Perfect Reflection of Light by an Oscillating Dipole

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We show theoretically that a directional dipole wave can be perfectly reflected by a single point-like oscillating dipole. Furthermore, we find that in the case of a strongly focused plane wave up to 85% of the incident light can be reflected by the dipole. Our results hold for the full spectrum of the electromagnetic interactions and have immediate implications for achieving strong coupling between a single propagating photon and a single quantum emitter.

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Common treatments of light-matter interaction consider the excitation light to consist of a homogeneous field of area $A$, and often use the concept of a cross section $\sigma$ to arrive at the probability $\sigma/A$ for exciting an atom. In conventional spectroscopy experiments, this ratio is very small because either $\sigma$ is reduced by various broadening effects, or $A$ is large for technical reasons. However, recent experiments have shown that it is possible to overcome these difficulties for the optical excitation of single molecules, quantum dots, or atoms [1, 2, 3, 4, 5]. The intriguing question that arises is whether the experimentally observed coupling efficiencies are close or far from theoretical limits. In particular, is it possible to excite an atom with probability equal to one by a single photon [6]? Is it possible for an atom to imprint a large phase shift on a photon that passes by?

On the theoretical side, the interaction of freely propagating photons with the dipolar transition of a two-level system (TLS) has been addressed for a quasi-one-dimensional case with emphasis on the quantum statistics of the incident light [7]. In three dimensions, methods of expansion of the focused beam in terms of vectorial mode functions and decomposition of the focused beam on resonance (\(\Delta = 0\)) into plane waves and displays an attenuation of $T \approx 80$% at resonance for $\alpha = \beta = \pi/3$. The solid curve shows that a directional dipolar wave can be completely attenuated for $\alpha = \beta = \pi/2$.

The classical interaction of light with an oscillating point-like dipole located at the origin $O$ is described by the Abraham-Lorentz equation [13]. After calculating the differential scattering cross section, one can arrive at the total scattered power $\frac{1}{2} \epsilon_0 \int_{4\pi} r^2 |E_{sca}(r)|^2 \, d\Omega = 2 c W_{sca}(O)\sigma$, (1)

where $E_{sca}$ is the field scattered by the dipole, and the distance $r$ lies in the far field, $kr \gg 1$. $W_{sca}(O) = \epsilon_0 |E_{inc}(O)|^2/4$ is the time-averaged electric energy density at $O$. The parameter

$$\sigma = \sigma_0 \frac{\Gamma^2}{2(\Delta^2 + \Gamma^2)} ,$$

denotes the total scattering cross section of the oscillator where $\Gamma$ is the damping rate dictated by radiation reaction, and $\Delta = \omega_L - \omega_0$ is the detuning between the incident light and oscillator frequencies $\omega_L$ and $\omega_0$, respectively. The quantity $\sigma_0 = 3\lambda^2/(2\pi)$ denotes the cross section on resonance (\(\Delta = 0\)). We now consider the scattering ratio [15],

$$K_0 = \frac{P_{sca}}{P_{inc}} = \frac{2 c W_{sca}^l(O)\sigma_0}{\int \Delta S(r) \cdot n \, d^2r} = \frac{\sigma_0}{A} ,$$

FIG. 1: a) Incident light propagating along the z-axis is focused with a spherical phase front onto a dipole placed in vacuum. GRS: Gaussian reference sphere, $\alpha$: entrance-aperture radius, $\beta$: collection half angle, $f$: focal length. b) The dashed curve plots the transmittance for the totally scattered power $W_{sca}$ of the focused plane wave and displays an attenuation of $T \approx 80$% for $\alpha = \beta = \pi/3$. The solid curve shows that a directional dipolar wave can be completely attenuated for $\alpha = \beta = \pi/2$. The classical interaction of light with an oscillating point-like dipole located at the origin $O$ is described by the Abraham-Lorentz equation [13]. After calculating the differential scattering cross section, one can arrive at the total scattered power $\frac{1}{2} \epsilon_0 \int_{4\pi} r^2 |E_{sca}(r)|^2 \, d\Omega = 2 c W_{sca}(O)\sigma$, (1)

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at resonance. Here $P_{\text{inc}}$ is the incident power, $\mathbf{S}$ is the time-averaged Poynting vector of the incident field, and $\mathbf{n}$ is a unit vector normal to the integration surface. The integration can be taken over a plane at the incident aperture, over the Gaussian reference sphere (GRS), or over the focal plane.

The derivation of Eqs. (11) and (12) is based on the fact that the oscillator interacts only with the electric field at the location of the oscillator, irrespective of whether the field is homogeneous as for a plane wave, or inhomogeneous as in the focal region of a strongly focused beam [10]. Thus, $\sigma$ can be treated as a universal quantity for a point-like oscillator regardless of the modal properties of the excitation light. The quantity $\mathcal{A}$ introduced in Eq. (1) represents an effective focal area and depends implicitly on $\lambda$ through the diffraction phenomenon. It is closely related to the normalized energy density $W_{\text{inc}}^\text{el}/P_{\text{inc}}$, which has been studied for various focal systems [17, 18, 19]. The peculiarities of the incident field enter $\mathcal{K}_0$ via $\mathcal{A}$. Consequently, as we will show below, the problem of minimal transmittance is shifted to that of a minimal $\mathcal{A}$, and a strong photon-oscillator interaction is reachable for $\mathcal{K}_0 \gtrsim 1$ [11].

We first consider an incident $x$-polarized plane wave of amplitude $E_0$. The integration in Eq. (3) over the incident aperture is straightforward and yields $P_{\text{inc}} = \frac{1}{2} \varepsilon_0 E_0^2 \pi a^2 [12]$, where $a$ is the radius of the entrance aperture. We also have $W_{\text{inc}}^\text{el}(O) = \varepsilon_0 \pi f E_0 |I_0(O)| / (2\lambda)^2$ where $f$ is the focal length of the focusing system and $I_0(O)$ is a diffraction integral [13, 14]. The resulting value of $\mathcal{A}$ then yields

$$\mathcal{K}_0 = \frac{128}{75} \frac{1}{\sin^2 \alpha} \left( 1 - \frac{1}{8} (5 + 3 \cos \alpha) \cos^{3/2} \alpha \right)^2, \quad (4)$$

where $\alpha$ specifies the incident solid angle $\Omega_\alpha$ (see Fig. 1a). For $\alpha = \pi/2$, $\mathcal{K}_0$ reaches the maximum value of $128/75 \simeq 1.7$. Assuming a backward and forward half space and accounting that half of the power is scattered in each direction, it follows that up to $85\%$ of the incident light is reflected into the backward half space. For this configuration the reflectance and transmittance are thus limited to $\mathcal{R} \lesssim 0.85$ and $\mathcal{T} = 1 - \mathcal{R} \gtrsim 0.15$, respectively.

An alternative way of performing the integration in Eq. (3) is to consider the FP. Because the intensity has cylindrical symmetry about the optical axis, the electric and the magnetic energy densities are equal at the focal spot [10] so that $2eW_{\text{inc}}^\text{el}(O) = S_z(O)$. The calculation of $\mathcal{A}$ then becomes

$$\mathcal{A} = \int_{\mathcal{FP}} S_z d^2 r = \int_{\mathcal{FP}} \left( |I_0|^2 - |I_2|^2 \right) d^2 r / |I_0(O)|^2, \quad (5)$$

where $I_2$ is again a diffraction integral [16]. The integration in the numerator turns out to be straightforward when an orthogonality relationship for Bessel functions is considered [15]. We note that the fields in a strongly focused beam show vortices in the FP [20] so that $S_z$ takes on positive and negative values as shown in Fig. 2a [17]. Thus, in general $S_z(r)$ cannot be substituted by $2eW_{\text{inc}}^\text{el}(r)$, which is a positive quantity. We remark in passing that Ref. [8] predicts a much lower value than $1.7$ for a quantity equivalent to our parameter $\mathcal{K}_0$. We believe one of the origins of this discrepancy is that Ref. [8] takes the integrand in the definition of $\mathcal{A}$ to be $W_{\text{inc}}^\text{el}(r)$.

In order to derive an upper limit of $\mathcal{K}_0$ for the general class of transverse axially symmetric systems, we consider the field produced by the combination of an electric and a magnetic dipole which has been suggested for optimal focusing [19, 21]. To emulate such a field, one considers the emission field patterns at the GRS of virtual electric and magnetic dipoles orthogonal to each other and placed at $O$ and then reverses the field propagation. Using Eq. (3) for the calculation of $\mathcal{A}$ we obtain [15]

$$\mathcal{K}_0 = \frac{1}{4} (7 - 3 \cos \alpha - 3 \cos^2 \alpha - \cos^3 \alpha) \quad (6)$$

At $\alpha = \pi/2$, $\mathcal{K}_0 = 7/4$ establishes the maximum value for transverse axially symmetric systems. This is only slightly larger than $128/75$ obtained for the plane wave.

We next abandon the restriction of axial symmetry and search for an upper limit of $\mathcal{K}_0$ in general. Guided by a mode matching argument [6, 8], we consider a directional dipolar incident wave. In this case the incident field stems from the emission pattern at the GRS of a virtual dipole parallel to the $x$-axis and placed at the origin [22]. Following Eq. (3), we obtain [15]

$$\mathcal{K}_0 = \frac{1}{2} (4 - 3 \cos \alpha - \cos^3 \alpha) \quad (7)$$

We remark that $\mathcal{A}$ deduced from Eqs. (3)-(7) is equivalent to the corresponding expression for the normalized energy density in Ref. [17, 18]. At $\alpha = \pi/2$, $\mathcal{A}$ reaches its minimum value of $\mathcal{A} = \sigma_0/2$ and $\mathcal{K}_0$ its ultimate maximum value of $2$, respectively. This value is consistent with the limit $W_{\text{inc}}^\text{el}(O)/P_{\text{inc}} \leq k^2/(3\pi c)$ given by Bassett for the sum $W_{\text{inc}}^\text{el}$ of the time-averaged electric and magnetic energy densities at the focal spot [22]. As a last case study we consider the interaction of an oscillating dipole oriented along the $z$-axis with a radially polarized dipolar incident field obtained from the radiation of a virtual dipole oriented along the $z$-axis and located at $O$ [19]. Here too, we find that $\mathcal{K}_0$ reaches the maximum value of $2$ at $\alpha = \pi/2$.

Fig. 2b displays $\mathcal{K}_0$ as a function of $\alpha$ for various illuminations considered above. In all cases, $\mathcal{K}_0 \gtrsim 1$ is met for realistic numerical apertures. We are, thus, facing
the paradoxical seeming situation that the power emitted by the oscillator may be larger than the incident power. However, this finding does not violate the law of power conservation because there is destructive interference in the forward direction. We analyze this interference by determining now the incident and scattered fields at the GRS for $z > 0$. A particularly insightful approach is to expand an arbitrary excitation field in terms of vectorial multipoles [24, 25, 26]. All multipoles become zero at the origin except the electric dipole mode, which for a transverse system reads [24]

$$N_{e11} = \begin{cases} \frac{2}{\beta} \hat{e}_x, \\ (\cos \vartheta \cos \varphi \hat{e}_\vartheta - \sin \varphi \hat{e}_\varphi) \frac{e^{i(kr-\pi/2)}}{kr}, \quad kr \gg 1. \end{cases}$$

We note that here the field for $kr \gg 1$ is given only for the outgoing wave. The electric dipole-wave component $\Psi$ of the excitation field can be written as

$$\Psi(r) = \frac{E_{\text{inc}}(O)}{|N_{e11}(O)|} N_{e11}(r),$$

where $E_{\text{inc}}(O)$ is taken from the Debye diffraction approach [13]. The field scattered by the oscillator also forms a dipole wave [14]

$$E_{\text{sca}}(r) = -\frac{3\Gamma E_{\text{inc}}(O)}{2(2\Delta + i\Gamma)} \frac{e^{ikr}}{kr} \left[\hat{e}_x - (\hat{e}_x \cdot \hat{r}) \hat{r}\right],$$

where $\hat{r}$ is the unit vector along $r$, and the polarization of $E_{\text{inc}}(O)$ is along the $x$-axis. At resonance one finds $E_{\text{sca}} = -\Psi$ for $kr \gg 1, z > 0$. Therefore, the dipole wave component of the excitation field is completely reflected just as in the reflection of a collimated beam from a perfect metal. The $\pi$ phase shift of $E_{\text{sca}}$ with respect to $E_{\text{inc}}$ results from the sum of two effects. First, the comparison of Eqs. (8) and (10) reveals a relative Gouy phase shift of $-\pi/2$ [27]. Second, the denominator of the Lorentzian term in Eq. (10) yields a phase shift of $\pi/2$ for $\Delta = 0$, as is common for an oscillator driven at resonance.

This approach allows for an easy calculation of the transmittance $T$ as a function of the angles $\alpha$ and $\beta$. For a focused incident plane wave, we find [15]

$$T(\alpha, \beta) = 1 + \frac{3\Gamma_0(\alpha)}{2\sin^2 \gamma} \left[ X(\beta) \mathcal{I}_0(\alpha) - \mathcal{I}_0(\gamma) \right],$$

where $\Omega_\alpha$ and $\Omega_\beta$ are the incident and collection solid angles, respectively. The numerical data in Fig. 3 display a rapid decrease of $T$ with increasing $\alpha$, while the dependence on $\beta$ is less pronounced. Of particular experimental relevance is the geometrical shadow boundary where $\beta = \alpha$, i.e. one collects all the incident light. Along this line the transmittance experiences a minimum of $T \approx 0.1$ at $\alpha \approx 0.43\pi$. The transmittance is plotted as a function of detuning in Fig. 1b for $\alpha = \beta = \pi/3$. We point out that more complicated expressions are expected if the dipole is displaced from the focal spot. Particularly, the phase fronts of the scattered and excitation fields no longer match at the GRS. The different phase fronts can give rise to dispersive shapes of the transmission spectra depending on the position of the detector at the GRS.

In this work, we have shown that a classical point-like oscillating dipole can undergo a strong coupling with a
confined incident beam, reaching a 100 % efficiency when the illumination consists of a directional dipolar field. In fact, in the limit of weak excitation many essential features of light-matter interaction are shared by the quantum electrodynamic and classical formalisms alike [28]. A central underlying reason for this phenomenon is that both treatments use the same spatial description of the electromagnetic field. To this end, our classical results can be readily extended to the interaction of light with a two-level system. The scattering cross section of a quantum mechanical TLS is known to be [29],

\[ \sigma_{TLS} = \sigma_0 \frac{\Gamma^2}{4 \Delta^2 + \Gamma^2 + 2 \gamma^2} , \]  
(12)

where \( \sigma_0 \) is the same quantity as in Eq. [2], \( \Gamma \) stands for the spontaneous emission rate of the upper level, \( \mathcal{V} = -d_{12} \cdot \mathbf{E}_{inc}(O)/\hbar \) is the Rabi frequency, and \( d_{12} \) denotes the vectorial transition dipole moment. In a semiclassical treatment, the coherently scattered field by an atom is [29]

\[ \mathbf{E}_{sca}^{coh} = \frac{-3\Gamma (\Delta - i\Gamma/2) \mathbf{E}_{inc}(O) e^{i k r} \hat{\mathbf{r}}}{4 \Delta^2 + \Gamma^2 + 2 \gamma^2} (\hat{\mathbf{e}}_z - (\hat{\mathbf{e}}_z \cdot \hat{\mathbf{r}}) \hat{\mathbf{r}}) . \]  
(13)

At weak excitation, \( |\mathcal{V}| \ll \Gamma \), Eqs. (12) and (13) become equivalent to Eqs. (2) and (10). Therefore, the results obtained for the classical oscillator also hold for a TLS. We thus conclude that a directional dipole wave can be perfectly reflected from a TLS under weak excitation. However, in the saturation regime, \( |\mathcal{V}| \gg \Gamma \), \( \sigma_{TLS} \) decreases with increasing excitation.

Considering a quantized field, we are led to conclude that a few or even single photon pulses can be fully reflected by a single TLS if the coherence time of the photon is sufficiently long compared to the excited state lifetime [11]. The modal formalism developed in this Letter can be extended in the context of QED to analyze such phenomena and will be the subject of a future study. 

In conclusion, we have shown that a single point-like oscillating dipole can fully reflect an incident light field. For the experimentally important case of a focused plane wave we have found that the transmission can be attenuated by up to 85%. Our findings readily hold for the whole electromagnetic spectrum and we expect interesting applications in the detection and spectroscopy of subwavelength objects in the infrared to radiowave domains. In the optical range, we anticipate that a strong coupling between a single photon and a single quantum system can be realized in a directional focal system without the need for high finesse cavities. Such an arrangement would open new doors for quantum information processing using photons as information carriers.

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