Evaluation of $\Delta J$-integral for a shallow crack in steel for pressure vessels under large scale yielding (LSY) condition

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Abstract. Fracture mechanics studies on fatigue crack growth under elastic-plastic condition is usually characterized by $J$ integral range, $\Delta J$ determined by an approximation from area under load-displacement hysteresis loop developed by Dowling and Begley. Despite being supported by some experimental data which reported that fatigue crack growth rates under elastic-plastic conditions have good correlations with $\Delta J$, the applicability of $\Delta J$ on crack growth in a material subjected to cyclic bending remains inconclusive. In this study, finite element analysis of a four-point-bending rectangular plate model with an edge crack length $a/W$ between 0.1 to 0.5 was carried out under displacement amplitude $\delta_a$ between 0.1 to 1.5 mm to represent small to large scale yielding conditions. The elastic component was found to be the driven force of fatigue crack growth parameter in small scale yielding (SSY) condition while in large scale yielding (LSY) condition, the plastic component must be considered. It was also found that the simple method of $\Delta J$-integral calculation using load-displacement approach is overestimated for shallow crack under LSY condition.

1. Introduction

The $J$-Integral range, $\Delta J$ calculated from a load-displacement approach which was introduced by Dowling and Begley has been used as a parameter to correlate with fatigue crack growth rate under elastic-plastic condition [1]. Several studies have been conducted to discussed $\Delta J$ from theoretical point of view [2]-[4]. It was found that $\Delta J$ values from finite element analysis of elastic-plastic crack growth under cyclic loading shows path independency using path-integral method [2]. The validity of simple estimation technique by Dowling and Begley on $\Delta J$ calculation has been discussed by Azmi et al. [5]. They compare $\Delta J$ value obtained from path-integral ($\Delta J_{\text{path}}$) and simple estimation method ($\Delta J_{\text{p,\delta}}$) on cyclic bending deformation and found that the stress distribution ahead of crack tip could be characterized using $\Delta J$ calculated and elastic-plastic fatigue crack growth rate would be evaluated using $\Delta J$. In recent studies, fatigue crack growth tests have been conducted using CT specimen as well as finite element analysis to establish an expression of $\Delta J$ [6][7]. This show that the evaluation of $\Delta J$ on the fatigue crack growth property remains controversial. Furthermore, the applicability of the simple estimation technique to calculate $\Delta J$ on shallow crack conditions on cyclic bending remains unclear. Thus, finite element analysis of a four-point bending rectangular plate specimen with various edge crack length was carried out in this study under small to large scale yielding conditions.
2. Methodology

2.1. Material

The material to be analyzed in this study was hot rolled SPV235 which is a steel for pressure vessels used for intermediate temperature services. The mechanical properties of the material are as follows; the Young’s Modulus $E = 215$ GPa, Poisson’s Ratio $\nu = 0.32$, yield stress $\sigma_y = 247$ MPa and work-hardening exponent $n = 0.21$. For the sake of simplicity, the monotonic stress-strain relation is used.

2.2. Analytical Procedure

Figure 1 shows finite element mesh of two-dimensional model for half region of rectangular plate specimen with an edge crack of $a$, having a width of $W = 15$ mm and length of $L = 70$ mm. An elastic-plastic finite element analysis was conducted on a four-point bending rectangular model using an outer and inner span of $L_o = 60$ mm and $L_i = 30$ mm, respectively. The edge crack length, $a$ is modelled in range between 1.5 to 7.5 mm ($a/W = 0.1$ to 0.5) for the simulation processes. Due to symmetry, only half region of the model was analysed. The 8-noded isoparametric elements were used. The simulation was conducted under plain strain condition using ANSYS 15.0 Software [8]. In the symmetric plane, a rigid plane as a contactor was introduced to simulate crack open and closure behaviour as shown in Fig. 1(b). The coefficient of friction was set to be zero as the contact behaviour at the crack surface. The contact simulation was conducted using the augmented lagrangian method [9].

Fully reversed loading is used for the loading process. First, the displacement was applied up to $+\delta_a$, then the reversal displacement was applied up to $-\delta_a$. Finally, the displacement was again applied up to $+\delta_a$. The applied displacement amplitude $\delta_a$ was change in the range between 0.1 mm to 1.5 mm. Note that the direction of displacement $\delta$ to open the crack were set to be positive.

![Figure 1. Finite Element Mesh for half region of four-point bending rectangular specimen.](image)

2.3. $\Delta J$ Calculation Method

Using the load-displacement hysteresis loop from a simulation result, $\Delta J_{p,\delta}$ was calculated as in eq. (1) [10].

$$\Delta J_{p,\delta} = \Delta J_e + \Delta J_p$$  \hspace{1cm} (1)

where, $\Delta J_e$ and $\Delta J_p$ represent an elastic and plastic component, respectively. The value of $\Delta J_e$ was calculated by following equation.

$$\Delta J_e = \frac{(1-\nu^2)\Delta K_I}{E}$$  \hspace{1cm} (2)
where, Δ\(K_I\) is stress intensity factor range, and the Δ\(K_I\) was calculated using the handbook [11]. Also, Δ\(J_p\) was calculated by

\[
\Delta J_p = \frac{2\pi}{\tau(W-a)}
\]

where, \(A_p\) corresponding to plastic work obtained from load-displacement curve as defined by Azmi et al. [5] and \(t\) is the specimen thickness.

Following the original definition of \(J\)-integral for monotonic loading [12], the Δ\(J_{path}\) was defined by

\[
\Delta J_{path} = \int_{R} (\Delta W_n_x - \Delta T_i \cdot \frac{\partial \Delta u_i}{\partial x}) \, ds
\]

where, \(\Delta T_i\) is the range of traction vector and \(\Delta u_i\) is the difference in displacement. The value of \(\Delta W\) is the strain energy density given by following equation,

\[
\Delta W = \int_0^{\sigma_{ij}} \Delta \sigma_{ij} \, d\Delta \varepsilon_{ij}
\]

where, \(\Delta \sigma_{ij}\) and \(\Delta \varepsilon_{ij}\) are the range of stress and strain from the corresponding current values to the opening point as a reference point, respectively.

3. Results and Discussions

Figure 2(a) and (b) show typical load-displacement hysteresis loop for \(a/W = 0.1\) with an applied displacement amplitude \(\delta_a\) of 0.3 and 1.5 mm, respectively. In the first step, crack opening displacement was simulated followed by reversal displacement in second step with crack closure process. Then, crack opening deformation was analysed again in the last step. It was found that for small scale applied-displacement as in Fig. 2(a), the width of the hysteresis loop was very small and elastic deformation was dominant. On the other hand, for large scale applied-displacement shown in Fig. 2(b), the hysteresis loop was wide and plastic deformation was dominant. From Fig. 2, it is difficult to determine the crack opening and closure point based on load-displacement hysteresis loop in shallow crack condition regardless of applied-displacement condition. Therefore, a crack opening point (R-point) base on crack opening point were estimated on the third step of loading process as defined by Azmi et al. and the Δ\(J\)-integral were calculated [5].

![Figure 2](image_url)

\(\delta_a = 0.3\) mm  \(\delta_a = 1.5\) mm

**Figure 2.** Load-displacement hysteresis loop for \(a/W = 0.1\).
Figure 3 shows the change in $\Delta J_{p}/\Delta J_{\text{path}}$ as a function of displacement amplitude by three different crack length i.e. $a/W = 0.1, 0.3$ and $0.5$. It was found that in the case of shallow crack, the value of $\Delta J$ calculated based on the simple method proposed by Dowling and Begley ($\Delta J_{p}$) did not agree with the $\Delta J$ calculated based on path integral ($\Delta J_{\text{path}}$). However, the difference between these two approaches become small with increasing of crack length. In addition, regardless of crack length the difference also become smaller when the applied displacement $\delta_{a}$ is decrease.

Figure 4 shows the value of elastic and plastic component of $\Delta J$ ($\Delta J_{e}$ and $\Delta J_{p}$) versus applied displacement $\delta_{a}$. It was found that the elastic component is dominant when the applied displacement $\delta_{a}$ is below $0.2$ mm while plastic component is nearly zero in this condition. This shows that in this condition, the driving force of fatigue crack growth parameter is only affected by $\Delta J_{e}$ while $\Delta J_{p}$ can be neglected. In other words, this condition is under small scale yielding (SSY) condition. On the other hand, plastic component $\Delta J_{p}$ has major contribution than elastic component $\Delta J_{e}$ when the applied displacement $\delta_{a}$ is above $0.2$ mm. This condition is also known as large scale yielding (LSY) condition.
4. Conclusions
In this study, finite element analysis of a four point-bending rectangular plate model with various edge crack length was carried out under fully reversed cycling loading from small to large scale yielding conditions. It was found that the load-displacement approach proposed by Dowling and Begley cannot be used for $\Delta J$ calculation in shallow crack under LSY conditions. Moreover, plastic component $\Delta J_p$ has major contribution towards $\Delta J$ values under LSY conditions while in SSY condition, it can be considered that the fatigue crack growth parameter is driven by only elastic component $\Delta J_e$ since the plastic component $\Delta J_p$ is almost zero in this state.

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