ANTI-BARYON PUZZLE IN ULTRA-RELATIVISTIC HEAVY-ION COLLISIONS

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Abstract

The evolution of (non-strange) antibaryon abundances in the hadronic phase of central heavy-ion collisions is studied within a thermal equilibrium framework, based on the well-established picture of subsequent chemical and thermal freezeout. Due to large annihilation cross sections, antiprotons are, a priori, not expected to comply with this scheme. However, we show that a significant regeneration of their abundance occurs upon inclusion of the inverse reaction of multipion fusion, $n\pi \rightarrow p\bar{p}$ (with $n\pi=5-6$), necessary to ensure detailed balance. Especially at SPS energies, the build-up of large pion-chemical potentials between chemical and thermal freezeout reinforces this mechanism, rendering the $\bar{p}/p$ ratio in reasonable agreement with the observed one (reflecting chemical freezeout). Explicit solutions of the pertinent rate equation, which account for chemical off-equilibrium effects, corroborate this explanation.

1 Introduction

The combined experimental and theoretical study of (ultra-) relativistic heavy-ion collisions has lead to impressive progress in our understanding of the mechanisms underlying the formation and evolution of strongly interacting matter. A key issue concerns the question whether one is indeed producing systems of sufficient macroscopic extent with frequent enough re-interactions among the constituents that justify the use of (thermal-) equilibrium techniques. This, after all, constitutes an essential prerequisite for a meaningful investigation of the phase diagram of QCD in these reactions, including the identification of (the approach towards) phase transitions and Quark-Gluon Plasma (QGP) formation. A successful interpretation of experimental observables has to be based on a comprehensive overall picture of the collision dynamics. For SPS energies, this has been accomplished to a remarkable degree, as evidenced by, e.g., (i) hadro-chemical equilibration as inferred from produced particle ratios, (ii) thermal equilibration with collective transverse expansion, as inferred from transverse momentum spectra combined with two-particle correlation
measurements, (iii) a substantial excess of electromagnetic radiation (photons and dileptons) ascribed to thermal emission, (iv) the ‘anomalous’ suppression pattern of charmonium states. An important feature of this picture is the distinction between chemical and thermal freezeout, implying a significant duration of an expanding hadronic phase (also essential to explain (ii) and (iii)).

A solid understanding of the majority of observables then allows to go beyond and investigate well-defined deviations. In this talk we will focus on antiproton production, which, upon closer inspection, is not easily understood within the above picture, but, as we argue below, can be reconciled with it.

The paper is organized as follows: in Sect. 2 we briefly recall the essential elements of hadronic equilibration needed to formulate the ‘antiproton puzzle’. In Sect. 3 we address the problem within a thermal rate equation, first for the more transparent equilibrium case, then including off-equilibrium effects via its explicit solution. We summarize in Sect. 4.

2 Equilibration Times and Antiproton Puzzle

2.1 Two Types of Hadronic Freezeout

The notion of chemical and thermal freezeout stages in central collisions of heavy nuclei can be given a clear physical meaning (and applies equally well to hadronic and partonic degrees of freedom). It is based on a hierarchy of time scales related to the underlying cross sections.

Thermal equilibrium in the hadronic fireball can be (locally) maintained as long as elastic collisions are able to keep up with its expansion (dilution) rate. This criterion can be quantified as

\[ R(\tau) \equiv \int_{\tau}^{\infty} \frac{d\tau'}{\tau_{th}(\tau')} \leq R_{fo} \]

with \( R_{fo} \approx 1/2 \) (almost independent of collision energy from SIS/BEVALAC to SPS) as extracted from hydrodynamic fits to single-particle spectra [1, 2]. The thermal relaxation time for a hadron species \( i \) is given by

\[ (\tau_{th,i})^{-1} = \sum_{h} \left\langle \sigma_{ih} v_{rel} \right\rangle \rho_{h} \]

in terms of the density \( \rho_{h} \) of hadrons which serve as scattering centers, and the pertinent (thermally averaged) cross section \( \left\langle \sigma_{ih} \right\rangle \) (\( v_{rel} \): relative velocity between hadron \( i \) and \( h \)). Standard (resonance dominated) hadronic reactions, such as \( \pi\pi \rightarrow \rho \rightarrow \pi\pi, \pi K \rightarrow K^{*} \rightarrow \pi K, \pi N \rightarrow \Delta \rightarrow \pi N \) (or
even nonresonant NN scattering), lead to typical values around $\langle \sigma \rangle \simeq 50$ mb. Even for a rather dilute hadron gas of normal nuclear matter density ($\rho_h = \rho_0 = 0.16$ fm$^{-3}$) one finds a thermal equilibration time as short as 2 fm/c. A simplified freezeout criterion, which does not explicitly involve the expansion time scale, is given by the condition that the (thermally averaged) mean free path becomes equal to the system size, $\lambda_{MFP} \simeq r_{FB}$ (r$_{FB}$: fireball radius). Both criteria, when applied to central heavy-ion collisions at full SPS energy ($E_{lab} = 158$ AGeV), indicate thermal freezeout temperatures of about $T_{th} = 110 - 120$ MeV. This is consistent with combined experimental analysis of (light-) hadron $p_t$-spectra and HBT correlations [3], which can be schematically summarized by (nonrelativistic) slope parameters $T_{slope,i} = T_{th} + m_i v_t^2$ with an average transverse flow velocity $v_t \simeq 0.4-0.5$ c at thermal freezeout.

Chemical equilibrium is maintained by inelastic collisions which change the particle composition of the system. Corresponding hadronic cross sections, e.g., for $\pi\pi \leftrightarrow K\bar{K}, \rho\rho$, are typically much smaller than elastic ones, $\langle \sigma_{inel} \rangle \simeq 1$ mb after thermal averaging. Consequently, chemical equilibration times are around $\tau_{chem} \simeq 100$ fm/c, well above possible lifetimes of hadronic fireballs created in high-energy nuclear collisions. If anything, the chemical composition of the hadronic phase must frozen early in the evolution. At full SPS energy essentially all hadron abundances can be accommodated by a common chemical freezeout at $(T_{ch}, \mu_B^{ch}) \simeq (170, 260)$ MeV [4, 5]. This result is, in fact, difficult to understand in purely hadronic scenarios and has been interpreted as ‘prehadronic’ flavor equilibration, possibly in a QGP.

A particular consequence of the sequential freezeouts that will be important below is the effective conservation of particle numbers for species that are not subject to strong decays (e.g., $\pi, K, \eta$; after all, the abundances at thermal freezeout have to agree with the measured ones). In statistical mechanics language this is expressed by finite meson-chemical potentials, which, under SPS conditions, reach appreciable values of $\mu_{ch}^{\pi} \simeq 60-80$ MeV, $\mu_{ch}^{K} \simeq 100-130$ MeV (similar for $\mu_{ch}^{\eta}$), etc. Resonances that are regenerated via strong interactions acquire the chemical potentials of their decay products, i.e., $\mu_\Delta = \mu_N + \mu_\pi$, $\mu_\rho = 2\mu_\pi$, $\mu_{K^*} = \mu_K + \mu_\pi$, etc.\footnote{Strongly decaying resonances with narrow widths (\leq 50 MeV or so) are in between the two cases. However, for the vector mesons $\omega$ and $\phi$, e.g., it turns out that the two limiting cases – full chemical equilibrium ($\mu_\omega = 3\mu_\pi$ and $\mu_\phi = 2\mu_\pi$) versus chemical off-equilibrium (individual $\mu_\omega$ and $\mu_\phi$ adjusted to their conserved number) – give numerically very similar results. We also note that, if reactions of the type $\omega\pi \leftrightarrow \pi\pi$ are relevant, the $\omega$ chemical potential would be reduced.}
2.2 Antiproton Systematics

An enhanced production of antiprotons in central heavy-ion collisions (over the extrapolation from \( pp \) or \( pA \) reactions) has been among suggested signatures for QGP formation [4, 8] (which is expected to facilitate copious antiquark production). The experimental results for \( \bar{p}/p \) ratios at AGS [9], SPS [10, 11] and RHIC [12, 13] are, however, consistent with the chemical freezeout systematics at the respective energies [4, 5, 14]. In Pb\((158 \text{ AGeV})+\text{Pb} \), one finds [10] \( \bar{p}/p = (5.5 \pm 1) \% \), to be compared to \( \bar{p}/p = \exp(-2\mu_N^{ch}/T_{ch}) \simeq 5\% \). Moreover, the \( \bar{p}/p \) ratio does not exhibit a pronounced centrality dependence [11], which also borne out of hadro-chemical model fits [15].

Despite this apparent agreement it is nevertheless puzzling why the number of antiprotons should to be ‘frozen in’ early in the hadronic phase: large \( \bar{p}p \) annihilation cross section of around 50 mb at the relevant thermal energies (\( E_p^{th} = m_p + \frac{3}{2}kT \)), together with the sizable baryon densities under SPS conditions, seem to imply a chemical freezeout for antibaryons at a much lower temperature (i.e., later time) than \( T_{ch} \simeq 170 \text{ MeV} \); e.g., for \( T = 150 \text{ MeV} \), where according to thermal fireball calculations [10] \( \rho_B \simeq 0.75 \rho_0 \), one obtains for the chemical relaxation time,

\[
\tau_{\bar{p}}^{chem} = \frac{1}{\langle \sigma_{ann}^{pp}(s)v_{rel} \rangle \rho_B}, \tag{3}
\]

a value of about 3 fm/c, which is significantly smaller than the fireball lifetime. A naive evaluation of the \( \bar{p}/p \) ratio using thermal freezeout conditions, \( (T_{th}, \mu_N^{th}) \simeq (120, 400) \text{ MeV} \) [10], yields \( \exp(-2\mu_N^{th}/T_{th}) \simeq 0.1 \% \), a factor \( \sim 50 \)
below the measured value! This constitutes the ‘antiproton puzzle’.

There have been attempts to resolve this puzzle within sequential scattering (transport) simulations \[17, 18\]. The latter are, in principle, suited for a realistic description of the late (dilute) hadronic stages in heavy-ion collisions. Nevertheless, extra assumptions, such as a strong in-medium suppression (‘shielding’) of the annihilation cross section \[17\] or an enhanced production via an increased string tension in the early phases \[18\], had to be invoked to avoid or compensate annihilation losses in the hadron gas. However, a notorious problem within these approaches is the treatment of multi-body collisions (3 or more), which are usually neglected, thus violating detailed balance. For the case at hand, these are multi-meson fusion reactions, \( n \pi \rightarrow p\bar{p} \) with an average of \( n \pi \simeq 5-6 \) pions in the incoming channel. In the following we will show that the inclusion of these reactions, treated within a thermal rate equation, is indeed capable of regenerating a significant number of antiprotons \[6\]. The notion of detailed balance alone, however, is not enough. A second crucial ingredient \[6\] is the over-saturation of pion phase-space, encoded in nonzero pion-chemical potentials, which enhances the equilibrium abundance of antiprotons substantially (see also ref. \[19\] for a recent transport approach to the problem).

### 3 Thermal \( \bar{p} \)-Production

#### 3.1 Rate Equation and Chemical Potentials

In thermal equilibrium, the (net) rate per unit 4-volume for producing antiprotons via \( n_\pi \pi \)-pion fusion and its back-reaction, \( n_\pi \pi \leftrightarrow p\bar{p} \), is given in terms of the corresponding invariant matrix element \( \mathcal{M}_{n_\pi} \) as (see, \( e.g. \), ref. \[20\])

\[
R_{\bar{p}}^{th} = \int d^3k_p d^3k_{\bar{p}} \prod_{i=1}^{n_\pi} d^3k_i \delta^{(4)}(K_{\text{tot}}) |\mathcal{M}_{n_\pi}|^2 \left\{ z_p z_{\bar{p}} e^{-\frac{E_p + E_{\bar{p}}}{T}} - z_{n_\pi} e^{-\sum_{i=1}^{n_\pi} \omega_i} \right\} . \tag{4}
\]

The thermal factors are in Boltzmann approximation, which allows to factorize the fugacity factors \( z_i = \exp(\mu_i/T) \). The rate equation can then be written in the compact form

\[
\frac{d\rho_{\bar{p}}}{dt} \simeq \frac{\rho_{\bar{p}}}{\tau_{\bar{p}}^{\text{chem}}} \left( \frac{\langle z^n_\pi \rangle}{z_p z_{\bar{p}}} - 1 \right) , \tag{5}
\]

which represents a first order differential equation in the antiproton density \( \rho_{\bar{p}} \). The relaxation time \( \tau_{\bar{p}}^{\text{chem}} \) is given in eq. \[3\]: the use of the total baryon density \( \rho_B \) therein (rather than just the proton density \( \rho_p \), which is much smaller)
is based on the assumption that the annihilation cross section of antiprotons on other baryons is similar to the one on protons. Eq. (3) furthermore presumes that $\bar{p}$ annihilation on excited resonances yields (on average) as many additional pions as the resonance itself decays into (e.g., $\bar{p}$ annihilation on a $\Delta(1232)$ gives one pion more than on a nucleon). This leads to a cancellation of additional fugacity factors multiplying $\langle z_n^{\pi\pi} \rangle$ and $z_{\bar{p}}$ in eq. (5). The use of the total annihilation cross section in $\tau_{\text{chem}}^{\bar{p}}$ implies a sum over all possible $n_\pi$-pion final states ($s\bar{s}$ production is suppressed by the OZI rule). Again, this automatically incorporates mesonic resonance states such as $\rho$ and $\omega$ to the extent that they contribute with a pion-fugacity factor according to their pion final states. In a heavy-ion environment this is obviously satisfied for $\rho$ mesons, but to a good approximation also for $\omega$ (and even $\eta$) mesons, cf. footnote 1.

Under SPS conditions, where the relative amount of antiprotons in the system is small, baryon and pion densities are not significantly altered by the reactions under consideration. The rate equation can thus be considered with the antiproton density as its only variable. In equilibrium one has $R_{\bar{p}}^{\text{th}} = 0$, so that the antiproton fugacity can be determined as

$$z_{\bar{p}}^{\text{eq}} = z_p \langle z_n^{\pi\pi} \rangle.$$ (6)

Here, as in eq. (3), the averaging of $z_n^{\pi\pi}$ is over the initial (or final) pion numbers. Since the pion fugacities enter with large powers, a reasonable estimate of $z_{\bar{p}}$ requires a reliable description of the final state multiplicities in $p\bar{p}$ annihilation, which we will turn to in the following section.

### 3.2 Properties of $\bar{p}p$ Annihilation

Fig. 3 shows the experimental results for pion multiplicities in $p\bar{p}$ annihilation at rest (see, e.g., the review [21] and references therein). The distribution can be reasonably well reproduced by a Gaussian $P(n_\pi)$ with an average $\langle n_\pi \rangle = 5.06$ and a width $\delta n_\pi = 0.9$.

In a thermal heat bath, the annihilation processes occur, on average, at somewhat larger CMS energies, $\sqrt{s} \approx 2E_{\text{th}}^{\bar{p}}$. Experimental data on the $s$-dependence of $\langle n_\pi \rangle$ are displayed in Fig. 4. A linear increase accurately describes $\langle n_\pi \rangle(s)$ in the relevant (thermal) energy range. At the same time, also the width of the Gaussian moderately increases. Put together, the averaged multi-pion fugacity at given temperature takes the form

$$\langle z_n^{\pi\pi} \rangle(T) = \sum_{n_\pi=2}^{n_{\pi}^{\text{max}}} P(n_\pi) e^{n_\pi \mu_\pi/T},$$ (7)
where for any practical purposes $n_{\pi}^{max} = 9$.

For the treatment of off-equilibrium effects (cf. Sect. 3.4) one also needs to account for the energy-dependence of the total annihilation cross section, which figures into the equilibration time, eq. (3). Its decrease with increasing $s$ can be fitted by the empirical formula

$$\sigma_{\bar{p}p}^{ann}(s) = (40p_{lab}^{-0.5} + 24 p_{lab}^{-1.1}) \text{ mb}.$$
3.4 Off-Equilibrium Evolution

For an explicit solution of the rate equation describing the evolution of the antiproton number in an expanding hadron gas we rely on a hierarchy of time scales $\tau_{\text{therm}} < \tau_{\bar{p}}^{\text{chem}} \ll \tau_{\pi}^{\text{chem}}$. This implies an equilibrium $\bar{p}$ density

$$\rho_{\bar{p}}^{eq} = \langle \pi^{n=0} \rangle \int \frac{d^3k}{(2\pi)^3} \exp\left[- \frac{E_{\bar{p}} - \mu_{\bar{p}}}{T}\right].$$

(8)

The finite system size in a heavy-ion collision requires the inclusion of volume expansion effects. Using $dN_{\bar{p}} = \mathcal{R}_{\bar{p}}^{\text{th}} V_{FB}(t) \, dt$ where $N_{\bar{p}} = \rho_{\bar{p}} V_{FB}(t)$ the antiproton number and $V_{FB}(t)$ the expanding fireball volume, one obtains

$$\frac{d\rho_{\bar{p}}}{dt} = -\rho_{\bar{p}} \left( \frac{1}{\tau_{\bar{p}}(t)} + \frac{1}{V_{FB}(t)} \frac{dV_{FB}}{dt} \right) + \rho_{\bar{p}}^{eq}(t) \frac{1}{\tau_{\bar{p}}(t)} \equiv -\rho_{\bar{p}}(t)L(t) + G(t)$$

(9)

with gain and loss terms $G(t)$ and $L(t)$, respectively. Eq. (9) is an ordinary first order differential equation which has the solution

$$\rho_{\bar{p}}(t) = \varphi(t) \left( \rho_{\bar{p}}(t_0) + \int_{t_0}^{t} dt' G(t') / \varphi(t') \right), \quad \varphi = \exp\left[-\int_{t_0}^{t} dt' L(t')\right].$$

(10)

As shown in Fig. 4, the off-equilibrium results confirm the importance of both the pion-fugacities as well as the backward reaction of multi-pion fusion in
Figure 7: Solid lines: evolution of the $\bar{p}/p$ ratio according to the solution, eq. (10), of the rate eq. (9) for an expanding fireball using the free (left panel) and a 50% reduced (right panel) $\sigma_{\text{ann}}^{\bar{p}p}$; long-dashed lines: equilibrium ratio; short-dashed lines: solution when switching off the gain term $G(t)$.

maintaining an antiproton abundance close to the observed value. The effective freezeout temperature $T_{\text{chem}}^{\bar{p}}$ turns out to be close to 130 MeV (slightly higher if one assumes a, e.g., 50% smaller in-medium annihilation cross section).

4 Summary

We have shown that antiproton production in central heavy-ion collisions at SPS energies can be reconciled with the observed value corresponding to standard chemical freezeout: annihilation losses in the subsequent hadron gas phase are compensated by the back-reaction of multipion fusion, reinforced by the build-up of large pion-chemical potentials. We have corroborated our earlier findings for the equilibrium case by an explicit solution of the underlying rate equation. An interesting test of the regeneration mechanism could be provided by the recently suggested balance function technique [22], which has significant sensitivity to the time of (pair) production especially for heavy particles (such as nucleons). The off-equilibrium framework is more involved under RHIC conditions, as $\rho_B$ is of the same order as $\rho_{\bar{B}}$, so that feedback effects on the baryon densities cannot be neglected. This will be addressed in future work, as it promises valuable insights into hadro-production at collider energies.

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References

[1] I. Mishustin and L. Satarov, Sov. J. Nucl. Phys. 37 (1983) 532.
[2] C.M. Hung and E.V. Shuryak, Phys. Rev. C57 (1998) 1891.
[3] R. Stock, Nucl. Phys. A661 (1999) 282c.
[4] P. Braun-Munzinger, I. Heppe and J. Stachel, Phys. Lett. B465 (1999) 15.
[5] F. Becattini, J. Cleymans, A. Keranen, E. Suhonen and K. Redlich, LANL e-print hep-ph/0011322.
[6] R. Rapp and E.V. Shuryak, Phys. Rev. Lett. 86 (2001) 2980; Nucl. Phys. A698 (2002) 587c.
[7] U. Heinz, P.R. Subramanian, H. Stöcker and W. Greiner, J. Phys. G12 (1986) 1237.
[8] P. Koch, B. Müller, H. Stöcker and W. Greiner, Mod. Phys. Lett. A3 (1988) 737.
[9] D.J. Hofman for the E917 Collaboration, Nucl. Phys. A661 (1999) 75c.
[10] M. Kaneta for the NA44 Collaboration, Nucl. Phys. 638 (1998) 419c.
[11] G.I. Veres for the NA49 Collaboration, Nucl. Phys. 661 (1999) 383c.
[12] STAR Collaboration (C. Adler et al.), Phys. Rev. Lett. 86 (2001) 4778.
[13] PHENIX Collaboration (K. Adcox et al.), LANL e-print nucl-ex/0112006.
[14] P. Braun-Munzinger, J. Stachel, J.P. Wessels and N. Xu, Phys. Lett. B344 (1995) 43.
[15] J. Cleymans, B. Kämpfer and S. Wheaton, Phys. Rev. C65 (2002) 027901; B. Kämpfer, these proceedings.
[16] R. Rapp and J. Wambach, Eur. Phys. J. A6 (1999) 415.
[17] Y. Pang, D.E. Kahana, S.H. Kahana and H. Crawford, Phys. Rev. Lett. 78 (1997) 3418.
[18] M. Bleicher et al., Phys. Lett. B485 (2000) 133.
[19] W. Cassing, LANL e-print nucl-th/0105069.
[20] P. Koch, B. Müller and J. Rafelski, Phys. Rep. 142 (1986) 167.
[21] C.B. Dover, T. Gutsche, M. Maruyama and A. Faessler, Prog. Part. Nucl. Phys. 29 (1992) 87.
[22] S. Bass, P. Daneliewicz and S. Pratt, Phys. Rev. Lett. 85 (2000) 2689.