On Superpotential and BPS domain wall in SQCD and MQCD

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Abstract

We examine the decoupling properties of the N=2 SQCD vacua when the adjoint mass term is turned on and then the N=1 limit is taken. The BPS domain wall tension in N=1 MQCD and SQCD is also examined. The correspondence of the MQCD integrals with the superpotential and the gaugino condensate is shown.

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1 Introduction

Supersymmetric gauge theories have the special property that certain quantities are holomorphic and can often be obtained exactly. The exact results so obtained provide us an insight into the dynamics of strongly coupled gauge theories, revealing a variety of interesting phenomena.

In the case of $SU(N_c)$ $N=1$ supersymmetric QCD with $N_f$ flavors (SQCD), the exact superpotential is calculated for $N_f < N_c$ and the holomorphic constraint is obtained for $N_f = N_c$. These theories are sometimes related to the $N=2$ derived $N=1$ SQCD that is constructed by adding the explicit mass term $\mu \Phi^2$ for the adjoint matter field $\Phi$. Naively, $N=2$ derived SQCD seems to be connected smoothly to ordinary $N=1$ SQCD when the decoupling limit $\mu \to \infty$ is taken. In section 2, however, we show that this naive picture does not always give a correct answer for the massless limit of $N=2$ and $N=1$ SQCD because $\mu \to \infty$ and $m_f \to 0$ do not commute. First we find a new (but naive) solution for the massive SQCD that has the required properties of the $N=1$ limit such as the $N_c$ degenerated vacua and the decoupling of the extra $N_c - N_f$ vacua. Introducing this solution that was missed in ref. [1], we find that there exist three different types of $N=1$ massless limits for $N=2$ massive SQCD, corresponding to what limit we consider for $\mu m_f$ and $\mu m_f^2$. These solutions are characterized by their discrete symmetries and the decoupling properties. Decoupling properties are discussed by means of the BPS bound for the wall configuration. Although we are not sure if the BPS domain wall can exist for these configurations, the BPS bound is plausible as is discussed in ref. [2].

In section 3 we examine the MQCD calculation of the BPS domain wall tension for $N=1$ massive theory and discuss its properties. Our main concern is to examine the discrepancy between the MQCD brane explanation and the SQCD domain wall derived from the effective Lagrangian. In this process, we find the integrals which correspond to the superpotential and the gaugino condensate. These quantities are important when one considers the relation between SQCD and MQCD.
2 Field theory analysis

In this section we consider a field theory analysis of N=2 SQCD and examine the vacuum structure obtained by breaking the N=2 supersymmetry to N=1 by adding a mass term for the adjoint chiral multiplet. We find a new (but naive) solution which has a smooth limit to ordinary N=1 SQCD in the sense that the discrete symmetry and the decoupling property are properly satisfied. Several types of the massless limits are also considered and characterized by their discrete symmetries and the decoupling properties.

The discrete symmetry and the decoupling of the $N_c - N_f$ vacua

First we consider the N=2 $SU(N_c)$ gauge theory with $N_f$ quark hypermultiplets in the fundamental representation. In terms of N=1 superfields the vector multiplet consists of a field strength chiral multiplet $W^\alpha$ and a scalar chiral multiplet $\Phi$ both in the adjoint representation of the gauge group. The N=2 superpotential takes the form

$$W = \sqrt{2} \tilde{Q} \Phi Q + \sqrt{2} m_f \tilde{Q} Q.$$  (2.1)

To break the N=2 supersymmetry to N=1, a bare mass term is added to the adjoint chiral multiplet $\Phi$

$$W = \sqrt{2} \tilde{Q} \Phi Q + \sqrt{2} m_f \tilde{Q} Q + \mu Tr(\Phi^2).$$  (2.2)

When the mass $\mu$ for the adjoint chiral multiplet is increased beyond $\Lambda_{N=2}$, the renormalization group flow below the scale of $\mu$ is the same as in N=1 SQCD with the dynamical scale $\Lambda$ given by

$$\Lambda^{3N_c-N_f} = \mu^{N_c} \Lambda_{N=2}^{2N_c-N_f}.$$  (2.3)

If the adjoint multiplet is much heavier than the dynamical scale, we can first integrate out the heavy field $\Phi$ obtaining the superpotential proportional to $1/\mu$

$$W_\Delta = \frac{1}{2\mu} \left[ Tr(M^2) - \frac{1}{N_c} (Tr M)^2 \right].$$  (2.4)

On the other hand, in N=1 SQCD with $N_f < N_c$, it is well known that a superpotential is dynamically generated and takes the form

$$W_{ADS} = (N_c - N_f) \left( \frac{\Lambda^{3N_c-N_f}}{det M} \right)^{1/(N_c-N_f)}.$$  (2.5)
For large but finite $\mu$, we can expect that the effective superpotential is just the sum of these terms,
\[
W_{\text{eff}} = W_{\text{ADS}} + W_\Delta
= (N_c - N_f) \left( \frac{\Lambda^{3N_c-N_f}}{\det M} \right)^{1/(N_c-N_f)} + \frac{1}{2\mu} \left[ \text{Tr}(M^2) - \frac{1}{N_c} \text{Tr}(TM^2) \right].
\] (2.6)

Extremizing the superpotential, one can find that the vacuum structure of this superpotential is determined by at most two different values of $m_i$ that appears in the diagonal part of $M$. The equation determining $m_1$ and $m_2$ is
\[
m_i^2 - \frac{1}{N_c} \left[ \sum_{l=1}^{N_f} m_l \right] m_i + \mu m_i m_f = \mu \left( \frac{\Lambda^{3N_c-N_f}}{\Pi l m_l} \right)^{1/(N_c-N_f)}. \tag{2.7}
\]

Subtracting the eq.(2.7) for $i = 1$ from the one for $i = 2$, one obtains an equation
\[
(m_1 - m_2) \left[ \left( 1 - \frac{r}{N_c} \right) m_1 + \left( 1 - \frac{N_f - r}{N_c} \right) m_2 + \mu m_f \right] = 0 \tag{2.8}
\]
where $r$ denotes the number of $m_i$ that appears in the diagonal part of $M$. Although there are two types of solutions ($m_1 \neq m_2$ or $m_1 = m_2$ corresponding to $r \neq 0$ or $r = 0$) as seen from eq.(2.8), it is known\cite{1} that $m_1 \neq m_2$ and $r \neq 0$ is not suitable for the analysis of the $N=1$ limit. As we are concerned about the $N=1$ limit, we focus our attention only on the solution $m_1 = m_2 \equiv m$ and $r = 0$. The equation for $m$ is now given by
\[
\frac{1}{\mu} \left( 1 - \frac{N_f}{N_c} \right) m^{2N_c-N_f} + m f m^{N_c-N_f} - \Lambda^{3N_c-N_f} = 0. \tag{2.9}
\]

Taking the $\mu \to \infty$ limit, the contribution from the first term becomes negligible and the $N_c$ solutions remain finite. The finite solutions are given by,
\[
m = \left( \frac{\Lambda^{3N_c-N_f}}{m_f^{N_c-N_f}} \right)^{1/N_c}. \tag{2.10}
\]

Here the $N_c$ phase factors are implemented. These solutions correspond to the well-known $N_c$ degenerated vacua of $N=1$ SQCD. In this paper, we call these $N_c$ solutions by $m_{N=1}$ and the remaining $N_c - N_f$ solutions by $m^{\text{decouple}}_{N=2}$. From eq.(2.10), it is easy to estimate the remaining solutions $m^{\text{decouple}}_{N=2}$. They are given by
\[
m^{\text{decouple}}_{N=2} \sim -\left( \mu m_f \right)^{N_c-N_f}/N_c-N_f \sim -\mu m_f \tag{2.11}
\]

\footnote{Note that this naive solution was misses in ref.\cite{1}}
where $N_c - N_f$ solutions are degenerated. In the $\mu \to \infty$ limit, the latter solution goes away to infinity and expected to be decoupled from $m_{N=1}$. To ensure the decoupling of these vacua, it should be important to examine the tension of the domain wall which interpolates between $m_{N=1}$ and $m_{N=2}^{\text{decouple}}$. The BPS bound for the tension of the domain wall that interpolates between $m_{N=1}$ and $m_{N=2}^{\text{decouple}}$ is

$$T_D \geq W_{\text{eff}}|_{m=m_{N=1}} - W_{\text{eff}}|_{m=m_{N=2}} \sim O(\mu)$$  \hspace{1cm} (2.12)$$

which diverges in the $N=1$ ($\mu \to \infty$) limit. The divergence of the domain wall tension indicates the decoupling of the $m_{N=2}$ solution. This decoupling property does not appear when one considers the solution given in ref. [1].

**Massless limits**

Let us consider the massless limit of the solution and see how the massless limit is realized in $N=1$ and $N=2$ SQCD. We are not interested in the massless theory itself but the symmetries and the decoupling properties of the vacua that appears when one takes the small mass limit. We examine the $N=1$ ($\mu \to \infty$, $\Lambda = \text{fixed}$) and massless ($m_f \to 0$) limit which have several branches corresponding to what value one chooses for $\mu m_f$ or $\mu m_f^2$.

First let us consider the $\mu m_f \to \infty$ limit. In this limit the first term in eq.(2.9) is negligible, indicating the separation between two solutions, $m_{N=1}$ and $m_{N=2}^{\text{decouple}}$. Although these two vacua are separated, the decoupling is still not clear because the tension of the domain wall which interpolates between $m_{N=1}$ and $m_{N=2}$ is estimated as

$$T_D \geq W_{\text{eff}}|_{m=m_{N=1}} - W_{\text{eff}}|_{m=m_{N=2}} \sim O(\mu m_f^2)$$  \hspace{1cm} (2.13)$$

and the value is not determined. When $\mu m_f^2 \to 0$, the bubbles of the $m_{N=2}^{\text{decouple}}$ vacua can be formulated in the $m_{N=1}$ vacua without costing any energy, thus making the decoupling

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3 If one solves the eq.(2.7) for $m_1 \neq m_2$ (see eq.(2.8)) and assume that the obtained solution is also applicable for $r = 0$, one may find two solutions that looks very similar to (2.10) and (2.11). In this case, however, $m_1$ is not a physical solution and does not appear in the Lagrangian even before one considers the decoupling limit, since $m_1$ is not a component of the meson matrix $M$ when $r = 0$. 

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uncertain. To ensure the decoupling of the extra vacua, an additive requirement \( \mu m_f^2 \to \infty \) is needed. When the \( \mu m_f^2 \to \infty \) limit is taken, \( N_c \) vacua are related by the discrete symmetry \( Z_{N_c} \) while the other \( N_c - N_f \) vacua are decoupled. The symmetry and the decoupling property show that this limit corresponds to the massless limit (to be more precise, what we consider is the small mass limit) of the ordinary \( N=1 \) SQCD. Although both vacua run away as one takes the small mass limit, one should not neglect these characteristic properties.

The next branch appears when one considers the limit \( \mu m_f \to 0 \). In this limit the second term in eq.(2.9) becomes negligible and the \( Z_{2N_c-N_f} \) degeneracy is restored. The resulting equation is precisely the one which appears in the analysis of the \( \mu \to \infty \) limit of \( N=2 \) massless SQCD. It is important that the naive \( \mu \to \infty \) limit of \( N=2 \) massless SQCD lies at the opposite limit to the massless limit of ordinary \( N=1 \) SQCD.

There is also a branch with \( \mu m_f \to \text{const} \). In this case, either \( Z_{N_c} \) nor \( Z_{N_c-N_f} \) symmetry is restored and no decoupling is observed. However, it is also easy to find that some other symmetry should be restored. Multiplying the eq.(2.9) by \( \mu \) and defining \( x = m^{N_c-N_f} \), we can find

\[
ax^{2N_c-N_f} + bx^{N_c} - \mu c = 0 \tag{2.14}
\]

where \( a, b \) and \( c \) are finite constants. Since the last term depends on \( \mu \), there should be at least one vacuum that runs away. On the other hand, to keep the coefficient of \( x^{N_c} \) finite, some symmetry should be restored to ensure the cancellation. It is very easy to find the explicit example for \( N_c = 4, N_f = 2 \).

It should be useful to comment on a remaining problem. Because we do not know how to realize the new (and naive) \( r = 0 \) solution in MQCD, the relation between \( N=2 \) and \( N=1 \) MQCD is still unclear. In this respect, the construction of the curve that realizes the new \( r = 0 \) solution seems an urgent task.

### 3 SQCD and MQCD analysis of the BPS domain wall

In this section we review and examine the calculation of BPS domain wall tension in \( N=1 \) SQCD and MQCD to solve the discrepancy between these two calculations. We hope this provides us a key to understand more about the correspondence between SQCD
and MQCD. These analyses give us an important information on the superpotential and the gaugino condensate.

Although the configuration of the five-brane for finite $\mu$ is already found in [1], the equations followed by the curve does not contain the $r = 0$ solution of which we have discussed in the previous section. Here we suspect the validity of the solution with $m_1 \neq m_2$ and $r = 0$ (See the footnote in the previous section) and consider only the N=1 curve that can be obtained without referring to the finite $\mu$ behavior. According to ref.[2], the curve for N=1 SQCD is

$$
v = -\frac{\zeta}{\lambda},
$$

$$
w = \lambda
$$

$$
t = \lambda^{N_c-N_f} \left( \lambda - \frac{\zeta}{m_f} \right)^{N_f}.
$$

(3.1)

The BPS domain wall tension for this curve is already calculated in ref.[3] and given by:

$$
T_D^{MQCD} = \zeta(2N_c - N_f).
$$

(3.2)

Here we follow the notations and the calculations of ref.[3]. Although the constant ambiguity is implemented, $N_c$ and $N_f$ dependence should be exact provided that the physical observations given by E.Witten[5] are correct.

On the other hand, in the field theory analysis that was given in ref.[4], the tension of the BPS domain wall is calculated to be proportional to the vacuum expectation value of the effective superpotential. If this observation is correct, the tension should have only the $N_c$ dependence and must not depend on $N_f$.

One way to solve this discrepancy between MQCD brane configuration and the effective Lagrangian approach is to modify the original integral for MQCD domain wall tension. In ref.[3], a modification of the integral is proposed. The main idea of ref.[3] is to divide the integral into two parts, namely the mesonic $w$-integral

$$
W_2^w = \frac{1}{2\pi i} \zeta \int_{C_{w=\infty}} dt(w)/t
$$

$$
= \zeta([\text{No. of zeros}] - [\text{No. of poles of } t(w)])
$$

(3.3)

4In the mesonic integral, $t$ is written by $w$ so that $W_2^w$ depends only on the mesonic variable $w$. This is why we call $W_2^w$ mesonic integral.
and the $v$-integral
\[
W_v^w = \frac{-1}{2\pi i} \zeta \int_{C_{v=\infty}} dt(v)/t = \zeta([\text{No. of zeros}] - [\text{No. of poles of } t(v)]) \quad (3.4)
\]

and assume that the domain wall tension $T_D$ should be written as $T_D = W_2^w$. Although this modification ($T_D = W_2^w + W_2^v \rightarrow T_D = W_2^w$) gives us the desired dependence on $N_c$, it is clear that such a modification ruins the physical observations given by E. Witten in ref. [5]. The author of ref. [3] claimed that such an ambiguity in defining the integral may appear as the result of the $\Sigma_0$ dependence of the Witten’s definition and proposed a new integral that does not depend on $\Sigma_0$. However, what he did to obtain the correct $N_c$ dependence was to divide the new integral which he claimed to be $\Sigma_0$ independent. We should also remember that the domain wall can be constructed as the brane configuration that interpolates $N_c$ degenerated vacua. If such a configuration is considered, it does not depend on the auxiliary $\Sigma_0$ curve.

The other way to understand this discrepancy between MQCD and the effective Lagrangian approach is to examine the field theory analysis. To show that the MQCD gives us the correct answer, first we try to explain why this discrepancy appears in these two different approaches and then discuss more details on SQCD calculation.

The basic idea revisited

The key ingredient of the BPS domain wall is the central extension of the N=1 superalgebra
\[
\{\overline{Q}_\dot{a}, \overline{Q}_\dot{\beta}\} = -2i(\sigma^0)_{\dot{\alpha}} \int d^3x \{\overline{D}_{\dot{\beta}} \overline{D}^\dagger_{\dot{\gamma}} J_{\gamma\dot{\delta}}\} \theta=0. \quad (3.5)
\]
The contribution to the central extension is due to the non-conservation (classical or quantum anomaly) of the current multiplet $J$. The lowest component of $J$ is the $U(1)_R$ current that is broken classically (in Wess-Zumino model) or by quantum anomaly ($U(1)_R$ anomaly which is induced by gaugino).

The simplest example for the Kovner-Shifman domain wall is the Wess-Zumino model with only a superfield $\Phi$ in which the presence of the central extension can be seen at

\footnote{It should be noted that the definitions of the integral (3.3) and (3.4) are not new. Integrating by parts and picking up surface terms, the integral in ref. [5] can be divided into two surface terms that correspond to (3.3) and (3.4).}
the tree level. In this case, the \( U(1)_R \) symmetry is broken classically. The Wess-Zumino Lagrangian in terms of a superfield \( \Phi \) has the following form,

\[
L = \frac{1}{4} \int d^4\theta \Phi \bar{\Phi} + \left[ \frac{1}{2} \int d^2\theta W(\Phi) + h.c. \right],
\]

\[
W(\Phi) = \mu^2 \Phi - \frac{\lambda}{3} \Phi^3.
\] (3.6)

When \( \mu = 0 \), the Lagrangian is invariant under the \( U(1)_R \) rotation

\[
d\theta \to e^{-i\alpha}d\theta,
\]

\[
\Phi \to e^{\frac{2i\alpha}{3}}\Phi.
\] (3.7)

However, if \( \mu \neq 0 \), this \( U(1)_R \) invariance is broken and only the discrete part \( Z_2^R \) persists,

\[
d^2\theta \to -d^2\theta,
\]

\[
\Phi \to -\Phi.
\] (3.8)

When \( \langle \Phi \rangle \neq 0 \), the \( Z_2^R \) symmetry is spontaneously broken and the corresponding domain wall which interpolates between two degenerate vacua

\[
\langle \Phi \rangle = \pm \mu/\sqrt{\lambda},
\]

\[
\langle W \rangle = \mp \frac{2}{3} \frac{\mu^3}{\sqrt{\lambda}}
\] (3.9)

appears. From the definition of the \( U(1)_R \) symmetry and the superpotential, the superpotential must have the \( U(1)_R \) charge that is broken if the vacuum expectation value of \( W \) becomes non-zero. In this respect, the vacuum expectation value of the superpotential parameterizes the classical (explicit) breaking of the \( U(1)_R \) current. The supersymmetric current multiplet \( J \) has the anomaly of the following form,

\[
\mathcal{D}^\dagger J_{\alpha\dot{\alpha}} = \frac{1}{3} D_\alpha \left[ 3W - \Phi \frac{\partial W}{\partial \Phi} \right].
\] (3.10)

Substituting the anomaly equation into the central extension of the superalgebra(3.3), one obtains

\[
\{Q_{\alpha}, Q_{\beta}\} = 4(\bar{\sigma})_{\alpha\beta} \int d^3x \nabla \left[ W - \frac{1}{3} \Phi \frac{\partial W}{\partial \Phi} \right]_{\tau=0}
\] (3.11)

where the right hand side of the equation is related to the central charge of the wall configuration therefore it represents the surface energy density of the wall. It is apparent
that the contribution is non-zero when the domain wall interpolates the two different vacuum configurations of eq.\((3.9)\). The domain wall solution for such non-gauge theories can easily be extended to more general theories such as an effective theory of SQCD or some other complicated theories.

Now let us discuss the SQCD with the superpotential. As we have stated above, this theory has both the classical and the quantum anomaly that contributes to the central extension. The Lagrangian is

\[
L = \left[ \frac{1}{4g^2} \int d^2\theta Tr W^2 + h.c. \right] + \frac{1}{4} \int d^2\theta \left[ \bar{Q} e^V Q^i + \bar{\tilde{Q}} e^V \tilde{Q}^i \right].
\] (3.12)

In superfield notation the anomaly is

\[
\bar{D}^{\hat{\alpha}} J_{\alpha\hat{\alpha}} = \frac{1}{3} D_\alpha \left\{ \left[ 3W - \sum_i Q_i \frac{\partial W}{\partial Q_i} \right] \right. \\
\left[ \frac{3T(G) - \sum_i T(R_i)}{16\pi^2} Tr W^2 + \frac{1}{8} \sum_i \gamma_i Z_i (D)^2 (\bar{Q}_i e^V Q_i) \right] \right\}
\] (3.13)

where in SQCD, \(T(G) = N_c\) and \(T(R_i) = 1\) for each flavor. Substituting the anomaly equation into the superalgebra\((3.5)\) and using the Konishi anomaly equation, one gets

\[
\{ \bar{Q}_\alpha, Q_\beta \} = 4 (\vec{\sigma})_{\hat{\alpha}\hat{\beta}} \int d^3x \vec{\nabla} \left[ W - \frac{T(G) - \sum_i T(R_i)}{16\pi^2} Tr W^2 \right]_{\theta=0}.
\] (3.14)

For \(N_f < N_c\) SQCD the domain wall solution is usually formulated for the effective superpotential

\[
W_{eff} = tr(m_f M) + c_{N_c,N_f} \left( \Lambda^{3N_c-N_f} \right)^{N_c-N_f} \frac{\Lambda^{3N_c-N_f}}{det M}
\] (3.15)

which looks like an extended Wess-Zumino model. From eq.\((3.14)\), it is easy to understand that there is always a contribution to the central charge from the quantum \(U(1)_R\) anomaly as far as gaugino condensation takes place. (Note that the coefficient in eq.\((3.14)\) in front of gaugino condensation \(Tr W^2\) does not vanish for \(N_f \neq N_c\). Although the contribution from anomaly vanishes for \(N_f = N_c\) massless SQCD, the central charge does not vanish when the mass term is switched on since a contribution from the superpotential appears\([8]\).) On the other hand, from eq.\((3.15)\), one can see that gaugino is already integrated out and thus the quantum anomaly that is induced by gaugino is decoupled from the effective mesonic Lagrangian\((3.13)\). In other words, the contribution from the quantum anomaly that
appears in the original Lagrangian is not included in the central charge when it is derived from the mesonic Lagrangian. As we have mentioned in the last paragraph, $< W >$ parameterizes the classical anomaly that comes from the explicit breaking of the $U(1)_R$ symmetry. On the other hand, there is another contribution from the vector superfield, which appears as a quantum anomaly of the theory. Adding these contributions, one can find

$$T_{SQCD}^D = |(2N_c - N_f)\Lambda_0^3|$$

(3.16)

where $\Lambda_0^3$ is the scale for the gaugino condensate.

In general, terms like $S\log S$ are required to include the anomaly in the effective Lagrangian since the imaginary part of the highest component of $S$ contains $F\tilde{F}$. Although the glueball superfield $S$ is already integrated out, one may suspect that the shift of the classical anomaly that is induced by the dynamical superpotential could be identified with the shift by the quantum anomaly in the original theory thus making the total central charge $c \sim N_c\Lambda^3$. To answer this question, it will be useful to think about the theory where the dynamical superpotential is calculated without using the effective Lagrangian. In this respect, it should be useful to consider SQCD with $N_f = N_c - 1$. In this case, one may calculate the dynamical superpotential by using the perturbative instanton calculation. The dynamical superpotential appears as one instanton contribution and is calculated explicitly within the original (constituent) Lagrangian. It shifts the vacuum expectation value of the superpotential and contributes to the central charge. At the same time, quantum anomaly is calculated from the triangle diagram or by using the well-known Fujikawa method. It is apparent that these two (the shift from the dynamical superpotential and the one from the quantum anomaly) appear as the independent contributions to the central charge. Adding these contributions, one can find

$$T_{SQCD}^D = |(2N_c - N_f)\Lambda_0^3|$$

(3.17)

which agrees with the MQCD calculation when $\zeta$ in eq.(3.2) is identified with $\Lambda_0^3$. $^6$

MQCD correspondence

$^6$ Some people may claim that there should be some unknown mechanism that makes these two contributions identical, or claim that the dynamical superpotential does not contribute to the central change from unknown reason. Here we respect MQCD calculation and do not consider such arguments.
As we have mentioned in the last paragraph, we may consider two independent contributions (the classical and the quantum anomaly) in the field theory analysis. When one considers the MQCD integral for the domain wall tension, mesonic $w$-integral \([3.3]\) is likely to pick up the contribution from the mesonic field as the mesonic Lagrangian picked up the classical anomaly. In this sense, it is plausible that the contribution from the mesonic Lagrangian eq.(3.15) appears as the $w$-integral and is identified with the contribution from the classical anomaly. On the other hand, the other contribution, namely the $v$-integral, appears as the contribution from the quantum anomaly that is decoupled in the mesonic picture.

We think it is very hard to prove this correspondence, but it is very important to examine if these correspondence can survive in several cases. Next we will examine them by taking the massless and the decoupling limits.

Massless limit

In the field theory language, the perturbative calculation at the intermediate scale suggests that the low energy effective Lagrangian for N=1 massless SQCD is \([3]\)

\[
L_{\text{eff}} = L_{SU(N_c-N_f)SYM} + L_{\text{singlets}} + O(1/M)
\]  

(3.18)

where \(O(1/M)\) denotes the higher order corrections that interpolate between the effective pure SYM sector \(L_{SU(N_c-N_f)SYM}\) and the singlet sector \(L_{\text{singlets}}\). If one neglects the higher terms, the effective theory looks like a pure SYM with the gauge group \(SU(N_c-N_f)\) and likely to have the BPS domain wall whose tension is \(T_D = 2(N_c-N_f)A_0^3_{SU(N_c-N_f)}\). (Of course this result is not exact because the higher terms are important when one discusses the runaway. We will find the gap appears also in MQCD.)

On the other hand, when massive SQCD is considered the wall tension \(T_D\) is always \(T_D = (2N_c-N_f)A_0^3\) for any value of \(m_f\). This is because the expectation value of \(W\) at its minimum does not depend on the mass \(m_f\) as far as \(m_f \neq 0\).

It should be important to examine these properties in the light of MQCD and explain what happens in the massless limit. As we have discussed in the last paragraph, we consider the MQCD integral \(W_2^w\) and \(W_2^w\). As is discussed above, \(W_2^w\) corresponds to the mesonic superpotential and is always \(W_2^w = \zeta N\) as far as the variations of the parameters
are smooth. The curve (3.1) in the $v$-picture is,

\begin{align*}
vw &= -\zeta \\
\tau &= \zeta^{N_c} v^{-N_c} (m_f - v)^{N_f}.
\end{align*}

(3.19)

When $m_f \to 0$ limit is taken, eq.(3.19) looks like a SYM with the gauge group $SU(N_c - N_f)$, which coincides with the field theory analysis. Although the domain wall tension $T_D = (2N_c - N_f)\zeta$ remains the same as far as we keep $m_f \neq 0$, a discrepancy appears at $m_f = 0$. When $m_f = 0$ limit is taken, the $N_f$ zeros of $t(v)$ at $v = m_f$ join the poles at $v = 0$. On the other hand, in the mesonic picture, the $N_f$ zeros of $t(w)$ at $w = \zeta/m_f$ run away to infinity and finally decouples from $W_2^w$. As a result, the domain wall tension seems to have a gap at $m_f = 0$, which is the consequence of the runaway. The gap found for the superpotential also indicates that the theory is ill-defined in this limit.

It should be useful to examine further on $N_c = N_f$ in which the vacua is well defined in the massless limit and does not have a runaway behavior. In this case, the curve (3.1) becomes

\begin{align*}
vw &= -m_f \Lambda^2 \\
\tau &= (w - \Lambda^2)^{N_c}.
\end{align*}

(3.20)

where we set $\Lambda^2 = \frac{\zeta}{m_f}$. It is easy to calculate the domain wall tension $T_D$ for the curve (3.20). The result is,

$$T_D = N_c m_f \Lambda^2.$$  \hfill (3.21)

In the field theory, the same result is already obtained in ref.[8] by using the holomorphic constraint. As is discussed in the field theory in ref.[8], the contribution comes solely from the mesonic integral. In the field theory, it is apparent that $T_D$ has a smooth massless limit. In MQCD, the curve (3.20) in the massless limit becomes

\begin{align*}
vw &= 0 \\
\tau &= (\lambda - \Lambda^2)^{N_c}.
\end{align*}

(3.22)

In this case we should think that the curve factorizes into two different components,

\begin{align*}
v &= 0 \\
\tau &= (\lambda - \Lambda^2)^{N_c}
\end{align*}

(3.23)
and

\[ w = 0 \]
\[ t = \Lambda^{2N_c}. \]  \hspace{1cm} (3.24)

Both give us the vanishing domain wall tension \( T_{D}^{MQCD} \), which is consistent with the field theory analysis. In this case, both \( T_D \) and the superpotential do not have a gap in this smooth massless limit.

**Decoupling limit**

When the infinite mass limit is taken, a fundamental matter superfield decouples. If this decoupling is completed, the effective Lagrangian should be the N=1 SQCD with \( N_f - 1 \) matter fields. In this limit, the contribution from the quantum anomaly \((N_c - N_f)\Lambda_0^3\) should be replaced by \((N_c - N_f + 1)\Lambda_0^3\). Then the BPS domain wall tension is changed from \( T_D = (2N_c - N_f)\Lambda_0^3 \) to \((2N_c - N_f + 1)\Lambda_0^3\) because of the gap in the quantum anomaly. Is it possible to explain this in the light of MQCD?

Here we consider the case one of the matter field (\(Q^{\text{heavy}}\)) acquires large mass \((m^{\text{heavy}})\) and decouples. In \(w\)-picture, when the decoupling limit is taken, the zero at \(\zeta/m_f^{\text{heavy}}\) join the zeros at \(w = 0\) but the \(w\)-integral itself is not changed. In \(v\)-picture, the zero at \(v = m_f^{\text{heavy}}\) decouples and then the \(v\)-integral is changed. As a result, \(T_D\) becomes \(T_D = (2N_c - N_f + 1)\Lambda_0^3\). This result agrees with the field theory analysis if \(v\)-integral is identified with the contribution from the quantum anomaly. The gap appears because the domain wall configuration cannot survive the decoupling process. Of course, the contribution from the superpotential \((W_2^w)\) is not changed. This gives us an easy explanation why \(T_D\) looks different in the decoupling limit.

**Summary**

In this section we analyzed the correspondence between the SQCD \(U(1)_R\) anomaly and the MQCD integrals. From these analyses we can confirm that \(W_2^w\) and \(W_2^v\) should have important physical meanings when they are compared to the field theory.
4 Conclusion and discussions

In this paper we have found a new solution for \( r = 0 \) and discussed its properties. First we examined the N=1 limit with massive quarks. We have shown that the \( Z_{N_c} \) degeneracy and the decoupling of the extra \( N_c - N_f \) vacua are realized in this limit. Second we examined the N=1 and massless limit. Keeping our attention on the symmetry and the decoupling property of the vacua, we found three different limits each of which are characterized by their discrete symmetries of the degenerated vacua and decoupling properties.

We also examined the discrepancy between MQCD and SQCD domain wall calculation and found that the result given in ref.\cite{4} should be modified to include the decoupled quantum anomaly. In general, the integral for the MQCD domain wall surface energy picks up two surface integrals which we call \( w \) and \( v \)-integrals in this paper. Because the \( w \)-integral is given by the mesonic variable \( w \), it should be natural to identify it with the mesonic superpotential. If we may think that the \( w \)-integral corresponds to the superpotential, the \( v \)-integral should correspond to the contribution that comes from the quantum anomaly. We examined this correspondence in several cases and found that they are correctly realized. These quantities must be important when one discusses the relation between MQCD and SQCD.

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