STRUCTURE FUNCTION SCALING IN THE TAURUS AND PERSEUS MOLECULAR CLOUD COMPLEXES

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ABSTRACT

We compute the structure function scaling of the integrated intensity images of two $J = 1–0$ $^{13}$CO maps of Taurus and Perseus. The scaling exponents of the structure functions, normalized to the third order, follow the velocity scaling of supersonic turbulence, suggesting that turbulence plays an important role in the fragmentation of cold interstellar clouds. The data also allow one to verify the validity of the two basic assumptions of the hierarchical symmetry model, originally proposed for the derivation of the velocity structure function scaling. This shows that the same hierarchical symmetry holds for the projected density field of cold interstellar clouds.

Subject headings: ISM: clouds — ISM: individual (Perseus Molecular Cloud, Taurus Molecular Cloud) — ISM: kinematics and dynamics — radio lines: ISM — turbulence

1. INTRODUCTION

Due to the complexity of the Navier-Stokes equations, mathematical work on turbulence is often inspired by experimental and observational measurements. Since geophysical and laboratory flows are predominantly incompressible, turbulence studies have been limited almost entirely to incompressible flows (or to infinitely compressible ones, described by the Burgers equation). Little attention has been paid to highly compressible, or supersonic, turbulence.

The cold interstellar medium (ISM) of galaxies such as the Milky Way is both highly turbulent and highly supersonic, within a range of scales from hundreds of parsecs to approximately a tenth of a parsec. Large observational surveys of the cold ISM of our Galaxy and of dust within it have become available in the last few years, and new surveys of unprecedented sensitivity and resolution will be obtained in the near future by FIR satellites such as SIRTF and Herschel.

The cold ISM provides a good laboratory for supersonic turbulence, and results from new observational surveys should motivate new mathematical work. Furthermore, the interpretation of the astronomical data requires a knowledge of basic properties of supersonic turbulence.

Previous works have tried to investigate the properties of ISM turbulence by estimating the second-order structure function or the power spectrum of the velocity field in dark clouds (Kleiner & Dickman 1987; Hobson 1992; Miesch & Bally 1994; Stutzki et al. 1998; Brunt & Heyer 2002a, 2002b), as sampled by molecular emission lines, with a number of different methods. The centroid velocity at each map position is used as an estimate of the local radial velocity. This centroid velocity results from a convolution of density, velocity, and excitation temperature along the line of sight and is not easily related to the three-dimensional velocity structure. Other works have instead studied the column density distribution of dark clouds, estimated from the integrated intensity of molecular emission lines or from dust thermal emission. Fractal dimensions have been estimated (Beech 1987; Bazell & Desert 1988; Scalo 1990; Dickman, Margulis, & Horvath 1990; Falgarone, Phillips, & Walker 1991; Zimmermann, Stutzki, & Winningwer 1992; Henrikson 1991; Hetem & Lepine 1993; Vogelaar & Wakker 1994; Elmegreen & Falgarone 1996; Stutzki et al. 1998). Multifractal (Chappell & Scalo 2001) and wavelet (Langer, Wilson, & Anderson 1993) analysis have also been proposed as a way of characterizing the projected density structure of molecular clouds.

The multifractal analysis applied by Chappell & Scalo (2001) to dust continuum images is related to the scaling of the moments of the projected density. In the present work, we study the scaling of integrated intensity differences (structure functions) of the $J = 1–0$ transition of $^{13}$CO from the Taurus and the Perseus molecular cloud complexes. If we had computed the moments of intensity, instead of intensity differences, then our analysis would be equivalent to the multifractal analysis by Chappell & Scalo (2001).

We have already shown in previous works that supersonic turbulence in a roughly isothermal gas, such as the cold ISM, generates a complex density field with density contrasts of several orders of magnitude and with statistical properties consistent with observational data from molecu-
lar clouds (Padoan et al. 1998, 1999; Padoan & Nordlund 1999). We usually refer to this process as turbulent fragmentation. Turbulent fragmentation of star-forming clouds is unavoidable, since supersonic turbulent motions are ubiquitously observed. The results of this work provide further evidence of the importance of turbulent fragmentation in star-forming clouds.

In § 2 we compute the relative scaling of the structure functions, and in § 3 we discuss our results. The hierarchical structure model is briefly presented in § 4 and its validity for our data is verified with the so-called /C₁₂ and /C₁₃ tests. We draw our conclusions in § 5.

2. STRUCTURE FUNCTIONS OF PROJECTED DENSITY FIELD

We use two observational $J = 1–0$ $^{13}$CO spectral maps of the Perseus molecular cloud (MC) complex (Padoan et al. 1999) and of the Taurus MC complex (Mizuno et al. 1995). These MC complexes have an extension of approximately 30 pc and rms radial velocity $\sigma_v = 2.0 \text{ km s}^{-1}$ in Perseus and $\sigma_v = 1.0 \text{ km s}^{-1}$ in Taurus. The rms sonic Mach number is therefore approximately 14 in Perseus and 7 in Taurus, assuming a gas kinetic temperature of approximately 10 K. The angular resolution is approximately 0.09 pc in the Perseus map, assuming a distance of 300 pc, and 0.08 pc in the Taurus map, assuming a distance of 140 pc.

In each map we compute an image of the integrated intensity, $I(r)$, defined as

$$I(r) = \sum_v T(v,r) \, dv,$$

where $T(v,r)$ is the antenna temperature at the velocity channel $v$ and map position $r$. The integrated intensity is set to zero in each map position, with peak antenna temperature smaller than 5 times the rms noise averaged over the whole map. The integrated intensity of $J = 1–0$ $^{13}$CO is roughly proportional to the total gas column density since, in most lines of sight of the two maps analyzed here, this transition is not optically thick. However, the conversion from $J = 1–0$ $^{13}$CO integrated intensity and gas column density also depends on the abundance ratio between $^{13}$CO and H$_2$ and on the precise spatial distribution of the excitation temperature (see Padoan et al. 2000 for a detailed computation of the effect of the spatial distribution of excitation temperature on the estimation of the $J = 1–0$ $^{13}$CO column density).

The structure functions of the integrated intensity image $I(r)$ are defined as

$$S_p(r) = \langle |I(r') - I(r + \Delta r')|^p \rangle = \langle |\Delta I|^p \rangle,$$

where $p$ is the order, and the average is extended to all map positions $r'$ and all position differences $\Delta r'$ such that $|\Delta r'| = r$.

It is possible to compute the moments by first deriving the probability density function (PDF) of $\Delta I$, $P(\Delta I, r)$, from the observational map and then computing

$$S_p(r) = \int_{\Delta I} \Delta I^p P(\Delta I, r) \, d(\Delta I).$$

The PDF can be obtained by appropriately smoothing the histogram obtained from the observational data, as suggested by Léveque & She (1997). Here, however, we prefer to average the moments directly over the whole observational sample, as in equation (2), without first deriving the PDF.

The structure functions are plotted in Figure 1, up to the order $p = 20$. They are well approximated by a power law, at least at low orders, between approximately 0.3 and 3 pc, for both Taurus and Perseus. We have performed a least-squares fit in that range of scales for all orders. The least-

![Fig. 1.—Structure functions of orders $p = 1–20$ (bottom to top) of the $J = 1–0$ $^{13}$CO integrated intensity maps of Taurus (Mizuno et al. 1995) and Perseus (Padoan et al. 1999). The solid lines are least-squares fits to the structure functions in the approximate range 0.3–3 pc, where the structure functions are well described as power laws.](image)
and within 10% accuracy for Perseus. Bottom panel: integrated intensity images of Taurus and Perseus follow Boldyrev’s velocity dotted line scaling (incompressible turbulence (solid line), She & Lévêque’s (1994) velocity scaling for super-sonic turbulence (dotted line), and Kolmogorov’s (1941) velocity scaling (dashed line). The moments of the structure functions of the integrated intensity images of Taurus and Perseus follow Boldyrev’s velocity scaling, within 5% accuracy up to the 20th order (and beyond) for Taurus, and within 10% accuracy for Perseus. Bottom panel: Same as top panel, but normalized to Boldyrev’s scaling, $\eta(p)$.

![Diagram](image)

Fig. 2.— *Top panel:* Structure function scaling exponents normalized to the third order, $\eta(p) = \eta(p)/\eta(3)$, up to the 20th order, for Taurus (asterisks), Perseus (diamonds), Boldyrev’s (2002) velocity scaling for supersonic turbulence (solid line), She & Lévêque’s (1994) velocity scaling for incompressible turbulence (dashed line), and Kolmogorov’s (1941) velocity scaling (dotted line). The moments of the structure functions of the integrated intensity images of Taurus and Perseus follow Boldyrev’s velocity scaling, within 5% accuracy up to the 20th order (and beyond) for Taurus, and within 10% accuracy for Perseus. *Bottom panel:* Same as top panel, but normalized to Boldyrev’s scaling, $\eta(p)$.

Table 1: Exponents of the power-law fit of the moment of order $p$.

| Order | Taurus | Perseus |
|-------|--------|---------|
| 2     | 0.57   | 0.53    |
| 3     | 1.02   | 1.00    |
| 4     | 1.32   | 1.29    |
| 5     | 1.62   | 1.58    |
| 6     | 1.89   | 1.85    |
| 7     | 2.12   | 2.09    |
| 8     | 2.35   | 2.32    |
| 9     | 2.56   | 2.53    |
| 10    | 2.75   | 2.72    |
| 11    | 2.93   | 2.90    |
| 12    | 3.09   | 3.06    |
| 13    | 3.24   | 3.21    |
| 14    | 3.38   | 3.35    |
| 15    | 3.51   | 3.48    |
| 16    | 3.63   | 3.60    |
| 17    | 3.75   | 3.72    |
| 18    | 3.86   | 3.83    |
| 19    | 3.97   | 3.94    |
| 20    | 4.07   | 4.04    |

We then compute the scaling exponents normalized to the third order, $\eta(p) = \eta(p)/\eta(3)$, following the idea of extended self-similarity (Benz et al. 1993; Dubrulle 1994).

Values of $\eta(p)/\eta(3)$ for both Taurus and Perseus are plotted in Figure 2, up to the 20th order, and are compared with those predicted for the scaling of the structure functions of the velocity field $[\zeta(p)/\zeta(3)]$ by the Kolmogorov model (Kolmogorov 1941), the She & Lévêque model (1994; our eq. [7], below), and the model by Boldyrev (2002). The latter is an extension of the She & Lévêque model (1994) to supersonic turbulence, verified with numerical simulations of supersonic turbulence by Boldyrev, Nordlund, & Padoan (2002a, 2002b). The scaling exponents of the structure functions of the integrated intensity images of Taurus and Perseus follow the velocity scaling found in Boldyrev (2002), within 5% accuracy up to the 20th order for Taurus and within 10% accuracy for Perseus.

For practical purposes, it may be useful to know the intrinsic value of the scaling exponents, not just the value normalized to the third order. For Taurus we find $\eta(3) = 1.10$, and for Perseus $\eta(3) = 1.18$, both close to the Kolmogorov value for the velocity structure function, $\zeta(3) = 1.0$. The second-order exponents are $\eta(2) = 0.77$ for Taurus and $\eta(2) = 0.83$ for Perseus, not far from the value of $\eta(2) = 0.7$, predicted analytically (Boldyrev et al. 2002a) from the velocity scaling in Boldyrev (2002).

3. STATISTICAL SIGNIFICANCE OF HIGH-ORDER MOMENTS

The velocity scaling of laboratory, geophysical, or numerical flows are hardly verified up to the 10th order (Müller & Biskamp 2000; She et al. 2001). In the present work, we have used astronomical data to compute moments beyond the 10th order. A way to verify the statistical significance of high-order moments is to plot the integrand of equation (3), $\Delta I^p (\Delta I, r)$, as a function of $\Delta I$ (for a fixed value of $r$). If this function has a well-defined peak at a value of $\Delta I$, where the PDF is defined by a significant number of samples, then the moment of order $p$ is statistically significant.

The peak position of the integrand of equation (3) as a function of $\Delta I$ grows with increasing order $p$ and converges to the value of the cutoff of the PDF. For a finite sample size, the cutoff of the PDF may depend on the sample size (larger sample giving a more populated and extended tail of the PDF). High-order moments converging to the cutoff of the PDF would then be inaccurate, as their convergence would be due to the finite sample size. However, as pointed out in Lévêque & She (1997), if a real PDF cutoff is present, for example, if it is well defined by a significant number of samples, then the moment calculation up to any order should be accurate, and the fast convergence of high-order moments would not be an artifact of the finite sample size.

We have plotted in Figure 3 (left panel) the tail of the PDF of $|\Delta I| = |I(r') - I(r + \Delta r')|$ from the Taurus data, for $|\Delta r'| = 2.6$ pc. Vertical dotted segments mark the peak position of the integrand of equation (3) for moments of increasing order, from $p = 3$ to 15. These peaks are well defined for all values of $|\Delta r'|$, where the structure functions are well approximated by power laws.

The moments converge at $p = 15$ and are therefore accurate up to that order. However, the PDF shows a very sharp cutoff. The cutoff is definitely significant, since it is defined by a few hundred samples. The “real” PDF describing the physical process may therefore have a similarly sharp cutoff, in which case the convergence around the 15th order found in the observational data would be significant, and orders above the 15th would be accurately estimated by the present data set. The moment convergence corresponds to the asymptote of the function $\eta(p)/\eta(3)$ plotted in Figure 2.

On the other hand, it is possible that the sharp cutoff in the tail of the PDF is due to the saturation of the $J = 1\rightarrow 0$ $^{13}$CO transition in the regions of largest gas column density and optical thickness. In that case, the convergence of the moments above the order $p = 15$ would be an artifact of the observational method, and the structure function scaling would be uncertain above $p = 15$.

A similar result regarding the PDF cutoff is obtained for the Perseus data (Fig. 3, right panel). However, in this case, the peak of the integrand of equation (3) for every order is very shallow. Although formally the peak position has already converged at the fifth order (Fig. 3, right panel), the moments converge around the eighth order. The convergence may again be well defined, as for the Taurus data,
since the PDF cutoff is defined by a large sample size or may result from the observational method (saturation of the $J=1-0^{13}$CO transition).

4. THE HIERARCHICAL STRUCTURE MODEL FOR SUPersonic TURBULENCE

The scaling of the velocity structure functions in incompressible turbulence is best described by the She & Lévêque (1994) formula

$$\frac{\zeta(p)}{\zeta(3)} = \gamma p + C(1 - \beta^p),$$

where

$$C = \frac{1 - 3\gamma}{1 - \beta^p}$$

is interpreted as the Hausdorff codimension of the support of the most singular dissipative structures. In incompressible turbulence, the most dissipative structures are organized in filaments along coherent vortex tubes with Hausdorff dimension $D=1$, and so $C=2$. Furthermore, $\beta^p = 2/3$ (She & Lévêque 1994), which yields the She-Lévêque velocity scaling for incompressible turbulence,

$$\frac{\zeta(p)}{\zeta(3)} = \frac{p}{9} + \frac{2}{3} \left[1 - \left(\frac{2}{3}\right)^{p/3}\right].$$

Boldyrev (2002) has proposed applying the scaling of equation (5) to supersonic turbulence, with the assumption that the Hausdorff dimension of the support of the most singular dissipative structures is $D=2$ ($C=1$), since dissipation of supersonic turbulence occurs mainly in sheetlike shocks. Using the physical interpretation of equation (5) by Dubrulle (1994), $\gamma = 1/9$, and $C = (2/3)/(1 - \beta^p)$. With $C = 1$, therefore, $\beta^3 = 1/3$, and one obtains the Boldyrev’s velocity scaling,

$$\frac{\zeta(p)}{\zeta(3)} = \frac{p}{9} + 1 - \left(\frac{1}{3}\right)^{p/3}.$$  

This velocity scaling has been found to provide a very accurate prediction for numerical simulations of supersonic and super-Alfvénic turbulence (Boldyrev et al. 2002a, 2002b). The computation of the corresponding structure functions for the gas density distribution is discussed in Boldyrev et al. (2002a). The scaling exponents, $\xi(p)$, of the density correlators, $\langle\rho(r' + \Delta r')\rho(r')^p\rangle$, are found to depend on higher order exponents of the velocity scaling, $\zeta(p')$,

$$\xi(p) = \zeta(2p_0) - 2p \zeta(p_0)\,.$$

where $p_0 = 2.28$. For the second-order structure function of the projected density, the exponent is $\eta(2) = 1 + \xi(1) = 0.7$.

In this paper, we have found that the structure function of the projected density field of supersonic turbulence in the ISM is almost indistinguishable from Boldyrev’s scaling of velocity in supersonic turbulent flows. This result should inspire the mathematical work. As shown by Dubrulle (1994) and by She & Waymire (1995), a hierarchy of structures producing the scaling relation (5) can be obtained by a random multiplicative process with log-Poisson statistics. The result of this paper suggests that the density field of supersonic turbulence is the result of a multiplicative process with log-Poisson statistics.

4.1. The $\beta$ and $\gamma$ Tests for the Hierarchical Model

In § 4 the scaling of the structure functions of integrated intensity of two maps of the Taurus and Perseus molecular cloud complexes has been compared with the velocity scal-
She & Léveque (1994) is the existence of the universal hierarchical structure model at the same time. This method is also tests the validity of two major assumptions of the hierarchical structure model at the same time. This method is presented in She et al. (2001); we apply it in the following.

The basic assumption in the derivation of equation (5) by She & Léveque (1994) is the existence of the universal scaling behavior,

\[ F_{p+1}(r) = A_p F_p(r)^\beta F(\infty)(r)^{1-\beta}, \]

(10)

where

\[ F_p(r) = S_{p+1}(r)/S_p(r) = \langle |I|^{p+1}\rangle/\langle |I|^p\rangle \]

(11)

is usually referred to as the \(p\)th-order intensity of fluctuations, and \(A_p\) are constants independent of \(r\) and are also found to be independent of \(p\) in a number of cases.

The test verifies the validity of this basic assumption, with a log-log plot of \(F_{p+1}/F_2\) versus \(F_p/F_1\). If the plot is a straight line, the assumed hierarchical symmetry (eq. [10]) is satisfied, and the data pass the \(\beta\) test. The plot is shown in Figure 4 (left panel) for both Perseus (diamonds) and Taurus (asterisks). We have combined values of \(F_p(r)\) for all values of \(r\) used to compute the moment scaling exponents \(\eta(p)\) (approximately the range of scales between 0.3 and 3 pc). Both Taurus and Perseus data apparently pass the \(\beta\) test.

The constants \(A_p\) are independent of \(p\), as for the hierarchy of intensity of fluctuations of velocity in the shell model analyzed by Léveque & She (1997), in the turbulent Couette-Taylor flow studied by She et al. (2001), and in laboratory data obtained by Chavarria, Baudet, & Ciliberto (1995). The slope of the intensity of fluctuation function plotted in Figure 4 provides an estimate of the value of \(\beta = 0.79\) for Taurus and \(\beta = 0.78\) for Perseus. This is to be compared with the value of \(\beta = 0.69\) of the velocity scaling of supersonic turbulence in Boldyrev (2002).

The second basic assumption by She & Léveque (1994) is

\[ F_\infty \sim S^\gamma, \]

(12)

which allows them to derive equation (5). The \(\gamma\) test directly verifies the validity of equation (5) by first assuming the value of \(\beta\) obtained from the \(\beta\) test and by then plotting \(\eta(p) - \chi(p, \beta)\) versus \(p - 3\chi(p, \beta)\), where \(\chi(p, \beta) = (1 - \beta^p)/(1 - \beta^3)\). If the plot is a straight line, the data pass the \(\gamma\) test, and the slope of the plot provides an estimate of the value of \(\gamma\). The plot is shown in Figure 4 (right panel). We are able to fit the plot to a straight line for orders \(p > 11\) for both Taurus and Perseus. Since \(\gamma\) is a property of the very high order moments (see eq. [12]), and since we have obtained a straight line for large moments, we can say that the Taurus and Perseus data pass the \(\gamma\) test. We estimate \(\gamma = 0.11\) for Taurus and \(\gamma = 0.06\) for Perseus.

The value of the parameter \(\beta\) is a measure of intermittency. In the Kolmogorov (1941) model, \(\beta = 1\) and \(\zeta(p) = p/3\). This is the limit of no intermittency. Lower \(\beta\) corresponds to a higher degree of intermittency. The value of the parameter \(\gamma\) is related to very high order moments and is therefore more difficult to estimate accurately (especially from the Perseus data). Kolmogorov’s turbulence corresponds to \(\gamma = 1/3\). A full mathematical description of the relation between velocity and projected density structure functions is not available yet, and comparison of these preliminary values of \(\beta\) and \(\gamma\) of molecular cloud images with those of the velocity scaling of turbulent flows is premature.

5. CONCLUSIONS

We have computed the structure functions of the integrated intensity images of two \(J = 1-0\) \(^{13}\)CO maps of Taurus and Perseus. The structure functions scale as power laws within the range of scales 0.3–3 pc. The scaling exponents have been computed up to the 20th order and are statistically significant at least up to the 15th order in Taurus and the eighth order in Perseus. They are found to follow the velocity scaling of supersonic turbulence proposed by
Boldyrev (2002), within 5% and 10% accuracy for Taurus and Perseus, respectively.

We have verified that the projected density field (or integrated intensity) of the Taurus and Perseus molecular cloud complexes can be described by a hierarchical model, such as the one proposed by She & Léveque (1994), for the velocity structure functions of incompressible turbulence. We have done so by testing the validity of the two basic assumptions of the hierarchical model for our data. The validity of the assumptions of the hierarchical model means that the integrated intensity images we have analyzed (an approximate estimate of the projected density of the Taurus and Perseus molecular cloud complexes) are the result of a multiplicative process with log-Poisson statistics (Dubrulle 1994).

The complete derivation of the relation between the structure functions of velocity and projected density is a subject for future works. However, the close similarity of the structure functions of projected density in Taurus and Perseus with that of the velocity field of turbulence provides additional evidence that supersonic turbulence is the major factor controlling the density field in the range of densities and scales sampled by the maps we have analyzed.

It is well established that supersonic turbulence plays an important role in the dynamics of the cold ISM (Larson 1981; Padoan et al. 1998, 1999, 2001; Padoan & Nordlund 1999). The statistical properties of this ISM turbulence need to be discovered and understood in order to elaborate a statistical theory of star formation (Padoan & Nordlund 2002). We have shown in the present paper that this can be achieved with existing observational data by studying the projected density field of molecular clouds. This type of study will be greatly improved by far-infrared imaging of the dust thermal emission from turbulent ISM clouds, obtained by future satellite missions such as SIRTF.

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