Research Article

Advances in the Measurement of the Lense-Thirring Effect with Planetary Motions in the Field of the Sun

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By processing more than 400,000 planetary observations of various types with the dynamical models of the EPM2006 ephemerides, E.V. Pitjeva recently estimated a correction to the canonical Newtonian-Einsteinian Venus' perihelion precession of −0.0004 ± 0.0001 arseconds per century. The prediction of general relativity for the Lense-Thirring precession of the perihelion of Venus is −0.0003 arseconds per century. It turns out that neither other mismodelled/unmodelled standard Newtonian/Einsteinian effects nor exotic ones, postulated to, for example, explain the Pioneer anomaly, may have caused the estimated extra-precession of the Venus orbit which, thus, can be reasonably attributed to the gravitomagnetic field of the Sun, not modelled in the routines of the EPM2006 ephemerides. However, it must be noted that the quoted error is the formal, statistical one; the realistic uncertainty might be larger. Future improvements of the inner planets' ephemerides, with the inclusion of the Messenger and Venus-Express tracking data, should further improve the accuracy and the consistency of such a test of general relativity which would also benefit from the independent estimation of the extra-precessions of the perihelia (and the nodes) by other teams of astronomers.

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In the weak field and slow motion approximation, the Einstein field equations of general relativity get linearized resembling the Maxwellian equations of electromagnetism. As a consequence, a gravitomagnetic field arises [1, 2]; it is induced by the off-diagonal components $g_{0i} = 1, 2, 3$ of the space-time metric tensor related to the mass-energy currents of the source of the gravitational field. It affects orbiting test particles, precessing gyroscopes, moving clocks and atoms, and propagating electromagnetic waves [3, 4]. The most famous gravitomagnetic effects are, perhaps, the precession of the axis of a gyroscope [5, 6], whose detection in the gravitational field of the rotating Earth is the goal of the space-based GP-B experiment [7] (see http://einstein.stanford.edu/), and the Lense-Thirring precessions [8] of the orbit of a test particle for which some disputed satellite-based tests in the gravitational fields of the spinning Earth [9–14] and Mars [15–17] have been reported. (According to an interesting historical analysis recently performed in [18], it would be more correct to speak about an Einstein-Thirring-Lense effect.)

We focus on the detection of the solar gravitomagnetic field through the Lense-Thirring planetary precessions of the longitudes of perihelia $\omega = \omega + \cos i \Omega$, where $G$ is the Newtonian gravitational constant, $S$ is the proper angular momentum of the Sun, $c$ is the speed of light in vacuum, $a$ and $e$ are the semimajor axis and the eccentricity, respectively, of the planet's orbit. (Here $\omega$ is the argument of pericentre, reckoned from the line of the nodes, $i$ is the inclination of the orbital plane to the equator of the central rotating mass and $\Omega$ is the longitude of the ascending node.) It may be interesting to know that in [19] it was proposed to measure the solar gravitomagnetic field through the Schiff effect with a drag-free gyroscope orbiting the Sun in a polar orbit.

The impact of the Sun’s rotation on the Mercury’s longitude of perihelion was calculated for the first time with general relativity by de Sitter [20] who, by assuming a homogeneous and uniformly rotating Sun, found a secular rate of −0.01 arseconds per century ("cy"−1 in the following). This value is also quoted in [21, page 111]. Cugusi and Proverbio [22] yield −0.02 "cy"−1 for the argument of perihelion of Mercury. Instead, recent determinations of the Sun's proper angular momentum $S_\odot = (190.0 \pm 1.5) \times$
in which the quoted uncertainty is the formal, statistical one (personal communication by Pitjeva to the author, June 2008). By looking at Table 1, it turns out that such an extra-precession can be well accommodated by the general relativistic prediction for the Lense-Thirring rate of the Venus’ perihelion whose existence would, thus, be confirmed at 25%. It may be objected that the gravitomagnetic force should have been explicitly modelled, and an ad-hoc parameter accounting for it should have been inserted in the set of parameters to be estimated. Certainly, it may be an alternative approach which could be implemented in future. In addition, we note that the procedure followed by Pitjeva may be regarded, in a certain sense, as safer for our purposes because it is truly model-independent. Since her goal in estimating \( \Delta \dot{\omega} \) was not the measurement of the Lense-Thirring effect, there is a priori no risk that, knowing in advance the desired answer, something was driven just toward the expected outcome.

The main question to be asked is, at this point, the following: can the result of (2) be explained by other unmodelled/mismodelled canonical or nonconventional dynamical effects? Let us, first, examine some standard candidates like the residual precession due to the still imperfect knowledge of the Sun’s quadrupole mass moment \( J_2 \) [31] whose action was, in fact, modelled by Pitjeva [26] by keeping it fixed to \( J_2^0 = 2 \times 10^{-7} \) in the global solution in which she estimated the corrections to the perihelion precessions. The answer is negative since the Newtonian secular precession due to the Sun’s oblateness, (for an oblate body \( J_2 > 0 \)) whatever magnitude \( J_2 \) may have, is positive. Indeed, it is [28, 32]

\[
\dot{\omega}_p = \frac{3}{2} \frac{n J_2}{(1-e^2)^2} \left( \frac{R}{a} \right)^2 \left( 1 - \frac{3}{2} \sin^2 i \right),
\]

where \( n = \sqrt{GM/a^3} \) is the Keplerian mean motion and \( R \) is the Sun’s mean equatorial radius. The angle \( i \) between the Venus’ orbit and the Sun’s equator amounts to 3.4 deg only. (Indeed, the orbit of Venus is tilted by 3.7 deg to the mean ecliptic of J2000 (http://ssd.jpl.nasa.gov/txt/aprx_pos_planets.pdf), while the Carrington’s angle between the Sun’s equator and the ecliptic is 7.15 deg [33]). For \( J_2^0 = 2 \times 10^{-7} \), the nominal value of the Venus’ perihelion precession induced by the solar quadrupole mass moment amounts to +0.0026”/cy^{-1}.

By assuming an uncertainty of about \( \delta J_2 \approx 10\% \), [34], if \( \Delta \dot{\omega}_{\text{Venus}} \) was due to such a mismodeled effect, it should amount to +0.0002”/cy^{-1}, which is, instead, ruled out at \( 6 - \sigma \) level. Concerning the precession due to the solar octupole mass moment \( J_3 \), it is [32]

\[
\dot{\omega}_p = -\frac{15}{16} n J_3 \left( \frac{R}{a} \right)^4 \left[ \frac{3}{(1-e^2)^2} + 7 \left( 1 + (3/2) e^2 \right) \right] \left( 1 - e^2 \right)^4 \left( \frac{7}{4} \sin^4 i - 2 \sin^2 i + \frac{2}{5} \right).
\]

For Venus, it amounts to \(-1.2 J_3^0 \)”/cy^{-1}. Since \( J_3^0 \approx -4 \times 10^{-9} \), we conclude that the second even zonal harmonic of the multipolar expansion of the solar gravitational potential cannot be responsible for (2). More generally, it

\[
\Delta \dot{\omega}_{\text{Venus}} = -0.0004 \pm 0.0001 \text{”/cy}^{-1},
\]
does not represent a potentially relevant source of systematic error for the measurement of the Lense-Thirring planetary precessions. Similar arguments hold also for other potential sources of systematic errors, for example, the asteroid ring and the Kuiper Belt objects, both modelled in EPM2006. The precessions induced by them are positive. Indeed, a Sun-centered ring of mass \( m_{\text{ring}} \) and inner and outer radius \( R_{\text{min}}/R_{\text{max}} \gg a \) induces a perihelion precession [37]:

\[
\dot{\omega}_{\text{ring}} = \frac{3}{4} \sqrt{G a^3 (1 - e^2)} \frac{m_{\text{ring}}}{M R_{\text{min}} R_{\text{max}} (R_{\text{min}} + R_{\text{max}})} > 0. 
\]

(5)

According to (5), the precession induced by the asteroids’ ring on the Venus’ perihelion amounts to \(+0.0007 \pm 0.0001'' \)cy\(^{-1} \) by using \( m_{\text{ring}} = (5 \pm 1) \times 10^{-10} \)M\(_{\odot} \). The lowest value \(+0.0006'' \)cy\(^{-1} \) is incompatible with (2) at 10 – \( \sigma \) level. In the case of the Kuiper belt objects, (5) yields a precession of the order of \(+0.00006'' \)cy\(^{-1} \) with \( m = 0.052 m_{\oplus} \) [37]. Thus, we can rule out such modelled classical features of the Sun and the solar system as explanations of \( \Delta \dot{\omega}_{\text{Venus}} \). General relativistic terms of order \( \mathcal{O}(c^{-4}) \) were not modelled by Pitjeva. However, the first correction of order \( \mathcal{O}(c^{-4}) \) to the perihelion precession [39] can be safely neglected because for Venus, it is

\[
\dot{\omega}_{c^4} \approx \frac{n (GM)^2}{c^3 a^2 (1 - e^2)} \approx 10^{-7}'' \text{cy}^{-1}. \tag{6}
\]

Concerning possible exotic explanations, that is, due to some modifications of the currently known Newton-Einstein laws of gravity, it may have some interest to check some of the recently proposed extra-forces [40] which would be able to phenomenologically accommodate the Pioneer anomaly [41]. All of such hypothetical new forces have not been modelled by Pitjeva, so that if they existed in nature, they would affect \( \Delta \dot{\omega}_{\text{Venus}} \). A central acceleration quadratic in the radial component \( v_r \) of the velocity of a test particle [40, 42]

\[
A = -v_r^2 \mathcal{H}, \quad \mathcal{H} = 6.07 \times 10^{-18} \text{m}^{-1} \tag{7}
\]

would induce a retrograde perihelion precession according to [43]

\[
\dot{\omega} = \frac{\mathcal{H} n a \sqrt{1 - e^2}}{e^2} \left( -2 + e^2 + 2\sqrt{1 - e^2} \right) < 0. \tag{8}
\]

(The quoted numerical value of \( \mathcal{H} \) allows to reproduce the Pioneer anomaly). However, (8) predicts a precession of \(-0.0016'' \)cy\(^{-1} \) for Venus, which is ruled out by (2) at 12 – \( \sigma \) level. Another possible candidate considered in [40] is an acceleration linear in the radial velocity

\[
A = -|v_r| \mathcal{K}, \quad \mathcal{K} = 7.3 \times 10^{-14} \text{s}^{-1} \tag{9}
\]

which yields a retrograde perihelion precession [43]:

\[
\dot{\omega} = \frac{\mathcal{K} \sqrt{1 - e^2}}{\pi} \left[ 2 e - (1 - e^2) \ln \left( (1 + e)/(1 - e) \right) \right] < 0. \tag{10}
\]

The prediction of (10) for Venus is \(-0.1'' \)cy\(^{-1} \), clearly incompatible with (2). Should one consider a central uniform acceleration with the magnitude of the Pioneer anomalous one, that is, \( A = -8.74 \times 10^{-10} \)ms\(^{-2} \), the exotic precession induced by it [44, 45] on the perihelion of Venus would be

\[
\dot{\omega}_{\text{V}} = A \sqrt{a(1 - e^2)} \frac{c}{GM} = -16'' \text{cy}^{-1}. \tag{11}
\]

Another nonconventional effect which may be considered is the precession predicted by Lue and Starkman [46] in the framework of the DGP multidimensional braneworld model by Dvali et al. [47] which is proposed to explain the cosmic acceleration without invoking dark energy. It is

\[
\dot{\omega}_{\text{LS}} = \frac{3c}{8r_0^2} + \mathcal{O}(c^{-4}) \approx \pm0.0005'' \text{cy}^{-1}, \tag{12}
\]

where the plus sign is related to the self-accelerated branch, while the minus sign is for the standard, Friedmann-Lemaître-Robertson-Walker (FLRW) branch; \( r_0 = 5 \) Gpc is a threshold characteristic of the DGP model after which gravity would experience neat deviations from the Newtonian-Einsteinian behavior. As can be noted, the self-accelerated branch is ruled out at 9 – \( \sigma \) level by (2), while the FLRW case is still compatible with (2) (1 – \( \sigma \) discrepancy). By the way, apart from the fact that there are theoretical concerns with the DGP model (see, e.g., [48] and references therein), the existence of both the Lue-Starkman FLRW precession and the Lense-Thirring one, implying a total unmodelled effect of \(-0.0008'' \)cy\(^{-1} \), would be ruled out by (2) at 4 – \( \sigma \) level. As a consequence, we can conclude not only that the examined exotic modifications of the standard laws of gravity, not modelled by Pitjeva, are not responsible for the estimated \( \Delta \dot{\omega}_{\text{Venus}} \), but also that their existence in the inner regions of the solar system is falsified by the observations. Moreover, given the magnitudes of the hypothetical effects with the negative sign, it is not possible that reciprocal cancelations with the positive classical mismodelled precessions can explain (2). Indeed, the sum of the latter ones is \(+0.0004'' \)cy\(^{-1} \), the sum of, for example, (8) and (12) (FLRW) is \(-0.0021'' \)cy\(^{-1} \), while the sum of (8) and (12) (self-accelerated branch) is \(-0.0011'' \)cy\(^{-1} \). Thus, we conclude that the most likely explanation for (2) is just the general relativistic Lense-Thirring effect. However, caution is in order in assessing the realistic uncertainty in such a test because, as already stated, the released error of 0.0001'' cy\(^{-1} \) is the formal, statistical one; the realistic uncertainty might be larger. By the way, we can at least firmly conclude that now also in the case of Venus, the general relativistic predictions for the Lense-Thirring effect on \( \dot{\omega} \) are compatible with the observational determinations for the unmodelled perihelion precessions, contrary to the case of [28]. Moreover, future modelling of planetary motions should take into account the relativistic effects of the rotation of the Sun as well. The steady improvement in the planetary ephemerides, which should hopefully benefit of the radiometric data from Messenger and Venus-Express as well, should allow for more accurate and stringent test in the
near-mid future. It would be of great significance if also other teams of astronomers would estimate their own corrections to the canonical perihelion (and also node) precessions in order to enhance the statistical significance and robustness of this important direct test of general relativity.

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