Normal Type-2 Fuzzy Geometric Curve Modeling: A Literature Review

R S Adesah and R Zakaria
Mathematics, Graphics and Visualization Research Group (M-GRAVS), Faculty of Science and Natural Resources, Universiti Malaysia Sabah, 8800 Kota Kinabalu, Sabah, Malaysia
E-mail: reedzialade@gmail.com and rozaimi@ums.edu.my

Abstract. Type-2 Fuzzy Set Theory (T2FST) is widely used for defining uncertainty data points rather than the traditional fuzzy set theory (type-1) since 2001. Recently, T2FST is used in many fields due to its ability to handle complex uncertainty data. In this paper, a review of normal type-2 fuzzy geometric curve modeling methods and techniques is presented. In particular, there have been recent applications of Normal Type-2 Fuzzy Set Theory (NT2FST) in geometric modeling, where it has helped improving results over type-1 fuzzy sets. In this paper, a concise and representative review of the processes in normal type-2 fuzzy geometrical curve modeling such as the fuzzification is presented.

1. Introduction
Uncertainty has become such a major problem in many fields. The communities determine that it is hard to deal with uncertainty and fuzziness of the data to obtain a perfect solution or model. In 1965, Zadeh has introduced an idea in the handling of uncertainty and fuzzy data [1]. It is called Fuzzy Set Theory or Type-1 Fuzzy Set Theory (T1FST). Type-1 Fuzzy Number (T1FN) concept which established from the T1FST is used in defining the problem in uncertainty. The uncertainty problem involving conducting the uncertainty in the form of the real number is defined by using the concept of T1FN [2].

However, T1FST have limited efficiency to directly handle the uncertainties because it comes in many guises [3]. Furthermore, T2FST is introduced ten years later to handle complex uncertainty data which T1FST cannot handle. It is known that T2FST have the ability to handle complex uncertainty data rather than T1FST which it cannot define higher uncertainty data. Therefore, T2FST is defined for a higher level definition of T1FST. Zadeh has proposed the concept of Type-2 Fuzzy Number (T2FN) based on T2FST where T2FST is used in defining the problem in uncertainty. The uncertainty problem involving conducting the uncertainty in the form of the real number is defined by using the concept of T1FN [2].

Recently, T2FST is widely used for defining uncertainty data points rather than T1FST and T2FST immediately becomes popular. However, T1FST was introduced more than fifty years ago [1]. T2FST starts to emerge in 2001 [7] and unfortunately, it takes so long to develop due to T2FST are more difficult to use and understand [8, 9]. The researchers are now embracing T2FST to handle more imprecise or vagueness of the data [9]. Mendel and John [8] explained briefly about T2FST which presented in [7] to make researchers more understand the concept of T2FST. The evolution and explanation of T2FST are continued by [3, 10, 11].

Fuzzy set theory is also used to define uncertainty data to construct B-spline curves or surfaces due to uncertainty data on the control points of B-spline. The Fuzzy Set Theory is used in geometrical
modeling due to geometric function cannot handle the uncertainty and vagueness of data. Usually, the uncertainty data is rejected to produce a perfect model. The concept of T1FST is used in geometrical modeling in a certain research [12-19] and T2FST in geometrical modeling is presented in [2, 20-24]. The concept of the fuzzy set theory is implemented in the interpolation of geometry in [18, 22-26] and also implemented in rational B-spline curve [21] and rational Bézier curve [25].

This paper will present a review on NT2FST in Geometrical Curve Modeling and an NT2FST is defined by Bézier function which is called Normal Type-2 Fuzzy Bézier Curve (NT2FBC) model will be given as example. Fuzzification process also presented in this paper which used the alpha-cut operation.

2. Preliminaries
The definition of T1FST is not presented here due to existing T1FST introduced by Zadeh [1] and there are most of the researches in the fuzzy field are defined T1FST. T1FST is the core in defining the uncertainty problems [11]. The definition of the fundamentals of the fuzzy number or T1FN, T2FST and T2FN is presented in this section.

2.1. Definition 1: Fuzzy Number (T1FN) [1]
A fuzzy set \( FS \) is called a fuzzy number if the following conditions are fulfilled
i. There exist \( x \in \mathbb{R} \), such that \( \mu_{FS}(x) = 1 \).
ii. For any \( \alpha \in (0,1] \), the set \( \{ x : \mu_{FS}(x) \geq \alpha \} \) is a closed interval denoted by \( \overline{FS_{\alpha}}, \underline{FS_{\alpha}} \).

Then, the fuzzy number is literally fuzzy set defined on the real number set that described by mean of the membership function which produces information either “about” or “around” such number which has decreasing, increasing, unique modal crisp value, convex and left-right continuous [25].

2.2. Definition 2: T2FST [10,20,21,23,24]
\( FS \) is denoted as Type-2 Fuzzy Sets (T2FS), is described by a type-2 membership function \( \mu_{FS}(x,u) \), where \( x \in X \) and \( u \in U \), \( \alpha \in [0,1] \) that is,
\[
FS = \{ (x,u), \mu_{FS}(x,u) \forall x \in X, \forall u \in U, \alpha \in [0,1] \}
\] (1)
where, \( 0 \leq \mu_{FS}(x,u) \leq 1 \) [7]

2.3. Definition 3: T2FN [10, 11]
T2FN is widely defined as a T2FS which has a numerical domain. The following four constraints are used to define an interval T2FS, where \( FS_{\alpha} = \left[ \left[ d^a, e^a \right], \left[ f^a, g^a \right] \right], \forall \alpha \in [0,1], \forall d^a, e^a, f^a, g^a \in \mathbb{R} \) (Figure 1) [27]:
i. \( d^a \leq e^a \leq f^a \leq g^a \).
ii. \( \left[ d^a, e^a \right] \) and \( \left[ e^a, f^a \right] \) develop a function that is convex and \( \left[ d^a, g^a \right] \) develop a function is normal.
iii. \( \forall \alpha_1, \alpha_2 \in [0,1]: (\alpha_2 > \alpha_1) \Rightarrow \left[ \left[ d^a, e^a \right], \left[ f^a, g^a \right] \right] \Rightarrow \left[ \left[ e^a, f^a \right], \left[ d^a, g^a \right] \right], \forall e^a \geq f^a \).
iv. If the maximum of the membership function developed by \( \left[ e^a, f^a \right] \) is the level \( \alpha_1 \), which is, \( \left[ e^a, f^a \right] \), then \( \left[ e^a, f^a \right] \subset \left[ d^{a_1}, g^{a_1} \right] \).

2
3. Normal Type-2 Fuzzy Set Theory

There are several definitions are discussed in recent researches to define Normal Type-2 Fuzzy Set Theory (NT2FST) [10, 20, 21, 23, 24]. Normal type-2 fuzzy number (NT2FN) is defined based on NT2FST as stated before. In this section, the definition of NT2FN and alpha-cut operation of NT2FN is presented.

3.1. Definition 4: [10, 11]

It is given $\mathcal{FS}$ that is T2FN, which $H(F_{S\uparrow})$ and $H(F_{S\downarrow})$ are the height for the Upper part of Membership Function (UMF) and the height of the Lower part of Membership Function (LMF) respectively, then T2FN is called Normal Type-2 Triangular Fuzzy Number (NT2TFN) if $H(F_{S\downarrow}) < H(F_{S\uparrow}) = 1$. This definition 4 can be interpreted through Figure 2.

![Figure 2. The interpretation of NT2FN.](image)
3.2. Definition 5: [10, 11]
Based on Definition 4, let \( \overline{FS} \) be the set of T2FN in a triangular form with \( \overline{FS_i} \in \overline{FS} \) in which \( i = 0,1,...,n-1 \). Then, the operation of alpha-cut NT2TFN is \( \overline{FS_\alpha} \) which is given as follows.

\[
\overline{FS_\alpha} = \left\{ \overline{FS_{l_i}}, \overline{FS_{u_i}} \right\} \in \overline{FS} \quad \alpha \in [0,1]
\]

\[
= \left\{ \left( \overline{FS_{l_i}} - \overline{FS_{u_i}}, \overline{FS_{l_i}} + \overline{FS_{u_i}} \right), \alpha, \overline{FS_{l_i}}, \overline{FS_{u_i}} \right\} \quad \alpha \in [0,1]
\]

\[
= \left\{ \left( \overline{FS_{l_i}} - \overline{FS_{u_i}}, \overline{FS_{l_i}} + \overline{FS_{u_i}} \right), \alpha, \overline{FS_{l_i}}, \overline{FS_{u_i}} \right\} \quad \alpha \in [0,1]
\]

(2)

where \( \alpha \) and \( \alpha_{c_i} \) are alpha values of LMF and crisp LMF of NT2TFN respectively. This explanation is interpreted through Figure 3.

![Figure 3. The operation of the alpha-cut NT2TFN.](image)

4. Normal Type-2 Fuzzy Bézier Curve Model
A Bézier curve model is constructed based on the definition given. The T2FST is used to define uncertainties of the control points. Then, NT2TFN and alpha-cut operation of NT2TFN are implemented into a geometric modeling function curve [20, 23]. NT2FBC is obtained by blended the given definition with Bézier function. Definition 2, 3 and 4 is blended with the control points of Bernstein polynomial, which is given as follows.

\[
\overline{B}(t) = \sum_{i=0}^{n} B^*_i(t) \overline{FS_i}
\]

(3)

where \( B^*_i(t) = \binom{n}{i} t^i (1-t)^{n-i} \) is the Bernstein’s polynomial and \( \overline{FS_i} \) is the fuzzy control points [20, 25].

After NT2FBC is constructed, the definition 5 is applied against NT2FBC. The example model of this process is illustrated in Figure 4.
Figure 4. The modeling of NT2FBC in (a) and the operation of alpha-cut NT2FBC in (b).

5. Conclusion
In this paper, the process of NT2FBC used by researchers has been discussed. The flows from T1FST to the alpha-cut of T2FST have been presented. The reason of using T2FST is used instead of T1FST is because of the complexity of the uncertainty of the data and it has been explained. In creating a perfect geometric curve modeling, T2FST is needed to define the uncertainty of data due to some corrupted data by using existing function which is Bézier function in this research. However, there doesn’t have many researches to refer about Type-2 Fuzzy Geometrical Modeling. Therefore, our future research direction will implement the B-spline concept in NT2FBC on lakebed topography and also will continue with the reduction and defuzzification processes to obtain a best fit model.

Acknowledgements
The authors would like to gratitude Mathematics, Graphics and Visualization Research Group (M-GRAVS), Faculty of Science and Natural Resources of University of Malaysia Sabah (UMS) and Ministry of Higher Education Malaysia (MOHE) for funding (RAG0062-SG2015) and grant the facilities to accomplish this research.

References
[1] Zadeh L A 1965 Fuzzy sets Information and control 8 338-53.
[2] Zakaria R and Wahab A F 2013 Int. Journal of Math Analysis 7 1285-1300.
[3] Mendel J M 2007 Type-2 fuzzy sets and systems: An overview IEEE Computational Intelligence Magazine 2 20-29.
[4] Zadeh L A 1975 The concept of a linguistic variable and its application to approximate reasoning-I Information sciences 8 199-249.
[5] Zadeh L A 1975 The concept of a linguistic variable and its application to approximate reasoning-II *Information sciences* 8 301-357.

[6] Zadeh L A 1975 The concept of a linguistic variable and its application to approximate reasoning-III *Information sciences* 9 43-80.

[7] Karnik N N and Mendel J M 2001 *Fuzzy sets and systems* 122 327-348.

[8] Mendel J M and John R B 2002 *IEEE Transactions on fuzzy systems* 10 117-127.

[9] John R and Coupland S 2007 Type-2 fuzzy logic: A historical view *IEEE computational intelligence magazine* 2 57-62.

[10] Wahab A F and Zakaria R 2013 *Applied Mathematical Sciences* 7 2239-2252.

[11] Wahab A F and Zakaria R 2013 *Applied Mathematical Sciences* 7 2253-2263.

[12] Gallo G and Spagnuolo M 1998 *Uncertainty coding and controlled data reduction using fuzzy-B-splines* Proc. Int. on IEEE Computer Graphics pp 536-542.

[13] Anile A M Falcidieno B Gallo G Spagnuolo M and Spinello S *Fuzzy sets and systems* 113 397-410.

[14] Gallo G Spagnuolo M and Spinello S 2000 Fuzzy B-splines: a surface model encapsulating uncertainty *Graphical Models* 62 40-55.

[15] Wahab A F et al 2004 *Fuzzy set in geometric modeling* Proc. on IEEE Computer Graphics, Imaging and Visualization CGIV pp 227-232.

[16] Wahab A F Ali J M and Majid A A 2009 *Fuzzy Geometric Modeling* 6th Int. Conf. IEEE Computer Graphics, Imaging and Visualization 2009 CGIV'09 pp 276-280.

[17] Wahab A F and Ali J M 2010 Penyelesaian Masalah Data Ketakpastian Menggunakan Splin-B Kabur *Sains Malaysiana* 39 661-670.

[18] Behforooz H Ezzati R and Abbasbandy S 2010 *International Journal of Pure and Applied Mathematics* 60 383-392.

[19] Zakaria R and Wahab A F *Sains Malaysiana* 43 799-805.

[20] Zakaria R et al 2013 Type-2 fuzzy Bezier curve modeling ed Ishak I Hashim E S Ismail and R Nazar *Proc. Conf. on AIP* 1522 pp 945-952.

[21] Zakaria R Wahab A and Gobithaasan R U 2013 Normal Type-2 Fuzzy Rational B-spline Curve *Preprint gr-qc/1304.7868*.

[22] Zakaria R Wahab A and Gobithaasan R U 2013 Perfectly normal type-2 fuzzy interpolation B-spline curve *Preprint gr-qc/1305.0001*.

[23] Zakaria R Wahab A F and Gobithaasan R U 2013 The Representative Curve of Type-2 Fuzzy Data Point Modeling *Modern Applied Science* 7 60.

[24] Zakaria R et al 2014 Normal type-2 fuzzy interpolating B-spline curve Ismail S Ahmad and R A Rahman *Proc. Conf. on AIP* 1605 pp 476-481.

[25] Wahab A F Zakaria R and Ali J M 2010 Fuzzy interpolation rational bezier curve *7th Int. Conf. Computer Graphics, Imaging and Visualization (CGIV)* pp 63-67.

[26] Zakaria R and Wahab A F 2012 *Applied Mathematical Sciences* 6 6971-6991.

[27] Aguero J R and Vargas A 2007 *IEEE Transactions on Fuzzy systems* 15 31-40.