Length contraction puzzle solved?

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Abstract
It is argued that the ‘perspectival change’ of a physical object in special relativity may be given a natural dynamical explanation in terms of a change in the object under the action of certain forces in a rest properties-preserving way.

1. Introduction
In a recent discussion about the meaning of Lorentz contraction, Franklin [1] stressed that in special relativity (SR) there is no change in the object, it is only the reference frame that is changed. In another recent paper, Miller [2] expressed essentially the same point arguing that ‘when an observer changes frames (or when we compare the results of observers in different frames) there are no dynamical effects in the physical object being observed ... The different inertial observers have no dynamical explanation [of the differences among their observations] in terms of a change in the object, nor do they require one.’

The statement that there is no change in the object in SR is a commonplace in the literature. (The length of a uniformly moving rod is reduced as compared with its length in the rod’s rest frame, ‘but of course nothing at all has happened to the rod itself’ [3].) The statement appears to be truism in the sense that apparently no action was exerted upon the object by a mere different choice of reference frame. The differences among the observations of the different observers are interpreted as natural consequences of a change in perspective and thus outside the scope of the concepts of cause and effect.
Recall briefly where does the ‘perspectival’ interpretation of SR come from. SR is a ‘principle’ theory, based on the relativity postulate and the light postulate. From the two postulates it is deduced that the space-time coordinates of two inertial frames in relative motion are related by a Lorentz transformation and, consequently, that any mathematical relationship which is a candidate to be a law of physics must be Lorentz-covariant. Stress that in the principle approach to SR it is taken (more or less tacitly) that measuring instruments (metre sticks and clocks) in any inertial frame can be constructed in the same way ‘from scratch’. Since the above argument is ‘independent of any special assumptions about the constitution of matter’ and no explicit mention of forces is found in it, it is inferred that the Lorentz transformation, as well as its simple consequences length contraction and time dilation, belong to geometry, i. e. kinematics of SR. This attitude was concreted when the Minkowski spacetime with its geometry is taken to be the fundamental entity that determines the kinematics in all inertial frames. Thus a change of reference frame (rotation in the Minkowski spacetime) is naturally interpreted as a change in perspective, the object being perfectly passive in the process, without any forces acting on it.

Now, the notorious problem with SR is that ‘very many people understand nothing in the beginning but become accustomed to it in the end’. Einstein’s original definition of time [4] and the light postulate, on its own completely benign, in combination with the relativity postulate always gives rise to the same dramatic effect: the feeling of loosing all the ground under one’s feet, disbelief and insecurity, and a perennial question if it is possible that everything could really be so. Even when this new concept of time is somehow ‘swallowed’ and the student of relativity yielded to his or her destiny expects new relativistic wonders, the disbelief and insecurity stay.

\[\text{1It should be noted that the author of SR originally had not seen provision of measuring instruments in various frames in this way [4], but it seems that later he fully embraced this point of view.}\]

\[\text{2That includes the kinematic relations between different inertial frames, the Lorentz transformations corresponding to rotation in the Minkowski spacetime. No dynamical explanation of these relations is needed as these are encoded in the Minkowski geometry that encapsulates the theory’s fundamental principles.}\]
It seems that the discomfort that physicists (and laymen) feel about the contraction of a rod in motion and the slowing down of a clock in motion is a consequence of the opacity of the usual relativistic method of inference. Namely, the features of a certain physical system (e.g. a specific moving clock) are deduced not from the structure of the system described in the inertial frame relative to which the clock moves (‘the lab’), but from the Lorentz transformations that connect the lab frame and the clock’s rest frame. Naturally, a question arises of what is the role of the clock frame, with all of its Einstein-synchronized clocks (which, while mutually identical, may be different from the observed clock in motion). Is the lab frame not quite sufficient? The Lorentz transformations appear as the Fates whose power over the destiny of all physical systems (our moving clock included) is indubitable (as proven by experiments), but quite puzzling. Einstein himself pointed out this fundamental limitation of ‘the principle of relativity, together with the principle of constancy of the velocity of light’ [5].

From its advent until today, various authors argued with various degrees of sophistication that one of barriers to understanding of SR is due to the neglect of its dynamical content, which remains hidden or implicit in a purely principle approach [6-13], [2]. Namely, despite its precision and power (and perhaps just because of that), the principle approach to SR which apparently excludes dynamics may give rise to fundamental misconceptions. For example, the fascinating simplicity and universality of Einstein’s original derivation of length contraction was a kind of red herring: the derivation of the phenomenon is taken to be its root. Thus length contraction is interpreted as a kinematical effect whereas in fact dynamical concepts are indispensable for the right interpretation.\(^3\)

\[^3\]The length \(L'_0\) of a rod in its rest frame \(S'\), and the length \(L_v\) of the rod in the frame \(S\) relative to which it moves along its length at the speed \(v\), are related by

\[
L_v = L'_0 \sqrt{1 - v^2/c^2}.
\] (A)

The length \(L_v\) of the rod in motion is determined by equilibrium of internal forces in \(S\) governing its structure; mutatis mutandis the same remark applies to \(L'_0\). Since \(L_v\) and \(L'_0\) are determined by the forces, the simple relationship (A) between the two lengths is due to the fact that the forces must be Lorentz-covariant [7]. (A witty model of a measuring
As a remedy, Bell [9] advocated the use of a constructive (dynamical) approach in teaching SR (‘Lorentzian pedagogy’): starting from known and conjectured laws of physics in any one inertial frame, one can account for all physical phenomena, including the experience of moving observers. He attempted to illustrate this programme by considering a simple model of the hydrogen atom in the framework of Maxwell’s theory of electromagnetism, anticipating from the outset the relativistic form of Newton’s second law. Unfortunately, even the simple model of the atom is too complex to be solved analytically, and effects of accelerating the atom can be painfully recognized only through a numerical solution. Also, anyone who has ventured to show Lorentz-covariance of Maxwell’s equations and the Lorentz force equation in Einstein’s original 1905 way, without the luxury of tensors in the Minkowski spacetime, knows well that the task is a true tour de force [14-16]. Thus

As Feinberg [10] pointed out, it is a miracle that Maxwell had written his equations in a Lorentz-covariant form straightaway, luckily not adding some extra terms in them. It is clear that Bell’s constructive approach, following the path made by Fitzgerald, Larmor, Lorentz and Poincaré, was possible due to the happy circumstance that Maxwell’s equations were Lorentz-covariant (even better, they were Lorentz-covariant when nobody was aware of that). The relativistic equation of motion is indispensable in a constructive approach to SR. Namely, one can infer properties of a physical system in motion after accelerating it starting from rest until reaching a steady state only if physical laws governing its structure (Maxwell’s equations) and the correct relativistic equation of motion (the Lorentz force equation) are known from the outset. Then, as Bell outlined, in the long run Lorentz-covariance of Maxwell’s equations follows as the exact mathematical fact which can be given a natural physical interpretation. Thus Bell’s anticipation of the relativistic equation of motion can not be considered as a limitation of Bell’s approach, contrary to Miller’s statement in [2]. In his recent attempt to derive SR constructively, Miller [2] avoided the use of the Lorentz force equation. Instead, he tacitly postulated that Maxwell’s equations apply not only in the original rest frame of a physical object but also in its final rest frame, cf the argument leading to equation (6) in [2]. Thus Miller postulated the observations of a moving observer instead of deducing them, which is hardly a constructive approach to SR. It is rather a combination of a constructive and a principle approach.
Bell’s seminal essay gives only an outline of a constructive approach to SR. One of the key insights of the paper is the recognition that at one point of the constructive argument one must postulate Lorentz-covariance of the complete theory (since the Maxwell-Lorentz theory provides a very inadequate model of matter). While the constructive approach must eventually be complemented by the principle approach, it does feed our lust for meaning: unexpected properties of rods and clocks in motion do not appear as a dry consequence of certain abstract mathematical transformations, achieved from logically entangled postulates, as is the case in Einstein’s approach, but as a natural offspring of earlier physical ideas.

The purpose of the present note is to point out a dynamical aspect of SR which seems to have been overlooked or perhaps not sufficiently stressed in the literature. It will be argued that the differences among observations of observers in different inertial frames (or when a single observer changes frames) can be interpreted in a legitimate way in terms of the action of certain forces on the physical object being observed, regardless of its nature. In other words, as if something has happened to the object in a change of reference frame; the so-called ‘change in perspective’ may be understood as hiding a complex dynamical process. Hopefully, our treatment could to some extent dispel ‘the mystical mist’ which surrounds length contraction and time dilation from the advent of SR.

2. Where do the ‘perspectival’ effects come from?
Consider two inertial reference frames $S$ and $S'$ in standard configuration, $S'$ is uniformly moving at speed $v$ along the common positive $x, x'$-axes, and the $y$- and $z$-axis of $S$ are parallel to the $y'$- and $z'$-axis of $S'$, respectively. As was noted above, in a purely principle approach to SR, the two frames are introduced by fiat, taking that measuring instruments (metre sticks and

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*Even with an oversimplified model of matter such as the Maxwell-Lorentz theory of electromagnetism, it is a revealing small exercise for the student to recognize that using his or her FitzGerald-Lorentz contracted measuring rods and Larmor dilated clocks (e. g., those proposed in [2]) a moving observer would measure that one clock-two way speed of light is again $c$, the same that was found when the observer, the rods and the clock were at rest.*
clocks) in both frames are constructed \textit{in the same way} ‘from scratch’. On the other hand, according to Einstein’s original 1905 argument [4], we have initially two inertial frames in relative rest, each frame being provided with its own set of measuring instruments, the frames (including instruments) being identical to one another. Then say the $S'$ frame has been created by accelerating one of the copy frames (with its set of instruments), whereas the other copy, its instruments included, remained at rest (the $S$ frame). Stress that the two ways of introducing the $S$ and $S'$ frames are equivalent under the proviso that accelerations were rest properties-preserving.\footnote{Starting from Einstein [4], various presentations of SR introduce, often tacitly, the assumption that rest properties of an initially free connected object in a steady state are preserved under arbitrary accelerations, if the object is free in the final steady state (after all transient effects of acceleration have died out). It seems, however, that construction of SR requires only rest properties-preserving accelerations [7], [9], [13].}

Consider now a free connected object in an equilibrium internal state at rest in the $S$ frame, and assume that its rest properties are known to us. Let the object be accelerated in an arbitrary way along the $x$-axis in the direction of the increasing $x$ until reaching steady speed $v$, all with respect to $S$, so that $S'$ is its new rest frame. Properties of the object in uniform motion relative to $S$ could be found in the following way, at least in principle: From known laws governing the structure of the object, and from known fields of forces accelerating it, using the correct equations of motion, one could deduce exactly what changes happen in the object during its acceleration, under the action of external and internal forces, until reaching the final equilibrium state. In this, dynamical approach to the problem, all we need are the \textit{true} laws of physics in the $S$ frame and an omnipotent mathematician. The dynamical approach clearly shows that properties of the object change until reaching a persistent final state, all relative to $S$, due to the interplay of external and internal forces.

In case the object is accelerated in a rest properties-preserving way, there is another method for finding final steady properties of the object in uniform motion relative to $S$. As is well known, the method is provided by SR: starting from known properties of the object in its rest frame $S'$ one can deduce
required properties of the moving object in $S$ using the laws of transformation of the relevant physical quantities with respect to the Lorentz transformation. This principle approach circumvents too cumbersome (in fact impracticable) calculations appearing in the dynamical approach. The price to be paid is a potential loss of understanding.

In the preceding paragraph a physical object was transferred from its initial rest frame $S$ to its final rest frame $S'$ through a rest properties-preserving acceleration. Feinberg [10] pointed out, revitalizing Einstein’s original 1905 argument, that the same final state of the object in motion relative to $S$ could be reached in a different way. Let the object be initially at rest in $S'$ and let there be another inertial frame also at rest relative to $S'$, the two frames (including their measuring instruments) being identical to one another. Now accelerate the copy frame (as well as its set of measuring instruments) in a rest properties-preserving way with respect to $S'$ in the direction of the decreasing $x'$ until reaching the steady speed $v$, without touching the object (which remains at rest relative to the inertial frame $S'$). Since the copy frame eventually becomes the frame $S$, we have again the object in the same uniform motion with respect to $S$; moreover, according to SR, its properties with respect to $S$ are the same as in the preceding case, when instead of accelerating the copy frame, the object has been accelerated.

Now in this frame-acceleration procedure the problem arises of why different properties of the object are observed in the $S$ frame, as compared with its rest properties, apparently without action of any forces on the object. Feinberg [10] asked the simple question: why does the action on the measuring system of the rods and clocks cause a contraction of the measured rod? The author stated that the answer is ‘almost trivial: clearly, if the measur-

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8The relativity postulate together with the light postulate plays a role analogous to that of the law of conservation of energy in mechanics. Namely, certain aspects of a mechanical problem can be reached in a simple and elegant way without entering complex dynamical analyses. This point was discussed in detail in [10].

9Note that the question is based on the presupposition that just the action on the measuring rods and clocks of the copy frame is the cause of a contraction of the measured rod. By the way, in references [10] a poor translation of Russian originals is found at some places.
ing instruments are changed somehow under the action of forces, then the result of the measurement may be changed.’ However, after an explanation that I found somewhat obscure, Feinberg stressed that ‘one may naturally still wonder why a symmetric result is obtained when there is such an enormous asymmetry in the transition to the final state of motion with the same relative velocity.’

The preceding considerations brings us naturally to what is perhaps the key question of SR. When a single physical object is observed by two different inertial observers, where do the differences between their observations come from? Particularly, when an object which is at rest relative to \( S' \) and thus in uniform motion relative to \( S \) is observed from the two frames, why the results of observations differ? (Note that the question has nothing to do with the object’s history either in \( S' \) or \( S \), the history may be unknown to us. Note also that the object considered need not be free nor connected.)

Miller in [2] argued that the differences among observations of different inertial observers ‘are due to the differences in their respective measuring instruments [the measuring rods and clocks] and will be referred to as perspectival effects.’ Now the problem arises of where do the differences among their observations come from when the measuring instruments themselves are observed (say, when a measuring rod at rest in \( S' \) is observed from \( S \)). The author explained that ‘these perspectival effects ultimately have a dynamical origin because the properties of measuring instruments are determined by the forces that keep them in equilibrium in their respective frames.’

At first sight, Miller’s explanation appears to be convincing. Indeed, properties of measuring instruments are determined by forces that keep them in equilibrium in their respective frames. Since the forces are generally velocity dependent, equilibrium conditions are velocity dependent too. (Note that this applies in any inertial frame.) It follows that equilibrium properties (the length of the measuring rod and the rate of the clock) are frame dependent, and thus a change of reference frame involves differences among the observed properties of the measuring rod and clock (‘perspectival changes’)[10]

[10]This explanation differs from that given in [2]. Namely, Miller argued, following
While Feinberg and Miller advocate a force interpretation of the so-called kinematical effects of SR, a common thread in their discussions is that there is no change in the object being observed; the differences among observations of different inertial observers are due to the differences in their respective measuring instruments and the latter are ultimately due to a change in perspective (since equilibrium of forces changes in character with a change of velocity). Thus a change of reference frame is all that matters, nothing at all has happened to the object being observed.

It seems however that there are some confusing points in the authors’ arguments. First, each inertial observer possesses his or her own set of measuring instruments which are perfectly identical to one another\footnote{Feinberg \cite{10}, that ‘when the measuring rods and clocks are moved between inertial observers, they suffer dynamical changes. When the observers use their dynamically altered rods and clocks to make measurements, it is not surprising that their results differ and that they differ by the same factors that are involved in the dynamical changes.’ Note however that measuring instruments need not be transferred between frames; as was pointed out above, they could be constructed in each frame ‘from scratch’. Moreover, even when measuring instruments are transferred between inertial frames, there always remains the problem of why the results of observations of the measuring instruments by two observers differ (either in initial or final states). Also, the measuring instruments are not altered as observed in their respective rest frames.\footnote{This is the content of Born’s ‘principle of the physical identity of the units of measure’\cite{17}, cf also \cite{13}, footnote 12.}}. A measuring rod at rest in \(S\) is in all respects identical to a measuring rod of the same construction at rest in \(S'\) under identical physical conditions; the rods embody the same length in their respective rest frames. Therefore it is somewhat perplexing to explain the differences between the observations of the \(S\)- and \(S'\)-observer in terms of the differences in their respective measuring instruments. Deducing, e. g., the observations of the \(S'\)-observer through the corresponding observations of the \(S\)-observer may lead to misunderstandings, due to the fact that the relativity of simultaneity may remain hidden in such deductions. (A metre stick at rest in \(S'\) and parallel to the \(x'\)-axis is observed in \(S\) to have the length \(\sqrt{1 - v^2/c^2}\)m but this does not mean that this reduced length represents a unit of length in \(S'\).) Second, it is rather strange that different dynamical phenomena in a physical object (for example, dif-
ferent equilibrium configurations in a measuring rod$^{12}$ are observed as the result of a mere change of reference frame, apparently without exerting any action upon the object. Since the $S$ and $S'$ frames (including their respective sets of measuring instruments) are perfectly equivalent, it seems natural to look for the root of the differences between the observations in terms of a change in the object being observed i.e. as the result of the action of certain forces on it.

4. **Length contraction puzzle solved?**

Consider a physical object at rest in $S'$. The object need not be free nor connected. For example, it may consist of two unconnected stationary material points lying on the $x'$-axis. If the object is connected, assume that it is in a persistent state. Let there be another reference frame with its own set of measuring instruments, perfect copies of $S'$ and its instruments, all at rest relative to $S'$. Now, following Feînberg’s procedure described above, accelerate the copy frame together with its instruments in a rest properties-preserving way with respect to $S'$ in the direction of the decreasing $x'$ until reaching the steady speed $v$ in all of its parts (after all transient effects have died away). Thus the accelerating copy frame eventually becomes our inertial frame $S$ (perhaps after re-synchronizing its clocks, if necessary). Assume that during the acceleration and after that no action was exerted upon the physical object being considered from the point of view of the $S'$-observer. What are the final properties of the object from the point of view of an observer attached to the copy frame (‘the $C$-observer’)?

Construction of the reference frame of an accelerated observer in SR is somewhat tricky even in the simple case of an observer with a constant rest acceleration [19]. It seems however that main conclusions could be reached

$^{12}$ Miller gave instructive and simple enough models of a measuring rod and clock in the framework of the Maxwell-Lorentz theory of electromagnetism that show clearly that the structure of the measuring instruments is velocity dependent ([2], cf also [18]). Note that Miller's measuring rod is modeled as a system of point charges which has only one equilibrium configuration in its rest frame, and thus only one rest frame length. However, a real connected standard of length may have various equilibrium configurations in its rest frame and thus various rest frame lengths.
without entering complex analyses. Initially, the object was at rest with respect to the inertial copy frame and its \( C \)-observer. Then the object was accelerated with respect to the \( C \)-observer in the direction of the increasing \( x \). Finally, the object in a persistent state is in uniform motion at the velocity \( \mathbf{v} = v\hat{x} \) with respect to the again inertial copy frame (now the \( S \) frame). Moreover, the object has \textit{a fortiori} the same rest-properties in its final state as it had in its initial state. (This information could be communicated to the \( C \)-observer by a radio transmission.) Since the \( C \)-observer finds no differences between the initial and final inertial copy frame (coinciding with \( S' \) and \( S \), respectively) he or she rightly infers that the final properties of the object being considered are exactly the same as if its complete history developed in the inertial frame \( S \), i.e. as if the object was accelerated with respect to \( S \) starting from rest under the action of certain forces in a rest properties-preserving way.\textsuperscript{13}

What actually happened to the copy frame in between is irrelevant for the final properties of the object. Assume, e.g., that the \( C \)-observer had fallen into a deep sleep before the acceleration of the copy frame began and awoke only after all transient effects of the acceleration have died away. Thus he or she slept away the intermediate (non-inertial) stages of the copy frame. Assume also that the copy frame accelerometer was broken all the time. Then the \( C \)-observer would be most inclined to ascribe the change in velocity of the object (and all related changes in its properties) to the action of some real external forces upon the object rather than acceleration of the copy frame. On the other hand, if the \( C \)-observer is aware that just the copy frame was accelerated (either the observer was fully awaken or the accelerometer was in function during the intermediate stages), he or she could explain the corresponding acceleration of the object as the result of the action of some (conditionally speaking) inertial forces (or a temporarily ‘switching on’ of a gravitational field) as the classical observer could do. (Needless to say, both

\textsuperscript{13}Franklin [1] recently analysed the case of two unconnected material points that move relative to an inertial frame with constant rest accelerations starting from rest in a restlength-preserving way. Some weak points of [1] are pointed out in [20].
the inertial forces and the gravitational field are just convenient vehicles for
describing the experience of the $C$-observer, without a physical reality.) Thus
‘the enormous asymmetry’ pointed out by Feinberg [10] seems to be removed.

4. Conclusions

When a connected physical object in a persistent state is observed by ob-
servers in two different inertial frames, the differences between their observ-
ations are due to changes in character of the equilibrium of forces which
determines the structure of the object with a change of its velocity, provided
that the velocity change was performed in a rest properties-preserving way.
This ‘perspectival change’ in the object being observed has nothing to do
with actual history of the object in any of the two inertial frames. However,
the perspectival change may be given a natural dynamical interpretation in
terms of a change in the object under the action of certain forces, either
with respect to any one inertial frame or with respect to the frame of an
accelerated observer. The last statement applies also to a system consisting
of two or more unconnected material points at permanent rest relative to an
inertial frame. Thus, a change of reference frame (‘a change in perspective’)
may be understood as involving a change in the object being observed. The
different inertial observers do have a dynamical explanation of the differences
between their observations in terms of a change in the object.

It is irrelevant whether two different states of motion of the object are
observed from two different inertial frames, respectively, or from only one
inertial frame, if in the latter case the two states of motion are related by
a rest properties-preserving acceleration. The results of observations of the
object in the two states of motion are identical in both cases.
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