Aspect ratio dependence of charge transport in turbulent electroconvection

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We present measurements of the normalized charge transport or Nusselt number $Nu$ as a function of the aspect ratio $\Gamma$ for turbulent convection in an electrically driven film. In analogy with turbulent Rayleigh-Bénard convection, we develop the relevant theoretical framework in which we discuss the local-power-law-scaling of $Nu$ with a dimensionless electrical forcing parameter $\mathcal{R}$. For these experiments where $10^4 \lesssim \mathcal{R} \lesssim 2 \times 10^5$ we find that $Nu \sim F(\Gamma)/\mathcal{R}$ with $\gamma = 0.26 \pm 0.02$ or $0.20 \pm 0.03$, in excellent agreement with the theoretical predictions of $\gamma = 1/4$ and $1/5$. Our measurements of the aspect ratio-dependence of $Nu$ for $0.3 \leq \Gamma \leq 17$ compares favorably with the function $F(\Gamma)$ from the scaling theory.

Heat transport has for many years been the cornerstone in the study of turbulent Rayleigh-Bénard convection (RBC) [1]. Over the past three years there has been remarkable experimental [2, 3, 4, 5, 6, 7, 8] and theoretical [9, 10, 11] progress in characterizing the properties and mechanisms of heat transfer in fluids heated from below. So far, experiments and theory are relevant only to approximately unit aspect ratio systems i.e. geometries that have comparable lateral and vertical dimensions. Turbulent convection flows in nature, however, are laterally extended and the transport of heat across a layer of fluid has been inadequately studied as a function of the aspect ratio $\Gamma = \text{lateral dimension/vertical dimension}$. In particular, experiments at $\Gamma > 2$ have seldom been performed primarily because it is extremely difficult to achieve the strong forcing that is readily reached in $\Gamma \approx 1$ containers.

In this Letter, we present a study of an analogous electrically driven convecting system which allows a broad range of $\Gamma$ to be explored at moderate levels of forcing. We show that measurements of the normalized charge transport $Nu$ varies with the aspect ratio $\Gamma$ consistent with a function $F$ given by the scaling theory. The strong dependence on $\Gamma$ especially for $\Gamma \approx 1$ implies that experiment and theories relevant to this regime are restrictive. The data and the function $F$ become increasingly independent of $\Gamma$ for $\Gamma \gg 1$. This observation highlights the importance of developing experiments and theory for laterally extended systems where it appears that universal behavior, often lacking in $\Gamma \approx 1$ systems, may be restored [12]. Our data also reveal that $Nu \sim \mathcal{R}^{-\gamma}$ with $\gamma = 0.26$ and $0.20$ in agreement with a local-power-law-scaling theory developed in a manner identical to the Grossmann-Lohse (GL) model for turbulent RBC [9, 10].

In previous work, it has been demonstrated that an electrically conducting, freely suspended liquid crystal film between parallel wires could be driven to convect when a sufficiently large potential drop was applied across its edges [13, 14, 15, 16, 17]. Experiments [17, 19, 21] and theory [18, 19] were then extended to the naturally periodic geometry of an annular film shown schematically in Fig. 1a. Analogous to RBC where an inverted mass density distribution is unstable to buoyant forcing, the thin film has an inverted surface charge density distribution that is unstable to electrical forcing. In this system, an initial bifurcation to convection rolls occurs when the applied voltage $V_o$ exceeds a critical voltage $V_c$, corresponding to the critical temperature difference in RBC. Secondary bifurcations occur at higher forcing and result in changing the number of counter-rotating vortex pairs that are arranged around the annulus. At much higher forcing the fluid becomes turbulent but retains the large scale structure of the rolls or convection cells.

Electroconvecting smectic films have several advantages and disadvantages relative to traditional RBC experiments. In turbulent RBC experiments, the accurate measurement of heat transport requires detailed accounting of the heat conducted through the sidewalls [3]. The annular geometry of the smectic film (see Fig. 1a) is free of lateral boundaries and consequently without sidewall losses, which facilitates precise measurement of the charge transport between the inner and outer electrodes. The film is an annular disk of width $d = r_o - r_i \sim 5 \text{ mm}$ and thickness $s \sim 0.1 \mu \text{m}$. Here $r_i$ ($r_o$) are the radii of the inner (outer) electrodes that support the film (see Fig. 1b). These radii can be varied so as to achieve different values of $\Gamma$. The tiny size of the film, which contains many orders of magnitude less working fluid than required for an RBC experiment, results in data acqui-

![Fig. 1](image-url)
sition timescales of minutes, rather than days. On the other hand, the smectic film is delicate and while many films survive vigorous forcing without thickness change, some do suffer sudden thickness variations. Since $V_c$ and other properties of the flows depend on thickness, these data must be discarded. The dc electrical forcing also results in conductivity drifts in the films, which result in systematic uncertainties. The experiments we describe here were carried out at atmospheric pressure, which increases the film stability. But air drag, which is known to produce a quantitative but not qualitative change to the dissipation, is not properly accounted for in the theory. Nevertheless, notwithstanding these caveats, we are able to efficiently explore the turbulent scaling regime over a broad range of $\Gamma$, complementing RBC experiments.

Our experiment consists of a temperature controlled film of octylcyanobiphenyl (8CB), a smectic-A liquid crystal, suspended between two concentric gold-plated electrodes. In the smectic-A phase, the film flows as an isotropic, incompressible and Newtonian fluid in the plane of the film. The flow is strictly two-dimensional. The film is driven to electroconvect by a dc voltage applied to its inner edge while holding the outer edge at ground potential. The annular assembly is enclosed in a Faraday cage. An experiment consists of imposing a voltage $V$ to the inner electrode and measuring the current $I$ transported through the film with a sensitive electrometer. The applied voltage was varied between 0 and 1000 volts in a sequence of small incremental and decremental steps resulting in a current-voltage characteristic. Further details of the apparatus and procedure can be found in Refs. [17, 18, 19, 20, 21]. Figure 2 shows a representative current-voltage (IV) characteristic. For $V < V_c$, the fluid is quiescent with the current sustained by ohmic conduction. When $V > V_c$ the fluid is organized in counter-rotating vortices and additional charge is transferred by convection; this is seen by the increase in slope of the IV-characteristic. At higher voltages, the fluid becomes turbulent and the transition is marked by a sudden increase in the rms fluctuations of the current, as shown in the inset of Fig. 2.

A film is a 2D annular sheet. The film has thickness/width $s/d \sim 10^{-4}$. Geometrically, a film is described by its radius ratio $\alpha = r_i/r_o$ and the aspect ratio $\Gamma_r = \pi (r_i + r_o)/(r_o - r_i)$, which is the ratio of the mid-radius circumference to the film width. So defined, $\pi < \Gamma_r < \infty$. To make correspondence with the conventional aspect ratio $\Gamma$ for RBC, we define $\Gamma = \Gamma_r - \pi$ for the annulus. Two other dimensionless parameters describe the experimental system: $\mathcal{R} = e_0^2 V^2/\sigma^2 \eta s^2$ and $\mathcal{P} = \epsilon_0 \eta / \rho s d$. Here $\mathcal{R}$ is the control parameter and is a measure of the external electrical forcing and $\mathcal{P}$, the Prandtl-like parameter, is the ratio of the time scales of electrical and viscous dissipation processes in the film. In the above the fluid density, molecular viscosity, and conductivity are denoted by $\rho$, $\eta$, and $\sigma$, and $e_0$ is the permittivity of free space.

We normalize the measured total electric current $I$ by the portion due to conduction $I_{cond} = cV$ where $c$ is the ohmic conductance of the film obtained from the IV-characteristic when the film is not convection, i.e., when $V < V_c$. We define the normalized charge transport or Nusselt number $Nu = I/I_{cond}$. Figure 2 shows representative sets of $Nu$ vs $\mathcal{R}$ data at $\Gamma = 0.33 \pm 0.01$ and $6.60 \pm 0.02$. We find that $Nu \sim \mathcal{R}^\gamma$ for $10^{4} \leq \mathcal{R} \leq 2 \times 10^{5}$ with either $\gamma = 0.26 \pm 0.02$ or $\gamma = 0.20 \pm 0.03$, depending on the size of $\mathcal{P}$. The error bar is obtained from the variation in the best-fit value of $\gamma$ over 26 data sets for all four values of $\Gamma$.

We developed a theoretical model describing turbulent electroconvection by borrowing from turbulent RBC. The equations of motion that describe electroconvection consist of the incompressible Navier-Stokes equation supplemented with an electrical body force, the charge conservation equation that accounts for ohmic conduction and advection of charge and Maxwell’s equation that relates the surface charge density to the electric potential:

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla \frac{\mathcal{P}}{\rho} + \nabla^2 \mathbf{u} - \frac{2}{\rho} \nabla \psi,$$

(1)

$$\partial_t q + \mathbf{u} \cdot \nabla q = \sigma \nabla^2 \psi,$$

(2)

$$q = -2\epsilon_0 \partial_z \psi.$$

(3)

These equations are constrained by the no-slip and applied electric potential boundary conditions. In the above equations $\nabla$, $\mathbf{u}$, $p$ and $\nu$ are the 2D gradient, velocity, pressure and kinematic viscosity respectively. The electric potential $\psi$ is three dimensional and extends outside the film, where $(\nabla^2 + \partial_z^2)\psi = 0$. The Maxwell equation relates the perpendicular gradient of $\psi$ to the surface
charge $q$. The factor of 2 arises from the film’s two free surfaces. The relation between $v$ and $q$ is thus nonlocal and somewhat complex. It can be considerably simplified by making a local approximation, setting $q = 2\beta \psi$, where $\beta$ is a constant. This local approximation has been shown to be adequate in describing the onset of electroconvection. See Ref. [13] for a detailed discussion of the theoretical model.

The Eqsns. 4 are similar to the Boussinesq equations for turbulent RBC. In particular, in the local approximation, there is an almost precise correspondence between the temperature and the electric potential. The periodical of convection cells that tile the annulus as shown in Figure 3 are in reasonable agreement with the theoretical predictions from the GL model, if we suppose that we traverse the appropriate regimes as $\Gamma$ varies. A detailed validation of the theory would depend crucially on either directly demonstrating the correctness of the assumptions about the dominant contributions to the dissipation or on systematically showing that the right local power-law scalings are indeed obtained as one varies $P$ and $R$. Our experiments currently span only relatively low $R < 2 \times 10^3$ and sparsely cover the rather wide range $5 < P < 250$. Many more experiments will be needed to adequately test the GL theory for turbulent electroconvection.

In the theoretical treatment described above, we have taken into account the dependence on the aspect ratio of the system, a consideration that was missing in the GL theory which treated only $\Gamma \approx 1$ systems [9, 10, 11]. Following Ref. [2], we choose this normalization so that $F(\Gamma) \approx 0$ for $\Gamma = 1$. Then $F(\Gamma = 2) \approx 8$ and $F(\Gamma = 3) \approx 60$. The function $kF(\Gamma)$ decreases monotonically with $\Gamma$ with the greatest variation for $\Gamma = 2, 3$ and is within 2% of the finite annular geometry. We can make a direct comparison with previous turbulent RBC experiments by making a correspondence between the different cell geometries, using $\Gamma = \Gamma_r - \pi$ as described above. We plot $kF(\Gamma)$ vs. $\Gamma$, with the constant $k = \pi / ((\pi + 1) \ln(2\pi + 1)) = 0.382$. Following Ref. [2], we choose this normalization so that $kF(\Gamma = 1) = 1$. Then $kF \to 0.764$ in the $\Gamma \to \infty$ limit.

Our experimental data span the range $0.3 \leq \Gamma \leq 17$. From power-law fits to $\gamma = B$ vs. $\Gamma$ we have extracted the exponents $\gamma$. To determine $F(\Gamma)$ we then divide $B$ by $\Gamma^{\gamma + \delta}$. Because our data extend over a rather wide range of $\Gamma$, we expect to cross regimes with differing $\gamma$ and $\delta$. To extract the aspect ratio dependence alone, we used the fitted value of $\gamma$ and where $\gamma \approx 0.25 (0.20)$, we used the GL-theory prediction of $\delta = 1/2 (1/4)$. Using one free parameter for all the data,
we again scaled these results so that $F(\Gamma = 1) = 1$. Our data for 4 different $\Gamma$ are in reasonable quantitative agreement with the theoretical function $F$, as shown in Fig. 4. Each data point is an average over 4–10 runs. The error estimates are representative of the scatter in $F$ over these data. In particular, the largest contribution to the error at $\Gamma = 16$ is systematic and arises from the ambiguity in the correct exponent for $P$.

Data from several turbulent RBC experiments \cite{2, 3, 4, 5} for 4 values of $\Gamma$ are also broadly in agreement with the function $F$, in spite of the difference in geometry and the higher range of Rayleigh numbers. Earlier RBC experiments \cite{6, 7} used gases as the working fluid and deviate significantly from $F$.

Several important questions are raised by our results on the aspect ratio dependence of charge transport in turbulent electroconvection and, by extension, that of heat transport in turbulent RBC. The strong dependence of $F$ on $\Gamma$ for $0 \leq \Gamma \lesssim 7$, suggests that the dynamics of turbulent convection is dependent on the system's lateral extent. That the charge or heat transport, a global property, is sensitive to the aspect ratio further suggests that local properties such as temperature and charge fluctuations may be even more strongly dependent on $\Gamma$. Experiments in turbulent RBC clearly show that while heat transport is insensitive to cell geometry at fixed $\Gamma \approx 1$, the fluctuations are strongly affected by the shape of the lateral boundary \cite{12}. The relative independence of $F$ on $\Gamma$ for large aspect ratio, $\Gamma \gtrsim 7$ suggests that the normalized charge and heat transport approach a universal value in laterally extended systems. We conjecture that fluctuations in the interior may also become universal in large aspect ratio systems.

Our results emphasize the importance of extending convective turbulence experiments to larger aspect ratios. For turbulent smectic electroconvection, we would like to make a systematic study of the scaling of $Nu$ in the parameter space of $R$ and $P$. $P$ can be varied in principle by changing the thickness and/or width of the film. We would also like to study long time series of $Nu$ in order to infer the characteristic size and duration of the fluctuations. It is a challenge to extend the forcing parameter $R$ to values much beyond $10^6$, since this requires applying large voltages across the film that may result in dielectric breakdown. A first step would be to better control the electrochemistry of the liquid crystal film to reduce drifts and push down the critical voltage $V_c$. It would also be interesting to extend our previous studies of electroconvection under shear \cite{14, 15, 16, 17} to the turbulent regime. Finally, we would like to develop local probes to study the fluctuations of the velocity and electric potential over the film.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.png}
\caption{The function $kF$ vs $\Gamma$. $kF(\Gamma)$ is normalized to unity at $\Gamma = 1$. The solid circles (●) are from the present work. Data from RBC experiments in Acetone with (◇) and without (□) sidewall correction are from Refs. 2 and 3. Also shown are data in Air (△), from Ref. 24. Helium (▽) from Ref. 25 and (○) from Refs. 4, 5 and water (○) from Refs. 26. The inset shows the same data on a logarithmic scale.}
\end{figure}

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