1. INTRODUCTION

The high-state spectral energy distribution (SED) of black hole X-ray binaries is dominated by a soft thermal component that is generally believed to be emission from an optically thick accretion flow. It is often hypothesized that the geometry of this flow is a radiatively efficient, geometrically thin disk (Shakura & Sunyaev 1973, hereafter SS73), or at higher luminosities a less radiatively efficient slim disk (Abramowicz et al. 1988). Most sophisticated spectral modeling of black hole accretion disks has focused on the case of supermassive black holes. However, models that compute vertical structure and radiative transfer have been constructed for Galactic black hole candidates assuming thin disk (Shimura & Takahara 1995, hereafter ST95; Merloni et al. 2000) and slim disk (Wang et al. 1999) relations.

ST95 argue that a simple diluted blackbody provides adequate fits to the local specific flux $F_\nu$ for regions of the disk that produce most of the X-ray photons. They conclude this for disks with sufficiently high luminosity over a range of black hole mass and black hole spin. The local specific flux is then given by

$$F_\nu = \frac{\pi}{f_{\text{col}}} B_\nu (f_{\text{col}} T_{\text{eff}}),$$

where $B_\nu$ is the Planck function, $T_{\text{eff}}$ is the effective temperature of the emitting surface, and $f_{\text{col}}$ is the color correction. They find that in many cases most of the disk spectrum is well approximated by a multitemperature diluted blackbody with a single value of $f_{\text{col}}$. Furthermore, the value of $f_{\text{col}}$ is not a strong function of the disk parameters. They attribute these results to saturated Comptonization playing a dominant role and producing Wien-like spectral profiles. The relative constancy of $f_{\text{col}}$ in these model annuli has been used to justify fitting high-state SEDs with the multicolor blackbody model ($\text{diskbb}$ in XSPEC) of Mitsuda et al. (1984). Although this model assumes a simple profile of effective temperature with radius, $T_{\text{eff}} \propto R^{-3/4}$ and does not account for relativistic transfer effects, it has been used to infer inner radii and black hole spins for disks believed to extend deep within the gravitational potential of the black hole.

Merloni et al. (2000) caution against assuming a nearly constant value for $f_{\text{col}}$. They construct several constant density disk models with a range of accretion rates. They assume that some fraction of the released gravitational energy dissipates directly into an optically thin corona above the disk (Svensson & Zdziarski 1994) but do not account for the irradiation of the disk surface by the coronal emission. They fit the resulting SEDs with the $\text{diskbb}$ model, apply relativistic corrections to the best-fit parameters, and use the results to infer an inner radius for the disk. They find that the value of $f_{\text{col}}$ needed to obtain the correct radius varies significantly from model to model.

Gierliński & Done (2004, hereafter GD04) investigate temperature and luminosity evolution in a sample of black hole binaries observed in the high state with the Rossi X-Ray Timing Explorer (RXTE). They fitted their X-ray spectra with a model that includes a $\text{diskbb}$ component and accounts for Comptonized emission from a corona, focusing on observations in which the Comptonized emission is less than 15% of the bolometric luminosity. They find that many of the well-observed sources are consistent with a weak evolution of $f_{\text{col}}$ in which $f_{\text{col}}$ increases with higher luminosity or temperature. Furthermore, they tentatively conclude that such evolution is inconsistent with a modified stress prescription in which the accretion stress scales with only the gas pressure and not the total pressure.

Subject headings: accretion, accretion disks — black hole physics — radiative transfer — X-rays: binaries
To our knowledge, all the detailed theoretical models of thermal disk spectra of black hole X-ray binaries have so far neglected the effects of bound-free opacity of metal ions. (This is in marked contrast to models of reflection spectra in these systems.) Bound-free opacity can often dominate free-free opacity at observed X-ray energies, reducing the relative importance of electron scattering and therefore altering the expected color correction factors. This may therefore affect the fits to high/soft state spectra and their interpretation. Bound-free opacity also impacts absorption/emission edges in the spectrum that are potentially observable, particularly with the high-throughput spectra being produced by modern X-ray observatories.

In addition, significant theoretical advances have been made recently in understanding the nature and vertical distribution of turbulent dissipation (Turner 2004), as well as highlighting the possibility of significant external torques on the disk at the innermost stable circular orbit (e.g., Gammie 1999; Krolik 1999; Hawley & Krolik 2002). These results imply substantial modifications of the basic assumptions underlying the disk models on which all spectral calculations have been based thus far.

In this paper, we construct fully relativistic accretion disk model atmospheres and examine the integrated SEDs from these disks. We fully account for bound-free opacity by incorporating non-LTE rate equations for the ground-state level populations of all abundant metal ions. We focus on understanding the sensitivity of these SEDs to the underlying assumptions in our models. We compare our results with previous theoretical investigations and recent observations. In § 2 we review the method used to construct our models. We describe our standard models in detail in § 3. We report our results from altering the stress prescription in § 4, adding an inner torque in § 5 and modifying the dissipation profile in § 6. In § 7 we compare our model SEDs with the work of GDO4. We provide further discussion of all these results and a summary of our conclusions in §§ 8 and 9. Readers primarily concerned with the observational implications of our models may wish to skip directly to §§ 7, 8, and 9.

2. METHOD

We construct models using the methods described in a series of papers (Hubeny & Hubeny 1997, 1998, hereafter HH98; Hubeny et al. 2000, 2001, hereafter HBKA). We construct thin disk models by solving the fully relativistic one-zone disk structure equations in the Kerr metric (Novikov & Thorne 1973, hereafter NT73; Page & Thorne 1974; Riffert & Herold 1995). Next, we compute the vertical structure and local spectra for annuli evenly spaced in log \( r \), where \( r = R/R_g \), \( R \) is the Boyer-Linquist radial coordinate, and \( R_g = GM/c^2 \) is the gravitational radius. We compute 10 annuli per decade in radius and assume that the disk extends from the radius of marginal stability \( r_{ms} \) to \( r_{out} \approx 1000 \). Emission from radii larger than \( r_{out} \) is assumed to be blackbody and contributes little to the integrated flux at photon energies \( \geq 0.1 \) keV.

At each annulus, we use the TLUSTY stellar atmospheres code (Hubeny & Lanz 1995) to simultaneously solve the equations for the vertical structure and angle-dependent radiative transfer. At X-ray temperatures, metal opacities are important, and we incorporate fully non-LTE ground-state level populations for all ions of H, He, C, N, O, Ne, Mg, Si, S, Ar, Ca, Fe, and Ni assuming solar abundances. Bound-free opacity due to each ion is included but bound-bound transitions are neglected. We account for Comptonization with an angle-averaged Kompaneets treatment of the electron-scattering source function. Finally, we calculate the integrated disk spectrum seen by an observer at infinity by using a fully general relativistic transfer function (Agol 1997).

We make several approximations that may be violated in nature. The disk models are time steady and azimuthally symmetric. All heat is transported vertically by radiative energy flux. There is no mass loss from the disk, so the accretion rate is independent of radius. All radiative flux is assumed to originate within the disk and its atmosphere; irradiation of the disk surface due to an external X-ray source is neglected. Irradiation of the disk surface by returning radiation (Cunningham 1976) is also ignored. The atmosphere calculations are all one-dimensional despite evidence that radiation pressure–supported accretion disks have significant inhomogeneity (Turner et al. 2003; Turner 2004). Only the gradients in the gas and radiation pressure are considered in constructing the hydrostatic equilibrium. Specifically, a contribution from magnetic pressure is not included, even though the magnetic energy density may dominate the internal gas energy density near the disk surface (Turner 2004).

For our base model (hereafter the standard model) we make additional assumptions that we relax later. First, we assume that there is no torque on the inner edge of the disk. Second, we assume that the energy dissipation rate per unit volume \( \dot{\epsilon} = \nabla \cdot \mathbf{F} \) is locally proportional to the density \( \rho \). Third, we relate the vertically averaged stress, \( \tilde{\tau}_{ro} \), to the vertically averaged total pressure, \( P \), using the prescription (SS73)

\[
\tilde{\tau}_{ro} = \alpha \rho_P.
\]

This standard model is parameterized by only four quantities: black hole mass \( M \), spin parameter \( a \), the stress parameter \( \alpha \), and accretion rate (SS73; NT73). The accretion rate is often expressed relative to the Eddington accretion rate as \( \dot{m} = \dot{m}_{edd} / \dot{m}_{edd} \), where \( \dot{m}_{edd} = 1.5 \times 10^{38} (M/M_\odot) \text{ ergs s}^{-1} \) is the Eddington luminosity for completely ionized H. We prefer to use the quantity \( I \equiv L/L_{edd} = \eta \dot{m} \), since it relates more directly to the observed luminosity. Here \( L \) is the bolometric luminosity of the disk and \( \eta(M, a) \) is the fraction of the energy at infinity that is radiated before crossing \( r_{ms} \). The mass is fixed at \( 10 M_\odot \) for all models considered here.

The atmospheres of the individual annuli are entirely determined by specifying the midplane column mass, the radiative flux, the local gravity, the composition, and the vertical dissipation profile \( \dot{\epsilon}(z) \). The midplane column mass \( m_0 \) is equal to half the surface density of the disk. The flux is parameterized by the effective temperature, \( T_{eff} \), of the corresponding blackbody emitter. The vertical component of the gravitational acceleration \( g = \nabla z \) is proportional to the distance \( z \) above the midplane and the constant of proportionality, \( Q \), is a function only of \( R \). These quantities \( (m_0, T_{eff}, Q) \) are uniquely determined at a radius \( r \) in a one-zone model by a given set of disk parameters \( (M, I, \alpha, a) \). As noted above, we consider \( \dot{\epsilon}(z) \propto \rho(z) \) for the standard models but explore other prescriptions in § 6.

Except for the method of calculating \( m_0 \), the standard models are equivalent to the supermassive black hole models described in HBKA, and the reader is referred there for further details. HBKA solved a one-zone, algebraic relation for \( m_0 \) in terms of \( M, a, \alpha, I \), and \( r \). This \( m_0 \) is then used throughout the stellar atmospheres calculation. Because of the simplifying assumptions of the one-zone model, there is no guarantee that equation (2) will hold for the converged solution. The vertically integrated pressure \( P \) can be calculated from the converged non-LTE model, so the consistency of this prescription can be checked after the fact using equation (2). The one-zone calculation provides
a surprisingly accurate approximation, but order unity discrepancies do typically arise between the final $\alpha$ calculated from the converged non-LTE model and the input $\alpha$.

We alter this prescription by using the initial $m_0$ to construct the LTE-gray model (Hubeny 1990) from which we calculate an $\alpha'$ with equation (2). If $\alpha'$ is within 10% of $\alpha$, we continue the full-atmosphere calculation using the initial $m_0$. Otherwise, we iterate by choosing a new $m_0$, recomputing the LTE-gray model and $\alpha'$ until this condition is satisfied. We use Newton-Raphson to calculate the change in $m_0$ for the next iteration: $\Delta m_0 = dm_0/\delta \alpha (\alpha - \alpha')$, with the derivative evaluated analytically assuming the one-zone scaling holds. The resulting LTE-gray model is then used as the starting point for constructing the full non-LTE atmosphere. For most annuli, this method rapidly converges and provides a final $\alpha$ consistent (within $\sim 10\%$) with the input value. However, in some of the hottest annuli of the torqued disk model the method fails to converge because of inaccuracies in the LTE-gray model. In this case, we default to the one-zone calculation for all annuli in the disk, but in all other cases we utilize the iterative procedure.

3. THE STANDARD MODEL

3.1. The Local Spectrum

The disks in X-ray binaries with a black hole accretor can be much hotter than a disk in a cataclysmic variable or quasar. The gas temperatures at the surface of these disks are sufficiently high ($\sim 10^7-10^8$ K) that electron scattering dominates the opacity at typical photon energies and the emitted spectrum may deviate significantly from that of a blackbody (SS73).

We define the effective optical depth as

$$\tau_{\nu}^{\text{eff}} \equiv \int_0^m \left[ 3\kappa_{\nu}^{\text{th}} (\kappa_{\nu}^{\text{es}} + \kappa_{\nu}^{\text{th}}) \right]^{1/2} dm',$$

where $\kappa_{\nu}^{\text{es}} = \kappa_{\nu}^{\text{th}} + \kappa_{\nu}^{\text{ff}}$ is the sum of the total bound-free and free-free absorption opacity, $\kappa_{\nu}^{\text{es}}$ is the electron scattering opacity, and $m$ is column mass measured from the surface. A closely related quantity is the depth of formation,

$$\tau_{\nu}^* = \int_0^{m_*} (\kappa_{\nu}^{\text{es}} + \kappa_{\nu}^{\text{th}}) dm',$$

where $m_*^{\nu}$ is the frequency-dependent column mass at which $\tau_{\nu}^{\text{eff}} = 1$. With one exception, all of the disk models considered in this paper are effectively thick (i.e., $m_*^{\nu} < m_0$) at each annulus for frequencies of interest. The exception is the $a = 0, I = 0.3$ model, in which the hottest inner annuli are only marginally effectively thick. In an electron-scattering–dominated atmosphere $\kappa_{\nu}^{\text{th}} \ll \kappa_{\nu}^{\text{es}}$ for typical photon energies, implying that the photon destruction probability, $\epsilon_{\nu} = \kappa_{\nu}^{\text{th}}/(\kappa_{\nu}^{\text{th}} + \kappa_{\nu}^{\text{es}}) \approx 1$ and $\tau_{\nu}^* \gg 1$. Because of the temperature gradients in the atmosphere, the temperature at the depth of formation $\tau_{\nu}^*$ is generally greater than $T_{\text{eff}}$. In the absence of Comptonization, the frequency dependence of $\epsilon_{\nu}$ and $T_{\nu}$ alter the spectrum and produce a modified blackbody.

Because of the potentially large number of scatterings ($n_{\nu} \approx \tau_{\nu}^{2} \gg 1$), Comptonization may also be important. To account for its effect on our model spectra we always solve the radiative transfer equation including the Compton-scattering source term. However, we still wish to have an indicator of the impact of electron scattering on the SED. For a homogeneous electron distribution, the $\gamma$-parameter provides a simple means of characterizing the importance of Compton scattering. To gain similar insight, we generalize this prescription by defining a frequency dependent $\gamma$-parameter (HBKA),

$$\gamma_{\nu}^{\gamma} \equiv \frac{4k_B T_{\nu}^*}{m_e c^2} \max (\tau_{\nu}^{2/3}, \tau_{\nu}^*),$$

where $k_B$ is Boltzmann’s constant, $T_{\nu}^*$ is the temperature at the depth of formation, $m_e$ is the electron rest mass, and $c$ is the speed of light.

In Figure 1, we plot the locally emitted SED from a single annular with the frequency-dependent quantities defined above. The model atmosphere was calculated for an annulus at $r = 12.6$ in a disk with $I = 0.1$ and $a = 0.1$, and $u = 1$. We compare the specific intensity viewed from 45° (solid curve) with a diluted blackbody at the same $T_{\text{eff}}$ and $f_{\text{bol}} = 1.56$ (dotted curve). The units of the ordinate are ergs cm$^{-2}$ sr$^{-1}$. (b) We show the fraction of the total opacity provided by electron scattering (solid curve), free-free absorption (dotted curve), and bound-free absorption (dashed curve) evaluated at $m_*^{\nu}$. (c) The depth of formation (eq. [4]) is plotted. (d) The frequency-dependent $\gamma$-parameter (eq. [5]) is plotted.
formation from exceeding 10. If we had neglected metal opacities, the depth of formation would continue to grow as the ratio of free-free to scattering opacity declined. In that case, the photons reaching an observer are created deeper in the atmosphere, where the temperatures and densities are higher. A direct consequence is seen in Figure 1d, where \( \gamma^* > 1 \) for all frequencies of interest. Compton scattering may contribute to the broadening of photoionization edges, but does not significantly alter the continuum. If we had neglected metals, the depth of formation would have been larger, the number of scatterings would have been higher, \( \gamma^* \) would have been greater than unity for \( h\nu \geq 2 \) keV, and Comptonization would have had a larger effect.

Despite the limited role of Comptonization, the specific intensity is still reasonably well approximated by the diluted blackbody spectrum. A modified blackbody caused by dominant electron-scattering opacity, strong temperature gradients, or both, is generally not well characterized by a single temperature, so this result is somewhat surprising. It seems to be due to the relative constancy of \( \gamma^* \) over the range of photon energies near the peak. This means that most of the photons are being created by gas at a narrow range of temperatures and that the photon destruction probability is only weakly dependent on frequency. Both of these effects lead to a more blackbody-like spectral profile, but \( T^* \) is still larger than \( T_{\text{disk}} \). In this case, \( T^* \approx 1.5 - 1.6 \) for the peak-consistent with the \( f_{\text{disk}} = 1.56 \) used for the plot. If we had neglected bound-free metal opacity and Comptonization, the absorption opacity would be more frequency dependent and the diluted blackbody would be a poorer match to the emission.

Electron scattering and temperature gradients also affect the angular dependence of the radiation field. As shown in Figure 2, the spectra are significantly limb darkened at frequencies above 0.1 keV and the degree of limb darkening is frequency dependent. The curves in Figure 2 show the normalized specific intensity for emission at distance \( r = 12.6 \) from an \( l = 0.1 \), \( \alpha = 0.1 \), and \( a = 0 \) disk model. The curves are normalized by the specific flux so that isotropic emission would have an ordinate of unity. The curves for photon energies of 0.21, 2.2, and 8.4 keV are plotted at 81% of their intrinsic luminosity to account for limb darkening in the non-LTE atmosphere models. The lower panel shows the ratio of the non-LTE model spectra to the diluted blackbody spectra.

2.2., and 8.4 keV. In this case, the emission near the peak (2.2 keV) is well approximated by the results for a semi-infinite Thomson scattering atmosphere assuming a Rayleigh phase function (Chandrasekhar 1960). It is less limb darkened at lower frequencies and more limb darkened at higher frequencies because of the temperature gradient in the atmosphere.

3.2. Disk Integrated Spectra

We calculate four standard model disks with \( \alpha = 0.1 \) accreting onto a Schwarzschild black hole with \( l = 0.01, 0.03, 0.1, \) and 0.3. Although larger values of \( l \) are astrophysically interesting, we consider only models with \( l \leq 0.3 \). For \( l \sim 1 \), radial advection begins to become important and the assumption that all locally dissipated accretion power is radiated locally ceases to be accurate. In this regime, a slim-disk model should be used in place of the thin-disk model, which is no longer self-consistent.

In Figure 3 we plot the standard model SEDs (solid curves) observed from infinity at \( i = 70^\circ \). At each value of \( l \), we also plot best-fit, fully relativistic spectra from Schwarzschild disks in which the local SED at each annulus is assumed to be a diluted blackbody. These diluted blackbody disks have the same flux and utilize the same transfer function as the standard model disks. Therefore, they do not directly match the diskbb model, which assumes a simplified form for the radial dependence of the flux and neglects relativistic transfer effects. The best-fit values of \( f_{\text{disk}} = 1.4, 1.46, 1.56, \) and 1.62 for the \( l = 0.01, 0.03, 0.1, \) and 0.3 models, respectively. Because of limb darkening, the fully non-LTE atmosphere spectra have a lower apparent luminosity than the isotropic diluted blackbodies at \( i = 70 \). To account for this, the diluted blackbody spectra are plotted at 81% of their intrinsic luminosity.

**Fig. 2.—Normalized specific intensity of the emission at \( r = 12.6 \) in an \( l = 0.1, \alpha = 0.1, \) and \( a = 0 \) disk model. The curves are normalized by a factor of \( F_{\nu}/r^2 \) so that isotropic emission would have an ordinate of unity. The curves are plotted for photon energies of 0.21 (dashed curve), 2.2, (dotted curve), and 8.4 keV (solid curve). For comparison, we plot the limb-darkening law (crosses) for a semi-infinite Thomson scattering atmosphere assuming a Rayleigh phase function (Chandrasekhar 1960).**
We performed the fits shown in Figure 3 by creating XSPEC table models from diluted blackbody spectra and the fully non-LTE spectrum at the same $l$ and $i$. For each $l$ and $i$, we use the non-LTE model SED to generate an artificial PHA data set using a diagonal response matrix with constant effective area. This data set is then fitted over the 0.1–10 keV range with the corresponding diluted blackbody table model, which has two parameters, $f_{\text{out}}$ and the normalization. We account for the effects of limb darkening in the non-LTE atmospheres by fitting for the normalization of the diluted blackbody model and find that the best-fit models deviate from the artificial spectra by less than 10%–20% except out in the high-energy tail. (In an observed source, this tail would have low photon statistics or would be swamped by hard, nonthermal emission.) If we had not fitted the normalization, the deviations would be as large as 50% at higher inclinations.

Because $a$ and $M$ are fixed, the emitting area of the disk is fixed. An increase in luminosity can only result from an increase in the flux of the disk surface. For blackbody emission, $\nu_{\text{peak}} \propto T_{\text{eff}}(r_{\text{max}}) \propto l^{1/4}$. The same general trend is observed for our models, but the scaling does not strictly hold because of deviations from isotropic blackbody emission in the atmosphere calculations. As $l$ increases, it can be seen that standard model SEDs become increasingly hard relative to a blackbody disk at the same luminosity. That is, the solid curves with high (low) $l$ are better fitted with diluted blackbodies with higher (lower) values of $f_{\text{out}}$. This relative hardening is largely due to the increased dominance of electron scattering in the more luminous models. The trend can be seen in Figure 4, where we plot the relative contributions of the opacity mechanisms at $\tau_{\nu}^c$ in the $r = 12.6$ annuli. As $l$ increases, the radiative flux increases and the metals become highly ionized. The corresponding decrease in the bound-free opacity at lower photon energies moves $\tau_{\nu}^c$ nearer the midplane, where the temperatures are larger. This produces a modified blackbody spectrum that is harder for higher values of $l$. For $l = 0.3$, only the highest ionization states of iron are sufficiently populated to provide significant bound-free opacity. This produces a larger value for $\tau_{\nu}^c$ and $y_{\nu}^r \gtrsim 1$, implying that Comptonization is saturated.

ST95 attribute the effectiveness of the diluted blackbody approximation for their higher luminosity models to the presence of saturated Comptonization producing Wien-like spectra in the inner annuli of their disks. The impact of Compton scattering on our spectral calculations can be approximately estimated from Figure 5, in which we plot $y_{\nu}^r$ for models with four different values of $l$ evaluated in the local frame of the disk at $r = 12.6$. The curves correspond to $l = 0.3$ (solid curve), 0.1 (dotted curve), 0.03 (dashed curve), and 0.01 (dot-dashed curve).

3.3. Dependence on Spin

In an X-ray binary, the case of nonzero black hole spin seems particularly plausible. Here we consider the “maximally spinning” $a = 0.989$ case (Thorne 1974) as an upper bound on the disk spin, with the expectation that angular momentum extraction mechanisms limit real black holes to lower spin parameters.

Changing $a$ modifies the one-zone radial disk structure equations and the location of $r_{\text{in}}$ (NT73). Near the event horizon, the...
relativistic effects on the photon geodesics are amplified, altering the SED observed from infinity. At fixed $l$, the accretion rate drops because of the increased efficiency. However, the innermost annuli are still hotter than in the Schwarzschild case and the SED is expected to be harder modulo relativistic effects on the geodesics.

We construct four maximally spinning Kerr models with $\alpha = 0.1$ accreting at $l = 0.01$, 0.03, 0.1, and 0.3. In Figure 7, we plot the integrated SEDs for these models observed at $i = 70^\circ$ by an observer at infinity. At each value of $l$, we also plot the best-fit, fully relativistic spectra from maximally spinning disks in which the spectrum at each annulus is assumed to be a diluted blackbody. We fit them with the method described in § 3.2. Comparison with Figure 3 shows that both the full-atmosphere models and the diluted blackbody curves peak at higher energies than their Schwarzschild counterparts at the same $l$. Increasing $a$ modifies the one-zone radial disk structure equations, yielding larger fluxes and correspondingly higher $T_{\text{eff}}$ in the innermost annuli.

Relativistic broadening also accounts for the hardening of the SED. As $a$ increases from zero to 0.998, $r_{\text{ms}}$ decreases from 6.0 to 1.23. Moving closer to the event horizon amplifies the relativistic effects on the photon geodesics, altering the SED observed from infinity. At $i = 70^\circ$, strongly blueshifted emission from matter moving toward the observer pushes the high-energy tail to larger frequencies. In Figure 8, we plot the same set of models as viewed from a lower inclination of $i = 45^\circ$. The relativistic broadening is clearly reduced and both the spectral peak and the high-energy tail occur at lower energies.

In the hot inner annuli, the metals are highly ionized. Even for iron, a substantial fraction of the atoms are completely stripped of electrons in the $l = 0.1$ and 0.3 models. This reduces the bound-free opacity, and electron scattering plays a more dominant role than in the Schwarzschild case. As a result, the diluted blackbody models that best approximate the solid curves generally have higher values of $f_{\text{col}}$ and the breaks near the metal edges are not as prominent. A calculation for these hotter annuli finds $\gamma_{\nu_{\text{c}}} \gtrsim 1$ for most relevant photon energies in the $l \geq 0.03$ models. The agreement between the diluted blackbody and atmosphere model SEDs generally improves with increasing $l$. For $i = 70^\circ$, the maximum deviations are always less than or equal to those in the corresponding Schwarzschild model. Since

Fig. 8.—Integrated disk SEDs for three values of $l$ (solid curve) observed from an $i = 45^\circ$ by an observer at infinity. At each $l$, we also plot the best-fit, fully relativistic disk models in which the local flux is assumed to be an isotropic diluted blackbody (dashed curve). Each model has $\alpha = 0.1$ and $a = 0.998$, and from lower left to upper right the curves correspond to $l = 0.01$, 0.03, 0.1, and 0.3 with best-fit $f_{\text{col}} = 1.4$, 1.41, 1.47, and 1.57, respectively. The diluted blackbody spectra are plotted at 114% ($l = 0.01$) and 112% ($l = 0.03, 0.1$, and 0.3) of their intrinsic luminosity to account for limb darkening in the non-LTE atmosphere models.
determine the structure and spectrum of an annulus in our models. Of these quantities, only $m_0$ depends on $\alpha$. If all other quantities are fixed, an increase (decrease) in $\alpha$ produces a larger (smaller) stress. For the accretion rate to remain constant, there must be a corresponding reduction (increase) in the surface density and therefore in $m_0$. In a constant density, radiation pressure–dominated one-zone model, a change in $m_0$ leads to an equivalent fractional change in the density because the scale height is independent of $\alpha$ (SS73).

In Figure 9 we compare the vertical temperature and electron number density $n_e$ profiles in disks with different values of $\alpha$. The solid and dashed curves correspond to $\alpha = 0.1$ and 0.01, respectively. We plot results for annuli at $r = 12.6$ in disks with $a = 0$ and $l = 0.1$. Both of these atmospheres are expected to be radiation pressure dominated, so we scale the $z$-coordinate with the appropriate one-zone model $h$ (HH98, eq. [53]). The crosses in Figure 9 mark the location of $\tau_v^* \equiv \int n_e \sqrt{P_\text{gas}} dz$ evaluated at a frequency near the spectral peak ($\nu_{\text{peak}} \approx 5 \times 10^{17}$ Hz$^{-1}$).

Below one scale height the models are consistent with the predictions of a one-zone model. The densities are nearly constant and $n_e$ in the $\alpha = 0.01$ atmosphere is about a factor of 10 larger than in the $\alpha = 0.1$ case. The flux rises linearly with $z$, providing a force that balances gravity. However, above $z = h$ the flux rises less steeply and begins to asymptote to $\sigma T_\text{eff}^4$. Gas pressure gradients are therefore needed to balance the continued increase in gravity and the density decreases exponentially. The crosses indicate that most of the spectrum is formed where the density is dropping rapidly and not where the density is relatively constant. Disk models that neglect vertical gradients in density will therefore overestimate the ratio of absorption to scattering opacity and underestimate the effective Compton $\gamma$-parameter.

In Figure 10 we compare the integrated SEDs from these disks viewed at $i = 70^\circ$, and they are nearly indistinguishable. Despite the differences in the models for $z \leq h$, the temperature and density profiles seen from the surface down to $\tau_v^*$ are nearly identical in the two annuli. In this annulus and other annuli at larger $r$ and smaller $T_\text{eff}$, the $\tau_v^*$ surfaces always occur at relatively low column depth because of the significant absorption opacity. In general, the temperature, density, and ion level population profiles of the two models do not differ significantly in these surface regions at any $r$.

4. THE STRESS PRESCRIPTION

Most of the standard model disks ($\alpha$-disks) presented in § 3 are radiation pressure dominated in the innermost annuli. Such disks have long been known to be "viscously" and thermally unstable. One way to avoid these instabilities is to construct models (hereafter $\beta$-disks) in which the accretion stress scales with gas pressure $P_\text{gas}$ rather than total pressure $P$ in equation (2) (Lightman & Eardley 1974). It has been argued that such a scaling could be the consequence of magnetic field buoyancy limiting magnetic stresses (Sakimoto & Coroniti 1981; Stella & Rosner 1984).

As discussed in § 3.4, altering the stress prescription only changes $m_0$ in a one-zone model. A process that limits the magnetic stress (e.g., magnetic field buoyancy) may also alter the vertical dissipation profile, but we discuss the impact of the dissipation profile separately in § 6. For fixed $\alpha$, replacing $P$ by $P_\text{gas}$ in equation (4) results in a lower stress. Thus, a $\beta$-disk requires a larger $m_0$ to produce the same accretion rate as an $\alpha$-disk. The larger $m_0$ results in greater midplane densities and temperatures.

We construct $\beta$-disk models with $\alpha = 0.1$ for Schwarzschild black holes and maximally spinning Kerr black holes ($a = 0.998$) at $l = 0.03$, 0.1, and 0.3. At $l = 0.01$ the $\alpha$-disk models are gas pressure dominated down to $r_{\text{in}}$ and are therefore equivalent to $\beta$-disks. We calculate $m_0$ by integrating the LTE-gray model until equation (2) is satisfied for $P = P_\text{gas}$. In Figure 11, we compare the integrated SEDs of the spinning black hole models with the equivalent $\alpha$-disk spectra for an observer at $i = 70^\circ$. We find that the stress prescription has little effect on the black hole disk SEDs for $l < 0.3$ for the same reasons as in § 3.4. Specifically, $\tau_v^*$ lies near the surface of the disk where the vertical structure of the two types of annuli are always similar regardless of the conditions at the midplane. Photons emitted at $\tau_v^*$ in the $\beta$-disks encounter nearly identical profiles of density, temperature, and level populations as those in an $\alpha$-disk. The same result hold for the Schwarzschild models at all four values of $l$ considered here.

The $\alpha$-disk spectrum is slightly harder than the $\beta$-disk spectrum for the $l = 0.3$, $a = 0.998$ black hole model. In this case, the innermost annuli of the $\alpha$-disk are less effectively thick at the midplane than annuli in the other models because $m_0$ is
lower and $T_{\text{disk}}$ is higher, reducing the absorption opacity. Therefore, the $\tau_{\alpha}$ surfaces occur deeper in the annulus where the density of the $\alpha$-disk is less dense than $\beta$-disk. This produces two competing effects. First, it reduces the importance of absorption opacity relative to electron scattering opacity. This increases the average number of scatterings of a photon before it escapes the atmosphere and therefore increases the $y$-parameter, driving the photons toward a softer spectrum. However, the lower density of the $\alpha$-disk also moves the $\tau_{\alpha}$ surface deeper into the atmosphere, where the temperature is larger. The photons are then emitted with a higher average energy, which tends to produce a harder spectrum. In this model, the latter effect dominates and the resulting spectrum from the $\alpha$-disk is slightly harder.

5. TORQUES ON THE INNER BOUNDARY

The no-torque boundary condition may be invalid if magnetic fields keep the material in the plunging region causally connected to the disk (Gammie 1999; Krolik 1999). In that case, the surface density and brightness of the disk are modified from the standard model. The corresponding changes in $T_{\text{eff}}(r)$ and $m_{\text{f}}(r)$ can be parameterized in terms of $\Delta \eta$, the change in efficiency of converting energy into radiative flux (Agol & Krolik 2000). For a Schwarzschild black hole with no inner torque, $\eta = 0.057$. We calculate a torqued Schwarzschild disk with $l = 0.1$, $\alpha = 0.1$, and $\Delta \eta = 0.05$ so that the efficiency is nearly double the no-torque case. Since we fix the luminosity of the two disks, the accretion rate of the torqued disk is lower. Our method of integrating the LTE-gray model fails to converge for some of the hottest annuli, so we use the one-zone calculation to find $m_{\text{f}}$ throughout the disk. We computed the values of $\alpha$ required to satisfy equation (2) in the converged non-LTE annuli, and they differed from the assumed value by less than 50%. For the reasons discussed in § 3.4, we do not expect the resulting spectrum to be sensitive to this small discrepancy. In the disk without a torque, $T_{\text{eff}}$ increases with decreasing $r$ until reaching a maximum at $r_{\text{max}} > r_{\text{ms}}$ and then falls to zero at $r_{\text{ms}}$. For the torqued disk, $r_{\text{max}} = r_{\text{ms}}$ and $T_{\text{eff}}$ continues to rise all the way down to the inner edge of the disk. Despite the decrease in $\dot{m}$, the fluxes in the innermost annuli are higher than in a standard disk at the same luminosity.

In Figure 12, we compare the SEDs of the torqued (solid curve) and standard model (dotted curve) disks viewed at $i = 70^\circ$. Using the same flux and folding through the same transfer function, we also calculate spectra assuming that the local emission produces an isotropic diluted blackbody with $f_{\text{col}} = 1.61$ (dashed curve). The diluted blackbody spectra are folded through the same transfer function and viewed at the same inclination as the solid curve. The SED of the torqued disk is considerably harder than the SED of the disk without a torque (dotted curve).

![Integrated disk SEDs for three values of $l$ observed at $i = 70^\circ$ by an observer at infinity. Each model has $\alpha = 0.1$ and $\dot{a} = 0.998$, and from lower left to upper right the curves correspond to $l = 0.03$, $0.1$, and $0.3$. The $\alpha$-disks (solid curve) are equivalent to the curves plotted in Fig. 7. The $\beta$-disks (dashed curve) show little deviation from the $\alpha$-disks at lower luminosities. However, the $l = 0.3$ $\alpha$-disk model is slightly harder.](image1)

![SED (solid curve) from a torqued disk with $l = 0.1$, $\alpha = 0$, $\alpha = 0.1$, and $\Delta \eta = 0.05$ viewed at $i = 70^\circ$ by an observer at infinity. We also plot the best-fit, fully relativistic disk models in which the local flux is assumed to be an isotropic diluted blackbody with $f_{\text{col}} = 1.61$ (dashed curve). The diluted blackbody spectra are folded through the same transfer function and viewed at the same inclination as the solid curve. The SED of the torqued disk is considerably harder than the SED of the disk without a torque (dotted curve).](image2)
model is narrower than our model, providing a deficit of flux at the high- and low-energy ends of the band and an excess near the spectral peak. This is in part due to the departures from blackbody emission, but is due mostly to relativistic transfer effects and the remaining differences in the flux profile. As a result, the quality of the diskbb fits to our torqued disk spectral model is not substantially better than similar fits to the torque-free disks that we discussed in § 7. Considering a larger torque would increase the similarity in the flux profiles. This should improve the quality of the fit, but discrepancies would remain. Even when we assume locally isotropic blackbody emission with a flux identical to the diskbb model, the resulting SED is intrinsically broader than the diskbb because of the effects of relativistic transfer.

6. THE DISSIPATION PROFILE

In § 2 we stated that our standard model assumes that the energy dissipation rate per unit volume is locally proportional to the density. Before exploring other dissipation profiles, it is worthwhile reminding the reader what this implies for a radiation pressure-dominated disk. In that case, hydrostatic balance reduces to

\[
\frac{-1}{\rho} \frac{dP_{\text{rad}}}{dz} = \frac{\kappa_F F}{c} = \frac{GM C}{R^3 B^2 z},
\]

where \(P_{\text{rad}} \sim P\) is the radiation pressure, \(C\) and \(B\) are general relativistic correction factors (Riffert & Herold 1995), and \(\kappa_F\) is the flux mean opacity. In these scattering-dominated disks, \(\kappa_F \approx \kappa_{ee}\), which is only weakly dependent on \(z\) through the free electron fraction. Therefore, \(F \propto z\), implying that \(\varepsilon\) is constant when the gradient in radiation pressure balances gravity. Our assumption that \(\varepsilon \propto \rho\) then implies that the density is constant so long as gas pressure gradients are negligible. From the definition of column mass \(dm = -\rho dz\), it is easy to see that our assumption reduces to

\[
\varepsilon = \rho \left( -\frac{dF}{dm} \right) = \rho \frac{\sigma T_{\text{eff}}^4}{m_0},
\]

where \(\sigma\) is the Stefan-Boltzmann constant. The first equality holds by definition and the second follows from our assumption that implies that \(dF/dm\) is independent of \(m\). We have set \(F(m) = \sigma T_{\text{eff}}^4(1 - m/m_0)\) in all the models discussed above.

6.1. Physically Motivated Dissipation Profiles

We would like to examine how the choice of \(F(m)\) affects the vertical structure of the accretion disk, but there is no firm theoretical basis to guide our analysis. It is commonly accepted that the magnetorotational instability (MRI; Balbus & Hawley 1991) can provide a means for tapping the free energy of the differential rotation and producing turbulence in which energy may cascade irreversibly from large to small scales. However, the nature of nonradiative vertical energy transport and the mechanisms for dissipation in these disks are still not well understood. Despite these uncertainties, there is a rather robust prediction of magnetic field buoyancy in radiation dominated annuli (Sakimoto & Coroniti 1981; Stella & Rosner 1984). It is therefore quite reasonable to expect that a larger fraction of the dissipation occurs nearer the surface than is assumed in equation (7).

Numerical simulations that include radiative diffusion in a vertically stratified disk (Turner 2004) bear out this prediction. The radiation-MHD equations are solved for a patch of disk centered at 200 \(R_g\) from a 10\(^8\) \(M_\odot\) black hole. Magnetic field energy is produced fastest by the MRI 2–3 density scale heights away from the midplane. The magnetic field is buoyant and rises toward the surface at approximately the Alfvén speed. The density is more centrally concentrated than in a standard model disk with the same surface density and radiative flux. The time-averaged dissipation is not at all consistent with the \(\varepsilon \propto \rho\) assumption, since much of the dissipation takes place in the low-density surface layers.

Similar calculations are not yet available for disks accreting onto 10 \(M_\odot\) black holes. In order to explore the possible consequences of magnetic buoyancy, we choose a form for \(dF/dm\) that approximates the Turner (2004) results for a 10\(^8\) \(M_\odot\) black hole. We then use this form to calculate \(F(m)\) with choices of \(m_0\) and \(T_{\text{eff}}\) appropriate for an annulus around a 10 \(M_\odot\) black hole. Although these simulations are the best quantitative model for \(F(m)\) available, this choice should not be considered overly restrictive. There is still considerable uncertainty in the simulation itself. Although 29% of the dissipation is due to Silk damping (Agol & Krolik 1998), most is still the result of numerical reconnection. Furthermore, the outermost grid zone is quite optically thick but our models extend to \(\tau \ll 1\), so there are several decades in \(m\) for which \(F(m)\) is not constrained by the simulation results. Following HH98, we approximate \(dF/dm\) as a broken power law with \(m\) being the dependent variable. The broken power law is parameterized by a division point \(m_f = f_d m_0\) and exponents above \((\zeta_1)\) and below \((\zeta_0)\) the division. For the standard model, \(\zeta_0 = \zeta_1 = 0\). In the simulation, the dissipation and density are functions of position and time, so we average the domain horizontally and average over time from 20 to 40 orbits. The broken power law provides a reasonable approximation with \(\zeta_0 = -0.9, \zeta_1 = 0\), and \(f_d = 0.004\), so we do not consider more complicated profiles.

Next we insert this choice of \(dF/dm\) in an annulus with \(m_0\) and \(T_{\text{eff}}\) appropriate for \(r = 12.6\) in an \(l = 0.1, a = 0, \alpha = 0.1\), and 10 \(M_\odot\) standard model disk. We had difficulties obtaining convergence for non-LTE models in which the Eddington factors were updated after each solution of the radiative transfer equation. If we instead fix the Eddington factors after the first radiative transfer solution, the models converge. We believe the lack of convergence in the first case is a result of our calculation method rather than an instability in the model. Furthermore, we doubt that the assumption in the second case has a strong effect on the resulting spectra. The maximum fractional change in the Eddington factors after a global solution of the radiative transfer is less than 3% for all frequencies and depths and is generally significantly smaller.

The resulting annulus structure (dotted curve) is compared with the corresponding standard model (solid curve) in Figure 13. The two are quite different. The radiative flux is plotted in the top panel. For \(z < h\), the solid curve rises linearly because the radiative flux must balance the linearly increasing vertical gravity. The dotted curve falls below this line, so the gradient in the gas pressure also provides significant support against gravity in the modified annulus. The temperature is shown in the middle panel. The surface layers above \(\tau = 1\) (marked by squares) are somewhat hotter in the modified annulus owing to the increased fraction of dissipation in that model. The electron number density is shown in the bottom panel. The density is more centrally concentrated than in the standard model annulus, since a density gradient is necessary to compensate for the reduced radiation pressure support. The temperatures and densities at \(\tau_{\text{m}}\) (marked by crosses) are nearly equal in the two models, although the shape of the temperature and density profiles are more dissimilar from each other than in the models with differing stress prescriptions.
The specific intensities at $i = 55^\circ$ are plotted in Figure 14. The SED of the modified annulus is slightly harder, giving an $f_{\text{eq}}$ that is $\sim 10\%$ higher, and the He-like Fe absorption edge at 8.8 keV becomes an emission edge. Despite these differences, the model spectra are still qualitatively quite similar. The strong temperature inversion in the modified annulus is above the photosphere for frequencies redward of the 8.8 keV edge and has little effect on the resulting SED.

6.2. Dissipation and Coronae

The geometry and origin of the coronae in accretion disks remain uncertain. One plausible explanation is that the coronae are due to hot electrons heated by the reconnection of magnetic fields in the optically thin layers at the disk surface. This is the essence of the Svensson & Zdziarski (1994) model, where it is assumed that some fraction of released energy is transported without dissipation to a corona above the disk. It can be seen from Figure 13 (top) that very little dissipation occurs above the Thomson photosphere in the models described in 6.1. We have considered several different prescriptions for $F(m)$ in which larger fractions of the dissipation occur at low $m$. The choice of a broken power law for $df/dm$ provides a smooth, continuous dissipation profile that is different from the Svensson & Zdziarski (1994) model in which $df/dm$ would be a step function.

In Figure 15, we show two annuli in which almost half of the dissipation occurs above the Thomson photosphere. The first case (solid curve) is similar to a Svensson & Zdziarski (1994) model in that $df/dm$ is constant below the photosphere as in the standard model disks but $df/dm$ is discontinuous at the photosphere. Nearly 50% of the dissipation occurs above a Thomson depth of unity and the resulting annulus has a sharp rise (drop) in the temperature (density) just above the scattering photosphere. In order to better approximate these gradients, we increase our number of depth points, but memory restrictions then limit us to just considering H, He, C, N, O, and Fe for this annulus only. The neglect of less abundant species seems to have little effect on the vertical structure or spectrum. In the second case (dotted curve) $df/dm$ is a continuous broken power law with $\zeta_0 = -1.1$, $\zeta_1 = 0$, and $f_d = 0.0001$. About 40% of the dissipation occurs above the Thomson photosphere and another 20% occurs above $\tau^*_e$ (measured near the peak). We consider the same $m_0$ and $T_{\text{eff}}$ used for the models plotted in Figures 13 and 14. The profiles in the discontinuous model closely resemble those of a standard model disk below the photosphere, but above there is a sharp rise in the...
temperature and drop in the density. In the second case, the shape of the density and temperature profiles are similar to the modified annulus discussed in \( \S 6.1 \) but on a more exaggerated scale. The surface layers are much more extended, the surface temperature and central density are higher, and the densities at the surface are lower.

We plot the SEDs of these annuli in Figure 16. In the top panel we plot the specific intensity at 55'. In both cases, Compton scattering by the hot thermal electrons above the photosphere produces a steep tail with emission up to \( \sim 100 \text{ keV} \). These tails are only crudely approximated by a power law, since there is curvature at low energies near the peak and at high energies due to the thermal cutoffs. The discontinuous model (solid curve) produces a slightly harder spectrum. We also include power-law curves with photon indices of \( \Gamma = 3.4 \) and 3.5 to approximate the two spectra over the \( \sim 5-50 \text{ keV} \) band. Although each spectra only accounts for the emission from a single annulus it is suggestive that they bear some qualitative resemblance to observations of the very high state (e.g., Kubota & Done 2004). The nonthermal power laws in these sources are typically flatter (smaller \( \Gamma \)), but a harder tail could presumably be produced by choosing a more top heavy \( f(\theta) \). However, the surface temperatures become too large and our treatment of Compton scattering is no longer valid.

In the bottom panel we plot the ratio of the model SEDs to these power laws. For the discontinuous model there is curvature due to the reduction in the scattering albedo from bound-free opacity above \( \sim 10 \text{ keV} \) and the spectrum resembles the putative Compton reflection hump. The high-energy side of the hump is due to both Compton downscattering by the cooler electrons near the photosphere and the thermal cutoff of the hot electrons. There is a strong emission feature at the He-like Fe edge. This feature is likely enhanced because our treatment only allows recombinations to the ground state. Some fraction of this power would be shifted to lower energies if we had considered recombinations to excited levels. In the second case, the spectrum simply falls off at high energies and there is no structure in the residuals that could be characterized as a reflection hump. A large fraction of dissipation occurs in the spectral formation regions just below the photosphere, so the temperature there is high and the density is relatively low. Almost all iron is completely ionized so that the bound-free opacity that produces the low-energy side of the hump in the previous model is insignificant.

7. LUMINOSITY-TEMPERATURE RELATION

Detailed fits to individual observations are beyond the scope of this paper. Because we neglect irradiation of the disk surface, we focus on comparison with the results of GD04. Their sample is particularly relevant because they select observations in which the coronal emission is less than 15% of the bolometric luminosity of the source.

GD04 construct plots of the temperature-luminosity relation for each source in their sample. The luminosity of the disk model \( L_{\text{disk}} \) is plotted as a fraction of \( L_{\text{edd}} \) versus the maximum color temperature \( T_{\text{max}} \). A difficulty with inferring the physical properties of accretion disks is that the distance, inclination, and black hole mass of the source are not all known with great precision. Therefore, the vertical and horizontal positions of the locus of points are only known to varying degrees of accuracy depending on the source. However, the shape of the locus of points for any individual source should be robust to such uncertainties.

If the disk spectrum is well approximated by a multi-temperature diluted blackbody with fixed \( f_{\text{col}} \), \( L_{\text{disk}} \) should be proportional to \( T_{\text{max}}^4 \). Specifically, Gierliński et al. (1999) assume that the gravitational field may be approximated with a pseudo-Newtonian potential (Paczyński & Witt 1980) and find the relation

\[
\frac{L_{\text{disk}}}{L_{\text{edd}}} \approx 0.583 \left( \frac{1.8}{f_{\text{col}}} \right)^4 \left( \frac{M}{10 M_\odot} \right)^{3/2} \left( \frac{kT_{\text{max}}}{1 \text{ keV}} \right)^4,
\]

where \( T_{\text{max}} = T_{\text{eff}, \text{max}} f_{\text{col}} \). GD04 find that most of the sources roughly follow this scaling, although several show some degree of relative hardening (increasing \( f_{\text{col}} \)) as \( L_{\text{disk}} \) increases.

One obstacle to comparing with their results is that neither \( T_{\text{max}} \) nor \( f_{\text{col}} \) is a well-defined theoretical quantity in our models. In principle, we could use the \( f_{\text{col}} \) values from the best-fit models shown in Figure 3, but that fitting procedure is quite different from the method implemented by GD04. In constructing their temperature-luminosity diagram, they fitted the diskbb model to their spectra to account for the presumed disk component. They then use the best-fit inner temperature and total model flux to calculate \( L_{\text{disk}}/L_{\text{edd}} \) and \( T_{\text{max}} \). In doing so they apply relativistic correction factors (Zhang et al. 1997) for a Schwarzschild black hole at the appropriate inclination. They also apply an additional correction factor to account for the mismatch between the flux profile of the diskbb model and the pseudo-Newtonian potential used to derive equation (8).

To facilitate comparison, we try to reproduce this procedure as closely as possible. We produce an artificial EPIC pn PHA data set as described in \( \S 5 \). We also generate a second set of artificial spectra by the same method but employing the RXTE Proportional Counter Array (PCA) response matrices. Galactic absorption has less impact on the PCA photon statistics, so we do not include it in those artificial spectra and fits. The EPIC pn spectra are useful because of the high signal-to-noise ratio and
the low-energy coverage, while the PCA spectra are necessary for comparing with GD04. Next, we fit diskbb models over the 0.3–10 and 3–20 keV energy ranges for the EPIC pn and PCA data sets, respectively. We also generate artificial spectra, including a \textit{compTT} (Titarchuk 1994) component that is 10% of the bolometric flux to account for possible Comptonized coronal emission. These spectra are then fitted with a combined diskbb + \textit{compTT} model. We find that the best-fit parameters of the diskbb models are only weakly sensitive to the presence or absence of the \textit{compTT} component as long as we fit for this component with another \textit{compTT} model and not a power law. Finally, we apply the same prescription outlined in GD04 to calculate $L_{\text{disk}}/L_{\text{Edd}}$ and $T_{\text{max}}$ from the model temperature and flux.

We show the results of this procedure for the four Schwarzschild models with $l = 0.01, 0.03, 0.1,$ and $0.3$ viewed at $i = 45^\circ$ and $70^\circ$ in Figure 17. The solid curves represent the luminosity-temperature relation of equation (8) for $f_{\text{col}} = 1.4, 1.6, 1.8,$ and $2.0$. The triangles and squares mark the XMM-Newton EPIC pn and the RXTE PCA measurements, respectively. Only the highest three luminosities are plotted for the PCA measurements because the model with $l = 0.01$ emits only a small fraction of its luminosity above 3 keV and is not well constrained by a PCA observation. The diskbb model provides a poor fit to the artificial XMM-Newton data, so reliable uncertainties cannot be estimated for best-fit parameters. The uncertainties (90% confidence for one parameter) on PCA depend on the signal-to-noise ratio of the simulated data set, but they are of order the symbol size or smaller for a 1.5 ks exposure of a source at 5 kpc.

First, we focus on the EPIC pn data sets. The filled and open triangles mark the $i = 45^\circ$ and $70^\circ$ measurements, respectively. Both sets of measurements show a similar degree of spectral hardening with increasing luminosity but with approximately the same fractional offset in $T_{\text{max}}$ and $L_{\text{disk}}/L_{\text{Edd}}$ for each $l$. The difference in apparent luminosity between the two sets of measurements and the known $l$ values are predominantly due to limb darkening in the atmospheres (compare with Fig. 2), but there is also a small mismatch between our relativistic transfer function and the $g$-correction factors we interpolate from Zhang et al. (1997) Table 1. Comparing the symbols with the solid lines shows that the derived values of $f_{\text{col}}$ are inclination dependent. We compare the best-fit diskbb spectra to the $i = 70^\circ$ non-LTE model SEDs in Figure 18. When folded through the spectral response, the best-fit diskbb models disagree with artificial spectra by $\sim 5\%$–$10\%$ at lower energy and less than 5% near the peak. These deviations can be significant for the high signal-to-noise ratio obtained in a long exposure of the bright high/soft state X-ray binaries. The model provides poor fits (reduced $\chi^2 \gg 1$) to the data for the simulated 75 ks exposures of sources at 5 kpc.

The PCA measurements at $l = 0.1$ and $0.3$ agree reasonably well with the EPIC pn data sets. The $T_{\text{max}}$ values are larger because the PCA band only extends down to 3 keV in our fits and does not include the spectral peak. The Doppler-broadened high-energy tail is consistent with a slightly higher inner temperature than the emission near the spectral peak, which is given more weight in the fits to the EPIC pn artificial spectra. The $l = 0.03$ case is slightly more complicated because there is a spectral break above 3 keV due to the presence of bound-free opacity. This produces a slightly steeper falloff in the high energy tail. This can be more easily fitted by a lower temperature and correspondingly higher normalization leading to a larger $L_{\text{disk}}/L_{\text{Edd}}$ and lower $T_{\text{max}}$. These shifts overestimate the emission near the peak that is outside of the PCA band but must be accounted for in the EPIC pn fits.

8. DISCUSSION
8.1. Comparison with Previous Work

Our models are qualitatively different from those of ST95 in several important ways. We examine the specific intensities and account for relativistic effects on photon geodesics, while ST95 compare the specific flux in the local frame of the disk. Calculating specific fluxes in the local frame simplifies the comparison with diluted blackbodies by avoiding inclination-dependent effects. However, this has the drawback of neglecting the nontrivial coupling between frequency and inclination caused by the limb darkening and relativistic transfer that is presumably present in real accretion disks. Another important difference between our models is our inclusion of metals. We account for the bound-free opacity of all important ions in addition to free-free opacity, which was the only absorption opacity included in the calculations of ST95. The addition of metal opacity reduces the range of $l$ in which saturated Comptonization is important and produces
modified blackbodies that are slightly softer (more consistent with lower values of \( f_{\text{col}} \)) than the model SEDs of ST95.

ST95 conclude that multitemperature, diluted blackbody spectra adequately approximate their models at higher luminosities but are inadequate at low luminosity. When they compare diluted blackbodies with their \( \alpha = 0.1 \), Schwarzschild black hole integrated disk spectra at energies above 0.1 keV, the deviations are less than 20%, 15%, and 30% for the \( l = 0.0057 \), 0.057, and 0.57 models, respectively. For the two more luminous models, the largest disagreement is at the low-energy end of the band and the agreement is better at higher energies. For \( l = 0.0057 \), the integrated spectra show larger deviations at higher energies and the individual model annuli are not well represented by diluted blackbodies because the \( \gamma \)-parameter is small. They conclude that color-corrected blackbodies are good approximations only at the higher luminosities.

We focus on the integrated emission from the whole disk. For all the \( l = 0.01 \) to 0.3 models, the comparisons between diluted blackbody and non-LTE atmosphere model SEDs depend on the viewing angle. In § 3.2, we presented fits to our non-LTE models with fully relativistic diluted blackbody models with constant color correction \( f_{\text{col}} \). Provided one fits for both \( f_{\text{col}} \) and the normalization (in order to account for limb darkening), these fits are surprisingly good. The disagreement between the models is less than 20% at most photon energies regardless of inclination. As seen in Figure 3, the deviations in the high-energy tails can eventually become much larger. However, by that point the statistical weight of these energy bins is typically low because the integrated photon flux falls off rapidly in the tail. The nonthermal emission component typically dominates at these energies and the deviations would be very low contrast features. The fitted values of \( f_{\text{col}} \) at \( l = 70^\circ \) are summarized in the second column of Table 1. These best-fit values are weakly dependent on inclination, but they are always lower than the 1.7–1.9 values reported by ST95.

A direct comparison of our models with those of Merloni et al. (2000) is complicated by their assumption that a fraction \( f \) of the dissipation occurs in the corona. Inspection of their Figure 2 suggests that the value of \( f_{\text{col}} \) inferred from the spectral models of Merloni et al. is mostly determined by the corrected disk luminosity parameterized by \( (1 - f) \). Specifically, models with different \( f \) but the same \( (1 - f) \) yield similar values for \( f_{\text{col}} \). Their \( m \) accounts for the disk efficiency so this disk luminosity parameter is the rough equivalent of our \( l \). Merloni et al. (2000) also construct artificial spectra from their model SEDs using an RXTE response matrix. They fit these spectra with diskbb models and derive values of \( f_{\text{col}} \) by a method that is similar to, but differs in detail from, the procedure implemented by us in § 7 and in GD04. They find a trend toward increasing \( f_{\text{col}} \) with decreasing \( (1 - f) \). If we take the diskbb fits of § 7 and use their method for determining \( f_{\text{col}} \), we find the values listed in the fourth column of Table 1. (For completeness, we also show values appropriate for the XMM-Newton EPIC pn camera in the third column.) We find a weak increase in \( f_{\text{col}} \) with increasing \( l \) that is inconsistent with the results of Merloni et al. (2000). We suspect that this discrepancy is largely due to their neglect of bound-free metal opacity, which causes them to overestimate the ratio of scattering to absorption opacity. Therefore, \( \epsilon_b \), is lower and \( \tau_e \) is greater, providing a higher \( T_e^* \). The higher \( T_e^* \) and lower \( \epsilon_b \) would both give rise to harder, more modified spectra, consistent with larger values of \( f_{\text{col}} \). These effects are most significant in the low accretion rate models where Comptonization is negligible at most radii. Their assumption of constant density likely overestimates the density in the spectral formation regions in both gas pressured and radiation pressure–dominated annuli, increasing the ratio of absorption to scattering opacity at \( \tau_e^* \). This reduces the \( \gamma \)-parameter but may somewhat mitigate the increased spectral hardening that comes from neglecting metal opacity.

### 8.2. Stress Prescription and Dissipation Profile

For the range of \( l \), \( \alpha \), and \( \rho \) explored here, altering \( m_0 \) by varying \( \alpha \) or changing the stress prescription has little effect on the SED. Even changes in the vertical dissipation profile do not matter very much unless a substantial fraction of the heating occurs above the effective photosphere. It therefore appears that the spectra are insensitive to the details of the very uncertain vertical structure of disks, which might be encouraging from a theoretical point of view but is disappointing if one wants to use spectral observations to probe disk physics.

However, it is important to bear in mind how this robustness arises. The innermost parts of luminous black hole accretion disks are both radiation pressure supported and electron-scattering dominated. Provided that heat is vertically transported by radiative diffusion, then the scale height \( h \), which determines the surface gravity \( g \), is completely independent of the stress prescription. If there is little dissipation in the surface layers the radiative flux is constant, but the gravity still increases with height. As a result, gradients in gas pressure are always important in maintaining hydrostatic equilibrium near the surface, and this always produces a very steeply declining density profile there. This is in contrast to the temperature profile, which is relatively constant in the surface layers (cf. Fig. 9), and close to the effective temperature. The steep decline in density occurs over a gas pressure scale height \( H_g \sim P_g/(\rho g) \) (Hubeny 1990), which is independent of the details of the overall vertical structure, since it only depends on the surface temperature and the local gravity. The effective photosphere at characteristic frequencies is determined by

\[
\tau_{\text{eff}} = \int [3k^{\text{th}}(\kappa^{\text{es}} + \kappa^{\text{th}})]^{1/2} \rho \, dz = 1. \tag{9}
\]

Because \( \kappa^{\text{th}} \propto \rho \) and \( \kappa^{\text{es}} \propto \rho^0 \), this integral is approximately proportional to \( \rho^{3/2} H_g \). Hence, the density \( \rho \) at the effective photosphere is approximately independent of the details of the vertical structure. The column mass down to the effective photosphere \( m^* \propto \rho^0 H_g \) and the temperature at the effective photosphere \( T^* \propto T_{\text{eff}}(\kappa^{\text{th}} m^*)^{1/4} \) will also be approximately independent of the vertical structure. The resulting spectrum will therefore be quite robust. This all fails, however, if heat is transported by other means, such as convection, rather than radiative diffusion. It may also fail...
if substantial density inhomogeneities are present within the disk turbulence, as we discuss below.

The robustness of the SEDs to changes in the stress is likely to break down at higher Eddington ratios and spins, when the innermost disk annuli become increasingly effectively thin. We have already started to observe this effect for the $l = 0.3$ and $a = 0.998$ model. In these disks, the effective optical depth of the innermost annuli is reduced and the effective photosphere at all frequencies moves deeper into the disk atmosphere. Disks with high values of $\alpha$ or an $\alpha$-disk prescription will have lower densities at the depth of formation than disks with low values of $\alpha$ or a $\beta$-disk. The depth of formation lies even deeper in these less dense annuli, where the temperatures are hotter. Therefore, the resulting spectra from these less dense disks are harder than their denser counterparts.

The weak dependence of the model SEDs on the choice of stress prescription casts some doubt on the speculation of GDO4 that their luminosity-temperature relations are inconsistent with $\beta$-disk models. Their reasoning relies on the assumption that Comptonization is not important in $\beta$-disks because of the larger midplane densities inferred from a one-zone model (Nannurelli & Stella 1989). However, the effective photospheres in our models usually occur in the surface layers, where the densities are much less than the midplane density. Therefore, Comptonization is equally important (or unimportant depending on $l$) for both $\alpha$-disks and $\beta$-disks over most of the range of parameters we consider. Our model with the highest combination of Eddington ratio and spin is the only exception. In that case, the $\alpha$-disk is slightly hard compared to the corresponding $\beta$-disk but the deviation is still too small to be constrained by the current observations.

In an attempt to inject first-principles physics that is not present in the standard model, we also considered modifications to the vertical dissipation profile. The standard model dissipation profile produces a disk that is hydrodynamically unstable to convection in the radiation pressure–dominated regions (Bisnovatyi-Kogan & Blinnikov 1977; HBKA), although it is far from clear whether significant convective transport of heat would occur in the presence of MRI turbulence. Magnetic field buoyancy suggests that the dissipation may be more concentrated near the surface than in the standard model. An annulus with a modified dissipation profile motivated by simulations of accretion disk turbulence (Turner 2004) produces a vertical structure that is increasingly gas pressure supported in its interior and hydrodynamically stable to convection.

Adopting this modified dissipation profile only produces slight changes to the resulting spectrum, however. The reason is that the considerations of the first paragraph of this section still apply. The surface density profile is still very steep, and the physical properties of the plasma at the effective photosphere are largely independent of the dissipation profile, provided that most of the dissipation occurs deeper than the effective photosphere. The impact is not completely negligible, since the modified annulus SED is slightly harder than in the standard model, although the inferred $f_{\text{bol}}$ only differs by $\sim 10\%$. However, if we consider more top-heavy dissipation profiles in which a significant fraction of the dissipation occurs near or above the photosphere we can produce atmospheres with very different SEDs even though the surface radiative flux, gravity, and surface density are the same. The “corona”-producing annuli discussed in § 6.2 are extreme examples. These calculations may allow for a self-consistent treatment of the hard and soft spectral components in models in which the coronal emission is provided by flaring regions above the disk although there are several caveats to our treatment that remain to be addressed. These include the assumptions that the temperatures of the ions and electrons are equivalent, the electrons are entirely thermal, and the corona is time steady.

Most of the uncertainties in the vertical distribution and nature of the dissipation are relatively easy to address with the methods presented here. However, other potentially important aspects of the dynamic nature of these disks remain unaccounted for in our models. Simulations (e.g., Turner 2004) suggest the presence of substantial inhomogeneities in density due to the compressible nature of the turbulence (Turner et al. 2003) and radiatively driven instabilities (Gammie 1998). Modeling of the emergent spectrum from such structures requires three-dimensional radiative transfer. We have reported on Monte Carlo radiative transfer calculations on inhomogeneous structures created by simulations elsewhere (Davis et al. 2004). The temperature regime was more appropriate for a quasar disk than an X-ray binary, and thermal Comptonization was negligible. The main effect of the inhomogeneities was to reduce the modified blackbody effects and produce a softer, more thermal spectrum. The emergent flux through an inhomogeneous medium is also larger than the spatially averaged flux required to support the medium against gravity (Begelman 2001). This may reduce the vertical scale height of the disk compared to the standard model and make the emergent spectrum more sensitive to the stress prescription and dissipation profile. Because the turbulence is compressible and supersonic, bulk Comptonization can also significantly affect the emergent spectrum (Socrates et al. 2004). In addition to the effects of Silk damping, bulk Comptonization may also provide a radiative dissipation mechanism for the turbulence itself. It is also possible that the time-averaged spectrum emerging from a time-dependent, turbulent annulus differs from the spectrum emerging from a time-average of the vertical structure.

### 8.3. Observational Implications

Our luminosity-temperature relation agrees reasonably well with the results of GDO4. For example, between $l = 0.3$ and 0.1 we find weak evolution toward lower $f_{\text{bol}}$, consistent with that observed in XTE J1550–564. The larger decrease in $f_{\text{bol}}$ in our models from $l = 0.1$–0.03 is not clearly observed in their data, but it is not inconsistent because of large uncertainties in the fits at lower luminosities. The magnitudes of $f_{\text{bol}}$ derived using the procedure of GD04 depend on the inclination due to limb darkening. In this case, they are in approximate agreement with the observed values for $i = 70$, which is near the measured inclination of $73.5^{+1.9}_{-1.7}$ for XTE J1550–564 (Orosz et al. 2002). The values of $f_{\text{bol}}$ are probably also sensitive to our assumptions about inner torques, dissipation profiles, and metal abundances. Despite these concerns, we expect the generally weak trend toward decreasing $f_{\text{bol}}$ with decreasing $l$ to be robust because it is largely due to the increasing ratio of absorption to scattering opacity as the temperature of the disk drops. This reduces $\epsilon$, and moves $r_{\text{out}}$ closer to the photosphere, lowering $T_{\text{in}}$. The spectra soften and the associated $f_{\text{bol}}$ values decrease.

Although we employ the concept of a color correction throughout this paper for the purposes of comparison, we caution that isotropic, diluted blackbodies are not perfect approximations to our models. Given the high signal-to-noise ratio spectra afforded by an observation with XMM-Newton, the $\leq 20\%$ discrepancies between the best-fit, fully relativistic diluted blackbodies and the non-LTE atmosphere spectra are potentially significant. Any assumption of isotropy is clearly poor as limb darkening can alter the inferred luminosity of a source by up to 50% at high inclinations. We further caution that the diskbb model is an even
poorer match to our model SEDs. For high signal-to-noise ratio observations we do not expect it to provide adequate fits unless the disk is truncated at large radius. There are three effects of roughly equal importance that lead to differences between our model SEDs and the diskbb model. These are the deviations from blackbody shape in the local spectrum, the differences in the flux profile as a function of radius, and the effects of the relativistic transfer on the photon geodesics. The best-fit diskbb spectra are always significantly narrower than our models because of differences in the flux profiles and relativistic broadening. To accurately model all the emission from a disk component, one needs to include the relativistic effects on the disk structure and transfer function in the spectral model before fitting it to the observed SED (e.g., Ebisawa et al. 1991; Gierlinski et al. 2001). The data in Table 1 indicate that different but seemingly consistent methods of calculating $f_{\text{col}}$ with diskbb model fits give differing results. Furthermore, the results depend on the detector response and band.

These concerns are particularly relevant for higher signal-to-noise ratio observations that have inferred relativistically broadened Fe Kα lines but fit the disk component with an unbroadened diskbb model (e.g., Miller et al. 2004a, 2004b). In some cases the iron line profiles are so broad and asymmetric that they require a near maximally spinning black hole with a very steep iron line radial emissivity profile. Contrary to our expectations of 10%–15% discrepancies, the diskbb model seems to account surprisingly well for the presumed disk component in these observations. As these are very high state observations, our models may not be applicable because they do not capture the effects of irradiation of the disk surface by the hard component. Alternatively, it may be the case that this disagreement is due to their use of a power law to model the emission from the hard component. If the disk provides the seed photons for the corona, a power law will likely overestimate the flux of this component at the low-energy end of the band where we predict the discrepancies are largest. This extra emission could account for the difference between our model SEDs and the diskbb models. We test this hypothesis by constructing an artificial spectrum with our $l = 0.1$, $a = 0.998$, and $i = 45^\circ$ model SED, assuming it is located at 5 kpc and accounting for an intervening H column of $5 \times 10^{21}$ cm$^{-2}$. We also include a compTT component whose flux is $\sim$10% of the model flux. First, we fitted these data with a model that includes a diskbb component, a compTT component with the seed temperature fixed at the input value, and neutral absorption at the input H column. We get a poor fit with residuals of $\sim$15%. When we replace the compTT model with a power law, the residuals of the best fit are reduced to less than 5% and the quality of fit improves dramatically. It therefore seems plausible that the combinations of a diskbb and power law may effectively reproduce the SED of a real disk even though the diskbb model does not properly account for the accretion disk emission.

9. CONCLUSIONS

We have calculated several representative non-LTE, fully relativistic models for geometrically thin disks accreting onto a 10 $M_\odot$ black hole. We first explored the behavior of standard model (SS73; NT73) accretion disks. We found that the inclusion of metals with solar abundance has a significant impact on the model SEDs because of bound-free absorption opacity at high photon energies. The inclusion of this additional opacity generally decreases the importance of Comptonization, but still produces softer, more Wien-like spectra than would be expected from free-free opacity alone. The associated diluted blackbody color corrections are generally lower than found by previous authors (ST95; Merloni et al. 2000).

The frequency dependence of the disk integrated-model SEDs is qualitatively similar to that of spectra produced by assuming isotropic diluted blackbodies with constant color correction for the local emission, provided that these local spectra are folded through a relativistic transfer function. Quantitative differences still exist that should be discernible for some observations with modern X-ray observatories. The effects of limb darkening are significant and the assumption of isotropy of the local emission must be compensated by fitting the normalization as well as the color correction. The best-fit $f_{\text{col}}$ are weakly dependent on the black hole spin and accretion rate, generally increasing as either parameter increases.

We have found that the standard model SEDs are strongly affected by changes in accretion rate and black hole spin, but they are only weakly dependent on the value of $\alpha$ for the parameter space explored here. Modifying the stress prescription by replacing $P$ with $P_{\text{gas}}$ in equation (2) also has little effect on the model SEDs, contrary to the supposition of GD04. The reason for this is that the overall disk scale height in the inner, radiation pressure–supported regions is largely independent of the stress prescription, and so are the resulting physical conditions at the effective photosphere. Higher Eddington ratio models, particularly those around maximally spinning holes, may be more sensitive to the treatment of the stress. Steep density gradients generally exist in the surface layers, and modeling the disk annuli with constant density slabs is also a poor approximation.

We found that our model SEDs are generally not well matched by a simple diskbb model over the whole 0.3–10 keV band. Fits using the XMM-Newton EPIC pn response have discrepancies of greater than 10%. Such large discrepancies are generally not seen in diskbb fits to high signal-to-noise ratio observations of X-ray binaries with relativistically broadened Fe Kα lines. We speculate that the lack of residuals in these fits may be due to overestimation of the coronal emission at low energies. A resolution of this issue will require detailed fits of our models to real observations.

We used the method of GD04 to construct a luminosity-temperature relation for our Schwarzschild disk models. The luminosity scales roughly as the fourth power of the temperature, but we also found a trend toward larger $f_{\text{col}}$ with increasing $l$, which is generally consistent with their analysis for several sources. The agreement with their measured values of $f_{\text{col}}$ is somewhat more uncertain and our $f_{\text{col}}$ measurements depend on the inclination due to limb-darkening effects. The values of $f_{\text{col}}$ that we infer using the method of GD04 are higher than those we derive from fits with the relativistic diluted blackbody models (see Table 1).

We have also looked at modifications of the standard model on the basis of recent theoretical progress in understanding magnetohydrodynamic stresses in the flow. Adopting a vertical dissipation profile derived from numerical simulations produces a large change in the vertical disk structure but has a weaker effect on the SED, consistent with a $\sim$10% increase in the best-fit $f_{\text{col}}$. Profiles that have a greater fraction of the dissipation near the surface can produce much larger changes. Thus, our choice of dissipation profiles represents a large uncertainty in the models.

The addition of an inner torque at fixed fraction of the Eddington luminosity produces a significant hardening of the spectrum similar to that made by increasing the black hole spin. Unless the two types of models are distinguished by fits with detailed continuum models or relativistic line profiles, it may be difficult to infer either parameter without making an assumption
about the other. Additional uncertainties in the disk physics that we have not explored here are the possibility of bulk vertical transport of heat, e.g., convection and the effects of density inhomogeneities and bulk Comptonization in the supersonic disk turbulence.

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