I. INTRODUCTION

In quasi-one-dimensional structures, such as quantum wires or quantum point contacts (QPCs), an in-plane magnetic field induces a Zeeman splitting of different spin subbands. When this splitting equals the one-dimensional level spacing introduced by the lateral confinement of the structure, one finds crossing features similar to the 0.7 anomaly, as observed at zero magnetic field. Therefore, these features are called 0.7 analogs [1]. The most prominent of these 0.7 characteristics is the development of a shoulderlike structure in the conductance with increasing magnetic fields. In Fig. 1, one can see this shoulder in the original 0.7 regime (dashed ellipse) as well as the similar feature at the 0.7 analog (solid ellipse). The apparent similarities have intertwined the explanation attempts of 0.7 anomaly and 0.7 analogs, prominently featuring spontaneous spin-polarization [2], and quasilocalized states [3].

However, despite observed similarities, there are also features specific to the 0.7 analog that have no counterpart for the 0.7 anomaly. A striking example is the asymmetry in the magnetic field dependence of the conductance, depending on whether the analog is approached from higher or lower fields, see Fig. 1, which is a annotated version of Fig. 1 in Ref. [1]. While the 0.7 analog resembles the 0.7 anomaly at higher magnetic fields (green curve), the conductance curves at lower fields (red curve) are much more symmetric and show no sign of a 0.7 shoulder.

Some years ago, an interpretation of the 0.7 anomaly was introduced in Ref. [4] that traces its origins back to the structure of the noninteracting van Hove ridge in the local density of states. This interpretation has been supported by direct conductance calculations of the QPC via one-dimensional spinful tight-binding structure of the noninteracting van Hove singularity in the local density of states. However, in a similar manner as the 0.7 anomaly, evoking a smeared van Hove singularity in the local density of states. However, the effects of the electrons in the lowest spin subband are of critical importance. We demonstrate that these electrons cause the above-mentioned asymmetry in the magnetic field dependence of the conductance and study its dependence on the ratio of intra- to interband interaction strength.

II. THEORETICAL MODEL AND METHOD

A. Model

Since our goal is a qualitative understanding of the 0.7 analog physics, we use here the simplest model that should be able to give us the relevant features. We model the lowest two bands of the QPC via one-dimensional spinful tight-binding chains with an intra- and interband short-ranged interaction. The external magnetic field is modeled by a Zeeman term, splitting the energies of spin up and spin down electrons. We point out that, in experiments, one observes additionally to the Zeeman effect also a diamagnetic shift with increasing magnetic field. This shift is understood analytically [6], and is expected not to be relevant for the qualitative physics of interest here [1]. Therefore, we will omit this effect in the present qualitative study, and concentrate on the physics caused by the interactions. Our Hamiltonian will thus be of the form

$$H = -\tau \sum_{i,\sigma} [c_{i\sigma}^\dagger c_{i+1\sigma} + H.c.] + \sum_{i,\sigma} V_{i\sigma} n_{i\sigma} + \sum_{i} U_{i}^{\text{inter}} n_{i\uparrow} n_{i\downarrow} + \sum_{i,\sigma_1,\sigma_2} U_{i}^{\text{inter}} n_{i\sigma_1} n_{i\sigma_2},$$

where $c_{i\sigma}$ annihilates an electron at site $i$ in band $s$ with spin $\sigma \in \{+, -\} = \uparrow, \downarrow$, and $n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}$ is the corresponding number operator. In our calculations, we will use the hopping amplitude $\tau$ as unit of energy, i.e., we measure the on-site energy, $V_{i\sigma}$, as well as the intraband interaction, $U_{i}^{\text{ intra}}$, and the interband interaction, $U_{i}^{\text{ inter}}$, in units of $\tau$. Within a
central region, \( i \in [-N, N] \), we use the following form for the potential term:

\[
V_{i\sigma} = V_g \exp \left[ -\frac{(i/N)^2}{1 - (i/N)^2} \right] + V_{\text{off}} + \sigma \frac{B}{2}.
\]  (2)

Here the first summand leads to a quadratic barrier top in the middle of the QPC with curvature \( \Omega_s = 2\sqrt{V_g/N} \) and corresponding characteristic length \( l_s = a/\sqrt{\varepsilon_F} \Omega_s \), with \( a \) being the lattice constant. The second term constitutes the band offset (we choose \( V_{\text{off}} = 0 \), and therefore use the abbreviation \( V_{\text{off}} := V_{2} = V_{\text{off}} \)) and the third term is the Zeeman splitting. To illustrate these settings, we have plotted the potential structure in Fig. 2.

Analogous to Ref. [4], we take both \( V_g \) as well as \( U_i^{\text{intra}} \), and \( U_j^{\text{inter}} \) to be zero outside of the central region, where we thus have two noninteracting tight-binding leads with the site independent energy offset

\[
V_{i\sigma} = V_{\text{off}} + \sigma \frac{B}{2}.
\]  (3)

Those can be integrated analytically and their contribution absorbed in the self-energy \( \Sigma \) of the central region. Note that this contribution will, however, depend on \( V_{\text{off}} \), as well as \( B \). The short-ranged interactions \( U_i^{\text{intra}} \) and \( U_j^{\text{inter}} \) are treated as free parameters, chosen as site independent within the middle of the central region, and reduced smoothly to zero at its edges. All our calculations will be carried out in thermal equilibrium at zero temperature, implying that all states below the chemical potential \( \mu \) are filled, all states above empty.

Our typical observable will be the linear response conductance through the system, and its dependence on the chemical potential \( \mu \), as well as on the magnetic field \( B \).

Note that to keep things simple and clear, we have made here several simplifying assumptions. We omit any hopping terms between the two bands, keep the offset between the bands a site independent constant throughout the whole system (in particular the barrier curvature for both bands is the same) and omit any longer-ranged interactions. Furthermore, in all our calculations we will keep \( V_g \) constant and vary \( \mu \) instead. In terms of the Fermi energy on the central site, \( \epsilon_F = \mu - V_g \), this is the same as varying \( V_g \) with constant \( \mu \), but has the advantage that the bare curvature \( \Omega_s(V_g) \) of the barrier does not change.

**B. Method**

To determine the interaction-induced self-energy, \( \Sigma \), and two-particle vertex, \( \gamma \), we use the recently introduced eCLA fRG scheme [5] within a static implementation. This scheme was originally designed to treat longer-ranged interactions. It enables the treatment of our two-band model, since it is possible to map the Hamiltonian Eq. (1) onto a one-dimensional chain model with longer-ranged interactions. For this, we simply interleave the different bands, as sketched in Fig. 3, leading to a new effective one-dimensional Hamiltonian, containing interactions between neighboring sites:

\[
H_{\text{eff}} = -\tau \sum_{j,\sigma} \left[ c_j^{\dagger} c_{j+1,\sigma} + \text{H.c.} \right] + \sum_{j,\sigma} \tilde{V}_{j\sigma} n_{j\sigma}
\]

\[
+ \sum_j U_i^{\text{intra}} n_j n_{j+1} + \sum_{j,\sigma_1,\sigma_2} \tilde{U}_j^{\text{inter}} n_{2j\sigma_1} n_{2j+1\sigma_2}.
\]  (4)

Here the new index is given by \( j = 2i + s - 1 \) (\( s = 1 \) is band 1, \( s = 2 \) is band 2), and the coefficients are \( \tilde{V}_{j\sigma} = V_{i\sigma} \), \( \tilde{U}_i^{\text{intra}} = U_i^{\text{intra}} \), and \( \tilde{U}_j^{\text{inter}} = U_j^{\text{inter}} \). We will sometimes use \( \alpha = (s, \sigma) \) as composite species index.

This Hamiltonian is now in a form suitable for the eCLA approach. Without going into detail, we just point out that this method depends crucially on a dimensionless parameter, \( L \), called the feedback length in Ref. [5], which determines the spatial extent of the renormalized vertex, \( \gamma \). This \( L \) has to

---

**FIG. 2.** Schematic illustration of the potential structure for the two spin-split bands, as given by Eq. (2). Note that the curvature of the barrier, \( \Omega_s \), is the same for all four subbands.
be chosen large enough to reach convergence, and we will comment on the convergence properties in the beginning of the next section.

Finally, the calculation of the zero-temperature linear response conductance, \( g = \frac{\hbar}{2e^2V} \), from the self-energy obtained with our fRG method, is carried out via the formula [7–9]

\[
g = \frac{1}{2} \sum_{\sigma,s} |2\pi \rho^{\sigma s}(\mu + i0^+)G_{\sigma NN}^{-}(\mu + i0^+)|^2, \tag{5}
\]

where \( \rho^{\sigma s} \) is the density of states on the first lead site for spin \( \sigma \) and band \( s \), and \( G_{\sigma NN}^{-} \) is the propagator for a electron in band \( s \) with spin \( \sigma \) from the leftmost to the rightmost site of the central region.

III. RESULTS

We use the following general settings in this section: The band offset is chosen as \( V_{\text{off}} = 0.1\tau \) and \( N = 30 \), therefore the total number of spatial sites in the central region is \( N_{\text{tot}} = 61 \) and correspondingly the total number of effective sites in Eq. (4) is \( N_{\text{eff-tot}} = 122 \). Furthermore, except for Fig. 6, we set \( V_{g} = 0.5\tau \), implying a curvature \( \Omega_1 \approx 0.05\tau \).

In Fig. 4(a), we show the noninteracting, as well as the fully interacting conductance for our two-band model, with the simplest nontrivial interaction configuration, \( U_{\text{intra}} = U_{\text{inter}} = 0.7\tau \). These values correspond to a typical value for the onsite interactions in a one-band QPC used in Ref. [4].

The main changes caused by the interaction are the slightly more asymmetric shape of the conductance steps, and the shift to larger chemical potentials observed for the second step. Qualitatively, this shift is caused by the additional interaction energy between the electrons of the two bands (Hartree shift).

A. Convergence in \( L \)

Before we proceed, let us first discuss the convergence of our method with respect to the feedback length \( L \). For a one-band QPC with onsite interactions, \( L \) has to be of the order of the characteristic length of the harmonic barrier top to achieve convergence: \( L \approx l_x/a \), with the lattice spacing \( a \). For our interleaved two-band system, we would thus simply expect \( L \approx 2l_x/a \), since the effective distance between two points of the same band is doubled and the effect of the now-finite interaction range on the convergence should be negligible, since the introduced nearest-neighbor interaction is still much shorter than \( l_x \). In Fig. 4(b), the convergence behavior in \( L \) is shown. We see that the convergence for the two-band model is achieved around \( L \approx 5 \). Since in our system \( l_x \approx 4.6a \), this shows that \( L \) can in fact be chosen smaller than the naive guess, \( L \approx 2l_x/a \), indicating stabilizing feedback effects between the two bands.

As a side remark, we point out that the finite extent of the renormalized vertex beyond the lowest value (i.e., \( L > 1 \)) is actually important to treat the screening properties between the two bands. This will be seen in the next section when we study the magnetic-field dependence of the conductance.

B. Small magnetic field

Before we look at the 0.7 analog, we want to take a brief look at the properties of the conductance at magnetic fields much smaller than the band spacing, \( B \ll V_{\text{off}} \), see Fig. 5, solid curves.

There are two main observations we make here: First, we see that the magnetic-field dependence of the second step is more symmetric, indicating that the interaction of the electrons in the second band is screened by electrons in the first band. Second, we see that the second conductance step is broader than the first one. This feature can be qualitatively understood in a simple Hartree picture: While increasing \( \mu \) during the second step, electrons are still filling up the lowest band, leading to a increasing Hartree shift for the...
FIG. 5. Solid curves: Conductance at low magnetic fields, i.e., with $B \ll V^{\text{off}} = 0.1 \tau \approx 2.12 \Omega$, and $U^{\text{intr}} = U^{\text{inter}} = 0.7 \tau$. The second spin-split double step is more symmetric and broader than the first. Dotted curves: Spin resolved electron densities $n_{\alpha}$ on the central QPC site for $B/\Omega_x = 0$ (blue) and $B/\Omega_x = 1.06$ (black). Note the damping that appears in $n_{\alpha}$ whenever a different particle species enters the QPC.

electrons in the higher band. As a result, the second step gets broadened.

To further validate this explanation, we can compute an estimate for the observed broadening via

$$\Delta E = \mu (\Delta n_{1\downarrow} + \Delta n_{1\uparrow}) a,$$

(6)

where $\Delta n_{1\sigma}$ is the total change of the first-band spin-$\sigma$ density at the center of the QPC during the second conductance step. Instead of considering a “pure” Hartree effect where one would use $n_{1\sigma}$ only the density of a system without interband interaction, we can improve on that by using the actual fully interacting densities that we obtained from our fRG calculation. These densities are given by

$$n_{i\sigma} = \int_{-\infty}^{\mu} d\omega A_i^\sigma(\omega),$$

(7)

where the local density of states at site $i$ for particle species $\sigma$, $A_i^\sigma(\omega) = -\text{Im} G_i^\sigma(\omega)/\pi$ is given by the imaginary part of the fully interacting retarded electron propagator $G_i^\sigma(\omega)$. It is instructive to take a quick look at these densities themselves: In Fig. 5, we have plotted the density for the different particle species in the center of the QPC, $n_{\alpha} \equiv n_{0}^\alpha$ (dotted curves). Each time a new particle species enters the QPC, the increase of any other species $n_{\sigma}$ is slowed down, or “damped,” due to the corresponding interaction. The damping of $n_{1\sigma}$ during the second conductance step will lead (starting in second order in $U^{\text{inter}}$) to a reduction of the pure Hartree broadening of that step. We see that the damping in $n_{1\sigma}$ is most pronounced at $B = 0$, when both $n_{2\uparrow}$ and $n_{2\downarrow}$ particles enter the QPC at the same time. Correspondingly, the width of the second conductance step is only slightly larger than that of the first. On the other hand, for $B/\Omega_x = 1.06$, the damping in $n_{1\sigma}$ is relatively small, leading to a pronounced broadening of the second conductance step. Using Eq. (6), the effective Hartree broadening can be obtained from the change of the densities $n_{1\sigma}$ during the second conductance step. For example, in the $B/\Omega_x = 1.06$ case, the second conductance step occurs between $\mu/\Omega_x \approx 3.3$, with densities $n_{1\downarrow} \approx 0.12/a$, $n_{1\uparrow} \approx 0.08/a$, and $\mu/\Omega_x \approx 7.8$, with densities $n_{1\downarrow} \approx 0.15/a$, $n_{1\uparrow} \approx 0.13/a$. Therefore, the effective Hartree broadening given by Eq. (6) is

$$\Delta E \approx 0.7 \tau \cdot [(0.15 - 0.12) + (0.13 - 0.08)]$$

$$\approx 0.056 \tau \approx 1.1 \Omega_x.$$
The qualitative behavior of this equations is shown in Fig. 8, and provides a good explanation for the observed phenomena: In contrast to the noninteracting case [Fig. 8(a)], we obtain for $U_{\text{intra}} = U_{\text{inter}}$ a pinch-off asymmetry, $\Delta \mu$, between the pinch-offs at magnetic fields above and below the analog, see Fig. 8(b). Taking into account the interaction between 1↑ and 2↓ (whose main effect is a broadening of the second half-step), this results in the more symmetric arrangement of the two half-steps around the crossing curve for $\Delta B < 0$, and to a more asymmetric situation in the $\Delta B > 0$ case. However, we see that the shape of corresponding curves is the same since the $\mu$-width of the half-steps, $\Delta \mu_{1\uparrow}$ and $\Delta \mu_{2\downarrow}$, is equal.

If we compare this to experiment [1], we see that this setting reflects only partially the experimental situation: While the half-steps are indeed arranged more symmetrically for $\Delta B < 0$ than for the $\Delta B > 0$ case, also the form of the corresponding curves themselves differs substantially in experiment. For $\Delta B > 0$, the conductance curves are much more asymmetric in the $\mu$ behavior, developing a 0.7 analog plateau, while for $\Delta B < 0$ they are not. To analyze this quantitatively in our calculation, we introduce the “conductance asymmetry” $\Delta g(\mu) = g_m(\mu) - g(\mu)$, where $g_m(\mu)$ is the mirror image of $g(\mu)$ around the point $g(\mu)/g_0 = 1.0$ under reflection in both the horizontal and vertical direction. The more asymmetric the conductance curve is in $\mu$, the larger gets the modulus of $\Delta g$. This is illustrated in Fig. 9(a). Figure 9(b) shows the dependence of this asymmetry $\Delta g$ on the magnetic field. We see that contrary to the experiment the asymmetry is equally strong above and below the crossing value $B_c$.

This indicates that our description up to now lacks an important ingredient. We will argue in the following that this is due to the unphysical choice $U_{\text{intra}} = U_{\text{inter}}$. Generically, one would expect $U_{\text{inter}} < U_{\text{intra}} < U_{\text{intra}}$. The first statement is due to the smaller overlap of the transversal wave functions between different bands, the second because the transversal wave function in the second band is spread out wider than in the first band. Both effects lead to a weakening of the effective one-dimensional interaction strength. Estimates for the ratios of this different interaction strengths can be obtained in a similar manner as in Ref. [5], see Appendix, and yield $U_{\text{intra}}^2/U_{\text{intra}} < 0.77$ and $U_{\text{inter}}/U_{\text{intra}} < 0.36$. Keeping our previous $U_{\text{intra}}^2$ fixed, this leads approximately to $U_{\text{intra}} < 0.5\tau$ and $U_{\text{inter}} < 0.3\tau$.

To investigate the influence of these differences in interaction strength, we proceed in two steps. In the ideal case where the analog region is well separated from the 2↑ conductance step, we expect that the influence of $U_{\text{intra}}^2$ at the analog is not important, since the barrier for the 2↑ electrons is way above the chemical potential. Therefore, we will first keep $U_{\text{intra}}^2$ equal to $U_{\text{intra}} = 0.7\tau$ and investigate the influence of a reduction of $U_{\text{intra}} = 0.3\tau$ alone. In Fig. 10, we show the resulting conductance curves. Again, we encounter a pinch-off shift of the higher spin subband steps; however, due to
FIG. 8. Schematic behavior of the Hartree renormalized barriers of the $1\uparrow$ (red) and $2\downarrow$ (blue) particles as function of $\mu$ and $B$. The colored regions indicate where $|b_\mu - \mu| < \Omega_c/2$, i.e., the regions within which the conductance steps occur. (a) Noninteracting case: $B_c = V^{\text{off}}$, no pinch-off asymmetry, no shape asymmetry. (b) $U^{\text{intra}} = U^{\text{inter}}$: $B_c = V^{\text{off}}$, pinch-off asymmetry ($\Delta p > 0$), no shape asymmetry. (c) $U^{\text{intra}} > U^{\text{inter}}$: $B_c < V^{\text{off}}$, pinch-off asymmetry ($\Delta p > 0$) and shape asymmetry ($\Delta \mu_{1\uparrow} > \Delta \mu_{2\downarrow}$).

the different interaction strengths, the crossing point $B_c$ is now shifted, too. More importantly, we see that in addition to the pinch-off asymmetry, also the shape of corresponding curves for $\Delta B < 0$ and $\Delta B > 0$ differ, the curves for $\Delta B < 0$ being much more symmetric than the $\Delta B > 0$ curves. This is the behavior also observed in experiment and for further reference, we will call it the “shape asymmetry.”

These features can be readily explained with our Hartree picture for the renormalized barrier positions Eq. (9). Their behavior for $U^{\text{intra}} > U^{\text{inter}}$ (i.e., the Hartree shift for the $2\downarrow$ subband is smaller than for the $1\uparrow$ subband) is shown in Fig. 8(c). We see two immediate effects: (i) The $2\downarrow$ subband is shifted to lower values of $\mu$ and therefore the value of the magnetic field $B_c$, where the two subbands cross is shifted to lower magnetic fields, as encountered in the Fig. 10, and (ii) the width $\Delta \mu_{2\downarrow}$ of the $2\downarrow$ half-step is decreased, therefore yielding the shape asymmetry: For $\Delta B < 0$, the first half-step ($1\uparrow$) is broader than the second half-step ($2\downarrow$), thus counteracting the asymmetry introduced by the interband interaction between the competing particles themselves and leading in total to a more symmetric curve. For $\Delta B > 0$, the effect is reversed, leading to a more asymmetric curve.

Furthermore, Fig. 8(c) exhibits a third interesting, albeit less pronounced feature: Due to the smaller interaction with the lowest electrons, the $\mu$-width of the $2\downarrow$-strip is smaller than the width of the $1\uparrow$-strip and therefore the two middle corners of the intersecting diamond [light pink region in

FIG. 9. Illustration of the asymmetry in $\mu$. (a) Conductance curve $\Delta B/\Omega_c = -0.64$ from Fig. 7 (black), together with its mirror image (red) under inversion around the $g = 1.0$ point, and the difference $\Delta g$ between the two curves (blue). (b) Colorplot of $\Delta g$ as a function of magnetic field and chemical potential. We see that the strength of the curve asymmetry is symmetric around $B_c$.

FIG. 10. (a), (b) same plot as in Fig. 7, but for larger intra- than interband interaction ($U^{\text{intra}} = 0.7\tau$, $U^{\text{inter}} = 0.3\tau$), resulting in $B_c \approx 1.48\Omega_c$. In (b), the dashed curve is again the $\Delta B/\Omega_c = -0.64$ curve from (a), manually shifted such that it intersects the corresponding $\Delta B/\Omega_c = 0.64$ curve at the $g/g_0 = 1$ point. However, contrary to Fig. 7(b), the shape of the two curves does not coincide. (c) Colorplot of the shape asymmetry. In contrast to Fig. 9(b), we see that the asymmetry is clearly stronger for $\Delta B > 0$ than for $\Delta B < 0$. 

FIG. 10. (a), (b) same plot as in Fig. 7, but for larger intra- than interband interaction ($U^{\text{intra}} = 0.7\tau$, $U^{\text{inter}} = 0.3\tau$), resulting in $B_c \approx 1.48\Omega_c$. In (b), the dashed curve is again the $\Delta B/\Omega_c = -0.64$ curve from (a), manually shifted such that it intersects the corresponding $\Delta B/\Omega_c = 0.64$ curve at the $g/g_0 = 1$ point. However, contrary to Fig. 7(b), the shape of the two curves does not coincide. (c) Colorplot of the shape asymmetry. In contrast to Fig. 9(b), we see that the asymmetry is clearly stronger for $\Delta B > 0$ than for $\Delta B < 0$. 

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Fig. 8(c) do not lie on the $B_c$ line, but are shifted slightly upward or downward from it. Directly at the $B_c$ line, i.e., at the 0.7 analog, the broader $1\uparrow$ and the thinner $2\downarrow$ steps superimpose symmetrically, which leads to a conductance curve with slightly less steep parts at the onset and at the end in $\mu$ direction. Since the Hartree picture we use in Fig. 8 takes only the interaction with the $1\downarrow$ electrons and not the interaction between the $1\uparrow$ and $2\downarrow$ electrons themselves into account, we expect this small effect to be most prominent at the onset of the second conductance step. In the experimental data, one might interpret the slight kink that occurs at the onset of the blue 0.7 analog step, compared to the smooth $B = 0$ curve, as a result of the described effect, compare circled onsets in Fig. 11. However, this feature is quite weak and could also be caused by other causes, e.g., a gate-dependent deformation of the QPC potential. Furthermore, we do not observe any visible effect of this kind in our fRG calculations, see Fig. 10.

As a last step, we finally also reduce $U_{1\text{ intra}}^{\text{ intra}} = 0.5\tau < U_{1\text{ intra}}^{\text{ intra}}$. The results are shown in Fig. 12. We see that the reduction of $U_{1\text{ intra}}^{\text{ intra}}$ slightly shifts the crossing point $B_c$ to lower values of the magnetic field; however, the shape asymmetry introduced by the lowering of $U_{\text{ inter}}^{\text{ inter}}$ stays intact. Thus, in terms of Fig. 8, the net effect of the reduction of $U_{1\text{ intra}}^{\text{ intra}}$ is simply a slight shift of the blue $2\downarrow$ barrier top position stripe to the left, i.e., to lower values of $\mu$, without changing its slope.

D. Limitations

A limitation of our static zero temperature calculation is that we have no access to inelastic processes. We suspect that this leads to a main difference between our results and experimental observations, namely that we do not see a pronounced finite temperature plateau in the conductance. This can be clearly seen by comparing the transconductances $dg/d\mu$, see Fig. 13, where we do not observe the “gap” at $\Delta B > 0$ as in the experimental data, cf. Fig. 2(a) in Ref. [11] or Fig. 1(b) in Ref. [11]. However, we also see in the transconductance, that for $\Delta B > 0$ the broadening of the conductance curve in the second half-step is more pronounced than for $\Delta B < 0$, where the half-steps are more symmetric in position as well as slope.

IV. CONCLUSION

We have studied the 0.7 analog in QPCs using a two-band model with intra- and interband onsite interactions and found that we could qualitatively reproduce the magnetic field dependence of the conductance around the analog. In particular, we could reproduce the asymmetry in the conductance, depending on whether the analog is approached from higher or lower magnetic fields.

Due to our use of a static fRG scheme, we were not able to investigate finite temperature properties of the analog, which is an interesting direction for further research.
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APPENDIX: ESTIMATE OF THE QPC INTERACTION STRENGTHS

Following the approach of Ref. [12], we calculated in Ref. [5] the intraband interaction for a QPC with a single band that resulted from a screened Coulomb interaction. This was done by taking only the ground state $\phi_1$ of the transversal $y$ direction (in the two-dimensional electron gas plane) into account. Since the confinement in $y$ direction can be approximated by a harmonic potential, $\phi_1$ is simply the ground state of a harmonic oscillator. In a QPC with two bands, we additionally also take the first excited state of the harmonic confinement into account. The computation of the resulting matrix elements for the interaction between two effective one-dimensional states at $x_0$ and $x_1$ can be done analogously to the one-dimensional case and yields in terms of integrals over the relative coordinate $r$ in the transversal direction:

\[
U_{\text{intra}}^{0}(x_0, x_1) = \left( I_y^2(x_0) + l_y^2(x_1) \right)^{1/2} \int dr \, g(r),
\]

\[
U_{2}^{\text{intra}}(x_0, x_1) = \left( I_y^2(x_0) + l_y^2(x_1) \right)^{9/2} \int dr \, g(r) \left[ 3l_y^2(x_0)l_y^2(x_1)\left( I_y^2(x_0) + l_y^2(x_1) \right)^2 + l_y^2(x_0) + l_y^2(x_1) \right]
\times \left[ I_y^0(x_0) - 4l_y^0(x_0)l_y^2(x_1) + l_y^2(x_1) \right] r^2 + l_y^2(x_0)l_y^2(x_1) r^4 \right],
\]

\[
U_{\text{inter}}^{0}(x_0, x_1) = \left( I_y^2(x_0) + l_y^2(x_1) \right)^{3/2} \int dr \, g(r) \left[ l_y^0(x_1) + l_y^2(x_0) \left( l_y^2(x_1) + r^2 \right) \right],
\]

where $l_y(x)$ is the (x dependent) characteristic length in $y$ direction, $e$ the electron charge, $\kappa$ the dielectric constant, and $g(r)$ (which consists of the screened Coulomb interaction, as well as the lateral confinement) is given by

\[
g(r) = \frac{e^2}{\kappa} \left[ \frac{1}{\sqrt{(x_0 - x_1)^2 + r^2}} - \frac{1}{\sqrt{(x_0 - x_1)^2 + r^2 + l_y^2}} \right] e^{-r^2/(2l_y^2(x_0)+l_y^2(x_1))},
\]

where $l_y$ is the screening length. All these contributions are logarithmically divergent for $x_0 \to x_1$. In this work, we make the simplest approximation and ignore the position dependence of the $U$’s, by setting them to their value in the QPC center. Then we obtain for the ratios of the different effective interaction strengths used in Sec. III C:

\[
\frac{U_{2}^{\text{intra}}}{U_{1}^{\text{intra}}} = \lim_{x_1 \to 0} \frac{U_{2}^{\text{intra}}(0, x_1)}{U_{1}^{\text{intra}}(0, x_1)} \approx 0.77,
\]

\[
\frac{U_{\text{inter}}}{U_{1}^{\text{intra}}} = \lim_{x_1 \to 0} \frac{U_{\text{inter}}(0, x_1)}{U_{1}^{\text{intra}}(0, x_1)} \approx 0.36,
\]

where in the last step we used a ratio $l_y/l_x(0) = 3$, which could, for example, be realized in a QPC with $l_x = 50 \text{ nm}$ and $l_y = 17 \text{ nm}$, which corresponds in a GaAs 2DEG to a curvature $\Omega_y = 2 \text{ meV}$.

[1] A. C. Graham, K. J. Thomas, M. Pepper, N. R. Cooper, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. Lett. 91, 136404 (2003).
[2] K.-F. Berggren, P. Jaksch, and I. Yakimenko, Phys. Rev. B 71, 115303 (2005).
[3] Y. Meir, J. Phys.: Condens. Matter 20, 164208 (2008).
[4] F. Bauer, J. Heyder, E. Schubert, D. Borowsky, D. Taubert, B. Bruogno, D. Schuh, W. Wegscheider, J. von Delft, and S. Ludwig, Nature 501, 73 (2013).
[5] L. Weidinger, F. Bauer, and J. von Delft, Phys. Rev. B 95, 035122 (2017).
[6] F. Stern, Phys. Rev. Lett. 21, 1687 (1968).
[7] S. Datta, Electronic Transport in Mesoscopic Systems, Cambridge Studies in Semiconductor Physics (Cambridge University Press, Cambridge, 1997).
[8] A. Oguri, J. Phys. Soc. Jpn. 70, 2666 (2001).
[9] D. Schimmel, Transport through Inhomogeneous Interacting Low-Dimensional Systems, Ph.D. thesis, Ludwig-Maximilians-Universität, München, 2017.
[10] K. Eissing, Functional Renormalization Group Applied to a Multimode Quantum Point Contact, Master’s thesis, LMU München, 2013.
[11] A. C. Graham, D. L. Sawkey, M. Pepper, M. Y. Simmons, and D. A. Ritchie, Phys. Rev. B 75, 035331 (2007).
[12] A. M. Lunde, A. D. Martin, A. Schulz, R. Egger, and K. Flensberg, New J. Phys. 11, 023031 (2009).