NON-GAUSSIAN SCATTER IN CLUSTER SCALING RELATIONS

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ABSTRACT

We investigate the impact of non-Gaussian scatter in the cluster mass-observable scaling relation on the mass and redshift distribution of clusters detected by wide area surveys. We parameterize non-Gaussian scatter by incorporating the third and fourth moments (skewness and kurtosis) into the distribution $P(M_{\text{obs}}|M)$. We demonstrate that the effect of the higher order moments becomes important when the product of the standard deviation of $P(M_{\text{obs}}|M)$ and the slope of the mass function is greater than unity. For high scatter mass indicators it is therefore necessary for the survey, limiting mass threshold to be less than $10^{14} h^{-1} M_\odot$, to prevent the skewness from having a significant impact on the observed number counts, particularly at high redshift. We also show that an unknown level of non-Gaussianity in the scatter is equivalent to an additional uncertainty on the variance in $P(M_{\text{obs}}|M)$ and thus may limit the constraints that can be placed on $\sigma_8$ and the dark energy equation of state parameter $w$.

Key words: dark matter – galaxies: clusters: general – intergalactic medium

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1. INTRODUCTION

The evolution of the number density of galaxy clusters is a sensitive cosmological probe (Bahcall & Fan 1998; Eke et al. 1998). As an indicator of the expansion rate as a function of time, the galaxy cluster number density is sensitive to the dark energy equation of state (Haiman et al. 2001; Weller et al. 2001). This provides a growth-based dark energy test, an important complement to the distance-based tests that have provided the most compelling evidence for dark energy up to this point (Perlmutter et al. 1999; Schmidt et al. 1998).

Galaxy clusters can be selected by many diverse methods, including (but not limited to) optical richness, X-ray thermal bremsstrahlung flux, weak lensing shear, and the Sunyaev–Zel’dovich (SZ) effect. The key challenge for using galaxy clusters as precise cosmological probes is in understanding how to relate observables to a quantity that can be well predicted by theory, for example, mass. The ultimate goal is to produce theoretical predictions of the distributions of observables as a function of redshift and cosmological parameters. Short of this, one approach is to theoretically model the evolution of number density as a function of mass, and then estimate the mapping between observables and mass in order to predict the observed evolution. This mapping can either be estimated from theoretical considerations or be determined directly from the data, assuming some regularity in the mapping (Majumdar & Mohr 2003; Hu 2003; Lima & Hu 2004, 2005).

It is important to understand the statistics of the relevant mass-observable scaling relation. Although clusters may follow a mean relation, individual clusters will deviate from it. If the level of scatter around the mean relation is not small, the shape and amplitude of the observed mass function can change significantly. At cluster scales, the mass function is a steeply declining function of mass. Therefore, a larger number of low-mass clusters will scatter over the detection threshold of the survey than those higher mass clusters that scatter below it. The net increase in the total number of clusters in the sample is thus dependent on the slope of the mass function at the threshold mass, and the magnitude of the scatter around the mass-observable scaling relation. If the latter is well constrained then the measurement of the cluster mass function is actually improved statistically due to the reduction in shot noise. However, in practice it is difficult to precisely measure the scatter, and theoretical estimates can vary substantially. Furthermore, as we shall demonstrate, for large intrinsic scatter, the magnitude of the higher order moments (skewness and kurtosis) can become significant.

Previous works forecasting the constraints on cosmological parameters that can be achieved by cluster surveys have assumed the scatter in scaling relations to be lognormally distributed around the mean relation (that is, normally distributed in the logarithm of the mass). However, cosmological simulations and observations of large samples of clusters have demonstrated deviations from lognormal behavior. In general, the cause of these deviations can be separated into two categories: dynamical state and projection effects.

The former refers to the impact of a sub-population of clusters that are systematically offset from the mean scaling relation due to their dynamical state. Shaw et al. (2006) and Evrard et al. (2008) demonstrated using N-body simulations that dark matter halos undergoing a major merger have a systematically higher dark matter velocity dispersion ($\sigma_{\text{DM}}$) than their more relaxed counterparts (of the same mass). Evrard et al. (2008) showed that the small fraction of interacting halos cause positive skewness in the distribution of the residuals around the mass–$\sigma_{\text{DM}}$ relation. Stanek et al. (2010) analyze the covariance of bulk cluster properties using a large sample of halos extracted from hydrodynamical simulations. They find small deviations from Gaussian scatter for some properties, most notably in the mass-weighted temperature–mass relation. Pratt et al. (2009) analyzed X-ray luminosity scaling relations using a sample of 31 nearby clusters observed by the XMM-Newton X-ray observatory. They find that the scatter in the $L_X$–$Y_X$ relation (where $Y_X$ was assumed to be a robust, low-scatter proxy for cluster mass) was significantly non-Gaussian due to the systematically above-average luminosity of cooling core clusters in their sample. They also note that the scatter becomes more Gaussian when the central regions of the clusters are excluded in the luminosity measurements.
The second cause of non-Gaussianity in observed scaling relations is confusion in cluster selection due to projection effects. Several studies have demonstrated that lower mass, unresolved clusters—as well as gas outflow with cluster regions—can contribute significantly to the measured integrated SZ flux ($Y$). This generates additional scatter in the $Y-M$ relation and introduces a tail toward high flux in the distribution of $Y$ at constant mass (White et al. 2002; Holder et al. 2007; Hallman et al. 2007; Shaw et al. 2008). A similar effect is found for optically selected clusters. Cohn & White (2009) demonstrated using mock-galaxy catalogs that a non-negligible fraction of clusters (10% at $z = 0.4$ to 22% at $z = 1$) identified using the red sequence are “blends”—cluster candidates in which a large number of different halos have contributed galaxies. Blends thus cause a tail toward high richness in the distribution of optical richness at fixed mass.

In this work, we relax the assumption of lognormal scatter and investigate the effect of non-Gaussian scatter around the mean mass-observable scaling relation on the observed mass and redshift distribution of clusters. Specifically, we quantify the impact of non-zero skewness and kurtosis—the third and fourth standardized moments of a generalized probability distribution—as perturbations to the purely Gaussian case.

Throughout this paper, we make a distinction between true mass $M$ and observed mass $M_{\text{obs}}$. The former is the actual cluster mass, as defined by a spherical overdensity, $\Delta$, and measured in cosmological simulations of structure formation (Jenkins et al. 2001; Warren et al. 2006; Tinker et al. 2008). $M_{\text{obs}}$ is the cluster mass that would be inferred from observations through application of a scaling relation (e.g., $Y-M$, $L_x-M$, etc.), or via self-calibration of the mass function (Lima & Hu 2005; Cunha 2009).

2. SCATTER IN THE MASS-OBSERVABLE SCALING RELATION

The predicted redshift distribution of clusters observed by a given survey is given by

$$\frac{dN}{dz} = \frac{\Delta \Omega}{dz d\Omega(z)} \int_0^{\infty} \frac{dM}{M} \frac{d\bar{n}}{d \ln(M)} f(M_{\text{lim}}, z),$$

where $M$ is the cluster mass and $d\bar{n}/d \ln M$ is the mean comoving number density of clusters (the mass function). The function $f(M_{\text{lim}}, z)$ represents the survey selection function which accounts for the limiting mass of the survey, $M_{\text{lim}}$, defined by some threshold in the mean observable (e.g., optical richness, X-ray luminosity, integrated SZ flux), and the statistics of the mapping between $M_{\text{obs}}$ and $M$. In the limit of perfect (zero scatter) mass measurements, this is simply a step function at the limiting mass of the survey.

If one assumes lognormal scatter around the mean scaling relation (Gaussian scatter in $\ln M$) then the probability $P(M_{\text{obs}} | M)$ of observing the mass $M_{\text{obs}}$ given the “true” underlying mass $M$ is

$$P(M_{\text{obs}} | M) = \frac{1}{\sqrt{2\pi \sigma_{\ln M}^2}} \exp[-x^2(M_{\text{obs}})],$$

with

$$x(M_{\text{obs}}) = \ln M_{\text{obs}} - \ln M - \ln M_{\text{bias}}.$$  

This parameterization allows for redshift-dependent scatter $\sigma_{\ln M}$ and bias $M_{\text{bias}}$ in the mass-observable scaling relation (Lima & Hu 2005; Cunha 2009). For simplicity, we henceforth ignore the bias term (e.g., Francis et al. 2005) and concentrate on the impact of scatter alone. The distribution of observed cluster masses is just a convolution of the true mass function with $P(M_{\text{obs}} | M)$,

$$\frac{d\bar{n}}{d \ln M_{\text{obs}}} = \int_0^{\infty} \frac{d\bar{n}}{d \ln M} P(M_{\text{obs}} | M) d \ln M.$$  

Plugging in Equation (2) and assuming an intrinsic power-law distribution in mass, $d\bar{n}/d \ln M \propto M^{-\alpha}$, it is straightforward to show that the observed mass distribution is

$$\frac{d\bar{n}}{d \ln M_{\text{obs}}} = \left(\frac{d\bar{n}}{d \ln M}\right)_{\circ} e^{\alpha(\sigma_{\ln M}^2/2)},$$

where $\circ$ denotes the true mass distribution evaluated at $M_{\text{obs}}$ by applying the mean $M-M_{\text{obs}}$ scaling relation.

The extent of the deviation of the observed mass function from the true mass function is clearly controlled by $\alpha$ and the product of the standard deviation of the distribution $P(M_{\text{obs}} | M)$ with the slope of the mass function, $\alpha$. Evidently, a constant $\sigma_{\ln M}$ results in an observed mass function that has a constant and positive vertical offset from the true mass function. For more realistic mass functions (Jenkins et al. 2001; Tinker et al. 2008), the slope $\alpha$ is approximately 1 at the group mass scale and increases with increasing mass and redshift, exponentially so at very high mass and redshift. The impact of scatter on the observed mass function is thus significantly greater at high masses/redshifts (see Section 3).

Above a limiting threshold in $M_{\text{obs}}$, scatter in a cluster scaling relation causes a net increase in the number of detected clusters. An unknown amount of scatter in the scaling relation thus degrades the cosmological constraints that can be obtained from cluster number counts as one must also marginalize over $\sigma_{\ln M}$. Lima & Hu (2005) demonstrated that for a survey with a fixed limiting mass of $10^{14.2} h^{-1} M_{\odot}$ and $\sigma_{\ln M}^2 = 0.25^2$, a 1σ uncertainty of $0.25^2$ on $\sigma_{\ln M}$ would produce a 10% uncertainty in the number counts at $z = 0.5$, a 20% uncertainty at $z = 1$, and a 50% uncertainty at $z = 2$.

2.1. Non-Gaussian Scatter

We now determine the impact on the observed mass function of non-Gaussian scatter in the mass-observable scaling relation. We proceed by using the Edgeworth series to approximate a non-Gaussian $P(M_{\text{obs}} | M)$ (e.g., Bernardeau & Kofman 1995; Blinnikov & Moessner 1998), thus taking it to be a perturbation to the Gaussian case. It is parameterized by the third and fourth moments of the distribution (gamma ($\gamma$) and kurtosis ($\kappa$), of the probability distribution $P(M_{\text{obs}} | M)$. The Edgeworth expansion is particularly useful for convolutions when expressed as a series of derivatives of Gaussians,

$$P(M_{\text{obs}} | M) \approx G(x) - \frac{\gamma}{6} \frac{d^3G}{dx^3} + \frac{\kappa}{24} \frac{d^4G}{dx^4} + \frac{\gamma^2}{72} \frac{d^6G}{dx^6},$$

where the skewness, $\gamma$, is defined as

$$\gamma = \frac{\langle (M_{\text{obs}} - M)^3 \rangle}{\sigma^3},$$

and the kurtosis, $\kappa$ as

$$\kappa = \frac{\langle (M_{\text{obs}} - M)^4 \rangle}{\sigma^4} - 3.$$
and $G(x)$ is a Gaussian distribution in $x$ (equal to $P(M_{\text{obs}} | M)$ in Equation (2)).

By plugging Equation (6) into Equation (4) and doing some integration by parts, the observed mass function can be calculated for any true mass function,

$$\frac{d\bar{n}}{d\ln M_{\text{obs}}} = \int \frac{d\bar{n}}{d\ln M} G(x) dx - \frac{\gamma}{6} \int \frac{d^3}{dx^3} \left( \frac{d\bar{n}}{d\ln M} \right) G(x) dx$$

$$+ \frac{\kappa}{24} \int \frac{d^4}{dx^4} \left( \frac{d\bar{n}}{d\ln M} \right) G(x) dx$$

$$+ \frac{\gamma^2}{72} \int \frac{d^6}{dx^6} \left( \frac{d\bar{n}}{d\ln M} \right) G(x) dx + \cdots$$

(9)

Assuming again an intrinsic power-law distribution of mass with slope $\alpha$, $d\bar{n}/d\ln M_{\text{true}} = M^{-\alpha}$, it is straightforward to show that the observed mass distribution is now

$$\frac{d\bar{n}}{d\ln M_{\text{obs}}} = \left( \frac{d\bar{n}}{d\ln M} \right)_o \times \left[ \frac{\alpha^2 \sigma^2}{6} - \frac{\alpha^4 \sigma^4}{24} + \frac{\alpha^6 \sigma^6}{72} \kappa + \cdots \right]$$

(10)

where the subscript “$o$” denotes the true mass distribution evaluated at $M_{\text{obs}}$.

The relevant parameter here is again clearly $\alpha \sigma_{\text{in}} M$. If this parameter is large, there are two important effects: the number of objects at any given mass scale is increased substantially (as for the Gaussian case), and the higher order moments of the distribution, $\gamma$ and $\kappa$, become important. The transition at which the latter occurs is clearly when $\alpha \sigma_{\text{in}} M$ becomes greater than unity. For example, if $\alpha \sigma_{\text{in}} M = 1$, then $\gamma = 1$ produces an $17\%$ increase on the number counts compared to the purely Gaussian case at any given mass scale. Assuming $\gamma = 1$ provides an additional $4\%$ correction. Note that Equation (10) demonstrates that both (positive) $\gamma$ and $\kappa$ cause an upscattering of clusters. For skewness this is due to the tail toward large mass that increases the probability of a cluster of having $M_{\text{obs}} \gg M$. For kurtosis, the upscattering is due to the wider wings of the distribution.

3. IMPLICATIONS OF NON-GAUSSIAN SCATTER FOR SZ AND OPTICAL CLUSTER SURVEYS

We now evaluate the impact of non-Gaussian scatter in the mass-observable scaling relation on the predicted mass and redshift distribution of clusters observed by a South Pole Telescope (SPT)-like SZ survey and a Dark Energy Survey (DES)-like optical survey. For the SZ survey, we assume that the scatter in the mass-SZ flux relation is $\sigma_{\text{SZ}} = 0.25$ (Shaw et al. 2008), and the limiting mass is $3 \times 10^{14} h^{-1} M_\odot$. For the optical survey, we assume the scatter in the optical richness–mass relation is $\sigma_{\text{opt}} = 0.5$, in keeping with the results of Rykoff et al. (2008a, 2008b), Becker et al. (2007), Rozo et al. (2009a, 2009b), and a constant limiting mass of $5 \times 10^{13} h^{-1} M_\odot$. Note that Becker et al. (2007) found that the scatter in the optical richness–mass relation is $0.5 \pm 0.75$, and we have taken the value at the lower end of this range.

We have demonstrated that the impact of the higher order moments, $\gamma$ and $\kappa$, becomes significant when $\alpha \sigma_{\text{in}} M$, the product of the slope of the mass function and the standard deviation of $P(M_{\text{obs}} | M)$, becomes greater than unity. Under the simplifying assumption that $\sigma_{\text{in}} M$ is independent of mass and redshift for a given observable, the higher order moments thus become relevant when the effective slope of the mass function exceeds the threshold value, $\alpha_c > 1/\sigma_{\text{in}} M$. This critical slope is reached above a redshift (and cosmology) dependent mass threshold $M_c(z)$. For SPT, $\alpha_c \text{SZ} = 1/0.25 = 4$, for DES $\alpha_{\text{opt}} = 2$.

In Figure 1, we plot the slope of the mass function as a function of mass and redshift. We adopt the mass function of Tinker et al. (2008), and assuming $d\bar{n}/d\ln M = M^{-\alpha}$ at any given mass and redshift, calculate $\alpha$ over a wide range of mass and redshift. The mass function is defined in terms of $M_{\text{obs}}$, the mass enclosed in a sphere of mean overdensity 200 times the mean density of the universe (at the relevant redshift). For our fiducial cosmology, we assume parameters consistent with the WMAP 5 year results ($\Omega_M = 0.27$, $\sigma_8 = 0.8$; Dunkley et al. 2009). The gray lines denote contours of constant $\alpha$, from 10 (top right corner) to 1 (bottom left corner). The thick black solid and dashed contours denote $\alpha = 2$ and 4, the critical slopes $\alpha_c$, for DES and SPT, respectively. These contours thus give $M_c(z)$ for each survey. The vertical solid and dashed lines represent the limiting mass for each survey.

Figure 1 demonstrates that for the SZ survey $\alpha \sigma_{\text{in}} M$ becomes greater than unity above $8 \times 10^{14} h^{-1} M_\odot$ at $z = 0.5$ and above $4 \times 10^{14} h^{-1} M_\odot$ at $z = 1$. Given the scarcity of objects above these thresholds, it is clear that $\gamma$ and $\kappa$ will not significantly impact the SPT cluster number counts. For the optical survey, $M_c(z) = 1.7 \times 10^{14} h^{-1} M_\odot$ at $z = 0.5$ and $7.3 \times 10^{13} h^{-1} M_\odot$ at $z = 1$. As $M_c(z)$ remains greater than the expected limiting mass for DES, $\gamma$ and $\kappa$ will not strongly affect the total number of detected clusters. However, a sizable number of clusters will be detected in mass bins that exceed $M_c(z)$. Non-Gaussian scatter in the mass–optical richness relation may therefore cause a detectable redistribution of clusters in mass bins greater than $M_c(z)$.

In Figure 2, we demonstrate the impact of skewness and kurtosis in $P(M_{\text{obs}} | M)$ on the observed redshift distribution of clusters. In the upper panel, we plot the redshift distribution of clusters $dN/dz$ for the SPT-like SZ survey (black lines) and the DES-like optical survey (red lines). As described above, for SPT we assume a limiting mass threshold of $3 \times 10^{14} h^{-1} M_\odot$ and intrinsic scatter $\sigma_{\text{in}} M = 0.25$, while for DES we assume a limiting mass of $5 \times 10^{13} h^{-1} M_\odot$ and $\sigma_{\text{in}} M = 0.5$. For each survey, the solid lines represent $\gamma = \kappa = 0$, dashed
we assume a limiting mass of $5 \times 10^{15} M_\odot$ and intrinsic scatter $\sigma_{\text{in}} M = 0.25$. For the latter we assume a limiting mass of $5 \times 10^{14} M_\odot$ and $\sigma_{\text{in}} M = 0.5$. The solid lines represent $\gamma = \kappa = 0$, the dashed $\gamma = 0.5, \kappa = 0$, dot-dashed $\gamma = 0.5$ ($\gamma = 0$), and the dotted line $\gamma = 1, \kappa = 1$. Lower panel: fractional increase in the number counts due to non-Gaussian scatter relative to the purely Gaussian case (i.e., $\kappa = \gamma = 0$). The line type and color are the same as for the upper panel.

(A color version of this figure is available in the online journal.)

Table 1

Percentage Increase in the Number Counts at $z = 0.5$ and $z = 1$ Due to Non-Gaussian Scatter (Relative to the Gaussian Case) for the DES and SPT-like Surveys

| $\gamma$ | $\kappa$ | $\%$ Increase in $N(z)$ (equiv. $\sigma_{\text{in}} M$) |
|----------|----------|-------------------------------------------------------|
| $\gamma$ | $\kappa$ | at $z = 0.5$ | at $z = 1$ |
|----------|----------|----------------|----------------|
| 0.5      | 0        | 5 (0.28)      | 7 (0.56)      |
| 0.5      | 0.5      | 0.5 (0.25)    | 1 (0.51)      |
| 1.0      | 1.0      | 12 (0.31)     | 17 (0.64)     |

Notes. Values in brackets give the value of $\sigma_{\text{in}} M$ that produces an equivalent increase in the number counts for purely Gaussian scatter. The fiducial values of $\sigma_{\text{in}} M$ are 0.25 for SPT and 0.5 for DES.

$\gamma = 0.5$, $\kappa = 0$, dot-dashed $\gamma = 0$, $\kappa = 0.5$, and dotted $\gamma = 1$, $\kappa = 1$ (as a more extreme scenario). In the lower panel, we plot the fractional increase in the number counts due to non-Gaussian scatter relative to the Gaussian case (i.e., $\kappa = \gamma = 0$), where the line type and color are same as for the upper panel. The results are also summarized in Table 1. As predicted by Equation (10), the impact of skewness and kurtosis on the redshift distribution is dependent on the value of $\sigma_{\text{in}} M$ and the local slope of the (true) mass function at the limiting mass of the survey. Thus, at higher redshifts the impact of $\gamma$ and $\kappa$ becomes greater. This is clearly demonstrated in the lower panel of Figure 2, which shows that toward higher redshift a positive $\gamma$ and $\kappa$ increasingly boost the observed number counts relative to purely Gaussian scatter.

For the SPT-like SZ survey (black lines), increasing $\gamma$ from 0 to 0.5 results in a 5% increase in the number counts at $z = 0.5$ and a 14% increase at $z = 1$. This is approximately equivalent to an increase in $\sigma_{\text{in}} M$ for Gaussian scatter from 0.25 to 0.28 at $z = 0.5$ and 0.29 at $z = 1$. The same increase in kurtosis has only a small impact on the number counts. The $\gamma = \kappa = 1$ case provides a 12% (34%) increase in the number counts at $z = 0.5$ (1), equivalent to increasing $\sigma_{\text{in}} M$ to 0.31 (0.33) for purely Gaussian scatter.

For the DES-like survey, a skewness of $\gamma (\kappa) = 0.5$ increases the mean number of clusters by 7% at $z = 0.5$ and 14% at $z = 1$. The increase in $dN/dz$ for $\gamma = 0.5$ is equivalent to increasing $\sigma_{\text{in}} M$ (for Gaussian scatter) from 0.5 to 0.56 and to 0.58 at $z = 1$. The more extreme case of $\gamma = \kappa = 1$ produces a 17% increase at $z = 0.5$ and 34% at $z = 1$, equivalent to increasing $\sigma_{\text{in}} M$ to 0.64 and 0.67, respectively.

The lower panel of Figure 2 indicates that non-Gaussian scatter has greater impact on the optical number counts for $z < 1$, but on the SZ counts at higher redshift. Comparing the redshift evolution of $\sigma_{\text{in}} M$ for the optical and SZ surveys, we find that they converge at $z = 1$. At lower redshifts, $\sigma_{\text{in}} M$ is greater for the DES-like survey, whereas for $z > 1$ it is greater for the SPT-like survey. Thus, at low redshift, the larger intrinsic scatter of the mass–optical richness relation drives the impact of non-Gaussian scatter. However, at high redshift, the steeper slope of the mass function at the SPT limiting mass is the dominant parameter.

In Figure 3, we plot the observed mass distribution of clusters, $M_{\text{obs}}(d\bar{n}/dM_{\text{obs}})$ (scaled to reduce the dynamic range, where $\rho_m$ is the mean matter density of the universe at each redshift) at $z = 0.5$. The line types and colors denote the same values of $\sigma_{\text{in}} M$, $\gamma$, and $\kappa$ as in Figure 2. For integrated SZ flux (for which $\sigma_{\text{in}} M = 0.25$), the higher order moments do not have a significant effect on the shape of the mass function. The impact of $\gamma$ and $\kappa$ is more evident for higher scatter mass indicators such as optical richness ($\sigma_{\text{in}} M = 0.5$, red lines) flattening the mass distribution above $2 \times 10^{14} h^{-1} M_\odot$. This roughly corresponds to the predicted values of $M_\gamma (z)$ at $z = 0.5$ in Figure 1 ($\omega = 2$ contour).

Overall, the results in this section demonstrate several points. A skewness of $\gamma = 0.5$ has the same effect on $dN/dz$ (at $z = 0.5$) as increasing the fiducial value of the variance $\sigma_{\text{in}} M$ by 25% for both the SZ and optical surveys. For the more non-Gaussian distribution parameterized by $\gamma = \kappa = 1$, the equivalent increase in $\sigma_{\text{in}} M$ is 54% and 64% for SPT and DES, respectively. The impact of non-Gaussian scatter on the cluster number counts for DES and SPT is similar because the
shallow slope of the mass function at the limiting mass of the optical survey compensates for the larger intrinsic scatter in the mass–optical richness relation (compared to the mass–SZ flux relation).

Second, an unknown level of non-Gaussianity in the scatter is equivalent to an uncertainty in the fiducial value of $\sigma^2_{\ln M}$. This uncertainty may provide a limit to the accuracy with which $\sigma_\gamma$ and the dark energy equation of state parameter $w$ can be measured using cluster number counts due to the degeneracies between these parameters and $\sigma^2_{\ln M}$ (Lima & Hu 2005).

Finally, if one aims to use the shape of the mass function to self-calibrate a survey to obtain information on the slope, normalization, and variance of the mass-observable scaling relation, then care must be taken to ensure that the parameters are not biased due to the increasing impact of the higher order moments on the shape of the mass function toward high masses.

4. CONCLUSION

We have investigated the impact of non-Gaussian scatter in the mass-observable scaling relation on the cluster mass function by incorporating the third and fourth moments (skewness and kurtosis) into the probability distribution $P(M_{\text{obs}} | M)$ via the Edgeworth expansion. We have demonstrated that a level of non-Gaussian scatter—i.e., the product of the standard deviation in $P(M_{\text{obs}} | M)$ and the slope of the mass function at the limiting mass of a survey—is greater than unity, then positive skewness and kurtosis will increase the number of clusters detected and flatten the high-mass end of the observed mass function. For low scatter mass proxies like integrated SZ flux, higher order moments do not strongly affect the mass and redshift distribution of clusters. However, for surveys utilizing a high scatter mass proxy such as optical richness, the limiting mass threshold must be less than $10^{14} h^{-1} M_\odot$ to ensure that the skewness does not significantly affect $dN/dz$, especially at high redshift.

We have demonstrated that the impact of non-Gaussian scatter on the observed cluster number count distribution should be similar for the SPT and DES surveys. While the scatter in the mass–optical richness relation is expected to be at least twice that of the mass–SZ flux relation, the slope of the mass function at the DES limiting mass threshold is significantly shallower than at the SPT mass threshold. Therefore, the values of $\sigma_{\ln M}$ and thus the impact of non-Gaussian scatter are similar for the two surveys.

We have also found that an unknown level of non-Gaussian scatter is roughly equivalent to an additional uncertainty on the variance $\sigma^2_{\ln M}$ in $P(M_{\text{obs}} | M)$ and thus may limit the constraints that can be placed on the dark energy equation of state parameter $w$. Furthermore, if one wishes to use the shape of the mass function in each redshift bin to self-calibrate for cluster scaling relation parameters then it will be necessary to account for non-Gaussian scatter on the shape of the mass function by marginalizing over the skewness and kurtosis parameters, $\gamma$ and $\kappa$, in addition to the variance $\sigma^2_{\ln M}$, slope, and normalization in each redshift bin.

We note that the values of $\gamma$ and $\kappa$ used in the examples given in this work were chosen arbitrarily (and to be small enough to ensure that the Edgeworth expansion remains an appropriate approximation to a non-Gaussian $P(M_{\text{obs}} | M)$). Recently, Stanek et al. (2010) investigated the Gaussianity of the intrinsic scatter around cluster scaling relations for a sample of clusters identified in hydrodynamical simulations (ignoring projection effects). They find that most cluster observables have effective Gaussian scatter. With the exception of the dark matter velocity dispersion, we find that $|\gamma| < 0.2$ for the observables they analyze (with several having negative values of $\gamma$). Cohn & White (2009) investigate the scatter in the mass–optical richness and mass–SZ flux relation, measuring the optical richness from mock-galaxy catalogs and the SZ flux from simulated maps (i.e., including projection effects). Their Figure 8 implies that $\gamma \approx 0.3$ for both the optical and SZ relations, with the non-Gaussianity driven by confusion between multiple objects along the same line of sight.

These results suggest that the level of non-Gaussian scatter in optical and SZ scaling relations may be less than that investigated in this paper. Large-volume simulations and mock-galaxy catalogs containing several thousands of clusters will be necessary to obtain more robust predictions of $\gamma$ and $\kappa$. Measuring $\gamma$ and $\kappa$ would require a large observational sample with precisely measured masses (for example, using X-ray spectroscopic temperatures). However, the higher order statistics are strongly influenced by rare, outlying objects (e.g., major mergers), or the impact of observational selection effects such as cluster–cluster confusion. For this reason, it is unlikely that it will be possible to place tight priors on these parameters.

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