Maximally Natural Supersymmetry

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We consider 4D weak scale theories arising from 5D supersymmetric (SUSY) theories with maximal Scherk-Schwarz breaking at a Kaluza-Klein (KK) scale of several TeV. Many of the problems of conventional SUSY are avoided. Apart from 3rd family sfermions the SUSY spectrum is heavy, with only ∼50% tuning at a gluino mass of ∼2 TeV and a stop mass of ∼650 GeV. A single Higgs doublet acquires a vacuum expectation value, so the physical Higgs is automatically Standard-Model-like. A new $U(1)'$ interaction raises $m_h$ to 126 GeV. For minimal tuning the associated $Z'$, as well as the 3rd family sfermions, must be accessible to LHC13. A gravitational wave signal consistent with BICEP2 is possible if inflation occurs when the extra dimensions are small.

PACS numbers: 12.60.Jv, 12.15.-y, 14.80.Da, 14.80.Rt

The LHC has set stringent limits on the masses of SUSY particles and deviations in Higgs properties, implying a tuning of electroweak symmetry breaking (EWSB) at the percent level or worse for traditional SUSY models [1–6]. This undermines the motivation for SUSY as the solution to the hierarchy problem and the case for discovery of SUSY at the LHC or proposed future colliders. Given the importance of this issue for current and future searches for new physics we examine the possibility of constructing natural, untuned theories. Specifically, we consider 4D theories of the weak scale that arise from 5D SUSY theories with Scherk-Schwarz SUSY breaking (SSSB) at a KK scale $1/R$ of several TeV [7–20].

The key features are:

- The theory is never well approximated by a 4D softly-broken $N = 1$ SUSY limit. Many of the problems of the MSSM and its extensions are avoided.
- Higgsinos, gauginos, and the 1st and 2nd family sfermions get (mainly Dirac) $S$-SSB masses of size $1/2R$.
- A natural SUSY spectrum [21,22] is obtained through localization of the 3rd family on a 4D brane. The absence of large logs due to the super-softness of SSSB [24–30] protects the weak scale and suppresses the tendency of the gluino to pull up the stop mass [2,3].
- The $\mu$ term neither exists nor is needed. Only $H_u$ acquires a VEV, and the down-like quark and lepton masses are generated by Kahler couplings to $H_u^+$ [31]. The physical Higgs is automatically SM-like.
- An additional SUSY breaking sector is necessarily present for radius stabilization with zero cosmological constant (CC), and SUSY breaking in this sector can naturally be driven by SSSB. Higher dimensional couplings of the MSSM fields to this sector play a crucial role in EWSB and collider phenomenology.
- A $U(1)'$ broken in this additional sector raises the Higgs mass to 126 GeV through an unusual non-decoupling D-term, with a $Z'$ mass of order $1/R$.

The pattern of localization of matter and Higgs multiplets and the mechanism driving EWSB, generating Yukawa couplings, and accommodating the observed physical Higgs mass lead to important differences from previously studied models of SSSB [7–20].

NATURAL SPECTRUM FROM SCHERK-SCHWARZ

Symmetries may be broken in a way preserving 4D Poincare invariance by imposing boundary conditions (bc’s) on bulk fields involving a symmetry twist. If the twist includes an $R$-symmetry group action, then SUSY is softly broken by the SSSB mechanism [32,33]. This SSSB is non-local from the higher-dimensional perspective, and is of an exceptionally soft type, similar to finite-temperature breaking of SUSY. In our case the twists will be maximal, ±1, and the underlying non-gravitational sector can be described as a 5D gauge theory compactified on a $S^1/(Z_2 \times Z_2)$ orbifold. The 5th dimension, of physical length $\pi R$, is parameterised by $y \in [0, \pi]$, and branes sit at the inequivalent fixed points at $0, \pi R$.

Our 5D bulk theory is a SUSY theory containing the SM gauge fields, the first two families, and a pair of distinct Higgs hypermultiplets, $H_u, H_d$, (see Fig.1a). As the minimal SUSY in 5D corresponds to $N = 2$ 4D SUSY, the superpartners of these bulk states fill out $N = 2$ 4D multiplets, with each 5D vector implying both a 4D vector and chiral supermultiplet in the adjoint representation, $V_5^a = \{V^a_{4D}, \Sigma^a\}$ (with physical fields $V^a_{4D}, \Sigma^a$ and $\Sigma^a, \Sigma^a$) while the matter fields are hypermultiplets consisting of 4D chiral and anti-chiral multiplets $\Phi_{SSB}^{5D} = \{\phi^i, \phi_i\}$ (physical fields $\phi_i, \psi_i$ and $\bar{\phi}_i, \bar{\psi}_i$) [24,25].

The two $Z_2$ actions, at their respective fixed points at $0, \pi R$, break 5D SUSY to two different and incompatible $N = 1$ 4D SUSYs thus breaking SUSY completely in the 4D effective theory; the component field bc’s are summarised in Table II. Due to the non-local nature of SSSB there are no cutoff-dependent log enhancements of the effective 4D soft terms. At $y = 0$ we localise the
TABLE I: Bc’s at y = (0, π) for bulk fields of complete model with ± corresponding to Neumann/Dirichlet. Only the (±, +) fields have a zero mode, and the KK mass spectrum (n ≥ 0) is: \( m_n = n/R \) for (±, +) fields; (2n + 1)/2R for (±, −) and (−, +); and (n + 1)/R for (−, −). \( \psi_{F_3} \) stands for all 1st/2nd generation fermions; \( \varphi_F \) their 4D \( N = 1 \) sfermion partners; barred states are the extra 5D \( N = 1 \) SUSY partners.

3rd generation fields. As the fixed points preserve only \( N = 1 \) 4D SUSY, these states are simply 4D chiral multiplets with no additional partners, and a localised Yukawa superpotential for up-like states is allowed

\[
\delta(y)H_u(y) \left( \frac{\tilde{y}_u}{M_5^{1/2}} Q_3 U_3^c + \frac{\tilde{y}_c}{M_5^{3/2}} Q_2(y) U_2^c(y) + \ldots \right),
\]

where \( \tilde{y}_u \) are dimensionless and the Yukawa couplings to bulk 1st/2nd generations are naturally suppressed compared to the brane-localized 3rd generation. We later return to the down-type Yukawas.

There is no need for a \( \mu \) term linking \( H_u H_d \) to lift the higgsinos. Instead, SSB gives the higgsinos a large 1/2R mass by marrying \( \psi_{h_u} \) with \( \bar{\psi}_{h_u} \). The SSB bc’s lift the Higgsinos while making no contribution to the scalar Higgs masses, avoiding the usual source of tree-level tuning.

After SSSB the brane-localised scalars pick up, at 1-loop, finite positive soft SUSY-breaking masses

\[
\delta\tilde{m}^2_i \simeq \frac{7\zeta(3)}{16\pi^4 R^2} \left( \sum_{l=1,2,3} C_l(i)\tilde{y}^2_l + C_l(i)\bar{\tilde{y}}^2_l \right),
\]

with \( C(U_3) = \{4/9, 0, 4/3, 1\} \), \( C(D_3) = \{1/9, 0, 4/3, 0\} \), \( C(E_3) = \{1, 0, 0, 0\} \), \( C(L_3) = \{1/4, 3/4, 0, 0\} \), \( C(Q_3) = \{1/36, 3/4, 4/3, 1/2\} \), and for the Higgs bulk scalar zero mode \( C(H_{u,d}) = \{1/4, 3/4, 0, 0\} \) [7].

In addition to the positive 1-loop EW contribution Eq. (2), the Higgs soft mass \( \tilde{m}^2_{H_u} \) receives a comparable negative contribution at 2-loops from the \( t\bar{t} \) sector. Ref. [12] performed a 2-loop 5D calculation of this term, and we have used RG methods to determine the leading 3-loop \( \log(m_t^2/R^2), \log(m_{\tilde{t}}/m_t) \)-enhanced corrections, which are numerically important in determining the fate of EWSB [39]. As shown in Fig. 2, these minimal contributions do not so far lead to EWSB. Nevertheless, the model has attractive features: Compared to 4D theories the Higgs soft mass is more screened from SUSY-breaking as Eq. (2) involves a finite 1-loop factor with no log enhancement, SUSY breaking for all but the 3rd generation and Higgs scalar zero mode is direct and universal, and higgsinos are heavy without a large \( \mu \) term.

**SUCCESSFUL EWSB AND HIGGS MASS**

Other faults remain in this model, and we find their solution plays a major role for EWSB and experimental signatures. First, our 5D theory is an effective theory which must be cutoff at a scale \( M_5 \). The bulk 5D
gauge couplings are dimensionful \((1/g^2_{FA} = \pi R/g^2_{1,5} \text{ up to small brane-kinetic-term corrections})\), and 5D perturbative unitarity bounds for \(g_5\) require \(\pi M_5 R \lesssim 25\) \([40, 41]\). NDA strong coupling estimates for the brane-localized Yukawas give a similar bound \([42]\).

This cutoff is large enough to justify the 5D viewpoint and the parameterization of UV effects in terms of higher dimensional operators, but the weakness of gravity in the low energy 4D theory, \(M_{pl} \gg M_5\), must still be explained. The two controllable possibilities of which we are aware are: (a) Embed the 5D theory in a 10 or 11D string/M-theory where some or all of the extra 5 or 6 purely gravitational dimensions are ‘large’, similar to the original brane-world proposal of Refs. \([43-46]\) (see Fig. 1b). Since our fundamental scale is \(M_5 \gtrsim 30\, \text{TeV}\), \(n \gtrsim 2\) extra dimensions is safe from cosmological, astrophysical, and laboratory constraints. (b) Utilise a little-string-theory construction with tiny string coupling \([47]\).

Second, the radius \(R\) is unstabilized. Moreover, SSSB without radius stabilization is of no-scale type with zero CC at tree level \([32, 38, 39]\), and generally radius stabilization yields a deep negative CC of order \(\sim -\pi R M_5 \gtrsim 126\, \text{GeV}\) \([61]\). For lighter \(h\)-particles, we obtain \(m_h \approx 126\, \text{GeV}\) with a non-decoupling D-term (as only \(\langle H_u \rangle \neq 0\), a NMSM-like singlet interaction \(SH_uH_d\) can not be employed as in ref. \([62]\)). Specifically, we introduce a bulk \(U(1)\) gauging a subgroup of right-handed \(SU(2)\) generated by \(T_{3R}\) under which \(H_u\) and \(H_d\) transform (the \(U(1)\) is anomaly-free if three light RHD neutrino superfields are introduced in the bulk; we find our theory allows a novel theory of neutrino mass generation \([39]\)). To avoid suppression of the quartic, the breaking of the gauge group must couple to large SUSY breaking F-terms \([63, 65]\). It is natural to associate the breaking of the \(U(1)\) with the same dynamics that generates \(F_X\) with the resulting \(Z'\) mass \(\sim 1/R\).

A simple model where \(F_X\) is induced by SSSB and is associated with the breaking of the \(U(1)\) is obtained by introducing bulk hypermultiplet fields \(\phi_{1,2}\) charged \(\pm \frac{1}{2}\) under the \(U(1)\) with SSSB bc’s given in Table. 1 and a brane-localized superpotential \(\lambda M_5 / \pi R\). This positive SUSY breaking contribution to the radion potential can be tuned to allow stabilization with zero CC. We find that for \(m_{\tilde{t}} \gtrsim 650\, \text{GeV}\) \((m_{\tilde{t}} \gtrsim 1\, \text{TeV})\) and \(m_{\tilde{\chi}_2^0} \lesssim 2/R\), the \(U(1)\) D-term can yield \(m_h \approx 126\, \text{GeV}\) with \(g_x < 1\) \((g_x < g_2)\). The \(U(1)\) sector also contributes to the Higgs soft mass. The contribution is not well-approximated by the truncated lightest KK modes; we evaluate it in the 5D theory and find for \(m_{\tilde{\chi}_2^0} \gtrsim 1/R\) the contribution favors EWSB and numerically approaches \(\delta m_{H_u}^2 (U(1)) \approx -10^{-3} g_X^2 m_{\tilde{\chi}_2^0}^2\). \(\text{(7)}\)

**PHENOMENOLOGY AND VARIATIONS**

The theory has a rich phenomenology, and a variety of new physics signatures are accessible to LHC14 in the low-fine-tuning parameter region. Here we provide just a brief summary of the main features \([39]\). The spectrum of new (non-gravitational) states is illustrated in Fig. 3 where we have shown values with minimal fine-tuning consistent with current bounds.
The theory is mostly protected from precision, flavor and CP observables, although signatures are possible. While SUSY flavor problems are suppressed by the automatic near-degeneracy of 1st/2nd generation squarks and the near-Dirac masses of higgsinos and gauginos, KK gauge boson exchange can lead to deviations in kaon and especially $B_s$ mixing and rare decays depending on model-dependent details \cite{66}. The high scale of the KK states and $U(1)'$ sectors, $1/R \sim m_{Z'} \gtrsim 4$ TeV protects from EWPT\cite{72,73}. Higgs properties are automatically SM-like since only $H_u$ obtains a VEV, and the inert $H_d$ is easily made consistent with limits.

The presence of additional large gravitational dimensions constrains models of inflation and reheating. A detailed treatment is left to future work \cite{39}, but we note that a small inflationary energy scale $V_\phi < M_3^2 < M_{pl}^2$ can be consistent with recent evidence for tensor perturbations \cite{72} if the extra gravitational dimensions and thus the corresponding 4D Planck mass are small during inflation, as in models of rapid asymmetric inflation \cite{68}.

The leading signature of this model is sparticle production at the LHC and future colliders. Two important differences from generic natural SUSY phenomenology occur. First, $m_{\tilde{g}} \sim (3\div5)m_t$ arises without extra tuning, and tuning limits will likely be driven by direct production of 3rd generation sparticles, not gluino production. Second, the absence of a light higgsino leads to unusual stop and sbottom decay chains. The brane-localized 3rd generation sleptons masses are dominantly from higher dimensional operators Eq. (1), so either $\tilde{\tau}_R$ or $\tilde{\nu}_{\tau L}$ could be the lightest ordinary superpartner (LOSP). Three-body decays of $\tilde{t}$ and $\tilde{b}$ to the LOSP can dilute missing energy signatures and lead to $\tau$-rich final states. Depending on the embedding of the 5D theory in the gravitational dimensions, the LOSP can be collider stable, or decay through prompt or displaced vertices to extradimensional-gravitini or other $R_L$-odd states in the bulk. In another variation, if $F_X$ is generated independently of SSSB, the associated goldstino remains light \cite{69} and ordinary superpartners will decay directly to this state, mimicking more standard natural susy signatures. For this short work we take the LHC8 bounds on $\tilde{t} \to t + \text{MET}$ of $m_t \gtrsim 650$ GeV \cite{70,71} as a guideline, but this can potentially be eased.

The mass and couplings of the new $Z'$ are restricted by the requirement $m_{\tilde{g}} \approx 126$ GeV, suggesting this state is also likely to be accessible; 8 TeV limits require $m_{Z'} \gtrsim 3$ TeV \cite{72} \cite{73}. The tuning of EWSB in this theory can be quantified by the sensitivity of $v$ to shifts at the scale $1/R$ of the stop mass (through the operator Eq. (4)) and the $Z'$ mass,

$$\Delta = \left( \frac{\partial \ln v^2}{\partial \ln m_{\tilde{t}}^2} \right)^2 + \left( \frac{\partial \ln v^2}{\partial \ln m_{\tilde{Z}'}^2} \right)^2,$$

where for simplicity we set $m_{\tilde{b}_3}^2 = m_{\tilde{e}_3}^2 \equiv m_{\tilde{b}}^2$. The fine-tuning is shown in Fig. 1 where the stop mass has been fixed as a function of $1/R$ and $m_{Z'}$ to give successful EWSB. For $m_{Z'} \lesssim 1.5/R$, the stop contribution is the dominant source of tuning. Remarkably at current LHC8 limits the theory is natural with a tuning of $\sim 50\%$. LHC14 can discover stops at $m_{\tilde{t}} \sim 1.2$ GeV \cite{74},

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Schematic spectrum of new states of primary experimental interest.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Fine-tuning $\Delta^{-1}$ (solid lines) as function of $1/R$ and the $Z'$ mass, Eq. (3). Iso-contours of stop mass are dashed. Limits from LHC8 searches for $t \to t + \text{MET}$ \cite{70,71} (red) and $Z'$ resonance searches \cite{72} \cite{73} (green) are shaded. Subdominant limits $m_{\tilde{g}} \approx 1/(2R) \gtrsim 1.3$ TeV from $g \to t\bar{t}/b\bar{b} + \text{MET}$ searches (blue) are also shaded \cite{75} \cite{76}.}
\end{figure}
for which the theory is $\sim 20\%$ tuned. For $m_{1/2} \gtrsim 3.5$ TeV, the tuning is still only at the few percent level and $m_{h} = 126$ GeV might be obtained radiatively [61] without the complications of an extra $U(1)^{'}$ sector – an attractive target for a 100 TeV proton collider.

The production of KK excitations of SM particles would be an important signature of the extra-dimensional nature of this model, but their large mass $\sim 1/R$ and an approximate KK-parity make these particles difficult to reach at LHC14. Observing the near degeneracy of gauginos, higgsinos, and 1st/2nd generation sfermions would be an important signature of the extra-dimensional nature of the theory.

In summary, we have presented a model where SSSB accompanied by a simple mechanism driving EWSB leads to a natural spectrum consistent with Higgs properties and sparticle bounds with fine-tuning better than $\sim 50\%$ even after LHC8 limits. Variations involving different field content and localizations, including interplay with other mechanisms for driving EWSB in SSSB via different bc’s [10, 11] or Higgs [17–20] deserve further attention as leading candidates for natural theories at LHC14 and future colliders. In an aesthetic direction, the extended gauge structure and extra dimensions suggest interesting possibilities for gauge unification in this model [39].

We thank N. Arkani-Hamed, A. Arvanitaki, M. Baryakhtar, N. Craig, I. Garcia Garcia, T. Gherghetta, E. Hardy, X. Huang, D. E. Kaplan, K. Van Tilburg, J. Wacker, and Y. Zhao for useful discussions. We especially thank DEK for hospitality at JHU during a portion of this work. This work was supported by ERC grant BSMOXFORD no. 228169. KH is supported by an NSF Graduate Research Fellowship under Grant number DGE-0645962 and by a portion of this work. This work was supported by the US DoE under contract DE-AC02-76SF00515.

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