Modelling of a semi-active vehicle suspension implemented with magneto-rheological dampers

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Abstract. This paper describes a procedure for a simulation design aimed to achieve improved performance of the vehicle suspension, using magneto-rheological (MR) semi-active dampers. It is used a spatial so-called “Full car”-model of the vehicle. The model of the kinematic excitations from the unevenness of the road pavement is described in detail. The function of the MR damper is represented by linear and non-linear analytical models. Based on them, is obtained and its inverse neural-network model needed for the suspension control purposes.

1. Introduction
When the vehicle’s suspension is synthesized, the simultaneous provision of ride comfort and stability of the vehicle on the road surface is required. The ride comfort assesses the degree of the impact of the road roughness on the driver’s health and the functional abilities as well as the degree of vibrational and shock protection of the transported cargo and transport equipment. Evaluatively, the ride comfort is related to the reduction of the vibrational displacement and especially of the vibrational acceleration of the sprung masses (chassis, coupe, and driver). The stability of the vehicle has several indicators, and in this study is considered in the context of this one, linked to the presence of sufficiently large forces of interaction between the tires and the pavement, providing the necessary level of adhesion. In this regard, stability requires a reduction in the vibrational characteristics of the unsprung masses (mainly the car’s tires and some suspension elements) and, above all, the reduction of the dynamic component of the normal reaction between the tire and the road pavement. Additionally, this component takes an effect and on the operational condition of the road. These two criteria are contradictory, as in principle the first one requires a softer suspension and the other one requires a stiffer. The compromise between them in the passive suspension is limited. Furthermore, in the different frequency ranges of the excitations, these requirements having changes quantitatively. The relationship between the elasticity of the front and the rear suspension with respect to both criteria is also ambiguous and also changes in the different frequency ranges [1-11]. In this regard, the use of controllable dampers allows for a much higher quality of the compromise between them. Semi-active dampers, although more limited in function than the active dampers, do not require extra energy (except this for the control) and are more reliable, making them more widely applicable.

2. The full car model
The model is shown in figure 1. The CY and CZ axles are central inertia axes, and CX is the principal inertia axis of the model’s sprung mass. It is with seven degrees of freedom: the displacements of the unsprung masses in a vertical direction – \(z_1, z_2, z_3, z_4\) and the bounce \(z_c\), the pitch angle \(\theta_x\) and the roll angle \(\theta_y\) of the vehicle’s sprung mass, denoted from the situation of the static equilibrium:
\[
q = \begin{bmatrix} q^m \\
q^r 
\end{bmatrix}, \quad q^r = \begin{bmatrix} z_1, \theta_1, \theta_2 
\end{bmatrix}^T, \quad q^m = \begin{bmatrix} z_3, z_4, z_5 \n \end{bmatrix}^T
\] (1)

Figure 1. Full car model of the vehicle’s suspension.

2.1. The parameters of the model

- \( m_{\text{usf}}, m_{\text{usr}}, m_s \) – masses of the unsprung part of the front and rear suspension and of the sprung parts;
- \( I_{cx}, I_{cy} \) – mass inertia moments towards the axis \( CX \) and \( CY \);
- \( k_f, k_r, c_f, c_r \) – coefficients of elasticity and viscous damping of the front and rear tires;
- \( k_{fs}, k_{rs} \) – coefficients of elasticity of the front and rear suspension;
- \( F_{id}, i = 5, \ldots, 8 \) – forces in the semi-active dampers;
- \( \xi_{fr}, \xi_{rr}, \xi_{fl}, \xi_{rl} \) – kinematic excitations from the road roughness;
- \( l_1, l_2 \) – distances between the centre of gravity (CG) of the sprung part and the axes of the front and rear suspension respectively;
- \( 2l_3 \) – length of the wheel track (it is accepted that \( C_yz \) is a plane of a mass-geometrical symmetry);
- \( l_4, l_5 \) – coordinates of the driver’s seat position from \( CG \), \( q_D = [-l_4, l_5] \).

2.2. Mechano-mathematical model

The displacements of the suspension points \( A_i(x_i, y_i, z_i), i = 1, \ldots, 4 \) of the sprung mass are:

\[
\bar{r}_i = \bar{r}_C + \bar{\theta} \times \bar{\rho}_i, \quad i = 1, \ldots, 4,
\] (2)
where: \( \vec{r} = [0,0,z] \), \( \vec{j}, \vec{k} \),

\[
\vec{\theta} = [\theta_x, \theta_y, 0] \vec{i}, \vec{j}, \vec{k} \),
\]

\[
\vec{\rho} = \begin{bmatrix}
\vec{\rho}_{A_x} \\
\vec{\rho}_{A_y} \\
\vec{\rho}_{A_z}
\end{bmatrix} = \begin{bmatrix}
\vec{l}_1 \\
\vec{l}_2 \\
\vec{l}_3 \\
\vec{l}_4
\end{bmatrix}
\]

and in the vertical direction: \( z_A = [z_A, z_A, z_A, z_A] \),

\[
\begin{bmatrix}
\rho_{A_x} \\
\rho_{A_y} \\
\rho_{A_z}
\end{bmatrix} = \begin{bmatrix}
1 & -l_3 \\
-l_2 & 1 \\
-l_1 & -l_4
\end{bmatrix}
\]

The dynamic deformations of sprung elements are:

\[
\delta_i^s = z_i - z_A, \quad i = 1,4
\]

The dynamic components of the forces acting on the sprung mass are:

\[
F^s = \begin{bmatrix}
F_{1}^s \\
F_{2}^s \\
F_{3}^s \\
F_{4}^s
\end{bmatrix} = \begin{bmatrix}
F_{1}^{sp, dyn} + F_{1}^{dd} \\
F_{2}^{sp, dyn} + F_{2}^{dd} \\
F_{3}^{sp, dyn} + F_{3}^{dd} \\
F_{4}^{sp, dyn} + F_{4}^{dd}
\end{bmatrix}
\]

where: \( f_{p}(I,q,\dot{q}), f_{n}(I,q,\dot{q}), f_{m}(I,q,\dot{q}), f_{s}(I,q,\dot{q}) \) are the forces in controllable dampers, functionally depend from \( q, \dot{q} \) and from the controlling signal (current in this case) \( I \); \( \text{diag}(.) \) – function that denotes the diagonal matrix with the argument matrix at the main diagonal.

Their sum is:

\[
F^s = \sum_{i=1}^{4} F_i = [k\alpha_g, 0_{1x4}] \begin{bmatrix}
q \\
\dot{q}
\end{bmatrix} + 1_{4x4}U, \quad U = [f_{p}(I,q,\dot{q}), f_{n}(I,q,\dot{q}), f_{m}(I,q,\dot{q}), f_{s}(I,q,\dot{q})]^T
\]

For linear elastic and damping forces in the tires it is obtained:

\[
F^{\text{tire}} = \begin{bmatrix}
F_{1}^{\text{tire}} \\
F_{2}^{\text{tire}} \\
F_{3}^{\text{tire}} \\
F_{4}^{\text{tire}}
\end{bmatrix} = \begin{bmatrix}
F_{1}^{sp, dyn} + F_{1}^{dd} \\
F_{2}^{sp, dyn} + F_{2}^{dd} \\
F_{3}^{sp, dyn} + F_{3}^{dd} \\
F_{4}^{sp, dyn} + F_{4}^{dd}
\end{bmatrix}
\]

and the acting on the unsprung masses forces are:
The differential equations of movement in a matrix presentation are:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\vdots \\
\dot{x}_n
\end{bmatrix} = \begin{bmatrix}
F^{\text{aux}}_1 \\
F^{\text{aux}}_2 \\
\vdots \\
F^{\text{aux}}_n
\end{bmatrix} - F^{\text{free}},
\]

where:

\[
m = \text{diag}([m_{\text{usf}}, m_{\text{usr}}, m_{\text{usf}}, m_{\text{usr}}, m_s, I_c X, I_c Y]).
\]

With introducing the state and the outputs variables vectors:

\[
X = [q, \tilde{q}]^T, \quad Y = [q, \tilde{q}, -F^{\text{free}}]^T \in \mathbb{R}^{12 \times 1},
\]

it is obtained the state-space presentation:

\[
\begin{align*}
\dot{X} &= AX + B_x \Lambda + B_x U \\
Y &= CX + D_x \Lambda + D_x U,
\end{align*}
\]

where:

\[
A = \begin{bmatrix}
0_{7 \times 7} & E_7 \\
-m^{-1}k & -m^{-1}c
\end{bmatrix} \in \mathbb{R}^{14 \times 14}, \quad B_x = \begin{bmatrix} 0_{7 \times 8} \end{bmatrix} \in \mathbb{R}^{14 \times 8}, \quad B_u = \begin{bmatrix} 0_{7 \times 4} \end{bmatrix} \in \mathbb{R}^{14 \times 4},
\]

\[
C = \begin{bmatrix}
I_7, 0_{7 \times 7} \\
A_{(12,::)} + \ell_4 A_{(13,::)} + \ell_4 A_{(14,::)}
\end{bmatrix} \in \mathbb{R}^{12 \times 14}, \quad D_x = \begin{bmatrix} 0_{7 \times 8} \end{bmatrix} \in \mathbb{R}^{12 \times 8},
\]

\[
D_u = \begin{bmatrix} B_x (12,::) + \ell_4 B_x (13,::) + \ell_4 B_x (14,::) \\
0_{4 \times 4}
\end{bmatrix} \in \mathbb{R}^{12 \times 14}.
\]

### 2.3. Modelling of the kinematic excitations

In order to establish a correlation between disturbances in the right and the left track, is proposed an approach similar to this in [12, 13], and based on the summing of two uncorrelated processes, obtained by white noise filtration. The algorithm is next:

- It is formed a base signal \( \xi_b \) with spectral density \( S_{\xi_b} \) with three components \( S_{\xi_b}, S_{\xi_b} \) and \( S_{\xi_b} \) and with the next requirements for them:

\[
S_{\xi_b} (\Omega) = \text{const} = S_{\xi_b}, \quad S_{\xi_b} (\Omega) = \text{const} = S_{\xi_b}, \quad S_{\xi_b} (\Omega) = \text{const} = S_{\xi_b} \Rightarrow
\]

\[
S_{\xi_b} (\Omega) = S_{\xi_b} + \Omega^2 S_{\xi_b} + \Omega^4 S_{\xi_b},
\]

where \( \Omega \) is a spatial frequency.
Table 1. Values of the spectral density for different pavement types [14].

| Pavement’s type    | $S_x.10^3$, m$^3$ | $S_s.10^6$, m | $S_a.10^6$, m$^{-1}$ |
|--------------------|-------------------|---------------|---------------------|
| smooth asphalt     | 0                 | 0.955         | 0                   |
| asphalt            | 0                 | 1.910         | 0.027               |
| rough asphalt      | 0.04775           | 3.183         | 0.032               |
| smooth concrete    | 0                 | 0.159         | 0                   |
| concrete           | 1.59              | 3.183         | 0.040               |
| rough concrete     | 1.59              | 5.570         | 0.048               |

- The obtained signal is filtered by low- and high-pass first-order filters:
  \[
  W_p(s) = \Omega_s (\Omega_c + s)^{-1}, \quad W_{hp}(s) = s(\Omega_c + s)^{-1},
  \]
  where the cut-off frequency is $\Omega_c = \sqrt{2\pi / L}$ (for length of the wheel track $2l_1 = 1.5$ m, $L = 5$ m [15]).

- By the spectral densities of the filtered signals:
  \[
  S^{lp}(\Omega) = \Omega_s^2 \left(\Omega_c + \Omega^2\right)^{-1} S_{ss}(\Omega) \quad \text{and} \quad S^{hp}(\Omega) = \Omega^2 \left(\Omega_c + \Omega^2\right)^{-1} S_{as}(\Omega)
  \]
  are obtained these for the forming signals:
  \[
  S_{sx}(\Omega) = S_{ss}(\Omega) + S^{lp}(\Omega), \quad S_{asx}(\Omega) = S^{hp}(\Omega),
  \]
  and themselves signals $\xi_c$ and $\xi_{ap}$ by applying an inverse Fourier transform:
  \[
  \xi_c(s) = F^{-1}\{S_{sx}(\Omega)\}, \quad \xi_{ap}(s) = F^{-1}\{S_{asx}(\Omega)\}
  \]
  where with $s(t)$ is denoted the vehicle’s position on the road.

- Finally the kinematic excitations are obtained as:
  \[
  \xi_{ls}(s) = \xi_c(s) + \xi_{ap}(s) \Rightarrow \xi_{ls}(t) = \xi_c(s(t) + l_1 + l_2), \quad \xi_{rr}(t) = \xi_c(s(t) + l_1 + l_2),
  \]
  \[
  \xi_{ls}(s) = \xi_c(s) + \xi_{ap}(s) \Rightarrow \xi_{ls}(t) = \xi_c(s(t)), \quad \xi_{rr}(t) = \xi_c(s(t)).
  \]

On the figure 2 are shown the excitations under the left and right vehicle’s track.

![Figure 2. Excitations $\xi_{ls}(s)$ and $\xi_{rr}(s)$ under the left and right vehicle’s track.](image-url)
3. Model of the magneto-rheological fluid damper

The semi-active MR dampers have controlled rheology that is due to the presence in their basic fluid of microscopic polarizable particles. Under the impact of the magnetic field, arisen by the controlling current signal, those particles form chain-domains, which leads to a significant increase of the damping coefficient.

The controlling current \( I_{\text{cont}} \) in MR dampers varying in the diapason \([0, I_{\text{max}}]\), and usually \( I_{\text{max}} = 1\) A.

3.1. Linear model

The linearized model of the damper has the next presentation:

\[
F^d = \left[c^l + \left(c^u-c^l\right)I_{\text{cont}}^{-1}I_{\text{max}}\right]\left(\dot{z}_B - \dot{z}_A\right),
\]

where \( c^l \) and \( c^u \) are the minimal and maximal values of the damping coefficient.

3.2. Phenomenological model

The experimental characteristics of the MR damper are shown on figure 3. They have such nonlinearities as hysteresis, saturation and others. A nonlinear mechanical secondary model proposed in [16] based on the hysteresis presentation in [17, 18] is shown on figure 4.

The damper’s force is

\[
F^d = k_0(z_0 - z_A) + c_0(\dot{z}_0 - \dot{z}_A) - k_h\dot{\zeta} + k_0(z_B - z_A + \delta_0) = k_0(z_B - z_A + \delta_0) + c_0^*(\dot{z}_B - \dot{z}_A),
\]

where \( \delta_0 \) accounting the presence of a hydraulic accumulator.

From the balance of the forces on the \( z_0 \) is obtained:

\[
\ddot{z}_0 = \left[c_0^*\dot{z}_B + c_0\dot{z}_A - k_0(z_0 - z_A) + k_h\dot{\zeta}\right] / (c_0 + c_0^*).
\]

The evolutionary variable \( \zeta \) is determined by the dependence:

\[
\dot{\zeta} = -\gamma |\dot{z}_0 - \dot{z}_A| |\zeta|^{\gamma - 1} + \beta(\dot{z}_0 - \dot{z}_A)|\zeta|^{\gamma} - \nu(\dot{z}_0 - \dot{z}_A).
\]

The variables \( k_h, c_0 \) and \( c_0^* \), in first approximation, are presented by the linear dependences:

\[
k_h = k_h^* + k_h^\prime I, \quad c_0 = c_0^c + c_0^\prime I, \quad c_0^* = c_0^{c*} + c_0^{\prime*} I, \quad I = (I - I') / \tau_r.
\]

The obtained values of the model’s parameters for MR damper RD-1005-3 of “Lord Corporation” Ltd USA [19] are given in table 2. These values are averaged from those identified from a set of experiments at different parameters of the excitation varying in the ranges: frequency \([0,1+15]\) Hz, control current \([0+1]\) A, and amplitude of \([0,5+4]\) mm, by processing of the experimental data. The identification is based on an optimization procedure.
Table 2. Parameter’s values of the MR damper model RD1005-3 of Lord Corporation Ltd.

| Parameter | Value |
|-----------|-------|
| $k_h$     | 14500 N/m |
| $k_a$     | 71500 N/m |
| $k_0$     | 1400 N/m |
| $c_v$     | 780 Ns/m |
| $c_i$     | 2230 Ns/m |
| $k'_h$    | 540 N/m |
| $c'_v$    | 28000 Ns/m |
| $c'_i$    | 500 Ns/m |
| $\chi$    | 2.5 |
| $\tau$    | 5.3 $10^{-3}$ s |
| $\gamma$  | 0.5 $10^6$ m^{-2} |
| $\beta$   | 0.8 $10^6$ m^{-2} |
| $\nu$     | 170 |

The vector of the identification parameters is

$$\Omega = [k_h, k_a, k_0, c_v, c_i, k'_h, c'_v, c'_i, \chi, \tau, \gamma, \beta, \nu].$$  \hspace{2cm} (25)

If $F^d(t_i), \delta(t_i), \dot{\delta}(t_i)$ are the discrete in time values respectively of the damper force, deformation and its velocity, the identification of $\Omega$ is reduced to the least-squares optimization problem:

$$\min_{\Omega \in \Theta} \left\{ J(\Omega) = \sum_{i=1}^{n} \left[ F^d(t_i) - \hat{F}^d(t_i) \right]^2 \right\},$$  \hspace{2cm} (26)

where $t_i$ is a discrete time and $\hat{F}^d(t_i)$ is the damper’s force calculated by the relation (21).

The solution to the task is difficult due to the large size of the vector $\Omega$. Therefore, a Monte Carlo optimization method is used, based on the generation of pseudo-random solutions (points) evenly distributed in the hypercube $\Theta$ based on the Sobolev’s Lp Tau-sequences [20]. The best results obtained are for the values shown in table 2.

3.3. Neural networks (NN) model

From the experimental investigation becomes clear that the upper values are not constant and varying with the amplitude, frequency of the vibrations and with the level of the control signal. For this reason, the use of a model based on neural networks for the approximation of the experimental data is very successfully. It also may be used and as an approximation of the analytical model of the damper.

The structure of the used neural model is shown on figure 5. The Input layer is formed by the values: of the damper’s deformation and its first and second derivatives $d(i), \dot{d}(i), \ddot{d}(i)$, for the current discrete-value of the time, the control current $I_{\text{cont}}(i)$ and the damper’s force $F^d(i-1)$ in the previous step.

The hidden layer consist of 14 neurons with non-linear logical sigmoid activation functions $g^{(i)}(x) = 2(1 + e^{-2x})^{-1} - 1$.

The Output layer defines the damper force $F^d(i)$ for the current step. The index $i$ is related to the discrete time moment $t_i = i\Delta t, i = 0, 1, \ldots$, where $\Delta t$ is the sampling period. This layer is with a linear activation functions $g^{(2)}(x) = x$.

For the training of the artificial neural network the “Back-propagation” algorithm is used. For the better results is used the data normalization with regard to their minimal and maximal values.

The training is going through in 300 epochs. From the training dataset 60% are used for the training, and 40% for the verification of the trained artificial neural network. The obtained results are given on figure 6.
3.4. Inverse model of MR damper.

For the purposes of the control, it is needed an inverse damper’s model.

3.4.1. Linear model. Using the presentation (20) of the linearized model, the inverse model has the form:

$$
\tilde{I} = \frac{I_{\text{max}}}{(c' - c)} \left[ 1 - \left( \frac{c}{c'} \right) F^d - c' \right], \quad I_{\text{cont}} = \begin{cases} 
\tilde{I}, & \text{za } \tilde{I} \in [0, I_{\text{max}}] \\
0, & \text{za } \tilde{I} < 0 \\
I_{\text{max}}, & \text{za } \tilde{I} > I_{\text{max}} 
\end{cases}
$$

(28)

Figure 5. Neural model of MR Damper.

Figure 6. (a) Results the obtained NN model of MR damper; (b) The error in comparison with the experimental results.
3.4.2. Nonlinear model. The inversion of the phenomenological model is rather complicated and in most cases is in a contradiction with the condition for real-time control. For this reason, it is more felicitous the using on inverse NN model. Its structure is similar to the forward model – figure 7. The experimental control voltage and this calculated from the obtained inverse model are shown on figure 8. It can be seen from figure 8 that a very good approximation of the experimental data has been obtained. Some of the training data are shown in figure 9.

Figure 7. Inverse NN model of MR Damper.

Figure 8. The control voltage and this calculated from the obtained inverse model.

Figure 9. The training data.
4. Conclusions
The paper studies a spatial model of the vehicle’s suspension. The model of the kinematic excitations from the unevenness of the road pavement is described in detail. Correlation between the disturbances under the left and right wheel-trucks, as well as the time delay between the front and rear axles of the vehicle, are taken into account. The linearized, phenomenological and neural network (NN) forward and inverse models of the magneto-rheological damper used in the vehicle’s semi-active suspension are discussed. Appropriately selected neural structure and the learning variables allow the obtained artificial neuronal model to approximate the behaviour of the MP damper to a very good extent. For the purpose of the control is obtained and its inverse NN model.

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