VALIDATION OF A SOURCE MODEL FOR THE 2011 TOHOKU EARTHQUAKE USING RECORDS FROM THE SMALL-TITAN

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To investigate the safety of structures for future mega-thrust earthquakes, it is important to develop a reliable source model for the simulation of strong ground motions from such earthquakes. Nozu et al. (2012) developed a source model called the “SPGA model” for the purpose of simulating strong ground motions from mega-thrust earthquakes in the frequency range relevant to structural damage. It is, however, necessary to further investigate the reliability of the source model especially by using records from dense strong-motion arrays such as the Small-Titan. In this article, the SPGA model is applied to the records of the Small-Titan and its performance is discussed.

Key Words : the 2011 Tohoku earthquake, strong ground motion, source model, SPGA model, Small-Titan

1. INTRODUCTION

After the 2011 off the Pacific coast of Tohoku earthquake (hereinafter referred to as the “Tohoku earthquake”) in Japan, the importance of reliable predictions of strong ground motions for mega-thrust earthquakes has become more widely recognized than ever. For this purpose, it is essential to validate source models and simulation methods using strong-motion data of the Tohoku earthquake, an actual mega-thrust earthquake event, before those models are applied to future earthquakes.

From the engineering point of view, one of the most striking features of strong ground motions of the Tohoku earthquake was the generation of pulses; strong ground motions in the frequency range from 0.2 to 1 Hz observed at stiff stations along the coast of Miyagi through Ibaraki were characterized by distinctive pulses1). The importance of the pulses is that they appeared in the frequency range relevant to structural damage. The pulses will be referred to as the “strong-motion pulses” in this article. It is significantly important to consider the generation of such pulses in the prediction of strong ground motions for mega-thrust earthquakes, especially when the prediction is aimed at seismic design of structures.

A source model called the “SPGA (Strong-motion Pulse Generation Area) model” was developed to explain strong ground motions from the Tohoku earthquake with special attention to those pulses1), 2). The source model involved nine sub-events with relatively small size (in the order of several kilometers), located off the coast of Miyagi through Ibaraki. This source model satisfactorily reproduced strong ground motions along the coast of Miyagi through Ibaraki including strong-motion pulses. Source models with larger sub-events (SMGAs) with a size of tens of kilometers were also developed for the same earthquake5)–8). However, sub-events with small size were required to reproduce strong-motion pulses in the frequency range from 0.2 to 1 Hz.

Although the SPGA model was validated using strong motion records along the coast of Miyagi through Ibaraki, it is necessary to further investigate the reliability of the source model especially by using records from dense strong-motion arrays such as the Small-Titan9), 10). In this article, the SPGA model is applied to the records of the Small-Titan and its performance is discussed. A source model that is optimized for one set of data sometimes fails to explain another set of data. In that case, the source
model is not reliable. On the other hand, when a source model developed for one set of data can explain an independent set of data, the source model is really reliable. From such a point of view, the validation of a source model using an independent set of data is meaningful.

In the following, first, the source model to be validated will be explained in Chapter 2. Secondly, the outline of the Small-Titan system and the records for the 2011 Tohoku earthquake will be reviewed in Chapter 3. Then, the simulation method will be described in Chapter 4. The results will be presented.

**Figure 1** The SPGA model to be validated in this study\textsuperscript{1-4}. Green plots indicate the sub-events that generate strong-motion pulses. Black cross indicates the location of the small event used in the simulation. Small rectangle shows the region where the Small-Titan is located and shown in Figure 2.

**Table 1** Parameters for the SPGA model\textsuperscript{1-4}.

|   | Rupture time (h:m:s) | Length (km) | Width (km) | Area (km\textsuperscript{2}) | $M_0$ (Nm) | Rise time (s) |
|---|----------------------|-------------|------------|-------------------------------|------------|---------------|
| SPGA1 | 14:46:43.5 | 3.0 | 2.0 | 6.0 | 8.00E+18 | 0.17 |
| SPGA2 | 14:46:46.9 | 4.0 | 3.0 | 12.0 | 8.00E+18 | 0.25 |
| SPGA3 | 14:47:33.4 | 4.0 | 2.0 | 8.0 | 4.00E+18 | 0.17 |
| SPGA4 | 14:47:26.3 | 3.5 | 3.0 | 10.5 | 2.10E+19 | 0.25 |
| SPGA5 | 14:47:57.1 | 3.0 | 4.0 | 12.0 | 3.00E+18 | 0.33 |
| SPGA6 | 14:48:04.4 | 3.0 | 4.0 | 12.0 | 3.00E+18 | 0.33 |
| SPGA7 | 14:48:15.0 | 6.0 | 2.0 | 12.0 | 5.00E+18 | 0.17 |
| SPGA8 | 14:48:25.8 | 8.0 | 3.0 | 24.0 | 9.00E+18 | 0.25 |
| SPGA9 | 14:48:30.9 | 7.0 | 7.0 | 49.0 | 2.00E+19 | 0.58 |
Fig. 2 Strong-motion stations of the Small-Titan system\(^9,^{10}\). Black circle indicates the K-NET\(^{11}\) station MYG013.

| Site No. | Site name       | Code | Longitude (deg.) | Latitude (deg.) | Geological condition       |
|----------|-----------------|------|------------------|-----------------|-----------------------------|
| S1(AKA001) | Shokei Univ.     | SHOK | 140.832          | 38.192          | Diluvial plateau            |
| S2(AKA002) | Yanagiu         | YAGI | 140.876          | 38.186          | Alluvial lowland            |
| S3(AKA003) | Higashi-Shiroumaru | HSHR | 140.927          | 38.191          | Alluvial lowland            |
| S4(AKA004) | Arahama         | ARAH | 140.983          | 38.220          | Alluvial lowland            |
| S5(AKA005) | Sendai Higashi HS | SENH | 140.936          | 38.217          | Alluvial lowland            |
| S6(AKA006) | Higashi-Nagamachi | HNAG | 140.894          | 38.217          | Alluvial lowland            |
| S7(AKA007) | Taihaku         | TAIH | 140.821          | 38.228          | Diluvial plateau            |
| S8(AKA008) | Kuriu           | KURI | 140.789          | 38.264          | Diluvial plateau            |
| S9(AKA009) | TIT-Kasumichio   | TITK | 140.854          | 38.243          | Diluvial plateau            |
| S10(AKA010) | TIT-Futatetsawa | TITF | 140.874          | 38.231          | Diluvial plateau            |
| S11(AKA011) | Shichigou       | CCHG | 140.949          | 38.234          | Alluvial lowland            |
| S12(AKA012) | Sendai-Tech-HS  | SIKO | 140.921          | 38.256          | Alluvial lowland            |
| S13(AKA013) | Renbou          | RENB | 140.891          | 38.249          | Diluvial plateau            |
| S14(AKA014) | Sakuragaoka     | SAKR | 140.854          | 38.300          | Diluvial plateau            |
| S15(AKA015) | Mougakkou       | MOGA | 140.882          | 38.275          | Diluvial plateau            |
| S16(AKA016) | Nankoudai-Higashi | NANK | 140.916          | 38.296          | Diluvial plateau            |
| S17(AKA017) | Tago            | TAKA | 140.961          | 38.271          | Alluvial lowland            |
| S18(AKA018) | Tagajyou2       | TGNI | 140.983          | 38.298          | Alluvial lowland            |
| S19(AKA019) | Iwakirichu      | IWAK | 140.949          | 39.299          | Alluvial lowland            |
| S20(AKA020) | Nanakitachu     | NAKI | 140.896          | 38.321          | Diluvial plateau            |

and discussion will be made in Chapter 5. Conclusions will be given in Chapter 6.
2. SOURCE MODEL

Figure 1 shows the source model to be validated in this study. The source model involves nine SPGAs, located off the coast of Miyagi through Ibaraki. Table 1 shows the model parameters. The model can basically explain time histories of observed strong ground motions at stiff sites along the coast of Miyagi through Ibaraki, especially in the frequency range relevant to structural damage.

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As well known, observed ground motions in Miyagi Prefecture consisted of two wave packets. The former wave packet can be attributed to the rupture of SPGA1 and SPGA2 in this model. On the other hand, the latter wave packet can be attributed to the rupture of SPGA4. SPGA4 had the largest seismic moment and it determined the peak amplitude of strong ground motions in Miyagi Prefecture.

3. OUTLINE OF THE SMALL-TITAN AND RECORDS DURING THE 2011 TOHOKU EARTHQUAKE

(1) Outline of the Small-Titan

The Small-Titan (Strong Motion Array of Local Lots by the Tohoku Institute of Technology Area Network) system covers the Greater Sendai, Japan, with 20 strong-motion stations as shown in Figure 2. Table 2 shows the list of the stations. Each station consists of a digital accelerograph equipped with a 24-bit A/D converter, with a capacity for observing acceleration up to 20 m/s². Since its installation in June 1998, it has successfully obtained more than 500 strong-motion records.

(2) Outline of strong-motion records by the Small-Titan during the Tohoku earthquake

Strong ground motions of the 2011 Tohoku earthquake were successfully recorded at 17 out of 20 Small-Titan stations. Table 3 shows the outline of the observed records. As can be seen in Table 3, the instrumental seismic intensity ranged from 5.1 to 6.5, depending on local site conditions.

Velocity waveforms in the frequency range from 0.2 to 1 Hz, which will be the target of the ensuing analysis, are shown in Figure 3. The bottom axis indicates the time from the initial rupture at the hypocenter. The left axis indicates the distance from SPGA4, which generated the largest pulse during the earthquake. In the figure, we can clearly see the propagation of a large pulse generated by SPGA4 from east to west. The red dotted line indicates the theoretical arrival times of the pulse for the assumed rupture time of SPGA4 at 14:47:26.3 and the assumed S-wave velocity of 3.9 km/s. The theoretical arrival times are consistent with the actual ones, indicating the appropriateness of these assumptions.

4. SIMULATION OF STRONG GROUND MOTION

(1) Outline of the simulation method

Strong ground motions at the Small-Titan stations were calculated with the same method as that used for the development of the SPGA model; they were simulated based on site amplification and phase characteristics, to take into account the effect of sediments on both Fourier amplitude and phase of strong ground motions. An outline of the method follows.

The first step is to evaluate ground motions from a small event (Green’s function). The Fourier amplitude of the Green's function is evaluated as a product of the source spectrum |S(f)|, the path effect |P(f)| and the site amplification factor |G(f)|. The source spectrum was assumed to follow the $\omega^{-2}$ model. Geometrical spreading and non-elastic attenuation were considered for the path effect. The details of the path effect will be explained in (2). The site amplification factors at the Small-Titan stations were obtained from Table 3.

| Site No. | Code | Peak acceleration (Gal) | Instrumental Seismic Intensity |
|----------|------|-------------------------|-------------------------------|
| S1(AKA001) | SHOK | 215 | 5.1 |
| S2(AKA002) | YAGI | 764 | 5.9 |
| S3(AKA003) | HSHR | 521 | 5.9 |
| S4(AKA004) | ARAH | 542 | 5.9 |
| S5(AKA005) | SENH | Not observed | |
| S6(AKA006) | HNAG | 720 | 6.0 |
| S7(AKA007) | TAII | 709 | 5.6 |
| S8(AKA008) | KURI | 564 | 5.5 |
| S9(AKA009) | TITK | 471 | 5.8 |
| S10(AKA010) | TITF | 429 | 5.6 |
| S11(AKA011) | CCHG | 1074 | 6.5 |
| S12(AKA012) | SIKO | 542 | 5.8 |
| S13(AKA013) | RENB | 521 | 5.7 |
| S14(AKA014) | SAKR | 681 | 5.8 |
| S15(AKA015) | MOGA | 700 | 5.6 |
| S16(AKA016) | NANK | 699 | 5.8 |
| S17(AKA017) | TAKA | Not observed | |
| S18(AKA018) | TJNI | Not observed | |
| S19(AKA019) | IWA barrier | 859 | 6.4 |
| S20(AKA020) | NAKI | 1853 | 6.1 |
The Fourier phase of an actual record of a small earthquake at the site of interest was used as the Fourier phase of the Green's function. Thus, we obtained a frequency-domain Green's function $F(f)$, which incorporates the effects of sediments both on Fourier amplitude and Fourier phase as follows:

$$F(f) = \frac{|S(f)|}{|P(f)|} \frac{|G(f)|}{|O_s(f)|_p}, \quad (1)$$

where $O_s(f)$ is the Fourier transform of an actual record at the site of interest and $|O_s(f)|_p$ is its Parzen-windowed amplitude (a band width of 0.05 Hz was used). Thus, $O_s(f) / |O_s(f)|_p$ corresponds to a unit phase spectrum. If several records are available for the site, it is preferable to choose an event that has a similar incident angle and a similar back-azimuth to the target event. In this application, the records of the

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**Table 4** Simulation cases with different modeling of geometrical spreading, Q value and nonlinear soil response.

| Case | Geometrical spreading | Q value | Nonlinear soil response |
|------|-----------------------|---------|-------------------------|
| Case000 | \( \frac{1}{r} \) for \( r < 80 \text{ km} \), \( \frac{1}{\sqrt{80}r} \) for \( r > 80 \text{ km} \) | constant below 0.7 Hz | – |
| Case100 | Yes | – | – |
| Case010 | – | Yes | – |
| Case110 | Yes | Yes | – |
| Case001 | – | – | Yes |
| Case101 | Yes | – | Yes |
| Case011 | – | Yes | Yes |
| Case111 | Yes | Yes | Yes |
December 17, 2005 earthquake ($M_{J}6.1$) were used to determine the phase characteristics, because its incident angle and back-azimuth were preferable. This event was also used during the development of the source model\cite{1,2}. The location of the event is shown in Fig. 1. The Green’s function in the time domain can be obtained by inverse Fourier transform.

The second step is to superpose Green’s functions to obtain strong ground motions from a large event (or a sub-event of a large event) in the same way as the conventional EGF method\cite{18,19}. When soil non-linearity is considered, the time-domain Green’s function is corrected with the method described in (4).

\section*{(2) Path effect}

During the development of the SPGA model\cite{1,2}, the path effect was simply evaluated with the following formula:

$$|P(f)|=(1/r) \exp(-\pi f r /Q(f) \beta),$$  \hspace{0.5cm} (2)

where $r$ is the distance from the source to the site, $\beta$ is the shear wave velocity, and $Q(f)$ stands for the Q value along the propagation path that is proportional to some power of frequency. More specifically, the Q value estimated for the region ($Q(f)=114 f^{0.92}$)\cite{20} was used. However, in this study, four different path models were used as shown in Table 4.

As discussed by Wakai and Nozu\cite{21}, the path model in Equation (2) involves two factors that could potentially lead to underestimation of strong ground motions at greater distances. One of the factors is related to geometrical spreading. Although Equation (2) implicitly assumes the predominance of body waves, the geometrical spreading term is expected to become inversely proportional to the square root of distance at greater distances, where surface waves are expected to predominate. Although such an effect has been suggested particularly for shallow crustal earthquakes\cite{20,22}, the same effect could be expected more or less for subduction earthquakes because, even for subduction earthquakes, surface waves predominate at great distances. Therefore, assuming a geometrical spreading term inversely proportional to distance may result in underestimation of strong ground motions at great distances. The other factor associated with underestimation is related to the Q value. Although it is well known that the Q value is proportional to some power of frequency at high frequencies, it has been suggested that the Q value has a minimum value at low frequencies\cite{23}. Therefore, using a Q value that is proportional to some power of frequency at low frequencies may also result in underestimation of strong ground motions.

To account for these effects in this study, several models were used to represent the path effect. In terms of geometrical spreading, Satoh and Tatsumi\cite{20} proposed a geometrical spreading term expressed as $1/r$ for $r < 80$ km and $1/\sqrt{80r}$ for $r > 80$ km. This model was used in four simulation cases.

| No. | Date     | Time | $M_J$ | Depth (km) | $\Delta$ (km) |
|-----|----------|------|-------|------------|---------------|
| 1   | 1998/11/24 | 4:48 | 5.1   | 82         | 64            |
| 2   | 1999/02/01 | 1:52 | 5.1   | 49         | 138           |
| 3   | 1999/11/15 | 10:35| 5.5   | 49         | 132           |
| 4   | 2000/01/09 | 13:02| 5.0   | 46         | 129           |
| 5   | 2000/03/20 | 6:26 | 5.0   | 78         | 59            |
| 6   | 2000/11/16 | 18:31| 5.0   | 51         | 105           |
| 7   | 2001/04/12 | 16:02| 5.0   | 44         | 131           |
| 8   | 2001/10/02 | 17:20| 5.4   | 41         | 98            |
| 9   | 2002/05/06 | 17:12| 5.0   | 40         | 109           |
| 10  | 2003/03/03 | 7:47 | 5.3   | 41         | 99            |
| 11  | 2003/07/26 | 16:56| 5.3   | 12         | 34            |
| 12  | 2004/01/23 | 18:01| 5.3   | 66         | 113           |
| 13  | 2004/05/29 | 12:47| 5.9   | 38         | 117           |
| 14  | 2004/12/29 | 22:59| 5.5   | 39         | 111           |
| 15  | 2004/12/30 | 22:29| 5.0   | 73         | 97            |
| 16  | 2005/10/22 | 22:12| 5.6   | 52         | 133           |
| 17  | 2006/01/18 | 23:25| 5.7   | 36         | 123           |
| 18  | 2008/06/16 | 23:24| 5.3   | 11         | 81            |
| 19  | 2008/10/30 | 0:48 | 5.1   | 86         | 75            |

Fig. 4 Fourier spectral ratios at the Small-Titan station AKA001 with respect to the K-NET station MYG013 for 19 small events (gray traces) and their geometric mean (red trace).
Fig. 5 The site amplification factors estimated for the Small-Titan stations (red traces). The site amplification factor at the K-NET station MYG013 is also shown (black traces).
as shown in Table 4. In terms of the Q value, the application of a Q value having a minimum value at low frequencies was investigated. Although path models with such effects have not been used frequently in conventional studies, some studies have employed Q values that are constant below 1.0 Hz22 or 0.64 Hz24. In this study, referring to the work of Wakai and Nozu25, a Q value that is constant below 0.7 Hz was used in four simulation cases as shown in Table 4.

(3) Site amplification factor

The site amplification factors at the Small-Titan stations were evaluated empirically using weak-motion records before the Tohoku earthquake as follows. First, all the events that satisfy the following conditions were selected from the Small-Titan database.

i) The event was also recorded at the K-NET station MYG013 (see Fig. 2).
ii) 5≤MJ<6.
iii) 30 km≤Δ<150 km at MYG013, where Δ is the epicentral distance. Nineteen events were selected as shown in Table 5. Events with 5≤MI were selected for the purpose of selecting records with a good S/N ratio.

Then, for each station, the Fourier spectral ratios with respect to MYG013 were calculated for the 19 records, where the Fourier spectra were the composition of two horizontal components and processed through a Parzen window with a bandwidth of 0.05 Hz. Then, the geometric mean of the spectral ratios was calculated. An example for the station AKA001 is shown in Fig. 4, where the spectral ratios for the 19 small events are indicated by the gray traces and their geometric mean is indicated by the red trace. The event No.1 was not used at AKA009 considering the S/N ratio. Similarly, the event No.5 was not used at AKA004. The event No.6 was not used at AKA006. The event No.19 was not used at AKA015, AKA017, AKA018, and AKA019.

Finally, the empirical site amplification factor at the K-NET station MYG01326 was multiplied by the geometric mean of the spectral ratio to obtain the empirical site amplification factor at a Small-Titan station. The results are shown in Fig. 5 for all the Small-Titan stations. These results were used in the simulation.

(4) Nonlinear soil response

In this study, nonlinear soil response was considered in four simulation cases as shown in Table 4, using the simple method proposed by Nozu and Morikawa27. One of the key concepts involved in the method is the multiple nonlinear effects. In general, a seismic ray connecting the source and the site usually crosses the soft soil layers several times except for the direct S wave. Therefore, the seismic wave is affected by soil nonlinearity several times during the propagation from the source to the receiver. This phenomenon is referred to as “the multiple nonlinear effects”27). The simplified method uses two parameters to consider the multiple nonlinear effects: one representing the reduction of averaged shear wave velocity within the sediment (v1) and the other representing the increase of averaged damping factor within the sediment (v2). Using these parameters, the time-domain Green’s function can be modified as follows:

\[ g_n(t) = g(t) \] for \( t \leq t_0 \)
\[ g_n(t_0 + (t-t_0)/\nu_1) = g(t) \exp[-\nu_2 \omega (t-t_0)] \] for \( t > t_0 \) \hspace{1cm} (3)

where \( g(t) \) is the original Green’s function and \( g_n(t) \) is the modified Green’s function. The parameters \( \nu_1 \) and \( \nu_2 \) are referred to as “the nonlinear parameters”27). Because \( \omega \) is involved in Equation (3), the Green’s function should be, first, decomposed into components having different frequencies and then each component should be modified based on Equation (3). Finally, the modified components should be summed up.

The nonlinear parameters could be determined in such a way that the synthetic ground motions become consistent with the observed ones. However, such a method cannot be applied to prediction problems. Therefore, in this study, an iterative scheme28 was used to determine the nonlinear parameters. In this scheme, in each iteration step, the parameters can be determined from the peak ground velocity (PGV) (cm/s) as follows:

\[ v_1 = 1/(1+0.0082 \times \text{PGV}) \]
\[ v_2 = 0.020 (1 - v_1^2) \] \hspace{1cm} (4)

The iteration is terminated when the error involved in \( v_1 \) becomes less than 5\%28). The iteration is also terminated when \( v_1 \) becomes smaller than 0.70, because the iteration scheme was proposed based on data with \( v_1 \) greater than 0.70.

The treatment of nonlinear soil response in this study utilizes the concept of equivalent linear analysis27). This fact may imply that the method cannot be applied to a case with very strong nonlinearity.

(5) Other parameters

In the simulation, the averaged radiation coefficient of 0.63 was used. For the parameter PRITTN17, which represents the partition of S-wave energy into two horizontal components, the same value of 0.71 was used for the EW and NS components. For the S-wave velocity and the density in the source region,
Fig. 6 Errors involved in velocity waveforms (0.2–1 Hz), velocity envelopes (0.2–10 Hz) and Fourier spectra (0.2–1 Hz) in each simulation case where nonlinear soil response was not considered. Equations (5)–(7) were used to evaluate the errors.

Fig. 7 Errors involved in velocity waveforms (0.2–1 Hz), velocity envelopes (0.2–10 Hz) and Fourier spectra (0.2–1 Hz) in each simulation case where nonlinear soil response was considered. Equations (5)–(7) were used to evaluate the errors.
Fig. 8 Comparison of observed (black) and synthetic (red) velocity waveforms at 17 Small-Titan stations for Case001, where the original path model was used and nonlinear soil response was considered. Converged values of $\nu_1$ and $\nu_2$ are also shown.
Fig. 8 (contd.)
Fig. 8 (contd.)
Fig. 8 (contd.)
3.9 km/s and 3.1×10^3 kg/m^3 were used\(^{29}, \ 30\).  

5. RESULTS AND DISCUSSION  

(1) Error evaluation  
To evaluate the errors involved in velocity waveforms (0.2–1 Hz), velocity envelopes (0.2–10 Hz) and Fourier spectra in each simulation case, Equations (5)–(7) were used:

\[
\int_{t_1}^{t_2} [v_{\text{syn}}(t) - v_{\text{obs}}(t)]^2 dt / \int_{t_1}^{t_2} v_{\text{obs}}^2(t) dt \quad (5)
\]

\[
\int_{t_1}^{t_2} [VE_{\text{syn}}(t) - VE_{\text{obs}}(t)]^2 dt / \int_{t_1}^{t_2} VE_{\text{obs}}^2(t) dt \quad (6)
\]

\[
\int_{\log_{10} f_1}^{\log_{10} f_2} \left( \log_{10} FS_{\text{syn}}(f) - \log_{10} FS_{\text{obs}}(f) \right)^2 d(\log_{10} f) \quad (7)
\]

where \(v_{\text{syn}}(t)\) is the synthetic velocity wavefrom, \(v_{\text{obs}}(t)\) is the observed velocity waveform, \(VE_{\text{syn}}(t)\) is the synthetic velocity envelope, \(VE_{\text{obs}}(t)\) is the observed velocity envelope, \(FS_{\text{syn}}(f)\) is the synthetic acceleration Fourier spectrum and \(FS_{\text{obs}}(f)\) is the observed acceleration Fourier spectrum. The Fourier spectra are the composition of two horizontal components and processed through a Parzen window with a band width of 0.05 Hz. In Equations (5) and (6), the integration was made for 150 s starting from the beginning of the observed data (\(t_1=0\) s and \(t_2=150\) s). In Equation (7), the integration was made in the frequency range of 0.2 – 1 Hz (\(f_1=0.2\) Hz and \(f_2=1\) Hz).

The results are shown in Fig. 6 and Fig. 7. Figure 6 corresponds to the cases where nonlinear soil response was not considered. Figure 7 corresponds to the cases where nonlinear soil response was considered. At each station, the RMS error for the two horizontal components for the velocity waveforms and the velocity envelopes. Within the box at the top-right corner of each panel, the RMS error for all the stations is shown within parentheses.

A comparison of Fig. 6 and Fig. 7 reveals that the error can be reduced by considering the nonlinear soil response with the simple scheme described in the previous chapter. Among all the cases, Case001, where the original path model was used and nonlinear soil response was considered, yielded the best result in terms of velocity waveforms (0.2–1 Hz) and velocity envelopes (0.2–10 Hz). On the other hand, Case011, where the path model was revised for the Q value and nonlinear soil response was considered, yielded the best result in terms of Fourier spectra (0.2–1 Hz). In addition, Case011 yielded the “second best” result for the velocity waveforms (0.2–1 Hz) and velocity envelopes (0.2–10 Hz). In the following, these two cases will be examined in more detail.

(2) Velocity waveforms and Fourier spectra  
In Fig. 8, the observed and synthetic velocity waveforms (0.2–1 Hz) are compared for Case001 for all the Small-Titan stations where the Tohoku earthquake was observed. The main features of the observed waveforms were well captured by the synthetic waveforms. In particular, the largest pulse around 100 s originated from SPGA4 were reproduced well in this case. In addition, the underestimation of the pulse amplitude was mitigated at AKA011, AKA016 and AKA020, although the EW component at AKA019 was still underestimated and, at some stations, waveforms were overestimated. Although the error defined by
Fig. 9 Comparison of observed (back) and synthetic (red) velocity waveforms at 17 Small-Titan stations for Case011, where the path model was revised for the Q value and nonlinear soil response was considered. Converged values of $\nu_1$ and $\nu_2$ are also shown.
Fig. 9 (contd.)
Fig. 9 (contd.)
Fig. 9 (contd.)
Equation (5) suggests that Case001 is the best case, from such a point of view that underestimation is more unfavorable than overestimation, it would be more appropriate to conclude that Case011 is the best case.

Figure 10 compares the observed and synthetic Fourier spectra at selected stations. Case011 considers the effects of soil nonlinearity, whereas Case010 does not consider those effects. Characteristics of synthetic Fourier spectra approached to the observed ones, although the coincidence was not perfect because the nonlinear parameters were determined through an iterative scheme in this study.

6. CONCLUSIONS

In this study, to investigate the reliability of the source model to explain strong ground motions from the Tohoku earthquake, the SPGA model was applied to simulate strong ground motions at 17 Small-Titan stations. Findings of this study can be summarized as follows:

- The SPGA model can simulate strong ground motions during the Tohoku earthquake quite accurately especially in the frequency range from 0.2 to 1 Hz. Because this frequency range tends to cause significant damage to structures, the SPGA model is favorable for generating design ground motions for structures.
- The error can be reduced by considering the nonlinear soil response with a simple scheme.
- In terms of the path model, the original simple path model yielded the smallest error. However, a revised path model with a constant Q below 0.7 Hz was effective in avoiding underestimation.

It is not easy to show the meaning of the absolute values of the errors presented in this article. However, the error associated with the present source model and that associated with other source models for the same earthquake were compared in another study for the K-NET and KiK-net stations. In general, the present source model yielded smaller errors. In the future work, similar comparison should be made for the Small-Titan stations.

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