Intraday Optical Variability of BL Lacertae

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Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT

We monitored BL Lacertae simultaneously in the optical B, V, R and I bands for 13 nights during the period 2012-2016. The variations were well correlated in all bands and the source showed significant intraday variability (IDV). We also studied its optical flux and colour behaviour, and searched for inter-band time lags. A strong bluer-when-brighter chromatism was found on the intra-night time-scale. The spectral changes are not sensitive to the host galaxy contribution. Cross-correlation analysis revealed possible time delay of about 10 min between variations in the V and R bands. We interpreted the observed flares in terms of the model consisting of individual synchrotron pulses.

Key words: galaxies: active - BL Lacertae Objects: individual: BL Lacertae -galaxies: photometry.

1 INTRODUCTION

BL Lacertae is the archetype of BL Lac class, which together with flat spectrum radio quasars (FSRQs), constitutes a violently variable class of active galactic nuclei (AGN) known as blazars. Blazars are characterized by high and variable polarization, synchrotron emission from relativistic jets, core-dominated radio morphology and intense flux and spectral variability in all wavelengths ranging from radio to γ-ray on a wide variety of time-scales (Urry & Padovani 1995; Wagner & Witzel 1995; Böttcher et al. 2003). BL Lacertae, located at redshift value of \( z = 0.0668 \pm 0.0002 \) (Miller & Hawley 1977), is hosted by a giant elliptical galaxy with \( R = 15.5 \) (Scarpa et al. 2000). As its first spectral component peaks in near-IR (NIR)/optical region, BL Lacertae is a typical low-frequency peaked blazar (LBL) with the radio to X-ray spectral index equal to 0.84 (Fossati et al. 1998; Fiorucci et al. 2004).

BL Lacertae was continuously observed by several multiwavelength campaigns carried out by the Whole Earth Blazar Telescope (WEBT/GASP) (Villata et al. 2003, 2004; Bach et al. 2006; Raiteri et al. 2009). Numerous investigations have been carried out to search for the flux variations, spectral changes and periodicities (Epstein et al. 1972; Carini et al. 1992; Villata et al. 2002; Papadakis et al. 2003; Agarwal & Gupta 2015; Gaur et al. 2015). The majority of the observations revealed that its IDV amplitude is larger at higher frequencies and decreased as the flux increased. These studies confirmed the presence of bluer-when-brighter (BWB) trend, but yielded no evidence for periodicities (Carini et al. 1992; Villata et al. 2002; Gaur et al. 2015). Hagen-Thorn et al. (2002) proposed the evidence for periodicity of 308 d for total flux variations in 22 yr. Most authors did not find any significant time lags between different optical bands during a single night observation (Nesci et al. 1998). However, Papadakis et al. (2003) found that the delay between B- and I-band light curves was \( \sim 0.4 \) h. In addition, a possible time-lag between \( \varepsilon \) and m bands was reported as \( \sim 11.6 \) min by Hu et al. (2006).

Polarimetric monitoring can offer information about the physical processes in blazar. Unlike radio polarization, optical polarimetry probes the central nuclear regions of blazar jets (Falomo et al. 2014). The optical polarized emission, as a potential good tracer of the high-energy emission, is being widely studied (Marscher et al. 2008; Raiteri et al. 2013; Sorcia et al. 2013; Covino et al. 2015). Since blazar optical radiation is dominated by the synchrotron mechanism, this implies the presence of highly ordered large scale magnetic fields (Westfold 1959). Observation of optical polarizations of BL Lac objects has shown the degree of polarization varies on time-scales from IDV to years (Impey et al. 2000; Tommasi et al. 2001; Sasada et al. 2008). Occasionally, the correlation (Hagen-Thorn et al. 2008) and anticorrelation (Gaur et al. 2014) between the total and polarized fluxes is observed, while in general no clear relation is found.

In this paper, we studied the IDV and colour behaviour of this object, and searched for the inter-band time lags with...
data collected during the period 2012–2016 with high temporal resolution. The results can give us insight into theoretical causes of variability. This paper is organized as follows: Section 2 describes the observations and data reductions, and Section 3 provides a brief introduction to various analysis techniques, followed by the results in Section 4. The discussion and conclusions are in Sections 5 and 6, respectively.

2 OBSERVATIONS AND DATA REDUCTIONS

The monitoring was performed with three telescopes at Xinglong Station of the National Astronomical Observatories Chinese Academy of Sciences (NAOC). The details of the telescopes are shown in Table 1. We observed BL Lacertae in the $B, V, R$ and $I$ bands for 13 nights covering the period from 2012 November 5 to 2016 January 17. During 12 of those nights, we observed simultaneously in different bands, providing a total of 24 intraday light curves. The longest individual duration of observation was about 7.7 h. The entire observation log together with all the available results is presented in Table 2. More than ten thousand original data points were collected on the 13 nights.

The data reduction procedures included bias subtraction, flat-fielding, extraction of instrumental magnitudes and flux calibration. The pre-processing of the raw data was accomplished by using standard procedures in the IRAF \(^1\) software. For each night, photometry was carried out with five different aperture radii, i.e., $\sim 1 \times \text{FWHM}$, $1.5 \times \text{FWHM}$, $2 \times \text{FWHM}$, $3 \times \text{FWHM}$, $4 \times \text{FWHM}$. The minimum standard deviation of photometric error corresponds to the best aperture and we finally selected the best aperture data for our analysis. Three local comparison stars (B, C, H in Fig. 1) were observed in the same field. The standard magnitudes of these stars in the $B, V, R$ and $I$ bands are given by Smith et al. (1985). The brightness of BL Lacertae was calibrated relative to the average brightness of stars B and C. They have similar magnitude and colour to BL Lacertae. Star H acted as a check star.

The host galaxy of BL Lacertae is relatively bright and its contribution to the magnitudes was subtracted after flux calibration in order to avoid contamination. Hyvönen et al. (2007) derived a $B$ magnitude of 17.35 for the host galaxy of BL Lacertae. The host galaxy contribution in $V, R$ and $I$ bands was inferred by adopting the elliptical galaxy colours of $B-V = 0.99$, $V-R = 0.59$ and $V-I = 1.22$ from Mannucci et al. (2001). Villata et al. (2002) estimates that the host galaxy contribution to the observed flux is about 60% of the whole galaxy flux. The magnitudes were transformed into flux and the host galaxy contribution was removed.

Given that short exposure times cause the data dispersion, a smoothing algorithm was implemented for the data in 2013, 2014 and 2015. The data were smoothed using the average of each 3 mins bin. The overall light curves of this source are displayed in Fig. 2. Small black dots denote original data and different colour dots denote smoothed data in different passbands. There are about 2000 smoothed data points used for further calculation. For clarity, several band light curves are shifted correspondingly.

3 VARIABILITY DETECTION CRITERIA

To quantify the IDV of BL Lacertae, two statistical analysis techniques were adopted, the $\chi^2$ test and ANOVA test.

3.1 $\chi^2$ test

The $\chi^2$ statistic is defined as:

$$
\chi^2 = \sum_{i=1}^{N} \frac{(V_i - \overline{V})^2}{\sigma_i^2},
$$

where, $\overline{V}$ is the mean magnitude of all $i$th observation $V_i$ with a corresponding error $\sigma_i$. The exact errors from the IRAF reduction package are smaller than the real ones by a factor of 1.3 to 1.75 (Gupta et al. 2008; Agarwal & Gupta 2015). We chose the factor of 1.5 for data processing to get a better estimate of the actual photometric errors. If the actual variability is greater than the critical value at the $N-1$ degree of freedom and selected significance level, then the presence of variability can be claimed.

3.2 ANOVA test

de Diego et al. (1998) used the one-way ANOVA to investigate the variability of quasars. The mathematical description of the one-way ANOVA test is as followed: if $y_{ij}$ represents the $i$th (with $i = 1, 2, \ldots, n_j$) observation on the $j$th

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\(^1\) IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

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Table 1. Parameters of three telescopes.

| Telescope        | CCD resolution | CCD view             |
|------------------|----------------|----------------------|
| 60cm reflector   | 1".06 /pixel   | 11" × 11"            |
| 85cm reflector   | 1" /pixel      | 33" × 33"            |
| 216cm reflector  | 0".305 /pixel  | 6.5 × 5.8            |

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Figure 1. Finding chart of BL Lacertae in the $B$ band on 2015 October 19.
Figure 2. Light curves of BL Lacertae in the $B, V, R$ and $I$ bands in 5 yr. Small black dots denote original data. Different colour dots denote smoothed data in different passbands. For clarity, several band light curves are shifted correspondingly.

(with $j = 1, 2, \ldots, k$) group, the linear model describing every observation is

$$y_{ij} = \bar{y} + g_j + \varepsilon_{ij}, \quad (2)$$

where, $\bar{y}$ represents the mean value of the whole data set, $g_j = \bar{y}_j - \bar{y}$ the between-groups deviation and $\varepsilon_{ij} = y_{ij} - \bar{y}_i$ the within-groups deviation. Our observations on each night were divided into groups of five consecutive data points. The total sample variation can be separated between and within group deviations:

$$SS_T = \sum_{j=1}^{k} \sum_{i=1}^{n_j} (y_{ij} - \bar{y})^2 = \sum_{j=1}^{k} (y_j - \bar{y})^2 + \sum_{j=1}^{k} \sum_{i=1}^{n_j} (y_{ij} - \bar{y}_j)^2. \quad (3)$$

Equation (3) can be shortened to $SS_T = SS_G + SS_R$. $SS_T$ stands for the total sum of squares that describes the total deviation of the data with respect to the mean. $SS_G$ stands...
for the right-hand side of equation (3) and $SS_R$ is the total error.

The statistics corresponds to the $F$ distribution with $k - 1$ and $N - k$ degrees of freedom.

$$F = \frac{SS_G/(k-1)}{SS_R/(N-k)}.$$  

(4)
4 RESULTS

4.1 Light Curves and Flux Variations

The light curves we obtained are shown in Fig. 2. We adopted two statistical analysis techniques described in Section 3 to search for variability and the results are listed in Table 2. The first column of Table 2 shows the observation date, while the second column records the telescope. The band and number of the data points are given in the 3rd and 4th columns respectively. The observation durations are listed in the 5th column and the test results of $\chi^2$ and ANOVA are in the 6th and 7th columns. If $F$ are listed in the 5th column and the test results of $\chi$ are calculated by using $\chi^2$ and ANOVA are in the 6th and 7th columns. If $F$ exceed the critical value (CV) at the 99% significant level, the null hypothesis that there is no variations will be rejected. BL Lacertae is marked as Y if the variability conditions for each test are satisfied, while N means no variations. The final column is IDV amplitude.

The IDV amplitudes are given by Heidt & Wagner (1996):

$$A = \sqrt{(m_{\text{max}} - m_{\text{min}})^2 - 2\sigma^2},$$

where $m_{\text{max}}$ and $m_{\text{min}}$ are the maximum and minimum magnitudes, and $\sigma$ is the standard deviation.

The variations were well correlated in all bands. As can be seen, there are variations on four nights that are detected by both tests. We calculated their amplitudes using the above equation. The maximum amplitude of IDV is 15.85% in the $B$ band on 2015 October 19. The amplitude of IDV is greater in higher energy bands. This has been observed in BL Lacertae (Webb et al. 1998; Nesci et al. 1998; ?). The amplitude of variability in different bands is changed during different nights. The comparison results of two tests indicate that ANOVA shows efficient detection of IDV with small flares. Several small flares can be seen in the light curves on four of these nights, which can be claimed as the IDV by ANOVA test. We interpreted the observed flares in terms of the model consisting of individual synchrotron pulses in another section below.

4.2 colour behaviour

We investigated the colour behaviour with respect to the brightness of BL Lacertae for each separate night. The colour indices of $B - R, V - R, V - I$ and $B - I$ are calculated by using the almost simultaneous $B, V, R$ and $I$ magnitudes. Because of the existence of gaps in the light curves on 2015 October 18, we plotted two separate diagrams and the results are displayed in Fig. 3. We fitted the colour-magnitude diagrams with a linear model (where $r$ is the correlation coefficient).
The spectral indices of $\alpha_{BR}$, $\alpha_{BI}$ and $\alpha_{VR}$ changed by only 0.21, 0.02 and 0.45, respectively. The $\alpha_{VI}$ even remained unchanged due at least partly to the few data points. The accretion disc radiation is expected to be overwhelmed by that from the strongly Doppler-boosted jets, so the observed spectral variations in blazars cannot be explained by the accretion disc. The relatively steep spectral indices indicate strong synchrotron emission from the blazar jet and small accretion disc contribution (Agarwal et al. 2016).

### 4.3 Cross-correlation analysis and time lags

We performed the correlation analysis to search for the possible inter-band time lags by using two cross-correlation methods. The first one is the $z$-transformed discrete correlation functions (ZDCFs) method (Alexander 1997). ZDCF deals with under-sampled light curves and divides all observation points into equal bins. It uses Fisher’s $z$-transform to stabilize the highly skewed distribution of the correlation coefficient. The Gaussian fitting (GF) is made to the central ZDCF results. Meanwhile, we try another way to measure the lags and errors by interpolated cross-correlation function (ICCF) method (Gaskell & Peterson 1987). The error was estimated with a model-independent Monte Carlo method, and the lag was taken as the centroid of the cross-correlation functions that were obtained with a large number of independent Monte Carlo realizations. This is the flux-randomization/random-subset selection (FR/RSS) approach described by Peterson et al. (1998, 2004). Five thousand independent Monte Carlo realizations were performed on each light curve.

One problem in the GF is that it usually underestimates the error for time delay (Wu et al. 2012). The results for ZDCF+GF are only for reference. The FR/RSS lags have significance lower than 3$\sigma$ except for the $V-R$ lag on 2013 October 20. On that night, the variability in the $R$ band led that in the $V$ band by 11.8 min. The correlation analysis plot is displayed in Fig. 6. Date and the correlated passbands are given at left side. The peak of the Gaussian profile (the dashed line) is marked with a vertical dotted line. A negative lag ($\tau$) means that the later variation leads the former one. Because of large errors, no time delays were found on other nights. Since the data were binned at 3-min intervals and we used the binned data to estimate the time lags, so the result of 11.8 mins should be reasonable.

Wu et al. (2012) discussed the possible key factors that determine the detectability of the optical time lags. Because of large errors, no time delays were found on other nights. Since the data were binned at 3-min intervals and we used the binned data to estimate the time lags, so the result of 11.8 mins should be reasonable.

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$\chi^2$ test with those of the ANOVA test, we extracted five IDV light curves with several

\[ \chi^2 = \frac{1}{N-p} \sum_{i=1}^{N} (y_i - y) \]
Table 4. Pulse parameters used to fit the data.

| Date         | Band | Pulse | Centre (h) | Amp (mJy) | $\tau_{\text{pulse}}$ (h) | $S_{\text{cell}}$ (AU) |
|--------------|------|-------|------------|-----------|---------------------------|------------------------|
| 2013 10 20   | V    | 1     | 22.80      | 0.19      | 0.84                      | 4.40                   |
|              |      | 2     | 24.50      | 0.24      | 1.17                      | 6.16                   |
|              |      | 3     | 25.50      | 1.54      | 1.03                      | 5.43                   |
|              |      | 4     | 27.38      | 0.24      | 1.39                      | 7.34                   |
|              |      | 5     | 28.41      | 0.18      | 0.56                      | 2.93                   |
| 2013 10 20   | R    | 1     | 22.64      | 0.23      | 1.11                      | 5.87                   |
|              |      | 2     | 24.39      | 0.39      | 1.20                      | 6.31                   |
|              |      | 3     | 25.39      | 1.63      | 1.06                      | 5.57                   |
|              |      | 4     | 27.25      | 0.38      | 1.34                      | 7.04                   |
| 2014 11 03   | R    | 1     | 24.00      | 0.05      | 1.36                      | 7.19                   |
|              |      | 2     | 27.12      | 0.12      | 3.34                      | 17.60                  |
| 2015 10 16   | I    | 1     | 22.65      | 0.20      | 0.28                      | 1.47                   |
|              |      | 2     | 23.82      | 0.28      | 1.53                      | 8.07                   |
|              |      | 3     | 26.00      | 0.35      | 2.48                      | 13.06                  |
|              |      | 4     | 28.30      | 0.50      | 1.75                      | 9.24                   |
|              |      | 5     | 29.90      | 0.60      | 1.67                      | 8.80                   |
| 2015 10 17   | I    | 1     | 2.10       | 0.58      | 2.62                      | 13.79                  |
|              |      | 2     | 6.00       | 0.32      | 1.87                      | 9.83                   |

As the strong shock hits each stochastic cell, particle acceleration and subsequent cooling by synchrotron emission produce a pulse. The convolution of these individual pulse emissions from inhomogeneous cells of various sizes and density enhancements leads to the observed microvariability. For every local peak in the light curve, its location was taken as the centre position of the cell, its amplitude as the degree of the density enhancement and its width as its spatial extension. We applied this model to five intraday light curves by using their pulse code. We used the Doppler factor of 7.3 to calculate the pulse shape (Hovatta et al. 2009).

By varying the width and amplitude of the standard pulse, we have fitted each significant flare of the light curves. The resulting parameters for the pulses used in modelling the light curves are listed in Table 4. The first column shows the observation date, while the second column records the band. The pulse IDs and the centre time of the pulse are given in the 3rd and 4th columns respectively, and the amplitude is in the 5th column. Column 6 gives width ($\tau_{\text{pulse}}$) of each pulse. The number quoted in Column 7 is an estimate of the size of the cell in AU based on the assumed shock speed ($u_s = 0.1c$) and the duration of the pulse. Fig. 7 shows the light curves fitted with the convolved pulses. Although the fit is not unique, it is representative of how well the model compares to the data. The correlation coefficient ($r$) of each fitting is calculated. On 2013 October 20, the peaks were missing in both bands; therefore, the correlation coefficients of fitting are less than 0.9. Based on the centre time of each pulse on 2013 October 20, the time lag is estimated to be about 8 min, which is consistent with the FR/RSS result. The other 3 d are well fitted with the model.

Figure 6. ZDCF correlation and fitting result on 2013 October 20. The dashed line shows Gaussian fitting to the points, and the peak is marked with the vertical dotted line. $\tau$ gives the lag result.

5 DISCUSSION

There are various models to explain the IDV flux of blazars. Intrinsic ones include the instabilities in accretion disc (Wiita 1996) and the shocks travelling down the jet (e.g. Marscher 1996, 2014, and references therein). Extrinsic ones involve gravitational microlensing (Schneider & Weiss 1987) and interstellar scintillation (Bignall et al. 2003).

Following the turbulent jet model, Bhatta et al. (2013) interpreted the microvariability as emission from individual synchrotron cells, which are energized by a plane shock propagating down the jet. This results in an increase in flux resembling a pulse. Since turbulence is a stochastic process, each microvariability curve is a realization of it. We use five IDV light curves with several flares to test their theoretical model and get the turbulent parameters from our observations. There is a large range of length scales for the turbulent vortices. The largest cell size is ~17.6 AU that could correspond either to the correlation length scale or to the physical width of the jet, while the smallest cell size is...
around 1.5 AU, could correspond to the Kolmogorov scale-length of the turbulent plasma. We can get a picture of the underlying turbulent structure. Fitting pulses to BL Lacertae microvariability curves give us a much better indication of the turbulent nature of the plasma in these sources.

Miller et al. (1989) argue that the microvariations are produced very close to the central supermassive black hole (BH). Several works have attempted to estimate the mass of the BH in BL Lacertae (Fan et al. 1999; Woo & Urry 2002; Ghisellini et al. 2010; Capetti et al. 2010; Gupta et al. 2012), which seems to be $0.1 - 6 \times 10^8 M_\odot$. If we assume fluctuations in the inner portions of the accretion disc, the observed minimum time-scale $\Delta t_{\text{obs}}$ will provide an upper limit to the mass of BH. Emmanoulopoulos et al. (2010) pointed out that the first-order structure function (SF) sometimes leads to incorrect claims of time-scales. Hence, we adopt the ZDCF (in autocorrelation mode) method to get a possible IDV time-scale. We choose the minimum zero-crossing time of the DCF as the correlation time-scale and get the time-scale of variability of 42.5 mins on 2013 October 20 (in Fig. ??). Then according to Gupta et al. (2012), the mass of BH can be estimated by,

$$M_{\text{BH}} = \frac{c^3 \Delta t_{\text{obs}}}{16G(1+z)}.$$  

For our target, $M_{\text{BH}}$ is calculated to be $0.49 \times 10^8 M_\odot$. If the variations arise in the jets and are not explicitly related to the inner region of the accretion disc, the BH mass estimation is invalid.

We might try to launch possible multiwavelengths observation campaign in the future to gain a much more comprehensive understanding of the physical model of blazars.

6 CONCLUSIONS

Our conclusions are summarized as follows:

- We carried out a four-colour monitoring programme on BL Lacertae from 2012 to 2016. The variations were well correlated in all bands.
- The amplitude of IDV is greater in higher energy bands.
- After the host galaxy contribution was removed, the source exhibits a BWB trend. The spectral indices varied slightly.
- The possible time delays are about 10 min between variations in the $V$ and $R$ bands. The IDV light curves with flares are helpful to study time-lag detection. Further observations programme with a temporal resolution of about 1 min are needed to validate our time-lag results.
- Our data can be well fit by the model of individual synchrotron pulses.

ACKNOWLEDGEMENTS

We would like to thank the anonymous referee for insightful comments and suggestions to improve this manuscript. This work has been supported by the National Basic Research Programme of China 973 Program 2013CB834900; Chinese National Natural Science Foundation grants 11273006 and U1531242; and the fundamental research funds for the central universities and Beijing Normal University.
REFERENCES

Agarwal, A., & Gupta, A. C. 2015, MNRAS, 450, 541
Agarwal, A., Gupta, A. C., Bachev, R., et al. 2016, MNRAS, 455, 680
Alexander, T. 1997, Astronomical Time Series, 218, 163
Böttcher, M., Marscher, A. P., Ravasio, M., et al. 2003, ApJ, 596, 847
Bach, U., Villata, M., Raiteri, C. M., et al. 2006, A&A, 456, 105
Bessell, M. S., Castelli, F., & Plea, B. 1998, A&A, 333, 231
Bhatta, G., Webb, J. R., Hollingsworth, H., et al. 2015, A&A, 558, A92
Bignall, H. E., Jauncey, D. L., Lovell, J. E. J., et al. 2003, ApJ, 585, 653
Capetti, A., Raiteri, C. M., & Buttiglione, S. 2010, A&A, 516, A59
Carini, M. T., Miller, H. R., Noble, J. C., & Goodrich, B. D. 1992, AJ, 104, 15
Covino, S., Baglio, M. C., Foschini, L., et al. 2015, A&A, 578, A68
D’Ammando, F., Raiteri, C. M., Villata, M., et al. 2013, A&A, 558, A92
Gaskell, C. M., & Peterson, B. M. 1987, ApJS, 65, 1
Gaur, H., Gupta, A. C., Bachev, R., et al. 2015, MNRAS, 452, 4263
Gaur, H., Gupta, A. C., Wiita, P. J., et al. 2014, ApJ, 781, L4
Ghisellini, G., Tavecchio, F., Foschini, L., et al. 2010, MNRAS, 402, 497
Gupta, A. C., Fan, J. H., Bai, J. M., & Wagner, S. J. 2008, AJ, 135, 1384
Gupta, S. P., Pandey, U. S., Singh, K., et al. 2012, New Astron., 17, 8
Hagen-Thorn, V. A., Larionov, V. M., Horvat, T., et al. 2009, A&A, 494, 527
Howard, E. S., Webb, J. R., Pollock, J. T., & Stencil, R. E. 2004, AJ, 127, 17
Hovatta, T., Valtaoja, E., Tornikoski, M., & Lähteenmäki, A. 2009, A&A, 499, 433
Ikejiri, Y., Uemura, M., Sasada, M., et al. 2011, PASJ, 63, 639
Impey, C. D., Bychkov, V., Tapia, S., Gnedin, Y., & Pustilnik, S. 2000, AJ, 119, 1542
Kirk, J. G., Rieger, F. M., & Mastichiadis, A. 1998, A&A, 333, 452
Mannucci, F., Basile, F., Poggianti, B. M., et al. 2001, MNRAS, 326, 745
Marscher, A. P. 1996, Blazar Continuum Variability, 110, 248
Marscher, A. P. 2014, ApJ, 780, 87
Marscher, A. P., Jorstad, S. G., D’Arcangelo, F. D., et al. 2008, Nature, 452, 966
Miller, H. R., Carini, M. T., & Goodrich, B. D. 1989, Nature, 337, 627
Miller, J. S., & Hawley, S. A. 1977, ApJ, 212, L47
Nesci, R., Maesano, M., Massaro, E., et al. 1998, A&A, 332, L1
Papadakis, I. E., Boumis, P., Samarakis, V., & Papamastorakis, J. 2003, A&A, 397, 565
Papadakis, I. E., Villata, M., & Raiteri, C. M. 2007, A&A, 470, 857
Peterson, B. M., Ferrarese, L., Gilbert, K. M., et al. 2004, ApJ, 613, 682
Peterson, B. M., Wanders, I., Horne, K., et al. 1998, PASP, 110, 660
Racine, R. 1970, ApJ, 159, L99
Raiteri, C. M., Villata, M., Capetti, A., et al. 2009, A&A, 507, 769
Raiteri, C. M., Villata, M., D’Ammando, F., et al. 2013, MNRAS, 436, 1530
Sasada, M., Uemura, M., Arai, A., et al. 2008, PASJ, 60, L37
Scarpa, R., Urry, C. M., Falomo, R., Peace, J. E., & Treves, A. 2000, ApJ, 532, 740
Schneider, P., & Weiss, A. 1987, A&A, 171, 49
Smith, P. S., Balonek, T. J., Heckert, P. A., Elston, R., & Schmidt, G. D. 1985, AJ, 90, 1184
Soria, M., Benitez, E., Hiriart, D., et al. 2013, ApJS, 206, 11
Speziali, R., & Natali, G. 1998, A&A, 339, 382
Stalin, C. S., Gopal-Krishna, Sagar, R., et al. 2006, MNRAS, 366, 1337
Tommasi, L., Palazzi, E., Pian, E., et al. 2001, A&A, 376, 51
Urry, C. M., & Padovani, P. 1995, PASP, 107, 803
Vagnetti, F., Trevese, D., & Neschi, R. 2003, ApJ, 590, 123
Villata, M., Raiteri, C. M., Aller, H. D., et al. 2004, A&A, 424, 497
Villata, M., Raiteri, C. M., Kurtanidze, O. M., et al. 2002, A&A, 390, 407
Villata, M., Raiteri, C. M., Kurtanidze, O. M., et al. 2003, High Energy Blazar Astronomy, 299, 221
Wagner, S. J., & Witzel, A. 1995, ARA&A, 33, 163
Webb, J. 2016, Galaxies, 4, 15
Webb, J. R., Freedman, I., Howard, E., et al. 1998, AJ, 115, 2244
Westfold, K. C. 1959, ApJ, 130, 241
Wierzcholska, A., Ostrowski, M., Stawarz, L., Wagner, S., & Hauser, M. 2015, A&A, 573, A69
Wiita, P. J. 1996, Blazar Continuum Variability, 110, 42
Woo, J.-H., & Urry, C. M. 2002, ApJ, 579, 530
Wu, J., Böttcher, M., Zhou, X., et al. 2012, AJ, 143, 108

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