Like-sign dimuon charge asymmetry in the
decay of $B \bar{B}$ pairs

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Abstract

We discuss the CP-violating dimuon charge asymmetry of events $p \bar{p} \to B \bar{B} X \to \mu^+ \mu^+ X$ in the Standard Model. Our conclusion is that the asymmetry is larger than previously expected and may reach a few percent for the $(B^0_d, \bar{B}^0_d)$ system. The analysis is also extended to the Two Higgs Doublet Model (Model II).

1 Introduction

We discuss the inclusive CP-violating dimuon charge asymmetry $A \equiv (N_{++} - N_{--})/(N_{++} + N_{--})$ in the Standard Model of events $p \bar{p} \to B \bar{B} X \to \mu^+ \mu^+ X$ where the $B \bar{B}$ pair is either $B_d \bar{B}_d$ or $B_s \bar{B}_s$. $N_{++}$ is the number of events $p \bar{p} \to B \bar{B} X \to \mu^+ \mu^+ X$. In particular we consider “indirect CP violation (or CP violation in the mixing)” due to complex effects in $B \leftrightarrow \bar{B}$ mixing and decay.

2 Indirect CP violation

Let us review the standard formalism of indirect CP violation but without making the usual approximations valid when CP violation is “small”. We take the hamiltonian in the $B^0_d \leftrightarrow \bar{B}^0_d$ (or $B^0_s \leftrightarrow \bar{B}^0_s$, $D^0 \leftrightarrow \bar{D}^0$, or $K^0 \leftrightarrow \bar{K}^0$) basis as

$$H \equiv M - \frac{i}{2} \Gamma \equiv \left[ \begin{array}{cc} m & M_{12} \\ M_{12}^* & m \end{array} \right] - \frac{i}{2} \left[ \begin{array}{cc} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{array} \right],$$

(1)
where the matrices $M$ and $\Gamma$ are hermitian. The hamiltonian $H$ itself is not hermitian since the $B$ mesons do decay. The matrix elements of the hamiltonian are obtained from second order perturbation theory:

$$M_{\alpha\beta} = m \delta_{\alpha\beta} + \langle \alpha | H_{SW} | \beta \rangle - P \sum_{\xi \neq 1,2} \frac{\langle \alpha | H_W | \xi \rangle \langle \xi | H_W | \beta \rangle}{E_\xi - m},$$

(2)

$$\Gamma_{\alpha\beta} = 2 \pi \sum_{\xi \neq 1,2} \langle \alpha | H_W | \xi \rangle \langle \xi | H_W | \beta \rangle \delta (E_\xi - m),$$

(3)

where $H_W$ is the Standard Model weak interaction, $H_{SW}$ is a superweak interaction (which we consider no further), and $P$ denotes “principal part”.

The diagonal elements of $H$ are assumed equal due to CPT invariance. The argument goes as follows: $M_{11}$ is the amplitude for a $B_0$ to remain a $B_0$. Applying CPT we obtain the amplitude for a $\bar{B}_0$ to remain a $\bar{B}_0$, i.e. $M_{22}$. So, if CPT invariance holds, we obtain $M_{11} = M_{22} \equiv m$. Likewise, $\Gamma_{11}$ is the probability per unit time for the decay $B_0 \to \sum \xi$. Applying CPT we obtain the probability per unit time for $\sum \bar{\xi} \to \bar{B}_0$. From Equation (3) for this process, and changing the order of the two brackets, we obtain the probability per unit time for $\bar{B}_0 \to \sum \bar{\xi}$, i.e. $\Gamma_{22}$. So, if CPT invariance holds, we obtain $\Gamma_{11} = \Gamma_{22} \equiv \Gamma$.

The solution to the equation $i \partial \psi / \partial t = H \psi$ with $\psi^T \equiv (B^0(t), \bar{B}^0(t))$ is

$$B^0(t) = \frac{1}{2} \{ s_+(t) + s_-(t) \} B^0(0) + \frac{1 - \varepsilon}{1 + \varepsilon} \cdot \frac{1}{2} \{ s_+(t) - s_-(t) \} B^0(0),$$

$$\bar{B}^0(t) = \frac{1 + \varepsilon}{1 - \varepsilon} \cdot \frac{1}{2} \{ s_+(t) - s_-(t) \} B^0(0) + \frac{1}{2} \{ s_+(t) + s_-(t) \} \bar{B}^0(0),$$

(4)

where

$$s_-(t) = \exp(-imt) \exp(-\Gamma t/2) \exp(i\Delta M t/2) \exp(\Delta \Gamma t/4),$$

$$s_+(t) = \exp(-imt) \exp(-\Gamma t/2) \exp(-i\Delta M t/2) \exp(-\Delta \Gamma t/4),$$

(5)

$$\frac{1 - \varepsilon}{1 + \varepsilon} = \frac{\Delta M - i \frac{\Delta \Gamma}{2}}{2 \left( M_{12}^* - \frac{i}{2} \Gamma_{12} \right)} = \frac{2 \left( M_{12} - \frac{i}{2} \Gamma_{12} \right)}{\Delta M - \frac{i}{2} \Delta \Gamma}. $$

(6)

The phase of $(1 - \varepsilon)/(1 + \varepsilon)$ is arbitrary: it can be changed by redefining the phase of $\bar{B}^0(0)$. Observables depend on the absolute value of $(1 - \varepsilon)/(1 + \varepsilon)$, or equivalently on

$$\alpha \equiv \frac{Re(\varepsilon)}{1 + |\varepsilon|^2}. $$

(7)
For the same reason, we can multiply \( M_{12} \) and \( \Gamma_{12} \) by a common phase-factor. Only the relative phase is observable:

\[
\angle \frac{\Gamma_{12}}{M_{12}} \equiv \varphi. \tag{8}
\]

We introduce the notation

\[
x \equiv \frac{\Delta M}{\Gamma}; y \equiv \frac{\Delta \Gamma}{2\Gamma}. \tag{9}
\]

Then the probability that a \( B_0 \) decays as a \( B_0 \) is

\[
\chi = \frac{\int_0^\infty \left| \frac{1}{1+\varepsilon} \right|^2 \frac{1}{4} |s_+ - s_-|^2 dt}{\int_0^\infty \left| \frac{1}{1+\varepsilon} \right|^2 \frac{1}{4} |s_+ - s_-|^2 dt + \int_0^\infty \frac{1}{4} |s_+ + s_-|^2 dt} = \frac{(x^2 + y^2) \left( \frac{1}{2} - \alpha \right)}{1 + x^2 + 2\alpha (1 - y^2)}. \tag{10}
\]

Similarly, the probability that a \( B_0 \) decays as a \( \bar{B}_0 \) is

\[
\bar{\chi} = \frac{(x^2 + y^2) \left( \frac{1}{2} + \alpha \right)}{1 + x^2 - 2\alpha (1 - y^2)}. \tag{11}
\]

Finally, let us relate \( M_{12} \) and \( \Gamma_{12} \) with \( \Delta M \) and \( \Delta \Gamma \):

\[
\alpha \frac{1}{1 + 4\alpha^2} = -\frac{Im \{ \Gamma_{12}/M_{12} \}}{4 + |\Gamma_{12}/M_{12}|^2}; \tag{12}
\]

\[
Im \left( \frac{\Gamma_{12}}{M_{12}} \right) = -4\alpha \frac{1 + (\Delta \Gamma/(2\Delta M))^2}{1 + (\alpha \Delta \Gamma/\Delta M)^2}; \tag{13}
\]

\[
\tan(\varphi) = \frac{-\alpha}{1 - 4\alpha^2} \cdot \frac{4\Delta M^2 + \Delta \Gamma^2}{\Delta M \Delta \Gamma}; \tag{14}
\]

\[
|M_{12}|^2 = \frac{1}{4} \frac{1}{1 - 4\alpha^2} \Delta M^2 + \frac{1}{4} \frac{\alpha^2}{1 - 4\alpha^2} \Delta \Gamma^2; \tag{15}
\]

\[
|\Gamma_{12}|^2 = \frac{4\alpha^2}{1 - 4\alpha^2} \Delta M^2 + \frac{1}{4} \frac{1}{1 - 4\alpha^2} \Delta \Gamma^2. \tag{16}
\]

These equations are exact for the \((B_d^0, \bar{B}_d^0)\) and \((B_s^0, \bar{B}_s^0)\) systems separately. As a cross check, for \( |\alpha| \ll 1 \) we recover the well known results \( \Delta M \approx 2|M_{12}| \) and \( A \approx -4\alpha \approx Im \left( \Gamma_{12}/M_{12} \right) \).
Figure 1: A tree level Feynman diagram contributing to the $B_s^0$ meson decay rate $\Gamma_{11} \equiv \Gamma \propto \langle B_s^0 | H_W | \xi \rangle \langle \xi | H_W | B_s^0 \rangle$. The intermediate quarks can also be $\bar{d}, u$ or $\bar{u}$. There are also lepton channels.

3 $\Gamma_{12}$ and $M_{12}$ in the Standard Model

From the box diagrams[1] of $B^0 \leftrightarrow \bar{B}^0$ mixing and Equation (2) for the Standard Model, we obtain[2]

$$x_q \equiv \frac{\Delta M}{\Gamma} \approx \frac{2|M_{12}|}{\Gamma} \approx \frac{\eta' f_B^2 G_F^2 m_{\pi^0}^2 m_B}{8\pi^2 \Gamma} |V_{tq}|^2 \alpha \left( \frac{m_t^2}{m_W^2} \right) \quad (17)$$

where

$$\alpha (x_t) \equiv \frac{x_t^3 - 11x_t^2 + 4x_t}{4(1 - x_t)^2} - \frac{3x_t^3 \ln(x_t)}{2(1 - x_t)^3}. \quad (18)$$

Let us now consider the absorptive matrix elements given by Equation (3). For $\alpha = \beta$ Equation (3) is “Fermi’s golden rule”. The tree level Feynman diagrams are shown in Figures 1 and 2.

We arrive at three different predictions for the CP-violating asymmetries $A_d$ and $A_s$ in the Standard Model:

1. If we require that intermediate quark spins and momenta match in the diagrams of Figure 2, then we obtain negligible asymmetries as discussed by J. Hagelin[3]. This result would be correct if quarks were observable.

2. However quarks are not observable. Hadrons are observable. Due to hadronization, i.e. due to gluons, the momenta need not match at the quark level and we obtain the approximate results shown in Table 1.
Figure 2: A tree level Feynman diagram contributing to $\Gamma_{12} \propto \langle B^0_s \mid H_W \mid \xi \rangle \langle \xi \mid H_W \mid \bar{B}^0_s \rangle$. The intermediate quarks can also be $\bar{u}$ or $u$.

(All input numerical data for these calculations not specified in the Tables were obtained from [1].) A similar point was emphasized by T. Altomari, L. Wolfenstein, and J.D. Bjorken [4] so that their “conclusion is that a reasonable estimate of the asymmetry lies between $10^{-3}$ and $10^{-2}$ but that neither the sign nor the magnitude can be reliably calculated”. That conclusion is in agreement with Table 1.

3. If, due to gluon exchange in the hadronization process, it were a good approximation to neglect both the requirements of spin and momentum match, then we obtain approximately the results shown in Table 2. As an example, suppose that the c-quark in the left hand diagram of Figure 2 has “spin up”, and the c-quark in the right hand diagram has “spin down”. This miss-match of spins of the quarks need not imply a miss-match at the hadron level, as can be seen, for example, in the case in which both quarks hadronize into a scalar meson.

Which of the three predictions, if any, is correct?

4 Discussion

Consider diagrams of the form shown in Figure 1. Requiring match of spins of the quarks (and weighting hadronic modes by a factor 3 for color) we obtain an inclusive branching fraction $B(b \rightarrow \mu X) = 0.16$ to be compared with the experimental value $B(b \rightarrow \mu X) = 0.103 \pm 0.005$ [1]. This is the well known “baffling semi-leptonic branching fraction of B mesons” [4]. But quark
spins are not observable. Only hadrons are observable. So the hadronization process, \(i.e.\) gluons, enhance the decay rate \(\Gamma_{11}\) by a factor \(\approx 2\) to account for the observed drop in semi-leptonic branching fraction.

Let us now consider the momentum miss-match of the spectator quarks in the diagrams of Figure 2. To obtain matching of momenta we need at least one gluon as shown in Figure 3. The amplitude of the diagram of Figure 3 is “enhanced” with respect to the amplitude of the diagram of Figure 2 by a factor of order

\[
\frac{\Gamma_{12}(3)}{\Gamma_{12}(2)} \approx \frac{A_x(3)}{A_x(2)} \approx \frac{\alpha_s m_B f_B^2}{8} \cdot P_x \cdot P_g. \tag{19}
\]

The numbers in parenthesis refer to the Figures, and \(x \equiv s\) or \(d\) for the \(B^0_s\) or \(B^0_d\) systems respectively. The factor \(m_B f_B^2\) was introduced (somewhat arbitrarily) on dimensional grounds. The gluon propagator is \(P_g \approx 1/(2m_x|\vec{p}_2|)\). We take \(|\vec{p}_2|\) to be of order \(m_B/3\) (see definition of \(\vec{p}_2\) and \(x\) in Figure 3). If the outgoing quarks were free, the quark propagator \(P_x\) would be on-shell and would diverge. Since the outgoing quarks are confined to dimensions of order 1 Fermi, we replace the quark propagator by \(P_x \approx 1/m_\pi\). Then the order of magnitude estimate of the “enhancement factor” (19) is

\[
\frac{\Gamma_{12}(3)}{\Gamma_{12}(2)} \approx \frac{A_x(3)}{A_x(2)} \approx \frac{3\alpha_s f_B^2}{16m_\pi m_x} \tag{20}
\]
Table 1: Standard Model calculation of the dimuon charge asymmetries $A_d$ and $A_s$, and other parameters, as a function of the angles $\gamma$ and $\beta$ of the unitarity triangle\([1]\), for $V_{ub} = 0.0036$, $\eta' = 1.09$, and $f_B = 0.18\text{GeV}$. The following parameters define the Cabbibo-Kobayashi-Maskawa (CKM) matrix in the Wolfenstein parametrization\([1]\): $\lambda = 0.2235$, $A = 0.8357$, $[\rho^2 + \eta'^2]^{1/2} = 0.3854$ and $\gamma \equiv \arctan(\eta'/\rho)$ listed above. Momenta of the $s$-quarks in the diagram of Figure 2 are not required to match. Spins are required to match. $m_b = 4.25\text{GeV}/c^2$, $m_c = 1.25\text{GeV}/c^2$, $m_u = 0.00325\text{GeV}/c^2$, $m_d = 0.006\text{GeV}/c^2$, and $m_s = 0.115\text{GeV}/c^2$.

| $\gamma$ | $22.5^0$ | $45^0$ | $67.5^0$ | $90^0$ | $112.5^0$ | $135^0$ | $157.5^0$ |
| $\beta$ | $12.9^0$ | $20.5^0$ | $22.7^0$ | $21.1^0$ | $17.2^0$ | $12.1^0$ | $6.2^0$ |
| $\varphi_d$ | $-41^0$ | $-78^0$ | $-106^0$ | $-127^0$ | $216^0$ | $203^0$ | $191^0$ |
| $A_d$ | $-18\%$ | $-33\%$ | $-36\%$ | $-32\%$ | $-25\%$ | $-17\%$ | $-08\%$ |
| $A_s$ | $04\%$ | $04\%$ | $05\%$ | $05\%$ | $04\%$ | $04\%$ | $03\%$ |
| $x_d$ | $0.48$ | $0.66$ | $0.93$ | $1.26$ | $1.58$ | $1.85$ | $2.04$ |
| $x_s$ | $22$ | $22$ | $22$ | $22$ | $22$ | $22$ | $22$ |
| $\Delta \Gamma_d/\Gamma$ | $0.1\%$ | $0.0\%$ | $0.1\%$ | $0.3\%$ | $0.5\%$ | $0.7\%$ | $0.9\%$ |
| $\Delta \Gamma_s/\Gamma$ | $3.3\%$ | $3.2\%$ | $3.0\%$ | $2.8\%$ | $2.7\%$ | $2.5\%$ | $2.5\%$ |

which is $\approx 0.08$ for the $B^0_s$ system, and $\approx 1.6$ for the $B^0_d$ system.

So, which of the three predictions, if any, is correct? Prediction 1 by Hagelin includes a restriction on $|\vec{p}_2|$ due to a miss-match of momenta of the spectator quark, and also requires matching spins. We have seen that this momentum miss-match can be “fixed” by gluons (at no cost for the $B^0_d$ system). Therefore we arrive at Prediction 2 given in Table 1 which does not require momenta to match but still requires matching spins. We have seen that hadronization relaxes the need to match spins (as required by the semi-leptonic branching fraction of $B$ mesons). Then we arrive at Prediction 3 given in Table 2. Since $x_d = 0.723\pm 0.032$\([1]\) we obtain from Table 2 $A_d \approx 1\%$ to $3\%$ for the allowed range of $V_{ub}$\([1]\). But for the $B^0_d$ system the gluon in the diagram of Figure 3 can even enhance the asymmetry above the prediction given in Table 2. Note that relaxing the requirement of matching spins we obtain a large $\Delta \Gamma_s/\Gamma$ as predicted in \([3]\), leading to interesting experimental consequences\([7]\).
Table 2: Same as Table 1 except that the spins (in addition to the momenta) of the intermediate quarks in Figure 2 are not required to match. For the first half of the Table $\rho^2 + \eta^2 = 0.3854$ and $V_{ub} = 0.0036$. For the second half of the Table $\rho^2 + \eta^2 = 0.54$ and $V_{ub} = 0.0050$. At $\gamma = 0^0$ and $180^0$, $A_d = 0$ and $A_s = 0$.

### 5 The Two Higgs Doublet Model

We have repeated the calculations using the Two Higgs Doublet Model (Model II) [8]. The results are shown in Table 3 (which requires matching intermediate quark spins but not momenta). Comparing with Table 1 we note that the predictions for the asymmetry $A_d$ are not significantly changed by this extension of the Standard Model.

### 6 Conclusions

Our conclusion is that the inclusive CP-violating like-sign dimuon charge asymmetry in the Standard Model is larger than previously expected [4]. For the $(B_d^0, \bar{B_d}^0)$ system the asymmetry is probably between $\approx 1\%$ and $3\%$, and may even be enhanced by gluons above this value. Such a large asymmetry would be within the reach of the next run of the D$\Phi$ and CDF experiments at the FERMILAB Tevatron collider.
| $\gamma$ | $22.5^0$ | $45^0$ | $67.5^0$ | $90^0$ | $112.5^0$ | $135^0$ | $157.5^0$ |
|------|--------|--------|--------|--------|--------|--------|--------|
| $\beta$ | $12.9^0$ | $20.5^0$ | $22.7^0$ | $21.1^0$ | $17.2^0$ | $12.1^0$ | $6.2^0$ |
| $\varphi_d$ | $-43^0$ | $-80^0$ | $-109^0$ | $-130^0$ | $248^0$ | $200^0$ | $187^0$ |
| $A_d$ | $-17\%$ | $-31\%$ | $-33\%$ | $-29\%$ | $-22\%$ | $-14\%$ | $-0.05\%$ |
| $A_s$ | $0.28\%$ | $0.27\%$ | $0.26\%$ | $0.25\%$ | $0.24\%$ | $0.23\%$ | $0.23\%$ |
| $x_d$ | $0.48$ | $0.66$ | $0.93$ | $1.26$ | $1.58$ | $1.85$ | $2.04$ |
| $x_s$ | $22$ | $22$ | $22$ | $22$ | $22$ | $22$ | $22$ |
| $\Delta \Gamma_d/\Gamma$ | $0.1\%$ | $0.0\%$ | $0.1\%$ | $0.3\%$ | $0.5\%$ | $0.7\%$ | $0.8\%$ |
| $\Delta \Gamma_s/\Gamma$ | $2.3\%$ | $2.4\%$ | $3.0\%$ | $2.6\%$ | $2.5\%$ | $2.3\%$ | $2.1\%$ |

Table 3: Same as Table 1 but for the Two Higgs Doublet Model (Model II) with $m_H^\pm = 45\text{GeV}/c^2$ and $\tan \beta = 30$, instead of the Standard Model.

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