Fates of the dense cores formed by fragmentation of filaments: do they fragment again or not?

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ABSTRACT
Fragmentation of filaments into dense cores is thought to be an important step in forming stars. The bar-mode instability of spherically collapsing cores found in previous linear analysis invokes a possibility of re-fragmentation of the cores due to their ellipsoidal (prolate or oblate) deformation. To investigate this possibility, here we perform three-dimensional self-gravitational hydrodynamics simulations that follow all the way from filament fragmentation to subsequent core collapse. We assume the gas is polytropic with index γ, which determines the stability of the bar-mode. For the case that the fragmentation of isolated hydrostatic filaments is triggered by the most unstable fragmentation mode, we find the bar mode grows as collapse proceeds if γ < 1.1, in agreement with the linear analysis. However, it takes more than ten orders-of-magnitude increase in the central density for the distortion to become non-linear. In addition to this fiducial case, we also study non-fiducial ones such as the fragmentation is triggered by a fragmentation mode with a longer wavelength and it occurs during radial collapse of filaments and find the distortion rapidly grows. In most of astrophysical applications, the effective polytropic index of collapsing gas exceeds 1.1 before ten orders-of-magnitude increase in the central density. Thus, supposing the fiducial case of filament fragmentation, re-fragmentation of dense cores would not be likely and their final mass would be determined when the filaments fragment.

Key words: stars: formation – galaxies: star formation – galaxies: evolution.

1 INTRODUCTION
What determines the mass of stars? In the standard scenario of star formation, stars are formed inside dense cores with the mass conversion efficiency of about 30%, as supported by both simulations (e.g., Machida & Matsumoto 2012) and observations (e.g., André et al. 2010). Dense cores, in turn, are thought to be formed by fragmentation of filamentary molecular clouds, or simply filaments. While observations with the Herschel satellite have revealed that the dense cores are located along filaments (e.g., André et al. 2010; Arzoumanian et al. 2011; Roy et al. 2015), simulations have shown that filaments fragment into cores that subsequently collapse in a run-away fashion (e.g., Inutsuka & Miyama 1997). At the end of the collapse, which is well approximated with the self-similar solution of spherical collapse (so-called Larson-Penston solution; Larson 1969; Penston 1969; Yahil 1983), protostars are formed when the central parts of the dense cores become optically thick.

The theoretical works on filament fragmentation, both analytical (e.g., Stodolkiewicz 1963; Larson 1985; Naga- sawa 1987; Inutsuka & Miyama 1992; Fischera & Martin 2012) and numerical (e.g., Inutsuka & Miyama 1997; Heitsch 2013a,b; Heigl et al. 2016; Clarke et al. 2016; Gritschneder et al. 2017) ones, have shown that the typical fragmentation mass is given by the Jeans mass at fragmentation of the filaments. This gives reasonable estimate for the initial mass of dense cores, but the final mass can be dramatically altered if the cores will fragment again in later evolution.

Such re-fragmentation can be caused by deformation of the cores due to the bar-mode instability of the Larson-Penston solution found in the previous linear stability analyses (Hanawa & Matsumoto 2000; Lai 2000). They have shown that the bar (l = 2) mode is unstable if γ < 1.1, where γ is the effective polytropic index of gas. This instability deforms cores into a prolate or oblate shape depending on the seed perturbation for the instability. Unfortunately, the role of this instability in star formation was unclear,
because the linear analysis can predict neither the initial amplitude of the unstable mode nor the fate of the instability in the non-linear regime. In order to address this issue, it is necessary to perform numerical simulations.

In this paper, we investigate the possibility of re-fragmentation of dense cores formed by filament fragmentation, focusing on the role of the bar-mode instability. We continuously follow filament fragmentation and subsequent core collapse with three-dimensional self-gravitational hydrodynamics simulations. We study the $\gamma$ dependence of the evolution of cores, as well as the dependence on the way in which filaments fragment. Although there have been a number of simulations for core collapse after filament fragmentation (e.g., Nakamura et al. 1993; Nakamura 2000; Matsumoto et al. 1997; Inutsuka & Miyama 1997), none of them has focused on the possibility of re-fragmentation or the role of the bar-mode instability.

The paper is organized as follows. In Sec. 2, we describe our models and numerical methods. In Sec. 3, we present the result of our simulations. The conclusion and discussion are given in Sec. 4.

2 MODELS & METHODS

2.1 Basics

Below, we briefly summarize the $\gamma$ dependence of the two types of instabilities, namely the fragmentation-mode and bar-mode instabilities, which motivates the models of this work. We first introduce some useful variables and then review the fragmentation-mode instability of filaments and the bar-mode instability of the Larson-Penston solution.

The polytropic gas is characterized by the equation of state,

$$P = K \rho^\gamma,$$

and the sound speed is obtained as

$$c_s = \sqrt{K \gamma \rho^{\gamma-1}}.$$

We define the free-fall time\(^1\) as

$$t_{\text{ff}} = \frac{1}{\sqrt{4\pi G \rho}},$$

and the Jeans length as

$$\lambda_J = \frac{3}{G \rho} c_s = 2\pi c_s t_{\text{ff}}.$$

Once a reference density $\rho_0$ is given, we can obtain $c_{s,0}$, $t_{\text{ff},0}$, and $\lambda_{J,0}$ for $\rho_0$ with equations (2) – (4). Using these quantities and their combinations, all dimensional quantities in this work can be made dimensionless.

Infinitely long and static filaments are subject to gravitational instability of axisymmetric modes that leads to fragmentation into cores. The linear stability analysis of static polytropic filaments (e.g., Larson 1985; Inutsuka & Miyama 1992) has shown that amplitude of the most unstable mode $\delta_{\text{F,max}}$ grows exponentially with the growth rate $\sigma_{\text{F,max}}$, as

$$\delta_{\text{F,max}} \propto \exp\left[\sigma_{\text{F,max}} \frac{t}{t_{\text{ff},0}}\right]. \quad (5)$$

We take the data for the wave number $k_{\text{max}}$ and squared growth rate $\mu_{\text{max}}$ (= $\sigma_{\text{F,max}}^2$) of the most unstable mode from Fig. 9 of Inutsuka & Miyama (1992) and fit them as functions of $\gamma$ for $1 \leq \gamma \leq 1.5$. As a result, we obtain the fitting formulae,

$$k_{\text{max}} H_0 = 2.30 - 2.89 \gamma + 1.77 \gamma^2 - 0.374 \gamma^3, \quad (6)$$

and

$$\mu_{\text{max}} = 0.0688 + 0.236 \gamma - 0.268 \gamma^2 + 0.0781 \gamma^3, \quad (7)$$

where the central density of the filament is assumed to be $\rho_0$ and $H_0 = 2c_{s,0}/\sqrt{2\pi G \rho_0} = (\sqrt{2}/\pi) \lambda_{J,0}$ is the width of filaments (e.g., Inutsuka & Miyama 1992). Table 1 presents the wavelength of the most unstable mode $\lambda_{\text{F,max}} (= 2\pi/k_{\text{max}})$ and $\sigma_{\text{F,max}}$ for selected values of $\gamma$, computed with the above fitting formulae.

The linear analyses (Hanawa & Matsumoto 2000; Lai 2000) have shown the Larson-Penston solution is unstable if $\gamma < 1.1$ due to the bar-mode instability. The amplitude of the bar mode $\delta_B$ grows in a power-law fashion with the growth rate $\sigma_B$, as

$$\delta_B \propto (t_{\text{col}} - t)^{-\sigma_B} \propto (\rho_{\text{max}})^{3\sigma_B}.$$

To derive the second relation, we have used the relation $\rho_{\text{max}} \propto (t_{\text{col}} - t)^{-2}$ for the Larson-Penston solution, where $t_{\text{col}}$ is the time when $\rho_{\text{max}}$ formally diverges. Using the data for $\sigma_B$ taken from Fig. 2 of Hanawa & Matsumoto (2000), we obtain the fitting formula,

$$\sigma_B = -2.84 + 9.39 \gamma - 6.20 \gamma^2,$$

for $0.9 \leq \gamma \leq 1.1$. This formula is evaluated for some values of $\gamma$ and shown in Table 2.\(^2\)

| $\gamma$ | 0.9 | 0.95 | 1 | 1.05 | 1.1 | 1.2 |
|---------|-----|------|---|------|----|----|
| $\lambda_{\text{F,max}} [H_0]$ | 7.3 | 7.6 | 7.8 | 8.0 | 8.2 | 8.6 |
| $\sigma_{\text{F,max}}$ | 0.35 | 0.34 | 0.34 | 0.33 | 0.33 | 0.32 |

2 Note that there is a room for numerical error even in the linear analysis because the eigenmode is numerically obtained. As a result, the reported values of the critical $\gamma$ for the bar-mode instability, 1.097 in Hanawa & Matsumoto (2000) and 1.09 in Lai (2000), are similar but not exactly the same.

\(^1\) For our convenience, we adopt the definition of $t_{\text{ff}}$ in equation (3), instead of $(3/32\pi G \rho)^{1/2}$, which is also used in the literature.

\(^2\) Table 2. $\gamma$ dependence of the bar mode

| $\gamma$ | 0.9 | 0.95 | 1 | 1.05 | 1.1 | 1.2 |
|---------|-----|------|---|------|----|----|
| $\sigma_B$ | 0.59 | 0.48 | 0.35 | 0.18 | stable | stable |
is polytropic (equation 1). The initial conditions are generated by adding velocity perturbations to filaments. Below, we will describe the models studied in our simulations.

The initial density profile is given by

\[ \rho_{\text{ini}}(r) = f \rho_{\text{st}}(r), \]

where \( \rho_{\text{st}} \) is the density profile of a hydrostatic polytropic filament axisymmetric about the z axis (Stodolickiewicz 1963; Ostriker 1964), \( r = \sqrt{x^2 + y^2} \) the cylindrical radius and \( f \) a density enhancement factor. Here, we take \( \rho_0 = \rho_{\text{st}}(0) \).

For reference, the central density of filaments in local star forming regions is about \( 10^{-20} - 10^{-18} \) g cm\(^{-3} \) (see, e.g., Arzoumanian et al. 2011). We numerically obtain \( \rho_{\text{st}} \) except for \( \gamma = 1 \), when \( \rho_{\text{st}} \) is obtained analytically as \( \rho_{\text{st}} = \rho_0 (1 + r^2/H_0^2)^{-2} \) with \( H_0 = 2c_s^2/\sqrt{2\pi G \rho_0} \). We assume that the filament is under the external pressure of an ambient gas with density \( \rho_{\text{ext}} \).

Fragmentation of filaments is triggered by the initial velocity perturbations. We assume a perturbation of \( v_z \) with the sinusoidal \( z \) dependence and the initial velocity field is given by

\[ v_{\text{ini}}(x) = \begin{pmatrix} 0 \\ 0 \\ v_0 \sin(2\pi z/\lambda) \end{pmatrix}, \]

where \( v_0 \) and \( \lambda \) are the amplitude and wavelength of the perturbation, respectively.

The simulations are run with the model parameters summarized in Table 3. We first study the fiducial case where the fragmentation of isolated hydrostatic filaments is triggered by the most unstable mode, as often supposed in the literature (e.g., Larson 1985). To investigate the \( \gamma \) dependence of the evolution, we perform the simulations of sets “G” (for “gamma”) with different \( \gamma = 0.9, 0.95, 1, 1.05, 1.1 \) and 1.2). In this case, we take \( f = 1, v_0 = 10^{-5} c_s, \lambda = \lambda_{\text{max}} \) and \( \rho_{\text{ext}} = 0 \). (See Sec. 2.1). The very small perturbation adopted just to follow the fragmentation triggered purely by the most unstable eigenmode and the evolution is astrophysically relevant only after the mode grows up to have a certain amplitude.

In the actual astrophysical situations, however, fragmentation can proceed in a non-fiducial way. Thus we perform four sets of simulations with the model parameters different from the fiducial ones. The parameters are set to the fiducial values unless otherwise stated. First, we study the effect of the deviation of the initial velocity perturbation from the most unstable eigenmode with the simulations of set “V” (for “velocity”). For this set, we enhance \( v_0 \) to 0.1, 0.3 and 0.5 \( c_s \) (see equation 11) and emphasize the converging nature of the initial velocity field. Second, we study fragmentation triggered by modes with various wavelength by performing the simulations of set “L” (for “lambda”), for which we change \( \lambda = 0.6, 1.5 \) and 2 \( \lambda_{\text{max}} \) (see equation 11). Third, to study fragmentation of radially collapsing filaments, we perform the simulations of set “D” (for “density”), for which we enhance the initial density taking \( f = 1.05, 1.1 \) and 1.2 (see equation 10), with \( v_0 \) also increased to 0.1 and 0.5 \( c_s \). In the above three cases, we take \( \gamma = 1, 1.05 \) and 1.2 to see the \( \gamma \) dependence. Finally, to study fragmentation of filaments under external pressure of an ambient gas, we perform the simulations of set “E” (for “external”), for which we change the density of an ambient gas by taking \( \rho_{\text{ext}} = 0.01, 0.04 \) and 0.09. We take \( \gamma = 1 \) for set E.

### Table 3. Model parameters for runs studied

| Set | \( \gamma \) | \( \lambda/\lambda_{\text{max}} \) | \( f \) | \( v_0/c_s \) | \( \rho_{\text{ext}}/\rho_0^a \) | shape \( b \) |
|-----|-------------|----------------------|-------|-------------|----------------------|-------|
| 0.9 | 1           | 1                    | 10^-3 | 0           | P                    |       |
| 0.95| 1           | 1                    | 10^-3 | 0           | P                    |       |
| G   | 1           | 1                    | 10^-3 | 0           | O                    |       |
| 1.05| 1           | 1                    | 10^-3 | 0           | O                    |       |
| 1.1 | 1           | 1                    | 10^-3 | 0           | S                    |       |
| 1.2 | 1           | 1                    | 10^-3 | 0           | S                    |       |

\[ ^a \text{The density at the boundary of the computational box is finite even when } \rho_{\text{ext}}/\rho_0 = 0 \text{ for computational reason (see text).} \]

\[ ^b \text{Shape of core at the end of simulation: “P”, “O” and “S” indicate prolate, oblate and spherical shapes, respectively.} \]

2.3 Numerical methods

We use the self-gravitational magneto-hydrodynamics code with adaptive mesh refinement (AMR), SFUMATO (Matsumoto 2007), but with the magnetic module switched off.
The hydrodynamical solver adopts the total variation diminishing cell-centred scheme with second-order accuracy in space and time.

Our computational domain is a cube with the side length $L_{\text{box}} = \lambda$ (see equation 11). We solve the three-dimensional (3D) hydrodynamics with the Cartesian coordinate set, without using the axisymmetry of the system. We initially set a uniform grid with $N_{\text{ini}} = 256$ meshes in each direction (256$^3$ cells). The Jeans condition is employed as a refinement criterion in the block-structured AMR technique of the code. Blocks are refined to resolve one Jeans length $\lambda_J$ (equation 4) with at least $N_{\text{cell}} = 32$ meshes. We test the convergence of the numerical results in Appendix A.

We assume the periodic boundary condition in the $z$ direction. In the $r$ direction, however, we fix the density to the boundary value $\rho_b$ and the velocity to zero outside the boundary at $r_b$. We take $r_b$ as the radius where the initial density is equal to the ambient density, i.e., $\rho_{\text{ini}}(r_b) = \rho_{\text{ext}}$, and $\rho_b$ as $\rho_{\text{ext}}$. If the cylinder with $r_b$ is larger than the computational box, $r_b$ and $\rho_b$ are replaced with $L_{\text{box}}/2$ and $\rho_{\text{ini}}(L_{\text{box}}/2)$, respectively. For the case $\rho_b = 0$, we take a finite but sufficiently small value (e.g., $\rho_b = 10^{-8}\rho_0$) in the actual calculations for computational reasons. We assume the gravitational potential for the isolated filament on the surfaces of the computational box in the $x$ and $y$ directions.

3 RESULTS

3.1 Evolution in a typical case

In this section, we describe the time evolution in the $\gamma = 1.05$ model of the fiducial case (fourth line in Table 3), as a typical example of our simulations. Although the shapes of the cores at the end of simulations are greatly different depending on the models, the evolution generally proceeds in a way similar to that described below.

Fig. 1 shows the density and velocity distributions in the $xz$-plane at the four different stages of evolution: (a) the maximum (central) density $\rho_{\text{max}}/\rho_0 = 1$ (initial time), (b) 10, (c) $10^6$ and (d) $10^{16}$. Note that we use $\rho_{\text{max}}/\rho_0$ as a time variable since it monotonically increases with the time as collapse proceeds. The panels in Fig. 1 show that filament fragmentation and subsequent collapse of the core proceed as follows:

(a) A small velocity perturbation to the filament is seen in the initial condition. It is a seed for the most unstable fragmentation mode, which later grows and leads to the fragmentation of the filament.

(b) Left: A high density region, or core, is formed due to the fragmentation of the filament and starts collecting the surrounding gas gravitationally. Right: The core is initially prolate along the filament.

(c) Left: The core collapses in a run-away fashion. Right: The gas dynamics near the centre approaches the Larson-Penston solution. The formerly prolate core becomes nearly spherical, although slightly oblate.

(d) Left: The run-away collapse continues. Right: Once the gas dynamics becomes sufficiently close to the Larson-Penston solution, the bar-mode instability begins to grow. Accordingly, the core becomes more and more oblate and the distortion finally becomes non-linear.

Below, we will examine the evolution in more detail, focusing on the fragmentation of the filament and the collapse and deformation of the core.

First, we see how fragmentation occurs. To quantify the degree of fragmentation, we define the overdensity $\delta$ normalized by $\rho_0 = \rho_{\text{ini}}(0)$ (equation 10) as

$$\delta = \frac{\rho_{\text{max}}(t) - \rho_0}{\rho_0},$$

where $\rho_{\text{max}}(t)$ is the maximum density at $t$. The density is largest at the centre of the core since the beginning of the fragmentation. The fragmentation is roughly completed when $\delta \sim O(1)$. Fig. 2 shows the time evolution of $\delta$, along with the linear growth rate of the most unstable fragmentation mode ($\sigma_{\nu,\text{max}} = 0.33$; Table 1). The agreement of the two slopes for $10^{-2} \lesssim \delta \lesssim 1$ indicates that the fragmentation is caused by the growth of the most unstable mode, as we expect for the fiducial case. Note that only the result for $\delta \gtrsim 10^{-2}$ is reliable because that for lower $\delta$ depends on the resolution (see Appendix A).

Second, we examine the convergence of the core collapse to the Larson-Penston solution. To quantify how much the central dynamics is close to the Larson-Penston solution, we introduce the ratio $t_{\text{dyn}}/t_{\text{ff}}$, where the dynamical time scale $t_{\text{dyn}}$ is defined as

$$t_{\text{dyn}} = \frac{\rho_{\text{max}}}{\rho_{\text{max}}},$$

and the free-fall time scale $t_{\text{ff}}$ is given by equation (3) with $\rho = \rho_{\text{max}}$. This ratio indicates the rapidness of the collapse: $t_{\text{dyn}}/t_{\text{ff}} = 0.41$ for the homogeneous gravitational collapse (see, e.g., Tsuribe & Inutsuka 1999) and it increases to $t_{\text{dyn}}/t_{\text{ff}} = 0.76$ for the Larson-Penston solution with $\gamma = 1.05$ (see, e.g., Lai 2000), because the pressure delays the collapse. The evolution of $t_{\text{dyn}}/t_{\text{ff}}$ is shown in Fig. 3(a). Soon after the fragmentation, the collapse is still slow and $t_{\text{dyn}}/t_{\text{ff}}$ is large. Later on, the dynamics of core approaches the Larson-Penston solution (e.g., Inutsuka & Miyama 1997), as indicated by the decrease of $t_{\text{dyn}}/t_{\text{ff}}$ toward the value for the Larson-Penston solution (horizontal dashed line). However, the approach is rather slow and it takes about five orders of magnitude in density increase for the collapse to become close to the Larson-Penston solution. Note that the features of $t_{\text{dyn}}/t_{\text{ff}}$ appearing every four-time increase in the density are originated from numerical errors at refinement, although they hardly affect the result (Appendix A).

Finally, we see how the core deforms. We use the axial ratio to quantify the deformation, where the axes of the core are defined using the inertia tensor (see, e.g., Matsumoto & Hanawa 1999), as below. We regard the central dense region with $\rho > \rho_{\text{th}} = 0.1\rho_{\text{max}}$ as the core, and then its inertia tensor and total mass are given respectively by

$$I_{ij} = \int_{\rho > \rho_{\text{th}}} dx x^i x^j \rho(x),$$

for the same purpose, Tsuribe & Inutsuka (1999) introduced the normalized central density $z_0$ using a relation $\rho_{\text{max}} = z_0/(t_{\text{col}} - t)^2$. We can show $t_{\text{dyn}}/t_{\text{ff}} = \sqrt{\pi}/2$ with equation (13).
Figure 1. Time evolution of density (colour) and velocity (arrows) distributions in the \(xz\)-plane for the \(\gamma = 1.05\) model of the fiducial case (set G in Table 3). The whole computational domains (left) and the central boxes on a Jeans length scale (right) are shown. The maximum (central) density is (a) \(\rho_{\text{max}}/\rho_0 = 1\) (initial time), (b) 10, (c) 10\(^6\) and (d) 10\(^{16}\). The velocity and length scales are shown on the bottom-right corner of each panel, with \(c_{\alpha,n} = 10^{n(\gamma-1)/2} \tilde{c}_{\alpha,0}\) (equation 2) and \(\lambda_{J,n} = 10^{n(\gamma-2)/2} \tilde{\lambda}_{J,0}\) (equation 4) for \(\rho = 10^n \rho_0\).
The dashed line represents the linear growth rate (see equation 8). Time evolution of (a) \( t_{\text{dyn}}/t_{\text{ff}} \), (b) \( a_x/a_z \) and (c) \( \Delta = 2(a_x - a_z)/(a_x + a_z) \) for the model in Fig. 1. In panel (a), the dashed line represents \( t_{\text{dyn}}/t_{\text{ff}} \) of the Larson-Penston solution for \( \gamma = 1.05 \). In panel (c), the solid (dot-dashed) line corresponds to prolate, spherical and oblate shapes, respectively. Note the definition of the axes of the core is not valid as the distortion is caused by the bar mode. We emphasize here that the bar mode grows only when the col- lapse becomes sufficiently close to the Larson-Penston solution. To analyze the growth of the bar mode in more detail, we define the oblateness (see Matsumoto & Hanawa 1999) as

\[
\Delta = \frac{a_x - a_z}{(a_x + a_z)/2},
\]

where \( a_x \) and \( a_z \) are defined in equation (16). Fig. 3(c) shows the evolution of \( \Delta \). The oblateness \( \Delta \) changes its sign at \( \rho_{\text{max}}/\rho_0 \sim 10^2 \), as the axial ratio changes from \( a_x/a_z < 1 \) to \( a_x/a_z > 1 \) (Fig. 3b), and begins to increase in a power-law fashion with a small initial amplitude of \( \Delta \lesssim 0.1 \) at \( \rho_{\text{max}}/\rho_0 \sim 10^5 \), when the collapse becomes sufficiently close to the Larson-Penston solution (Fig. 3a). The rate of the power-law increase is consistent with the linear analysis of the bar-mode instability (\( \sigma_B = 0.089 \); Table 2),\(^4\) indicating that the distortion is caused by the bar mode. We emphasize here that the bar mode grows only when the collapse is sufficiently close to the Larson-Penston solution.

The growth rate is slightly smaller than in the linear analysis due to several reasons. First, the background collapse is not exactly the Larson-Penston solution. Second, not only the pure bar mode but also other modes contribute to \( \Delta \). Third, the growth rate tends to be smaller in the non-linear regime owing to the small dynamic range of \( \Delta (\sim 2 < \Delta < 2) \). Note also that the growth rate can be overestimated in the linear analysis (Hanawa & Matsumoto 2000; Lai 2000) due to numerical error (see the footnote in Sec. 2.1).

### 3.2 \( \gamma \) dependence

Here we investigate the \( \gamma \) dependence of the evolution in the fiducial case, where fragmentation of isolated hydrostatic

\[ M = \int_{\rho > \rho_{\text{th}}} \rho(x) \, dx. \]
filaments is triggered by the most unstable mode, as often considered in the literature (e.g., Larson 1985). Below, we present the results for the models with $\gamma = 0.9, 0.95, 1, 1.05, 1.1$ and 1.2 (set G in Table 3).

Fig. 4 shows the final shapes of the cores at $\rho_{\text{max}}/\rho_0 = 10^{12}$. Although the evolution generally proceeds in a way similar to that explained in Sec. 3.1 irrespective of $\gamma$, the final shapes are greatly different depending on $\gamma$, as summarized in Table 4. Below, we will see how this dependence arises.

Fig. 5(a) shows the evolution of the axial ratio $a_x/a_z$. Although $a_x/a_z$ generally increases in the initial phase ($\rho_{\text{max}}/\rho_0 \leq 10^2$), it evolves differently later on depending on the value of $\gamma$. In the $\gamma = 0.9$ model, $a_x/a_z$ begins to decrease before reaching unity due to the rapid growth of bar-mode instability; and thus the core becomes prolate. In the less unstable case with $\gamma = 0.95$, the evolution is similar but the final distortion is weaker. In the models with $\gamma = 1$ and 1.05, the bar-mode instability is so weak that $a_x/a_z$ exceeds unity before the instability begins to grow and thus the cores become oblate. The oblateness is stronger in the model with $\gamma = 1$ than with $\gamma = 1.05$ because the growth rate of the bar-mode instability is larger for smaller $\gamma$. In the models with $\gamma = 1.1$ and 1.2, the cores become spherical because the spherical collapse is stable and the bar-mode perturbation damps.

How the growth of the bar-mode instability depends on $\gamma$ is clearly seen in Fig. 5 (b), where we plot the evolution of the oblateness $\Delta$ (equation 17) for the models with $\gamma \leq 1.05$. Except for the $\gamma = 0.9$ model, the bar mode begins to grow in a power-law fashion with a small initial amplitude of $\Delta \lesssim 0.1$ at $\rho_{\text{max}}/\rho_0 \sim 10^7-10^8$, when the background spherical collapse becomes close to the Larson-Penston solution, as seen for the $\gamma = 1.05$ case in Sec. 3.1. Because of this, in astrophysically interesting cases of $1 \leq \gamma < 1.1$ (see Sec. 4), it needs about ten orders-of-magnitude increase in the density before the distortion becomes significant.

![Figure 4](image_url)

**Figure 4.** Same as the right column of Fig. 1 but for the models of the fiducial case with (a) $\gamma = 0.9$, (b) 0.95, (c) 1, (d) 1.05, (e) 1.1 and (f) 1.2. The maximum (central) density is $\rho_{\text{max}}/\rho_0 = 10^{12}$ in all panels.

| $\gamma$ | Shape                  |
|---------|------------------------|
| (a) 0.9 | strongly prolate       |
| (b) 0.95| weakly prolate         |
| (c) 1   | moderately oblate      |
| (d) 1.05| weakly oblate          |
| (e) 1.1 | spherically symmetric  |
| (f) 1.2 | spherically symmetric  |
the $\gamma = 0.9$ model, the bar mode is so strong that it begins to grow in the relatively early stage of the evolution ($\rho_{\text{max}}/\rho_0 \sim 10^4$) with a certain initial amplitude ($\Delta \gtrsim 0.1$). It is also seen that the bar mode grows faster for smaller $\gamma$, as expected from the linear analysis (Hanawa & Matsumoto 2000; Lai 2000). The growth rates agree well with those obtained by the linear analysis (dashed lines), confirming that the distortion is caused by the bar-mode instability.

In summary, the $\gamma$ dependence of the final shapes of the cores can be understood from the $\gamma$ dependence of the bar-mode instability. The core becomes spherical if the bar mode is stable ($\gamma \geq 1.1$), whereas it tends to deform if the bar mode is unstable ($\gamma < 1.1$). In the case with strong instability ($\gamma < 1$), the deformation begins before the axial ratio reaches unity and the core becomes prolate. In the case with weak instability ($1 \leq \gamma < 1.1$), however, the deformation begins after the axial ratio exceeds unity and the core becomes oblate.

### 3.3 Non-fiducial cases

Here, we investigate the fragmentation of filaments proceeding in a non-fiducial way, i.e., the cases different from the fragmentation of isolated hydrostatic filaments triggered by the most unstable fragmentation mode. Astrophysically, various situations can be encountered: for example, unstable modes other than the most unstable one can trigger fragmentation, fragmentation can occur during radial collapse of filaments, filaments are not isolated but under the external pressure of an ambient gas, etc.

Considering these possibilities, we perform simulations with the different amplitude and wavelength of the initial velocity perturbation in Sec. 3.3.1 and Sec. 3.3.2, respectively. We then perform simulations with the enhanced filament density in Sec. 3.3.3. To see the $\gamma$ dependence, we take $\gamma = 1$, 1.05 and 1.2 for the cases above. Finally, we study fragmentation of filaments under external pressure in Sec. 3.3.4. The parameters are fixed to the fiducial values, unless otherwise stated.

As will be shown below, the results in this section suggest that the evolution of the cores can be largely affected by how filament fragmentation proceeds. Thus, the results obtained for the fiducial case should be treated with caution in astrophysical applications.

#### 3.3.1 Fragmentation by perturbation with large amplitude

Here, we present the results of our simulations for the models with different amplitudes of the initial velocity perturbation. Since the initial perturbation is not exactly the most unstable eigenmode, the converging nature of the initial velocity field becomes important as the initial amplitude increases. The amplitude is taken to be $\delta v/c_{s,0} = 10^{-3}$ (fiducial), 0.1, 0.3 and 0.5 (set V and a part of set G in Table 3).

Fig. 6 shows the evolution of $a_x/a_z$ for (a) $\gamma = 1.2$, (b) 1.05 and (c) 1. For all $\gamma$, the dependence on the initial amplitude is small unless $\delta v/c_{s,0}$ is as large as 0.5. In the case $\delta v/c_{s,0} = 0.5$, the core is compressed due to the converging initial velocity field (equation 11) and becomes more oblate than in the other cases in the early phase ($\rho_{\text{max}}/\rho_0 \lesssim 10^2$). The subsequent evolution is similar to the other cases for (a) $\gamma = 1.2$ and (b) 1.05. For (c) $\gamma = 1$, however, the distortion becomes non-linear at somewhat smaller $\rho_{\text{max}}/\rho_0$ due to the larger oblateness in the early phase.

These results suggest that the evolution is almost the
Fates of the dense cores formed by fragmentation

same as the fiducial case, where the most unstable eigenmode triggers the fragmentation, as long as $\delta v/c_s,0 \lesssim 0.3$.

This is because the most unstable mode grows and dominates the other modes before fragmentation, although the initial perturbation given by equation (11) is not exactly the most unstable eigenmode. We conclude that the dependence on the amplitude of the initial velocity perturbation is weak as long as $\delta v/c_s,0 \lesssim 0.3$.

3.3.2 Fragmentation by perturbation with different wavelength

Having seen the cases with various amplitudes of the initial perturbation, now we see the cases with various wavelengths of it. Here, we present the results for the models with $\lambda/\lambda_{\text{max}} = 0.6$, 1 (fiducial), 1.5 and 2 (set L and a part of set G in Table 3). We see in all models that the filaments initially fragment into the cores. This is expected because the linear analysis shows that modes with $\lambda/\lambda_{\text{max}} \gtrsim 0.5$ are unstable (see e.g., Larson 1985; Inutsuka & Miyama 1992).

Fig. 7 shows the evolution of $a_x/a_z$ for (a) $\gamma = 1.2$, (b) 1.05 and (c) 1. Below, we examine each case separately. Firstly, for (a) $\gamma = 1.2$, the $\lambda$ dependence is weak and $a_x/a_z$ finally converges to unity irrespective of $\lambda$. Secondly, for (b) $\gamma = 1.05$, however, the $\lambda$ dependence is strong and $a_x/a_z$ becomes larger than unity if $\lambda/\lambda_{\text{max}} \lesssim 1$ but becomes less than unity if $\lambda/\lambda_{\text{max}} \gtrsim 1.5$ at the end of the simulations. In the early stage ($\rho_{\text{max}}/\rho_0 \lesssim 10^2$), the core collects gases from more distant regions in the $z$ direction and thus tends to be more prolate with larger $\lambda$. As a result, the bar mode begins to grow before $a_x/a_z$ reaches unity and thus the core becomes prolate if $\lambda/\lambda_{\text{max}} \gtrsim 1.5$, while the bar mode begins to grow after $a_x/a_z$ exceeds unity and thus the core becomes oblate if $\lambda/\lambda_{\text{max}} \lesssim 1$. In the case $\lambda/\lambda_{\text{max}} = 0.6$, the core becomes spherical because the pressure is relatively strong compared to the gravitational force in the core with small mass. Finally, for (c) $\gamma = 1$, we see a trend similar to (b) $\gamma = 1.05$ case, although the core is more easily distorted due to the stronger bar-mode instability. In the case with $\lambda = 2\lambda_{\text{max}}$, the core is always prolate with $a_x/a_z \lesssim 0.5$ and evolves into a very elongated shape.

These results indicate that the evolution of the core...
strongly depends on the wavelength of perturbation, or equivalently the interval of fragments under our periodic boundary condition in the $z$ direction. If the wavelength is longer than that of the most unstable mode, the core evolves differently from the fiducial case and tends to become prolate.

### 3.3.3 Fragmentation during radial collapse of filament

Here, we present the results for filaments fragmenting during their cylindrical radial collapse. To induce the radial collapse of filaments, we enhance the initial density with the density enhancement factor $f$ (equation 10). Meanwhile, we add a certain amplitude of the initial velocity perturbation, $v_{0}$, to begin the fragmentation. If the amplitude were extremely small, the cylindrical radial collapse would be indistinguishable from the hydrostatic state ($\gamma > 1$). In this section, we perform simulations with $f = 1$ (fiducial), 1.05, 1.1 and 1.2 and $v_{0}/c_{s,0} = 0.1$ and 0.5 (set D and some of set V in Table 3). We see in all models that filaments initially fragment into the cores, although the fragmentation cannot be well discriminated from the cylindrical radial collapse in the model with $\gamma = 1$, $f = 1.2$ and $v_{0}/c_{s,0} = 0.1$.

For $s_{0}/a_{s}$ for (a) ($\gamma = 1.05$, $v_{0}/c_{s,0} = 1.2$, 0.1), (b) (1.05, 0.1), (c) (1, 0.1), (d) (1.2, 0.5), (e) (1.05, 0.5) and (f) (1, 0.5), which we explain below. Firstly, for (a, d) $\gamma = 1.2$ and $v_{0}/c_{s,0} = 0.1$ and 0.5, $s_{0}/a_{s}$ finally converges to unity in all cases. Secondly, for (b) $\gamma = 1.05$ and $v_{0}/c_{s,0} = 0.1$, however, the overall motion of the collapsing filaments induces prolate deformation of the core in the early phase, resulting in a more prolate shape in the subsequent evolution with larger $f$. Thirdly, for (d) $\gamma = 1.05$ and $v_{0}/c_{s,0} = 0.5$, the trend is the same as (b) $v_{0}/c_{s,0} = 0.1$ but weaker, because the effect of overall motion is less significant due to the larger flow velocity in the $z$ direction. Finally, for (c, f) $\gamma = 1$ and $v_{0}/c_{s,0} = 0.1$ and 0.5, we again see a similar trend to that seen for (b, d) $\gamma = 1.05$ but with larger distortion due to the stronger bar-mode instability, as seen in Sec. 3.3.2.

In summary for this section, cylindrical radial collapse of filaments can have a significant impact on the evolution of the cores, although its effect is reduced if $v_{0}/c_{s,0}$ is large. If the filament fragments during its radial collapse, the core tends to become prolate.

### 3.3.4 Fragmentation of filaments under external pressure

Finally, we show the results for fragmentation of static filaments under the external pressure by an ambient medium. Although we have so far studied the ideal cases with isolated filaments, filaments are indeed embedded in an interstellar medium with finite pressure. Here, we study only the cases of $\gamma = 1$ and perform simulations with $\rho_{\text{ext}}/\rho_{0} = 0.5, 0.01, 0.04$ and 0.09 (set E and one of set V in Table 3).

In practice, the density at the boundary is taken as $\rho_{0}/\rho_{0} = 3.8 \times 10^{-3}$ even though $\rho_{\text{ext}}/\rho_{0} = 0$, because of the finite size of our computational box (see Sec. 2.3).

The external pressure corresponds to the line mass of the filaments. The line mass is defined as $M_{\text{line}} = \int_{0}^{r_{0}} \rho(r) 2\pi r dr$, with the filament radius $r_{\text{fil}}$ given by $\rho(r_{\text{fil}}) = \rho_{\text{ext}}$. The density profile of the static filament with $\gamma = 1$ is given by $\rho(r) = \rho_{0}(1 + r^{2}/H_{0}^{2})^{-1}$ and $M_{\text{line}}$ takes its maximum when the filament is isolated, i.e., $\rho_{\text{ext}} = 0$ and $r_{\text{fil}} = \infty$. This maximum value, $M_{\text{line,cr}} = 2c_{s}^{2}/G$ (e.g., Ostriker 1964), is called the critical line mass because only sub-critical filaments can be hydrostatic and super-critical ones are always gravitationally unstable. The line mass of filaments embedded in an ambient gas with finite $\rho_{\text{ext}}$ is smaller than $M_{\text{line,cr}}$, as $M_{\text{line}} = M_{\text{line,cr}} - (\rho_{\text{ext}}/\rho_{0})^{1/2}$ (see, e.g., Fischera & Martin 2012). Thus, the cases studied here with $\rho_{\text{ext}}/\rho_{0} = 0, 0.01, 0.04$ and 0.09 correspond to $M_{\text{line}}/M_{\text{line,cr}} = 1, 0.9, 0.8$ and 0.7, respectively.

We see in all cases that the filaments fragment into the cores, which subsequently become oblate as collapse proceeds (Fig. 9). The dependence on the external pressure is weak in our cases examined, although the cores tend to be more oblate as the external pressure increases. Such a trend can be understood as follows. Since we have already studied fragmentation to the case with static sub-critical filaments under the external pressure. Since we have already studied fragmentation of radially collapsing super-critical filaments in Sec. 3.3.3, our results encompass the cases with sub- and super-critical filaments, both of which are found in observations (see, e.g., Arzoumanian et al. 2011; Fischera & Martin 2012).
CONCLUSION AND DISCUSSION

We have studied the collapse of dense cores formed by fragmentation of filaments assuming a polytropic gas with $\gamma$. By employing the adaptive mesh refinement (AMR) technique, we are able to follow the filament fragmentation and subsequent core collapse continuously in a single run of simulation. Since the self-similar spherical collapse solution, the so-called Larson-Penston solution, is known to be unstable due to the bar-mode instability if $\gamma < 1.1$, we have focused on how this instability affects the evolution of the cores.

We have found that the cores formed by fragmentation of filaments tend to become spherical in the early phase of the collapse but later begin to distort due to the bar-mode instability, if exists. In this paper we regard the fragmentation of an isolated hydrostatic filament triggered by the most unstable fragmentation mode (e.g., Larson 1985) as a fiducial case. For the fiducial case with $1 \leq \gamma < 1.1$, we have found the distortion becomes significant only after the central density increases by more than ten orders of magnitude. This is because the distortion begins to grow out of a small seed when the background spherical collapse becomes sufficiently close to the Larson-Penston solution, which takes about five orders of magnitude increase in the central density. For the fiducial case with the other $\gamma$, we see the core becomes strongly distorted if $\gamma \leq 1$ while the core always becomes spherical if $\gamma \geq 1.1$. The $\gamma$ dependence of the evolution can be understood from the fact that the bar-mode instability exists for $\gamma < 1.1$ and becomes stronger with decreasing $\gamma$.

In addition, we have studied the filament fragmentation that occurs in a non-fiducial way and have found the evolution of the cores can be largely affected by the way of filament fragmentation. The distortion grows much faster than in the fiducial case, if the fragmentation is triggered by a perturbation with wavelength longer than that of the most unstable mode or proceeds during the cylindrical radial collapse of filaments. We caution that it is necessary to check whether the fragmentation proceeds in the fiducial way when applying our results for the fiducial case in an astrophysical context.

How the filament fragmentation proceeds is determined by the initial condition. Theoretically, it can be addressed by simulations of filament formation in a turbulent medium, where the perturbation is automatically provided at the time of filament formation. However, although many authors have performed such simulations (e.g., Gammie et al. 2003; Inoue & Inutsuka 2012; Matsumoto et al. 2015; Federrath 2016), none of them have focused on the subsequent collapse of the fragmented cores. To reduce the uncertainty coming from the initial condition, it is important to perform a simulation similar to this work but starting from filament formation in future.

Let us discuss an astrophysical implication of our results on the mass of dense cores. As in the literature (e.g., Omukai et al. 2005), the core mass at fragmentation of filaments can be estimated as follows. In forming stars from the interstellar medium, the temperature evolution with the increasing density draws a evolutionary path in a density-temperature plane that is determined by environmental conditions, such as metallicity and external radiation field (see, e.g., Omukai et al. 2005; Chiaki et al. 2016). Using the effective polytropic index $\gamma$ defined with this path, we estimate the physical state of the gas at each stage of the evolution. Suppose a filamentary gas initially collapses with $\gamma < 1$. It stops its radial collapse when $\gamma$ exceeds unity, because the critical $\gamma$ for filaments is $\gamma_{cr} = 1$, i.e., the pressure increases more (less) rapidly than the gravitational force if $\gamma > 1$ ($\gamma < 1$). Then, the pressure-supported filament fragments into dense cores (Tsuribe & Omukai 2006). The cores subsequently collapse and form stars inside. Here, the mass of the fragments can be estimated as the Jeans mass for the density and temperature when $\gamma$ exceeds unity (e.g., Larson 1985; Inutsuka & Miyama 1992) and gives a good estimate for the initial mass of the cores.

The final mass of the star-forming cores, however, can be largely altered if the distortion of collapsing cores results in their re-fragmentation. We have shown for the fiducial case that it takes more than ten orders-of-magnitude increase in the central density for the distortion to become non-linear. Thus, such re-fragmentation is not likely in most cases, because the phase with $\gamma < 1.1$ does not last such long. This can happen, however, in the following situations in the early Universe: in supermassive ($\sim 10^5 M_\odot$) star formation in strongly irradiated pristine clouds (e.g., Omukai 2001; Bromm & Loeb 2003; Sugimura et al. 2014), the gas collapses with almost constant temperature of $\sim 10^4$ K due to the Ly$\alpha$ cooling; in Pop II star formation in the very high-redshift (z $\geq 20$) Universe, the gas evolves with the temperature of the cosmic microwave background at that time (e.g., Omukai et al. 2005; Safranek-Shrader et al. 2014). In these exceptional cases, the cores can be significantly distorted and finally re-fragment before forming stars. In addition, it should be emphasized again that in non-fiducial cases, i.e., if the filament is not hydrostatic or fragmentation is not triggered by the most-unstable mode, the re-fragmentation of the cores can be important.

Here, we give qualitative estimate of the condition for re-fragmentation, although its numerical investigation is out of the scope of this work. Suppose that the collapse of distorted cores is delayed at some moment, possibly due to the increase of $\gamma$. In such a case, a rough estimate can be made using the critical wavelengths for unstable modes of static filaments and sheets, about four and six times the scale length, respectively (e.g., Larson 1985). If the axial ratio of the cores is larger than twice the ratio of the critical wavelengths to the scale length, i.e., eight for prolate cores and twelve for oblate ones, they are able to fragment into more than two depending on the initial amplitude of the unstable modes. For more realistic estimate, however, numerical simulations dedicated to the re-fragmentation of distorted cores are needed.

The initial condition dependence found in this work suggests an observational relation between the physical state of filaments and the shapes of the cores within them (e.g., Myers et al. 1991; Ryden 1996). We suggest that the cores tend to be more prolate along the filaments if the interval of the cores is longer than the wavelength of the most-unstable mode, i.e., four times the diameter of the filaments (e.g., Inutsuka & Miyama 1992), or the filaments show the sign of overall cylindrical radial collapse. These relations can be observationally tested.

In the current calculation, we have neglected the effects of a magnetic field and a rotational and turbulent velo-
ity field, in order to extract only the effect of the bar-mode instability on the evolution of cores. We should ultimately account for them in studying the evolution of the dense cores formed from filaments, as it has been suggested that a rotational velocity field (e.g., Matsumoto et al. 1997) and a magnetic field (e.g., Nakamura et al. 1993) affect the evolution of cores. It is also known that the turbulence generated during gravitational collapse of cores can play an important role in the evolution of the cores (e.g., Federrath et al. 2011). We would like to address these issues in future publication.

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APPENDIX A: RESOLUTION CHECK

To check the resolution dependence of our results, here we repeat the simulation shown in Sec. 3.1 (γ = 1.05 model of set G in Table 3) but with different resolutions. In our AMR simulation, there are two parameters controlling the resolution: the initial number of meshes in each direction, Nini, and the minimum number of meshes per one Jeans length λJ (equation 4), Nref. The former controls the resolution during the initial fragmentation phase while the latter does during the subsequent collapse phase. We take Nini = 256 and Nref = 32 as the fiducial resolution in this paper, but here we change Nini to 128, 512 or 1024 or 2048 and Nref to 8, 16 or 32.

Fig. A1 shows the Nini dependence of the time evolution of δ = (ρmax − ρ0)/ρ0 (equation 12). We see that the evolution of δ can be correctly followed from the earlier stage by adopting larger Nini, although Nini hardly affects the evolution at the time of fragmentation. With fiducial Nini = 256, the evolution is reliable after δ ≥ 10−2.

We plot the Nref dependence of (a) tdyn/tff (see above equation 13), (b) a2/a2 (equation 16) and (c) Δ = 2(a2 − a2)/(a2 + a2) (equation 17) in Fig. A2. We see in panel (a) that the features of tdyn/tff appearing every four-time density increase are generated by numerical errors at refinement, which can be suppressed by adopting larger Nref. These features, however, affect the overall evolution only slightly. Panels (b) and (c) show the Nref dependence of the growth of the bar-mode instability. We see that the Nref dependence is substantial for Nref = 8, but becomes weaker for Nref = 16 and cannot be seen by eyes for Nref ≥ 32. This suggests that the minimum resolution is Nref ∼ 16 and that our fiducial resolution with Nref = 32 is sufficient.

Let us discuss the minimum resolution required to correctly solve the dynamics of self-gravitating gas. We here
obtain the condition $N_{\text{ref}} \geq 16$, required to correctly follow the growth of the bar mode. The most often-used condition in the literature is the so-called Truelove condition, $N_{\text{ref}} \geq 4$ (Truelove et al. 1997), which is required to avoid artificial fragmentation. More recently, Federrath et al. (2011) suggest to use a condition $N_{\text{ref}} \geq 32$ to resolve the turbulence generated during gravitational collapse of cores. Note that these three conditions are derived to follow the different physical processes. In performing simulations, either one of the above three conditions should be used depending on the process to be followed.

**Figure A1.** Same as Fig. 2 but with $N_{\text{ini}}$ taken as 1024 (red), 512 (orange), 256 (green; fiducial) and 128 (blue).

**Figure A2.** Same as Fig. 3 but with $N_{\text{ref}}$ taken as 64 (red), 32 (orange; fiducial), 16 (green; fiducial) and 8 (blue). The lines for $N_{\text{ref}} = 64$ and 32 are overlapped and difficult to be separately seen.