How 'New Horizons' will see the Pluto-Charon system

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Abstract. We present a detailed survey of the dynamical structure of the phase space around the new moons of the Pluto–Charon system. We investigated the system in the framework of the spatial elliptic restricted three-body problem. Stability maps were created using chaos indicators both on the semimajor axis - eccentricity and semimajor axis - inclination plane. The structures related to the 4:1 and 6:1 mean motion resonances are clearly visible on the maps, but the detailed investigation of the resonant arguments shows no evidence of the resonances. We showed the possibility that Nix might be in the 4:1 resonance if its argument of pericenter or longitude of node falls in a certain range. Recently we improved our model to the general four-body problem, which can provide more accurate information about the stability, motion and possible resonances between the moons and Charon, which is relevant, since the 'New Horizons' mission will reach and study the system in 2015.

1. Introduction

Pluto’s first moon, Charon was found by Christy & Harrington (1978), which greatly facilitated the study of Pluto. The Pluto–Charon system is remarkable, since in the Solar system Charon is the largest moon relative to its primary, with the highest mass ratio of 0.1166 (Tholen 2008, hereafter referenced to as T08).

The discovery of Pluto’s new moons (Weaver 2005, Weaver 2006), Nix and Hydra rendered the system even more interesting. Thank to this, in latter years numerous studies were devoted to the system.

In 2006 two-body orbit solutions for Nix and Hydra were computed by Buie using the Hubble Space Telescope (Buie 2006, hereafter referenced to as B06). According to the results, the orbital periods of Nix and Hydra are close to the ratio of 4:1 and 6:1 with that of Charon, respectively, indicating mean-motion resonances. In this paper two bodies are in mean-motion resonance, when $n/n' = p/(p + q)$, where $n$ and $n'$ are the mean motions, $p$ and $q$ are small integers, where $q$ is the order of the resonance. Some of Buie’s major conclusions are: (i) Nix and Hydra are in nearly circular orbits, with eccentricities of 0.0023 and 0.0052, respectively, (ii) the orbits of the small moons are almost coplanar with Charon’s orbit, (iii) the orbital periods of Nix and Hydra are nearly commensurate with the period of Charon, but differ significantly from the exact ratios of 4:1 and 6:1, respectively. We note that in B06 the eccentricity of Charon was assumed to be zero. We demonstrated that the eccentricity of Charon substantially influences the phase space of the Pluto–Charon system; for details see Süli & Zsigmond (2009, hereafter referenced as S09).
In 2008 four-body orbit solution was computed by T08 fitting all the 22 parameters simultaneously. They studied whether the direct perturbations might be too strong to permit a sufficiently accurate extrapolation forward to the ‘2015 - New Horizons’ spacecraft encounter with Pluto. Also, with adequate data, this four-body orbit solution should yield the mass for each member of the system. Their major conclusions are the follows: (i) Pluto’s three moons are not quite coplanar, (ii) the new moons’ orbit planes precess around the system’s invariable plane with periods of $\approx 5$ and $\approx 15$ years, (iii) the orbital eccentricities are nonzero, but small for all three moons, (iv) there is no evidence of any mean motion resonances.

Nagy et al. (2006) studied the phase space of the Pluto–Charon system in the framework of the spatial circular restricted problem. The moons were treated as test particles and their semimajor axes, eccentricities and inclinations were varied. Summarizing their results: (i) the region inside 42 000 km is unstable, thus no moon could exist there, (ii) both moons reside in the stable region and the upper limit for the eccentricities are 0.17 for Hydra and 0.31 for Nix, far greater, than the current values of the moons, (iii) in the semimajor axis–inclination plane the 4:1 and 6:1 resonances are clearly visible above $\approx 20^\circ$ and $\approx 35^\circ$, respectively. Our main goal was to extend the previous investigations using the spatial elliptic restricted and general four body problems. Detailed analysis can be found in the work of S09. This paper is organized as follows: Section 2 describes the model and the initial conditions; Section 3 is devoted to the methods and in Section 4 we present the new results.

2. Model and initial conditions
Since the orbital radii of the moons are much smaller than the Hill radius ($\approx 8.0 \times 10^6$ km) of the Pluto-Charon system, the moons are deep in Pluto’s gravitational well, so the perturbations by the Sun can be ignored, as did T08.

First we applied the model of the spatial elliptic restricted three-body and later the general four-body problem. We integrated the dimensionless equations of motion. An obvious advantage of using such equations is that the results are independent of the exact value of the semimajor axis of Charon. The unit of length was chosen such that the separation of Pluto and Charon (the primaries) is unity, i.e. the semimajor axis of Charon $a_1 = 1$ A in all computations.

Let the unit of mass be the sum of the primaries, and the unit of the time the orbital period of Charon. The orbital plane of the primaries was used as reference plane, in which the line connecting the primaries at $t = 0$ defines a reference $x$-axis. The initial orbital elements are given in S09.

We study the problem more generally by considering the effect of non-zero inclinations on the orbital stability. The mass parameter $\mu = 0.104424$ was chosen according to the mass ratio of 0.1166, published in T08. To examine the phase space in the vicinity of Nix and Hydra separately we varied the initial orbital elements of the test particles. Stability maps were created for the $(a - e)$ and $(a - i)$ orbital element space for both moons for different eccentricity $e$, inclination $i$, argument of pericenter $\omega$ and longitude of node $\Omega$ values (for details see S09).

3. Methods
To compute the stability maps, the method of the maximum eccentricity (ME), the Lyapunov characteristic indicator (LCI) and the relative Lyapunov indicator (RLI) were used as tools for stability investigations of the massless bodies representing the small moons.

The ME method uses as an indication of stability a straightforward check based on the eccentricity. This action-like variable shows the probability of orbital crossing and close encounter of two bodies and therefore its value provides information on the stability of orbits. This simple check was found to be a powerful indicator of the stability character of orbits in previous investigations (Dvorak et al. 2003; Süli et al. 2005; Nagy, Süli & Erdi 2006). In this
work we define $\text{ME}$ as follows:

$$\text{ME} = \max_{t \in [0, 10^4]} T_C(e).$$

We plotted $\text{ME}_-$, where:

$$\text{ME}_- = \begin{cases} 1 & \text{if } \text{ME} = 1 \\ \text{ME} - e_0 & \text{otherwise} \end{cases},$$

where $e_0$ is the initial eccentricity of the test particle.

As a complementary tool, we computed also the RLI (Sándor et al. 2006) and the LCI (Froeschlé 1984), which are well-known chaos indicators.

Figure 1. The $(a-e)$ stability map for 8 $\omega$ values for Nix. Along the contour curves $\text{ME}_- = 0.04$. The location of the plus signs corresponds to the orbital elements published by B06 and T08.

4. Results

Here we show the $\text{ME}_-$ stability maps, presenting one phase space structure for Nix. On Figure 1 white solid lines are contour lines: along them $\text{ME}_-$ has a constant value, this way they can draw the approximate boundary of the most prominent structures. The white numbers are the corresponding $\omega$ values.

4.1. Mass parameter

We performed two different runs to estimate the effect of the different values of the mass parameter used by Nagy et al. (2006) and by T08. The structures for the two mass parameter are almost identical, for $\mu = 0.104424$ they are only shifted along the horizontal axis. This is a consequence of the changes in the coordinates of the barycenter. This shift could be important in the close vicinity of mean motion resonances.
4.2. Charon’s eccentricity
In order to visualise the effect of the absence of the Jacobi-integral, we plotted also the circular and the elliptic case for both moons. The differences are: in the elliptic case the unstable zone is much larger and the center of the resonance is shifted to larger semi-major axis, and the 4:1 and 6:1 resonances became stronger for low eccentricities. The V-shape did not change significantly.

4.3. The stability maps
We investigated the orbits systematically by changing the initial orbital elements of the test particle. In Figure 1, the results are summarized for the 8 values of $\omega$. The figure is dominated by a V-shaped gray structures, corresponding to the 4:1 mean motion resonance between Charon and Nix. These resonances can represent either ordered or weakly chaotic behaviour. The present positions of the moons are in stable region both on the ($a-e$) and ($a-i$) parameter spaces. On the maps the structures related to the 4:1 and 6:1 resonances are clearly visible, but none of them contains any of the moons. As you can see in Figure 1, in the case of $\omega = 157.9$ for Nix it can show the possibility of active resonance. But for Hydra we can not find any value, where the 6:1 resonance would appear, since it is very weak in the vicinity of the moon. The ($a-i$) maps give the same results. Mention must be made that the planar and spatial cases are almost identical, when the inclinations are very small (as they are for the moons).

4.4. Results using the general problem
Newly we started to investigate the system using the framework of the general four-body problem. The masses and their error limits were from work of T08. Using the maximal margin of error, our initial results show that the moons are on orbits those very hard to estimate for larger time. First we calculated the well-known Lyapunov-time ($T_L$). The Lyapunov time reflects the limits of the predictability of the system. By convention, it is measured as the time for nearby trajectories of the system to diverge by $e$ (Euler-number). We found that for the new moons $T_L = 20$ day. Since the ‘New Horizons’ mission will reach and study the system in 2015 (Stern 2002), it is very relevant to know the precise positions of the system’s members. This claims further investigations which are under way.

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