The properties of $\Lambda$-hyperons in pure $\Lambda$ matter are studied with the finite-density QCD sum rule approach. The first order quark and gluon condensates in $\Lambda$ nuclear matter are deduced from the chiral perturbation theory. The sum rule predictions are sensitive to the four-quark condensates, $\langle \bar{q}q \rangle_\rho$ and $\langle \bar{q}q \rangle_\rho (\bar{s}s)_\rho$, and the $\pi N$ sigma term. When $\langle \bar{q}q \rangle_\rho$ is nearly independent of density and $\langle \bar{q}q \rangle_\rho (\bar{s}s)_\rho$, depends strongly on density, we can obtain weakly attractive $\Lambda\Lambda$ potentials (about several MeV) in low $\Lambda$ density region, which agree with the information from the latest double $\Lambda$ hyper-nucleus experiments. The nearly density dependence of $\langle \bar{q}q \rangle_\rho$ and strong density dependence of $\langle \bar{q}q \rangle_\rho (\bar{s}s)_\rho$ can be explained naturally if the properties of $\langle \bar{q}q \rangle_\rho$ and $\langle \bar{q}q \rangle_\rho (\bar{s}s)_\rho$ are assumed to be similar to those of $\pi\pi$ and $KK$ in nuclear medium, respectively.

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I. INTRODUCTION

In neutron stars or other dense nuclear matter, $\Lambda$-hyperons may play an important role. How to understand the interactions between $\Lambda$-hyperons is one of interesting topics in nuclear physics [1,2]. One can extract $\Lambda\Lambda$ interactions from experiments on double $\Lambda$ hypernuclei [3,4,5,6]. According to the latest measured data [3,4,5,6,7,8], a weak $\Lambda\Lambda$ potential (about several MeV) is predicted [7,8,9], which is much weaker than the previous predicted one, $U_\Lambda \simeq -20$ MeV, at the $\Lambda$ nuclear density $0.5\rho_0$ [8] with the old measured data [3,4,5,6,7]. Theoretically, there have been a few attempts towards a dynamical understanding of the $\Lambda\Lambda$ interactions, such as the chiral quark model [9] and the latest chiral unitary approach [10]. However, the theoretical predictions have strong model dependence. Further double $\Lambda$ hyper-nucleus experiments are being carried out at KEK. There will be more accurate experimental data in the future, which will not only give a test to the existing theoretical models, but also help us to develop new methods to understand the non-perturbative $\Lambda\Lambda$ interactions at low energies.

In this paper, we attempt to study the $\Lambda\Lambda$ interactions with the finite-density QCD sum rule method (QCDSR), which has been developed in the serial papers [11,12,13,14,15,16,17]. In [16], the $\Lambda N$ interactions are investigated with the $\Lambda$-hyperon in nucleonic nuclear matter. Similarly, we study the $\Lambda\Lambda$ interactions with the $\Lambda$-hyperon in nuclear matter. We extend the $\Lambda$ sum rules [16] to the study of the $\Lambda\Lambda$ interactions by substituting the in-medium condensates in nucleonic nuclear matter with those in $\Lambda$ nuclear matter.

The focus of the finite-density QCDSR is the correlation function of interpolating fields, which is made up of quark fields. Other than the usual sum rules, the ground states in nuclear medium are used for the finite-density QCDSR rather than those in the QCD vacuum. The correlation function can be evaluated at large space-like momenta with an operator product expansion. On the other hand, one can obtain another presentation of the correlation function by introducing a simple phenomenological ansatz for the spectral densities. Finally, the sum rules can be deduced by equating the two different presentations of the correlation function with appropriate weighted integrals. With the obtained sum rules, the baryon self-energies in nuclear matter are related to the in-medium condensates.

In this work, we only take into account the leading order in-medium condensates. The calculations include all the condensates up to dimension four and the first order of the strange quark mass $m_s$. The dimension six scalar-scalar four quark condensates are retained for they are important in the calculations. In this work, the leading order in-medium condensates, $\langle \bar{q}q \rangle_\rho$, $\langle \bar{s}s \rangle_\rho$, $\langle \bar{s}s \rangle_\rho (G^2)_\rho$, $\langle q^4 \rangle_\rho$, $\langle s^4 \rangle_\rho$, and $\langle \pi^2 \rangle_\rho$, are derived from the chiral perturbation theory (ChPT). We deal with the unknown in-medium four-quark condensates $\langle \bar{q}q \rangle_\rho^2$ and $\langle \bar{q}q \rangle_\rho \langle \bar{s}s \rangle_\rho$, as did in [10], by introducing two arbitrary numbers $f_1$ and $f_2$ to describe their density dependence, respectively.

When $\langle \bar{q}q \rangle_\rho^2$ is nearly independent of density (i.e., $f_1 \to 0$) and $\langle \bar{q}q \rangle_\rho \langle \bar{s}s \rangle_\rho$ depends strongly on density (i.e., $f_2 \to 1$), the $\Lambda\Lambda$ potential $U_\Lambda$ is weakly attractive, with a value about several MeV in the low $\Lambda$ nuclear density region (i.e., $\rho \lesssim 0.8\rho_0$). The weakly attractive $\Lambda\Lambda$ potential is compatible with the prediction from the latest double $\Lambda$ hyper-nucleus experiments. The weak density dependence of $\langle \bar{q}q \rangle_\rho^2$ and strong density dependence of $\langle \bar{q}q \rangle_\rho (\bar{s}s)_\rho$ can be explained naturally by assuming that the properties of $\langle \bar{q}q \rangle_\rho^2$ and $\langle \bar{q}q \rangle_\rho (\bar{s}s)_\rho$ are similar to those of $\pi\pi$ and $KK$ in nuclear medium, respectively.

In the subsequent section, the $\Lambda\Lambda$ interactions in sum rules are given. The in-medium quark and gluon condensates are deduced in Sec. III and the parameters are analyzed in Sec. IV. Then the in-medium properties versus the nuclear density are shown in Sec. V. Finally, the summary and conclusions are given in Sec. VI.
II. THE ΛΛ INTERACTIONS IN QCDSR

Based on the finite-density QCDSR for the study of the ΛN interactions in nucleonic nuclear matter, we can conveniently extend it to the investigation of the ΛΛ interactions in pure Λ matter (see appendix [13]). With the obtained sum rules, the scalar self-energy $\Sigma_s$ (i.e., effective mass $M_s^2$) and vector self-energy $\Sigma_v$ of Λ-hyperons in nuclear medium are related to the in-medium quark and gluon condensates at finite density. If only the in-medium quark and gluon condensates in pure Λ matter are determined, the self-energies ($\Sigma_s$ and $\Sigma_v$) can be obtained. Then the ΛΛ nuclear potential is related to the two self-energies by the simple relation

$$U_\Lambda = \Sigma_s + \Sigma_v.$$  \hspace{1cm} (1)

Thus, in the subsequent section we will attempt to deduce these quark and gluon condensates in pure Λ matter.

III. QUARK AND GLUON CONDENSATES

A. The vacuum condensates

Neglecting the isospin breaking effects, the vacuum condensates of $u$, $d$ quarks are denoted by

$$\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 \equiv \langle \bar{q}q \rangle_0.$$  \hspace{1cm} (2)

The numerical value of $\langle \bar{q}q \rangle_0$ can be determined by the Gell-Mann-Oakes-Renner relation [14]

$$\langle m_u + m_d \rangle \langle \bar{q}q \rangle_0 = -m_\pi^2 f_\pi^2 \left[ 1 + \mathcal{O}(m_\pi^2) \right],$$  \hspace{1cm} (3)

where $m_u$ and $m_d$ are the up and down current quark masses, respectively; $m_\pi$ and $f_\pi$ are the pion mass and pion decay constant, respectively. In the calculations, $m_\pi = 138$ MeV, $f_\pi = 93$ MeV and $m_\pi \equiv (m_u + m_d)/2 = 5.5$ MeV are adopted.

For the strange quark condensates in vacuum, we take [18, 19]

$$\langle \bar{s}s \rangle_0 = 0.8 \langle \bar{q}q \rangle_0,$$  \hspace{1cm} (4)

the gluon condensates in vacuum are given by [18, 20]

$$\frac{\alpha_s}{\pi} G^2 \langle \bar{q}q \rangle_0 = (0.33 \text{GeV})^4,$$  \hspace{1cm} (5)

and the dimension-four quark condensates in vacuum, $\langle q^4 iD_0 q \rangle_0$ and $\langle s^4 iD_0 s \rangle_0$, are given by [14, 16]

$$\langle q^4 iD_0 q \rangle_0 = \frac{m_q}{4} \langle \bar{q}q \rangle_0,$$  \hspace{1cm} (6)

$$\langle s^4 iD_0 s \rangle_0 = \frac{m_s}{4} \langle \bar{s}s \rangle_0,$$  \hspace{1cm} (7)

where $m_s$ is the strange quark mass. In this work we adopt $m_s = 25 m_q$ [21].

B. In-medium condensates

The first order in-medium condensates of any operator $\hat{O}$ can be generally written as

$$\langle \hat{O} \rangle_\rho = \langle \hat{O} \rangle_0 + \langle \hat{O} \rangle_\Lambda \rho + \cdots,$$  \hspace{1cm} (8)

where the ellipses denote the corrections from higher orders in density, and $\langle \hat{O} \rangle_\Lambda$ stands for the spin-averaged Λ matrix element.

1. $\langle q^4 q \rangle_\rho$ and $\langle s^4 s \rangle_\rho$.

The simplest in-medium condensates are $\langle q^4 q \rangle_\rho$ and $\langle s^4 s \rangle_\rho$. According to the conservations of the baryon current, one has

$$\langle q^4 q \rangle_\rho = \langle u^4 u \rangle_\rho = \langle d^4 d \rangle_\rho = \langle s^4 s \rangle_\rho = \rho.$$  \hspace{1cm} (9)

2. $\langle \bar{q}q \rangle_\rho$ and $\langle \bar{s}s \rangle_\rho$.

The in-medium condensates $\langle \bar{q}q \rangle_\rho$ and $\langle \bar{s}s \rangle_\rho$ can be deduced from ChPT. In the QCD Hamiltonian density $\mathcal{H}_{\text{QCD}}$, the chiral symmetry is explicitly broken by the current quark mass terms. This part of the Hamiltonian is given by [12]

$$\mathcal{H}_{\text{mass}} \equiv m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + \cdots.$$  \hspace{1cm} (10)

Neglecting the isospin breaking effects, the Hamiltonian becomes

$$\mathcal{H}_{\text{mass}} \equiv 2m_q \bar{q}q + m_s \bar{s}s + \cdots.$$  \hspace{1cm} (11)

Considering the Hamiltonian $\mathcal{H}_{\text{mass}}$ as a function of $m_q$, in the Hellmann-Feynman theorem, one obtains

$$2m_q \langle \psi(m_q) \rangle \int dx^3 \bar{q}q |\psi(m_q)\rangle = m_q \frac{d}{dm_q} \langle \psi(m_q) \rangle \int dx^3 \mathcal{H}_{\text{mass}} |\psi(m_q)\rangle.$$  \hspace{1cm} (12)

In Eq. (12), replacing $|\psi(m_q)\rangle$ with the vacuum state $|0\rangle$ and the ground state of Λ matter $|\rho\rangle$, respectively, then taking the difference of the two cases, and taking into account the uniformity of the system, we have

$$2m_q (\langle \bar{q}q \rangle_\rho - \langle \bar{q}q \rangle_0) = m_q \frac{dE}{dm_q},$$  \hspace{1cm} (13)

where $E$ is the energy density of Λ matter, it can be written as

$$E = M_{\Lambda \rho} + \delta E.$$  \hspace{1cm} (14)

Here $\delta E$ is the contribution from the Λ kinetic energy and ΛΛ interactions. Because $\delta E$ is of higher order in
the Λ density and empirically small at low densities, it is neglected in the calculations.

Similarly, considering the Hamiltonian $H_{\text{mass}}$ as a function of $m_s$, we have

$$m_s(\langle \bar{s}s \rangle_\rho - \langle \bar{s}s \rangle_0) = m_s \frac{d\mathcal{E}}{dm_s}. \quad (15)$$

In ChPT, the Λ mass is given by (see appendix B)

$$M_\Lambda = M_N + \frac{4}{3} [(4b_D + 3b_F)m_q + (2b_D - 3b_F)m_s]B_0, \quad (16)$$

where $b_D$, $b_F$ and $B_0$ are real parameters in the chiral Lagrangian, which can be well determined in the ChPT. Then we obtain

$$m_q \frac{dM_\Lambda}{dm_q} = \left[ \sigma_{\pi N} + 4m_q \frac{4}{3} (b_D + b_F)B_0 \right], \quad (17)$$

the πN sigma term $\sigma_{\pi N}$ is defined as

$$\sigma_{\pi N} = m_q \frac{dM_N}{dm_q}. \quad (18)$$

Thus, the first-order in-medium condensate of $\langle \bar{q}q \rangle_\rho$ is obtained as

$$\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_0 + \langle \bar{q}q \rangle_\Lambda \rho, \quad (19)$$

where the spin averaged Λ matrix element is

$$\langle \bar{q}q \rangle_\Lambda = \frac{1}{2m_q} \left[ \sigma_{\pi N} + 4m_q \frac{4}{3} (b_D + b_F)B_0 \right]. \quad (20)$$

Comparing the condensates of $\bar{q}q$ in Λ matter with those in nucleonic nuclear matter, we find that there is an additional term, $4m_q \frac{4}{3} (b_D + b_F)B_0$, in Eq. (20).

Furthermore, from the Eq. (14), one has

$$m_s \frac{dM_\Lambda}{dm_s} = S + m_s \left[ \frac{8}{3} b_D - 4b_F \right]B_0, \quad (21)$$

where $S$ is the strangeness content in a nucleon, which is given by

$$S = m_s \frac{dM_N}{dm_s} = \frac{y}{2} \left( \frac{m_s}{m_q} \right) \sigma_{\pi N}, \quad (22)$$

with a dimensionless quantity $y = \langle \bar{s}s \rangle_N/\langle \bar{q}q \rangle_N$. Analyzing the baryonic spectrum in the context of SU(3)-flavor symmetry suggests that the strangeness content $y$ is related to the πN sigma term in the following manner

$$y = 1 - \sigma_{\pi N}^0/\sigma_{\pi N}, \quad (23)$$

where $\sigma_{\pi N}^0$ is the πN sigma term in the limit of the vanishing strangeness, its value is in the range of $\sigma_{\pi N}^0 = 36 \pm 7 \text{ MeV} \ [23]$. The recent determinations of the πN sigma term suggest larger values for it, i.e., $\sigma_{\pi N} = 64 \pm 8,$ $(79 \pm 7) \text{ MeV}$, hence $y \sim 0.5 \ [24, 25, 26]$. Thus, in this work, we adopt the new determination of the strangeness content $y \sim 0.5$, and constrain our discussions in the new determined region of $\sigma_{\pi N}$, i.e., $\sigma_{\pi N} = 56 \sim 86 \text{ MeV}$.

Finally, the first-order in-medium condensates of $\langle \bar{s}s \rangle_\rho$ are obtained as

$$\langle \bar{s}s \rangle_\rho = \langle \bar{s}s \rangle_0 + \langle \bar{s}s \rangle_\Lambda \rho, \quad (24)$$

with the spin averaged Λ matrix element

$$\langle \bar{s}s \rangle_\Lambda = \frac{1}{m_s} \left[ S + 4m_s \frac{2}{3} (b_D - b_F)B_0 \right]. \quad (25)$$

3. $\langle q^iD_0q \rangle_\rho$ and $\langle s^iD_0s \rangle_\rho$

In light of Eq. (15), the first order dimension-four quark condensates in medium are written as

$$\langle q^iD_0q \rangle_\rho = \langle q^iD_0q \rangle_0 + \langle q^iD_0q \rangle_\Lambda \rho, \quad (26)$$

$$\langle s^iD_0s \rangle_\rho = \langle s^iD_0s \rangle_0 + \langle s^iD_0s \rangle_\Lambda \rho. \quad (27)$$

Following Refs. [14, 16], the Λ matrix elements $\langle q^iD_0q \rangle_\Lambda$ and $\langle s^iD_0s \rangle_\Lambda$ can be related to the familiar moments of parton distribution functions $A_2^u(\mu^2)$, $A_2^d(\mu^2)$ and $A_2^s(\mu^2)$ in a Λ-hyperon as such

$$\langle q^iD_0q \rangle_\Lambda = \frac{m_q}{4} \langle q^iD_0q \rangle_\Lambda + \frac{3}{8} M_\Lambda A_2^{u+d}(\mu^2), \quad (28)$$

$$\langle s^iD_0s \rangle_\Lambda = \frac{m_s}{4} \langle \bar{s}s \rangle_\Lambda + \frac{3}{4} M_\Lambda A_2^s(\mu^2), \quad (29)$$

with $A_2^{u+d}(\mu^2) = A_2^u(\mu^2) + A_2^d(\mu^2)$. The previous studies with QCDSR predicted that $A_2^u : A_2^d : A_2^s = 0.31 : 0.17 : 0.52$ in a Λ-hyperon at $\mu^2 = 1 \text{ GeV}^2 \ [27]$, which indicates that the sum of the moments for $u, d$ quarks are approximately equal to those of strange quarks, i.e., $A_2^{u+d} \simeq A_2^s$. However, the moments do not include the contributions of gluons, which are important in a hadron. According to the recent predictions, the gluonic contributions to the moments are very large, which can even reach to $\sim 0.47$ in a pion meson $\ [28]$, and $\sim 0.39$ in a proton $\ [29]$. The moments of gluons in a Λ-hyperon should be similar to those in the pion and proton, which are about $A_2^g \simeq 0.4$. Momentum conservation within the Λ-hyperon is enforced by requiring

$$A_2^u + A_2^d + A_2^s + A_2^g = 1, \quad (30)$$

immediately we obtain

$$A_2^{u+d} \simeq A_2^s \simeq 0.3. \quad (31)$$

Although there are some uncertainties in these moments, fortunately, they are less important to our predictions.
4. In-medium gluon condensates

The gluon condensates \( \langle \frac{\alpha_s}{\pi} [(u' \cdot G)^2 + (u' \cdot \tilde{G})^2] \rangle_\rho \) are given by \[14, 16\]

\[
\langle \frac{\alpha_s}{\pi} [(u' \cdot G)^2 + (u' \cdot \tilde{G})^2] \rangle_\rho = -\frac{3}{2\pi} C(\mu^2) M_\Lambda \rho, 
\]

(32)

where \( C(\mu^2) = \alpha_s(\mu^2) A_{2\Lambda}^2(\mu^2) \). We take \( C(\mu^2) = 0.22 \) as did in \[16\] approximately. Fortunately, the predictions are nearly independent of this kind of in-medium gluon condensate.

Another gluon condensate \( \langle \frac{\alpha_s}{\pi} G^2 \rangle_\rho \) can be related to the trace of the energy-momentum tensor \[12\]:

\[
T^\mu_\mu = -\frac{9\alpha_s}{8\pi} G^2 + 2m_q\bar{q}q + m_s\bar{s}s. \quad (33)
\]

For nuclear matter in equilibrium, the ground-state expectation value of the trace of the energy-momentum tensor is

\[
\langle T^\mu_\mu \rangle_\rho = \langle T^\mu_\mu \rangle_0 + \mathcal{E}. \quad (34)
\]

Combining Eqs. (34), (19) and (24), we easily obtain

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle_\rho = \langle \frac{\alpha_s}{\pi} G^2 \rangle_0 - \frac{8}{9} (M_\Lambda - \sigma - N + S + D_1 + D_2) \rho, \quad (35)
\]

with

\[
D_1 = 4m_q \left\{ \frac{4}{3} b_D + b_F \right\} B_0, \quad D_2 = 4m_s \left\{ \frac{4}{3} b_D - b_F \right\} B_0.
\]

5. \( \langle \bar{q}q \rangle_\rho^2 \) and \( \langle \bar{q}q \rangle_\rho \bar{s}s_\rho \)

Finally, the in-medium four-quark condensates \( \langle \bar{q}q \rangle_\rho^2 \) and \( \langle \bar{q}q \rangle_\rho \bar{s}s_\rho \) must be considered justly in the \( \Lambda \) sum rules, for they are numerically important in the calculations. However, the in-medium four-quark condensates in the \( \Lambda \) sum rules [see Eqs. (A1–A3)] are their factorized forms, which may not be justified in nuclear matter \[13, 14, 15, 10\]. Thus, following Refs. \[13, 10\] the scalar-scalar four-quark condensates are parameterized so that they interpolate between their factorized form in the QCD vacuum and in nuclear medium. That is, in the calculations, \( \langle \bar{q}q \rangle_\rho^2 \) and \( \langle \bar{q}q \rangle_\rho \bar{s}s_\rho \) in Eqs. (A1–A3) are replaced with the modified forms \( \langle \bar{q}q \rangle_\rho^2 \) and \( \langle \bar{q}q \rangle_\rho \bar{s}s_\rho \):

\[
\langle \bar{q}q \rangle_\rho^2 = (1 - f_1) \langle \bar{q}q \rangle_0^2 + f_1 \langle \bar{q}q \rangle_\rho^2, \quad (36)
\]

\[
\langle \bar{q}q \rangle_\rho \bar{s}s_\rho = (1 - f_2) \langle \bar{q}q \rangle_0 \bar{s}s_0 + f_2 \langle \bar{q}q \rangle_\rho \bar{s}s_\rho, \quad (37)
\]

where \( f_1 \) and \( f_2 \) are real parameters. Theoretically, both \( f_1 \) and \( f_2 \) are in the range of \( 0 \sim 1 \). However, the studies of the \( \Lambda \) in nucleonic nuclear matter suggest that \( 0 \leq f_1 \leq 0.25 \) and \( 0.6 \leq f_2 \leq 1 \), i.e., \( \langle \bar{q}q \rangle_\rho^2 \) depends weakly, while \( \langle \bar{q}q \rangle_\rho \bar{s}s_\rho \) depends strongly on the nuclear density \[16\].

IV. THE ANALYSES OF THE PARAMETERS

In the calculations, we use the logarithmic measure \[13, 16, 17, 30, 31\].

\[
\delta(M_\rho^2) = \ln \left[ \frac{\max \left\{ \lambda^2 e^{-\left( E_\rho^2 - Q^2 \right)/M^2}, \Pi_{\rho}^\prime(M_\rho^2)/M_{\rho}^\prime, \Pi_{\rho}^\prime(M_\rho^2)/M_{\rho}^\prime, \Pi_{\rho}^\prime(M_\rho^2)/\Sigma_{\rho} \right\}} \right] 
\]

\[
\min \left\{ \lambda^2 e^{-\left( E_\rho^2 - Q^2 \right)/M^2}, \Pi_{\rho}^\prime(M_\rho^2)/M_{\rho}^\prime, \Pi_{\rho}^\prime(M_\rho^2)/M_{\rho}^\prime, \Pi_{\rho}^\prime(M_\rho^2)/\Sigma_{\rho} \right\} \right] \quad (38)
\]

to quantify the fit of the left- and right- sides of the \( \Lambda \) sum rules. Here \( \Pi_{\rho}^\prime(M_\rho^2) \), \( \Pi_{\rho}^\prime(M_\rho^2) \) and \( \Pi_{\rho}^\prime(M_\rho^2) \) stand for the right-hand sides of the Eqs. (A1–A3), respectively. The values of \( \lambda^\prime, s_0^\prime, M_{\rho}^\prime \) and \( \Sigma_{\rho} \) are predicted by minimizing the measure \( \delta \). In the zero-density limit, we can obtain the \( \Lambda \) vacuum mass by applying the same procedure to the sum rules.

A. About the Borel mass \( M^2 \)

Firstly, we should choose a proper Borel mass \( M^2 \) in the calculation. In principle the predictions should be independent of the Borel mass \( M^2 \). However, in practice one has to truncate the operator product expansion and use a simple phenomenological ansatz for the spectral density, which cause the sum rules to overlap only in some limited range of \( M^2 \). The previous studies for the octet baryons show that the sum rules truncated at dimension-six condensate do not provide a particulary convincing plateau. Nevertheless, we can assume that the sum rules actually has a region of overlap, although it is imperfect. Thus, in the following we will try to find an optimization region for \( M^2 \), in this region the predictions should be less sensitive to \( M^2 \) than those in other regions. In Refs. \[15, 10\], the optimization region of \( M^2 \) is suggested as \( 0.8 \leq M^2 \leq 1.4 \) GeV², thus, in this work we choose the proper Borel mass \( M^2 \) around this region.

To study the sensitivities of \( \Lambda \) vacuum mass to \( M^2 \), we plot the \( \Lambda \) vacuum mass as a function of \( M^2 \) in the range of \( 0.8 \leq M^2 \leq 1.5 \) GeV² in Fig. 4. From the figure, we see that the predicted \( \Lambda \) vacuum masses are in the range of \( 1036 \sim 1222 \) MeV, which increase monotonously with the increment of \( M^2 \). In the region of \( 1.1 \leq M^2 \leq 1.5 \)
GeV², the predicted Λ vacuum masses are less sensitive to $M^2$ than those in $0.8 \leq M^2 \leq 1.1$ GeV², which indicates that the optimization region of $M^2$ should be $1.1 \leq M^2 \leq 1.5$ GeV².

Furthermore, we also study the sensitivities of the in-medium properties of Λ-hyperons (e.g., $M^∗_Λ$, $Σ_v$ and $U_Λ$) to the Borel mass $M^2$ at the normal nuclear density. In the calculations, we set $f_1 = 0.25$, $f_2 = 0.8$ and $σ_{πN} = 56$ MeV. To cancel the systematic discrepancies, $M^∗_Λ$, $Σ_v$ and $U_Λ$ are normalized to the predicted Λ vacuum masses.

In Fig. 2, $M^∗_Λ/M_Λ$, $Σ_v/M_Λ$ and $U_Λ/M_Λ$ as functions of $M^2$ are plotted. From the figure, we see that all the predictions are insensitive to $M^2$ in the region of $1.1 \leq M^2 \leq 1.4$ GeV², however, in the region of $0.8 \leq M^2 \leq 1.1$ GeV² they depend obviously on $M^2$, which also indicates that $1.1 \leq M^2 \leq 1.4$ GeV² is the optimization region.

Finally, we must point out that we had better choose the lower limit of $1.1 \leq M^2 \leq 1.4$ GeV² (i.e., $M^2 = 1.12$ GeV²) in the following calculations, for the increment of $M^2$ will enlarge the differences between the predicted Λ vacuum mass and its experimental value.

**B. The sensitivity to $f_1$, $f_2$**

In the calculations, $f_1$ and $f_2$ in the parameterized four-quark condensates are not well determined. There have been a few discussions of them in Refs [16, 17]. The studies suggest that it requires a small value of $f_1$ and a large value of $f_2$ to obtain reasonable results. The possible regions for $f_1$ and $f_2$ are $0 \leq f_1 \leq 0.25$ and $0.6 \leq f_2 \leq 1.0$, respectively. In the following, the sensitivities of the predictions to $f_1$ and $f_2$ in their possible regions are studied.

$M^∗_Λ/M_Λ$, $Σ_v/M_Λ$ and $U_Λ/M_Λ$ as functions of $f_2$ are plotted in Fig. 3. The predictions of two cases, $f_1 = 0.0$ and $f_1 = 0.25$, are shown in the same figure, which are denoted by circles and the squares, respectively. In the calculations, $M^2 = 1.12$ GeV² and $σ_{πN} = 56$ MeV are adopted.

From the figure, we find that $M^∗_Λ$ and $U_Λ$ depend strongly on $f_1$ and $f_2$, however, $Σ_v$ is insensitive to these parameters. The effective mass $M^∗_Λ$ increases (decreases), while the absolute value of the potential $|U_Λ|$ decreases. 

![FIG. 1: Λ mass in vacuum as a function of $M^2$.](image1)

![FIG. 2: The effective mass $M^∗_Λ$, vector self-energy $Σ_v$ and the potential $U_Λ$ as functions of $M^2$ at the density $ρ = ρ_0$.](image2)

![FIG. 3: The effective mass $M^∗_Λ$, vector self-energy $Σ_v$ and the potential $U_Λ$ as functions of $f_2$ at the density $ρ = ρ_0$.](image3)
can obtain the upper limit of the potential $U$ that $\Sigma$ increases monotonously with the increment of $f_2$ ($f_1$). If we fix $f_1 = 0.25$ and vary $f_2$ from 0.6 to 1.0, $M_\Lambda^*$ increases from $0.68M_\Lambda$ to $0.88M_\Lambda$, and $U_\Lambda$ decreases from $U_\Lambda \approx -0.015M_\Lambda$ to $U_\Lambda \approx -0.227M_\Lambda$, there is a large change $\sim 0.2M_\Lambda$ for both $M_\Lambda^*$ and $U_\Lambda$, however, the vector self-energy has trivial changes, with a value about $\Sigma_v = 0.1M_\Lambda$.

From the above analyses, we know that on condition that $f_1 \to 0$ (i.e., $(q\bar{q})_\rho^2$ is independent of density) and $f_2 \to 1$ (i.e., $(q\bar{q})_\rho(\bar{s}s)_\rho$ depends strongly on density), we can obtain the upper limit of the potential $U_\Lambda$ (i.e., the weakest potential).

### C. The sensitivity to $\sigma_{\pi N}$

![ FIG. 4: The effective mass $M_\Lambda^*$, vector self-energy $\Sigma_v$ and the potential $U_\Lambda$ as functions of the $\pi N$ sigma term $\sigma_{\pi N}$ at the density $\rho = \rho_0$ and $\rho = 0.5\rho_0$, respectively. ]

The recent determinations of the $\pi N$ sigma term have obtained large values: $\sigma_{\pi N} = 64 \pm 8, (79 \pm 7) \text{ MeV}$. To see the sensitivities of the predictions to $\sigma_{\pi N}$, we plot $M_\Lambda^*/M_\Lambda$, $\Sigma_v/M_\Lambda$ and $U_\Lambda/M_\Lambda$ as functions of $\sigma_{\pi N}$ in the range of $(56 - 76)$ MeV. In the calculations, we set $f_1 = 0.25$ and $f_2 = 0.8$. In Fig. 4, the predictions of two cases, $\rho = \rho_0$ and $\rho = 0.5\rho_0$, are shown (denoted by circles and squares, respectively).

From the figure, we find that $M_\Lambda^*$, $\Sigma_v$, and $U_\Lambda$ are more and more sensitive to $\sigma_{\pi N}$ with the increment of the nuclear density. For example, at $\rho = \rho_0$ obvious changes of $\Sigma_v/M_\Lambda$ can be seen in the region of $\sigma_{\pi N} = (56 - 76)$ MeV, however, at lower density, $\rho = 0.5\rho_0$, trivial changes can be seen. The effective mass $M_\Lambda^*$ decreases monotonously with the increment of $\sigma_{\pi N}$, while the vector self-energy $\Sigma_v$ and the potential $|U_\Lambda|$ increase monotonously with $\sigma_{\pi N}$. At $\rho = \rho_0$, if $\sigma_{\pi N}$ increases 2 MeV, $M_\Lambda^*$ will decrease a value of $\sim 20$ MeV, $\Sigma_v$ and $|U_\Lambda|$ will increase $\sim 10$ MeV, respectively. At lower density $\rho = 0.5\rho_0$, if $\sigma_{\pi N}$ increases 2 MeV, $M_\Lambda^*$ and $|U_\Lambda|$ will decrease $\sim 10$ MeV, respectively, while $\Sigma_v$ only increases $\sim 1$ MeV.

Finally, it should be noted that when we set $\sigma_{\pi N} \to 56$ MeV (i.e., the lower limit of the new determinations), the upper limit of the potential $U_\Lambda$ (i.e., the weakest potential) is obtained.

### D. The sensitivity to $|q|$.

The effective mass $M_\Lambda^*/M_\Lambda$ and the vector self-energy $\Sigma_v/M_\Lambda$ as functions of three momentum $|q|$ at $\rho = \rho_0$ are plotted in Fig. 5. The squares and circles correspond to the predictions of two cases: $f_1 = 0.25$, $f_2 = 0.8$ and $f_1 = 0.0$, $f_2 = 1.0$, respectively.

It is shown that $M_\Lambda^*/M_\Lambda$ and $\Sigma_v/M_\Lambda$ depend weakly on $|q|$. When $|q|$ changes from zero to 500 MeV, the predicted values of $M_\Lambda^*$ and $\Sigma_v$ decrease $\sim 0.06M_\Lambda$ and $\sim 0.03M_\Lambda$, respectively. Usually, $|q| = 270$ MeV is adopted in the QCDSR calculations.

### E. Summary

Now, we have known that the predictions are mainly determined by three parameters, $f_1$, $f_2$ and $\sigma_{\pi N}$. The scalar self-energy $\Sigma_v$ and the potential $U_\Lambda$ are sensitive to $f_1$, $f_2$ and $\sigma_{\pi N}$. The vector self-energy $\Sigma_v$ is insensitive to $f_1$ and $f_2$, but it is sensitive to $\sigma_{\pi N}$ around $\rho = \rho_0$.

As a whole, at $\rho = \rho_0$, if we set $0 \leq f_1 \leq 0.25$, $0.6 \leq f_2 \leq 1$ and $56 \leq \sigma_{\pi N} \leq 76$ MeV, the effective mass (scalar self-energy) and the potential $U_\Lambda$ have large possible regions, which are $M_\Lambda^* \approx (0.73 \pm 0.1 \pm 0.05)M_\Lambda$ and $U_\Lambda \approx (0.76 \pm 0.05 \pm 0.02)M_\Lambda$, respectively; the vector self-energy $\Sigma_v$ is reasonably determined, which is $\Sigma_v \approx (0.14 \pm 0.04)M_\Lambda$.

The information from the latest measured data of double $\Lambda$ hyper-nuclei indicates that the potential $U_\Lambda$ is weakly attractive, with a value about several MeV. Therefore, the potentials around the lower limit of $U_\Lambda (\approx -0.12M_\Lambda)$ are unreasonable. The physical predictions should be close to the upper limit of $U_\Lambda$, where the parameters $f_1 \to 0$, $f_2 \to 1$ and $\sigma_{\pi N} \to 56$ MeV. The reasons why the parameters $f_1 \to 0$, $f_2 \to 1$ will be discussed later.

### V. IN-MEDIUM PROPERTIES VERSUS NUCLEAR DENSITY

To further study the in-medium properties of $\Lambda$-hyperons, we plot $M_\Lambda^*$, $\Sigma_v$, and $U_\Lambda$ as functions of nuclear density in Fig. 6. On condition that $f_1 = 0.0$, $f_2 = 1.0$...
and \( \sigma_{\pi N} = 56 \) MeV, the upper limits of \( M_\Lambda^* \) and \( U_\Lambda \) at finite-density are obtained (denoted by squares in Fig. 6). For comparison, the predictions with another set of parameters are also presented in the same figure, which are denoted by circles.

From Fig. 6 we see that the effective mass \( M_\Lambda^* \) decreases, while the vector self-energy \( \Sigma_v \) increases monotonously with the increment of \( \Lambda \) density. The changed tendencies agree with the predictions for baryons in the usual Dirac Phenomenology. The two parameter sets, \( f_1 = 0.0, f_2 = 1.0 \) and \( f_1 = 0.25, f_2 = 0.8 \), give very different predictions for the effective mass \( M_\Lambda^* \) and potential \( U_\Lambda \). However, \( \Sigma_v \) is nearly independent of parameter in the lower density region (0 \( \leq \rho \leq 0.6 \rho_0 \)), when \( \rho > 0.6 \rho_0 \) only weak parameter dependence can be seen.

To see the variations of potentials with \( \Lambda \) nuclear density more clearly, \( U_\Lambda \) as a function of \( \Lambda \) nuclear density is plotted in Fig. 7 alone. From the figure, we find that the potentials \( U_\Lambda \) have obvious parameter dependence in the whole density region (0 \( \leq \rho \leq \rho_0 \)). The differences between the two sets of parameters are more and more obvious with the increment of \( \Lambda \) nuclear density.

When we set \( f_1 = 0.25 \) and \( f_2 = 0.8 \), the potential \( |U_\Lambda| \) increases monotonously with the increment of \( \Lambda \) density. From Fig. 7 we find that with \( f_1 = 0.25 \) and \( f_2 = 0.8 \), it gives too strong attractive potentials \( U_\Lambda \) in the whole density region, which are inconsistent with the information from the latest double \( \Lambda \) hyper-nucleus experiments.

However, with the parameters \( f_1 = 0.0 \) and \( f_2 = 1.0 \), there is a large cancellation of the self-energies, \( \Sigma_s \) and \( \Sigma_v \). In this case, we obtain the upper limit potentials, which are weakly attractive (on the order of several MeV) in the lower density region \( \rho < 0.8 \rho_0 \). There is an extremum

\[
U_\Lambda \simeq -0.008M_\Lambda \simeq -9 \text{ MeV}
\]

around \( \rho = 0.5 \rho_0 \). Our predictions are compatible with the latest experimental observation of the double \( \Lambda \) hyper-nucleus \( ^6\Lambda\Lambda\text{He} \). From the measured data, the bound energy of \( \Lambda \Lambda \) is deduced, \( \Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20_{-0.11}^{+0.18} \) MeV [5]. Using the value \( \Delta B_{\Lambda\Lambda} \simeq 1.01 \) and following the method of Schaffner et al. [6], one obtains the \( \Lambda \) potential \( U_\Lambda \simeq -5 \) MeV at the density \( \rho = 0.5 \rho_0 \).

From the above analyses, we know that, in order to obtain compatible results with experiments, \( \langle \bar{q}q \rangle_\rho^2 \) should be nearly independent of density (i.e., \( f_1 \to 0 \)), and \( \langle \bar{q}q \rangle_\rho^2 \langle ss \rangle_\rho \) should depend strongly on density (i.e., \( f_2 \to 1.0 \)).

It is no accident that \( \langle \bar{q}q \rangle_\rho^2 \) depends only weakly, while \( \langle \bar{q}q \rangle_\rho^2 \langle ss \rangle_\rho \) depends strongly on the nuclear density. All the finite-density QCD calculations indicate that \( \langle \bar{q}q \rangle_\rho^2 \) should depend weakly on density [12, 13, 14, 17], and \( \langle \bar{q}q \rangle_\rho^2 \langle ss \rangle_\rho \) should depend strongly on density [16, 17]. The reasons why \( \langle \bar{q}q \rangle_\rho^2 \) has nearly no density dependence, however, \( \langle \bar{q}q \rangle_\rho^2 \langle ss \rangle_\rho \) has strong density dependence may be explained as follows:

In Ref. [22], Jaffe have studied the possible four-quark states. He predicted that the lowest nonet of \( Q^2Q^2 \) states, \( C^0(\bar{q}q, 0^+), C^*(\bar{q}q, 0^+) = (u\bar{u}+d\bar{d})ss/\sqrt{2} \) (just corresponding to \( \langle \bar{q}q \rangle_\rho^2 \) and \( \langle \bar{q}q \rangle_\rho^2 \langle ss \rangle_\rho \), couple strongly to \( \pi \pi \) (i.e., \( C^0(\bar{q}q, 0^+) \) falls apart into \( \pi \pi \) dominantly) and \( \bar{K}K \) (i.e., \( C^*(\bar{q}q, 0^+) \) falls apart into \( \bar{K}K \) dominantly), respectively. According to these predictions, we can conclude that \( \langle \bar{q}q \rangle_\rho^2 \) and \( \langle \bar{q}q \rangle_\rho^2 \langle ss \rangle_\rho \) should also couple strongly to \( \pi \pi \) and \( \bar{K}K \) in nuclear medium,

![FIG. 5: The effective mass \( M_\Lambda^* \) and vector self-energy \( \Sigma_v \) as functions of three momenta \( |q| \) at \( \rho = \rho_0 \).](image)

![FIG. 6: The effective mass \( M_\Lambda^* \), the vector self-energy \( \Sigma_v \) and the potential \( U_\Lambda \) as functions of the \( \Lambda \) density \( \rho \).](image)
respectively. That is, the properties of the two kinds of in-medium four-quark condensates, \((\bar{q}q)_{\rho}^2\) and \((\bar{q}q)_{\rho}(\bar{s}s)_{\rho}\), should be similar to the in-medium properties of \(\pi\pi\) and \(\bar{K}K\), respectively. As we know, the \(\pi\) mesons, as Goldstone bosons, do not change their properties in the nuclear medium \([33]\), hence \((\bar{q}q)_{\rho}^2\) should be independent of the nuclear density; however, the in-medium properties of kaon-mesons depends strongly on the nuclear density \([34, 35, 36, 37, 38, 39, 40]\), hence \((\bar{q}q)_{\rho}(\bar{s}s)_{\rho}\) has strong density dependence.

In the calculations, the minimum measure \(\delta\) is at the order of \(10^{-6} \sim 10^{-5}\). The parameters \(\lambda^*\) and \(s_0^\rho\) have density dependence. For example, with \(f_1 = 0.25\), \(f_2 = 0.8\), when the density increases from zero to \(\rho_0\) the optimized value for the residue \(\lambda^*\) will decrease from \(3.27 \times 10^{-2}\) GeV\(^3\) to 1.66 \times 10^{-2}\) GeV\(^3\), and the optimized value for the continue threshold \(s_0^\rho\) will decrease from 2.86 GeV\(^2\) to 1.94 GeV\(^2\).

VI. SUMMARY AND CONCLUSIONS

Based on the finite-density QCDSR for describing the \(\Lambda N\) interaction in nucleonic nuclear matter, we conveniently extend it to the study of the \(\Lambda\Lambda\) interactions in \(\Lambda\) nuclear matter. The in-medium condensates of \((\bar{q}q)_{\rho}\), \((\bar{s}s)_{\rho}\), \((\bar{q}q)_{\rho}(\bar{s}s)_{\rho}\), and \((s^1 i D_0q)_{\rho}\) and \((s^1 i D_0s)_{\rho}\) are derived from the ChPT.

The \(\Lambda\) potentials \(U_{\Lambda}\) are sensitive to the in-medium four quark condensates and the \(\pi N\) sigma term (i.e., three parameters \(f_1\), \(f_2\) and \(\sigma_{\pi N}\)), for the scalar self-energies \(\Sigma\) are sensitive to them. There is a large cancelation of the scalar self-energy \(\Sigma\) and vector self-energy \(\Sigma_v\), each is on the order of a few hundred MeV around \(\rho = \rho_0\). On condition that \(f_1 \to 0\) (i.e., \((\bar{q}q)_{\rho}^2\) is independent of density), \(f_2 \to 1\) (i.e., \((\bar{q}q)_{\rho}(\bar{s}s)_{\rho}\) depends strongly on density) and \(\sigma_{\pi N} \to 56\) MeV (i.e., the lower limit of the new determinations), the upper limit of the \(\Lambda\) nuclear potentials are predicted, which are weakly attractive (about several MeV) in low density region \(\rho < 0.8\rho_0\). In this case, the predicted \(\Lambda\) nuclear potentials agree well with the latest experimental observation of a double \(\Lambda\) hyper-nucleus \(\Lambda\Lambda\)He.

The nearly density independent \((\bar{q}q)_{\rho}^2\) and strongly density dependent \((\bar{q}q)_{\rho}(\bar{s}s)_{\rho}\) can be explained naturally by assuming the properties of \((\bar{q}q)_{\rho}^2\) and \((\bar{q}q)_{\rho}(\bar{s}s)_{\rho}\) are similar to those of \(\pi\pi\) and \(\bar{K}K\) in nuclear medium, respectively.

In this work, the \(\pi N\) sigma term in the new determinations (i.e., \(\sigma_{\pi N} = 56\) MeV) are adopted, and hence a large strange content of the nucleon (i.e., \(y = 0.5\)) are obtained according to the overviews in \([24]\). The reasonable results predicted by us support these new determinations.

It is a preliminary attempt to study the \(\Lambda\Lambda\) interactions in finite \(\Lambda\) density with QCDSR. More studies are needed to extend QCDSR to finite density. The four-quark condensates in medium should be studied further. In light of our predictions, how to relate the four-quark condensates to two pseudoscalar mesons in the practical calculations should be considered carefully in our later work.

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APPENDIX A: THE \(\Lambda\) SUM RULES

The sum rules for the \(\Lambda\) hyperon propagating in the nuclear matter had been deduced by Xuemin Jin and R. J. Furnstahl\([16]\), which are given by

\[
\lambda^2 M_{\Lambda}^2 e^{-\left(E^2 - q^2\right)/M^2} = -\frac{m_s}{48\pi^4} M^6 E_2 L^{-8/9} - \frac{M^4}{12\pi^2} E_1 (\Lambda(\bar{q}q)_{\rho} - (\bar{s}s)_{\rho}) - \frac{m_s}{6\pi^2} E_0 M^2 E_0 (\langle q^4 i D_0q \rangle_{\rho} + \langle s^1 i D_0s \rangle_{\rho}) L^{-8/9} \nonumber \\
- \frac{2m_s}{9\pi^2} q^2 (2\langle q^4 i D_0q \rangle_{\rho} + \langle s^1 i D_0s \rangle_{\rho}) L^{-8/9} + \frac{4m_s}{3} (\bar{q}q)_{\rho}^2.
\]
for the dimension-four quark condensate,
\[ \langle \bar{s}s \rangle, \]
and we can conveniently extend these sum rules to the study of the \( \Lambda \)-hyperon in pure \( \Lambda \) matter by changing the in-medium condensates in nucleon matter to the in-medium condensates in \( \Lambda \) matter. In Eqs. \((A1-A3)\), we have defined
\[ \lambda \equiv \frac{\lambda^0}{\alpha}, \] where \( \lambda^0 \) is the energy at the continuum threshold, and \( \alpha \) is the dimension-three quark condensate, \( \langle \bar{q}q \rangle \) stands for the dimension-four quark condensate, \( \langle \bar{q}q \rangle' \) stands for two gluon condensates, \( \lambda^* \) is the residue at the quasi-lambda-hyperon pole, and \( M \) is known as the Borel mass. The quantities account for continuum corrections to the sum rules are defined as:

\[ E_0 = \left( 1 - e^{-s^*_0/M^2} \right), \]  
\[ E_1 = \left[ 1 - e^{-s^*_0/M^2} \left( \frac{s^0_0}{M^2} + 1 \right) \right], \]  
\[ E_2 = \left[ 1 - e^{-s^*_0/M^2} \left( \frac{s^0_0}{2M^4} + \frac{s^0_0}{M^2} + 1 \right) \right], \] with the continue threshold
\[ s^*_0 = \omega_0^2 - q^2, \] where \( \omega_0 \) is the energy at the continuum threshold, and \( q \) is the three-momentum of the quasi-Lambda-hyperon. In Eqs. \((A1-A3)\) we have defined

\[ M^*_{\Lambda} \equiv M_{\Lambda} + \Sigma_s, \] where \( \Sigma_s \) and \( \Sigma_v \) are the scalar and vector self-energies of the \( \Lambda \) hyperon in \( \Lambda \) matter, respectively. \( M^* \) is the \( \Lambda \) effective mass in \( \Lambda \) matter. \( E_q \) and \( \bar{E}_q \) correspond to the positive- and negative-energy poles, respectively.

The factor \( L \) in Eqs. \((A1-A3)\) is defined as
\[ L \equiv \ln \frac{M/\Lambda_{QCD}}{\ln \mu/\Lambda_{QCD}}, \] where \( \mu \) is the normalization point of the operator product expansion, and \( \Lambda_{QCD} \) is the QCD scale parameter. In numerical calculations, one takes \( \Lambda_{QCD} = 0.1 \text{ GeV} \), \( \mu = 0.5 \text{ GeV} \).

**APPENDIX B: THE HADRON MASS IN CHPT**

In chiral perturbation theory, the hadron masses originate in the chiral breaking. The leading term of the explicitly chiral breaking Lagrangian for mesons is
\[ \mathcal{L}^0_{ab} = B_0 \frac{f^2}{2} \langle \mathcal{M}(U + U^\dagger) \rangle, \] where \( B_0 = -\langle \bar{q}q \rangle / f^2 \) is the order parameter of spontaneous symmetry breaking. \( \mathcal{M} \) is the quark mass matrix \( \mathcal{M} = \text{diag}(m_q, m_q, m_s) \). The pseudoscalar meson decay constants are equal in the \( SU(3)_V \) limit and denoted
by $f = f_s$. $U = \exp(i\sqrt{2}\phi/2)$, in which $\phi$ collects the pseudoscalar meson octet. The explicitly chiral breaking Lagrangian for baryons is given by

$$L^{\Lambda}_{bs} = M_0\langle BB\rangle + 4B_0b_0\langle BB\rangle\langle M \rangle + 4B_{bD}\langle B[M,B] \rangle + 4B_{bF}\langle B[M,B] \rangle, \quad (B2)$$

where $M_0$ is the common octet baryon mass in the chiral limit, $B$ is the ground state SU(3) baryon octet consisting of the nucleons and hyperons which are collected in a $3 \times 3$ matrix, and $b_0$, $b_D$ and $b_F$ are the parameters to be determined. From the chiral Lagrangian, we can get the masses for $\pi, K$ and $\eta$

$$m^2_{\pi} = 2m_qB_0, \quad (B3)$$

$$m^2_{K} = (m_q + m_s)B_0, \quad (B4)$$

$$m^2_{\eta} = \frac{2}{3}(m_q + 2m_s)B_0, \quad (B5)$$

and the masses of different baryons

$$M_\Lambda = \tilde{M}_0 - \frac{4}{3}(m^2_K - m^2_{\pi})b_D, \quad (B6)$$

$$M_N = \tilde{M}_0 - 4m^2_Kb_D + 4(m^2_K - m^2_{\pi})b_F, \quad (B7)$$

$$M_\Sigma = \tilde{M}_0 - 4m^2_Kb_D, \quad (B8)$$

$$M_\Xi = \tilde{M}_0 - 4m^2_Kb_D - 4(m^2_K - m^2_{\eta})b_F. \quad (B9)$$

From the relations, $B6$ and $B8$, we can obtain the parameters $b_D$ and $b_F$, which are determined by

$$b_D = \frac{3}{4}(M_\Sigma - M_\Lambda)/(m^2_K - m^2_{\pi}), \quad (B10)$$

$$b_F = b_D + \frac{1}{4}(M_N - M_\Sigma)/(m^2_K - m^2_{\eta}). \quad (B11)$$

Combining Eqs. $B8$ and $B7$, one has

$$M_\Lambda = M_N + \frac{4}{3}(4b_D + 3b_F)m_q$$

$$+ (2b_D - 3b_F)m_sB_0. \quad (B12)$$

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