Probing the boundary of phase transition of nuclear matter using proton flows in heavy-ion collisions at 2-8 GeV/nucleon

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Based on a relativistic transport model ART, nuclear many-body approach and the MIT bag model, properties of phase transition of dense nuclear matter formed in relativistic heavy-ion collisions are reinvestigated. Proton average transverse momentum and directed flow are calculated with different equation of states in Au + Au collisions at beam energies of 2, 4, 6 and 8 GeV/nucleon. Compared with AGS experiment data in existence, the boundary of the first-order phase transition is roughly confined to be 2.5-4 times saturation density with temperature about 64-94 MeV. This constraints may be considered as a cross verification of searching for landmarks of the QCD matter phase diagram at ongoing RHIC Beam Energy Scan-II program.

I. INTRODUCTION

Studying phase transition from nuclear matter to quark-gluon plasma (QGP) is a main purpose of heavy ion collisions at relativistic energies \cite{1-3}. Phase transition is a smooth crossover at finite temperature and small baryon chemical potential according to lattice QCD calculation \cite{4-6}. At large chemical potential, it changes to the first-order phase transition, and the point of change is named as critical point \cite{7-10}. It is important to understand the phase structure of QCD which showing the critical point and phase boundary in the QCD phase diagram. However, the exact value of the transition density from hadronic matter to quark matter is still a matter of long-standing debate in both nuclear physics and astrophysics \cite{11-17}. Heavy ion collisions at relativistic energies are the only practical method to study the QCD phase transition on earth. Both experimental measurements and transport model calculations demonstrate that at alternating-gradient synchrotron (AGS) energies, heavy-ion collisions can form hot and dense matter with density beyond 3\textit{\rho}_0, and temperature above 50 MeV \cite{18-20}. There is a large variety of theory calculations and experimental observations dedicating to finding the signal of phase transition \cite{21-32}. Unfortunately, up to now, the critical point and the phase-transition boundary are still not clearly demonstrated.

As a result of the phase transition, the Equation of State (EoS) of nuclear matter is soften when changing from hadronic matter to quark matter. Directed nucleon flow caused by the directed increasing pressure in semi-central heavy-ion collisions thus should be affected the softened EoS. In this study, to search for the phase-transition boundary, we reinvestigate the mean transverse momentum and direct flow of proton in Au + Au collisions at beam energies of 2, 4, 6 and 8 GeV/nucleon in the framework of A Relativistic Transport (ART) model \cite{18, 19} by varying the EoS of hadronic and quark matter. The first-order phase transition of nuclear matter in Au+Au heavy-ion collisions at 2-8 GeV/nucleon beam energy is deduced to occur at 2.5-4 times saturation density with temperature about 64-94 MeV. The obtained boundary of phase transition here can be considered as cross-checking with that from the beam energy scan phase II (BES-II) program \cite{15}.

II. MODEL AND METHOD DESCRIPTIONS

The relativistic transport model ART \cite{18, 19} stems from the original Boltzmann-Uehling-Uhlenbeck (BUU) model \cite{34, 35}, but includes more baryons and meson resonances as well as their interactions. More detailedly, the following baryons \textit{N}, \textit{\Delta}(1232), \textit{N}*(1440), \textit{N}*(1535), \textit{\Lambda}, \textit{\Sigma}, and mesons \textit{\pi}, \textit{\rho}, \omega, \eta, \textit{K} with their explicit isospin degrees of freedom are included. The model has been successfully used in studying many features of heavy-ion reactions at AGS energies up to a beam momentum of about 15 GeV/c \cite{19}. An extended version of ART is now used as a hadronic afterburner in the AMPT (A Multi-Phase Transport Model) model for heavy-ion collisions mainly at RHIC and LHC energies \cite{36}.

It is undeniable that the single nucleon potential at high energies (above 1 GeV kinetic energy) and high densities (above 3 times saturation density) is still unknown to date \cite{37}. The density and momentum-dependent single nucleon potential given by Danielewicz in Ref. \cite{38} is effective only at low energies (below 1 GeV kinetic energy) and low densities (below 2-3 times saturation density). The experimentally confirmed Hama potential \cite{39} at saturation density does not evidently affects the directed flow originating from high baryon densities in heavy-ion collisions at several GeV beam energies. To make minimum assumption, the single nucleon mean-field potential at hadron phase (HP) uses the density-
tained by fitting the ground state properties of nuclear density [41–43]. Energy per nucleon of nuclear matter is given with $B= 150$ and 90 MeV/fm$^3$ with first-order (a) and crossover phase (b) transitions are shown in Figure 1 stand for the EoSs of first-order and crossover phase transitions, respectively. In the ART model, the EoSs of nuclear matter with first-order and crossover phase transitions are obtained via variable $\alpha$, $\beta$, $\gamma$ parameters in Eq. (1) by fitting the EoSs of hadronic matter at low densities and quark matter at high densities. Since the chance of the crossover of phase transition occurs at AGS energies with high baryochemical potential and low temperature is rare [13], in the following studies, to probe the signal of phase transition, besides the EoS with phase transition crossover, we frequently compare the results with hadronic matter EoS to those with first-order phase transition.

Dependent form [40], i.e.,

$$U(\rho) = \alpha \frac{\rho}{\rho_0} + \beta \left(\frac{\rho}{\rho_0}\right)^\gamma,$$

where $\rho_0$ stands for saturation density. The parameter values $\alpha = -232$ MeV, $\beta = 179$ MeV and $\gamma = 1.3$ are obtained by fitting the ground state properties of nuclear matter, i.e., the binding energy $E_0 = -16$ MeV, pressure $P_0 = 0$ MeV/fm$^3$ and the most probable incompressibility of symmetric nuclear matter $K_0 = 230$ MeV at saturation density. Energy per nucleon of nuclear matter is thus expressed as

$$E(\rho) = \frac{8\pi}{5m_{\pi}^3} \rho^{5/3} + \frac{\alpha}{2} \frac{\rho}{\rho_0} + \frac{\beta \rho}{1 + \gamma \left(\frac{\rho}{\rho_0}\right)^\gamma}.$$ (2)

The pressure $P_H$ in hadron sector is given by

$$P_H = \rho^2 \frac{\partial E}{\partial \rho} = \frac{16\pi}{15m_{\pi}^3} \rho^{5/3} + \frac{\alpha}{2} \frac{\rho}{\rho_0} + \frac{\beta \rho}{1 + \gamma \left(\frac{\rho}{\rho_0}\right)^\gamma}.$$ (3)

We describe the properties of quark matter with the MIT bag model, and consider massless $u$ and $d$ quarks [44,45]. The pressure $P_Q$ in the quark sector at zero temperature reads

$$P_Q = \frac{3}{4} \frac{\pi}{15} \rho^{4/3} - B,$$ (4)

where $B$ is the bag constant, $\alpha_s = 0.1$ is the QCD coupling constant [19,45,46].

Figure 4 shows pressure as a function of baryon density with different phases. The solid lines denote the pressure $P_H$ at hadron phase (HP) with $K_0 = 230$ MeV, and the dash lines denote the pressure $P_Q$ at quark phase (QP) with $B = 150$ MeV/fm$^3$ (panels (a)) and $B = 90$ MeV/fm$^3$ (panel (b)), respectively. We use isobaric phase transition EoS at 2.5-4 times saturation density representing the *first-order* phase transition to connect hadron phase with quark phase (panels (a)), and continuously smooth change of EoS to represent the phase transition *crossover* (panel (b)). The dash dotted lines shown in Figure 4 stand for the EoSs of first-order and crossover phase transitions, respectively. In the ART model, the EoSs of nuclear matter with first-order and crossover phase transitions are obtained via variable $\alpha$, $\beta$, $\gamma$ parameters in Eq. (1) by fitting the EoSs of hadronic matter at low densities and quark matter at high densities. Since the chance of the crossover of phase transition occurs at AGS energies with high baryochemical potential and low temperature is rare, in the following studies, to probe the signal of phase transition, besides the EoS with phase transition crossover, we frequently compare the results with hadronic matter EoS to those with first-order phase transition.

### III. RESULTS AND DISCUSSIONS

Transverse flow as a probe of nuclear matter EoS is derived from anisotropic pressure gradient with finite impact parameters at high energies heavy ion collisions [47]. Isobaric phase transition can lead to a soft region in the nuclear EoS, which could generate a minimum in the pressure-driven sideward flow. Thus, the change of the strength of sideward flow may be a signal of the first-order phase transition. The collective sideward flow has already been measured by E895 experimental group at Brookhaven in Au + Au collisions at beam energies of 2, 4, 6, and 8 GeV/nucleon [22]. In such collisions, as shown in Figure 2, the maximum densities reached are approximately 3-5 times saturation density, thus providing the ideal condition to survey the phase transition of dense nuclear matter.

Figure 5 shows the proton average transverse momen-
Figure 3 shows proton directed flows given by the ART model with different EoSs, together with the E895 experimental data [23]. It is first seen from the proton sideward flow experimental data that, as increase of the incident beam energy, the slope of proton flow at mid-rapidity does not enhance at all, perhaps indicating occurrence of phase transition in dense nuclear matter. Theoretically, at such beam energy range, as increase of incident beam energy, compression density reached in heavy-ion collisions is also enhanced (as shown in Figure 2), the enhanced compression would cause a large pressure gradient. Therefore the slope of proton flow should be elevated if the length of interaction time is the same. However, one could not judge which is the main factor, the enhanced compression or the shortened interaction time as increase of beam energy. Therefore, one must carry out theoretical simulations based on transport model with different EoSs. Shown in the four panels of Figure 2 one can see that, as increase of incident beam energy, results of proton sideward flows given by the ART with hadronic EoS gradually deviate from experimental data while results with first-order phase transition EoS are well in agreements with experimental data. Figure 4 indicates the occurrence of first-order phase transition in the compression nuclear matter from saturation to 5.5 time saturation density in heavy-ion collisions at 2-8 GeV/nucleon.

The proton directed flow $v_1$ is the first Fourier coefficient of nucleon azimuthal distribution [24], i.e.,

$$v_1 = \langle \cos \phi \rangle = \frac{\langle p_x \rangle}{p_T}.$$  \hspace{1cm} (5)

Figure 4 shows proton directed flows given by the ART model with different EoSs. One sees that results with hadronic matter EoS and crossover phase transition EoS evidently deviate from experimental data while the results with first-order phase transition fit the data quite well. Compared with experimental data, the deviation of results with hadronic matter EoS and crossover phase transition EoS becomes larger as increase of incident beam energies, indicating larger probability of occurrence of first-order phase transition at compression densities above 2.5 times saturation density. The resulting proton directed flows with hadronic matter EoS and crossover phase transition EoS are quite similar simply because pressures from the two EoSs in the compression density range 3-5.5 times saturation density are quite close to each other (as shown in Figure 4).

While shifting the starting density of isobaric first-order phase transition EoS (shown in Figure 1) from 2.5 to 2 or 3 $\rho_0/\rho_0$, it is found that our results of proton directed flow become worse at 2-4 GeV/nucleon beam energies but better at 6-8 GeV/nucleon beam energies when using $2\rho_0/\rho_0$ starting density of isobaric first-order
phase transition, as shown in Figure 5. Whereas with 3$\rho_0$ starting density does not improve (mostly worsen) our fits to data except at 2 GeV/nucleon beam energy (technically, to match the MIT bag model calculations, the ending densities are slightly shifted). When changing the ending density of isobaric first-order phase transition EoS from 4 to 3$\rho_0$, it is found that the strengths of proton directed flow at 2 GeV/nucleon are almost unchanged. However, the slopes of proton directed flow at 4-8 GeV/nucleon are evidently enhanced, becoming far from the experimental data. Shifting the ending density from 4 to 3.5$\rho_0$ causes an inapparent discrepancy in proton directed flows at 2-4 GeV/nucleon, but slight enhancement at 6-8 GeV/nucleon. Setting even larger ending density of first-order phase transition needs more larger MIT bag model $B$ parameter than 150 MeV, which seems not supported by relevant theories. Therefore, it is deduced that the boundary of first-order phase transition of nuclear matter is around 2.5-4$\rho_0$.

It is highly necessary to survey whether the in-medium correction of nucleon-nucleon scattering cross section affects the strength or slope of proton transverse flows studied here. From Refs. [50, 52], it is found that the in-medium correction factor of nucleon-nucleon scattering is not only density-dependent, but also vitally momentum-dependent, especially at higher momenta. At beam energies of 2-8 GeV/nucleon, the reduced factors in-medium correction of nucleon-nucleon scattering cross section are expected to trend to unit since nucleon momentum at nucleon-nucleon center-of-mass is usually close to or greater than 1 GeV/c. Therefore, the in-medium correction of nucleon-nucleon scattering cross section should have negligible effects on the proton directed flows studied here.

In the light of the excitation function of elliptic flow in Au+Au collisions at beam energies of 1-11 GeV/nucleon exhibits characteristic signatures which could signal the onset of a possible phase transition [14], we in fact also investigated the excitation function of elliptic flow in Au+Au collisions at beam energies of 2-8 GeV/nucleon. Although our theoretical calculations overall fit relevant experimental data [22] in existence well, they are unfortunately not sensitive to EoSs with and without first-order phase transition.

Finally, it should be pointed that in Au+Au collisions at 2-8 GeV/nucleon, the temperature reached in formed compression matter are about 64-94 MeV [20], thus the above discussed boundary of phase transition of nuclear matter should be at such temperature range.

IV. CONCLUSIONS

In summary, with a relativistic transport model ART, nuclear many-body theory and the MIT bag model, the boundary of phase transition of dense nuclear matter is studied using proton transverse and directed flows formed in Au + Au collisions at beam energies of 2, 4, 6 and 8 GeV/nucleon. The results indicate that first-order phase transition of dense nuclear matter occurs at densities about 2.5-4 times saturation density with temperature about 64-94 MeV. More accurate phase-transition boundary exploration by proton transverse or directed flows needs more knowledge about nuclear many-body theories and quark model theories besides relativistic transport approaches. The ongoing BES-II program is expected to get more insight into landmarks of the QCD matter phase diagram.

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[1] S. Gupta, X. F. Luo, B. Mohanty, H. G. Rüttter, and N. Xu, Science 322, 6037 (2011).
[2] M. A. Braun, J. Dias de Deus, A. S. Hirsch, C. Pajares, R. P. Scharenberg, and B. K. Srivastava, Phys. Rep. 599, 1 (2015).
[3] P. Braun-Munzinger, V. Koch, T. Schäfer, and J. Stachel, Phys. Rep. 621, 76 (2016).
[4] F. Karsch, E. Laermann, and A. Peikert, Nucl. Phys. B 605, 579 (2001).
[5] C. Bernard, T. Burch, C. DeTar, J. Osborn, S. Gottlieb, E. Gregory, D. Toussaint, U. Heller, and R. Sugar (MILC Collaboration), Phys. Rev. D 71, 034504 (2005).
[6] A. Bazavov et al., Phys. Rev. D 85, 054503 (2012).
[7] M. Asakawa and K. Yazaki, Nucl. Phys. A 504, 668 (1989).
[8] M. A. Stephanov, Prog. Theor. Phys. Suppl. 153, 139 (2004).
[9] S. Carignano, D. Nickel, and M. Buballa, Phys. Rev. D 82, 054009 (2010).
[10] N. Bratovic, T. Hatsuda, and W. Weise, Phys. Lett. B 719, 131 (2013).
[11] K. Schertler, C. Greiner, and M. H. Thoma, Nucl. Phys. A 616, 659 (1997).
[12] K. Schertler, C. Greiner, J. Schaffner-Bielich, and M. H. Thoma, Nucl. Phys. A 677, 463 (2000).
[13] M. Di Toro et al., Phys. Rev. C 83, 014911 (2011).
[14] P. Danielewicz, Roy A. Lacey, P.-B. Gossiaux, C. Pinkenburg, P. Chung, J. M. Alexander, and R. L. McGrath, Phys. Rev. Lett. 81, 2438 (1998).
[15] Tetyana Galatyuk, Nucl. Phys. A 982, 163 (2019).
[16] Agnieszka Sorensen, Volker Koch, arXiv:2001.06635 (2020).
[17] Wen-Jie Xie and Bao-An Li, arXiv:2009.13653 (2020).
[18] B. A. Li and C. M. Ko, Phys. Rev. C 52, 2037 (1995).
[19] B. A. Li, A. T. Sustich, B. Zhang, and C. M. Ko, Int. J. Mod. Phys. E 10, 267 (2001).
[20] A. Andronic, P. Braun-Munzinger, J. Stachel, Nucl. Phys. A 772, 167 (2006).
[21] J. Barrette, R. Bellwied, S. Bennett et al., Phys. Rev. C 56, 3254 (1997).
[22] C. Pinkenburg, N. N. Ajitanand, J. M. Alexander et al., Phys. Rev. Lett. 83, 1295 (1999).
[23] H. Liu, N. N. Ajitanand, J. Alexander et al., Phys. Rev. Lett. 84, 5488 (2000).
[24] L. Ahle, Y. Akiba, K. Ashktorab et al., Phys. Lett. B 476, 1 (2000).
[25] B. B. Back, R. R. Betts, J. Chang, W. C. Chang et al., Phys. Rev. Lett. 87, 242301 (2001).
[26] P. Chung, N. N. Ajitanand, J. M. Alexander et al., Phys. Rev. C 66, 021901(R) (2002).
[27] C. A. Ogilvie, Nucl. Phys. A 698, 3 (2002).
[28] M. Isse, A. Ohnishi, N. Otuka, P. K. Sahu, and Y. Nara, Phys. Rev. C 72, 064908 (2005).
[29] G. Kestin and U. Heinz, Eur. Phys. J. C 61, 545 (2009).
[30] X. Luo, N. Xu, Nucl. Sci. Tech. 28, 112 (2017).
[31] K. J. Sun, L. W. Chen, C. M. Ko, and Z. B. Xu, Phys. Lett. B 774, 103 (2017).
[32] Edward Shuryak, Juan M. Torres-Rincon, Phys. Rev. C 100, 024903 (2019).
[33] A. Bzdak, S. Esumi, V. Koch, J. Liao, M. Stephanov, N. Xu, Phys. Rep. 853, 1 (2020).
[34] G. F. Bertsch and S. Das Gupta, Phys. Rep. 160, 189 (1988).
[35] B. A. Li, W. Bauer, and G. F. Bertsch, Phys. Rev. C 44, 2095 (1991).
[36] Z. W. Lin, C. M. Ko, B. A. Li, B. Zhang, S. Pal, Phys. Rev. C 72 (2005) 064901.
[37] Yasushi Nara, Tomoyuki Maruyama, and Horst Stoecker, Phys. Rev. C 102 (2020) 024913.
[38] P. Danielewicz, Nucl. Phys. A 673, 375 (2000).
[39] S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif and R. L. Mercer, Phys. Rev. C 41, 2737 (1990).
[40] C. Gale, G. M. Welke, M. Prakash, S. J. Lee, and S. Das Gupta, Phys. Rev. C 41, 1545 (1990).
[41] D. H. Youngblood, H. L. Clark, and Y. W. Lui, Phys. Rev. Lett. 82, 691 (1999).
[42] S. Shlomo, V.M. Kolomietz, G. Colò, Eur. Phys. J. A 30, 23 (2006).
[43] J. Piekarewicz, J. Phys. G 37, 064038 (2010).
[44] A. Chodos, R. L. Jaffe, K. Johnson, C. B. Thorn, and V. F. Weisskopf, Phys. Rev. D 9, 3471 (1974).
[45] G. F. Burgio, M. Baldo, P. K. Sahu, and H. J. Schulze, Phys. Rev. C 66, 025802 (2002).
[46] O. E. Nicotra, M. Baldo, G. F. Burgio, and H.-J. Schulze, Phys. Rev. C 74, 123001 (2006).
[47] B. A. Li and C. M. Ko, Phys. Rev. C 58, 1382(R) (1998).
[48] J. Y. Ollitrault, Nucl. Phys. A 638, 195c (1998).
[49] Bao-An Li, Lie-Wen Chen, Phys. Rev. C 72, 064611 (2005).
[50] Qingfeng Li, Zhuxia Li, Sven Soff, Marcus Bleicher and Horst Stöcker, J. Phys. G: Nucl. Part. Phys. 32, 407 (2006).
[51] Ying Yuan, Qingfeng Li, Zhuxia Li, and Fu-Hu Liu, Phys. Rev. C 81, 034913 (2010).
[52] Wen-Mei Guo, Gao-Chan Yong, Yongjia Wang, Qingfeng Li, Hongfei Zhang, Wei Zuo, Phys. Lett. B 726, 211 (2013).