GPDs and underlying spin structure of the nucleon

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Abstract

It is shown that, based only upon two empirically known facts besides two reasonable theoretical postulates, we are inevitably led to a model-independent conclusion that the quark orbital angular momentum carries nearly half of the total nucleon spin at the low energy scale of nonperturbative QCD. Also shown are explicit model predictions for the forward limit of the unpolarized spin-flip GPDs, which are believed to give valuable information on the distributions of quark angular momentum inside the nucleon.

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1 Introduction

The so-called nucleon spin puzzle raised more than 15 years ago is still an unsolved fundamental puzzle in hadron physics [1]. If intrinsic quark spin carries little of total nucleon spin, what carries the rest of the it? That is the question to be answered. Admitting that QCD is a correct theory of strong interactions, the answer must naturally be searched for in some one of the following three, i.e. the quark orbital angular momentum (OAM), the gluon polarization or the gluon orbital angular momentum.

An important remark here is that it has little meaning to talk about the spin contents of the nucleon without reference to the energy scale of observation. In fact, it is a widely known fact that the gluon polarization grows rapidly as $Q^2$ increases, even if it is small at low energy. Conversely, the gluon orbital angular momentum decreases rapidly to partially compensate the increase of $\Delta g$. Hence, when we talk about the nucleon spin contents naively, we should understand that we are thinking of it at low energy scale of nonperturbative QCD.

Roughly speaking, there exist two opposing or contrasting standpoints to try to answer the above question. The chiral soliton picture of the nucleon emphasizes the importance of the quark orbital angular momentum [2]. We recall that, the dominance of the quark OAM in these unique models can be traced back the collective motion of quark fields in the rotating hedgehog mean field [3].

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On the other hand, the possible importance of gluon polarization was emphasized by several authors on the basis of the axial anomaly of QCD \[6\]–\[8\]. Later, the role of QCD anomaly was understood more precisely at least within the framework of perturbative QCD. That is, the perturbative aspect of axial anomaly is understood as a factorization scheme dependence of the longitudinally polarized PDF in the flavor singlet channel. However, the nonperturbative aspect of it is left totally unresolved. As a consequence, no one can give any reliable theoretical prediction for the actual magnitude of $\Delta g$. Probably, one of the most promising attempts aiming at a direct measurement of $\Delta g$ is to use photon-gluon fusion processes. For instance, the Compass group recently extracted the value of $\Delta g/g$ from the analysis of the asymmetry of high $p_T$ hadron pairs \[9\]. Their first result for $\Delta g/g$ has turned out fairly small, $\Delta g/g \sim 0.06 \pm 0.31$, although it would be premature to draw any decisive conclusion only from this result.

On the other hand, the key quantity for the direct measurement of $J_q$ and/or $L_q$ is the generalized parton distributions (GPDs) appearing in the cross sections of deeply virtual Compton scattering and deeply virtual meson productions. As is widely known, what plays the central role here is Ji’s quark angular momentum sum rule \[10\].

## 2 Generalized form factors and quark orbital angular momentum

Here, let us start with the familiar definition of generalized form factors $A_{20}(t)$ and $B_{20}(t)$ of the nucleon, which is given as a nonforward nucleon matrix element of QCD energy momentum tensor:

$$
\langle N(P') | T_{\mu\nu}^{q,g} | N(P) \rangle = \bar{U}(P') \left[ A_{20}^{q,g}(t) \gamma^{(\mu} P^{\nu)} + B_{20}^{q,g}(t) \frac{P^{(\mu} i(\sigma^{\nu)} \Delta_\alpha)}{2M} \right] U(P).
$$

The famous Ji’s sum rule relates the total angular momentum carried by quarks and gluons to the forward limit of these generalized form factors \[10\] :

$$
J^{u+d} = \frac{1}{2} \left[ A_{20}^{u+d}(0) + B_{20}^{u+d}(0) \right],
$$

$$
J^g = \frac{1}{2} \left[ A_{20}^g(0) + B_{20}^g(0) \right].
$$

Here, the first $A_{20}(0)$ parts reduce to the total momentum fractions of quarks and gluons as

$$
A_{20}^{u+d}(0) = \int_0^1 \left[ u(x) + \bar{u}(x) + d(x) + \bar{d}(x) \right] dx \equiv \langle x \rangle^{u+d},
$$

$$
A_{20}^g(0) = \int_0^1 x g(x) dx \equiv \langle x \rangle^g,
$$

while the second $B_{20}(0)$ parts are sometimes called the anomalous gravitomagnetic moment. More precisely, $B_{20}^{u+d}(0)$ and $B_{20}^g(0)$ respectively stand for the quark and gluon contribution to the anomalous magnetic moment of the nucleon.

Now, our first important observation is that total nucleon AGM vanishes identically :

$$
B_{20}^{u+d}(0) + B_{20}^g(0) = 0.
$$
This is an exact field theoretical identity, since it just follows from the familiar total momentum and spin sum rules of the nucleon:

\[ A_{20}^{u+d}(0) + A_{20}^g(0) = \langle x \rangle^{u+d} + \langle x \rangle^g = 1 : \text{momentum sum rule,} \quad (7) \]
\[ A_{20}^{u+d}(0) + B_{20}^{u+d}(0) + A_{20}^g(0) + B_{20}^g(0) = 1 : \text{spin sum rule.} \quad (8) \]

To proceed further, we must distinguish 3 possibilities.

1. \[ B_{u}^{u+d}(0) = -B_{d}^g(0) \neq 0, \]
2. \[ B_{u}^{u+d}(0) = B_{d}^g(0) = 0, \]
3. \[ B_{u}^g(0) = B_{d}^g(0) = B_{20}^g(0) = 0. \]

It is interesting to see that the recent lattice simulation by the LHPC Collaboration support the 2nd possibility, i.e. the absence of the net quark contribution to the nucleon AGM [11]:

\[ B_{20}^{u+d}(0) = 0, \quad (\text{and} \quad B_{20}^g(0) = 0), \quad (9) \]

while it at least denies the 3rd possibility, which was indicated by Teryaev on the basis of the equivalence principle some years ago [12]. In fact, the result of the LHPC Collaboration for the difference of the u- and d-quark contributions to the nucleon anomalous gravitomagnetic moment shows that it is clearly nonzero and has a sizable magnitude. However, they also find that sum of the u- and d-quark contributions, i.e. the net quark contribution to the nucleon AGM is consistent with zero within the numerical errors.

In the following argument, we accept the relation (9) as a theoretical postulate. Once accepting it, Ji’s sum rule reduces to an extremely simple relation as follows,

\[ 2J^{u+d} = \langle x \rangle^{u+d}, \quad (10) \]

which dictates the equal partition of the momentum and total angular momentum of quark fields in the nucleon as advocated by Teryaev [12].

Now we can reach more surprising conclusion, based only upon two already known empirical information at low energies [13]. The 1st observation is that the quark and gluon fields shares about 70\% and 30\% of the total nucleon momentum at low energy scale of nonperturbative QCD:

\[ \langle x \rangle^{u+d} \simeq 0.7, \quad \langle x \rangle^g \simeq 0.3. \quad (11) \]

This can, for example, be convinced from the famous GRV fit of the unpolarized PDF at the NLO [14]. Given below is their gluon density given at the low energy scale around 600 MeV:

\[ xg(x, \mu_{NLO}^2) = 20.8x^{1.6}(1 - x)^{4.1} \quad \text{at} \quad Q_{ini}^2 = \mu_{NLO}^2 \simeq (630 \text{MeV})^2. \quad (12) \]

Using it, one finds that the momentum fraction carried by gluons is just about 30\% at this energy scale:

\[ \langle x \rangle^g = \int_0^1 xg(x, \mu_{NLO}^2)dx \simeq 0.3 \quad (30\%). \quad (13) \]

This conversely means that, at this low energy, the quark fields carry about 70\% of total nucleon momentum and also the total angular momentum:

\[ 2J^{u+d} = \langle x \rangle^{u+d} \simeq 0.7. \quad (14) \]
The 2nd observation is nothing but the celebrated EMC observation combined with the results of the subsequent polarized DIS experiments, which revealed that the quark spin fraction is only from 20% to 35%:

$$\Delta \Sigma \simeq (0.2 \sim 0.35) : \text{weakly scale-dependent.} \quad (15)$$

Combining these two observations, we are then inevitably led to the conclusion that the quark orbital angular momentum carries nearly half of the nucleon spin at the low energy scale around $Q^2 \simeq (600 \text{MeV})^2$!

$$2 L^{u+d} = 2 J^{u+d} - \Delta \Sigma \simeq (0.35 \sim 0.5). \quad (16)$$

3 Unpolarized GPDs and quark angular momentum distributions

Next, we turn to the discussion of the unpolarized GPD, which contains more rich information than the corresponding generalized form factors. Given below is the standard definition of the unpolarized GPDs $H(x, \xi, t)$ and $E(x, \xi, t)$:

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P', s' | \bar{\psi} \left( -\frac{\lambda n}{2} \right) \not{n} \psi \left( \frac{\lambda n}{2} \right) | P, s \rangle \quad (17)$$

$$= \bar{U}(P', s') \left[ H(x, \xi, t) \not{n} + E(x, \xi, t) \frac{i\sigma^{\mu\nu} n_{\nu} \Delta_{\mu}}{2M} \right] U(P, s). \quad (18)$$

As is widely known, the spin decomposition of the above amplitude is most conveniently carried out in the Breit frame:

$$H_E(x, \xi, t) \equiv H(x, \xi, t) + \frac{t}{4 M_N^2} E(x, \xi, t), \quad (19)$$

$$E_M(x, \xi, t) \equiv H(x, \xi, t) + E(x, \xi, t). \quad (20)$$

It corresponds to the Sachs decomposition of the nucleon electromagnetic form factors. In fact, the 1st moment of $H_E$ gives the Sachs electric form factor, while the 1st moment of $E_M$ does the Sachs magnetic form factor:

$$\int_{-1}^1 H_E(x, \xi, t) \, dx = G_E(t) : \text{electric F.F.,} \quad (21)$$

$$\int_{-1}^1 E_M(x, \xi, t) \, dx = G_M(t) : \text{magnetic F.F.} \quad (22)$$

Very recently, the forward limit of $E_M(x, \xi, t)$ was predicted within the framework of the chiral quark soliton model (CQSM). The isoscalar part was investigated by Ossmann et. al. [15], while the isovector part was studied by us [16]. We first look into the isoscalar part, which is directly related to the total quark contribution to the nucleon spin. One can verify that the model satisfies the following 1st and 2nd moment sum rules:

$$\int E_M^{u+d}(x, 0, 0) \, dx = 3 (\mu_p + \mu_n), \quad (23)$$

$$\int x E_M^{u+d}(x, 0, 0) \, dx = 2 J^{u+d} = 1. \quad (24)$$
That is, the 1st moment of $E_{M}^{u+d}$ gives the isoscalar magnetic moment of the nucleon. This is an important relation, because it means that the forward limit of the $E_{M}^{u+d}$ gives a distribution of nucleon isoscalar magnetic moment in Feynman momentum $x$-space (not in ordinary coordinate space). On the other hand, we can prove that the 2nd moment of $E_{M}^{u+d}$ is reduced to twice the total quark angular momentum, which turns out just unity in the model. This is only natural, because the CQSM is an effective quark model containing quark fields alone.

Fig.1(a) shows the CQSM prediction for $E_{M}^{u+d}(x,0,0)$. Here, the distribution in the negative $x$ region should be interpreted as that of antiquarks as $E_{M}^{u+d}(-x,0,0) = -E_{M}^{u+d}(x,0,0)$ with $x > 0$. What is remarkable here is the $1/x$ behavior of the contribution of Dirac sea quarks, first pointed out by Ossmann et.al. [15]. It is interesting to see that, because of the peculiar antisymmetric behavior with respect to $x$, the Dirac sea part gives no contribution to the 1st moment sum rule, while it gives a significant contribution to the 2nd moment, i.e. the nucleon spin sum rule.

Here we look into the relation between the quark angular momentum distribution and the momentum distribution in more detail. As mentioned before, the distribution $E_{M}$ consists of two parts, i.e. the familiar unpolarized distribution $f(x)$ and the genuine or anomalous part as,

$$E_{M}^{u+d}(x,0,0) \equiv f^{u+d}(x) + E_{M}^{u+d}(x,0,0). \quad (25)$$

Using Ji’s unintegrated sum rule, the quark spin and momentum distributions, i.e. $J^{u+d}(x)$ and $xf^{u+d}(x)$, are then related as,

$$2J^{u+d}(x) = xf^{u+d}(x) + xE_{M}^{u+d}(x,0,0). \quad (26)$$

That is, the anomalous part gives the measure of the difference of these two distributions. Here, we recall the important constraints for the anomalous part of distribution. Its first
moment is proportional to the isoscalar anomalous magnetic moment of the nucleon, which is empirically known to be quite small:

$$\int E^{u+d}(x, 0, 0) \, dx = 3 (\kappa^p + \kappa^n) : \text{small.}$$ (27)

On the other hand, its 2nd moment gives the isoscalar AGM, which vanishes exactly within the CQSM:

$$\int x E^{u+d}(x, 0, 0) \, dx = 0 : \text{absence of AGM.}$$ (28)

Very interestingly, one observes that, while the \(x\)-distribution of isoscalar anomalous magnetic is nonzero (though small), it gives no net contribution to the total nucleon spin. In other words, the net quark contribution to the nucleon spin is solely determined by the familiar unpolarized quark distribution \(f^{u+d}(x)\), which can also be interpreted as the canonical part of the isoscalar magnetic moment distribution in the Feynman \(x\)-space. We emphasize that this conclusion is never restricted to the CQSM. It would be intact also in real QCD, since it is equivalent to assuming Eq.(9), i.e. the absence of the net quark contribution to the nucleon AGM. One can also convince from Fig.1(b) that, because of the smallness of anomalous part of distribution \(E^{u+d}(x, 0, 0)\), the difference of the quark spin and momentum distributions is not very large.

Now we turn to the discussion of the isovector part. The model expression for the isovector distribution satisfies the desired 1st moment sum rule, that is, it reproduces the known theoretical expression for the nucleon isovector magnetic moment:

$$\int_{-1}^{1} E_{M}^{u-d}(x, 0, 0) \, dx = -\frac{M_N}{9} N_c \sum_{n \in \text{occ}} \langle n | (\mathbf{x} \times \mathbf{\alpha}) \cdot \mathbf{\tau} | n \rangle = \mu_p - \mu_n.$$ (29)

Fig.2(a) shows the CQSM prediction for the distribution \(E_{M}^{u-d}(x, 0, 0)\). The long-dashed curve is the contribution of three valence quarks, while the dash-dotted curve stands for the contribution of the polarized Dirac-sea quarks, while their sum is represented by the solid curve. Here, a prominent feature of the CQSM predictions for the isovector distribution is that the contribution of polarized Dirac-sea quarks has a large and sharp peak around \(x = 0\). What does it mean? Since the partons with \(x\) being 0 are at rest in the longitudinal direction, its large contribution to the magnetic moment must come from the motion of quarks and antiquarks in the transverse plane. If one remembers the important role of the pion clouds in the isovector magnetic moment of the nucleon, the above transverse motion can be interpreted as simulating the pionic quark-antiquark excitation with long-range tail. The validity of the proposed physical picture would be verified more clearly if one can experimentally determine the so-called impact parameter dependent parton distribution proposed by Burkardt and others [17].

Next, we compare the spin and momentum distribution in the isovector case. Assuming Ji’s relation also in this case, the measure of the difference between the spin and momentum distribution is again given by the genuine or anomalous part of the distribution \(E^{u-d}(x, 0, 0)\) as

$$2 J^{u-d}(x) = x f^{u-d}(x) + x E^{u-d}(x, 0, 0).$$ (30)
Figure 2: The theoretical prediction for $E_{M}^{u-d}(x,0,0)$ (a) and a comparison of the isovector quark spin and momentum distributions, $x E_{M}^{u-d}(x,0,0)$ and $x f^{u-d}(x)$ (b).

Here, we find a big difference with the isoscalar case. As is clear from the 1st moment sum rules or the magnetic moment sum rules,

$$\int f^{u-d}(x) \, dx = 1 : \text{small},$$

$$\int E_{M}^{u-d}(x,0,0) \, dx = \kappa_p - \kappa_n : \text{large},$$

the magnitude of the anomalous part is much larger than the canonical charge part here. Accordingly, one would expect that the difference of the spin and momentum distribution is fairly large in the isovector case. As shown in Fig.2(b), our theoretical calculation confirms that this is indeed the case.

4 Summary

$L_q$ or $\Delta g$? There has been long-lasting dispute over this issue. Here we advocated a viewpoint which favors the importance of $L_q$. In fact, relying only upon the following information, i.e. Ji’s quark angular momentum sum rule, the probable absence of the flavor singlet quark AGM, and the empirical PDF information evolved down to the low energy scale, we are inevitably led to the conclusion that the quark orbital angular momentum carries nearly half of the total nucleon spin at the low energy scale of nonperturbative QCD. Note that this is a model-independent conclusion, although the result is consistent with the prediction of the CQSM.

Naturally, for more definite confirmation, experimental extraction of the unpolarized spin-flip GPD, at least its forward limit, is indispensable. I stress that these forward distributions are interesting themselves, because they give the distribution of the nucleon anomalous magnetic moments in Feynman momentum space. Also desirable is experimental extraction
of impact-parameter dependent parton distributions, which would certainly contain more
detailed information not only on the origin of nucleon spin but also on the origin of the
anomalous magnetic moments of a relativistic composite particle. Can we really see chiral
enhancement near $x = 0$ or large $b_\perp$?

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