Numerical Flexibility Determination Method of Stress Intensity Factor for Concrete

Yongjin Wu1,a, Hu Chen2 and Xiangdong Wang3
1 Key Laboratory of Geotechnical Mechanics and Engineering of Ministry of Water Resources, Yangtze River Scientific Research Institute, Wuhan 430010, China
2 College of Civil and Transportation Engineering, Hohai University, Nanjing 210098, China
3 College of Mechanics and Materials, Hohai University, Nanjing 210098, China
Email: a wuyj@hhu.edu.cn

Abstract. Flexibility method is a commonly used method to determine fracture toughness. In the experiment, it is necessary to prepare specimens with different crack lengths and other exactly same conditions, and carry out a large number of repeated experimental works. Given the above problems, this paper develops a flexibility determination method based on numerical simulation method to calculate the stress intensity factor and fracture toughness of concrete specimens. The results of the test example show that the use of the numerical simulation experiment method is practically feasible and effective to obtain the relation curve between the flexibility C and the crack length a of the concrete specimens and further get the stress strength factor and the fracture toughness under the ultimate load.

1. Introduction
Concrete is a kind of quasi-brittle material with low tensile properties. Under the external factors such as load, temperature change and chemical action, the occurrence of cracks in a concrete structure is unavoidable. Under the action of certain external factors (load, temperature change and so on), the internal defects of the concrete structure will produce a high degree of stress concentration, and gradually expand the development to form micro-cracks in the concrete body. Besides, the joint surfaces between phases will be separated under the influence of external factors to form interface cracks and develop into micro-cracks. Micro-cracks continue to expand, converge and pass through to for macro-cracks. The macro-cracks continue to expand, and ultimately may lead to complete failure of concrete structure [1-2]. Crack is the main reaction of aging and lesion of the concrete structure, which is very harmful to its safety. Both the failure modes of complex structures such as high arch dam and engineering fracture problems of the conventional buildings such as piers, foundations, and chimneys are urgent and challenging problems to be solved in design. In the concrete structures there are many examples of cracks that affect the operation and influence the benefits.

At present, the stability analysis of cracks is mainly from two aspects. Firstly, according to the influencing factors, establish a mathematical monitoring model using the monitoring data and analyze the development trend of cracks. Secondly, calculate the stress strength factor at the crack tip or other fracture parameters using the fracture mechanics principles and methods, and determine whether the cracks will expand according to the corresponding fracture criteria [3]. In determining whether the cracks will expand, the fracture criteria of crack-tip in the structure criteria is essential. The stress intensity factor K of the fracture parameters is a useful parameter to characterize the strength of the stress field near the crack tip. The methods of determining the stress intensity factor K include the
analytical method, numerical method, and experimental method. Flexibility method is commonly used in the experiment to determine the fracture toughness. The specimen with the different crack lengths and other exactly same conditions should be prepared in the experimental process, and a large number of repeated experimental works should be carried out. Given the above problems, this paper develops a flexibility determination method based on numerical simulation method to calculate the stress intensity factor and fracture toughness of concrete specimens.

2. Flexibility Expression of Stress Intensity Factor
Flexibility is the amount of displacement caused by the unit force, only related to the structural geometry, \( E \) and \( \nu \). The expression of flexibility is:

\[
C = \frac{\Delta}{P}. \tag{1}
\]

The energy release rate \( G \) is an important index of fracture mechanics related to the external load and the structure type (including the length, shape, position and structural geometries of the crack). The dimension is \([\text{force}] \times [\text{length}]^{-1}\), which can be regarded as the necessary force of unit length for crack propagation. There is a particular relationship between the stress intensity factor \( K \) and the energy release rate \( G \), assuming that the crack expands a certain length \( \Delta a \) along its propagation line, calculate the stress relaxation work of the crack tip, we can get the relationship between the energy release rate and stress intensity factor \([4-5]\):

\[
G = \frac{1}{E'}(K_1^2 + K_{II}^2) + \frac{1}{2\mu}K_{III}^2. \tag{2}
\]

Where \( E \) is the elastic modulus, \( E' = E \) during the plane stress state, \( E' = E/(1 - \nu^2) \) during the plane strain state; \( \mu \) is the shear modulus.

The energy release rate is independent of the nature of the loading system. Whether it is constant displacement or constant load, \( G \) is equal to \( \frac{dU_e}{(B \cdot da)} \). Then the relationship between the energy release rate \( G \) and the flexibility \( C \) is deduced as follows:

\[
G = \frac{P^2}{2B} \frac{dC}{da}. \tag{3}
\]

Formula (3) is often used for the energy release rate of experimental determination. \( dc/da \) can be determined by experimental determination of flexibility of multiple specimens with different crack lengths and other exactly same conditions.

The stress intensity factor \( K \) of the specimen with certain geometric type and crack shapes can be obtained by combining formula (2) and formula (3), and furthermore, the ultimate load \( P_{\text{max}} \) obtained by the experiment can determine the fracture toughness \( K_{\text{ic}} \) of the specimen. Fracture toughness is a mechanical property parameter of the material and a toughness index in which the material resists the crack \([2]\).

3. Implementation Process of Numerical Flexibility Method
It is necessary to carry out a large number of repeated experimental works to obtain \( dc/da \) through the experimental method, including the preparation of specimens with different crack lengths and other exactly same conditions. Each specimen should be loaded to obtain \( P-\Delta \) relation curve, consuming a lot of time and materials. With the advancement of computer technology and numerical method, it is possible to carry out the numerical simulation of the fracture process of concrete materials. Under the approximate physical conditions of numerical simulation structure and actual structure, the numerical simulation method saves time, effort, without instrument limit, and may repeat and done a lot of experiments. It not only can simulate complex loading paths that are difficult to implement by many loading devices but also is immune to the effects of various factors in the experiment. Therefore, it is of importance to study the mechanical properties of concrete by numerical simulation method \([6-7]\).
The load - displacement curve of the concrete specimen under different crack lengths during fracture failure process is obtained by numerical simulation experiment method, and the relation curve between the flexibility C and the crack length a is established to obtain $dC/da$. The numerical simulation of this paper is carried out based on ANSYS. The solid65 element in ANSYS is a three-dimensional 8-node solid isoparametric element which is usually used to simulate concrete structure. It is characterized by plastic deformation, which can be cracked in three directions and crushed. The detailed process of the numerical implementation is as follows:

1. Establish the geometric model of the specimen in the ANSYS, the crack length of the model is a.

2. Define the element, material properties and divide the grid. The solid65 element is used as the element, and the failure criterion of William-Warnke five parameters is adopted. The followings should be defined: uniaxial tensile strength $f_t$, uniaxial compressive strength $f_c$, shear strength reduction coefficient of open and closed slip surface, two-way compression strength $f_{cb}$, and uniaxial compressive strength $f_1$ and biaxial compressive strength $f_2$ under a certain confining pressure.

3. Add the boundary condition and the external load to solve and extract the displacement $\Delta$ of the load point.

4. Change the crack length a, repeat steps (1) ~ (3).

5. Establish the relation curve between the flexibility C and the crack length a, and get $dC/da$ through derivation.

6. Calculate the stress intensity factor $K_I$, for example complete type I cracks:

$$K_I^2 = \frac{E}{2B^2} \frac{dC}{da}.$$  \hspace{1cm} (4)

4. Applications

4.1 Complete Type I Crack Problem

The geometry of the three-point bending beam specimen and loading method is shown in Fig.1. As shown in Fig.1, there will be only normal stress on both sides of the crack including the crack surface, so it's a complete type I crack problem. The specimen size is $L \times h \times b=2100mm \times 500mm \times 200mm$, $S=2000mm$, span ratio $s/h=4$. Elastic modulus $E=30GPa$, Poisson's ratio $\nu=0.194$, compressive strength $f_c=25.2MPa$, tensile strength $f_t=2.37MPa$.

![Figure 1](image)

**Figure 1.** Three-point bend beam specimen.

The results obtained according to the method in Section 3.2 are shown in Fig.2. It is observed that the $C$-$a$ curve of the complete type I crack is gentle while the seam height ratio is less than 0.4, but the amplitude of change is significantly increased when the ratio is greater than 0.4. At this point, the specimen has been approaching the destruction state. This is consistent with the change of the formula $f(a)$ proposed by Chen Chi[8s]. The relationship between C and a of this three-point bending beam is obtained by quadratic polynomial fitting:

$$C = 2 \times 10^6 a^3 - 8 \times 10^{-7} a^2 + 2 \times 10^7 a - 6 \times 10^9.$$  \hspace{1cm} (5)

By deriving the formula (5), we can get $dC/da$. Substituting into formula (4), the value of the stress intensity factor $K_I$ can be obtained. The comparison results between numerical flexibility method and Chenchi’s formula are shown in Fig.3. It can be seen that the error is basically less than 15%.
4.2 Complete Type II Crack Problem

The geometry of the four-point shear specimen and loading method is shown in Fig.4. According to the illustrated loading method, there will be only shear stress on both sides of the crack including the crack surface, and the bending moment on the crack plane is zero. The specimen is a complete type II crack under this loading method. The specimen size is \( L \times h \times b = 500\text{mm} \times 100\text{mm} \times 100\text{mm} \), \( S = 200\text{mm} \), \( S' = 20\text{mm} \). Elastic modulus \( E = 30\text{GPa} \), Poisson's ratio \( \nu = 0.17 \), compressive strength \( f_c = 25.2\text{MPa} \), tensile strength \( f_t = 2.37\text{MPa} \).

The flexibility \( C \) corresponding to the crack length \( a \) obtained by numerical simulation is shown in Fig.5. Compared with complete type I crack, the \( C-a \) curve of the complete type II crack is quite gentle. This also proves that type I cracks are the most dangerous. The relationship between \( C \) and \( a \) of this four-point shear specimen is obtained by quadratic polynomial fitting:

\[
C = 0.0719a^4 - 0.0156a^3 + 0.0014a^2 - 6 \times 10^{-4} a^3 + 2 \times 10^{-5} a^2 - 2 \times 10^{-6} a + 2 \times 10^{-7}.
\]  (6)

By deriving the formula (6), we can get \( dC/da \). Then the value of the stress intensity factor \( K_{II} \) can be obtained. The comparison results between numerical flexibility method and the finite element displacement method are shown in Fig.6. It can be seen that under the same computing conditions the error is basically less than 10%.
5. Conclusions

The results of the test example show that the use of numerical simulation experiment method is a practically feasible and effective method to obtain the relation curve between the flexibility $C$ and the crack length $a$ of the concrete specimens, and further get the stress strength factor and the fracture toughness of the specimen under the ultimate load. Compared with the experimental determination method, it is not necessary to carry out a large number of repetitive experimental works, and compared with the traditional finite element method; it also can meet the precision needed for the engineering without dividing the grid of the crack tip region into small scale. But the concrete is a heterogeneous composite material, including coarse aggregate, fine aggregate, cement hydrate, pores, and cracks, etc. The numerical simulation of the loading failure process has certain limitations, and the accurate numerical simulation has room for further improvement.
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7. References
[1] Wang, D.: Fracture Mechanics (Volume 1). Harbin: Harbin Institute of Technology Press. (1989)
[2] Yu, X.Z., Qiao, C.X., Zhou, Q.L.: Rock and Concrete Fracture Mechanics. Changsha: Central South University of Technology Press (1988)
[3] Xu, S.L., Reinhardt H.W.: Crack extension resistance and fracture properties of quasi-brittle softening materials like concrete based on the complete process of fracture. Int. J. Fracture. 92, 71--99 (1998)
[4] Irwin, G.R.: Analysis of stresses and strains near the end of a crack traversing a plate. J. App. Mech. 24, 361--365 (1957)
[5] Orown E.: Fracture and strength of solids. Reports on Progression Physical, the Physical Society.12, 185--232 (1949)
[6] Deng, A.M., Cao L., Xu D.Y.: Studies on Size Effect of Concrete Fracture Toughness. J. Yangzhou University. 10, 63--66 (2007)
[7] Deng A.M.: Study on Fracture Criterion Series of Concrete Considering the Influence of Fracture Process Zone. Nanjing: Hohai University. (2007)
[8] Chen C.: Research on Metal Fracture. Beijing: Metallurgical Industry Press. (1978)