Distributive Stochastic Learning for Delay-Optimal OFDMA Power and Subband Allocation

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Abstract—In this paper, we consider the distributive queue-aware power and subband allocation design for a delay-optimal OFDMA uplink system with one base station, $K$ users and $N_F$ independent subbands. Each mobile has an uplink queue with heterogeneous packet arrivals and delay requirements. We model the problem as an infinite horizon average reward Markov Decision Problem (MDP) where the control actions are functions of the instantaneous Channel State Information (CSI) as well as the joint Queue State Information (QSI). To address the distributive requirement and the issue of exponential memory requirement and computational complexity, we approximate the subband allocation Q-factor by the sum of the per-user subband allocation Q-factor and derive a distributive online stochastic learning algorithm to estimate the per-user Q-factor and the Lagrange multipliers (LM) simultaneously and determine the control actions using an auction mechanism. We show that under the proposed auction mechanism, the distributive online learning converges almost surely (with probability 1). For illustration, we apply the proposed distributed stochastic learning framework to an application example with exponential packet size distribution. We show that the delay-optimal power control has the multi-level water-filling structure where the CSI determines the instantaneous power allocation and the QSI determines the water-level. The proposed algorithm has linear signaling overhead and computational complexity $O(KN)$, which is desirable from an implementation perspective.

I. INTRODUCTION

There are plenty of literature on cross-layer optimization of power and subband allocation in OFDMA systems [1], [2]. Yet, all these works focused on optimizing the physical layer performance and the power/subband allocation solutions derived are all functions of the channel state information (CSI) only. On the other hand, real life applications are delay-sensitive and it is critical to consider the bursty arrivals and delay performance in addition to the conventional physical layer performance (such as sum-rate or proportional fair) in OFDMA cross-layer design. A combined framework taking into account of both queueing delay and physical layer performance is not trivial as it involves both the queueing theory (to model the queue dynamics) and information theory (to model the physical layer dynamics). The first approach converts the delay constraint into average rate constraint using tail probability at large delay regime and solve the optimization problem using information theoretical formulation based on the rate constraint [3], [4]. While this approach allows potentially simple solution, the derived control policy will be a function of the CSI only, which is good only for large delay regime where the probability of buffer empty is small. In general, delay-optimal control actions should be a function of both the CSI and queue state information (QSI). In [5], the authors showed that the Longest Queue Highest Possible Rate (LQHP) policy is delay optimal for multiaccess fading channels. However, the solution utilizes stochastic majorization theory which requires symmetry among the users and is difficult to extend to other situations. In [6], [7], the authors studied the queue stability region of various wireless systems using Lyapunov drift, which is good only for large delay region. While all the above works addressed different aspects of the delay sensitive resource allocation problem, there are still a number of first order issues to be addressed to obtain decentralized resource optimization for delay-optimal uplink OFDMA system.

• The Curse of Dimensionality A more general approach is to model the problem as a Markov Decision Problem (MDP) [8], [9]. However, a primary difficulty in determining the optimal policy using the MDP approach is the huge state space involved. For instance, the state space is exponentially large in the number of users. Consider a simple example. For a system with 4 users, 6 independent subbands, a buffer size of 50 per user and 4 channel states, the state space state contains $4^{4 \times 6} \times (50 + 1)^4$ states, which is already unmanageable.

• Decentralized Solution Most of the solutions in the literature are centralized in which the processing is done at the base station [10] requiring global knowledge of CSI and QSI from $K$ users. However, in the uplink direction, the QSI is only available locally at each of the $K$ users. Hence, centralized solution at the BS requires all the $K$ users to deliver their QSI to the BS, which consumes enormous signaling overhead, and the BS to broadcast the allocation results for the resource allocations at the Mobile side in the uplink system. In addition, the centralized solution also leads to an exponential computational complexity to the BS.

• Convergence of Stochastic Iterative Solution There are a number of works on decentralized OFDMA control using deterministic game [11] or primal-dual decomposition theory for solving deterministic NUM [12]. The derived

1As illustrated later, we could derive the closed form action given the Q-factor. Therefore, the curse of the dimension refers to the exponential growth of state space only. The dimensionality of the action space is not an issue.
distributive algorithms are iterative in nature where all the nodes exchange some messages explicitly in solving the master problem. However, the CSI is always assumed to be quasi-static during the iterative updates with message passing. When we consider delay-optimization, the problem is stochastic in nature and is quite challenging because the game is played repeatedly and the actions as well as the payoffs are defined over ergodic realizations of the system states (CSI, QSI). During the iterative updates, the system state will not be quasi-static anymore.

In this paper, we consider an OFDMA uplink system with one base station (BS), $K$ users and $N_F$ independent subbands. The delay-optimal problem is cast into an infinite horizon average reward constrained Markov Decision Process (MDP). To address the distributive requirement and the issue of exponential memory requirement and computational complexity, we propose a distributive online stochastic learning algorithm, which only requires knowledge of the local QSI and the local CSI at each of the $K$ mobiles and determine the resource control actions using a per-stage action. Using separation of time scales, we show that under the proposed auction mechanism, the distributive online learning converges almost surely. For illustration, we apply the proposed distributive stochastic learning framework to an application example with exponential packet size distribution. We show that the delay-optimal power control has the multi-level water-filling structure where the CSI determines the instantaneous power allocation and the QSI determines the water-level. We show that the proposed algorithm converges to the global optimal solution for sufficiently large number of users. The proposed algorithm has linear signaling overhead and computational complexity $O(KN)$, which is desirable from an implementation perspective.

Fig. 1. OFDMA physical layer and queueing model.

II. SYSTEM MODELS

In this section, we shall elaborate the system model, the OFDMA physical layer model as well as the underlying queueing model. There are one BS and $K$ mobile users (each with one uplink queue) in the OFDMA uplink system with $L$ subcarriers over a frequency selective fading channel with $N_F$ independent multipaths as illustrated in Figure 1. The BS has a cross-layer controller which takes the joint channel state information (CSI) and joint queue state information (QSI) as the inputs and produces a power allocation and subband allocation actions as output.

We first list the important notations in this paper in Table I.

| Symbol | Description |
|--------|-------------|
| $K$    | number of users |
| $N_F$  | number of independent subbands |
| $N_Q$  | buffer size |
| $k$, $n$ | user, subband index |
| $\Omega$ | mean packet size of user $k$ |
| $t$ | slot index |
| $s_{k,n}$, $p_{k,n}$ | subband, power allocation action |
| $\Omega = (\Omega_p, \Omega_s)$ | power and subband allocation policy |
| $H = \{[H_{k,n}]\}$ | joint CSI |
| $Q = (Q_k)$ | joint QSI |
| $A = (A_k)$ | bit/packet arrival vector |
| $\chi$ | global system state |
| $\tau$ | frame duration |
| $\lambda_k$ | average arrival rate of user $k$ |
| $\mu_k(\chi)$ | conditional mean departure rate of user $k$ |
| $P_k$, $P_k^d$ | total power, packet drop rate of user $k$ |
| $\{V(\chi)\}$ | system potential function on $\chi$ |
| $\{Q(\chi, s_k)\}$ | subband allocation Q-factor |
| $\{q_k^q(\chi, H, s)\}$ | per-user subband allocation Q-factor |
| $\gamma_k$ | per-user per-subband allocation Q-factor |
| $\{\xi_k\}$ | LM w.e.l. average power constraint of $k$ |
| $\{\epsilon_k\}$ | LM w.e.l. average pck drop constraint of $k$ |
| $\{\xi_k\}$ | stepsize sequence for per-user potential update |
| $\{\epsilon_k\}$ | stepsize sequence for per-user 2 LMs update |

Table I

LIST OF IMPORTANT NOTATIONS.

A. OFDMA Physical Layer Model

Let $s_{k,n} \in \{0,1\}$ denote the subband allocation for the $k$-th user at the $n$-th subband. The received signal from the $k$-th user at the $n$-th subband of the base station is given by

$$Y_{k,n}^r = s_{k,n}(H_{k,n}X_{k,n}^l + Z_{k,n}),$$

where $X_{k,n}^l$ is the transmitted symbol, $H_{k,n}$ and $Z_{k,n}$ are the random fading and the channel noise of the $k$-th user at the $n$-th subband respectively. The data rate of user $k$ can be expressed as:

$$R_k = \sum_{n=1}^{N_F} R_{k,n} = \sum_{n=1}^{N_F} s_{k,n} \log \left(1 + \xi p_{k,n}|H_{k,n}|^2\right)$$

for some constant $\xi$. Note that the data rate expression in (1) can be used to model both the uncoded and coded systems. For uncoded system using MQAM constellation, the BER of the $n$-th subband and the $k$-th user is given by

$$BER_{k,n} \approx c_1 \exp\left(-c_2 \frac{\Gamma_{k,n}}{\gamma_{k,n}}\right),$$

where $\Gamma_{k,n}$ is the received SNR of the $k$-th user at the $n$-th subband, and hence, for a target BER $\epsilon$, we have $\xi = -\frac{c_2}{m(e/c_1)}$. On the other hand, for system with powerful error correction codes such as LDPC with reasonably large block length (e.g. 8Kbyte) and target PER of 0.1%, the maximum achievable data rate is given by the instantaneous

\[\text{max}\]
mutual information (to within 0.5dB SNR). In that case, \( \xi = 1 \).

B. Source Model, Queue Dynamics and Control Policy

In this paper, the time dimension is partitioned into scheduling slots indexed by \( t \) with slot duration \( \tau \).

**Assumption 1:** The joint CSI of the system is denoted by \( \mathbf{H}(t) = \{H_{k,n}(t)\forall k,n\} \), where \( H_{k,n}(t) \) is a discrete r.v. and distributed according to \( \Pr[H] \). The CSI is quasi-static within a scheduling slot and i.i.d. between scheduling slots.

Let \( \mathbf{A}(t) = (A_1(t), \ldots, A_K(t)) \) be the random new arrivals (number of bits) at the end of the \( t \)-th scheduling slot.

**Assumption 2:** The arrival process \( A_k(t) \) is i.i.d. over scheduling slots according to a general distribution \( \Pr(A_k) \) with average arrival rate \( \mathbb{E}[A_k] = \lambda_k \).

Let \( \mathbf{Q}(t) = (Q_1(t), \ldots, Q_K(t)) \) be the joint QSI of the \( K \)-user OFDMA system, where \( Q_k(t) \) denotes the number of bits in the \( k \)-th queue at the beginning of the \( t \)-th slot. \( N_Q \) denotes the maximum buffer size (number of bits). Thus, the cardinality of the joint QSI is \( |\mathbf{Q}(t)| = (N_Q + 1)^K \), which grows exponentially with \( K \). Let \( N_H \) denote the cardinality of \( H_{k,n} \) (\( \forall k,n \)). Hence, the cardinality of the global CSI is given by \( I_H = N_H^{NK} \).

Let \( \mathbf{R}(t) = (R_1(t), \ldots, R_K(t)) \) (bits/second) be the scheduled data rates of the \( K \) users, where \( R_k(t) \) is given by (1). We assume the controller is causal so that new bit arrivals \( \mathbf{A}(t) \) is observed only after the controller’s actions at the \( t \)-th slot. Hence, the queue dynamics is given by the following equation:

\[
Q_k(t + 1) = \min \left\{ \left[ Q_k(t) - R_k(t)\tau \right]^+ + A_k(t), N_Q \right\},
\]

where \( x^+ \triangleq \max\{x, 0\} \) and \( \tau \) is the duration of a scheduling slot.

For notation convenience, we denote \( \chi(t) = (\mathbf{H}(t), \mathbf{Q}(t)) \) to be the global system state at the \( t \)-th slot. Therefore, the cardinality of the state space of \( \chi \) is \( I_{\chi} = I_H \times I_Q = (N_H^{NK} (N_Q + 1)^K) \). Given the observed system state realization \( \mathbf{H}(t) \) at the beginning of the \( t \)-th slot, the transmitter may adjust the transmit power and subband allocation (equivalently data rate \( \mathbf{R}(t) \)) according to a stationary power control and subband allocation policy defined below.

**Definition 1:** (Stationary Power Control and Subband Allocation Policy) A stationary transmit power and subband allocation policy \( \Omega = (\Omega_p, \Omega_s) \) is a mapping from the system state \( \chi \) to the power and subband allocation actions. A policy \( \Omega \) is called feasible if the associated actions satisfy the average total transmit power constraint and the subband assignment constraint. Specifically, \( \Omega_p(\chi) = p = \{p_{k,n} \geq 0 : \forall k,n\} \) and \( \Omega_s(\chi) = s = \{s_{k,n} \in \{0, 1\} : \forall k,n\} \) satisfy

\[
\sum_{n=1}^{N_P} \mathbb{E}[p_{k,n}] \leq P_k, \forall k \in \{1, K\},
\]

\[
\sum_{k=1}^{N_K} s_{k,n} = 1, \forall n \in \{1, N_P\}
\]

Furthermore, \( \Omega \) also satisfies an average packet drop rate constraint for each queue as follows

\[
\Pr[Q_k = N_Q] \leq P_k^t, \forall k \in \{1, K\}
\]

From (1), the vector queue dynamics is Markovian with the transition probability given by

\[
\Pr[\mathbf{Q}(t+1)|\mathbf{Q}(t), \Omega(\chi(t))]
\]

\[
= \Pr[\mathbf{A}(t) = Q(t+1) - \mathbf{Q}(t) - \mathbf{R}(t)\tau^+]
\]

\[
= \prod_k \Pr[A_k(t) = Q_k(t+1) - [Q_k(t) - R_k(t)\tau]^+]
\]

(6)

Note that the \( K \) queues are coupled together via the control policy \( \Omega \) and the constraint in (4). From Assumption 2, the induced random process \( \chi(t) = (\mathbf{H}(t), \mathbf{Q}(t)) \) is Markovian with the following transition probability:

\[
\Pr[\chi(t+1)|\chi(t), \Omega(\chi(t))]
\]

\[
= \Pr[\mathbf{H}(t+1)|\chi(t), \Omega(\chi(t))] \Pr[\mathbf{Q}(t+1)|\chi(t), \Omega(\chi(t))]
\]

\[
= \Pr[\mathbf{H}(t+1)] \Pr[\mathbf{Q}(t+1)|\chi(t), \Omega(\chi(t))]
\]

(7)

where \( \Pr[\mathbf{Q}(t+1)|\chi(t), \Omega(\chi(t))] \) is given by (6). Given a unichain policy \( \Omega \), the induced Markov chain \( \{\chi(t)\} \) is ergodic and there exists a unique steady state distribution \( \pi_\chi \) such that \( \pi_\chi(\chi) = \lim_{t \to \infty} \Pr[\chi(t) = \chi] \). The average utility of the \( k \)-th user under a unichain policy \( \Omega \) is given by:

\[
\mathbb{T}_k(\Omega) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}[f(Q_k(t))] = \mathbb{E}_{\pi_\chi}[f(Q_k)],
\]

(8)

where \( f(Q_k) \) is a monotonic increasing function of \( Q_k \) and \( \mathbb{E}_{\pi_\chi} \) denotes expectation w.r.t. the underlying measure \( \pi_\chi \). For example, when \( f(Q_k) = \frac{Q_k}{Q_k^0} \), \( \mathbb{T}_k(\Omega) = \frac{1}{\mathbb{E}_{\pi_\chi}[Q_k]} \) is the average delay of the \( k \)-th user. Another interesting example is the queue outage probability \( \mathbb{T}_k(\Omega) = \Pr[Q_k \geq Q_k^0] \), in which \( f(Q_k) = 1 \) \( Q_k \geq Q_k^0 \), where \( Q_k^0 \in \{0, N_Q\} \) is the reference outage queue state. Similarly, the average transmit power constraint in (3) and the packet drop constraint in (5) can be written as

\[
\mathbb{P}_k(\Omega) = \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E}\left[ \sum_{n} p_{k,n}(t) \right] = \mathbb{E}_{\pi_\chi}\left[ \sum_{n} p_{k,n} \right] \leq P_k,
\]

(9)

where \( \mathbb{E}_{\pi_\chi} \) denotes expectation w.r.t. the measure \( \pi_\chi \). For example, when \( f(Q_k) = Q_k^0 \), \( \mathbb{P}_k(\Omega) = \mathbb{E}_{\pi_\chi}[Q_k] \) is the average power of the \( k \)-th user. Another interesting example is the queue outage probability \( \mathbb{T}_k(\Omega) = \Pr[Q_k \geq Q_k^0] \), in which \( f(Q_k) = 1 \) \( Q_k \geq Q_k^0 \), where \( Q_k^0 \in \{0, N_Q\} \) is the reference outage queue state. Similarly, the average transmit power constraint in (3) and the packet drop constraint in (5) can be written as
Thus, the corresponding unconstrained MDP for a particular
of a
problem is formulated as
A CMDP, which is summarized below.

A. CMDP Formulation

A MDP can be characterized by a tuple of four objects,

- **State Space**: The state space for the MDP is given by
  \( \{ \chi^1, \cdots, \chi^L \} \), where \( \chi^1 = (H^1, Q^1) \) (1 \( \leq i \leq I_K \)) is a realization of the global system state.

- **Action Space**: The action space of the MDP is given by
  \( \{ \Omega(\chi^1), \cdots, \Omega(\chi^L) \} \), where \( \Omega \) is a unichain feasible policy as defined in Definition 1.

- **Transition Kernel**: The transition kernel of the MDP
  \( \Pr[ \chi^j | \chi^i, \Omega(\chi^i) ] \) is given by (7).

- **Per-stage Reward**: The per-stage reward function of the
  MDP is given by \( d(\chi, \Omega(\chi)) = \sum_k \beta_k f(Q_k) \).

As a result, the delay-optimal control can be formulated as
a CMDP, which is summarized below.

**Problem 1 (Delay-Optimal Constrained MDP)**: For some positive constants \( \beta = (\beta_1, \cdots, \beta_K) \), the delay-optimal problem is formulated as

\[
\min_{\Omega \in \Omega} J_{\beta}(\Omega) = \sum_{k=1}^{K} \beta_k T_k(\Omega)
\]

\[
= \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E} [d(\chi(t), \Omega(\chi(t)))]
\]

s.t. the power and packet drop rate constraints in (9), (10).

B. Lagrangian Approach to the CMDP

For any LMs \( \chi^k, \gamma^k > 0 \), define the Lagrangian
as

\[
L_{\beta}(\Omega, \gamma) = \lim_{T \rightarrow \infty} \frac{1}{T} \sum_{t=1}^{T} \mathbb{E} [g(\gamma, \chi, \Omega(\chi))],
\]

where \( \gamma = (\gamma^1, \cdots, \gamma^K) \) with \( \gamma^k = (\gamma^k, \gamma^k) \), \( g(\gamma, \chi, \Omega(\chi)) = \sum_k (\beta_k f(Q_k) + \sum_k \beta_k \gamma^k (1 | Q_k = N_Q - P_k^i)) \).

Thus, the corresponding unconstrained MDP for a particular
LM \( \gamma \) is given by

\[
G(\gamma) = \min_{\Omega, \gamma} L_{\beta}(\Omega, \gamma)
\]

where \( G(\gamma) \) gives the Lagrangian dual function. The dual problem of the primal problem in Problem 1 is given by \( \max_{\gamma > 0} G(\gamma) \). The general solution to the unconstrained MDP in (12) is summarized in the following lemma.

**Lemma 1**: (Bellman Equation and Subband Allocation Q-factor) For a given \( \gamma \), the optimizing policy for the unconstrained MDP in (12) can be obtained by solving the Bellman equation (associated with the MDP in (11)) w.r.t. \((\theta, \{Q(\chi, s)\})\) as below:

\[
Q(\chi^i, s) = \min_{\Omega(\chi^i)} \left[ \sum_{s^i} \Pr[\chi^j | \chi^i, s, \Omega(\chi^i)] \min_{\Omega(\chi^i)} Q(\chi^i, s^i) \right] - \theta
\]

\[
\forall 1 \leq i \leq I_K, \forall s (13)
\]

Proof: Please refer to Appendix A for the proof.

Using standard optimization theory [14], the problem in (12) has an optimal solution for a particular choice of the LM \( \gamma = \gamma^* \), where \( \gamma^* \) is chosen to satisfy the average power constraint in (9) and packet drop constraint in (10). Moreover, it is shown in [15] that the following saddle point condition holds:

\[
L(\Omega^*, \gamma) \leq L(\Omega^*, \gamma^*) \leq L(\Omega, \gamma^*)
\]

(14)

In other words, \( (\Omega^*, \gamma^*) \) is a saddle point of the Lagrangian, then \( \Omega^* \) is the primal optimal (i.e. solving Problem 1), \( \gamma^* \) is the dual optimal (solving the dual problem) and the duality gap is zero. Therefore, by solving the dual problem, we can obtain the primal optimal \( \Omega^* \).

Remark 1: The optimal control actions are functions of the
subband allocation Q-factor \( \{Q(\chi, s)\} \) and the 2\( K \) LMs. Unfortunately, for any given LMs, determining the subband allocation Q-factor involves solving the Bellman equation in (13), which is a fixed point problem over the functional space with exponential complexity. In other words, it is a system of \( K^{N_F} I_K = K^{N_F} (N_F(N_Q+1)K) \) non-linear equations with \( K^{N_F} I_K + 1 \) unknowns \( (\theta, \{Q(\chi, s)\}) \). Furthermore, even if we could solve it, the solution will be centralized and the joint CSI and QSI knowledge will be required, which is highly undesirable.

\( ^{\text{It is known that for CDMP, the optimal policy may be randomized policy. However, for implementation consideration, we are interested in deterministic policy in this paper.}} \)

\( ^{\text{For sufficiently large total transmit power \( \{P_1, \cdots, P_K\} \) so that the optimization problem in (11) is feasible, and the state \( \chi = (H, Q) (\forall H \text{ and } Q = (0, \cdots, 0)) \) is recurrent.}} \)
IV. GENERAL DECENTRALIZED SOLUTION VIA LOCALIZED STOCHASTIC LEARNING AND AUCTION

The key steps in obtaining the optimal control policies from the R.H.S. of the Bellman equation in (13) rely on the knowledge of the subband allocation Q-factor \( Q(\chi, s) \) and the \( 2K \) LMs \( \{\gamma^k(\chi, s)\} (1 \leq k \leq K) \), which is very challenging. Brute-force solution of \( Q(\chi, s) \) and \( 2K \) LMs has exponential complexity and requires centralized implementation and knowledge of the joint CSI and QSI (which also requires huge signaling overheads). In this section, we shall approximate the subband allocation Q-factor \( Q(\chi, s) \) by the sum of per-user subband allocation Q-factor \( Q^k(\chi_k, s_k) \), i.e. \( Q(\chi, s) \approx \sum_k Q^k(\chi_k, s_k) \). Based on the approximate Q-factor, we shall derive a per-stage decentralized control policy using a per-stage auction. Next, we shall propose a localized online stochastic learning algorithm (performed locally at each MS) to determine the per-user Q-factor \( Q^k(\chi_k, s_k) \) as well as the two local LMs \( \gamma^k = (\gamma^k_1, \gamma^k_2) \) based on observations of the local CSI and local QSI as well as the auction result. Furthermore, we shall prove that under the proposed per-stage auction, the local online stochastic learning algorithm converges almost surely (with probability 1).

A. Linear Approximation on the Subband Allocation Q-Factor and Distributive Power Control

Denote the per-user system state, channel state, subband allocation actions and power control actions as \( \chi_k = (Q_k, H_k) \), \( H_k = \{[H_{k,n}] : \forall n\} \), \( s_k = \{s_{k,n} : \forall n\} \) and \( p_k = \{p_{k,n} : \forall n\} \), respectively. To reduce the size of the state space and to decentralize the resource allocation, we approximate \( Q(\chi, s) \) by the sum of per-user subband allocation Q-factor \( Q^k(\chi_k, s_k) \), i.e.

\[
Q(\chi, s) \approx \sum_k Q^k(\chi_k, s_k)
\]

(15)

where \( Q^k(\chi_k, s_k) \) satisfies the following per-user subband allocation Q-factor fixed point equation for each MS \( k \):

\[
Q^k(\chi_k, s_k) = \min_{p_k} \left[ g_k(\gamma^k, \chi_k, s_k, p_k) + \sum_{s_k} \Pr[\chi_k|\chi_k, s_k, p_k] W^k(\chi_k) \right] - \theta^k,
\]

\(1 \leq i \leq I^k_\chi, \forall s_k \) (16)

where \( g_k(\gamma^k, \chi_k, s_k, p_k) = \beta_k f(Q_k) + \gamma^k (\sum_{k} p_{k,n} - p_k) + \gamma^k(1 - N_Q - P_k) \) and \( W^k(\chi_k) = \mathbb{E}[Q^k(\chi_k, \{s_{k,n} = 1\}|H_{k,n} \geq H_{k,n}^{k-1})] |\chi_k | \) (\( H_{k,n}^{k-1} \) denotes the largest order statistic of the \( k-1 \) i.i.d. random variables with the same distribution as \( |H_{k,n}| \)), and \( I^k_\chi = N^N_H(N_Q + 1) \) is the cardinality of the space of per-user system state. Note that under the subband allocation Q-factor approximation, the state space of \( K \) users is significantly reduced from \( I_\chi = (N^N_H(N_Q + 1))^K \) to \( K I^k_\chi = K N^N_H(N_Q + 1) \).

B. Per-Stage Subband Auction

The subband allocation control can be obtained by minimizing the original subband allocation Q-factor in (13) over subband allocation actions. Using the approximate Q-factor, the subband allocation control is given by \( Q^*_s(\chi) = \arg \min_{s} Q(\chi, s) \approx \arg \min_{s} \sum_k Q^k(\chi_k, s_k) \). This can be obtained via a per-stage subband auction with \( K \) bidders (MSs) and one auctioneer (BS) based on the observed realization of the system state at each MS \( \chi_k \). The Per-Stage Subband Auction among \( K \) MSs is as follows:

- **Bidding**: Based on the local observation \( \chi_k \), each user \( k \) submits his bid \( Q^k(\chi_k, s_k) \).
- **Subband Allocation**: The BS assigns subbands to achieve the maximum sum bids, i.e.

\[
s^* = Q^*_s(\chi) = \arg \min_{s} \sum_k Q^k(\chi_k, s_k)
\]

(17)

and then broadcasts the allocation results \( s^* = \{s_k^* : \forall k\} \) to \( K \) users.
- **Power Allocation**: Based on the subband allocation result \( s_k^* \), each user \( k \) determines the transmit power, which minimizes the R.H.S. of (16), i.e.

\[
p_k^* = \Omega_{\mathcal{P}_k}(\chi) g_k(\gamma^k, \chi_k, s_k, p_k)
\]

\[
= \arg \min_{p_k} \left[ + \sum_{s_k} \Pr[\chi_k|\chi_k, s_k, p_k] W^k(\chi_k) \right] - \theta^k
\]

\(18\)

**Remark 2**: (Optimal Subband and Power Allocation under Q-factor Approximation) In proposed per-stage subband auction, the subband allocation actions minimize \( \sum_k Q^k(\chi_k, s_k) \), and the power allocation actions at each MS minimizes the R.H.S. of the per-user subband allocation Q-factor fixed point equation in (16). Therefore, the proposed per-stage subband auction achieves the solution of the Bellman equation in (13) under the linear Q-factor approximation in (15).

**Remark 3**: (Computational Complexity and Memory Requirement Reduction at BS) With the per-stage subband auction mechanism, the BS does not need to store the per-user subband allocation Q-factor \( \{Q^k(\chi_k, s_k)\} (\forall k) \) and \( 2K \) LMs for all the MSs, which greatly reduced the memory requirement at the BS. On the other hand, the BS does not need to perform power allocation for each MS on each subband \( p_{k,n} (\forall k, n) \), which significantly reduces the computational complexity at the BS.

C. Online Per-user Primal-Dual Learning Algorithm via Stochastic Approximation

Since the derived power and subband allocation policies are all functions of the per-user subband allocation Q-factor and LMs, we shall propose an online localized learning algorithm to estimate \( \{Q^k(\chi_k, s_k)\} \) and LMs \( \gamma^k \) at each MS \( k \). For notation convenience, we denote the per-user state-action combination as \( \varphi = (\chi_k, s_k) (\forall k) \) be the dummy indices enumerating all the per-user state-action combinations of each user with cardinality \( I_\chi = 2^{N^N_H} I^k_\chi \). Let \( Q_k^\varphi = (Q^k(\varphi^1), \ldots, Q^k(\varphi^{I_\chi}))^T \) be the vector of per-user Q-factor for user \( k \). Let \( \varphi_k(t) \triangleq (\chi_k(t), s_k(t)) \) be the state-action pair observed at MS \( k \) at the \( t \)-th slot, where \( \chi_k(t) = (Q_k(t), H_k(t)) \) is the system state realization observed at MS.
Based on the current observation \( \varphi_k(t) \), user \( k \) updates its estimate on the per-user Q-factor and the LMs according to:

\[
\begin{align*}
Q_{t+1}^k(\varphi') &= Q_t^k(\varphi') + \epsilon_t^k(\varphi, t) \left[ g_k(\gamma_t^k, \varphi', \mathbf{p}_k(t)) 
+ \hat{W}_t^k(Q_k(t+1)) - (g_k(\gamma_t^k, \varphi', \mathbf{p}_k(t)) 
+ \hat{W}_t^k(Q_k(t+1)) - Q_t^k(\varphi')) \right] \\
\epsilon_t^{\pi k}(t) &= \epsilon_t^k(\mathbf{p}_k(t) - P_k) \\
\gamma_t^{p k}(t) &= \gamma_t^k(1|Q_t(t) - N_Q - P_k) \\
\end{align*}
\]

where \( l_k(\varphi', t) \triangleq \sum_{m=0}^t [\varphi_k(m) = \varphi'] \) is the number of updates of \( Q_t^k(\varphi') \) till \( t \). \( \mathbf{p}_k(t) = \{p_{k,n}(t) : \forall n\} \) is the power allocation actions given by the per-stage auction, \( \tilde{W}_t^k(Q_k(t)) \triangleq \mathbb{E}[\tilde{W}_t^k(\mathbf{x}_k)|Q_k] \) with \( \tilde{W}_t^k(\mathbf{x}_k) = \mathbb{E}[Q_t^k(\mathbf{x}_k; \{s_{k,n} = 1|H_{k,n} \geq H_{k-1}^*\})] \) and \( T \triangleq \sup \{ t : \varphi_k(t) = \varphi' \} \). \( \varphi' \) is the reference per-user state-action combination. \( \Gamma(\cdot) \) is the projection onto an interval \([0,B]\) for some \( B > 0 \) and \( \{\epsilon_t^k\}, \{\epsilon_t^{\pi k}\} \) are the step size sequences satisfying the following conditions:

\[
\begin{align*}
\sum_{t} \epsilon_t^k &= \infty, \epsilon_t^k \geq 0, \epsilon_t^k \to 0, \sum_{t} \epsilon_t^{\pi k} &= \infty, \epsilon_t^{\pi k} \geq 0, \epsilon_t^{\pi k} \to 0, \\
\epsilon_t^{(\epsilon_t^k)^2 + 2(\epsilon_t^{\pi k})^2} &< \infty, \frac{\epsilon_t^{\pi k}}{\epsilon_t^k} \to 0
\end{align*}
\]

The above distributive per-user potential learning algorithm requires knowledge on local QSI and CSI. The following analysis shows the convergence of the algorithm.

**Remark 4 (Comparison to the Deterministic NUM):** In conventional iterative solutions for deterministic NUM \([12]\), the iterative updates (with message exchange) are performed within the CSI coherence time and hence, this limits the number of iterations and the performance. However, in the proposed online algorithm, the updates evolve in the same time scale as the CSI and QSI. Hence, it could converge to a better solution because the number of iterations is no longer limited by the coherence time of CSI.

**Remark 5 (Comparison to the Conventional Reinforced Learning):** There are two key novelties in the proposed per-user online update algorithms. Firstly, most of the existing literature regarding online learning addressed unconstrained MDP only \([9]\). In the case of CMDP, the LM are determined offline by simulation \([17]\). In our case, both the LM and the per-user Q-factor are updated simultaneously. Secondly, conventional online learning are designed for centralized solution where the control actions are determined entirely from the potential or Q-factor update. However, in our case, the control actions for user \( k \) are determined from \( \{Q_t^k(\varphi') \} \) via a per-stage auction. During the iterative updates, both the per-user Q-factor/LMs as well as the control actions are changed dynamically and the existing convergence results (based on contraction mapping argument) cannot be applied directly to our distributive stochastic learning algorithm.

**D. Convergence Analysis**

In this section, we shall establish technical conditions for the almost-sure convergence of the online distributive learning algorithm. For any LM \( \gamma_k(\gamma_k^k, Q_k) \triangleq \min_{p_k} g_k(\gamma_k^k, \varphi', p_k) + \sum_{\varphi_k} Pr[\varphi | \varphi', p_k]Q_k(\varphi') \), where \( Pr[\varphi | \varphi', p_k] = Pr[\chi_k | \varphi', p_k] Pr[s_{k,n} | \varphi'] \) is the power allocation for \( \varphi' \) obtained by per-stage subband auction at the \( t \)-th iteration, and \( \varphi_k \) is the identity matrix.

Since we have two different step size sequences \( \{\epsilon_t^k\} \) and \( \{\epsilon_t^{\pi k}\} \), the LM updates and the per-user Q-factor updates are done simultaneously but over two different time scales. During the per-user Q-factor update (timescale I), we have \( \tau_{t+1} - \tau_t = e(t) \) and \( \tau_{t+1} - \gamma^k_t = e(t) - \gamma^k_t \) for \( \forall k \). Therefore, the LM appears to be quasi-static \([18]\) during the per-user Q-factor update in \([19]\). We shall have the following lemma.

**Lemma 2:** (Convergence of Per-user Potential Learning Algorithm) Assume for all the feasible policies \( \Omega \) in the policy space, there exists a \( \delta_m \triangleq O(\epsilon_m^k) > 0 \) and some positive integer \( m \) such that

\[
[A_{m}^{k} \cdots A_{1}^{k}]_{ir} \geq \delta_m, \quad [B_{m}^{k} \cdots B_{1}^{k}]_{ir} \geq \delta_m, \quad 1 \leq i \leq I \varphi
\]

where \( [\cdot]_{ir} \) denotes the element of the \( i \)-th row with \( r \)-th column of the corresponding \( I \varphi \times I \varphi \) matrix \( P^k \) which contains the aggregate reference state \( \varphi' \). For stepsize sequence \( \{\epsilon_t^k\}, \{\epsilon_t^{\pi k}\} \) satisfying the conditions in \([22]\), we have \( \lim_{t \to \infty} Q_t^k = Q_\infty^k \forall k \) a.s. for any initial per-user subband allocation Q-factor vector \( Q_0^k \) and LM \( \gamma_k \), where the converged per-user subband allocation Q-factor vector \( Q_\infty^k \) satisfies:

\[
(T^k(\gamma^k, Q_\infty^k) - Q_\infty^k(\varphi'))e + Q_\infty^k(\gamma = T^k(\gamma^k, Q_\infty^k)(25)
\]

**Proof:** Please refer to Appendix B.

On the other hand, during the LM update (timescale II), we have \( \lim_{t \to \infty} [Q_t^k - Q_\infty^k(\gamma)] = 0 \) a.s. 1 by the Corollary 2 of \([19]\). Hence, during the LM updates in \([20]\) and \([21]\), the per-user subband allocation Q-factor update is seen as almost equilibrated. The convergence of the LM is summarized below.

**Lemma 3** (Convergence of the LM over Timescale II): The iterates \( \lim_{t \to \infty} \gamma_t = \gamma_\infty \) a.s., where \( \gamma_\infty \) satisfies the power and packet drop rate constraints in \([40]\) and \([41]\).

**Proof:** Please refer to Appendix C.

Based on the above lemmas, we shall summarize the per-user Q-factor convergence of the online per-user Q-factor and LM learning algorithm in the following theorem.
Theorem 1 (Convergence of Online Per-user Learning Algorithm): Given by (11).

For the same conditions as in Lemma 2, we have $(Q^k_1, \gamma^k_1) \rightarrow (Q^\infty_1, \gamma^\infty_1)$ a.s. for all $k$, where $Q^\infty_1$ and $\gamma^\infty_1$ satisfy

$$T^k_1(\gamma^k_1, Q^\infty_1) - Q^\infty_1(\gamma^k_1)e + Q^\infty_1 = T^k(\gamma^k_1, Q^k_1)$$

and $\gamma^\infty_1$ satisfies the power and packet drop rate constraints in (9) and (10).

V. APPLICATION TO THE OFDMA SYSTEMS WITH EXPONENTIAL PACKET SIZE DISTRIBUTION

In this section, we shall illustrate the application of the proposed stochastic learning algorithm by an example with exponential packet size distribution.

A. Dynamics of System State under Exponential Distributed Packet Size

Let $A(t) = (A_1(t), \cdots, A_K(t))$ and $N(t) = (N_1(t), \cdots, N_K(t))$ be the random new packet arrivals and the packet sizes for the $K$ users at the $t$-th scheduling slot, respectively. $Q(t) = (Q_1(t), \cdots, Q_K(t))$ and $N_Q$ denote the joint QSI (number of packets) at the end of the $t$-th scheduling slot and the maximum buffer size (number of packets).

Assumption 3: The arrival process $A_k(t)$ is i.i.d. over scheduling slots according to a general distribution $P_r(A_k)$ with average arrival rate $E[A_k] = \lambda_k$. The random packet size $N_k(t)$ is i.i.d. over scheduling slots following an exponential distribution with mean packet size $N_k$.

Given a stationary policy, define the conditional mean departure rate of packets of user $k$ at the $t$-th slot (conditioned on $\chi(t)$) as $\mu_k(\chi(t)) = R_k(\chi(t))/N_k$.

Assumption 4: The slot duration $\tau$ is sufficiently small compared with the average packet service time, i.e. $\mu_k(\chi(t))\tau \ll 1$. Given the current system state $\chi(t)$ and the control action, and conditioned on the packet arrival $A(t)$ at the end of the $t$-th slot, there will be a packet departure of the $k$-th user at the $(t+1)$-th slot if the remaining service time of a packet is less than the current slot duration $\tau$. By the memoryless property of the exponential distribution, the remaining packet length (also denoted as $N(t)$) at any slot $t$ is also exponential distributed. Hence, the transition probability to $Q_k(t+1)$ at the $(t+1)$-th slot corresponding to a packet departure event.

12This assumption is reasonable in practical systems. For instance, in the UL WiMAX (with multiple UL users served simultaneously), the minimum resource block that could be allocated to a user in the UL is $8 \times 16$ symbols $- 12$ pilot symbols $\Rightarrow 116$ symbols. Even with 64QAM and rate $\frac{3}{4}$ coding, the number of payload bits it can carry is $116 \times 3 = 348$ bits. As a result, when there are a lot of UL users sharing the WiMAX AP, there could be cases that the MPEG4 packet (around 10K bits) from an UL user cannot be delivered in one frame. In addition, the delay requirement of MPEG4 is 500ms or more, while the frame duration of WiMax is 5ms. Hence, it is not necessary to serve one packet during one scheduling slot so that the scheduler has more flexibility in allocating resource. Therefore, in practical systems, an application level packet may have mean packet length spanning over many time slots (frames) and this assumption is also adopted in [20]–[23].

13Since $N_k(t)$ is exponentially distributed and is memoryless, we have the probability in (27) (conditioned on the current state $\chi(t)$ and the associated action $\Omega(\chi(t))$) independent of the previous states $\{\chi(t-1), \chi(t-2), \cdots\}$. 

$$\Pr[Q_k(t+1) = A_k(t) + Q_k(t) - 1|\chi(t), A(t), \Omega(\chi(t))]$$

$$\Pr[\frac{N_k(t)}{R_k(t)} < \tau|\chi(t), A(t), \Omega(\chi(t))]$$

$$\Pr[\frac{N_k(t)}{N_k} < \mu_k(\chi(t))\tau]$$

$$1 - \exp(-\mu_k(\chi(t))\tau) \approx \mu_k(\chi(t))\tau$$

where the last equality is due to Assumption 4. Note that the probability for simultaneous departure of two or more packets from the same queue or different queues in a slot is $O((\mu_k(\chi(t))\tau)^2)$, which is asymptotically negligible. Therefore, the vector queue dynamics is Markovian with the transition probability given by

$$\Pr[Q(t+1)|\chi(t), \Omega(\chi(t))]$$

$$\sum_k \Pr[A(t) = Q(t+1) - Q(t) + e_k]\mu_k(\chi(t))\tau$$

$$+ \Pr[A(t) = Q(t+1) - Q(t)](1 - \sum_k \mu_k(\chi(t))\tau)$$

where $e_k$ denotes the standard basis vector with 1 for its $k$-th component and 0 for every other component.

B. Decomposition of the Per-user Subband Allocation $Q$-factor

In the following lemma, we shall show that the per-user subband allocation $Q$-factor $Q^k(\chi_k, s_k)$ can be further decomposed into the sum of per-user per-subband $Q$-factor, which further simplifies the learning algorithm.

Lemma 4 (Decomposition of Per-user $Q$-factor): The per-user $Q$-factor $Q^k(\chi_k, s_k)$ (defined by the fixed point equation in (16)) can be decomposed into the sum of the per-user per-subband $Q$-factor $q^k(Q_k, |H_k, s|)$, i.e. $Q^k(\chi_k, s_k) = \sum_{n} q^k(Q_k, |H_k, s|, |s_k, n|)$, where

$$q^k(Q_k, |H_k, s|, s_k, n) \triangleq \min_{p_{k,n}} \left\{g_{k,n}(\gamma^k_Q, Q_k, |H_k, n|, s_k, n, p_{k,n}) \right\}$$

$$- \frac{N_F - \delta q^k(Q_k)|Q_k|}{N_k} s_{k,n} \log(1 + p_{k,n}|H_k, n|^2)$$

$$+ E[\tilde{w}^k(Q_k + A_k)|Q_k] - \frac{g^k}{N_F}$$

$$g_{k,n}(\gamma^k_Q, Q_k, |H_k, s_k, n, p_{k,n}) = \gamma^k_p p_{k,n} + \frac{1}{N_F} (\beta_k f(Q_k)$$

$$- \gamma^k P_k + \gamma^k (1 |Q_k = Q_f - P_f|)$$

$$\tilde{w}^k(Q_k) = E[q^k(Q_k, |H_k, s|, s_k, n = 1 |H_k, n| \geq H^k_k - 1)]|Q_k$$

$$\delta q^k(Q_k) = E[\tilde{w}^k(Q_k + A_k) - \tilde{w}^k(Q_k + A_k - 1)|Q_k]$$

Furthermore, we have $\tilde{W}^k(Q_k) = N_F \tilde{w}^k(Q_k)$.

Proof: Please refer to Appendix D for the proof.

Based on the per-user per-subband $Q$-factor $q^k(Q_k, |H, s|)$, we can obtain the closed-form power
allocation actions minimizing the R.H.S. of the per-user subband allocation Q-factor fixed point equation in (16), which is summarized in the following lemma:

**Lemma 5 (Decentralized Power Control Actions):** Given subband allocation actions $s_k$, the optimal power control actions of user $k$ under the linear approximation on subband allocation Q-factor in (15) are given by

$$p_{k,n}(Q_k,H_{k,n}) = s_{k,n} \left( \frac{\bar{\gamma}_k N_F \delta \tilde{w}_k(Q_k)}{|H_{k,n}|^2} - \frac{1}{|H_{k,n}|^2} \right), \forall k,n$$

\( (33) \)

**Proof:** Please refer to Appendix E for the proof.

**Remark 6:** (Multi-level Water-filling Structure of the Power Control Action) The power control action in (33) of Lemma 5 is both function of CSI and QSI (where it depends on the QSI indirectly via $\delta \tilde{w}_k(Q_k)$, which is function of $\{q^k(Q_s | H_s, s)\}$). It has the form of multi-level water-filling where the power is allocated according to the CSI across subbands but the water-level is adaptive to the QSI.

### C. Per-Stage Per-Subband Auction

Applying the per-stage subband auction in Section V-C to the system dynamics setup in this section, we obtain a low computational complexity and signaling overhead Scalarized Per-Subband Auction ($\forall n \in \{1, N_F\}$) as illustrated in Fig. 2 which is based on the per-user subband allocation Q-factor decomposition in Lemma 4 and the closed-form power allocation actions in Lemma 5 as follows:

- **Bidding:** For the $n$-th subband, each user submits a bid

$$X_{k,n} = \frac{N_F \delta \tilde{w}_k(Q_k) \tau}{N_k} \log \left( 1 + \frac{N_F \delta \tilde{w}_k(Q_k)}{|H_{k,n}|^2} \right) - \frac{1}{|H_{k,n}|^2} - \bar{\gamma}_k \left( \frac{N_F \delta \tilde{w}_k(Q_k) \tau}{N_k} - \frac{1}{|H_{k,n}|^2} \right)$$

- **Subband Allocation:** The BS assigns the $n$-th subband according to the highest bid:

$$s_{k,n}(H_{n}, Q) = \begin{cases} 1, & \text{if } k = k^*_n \text{ and } X_{k^*_n,n} > 0 \\ 0, & \text{otherwise} \end{cases} \quad (34)$$

where $k^*_n = \arg \max_k X_{k,n}$ denotes the user with the highest bid and then broadcasts the allocation results to $K$ users.

- **Power Allocation:** Each user determines the transmit power according to:

$$p_{k,n}(H_{n}, Q) = s_{k,n}(H_{n}, Q) \left( \frac{N_F \delta \tilde{w}_k(Q_k)}{|H_{k,n}|^2} - \frac{1}{|H_{k,n}|^2} \right) + \left( \frac{1}{|H_{k,n}|^2} \right)^+$$

\( (35) \)

**Remark 7 (Comparison to Brute-Force (CSI,QSI)-Feedback):** In the brute-force (CSI,QSI)-feedback scheme, each MS $k$ needs to feedback CSI $|H_{k,n} | (\forall n)$, QSI $Q_k$ and the LMs $\gamma_k$. BS needs to solve the subband allocation $s_{k,n}$ and power allocation $p_{k,n}$, and broadcast the (real number) power allocation $p_{k,n}$ to the MSs. Note that for the signaling from MS to BS, the quantization bits used in signaling for the bid $X_{k,n}$ versus for the CSI $|H_{k,n} |$ is similar. However, the proposed per-subband auction does not need to feedback QSI and LM. For the signaling from BS to MS, the proposed per-stage auction only needs 1 bit per subband for $s_{k,n}$. However, the brute-force (CSI,QSI)-feedback scheme needs much more bits per subband for a relatively accurate $p_{k,n}$ to ensure acceptable performance. Therefore, compared with the brute-force (CSI,QSI)-feedback scheme for uplink OFDMA systems, the proposed scalarized per-subband auction greatly reduces the signaling overhead and computation complexity (at the BS) for subband allocation and power allocation in the decentralized solution.

### D. Online Per-user Primal-Dual Learning Algorithm via Stochastic Approximation

In this part, we shall apply the online localized primal-dual learning algorithm in Section V-D to estimate $\{q^k(Q_s | H_s, s)\}$ and LMs. The update equations for LMs are the same as (20) and (21), and hence, we shall focus on the online learning of per-user per-subband Q-factor $\{q^k(Q_s | H_s, s)\}$ in the following.

For notation convenience, we denote the per-user per-subband state-action pair as $\phi \triangleq (Q_s | H_s, s)$. Let $i (1 \leq i \leq I)$ be a dummy index enumerating over all the possible state-action pairs of each user over one subband with cardinality $I_k = 2N_H(N_Q + 1)$ and $\phi_{k,n}(t) \triangleq (Q_k(t), |H_{k,n}(t)|, s_{k,n}(t))$ be the current state-action pair observed at MS $k$ on subband $n$ at the $t$-th slot. Based on the current observation $\phi_{k,n}(t)$, user $k$ updates its estimate on the per-user per-subband Q-factor according to:

$$q_{i+1}^k(\phi^i) = q_i^k(\phi^i) + c_{i,\phi^i}(t) \left[ g_{k,n}(t - \bar{\gamma}_k^k \phi^i, p_{k,n}(t)) + \bar{\gamma}_k^k(Q_k(t + 1)) - (g_{k,n}(t - \bar{\gamma}_k^k \phi^i, p_{k,n}(t)) + \bar{\gamma}_k^k(Q_k(t + 1)) - q_i^k(\phi^i) - q_i^k(\phi^i)) \cdot 1 \left( \cup_n \{\phi_{k,n}(t) = \phi^i\} \right) \right]$$

\( (36) \)
I (aggregated over 2 subbands). Without loss of generality, we initialize the
each user
tative stochastic learning algorithm. For instance, we are inter-
is visited at the steady state. We define the
corresponding constraints in (9), (10).

\[ I_k(t, \phi) \triangleq \sum_{m=0}^{t} 1 \{ \cup_n \{ \phi_{k,n}(m) = \phi_k \} \} \]
where \( I_k(t, \phi) \) is the number of updates of \( q^k(\phi_k) \) till \( t \) \[16\]. \( q^k \in \{ n : \phi_{k,n}(t) = \phi_k \} \]
\( t \triangleq \sup \{ t : \phi_{k,n}(t) = \phi_k \} \). \( \phi_k \) is the reference (per-subband) state-action combination \[13\] (per-user per-subband), \( \phi_k \in \{ n : \phi_{k,n}(t) = \phi_k \} \).

E. Rate of Convergence and Asymptotic Performance

In this section, we shall discuss the convergence speed as well as the asymptotic performance of the proposed distribu-
tive stochastic learning algorithm. For instance, we are inter-
est in how the convergence speed scales with the number of
MS \( K \) and the number of subbands \( N \). In the asynchronous
per-user per-subband Q-factor learning algorithm, at slot \( t \),
each user \( k \) updates the Q-factor of all the per-user per-
subband state-action pairs observed in \( N \) subbands. Thus,
the convergence speed of the asynchronous per-user per-
subband Q-factor learning algorithm depends on the speed
that every per-user per-subband state-action pair of each user
\( k \) is visited at the steady state. We define the ergodic visiting speed
each MS \( k \) as \( V_k = \lim_{t \to \infty} \min_t I_k(t, \phi) \), where

\[ I_k(t, \phi) \triangleq \sum_{m=0}^{t} 1 \{ \cup_n \{ \phi_{k,n}(m) = \phi_k \} \} \]

is the number of updates of \( q^k(\phi_k) \) up to slot \( t \). The following lemma summarizes the main results regarding the ergodic visiting speed.

Lemma 6 (Ergodic Visiting Speed w.r.t. K and N): The

ergodic visiting speed for each MS \( k \) of the per-user per-subband
Q-factor stochastic learning algorithm in \[56\] is given by

\[ V_k = O(N/K) \] (\( \forall k \)).

Proof: Please refer to Appendix F.

Remark 8 (Interpretations): Note that the convergence rate of the learning algorithm is related to \( V_k = O(N/K) \). Observe
that the convergence speed increases as \( N \) increase. This
is because in the asynchronous update process in \[56\], each user
\( k \) updates the Q-factor of all the per-user per-subband state-
action pair observed in \( N \) subbands in a single time slot.
Hence, there is intrinsic parallelism in the learning process
across different subbands.

Finally, we shall show that the performance of the distrib-
tive algorithm is asymptotically global optimal for large
number of users.

Theorem 2 (Asymptotically Global Optimal): For
sufficiently large \( K \) such that the optimization Problem
\[1\] is feasible, the performance of the online distributive
per-user primal-dual learning algorithm is asymptotically
global optimal, i.e. \( \sum_{k=1}^{K} \gamma^k(\chi_k, s_k) \to \gamma^* (\chi, s) \) and \( \gamma^\infty \to \gamma^* \) as \( K \to \infty \), where \( \gamma^* (\chi, s) \) and \( \gamma^\infty \) are the solution of the centralized Bellman equation in \[13\]
satisfying the corresponding constraints in \( \Theta \), \( \Theta_0 \).

Proof: Please refer to Appendix G.

\[14\] is equal.

The reference (per-user) state-action combination \( \phi^* \) is composed of the
(per-subband) state-action combination \( \phi^k \). For example, say \( N_P = 2, Q = \{ 0, 1 \}, H = \{ \text{Good (G), Bad (B)} \}, s = \{ 0, 1 \}, I_g = 2 \times 2^2 \times 2^2 = 48 \), \( I_b = 2 \times 2 \times 2 = 8 \). Let \( \phi^k = (0, B, 0) \), then \( \phi^* = (0, B, B, 0) \) (aggregated over 2 subbands). Without loss of generality, we initialize the
per-user per-subband Q-factor as 0. \( \phi_k^* = 0 \) for all \( k \).

VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, we shall compare our proposed per-user
online learning algorithm via stochastic approximation to the
delay optimal problem for OFDMA uplink systems with the
centralized subband allocation Q-factor \( \{ \Omega(\chi, s) \} \) learning
algorithm and three other reference baselines. Baseline 1 refers
to a throughput optimal policy \[14\], namely the Modified Largest
Weighted Delay First (M-LWDF) \[22\], in which the subband
and power control are chosen to maximize the weighted
delay. Baseline 2 refers to the CSIT Only Scheduling, in
which optimal subband and power allocation is performed
purely based on CSIT. Baseline 3 refers to the Round Robin
Scheduling, in which different users are served in TDMA
fashion with equally allocated time slots and water-filling
power allocation across the subbands. In the simulation, we
consider Poisson packet arrival with average arrival rate \( \lambda_k \)
(pck/s) and exponential packet size distribution with mean \( \bar{N}_k \).

We consider average delay as our utility \((f(Q_k) = \frac{Q_k}{\lambda_k})\).
We assume there are 64 subbands with total BW 10MHz, and
the number of independent subbands \( N_P \) is 4. The scheduling slot
duration \( \tau \) is 5ms. The buffer size \( N_Q \) is 10.

Figure 3 illustrates the average delay per user versus SNR of
2 users. It can be observed that both the centralized solution
and the distributive solution have significant gain compared
with the three baselines (e.g. more than 7.5 dB gain over M-
LWDF when average delay per queue is less than 9 packets).
In addition, the delay performance of the distributive solution,
which is asymptotically global optimal in large number of
users, is very close to the performance of the optimal solution
even in \( K = 2 \). Similar observations could be made in Figure 4
where we plot the average weighted delay versus SNR of two
heterogeneous users.

Figure 5 illustrates the average delay per user versus the
average delay per user of the

Fig. 3. Average delay per user versus SNR. The number of users \( K = 2 \),
the buffer size \( N_Q = 10 \), the mean packet size \( \bar{N}_k = 305.2 \) Kbyte/pck, the
average arrival rate \( \lambda_k = 20 \) pck/s, the queue weight \( \beta_1 = \beta_2 = 1 \). The
packet drop rate of the proposed scheme is 5\% while the packet drop rate of the
Baseline 1 (M-LWDF), Baseline 2 (CSIT Only) and Baseline 3 (Round Robin)
are 5\%, 8\%, 9\% respectively.
The number of users $K = 2$, the buffer size $N_Q = 10$, the mean packet size $N_k = 305.2$ Kbyte/pck, the average arrival rate $\lambda_k = 20$ pck/s, the queue weight $\beta_1 = 1$, $\beta_2 = 4$. The packet drop rate of the proposed scheme is 7% while the packet drop rate of the Baseline 1 (M-LWDF), Baseline 2 (CSIT Only) and Baseline 3 (Round Robin) are 7%, 9%, 9% respectively.

For the proposed scheme, the average delay per user is 6.5 pck, which is much smaller than the other baselines.

The packet drop rate of the proposed scheme is 4%, 8%, 9% respectively.

Figure 6 illustrates the cumulative distribution function (cdf) of the queue length. The buffer size $N_Q = 10$, the mean packet size $N_k = 78.125$ Kbyte/pck, the average arrival rate $\lambda_k = 20$ pck/s, the queue weight $\beta_k = 1$, the number of users $K = 6$ at a transmit SNR = 10dB. The packet drop rate of the proposed scheme is 4% while the packet drop rate of the Baseline 1 (M-LWDF), Baseline 2 (CSIT Only) and Baseline 3 (Round Robin) are 2%, 8%, 8% respectively.

The average delay corresponding to the average $\{\bar{W}^k(Q_k)\}$ at the 500-th scheduling slot is 5.9 pck, which is much smaller than

In conventional iterative algorithms for deterministic NUM, there is message passing between iterative steps within a CSI realization and these iterative steps (before convergence) are overheads because they do not carry useful payload. On the other hand, the proposed algorithm is an online distributive algorithm and hence, the slots before “convergence” also carry useful payload and they are not “wasted”.    

VII. SUMMARY

In this paper, we consider a distributive delay-optimal power and subband allocation design for uplink OFDMA system, which is cast into an infinite-horizon average-reward CMDP. To address the distributive requirement and the issue of exponential memory requirement and computational complexity, we proposed a per-user online learning with per-stage auction, which requires local QSI and local CSI only. We show that under the auction, the distributive online learning converges with probability 1. For illustration, we apply the proposed learning algorithm to an application example with exponential packet size distribution. We show that the delay-optimal power control has the multi-level water-filling structure.  

17In conventional iterative algorithms for deterministic NUM, there is message passing between iterative steps within a CSI realization and these iterative steps (before convergence) are overheads because they do not carry useful payload. On the other hand, the proposed algorithm is an online distributive algorithm and hence, the slots before “convergence” also carry useful payload and they are not “wasted”.
that the proposed algorithm converges to the global optimal solution for sufficiently large number of users. Numerical results illustrated significant delay performance gain over various baselines.

**APPENDIX**

**APPENDIX A: PROOF OF LEMMA 1**

For a given $\gamma$, the optimizing policy for the unconstrained MDP in (1.2) can be obtained by solving the Bellman equation w.r.t. $(\theta, \{V(\chi)\})$ as below [9]:

$$\theta + V(\chi^i), \quad \forall 1 \leq i \leq I_\chi \quad (37)$$

$$\quad = \min_{\Omega(\chi^i)} \left[ g(\gamma, \chi^i, \Omega(\chi^i)) + \sum_{\chi^i} \Pr[\chi^j | \chi^i, \Omega(\chi^i)] V(\chi^j) \right]$$

where $\Omega(\chi^i) = (p, s)$ is the power control and sub-band allocation actions taken in state $\chi^i$, $\theta = L_\delta(\gamma)$ is the optimal average reward per stage, $V(\chi)$ is the potential function of the MDP. Since $\Omega(\chi^i) = (\Omega^i(\chi^i), \Omega_p(\chi^i))$, we define the subband allocation $Q$-factor of state $\chi^i$ under subband allocation action as $Q(\chi^i, s) \triangleq \min_{\Omega_p(\chi^i)} \left[ g(\gamma, \chi^i, s, \Omega_p(\chi^i)) + \sum_{\chi^i} \Pr[\chi^j | \chi^i, s, \Omega_p(\chi^i)] V(\chi^j) \right]$. Thus, $V(\chi) = \min_{Q(\chi^i, s)} (\forall \chi)$ and $\{Q(\chi^i, s)\}$ satisfy the Bellman equation in (1.2).

**APPENDIX B: PROOF OF LEMMA 2**

Since $\forall k$, each state-action pair $\phi^i$ is updated comparably often [16], the only difference between the synchronous update and asynchronous update is that the resultant ODE of the asynchronous update is a time-scaled version of the synchronous update [16]. However, it does not affect the convergence behavior. Therefore, we consider the convergence of related synchronous version for simplicity in the following.

Due to symmetry, we only consider the update for user $k$. It can be easily proved that the synchronous version of the per-user $Q$-factor update in [19] is equivalent to the per-user $Q$-factor update given by

$$Q_{t+1}^k(\phi^i) = Q_t^k(\phi^i) + \epsilon_t^i Y_t^k(\gamma^k, \phi^i), \quad 1 \leq i \leq I_\phi \quad (38)$$

where $Y_t^k(\gamma^k, \phi^i) \triangleq g_{k}^i(\gamma^k, \phi^i, p^k(t)) + \overline{W}_t^k(Q_t, t + 1) - (g_{k}^i(\gamma^k, \phi^i, p^k(t)) + W_t^k(Q_t^k) - Q_t^k(\phi^i)) - Q_t^k(\phi^i)$. Denote $Y_t^k \triangleq (Y_t^k(\gamma^k, \phi^i), \ldots, Y_t^k(\gamma^k, \phi^i)^T)$. Let $Q_t^k \triangleq (Q_t^k, \ldots, Q_t^k)$ and $Y_t \triangleq (Y_t^1, \ldots, Y_t^k)$ be the aggregate vector of per-user $Q$-factor and $Y_t^k$ (aggregate across all $k$ users in the system). We shall first establish the convergence of the martingale noise in the $Q$-factor update dynamics. Let $E_t$ and $Pr_t$ denote the expectation and probability conditioned on the $\sigma$-algebra $F_t$, generated by $\{\Omega_t, Y_t, i < t\}, i.e. \ E_t[\cdot] = E[\cdot | F_t]$ and $Pr_t[\cdot] = \Pr[\cdot | F_t]$. Define $R_t^k(\gamma^k, \phi^i) \triangleq E_t [Y_t^k(\gamma^k, \phi^i)] = T_t^k(\gamma^k, Q_t^k) - Q_t^k(\phi^i) - (T_t^k(\gamma^k, Q_t^k) - Q_t^k(\phi^i))$, and $\delta M_t^k(\phi^i) \triangleq Y_t^k(\gamma^k, \phi^i) - E_t [Y_t^k(\gamma^k, \phi^i)]$. Thus, $\delta M_t^k(\phi^i)$ is the martingale difference noise satisfying the property that $E_t[\delta M_t^k(\phi^i) = 0$ and $E_t[\delta M_t^k(\phi^i)\delta M_t^k(\phi^i)] = 0 \ (\forall t \neq t')$. For some $j$, define $M_t^k(\phi^i) = \sum_{t = j}^t \delta M_t^k(\phi^i)$. Then, from (38), we have

$$Q_{t+1}^k(\phi^i) = Q_t^k(\phi^i) + \epsilon_t^i (R_t^k(\gamma^k, \phi^i) + \delta M_t^k(\phi^i))$$

$$= Q_t^k(\phi^i) + \sum_{i = j}^t \epsilon_t^i R_t^k(\gamma^k, \phi^i) + \sum_{i = j}^t \delta M_t^k(\phi^i) + \sum_{i = j}^t \delta M_t^k(\phi^i) \quad (39)$$

Since $E_t[M_t^k(\phi^i)] = E_t[\sum_{j = 1}^t \delta M_t^k(\phi^i)]$, $M_t^k(\phi^i)$ is a Martingale sequence. By martingale inequality, we have $\Pr_t \left( \sup_{j \leq t} \left| M_j^k(\phi^i) \right| \geq \lambda \right) \leq \frac{\mathbb{E}_t[M_t^k(\phi^i)]^2}{\lambda^2}$. By the property of martingale difference noise and the condition on the stepsize sequence, we have $\mathbb{E}_t[M_t^k(\phi^i)]^2 = E_t[\sum_{i = j}^t \epsilon_t^i \delta M_t^k(\phi^i)^2] = \sum_{i = j}^t E_t[(\epsilon_t^i)^2(\delta M_t^k(\phi^i))^2] \leq M \sum_{i = j}^t (\epsilon_t^i)^2 < \infty$, where $M = \max_{j \leq t \leq t} \left( \delta M_t^k(\phi^i) \right)^2 < \infty$. Hence, we have $\lim_{t \to \infty} \Pr_t \left( \sup_{j \leq t} \left| M_j^k(\phi^i) \right| \geq \lambda \right) \to 0$. Thus, from (39), we have $Q_{t+1}^k(\phi^i) = Q_t^k(\phi^i) + \sum_{i = j}^t \epsilon_t^i R_t^k(\gamma^k, \phi^i)$ a.s. with the vector form

$$Q_{t+1}^k = Q_t^k + \sum_{i = j}^t \epsilon_t^i R_t^k$$

where $R_t^k = T_t^k(\gamma^k, Q_t^k) - Q_t^k - (T_t^k(\gamma^k, Q_t^k) - Q_t^k(\phi^i)) e$ and $e = [1, \ldots, 1]^T$ is the $I_\phi \times 1$ unit vector.

Next, we shall establish the convergence of the dynamic equation in (19) after the martingale noise are averaged out. Let $g_t^k$ and $p_t^k$ denote the reward column vector and the transition probability matrix under the power allocation $p_t^k$, which attains the minimum of $T_t^k$ of the $t$-th iteration. Denote $z_t^k = T_t^k(\gamma^k, Q_t^k) - Q_t^k(\phi^i)$. Then, we have

$$R_t^k = g_t^k + p_t^k Q_t^k - Q_t^k - z_t^k e$$

$$\leq g_t^k + p_t^k Q_t^k - L_{t+1} Q_t^k - z_t^k e$$

$$\leq g_t^k + p_t^k Q_t^k - L_{t} Q_t^k - z_t^k e$$

$$\leq g_t^k + p_t^k Q_t^k - L_{t} Q_t^k - z_t^k e$$

$$\Rightarrow A_t^k R_t^k - (z_t^k - z_t^k) e \leq R_t^k$$

$$\leq B_t^k - g_t^k + p_t^k Q_t^k - L_{t} Q_t^k - z_t^k e$$

$$\leq B_t^k - g_t^k + p_t^k Q_t^k - L_{t} Q_t^k - z_t^k e$$

$$\Rightarrow A_t^k R_t^k - (z_t^k - z_t^k) e \leq R_t^k$$

$$\leq B_t^k - g_t^k + p_t^k Q_t^k - L_{t} Q_t^k - z_t^k e$$

Since $R_t^k(\gamma^k, \phi^i) = T_t^k(\gamma^k, Q_t^k) - Q_t^k(\phi^i) - (T_t^k(\gamma^k, Q_t^k) - Q_t^k(\phi^i)) = 0 \ \forall t$, by (24), we have

$$1 - \delta_m \left( R_t^k(\gamma^k, \phi^i) - (z_t^k - z_t^k) e \right) \leq R_t^k(\gamma^k, \phi^i)$$

$$\leq 1 - \delta_m \left( R_t^k(\gamma^k, \phi^i) - (z_t^k - z_t^k) e \right) \forall i$$

$$\Rightarrow \min_{\gamma, \phi^i} R_t^k(\gamma^k, \phi^i) \geq (1 - \delta_m) \min_{\gamma, \phi^i} R_t^k(\gamma^k, \phi^i)$$

$$- (z_t^k - z_t^k) e$$

$$\Rightarrow \max_{\gamma, \phi^i} R_t^k(\gamma^k, \phi^i) \leq (1 - \delta_m) \max_{\gamma, \phi^i} R_t^k(\gamma^k, \phi^i)$$

$$- (z_t^k - z_t^k) e$$
\[ \Rightarrow \max_{\gamma'} R^k_t(\gamma', \varphi'') - \min_{\gamma'} R^k_t(\gamma', \varphi'') \leq (1 - \delta_m) \left( \max_{\gamma'} R^k_{t-m}(\gamma', \varphi'') - \min_{\gamma'} R^k_{t-m}(\gamma', \varphi'') \right) \]

where \( \delta_m > 0 \). Since \( R^k_t(\gamma', \varphi'') = 0 \) \( \forall t \), we have \( \max_{\gamma'} R^k_t(\gamma', \varphi'') \leq 0 \) and \( \min_{\gamma'} R^k_t(\gamma', \varphi'') \leq 0 \). Thus, \( \forall i, j \), we have \( |R^k_t(\gamma', \varphi'')| \leq \max_{\gamma'} R^k_t(\gamma', \varphi'') - \min_{\gamma'} R^k_t(\gamma', \varphi'') \leq \delta_j \prod_{i=1}^{\infty} (1 - \delta_j + m) \). Therefore, as \( t \to \infty, R^k_t \to 0, \) i.e. \( \Omega^\infty(\gamma) \) satisfies equation in \( (25) \). Similar to the potential function of Bellman equation (Proposition 1 in Chapter 7 of [13]), the solution to \( (25) \) is unique only up an additive constant. Since \( Q^k_t(\varphi'') = Q^0_k(\varphi'') \) \( \forall t \), we have the convergence of the per-user subband allocation Q-factor limit \( \lim_{t \to \infty} Q^k_t = Q^\infty(\gamma) \) almost surely.

**APPENDIX C: PROOF OF LEMMA 5**

Due to the separation of time scale, the primal update of the Q-factor can be regarded as converged to \( Q^\infty(\gamma_k) \) w.r.t. the current LMs \( \gamma_k \). Using standard stochastic approximation theorem [18], the dynamics of the LMs update equation in \( (20) \) and \( (21) \) can be represented by the following ODE:

\[ \gamma(t) = \mathbb{E}[\Omega^t(\gamma(t)) \left( \sum_{n} p_{1n} - P_1 \right), (1|Q_k = N_Q) - P^1_{d_k})] \]

where \( \Omega^t(\gamma(t)) = \Omega^t(\gamma(t)) \) is the converged control policies in \( (13) \) and \( (17) \) w.r.t. the current LM \( \gamma(t) \), and \( \mathbb{E}[\Omega^t(\gamma(t))] \) denotes the expectation w.r.t. the measure induced by \( \Omega^t(\gamma) \). Define \( G(\gamma) = \mathbb{E}[\Omega^t(\gamma)] \left( \sum_k g_k(\gamma, \chi_k, s_k, p_k) \right) \). Since subband allocation policy is discrete, we have \( \Omega^t(\gamma) = \Omega^t(\gamma + \delta) \). Hence, by chain rule, we have \( \frac{\partial G}{\partial \gamma} = \sum_{k,n} \frac{\partial g_k}{\partial \gamma} \frac{\partial \Omega^t(\gamma)}{\partial \gamma} + \mathbb{E}[(\Omega^t(\gamma), \Omega^t(\gamma)) \left( \sum_k p_{kn} - P_k \right)] \) and \( \Omega^t(\gamma) = \arg \min_{\Omega^t(\gamma)} \mathbb{E}[\Omega^t(\gamma, \Omega^t(\gamma)) \left( \sum_k p_{kn} - P_k \right)] \) with \( \frac{\partial G}{\partial \gamma} = 0 + \mathbb{E}[(\Omega^t(\gamma), \Omega^t(\gamma)) \left( \sum_k p_{kn} - P_k \right)] = \gamma^t(t) \). Similarly, \( \frac{\partial G}{\partial \gamma} = \mathbb{E}[(\Omega^t(\gamma), \Omega^t(\gamma)) \left( 1|Q_k = N_Q) - P^1_{d_k})] = \gamma^t(t) \). Therefore, we show that the ODE in \( (41) \) can be expressed as \( \gamma(t) = \nabla G(\gamma(t)) \). As a result, the ODE in \( (41) \) will converge to \( \nabla G(\gamma) = 0 \), which corresponds to \( (9) \) and \( (10) \).

**APPENDIX D: PROOF OF LEMMA 4**

Let \( q^k(Q_k, |H_k, n|, s_k, n) = \min_{p_k,n} \left\{ g_{k,n}(\gamma, Q_k, |H_k, n|, s_k, n, p_k) : \right\} \)

\[ \begin{align*}
&= \frac{\Delta \bar{W}_k(Q_k)}{N_k} s_k, n \log(1 + p_k,n|H_k, n|) \\
&+ \frac{\mathbb{E}[\bar{W}_k(Q_k) + A_k]|Q_k|}{N_F} - \frac{\theta_k}{N_F}
\end{align*} \]

where \( \Delta \bar{W}_k(Q_k) \) \( \triangleq \mathbb{E}[\bar{W}_k(Q_k)|Q_k] \) and \( \Delta \bar{W}_k(Q_k) = \mathbb{E}[\bar{W}_k(Q_k + A_k) - \bar{W}_k(Q_k + A_k - 1)|Q_k] \). Then, we have \( Q^k(\chi_k, s_k) = \sum_k q^k(Q_k, |H_k, n|, s_k, n) \). Thus, we can derive

\[ \begin{align*}
\bar{W}_k(Q_k) &= \mathbb{E}[Q^k(\chi_k, |H_k, n|, s_k, n)] \\
&= \mathbb{E} \left[ \sum_n q^k(Q_k, |H_k, n|, s_k, n) = \mathbb{E}[\bar{W}_k(Q_k) + A_k - \bar{W}_k(Q_k + A_k - 1)|Q_k] \right] \]

\[ \Delta \bar{W}_k(Q_k) = \mathbb{E}[\bar{W}_k(Q_k + A_k) - \bar{W}_k(Q_k + A_k - 1)|Q_k] \]

\[ = N_F \mathbb{E}[w_k(Q_k + A_k) - w_k(Q_k + A_k - 1)|Q_k] \]

Therefore, from \( (42) \), we can obtain \( (29) \).

**APPENDIX E: PROOF OF LEMMA 5**

The conditional transition probability of user \( k \) is given by \( \Pr[\chi_k^i|\chi_k, s_k, p_k] = \Pr[H_k^i | H_k^i, \chi_k, s_k, p_k] \), where \( \Pr[H_k^i | H_k^i, \chi_k, s_k, p_k] = \Pr[A_k = Q_k^i = Q_k + 1] \mu(\chi_k, s_k, p_k) \tau + \Pr[A_k = Q_k^i = Q_k - 1] \mu(\chi_k, s_k, p_k) \tau \).

\[ Q^k(\chi_k, s_k) \]

\[ + \sum_{H_k^i} \Pr[H_k^i] \Pr[Q_k^i | H_k^i, \chi_k, s_k, p_k] \mathbb{E}[\bar{W}_k(Q_k)] - \theta_k \]

\[ = \min_{p_k,n} \left\{ g_{k,n}(\gamma, \chi_k, s_k, p_k) \right\} \]

\[ + \sum_{H_k^i} \Pr[H_k^i] \Pr[Q_k^i | H_k^i, \chi_k, s_k, p_k] \mathbb{E}[\bar{W}_k(Q_k)] - \theta_k \]

\[ = \min_{p_k,n} \left\{ g_{k,n}(\gamma, \chi_k, s_k, p_k) \right\} \]

\[ + \mu(\chi_k, s_k, p_k) \tau \mathbb{E}[\bar{W}_k(Q_k + A_k - 1)|Q_k] \]

\[ = N_F \mathbb{E}[\bar{W}_k(Q_k + A_k) - \bar{W}_k(Q_k + A_k - 1)|Q_k] \]

\[ \Delta \bar{W}_k(Q_k) \]

\[ = N_F \mathbb{E}[w_k(Q_k + A_k) - w_k(Q_k + A_k - 1)|Q_k] \]

Therefore, from \( (42) \), we can obtain \( (29) \).

**APPENDIX F: PROOF OF LEMMA 6**

We first fix \( K \) and consider the growth of the ergodic visiting speed w.r.t. \( N \). As \( N \) increases, the number of per-user per-subband state-action pair observations made at each
time slot increases (this "parallelism" helps to speed up the convergence rate). Thus, the chance that all per-user per-subband state-action pair of each user are visited grows like \( O(N) \), and hence, the ergodic visiting speed of each user grows like \( O(N) \). Next, we fix \( N \) and consider the growth of the ergodic visiting speed w.r.t. \( K \). Each subband can only be allocated to one user. Thus, the chance of the bottleneck state-action pair with \( s = 1 \) for each user being visited decreases like \( O(K) \), and hence, the ergodic visiting speed of each user grows like \( O(1/K) \). Combine the above two cases, we conclude Lemma 6.

**APPENDIX G: PROOF OF THEOREM 7**

For given \( \gamma \), we shall prove that under a Best-CSI subband allocation policy, the Q-factor satisfying the Bellman equation (13) can be decomposed into the additive form in (15). Based on that, we shall show that for large \( K \), the linear Q-factor approximation in (15) is indeed optimal.

**Definition 2:** [Best-CSI Subband Allocation Policy] A Best-CSI subband allocation policy is defined as \( \Omega_s(H) = \{s_{k,n}(H_n) \in \{0, 1\} \mid \sum_{k=1}^{K} s_{k,n} = 1 \forall n \} \), where

\[
s_{k,n}(H_n) = 1[H_{k,n} = \max_j |H_{j,n}| = 1[H_{k,n} \geq \max_j |H_{j,n}|]
\]

We first establish a property of the Q-factor in the original Bellman equation in (13) under the Best-CSI subband allocation policy, which is summarized in Lemma 7.

**Lemma 7:** (Additive Property of the Subband Allocation Q-Factor) Under the Best-CSI subband allocation policy, the solution to the original Bellman equation in (13) can be expressed into the form \( Q(\chi, s) = \sum_k Q_{\Omega_s}(\chi, s_k) \), where \( \{Q_{\Omega_s}(\chi, s_k)\} \) is the converged per-user Q-factor, which is also the solution of the \( k \)-th user’s per-user subband allocation Q-factor fixed point equation given by (13).

**Proof:** Under the Best-CSI subband allocation policy, the Bellman equation in (13) becomes

\[
Q(\chi^i, s) = \min_{\Omega_p(\chi^i)} \left[ g(\gamma, \chi^i, s, \Omega_p(\chi^i)) + \sum_{Q_i} \Pr[Q^i | \chi^i, s, \Omega_p(\chi^i)] \left( \sum_{H_i} \Pr[H_i | Q^i] \frac{Q(\chi^i, \Omega_p(\chi^i))}{\tilde{V}(Q^i)} \right) \right] - \theta \forall 1 \leq i \leq I_x, \forall s
\]

\[
\tilde{V}(Q^i) = \sum_{H_i} \Pr[H_i] \min_{\Omega_p(\chi^i)} \left[ g(\gamma, \chi^i, \Omega_p(\chi^i), \Omega_s(H^i)) + \sum_{Q_i} \Pr[Q^i | \chi^i, \Omega_s(H^i), \Omega_p(\chi^i)] \tilde{V}(Q^i) \right] - \theta, 1 \leq i \leq I_Q
\]

where (a) is due to (7) and the definition \( \tilde{V}(Q^i) \triangleq \mathbb{E}[Q(\chi, \Omega_s(H)) | Q] \), (b) is obtained by taking conditional expectation (conditioned on \( Q_i^i \) on both sides of (45) and the definition of \( \tilde{V}(Q) \). In addition, denote \( \Delta_k \tilde{V}(Q) \triangleq \mathbb{E}[\tilde{V}(Q + A) - \tilde{V}(Q + A - e_k) | Q] \).

From (45), we know that \( \{Q(\chi^i, s) \} \) is determined by \( \{\tilde{V}(Q^i) \} \). Next, we shall try to solve \( \{\tilde{V}(Q^i) \} \) by the \( I_Q \) equations in (46). First, assume the linear approximation \( Q(\chi, \Omega_s(H)) = \sum_k Q_k(\chi, \Omega_s(H)) \) holds under the best-CSI subband allocation policy, we have

\[
\tilde{V}(Q) = \mathbb{E}[\sum_k Q_k(\chi, \Omega_s(H)) | Q] = \sum_k \mathbb{E}[Q_k(\chi, H_k, \Omega_s(\tilde{H})) | Q]
\]

\[
= \sum_k \mathbb{E}[\Omega_s(\tilde{H}) | Q] - \mathbb{E}[Q_k(\chi, H_k, \Omega_s(H)) | Q] = \sum_k \mathbb{E}[\Omega_s(\tilde{H}) | Q] - \mathbb{E}[Q_k(\chi, H_k, H, \Omega_s(H)) | Q]
\]

\[
= \sum_k \mathbb{E}[\tilde{V}(Q_k) | Q] = \sum_k \mathbb{E}[\tilde{W}_k(Q_k) | Q] = \Delta_k \tilde{V}(Q) = \mathbb{E}[\tilde{W}_k(Q_k) + \tilde{W}_k(Q_k + A_k)] - \mathbb{E}[\tilde{W}_k(Q_k) + \tilde{W}_k(Q_k + A_k)]
\]

\[
\Delta_k \tilde{V}(Q) = \mathbb{E}[\tilde{W}_k(Q_k + A_k)] - \mathbb{E}[\tilde{W}_k(Q_k)] = \mathbb{E}[\tilde{W}_k(Q_k)] - \mathbb{E}[\tilde{W}_k(Q_k + A_k)]
\]

\[
\theta = \mathbb{E}[\tilde{W}_k(Q_k)] - \mathbb{E}[\tilde{W}_k(Q_k + A_k)]
\]

Thus, the optimal power allocation and corresponding conditional departure rate to \( \min_{\Omega_p(\chi^i)}[\cdot] \) part in (46) are as follows

\[
\mu_k(Q_k, H_k, \tilde{s}_{k,n}(H_n)) \triangleq \tilde{s}_{k,n}(H_n) \log(1 + \tilde{p}_{k,n}(Q_k, H_k, \tilde{s}_{k,n}(H_n)[H_{k,n}^2])
\]

Therefore, from (46), we have

\[
\sum_k \tilde{W}_k(Q_k) = \sum_k \left( \tilde{g}_k(\gamma, Q_k) + \mathbb{E}[\tilde{W}_k(Q_k + A_k)] | Q_k \right)
\]

\[
= \tilde{g}_k(\gamma, Q_k) + \mathbb{E}[\tilde{W}_k(Q_k + A_k) | Q_k] - \tilde{W}_k(Q_k) + \tilde{W}_k(Q_k + A_k)
\]

\[
\rightarrow \theta = \sum_k \theta_k = \sum_k \left( \tilde{g}_k(\gamma, Q_k) + \mathbb{E}[\tilde{W}_k(Q_k + A_k)] | Q_k \right)
\]

\[
\tilde{V}(Q) \triangleq \sum_k \tilde{W}_k(Q_k) = \sum_k \tilde{W}_k(Q_k + A_k)
\]

\[
\tilde{V}(Q) \triangleq \sum_k \tilde{W}_k(Q_k) = \sum_k \tilde{W}_k(Q_k + A_k)
\]

\[
\Delta_k \tilde{V}(Q) = \mathbb{E}[\tilde{W}_k(Q_k)] - \mathbb{E}[\tilde{W}_k(Q_k + A_k)]
\]

\[
\theta = \sum_k \theta_k
\]

where \( \tilde{g}_k(\gamma, Q_k) \triangleq \mathbb{E}[\tilde{W}_k(Q_k)] + \gamma \mathbb{E}[\sum_{k=1}^{K_p} p_k(Q_k, H_k, \tilde{s}_{k,n}(H_n) | H_{k,n}^2)] - \tilde{W}_k(Q_k) + \gamma \mathbb{E}[1 | Q_k]
\]

Since there are only \( (N_Q + 1) \) QSI states for each user and the structure in (49) is decoupled under the additive assumption, for each user \( k \), there are only \( (N_Q + 1) \) independent Poisson equations with \( N_Q + 2 \) unknowns \( \{\theta_k, \tilde{W}_k(Q_k)\} \). \( \theta_k \) is unique and \( \{\tilde{W}_k(Q_k)\} \) is unique up to an additive constant (13). Therefore, \( \{\theta, \tilde{V}(Q)\} \) is the solution to (46), where

\[
\theta = \sum_k \theta_k
\]
Next, we shall show \( Q(\chi, s) = \sum_k Q_k^\infty(\chi_k,s_k) \). Substitute \( \theta = \sum_k \theta_k \) and \( \bar{V}(Q) = \sum_k \bar{W}_k(Q_k) \) into (45), we have

\[
Q(\chi, s) = \min_{\Omega_p(\chi')} \left[ g(\gamma, \chi, s, \Omega_p(\chi')) + \sum_{Q_k} \Pr\left( Q_j^k | \chi, s, \Omega_p(\chi') \right) \right] - \sum_k \theta_k
\]

\[
= \sum_k Q_k^\infty(\chi_k, s_k)
\]

where \( Q_k^\infty(\chi_k, s_k) = \min_{p_k} \left[ g_k(\gamma, \chi, s_k, p_k) + \sum_{Q_k} \Pr\left( Q_j^k | \chi_k, s_k, p_k, \Omega_p(\chi') \right) - \theta_k \right] \), which is equivalent to (16). By Lemma 2 the converged \( \{ Q_k^\infty(\chi_k, s_k) \} \) satisfy (16) and this completes the proof.

Next, we shall consider the asymptotic subband allocation results for large \( K \). The optimal allocation actions to (13) are given by

\[
p_{k,n}(h_n, Q) = s_{k,n}(h_n, Q) \left( \frac{\sum_{j=1}^{\infty} \bar{V}^*(Q)}{\gamma} - \frac{1}{|H_{k,n}|^2} \right)^+ + s_{k,n}(h_n, Q) \left( \frac{1}{|H_{k,n}|^2} \right)^+
\]

\[
s_{k,n}(h_n, Q) = \begin{cases} 1, & \text{if } X_{k,n} = \max_j \{ X_{j,n} \} > 0 \\ 0, & \text{otherwise} \end{cases}
\]

For large \( K \), \( |H_{k,n}|^2 \) grows with \( \log(\gamma) \) by extreme value theory. Because the traffic loading remains unchanged as we scale up \( K \), \( \max_{j \neq k} |\Delta_k \bar{V}^*(Q) - \Delta_j \bar{V}^*(Q)| = O(1) \). Hence, \( X_{k,n}^c \) grows like \( \log(\gamma) \). As \( K \to \infty \), \( \Pr[k_n^c = \arg \max_k |H_{k,n}|^2] = 1 \). Thus the subband allocation result of optimal subband allocation in (44) and the best CSI subband allocation in (45) will be the same for large \( K \). Using the result in Lemma 2, the linear Q-factor approximation is therefore asymptotically accurate for given \( \gamma \). Combining the results of Theorem 1 we can prove Theorem 2.

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