Research Article

Application of VMD in Pipeline Leak Detection Based on Negative Pressure Wave

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Leakage problems are common in the water supply pipeline system, which will threaten the health of residents and cause economic losses. Negative pressure wave (NPW) technology calculates the time difference through the inflection point to locate the leak. However, due to the nonlinear and nonstationary characteristics of the pressure signal, it is difficult to obtain an accurate inflection point of the NPW by the traditional method. Therefore, the advantages of applying variational mode decomposition (VMD) in NPW technology are explored. Firstly, the correlation coefficient and permutation entropy (PE) are used for effective intrinsic mode function (IMF) component selection and parameter optimization. Thus, an adaptive denoising method based on VMD (AD-VMD) is presented. Then, to effectively separate the detail features of the NPW, a novel inflection point extraction method based on VMD (IPE-VMD) is proposed. Simulation and experimental results show that AD-VMD can effectively suppress noise interference and conserve the mutation characteristic of the leakage. IPE-VMD can obtain a distinct maximum peak at the inflection point and has good robustness to noise interference. This method can calculate the time difference precisely and stably. In addition, the accuracy of the leak location is verified. The average relative positioning error is 5.13%.

1. Introduction

With the rapid advancement of urbanization, water supply pipelines play an important role in the survival and development of modern urban civilization. Nowadays, water supply pipelines have provided the basic guarantee for the survival of more than 4 billion people worldwide. However, the pipeline may be affected by water hammering, aging, and weld defects during operation [1, 2]. These factors will inevitably cause pipeline failures, which will lead to leakage. Many countries are currently facing varying degrees of pipeline leakage problems. For example, in Europe, about 20% of transport water is lost every year due to leaks or other factors [3]. The loss rate of water supply in some Asian cities has even exceeded 50%, such as Manila (PH) [4]. Pipeline leakage will seriously waste water resources and cause additional economic losses. Moreover, the surrounding contaminants entering the pipe through the leakage point will pollute the water quality and endanger people’s health [5]. Therefore, to reduce the hazards and losses caused by leakage, exploring some effective methods to accurately detect the leakage location is meaningful.

Among the numerous pipeline leak detection technologies, non-real-time detection and real-time detection can be divided according to the way of implementation. Manual patrol, leakage listening detector, gas tracer, smart ball, and robot are examples of non-real-time detection. Among them, manual patrol does not require any additional equipment, but this method cannot detect buried pipelines, and the efficiency is low [6]. Leakage listening detector has been commercialized, and the technology is simple in principle and easy to operate. However, this method is sensitive to background noise and is only suitable for specific detection environments [7]. Gas tracer, smart ball, and robot have high detection accuracy, but they are intrusive and difficult for users to accept [8, 9]. Moreover, non-real-time detection technology cannot detect leakage online and is inefficient. With the development of sensor technology and computer...
science, methods for real-time monitoring of pipeline status have been widely studied and applied. For example, mass balance, acoustic emission, optical fiber sensor, and negative pressure wave (NPW). The principle of using mass balance to judge leakage is simple, but this method cannot detect the information containing the leakage location and cannot be used for positioning [10]. Although acoustic emission and optical fiber sensor technologies have high accuracy for leak location, these methods also have high requirements for sensor configuration, and the equipment is expensive [11]. NPW is actually a technique based on transient pressure, which is produced by a rapid pressure drop caused by leakage. The NPW method has a long detection distance, even reaching dozens of kilometers [12]. This technology has the advantages of fast response, low cost, and relatively high positioning accuracy. The real-time detection technology can monitor the pipeline status online with high efficiency. NPW is the preferred technology among them [13], and it has an excellent application prospect in the leakage detection of water supply pipelines.

In actual engineering, the water supply piping system is a complex nonlinear system. When the NPW is used to detect leakage, the effective information carried in the signal will inevitably be interfered by various noises, for example, noise caused by pump adjustment, valve operation, and factory construction. These noises will increase the error of the inflection point calculation and make it difficult to locate leakage. Therefore, when applying NPW to locate the leakage, it is essential to adopt an appropriate method to denoise the signal.

The traditional filter-based method can suppress the noise in the specified frequency band well, but when the pressure signal and noise are random signals with unknown characteristics, it is difficult to determine the frequency band of filtering. Compared with the filter-based method, the wavelet threshold method can obtain a better noise reduction effect [14]. However, the pressure signal and noise in the pipeline are nonlinear and nonstationary signals, and it is difficult to select the appropriate wavelet type and threshold [15]. Empirical mode decomposition (EMD) can decompose the signal adaptively, but it lacks rigorous mathematical derivation. Moreover, the end effect and mode aliasing will affect the signal separation effect [16]. To solve the mode mixing problem, EEMD (Ensemble EMD) and CEEMD (Complementary EEMD) are proposed. Nevertheless, these methods for improving EMD have insufficient definitions of extreme points and still lack a solid mathematical foundation [17]. Variational mode decomposition (VMD) is a new adaptive signal decomposition method proposed by Dragomiretskiy and Zosso in 2014 [18]. The algorithm determines the center frequency and bandwidth of each mode by iteratively searching for the optimal solution of the variational problem, thereby realizing the division of the signal in the frequency domain and the separation of the components. VMD has rigorous mathematical derivation and can effectively avoid the problem of mode aliasing.

The signal collected by the sensor is decomposed into multiple intrinsic mode functions (IMFs) by VMD. The effective IMFs mainly contain pressure signals, while the invalid ones mainly contain noise. Only when effective IMFs are selected for reconstruction can noise reduction be realized. Although the VMD algorithm has an excellent signal decomposition performance, it needs to determine the number of decomposition layers $k$ in advance. The traditional methods select the optimal $k$ through trial and error. These methods depend on experience and lack adaptability [19]. Therefore, AD-VMD is presented in this paper. This method uses a correlation coefficient to screen effective IMFs and determines the optimal $k$ through permutation entropy. Finally, the adaptive denoising of NPW signal is realized.

The key to using the denoised signal for leak location is to accurately capture the NPW’s inflection point (transient information). The traditional fault detection method is based on the Fourier transform, which can characterize the frequency characteristics of the signal, but it does not have the ability of localized analysis in the time domain. The short-time Fourier transform (STFT) has the ability of time-frequency localized analysis, but it is usually based on a fixed short data window [20]. This method cannot sensitively reflect the abrupt change of the signal, and it has limitations in analyzing NPW. Wavelet transform has an adjustable resolution in the time domain and frequency domain, which can effectively obtain the detailed information of the signal. This method has the ability to extract the position of signal mutation [21]. However, the process of wavelet analysis is not adaptive and the results may be unstable. VMD is suitable for the separation of multicomponent nonstationary signals [22]. This algorithm can well extract the transient mutation characteristics of the signal [23]. Based on this, IPE-VMD is proposed to self-adaptively extract the inflection point of the NPW.

VMD has been widely studied and applied in the denoising of vibration signals and the fault feature extraction of rotating machinery [24–26]. In this paper, VMD is introduced into the research of the NPW signal denoising and inflection point extraction, and the effectiveness of the proposed method is verified by simulation and actual experiments. The structure of this paper is as follows: In Section 2, the noise reduction principle of AD-VMD and the inflection point extraction principle of IPE-VMD are introduced in detail. In Section 3, the advantages of IPE-VMD are analyzed through simulation experiments. Section 4 is the experimental environment and leak point layout. Section 5 is the results and discussion. The practicability and advantages of AD-VMD and IPE-VMD are verified. Finally, conclusions are given in Section 6.

2. Methodology

2.1. Principle of NPW-Based Leak Location. When a leak occurs in a pipe, the pressure at the leak point will drop rapidly. The pressure drop will propagate to the sensors at both ends of the pipeline in the form of a NPW. Furthermore, the turning point at which pressure suddenly drops is known as the inflection point of the NPW. Through this inflection point, the time difference between the NPW propagation to the two sensors can be calculated. If the pipe length and wave velocity are known, then the leak can be located by this time difference.

When there is a sensor on both sides of the leakage point, the principle of NPW positioning leakage is shown in Figure 1. Assume that the distance between sensor A and
Sensor B is \( L \); the distance between the leak point and sensor A is denoted as \( X_A \); the NPW velocity is \( v \). Correspondingly, the time of the NPW to reach sensor A is \( t_1 = X_A / v \), and the time to reach sensor B is \( t_2 = (L - X_A) / v \). \( X_A \) can be obtained by

\[
X_A = \frac{1}{2} (L + v \Delta t),
\]

where the time difference \( \Delta t = t_1 - t_2 \), and the velocity \( v \) can be given by Equation (2) [27]:

\[
v = \frac{K/\rho}{1 + (K/E)(D/e)C_4}
\]

where \( K \) is the bulk modulus of water, \( \text{Pa} \); \( \rho \) is the density of water, \( \text{kg/m}^3 \); \( E \) is the modulus of elasticity of pipeline materials, \( \text{Pa} \); \( e \) is the tube thickness, \( \text{m} \); \( D \) is the pipe inner diameter, \( \text{m} \); and \( C_4 \) is the correction factor.

When calculating the leak location, the pipe length \( L \) can be obtained from field measurements or design drawings. Therefore, in Equation (1), the key factors that determine leakage location are as follows: NPW velocity \( v \) and time difference \( \Delta t \), where \( v \) is approximately constant in the same pipeline. \( \Delta t \) is mainly calculated by the inflection point of the NPW. However, noise will weaken the features of the inflection point, leading to large errors in the calculation results and affecting the accuracy of leakage location. In order to improve the precision of the inflection point extraction, this paper will investigate the noise reduction method of the pressure signal. Furthermore, traditional inflection point extraction methods rely on the setting of empirical parameters, and their effects are easily affected by the characteristics of the signal itself. Thus, this paper will investigate the method of adaptively extracting the inflection point of the NPW to improve the accuracy of leakage location.

2.2. Methodology of AD-VMD

2.2.1. VMD. VMD regards the signal decomposition process as the solution process of the constrained variational problem. It decomposes the input signal into \( k \) IMF components. Using \( H^1 \) Gaussian smoothness of the demodulated signal, the bandwidth of each mode is estimated. An optimal solution processing for a constrained variational problem can be expressed as follows [28]:

\[
\min_{\{u_k\}, \{\omega_k\}} \left\{ \sum_k \left\| \partial_t \left[ (\delta(t) + \frac{j}{\pi t}) \ast u_k(t) \right] e^{-j\omega_k t} \right\|^2 \right\},
\]

where \( \{u_k\} = \{u_1, \cdots, u_T\} \) represents the \( T \) IMF components obtained by decomposing the signal \( f \), and \( \{\omega_k\} = \{\omega_1, \cdots, \omega_T\} \) is the center frequency corresponding to each component.

When VMD is adopted to decompose the signal, the augmented Lagrangian function is introduced to convert the constrained variational problem of Equation (4) into an unconstrained variation problem. The augmented Lagrangian is expressed as follows:

\[
L(\{u_k\}, \{\omega_k\}, \lambda) = \alpha \sum_k \left\| \partial_t \left[ \left( \delta(t) + \frac{j}{\pi t} \right) \ast u_k(t) \right] e^{-j\omega_k t} \right\|^2
+ \left\| f(t) - \sum_k u_k(t) \right\|^2 + \left\langle \lambda(t), f(t) - \sum_k u_k(t) \right\rangle,
\]

where \( \alpha \) is the penalty factor and \( \lambda \) is the Lagrangian multiplier.

The saddle point of Equation (5) is obtained by using the alternating direction multiplier method (ADMM) and iteratively updates \( u_k, \omega_k, \) and \( \lambda \) in the frequency domain to obtain the optimal solution, thereby obtaining each IMF \( \{u_k\} \) and its corresponding center frequency \( \{\omega_k\} \). The detailed theoretical derivation and implementation of VMD can be found in Ref. [18].

When using VMD for denoising, the optimal denoising signal and the optimal \( k \) are determined according to the minimum value of the reconstructed signal’s permutation entropy. Therefore, the proposed AD-VMD first needs to complete the selection of effective IMFs to obtain the reconstructed signals corresponding to different \( k \) values. Then, calculate the permutation entropy of each reconstructed signal to achieve parameter optimization. The detailed analysis is shown below.

2.2.2. Selection of Effective IMFs. In pipeline leak detection, the pressure signal and noise collected by the sensor are random signals with unknown characteristics. When VMD decomposes the signal into multiple IMF components, it is necessary to retain the effective components and eliminate the noise components to achieve noise reduction. Traditional methods rely on empirical parameters to select effective components, and the characteristics of signal and noise need to be known in advance. Considering the deficiencies of these methods, this paper uses a correlation coefficient to screen effective IMFs. When there is no fixed interference around, it can be considered that the noise collected by the two sensors is not correlated, but the pressure signal is correlated [29]. The signal collected by one sensor is used as the detection signal, and the signal collected by another sensor is used as the reference signal. Firstly, the detection signal is decomposed into multiple IMFs through VMD. Then, the correlation coefficient between each IMF and the reference signal is calculated. Finally, the effective components that mainly contain the pressure signal are selected for reconstruction.
Assuming that the signals collected by the sensors at both ends of the pipeline are \( x_A(t) \) and \( x_B(t) \), respectively, the relationship between them can be expressed by

\[
\begin{align*}
    x_A(t) &= s^*(t) + n_1(t), \\
    x_B(t) &= a^*(t - \Delta t) + n_2(t),
\end{align*}
\]

where \( s^*(t) \) is the pressure signal, \( a \) is the attenuation factor, \( \Delta t \) is the time difference, and \( n_1(t) \) and \( n_2(t) \) are noise.

In the correlation analysis, the IMF that mainly contains the pressure signal has a higher correlation with the reference signal, while the IMF that mainly contains noise has a lower correlation with the reference signal. Therefore, when \( x_B(t) \) is used as the reference signal to select the effective IMF of \( x_A(t) \) (and vice versa), only the components with higher correlation will be retained. Assuming that \( x_A(t) \) is decomposed into \( u_1(t), u_2(t), \ldots, u_m(t), \ldots, u_k(t) \) by VMD, the correlation coefficient of the \( m \)th component \( u_m(t) \) and \( x_B(t) \) can be given by

\[
R = \frac{\sum_{i=1}^{n} (u_{mi} - \bar{u}_m)(x_{B,i} - \bar{x}_B)}{\left\{ \sum_{i=1}^{n} (u_{mi} - \bar{u}_m)^2 \sum_{i=1}^{n} (x_{B,i} - \bar{x}_B)^2 \right\}^{1/2}},
\]

where \( \bar{u}_m = \sum_{i=1}^{n} u_{mi} / n \), \( \bar{x}_B = \sum_{i=1}^{n} x_{B,i} / n \), and \( n \) is the length of the signal.

Take the absolute value of \( R \) in Equation (7) and normalize it to 0-1, which is denoted as \( R_{ux} \). According to the definition of a correlation coefficient, IMF with higher correlation (\( R_{ux} \geq 0.3 \)) is selected as the effective component and denoted as \( u_m(t) \). The \( u_M(t) \) is reconstructed to obtain the signal \( s(t) \), as shown in

\[
s(t) = \sum_{M} u_M(t).
\]

2.2.3. Parameter Optimization Based on Permutation Entropy. Permutation entropy (PE) can reflect the complexity of one-dimensional time series, and can quantitatively evaluate the random noise contained in the signal. The larger the value of PE, the more complex and random the corresponding time series. On the contrary, the corresponding time series are simpler and more regular. When using VMD to denoise the collected signal, the noise in the reconstructed signal is more disordered than the pressure signal. Therefore, PE can be used as a characterization of the noise content in the signal. When PE is small, it can be considered that there is less noise in the reconstructed signal, and the noise reduction effect is better at this time. Based on this, this paper determines the optimal denoising signal and the optimal decomposition layers \( k \) according to the minimum value of PE. The detailed principle and calculation steps of PE can be found in Ref. [30]. When calculating PE, embedding dimension \( m \), time delay \( \tau \), and length \( N \) of time series need to be determined in advance, where \( m \) is the key factor affecting the calculation results. If \( m \) is too small, the algorithm will lose its meaning. Nevertheless, if \( m \) is too large, the calculation time will increase, and it will be difficult to reflect small changes in the sequence [31]. Ref. [31] analyzed the parameters of PE. According to the research results, this paper sets the initial parameter \( m = 6 \), \( \tau = 1 \), \( N \geq 1024 \). After setting the basic parameters of PE, the next step is to determine the decomposition layer number of VMD and the optimal denoising signal.

When VMD is used to decompose the signal, if \( k \) is too large, the amount of calculation will increase, but if \( k \) is too small, the noise will be difficult to separate from the signal. Ref. [22] discussed the search range for optimizing \( k \). According to the research results, this paper sets the search range of \( k \) to [2, 15], and the step size is 1; the penalty factor \( \alpha \) is set to 2000.

VMD decomposes the signal into \( k \) IMF components. The effective components are screened by the method in Section 2.2.2, and the reconstructed signal \( s_k(t) \) is obtained when the number of decomposition layers is \( k \). The search range of \( k \) is [2, 15]. Therefore, 14 reconstructed signals can be obtained. As shown in

\[
\tilde{s}_k(t) = (s_2(t), s_3(t), \ldots, s_{15}(t)), \quad k = 2, 3, \ldots, 15,
\]

denote the value of PE as \( H_{pe} \). Calculate the \( H_{pe} \) of all signals in \( \tilde{s}_k(t) \). Thus, we can get

\[
H_{pe}^k = (H_{pe}^2, H_{pe}^3, \ldots, H_{pe}^{15}), \quad k = 2, 3, \ldots, 15,
\]

where \( H_{pe}^k \) represents the permutation entropy of the reconstructed signal when \( k \) is 2, and so on.

Then, the minimum permutation entropy \( H_{pe}^{min} \) can be obtained:

\[
H_{pe}^{min} = \min \left( H_{pe}^k \right) = \min \left( H_{pe}^2, H_{pe}^3, \ldots, H_{pe}^{15} \right).
\]

Finally, the optimal \( k \) is denoted by \( k' \), and it can be calculated by

\[
k' = \arg\min \left( H_{pe} \right).
\]

The reconstructed signal corresponding to \( k' \) is the optimal denoising signal.

Figure 2 shows the detailed flow chart of denoising the NPW signal through the above steps, where \( \tilde{s}_{k_A}(t), H_{pe}^{k_A}, H_{pe}^{min} \), and \( k' \) are corresponding to \( s_{k_A}(t), H_{pe}^{A}, H_{pe}^{min} \), and \( k' \) in Equations (9), (10), (11), and (12), respectively. \( s_{k_A}(t) \) is the optimal denoising signal corresponding to sensor \( A \). \( \tilde{s}_{k_B}(t), H_{pe}^{B}, H_{pe}^{min} \), and \( s_{B}(t) \) have similar definitions.

2.3. Methodology of iPE-VMD

2.3.1. Principle of NPW Inflection Point Extraction. After the original NPW signal is denoised by AD-VMD, it is necessary
to adopt an appropriate method to extract the inflection point. The inflection point of NPW is the sudden drop point of the pressure signal after leakage. However, the characteristic information of the abrupt part of the pressure signal is often contained in its high-frequency components. Therefore, it is requisite to separate the high-frequency components to extract the information containing the inflection point features.

The characteristics of pressure signal in pipeline change rapidly with time, and it belongs to nonstationary and nonlinear signal. The traditional method has problems such as mode aliasing and excessive dependence on empirical parameters. Therefore, this paper uses VMD to decompose the denoised NPW signal to extract the detail features of the inflection point. Taking the 3-layer decomposition as an example, the frequency of IMF1 to IMF3 decreases successively. As shown in Figure 3, the lowest frequency IMF3 reflects the overall profile of the NPW, while the higher frequency components IMF1-IMF2 reflect the details of the signal. At the position of the signal mutation, the high-frequency components show a series of peaks. At the inflection point, the abrupt change of the NPW signal is the most drastic, and it appears as the maximum peak in the high-frequency component. Therefore, the inflection point of the NPW can be obtained as long as the maximum peak position is calculated. The steps of inflection point extraction are as follows.

First, remove the lowest frequency IMF that reflects the signal profile (in this case, IMF3). Then, in order to retain the details of the inflection point adequately, the remaining components are directly reconstructed. Finally, the detail signal that can represent the feature of the inflection point is obtained. As shown in Figure 4, the maximum peak of the detail signal accurately reflects the position of the inflection point. Therefore, the NPW inflection point extraction method proposed in this paper is workable.

When VMD is applied to inflection point extraction, the optimal number of decomposition layers $k$ also needs to be determined. If the value of $k$ is too small, the information containing the details of the inflection point is difficult to separate. Conversely, too much invalid information may be introduced. Therefore, it is necessary to select the optimal $k$ to obtain an exact inflection point of the NPW.

2.3.2. Parameter Optimization Based on Kurtosis. Kurtosis is a numerical index used to describe the characteristics of signal distribution, and it is a dimensionless parameter that measures the sharpness of a waveform. It is defined as follows [32]:

$$K = \frac{E(x - \mu)^4}{\sigma^4}, \tag{13}$$

where $\mu$ is the mean value of signal $x$ and $\sigma$ is the standard deviation of $x$.

When $K$ is approximately equal to 3, the signal amplitude is close to a normal distribution, and the signal curve has normal kurtosis at this time. When $K$ is less than 3, the signal curve tends to be flat, and its peak is lower than the normal distribution curve. When $K$ is greater than 3, the signal curve tends to be steep, and its peak is higher than the normal distribution curve. It can be concluded that the larger $K$ is, the steeper the signal curve will be, and the peak will be more obvious. Similarly, with the increase of kurtosis, the detail signal reconstructed in this paper will have a more obvious maximum peak at the inflection point. Therefore, this paper takes kurtosis as a characterization of the signal reconstruction effect. When the detail signal has the maximum kurtosis, the reconstruction effect is considered to be the best at this time, which is most conducive to the accurate extraction of the inflection point. Moreover, the number of optimal decomposition layers corresponds to the maximum kurtosis.

Similar to the process of optimizing $k$ in Section 2.2.3. Firstly, set the search range of $k$ as $[2, 15]$, and the step size is 1; the penalty factor $\alpha$ is set to 2000. Then, the signal is decomposed by VMD. Remove the lowest frequency IMF

![Figure 2: Detailed implementation process of AD-VMD.](image-url)
and reconstruct the remaining components. After the search is completed, 14 reconstructed detail signals are obtained. Calculate the kurtosis of these signals, and the optimal number of decomposition layers $k$ is obtained according to the maximum kurtosis.

In order to objectively analyze the advantages of IPE-VMD. In Section 3 and Section 5, the effectiveness of the wavelet method, EMD method, and IPE-VMD for the inflection point extraction are compared. Ref. [20] studied the effect of wavelet analysis to extract the inflection point of...
the NPW. The initial parameter sets in this paper are the same as in Ref. [20] when applying the wavelet method. Initially, db2 wavelet is used for 5-level decomposition. Subsequently, the detail signal is reconstructed. Eventually, the inflection point is calculated according to the maximum of modulus [33]. The method of EMD to extract the inflection point is similar to that in Section 2.3.1.

2.4. Overall Implementation Process for Leak Location. The overall implementation process of locating water supply pipeline leaks is shown in Figure 5. The specific steps are as follows:

1. Input the pressure signals \( x_A(t) \) and \( x_B(t) \) collected by sensor A and sensor B. Initialize the parameters of PE and VMD

2. Take \( x_A(t) \) as an example. \( x_A(t) \) is decomposed by VMD, effective components are screened and reconstructed according to the method in Section 2.2.2, and the vector \( \hat{s}_{k_1}(t) \) composed of 14 reconstructed signals is obtained. The PE values of all signals \( \hat{s}_{k_1}(t) \) are calculated to obtain the minimum PE value \( H_{pe}^{min,A} \). The optimal denoising signal \( s_A(t) \) and the optimal k are calculated according to \( H_{pe}^{min,A} \). The detailed calculation steps are shown in Section 2.2.3

3. \( s_A(t) \) is decomposed by VMD, and the signal is reconstructed according to the method in Section 2.3.1. After completing the search, 14 reconstructed detail signals are obtained. Calculate the kurtosis of all detail signals, and obtain the optimal detail signal \( \hat{s}_{dA}(t) \) and the optimal k according to the maximum kurtosis. Similarly, the optimal detail signal \( \hat{s}_{dB}(t) \) corresponding to \( x_B(t) \) can be obtained. The detailed analysis is shown in Section 2.3.2

4. Calculate the maximum peak point of \( \hat{s}_{dA}(t) \) and \( \hat{s}_{dB}(t) \) to obtain the inflection point of the NPW. According to the inflection point, the time difference between the NPW reaching the two sensors is calculated. Finally, the leak is located by substituting the time difference into Equation (1)

In the leakage location, the method proposed in this paper adaptively realizes the denoising and inflection point extraction of the NPW signal. Accordingly, this method can effectively improve the lack of adaptability of traditional methods. The entire algorithm ran 40 s in a computer configured with AMD Ryzen 31300X Quad-Core Processor, 3.50 GHz. The processing speed can meet the needs of practical applications. Therefore, the leak location method of the water supply pipeline proposed in this paper has good practicability.

3. Simulation Experiment

In order to verify the advantages of IPE-VMD, a simulation signal is constructed through Equation (14), as shown in Figure 6. The inflection point of this signal is set at 1800, and there is also a mutation at 2000. Wavelet analysis, EMD analysis, and IPE-VMD are used to extract the inflection point, and the results are presented in Figure 6.

![Figure 5: The overall implementation flow chart of leak location.](Image)

The maximum modulus value of wavelet analysis is at point 2018, which is quite different from the actual two mutation positions. In addition, there is an extreme value at point 1801, which is very close to the actual inflection point. Nevertheless, it is not in the position of the maximum value of the modulus, and it is difficult to set the criterion in practical applications. When using wavelet analysis to extract the inflection point, the calculation result has some deviations due to the influence of empirical parameters. Furthermore, wavelet analysis is susceptible to interference from signal mutations. The reconstructed detail signal has multiple extreme values with similar amplitudes. Accordingly, the obtained inflection point may be false.

The maximum peak obtained by EMD is at point 1800, and the inflection point is accurately extracted. However, the result obtained by EMD has end effect and modal aliasing. When processing actual signals, these distortion phenomena may cause large errors. IPE-VMD effectively overcomes the deficiency of EMD and obtains the unique and clear maximum peak. This peak is located at point 1800, which is consistent with the actual inflection point. Therefore, compared with EMD and wavelet analysis, IPE-VMD is more suitable for extracting the inflection point of the NPW.

4. Laboratory Experiment

4.1. Experimental Environment. In order to investigate the practical application effect of the proposed leak location method, a pipeline leakage experimental system was established, as shown in Figure 7. The experimental pipeline is connected to the external water supply pipeline through a valve. The pipe material is carbon steel with a diameter of 0.08 m. The leakage was simulated by quickly opening the valve. The simulated leakage area is about 25 mm$^2$ and
15 mm², respectively. The name of the pressure sensor is HM90, with the full scale (F.S) from 0 MPa to 0.6 MPa and the precision of 0.1% F.S. The data acquisition card collects the voltage signal of the sensor. The computer is used for signal processing and powering the data acquisition card. The battery is charged by the solar panel and supplies power to the sensor. The sampling frequency of the signal is 500 Hz. The pressure in the pipe fluctuates slowly between 0.26 and 0.28 MPa. The NPW velocity is 1100 m/s [34, 35]. Some main experimental parameters are summarized in Table 1. The field experimental equipment is represented in Figure 8.

4.2. Leak Point Layout. Suppose the length of the pipeline between the two sensors is \( L \), and the real distance between the leak point and sensor A is \( L_A \). The experimental pipeline is assembled by several short pipes, so \( L \) can be adjusted flexibly. In this paper, leak points 1 and 2 are set when \( L \) is 17.34 m, and leak points 3 and 4 are set when \( L \) is 23.34 m. The \( L_A \) corresponding to each leak point is listed in Table 2.

5. Results and Discussion

5.1. Inflection Point Extraction of the Original Signal. In order to investigate the influence of noise on leak location, the effectiveness of inflection point extraction of the original NPW signal is analyzed in this section. The extraction results of the inflection point by the three methods are shown in Figure 9. Figure 9(a) is the signal processing results of sensor A, and Figure 9(b) is the corresponding results of sensor B.

In Figure 9(a), the mutations obtained by wavelet analysis and IPE-VMD are all at the burst interference of the original signal (e.g., a clear mutation value can be observed between 2000 and 3000), and the actual inflection point of the NPW has been entirely covered by noise. The detail signal obtained
by EMD can observe the area where the inflection point is located. Nevertheless, this is the result of mode aliasing, and the purpose of extracting the detail of the inflection point is not achieved, similarly, as shown in Figure 9(b).

The burst interference in the original signal has more obvious mutation characteristics than the inflection point. The real inflection point of the NPW may be drowned by these disturbances and generates false inflection point. The false inflection point will increase the error of leakage positioning and even make the positioning impossible. Therefore, in order to extract the inflection point of the NPW exactly, it is necessary to denoise the signal.

5.2. Signal Denoising

5.2.1. Signal Decomposition and Reconstruction. Take the signal \( x_A(t) \) of leak point 1 collected by sensor A as an example. In the process of optimizing the number of decomposition layers of VMD, when \( k = 5 \), the signal is decomposed into five IMFs. The results are presented in Figure 10.

After the signal decomposition is completed, the correlation coefficient between each IMF and the signal \( x_B(t) \) of sensor B will be calculated. The normalized correlation coefficient is shown in Table 3.

According to the method in Section 2.2, when \( k = 5 \), IMFs with \( R_{ux} \geq 0.3 \) are selected to obtain the reconstructed signal. Accordingly, 14 reconstructed signals are obtained after the search is completed in the range of \([2, 15]\). Then, the permutation entropy of each reconstructed signal is calculated, and the results are shown in Figure 11. Where when \( k = 15 \), the permutation entropy has the minimum value of 0.3896. At this point, the reconstructed signal has relatively less noise and the denoising effect is optimal. Similarly, signal denoising of sensor B can be realized.

5.2.2. Analysis of Denoising Results. In order to verify the advantages of AD-VMD, the signals under normal-running and leak conditions are, respectively, processed by wavelet denoising, EMD denoising, and AD-VMD. The results are presented in Figures 12 and 13, respectively. Ref. [36] achieves effective denoising of the NPW signal by the wavelet method. Accordingly, when wavelet denoising is applied in this paper, the same parameters are set as in Ref. [36]. The EMD method adopts the same effective IMF selection steps as in Section 2.2.2 to achieve denoising.
In Figure 12, the green part represents the normal-running signal. The fluctuation trend of the signal after wavelet denoising is consistent with the original pressure signal. Nevertheless, the denoised signal still contains a lot of small noises, which may lead to misjudgment of leakage. The signal after EMD denoising is very smooth. However, affected by the end effect, the denoised signal deviates from the original signal. Compared with wavelet denoising, the signal processed by AD-VMD is much smoother. Compared with the EMD method, AD-VMD better retains the fluctuation trend of the original signal. Therefore, AD-VMD achieves a more effective denoising for the normal-running pressure signal.

In terms of accurately extracting the inflection point of the NPW, the denoised leak signal should meet the following requirements: First of all, small fluctuations and burst interferences need to be suppressed, and the pressure curve after denoising should be smooth. Secondly, the mutation

Figure 9: Inflection point extraction of the original signal: (a) the results of sensor A; (b) the results of sensor B.
characteristic caused by leakage (i.e., the NPW inflection point) should be conserved. Finally, distortion is not allowed.

In Figure 13, the green part represents the leak signal. The signal after wavelet denoising still has many small fluctuations and burst interferences. When locating leaks, these disturbances may cover the real inflection point and generate a false one. The result of the EMD method is extremely smooth, but the signal is distorted and difficult to be applied to leak location. The signal processed by AD-VMD is very smooth, and the interference of small noise is effectively eliminated. Furthermore, the mutation characteristic of the leakage is well conserved, which lays a foundation for improving the accuracy of inflection point extraction.

5.3. Inflection Point Extraction of the Denoised Signal. In this paper, the signal has distortion after denoising by the EMD method, which is difficult to be applied to the inflection point extraction of the NPW. Consequently, in this section, only the signals processed by wavelet denoising and AD-VMD are investigated for inflection point extraction.

5.3.1. Inflection Point Extraction after Wavelet Denoising. The signal of sensor A after wavelet denoising is denoted as $s_A'(t)$. According to the application process in Section 2.3, IPE-VMD is used to extract the inflection point of $s_A'(t)$. Initially, the detail signal corresponding to each $k$ is reconstructed. Then, the kurtosis of these signals is calculated. As shown in Figure 14, when $k$ is 2, the kurtosis is maximum. According to the analysis in Section 2.3.2, the reconstructed detail signal is optimal at this time.

The signal of sensor B after wavelet denoising is denoted as $s_B'(t)$. Similarly, the result corresponding to sensor B can be obtained. In the next step, the inflection point of the NPW...
will be extracted according to the maximum peak of the optimal detail signal.

In order to verify the advantages of IPE-VMD in extracting the inflection point, the results obtained by wavelet analysis, EMD analysis, and IPE-VMD are compared, as shown in Figure 15. Figure 15(a) is the inflection point extraction results of $s_A'(t)$, and Figure 15(b) is the results corresponding to $s_B'(t)$.

In Figure 15(a), due to the interference of small noise, there are a large number of amplitude mutations in the result obtained by wavelet analysis. Moreover, there is no obvious peak at the inflection point of the NPW, and the leakage feature has been entirely covered by noise. The EMD method exists mode aliasing and does not extract the detail of the inflection point. Under the condition that $s_A'(t)$ contains a lot of small noise, IPE-VMD still obtains a distinct maximum
peak at the inflection point. The correct inflection point of the NPW can be extracted easily. Consequently, compared with the other two methods, IPE-VMD is more robust to small noise and can avoid mode aliasing.

In Figure 15(b), the signal of sensor A after AD-VMD denoising is denoted as $s_A(t)$, and the signal of sensor B after AD-VMD denoising is denoted as $s_B(t)$. The results of the two signals processed by wavelet analysis, EMD analysis, and IPE-VMD are shown in Figure 16. Figure 16(a) is the inflection point extraction results of $s_A(t)$, and Figure 16(b) is the results corresponding to $s_B(t)$.

In Figure 16(a), the maximum of modulus obtained by wavelet analysis is very clear, which could effectively extract the inflection point. However, in Figure 15(a), this method cannot identify the inflection point of the NPW after wavelet denoising. The detail of the inflection point is not extracted by the EMD method. IPE-VMD obtains a very clear maximum peak at the inflection point, which can exactly extract the inflection point of the NPW. Furthermore, compared with the result of IPE-VMD in Figure 15(a), the detail signal suffers less interference, which is more conducive to the correct extraction of the inflection point. It can be concluded that AD-VMD can effectively eliminate the interference of noise.

In Figure 16(b), several extreme values whose amplitude is close to the maximum value of the modulus are obtained by wavelet analysis. In practical applications, it is probable to extract a false inflection point due to the influence of these extreme values. The EMD method cannot extract the inflection point. IPE-VMD obtains a distinct maximum peak at the inflection point, and it is easy to extract the real inflection point. Therefore, compared with the other two methods, IPE-VMD has better stability when extracting the inflection point of the NPW.

5.4. Analysis of the Synthesized Signal. In order to investigate the application effect of the proposed method under the condition of relatively low SNR, Gaussian white noise was added to the original signals of the two sensors in Figure 9 to obtain two synthesized signals with lower SNR. As shown in Figure 17, the black part is the original signal collected by the sensor, and the green part is the synthesized signal. It can be seen that compared with the original signal, the noise in the synthesized signal increases greatly, and the inflection point of the NPW is more difficult to identify.

Take the synthesized signal of sensor A as an example. The signal is denoised by three methods, and the results are shown in Figure 18. Where the green part is the synthesized signal, and the black part is the denoising result of the corresponding method. In Figure 18, the signal after wavelet denoising still has a lot of noise, and the result obtained by EMD has been distorted. Compared with Figure 13, the result of AD-VMD increased some small fluctuations, but the changing trend and the inflection point of the NPW signal are still clearly visible. In order to study the influence caused by the added noise, the inflection point of the synthesized signal after AD-VMD denoising is extracted. The results obtained by wavelet analysis and IPE-VMD are shown in Figure 19. EMD cannot extract the details of the signal, and it is not shown in the figure.

In Figure 19(a), the maximum modulus value of wavelet analysis is between 8000 and 9000, and the real inflection point of the NPW has been completely covered by interference. In Figure 19(b), the result of the wavelet can observe an extreme value near the inflection point. However, there are a large number of extreme points with similar amplitude in this result, and the real inflection point is easily covered by these extreme points. In Figure 19, IPE-VMD obtains a clear maximum peak at the inflection point of the signals. The inflection point of the NPW is effectively extracted. This further reveals that compared to wavelet analysis, IPE-VMD has certain advantages in stability and robustness to noise.

It can be concluded that the proposed method still has a certain degree of applicability under the condition of relatively low SNR. After the inflection point of the NPW signal is extracted, the next step will be to locate the leak.

5.5. Leakage Location. The processing results of the original signal correspond to Figure 16. These results are used to extract the inflection point and analyze the location of the leakage. In Figure 16, suppose the inflection point of $s_A(t)$ is $n_1$, and the inflection point of $s_B(t)$ is $n_2$. The sampling frequency is $f$ ($f = 500$ Hz). The time difference between the two signals is $\Delta t$, and it can be obtained by
In Figure 16, $n_1 = 2690$ and $n_2 = 2690$ are obtained by wavelet analysis. Substituting $n_1$ and $n_2$ into Equation (15), we get $\Delta t = 0$ s. The length of the pipeline between the two sensors is 17.34 m, and the distance $X_A$ between the leak point and sensor A is calculated by Equation (1) to be 8.67 m. The real distance between the leak point and sensor A is 2.47 m. Therefore, the absolute positioning error is 6.2 m, and the relative positioning error is 35.76%. This error makes it difficult to locate the leak. In this paper, when the EMD method is used to extract the inflection point, the obtained detail signal has no obvious peak, which is difficult to be applied to the leak location. The $n_1$ and $n_2$ obtained by IPE-VMD are 2669 and 2674, respectively.

$$\Delta t = \frac{n_1 - n_2}{f}. \quad (15)$$
Similarly, we get $X_A$ which is 3.17 m. The absolute positioning error is 0.7 m, and the relative positioning error is 4.04%. After the analysis of the original signal is completed, the following will analyze the leak location effect of the synthesized signal.

Figure 19 shows the processing results of the synthesized signal. The result of wavelet analysis is seriously disturbed, and the effective inflection point cannot be extracted. The extraction results of IPE-VMD are $n_1 = 2671$ and $n_2 = 2678$. Then, we can get $X_A$ which is 0.97 m. The absolute error is 1.5 m, and the relative error is 8.65%. Although the error of IPE-VMD increases when processing the synthesized signal, the result still has certain application value in practical engineering.

In order to further investigate the effectiveness of the proposed method applied to leak location, for each leak point in Table 2, two leakage experiments were carried out, respectively, under the condition that the leakage areas are 25 mm$^2$. 

![Figure 16: Inflection point extraction after AD-VMD denoising: (a) the results of $s_A(t)$; (b) the results of $s_B(t)$.

\[ s_A(t) \]

\[ s_B(t) \]

\[ 0.273 \]

\[ 0.265 \]

\[ 0.257 \]

\[ 1.8 \times 10^{-4} \]

\[ 1.5 \times 10^{-3} \]

\[ 6.2 \times 10^{-4} \]

\[ 0 \]

\[ 1000 \]

\[ 2000 \]

\[ 3000 \]

\[ 4000 \]

\[ 5000 \]

\[ 6000 \]

\[ 7000 \]

\[ 8000 \]

\[ 9000 \]

\[ 10000 \]

\[ 0 \]

\[ 1000 \]

\[ 2000 \]

\[ 3000 \]

\[ 4000 \]

\[ 5000 \]

\[ 6000 \]

\[ 7000 \]

\[ 8000 \]

\[ 9000 \]

\[ 10000 \]

\[ 0 \]

\[ 1000 \]

\[ 2000 \]

\[ 3000 \]

\[ 4000 \]

\[ 5000 \]

\[ 6000 \]

\[ 7000 \]

\[ 8000 \]

\[ 9000 \]

\[ 10000 \]

\[ 0 \]

\[ 1000 \]

\[ 2000 \]

\[ 3000 \]

\[ 4000 \]

\[ 5000 \]

\[ 6000 \]

\[ 7000 \]

\[ 8000 \]

\[ 9000 \]

\[ 10000 \]

\[ 0 \]

\[ 1000 \]

\[ 2000 \]

\[ 3000 \]

\[ 4000 \]

\[ 5000 \]

\[ 6000 \]

\[ 7000 \]

\[ 8000 \]

\[ 9000 \]

\[ 10000 \]
and $15 \text{ mm}^2$. First, the collected signal is denoised by AD-VMD. Secondly, wavelet analysis, EMD analysis, and IPE-VMD are used to calculate the inflection point of the NPW. Finally, the time difference is calculated to locate the leak.

The results of leak location are listed in Table 4, where $L$ is the distance between the two sensors; $L_A$ is the real distance between the leak point and sensor $A$; $X_1$ is the positioning result after the inflection point is extracted by wavelet analysis,
and the relative positioning error is $E_1$; $X_2$ is the positioning result after the inflection point is extracted by IPE-VMD, and the relative positioning error is $E_2$; "No" means the leak cannot be located. In this paper, the EMD method does not achieve leak location. Thus, the results are not given in Table 4.

Wavelet analysis is susceptible to interference when extracting the inflection point, and its adaptability is poor. Therefore, in Table 4, the minimum relative error of the wavelet method is 21.57%. When the error is the largest, this method is difficult to locate the leak, and the positioning result is unstable. In this paper, the EMD-based method is difficult to be applied to leak location. IPE-VMD can effectively avoid mode aliasing, and it has good robustness to noise interference. In Table 4, the minimum relative error of IPE-VMD positioning is 0.6% and the maximum is 10.38%. The error is smaller when the leakage area is 25 mm$^2$. When the leakage area is larger, the pressure drop is more obvious, which may be more conducive to the accurate extraction of the inflection point. Compared with the other two methods, IPE-VMD can

**Figure 19:** Inflection point extraction of the synthesized signal: (a) the results of sensor A; (b) the results of sensor B.
extract a relatively accurate inflection point. The leakage location error of this method is smaller, and the location result is more stable.

6. Conclusions

This paper investigates the application effects of VMD-based NPW signal denoising and inflection point extraction methods in the leak location of the water supply pipeline. The main results and findings are summarized as follows:

1. Correlation coefficient and permutation entropy are used for effective IMF selection and parameter optimization. AD-VMD realizes adaptive denoising of the NPW signal. Experimental results show that AD-VMD obtains a very smooth pressure signal, and it can effectively eliminate the noise interference. Furthermore, the mutation characteristic of the leakage is well conserved by this method.

2. Wavelet analysis obtains a better inflection point extraction effect than EMD, but it is susceptible to noise interference and lacks adaptability. After the number of decomposition layers is optimized by kurtosis, IPE-VMD adaptively extracts the inflection point of the NPW. Simulation and experimental results show that IPE-VMD has good robustness to noise interference. Furthermore, compared with the traditional method, this method has a better processing effect on pipeline pressure signal, and it can obtain a very clear maximum peak at the inflection point, which is more suitable for precisely extracting the inflection point of the NPW.

3. In order to verify the effectiveness of the leak location method proposed in this paper, four leak points were located. The results demonstrate that the proposed method has relatively higher positioning accuracy and better stability. The minimum relative error is 0.6%, and the maximum is 10.38%. Therefore, the presented method has tremendous application prospects in the leak location technology based on NPW

4. Conventional valve operation or pump adjustment can also produce NPWs, which may cause false alarms of leakage. Consequently, it is requisite to carry out further research on the method of accurate identification of the leakage NPW in practical engineering applications. In addition, data measurement is the key factor affecting the NPW technology [37]. In order to reduce the positioning error caused by this factor, it is necessary to select measuring equipment with appropriate accuracy and measurement range.

Data Availability

The [data type] data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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