Inertial spin alignment in a circular magnetic nanotube

G. Bergmann, R. S. Thompson, and J. G. Lu
Department of Physics and Astronomy
University of Southern California
Los Angeles, California 90089-0484
e-mail: bergmann@usc.edu
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Abstract

In Co-nanotubes with a curling magnetization, the orbital motion of the conduction electrons interacts with their spin. We predict that the (absolute) value of the magnetic energy of the spin $\mu \cdot B$ is strongly reduced. The new precession axis for the spin is almost parallel to the axis of the nanotube and precesses with the angular velocity of the electron. The physics of the ferromagnet is considerably modified.

Nanotubes and nanowires of both metals and semiconductors have been extensively studied for electric charge transport. However, the electron spin has been often ignored. How to control and manipulate the spin degree of freedom in nanostructures is of vital importance not only for fundamental science, but also for technological applications in micromagnetism and spintronics. This has stimulated much research effort in the synthesis and characterization of ferromagnetic nanowires and nanotubes. These quasi-one-dimensional magnetic nanostructures have exhibited unique and intriguing physical properties. As an example a number of magnetic nanotubes of different materials show the remarkable property that their magnetic polarization is circumferential around the axis of the tube [1], [2], [3], [4], [5], [6] (see Fig.1).
Fig.1: a) A magnetic Co-nanotube with the magnetization circular about the axis of the tube. b) The cross section in the limit of zero thickness.

In such a circular magnetic nanotube (CMNTB) the conduction electrons experience a change of direction of the internal exchange field $B$ during their propagation. This yields some interesting effects on the spin of the electrons. For a theoretical discussion we treat the CMNTB as a tube with zero thickness and large (infinite) mean free path of the electrons. We consider an electron with the velocity $v = v_z \hat{z} + v_\phi \hat{\phi}$ where $v_z$ is the component of the Fermi velocity $v_F$ parallel to the axis and $v_\phi$ is the circular velocity. The z-component of the electron velocity $v_z$ has no bearing on the results of the following consideration. Therefore we set for simplicity $v_z = 0$ and treat the electron propagation as circular. The radius of the tube is $R$. Then the electron circles the CMNTB with the angular frequency $\omega_e = (v_\phi / R) \hat{z}$.

The electron has a spin $s$ and a magnetic moment of $\mu$ where $\mu = \gamma s$ with $\gamma = -2\mu_B / \hbar = -e/m$ (We set the Lande factor $g$ for the conduction electrons equal to 2). The circular magnetization acts as a magnetic field of strength $B_0$ on the magnetic moment of the electron. In the inertial lab frame $S_0$ the magnetic field causes a torque $\tau$ on the magnetic moment of the electron

$$\tau = \mu \times B = \gamma s \times B$$  \hfill (1)

Along the circular path of the electron with the angular velocity $\omega_e$ the direction of the magnetic field changes. At the position $(R, \phi, z)$ (in cylinder coordiantes) the magnetic field is given by

$$B = B_0 (-\sin \phi, \cos \phi, 0)$$  \hfill (2)

As a consequence the torque constantly changes its direction for an electron whose position is given by $(R, \omega_e t, z)$ with $\phi = \omega_e t$. The fast angular velocity does not give the electron enough time to precess about the direction of the local magnetic field.
In the following we treat the motion of the electron in the frame $S$ that rotates with the electron, i.e. with the frequency $\omega_e$. We assign coordinate axes $(\hat{x}, \hat{y}, \hat{z})$ to the electron position which, at $t = 0$, are equal to $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$ of the lab system. We attach these axes $(\hat{x}, \hat{y}, \hat{z})$ rigidly to the cylindrical surface of the tube (at the position of the electron). In the next step the cylinder, electron and local axes $(\hat{x}, \hat{y}, \hat{z})$ rotate together with frequency $\omega_e$ (so that $(\hat{x}, \hat{y}, \hat{z}) = (\hat{r}, \hat{\phi}, \hat{z})$).

In the rotating system $S$ any vector $Q$ that is constant in the inertial system $S_0$ changes its direction according to
\[
\left( \frac{dQ}{dt} \right)_S = -\omega_e \times Q
\] (3)
(An analogous consideration yields the Coriolis and centrifugal forces on the surface of the earth).

For the spin this means that $ds/dt$ in the rotating system is given by
\[
\left( \frac{ds}{dt} \right)_S = \left( \frac{ds}{dt} \right)_{S_0} - \omega_e \times s
\] (4)
Here $(ds/dt)_{S_0}$ is the change of the spin due to the torque in the inertial system $S_0$, i.e.
\[
\left( \frac{ds}{dt} \right)_{S_0} = \tau = \gamma s \times B
\] (5)
In the rotating system the magnetic field is constant $B = (0, B_0, 0)$, and we obtain the for $ds/dt$
\[
\left( \frac{ds}{dt} \right)_S = \gamma s \times B + s \times \omega_e = \gamma s \times \left( B + \frac{\omega_e}{\gamma} \right)
\] (6)
yielding
\[
\left( \frac{ds}{dt} \right)_S = \gamma s \times B_{eff}
\] (7)
with
\[
B_{eff} = B + \frac{1}{\gamma} \omega_e = (0, B_0, \omega_e/\gamma)
\] (8)
The solutions to equ. (7) (in the system $S$) are those for a free electron spin in a constant field $B_{eff}$. The spin has a stable constant solution (when $\mu$ is parallel to $B_{eff}$) and a meta-stable solution (when $\mu$ is anti-parallel to $B_{eff}$). For a finite angle between $\mu$ and $B_{eff}$ the spin performs a precession about the direction of $B_{eff}$.

In components this yields
\[
\left( \frac{d}{dt} \right)_S \begin{pmatrix} s_x \\ s_y \\ s_z \end{pmatrix} = \begin{pmatrix} \omega_e s_y - \gamma B_0 s_z \\ -\omega_e s_x \\ \gamma B_0 s_x \end{pmatrix}
\] (9)
For the stationary solutions in the rotating system we set \((ds/dt)_S = 0\). This yields
\[
\begin{align*}
s_y &= \frac{\gamma B_0}{\omega_e} s_z \\
s_x &= 0
\end{align*}
\] (10)
(11)

So the unit vector of the spin is in \(S\)
\[
\hat{s} = \pm \frac{1}{X} (0, -\omega_B, \omega_e)
\] (12)

with \(\hbar \omega_B = 2 \mu_B B_0\) and \(X = \sqrt{\omega_e^2 + \omega_B^2}\). The magnetic moment \(\mu\) is parallel to \(B_{\text{eff}}\) for the stable solution or anti-parallel in the meta-stable solution.

If the spin is not parallel or anti-parallel to \(B_{\text{eff}}\) then it precesses about the effective field \(B_{\text{eff}}\) with a precession frequency of
\[
\omega_{\text{pre}} = \gamma |B_{\text{eff}}| = \sqrt{\omega_e^2 + (\gamma B_0)^2} = \sqrt{\omega_e^2 + \omega_B^2}
\] (13)

In the rotating system the effective field \(B_{\text{eff}}\) is fixed with the coordinates given by equ. \(\text{(8)}\). In the inertial lab frame \(B_{\text{eff}}\) rotates with \(\omega_e\) about the z-axis. This rotation and the precession about \(B_{\text{eff}}\) have opposite senses.

This is drawn in Fig.2. Here \(B_{\text{eff}}\) is the constant effective field in the rotating frame \(S\). For \(s\) parallel or anti-parallel to \(B_{\text{eff}}\) the spin is stationary in a stable or metastable
orientation (in \( S \)). If \( s \) has an arbitrary angle with \( \mathbf{B}_{\text{eff}} \) then \( s \) precesses about the axis \( \mathbf{B}_{\text{eff}} \) with the precession frequency \( \sqrt{\omega_e^2 + \omega_B^2} \). (To simplify the drawing we treated \( s \) and \( \mu \) as parallel.)

The physical picture in the inertial lab system \( S_0 \) is the following. The axis of precession \( \mathbf{B}_{\text{eff}} \) rotates itself with the frequency \( \omega_e \) about the \( z \)-axis. Furthermore the spin \( s \) precesses about \( \mathbf{B}_{\text{eff}} \) (see Fig.2). The rotation of the \( \mathbf{B}_{\text{eff}} \) axis with frequency \( \omega_e \) and the precession about this axis with frequency \( \sqrt{\omega_e^2 + \omega_B^2} \) have opposite senses.

For \( \omega_e \gg \omega_B \) the actual precession in the lab system is approximately the difference

\[
\omega_{\text{pcn}} \approx \sqrt{\omega_e^2 + \omega_B^2} - \omega_e \approx \frac{\omega_B^2}{2\omega_e} \tag{16}
\]

This is a much smaller precession frequency than \( \omega_B = |\gamma| B_0 = 2\mu_B B_0/\hbar \) which one would observe for a constant magnetic field of \( B_0 \hat{z} \) in \( z \)-direction.

In addition the lowest magnetic energy of the electron magnetic moment in the field \( \mathbf{B} \) is reduced to

\[
E_{\text{mag}} = -\mu \cdot \mathbf{B} = -\mu_B B_0 \cos \theta \tag{17}
\]

where \( \theta \) is the angle between \( \mathbf{B}_{\text{eff}} \) and \( \mathbf{B} \) with

\[
\tan \theta = \frac{\omega_e}{\omega_B}, \quad \cos \theta = \frac{\omega_B}{\sqrt{\omega_e^2 + \omega_B^2}} \tag{18}
\]

A quantum theoretical treatment of this effect is desirable. The simplest approach in the inertial lab system would be to find the right Hamiltonian. The potential energy is straightforward \( U = -\mu \cdot \mathbf{B}(\phi) \). There is no kinetic energy in the spin precession. But using \( U(\phi) \) as the Hamiltonian does not include the dynamics of the orbiting electron. The \( \phi \)-dependent magnetic exchange field \( \mathbf{B} \) complicates the calculation. We apply two different approaches in the rotating system, (i) with the local axes \((\hat{x}, \hat{y}, \hat{z})\) fixed parallel to the axis of the inertial frame and (ii) the local axes fixed to the rotating system. The preliminary results confirm the conclusions of the classical approach. But naturally one has to include the quantization of \( \omega_e \) and of \( \omega_{\text{pcs}} \).

**Thomas precession:** In our non-relativistic calculation we had two coordinate systems, \((\hat{x}, \hat{y}, \hat{z})\) for the rotating system \( S \) and \((\hat{e}_1, \hat{e}_2, \hat{e}_3)\) for the inertial system \( S_0 \). Classically the system \( S \) is rotating with \( \omega_e \) with respect to system \( S_0 \). However, the rotating system \( S \) experiences a time-dependent acceleration \( \mathbf{a} \). In a relativistic calculation this acceleration yields an additional precession of the axes of the orbiting system which was first calculated by Thomas [7] and is given by

\[
\omega_{\text{Th}} = -\frac{1}{2c^2} \mathbf{v} \times \mathbf{a} \tag{19}
\]

In our case we have

\[
\mathbf{a} = (-R\omega_e^2 \cos(\omega_e t), -R\omega_e^2 \sin(\omega_e t), 0)
\]

which yields a Thomas precession about the axis of the nanotube of

\[
\omega_{\text{Th}} = \frac{R^2 \omega_e^3}{2c^2} \tag{20}
\]
The Thomas precession changes the observed precession of the electron spin in the inertial lab system by $\omega_{Th}$. Below we estimate the contribution of the different terms and conclude that the Thomas precession can be neglected.

For a quantitative discussion essentially two parameters are required, the (maximal) angular frequency $\omega_e$, given by the radius and the Fermi velocity and the magnetic field acting on the conduction electrons. Here one has two extremes cases:

1) Ferromagnets which are described by the Stoner model. Here one assumes essentially only one band which is generally the d-band of transition metal ferromagnets. The Stoner model connects the Stoner field $B_0$ with the Curie temperature $T_C$ through the relation $B_0 \approx k_B T_C / \mu_B$. This yields fields in the range of a few $10^3 T$. Due to the flat d-bands the Fermi velocity is generally a factor of 10 smaller than in (s,p)-metals.

2) A two-band ferromagnet where the magnetic properties are defined by the d-electrons and the conduction electrons are (s,p)-electrons. In this case it is more difficult to estimate the $B_0$ field and the Fermi velocity. In the literature values for the Fermi energy of spin-up and down conduction electrons are given. In ref. [8] tunnel experiments into CoFe and NiFe alloys are evaluated with values for the Fermi energy of spin-up and down conduction electrons.

We use for the following estimate the value $v_F \approx 10^6 m/s$. This yields for a Co nanotube with the radius $R = 25 nm$ the (maximal) value $\omega_e \approx 4.0 \times 10^{13} s^{-1}$. Only when the angle between $B_{eff}$ and the z-axis is small can one use the simple relation (16) for the precession frequency in the lab frame. In Table I this angle $\alpha = \angle (B_{eff}, \hat{z})$ is given in degrees for different values of $B_0$. For the larger values the superposition of the rotation and precession in the rotating system $S$ yields a complicated wobbling motion in the lab system $S_0$.

| $B_0$  | $\omega_B/\omega_e$ | $\alpha$ |
|--------|----------------------|---------|
| $10T$  | $4.4 \times 10^{-2}$  | 2.5°    |
| $10^2T$| $4.4 \times 10^{-1}$  | 24°     |
| $10^3T$| 4.4                  | 77°     |

Table I: The ratio $\omega_B/\omega_e$ and the resulting angle $\alpha$ between $B_{eff}$ and the z-axis are calculated for different values of $B_0$.

For the Thomas precession frequency we obtain a value of $\omega_{Th} \approx -2.2 \times 10^8 s^{-1}$. This value is much smaller than precession frequency $\omega_{prc}$ and can be neglected.

To summarize our conclusion: The odd alignment of the electron spins in a magnetic nanotube with circular magnetization has a number of interesting effects which modify the magnetic properties. A few shall be considered here qualitatively. In the following we assume...
that the mean free path of the conduction electrons is sufficiently long so that the conduction electrons can circle the nanotube several times before they are scattered.

(1) The ground-state energy of the circular magnetic state is increased. In a regular ferromagnetic metal the conduction electrons align parallel or anti-parallel to the exchange field $B$ and lower their energy by $N_0 (\mu_B B_0)^2$ where $N_0$ is the (conduction) electron density of states per spin. In the nanotube with circular magnetization this energy reduction is much smaller and therefore the ground-state energy is increased by almost the same amount.

(2) This energetic effect should be particularly important for Stoner magnets. Here the magnetic moments are band electrons (generally d-electrons) which are not localized and possess a finite (group) velocity $v_d(k) = (1/\hbar) \partial \varepsilon_d(k) / \partial k$. In a magnetic field the spin-up and -down d-electrons are shifted in opposite directions on the energy scale. The resulting magnetization acts back on the d-moments through the Coulomb exchange field, and the magnetization becomes Stoner-enhanced. For a sufficiently large product of $N_d U$ ($N_d=$d-electron density of states, $U=$Coulomb exchange energy) the d-band makes a transition into a Stoner band magnet. This mechanism would be dramatically disturbed if the propagating d-electrons don’t align their moments in the direction of the circular magnetization but (almost) parallel and anti-parallel to the cylinder axis. If the $\phi$-component of $v_d$ is sufficiently large then half the d-electrons align their moments (roughly) parallel and the other half anti-parallel to the cylinder axis, cancelling the exchange field. A conclusive answer requires, of course, a detailed band structure calculation for the Stoner system under consideration.

(3) The interaction between spin waves and the conduction electrons will be altered. The excitation of a spin wave means the transfer of an angular momentum $\hbar$ from a conduction electron into the spin wave. Normally this is a simple transfer because the electron spin and the magnetization have the same quantization direction. However, in the CMNTB the two quantization directions are almost orthogonal to each other. The investigation of this interaction is to be considered in the future.

(4) By covering the circular magnetic Co nanotube with another ferromagnet or a superconductor one can investigate a cylindrical proximity effect. We expect a considerable potential for new and interesting effects.

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