Sensitivity of Carbon and Oxygen Yields to the Triple-Alpha Resonance in Massive Stars

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Abstract

Motivated by the possible existence of other universes, this paper considers the evolution of massive stars with different values for the fundamental constants. We focus on variations in the triple alpha resonance energy and study its effects on the resulting abundances of \(^{12}\text{C}\), \(^{16}\text{O}\), and larger nuclei. In our universe, the \(0^+\) energy level of carbon supports a resonant nuclear reaction that dominates carbon synthesis in stellar cores and accounts for the observed cosmic abundances. Here we define \(\Delta E_R\) to be the change in this resonant energy level, and show how different values affect the cosmic abundances of the intermediate alpha elements. Using the state of the art computational package MESA, we carry out stellar evolution calculations for massive stars in the range \(M_\star = 15 - 40 M_\odot\), and for a wide range of resonance energies. We also include both solar and low metallicity initial conditions. For negative \(\Delta E_R\), carbon yields are increased relative to standard stellar models, and such universes remain viable as long as the production of carbon nuclei remains energetically favorable, and stars remain stable, down to \(\Delta E_R \approx -300 \text{ keV}\). For positive \(\Delta E_R\), carbon yields decrease, but significant abundances can be produced for resonance energy increments up to \(\Delta E_R \approx +500 \text{ keV}\). Oxygen yields tend to be anti-correlated with those of carbon, and the allowed range in \(\Delta E_R\) is somewhat smaller. We also present yields for neon, magnesium, and silicon. With updated stellar evolution models and a more comprehensive survey of parameter space, these results indicate that the range of viable universes is larger than suggested by earlier studies.
1. Introduction

Over the past few decades, a detailed paradigm for the evolution of our universe has been developed, and this framework provides a successful explanation for many observed cosmic features (e.g., see [1] for a recent review). Some versions of this theory also argue that our universe could be one portion of a much larger region of space-time, sometimes known as the “multiverse,” i.e., our local region could represent one member of a vast collection of other universes [2, 3, 4, 5, 6, 7]. Moreover, these alternate universes could have different realizations of the laws of physics. Specifically, the constants of nature, including the strengths of the fundamental forces and the masses of the fundamental particles, could vary from region to region [8, 9, 10, 11]. Many authors have studied the effects of these possible variations in the laws of physics and find that only certain ranges for the parameters allow for universes to form cosmic structure and support working stars [12, 13, 14, 15, 16, 17, 18]. Related work studies the possible time variations of the laws of physics in our universe [19, 20]. Since carbon is generally considered a prerequisite for biology — at least for life in familiar forms — the synthesis of carbon provides an important constraint for habitable universes.

Carbon production in stellar interiors takes place through a rather complicated nuclear process known as the triple alpha reaction [21, 22, 23]. Because of the intricate landscape of nuclear binding energies and reaction rates, this process takes place through a resonant reaction, which is enabled by a particular excited state of the carbon nucleus. The resulting reaction rate for carbon production depends sensitively on the value of the resonant energy level (for greater detail, see Section 2 and references therein). As a result, if the laws of physics take different forms in other regions of space-time, the resonance energy level could be different, and the amount of carbon produced would vary accordingly. The objective of this paper is to determine how variations in the triple alpha resonance energy affect the abundances of the alpha elements produced in massive stars, with a focus on $^{12}\text{C}$ and $^{16}\text{O}$. The overall goal is to specify the range in resonance energy, characterized the change $\Delta E_R$ (see equation [8]), that allows the universe to be viable.

Possible variations to the energy level of the carbon resonance, and their effects on stellar evolution, have been explored previously [24, 25, 26, 27].
This paper generalizes earlier work by taking advantage of continuing developments in computational capabilities. This contribution uses the stellar evolution package MESA (Modules for Experiments in Stellar Astrophysics), a state of the art coding package that has been recently developed for a host of applications and is publicly available [28, 29]. The standard MESA package does not include changes to the triple alpha resonance level, so these modifications must be implemented at the code-level. In addition to using an updated stellar evolution code, this work also explores a much larger regime of parameter space. Whereas previous papers were limited to relatively few values for the stellar mass $M_*$ and the resonance energy increment $\Delta E_R$, this work considers much wider ranges for $M_*$ and $\Delta E_R$, along different choices for the stellar metallicity $Z$ (our results are broadly consistent with earlier work [24, 25, 26] for given parameter values). In addition to considering carbon and oxygen, we also compile yields for larger alpha elements (neon, magnesium, and silicon). Altogether, this paper reports the results from $\sim 2400$ stellar evolution simulations.

In assessing changes to the triple alpha process, the standard approach, which we also follow, is to vary the energy level of the $^{12}\text{C}$ nucleus, but keep all other parameters the same. At the fundamental level, however, variations in the excited state of nuclei are determined by changes in the strengths of the fundamental forces, especially the strong and electromagnetic interactions [30, 31, 32, 33, 34]. In principle, changes in these interaction strengths would affect all nuclear characteristics, including binding energies and reaction rates, not just the energy level of the $^{12}\text{C}$ resonance of interest here. In practice, however, small changes to the resonance energy lead to large changes in carbon production. Because of this extreme sensitivity, we can implement variations to the carbon resonance energy while keeping other nuclear parameters fixed. More specifically, the required changes to the resonance energy are of order 300 keV, whereas the binding energies of the nuclei are much larger and fall in the range 28 – 92 MeV.

This paper is organized as follows. We start with a brief review of the triple alpha reaction in Section 2, which also shows how changes to the process are implemented. The stellar evolution calculations are presented in Section 3, including a description of numerical considerations, basic evolution for massive stars, the effects of changing the triple alpha resonance energy, and the resulting carbon and oxygen yields over a wide range of parameter space. The abundance of carbon required for a viable universe is addressed in Section 4, along with the possibility that $^8\text{Be}$ can be stable and obviate the need for the triple alpha process. The relationship between the triple alpha resonance energy and the fundamental parameters of particle physics is briefly discussed. The paper concludes, in Section 5, with a summary of our results and a discussion of their implications.
2. The Triple Alpha Reaction

After a star burns through the hydrogen fuel in its central core, which is then composed primarily of helium, the star adjusts its internal structure. The central regions condense so that core becomes hotter and denser. Under these conditions, helium becomes the stellar fuel and leads to the production of heavier elements. Given the tight binding energy of helium nuclei — alpha particles — the natural progression is for the helium nuclei to fuse together to synthesize the so-called alpha elements: carbon, oxygen, and neon. In fact, after hydrogen and helium, these three species are the most abundant elements in our universe \[35, 36\], with magnesium, silicon, and sulfur close behind.

This nuclear chain is complicated by the fact that \(^{8}\)Be (and all other nuclei with atomic mass number \(A = 8\)) are unstable in our universe. In the absence of stable \(^{8}\)Be, which provides a stepping stone on the path to heavier alpha elements, the fusion of helium takes place through the triple alpha reaction \[21, 22, 23\], where three helium nuclei combine to make carbon. The net result of this process can be written in the form

\[
3 \; (^{4}\text{He}) \rightarrow ^{12}\text{C} + \gamma, \tag{1}
\]

but intermediate steps are required. In order to facilitate the reaction (1), the stellar core maintains a transient population of unstable \(^{8}\)Be nuclei \[37\]. In this setting, the alpha particles fuse to produce \(^{8}\)Be, which decays back into its constituent alpha particles with a half-life of approximately \(\tau_{1/2} \sim 10^{-16} \text{ sec}\),

\[
^{4}\text{He} + ^{4}\text{He} \leftrightarrow ^{8}\text{Be}, \tag{2}
\]

The forward reactions occur fast enough that the stellar core maintains nuclear statistical equilibrium (NSE), which determines the abundances of the relevant nuclear species. The resulting transient population of \(^{8}\)Be is large enough for some of these unstable nuclei to interact during their short lifetimes through the reaction

\[
^{4}\text{He} + ^{8}\text{Be} \rightarrow ^{12}\text{C}. \tag{3}
\]

Given the densities and temperatures of helium-dominated stellar cores, the nonresonant reaction does not take place fast enough to explain the observed carbon and oxygen abundances found in our universe. However, the \(^{12}\)C nucleus has an excited state at an accessible energy so that this reaction can operate in a resonant manner, which increases the reaction rate and allows stars to produce the observed cosmic abundances of carbon. Both the existence and the particular energy level of this excited state were predicted by Hoyle \[38\], and subsequent laboratory experiments \[39\] measured the resonance with the anticipated properties (see also
the reviews of [40, 41]). The relevant excited state has an energy of 7.6444 MeV, and corresponds to a $0^+\,^{12}$C nuclear state. Significantly, this energy is somewhat larger than the energy of the alpha particle and the $^8\text{Be}$ nucleus considered as separate particles (see equation [3]). The efficacy of carbon production is highly sensitive to the energy of this resonance.

As outlined above, the production of carbon relies, in part, on the intermediate reaction from equation (2), even though the product $^8\text{Be}$ is unstable. The reaction rate for this process [21, 22, 30] depends on the energy difference

\[ (\Delta E)_b \equiv E_8 - 2E_4. \tag{4} \]

The ground state energies of $^4\text{He}$ and $^8\text{Be}$ are denoted as $E_4$ and $E_8$, respectively. Similarly, the ground state of carbon is denoted as $E_8$ and the excited state (the $0^+$ resonance) as $E_1^*$. In the reaction [3], the energy difference between the excited carbon nucleus and the interacting nuclei is then given by

\[ (\Delta E)_h = E_1^* - E_8 - E_4. \tag{5} \]

The energy scale $E_R$ of the resonant reaction can then be defined as follows:

\[ E_R \equiv (\Delta E)_b + (\Delta E)_h = E_1^* - 3E_4. \tag{6} \]

This energy level has been experimentally measured to be $(E_R)_0 \approx 379.5$ keV. Given the above definitions, the resonant reaction rate $R_{3\alpha}$ for the triple alpha process at temperature $T$ can be written in the form

\[ R_{3\alpha} = 3^{3/2}n_\alpha^3 \left( \frac{2\pi \hbar^2}{E_4 kT} \right)^3 \frac{\Gamma_{\gamma}}{\hbar} \exp \left[ -\frac{E_R}{kT} \right], \tag{7} \]

where $n_\alpha$ is the number density of alpha particles and $\Gamma_{\gamma} \approx 0.0037$ eV is the radiative width of the Hoyle state [42]. Note that the energy scale $E_R$ appears in the exponential term. As a result, the net reaction rate for carbon production is exponentially sensitive to the value of $E_R$. Notice also that the argument of the exponential function varies as $T^{-1}$. For non-resonant reactions, Coulomb barrier penetration convolved with a thermal distribution of particle velocities leads to the usual $T^{-1/3}$ dependence of the reaction rate [21, 22, 23]; for resonant reactions, the convolution picks out a single energy (here $E_R = E_1^* - 3E_4$) and hence the factor $\exp[-E_R/kT]$.

In this treatment, we allow the energy of the resonance to take different values than that of our universe. Specifically, we define the energy increment

\[ \Delta E_R \equiv E_R - (E_R)_0, \tag{8} \]
where the subscript ‘0’ denotes the value in our universe. The energy difference $\Delta E_R$ represents the most important variable in the problem.

Equation (7) shows that higher energies for the $^{12}\text{C}$ resonance lead to suppression of the triple alpha reaction. Specifically, for a given temperature, increasing the resonance level ($\Delta E_R > 0$) results in a lower reaction rate for carbon production (helium burning). However, the star must generate enough energy to produce the pressure necessary to support itself against gravity. For $\Delta E_R > 0$, the stellar core must increase its temperature to compensate. Since the reaction rate is exponentially sensitive to the central temperature, only modest increases are necessary. However, these higher temperatures allow carbon nuclei in the core to fuse into oxygen through the reaction

$$^{4}\text{He} + ^{12}\text{C} \rightarrow ^{16}\text{O}. \quad (9)$$

In this reaction, the $^{16}\text{O}$ nucleus has an energy level of 7.1187 MeV, which is below the combined energy of the reactants $^{12}\text{C}$ and $^{4}\text{He}$ (which have energy 7.1616 MeV). The reaction (9) thus takes place in a non-resonant manner. In addition, a sizable Coulomb barrier between the reacting nuclei must be overcome, and this barrier depends sensitively on temperature. These features allow some fraction of the carbon to survive.

The basic problem that the triple alpha reaction poses for carbon production can be stated as follows: If the resonance energy level is raised ($\Delta E_R > 0$), then the temperature required for carbon production increases. But this hotter temperature also increases the rate of carbon depletion via equation (9). As a result, for sufficiently large increases in $\Delta E_R$ — and hence higher operating temperatures — the carbon produced by stellar nucleosynthesis can be immediately be transformed into oxygen and heavier elements. In this regime, relatively little carbon would be left behind for making life forms and other interesting structures.

The extreme sensitivity of the triple alpha reaction rate to both temperature and the resonance energy represents an important aspect of the problem. This sensitivity is often characterized by the indices defined by

$$\Xi_T \equiv \frac{R_{3\alpha}}{R_{3\alpha} \frac{dR_{3\alpha}}{dT}} = -3 + \frac{E_R}{kT}, \quad (10)$$

where the second equality uses equation (7) to evaluate the index, and

$$\Xi_E \equiv \frac{\Delta E_R}{R_{3\alpha} \frac{dR_{3\alpha}}{d\Delta E_R}} = -\frac{\Delta E_R}{kT}. \quad (11)$$

For our universe, $E_R \approx 380$ keV, whereas typical operating conditions for carbon production correspond to temperatures $kT \approx 9 - 17$ keV. As a result, the index
Ξₜ ≈ 20 – 40, so that the triple alpha reaction rate varies rapidly with changes in temperature. Stellar cores thus have temperatures of order kT ≈ 10 keV during helium burning. Stability of the star requires that the index Ξₜ > 0, so we obtain a lower bound on the possible values of the resonance energy, equivalently, on the scale ER:

$$E_R > 3(kT) \gtrsim 30 \text{ keV}.$$  \hspace{1cm} (12)

In other words, the resonance energy cannot be made arbitrarily small, but this correction (∼ 30 keV) is small compared to the observed value of ER (∼ 380 keV). For completeness, note that carbon can also be produced by a non-resonant reaction [37]. This alternative channel is dominant for sufficiently low temperatures, T ≲ 8 × 10⁷ K for canonical values of the parameters used in MESA. For the stellar models of this paper, however, the operating temperatures for helium burning are larger than this benchmark value.

3. Stellar Evolution Simulations

This section presents results from stellar evolution simulations carried out using MESA. Numerical considerations are discussed in Section 3.1 along with specification of the initial conditions and the extent of the parameter space. For comparison, Section 3.2 reviews basic trends for the evolution of massive stars in our universe. The effects of changing the energy of the triple alpha resonance are addressed next. Evolutionary tracks in the H-R diagram and the central density-temperature plane are described in Section 3.3 along with the time evolution of the nuclear inventory. The yields for carbon, oxygen, and other alpha elements are then given in Section 3.4.

3.1. Numerical Considerations and Initial Conditions

This study uses the computational package MESA to follow the time evolution of stars [28]. In order to include varying values for the triple alpha resonance energy, we had to modify the standard software. The MESA code controls nuclear reaction rates within a particular module, which creates a table of integrated cross-sections for each simulation. Note that these cross sections are the thermally-averaged quantities ⟨σv⟩. Within this module, the cross section for the triple-alpha reaction ⟨ααα⟩ includes the exponential factor given in equation (7), so that

$$\langle ααα \rangle \propto \langle αα \rangle \langle α⁸\text{Be} \rangle \propto \exp \left[ -\frac{E_R}{kT} \right],$$  \hspace{1cm} (13)

where ⟨αα⟩ and ⟨α⁸Be⟩ denote the cross sections for the sub-processes given in equations (2) and (3), respectively, and Eₐ denotes the energy of the triple alpha
resonance [42]. We allow the resonance energy to vary by an increment $\Delta E_R$, as defined in equation (8).

In addition to variations in the resonance energy for the triple alpha reaction, as determined by the variable $\Delta E_R$, we also explore a range of stellar masses $M_*$ and metallicities $Z$. The parameter space for this study thus has three variables and can be defined by $\mathcal{S} = \{\Delta E_R, M_*, Z\}$.

Here we are interested in massive stars that eventually explode as supernovae. In such stars, the stellar nucleosynthesis continues until the star produces a degenerate iron core. Since iron is the most tightly bound nucleus, no further nuclear processing is energetically favorable, and the star subsequently explodes. In the present application, we are interested in the final abundances of carbon, oxygen, and other heavy elements. We evolve the stellar models until the cores start to produce iron. At this point in evolution, the abundances of carbon and oxygen become fixed and we can determine their elemental abundances. For computational convenience, the simulations are stopped once the core begins to produce iron (specifically, when the iron mass exceeds $0.1 \, M_\odot$). After this milestone, the time-step in the code becomes increasingly small, and further evolution becomes computationally expensive.

The stellar mass range is taken to be $M_* = 15 - 40 M_\odot$. The lower end of the mass range ($15M_\odot$) is chosen for computational convenience: Since we are interested in massive stars that experience supernovae, the stellar mass must be larger than the minimum value $\sim 8-10 M_\odot$ required to produce an iron core (see [43] for a detailed discussion). We choose the slightly larger lower bound of $15 M_\odot$ because the lower mass cutoff can vary with changes in the input physics (e.g., the value of $\Delta E_R$) and because stellar models near the minimum supernova mass can have difficulty converging. We also impose an upper mass cutoff ($M_* = 40 M_\odot$). In our universe, the stellar initial mass function is steep, $dN/dM_* \propto M_*^{-2.3}$ [44], so that stars with masses beyond the cutoff at $40 M_\odot$ are rare. Moreover, such high mass stars are subject to pulsations and other instabilities, and they experience significant mass loss before exploding as supernovae. Such complications require specification of a large set of (uncertain) parameters. Notice also that the stellar mass distribution could be different in other universes, so that the proper mass range is not known. In any case, we focus on the confined range $M_* = 15 - 40 M_\odot$, sampled at integer values of $M_*/M_\odot$, to represent typical stars of high mass. Although this approach does not fully determine the cosmic abundances, it provides a good description of how nuclear yields vary with $\Delta E_R$.

For the stellar metallicity, we use two values: First, we adopt the Solar value $Z = 0.02$ because it allows for comparison with nearby stars in our universe. Since the primordial abundances of heavy elements are expected to be small, we also consider a much lower metallicity. Here we adopt the value $Z = 10^{-4}$ charac-
teristic of the lowest metallicity stars observed in our universe. With such low metallicity, stellar evolution proceeds similar to the case of zero metallicity, but the numerical code has better convergence properties. With different metallicities, stars have different starting carbon abundances, which in turn affects the relative importance of the CNO cycle versus the \( p-p \) reaction chain. As a result, stars with different metallicities can enter into their helium burning phase with different configurations. In practice, however, the differences are modest, and our results show the same general trends for both metallicities (see below and the discussions in [25, 26]).

This paper implicitly assumes that Big Bang Nucleosynthesis (BBN) is relatively unchanged by variations in the triple alpha reaction. As a result, the universe emerges from its early epochs with a composition dominated by hydrogen and helium, and relatively little mass locked up in heavier isotopes. The early universe does not produce appreciable amounts of carbon even if the triple alpha reaction rate is much larger. The universe goes through its hot early phase quickly — and with high entropy [45, 46, 47]. By the time the universe builds up a substantial mass fraction of helium, so that the triple alpha process has enough raw material to operate, both the temperature and density are below the values required to produce substantial amounts of carbon (and below values found in stellar interiors). This same argument applies to the production of lithium, which is more easily synthesized than carbon, and has a mass fraction of only about \( \text{Li/H} \approx 10^{-10} \). For comparison, typical abundances for CNO elements are \( \sim 10^{-16} \), which is five orders of magnitude too small to affect Population III stars [47].

3.2. Stellar Evolution for Massive Stars

The general picture for the time development of massive stars is well known. Here we provide only a brief overview (see [22] for a textbook treatment and [48] for a detailed review). The majority of time is spent on the main sequence, where stars have the proper configuration to burn hydrogen into helium. After hydrogen fuel in the stellar core is exhausted, the star adjusts its structure and starts to burn helium into carbon through the triple alpha process. As outlined above, oxygen is also produced through the reaction (9). Changes to the resonance energy affect the rate of carbon production, and the temperature at which it occurs, which in turn affects the rate of transforming carbon into oxygen. After the end of helium burning, the star condenses further and heats up, so that carbon is burned into neon, primarily through the reaction \( ^{12}\text{C}(^{12}\text{C},\alpha)^{20}\text{Ne} \). The alpha particles released through this process are (mostly) absorbed by \( ^{16}\text{O} \) nuclei to make more neon. The relative amounts of carbon and oxygen produced during the main helium burning phases determine the radial extent of the star over which these subsequent reactions take place, and how long they last.
Figure 1: Heavy element abundances a function of time for a massive star with \( M_* = 15 \, M_\odot \), metallicity \( Z = 10^{-4} \), and the triple alpha resonance properties of our universe. The yields are given as mass fractions \( X_i \). The time variable is a logarithmic measure of the time remaining before the end of the simulation at \( t_{\text{end}} \) when an iron core develops. As time elapses, progressively heavier nuclei are synthesized and their abundances grow, including carbon (blue), oxygen (orange), neon (green), magnesium (red), silicon (purple), sulfur (brown), iron (pink), and all other metals (gray). The abundances of the alpha elements increase, reach a maximum, and subsequently decline back down to an essentially constant value.

The general evolutionary trend is for the stellar core to become both hotter and denser with time, as one nuclear fuel supply is used up, and the products from the previous reactions become the fuel for the next phase. In the later stages, the core is hot enough for the background photons to photo-dissociate \(^{20}\text{Ne}\) into \(^{16}\text{O}\) and more alpha particles. Additional alpha elements are then produced, especially \(^{24}\text{Mg}\) and \(^{32}\text{S}\). Another significant process is that \(^{16}\text{O}\) nuclei combine to form \(^{32}\text{S}\), \(^{28}\text{Si}\), and similar nuclei. The eventual formation of a degenerate iron core marks the end of this chain of nuclear creation. As this process plays out, the carbon and oxygen mass fractions increase during the epoch of helium burning, and generally decrease afterwards (with some additional production, especially oxygen). In the later stages of the evolution, the central region, which becomes the iron core, is largely devoid of carbon and oxygen. Moreover, the abundances of these nuclei reach a plateau.

This behavior is illustrated by Figures 1 and 2 which show the mass fractions of the most important nuclei as a function of time for stellar models with mass \( M_* = 15 \, M_\odot \) and resonance increment \( \Delta E_R = 0 \) and +100 keV, respectively. Because the time for each subsequent nuclear burning stage is shorter than the previous
Figure 2: Heavy element abundance as a function of time for a massive star with $M_\star = 15 \, M_\odot$, metallicity $Z = 10^{-4}$, and resonance increment $\Delta E_R = 100$ keV. The yields are given as mass fractions $X_i$. The time variable is a logarithmic measure of the time remaining before the end of the simulation at $t_{\text{end}}$ when an iron core develops. As time elapses, progressively heavier nuclei are synthesized and their abundances grow, including carbon (blue), oxygen (orange), neon (green), magnesium (red), silicon (purple), sulfur (brown), iron (pink), and all other metals (gray). The abundances of the alpha elements increase, reach a maximum, and subsequently decline back down to an essentially constant value.
one, we present the results in terms of the time variable

\[ \xi \equiv -\log_{10} \left[ 1 - \frac{t}{t_{\text{end}}} \right], \]

where the time \( t_{\text{end}} \) marks the formation of the iron core and end of the numerical simulation. The variable \( \xi \) thus provides a logarithmic measure of the time remaining before the star explodes. Note that the variable \( \xi \to \infty \) in the limit \( t \to t_{\text{end}} \). For the curves shown in Figures 1 and 2, the value of \( \xi_n \) at the final (\( n \)th) time-step is plotted as \( \xi_n = \xi_{n-1} + 1 \).

As time increases, the mass fractions \( X_i \) of progressively heavier elements start to grow. The fraction of mass contained in carbon (blue), oxygen (orange), neon (green), magnesium (red), and silicon (purple) all increase, reach a maximum value, and then decrease down to an essentially constant value. Near the end of its life, the star produces a substantial amount of sulfur (brown), which is then transformed into iron (pink). Significantly, by the end stages of evolution when iron starts to accumulate in the core (\( \xi \gtrsim 10 \)), the mass fractions for all of the alpha elements are nearly flat/constant. As a result, the carbon and oxygen yields reported in this paper correspond to the abundances of the star at the time when the iron core develops.

The stellar evolution code (MESA) does not follow the supernova explosion. However, the stellar core itself contains little carbon/oxygen at this point, so that these elements are not lost to the neutron star remnant that forms out of the iron core. We also assume that the supernova explosion successfully detonates, so that no carbon and oxygen in the outer stellar layers are lost due to fallback onto the remnant. On the other hand, some additional nuclear processing does take place in the supernova ejecta, but these modifications to the carbon and oxygen inventories are beyond the scope of this present treatment. Earlier work argues that the carbon abundances are not altered appreciably by the supernova shock wave [26], whereas the abundance of oxygen can be depleted by \( \sim 10\% \). The carbon yields are determined to higher accuracy.

3.3. Stellar Evolution with Varying Carbon Resonance Energy

Variations in the energy of the triple alpha resonance have important implications for the final yields of carbon and oxygen produced by massive stars, but relatively modest effects on the overall trajectory of stellar evolution. Before determining the carbon and oxygen yields as function of \( \Delta E_R \) (see Section 3.4), we consider how changes in the resonance energy affect the evolution of massive stars. All of the stars in the mass range of interest tend to exhibit qualitatively similar behavior, so this discussion includes only a few representative examples.
The Hertzsprung-Russell (H-R) diagram is shown in Figures 3 and 4 for massive stars with $M_\ast = 15 M_\odot$ and metallicity $Z = 10^{-4}$. The two figures show evolutionary tracks for stars with varying triple alpha resonance energies, corresponding to the range $\Delta E_R = -200 \text{ keV}$ to $+250 \text{ keV}$. For all of the stars, evolutionary tracks start on the zero-age main-sequence, on the left side of the diagram. The stars then move upward in the diagram as their hydrogen fuel becomes depleted. The symbols on the tracks mark the points where alpha elements of interest start to be produced, including carbon (triangles), oxygen (squares), neon (plus signs), magnesium (stars), and silicon (circles). The symbols denote the points where the mass contained in these elements doubles from its initial value. Note that the tracks start to diverge with the onset of helium burning (carbon production). The models with the lowest values of $\Delta E_R$ branch off first, and the tracks continue to branch in order of increasing $\Delta E_R$, corresponding to higher temperatures required for the triple alpha reaction to operate. At late times, the tracks tend to converge, although the stellar models with $\Delta E_R \sim 0$ have lower luminosity at the endpoint (given by the formation of a degenerate core). Nonetheless, all of stars have similar internal configurations at the end of their tracks as determined by their central temperatures and densities (see Figure 5 below).

At intermediate times, the tracks in the H-R diagram show more complicated
Figure 4: H-R diagram for the evolution of a star with mass $M_* = 15 \, M_\odot$ and metallicity $Z = 10^{-4}$. Tracks for shown for positive increments of the triple alpha resonance energy $\Delta E_R = 0$ (red), 50 keV (blue), 100 keV (green), 150 keV (orange), 200 keV (violet), and 250 keV (cyan). The symbols on the tracks mark the points where the alpha elements start to be produced, including carbon (triangles), oxygen (squares), neon (plus signs), magnesium (stars), and silicon (circles).

behavior. In Figure 3, the tracks for stellar models with $\Delta E_R = -200$, $-150$, and $-100$ keV execute additional loops compared to the others, i.e., the tracks go back and forth in the H-R diagram. Additional loops are also seen in Figure 4 for stellar models with large $\Delta E_R = 150 – 250$ keV. Such loops are common signatures of the later stages of nuclear burning in massive stars, and are extremely sensitive to the input physics (see [22] for an in-depth discussion). Under the standard ordering, the production of nuclei proceeds in order of increasingly atomic number: carbon, oxygen, neon, magnesium, and then silicon. The symbols in Figures 3 and 4 generally follow this pattern, but exceptions arise for extreme values of $\Delta E_R$. For example, for $\Delta E_R = -200$ keV (the most negative value shown in Figure 3), oxygen production (marked by the blue square) is delayed. For $\Delta E_R$ large and negative, the temperature for the triple alpha reaction is low, and little oxygen is produced. The carbon produced is then processed into neon (plus symbol), with oxygen produced much later during shell burning in the later stages of evolution. As another example, for $\Delta E_R = +250$ keV (Figure 4), the temperature for the triple alpha process is so high that oxygen production (cyan square) occurs before carbon (cyan triangle).

Another way to illustrate stellar evolution is to plot the central temperature of the star versus the central density as the star evolves. The general trend is for
stellar cores to become both hotter and denser as they burn through one nuclear fuel source and move on to the next heavier one. Figure 5 illustrates this type of behavior for massive stars with $M_*=30M_\odot$ and low metallicity $Z=10^{-4}$. Tracks are shown for a range of triple alpha resonance energies corresponding to $\Delta E_R = –200$ (blue), $–100$ (green), $0$ (red), $100$ (gold), $200$ (purple), and $300$ keV (cyan). The evolution of the central stellar conditions in this diagram displays the same basic evolutionary behavior for all values of the resonance energy. For the tracks shown here, the stars start on the zero-age main-sequence. Stellar cores reside in the lower left portion of the diagram over most of their lifetimes as they burn hydrogen into helium, with central temperature $T_c \sim 3 \times 10^7$ K. The stars then move toward the upper right as they produce ever larger nuclei. In our universe, stellar cores typically have temperature $T_c \sim 2 \times 10^8$ K and density $\rho_c \sim 10^3$ g cm$^{-3}$ during their helium burning phases [49], when the triple alpha process is active. This epoch is marked by the triangles in the figure, which shows how the central temperature for carbon production becomes progressively hotter as $\Delta E_R$ increases. The density increases as well, so that the stars stay on nearly the same trajectory in the diagram. As a result, the curves shown in Figure 5 with varying values of $\Delta E_R$, show relatively little spread as the stars sequentially produce helium, carbon, oxygen, and then neon. The stars evolve until they reach the upper right portion of the diagram, as they eventually develop degenerate iron cores. Superimposed on this general evolutionary trend, the tracks in the diagram show minor excursions from simple monotonic behavior, primarily during the later stages of nuclear burning.

The behavior depicted in Figure 5 can be understood in terms of the stellar structure equations [21, 23]. In order for the star be be in hydrostatic equilibrium, the central pressure is given by

$$P_c \approx \left( \frac{\pi}{36} \right)^{1/3} GM_*^{2/3} \rho_c^{1/3}. \quad (15)$$

While the star remains operational, its pressure support is dominated by the ideal gas law, so that the central pressure is also given by

$$P_c \approx \frac{1}{\langle m \rangle} \rho_c kT_c, \quad (16)$$

where $\langle m \rangle$ is the mean mass of the ions. These two considerations imply a temperature-density relation of the form

$$kT_c = \left( \frac{\pi}{36} \right)^{1/3} \langle m \rangle GM_*^{2/3} \rho_c^{1/3}. \quad (17)$$

This result provides an approximate description of the curves shown in Figure 5. The numerical results are slightly less steep than $T_c \sim \rho_c^{1/3}$, primarily due to
Figure 5: Central density $(g \text{ cm}^{-3})$ and central temperature (K) of a massive star over its lifetime. The stellar mass $M_* = 30 \, M_\odot$ and the initial metallicity $Z = 10^{-4}$. The curves show the results obtained from stellar evolution simulations using different resonance energies for the triple alpha reaction corresponding to $\Delta E_R = -200 \rightarrow +300$ keV. Stars start in the lower left part of the diagram with the right configuration to burn hydrogen and end up in the upper right part of the diagram when they develop degenerate iron cores. The triangles mark the points where helium burning through the triple alpha process begins.
the inclusion of radiation pressure at high temperatures. The nuclear composition determines the mean ionic mass \( \langle m \rangle \), which also changes the slope of the relation and produces the departures from monotonic behavior, as shown in the diagram.

3.4. Yields for Carbon, Oxygen, and Larger Nuclei

We have carried numerical simulations for stellar models corresponding to the parameter space outlined in Section 3.1. The resulting carbon and oxygen yields are shown as a function of the change \( \Delta E_R \) in the triple alpha energy level in Figure 6 (for low metallicity \( Z = 10^{-4} \)) and Figure 7 (for solar metallicity \( Z = Z_\odot \)). In these figures, the mass in carbon (blue symbols) and oxygen (red symbols) are plotted for each stellar evolution model (specified by stellar mass \( M_* \) and resonance energy increment \( \Delta E_R \)). The stellar models are run until the stars begin to produce iron cores, at which time no further processing of carbon and oxygen occurs. The expectation value of the yields (in units of \( M_\odot \) per star) is shown as the thick black curves, which were obtained by integrating over the range of stellar masses, weighted by the stellar initial mass function. The curves were also smoothed by averaging over adjacent bins. The horizontal lines in the figures show the expectation values for carbon (lower gold line) and oxygen (upper purple line) appropriate for the starting metallicity.

Figures 6 and 7 show a number of interesting trends: When the resonance energy is lowered, so that \( \Delta E_R < 0 \), the carbon yields are higher than those of our universe. Our numerical simulations are only carried out over the full mass range for the values of \( \Delta E_R \) shown, but the carbon yields are larger than those in our universe for all \( \Delta E_R < 0 \). Moreover, although this finding is consistent with previous results [25, 26], the fact that lower resonance levels lead to more carbon production is not widely appreciated. On the other hand, the oxygen abundances decrease for lower values of \( \Delta E_R \). For both metallicities under consideration, the expectation values for carbon and oxygen yields are equal for \( \Delta E_R \approx -35 \) keV. Notice also that the resonance level cannot be made arbitrarily low: If the resonance is lowered by more than \( \sim 380 \) keV, the reaction is no longer energetically favorable (see equation [6]). In practice, the resonance energy must be moderately higher in order for the star to remain stable (see equation [12]), so that the lower limit becomes \( \Delta E_R > \sim -300 \) keV.

The carbon yields decrease with increasing values of \( \Delta E_R \), as expected. For higher resonance energies, the temperature required for the triple alpha reaction to operate increases, and much of the carbon produced can be immediately burned into oxygen. For low metallicity stars (Figure 6), the carbon yields steadily decrease with increasing \( \Delta E_R \) and cross the starting values at \( \Delta E_R \approx +500 \) keV. The oxygen yields increase with moderate increases in the resonance energy. For \( \Delta E_R \gtrsim 130 - 150 \) keV, however, the oxygen yields steadily decrease with further
Figure 6: Carbon and oxygen production yields in massive stars as a function of the $0^+$ resonance energy of the carbon-12 nucleus (for initial metallicity $Z = 10^{-4}$). The resonance energy is specified on the horizontal axis by the difference $\Delta E_R$ from the value in our universe (in keV). Results are shown for stellar evolution simulations with stellar masses in the range $M_* = 15 - 40 M_\odot$. The red circles show the resulting yields for carbon (in $M_\odot$), whereas the blue circles show the corresponding yields for oxygen. The heavy black curves show the expectation value (in $M_\odot$ per star) obtained with a weighted average over the stellar initial mass function in the specified range. The horizontal lines show the expectation values for the starting abundances of carbon (lower) and oxygen (upper) corresponding to $Z = 10^{-4}$. The yields for carbon and oxygen fall below their starting values for $\Delta E_R \approx +480$ keV and $\Delta E_R \approx +250$ keV, respectively.
Figure 7: Carbon and oxygen yields in massive stars as a function of the $0^+$ resonance energy of the carbon-12 nucleus (for solar metallicity). The resonance energy is specified by the difference $\Delta E_R$ from the value in our universe (in keV). Results are shown for stellar evolution simulations with stellar masses in the range $M_*= 15 - 40 M_\odot$. The red circles show the resulting yields for carbon (in $M_\odot$), whereas the blue circles show the corresponding yields for oxygen. The heavy black curves show the expectation value (in $M_\odot$ per star) obtained with a weighted average over the stellar initial mass function in the specified range. The horizontal lines show the expectation values for the starting abundances of carbon (lower) and oxygen (upper) corresponding to $Z = Z_\odot$. The yields fall below the starting values for $\Delta E_R \approx +180$ keV for oxygen and $\Delta E_R \approx +120$ keV for carbon.
increases in the resonance energy. The oxygen abundances fall below that of carbon for $\Delta E_R \sim 200$ keV, and become less than the starting value for $\Delta E_R \sim 300$ keV. To summarize, stars in other universes can support carbon production over the range in resonance energy increment given by

$$-300 \text{ keV} \lesssim \Delta E_R \lesssim 500 \text{ keV}.$$  \hspace{1cm} (carbon) \hspace{1cm} (18)

A moderately smaller range in $\Delta E_R$ allows for oxygen production,

$$-300 \text{ keV} \lesssim \Delta E_R \lesssim 300 \text{ keV}.$$  \hspace{1cm} (oxygen) \hspace{1cm} (19)

The total ranges in $\Delta E_R$ are thus $\sim 800$ keV for carbon and $\sim 600$ keV for oxygen.

For solar metallicity stars $Z = Z_\odot$ (Figure 7), the ranges in resonance energy for which stars produce more carbon and oxygen than their starting values are somewhat smaller than for low metallicity. Carbon production exceeds the initial supply for all negative $\Delta E_R$ and for positive resonance energy increments up to $\Delta E_R \approx +160$ keV. Oxygen production exceeds the starting value for resonance increments in the range $-150$ keV $\lesssim \Delta E_R \lesssim +200$ keV.
Figure 9: Yields from massive stars for an ensemble of alpha elements as a function of the triple alpha resonance energy (specified by the energy increment $\Delta E_R$). These simulations have starting metallicity $Z = Z_\odot$. Each curve shows the weighted-mean mass per star for a given alpha element. Yields are shown here for carbon (red), oxygen (blue), neon (green), magnesium (orange), and silicon (dashed purple). Changes to the resonance energy $\Delta E_R$ are given in keV, and the expectation values for the yields are given in $M_\odot$.

The nuclear abundances for a wider range of elements synthesized in massive stars are shown in Figures 8 and 9 for initial metallicities $Z = 10^{-4}$ and $Z = Z_\odot$, respectively. For each star, the numerical results determine the mass locked up in each element at the end of the simulation. These masses are then averaged over stellar mass, for the range $M_\ast = 15 - 40 M_\odot$, weighted by the stellar initial mass function. The functions are then smoothed over adjacent bins. The resulting expectation value (in $M_\odot$ per star) for each element is shown as a function of the change $\Delta E_R$ in the triple alpha resonance energy for carbon (red), oxygen (blue), neon (green), magnesium (orange), and silicon (purple). These simulations are run until the stars begin to develop iron cores. At this juncture, the abundances of the first four of these elements have reached their asymptotic values, but some of the silicon could still be processed into iron at later times. As a result, the yields for silicon are more uncertain than those of the other elements, so that these estimates are depicted by dashed curves. The lower horizontal lines in Figures 8 and 9 show the expectation values for carbon (lower red) and oxygen (upper blue) at the start of the simulations. As outlined above, they determine the ranges in $\Delta E_R$ for which stars can produce carbon and oxygen (see equations 18 and 19).

The results depicted in Figures 8 and 9 show the same basic trends: As noted above, the carbon yields steadily decrease as the triple alpha energy increment
\( \Delta E_R \) increases. In contrast, the oxygen yields increase with \( \Delta E_R \) until the resonance energy is higher than that of our universe, then saturate as they reach a broad peak, and decrease with further increases in the resonance energy. The neon yields are primarily decreasing functions of the resonance energy, but show more structure than the results for carbon. In particular, both neon curves have a local maximum for a small negative value of \( \Delta E_R \), and another local maximum near \( \Delta E_R \approx 100 \text{ keV} \). To leading order, the magnesium yields are anti-correlated with those of oxygen, with a minimum near the resonance level of our universe, and larger values for both positive and negative values of the resonance increment \( \Delta E_R \). Silicon yields generally grow with increasing \( \Delta E_R \), especially for large and positive values, but show a moderate deficit near \( \Delta E_R \sim 0 \). All of these trends are present in the results for both metallicities, although the detailed shape of the abundance curves vary (notice also that the axes are different in Figures 8 and 9).

To a rough approximation, the abundances of these alpha elements exhibit zero-sum behavior, with their sum nearly constant across the range in \( \Delta E_R \) of interest. Each of the five species is the most abundant element (by mass) for a small range of \( \Delta E_R \). As the resonance energy increases, the isotope that has the peak abundance varies from carbon to neon, oxygen, magnesium, and then silicon. With the exception of neon — and the neon peak is also the least prominent — this ordering follows that of increasing atomic number. Under typical stellar conditions, however, the temperature required for neon burning is slightly lower than that of oxygen burning, so the usual ‘onion-skin’ structure of high mass stars also follows this ordering [21, 22]. The basic trend shown in Figures 8 and 9 is thus expected: As \( \Delta E_R \) increases, the triple alpha process requires a higher central temperature, which favors the production of heavier nuclei.

4. Implications and Interpretation

Results from the previous section show that the resonance energy for the triple alpha reaction can vary over a range of \( \sim 800 \text{ keV} \) and still allow carbon to be produced. With this viable range specified, this section discusses how much carbon is necessary for habitability, alternate paths for carbon production via stable beryllium-8, and implications for the fundamental constants of physics.

4.1. Observed Carbon Abundances

With the value observed for the triple alpha resonance energy in our universe (corresponding to \( \Delta E_R = 0 \)), stellar nucleosynthesis can account for the observed carbon abundances. As shown in the previous section, massive stars produce carbon-to-oxygen ratios [C/O] of order unity. This finding is consistent with the value found for the Galaxy as a whole, [C/O] \( \approx 0.67 \), and that for the Solar
The observed range of carbon-to-oxygen ratios varies from region to region, with \([\text{C/O}] \approx 0.25 - 0.75\). Although observed [C/O] values are consistent with predictions from stellar evolution calculations for massive stars, the carbon inventory of the Galaxy also gets substantial contributions from intermediate mass stars.

In contrast to the [C/O] ratios quoted above, the observed value for Earth is substantially lower, with estimates falling in the range \([\text{C/O}] \approx 0.002 - 0.01\). As a result, the carbon abundance of Earth is about two orders of magnitude lower than that of the Solar System, the Galaxy, and the universe as a whole. The minimum carbon abundance for habitability remains unknown at the present time. Nonetheless, given that Earth is the only astrophysical environment known to support life, and that its carbon supply is highly depleted, this carbon threshold could be as much as \(\sim 100\) times lower than the observed cosmic abundance. From the results shown in Figure 6, stars in other universes could produce carbon at Earth-like levels for triple alpha resonance energies corresponding to increments as large as \(\Delta E_R \sim +300\) keV.

On the other hand, the estimated [C/O] ratio for Venus is comparable to that of Earth and hence also sub-solar. One interpretation of this finding is that rocky planets are ineffective at capturing and holding onto carbon, so that the formation of terrestrial planets generically results in [C/O] ratios that are similarly low. However, the formation location is also important: C-type (carbonaceous) asteroids, which represent the majority of these rocky bodies, contain a large abundance of carbon, with chemical compositions comparable to that of the Sun (after accounting for the depletion of hydrogen, helium, and other volatiles). Such asteroids are thought to be left-over building blocks from the planet formation process, and could in principle build planets with higher carbon inventories than Earth. With the wide range of carbon abundances inferred for bodies in our Solar System, and the diversity of worlds now being discovered around other stars, the expected carbon abundance for planets in systems with a given stellar composition remains to be determined. For completeness, we also note that low [C/O] ratios could in principle be produced through the enhancement of oxygen on terrestrial planets.

4.2. The Possibility of Stable Beryllium-8

The above discussion indicates that the energy level of the carbon resonance can vary over a range of \(\sim 800\) keV while allowing massive stars to synthesize carbon at acceptable levels. Much larger variations could render the universe sterile, by making carbon energetically disfavored for large negative \(\Delta E_R\) (see equation 6), and by shutting down carbon production for large positive \(\Delta E_R\). For comparison, the \(^8\text{Be}\) nucleus fails to have a bound state by only 92 keV. In
order words, the changes to nuclear binding energies required to render carbon production untenable is much larger than the changes necessary for the universe to have stable $A = 8$ nuclei.

With the existence of a stable isotope with mass number $A = 8$, universes no longer need the triple alpha process to produce carbon $^{12}$C. Helium burning can proceed through the arguably more natural reaction of equation (2), which would primarily proceed in the forward direction with stable $^8$Be as the end product. The beryllium could then be processed into carbon through equation (3) without the need for an enhanced reaction rate. The subsequent carbon production could take place in later evolutionary states of the same star, or much later in future stellar generations.

In order for a stable $^8$Be nuclei to exist, the required changes to the fundamental constants are only about $1 - 2\%$. The size of these variations can be estimated by detailed calculations using Lattice Chiral Effective Field Theory [30], by simpler models of the nucleon-nucleon potential [27], and/or by straightforward analytic arguments [56, 58]. Moreover, the necessary changes to the strengths of the nuclear and electromagnetic forces are roughly comparable. If the fundamental forces have different strengths than in our universe, then both the binding energy of $^8$Be and that of the constituent $^4$He nuclei will change. Significantly, the aforementioned effective field theory calculations show that the binding energies of $^8$Be and its building blocks do not change at the same rate as the fundamental constants are varied [30, 31]. This mismatch is necessary for $^8$Be to become stable: If the derivative of the $^4$He binding energy with respect a fundamental parameter is equal to half the derivative of the $^8$Be binding energy with respect to the same parameter, then both sides of equation (2) would scale together, and the forward reaction would not be energetically favored.

4.3. Triple Alpha Resonance and the Fundamental Constants

The focus of this paper is to explore how changes to the triple alpha resonance energy affect the evolution of massive stars and their yields of carbon, oxygen, and other elements. From a phenomenological viewpoint, the most important variable is the energy increment $\Delta E_R$ defined in equation (8). However, this quantity is not a fundamental parameter of the Standard Model of Particle Physics (see, e.g., [9] for one accounting of the constants), but rather is a complicated function of the basic parameters [59]. Small changes to the strength of the nucleon-nucleon potential lead to large changes to the triple alpha reaction rate [60], primarily due to the exponential dependence displayed in equation (7). More specifically, previous work has shown that variations in the resonance energy of order $\Delta E_R \sim 100$ keV correspond to variations in the Higgs vacuum expectation value of order $\sim 10\%$ [61]. The requirement that stable complex nuclei exist places a weaker
bound on this expectation value [62]. For the same increment $\Delta E_R \sim 100$ keV, the required changes to the nuclear potential and/or the electromagnetic interaction are estimated to be of order 0.5% and 2–4%, respectively [26, 27].

One can also assess the triple alpha constraint by considering the energy levels of nuclei: The triple alpha reaction rate is enhanced because the particle energies (under the relevant stellar conditions) happen to be comparable to an excited state of the carbon nucleus. Resonances correspond to excited states (energy levels) of the nucleus. In general, the excited states of nuclei are spaced with energy intervals of order $\sim 1$ MeV [63]. For the specific nucleus of interest, however, five of the first excited states of carbon have energies $E = 4.44, 7.65, 9.64, 12.7, \text{ and } 15.1$ MeV (many additional resonances are also present [64]). The energy separation between adjacent resonances is $\sim 3$ MeV. The allowed range for $\Delta E_R$ ($\sim 800$ keV) corresponds to about one fourth of the spacing interval. Given that the life-permitting range ($7.65 \pm 0.3$ MeV) lies in the range of carbon energy levels, the chances for particle energies to be sufficiently near a resonance are about 1 part in 4. Being near a resonance is neither necessary nor sufficient for carbon production, so the odds of a successful universe are not this favorable, but the chances of being near a resonance are relatively high. All strong and electromagnetic nuclear reactions must obey angular momentum and parity conservation laws, which in turn lead to selection rules. One must also take into account the widths of the resonances, and their proximity to other excited states, and these considerations affect the nuclear reaction rates [65]. As a result, a wide range of possible reactions could play a role in carbon production in other universes, i.e., the available parameter space is large. A full discussion of these complications is beyond the scope of this current paper, but should be addressed in the future.

This work has assumed that the triple alpha resonance energy varies, but the binding energies for the relevant nuclei and the reaction rates for other processes are unchanged. Given the allowed range for the resonance energy from equations (18) and (19), we can consider the consistency of this assumption. The binding energies for carbon and helium are 92.2 MeV and 28.3 MeV. These energies are larger than the energy increments of interest, $|\Delta E_R| \approx 300 - 400$ keV, by almost two orders of magnitude. Although a fully self-consistent treatment should be carried out in the future — where all of the binding energies, resonance levels, and reaction rates are re-calculated as a function of the fundamental parameters — the current approach provides a good working approximation. Notice also that the nucleon binding energy and the nuclear energy levels scale as $E \sim \alpha_s^2 m_p$ [14], so that the allowed range in $\Delta E_R$ corresponds to relatively modest changes ($\sim 5\%$) in the strong coupling constant $\alpha_s$. 

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5. Conclusion

This paper presents results from a large ensemble (\(\sim 2400\)) of stellar evolution simulations for massive stars in other universes with varying values for the triple alpha resonance energy. These findings indicate that the sensitivity of our universe — and others — to the triple alpha reaction for carbon production is more subtle and less confining than previously reported.

5.1. Summary of Results

The results of our stellar evolution calculations add to our understanding of the triple alpha fine-tuning problem, and can be summarized as follows:

- The change in the triple alpha resonance energy \(\Delta E_R\) is the defining variable in the problem. For lower values of the resonance energy, \(\Delta E_R < 0\), massive stars produce more carbon than those in our universe (not less). The allowed parameter space for viable universes extends down to \(\Delta E_R \approx -300\) keV. For larger values of the resonance energy, \(\Delta E_R > 0\), the carbon yields decrease steadily, but substantial carbon production continues up to \(\Delta E_R \approx +500\) keV. The total allowed range is \(\sim 800\) keV.

- Oxygen abundances decrease with decreasing \(\Delta E_R\), and increase as the resonance energy increases up to \(\Delta E_R \sim 150\) keV. Oxygen abundances then decline with further increases in \(\Delta E_R\). The range of \(\Delta E_R\) for which massive stars can produce significant oxygen is comparable to, but somewhat smaller than, that for carbon. As a result, the requirement that universes produce sufficient oxygen may be more restrictive than the constraint from carbon.

- Although the ability of stars to produce carbon and oxygen depends sensitively on the triple alpha resonance energy, the yields do not show a strong dependence on stellar mass or metallicity. Similarly, the overarching trajectory of stellar evolution, from main-sequence hydrogen burning to the development of an iron core, is not greatly affected by changes to the triple alpha process and yields of the intermediate elements.

- The allowed range (\(\sim 800\) keV) for the triple alpha resonance energy corresponds to \(\sim 10 - 20\%\) changes to the parameters of fundamental physics. For comparison, the \(^8\)Be nucleus fails to be bound by only 92 keV, and could become stable with 1 – 2% changes to the fundamental parameters. If \(^8\)Be is stable, then stars can burn helium and make carbon through alternate reaction chains \[^56\]. The range of parameters for which the triple alpha process can operate effectively is thus much larger than the range over which it is necessary for carbon production.
5.2. Discussion

Enforcing the requirement that other universes produce sufficient inventories of carbon is subject to a number of complications: Although increasing the triple alpha resonance energy leads to lower carbon yields, as generally claimed, the allowed change $\Delta E_R$ can be as large as $+500$ keV. On the other hand, if the resonance energy is lower, then stars produce more carbon than in our universe, where this finding is consistent with previous results [24, 25, 26]. In this regime, the increased carbon abundances are accompanied by decreased oxygen abundances, so that a smaller range of $\Delta E_R$ remains viable if the [C/O] ratio is required to be of order unity [9, 14, 17]. Universes produce nearly equal abundances for $\Delta E_R \sim 0$, and also for $\Delta E_R \sim +200$ keV, although the latter regime has lower absolute abundances (see Figure 6). On another front, the changes to nuclear physics required to compromise the triple alpha reaction are much smaller than the changes required for $^8\text{Be}$ to become stable, which would allow stars to produce carbon through a different set of nuclear reactions. Yet another possible pathway for carbon synthesis arises if the excited states of the carbon nucleus are altered so that a different resonance is active. Finally, one should keep in mind that the carbon abundance of Earth, the only place in the universe where life is known to exist, is depleted by a factor of $\sim 100$ relative to cosmic and solar abundances. These generalizations enlarge the parameter space for viable universes.

The main result of this study is that massive stars can provide universes with significant amounts of carbon over a wider range of parameter space than is often considered viable. This finding adds to a growing body of work showing that stars and stellar evolution are relatively robust to changes in the fundamental parameters of physics and astrophysics. Stars exist as stable nuclear burning entities while the fine structure constant and the gravitational constant vary over many orders of magnitude [66, 67]. Similarly, the coefficients that set nuclear reaction rates can vary over even wider ranges [66] and can allow stellar evolution to proceed in universes where diprotons are stable [68]. On a related note, stars can also function in universes where deuterium is unstable through the combined action of a triple nucleon reaction, the CNO cycle, explosive nucleosynthesis, and power generation by gravitational contraction [69] (cf. [70]). Stars can even operate in universes without the weak interaction [71], where deuterium burning largely replaces hydrogen burning as the principle nuclear reaction for stellar energy generation [72], or in universes where the weak interaction is stronger [73]. Taken together, these results suggest that stars and stellar evolution are not the limiting factor for universes to remain viable as the fundamental constants are changed.

In addition to the stellar constraints of this paper, universes are limited by additional considerations, including nuclear stability and atomic structure. As one example, the Higgs vacuum expectation value $\mathcal{V}$ must fall in the range $0.39 <$
$V/V_0 < 1.64$, where $V_0$ is the observed value and the quark Yukawa couplings are held constant [74]. Hydrogen becomes unstable (through the reaction $p + e^- \rightarrow n + \nu$) if the expectation value $V$ is too small, whereas nuclear binding is compromised if $V$ is too large. For a full assessment of fine-tuning, the relevant question is the overall size of the life-permitting region of parameter space, which is beyond the scope of this present discussion (e.g., see also [9, 10, 11, 12, 14, 15]).

This paper extends our understanding of universes with varying values of the triple alpha resonance energy, but a great deal of work remains to be done. The simulations presented here show the requirements for stars with $M_* = 15 - 40 M_\odot$ to produce carbon. This mass range can be extended to include both more massive and less massive stars, especially intermediate mass stars with $M_* \approx 2 - 10 M_\odot$. Inclusion of an extended stellar mass distribution could allow for carbon production over an even wider range of $\Delta E_R$ than presented herein. In particular, the latter mass range includes progenitors that become Asymptotic Giant Branch stars, which provide significant contributions to the carbon inventory in our universe [75]. Nucleosynthesis should also be studied in other astrophysical environments, including supernova blast waves, white dwarf explosions, and neutron star mergers. Carbon survives in our universe because it is not destroyed through the reaction $^{12}\text{C}(\alpha, \gamma)^{16}\text{O}$, which is non-resonant. Just as the excited states of the carbon nucleus could be different in other universes, those of the oxygen nucleus could also vary. Future work should consider oxygen resonances and the ramifications for the loss of carbon.

Another important issue is to improve our understanding of the relationship between the fundamental parameters, those that appear in the Standard Model of particle physics, and variations in nuclear properties. As the energy of the triple alpha resonance changes, the reaction rates and binding energies must also vary at some level. A self-consistent treatment remains to be carried out. On a more global scale, the question of whether or not the universe is fine-tuned is subject to a number of uncertainties. In spite of the extensive literature on this topic, we still do not have a detailed model of the multiverse, with a definitive determination of which fundamental parameters actually vary, how such variations are coupled or correlated, and a determination of the possible ranges of parameters that allow for viable universes.

Although the consideration of parameter variations in other universes is necessarily counterfactual, calculations of this type are useful in many ways. In addition to evaluating the degree of fine-tuning required for a universe to be habitable, another important goal is to increase our understanding of how the universe works. For example, this paper shows that variations in the triple alpha process lead to different abundances of carbon and oxygen, but do not greatly change the overall trajectory of stellar evolution. Massive stars always start their lives with configu-
rations capable of burning hydrogen, then produce increasingly larger nuclei until they develop degenerate iron cores, and end their lives with supernova explosions. Most of the time is spent during the production of $^4\text{He}$, while the total amount of energy generated is determined by the binding energy of $^{56}\text{Fe}$. The inventory of intermediate elements produced along the way, while vital to biology, is largely incidental to stellar operations. This perspective is useful for understanding stars in our universe, whether or not variations in the fundamental parameters are realized in other regions of space-time.

**Acknowledgments:** We would like to thank Juliette Becker, George Fuller, Alex Howe, Mark Paris, and Frank Timmes for useful discussions. We also thank an anonymous referee for many comments that improved the manuscript. The computational resources used for this project were provided by Advanced Research Computing–Technology Services (ARC-TS) at the University of Michigan, where we used the FLUX high-performance computing cluster. This work was supported by the University of Michigan and in part by the John Templeton Foundation through Grant ID55112 *Astrophysical Structures in Other Universes*. EG acknowledges additional support from the National Science Foundation, Grant PHY-1630782, and the Heising-Simons Foundation, Grant 2017-228.

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