Optimization of Tool Path for Surface Machining Based on Accuracy

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Abstract. Surface machining is the key technology in numerical control (NC) machining, and the planning of tool path is the basis and key of NC programming. Based on the improvement of the precision and efficiency of surface machining, this work analyzes the approximation machining error of NC machining of multi-coordinate surface. The tool step and calculation of cutting distance for tool path are discussed. The principle of tool path planning during surface NC machining programming is proposed, and the track verification is carried out with examples.

1. Introduction
With the rapid development of intelligent manufacturing and information technology, free-form surface has been widely used in aviation, national defense, new energy, transportation and other industries. Furthermore, the requirements of design quality and processing quality are increasingly demanding, which makes processing more difficult and time-consuming. Therefore, it is the important to find an efficient and accurate method for tool path generation in NC surface machining. In tool path planning, machining efficiency and quality are contradictory. The solution is to find the optimal cutting step and cutting line spacing [1].

There are many free-form surfaces, represented by parameter equation \( r(u, v) \), in NC machining. The processing of surface has always been the main object and research focus of NC machining. In order to realize NC machining of free-form surface, the tool path of free-form surface must be generated, and then the NC code needed can be obtained by post-processing [2]. Therefore, multi-coordinate surface machining is the key technology in NC machining. Moreover, the tool path planning of surface machining is the basis for NC programming. Multi-coordinate face milling of parametric free-form surface often uses ball-end cutter, flat-bottom cutter or ring cutter [3]. In this paper, based on the error analysis of multi-coordinate ball-end cutter and the rational planning of tool path, the effective method and way to improve the machining accuracy and efficiency of curved surface are proposed. Reasonable increase of tool step and width and optimization of tool path are effective ways to improve surface machining accuracy and efficiency [4].

2. Some basic concepts of tool path calculation
Cutter-contact (CC) point: It refers to the ideal contact point between the tool and the machined surface in the process of machining. From the point of view of geometry, the contact relationship between tool and machined surface in surface machining is point contact [5].
Cutter-contact point curve: It refers to the curve formed by the contact points of the cutter in the process of cutting, which is the basic element of tool path generation. A number of small linear segments of tool linear interpolation forms it, which is approximative approaching the real curve on the surface.

Cutter location (CL) point: It refers to the point at which the position of the tool is determined in the process of machining [6]. The cutter location is generally the center point of the cutter, which can be either the cutter center point or the cutter tip point. The cutter location of spherical cutter NC multi-coordinate machining is the cutter center point, that is, the ball center of ball-end milling cutter.

Cutter location data: It refers to the data needed to accurately determine each position of the tool in the process of machining. The accurate position of tool in workpiece coordinate system is usually described by tool center point and tool vector.

Cutter location data: A curve consisting of cutter locations in the process of machining, i.e. each point on the curve contains a cutter axis vector [7].

3. Surface processing point calculation and machining error analysis

3.1 Surface machining tool position calculation

As shown in Figure 1, a machined surface is a free-form surface and its equation is 
\[ r = r(u, v) = (x(u, v), y(u, v), z(u, v)) \] (0 ≤ u, v ≤ 1), The cutter center \( CL_0 \) of spherical cutter is the cutter location of NC multi-coordinate machining, and the point \( P \) is any cutter contact point on the machined surface when the machined surface is machined [8]. Then the unit normal vector at point \( P \) on the surface \( r(u, v) \) is:

\[ n = \frac{r_u \times r_v}{|r_u \times r_v|} \] (1)

The normal distance between tool center and surface is tool radius \( R \), and the vector equation of tool center is:

\[ r_{CL_0} = r(u, v) + Rn \] (2)

Where, \( r_{CL_0} \) is the point vector of the tool center and \( r(u, v) \) is the point vector of the tool contact point \( P \) on the machined surface. Then

\[ x_{CL_0} = x(u, v) + Rn_x \] (3)

\[ y_{CL_0} = y(u, v) + Rn_y \] (4)

\[ z_{CL_0} = z(u, v) + Rn_z \] (5)

3.2 Surface machining error analysis

In multi-coordinate NC machining, tool motion is linear interpolation, so there is an approximation error between the envelope of tool motion and the machined surface [9]. Fig. 2 shows the local geometric relationship between the tool and the surface when the tool center moves in a straight line interpolation along the parametric line on the equidistant surface of the machined surface. Let \( k \) be the
normal curvature of the surface along the feed direction in the interpolation section, $L$ the length of the interpolation line section, and $\theta$ the rotation angle of the normal vector of the surface along the interpolation line direction in the interpolation section. Approximation error of spherical tool center with radius $R$ from $CL_0$ to $CL_1$ by linear interpolation:

$$\delta = \delta_i + \delta_n$$

(6)

where, $\delta_n$ is the processing error caused by the normal vector rotation of the machined surface, which is called normal vector rotation error, $\delta_i$ is the and the linear approximation error.

Fig. 2 Approximation error of linear interpolation moving cutter and surface

From Fig. 2, it can be found:

$$\delta_n = R - R \cos \frac{\theta}{2} = R \left(1 - \cos \frac{\theta}{2}\right)$$

$$= 2 R \sin^2 \frac{\theta}{4} \leq \frac{1}{8} R \theta^2$$

(7)

From differential geometry:

$$\theta = k_f (\Delta S) \quad \Delta S \text{ is the arc length of the curve in the interpolation section}$$

$$\delta_n \leq \frac{1}{8} R k_f^2 (\Delta S)^2$$

(8)

As shown in Figure 3, according to differential theory:

Fig. 3 Linear approximation error of tool linear approximation curve

$$\rho = OP_0 = OB = \frac{\Delta S}{2 \sin \frac{\theta}{2}}$$

(9)

then

$$\delta_i = \rho - \rho \cos \frac{\theta}{2} = \frac{\Delta S}{2 \sin \frac{\theta}{2}} - \frac{\Delta S}{2 \sin \frac{\theta}{2}} \cos \frac{\theta}{2}$$

$$= \frac{\Delta S}{2 \sin \frac{\theta}{2}} \left(1 - \cos \frac{\theta}{2}\right)$$

$$= \frac{\Delta S}{2 \sin \frac{\theta}{2}} \cdot \sin \frac{\theta}{4}$$
\[ \frac{1}{8} \Delta S \cdot \theta = \frac{1}{8} k_f (\Delta S)^2 \]

then

\[ \delta_i = \frac{1}{8} k_f (\Delta S)^2 \]

(10)

\[ \delta = \delta_i + \delta_n \leq \frac{1}{8} k_f (\Delta S)^2 + \frac{1}{8} Rk_f^2 (\Delta S)^2 \]

(11)

\[ L \text{ can be used to replace } \Delta S \text{ in the small interpolation segment} \]

\[ \delta \leq \frac{1}{8} k_f L^2 + \frac{1}{8} Rk_f^2 L^2 \]

(12)

From the above analysis, it can see that the maximum machining error in the interpolation section occurs near the midpoint. The approximation error in the processing includes the linear approximation error and the normal vector rotation error, which are proportional to the square of the length L of the interpolation line segment. The vector rotation error \( \delta_n \) is caused by the rotation of the processing surface vector along the interpolation line direction, which is proportional to the tool radius R.

3.3 Error compensation

(1) Normal vector rotation error compensation

As shown in Fig. 4(a), in order to compensate \( \delta_n \) for the machining of convex surfaces, the distance \( \Delta d = \frac{1}{8} Rk_f^2 L^2 \), which from the tool centers \( CL_0 \) and \( CL_1 \), is moved along the normal vector direction of the machined surface. The tolerance of the machined surface is not exceeded. The position of the tool centers after moving is the \( CL^*_0 \) and \( CL^*_1 \). To compensate the rotation error of normal vector, the distance between the tool centers and the interpolation line is very close to the tool radius.

![Fig. 4 Compensation of tool normal vector rotation error](image)

For concave surface processing, the tool center movement compensation of \( \delta_n \) is not done. As shown in Fig. 4(b), because of \( \delta_n < \delta_t \), approximation error of concave Surface is \( \delta = \delta_t - \delta_n \), so the processing accuracy has been improved.

(2) Linear approximation error compensation

Because the linear approximation error \( \delta_i \) is proportional to \( k_f \) and \( L \) square. Therefore, for a certain \( k_f \), \( \delta_i \) can be effectively reduced by reducing the interpolation step (tool walking step) \( L \). However, the number of interpolation segments will be increased, which will increase the length of NC processing program and reduce the processing efficiency. The actual production is to calculate interpolation tool step \( L \) according to the requirement of surface processing accuracy.
4. Calculation of tool step and cutting line width in surface machining

The calculation basis of tool step length and cutting line width (or cutting line spacing) is to control the size of machining error. The higher the requirement of machining accuracy, the smaller the step length and width of cutting line, the lower the programming efficiency and processing efficiency. Therefore, on the premise of meeting the requirement of machining accuracy, we should increase the step length and cutting line width as far as possible to improve the programming efficiency and processing efficiency [10].

4.1 Calculation of tool step in surface machining

After vector rotation error $\delta_n$ is compensated by moving tool centers, the interpolation approximation error of tool relative to workpiece motion is only linear approximation error (chord difference for short, cutting tolerance for programming). Therefore, $\delta_n$ can be used as a basis for calculating tool step size.

For any given line approximation error $\epsilon_T (\epsilon_T=0.1\sim0.2T, T$ is the tolerance of surface machining), while $|\delta_t| \leq \epsilon_T$, there is

$$\delta_t = \frac{1}{8}k\frac{L^2}{2} \leq \epsilon_T \quad (13)$$

The tool step length is as follows:

$$L \leq 2\sqrt{\frac{2\epsilon_T}{k}} \quad (14)$$

4.2 Calculation of cutting line width (machining bandwidth) in surface machining

Cutting line width refers to the line distance between two tool paths (the contact line between the tool and the machined surface), which is closely related to tool radius $R$ and residual height $h$, as shown in Fig. 5. When surface is machined with spherical cutter, the residual height $h$ is formed between cutting lines. It can be seen from the geometric relationship and differential theory of Fig. 5

$$h_y = R - h = R - \sqrt{R^2 - \left(\frac{d_w}{2}\right)^2} \quad (15)$$

Same as the $\delta_t$ algorithm, the linear error between $Q_0$ and $Q_1$ is

$$\delta_b = \frac{1}{2}K_b\left(\frac{d_w}{2}\right)^2 \quad (16)$$

Where, $K_b$ is the normal curvature of the machined surface along the cutting feed direction. The relationship between the residual height $h$ and the width $d_w$ of the cutting line is as follows,

$$h = h_y - \delta_0 \quad (17)$$

$$= R - \sqrt{R^2 - \left(\frac{d_w}{2}\right)^2} - \frac{1}{2}K_b\left(\frac{d_w}{2}\right)^2$$
If the maximum allowable residual height is $\varepsilon_h$, the width of cutting line can be deduced as follows when the inequality is satisfied $h \leq \varepsilon_h$:

\[
d_w \leq 2\sqrt{2R\varepsilon_h/(1-2Rk)}
\]  

(19)

5. Principle of tool path planning for surface machining

The fixed tool step $L$ and cutting line width $d_w$ can be used for tool path planning while NC programming is used for tool path planning. The fixed linear approximation error (chord difference or cutting tolerance) $\delta_t$ and residual height $h$ can also be used for tool path planning [11]. The general principles used in planning are as follows. When the curvature radius of the machined surface is large and there is no sharp angle or the machining accuracy of the surface is not high, it is advisable to use fixed tool step $L$ and cutting line width $d_w$ to program. This method is simple to calculate, efficient to program, and the reliability of the program is high. When the curvature radius of the machined surface is small and has sharp angle or the requirement of working accuracy is very high, programming should be carried out in the way of fixed chord difference (or cutting tolerance) $\delta_t$ and residual height $h$ to ensure the processing accuracy [12].

The radius of spherical cutter should be less than the minimum radius of curvature at the concave part of the machined surface. Namely, $R < 1/k_{\text{max}} (k_{\text{max}} > 0)$, where $k_{\text{max}}$ is the maximum normal curvature of concave surface. In addition, the following factors need to be considered: (1) processing efficiency. The bigger the tool radius is, the bigger the cutting width is and the higher the machining efficiency is at the same residual height. (2) Normal vector rotation error is proportional to tool radius. For convex surfaces, the bigger the tool radius is, the better. However, the appropriate tool radius must be chosen. Especially when the normal vector rotation error is not compensated, the normal vector rotation error should be checked. If the tool radius is excessive, the tool radius should be reduced as appropriate to reduce the normal vector rotation error. (3) The size of the tool should match the size of the machined surface.

Fig. 6 shows an example of tool path for a curved surface. Fig. 6(a) is the tool path planned by the fixed step length and cutting line width. Fig. 6 (b) is the tool path planned by fixed tolerance and residual height.

Fig. 6 Planning of the surface machining tool path

6. Conclusions

In the free-form surface NC machining, the reasonable tool path is one of the most important method to reduce machine tool energy consumption and improve machining efficiency [13]. In the process of tool path planning for multi-coordinate free-form surface NC machining, the relationship between machining accuracy, programming efficiency and machining efficiency should be correctly handled [14]. The higher the machining accuracy requirement, the smaller the step size and the width of the processing belt, the smaller the machining error, and the lower the programming efficiency and processing efficiency. Therefore, under the premise of meeting the processing precision requirements,
the step size and the cutting line width should be increased as much as possible to optimize the surface machining tool path and improve the programming and processing efficiency [15].

Acknowledgements

The project funded by Chongqing education committee science and technology research projects (KJ1401208, KJ15012025) and Fu Ling science and technology committee key research project (FLKJ2013XJYB004, FLKJ2013XJYB005).

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