Vacuum instability and gravitational collapse

Michael Hewitt
Computing Services, Canterbury Christ Church University College,
North Holmes Road, Canterbury, CT1 1QU, U.K.
email: mike@cant.ac.uk, tel:+44(0)1227-767700.

11 December 2002

Abstract

We argue that (0,1) heterotic string models with 4 non-heterotic spacetime dimensions may provide an instability of the vacuum to gravitational polarization in globally strong gravitational fields. This instability would be triggered during gravitational collapse by a non-local quantum switching process, and lead to the formation of regions of a high temperature broken symmetry phase which would be the equivalent in these string models of conventional black holes.

PACS numbers: 11.25.Sq, 11.25.Mj, 04.70Dy.

1 Introduction

In this paper we will argue that in heterotic string theory [1] with 4 space-time dimensions in non-heterotic form (i.e that can be bosonised without twisted boundary conditions, see [2]), the equivalent of conventional black holes are regions of a high temperature broken symmetry phase characterised by a condensate of strings. These would be the ultimately compressed states formed in gravitational collapses unchecked in any other way. The thermodynamic properties of these regions would closely match those of the corresponding Hawking black holes [3]. The analysis presented here has its origin in an attempt to make sense of the interacting string spectrum in the context of thermal duality [4]. It is intended that this paper will give an improved
analysis both of the nature of the final state and of the nucleation mechanisms that can lead to the formation of such states, remedying some of the shortcomings of the scenario proposed in [4]. In section 2 we treat the static case and the region of string condensate equivalent to a black hole, and in section 3 we discuss a nucleation mechanism that could lead to the formation of such regions in the weak tidal gravity of a large collapsing aggregate of matter. Our approach differs from that of Leonard Susskind [5,6] who argues that the relationship between strings and black holes may be understood without introducing a modification of the gravitational field structure around black holes. The scenario outlined below would lead to the formation of a new kind of object which we may call ‘string stars’ in extreme astrophysical collapses. The formation such an object would take place through a non-local quantum effect producing a phase transition. The argument below does not demonstrate conclusively that such a process actually takes place - rather, the intention is to alert the reader to the possibility that such a mechanism may be latent within string theory.

2 Broken Symmetry Phase

At the Hagedorn temperature the partition sum $Z$ for the spectrum of a free string diverges [7]. This is associated with the appearance of a tachyon [8,9] in the string spectrum which we will call the thermalon $\phi$. For free strings, this appears as a pole in the one loop diagram. This pole represents the contribution of long string loops propagating around the thermal circumference $\beta$ in imaginary time, and by modular invariance this has a dual interpretation as the contribution of thermalons propagating around the long loops represented by the corresponding string configurations. In this picture, the thermalon summarises the thermal behaviour of the whole string spectrum. This suggests that the condensation of strings at the Hagedorn temperature, when quartic self interactions are included, would be reminiscent of the condensation of Higgs bosons, and that a high temperature broken symmetry phase should exist [10]. Although the thermalon itself exists only in the Euclidean regime, on continuing amplitudes back to Minkowski spacetime the contribution of the thermalon represents the effects of condensed strings in the high temperature phase. Now it would appear that to produce a region of this high temperature phase would be very difficult - a very large amount of energy would need to be expended to bring thermal radiation in the region.
up to the temperature where long strings would condense. However, we will argue that the required temperatures could be realised by geometric thermal effects in a strong gravitational field. An observer suspended against the strong gravitational acceleration $\alpha$ near an event horizon would experience a heat bath of temperature $2\pi/\alpha$ [11], while the energy density around a black hole of mass $M$ is $O(1/M^4)$ [12], which is small and independent of $\alpha$. As this geometric temperature reaches the Hagedorn point, the observer’s interaction with the heat bath could have the effect of precipitating the condensation transition. Thus, where the geometric temperature of a static observer is at or above the transition temperature, the usual vacuum state may be modified by a symmetry breaking phase transition, and the difference $\Delta \ln Z$ per unit volume between the two phases will have a finite, non-zero value. The broken symmetry phase (BSP) exists between $\beta = \pi/(1 + \sqrt{2})$, and $\beta = \pi(1 + \sqrt{2})$, which we will call the upper and lower Hagedorn points $\beta_+, \beta_-$ respectively.

The formula for the free energy as a function of $\phi$, the thermalon scalar field and $\beta$ in quartic approximation is [10]:

$$F = (-6 + \frac{\pi^2}{\beta^2} + \frac{\beta^2}{\pi^2})\phi^*\phi + \lambda\phi^2\phi^2$$  \hspace{1cm} (1)

where $\lambda \sim g_3^2$. This depends upon $\beta$ through the combination

$$\gamma^2 = (\beta/\pi)^2 + (\beta/\pi)^{-2}$$  \hspace{1cm} (2)

which is invariant under the duality transformation

$$\beta \to \pi^2\beta^{-1},$$  \hspace{1cm} (3)

as a consequence of a compactification duality of imaginary time. Strings at finite temperature have a lattice of heterotic winding numbers in imaginary time, where the winding numbers $(a, b)$ represent null coordinates of a point in an even self dual Lorentzian lattice. These winding numbers give a conformal weight of

$$a^2\pi^{-2}\beta^2 + b^2\pi^2\beta^{-2}$$  \hspace{1cm} (4)

The thermalon itself has winding numbers $(1, 1)$ in the heterotic string theory [13] and so is invariant up to a phase under the thermal duality arising from modular invariance:

$$a \to b, b \to a, \beta \to \pi^2/\beta$$  \hspace{1cm} (5)
which is realised as a reflection in heterotic imaginary time. A change of metric $g_{00} \rightarrow \exp(2\tau)g_{00}$ is represented by the Lorentzian transformation

$$\beta \rightarrow \exp(\tau)\beta.$$  \hfill (6)

From the viewpoint of quantum geometry, the effective circumference of imaginary time for the thermalon, is $\gamma$. The overall physics of the BSP however will not be dual symmetric as spacetime is described by the level zero field $g_{\mu\nu}$ and the coupling of $g_{\mu\nu}$ to $F$ is not dual symmetric. The contribution to $\ln Z$ from the thermalon in the BSP is

$$\Delta \ln Z(\beta, \phi) = -\beta \Delta F.$$  \hfill (7)

The difference in energy density produced by the high temperature phase is given by

$$\Delta \rho = -\frac{\partial}{\partial \beta} \Delta \ln Z$$  \hfill (8)

obtained by varying $g_{00}$. We may note that in thermal equilibrium the value of $\beta$ will vary from place to place according to

$$\sqrt{g_{00}}\beta = \text{const.}$$  \hfill (9)

We might expect that the change in entropy density due to the phase transition is given by

$$\Delta S = \left(1 - \frac{\partial}{\partial \ln \beta}\right) \Delta \ln Z = \beta^2 \frac{\partial \Delta F}{\partial \beta}$$  \hfill (10)

i.e.

$$\Delta S = \beta(\Delta \rho - \Delta F)$$  \hfill (11)

However, a modification of this formula is needed due to the conservation properties of energy in a gravitational field. The essential property of eq.(11) is that because usually

$$\int \beta \rho dv = \text{const}$$

where $dv$ is the 3 dimensional volume element, we have $\int \Delta Sdv$ maximised when $\int \Delta Fdv$ is minimised. The point which we now want to make is that the integral of the gravitational source density $\epsilon = \rho + \text{Tr}(p) = T_{00} + T_{11} + T_{22} + T_{33}$ is the quantity which must be constrained to remain the same to give the same
external gravitational field. For example, the Schwarzschild metric for mass $M$ may be truncated to a flat interior at a surface where the gravitational acceleration is $O(1)$ in string tension units. If the transition is concentrated in a region with thickness $O(1)$, the transverse pressure will be $O(1)$ but the total local energy will only be $O(M)$, or $O(1)$ as viewed by a distant observer. (The energy is supported against gravity by the pressure as at the top of a dome.) The gravitational source in this case is almost entirely converted to pressure. The distinction between $\epsilon$ and $\rho$ only matters if the pressure is contained by gravity - otherwise the flow of momentum is returned by the containment vessel and the integral of $\beta \text{Tr}(p)dv$ vanishes. Thus we require

$$\int \beta \epsilon dv = \text{const} = M. \tag{12}$$

and therefore that $\int \Delta S dv$ should be maximised when $\int \beta \Delta F dv$ is minimised subject to eq.(12). If we write

$$\Delta \epsilon = \Delta F + \Delta S / \beta \tag{13}$$

so that $\Delta S \equiv \beta (\Delta \epsilon - \Delta F)$, replacing eq.(11), then $\Delta S$ has the required property. Note that eq.(13) is consistent with the degeneracy of states needed with a Boltzmann factor based on $\epsilon$ rather than $\rho$, i.e.

$$Z = \sum \exp - \int \beta \epsilon dv \tag{14}$$

as would be expected for the statistical distribution of a conserved quantity.

Now $\Delta S$ represents the difference in entropy between the BSP and the conventional vacuum state. Thus $\ln \Delta S$ represents the effective degeneracy of states in the BSP with $< \phi > \neq 0$. The origin of this degeneracy will be discussed below. Now $\Delta F$ has a minimum at $\beta = \pi$, so $\Delta \ln Z(\beta, \phi) = -\beta \Delta F(\beta, \phi)$ will have a maximum at some $\beta_p(\phi)$ with $\pi < \beta_p(\phi) < \beta_-$. For a region in which $\phi$ is approximately constant, terms involving $\partial_a \phi$ may be neglected. Now a space-filling solution with $\phi$ and $\beta$ constant should exist by an application of the mean value principle. As $\beta$ varies, let $\phi_m(\beta)$ be the circle of values for which $\Delta \ln Z(\beta, \phi)$ is maximised with $\phi$ constant in space and time. Now let $\beta_M$ be the value of $\beta$ for which $\Delta \ln Z(\beta, \phi_m(\beta))$ is maximal, i.e. representing the global maximum of $\Delta \ln Z$. At $\beta = \beta_M$ we have $\Delta \rho = 0, \Delta \text{Tr}(P) > 0$ whereas just below the upper Hagedorn point

$$\Delta \text{Tr}(P)(\beta, \phi_m(\beta)) < -\Delta \rho(\beta, \phi_m(\beta)) \tag{15}$$
since $\Delta \text{Tr}(P) = -3\Delta F$ when $\phi$ is constant and $\ln \Delta Z$ will have a lower order zero at $\beta_+$ than its derivative w.r.t. $\beta$, so that for some $\beta_C$ with $\beta_+ < \beta_C < \beta_M$ we have $\epsilon(\beta_C, \phi_m(\beta_C)) = 0$. These values give a solution with $\phi$ and $g_{00}$ constant. Such a solution may be seen as the interior of a gravitational conductor from which the gravitational acceleration field has been expelled. The energy density has a negative value and the pressure has a positive value, both contributing to a negative spatial curvature: the spatial geometry will be a Bolyai-Lobachevski 3-space with line element

$$du^2 = \frac{dr^2}{1 + a^{-2}r^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2)$$

(16)

giving space-time interval

$$ds^2 = dt^2 - du^2$$

(17)

with

$$a^{-2} = 8\pi G(p_C - \rho_C) \sim O(1)$$

(18)

and $a^{-2}$ is positive since the constant values $p_C, \rho_C$ for pressure and density satisfy $p_C > 0$ and $\rho_C < 0$. This geometry provides an implementation of the holographic principle [14,15], $\delta V \sim \delta S$. By eq.(13) we have $\Delta S \sim -\Delta F > 0$ for the quasi-constant region where $\Delta \epsilon \sim 0$. The excitations in this phase are strings with a higher energy per unit length than free strings due to the condensate $<\phi>$. The thermalon correlation length is given by

$$\lambda = \frac{1}{\mu_{\text{eff}}}$$

(19)

where $\mu_{\text{eff}}$ is the effective thermalon mass. Now because of spontaneous symmetry breaking, the thermalon in the BSP is represented by two different modes, a Goldstone boson $\phi_G$ with $\delta \phi_G/ <\phi>$ imaginary, and a ‘Higgs’ mode $\phi_H$ with $\delta \phi_G/ <\phi>$ real. The Goldstone mode corresponds to a spectrum of strings at the critical temperature, equivalent to free strings with

$$\gamma_F^2(G) = 6$$

(20)

i.e. $\lambda(G) \rightarrow \infty$, whereas the Higgs mode gives a spectrum which is equivalent to that for free strings at a value of

$$\gamma_F^2(H) = 18 - 2\gamma^2$$

(21)
for the quartic model. As $\lambda(H)$ is shortest for $\beta = \pi$ we can see that Higgs mode strings are relatively excluded around the duality point, an effect which is essentially due to the increased energy density of the strings. The effect of the condensate $\langle \phi \rangle$ on the string spectrum is equivalent to a temporal dilatation effect. Strings propagate as if $g_{00}$ were replaced by $g_{00}(G, H)$ with

$$g_{00}(G) = \frac{\beta F(G)^2}{\beta^2} g_{00} = \frac{\beta^2}{\beta^2} g_{00} \quad (22)$$

and

$$g_{00}(H) = \frac{\beta F(H)^2}{\beta^2} g_{00}, \quad (23)$$

where $\beta F(G, H)$ is the largest root of eq.(4) for $\gamma = \gamma F(G, H)$. Another equivalent solution exists by thermal duality by choosing the smaller roots for $\beta$, showing that the physics is effectively continuous at both upper and lower transition temperatures. The effective temporal dilatation produced by $\langle \phi \rangle$ applies as a Lorentzian boost to the heterotic time lattice discussed above, preserving the self duality of the lattice that is needed for consistent propagation of the Goldstone and Higgs strings. The critical Goldstone strings have neutral buoyancy due to the condensate, and are effectively maintained at the Hagedorn transition throughout the BSP. As noted above, the space-filling solution will have a temperature $T_C = 1/\beta C$. Because of the effect of gravitational screening, highly excited string states can exist within the BSP, whereas gravity excludes most of them from the normal phase [4]. Thus the greater entropy of the BSP can be understood as being due to the gravitational screening within the BSP allowing modes such as longer strings, which cannot exist within the normal phase because of gravitational collapse. The next step is to examine the possibility of solutions for finite regions of the high temperature phase. This will allow a separation of space into normal and BSP regions, with long strings effectively confined to the latter. The idea is that the solution above would represent limiting values for the interior region, and that a solution for a finite extent would interpolate between this and an exterior region of conventional vacuum, described by the same solution as the exterior region of a black hole. At the surface where the geometric temperature reaches the lower Hagedorn point, $\phi = 0$, and $\beta$ will interpolate inwards according to the constraint eq.(4).

This condition gives the possibility of thermal equilibrium between the interior region of broken symmetry and an external vacuum region.
would this be possible without an event horizon? In the Euclidean continuation, the region external to a horizon can be described in radial coordinates. The radius \( r \) represents distance from the horizon \( r = 0 \), while imaginary time wraps around circles of constant \( r \) with period \( \beta = 2\pi r \). We envisage a solution where the interior tends to a constant temperature, which would be equivalent to excising a disc around \( r = 0 \) and replacing this with a tube tending to a constant circumference \( \beta C \). This tube would take us in past the point where the horizon would have been, indeed right across the interior region to open out to join another almost flat section at the far boundary of the BSP. Thus a consistent geometric temperature can be maintained without an event horizon because the Euclidean continuation of the space-time is not simply connected. The equilibrium will be typical of phases connected by a first order phase transition coexisting at the transition temperature. First consider a region with zero gauge charges. The degrees of freedom within this surface will be \( g_{ab} \), the spatial components of the metric, \( g_{00} \), the metric time component (\( g_{\mu\nu} \) is the tensor string zero mode), and \( \phi \). Some configuration \( C \) of these fields will give a global minimum of

\[
-\ln Z = -\int \sqrt{\text{det} g_{ab}} d^3x (g_{00}^{-2/3} \beta R + \Delta \ln Z(\beta, \phi)).
\]  

(24)

For 4 dimensional string models where the additional worldsheet degrees of freedom are subject to twisted boundary conditions [2], there is no breathing mode dilaton \( \phi_6 \), and the 4 dimensional dilaton \( \phi_4 \) decouples for the heterotic string, hence they need not be included in eq.(24). (The suggestion in [4] that a modification of the theory needs to be made to enable \( \phi_4 \) to decouple is mistaken.) This feature gives a stable value for the gauge and gravitational couplings, and indicates a special role for 4 dimensional heterotic models. The configuration \( C \) will also be a local minimum, so that it is a solution of the field equation derived from eq.(24). Consider the spherically symmetric (zero angular momentum) case. At the centre, the partial derivatives of the fields \( \phi \) and \( g \) all vanish. As the components of \( g \) will all be constrained by continuity with the external values, there is essentially one degree of freedom: the variation \( \delta \phi \) of the thermalon field from its limiting value, corresponding to different values of \( M \), the total mass of the region. Working inwards from the outer surface, the variations of the fields from their limiting values decay exponentially over a distance of \( O(\ln M) \) to the centre, giving power law dependences on \( M \) there. Again this behaviour is suggestive of a gravitational conductor, and there is a screening mass in the boundary layer \( O(1) \) per unit
area, and $O(1)$ in thickness in their dependence on $M$. For a region carrying
gauge charges, there will be a configuration which minimises
\[
- \ln Z = - \int \sqrt{\det g_{ab}} d^3 x (g^{-2} \beta (R + F^2) + \Delta \ln Z (\beta, \phi)),
\]
where $F$ represents the gauge fields, interpolating inwards from a Reissner-
Nordstrom type solution. It is likely that the gauge charges will also be
concentrated in the boundary layer of the region. We anticipate that a gen-
eralisation to the rotating case will also be possible, perhaps by giving a
uniform angular velocity to the interior. We may also note that although
there will be higher order corrections in $R$ and $F$ to eq.\((23)\), the mean value
argument above should still apply. As a further investigation, for the sim-
plest form of $\Delta \ln Z$ based on eq.\((1)\) an attempt may be made to solve the
non-linear differential equations generated by eqs.\((24,25)\) either numerically
or in closed form, if this is possible.

3 Nucleation Mechanism

In this section we will consider how regions of the BSP could form during
gravitational collapse. The mechanism that we propose is a kind of ‘diagram-
gravitational’ breakdown of the normal phase in globally strong gravitational
fields. There are significant differences with the dielectric breakdown of an
insulator in a strong electric field. One is the unexpected sign of the effect
(as like gravitational charges attract rather than repel). The other is that,
due to Einstein’s equivalence principle, there is no local trigger in the form
of a strong local invariant field strength. Rather, the transition must begin
simultaneously around a critical gravity surface. Such a surface is found by
the physical world ‘trying’ all possible Feynman paths, acting in this as a
quantum-parallel computer. If such a surface is found, the probability of nu-
cleation rapidly switches to almost 1, so that the nucleation process acts as a
non-local ‘everywhere or nowhere’ quantum switch. The long-distance corre-
lations of nucleation vs. non-nucleation would be of Einstein-Podolsky-Rosen
(EPR) type, and would not represent any form of super-luminal causality.
Consider a family of spacelike surfaces $S(t)$ with $S^2$ topology connected by
timelike curves generated by a unit vector field $T$. Choose a frame field $E_i$
with $E_0 = T$, $E_2$ and $E_3$ tangent to $S(t)$ (using two or more patches) and $E_1$
an inward normal. Suppose that each curve has acceleration $\alpha$ orthogonal
to \( S \) where the associated inverse temperature \( \beta = 2\pi/\alpha \) lies in the range \( \beta_- > \beta > \beta_+ \). Applying the thermodynamic equation \( dS = \beta dQ \) and assuming for now the Hawking relationship \( dS = dA/4G \) (we will argue below that this is needed for consistency) gives

\[
d\dot{A} = T(dA) = dS \frac{dA}{dS} = \beta T_0 \sigma dA \frac{4G}{\alpha} = \frac{8\pi G}{\alpha} T_0 \sigma dA = \kappa T_0 \sigma dA 
\]  

where \( 0 \leq \sigma \leq 1 \) is an opacity factor for the absorption of energy at the surface and \( \kappa \) is the constant appearing in Einstein’s field equations. Initially, we will have \( \sigma = 0 \), and we are led to consider surfaces with \( d\dot{A} = 0 \) and \( \beta = \beta_P \) as nucleation sites, where we take \( \beta_P = \beta_P(\phi \sim 0) \). The polarization point value for \( \beta \) is chosen so that polarization may begin. For notation, we will indicate a family with \( d\dot{A} = 0 \) as \( S_0^t \) and if \( \beta = \beta_P \) also, as \( S_0^t(\beta_P) \). An element of a \( S_0^t(\beta_P) \) family may be regarded as a string regulated version of a Penrose closed trapped surface [16], with such a Penrose surface being obtained in the infinite tension limit. Now given such a family, a condensate \( < \phi > \neq 0 \) around \( S_0^t(\beta_P) \) would produce gravitational polarization with \( \rho > 0 \) outside \( S_0^t(\beta_P) \) and \( \rho < 0 \) inside \( S_0^t(\beta_P) \). To produce such a condensate requires a continuous phase for the condensate \( < \phi > \) all around the spherical topology surfaces \( S_0^t(\beta_P) \). This condensate would be self-sustaining as the outward gravitational force on the inner part with \( \beta < \pi \) would balance the inward force on the outer part with \( \beta > \pi \), sustaining the acceleration relative to free-fall needed to maintain the geometric temperature. Nucleation is now possible, and with rising values of \( \sigma \) the outer boundary of the condensate follows eq. (23), with the \( \sigma \) value of the condensate driven closer to 1, while the inner boundary will be at the innermost \( S_0^t(\beta_P) \) surface to have formed. The existence of a \( S_0^t(\beta_P) \) family gives the coherence needed to produce a stable BSP region. Because the boundary of the BSP region is necessarily closed, the nucleation is a holistic effect requiring the existence of such a family of closed surfaces. Because the conditions defining a \( S_0^t(\beta_P) \) family apply locally (including continuity of \( < \phi > \)), in computational terms the question of whether such a family exists represents a non-local problem for which a candidate solution may be confirmed by parallel local checking, and may be called a \textit{locally checkable problem}.

As the gravitational polarization progresses, pressure builds within the shell. The spatial geometry is also significantly affected by the energy needed to produce gravitational screening, with the coefficient of \( dr^2 \) reaching the
Bolyai-Lobachevsky value $O(M^{-2})$ (see eq. (16)) at the zero active gravity (ZAG) point where $\epsilon$ vanishes. The average value of curvature in the $r, \theta$ plane between the screen boundaries and the ZAG surface is given by

$$R_{r,\theta} = \frac{\Delta \pi_{\text{eff}}}{\pi \delta s}$$

(27)

where $\delta s$ is the radial distance, and $\pi_{\text{eff}}$ is the effective value of $\pi$ in the $r, \theta$ plane,

$$\pi_{\text{eff}} = \frac{\delta r}{2 \delta s}$$

(28)

which is $(1 - a^{-2}r^2)^{1/2}$ in the BL region, or $\sim O(1/M)$ for the boundaries of a static shell. This gives positive spatial curvatures of $O(1)$ in the outer layers and negative spatial curvatures of $O(1)$ on the inner layers in agreement with the magnitude and changing sign of $\rho - p$, where $p$ is appropriate component of the pressure matrix. The time needed to produce the radial deflation that produces the BL geometry will be $O(\ln M)$. From the viewpoint of an observer at a constant area surface, the radial contraction is initiated by the pulse of positive energy represented by the ultra-relativistic in-falling matter, and then the radial metric follows an exponential decay as the negative energy screen passes through, stabilizing at the BL geometry. The formation of gravitational screens may occur many times during the collapse process, depending on details of the in-falling matter distribution. Thus, in a local time $O(\ln M)$ an outer gravitational screen like the boundary of the BSP regions of Section 2 will have formed, and the inner and outer gravitational screens will separate, with a $\beta \sim \beta_C$ region between them. The surface gravity in this region will be effectively zero, rising to $2\pi/\beta_P$ on the inside and to a somewhat smaller value $\sim 2\pi/\beta_-$ on the outside, giving a characteristic double peak pattern. The negative gravitational energy formed at the inner part of the condensate will cancel just enough of the energy of the in-falling matter (which will be absorbed into the BSP) to maintain a value of $2\pi/\beta_P$ as the maximum acceleration of $\mathcal{S}^0$ surfaces, so that the conversion of the interior to BSP proceeds by a process of inward induction. Note that thermal equilibrium is maintained at the outer surface of the BSP, but not at the inner boundary which is intrinsically unstable, as the transition to the BSP is in progress there. (Near the inner boundary $\Delta S$ may become negative, suggesting that the inner screen may act as a heat pump moving entropy to the $\Delta S > 0$ part of the BSP.) The inner surface of the BSP will have $\beta < \beta_C$, whereas the nascent $\mathcal{S}^0_\gamma(\beta_P)$ surfaces have $\beta = \beta_P > \beta_C$ and as the boundary
moves further inward, $\beta$ settles to $\beta_c$. Inward induction will be complete when the whole of the interior is converted to a BSP region, which will take a further time of at least $O(\ln M)$ as seen from the outer surface since the boundary between normal and BSP regions is timelike.

During conversion to BSP, space from the normal phase will undergo a radical radial contraction, and its contents will be thermalised. In cases where there is a multiple concentric nucleation of BSP regions, the negative energy inner screen of each region will eventually encounter the positive energy outer screen of the next region in, leading to coalescence. Eventually, all matter will be absorbed and all regions of BSP will coalesce, leading to the formation of a quasi-stable region. The time to complete process of in-falling matter being absorbed at the outer screen, causing this to expand outwards, will be comparable to the time taken for matter to fall to a critical gravity surface in the Schwarzschild metric. This will be $O(\ln M)$ as seen from such a surface, or $O(M\ln M)$ as seen from outside the strong field region - see [17], section 15. This is comparable to the conversion time for the interior by inward induction. The pressure on the outer surface of the BSP region due to accretion will be relatively small, at $O(1/\ln M)$. A small amount of thermal radiation will escape the very strong gravitational field of this hot region, leading to a slow decay like that of a Hawking black hole for an isolated BSP region. In a realistic astrophysical context, further accretion would overwhelm this effect.

We will next make some observations about the relationship of the nucleation process to the framework of quantum mechanics. The appearance of large closed surfaces as nucleation sites is an indication of that nucleation is a non-local quantum process. There does not appear to be a problem with superluminal transmission of information, as a strong gravitational field on the same spatial scale is required to provoke a phase transition, and this would take time $O(M)$ to set up, and the initial nucleation process takes time $O(M)$ as seen from the outside. The conversion of a collapsing object to a BSP region would be analogous to the operation of a quantum information processing device. If we separate space into an inner region $I$ and outer region $I'$ by a closed surface $S = \delta I$, a quantum state may be represented as

\[ |\psi > = |\psi, I' >^i |I >_i . \]

The Hilbert space is decomposed as $H = H_I \otimes H_I$ and the states $|I >_i$ form a basis for $H_I$ while the coefficient states $|\psi, I' >^i$ lie in $H_{I'}$. Here the index $i$ is
specific to \( S \) (an abbreviation for \( i(S) \)). If our knowledge of \( |\psi\rangle \) is restricted to \( I \), or we have no control over the relative phases of the coefficient states, this may be represented by the density matrix

\[
\rho^{j*} = \langle \psi, I' | \psi, I' \rangle^{j*}
\]

(30)

where the index \( j* \) refers to \( <\psi, I'| \), or

\[
\rho = \langle \psi, I'| \psi, I' \rangle^{j*} |I \rangle < I| \rangle^j ,
\]

giving an entropy of entanglement \( S = Tr \rho \ln \rho \). Now the relationship of entanglement between \( I \) and \( I' \) is mutual, and reversing the roles of \( I, I' \) in the above will give a density matrix \( \rho' \) over \( I \) giving the same value of \( S \). The proof is easy: if \( \dim(H_I) = N \), then the \( |\psi, I' >^i \) span a space \( V \) of dimension \( K \leq N \). Let \( |W >^k \) be an orthonormal basis of \( V \) made up, if necessary, to \( N \) elements. Then we can write

\[
|\psi > = a^{k,i} |W >^k |I >^i .
\]

(32)

Now \( \rho = a^\dagger a \) while \( \rho' = aa^\dagger \), and both matrices are evidently Hermitian. The spectra of eigenvalues of \( \rho \) and \( \rho' \) will be the same, since \( Tr(a^\dagger a)^n = Tr(aa^\dagger)^n \) for any \( n \). In fact, if we choose the bases \( |I >^i \) and \( |W >^k \) to consist of eigenvectors of \( \rho \) and \( \rho' \) respectively, then \( \rho = \rho' \). (If the entanglement between \( I \) and \( I' \) is produced by a measurement process, these bases represent the eigenstates of the measured quantities.) The equality of the entropies calculated from \( \rho \) and \( \rho' \) follows. Thus \( S \) also represents the entropy of ignorance of an observer in \( I' \) who has no control over the phases of coefficient states within \( I \). We will now suppose a form of the holographic principle, which is that the entropy of entanglement for any state \( |\psi > \) between \( I \) and \( I' \) is bounded by \( \sim \exp \kappa A(S) \) for some constant \( \kappa \). The two special cases that we need to consider to generate a calculation for the general case are that \( I \) represents (1) a region of the normal phase and (2) a region of BSP. In case (1), the entropy bound is supported by the analysis of Srednicki [18]. The following (very) heuristic argument points in the same direction. To measure and record information \( U \) within \( I \) by generating entanglements with uncontrolled phases to the environment \( I' \) requires energy \( E \geq U/\beta \) at inverse temperature \( \beta \). (At large \( \beta \) there will be a shortage of low energy modes within \( I \) and an inequality will definitely be needed.) Now at \( S \), \( E \) will produce an average gravitational acceleration of \( 4\pi GE/A(S) \) giving
a geometric $\beta = A(S)/2GE$, and using this to solve $U \leq \beta E$ gives $U \leq A(S)/2G$. Presumably, a careful treatment would give consistency with the Hawking value $A(S)/4G$ for black holes. For case (1) it seems that the ‘strong’ holographic principle holds, i.e $\dim(H_I)$ is bounded by a multiple of $A(S)$ as highly excited states are liable to gravitational collapse. Thus, for example, very high excitation levels of bosonic field modes within $I$ would be cut off by the transition to the BSP. For case (2), the combination of hyperbolic spatial geometry and the lower bound on $\gamma$ give a finite upper bound on $\Delta S$ consistent with the ‘weak’ holographic principle, even though gravitational screening means that there is no further collapse and so the dimension of $H_I$ is not necessarily finite in this case. The contribution of the Goldstone strings is understood to be included within $\Delta S$.

For concentric regions $I, J$ with $I \supset J, R = I - J$ we may write

$$|I >_i = R >^j_i |J >_j$$

so that

$$|\psi > = |\psi, I' >^i_i |I >_i = |\psi, I' >^i_i |R >^j_i |J >_j.$$  

(34)

Here $H_I$ is decomposed as $H_R \otimes H_J$ with $|R >^j_i$ in $H_R$. $H_R$ may be viewed as a kind of quotient space with

$$\dim(H_R) = \frac{\dim(H_I)}{\dim(H_J)}.$$  

(35)

Resolving gives

$$|R >^j_i = C^j_r_x^i |R >_r$$

(36)

where the combined index $j, r$ takes as many values as $i$. Thus

$$|\psi > = |\psi, I' >^i_i C^j_r_x^i |R >_r |J >_j.$$  

(37)

During the transition we may take $R$ to be the region of BSP, bounded on the outside by $S_+$ and on the inside by $S_-$. Now $\exp(f_R \Delta \ln Zdv)/Z(R)_{\text{infalling}}$ represents the relative probability of nucleation over no nucleation ($dv$ is the 3-dimensional BSP volume element), and as $f_R \Delta \ln Zdv \sim O(A(S_+ - A(S_-))$, this will usually be overwhelming in gravitational collapse where the conditions for nucleation are met. Thus we have an automatic inter-phase conversion process of the states in $R$

$$M_\phi : |0, R >_r \rightarrow |< \phi >, R >_r.$$  

(38)
where the states \( |< \phi >, R > \), have the condensate \( < \phi > \), the states on
the left having \( < \phi > = 0 \). As noted above, these states will an effective
degeneracy of \( \exp(\int_R \Delta Sdv) \). What are the ‘microstates’ being counted by
this formula? The answer to this question is provided by the Goldstone string
modes of the BSP. They provide a heat bath giving a zero contribution to
\( \Delta \ln Z \) but a positive contribution to \( \Delta S \), and can adjust the contribution
of the Higgs strings to give the correct value for \( \Delta S \). Because the states of the
normal phase are converted rather than copied, the transition process does
not conflict with the ‘Xerox principle’ [6]. Note that because

\[
\frac{\partial \beta(G)}{\partial \beta} = 0
\]

the Goldstone strings have zero effective gravitational energy, (i.e. they dis-
place an equal amount of energy from the condensate) and because there
is no gravitational gradient for them, there is no build up of pressure, and
their contribution to the gravitational source is therefore zero. The absence
of pressure due to the Goldstone strings indicates that this component of the
BSP behaves more like a single long string than a gas of short strings. In-
roducing Goldstone string excitations to reconcile \( \Delta S \) thus does not modify
the spacetime metric, while any modification due to Higgs strings should not
alter the qualitative features of the metric. The negative value of \( \Delta \ln Z \) is
provided by the symmetry breaking phase transition (the potential energy
of \( < \phi > \)), while \( \Delta S \) is provided by the extra string modes of the BSP. At
the end of the nucleation process, when \( R \) no longer has an inner boundary,
\( R = I \) and since this region is in thermal equilibrium with the surrounding
vacuum, consistency seems to require

\[
\int_I \Delta Sdv = S_{\text{Hawking}}
\]

as information can be freely exchanged between the two phases. The effective
potential for \( \phi \) should therefore be consistent with eq. (11). The quantity \( \Delta S \)
may be divided into a ‘bulk’ component \( \Delta S_{\text{int}} \) where \( \beta \sim \beta_C \) and a ‘surface’
boundary layer component \( \Delta S_{\text{surf}} \). Due to the hyperbolic geometry of the
interior, both are proportional to the surface area. We would expect that
\( \Delta S/\Delta A \) would be less for the surface components (i.e the average for the
outer and inner screens) than for the bulk, so that the coalescence process
for BSP shells would be thermodynamically irreversible (i.e. the reverse
process would be very unlikely). Due to the onset of hyperbolic geometry,
the inner area of the surface component would be a fraction $0 < r < 1$ of the outer area, so the quantity $\Delta S/\Delta A$ would tend to a constant value for large $A$, so that this condition would be reasonable. The two phases would coexist locally at the Hagedorn temperature, with the latent heat of the transition used to produce more Goldstone and Higgs strings. These would act as an inner heat bath in equilibrium with the external Unruh/Hawking radiation (except for the small rate of escape). Thus the nucleation mechanism acts as a surface gravity regulator, and is the mechanism which implements the gravitational bound on the information content of the normal phase. If we try to force too much information into a region $I$, the gravitational field of the energy used to store this information will cause BSP nucleation from the boundary surface $S$. Because the holographic principle applies in both the normal and BSP cases it is possible for $M_\phi$ to be one-to-one so that the information content of a collapsing body can be preserved. It is also worth noting that the outer gravitational screen will have states that are strongly entangled with the states of the inner gravitational screen and, as the inner screen rolls inwards, with the interior of $R$, so that it will also act as a thermal information screen, or quantum encryption device, in analogy to the event horizon of a Hawking black hole. Thus $\int_I \Delta S dv$ will also represent the apparent change of entropy to an observer who has access only to $I'$. The BSP will act as a heat bath for the surrounding space, much as the horizon does in Hawking’s theory. As in Hawking’s theory, a small amount of radiation will escape the strong gravitational field of the BSP. During the collapse and radiative decay, the BSP acts as an information bank to hold quantum information about the collapsed object. The nucleation process acts as an extremely sensitive detector which captures essentially all details of the local environment and records them in a region of the quasi 2-dimensional BSP. This process could therefore be viewed as a form of flash holography, converting the collapsing region to a holgram of itself, given the rapid and non-local nature of the transition.

4 Conclusions

If the scenario presented in this paper turns out to be viable, some of the unsatisfactory aspects of the physics of black holes may be avoided. These include the presence of singularities and the problem of time reversibility if black holes are allowed but their time reversals (white holes) are not. The
time asymmetry in forming BSP regions is statistical only, and there is no clash with the CPT theorem. The formation of quasi two-dimensional BSP regions can be viewed as a real hologram formation mechanism, preserving the information content of the collapsed body. Finally we may note an analogy with the physics of the strong interaction, with long string modes confined within BSP regions, where gravity is effectively weak, much as quarks are confined within hadrons where they are asymptotically free.

The author would be very grateful for any comments or suggestions on the contents of this paper.

References

[1] D.Gross, J.Harvey, E.Martinec and R.Rohm, Nucl. Phys. B256 (1985) 253; 267 (1986) 75.
[2] H.Kawai, D.C.Lewellen and S.H.H.Tye, Nucl. Phys. B288 (1987) 1.
[3] J.Hartle and S.W.Hawking, Phys. Rev. D13 (1976) 2108.
[4] M.Hewitt, Phys. Lett. B309 (1993) 264.
[5] L.Susskind, hep-th/9309145.
[6] D.Bigatti and L.Susskind hep-th/0002044.
[7] R.Hagedorn, Nuovo Cim.64 A (1965).
[8] B.Sathiapalan, Phys. Rev. D35 (1987) 3277.
[9] K.H.O’Brien and C.I.Tan, Phys. Rev. D36 (1987) 1184.
[10] J.J.Atick and E.Witten, Nucl. Phys. B310 (1988) 291.
[11] W.G.Unruh, Phys. Rev. D14 (1976) 870.
[12] N.D.Birrell and P.C.W.Davies, Quantum fields in curved space (CUP, Cambridge 1982).
[13] R.Rohm, Nucl. Phys. B237 (1984) 553.
[14] L.Susskind, hep-th/9409089.
[15] C.R.Stephens, G.’t Hooft and B.F.Whiting, gr-qc/9310006.
[16] R.Penrose, Phys. Rev. Lett.14(1965) 57.
[17] A.P.Lightman, W.H.Press, R.H.Price and S.A.Teukolsky, Problem book in relativity and gravitation (Princeton University Press, Princeton 1975)
[18] M.Srednicki, Phys.Rev.Lett71 (1993) 666.