Simplified quartessence cosmology

J. A. S. Lima\textsuperscript{1}$^\dagger$ J. V. Cunha\textsuperscript{1}$^\ddagger$ and J. S. Alcaniz\textsuperscript{2}$^\star$

\textsuperscript{1}Instituto de Astronomia, Geofísica e Ciências Atmosféricas, USP, 05508-900 São Paulo, SP, Brasil and
\textsuperscript{2}Departamento de Astronomia, Observatório Nacional, 20921-400 Rio de Janeiro, RJ, Brasil

(Dated: May 20, 2021)

We propose a new class of accelerating world models unifying the cosmological dark sector (dark matter and dark energy). All the models are described by a simplified version of the Chaplygin gas quartessence cosmology. It is found that even for $\Omega_b \neq 0$, this quartessence scenario depends only on a pair of parameters which can severely be constrained by the cosmological tests. As an example we perform a joint analysis involving the latest SNe type Ia data and the recent Sloan Digital Sky Survey measurement of baryon acoustic oscillations. In our analysis we have considered the SNe type Ia Union sample compiled by Kowalski et al. (2008). At 95.4\% (c.l.), we find for BAO + Union sample, $\alpha = 0.81^{+0.04}_{-0.04}$ and $\Omega_{Q4} = 1.15^{+0.16}_{-0.17}$. The best fit for this simplified quartessence scenario is a spatially closed Universe and its reduced $\chi^2$ is exactly the same of the flat concordance model (ΛCDM).

PACS numbers: 98.80.Es; 95.35.+d; 98.62.Sb

I. INTRODUCTION

The most plausible picture for the observed Universe seems to be represented by a nearly flat scenario dominated by cold dark matter (CDM) and a relativistic component endowed with large negative pressure, usually named dark energy \cite{1,2,3}. Although having different status from a theoretical and observational viewpoints, the actual nature of these dominant components remains unknown until the present. Therefore, in certain sense, one may say that the modern general relativistic cosmology is plagued with the so-called cosmological “dark sector problem”.

Recently, many cosmological models driven by dark matter and dark energy have been proposed in the literature aiming to explain the late time cosmic acceleration \cite{8,9,10,11}. Among these scenarios, a very interesting one was suggested by Kamenshchik et al. \cite{9} and further developed by Bilic et al. \cite{10} and Bento et al. \cite{11}. It corresponds to a class of world models dominated by an exotic fluid, named Chaplygin gas (C-gas), which can be macroscopically characterized by the equation of state (EoS)

\[
p_C = -A/\rho_C^\alpha,
\]

where $\alpha = 1$ and $A$ is a positive constant related to the present-day Chaplygin adiabatic sound speed, $v_s^2 = \alpha A/\rho_C^{1+\alpha}$ (where $\rho_C$ stands for the current C-gas density).

The above equation for $\alpha \neq 1$ constitutes a generalization of the original C-gas EoS proposed by Bento et al. in Ref. \cite{11}. One of its fundamental features comes from the fact that the C-gas becomes pressureless at high redshifts, which suggests a possible unification scheme for the cosmological “dark sector” (CDM plus dark energy). Scenarios driven by a C-gas (without an extra CDM component) are usually termed quartessence models and have been largely explored in the literature \cite{12}.

In most of these quartessence analyses, besides the present value of the C-gas density parameter ($\Omega_C$), the above barotropic EoS implies that one needs to constrain two additional free parameters, namely, $\Lambda$ and $\alpha$ since the baryonic density ($\Omega_b$) may be fixed a priori by using, for instance, nucleosynthesis or the recent Cosmic Microwave Background (CMB) observaions \cite{13}. Therefore, in the context of a general Friedman-Robertson-Walker (FRW) cosmologies quartessence scenarios require at least 3 parameters to be constrained by the data (see for instance, Bertolami et al. \cite{14}). In other words, there are so many parameters to be constrained by the data, that a high degree of degeneracy on the parametric space becomes inevitable. The common solution in the literature to reduce the number of free parameters (motivated by current CMB results) is to assume a flat geometry, i.e., $\Omega_{Q4} = 1 - \Omega_b$, where $\Omega_b$ and $\Omega_{Q4}$ stand, respectively, for the baryons and C-gas density parameters.

In the last few years, some generalizations of the original C-gas \cite{13,15,16,17,18}, or even of its extended version \cite{19} have appeared in literature. In these cases, the number of free parameters is usually increased, and, as consequence, the models become mathematically richer although much less predictive from a physical viewpoint.

In a recent paper (from now on paper I) we took the opposite way, that is, we have proposed a large set of cosmologies driven by dark energy plus a CDM component where the dark energy component was represented by a simplified version of the Chaplygin gas whose equation of state is described by just one free parameter \cite{20}. By adding the flatness condition the resulting cosmology depends only of two free dimensionless quantities, namely: the density parameter, $\Omega_m$, and the equation of state.
parameter, \( \alpha \).

In this work we explore the results of paper I now for a Quartessence version thereby reducing still more the parameter space. In this way, we discuss what we believe to be the simplest Quartessence scenario, that is, the one with the smallest number of free parameters. As we shall see, by an additional physical condition, the allowed range of the \( \alpha \) free parameter is also restricted a priori, which makes not only the relevant parametric space biderimensional - even for nonflat spatial sections - but also (and more important) the model can be more easily discarded or confirmed by the present set of observations since the range of its free parameter is physically limited from causality considerations. We test the viability of this simplified Quartessence approach by discussing the constraints imposed from current SNe Ia observations, compilation obtained by Supernovae Cosmology Project (SCP) group, and Large Scale Structure (LSS) data. As we shall see the model passes the background tests discussed here and its reduced \( \chi^2 \) test is slightly smaller than the one of the \( \Lambda CDM \) model.

II. A SIMPLIFIED QUARTESSENCE SCENARIO

Let us now consider that the geometrical properties of the observed Universe are described by the general FRW line element

\[
ds^2 = dt^2 - a(t)^2 \left( \frac{dr^2}{1 - kr^2} + r^2 d\Sigma^2 \right),
\]

where \( a(t) \) is the scale factor, \( d\Sigma^2 \) is the area element on the unit 2-sphere, \( k = 0, \pm 1 \) is the curvature parameter and we have adopted the metric signature convention \((+,−,−,−)\). Throughout this paper we adopt units such that \( c = 1 \). The matter content of the Universe is assumed to be composed of a baryonic component plus the Quartessence C-gas fluid.

Since each component is separately conserved, one may integrate out the energy conservation for the C-gas, \( \dot{\rho}_C = -3H(\rho_C + p_C) \), to obtain the following expression for its energy density \([11, 13, 21]\)

\[
\rho_C = \rho_{C_0} \left[ A_s + (1 - A_s)a^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}},
\]

where \( A_s = A/\rho_{C_0}^{1+\alpha} \) is a convenient dimensionless constant (as usual, the subscript "0" denotes present-day quantities). In the background defined by (2), the Friedmann equation for a conserved C-gas plus the baryonic component reads

\[
\mathcal{H} = \left[ \Omega_h \left( \frac{a_0}{a} \right)^3 + \Omega_Qf(A_s, \alpha) + \Omega_k \left( \frac{a_0}{a} \right)^2 \right]^{1/2},
\]

where \( \mathcal{H} \equiv H/H_0 \) (\( H \) is the Hubble parameter), the function \( f(A_s, \alpha) \) is given by \( f(A_s, \alpha) = \left[ A_s + (1 - A_s)(\frac{a_0}{a})^{3(\alpha+1)} \right]^{1/(1+\alpha)} \) and \( \Omega_k \) is the fractional contribution of the spatial curvature to \( \mathcal{H} \). Note that, besides the Hubble parameter \( H_0 \), we still have 3 additional parameters in this case \((\alpha, A_s, \Omega_{Q4})\), since the baryonic contribution is defined to be \( \approx 4.6\% \) from current CMB experiments \([22]\). This is the standard treatment. Therefore, the important aspect to be discussed at this point is how to reduce the quartessence C-gas parameters based on reasonable physical requirements?

In order to answer the above question, we follow the arguments of Ref. \([20]\). Note that the C-gas adiabatic sound speed reads

\[
v_s^2 = \frac{dp}{d\rho} = \frac{\alpha A}{\rho C_0^{1+\alpha}},
\]

which must be positive definite for a well-behaved gas (zero in the limit case of dust). Note also that the present-day C-gas adiabatic sound speed is \( v_{s0}^2 = \alpha A/\rho_{C_0}^{1+\alpha} \), or still

\[
v_{s0}^2 = \alpha A/\rho_{C_0}^{1+\alpha} = \alpha A_s.
\]

Therefore, from the above equation one clearly see that if the parameter \( A_s \) is a function of the index \( \alpha \), i.e., \( A_s \rightarrow A_s(\alpha) \), the number of free parameters is naturally reduced, and, as an extra bonus, the positiveness of \( v_s^2 \) at any time, as well as its thermodynamic stability, is naturally guaranteed. Clearly, among many possibilities the simplest choice is \( A_s \propto \alpha \), which we assume in this paper. In this case, \( v_{s0}^2 = \alpha^2 \), or more generally, \( v_s^2 = \alpha^2(\rho C_0/\rho)^\alpha \). Note also that, since the light speed is a natural cutoff for the sound propagation, it follows that \( v_{s0} = |\alpha| \leq 1 \), thereby restricting \( \alpha \) to the interval \([-1,1]\]. An additional constraint can still be imposed to this parameter. In fact, with \( A_s \propto \alpha \), the simplified C-gas EoS (1) becomes

\[
\rho_C = -\alpha \rho_{C_0} \left( \frac{\rho C_0}{\rho C} \right)^\alpha,
\]

so that a negative pressure is obtained only for positive values of \( \alpha \). In other words, this accounts to saying that the combined requirements from causality along with the observed accelerating stage of the Universe naturally restrict the parameter \( \alpha \) to the interval \( 0 < \alpha \leq 1 \).[23]

Note that the simplified quartessence component preserves the unifying character of the original C-gas, i.e., it behaves as a pressureless fluid (non-relativistic matter) at high-z while, at late times, it approaches the quintessence behavior, which now is fully characterized by the \( \alpha \) parameter. However, note also that, even in this limiting case, the sound speed is positive.

In this simplified approach, Eq. (4) is rewritten as

\[
\mathcal{H} = \left[ \Omega_h \left( \frac{a_0}{a} \right)^3 + \Omega_Q g(\alpha) + \Omega_k \left( \frac{a_0}{a} \right)^2 \right]^{1/2},
\]

where the function \( g(\alpha) \) is simply given by \( g(\alpha) = \alpha + (1 - \alpha)(\frac{a_0}{a})^{3(\alpha+1)} \). So, that the only remaining
parameters to be determined in this unified dark matter/energy scenario are \( \alpha \) and \( \Omega_{Q4} \). In what follows, we confront this simplified quartessence scenario with some of the most recent SNe Ia and Large Scale Structure (LSS) data.

III. OBSERVATIONAL CONSTRAINTS

A. SNe Ia

Let us first investigate the bounds arising from SNe Ia observations on the SC-gas scenario described above. To this end we use the most recent SNe Ia observations, namely, the Union compilation \cite{3}. It includes 13 independent sets with SNe from the SCP, High-z Supernovae Search (HZSNS) team, Supernova Legacy Survey and ESSENCE Survey, the older datasets, as well as the recently extended dataset of distant supernovae observed with HST. After selection cuts, the robust compilation obtained is composed by 307 SNe Ia events distributed over the redshift interval \( 0.015 \leq z \leq 1.55 \). Figure 1 shows residual magnitude versus redshift for 307 SNe Ia from SCP Union compilation. The Union sample is illustrated on a residual Hubble Diagram with respect to the empty universe model (\( \Omega_{total} = 0 \)).

The predicted distance modulus for a supernova at redshift \( z \), given a set of parameters \( \mathbf{p} \), is

\[
\mu_p(z|\mathbf{p}) = m - M = 5 \log d_L + 25,
\]

where \( m \) and \( M \) are, respectively, the apparent and absolute magnitudes, the complete set of parameters is \( \mathbf{p} \equiv (H_0, \Omega_{Q4}, \alpha) \) and \( d_L \) stands for the luminosity distance (in units of megaparsecs),

\[
d_L = H_0^{-1}(1+z) \frac{1}{\sqrt{\Omega_k}} \xi \left( \sqrt{\Omega_k} \int_{x'}^1 \frac{dx}{x^2 \mathcal{H}(x; \mathbf{p})} \right),
\]

with \( x' = (1+z)^{-1} \), \( \mathcal{H}(x; \mathbf{p}) \) the expression given by Eq. (8), and the function \( \xi(x) \) is defined as \( \xi(x) = \sin(x) \) for a closed universe, \( \xi(x) = \sinh(x) \) for an open universe and \( \xi(x) = x \) for a flat universe.

We estimated the best fit to the set of parameters \( \mathbf{p} \) by using a \( \chi^2 \) statistics

\[
\chi^2 = \sum_{i=1}^{N} \frac{[\mu_p^i(z|\mathbf{p}) - \mu_o^i(z|\mathbf{p})]^2}{\sigma^2_i},
\]

with the parameters \( \Omega_{Q4} \) and \( \alpha \) spanning the interval \([0,1]\) in steps of 0.01. In the above expression, \( N = 307 \), \( \mu_o^i(z|\mathbf{p}) \) is given by Eq. (9), \( \mu_p^i(z|\mathbf{p}) \) is the distance modulus for a given SNe Ia at \( z_i \), and \( \sigma_i \) is the uncertainty in the individual distance modulus. In our analysis, \( H_0 \) is considered a nuisance parameter so that we marginalize over it.

In Figures (2a) we plot the results of our statistical analysis. Contours of constant likelihood (99.73%, 95.4% and 68.3%) are shown in the parametric space \( \alpha - \Omega_{Q4} \). It displays the results for the Union SCP compilation. Note that although degenerate in \( \Omega_{Q4} \), the parameter \( \alpha \) is now considerably more restricted than in the standard C-gas approach (see, e.g., Fig. 4 of Ref. \cite{23}). In particular, note also that for any value of the C-gas density parameter, models with \( \alpha \lesssim 0.73 \) are ruled out at 99.73% level. The best-fit model for this analysis occurs for \( \Omega_{Q4} = 1.02 \) and \( \alpha = 0.83 \) with \( \chi^2_{\text{min}} = 310.4 \) (\( \chi^2_{\text{min}}/\nu = 1.01 \), where \( \nu = \text{degrees of freedom} \)). At 95.4% c.l. we also find \( 0.57 \leq \Omega_{Q4} \leq 1.40 \) and \( 0.77 \leq \alpha \leq 0.92 \).

B. SNe Ia + LSS analysis

In order to break possible degeneracies in the \( \Omega_{Q4} - \alpha \) space, we study now the the joint constraints on this plane from SNe Ia and LSS data. For the LSS data, we use the recent measurements of the BAO peak in the large scale correlation function detected by Eisenstein et al. \cite{24} using a large sample of luminous red galaxies from the SDSS Main Sample. The SDSS BAO measurement provides \( A = 0.469(n_s/0.98)^{-0.39} \pm 0.017 \), with \( A \) defined as

\[
A = \frac{\Omega_M^{1/2}}{\mathcal{H}(z_{\text{BAO}}; \mathbf{p})^{1/3}} \frac{1}{z_{\text{BAO}} \sqrt{\Gamma(\Omega_k; \mathbf{p})}} \frac{1}{\sqrt{\xi^2(\sqrt{\Omega_k} \int_{x'}^1 \frac{dx}{x^2 \mathcal{H}(x; \mathbf{p})})^{1/3}}},
\]

where \( z_{\text{BAO}} = 0.35 \), \( \mathcal{H}(z_{\text{BAO}}; \mathbf{p}) \) is given by Eq. (8), and we take the scalar spectral index \( n_s = 0.96 \), as.
given by Komatsu et al. \cite{22}. In the above expression, \( \Gamma(z_{\text{BAO}}) \) is the dimensionless comoving distance to \( z_{\text{BAO}} \), and \( \Omega_M = \Omega_\Lambda (1 - \alpha) \Omega_{Q4} \), where \( \Omega_\Lambda \) is the baryonic component and \( (1 - \alpha) \Omega_{Q4} \) is the portion of the Chaplygin gas that acts like dark matter. It should be noticed that the dark matter contribution was derived here by using the separation proposed in Ref. \cite{25} (see also \cite{26}).

As shown in Panel (2b) the regions representing the constraints from SDSS BAO measurements on the parameter space \( \Omega_{Q4} - \alpha \) are approximately orthogonal to those arising from SNe Ia data, which indicates that possible degeneracies in this plane may be broken by this combination of observational data. Figure (2c) shows the results of our joint analyses for the BAO + Union sample. Note that the available parametric plane in both cases are considerably reduced relative to the former analyses (Figs. 2a, 2b and 2c). For the BAO + Union sample we find \( \alpha = 0.81^{+0.04}_{-0.04} \) and \( \Omega_{Q4} = 1.19^{+0.16}_{-0.17} \) at 95.4\% (c.l.) with \( \Omega_k = -0.19^{+0.17}_{-0.16} \). This best-fit scenario corresponds to a closed accelerating universe with \( q_0 \simeq -0.7 \), a total age of the Universe of \( t_0 \simeq 10^{10} h^{-1} \) Gyr, and a D/A transition redshift (from deceleration to acceleration) \( z_{\text{D/A}} \simeq 0.5 \).

At this point, it is interesting to compare the above constraints with some independent analyses. In the context of the \( \Lambda \)CDM model, for instance, the WMAP 5y constrains the curvature parameter to be \( \Omega_k = -0.099^{+0.085}_{-0.100} \) (95\%) \cite{22} in nice agreement with our result. In the same vein, the age of the Universe falls on the interval 13.5-14.0 Gyr, or equivalently, \( 9.3 h^{-1} < t_0 < 10.5 h^{-1} \) Gyr which is also comparable with the above values. In addition, recent kinematic studies (with no gravity theory) using SNe type Ia also leads to the constraints \(-0.5 \lesssim q_0 \lesssim -1 \) and \( 0.3 \lesssim z_{\text{D/A}} \lesssim 0.9 \) (1\%) \cite{27}. Finally, it is also worth noticing that many alternative scenarios unifying dark matter and dark energy and even accelerating cosmologies with no dark energy have been proposed in the literature \cite{8, 28}. Usually, such models are able to explain not only the present accelerating expansion, but also the majority of the so-called background tests (see the paper by Ellis et al. \cite{8} for a general analysis involving different scenarios).

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2}
\caption{(a) Contours represent confidence regions for Union data \cite{3}. Our analysis furnishes the regions on \( \Omega_{Q4} \) and \( \Omega_k \) with \( \Omega_k = 1.02^{+0.38}_{-0.45} \) (2\%). (b) Contours on the space parameter \( (\Omega_{Q4}, \alpha) \) from a joint analysis involving the Sloan Sky Digital Survey (SDSS) baryon acoustic oscillations. The corresponding 68\%, 95.4\% and 99.73\% c.l. are shown. (c) We display the results for the SNe sample plus BAO. The best fit and confidence regions are \( \alpha = 0.81^{+0.04}_{-0.04} \) and \( \Omega_{Q4} = 1.15^{+0.17}_{-0.17} \) (2\%).}
\end{figure}

\section{IV. Final Remarks}

A considerable amount of observational evidence suggests that the current evolution of our Universe is fully dominated by two dark components, the so-called dark matter and dark energy. The nature of these components, however, is tantalizing mystery at present, and it is not even known if they constitute two separate substances. In this paper, we have argued that one of the candidates for a unifying dark matter/dark energy scenario, a C-gas quartessence whose EoS is given by Eq. (1), may have a very simplified description. We have postulated that if \( A_s \) is a function of the index \( \alpha \) the resulting FRW cosmology (with arbitrary curvature) can be completely described by a pair of parameters \( (\alpha, \Omega_{Q4}) \). For the sake of simplicity, we have considered \( A_s \propto \alpha^n \) with \( n = 1 \).

By considering this class of parameterization we have investigated the constraints from current SNe Ia and LSS data. We have shown that, differently from the original C-gas models (in which the value of the index \( \alpha \) is completely degenerated) a joint analysis involving these data sets restricts considerably the \( \Omega_{Q4} - \alpha \) parametric space [Fig. (2c)] with \( \alpha = 0.81^{+0.04}_{-0.04} \) and \( \Omega_{Q4} = 1.15^{+0.16}_{-0.17} \). At the level of SNe Ia data and BAO, we may conclude that this class of quartessence scenario passes this combination of tests, thereby providing an interesting possibility to a dark matter/dark energy unification.
noticing that the best-fit for this simplified quartessence scenario corresponds to a spatially closed universe and, with the same number of parameters, the $\chi^2_{\text{min}} = 311.1$ is slightly smaller than the one of the flat concordance model ($\Lambda$CDM). Still more important, the reduced $\chi^2$ (by degree of freedom) in our curved scenario, $\chi^2/dof = 1.02$, is exactly the same of the cosmic concordance flat model.

Acknowledgments

JASL and JSA are partially supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brazil). JVC and JASL are also supported by Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP), 2005/02809-5 and 04/13668-0, respectively.

[1] S. Perlmutter et al., Astrophys. J. 517 (1999) 565; A. Riess et al., Astrophys. J. 116 (1998) 1009; C. L. Bennett et al., Astrophys. J. Suppl. 148 (2003) 1.
[2] A. G. Riess et al., Astrophys. J. 607 (2004) 665; P. Astier et al., Astron. and. Astrophys. 447 (2006) 31.
[3] M. Kowalski et al., Astrophys. J. 686 (2008) 749. arXiv:0804.1142
[4] T. Padmanabhan, Phys. Rep. 380 (2003) 235; P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75 (2003) 559; J. A. S. Lima, Braz. J. Phys. 34 (2004) 194; E. J. Copeland, M. Sami and S. Tsujikawa, Int. Jour. Mod. Phys. D 15 (2006) 1753.
[5] W. Chen and Y-S. Wu, Phys. Rev. D 41, (1990) 695; D. Pavón, Phys. Rev. D 43 (1991) 375; J. C. Carvalho, J. A. S. Lima and I. Waga, Phys. Rev. D 46 (1992) 2404; J. A. S. Lima and J. M. F. Maia, Phys. Rev. D 49 (1994) 5597; J. C. Carvalho and J. A. S. Lima, Gen. Rel. Grav. 26, 909 (1994); J. A. S. Lima, Phys. Rev. D 54, 2571 (1996), gr-qc/9605055; I. L. Shapiro, J. Sola and H. Stefancic, JCAP 0501, 012 (2005); J. F. Jesus et al., Phys. Rev. D 78 (2008) 063514, arXiv:0806.1366 [astro-ph].
[6] M. S. Turner and M. White, Phys. Rev. D 56, R4439 (1997); T. Chiba, N. Sugiyama and T. Nakamura, Mon. Not. Roy. Astron. Soc. 289, L5 (1997); J. A. S. Lima and J. S. Alcaniz, Astron. Astrophys. 357, 393 (2000), astro-ph/0003189; J. V. Cunha, L. Marassi and R. C. Santos, JMP D 16, 403 (2007); R. C. Santos and J. A. S. Lima, Phys. Rev. D 77 (2008) 083505, arXiv:0803.1865 [astro-ph].
[7] B. Ratra and P. J. E. Peebles, Phys Rev D37, 3406 (1988); C. Weterrich, Nucl. Phys. B 302 (1988) 668; R. R. Caldwell, Phys. Lett. B 545, 23 (2002); S. M. Carroll, M. Hoffman and M. Trodden, Phys. Rev. D 68, 023509 (2003); F. C. Carvalho et al., Phys. Rev. Lett. 97, 081301 (2006), astro-ph/0608439; Europhys. Lett. 83, 2901 (2008).
[8] H. Kim, Mon. Not. Roy. Astron. Soc. 364, (2005) 813; A. Arbey, Phys. Rev. D 74 (2006) 043516; R. R. Khuri, Phys. Lett. B 568 (2005) 8; J. R. Ellis et al., Astropart. Phys. 27 (2007) 185; N. E. Mavromatos et al., Astropart. Phys. 29 (2008) 442.
[9] A. Kamenshchik, U. Moschella and V. Pasquier, Phys. Lett. B 511 (2001) 265.
[10] N. Bilić, G. B. Tupper and R. D. Viollier, Phys. Lett. B 535 (2002) 17.
[11] M. C. Bento, O Bertolami and A. A. Sen, Phys. Rev. D 66 (2002) 043507.
[12] T. Matos and L. A. Ureña-Lopez, Class. Quantum Grav. 17 (2000) L75; Phys. Rev. D 63 (2001) 063506; A. Davidson, D. Karnsik and Y. Lederer, gr-qc/0111107; C. Wetterich, Phys. Rev. D 65 (2002) 123512; S. Kasuya, Phys. Lett. B 515 (2001) 121; M. Makler, S. Q. de Oliveira and I. Waga, Phys. Lett. B 68 (2003) 123521; L. M. G. Beça et al., Phys. Rev. D 67 (2003) 103501(R); J. S. Alcaniz and J. A. S. Lima, Astrophys. J. 618 (2005) 16; L. Amendola, I. Waga e F. Finelli, JCAP 0511 (2005) 009.
[13] J. P. Kneller and G. Steigman, Phys. Rev. D 67 (2003) 063501; G. Steigman, Int. J. Mod. Phys. E 15 (2006) 1.
[14] O. Bertolami, A. A. Sen, S. Sen and P. T. Silva, Mon. Not. Roy. Astron. Soc. 353 (2004) 329.
[15] X. Zhang, F.-Q. Wu and J. Zhang, JCAP 0601 (2006) 003.
[16] L. P. Chimento, Phys. Rev. D 69 (2004) 123517; L. P. Chimento and R. Lazkoz, Phys. Lett. B 615 (2005) 146.
[17] M. K. Mak and T. Harko, Phys. Rev. D 71 (2005) 104022; S. S. E Costa, M. Ujevic, and A. F. Dos Santos, General Relativity and Gravitation 40 (2008) 1683; H. Benaoum, arXiv:hep-th/0205140.
[18] Z.-K. Guo and Y.-Z. Zhang, Phys. Lett. B 645 (2007) 326; W. Wang et al., Mod. Phys. Lett. A 20 (2005) 1443.
[19] A. A. Sen and R. J. Scherrer, Phys. Rev. D 72 (2005) 063511.
[20] J. A. S. Lima, J. S. Alcaniz and J. V. Cunha, Astropart. Phys. 30 (2008) 196, astro-ph/0608469.
[21] J. V. Cunha, J. S. Alcaniz and J. A. S. Lima, Phys. Rev. D 69 (2004) 083501, astro-ph/0306319.
[22] E. Komatsu et al., arXiv:0803.0547 [astro-ph]; D. N. Spergel et al., Astrophys. J. Suppl. 170 (2006) 377.
[23] J. S. Alcaniz, D. Jain and A. Dev, Phys. Rev. D 67 (2003) 043514.
[24] D. J. Eisenstein et al., Astrophys. J. 633 (2005) 560.
[25] M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev D 70 (2004) 083519.
[26] M. Makler, S. Q. de Oliveira and I. Waga, Phys. Lett. B 555 (2003) 1.
[27] J. V. Cunha and J. A. S. Lima, Mon. Not. Roy. Astron. Soc. 390 (2008) 210, arXiv:0805.1261 [astro-ph]. See also, J. V. Cunha, arXiv:0811.2379 [astro-ph].
[28] W. Zimdahl and D. Pavon, Int. J. Mod. Phys. D 3, 1994 327; J. A. S. Lima, A. S. M. Germano and L. R.
W. Abramo, Phys. Rev. D 53 (1996) 4287; J. A. S. Lima and J. S. Alcaniz, Astron. Astrop. 348 (1999) 1, astro-ph/9902337; E. W. Kolb, S. Mattarrese and A. Riotto, Phys. Rev. D 71 2005 023524; J. A. S. Lima, F. E. Silva and R. C. Santos, Class. Quant. Grav. 25 (2008) 205006, arXiv:0807.3379 [astro-ph].