Is there a spectral turnover in the spin noise of millisecond pulsars?

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ABSTRACT

Pulsar timing arrays provide a unique means to detect nanohertz gravitational waves through long-term measurements of pulse arrival times from an ensemble of millisecond pulsars. After years of observations, some timing array pulsars have been shown to be dominated by low-frequency red noise, including spin noise that might be associated with pulsar rotational irregularities. The power spectral density of pulsar timing red noise is usually modeled with a power law or a power law with a turnover frequency below which the noise power spectrum plateaus. A possible physical origin for the spectral turnover is neutron star superfluid turbulence. In this work, we search for a spectral turnover in spin noise using the first data release of the International Pulsar Timing Array. Through Bayesian model selection, we find no evidence of a spectral turnover. Our analysis also shows that data from pulsars J1939+2134, J1024−0719 and J1713+0747 prefers the power-law model to the superfluid turbulence model.

Key words: stars: neutron – pulsars: general – methods: data analysis

1 INTRODUCTION

It has long been proposed that pulsars can be used to detect gravitational waves in the nHz band (Sazhin 1978; Detweiler 1979; Hellings & Downs 1983). Millisecond pulsars, first discovered in 1982 (Backer et al. 1982), provide promising prospects for gravitational wave detection thanks to their exceptional rotational stability. The concept of a pulsar timing array (PTA), long-term monitoring of pulse arrival times from a spatial array of millisecond pulsars, was conceived three decades ago (Romani 1989; Foster & Backer 1990). Currently, several collaborations are conducting PTA observations, including the Parkes Pulsar Timing Array (PPTA) (Manchester et al. 2013), the European Pulsar Timing Array (EPTA) (Kramer & Champion 2013) and the North American Nanohertz Observatory for Gravitational Waves (NANOGrav) (McLaughlin 2013). A consortium of these collaborations is called the International Pulsar Timing Array (IPTA) (Hobbs et al. 2010a; Perera et al. 2019).

The first gravitational-wave signal detected with PTAs is likely to be a stochastic gravitational-wave background, formed by a cosmic population of supermassive binary black holes (Rosado et al. 2015). Apart from the detection of gravitational waves, PTAs also offer the opportunity to establish a pulsar-based time standard (Hobbs et al. 2012), to study the Solar System (Caballero et al. 2018), the interstellar medium (Coles et al. 2015) and the Solar wind (Madison et al. 2019), and to constrain ultradark light matter candidates (Porayko et al. 2018).

The science output of PTA data relies on how well we model noise. Incorrect noise models can also lead to false detection in gravitational-wave searches (Arzoumanian et al. 2018b; Hazboun et al. 2019). At low frequencies, where we are most sensitive to the stochastic gravitational-wave background, some millisecond pulsars show evidence of red noise (Hobbs et al. 2010b; Lentati et al. 2016). One particular source of red noise is the spin noise, which might be associated with pulsar rotational irregularities (see, e.g., Shannon & Cordes 2010). In PTA noise analyses (Coles et al. 2011; Reardon et al. 2015; Lentati et al. 2016; Caballero et al. 2016; Arzoumanian et al. 2015, 2018a), the red noise power spectrum is modelled by either a power law, or the broken power law, which introduces a corner frequency below which the noise power spectrum plateaus. While some young pulsars show hints of a spectral turnover at low frequencies (Parthasarathy et al. 2019), it has not yet been found for millisecond pulsars. If the typical time scale of a spectral turnover for millisecond pulsars is on the order of years or shorter, it reduces the noise in the most sensitive frequency band of PTAs, yielding a faster detection of a stochastic gravitational-wave background.

Moreover, pulsar timing red noise provides interesting prospects for studying neutron star physics. A range of mechanisms have been proposed to explain pulsar red noise, including switching between two different spin-down rates (Lyne et al. 2010), recovery from a glitch—a sudden increase in the rotational frequency (Johnston & Galloway 1999), a cumulative effect of frequent micro-glitches (Cordes & Downs 1985; D’Alessandro et al. 1995; Melatos et al. 2008), variable coupling between the crust and liquid interior (Alpar et al. 1986; Jones 1990), influence of planets (Cordes 1993) and asteroids (Shannon et al. 2013). Nevertheless, there are not many models that link power spectral density model parameters to physical features. One such model by Melatos & Link (2013), which we explore in this paper, predicts a superfluid turbulence in neutron star interiors as the origin of red noise. The turbulent process exerts a torque on star’s crust, where external magnetic field of the star is

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produced. The model features a spectral turnover. Implications of how a spectral turnover in this model will affect times to detection of a stochastic background were discussed in Lasky et al. (2015).

In this work we employ Bayesian inference to search for evidence of spectral turnover in pulsar spin noise in the first data release (DR1) of the IPTA (Verbiest et al. 2016). We discuss our data analysis methods in Section 2. Our simulation study is presented in Section 3. We describe the noise processes of the first IPTA data release in Section 4. We present the results in Section 5, and discuss our conclusions in Section 6.

2 METHOD

2.1 Bayesian methodology in pulsar timing

First, following Van Haasteren et al. (2009), we assume a multivariate Gaussian likelihood function to describe pulsar timing residuals \( \delta t \) after fitting for the timing model:

\[
L(\delta t|\theta, \xi) = \frac{1}{(2\pi)^{n/2} |\det(C)|^{1/2}} \exp\left(-\frac{1}{2} \delta t^T C^{-1} \delta t\right).
\]

Stochastic signals are modeled using a covariance matrix \( C \), while \( s \) is a deterministic signal vector. Parameters of our models are \( \theta \). The vector \( \xi \) contains timing model parameters and \( M \) is a design matrix, describing the contribution of \( m \) timing model parameters to \( t \) times of arrivals. Throughout our study, we work with ToA and residuals, referenced to the Solar System Barycenter. Assuming uniform prior on timing model parameters, the likelihood is marginalized over these parameters (Van Haasteren et al. 2009):

\[
L(\delta t|\theta) = \frac{\sqrt{\det(M^T C^{-1} M)^{1/2}}}{(2\pi)^{n/2} \det(C)} \exp\left(-\frac{1}{2} \delta t^T C^{-1} \delta t\right),
\]

where we have defined

\[
C' = C^{-1} - C^{-1} M (M^T C^{-1} M)^{-1} M^T C^{-1}.
\]

To speed up the calculation, we employ the singular value decomposition of the design matrix in the form \( M = USV^* \), where \( S \) contains singular values of \( M \), \( U \) and \( V \) are unitary matrices with dimensions \( n \times n \) and \( m \times m \) respectively. Then we obtain the likelihood function in a form (van Haasteren & Levin 2012)

\[
L(\delta t|\theta) = \frac{1}{(2\pi)^{m/2} \det(G^T CG)} \exp\left(-\frac{1}{2} \delta t^T G(G^T CG)^{-1} G^T \delta t\right),
\]

so that \( U = U_1 G \) with \( U_1 \) and \( G \) consisting of the first \( m \) and remaining \( n - m \) columns of \( U \). Our prior probability distribution is \( \pi(\theta) \). The integral of the likelihood over the prior parameter range is the Bayesian evidence for our model:

\[
Z(\theta, \delta t) = \int L(\delta t|\theta) \pi(\theta) d\theta.
\]

To infer our model parameters \( \theta \), given observational data, we employ the Bayes’ theorem:

\[
P(\theta|\delta t) = \frac{L(\delta t|\theta)\pi(\theta)}{Z(\theta, \delta t)}.
\]

Using two different models \( A \) and \( B \) with parameters \( \theta_A \) and \( \theta_B \), we employ the Bayes factor as a measure of which model better fits the data:

\[
B_{AB} = \frac{Z_{\theta_B}(\delta t)}{Z_{\theta_A}(\delta t)}, \quad i \in [1, N_{psr}],
\]

where \( N_{psr} \) is the number of pulsars. For simulation studies, we calculate the Bayes factors from evidence, which is obtained with nested sampling (Skilling 2004). For real data, we calculate our Bayes factors using the product-space sampling method (Hee et al. 2015; Carlin & Chib 1995). Assuming timing data for each pulsar are independent measurements, we combine all available data:

\[
B_{A} = \prod_{i=1}^{N_{psr}} B_{AB}^i,
\]

which provides a metric to determine whether the spectral turnover is a real physical feature of pulsar spin noise.

2.2 Modelling stochastic processes

We model stochastic red noise processes as a power-law power spectral density \( P(f) \). We include \( P(f) \) in our likelihood function using the Fourier-sum method from Lentati et al. (2013), described briefly below. We represent the covariance matrix as \( C = N + K \), where \( N \) is a diagonal matrix for white noise component, and \( K \) is a red noise component. A Woodbury lemma is used to simplify the inversion of a covariance matrix, decomposed into \( N \) and \( K \) (Hager 1989; van Haasteren & Vallisneri 2014). We define a Fourier basis \( F \) with elements:

\[
F_{i,j} = \begin{cases}
\kappa a_i \sin(2\pi f_i \Delta t), & \text{if } i \text{ is even} ; \\
\kappa b_j \cos(2\pi f_i \Delta t), & \text{if } i \text{ is odd} ; \\
& i \in [1, 2N_f], \quad j \in [1, N_{\text{ToA}}].
\end{cases}
\]

The parameter \( \kappa \) is a constant that is set to one for spin noise. The multiplicative factors \( a_i \) and \( b_j \) are Fourier coefficients which follow the standard Gaussian distribution. Each \( \Delta t_f = (t_f - t_1) \) is the difference between the first ToA and the \( j \)th ToA. The elements \( f_i \) are components of a frequency vector that depend on the total observation span \( T_{obs} \). They are defined as

\[
f_i = \begin{cases}
\frac{i}{2f}, & \text{if } i \text{ is odd} ; \\
\frac{i}{2f}, & \text{if } i \text{ is even} .
\end{cases}
\]

The variable \( N_f \) determines the number of Fourier basis components in the frequency domain, with a minimum of \( 1/T_{obs} \) and spacing \( \Delta f = 1/T_{obs} \). In our analysis we choose \( N_f = 30 \). Next, we obtain a diagonal matrix \( \Phi(\Phi^T \Phi)^{-1} \) with elements \( \Phi_{ij} = P(f_i) \), which depends on our red noise model with parameters \( \theta_{\text{red}} \). The red noise component in our likelihood function, marginalized over Fourier coefficients \( a_i \) and \( b_j \) (van Haasteren & Vallisneri 2014), is

\[
K = F \Phi F^T \Delta f.
\]

The white-noise covariance matrix \( N \) is diagonal with elements

\[
\sigma_i^2 = (E Q A C \sigma_i^{\text{ToA}})^2 + \text{EQUAD}^2.
\]
where EFAC and EQUAD are factors to account for the excess of white noise, in addition to ToA error bars, \( \sigma^2_{\text{ToA}}. \)

### 2.3 Red noise models

Some pulsars in real data do not show evidence of red noise (Lentati et al. 2016). We refer to the model without red noise as “Model PL”. Next, we employ the two following phenomenological models for red noise. The power-law model

\[
P_{\text{PL}}(f) = \frac{A^2}{12 \pi^2} f^3 (f \text{ yr})^{-\gamma},
\]

which we refer to as the “Model PL”. And the broken power-law model

\[
P_{\text{BPL}}(f) = \frac{A^2}{12 \pi^2} f^3 (\sqrt{f^2 + f_c^2} \text{ yr})^{-\gamma},
\]

which we refer to as “Model BPL”. In the above two equations, model parameters are: the red noise amplitude, \( A \), the slope, \( \gamma \), the corner frequency, \( f_c \).

We also study the superfluid turbulence model from (Melatos & Link 2013)

\[
P_{\text{M}}(f) = \frac{15 \pi^2}{8 \pi^2 \eta(R^{-1})} \int_{2\pi} f^3 \left[ \frac{2 \pi^2}{\eta(R^{-1})} \right] + \frac{1}{3} f^{31/3} \, df.
\]

The model depends on parameters \( \eta(R^{-1}) \) and \( \lambda \). In accordance with Melatos & Link (2013), pulsar spin period squared \( p^2 \) is added to Equation 15. This is done in order to obtain the power spectral density in units of \( \text{s}^3 \), to be consistent with Equations 13 and 14. Parameter \( \lambda \) is a non-condensate fraction of the moment of inertia, which affects the amplitude of red noise. Parameter \( \eta(R^{-1}) \) is a decorrelation frequency, which determines the spectral turnover. For convenience, we reparametrize Equation 15, in the form of parameters \( \lambda \) and \( \lambda_c \), using Equation A1. The integral in Equation 15 yields an analytical solution, given by Equation A2.

In Figure 1, we plot examples of models of spin noise power spectral density. Note that, at high frequencies, Model M asymptotically approaches Model PL with \( \gamma = 2 \), and that parameters \( \eta(R^{-1}) \) and \( \lambda \) are degenerate. In order to break this degeneracy, and to distinguish Models PL and M, one must observe a spectral turnover.

### 2.4 Software

We estimate the design matrix using the designmatrix plugin in TEMPO2 (Hobbs et al. 2006). We simulate data and access TEMPO2 using libempo (Vallisneri 2013). We construct our models and likelihood, and do parameter estimation using Enterprise (Ellis et al. 2017). We perform likelihood sampling using the PMCMCSampler (Ellis & van Haasteren 2017) for IPTA DR1 data. For simulations we use a nested sampler Dynesty (Speagle & Bary 2018), and we use Bilby (Ashton et al. 2019) to access the Dynesty sampler.

### 3 SIMULATION STUDY

We perform a simulation study to demonstrate our ability to do Bayesian model selection. We also demonstrate some potential subtleties in recovering a low-frequency turnover. We simulate ToAs, ToA errors, and timing residuals for the pulsar J0711–6830, using ephemerides from the ATNF Pulsar Catalogue (Manchester et al. 2005). We simulate ToAs evenly sampled once every 30 days between MJD 53000 and 56650. We assume ToA errors to be 0.5 \( \mu \text{s} \). These parameters are applied to all simulations described in this section of the paper. In our noise simulations we only assume one observing system, one observed radio frequency, and only red and white noise. The noise parameters are described in the following subsections.

#### 3.1 Red noise in an ensemble of pulsars

We simulate 50 mock pulsars with different random realisations of Model PL red noise and white noise. Then we perform model selection between Model PL and Model BPL. The simulated white noise parameters throughout the subsection are \( \text{EFAC} = 1 \) and \( \text{EQUAD} = 0.1 \mu \text{s} \). The simulated red noise amplitude is different for three cases we describe in this subsection, while the priors for red noise power-law index and corner frequency are \( \pi(f_c) = U(2, 5) \) and \( \pi(\eta) = U(10^{-10}, 10^{-8}) \). Here \( U \) stands for a uniform distribution, and \( \log_{10} U \) stands for a uniform in log distribution. We use the same red noise priors for \( A \) and \( \eta \) for models PL and BPL, for both injection and recovery.

First, we simulate Model PL with a prior \( \pi(A) = \log_{10} U(10^{-14}, 10^{-11}) \). The prior range for noise amplitude is chosen such that red noise is overall stronger than white noise. As a result, with all simulated pulsars, we obtain red noise power-law model parameters are: the red noise amplitude, \( A \), the slope, \( \gamma \), the corner frequency, \( f_c \), and parameters \( \eta(R^{-1}) \) and \( \lambda \) are degenerate. In order to break this degeneracy, and to distinguish Models PL and M, one must observe a spectral turnover.

Second, we demonstrate that we do not recover the wrong model if the red noise is overall much weaker than white noise. The prior for simulation and recovery of red noise amplitude is reduced to \( \pi(A) = \log_{10} U(10^{-17}, 10^{-14}) \). Now, log red noise power-law model parameters are: the red noise amplitude, \( A \), the slope, \( \gamma \), the corner frequency, \( f_c \), and parameters \( \eta(R^{-1}) \) and \( \lambda \) are degenerate. In order to break this degeneracy, and to distinguish Models PL and M, one must observe a spectral turnover.

Finally, we demonstrate that we can successfully inject and recover Model BPL. To do this, we use the following prior on red noise amplitude \( \pi(A) = \log_{10} U(10^{-14}, 10^{-11}) \). Now we obtain log red noise power-law model parameters are: the red noise amplitude, \( A \), the slope, \( \gamma \), the corner frequency, \( f_c \), and parameters \( \eta(R^{-1}) \) and \( \lambda \) are degenerate. In order to break this degeneracy, and to distinguish between two models, as expected.

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by including in Equation 8 only pulsars having log $B_{\odot,i}$ by white noise, can lead to a recovery of the incorrect model, if we do not have a data span long enough to effectively probe residuals spectra at frequencies around the turnover.

To demonstrate this, we simulate 1000 pulsars with red noise Model PL amplitude $\pi(A) = \log_{10} \mathcal{U}(10^{-15}; 10^{-11})$ and $\gamma = 3$, and simulate additional 1000 pulsars with red noise Model BPL with the same parameters and a corner frequency $f_c = 10$ nHz. As the amplitude of the red noise in the set of simulated pulsars increases, the average log $B_{\odot,i}$ in favor of the correct model plateaus. This is demonstrated in Figure 2. We can see that, at some point, increasing log $B_{\odot,i}$ starts slightly favouring the correct model, but then saturates, so that increasing the amplitude of the red noise does not help to resolve a low-frequency turnover. In this medium-to-strong red noise regime, some realisations of Model PL may favour the Model BPL hypothesis, and vice versa. However, the mean log $B_{\odot,i}$ (red line in Figure 2) favours the correct model.

### 3.3 The effect of sample variance in recovery of high amplitude red noise

In this subsection we find that with a PTA observation time of 10 years, we are unlikely to resolve a turnover in the red noise process of any particular pulsar, assuming a fiducial $f_c = 10$ nHz. This is because factors $a_i$, $b_i$ in Equation 9 become a source of noise themselves, and we do not have a data span long enough to effectively probe residuals spectra at frequencies around the turnover.

### 4 SOURCES OF NOISE IN THE FIRST IPTA DATA RELEASE

In this paper we perform model selection on IPTA DR1 dataset, where noise modeling has been performed by Lentati et al. (2016). In order to do model selection for spin noise, we include known sources of noise to our model for each pulsar, as prescribed by Lentati et al. (2016). In this section we review noise properties of the data release. In Table 2 we list the prior distributions for parameters used in our models.

#### 4.1 White noise

IPTA pulsars are often monitored by several radio observatories. The raw voltages from each telescope are processed by different hardware. Each radio observatory has different measurement errors. Moreover, radio pulse profiles exhibit stochastic variations in both phase and amplitude, which is called “pulse jitter”. Telescope-related radiometer noise, pulse jitter, and other factors contribute to white noise. For each pulsar we describe the white noise with EFAC and EQUAD parameters for each backend system that processes raw telescope data. In NANOGrav data, many ToAs can be present in each observing epoch, which requires an ECORR parameter to account for correlations between ToAs at each epoch (Arzoumanian et al. 2018b).

#### 4.2 DM noise

Dispersion measure (DM) is the electron column density, integrated along the line of sight to a pulsar. Stochastic variations in dispersion measure result in DM noise. We model DM noise as a power law with $A_{\text{DM}}$ and $\gamma_{\text{DM}}$, which depends on the radio frequency $\nu$ (Hz): $\kappa = K^2 \nu^{-2}$ in Equation 9. A constant $K = 1400$ MHz can be thought

**Figure 2.** The demonstration of the effect of sample variance on the recovery of a spectral turnover. Each point represents log $B_{\odot,i}$ BPL. The top plot with blue points is for different realisations of a power law, Model PL (Equation 13), while the bottom plot with orange points is for different realisations of a broken power law, Model BPL (Equation 14). The injection parameters, except red noise injection amplitude $A$ (horizontal axes), are the same for both plots. As the amplitude of the red noise is increased, the evidence is in favour (bottom plot) and against (top plot) the spectral turnover plateaus.

**Table 1.** Priors for the injection study in Section 3.1. Here $\mathcal{U}$ stands for a uniform distribution, and log $B_{\odot,i}$ stands for a uniform in log $B_{\odot,i}$ distribution.

| Injected model | $\pi(A)$ | log $B_{\odot,i}^{\text{BPL}}$ | Recovered model |
|----------------|---------|-------------------------------|-----------------|
| PL             | log $\mathcal{U}(10^{-14}, 10^{-11})$ | -3.00 | PL |
| PL             | log $\mathcal{U}(10^{-17}, 10^{-14})$ | 1.00  | N/A |
| BPL            | log $\mathcal{U}(10^{-14}, 10^{-12})$ | 95.6  | BPL |

**3.2 Prior mismatch in simulations**

Most of the IPTA pulsars from DR1 are dominated by white noise Lentati et al. (2016). In this subsection, we perform simulations that demonstrate that model selection for red noise in data, dominated by white noise, can lead to a recovery of the incorrect model, if we do not carefully choose our prior. We perform simulations of only white noise with EFAC = 1 and EQUAD = 0.1 $\mu$s. We perform model selection between models BPL and PL. We observe that evidence for the absence of red noise (Model PL) is always the strongest, while either Model PL or BPL may be preferred, depending on our prior on $f_c$ parameter. As we allow our prior on $f_c$ to include only low values less than around $1/T_{\text{obs}}$, we can not distinguish models PL and BPL. As we allow our prior on $f_c$ to include only frequencies higher than our sampling frequency, we cannot distinguish between models BPL and PL, and model selection between PL and BPL prefers BPL. This is not surprising, as white noise and Model PL are limiting cases of Model BPL. Therefore, for the case of the DR1 analysis, when the true prior is unknown, we propose to account for this effect by including in Equation 8 only pulsars having log $B_{\odot,i}^{\text{BPL}} > 0$ and log $B_{\odot,i}^{\text{PL}} > 0$. This way we exclude pulsars with no evidence of any red noise, and do not obtain false positives in favour of either a spectral turnover or its absence. Another solution to this problem is to fit the priors using the hierarchical inference (MacKay 2003), which we defer to a future work.
of as a reference radio frequency. We account for DM variations for every pulsar in IPTA analysis.

4.3 Band noise and system noise

Lentati et al. (2016) found that specific IPTA pulsars show evidence of band noise and system noise, which introduces additional red noise in some observing systems and radio frequency bands. In order to separate band noise and system noise from spin noise, we add a separate power law with \(A_{\text{BS}}\) and \(\gamma_{\text{BS}}\) on specific radio frequency bands and systems for specific pulsars where band and system noise for IPTA data release 1 has been found (see Table 4 in Lentati et al. 2016, for details).

4.4 Spin noise

We model spin noise as a common red noise process between all observing systems and radio frequencies. Model PL depends on parameters \(A_{\text{SN}}\) and \(\gamma_{\text{SN}}\). Model BPL depends on an additional parameter \(f_c\). We refer to a hypothesis that no spin noise is present in the data, as to Model \(\emptyset\). In this work, we are mostly interested in resolving a spectral turnover in spin noise, characterized by the parameter \(f_c\) in Model BPL. We are also interested in Model M with parameters \(A_{\text{BS}}\) and \(\gamma_{\text{BS}}\). When carrying out model selection between Model M and Model PL, we chose our prior on Model PL amplitude \(A\) to match the range of spin noise amplitudes that is allowed by our priors for \(\eta(R^{-1})\) and \(\lambda\) in Model M. Otherwise, the model with a wider prior range on spin noise amplitude would be incorrectly penalized when calculating a Bayes factor.

4.5 Transient noise events

Pulsars J1713+0747 and J1603–7202 show evidence of a sudden change in dispersion measure (Lentati et al. 2016). We take these events into account using the same empirical models that were used in Lentati et al. (2016). For J1713+0747 we model the event as a frequency-dependent sudden decrease followed by an exponential increase in timing residuals:

\[
s_E(t|A_{\text{PL}}, \tau_E) = K^2 \nu^{-2} \left\{ \begin{array}{ll}
0, & t < \tau_E; \\
A_{\text{PL}} e^{-\nu t/\tau_E}, & t \geq \tau_E
\end{array} \right.
\]  

(16)

where \(\nu\) is a radio frequency, and \(K = 1400\) MHz is the same reference frequency as we use to model DM noise. We model the DM event in pulsar J1603–7202 as a Gaussian function in the time domain:

\[
s_G(t|A_G, \tau_G, \sigma_G) = K^2 \nu^{-2} A_G e^{-\frac{(t-\xi)^2}{2\sigma_G^2}}.
\]  

(17)

DM event models in Equation 16 and Equation 17 are added to the signal vector \(s\) in the likelihood.

5 RESULTS

We perform parameter estimation and model selection for pulsars from the first IPTA data release. A summary of our analysis for individual pulsars is given in Table 3. The first six columns describe the spin noise in pulsars. The last two columns demonstrate the evidence for Model PL and Model BPL in pulsars. From Column 7, we see that specific pulsars do not show support in favour of a spectral turnover because \(|\log B_{\text{PL}}| < 2\) for all pulsars.

Next, we employ Equation 8, in order to use all available data for model selection. Performing our analysis consequentially with five different Solar System ephemeris models, we find that Model PL has a weak preference over Model BPL. This result is summarized in Table 4. Note, Tables 3 and 4 contain only results from pulsars where \(\log B_{\text{PL}} > 0\) and \(\log B_{\text{BPL}} > 0\) are obtained for all five different Solar System ephemeris models.

The last column in Table 3, \(\log B_{\text{PL},i}\), presents log Bayes factors in favour of Model M over Model PL. We find that no pulsars show a strong support for Model M. However, pulsars J1939+2134, J1024–0719 and J1713+0747 disfavour Model M with \(\log B_{\text{PL},i} < -4\).

We also consider that our data may contain a mixture of pulsars from two models. For this case, we define a likelihood:

\[
L_B^A(\xi) = \prod_{i=1}^{N_{ps}} \left( \xi z_{i1}^A + (1-\xi) z_{i2}^B \right).
\]  

(18)

where \(\xi\) is a hyper-parameter that determines the fraction of pulsars that are described by model A. The rest of the pulsars are described by model B. Using Equation 18, we estimate the fraction of pulsars that are consistent with a superfluid turbulence origin and a spectral turnover. The results are summarized in Figure 3. We estimate that the fraction of pulses with the spectral turnover is consistent with zero. Our analysis also shows that around 40% of spin noise in the data might be originated from a superfluid turbulence. However, we also find that \(\log B_{\text{PL},i} = 0.6\), where PL2 is the power-law model with \(\gamma = 2\), which is what Model M approaches at frequencies above the spectral turnover. Therefore, we cannot exclude other models that may yield a phenomenological Model M with \(\gamma = 2\).
We perform Bayesian model selection to search for a spectral turnover in pulsar spin noise using the first data release of the IPTA. We find strong support, with a log Bayes factor above 3, for spin noise in nine pulsars, which is consistent with Lentati et al. (2016). However, we find no evidence for a spectral turnover either in individual pulsar data or by combining different pulsars. We also fit the data to the superfluid turbulence model proposed by Melatos & Link (2013). Our results show that whereas this model is indistinguishable from the power-law model for most pulsars, it is strongly disfavored by three pulsars, especially PSR J1939+2134 with a log Bayes factor of 110.

Based on a range of simulations, we find that it is unlikely to resolve a spectral turnover with a fiducial corner frequency of 10 nHz in any pulsar with ≈ 10 years of observations. Longer data spans are required to increase the detection confidence of a spectral turnover in individual pulsars, while a larger number of pulsars with red noise help resolve the presence of a spectral turnover in a population of pulsars. A follow-up study using longer data sets and a larger sample of pulsars, e.g., the IPTA second data release (Perera et al. 2019), will prove useful in not only understanding the nature of red noise in millisecond pulsars but also in evaluating the realistic prospect of gravitational-wave detection.

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\[ M = \frac{15}{14}\pi^{3} ; \]
\[ t_{c} = \frac{\pi 3202}{6\sqrt{5 \pi} a^{3}} \]

Next, we obtain the analytical solution in a form:

\[
P(f) = \frac{3mp^{2}}{4c^2f^2} \left( \frac{1}{128\sqrt{2}\pi^{16/3}} + \frac{3}{704\sqrt{2}\pi^{22/3}} + \frac{9}{3584\sqrt{2}\pi^{28/3}} - \frac{1}{f^2t_{c}^{2}} \left( \frac{1}{48\pi^4} - \frac{1}{96\pi^6} + \frac{3}{512\pi^8} \right) \right) + 
\frac{1}{f^{4t_{c}^{4}}} \left( \frac{1}{82^{3/2}\pi^{8/3}} + \frac{1}{562^{3/2}\pi^{14/3}} + \frac{9}{3202^{3/2}\pi^{20/3}} \right) - 
\frac{1}{f^{6t_{c}^{6}}} \left( \frac{1}{2\sqrt{2}\pi^{4/3}} + \frac{3}{20\sqrt{2}\pi^{10/3}} + \frac{9}{128\sqrt{2}\pi^{16/3}} \right) + 
\frac{1}{f^{8t_{c}^{8}}} \left( \frac{1}{2\pi^{2}} - \frac{3}{16\pi^{4}} - \frac{4\log(\pi)}{3} - \log 4 \right) - 
\frac{1}{f^{10t_{c}^{10}}} \left( \frac{3\sqrt{2}}{\pi^{2/3}} + \frac{9}{82^{3/2}\pi^{8/3}} \right) + 
\frac{1}{f^{12t_{c}^{12}}} \left( \frac{1}{2\pi^2} - \frac{3\log(\pi)}{3} \right) + 
\frac{1}{f^{14t_{c}^{14}}} \left( \log(\pi) + 3 \log(64) \right) \]

\[ (A2) \]

APPENDIX A: EXPLICIT FORM OF MODEL M POWER SPECTRAL DENSITY

The definite integral in Equation 15 yields an analytical solution. First, we reparametrize Equation 15:

\[
M = \frac{15}{14}\pi^{3} ;
\]
\[
t_{c} = \frac{\pi 3202}{6\sqrt{5 \pi} a^{3}} .
\]