A Comparative Study of Unipolar OFDM Schemes in Gaussian Optical Intensity Channel

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Abstract—We study the information rates of unipolar orthogonal frequency division multiplexing (OFDM) in discrete-time optical intensity channels with Gaussian noise under average optical power constraint. Several single-, double-, and multi-component unipolar OFDM schemes are considered under the assumption that i.i.d. Gaussian or complex Gaussian codebook ensemble and nearest neighbor decoding (minimum Euclidean distance decoding) are used. Results for single-component schemes are obtained by considering the frequency domain equivalent models; results for double- and multi-component schemes are obtained based on successive decoding, interference cancellation, and results for single-component schemes. For double- and multi-component schemes, the component power allocation strategies that maximize the information rates are investigated. In particular, we prove that several multi-component schemes achieve the high-SNR capacity of the discrete-time Gaussian optical intensity channel under average power constraint to within 0.07 bits, with optimized power allocation.

Index Terms—Channel capacity, information rate, intensity modulation and direct detection (IM/DD), optical wireless communications (OWC), orthogonal frequency division multiplexing (OFDM).

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) has been widely used in wireline and wireless communications. In optical communications including optical fiber systems and optical wireless communications (OWC), OFDM is a promising technique [1]. Many optical systems are based on intensity modulation and direct detection (IM/DD), which carries information by varying optical intensity, which is the optical power transferred per unit area. Therefore in IM/DD based systems the transmitted signal must be nonnegative, and only unipolar signaling can be used. In recent years, several kinds of unipolar OFDM schemes have been proposed for IM/DD based optical communications, especially for OWC, e.g., [2]–[14]. A summary of these schemes is given in Table I.

The studies listed in Table I mainly focus on designing unipolar signals. However, the information theoretic performance measures, e.g., the information rate, which is the highest rate achieved by a specific communication scheme with arbitrarily low error probability as the channel coding length grows without bound, are not considered. The capacity of optical channels has been studied in several works; e.g., for Gaussian optical intensity channels see [15]–[20]. While capacity is a fundamental limit on the rate of reliable communication over a channel, the information rate of a communication scheme (typically specified by some constraints on the transceiver) in that channel, although not necessarily achieving capacity, is sometimes more useful for evaluating performance of the corresponding practical systems, e.g., giving an approximation of the maximum data rate achieved by a coded system using capacity-achieving codes, with a sufficiently low error probability. There are several existing information-theoretic works on IM/DD based unipolar OFDM schemes. The information rate of the ACO-OFDM in optical intensity channels was derived in [21] and [22]. In [23] and [24], the information rates of the DCO-OFDM, the ACO-OFDM, and the Flip-OFDM were analyzed under both average optical power and dynamic optical power constraints. In [25], the information rates of the DCO-OFDM and the ACO-OFDM were studied in a OWC channel model with nonlinear distortion. In [26], lower bounds on the capacity of the ASCO-OFDM and the ADO-OFDM were derived by calculating the input-output mutual information for given input constellations.

In this paper, we study the information rates of unipolar OFDM schemes given in Table I in the discrete-time Gaussian optical intensity channel (OIC) [16]–[18]

\[ r_n = s_n + z_n, \quad s_n \geq 0, \]

under an average power constraint on the optical power of the input as

\[ E[s_n^2] \leq \mathcal{E}, \]

where \( s_n \) is the transmitted optical intensity. The additive Gaussian noise \( z_n \) is independent and identically distributed (i.i.d.) with variance \( \sigma_z^2 \). The OIC model has been widely used in the information theoretic studies on the IM/DD based OWC including free-space optical communications [16]–[19] and visible light communications (VLC) [27]. Note that in (1) we normalize the channel gain (including, e.g., the responsivity of the photodiode and the optoelectronic conversion factor in IM/DD systems) to unity without loss of generality.

Our contributions in this paper are as follows.

- For each unipolar OFDM scheme listed in Table I, we derive its information rate or lower bound on its information rate, under the assumption that i.i.d. Gaussian or
TABLE I
UNIPOLAR OFDM SCHEMES FOR IM/DD

| Name                        | Basic Idea                                                | OFDM Input Constraint | Frame Length\(^1\) | DoF Efficiency | Reference | Result |
|-----------------------------|-----------------------------------------------------------|-----------------------|---------------------|---------------|-----------|--------|
| Direct current offset OFDM  | Adding direct current bias (peak clipping is usually needed) | Hermitian symmetry of \(X\) | \(N\)               | 1             | [2]       | [3]    |
| (DCO-OFDM)                  |                                                           |                       |                     |               |           |        |
| Asymmetric clipped optical OFDM (ACO-OFDM) | Clipping the negative \(x_i\) to zero | Hermitian symmetry of \(X, X_n \equiv 0\) for even \(n\) | \(N\) | \(\frac{1}{2}\) | [3] | [20] |
| Pulse-amplitude-modulated discrete multitone modulation (PAM-DMT) | Clipping the negative \(x_i\) to zero | Hermitian symmetry of \(X, \text{Re}\{X_n\} \equiv 0\) | \(N\) | \(\frac{1}{2}\) | [4] | [21] |
| Flip-OFDM, or Unipolar OFDM (U-OFDM) | Transmitting the nonnegative and the flipped negative part separately | Hermitian symmetry of \(X\) | \(2N\) | \(\frac{1}{2}\) | [51]–[57] | [28] |
| Position modulating OFDM (PM-OFDM) | An analog of Flip-OFDM | None | \(4N\) | \(\frac{1}{2}\) | [8] | [36] |
| Asymmetrically clipped DC biased optical OFDM (ADG-OFDM) | Frequency division multiplexing of ACO-OFDM and DCO-OFDM | For DCO-OFDM, \(X_n \equiv 0\) for odd \(n\) | \(N\) | 1 | [9] | [29] |
| Hybrid asymmetrically clipped OFDM (HACO-OFDM) | Frequency division multiplexing of ACO-OFDM and PAM-DMT | For PAM-DMT, \(X_n \equiv 0\) for odd \(n\) | \(2N\) | \(\frac{3}{4}\) | [10] | Corollary 1 |
| Asymmetrically and symmetrically clipped OFDM (ASCO-OFDM) | Frequency division multiplexing of ACO-OFDM and Flip-OFDM | For Flip-OFDM, \(X_n \equiv 0\) for odd \(n\) | \(2N\) | \(\frac{3}{4}\) | [11] | Corollary 1 |
| Spectrally and energy efficient OFDM (SEE-OFDM), or layered ACO-OFDM | Frequency division multiplexing of \(L\) ACO-OFDM components | Component \(l\) uses the \(2^{l-1}(2n+1)\)-th subcarriers \(\left(0 \leq n \leq \frac{N}{2} - 1\right)\) | \(N\) | \(1 - 2^{-L}\) | [12], [13] | Corollary 2 |
| Enhanced U-OFDM (eU-OFDM) | Code division multiplexing of \(L\) Flip-OFDM components | No additional constraint | \(2LN\) | \(1 - 2^{-L}\) | [14] | Theorem 9, Corollary 2 |

\(^2\)In Table I, the frame length stands for the length of a complete transmission period in terms of the number of channel use in time domain; the DoF (degrees of freedom) efficiency stands for the number of independent complex symbols transmitted per channel use in time domain, divided by the maximum symbol rate of the discrete-time OIC (\(\frac{1}{2}\) complex symbols per channel use). Note that the DoF efficiency determines the pre-log factor of the information rate.

complex Gaussian codebook ensemble and nearest neighbor decoding (minimum Euclidean distance decoding) are used. Some asymptotic results are also obtained.

- We provide results and engineering insights on the design of unipolar OFDM schemes including the parameter optimization of the DCO-OFDM and the power allocation of the double- and multi-component schemes. In particular, we prove that with appropriate power allocation, several multi-component schemes achieve the high-SNR capacity of the discrete-time Gaussian OIC with average power constraint to within 0.07 bits.

The remaining part of this paper is organized as follows. Sec. II gives some preliminaries of our study. Sec. III, IV, and V study the single-, double-, and multi-component schemes, respectively. Sec. V presents numerical results and their discussions. Sec. VII comments on the design of unipolar OFDM in bandlimited optical intensity channels.

II. Preliminaries

Let the number of subcarriers of OFDM be \(N\). In this paper, we focus on the performance when \(N\) is large. We use \(F\) and \(F^{-1}\) to denote the discrete Fourier transform (DFT) matrix and the inverse discrete Fourier transform (IDFT) matrix, respectively, i.e., \([F]_{n,k} = N^{-\frac{1}{2}} \exp\left(-j2\pi nk \frac{1}{N}\right)\), \([F^{-1}]_{k,n} = N^{-\frac{1}{2}} \exp\left(j2\pi nk \frac{1}{N}\right)\), where \(0 \leq k \leq N - 1\) and \(0 \leq n \leq N - 1\). Some upper case Roman letters including \(X, S, C, D\), and their bold forms, are used to denote scalar and vector signals in discrete frequency domain, respectively; the corresponding lower case Roman letters \(x, s, c, d\), and their bold forms, are used to denote scalar and vector signals in discrete time domain, respectively (where \(s\) denotes the optical intensity, e.g., in (1)). The two domains are connected by DFT/IDFT, e.g., \(x = FX\), where \([X]_n = x_n\) and \([X]_k = X_k\).

We use \(\mathcal{N}(a, b)\) and \(\mathcal{CN}(a, b)\) to denote Gaussian and complex Gaussian distributions, respectively, with mean \(a\) and variance \(b\). The truncated Gaussian distribution with probability density function \(f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right), x > 0\), \(\Pr(x = 0) = \frac{1}{2}\).
is denoted as $\mathcal{C}_{AWGN}(\gamma)$ to denote the capacity of a complex-valued discrete-time additive Gaussian noise (AWGN) channel $y = x + w$ with signal-to-noise ratio (SNR) $\gamma = \frac{E[x^2]}{E[w^2]}$, i.e., $\mathcal{C}_{AWGN}(\gamma) = \log(1 + \gamma)$. The complex conjugate of a complex number $A$ is denoted as $A^\dagger$. For a matrix (or vector) $A$, $A^\dagger$ stands for its transpose, $A^H$ stands for its conjugate transpose, and $A$ stands for its element-by-element conjugation.

Throughout the paper we have the following assumptions on the transceiver of unipolar OFDM.

- The codebook of the unipolar OFDM schemes (i.e., the OFDM input in discrete frequency domain denoted as $X$) is an i.i.d. Gaussian or complex Gaussian codebook ensemble (denoted as IG and ICG codebook ensemble, respectively).

- The receiver uses decoding techniques as that commonly used in practical OFDM based IM/DD systems (not necessarily optimal). For example, for the ACO-OFDM, we discard all the received signal on even subcarriers which contains useful information, and perform nearest neighbor decoding on odd subcarriers.

We denote the information rate of unipolar OFDM schemes under these assumptions by $R_{OFDM}$. The corresponding input-output mutual information $I_{OFDM}$, which may be higher than $R_{OFDM}$, is not achievable since practical decoders are not optimal. The single-letter input-output mutual information $I(s;r)$ obtained based on the distribution of the transmitted optical intensity $s$ (e.g., $[20]$), which has the form $\frac{1}{2} \log(1 + a s^2)$, is also not achievable because $s$ is not i.i.d. and the single-letter characterization of information rate is not valid from an information theoretic perspective. In fact, when the DoF efficiency of a unipolar OFDM scheme is smaller than one, the pre-log factor of $R_{OFDM}$ should be smaller than $\frac{1}{2}$.

In general, our proofs of results include two steps: 1) determining the optical power cost of a unipolar OFDM scheme with given input; 2) studying the information rate of the frequency domain equivalent model of that scheme. In most cases the first step is just determining the $\sigma_X^2 \cdot E$ relationship, i.e., the relationship between the variance of $X$ and the average optical intensity $E[s]$, which is always assumed to be equal to $E$ to maximize the achievable information rate.

For brevity, we combine the detailed description of each unipolar OFDM scheme into the proof of the corresponding result. For more details of these schemes see references listed in Table I.

### III. SINGLE-COMPONENT SCHEMES

This section considers the first five unipolar OFDM schemes in Table I. We use the theoretical framework for transmission with transceiver distortion proposed in $[28]$ to study the information rate of the DCO-OFDM. This framework is based on the generalized mutual information (GMI), denoted as $I_{GMI}$, which is a lower bound on the information rate of a communication scheme in a channel with a given decoding metric $[29]$, $[30]$. Moreover, the GMI is the highest information rate below which the average probability of error with that decoding metric, averaged over the chosen i.i.d. ensemble of codebooks, converges to zero as the code length tends to infinity. Appendix A gives a brief introduction on this GMI-based framework. We will show that with ICG/IG codebook ensemble and nearest neighbor decoding, the ACO-OFDM, the Flip-OFDM, and the PM-OFDM have the same information rate, and the PAM-DMT performs worse than them.

**Theorem 1**: The information rate of the DCO-OFDM with ICG codebook ensemble and nearest neighbor decoding is lower bounded by (3).

**Proof**: Consider a block of the input of the DCO-OFDM as

$$X = \begin{bmatrix} 0, X_1, \ldots, X_{N-1}, 0, \bar{X}_{N-1}, \ldots, \bar{X}_1 \end{bmatrix}^T$$

which is a length $N$ complex vector with Hermitian symmetry. For $1 \leq k \leq N-1$, let $X_k$ be i.i.d. and $X_k \sim CN(0, \sigma_X^2)$. Taking IDFT of $X$, we obtain $x = [x_1, \ldots, x_N]^T = F^{-1}X$. Since IDFT is unitary, we have $\|x\| = \|X\|$. Moreover, it can be shown that

$$E[x_n x_{n'}] = \begin{cases} \frac{N-2}{2N} \sigma_X^2, & n = n', \\ 0, & n - n' \text{ is odd}, \\ -\frac{1}{N} \sigma_X^2, & n - n' = \text{even}. \end{cases}$$

So $x_n$ satisfies $x_n \sim N(0, \sigma_X^2)$ for given $n$ since it is a linear combination of real and imaginary parts of $X_k$ (both are i.i.d. Gaussian variables), where $\sigma_X^2 = \frac{N-2}{N} \sigma_X^2$, and $x_n$ is asymptotically i.i.d. as $N \to \infty$. The unipolar input to the OIC, $s = [s_1, \ldots, s_{N-1}]^T$, is obtained by clipping the signal peaks symmetrically as

$$c_n = \begin{cases} A, & x_n > A \\ x_n, & |x_n| \leq A \\ -A, & x_n < -A \end{cases}$$

and adding a direct current (DC) bias $A$ on $c_n$. Apparently $E[s_n] = A$, and we let $A = E$. The output of the OIC after removing the DC bias is $y = c + z$ where $|y_n| = y_n$. By taking DFT of $y$ we obtain $Y = C + Z = Fe + Z$ where $Y_k = Y_k, 1 \leq k \leq N$. Note that $C$ is determined by $X$ via (6). Without loss of optimality, we consider the $(\frac{N}{2} - 1)$ dimensional equivalent channel

$$Y = C + Z$$

where $Y = [Y_1, \ldots, Y_{\frac{N}{2}-1}]^T$, $C$ is determined by $X = [X_1, \ldots, X_{\frac{N}{2}-1}]^T$ and consists of i.i.d. elements, and so does the noise $Z$.

2in this paper we do not consider channels with time dispersion, so the cyclic prefix (CP) is not considered

$$R_{DCO-OFDM} \geq \max_{\nu > 0} \frac{1}{2} \log \left( 1 + \frac{\text{erf}(\nu \mathcal{E})}{\text{erf}(\nu \mathcal{E}) - \text{erf}^2(\nu \mathcal{E}) - 2\pi^{-\frac{1}{2}} \nu \mathcal{E} \exp(-\nu^2 \mathcal{E}^2) + 2\nu^2 \mathcal{E}^2 \text{erfc}(\nu \mathcal{E}) + 2\nu^2 \sigma_s^2} \right).$$

(3)
For transmission at rate $R$, assume that a message $m$ is selected from $\mathcal{M} = \{1, \ldots, \lfloor \exp^R \rfloor \}$ uniformly randomly. The encoder maps $m$ to a length-$\ell$ codeword $[x^{(i)}(m), \ldots, x^{(\ell)}(m)]^T$ in an ICG codebook ensemble, where $x^{(i)}(m) = [x_1^{(i)}(m), \ldots, x_{N/2-1}^{(i)}(m)]^T$ for $1 \leq i \leq \ell$. At the receiver, we let the decoder follow a (scaled) nearest neighbor decoding rule as

$$\hat{m} = \arg \min_{m \in \mathcal{M}} \frac{1}{\ell} \sum_{i=1}^{\ell} \left\| Y^{(i)} - aX^{(i)}(m) \right\|^2, \ m \in \mathcal{M},$$

(8)

where $\| \cdot \|$ is the $L_2$ norm. In (8), $a$ is a decoding scaling parameter to be optimized. Under the above assumption, the GMI achieved by the channel $Y = C + Z$ in (7) is (see Appendix A)

$$I_{\text{GMI}} = \left( \frac{N}{2} - 1 \right) \log \left( 1 + \frac{\Delta}{1 - \Delta} \right),$$

(9)

where

$$\Delta = \frac{\left| E [Y^H X] \right|^2}{(N/2 - 1) \sigma_X^2 \left( E [\|C\|^2] + E [\|Z\|^2] \right)},$$

(10)

and the expectation $E [\cdot]$ is taken with respect to $X$ and $Z$. Here the decoding scaling parameter $a$ is optimally set as

$$a_{\text{opt}} = \frac{E [Y^H X]}{(N/2 - 1) \sigma_X^2}.$$  

(11)

To evaluate $\Delta$ and $a_{\text{opt}}$, we write $c_n$ as $c_n = \alpha x_n + d_n = \frac{E [x_n]}{\sigma_x^2} x_n + d_n$, where $d_n$ satisfies $E [d_n x_n] = 0$. By taking DFT we obtain $C_k = \alpha X_k + D_k$, where $D_k = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} d_n \exp (-j2\pi n k/N)$ which satisfies $E [\|D_k\|^2] = E [d_n^2]$. Moreover, we have $E [\bar{D}_k X_k] = 0$, which is because

$$E [X_{\text{DFT}}^H] = E [F_X F_d^H] = E [F_X F_d^H F^{-1}] = 0,$$

(12)

since $E [x_n d_n] = 0$ for any $n, n'$. Substituting (7) into (11) and (10) yields

$$a_{\text{opt}} = \frac{E [(C + Z)^H X]}{(N/2 - 1) \sigma_X^2} = \frac{\sum_{k=1}^{N/2-1} E [(\alpha X_k + D_k) X_k + \bar{Z}_k X_k]}{(N/2 - 1) \sigma_X^2} = \frac{\alpha \sum_{k=1}^{N/2-1} E [\bar{X}_k X_k]}{(N/2 - 1) \sigma_X^2} = \alpha,$$

(13)

and

$$\Delta = \frac{\left| E [(C + Z)^H X] \right|^2}{(N/2 - 1) \sigma_X^2 \left( E [\|C\|^2] + E [\|Z\|^2] \right)} = \frac{\left( \sum_{k=1}^{N/2-1} E [(C_n^2 + D_n^2 + E [Z_n^2]) \right] / (N/2 - 1) \alpha^2 \sigma_X^2} = \frac{\sum_{n=1}^{N/2-1} (\alpha^2 E [\|X_n^2\|^2] + E [\|D_n\|^2] + E [\|Z_n\|^2])}{\alpha^2 \sigma_X^2} = \frac{\alpha^2 \sigma_X^2 + E [\|D_n\|^2] + \sigma_z^2}{\alpha^2 \sigma_X^2 + \sigma_z^2}.$$  

We thus obtain

$$\Delta = \frac{\alpha^2 \sigma_X^2}{1 - \Delta} + \frac{\sigma_z^2}{E [d_n^2] + \sigma_z^2}.$$  

(15)

Note that $I_{\text{GMI}}$ measures the information rate per frequency domain channel use, and in time domain the channel is used for $N$ times. So we have

$$R_{\text{DCO-OFDM}} \geq \frac{1}{N} I_{\text{GMI}} = \frac{N - 2}{2N} \log \left( 1 + \frac{\alpha^2 \sigma_X^2}{E [d_n^2] + \sigma_z^2} \right).$$  

(16)

According to (6), by simple calculation we obtain $E [c_n x_n] = \text{erf} \left( \frac{\epsilon}{\sqrt{2} \sigma_x} \right) \sigma_x^2$ which implies

$$\alpha = \frac{\text{erf} \left( \frac{\epsilon}{\sqrt{2} \sigma_x} \right) \sigma_x^2}{\sqrt{2} \sigma_x},$$  

(17)

and the variance of $c_n$ as

$$E [c_n^2] = \sigma_x^2 \left( \text{erf} \left( \frac{\epsilon}{\sqrt{2} \sigma_x} \right) \frac{\epsilon}{\sqrt{2} \sigma_x} \sigma_x^2 \right) + \epsilon^2 \text{erfc} \left( \frac{\epsilon}{\sqrt{2} \sigma_x} \right).$$  

(18)

We can thus obtain $E [d_n^2]$ by noting that

$$E [c_n^2] = E [\alpha^2 x_n^2 + d_n^2 + 2 \alpha x_n d_n] = \alpha^2 \sigma_x^2 + E [d_n^2].$$  

(19)

Combining (16)-(19), letting $\nu = \frac{\epsilon}{\sqrt{2} \sigma_x}$ noting that $\sigma_x^2 = N \tilde{\sigma}_x^2$, letting $N \rightarrow \infty$, and choosing the $\nu$ that maximizes the RHS of (16), (3) is obtained. 

Note: An alternative way to (3) is outlined as follows: 1) Apply the decomposition $c_n = \frac{E [x_n]}{\sigma_x^2} x_n + d_n$, and obtain the corresponding decomposition on $Y_k$ as $Y_k = \alpha X_k + D_k + Z_k$; 2) Show that $D_k$ is uncorrelated with $X_k$; 3) Treating $D_k + Z_k$ as noise, using the result that the Gaussian noise is the worst uncorrelated noise [31], the information rate of subcarrier $k$ can be lower bounded by the capacity of a complex AWGN channel whose SNR is $\gamma_k = \frac{\alpha^2 \tilde{\sigma}_x^2}{E [\|D_k\|^2] + \sigma_z^2}$ and then (3) can be obtained. This lower bounding approach for information rate was used in [32] for clipped OFDM in Gaussian channels, and in [23] for unipolar OFDM in VLC under both average optical power and dynamic optical power constraints. Our proof, however, provides more engineering insights since it connects the information rate with the input codebook and the decoding rule, and gives the exact information rate result when a “typical” codebook from the ICG codebook ensemble is used (see Appendix A).

**Theorem 2:** The information rate of the ACO-OFDM with ICG codebook ensemble and nearest neighbor decoding is

$$R_{\text{ACO-OFDM}} = \frac{1}{4} \log \left( 1 + \frac{\pi \epsilon^2}{\sigma_z^2} \right),$$  

(20)

and the information rate of the PAM-DMT with IG codebook ensemble and nearest neighbor decoding is

$$R_{\text{PAM-DMT}} = \frac{1}{4} \log \left( 1 + \frac{\pi \epsilon^2}{2\sigma_z^2} \right).$$  

(21)

Proof: Eqn. (20) was proved in [21] and [22], by determining the capacity of the frequency domain equivalent channel which is a complex AWGN channel for each subcarrier.
Since the capacity of a complex AWGN channel is achieved by employing the ICG codebook ensemble and the nearest neighbor decoding, we can get (20) under the assumption on the transceiver here.

To prove (21), consider a block of the input of the PAM-DMT as

\[ \mathbf{X} = \left[ 0, X_1, \ldots, X_{N-1}, 0, \tilde{X}_{N-1}, \ldots, \tilde{X}_1 \right]^T \]  \hspace{1cm} (22)

which is a length \( N \) complex vector with Hermitian symmetry, and \( \text{Re}[X_k] = 0 \). For 1 \( \leq k \leq \frac{N}{2} - 1 \) we let \( X_k \) be i.i.d. and \( \text{Im}[X_k] \sim \mathcal{N}(0, \sigma_X^2) \). Taking IDFT of \( \mathbf{X} \), we obtain \( \mathbf{x} \) satisfying \( ||\mathbf{x}|| = ||\mathbf{X}|| \) and \( x_n \sim \mathcal{N}(0, \sigma_x^2) \), where \( \sigma_x^2 = \frac{N-2}{N} \sigma_X^2 \). The unipolar input to the OIC, denoted as \( \mathbf{s} = [s_1, \ldots, s_{N-1}]^T \), is obtained by asymmetrically clipping as

\[ s_n = \max(x_n, 0) \]  \hspace{1cm} (23)

So \( s_n \sim \mathcal{T}_G(0, \sigma_x^2) \) and the transmitted optical power is thus \( E[s_n] = \frac{\sigma_x^2}{\sqrt{\pi}} \). Letting \( E[s_n] = \mathcal{E} \), we get the \( \sigma_X - \mathcal{E} \) relationship as

\[ \sigma_X^2 = \frac{N}{N-2} 2\pi \mathcal{E}^2. \]  \hspace{1cm} (24)

Taking DFT of the output of the OIC yields \( \frac{N}{2} - 1 \) parallel channels as (as in the proof of Theorem 1, we can discard \{\( Y_k, k \geq \frac{N}{2} \)\} here and in other proofs in this paper)

\[ Y_k = S_k + Z_k, \hspace{0.5cm} 1 \leq k \leq \frac{N}{2} - 1, \]  \hspace{1cm} (25)

and taking the imaginary parts we obtain

\[ \text{Im}[Y_k] = \frac{1}{2} \text{Im}[X_k] + \text{Im}[Z_k], \hspace{0.5cm} 1 \leq k \leq \frac{N}{2} - 1, \]  \hspace{1cm} (26)

since the distortion due to clipping exists only in the real part of each subcarrier [4]. The information rate of each subchannel in (26) when \( \text{Im}[X_k] \sim \mathcal{N}(0, \sigma_Y^2) \) is \( \frac{1}{2} C_{\text{AWGN}} \left( \frac{\sigma_Y^2}{\sigma_z^2} \right) \). So the information rate of the PAM-DMT with ICG codebook ensemble and nearest neighbor decoding is

\[ R_{\text{PAM-DMT}} = \frac{N}{2} - 1 \frac{1}{2N} C_{\text{AWGN}} \left( \frac{\sigma_X^2}{4 \sigma_z^2} \right). \]  \hspace{1cm} (27)

Combining (24) and (27) and letting \( N \to \infty \) complete the proof.

**Theorem 3:** The information rate of the Flip-OFDM with ICG codebook ensemble and nearest neighbor decoding is

\[ R_{\text{Flip-OFDM}} = \frac{1}{4} \log \left( 1 + \frac{\pi \mathcal{E}^2}{\sigma_z^2} \right). \]  \hspace{1cm} (28)

**Proof:** Consider a block of the input of the Flip-OFDM as

\[ \mathbf{X} = \left[ X_0, X_1, \ldots, X_N \right]^T \]  \hspace{1cm} (29)

which is a length \( N \) complex vector with Hermitian symmetry. For 1 \( \leq k \leq \frac{N}{2} - 1 \), let \( X_k \) be i.i.d. and \( X_k \sim \mathcal{C}N(0, \sigma_X^2) \).

Taking IDFT of \( \mathbf{X} \), we obtain \( \mathbf{x} \) satisfying \( ||\mathbf{x}|| = ||\mathbf{X}|| \) and \( x_n \sim \mathcal{N}(0, \sigma_x^2) \), where \( \sigma_x^2 = \frac{N-2}{N} \sigma_X^2 \). A frame of the input to the OIC is

\[ \mathbf{s} = \left[ s_1^T, s_2^T \right]^T = [s_{1,0}, \ldots, s_{1,N-1}, s_{2,0}, \ldots, s_{2,N-1}]^T, \]  \hspace{1cm} (30)

which is obtained by

\[ s_{1,n} = \max(x_n, 0), \hspace{0.5cm} s_{2,n} = -\min(x_n, 0). \]  \hspace{1cm} (31)

So \( s_{i,n} \sim \mathcal{T}_G(0, \sigma_x^2) \) and the transmitted optical power is \( E[s_{i,n}] = \frac{\sigma_x^2}{\sqrt{2\pi}} \) for \( i = 1 \). Letting \( E[s_{i,n}] = \mathcal{E} \), we get the \( \sigma_X - \mathcal{E} \) relationship as

\[ \sigma_X^2 = \frac{N}{N-2} 2\pi \mathcal{E}^2. \]  \hspace{1cm} (32)

At the receiver, we perform the following combining,

\[ u_n = y_{1,n} - y_{2,n} = s_{1,n} + z_{1,n} - (s_{2,n} + z_{2,n}) = \max(x_n, 0) + \min(x_n, 0) + z_{1,n} - z_{2,n} = x_n + w_n, \]  \hspace{1cm} (33)

where \( 0 \leq n \leq N - 1 \), \( w_n \sim \mathcal{N}(0, 2\sigma_x^2) \). Taking DFT of \( u \) yields \( \frac{N}{2} - 1 \) parallel channels as

\[ U_k = X_k + W_k, \hspace{0.5cm} 1 \leq k \leq \frac{N}{2} - 1. \]  \hspace{1cm} (34)

The information rate of the \( k \)-th subchannel of (34) when \( X_k \sim \mathcal{C}N(0, \sigma_X^2) \) is \( C_{\text{AWGN}} \left( \frac{\sigma_X^2}{4 \sigma_z^2} \right) \). So the information rate of the Flip-OFDM with ICG codebook ensemble and nearest neighbor decoding is

\[ R_{\text{Flip-OFDM}} = \frac{N}{2} - 1 \frac{1}{2N} C_{\text{AWGN}} \left( \frac{\sigma_X^2}{2 \sigma_z^2} \right). \]  \hspace{1cm} (35)

Combining (24) and (35), and letting \( N \to \infty \) complete the proof.

**Theorem 4:** The information rate of the PM-OFDM with ICG codebook ensemble and nearest neighbor decoding is

\[ R_{\text{PM-OFDM}} = \frac{1}{4} \log \left( 1 + \frac{\pi \mathcal{E}^2}{\sigma_z^2} \right). \]  \hspace{1cm} (36)

**Proof:** Consider a block of the input of the PM-OFDM as

\[ \mathbf{X} = \left[ X_0, X_1, \ldots, X_N \right]^T \]  \hspace{1cm} (37)

which is a length \( N \) complex vector in which \( X_k \) is i.i.d. and \( X_k \sim \mathcal{C}N(0, \sigma_X^2) \). Taking IDFT of \( \mathbf{X} \), we obtain \( \mathbf{x} \) satisfying \( ||\mathbf{x}|| = ||\mathbf{X}|| \) and \( x_n \sim \mathcal{C}N(0, \sigma_x^2) \). A frame of the input to the OIC is

\[ \mathbf{s} = \left[ s_1^T, s_2^T, s_3^T, s_4^T \right]^T \]  \hspace{1cm} (38)

where \( s_i = [s_{i,0}, \ldots, s_{i,N-1}]^T \), \( s_{1,n} = \max(\text{Re}[x_n], 0) \), \( s_{2,n} = -\min(\text{Re}[x_n], 0) \), and \( s_{3,n} \) and \( s_{4,n} \) are obtained with respect to \( \text{Im}[x_n] \) similarly. So \( s_{i,n} \sim \mathcal{T}_G(0, \sigma_x^2) \) and the transmitted
optical power is $E[s_{1,n}] = \frac{\sigma_s^2}{2}$. At the receiver we first perform combining similar to (33) and obtain $u_n = x_n + w_n$, where $0 \leq n \leq N - 1$, $w_n \sim \mathcal{N}(0, 4\sigma_w^2)$. Taking DFT of $u$ yields $N$ parallel channels as $U_k = X_k + W_k$. The information rate of the $k$-th subchannel when $X_k \sim \mathcal{CN}(0, \sigma_x^2)$ is $C_{\text{AWGN}}\left(\frac{\sigma_x^2}{2\sigma_w^2}\right)$. So the information rate of the PM-OFDM with ICG codebook ensemble and nearest neighbor decoding for components, interference cancellation, and results for single-component schemes. In addition, we utilize a result due to (43) to determine the information rate of the ACO-OFDM component of the ADO-OFDM.

**Theorem 5:** The information rate of the ADO-OFDM with ICG codebook ensemble and nearest neighbor decoding for both components is lower bounded by (39).

**Proof:** Consider a block of the input of the ADO-OFDM component as

$$X_1 = [0, X_{1,1}, 0, X_{1,3}, \ldots, 0, X_{1,\frac{N}{2} - 1}, 0, \tilde{X}_{1,\frac{N}{2} - 1}, 0, \tilde{X}_{1,3}, 0, \tilde{X}_{1,1}]^T$$

which is a length $N$ complex vector with Hermitian symmetry, and $N$ is divisible by $4$. For odd $k$ satisfying $1 \leq k \leq \frac{N}{2} - 1$, we let $X_{1,k}$ be i.i.d. and $X_{1,k} \sim \mathcal{CN}(0, \sigma_x^2)$. Taking IDFT of $X_1$, we obtain $x_1$ satisfying $\|x_1\| = \|X_1\|$ and $x_{1,n} \sim \mathcal{N}(0, \sigma_x^2)$, where $\sigma_x^2 = \frac{1}{2}\sigma_x^2$. The unipolar ACO-OFDM component, denoted as $s_1 = [s_{1,1}, \ldots, s_{1,N-1}]^T$, is obtained by asymmetrically clipping like (42). So $s_{1,n} \sim T \mathcal{G}(0, \sigma_{s1}^2)$ and its optical power is $E[s_{1,n}] = \frac{\sigma_s^2}{4\sqrt{3}}$. Letting $E[s_{1,n}] = (1 - \lambda)E$ we obtain the $\sigma_x$-$E$ relationship of the ACO-OFDM component as $\sigma_x^2 = (1 - \lambda)^24\pi\sqrt{2}E$.

Consider a block of the input of the DCO-OFDM component as

$$X_2 = [0, 0, X_{2,2}, 0, X_{2,4}, \ldots, X_{2,\frac{N}{2} - 2}, 0, 0, X_{2,\frac{N}{2} - 2}, \ldots, X_{2,4}, 0, X_{2,2}, 0]^T$$

which is a length $N$ complex vector with Hermitian symmetry. For even $k$ satisfying $2 \leq k \leq \frac{N}{2} - 2$, let $X_{2,k}$ be i.i.d. and $X_{2,k} \sim \mathcal{CN}(0, \sigma_x^2)$. Taking IDFT of $X_2$, we obtain $x_2$ which satisfies $\|x_2\| = \|X_2\|$ and $x_{2,n} \sim \mathcal{N}(0, \sigma_{s2}^2)$, where $\sigma_{s2}^2 = \frac{N}{4\pi}\sigma_x^2$. The unipolar DCO-OFDM component, denoted as $s_2 = [s_{2,1}, \ldots, s_{2,N-1}]^T$, is obtained by clipping the peaks of $x_2$ symmetrically like (6) (the obtained signal is denoted as $c_2$), and adding a DC bias, i.e., $s_2 = c_2 + A$. Apparently $E[s_{2,n}] = A$, and we let $A = \lambda E$.

A frame of the unipolar ADO-OFDM input to the OIC is thus $s = s_1 + s_2$ where $E[s_{1,n}] = E$. Removing the DC bias from the received signal and then taking its DFT, we obtain $\frac{N}{2} - 1$ parallel channels as

$$Y_k = \begin{cases} \frac{1}{\lambda}X_{1,k} + Z_k + D_{2,k}, & k \text{ odd}, \\ C_{2,k} + Z_k + D_{1,k}, & k \text{ even} \end{cases} \quad (42)$$

where $1 \leq k \leq \frac{N}{2} - 1$, $D_{1,k}$ and $D_{2,k}$ are the distortion terms introduced by the ACO-OFDM component and the DCO-OFDM component, respectively. The distortion $\{D_{2,k}\}$ are i.i.d. and satisfy $E[|D_{2,k}|^2] = E[d_{2,n}^2]$, where $E[d_{2,n}^2]$ is given in (43). Note that there are $N/2$ odd numbered channels and $N/2 - 1$ even numbered channels in (42).

We first consider the information rate of the ACO-OFDM component. For the $k$-th channel in (42) where $k$ is odd, for transmission of rate $R_{1,k}$, assume that a message $m$ is selected from $\mathcal{M} = \{1, \ldots, |\exp(\gamma R_{1,k})|\}$ uniformly randomly. The encoder maps $m$ to a length-$\ell$ codeword $X_k = [X_{1}^{(1)}(m), \ldots, X_{1}^{(N)}(m)]^T$ in an ICG codebook ensemble. In (42), $X_{1,k}$ and $D_{2,k}$ are independent since the ACO-OFDM component and the DCO-OFDM component are independent. So we can treat $D_{2,k} + Z_k$ as additive noise in the decoding of the ACO-OFDM component, and let the decoder follow a nearest neighbor decoding rule as

$$\hat{m} = \arg \min_{m \in \mathcal{M}} \frac{1}{\ell} \sum_{i=1}^{\ell} \left( Y_k - \frac{1}{2}X_{1}^{(i)}(m) \right)^2, \quad m \in \mathcal{M}. \quad (44)$$

According to a result on the information rate of nearest neighbor decoding in additive non-Gaussian noise channels [33], the information rate of a subchannel of the ACO-OFDM component under the above assumption is $R_{1,k} = C_{\text{AWGN}}(\gamma_2)$ where $\gamma_2 = \frac{\mu^2}{E[d_{2,n}^2] + \sigma_{s2}^2}$. Since $E[|D_{2,k}|^2] = E[d_{2,n}^2]$ and $\sigma_{s2}^2 = (1 - \lambda)^24\pi\sqrt{2}E^2$, by noting that there are $N/2$ equivalent channel uses in discrete frequency domain for the ACO-OFDM component (since there are $N/2$ odd numbered channels in (42)) per $N$ channel uses in time domain, we get the information rate of the ACO-OFDM component under the ICG codebook ensemble and the nearest neighbor decoding as

$$R_1 = \frac{N/4}{N}R_{1,k} = \frac{1}{4} \log \left( 1 + \frac{\pi(1 - \lambda)^2E^2}{E[d_{2,n}^2] + \sigma_{s2}^2} \right). \quad (45)$$

For the decoding of the DCO-OFDM component, we first perform an interference cancellation process as follows. We reconstruct the ACO-OFDM component $s_1$ according to the corresponding decoding output of the nearest neighbor decoder given by (44), and subtract the reconstructed $\hat{s}_1$ from $s$. For the ACO-OFDM component, for any transmission rate below $R_1$, the decoding error probability (i.e., the probability of $s_1 \neq \hat{s}_1$) tends to zero as the code length $\ell$ grows without
bound. So we can analyze the performance of the DCO-
OFDM component based on \( y_2 = y - s_1 - A = c_2 + z \)
where the DC bias has been removed, or \( Y_2 = C_2 + Z \)
in the discrete frequency domain. We rewrite the channel model
as \( Y_2 = C_2 + Z \), where \( Y_2 = [Y_{2,2}, Y_{2,4}, \ldots, Y_{2,\frac{N}{2} - 2}]^T \).
\( C_2 = [C_{2,2}, C_{2,4}, \ldots, C_{2,\frac{N}{2} - 2}]^T \), and \( C_{2,k} = \mathbb{F}c_2[k] \).
Based on Appendix A, the GMI achieved by employing an ICG
codebook ensemble and using a nearest neighbor decoding rule
as (71) is
\[
\mathcal{I}_{\text{GMI}} = \left( \frac{N}{4} - 1 \right) \log \left( 1 + \frac{\Delta}{1 - \Delta} \right)
\]
where
\[
\Delta = \frac{\left[ \mathbb{E} \left[ Y_2^2 \right] \right]^2}{\left( \frac{N}{4} - 1 \right) \sigma_X^2 \left( \mathbb{E} \left[ \left| C_2 \right|^2 \right] + \mathbb{E} \left[ \left| Z \right|^2 \right] \right)}.
\]
Evaluating (46) by essentially the same derivations as the proof of
Theorem 1, noting that \( \mathbb{E} \left[ \left| D_{2,k} \right|^2 \right] = \mathbb{E} \left[ a_{2,n}^2 \right] \), and noting that there are
\( \frac{N}{4} - 1 \) equivalent channel uses in discrete frequency domain
for the DCO-OFDM component per \( N \) channel uses in time
domain, the information rate of the DCO-OFDM component
using the ICG codebook ensemble and the nearest neighbor
decoding is lower bounded by
\[
\mathcal{R}_{2} \geq \frac{N}{4} - 1 \frac{1}{N} \log \left( 1 + \left( \text{erf} \left( \frac{\lambda \mathcal{E}}{\sqrt{2} \sigma_{x_2}} \right) \right)^2 \frac{2\sigma_{x_2}^2}{\mathbb{E} \left[ a_{2,n}^2 \right] + \sigma_z^2} \right).
\]
Combining (43) and (48), letting \( \nu = \frac{1}{\sqrt{\lambda \mathcal{E}}} \) and noting that
\( \sigma_{x_2}^2 = \frac{N - 4}{2N} \sigma_x^2 \), letting \( N \to \infty \), and jointly choosing \( \lambda \) and \( \nu \) that maximize the information rate, we obtain (39). ■

**Theorem 6:** The information rate of the HACO-OFDM
with ICG and IG codebook ensembles for the ACO-OFDM
component and the PAM-DMT component, respectively, and
nearest neighbor decoding for both components, is
\[
\mathcal{R}_{\text{HACO-OFDM}} = \max_{0 \leq \lambda \leq 1} \left( \frac{1}{4} \log \left( 1 + \left( \frac{\pi(1 - \lambda)^2 \mathcal{E}^2}{\sigma_x^2} \right) \right) + \frac{1}{8} \log \left( 1 + \frac{\pi \lambda^2 \mathcal{E}^2}{\sigma_x^2} \right) \right).
\]

**Proof:** An HACO-OFDM frame \( s = s_1 + s_2 \) is the sum of
an ACO-OFDM component and a PAM-DMT component
which occupies only even subcarriers. Apparently the decoding
of the ACO-OFDM component is not disturbed by the
PAM-DMT component since the clipping noise from the PAM-
DMT component occurs only in the real part of odd subcarriers.
So the information rate of the ACO-OFDM component
can be obtained by Theorem 2 directly. Since the total transmit
optical power is the sum of those of the two components,
we let the optical power of the ACO-OFDM component be
\( (1 - \lambda) \mathcal{E} \). The information rate of the ACO-OFDM component
with the ICG codebook ensemble and the nearest neighbor
decoding is thus \( \mathcal{R}_{1} = \frac{1}{4} \log \left( 1 + \frac{\pi(1 - \lambda)^2 \mathcal{E}^2}{\sigma_x^2} \right) \).

Now consider the PAM-DMT component. Let a block of the input be
\[
X_2 = [0, 0, X_{2,2}, 0, X_{2,4}, \ldots, X_{2,\frac{N}{2} - 2}, 0, 0, X_{2,\frac{N}{2} - 2}, \ldots, \hat{X}_{2,4}, 0, \hat{X}_{2,2}, 0]^T
\]
which is a length \( N \) complex vector with Hermitian symmetry.
For even \( k \) satisfying \( 1 \leq k \leq \frac{N}{2} - 1 \) we let \( X_2[k] \) be i.i.d.
and \( X_k \sim \mathcal{C}\mathcal{N} \left( 0, \sigma_{X_2}^2 \right) \). Taking IDFT of \( X_2 \), we obtain
\[
0, 0, \hat{X}_{2,\frac{N}{2} - 2}, \ldots, \hat{X}_{2,4}, 0, \hat{X}_{2,2}, 0]^T
\]
we obtain

\[ \sigma_{x_2}^2 = \frac{N-2}{2N} \sigma_{x_2}^2. \]

The unipolar Flip-OFDM component is obtained by \( T \frac{N}{2} \). So each element of \( s_{\text{FP}} \) and \( s_{\text{EN}} \) has a distribution as \( T \mathcal{G} (0, \sigma_{x_2}^2) \) and the optical power it costs is

\[ E [s_{2,n}] = \frac{\lambda E}{\sqrt{2N}}. \]

We let \( E [s_{2,n}] = \lambda E \), i.e., \( \sigma_{x_2} = \sqrt{2\pi E}. \) At the receiver, we first perform the decoding of the ACO-OFDM component and the clipping noise cancellation process like that described in the proof of Theorem 6 so that the clipping noise from the ACO-OFDM component can be ideally cancelled. Then we get the clipping-noise-free Flip-OFDM component

\[ X_l = \begin{bmatrix} 0, 0^{d-1}, X_{l,1}, 0, \ldots, X_{l,d-1}, 0^{d-1}, X_{l,1}^{d-1}, 0 \end{bmatrix}. \]

in general the\( -\text{C} \) relationship, and the information rate of the ACO-OFDM component, letting \( N \rightarrow \infty \), and choosing the optimal \( \lambda \), we obtain \( \mathcal{R}_{\text{ASC} \text{O}-\text{OFDM}} \).

\[ \mathcal{R}_{\text{HAC} \text{O}-\text{OFDM}} \sim \frac{3}{8} \log \left( \frac{\pi E^2}{9 \sigma_x^2} \right), \quad \text{and} \]

\[ \mathcal{R}_{\text{ASC} \text{O}-\text{OFDM}} \sim \frac{3}{8} \log \left( \frac{\sqrt{T} E^2}{9 \sigma_x^2} \right), \quad \text{(55)} \]

respectively.

Proof: Consider the ASCO-OFDM case. Since for \( \frac{E}{\sigma_x^2} \gg 1 \),

\[ \frac{1}{4} \log \left( 1 + \frac{\pi (1 - \lambda)^2 E^2}{\sigma_x^2} \right) + \frac{1}{8} \log \left( 1 + \frac{2 \pi \lambda^2 E^2}{\sigma_x^2} \right) \]

\[ \approx \frac{1}{4} \log \left( 1 + \frac{\pi (1 - \lambda)^2 E^2}{\sigma_x^2} \right) \left( 1 + \frac{2 \pi \lambda^2 E^2}{\sigma_x^2} \right) \]

\[ \approx \frac{1}{4} \log \frac{\sqrt{2\pi} \lambda (1 - \lambda)^2 E^3}{\sigma_x^2}, \quad \text{(56)} \]

the RHS of (56) is maximized when \( \lambda (1 - \lambda)^2 \) is maximized, which implies the optimal choice to be \( \lim_{\lambda \rightarrow \infty} \lambda^* = \frac{1}{2} \). The proof for the HACO-OFDM case is similar. The corresponding asymptotic information rates can be obtained straightforwardly.

V. MULTI-COMPONENT SCHEMES

This section studies the information rates of two multi-component unipolar OFDM schemes in Table I, namely, the SEE-OFDM and the eU-OFDM. Our results show that with ICG codebook ensemble and nearest neighbor decoding, these schemes have the same information rate, and achieve the high-SNR capacity of the discrete-time Gaussian OIC with average power constraint to within 0.07 bits. Note that the SEE-OFDM does not enlarge the frame length.

Theorem 8: The information rate of the SEE-OFDM with \( L \) ACO-OFDM components, each employing ICG codebook ensemble and nearest neighbor decoding, is

\[ \mathcal{R}_{L, \text{SEE-OFDM}} = \max_{\lambda_1, \ldots, \lambda_L: \sum \lambda_i = 1} \sum_{\lambda_i = 1}^{L} \frac{1}{2^{l+1}} \log \left( 1 + \frac{2^{l+1} \pi \lambda_i^2 E^2}{\sigma_x^2} \right). \quad \text{(57)} \]

Proof: Let \( N \) be divisible by \( 2^L \) where \( L \leq \log_2 N - 1 \). The input of the first ACO-OFDM component of the SEE-OFDM is

\[ X_1 = \begin{bmatrix} 0, X_{1,1}, 0, X_{1,3}, \ldots, 0, X_{1,2^{l-1}}, 0, \ldots, X_{1,2^{l-1}} \end{bmatrix}, \]

\[ 0, \bar{X}_{1,2^{l-1}}, 0, \ldots, \bar{X}_{1,2^{l-1}}, 0 \]

\[ X_{l,k} = \begin{bmatrix} 0, X_{l,1}, 0, X_{l,3}, \ldots, 0, \bar{X}_{l,2^{l-1}}, 0, \ldots, \bar{X}_{l,2^{l-1}} \end{bmatrix}, \]
the transmission rate of the first ACO-OFDM component is below $R_{\text{SEE}, l}$, we can perform the clipping noise cancellation process like that described in the proof of Theorem 6 so that the clipping noise from the first ACO-OFDM component can be ideally cancelled and the decoding of the second ACO-OFDM component is clipping-noise-free. This process can be performed recursively so that the decoding of each ACO-OFDM component is also clipping-noise-free. We can obtain $2^{-(l+1)}N$ parallel channels for the $l$-th ACO-OFDM component as $Y_{l,k} = \frac{1}{\sqrt{L}}X_{l,k} + Z_{l,k}$, where $k$ belongs to the set $K$. The information rate for each $k$ when $X_{l,k} \sim \mathcal{CN}(0, \sigma_{X_l}^2)$ is $C_{\text{AWGN}}(\frac{\mathbb{E}[Y_{l,k}^2]}{2\sigma_z^2})$. The information rate of this component with ICG codebook ensemble and nearest neighbor decoding is thus $R_{\text{SEE}, l} = 2^{-(l+1)}C_{\text{AWGN}}(\frac{\mathbb{E}[X_{l,k}^2]}{2\sigma_z^2})$. Combining the $\sigma_X$-$\mathcal{E}$ relationship, this yields

$$R_{\text{SEE}, l} = \frac{1}{2^{l+1}} \log \left( 1 + \frac{\sigma_l^2 \lambda_l^2 E^2}{4\sigma_z^2} \right).$$ (61)

The proof is completed by summing up $\{R_{\text{SEE}, l}\}$ over $l$ and choosing $\{\lambda_l\}$ maximizing the information rate.

**Theorem 9:** The information rate of the eU-OFDM with $L$ Flip-OFDM components, each employing ICG codebook ensemble and nearest neighbor decoding, is

$$R_{\text{L,eU-OFDM}} = \max_{\lambda_1, \ldots, \lambda_L; \gamma} \sum_{l=1}^{L} \frac{1}{2^{l+1}} \log \left( 1 + \frac{\sigma_l^2 \lambda_l^2 E^2}{4\sigma_z^2} \right).$$ (62)

**Proof:** Let the $m$-th block of the input of the $l$-th Flip-OFDM component be

$$X_{l,m} = \begin{bmatrix} 0, X_{l,m,1}, \ldots, X_{l,m,2^l-1}, 0, \tilde{X}_{l,m,1}, \ldots, \tilde{X}_{l,m,1} \end{bmatrix}^T$$

which is a length $N$ complex vector with Hermitian symmetry. For $1 \leq k \leq 2^l - 1$, let $X_{l,m,k}$ be i.i.d. and $X_{l,m,k} \sim \mathcal{CN}(0, \sigma_{X_l}^2)$. Taking DFT of $X_{l,m}$ we obtain $x_{l,m}$ satisfying $\|x_{l,m}\| = \|X_{l,m}\|$ and $x_{l,m,1} \sim \mathcal{N}(0, \sigma_{X_l}^2)$. Also, $\sigma_{X_l} = \sqrt{\frac{eU-\text{OFDM}}{2}}X_l$. Denote a corresponding Flip-OFDM symbol as $[s_{l,m,1}^T, s_{l,m,2}^T]$. The $l$-th Flip-OFDM component of a frame of the eU-OFDM is

$$s_l = \begin{bmatrix} [1_{2^l-1} \otimes s_{l,1,1}]^T, [1_{2^l-1} \otimes s_{l,1,1}]^T, \ldots, [1_{2^{l-1}} \otimes s_{l,2^{l-1}}, 1_{2^{l-1}} \otimes s_{l,2^{l-1},1}]^T \end{bmatrix}^T$$

where $\otimes$ denotes the Kronecker product and $1_l$ denotes a length $l$ all-one vector. The length of such a frame is $2^{L+1}N$, which is determined by the $L$-th component. A frame of the eU-OFDM is thus $s = \sum_{l=1}^{L} s_l$. Since the Flip-OFDM components are mutually independent we have $E[s_{nl}] = \sum_{l=1}^{L} E[s_{l,n}]$ where $s_{l,n} \sim \mathcal{N}(0, \sigma_{X_l}^2)$ and the optical power it costs is thus $E[s_{l,n}] = \frac{\sigma_{X_l}^2}{2\sigma_z}$. We let $E[s_{l,n}] = \gamma \mathcal{E}$, i.e., $\sigma_{X_l} = \sqrt{\frac{\gamma \mathcal{E}}{2\sigma_z}}$. Where $\{\lambda_l\}$ satisfy $\sum_{l=1}^{L} \lambda_l = 1$.

At the receiver the first Flip-OFDM component can be decoded as that described in the proof of Theorem 3 because the combining as (33) also cancels all other components since they are spreaded by the all-one vector. The information rate of the first Flip-OFDM component with ICG codebook ensemble and nearest neighbor decoding is thus $R_{\text{eU}, l} = \frac{1}{2} \log \left( 1 + \frac{\lambda_l^2 \sigma_l^2 E^2}{\sigma_z^2} \right)$. When the transmission rate of the first Flip-OFDM component is below $R_{\text{eU}, l}$, we can perform the clipping noise cancellation process like that described in the proof of Theorem 5 so that the clipping noise from the first Flip-OFDM component can be ideally cancelled and the decoding of the second Flip-OFDM component is clipping-noise-free. This process can be performed recursively so that the decoding of each Flip-OFDM component is also clipping-noise-free. Moreover, the decoding of the $l$-th Flip-OFDM component includes a despreading process which introduces a spreading gain of $2^{l-1}$ on the SNR of the equivalent channel as (33), and a rate loss factor of $2^{-(l-1)}$. So the information rate of the $l$-th Flip-OFDM component with ICG codebook ensemble and nearest neighbor decoding is

$$R_{\text{eU}, l} = \frac{1}{2^{l+1}} \log \left( 1 + \frac{\sigma_l^2 \lambda_l^2 E^2}{4\sigma_z^2} \right)$$ (65)

and the proof is completed by summing up $\{R_{\text{eU}, l}\}$ over $l$ and choosing $\{\lambda_l\}$ maximizing the information rate.

**Corollary 2:** The asymptotic information rates of the $l$-component SEE-OFDM and eU-OFDM as $L \to \infty$, with ICG codebook ensemble and nearest neighbor decoding for each component, are both lower bounded by $\frac{1}{2} \log \left( \frac{\mathcal{E}}{8\sigma_z^2} \right)$.

**Proof:** We only prove the case of the SEE-OFDM, and the other case can be proved similarly. By letting $\lambda_l = 2^{-l}$ (i.e., $\sigma_{X_l}^2 = 2^{-l} 2\mathcal{E}^2$, noting that $\sum_{l=1}^{L} 2^{-l} < 1$) we obtain

$$\lim_{L \to \infty} R_{\text{L,SEE-OFDM}} \geq \frac{1}{2} \log \left( \frac{\mathcal{E}^2}{2\pi \sigma_z^2} \right)$$

The capacity of the discrete-time OIC satisfies [15], [16]

$$\frac{1}{2} \log \left( 1 + \frac{e}{2\pi \sigma_z^2} \right) \leq C_{\text{DTOIC}} \leq \frac{1}{2} \log \left( \frac{e}{2\pi} \left( \frac{\mathcal{E}}{\sigma_z} + 2 \right)^2 \right).$$ (67)

According to Corollary 2, at high SNR the asymptotic gap between the information rate of multi-component schemes and the capacity of the discrete -time OIC is at most $10 \log_{10} \frac{2\sqrt{\mathcal{E}}}{\sigma_z} \approx 0.21$ dB. In other words, the multi-component schemes can approach the high-SNR capacity of the discrete-time OIC to within 0.07 bits. In [37] it is shown that for uncoded transmission, when $L = 2$, equal optical power
allocation may perform better than unequal optical power allocation. However, according to our results, the information rates of multi-component schemes obtained by equal optical power allocation in each component (i.e., $\lambda_l = 1/L$ for all $1 \leq l \leq L$) tend to zero as $L$ tends to infinity.

**Remark:** The eU-OFDM is a kind of code division multiplexing employing orthogonal variable spreading factor (OVSF) codes [36], where . By using components other than the Flip-OFDM, we can obtain new multi-component schemes which are analogs of eU-OFDM. For example, if the ACO-OFDM components are used, the obtained scheme has a frame length of $2^{L-1}N$, a DoF efficiency of $1 - 2^L$, and the same information rate with the eU-OFDM.

**Note:** If there exist further input constraints, e.g., a peak power constraint or a dynamic optical power constraint, the information rates of the schemes considered in Sec. III, under the assumptions given in Sec. II, can also be lower bounded by clipping the peaks of the transmitted optical intensity signals, and then evaluating the GMI. The information rates of the schemes considered in Sec. III and IV can also be investigated in this way, where the clipping is performed in each component.

VI. **Numerical Results and Discussions**

A. **Comparison of Different Schemes**

Fig. 1 shows some of our results on the information rate of unipolar OFDM in the discrete-time OIC with average power constraint, where SNR $\triangleq \frac{E}{\sigma_z}$ is the optical SNR. The sphere packing based upper bound (SP UB) and the geometrically distributed input based lower bound (Geom LB) on the capacity of the discrete-time OIC (see (67) for their expressions) are shown as benchmarks (also shown in Fig. 2, 4, and 5), which bound the channel capacity to within a small gap. It is shown that the multi-component schemes including the SEE-OFDM and the eU-OFDM approach the capacity of the discrete-time OIC at high SNR, where the corresponding result, obtained by letting $\lambda_l = 2^{-l}$ in e.g. (57), is denoted as Multi-Component-OFDM LB in Fig. 1.

At low SNR three single-component schemes including the ACO-OFDM, the Flip-OFDM, and the PM-OFDM, and two double-component schemes including the ADO-OFDM and the ASCO-OFDM, have the same information rate which is better than the information rates of other schemes. However, all our achievability results have considerable gaps to the low-SNR capacity. For the ADO-OFDM and the ASCO-OFDM, when SNR is blow 3.36 dB and 5.71 dB (see part C of this section), respectively, the information rates are the same as that of the ACO-OFDM, because the optimal strategy then is allocating all power to the ACO-OFDM component. For clarity, the information rate of the PAM-DMT and the HACO-OFDM are not shown in Fig. 1 since the former is 1.5 dB worse than the information rate of the ACO/Flip/PM-OFDM, and the latter is asymptotically 0.5 dB (10$\log_{10}$ $\sqrt{2}$, see Corollary 1) worse than the information rate of the ASCO-
OFDM.

In Fig. 2 results on the information rates of the three multi-component schemes with $L$ components are given, where we set $\lambda_l = 2^{-l}$ for $1 \leq l \leq L-1$ and $\lambda_L = 2^{-(L-1)}$. It is shown that a relatively small $L$ is already good enough. For example, multi-component schemes approach the high-SNR capacity of the discrete-time OIC to within 2 dB when $L$ is 4.

B. DCO-OFDM Parameter Optimization

The performance of the DCO-OFDM and the ACO-OFDM have been compared in the literature, e.g., [34], [35]. In Fig. 1 it is shown that when SNR is lower than 9 dB the ACO-OFDM performs better, otherwise the DCO-OFDM performs better. However, the optimal value of $\sigma_X$ (or $\nu$, equivalently) in the DCO-OFDM varies as SNR increases, which renders challenges for the design of practical systems. Fig. 3 shows the optimal $\sigma_X$, denoted as $\sigma_X^*$, that maximizes the information rate of the DCO-OFDM under our assumption on transceiver. Obviously, neither a fixed $\sigma_X$ nor a $\sigma_X$ being proportional to SNR (with fixed ratio) can achieve the best performance at low and high SNR simultaneously. Fig. 4 and Fig. 5 show our results on the information rate of the DCO-OFDM for different $\sigma_X$ where the result on the ACO-OFDM is included for comparison. In Fig. 4 it is shown that when $\sigma_X$ is fixed, using a relatively small $\sigma_X$ (e.g., $\sigma_X < 4\sigma_z$) approaches the best performance of the DCO-OFDM when SNR < 10 dB. At high SNR a larger $\sigma_X$ must be used. However, in this case a fixed $\sigma_X$ causes performance loss which becomes larger as SNR increases. In Fig. 5, $\sigma_X$ is increased as SNR increases. It is shown again that a fixed ratio between $\sigma_X$ and SNR cannot achieve the best performance at low and high SNR simultaneously. Note that for given SNR, a larger $\sigma_X$ causes a larger clipping noise. However, when the ratio $\frac{\sigma_X}{\sigma_z}$ tends to infinity (i.e., $\nu \to 0$), the high-SNR information rate of the DCO-OFDM does not tend to zero although the probability that the IDFT output $x$ being clipped tends to one. In fact, we can show that

$$\lim_{\nu \to 0, \xi \to \infty} R_{DCO-OFDM} = \frac{1}{2} \log \frac{\pi}{\pi - 2} \approx 0.73 \text{ bit/channel use.}$$

(68)
The interpretation of this fact is as follows: in this case the DCO-OFDM scheme tends to a scheme employing IG codebook ensemble, nearest neighbor decoding, and binary output quantization in Gaussian channel, so $R_{\text{DCO-OFDM}}$ tends to the corresponding GMI which is 0.73 bit/channel use, as shown in [28] Sec. IV).

C. Power Allocation for Double-Component Schemes

Fig. 6 provides results on the optimal power allocation parameters (i.e., $\lambda^*$) for double-component schemes. It is shown that all the optimal parameters have a jump, which is at 5.71 dB, 5.05 dB, and 3.36 dB for the ADO-OFDM, the HACO-OFDM, and the ASCO-OFDM, respectively. In [10], it was shown that for uncoded transmission with M-QAM and M-PAM constellations in the ACO-OFDM component and the PAM-DMT component, respectively, the normalized optical bit energy to noise power $E_b/N_0$ required to achieve the bit error rate performance $\text{BER} = 10^{-3}$ is minimized when $\lambda = 0.6$ (the simulation step size is 0.1). However, our results demonstrate that $\lambda^*$ is always smaller than $\frac{2}{3}$ in terms of information rate. For the HACO-OFDM and the ASCO-OFDM, the numerical results demonstrate the validity of Corollary 1, i.e., $\lim_{\varepsilon \rightarrow \infty} \lambda^* = \frac{1}{2}$. However, for the ADO-OFDM the asymptotically optimal choice of $\lambda$ at high SNR is unknown. These results imply that we can introduce the following simple switch strategy in a practical double-component unipolar OFDM scheme, without significant performance loss. Take the ASCO-OFDM as an example. At low SNR, the Flip-OFDM component is inactive (i.e. no power is allocated) and the ASCO-OFDM reduces to the ACO-OFDM. When SNR is higher than a threshold (e.g. 4 dB), the Flip-OFDM component becomes active, and its power is allocated according to a fixed power allocation parameter (e.g., $\lambda = 1/3$).

VII. DISCUSSION ON UNIPOLAR OFDM IN BANDLIMITED GAUSSIAN OPTICAL INTENSITY CHANNELS

The results in this paper can be extended to other channel models, e.g., frequency selective channels. Here we consider how to apply a unipolar OFDM scheme in continuous-time Gaussian OICs with bandwidth constraint.

For a unipolar OFDM symbol $s$, the corresponding optical intensity waveform obtained by the digital-to-analog converter (DAC) is

$$s(t) = \sum_{n=0}^{N-1} s_n g(t - nT) + A, \quad s(t) \geq 0$$

where $g(t)$ is typically a Nyquist pulse which eliminates the intersymbol interference (ISI). The ISI-free property guarantees that key properties of the discrete-time OFDM model (e.g., IDFT/DFT transceiver structure and inter-carrier interference free property) still hold valid in this continuous-time model. Note that a DC bias $A \geq 0$ may be needed since the first term in (69) is not necessarily nonnegative. For i.i.d. codebook ensemble, the bandwidth of $s(t)$ is the bandwidth of $g(t)$. We can study the information rates of unipolar OFDM schemes in bandlimited Gaussian OICs using the bounding technique for ISI-free signaling in [20]. Optimizing $g(t)$ to boost the information rate of $s(t)$ is an interesting problem with practical importance.

A practical OFDM system typically has a given symbol period, a given bandwidth constraint with a maximum out-of-band power, and a peak-to-average-power ratio (PAPR) constraint. So the design of $g(t)$ with a good tradeoff among its time length (which is related to the CP length), nominal bandwidth (which is related to the bandwidth/DoF efficiency), sidelobe decaying property (which is related to the DC bias needed and the PAPR), spectrum (which is related to the information rate [20]), and out-of-band energy, is very important. In Sec. II to Sec. V we considered clipping of discrete-time signals. A further clipping on $s(t)$ can also be used to reduce the PAPR if the clipping probability is so low that the out-of-band energy caused by clipping is sufficiently small.

APPENDIX A

A General Framework for Transmission over Complex-Valued Vector Channels with Transceiver Distortion

In this appendix we extend the general framework for transmission with transceiver distortion in [28] to complex-valued vector channels.

Consider a complex-valued vector channel with input $\mathbf{X}^{(i)} = \left[ X_1^{(i)}, \ldots, X_N^{(i)} \right]^T$ and additive noise $\mathbf{Z}^{(i)} = \left[ Z_1^{(i)}, \ldots, Z_N^{(i)} \right]^T$, where $\mathbf{Z}^{(i)}$ is ergodic in terms of blocks and is independent of $\mathbf{X}^{(i)}$. The channel output is a deterministic mapping $f(\cdot)$ which transforms a pair of channel input vector and noise vector $\{\mathbf{X}^{(i)}, \mathbf{Z}^{(i)}\}$ into a vector

$$\mathbf{Y}^{(i)} = f(\mathbf{X}^{(i)}, \mathbf{Z}^{(i)}) = f_0 \left( f_1(\mathbf{X}^{(i)}) + \mathbf{Z}^{(i)} \right),$$

(70)

where $i = 1, \ldots, \ell$, $f_1$ and $f_0$ are the mappings in the transmitter and the receiver, respectively, $\ell$ denotes the codeword length in terms of vectors. Eqn. (70) models the channels with nonlinear transceiver distortions which are memoryless in terms of vectors/blocks, e.g., OFDM with clipping, the
analog-to-digital conversion (i.e., quantization) at receiver, I/Q imbalances, and so on.

For transmission of rate $R$ nats per block, assume that a message $m$ is selected from $\mathcal{M} = \{1, \ldots, \exp(R)\}$ uniformly randomly. The encoder maps $m$ to a length-$\ell N$ codeword $\{X^{(i)}(m)\}_{i=1}^{\ell}$ in an ICG codebook ensemble, where $X^{(i)}(m) = [X_1^{(i)}(m), \ldots, X_N^{(i)}(m)]^T$, $1 \leq i \leq \ell$. At the receiver, we let the decoder follow a nearest neighbor decoding rule as

$$\hat{m} = \arg\min_{m \in \mathcal{M}} d_{\ell}(m)$$

(71)

where

$$d_{\ell}(m) = \frac{1}{\ell} \sum_{i=1}^{\ell} \|Y(i) - aX^{(i)}(m)\|^2$$

(72)

i.e., in the codebook, the decoder finds the codeword that minimizes its (scaled) Euclidean distance to the received vector. In (71), $a$ is a decoding scaling parameter to be optimized.

For any transmission rate $R$ less than the GMI, the average probability of decoding error of the decoder described above decreases to zero as the channel coding length grows without bound 29, 28. Following essentially the same steps as 28 Appendix Cj, the GMI of the channel 70 achieved by an ICG codebook ensemble and the decoding metric 71 can be obtained as

$$\mathcal{I}_{\text{GMI}} = N \log \left(1 + \frac{\Delta}{1 - \Delta}\right)$$

(73)

where

$$\Delta = \frac{\mathbb{E} \left[ (f(X, Z))^H X \right]^2}{\mathbb{E} \left[ \|X\|^2 \right] \mathbb{E} \left[ \|f(X, Z)\|^2 \right]}$$

(74)

and the expectation operation $\mathbb{E} \cdot$ is taken with respect to $X$ and $Z$. Here, the decoding scaling parameter $a$ should be set as

$$a_{\text{opt}} = \frac{\mathbb{E} \left[ (f(X, Z))^H Z \right]}{\mathbb{E} \left[ \|X\|^2 \right]}$$

(75)

to achieve $\mathcal{I}_{\text{GMI}}$ given in (73). In this paper, the distortion occurs only at the transmitter. So we have

$$Y = f_1(X) + Z$$

(76)

$$\Delta = \frac{\mathbb{E} \left[ (f_1(X))^H X \right]^2}{\mathbb{E} \left[ \|X\|^2 \right] \mathbb{E} \left[ \|f_1(X)\|^2 \right] + \mathbb{E} \left[ \|Z\|^2 \right]}$$

(77)

and

$$a_{\text{opt}} = \frac{\mathbb{E} \left[ (f_1(X))^H X \right]}{\mathbb{E} \left[ \|X\|^2 \right]}$$

(78)

In addition, for i.i.d. codebook ensembles, for any transmission rate $R$ above the GMI, the average probability of decoding error of the decoder described above tends to one as the channel coding length grows without bound 29, 30. So the GMI can be interpreted as the maximally achievable information rate of a “typical” codebook which is generated without regard to the special operations in unipolar OFDM schemes such as clipping and nearest neighbor decoding.

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