Computed efficiently predictive torque control for induction motor drives based on flux positional errors and extended Kalman filter

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Abstract
This paper proposes an improved model based predictive torque control (MPTC) method based on positional error between reference and estimated stator flux vectors. The main advantages of the proposed method are: the improved computational efficiency which requires minimum hardware resources and weighting-factor-free cost function. Weighting factor is removed by using modified reference transformation, which converts torque reference into equivalent stator flux reference. Improvement in computational performance is achieved by using decreased number of voltage vectors for prediction. An admissibility criterion based on flux positional errors is introduced to reduce the number of voltage vectors. The computational time saved is utilised to incorporate extended Kalman filter for better estimation of flux and torque. The validity of the proposed method is tested on a two-level three-phase inverter fed induction motor drive with dSpace DS1104 as controller board. The dynamic response and computational cost of the proposed method is compared to other established MPTC methods. The superiority of the proposed technique is confirmed by experimental results, which show an average of 32% reduction in computational time when compared to conventional MPTC while comparable dynamic response in terms of torque ripple, flux ripple and load current harmonics, is also maintained.

1 | INTRODUCTION

Finite Control Set Model-based Predictive Torque Control (FCS-MPTC)—an emerging type of model predictive control (MPC)—has recently gained wide attention in research communities [1–4]. MPC control offers numerous advantages such as effective handling of system constraints and non-linearities, easier implementation, and the ability to accommodate several control objectives in a single cost function. Like DTC, FCS-MPTC is a direct control method that manipulates a finite number of switching states of the inverter to achieve its control objectives. As the first step of its implementation, stator flux and torque are predicted for a predefined horizon and then compared with their references to compute the errors. The predictions are made for each switching combination or voltage vector (VV) and the errors are captured in a cost function. The VV that produces minimum value of the cost function is chosen and applied to the inverter. In 2L-VSI fed IM, based on the admissible switching states, there are six active and two null VVs. Each variable in cost function is predicted for all the admissible VVs for a prediction horizon normally taken as 1. The restriction on prediction horizon is governed by the higher number of calculations to be performed in a small sampling time (30–100 μs). In spite of its convincing advantages, MPTC however, has two main drawbacks: challenging weighting factor design for satisfactory performance and higher computational burden [5–10].

The weighting factor assigns relative importance to torque and stator flux errors that differ in units of measurement and amplitudes. Therefore, its appropriate selection has a direct effect on the controller performance. Currently, the selection of weighting factor for MPTC is considered to be an open
research problem. As an empirical solution to this problem, some general guidelines are given in [11]. Other recent solutions can be classified into two main approaches: online adaptation of the weighting factor and MPTC without the weighing factor. For online weighting factor adaptation, methods based on torque or current ripples have been suggested in [12, 13]. These methods depend on the system parameters—such as stator resistance—which may vary during the operation. To compensate the variations, some parameter estimation mechanism is required in the solution, which increases the complexity of the controller and requires higher computational effort. For the second approach, which is based on weighting factor removal, a ranking-based multi-objective optimisation approach is suggested in [14]. Although this procedure works well for single weighting factor selection problems, it becomes more complicated and computationally heavier for cost functions with multiple weighting factors. Another weighting factor removal solution known as the “fuzzy-based multi-objective method” is suggested in [15], which is also computationally intensive and difficult to implement in real time. An alternative solution has been proposed recently by [16] which transforms the torque reference into equivalent flux reference, effectively removing the weighting factor from the cost function. This solution is computationally challenging, and for faster speed it may require additional hardware resources because of the presence of inverse trigonometric functions.

A major part of the computational effort in FCS-MPTC involves optimisation and selection of the VVs. If the available VVs are large in number, this process becomes more challenging and demands even higher computational effort. Therefore, to reduce the computational burden, it becomes imperative to reduce the admissible VVs. The simplest way for reduction of VVs, is the use of a so-called “graph algorithm”, which allows only one switch position change during a sampling interval, hence reducing the number of admissible VVs to almost half. Similarly, a branch and bound (BB) algorithm is used in [17] to achieve longer prediction horizons for better steady-state performance of MV drives where sampling times are larger. This method effectively reduces the computational burden and only certain set of VVs that satisfies the bound limits, is tested. However, this method is difficult to implement on small voltage drives where sampling times are much shorter. Another method to reduce computational burden is to use the concept of “sectorisation” to reduce the number of VVs and implement it as a lookup table. Recent works on a sectorisation-based approach include [2,18–21]. However, in these solutions, the weighting factor designed for conventional MPTC with full VVs still remains part of the cost function and it deteriorates controller performance when a reduced number of VVs are used for optimisation.

In this work, reference transformation is adapted to remove the weighting factor and in line with the sectorisation method, a novel approach based on the positional difference between reference and estimated stator flux is introduced to determine a sub-group of VVs. It is further proved that the sign of the angle error of estimated and reference flux is directly proportional to the torque error. Based on angle error and estimated flux position, a new lookup table is established to minimise the number of VVs to be tested for optimality. The time saved by a reduction in the admissible VVs is used to employ extended Kalman filter estimator for flux and torque. The resulting method offers improved computational efficiency while maintaining comparable dynamic performance.

2 | DYNAMIC MODEL OF AN INDUCTION MOTOR (IM)

The equations of an induction motor using space vector notation in stationary reference frame can be represented as [11]:

\[ v_s = R_s i_s + \frac{d\psi_s}{dt} \]

\[ 0 = R_s i_r - j\omega \psi_r + \frac{d\psi_r}{dt} \]

\[ \psi_s = L_s i_s + L_m i_r \]

\[ \psi_r = L_r i_r + L_m i_s \]

\[ T_e = 1.5p \Re \{ \psi_s, i_s \} = -1.5p \Re \{ \psi_r, i_r \} \]

\[ J \frac{d\omega_m}{dt} = T_e - T_l \]

where \( R_s \) and \( R_r \) are stator and rotor resistances; \( L_m, L_s, \) and \( L_r \) represent mutual, rotor, and stator inductance; \( \psi_s \) and \( \psi_r \) are stator voltage and current vectors; \( i_r \) is rotor current vector and rotor voltage is assumed to be zero; \( \psi_s \) and \( \psi_r \) are flux vectors; \( p \) is the number of pole pairs of motor; \( J \) is the moment of inertia; \( T_e \) and \( T_l \) denote electromagnetic and load torques; and \( \omega \) and \( \omega_m \) are electrical and mechanical rotor angular speeds.

Selecting stator current \( i_s \) and rotor flux \( \psi_r \), vectors as the state variables, induction motor model in state-space can be represented as:

\[ \begin{bmatrix} \frac{d}{dt} (i_s) \\ \frac{d}{dt} (\psi_r) \end{bmatrix}_{x(i)} = \begin{bmatrix} \frac{1}{\tau_s} & \frac{k_r}{\tau_r \tau_s} (\frac{1}{\tau_r} - j\omega) \\ \frac{L_m}{\tau_r} & \frac{1}{\tau_r} - j\omega \end{bmatrix} \begin{bmatrix} i_s \\ \psi_r \end{bmatrix}_{x(i)} + \begin{bmatrix} \frac{1}{\tau_s \tau_r} \\ 0 \end{bmatrix} v_s \]

where \( \tau_s = \frac{L_r L_m}{L_s (R_s + R_r)} \) is stator transient time constant; \( \tau_r = \frac{L_r}{R_r} \) is rotor time constant; and \( k_r = \frac{L_m}{L_s} \) is rotor coupling factor. The model given in Equation (7) can be discretised using Euler’s first order rule and is given by:

\[ x(k+1) = \begin{bmatrix} I + T_s A \\ \frac{A_i}{b_i} \end{bmatrix} x(k) + \begin{bmatrix} T_s B \\ b_i \end{bmatrix} v_s \]

where \( T_s \) is the discretisation sampling time. Finally the discrete model of the induction motor can be obtained as:
\[
\begin{align*}
\left( \begin{array}{c} i_s(k+1) \\ \psi_s(k+1) 
\end{array} \right) &= A_d \left( \begin{array}{c} i_s(k) \\ \psi_s(k) 
\end{array} \right) + B_d v_s \\
\end{align*}
\]

3 CONVENTIONAL MPTC AND EKF

3.1 Model-based predictive torque control

The conventional MPTC controller consists of three different steps: (i) estimation of stator flux \( \psi \) and electromagnetic torque \( T_e \), (ii) prediction of estimated variables \( \psi \) and \( T_e \) based on the finite number of voltage vectors defined by the inverter topology, and (iii) determination of optimal voltage vector by minimising an objective function which consists of the weighted sum of torque and flux errors. The reference value of torque is generated by an outer speed loop, which has a larger time constant as compared to the inner control loop. Therefore, within a very small time interval it can be assumed that \( T_e(k+1) \approx T_e(k) \).

From Equation (1), the stator flux prediction is obtained as follows:

\[
\psi_s^p(k+1) = \psi_s(k) + T_e(v_s(k) - R_s i_s(k))
\]

and the torque prediction is given as:

\[
T_e(k+1) = \frac{3}{2} p r \text{Im} \{ \psi_s^p(k+1) i_s^p(k+1) \}
\]

The stator current prediction used in the above relation can be derived from the dynamic model of IM [7] and (9) as given in (12):

\[
i_s^p(k+1) = \left( 1 + \frac{T_s}{R_s} \right) i_s(k) + \frac{T_s}{R_s(T_s + \tau_s)} \left( \psi_s(k) + v_s(k) \right)
\]

For the selection of optimal VV, a cost function \( g \) is formulated as the weighted sum of stator flux and torque error:

\[
g = \left| T^*(k+1) - T^p(k+1) \right| \\
+ \lambda_p \left| \psi_s^*(k+1) - \psi_s^p(k+1) \right|
\]

where \( \lambda_p \) is the weighting factor which sets the relative importance of each term in cost function. In conventional MPTC, the weighting factor remains constant throughout the operating range and is given by the ratio between rated torque \( T_{\text{nom}} \) and rated flux \( \psi_{\text{nom}} \) [11]:

\[
\lambda_p = \frac{T_{\text{nom}}}{\psi_{\text{nom}}}
\]

The flux reference is normally a constant quantity, therefore it is assumed that \( \psi_s^*(k+1) = \psi_s^*(k) \). Moreover, torque reference is generated from the outer speed loop, which has a larger time constant as compared to the inner control loop.

The major drawbacks of this MPTC approach are:

- Since all the admissible VVs are tested for optimality, a higher computational burden makes MPTC a practically unattractive and expensive solution. If additional objective terms are included in the cost function, the computational burden increases further.
- Weighting factor selection is a challenging task to obtain satisfactory performance. Some of the solutions have been reported in the literature but most work only for two term cost functions, that is torque error and flux error. Weighting factor tuning becomes more challenging when extra objectives are included in the cost function, such as switching frequency reduction term.

3.2 Extended Kalman filter for state estimation

The basic current model (CM) for estimation is used in the proposed MPTC to verify its computational superiority over conventional MPTC. However, EKF is also tested with it to show that the saved computational effort can be utilised for incorporation of better estimation techniques. The ubiquitous EKF is an optimal recursive estimator which is used for obtaining estimates of immeasurable states [22]. The estimation process is based on the assumption that system variables are prone to stochastic uncertainties known as measurement noise \( \omega_s(k) \) and state or process noise \( \omega_x(k) \), respectively. These noises are also assumed to be uncorrelated zero-mean (white) noises. Under these assumptions, the discrete model of IM given in Equation (9) can be represented as:

\[
\begin{align*}
x(k+1) &= A_d x(k) + B_d u(k) + \omega_s(k) \\
y(k) &= C_d x(k) + \omega_x(k)
\end{align*}
\]
The system variables and matrices can be extended for real and imaginary parts of the complex space vector representation of the discrete IM model given in Equation (9) as follows:

\[
\begin{bmatrix}
    x = \begin{pmatrix} i_a & i_b & \psi_{ra} & \psi_{rb} \end{pmatrix}^T \\
    u = \begin{pmatrix} v_a & v_b \end{pmatrix}^T \\
    y = \begin{pmatrix} i_a & i_b \end{pmatrix}^T
\end{bmatrix}
\]

\[
A_d = \begin{bmatrix}
    1 - \frac{T_s}{\tau_\sigma} & 0 & \frac{k_r T_s}{\tau_\sigma} & \frac{\omega k_r T_s}{\tau_\sigma} \\
    0 & 1 - \frac{T_s}{\tau_r} & \frac{k_r T_s}{\tau_r} & \frac{\omega k_r T_s}{\tau_r} \\
    \frac{L_m T_s}{\tau_r} & 0 & 1 - \frac{T_s}{\tau_r} & -T_s \omega \\
    0 & \frac{L_m T_s}{\tau_r} & T_r \omega & 1 - \frac{T_s}{\tau_r}
\end{bmatrix}
\]

\[
B_d = \begin{bmatrix}
    \frac{T_s}{\tau_\sigma} & 0 \\
    0 & \frac{T_s}{\tau_r} \\
    0 & 0 \\
    0 & 0
\end{bmatrix}
\]

\[
C_d = \begin{bmatrix}
    1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0
\end{bmatrix}
\]

The prediction, innovation, and correction processes of the EKF can be summarised as follows [22]:

\[
\hat{x}(k|k-1) = A_d x(k-1|k-1) + B_d u(k)
\]

\[
\hat{y}(k) = C_d \hat{x}(k|k-1)
\]

\[
P(k|k-1) = F(k) P(k-1|k-1) F^T(k) + Q
\]

where \(P\) is the covariance matrix of the state estimate error, \(Q\) is the covariance matrix of the process noise, and \(F\) is the Jacobian matrix involving linearised state space model of IM.

\[
Q = E(w_w w_w^T)
\]

\[
F(k) = \frac{\partial}{\partial x} \left( \hat{x}(k|k-1) \right) \bigg|_{x(k)=\hat{x}}
\]

The prediction of the discrete IM model given in Equation (9) as follows:

\[
R = E \left( \omega y \omega_y^T \right)
\]

\[
H = \frac{\partial}{\partial x} (C_d x(k)) \bigg|_{x=\hat{x}(k|k-1)}
\]

Once the state variables \(i_s\) and \(\psi_s\) are estimated, stator flux can be calculated from Equations (3) and (4) by the following relation:

\[
\psi_s = \left( L_r - \frac{L_m^2}{L_r} \right) i_s + \frac{L_m}{L_r} \psi_r
\]

### PROPOSED MPTC

The electromagnetic torque equation, in terms of rotor and stator flux magnitudes, can also be expressed as [16]:

\[
\hat{T} = \frac{3}{2} \eta n L_m \left( \psi_r \times \psi_s \right) = \frac{3}{2} \eta n L_m \left| \psi_r \right| \left| \psi_s \right| \sin \theta_{rs}
\]

where \(\eta = \frac{1}{L_r - L_m}\) is a constant. Let \(\psi_r = \left| \psi_r \right| e^{j\theta_r}\), \(\psi_s = \left| \psi_s \right| e^{j\theta_s}\) and \(\theta_{rs} = \theta_s - \theta_r\). If rotor flux is known, then Equation (30) also remains valid at reference values of the torque and stator flux:

\[
T^* = \frac{3}{2} \eta n L_m \left( \psi_r \times \psi_s^* \right) = \frac{3}{2} \eta n L_m \left| \psi_r \right| \left| \psi_s^* \right| \sin \theta_{rs}^*
\]

where \(\psi_s^* = \left| \psi_s^* \right| e^{j\theta_{rs}^*}\) is the reference stator flux and \(\theta_{rs}^* = \theta_{rs}^\star - \theta^\star\). Since rotor flux is known in advance and stator flux magnitude is normally set to the rated flux of the IM, therefore angle \(\theta_{rs}^*\) can easily be evaluated from Equation (31) as

\[
\theta_{rs}^* = \arcsin \left( \frac{T^*}{\frac{3}{2} \eta n L_m \left| \psi_r \right| \left| \psi_s^* \right|} \right)
\]

Once the angle \(\theta_{rs}^*\) is known, the reference stator flux \(\psi_s^* = \left| \psi_s^* \right| e^{j\theta_{rs}^*}\) is formulated and a new cost function to select the optimal VV is formed and is given as:

\[
g = \left| \psi_s^*(k+1) - \psi_s^p(k+1) \right|
\]

The above process of transforming torque reference into flux reference is called "reference transformation" and the resulting technique is termed model predictive flux control (MPFC) [23]. The advantage of reference transformation is clearly evident from the comparison of Equations (13) and (33) where there is no weighting factor involved in the cost function formulation of Equation (33) and the prediction variables are
also decreased. The drawbacks of MPFC include (i) increased computational burden and (ii) difficulty of applying it to a larger number of control objectives in cost function.

To reduce the higher computational burden involved in MPTC, a method based on the concepts of direct torque control (DTC) is presented in [2] and is termed as “simplified FCS-PTC”. This simplified version of MPTC (S-MPTC) manipulates torque deviations to reduce the prediction VVs and implements them in the form of a lookup table. The number of VVs in S-MPTC are reduced to three (two active and one null) from a total number of eight VVs. However, when reference transformation is applied to MPTC, the torque deviations cannot be easily incorporated into single objective-based cost function to reduce the number of prediction VVs because torque reference is merged within the reference flux. The proposed method, however, shows that there is a direct relationship between torque deviations and the reference flux vector position. Based on reference and estimated flux deviations, the prediction vectors are effectively reduced to three active VVs and one null vector.

The position of the reference stator flux, after applying reference transformation, is given as:

\[ \theta_r^* = \theta_r + \theta_{rs}^* \]  

\[ \theta_s^* = \text{arctan} \left( \frac{\psi_{rm}}{\psi_{ma}} \right) + \arcsin \left( \frac{T^*}{2p|L_m|\psi_r \psi_s^*} \right) \]  

\[ \theta_i^* = \text{arctan} \left( \frac{\psi_{ri}}{\psi_{ma}} \right) + \arctan \left( \frac{T^*}{\sqrt{\left(\frac{2p|L_m|\psi_r \psi_s^*}{\psi_i^*}\right)^2 - (T^*)^2}} \right) \]  

\[ \theta_s^* = \text{arctan}(u) + \text{arctan}(v) \]  

\[ \theta_{rs}^* = \text{arctan} \left( \frac{u + v}{1 - uv} \right) \]  

Then the torque deviation can be calculated from Equations (30) and (31) as:

\[ \delta T = \frac{3}{2} \frac{p}{\eta L_m} \left| \psi_r \right| \left( \left| \psi_s^* \sin \theta_{rs} - \left| \psi_s^* \sin \hat{\theta}_{rs} \right| \right) \]  

Assuming that rotor flux magnitude remains constant and stator flux magnitude error is very small, that is |\psi_s^*| \approx |\psi_s^|, it can be deduced that:

\[ \delta T \propto \left( \sin \theta_{rs} - \sin \hat{\theta}_{rs} \right) \]  

\[ \delta T \propto \left( \sin \theta_{rs} (\cos \delta - 1) + \sin \delta \cos \hat{\theta}_{rs} \right) \]  

For very small deviations of \( \delta, \cos \delta \approx 1 \) and \( \sin \delta \approx \delta \) and the relation in Equation (44) becomes:

\[ \delta T \propto \delta \cos \hat{\theta}_{rs} \]  

Under normal operation \( \hat{\theta}_{rs} \leq \pm \pi \) and \( \cos \hat{\theta}_{rs} \) remain positive. Therefore, the torque deviations are directly translated in terms of angle deviations as:

\[ \text{sign}(\delta T) = \text{sign}(\delta) \]  

The above relationship provides a solution to use a reduced number of VVs even if there is no torque error available. In contrast to the approach adapted in [2], the above relation can be used with weighting factor-less formulation of FCS-MPTC. The angle deviation indirectly combines both torque and flux variations and removes the need for re-tuning of the weighting factor when reduced enumeration is used instead of exhaustive or full enumeration. Based on this approach and current sector, two VVs are sufficient to produce optimal VVs. However, to cater to special cases where direction of rotation is changed, three vectors are included in the admissibility vector space as shown in Table 1.

The complete block diagram of the proposed method is shown in Figure 1. Once the reference torque is translated to equivalent stator flux reference position \( \theta_r^* \), the position of estimated stator flux \( \hat{\theta}_r \) is compared to it and the angle deviation \( \delta \) is determined. If \( \delta > 0 \), it means that stator flux is lagging reference stator flux and needs to be increased. Since angle deviation \( \delta \) indirectly represents torque deviation \( \delta T \), the direction of rotation of stator flux is considered to be anti-clockwise for this situation. To choose the optimal voltage vector that will bring angle deviation \( \delta \) closer to zero, the next three VVs in the counter-clockwise direction are tested in the following cost function:

\[ g = |\psi_s^*(k+1) - \psi_s^0(k+1)| \]
If the angle deviation $\delta < 0$, then the next three VVs in a clockwise direction are tested in Equation (47). The testing of the next three VVs also guarantees the optimisation of the flux error for $\delta \psi_s > 0$ or $\delta \psi_s < 0$. If the deviation $\delta = 0$, then a null vector $v_1$ or $v_7$ can be applied to maintain the situation. The null vector is chosen for the minimum switch position change condition and depends upon the current VV. If the current switch positions are $10\{\psi_0\}$, then the null VV that fulfils minimum position change is $11\{\psi_0\}$ (only one switch position change).

The proposed method provides the following advantages:

- Weighting factor is removed and an optimal balance between flux and torque tracking is obtained based on reference transformation
- Instead of predicting current, torque, and stator flux as in MPTC and S-MPTC, only flux prediction is involved, which greatly reduces the computational burden
- Based on angle deviations, the number of VVs is reduced to half, which also contributes to significant computational burden reduction.

## 5 | PROPOSED ALGORITHM

The complete process of the proposed control algorithm can be described in following steps.

1. Measure current $i_s(k)$ and rotor speed $\omega(k)$

### TABLE 1 Sub-prediction vectors for optimisation based on flux positional errors

| Sector/ $\delta$ | $\delta > 0$ | $\delta < 0$ |
|------------------|-------------|-------------|
| I                | $v_1, v_2, v_3$ | $v_1, v_6, v_5$ |
| II               | $v_2, v_3, v_4$ | $v_2, v_1, v_6$ |
| III              | $v_3, v_4, v_5$ | $v_3, v_2, v_1$ |
| IV               | $v_4, v_5, v_6$ | $v_4, v_2, v_5$ |
| V                | $v_5, v_6, v_1$ | $v_5, v_3, v_6$ |
| VI               | $v_6, v_1, v_2$ | $v_6, v_5, v_4$ |

2. Estimate stator flux vector $\mathbf{\psi}_s(k)$ and rotor flux vector $\mathbf{\psi}_r(k)$ from extended Kalman filter (EKF) estimation model [25]

3. Calculate reference flux position $\mathbf{\theta}_s^d(k)$ and estimated flux position $\mathbf{\theta}_s(k)$ by using Equation (38) and compute the sign of angle deviation $\delta(k)$ from Equation (39)

4. Depending upon the sign of $\delta(k)$ and flux position $\mathbf{\theta}_s(k)$ choose appropriate prediction vectors as given in Table 1

5. Predict stator flux vector $\mathbf{\psi}_s^d(k + 1)$ from Equation (10) for the selected VVs

6. Optimise the cost function in Equation (47) to choose the optimal VV.

## 6 | EXPERIMENTAL RESULTS

A 2L-VSI-fed induction motor drive is used to investigate and compare the performance of the proposed MPTC method. The experimental setup is shown in Figure 2. The setup consists of a dSpace DS1104 controller board, an FPGA board for generating dead-time or blanking time for IGBTs, gate driver circuits, IGBT modules acting as two-level three-phase inverter along with a DC voltage source, and an induction motor along with incremental encoder for speed measurement and current sensors. The controllers are programmed in C language using dSpace DS1104 function library and the accompanied software to capture results. Different built-in subroutines for measuring execution time are used to assess the computational performance of various MPTC methods along with the proposed MPTC. A hysteresis brake is attached with the motor setup which acts as the load. This brake is controlled through a proportional amplifier. Two different timers are used to control the outer speed loop and inner torque loop of the implemented controllers with two different sampling times. The parameters of the IM and controller are given in Table 2.

To emphasise the advantages of the newly proposed scheme, it will be compared to two different MPTC methods, namely conventional MPTC and simplified MPTC (S-MPTC) [2].

![FIGURE 1 Block diagram of the proposed model-based predictive torque control](image-url)
6.1 Computational time

For a fair comparison of execution times, a basic current model (CM) for flux estimation is used so that a minimum computational and sampling time could be achieved. Figure 3 shows the execution times of the proposed MPTC (P-MPTC) and conventional MPTC for prediction, estimation, and optimisation stages. The minimum sampling time achieved with CM is 40 μs, whereas the sampling time of the speed loop is kept at 4 ms. The execution times are slightly higher after every 4 ms. At that particular time instant both the inner and outer loops run simultaneously and the execution time increases. On the average, the proposed method takes approximately 9.3 μs less time as compared to conventional MPTC.

Table 3 presents average execution times of three methods for comparison. The proposed method only takes an average of 19.7 μs in total to perform all the tasks. In fact, the proposed algorithm was also tested for a sampling time of 30 μs, which was not possible for other schemes due to execution time overrun. As can be seen from the table, the reduction in execution time of the proposed method is 32.3% and 19.4% when compared to conventional MPTC and S-MPTC, respectively. The results show that P-MPTC has not only reduced the number of VVs and prediction variables but also simplifies the remaining computations. The additional computational times for stator flux angles (estimated and reference) are reduced due to the use of Lagrange interpolation instead of using built-in arctan function, which requires more computational time and hardware resources. It was observed that the use of Lagrange interpolation reduces computational time of arctan from 2.73 μs to only 0.66 μs on the average (75.8% less). Although angle approximations can also be used in S-MPTC to further reduce its execution time, the presence of weighting factor and comparatively higher number of prediction variables makes it inferior to the proposed method.

6.2 Transient response

To compare the transient response of the proposed scheme, a speed reversal test at a rated speed of 65 rad/sec and without load was conducted on the drive with CM estimator. The results of this test for MPTC, S-MPTC, MPTC-RT, and proposed MPTC are presented in Figures 4(a), (b), (c) and (d), respectively. Each plot represents speed, estimated torque, stator flux magnitude, and \( i_r \) current of the induction motor. The stator flux reference is assumed constant at its rated value 0.75 Wb. It is evident from these plots that the proposed method gives not only comparable performance to all other methods but also outperforms S-MPTC in flux regulation. Since S-MPTC is a weighting factor-based method, it is affected severely when the number of VVs is reduced, especially at speed reversal points [2]. By the empirical rules given in [11], a best suited weighting factor value of 21.8 was chosen.

![Figure 2](image-url) Actual hardware setup for real-time implementation of the proposed work

| Parameter                      | Value | Parameter                      | Value |
|--------------------------------|-------|--------------------------------|-------|
| Rated torque, \( T_{\text{nom}} \) | 5 Nm  | Stator resistance, \( R_s \)  | 3 Ω   |
| Rated stator flux, \( \phi_{r,\text{nom}} \) | 0.75 Wb | Rotor resistance, \( R_r \)  | 4.1 Ω |
| Base speed, \( \omega_{\text{base}} \) | 65 rad/sec | Rotor inductance, \( L_r \)  | 351 mH |
| Inverter DC source, \( V_d \) | 240 V | Stator inductance, \( L_m \)  | 341.9 mH |
| Total number of pole pairs, \( p \) | 2 | Mutual inductance, \( L_m \)  | 324 mH |
| Sampling time (CM), \( T_s \) | 40 μs | Total viscous friction, \( B \) | 0.0019 N.m.s |
| Sampling time (EKF), \( T_s \) | 80 μs | Total inertia, \( J \)  | 0.0031 kg.m² |

Abbreviations: CM, current model; EKF, extended Kalman filter.
and used in both MPTC and S-MPTC. However, when the number of VVs is reduced in S-MPTC, decoupling of stator flux magnitude and torque becomes poor and the flux regulation is deteriorated, resulting in increased flux ripple. This happens because a vector that increases torque and decreases flux is usually selected and the weighting factor emphasis is imbalanced. At this stage, readjustment of the weighting factor is required to improve the flux magnitude response. This problem does not exist in the newly proposed method since the weighting factor is no longer present. Therefore, it can be concluded that the proposed method offers a better flux response as compared to S-MPTC and comparable transient behaviour to MPTC and MPTC-RT with significantly reduced computational burden.

However, due to the poor estimation performance of CM, a better technique based on EKF is used to demonstrate and compare the performance of P-MPTC to S-MPTC and conventional MPTC. Another reason for using EKF estimation is to show the computational efficiency of the proposed MPTC for complex estimation algorithms. Since EKF is computationally heavier than the other estimation techniques such as voltage model (VM) and full state observer, therefore, it is employed here to demonstrate the computational capabilities of the proposed MPTC. To keep the EKF algorithm simple, all the tuning matrices are chosen as diagonal matrices [25] where elements are chosen by a hit-and-trial method. The initial values of state vector and P are $X_0 = [0 \ 0 \ 0 \ 0]^T$ and $P = \text{diag}[0.1 \ 0.1 \ 0.01 \ 0.01]$, whereas $Q = 10^{-6} \text{diag}[0.1 \ 0.1 \ 0.001 \ 0.001]$, $R = \text{diag}[0.0135 \ 0.0135]$, and $H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. The sampling time for EKF experiments is set to 80 $\mu$s for all the methods. EKF involves a significant increase in computational time for flux estimation. However, this does not change the basic execution times for other steps, such as prediction and actuation. Based on the experimental results, the average execution time for EKF was observed to be around 34.7 $\mu$s.

### 6.3 Speed reversal test

The results of a speed reversal test similar to the test conducted for Figure 4 are shown in Figure 5. The use of an EKF estimator has significantly improved the regulation of both torque and flux as is evident from the figure. To further emphasise the performance comparability of P-MPTC, steady-state results are obtained and compared to S-MPTC and MPTC.

### 6.4 Steady-state response

Steady-state responses for torque, current $i_{\alpha}$, and stator flux magnitude of the three methods were recorded at the rated base speed of 65 rad/s with a constant load torque of 4 Nm.
applied to the motor. These results are shown in Figures 6a–c for conventional MPTC, S-MPTC, and P-MPTC, respectively. As can be seen from the plots, the torque response for P-MPTC is almost the same as that of the others. However, as observed in transient response, flux regulation for S-MPTC is poor (flux ripple 0.012 Wb) due to weighting factor adaptation challenges for a reduced number of VVs as explained previously. The proposed method works satisfactorily and the flux ripple is 0.007 Wb on the average. The flux regulation response explains the importance of weighting factor tuning. The weighting factor can be re-tuned to improve the flux regulation in the conventional and S-MPTC but the torque ripple has to be compromised. The weighting factor selection herein is made to keep the torque ripple equal in all the investigated methods. Total harmonic distortion (THD) of $i_a$ for the proposed method is 6.278% at the fundamental component of 20.7 Hz as compared to 6.174% and 8.57% for conventional and S-MPTC, respectively. The main results of steady-state investigation are summarised in Tables 4 and 5 for CM and EKF estimators. A quick comparison of these tables reveals that performance has significantly improved with EKF but at the cost of increased computational burden and decreased sampling time.

To test the step load change response of the proposed scheme, a load of 4.2 Nm was applied under a rated speed of 65 rad/sec when the motor was running at no load. The waveform comparison of the proposed method to only conventional MPTC is shown in Figures 7a,b. It is again evident that the dynamic response of the proposed method is very similar to the conventional MPTC scheme but with the advantage of a significantly reduced computational burden.

Like all other MPC methods, the proposed method also relies on precise motor parameters for accurate controller performance and optimal VV selection. Therefore, it is essential to evaluate the performance of the proposed method for parameter mismatching or variations. Practically, stator

**FIGURE 4** Experimental results for speed, estimated torque, and flux under a speed reversal test at rated speed of 65 rad/s with CM estimation: (a) conventional MPTC, (b) S-MPTC, (c) P-MPTC
resistance can vary up to ±50% of its modelled value due to heating of the machine during operation, whereas a change in inductance is even lower due to saturation [26]. However, a wider range for resistance and inductance variation is adopted here to show the wider range of parameter mismatch robustness of the proposed method. The speed and torque waveform are presented in Figures 8 and 9 under stator parameter variations. The magnitudes of $L_s$ and $R_s$ are suddenly increased to 1.3$L_s$ and 5.5$R_s$ at $t = 1$. The load torque is kept at 4 Nm during the parameter variations, while the motor is operating at the rated speed. The results show that parameter variation affects the stability of torque and speed tracking. In the $L_s$ variation case, the system undergoes a transient during sudden change in parameter value. However, the proposed MPTC recovers from the transient after 0.3 s for speed and 0.05 s for torque and tracks the controlled variables normally. For $R_s$ variation, a small increase in torque ripple is observed, whereas speed is decreased slightly and recovers soon after. The $R_s$ variation lower than 5.5$R_s$ values do not have a noticeable impact on the performance. The wider $R_s$ robustness is due to the use of CM estimation. If we employ other estimation methods, the sensitivity range will change. From the presented results, it can be seen that the proposed method has relatively good robustness against parameter variations.

6.5 | Difference between S-MPTC and P-MPTC

Both S-MPTC and P-MPTC make the use of a reduced number of VVs on the basis of direct and indirect torque deviations, respectively. However, there is a significant difference in which both methods choose the optimal VV. In S-MPTC, the selection of optimal VV is dictated by the optimisation of cost function, which involves a weighting factor, whereas P-MPTC does not have such a restriction. Owing to this limitation, the optimal performance with the conventional method of weighting factor design is difficult to achieve without rigorous hit-and-trial experiments to obtain the best value of the weighting factor. The selection of VV is mostly random in S-MPTC to cater to the requirements of regularising both torque and flux. This leads to increased switching frequency and ripples. In comparison, P-MPTC selects the optimal vector indirectly on the basis of flux position and
without the use of a weighting factor. This method results in smooth response and selection of neighboring VV, hence avoiding random switching patterns. With CM estimation, the frequency spectra of both methods are shown in Figure 10. The harmonic contents are presented as the magnitude percentage of the fundamental component at $f = 20.7$ Hz for both methods. Due to weighting factor presence in S-MPTC, more dominant lower frequency harmonics are present at lower frequencies as compared to P-MPTC. Although the frequency response can be improved for S-MPTC, it requires rigorous weighting factor tuning.

For a similar torque performance, the comparison of flux regulation achieved with S-MPTC and P-MPTC with respect to flux vector positions is shown in Figure 11. The random selection of optimal VVs and its effect are particularly visible during flux plane sector boundary crossings at 30°, 90°, and so on. S-MPTC has larger ripples during boundary crossings due to improper selection of the VVs. However, in P-MPTC, since selection of optimal VV is directly linked with flux angle, the response is much smoother and there are no ripples at sector boundaries. In the same figure a zoomed-in lower subplot
around flux sector at 150° is also shown which clearly contrasts the flux regulation performance of both methods.

It can be argued that poor performance of S-MPTC at sector boundaries is due to a poor choice of weighting factor and can be improved by employing a better weighting factor design method. Such a choice however does not come without a compromise on the computational burden. To compare the computational costs of both S-MPTC and P-MPTC under exactly the same dynamic performance, one of the well-known online weighting factor methods called the fuzzy decision making (FDM) technique [15] is used to compare the computational costs of both S-MPTC and P-MPTC.

Figure 12a,b shows polar plots of the flux vectors for both methods under offline nominal weighting factor design method and online FDM. In Figure 12a, P-MPTC outperforms S-MPTC under the same torque performance with average execution times ($t_{exec}$) of 19.7 and 24.43 μs. Note that $t_{exec}$ excludes the time taken for the estimation. Under the FDM method, the dynamic performance of S-MPTC is almost identical to P-MPTC as shown in Figure 12(b). However, this improvement in performance also requires additional computational time for implementing the FDM algorithm which increases $t_{exec}$ for S-MPTC to 36.19 μs. Therefore, it can be concluded that for a comparable dynamic performance of both methods, S-MPTC requires 48.14% additional computational time.

The difference between the performances of S-MPTC and P-MPTC can also be explained through results obtained under load and reference speed variations. Figure 13a–d is for current THD, switching frequency, torque, and flux ripples. The load torque $T_l$ is varied from no-load condition to 80% of the rated load, that is 4 Nm, whereas the reference speed is varied from
estimators and purely understand the performance of controllers. The results are obtained with a nominal value of weighting factor, that is $\lambda_w = 21.8$. The THD is higher at low speed and higher loads for both methods, whereas it drops to lower values at higher speed. However, the manner in which THD changes under these variations is entirely different, as is evident from the figure. Changes in THD for S-MPTC are highly irregular and heavily affected by the operating point due to the presence of the same weighting factor during variations. On the other hand, THD for P-MPTC follows a smooth pattern and overall remains lower than the THD in S-MPTC. Similarly, the plots for $f_{\text{ripple}}$ show a similar performance with P-MPTC offering overall lower switching frequency at all operating points. Switching frequency increases almost linearly with speed and load with its maximum at highest speed and load. Since the weighting factor is chosen to maintain a similar torque performance for both methods, the torque ripple plots are almost identical. Torque ripples are higher for lower loads. (This is called the zero torque regulation problem in MPTC algorithms.) However, the flux ripple waveform shows that P-MPTC is much superior to S-MPTC, which suffers from severe ripples ranging up to 8% randomly at all operating points, whereas these ripples remain under 2% for the entire variation range.

The main advantage offered by the proposed method is significantly reduced computational time, which makes room for the employment of more sophisticated estimators such as extended Kalman filters (EKF) for improving flux estimation

FIGURE 10 Frequency spectra of stator current phase $a$ current $i_a$: (a) S-model-based predictive torque control (MPTC), (b) P-MPTC

FIGURE 11 Comparison of flux regulation performance of S-model-based predictive torque control (MPTC) and P-MPTC during flux sector boundary crossings

25 rad/s (38.46% of the rated speed) to 65 rad/s (100% of the rated speed) to record the results under CM estimator. The medium speed range is chosen to avoid the effects of

FIGURE 12 Flux regulation comparison of S-model-based predictive torque control (MPTC) and P-MPTC under different weighting factor tuning methods: (a) nominal weighting factor (offline), (b) fuzzy decision method (FDM) (online)
for wider speed ranges. While the computational time is reduced, the performance in terms of torque, flux ripples, and current THD remains comparable or noticeably improved, as shown in the experimental results.

7 | CONCLUSION

Weighting factor design and computational burden in MPTC remain as two of the major issues in real-time IM control. Reference transformation is known to remove the problem of weighting factor by translating reference torque into reference stator flux vector and the formation of a new cost function consisting only of flux error. Although reference transformation solves the weighting factor problem, it comes at the cost of increased computational burden. A novel MPTC based on modified reference transformation and reduced number of prediction voltage vectors on the basis of angular position difference between reference flux and estimated flux is proposed herein. The torque reference is converted to equivalent flux reference using standard reference transformation and a modified formulation is obtained by merging the rotor flux angle into the stator flux relative angle. The angle relation between reference flux and estimated flux is used to form new prediction tables containing only three active voltage vectors and a null vector significantly reducing the computational cost while improving the dynamic behaviour of the controller. Experimental results validate the superiority of the proposed method and show a reduced execution time and comparable dynamic response. Future works may include a frequency reduction term in the cost function, use extended prediction to further improve steady state, and use of improved EKF estimation techniques.

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