Solar differential rotation reproduced with high-resolution simulation

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The Sun rotates differentially with a fast equator and slow pole\(^1\). Convection in the solar interior is thought to maintain the differential rotation. However, although many numerical simulations have been conducted to reproduce the solar differential rotation\(^2\)–\(^6\), previous high-resolution calculations with solar parameters fall into the antisolar (fast-pole) differential rotation regime. Consequently, we still do not know the true reason why the Sun has a fast-rotating equator. While the construction of the fast equator requires a strong rotational influence on the convection, the previous calculations have not been able to achieve the situation without any manipulations. The problem is called the convective conundrum\(^7\). The convection and the differential rotation in numerical simulations were different from the observations. Here, we show that a high-resolution calculation succeeds in reproducing the solar-like differential rotation. Our calculations indicate that the strong magnetic field generated by a small-scale dynamo has a significant impact on thermal convection. The successful reproduction of the differential rotation, convection and magnetic field achieved in our calculation is an essential step to understanding the cause of the most basic nature of solar activity, specifically, the 11 yr cycle of sunspot activity.

In this study, we markedly increase the resolution using the supercomputer Fugaku to investigate the possible influence of the magnetic field on the differential rotation. We consider three cases, low, middle and high, where the numbers of grid points are \((N_r,N_\theta,N_\phi,N_{\text{YY}}) = (96, 384, 1,536, 2), (192, 768, 2,312, 2)\) and \((384, 1,536, 4,608, 2)\), respectively. \(N_r, N_\theta\) and \(N_\phi\) are the radial, latitudinal and longitudinal grid points, respectively. \(N_{\text{YY}}\) is the factor from the Yin–Yang grid\(^8\). In the ordinary spherical grid, the numbers of grid points are \((N_r,N_\theta,N_\phi) = (96, 768, 1,536), (192, 1,536, 3,072)\) and \((384, 1,536, 6,144)\) in the low, middle and high cases, respectively. Note that the resolution is fairly high even in the low case compared with previous studies. We adopt solar stratification\(^9\), solar rotation and solar luminosity, and exclude any type of explicit diffusivity to maintain high resolution. Details of the numerical method are found in Methods. We continue these calculations for 4,000 d. The temporal evolutions of the energies are shown in Supplementary Fig. 1.

Figure 1a,b shows three-dimensional volume renderings of the normalized entropy and the magnetic field strength, respectively, in the high case. The maximum magnetic field strength exceeds 80 kG, which is a significant superequipartition magnetic field.

Figure 2 shows the dependence of the differential rotation on the resolution. Panels a, b and c show the results for the low, middle and high cases, respectively. In the low case (Fig. 2a), we obtain a fast pole and slow equator, which is consistent with previous studies. In the middle case (Fig. 2b), the differential rotation becomes more solar-like, while we still see a significant decrease of the angular velocity in the near-surface region around the equator. Even in the middle case, the resolution is high as a long-term full spherical dynamo calculation. In the high case (Fig. 2c), we nicely reproduce the solar-like differential rotation: specifically, the equator region is rotating faster than the pole. Similar to the real Sun, the topology of the differential rotation deviates from the Taylor–Proudman-like profile, that is, the contour lines of our differential rotation are not aligned to the rotational axis. We do not force the entropy gradient at the bottom boundary\(^7\), but the efficient small-scale dynamo increases the latitudinal entropy gradient as found by Hotta\(^7\). A differential rotation in a hydrodynamic calculation without the magnetic field is also shown in Supplementary Fig. 3.

Figure 3 shows the convection and magnetic field properties. Figure 3a shows root-mean-square (r.m.s.) velocity \(v_{\text{RMS}}\). Higher resolution tends to show a smaller amplitude of the convection. The decrease in the convection velocity reduces the Rossby number. Figure 3b shows the r.m.s. magnetic field \(B_{\text{RMS}}\) (solid line) and equipartition magnetic field \(B_{\text{eq}}\) (dotted line), where \(B_{\text{eq}} = \sqrt{4 \pi \rho_0 v_{\text{RMS}}^2}\) and \(\rho_0\) is the background density. The r.m.s. magnetic fields monotonically increase with the resolution. In the low case, the magnetic field is always smaller than the equipartition magnetic field. In previous studies, the r.m.s. magnetic field has reached 10–20% of the equipartition magnetic field\(^7\). The system reaches an efficient small-scale dynamo regime even in the low case because the magnetic field achieves an almost equipartition level. In the middle case, the superequipartition magnetic field \((B_{\text{RMS}} > B_{\text{eq}})\) is reproduced in the bottom half of the convection zone. In the high case, the magnetic energy exceeds the kinetic energy in all layers in the convection zone. This strong magnetic field suppresses the convection velocity significantly. In the high case, the stretching becomes weaker and the compression increases. The generation mechanism of the magnetic field is discussed also in Supplementary Figs. 4 and 5.

Figure 4 shows the kinetic (solid line) and magnetic (dotted line) energy spectra at \(r = 0.83 R_\odot\). In the low case, the magnetic energy exceeds the kinetic energy only on a small scale \(\ell > 100\). This is a clear sign of the efficient small-scale dynamo\(^7\). In the middle case, the turnover of the superequipartition magnetic field moves to a larger scale \(\ell \sim 45\). While on the small scale \(\ell > 10\) the kinetic energy in the middle case is smaller than that of the low case because of stronger Lorentz force feedback, the kinetic energy does not change on the large scale \(\ell < 10\). In the high case, the magnetic energy exceeds the kinetic energy on almost all the scales. The kinetic energy is also reduced on all the scales. Because of this kinetic energy suppression in the high case, the peak of the kinetic energy is shifted from \(\ell \sim 6\) (low and middle cases) to \(\ell \sim 30\). These spectral variations indicate that the dynamo in the high case is qualitatively different from the others.

In this study, we reproduce the solar-like differential rotation in a high-resolution calculation with solar parameters, such as the

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stratification, the luminosity and the rotation rate. Our calculation results show that the Sun has a marginal Rossby number between antisolar (fast-pole) and solar-like (fast-equator) differential rotation, and we need a sophisticated treatment of the thermal convection and the magnetic field to reproduce the differential rotation. We offer a path forward in resolving the convective conundrum. Significant reduction in the convective energy on a large scale ($\ell < 30$) is a promising trend to solve the other part of the convective conundrum, that is, the energy spectra obtained by helioseismology.$^{14}$ Because the observational result for the energy spectra is still controversial, detailed comparisons between numerical simulations and observations are needed to solve the problem. Furthermore, by appropriately reproducing the differential rotation, the convection will lead to a
correct understanding of the generation of the large-scale magnetic field and cycle. In this study, the dynamo has not constructed the large-scale magnetic field (Supplementary Fig. 2), probably due to lack of calculation time for the large-scale dynamo or insufficient suppression of the convection velocity. It is possible that the longer-time calculation changes the amplitude of the magnetic field. This is still out of our reach. In addition, we have not reached numerical convergence, where the result does not change on doubling the resolution. We cannot rule out further change of the differential rotation in higher-resolution simulations. We expect greater resolution to lead to a stronger magnetic field, which will further suppress the convection velocity. This is a good factor for the construction of the large-scale magnetic field as well. Higher-resolution simulation is still desired.

Methods
Numerical simulation. We solve the three-dimensional magnetohydrodynamic equations in spherical geometry \((r, \theta, \phi)\) with an extended version of the R2D2 code17. The equations for the calculation are

\[
\begin{aligned}
\frac{\partial \rho_1}{\partial t} &= \frac{-1}{r^2} \nabla \cdot \left( \rho \mathbf{v} \right), \\
\frac{\partial \rho \mathbf{v}}{\partial t} &= -\nabla \cdot \left( \rho \mathbf{v} \mathbf{v} - \frac{1}{2} \nabla \rho + \frac{1}{4 \rho} \left( \nabla \times \mathbf{B} \right) \times \mathbf{B} + 2 \rho \mathbf{v} \right) - \mathbf{v} \times \nabla \times \mathbf{B} \mathbf{1}, \\
\frac{\partial \mathbf{B}}{\partial t} &= \nabla \times \left( \mathbf{v} \times \mathbf{B} \right), \\
\frac{\rho T}{\partial t} &= \nabla \cdot \left( \mathbf{v} \cdot \nabla \right) T + Q_{\text{rad}}, \\
\rho_1 &= \frac{1}{2} \left( 1 + \frac{3}{\gamma} \right) \rho + \frac{2}{3} \left( \frac{\gamma}{\gamma - 1} \right) \rho s_i,
\end{aligned}
\]

where \(\rho, \mathbf{v}, T, \mathbf{B}, \mathbf{1}, \nabla, \mathbf{Q}_{\text{rad}}\), and \(Q_{\text{rad}}\) are the density, the fluid velocity, the temperature, the magnetic field, the specific entropy, the rotational heating, \(\mathbf{1}\) is the factor from the reduced speed of sound technique17, \(\nabla\) is the radial unit vector. The subscript 1 indicates the perturbation from the zeroth-order spherically symmetric background from Model S17. For the detailed method, the readers can find information in our previous publications17. In this study, the calculation domain extends from 0.71\(R_\odot\) to 0.96\(R_\odot\), in the radial direction. The whole sphere is covered with the Yin–Yang grid15. The equations are solved with the fourth-order space-centred method and four-step Runge–Kutta method for time integration. We do not include any explicit diffusivity, and only artificial viscosity with a slope limiter18 is used. Non-penetration and stress-free boundary conditions are used for the top and bottom boundaries. The horizontal and vertical magnetic field boundary conditions are used at the bottom and top boundaries, respectively.

We averages quantities in a period from 3,600 to 4,000 d to show the results.

Normalization of energy spectra. We adopt a standard way to normalize the energy spectra adopted in our field. When \(\nu_{\text{rms}}(\ell)\) and \(B_{\text{rms}}(\ell)\) are defined, the kinetic \(E_{\text{k}}\) and magnetic \(E_{\text{B}} \) energy spectra are normalized to satisfy the relation

\[
\begin{aligned}
\frac{1}{\tau} \nu_{\text{rms}}^2 &= \sum_{\ell=0}^\ell \frac{E_k(\ell)}{\ell}, \\
\frac{B_{\text{rms}}^2}{\delta E} &= \sum_{\ell=0}^\ell \frac{E_B(\ell)}{\ell}.
\end{aligned}
\]

Data availability
The data generated, analysed and presented in this study are available at https://doi.org/10.5281/zenodo.5003258.

Code availability
We have opted not to make R2D2 code publicly available. Running R2D2 code requires expert assistance and an appropriate computer system. The numerical method is explained in our previous publications in detail17.

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Author contributions
H.H. contributed to the design of the project, developed the numerical code, carried out simulations, performed analysis and wrote the first draft of the paper. K.K. contributed to the design of the project, interpretation of the result and writing of the final draft.

Competing interests
The authors declare no competing interests.

Additional information
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