Decoherence-Free Quantum Information Processing with Four-Photon Entangled States

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Quantum information processing enables secure classical communication, powerful quantum communication schemes, and speedup in computation \cite{1}. These methods rely on the preparation, manipulation, and detection of the superposition of quantum states. Superpositions, however, are very fragile and easily destroyed by the decoherence processes due to unwanted coupling with the environment \cite{2}. Such uncontrollable influences cause noise in the communication or errors in the outcome of a computation, and thus reduce the advantages of quantum information methods.

Several strategies have been devised to cope with decoherence, each of them appropriate for a specific type of coupling with the environment. For instance, if the interaction with the environment is weak enough such that qubits are affected only with a very low probability, a good strategy would be to add redundancy when encoding the quantum information in order to detect and correct the errors by active quantum error correction methods \cite{3}.

If the qubit-environment interaction, no matter how strong, exhibits some symmetry, then there exist quantum states which are invariant under this interaction. These states are called decoherence-free (DF) states, and allow protection of quantum information \cite{4,5,6,7,8}. A particularly relevant symmetry arises when the environment couples the qubits without distinguishing between them, resulting in the so-called collective noise. This situation occurs, for instance, when the spatial (temporal) separation between the carriers of the qubits is small relative to the correlation length (time) of the environment. Typical examples arise in ion-trap or nuclear magnetic resonance (NMR) experiments, which are susceptible to fluctuations of magnetic or electrostatic fields, but also in quantum communication, e.g., when the qubits carried by polarized photons are successively sent via the same optical fiber and therefore experience the same birefringence.

Experimental efforts investigating features of DF systems so far have been limited to two-qubit systems only. For two qubits, however, the singlet state is the only DF state and thus, it is not sufficient to fully protect an arbitrary logical qubit against collective noise. The experiments so far have demonstrated the features of the single DF state or the immunity against restrictive types of noise \cite{8}. For three qubits there is no DF state immune to collective noise. However, quantum information can be preserved in an abstract subsystem known as a noiseless subsystem, which was demonstrated in NMR experiments \cite{9}.

In this Letter we report on the production of various decoherence-free four-photon polarization-entangled states using spontaneous parametric down conversion (SPDC). The immunity of these states against collective noise is experimentally verified by showing their invariance when passing the four photons through a noisy environment simulated by birefringent media. Moreover, we show that one can both distinguish two orthogonal four-photon DF states and perform state tomography by local polarization measurements only. We are thus able to demonstrate that quantum information can be reliably extracted from qubits communicated through a noisy environment, and also between parties who do not even share a common reference frame.

For the construction of DF states formed by $N$ qubits, we note that for collective noise, all $N$ qubits undergo the same (unknown) unitary transformation $U$. States are decoherence-free if they are invariant under such an $N$-lateral unitary transformation, i.e., $U^\otimes N |\psi\rangle = |\psi\rangle$, where $U^\otimes N = U \otimes \cdots \otimes U$ denotes the tensor product of $N$ unitary transformations $U$ \cite{7}. The amount of quantum information that a given DF subspace is able to protect is determined by the number $N$ of qubits used \cite{8}. For $N = 2$ qubits there is only one DF state, the singlet state, $|\psi^-\rangle_{ab} = (1/\sqrt{2})(|01\rangle - |10\rangle)_{ab}$, where $|01\rangle_{ab} = |0\rangle_a \otimes |1\rangle_b$. The smallest useful DF subspace is spanned by two four-
subspace requiring non-trivial quantum gates. However, measurements: It suffices to project the first two qubits onto "1\( \Sigma \)\
\[ \equiv V \]

\[ \sigma_{x} \otimes \sigma_{x} \otimes \sigma_{x} \otimes \sigma_{x} \]

because they can be determined again by local measurements on the four qubits. Here, \( \{ \sigma_{x}, \sigma_{y}, \sigma_{z} \} \) denote the Pauli matrices, and \( \mathbf{1} \) is the identity. The results of these measurements allow us to perform the tomographic reconstruction of the density matrix \( \rho \), since its elements can be expressed as \( \rho_{zz} = (3\langle \Sigma_{z} \rangle + 1)/4 \), \( \rho_{xy} = \sqrt{3}/2 \langle \Sigma_{x} \rangle + \langle \Sigma_{y} \rangle - 1)/4 \), and \( \text{Im}(\rho_{zz}) = \sqrt{3}/2 \langle \Sigma_{y} \rangle /2 \), where \( \langle \Sigma_{i} \rangle = \text{Tr}(\rho \sigma_{i}) \) describes the expectation value of \( \Sigma_{i} \).

In our experiment the physical qubits are polarized photons, where the computational basis corresponds to horizontal and vertical linear polarization, "0\( \equiv H \) and "1\( \equiv V \). The four-photon polarization-entangled state \( |\Phi_{0}\rangle \) can be obtained from two synchronized (first order) SPDC sources for photon pairs in the singlet state \( |\psi^{\ominus}\rangle \).

More practically, for the measurements shown in Figs. 2 and 3, \( |\Phi_{0}\rangle \) was generated by using the product state of two polarization-entangled photon pairs created from two consecutive pump pulses and swapping the modes \( b \leftrightarrow c \). The four-photon polarization-entangled state \( |\Phi_{1}\rangle \) was observed using the second order SPDC process \[ |\Phi_{1}\rangle \]

We used the UV pulses of a frequency-doubled mode-locked Ti:sapphire laser (pulse length 130 fs) to pump a 2 mm thick BBO (barium betaborate) crystal at a wavelength of 390 nm and a repetition rate of 82 MHz with an average power of 750 mW (see Fig. 1). The pump beam was focused to a waist of 100 \( \mu \)m inside the crystal. The degenerate down-conversion emission into the two characteristic type-II crossing directions \( |\Phi_{L}\rangle \) was coupled into single mode optical fibers and passed through narrowband interference filters (\( \Delta \lambda = 3 \) nm) to exactly define the spatial and spectral emission modes. To observe \( |\Phi_{0}\rangle \) and \( |\Phi_{1}\rangle \), two polarization-independent 50:50 beam splitters were used to split the four photons into the four modes \( a, b, c, \) and \( d \). Next, the photons were sent through the quantum channel, where the noisy environment was simulated by a combination of birefringent quarter- (QWP) and half- (HWP) wave plates in each arm. The polarization analysis was performed using fur-

FIG. 1: Experimental setup to show the invariance of four-photon entangled states under collective noise. The photons are emitted from spontaneous parametric down-conversion in a BBO crystal followed by birefringence compensation into two spatial modes \( a_{0} \) and \( b_{0} \). They are distributed into the four modes \( a, b, c, d \) by 50:50 beam splitters (BS) behind interference filters (F). The noisy quantum channel causing the unitary transformation \( U^{\oplus 4} = U \otimes U \otimes U \otimes U \) is simulated by equal combinations of quarter- (QWP) and half-wave plates (HWP). Additional waveplates and polarizing beam splitters (PBS) are employed for the polarization analysis of the four photons. For the registration of the decoherence-free states, events are selected where one photon was detected by single-photon avalanche diodes (SPAD) in each of the four modes.
ther waveplates and polarizing beam splitters followed by silicon avalanche single photon detectors. Only events with one photon detected in each of the four arms have been selected.

Figure 2 shows the 16 possible fourfold coincidences for polarization analysis of one photon in each of the four outputs of the beam splitters exhibiting the characteristic statistics of the states \(|\Phi_0\rangle\) (A) and \(|\Phi_1\rangle\) (B). As a measure of the quality of the state preparation we use the quantum bit error rate (QBER), which is defined as the ratio of false events over total events or, in terms of the four-photon visibility \(V\), as \(QBER = (1 - V)/2\). For the data shown, we obtain \(QBER = 3.91\% \pm 0.44\%\) (A) and \(QBER = 4.30\% \pm 0.25\%\) (B). The ratio of total events observed upon encoding \(|\Phi_1\rangle\) and \(|\Phi_0\rangle\) is expected to be 3 for otherwise similar pumping conditions. Within the variation in the observed four-photon rate, this ratio is also reflected in the experiment.

To demonstrate the invariance of the four-photon states \(|\Phi_0\rangle\) and \(|\Phi_1\rangle\) under collective decoherence, i.e., under phase and bit flip errors caused by a birefringent quantum channel, we have arbitrarily chosen the unitary transformation \(U = 0.012i\mathbb{1} - 0.332\sigma_z - 0.707\sigma_y + 0.624\sigma_x\). We implement this unitary transformation here by the addition of a HWP set at an angle of 59° and a QWP at an angle of 13.5°.

Figures 2(C) and 2(D) show that the distribution of detection events is not changed for the states \(|\Phi_0\rangle\) and \(|\Phi_1\rangle\) under the unitary transformation \(U^\otimes 4\) (i.e., when the four photons are subject to collective noise). Here, we obtain similar error rates, with \(QBER = 7.11\% \pm 0.50\%\) (C), and \(QBER = 6.41\% \pm 0.28\%\) (D) for \(|\Phi_0\rangle\) and \(|\Phi_1\rangle\), respectively. From these measurements one can deduce the diagonal elements of the four-photon density matrices. Obviously, no additional elements are populated under the action of the collective noise indicating that the states \(|\Phi_0\rangle\) and \(|\Phi_1\rangle\) do not leave the DF subspace.

To distinguish between the states \(|\Phi_0\rangle\) and \(|\Phi_1\rangle\) by local measurements, we have projected the photon polarizations in paths \(a\) and \(b\) on the \(\{H, V\}\) basis, and in paths \(c\) and \(d\) on the \(\{+, -\}\) basis, i.e., \(\pm 45°\) polarization basis. Figures 3(A) and 3(B) show the fourfold coincidence counts corresponding to a detection of \(|\Phi_0\rangle\) and \(|\Phi_1\rangle\), respectively, for a noiseless environment. For \(|\Phi_1\rangle\), we clearly observe that the false fourfold coincidence counts, i.e., the terms \(HV+, HV-, VH+, VΗ-\), and \(VΗ-+\), are negligible compared to the other terms, and vice versa for \(|\Phi_0\rangle\). We observe an error rate of \(QBER = 5.23\% \pm 0.46\%\) for \(|\Phi_0\rangle\) (A) and \(QBER = 2.56\% \pm 0.22\%\) for \(|\Phi_1\rangle\) (B). Analyzing the DF states in presence of the collective noise \(U^\otimes 4\) shows that one is still able to distinguish reliably the two DF states, now with \(QBER = 6.82\% \pm 0.75\%\) (C) and \(QBER = 3.99\% \pm 0.26\%\) (D), respectively.

In order to encode any arbitrary logical qubit, one could use two sources, one for \(|\Phi_0\rangle\) and the other for \(|\Phi_1\rangle\), and coherently overlap the generated photons. Yet, the technical requirements go beyond a first proof of principle. For a demonstration of the invariance of a logical qubit encoded in DF states, we prepared the state \(|\Psi_L\rangle = (\sqrt{3}|\Phi_0\rangle - |\Phi_1\rangle)/2\) and performed quantum state tomography of the encoded qubit before and after passage through a noisy quantum channel. We choose this state as it can be also obtained from the set-up shown in Fig. 1 by swapping modes \(b \leftrightarrow c\). Figure 4 shows the elements of the density matrices \(\rho_{in}\) (A) and \(\rho_{out}\) (B) in
F

tain an overlap between the initial and the outgoing state 

affect qubits which were sent. Yet, such an operation does not 
cation of the same unitary transformation to each of the 

the receiver's reference frame corresponds to the appli-

cence frame [13]. In such a scenario, any realignment of 

tecting against collective noise, the DF states are useful 

time 12 h).

the \{ |\Phi_0\rangle, |\Phi_1\rangle \} basis of the logical qubit. The imaginary 

parts of the density matrices obtained are negligible. These results show that the diagonal elements before 

and after the interaction are in good agreement, and that 

in order to read the quantum information encoded in the 

gence matrices before (|\rho_{in}\rangle) and after (|\rho_{out}\rangle) pass through a 

oisy quantum channel. The encoding in a DF subspace pro-
tected the transmission, leading to a fidelity of \( F_{\rho_{in},\rho_{out}} = 0.9958 \pm 0.0759 \) in the presence of noise (overall measurement 

time 12 h).

FIG. 4: Propagation of the logical qubit \(|\Psi\rangle = (\sqrt{3}|\Phi_0\rangle - |\Phi_1\rangle)/2\): (A) and (B) show the experimentally obtained den-
sity matrices before (|\rho_{in}\rangle) and after (|\rho_{out}\rangle) which are accessible by local measurements of the four photons, 

without two-qubit quantum logic gates being necessary, and thus realizable with state-of-the-art technology. This 
is relevant for possible applications of quantum communication [14]. We have performed a tomographic recon-
struction of the density matrix of a logical qubit en-
coded in the DF subspace showing its immunity against a noisy environment. Our measurements also show that 

DF states permit the communication of quantum infor-
mation even if the sender and the receiver do not share 
a reference frame. This is of great importance for future 

experiments studying quantum nonlocal effects between distant observers [13, 15].

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