its radio jets (Mirabel & Rodríguez 1999). Zhang et al. (1997) were the first to report a very high spin parameter of this black hole from the analysis of the thermal spectrum of the disk, later confirmed by McClintock et al. (2006) using RXTE data and a more sophisticated theoretical model. The discovery of a relativistically broadened iron Kα emission line was reported in Martocchia et al. (2002) using BeppoSAX data. The very high value of the black hole spin parameter has been confirmed by studies of the reflection features using Suzaku (Blum et al. 2009) and NuSTAR data (Miller et al. 2013). In our study, we re-analyze the Suzaku observation, which is more suitable for accurate and precise measurements of the properties of the source (see, e.g., Zhang et al. 2019a,b). In particular, the NuSTAR spectrum analyzed in Miller et al. (2013) presents a non-relativistic reflection

![Figure 2](image-url)
Ionization and electron density gradients in reflection spectra

Table 1. Summary of the sources and the observations analyzed in the present work.

| Source       | Satellite | Observation ID | Observation Date | Exposure (ks) |
|--------------|-----------|----------------|------------------|--------------|
| GS 1354–645  | NuSTAR   | 90101006004    | 2015 July 11     | 30.0         |
| GRS 1739–278 | NuSTAR   | 80002018002    | 2014 March 26    | 29.7         |
| GRS 1915+105 | Suzaku   | 402071010      | 2007 May 7       | 117          |

component and from its analysis we cannot get an accurate measurement of the properties of the source (Zhang et al. 2019a).

4 DATA REDUCTION

We note that the NuSTAR observations of GS 1354–645 and GRS 1739–278 and the Suzaku observation of GRS 1915+105 were already analyzed with an earlier version of our reflection model reflxill_nk in Tripathi et al. (2021a) and Tripathi et al. (2020b), respectively. We follow Tripathi et al. (2021a) for the data reduction of the NuSTAR spectra and Tripathi et al. (2020b) for the data reduction of the Suzaku data, and here we only report the main steps.

4.1 Reduction of NuSTAR data

The data from the NuSTAR detectors (FPMA and FPMB) were processed to get clean events using the script nupipeline of NuSTAR data analysis Software NusstarDAS v2.0.0, which was a part of spectral analysis software HEASOFT v6.28. We used the latest calibration database CALDB v20200912. The source region was selected to get 90% of its photons. A background region of the same size as the source region was selected far from the source to avoid inclusion of source photons. The redistribution matrix file (RMF) and ancillary response file (ARF) were created using the module nuproducts of NuSTARDAS.

4.2 Reduction of Suzaku data

Suzaku observed GRS 1915+105 on 2007 May 7 (Obs. ID 402071010) for approximately 117 ks. The event files from the XIS1 were processed with aepipeline to create a clean event file, using XIS CALDB version 20160616. The source region was selected by an annulus region due to photon pile up in the center of the detector (Blum et al. 2009). The background region was also selected by an annulus region. The XIS redistribution matrix file (RMF) and ancillary response file (ARF) were created respectively using the tools xisrmfgen and xissimarfgen available in the HEASOFT version 6.24 data reduction package. HXD/PIN data were reduced similarly, employing aepipeline and then hxdpinxbpi using the latest CALDB version 20110915.

5 BROADBAND X-RAY SPECTRAL ANALYSIS

For the spectral analysis, we use XSPEC v12.9.1 (Arnaud 1996). We employ the models discussed in Section 2 with tbabs, powerlaw, cutoffpl, and xillver. tbabs describes the Galactic absorption (Wilms et al. 2000). It has one parameter, the hydrogen column density \( N_H \). powerlaw describes a power law component and has two parameters: the photon index \( \Gamma \) and its normalization. cutoffpl describes a power law component with an
Table 2. Summary of the best-fit values from the analysis of the spectrum of GS 1354–645. * indicates that the parameter is frozen in the fit. The reported uncertainties correspond to the 90% confidence level for one relevant parameter ($\Delta \chi^2 = 2.71$).

| Parameter | relxill_nk | relxillion_nk | relxillgrad_nk |
|-----------|------------|---------------|----------------|
| $N_H$ [10^{22} cm^{-2}] | $0.7^*$ | $0.7^*$ | $0.7^*$ |
| cutoffpl | $\Gamma$ | $R_{\text{in}} [r_g]$ | $E_{\text{cut}}$ [keV] | $\log \xi$ [erg cm s^{-1}] | $A_F$ | $E_F$ [keV] | $\log n_e$ [cm^{-3}] | $\log n_{\text{in}}$ [cm^{-3}] | $\alpha_n$ | $R_{\text{out}}$ | norm [10^{-3}] | $\chi^2/\nu$ |
| $q_{\text{in}}$ | $10^{-0.21}$ | $10^{-0.6}$ | $10^{-5}$ | $0.90 \pm 0.05$ | $1.00 \pm 0.22$ | $0.80 \pm 0.09$ | $4.1 \pm 0.4$ | $4.1 \pm 0.8$ | $3.18 \pm 0.08$ | $0.991 \pm 0.004$ | $0.991 \pm 0.004$ | $81.9 \pm 0.5$ | $2.18 \pm 0.04$ | $\Gamma$ | $3.60 \pm 0.13$ | $0.22 \pm 0.03$ | $2.24 \pm 0.16$ | $0.74 \pm 0.06$ | $0.59 \pm 0.09$ | $0.65 \pm 0.02$ | $128 \pm 3$ | $323 \pm 17$ | $300^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*$ | $15^*
Table 3. Summary of the best-fit values from the analysis of the spectrum of GRS 1739–278. * indicates that the parameter is frozen in the fit. The reported uncertainties correspond to the 90% confidence level for one relevant parameter ($\Delta \chi^2 = 2.71$).

| Parameter | relxill_nk | relxillion_nk | relxilldgrad_nk |
|-----------|------------|---------------|-----------------|
| $N_{\text{H}}$ [$10^{22} \text{ cm}^{-2}$] | 1.58 ± 0.15 & 1.04 ± 0.23 & 2.99 ± 0.19 |
| cutoffpl | | | |
| $\Gamma$ | | | |
| $E_{\text{cut}}$ [keV] | | | |
| $\alpha_{\text{in}}$ | | | |
| $q_{\text{in}}$ | | | |
| $q_{\text{out}}$ | 6.7 ± 0.6 & 10.2 ± 5 & 7.56 ± 0.02 |
| $r_{\text{in}}$ [$r_g$] | 2.12 ± 0.09 & 2.73 ± 0.08 & 3.09 ± 0.09 |
| $\alpha_{\text{in}}$ | 5.5 ± 0.8 & 3.55 ± 0.65 & 4.52 ± 0.09 |
| $\alpha_{\text{out}}$ | 0.97 ± 0.017 & 0.946 ± 0.015 & 0.975 ± 0.005 |
| $i_{\text{deg}}$ | 16 ± 5 & 23 ± 4 & 47 ± 12 |
| $\Gamma_{\text{c}}$ | 1.201 ± 0.0027 & 1.18 ± 0.04 & 1.18 ± 0.04 |
| $\log \xi$ [erg cm s$^{-1}$] | 3.48 ± 0.06 & - & |
| $\log \xi_{\text{in}}$ [erg cm s$^{-1}$] | 4.37 ± 0.10 & - & |
| $\alpha_{\text{f}}$ | 0.33 ± 0.04 & - & |
| $\log \xi_{\text{max}}$ [erg cm s$^{-1}$] | - & - & 3.687 ± 0.008 |
| $A_{\text{Fe}}$ | | | 3.09 ± 0.18 |
| $E_{\text{cut}}$ [keV] | 25.4 ± 0.7 & 43.8 ± 2.2 & 300$^*$ |
| $\log n_{\text{e}}$ [cm$^{-3}$] | 15$^*$ & 15$^*$ & - |
| $\log n_{\text{in}}$ [cm$^{-3}$] | - | - | 17.15 ± 0.09 |
| $\alpha_{\text{n}}$ | - | 7.908 ± 0.149 | |
| $R_{\text{f}}$ | 0.42 ± 0.07 & 2.4 ± 0.6 & - |
| norm [10$^{-3}$] | 9.2 ± 0.5 & 3.0 ± 0.3 & 12.4 ± 2.8 |
| xillver | | | |
| norm [10$^{-3}$] | - | - | 3.20 ± 0.06 |
| $\chi^2/\nu$ | 1344.41/1122 & 1298.27/1121 & 1334.66/1119 |
| =1.19823 | =1.15814 | =1.19273 |

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Reflaction features, with a broadened iron line at 5–8 keV and a Compton hump peaked around 20 keV.

We thus add our reflection models to fit the reflection features. When we use relxill_nk and relxillion_nk, the reflection fraction $R_f$ is left free in the fit and therefore our XSPEC models are

tbabsxrelxill_nk, tbabsxrelxillion_nk.

In the case of relxilldgrad_nk, the high-energy cutoff is frozen to 300 keV, which may be a limitation in the case of low temperature coronae, because our data are up to 50-80 keV. We thus prefer to set the reflection fraction to $-1$ and describe the coronal spectrum with cutoffpl. The XSPEC model is
tbabsxcutoffpl + relxilldgrad_nk.

Even if this choice is not self-consistent with the reflection spectrum calculated by relxilldgrad_nk, it can only provide a better fit than the more conservative choice tbabsxrelxilliondgrad_nk with free reflection fraction.

For the data of GS 1354–645 and GRS 1915+105, we obtain already acceptable fits with a relativistic reflection spectrum. For the NuSTAR data of GRS 1739–278, we still have some residual and we add xillver to describe the reflection radiation from some distant cold material. In XSPEC language, the final models for GRS 1739–278 are

tbabsx(relxill_nk + xillver),
tbabsx(relxillion_nk + xillver),
tbabsx(cutoffpl + relxilldgrad_nk + xillver).

The results of our fits are summarized in Tab. 2, Tab. 3, and Tab. 4, respectively for GS 1354–645, GRS 1739–278, and GRS 1915+105. * is used to indicate that the parameter is frozen in the fit. The reported uncertainties are obtained with the error command in XSPEC and correspond to the 90% confidence level for one relevant parameter ($\Delta \chi^2 = 2.71$). If there is no lower/upper uncertainty, it means that the parameter is stuck at one of the boundaries of the allowed range. $q_{\text{in}}$ and $q_{\text{out}}$ are allowed to vary in the range 0 to 10 in the fit. The maximum value of the black hole spin parameter allowed by the model is 0.998. The iron abundance is allowed to vary in the range 0.5 to 10. Fig. 6, Fig. 7, and Fig. 8 show the residuals for the final fits of, respectively, GS 1354–645, GRS 1739–278, and GRS 1915+105. Figs. 9, 10, and 11 show the corner plots for the key-parameters of the fits with, respectively, relxill_nk, relxillion_nk, and relxilldgrad_nk of
Here we focus the discussion on the differences among the fits obtained for the first time, as well as in Tripathi et al. (2021a) and Tripathi et al. (GRS 1915+105), where these observations were analyzed for the first time, as well as in El-Batal et al. (2016) (GS 1354–645), Miller et al. (2015a) (GRS 1739–278), and Blum et al. (2009) (GRS 1915+105), where these observations were analyzed for the first time, as well as in Tripathi et al. (2021a) and Tripathi et al. (2020b), where these data were re-analyzed with relxill_nk. Here we focus the discussion on the differences among the fits obtained with different assumptions on the ionization and/or electron density profiles.

The first and most important consideration is that, in general, there is no significant difference among the measurements of the parameters of the sources obtained with the three models. In particular, the measurements of the black hole spin parameters $a_*$ and of the inclination angles of the disks $i$ are all consistent.

The estimates of the emissivity profiles are consistent too, and, in particular, we do not find that the inner emissivity index is overestimated by the reflection model with constant ionization and electron density. We note that the emissivity profile of GRS 1739–278 is steep in the inner part of the accretion disk and that the outer emissivity index is $q_{\text{out}} \approx 3$. This is the typical emissivity profile of a compact corona close to the black hole (Martocchia & Matt 1996; Dauzer et al. 2013; Riaz et al. 2022). For the other two sources, GS 1354–645 and GRS 1915+105, we find instead a steep emissivity profile in the inner part of the accretion disk and an almost flat profile for the outer part. This is the emissivity profile that can be expected from an extended corona covering the accretion disk (Miniutti et al. 2003; Wilkins & Fabian 2012; Gonzalez et al. 2017). If we model the emissivity profile with a twice broken power law, we find a steep emissivity profile in the inner region ($r < a_{\ast} r_g$), an almost flat profile in the central region (between a few $r_g$ and a few hundred $r_g$), and $q_{\text{out}} \approx 3$ in the outer part (Liu et al. 2022).

6 DISCUSSION AND CONCLUSIONS

The goal of our study is to understand if the available X-ray data of accreting black holes require reflection models with non-trivial ionization and/or electron density profiles and if fitting the data with reflection models with constant ionization and electron density can introduce unacceptably large systematic uncertainties in the estimate of the parameters of the systems. To do this, we have selected three high-quality observations of Galactic black holes with strong relativistic reflection features and we have fit every observation with three models with different assumptions about the ionization and/or electron density profiles. The fits of these observations have already been discussed in El-Batal et al. (2016) (GS 1354–645), Miller et al. (2015a) (GRS 1739–278), and Blum et al. (2009) (GRS 1915+105), where these observations were analyzed for the first time, as well as in Tripathi et al. (2021a) and Tripathi et al. (2020b), where these data were re-analyzed with relxill_nk. Here we focus the discussion on the differences among the fits obtained with different assumptions on the ionization and/or electron density profiles.

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The estimates of the emissivity profiles are consistent too, and, in particular, we do not find that the inner emissivity index is overestimated by the reflection model with constant ionization and electron density. We note that the emissivity profile of GRS 1739–278 is steep in the inner part of the accretion disk and that the outer emissivity index is $q_{\text{out}} \approx 2.3$. This is the typical emissivity profile of a compact corona close to the black hole (Martocchia & Matt 1996; Dauzer et al. 2013; Riaz et al. 2022). For the other two sources, GS 1354–645 and GRS 1915+105, we find instead a steep emissivity profile in the inner part of the accretion disk and an almost flat profile for the outer part. This is the emissivity profile that can be expected from an extended corona covering the accretion disk (Miniutti et al. 2003; Wilkins & Fabian 2012; Gonzalez et al. 2017). If we model the emissivity profile with a twice broken power law, we find a steep emissivity profile in the inner region ($r < a_{\ast} r_g$), an almost flat profile in the central region (between a few $r_g$ and a few hundred $r_g$), and $q_{\text{out}} \approx 3$ in the outer part (Liu et al. 2022).

Table 4. Summary of the best-fit values from the analysis of the spectrum of GRS 1915+105. * indicates that the parameter is frozen in the fit. The reported uncertainties correspond to the 90% confidence level for one relevant parameter ($\Delta \chi^2 = 2.71$). † indicates that the parameter cannot be constrained.

| Parameter               | relxill_nk | relxillnon_k | relxilldgrad_nk |
|-------------------------|------------|-------------|-----------------|
| $N_{\text{H}}$ [10$^{22}$ cm$^{-2}$] | 7.57 $\pm$ 0.03 | 7.86 $\pm$ 0.03 | 8.35 $\pm$ 0.07 |
| $r_{\text{out}}$ [r$_g$] | 2.605 $\pm$ 0.022 | 3921 | 7.251 $\pm$ 0.215 |
| $q_{\text{in}}$ | 8.14 $\pm$ 0.51 | 8.78 $\pm$ 0.30 | 10.92 $\pm$ 0.82 |
| $q_{\text{out}}$ | 0.97 $\pm$ 0.10 | 0.985 $\pm$ 0.004 | 0.996 $\pm$ 0.002 |
| $\Gamma$ | 67.5 $\pm$ 4.8 | 69.5 $\pm$ 4.6 | 79.4 $\pm$ 4.2 |
| $\log \xi$ [erg cm s$^{-1}$] | 3.07 $\pm$ 0.02 | 3.14 $\pm$ 0.02 | 3.16 $\pm$ 0.02 |
| $\log \xi_{\text{in}}$ | 2.079 $\pm$ 0.013 | 2.14 $\pm$ 0.02 | 2.16 $\pm$ 0.02 |
| $r_{\text{in}}$ [cm$^{-3}$] | 15$^{+4}_{-3}$ | 15$^{+4}_{-3}$ | 15$^{+4}_{-3}$ |
| $\alpha_{\text{in}}$ | 2.33 $\pm$ 0.09 | 2.33 $\pm$ 0.09 | 2.33 $\pm$ 0.09 |
| $\log \xi_{\text{max}}$ [erg cm s$^{-1}$] | 0.5 $\pm$ 0.03 | 0.5 $\pm$ 0.03 | 0.5 $\pm$ 0.03 |
| $\log n_{\text{e}}$ [cm$^{-3}$] | 19$^{+10}_{-9}$ | 19$^{+10}_{-9}$ | 19$^{+10}_{-9}$ |
| $\alpha_{\text{e}}$ | 0.75 $\pm$ 0.018 | 0.75 $\pm$ 0.018 | 0.75 $\pm$ 0.018 |
| $R_{\text{in}}$ | 3.07 $\pm$ 0.03 | 3.14 $\pm$ 0.02 | 3.16 $\pm$ 0.02 |
| $\log \chi^2/\nu$ | 2342.97/2209 | 2338.61/2208 | 2300.77/2207 |

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Table 5. AICc values of the fits with relxill_nk, relxillion_nk, and relxilldgrad_nk of the three sources in our study.

| Source       | relxill_nk | relxillion_nk | relxilldgrad_nk |
|--------------|------------|---------------|-----------------|
| GS 1354–645  | 2923.0     | 2911.2        | 2918.8          |
| GRS 1739–278 | 1370.7     | 1327.1        | 1367.1          |
| GRS 1915+105 | 2369.1     | 2366.8        | 2331.0          |

From the residuals of the three models (Fig. 6, Fig. 7, and Fig. 8), we do not see any significant difference among the fits of relxill_nk, relxillion_nk, and relxilldgrad_nk. Blue and green crosses are for NuSTAR/FPMA and NuSTAR/FPMB data, respectively.

Comparing the minimum of $\chi^2$ of different models is not a particularly robust method to determine which model is favored by the data. A more robust method is the Akaike information criterion (AIC) (Akaike 1974). Here we employ the Akaike information criterion corrected for small sample sizes (AICc) (see, e.g., Burnham & Anderson 2002), which is more appropriate in our case because the sample size is not large with respect to the number of
Figure 8. Data to best-fit model ratios for GRS 1915+105 when we use \texttt{relxill\_nk}, \texttt{relxillion\_nk}, and \texttt{relxilldgrad\_nk}. Green and blue crosses are for Suzaku/XIS and Suzaku/HXD data, respectively.

Figure 9. Corner plot for the key-parameter pairs (spin parameter $a_*$, inclination angle of the disk, photon index $\Gamma$, high-energy cutoff $E_{\text{cut}}$, and ionization log $\xi$) of the \texttt{relxill\_nk} fit of GS 1354–645 after the MCMC run. The 2D plots report the 1, 2, and 3\,$\sigma$ confidence contours.

Figure 10. Corner plot for the key-parameter pairs (spin parameter $a_*$, inclination angle of the disk, photon index $\Gamma$, high-energy cutoff $E_{\text{cut}}$, and ionization log $\xi$) of the \texttt{relxillion\_nk} fit of GS 1354–645 after the MCMC run. The 2D plots report the 1, 2, and 3\,$\sigma$ confidence contours.

free parameters. Since we have already the minimum of $\chi^2$ for every model, $\chi^2_{\text{min}}$, AICc is straightforward to calculate

$$\text{AICc} = \chi^2_{\text{min}} + 2N_p + \frac{2N_p(N_p + 1)}{(N_b - N_p - 1)},$$

where $N_p$ is the number of free parameters and $N_b$ is the number of bins. Tab. 5 shows the values of AICc for every model for the three sources. As a general and empirical rule, we can say that a model with $\Delta \text{AICc} > 5$ (where $\Delta \text{AICc}$ is the difference between the AIC value of the model and the AIC value of the model with the lowest AIC) is less favored by the data, and that a model with $\Delta \text{AICc} > 10$ can be ruled out and omitted from further consideration (Burnham & Anderson 2002). With such a criterion, we can confirm that \texttt{relxillion\_nk} fits the data of GS 1354–645 and GRS 1739–278 better than \texttt{relxill\_nk} and \texttt{relxilldgrad\_nk}, and that \texttt{relxilldgrad\_nk} is the best model for the data of GRS 1915+105.

The fact that \texttt{relxillion\_nk} can fit better the data than \texttt{relxill\_nk} is understandable: even if the ionization gradient

in the disk is modest in these sources, a non-vanishing ionization gradient is required. The fact that \texttt{relxillion\_nk} can provide a lower $\chi^2$ than \texttt{relxilldgrad\_nk} for GS 1354–645 and GRS 1739–278 may be related to the limitations imposed by the \texttt{xillverD} table, in which $E_{\text{cut}}$ is fixed to 300 keV. In particular for the observation of GRS 1739–278, where the fit requires a low value of $E_{\text{cut}}$, this may be an important limitation and explain a larger difference between $\chi^2$ of \texttt{relxillion\_nk} and...
relxilldgrad_nk (even if we do not see clear residuals in the bottom panel of Fig. 7 to support such a conclusion). However, for a conclusive answer we should wait for the next version of relxilldgrad_nk, which will use a reflection table with free $E_{\text{cut}}$. We note that if we had used a free reflection fraction in relxilldgrad_nk rather than adding cutoffpl, as it would have been required in a consistent model, the difference of $\chi^2$ between the models with relxillion_nk and relxilldgrad_nk would have been even larger, because we would have one less free parameter.

We note that the only discrepancy among the reflection models is in the estimate of the high-energy cutoff in the spectrum of the corona. For GS 1354–645 and GRS 1739–278, the value of $E_{\text{cut}}$ inferred by relxill_nk is about half the value obtained by relxillion_nk, while the comparison with the best-fit found by relxilldgrad_nk is not straightforward because of the al-

Figure 11. Corner plot for the key-parameter pairs (spin parameter $a_*$, inclination angle of the disk, photon index $\Gamma$, ionization log $\xi_\text{i}$, and electron density log $n_\text{e}$) of the relxilldgrad_nk fit of GS 1354–645 after the MCMC run. The 2D plots report the 1, 2, and 3$\sigma$ confidence contours.

Figure 12. Corner plot for the key-parameter pairs (spin parameter $a_*$, inclination angle of the disk, photon index $\Gamma$, ionization log $\xi_\text{i}$, and electron density log $n_\text{e}$) of the relxill_nk fit of GRS 1739–278 after the MCMC run. The 2D plots report the 1, 2, and 3$\sigma$ confidence contours.

Figure 13. Corner plot for the key-parameter pairs (spin parameter $a_*$, inclination angle of the disk, photon index $\Gamma$, high-energy cutoff $E_{\text{cut}}$, and ionization log $\xi_\text{i}$) of the relxillion_nk fit of GRS 1739–278 after the MCMC run. The 2D plots report the 1, 2, and 3$\sigma$ confidence contours.

Figure 14. Corner plot for the key-parameter pairs (spin parameter $a_*$, inclination angle of the disk, photon index $\Gamma$, ionization log $\xi_\text{i}$, and electron density log $n_\text{e}$) of the relxilldgrad_nk fit of GRS 1739–278 after the MCMC run. The 2D plots report the 1, 2, and 3$\sigma$ confidence contours.
present work. The measurements of the parameters of the sources, and in particular the measurements of the black hole spin parameter, the inclination angle of the disk, and the emissivity profiles, were consistent among the three models, with the only exception of the high-energy cutoff \( E_{\text{cut}} \) (but for EXO 1846–031 the value found with \texttt{relxill\_nk} was higher than the value inferred by \texttt{relxillion\_nk}). The lowest \( \chi^2 \) was obtained in the fit with \texttt{relxillion\_nk}, followed by the \( \chi^2 \) of \texttt{relxillgrad\_nk} and the highest \( \chi^2 \) was found with \texttt{relxill\_nk}. It is remarkable that for EXO 1846–031 the fit with \texttt{relxill\_nk} required a Gaussian and was unable to constrain the reflection fraction \( R_t \), while the fits with \texttt{relxillion\_nk} and \texttt{relxillgrad\_nk} did not require any Gaussian and we were able to constrain the reflection fraction or the normalization of \texttt{cuttloffpl}.

\textcite{Wilkins+22} fit the broadband (0.3–50 keV) reflection spectrum of the Seyfert galaxy 1 Zwicky 1 and find that an ionization gradient is required, but they do not report the parameter estimates without ionization gradient for a comparison. In their case, the difference between the residuals of the model with and without ionization gradient is clear. Since their analysis includes even the very soft X-ray band, which is not our case here, their fit may be more sensitive to an ionization gradient (see Figs. 1 and 2). \textcite{Wilkins+22} argue that the ionization parameter may fall as \( r^{-3/2} \) in the inner part of the accretion disk for a compact corona close to the black hole and as \( r^{-3/2} \) at large radii. This is not what we find in our fits with \texttt{relxillion\_nk}, where \( \alpha_{\xi} < 0.4 \) at 90% confidence level in all our sources.

To conclude, our study based on a small number of high-quality spectra of Galactic black holes suggests that in many sources the ionization gradient is probably modest and, even if the models with non-vanishing ionization gradient provide better fits, the models with constant ionization provide reliable estimates of the main parameters of the system. However, it is certainly possible that some sources present steep ionization gradients and that their analysis strictly requires models with ionization gradients.
ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (NSFC), Grant No. 11973019, the Natural Science Foundation of Shanghai, Grant No. 22ZR1404300, the Shanghai Municipal Education Commission, Grant No. 2019-01-07-00-07-E00035, and Fudan University, Grant No. J1H1512604. GM acknowledges also the support from the China Scholarship Council (CSC), Grant No. 2020GZX1016474. This work used the data and software provided by the High Energy Astrophysics Science Archive Research Center (HEASARC), which is a service of the Astrophysics Science Division at NASA/GSFC.

DATA AVAILABILITY

The NuSTAR and Suzaku raw data analysed in this work are available at download at the HEASARC Data Archive website. The reflection models used in this work are available from the corresponding author (C.B.) upon reasonable request and will be soon public at https://github.com/ABHModels.

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