An Efficient Method for Characterizing the Operational Feasible Region of Virtual Power Plant

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Abstract. The penetration of distributed energy resources (DERs) in the power system is increasing considerably in the last years. Virtual power plant (VPP) which provides services for power market and system dispatching is an important method to aggregate DERs. In this paper, the virtual power plant is studied from the perspective of “region”. The operational feasible region (OFR) of the virtual power plant is defined as the set of all active and reactive power output by the virtual power plant, in which the network constraints and the operation constraints of DERs will not be violated. A new approximation method is proposed to characterize the operational feasible region of a VPP. Based on iterative linearization, a strategy is proposed to improve the accuracy of the region description. The effectiveness and efficiency of the proposed method is tested on the IEEE 30-bus system.

Keywords: virtual power plant; aggregation; distributed energy resources; feasible region.

1. Introduction

Distributed energy resources (DERs) represented by renewable energy have been developed rapidly in recent years. But the increasing integration of DERs has brought challenges to the power system operation and scheduling [1]. The uncertainty of DERs will have certain impact on the safety, stability, economy operation of the grid [2]. The virtual power plant (VPP) is believed to be a promising method to solve the new challenges brought by DERs. VPP can be used to aggregating distributed generations, energy storage systems and controllable loads. Hence, VPP can functions as a dispatchable unit to provide services to the grid operations [3].

In the existing researches, the optimal scheduling of the VPP and bidding strategies for participating in the market have been studied extensively. In [4], an agent-based approach is proposed to optimize its energy supply schedule to the grid and maximize profit using storage to cope with the uncertainty. In [5], a new algorithm is proposed to optimize the day-ahead thermal and electrical scheduling of a large scale VPP. In [6], an optimal operation of VPP is modelled in both day ahead and balancing markets as a two-stage stochastic mixed integer linear programming in order to maximize a generation company expected profit. In [7], a robust optimization approach is proposed to deal with uncertainties in wind-power production and market prices. VPP submits the feasible region to the system operator or the wholesale electricity market and then receives the scheduling commands.
To ensure the scheduled output in the boundary node of a VPP is executable, it is necessary for the VPP to consider operational constraints of DER coupled with network restrictions. In essence, the OFR of a VPP is the mapping of the internal network constraints and operational constraints on the $P$-$Q$ plane of the tie-line of a VPP. However, these researches do not consider the reactive power and the topology of internal network. The coupling of the active power and reactive power and the physical topology will impact the system scheduling and security. The region description cannot ensure convergence and accuracy in the limited time \cite{8}. Hence, the security constraints of the VPP need to be formulated, which include the operational constraints of DERs and internal network constraints. The operational feasibility region (OFR) of a VPP describes the set of all active and reactive power output by the VPP after aggregating various resources, in which the network constraints and the operation constraints of DERs will not be violated. Regarding the OFR of a VPP as a boundary condition for system scheduling and market clearing can guide system operator to make decisions. Dispatch commands satisfying the OFR are guaranteed to be executable for the VPP.

There are two fundamental methodologies to characterize the OFR of a VPP \cite{9}:

- a. Monte Carlo estimation: In Monto Carlo based simulations, we need to calculate a large number of power flow scenarios under the randomly generated operating points. The drawback is the fact that massive calculations are required to get an accurate estimate.

- b. Optimization approach: The core idea of this method is to search the boundary of the feasible region \cite{10}. The determination of feasible region only depends on the distribution of extreme points. This approach avoids enumerating feasible points within the feasible region which have little effect on the determination of the feasible region. However, the method in \cite{10} cannot mathematically ensure that the global optimum and the required processing time of the method may be too large in some scenarios. In \cite{11}, an improvement method is proposed to reduce the computing time. But the accuracy of this method cannot satisfy practical applications.

To deal with these difficulties, this paper proposes a new approximation method to calculate the operational feasible region of a VPP. The proposed method guarantees the calculation accuracy and efficiency. The case studies demonstrate that this method can reduce the calculation time while ensuring the calculation accuracy. The contributions of this paper are as follows:

Based on searching boundary points, this paper proposes a new approximation method to characterize the operational feasible region of a VPP.

To speed up the computational time and improve the accuracy of result, this paper proposes an improvement strategy using the iterative linearization.

The rest of this paper is organized as follows: Section II presents the mathematical model of the problem. Section III introduces the proposed approximation method to calculate the OFR of the VPP. Section IV validates the efficiency and effectiveness of the proposed method based on the IEEE 30-bus system. Finally, conclusions are drawn in Section V.

### 2. Mathematical Model

#### 2.1. Security Constraints of VPP

In this section, the security constraints of VPP are introduced. The model considers single time interval. We consider the decoupled $P$-$Q$ operational constraints of DERs.

$$P_{G_{\min}} \leq P_G \leq P_{G_{\max}}, \quad (1)$$

$$Q_{G_{\min}} \leq Q_G \leq Q_{G_{\max}}. \quad (2)$$

Constraints (1) (2) enforce power output bounds of DERs. These two constraints can also be used when considering flexible load, if the adjustable range of active load is determined.

In this paper, we assume the three-phase balance of the internal distribution system for ease of analysis. Constraints (3)-(9) are the network constraints.
\[ P_i = g_i (v_j^2 - v_i v_j \cos \theta_j) - h_i v_i v_j \sin \theta_j, \]  
\[ Q_i = -h_i (v_j^2 - v_i v_j \cos \theta_j) - g_i v_i v_j \sin \theta_j, \]

(3)

(4)

\[ e_G P_D - e_D P_G - e_B P_B = \sum_{j=i} P_{ij}, \]

(5)

\[ e_G Q_D - e_D Q_G - e_B Q_B = \sum_{j=i} Q_{ij}, \]

(6)

\[ v_{i,\min} \leq v_j \leq v_{i,\max}, \]

(7)

\[ \theta_{i,\min} \leq \theta_j \leq \theta_{i,\max}, \]

(8)

\[ P_{ij}^2 + Q_{ij}^2 \leq S_{ij,\max}^2. \]

(9)

The index \( i \) denotes a node in the VPP and the index \( j \) denotes the adjacent node of node \( i \). Eq. (3) (4) represent the alternating current (AC) power flows on each branch. These two formulas are highly nonlinear and increase the computational burden. Eq. (5) (6) refer to active and reactive power balance at each node. \( e_G, e_D \) and \( e_B \) are the incidence matrices. Constraints (7) (8) enforce node voltage magnitude and angle bounds and constraint (9) enforces capacity limits of branches.

2.2. The Mathematical Model of the OFR of a VPP

The main aim to calculate the OFR of a VPP is to avoid excessive burden on the calculation of system scheduling. Hence, the accurate operational feasible region of the VPP must reflect the topology and operating constraints of the internal network, namely, the projection of the high-dimensional operating region on the \( P-Q \) plane of the tie-line of the VPP. The mathematical model of OFR of VPP can be formulated in Eq. (10).

\[ \Theta_{OFR} = \{ w_B \in \mathbb{R}^2 | \forall x \in X, w_B \in W \}. \]

(10)

In Eq. (10), \( w_B = (P_B, Q_B) \), \( x = (v, \theta, P_G, Q_G) \), \( X \) denotes the feasible set of \( x \) subject to the security constraints (1)-(4) and (7)-(9), \( W \) denotes the constraints (5) (6). From this model, \( \Theta_{OFR} \) depicts the feasible set of \( w_B \), in which \( x \) will not violate operational limits and guarantee the active and reactive power balance.

3. Approximation Method

3.1. Algorithm

In this section, the algorithm to calculate the OFR of a VPP will be introduced. It should be noted that due to the coupling of the active and reactive power flows, the perimeter of \( \Theta_{OFR} \) is nonconvex. A nonconvex representation of the OFR of a VPP is not compatible with the existing system dispatch and market clearing algorithms, which are based on convex programming [12]. Hence, after finding the boundary points through the proposed algorithm, we use the convex hull of these points to characterize the operational feasible region. The proposed algorithm is based on optimization approach. By solving an appropriate number of optimal power flow (OPF) problems, the approximation method can find enough boundary points to characterize the OFR of a VPP. Fig. 1 illustrates the proposed algorithm. The calculation steps are as follows:
**Step 1:** Find four initial boundary points. Calculate the minimum and maximum values $P_{B,min}$ and $P_{B,max}$ of $P_B$ as well as the corresponding reactive power. Calculate the minimum and maximum values $Q_{B,min}$ and $Q_{B,max}$ of $Q_B$ as well as the corresponding active power. The four OPF problems are:

$$
\min \eta^T w^T
\text{s.t.} \quad (1)-(9).
$$

$\eta$ equals $(1, 0), (-1, 0), (0, 1), (0, -1)$. $w_B = (P_B, Q_B)$. The four initial boundary points are $w_1, w_2, w_3$ and $w_4$ (the four orange points in Fig. 1).

**Step 2:** Use the formula (12) to find the center point $w_0$ (the blue circular point in Fig. 1) of these four initial boundary points.

$$
w_0 = (w_1 + w_2 + w_3 + w_4)/4.
$$

**Step 3:** Generate new search direction and new boundary points: a) calculate the midpoints of adjacent boundary points separately (the blue triangle points in Fig. 1); b) ray with $w_0$ and the midpoints ($w_0$ is the endpoint); c) search new boundary points on the generated rays (the dotted line in Fig. 1). The new OPF problems are:

$$
\min \eta_n^T w_n^T
\text{s.t.} \quad (1)-(9),
$$

$$
\alpha \cdot w_n^T = \beta.
$$

$\alpha$ and $\beta$ are the coefficient row vector and constant of the linear equation. Row vector $\eta_n$ equals $(1, 1), (-1, -1), (-1, 1), (1, -1)$, respectively. The choice of $\eta_n$ depends on the direction of the ray.

**Step 4:** The convergence criteria used in the proposed algorithm is based on the distance between the new found points and the corresponding midpoint (name the distance $d$). When all $d$ are less than the given value $\varepsilon$, the procedure stops. Note that in the search process, once the distance is less than the given value, no longer search for any point between the two adjacent points found in the previous iteration.

$$
d \leq \varepsilon.
$$

The reason for using the condition (14) as the convergence criteria is that these points that satisfy condition (14) will not contribute to significant changes in the feasible region shape of $w_B$.

The flowchart summarizes the proposed approximation method just described can be seen in Fig. 2.
3.2. Improvement Strategy
The nonlinearity of the AC power flow equations and capacity limits of branches aggravate the computational burden of characterizing the OFR of a VPP [13]. To speed up the calculation time, we use iterative linearization technology as the improvement strategy. On the one hand, replacing nonlinear equations with linear equations can reduce calculation time. On the other hand, iterative calculation can improve calculation accuracy.

3.3. Linear power flow model:
The linear power flow model proposed in [14] is employed. This model regards $v_i^2$ and $\theta_{ij}$ as independent variables. To simplify the losses, we use the simplified loss models in [15].

$$P_{ij}^L = g_{ij} \frac{v_i^2 - v_j^2}{2} - b_{ij} \theta_{ij} + \frac{1}{2} g_{ij} [(\theta_{ij})^2 + (v_i^h - v_j^h)^2].$$  \hspace{1cm} (15)

$$Q_{ij}^L = -b_{ij} \frac{v_i^2 - v_j^2}{2} - g_{ij} \theta_{ij} - \frac{1}{2} b_{ij} [(\theta_{ij})^2 + (v_i^h - v_j^h)^2].$$  \hspace{1cm} (16)

In linear Eq. (15) (16), the loss term is constant related to the hot-start point $(v_i^h, \theta_i^h)$). With the square of nodal voltage magnitudes $v_i^2$ as independent variables, the constraint (7) needs to be reformulated as constraint (17).

$$v_i^{\min} \leq v_i^2 \leq v_i^{\max}.$$  \hspace{1cm} (17)

3.4. Linear branch flow limits:
Constraint (9) is quadratic and represents a circular area. In this paper, the polygonal area covered by two circumscribed squares of a circle is used to approximate the circle area. Constraint (18)-(21) describes the polygonal area.

$$-S_{ij,\max} \leq P_{ij} \leq S_{ij,\max}.$$  \hspace{1cm} (18)
\[-S_{ij}\text{max} \leq Q_{ij} \leq S_{ij}\text{max},\]  
(19)

\[-\sqrt{2}S_{ij}\text{max} \leq P_{ij} + Q_{ij} \leq \sqrt{2}S_{ij}\text{max},\]  
(20)

\[-\sqrt{2}S_{ij}\text{max} \leq P_{ij} - Q_{ij} \leq \sqrt{2}S_{ij}\text{max},\]  
(21)

3.5. Iterative linearization:
The choice of loss constant can introduce inevitable approximation errors in power flow equations. The inappropriate choice of loss constant may lead the AC infeasible dispatch command for the VPP. Hence, the accuracy of the linearized network model needs to be further improved. Considering that network loss modeling has a huge impact on linearization errors, we adopted an iterative correction method to correct the network loss value. The procedure is shown as follows:

**Step 1:** Use cold-start linear power flow model which can be obtained by choosing \(v_0^h\) equals 1.0 p.u., \(\theta_0^h\) equals 0. Store the voltage amplitude and angle \((v_1^h, \theta_1^h)\) after calculation.

**Step 2:** Use the results \((v_k^h, \theta_k^h)\) from the last iteration as the hot-start points to compensate for network loss. Continue the OPF calculation.

**Step 3:** Judge whether the objective function value difference between two iterations is less than the set value, if yes, stop the iteration, otherwise return to step 2.

A flowchart of the iterative linearization can be seen in Fig. 3.

![Flowchart of iterative linearization](image)

4. Case Study
The effectiveness of the proposed algorithm and the computational efficiency are verified based on the IEEE 30-bus system. Optimizations are solved in the MATLAB 2015b environment with Cplex 12.4 on a computer with an Intel(R) Core(TM) i5-4460 CPU.

4.1. Effectiveness validation
The effectiveness of the proposed algorithm is verified in this section. Set the first node of the IEEE 30-bus system as the boundary bus of the VPP. The output active and reactive power range of the boundary node is the OFR of the VPP. Fig. 4 shows the OFR of the VPP. Due to the topology and operational constraints of the internal network, the operational feasible region is an irregular polygon. This shows that it is not accurate to simply use the capacity limits of the tie line as the operational feasible region of the VPP. Using the capacity limits of the tie line as OFR will lead to unexecutable dispatch command. Hence, using the OFR that considering the internal network constraints can guarantee the feasibility of the system dispatch.

To verify the effectiveness of the proposed approximation method, we compare our result with the one obtained by Monte Carlo estimation. The region enclosed by the dotted line can be considered as
the accurate result. The region enclosed by the solid line is the result of the proposed approximation method. In comparison, the OFR of the proposed approximation method is slightly smaller than the accurate feasible region. Considering that the feasible region edge is rarely selected in actual scheduling plan, the result of the proposed method is effective in system dispatch and market clearing.

4.2. Efficiency comparison
In this section, we verify the efficiency of the proposed improvement strategy. Based on the proposed approximation algorithm, we calculate the nonlinear model and iterative linearization model respectively, and compare the solution time. Table I shows the results. By using linearization technology, the proposed method can avoid solving nonlinear models and speed up the optimization solution. As can be seen, the computational efficiency of iterative linearization model is improved by 3 times compared to that of nonlinear model.

| Table 1. Comparison of solution time |
|-------------------------------------|
| **Nonlinear Model** | **Iterative Linearization Model** |
| Computational Time (s) | 102.7960 | 31.2490 |

In summary, the proposed approximation method is effective and efficient.

4.3. Impact of network losses
In this section, the impact of network losses on the operational feasible region of the VPP is researched. Fig. 5 shows the impact of network losses. The feasible region enclosed by the solid line is the result of not considering the network losses. The feasible region enclosed by the dot-dash line is the result of considering the network losses using iterative linearization. Due to not considering the network losses, the operational feasible region has shifted. The existence of network losses can affect the distribution of power flow. Hence, to accurately describe the feasible region requires consideration of the network loss in the linearization model.

4.4. Impact of flexible demand and network topology
In this section, the impact of flexible demand and network topology on the operational feasible region of the VPP is researched. Fig. 6 shows the impact of flexible demand. Fig. 7 shows the impact of network topology.

The existence of flexible demand can increase the output of a VPP and results in larger OFR. We set the system active loads as flexible loads. Assume that the load variation range is 80%–100%. As shown in Fig. 6, since the active load can be reduced according to the needs of users, the active output of VPP increases accordingly. Hence, additional flexible demand will increase the output capacity of the virtual power plant, so that the VPP can provide more services to the system or increase market competitiveness.
Fig. 4 The OFR of the VPP

Fig. 5 The impact of network losses on OFR of the VPP

Fig. 6 The impact of flexible demand on OFR of the VPP
The change of network topology will have an impact on the power flow distribution, and then change the operational feasible region of a VPP. In this section, the N-1 scenario is considered to represent the change of network topology. In IEEE 30-bus system, branch {6, 9} is broken. As is shown in Fig. 7, the shape of the operational feasible region is changed compared with the OFR without N-1 scenario.

From this section, we can conclude that the internal network topology constraints and operational constraints, such as the line limits, the structure of the network and the operational constraints of resources, can have great impact on the shape of OFR of a VPP.

5. Conclusion
To ensure the scheduled output is executable when VPP participate in system dispatch and market clearing, it is necessary for the VPP to consider operational constraints of DER coupled with network restrictions. This paper proposes a new approximation method to characterize the operational feasible region of a VPP. To speed up the calculation time, we use iterative linearization technology as the improvement strategy. Case study based on IEEE 30-bus system verifies the effectiveness and efficiency the proposed approximation method. Additionally, the impact of flexible demand and network topology on OFR of a VPP is also discussed in case study.

The proposed approximation method is based on deterministic model and the uncertainty is neglected. In our future work, the volatility and limited predictability of intermittent DERs will be incorporated in the proposed method.

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