Recovery of information from black hole radiation by considering stimulated emission

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Abstract

We deal with the black hole information loss paradox by showing that the stimulated emission component of the black hole radiation contains information about the initial state of the system. The nonlocal behaviour that allows the recovery of information about the matter that falls behind the horizon appears in a natural way. We calculate the expectation value and probability distribution of particles at $J^+$ for a non-vacuum initial state. The entropy of the final state is compared to that of a thermal state with the same energy per mode. We find that the information recovered about the initial state increases with the number $r$ of the initially incoming particles, reaching for example over 30% for $r = 1000$.

We point out that recovering information about the initial state, nevertheless, does not automatically imply the purity of the final state.

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1 Introduction

Currently there is a vivid discussion about the so-called information-loss paradox that appears when a matter distribution collapses to form a black hole and subsequently evaporates into Hawking radiation. Consider a matter distribution that is described quantum mechanically by a pure state (zero entropy) and collapses, forming a black hole. The black hole will emit Hawking radiation and, taking gravitational back-reaction into account, is believed to evaporate almost completely. After the evaporation of the black hole, only the thermal Hawking radiation is left behind, which is described by a density matrix. Hence it appears as if the system had evolved from an initial pure state to a final mixed state so that information must have been lost.

Let us examine what kind of information has been lost. There are two sources of information loss, which may be related, but which are not always properly distinguished in the literature: (1) After the complete black hole evaporation the total system consists solely of Hawking radiation. Thermal radiation cannot carry any information except for its temperature. It depends only on the geometry of the black hole and does not reveal anything about the matter that formed the black hole (no hair). Thus, even if the initial state of the collapsing matter that fell behind the horizon was precisely known, this information will not be present in the thermal Hawking radiation of the final state. (2) When the Hawking radiation is created, the total system consists of two subsystems: The interior and the exterior region of the black hole. In such cases it is known that, although the state of the total system is pure, information can be transferred from each of the subsystems into correlations between the subsystems. The state of each subsystem is then described by a density matrix. This is also the case for a black hole and the radiation it emits. When the black hole evaporates, it is not clear what happens to the information contained in correlations.

There have been several attempts to resolve this information loss paradox. Each of them is plagued of serious objections, however. Review of the subject are given in Refs. There are three main approaches to the problem:
The first possibility is to assume that the black hole does not evaporate completely but leaves behind a Planck mass remnant [10]. This proposal has problems with the infinite number of remnant species and their specific entropy and is therefore disregarded by most authors.

A second alternative, advocated by Hawking [11], is to accept the information loss and the evolution from pure into mixed states. This would mean that the usual rules of quantum mechanics would not apply to the problem of black hole evaporation (in particular there are problems with locality and energy conservation [12]). Unitary evolution seems to be lost at least at the level of quantum gravity.

The third possibility is that the information is not lost and the final state is pure. This has been put forward mainly by Page [13, 5] and ‘t Hooft [7]. In this case, the information should be completely encoded in the Hawking radiation. One is then faced with the problem that the information must be extracted from behind the horizon in a nonlocal manner [14]. Up to now, no explicit mechanism has been given for this nonlocal process.

In the present paper we study a mechanism that encodes information about the initial state into the Hawking radiation. We found already in a previous paper [15] that the outgoing Hawking radiation can be influenced by ingoing particles via stimulated emission, even if they fall behind the horizon. The imprints left by the stimulated emission on the black hole radiation carry information about the particles that formed the black hole. They can, in principle, be measured by an external observer that could thus estimate the initial state of the system. It is remarkable that the nonlocal behaviour that is necessary for obtaining information about the matter that crosses the horizon appears in the stimulated emission process in a natural way.

Since our calculation is valid only near the time of horizon formation, we are not able to treat infalling matter at late times. The role of stimulated emission in this situation has been recently investigated by Bekenstein [16] and Schiffer [17].

We will show, however, that although the stimulated emission process provides a mechanism for information extraction, this does not necessarily imply the purity of the final state. The problems of recovering information and obtaining a pure state must therefore be carefully distinguished.

In Sec. 2, we review the main results of the computation of the stimulated emission component of the black hole radiation. We then compute in Sec. 3 the entropy associated with this radiation and compare it with the entropy of a thermal state with the same mean energy to have a measure of the information carried by the Hawking radiation. We find that, for example, for 100 incoming particles per mode as much as 28% of the information can be recovered. The paper ends with a discussion of the results and the Appendix where we discuss explicitly the nonlocal terms of the stimulated emission.

2 Modification of Hawking radiation by stimulated emission

We consider the spacetime generated by a spherically symmetric matter distribution that collapses to form a black hole (Fig. 1). Outside the matter the metric is the Schwarzschild metric which can be covered by the Eddington-Finkelstein null coordinates \( u = t - r^* \) and \( v = t + r^* \) where \( r^* = r + 2M \ln \left| \frac{r}{2M} - 1 \right| \). Radially ingoing null geodesics \( v = \text{const} \) and outgoing null geodesics \( u = \text{const} \) are connected inside the matter distribution by Hawking’s ray tracing formula \( \text{(1)} \)

\[
v = v_0 - CD e^{-\kappa u},
\]

where \( C, D \) are constants and \( \kappa = (4M)^{-1} \) is the black hole’s surface gravity. The ingoing ray that reaches the horizon at the moment of its formation is denoted by \( v_0 \). Eq. \( \text{(1)} \) is valid for \( v - v_0 \) small and positive. Therefore we restrict our considerations to late times (large \( u \)) at \( J^+ \). It is possible to derive the redshift suffered by a null ray that follows the trajectory \( \text{(1)} \) in the limit of geometrical optics. It is given by the redshift formula \( \text{(2)} \)

\[
\omega = \kappa CD e^{-\kappa u} \omega'.
\]

Throughout the paper, primed quantities refer to observables at \( J^- \), unprimed ones to \( J^+ \).

Let us consider a real massless scalar field in this spacetime. In order to quantize the field we need a complete set of mode functions. We choose ingoing and outgoing wave packets which are positive frequency
whose angular dependence is given by the spherical harmonics $Y_{\omega,n}$ neglected. The parameters $(\omega, n, l, m)$ are solutions of the Klein-Gordon equation in the Schwarzschild geometry if gravitational backscattering is neglected. The parameters $(\omega', n')$ of the equivalent mode of a wave packet (3) specify their mean energy and their trajectory according to

$$v = -n', \quad u = -n. \quad (5)$$

With trajectory we refer to the path of the wave packet maximum. One must not forget, however, that a wave packet inevitably has tails that extend over the whole spacetime.

The quantization is carried out with respect to the wave packet mode functions (3) on $\mathcal{J}^-$ and $\mathcal{J}^+$.

$$\phi(x) = \sum_{\omega'u'n'lm'} \left( b^{in}_{\omega'u'n'lm'} f_{\omega'u'n'lm'}(x) + b^{in\dagger}_{\omega'u'n'lm'} f^{\dagger}_{\omega'u'n'lm'}(x) \right), \quad (6)$$

and on $\mathcal{J}^+$

$$\phi(x) = \sum_{\omega'nlm} \left( a^{out}_{\omega'nlm} p_{\omega'nlm}(x) + a^{out\dagger}_{\omega'nlm} p^{\dagger}_{\omega'nlm}(x) \right), \quad (7)$$

resulting in wave packet creation and annihilation operators. To describe particles at the future event horizon $H^+$, we choose wave packets formed from Wald’s horizon modes $20$. The choice of these mode functions does not affect observables at $\mathcal{J}^+$.

In Ref. $15$, we have obtained the Bogoliubov transformation that connects creation and annihilation operators on $\mathcal{J}^-$ and $\mathcal{J}^+$. It can be written as

$$a^{out}_{\omega'nlm} = \alpha_{\omega'nlm} b^{in}_{\omega'n'lm'} - \beta_{\omega'nlm} b^{in\dagger}_{\omega'n'lm'} \quad (8)$$

The approximations contained in the derivation of (8) are discussed in $15$. In (8) we introduced the notion of the equivalent mode $21$ of a wave packet $(\omega, n, l, m)$ at $\mathcal{J}^+$:

$$(\omega'_x, n'_x, l, m) = \left( \frac{1}{\kappa CD} e^{-\kappa \omega}, -(v_0 - CD e^{\kappa n}), l, m \right). \quad (9)$$

It is given by the wave packet $(\omega'_x, n'_x, l, m)$ at $\mathcal{J}^-$ which is connected to $(\omega, n, l, m)$ by means of the classical trajectory $\mathbb{1}$ and redshift relation $\mathbb{2}$ (cf. Fig. 1). The mirror mode

$$(\omega'_2, n'_2, l, -m) = \left( \frac{1}{\kappa CD} e^{-\kappa \omega}, -(v_0 + CD e^{\kappa n}), l, -m \right) \quad (10)$$

of a wave packet $(\omega, n, l, m)$ which enters the horizon is obtained from the equivalent mode by ‘reflection’ at the null line $v_0$. Furthermore we used in (8) the Bogoliubov parameters

$$\begin{align*}
\alpha_{\omega'lm} &= \mp i \left( \frac{\kappa}{2\pi \omega} \right)^{\frac{1}{2}} e^{\pm \omega \pi / 2 \kappa} \Gamma \left( 1 + i \frac{\omega}{\kappa} \right) \left( -1 \right)^l (\pm 1)^m. \quad (11)
\end{align*}$$

Their squares are given by

$$|\alpha_{\omega'lm}|^2 = 1 + \frac{1}{e^{\omega / T} - 1}, \quad |\beta_{\omega'lm}|^2 = \frac{1}{e^{\omega / T} - 1}. \quad (12)$$
where \( T = \kappa / (2\pi) \) is the black hole temperature.

With these results, it is possible to discuss the spontaneous and stimulated emission of the Hawking radiation. To this end, we calculate the expectation value of the outgoing particle number operator, \( \rho^\text{out}_{\omega nlm} = \langle \psi^\text{in} | a^\text{out}_\omega n^\text{in}_l m^\text{in} | \psi^\text{in} \rangle \), that is measured at \( J^+ \) for a given initial state \( \psi^\text{in} \) prepared at \( J^- \). We find

\[
r^\text{out}_{\omega nlm} = r^\text{in}_{\omega n l m} + |\beta^\text{in}_{\omega n l m}|^2 (r^\text{in}_{\omega n l m} + r^\text{in}_{\omega n l m,-m}) + |\beta_{\omega n l m}|^2.
\]  

Here \( r^\text{in}_{\omega n l m} = \langle \psi^\text{in} | b^\text{in}_\omega n^\text{in}_l m^\text{in} b^\text{in}_\omega n^\text{in}_l m^\text{in} | \psi^\text{in} \rangle \) denotes the mean number of ingoing particles in the state \( |\psi^\text{in} \rangle \) at \( J^- \). Eq. (13) shows how the number of particles in the black hole radiation depends on the particle number in the initial state.

We see that there are four contributions to the particle number at \( J^+ \). The last term is the thermal spectrum of the Hawking radiation, which is always present, regardless of the orientation of the initial state of the field. The remaining terms show that in addition to this thermal radiation, there are further contributions due to the presence of ingoing particles. The first term represents the particles that escaped the capture by the horizon \( (v < v_0) \) and reappear in the out region in the equivalent mode \( \tilde{J} \). They travel on the classical null geodesics \( \tilde{J} \) with the respective redshift \( \tilde{J} \). The second term shows that these particles stimulate the creation of additional particles on the same trajectory with the same energy. Finally, we see from the third term that the particle number in a wave packet on \( J^+ \) can also be enhanced by ingoing wave packets that travel behind the horizon \( (v > v_0) \) in the mirror mode \( \tilde{J} \). In the Appendix, we show the explicit nonlocality of this contribution.

It is possible to give a heuristic discussion of the physics contained in Eq. (13): The black hole radiation is created in pairs. The members of each pair are located on either side of the horizon \( \tilde{J} \). Ingoing wave packets that reach \( J^+ \) stimulate the creation of additional pairs in the same mode. The additional particles appear as the second term in (13). But the creation of additional pairs can also be stimulated by wave packets that travel behind the horizon in the mirror mode \( \tilde{J} \). The respective particles then reach \( J^+ \), leading to the nonlocal term in (13).

### 3 Entropy of the black hole radiation

Let us calculate the entropy of the black hole radiation when stimulated emission is taken into account. This can be compared with the entropy of thermal radiation with the same mean energy. The deviation of the black hole radiation from thermality is a measure for the amount of information that can be encoded in the black hole radiation.

To this end we transcribe an incoming many-particle state in terms of wave packet states at \( J^+ \) and the horizon \( H^+ \). This procedure has been carried out for a Bogoliubov transformation of the form (8) in the context of accelerated observers in Ref. [15] (cf. also [22]). We therefore give here only the relevant results. Consider a wave packet state \( |r^\text{in}_{\omega n l m} \rangle \) with \( r \) ingoing particles that cross the event horizon. With respect to particles at \( J^+ \) and \( H^+ \), this state can be written

\[
|r^\text{in}_{\omega n l m} \rangle = |r^\text{in}_{\omega n l m} \rangle \prod_{\omega n l m} \langle 0_{\omega n l m} | \prod_{\omega n l m} |0_{\omega n l m} \rangle,
\]  

where \( (\omega, n, l, m) \) and \( (\omega', n', l', -m) \) are connected according to (10) and the product extends over all remaining modes. Furthermore we have defined

\[
|r_{\omega n l m} \rangle = (\alpha_{\omega n l m}^* \beta_{\omega n l m})^{-r} \sum_{q=0}^{\infty} \left( \frac{r+q}{(r+q)!} \right) \left( \frac{\beta_{\omega n l m}}{\alpha_{\omega n l m}} \right)^r (r+q)_{\omega n l m} \langle r^\text{in}_{\omega n l m} | H^+ \otimes |q_{\omega n l m} \rangle_{J^+}.
\]  

The state (14) displays nonlocal correlations between particles at \( J^+ \) and \( H^+ \).

The state (14) is pure. To calculate the entropy of the black hole radiation at \( J^+ \) we need the reduced density matrix which is obtained by tracing over the horizon states. Since in our approximation ingoing
particles in the mode \((\omega_\beta, n_\beta, l, -m)\) influence only the mode \((\omega, n, l, m)\), we restrict our attention to this mode. Its reduced density matrix is

\[
\rho_{\omega nlm}^{J^+} = \text{Tr}_{H^+}(r_{\omega nlm}^+ (r_{\omega nlm}^-) \rho_{\omega nlm}^+ \rho_{\omega nlm}^-) \text{Tr}_{H^+}(r_{\omega nlm}^+ (r_{\omega nlm}^-) \rho_{\omega nlm}^+ \rho_{\omega nlm}^-) = |\alpha_{\omega nlm}|^{-2r-2} \sum_{q=0}^{\infty} \frac{(r+q)!}{r!q!} \frac{\beta_{\omega nlm}^{2q}}{|\alpha_{\omega nlm}|} |q_{\omega nlm}\rangle_J^+ \langle J^+ | q_{\omega nlm}^\rangle.
\] (16)

The entropy of the black hole radiation in the mode \((\omega, n, l, m)\) at \(J^+\) is given by

\[
S_{\omega nlm}^{bh} = -k \text{Tr}_{J^+}(\rho_{\omega nlm}^{J^+} \ln \rho_{\omega nlm}^{J^+}) = -k |\alpha_{\omega nlm}|^{-2r-2} \sum_{q=0}^{\infty} \frac{(r+q)!}{r!q!} \frac{\beta_{\omega nlm}^{2q}}{|\alpha_{\omega nlm}|} \ln \left( |\alpha_{\omega nlm}|^{-2r-2} \frac{(r+q)!}{r!q!} \frac{\beta_{\omega nlm}^{2q}}{|\alpha_{\omega nlm}|} \right).
\] (17)

The sum can be evaluated using the formula

\[
\sum_{q=0}^{\infty} \frac{(r+q)!}{r!q!} x^q = (1-x)^{-r-1},
\]

and the integral representation for the logarithm of the gamma function

\[
\ln \Gamma(z) = \int_0^\infty \frac{dt}{t} \left( (z-1)e^{-t} + \frac{e^{-zt} - e^{-t}}{1-e^{-t}} \right).
\] (18)

We obtain

\[
S_{\omega nlm}^{bh} = k(r+1) \left( |\alpha_{\omega nlm}|^2 \ln |\alpha_{\omega nlm}|^2 - |\beta_{\omega nlm}|^2 \ln |\beta_{\omega nlm}|^2 \right) + S_R^{bh},
\] (19)

where

\[
S_R^{bh} = -k |\alpha_{\omega nlm}|^{-2r-2} \int_0^\infty \frac{dt}{t} e^{-rt} - 1 + \frac{e^{-rt} - 1}{1-e^{-t}} \left[ \left( 1 - \frac{|\beta_{\omega nlm}|^2}{|\alpha_{\omega nlm}|^2} e^{-t} \right)^{-r-1} - \left( 1 - \frac{|\beta_{\omega nlm}|^2}{|\alpha_{\omega nlm}|^2} \right)^{-r-1} \right].
\] (20)

The remaining integral has to be treated numerically.

This expression for the entropy contained in the mode \((\omega, n, l, m)\) of the black hole radiation in the presence of incoming particles can be compared with the entropy \(S_{\omega nlm}^{th}\) of thermal radiation in this mode with the particle number expectation value \(r_{\omega nlm}^{th} = |\beta_{\omega nlm}|^2 \left( \sum_{n,l,m} (n_\alpha, l, -m) \right)\) that does not enter the horizon (cf. Fig. 1). In contrast to the behaviour of horizon-crossing modes, \(L_{\omega nlm}\) is in this case close to unity for small \(|\beta_{\omega nlm}|^2\). The wave packet propagates nearly undisturbed.
to $J^+$ so that almost all information it carries can be recovered. For larger values of $|\beta_{\omega lm}|^2$, $I_{\omega lm}$ decreases because of the disturbances that the black hole radiation introduces to the semiclassical propagation.

Let us turn now to the second aspect of the information loss paradox, the question about the information contained in correlations. A quantitative measure of this information can be defined by [2]

$$J_c = S_{J^+}^{bh} + S_{H^+}^{bh} - S_{tot}^{bh},$$

(22)

where $S_{J^+}^{bh} = \sum_{\omega_{nlm}} S_{\omega_{nlm}}^{bh}$ is the entropy of the black hole radiation at $J^+$, $S_{H^+}^{bh}$ is the entropy at the horizon, and $S_{tot}^{bh}$ is the entropy of the total state [14] which is zero in our case. Using the Araki-Lieb inequalities [25], this can be reduced to

$$J_c = 2 \sum_{\omega_{nlm}} S_{\omega_{nlm}}^{bh}.$$

(23)

This expression for the information contained in correlations in the case of black hole radiation can again be compared with the same quantity for thermal radiation which can be obtained by replacing in (23) $S_{\omega_{nlm}}^{bh}$ by $S_{\omega_{nlm}}^{r}$ of [25]. We see that the information contained in correlations is smaller for black hole radiation than for thermal radiation. Note however, that a pure state for the black hole radiation would require $J_c = 0$, which cannot be achieved by considering the stimulated emission process alone.

It is possible to compute not only the expectation value of the particle number, but also the probability distribution of particles at $J^+$ in the presence of incoming particles at $J^-$. This quantity can be easily derived from (14). The probability that $q$ particles are found in the mode $(\omega, n, l, m)$ at $J^+$ if the initial state is the vacuum is the well-known Bose-Einstein distribution (cf. [26])

$$P(q_{\omega_{nlm}}^{out}|0^{in}) = \frac{|\beta_{\omega_{nlm}}|^2q}{(1 + |\beta_{\omega_{nlm}}|^2)^{q+1}} = \left(1 - e^{-2\pi\omega/\kappa}\right) e^{-(2\pi\omega/\kappa)q},$$

(24)

which is independent of the trajectory and angular momentum of the particles.

The situation changes however when there are ingoing particles at $J^-$. Let us consider the case when there are $r$ incoming particles in the mode $(\omega', n', l, -m)$ that propagates behind the horizon. The probability for finding $q$ particles in the mirror mode $(\omega, n, l, m)$ at $J^+$ is then

$$P(q_{\omega_{nlm}}^{out}|r_{\omega'}^{in}, n', l, -m) = \frac{(r + q)!}{r!q!} \frac{|\beta_{\omega_{nlm}}|^2q}{(1 + |\beta_{\omega_{nlm}}|^2)^{r+q+1}} = \frac{(r + q)!}{r!q!} e^{-(2\pi\omega/\kappa)q} \left(1 - e^{-2\pi\omega/\kappa}\right)^{r+1}. $$

(25)

For the remaining modes, the probability distribution is the same as that for the vacuum (24). The result (25) shows that in agreement with the previous discussion the Hawking radiation at $J^+$ can be influenced by ingoing particles that propagate behind the horizon. The key point is that the probability distribution (25), which refers only to observables at $J^+$, depends explicitly on the number $r$ of ingoing particles.

It is possible to gain some further insight from (24). Let us first consider for fixed $q$ the maximum of the probability distribution as a function of $|\beta_{\omega_{nlm}}|^2$. We find $|\beta_{\omega_{nlm}}|^2_{max} = q/(r + 1)$. This shows that as $r$ increases the maximum of the probability distribution is located at lower values of $|\beta_{\omega_{nlm}}|^2$. This is in agreement with the behaviour of $I_{\omega_{nlm}}$ as shown in Fig. 2. The value of (25) at the maximum $|\beta_{\omega_{nlm}}|^2_{max}$ can be computed and in the limit of large $r$ we obtain $\frac{1}{q} q^q e^{-q}$. This is independent of $r$ and shows that for any $q$ the maximum of the probability distribution does not change as $r \to \infty$. Thus there seems to exist a finite limit for the amount of information that can be recovered when the number of ingoing particles is increased. This feature is also in agreement with our numerical studies.

4 Discussion

Let us examine to what extent the effect of stimulated emission contributes to the resolution of the black hole information loss paradox. The first problem is the loss of information about the initial state of the collapsing
matter. We have seen that when the effects of stimulated emission are taken into account, information concerning the early stages, close to the black hole formation, can be recovered in principle at late times by measuring the full Hawking radiation. In fact, we have shown that from the knowledge of the probability distribution (22) or the particle spectrum (13), which refer only to observables at $\mathcal{J}^+$, we can infer the initial state at $\mathcal{J}^-$. Black hole radiation has a lower entropy than thermal radiation with the same energy, allowing information to come out. Thus, if as is commonly said we form a black hole out of a pure quantum state $|\psi_{\text{in}}\rangle$, a part of the information it carries can be recovered at late times by measurement of the stimulated emission contribution to the Hawking radiation. Note that as always in quantum mechanics, one needs a series of measurements on identically prepared systems to obtain a mean value or a probability distribution.

We should mention that the energies of initial particles that produce stimulated emission at late times will suffer a very large redshift. This can be seen from Eqs. (2) and (10) and has been discussed previously in the literature (23). Accordingly, the stimulated emission contribution at late times will have very low energies, making its detection difficult, although the effect is present in principle.

The information loss paradox was put forward (1) by considering only the thermal component of the Hawking radiation (i.e. the last term in Eq. (15)). However, to have a consistent picture of the problem one must also consider the contribution of the stimulated emission. In fact, if one assumes that the total mass of the black hole formed from a pure state is a fraction $\epsilon$ of the total energy content of the initial state, i.e. $M = \sum_j \epsilon_j \omega_{lm}^2$, neglecting (as is usually made) the effects of the stimulated emission would mean to neglect the contribution of the initial state, and thus no black hole that could emit Hawking radiation (thermal or not) would in that case have formed.

In the previous sections, we restricted our investigations to massless bosonic particles. To obtain generally applicable results we must also consider massive particles and fermions. For massive bosonic particles the general structure of the stimulated emission process remains the same, although the details of particle creation will be modified (cf. the general formulas in Ref. [14]). For fermions, on the other hand, it is well known that there is an attenuation induced by ingoing particles instead of an amplification (27, 28). It does also represent a deviation from thermality in which information can be encoded. This point has been discussed recently by Schiffer (17).

In a recent paper Bekenstein (16) remarked that by exploiting the fact that the radiation emitted by a black hole is not perfectly that of a black body, but distorted by a barrier penetration factor $\Gamma(\omega)$, information can leak out from the hole in the course of its evaporation. Indeed, additional contributions in Eq. (13) can be incorporated in a $\Gamma(\omega)$ factor, and Bekenstein’s considerations would then apply. However, stimulated emission is an important effect in itself and we consider it in an independent way. Also recently, Schiffer (17) has considered the effects of stimulated emission for bosons and fermions that fall in at late times, thus making, in some sense, a complementary study to ours.

Whereas the stimulated emission mechanism allows in principle to recover information about the initial state of the field, it does not solve the second part of the information loss problem: The final state of the field at $\mathcal{J}^+$ is not pure because of our missing knowledge about the detailed state of the thermal part of the black hole radiation. The corresponding information is contained in correlations.

We can compare the situation with the problem of signal transmission through a noisy communication channel. The initial state of the field plays the role of the message, the thermal component of the black hole radiation represents the noise. Even if we could reconstruct the complete message after receiving the noisy signal (we are not interested in information about the noise), this would not be tantamount to eliminating the noise.

In the black hole context, this means that recovering information about the initial state does not imply automatically the purity of the final state. The problem is not the lack of information about the initial state but about the final state of the system. Note the double role played by $|\beta_{\text{atm}}|^2$: On one hand, it is responsible for the noise in the final state of the field (cf. (13)). On the other hand, the recovery of information about the particles that crossed the horizon is only possible for nonzero $|\beta_{\text{atm}}|^2$ (cf. (13)). The two aspects of the paradox mentioned in the introduction must therefore carefully be distinguished.

Probably, the definite answer to the information loss paradox has to be given in the context of a full quantum theory of gravitation. We can argue, however, that the effects of stimulated emission will play an important role in this resolution since, as we have shown, they can carry a non-negligible part of the information about the initial state of the system (see Fig. 2).
In the light of the above results, there remain a number of possibilities to solve the problem. One alternative is that the purity of the state is preserved at early times. One could rely for example on the existence of an environment that continuously ‘measures’ the system, thereby eliminating the lack of information about the final state. Possibly, the answer to the question has to be searched for at the late stages of evaporation. Perhaps a decrease in the number of degrees of freedom of the evaporating black hole results in a reduced ability to store information in correlations, thereby restoring the purity of the state. Finally there is the possibility that the nonunitarity appears only in our present description of black hole evaporation. This does not need to rule out the unitarity of quantum gravity, however, since we are dealing with a semiclassical theory. In fact, it has been shown recently that nonunitary corrections appear if the Wheeler-DeWitt equation is approximated by an effective equation for matter fields in a classical background geometry. Thus the full quantum gravitational description of black hole evaporation may well be unitary.

A Appendix: Angular distribution of the Hawking radiation

The wave packet mode functions are eigenfunctions of angular momentum. Because of their dependence, we were not able to obtain an angular localization of the Hawking radiation. Let us therefore introduce angular wave packet modes at $J^+$:

$$f_{\omega n^\prime \Theta \Phi^\prime}(v, \vartheta, \varphi) = \sum_{l'm'} Y_{lm}^{*^\prime}(\Theta', \Phi') f_{\omega n'v lm}(v, \vartheta, \varphi)$$

which carry angular quantum numbers $(\Theta', \Phi')$. The usual angular coordinates are denoted by $\vartheta$ and $\varphi$. Using the completeness relation of the spherical harmonics it is easy to show that

$$f_{\omega n' \Theta \Phi^\prime}(v, \vartheta, \varphi) \sim \frac{\delta(\vartheta - \Theta') \delta(\varphi - \Phi')}{\sin(\vartheta)}$$

showing that the angular wave packet modes are entirely concentrated in the direction $(\Theta', \Phi')$ indicated by their quantum numbers. In the same manner we define angular wave packet modes at $J^+$:

$$p_{\omega n \Theta \Phi}(u, \vartheta, \varphi) = \sum_{lm} Y_{lm}^*(\Theta, \Phi) p_{\omega nm}(u, \vartheta, \varphi)$$

which are also concentrated in the direction of their quantum numbers $(\Theta, \phi)$.

When the quantization at $J^-$ and $J^+$ is performed with respect to these bases, the creation and annihilation operators refer to the angular wave packet modes and (26). The corresponding Bogoliubov transformation can be derived easily from (8). Using some well-known properties of the spherical harmonics we find

$$a_{\omega \Theta \Phi}^{\text{out}} = \alpha_{\omega \Theta \Phi \Phi^\prime}^* b_{\omega n' \Theta \Phi^\prime}^{\text{in}} - \beta_{\omega \Theta \Phi \Phi^\prime}^* b_{\omega n' \Theta \Phi^\prime}^{\text{in} \dagger}$$

where the Bogoliubov parameters are now given by

$$\alpha_{\omega \Theta \Phi \Phi^\prime} = \frac{i}{\pi \kappa} \left( \frac{\omega}{2 \pi} \right)^{1/2} e^{\pm \omega \pi/2\kappa} \left| \Gamma \left( 1 + i \frac{\omega}{\kappa} \right) \right| \sin \Theta$$

Their angular part shows that the wave packet simply crosses the spatial origin of coordinates. The radial part remains unchanged. The formulas (13) and (25) are easily transcribed in terms of angular wave packets. We now see that ingoing particles in a mode $(\omega', n', \Theta', \Phi')$ that travels behind the horizon can influence the probability for finding particles in the mode $(\omega, n, \pi - \Theta', \Phi' - \pi)$ at $J^+$. Since they are spacelike separated this shows the nonlocality of the stimulated emission process.

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Figure captions:

Fig. 1. Penrose diagram for a Schwarzschild black hole. Radially ingoing (outgoing) null geodesics are characterized by $v = \text{const}$ ($u = \text{const}$). $v_0$ denotes the null ray that reaches the event horizon $H^+$ at the moment of its formation. The ingoing wave packet $(\omega'_\alpha, n'_\alpha, l, m)$ is the equivalent mode to the outgoing wave packet $(\omega, n, l, m)$. Its mirror mode $(\omega'_\beta, n'_\beta, l, -m)$, which propagates behind the horizon, is obtained by ‘reflection’ at the line $v_0$.

Fig. 2. The information $I_{\omega l m} = (S^{th}_{\omega l m} - S^{bh}_{\omega l m})/S^{th}_{\omega l m}$ contained in an outgoing mode $(\omega, n, l, m)$ about the initial wave packet $(\omega'_\beta, n'_\beta, l, -m)$ that crosses the horizon plotted as a function of $|\beta_{\omega l m}|^2 = (\exp(\omega/T) - 1)^{-1}$. The number of ingoing particles is $r = 10, 100, 1000$ (bottom to top).
