Earliest arrival dynamic flow model for emergency evacuation in fuzzy conditions

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Abstract. Tasks of emergency evacuation planning on dynamic networks in fuzzy conditions are becoming ubiquitous because of the imprecise time-dependent nature and arguments of network. Underlying basis for evacuation tasks are different types of flow algorithms. The paper aims to present the urgent problem of artificial intelligence, namely, the development of a method for finding the optimal evacuation plan in a transportation network in fuzzy conditions as the universal flow problem. The proposed method consists in transporting the maximum number of aggrieved in each time interval based on finding the maximum flow of the earliest arrival in a fuzzy dynamic transportation network. The process of flow conveying is implemented on the network with time-dependent parameters. The main contribution of this paper is to propose a method operating arc capacities and traversal times that can vary depending on the flow departure time and are represented in a fuzzy form. Technique of blurring fuzzy numbers is represented to facilitate calculations. A case-study in the area of Bolshoi Theatre Moscow, Russia and surroundings is conducted to illustrate proposed algorithm. Experiments on the designed network were conducted to check resistance of the network to topology changes.

1. Introduction

Search for solutions in conditions of incompleteness and inaccuracy of information is one of the major issues of artificial intelligence. The relevance of this class of problems is due to the complexity of the simulated systems, their complicated multi-link structure, the necessity to take into account human reasoning in terms of incompleteness and fuzziness of the initial information. In practice, the solution of such problem involves handling transportation networks with assigned arc capacities and traversal time parameters as networks conveying flows, and posing the flow problem statements on such networks. An urgent task in this regard is the task of evacuating the maximum number of aggrieved in each time phase within a given period of time. This type of evacuation is the safest one, as it guarantees the rescue of the maximum number of aggrieved in each period of time. The survey of approaches to the earliest arrival flow problem is presented in [1]. Proposed approaches are based on flow problem statements to solve evacuation task. Recent research tackles emergency evacuation task within a framework of modelling and computer simulation [2]. Lia at al. considered emergency evacuation problem of minimizing total travel time of evacuation [3]. Pyakurel et al. [4] implemented method of
designing contraflow configuration by reversing only necessary arc capacities to solve the earliest arrival contraflow problem, however, the modeled network possessed constant traversal times.

In real-world problems parameters assigned to networks are usually endowed with fuzziness [5]. However, vague nature of networks wasn’t considered in the studies. Our research is guided by related studies in theory of fuzzy sets [6-9], leading to the parameters in the form of fuzzy numbers. In the real-time evacuation scenarios, it is important to reckon in variations in arc capacities at different time periods depending on the changes in real-time situation, for instance, repairs on the roads, traffic jams, installation of new signs and traffic lights. Recently, development and extensions of fuzzy sets, for instance, intuitionistic and hesitant fuzzy sets induced novel approaches to emergency hesitant fuzzy decision making [10].

Inspired by above ideas, the paper proposes a method of evacuating the maximum number of agrieved in each time phase within a given period of time as a method of the maximum earliest arrival dynamic flow finding.

The rest of the paper is organized as follows. In Section 2, we observe preliminaries and problem statement. Proposed algorithm of the earliest arrival evacuation in a fuzzy dynamic network with underlying procedures is given in the Section 3. Section 4 observes the network of the area of Bolshoi Theatre and surroundings, Moscow, Russia as the case study for evacuation modelling and conducted experiments to check the resistance of the network to changes in its topology. Finally, Section 5 concludes the paper.

2. Preliminaries and problem statement

A network is considered to be a connected finite directed graph with allotted initial and terminal nodes – a source and a sink. The source node doesn’t possess any input flow, conversely, the sink node doesn’t hold any output flow. Let \( \tilde{G} = (\tilde{X}, \tilde{A}) \) be a fuzzy dynamic network, where \( \tilde{X} \) is a crisp set of nodes, \( \tilde{A} \) is a fuzzy set of arcs. Each fuzzy arc in the dynamic network has two assigned values: fuzzy transit arc capacity \( \tilde{u}_{ij}(\theta) \) and traversal time \( \tilde{\tau}_{ij}(\theta) \). In contrast to conventional networks, parameters of dynamic networks can vary depending on the flow departure time. Initially, the time frame \( T \) divided into \( p \) discrete time periods is given, \( T = \{0,1,...,p\} \).

The earliest arrival maximum flow problem (universal maximum flow problem) has multiple sources, which unity is denoted by \( S \) and the super-source node \( s' \) that integrates all sources. By analogy, our model contains multiple sinks, which union is denoted by \( R \) and the super-sink node \( r' \).

Non-negative fuzzy time-dependent transit arc capacities \( \tilde{u}_{ij}(\theta) \) [7] are assigned to each arc of the network. For arcs \( \forall (x_i, x_j) \in \tilde{A} \) and time periods \( \forall T' \in T \) the value \( \tilde{u}_{ij}(\theta) \) stands for capacity of the corresponding arc at the departure time \( \theta \). Time-varying capacity restricts the number of evacuees transporting along the arc \( (x_i, x_j) \) at the departure time \( \theta \). Due to the particular pattern of emergency, arc capacities are influenced by real traffic situation, that assumes traffic congestions or flow conflict of rescue and emergency flows at different departure time. Representing time-dependent arcs enables researcher to model real-time evacuation. Fuzzy nature of arc capacities is caused by the lack of relevant information about its value, including the impact of environment and emergency factors.

Traversal time parameters \( \tilde{\tau}_{ij}(\theta) \) are influenced by the flow departure time, pattern of emergency and other factors. As the flow leaves the node \( x_i \) at the time \( \theta \), and the traversal time for the arc is \( \tilde{\tau}_{ij}(\theta) \), then it reaches the node \( x_j \) at the time \( \theta = \theta + \tilde{\tau}_{ij}(\theta) \), as shown in figure 1.

![Figure 1. Traversal time \( \tilde{\tau}_{ij}(\theta) \) along the arc \( (x_i, x_j) \).](image-url)
Let us represent a model of the universal maximum dynamic flow problem (1-5) in a fuzzy network. Note that all parameters of the network vary due to the specific time period $\theta$ of the flow departure.

\[
\text{Maximize } \sum_{\theta=0}^{\tau'} \sum_{i \in E} \xi_{i\theta}(\theta), \quad \forall \tau' = 1, \ldots, T,
\]

Subject to:

\[
\sum_{\theta=0}^{\tau'} (\sum_{j \in E(x)} \xi_{ij}(\theta) - \sum_{j \in E^{-1}(x)} \xi_{ji}(\theta - \tau_{ij}(\theta))) = 0, \quad x_i \neq s', r', \theta \in T, \quad (2)
\]

\[
\sum_{\theta=0}^{\tau'} (\sum_{j \in E(x)} \xi_{ij}(\theta) - \sum_{j \in E^{-1}(x)} \xi_{ji}(\theta - \tau_{ij}(\theta))) = \sum_{\theta=0}^{\tau'} \sum_{i \in E} \xi_{i\theta}(\theta), x_i = s',
\]

\[
\sum_{\theta=0}^{\tau'} (\sum_{j \in E(x)} \xi_{ij}(\theta) - \sum_{j \in E^{-1}(x)} \xi_{ji}(\theta - \tau_{ij}(\theta))) = -\sum_{\theta=0}^{\tau'} \sum_{i \in E} \xi_{i\theta}(\theta), x_i = r',
\]

\[
0 \leq \xi_{ij}(\theta) \leq \tilde{u}_{ij}(\theta), \quad \forall (x_i, x_j) \in \tilde{A}, \theta \in T, \theta = 0, \ldots, T - \tau_{ij}(\theta).
\]

Hence, equation (1) of the evacuation model indicates maximizing the number of evacuees $(\tilde{\xi}_{p\tau})$ at each time period $T'$, $\forall T' = 1, \ldots, T$ and conveying them to the safe area (super sink node). Equations (2)-(4) are nodal flow conservation constraints: equation (2) displays that equal amount of flow both enters $\xi_{ij}(\theta - \tau_{ij}(\theta))$ and leaves $\xi_{ij}(\theta)$ any node $x_i$ except the initial and terminal. Equations (3) and (4) are flow conservation constraints for a super source and a super sink. The inequality (5) is a restriction on the upper bounds of flow values for each arc at each time period.

3. Proposed algorithm

In this section, we propose an algorithm for the universal maximum dynamic flow finding in a network in fuzzy conditions.

Step 1. Represent an initial dynamic fuzzy network $\tilde{G}$ in the form of a time-spaced network $\tilde{G}_p$ repeating each node $x_i \in X$ defined number of times depending on the given number of time period $\theta \in T$. Assign arc capacity and time traversal time due to the initial value at corresponding time period $[7,9]$. Introduce a super source $s'$ and a super sink $r'$.

Step 2. Construct a fuzzy residual network $\tilde{G}_p^{\mu}$ depending on the flow values of the initial network. Designing process occurs depending on the flow values conveying the arcs of the time-spaced graph according to the rule:

- decrease the residual arc capacity $\tilde{u}^{\mu}(x_i, x_j, \theta, \theta)$ of the direct arc by the value of current flow along the corresponding arc $\tilde{u}^{\mu}(x_i, x_j, \theta, \theta) = \tilde{u}^{\mu}(x_i, x_j, \theta, \theta) - \tilde{\xi}(x_i, x_j, \theta, \theta)$.
- increase the residual arc capacity $\tilde{u}^{\mu}(x_i, x_j, \theta, \theta)$ of the opposite artificial arc by the value of current flow along the corresponding arc $\tilde{u}^{\mu}(x_i, x_j, \theta, \theta) = \tilde{\xi}(x_i, x_j, \theta, \theta)$.

Step 3. Search an earliest arrival path $P_p^{\mu}$ as a minimum cost path from the super source to the super sink in the fuzzy residual network $\tilde{G}_p^{\mu}$.

1. If a path $P_p^{\mu}$ is found, turn to the step 4.

2. If a $P_p^{\mu}$ isn’t found, the maximum universal flow $\tilde{\xi}(x_i, x_j, \theta, \theta) + \delta^{\mu}_p \times P_p^{\mu}$ in the time-spaced network from the super source to the super sink is found and turn to the step 6.

Step 4. Convey the flow value equals the minimum value of arc capacity of the residual path $\delta^{\mu}_p = \min \{\tilde{u}^{\mu}(x_i, x_j, \theta, \theta)\}$.

Step 5. Recalculate the values of flows according to the rule:

1. for arcs in the opposite direction between nodes $(x_i^\mu, \theta)$ and $(x_j^\mu, \theta)$ modify the flow value $\tilde{\xi}(x_i, x_j, \theta, \theta)$ and decrease it on the value $\delta^{\mu}_p$. Final flow value will be $\tilde{\xi}(x_i, x_j, \theta, \theta) - \delta^{\mu}_p$. Return to the step 2.

2. for directed arcs between nodes $(x_i^\mu, \theta)$ and $(x_j^\mu, \theta)$, modify the flow value $\tilde{\xi}(x_i, x_j, \theta, \theta)$ and increase it on the value $\delta^{\mu}_p$. Final flow value will be $\tilde{\xi}(x_i, x_j, \theta, \theta) + \delta^{\mu}_p$. Return to the step 2.
Step 6. Transition to original dynamic network from the time-spaced one is executed by removing super nodes and artificial arcs connected with these nodes. Each walk in the time-spaced graph connecting nodes \((s, \theta)\) and \((r, \gamma = \theta + \tau_{st}(\theta))\) with the flow \(\tilde{\xi}(s, r, \theta, \gamma)\) corresponds to the flow \(\tilde{\xi}_{st}(\theta)\).

Consider a rule of handling fuzzy arc capacities. Current routines of calculating fuzzy numbers are elaborate – researcher has to perform iterative adding fuzzy flow values and subtracting flow values from residual arc capacities that leads to degradation of fuzzy numbers and lack of calculus efficiency. To manage this task, we propose a rule for tackling fuzzy parameters: throughout the algorithm operate centers of fuzzy numbers. Then, after the maximum flow finding determine deviation bodes as linear combinations of adjacent number borders. These basic values are set by experts. Technique is described in detail in related works of the authors [7].

4. Case study
In this section, area for the case study is presented. We consider area near the Bolshoi Theatre which includes a large number of roads with different characteristics, as presented in figure 2. Some of them are wide avenues, while others are a historical centre with low capacities.

To model real-world emergency, convert the considered real network to dynamic network with fuzzy parameters, as shown in figure 3. Designed network consists of 15 nodes and 18 arcs, 4 nodes-sources and 2 nodes-sinks. Arrays of fuzzy arc capacities and corresponding traversal times are assigned to each arc. Artificial source and sink with infinite arc capacities are introduced to combine real s,r-nodes.

To examine transportation networks using proposed algorithm, a web application that enables calculating the earliest arrival maximum flow value on any arbitrary network was developed. It provides arranging networks with graphic UI or in JSON format.

The final maximum flow value of the earliest arrival is 220 units. It is located between fuzzy numbers \((100,121,145)\) and \((200,240,285)\) set by experts. Let us define the uncertainty borders. Found result is between two basic neighboring values of arc capacities: 121 with the left deviation \(l^L = 21\), the right deviation \(l^R = 24\) and 240, with the left deviation \(l^L = 40\), the right deviation \(l^R = 45\), presented in the form of fuzzy triangular numbers. Therefore, we can represent the value of the maximum flow in the form of the fuzzy triangular number \((183,220,261)\).

![Figure 2. Road network for the case study.](image1)

![Figure 3. Dynamic representation of the initial network.](image2)
In order to check the resistance of the network to changes in its topology, a group of experiments was conducted. The essence of the experiment is to simulate the real situation that occurs during the evacuation. In a real-world emergency evacuation, some sources (evacuation points) become inaccessible due to various factors. We simulate this situation by sequentially removing sources in different combinations until only one source remains. For each set of remaining sources, we calculate the earliest arrival maximum flow, as shown in Table 1. We conclude, that the transportation network is quite resistant because the flow cannot be submitted only in the single case (see the last column in the Table 1).

Table 1. Results of experiments.

| Count | Source ID | Flow |
|-------|-----------|------|
| 1     | 1         | 220  |
| 1     | 2         | 120  |
| 1     | 3         | 160  |
| 1     | 4         | 160  |
| 2     | 2,1       | 160  |
| 2     | 3,1       | 160  |
| 2     | 3,2       | 160  |
| 2     | 4,1       | 160  |
| 2     | 4,2       | 160  |
| 2     | 4,3       | 160  |
| 3     | 3,2,1     | 160  |
| 3     | 4,2,1     | 160  |
| 3     | 4,3,1     | 160  |
| 3     | 4,3,2     | 100  |

a Number of removed sources

5. Conclusion

In this paper, the algorithm of the maximum earliest arrival flow in a fuzzy dynamic network is presented. The proposed algorithm can be applied in the emergency evacuation as a safest one and consists in transporting the maximum number of aggrieved in each time interval. Examined transportation network has dynamic structure, as arc capacities and traversal times depend on the flow departure time. Fuzzy nature of arc capacities enables to model real-world evacuation process. The case study was conducted and web-application was developed to examine the proposed algorithm. Series of experiments were conducted to check the resistance of the network to changes in its topology.

Future work will include considering extensions of fuzzy numbers, in particular, intuitionistic and hesitant fuzzy numbers. Furthermore, microscopic models of evacuation tackling human behaviour in emergency will be proposed.

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