Calculation of Moments of Nucleon Structure Functions

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Preliminary results are presented in our program to calculate low moments of structure functions for the proton and neutron on a $24^3 \times 32$ lattice at $\beta = 6.2$. A comparison is made for a variety of smeared nucleon sources and preliminary results for the calculation of the nucleon tensor charge are presented.

1. INTRODUCTION

There are now extensive experimental results on nucleon structure functions in deep inelastic scattering, providing a detailed empirical knowledge of the distribution of quarks and gluons in the nucleon. Thus a major theoretical challenge for lattice QCD is to understand this data from first principles and assess errors in current methods due to the quenched approximation and the finite lattice spacing in naive lattice operators. We envision a gradual improvement of lattice methodology to reduce these errors. Since structure functions cannot be calculated directly on a Euclidean lattice, we consider low moments which are related to matrix elements of local operators by the operator product expansion. The lattice matrix elements relevant to moments of $F_1$, $g_1$ and $h_1$ structure functions are being calculated on quenched $24^3 \times 32$ lattices at $\beta = 6.2$ at three quark masses, and initial results are presented below.

2. GENERAL FORMALISM ON THE LATTICE

In the continuum, the quark contribution to the moments of the unpolarized $F_1$, longitudinally polarized $g_1$ and transversely polarized $h_1$ structure functions are related to the matrix elements,

$$\sum_s \langle ps | \bar{\psi} \gamma_{\mu_1} D_{\mu_2} \ldots D_{\mu_n} \psi | ps \rangle \sim \bar{v}_n(\mu),$$

$$\langle ps | \bar{\psi} \gamma_{\nu} \gamma_5 D_{\mu_1} \ldots D_{\mu_n} \psi | ps \rangle \sim \bar{a}_n(\mu),$$

$$\langle ps | \bar{\psi} \sigma_{\nu\mu_1} D_{\mu_2} \ldots D_{\mu_n} \psi | ps \rangle \sim \bar{t}_n(\mu),$$

respectively, where we have indicated on the right the scalar coefficient in the notation of Refs. [1]. Our methodology for calculating these operators on the lattice is similar to Göckeler et al. [2]. The covariant derivative is replaced by a lattice finite difference and the operators are classified by representations of the surviving discrete subgroup $H_4$ of the continuum Euclidean Lorentz group $SO(4)$ [3]. The renormalization factors $Z_G$ are calculated using a new method [2] to one loop order in lattice perturbation series to account for a multiplicative shift of the Wilson coefficient due replacing the $\overline{MS}$ scheme at scale $\mu$ by the lattice spacing $a$. Our independently derived results confirm those in the literature [2,4] and extend them where necessary. For our current set of moments the results are given in the table.

Typical elements of an irreducible representation (IR) of $H_4$ are given in the second column, where the $\{ \cdots \}$ and $[ \cdots ]$ brackets imply symmetrized and anti-symmetries indices respectively. In many cases there is more than one IR corresponding to the continuum operator. We are exploring other representations to have a redundant set of measurements. The renormalization constants defined by the formula,

$$Z_G = 1 - \frac{g_0^2}{16\pi^2} C_F [\gamma_G \ln(\mu a) + B_G]$$

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are given in the third and the fourth column.

| Moment | $\gamma_G$ | $B_G$ |
|--------|-----------|-------|
| $v_2,a$ | $O_{(14)}$ | 16/3 | -3.160 |
| $v_3,a$ | $O_{(124)}$ | 25/3 | -19.012 |
| $v_4,b$ | $O_{(234)}$ | 157/15 | -33.206 |

| Spin-dependent |
|----------------|
| $a_0$ | $O_1^5$ | 8/3 | 4.094 |
| $a_1$ | $O_{(14)}^5$ | 25/3 | -19.562 |
| $a_2$ | $O_{(214)}^5$ | 157/15 | -33.582 |

| Tensor charge |
|---------------|
| $t_1$ | $O_{[24]}^5 - O_{[13]}^5$ | 2 | 16.237 |

3. PRELIMINARY RESULTS

On the CM-5 at MIT, we have generated 150 independent $24^3 \times 32$ lattices using the standard Wilson action. Dirac propagators for Wilson fermions with $r = 1$ are being calculated by conjugate gradient iterations with red-black preconditioning for the values of $\kappa = 0.15200, 0.15246, 0.15294$ corresponding to $m_q \approx 150, 98$ and 45 MeV. Our estimates for $\kappa_c$ and the lattice spacing derived from $\pi$ and $\rho$ masses are in a good agreement with those of UKQCD.

3.1. Hadronic Sources.

To compute the matrix elements we considered four types of sources: point sources (P), gauge invariant Wuppertal (W) sources (W) and both Coulomb gauge fixed Wuppertal (U) and Gaussian (G) smeared sources. To determine the most suitable form of the source, we investigated the plateau in $\ln(G(t)/G(t+1))$ for the two point functions for the pion, rho and nucleon sources. For the proton creation operator, we used $J_\mu(x) = \epsilon_{abc} u_\mu^a(x) u_\alpha(x) \Gamma^{\alpha\beta} d_\beta^c(x)$, with $\Gamma = C\gamma_5$ and with all quark operators truncated to 2 upper components.

Some comparisons are shown for the nucleon case in Fig. 1. The Gaussian (G) and the two Wuppertal (W & U) sources were adjusted so that $\sqrt{\langle x^2 \rangle} \approx 6.7a$ for each quark field, since this smearing produced the least noisy results in all three cases. For more localized sources the excited states are more prominent, whereas for less localized sources the signal becomes noisier at large distances.

As seen in Fig. 1, smearing both the sources and the sink results in substantially noisier behavior than smearing only the source. On the scale of the errors in Fig. 1, there is no significant difference between smeared source–point sink vs. point source–smeared sink. It is interesting to note that the gauge fixed Wuppertal (D) and gauge invariant Wuppertal (W) sources are essentially equivalent.

3.2. The Tensor Charge

We illustrate our preliminary results with a first (low statistics) measurement of the tensor charge $t_1$. (Also see results by Aoki et al.) To date we have analyzed 15, 10, 5 configurations with $p = (0,0,0)$ for $\kappa = 0.15200, 0.15246, 0.15294$ respectively and

\[
\frac{1}{p^2} \approx 6.7a
\]

for each quark field.
20, 15, 15 configurations at $p = (1, 1, 0)$ for the same kappa values. The Gaussian smeared source is placed at $t_0 = 8$ and the momentum projected point sink at $t_1 = 24$.

We measure three point functions in the following way. Once a gauge field is generated, it is gauge fixed to the Coulomb gauge and a set of quark propagators is computed. Then for every operator the corresponding 3-point function is constructed from the set of propagators. We calculate separately the 2-point function, comparing backward and forward propagators independently to verify convergence. Finally, we use the jackknife method to estimate statistical errors of the ratio of three- and two-point functions. Since we need to fix the final state momentum of the nucleon before calculating the backward propagators, the set of propagators has a built-in final state momentum.

The first moment $t_1$ of the transversity distribution $h_1(x)$ can be measured using the final state at rest $p = (0, 0, 0)$. In Fig. 2 the three upper plots show preliminary results for $t_1$ at the three kappas we used. Even for the low statistics (15 configurations for the smallest $\kappa$), the plateau in the signal is clearly visible. One can also see that as $\kappa$ approaches the chiral limit, the signal becomes noisier as expected.

Since other moments require non-zero momentum at the sink, it might be useful to consider a single set of propagators with non-zero momenta to measure all observables. As a comparison, the lowest plot in Fig. 2D gives the same tensor charge computed at $p = (1, 1, 0)$. To make this a fair comparison with the $p = (0, 0, 0)$ case, we have used the same set of configurations with propagators calculated to the same precision. It appears to have considerably stronger fluctuations, thus raising the question as to whether the exclusive use of non-zero momenta is an optimal strategy.

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