Stochastic motion of test particle implies that G varies with time

Davood Momeni

(Dated: March 8, 2011)

Abstract

The aim of this letter is to propose a new description to the time varying gravitational constant problem, which naturally implements the Dirac’s large numbers hypothesis in a new proposed holographic scenario for the origin of gravity as an entropic force. We survey the effect of the Stochastic motion of the test particle in Verlinde’s scenario for gravity. Firstly we show that we must get the equipartition values for $t \rightarrow \infty$ which leads to the usual Newtonian gravitational constant. Secondly, the stochastic (Brownian) essence of the motion of the test particle, modifies the Newton’s 2’nd law. The direct result is that the Newtonian constant has been time dependence in resemblance as.
I. GENERAL REMARKS ABOUT THE DIRAC LARGE NUMBER HYPOTHESIS (LNH)

During our survey of various physical laws of nature, we come across a number of constants entangled with those laws. Historically it was Weyl\textsuperscript{12} who initiated the idea of large numbers. But, it was Dirac who discovered an apparently unseen thread joining up those physical constants by a simple yet interesting law, viz., Law of Large Numbers. Using that law, Dirac arrived at his Large Number Hypothesis which has profound influence on the world of physics. In fact, a plethora of works have been done, both at theoretical and observational level with LNH as their starting point. Some works are centered around modification of Einsteins gravitational theory and related equations for adopting the idea of G variation\textsuperscript{3-4}. Another class of works counter arguments against LNH for justifying and refuting that hypothesis characterize the second category of works in favor of it\textsuperscript{5} and also Testing the validity of it\textsuperscript{6}. Even there is at least a formal link between LNH and cosmological constant term $\Lambda$. The Weyl’s large number is root square of Eddington’s one\textsuperscript{8} that was supported by observational data\textsuperscript{9}. Indeed Stewart showed that the ratio of the radius of the universe and electron was only two orders of magnitude smaller $10^{40}$ than that of Weyl’s number. Eddingtons magic number is $N = 1.7507 \times 10^{85}$ and Weyl’s number is $\sqrt{N}$. There are some other large numbers, Jordan’s number\textsuperscript{10} and recently the Shemi-zadeh’s number\textsuperscript{11}. There is a wide class of acclaim about the relation between LNH and other concepts of theoretical physics. For example as was claimed by Görnitz, it may be exist a close connection between the Bekenstein-Hawking entropy and Weizäckers ur theory\textsuperscript{12,13}. There is almost a new and updated review which contains some of these aclaims and theories\textsuperscript{14}.

A. Formulation of Dirac Large Numbers Hypothesis and time running of the $G$

There are three dimensionless numbers in the nature which can be constructed from the atomic and cosmological datas:

1- The ratio of the electric to the gravitational force between an electron and a proton $7 \times 10^{39}$

2- The age of the Universe $t$, expressed in terms of a unit of time provided by atomic constants $\frac{e^2}{m_e c^3}$

and finally
3- The mass of that part of the Universe that is receding from us with a velocity $v < c/2$ expressed in units of the proton mass of the order $10^{78}$

Dirac large number hypothesis in it’s orthodoxies form Germaned by himself says that ”...these numbers are related by equations in which the coefficients are close to unity”. Since the number in (2) varies with the age of the Universe, the L.N.H. requires that the other numbers must also vary, namely

$$\frac{e^2}{Gm_em_p} \propto t$$  \hspace{1cm} (1)

or

$$N \propto t^2$$  \hspace{1cm} (2)

$$G \propto t^{-1}$$  \hspace{1cm} (3)

There are two interpretations for the above relation both discussed by Dirac and the only one which was acceptable by himself as One can reconcile the relation (2) with conservation of mass by assuming that the velocity of recession of a galaxy is continually decreasing, so that more and more galaxies are continually appearing with velocity of recession $< c/2$.

This is the picture which was adopted in his first paper on the subject\textsuperscript{3,26}. There are a serious problem between (3) and GR: the Einstein’s theory requires G to be constant. As was noted by Dirac’s theory this inconsistency might be solved if”... we assume that the Einstein theory is valid in a different system of units from those provided by the atomic constants.” Consequence of Diracs LNH is the coexistence of a Variable G Cosmology.

II. ENTRISTIC APPROACH TO THE ORIGIN OF GRAVITY

In\textsuperscript{24} it is postulated that the change of entropy, related to the entropy that is saved on the holographic screen, satisfies the following relations

$$\Delta S = 2\pi k_B, \hspace{0.5cm} \Delta x = \hbar/mc.$$  \hspace{1cm} (4)

The coefficient $2\pi$ is stipulated by matching the correct expression for the force $F$

$$F \Delta x = T \Delta S.$$  \hspace{1cm} (5)

The temperature is associated with the acceleration through the famous Unruh formula\textsuperscript{25}

$$k_B T = \frac{\hbar a}{2\pi c}.$$  \hspace{1cm} (6)
Implying the homogeneous distribution of information on the holographic screen, for a particle approaching the screen one can write

\[ mc^2 = \frac{1}{2} n k_B T, \]  

(7)

where \( n \) is the number of bits. Together with the Unruh formula this gives

\[ \frac{\Delta S}{n} = k_B \frac{a \Delta x}{2c^2}. \]

(8)

In the general relativistic context one starts from a generalized form of the Newtonian potential

\[ \phi = \frac{1}{2} \log(-g^{\alpha\beta} \xi_\alpha \xi_\beta), \]

(9)

where \( e^\phi \) is the red-shift factor that is supposed to be equal to unity at the infinity (\( \phi = 0 \) at \( r = \infty \)), if the space-time is asymptotically flat. The background metric is supposed to be some static solution which admits a global time-like Killing vector \( \xi_\alpha \).

The acceleration is defined by the formula

\[ a^\alpha = -g^{\alpha\beta} \Delta_\beta \phi, \]

(10)

and the Unruh-Verlinde temperature on the screen is given by the formula

\[ T = \frac{\hbar}{2\pi} e^\phi n^\alpha \Delta_\alpha \phi, \]

(11)

where \( n_\alpha \) is a unit vector, that is normal to the holographic screen and the Killing time-like vector \( \xi_\beta \). In this approach it is supposed that the motion of the test particle is classical and no friction like force exists in the spacetime. In the next section we generalized the classical motion of test particle to a stochastic’s one, and we will show that the no-long time limit of the motion leads to a time dependent gravitational constant \( G \). This step is obtained by replacing the usual equipartition law with another time dependent formula which is valid not only in classical regime \( t \to \infty \) but for other finite time interval.

### III. ON THE STOCHASTIC MOTION OF TEST PARTICLE

The Roman Lucretius’s scientific poem “On the Nature of Things”\(^{15}\) has a remarkable description of Brownian motion of dust particles. He uses this as a proof of the existence of atoms:
"Observe what happens when sunbeams are admitted into a building and shed light on its shadowy places. You will see a multitude of tiny particles mingling in a multitude of ways... their dancing is an actual indication of underlying movements of matter that are hidden from our sight... It originates with the atoms which move of themselves. Then those small compound bodies that are least removed from the impetus of the atoms are set in motion by the impact of their invisible blows and in turn cannon against slightly larger bodies. So the movement mounts up from the atoms and gradually emerges to the level of our senses, so that those bodies are in motion that we see in sunbeams, moved by blows that remain invisible."

Although the mingling motion of dust particles is caused largely by air currents, the glittering, tumbling motion of small dust particles is, indeed, caused chiefly by true Brownian dynamics. The first person to describe the mathematics behind Brownian motion was Thiele in a paper on the method of least squares. However, it was Einstein and Smoluchowski who independently brought the solution of the problem to the attention of physicists, and presented it as a way to indirectly confirm the existence of atoms and molecules. Specifically, Einstein predicted that Brownian motion of a particle in a fluid at a thermodynamic temperature $T$ is characterized by a diffusion coefficient

$$D = \frac{k_B T}{b}$$

(12)

where $k_B$ is Boltzmann’s constant and $b$ is the linear drag coefficient on the particle (in the Stokes/low-Reynolds regime applicable for small particles). As a consequence, the root mean square displacement in any direction after a time $t$ is

$$\bar{s}^2 = 2Dt = \frac{2k BT}{b}t$$

(13)

At first the predictions of Einstein’s formula were seemingly refuted by a series of experiments, which gave displacements of the particles as 4 to 6 times the predicted value. But Einstein’s predictions were finally confirmed in a series of experiments carried out by Chaidesaignes and Perrin. The confirmation of Einstein’s theory constituted empirical progress for the kinetic theory of heat. In essence, Einstein showed that the motion can be predicted directly from the kinetic model of thermal equilibrium. The importance of the theory lay in the fact that it confirmed the kinetic theory’s account of the second law of thermodynamics as being an essentially statistical law. For more physical examples specially
for applications of stochastic problems in physics we refer the reader to the classical review of Chandrasekhar which is the best one even after near 70 years\textsuperscript{20}.

\section{Modeling using differential equations}

In mathematics, Brownian motion is described by the Wiener process; a continuous-time stochastic process named in honor of Norbert Wiener\textsuperscript{21}. The Wiener process \( W_t \) is characterized by this fact:

\begin{equation}
W_0 = 0
\end{equation}

\( W_t \) is almost surely continuous Also \( W_t \) has independent increments. An alternative characterization of the Wiener process is the so-called Levy characterization that says that the Wiener process is an almost surely continuous martingale with \( W_0 \) and quadratic variation

\[[W_t, W_t] = t.\]

A third characterization is that the Wiener process has a spectral representation as a sine series whose coefficients are independent \( \mathcal{N}(0, 1) \) random variables. This representation can be obtained using the KarhunenLoeve theorem.

The Wiener process can be constructed as the scaling limit of a random walk, or other discrete-time stochastic processes with stationary independent increments. This is known as Donsker’s theorem. Like the random walk, the Wiener process is recurrent in one or two dimensions (meaning that it returns almost surely to any fixed neighborhood of the origin infinitely often) whereas it is not recurrent in dimensions three and higher. Unlike the random walk, it is scale invariant.

The time evolution of the position of the Brownian particle itself can be described approximately by a Langevin equation, an equation which involves a random force field representing the effect of the thermal fluctuations of the solvent on the Brownian particle. On long timescales, the mathematical Brownian motion is well described by a Langevin equation. On small timescales, inertial effects are prevalent in the Langevin equation. However the mathematical Brownian motion is exempt of such inertial effects. Note that inertial effects have to be considered in the Langevin equation, otherwise the equation becomes singular, so that simply removing the inertia term from this equation would not yield an exact description, but rather a singular behavior in which the particle doesn’t move at all.
B. Ornstein’s approach to the Brownian motion

Following the Ornstein and Uhlenbeck method for surveying the motion of the Brownian particle, we know that such a particle obeys from the famous Einstein-Langevin equation:

\[
\frac{du}{dt} = -\beta u + w(t)
\]  

(15)

Here \( u(t) \) is the velocity of the particle. The influence of the surrounding medium is split into two distinct parts:

1. A friction part \(-\beta u\)
2. A fluctuating part \(w(t)\).

It must be understanding as a stochastic differential equation and not a commonplace one. The mean (average) is taken over an ensemble of particles which have started at \( t = 0 \) with the same velocity \( u_0 \) as the initial velocity at \( t = 0 \). The force (per unit mass) of the particle is restricted such that it is a random distributed function of time as it’s average vanishes and also it is momentous only for two neighboring correlation for small time’s intervals. The interaction of the particle with the medium creates from a dissipative velocity dependence term \(-\beta u\) and a Random force \(w(t)\). The first method to solve the problem is by calculating all the mean values \( \bar{u}^k \) for given \( u_0 \). As has first been shown by Ornstein for \( \bar{u} \) and \( \bar{u}^2 \), this is possible by integrating the equation of motion (15). Of course, the next assumptions hold for the fluctuating acceleration \( w(t) \):

\[
\bar{w}(t)u_0 = 0
\]

(16)

\[
\bar{w}(t_1)\bar{w}(t_2)u_0 = \phi_1(t_1 - t_2)
\]

(17)

where \( \phi_1(x) \) is a function with a very sharp maximum at \( x = 0 \). More generally, when \( t_1, t_2, ..., t_{n+1} \) are all lying very near each other, we assume:

\[
\prod_{i=1}^{n+1} \bar{w}(t_i) = \phi_n(r, \theta_1, \theta_2, ..., \theta_{n-1})
\]

(18)

where \( r \) is the distance perpendicular to the line \( t_1 = t_2 = ... = t_{n+1} \) in the \((n+1)\) dimensional \((t_1, t_2, ..., t_{n+1})\) space, and \((\theta_1, \theta_2, ..., \theta_{n-1})\) are \((n-1)\) angles to determine the position of \( r \) in the subspace perpendicular to this line. The function \( \phi_n \) has again a very sharp maximum for \( r = 0 \).
In brief

\[ \int_{-\infty}^{\infty} w(t) dt = 0 \] (19)

\[ \int_{-\infty}^{\infty} w(\xi) w(\xi + \psi) d\xi = \theta \] (20)

We know that the distribution function for such particles must be Gaussian with mean and variance

\[ \bar{u}(t) = u_0 e^{-\beta t} \] (21)

\[ \bar{u}(t)^2 = u_0^2 e^{-2\beta t} + \left(1 - e^{-2\beta t}\right) \theta \] (22)

The long time limit of the distribution function of particles must be Maxwellian with temperature \( T \). Thus we obtain the following alternative form for equipartition theorem

\[ \frac{1}{2} m \bar{u}(t)^2 = \frac{1}{2} m u_0^2 e^{-2\beta t} + \frac{k_B T}{m} (1 - e^{-2\beta t}) \] (23)

Which it has the common form only for long times and differs very strangely for finite times chiefly for short times after beginning the motion.

IV. BROWNIAN CORRECTION TO THE NEWTON’S GRAVITY VIA VERLINDE’S APPROACH

In this section we replace (7) with (23) in section (2). Following the Verlinde’s nice idea about the gravity as an entropic force and gravity as an emergent phenomena we know that if a test particle accedes neat to a holographic screen (in Verlinde original proposal, a collection of equipotential surfaces in spacetime) which has the mass \( M \) and the test particle seances himself in a bath with Unruh temperature \( T \) and by assuming that the holographic screen has \( N \) bits \( N = \frac{A}{l_p} \) (The horizon, if we take the equipotential surface as the surface of the black-hole) is a sphere with radius \( R \) and by replacing the alternative-Brownian analogous of the equipartition theorem instead of the infinite time approximation we obtain the next expression for gravitational acceleration

\[ a = \frac{G_{\text{eff}} M}{R^2} - \frac{2\pi u_0^2}{\lambda_c} \frac{e^{-2\beta t}}{1 - e^{-2\beta t}} \] (24)

Where in it

\[ G_{\text{eff}} = \frac{G_N}{1 - e^{-2\beta t}} \] (25)
is going to be identified with Newton’s constant, thus only for long times. The $G_{\text{eff}}$ must be understood as an effective time varying Newton’s constant. $\lambda_c$ is the Compton’s wavelength of the test particle. For small values of $\beta$ we have

$$G_{\text{eff}} = \frac{G_N}{2\beta t} \propto t^{-1} \quad (26)$$

Comparing it with the Dirac hypothesis about the large numbers is very surprising. Again it seems that there is a delicate relation between running of the Newtonian constant, Dirac’s Large Numbers Hypothesis and Verlinde scenario for the gravity, for by the Brownian motion hypothesis for test particle. As I think that this running scheme must be related to the quantum corrections of the Verlinde’s idea. Dirac interpreted this to mean that $G$ varies with time as, and thereby pointed to a cosmology that seems ‘designer-made’ for a theory of quantum gravity. According to General Relativity, however, $G$ is constant, otherwise the law of conserved energy is violated. Dirac met this difficulty by introducing into the Einstein equations a gauge function $\beta$ that describes the structure of spacetime in terms of a ratio of gravitational and electromagnetic units. He also provided alternative scenarios for the continuous creation of matter, one of the other significant issues in LNH are

1- ‘additive’ creation (new matter is created uniformly throughout space)
2- ‘multiplicative’ creation (new matter is created where there are already concentrations of mass).

In above we observed that this effective $G$ arisen when we take the motion of the test particle as a Stochastic one. Attending that at sufficiently long times $t >> 1/\beta$ the second term is negligible and we recover the usage form of Newton’s gravity. There is an unsolved problem about the appearance of the dissipation term $\beta$ in this equation. In the classical theory of the Brownian motion we assume that the medium obeys from the Stoke’s formula for a test particle in an emulsion medium. We can attribute this property to the nature of the gravitational attraction or other unlikely feature with an unknown pedigree.

V. RATE OF CHANGE OF $G_{\text{eff}}$

A volume of works has been centered around the act of calculating the amount of variation of the gravitational constant. See for example the references.
From (25) we get,

\[
\frac{\dot{G}_{\text{eff}}}{G_{\text{eff}}} = -\frac{2\beta}{e^{2\beta t} - 1}
\]  

(27)

For small values of $\beta$, equation (27) tells us that the rate of change of $G$ is of the order of $t^{-1}$. Also, the hypothesis demands that creation of matter occurs continuously in the universe. This creation of matter can occur in two possible ways, viz., *additive creation* and *multiplicative creation*. According to *additive creation theory*, matter is created through the entire space and hence in intergalactic space also. In *multiplicative creation theory*, creation of matter occurs only in those places where matter already exists and this creation proceeds in proportion to the amount and type of atoms already existing there. According to general relativity, $G$ is constant and hence we cannot readily consider $G$ as a variable quantity in Einstein equation. To overcome this difficulty, Dirac considered two metrics. The equations of motion and classical mechanics are governed by the Einstein metric which remains unaltered while the other metric, known as atomic metric, includes atomic quantities and the measurement of distances and times by laboratory apparatus. The interval $ds(A)$ separating two events as determined by apparatus in atomic system of units (a.s.u.) will be different from the interval $ds(G)$ between the same two events as measured in the gravitational system of units (g.s.u.). This implies that equations written in g.s.u. and a.s.u. cannot be used at a time until one of them is converted to the other system of units. The velocity of light is unity for both metrics. Considering the case of a planet orbiting the sun, Dirac showed that the relationship of Einstein and atomic metric was different for additive and multiplicative creation theory. In terms of the atomic distance scale, the solar system is contracting for the additive creation model while it is expanding in multiplicative creation.

If we take the present age of the Universe as 14 Gyr, then the value of $\frac{\dot{G}_{\text{eff}}}{G_{\text{eff}}}$ is of the order of $10^{-11}$ per year which is supported by various theoretical and observational results even in higher dimensional Dark Energy Investigation with Variable $A$ and $G$ in GR. In context of a formalism has been developed for discussing the symmetries of Galilei-invariant classical and quantum mechanical systems associated with the nonrelativistic spacetime picture, in reference it was shown that the metric associated to a time-varying gravitational constant $G(t)$ is conformally related to the $G_0$ case if and only if $G(t)$ changes according to the prescription of Vinti, whose particular case is Diracs suggestion. Concerning the observationally determined increase of the Astronomical Unit, more recent estimates from
processing of huge planetary data sets by Pitjeva point towards a rate of the order of $10^{-1} myr^{-1}$. It may be noted that my result for the secular variation of the terrestrial radial position on the line of the apsides would agree with such a figure by either assuming a mass loss by the Sun of just $-9 \times 10^{-14} yr^{-1}$ or a decrease of the Newtonian gravitational constant $\dot{G}$ is $-1 \times 10^{-13} yr^{-1}$. Such a value for the temporal variation of $G$ is in agreement with recent upper limits from Lunar Laser Ranging $\dot{G} = (2 \pm 7) \times 10^{-13} yr^{-1}$. The main main result of Pitjeva is $\dot{G} = (-5.94.4) \times 10 -14 yr^{-1}$. Gaztanaga et al., relying on data provided by SN Ia have shown that the best upper bound of the variation of $G$ at cosmological ranges is given by

$$-10^{-11} \leq \frac{\dot{G}}{G} \leq 0$$

(28)

where $z$, the red-shift, assumes the value nearly equal to 0.5. Observation of spinning-down rate of pulsar PSR J2019+2425 provides the result

$$|\frac{\dot{G}}{G}| \leq (1.4 - 3.2) \times 10 -11 yr^{-1}$$

(29)

VI. SUMMARY

In this note we assumed that the test particle in the Verlinde’s scenario obeys from the Einstein-Langevin equation, i.e. has a Brownian motion. This assumption modified the equipartition theorem for finite times. After inserting this modification in the entropic expression for Newton 2’nd law, we observe that the usual formal acceleration suddenly voids the stationary form and the gravitational constant $G$ been time dependent. For long times the common gravity recovered but for small values of $\beta$, $G \propto t^{-1}$. This is the repetition of the Dirac’s hypothesis about large numbers. Thus we can state that Dirac large number hypothesis is a direct result from basic Holographic scenario of gravity with this further assumption about the Stochastic nature of the test particle’s motion. Since the Brownian treatment is completely true for all times, and we must get the equipartition values for $t \to \infty$ then it is putting that we can treat the test particle in spacetime as a particle in a medium with the same viscosity properties as emulsion.
VII. ACKNOWLEDGMENTS

We are indebted to P. A. Horváthy and E. V. Pitjeva for their interests and helps.

* Electronic address: d.momeni@yahoo.com

1 Weyl, H.: Annals of Physics. 54, 117 (1917)
2 Weyl, H.: Annals of Physics. 59, 101(1919)
3 Dirac, P. A. M.: Proc. R. Soc. Lond. 333, 403 (1973)
4 Camuto, V., Hsieh, S. H., Adams, P. J.: Phy. Rev. Lett. 39, 8 (1977)
5 Bousso, R.: JHEP 0011, 038 (2000)
6 Blake, G. M.: Mon. Not. R. Astron. Soc. 185, 399 (1978)
7 Peebles, P. J. E., Ratra, B.: Rev. Mod. Phys. 75, 559 (2003)
8 Eddington, A.: Proc. Cam. Phil. Soc. 27, (1931)
9 Stewart, J.: The Deadbeat Universe Lars Wijahlin, Colutron research, Boulder, Colorado, chap. 8, pp 104 (1931)
10 Jordan, P.: Die Herkunft der Sterne (1947)
11 Shemi-zadeh, V. E.: [gr-qc/0206084]
12 Thomas Görnitz, International Journal of Theoretical Physics. Vol. 25, Number.8, (1986)
13 Weizscker, C. F. v. (1971a). The unity of physics, in Quantum Theory and Beyond, T. Bastin, ed., University Press. Cambridge.
14 Saibal Ray, Utpal Mukhopadhyay, Partha Pratim Ghosh: Large Number Hypothesis: A Review, arXiv:0705.1836v1 [gr-qc]
15 Lucretius, 'On The Nature of Things.', translated by William Ellery Leonard.
16 Theile, T. N. Danish version: "Om Anvendelse af mindste Kvadraters Methode i nogle Tilfælde, hvor en Komplikation af visse Slags uensartede tilfældige Fejlkilder giver Fejlene en systematisk Karakter". Vidensk. Selsk. Skr. 5. Rk., naturvid. og mat. Afd., 12:381408, 1880.
17 Smoluchowski, M. (1906), "Zur kinetischen Theorie der Brownschen Molekularbewegung und der Suspensionen", Annalen der Physik 21: 756780
18 Chaudesaigues, M. (1908) 'Le mouvement brownien et le formula d'Einstein' Comptes Rendus, 147 pp 10446
19 J. Perrin. : Ann. Chim. Phys. Série 18, 5114 (1909).

20 S. Chandrasekhar, Stochastic problems in physics and astronomy, Rev. Mod. Phys. 15, 1 (1943).

21 Durrett, R. (2000) Probability: theory and examples, 4th edition. Cambridge University Press, ISBN 0521765390.

22 Ornstein, L.S., On the Brownian Motion, Proc. Acad. Amst, 21, pp. 96-108 (1919).

23 A. Einstein, Ann. d. Physik 17, 549 (1905).

24 E. P. Verlinde, On the Origin of Gravity and the Laws of Newton, [arXiv:1001.0785] [hep-th].

25 W. G. Unruh, Phys. Rev. D 14, 870 (1970).

26 P. A. M. Dirac, Proc. R. Soc. Lond. A, Mathematical Vol. 365, No. 1720 (Feb. 19, 1979), pp. 19-30.

27 M. R. Setare, D. Momeni, [arXiv:1004.0589] [physics.gen-ph].

28 Subir Ghosh, [arXiv:1003.0285] [hep-th].

29 I. V. Vancea, M. A. Santos, [arXiv:1002.2454] [hep-th].

30 P. Nicolini, Phys. Rev. D 82, 044030 (2010) [arXiv:1005.2996] [gr-qc].

31 Gaztanaga, E. et. al.: Phys. Rev. D 65, 023506 (2002).

32 Arzoumanian, Z.: Ph. D. Thesis, Princeton University Press, Princeton, New Jersey, USA (1995).

33 Stairs, I. H.: Living Rev. Rel. 6, 5 (2003).

34 Faulkner, D. F.: Mon. Not. R. Astron. Soc. 176, 621 (1976).

35 Rogachev, A. V.: [gr-qc/0606057].

36 Dirac, P. A. M.: Proc. R. Soc. Lond. A 338, 439 (1974).

37 Utpal Mukhopadhyay, Partha Pratim Ghosh, Saibal Ray, Int J Theor Phys (2010) 49: 16221627.

38 Ray, S., Mukhopadhyay, U., Dutta Choudhury, S.B.: Int. J. Mod. Phys. D 16, 1791 (2007).

39 J. M. Levy-Leblond, Comm. Math. Phys. 6, 286 (1967); Galilei Group and Galileian Invariance in Group Theory and its Applications, Vol. 2, edited by E. M. Loebl (Academic Press, N.Y., 1971), p. 221.

39 J. M. Souriau, Structure des systèmes dynamiques (Dunod, Paris, 1969).

40 C. Duval, G. Burdet, H. P. Kunzle and M. Perrin, Phys. Rev. D 31, 1841 (1985).
E. Prugovecki, Class. Quantum Grav. 4, 1659 (1987).

40 C. Duval, Gary Gibbons and P.A. Horvathy, [arXiv:hep-th/0512188v1]

41 J. P. Vinti, Mon. Not. R. astr. Soc. 169, 417 (1974).

42 Pitjeva, E.V. (2005) The astronomical unit now. Transits of Venus: New Views of the Solar System and Galaxy. Proceedings of the IAU Colloquium 196, 2004, Kurtz, D. W., Ed., Cambridge University Press, Cambridge, 177.

43 Pitjeva, E.V. (2008) Personal communication to P.Noerdlinger.

44 Mller, J. and Biskupek, L. (2007) Variations of the gravitational constant from lunar laser ranging data. Classical and Quantum Gravity, 24, 4533-4538.

45 Pitjeva, E.V. (2009). Relativity in Fundamental Astronomy Proceedings IAU Symposium No. 261, 2009 S. A. Klioner, P. K. Seidelman and M. H. Soffel, eds.

46 Gaztanaga, E. et. al.: Phys. Rev. D 65, 023506 (2002)

47 Perlmutter, S. et al.: Nat. 391, 51 (1998) 39.

48 Riess, A. G. et al.: Astron. J. 116, 1009 (1998)

49 Arzoumanian, Z.: Ph. D. Thesis, Princeton University Press, Princeton, New Jersey, USA (1995) 87.

50 Stairs, I. H.: Living Rev.Rel. 6, 5 (2003)