Strings and QCD?

Joseph Polchinski

Theory Group
Department of Physics
University of Texas
Austin, Texas 78712
bitnet:joe@utaphy

Abstract: Is large-$N$ QCD equivalent to a string theory? Maybe, maybe not. I review various attempts to answer the question.

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It is an old and seductive idea that large-$N$ QCD might be exactly reformulated as a string theory[1]. It is probably too much to hope that the large-$N$ theory can be solved directly, but the topological structure of the perturbation theory and the success of Regge phenomenology make it plausible that one could first recast it as a string theory, and then apply string methods to determine the spectrum and amplitudes. Over the years a fair amount of work has gone into this, but it is not clear that we have learned a single thing about QCD. Fortunately the effort has not been wasted, because it has led to many discoveries which have been important to the string theory of the fundamental interactions.

1Shortly after writing these words, I learned of the recent work by Migdal and Kazakov, reducing gauge theory to a scalar matrix theory which can be solved by matrix model methods. This is a very clever idea, though my initial reaction is skepticism that the model that has been solved is really QCD: the non-Abelian information that one expects in the master field is not there. But perhaps this objection is naive; certainly this deserves a closer look, and might render my talk irrelevant!
Since 1984, several thousand physicist-years have gone into the development of string theory. It therefore seems timely to reexamine the question, “What, if anything, is the relation between strings and QCD?” In recent months I have thought about this from a number of points of view. The result is something like the parable of the blind men and the elephant: there is very little overlap between the different perspectives. In the next six sections I will describe the elephant as seen by six of these blind men.

1 Regge Phenomenology

One often encounters the statement “Regge phenomenology works better than it ought to.” As one example, the trajectories remain fairly straight down to small angular momentum. As a more detailed example, applying dual resonance theory and PCAC to the process \( \pi + A \rightarrow A^*(\text{resonance}) \), leads to the mass relations\(^2\)

\[
m^2(A^*) - m^2(A) = \frac{1}{2\alpha'} \times \text{odd integer}. \tag{1}
\]

A number of relations of this form hold to within a few per cent, for example with \( A^* = \rho \) and \( A = \pi \). This is striking evidence that the string picture is good even down to the pion.

Llewellyn has recently updated the data and tried to find an explanation of the result (1) within modern conformal field theory\(^3\). He makes a nice try: if one has an \( SU(2)_R \times SU(2)_L \) current algebra on the string worldsheet, boundaries will break this down to \( SU(2)_D \). From the spacetime point of view, this is spontaneous breaking, and gives rise to a massless Goldstone state. Now, by the usual rules, the \( SU(2)_L \times SU(2)_R \) symmetry is a local symmetry in spacetime, with gauge bosons from the closed string sector, and the Goldstone boson is eaten. So we must imagine that there is some way to change the rules and get rid of these gauge bosons, in which case there is now a Goldstone pion. Llewellyn argues that the mass relations (1) then follow from an operator product locality argument. It seems very reasonable to play with the idea of keeping some of the usual rules while ignoring others, but it is hard to reconcile this argument with the picture one gets from large-\(N\) QCD. The \( SU(2) \times SU(2) \) chiral symmetry is carried by the quarks but not
the gluons. In the usual string picture, based on the large-$N$ approximation, the quarks live only at the endpoints of the string, so that the $SU(2) \times SU(2)$ should be an endpoint (Chan-Paton) symmetry, not an interior (Kac-Moody) symmetry: these are quite different, and the former does not seem to work for the present purpose.

It is interesting to consider the splitting (1) further. The $\rho$ and $\pi$, differ only in the relative orientation of the quark and antiquark spins. These live at opposite ends of the string and communicate only through their couplings to the string between. Evidently there is something in this coupling that gives rise to universal splittings. How do the quark spins couple to the string variables? Unfortunately, no one seems to have found a string model in which the endpoints carry spin quantum numbers, and I have not found anything simple. This would seem to be a prerequisite for string Regge phenomenology, since the data is all for mesons and baryons, not glueballs.

2 Loop Equations

Loop expectation values would seem to be the natural observables both in gauge theory and string theory, and an obvious starting point in trying to relate the two[4]. In gauge theory the Wilson loop is the basic geometric observable, measuring the curvature of the field. In string theory, the sum over world-sheets with fixed boundary loops defines a natural correlation function. Unfortunately, this plausible approach quickly becomes mired in technicalities. In gauge theory, the field equations for Wilson loops are notoriously ill defined. In string theory, the path integral with fixed boundary loops turns out to be difficult to evaluate, and to have a complicated and poorly understood analytic structure. Other string observables—the vertex operators and the BRST-invariant string fields—are far simpler, and their world-sheet and spacetime analytic and symmetry properties much clearer. Perhaps with some insight these difficulties can be resolved. Even in matrix models, where in principle everything is solvable, the dictionary between loop observables and string variables has proven quite complicated[5]. This is perhaps a place where matrix models will eventually teach us something useful to higher-dimensional physics.

The loop equations for gauge theory have sometimes been argued to be
equivalent to those for one or another specific string theory, but the above complications make it difficult to evaluate these claims. Some of these ideas can be ruled out by study of the high energy behavior of the partition function, to be discussed below.

3 Lattice Strong Coupling Expansion

Wilson’s strong coupling expansion for lattice gauge theory strongly resembles a string theory[6]. Terms involving non-intersecting sets of plaquettes look exactly like string theory: a sum over surfaces weighted by the exponential of the area. This led to some optimistic conjectures, but Weingarten pointed out that the weight for self-intersecting surfaces is more complicated: it implies a contact interaction[7]. Contact interactions, such as self-avoidance

\[ X^\mu(\sigma) \neq X^\mu(\sigma') \quad \text{for all } \sigma, \sigma' \]  

are non-local on the world-sheet and so represent a great complication of the theory. Subsequently it was shown that, as for the two- and three-dimensional Ising models, the theory could be written as a sum over surfaces with a local weight but with additional world-sheet variables[8].

This result is very suggestive. Essentially it is an existence proof for a string theory of QCD. Of course, the strong coupling expansion is far from the continuum limit, but because the strong coupling expansion has a finite radius of convergence, it seems reasonable that an equality which holds to all orders in the strong coupling expansion will hold in some form when continued to the continuum limit. The new world-sheet variables are infinite in number, and the modified theory rather complicated, so it is hard to guess the form of the continuum string theory. At least some of the new variables seem to be associated with the weight given to special points, like saddles, on the string world-sheet. (Also, to answer a question raised by A. Strominger, the \(1/N\) expansion is more complicated than might be expected. The new variables are classical at \(N = \infty\) but fluctuate at finite \(N\), so there is a \(1/N\) expansion on the world-sheet in addition to the topological expansion.)
Two-dimensional gauge theory is an interesting arena for string ideas. Consider a Wilson loop $W_R(L)$, where $L$ is a non-self-intersecting loop and $R$ denotes the representation. The expectation value is readily found to be

$$< W_R(L) > = e^{-C_2(R)A_L/2g^2}$$

from the energy of the electric flux, with $C_2(R)$ being the Casimir and $A_L$ the area of $L$. This looks rather stringy. For two such loops, separated from one another, the expectation value simply factorizes (for all $N$),

$$< W_R(L_1)W_R(L_2) > = e^{-C_2(R)A_1/2g^2 - C_2(R)A_2/2g^2}.$$ 

This factorization occurs because there are no propagating gluons. However, from the string point of view it is a bit strange, because the cylindrical world-sheet bounded by $L_1$ and $L_2$ might be expected to make a non-zero contribution and therefore give a correlation. Noting that the cylinder must be squashed to fit into two dimensions, we might imagine that the string is self-avoiding, thus forbidding this world-sheet. This is not quite right, though, because there are examples (overlapping loops, and self-intersecting loops), where this rule would not work. Also, we have noted in the previous section that self-avoidance is non-local on the world-sheet, and a strictly local interaction is both desirable and possible. The squashed cylinder can be forbidden instead by the local rule that folds on the world-sheet are not permitted. Kazakov and Kostov made a detailed study of expectation values of Wilson loops in two dimensions[9], and Kostov showed that on the lattice the results can be obtained from the strong coupling sum over surfaces theory described in the previous section[10] (except that the proof of the no-folds rule is not complete). Recently, gauge theories on topologically non-trivial two-dimensional spacetimes have also been considered[11]. It would be an interesting exercise to see how the lattice strong-coupling expansion reproduces these.
5 Long Strings at Low Energy

In this section and the next I wish to consider a gedanken experiment.\footnote{For further details and references, see ref. [12] for the present section and [13] for the next.} The experiment is rather tame by the standards of this meeting: no black holes will be involved. I simply wish to grab hold of a QCD string and stretch it out (imagining that quarks are absent or heavy, so that the string cannot break) and then to shake it and squeeze it, and to compare the results with the same experiments on a fundamental string. A basic question is the number of degrees of freedom in each case. In the present section I will consider low frequencies (compared to the scale set by the string tension) and in the next section high frequencies.

Obviously, low frequencies are less revealing than high, but there is an old and interesting paradox to deal with. A long QCD (or Nielsen-Olesen) string should be described to first approximation by the Nambu-Goto action, and its excitations by some quantization of this action. The QCD and Nielsen-Olesen (NO) strings exist within consistent quantum field theories—how does this fit with the problem of quantizing the Nambu-Goto action outside the critical dimension? First we must sharpen the paradox, and see that the problem of the critical dimension arises even in the long-string limit where the Nambu-Goto description should be valid. The basic problem is that none of the standard quantizations gives the correct spectrum for a QCD or NO string in this limit. The old covariant (Virasoro) quantization starts with 4 oscillators, where I have specialized to $D = 4$ where the QCD and NO strings live. Classically, world-sheet coordinate invariance reduces this to 2, but below the critical dimension an anomaly in this invariance reduces the number of null states and leaves roughly 3 sets of oscillators. In contrast, the light-cone quantization gives only the 2 transverse oscillators, but the spectrum is not Lorentz invariant outside the critical dimension. Polyakov introduced two new ideas\cite{14}: an independent world-sheet metric field, and a careful treatment of the path integral measure. These nearly offset one another: after gauge-fixing there is one additional world-sheet scalar, the Liouville field, for 5 in all, but now the coordinate algebra is non-anomalous reducing the number to 3. In fact, except for the subtlety of the Liouville
zero mode, the old covariant and Polyakov quantizations are the same [15].

How do these results compare with what is expected for the QCD and NO strings? From the point of view of the string world-sheet, the oscillators correspond to massless scalars. On general field-theoretic principle, massless scalars are natural only if they are Goldstone bosons. The 2 transverse oscillations are indeed of this type, but there is no place for the 3'rd to come from. For the NO string, one can see explicitly from the semiclassical expansion that there are only 2 degrees of freedom. For the QCD string one cannot, but it seems overwhelmingly likely on grounds of naturalness. Of the standard quantizations, only the light-cone quantization gives the correct number of degrees of freedom, but it is noncovariant while the QCD and NO strings live within covariant theories [16].

For the NO string, one can in principle find the correct quantization, starting from the quantization of the underlying gauge theory, by careful treatment of the collective coordinate quantization. This has not been done, but it is not too hard to guess what the result should be. The old covariant and light-cone quantizations have always seemed suspect from a path integral point of view, because one carries out classical manipulations without regard for measure factors before quantizing. One should be careful about the measure throughout, as did Polyakov. On the other hand, his second innovation, the independent metric, will not come out of the collective coordinate method: lengths should be set by the spacetime metric, through the induced world-sheet metric. The guess is therefore that one will obtain Polyakov's determinants but in terms of the induced rather than Liouville metric—let me call this the effective string. One can then check that this gives the expected spectrum: the coordinate invariance is non-anomalous (matter central charge = 26) but there are only 4 fields and so 2 physical sets of oscillators. One can also confirm this result in another way, writing the most general long-string CFT with 4 $X^a$ fields and requiring $c = 26$.

Notice that this is certainly not the same as the Polyakov/Liouville quantization, as in the latter the Liouville field is decoupled from the embedding. Rather it has in effect a Lagrange multiplier setting the induced and intrinsic metrics equal.

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3The spontaneous breaking of Lorentz invariance by a long string should not be confused with the explicit breaking in the light-cone quantization. See ref. [16] for further discussion.
What do we learn? First, that the Liouville theory for $c > 1$, while an interesting problem in field theory, is not relevant to physical (QCD or NO) strings. This has also been argued by others, such as Banks, from other points of view. Second, it is interesting that the result, a covariant Poincaré invariant $c = 26$ CFT constructed from $4$ $X^\mu$ fields, can be proved not to exist! To see this, consider the world-sheet current of spacetime translation invariance, $j_\mu^a$. Conformal invariance determines

$$<j_\mu^a(z) j_\nu^a(-z)> \propto z^{-2},$$

whence

$$0 = <\partial_z j_\mu^a(z) \partial_{\bar{z}} j_\nu^a(-z)> = ||\partial_z j_\mu^a(z)|0||^2.$$ 

But a local operator which annihilates the vacuum is zero, which means that the current is analytic\(^4\). This is sufficient to show that the $X^\mu$ CFT is free; together with Lorentz invariance, this fixed the energy-momentum tensor to the standard form $-\partial_z X^\mu \partial_{\bar{z}} X_\mu / 2$ and so has $c = 4$. In the present case, it is the first step \(^5\) which goes wrong, because conformal invariance is spontaneously broken in the long-string background by the expectation value of $\partial_a X^\mu$. Thus, the translation currents are not analytic, and the $X^\mu$ are not free.

The effective string action involves the inverse of $\partial_a X^\mu \partial_b X_\mu$ and so only makes sense in the long-string background. It breaks down for short (low-lying) string states. Also, because it is nonrenormalizable, it breaks down for long strings shaken at high frequency. So it cannot be a complete theory, but only a clue. Incidentally, it is hard to violate the above theorem: I am aware of two renormalizable string theories whose low energy limit is the effective string, and both are pathological: the rigid string[18], which as I will discuss below is non-unitary, and a long string in a certain class of sigma model backgrounds, considered by Natsuume[19], which necessarily has world-sheet tachyons.

\(^4\)Spontaneous breaking of conformal invariance is the identifying feature of non-critical strings.
6 Long Strings at High Energy

As we shake the string at higher and higher frequencies, we would not be surprised to excite new world-sheet degrees of freedom beyond the 2 transverse oscillators. One thing we have learned in recent years is that the string partition function is very revealing, and often simpler to evaluate than the interactions. Let us try to count the number of degrees of freedom of the string via the partition function. In QCD at very high temperatures, we might expect that this would be possible due to asymptotic freedom. The problem is that there is a transition to a deconfining phase at high temperature. This transition is very similar to the Hagedorn transition in string theory: a winding state in the Euclidean temperature direction becomes tachyonic and acquires an expectation value.\footnote{We are concerned here with analytic continuation of the canonical ensemble; the actual physics at the string Hagedorn temperature may be more complicated.} Now, in string perturbation theory we routinely ignore tachyons, expanding around the maximum in the potential. It therefore seems plausible that this should also be possible in QCD, corresponding to analytic continuation of the confined-phase partition function to high temperature. Indeed, this turns out to be extremely easy, given existing results on the high-temperature effective potential. I will not repeat here the details which appear in ref. [13]. For the string free energy per unit length, one finds in QCD

$$\mu^2(\beta)/L^2 \sim -\frac{2g^2(\beta)N}{\pi^2\beta^4},$$

(7)

to be compared with the Nambu-Goto spectrum with $n$ sets of oscillators,

$$\mu^2(\beta)/L^2 \sim -\frac{n}{6\alpha'\beta^2}.$$  

(8)

These do not agree unless more and more fields are excited as the temperature increases, $n_{\text{eff}}(\beta) \propto \beta^{-2}$.

The are several possible interpretations. The simplest is that the QCD string is simply a fat object. The number of internal shape excitations would indeed be expected to scale as energy-squared (like the spectrum of a field in a two-dimensional cavity). This is really the essential issue: large-$N$ perturbation theory is planar in index space, but is there any sense in which it is
two-dimensional (= thin string) in its spacetime structure? If not, the string picture is unlikely to be useful outside of the long-string and Regge limits. Fortunately, there are indications that the QCD string is not as complicated as it might be. The first is the lattice strong-coupling expansion, which is at least formally a representation of QCD in terms of sums over thin surfaces with additional fields. It is tempting to identify the extra fields of the strong coupling expansion with those found in the partition function, but I do not have a direct connection between these.

The prospect of an infinite number of fields might in any case make one worry that a string description will be too complicated to be useful; however, if one is interested in low-lying hadrons it may be that all but a few of the fields decouple.

Another indication that the large-$N$ QCD string is thin comes from further application of the high-temperature continuation. If the string is squeezed by imposing transverse periodic boundary conditions, its spectrum and correlations do not change (this follows from the planarity of the large-$N$ approximation). The simplest interpretation is that it does not feel the squeezing because it is a thin object. Incidentally, this is equivalent to the Eguchi-Kawai reduction: by compactifying all dimensions spacetime is reduced to a point without changing the correlations[19]! (The high-temperature continuation is equivalent to what is referred to as ‘quenching.’) This seems remarkable, but the mechanics of it is quite simple. Focus on the case of one compact dimension ($0 < \tau < \beta$). In a gauge in which the $\tau$ component of the vector potential is diagonal and $\tau$-independent,

$$A^\tau(\tau) = \text{diag}(A_i^\tau),$$

the confined phase is one in which the Polyakov loops average to zero, so the phases $e^{i\beta A_i^\tau}$ are distributed uniformly around the circle. Now, if we take $\beta \to 0$ and dimensionally reduce, the covariant derivative is

$$D_\tau = \partial_\tau + iA^\tau \to A^\tau.$$ 

In particular, an $ij$ gluon has covariant momentum $A_i^\tau - A_j^\tau$. Because the $A_i^\tau$ are uniformly distributed, the index sum $i$ around each loop effectively

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6Note, though, that the partition function rules out various proposals that large-$N$ QCD is equivalent to a Nambu-Goto string with a finite number of additional degrees of freedom.
produces a momentum integral! By taking $N$ sufficiently large, that is, the momentum integral can be hidden in the color sum. Once this is understood, we see that the reduction is bit of a swindle, and indeed it has not as yet led to useful progress.

Unfortunately, what we have calculated for QCD cannot be directly interpreted in terms of a number of degrees of freedom because it has been continued to an unphysical region. Above, we used the Nambu-Goto spectrum for comparison, but there are modifications of string theory which change the high-energy behavior. (This point was made to me independently by Zhu Yang and Mike Green). One is the ‘rigid string,’ where the world-sheet ultraviolet behavior is dominated by an extrinsic curvature-squared term. Yang and I have calculated the continued partition function for the rigid string, and find that it gives the same power of temperature as QCD, eq. (7), though the sign is wrong [21]. However, the rigid string has a serious and basic problem which precludes it from being of any relevance to QCD: the curvature term gives the $X^\mu$ a fourth derivative action, so the spectrum is not unitary. As far as we can determine, this problem has been almost totally ignored in the literature (except of ref. [22]), but if the curvature term dominates at any scale, then there must be negative norm (or negative energy) modes at that scale. So this theory cannot apply to strings in Minkowski space. Of course, it could (and evidently does) apply to the statistical mechanics of surfaces, because there is no Minkowski continuation here.

Another short-distance modification is the introduction of boundary loops with Dirichlet conditions ($X^\mu$ constant on a given boundary, the value of the constant being integrated over). These are pointlike in spacetime but extended in terms of the world-sheet conformal structure, and have a substantial effect on the behavior of amplitudes. They were proposed in order to introduce partonic structure into string theory, and Mike Green in particular has pursued this over the years [23]. Recently he has shown that these appear to give the correct behavior for the continued partition function. I find this idea very intriguing: it has some of the correct properties, is nontrivial enough that it could be correct, but at the same time is simple enough that it might be useful. In any case, consistent modifications of string theory are few and far between, so it is of interest to see whether this is consistent, and what the physics of it is.
7 Conclusions

The initial goal was to see whether recent years’ progress in string theory enables us to evaluate the conjecture that large-$N$ QCD is a string theory. Indeed, we have learned a few things, though the results are not especially favorable. The study of the long string low energy limit revealed that the embedding coordinates $X^\mu$ cannot be free fields. By contrast, for the reason discussed in section 5 they are free in all Poincaré invariant four-dimensional critical string theories. This is a great complication: the simplicity of dual models comes largely because they are built from free oscillators. The study of the long string high temperature limit highlights the differences between string theory and QCD at short distance. Of course this difference is well-known in the interactions (soft versus partonic), but this shows that it can also be seen in the partition function, which has been a very useful object in string theory.

Perhaps these differences are a sign that we are at a dead end. However, I think that there is still tantalizing evidence that there is something more to be said. The best approach I see is to try to guess the continuum limit of the strongly coupled lattice theory, perhaps aided by the continuum high temperature continuation/Eguchi-Kawai reduction. Also, I would like to better understand the physics of Dirichlet boundaries.

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McGuigan and Thorn have recently made the clever observation that Regge trajectories at large negative $t$ can be evaluated in QCD perturbation theory[24], somewhat similar in spirit to the idea in section 6. They do not remain linear, but approach constants. This lack of linearity also shows that the $X^\mu$ are not free.
References

1. G. 't Hooft, Nucl. Phys. B72, 461 (1974).

2. C. Lovelace, Phys. Lett. B34, 500 (1971); M. Ademollo, G. Veneziano, and S. Weinberg, Phys. Rev. Lett. 22, 83 (1969).

3. D. C. Lewellyn, “Effective String Amplitudes for Hadronic Physics,” ITP preprint NSF-ITP-91-105 (1991).

4. For a review see A. A. Migdal, Phys. Rep. 102, 199 (1983).

5. See, for example, G. Moore, N. Seiberg, and M. Staudacher, Nucl. Phys. B362, 665 (1991).

6. K.G. Wilson, Phys. Rev. D10, 2445 (1974).

7. D. Weingarten, Phys. Lett. B90, 285 (1980).

8. V. A. Kazakov, Phys. Lett. B128, 316 (1983); V. I. Kostov, Phys. Lett. B138, 191 (1984); K. H. O’Brien and J.-B. Zuber, Nucl. Phys. B253, 621 (1985).

9. V. Kazakov and I. Kostov, Nucl. Phys. B220, 167 (1983).

10. I. K. Kostov, Nucl. Phys. B265, 223 (1986).

11. M. Blau and G. Thompson, “Quantum Yang-Mills Theory on Arbitrary Surfaces,” preprint NIKHEF-H/91-09, MZ-TH/91-17; E. Witten, “On Quantum Gauge Theories in Two Dimensions,” Comm. Math. Phys. 141, 153 (1991); A. Strominger, unpublished.

12. J. Polchinski and A. Strominger, Phys. Rev. Lett. 67, 1681 (1991).

13. J. Polchinski, Phys. Rev. Lett. 68, 1267 (1992).

14. A. M. Polyakov, Phys. Lett. B40, 235 (1972).

15. R. Marnelius, Phys. Lett. B172, 337 (1986).
16. J. D. Cohn and V. Periwal, “Lorentz Invariance of Effective Strings,” IAS/Fermilab preprint IASSNS-HEP-92-18, Fermilab-92/126-T (1992).

17. R. Dashen and Y. Frishman, Phys. Rev. D11, 2781 (1975); I. Affleck, Phys. Rev. Lett. 55, 1355 (1985).

18. A. M. Polyakov, Nucl. Phys. B268, 406 (1986); H. Kleinert, Phys. Lett. B174, 335 (1986).

19. M. Natsuume, “Nonlinear Sigma Model for String Solitons,” Texas preprint UTTG-10-92 (1992).

20. T. Eguchi and H. Kawai, Phys. Rev. Lett. 48, 1063 (1982); D. J. Gross and Y. Kitazawa, Nucl. Phys. B206, 440 (1982); A. A. Migdal, Phys. Lett. B116, 425 (1982); S. Das and S. Wadia, Phys. Lett. 117B (1982) 228; see also ref. [4].

21. J. Polchinski and Z. Yang, “High-Temperature Partition Function of the Rigid String,” Texas/Rochester preprint UTTG-08-92, UR-1254, ER-40685-706 (1992).

22. E. Braaten and C. K. Zachos, Phys. Rev. D34, 1512 (1987).

23. M. B. Green, “Temperature Dependence of String Theory in the Presence of World-Sheet Boundaries,” Queen Mary College preprint QMW-91-24 (1991).

24. M. McGuigan and C. T. Thorn, “Quark-Antiquark Regge Trajectories in Large-\(N_c\) QCD,” Florida preprint UFIFT-92-12 (1992).