Steady helix states in a resonant XXZ Heisenberg model with Dzyaloshinskii-Moriya interaction

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We systematically investigate possible helix states in XXZ Heisenberg model with Dzyaloshinskii-Moriya (DM) interaction. Exact solutions show that a set of precession helix states can be constructed by deliberate superposition of degenerate eigenstates of the Hamiltonian under the resonant condition. When a non-Hermitian balance boundary term is imposed as a quenching action, the quench dynamics shows that a steady helix state emerges from some easily prepared initial states, including saturate and maximally mixed ferromagnetic states, according to the analysis of perturbation method. The corresponding dynamics for near resonant cases is also investigated numerically, indicating the robustness of the scheme. Our findings highlight the cooperation of non-Hermiticity and the DM interaction in quantum spin system, suggesting a way for preparing steady helix state in non-Hermitian quantum spin system.

I. INTRODUCTION

The quantum Heisenberg model, as a simple model of interacting spins, takes an important role in physics. It not only captures the properties of many magnetic materials, but also provides a tractable theoretical example for understanding fundamental concepts in physics. Although the one-dimensional Heisenberg chain is an old topic, quantum dynamics of the system is still an active frontier of research, especially after the quantum simulator is realized in experiment [1–7]. Recently, the discovery of highly excited many-body eigenstates of the Heisenberg model, referred to as Bethe phantom states, has received much attention from both theoretical [8–11] and experimental approaches [12–15].

In this work, we investigate possible helix states in XXZ Heisenberg model under two considerations. One corresponds to the introduction of Dzyaloshinskii-Moriya (DM) interaction. The DM interaction is an antisymmetric exchange interaction that appears in inversion asymmetric structures and favors perpendicular alignment of neighboring spins in a magnetic material [16–18]. The other is the imposed non-Hermitian balance boundary condition, which takes the role of source and drain of spin flip. Under a resonant condition on the DM and anisotropic terms, the modified Heisenberg model obeys the SU(2) symmetry, and then possesses a set of degenerate eigenstates. It allows the existence of spin helix state as exact solution obtained by deliberate superposition of these degenerate eigenstates. We are interested in the dynamic preparation of the spin helix state. Based on the analysis of perturbation method, it is shown that a steady helix state emerges from some easily prepared initial states, including saturate and maximally mixed ferromagnetic states, when a non-Hermitian balance boundary is imposed as a quenching action. For near resonant cases, the corresponding dynamics is also investigated numerically and the results indicate that the scheme works well at certain time window. It relates to an exclusive concept in a non-Hermitian system, exceptional point (EP), which has no counterpart in a Hermitian system. The EP in a non-Hermitian system occurs when eigenstates coalesce [19–21], and usually associates with the non-Hermitian phase transition [22, 23]. In a parity-time (PT) symmetric non-Hermitian coupled system, the PT-symmetry of eigenstates spontaneously breaks at the EP [24–29], which determines the exact PT-symmetric phase and the broken PT-symmetric phase in this system.

In this work, we will impose a pair of balance non-Hermitian impurities [30, 31] to the ends of the spin chain, as non-Hermitian boundary condition. The corresponding dynamics is also investigated analytically and numerically. The approximate solutions for the quantum spin chain with finite length provide valuable insights for the description of the non-equilibrium dynamics. Our findings highlight the cooperation of non-Hermiticity and the DM interaction in quantum spin system, suggesting a way for preparing steady helix state in non-Hermitian quantum spin system.

The rest of this paper is organized as follows: In Sec. II, we introduce the model Hamiltonian and the corresponding SU(2) symmetry. With these preparations, in Sec. III we demonstrate that two types of helix states can be constructed by a set of degenerate eigenstates. Based on these results, the dynamic generation of spin helix state are proposed in Sec. IV by means of three kinds of imposed fields. Sec. V concludes this paper.

II. MODEL HAMILTONIAN AND SYMMETRIES

We begin this section by introducing a general Hamiltonian

\[ H = H_0 + H_1 \]  

(1)

where \( H_0 \) and \( H_1 \) describe quantum spin Heisenberg chain with DM interaction and external interaction re-
spectively

\[ H_0 = - \sum_{j=1}^{N-1} (J_x s_j^x s_{j+1}^x + J_y s_j^y s_{j+1}^y + J_z s_j^z s_{j+1}^z) \]

\[ + \frac{D}{2} \sum_{j=1}^{N-1} (s_j^z s_{j+1}^z - s_{j+1}^z s_j^z), \]

where \( k_0 = \text{arctan}(D/J) \) is a crucial factor for helix state arising from \( D \).

For eigenstates \( |\psi_n\rangle \), straightforward derivation of \( h^\alpha_i(n) = \langle \psi_n | s^\alpha_i | \psi_n \rangle \) show that

\[ h^x_i(n) = h^y_i(n) = 0, h^z_i(n) = \frac{n}{N} - \frac{1}{2}, \]

which is uniform, indicating that \( |\psi_n\rangle \) is not a helix state. Nevertheless, in the following we will show that their superposition can be helix states. And these states can be classified as two types of helix states: precession and entanglement helix states.

A. Precession helix state

We consider a superposition eigenstates in the form

\[ |\phi(\theta)\rangle = \sum_n d_n |\psi_n\rangle, \]

where

\[ d_n = \sqrt{C^N}\sin^n(\theta/2)\cos^{(N-n)}(\theta/2). \]

The corresponding helix vector is

\[ h_\alpha_i = \frac{1}{2} [\sin \theta \sin(k_0 l), \sin \theta \cos(k_0 l), -\cos \theta], \]

which indicates that \( |\phi(\theta)\rangle \) is a helix state for nonzero \( \sin \theta \). Here \( \theta \) is an arbitrary angle and determines the profile of the state. This can be obtained easy when we express it in the form.

\[ |\phi(\theta)\rangle = \prod_{j=1}^{N} \left(-ie^{ik_0 j} \sin(\theta/2)|\downarrow\rangle_j + \cos(\theta/2)|\uparrow\rangle_j \right). \]

It represents a tensor product of the precession states of all spins, which is an unentangled state. It accords with the result \( |\psi_i\rangle = 1/4 \).

III. TWO TYPES OF HELIX STATES

In this section, we will introduce two types of helix states based on the eigenstates of \( H_0 \) with \( \Delta = 1 \). We start by the ferromagnetic eigenstate of \( H_0 \)

\[ |\psi_0\rangle = |\psi\rangle = \prod_{j=1}^{N} |\downarrow\rangle_j, \]

satisfying the equation \( H_0 |\psi_0\rangle = -(N-1)/4 |\psi_0\rangle \), with \( s_j^z |\downarrow\rangle_j = -1/2 |\downarrow\rangle_j \). Based on the symmetry of \( H_0 \) mentioned above, a set of eigenstates \( \{ |\psi_n\rangle, n \in [1, N]\} \) can be constructed as

\[ |\psi_n\rangle = \frac{1}{\Omega_n} (s^+_k)^n |\downarrow\rangle, \]

where the normalization factor \( \Omega_n = (n!)^{1/N} \).

Obviously, we have \( |\psi_n\rangle = e^{ik_0 (1+N)/2 |\downarrow\rangle_j} \). We introduce a local vector \( h_\alpha_i = (h^x_i, h^y_i, h^z_i) \) with \( h^\alpha_i = \langle \psi | s^\alpha_i | \psi \rangle \) (\( \alpha = x, y, z \)) to characterize the helicity of a given state \( |\psi\rangle \).

For eigenstates \( |\psi_n\rangle \), straightforward derivation of \( h^\alpha_i(n) = \langle \psi_n | s^\alpha_i | \psi_n \rangle \) show that

\[ h^x_i(n) = h^y_i(n) = 0, h^z_i(n) = \frac{n}{N} - \frac{1}{2}, \]

which is uniform, indicating that \( |\psi_n\rangle \) is not a helix state. Nevertheless, in the following we will show that their superposition can be helix states. And these states can be classified as two types of helix states: precession and entanglement helix states.
FIG. 1. Plots of helix vector from Eq. (11) for several representative values of $\theta$, with parameters $k_0 = \arctan(0.5)$ and $N = 10$. For $\theta = 0$ and $\pi$, all spins align in the $z$-direction. For $\theta = \pi/2$ and $3\pi/2$, all spins lie in the $xy$-plane.

We note that
\[(R_0^\pm)^m |\phi(\theta)\rangle \propto |\phi(\theta')\rangle,\]
with $\tan(\theta'/2) = (m + 1) \tan(\theta/2)$, which indicates that the action of operator $(R_0^\pm)^m$ is a shift of the angle $\theta \rightarrow \theta'$, referred to as angle shift operator.

B. Entanglement helix state

Here is an example for entanglement helix state. We construct a state by a simple superposition
\[|\psi_E\rangle = \frac{1}{\sqrt{N^2 + N}} \sum_j \left(e^{ik_0 s_j^+} + 1\right) |\downarrow\rangle.\] (18)

The corresponding helix vector is
\[h_l = \frac{1}{2(N+1)}[2\cos(k_0 l) - 2\sin(k_0 l) , \frac{2}{N} - N - 1],\]
which indicates that $|\psi_E\rangle$ is a weak helix state for finite $N$. In addition, we note that
\[|h_l|^2 = \frac{4N^2 + (N^2 + N - 2)^2}{4N^2(N+1)^2},\]
and $|h_l|^2 < 1/4$ for finite $N$. It indicates that $|\psi_E\rangle$ cannot be written as a tensor product, in the form of $|\phi(\theta)\rangle$.

Helix state $|\psi_E\rangle$ is an entangled state. This example indicates that if the coefficients $\{d'_n\}$ of superposition $\sum_n d'_n |\psi_n\rangle$ are deviated from the set $\{d_n\}$ a little, the quasi-helix state is probably entangled.

In comparison with the helix states presented in previous work [11, 15], the existence of the set of states $\{|\psi_n\rangle\}$ are well understood on the basis of the modified SU(2) symmetry of $H_0$. In the presence of $H_1$, the SU(2) symmetry is broken, $(N + 1)$-fold degeneracy is left and the set of states $\{|\psi_n\rangle\}$ are no longer the eigenstates. Nevertheless, certain appropriately designed external field $H_1$ may provide a pathway to hybrid the $(N + 1)$-fold degenerate states, forming the helix state on demand. Similar to the helix state in XXZ chain, the present helix states contain the information of $H_0$, the strength of DM interaction $D$.

IV. DYNAMIC GENERATION OF HELIX STATE

In this section, we focus on the preparation of a helix state through a dynamic way, which is a crucial step in coherent experimental protocol. The strategy is to take an easily prepared eigenstate of $H_0$ as the initial state, and then add $H_1$. It is expected that the evolved state to be a helix state at certain instant. In the following, we consider three kinds of $H_1$, which are spatially modulated Hermitian, non-Hermitian fields, and balanced non-Hermitian boundary respectively.
A. Hermitian field

We consider the situation that the system is exerted by a resonant field

\[ B_j = B_0(t) \{ \cos (k_0 j), - \sin (k_0 j), 0 \}, \]

(21)

where \( B_0(t) \) is an arbitrary function of time, but is taken as a pulse function in our scheme. Here the word resonance does not mean in the magnitude or frequency but the matching distribution of the field with coupling strength in the spin chain. We will show that such a spatially modulated pulse field can drive a simple ferromagnetic state to a precession helix state.

In general, the time evolution of a given initial state \(| \psi (0) \rangle \) under a time dependent Hamiltonian \( H(t) \) can be expressed as

\[ | \psi (t) \rangle = \mathcal{T} \exp \left[ -i \int_0^t H(t')dt' \right] | \psi (0) \rangle, \]

(22)

with \( \mathcal{T} \) being the time-ordered operator. The merit of a resonant field is the commutative relation

\[ [H_0, H_1] = 0, \]

(23)

which ensures the analytical expression

\[ | \psi (t) \rangle = e^{it(N-1)/4} e^{-i \int_0^t H_0(t')dt'} | \psi (0) \rangle = e^{it(N-1)/4} \prod_{j=1}^N \left\{ -ie^{i \theta} \sin \left( \int_0^t \frac{1}{2} B_0(t')dt' \right) | \uparrow \rangle \right\}, \]

(24)

for the initial state \(| \psi (0) \rangle = | \uparrow \rangle \). Obviously, it is a precession helix state with the vector

\[ h_1(t) = \frac{1}{2} [\sin \theta \sin (k_0 l), \sin \theta \cos (k_0 l), - \cos \theta], \]

(25)

where \( \theta \) is a function of time

\[ \theta (t) = \int_0^t B_0(t')dt'. \]

(26)

One find that \(| \psi (t) \rangle \) is a helix state at every fixed time point satisfying \( \theta = n \pi + \pi /2, (n \in Z) \). Specifically, when we take \( B_0(t) \) as a pulse field satisfying \( B_0(t) = 0 \) for \( t > T \), and \( \int_0^T B_0(t)dt = \pi /2 \), we have a stable state with maximal helicity

\[ h_1(t > T) = \frac{1}{2} [\sin (k_0 l), \cos (k_0 l), 0]. \]

(27)

As an example, we consider a Gaussian pulse driving field

\[ B_0(t) = \sqrt{\frac{2 \alpha}{\pi}} \exp [-\alpha (t - T/2)^2], \]

(28)

where the internal \( T \) is taken sufficiently long as \( \alpha \gg T^{-2} \) to meet \( \int_0^T B_0(t)dt \approx \pi /2 \). Note that the conclusion is obtained under the resonant condition \( \Delta = 1 \). It is expected that a similar helix state can still be obtained when \( \Delta \) deviates a little from 1. The computation is performed by using a uniform mesh in the time discretization for the time-dependent Hamiltonian \( H(t) \). We consider the case with initial state \(| \psi (0) \rangle = | \phi (0) \rangle \). We introduce the quantity

\[ p(t) = | \langle \psi (t) | \phi (\theta) \rangle |^2 \]

(29)

to characterize the fidelity of the scheme. The plots of \( p(t) \) in Fig. 2 for several typical cases show that the scheme works well even for the case with \( \Delta \neq 1 \). However, the flaw of this scheme is that the prior knowledge of system parameter \( k_0 \) and a time-dependent field is required.

B. Non-Hermitian field

Now we turn to alternative scheme to prepare helix state by non-Hermitian \( H_1 \). It is a crossover scheme for the case that \( k_0 \) is unknown. We start with the investigation for an exactly solvable case, in which the external field is a complex spatially modulated field

\[ B_j = B_0 e^{ik_0 j} (1, i, 0), \]

(30)
with which we still have $[H_0, H_I] = 0$. Importantly, we have
\[ H_I |\psi_n\rangle = B_0 \sqrt{(n+1)(N-n)} |\psi_{n+1}\rangle, \tag{31} \]
with $n \in [0, N-1]$, which ensures the existence of an invariant $(N+1)$-D subspace spanned by the set of states \([|\psi_n\rangle]\). The matrix representation of Hamiltonian $H$ is an $(N+1) \times (N+1)$ matrix $M$ with nonzero matrix elements
\[ (M)_{N+1-n, N-n} = B_0 \sqrt{(n+1)(N-n)}, \tag{32} \]
with $n = [0, N-1]$, and
\[ (M)_{N+1-n, N+1-n} = -(N-1)/4, \tag{33} \]
with $n = [0, N]$. It is obviously $M + (N-1)/4$ is a nilpotent matrix, i.e.
\[ [M + (N-1)/4]^{N+1} = 0, \tag{34} \]
or an $(N+1)$-order Jordan block. The dynamics for any states in this subspace is governed by the time evolution operator
\[ U(t) = e^{-iMt} = \sum_{l=0}^{N} \frac{1}{l!} (-iMt)^l. \tag{35} \]

Then for the initial state $|\psi(0)\rangle = |\uparrow\rangle$, we have the normalized evolved state
\[ |\psi(t)\rangle = \frac{e^{it(N-1)/4}}{\sqrt{(1+B_0^2t^2)^N}} \prod_{j=1}^{N} (-itB_0e^{ik_0j} |\uparrow\rangle_j + |\downarrow\rangle_j), \tag{36} \]
which turns to the coalescing state, i.e., $|\psi(\infty)\rangle \rightarrow |\uparrow\rangle$. Accordingly, we have
\[ h(t) = \frac{B_0t}{1+B_0^2t^2} \left[ \sin (k_0t), \cos (k_0t), \frac{B_0^2t^2-1}{2B_0t} \right], \tag{37} \]
which indicates that $|\psi(t)\rangle$ is a helix state at finite time. At instant $t = B_0^{-1}$, it reaches the maximal helicity
\[ h(B_0^{-1}) = \frac{1}{2} \left[ \sin (k_0t), \cos (k_0t), 0 \right]. \tag{38} \]
The above analysis is still true when we take $B_j = B_0e^{-ik_0j} (1,-i,0)$ and $|\psi(0)\rangle = |\uparrow\rangle$, which corresponds to a time reversal process.

\section*{C. Non-Hermitian boundary}

So far, it seems that the introduction of the complex field does not improve the scheme since it still requires a specific field distribution. The only difference is that the time evolution under $U(t)$ is unidirectional, rather than periodic in the Hermitian system. However, there is a key fact that the Jordan block still exists when we take a local complex field at $l$th site
\[ B_j = B_0\delta_{jl} (1, i, 0). \tag{39} \]

Actually, in the case of $\Delta \neq 1$, states $|\uparrow\rangle$ and $|\uparrow\rangle$ are two degenerate states of the Hermitian Hamiltonian $H_0$, and we have
\[ H |\uparrow\rangle = -(N-1)/4 |\uparrow\rangle, H^\dagger |\downarrow\rangle = -(N-1)/4 |\downarrow\rangle, \tag{40} \]
due to the facts
\[ H_1 |\uparrow\rangle = 0, (H_1)^\dagger |\downarrow\rangle = 0. \tag{41} \]

It means that two states $|\uparrow\rangle$ and $|\downarrow\rangle$ are mutually biorthogonal conjugate and $|\downarrow\rangle |\uparrow\rangle$ is the biorthogonal norm of them. Importantly, the vanishing norm $|\downarrow\rangle |\uparrow\rangle = 0$ indicates that state $|\uparrow\rangle (|\downarrow\rangle$ is coalescing state of $H$ ($H^\dagger$), or Hamiltonians $H$ and $H^\dagger$ get an EP. From the perspective of dynamics, we have
\[ e^{-iHt} |\uparrow\rangle \rightarrow |\uparrow\rangle, e^{-iH^\dagger t} |\downarrow\rangle \rightarrow |\downarrow\rangle, \tag{42} \]
for a sufficiently long time $t$. Although both states $|\uparrow\rangle$ and $|\downarrow\rangle$ are not helix states, $e^{-iHt} |\uparrow\rangle$ and $e^{-iH^\dagger t} |\downarrow\rangle$ may have helicity at finite $t$ from the observation at the end of the previous subsection, for instance, Eq. (38).

This inspires us to consider a balanced local complex field
\[ B_j = B_0 [\delta_{1j} (1, i, 0) + \delta_{Nj} (1, -i, 0)], \tag{43} \]
which acts as non-Hermitian boundary and may result in stable helix state after a relaxation time. The physical intuition for this setup is simple. One complex field acts as a source of spin flips, while the other one takes the role of drain. It is expected that a stable helix state emerges when the source and drain are balanced. However, it is hard to get exact solution in this case due to the fact $[H_0, H_I] \neq 0$. In the following, we investigate this issue by perturbation method. In the subspace spanned by the set of degenerate ground states \([|\psi_n\rangle]\) of $H_0$, the matrix representation of Hamiltonian $H$ with $\Delta = 1$ is an $(N+1) \times (N+1)$ matrix $H$ with nonzero matrix elements
\[ (H)_{N+1-n, N-n} = \frac{B_0e^{-ik_0}}{N} \sqrt{(n+1)(N-n)}, \tag{44} \]
\[ (H)_{N-n, N+1-n} = \frac{B_0e^{ik_0N}}{N} \sqrt{(n+1)(N-n)}, \tag{45} \]
with $n = [0, N-1]$, and
\[ (H)_{N+1-n, N+1-n} = -(N-1)/4, \tag{46} \]
with $n = [0, N]$. In small $B_0$ limit, the eigenvalues and eigenvectors of matrix $H$ are the approximate solutions of
FIG. 3. Plots of numerical results of time evolution for three types initial states under the Hamiltonian $H$ with non-Hermitian boundary in Eq. (43). The initial states are (a) ferromagnetic state, (b) pure random state and (c) mixed state, which are defined in the text. The corresponding fidelity $F(t)$ defined in Eq. (59) is presented at several typical instants $t$. The complete plot of $F(t)$ is given in Fig. 4. The parameters are $B_0 = 0.005$, $N = 10$ and $k_0 = \arctan(0.5)$. It indicates that the evolved state for initial mixed state converges faster than that for the other two. The time is in units of $J^{-1}$ where $J$ is the scale of the Hamiltonian and we take $J = 1$.

The non-Hermitian Hamiltonian. We note that matrix $H$ is essentially related to the representation of the Hamiltonian $\mathcal{H}$ of a fictitious spin $S = N/2$ particle: $\mathcal{H} = \lambda S_z$, where $S_z$ is its angular momentum operator and $\lambda$ is some complex constant. Then the normalized approximate eigenstates can be obtained from states $\{ |\psi_n \rangle \}$

$$ |\tilde{\psi}_n \rangle = R |\psi_n \rangle = \prod_{j=1}^{N} R_j |\psi_n \rangle, \quad (47) $$

by a local transformation on spin at each site

$$ R_j = \frac{1}{\sqrt{2}} \left( e^{ik_0(2j-N-1)/2} \begin{pmatrix} 1 \\ -e^{-ik_0(2j-N-1)/2} \end{pmatrix} \right). \quad (48) $$

The corresponding eigenenergy is complex

$$ E_n = -\frac{N-1}{4} + \frac{B_0 e^{ik_0(N-1)/2}}{N} (2n-N), \quad (49) $$

with $n = [0, N]$ and its imaginary part is

$$ \text{Im} (E_n) = \frac{B_0}{N} (2n-N) \sin \left( \frac{k_0(N-1)}{2} \right). \quad (50) $$

Unlike a Hermitian system, the imaginary part of eigenenergy can amplify or reduce the corresponding amplitude of the eigenstate in the dynamic process. For the given initial state $|\psi(0)\rangle = |\uparrow\rangle$, when the evolution time
is long enough the final state is the eigenstate of $H$ with
the maximum imaginary part of eigenenergy. The corre-
sponding approximate eigenstate is

$$|\psi(\infty)\rangle = \begin{cases}
\sum_{n=0}^{N} p_n |\psi_n\rangle, & \sin \left[ \frac{k_0(N-1)}{2} \right] > 0 \\
\sum_{n=0}^{N} (-1)^n p_n |\psi_n\rangle, & \sin \left[ \frac{k_0(N-1)}{2} \right] < 0
\end{cases},$$

where the coefficient is

$$p_n = 2^{-N/2} \sqrt{C_n} e^{-ik_0(N+1)n/2}.$$  \(52\)

Accordingly, we have the helicity distribution along the chain

$$h_{l} = \frac{1}{2} [\cos (k_0 \ell) , -\sin (k_0 \ell) , 0],$$  \(53\)

for $\sin [k_0 (N - 1)/2] > 0$, and

$$h_{l} = \frac{1}{2} [-\cos (k_0 \ell) , \sin (k_0 \ell) , 0],$$  \(54\)

for $\sin [k_0 (N - 1)/2] < 0$, where $\ell = l - (N + 1)/2$ is a shifted coordinate. Obviously, the above two classes of state $|\psi(\infty)\rangle$ are standard helix states with opposite helicity, due to the fact $|h_{l}|^2 = 0.25$.

Numerical simulation is performed to verify our predictions. We compute the time evolution by exact diagonalization and present the dynamic process of the formation of the helix state through the time dependence of the helicity distribution $h_{l}$. In general, the time evolution of an arbitrary initial state $\rho(0)$ obeys the equation

$$i \frac{\partial}{\partial t} \rho(t) = H \rho(t) - \rho(t) H^\dagger,$$  \(55\)

which admits the formal solution

$$\rho(t) = e^{-iHt} \rho(0) e^{iH^\dagger t}.$$  \(56\)

Unlike the Hermitian case, the time evolution of the density matrix is no longer unitary. In order to get $h_{l}(t)$, with the definition

$$h_{l}^0 = \text{Tr} [\rho(t) s_\alpha^0], \ (\alpha = x, y, z),$$  \(57\)

we normalize $\rho(t)$ by taking \[52\] \[53\]

$$\rho(t) = e^{-iHt} \rho(0) e^{iH^\dagger t} / \text{Tr} \left[ e^{-iHt} \rho(0) e^{iH^\dagger t} \right],$$  \(58\)

in the following numerical calculation. We introduce the Uhlmann fidelity \[54\] \[55\]

$$F(t) = \left[ \text{Tr} \sqrt{\rho_h(t) \sqrt{\rho_h}} \right]^2,$$  \(59\)

to characterize the degree of similarity between the evolved state $\rho(t)$ and the target state

$$\rho_h = |\psi(\infty)\rangle \langle \psi(\infty)|.$$  \(60\)

The value of $F(t)$ after a sufficient long time can be estimated intuitively. In general, an initial mixed state $\rho(0)$ contains equal-amplitude components in each state of $\{|\psi_n\rangle\}$. Then we always have $F(\infty) \approx 1$.

We focus on three types of initial states: (i) ferromagnetic state $\rho(0) = |\uparrow\rangle \langle \uparrow|$; (ii) random pure state $\rho(0) = |\psi(0)\rangle \langle \psi(0)|$, where

$$|\psi(0)\rangle = \left[ \sum_{n=1}^{2N} (\alpha_n)^2 \right]^{-1/2} \sum_{n=1}^{2N} \alpha_n |n\rangle.$$  \(61\)

Here coefficient $\alpha_n$ is taken as a uniform random number within the interval $(-1,1)$, and $\{|n\rangle\}$ is the complete set of eigenstates of $H_0$; (iii) maximally mixed ferromagnetic state

$$\rho(0) = \frac{1}{N + 1} \sum_{n=0}^{N} |\psi_n\rangle \langle \psi_n|.$$  \(62\)

The plots of $h_{l}$ and $F(t)$ in Figs. 3 and 4 show the dy-
amic behaviors of the evolved states of above three types of initial states, induced by the non-Hermitian boundary. It indicates that the evolved states for initial mixed state and ferromagnetic state converge fastly. Importantly, the final states for all three different initial states turn to the target state after sufficient long time. Notably, the initial states, as well as the selected non-Hermitian boundary, do not contain any information of the prequench Hamiltonian.

![fig4.png](fig4.png)

**FIG. 4.** Plots of $F(t)$ in Eq. \[59\] as a function of time for same time evolution process in Fig. 3. We can see that the final states for three different initial states turn to the target state eventually. The time is in units of $J^{-1}$ where $J$ is the scale of the Hamiltonian and we take $J = 1$.  

V. SUMMARY

In summary, we have studied the possible helix states in the XXZ Heisenberg model with DM interaction. Unlike previous works on this topic, the existence of spin helix state in this work is the direct result of the resonant DM interaction. Our findings offer a method for the efficient preparation of a spin helix state as the ground state of a spin chain by the quench dynamic process with the aid of non-Hermitian balanced perturbation. It is expected to be insightful for quantum engineering by non-Hermitian boundary.

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