Enhanced sensitivity of optical gyroscope in a mechanical parity-time-symmetric system based on exceptional point

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Abstract

High-sensitivity gyroscopes are widely used for rotation detection in several practical applications. Recently, exceptional points (EPs) have garnered considerable attention for enhancing the sensitivity of sensors based on optical cavities. Here, we propose an EP-enhanced optical gyroscope based on mechanical parity-time (PT) symmetry in a microcavity system. We demonstrate that by pumping the two optical modes with different colors, i.e., blue and red detuning, an effective mechanical PT-symmetric system can be obtained, and the system can be prepared at EP with appropriate parameters. The sensitivity of gyroscope at EP was enhanced by more than one order of magnitude in the weak perturbation regime as compared to that at diabolic point. This indicates that the sensitivity of gyroscope can be effectively enhanced by monitoring the mechanical modes rather than the optical modes. Overall, our work provides a promising approach to design high-sensitivity gyroscopes in optical microcavities and is potentially useful in a variety of research fields including fundamental physics and precision measurement.

1. Introduction

High-accuracy gyroscopes based on Sagnac effect [1, 2] play a crucial role in precision measurements. Gyroscopes comprise of various systems including optomechanical systems [3–5], photon and matter-wave interferometers [6, 7], and solid spin systems [8–12]. High-quality optical microcavities [13] are a promising platform to investigate both fundamental physics and applications [14–25] owing to their ability to enhance light–matter interaction in an ultra-small volume. Several earlier studies have demonstrated that exceptional point (EP) [26–36] associated with non-Hermitian Hamiltonian governs the dynamics of open systems such as parity-time (PT) symmetric system, and the applications of EPs in quantum sensing [37, 38], quantum metrology [39], low-threshold lasers [40–43], etc have been presented.

Recently, sensitivity enhancement based on EP has been proven both theoretically [27, 44–46] and experimentally [3, 28, 47–49] in many detection schemes such as nanoparticle detector [27, 28, 44], mass sensor [45], and gyroscope [3, 46–48, 50, 51], which is attributed to the complex square-root topological behavior near an EP. Besides, optomechanical systems [52–70] are considered as a potential candidate for precision sensing [71–77]. Generally, in these systems, the optical mode response is investigated before and after the introduction of perturbation, which causes a mode frequency shift or mode splitting. However, the narrower effective linewidth of mechanical modes has facilitated an enhancement in the detection resolution by several orders of magnitude [75] in optomechanical systems. The advantage of measuring the response of the mechanical modes rather than the frequency shift or splitting in the optical modes lies in the fact that the minimal response required to discern the signal in the sensing scheme is reinforced in the former case.
In this study, to utilize the combined benefits of sensitivity enhancement at the EP and the narrow linewidth of mechanical modes, we theoretically propose an effective gyroscope scheme based on mechanical PT-symmetric system. To obtain a balance between the gain and loss of mechanical modes, the system exhibits EP in its mechanical spectrum, which ensures a giant enhancement in the sensitivity near EP. The proposed scheme differs from the conventional scheme in the following aspects: (i) it operates at mechanical EP rather than at optical EP; (ii) the gain and loss of the mechanical modes are self-engineered, while the cavity modes are driven. By pumping the two optical modes with different colors, i.e., blue and red detuning, the gain and loss of the mechanical modes are engineered and the effective mechanical PT-symmetric system can be obtained. In the absence of rotation, the system is controlled in the EP state. In the presence of rotation, the eigenvalues of the system are shifted away from the EP. Overall, our results reveal that the sensitivity of gyroscope can be effectively enhanced by monitoring the mechanical response rather than the variation in the optical modes. Therefore, we believe that our work offers an alternative approach to design high-sensitivity gyroscopes in optomechanical systems, and it is potentially useful in various research fields including fundamental physics and precision measurements.

This article is organized as follows: in section 2, we demonstrate the basic model and the dynamical equations. We study the PT-symmetric system and realize EP in section 3. We show the enhanced performance of EP and the superiority of monitoring mechanical modes response in section 4. In section 5, different transmission spectra are presented. Conclusion is given in section 6.

2. Model and dynamic equations

The mechanical PT-symmetric system can be realized by several methods, such as embedding a nanomechanical beam inside a superconducting transmission-line microwave cavity [61]. For the application in gyroscope, it is practicable to couple the fiber cavity with a one-end-vibrating cantilever. The coupling between the two cantilevers can be tuned by applying stress to the coupling overhang by piezoelectric effect or photothermal effect [61, 78–81], by applying the electrostatic force [82], and can be adjusted by a more direct way, i.e., modifying their separation [83] experimentally. Without losing the generality of the model, we do not restrict our model to any particular system. The proposed model is illustrated in figure 1, which contains two optical modes and two coupled mechanical modes. The first cavity is in the red-sideband regime, while the other one is driven with a blue-detuned laser. The two optical modes are coupled to the corresponding mechanical mode, and the two mechanical modes are simultaneously coupled with each other. Since the two cavities are symmetrically driven, it is feasible to control either mechanical gain or loss. For non-rotation case, the Hamiltonian describing the dynamics of coupled optomechanical system is

\[ H = H_{\text{free}} + H_{\text{int}} + H_{\text{drive}} + H_{\text{probe}}, \tag{1} \]

where \((h = 1)\)

\[ H_{\text{free}} = \omega_1 a_1^\dagger a_1 + \omega_2 a_2^\dagger a_2 + \omega_{m1} b_1^\dagger b_1 + \omega_{m2} b_2^\dagger b_2, \]

\[ H_{\text{int}} = g_1 a_1^\dagger a_1 \left( b_1^\dagger + b_1 \right) + g_2 a_2^\dagger a_2 \left( b_2^\dagger + b_2 \right) + J \left( b_1^\dagger b_2 + b_1 b_2^\dagger \right), \]

\[ H_{\text{drive}} = i \sqrt{\kappa_{\text{ex1}}} \epsilon_{p1} e^{-i\omega_{p1} t} a_1^\dagger + i \sqrt{\kappa_{\text{ex2}}} \epsilon_{p2} e^{-i\omega_{p2} t} a_2^\dagger + \text{h.c.}, \]

\[ H_{\text{probe}} = i \sqrt{\kappa_{\text{ex}}} S e^{-i\omega_{\text{p}} t} a_1^\dagger + \text{h.c.}. \tag{2} \]

\(H_{\text{free}}\) represents the free Hamiltonian of the optomechanical system; \(a_j(b_j)\) and \(a_j^\dagger(b_j^\dagger)\) \((j = 1, 2)\) are the annihilation and creation operators of the \(j\)th optical (mechanical) mode. The frequency and total decay rate of the optical modes are \(\omega_{ij}\) and \(\kappa_j\), respectively. The resonant frequency, effective mass, and decay rate of the mechanical resonators are denoted by \(\omega_{m}, m, \) and \(\gamma_j\), respectively. \(H_{\text{int}}\) represents the interaction Hamiltonian of the model; the first two terms signify the coupling of the cavity modes to the corresponding mechanical modes with single-photon optomechanical coupling strength of \(g_1\) and \(g_2\). Further, the third term of \(H_{\text{int}}\) denotes the mechanical–mechanical coupling with coupling strength \(J\). \(H_{\text{drive}}\) indicates that the two optical modes are driven by external fields with strength \(\epsilon_{pj}\) and frequency \(\omega_{pj}\). \(\kappa_{\text{ex}}\) denotes the external loss rate between the optical mode \(a_j\) and the fiber. \(H_{\text{probe}}\) describes the energy of the probe laser characterized by strength \(S\) and frequency \(\omega_p\). In the rotating frame of the driving fields (to be simplified, we consider the case of \(\omega_{p1} = \omega_{p2} = \omega_{p}\)), the dynamic equations describing the optomechanical system thus
The linearization approach is introduced i.e. divide the operators into steady-state value and the fluctuation parts as mentioned above. By substituting these ansatzs into equations (3)–(6), we can easily get the classical steady-state values

\[
\begin{align*}
\alpha_1 &= \frac{\sqrt{g_{11} \kappa_1'}}{i \Delta_1 + \kappa_1/2 + i g_1 \beta_1}, \\
\alpha_2 &= \frac{\sqrt{g_{12} \kappa_2'}}{i \Delta_2 + \kappa_2/2 + i g_2 \beta_2}, \\
\beta_1 &= -\frac{ig_1 \alpha_1^* \beta_1^* + i \beta_1}{\omega_m + \gamma_1/2}, \\
\beta_2 &= -\frac{ig_2 \alpha_2 \beta_2^* + i \beta_2}{\omega_m + \gamma_2/2}.
\end{align*}
\]

Here, \( \Delta_j = \omega_{pl} - \omega_{j} \) \((j = 1, 2)\) represents the detuning between the optical mode and the driving field. \( \delta = \omega_r - \omega_p \) is the detuning between the probe laser and the control field. To solve these nonlinear equations, the linearization approach is introduced i.e. \( a_j \rightarrow a_j + \alpha_j \) and \( b_j \rightarrow \beta_j + b_j \) \((j = 1, 2)\). We divide the operators into steady-state value and the fluctuation parts as mentioned above. By substituting these ansatzs into equations (3)–(6), we can easily get the classical steady-state values

\[
\begin{align*}
\alpha_1 &= \frac{\sqrt{g_{11} \kappa_1'}}{i \Delta_1 + \kappa_1/2 + i g_1 \beta_1}, \\
\alpha_2 &= \frac{\sqrt{g_{12} \kappa_2'}}{i \Delta_2 + \kappa_2/2 + i g_2 \beta_2}, \\
\beta_1 &= -\frac{ig_1 \alpha_1^* \beta_1^* + i \beta_1}{\omega_m + \gamma_1/2}, \\
\beta_2 &= -\frac{ig_2 \alpha_2 \beta_2^* + i \beta_2}{\omega_m + \gamma_2/2}.
\end{align*}
\]

In the regime of strong driving, the photon number inside the cavities is more than one. Therefore the linearized equations of the fluctuation components are obtained as

\[
\begin{align*}
\frac{da_1}{dt} &= -i \Delta_1 a_1 - \frac{\kappa_1}{2} a_1 - i G_1 b_1 + \sqrt{g_{11} \kappa_1'} e^{-i \delta t}, \\
\frac{da_2}{dt} &= -i \Delta_2 a_2 - \frac{\kappa_2}{2} a_2 - i G_2 b_2^*, \\
\frac{db_1}{dt} &= -i \omega_m b_1 - \frac{\gamma_1}{2} b_1 - i G_1 \alpha_1 - i \beta_2, \\
\frac{db_2}{dt} &= -i \omega_m b_2 - \frac{\gamma_2}{2} b_2 - i G_2 \alpha_2 - i \beta_1.
\end{align*}
\]

\( G_j = g_j \alpha_j \) \((j = 1, 2)\) are the effective optomechanical coupling strengths. If we focus on the transmission rate of the probe field, the above equations can be solved to obtain the steady-state solution for the fluctuation component \( a_1 \) as follows:

\[
a_1 = \frac{-\sqrt{g_{11} \kappa_1'}}{PF + G_1^2},
\]

\( P \) and \( F \) are the transmission and reflection rate of the probe field, respectively.
where, $P = -[A + MP^2/(MB - G^2)]$, $A = i(\omega_m - \delta) + \gamma_1/2$, $B = i(\omega_m - \delta) + \gamma_2/2$, $F = -[i(\Delta_1 - \delta) + \kappa_1/2]$ and $M = \kappa_2/2 - i(\Delta_2 - \delta)$. According to the input–output formula, the output of the probe laser is expressed as $\eta_{\text{out}} = S - \sqrt{\eta_{\text{in}}}$. Without considering the higher-order sidebands, the normalized transmission coefficient is $\eta_{\text{tran}} = 1 - \sqrt{\eta_{\text{in}}}/S$ and the transmission rate of the probe field is $T = |\eta_{\text{tran}}|^2$. On the other hand, under slowly varying amplitude approximation and assuming that the evolution of mechanical modes is much slower than that of the optical modes, we can derive the PT-symmetric dynamical equations for the coupled mechanical modes as follows:

$$\begin{align*}
\frac{db_1}{dt} &= -\left(i\Omega_{m1} + \frac{\Gamma_1}{2}\right)b_1 - i\sigma b_2, \\
\frac{db_2}{dt} &= -\left(i\Omega_{m2} + \frac{\Gamma_2}{2}\right)b_2 - i\sigma b_1, \\
\end{align*}$$

where

$$\begin{align*}
\Gamma_1 &= \gamma_1 + \frac{4\kappa_1 G_1^2}{\kappa_1^2 + 4(\Delta_1 - \omega_m)^2}, \\
\Gamma_2 &= \gamma_2 + \frac{4\kappa_2 G_2^2}{\kappa_2^2 + 4(\omega_m + \Delta_2)^2}, \\
\Omega_{m1} &= \omega_m - \frac{4G_1^2(\Delta_1 - \omega_m)}{\kappa_1^2 + 4(\Delta_1 - \omega_m)^2}, \\
\Omega_{m2} &= \omega_m - \frac{4G_2^2(\Delta_2 + \omega_m)}{\kappa_2^2 + 4(\Delta_2 + \omega_m)^2}.
\end{align*}$$

Here $\Omega_m$ and $\Gamma_j$ represent the effective frequency and the effective decay rate of the $j$th mechanical mode ($j = 1, 2$), respectively. Further, the effective Hamiltonian of the coupled mechanical resonators can be expressed as

$$H_{\text{eff}} = \begin{bmatrix}
\Omega_{m1} - i\frac{\Gamma_1}{2} & J \\
J & \Omega_{m2} - i\frac{\Gamma_2}{2}
\end{bmatrix}.$$  

The eigenvalues of the above Hamiltonian $H_{\text{eff}}$ are given by

$$\Lambda_{\pm} = \omega_{\pm} + i\gamma_{\pm} = \frac{2(\Omega_{m1} + \Omega_{m2}) - i(\Gamma_1 + \Gamma_2) \pm \sqrt{\sigma}}{4},$$

where

$$\sigma = 16\Omega^2 - (\Gamma_1 - \Gamma_2)^2 + 4(\Omega_{m1} - \Omega_{m2})^2 - 4i(\Omega_{m1} - \Omega_{m2})(\Gamma_1 - \Gamma_2).$$

The real parts $\omega_{\pm}$ and the imaginary parts $\gamma_{\pm}$ of the eigenvalues correspond to the effective frequency and linewidth of the mechanical modes, respectively. The EP of the effective mechanical system appears only if $\sigma = 0$. If $\Omega_{m1} = \Omega_{m2}$, the EP condition is equivalent to $4J = \Gamma_1 - \Gamma_2$. At this specific point, the two eigenvalues of the system coalesce. After introducing the angular frequency $\Omega$ into the whole system rather than one of the cavities, due to the Sagnac effect, the effective frequency of the optical modes experience a Sagnac–Fizeau shift $[84]$, which is expressed as

$$\omega_{\pm} \rightarrow \omega_{\pm} - \Delta_{\text{Sagnac}},$$

where $n$ and $r$ are the refractive index and the cavity radius, respectively. $\lambda$ and $c$ are the wavelength and the speed of light in vacuum, respectively. If the rotation speed $v = \Omega r$ is smaller than the light speed, the relativistic effects can be ignored, and the dispersion term $dn/d\lambda (\sim 1\%$ for typical material $[85]$) becomes negligible. In this paper, the case of $\Omega > 0$ is corresponding to that the rotation direction is consistent with the direction of the cavity mode. According to Sagnac effect, the optical mode frequency should decrease by an amount of $|\Delta_{\text{Sagnac}}|$. On the contrary, if $\Omega < 0$, the resonant frequency is increased by an amount of $|\Delta_{\text{Sagnac}}|$. For our system, one can easily replace $\Delta_j$ by $\Delta_j - \Delta_{\text{Sagnac}}$, and the remaining calculation process is similar.
3. Mechanical PT-symmetric system and EP

Firstly, we focus on the case of $\Omega = 0$. According to equations (22)–(24), the proposed optomechanical system can be regarded as an effective mechanical PT-symmetric system. Further, for some specific parameter values, the mechanical system can reach EP. The two eigenvalues of the system coalesce at this specific point. Figure 2 shows the real and imaginary parts of the eigenvalues vs the mechanical–mechanical coupling strength $J$ before (solid lines) and after (dashed lines) the perturbation introduced by rotation, where the corresponding optical frequency shift $\Delta_{\text{Sagnac}} = 2 \times 10^{-4} \omega_m$. The other parameters we used here are $\kappa_1 = \kappa_2 = 6 \text{ MHz}$, $\omega_m1 = \omega_m2 = \omega_m = 600 \text{ MHz}$, $\gamma_1 = \gamma_2 = 70 \text{ kHz}$, $G_1 = 3 \text{ MHz}$, $n = 1.44$, $r = 9 \text{ nm}$, $\omega_3 = 2 \times 10^{14} \text{ Hz}$, and $\lambda = 1550 \text{ nm}$. The Q factor of the optical mode and the mechanical mode we used in this paper are $3.3 \times 10^7$ and $8.6 \times 10^3$. These parameters are feasible in experiments [5, 86–88]. Considering the experimental generality, we choose $n_{\text{ex}} = n_i/2$ (for $i = 1, 2$), which indicates that the coupling between the cavity and fiber taper is in the critical coupling regime. As the cavity mode $a_2$ is in the blue sideband, it is necessary to mention that the effective coupling strength $G_2$ cannot be too strong. In our system, $G_2$ satisfies $c_2 = 4G_2^2/(\kappa_2\gamma_2) = 0.9999$ to avoid any instability.

It is evident in figure 2 that one can modulate the effective mechanical PT-symmetric system, which exhibits a transition from PT-symmetric region to PT-symmetric broken region, by modifying the system parameters such as the effective optomechanical coupling strength $G_1$ and mechanical–mechanical coupling strength $J$. Here, the two regions with PT-symmetry and PT-symmetry broken are indicated with different colors. In the PT-symmetric region, the imaginary parts of the eigenvalues are equal, but the real parts are different, which implies that the two eigenvalues of the system have the same linewidth but different frequencies. On the contrary, the eigenvalues have the same frequency with different linewidths when the system is in the PT-symmetric broken region. The transition point between these two regions is $J = 1.52 \text{ MHz}$. This indicates that the effective mechanical system exhibits an EP.

For our effective mechanical system, the two eigenvalues coalesce at EP and any perturbation can induce the splitting of eigenvalues. For a conventional gyroscope, the response of the system is proportional to the strength of the rotation. However, the situation is entirely different in our system. For e.g., figure 2 shows that for the rotation-induced optical frequency shift $\Delta_{\text{Sagnac}} = 2 \times 10^{-4} \omega_m$, the real and imaginary parts of the eigenvalues are changed with the variation in the magnitude of $J$ before (solid lines) and after (dashed lines) introducing the rotation. According to equation (22) the real and imaginary parts of the eigenvalues correspond to the frequency and linewidth of the mechanical modes, respectively. In figure 2(a), the frequency of the mechanical modes exhibit a transition from degenerate regime to non-degenerate one after introducing the perturbation at EP. Figure 2(c) indicates that the gap between the solid and dashed lines is maximum at EP for the same perturbation strength and similar conclusion is obtained for the linewidth of the mechanical mode in figure 2(d). It should be noted that the frequencies of mechanical modes move toward the opposite directions, which validates our notion of utilizing the EP in the gyroscope. Another interesting feature observed at EP is the damping of the degenerate mechanical modes, as depicted by solid lines in figure 2(b). The introduction of rotation into the system lifts this degeneracy, which can be confirmed from the difference between the solid and dashed lines. The imaginary part of the eigenvalues in equation (23) diverge when introducing rotation into the whole system, which implies a limit over the maximum detectable angular frequency to avoid any instability. Considering the giant enhancement near EP in the weak perturbation regime, the proposed gyroscope scheme based on the mechanical PT-symmetric system aims to detect small rotation frequency. Compared to the gap between the real parts, the linewidth of the mechanical modes vary in a similar way, but they change along the opposite directions. This is attributed to the fact that one of the mechanical modes experiences gain whereas the other one experiences loss in this process. This proves that the proposed system is actually a PT-symmetric system, and either the mechanical gain or the loss can be symmetrically controlled. By further increasing the angular frequency $\Omega$, the linewidth of the mechanical modes should push the system away from the EP.

To characterize the effective mechanical system under different rotation speeds, the trajectories of the eigenvalues in the perturbation plane are shown in figure 3. For different perturbation strengths, the evolution behaviors of the eigenvalues are similar except the distance between the associated curves (indicated by the same color), which increases with the increase in the rotation speed. The arrows indicate the direction of increasing $J$, and the EP of the system is located at the center of the plane (marked by the green dot). As expected, for the same perturbation strength (lines of same color in the figure), the distance from the curves to the unperturbed system (shown by blue lines) is maximum at $J = 1.52 \text{ MHz}$ under the same parameter values.
Figure 2. (a) and (b) Real and imaginary parts of the eigenvalues vs the mechanical–mechanical coupling strength $J$ before (solid lines) and after (dashed lines) the perturbation introduced by rotation, where the corresponding optical frequency shift $\Delta_{\text{Sagnac}} = 2 \times 10^{-4} \omega_m$. The two eigenvalues of the effective Hamiltonian are marked with red and blue lines. (c) and (d) Gap between the solid and dashed lines shown in (a) and (b). The regions with PT-symmetry and PT-symmetry broken are indicated with different colors, i.e., white and green. The other parameters used in this system are $\kappa_1 = \kappa_2 = 6$ MHz, $\omega_{\text{a}} = \omega_{\text{p}} = \omega_m = 600$ MHz, $\gamma_1 = \gamma_2 = 70$ kHz, $G_1 = 3$ MHz, $n = 1.44$, $r = 9$ mm, $\omega_m = 2 \times 10^{14}$ Hz, $\lambda = 1550$ nm, and $c_2 = 4G_1^2/(\kappa_2 \gamma_2) = 0.9999$.

Figure 3. Trajectories of the eigenvalues in the perturbation plane for different rotation speeds characterized by different $\Delta_{\text{Sagnac}}$. For the perturbed systems, the distance between the intersections of the associated curves (depicted by the same color) increases as the rotation speed increases. The arrows indicate the evolution of the eigenvalues with the increase in $J$. Further, the black dashed line represents $4J = \Gamma_1 - \Gamma_2$ condition. Note that the EP is located at the center of the plane and is marked by the green dot. The parameters are the same as that in figure 2.

4. Enhancement of sensitivity at EP

A typical approach for detection is to observe the related mode response, usually the frequency shift or the mode splitting, before and after the introduction of perturbation. If one intends to detect the rotation speed or nanoparticles, it is essential to measure the frequency shift or the variation in the linewidth of the optical modes. In parallel, the shift in the mechanical modes is a crucial parameter for mass and displacement sensing schemes. In our study, we observed that frequency of the mechanical modes shifts under the rotation frame even though it only induces an optical frequency shift. The motivation of the proposed model is to utilize the advantage of the fact that the linewidth of mechanical modes is much narrower than that of the optical modes. In this paper, we pay our attention to the frequency shift that the mechanical modes experience as it is the monitored signal and we have built the connection between the frequency shift and the rotation frequency in the weak perturbation regime as shown in the discussion below. In contrast to EP, some special microcavities such as microtoroids support modes with degenerate eigenvalues, but the associated eigenvectors can always be chosen to be orthogonal to each other, also known as DP.
traditional cavity-based gyroscope schemes, the DP is utilized and the perturbation-induced frequency shift in response to the perturbation strength $\epsilon$ is proportional to the perturbation strength $\epsilon^{1/2}$. Consequently, it is easy to infer that the sensitivity is considerably enhanced by using EP rather than DP for sufficiently small perturbations. This enhancement is attributed to the intrinsic properties of EP and does not depend on the type of cavity, materials, or the perturbation properties. As mentioned above, the frequency shift in the mechanical modes for our system, i.e., $|\Re \Delta \Lambda| = |\Re \Lambda^+ - \Re \Lambda^-|$, is proportional to the square root of perturbation strength near the EP, which implies that the frequency shift caused by the angular frequency can be fitted using

$$|\Re \Delta \Lambda| = \zeta \Omega^{1/2}. \tag{27}$$

Here, $\zeta$ is the fitting coefficient. To validate the superiority of utilizing EP and mechanical modes, the optical shift is compared with the mechanical shift in figure 4(a) as a function of the angular frequency $\Omega$. Further, the EP and DP sensitivities are compared in figure 4(b). For the same $\Omega$, the mechanical shift (red line) is always higher than the optical shift (blue line). It is important to note the difference between the blue and red $y$-axis labels in figure 4(a). The red axis indicates the difference between the real part of the two eigenvalues at EP, while the blue axis indicates the product of the optical shift at EP and 50, which suggests that the mechanical response is 50 times stronger than the optical response when the angular frequency is below 0.1 Hz under the same parameter values. The performance of the mechanical modes is even better if the angular frequency is sufficiently small (red curve). This is expected due to the properties of EP in our system. The open circles represent the fitting trend according to equation (27) with $\zeta = 1.17 \times 10^5 \text{ Hz}^{1/2}$, which is consistent with the mechanical shift in our model. Therefore, it can be inferred that the mechanical shift in response to the external angular frequency $\Omega$ obeys the square root behavior, which is a direct consequence of utilizing EP rather than DP in our scheme.

It is clear that the sensitivity of rotation sensing for the optomechanical system operating at EP is considerably enhanced. The corresponding enhancement factor quantitatively characterizes the superior performance of the sensor operating at EP over that operating at DP. The enhancement factor can be defined as

$$\eta \equiv \frac{|\Re \Delta \Lambda|}{\Delta_{\text{Sagnac}}}.$$

For conventional sensors utilizing DP rather than EP, where the eigenvalues are degenerate while the corresponding eigenvectors are orthogonal, the response is proportional to the perturbation strength and the enhancement factor is always 1. However, for the sensors based on EP, in the absence of rotation, the system is in the vicinity of EP. In the presence of rotation, the eigenvalues of the effective mechanical system move away from the EP, which facilitates the detection of applied external perturbation at EP. Figure 4(b) illustrates that the EP sensitivity is much better than the DP sensitivity for weak perturbation. As the angular frequency decreases, the performance of the EP-based gyroscope becomes better, while that of the DP-based sensor remains the same irrespective of the value of angular frequency. This validates the efficiency of the sensors based on EP in weak perturbation regime. As shown in figure 4(b), EP enhances the sensitivity of gyroscope by more than 20 times when the angular frequency is below 1 Hz. If the perturbation strength is large enough, the response of the EP-based sensor is no longer proportional to $\epsilon^{1/2}$ and approaches linear limit. Consequently, the enhancement factor approaches the limit $\eta \sim 1$.

It should be noted that the one of the mechanical modes becomes narrower and the other one becomes wider due to the pumping of two optical modes with different colors. The linewidth of the
Figure 5. (a)–(c) Transmission of the cavity modes under different parameters with rotation angular frequency $|\Omega| = 6$ kHz. (a) Resonant frequency of the cavity modes can be modulated by both $\Omega$ and the rotation direction. (b) For $\Omega = 0$, OMIT occurs when the effective optomechanical coupling strength $G_1$ increases. (c) A dip is observed within the OMIT window when $J$ reaches to 1.52 MHz. (d)–(f) Transmission rate as a function of $\Omega/\Omega_0$ and $\delta/\omega_m$ with $\Omega_0 = 1.2$ kHz. The other parameters are the same as that in figure 2.

5. Transmission analysis

From the perspective of experiments, the frequency shift in the mechanical modes can alter the optical response. Therefore, for our system demonstrated in figure 1, it is necessary to investigate the transmission spectra under different values of the mechanical–mechanical coupling strength $J$, the effective optomechanical coupling strength $G_1$, and the angular frequency $\Omega$. Figure 5 illustrates the transmission $T$ as a function of the detuning between the probe laser and the control field $\delta$ in the unit of mechanical frequency $\omega_m$.

First, we consider the simplest case in which $J = 0$ and $G_1$ is extremely small. As expected, the transmission exhibits a Lorenz curve, which is shown in figure 5(a). The resonant frequency of the cavity mode can be modulated by both the magnitude of the angular frequency $\Omega$ and the rotation direction. If $\Omega > 0$, the rotation direction is consistent with the direction of the cavity mode. According to Sagnac effect, the optical mode frequency should decrease by an amount of $|\Delta_{\text{sagnac}}|$. As the rotation speed increases, the frequency shift also increases. On the contrary, if $\Omega < 0$, the resonant frequency is increased by an amount of $|\Delta_{\text{sagnac}}|$. Based on the case (a), the magnitude of $G_1$ is increased to 3 MHz. Optomechanical induced transparency (OMIT) is observed due to the interference between different pathways, as shown by the orange line in figure 5(b). In the first pathway, the probe photons excite the cavity mode $a_1$ and couple to the output port, and in the other one, the photons generated by the sideband transition through the optomechanical interaction in $a_1$ are coupled out of the cavity. When the angular frequency is small, there are still two dips in the transmission spectrum. As the angular frequency increases, the width of the transparency window becomes larger, and the depth of one of the dips reduces. Note that the frequency shift of the other dip is exactly the same as that shown in figure 5(a) under the same rotation speed.

Furthermore, when the mechanical–mechanical coupling strength $J$ reaches to 1.52 MHz, a narrow dip occurs within the transparency window, as shown in figure 5(c). Similarly, as the rotation speed $\Omega$ increases, the transparency window becomes wider and the two dips in the spectrum become shallower. Meanwhile, the other dip maintains the same frequency shift under the same value of $\Omega$. To investigate the transmission
rate of the system under different rotation frequencies, we plot the transmission rate as a function of \( \Omega / \Omega_0 \) and \( \delta / \omega_m \) in figures 5(d)–(f). The three dashed lines in figures 5(d)–(f) correspond to the three curves in figures 5(a)–(c) according to the colors. In expectation, the shift of the optical mode is proportional to the rotation frequency \( \Omega \) as depicted in figure 5(d). As indicated in figure 5(e), as the rotation frequency increases, one of the dips becomes narrower and narrower till OMIT vanishes and the transmission spectrum again appears as a Lorenz curve when \( \Omega \) is extremely high. When the angular frequency becomes sufficiently high, the frequency shift of the optical mode becomes very large, which destroys the destructive interference effect. Therefore, it can be inferred in figure 5(f) that if the angular frequency is extremely high, the dip within the OMIT window and OMIT vanish. Notably, the optical response becomes recognizable if the magnitude of angular frequency is relatively high because the linewidth of optical mode is large. In comparison to the optical mode, the mechanical mode exhibits a smaller linewidth, and we have utilized this unique property of mechanical mode in this study.

6. Conclusion

We theoretically proposed an effective mechanical PT-symmetric scheme that consisted of two optical modes driven by red-detuned and blue-detuned pump lasers, and two mechanical modes were simultaneously coupled to the corresponding cavity modes and to each other. As the two cavities were symmetrically driven, either mechanical gain or loss could be controlled by tuning the system parameters such as the effective optomechanical coupling strength \( G_1 \) and the coupling strength between the two mechanical modes \( J \). In the absence of rotation, the system could be prepared at EP with appropriate parameters. In the presence of rotation, the eigenvalues of the system were driven apart from the EP and they were shifted toward opposite directions. We compared the optical frequency response with the mechanical frequency response to observe that the mechanical shift was 50 times higher than the optical shift when the angular frequency was sufficiently small. Furthermore, the linewidth of mechanical modes was narrower than that of optical modes, which boosted the sensitivity of the gyroscope to a new level. For traditional gyroscopes operating at DP, the response was proportional to the perturbation strength, and the enhancement factor was 1 irrespective of the value of perturbation strength. However, the sensitivity of the gyroscopes operating at EP depended on the angular frequency. As the rotation speed decreased, the sensitivity of the gyroscope was enhanced. As the perturbation strength increased, the enhancement factor approached the limit \( \eta \sim 1 \), which validated the efficacy of EP-based sensors for detecting weak perturbations such as nanoparticles, temperature variation, and angular frequency. The spectral features will be smeared by statistical averaging effect in ensembles of resonator pairs, which makes the sensitivity of sensors based on EP limited by noise and fluctuations [89–91]. Owing to the rapid progress in nanofabrication and experimental techniques, EP-based sensors were demonstrated in experiments [3, 28, 47, 49], which proves the enhancement of EP in practical applications. Overall, our effective mechanical scheme combines the advantages of the narrow linewidth of mechanical modes and the sensitivity enhancement at EP. Moreover, the parameter values were carefully chosen to ensure experimental feasibility. Therefore, our work provides useful insights on EPs and opens a new route toward utilizing them in optomechanical sensors.

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