THE $\phi \rightarrow \gamma K^0 \bar{K}^0$ DECAY

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Abstract

The branching ratio of the $\phi$ meson to $\gamma K^0 \bar{K}^0$ is calculated to be $5 \times 10^{-8}$ in a scheme which takes into account the different isospin channels involved, $I = 0, 1$ with the resonant, $f_0(980), a_0(980)$, and non resonant contributions.

The study of the process $\phi \rightarrow \gamma K^0 \bar{K}^0$ is an interesting subject since it provides a background to the reaction $\phi \rightarrow K^0 \bar{K}^0$. This latter process has been proposed as a way to study CP violating decays to measure the small ratio $\epsilon'/\epsilon$ [1], but since this implies seeking for very small effects a $\text{BR}(\phi \rightarrow \gamma K^0 \bar{K}^0) \gtrsim 10^{-6}$ will limit the scope of these perspectives. There are several calculations of this quantity [2, 3, 4, 5]. In [6] it is estimated for a non resonant decay process without including the $f_0$ and $a_0$ resonances. The issue is revisited in [7].

Here, a different way to treat the scalar meson-meson sector, and its related $f_0(980)$ and $a_0(980)$ resonances, is proposed. For this we use a recent approach [8] to the S-wave meson-meson interaction for isospin 0 and 1 which reproduces the experimental data for those processes up to about 1.2 GeV and generates dynamically the $a_0$ and $f_0$ resonances. In this way, we will consider their interference and the energy dependence of their widths and coupling constants to the $K \bar{K}$ system. Furthermore, other possible contributions, non resonant, are also taken into account. The ideas and amplitudes exposed there were used in [9] for the $\gamma \gamma \rightarrow \pi \pi, K \bar{K}$ and $\pi^0 \eta$ processes and a good agreement with the experiment was obtained.

As in former works [1–5] we consider the process $\phi \rightarrow \gamma K^0 \bar{K}^0$ through an intermediate $K^+ K^-$ loop which couples strongly to the $\phi$ and the scalar resonances, see Fig.1.

For calculating this loop contributions we use the minimal coupling to make the interaction between the $\phi$ and the $K^+ K^-$ mesons gauge invariant, then we have

$$H_{\text{int}} = (eA_{\mu} + g_\phi \phi_{\mu}) i(K^+ \partial^\mu K^- - \partial^\mu K^+ K^-) - 2eg_\phi A^\mu \phi_{\mu} K^+ K^-$$ (1)

Where $g_\phi$ is the coupling constant between the $\phi$ and the $K^+ K^-$ system.

An essential ingredient to evaluate the loop in Fig.1 is the strong amplitude connecting $K^+ K^-$ with $K^0 \bar{K}^0$. As we said before we will use the amplitude calculated in [8]. This
implies the sum of an infinite series of diagrams which is represented in Fig.2 for the diagram of Fig.1a, and the analogue corresponding to Figs.1b,c.

This series gives rise to the needed corrections due to final state interactions and in fact, from the vertex connecting the $K^+K^-$ with the $K^0\bar{K}^0$, this series is the same one that in [8] gives rise to the S-wave strong amplitude $K^+K^- \rightarrow K^0\bar{K}^0$. In this approach the vertex between the loops correspond to the lowest order chiral perturbation theory $\chi PT$. Note that an analogous series before the loop with the emission of the photon is absorbed in the infinite series of diagrams contained in the $\phi$ resonance propagator.

For this consider the diagrams in Fig.1 but with the $O(p^2)$ $\chi PT$ amplitude connecting the kaons. This amplitude is given by

$$<K^0\bar{K}^0|t|K^+K^-> = \frac{1}{2}[t_{I=0} - t_{I=1}] = \frac{1}{4f^2}[s + \frac{4m_K^2 - \sum_i p_i^2}{3}]$$  \hspace{1cm} (2)

where $f$ is the pion decay constant, $f \approx 93$ MeV, $I$ refers to the isospin channel of the amplitude and the subindex $i$ runs from 1 to 4 and refers to any of the four kaons involved in the strong interaction. If the particle is on-shell then $p_i^2 = m_K^2$. In our case $p_{K^0}^2 = p_{\bar{K}^0}^2 = m_K^2$ so we have

$$-\frac{1}{4f^2}[s + \frac{(m_K^2 - p_{K^+}^2) + (m_K^2 - p_{K^-}^2)}{3}]$$  \hspace{1cm} (3)

First of all, let us see that the strong amplitude connecting $K^+K^-$ with $K^0\bar{K}^0$ calculated in the way shown in Fig.2 [8] must factorize out of the integral.

For this consider the diagrams in Fig.1 but with the $O(p^2)$ $\chi PT$ amplitude connecting the kaons. This amplitude is given by
The important point for the sequel is that the off-shell part, which should be kept inside the loop integration, will not contribute.

In order to see this, note that, due to gauge invariance, the physical amplitude for \( \phi \rightarrow \gamma K^0 \bar{K}^0 \) has the form

\[
M(\phi(p) \rightarrow \gamma(q)K^0 \bar{K}^0) = [g^{\mu\nu}(p \cdot q) - p^\mu q^\nu] \epsilon_\mu^\gamma \epsilon_\nu^\phi H(p \cdot q, Q^2, q \cdot Q)
\]

where \( \epsilon_\mu^\gamma \) and \( \epsilon_\nu^\phi \) are the polarization vectors of the photon and the \( \phi \), \( Q = p_{K^0} + p_{\bar{K}^0} \) and \( H \) is an arbitrary scalar function. In the calculation of this loop contribution the problem is the presence of divergences in the loops represented in Fig.1. Following refs. \([2, 3, 4]\) we will take into account the contribution of \( p^\nu q^\nu \) of Figs. 1a,b, since Fig. 1c does not give such type of terms. Then, by gauge invariance, see formula \([2]\), the coefficient for \( (p \cdot q)g^{\mu\nu} \) is also fixed. In fact, as in ref. \([2, 3, 4, 7]\) it is shown, the \( (p \cdot q)g^{\mu\nu} \) contribution will be finite since the off shell part of the strong amplitudes do not contribute, as we argue below, and then we are in the same situation than in the latter references.

Take the diagrams of Figs.1a,b. These diagrams give the same contribution and this is the reason for the factor 2 in front of the following integral accounting for both contributions.

\[
M' = \epsilon_\mu^\gamma \epsilon_\nu^\phi \frac{2e^2\phi}{i} \int \frac{d^4k}{(2\pi)^4} \frac{(2k_\nu - p_\nu)(2k_\mu - q_\mu)}{(k^2 - m_K^2 + i\epsilon)(k-q)^2 - m_K^2 + i\epsilon)(k-p)^2 - m_K^2 + i\epsilon}
\]

\[
\cdot\left(2\frac{(2k_\nu - p_\nu)(2k_\mu - q_\mu)}{m_K^2 - p_{K^0}^2 - q_{K^0}^2 + i\epsilon} + \frac{2k_\mu k_\nu}{m_K^2 - p_{K^0}^2 - q_{K^0}^2 + i\epsilon}\right)
\]

The momentum for each particle in the loop is indicated in Fig.1a and so we have that \( p_{K^0} = q - k \), \( p_{\bar{K}^0} = k - p \). Concentrating in the off-shell part of the strong amplitude, we have the integral

\[
\int \frac{d^4k}{(2\pi)^4} \frac{(2k_\nu - p_\nu)(2k_\mu - q_\mu)}{(k^2 - m_K^2 + i\epsilon)(k-q)^2 - m_K^2 + i\epsilon)(k-p)^2 - m_K^2 + i\epsilon}
\]

\[
\times\left(2\frac{(2k_\nu - p_\nu)(2k_\mu - q_\mu)}{m_K^2 - p_{K^0}^2 - q_{K^0}^2 + i\epsilon} + \frac{2k_\mu k_\nu}{m_K^2 - p_{K^0}^2 - q_{K^0}^2 + i\epsilon}\right)
\]

Taking into account that

\[
\epsilon_\mu^\phi \cdot p^\mu = 0 ; \epsilon_\nu^\phi \cdot q^\nu = 0 \quad (\text{Feynman gauge})
\]

then we only have

\[
\int \frac{d^4k}{(2\pi)^4} \frac{4k_\mu k_\nu}{(k^2 - m_K^2 + i\epsilon)(k-q)^2 - m_K^2 + i\epsilon)} + \int \frac{d^4k}{(2\pi)^4} \frac{4k_\mu k_\nu}{(k^2 - m_K^2 + i\epsilon)(k-p)^2 - m_K^2 + i\epsilon)}
\]

The above integrals do not give contribution to \( q^\mu p^\nu \) since in each integral there is only one of the two vectors \( q \) or \( p \). In this way we see that the strong amplitude \( \mathcal{O}(p^2) \) factorizes out on-shell in \([3]\). Note that the important point in the former argumentation is the form of the off-shell part of the S-wave strong amplitude at \( \mathcal{O}(p^2) \) and this is common to any other S-wave meson-meson amplitude at this order, as one can see in \([4]\).

Next we want to sum all the infinite series represented in Fig.2. The intermediate loops also contain \( \pi \pi \) for \( I = 0 \) and \( \pi^0 \eta \) for \( I = 1 \), since in ref.\([5]\) coupled channel Lippmann-Schwinger equations were used with \( \pi \pi \), \( K \bar{K} \) in \( I = 0 \) and \( \pi^0 \eta \), \( K \bar{K} \) in \( I = 1 \). In ref.\([5]\) it is shown that the meson-meson amplitude factorizes on-shell outside the loop integrals and since we have also here the \( \mathcal{O}(p^2) \) strong amplitude factorizing we are then in the same situation as in \([5]\) and we can substitute the \( \mathcal{O}(p^2) \) strong amplitude by the one calculated
Then to all orders in the approach of [8], we have the amplitude

\[ \gamma \gamma \rightarrow K^0 \bar{K}^0 \]

Note that the amplitude obtained in [8] contains also the resonances \( f_0(980) \) and \( a_0(980) \), which are generated dynamically.

Then we have for the amplitude \( \phi(p) \rightarrow \gamma(q)K^0\bar{K}^0 \)

\[
M = e^\gamma e^{\phi} \frac{2g \phi}{t} \frac{(2k - p_\mu)(2k - g_\mu)}{(k_\nu + i\epsilon)(k - q)^2 - m_K^2 + i\epsilon)}
\]

This integral has been evaluated in [2] using dimensional regularization and confirmed in [9].

With a = \( M^2_{\phi}/m_K^2 \) and b = \( Q^2/m_K^2 \),

\[
I(a, b) = \frac{1}{2(a - b)} - \frac{2}{(a - b)^2} (f(\frac{1}{b}) - f(\frac{1}{a})) + \frac{a}{(a - b)^2} (g(\frac{1}{b}) - g(\frac{1}{a}))
\]

where

\[
f(x) = \begin{cases} 
-(\arcsin(\frac{1}{\sqrt{x}}))^2 & x > \frac{1}{4} \\
\frac{1}{4} \ln(\frac{a}{1-a}) - i\pi & x < \frac{1}{4}
\end{cases}
\]

\[
g(x) = \begin{cases} 
(4x - 1)\frac{1}{8}\arcsin(\frac{1}{\sqrt{x}}) & x > \frac{1}{4} \\
\frac{1}{2} (1 - 4x)^{\frac{1}{2}} \ln(\frac{\eta_+}{\eta_-}) - i\pi & x < \frac{1}{4}
\end{cases}
\]

\[ \eta_\pm = \frac{1}{\sqrt{2x}} (1 \pm (1 - 4x)^{\frac{1}{2}}) \]

After summing over the final polarisations of the photon, averaging over the ones of the \( \phi \) and taking into account the phase space for three particles [11] one obtains

\[
\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0) = \int \frac{dm_{12}^2 dQ^2}{(2\pi)^3 192 M_{\phi}^2} |g_\phi I(a, b)|^2 (M_{\phi}^2 - Q^2)^2 |t_S|^2
\]

where \( m_{12}^2 = (q + p_{K^0})^2 \).

Taking \( g_\phi^2 = 1.66 \) from its width to \( K^+K^- \), \( M_{\phi} = 1019.41 \) MeV, \( \Gamma(\phi) = 4.43 \) MeV, \( \text{BR}(\phi \rightarrow K^0\bar{K}^0) = 0.34 \) and using the mass of the \( K^0 \) for the phase space considerations, ref. [11], one gets

\[
\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0) = 2.22 \times 10^{-7} \text{MeV}
\]

\[
\text{BR}(\phi \rightarrow \gamma K^0 \bar{K}^0) = 0.50 \times 10^{-7}
\]

\[
\frac{\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0)}{\Gamma(\phi \rightarrow K^0 \bar{K}^0)} = 1.47 \times 10^{-7}
\]
The uncertainties coming from the range of the possible values for the cut-off give a relative error around 20%.

Taking only into account the $I = 0$ contribution

$$\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0) = 8.43 \times 10^{-7} \text{MeV}$$
$$BR(\phi \rightarrow \gamma K^0 \bar{K}^0) = 1.90 \times 10^{-7}$$

$$\frac{\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0)}{\Gamma(\phi \rightarrow K^0 \bar{K}^0)} = 5.58 \times 10^{-7}$$

and with only the $I = 1$

$$\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0) = 2.03 \times 10^{-7} \text{MeV}$$
$$BR(\phi \rightarrow \gamma K^0 \bar{K}^0) = 4.58 \times 10^{-8}$$

$$\frac{\Gamma(\phi \rightarrow \gamma K^0 \bar{K}^0)}{\Gamma(\phi \rightarrow K^0 \bar{K}^0)} = 1.35 \times 10^{-7}$$

We see that the process is dominated by the $I = 0$ contribution and that the interference between both isospin channels is destructive.

From the former results we see that the $\phi \rightarrow \gamma K^0 \bar{K}^0$ background will not be too significant for the purpose of testing CP violating decays from the $\phi \rightarrow K^0 \bar{K}^0$ process at DAΦNE in the lines of what was expected in [7]. All these calculations have been done in a way that both the resonant and non-resonant contributions are considered at the same time and taking into account also the different isospin channels.

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