Determination of Spacecraft Attitude and Source Position Using Non-aligned Detectors in Spin-stabilized Satellites

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1 Abstract

The modulation of high-energy transients’ (or steadily emitting sources’) light curves due to the imperfect alignment of the detector’s view axis with the spin axis in a spin-stabilized satellite is derived. It is shown how the orientation of the detector’s view axis with respect to the satellite’s spin axis may be estimated using observed light curves. The effects of statistical fluctuations are considered.

Conversely, it is shown how the attitude of a spin-axis stabilized satellite as well as the unknown position of a celestial source of high-energy photons may be determined using a detector whose view-axis is intentionally kept inclined and is known accurately beforehand. The case of three-axes stabilized satellites is also discussed.

Keywords: space vehicles:instruments, methods:analytical

2 Introduction:

In high-energy (X-ray and gamma ray) astronomy experiments photon detectors (scintillators such as Sodium Iodide, Cesium Iodide or gas-filled multi-wire proportional counters (MWPCs) or even solid-state detectors such as Si(PIN) or CdTe detectors) are flown onboard satellites. The satellites are either spin-axis stabilized (such as the Indian SROSS C-2 satellite) or three-axes stabilized. In the case of spin-axis stabilized satellites, normally, the detector’s view axis is aligned with the spin-axis of the satellite. However, sometimes this alignment is not perfect and the view axis of the detector makes a small but finite angle with the spin-axis of the satellite. In this case the observed light curve of a given celestial source of photons is modulated by the spin-period of the satellite. The amplitude of the modulation is a function of the polar angle ($\theta_v$) between the detector’s view axis and the satellite’s spin axis. It also depends on the relative direction of the source with respect to the spin axis ($\theta'_0$).
The spin-period is usually not strictly constant over the entire life time of the satellite. But over a small time period (during the entire duration of a high energy transient) it may be assumed to be almost constant.

In the present paper we derive the amplitude of the light curve’s modulation as a function of the space angle $\theta_v$. We take a high energy transient light curve and superimpose the modulation (using a given value of $\theta_v$). We show how the (if unknown) orientation of the view axis may be estimated reasonably accurately using the observed modulated light curve and the unmodulated light curve of the same high energy transient (say, a Solar X-ray flare) detected by another satellite.

Finally, we discuss how this apparently disadvantageous and undesirable phenomenon may be useful in determining the position of a celestial high energy photon source given that the orientation of the detector’s view axis (intentionally kept inclined) is known beforehand. Also it is described how this same phenomenon might be utilised in determining the (unknown) attitude of the spacecraft by placing two small detectors (one inclined to the spin-axis, the other aligned with the spin-axis).

3 Mathematical Formulation:

Let $XYZ$ (Fig. 1) denote the celestial equatorial co-ordinate system. Let $\alpha_S$ and $\delta_S$ denote the orientation of the satellite’s spin axis in the $XYZ$ system. We define $\theta_S = \pi - \delta_S$ and $\phi_S = \alpha_S$. Let the source direction be $\alpha_0$ and $\delta_0$ respectively.

![Figure 1: XYZ is the celestial polar co-ordinate system. OS is the spin-axis of the satellite. OO’ is the vector in the source direction. OV is the detector’s view axis. The detector’s view axis and the source rotate around the spin axis at the spin period of the satellite.](image)

Then, $\theta_0 = \pi - \delta_0$ and $\phi_0 = \alpha_0$. The direction cosines of the source with respect to the spin-axis system may be obtained as

$$\begin{pmatrix} \sin\theta'_0 \cos\phi'_0 \\ \sin\theta'_0 \sin\phi'_0 \\ \cos\theta'_0 \end{pmatrix} = \begin{pmatrix} \cos\theta_S \cos\phi_S & \cos\theta_S \sin\phi_S & -\sin\theta_S \\ -\sin\phi_S & \cos\phi_S & 0 \\ \sin\theta_S \cos\phi_S & \sin\theta_S \sin\phi_S & \cos\theta_S \end{pmatrix} \begin{pmatrix} \sin\theta_0 \cos\phi_0 \\ \sin\theta_0 \sin\phi_0 \\ \cos\theta_0 \end{pmatrix}$$

(1)

(see, for example, Arfken and Weber). Hence the unit vector in the source direction is

$$\vec{S} = \vec{i} \sin\theta'_0 \cos\phi'_0 + \vec{j} \sin\theta'_0 \sin\phi'_0 + \vec{k} \cos\theta'_0$$

(2)
Let the direction of the detector’s view axis be $\theta_v$ and $\phi_v$, respectively with respect to the spin-axis co-ordinate system. Then the unit vector along the view axis in the spin-axis system is given by

$$\vec{V} = \vec{i}' \sin \theta_v \cos \phi_v + \vec{j}' \sin \theta_v \sin \phi_v + \vec{k}' \cos \theta_v$$

(3)

If we assume $\phi_v = 0$, this becomes

$$\vec{V} = \vec{i}' \sin \theta_v + \vec{k}' \cos \theta_v$$

(4)

This essentially implies that the spin-axis system is given a trivial rotation by an amount $\phi_v$ in the opposite sense.

3.1 Determination of the inclination of the view axis:

The modulation factor (assuming $\phi_v = 0$) is given by

$$M = \sin \theta_v \sin \theta'_0 \cos (\omega t + \psi) + \cos \theta_v \cos \theta'_0$$

(5)

where we have put $\phi'_0 = (\omega t + \psi)$, $\psi$ being the epoch, $t$ the time and $\omega$ the angular velocity due to the spin.

The modulation affects only the signal and not the background counts. The modulated counts

$$D_i = C_i M = C_i (\sin \theta_v \sin \theta'_0 \cos (\omega t_i + \psi) + \cos \theta_v \cos \theta'_0)$$

(6)

where the $C_i$'s are the counts in the unmodulated light-curve.

To determine $\theta_v$ ($\theta'_0$ is given) one has to consider another unmodulated light curve (this light curve has to be multiplied by an appropriate factor $g$ such that the maxima (peaks) are the same for the two light curves.

Therefore, we have

$$\sin \theta_v \sin \theta'_0 \cos (\omega t_i + \psi) + \cos \theta_v \cos \theta'_0 = f_i$$

(7)

where $f_i = D_i / C_i$, $C_i$ being known from a detector which is aligned (on-board a different satellite). Here, of course, the modulated light curve has to be multiplied by a suitable factor such that the maxima of the two light curves are equal. This may be written as (since $\theta'_0$ is known, i.e. both $\sin \theta'_0$ and $\cos \theta'_0$ are known).

$$a_1 x_1 \cos (\omega t_i + \psi) + b_1 x_2 = f_i$$

(8)

where $x_1 = \sin \theta_v$, $x_2 = \cos \theta_v$. Also, $a_1 = \sin \theta'_0$, $b_1 = \cos \theta'_0$. Taking the time average of both sides (over an integral number of cycles),

$$< a_1 x_1 \cos (\omega t_i + \psi) > + < b_1 x_2 > = < f_i >$$

(9)

$$a_1 x_1 < \cos (\omega t_i + \psi) > + b_1 x_2 = < f_i >$$

(10)

Since $< \cos (\omega t_i + \psi) >$ is equal to zero for an integral number of cycles, the first term becomes equal to zero. Hence,

$$b_1 x_2 = < f_i >$$

(11)

or,

$$x_2 = (1/b_1) < f_i >$$

(12)

Since, $x_2 = \cos \theta_v$, $\theta_v = \cos^{-1}(1/b_1) < f_i >$. 

3
3.2 Determination of Source Location

If $\theta_v$ is known beforehand, it is possible to determine the location of a source in the sky using two detectors, one aligned, the other inclined (at a known angle) with the spin axis of the satellite.

The equation for the modulated light curve (counts vs. time) is

$$D_i = C_i (\sin \theta_v \cos \phi_v \sin \theta'_0 \cos \phi'_0 + \sin \theta_v \sin \phi_v \sin \theta'_0 \sin \phi'_0 + \cos \theta'_0 \cos \theta_v)$$

(13)

where $C_i$ is the counts detected during the ith time bin (when the view axis is aligned with the spin axis, i.e. $\theta_v = 0$). The other symbols have their usual meanings. Assuming $\phi_v = 0$ (reorientation of the spin-axis system), this reduces to

$$D_i = C_i (\sin \theta_v \sin \theta'_0 \cos \phi'_0 + \cos \theta'_0 \cos \theta_v)$$

(14)

Here, $\phi'_0 = \cos (\omega t_i + \psi)$, $\psi$ being the epoch. For a detector whose view axis is aligned with the spin axis, $\theta_v = 0$. Then

$$D_i = C_i (\cos \theta'_0)$$

(15)

From the last equation, for a given (say, the ith) time bin,

$$\cos \theta'_0 = D_i / C_i$$

(16)

which gives

$$\theta'_{0i} = \cos^{-1}(D_i / (C_i))$$

(17)

Let $a = \cos \theta'_0$ (known) and $\sin \theta'_0 = \pm \sqrt{1 - a^2}$. If, for a second detector (for which $\theta_v \neq 0$, i.e. $\sin \theta_v \neq 0$), then

$$D'_i = C'_i (b r \phi'_0 + ac)$$

(18)

This gives

$$\phi'_0 = \cos^{-1}((1 / br)(D'_i / C'_i - ac))$$

(19)

Here $b = \sin \theta_v$, $r = \sin \theta'_v$ and $c = \cos \theta_v$. Knowing the values of $\theta'_0$ and $\phi'_0$, the inverse transformation (corresponding to that given in eqn. (1)) may be used in order to obtain the values of $\theta_0$ and $\phi_0$.

$$\begin{pmatrix} \sin \theta_0 \cos \phi_0 \\ \sin \theta_0 \sin \phi_0 \\ \cos \theta_0 \end{pmatrix} = \begin{pmatrix} \cos \theta_S \cos \phi_S & -\sin \phi_S & \sin \theta_S \cos \phi_S \\ \cos \theta_S \sin \phi_S & \cos \phi_S & \sin \theta_S \sin \phi_S \\ -\sin \theta_S & 0 & \cos \theta_S \end{pmatrix} \begin{pmatrix} \sin \theta'_0 \cos \phi'_0 \\ \sin \theta'_0 \sin \phi'_0 \\ \cos \theta'_0 \end{pmatrix}$$

(20)

As evident there will be many such values (one set for each time bin) and their means and errors may be determined.

3.3 Determination of Spacecraft Attitude

If $\theta_v$ is known, it is possible to determine the attitude of the spacecraft as follows. This also requires two detectors—one aligned and the other inclined (at a known angle) with the satellite’s spin axis.
The equation connecting the unknown direction of the spin axis, $(\theta_S$ and $\phi_S$) to the known (true) direction cosines and known (apparent) direction cosines of the source is the following.

$$
\begin{pmatrix}
\sin\theta'_0 \cos\phi'_0 \\
\sin\theta'_0 \sin\phi'_0 \\
\cos\theta'_0
\end{pmatrix}
= 
\begin{pmatrix}
\cos\theta_S \cos\phi_S & \cos\theta_S \sin\phi_S & -\sin\theta_S \\
-\sin\theta_S & \cos\phi_S & 0 \\
\sin\theta_S \cos\phi_S & \sin\theta_S \sin\phi_S & \cos\theta_S
\end{pmatrix}
\begin{pmatrix}
\sin\theta_0 \cos\phi_0 \\
\sin\theta_0 \sin\phi_0 \\
\cos\theta_0
\end{pmatrix}
$$

(21)

Three equations are obtained from the above matrix equation. One of the equations is

$$
-l_1 \sin\phi_S + l_2 \cos\phi_S = k_2
$$

(22)

where $l_1 = \sin\theta_0 \cos\phi_0$, $l_2 = \sin\theta_0 \sin\phi_0$, and $k_2 = \sin\theta'_0 \sin\phi'_0$. This leads to the quadratic equation

$$(l_1^2 + l_2^2)x^2 + 2k_2 l_1 x + (k_2^2 - l_2^2) = 0
$$

(23)

Here $x = \sin\phi_S$. Solution of the last equation gives two values of $x$ from which the value of $\phi_S$ may be obtained as $\phi_{S_1} = \sin^{-1}(p_1)$ and $\phi_{S_2} = \sin^{-1}(p_2)$ where $p_1$ and $p_2$ are the two roots of equation (21).

The other two equations are, respectively

$$
l_1 \cos\theta_S \cos\phi_S + l_2 \cos\theta_S \sin\phi_S - l_3 \sin\theta_S = k_1
$$

(24)

and

$$
l_1 \sin\theta_S \cos\phi_S + l_2 \sin\theta_S \sin\phi_S + l_3 \cos\theta_S = k_3
$$

(25)

Here $k_1 = \sin\theta'_0 \cos\phi'_0$, $l_3 = \cos\theta_0$, and $k_3 = \cos\theta'_0$. The last two equations give rise to the following equation

$$(k_1^2 + k_2^2)y^2 + 2k_1 l_3 y + (l_3^2 - k_3^2) = 0
$$

(26)

Here $y = \sin\theta_S$. Solutions of the above equation gives two values of $y$ from which the values of $\theta_S$ may be obtained as $\theta_{S_1} = \sin^{-1}(q_1)$ and $\theta_{S_2} = \sin^{-1}(q_2)$ where $q_1 = \ldots$ and $q_2 = \ldots$ are the solutions of eqn. (24).

The correct values of $\theta_S$ and $\phi_S$ are to be decided based on physical considerations.

### 3.4 Three Axes Stabilized Spacecrafts

In the case of a three axes stabilized spacecraft one may have a small spinning platform on an extended boom on which the aligned and inclined detectors may be placed. The above mentioned procedures may be utilised to determine either source location or spacecraft attitude in this case as well.

### 4 Results:

#### 4.1 The Timing Data and the Light Curve:

Due to the non-availability of real data at hand, we take recourse to generating data artificially.

The total duration of the time series data (detected counts vs. time) is equal to 512 seconds with a time resolution of 256 ms (this is the width of each time bin).
During the leading 51.2 seconds and the trailing 51.2 seconds the detected counts are due only to the background. The background has a mean value of 1.8 counts and is fluctuated according to a Poisson distribution.

Between the leading and trailing background data, in the remaining time interval of 409.6 seconds a light curve is generated artificially. The light curve is essentially triangular in shape. The total duration (409.6 seconds) of the light curve is divided into two parts. The rising portion of the light curve has a duration of 153.6 seconds. The decaying portion has a duration of 256 seconds. Both the rising portion and the decaying portion of the light curve are linear in shape. The rising portion has a slope of +3.8 while the decaying portion has a slope of -2.28. Poissonian fluctuations are superimposed on the light curve generated by the Monte Carlo method (both in the rising as well as in the decaying portion). This simulated light curve is shown in Fig.2.

![UNMODULATED LIGHT CURVE](image-url)

Figure 2: The unmodulated light curve. The leading 51.2 seconds and the trailing 51.2 seconds consist of only the background counts with an average value of 1.8. Poissonian fluctuations are superimposed on the background counts. The intervening 409.6 seconds constitute the simulated light curve due to the high energy transient source. The rising portion has a duration of 153.6 seconds and has a slope of 3.8. The decaying part has a duration of 256 seconds and has a slope of -2.28.

### 4.2 The Modulated Light Curve:

Due to the spinning motion of the satellite and the fact that the detector’s view axis is not aligned with the satellite’s spin axis, the light curve of the high energy transient (it is true also in the case of a steadily emitting celestial object) will be modulated in amplitude. The modulation factor is calculated as described earlier.

The modulated light curve corresponding to the original (unmodulated) light curve (Fig.2) is shown in Fig.3. Here $\theta_S$ is the polar angle and $\phi_S$ is the azimuth angle of the source as seen from the spin-axis frame of reference. $\theta_v$ and $\phi_v$ are the corresponding parameters for the view axis. To simplify the problem we assume $\phi_v = 0$. This essentially means a trivial rotation of the spin axis coordinate system about its Z-axis by an amount equal to $\phi_v$.

The results of determination of $\theta_v$ are displayed in Table 1. The error on the mean of the estimated values of $\theta_v = 0.283$ degrees which is equal to 1.25 percent.
Figure 3: The simulated light curve in Fig.1 of the high energy transient modulated by the spin period of the satellite. The spin period is equal to 11.78 seconds. The counts in the light curve are fluctuated according to a Poisson distribution.

Table 1

Table 1: Estimated values of $\theta_v$ and their errors.

| Range of Data (in cycles) | True Value of $\theta_v$ (in degrees) | Estimated value of $\theta_v$ (in degrees) | Error on $\theta_v$ (in degrees) |
|--------------------------|----------------------------------------|------------------------------------------|-------------------------------|
| 1-2                      | 22.6                                   | 22.607                                   | 0.007                         |
| 3-4                      | 22.6                                   | 23.580                                   | 0.980                         |
| 5-6                      | 22.6                                   | 23.479                                   | 0.879                         |
| 7-8                      | 22.6                                   | 22.407                                   | -0.193                        |
| 9-10                     | 22.6                                   | 21.600                                   | -1.000                        |
| 11-12                    | 22.6                                   | 22.231                                   | -0.369                        |
5 Discussions:

Obviously the amplitude of the modulation of a high energy transient (or even a steady source) light curve depends on the parameters $\theta_v$, $\theta_0$ and $\phi_v$ etc (as shown in the figures 2 and 3). Since one assumes that the average of the term $\cos \omega t$, $<\cos \omega t>$ equals zero, one has to consider the time-history (light-curve) data only for an integral number of the spin period of the satellite. In the present work 2 cycles of data has been used for estimating each value of $\theta_v$.

It has been shown that even in the presence of fluctuations, this method is still viable and yields quite good results.

However, the accuracy in the estimated value of $\theta_v$ depends on the background. In another calculation where a background has been assumed which is 13 times larger, the errors in the estimated value of $\theta_v$ also becomes much larger, typically a few degrees. Therefore, this method may be used effectively in only those cases where the signal-to-noise ratios are quite large.

The spin period of the satellite and the integration time of the light curve are very much realistic. Actually, the GRBM (Gamma Ray Burst Monitor) on-board the SROSS C-2 satellite possessed an integration time of 256 ms (Sinha, S et al) although the spin-period was nearly 12 seconds (about half of the value used in the present work).

In the present work one assumes small (ideally point size) detectors since the mathematical analysis does not assume any finite value of $\Delta \phi$ (due to the finite size of the detector). It is quite fortunate that presently solid state (Si(PIN) and CdTe) detectors are available having size as small as 1mmX1mm that may be used to detect X-ray flares from the Sun.

6 Conclusions:

The effects of modulation of light curves of celestial high energy photon sources when the detector’s view axis is not aligned with the spin axis of the satellite has been described. It is shown how the orientation of the view axis with respect to the spin axis may be determined by comparing this modulated light curve with the light curve obtained from a detector whose view axis is perfectly aligned with the spin axis (either in the same satellite or in a different satellite). Finally, the usefulness of this apparently disadvantageous situation is described- how this effect may be utilised in order to estimate the attitude of the spin axis or to determine the unknown position coordinates of a celestial high energy photon source. Further work in this direction will be reported shortly.

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