Combining Direct & Indirect Kaon CP Violation to Constrain the Warped KK Scale

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Abstract

The Randall-Sundrum (RS) framework has a built in protection against flavour violation, but still generically suffers from little CP problems. The most stringent bound on flavour violation is due to $\epsilon_K$, which is inversely proportional to the fundamental Yukawa scale. Hence the RS $\epsilon_K$ problem can be ameliorated by effectively increasing the Yukawa scale with a bulk Higgs, as was recently observed in arXiv:0810.1016. We point out that incorporating the constraint from $\epsilon'/\epsilon_K$, which is proportional to the Yukawa scale, raises the lower bound on the KK scale compared to previous analyses. The bound is conservatively estimated to be 5.5 TeV, choosing the most favorable Higgs profile, and 7.5 TeV in the two-site limit. Relaxing this bound might require some form of RS flavour alignment. As a by-product of our analysis, we also provide the leading order flavour structure of the theory with a bulk Higgs.

1 Introduction

In generic RS models of a warped extra dimension with bulk fields, the flavour puzzle is solved by the split fermion mechanism, where the localization of fermions is determined based on their masses and mixing angles \cite{1, 2}. Within the RS, this yields extra protection against excess of flavour changing neutral current (FCNC) processes in the form of RS-GIM \cite{3}. A residual little CP problem is, however, still found in the form of too large contributions to the neutron electric dipole moment \cite{3} and sizable contributions to $\epsilon_K$ \cite{4, 5, 6, 7} (see also \cite{8} for some related recent RS flavour studies). Given an IR-localized Higgs field, a lower bound of $\mathcal{O}(20)$ TeV on the KK scale at leading order is obtained \cite{6, 7}.

Recently in \cite{9} it was pointed out, based on matching the full RS set-up onto a two site model (originally suggested in \cite{10}), that if the Higgs is in the bulk and one-loop matching of the gauge coupling is included, the KK scale can be lowered down to $\mathcal{O}(5)$ TeV. An important ingredient in that paper’s analysis is the ability to raise the overall size of the 5D down-type Yukawa coupling, $y_d$. The resulting weaker bound is actually controlled by simultaneously minimizing the contribution to $\epsilon_K$, which effectively falls like $1/(y_d^2)$, with the contribution to $b \to s\gamma$, which grows like $(y_d^2)$. In this paper we point that a contribution to $\epsilon'/\epsilon_K$, similar in structure to $b \to s\gamma$, actually yields a much stronger constraint on the 5D Yukawa size, which implies a strict conservative bound on the KK scale of 7.5 TeV in the two site case. The bound is weakened to 5.5 TeV if one allows the Higgs profile to saturate the AdS stability bound \cite{11}. This is still beyond the LHC reach \cite{12}, and
implies a rather severe little hierarchy problem. We also show that UV-sensitive operators raise the bound significantly, for instance in case the Higgs is localized on the IR brane.

2 Analysis

2.1 Flavour Structure with a Bulk Higgs

In [9] it was pointed out that when the Higgs is in the bulk, the light fermions can be made less composite while still keeping their masses constant, and also the overall Yukawa scale can be increased without violating the corresponding perturbative bound. Both effects allow to ameliorate the RS $\epsilon_K$ problem. In our analysis below we carefully analyze the flavour structure of the theory, allowing a rather general bulk Higgs profile. In most of the past studies, the flavour structure of RS was analyzed via the approximation that the Higgs and any relevant KK states are localized on the IR brane, where a transparent spurion structure can be formulated [3]. Here we consider the couplings by calculating full overlap integrals of wavefunctions, and parametrize these corrections by appropriate functions of the form:

$$r = \frac{\text{wavefunctions overlap}}{\text{approximate coupling on the IR}}.$$  (1)

This can be understood from some relevant sample terms in the 4D effective Lagrangian [3]:

$$L^{4D} \supset \sum_{i,j} Y_{ij}^d \mathcal{H} \left[ \psi_{Q_i}^{0\dagger} f_{Q_i} \psi_{d_j}^0 f_{d_j} r_{00}^H (\beta, c_{Q_i}, c_{d_j}) + \sqrt{2} \sum_n \psi_{Q_i}^{0\dagger} f_{Q_i} \psi_{d_j}^n r_{0n}^H (\beta, c_{Q_i}, c_{d_j}) \right. \\
+ \left. \sqrt{2} \sum_n \psi_{Q_i}^{0\dagger} \gamma_5 f_{d_j}^0 f_{d_j} r_{n0}^H (\beta, c_{Q_i}, c_{d_j}) + 2 \sum_{n,m} \psi_{Q_i}^{0\dagger} \gamma_5 \psi_{d_j}^m r_{nm}^H (\beta, c_{Q_i}, c_{d_j}) \right] + g_s \sum_i G_i \psi_i^{0\dagger} \psi_i^0 \left( -\frac{1}{k\pi R} + f_i^{2g} (c_i) \right).$$  (2)

The term with square brackets is the coupling of the Higgs, $\mathcal{H}$, to quarks of various zero/KK levels, $\psi^{0,n}$, respectively. The other term is the coupling of zero-mode quarks to the first KK gluon, $G_1$ and $i,j$ are flavour indices. For simplicity we only present the down type quark couplings, where $Q (d)$ stands for an SU(2) doublet (singlet) quark. The $f$’s parametrize the values of SM quarks’ profiles on the IR brane (note that in the convention we follow, the value of KK fermions’ wavefunction on the IR brane is $\sqrt{2}$) and the $c$’s are their bulk masses in units of $k$,

$$f(c) = \sqrt{\frac{1 - 2c}{1 - (z_v / z_h)^{2c-1}}}.$$  (3)

The coupling $Y_{ij}^d$ in Eq. (2) is the 5D anarchic down-type Yukawa matrix. We use $y^d$ to denote a generic entry in $Y^d$ (in units of $\sqrt{k}$). Note that in comparison to the notation of [9], $Y_{ij}^d = y^d r_{11}^H (\beta = 1, c_1 = 0.55, c_2 = 0.55)$.

The KK decomposition for the Higgs is $\mathcal{H}(x, z) = \tilde{v}(\beta, z) + \sum_n H^{(n)}(x) \phi_n(z)$ [9], where $\tilde{v}(\beta, z)$ is the Higgs VEV profile, which is very close to the physical Higgs profile when $m_h \ll M_{KK}$ (here
\[\beta = \sqrt{4 + \mu^2}, \text{ with } \mu \text{ being the bulk Higgs mass in units of } k.\] This profile can be chosen to peak near the IR brane:
\[\tilde{v}(\beta, z) = v z^\frac{1}{2} \left(1 + \frac{2z}{z_h} \right)^{2+\beta} \left(\frac{z}{z_v}\right)^{2+\beta}.\] \hspace{1cm} (4)

For the purposes of the following discussion, the only important parameter affecting the overlap corrections is \(\beta\). The \(\beta = 0\) case describes a Higgs maximally-spread into the bulk (saturating the stability bound), while \(\beta = 1\) corresponds to the two-site model considered in [9]. For a concrete comparison we take \(\beta = 1\), and add the case of the weakest expected bound on the KK scale, which is obtained for \(\beta = 0\).

Note that the case of an IR Higgs corresponds to setting the \(r^H\)'s to unity. Full definitions and discussion of the correction factors are presented in appendix A.

### 2.2 RS Contributions to \(\epsilon_K\) and \(b \rightarrow s\gamma\)

We start by considering the bound from \(\epsilon_K\). In this case the largest contribution is generated by left-right effective operators, and in particular by
\[Q_4^K = \tilde{d}_R^0 s_L^0 d_L^3 s_R^3.\] \hspace{1cm} (5)

In the RS framework the leading contribution to \(C_4^K\) (the effective coupling of \(Q_4^K\)) is generated by a tree-level KK-gluon exchange. Up to \(O(1)\) complex factors, this leads to
\[C_4^K \approx \frac{g_{ss}^2}{M_{KK}^2} f_Q f_d f_d f_d r_{00}^g(c_Q) r_{00}^g(c_d) \approx \frac{g_{ss}^2}{M_{KK}^2} \frac{\lambda_d \lambda_s}{y^d^2} r_{00}^g(c_Q) r_{00}^g(c_d).\] \hspace{1cm} (6)

Here \(M_{KK}\) is the scale of the first KK state, \(g_{ss}\) is the dimensionless 5D coupling of the gluon, and \(\lambda_i\) is the SM Yukawa coupling of the quark \(i\) (\(\lambda_i = m_{q_i}/v, v \approx 174\) GeV). SM and 5D Yukawa couplings are connected by the relation \(\lambda_i = y_i^4 f_Q f_d r_{00}^H(\beta, c_Q, c_d)\). Eq. (6) uses the fact that the mixing angles of the rotation matrices from the interaction basis to the mass basis are \(f_Q, f_d\) and \(f_d, f_d\) (\(i \leq j\)) for the quark doublets and singlets, respectively. We have verified numerically that the correction to these relations due to the presence of a bulk Higgs in the relevant range of parameters is a subleading effect. In principle, terms proportional to \(r_{00}^g(c_Q, c_d)\) also contribute, with the same \(f_L^3 s_R^3\) structure; however, since \(r_{00}^g(c_Q, c_d) < r_{00}^g(c_Q, c_d)\), the contribution shown in Eq. (6) is the dominant one.

The result in (6) is similar to the one given in [9]. Taking into account the chirally-enhanced \(\langle K^0|Q_4^K|\bar{K}^0\rangle\) matrix element, assuming an \(O(1)\) CP violating phase\(^1\) for \(C_4^K\), requiring that the NP contribution to \(|\epsilon_K|\) is 60% of the experimental value [13] and evaluating the resulting suppression scale [4] and the quark masses [14] at 5 TeV leads to
\[M_{KK} \gtrsim \frac{15 g_{ss}}{y^d^2} \text{ TeV,}\] \hspace{1cm} (7)

for \(\beta = 1\). The sources of difference from the result of [9] are a correction to the overlap of the quarks with the KK gluon and the 60% saturation requirement. This bound can be ameliorated

\(^1\)Note that here and below we assume a single maximal CP violating phase. Given the fact that each of the observables discussed by us actually receives contributions from multiple independent terms, this is a conservative assumption. A more reasonable approach might be to estimate the sum of the different contributions via a “random walk” approach, which will increase the amplitude by factor of roughly \(\sqrt{N/2}\), where \(N\) is the number of independent terms.
by taking the Higgs to be maximally spread into the bulk ($\beta = 0$), which enhances its coupling to the quarks (and raises the value of the mass corrections $r_{00}^H$ included in the calculation above). The result in this case is

$$M_{KK} \gtrsim \frac{8.5 g_{ss}}{y^d} \text{TeV}. \quad (8)$$

Next we consider $b \to s\gamma$. Here the largest contribution is generated by the effective operator (we follow the conventions of [9])

$$Q_7' = \frac{e m_b}{8\pi^2} b \sigma^{\mu\nu} F_{\mu\nu} (1 + \gamma_5) s.$$ \quad (9)

The effective coupling of $Q_7'$ is generated in RS by a loop diagram with a Higgs propagating in the loop [3], as shown in Figure 1. The corresponding Wilson coefficient, evaluated in appendix B, is

$$C_7' \approx \frac{1}{4\lambda_b M_{KK}^2} f_{Q_3} (y^d)^3 f_{d_2} \bar{r}_{bs} \approx \frac{1}{4M_{KK}^2} \frac{(y^d)^2 \lambda_s}{\lambda_b V_{ts}} \frac{r_{bs}'}{r_{00}^H(\beta, c_{Q_2}, c_{d_2})}, \quad (10)$$

where in the last equation we have used the relation $f_{Q_2}/f_{Q_3} \approx V_{ts}$ between the left-handed profiles ($f_{Q_i}$) and the CKM matrix elements ($V_{ij}$)$^2$. In Eq. (10) we grouped together the overlap corrections to the loop diagram under $\bar{r}_{bs}'$, which contains contributions from different quarks running in the loop. Under the assumption that the Yukawa is anarchical, so that in the bulk interaction basis the bulk masses are diagonal and the $c'$s are well defined [3], $\bar{r}_{bs}'$ is:

$$\bar{r}_{bs}' \approx \sum_{i,j} r_{0n}^H(\beta, c_{Q_3}, c_{d_i}) \left[ 2r_{n-m}^H(\beta, c_{Q_j}, c_{d_i}) - \frac{1}{3} r_{mm}^H(\beta, c_{Q_j}, c_{d_i}) \right] r_{m0}^H(\beta, c_{Q_j}, c_{d_2}), \quad (11)$$

where $i, j$ are flavour indices and $m, n$ are the KK levels of the fermions in the loop, and we only consider the first KK state, since at one-loop the above contribution is finite (or at most log-divergent) [3, 15, 16]. Following the analysis in [9] (allowing 20% departure from the SM value of $B(B \to X_s\gamma)$) and using the values of the quark Yukawa couplings at 5 TeV [14], we obtain for $\beta = 1$ by requiring $C_7'(5 \text{ TeV})/C_7^{SM}(M_W) < 1.4$:

$$M_{KK} \gtrsim 0.4 y^d \text{TeV}. \quad (12)$$

\footnote{We do not distinguish here between gluon and quark KK masses, which only slightly differ in the relevant range of parameters.}
This result is lower than the corresponding bound reported in [9], because of the overlap corrections considered here. Contrary to $\epsilon_K$, the case of $\beta = 0$ yields the same bound as for $\beta = 1$. As discussed in [3, 5, 9], the $b \rightarrow s\gamma$ operator with opposite chirality leads to a weaker condition.

In [9] the constraints from $\epsilon_K$ and $b \rightarrow s\gamma$ in (7) and (12) are combined to evaluate the value of $Y^d_\pi$ that minimizes the lower bound on $M_{KK}$, and the coupling $g_{ss}$ is matched to the measured 4D coupling at one-loop, resulting in $g_{ss} \approx 3$ ($g_{ss} \approx 6$ at the tree-level). Using this analysis for our results naively gives a bound of about 4 TeV, but the value of $y^d$ is above the perturbativity bound $(4\pi/\sqrt{N_{KK}})$ [9]. Taking $y^d$ equal to this bound with the minimal conceivable value $N_{KK} = 2$ gives for $\beta = 1$:

$$M_{KK} \gtrsim 5 \text{ TeV}. \quad (13)$$

### 2.3 The Constraint of $\epsilon'/\epsilon_K$

As anticipated in the introduction, here we show that the bound in (12) becomes substantially stronger after we include an additional constraint from $\text{Re}(\epsilon'/\epsilon)$, the direct CP violating observable of the $K^0 \rightarrow 2\pi$ system. As pointed out in [5], the constraint following from the contribution of the chromomagnetic operator to $\text{Re}(\epsilon'/\epsilon)$ is similar in structure to the $b \rightarrow s\gamma$ one, but is numerically more stringent.

Before analysing the extra contribution to $\text{Re}(\epsilon'/\epsilon)$ generated in the RS framework, it is worth to briefly recall the experimental status of this observable and its prediction within the SM:

- After a series of measurements by the KTeV and the NA48 collaborations, the present experimental world average is $\text{Re}(\epsilon'/\epsilon)_{\exp} = (1.65 \pm 0.26) \times 10^{-3}$ [17].
- $\text{Re}(\epsilon'/\epsilon)_{\text{SM}}$ is dominated by the contributions of two operators: the electroweak penguin, contributing to $\text{Im}(A_2) = \text{Im}[A(K \rightarrow (2\pi)_{I=2})]$, and the QCD penguin ($Q_6$ in the standard notations), contributing to $\text{Im}(A_0)$. The destructive interference of these two contributions is one of the reasons why it is difficult to obtain a precise estimate of $\text{Re}(\epsilon'/\epsilon)_{\text{SM}}$. The negative contribution to $\text{Re}(\epsilon'/\epsilon)_{\text{SM}}$ generated by the electroweak penguins is estimated with 20\%-30\% errors, both on the lattice and using analytic methods [18]. On the other hand, the chiral structure of $Q_6$ and the sizable final-state interactions in the $(2\pi)_{I=0}$ channel prevent, at present, a reliable estimate of this matrix element on the lattice [19]. Recent estimates based on analytic methods [20, 21] lead to values of $\text{Re}(\epsilon'/\epsilon)_{\text{SM}}$ in good agreement with $\text{Re}(\epsilon'/\epsilon)_{\exp}$, with errors ranging from 30\% to 50\%. As a conservative approach, in the following we assume a 100\% error, or $0 < \text{Re}(\epsilon'/\epsilon)_{\text{SM}} < 3.3 \times 10^{-3}$, consistently with the conservative range suggested in [22].

The potentially large new contribution to $\text{Re}(\epsilon'/\epsilon)$ in the RS framework is induced by the two effective chromomagnetic operators

$$Q_G = g_s H^\dagger s_R \sigma^{\mu\nu} T^a G^a_{\mu\nu} d_L, \quad Q'_G = g_s H s_L \sigma^{\mu\nu} T^a G^a_{\mu\nu} d_R. \quad (14)$$

3. The difficulty in estimating $(2\pi|Q_6|K^0)$ on the lattice is confirmed by the difficulty of reproducing the experimental value of $\text{Re}(A_0)$ on the lattice. The latter is affected by similar problems (in particular the large final-state interactions), but it is free from new-physics contaminations. In particular, lattice estimates tend to underestimate $\text{Re}(A_0)$: this provide a qualitative understanding of why lattice estimates of $\text{Re}(\epsilon'/\epsilon)_{\text{SM}}$ are typically smaller (or even negative) compared to the analytic ones.
Similar to the $b \to s\gamma$ case, these are generated by the Higgs-mediated one-loop amplitude in Fig. 1. The coefficients, evaluated in appendix B, are

\[ C_G \approx \frac{3}{16\pi^2 M_{KK}^2} f_{Q_1}(y^d)^2 f_{d} \tilde{r}_{sd} \approx \frac{3}{16\pi^2 M_{KK}^2} (y^d)^2 \lambda_s V_{us} \frac{\tilde{r}_{sd}}{r_{00}^H(\beta, c_{Q_2}, c_{d_2})}, \]

\[ C'_G \approx \frac{3}{16\pi^2 M_{KK}^2} f_{Q_2}(y^d)^2 f_{d} \tilde{r}'_{sd} \approx \frac{3}{16\pi^2 M_{KK}^2} (y^d)^2 \lambda_d V_{us} \frac{\tilde{r}'_{sd}}{r_{00}^H(\beta, c_{Q_1}, c_{d_1})}, \]

where $\tilde{r}_{sd}$ and $\tilde{r}'_{sd}$ group the overlap corrections (see Eq. (48)). Defining

\[ \delta_{\epsilon '} = \frac{\text{Re}(\epsilon'/\epsilon)_{\text{SM}} - \text{Re}(\epsilon'/\epsilon)_{\text{exp}}}{\text{Re}(\epsilon'/\epsilon)_{\text{SM}}}, \]

we obtain

\[ \delta_{\epsilon'} = \frac{\omega \langle (2\pi)_{I=0} | \lambda_s Q_G | K^0 \rangle}{\sqrt{2} \text{Re} A_0 \text{Re}(\epsilon'/\epsilon)_{\text{exp}} |\epsilon|_{\exp}} \left[ \frac{\text{Im}(C_G - C'_G)}{\lambda_s} \right] \approx (58 \text{ TeV})^2 B_G \left[ \frac{\text{Im}(C_G - C'_G)}{\lambda_s} \right], \]

where $B_G$ is the hadronic bag-parameter defined by [23]\(^4\)

\[ \langle 2\pi_{I=0} | \lambda_s Q_G | K^0 \rangle = \sqrt{\frac{3}{2}} \frac{11 m_\pi^2 m_K^2}{4 F_\pi^2} B_G . \]

The value $B_G = 1$ corresponds to the estimate of this hadronic matrix element in the chiral quark model and to the first order in the chiral expansion [24]. A similar numerical value is also obtained using different hadronization techniques [25]. The hadronic matrix element of the chromomagnetic operator is affected by the same difficulties appearing in $\langle 2\pi | Q_0 | K^0 \rangle$: beyond the lowest-order in the chiral expansion we expect large positive corrections from final-state interactions. Moreover, as pointed out in [23], higher order chiral corrections should remove the accidental $m_\pi^2$ suppression in Eq. (18). Therefore the estimate of $\delta_{\epsilon'}$ obtained with $B_G = 1$ can be considered as a conservative lower bound. The leading QCD corrections in running down the Wilson coefficients from the high scale ($\sim 5$ TeV) down to $\sim 1$ GeV are taken into account by the running of the (4D) Yukawa couplings (the residual effect is smaller than 15% [23]). As a result, the ratios $C_G / \lambda_s$ and the matrix element in (18) are, to a good approximation, scale-independent quantities.

Assuming $\mathcal{O}(1)$ CP violating phases for $C_G$ and $C'_G$, barring accidental cancellations among these two terms and imposing $|\delta_{\epsilon'}| < 1$, leads to

\[ M_{KK} \gtrsim 1.3 y^d \text{ TeV} \]

for $\beta = 1$, and

\[ M_{KK} \gtrsim 1.2 y^d \text{ TeV} \]

for $\beta = 0$.

The constraint in Eq. (19) is substantially stronger with respect to the one in Eq. (12) (note also that the former only depends on the down-type Yukawa, while the $b \to s\gamma$ amplitude, which

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\(^4\) Here we adopt the notation of [23], where $F_\pi = 131$ MeV and the $K^0 \to (2\pi)_I$ amplitudes are normalised such that $\text{Re}(A_0) = 3.3 \times 10^{-4}$ MeV (note that the analog of Eq. (18) reported in [5] has a missing factor 1/2). Additional numerical inputs are $\omega = |A_2/A_0| = 0.045$, and $|\epsilon|_{\exp} = 2.23 \times 10^{-3}$. 
Figure 2: The combination of the bounds on $M_{KK}$, as a function of $y^d$, for (a) $\beta = 1$, which corresponds to the two site model; (b) $\beta = 0$, which corresponds to the weakest bound.

is dominated by a charged Higgs contribution, implicitly depends on the up-type Yukawa, too). When combined with Eq. (7), the overall bound is obtained for $y^d \approx 5.9$ (assuming $g_{ss} \approx 3$) and is

$$M_{KK} \gtrsim 7.5 \text{ TeV}.$$  \hspace{1cm} (21)

The lowest possible bound comes from combining Eq. (8) and Eq. (20):

$$M_{KK} \gtrsim 5.5 \text{ TeV}.$$  \hspace{1cm} (22)

The combination of the bounds is shown in Fig. 2.

The reason why the $\epsilon'/\epsilon_K$ constraint is substantially stronger than $b \to s \gamma$ one (more than a factor of 10 at the amplitude level) can be understood by comparing the parametric dependence from quark masses and CKM factors of the $LR$ (chromo-)magnetic operators in the RS framework [5]:

$$A(b_L \to s_R \gamma(g))_{KK} \propto \frac{m_s}{m_b |V_{ts}|^2} \sim \frac{1}{|V_{us}|^2} \hspace{1cm} \text{vs.} \hspace{1cm} A(s_R \to d_L \gamma(g))_{SM} \propto \frac{|V_{ts}|}{|V_{td}|} \sim \frac{1}{|V_{us}|^4}.$$  \hspace{1cm} (23)

In principle, the large enhancement of the $s_R \to d_L$ magnetic transitions in the RS framework occurs also in the short-distance component of the $s \to d \gamma$ amplitude. In most $K$ decays this amplitude is unmeasurable, being obscured by long-distance contributions. The only case were it could be detected is the rare decay $K_L \to \pi^0 e^+ e^-$ [23, 26]. However, present experimental data on this decay mode are still far from the SM level [17], and the corresponding bound on $M_{KK}$ is not competitive even with Eq. (12) (see [27] for a study of rare decays in the context of RS).

2.4 The Case of an IR-Brane Higgs with UV Sensitive Operators

As already mentioned above, there are cases where the leading contributions are from UV sensitive operators. This is expected since the 5D theory is non-renormalizable with negative mass dimension operators. For instance, when the Higgs is localized on the IR brane, the above one-loop is divergent, and a counter-term in the form of a higher dimensional IR operator is included [3, 15, 16]. We can derive a bound on the corresponding cutoff of the theory required to satisfy the constraints
from \( b \to s\gamma \) and \( \epsilon'/\epsilon_K \) (similarly to the \( \mu \to e\gamma \) case [16]). The effective operator on the IR can be written as

\[
(O_{IR}^{bs, sd}) = \frac{g_s^2 Q_d}{(\Lambda_{IR}^{bs, sd})^2} Y_d H \bar{Q}^i \sigma_{\mu\nu} d^j K^\mu\nu,
\]

where for the case of \( b \to s\gamma \) \((\epsilon'/\epsilon_K)\), \( x = 1 \) \((x = 3)\), \( i, j = 3, 2 \) \((i, j = 2, 1 \text{ or } i, j = 1, 2)\), \( Q_d = 1/3 \) \((Q_d = 1)\) and \( K^{\mu\nu} = F^{\mu\nu} \) \((K^{\mu\nu} = G^{\mu\nu})\), the electromagnetic (gluon) field strength. The above expression is simplified when we switch from 5D fields to canonically normalized 4D ones. This cutoff operator can be rewritten (in terms of the zero-modes and 4D couplings) on the IR brane as [16]

\[
O_{IR}^{bs} \sim \frac{\epsilon}{6 \left(\Lambda_{IR}^{bs}\right)^2} \frac{m_s}{V_{ts}} F^{\mu\nu} b \sigma_{\mu\nu} (1 + \gamma_5)s, \quad O_{IR}^{sd} \sim \frac{g}{2 \left(\Lambda_{IR}^{sd}\right)^2} \frac{m_d}{V_{us}} G^{\mu\nu} s \sigma_{\mu\nu} d_R,
\]

where we have replaced the Higgs by its vev and, for simplicity, we only consider the case where \( i, j = 2, 1 \) for \( \epsilon'/\epsilon_K \). Repeating the above analysis, we find the following bounds for \( \Lambda_{IR}^{bs, sd} \)

\[
\Lambda_{IR}^{bs} \gtrsim 8 \text{ TeV}, \quad \Lambda_{IR}^{sd} \gtrsim 20 \text{ TeV}.
\]

### 3 Conclusion

Generic flavour models within the RS framework provide an elegant explanation of the fermion mass hierarchy; however, the resulting suppression of FCNC processes might not be enough. In this paper we have shown that the constraints stemming from \( \epsilon'/\epsilon_K \) yield a lower bound of at least \( \sim 5.5 \) TeV on the KK scale.

The numerical value of this bound is highly insensitive to the precise value of the quark bulk mass parameters. Moreover, there are various reasons to consider this result as very conservative:

- the bag parameter of the chromo-magnetic operator is likely to exceed the reference value we have adopted \((B_G = 1)\), and we have allowed the RS contribution to saturate the experimental measurement of \( \epsilon'/\epsilon_K \);
- we have only considered the first KK level of the quarks and the zero-mode of the Higgs;
- since the value of \( y^d \) is close to the perturbativity bound, the contribution from higher loops is probably not negligible and, a priori, does not need not to be suppressed by \( r_{1-1^-} \);
- the final bound is obtained with an “ideal” Higgs profile.

A more realistic evaluation should result in a stronger bound.

The bound thus obtained is stronger than the one derived from electroweak precision tests, and induces a rather severe little hierarchy problem. If taken a face value, it also implies that a direct LHC discovery of the relevant degrees of freedom is unlikely. This motivates the search for alternative solutions of the residual RS flavour problem, such as the inclusion of some form of flavour alignment of the fundamental down-type sector [16, 28].
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A Overlap Corrections

A common approach is to use the values of fermion zero-modes on the IR brane, parameterized by $f$’s, to evaluate their coupling to the Higgs. This is exact in case the Higgs is localized on the IR brane, but it is only an approximation for a bulk Higgs. A similar approximation is used in the coupling of fermions to the KK gluon, which is concentrated near the IR brane. Since we are trying to constrain the RS flavour model, a more careful treatment is required. In this appendix we estimate the corrections to these approximations by calculating full overlap integrals of the wavefunctions. For consistency, we follow the conventions of [9], and use the definitions given in appendix E of that paper.

The correction factor for the overlap of the Higgs zero-mode with two zero-mode fermions is given by

$$
\rho^H_{00}(\beta, c_L, c_R) \equiv \frac{\int_{z_h}^{z_v} dz (z_h/z)^5 \tilde{v}(\beta, z) \chi_0(c_L, z) \chi_0(c_R, z)/(v\sqrt{k})}{\chi_0(c_L, z_v) \chi_0(c_R, z_v) z_h^4/z_v^4 \chi_0(c_L, z_v) \chi_0(c_R, z_v) z_h^4/z_v^4} = \frac{1}{2 + \beta - c_L - c_R} \sqrt{2(1 + \beta)} \approx \sqrt{2(1 + \beta)} \frac{1}{2 + \beta - c_L - c_R},
$$

where $\tilde{v}(\beta, z)$ is the Higgs zero-mode wavefunction defined in Eq. (4) and $\chi_0(c, z)$ is the fermion zero-mode wavefunction (see also Eq. (3)):

$$
\chi_0(c, z) = \frac{f(c)}{\sqrt{z_h}} \left( \frac{z_h}{z_v} \right)^{1/2 - c} \left( \frac{z}{z_h} \right)^{2 - c}.
$$

The last approximate equality in Eq. (27) is valid to a very good accuracy for $2 + \beta > c_L + c_R$ (which is related to the “switching” behavior discussed in [29]). This result can be conveniently factorized to some approximation by

$$
\rho^H_{00} \approx \frac{4\sqrt{2 + 2\beta}}{2 + \beta - c_L - c_R} \frac{1}{2 - c_L} \frac{1}{2 - c_R},
$$

which is valid for $\beta \lesssim 5$ and when at least one of the $c$’s is close to 0.

Similarly, we define the correction factors for the overlap of the Higgs with a zero-mode fermion and a KK fermion, and with two KK fermions, respectively

$$
\rho^H_{01}(\beta, c_0, c_1) \equiv \frac{\int_{z_h}^{z_v} dz (z_h/z)^5 \tilde{v}(\beta, z) \chi_0(c_0, z) \chi_1(c_1, z)/(v\sqrt{k})}{\chi_0(c_0, z_v) \chi_1(c_1, z_v) z_h^4/z_v^4 \chi_1(c_0, z_v) \chi_1(c_1, z_v) z_h^4/z_v^4},
$$

$$
\rho^H_{11}(\beta, c_1, c_2) \equiv \frac{\int_{z_h}^{z_v} dz (z_h/z)^5 \tilde{v}(\beta, z) \chi_1(c_1, z) \chi_1(c_2, z)/(v\sqrt{k})}{\chi_1(c_1, z_v) \chi_1(c_2, z_v) z_h^4/z_v^4 \chi_1(c_1, z_v) \chi_1(c_2, z_v) z_h^4/z_v^4}.
$$


For simplicity, we focus only on the first KK state of the fermions $\chi_1(c, z)$, defined by

$$\chi_1(c, z) = \frac{1}{N_1 \sqrt{\pi R}} \left( \frac{z}{z_h} \right)^{5/2} [J_\alpha(m_1 z) + b_\alpha(m_1)Y_\alpha(m_1 z)] \tag{31}$$

with

$$-b_\alpha(m_1) = \frac{J_{a-1}(m_1 z_h)}{Y_{a-1}(m_1 z_h)} = \frac{J_{a-1}(m_1 z_v)}{Y_{a-1}(m_1 z_v)},$$

$$N_1 = \frac{1}{2\pi R} \left\{ z_v^2 \left[ J_{a-1}(m_1 z_v) + b_\alpha(m_1)Y_{a-1}(m_1 z_v) \right] - z_h^2 \left[ J_{a-1}(m_1 z_h) + b_\alpha(m_1)Y_{a-1}(m_1 z_h) \right] \right\},$$

and $\alpha \equiv |c + 1/2|$. $r_{01}^H(\beta, c_0, c_1)$ can be computed analytically, but the result is very complicated, while for $r_{11}^H(\beta, c_1, c_2)$ we could not find an analytic solution. Reasonable polynomial fits are given by

$$r_{01}^H(\beta, c_0, c_1) \approx 0.41 + 0.39c_1 - 0.04\beta c_1 + 0.15c_0c_1 + 0.10c_1^2 - 0.24c_1^3,$$

$$r_{11}^H(\beta, c_1, c_2) \approx 0.32 + 0.06\beta - 0.01\beta^2 + 0.08(c_1 + c_2) + 0.20c_1c_2 - 0.08c_1c_2(c_1 + c_2),$$

The first one is valid to an accuracy of about 10% for $\beta \lesssim 10$ (and breaks down in the region where $\beta \sim 0$ and $|c_1 - c_0| \gtrsim 1$) and the second for $\beta \lesssim 6$.

The wavefunction defined in Eqs. (31) and (32) describes a KK fermion with the same chirality as the zero-mode fermion, that is, with $\{++\}$ boundary conditions. There is also a KK fermion with opposite chirality ($\{-\}$ boundary conditions) $\tilde{\chi}_1(c, z)$, defined in the same way as in Eq. (31), but with

$$-\tilde{b}_\alpha(m_1) = \frac{J_{a}(m_1 z_h)}{Y_{a}(m_1 z_h)} = \frac{J_{a}(m_1 z_v)}{Y_{a}(m_1 z_v)},$$

$$\tilde{N}_1 = \frac{1}{2\pi R} \left\{ z_v^2 \left[ J_{a-1}(m_1 z_v) + b_\alpha(m_1)Y_{a-1}(m_1 z_v) \right] - z_h^2 \left[ J_{a-1}(m_1 z_h) + b_\alpha(m_1)Y_{a-1}(m_1 z_h) \right] \right\},$$

and the replacement $c \to -c$. Regarding the overlap of two KK fermions with the Higgs, we actually mostly use the opposite chirality states

$$r_{1-1}^H(\beta, c_1, c_2) \equiv \int_{z_h}^{z_v} dz_h (z_h / z)^5 \tilde{\chi}_1(c_1, z_\chi_1(c_2, z) / (v \sqrt{k}),$$

rather than $r_{11}^H(\beta, c_1, c_2)$. A polynomial fit to $r_{1-1}^H$ is given by

$$r_{1-1}^H(\beta, c_1, c_2) \approx 0.34 - 0.06\beta + 0.01\beta^2 + 0.05(c_1 + c_2) - 0.06(c_1^3 + c_2^3),$$

valid to $\beta \lesssim 5$.

The last correction factor we use is for the coupling of a KK gluon with two zero-mode fermions

$$r_{00}^g(c) \equiv \int_{z_h}^{z_v} dz_h (z_h / z)^4 (f_1(z) - f_1(z_h)) \chi_0^2(c, z) / \sqrt{k},$$

where $f_1(z)$ is the wavefunction of the first KK gluon, and we subtract its value on the UV brane because it represents the flavour-universal part. This formula has a useful approximation, obtained by neglecting the Y-type Bessel function in the KK gluon wavefunction [6, 30]:

$$r_{00}^g(c) = \frac{\sqrt{2}}{J_1(x_1)} \int_0^1 x^{1-2c} J_1(x_1 x) dx \approx \frac{\sqrt{2}}{J_1(x_1)} \frac{0.7}{6 - 4c} \left( 1 + e^{c/2} \right),$$

(38)
with \( x_1 \approx 2.4 \) being the first root of the Bessel function \( J_0(x) = 0 \).

In order to calculate realistic correction factors for the operators considered above, we employ the following procedure. First we choose values for three basic parameters: \( \beta \), \( y^d \) and \( c_{Q_3} \) (the bulk mass parameter of the third generation left-handed quark doublet). The masses of the other left-handed doublets are obtained by the relation \( f_Q/f_{Q_3} \sim V_{ij} \), and the masses of the right-handed quarks are extracted from \( \lambda_i \approx y^d f_Q f_{Q_3} \). Finally, the relevant correction factors for each operator are computed together (note that \( r_{10}^H \) appears only when the mass relation is used).

Using this procedure, it was found that the corrections are actually quite insensitive to the value of \( c_{Q_3} \) in the range of \( 0 - 0.6 \) and to \( y^d \) around the value that minimizes the overall bound, which makes this analysis robust (although for a third generation quark running in the loop the correction is a bit more sensitive to \( c_{Q_3} \)). The only important parameter is \( \beta \), as can be expected (e.g. the overlap of two light fermions with a bulk Higgs is quite different than with an IR brane-localized Higgs).

The main result is that for the operators responsible for \( b \to s \gamma \) and \( e'/e_K \), the coefficients are reduced by about an order of magnitude (for \( \beta = 0, 1 \)), lowering the bound on the KK scale. This primarily stems from the correction \( r_{11}^H - r_{11}^H \) and the mass correction \( r_{21}^H (r_{11}^H - r_{11}^H) \) is always smaller than \( r_{11}^H \), so the inclusion of the former in the dipole operators reduces their contribution relative to what might be naively expected). The \( \epsilon_K \) contribution is actually raised for \( \beta = 1 \), as a result of the KK gluon overlap correction \( r_{21}^g \).

An important comment is in order. The bulk Higgs zero-mode wavefunction is usually obtained by adding a bulk mass for the Higgs and kinetic terms on both branes (otherwise the zero-mode vanishes by boundary conditions). Hence, there is no smooth limit (e.g. \( \beta \to \infty \)) in which the bulk Higgs zero-mode wavefunction corresponds to an IR-localized Higgs.

**B One-Loop Coefficients of the Dipole Operators**

**B.1 The One-Loop Integral**

Up to the overall coupling dictated by the flavour structure and the wave-function overlaps, the amplitude for the diagram in Fig. 1 with an external gluon line (including only the contributions from the down-type flavour sector) is

\[
iA(s \to dg) = \int \frac{d^4k}{(2\pi)^4} \bar{u}(p') \left( \frac{\hat{p} + M_{KK}^{(i)}}{p'^2 - M_{KK}^{(i)2}} (g_s \gamma^\mu t^a G_\mu^a (1 + \gamma_5) u(p)) \frac{1}{k^2 - m_H^2} \right)
\]

\[
= \int \frac{d^4k}{(2\pi)^4} \bar{u}(p') \left[ g_s t^a G_\mu^a M_{KK}^{(i)} (p'^2 - M_{KK}^{(i)2})(p'^2 - M_{KK}^{(i)2})(k^2 - m_H^2) \right] (1 + \gamma_5) u(p),
\]

where \( \hat{p}^{(i)} = p^{(i)} + k \). Neglecting the Higgs mass, this leads to

\[
A(s \to d g) = g_s t^a G_\mu^a M_{KK} \int \frac{d^4l}{(2\pi)^4} \int_0^1 dx \int_0^{1-x} \frac{dym(p')}{y(1+y)} \left\{ p^{(i)2}(x + y) \right\} (1 + \gamma_5) u(p)
\]

\[
= \frac{g_s t^a G_\mu^a}{4(4\pi)^2 M_{KK}} \bar{u}(p') \sigma^{\mu\nu} q_\nu (1 + \gamma_5) u(p),
\]

(39)
where \( q \equiv p' - p \), we have used the equations of motion on the external spinors, and we have set the KK mass equal to the value of the first KK state.

### B.2 Diagonalization of the Quark Mass Matrix

In order to compute the overall coupling of the loop amplitude, we need to address the diagonalization of the quark mass matrix. In general, a zero-mode quark in the interaction basis mixes with the KK states. Restricting the discussion to the first KK level, there are two different states: one with the “right” chirality (\{++\} boundary conditions, similar to the zero-mode), the other with “wrong” chirality (\{-\} boundary conditions, projecting out the zero-mode). The actual contribution to a measurable quantity is then calculated after the mixing matrix is diagonalized to the mass basis (see appendix B in [9] and [31] for similar analyses).

For simplicity, we consider a one generation case, so the mass matrix \( M_q \) is given by

\[
\left( Q^{(0)}_d \quad Q^{(1)}_d \right) M_q \left( \begin{array}{c} d^{(0)}_L \\ Q^{(1)}_R \end{array} \right), \quad M_q = M_{KK} \left( \begin{array}{ccc} x f_Q f_d r_{00}^H & 0 & \sqrt{2} x f_Q r_{01}^H \\ 0 & 2 x r_{1-1}^H & 1 \\ \sqrt{2} x f_d r_{10}^H & 1 & 2 x r_{11}^H \end{array} \right), \tag{41}
\]

with \( x \equiv v y^d / M_{KK} \). \( M_q \) is diagonalized to first order in \( x \) by a bi-unitary transformation:

\[
M_q^{\text{mass}} = O_L^T M_q O_R = M_{KK} \times \text{diag}(x f_Q f_d r_{00}^H, 1 + x(r_{11}^H + r_{1-1}^H), 1 - x(r_{11}^H + r_{1-1}^H)), \tag{42}
\]

where

\[
O_L = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & \sqrt{2} x f_Q r_{01}^H & -x f_Q \\ -2 x f_Q r_{01}^H & 1 + x(r_{11}^H - r_{1-1}^H) & -1 + x(r_{11}^H - r_{1-1}^H) \\ 0 & 1 - x(r_{11}^H - r_{1-1}^H) & 1 + x(r_{11}^H - r_{1-1}^H) \end{pmatrix},
\]

\[
O_R = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & \sqrt{2} x f_d r_{10}^H & x f_d \\ -2 x f_d r_{10}^H & 1 & 1 \\ 0 & 1 & -1 \end{pmatrix}. \tag{43}
\]

The interaction matrix of the quarks with the Higgs in the interaction basis is

\[
\lambda = y^d \begin{pmatrix} f_Q f_d r_{00}^H & 0 & \sqrt{2} f_Q r_{01}^H \\ 0 & 2 r_{1-1}^H & 0 \\ \sqrt{2} f_d r_{10}^H & 0 & 2 r_{11}^H \end{pmatrix}, \tag{44}
\]

and in the quark-mass basis it is simply \( \lambda^{\text{mass}} = O_L^T \lambda O_R \).

The process that couples two opposite chirality zero-mode quarks is carried out through the interaction of one quark with a heavy mass eigenstate, a propagator that couples it to the opposite chirality mass eigenstate and a coupling to the other light quark, summing over the two heavy eigenstates. Specifically, the effective coupling of the dipole amplitude is

\[
A \propto \lambda^{\text{mass}}_{12} \lambda^{\text{mass}}_{13} / (M_{q}^{\text{mass}})_{22} + \lambda^{\text{mass}}_{31} \lambda^{\text{mass}}_{13} / (M_{q}^{\text{mass}})_{33}, \tag{45}
\]

which results into the overall coupling

\[
\frac{12 v (y^d)^3 f_Q f_d r_{00}^H r_{10}^H r_{11}^H}{M_{KK}}. \tag{46}
\]

\footnote{Here \( M_{q}^{\text{mass}} \) should actually be divided by \( M_{KK} \), to avoid double-counting of the propagator with the calculation of the previous appendix, and only consider the flavour structure that the propagator introduces.}
B.3 Coefficients of the Effective Operators

We are now ready to complete the calculation of the one-loop dipole amplitudes and derive the coefficients of the corresponding effective operators by a matching procedure.

In the case of the \( s \rightarrow d g \) amplitude, the complete result is obtained multiplying Eq. (40) and Eq. (46):

\[
A(s \rightarrow d g) = \frac{g_s t^a G_\mu^a}{4(4\pi^2)M_{KK}} \bar{u}(p') \sigma^{\mu\nu} q_\nu (1 \pm \gamma_5) u(p) \frac{12 v(y^d)^3 f_Q f_d r^H_{01} r^H_{10} r^H_{1-1}}{M_{KK}}.
\]

(47)

Inserting the appropriate projection operator, \( \bar{u}(p') \sigma^{\mu\nu} G_\mu q_\nu (1 - \gamma_5) u(p) \) can be identified with \( \bar{s}_R \sigma^{\mu\nu} G_{\mu\nu} d_L \). Hence the coefficient \( C_G \) of the operator in Eq. (14) is given by

\[
C_G = \frac{3}{16\pi^2 M_{KK}^2} f_Q(y^d)^3 f_d r^H_{01} r^H_{10} r^H_{1-1},
\]

(48)

and a similar expression is obtained for the opposite chirality coefficient \( C'_G \).

For \( b \rightarrow s \gamma \), the entire calculation follows in the same way. The coupling of a down-type KK quark to the photon adds a factor of 1/3. Moreover, there are two additional diagrams with a charged Higgs and up-type quarks. We verified that our calculation applies to one of these diagrams, with the photon attached to the KK quark (with another factor of 2, for the charge of an up-type quark relative to a down-type quark). The evaluation of the diagram in which the photon is emitted by the charged Higgs follows similarly. Here we simply use the result of [9], that the contribution of the latter is \( -1/6 \) of the former. Note that this part contains an overlap correction factor \( r^H_{11} \) instead of \( r^H_{1-1} \). Hence the final result for the matching with the operator in Eq. (9) is

\[
C'_7 = \frac{1}{4\lambda_b M_{KK}^2} f_Q(y^u)^2 y^d f_d r^H_{01} r^H_{10} (2 r^H_{1-1} - \frac{1}{3} r^H_{11}),
\]

(49)

where in our actual numerical calculations we assume that \( y^u \) and \( y^d \) are of similar magnitude.

References

[1] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D 61, 033005 (2000) [arXiv:hep-ph/9903417].
[2] S. J. Huber and Q. Shafi, Phys. Lett. B 498, 256 (2001) [arXiv:hep-ph/0010195]; T. Gherghetta and A. Pomarol, Nucl. Phys. B 586, 141 (2000) [arXiv:hep-ph/0003129].
[3] K. Agashe, G. Perez and A. Soni, Phys. Rev. D 71, 016002 (2005) [arXiv:hep-ph/0408134]; K. Agashe, G. Perez and A. Soni, Phys. Rev. Lett. 93, 201804 (2004) [arXiv:hep-ph/0406101].
[4] M. Bona et al. [UTfit Collaboration], arXiv:0707.0636 [hep-ph].
[5] S. Davidson, G. Isidori and S. Uhlig, Phys. Lett. B 663, 73 (2008) [arXiv:0711.3376 [hep-ph]].
[6] C. Csaki, A. Falkowski and A. Weiler, JHEP 0809, 008 (2008) [arXiv:0804.1954 [hep-ph]].
[7] M. Blanke, A. J. Buras, B. Duling, S. Gori and A. Weiler, JHEP 0903, 001 (2009) [arXiv:0809.1073 [hep-ph]].
[8] S. Casagrande, F. Goertz, U. Haisch, M. Neubert and T. Pfoh, JHEP 0810, 094 (2008) [arXiv:0807.4937 [hep-ph]]; M. E. Albrecht, M. Blanke, A. J. Buras, B. Duling and K. Gemmler, arXiv:0903.2415 [hep-ph].

[9] K. Agashe, A. Azatov and L. Zhu, [arXiv:0810.1016 [hep-ph]].

[10] R. Contino, T. Kramer, M. Son and R. Sundrum, JHEP 0705, 074 (2007) [arXiv:hep-ph/0612180].

[11] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B 115, 197 (1982).

[12] K. Agashe, A. Belyaev, T. Krupovnickas, G. Perez and J. Virzi, Phys. Rev. D 77, 015003 (2008) [arXiv:hep-ph/0612015]; B. Lillie, L. Randall and L. T. Wang, JHEP 0709, 074 (2007) [arXiv:hep-ph/0701166].

[13] K. Agashe, M. Papucci, G. Perez and D. Pirjol, [arXiv:hep-ph/0509117]; Z. Ligeti, M. Papucci and G. Perez, Phys. Rev. Lett. 97, 101801 (2006) [arXiv:hep-ph/0604112].

[14] Z. z. Xing, H. Zhang and S. Zhou, Phys. Rev. D 77, 113016 (2008) [arXiv:0711.419 [hep-ph]].

[15] K. Agashe, A. E. Blechman and F. Petriello, Phys. Rev. D 74, 053011 (2006) [arXiv:hep-ph/0606201].

[16] G. Perez and L. Randall, JHEP 0901, 077 (2009) [arXiv:0805.4652 [hep-ph]].

[17] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).

[18] V. Cirigliano, Eur. Phys. J. C 33, S333 (2004).

[19] W. Lee, PoS LAT2006, 015 (2006) [arXiv:hep-lat/0610058].

[20] S. Bertolini, J. O. Eeg and M. Fabbrichesi, Phys. Rev. D 63, 056009 (2001) [arXiv:hep-ph/0002234].

[21] A. Pich, [arXiv:hep-ph/0410215].

[22] A. J. Buras and J. M. Gerard, Phys. Lett. B 517, 129 (2001) [arXiv:hep-ph/0106104].

[23] A. J. Buras, G. Colangelo, G. Isidori, A. Romanino and L. Silvestrini, Nucl. Phys. B 566, 3 (2000) [arXiv:hep-ph/9908371].

[24] S. Bertolini, J. O. Eeg and M. Fabbrichesi, Nucl. Phys. B 449, 197 (1995) [arXiv:hep-ph/9409437].

[25] X. G. He and G. Valencia, Phys. Rev. D 61, 117501 (2000) [arXiv:hep-ph/0003399].

[26] G. Buchalla, G. D’Ambrosio and G. Isidori, Nucl. Phys. B 672, 387 (2003) [arXiv:hep-ph/0308008].

[27] M. Blanke, A. J. Buras, B. Duling, K. Gemmler and S. Gori, JHEP 0903, 108 (2009) [arXiv:0812.3803 [hep-ph]].
[28] A. L. Fitzpatrick, G. Perez and L. Randall, Phys. Rev. Lett. 100, 171604 (2008) [arXiv:0710.1869 [hep-ph]]; J. Santiago, JHEP 0812, 046 (2008) [arXiv:0806.1230 [hep-ph]]; C. Csaki, A. Falkowski and A. Weiler, [arXiv:0806.3757 [hep-ph]]; C. Csaki, Y. Grossman, G. Perez, A. Weiler, and Z. Surujon, in preparation.

[29] K. Agashe, T. Okui and R. Sundrum, Phys. Rev. Lett. 102, 101801 (2009) [arXiv:0810.1277 [hep-ph]].

[30] C. Csaki and D. Curtin, [arXiv:0904.2137 [hep-ph]].

[31] A. J. Buras, B. Duling and S. Gori, arXiv:0905.2318 [hep-ph].