Thermoelectric properties of tubular nanowires in the presence of a transverse magnetic field

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Abstract

We calculate the charge and heat current associate with electrons, generated by a temperature gradient and chemical potential difference between two ends of a tubular nanowire of 30 nm radius in the presence of an external magnetic field perpendicular to its axis. We consider a nanowire based on a semiconductor material, and use the Landauer-Büttiker approach to calculate the transport quantities. We obtain the variation of the Seebeck coefficient ($S$), thermal conductivity ($\kappa$), and the figure of merit ($ZT$), with respect to the temperature up to 20 K, and with the magnetic field up to 3 T. In particular we show that the Seebeck coefficient can change sign in this domain of parameters. In addition $\kappa$ and $ZT$ have oscillations when the magnetic field increases. These oscillations are determined by the energy spectrum of the electrons.

Keywords: thermoelectrics, nanowires, Seebeck coefficient, figure of merit $ZT$, thermal conductivity

(Some figures may appear in colour only in the online journal)

1. Introduction

Thermoelectric materials have attracted considerable attention due to their potential applications in electronics [1–4], as energy conversion devices [5–9], or as components of complex instruments used in medical science [10]. Thermoelectric devices display interesting properties such as being very reliable because they do not contain any moving part, being of very small size, and most importantly, capable of energy harvesting from waste heat of environment, that makes them very attractive for industry [5, 11, 12]. Semiconductor nanowires are promising candidates for thermoelectric applications, along with their rich and complex electrical, optical, and photovoltaics properties [13–16]. Nanowires have played an important role in this research direction due to their ability to provide efficient thermoelectric elements with low thermal conductivity and high figure of merit ($ZT$) [17–19].

In particular, core–shell nanowires based on III-V semiconductors enable the control of charge, and possibly heat transfer through the specific geometry and shell thickness. With a doped shell and undoped core one can obtain a tubular conductor [20] with conduction electrons captured inside the shell. Most often such nanowires have a hexagonal cross section and the charge density peaks at the shell corners [21–25]. A nanowire made of a single material, for example InAs, may also become a tubular conductor if the conduction electrons are pushed towards the nanowire walls due to a favorable band bending at the surface [26]. Assuming a tubular distribution of the electrons in the nanowire, anjournal localization mechanism, that we focus on in this paper, is produced by a magnetic field perpendicular to the nanowire axis, and in that case the electron density within the shell increases in the direction perpendicular to the field, where the so called snaking states are formed [27–30].

In a recent paper where two of the present authors where involved, it has been predicted theoretically that a temperature gradient can lead to reversal of thermoelectric current in tubular nanowires in the presence of transverse magnetic field, at low temperatures [31], meaning that the electrons can either flow from the hot to the cold lead, or vice versa. This prediction
indicates the importance of the magnetic field effect on the thermoelectric properties of a tubular conductor, but it still awaits an experimental validation.

In the present paper we want to address, still theoretically, the efficiency of a thermoelectric element based on a tubular conductor in a perpendicular magnetic field. Efficient thermoelectric devices are supposed to produce a considerable electric current, but at the same time to limit the heat flow [32, 33]. These characteristics are incorporated in the dimensionless figure of merit $ZT$, which is defined as

$$ZT = \frac{S^2 \sigma T}{\kappa},$$

where $S$ is the Seebeck coefficient, $\sigma$ is the electrical conductivity, $\kappa$ is the thermal conductivity, and $T$ is the temperature. Hence, there are several parameters that need to be optimized to reach maximum value of $ZT$. In our physical system we know that the thermoelectric current is a nontrivial function of the magnetic field and temperature [31], and the first step of the present paper will be to obtain the Seebeck coefficient. After that, we will look at the thermal conductivity and finally at $ZT$.

The thermal conductivity reported for crystalline nanowires is more than two orders of magnitude lower than the bulk values [34]. Also, the phonon scattering at the nanowire surface substantially reduces their thermal conductivity and increases the thermoelectric power factor ($S^2 \sigma$) [35]. The diversity of fabrication methods for introducing dopants or impurities into the lattice is an important reason that makes the semiconductor nanowires important for their thermoelectric characteristics [36–40]. The thermal transport in nanoscale systems, whose dimension is much smaller than the mean free path of electrons, cannot be explained by a simple law due to the presence of quantum-mechanical features and strong non-linear behavior [41]. At intermediate temperatures where ballistic and diffusive phonons coexist, the thermal conductance decreases non-linearly with the length. And especially at low frequency, where the acoustic phonons give the major contribution to the thermal conductance [42]. But at low temperatures charge carriers have an important role in thermal transport quantized in multiples of the universal value $\pi^2 k_B^2 T/3h$, also electrical conductance is quantized in multiples of universal value $G_0 = e^2/h$ which can be understood within Landauer’s formula [43, 44].

It has been shown that a magnetic field produces large changes in the thermoelectric properties, including the reduction of thermal conductivity of charge carriers [45]. This has been demonstrated experimentally for Bi$_2$Sb$_2$Te$_3$ at 78–295 K and magnetic fields up to 1.7 T. The magnetic field has been also studied for GaAs–Ga$_{1-x}$Al$_x$As heterojunction up to 20 T [46]. The results showed an oscillatory behavior of thermopower ($S$) with the applied magnetic field. The magnetic field has been also studied for Bi nanowires array at 50–300 K which showed there is an optimum magnetic field for power factor [47].

The material of the paper is structured in these steps: In section 2 we present the model and the energy spectra of our system, the tubular conductor in perpendicular magnetic field. Then, in section 3 we discuss and show the results for the Seebeck coefficient, for the thermal conductivity, and for the figure of merit. Finally, the conclusions are collected in section 4.

2. Model and methods

In this paper, we consider electronic transport in a tubular, cylindrical nanowire, in the presence of a longitudinal temperature difference and a uniform magnetic field transverse to the axis of the cylinder. The conduction takes place only in a narrow shell at the surface and not through the bulk [26].

The Hamiltonian of the system can be written as

$$H = \frac{(-i\hbar \nabla + eA)^2}{2m_{\text{eff}}} - g_{\text{eff}}\mu_B sB,$$

where $B$ is the magnetic field in the $z$ direction, i.e., perpendicular to the nanowire length which is oriented along the $z$ axis, and $A = (0, 0, By)$ is the corresponding vector potential. Also, $e$ is the electron charge, $m_{\text{eff}} = 0.023m_0$ and $g_{\text{eff}} = -14.9$ are the effective electron mass and bulk g-factor of InAs, $\mu_B$ is the Bohr’s magneton and $s = \pm 1$ is the spin label. We chose the effective mass and g-factor as for InAs because this is a relatively common material used for core–shell nanowires. We assume that electrons propagate along the nanowire without interacting with journal electrons.

System parameters are $R = 30$ nm, $B = 0 – 3$ T and also by considering material parameter for InAs, we can calculate the heat current and electrical current driven by the temperature bias and chemical potential difference between the two ends of the nanowire, where we assume external leads are contacted. We calculate the charge current $I_e$ and heat current $I_Q$ through the nanowire using the Landauer approach:

$$I_e = \frac{e}{\hbar} \int T(E)[f_R(E) - f_L(E)]dE,$$

$$I_Q = \frac{1}{\hbar} \int \left[ T(E) - \mu \right][f_R(E) - f_L(E)]dE,$$

where $T$ is the transmission function, and

$$f_{L,R}(E) = \frac{1}{1 - e^{(E - \mu_{L,R})/k_BT}}$$

is the Fermi function for the left (L) or right (R) reservoir with chemical potential $\mu_{L,R}$ and temperature $T_{L,R}$. We consider a temperature bias $\Delta T = T_R - T_L > 0$, with $T_L$ always fixed at 0.5 K, and a chemical potential bias $\Delta \mu = \mu_R - \mu_L$, with $\mu_L = \mu - \Delta \mu/2$ and $\mu_R = \mu + \Delta \mu/2$, where $\mu$ is fixed at 4.2 meV and $\Delta \mu$ is varied between 0 – 0.4 meV.

Ballistic transport of electrons in nanowires leads to a transmission $T$, as a function of energy, with a step behavior. Nanowires showing such step-like behaviour have been measured, and in the presence of a low, but achievable impurity
The eigenstates of the Hamiltonian (2) are calculated by numerical diagonalization in a basis set corresponding to plane waves $\exp(\text{i}kz)$, with $k$ the wave vector in the longitudinal direction, and angular momentum eigenfunction $\exp(\text{i}m\phi)$, with $m = 0, \pm 1, \pm 2, ...$, in the transversal plane $(x, y)$ where the electrons are confined on a circle of radius $R$ [27]. The resulting energy spectra for magnetic fields $B = 1.8$ T and $2.5$ T are shown in figure 1. These spectra have an interesting feature: they may not always monotonic functions of $k$ when it has a fixed sign. Meaning that the transport channels, i.e. the number of states with a fixed energy, which in general increases with increasing the energy, in this case may also decrease. Consequently, the thermoelectric current may change sign [31].

In the transport calculations we will consider that only electrons that are close to $\mu$ in energy contribute to the heat transport. The chemical potentials are chosen such that $\mu$ is close to a subband bottom. In figures 1(b) and (d) we show this energy interval for two values of the magnetic field.

3. Results and discussion

3.1. Seebeck coefficient

The Seebeck coefficient or the thermopower, $S$, is defined by the ratio $-\Delta V/\Delta T$, where the voltage difference $\Delta V$ is produced in presence of a small temperature difference $\Delta T$ between two points of a conductor, under an open circuit condition. Usually the thermopower consists of two additive contributions: diffusion $S^d$ and phonon drag $S^p$. The first one originates from the diffusion of carriers (electrons or holes) and second one comes from the momentum that is transferred to carriers via their coupling to non-equilibrium acoustic phonons in the presence of a temperature gradient [50–52]. For the total thermopower $S = S^d + S^p$ there is a very good agreement between theory and experiments at temperature below $21$ K for example in bulk silicon [53]. However, at this low temperatures, where normally $S^d$ dominates, we can neglect the phonon
The Seebeck coefficient is important for two reasons. First, this coefficient provides fundamental information about the electronic energy structure and the electron scattering mechanism in a system. Second, there is some evidence suggesting that thermoelectric energy conversion can be more efficient in low-dimensional systems [54]. For example, for a semiconductor, a large magnitude of Seebeck coefficient requires only one type of carrier, because mixed n-type and p-type conduction will send both carriers through contacts, leading to a reduced Seebeck voltage. The relation between the carrier concentration and Seebeck coefficient, for bulk states, is given by:

$$ S = \frac{8\pi^2 k_B^2 m^* T}{3e^2} \left( \frac{\pi}{3n} \right)^{2/3}, $$

where $n$ is the carrier concentration and $m^*$ is the effective mass of carriers. Although a low carrier concentration of insulators and semiconductors result in large Seebeck coefficient, it also leads to a low electrical conductivity, $1/\rho = \sigma = ne\mu$, where the electrical conductivity and electrical resistivity are related to $n$ through the carrier mobility $\mu_c$ [5, 55, 56]. There is also anjouctural conflict with the effective mass of the charge carriers, in a manner that large effective mass provide high thermopower but low electrical conductivity.

In the present work we consider ballistic transport such that the electronic energy spectra have the main role in the behavior of the thermopower. We assume that the scattering due to impurities have negligible effects, and that is a reasonable approximation in a sufficiently clean system [37, 40]. Using the transmission functions for the calculated energy spectra we determine the voltage in an open circuit condition, $V_{op}$, from the chemical potential bias necessary to bring to zero the electric current in the system [equation (3)]. That means we evaluate $\Delta \mu = \mu_R - \mu_L = eV_{op}$ as a function of the temperature of the right lead, for different values of magnetic field.

As one can see in figure 2 the open circuit voltage has a nonlinear dependence on the temperature bias, for magnetic fields between $2.0 - 2.3$ T. More remarkably though the change of sign as a function of the temperature, which occurs because of the nonmonotonic variation of the transmission function with respect to the energy [31].

We obtain numerically the Seebeck coefficient, $S = V_{op}/\Delta T$, as the linear coefficient of $V_{op}$ as function of the temperature gradient, $S = V_{op}/\Delta T$, by performing the procedure described above with a small temperature bias, $\Delta T = T_R - T_L = 0.1$ K. This time both $T_R$ and $T_L$ are varied. We calculate the Seebeck coefficient at the specific $\mu$ situated close to a subband minimum, where the insensitivity of $S^d$ to the energy dependence of electron relaxation time has a direct impact on the phonon-drag contribution to the magnet-journalpower tensor, that results in $S^d$ becomes dominating over $S^d$. 

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**Figure 2.** Open circuit voltage as function of right lead temperature in indicated values of magnetic field.

**Figure 3.** Seebeck coefficient with magnetic field for different values of $T_R$ where $\mu = 4.2$ meV and $T_L = T_R - 0.1$ K (i.e. $T_L \approx T_R$).

**Figure 4.** Heat current as function of the temperature of the right contact $T_R$ for the indicated magnetic field values and the chemical potential $\mu = 4.2$ meV.
One of the fundamental factors for high efficient thermoelectric conversion is the thermal conductivity, which needs to be minimized. There are many possibilities to reduce the thermal conductance of a nanosystem [34]. Our next step is to evaluate the heat transported by the electrons of our system, which accompany the transport of electric charge. We calculate the heat current as function of the temperature of the right lead, for different values of the magnetic field, using equation (34). The thermal conductance, \( \kappa \), and thermal conductivity, \( \kappa = L/(\pi R^2) \), as functions of the magnetic field, for different temperatures of the right lead, are shown in figure 5. Here we use the whole cross sectional area of the cylinder, although the transport of both heat and charge occurs through the shell defined by the outer surface. The full cross sectional area is however relevant if the core is also included in the heat transport with phonons, which are neglected at our low temperatures.

The figure clearly indicates two regions with (i) constant \( \kappa \) and \( \bar{\kappa} \) at low fields and (ii) a non-linear reduction at higher applied magnetic field. We can see almost the same trend for \( \kappa \) (or \( \bar{\kappa} \)) for different temperatures, but that is more evident at lower \( T_R \). It is obvious that the increase of the magnetic field leads to a reduction of the thermal conductance and thermal conductivity, but the amount of these changes are different for each temperature. The reduction of the contributions of charge carriers (electrons or holes) to the thermal conductivity has been observed in experimental studies for both bulk and nanowire arrays [45, 47].

Next, in order to find the optimum conditions for thermoelectric conversion, we need to calculate \( ZT \) using equation (1). High electrical conductivity and low thermal conductivity is required to maximize or optimize \( ZT \), that is achievable by considering lattice thermal conductance and materials characteristics [59–61]. In \( ZT \) formula the cross sectional area of the full cylinder \((\pi R^2)\) used in the thermal conductivity is going to be compensate by the area used in electrical conductivity. Electrical conductivity calculated from \( \sigma = dl_i/dV \), where for each specific temperature and magnetic field several values of \( I_3 \) and \( V = (\mu R - \mu L)/e \) were calculated separately and got carriers over the energy states such that more electrons are localized due to the closed cyclotron motion imposed by the field, and fewer of them are available for transport. Note also that the heat current does not change sign, as the charge current does, that is in agreement with the second law of thermodynamics.
differentiated. Figure 6 represents ZT as function of magnetic field for different temperatures of the right contact.

Again, the figure shows two regions, of constant and oscillating ZT, respectively. It is also clear that increasing the temperature shifts ZT to higher values, in both regions, non-linearly. Also, at low temperatures, the figure of merit shows limited changes with increasing magnetic field, but for temperatures more than 9 K we have some obvious variation of ZT values. There are two peaks for ZT in the figure, which is a specific result of magnetic field presence. At all temperatures studied here, the peaks are located at about $B = 1.8$ T and $2.5$ T with a slight shift to higher field at higher temperature. Thus, regardless of the temperature difference, there is an optimum magnetic field that leads to maximum ZT. In addition, although we are referring to low temperatures, doubling the value of ZT just by applying to the system an external magnetic field is interesting tuning possibility.

With increasing the temperature above $20$ K, one would expect an increase of ZT simply because of the temperature factor in the definition, Equation (1). But, of course, the phononic contribution to the heat transport increases with temperature, and the phonon drag and lattice vibrations will end up by dominating over the diffusive heat transport due to electrons. However, experimental values for the thermal conductivity $\kappa$ in nanowires with diameters between $20$–$100$ nm shows a saturation behavior for temperatures above $100$ K to values like $10$–$40$ W mK$^{-1}$ [34]. Therefore, for such a temperature we can expect the figure of merit of our system to become roughly ten times bigger than the values shown in figure 6.

4. Conclusions

In this paper we have calculated the most important thermoelectric parameters, such as heat and electric current, the open circuit voltage $V_{oc}$, Seebeck coefficient $S$, thermal conductivity $\kappa$, and figure of merit ZT, produced by electrons confined within a tubular nanowire due to a temperature bias, in presence of transverse magnetic field. To this end, heat current and electrical current variations are obtained in the temperature range between (0–20 K). Increasing the magnetic field leads to reduction in thermal conductivity, which is more pronounced at lower temperatures. The energy spectrum of electrons is a nonmonotonic function of the wave vector along the nanowire, and so is the transmission function with respect to the energy. Consequently $V_{oc}$ can change sign when the temperature gradient or the magnetic field increase. Both $S$ and ZT have a constant to oscillatory transition with increasing the magnetic field. For example for a cylinder radius of 30 nm, ZT presents two peaks at about 1.8 and 2.5 T which are independent of the temperature.

These feature allow a substantial tuning of the thermoelectric response of the nanowire with changing the temperature or with applying an external magnetic field. To the best of our knowledge, although several groups have performed thermolectric measurements of nanowires at low temperatures [9, 19, 25, 34–36], experimental investigations of tubular conductors based on core–shell geometry in a transverse magnetic field have not been reported yet. Therefore it is our hope that our theoretical results will stimulate such an experimental work.

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References

[1] Paulsson M and Datta S 2003 Phys. Rev. B 67 241403
[2] Weber J, Potje-Kamloeh K, Haase F, Detemple P, Vöklein F and Doll T 2006 Sensors Actuators A 132 325–30
[3] Suarez F, Nozariashmar Marz A, Vashaee D and Öztürk M C 2016 Energy Environ. Sci. 9 2099–113
[4] Suarez F, Parekh D P, Ladd C, Vashaee D, Dickey M D and Öztürk M C 2017 Appl. Energy 202 736–45
[5] Snyder G J and Toberer E S 2011 Complex thermoelectric materials Materials for Sustainable Energy: A Collection of Peer-Reviewed Research and Review Articles from Nature Publishing Group (Singapore: World Scientific) pp 101–10
[6] Wood C 1988 Rep. Prog. Phys. 51 459
[7] Fergus J W 2012 J. Eur. Ceram. Soc. 32 525–40
[8] Nolas G, Morelli D and Tritt T M 1999 Ann. Rev. Mater. Sci. 29 89–116
[9] Prete D et al 2019 Nano Lett. 19 3033–9
[10] Cosman E R 1990 Thermometric cardiac tissue ablation electrode with ultra-sensitive temperature detection US Patent 4,966,597
[11] Dudzinski L, Hamley J, McCallum P, Sandifer C, Sutliff T J and Zakrjasjeck J F 2014 Nasa’s radioisotope power systems program status 12th Int. Energy Conversion Conf. 3462
[12] Vining C B 2009 Nat. Mater. 8 83–5
[13] Zhao X, Wei C, Yang L and Chou M 2004 Phys. Rev. Lett. 92 236805
[14] Kateb M, Ahmadi V and Mohseni M 2013 Sol. Energy Mater. Sol. Cells 112 57–64
[15] Kateb M, Safarian S, Kolahdouz M, Fatipour M and Ahamdi V 2016 Sol. Energy Mater. Sol. Cells 145 200–5
[16] Tian B, Kempi T J and Lieber C M 2009 Chem. Soc. Rev. 38 16–24
[17] Rossella F, Pennelli G and Rondanini S 2018 Measurement of the thermoelectric properties of individual nanostructures Semiconductors and Semimetals vol 98 (Amsterdam: Elsevier) 409–44
[18] Hong M, Lyu W, Wang Y, Zou J and Chen Z G 2020 J. American Chem. Soc. 142 2672–81
[19] Diez G G, Gordillo J M S, Pujadó M P, Salleras M, Fonseca L, Morata A and Rubio A T 2020 Nano Energy 67 104191
[20] Göl O, Demarina N, Bömers C, Rieger T, Lüth H, Lepsa M I, Grützmacher D and Schäpers T 2014 Phys. Rev. B 89 045417
[21] Ferrari G, Goloni G, Bertoni A, Cuoghi G and Molinari E 2009 Nano Lett. 9 1631–5
[22] Sitek A, Thorgilsson G, Gudmundsson V and Manolescu A 2016 Nanotechnology 27 225202
[23] Sitek A, Serra L, Gudmundsson V and Manolescu A 2015 Phys. Rev. B 91 235429
[24] Torres M U, Sitek A, Erlingsson S I, Thorgilsson G, Gudmundsson V and Manolescu A 2018 Phys. Rev. B 98 085419
[25] Fuest S et al 2020 Adv. Mater. 32 190548
[26] Heedt S, Manolescu A, Nemnes G, Prost W, Schubert J, Grutzmacher D and Schapers T 2016 Nano Lett. 16 4569–75
[27] Manolescu A, Rosdahl T, Erlingsson S I, Serra L and Gudmundsson V 2013 European Phys. J. B 86 445
[28] Rosdahl T O, Manolescu A and Gudmundsson V 2015 Nano Lett. 15 254–8
[29] Manolescu A, Nemnes G A, Sitek A, Rosdahl T O, Erlingsson S I and Gudmundsson V 2016 Phys. Rev. B 93 205445
[30] Chang C H and Ortix C 2016 Int. J. Mod. Phys. 30 1630016
[31] Erlingsson S I, Manolescu A, Nemnes G A, Bardarson J H and Sanchez D 2017 Phys. Rev. Lett. 119 036804
[32] Mingo N 2004 Appl. Phys. Lett. 84 2652–4
[33] Li D, Wu Y, Kim P, Shi L, Yang P and Majumdar A 2003 Phys. Rev. Lett. 83 2934–6
[34] Wu P M, Gooth J, Zianni X, Svensson S, Nilsson H, Samuelson L and Linke H 2013 Nano Lett. 13 4080–6
[35] Pennelli G, Elyamny S and Dimaggio E 2018 Nanotechnology 29 505402
[36] Erlingsson S I, Bardarson J H and Manolescu A 2018 Beilstein J. Nanotechnol. 9 1156–61
[37] Dominguez-Adame F, Martin-González M, Sánchez D and Cantarero A 2019 Phys. E Low-dimensional Syst. Nanostruct. 113 213–25
[38] Vuttivorakulchai K, Luisier M and Schenk A 2018 J. Appl. Phys. 124 205101
[39] Thorgilsson G, Erlingsson S I and Manolescu A 2017 J. Phys. Conf. Series 906 012021
[40] Cahill D G, Ford W K, Goodson K E, Mahan G D, Majumdar A, Maris H J, Merlin R and Phillpot S R 2003 J. Appl. Phys. 93 793–818
[41] Yadav H K, Gupta V, Sreenivas K, Singh S, Sundarakannan B and Katiyar R 2006 Phys. Rev. Lett. 97 085502
[42] Yamamoto T, Watanabe S and Watanabe K 2004 Phys. Rev. Lett. 92 075502
[43] Yamamoto T and Watanabe K 2006 Phys. Rev. Lett. 96 255503
[44] Wolfe R and Smith G 1962 Appl. Phys. Lett. 1 5–7
[45] Fletcher R, Maan J, Ploog K and Weimann G 1986 Phys. Rev. B 33 7122
[46] Hasegawa Y, Ishikawa Y, Morita H, Komine T, Shirai H and Nakamura H 2005 J. Appl. Phys. 97 083907
[47] Kammhuber J et al et al 2016 Nano Lett. 16 3482–6
[48] Ferry D and Goodnick S M 1999 Transport in Nanostructures (Cambridge: Cambridge university press) vol 6
[49] Tsaoisidou M 2010 Front. Nanosci. Nanotechnol. 2 477
[50] Fletcher R, Pudalov V, Feng Y, Tsaoisidou M and Butler P 1997 Phys. Rev. B 56 12422
[51] Fletcher R, Harris J, Foxon C, Tsaoisidou M and Butler P 1994 Phys. Rev. B 50 14991
[52] Behnen E 1990 J. Appl. Phys. 67 287–92
[53] Tsaoisidou M 2010 Front. Nanosci. Nanotechnol. 2 477
[54] Hicks L and Dresselhaus M S 1993 Phys. Rev. B 47 16631
[55] Rowe D M 2018 CRC Handbook of Thermoelectrics (Boca Raton, FL: CRC Press)
[56] Beenakker C W J and Staring A A M 1992 Phys. Rev. B 46 9667
[57] Svensson S, Persson A, Hoffmann E, Nakpathomkun N, Nilsson H, Xu H, Samuelson L and Linke H 2012 New J. Phys. 14 033041
[58] Dresselhaus M S, Chen G, Tang M Y, Yang R, Lee H, Wang D, Ren Z, Fleurlia J P and Gogna P 2007 Adv. Mater. 19 1043–53
[59] Chen G, Dresselhaus M, Dresselhaus G, Fleurlia J P and Caillat T 2003 Int. Mater. Rev. 48 45–66
[60] Rosi F 1968 Solid State Electron 11 833–68