Non-Canonic Representation of the Random Process in Tasks of Simulating Seismic Impacts for Calculating Buildings and Structures

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Abstract. The seismic impact model is analyzed as a combination of simple functions with random parameters. A random process simulating seismic impact is defined by a simple non-canonical representation. A method for generating artificial accelerograms of local earthquakes is proposed, based on the representation of the correlation function as the sum of cosine-exponential terms. The approximation parameters of the correlation function were determined earlier based on the least squares method using the MATLAB package using nonlinear approximation methods. As a quality criterion, the initial and generated spectral curves coincide. Samples of random process realizations from 20 to 10,000 realizations (artificial accelerograms) are used. Algorithms are simply implemented in modern computer mathematics systems MATLAB, SCILAB. The mathematical model of the accelerogram of the earthquake in El Centro, 1940 is also considered. The area of application of the algorithms is calculations using Eurocode 8 and ISO 2394, ISO 8930 standards, including using probabilistic methods, determining the reliability of buildings and structures based on the statistical test method.

1. Introduction
For the tasks of assessing earthquake resistance and determining the seismic risk of buildings and structures, the universally applied spectral calculation method is insufficient. Therefore, seismic stimulation models again become attractive based on the application of random function theory methods [1-5]. Instrumental records of earthquakes are being processed, the results of which are used in constructing impact models [6,7].

Spectral-temporal models of seismic effects are very effective especially when regional features are taken into account, taking into account local soil conditions and local centers of earthquakes. All kinds of wave and spectral models, methods of using real accelerograms are used [8-16].

The most common and fruitful is the idea of seismic impact by stationary or non-stationary random processes (Bolotin V.V., Barshstein M.F.). Realizations of random processes can be constructed on the basis of the representation of random functions in the form of deterministic summations of random variables. Representations of random functions in the form of generalized Fourier series, Karunen or Kotelnikov series, canonical or non-canonical expansions are known.

The canonical decomposition of Pugacheva V.S. [5], for example, is exact in the case of an infinite number of members of the series, which is practically difficult to implement, since this requires the
presence of an infinitely large number of random variables \( v_i \) and deterministic functions \( \psi_i(t), i = 1,2,\ldots,\infty \). In order to overcome these difficulties, Chernetsky V.I. provides a non-canonical expansion of random functions in the form [1]

\[
X(t) \approx \Omega(t,v_1,v_2,\ldots,v_n),
\]

(1)

where \( \Omega(t,v_1,v_2,\ldots,v_n) \) — some deterministic function of time and independent random variables \( v_1, v_2,\ldots,v_n \).

The fulfillment of relation (1) can be replaced by the equalities of the moment functions of the left and right sides of it. If we confine ourselves to the framework of the correlation theory, then the equality of the moment functions up to the second order inclusive is sufficient. Functions \( \Omega(t,v_1,v_2,\ldots,v_n) \) can be selected from the possibility of decomposing any process into a series of harmonics whose amplitudes and frequencies are independent random functions of time.

The following theorem was proved in [1]: a random function \( X(t) \) can be represented absolutely exactly within the framework of the correlation theory in the form:

\[
X(t) = m_s(t) + \lambda_1 \sin \omega t + \lambda_2 \cos \omega t,
\]

(2)

if the following conditions are met:
1. The random function \( X(t) - m_s(t) \) is stationary.
2. \( M[X(t)]=m_s(t), \quad K_s(\tau) = \sigma_s^2 r_s(\tau), \)

(3)

where \( m(t) \) and \( r_s(\tau) \) — are known functions, \( \sigma_s \) — root mean square (rms) value.
3. Random variables \( \lambda_1, \lambda_2, \omega \) are independent.
4. The laws of probability distribution of random variables \( \lambda_1 \) and \( \lambda_2 \) are arbitrary; the distribution density of the random variables is determined by the formula:

\[
f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} r_s(\tau)e^{-ior} d\tau
\]

(4)

The formulated theorem is satisfied by the representation of stationary random functions \( \lambda \) in the form (2) providing absolute accuracy within the framework of the correlation theory, i.e. the identical coincidence of the first two points: mathematical expectations and correlation functions \( \lambda_1, \lambda_2, \omega \).

Thus, the non-canonical representation of the random process (2) is very economical in terms of the organization of the computing process. A simplicity of representation allows it to be used for various tasks of the theory of earthquake resistance and earthquake engineering.

The purpose of this work is to use representation (2) for modeling seismic impact.

2. The mathematical model of seismic impact

In order to assess the quality of the indicated method for modeling seismic effects, spectral curves were determined for each of the N realizations of the random process. Then, the averaged spectral curve was determined, which was then compared with the initial curve.
Consider the results of applying the seismic impact model in the form of a random function (1). We take the normalized correlation function of the random process in the cosine-exponential form:

\[ r_\nu(\tau) = e^{-\alpha |\tau|} \cos \nu \tau, \]

where \( \alpha, \nu \) – parameters of correlation function.

\( m_i(t) = 0 \) is set. The values of random variables \( \lambda_1 \) and \( \lambda_2 \) are distributed according to the normal law.

The values of \( \omega \) from (4) are determined by solving a nonlinear equation below

\[ \frac{1}{2} + \frac{1}{2\pi} \left[ \arctg \frac{\omega + \nu}{\alpha} + \arctg \frac{\omega - \nu}{\alpha} \right] = \frac{z + 1}{2}, \]

where random numbers have a uniform distribution over the interval \([-1, 1]\).

The following values were determined

\[ \beta = \left| \frac{\dot{\mathbf{X}} + \dot{\mathbf{X}}_0}{\mathbf{X}_0,\text{max}} \right|, \]

where \( \mathbf{X} \) – displacements of a single-mass linear oscillator \( \dot{\mathbf{X}}_0 \) – accelerogram of an earthquake (experimental record).

We note here that for N implementations it is necessary to generate 3N random numbers, where only N numbers are calculated by solving the nonlinear equation (5). However, we note that using the MATLAB computer mathematics system, a program in a high-level language that implements the above algorithms is compiled quite simply.

We take the expression for the correlation function in the form of segments of a series [2, 3]:

\[ K(\tau) = \sum_{i=1}^{N} A_i e^{-\alpha_i \tau} \cos(\nu_i \tau), \]

where \( A_i, \alpha_i, \nu_i \) – defined parameters, \( N \) – number of row entities.

There is a two-component earthquake record recorded by Kurmenta station at a distance of 35 km from the earthquake source. The maximum intensity in the epicenter is magnitude 8. Step of digitization is 0.008 s. The indicated record can be used to develop a model of seismic impact for the Almaty region [6]. This record is quite interesting - it contains two closely spaced peaks at the spectral density or reaction spectrum (N-S component with an acceleration maximum of 699 cm/s²).

The envelope of a stationary random process is adopted in the form of a fractional rational expression as in Aptikaev F.F. [7].

\[ A = \frac{3td}{9t^2 - 9td + 4d^2}. \]

where \( d \) – the effective duration of the seismic impact, equal to the duration of the oscillations with an amplitude equal to half the maximum value,

\( A_{\text{max}} \) – maximum value of acceleration.

In the case of a stationary random process, multiplication by a determinate envelope is not performed.
The approximation parameters of the correlation function (6) were previously determined based on the method of the least squares using the Curve Fitting Toolbox MATLAB package.

3. Main results and discussion

Table 1 shows the calculation results performed taking into account the vibration decrement $\delta = 0.3$. The maximum value of the spectral coefficient $\beta$, calculated from the real accelerogram of the Baysorun earthquake is 4.69. For the case $N = 1$, the discrepancy in the values of the spectral coefficient is up to 60%, which indicates the unsatisfactory approximation of the correlation function of the real accelerogram. Figure 1 shows the spectral curves obtained by averaging the calculation results for a different number of realizations of a random process for $N = 2$ (non-stationary case), and in Figure 2 - a stationary case. The spectral curve of the indicated accelerogram is also shown here. Compliance is very satisfactory. Even the shape of the spectral curve is maintained in the interval of periods of 0.2-0.3 s.

| Model          | Number of implementations | $N = 1$ (4.69) | $N = 2$ | $N = 3$ |
|----------------|---------------------------|---------------|--------|--------|
| Non-stationary | 20                        | 5.82          | 4.26   | 3.81   |
|                | 50                        | 6.16          | 4.27   | 4.96   |
|                | 100                       | 6.29          | 4.26   | 5.06   |
|                | 200                       | 5.72          | 4.00   | 4.94   |
|                | 500                       | 6.95          | 3.95   | 4.83   |
|                | 1000                      | 6.11          | 4.03   | 4.79   |
|                | 5000                      | -             | 3.93   | 4.91   |
|                | 20                        | 6.85          | 4.69   | 4.89   |
|                | 50                        | 7.19          | 4.73   | 4.96   |
|                | 100                       | 7.16          | 4.43   | 5.32   |
|                | 200                       | 6.63          | 4.36   | 5.14   |
|                | 500                       | 5.88          | 4.40   | 5.04   |
|                | 1000                      | 6.96          | 4.30   | 5.05   |
|                | 5000                      | -             | 4.38   | 5.11   |

For cases $N = 3$ the values of the spectral coefficient are also satisfactory, but the shape of the spectral curve is maintained with significant errors. The stationary model of seismic impact describes the seismic impact much better than non-stationary.

Thus, it is recommended that the implementation of the random process is calculated by the formula on the $K$th implementation for $N = 2$ in (6)

$$X(t) = \lambda_1^{(K)} \sin(\omega_1^{(K)} t) + \lambda_2^{(K)} \cos(\omega_1^{(K)} t) + \lambda_1^{(K)} \sin(\omega_2^{(K)} t) + \lambda_2^{(K)} \cos(\omega_2^{(K)} t).$$

(7)

Table 2. Maximum values of the spectral coefficient.

| Model          | Number of implementations | $N = 2$ | $N = 3$ |
|----------------|---------------------------|--------|--------|
| Non-stationary | 500                       | 4.16   | 4.14   |
|                | 5000                      | 4.00   | 4.21   |
|                | 10000                     | 4.04   | 4.21   |
|                | 500                       | 4.50   | 4.42   |
| Stationary     | 5000                      | 4.51   | 4.35   |
|                | 10000                     | 4.56   |        |
**Figure 1.** Spectral curves obtained for the non-stationary impact model with a different number of implementations.

**Figure 2.** Spectral curves obtained for the stationary impact model with a different number of implementations.

One of the acceleograms recommended for calculation according to Eurocode 8 is the instrumental recording of the El Centro earthquake, 1940 (component S00E), which is indicative of strong movements recorded near the center of the earthquake. The earthquake has a very complex spectral composition. Effective exposure time for use in the expression of the envelope $d = 10.18$ sec. The digitization step is 0.02 sec. The approximation parameters of the correlation function (6) are determined on the basis of the least squares method using the licensed Curve Fitting Toolbox.
MATLAB package. In Fig. 3, the correlation function and its approximation at N = 8 obtained by minimizing the sum of the residual moduli.

![Graph showing the correlation function and its approximation at N = 8.](image)

**Figure 3.** Values of the correlation function and approximation at N = 8.

**Table 3.** Maximum values of the spectral coefficient.

| Model       | Number of implementations | N = 4 | N = 6 | N = 8 |
|-------------|---------------------------|-------|-------|-------|
| Non-stationary | 50                        | 2.15  | 2.49  | 2.64  |
|             | 500                       | 2.05  | 2.57  | 2.68  |
| Stationary  | 50                        | 2.65  | 2.42  | 2.68  |
|             | 500                       | 2.53  | 2.09  | 2.76  |

Table 3 presents the calculation results. The accuracy of determining the spectral curves for N = 4 is insufficient. Figure 4 compares the “exact” spectral curves and those obtained using stationary and non-stationary models at N = 8. Correspondence is very satisfactory both in amplitude and spectral characteristics of the initial effect.
Figure 4. Spectral curves of a real accelerogram of an earthquake in El Centro and artificial models at N = 8 with 500 realizations.

4. Conclusion
1. The non-canonical representation of the random process (2), (7) is an acceptable alternative to the common methods of modeling seismic effects based on, for example, canonical spectral decompositions.

2. The random function (7) models the real accelerogram of the Baysorun earthquake well according to the criterion of correspondence between the initial and the generated spectral curves. For the case N = 1, the approximation of the effect is unsatisfactory. For the case N = 2, both the shape of the reference spectral curve and the maximum values are well approximated. The stationary model of seismic impact describes the impact much better than non-stationary.

3. For the case of a real accelerogram of an earthquake in El Centro, 1940 at N = 8, satisfactory accuracy was achieved with 500 implementations. A non-stationary model is preferable here.

4. The implementation of the non-canonical spectral representation of a random process is very beneficial from a computational point of view and very convenient for use, especially using computer mathematics systems MATLAB, SCILAB. We considered the methods of numerical integration of equations implemented in the above packages (various versions of the Runge-Kutta method, Adams forecast-correction, combined methods). In terms of the cost of computer time, it is preferable to use the five-stage Runge-Kutta method of the fourth order of accuracy.

5. Taking into account the simple form (2) of the expression for the non-canonical representation of a random process for solving the equations of dynamics of one-, two-mass nonlinear systems, one can use the whole well-developed field of nonlinear mechanics. The probabilistic characteristics of the reaction parameters can then be determined taking into account the form of the correlation function (6) by transforming random variables. It is possible to recommend the use of a non-canonical representation of seismic effects for the calculation of buildings with seismic isolation systems [17,18,19].

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