New anisotropic star solutions in mimetic gravity

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Abstract We extract new classes of anisotropic solutions in the framework of mimetic gravity, by applying the Tolman–Finch–Skea metric and a specific anisotropy not directly depending on it, and by matching smoothly the interior anisotropic solution to the Schwarzschild exterior one. Then, in order to provide a transparent picture we use the data from the 4U 1608-52 pulsar. We study the profile of the energy density, as well as the radial and tangential pressures, and we show that they are all positive and decrease toward the center of the star. Furthermore, we investigate the anisotropy parameter and the anisotropic force that are both increasing functions of the radius, which implies that the latter is repulsive. Additionally, by examining the radial and tangential equation-of-state parameters, we show that they are monotonically increasing, not corresponding to exotic matter. Concerning the metric potentials, we find that they have no singularity, either at the center of the star or at the boundary. Furthermore, we verify that all energy conditions are satisfied, we show that the radial and tangential sound speed squares are positive and sub-luminal, and we find that the surface redshift satisfies the theoretical requirement. Finally, in order to investigate the stability we apply the Tolman–Oppenheimer–Volkoff equation, we perform the adiabatic index analysis, and we examine the static case, showing that in all cases the star is stable.

1 Introduction

Astrophysical compact objects, such as neutron stars and black holes, can serve as a crucial laboratory to investigate gravitational fields in the strong-field regime and thus test General Relativity and its possible extensions [1–5]. Such extensions usually arise through the consideration of higher-order terms in the Einstein–Hilbert Lagrangian, such as in $f(R)$ gravity [6], in Gauss–Bonnet and $f(G)$ gravity [7, 8], in Weyl gravity [9], in Lovelock and $f(Lovelock)$ gravity [10, 11], in scalar–tensor theories [12–15], etc. (for a review, see [16]). Additionally, one may consider different classes of modifications by modifying the equivalent, torsional formulation of gravity, resulting in $f(T)$ gravity [17, 18], in $f(T, T_G)$ gravity [19], in scalar–torsion theories [20], etc. Hence, in the literature one may find many studies of spherically symmetric solutions in the framework of modified gravity [21–56].

One class of gravitational modification with interesting applications is mimetic gravity [57, 58], which can be obtained from general relativity through the isolation of the conformal degree of freedom in a covariant way, by applying the re-parametrization of the physical metric in terms of a mimetic field and an auxiliary metric. In this way, the field equations exhibit an additional term arising from the mimetic field. Since in a cosmological framework this term may be considered to correspond to a dust fluid component, the theory could be applied to describe cold dark matter in a “mimetic” way. Nevertheless, mimetic gravity can be extended in many ways, interpreted as a modification of gravity [58–85] (for a review see [86]). Since mimetic modified gravity has many interesting applications at the cosmological framework (among them the ability to alleviate the cosmological tensions [87], and to track possible quantum-related defects [88]), an amount of research has been devoted to the investigation of the spherically symmetric solutions too [89–105].

Mimetic theory does not exhibit any difference from general relativity in flat spacetime. Therefore, we will test the mimetic theory in the frame of stellar structure models using two equations of states and radial metric potential $g_{rr}$, and confront the output results with GR. Additionally, we are interested in extracting new anisotropic star solutions in mimetic gravity by imposing the Tolman–Finch–Skea metric [106], and in particular to examine the stability of the solutions as well as the behavior of the anisotropy. The plan of the work is as follows. In Sect. 2, we briefly review mimetic gravity, presenting the field equations. In Sect. 3, we extract...
new classes of anisotropic solutions. Then, in Sect. 4 we use the data from the 4U 1608-52 pulsar in order to investigate numerically the features of the obtained anisotropic stars, namely the profile of the energy density, as well as the radial and tangential pressures, the anisotropy factor and the radial and tangential equation-of-state parameters. In Sect. 5, we study the stability of the solutions, applying the Tolman–Oppenheimer–Volkoff (TOV) equation, the adiabatic index, and we examine the static case. Finally, in Sect. 6 we summarize the obtained results.

2 Mimetic gravity

In this section, we briefly present mimetic gravity. Starting from general relativity and parametrizing the physical metric $g_{\mu\nu}$ introducing an auxiliary metric $\bar{g}_{\alpha\beta}$ and a mimetic field $\phi$, we acquire \[ g_{\mu\nu} = \bar{g}_{\mu\nu} \bar{g}^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi, \] (1) which implies that the physical metric remains invariant under conformal transformations of the auxiliary metric. Hence, one can easily extract the expression \[ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi = -1, \] (2) which can then be applied as a Lagrange multiplier in an extended action, namely \[ S = \int d^4x \sqrt{-g} \left[ \frac{\kappa^2}{2} R - \lambda \partial_\mu \phi \partial_\mu \phi + 1 + \mathcal{L}_m \right], \] (3) where $\kappa^2$ is the gravitational constant, $R$ the Ricci scalar, and $\mathcal{L}_m$ the usual matter Lagrangian, in units where $c = 1$.

One can extract the field equations by varying the action in terms of the physical metric, incorporating additionally its dependence on the mimetic field as well as the auxiliary metric, resulting in

\[
\frac{1}{\kappa^2} G_{\mu\nu} = \lambda \partial_\mu \phi \partial_\nu \phi + T_{\mu\nu},
\]

(4)

where $G_{\mu\nu}$ is the Einstein tensor and $T_{\mu\nu}$ the standard matter energy–momentum tensor. Moreover, variation with respect to the Lagrange multiplier leads to the condition (2).

Taking the trace of equation (4), we find the Lagrange multiplier to be

\[ \lambda = \frac{G_{\mu\nu} \partial_\mu \phi \partial_\nu \phi}{G_{\mu\nu}} - T_{\mu\nu}, \]

where $G$ and $T$ are the traces of the Einstein tensor and the matter energy–momentum tensor, respectively. Varying (3) with respect to the mimetic scalar field leads to

\[ \nabla^\mu (\lambda \partial_\mu \phi) = 0. \]

(5)

Lastly, the above equations can be elaborated in a more compact form, namely

\[ G_\mu^\nu - (G - T) \partial_\mu \phi \partial^\nu \phi = \kappa^2 T_\mu^\nu, \]

(6)

\[ \nabla_\mu [(G - T) \partial^\mu \phi] = 0. \]

(7)

In this work, we consider $T_\mu^\nu$ to correspond to an anisotropic fluid; namely, we impose the form

\[ T_\mu^\nu = (p_r + \rho) u_\mu u_\nu - p_t \delta_\mu^\nu + (p_r - p_t) \xi_\mu \xi^\nu, \]

(8)

where $u_\mu$ is the time-like vector defined as $u^\mu = [1, 0, 0, 0]$ and $\xi_\mu$ is the space-like unit radial vector defined as $\xi^\mu = [0, 1, 0, 0]$, such that $u_\mu u^\mu = -1$ and $\xi_\mu \xi^\nu = 1$.

3 Novel classes of anisotropic solutions

We are interested in extracting new classes of spherically symmetric solutions of the field equations (6), (7). For this purpose, we introduce the metric

\[ ds^2 = [h(r) h_1(r)] dt^2 - h_1(r) dr^2 - r^2 (d\theta^2 + r^2 \sin^2 \theta d\phi^2), \]

(9)

where $h(r)$ and $h_1(r)$ are the two metric functions. Under this metric, the field equations (6), (7) give rise to the following nonlinear differential equations:

\[ \frac{h'_r + h_1^2}{r^2 h_1^2} = \kappa^2 \rho, \]

(10)

\[ ^1 \text{In the present study, we assume the Lagrangian multiplier to have a unit value.} \]
where \( \phi \equiv \phi(r) \) and with primes denoting derivatives with respect to \( r \). The system (10)-(12) consists of three independent equations for six unknown functions; \( h, h_1, \rho, p_r, p_t \) and \( \phi \). Hence, we require to impose three extra conditions. We will try to solve the above system assuming the two equations of state:

\[
p_r = \omega_1 \rho,
\]

and

\[
p_T = \omega_2 \rho,
\]

and by imposing the condition \( \phi = \int \sqrt{-g_{rr}} dr = \int \sqrt{h_1} dr \) we get the following system:

\[
-2 r h'_1 h^2 - 2 r h'' h_1^2 - 2 r h' h_1^2 - r h'' h_1 h_1' - 2 r h'_1 h^2 h_1 + r h^2 h_1^2 = -k^2 p_r,
\]

\[
\frac{2 r h'_1 h^2 - 2 r h'' h_1^2 - 2 r h' h_1^2 - r h'' h_1 h_1' - 2 r h'_1 h^2 h_1 + r h^2 h_1^2}{4 r h^2 h_1^3} = -k^2 p_t,
\]

(13)

where \( \omega_1 \) and \( \omega_2 \) are the parameters characterizing the fluid. The above differential equations, (13), have no analytical solution except for the case of dust, i.e., \( \omega_1 = \omega_2 = 0 \), the vacuum case; otherwise, we cannot find any analytical solution that can extract from it any physics.

Other way to solve equations (10)-(12) is to assume the expression of the metric potential \( g_{rr} \), i.e., \( h_1 \), as:

\[
h_1 = \left(1 + \frac{r^2}{k}\right)^s,
\]

(14)

where \( s \) is a real parameter and \( k \) is a constant. If \( s = 0 \), then Eq. (10) yields \( \rho = 0 \) which is not physically interesting. The gravitational potential (14) is well behaved and finite when \( r \to 0 \). For \( s = 1 \), the metric potential reduces to the well-known Tolman–Finch–Skea potential [106], which has been applied to model compact stars by using a proper choice of the radial pressure \( p_r \), compatible with observational data [107].

At this stage, it is important to introduce the anisotropy parameter \( \Delta \), defined as [83]

\[
\Delta = k^2 (p_t - p_r),
\]

(15)

which quantifies the amount of anisotropy present in the star, being zero in the isotropic case. Inserting (14) into (10)-(12), we can obtain the expression of anisotropy as

\[
\Delta = \frac{1}{r^2} - \frac{(1 + 3s) r^4 k^{2s} + r^2 (2 + s) k^{2+2s} + k^{4+2s}}{r^2 (k^2 + r^2)^{2+s}}
\]

\[
+ 2 r^2 \left[2 r h'' h'' h'[h(s-2) - r h']k^{2+2s} + (2 r h'' - 2 h''') k^{4+2s} + r^4 k^{2s} \left[2 r h'' + [2 h(s-1) - r h']h''\right]\right]
\]

\[
4 r (k^2 + r^2)^{s+2} h^2
\]

(16)

Imposing the condition

\[
\frac{2 r^2 \left[2 r h'' h'' h'[h(s-2) - r h']k^{2+2s} + (2 r h'' - 2 h''') k^{4+2s} + r^4 k^{2s} \left[2 r h'' + [2 h(s-1) - r h']h''\right]\right]}{4 r (k^2 + r^2)^{s+2} h^2} = 0,
\]

(17)

we find

\[
h = \left\{ c_2(s-2) - c_1 \frac{2^{3/2} (k^2 + r^2)^{(2 + s)} (2 + s)}{4(s-2)^2} \right\}^{2/s+2},
\]

(18)
where $c_1$ and $c_2$ are integration constants which can be determined from the matching conditions. Substituting (14) and (18) into (10)-(12), we obtain the energy density, as well as the radial and tangential pressures, respectively, as:

$$
\begin{align*}
\kappa^2 \rho(r) &= \frac{1 - \left(1 + \frac{c_2^2}{k^2}\right)^{-1}}{r^2} + \frac{2s(1 + \frac{c_2^2}{k^2})^{-1-s}}{k^2}, \\
\kappa^2 p_r(r) &= \frac{c_2 \left((k^2 + r^2)^{1-s} - k^2 (s-2) - c_1 \left((k^2 + r^2)^{1-s} - k^2 \left((k^2 + r^2)^{1-s} - k^2 (s-2)\right)\right)^{2s/2} (2 + s)^{-s/2} \right)}{\left(c_2(s-2) - c_1 \left((k^2 + r^2)^{1-s} - k^2 \left((k^2 + r^2)^{1-s} - k^2 (s-2)\right)\right)^{1+s} \right)}, \\
\kappa^2 p_t(r) &= -\frac{c_2 s \left((k^2 - r^2)(s-2) + c_1 \left((k^2 - r^2)(s-4) + r^2 (3s - 4)\right)\right)(k^2 + r^2)^{1-s/2} 2s/2 (2 + s)^{-s/2} \right)}{\left(c_1 2s/2 (k^2 + r^2)^{1-s/2} (2 + s)^{-s/2} - c_2 (s-2)\right) \left((k^2 + r^2)^{1+s} \right)},
\end{align*}
$$

Finally, the metric solution in the interior of the star is written as

$$
ds^2 = \frac{c_1 2s/2 (k^2 + r^2)^{1-s/2} (2 + s)^{-s/2} - c_2 (s-2)}{4k^2(s-2)^2} (k^2 + r^2)^{s} \sqrt{-g} d\xi^2 - \left(1 - \frac{r^2}{k^2}\right)^s dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2).
$$

We stress here that according to (2) the solution for the mimetic field can be obtained as $\phi(r) = \int dr \sqrt{g_{rr}} = F_1 \left(\frac{1}{2}, -\frac{s}{2}; \frac{s}{2}; -\frac{r^2}{k^2}\right) r$, with $F_1$ the hypergeometric function. Hence, this solution does not include general relativity result as a limit, since $\phi(r)$ is not constant; namely, it is a novel class of solutions.

The constraint (17) is important since in this case the anisotropy parameter does not directly depend on $h$, and it acquires the simple form

$$
\Delta = \frac{1}{r^2} - \frac{(1 + 3s)s^4 k^2 + r^2 (2 + s) k^2 + 4s}{r^2 (k^2 + r^2)^{2s}}.
$$

This form has the expected properties to vanish at the center of the star, i.e., $\Delta(r \to 0) = 0$, and it has no singularities and has a positive value inside the star [108–110]. Note that the anisotropic force, defined as $\frac{2\Delta}{\rho}$, is attractive for $\rho_r > 0$ and repulsive for $\rho_r < 0$.

From the above expressions, we deduce that $\rho, \rho_r, \rho_t$ are well defined at the center of the star, regular and singularity-free. In particular, we find

$$
\begin{align*}
\rho(r \to 0) &= \frac{3s}{k^2 k^2}, \\
\rho_r(r \to 0) &= \rho_t(r \to 0) = \frac{s c_2 (s-2) k^2 + 4s + c_1 2s/2 (s-4)}{k^2 c_2 (s-2) k^2 + 4s + c_1 2s/2 k^2},
\end{align*}
$$

and thus (22) implies that in physical cases $s > 0$. Additionally, note that for a large $s$-region $\rho, \rho_r, \rho_t$ are nonnegative, regular and singularity-free. Finally, note that calculating the gradients of the density and pressures from the above expressions, namely $\frac{\rho_r}{\rho}, \frac{\rho_t}{\rho}$ and $\frac{d\rho_t}{d\rho}$, we can immediately verify that they are negative, as required, and in particular $\rho$ is finite and monotonically decreasing toward the boundary.

We can now introduce the radial and tangential equation-of-state parameters $w_t$ and $w_r$ as

$$
w_t \equiv \frac{p_t}{\rho}, \quad w_r \equiv \frac{p_r}{\rho},
$$

while in cases of anisotropic objects it is convenient to introduce also the average equation-of-state parameter

$$
w_{av} \equiv \frac{p_r + 2p_t}{\rho}.
$$

Furthermore, we can calculate the radial and tangential sound speeds, $c_t^2 = \frac{dp_t}{d\rho}$ and $c_r^2 = \frac{dp_r}{d\rho}$, respectively, which are given in Appendix A. Finally, the mass contained within radius $r$ of the sphere is defined as:

$$
M(r) = \int_0^r \rho(\xi) \xi^2 d\xi.
$$

Inserting (19) into (26), we acquire

$$
M(r) = \frac{r \left(\frac{k^2 + r^2}{k^2}\right)^{1/2} - 1}{2 \left(\frac{k^2 + r^2}{k^2}\right)^{1/2}}.
$$
Thus, we can now introduce the compactness parameter of a spherically symmetric source with radius \( r \) as \([111]\)

\[
m(r) = \frac{2M(r)}{r} = \left( \frac{k^2 + r^2}{k^2} \right)^t - 1 \left( \frac{k^2 + r^2}{k^2} \right)^s.
\]

We proceed by determining the constants \( c_1, c_2 \) and \( k \). To achieve this, we match the interior solution (19) and the interior metric (20), with the exterior Schwarzschild solution\(^2\)

\[
ds^2 = \left( 1 - \frac{2M}{r} \right) dt^2 - \left( 1 - \frac{2M}{r} \right)^{\frac{1}{2}} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

for \( r > 2M \), and where \( M \) is the total mass of the compact star. (Note that the metric (9) reproduces the Schwarzschild solution in vacuum.) The junction condition of the metric potentials across the boundary is given by the first fundamental form, namely at the surface \( r \to l \), as \( g_{rr}^r = g_{rr}^l \), and \( g_{tt}^r = g_{tt}^l \), and the second fundamental form implies \( p_r (r \to l) = 0 \). These conditions lead to

\[
c_1 = \pm l^{s-3} \left\{ (s+2) \left[ \left( \frac{l}{l-2M} \right)^{1/s} - 1 \right]^{-1/2} \right\}^{s/2}
\cdot \left\{ 2^{-s/2} (2M-l) \left( \frac{l}{l-2M} \right)^{s/2} + 2^{-s/2} [2Ms + (M-sl)] \sqrt{\frac{l}{l-2M}} \right\},
\]

\[
c_2 = \pm \frac{2[2(1-2M)] - 4(l-2M)}{l} \left[ \left( \frac{l}{l-2M} \right)^{s/2} - 1 \right] (s-2),
\]

\[
k = \pm l^{2} \left[ \left( \frac{l}{l-2M} \right)^{s} - 1 \right]^{-1/2}.
\]

We close this section by analyzing the properties of the metric solution (20). Equations (14) and (18) imply that the metric potentials \( g_{tt} \) and \( g_{rr} \) at the center of the star \( r = 0 \) become

\[
g_{rr} |_{r \to 0} = 1
\]

\[
g_{tt} |_{r \to 0} = \frac{c_2^2(s-2)^2[k^2(2+s)]^{3s/2} + c_1}{4(s-2)^2[k^2(2+s)]^{3/2}}
\]

\[
\times \left\{ 2^{s/2} \left[ \left( k^2(2+s) \right)^s k^2c_1 - 2c_2 \left[ k^2(2+s) \right]^s (s-2) \right] \right\}^{2s/2k^2},
\]

and thus the star is free from a singularity at the center.

### 4 Physical features of the solutions

In this section, we proceed to the investigation of the physical features of the obtained anisotropic solutions. Any physical viable stellar model must satisfy the following conditions throughout the stellar configurations:

- The metric potentials, and all components of the energy–momentum tensor, must be well defined and regular throughout the interior of the star.
- The density must be finite and positive in the interior of the star and decrease monotonically toward the boundary.
- The radial and the tangential pressures must be positive inside the configuration of the fluid, and the derivatives of the density and pressures must be negative. Additionally, the radial pressure \( p_r \) must vanish at the boundary of the stellar model \( r \to l \); however, the tangential pressure \( p_t \) does not need to be zero at the boundary. Finally, at the center of the star the pressures should be equal, implying that the anisotropy vanishes, namely \( \Delta(r = 0) = 0 \).
- Any anisotropic fluid sphere must fulfill the energy conditions, namely the null energy condition (NEC): \( p_t + \rho > 0, \rho > 0 \), the strong energy condition (SEC): \( p_t + \rho > 0, p_t + \rho > 0, \rho - p_r - 2p_t > 0 \), the weak energy condition (WEC): \( p_t + \rho > 0, \rho > 0 \) and the dominant energy condition (DEC): \( \rho \geq |p_r| \) and \( \rho \geq |p_t| \).
- The interior metric potentials must join smoothly with the Schwarzschild exterior metric at the boundary.
- For a stable configuration, the adiabatic index must be greater than \( \frac{4}{3} \).

\(^2\)We have shown in \([105]\) that the only vacuum spherically symmetric solution, in the frame of mimetic gravitational theory, is the Schwarzschild one.
The anisotropy in the right graph of Fig. 3 we depict the anisotropic force we observe, $v_i^2$ and $v_t^2$ are the radial and tangential sound speed squares, respectively [112, 113].

The causality condition must be satisfied; namely, the sound speeds must be sub-luminal, i.e., $0 \leq \frac{dp_r}{d\rho} \leq 1, 0 \leq \frac{dp_t}{d\rho} \leq 1$.

The surface redshift $Z_R$ is defined as the value of $Z_R = \frac{1}{r_g(r)} - 1$ [14], where $g_{00}$ is the temporal component of the metric, and it must obey $Z_R \leq 2$.

Let us examine whether our solutions satisfy the above necessary physical conditions. In order to proceed, we need to give numerical values to the model parameters $c_1, c_2$ and $k$. This will be obtained by using as input values the mass and radius of the pulsar 4U 1608-52, estimated, respectively, as $M = 1.57_{-0.20}^{+0.3} M_\odot$ and $l = 9.8 \pm 1.8$ km [115–117]. Inserting these values into (30)-(32), we find

$$c_1 \approx \frac{2}{729} \left[ \frac{81(s + 2)}{2.1^{s-1} - 1} \right]^{1/2} 2^{-s/2} \left[ 5.3s(2.1^{\frac{s-2}{s}} - 1.2) + 3.4 \right],$$

$$c_2 \approx \frac{2(8.5 - 6.2 \cdot 2.1^{-s})}{9(2.1^{s-1} - 1)(s-2)}$$

$$k \approx 9 \sqrt{(e^{0.7s-1} - 1)},$$

where $(c_1)^{\frac{1}{s}}$ has units of km, $c_2$ is dimensionless, and $k$ has units of km$^2$. We mention that apart from 4U 1608-52 a similar analysis can be developed for other pulsars, such as 4U 1724-207 and J0030+0451, and for completeness, we provide the corresponding parameter values in Appendix 1. Adopting the above constants, the physical quantities extracted in the previous section can be plotted.

The profiles of the energy density, radial and tangential pressures, given by (19), are depicted in Fig. 1. As we observe, they are well defined at the center of the star, regular and singularity-free, and they are positive and monotonically decreasing toward the boundary.

In Fig. 2, we depict the radial, tangential and average equation-of-state parameters, $w_r$, $w_t$, and $w_{av}$, respectively. As we observe $w_r$ is monotonically increasing and $w_{av}$ is monotonically decreasing with $r$, while $w_t$ is monotonically decreasing for small $s$ and monotonically increasing for large $s$. Moreover, the values of $w_r$, $w_t$ and $w_{av}$ are positive and lie in the interval $[w_t, w_r, w_{av}] \in [0, 1]$, which implies that matter distribution is non-exotic in nature. Finally, in the left graph of Fig. 3 we depict the anisotropy $\Delta$, where we can see that it is positive, it vanishes at the center and it increases toward the surface of the star. Moreover, in the right graph of Fig. 3 we depict the anisotropic force $\Delta$, and the fact that it is positive implies that it is repulsive.

We proceed by investigating the behavior of the metric potentials. In Fig. 4, we present the temporal and the spatial components, for various choices of the model parameters. Furthermore, for transparency we additionally depict the smooth matching of the temporal component with the Schwarzschild exterior solution. As Fig. 4 shows, the metric potentials are both finite and positive at the center.

In Fig. 5, we depict the weak, null, strong and dominant energy conditions for $s = 15$, showing that they obtain positive values and thus are all satisfied, as required for a physically meaningful stellar model. (For other values of $s$, we obtain similar graphs.)
Fig. 2 The radial equation-of-state parameter $w_r$ (left graph), the tangential equation-of-state parameter $w_t$ (middle graph) and the average equation-of-state parameter (right graph) of the anisotropic star solution (19), as functions of the radial distance, for various values of the parameter $s$, using the 4U 1608-52 mass and radius values.

Fig. 3 The anisotropy $\Delta$ (left graph) and the anisotropic force $\Delta r$ (right graph) of the anisotropic star solution (19), as functions of the radial distance, for various values of the parameter $s$, using the 4U 1608-52 mass and radius values.

Fig. 4 The temporal component (left graph) and the spatial component (middle graph) of the metric of the anisotropic solution (20), as functions of the radial distance, for various values of the parameter $s$, using the 4U 1608-52 mass and radius values. In the right graph, we present the smooth matching of the temporal component with the Schwarzschild exterior solution.
Fig. 5 The weak, null, strong and dominant energy conditions of the anisotropic star solution (19), as functions of the radial distance, for \( s = 15 \), using the 4U 1608-52 mass and radius values.

Fig. 6 The radial sound speed square \( v_r^2 \) (left graph), the tangential sound speed square \( v_t^2 \) (middle graph) and the stability factor \( v_t^2 - v_r^2 \) (right graph) of the anisotropic star solution (19), as functions of the radial distance, for various values of the parameter \( s \), using the 4U 1608-52 mass and radius values.

In the left and middle graphs of Fig. 6, we present the radial and tangential sound speed squares, which indeed are positive and sub-luminal. Additionally, since a potentially stable configuration requires \( v_t^2 - v_r^2 < 0 \), in the right graph of Fig. 6 we depict the stability factor \( v_t^2 - v_r^2 \), and as we see it is negative and hence we conclude that our model is potentially stable everywhere within the stellar interior for various values of the parameter \( s \).

The mass function given by (27) is plotted in the left graph of Fig. 7, showing that it is a monotonically increasing function of the radial coordinate and \( M(r \to 0) = 0 \). Furthermore, the middle graph of Fig. 7 shows the behavior of the compactness parameter (28), which is increasing. Finally, the radial variation of the redshift (35) is plotted in the right graph of Fig. 7. We find that the surface redshift \( Z_R \approx 0.45 \) for all \( s \) choices, and since the theoretical requirement is \( Z_R \leq 2 \), we conclude that it is satisfied for solution (19).

5 Stability

In this section, we are going to discuss the stability issue using two different techniques: the Tolman–Oppenheimer–Volkoff (TOV) equations and the adiabatic index. For completeness, we will also examine the static case, too.

5.1 Equilibrium analysis through TOV equation

In this subsection, we are going to discuss the stability of a general stellar model. For this goal, we assume hydrostatic equilibrium through the TOV equation. Using the TOV equation [118, 119] as presented in [120], we obtain the following form:

\[
\frac{2[p_r - p_r]}{r} - \frac{M(r)[\rho(r) + p_r]\sqrt{H}}{r} - \frac{dp_r}{dr} = 0.
\]

(37)
Fig. 7 The mass $M(r)$ of (27) (left graph), the compactness $m(r)$ of (28) (middle graph) and the redshift (35) (right graph) of the anisotropic star solution (19), as functions of the radial distance, for various values of the parameter $s$, using the 4U 1608-52 mass and radius values. The surface redshift is $Z_R \approx 0.45$ for all $s$ choices.

Fig. 8 The gravitational, the anisotropic and the hydrostatic forces of the anisotropic star solution (19), as functions of the radial distance, for $s = 15$, using the 4U 1608-52 mass and radius values.

Here, $M_g(r)$ is the mass of the gravitational system at radius $r$ and is defined through the Tolman–Whittaker mass formula

$$M_g(r) = 4\pi \int_0^r \left( T_t^t - T_r^r - T_\theta^\theta - T_\phi^\phi \right) r^2 h_1 \sqrt{h_1} dr = \frac{r(h h_1)'}{2h \sqrt{h_1}}.$$  \hspace{1cm} (38)

Inserting (38) into (37), we find

$$\frac{2(p_t - p_r)}{r} - \frac{dp_r}{dr} - \frac{(h h_1)'}{2\sqrt{h h_1}} = F_g + F_a + F_h = 0,$$  \hspace{1cm} (39)

where $F_g = -\frac{(h h_1)'}{2\sqrt{h h_1}}(\rho + p_r)$, $F_a = \frac{2(p_t - p_r)}{r}$ and $F_h = -\frac{dp_r}{dr}$ are the gravitational, anisotropic and hydrostatic forces, respectively.

The behavior of the TOV equation for model (19) is shown in Fig. 8, in which the three different forces are plotted. (For other values of $s$, we obtain similar graphs.) As we observe, the hydrostatic and anisotropic forces are positive and are dominated by the gravitational force which is negative to maintain the system in static equilibrium.

5.2 Adiabatic index

The stable equilibrium configuration of a spherically symmetric system can be studied using the adiabatic index, which is a basic ingredient of the stability criterion. Considering an adiabatic perturbation, the adiabatic index $\Gamma$ is defined as [121–124]:

$$\Gamma = \left( \frac{\rho + p}{p} \right) \left( \frac{dp}{d\rho} \right).$$  \hspace{1cm} (40)
Fig. 9 The radial adiabatic index (C2) (left graph) and the tangential adiabatic index (C3) (right graph) versus the radius \( r \), for various values of \( s \), using the 4U 1608-52 mass and radius values.

A Newtonian isotropic sphere is in stable equilibrium if the adiabatic index satisfies \( \Gamma_1 > \frac{4}{3} \) \[125\], while for \( \Gamma_1 = \frac{4}{3} \) the isotropic sphere is in neutral equilibrium. As it was shown in \[124\], for the stability of a relativistic anisotropic sphere it is required that \( \Gamma_1 > \gamma \), where

\[
\gamma = \frac{4}{3} - \left[ \frac{4(p_r - p_t)}{3|p'_r|} \right]_{\text{max}}. \tag{41}
\]

Using Eq. (40) and the solution (19), we can find the expressions for the radial and tangential adiabatic indices, which are presented in Appendix C.

In Fig. 9, we draw \( \Gamma_r \) and \( \Gamma_t \) for various values of \( s \). As we can see, the profile of the radial and tangential adiabatic indices is monotonic increasing functions of \( r \) and acquires values greater than \( \frac{4}{3} \) everywhere within the stellar configuration for \( s \leq 10 \), and thus, the condition of stability is satisfied.

5.3 Stability in the static state

For completeness, in this subsection we discuss the stability in the static case. For a stable compact star, using the mass-central and mass-radius expression, as well as the relations for the energy density, Harrison, Zeldovich and Novikov \[126, 127\] stated that the gradient of the central density with respect to the mass should acquire positive values, namely \( \frac{\partial M}{\partial \rho_{\text{cen}}} > 0 \), in order to have stable configurations. More precisely, the stable or unstable region is satisfied for constant mass, namely \( \frac{\partial M}{\partial \rho_{\text{cen}}} = 0 \) \[111\].

Let us apply this procedure to our solution (19). In this case, the central density becomes \( \rho_{\text{cen}} = \frac{3s}{\kappa^2 k^2} \) and thus we find that

\[
M(\rho_{\text{cen}}) = l \left[ 1 - \left( 1 + \frac{\kappa^2 l^2 \rho_{\text{cen}}}{3s} \right)^{-3} \right], \tag{42}
\]

which finally leads to

\[
\frac{\partial M}{\partial \rho_{\text{cen}}} = \frac{4\pi l^3 \left( 1 + \frac{\kappa^2 l^2 \rho_{\text{cen}}}{3s} \right)^{-s-1}}{3}. \tag{43}
\]

The above expression implies that the solution (19) corresponds to a stable configuration since \( \frac{\partial M}{\partial \rho_{\text{cen}}} > 0 \) \[111\]. The behaviors of the mass and its gradient are shown in Fig. 10, and as we observe the mass decreases, while the gradient of mass increases, as the energy density becomes smaller.
6 Discussion and conclusions

Mimetic gravity is an interesting modification obtained from general relativity through the isolation of the conformal degree of freedom in a covariant way, by applying a re-parametrization of the physical metric in terms of a mimetic field and an auxiliary metric. The present work aimed to investigate new anisotropic compact solutions within mimetic gravity, since such solutions are known to be very interesting laboratories of gravity in the strong-field regime.

We derived new classes of anisotropic solutions, by applying the Tolman–Finch–Skea metric, and a specific anisotropy not directly depending on it. Thus, the anisotropy is positive, it vanishes at the center of the star and it has no singularities. Additionally, we determined the involved integration constants by matching smoothly the interior anisotropic solution to the Schwarzschild exterior one, requiring additionally that the pressure must be vanishing at the boundary of the star.

In order to provide a transparent picture, we used the data from the 4U 1608-52 pulsar and we investigated numerically the features of the obtained anisotropic stars. In particular, we studied the profile of the energy density, as well as the radial and tangential pressures, and we showed that they are all positive and decrease toward the center of the star. Furthermore, we investigated the anisotropy parameter and the anisotropic force, which are both increasing functions of the radius, which implies that the latter is repulsive. Additionally, by examining the radial and tangential equation-of-state parameters, we showed that they are monotonically increasing, and bounded in the interval [0,1], which implies that the matter in our star is not exotic.

Concerning the metric potentials we saw that they have no singularity, either at the center of the star or at the boundary, and the matching to the Schwarzschild exterior is indeed smooth. Moreover, we examined the weak, null, strong and dominant energy conditions, showing that they are all satisfied in the interior of the star. Additionally, we examined the radial and tangential sound speed squares, showing that they are positive and sub-luminal, while we found that the surface redshift satisfies the requirement $Z_R \leq 2$. Finally, we provided the profiles for the mass and compactness, which are monotonically increasing functions.

In order to investigate the stability of the new anisotropic solutions, we applied the Tolman–Oppenheimer–Volkoff (TOV) equation, we performed the adiabatic index analysis, and we examined the static case, providing the profiles of the gravitational, anisotropic and hydrostatic forces of the star, the radial and tangential adiabatic indices, as well as the mass and its gradient, showing that in all cases the star is stable. Lastly, for completeness in Appendix we provided the results for other pulsar data.

We mention that the obtained solutions do not have a trivial profile for the mimetic field, and therefore, they correspond to novel classes that do not exist for general relativity. Hence, the rich behavior of the aforementioned anisotropic solutions serves as an advantage of mimetic gravity. It would be interesting to investigate the gravitational wave structure of possible merges of such solutions, and whether this could provide signatures of mimetic gravity. Such an analysis will be performed in a future project.

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Appendix A: the radial and tangential sound speeds

The radial and tangential sound speed squares can be extracted from (19) as

\[
v_r^2 = \frac{dp_r}{d\rho} = \left\{4(k^2+r^2)^2c_1^2\left[5(1+s)-(k^2+r^2)^{2s}k^{-2s}r^4+k^2(s+2)r^2+k^4\right]((k^2+r^2)(s+2))^{-s} + (s-2)c_1c_2\left[2(k^2+r^2)^{2s}k^{-2s}r^4(2+s+12s)r^4 - 2k^2(s+2)r^2 - 2k^4\right]ight.\nonumber \\
\left.+ \frac{2s}{2}(k^2+r^2)\left[(k^2+r^2)(s+2)\right]^{-s/2}\right.\nonumber \\
\left.+ (2-s)^2c_2^2\left[(k^2+r^2)^2\left(\frac{k^2+r^2}{k}\right)^s + (-2s^2 - 3s - 1)r^4 - k^2(s+2)r^2 - k^4\right]\right\}^{-1},
\]

\[
v_t^2 = \frac{dp_t}{d\rho} = r^4\left\{(k^2+r^2)^2c_1^2\left[(k^2+3r^2)x^2 + (-5k^2-r^2)x - 4k^2 - 4r^2\right]2^s\left[(k^2+r^2)(s+2)\right]^{-s} + c_1c_2(2-s)(k^2+r^2)\left[(k^2+5r^2)x - 12k^2 - 4r^2\right]2^{s/2}\left[(k^2+r^2)(s+2)\right]^{-s/2}\right.\nonumber \\
\left.\left. - (s-2)^2c_2^2\left[(k^2-r^2)x - r^2 + 3k^2\right]\right\}
\]

\[
\left\{\frac{k^2+r^2}{k}\right\}^{-s/2} + c_2(2-s)\right\}^{-1}. \quad (A1)
\]

\[
v_r^2 = \frac{dp_r}{d\rho} = \left\{4(k^2+r^2)^2c_1^2\left[5(1+s)-(k^2+r^2)^{2s}k^{-2s}r^4+k^2(s+2)r^2+k^4\right]((k^2+r^2)(s+2))^{-s} + (s-2)c_1c_2\left[2(k^2+r^2)^{2s}k^{-2s}r^4(2+s+12s)r^4 - 2k^2(s+2)r^2 - 2k^4\right]ight.\nonumber \\
\left.+ \frac{2s}{2}(k^2+r^2)\left[(k^2+r^2)(s+2)\right]^{-s/2}\right.\nonumber \\
\left.+ (2-s)^2c_2^2\left[(k^2+r^2)^2\left(\frac{k^2+r^2}{k}\right)^s + (-2s^2 - 3s - 1)r^4 - k^2(s+2)r^2 - k^4\right]\right\}^{-1},
\]

\[
v_t^2 = \frac{dp_t}{d\rho} = r^4\left\{(k^2+r^2)^2c_1^2\left[(k^2+3r^2)x^2 + (-5k^2-r^2)x - 4k^2 - 4r^2\right]2^s\left[(k^2+r^2)(s+2)\right]^{-s} + c_1c_2(2-s)(k^2+r^2)\left[(k^2+5r^2)x - 12k^2 - 4r^2\right]2^{s/2}\left[(k^2+r^2)(s+2)\right]^{-s/2}\right.\nonumber \\
\left.\left. - (s-2)^2c_2^2\left[(k^2-r^2)x - r^2 + 3k^2\right]\right\}
\]

\[
\left\{\frac{k^2+r^2}{k}\right\}^{-s/2} + c_2(2-s)\right\}^{-1}. \quad (A2)
\]

Table 1 The values of the constants \(k\), \(c_1\) and \(c_2\) of
(30)-(32), using the compact star
4U 1724-207, whose observed mass and radius are
\((1.81^{+0.26}_{-0.37})M_\odot\) and \(12.2 \pm 1.4\)
km, respectively [128]

| \(s\) | \(k\) (km²) | \(\frac{1}{2}\) (km) | \(c_2\) |
|---|---|---|---|
| 1 | 15.09 | 161 | 1.207 |
| 5 | 38.263 | 133 | 0.597 |
| 15 | 67.619 | 268 | 0.717 |
| 20 | 78.273 | 335 | 0.727 |
| 30 | 96.104 | 466 | 0.737 |
| 40 | 111.11 | 633 | 0.741 |
| 50 | 124.316 | 724 | 0.744 |
| 60 | 136.249 | 852 | 0.746 |
| 70 | 147.218 | 980 | 0.747 |
| 80 | 157.425 | 1107 | 0.748 |
| 90 | 167.009 | 1234 | 0.748 |
| 100 | 176.072 | 1361 | 0.749 |
Table 2 Numerical values of physical quantities of the anisotropic star solution (19), for various values of the parameter $s$, using the 4U 1724-207 mass and radius values. The energy density is measured in MeV·fm$^{-3}$, while all other quantities are dimensionless in units where $c = 1$.

| $s$ | $\rho |_0$ | $\rho |_l$ | $\frac{dp}{d\rho} |_0$ | $\frac{dp}{d\rho} |_l$ | $\frac{d\rho}{d\rho} |_0$ | $\frac{d\rho}{d\rho} |_l$ | $\Gamma |_0$ | $\Gamma |_l$ |
|-----|--------|--------|----------------|----------------|----------------|----------------|--------|--------|
| 1   | 0.0132 | 0.005  | 0.56           | 0.33           | 0.76           | 0.179         | 2.5    | 3.4    |
| 5   | 0.01   | 0.0057 | 0.413          | 0.343          | 0.346          | 0.245         | 2.82   | 2.25   |
| 15  | 0.0098 | 0.0059 | 0.247          | 0.228          | 0.097          | 0.105         | 2.82   | 1.11   |
| 20  | 0.0097 | 0.0059 | 0.235          | 0.222          | 0.074          | 0.094         | 2.81   | 0.88   |
| 30  | 0.0097 | 0.0059 | 0.233          | 0.217          | 0.049          | 0.083         | 2.8    | 0.62   |
| 40  | 0.0097 | 0.0059 | 0.217          | 0.214          | 0.037          | 0.077         | 2.8    | 0.47   |
| 50  | 0.0097 | 0.0059 | 0.213          | 0.212          | 0.029          | 0.073         | 2.8    | 0.38   |
| 60  | 0.0097 | 0.0059 | 0.211          | 0.21           | 0.024          | 0.07          | 2.8    | 0.3    |
| 70  | 0.0097 | 0.0059 | 0.21           | 0.21           | 0.02           | 0.069         | 2.8    | 0.27   |
| 80  | 0.0097 | 0.0059 | 0.21           | 0.21           | 0.017          | 0.068         | 2.8    | 0.2    |
| 90  | 0.0097 | 0.0059 | 0.21           | 0.21           | 0.015          | 0.066         | 2.8    | 0.2    |
| 100 | 0.0097 | 0.0059 | 0.21           | 0.21           | 0.014          | 0.065         | 2.8    | 0.18   |

Table 3 The values of the constants $k$, $c_1$ and $c_2$ of (30)-(32), using the compact star J0030+0451, whose observed mass and radius are $(1.34^{+0.15}_{-0.16})M_\odot$ and $12.71^{+1.14}_{-1.19}$ km, respectively [129].

| $s$ | $k$ (km$^2$) | $(c_1) \sqrt{s}$ (km) | $c_2$ |
|-----|--------------|------------------------|-------|
| 1   | 17.46        | 43                     | 1.787 |
| 5   | 42.086       | 171                    | 0.521 |
| 15  | 73.786       | 302                    | 0.767 |
| 20  | 85.33        | 373                    | 0.786 |
| 30  | 104.665      | 514                    | 0.805 |
| 40  | 120.948      | 654                    | 0.814 |
| 50  | 135.285      | 794                    | 0.819 |
| 60  | 148.242      | 933                    | 0.823 |
| 70  | 160.154      | 1072                   | 0.825 |
| 80  | 171.239      | 1210                   | 0.827 |
| 90  | 181.65       | 1348                   | 0.828 |
| 100 | 191.495      | 1486                   | 0.83  |

Appendix B: analysis using 4U 1724-207 and J0030+0451 pulsars

In addition to 4U 1608-52, a similar analysis can be developed for other pulsars. In particular, using the pulsar 4U 1724-207, whose observed mass and radius given by $(1.81^{+0.25}_{-0.37})M_\odot$ and $12.2 \pm 1.4$ km, respectively [128], we obtain the model parameters displayed in Table 1, and using them, we obtain the physical quantities summarized in Table 2.

Additionally, in Table 3 we display the corresponding parameters for the pulsar J0030+0451, whose observed mass and radius are given by $(1.34^{+0.15}_{-0.16})M_\odot$ and $12.71^{+1.14}_{-1.19}$ km, respectively [129], and using them we obtain the physical quantities summarized in Table 4.

Appendix C: the radial and tangential adiabatic index

The adiabatic index of a spherically symmetric system is defined as [121, 123, 124]:

$$ \Gamma = \left( \frac{\rho + p}{p} \right) \left( \frac{dp}{d\rho} \right) \quad (C1) $$

and can be applied for the radial and tangential pressure separately. Hence, inserting the solution (19) into (40) we obtain the radial adiabatic index as

$$ \Gamma_r = 2r^2 \left[ c_1 \frac{2^{s/2}(k^2 + r^2)(s + 2)}{(k^2 + r^2)(s + 2)} \right]^{-1/2s} - 2s c_2(s - 2) $$

$$ \times \left( k^2 + r^2 \right)^{s/2} \left( \frac{k^2 + r^2}{k^2} \right)^s + (s + 2s^2 - 1)r^4 - k^2(s + 2)r^2 - k^4 \right]^{-1} $$
and the tangential adiabatic index as

$$\Gamma_r = r^2 \left\{ 4c_1^2 \left[ (s^2 - 4 - s)r^2 + (s^2 - 5s - 4)k^2 \right] \left[ (k^2 + r^2) \left[ (k^2 + r^2)(s + 2) \right] \right]^{-s} 
- c_2 2^{s/2} c_1 (k^2 + r^2) \left[ (5s - 4)r^2 + k^2(s - 12) \right] \left[ (k^2 + r^2)(s + 2) \right]^{-s/2} 
- c_2^2 s(s - 2)^2 \left[ k^2(s + 3) - (s + 1)r^2 \right] \right) \right\}$$

and the tangential adiabatic index as

$$\Gamma_r = r^2 \left\{ 4c_1^2 \left[ (s^2 - 4 - s)r^2 + (s^2 - 5s - 4)k^2 \right] \left[ (k^2 + r^2) \left[ (k^2 + r^2)(s + 2) \right] \right]^{-s} 
- c_2 2^{s/2} c_1 (k^2 + r^2) \left[ (5s - 4)r^2 + k^2(s - 12) \right] \left[ (k^2 + r^2)(s + 2) \right]^{-s/2} 
- c_2^2 s(s - 2)^2 \left[ k^2(s + 3) - (s + 1)r^2 \right] \right\}$$

$$\cdot \left\{ 2^{s/2} c_1 \left[ (k^2 + r^2)^{2s} k^{2s} + (3 - s)r^2 + k^2(s + 2)r^2 - k^4 \right] 
- (k^2 + r^2) \left[ (k^2 + r^2)(s + 2) \right]^{-s/2} 
- c_2(s - 2)^2 \left[ k^2(s^2 + 1)r^2 + (s - 1)r^2 \right] \right\}$$

$$\cdot \left\{ \left[ (k^2 + r^2)^{2s} k^{2s} + (s - 1 + 2s)r^2 - k^4 \right] \right\}$$

$$\cdot \left\{ (k^2 + r^2) \left[ (k^2 + r^2)^{2s} k^{2s} + (s - 1)r^2 \right] \left[ (k^2 + r^2)(s + 2) \right]^{-1/s} - c_2(s - 2) \right\}$$

$$\cdot \left\{ (3s - 4)r^2 + (s - 4)k^2 \right\}^{-1/s} c_1 \left[ (k^2 + r^2) \left[ (k^2 + r^2)(s + 2) \right]^{-1/s} \right. 
+ sc_2(k - r)(k + r)(s - 2) \right\} \right\}.$$
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