Dynamic D8-branes in IIA string theory

A. Chamblin¹ and M.J. Perry¹

¹DAMTP
Silver Street
Cambridge, CB3 9EW, England
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Abstract

In this paper we perform a detailed investigation of the Dirichlet eight-brane of the Type IIA string theory, when the effects of gravity are included. In particular, consider what happens when one allows the ten-form field strength $F_{10}$ to vary discontinuously across the worldvolume of the brane. Since the ten-form is constant on each side of the brane ($d * F_{10} = 0$), a variation in the bulk term $\int F_{10} * F_{10}$ gives rise to a net pressure acting on the surface of the brane. This means that the infinite ‘planar’ eight-brane is no longer a static configuration with these boundary conditions. Instead, a static configuration is found only when the brane ‘compactifies’ to the topology of an eight-sphere, $S^8$. These spherical eight-branes are thus bubbles which form boundaries between different phases of the massive Type IIA supergravity theory. While these bubbles are generically unstable and will want to expand (or contract), we show that in certain cases there is a critical radius, $r_c$, at which the (inward) tension of the brane is exactly counterbalanced by (outward) force exerted by the pressure terms. Intuitively, these ‘compactified’ branes are just spherical bubbles where the effective cosmological constant ‘jumps’ by a discrete amount as you cross a brane worldsheet. We point out that such configurations are very similar to Dirac’s original ‘membrane’ model for the electron, whereby an electron is thought of as a charged, spherical conducting surface such that $F^2 = 0$ inside the brane and $F^2 \neq 0$ outside the brane (where $F = F_{\mu\nu}$ is the Maxwell two-form field strength). For this reason, we propose that these spherical Dirichlet eight-branes can properly be known as ‘D(irac)-branes’. We argue that these Dirac branes will be unstable to various semi-classical decay processes. We discuss the implications of such processes for the open strings which have endpoints on the eight-brane.

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I. INTRODUCTION

In recent years, branes have emerged as an essential ingredient of string theory, which is our best hope for a ‘theory of everything’. Most importantly, a M( agical) theory has recently come to light which has taught us that all of string theory can be understood in a single framework, unified by the force of duality symmetries. This ‘M-theory’ is already quantized, in the sense that it has no coupling constant and therefore no classical limit. Of course, it does admit a low-energy limit, namely eleven dimensional supergravity. The bosonic sector of eleven dimensional supergravity contains just two fundamental fields, the graviton and a three-form potential, and it follows that there are two basic p-branes which solve the equations of motion: an (electric) 2-brane and a (magnetic) 5-brane. One can test what M-theory has to say about the world around us by compactifying combinations of these extended objects to lower dimensional configurations. For example, all of the p-branes of ten dimensional (Type IIA) supergravity theory can be obtained from either the membrane or the fivebrane of eleven dimensional supergravity by dimensional reduction [4].

Perhaps the most important thing which string duality teaches us is that in order to have a consistent string theory we have to include objects known as ‘D-branes’. These D-branes, which are just hyperplanes where open strings are allowed to end, were shown by Polchinski [3] to be the carriers of ten dimensional Ramond-Ramond (R-R) charge (i.e., D-branes are the solitonic p-branes of Type II supergravity which carry R-R charge). T-duality requires the existence of these D-branes [3], and once stringy effects are taken into account, the D-branes become dynamical objects. Indeed, the collective coordinates for the transverse fluctuations of a D-brane are just the massless open string excitations that live on the brane worldvolume.

Once you allow for the existence of these D-branes, where open strings can end, it is natural to start asking questions about their gross kinematical and/or dynamical properties. For example, is there any constraint on the topology of these branes? What happens when we push a brane away from extremality? Can these branes ‘rip’, or tear, by some semi-classical decay process analogous to the decay of cosmic strings (by black hole pair creation)? In this paper, we point out that it is possible to have ‘localized’ branes, of spherical topology, and that these compact branes will indeed want to decay through a semiclassical tunneling process rather similar to the mode described in [3].

Throughout this paper we are working in signature (−, +, +, …, +). Our convention for the sign of the curvature is

\[ \nabla_a \nabla_b u_c - \nabla_b \nabla_a u_c = R_{abcd} u^d \]

Before proceeding with the principal construction, it is useful if we first recall some basic facts about D-branes [3]. To begin, let us focus just on the bosonic terms in the effective field theory actions which arise from the IIA and IIB string theories. These field theories are of course just the type IIA and type IIB supergravity theories. In the Neveu-Schwarz-Neveu-Schwarz (NS-NS) sector these two theories have identical field content, consisting of a metric tensor \( g_{ij} \), an antisymmetric rank two tensor potential \( b_{ij} \) and a scalar dilaton field \( \phi \) (these fields arise from the solutions of string theory where the worldsheet fermions have anti-periodic ‘Neveu-Schwarz’ boundary conditions in both the right and left moving sectors of a closed string, and so that is why they are called NS-NS). On the other hand,
in the Ramond-Ramond (R-R) sector the field content of the two theories is quite different. Explicitly, the bosonic sector for the effective action of IIA supergravity is given as

\[
S_{IIA} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\phi}(R + 4(\nabla\phi)^2) - \frac{1}{2 \cdot 3!} H^2 \right] - \left( \frac{1}{2 \cdot 2!} F(2)^2 + \frac{1}{2 \cdot 4!} F(4)^2 \right] - \frac{1}{4\kappa^2} \int F(4) \wedge F(4) \wedge b ,
\]

(1.1)

whereas the effective action for IIB supergravity takes the form

\[
S_{IIB} = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left[ e^{-2\phi}(R + 4(\nabla\phi)^2) - \frac{1}{2 \cdot 3!} H^2 - \frac{1}{2} (\nabla A(0))^2 \right] - \frac{1}{2 \cdot 3!} (A(0) B + F(3))^2 ,
\]

(1.2)

Here, \( F(n) = ndA(n-1) \) are the antisymmetric \( n \)-form R-R field strengths, with \((n-1)\)-form potentials \( A_{(n-1)} \), and \( H = 3db \) is the antisymmetric NS-NS three-form field strength. The string coupling \( g_s \) is given in terms of the dilaton by the relation \( g_s = e^\phi \). Thus, in both of the above actions the NS-NS sector is multiplied by a factor of \( g_s^{-2} \); this means that NS-NS effects arise in ordinary string perturbation theory. On the other hand, the R-R terms are not multiplied by any such factor and so we know that fundamental string states do not carry R-R gauge charges. As we have already mentioned, the mystery of the R-R gauge fields was resolved when it was shown that R-R charge was carried by the topological defects known as D-branes [3].

Now these actions do not tell the whole story about D-brane configurations. This is because they really only tell us about what life is like for massless closed string states which exist “off” of the D-branes and how these states couple to the D-branes. For this reason, the supergravity effective actions are called the ‘bulk’ terms. There should also be an effective description of life on the brane worldvolume. Indeed, such an effective world-volume action (denoted \( S_{WV} \)) does exist and it is characterized by a D(p)-brane tension, \( T_p \), and R-R charge density form \( f_p \) as shown below:

\[
S_{WV} = -T_p \int d^{p+1}\sigma e^{-\phi/2} \sqrt{\det(h_{\mu\nu} + b_{\mu\nu} + 2\pi F_{\mu\nu})} - f_p \int d^{p+1}\sigma A_{(p+1)}
\]

(1.3)

Here, \( \sigma \) denotes the coordinates tangent to the brane worldvolume, \( h_{\mu\nu}, b_{\mu\nu} \) and \( \phi \) are the pullback of the (respective) spacetime fields to the brane worldvolume. \( F_{\mu\nu} \) is an ordinary Maxwell gauge field which lives on the brane worldvolume. Again, \( A_{(p+1)} \) is an antisymmetric \((p+1)\)-form R-R potential which couples naturally to the p-brane worldvolume (hence the appearance of the R-R charge density \( f_p \)). This form of the action is very useful if one is interested in studying ‘light’ branes, which have negligible gravitational fields (so that you can ignore the ‘bulk’ supergravity contributions). Indeed, several people have made a lot of mileage recently ([3], [4]) from the fact that the first term in (1.3) is more or less the old Born-Infeld action of electrodynamics (in the limit where you allow the Kalb-Ramond three-form and dilaton to vanish). In this limit a beautiful picture emerges, in which fundamental
strings ending on a D-brane appear, from the point of view of the brane worldvolume theory, as Coulomb-like point particle solutions of the Born-Infeld theory on the brane (Gibbons refers to all such solitonic configurations in Born-Infeld theory as ‘BIons’). Similarly, you can also think of an M2 brane ending on an M5 brane as a ‘vortex’-type BIon living on the M5 brane worldvolume, and so on. While these are certainly beautiful and promising results, it is still of some interest to understand how strings and branes in general will interact when they are ‘heavy’, and the effects of gravity are taken into account. Presumably, addressing this problem will have to involve somehow writing down an effective action which interpolates between the world-volume and bulk terms as gravity is turned up.

In this paper, we consider a slightly simpler problem, namely, how the world-volume and bulk terms ‘play off’ of each other, when you allow the brane to gravitate. What we find is that it is possible to exactly cancel the ‘tension’ term in the world-volume action with pressure-like bulk terms in scenarios where the D-brane is a compact spherical surface. These spherical eight-branes are ten-dimensional, gravitating analogues of Dirac’s original membrane model for the electron, and so we will refer to these spherical D8-branes as ‘Dirac branes’. In order to properly understand this appellation, it is useful if we briefly recall the basic facts about Dirac’s idea.

In 1962 Dirac presented his membrane model for the electron; in this picture, the electron is represented as a conducting sphere, with the property that inside the sphere the electromagnetic field vanishes ($F^2 = 0$) whereas outside of the sphere the field strength is non-vanishing. Thus in this scenario, to quote Dirac, “the electron may be pictured as a bubble in the electromagnetic field”. Of course, the discontinuous jump in $F^2$ as one moves across the membrane boundary of the bubble means that there will be a net pressure, which will cause the bubble to expand. In order to counteract this pressure, Dirac assumed that the membrane had some tension capable of holding the bubble in place. Indeed, Dirac showed that for generic values of the field parameters there would always be an ‘equilibrium radius’ for the bubble, at which the bubble would be perfectly static. Later, both classical and quantum instabilities were shown to exist for this model [2], although we feel it is worth pointing out that supersymmetry may imply that the quantum instability is not so bad. This membrane model, which may be regarded as the first suggestion that extended objects may play an important role in our attempts to describe subatomic phenomena, was clever but outside the bounds of experimental error: Certainly the mass ratio $m_\mu/m_e \sim 200$ was left unexplained.

Dirac’s motivations for introducing this idea were numerous. On the one hand, the idea of assigning a finite size to elementary particles in order to somehow regularize the total energy of the Coulomb field was an old one. On the other hand, evidence had emerged that the muon and the electron were very similar in many ways, and that perhaps the muon could be viewed as an excited electron. If the electron was a sphere, then perhaps the muon could be a ‘wiggly’ sphere.

In this paper, we will find gravitating spherical D-branes which are reminiscent of the old Dirac membrane model. As we have already said, the simplest situation where this occurs involves the eight-brane of Type IIA theory, and it is there that we now turn out attention.
II. THE D(8)-BRANE REVISITED

The Dirichlet eight-brane of Type IIA couples to the ten-form field strength $F_{10}$ of the R-R sector. As Polchinski [3] pointed out, this ten-form is not a dynamical variable; rather, it is just a constant field which generates a uniform physical energy density which permeates space. More explicitly, the bulk term generated by this field strength takes the form

$$S_{\text{bulk}} = -\frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \frac{1}{2!} F_{(10)}^2$$ \hspace{1cm} (2.1)

Since $F_{10}$ is constant, this bulk term corresponds to an effective cosmological constant term. This constant is precisely the mass term of the massive IIA supergravity theory. The first supersymmetric $m \neq 0$ lagrangian for the IIA theory was written down by Romans [10]. In his theory, the mass arises from a Higgs mechanism whereby the two-form ‘eats’ the vector. However, as Howe, Lambert and West (HLW) [11] have recently pointed out there are actually three distinct Higgs mechanisms, corresponding to whether the vector eats the scalar, the two-form eats the vector or the three-form eats the two-form. In this way, they have constructed a new massive IIA supergravity, which has the appealing property that it can be obtained by compactification of eleven-dimensional Minkowski space on a circle, with the introduction of a Wilson line. In this paper, we will always be assuming that the four-form field strength (coming from the eleven dimensional supergravity bosonic field sector) is turned off. With this caveat, we then obtain the below equations of motion in ten dimensions for the HLW theory:

$$R_{ab} - \frac{1}{2} g_{ab} R = -2(D_a D_b \phi - g_{ab} D^2 \phi + g_{ab} (D \phi)^2) + \frac{1}{2} (F^{ac} F^b_c - \frac{1}{4} g_{ab} F^2) - 18m(D_{(a} A_{b)} - g_{ab} D^c A_c) - 36m^2(A_a A_b + 4g_{ab} A^2) - 12mA_{(a} \partial_{b)} \phi - 30mg_{ab} A^c \partial_c \phi - 144m^2 g_{ab} e^{-2\phi} + \text{fermions}$$ \hspace{1cm} (2.2)

$$D^b F_{ab} = 18mA_b F^b_a + 72m^2 e^{-2\phi} A_a - 24m e^{-2\phi} \partial_a \phi$$ \hspace{1cm} (2.3)

$$6D^2 \phi - 8(D\phi)^2 = -R + \frac{1}{4} F^2 + 360m^2 e^{-2\phi} + 288m^2 A^2 + 96mA^b \partial_b \phi - 36mD^b A_b$$ \hspace{1cm} (2.4)

where $F_{ab} = \partial_a A_b - \partial_b A_a$ as usual. The mass parameter $m$ determines the effective cosmological constant generated by the ten-form field strength. These are the full bulk equations which we are free to play around with. In this paper, we will be simplifying things by truncating these equations of motion by setting the vector field to zero:

$$A_{(1)} = 0$$

With this assumption, things simplify considerably. Indeed, the only non-trivial remaining equations when the dust settles are the Einstein equation:

$$R_{ab} = 360m^2 e^{-2\phi} g_{ab}$$ \hspace{1cm} (2.5)
together with the ‘Maxwell’ equation (2.3), which implies that the dilaton $\phi$ is a constant. In other words, if we turn off all of the fields in this theory except for gravity, we simply recover de Sitter spacetime. The effective cosmological constant is then given explicitly in terms of the mass as

$$\Lambda = 1296m^2 e^{-2\phi}$$

Because this theory admits such a remarkably simple truncation, we will restrict our attention to this theory in this paper. Of course, everything which we will do here could in principle also be done for the old Romans massive IIA theory. However, in the Romans theory things are considerably more complicated; basically, the dilaton can never be constant, and in fact the scalar curvature is identically zero. It is only in the interest of simplicity that we do not consider the Romans theory further here.

Our plan now is simple. Our aim is to find all possible eight-branes in this theory since these are the objects that couple electrically to $F_{(10)}$. Away from the brane, spacetime obeys the source-free equation (2.5). The principle of equivalence tells us that all sources of energy and momentum gravitate. The brane is no exception to this rule, and its energy-momentum tensor can be calculated from the action (1.3). The brane itself is infinitely thin, and therefore its energy-momentum tensor can be regarded as a distributional object concentrated entirely on the brane itself. The physics of this situation is then rather similar to that of domain walls in the so-called “thin wall” approximation. Thus the spacetime energy-momentum tensor of the brane can be written as

$$T_{ab} = S_{ab} \delta(\eta)$$ (2.6)

where the argument of the delta-function is the single coordinate transverse to the brane evaluated in an normal coordinate system centered on the brane, and $S_{ab}$ is termed the surface energy-momentum tensor.

$$S_{ab} = \sigma h_{ab}$$ (2.7)

where $h_{ab}$ is the metric induced on the brane by its embedding into spacetime, and $\sigma$ is the brane’s energy density. The pullback of $h_{ab}$ to the world-volume is just $h_{\mu\nu}$, and it is this metric which appears in the worldvolume action for the brane. From equation (1.3), we see that the energy density scales in terms of the brane tension as

$$\sigma = T_p e^{-\phi/2} = \frac{1}{32} \pi^{-11/2} e^{-3\phi/2}$$ (2.8)

Thus, away from the brane we simply need to understand the solutions to equation (2.5). In other words, the eight-brane divides spacetime up into separate domains. As one moves from one domain to another, there will be conditions on how to match the spacetime in one region to the adjacent one. These conditions will depend on the geometry and the energy density in the brane.
III. BRANE COMPACTIFICATION, SPHERICAL SYMMETRY AND DIRAC BRANES

The fact that eight-branes divide ten-dimensional spacetime up into domains reminds one of domain walls in cosmology. As is well known, various cosmological models assume that a variety of phase transitions took place in the early universe. In such transitions, symmetries which are only valid at high temperatures are broken as the universe cools down. In the common picture, ‘bubbles’ of the new phase are nucleated in regions of the old phase and may expand; if the rate of production of these bubbles is not diluted by the rate of expansion of the universe, the process of bubble nucleation, expansion and amalgamation will continue until the universe is filled with new phase (with perhaps a few bubbles of old phase ‘remnants’ left over). When the new phase fills the entire universe, the transition is said to be complete. One of the principal conclusions of this work is that similar physics appears to occur in string theory.

The dynamical evolution of these bubbles, when the effects of gravity are included, has been studied by a number of authors [13], [14], [20]. These studies involve understanding the Einstein equations when the source is a ‘thin shell’, or domain wall. In such situations, the spacetime has low differentiability and so one has to regard the curvature as a distribution. It was shown long ago [16] that the correct formalism for studying such a problem involves constructing metric junction conditions for the thin shells. These junction conditions, commonly referred to as the ‘Israel matching conditions’, may be summarized as follows:

1. A domain wall hypersurface is totally umbilic; that is, the second fundamental form $K_{ij}$ is proportional to the induced metric $h_{ij}$ on the domain wall world-volume.

2. The discontinuity in the second fundamental form on the domain wall hypersurface is $[K_{ij}] = 4\pi \sigma h_{ij}$, where $\sigma$ denotes the energy density of the domain wall.

Thus, the energy density of an idealized domain wall measures the jump in the extrinsic curvature of surfaces parallel to the wall as you move through the wall. We will use these conditions to do ‘cut-and-paste’ constructions of eight-brane domain wall hypersurfaces.

Another point, worth bringing up, is that each side of these eight-branes will be regions where strings are free to propagate in the bulk, and to attach themselves to the worldvolume in the usual way, i.e., these branes will still be D-branes. Presumably, there will therefore be some minimal size for these objects (of order the string length). Any statements which we make about these objects, and how they interact with strings, must therefore assume that they are bounded in size in this simple way.

A. Homogeneous and isotropic branes in the HLW theory

We imagine the brane to be a surface sitting in spacetime. The simplest geometry we can expect for it is to be a surface of constant curvature. We therefore expect the brane to be spherical, planar or hyperbolic. Exterior to the brane, we expect the spacetime to reflect this geometrical property. We also suppose that the space-time orthogonal to the wall is static. This comes from the fact the observers co-moving with the wall would expect their
situation to be time independent. Furthermore, we assume that the directions parallel to the brane worldsheet are homogeneous and isotropic. Of course, this does not necessarily mean that the entire spacetime has to be static, it could well be that the orthogonal directions show some kind of cosmological expansion without violating these symmetry assumptions. With these assumptions in mind, it follows that the ten-dimensional metric takes the explicit form:

$$ds^2 = e^{2A(z)}(-dt^2 + dz^2 + f(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega_7 \right])$$

(3.1)

Here, $t$ denotes the time experienced by observers who move with the wall, so that $A$ is a function of $z$ only and $f$ is a function of $t$ only (i.e., we have used boost invariance and so $f$ measures the ‘scale’ of the brane), $d\Omega_7$ denotes the metric on a unit seven-sphere, and $k$ is the spatial curvature of the brane. In other words, we have manifestly decomposed the metric into a ‘normal to the brane’ part (the $(t, z)$ sector) and a ‘parallel to the brane’ part. In the directions parallel to the brane worldvolume, we have used our assumption that the brane is isotropic and homogenous to impose key constraints on the geometry. Indeed, homogeneity and isotropy imply that the brane worldvolume is an $(8 + 1)$-dimensional FLRW universe, and so that is why we have written that down for the tangential part of the metric (similar symmetry considerations were utilized in [20], where similar configurations were studied in four dimensions). For various reasons, we prefer to write the spatial part of the brane metric in conformally flat form. In other words, we introduce coordinate $\rho$ and function $g(\rho)$ such that

$$\frac{dr^2}{1 - kr^2} + r^2 d\Omega_7 = g(\rho)^2 [d\rho^2 + \rho^2 d\Omega_7]$$

where $r$ and $\rho$ are related in the usual way:

$$\int \frac{d\rho}{\rho} = \int \frac{dr}{r} (1 - kr^2)^{-1/2}$$

There are thus three basic cases to consider, corresponding to the three types of FLRW universe:

**Case 1, $k = 0$**

In this case, $k = 0$ and so the spatial section of the brane is a flat plane. For this case, $r = \rho$ and $g(\rho) = 1$.

**Case 2, $k > 0$**

In this case the brane has constant positive curvature, and so the spatial section is an eight-sphere. One calculates that

$$r = \frac{2(k)^{-1/2} \rho}{1 + \rho^2}$$

whence

$$g(\rho) = \frac{2(k)^{-1/2}}{1 + \rho^2}$$
Case 3, $k < 0$

In this case the brane has constant negative curvature, and so the spatial section is isometric to hyperbolic space. Here, the transformation between $r$ and $\rho$ takes the form

$$r = \frac{2(-k)^{-1/2} \rho}{1 - \rho^2}$$

so that one finds

$$g(\rho) = \frac{2(-k)^{-1/2}}{1 - \rho^2}$$

With all of this in mind, it follows that the metric on each side of the brane is given as

$$ds^2 = e^{2A(z)}(-dt^2 + dz^2 + f(t)^2 g(\rho)^2 [d\rho^2 + \rho^2 d\Omega_7])$$

(3.2)

In order to compute the curvature, we introduce the orthonormal basis of one-forms $\{E^i : i = 0, ..., 9\}$ related to the coordinates by:

$$E^0 = e^A dt, \quad E^1 = e^A dz, \quad E^i = e^A f(t) g(\rho) dx^i$$

whence

$$dt = e^{-A} E^0, \quad dz = e^{-A} E^1, \quad dx^i = \frac{e^{-A}}{g f} E^i$$

A straightforward calculation yields the connection one-form

$$\Gamma^0_i = \frac{\dot{f}}{f} e^{-A} E^i$$

$$\Gamma^0_1 = A' e^{-A} E^0$$

$$\Gamma^1_i = -A' e^{-A} E^i$$

$$\Gamma^i_j = e^{-A} \frac{\partial \rho g}{f g^2 \rho} (x^j E^i - x^i E^j)$$

(3.3)

where (for now), ‘dot’ denotes differentiation relative to $t$, and ‘prime’ denotes differentiation relative to $z$. Proceeding, one obtains the non-zero components of the Ricci tensor in the orthonormal frame, after using the specific functional form for $g(\rho)$:

$$R_{00} = e^{-2A} (A'' + 8A'^2 - 8 \frac{\dot{f}}{f})$$

$$R_{11} = e^{-2A} (-9A'')$$

$$R_{ij} = e^{-2A} (\frac{\ddot{f}}{f} + 7 \frac{\dot{f}^2}{f^2} + 7 \frac{k}{f^2} - A'' - 8A'^2) \delta_{ij}$$

(3.4)

From here we take the trace to obtain the Ricci scalar and so we construct the Einstein tensor $G_{\mu\nu}$. From what we said earlier about the massive HLW IIA theory, we are interested
in solving the vacuum Einstein equations with a cosmological constant, which take the form (in ten dimensions):

\[ e^{-2A}(A'' + 8A'^2 - 8\frac{\dot{f}}{f}) = \frac{-1}{4}\Lambda \]  
\[ e^{-2A}(-9A'') = \frac{1}{4}\Lambda \]  
\[ e^{-2A}(\frac{\ddot{f}}{f} + 7\frac{\dot{f}^2}{f^2} + 7\frac{k}{f^2} - A'' - 8A'^2) = \frac{1}{4}\Lambda \]  

(3.5)  
(3.6)  
(3.7)

Notice first that we have not specified the sign of the cosmological constant; this is because we are interested in finding all possible solutions, before restricting our consideration to the HLW sector (where the effective cosmological constant has to be greater than or equal to zero). The most obvious approach to solving these equations is to begin with (3.6), and to try and solve for \( A \). Proceeding in this way one obtains the integral

\[ \int \frac{dA}{(c^2 - \frac{\Lambda}{36} e^{2A})^{1/2}} = \pm(z - z_0) \]  

(3.8)

where \( c \) and \( z_0 \) are some constants of integration. There are three classes of solutions to this equation, corresponding to the three possible signs of \( \Lambda \). We enumerate them as follows:

**Class I, \( \Lambda = 0 \):**

In this case, to put it in the language of the massive supergravity theory, we are looking for the behaviour of the overall conformal factor \( e^{2A} \) in a region where the theory is in a massless phase. One easily integrates (3.8) and finds that

\[ e^A = e^{\pm c(z-z_0)} \]  

(3.9)

so that \( A' = \pm c \), i.e., this would seem to imply that \( c \) somehow measures the ‘acceleration’ of the brane. We will have more to say about this point later.

**Class II, \( \Lambda > 0 \):**

Here, we are looking at the conformal factor in a region of massive phase. Again, one integrates and obtains for \( A \) and finds that now

\[ e^A = \frac{6c}{\sqrt{\Lambda} \cosh c(z-z_0)} \]  

(3.10)

so that \( A' = -(c^2 - \frac{\Lambda}{36} e^{2A})^{1/2} \).

**Class III, \( \Lambda < 0 \):**

This case corresponds to regions where the effective cosmological constant is negative, or the mass is ‘imaginary’, and as such it has nothing to do with the HLW theory. Nevertheless, we include this case in the interest of presenting a complete classification of the solutions. Integrating with negative \( \Lambda \) one finds that

\[ e^A = \frac{6c}{\sqrt{-\Lambda} \sinh c(z-z_0)} \]  

(3.11)
Note that in this case $c^2$ can be less than zero, in which case $c$ is imaginary but the expression for $e^A$ is still valid.

These different classes summarize everything we need to know about the behaviour of the conformal factor $e^{2A}$. To understand completely the geometry of these branes, therefore, we just need to calculate the scale factor $f(t)$. In order to do this, we combine equations (3.5) and (3.7), and we use that fact that generically $A' = -(c^2 - \frac{\Lambda}{36} e^{2A})^{1/2}$, in order to derive the identity

$$c^2 = \frac{\dot{f}}{f}$$  \hspace{1cm} (3.12)

Using this identity, one can work out all of the possibilities for $f$. These possibilities can be classified according to the sign of $c^2$, as shown:

$c = 0$:

When $c = 0$, $f$ assumes the form

$$f(t) = \alpha t + \beta$$

so that $k$ satisfies $k = -\alpha^2 < 0$.

$c^2 > 0$:

When $c^2 > 0$ the scale factor assumes the form

$$f(t) = \alpha e^{ct} + \beta e^{-ct}$$

so that the curvature is given as $k = 4\alpha \beta c^2$.

$c^2 < 0$:

When $c^2 < 0$, $f$ can be written as

$$f(t) = \alpha \sinh(ct) + \beta \cosh(ct)$$

where $k = (\alpha^2 + \beta^2)c^2$.

Thus, we see that there are numerous possibilities for homogeneous, isotropic and boost invariant branes which bound regions which are solutions of the vacuum Einstein equations. We summarize all of these possibilities, in terms of the sign of the curvature of the brane, in order to emphasize the fact that spherical branes are preferred when $\Lambda > 0$ as shown below:

**Spherical branes ($k > 0$)**

Here, as we have seen, $c^2 > 0$ and so $k = 4\alpha \beta c^2$, where $\alpha$ and $\beta$ have the same sign and the scale factor $f(t)$ for the brane is given as

$$f(t) = \alpha e^{ct} + \beta e^{-ct}$$  \hspace{1cm} (3.13)

The overall conformal factor then depends on the sign of the cosmological constant as shown below:
Thus, we see that it is possible to have spherical eight-branes for any sign of the cosmological constant. When \( \Lambda = 0 \) these are just the ten-dimensional version of the old four-dimensional Vilenkin-Ipser-Sikivie domain walls [25]. When \( \Lambda < 0 \) we recover the ten-dimensional version of the non-extreme supergravity domain walls discussed by Cvetic et al [17]. When \( \Lambda > 0 \) we are left with two-sided de Sitter-de Sitter (dS-dS) type eight-brane ‘bubbles’. The energy density of these branes is calculated (using the Israel matching conditions) to be

\[
\sigma = \frac{2}{\kappa} \left( \sqrt{\pm l_1^2 + c^2} + \sqrt{\pm l_2^2 + c^2} \right) \tag{3.17}
\]

where \( l_i^2 = \pm \frac{\Lambda}{36} \) when \( \Lambda_i < 0 \) and \( l_i^2 = -\frac{\Lambda}{36} \) when \( \Lambda_i > 0 \). Thus, for the dS-dS type eight-branes of the HLW theory, the vacuum energy density on each side of the branes is bounded from above in terms of the parameter \( c^2 \):

\[
\Lambda_i \leq 36c^2
\]

Because these spherical Dirichlet eight-branes are reminiscent, from a ten-dimensional point of view, of the old Dirac membrane model for the electron, we shall henceforth refer to them as Dirac branes.

**Planar branes (\( k = 0 \))**

Here, there are two possibilities for the scale factor. We must take \( c^2 \geq 0 \) and hence either \( \alpha \) or \( \beta \) must vanish (when \( c^2 > 0 \)) or \( \alpha \) must vanish (when \( c^2 = 0 \)). In the \( c^2 > 0 \) case, the brane scale factor therefore takes the form

\[
f(t) = \alpha e^{\pm ct} \tag{3.18}
\]

The conformal factor in this case is then given as

\[
\Lambda = 0 \Rightarrow e^A = e^{-c(z-z_0)} \tag{3.19}
\]

\[
\Lambda > 0 \Rightarrow e^A = \frac{6c}{\sqrt{\Lambda}} \text{sech}[c(z - z_0)] \tag{3.20}
\]

\[
\Lambda < 0 \Rightarrow e^A = \frac{6c}{\sqrt{-\Lambda}} \text{cosech}[c(z - z_0)] \tag{3.21}
\]
In the $c^2 = 0$ case the brane scale function reduces to a constant, and the conformal factor is given as

$$\Lambda = 0 \Rightarrow e^A = \text{constant}$$

$$\Lambda < 0 \Rightarrow e^A = \frac{6}{\sqrt{-\Lambda}} \frac{1}{(z - z_0)}$$

There is no positive $\Lambda$ case when $c^2 = 0$. Thus, in the context of the HLW theory it is possible to have planar branes, as long as they are not static (i.e., as long as the scale factor is going like an exponential). At first glance, this might seem strange since it implies that it is possible to slice de Sitter space along planar timelike hypersurfaces. In fact, it is possible to find totally umbilic, timelike hypersurfaces which are isometric to Minkowski space in de Sitter spacetime, as was first demonstrated in [22].

**Hyperbolic branes ($k < 0$)**

Here, there are three possibilities for the scale function, corresponding to the three possible signs for $c^2$. For the case $c^2 > 0$, $f$ has the form

$$f(t) = \alpha e^{ct} + \beta e^{-ct}$$

where $\alpha$ and $\beta$ have the opposite sign, and the curvature is given as $k = 4\alpha\beta c^2$. The conformal factor is given as

$$\Lambda = 0 \Rightarrow e^A = e^{-c(z - z_0)}$$

$$\Lambda > 0 \Rightarrow e^A = \frac{6c}{\sqrt{\Lambda}} \text{sech}[c(z - z_0)]$$

$$\Lambda < 0 \Rightarrow e^A = \frac{6c}{\sqrt{-\Lambda}} \text{cosech}[c(z - z_0)]$$

When $c^2 = 0$, the scale factor has the form

$$f(t) = \alpha t + \beta$$

where $k = -\alpha^2$, and the conformal factor is

$$\Lambda = 0 \Rightarrow e^A = \text{constant}$$

$$\Lambda < 0 \Rightarrow e^A = \frac{6}{\sqrt{-\Lambda}} \frac{1}{(z - z_0)}$$

Again, there is no positive $\Lambda$ case when $c^2 = 0$. When $c^2 < 0$, the scale factor is

$$f(t) = \alpha \sinh(ct) + \beta \cosh(ct)$$

where $k = (\alpha^2 + \beta^2)c^2$, and so there is only one possibility for the conformal factor

$$e^A = \sqrt{-\frac{c^2}{\Lambda}} \text{cosech}[c(z - z_0)]$$
Thus, we see that it is possible to have branes of constant negative spatial curvature in the HLW theory, where the effective cosmological constant is bounded from below by zero, again as long as the branes are not static. This is interesting, since given a hyperbolic brane $H^8$ it is of course always possible to find some freely acting discrete group $G$ such that the quotient $H^8/G$ is a brane with non-trivial topology. In this way, one should be able to construct D8-branes of arbitrary topology. We will have nothing more to say about this issue, or hyperbolic branes in general, in this paper.

### B. A Dirac brane menagerie

With all of the above in mind we proceed with an analysis of the spherically symmetric solutions, which we dub the ‘Dirac branes’. To do this we first choose a gauge so that the spherically symmetric ten-dimensional metric takes the form

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\Omega_8^2$$

(3.33)

where $f$ is a function of $r$ only. The most general spherically symmetric vacuum metric in $9 + 1$ dimensions will have the form (3.33), where the potential $f$ is given as

$$f(r) = 1 - \frac{C}{r^7} - \frac{\Lambda}{36}r^2$$

(3.34)

This is of course just the metric of Schwarzschild-de Sitter (SdS) spacetime, where $\Lambda$ is the cosmological constant and $C$ is given in terms of the ADM mass $M$ mass of the black hole as follows:

$$C = \left(\frac{105G}{16\pi^3}\right)M$$

The reader may be wondering why we aren’t including any $U(1)$ charges at this point. Actually, we could have left $F_{\mu\nu}$ non-zero and considered the resulting Einstein-Maxwell-dilaton system of equations; we begin with the $F = 0$ truncation for simplicity.

Before we proceed with the construction of the equations of motion for the brane world-volume it is useful if we have a picture of the simplest Dirac brane. In the simplest scenario, a (static) Dirac brane is just a bubble of de Sitter space, bounded by the asymptotically flat region exterior to a Schwarzschild black hole. That is to say, we take one of our Schwarzschild solutions, and cut along the timelike (static) worldsurface that satisfies the Israel matching conditions. Then we throw away the ‘interior’ of the worldvolume of the brane, and instead we paste in a timelike cylinder which we have cut out of de Sitter space. Obviously, the Israel conditions can be satisfied at the junction hypersurface. Furthermore, it is not hard
to see that the brane tension will be positive. This construction is illustrated below:

![Diagram showing the simplest Dirac eight-brane.](image)

Figure 1
Schematic picture showing the simplest Dirac eight-brane.

With this simple picture in mind we can now proceed with the equations of motion. The equations of motion for a thin spherical shell, which is bounding a region of SdS from a region of a different SdS spacetime, were worked out for four dimensions in [14]. The analysis developed there will go through throughout this paper, because the assumption of spherical symmetry means that brane motion is described as a system in 1 + 1 dimensions. Put more simply, we are solving for one unknown function, \( r(t) \), which describes the radius of the spherical brane at time \( t \). As always, we will let \( \sigma \) denote the surface energy density of the brane. We will also distinguish between quantities which exist ‘inside’ of the brane, and those which are outside, by using subscripts. Thus, \( f_{\text{in}} \) denotes the metric function (3.34) inside of the brane, and similarly \( f_{\text{out}} \) denotes the same thing outside of the brane. As we approach the brane from the inside, spheres parallel to the brane can either be expanding or shrinking, and we introduce the symbol \( \epsilon_{\text{in}} \) to distinguish these two cases. That is to say, \( \epsilon_{\text{in}} = +1 \) if the spheres are growing as we move through the brane, \( \epsilon_{\text{in}} = -1 \) if the spheres are shrinking and similarly for \( \epsilon_{\text{out}} \). With all of this in mind, the equation of motion for a spherical bubble bounding two different (vacuum) phases of the HLW supergravity theory is given as

\[
\epsilon_{\text{in}} \left[ \dot{r}^2 + f_{\text{in}}(r) \right]^{1/2} - \epsilon_{\text{out}} \left[ \dot{r}^2 + f_{\text{out}}(r) \right]^{1/2} = \frac{\kappa \sigma}{2} r
\]

(3.35)

where \( \dot{r} = f^{-1/2} \partial_t r \) denotes the derivative relative to the time experienced by observers who co-move with the domain wall and \( \kappa = 8\pi G \) as always. Any solution of this equation automatically describes an eight-brane worldsheet which satisfies the Israel conditions. Let us begin our investigation of equation (3.35) by assuming that both the inside and outside
of the brane we turn off the Schwarzschild mass parameter, $M$. Thus, inside of the brane the metric potential is given as

$$f_{in} = 1 - \frac{\Lambda_{in}}{36} r^2$$

and similarly outside

$$f_{out} = 1 - \frac{\Lambda_{out}}{36} r^2$$

From equation (2.5) we know that the effective cosmological constants on each side of the brane are given in terms of the characteristic masses of the supergravity theory on each side by the relations $\Lambda_{in} = 1296 m_{in}^2 e^{-2\phi_{in}}$ and $\Lambda_{out} = 1296 m_{out}^2 e^{-2\phi_{out}}$. However, as we discussed above these masses are basically the squares of the ten-form field strengths on each side of the brane, and we know that R-R charge is quantized \([15]\). In other words, the masses must differ by some discrete multiple of the fundamental unit of R-R charge.

If we assume that each side of the eight-brane is a massive phase of the HLW theory, then we are just pasting together two portions of two distinct ten-dimensional de Sitter spacetimes. Thus, the total spacetime is closed, i.e., spatial sections are compact and homeomorphic to $S^9$ (with a ‘ridge’ of curvature running along where the location of the eight-brane). It follows that the distinction between ‘in’ and ‘out’ is purely formal.

Now, we are going to be assuming throughout this paper that the energy density $\sigma$ of any eight-brane is positive, on the grounds that the tension should be proportional to some R-R charge density. Following the analysis of \([14]\), it turns out that there are three basic cases, which are determined by the relative signs of $\epsilon_{out}$ and $\epsilon_{in}$. These cases are exhibited by first introducing the parameter

$$\Gamma = \frac{\Lambda_{out} - \Lambda_{in}}{576 \pi G \sigma^2}$$

which more or less measures the relative sizes of the ‘in’ and ‘out’ regions. The cases can then be classified as shown in the below table:
Different Cases | Value of $\Gamma$ | Value of $\epsilon_{\text{out}}$ | Value of $\epsilon_{\text{in}}$
--- | --- | --- | ---
Case 1 | $\Gamma > 1$ | $+1$ | $+1$
Case 2 | $-1 < \Gamma < 1$ | $-1$ | $+1$
Case 3 | $\Gamma < -1$ | $-1$ | $-1$

Table 2

In Case 1, the in region is small relative to the out region, whereas in Case 2 the regions are of comparable size. Case 3 is equivalent to Case 1, with the in and out regions interchanged. In all of these cases the motion of the eight-brane bubble, as seen by free-falling observers on either side, is qualitatively similar. Indeed, we completely analyzed this spacetime in our analysis above. For this class of spacetime, the brane is spherical ($k > 0$) and the cosmological constant is positive, so we know that the scale factor $f(t)$ goes like

$$f(t) = \alpha e^{ct} + \beta e^{-ct}$$

where $k = 4\alpha\beta c^2$ ($\alpha$ and $\beta$ have the same sign) and $t$ is the time experienced by observers who move with the brane. In other words, the brane expands exponentially with co-moving time. Thus, in a sense the eight-brane ‘contributes’ to the effects of inflation, because its positive tension generates a repulsive gravitational field. This is very interesting, especially when we recall that these Dirac branes are really just another type of supergravity domain wall \[17\]. In \[17\] a complete classification was performed for the homogeneous and isotropic domain walls which can arise in $N = 1$, $D = 4$ supergravity theory; there, the ground state of the theory had *negative* cosmological constant. Thus, in $N = 1$, $D = 4$ supergravity domain walls are generically constructed by pasting together sections of anti-de Sitter spacetime. It was also shown that in a certain limit the $D = 4$ domain walls are supersymmetric (we will discuss the supersymmetric aspects of these Dirac branes in more detail shortly). In \[18\] it was shown that black holes will generically be pair produced in the presence of repulsive, spherical domain walls, and indeed the arguments developed in those papers will go through here. That is to say, ten-dimensional black hole pairs will be spontaneously nucleated in the presence of the repulsive, de Sitter - de Sitter type Dirac branes which we have been
considering so far. As we shall see, this is just one of the semi-classical instabilities which can affect Dirac branes.

Now that we have considered the de Sitter - de Sitter type vacuum Dirac branes, let’s think about what happens when we turn the mass parameter $C$ back on.

We begin by considering the simplest situation where a Schwarzschild term arises, namely, the situation illustrated in Figure 1 above. In other words, we have a bubble of de Sitter bounded from an (asymptotically flat) region of Schwarzschild. Thus, inside the bubble we have

$$f_{in} = 1 - \frac{\Lambda}{36} r^2$$

and outside

$$f_{out} = 1 - \frac{C}{r^7}$$

We insert these functions into (3.35), and then rearrange terms, to obtain the below expression for the equation of motion:

$$\frac{C}{r^7} = r^2 \left( \frac{\Lambda}{36} + \frac{\kappa^2 \sigma^2}{4} \right) + \kappa \sigma r \sqrt{\dot{r}^2 + 1} - \frac{C}{r^7} \quad (3.36)$$

Following [13], let $\zeta = \frac{4 \sigma}{\sigma^2}$, $\frac{\Lambda}{36} = \xi^2$, and $R^2 = \zeta^2 + \xi^2$, then after a brief calculation (3.5) assumes the form

$$\left( \frac{dr}{d\tau} \right)^2 + V(r) = E \quad (3.37)$$

where the ‘potential energy’ $V(r)$ is given as

$$V(r) = -\frac{C}{r^7} - \frac{C^2}{4 \zeta^2 r^{16}} + \frac{C R^2}{2 \zeta^2 r^7} - \frac{R^4 r^2}{4 \zeta^2} \quad (3.38)$$

and the total energy $E$ is $E = -1$. As always, $\tau$ denotes co-moving time. Thus, the motion of the eight-brane reduces to the motion of a particle in a one-dimensional potential well. It is not too hard to show that the potential energy (3.38) generically has a unique maximum, at which

$$\frac{dV}{dr} = 0$$

For the edification of the reader, we include below a plot of $V(r)$ for generic values of $M$,
It is clear, then, that in general there will exist a unique critical radius \( r_c \) at which the Dirac brane will be static. It is also clear that this radius will be unstable, i.e., generic perturbations of a static Dirac brane will cause the brane to collapse or expand. A rather tedious calculation shows that \( r_c \) is given explicitly as

\[
    r_c = \left[ \frac{(7/2)C(\zeta^2 - \xi^2) + C\sqrt{9\xi^4 + 10\xi^2\zeta^2 + 44\zeta^4}}{R^4} \right]^{1/9}
\]

Thus, branes larger than the critical radius will want to expand, and branes smaller than the critical radius will want to collapse. It is not hard to generalize this analysis to the situation where there is a mass parameter and you are in a massive phase outside of the brane; in that situation, the exterior of the brane is just Schwarzschild-de Sitter space, and the basic equations are very similar with some minor changes.

Of course, an interesting generalization is to turn on the (massive) \( U(1) \) gauge field which comes from the dimensional reduction from eleven dimensions, so that you are basically looking for spherically symmetric black hole solutions in the Einstein-Maxwell-dilaton system described by (2.2)-(2.4). Naively, one might think that this would be important for understanding when these branes will bound supersymmetric vacua. However, we will not pursue further here the question of when these branes are SUSY, for the simple reason that there are some very subtle problems with the \( N = 2 \) SUSY algebra in ten dimensions when the cosmological constant is positive. We feel that it is better to postpone a thorough treatment of this issue for a future paper, which will be devoted to this subject.
IV. SEMICLASSICAL INSTABILITIES OF DIRAC BRANES

The nucleation and annihilation of bubbles of phase transition in the early universe has been studied by a number of authors [6], [13], [14], [19]. Here, we briefly sketch how these results will also go through for Dirac branes.

In [19], the authors calculated the probability that one could ‘create a universe in the laboratory’ by quantum tunneling. By this, they meant “what is the probability that a bubble of false vacuum (i.e., de Sitter space) can appear in a lab where we have attained a super-high mass density of the order $10^{76}$ g/cm$^3$ (i.e., the ‘lab’ is a black hole)”? The authors calculated the rate at which one could create new universes in this way by first finding an instanton, or imaginary time path, which interpolates between the initial state (Schwarzschild) and the final state (Schwarzschild with a bubble of de Sitter in it), then working out the Euclidean action $S_E$ for the instanton path and then using the standard semiclassical approximation for the probability $P$:

$$P \propto e^{-S_E}$$

Crudely, they found that the Euclidean action goes like

$$S_E \propto 1/H^2$$

where $H = (\Lambda^3)^{1/2}$ is the inverse of the Hubble radius. This form of the action follows from the fact that the domain wall sweeps out a three-sphere in imaginary time, and the action is basically the wall tension times the volume of the three-sphere.

In ten dimensions, a Dirac eight-brane sweeps out a nine-sphere in imaginary time; it follows that the Euclidean action for the nucleation of Dirac branes will go like $1/H^8$. In other words, the nucleation of isolated bubbles of massive phase in IIA supergravity will still be highly suppressed.

This shows that Dirac branes can be spontaneously nucleated, but what about the time reverse: How do they annihilate? In a recent and interesting paper, Kolitch and Eardley [6] studied the decay of Vilenkin-Ipser-Sikivie (VIS) domain walls in cosmology. These VIS domain walls are the Minkowski-Minkowski version of the de Sitter-de Sitter (dS-dS) Dirac branes discussed above. That is to say, a VIS domain wall is a repulsive spherical bubble which separates two (compact) portions of Minkowski space (just as a dS-dS Dirac brane separates two compact portions of de Sitter spacetime). It is not hard to see that their construction will in fact go through for the dS-dS Dirac branes. Again, the action for the instanton describing such an annihilation event will go crudely as $1/H^8$.

We have already mentioned that black hole pairs will be nucleated in the presence of the repulsive, dS-dS spherical Dirac branes. This is basically because these dS-dS branes are bubbles bounding two regions of inflationary phase, and we know that black holes will be produced in an inflating (or domain wall) background. Of course, just about anything can be produced in an inflationary background, simply because the repulsive gravitational energy is a natural source capable of pulling virtual loops of matter out of the vacuum. In particular, it is well known that topological defects [26] will also be nucleated in inflation. Typically, a defect nucleates at the Hubble radius $r = H^{-1}$. If the defect is much thinner than the scale of the universe at the moment of nucleation, it makes sense to model the defect
using the Nambu action. That is, it makes sense to assume that the defect is ‘infinitely thin’, and to define the action to be the area of the worldvolume swept out by the defect (multiplied by the characteristic tension, or mass, of the defect). In this limit, the instanton for a loop of string is a two-sphere of radius $H^{-1}$. Similarly, the instanton for a (closed) spherical domain wall is a three-sphere of radius $H^{-1}$, and so on. Thus, the Euclidean action for defect nucleation generically has the form

$$S_E = \mu \text{Vol}(S^n(1/H))$$

where $\text{Vol}(S^n(1/H))$ denotes the volume of an n-sphere of radius $1/H$, and $\mu$ denotes the mass of the monopoles (if $n = 1$), the tension of the string loop (if $n = 2$) or the energy density of the domain wall (if $n = 3$). One therefore finds that the rate of production of these defects is strongly suppressed if the defect tension is very large, or the cosmological constant is very small, as would be expected. Similarly, light defects are likely to be produced in a background with a large cosmological constant.

The point is, there is no reason why these arguments will not apply to the nucleation of loops of fundamental string in the presence of the dS-dS Dirac branes. This sort of string production is rather reminiscent of crossing symmetry. Indeed, fundamental string loops will be created by this process, and it would be interesting to understand the evolution of these strings after they appear.

It would also be interesting to compare the rate of brane annihilation to, say, the rate at which black holes, string loops, or other objects are nucleated in the presence of a brane.

V. CONCLUSION

Perhaps the most exciting thing which these results teach us is that it is possible to describe gravitating brane configurations without losing sight of the brane worldvolume. To put it another way, there is nothing to stop you from defining an effective worldvolume action for these gravitating Dirac branes (presumably the Born-Infeld-Dirac action will do), and studying these branes from the worldvolume point of view.

This should be contrasted to other ‘heavy’, or gravitating, brane configurations in supergravity theories, such as the super $p$-branes for instance [21]. There, the brane worldvolume ‘goes away’, and you are simply left with a geometry which looks rather like a black hole (extended in some extra dimensions), such that the solution ‘interpolates’ between different vacua (generically Minkowski spacetime ‘far’ from the brane, and $(\text{adS})_{p+2} \times S^{D-p-2}$ as you get ‘near’ the brane, as discussed in [21]). In these solutions, there is no brane to be found, and therefore it is meaningless to talk about worldvolume actions.

Of course, given the recent results ([1], [8]) concerning how a string ending on a D-brane will ‘tug’ on the brane, this begs the question: How will a ‘heavy’ string tug on a heavy D-brane? One has to be careful in posing this question, since it is not at all clear that the method of assigning distributional curvatures to spacetimes of low differentiability (i.e., this is what we did when we imposed the Israel matching conditions) will still make sense when one is dealing with extended objects which are not domain walls, that is, where the codimension of the worldvolume relative to the spacetime dimension is greater than one. Research into these problems is currently underway.
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