Finetuning Pretrained Vision-Language Models with Correlation Information Bottleneck for Robust Visual Question Answering

Jingjing Jiang, Ziyi Liu, Nanning Zheng
Institute of Artificial Intelligence and Robotics, Xi’an Jiaotong University
jingjingjiang2017@gmail.com, {liuziyi@stu, nnzheng@mail}.xjtu.edu.cn

Abstract

Benefiting from large-scale Pretrained Vision-Language Models (VL-PMs), the performance of Visual Question Answering (VQA) has started to approach human oracle performance. However, finetuning large-scale VL-PMs with limited data for VQA usually faces overfitting and poor generalization issues, leading to a lack of robustness. In this paper, we aim to improve the robustness of VQA systems (i.e., the ability of the systems to defend against input variations and human-adversarial attacks) from the perspective of Information Bottleneck when finetuning VL-PMs for VQA. Generally, internal representations obtained by VL-PMs inevitably contain irrelevant and redundant information for the downstream VQA task, resulting in statistically spurious correlations and insensitivity to input variations. To encourage representations to converge to a minimal sufficient statistic in vision-language learning, we propose the Correlation Information Bottleneck (CIB) principle, which seeks a tradeoff between representation compression and redundancy by minimizing the mutual information (MI) between the inputs and internal representations while maximizing the MI between the outputs and the representations. Meanwhile, CIB measures the internal correlations among visual and linguistic inputs and representations by a symmetrized joint MI estimation. Extensive experiments on five VQA benchmarks of input robustness and two VQA benchmarks of human-adversarial robustness demonstrate the effectiveness and superiority of the proposed CIB in improving the robustness of VQA systems.

1 Introduction

Visual Question Answering (VQA) [1] is a classical vision-language task. Recently, the large-scale Pretrained Vision-Language Models (VL-PMs) [2–4] have elevated the VQA performance to the level of human oracle. However, finetuning extremely large-scale VL-PMs with limited data for the downstream VQA task usually suffers from overfitting and poor generalization issues, making the improvement in VQA robustness brought by VL-PMs relatively limited and far less than the improvement in VQA accuracy.

In this paper, we explore two types of VQA robustness, namely, input robustness and human-adversarial robustness, and aim to improve the robustness when finetuning VL-PMs for VQA. The input robustness refers to the capability of VQA models to defend against visual variations in images (e.g., question-related object removal [5]) and linguistic variations in questions (e.g., word substitution and sentence rephrasing [6, 7]). The human-adversarial robustness is the ability of VQA models to defend against adversarial attacks by human [8, 9]. Practically, in the finetuning process, VQA is usually formulated as a multi-answer classification problem where VL-PMs act as representation extractors with rich knowledge, and the extracted vision-language representation are passed to
an additional VQA Head module for answer prediction. As such, improving the two robustness essentially makes the obtained representation more compact and task-related.

To this end, we propose to improve the robustness from an Information-theoretical perspective. From this view, one possible reason for poor VQA robustness is that representations yielded by VL-PMs inevitably contain irrelevant and redundant information for the downstream VQA task. Specifically, the irrelevant information will encourage VQA systems to learn statistically spurious correlations between representations and labels, while task-agnostic redundant information will reduce the sensibility of VQA systems to input variations. Both types of information will impair the VQA robustness. Therefore, to obtain more robust representations when finetuning VL-PMs for VQA, we expect to discard irrelevant and redundant information in representations while preserving the relevant information. The Information Bottleneck (IB) principle [10] can seek a tradeoff between representation compression and redundancy. Motivated by this, we exploit the IB principle to find the minimal sufficient statistic of the obtained representations for more robust VQA systems.

We propose Correlation Information Bottleneck (CIB) to improve the robustness of VQA systems when finetuning VL-PMs for VQA. Specifically, CIB promotes the vision-language representations to converge to a minimal sufficient statistic by minimizing mutual information (MI) between the representations and inputs while maximizing MI between the representations and outputs. In addition to the overall dependency between inputs and the vision-language representations, we utilize a symmetrized joint MI to measure the internal correlations among visual and linguistic representations and inputs, guiding VQA systems to better capture the actual correlations. Furthermore, to consider different architectures of VL-PMs (i.e., the single-stream and two-stream Transformer layers), we unify the internal representations of different VL-PMs for CIB estimation.

To demonstrate the proposed CIB principle, we first provide a rigorous theoretical derivation of the lower bound of CIB. Then, we apply it to finetune five baseline VL-PMs on five VQA benchmarks of input robustness and two VQA benchmarks of human-adversarial robustness. Extensive experiments consistently show that CIB can significantly improve the robustness of VQA systems and show the superiority of CIB compared with existing methods.

2 Method

In this section, we first state the preliminary of VQA and the general IB principle, then introduce the proposed Correlation Information Bottleneck (CIB) and explain how CIB is applied to finetune Pretrained Vision-Language Models (VL-PMs) for VQA.

2.1 Preliminary

Problem Setting. In the finetuning process, the VQA task is formulated as a multi-answer classification problem. Given a VQA dataset \( D = \{(I, Q, y) \in I \times Q \times Y\} \), where \( I \) is an image, \( Q \) is a question, and \( y \) is an answer, VL-PMs take image-question pairs as input, where the image is further represented as a set of image regions \( \{v_1, ..., v_K\} \) (\( K \) is the number of regions in one image) and the question is tokenized as a token sequence \( \{w_1, ..., w_L\} \) (\( L \) is the number of word tokens in a question), and output the answer probability distribution \( Y \) using a additional VQA Head module (i.e., two fully-connected layers sandwiched with GeLU activation and Layer Normalization operation).

IB View of Representation Learning. From an information-theoretical view, seeking a robust representation \( T \) for VL-PMs is equivalent to preserving information about the output \( Y \) while removing irrelevant and redundant information from the input \( X \). This is because for the given VQA task, the irrelevant and redundant information may encourage VL-PMs to learn superfluous correlations between answer labels and inputs. Formally, the IB principle [10, 11] formulates vision-language representation learning as an information-theoretic tradeoff and finds an optimal representation by maximizing the Lagrangian

\[
\mathcal{L}_\text{IB} := I(Y; T) - \beta I(X; T),
\]  

where \( \beta \geq 0 \) controls the tradeoff between compression and prediction, and \( I(\cdot; \cdot) \) denotes mutual information (MI).
2.2 Correlation Information Bottleneck

In vision-language representation learning, given the two modal inputs $X^v$ and $X^l$, $T^v$ and $T^l$ denote the corresponding internal visual and linguistic representations. To extend the general IB principle to this setting, we group the input sources and internal representations as $X = [X^v, X^l]$ and $T = [T^v, T^l]$. Therefore, the training objective is to learn the minimal sufficient representation $T$ that discards all irrelevant and redundant information from the input $X$ for the given VQA task.

Specifically, to derive a differentiable IB estimation in vision-language representation learning, we first focus on the term $I(Y; T)$ in Eq.(1), which can be rewritten as the following form using conditional probability definition:

$$I(Y; T) = \int p(y, t) \log \frac{p(y, t)}{p(y)} \, dy \, dt.$$  \tag{2}$$

Since the conditional probability $p(y|t)$ is intractable, we instead estimate $I(Y; T)$ with the BA [12] lower bound:

$$I(Y; T) \geq \int p(y, t) \log q(y|t) \, dy \, dt - \int p(y) \log p(y) \, dy , \tag{3}$$

where $q(y|t)$ is an accessible auxiliary distribution of $p(y|t)$, and the entropy of labels $H(Y) = - \int p(y) \log p(y) \, dy$ is independent for the optimization procedure. Ignoring $H(Y)$, the remaining term in Eq.(3) is equal to $-H(Y|T)$, meaning that maximizing the lower bound of $I(Y; T)$ is equivalent to minimizing the cross-entropy loss of the given task.

Next, we consider the mutual information between the input sources and their corresponding representations, i.e., $I(X; T)$ in Eq.(1). In addition to measuring the overall dependency between $X$ and $T$ (i.e., regarding visual and linguistic representations as one), we also concern the internal correlations among $X^v, X^l, T^v$, and $T^l$, which can guide VL-PMs to learn correlations between visual and linguistic representations. Therefore, we propose to maximize the Correlation Information Bottleneck (CIB) formula:

$$L_{\text{CIB}} := I(Y; T) - \beta I(X^v, X^l; T^v, T^l), \tag{4}$$

where, $I(X^v, X^l; T^v, T^l)$ can be regarded as a symmetrized variant of joint mutual information [13] that considers the relevance of the input set and the relevance of the representation set. To efficiently approximate $I(X^v, X^l; T^v, T^l)$, we first expand it conditioned on the properties of mutual information and the data processing inequality in representation learning [14]. The derivation can be formally stated by Theorem 1 (see Appendix for proof):

**Theorem 1.** (Upper Bound of $I(X^v, X^l; T^v, T^l)$) Given two groups of random variables $X = [X^v, X^l], T = [T^v, T^l]$, the mutual information $I(X^v, X^l; T^v, T^l)$ can be upper-bounded with

$$I(X; T) = I(X^v, X^l; T^v, T^l) \leq I(X^v; T^v) + I(X^l; T^l) - I(T^v; T^l) + D_{\text{SKL}}, \tag{5}$$

where $D_{\text{SKL}}$ denotes the Symmetric Kullback-Leibler divergence and can be obtained by averaging the two Kullback-Leibler divergences, $D_{\text{KL}}(p(t^v|x^v)||p(t^l|x^l))$ and $D_{\text{KL}}(p(t^l|x^l)||p(t^v|x^v))$.

Afterwards, we follow Wang et al. [15] to further upper-bound MI between the input and its representation with a localized formulation of IB. Formally, for the visual and linguistic inputs $X^v, X^l$, they are essentially two sets of random variables, i.e., $X^v = [X^v_1, X^v_2, \ldots, X^v_K], X^l = [X^l_1, X^l_2, \ldots, X^l_L], f_{\theta^v}$ and $f_{\theta^l}$ are the two functions that map $X^v$ and $X^l$ into visual and linguistic representations, i.e., $T^v = [T^v_1, T^v_2, \ldots, T^v_K] = [f_{\theta^v}(X^v_1), f_{\theta^v}(X^v_2), \ldots, f_{\theta^v}(X^v_K)]$ and $T^l = [T^l_1, T^l_2, \ldots, T^l_L] = [f_{\theta^l}(X^l_1), f_{\theta^l}(X^l_2), \ldots, f_{\theta^l}(X^l_L)]$. The MI between the visual/linguistic input and the visual/linguistic representation can be maximized by

$$I(X^v; T^v) \leq \sum_{i=1}^{K} I(X^v_i; T^v_i); \quad I(X^l; T^l) \leq \sum_{i=1}^{L} I(X^l_i; T^l_i). \tag{6}$$

Therefore, the lower bound of $L_{\text{CIB}}$ can be stated as Theorem 2.
Theorem 2. (Lower Bound of $\mathbb{L}_{\text{CIB}}$) Given two groups of random variables, $X = [X^v, X^l]$ and $T = [T^v, T^l]$, where $X^v, X^l, T^v,$ and $T^l$ are represented as the sets of random variables, i.e., $X^v = [X_1^v, X_2^v, \ldots, X_K^v]$, $X^l = [X_1^l, X_2^l, \ldots, X_L^l]$, $T^v = [T_1^v, T_2^v, \ldots, T_K^v]$, and $T^l = [T_1^l, T_2^l, \ldots, T_L^l]$. Two deterministic functions, $f_{\theta^v}$ and $f_{\theta^l}$, make $T^v = [T_1^v, T_2^v, \ldots, T_K^v] = [f_{\theta^v}(X_1^v), f_{\theta^v}(X_2^v), \ldots, f_{\theta^v}(X_K^v)]$, $T^l = [T_1^l, T_2^l, \ldots, T_L^l] = [f_{\theta^l}(X_1^l), f_{\theta^l}(X_2^l), \ldots, f_{\theta^l}(X_L^l)]$. Then, the formulation of Correlation Information Bottleneck (CIB) can be lower-bounded with

$$
\mathbb{L}_{\text{CIB}} = I(Y; T) - \beta I(X^v, X^l; T^v, T^l) 
\geq I(Y; T) - \beta \left[ \log p(T^v) - \log p(T^l) + K \sum_{i=1}^{K} I(X_i^v; T_i^v) + L \sum_{l=1}^{L} I(X_i^l; T_i^l) \right].
$$

The theorem indicates that in vision-language representation learning, once $I(Y; T)$ is regarded as a task-related objective, $-\beta I(X^v, X^l; T^v, T^l)$ can be used to constrain the representation compactness so that to seek robust representations by the tradeoff between redundancy and compression.

2.3 Finetuning VL-PMs for VQA with Correlation Information Bottleneck

As shown in Figure 1 (a) and (b), there are two typical transformer architectures of VL-PMs: the single-stream [16–18] and the two-stream [19, 20]. When finetuning VL-PMs for VQA, to unify the two typical architectures into one formulation, as shown in Figure 1 (c), we utilize the region-level features after the visual Embedding layer (i.e., $f_{\theta^v}$ is the parametric Embedding layer) as the internal visual representation $T^v$. Analogously, the token-level features after the linguistic Embedding layer (f_{\theta^l}) is regarded as the internal linguistic representation $T^l$. All the following Transformer layers (f_{\theta^\text{trans}}) and the VQA Head module (f_{\theta^Y}) serve as the parametric approximator to yield $Y$ given $T = [T^v, T^l]$. In summary, to utilize CIB to finetune VL-PMs for VQA, we regard $I(Y; T)$ in Eq.(7) as the task-related loss (i.e., the cross-entropy loss for answer prediction), and the remaining terms in Eq.(7) can be considered as regularizers.

Estimating MI Terms in $\mathbb{L}_{\text{CIB}}$. In the finetuning process, for sample pairs $\{(X_i^v, T_i^v)\}_{i=1}^{K}$ and $\{(X_i^l, T_i^l)\}_{i=1}^{L}$, the conditional probability distributions $p(t^v|x^v)$ and $p(t^l|x^l)$ are known. We thus adopt a sample-based differentiable MI estimator, CLUB [21], to approximate the upper bound of MI between the visual/linguistic inputs and the corresponding representations, i.e.,

$$
\hat{I}(X^v; T^v) = \frac{1}{K^2} \sum_{i=1}^{K} \sum_{j=1}^{K} \log \frac{p(t^v_i | x^v_j)}{p(t^v_i | x^v_i)},
$$

$$
\hat{I}(X^l; T^l) = \frac{1}{L^2} \sum_{i=1}^{L} \sum_{j=1}^{L} \log \frac{p(t^l_i | x^l_j)}{p(t^l_i | x^l_i)}.
$$

For $I(T^v; T^l)$, it is difficult to be estimated directly due to the different sequence lengths of $T^v \in \mathbb{R}^{K \times d}$ and $T^l \in \mathbb{R}^{L \times d}$. Thus, we first transform the sequence representations into global visual
Table 1: Quantitative results on input robustness and human-adversarial robustness. More complete comparisons with existing methods on corresponding benchmarks are available at Appendix.

| ID       | Models                      | Input Robustness | Adversarial Robustness |
|----------|-----------------------------|------------------|------------------------|
|          | VQA-Rep Pert | VQA P2 Pert | IV-QVA Pert | CV-QVA Pert | VQA-CE CE | AVQA test val | AdvQA test val |
| SoTA     | 56.59 58.95 | 68.10 74.40 | -            | 39.24       | 26.08 27.08 | 33.52 33.33 |
|          | VisualBERT +CIB     | 62.03 53.06 | 68.23 72.34 | 46.04 30.48 | 38.75 37.55 | 37.55 37.60 |
|          | UNITERB +CIB        | 62.68 55.66 | 70.36 74.36 | 75.71 42.60 | 40.64 38.00 | 37.50 37.60 |
|          | LXMERT +CIB         | 70.41 65.21 | 77.30 82.96 | 77.83 40.16 | 53.61 36.98 | 25.17 25.77 |
| +CIB     |                | 72.62 67.71 | 78.93 85.07 | 78.57 40.47 | 57.14 38.91 | 26.18 26.15 |
| Avg.     | +2.21 +2.50 | +1.63 +2.11 | +0.74 +0.31 | +3.53       | +1.01 +0.38 | +1.12 +1.93 |
| +CIB     | 59.16 51.22 | 67.18 71.39 | 72.37 32.24 | 36.94 36.54 | 25.27 25.83 | 36.45 37.11 |
| Average  | +1.83 +2.48 | +1.69 +1.82 | +2.28 +2.28 | +1.06 +0.63 | +1.10 +1.08 | +1.98 +1.18 |

and linguistic representations, $\hat{T}^v \in \mathbb{R}^d$ and $\hat{T}^d \in \mathbb{R}^d$, using two one-layer fully-connected (FC) networks. Then, to guarantee the inequality in Eq. (7) hold, we should approximate the lower bound of $I(\hat{T}^v; \hat{T}^d)$. Therefore, we lower-bound it with NWJ [22], i.e.,

$$\hat{I}(\hat{T}^v; \hat{T}^d) = \mathbb{E}_{p(\hat{t}^v, \hat{t}^d)}[\log f(\hat{T}^v, \hat{T}^d)] - \frac{1}{e} \mathbb{E}_{p(\hat{t}^v)p(\hat{t}^d)}[f(\hat{T}^v, \hat{T}^d)],$$

(10)

where $f$ is a discriminant function implemented by a two-layer FC network.

**Estimating $D_{\text{SKL}}$ in $\mathcal{L}_{CIB}$.** Since $p(t^v|t^d)$ and $p(t^d|t^v)$ have a known probability density, we directly compute the symmetric Kullback-Leibler divergence $D_{\text{SKL}}$.

### 3 Experiments

The goal of our experiments is to verify whether using CIB as a training objective when finetuning VL-PMs for VQA will make VQA systems more robust. Specifically, we consider the following questions: (1) Facilitated by CIB, does the VL-PMs learn more robust representations than vanilla VL-PMs? (2) How does each component of CIB contribute to the VQA robustness?

**Shared Implementation Details.** In all following experiments, the image region features are extracted using bottom-up attention Faster R-CNN [23] pre-trained on Visual Genome [24]. For all baseline VL-PMs, the number of word tokens $L$ is set as 20. The number of image regions $K$ for LXMERT and VisualBERT are 36 and 100, respectively. For UNITERB, ViLBERT, and VL-BERTA, $K$ is not fixed. The dimension $d$ of representation is 768. All experiments are implemented using PyTorch on one NVIDIA GTX2080 12GB GPU. As for optimization, we utilize AdamW optimizer with a linear warmup with linear decay, a warmup step of 1000, a batch size of 64, and a peak learning rate of 2e-5. The total number of training epochs is 10.

#### 3.1 Evaluation on Input Robustness

**Experimental Settings.** In this experiments, we utilize five typical VL-PMs as baselines (i.e., VisualBERT [16], UNITERB [18], LXMERT [19], VL-BERTA [17], and ViLBERT [20]) and evaluate our CIB objective on five VQA benchmarks of input robustness (i.e., VQA-Rep [6], VQA P2 [7], IV-VQA [5], CV-VQA [5], and VQA-CE [25]). Specifically, VQA-Rep and VQA P2 evaluate the robustness against linguistic variations, IV-VQA and CV-VQA evaluate robustness w.r.t. visual variations, and VQA-CE evaluates the robustness against shortcut learning involving inputs. Note that all these benchmarks of input robustness are built on VQA v2 val split, we thus only finetune
our models on VQA v2 training set. In addition, since the images of VQA v2 are derived from COCO [26], we follow the works [18] to divide these VL-PMs into ID (in-domain), ID+OOD (in-domain + out-domain), and OOD (out-domain) according to whether they use the COCO dataset in the pretraining process. The more detailed statistics of the benchmarks are shown in Appendix.

**Metrics.** In addition to the standard evaluation metric, *i.e.*, the VQA-Score, we follow Shah et al. [6] to evaluate the robustness against linguistic variations using Consensus Score (CS(k)). Specifically, 

$$CS(k) = \sum_{Q' \subset Q, |Q'| = k, q \in Q'} \frac{1}{nC_k} \Theta(q)$$  \hspace{1cm} (11)

where, $\Theta$ is a set where the answer of question $q$ is correct, and $1$ is an indicator function defined on $\Theta$. Obviously, the higher the average consensus score at higher values of $k$, the more robust the model is. Since the baseline VL-PMs partly use the original examples of robustness benchmarks, which would result in unreliable VQA-Score on original examples, we only test the VQA-Score of perturbed examples (Pert) and counterexamples (CE).

**Experimental Results.** The results are shown in Table 1. We see that compared with baselines (*i.e.*, finetuning VL-PMs for VQA with only the cross-entropy loss for answer prediction), using CIB as the additional training objective can significantly improve the input robustness of VQA systems. In general, the results on the five VQA benchmarks of input robustness consistently show that it is feasible to encourage VL-PMs to learn more compact and robust representations from an information-theoretic perspective. In addition, from the comparisons between different baseline VL-PMs, we see that the CIB objective is effective for different architectures and domains.

### 3.2 Evaluation on Human-Adversarial Robustness

**Experimental Settings.** In this part, we also consider the above mentioned five baseline VL-PMs and stress test the VQA models finetuned with the CIB objective on two VQA benchmarks of human-adversarial robustness (*i.e.*, AVQA [8] and AdVQA [9]). Specifically, AVQA is built on images from out-of-domains (excluding COCO images), with $\sim$142.1K/8.7K/26.4K image-question pairs in the train/val/test split. AdVQA is based on VQA v2 images from COCO, with $\sim$10K/36.8K image-question pairs for val/test split. We uniformly finetune all models on AVQA train split and evaluate them on val and test splits of the two benchmarks.

**Experimental Results.** The results are shown in Table 1. Overall, the results on AVQA and AdVQA consistently illustrate the effectiveness of CIB for improving the human-adversarial robustness of VQA systems, demonstrating that using CIB as the training objective can facilitate VL-PMs to learn more adversarially robust representations. In addition, we observe that the performance improvement of CIB for human-adversarial robustness is not significant as the performance improvement for input robustness. One possible reason is that compared to the VQA benchmarks of input robustness, there is less irrelevant and redundant information between the question token-wise representations of human-adversarial VQA benchmarks.

### 3.3 Evaluation on Standard VQA Benchmark

To analyze the impact of CIB on the standard VQA performance (*i.e.*, whether the compression of internal representations of VL-PMs impairs the general VQA performance), we utilize CIB as the objective to train the aforementioned baseline VL-PMs on the VQA v2 training and validation sets. The results on VQA v2 test-dev are shown in Table 2 (Since we do not use the additional question-answer pairs from Visual Genome like UNITER\textsubscript{B} [18] for data augmentation in our experiments and some other detailed differences, there are minor differences between our re-implementation of baseline VL-PMs and the original paper results). Overall, training baseline VL-PMs with the proposed CIB can slightly improve

| Models       | VQA-Score (%) | Baseline | + CIB       |
|--------------|---------------|----------|-------------|
| VisualBERT\textsubscript{B} [16]    | 70.80 (70.46\textsuperscript{†}) | 71.62 (+1.16) |
| UNITER\textsubscript{B} [18]       | 72.70 (71.63\textsuperscript{†}) | 72.11 (+0.48) |
| LXMERT\textsubscript{B} [19]       | 72.42 (72.58\textsuperscript{†}) | 72.99 (+0.41) |
| ViLBERT\textsubscript{B} [20]      | 70.55 (70.55\textsuperscript{†}) | 71.00 (+0.45) |
| VL-BERT\textsubscript{B} [17]      | 71.16 (71.20\textsuperscript{†}) | 71.59 (+0.39) |

Table 2: Results on VQA v2 test-dev set. \textsuperscript{†} denotes our re-implementations of baseline VL-PTMs.
the standard VQA performance. We thus find that a certain degree of compression of representations can make the obtained representations more compact and robust, and facilitate VL-PMs to learn more true correlation between representations and labels.

### 3.4 Ablation Study

In this section, we utilize UNITER$_b$ [18] and LXMERT [19] as representatives of the baseline VL-PMs, and conduct all ablated experiments on VQA-Rep [6] and AVQA [8].

#### Impact of CIB terms.
To analyze how different components of CIB contribute to the VQA robustness, we perform ablation study on different meaningful combinations of the terms in Eq. (5). Specifically, there are three other meaningful combinations: $\frac{1}{2} \left( I(X^v; T^v) + I(X^l; T^l) \right)$ (i.e., $\odot + \odot$), $-I(T^v; T^l) + D_{\text{SKL}} (\text{i.e., } \odot + \odot)$, and $I(X^v; T^v) + I(X^l; T^l) + D_{\text{KL}} (\text{i.e., } \odot + \odot + \odot)$. The results in Table 3 for different combinations of CIB terms are consistent in general, empirically demonstrating that the proposed CIB formula is a more tight upper bound of $I(X^v, X^l; T^v, T^l)$.

#### Impact of MI Estimator.
Practically, any sample-based upper bound estimator of MI can be utilized to approximate $I(X^v; T^v)$ and $I(X^l; T^l)$, and any differentiable MI lower bound estimator can be applied to approach to $I(T^v; T^l)$. To analyze the impact of different MI estimators on CIB, we consider the following settings: (i) alternately using L1Out [22] instead of CLUB [21] as the MI upper bound estimator to approximate $I(X^v; T^v)$ and $I(X^l; T^l)$, (ii) approximating $I(T^v; T^l)$ with the other three MI lower bound estimators, InfoNCE [28], NWJ [27], and MINE [29]. Table 4 shows the VQA-Score of perturbed examples on VQA-Rep and the results on AVQA val of different MI estimators, which consistently demonstrates that the effectiveness of CIB in improving the input robustness of VQA systems does not depend on a specific MI estimator.

#### Impact of different internal representations of two-stream VL-PMs.
For the two-stream VL-PMs, as shown in Figure 1 (b), there is an alternative form of the obtained internal representations $T = [T^v, T^l]$, i.e., the visual and linguistic representations after the Vision-Transformer layers ($f_{\text{vtran}}$) and Language-Transformer layers ($f_{\text{ltran}}$). When finetuning two-stream VL-PMs with CIB for VQA, to analyze the impact of different internal representations, we replace the original $T = [T^v, T^l]$ in $\mathcal{L}_{\text{CIB}}$ with $T = [T^v', T^l']$. The results shown in Table 5 indicate that for two-stream VL-PMs, using the visual/linguistic representations after Vision/Language Transformer layers as the internal representations to estimate $\mathcal{L}_{\text{CIB}}$ is also feasible.

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**Table 3: Ablation study on the impact of different terms in CIB.**

| CIB   | Pert | VQA-Rep | CS(1) | CS(2) | CS(3) | CS(4) | AVQA val |
|-------|------|---------|-------|-------|-------|-------|----------|
| @    | 62.68 | 71.45 | 63.72 | 59.01 | 55.66 | 25.49 |
| @ @ | 64.07 | 72.73 | 65.51 | 61.03 | 57.82 | 25.27 |
| @ @ | 64.11 | 72.82 | 65.61 | 61.18 | 57.99 | 25.55 |
| @ @ | 63.23 | 71.94 | 64.40 | 59.85 | 56.62 | 24.78 |
| @ @ @ | 64.45 | 73.18 | 66.21 | 61.88 | 58.75 | 25.89 |

**Table 4: Ablation study on the impact of different MI estimators.**

| Models  | MI Estimator | Upper Bound | Low Bound | VQA-Score | VQA-Rep | AVQA val |
|---------|--------------|-------------|-----------|-----------|---------|----------|
| UNITER$_b$ | L1Out [22] | NWJ [27] | 62.68 | 25.49 |
|          | CLUB [21] | InfoNCE [28] | 64.14 | 25.85 |
|          | CLUB [21] | MINE [29] | 64.32 | 25.53 |
|          | CLUB [21] | NWJ [27] | 64.45 | 25.89 |
| LXMERT | L1Out [22] | NWJ [27] | 70.41 | 25.77 |
|         | CLUB [21] | InfoNCE [28] | 72.31 | 26.03 |
|         | CLUB [21] | MINE [29] | 72.34 | 26.11 |
|         | CLUB [21] | NWJ [27] | 72.48 | 25.98 |
|         | CLUB [21] | NWJ [27] | 72.62 | 26.15 |

**Table 5: Ablation study on the impact of different internal representations obtained by the two-stream VL-PTMs.**

| Models  | $\mathcal{L}_{\text{CIB}}$ | VQA-Score | VQA-Rep | AVQA val |
|---------|---------------------------|-----------|---------|----------|
| LXMERT | $I(Y; T) - \beta I(X^v, X^l; T^v, T^l)$ | 70.41 | 25.77 |
|        | $I(Y; T) - \beta I(X^v, X^l, T^v, T^l)$ | 72.53 | 26.14 |
|        | $I(Y; T) - \beta I(X^v, X^l, T^v, T^l)$ | 72.62 | 26.15 |
Sensibility of tradeoff $\beta$. $\beta$ controls the tradeoff between representation redundancy and compression, which is the key hyperparameter of CIB. We thus perform a grid search for $\beta$. Specifically, we consider the following values: $\beta \in \{1 \times 10^{-6}, 1 \times 10^{-5}, 5 \times 10^{-5}, 1 \times 10^{-4}, 5 \times 10^{-4}, 1 \times 10^{-3}, 5 \times 10^{-3}, 1 \times 10^{-2}, 5 \times 10^{-2}\}$. Figure 3 shows the variation curve of the VQA-Score (Pert) on VQA-Rep and AVQA with increasing $\log \beta$. We observe that both on VQA-Rep and AVQA, the VQA-Scores start to boost when $\beta$ is a quite small value, which indicates the effectiveness of CIB. When $\beta$ increases to $5 \times 10^{-5}$ and $1 \times 10^{-4}$, UNITER$_R$ and LXMERT achieve the best performance, respectively. After that, the performance usually starts to drop, meaning that extremely compressed representations of VL-PMs may begin to compromise the robustness of VQA systems.

3.5 Qualitative Visualization

Why can CIB improve the VQA robustness when finetuning VL-PMs for VQA? To explore the possible reason, we first enumerate the image-question pairs of VQA-Rep whose answers are correctly predicted by LXMERT finetuned with CIB but incorrectly predicted by the baseline LXMERT (i.e., finetuned without CIB). Then, we compute the attention score between the final representation $Z \in \mathbb{R}^d$ for answer prediction and the input visual representation $X^v \in \mathbb{R}^{K \times d}$ of object regions by $\text{score}_{\text{att}} = \text{softmax}(Z \cdot (X^v)^T / \sqrt{d})$. Finally, we plot the top two objects with the highest attention scores in the image. From the results shown in Figure 2, we observe that compared with the baseline LXMERT, the attended two objects obtained by LXMERT finetuned with CIB are more consistent and question-related. This observation qualitatively illustrates that using CIB as an additional training objective to finetune VL-PMs for VQA can encourage the VQA systems to learn more discriminative representations for different answers and reduce the irrelevant information to questions.
4 Related Work

Robustness in VQA. Recently, in order to promote the practical application of VQA systems, many works have been proposed to study various VQA robustness, such as human-adversarial robustness [8, 9], input robustness [6, 7, 5, 30], and the robustness against answer distribution shift [31–35]. In this paper, we explore input robustness and human-adversarial robustness. The input robustness means the capability of VQA systems to defend against the visual and linguistic variations, such as rephrasing questions [6, 7], manipulating images [5]. The prevailing method to improve input robustness is data augmentation, generating additional data to train more robust VQA models. Besides, contrastive learning [30] and adversarial training [36] are also introduced to improve input robustness. While data augmentation is a feasible and effective solution, the quality of the generated data is uncontrollable (e.g., limited expressiveness and excessive verbosity), and the human-generated process is time-consuming. The human-adversarial robustness is the capability of VQA systems to defend against adversarial attacks by human. More specifically, Li et al. [8] and Sheng et al. [9] consider adversarial attacks by human annotators on state-of-the-art VQA models and introduce two adversarial VQA benchmarks, AVQA and AdVQA, which consist of adversarial samples that can be answered correctly by human but not by state-of-the-art models. All recent studies demonstrate state-of-the-art VQA models are still vulnerable to input variations and adversarial attacks. In this paper, we propose to improve the robustness of VQA systems from an information-theoretic perspective.

Information Bottleneck (IB). The IB principle is originally proposed by Tishby et al. [10] for information compression, and is later applied to analyze deep learning model architectures [11, 37]. Essentially, the IB objective is to seek a tradeoff between maximizing the predictive accuracy and minimizing the representation complexity. Some recent researches target exploiting the IB principle to improve the model robustness and generalization, especially in Domain Generalization [38, 39], OOD Generalization [40], Multiview Representation Learning [14, 41], and finetuning of Pretrained Language Models [42, 15]. Besides, some works [43–45] aim to learn the disentangled optimal representation from an IB perspective. Since IB can facilitate compact and meaningful representations learning, we extend it to vision-language learning and apply it to obtain robust VQA systems.

Pretrained Vision-Language Models (VL-PMs). Vision-Language pretraining aims to learn task-agnostic visiolinguistic representations for improving the performance of downstream tasks in a finetuning fashion [46–52]. From the perspective of model architecture, prevailing VL-PMs models can be roughly grouped into two types: single-stream models [17, 18, 53–55, 50] and two-stream models [20, 19, 56–58]. Specifically, the single-stream models first align image regions and text words and then apply a uniform transformer [59] to learn the contextualized representations. The two-stream models first use two separate transformers to learn high-level representations of image and text, and then integrate the two modalities with a cross-modal transformer. In this paper, we unify the two typical types of VL-PMs models and propose CIB to improve the VQA robustness when finetuning the two types of VL-PMs for VQA.

5 Conclusion and Discussion

Conclusion. In this paper, we propose to improve the robustness of VQA systems when finetuning VL-PMs for VQA from the Information Bottleneck perspective. Specifically, we first derive a new IB lower bound (CIB) for vision-language learning, and then apply CIB to finetune VL-PMs for VQA. Extensive experiments on five VQA benchmarks of input robustness and two VQA benchmarks of human-adversarial robustness consistently demonstrate the effectiveness and superiority of CIB. In the future, we plan to assess the effectiveness of CIB when tuning VL-PMs for VQA using parameter-effective strategies, such as adapter-based tuning and prompt-based tuning.

Discussion. Redundancy has two sides. One of the reason why VL-PMs can significantly improve the performance of the downstream tasks is that VL-PMs have learned rich and redundant knowledge in the pretraining process. Practically, for downstream tasks, especially in-domain tasks, task-related redundancy can help VL-PMs quickly adapt to the downstream tasks, while task-agnostic redundancy will simultaneously impair the robustness of systems. This paper investigates improving the robustness of systems while preserving their accuracy by seeking a tradeoff between representation compression and redundancy. Another potential study direction is to explore how to explicitly reduce task-agnostic redundancy and adequately exploit task-related redundancy when finetuning VL-PMs for the downstream tasks.
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Appendices

This Appendix provides additional experimental results and the proof of Theorem 1.

A  Additional Experimental Results

A.1 Details on VL-PTMs.

Table 6: A summary of baseline VL-PTMs. † denotes the length of RoI features in each image is not fixed. The results marked in orange are re-implementations using the officially released code.

| ID      | VL-PTMs      | Transformer Architecture | Pretraining Datasets          | Visual Feature | VQA v2 test-dev | VQA v2 test-std |
|---------|--------------|--------------------------|------------------------------|----------------|-----------------|----------------|
| ID+OOD ID | VisualBERT [16] | single-stream encoder | COCO                         | 100(RoI-2048) | 70.80           | 71.00           |
|         | UNITERB [18]  | single-stream encoder   | COCO,VG,GQA                  | †(RoI-2048+Loc-7) | 72.70           | 72.91           |
|         | LXMERT [19]   | two-stream encoders     | COCO,VG,GQA                  | 36(RoI-2048+Loc-4) | 72.42           | 72.54           |
|         | ViLBERT [20]  | two-stream encoders     | CC                           | †(RoI-2048+Loc-5) | 70.55           | 70.92           |
|         | VL-BERTB [17] | single-stream encoder   | CC                           | †(RoI-2048+Loc-4) | 71.16           | 71.20           |

Table 6 summarizes the details of baseline VL-PTMs in our experiments. Specifically, we list the Transformer architecture of VL-PTMs, the pretraining datasets, the visual features (i.e., image region features and location/position features), and the VQA-Score of the downstream VQA task on VQA v2. Pretraining datasets include MS COCO caption[26] (COCO), Visual Genome[24] (VG), VQA v2[31] (VQA v2), GQA balance version[60] (GQA), VG-QA[61] (VG-QA), Conceptual captions[62] (CC), and SBU captions[63] (SBU).

A.2 Evaluation on Input Robustness

A.2.1 Details on VQA Benchmarks of Input Robustness

A.2.2 Evaluation on Input Robustness

Table 7: Benchmark details. len(Q) is the average question length. #IQ, #Pert, and #Ori denote the number of total image-question pairs, perturbation samples, and original samples.

| Benchmark | Perturbation | Metric | QType | Trans&Val #IQ len(Q) | Trans&Val #IQ len(Q) | Test #Pert/CE #Ori/Easy |
|-----------|--------------|--------|-------|-----------------------|-----------------------|------------------------|
| VQA-Rep [6] | Rephrasing   | CS(k)  | All   | 444K 6.20              | 162K 7.15             | 121,516 40,504         |
| VQA P2 [7]  | Par&Syn&Ant  | CS(k)  | All   | 444K 6.20              | 52K 6.32              | 26,512 25,814          |
| IV-VQA [5]  | Invariant object | #flips | All   | 444K 6.20              | 120K 5.85             | 83,700 36,181          |
| CV-VQA [5]  | Covariant object | #flips | All   | 444K 6.20              | 4K 5.83               | 4,141 2,641            |
| VQA-CE [25] | Counterexample | -      | All   | 444K 6.20              | 214K 6.19             | 63,298 147,681         |

Table 7 shows the details on VQA benchmark of input robustness, including perturbation type, robustness evaluation metric, question type, and statistics on train&val and test data. Specifically, VQA-Rep averages 3 rephrasings for each of 40,504 questions sampled from the val set of VQA v2 [31], and obtains ~162K image-question pairs. VQA P2 creates three types of linguistic perturbations, i.e., Paraphrastic (Par), Synonymous (Syn), and Antonymous (Ant), for 25,814 sampled questions, finally obtains ~52K image-question pairs. IV-VQA is created using a GAN-based resynthesis technique to remove objects irrelevant to the QA pairs from the image (i.e., removal of objects does not lead to any change in answer). Conversely, CV-VQA targets counting questions (Num) and removes one relevant object that makes the model prediction on the quantity of such objects are expected to be subtracted by 1. Finally, IV-VQA and CV-VQA have about 120K and 4K image-question pairs, respectively. VQA-CE is an evaluation protocol for multimodal shortcuts involved in images and questions. It leverages the detected shortcuts on training set to obtain 63,298 Counterexamples (i.e., where all shortcuts provide an incorrect answer) from VQA v2 val set. Besides, it builds 147,681 easy examples on which at least one shortcut provides the correct answer.
### A.2.2 Comparisons with State-of-the-Arts

**Comparisons on VQA-Rep.** Table 8 shows the complete comparisons with existing methods on VQA-Rep in terms of the VQA-Scores as well as CS(k). The results in (I) are cited from the work [6]. The result of (II) is cited from the work [36]. Considering the five baseline VL-PTMs (i.e., VisualBERT, UNITER, LXMERT, ViLBERT, and VL-BERT), we evaluate the robustness of CIB against the linguistic variations, i.e., the question rephrasings. Overall, finetuning LXMERT and UNITER with our CIB respectively achieve the best and second best performance on the VQA-Score and CS(k). This results show that compared with existing methods, CIB is effective in finetuning VL-PTMs for VQA to improve the input robustness of the VQA model with respect to input linguistic variations. Since the VL-PTM, LXMERT, is pretrained on the VQA task, the best performance is far better than the second best performance.

**Comparisons on VQA P2.** Table 9 shows the results on VQA P2. Using VisualBERT, UNITER, LXMERT, ViLBERT, and VL-BERT as baseline VL-PTMs, we evaluate the robustness of the proposed CIB against the linguistic perturbation of Synonymous, Paraphrastic, and Antonymous. From the comparisons in the table, we can observe the advantage of CIB in improving the input robustness, especially the robustness against synonymous and paraphrastic perturbations.

**Comparisons on IV-VQA.** Table 10 shows the comparisons with exiting methods on IV-VQA and CV-VQA. Using VisualBERT, UNITER, LXMERT, ViLBERT, and VL-BERT as baseline VL-PTMs, we also evaluate the robustness of our method against visual variations on the test splits of IV-VQA and CV-VQA. Results in (I) and results in (I) are cited from the work [5].

| # | Methods | Synonymous Pert | Paraphrastic Pert | Antonymous Pert | All Pert | Synonymous Ori | Paraphrastic Ori | Antonymous Ori | All Ori | Synonymous CS(2) | Paraphrastic CS(2) | Antonymous CS(2) | All CS(2) |
|---|---------|----------------|------------------|----------------|---------|----------------|------------------|----------------|--------|----------------|------------------|----------------|----------|
| (I) | BAN [65] | 64.50 63.30 - | 63.50 56.70 - | 73.90 86.00 - | - - | 63.30 69.60 66.20 | - - | - - | - - | 60.37 61.63 64.34 | - - | - - |
| | StackNMN [66] | 61.20 63.50 64.70 | 53.20 53.60 53.80 | 74.80 84.90 76.10 | - - | 60.55 67.00 72.20 | - - | - - | - - | 70.03 | 55.60 56.80 60.70 | - - | - - |
| | + Q3R [7] | - - | 56.90 - | - 84.70 - | - 70.30 | 66.90 67.40 72.20 | - - | - - | - - | 52.58 62.44 | - - | - - |
| | HybridNet [7] | - - | 65.00 - | - 55.70 - | - 76.40 | 63.30 67.00 66.60 | - - | - - | - - | 84.70 | 68.10 68.90 74.40 | - - | - - |
| | XNM [67] | 62.80 65.20 67.60 | 65.60 58.60 60.70 | 74.30 85.10 76.00 | - - | 64.70 68.30 68.80 | - - | - - | - - | 82.01 75.46 71.05 | 67.71 | 66.04 |
| | + Q3R [7] | - - | 72.90 - | - 61.80 - | - 84.70 | 68.10 68.90 74.40 | - - | - - | - - | 74.52 |
| (II) | VisualBERT + CIB | 68.62 70.98 73.31 | 59.44 60.40 63.45 | 78.28 88.04 81.01 | - - | 69.92 73.15 73.83 | - - | - - | - - | 35.29 72.70 74.18 | - - | - - |
| | UNITER + CIB | 70.03 72.32 75.36 | 61.84 62.16 66.86 | 79.25 89.57 82.93 | - - | 71.30 74.52 75.91 | - - | - - | - - | 74.52 |
| | LXMERT + CIB | 78.98 81.66 85.51 | 73.23 76.17 80.39 | 80.21 94.17 85.72 | 78.93 83.38 85.97 | - - | - - | - - | - - |
| | ViLBERT + CIB | 68.35 70.99 73.11 | 60.31 61.70 65.32 | 79.25 88.64 82.48 | 69.92 73.70 73.98 | - - | - - | - - | - - |
| | VL-BERT + CIB | 68.21 70.52 72.97 | 60.37 61.63 64.34 | 79.17 89.08 82.59 | 69.82 73.04 73.88 | - - | - - | - - | - - |

| # | Methods | VQA-Score Pert | Robustness Metric |
|---|---------|---------------|------------------|
| (I) | BUTD [23] | 51.22 61.51 | 60.55 46.96 40.54 34.47 |
| | + CC [6] | 52.58 62.44 | 61.66 50.79 44.68 42.55 |
| | Pythia [64] | 54.20 64.08 | 63.43 52.03 45.94 39.49 |
| | + CC [6] | 55.65 64.52 | 64.36 55.45 50.92 44.30 |
| | BAN [65] | 55.87 64.97 | 64.88 53.08 47.45 39.87 |
| | + CC [6] | 56.59 65.87 | 65.77 56.94 51.76 48.18 |
| (II) | ConCAT [30] | - - | 68.62 64.12 57.08 53.99 |
| | UNITER [18] | - - | 71.29 63.95 59.48 56.31 |
| | MANGO [36] | - - | 72.66 66.03 61.92 58.95 |
| (III) | VisualBERT + CIB | 63.10 69.78 | 71.83 64.16 59.34 56.31 |
| | UNITER + CIB | 64.45 70.91 | 73.18 66.21 58.68 55.97 |
| | LXMERT + CIB | 72.62 80.93 | 82.01 75.46 71.08 67.71 |
| | ViLBERT + CIB | 62.28 69.15 | 71.05 63.54 59.04 55.89 |
| | VL-BERT + CIB | 60.86 68.74 | 70.52 63.46 58.75 53.89 |

| # | Methods | IV-VQA Pert | CV-VQA Pert | IV-VQA #flips | CV-VQA #flips |
|---|---------|-------------|-------------|---------------|---------------|
| (I) | CL [68] | - | 60.21 | 17.89 | - | 39.38 |
| | SNMN [66] | - | 66.04 | 6.52 | - | 47.95 |
| | SAAHA [69] | - | 70.26 | 7.85 | - | 49.90 |
| | UNITER [18] | - | 8.47 | - | - | 40.67 |
| | MANGO [36] | - | 7.32 | - | - | 38.11 |
| (II) | VisualBERT + CIB | 47.81 | 83.48 | 57.84 | 32.46 | 77.99 |
| | UNITER + CIB | 76.63 | 85.05 | 27.59 | 46.92 | 79.89 |
| | LXMERT + CIB | 78.57 | 89.15 | 23.10 | 40.47 | 93.90 |
| | ViLBERT + CIB | 74.67 | 83.35 | 30.04 | 35.33 | 71.11 |
| | VL-BERT + CIB | 73.66 | 83.37 | 30.41 | 35.29 | 72.70 | 74.18 |
and the work [36], respectively. The #flips is the ratio of the number of predictions mismatched before and after visual content manipulation to the total number of evaluation samples. Observed the results in Table 10, we find that compared with existing methods, CIB can better improve the ability of VQA models to defend against input visual variations.

### Comparisons on VQA-CE

The comparisons with state-of-the-art methods on VQA-CE are summarized in Table 11. All results in group (I) and (II) are cited from the work [25]. Using VisualBERT, UNITERB, LXMERT, ViLBERT, and VL-BERT as baseline VL-PTMs, we evaluate the robustness of our CIB against shortcut learning on VQA-CE. From the results in the table, we observe that the performance of finetuning baseline VL-PTMs with our CIB for VQA can surpass existing methods by a large margin, especially the VQA-Score on Counterexamples. This results suggest that the proposed CIB can better alleviate the spurious correlations between representations and the shortcut learning involved in inputs.

#### Table 11: Comparisons with state-of-the-arts on VQA-CE.

| # | Methods | VQA-Score |
|---|---|---|
| (I) | Shortcuts [25] | 0.00 61.13 42.26 |
| | SAN [70] | 26.64 68.45 55.61 |
| | BLOCK [71] | 32.91 76.69 63.52 |
| | BUTD [23] | 32.25 75.03 61.88 |
| | + RUBi [72] | 33.14 73.32 60.96 |
| | + LMH + RMFE [73] | 34.26 73.12 61.15 |
| | + ESR [74] | 34.27 76.60 63.57 |
| | + LMH [32] | 33.26 76.18 62.96 |
| | + RUBi [72] | 33.26 76.18 62.96 |
| | + LMH + CSS [76] | 34.36 62.08 53.55 |
| | + RandImg [33] | 34.41 76.21 63.34 |
| (II) | VisualBERT + CIB | 40.86 81.25 68.80 |
| | ViLBERT [20] | 39.24 80.50 67.77 |
| | LXMERT + CIB | 57.14 89.21 79.32 |
| | ViLBERT + CIB | 39.51 81.22 68.35 |
| | VL-BERT + CIB | 38.24 82.00 67.92 |

### Comparisons on AVQA and AdVQA

#### Table 12: Comparisons on AVQA.

| # | Methods | VQA-Score |
|---|---|---|
| (I) | Training on VQA v2+VG-QA+AVQA. | |
| | BUTD [23] | 22.78 23.91 |
| | ClipBERT [77] | 24.35 24.24 |
| | VILLA [53] | 26.08 27.08 |
| | VILLA [53] | 25.32 26.32 |
| | LXMERT [19] | 24.13 25.24 |
| | UNITERB [18] | 24.10 25.04 |
| | UNITERB [18] | 24.78 26.27 |
| (II) | Training on AVQA train split. | |
| | VisualBERT + CIB | 24.55 24.78 |
| | UNITERB + CIB | 25.71 25.89 |
| | LXMERT + CIB | 26.18 26.15 |
| | ViLBERT + CIB | 26.55 26.49 |
| | VL-BERT + CIB | 26.35 26.38 |

#### Table 13: Comparisons on AdVQA.

| # | Methods | VQA-Score |
|---|---|---|
| (I) | Training on VQA v2 train+val split. | |
| | MoViE+MCAN [78] | 26.64 26.37 |
| | MMBT [79] | 26.70 25.78 |
| | UniT [80] | 28.15 27.55 |
| | VisualBERT [16] | 28.70 28.03 |
| | ViLBERT [20] | 27.36 27.36 |
| | ViLT [50] | 27.11 27.19 |
| | UNITER-B [18] | 25.16 25.20 |
| | VILLA-B [53] | 25.14 25.17 |
| | UNITER-L [18] | 26.94 28.03 |
| | VILLA-L [53] | 25.79 26.18 |
| | M4C [81] | 33.52 33.33 |
| (II) | Training on AVQA train split. | |
| | VisualBERT + CIB | 38.20 38.45 |
| | UNITERB + CIB | 37.55 38.00 |
| | LXMERT + CIB | 37.24 38.91 |
| | ViLBERT + CIB | 37.63 38.05 |
| | VL-BERT + CIB | 37.43 39.39 |
A.4 Visualization

Figure 4 shows the t-SNE plot of the representation $Z$ obtained by LXMERT for answer prediction on VQA-Rep. To better visualize the differences, we only consider the representations with regard to the top-30 high-frequency and top-100 low-frequency answers. Compared to the vanilla implementation (LXMERT without pretrained weights) and the baseline (pretrained LXMERT without CIB), finetuning LXMERT with CIB can advance the discriminability of the obtained representation.

B Proofs

B.1 Proof of Theorem 1

To prove Theorem 1 in the main text, we first enumerate some of the properties of mutual information (MI) and then state two easily proven Lemmas.

B.1.1 Statements

Properties of MI. For any random variables $X, Y$ and $Z$:

(P1) Positivity:

$I(X; Y) \geq 0, I(X; Y|Z) \geq 0$.

(P2) Chain rule:

$I(X, Y; Z) = I(Y; Z) + I(X; Z|Y)$

$= I(X; Z) + I(Y; Z|X)$

$= \frac{1}{2}[I(Y; Z) + I(X; Z) + I(X; Z|Y) + I(Y; Z|X)]$.

(P3) Chain rule (Multivariate Mutual Information):

$I(X; Y; Z) = I(Y; Z) - I(Y; Z|X)$.

(P4) Positivity of discrete entropy (for discrete $X$):

$H(X) \geq 0, H(X|Y) \geq 0$.

(P5) Entropy and Mutual Information:

$H(X) = H(X|Y) + I(X; Y)$. 
**Lemma B.1.** In representation learning, given a random variable $X$, the random variable $Z$ is defined to be a representation of $X$, we can simply state that $Z$ is conditionally independent from any other variable in the model once $X$ is observed. That is, for any variable (or groups of variables) $T_1$ and $T_2$ in the model, we have

$$I(Z; T_1 | X, T_2) = 0.$$  

**Lemma B.2.** Given a sequence of random variables $X_1, X_2, ..., X_n$ and a deterministic function $f$, then $\forall i, j = 1, 2, ..., n$, we have

$$I(X_i; f(X_i)) \geq I(X_j; f(X_i)).$$

**Proof.** By the definition,

$$I(X_i; f(X_i)) = H(f(X_i)) - H(f(X_i) | X_i),$$

$$I(X_j; f(X_i)) = H(f(X_i)) - H(f(X_i) | X_j).$$

Since $f$ is a deterministic function,

$$H(f(X_i) | X_i) = 0,$$

$$H(f(X_i) | X_j) \geq 0.$$

Therefore,

$$I(X_i; f(X_i)) \geq I(X_j; f(X_i)).$$

**Lemma B.3.** Let $Z_1$ and $Z_2$ are the representation of $X_1$ and $X_2$, respectively, then

$$I_\theta(X_1; Z_1 | X_2) \leq D_{KL}(p_\theta(Z_1 | X_1) \| p_\psi(Z_2 | X_2)),$$

$$I_\psi(X_2; Z_2 | X_1) \leq D_{KL}(p_\psi(Z_2 | X_2) \| p_\theta(Z_1 | X_1)).$$

**Proof.** By the definition,

$$I_\theta(X_1; Z_1 | X_2)$$

$$= \mathbb{E}_{x_1, x_2 \sim p(x_1, x_2)} \mathbb{E}_{z \sim p_\theta(Z_1 | X_1)} \left[ \log \frac{p_\theta(Z_1 = z | X_1 = x_1)}{p_\theta(Z_1 = z | X_2 = x_2)} \right]$$

$$= \mathbb{E}_{x_1, x_2 \sim p(x_1, x_2)} \mathbb{E}_{z \sim p_\theta(Z_1 | X_1)} \left[ \log \frac{p_\theta(Z_1 = z | X_1 = x_1)}{p_\psi(Z_1 = z | X_2 = x_2)} \right]$$

$$- \mathbb{E}_{x_1, x_2 \sim p(x_1, x_2)} \mathbb{E}_{z \sim p_\theta(Z_1 | X_1)} \left[ \log \frac{p_\psi(Z_1 = z | X_2 = x_2)}{p_\psi(Z_2 = z | X_2 = x_2)} \right]$$

$$= D_{KL}(p_\theta(Z_1 | X_1) \| p_\psi(Z_2 | X_2)) - D_{KL}(p_\psi(Z_2 | X_1) \| p_\psi(Z_2 | X_2))$$

$$\leq D_{KL}(p_\psi(Z_2 | X_2)) \| p_\theta(Z_1 | X_1)).$$

If and only if $p_\psi(Z_2 | X_2)$ coincides with $p_\theta(Z_1 | X_2)$, the equality holds. Analogously, we can derive $I_\psi(X_2; Z_2 | X_1) \leq D_{KL}(p_\psi(Z_2 | X_2)) \| p_\theta(Z_1 | X_1)).$

**B.1.2 Proof of Theorem 1**

**Theorem 1.** (Upper Bound of $I(X^v, X^l; T^v, T^l)$) Given two groups of random variables $X = [X^v, X^l], T = [T^v, T^l]$, the mutual information $I(X^v, X^l; T^v, T^l)$ can be upper-bounded with

$$I(X; T) = I(X^v, X^l; T^v, T^l) \leq I(X^v; T^v) + I(X^l; T^l) - I(T^v; T^l) + D_{SKL}.$$  

(12)

where $D_{SKL}$ denotes the Symmetric Kullback-Leibler divergence and can be obtained by averaging the two Kullback-Leibler divergence, $D_{KL}(p(t^v | x^v)) \| p(t^l | x^l))$ and $D_{KL}(p(t^l | x^l)) \| p(t^v | x^v))$. 

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Proof.

\[ I(X; T) = I(X^l, X^v; T) \overset{(P_2)}{=} \frac{1}{2} \left[ I(X^l; T) + I(X^v; T) + I(X^l; T|X^v) + I(X^v; T|X^l) \right] = \frac{1}{2} \left[ I(X^l; T^l, T^v) + I(X^v; T^l, T^v) + I(X^l; T|X^v) + I(X^v; T|X^l) \right] \]

Since,

\[ I(X^l; T^l, T^v) \overset{(P_2)}{=} I(X^l; T^l) + I(X^l; T^v|T^l) \]
\[ \overset{(P_2)}{=} I(X^l; T^l) + I(X^l; T^v) - I(X^l; T^v; T^l) \]
\[ \overset{(P_2)}{=} I(X^l; T^l) + I(X^l; T^v) - I(T^l; T^v) + I(T^l; T^v|X^l) \]
\[ \overset{(L^A.1)}{=} I(X^l; T^l) + I(X^l; T^v) - I(T^l; T^v) \]
\[ \overset{(L^A.2)}{\leq} 2I(X^l; T^l) - I(T^l; T^v). \]

Analogously, \( I(X^v; T^l, T^v) \) is upper bounded by

\[ I(X^v; T^l, T^v) \leq 2I(X^v; T^v) - I(T^l; T^v). \]

And,

\[ I(X^l; T^l, T^v|X^v) = I(X^l; T^l, T^v|X^v) \overset{(P_2)}{=} I(X^l; T^l|X^v) + I(X^l; T^v|X^v, T^l) \]
\[ \overset{(L^A.1)}{=} I(X^l; T^l|X^v), \]
\[ I(X^v; T^l|X^l) = I(X^v; T^l, T^v|X^l) \overset{(P_2)}{=} I(X^v; T^v|X^l) + I(X^v; T^l|X^l, T^v) \]
\[ \overset{(L^A.1)}{=} I(X^v; T^v|X^l). \]

Let \( D_{SKL} = \frac{1}{2}(D_{KL}(p_0||p_\phi) + D_{KL}(p_\phi||p_0)) \), therefore,

\[ I(X; T) = I(X^l, X^v; T^l, T^v) \]
\[ \leq I(X^l; T^l) + I(X^v; T^v) - I(T^l; T^v) + \frac{1}{2} \left[ I(X^l; T^l|X^v) + I(X^v; T^v|X^l) \right] \]
\[ \overset{(L^A.3)}{\leq} I(X^l; T^l) + I(X^v; T^v) - I(T^l; T^v) \]
\[ + \frac{1}{2} \left[ D_{KL}(p_\phi(T_1|X_1)||p_\phi(T_2|X_2)) + D_{KL}(p_\phi(T_2|X_2)||p_\phi(T_1|X_1)) \right] \]
\[ = I(X^l; T^l) + I(X^v; T^v) - I(T^l; T^v) + D_{SKL}. \]

\[ \square \]

B.2 Proof of Alternatives in Section 3.4

In this section, we provide the derivations of the meaningful combinations of terms in Eq.(5), i.e., the three alternatives in Section 3.4.

Proof of \( I(X^l, X^v; T^l, T^v) \leq \frac{3}{2} [I(X^v; T^v) + I(X^l; T^l)] \).
Proof.

\[
I(X^l, X^v; T^l, T^v) \leq I(X^l; T^l) + I(X^v; T^v) - I(T^l; T^v) + \frac{1}{2} [I(X^l; T^l|X^v) + I(X^v; T^v|X^l)] \\
\leq I(X^l; T^l) + I(X^v; T^v) + \frac{1}{2} [I(X^l; T^l|X^v) + I(X^v; T^v|X^l)] \\
\leq I(X^l; T^l) + I(X^v; T^v) + \frac{1}{2} [I(X^l; T^l) + I(X^v; T^v)] \\
= \frac{3}{2} [I(X^l; T^l) + I(X^v; T^v)].
\]

\[\]

Proof of \(I(X^l, X^v; T^l, T^v) \leq -I(T^v; T^l) + D_{SKL}.\) Please see the work of Federici et al. [14].

Proof of \(I(X^l, X^v; T^l, T^v) \leq I(X^v; T^v) + I(X^l; T^l) + D_{SKL}.\)

Proof.

\[\]

\[
I(X^l, X^v; T^l, T^v) \leq I(X^l; T^l) + I(X^v; T^v) - I(T^l; T^v) + D_{SKL} \\
\leq I(X^l; T^l) + I(X^v; T^v) + D_{SKL}.
\]

\[\]