Two neutrino double-$\beta$ decay of $94 \leq A \leq 150$ nuclei for the $0^+ \rightarrow 2^+$ transition

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Abstract. Within the PHFB approach, the $0^+ \rightarrow 2^+$ transition of two neutrino double-$\beta$ decay of $^{94,96}$Zr, $^{100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes is studied employing wave functions generated with four different parametrizations of the pairing plus multipole type of two-nucleon interaction and the summation method. In comparison to the $0^+ \rightarrow 0^+$ transition, the nuclear transition matrix elements $M_{2\nu}(2^+)$ are quite sensitive to the deformation of the yrast $2^+$ state. Consideration of the available theoretical and experimental results suggest that the observation of the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay may be possible in $^{90}$Zr, $^{100}$Mo, $^{130}$Te and $^{150}$Nd isotopes. The effect of deformation on the $M_{2\nu}(2^+)$ is also studied.

PACS. 23.40.He Relation with nuclear matrix elements and nuclear structure – 21.60.Jz Hartree-Fock and random-phase approximations – 23.20.-g Electromagnetic transitions – 27.60.+j 90 $\leq A \leq 149$

1 Introduction

The nuclear double beta ($\beta\beta$) decay is a convenient tool to test the validity of models employed in the nuclear structure studies and probe the physics beyond standard model of electroweak unification (SM). Over the past years, the theoretical as well as experimental studies of the $\beta\beta$ decay has attracted a lot of attention and excellently reviewed in Ref. [1,2,3,4,5] and references therein. The two neutrino double-$\beta$ decay has attracted a lot of attention and excellently reviewed in theoretical as well as experimental studies of the electroweak unification (SM). Over the past years, the nuclear double beta ($2\nu\beta\beta$) decay is a second order process in weak interaction and is allowed in the SM. The neutrino-less double beta ($0\nu\beta\beta$) decay is far more interesting as it involves the Majorana neutrinos and violation of the lepton number conservation by two units. The observation of the $0\nu\beta\beta$ decay can not only establish the Majorana nature of neutrinos but also provide information on the physics beyond the SM [6].

The $\beta\beta$ decay can occur in four different modes namely, double-electron ($\beta^-\beta^-$) emission , double-positron ($\beta^+\beta^+$) emission, electron positron conversion ($e^+e^-$) and double-electron capture ($e\bar{e}$). The latter three processes are energetically competing. In the allowed approximation, the $0^+ \rightarrow 1^+$ transition is much less probable than the $0^+ \rightarrow 0^+$ and $0^+ \rightarrow 2^+$ transitions. The observation of $0\nu\beta\beta$ decay for the $0^+ \rightarrow 2^+$ transition can distinguish between the mechanisms involving the mass of the Majorana neutrinos and the right handed currents [7].

The theoretical implications and experimental aspects of the ground to the excited $2^+$ state transition of the $\beta\beta$ decay have been excellently reviewed over past years [8].

Out of 35 possible candidates, the $0^+ \rightarrow 0^+$transition of $2\nu\beta^-\beta^-$ decay has been observed for twelve nuclei [2] [12] and limits on the half-lives $T_{1/2}^{2\nu}$ of a number of isotopes for the $0^+ \rightarrow 0^+$ and $0^+ \rightarrow 2^+$ transitions have already been given [9]. The inverse half-life of $2\nu\beta^-\beta^-$ decay is a product of the phase space factor and model dependent nuclear transition matrix elements (NTMEs) $M_{2\nu}$. The phase space factors have been calculated employing the exact Dirac wave functions in conjunction with finite nuclear size and screening effects [10][11]. Using the observed experimental half-lives for the $0^+ \rightarrow 0^+$ transition, the NTMEs $M_{2\nu}$ has been extracted [12] and in all cases of $2\nu\beta^-\beta^-$ decay, it has been observed that the NTMEs $M_{2\nu}(0^+)$ are sufficiently quenched [13]. The main motive of all theoretical calculations is to understand the physical mechanism responsible for the observed suppression of $M_{2\nu}(0^+)$. Hence, the validity of different nuclear models can be tested by calculating $M_{2\nu}(0^+)$ and comparing them with the experimental value.

The $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay has not been experimentally observed so far. The marked variation in the theoretically calculated NTMEs $M_{2\nu}(2^+)$ for the $0^+ \rightarrow 2^+$ transition using different nuclear models is a general feature [8]. For example, the available results for $M_{2\nu}(2^+)$ of $^{96}$Zr show that the calculated NTMEs within QRPA [14][15][16], RQRPA(WS) [17], RQRPA (AWS) [17], and SRPA(WS) [18], differ by a factor of 341. Hence, the observation of the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ de-
cay can constrain the validity of different nuclear models employed in the calculation of NTMEs. Alternatively, a reliable theoretical prediction will supplement the experimental designing and planning to study this particular mode of $2\nu\beta^-\beta^-$ decay.

Employing the pnQRPA model, it has been shown by Raduta et al. [15] that the inclusion of deformation in the mean field can reduce the NTMEs $M_{2\nu}(2^+)$ up to a factor of 341. In the PHFB model, the pairing and deformation degrees of freedom are treated simultaneously [21]. This has motivated us to apply the same set of wave functions to study the $0^+ \rightarrow 0^+$ transition with the summation method [21]. This has motivated us to apply the same set of wave functions to study the $0^+ \rightarrow 0^+$ transition with the summation method [21].

Presently, the summation over the intermediate states is done by applying the summation method [21] to the study of $0^+ \rightarrow 0^+$ transitions of $2\nu\beta^-\beta^-$ decay. In sect. 2, the theoretical formalism to calculate the half-life $T_{1/2}^{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay is discussed. The results are presented in sect. 3. The final conclusions are given in sect. 4.

### 2 Theoretical Framework

The half life for the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay $T_{1/2}^{2\nu}(2^+)$ in 2n mechanism has been given in Refs. [19,20,22,23]. Using the summation method [21], the $0^+ \rightarrow 0^+$ and $0^+ \rightarrow 2^+$ transitions of $2\nu\beta^-\beta^-$ mode has already been studied by Hirsch et al. in the pseudo-SU(3) model [24,25]. Presently, the summation method has been extended to the study of $0^+ \rightarrow 0^+$ transition of $2\nu\beta^-\beta^-$ decay within the PHFB model [19,20].

In sect. 2, we outline the theoretical formalism to calculate the half life $T_{1/2}^{2\nu}(2^+)$ of $2\nu\beta^-\beta^-$ decay. The results are presented and discussed in sect. 3. The final conclusions are given in sect. 4.

$$T_{1/2}^{2\nu}(2^+) = G_{2\nu}(2^+) |M_{2\nu}(2^+)|^2$$

where the integrated kinematical factor $G_{2\nu}(2^+)$ has been calculated with good accuracy [26]. The model dependent NTME $M_{2\nu}(2^+)$ is given by

$$M_{2\nu}(2^+) = \sqrt{\frac{1}{2} \sum_N \langle \frac{2\nu}{2\nu} | \sigma | \frac{2\nu}{2\nu} \rangle \langle \frac{2\nu}{2\nu} | \sigma | \frac{2\nu}{2\nu} \rangle} \frac{|E_0 + E_N - E_l|^{3/2}}{E_0 + E_N - E_l}$$

where

$$E_0 = \frac{1}{2}(E_I - E_F) = \frac{1}{2}Q_{\beta\beta} + m_e$$

Presently, the summation over the intermediate states is performed using the summation method [21]. Extending the summation method already applied to the $0^+ \rightarrow 0^+$ transition of $2\nu\beta^-\beta^-$ decay [19,20] to the $0^+ \rightarrow 2^+$ transition, the NTME $M_{2\nu}(2^+)$ is written as

$$M_{2\nu}(2^+) = \sqrt{\frac{1}{2} \sum \langle \frac{2\nu}{2\nu} | \sigma | \frac{2\nu}{2\nu} \rangle \langle \frac{2\nu}{2\nu} | \sigma | \frac{2\nu}{2\nu} \rangle} \frac{|E_0 + E_N - E_l|^{3/2}}{E_0 + E_N - E_l}$$

and this expression is same as Hirsch et al. [25].

As each proton-neutron excitation is considered according to its spin-flip or non-spin-flip character, the use of the summation method in the present context goes beyond the closure approximation. The spin-orbit splitting is explicitly included in the energy denominator, and hence, the PHFB formalism in conjunction with the summation method goes beyond that previously employed in the pseudo SU(3) model [24,25]. In the PHFB model, the NTME $M_{2\nu}(2^+)$ for the $0^+ \rightarrow 2^+$ transition of $2\nu\beta^-\beta^-$ decay is calculated using

$$M_{2\nu}(2^+) = \sum_{\pi,\nu} \langle \nu | \pi \rangle \langle \pi | \nu \rangle \langle \nu | \pi \rangle \langle \pi | \nu \rangle$$

and the expressions for $n^d, n(Z,N), (Z+2,N-2)(\theta), f_{Z,N}$ and $F_{Z,N}(\theta)$ are given in Ref. [19,20].

### 3 Results and discussions

The model space, single particle energies (SPE’s), parameters of pairing plus multipolar type of effective two-body interaction have already been discussed in Refs. [19,20,27,28]. Specifically, the effective Hamiltonian is written as

$$H = H_{sp} + V(P) + \zeta_{qq} [V(QQ) + V(HH)]$$

where $H_{sp}$, $V(P)$, $V(QQ)$ and $V(HH)$ denote the single particle Hamiltonian, the pairing, quadrupole-quadrupole and hexadecapole-hexadecapole parts of the effective two-body interaction, respectively. The $\zeta_{qq}$ is an arbitrary parameter and the final results are obtained by setting the $\zeta_{qq} = 1$. The purpose of introducing $\zeta_{qq}$ is to study the role of deformation by varying the strength of the QQ and HH interactions.
interactions. In the $QQ$ part of the effective two-body interaction $V(QQ)$, the strengths of the proton-proton, the neutron-neutron and the proton-neutron interactions are denoted by $\chi_{2pp}$, $\chi_{2nn}$ and $\chi_{2pn}$, respectively. By reproducing the experimental excitation energies $E_{2+}$ of the $2^+$ state in two alternative ways provides two different parametrization of the $QQ$ interaction, namely PQQ1 [19], PQQ2 [20] and $PQQH2$ [28]. The inclusion of the hexadecapolar $HH$ part of the effective interaction adds two additional parametrizations, namely $PQQHH1$ [27] and $PQQHH2$ [28].

In Ref. [19,20,27,28], the reliability of wave functions generated with four different parametrizations of the effective two-body interaction, namely PQQ1, PQQHH1, PQQ2 and $PQQHH2$ was tested by comparing the theoretically calculated results for a number of spectroscopic properties, namely the yrast spectra, reduced $B(E2:0^+ \rightarrow 2^+)$ transition probabilities, quadrupole moments $Q(2^+)$ and $g$-factors $g(2^+)$ of $^{94,96,100}$Zr, $^{104}$Ru, $^{106,110}$Pd, $^{110}$Cd, $^{128,130}$Te, $^{128,130}$Xe, $^{150}$Nd and $^{150}$Sm isotopes with the available experimental data. In addition, the calculated $M2_{2\nu}$ and corresponding $T_{1/2}^{2\nu}$ for the $0^+ \rightarrow 0^+$ transition were compared with the available experimentally observed results. Presently, the same set of wave functions are employed to calculate the NTMEs $M_{2\nu}(2^+)$.

In table 1, the NTMEs $M_{2\nu}(2^+)$ calculated with wave functions generated with four different parametrizations of effective two-body interactions are presented. Although, there are only a set of four NTMEs $M_{2\nu}(2^+)$ for a statistical analysis, the estimated average NTMEs $\overline{M}_{2\nu}(2^+)$ and uncertainties $\Delta\overline{M}_{2\nu}(2^+)$ are given in the same table 1. The maximum uncertainty $\Delta\overline{M}_{2\nu}(2^+)$ in the average NTMEs $\overline{M}_{2\nu}(2^+)$ turns out to be about 45%, which implies that the NTMEs $M_{2\nu}(2^+)$ are highly sensitive to the deformation content of the intrinsic wave functions. The phase space factors $G_{2\nu}(2^+)$ have been calculated by Pahomi et al. [26] for most of the prospective $2\nu\beta-\beta$ emitters. However, the $G_{2\nu}(2^+)$ of $^{94,100}$Zr and $^{104}$Ru isotopes are not available. We calculate them by adopting the prescription of Suhonen and Civitarese [8], using axial vector coupling constant $g_A = 1.2701$ [29]. The calculated $G_{2\nu}(2^+)$ for the $0^+ \rightarrow 2^+$ transition of $2\nu\beta-\beta$ decay of $^{94}$Zr and $^{104}$Ru are $6.801 \times 10^{-30}$ y$^{-1}$ and $9.625 \times 10^{-25}$ y$^{-1}$, respectively.

A suppression of NTMEs $M_{2\nu}(0^+)$ for $2\nu\beta-\beta$ decay with respect to the spherical case has been reported when the parent and daughter nuclei have different deformations [27,30,31]. To investigate this effect for the $0^+ \rightarrow 2^+$ transition, we present the NTMEs $M_{2\nu}(2^+)$ for the $2\nu\beta-\beta$ decay of $^{94,96}$Zr, $^{100}$Mo, $^{104}$Ru, $^{106}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes in fig. 1 as a function of the difference in the deformation parameter $\Delta\beta_{2} = \beta_{2}(\text{parent}) - \beta_{2}(\text{daughter})$ between the parent and daughter nuclei. The NTMEs $M_{2\nu}(2^+)$ are calculated by keeping the deformation for parent nuclei fixed at $\zeta_{eq} = 1$ and changing the deformation of daughter nuclei by varying $\zeta_{eq}$ in the range 0.0 - 1.5. It can be observed that in all cases but for $^{128,130}$Te, the largest NTMEs correspond to the $|\Delta\beta_{2}|$ close to zero. With further increase in deformation, the NTMEs decrease with increase in $|\Delta\beta_{2}|$.

As already mentioned, it has been observed that the inclusion of deformation in the mean field can reduce the NTMEs $M_{2\nu}(2^+)$ calculated in the pnQRPA model up to a factor of 341 [15]. In table 2, we present the excitation energies $E_{2\nu}$, quadrupole moments $Q(2^+)$ of daughter nuclei along, $Q$-values of $0^+ \rightarrow 2^+$ transition $Q_{2\nu}$ and the $G_{2\nu}(2^+)$. According to the Grodzin’s rule [32], the excitation energies $E_{2\nu}$ and quadrupole moments $Q(2^+)$ are inversely related. Although, a smaller $E_{2\nu}$ can give a higher $Q$-value $Q_{2\nu}$ resulting in a larger phase space factor, the NTMEs $M_{2\nu}(2^+)$ are reduced due to a larger $Q(2^+)$. Thus, the $0^+ \rightarrow 2^+$ transition is intrinsically suppressed due to the nuclear structure effects in addition to the cubic dependence of the energy denominator.

A large number of experimental and theoretical studies have been carried out for the $0^+ \rightarrow 2^+$ transition of $2\nu\beta-\beta$ decay. Over the past years, the $0^+ \rightarrow 2^+$ transition of $2\nu\beta-\beta$ decay of $^{94}$Zr, $^{96}$Zr, $^{100}$Mo, $^{104}$Ru, $^{106}$Pd, $^{128,130}$Te, $^{128}$Xe, $^{150}$Nd and $^{150}$Sm isotopes has been experimentally investigated. However, the $2\nu\beta-\beta$ decay of $^{104}$Ru for the $0^+ \rightarrow 2^+$ transition has not been experimentally investigated so far. All the available theoretical and experimental results are compiled in table 3. We present only the theoretical $T_{1/2}^{2\nu}(2^+)$ for those models for which no direct or indirect information about $M_{2\nu}(2^+)$ is available to us. As already mentioned, there is a remarkable spread in the calculated NTMEs $M_{2\nu}(2^+)$ within different models. Specifically, the NTMEs $M_{2\nu}(2^+)$ calculated with the QRPA model without and with deformation vary by a factor of 2–341, corresponding to $^{100}$Te and $^{96}$Zr isotope, respectively. The average NTMEs $\overline{M}_{2\nu}(2^+)$ evaluated using the PHFBA approach are suppressed by a factor between 1 – 150 with respect to those of Raduta et al. [15] corresponding to $^{96}$Zr and $^{128}$Te isotopes, respectively. Consideration of the available theoretical and experimental results suggests that the prospective nuclei for the observation of the $0^+ \rightarrow 2^+$ transition of $2\nu\beta-\beta$ decay are $^{94,100}$Zr, $^{104}$Ru, $^{128,130}$Te and $^{150}$Nd.

4 Conclusions

Using a set of reliable wave functions generated with four different parametrizations of the effective two-body interaction namely, PQQ1, PQQHH1, PQQ2 and $PQQHH2$ [19,20,27,28], sets of four NTMEs $M_{2\nu}(2^+)$ have been calculated to study the $2\nu\beta-\beta$ decay of $^{94,96}$Zr, $^{100}$Mo, $^{104}$Ru, $^{106}$Pd, $^{128,130}$Te and $^{150}$Nd isotopes for the $0^+ \rightarrow 2^+$ transition. It is noticed that the $0^+ \rightarrow 2^+$ transition is intrinsically suppressed due to the cubic dependence of the energy denominator and nuclear structure effects. Specifically, a large phase space factor due a larger $Q$-value implies a smaller $E_{2\nu}$ resulting from a larger $Q(2^+)$, which results in the suppression of NTMEs $M_{2\nu}(2^+)$. The observation of Raduta et al. [15] that the inclusion of deformation in the mean field can reduce the NTMEs $M_{2\nu}(2^+)$ calculated within pnQRPA up to a factor of 341, motivated us to study the $0^+ \rightarrow 2^+$ transition of $2\nu\beta-\beta$
Fig. 1. NTMEs as a function of the difference in the deformation parameter $\Delta \beta_2$. "×" denotes the value of calculated NTMEs for $\Delta \beta_2$ at $\zeta = 1.0$. 

$^{94}$Zr$\rightarrow^{96}$Mo

$^{96}$Zr$\rightarrow^{96}$Mo

$^{100}$Mo$\rightarrow^{100}$Ru

$^{104}$Ru$\rightarrow^{104}$Pd

$^{110}$Pd$\rightarrow^{110}$Cd

$^{128}$Te$\rightarrow^{128}$Xe

$^{130}$Te$\rightarrow^{130}$Xe

$^{150}$Nd$\rightarrow^{150}$Sm
Table 1. Calculated NTMEs $M_{2\nu}(2^+)$ within the PHFB model and their average $\bar{M}_{2\nu}(2^+)$ along with standard deviation $\Delta M_{2\nu}(2^+)$. 

| Nuclei | \(PQQ\) | \(PQQHI\) | \(PQQ2\) | \(PQQHI2\) | \(\bar{M}_{2\nu}(2^+)\) | \(\Delta M_{2\nu}(2^+)\) |
|--------|---------|---------|---------|---------|----------------|----------------|
| \(^{94}\text{Zr}\) | 1.44×10^{-4} | 1.08×10^{-5} | 4.08×10^{-5} | 1.02×10^{-5} | 9.88×10^{-5} | 4.30×10^{-5} |
| \(^{96}\text{Zr}\) | 9.71×10^{-5} | 1.09×10^{-5} | 9.15×10^{-5} | 1.02×10^{-5} | 9.98×10^{-5} | 0.74×10^{-5} |
| \(^{100}\text{Mo}\) | 1.95×10^{-5} | 2.52×10^{-5} | 2.02×10^{-5} | 1.05×10^{-5} | 1.89×10^{-5} | 0.61×10^{-5} |
| \(^{104}\text{Ru}\) | 3.30×10^{-5} | 4.22×10^{-5} | 3.05×10^{-5} | 3.96×10^{-5} | 3.63×10^{-5} | 0.55×10^{-5} |
| \(^{110}\text{Pd}\) | 1.21×10^{-5} | 1.33×10^{-4} | 1.12×10^{-4} | 1.10×10^{-4} | 1.19×10^{-4} | 0.10×10^{-4} |
| \(^{128}\text{Te}\) | 1.19×10^{-6} | 2.89×10^{-6} | 1.54×10^{-6} | 2.65×10^{-6} | 2.07×10^{-6} | 0.83×10^{-6} |
| \(^{130}\text{Te}\) | 7.72×10^{-7} | 1.86×10^{-6} | 8.55×10^{-7} | 1.87×10^{-6} | 1.34×10^{-6} | 0.61×10^{-6} |
| \(^{156}\text{Nd}\) | 6.32×10^{-6} | 5.84×10^{-6} | 5.74×10^{-6} | 5.54×10^{-6} | 5.86×10^{-6} | 0.33×10^{-6} |

Table 2. Excitation energies \(E_{2^+}\), quadrupole moments \(Q(2^+)\) of daughter nuclei, Q-values of \(0^+ \rightarrow 2^+\) transition \(Q_{2^+}\) and the phase space factors \(G_{2\nu}(2^+)\) with \(Q_A = 1.2701\). 

| Transition | \(E_{2^+}\) (MeV) | \(Q(2^+)\) | \(Q_{2^+}\) (MeV) | \(G_{2\nu}(2^+)\) |
|------------|---------|---------|---------|---------|
| \(^{94}\text{Zr}\rightarrow^{94}\text{Mo}\) | 0.871099 | -0.13±0.08 | 1.145 | 6.801×10^{-30} |
| \(^{96}\text{Zr}\rightarrow^{96}\text{Mo}\) | 0.778213 | -0.20±0.08 | 2.572 | 1.494×10^{-18} |
| \(^{100}\text{Mo}\rightarrow^{100}\text{Ru}\) | 0.53959 | -0.54±0.07 | 2.494 | 1.460×10^{-18} |
| \(^{104}\text{Ru}\rightarrow^{104}\text{Pd}\) | 0.55579 | -0.47±0.10 | 0.743 | 9.625×10^{-25} |
| \(^{110}\text{Pd}\rightarrow^{110}\text{Cd}\) | 0.657751 | -0.40±0.04 | 1.360 | 1.228×10^{-20} |
| \(^{128}\text{Te}\rightarrow^{128}\text{Xe}\) | 0.4429 | | 0.425 | 1.429×10^{-24} |
| \(^{130}\text{Te}\rightarrow^{130}\text{Xe}\) | 0.5361 | | 1.989 | 4.632×10^{-19} |
| \(^{156}\text{Nd}\rightarrow^{156}\text{Sm}\) | 0.33395 | -1.32±0.19 | 3.037 | 3.253×10^{-17} |

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Table 3. Theoretically calculated NTMEs $M_{2ν}(2^+)$ and half-lives $T_{1/2}^{2ν}(2^+)$ for the $0^+ \rightarrow 2^+$ transition of $^{94,96}$Zr, $^{100}$Mo, $^{104}$Ru, $^{110}$Pd, $^{128,130}$Te and $^{150}$Nd nuclei along with experimental half-lives $T_{1/2}^{2ν}(2^+)$. "*" denotes the present calculation with the average NTME.

| Nuclei   | Model    | Ref. | Theory $|M_{2ν}(2^+)|$ | Experiment $T_{1/2}^{2ν}(y)$ | Ref. |
|----------|----------|------|----------------------|-------------------------------|------|
| $^{94}$Zr | PHFB     | *    | 9.88×10$^{-5}$       | 1.505×10$^{17}$               |      |
|          | QRPA$^\dagger$ | 33   | 0.0170               | 5.088×10$^{12}$               |      |
|          | QRPA$^\dagger$ | 33   | 0.0155               | 6.120×10$^{12}$               |      |
| $^{96}$Zr | PHFB     | *    | 9.98×10$^{-5}$       | 6.723×10$^{25}$               |      |
|          | QRPA     | 14   | (0.005-0.038)        | 2.677×10$^{22}$               |      |
|          | QRPA$^\dagger$ | 15   | 1.113×10$^{-4}$      | 5.403×10$^{25}$               |      |
|          | QRPA$^\dagger$ | 16   | 0.011                | 5.532×10$^{21}$               |      |
|          | RQRPA$^\dagger$ | 17   | 0.011                | 5.532×10$^{21}$               |      |
|          | RQRPA$^\dagger$ | 17   | 0.010                | 5.603×10$^{21}$               |      |
|          | SRPA     | 18   | 3.117×10$^{-4}$      | 6.889×10$^{24}$               |      |
| $^{100}$Mo | PHFB     | *    | 1.89×10$^{-5}$       | 1.924×10$^{27}$               |      |
|          | QRPA     | 19   | 0.033                | 6.290×10$^{20}$               |      |
|          | QRPA$^\dagger$ | 19   | 1.814×10$^{-4}$      | 2.081×10$^{25}$               |      |
|          | RQRPA    | 20   | 0.0078               | 1.126×10$^{22}$               |      |
|          | SRPA$^\dagger$ | 21   | (1.0-1.1)×10$^{22}$  | (1.0-1.1)×10$^{22}$           |      |
|          | SU(3)$^-$  | 21   | 1.82×10$^{-3}$      | 3.119×10$^{23}$               |      |
|          | SU(3)$^+$  | 21   | 7.3×10$^{-5}$       | 1.285×10$^{26}$               |      |
|          | SU(3)$^{++}$ | 21   | 1.53×10$^{-4}$      | 2.926×10$^{25}$               |      |
|          | MCM       | 21   |                     | (5.3-13)×10$^{20}$            |      |
| $^{104}$Ru | PHFB     | *    | 3.63×10$^{-5}$       | 7.867×10$^{22}$               |      |
|          | QRPA$^\dagger$ | 19   | 3.736×10$^{-3}$      | 7.444×10$^{28}$               |      |
|          | QRPA$^\dagger$ | 19   | 0.00792              | 1.656×10$^{28}$               |      |
|          | PFB$^\dagger$ | 19   | 0.00811              | 1.589×10$^{28}$               |      |
|          | PHFB$^\dagger$ | 19   | 1.19×10$^{-4}$      | 5.731×10$^{27}$               |      |
|          | QRPA$^\dagger$ | 15   | 6.671×10$^{-3}$      | 1.830×10$^{24}$               |      |
|          | QRPA$^\dagger$ | 15   | 0.0112               | 6.492×10$^{23}$               |      |
|          | SRPA$^\dagger$ | 15   | 0.00766              | 1.388×10$^{24}$               |      |
|          | SRPA$^\dagger$ | 14   | 5.621×10$^{-3}$      | 2.577×10$^{24}$               |      |
| $^{128}$Te | PHFB     | *    | 2.07×10$^{-6}$       | 1.636×10$^{25}$               |      |
|          | QRPA     | 15   | 3.055×10$^{-4}$      | 7.498×10$^{20}$               |      |
|          | QRPA$^\dagger$ | 16   | 0.00287              | 8.496×10$^{28}$               |      |
|          | SRPA     | 16   | 1.022×10$^{-3}$      | 6.700×10$^{29}$               |      |
|          | PHFB$^\dagger$ | 15   | 1.34×10$^{-6}$      | 1.201×10$^{30}$               |      |
|          | QRPA$^\dagger$ | 15   | 8.272×10$^{-5}$      | 3.155×10$^{26}$               |      |
|          | SRPA$^\dagger$ | 16   | 0.00016              | 8.433×10$^{25}$               |      |
|          | SRPA$^\dagger$ | 16   | 4.088×10$^{-3}$      | 1.292×10$^{23}$               |      |
| $^{150}$Nd | PHFB     | *    | 5.86×10$^{-6}$       | 8.940×10$^{26}$               |      |
|          | SU(3)    | 25   | 5.38×10$^{-5}$      | 1.062×10$^{25}$               |      |

\footnotesize{1WS basis; \(^\dagger\)AWS basis; \(\ast\)Spherical occupation wave functions; \(^{++}\)Deformed occupation wave functions}
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