Spin flip statistics and spin wave interference patterns in Ising ferromagnetic films: A Monte Carlo study

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Abstract

The spin wave interference is studied in two dimensional Ising ferromagnet driven by two coherent spherical magnetic field waves by Monte Carlo simulation. The spin waves are found to propagate and interfere according to the classic rule of interference pattern generated by two point sources. The interference pattern of spin wave is observed in one boundary of the lattice. The interference pattern is detected and studied by spin flip statistics at high and low temperatures. The destructive interference is manifested as the large number of spin flips and vice versa.

Keywords: Optics, Electromagnetism, Applied mathematics

1. Introduction

The spin wave excitation in the ferromagnets is a fundamental excitation which is well studied [1] in anisotropic Heisenberg ferromagnets. Several experimental studies were performed to observe the behaviours [2] of spin waves and the interference pattern of two coherent spin waves. The eigenmodes of spinwaves of square permalloy dots are studied [3] by Brillouin light scattering. The Brillouin scattering of lights by spin waves in ferromagnetic nanorods are studied [4]. To study the propagation of spin waves across thin magnetic film samples and experimental scheme has been proposed [5] which is based on the creation of picosecond pulses.
of strongly localized effective magnetic field via ultrafast optical irradiation of a spatially deposited exchange bias. The method of realising spin wave logic gate is investigated experimentally [6].

The experiments on the interference of spin waves are also performed recently. A few of those may be mentioned here. The modulation of propagating spin wave amplitude in Ni$_{81}$Fe$_{19}$(Py) films, resulting from constructive and destructive interference of spin waves has been studied [7]. The spin wave interference from rising and falling edges of electrical pulses has also been studied [8] to see the effect of the electrical pulses width of input excitation on the generated spin waves in a Ni$_{81}$Fe$_{19}$(Py) strip using pulse inductive time domain measurements. Current induced localized spin wave interference in a ferromagnetic nanowire with a domain wall is investigated [9] by simulation.

Several theoretical investigations are also done in past few years. The nonequilibrium behaviours of interference of spin waves, in the system of two dimensional electron gas with both Rashba and Dresselhaus (001) spin–orbit coupling, are investigated [10] by Green’s function within tight binding framework. Spin wave interference in microscopic ferromagnetic rings were studied and series of quantized modes in the vortex state were found [11].

The discussions made above reveals that the interference of spin waves in ferromagnetic materials becomes an important field of modern research. If the dynamical modes of spin waves are to be understood clearly, one has to study the time dependent response of spin in ferromagnetic models. The nonequilibrium response of ferromagnet driven by time dependent magnetic field would be helpful for this. What will be the right approach to study the spin wave excitation and its interference, in model ferromagnets, from statistical mechanical point of view? What will be the statistics of spin flips if the spin wave travels through the ferromagnetic sample and how does it reflected in the interference pattern of spin waves? These questions are not yet addressed in the literature (as far as the knowledge of this author is concerned) and this is the main motivation of this paper. Before going to the core of the problem, it would be convenient for the reader if a brief introduction is given. First of all, in the model ferromagnet, like Ising model, one has to know how the spin bands travel, if it is driven by propagating magnetic field wave. In the next paragraph, some recent studies, on the propagating wavelike spin bands in the Ising ferromagnet driven by propagating magnetic field wave, will be mentioned.

The nonequilibrium responses of model ferromagnets, driven by oscillating magnetic field are studied in details [12, 13]. The hysteresis and dynamic phase transition are two major responses giving rise to many nonequilibrium phenomena. However, the field oscillates in time and remains uniform over the space (at any particular instant) in all those above mentioned studies. If the field has both spatial and temporal
variations, it would lead to further interesting facts. It may be visualised as the propagating magnetic field wave with a well defined frequency and wavelength. If such wave passes through the ferromagnet, the coherent motion of the spin clusters are observed. These magnetic waves may generally be of two types, namely, plane wave and spherical wave.

The propagation of plane spin wave is observed [14, 15] and dynamic phase transition is found [16] in Ising ferromagnetic film driven by plane propagating magnetic wave. Recently the propagation of spin clusters and existence of various dynamic phases are observed [17] in random field Ising ferromagnet. In normal ferromagnet, the propagation of circular spin waves are observed [18] in Ising ferromagnet and multiple nonequilibrium phase transitions are found. The importance of all these studies mentioned above is not merely pedagogical. Recent experiments [19] on permalloy (50 nm thick) excited by ultrashort (150 fs) laser pulse give rise to the propagation of circular spin waves.

The question naturally arises in mind, what will happen if two coherent spin waves interfere? Can one study the interference pattern in this case using the tools of statistical mechanics? Being motivated by this idea the high field spin wave interference pattern, in driven Ising ferromagnetic film, is studied and reported in this paper. The paper is organised as follows: the Model and the Monte Carlo simulation method are described in section 2, the numerical results are presented in section 3, the letter ends with a summary in section 4.

2. Model

The Hamiltonian of a two dimensional Ising ferromagnet driven by magnetic field wave is:

$$H(t) = -J \sum_{x,y,x',y'} s(x, y, t)s(x', y', t) - \sum_{x,y} h(x, y, t)s(x, y, t)$$

(1)

Where, $s(x, y, t) = \pm 1$ is Ising spin at position $(x, y)$ and in time $t$. $J (> 0)$ is the ferromagnetic spin–spin interaction strength. The first term represents nearest neighbour spin–spin interaction. The prime over the summation indicates the sum over distinct nearest neighbour pairs. In this model, the time dependent Hamiltonian $H(t)$ (equation (1)) is measured in the unit of $J$. $h(x, y, t)$ is the magnetic field (measured in the unit of $J$) at position $(x, y)$ in time $t$ coming from a spherically propagating magnetic wave represented as:

$$h(x, y, t) = \frac{h_0 \cos 2\pi(f t - y/\lambda)}{r}$$

(2)

The amplitude, frequency and the wavelength are represented by $h_0$, $f$ and $\lambda$ respectively. $r$ is the distance from the source. If the source is placed at the origin
$r = \sqrt{x^2 + y^2}$. The model is defined on a square lattice of linear size $L$. Two coherent point sources of spherically propagating magnetic wave (represented in equation (2)) are placed on the $x$-axis. The periodic boundary conditions are applied in both directions.

### 3. Methodology

In the above mentioned model, the initial spin configuration is chosen as statistically and randomly chosen 50 percent spins take the value +1. This is a very high temperature configuration. Two coherent point sources of spherical magnetic waves are symmetrically placed in the middle of one end of the lattice with a separation $\delta$. In this simulation, one source is placed at a distance $\delta/2$ in the right side from the centre and the other one is placed in the left side of the centre. The Monte Carlo simulation is employed here with single spin flip Metropolis dynamics. The probability of spin flip is:

$$P(s(x, y, t) \rightarrow -s(x, y, t)) = \text{Min}[1, \exp\left(-\frac{\Delta E}{kT}\right)] \tag{3}$$

where $\Delta E$ is the change in energy due to spin flip, $k$ is Boltzmann constant and $T$ is the temperature of the system. Here, the temperature $T$ is measured in the unit of $J/k$. A spin is chosen randomly and flipped with the probability mention above in equation (3). $L^2$ such random move defines one Monte Carlo Step per Spin (MCSS) and defines the time unit in the problem. The steady state configuration is achieved after $2 \times 10^4$ MCSS and it is checked that this number is adequate.

In the opposite (to that where the sources are placed) side of the lattice the number of flips of spins at each site is calculated.

### 4. Results

Here, linear size of the lattice, $L = 100$ is considered. Two coherent point sources of spherical magnetic waves are placed symmetrically about the central site of one end (say bottom line). The separation $\delta$ is taken equal to 19 lattice unit. This choice is arbitrary, just to make it of the order of the wavelength of the sources. The wavelength ($\lambda$) of the source is taken 5 (lattice unit). The frequency $f = 0.01$ and the amplitude of each source is taken $h_0 = 2000$. As a result one complete cycle of the propagating magnetic wave requires 100 MCSS. The choice of this value of frequency $f = 0.01$ requires 100 MCSS. One can visualize the changes of the wave pattern (see Figure 1) within 25 MCSS which can be obtained in a few minutes in $L = 100$ in Intel Core i5 processor. The value of the amplitude of the field $h_0 = 2000$ is chosen quite large to see the significant effect of interference on the other side of the lattice. It may be noted that, the strength of field falls as $\frac{1}{r}$ as mentioned in eq. (2).
In Figure 1 the propagation of spin waves is shown by plotting the spin configurations for three different time instants. The propagation of spin waves is very similar to that of water waves for two coherent sources [14]. At the opposite end the interference pattern is observed. The propagating magnetic field will interfere and the position of destructive interference will be reflected in the number of flips (averaged over the full cycle of the propagating magnetic field) of the spins in that region. In this region the cooperatively interacting term will govern the probability of spin flip and the number of spin flip will be more. On the other hand, the square average of the superposed value of propagating magnetic field due to two sources, is very high in the region of constructive interference. In this region, the spins will follow the directions of the magnetic fields. As a result, the number of flips of the spin will be less. The number of flips of the spins are shown in Figure 2, as functions of position (in the top line of the lattice). Here, the positions of the minima of the square averaged magnetic field are same as the positions of the average number of flips of
Figure 2. The spin wave interference pattern. The number of flips of the spins are plotted against the position (along x-axis) on the top line of the lattice. The average number of flips of the spins is represented by green triangles and the square average (over the full cycle) of the superposed propagating field is shown by red bullets. The patterns at different temperatures, (a) at $T = 2.30$, (b) at $T = 1.00$ and (c) at $T = 0.20$. Here, $L = 100$, $h_0 = 2000$, $f = 0.01$ and $\lambda = 5$.

the Ising spins. These are studied for three different temperatures. As the temperature decreases the number of flips decreases (as the probability of flip decreases due to Metropolis rule described in eq. (3)), however the steady interference pattern of the spin waves is maintained.

5. Discussion

In this letter, the interference pattern of spin waves in Ising ferromagnet due to two coherent sources is studied by Monte Carlo simulation. The steady interference pattern of spin waves is recognized as the average number of flips of the spins. The destructive interference of magnetic wave gives rise to large averaged number of spin flips as the dynamics is governed by the cooperative term in the Hamiltonian. On the other hand, the constructive interference of superposed value of magnetic
waves keeps the spin along the direction of the field and hence reduce the number of flips of the spins. This steady interference pattern of spin waves persists over a wide range of temperature. It may be mentioned here that the circular ripples of spin waves are experimentally detected in ferromagnetic sample by ultrashort laser pulse [19]. It would be interesting to study the interference of propagating field driven spin waves experimentally.

The interference of spin waves in Ising ferromagnetic film studied in this paper is an appeal to the experimentalists to see the effect in real ferromagnetic samples irradiated by strong optical sources which may lead to interesting and technologically important features in the field of spintronics [20].

Declarations

Author contribution statement

Muktish Acharyya: Conceived and designed the analysis; Analyzed and interpreted the data; Contributed analysis tools or data; Wrote the paper.

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