Anisotropic Transport Properties of Ferromagnetic-Superconducting Bilayers.

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We study the transport properties of vortex matter in a superconducting thin film separated by a thin insulator layer from a ferromagnetic layer. We assume an alternating stripe structure for both FM and SC layers as found in [7]. We calculate the periodic pinning force in the stripe structure resulting from a highly inhomogeneous distribution of the vortices and antivortices. We show that the transport in SC-FM bilayer is highly anisotropic. In the absence of random pinning it displays a finite resistance for the current perpendicular to stripe and is superconducting for the current parallel to stripes. The average vortex velocity, electric field due to the vortex motion, Josephson frequency and higher harmonics of the vortex oscillatory motion are calculated.

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The interest in Heterogeneous ferromagnetic-superconducting systems has grown rapidly in recent years. This interest stems not only from their possible technological applications but also from new physical phenomena arising from the interaction between two order parameters. Typically such a system consists of a superconductor (SC) placed in close proximity with a ferromagnetic structure (FS) such as an array of ferromagnetic dots or holes. The two systems are separated by an infinitely thin layer of insulator oxide that guarantees the suppression of proximity effects. It was demonstrated experimentally [1, 2] that the interaction between the superconductor and the ferromagnet may lead to formation of superconductive vortices interacting with the FM film. Theoretical studies of such systems have been done in [3-7].

Recently, Erdin et.al. [7] studied the equilibrium structure of a FM-SC bilayer (FSB). They have proved that it represents a two-dimensional periodic stripe domain structure consisting of two equivalent sub-lattices, in which both the magnetization \( m(x, y) \) and the vortex density \( n_v(x) \) alternate. Thus, they predicted spontaneous violation of the translational and rotational symmetry in the bilayer. In this article we study the transport properties of the FSB. They are associated with the driving force acting on the vortex lattice from an external electric current. We show that the FSB exhibits strong anisotropy of the transport properties: the bilayer may be superconducting for the current parallel to the domain walls and resistive when the current is perpendicular to them.

Periodic pinning forces in the direction parallel to the stripes do not appear in continuously distributed vortices, their reappearance is associated with the discreteness of the vortex lattice. Therefore, we need to modify the theory [3] to incorporate the discreteness effects. Let us assume that the saturation magnetization per unit area of the FM film is \( m \) and its width is \( L \). The energy necessary to create a single Pearl vortex [8] in the superconductor is \( \epsilon_0 = \epsilon_0 \ln(\frac{\phi_0}{\phi}) \), with \( \epsilon_0 = \frac{\phi_0^2}{2\pi e} \), where \( \phi_0 \) is the flux quantum, \( \lambda = \frac{\phi_0}{\phi} \) is the effective penetration depth [9], \( \lambda_L = \frac{m_e c_2}{\epsilon s} \) is the London penetration depth, \( d_s \) is the thickness of the superconducting layer and \( \xi \) is its coherence length. It was shown in [3] that the interaction between the superconducting vortices and the magnetization in the stripe structure renormalizes the single-vortex energy to the value \( \epsilon_v = \epsilon_0 - m_0 \phi_0 \) which must be negative to allow development of the stripes. The density of the superconducting vortices increases when approaching the domain walls and can be expressed as \( n_v(x) = \frac{\tilde{m}}{L \phi_0 \sin(\frac{\phi}{\phi_0})} \), where \( \tilde{m} = m - \frac{e_0 \phi_0}{\phi} \) is the renormalized magnetization of the FM stripe. The vortices spontaneously appear in the superconductor. We assume that the vortices inside one stripe are arranged in parallel chains. Each chain is periodic with the same lattice constant \( b \) along the chain, whereas the distance between \( k \)-th and \( (k+1) \)-th chain \( a_k \) depends on \( k \). The correspondence between this discrete arrangement and continuous approximation [8] is established by the requirement that the local vortex density \( n_v(x_k) \) calculated in [8] must be equal to \( (a_k b)^{-1} \). The coordinate \( x_k \) is determined in terms of \( a_k \) as a sum: \( x_k = \sum_{k'=0}^{a_k-1} a_k \). For definiteness we choose the origin in the center of the stripe. We assume that the total number of the vortex chains \( 2N \) in a stripe is large. Then some of them are located very close to the domain walls. Let us remind that, in the continuous approximation [8], \( n_v = \frac{\pi}{L \phi_0 \sin(\frac{\phi}{\phi_0})} \), where \( L \) is the domain width. Considering the nearest to the domain wall vortex chain (with the number, \( N \)), we put \( n_v(x_N) = \frac{1}{2a_N} \), On the other hand \( x_N = L - a_N \). Since \( \frac{a_N}{L} \ll 1 \), we find: \( b = \frac{\phi_0}{\tilde{m}} \). The total number of chains in a stripe is \( 2N \), where \( N = b \int_0^{L-a} n_v(x) dx = \frac{1}{2} \ln(\frac{L}{a_N}) \). We cut the integration (and summation) at a distance \( \sim \lambda \) from the domain wall where the continuous approximation breaks. Thus the minimum value of \( a \) is \( \lambda \). When transport current passes through the superconducting...
The sum in (3) can be calculated using Poisson summation formula 12. Since the force is zero in the continuous approximation, it is possible to retain the lowest non-zero harmonic in the Poisson summation. Thus, we arrive at the following interaction energy of two chains:

\[ U(x_1, x_1', s) = \frac{N_0 \phi_0^2}{4\pi^2 b} \cos(2\pi s) \chi_{11} \]

where \( \chi_{11} = e^{-2\pi H_{c1} d_{11} / \rho_0 c} / \). The distance between two chains \(|x_1 - x_1'|\) is larger or equals \( \lambda \) hence \( \chi_{11} \sim \chi \sim e^{-2\pi H_{c1} d_{11} / \rho_0 c} / \). Typical value of \( \chi_{11} \) is \( e^{-2\pi H_{c1} d_{11} / \rho_0 c} / \) where \( \delta_m = \frac{2n_m d_m}{\pi v_s} \), with \( g \) is Landé factor, \( S \) is the ferromagnet elementary spin, \( n_m \) and \( d_m \) are the electrons density and thickness of the magnetic film respectively.

We conclude that the amplitude of the periodic potential for displacements parallel to the domains is exponentially small in units of \( \epsilon_0 \), near the transition temperature. Relative displacements in perpendicular direction have energy barrier \( \sim \epsilon_0 \) even in continuous approximation. We model the restoring forces by simple sines dependencies \( f_x = f_\perp \sin(2\pi a (x_1 - x_1')) \), \( f_y = f_\perp \sin(2\pi a (y_1 - y_1')) \), where \( f_\perp \sim \frac{\epsilon_0}{\alpha} \) and \( f_\perp \sim \frac{\epsilon_0}{\alpha} e^{-2\pi H_{c1} d_{11} / \rho_0 c} / \).

When the supercurrent is perpendicular to domains, the equations of motion for a vortex and antivortex

\[ \eta \dot{y}_+ = F - \frac{F}{v_s} \dot{x}_+ - f_\perp \sin(2\pi b (y_1 - y_1')) \]

\[ \eta \dot{y}_- = -F - \frac{F}{v_s} \dot{x}_+ + f_\perp \sin(2\pi b (y_1 - y_1')) \]

where \( F = \pi n_s d s v_s \), \( f_\perp = \frac{\epsilon_0}{\alpha} \) and \( f_\perp = \frac{\epsilon_0}{\alpha} e^{-2\pi H_{c1} d_{11} / \rho_0 c} / \). If the current is smaller than a critical value, Eq. 6,8 accept a static solution

\[ x_+ = x_- = F b \frac{4\pi \nu_s}{\arcsin(\frac{F}{f_\perp})} \]

\[ y_+ = y_- = -F b \frac{4\pi}{\arcsin(\frac{F}{f_\perp})} \]

It is valid at \( F \leq f_\perp = \frac{\epsilon_0}{\alpha} \). For \( F > f_\perp \) or equivalently, if the current is larger than its critical value, vortices and antivortices start to move. The solution of Eq. 6,8 for \( F > F_c \) reads:

\[ x_+ - x_- = 0 \]

\[ x_+ - x_- = F \frac{y_+ - y_-}{\nu_s} \]

\[ y_+ - y_- = \frac{b}{\pi} \arctan(\frac{f_\perp}{F} + \sqrt{1 - \frac{f_\perp^2}{F^2} (\tan(\omega_0 t)))} \]

\[ y_+ + y_- = 0 \]
where $\omega_0^2 = \frac{2\pi n_e^2 \sqrt{F^2 - \chi^2 v_0^2}}{\sigma (F^2 + \eta^2)}$ is the Josephson frequency. Thus the vortices and antivortices acquire the same velocity components $v_{+x} = v_{-x}$ in the direction of the current and opposite velocity components $v_{+y} = -v_{-y}$ in the direction perpendicular to the current. The domain walls do not interfere such a motion if they move in the direction of the current with the same velocity $v_{dw} = v_{+x} = v_{-x}$ as vortices and antivortices. Such a motion is a Goldstone mode. The solution displays an oscillatory motion of the vortices and antivortices in the direction parallel to the domain walls, in addition to their motion together with the domain walls along the direction of the current. Higher harmonics of the vortex motion can be calculated analytically. The distribution of vortices (antivortices) is inhomogeneous in the direction perpendicular to the domains. The local electric field $E$ due to the vortex motion is related to its time-average velocity $\langle v_+ \rangle$ as $E = -\frac{\eta \tilde{m}}{\rho}(\langle v_+ \rangle \times \hat{x})$.

Therefore, the local field produced by vortices in the direction parallel to the domains is equal but opposite to the one produced by antivortices, while the local field produced by vortices and antivortices in the direction perpendicular to the domains has both equal magnitude and sign. The time average of the vortex (antivortex) velocity over a period $T = \frac{2\pi}{\omega_0}$ is

$$
\langle v_{+y} \rangle = \pm \frac{\eta \sqrt{F^2 - f_0^2}}{(\eta^2 + \frac{2}{\omega_0})} (\eta^2 + \frac{2v_y}{\omega_0})
$$

$$
\langle v_{+x} \rangle = \frac{F \langle v_{+y} \rangle}{\eta v_x}
$$

The time-averaged local field components are

$$
E_x = -\frac{\eta \tilde{m}}{a c}(\frac{F^2 - f_0^2}{(\eta^2 + \frac{2v_y}{\omega_0})})
$$

$$
E_y = \frac{\eta \mu}{m c}(\frac{F^2 - f_0^2}{(\eta^2 + \frac{2v_y}{\omega_0})})
$$

The upper sign in Eq. 15 and Eq. 18 refer to the vortices velocity and produced field along the domain while the lower sign refer for those due to antivortices. Non-zero average electric field due to all vortices and antivortices in the FSB appears only in the direction perpendicular to the domains. The critical current $J_c$ is related to $F_c$ as $J_c = \frac{\sigma}{\rho_{sd}d_c} F_c$. Plugging $F_c = \frac{\sigma m}{\rho_{sd}d_c}$ into the expression for $J_c$ and accepting $\chi = 10^{-4} - 10^{-2}$, $b = 10^{-4} - 10^{-5} cm$, and $n_s = 10^{22} cm^{-3}$ we find: $J_c \sim 10^3 - 10^5 \frac{A}{cm^2}$.

When the current flows parallel to the stripes, the FM domain walls stay at rest while vortices and antivortices move both parallel and perpendicular to the domains. The solution of equations of motion for vortices and antivortices shows that they move opposite to one another both in $x$ and $y$ directions. Their motion along $x$ is oscillatory with fundamental frequency $\omega_0^x = \frac{2\pi n_e^2 \sqrt{F^2 - \chi^2 v_0^2}}{\sigma (F^2 + \eta^2)}$. The motion of vortices and antivortices in the parallel direction proceeds until the distance between them becomes half lattice spacing $\frac{a}{2}$. Once the vertical shift between the vortices and antivortices reaches $\frac{a}{2}$, their motion freezes. The critical current in this case is $J_c = \frac{\sigma_{sd} n_{sd}}{\rho_{sd} d_c}$, the lattice spacing $a$ is of the order of $\lambda \sim 10^{-5} - 10^{-4} cm$, hence the critical current $J_c$ is of the order $10^7 - 10^8 \frac{A}{cm^2}$, which is at least 10 times larger than the critical current for parallel current. Therefore, the system may be superconducting for the current parallel to the stripes and exhibit finite resistance for perpendicular current. The difference in the critical currents for parallel and perpendicular directions is due to the exponential factor $\chi$ which is small if $b \ll \lambda$. The anisotropy is pronounced when the density of the superconducting electrons, $\delta_m$ is temperature dependent and eventually decreases when temperature decreases starting from $T_c$. However, at the temperature of vortex disappearance $T_v < T_c$ the value $\tilde{m}$ turns into zero and $\chi$ again becomes exponentially small. Thus anisotropy has a minimum between $T_v$ and $T_c$

Kopnin and Vinokur considered a collection of superconducting grains with the washboard pinning potentials a model of random pinning. They obtained a similar result for vortex sliding in external magnetic field with a supercurrent applied. In contrast to their work (they considered vortices only), we consider vortices and antivortices in the field of periodic pinning and completely neglect the random pinning.

Let us discuss briefly how the magnetic field generated...
by supercurrent changes our result. In [16] it was shown that at sufficiently small critical magnetic field the domains vanish. Therefore, in general, magnetic field suppresses both the anisotropy and periodic pinning at a critical field for which domains disappear. At such critical field only random pinning prevails. However, the total per unit length current is proportional to the thickness of the SC film and can be kept small.

In conclusion, we studied the transport properties of the FM-SC bilayer in a state with stripe domains of alternating magnetization and vorticity. We showed that, in the absence of a driving force, the vortices and antivortices lines themselves up in straight chains configuration. We showed that the force between two chains of vortices falls off exponentially as a function of the distance separating the chains. We argued that the exponential decay of the pinning force in the direction parallel to the domains drops faster in the vicinity of the superconducting transition temperature $T_s$ and vortex disappearence temperature $T_v$. We solved the equations of motion for vortices and antivortices in two cases for the driving current direction, parallel or perpendicular to the domains. We calculated the critical current for both cases and found that its value for the parallel current is much higher than for perpendicular one. Our most important result is a strong anisotropy of the critical current. We expect the ratio of the parallel to perpendicular critical current is in the range $10^2 \div 10^4$ close to the superconducting transition temperature $T_s$ and to the vortex disappearence temperature $T_v$. The anisotropy decreases rapidly when the temperature goes from the ends of this interval reaching its minimum somewhere inside it. This anisotropic transport behavior could serve as a diagnostic tool to discover spontaneous topological structures in magnetic-superconducting systems.

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