Rapid temporal variability has been widely observed in the light curves of gamma-ray bursts (GRBs). One possible mechanism for such variability is related to the relativistic eddies in the jet. In this paper, we include the contribution of the inter-eddy medium together with the eddies to the gamma-ray emission. We show that the gamma-ray emission can either lead or lag behind the observed synchrotron emission, where the latter originates in the inter-eddy medium and provides most of the seed photons for producing gamma-ray emission through inverse Compton scattering. As a consequence, we argue that the lead/lag found in non-stationary short-lived light curves may not reveal the intrinsic lead/lag of different emission components. In addition, our results may explain the lead of gamma-ray emission with respect to optical emission observed in GRB 080319B.

**Key words:** gamma-ray burst: general – radiation mechanisms: non-thermal – turbulence

1. INTRODUCTION

Gamma-ray bursts (GRBs) are the most extreme explosive events in the universe. They are known to be highly variable and their temporal structure exhibits diverse morphologies (Fishman & Meegan 1995), which can vary from a single smooth large pulse to extremely complex light curves with many erratic short pulses. It is believed that such kinds of variability, especially fast variability, may provide an interesting clue to the nature of GRBs (e.g., Morsony et al. 2010).

The millisecond variabilities observed in the prompt phase have led to the development of the internal shock model (Rees & Mészáros 1994). In this model, an ultrarelativistic jet is ejected with a fluctuating velocity profile. When a fast, late-ejected portion of the jet catches up with a slow, early-ejected one, a pair of internal shocks is formed. Each pulse in the light curve of bursts corresponds to one such collision (Kobayashi et al. 1997; Maxham & Zhang 2009), accounting for the many erratic short pulses. It was found that only a small fraction of the total kinetic energy can be dissipated in this process (Kobayashi et al. 1997; Daigne & Mochkovitch 1998). However, a detailed study suggests that the radiative efficiency of some GRBs can be up to 90% (Zhang et al. 2007), which is difficult to reproduce within the straightforward internal shock model.

In the internal shock model, the variability of prompt emission is attributed to the history of central engine activity. However, the observed variability may originate in the emission region. This scenario requires that the emission region not be uniform. The external shocks, which are formed while the outflow is slowed down by extremely small circumburst clumps, can produce highly variable light curves (e.g., Dermer & Mitman 1999). However, this process is inefficient (Sari & Piran 1997; see Narayan & Kumar 2009 for details). Alternatively, Lorentz-boosted emission units in the jet, such as mini-jets (Lytikov & Blandford 2003; Giannios et al. 2009) or relativistic turbulent eddies (Narayan & Kumar 2009), can also produce strong variability of gamma-ray emission. According to this scenario and considering gamma-ray emission only from eddies, Lazar et al. (2009) reproduced the fast variability observed in a sub-sample of GRBs. However, the amplitude of the subjacent smooth component in the simulated light curve is too low compared with observations (e.g., Figure 2 of Lu et al. 2012), where the light curve is roughly divided into two components: a smooth component underneath the light curve and a rapidly varying component that is superimposed on the smooth component (e.g., Zhang & Yan 2011; Gao et al. 2012; Dichiara et al. 2013 and references therein). It should be noted that the light curves in a realistic situation are more complex; the relativistic turbulence model for fast variability is furthermore only applicable to a sub-sample of GRBs, such as GRBs with erratic symmetric short pulses (Lazar et al. 2009). On the other hand, Kumar & Narayan (2009) showed that the inter-eddy medium dominates the observed synchrotron emission and contributes significantly to the gamma-ray emission, which is produced by inverse Compton (IC) scattering of synchrotron photons from eddies and inter-eddies. The purpose of this work is to revisit the relativistic turbulence model by including the role of the inter-eddy medium in producing the light curve.

This paper is organized as follows. The relativistic turbulence and the kinematic toy model for jet radiation are described in Section 2. The simulated light curves for different conditions are presented in Section 3, in which the lead/lag of gamma-ray emission with respect to the observed synchrotron emission is our main focus. Our conclusions and discussion are presented in Section 4.

2. RELATIVISTIC TURBULENCE MODEL

The relativistic turbulence model is well described in the work of Narayan & Kumar (2009), Lazar et al. (2009), and Kumar & Narayan (2009). In this section, we present a brief description of this model. We refer the reader to the above papers for additional details.

We first introduce three frames related to the relativistic turbulence model: the lab frame; the shell’s frame, denoted by a prime, which is boosted radially with a Lorentz factor $\Gamma$ relative to the lab frame; and the frame of an eddy, denoted by two primes, which is boosted by $\gamma$ relative to the shell frame. In the relativistic turbulence model, the fluid in the jet consists of
eddies and an inter-eddy medium. The eddies are considered to have a typical Lorentz factor $\gamma'_e$ in the shell frame and a typical size $l'_e \sim R/(\gamma'_e \Gamma)$ in their respective frame, where $R$ is the distance between the emission region and the jet base. The filling factor of eddies in the jet is described by the parameter $f$. Around $f \gamma'_e^3$, eddies can be found in a causally connected volume ($\sim R^3/\Gamma^3$). Owing to collisions with other eddies, an eddy is not likely to travel along a perfectly straight line. In order to describe this behavior, $\tau' = R/(\gamma' \Gamma c)$ is introduced, which corresponds to the time required for an eddy to change its velocity orientation by an angle of $1/\gamma'$. The inter-eddy medium can be discretized into “inter-eddies,” which have the same size and are associated with a Lorentz factor $\gamma'_{it} = 1$ in the shell frame.

A kinematic toy model for jet radiation considers a shell that is divided into discrete randomly distributed emitters (eddies or inter-eddies); the emitters radiate as the shell moves from $R_0$ to $2R_0$. During this period, the directions and positions of eddies continuously change. In order to model this dynamic process, a set of successive shells between $R_0$ and $2R_0$ are introduced. Each new shell is constructed with randomly distributed emitters, representing the change in direction and position of the eddies. The time difference between two shells is $\tau = \tau T$ and the thickness $\Delta$ of shells is described with a parameter $d(\geq 1)$, i.e., $\Delta = dR/\Gamma T$. The thickness due to the intrinsic expansion of a shell is $\Delta \sim R/\Gamma^2$, i.e., $d = 1$. Then, the situation in which $d > 1$ reveals that the width of a shell is determined by the duration of the central engine activity. In the present work, we study the case of $d = 1$ and the emitters are described by their center positions rather than by their filling regions. The Doppler shift from an emitter is

$$\Lambda = \left[ \gamma \left( 1 - \frac{\gamma' c}{c} \cos \alpha \right) \right]^{-1},$$  

where $\gamma$, $\gamma' = c\sqrt{1 - 1/\gamma'^2}$, and $\alpha$ are the Lorentz factor, velocity, and the angle between the emitter velocity and the line to the observer (both in the lab frame), respectively. If the emitter velocity $\vec{v}'$ is in a direction with a polar angle $\theta'$ and an azimuthal angle $\phi'$ relative to the radial direction in the shell frame, the emitter would move with a Lorentz factor

$$\gamma = \gamma' \left( 1 + \frac{\gamma' v' c \cos \theta'}{c^2} \right)^{1/2},$$

in the lab frame. The polar angle $\theta$ and the azimuthal angle $\phi$ relative to the radial direction in the lab frame satisfy

$$\tan \theta = \frac{\gamma' \sin \theta'}{\Gamma (\gamma'_j + \gamma' \cos \theta')},$$

$$\phi = \phi',$$

where $\gamma'_j$ is the velocity of jet. Then, it is easy to find the relationship

$$\cos \alpha = -\sin \theta \cos \phi \sin \theta_j + \cos \theta \cos \theta_j,$$

where $\theta_j$ is the latitude of the emitter in the shell observed in the lab frame.

The radiation mechanism for gamma-ray emission observed in the prompt phase is the IC scattering of synchrotron photons from eddies and inter-eddies. Since the seed photon field, i.e., synchrotron photons from eddies and inter-eddies, is the same for IC scattering of eddies and inter-eddies, the observed peak frequency of the IC spectrum from eddies should be close to that from inter-eddies based on the relationship $\gamma''_e \gamma'_e = \gamma'_e$. Here, $\gamma''_e$ ($\gamma'_e$) is the thermal Lorentz factor of electrons in the eddies (inter-eddies) and the seed photon field is roughly isotropic and homogeneous in the shell’s frame. Therefore, the gamma-ray emission seen by an observer from an eddy or an inter-eddy can be described as (see Equations (52) and (53) in Kumar & Narayan 2009)

$$F_{\text{IC}} \propto \sigma_T (n_e \gamma'^3_0) f'_\text{syn} \Lambda^3,$$

where $f'_\text{syn}$ is the synchrotron flux as seen by a typical electron in the inter-eddy medium and $n_e$ is the number of electrons in an emitter. For the relativistic turbulence model, the parameters $n_{e,1}$, $n_{e,\text{it}}$, and $n_{a}$ should satisfy the relationship $n_{e,1}(1 - f) \gamma'^3_1 = n_{e,\text{it}}$ ($n_{e,a}$) is the number of electrons in an eddy (inter-eddy) and $n_{a}$ is the number of inter-eddies in the inter-eddy medium. In Equation (6), we adopt the assumption in Kumar & Narayan (2009) that at a fixed observer time, the observer receives radiation from only a fraction ($\sim \gamma'/\gamma''_e$) of the electrons in the eddy owing to the time dependence of the eddy velocity direction. Following the spirit of Lazar et al. (2009), we assume $f'_\text{syn} \propto 1/(\gamma' \gamma''_e)$ and $\alpha = 1$. We also examine the light curves of gamma-ray emission with other values of $\alpha$, which has a negligible effect on the profile of the light curves and on the lead/lag between the gamma-ray emission and the observed synchrotron emission. For simplicity, the pulse produced by a single eddy is assumed to have a Gaussian profile (e.g., Lazar et al. 2009; Maxham & Zhang 2009):

$$F(t) = F_{\text{IC}} \exp \left[ -\frac{(t - t_p)^2}{2(\delta t)^2} \right],$$

where $t_p$ is the time of peak flux and $\delta t$ takes the form (Lazar et al. 2009)

$$\delta t \approx R \psi / (\gamma' \Gamma c).$$

Norris et al. (1996) showed that pulses in some GRBs rise more quickly than they decay. This result is different from the short pulses produced by the eddies, which may be statistically symmetric (Lazar et al. 2009). However, the causes of variability in GRB light curves may be diverse (Gao et al. 2012) and therefore the shape of pulses may be different for different GRBs. Evidence, i.e., symmetric short pulses, is presented in the Appendix of Norris et al. (1996). It should also be noted that the IC radiation mechanism discussed above typically predicts $R_{\text{IC}} \gtrsim 1$ (Kumar & Narayan 2009; Beniamini et al. 2011; Guetta et al. 2011), where $R_{\text{IC}}$ is the ratio of the fluence in the second-order IC component to the fluence in the first-order IC component (i.e., the gamma-ray emission discussed in the present work). However, the observations of most GRBs do not support this behavior (Beniamini et al. 2011; Guetta et al. 2011; Ackermann et al. 2012; Ackermann et al. 2013). Overestimating the value of $R_{\text{IC}}$ may be an issue in the relativistic turbulence model at the current stage.

The light curves of gamma-ray emission discussed below are produced based on the kinematic toy model of jet radiation and Equations (6)–(8). In the simulations, the inter-eddies are uniformly distributed in the jet shell and a significantly large value of $n_a$ is chosen in order to produce a smooth light curve of gamma-ray emission from the inter-eddies. According to the relativistic turbulence model, the parameters $\gamma'_e$ and $f$,
and the distribution of eddy orientation will determine the main properties of the light curves. Since the typical value of \( V \equiv \beta_{\text{pram}}/\beta_{\text{var}} = \gamma_f^2 \) is around 100 (Narayan & Kumar 2009), we adopt \( \gamma_f' = 10 \) in the present work.

### 3. NUMERICAL SIMULATIONS OF THE LIGHT CURVES

We present simulated light curves considering the emission from inter-eddies in this section; we focus on the lead/lag of gamma-ray emission with respect to the observed synchrotron emission. In the realistic situations, the direction of eddies continuously changes with time and may be concentrated in some directions. Then, we model this behavior with different \( |\mu'| \) conditions, i.e., \( |\mu'| < a(\geq a) \), which is used in our simulations. Here, \( \mu' = \cos \theta' \), \( 0 \leq a \leq 1 \), and \( |\mu'| < a(\geq a) \) means that the value of \( \mu' \) for an arbitrary eddy varies with time and is randomly taken from \([-a, a] \) \( \cup \{a, 1\} \). As shown below, the different \( |\mu'| \) conditions may result in different lead/lag values of gamma-ray emission with respect to the observed synchrotron emission.

Figure 1 shows the simulated light curves of gamma-ray emission from the eddies (solid curve in the left panel), the inter-eddies (thick dashed line in the left panel), and both eddies and inter-eddies (right panel), with a filling factor \( f = 0.8 \).

![Figure 1. Simulated light curves of gamma-ray emission from eddies (solid curve in the left panel), inter-eddies (thick dashed line in the left panel), and both eddies and inter-eddies (right panel), with a filling factor \( f = 0.8 \).](image)

In order to suppress the fluctuations, we perform 40 simulations for each situation. Figure 2 shows that the contribution of the fast variability to the total gamma-ray emission is almost proportional to \( f \), i.e., the amplitude of the subjacent smooth component is large for low values of \( f \). Then, an appropriate value of \( f \) is required for producing a light curve that closely matches the observations.

Below, we focus on the lead/lag of gamma-ray emission with respect to the observed synchrotron emission, which is completely dominated by that from the inter-eddies (Kumar & Narayan 2009). In this work, we concentrate on the effects of gamma-ray emission from the eddies on the lead/lag. For simplicity, the lag of the gamma-ray emission from the inter-eddies with respect to the observed synchrotron emission is ignored. In other words, the light curve of the gamma-ray emission from the inter-eddies is used to represent that of the observed synchrotron emission. In addition, we point out that an additional lead/lag, which may appear when a light curve is plotted in a given energy range rather than over the total energy range of the gamma-ray emission (or synchrotron emission), is also neglected in this work. According to the above description, the lead/lag will depend on the value of \( f \), which describes the contribution of eddies to the gamma-ray emission. If \( f \leq 0.5 \), the gamma-ray emission is mainly from inter-eddies, and thus the absolute value of the lead/lag may be low. This behavior can be seen in Figure 3. In this figure, the circles correspond to the numerical results with \( |\mu'| \geq 0.8 \) and the squares correspond to the numerical results with \( |\mu'| \leq 1 \). The positive (negative) values of \( t_{\text{lag}} \Gamma^2 c/R_0 \) indicate that the gamma-ray emission leads (lags behind) the observed synchrotron emission. As shown in the figure, the absolute value of lead/lag between the gamma-ray emission and the synchrotron emission decreases as \( f \) approaches 0. We choose two cases, i.e., \( f = 0.6 \) and \( f = 0.8 \), to focus on for our study of the lead/lag behavior.

Figure 4 shows the lead/lag in different \( |\mu'| \) conditions. In this figure, the squares represent the numerical results with \( |\mu'| \geq a \) and the circles represent the numerical results with \( |\mu'| \leq a \). The empty symbols are for \( f = 0.8 \) and the filled symbols are for \( f = 0.6 \). As shown in the figure, the lead/lag for different \( |\mu'| \) conditions is quite different. The simulations with \( |\mu'| \geq a \) mainly produce a lead of gamma-ray emission with respect
to the observed synchrotron emission, whereas the simulations with $|\mu'| \leq a$ produce the opposite behavior. Obviously, the lead/lag in the simulations is related to the distribution of the eddy orientation rather than the radiation mechanism. We therefore argue that the lead/lag found in non-stationary, short-lived light curves may not reveal the intrinsic lead/lag of different emission components.

The physical reason for the lead/lag is related to the difference between the peak time of gamma-ray emission from the eddies and that from the inter-eddies. Figure 5 shows two examples of gamma-ray emission light curves illustrating the lead/lag behavior with $f = 0.8$, where the solid curves correspond to the gamma-ray emission from eddies and the dashed lines correspond to that from inter-eddies. In this figure, the left panel is for $|\mu'| = 1$, which is a typical example for the situation $|\mu'| \geq a$. The right panel is for $\mu' = 0$, which is a typical example for the situation $|\mu'| \leq a$. For $|\mu'| = 1$, the velocity of eddies is in the radial direction. Since the gamma-ray emission from eddies with $\mu' = 1$ is negligible compared with that from $\mu' = 1$, we use the situation of $\mu' = 1$ to represent that of $|\mu'| = 1$. In this case, the gamma-ray emission of eddies can be viewed as the emission from a jet with a Lorentz factor of $2\sqrt{\gamma'}$, according to Equation (2). In addition, the angular spreading time, which affects the peak time of the gamma-ray emission, is inversely proportional to the square of the jet Lorentz factor. Then, the gamma-ray emission from eddies will reach its peak luminosity ahead of the emission from the inter-eddies, owing to the fact that the Lorentz factor of the jet is less than that of the eddies. This behavior is clearly shown in the left panel. The corresponding result is that the peak time of the total gamma-ray emission is ahead of the peak time of the gamma-ray emission from inter-eddies, and therefore ahead of the peak time of the synchrotron emission. On the other hand, for $\mu' = 0$, the polar angle $\theta$ of eddies is $\sim 1/\Gamma'$ according to Equation (3). In this situation, eddies at high latitudes ($\theta_i > 1/\Gamma$) in the jet will contribute more to the gamma-ray emission than eddies at low latitudes. The corresponding result is that the peak time of the gamma-ray emission from eddies is behind that from the inter-eddies; therefore, the peak time of the total gamma-ray emission is behind that of the synchrotron emission. Thus, the simulation results in Figure 4, either for $|\mu'| \geq a$ or for $|\mu'| \leq a$, can be well understood.

4. DISCUSSION AND CONCLUSIONS

In this work, we have studied the light curves of GRBs in a relativistic turbulence model considering the role of inter-eddy emission. By ignoring the lag between gamma-ray emission from inter-eddies and the observed synchrotron emission, our numerical simulations for the light curves show that the gamma-ray emission can either lead or lag behind the observed synchrotron emission. The lead/lag is due to the different peak time of gamma-ray emission in different situations, which is related to the angular spreading time. We argue that the lead/lag found in non-stationary, short-lived light curves may not reveal the intrinsic lead/lag of different emission components.

For GRB 080319B, Figure 4 of Woźniak et al. (2009) implies $\zeta \sim 0.5$–0.7, which corresponds to a filling factor $f \sim 0.6$–0.8 according to our Figure 2. Since the duration of the main episode in this burst is $\Gamma^2c/R_0 \sim 28$ s (see Patricelli et al. 2012 and references therein), the lead/lag of gamma-ray emission with respect to the optical (synchrotron) emission is probably in the range of $[-7.3$ s, $3.7$ s] based on our Figure 4. On the other hand, Beskin et al. (2010) showed that the lead of gamma-ray
emission with respect to optical emission in this burst is around 2 s, which is well within the above range. Thus, such a lead may not rule out IC scattering as a radiation mechanism for producing the gamma-ray emission.

We thank the anonymous referee for helpful suggestions to improve the paper. We also thank Bing Zhang for helpful discussions. This work was supported by the National Basic Research Program (973 Program) of China under grant 2014CB845800, the National Natural Science Foundation of China under grants 11073015, 11103015, 11222328, 11233006, and 11025313, and the Guangxi Science Foundation under grant 2013GXNSFFA019001.

REFERENCES

Ackermann, M., Ajello, M., Asano, K., et al. 2013, arXiv:1303.2908
Ackermann, M., Ajello, M., Baldini, L., et al. 2012, ApJ, 754, 121
Beniamini, P., Guetta, D., Nakar, E., & Piran, T. 2011, MNRAS, 416, 3089
Beskin, G., Karpov, S., Bondar, S., et al. 2010, ApJL, 719, L10
Daigle, F., & Mochkovitch, R. 1998, MNRAS, 296, 275
Dermer, C. D., & Mitman, K. E. 1999, ApJL, 513, L5
Dichiara, S., Guidorzi, C., Amati, L., & Frontera, F. 2013, MNRAS, 431, 3608
Fishman, G. J., & Meegan, C. A. 1995, ARA&A, 33, 415
Gao, H., Zhang, B.-B., & Zhang, B. 2012, ApJ, 748, 134
Giannios, D., Uzdensky, D. A., & Begelman, M. C. 2009, MNRAS, 395, L29
Guetta, D., Pian, E., & Waxman, E. 2011, A&A, 525, A53
Kobayashi, S., Piran, T., & Sari, R. 1997, ApJ, 490, 92
Kumar, P., & Narayan, R. 2009, MNRAS, 395, 472
Lazar, A., Nakar, E., & Piran, T. 2009, ApJL, 695, L10
Lu, R.-J., Wei, J.-J., Liang, E.-W., et al. 2012, ApJ, 756, 112
Lyutikov, M., & Blandford, R. 2003, arXiv:astro-ph/0312347
Maxham, A., & Zhang, B. 2009, ApJ, 707, 1623
Morsony, B. J., Lazzati, D., & Begelman, M. C. 2010, ApJ, 723, 267
Narayan, R., & Kumar, P. 2009, MNRAS, 394, L117
Norris, J., Nemiroff, R. J., Bonnell, J. T., et al. 1996, ApJ, 459, 393
Patricelli, B., Bernardini, M. G., Bianco, C. L., et al. 2012, ApJ, 756, 16
Rees, M. J., & Mészáros, P. 1994, ApJL, 430, L93
Sari, R., & Piran, T. 1997, ApJ, 485, 270
Woźniak, P. R., Vestrand, W. T., Panaitescu, A. D., et al. 2009, ApJ, 691, 493
Zhang, B., Liang, E., Page, K. L., et al. 2007, ApJ, 655, 989
Zhang, B., & Yan, H. 2011, ApJ, 726, 90