Data Article

Estimation of upper and lower bounds of Gini coefficient by fuzzy data

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ABSTRACT

The data presented in this paper are used to examine the uncertainty in macroeconomic variables and their impact on the Gini coefficient. Annual data for the period 2017 - 1996 are taken from the Bank of Iran website https://www.cbi.ir. We used fuzzy regression with symmetric coefficients to calculate upper and lower bound data of Gini coefficient. Estimated data at this stage can be a very useful guide for policymakers, on the other hand, it is a benchmark for evaluating the effectiveness of government policies. The reason for using fuzzy regression to estimate data on Gini coefficients is the extra flexibility of this model.

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1. Data description

The Gini coefficient is one of the most important indicators introduced by Corrado Gini to measure inequality, which is always nonnegative and has values between zero and one [1,2]. A combination of

![Fig. 1. Graph inflation.](image)

![Fig. 2. Graph Real exchange rates.](image)
Fig. 3. Graph stock price index.

Fig. 4. Graph GDP.

Fig. 5. Graph bank interest rate.

Fig. 6. Graph Gini coefficient.
macroeconomic variables including GDP, bank interest rates, inflation rate, exchange rate and stock price index, an index called economic policy uncertainty (EPU) is introduced [1]. For estimate the upper and lower bound data, the Gini coefficient of EPU index data from 1991 to 2018 is taken from the Bank of Iran website https://www.cbi.ir. Also, all data is available in the Mendeley (https://data.mendeley.com/datasets/nr8cptwgf8/draft?aid=40f49de2-34d5-4bb1-bc98-5c59c5f02163). As can be seen in Fig. 1, inflation has been rising over time. Although it has been slow in some years, the trend of inflation is generally rising. Fig. 2 shows the trend of the real exchange rate fluctuation in different years. Figs. 3 and 4, respectively, show that GDP and stock price index have fluctuated over time, although they have fluctuated in some years. Fig. 5 also shows the fluctuating movement of bank interest rates. Fig. 6 shows the trend of the Gini coefficient of movement in different years. The fluctuations of this variable over time indicate that the instability of economic variables that will have a major impact on the Gini coefficient, depending on government policies.

2. Experimental design, materials, and method

In this section, the theoretical foundations of the regression model with symmetric and asymmetric fuzzy coefficients are briefly stated [3–5], where the general form of the regression model with fuzzy coefficients is as in Eq. (1) in order to compare the results of logistic smooth transition autoregressive model to the linear regression model with symmetric and asymmetric fuzzy coefficients.

\[ \tilde{Y} = f(x, A) = \tilde{A}_0 + \tilde{A}_1x_1 + \tilde{A}_2x_2 + \ldots + \tilde{A}_nx_n \]  

(1)

here \( \tilde{Y} \) is the dependent variable or fuzzy output, \( x = (x_1, x_2, \ldots, x_n) \) is the vector of independent variables or input vector, \( A = \{\tilde{A}_0, \tilde{A}_1, \ldots, \tilde{A}_n\} \) is a set of fuzzy numbers, and \( (y_1, x_1), (y_2, x_2), \ldots, (y_m, x_m) \) are the a set of regular data. The fuzzy parameters \( \tilde{A}_0, \tilde{A}_1, \ldots, \tilde{A}_n \) are meant to be determined in a way to

| Year | h = 0.1 | Left width | Right width | h = 0.2 | Left width | Right width |
|------|---------|------------|-------------|---------|------------|-------------|
| 1991 | 0       | 0.3996     | 0.3996      | 0       | 0.3996     | 0.3996      |
| 1992 | 0.016199| 0.370801   | 0.403199    | 0.018545| 0.368455   | 0.405545    |
| 1993 | 0.017828| 0.379772   | 0.415428    | 0.020438| 0.377162   | 0.418038    |
| 1994 | 0.015057| 0.384243   | 0.414357    | 0.01727 | 0.38203    | 0.41657     |
| 1995 | 0.017038| 0.390362   | 0.424438    | 0.019613| 0.387787   | 0.427013    |
| 1996 | 0.01587 | 0.37513    | 0.40687     | 0.018276| 0.372724   | 0.409276    |
| 1997 | 0.015122| 0.387778   | 0.418022    | 0.01742 | 0.38548    | 0.42032     |
| 1998 | 0.014368| 0.382132   | 0.410686    | 0.016556| 0.379944   | 0.413056    |
| 1999 | 0.013813| 0.387087   | 0.414713    | 0.015918| 0.384982   | 0.416818    |
| 2000 | 0.013698| 0.385402   | 0.412798    | 0.015782| 0.383318   | 0.414882    |
| 2001 | 0.010293| 0.388207   | 0.408793    | 0.011837| 0.386663   | 0.410337    |
| 2002 | 0.021313| 0.397778   | 0.440413    | 0.024422| 0.394678   | 0.443522    |
| 2003 | 0.020558| 0.395042   | 0.436158    | 0.023551| 0.392049   | 0.439151    |
| 2004 | 0.0203  | 0.3793     | 0.4199      | 0.023244| 0.376356   | 0.422844    |
| 2005 | 0.020616| 0.381684   | 0.422916    | 0.023589| 0.378711   | 0.425889    |
| 2006 | 0.018405| 0.381995   | 0.418805    | 0.021013| 0.379387   | 0.421413    |
| 2007 | 0.018075| 0.386425   | 0.422575    | 0.020614| 0.383886   | 0.425114    |
| 2008 | 0.024032| 0.361868   | 0.409932    | 0.027499| 0.358401   | 0.413399    |
| 2009 | 0.02052 | 0.37338    | 0.41442     | 0.023429| 0.370471   | 0.417329    |
| 2010 | 0.020241| 0.361059   | 0.401541    | 0.023065| 0.358235   | 0.404365    |
| 2011 | 0.027184| 0.347816   | 0.402184    | 0.031072| 0.343928   | 0.406072    |
| 2012 | 0.026864| 0.354716   | 0.412804    | 0.032733| 0.356067   | 0.416133    |
| 2013 | 0.023004| 0.363196   | 0.427204    | 0.036492| 0.358708   | 0.431692    |
| 2014 | 0.039243| 0.360657   | 0.439143    | 0.044699| 0.355201   | 0.445499    |
| 2015 | 0.043798| 0.355002   | 0.425938    | 0.049882| 0.348918   | 0.448682    |
| 2016 | 0.043458| 0.361142   | 0.448058    | 0.049499| 0.355101   | 0.454099    |
| 2017 | 0.048016| 0.352784   | 0.448816    | 0.054671| 0.346129   | 0.455471    |
| 2018 | 0.050172| 0.346828   | 0.447172    | 0.05703 | 0.33997    | 0.45403     |
have the best fitting on the above data in accordance with Eq. (1) based on some of the goodness-of-fit criteria. So, the general form of the membership function \( \tilde{A} \) can be written as Eq. (2) with respect to three parameters as the center \( a \), low width \( s^t \) and right width \( s^R \) [3]:

\[
\tilde{A}(x) = \begin{cases} 
1 \frac{a-x}{s^t} & a - s^t \leq x \leq a \\
1 \frac{x-a}{s^R} & a < x \leq a + s^R
\end{cases}
\]

This membership function can also be displayed in another way. That is, the high width is expressed based on a low width. Thus, \( k s^t = s^R \) is placed in the above membership function, in which \( k \) is a real and positive number known as the kurtosis coefficients [3]. Therefore, the asymmetric triangular fuzzy number \( \tilde{A} \) can also be described by \( \tilde{A} = (a, s^t, k) \). In this case, the membership function \( \tilde{A} \) is represented by Eq. (3)

\[
\tilde{A}(x) = \begin{cases} 
1 \frac{a-x}{s^t} & a - s^t \leq x \leq a \\
1 \frac{x-a}{ks^R} & a < x \leq a + ks^R
\end{cases}
\]

### 2.1. Model with symmetric coefficients

If \( \tilde{A}_i \) are fuzzy symmetric numbers \( i = 0, 1, 2, \ldots, n \), and \( x_i \) s are the real and positive numbers, then, according to Eq. (1), the fuzzy output \( \tilde{Y} \) will be a symmetric triangular fuzzy number.
\( \bar{Y} = (f_c(x), f_s(x)) \), in which \( f_c(x) \) is the center and \( f_s(x) \) is the width of \( \bar{Y} \), which are obtained by Eq. (4) and Eq. (5) as below [5].

\[
f_c(x) = a_0 + a_1 x_1 + \ldots + a_n x_n \quad (4)
\]

\[
f_s(x) = s_0 + s_1 x_1 + \ldots + s_n x_n \quad (5)
\]

In other words, the fuzzy output \( \bar{Y} \) membership function is:

\[
\bar{Y}(y) = \begin{cases} 
1 & -f_c(x) < y \leq f_c(x) \\
1 - \frac{f_c(x) - y}{f_c(x)} & f_c(x) - f_s(x) \leq y \leq f_c(x) \\
1 - \frac{y - f_c(x)}{f_s(x)} & f_c(x) < y \leq f_c(x) + f_s(x) 
\end{cases} \quad (6)
\]

### 2.2. Estimating the parameters of fuzzy regression model

For estimate the parameters of the fuzzy regression model Eq. (2), We consider two criterias. First, the membership value of each \( y_i \) in \( \bar{Y_i} \) should be a large number. In this case, it is ensured that the fuzzy model has a good fitting to the observations [3]. Thus, we are looking for a model that: 1) fuzzy output, \( \bar{Y} \) for all values \( Y_j \), has a membership degree as large as \( h \), that is:

| Year | h = 0.5 Width | Left width | Right width | h = 0.6 Width | Left width | Right width |
|------|----------------|------------|-------------|----------------|------------|-------------|
| 1991 | 0              | 0.3996     | 0.3996      | 0              | 0.3996     | 0.3996      |
| 1992 | 0.264048       | 0.122952   | 0.651048    | 0.036974       | 0.350026   | 0.423974    |
| 1993 | 0.25697        | 0.14063    | 0.65457     | 0.040645       | 0.356955   | 0.438245    |
| 1994 | 0.204595       | 0.194705   | 0.603895    | 0.034305       | 0.364995   | 0.433605    |
| 1995 | 0.148964       | 0.258436   | 0.556364    | 0.038702       | 0.368698   | 0.446102    |
| 1996 | 0.125547       | 0.265453   | 0.516547    | 0.036025       | 0.354975   | 0.427025    |
| 1997 | 0.110446       | 0.292454   | 0.513346    | 0.03431        | 0.36859   | 0.43721     |
| 1998 | 0.095386       | 0.301114   | 0.491886    | 0.03258        | 0.36392   | 0.42908     |
| 1999 | 0.082064       | 0.318836   | 0.482964    | 0.031296       | 0.369604   | 0.432196    |
| 2000 | 0.075887       | 0.323213   | 0.474987    | 0.031013       | 0.368087   | 0.430113    |
| 2001 | 0.064619       | 0.333881   | 0.463119    | 0.023289       | 0.375211   | 0.421789    |
| 2002 | 0.262183       | 0.156917   | 0.681283    | 0.048442       | 0.370658   | 0.467542    |
| 2003 | 0.242176       | 0.173424   | 0.657776    | 0.046686       | 0.369814   | 0.462286    |
| 2004 | 0.227954       | 0.171646   | 0.627554    | 0.04605        | 0.35355    | 0.44565     |
| 2005 | 0.222447       | 0.179853   | 0.624747    | 0.046713       | 0.355587   | 0.449013    |
| 2006 | 0.206512       | 0.194248   | 0.606552    | 0.041649       | 0.358751   | 0.442049    |
| 2007 | 0.182905       | 0.221595   | 0.587405    | 0.040808       | 0.363692   | 0.445308    |
| 2008 | 0.169834       | 0.216066   | 0.555734    | 0.054201       | 0.331699   | 0.440101    |
| 2009 | 0.153875       | 0.240025   | 0.547775    | 0.046221       | 0.347679   | 0.440121    |
| 2010 | 0.145722       | 0.235578   | 0.527022    | 0.045499       | 0.353801   | 0.426799    |
| 2011 | 0.146151       | 0.228849   | 0.521151    | 0.061123       | 0.313877   | 0.436123    |
| 2012 | 0.134874       | 0.248526   | 0.518274    | 0.064351       | 0.319049   | 0.447751    |
| 2013 | 0.170356       | 0.224844   | 0.565556    | 0.071808       | 0.323392   | 0.467008    |
| 2014 | 0.195609       | 0.204291   | 0.595509    | 0.087934       | 0.311966   | 0.487834    |
| 2015 | 0.203503       | 0.195297   | 0.602303    | 0.09809        | 0.30071    | 0.49689     |
| 2016 | 0.200922       | 0.203678   | 0.605522    | 0.097332       | 0.307268   | 0.501932    |
| 2017 | 0.212941       | 0.187859   | 0.613741    | 0.107483       | 0.293317   | 0.508283    |
| 2018 | 0.220873       | 0.176127   | 0.617873    | 0.112145       | 0.284855   | 0.509145    |
2) The fuzzy coefficients \( \tilde{A}_i \) are such that ambiguity of the fuzzy output \( \tilde{Y}_i \) is minimized.

In the case where \( \tilde{A}_i \)s are symmetric, the total fuzzy output width \( \tilde{Y} \) for all data is called the objective function. The Objective function for the case where \( \tilde{A}_i \)s are asymmetric is represented by Eq. (6).

\[
Z = 2m s_0 + 2 \sum_{i=1}^{n} \left( s_i \sum_{j=1}^{m} x_{ij} \right)
\]  

(8)

In Eq. (8), \( x_{ij} \) means the jth observation of the ith variable. In the case where \( \tilde{A}_i \)s are asymmetric, objective function [3] is represented by Eq. (9) [3].

\[
Z = m \left( s_0 + s_0^R \right) + \sum_{i=1}^{n} \left( \left( s_i^L + s_i^R \right) \sum_{j=1}^{m} x_{ij} \right)
\]  

(9)

The objective function \( Z \), by substituting \( k_i s_i^L = s_i^F \), can be written based on kurtosis coefficients as follows.
### Table 5
Calculation of the right and left widths of the Gini coefficient for membership degrees of 0.9.

| Year | Left width | Right width |
|------|------------|-------------|
| 1991 | 0.3996     | 0.3996      |
| 1992 | 0.237874   | 0.536126    |
| 1993 | 0.233604   | 0.561596    |
| 1994 | 0.260862   | 0.537738    |
| 1995 | 0.25105    | 0.56375     |
| 1996 | 0.245448   | 0.536552    |
| 1997 | 0.264263   | 0.541537    |
| 1998 | 0.264839   | 0.528161    |
| 1999 | 0.27442    | 0.52738     |
| 2000 | 0.273767   | 0.524433    |
| 2001 | 0.30442    | 0.49258     |
| 2002 | 0.223635   | 0.614565    |
| 2003 | 0.227224   | 0.603976    |
| 2004 | 0.213805   | 0.585395    |
| 2005 | 0.213853   | 0.590747    |
| 2006 | 0.232462   | 0.568338    |
| 2007 | 0.23998    | 0.56902     |
| 2008 | 0.167188   | 0.604612    |
| 2009 | 0.207481   | 0.580319    |
| 2010 | 0.197863   | 0.564737    |
| 2011 | 0.128388   | 0.621612    |
| 2012 | 0.123841   | 0.642959    |
| 2013 | 0.105626   | 0.684774    |
| 2014 | 0.045367   | 0.754433    |
| 2015 | 0.00332    | 0.79428     |
| 2016 | 0.01217    | 0.79703     |
| 2017 | 0.032528   | 0.834128    |
| 2018 | 0.054965   | 0.848965    |

\[
Z = (1 + K_0)m\sigma_0^4 + \sum_{i=1}^{n} \left[ \left( 1 + K_0 \right) s_i^4 \sum_{j=1}^{m} x_{ij} \right]
\]  
\[ (10) \]
2.3. FLR Model with symmetric coefficients

If $\tilde{A}_i$ in Eq. (2) are considered as symmetric with regard to Eq. (7) and Eq. (9) then the constraints Eq. (11) and Eq. (12) are obtained [3] as follows:

**Fig. 8.** Right and left widths Gini coefficient for degree of membership 0.2.

**Fig. 9.** Right and left widths Gini coefficient for degree of membership 0.3.

**Fig. 10.** Right and left widths Gini coefficient for degree of membership 0.4.
Fig. 11. Right and left widths Gini coefficient for degree of membership 0.5.

Fig. 12. Right and left widths Gini coefficient for degree of membership 0.6.

Fig. 13. Right and left widths Gini coefficient for degree of membership 0.7.
After describing the method of calculating the upper and lower bound data Gini coefficient, we have programmed the data of variables related to the EPU attribute using fuzzy regression using GAMS software and calculated the optimal probabilities for the relevant variables. Next, we calculate the upper and lower bound data of the Gini coefficient for the membership degree of 0.1–0.9. The data are presented in Tables 1–5. Figs. 7–15.

Ethics approval

Ethics approval is not applicable.
Consent for publication

The authors of the research have given their consent for the data to be used and published in this scientific article.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] A.B. Atkinson, On the measurement of inequality, J. Econ. Theor. 2 (3) (2015) 244–263.
[2] C. Gini, Variabilita’ e Mutabilita, Studio Economic Ogiuridici, Universita di Cagliari Anno III, Parte 2a, Reprinted in C, 1912, pp. 211–382.
[3] H. Tanaka, S. Uejima, K. Asai, Linear Regression Analysis with Fuzzy Institution, 1982.
[4] M.O. Lorenz, Methods of measuring the concentration of wealth, J. Am. Stat. Assoc. 9 (1905) 209–219.
[5] L.A. Zadeh, Fuzzy sets, Inf. Control 8 (3) (1965) 338–353.