$(g - 2)_\mu$ and CP Asymmetries in $B^0_{d,s} \to l^+l^-$ and $b \to s\gamma$ in SUSY models

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Abstract

We show that with a good fit to the muon $g - 2$ constraint, the CP asymmetry can be as large as 25% (15%) for $B^0_d \to \tau^+\tau^-$ ($B^0_d \to \mu^+\mu^-$) in SUGRA models with nonuniversal gaugino masses and MSSM. If tau events indentified at B factories the CP asymmetry in $B^0_d \to \tau^+\tau^-$ can be a powerful probe on physics beyond SM. An interesting case is that new physics gives a prediction on the branching ratio which is still in the uncertain region of the SM prediction. The CP asymmetry for $B^0_d \to l^+l^-$ in this case can still reach 20%. So it is powerful to shed light on physics beyond SM while the CP asymmetry of $b \to s\gamma$ in this case can only reach 2% at most which is too small to draw a definite conclusion on new physics effects at B factories.

The FCNC process $B^0_{d,s} \to l^+l^-$ has been shown in recent years to be a powerful process to shed light on new physics beyond SM \cite{1,2,3,4,5,6} especially for SUSY models which may enhance the decay amplitude by $\tan^3\beta$ \cite{2,3}, provided that $\tan\beta$ is large (say, $\gtrsim 20$). It became more interesting recently after the Brookhaven National Laboratory (BNL) reported the 2.6$\sigma$ excess of the muon anomalous magnetic moment $a_\mu = (g - 2)_\mu/2$ over its SM value: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (43 \pm 16) \times 10^{-10}$ \cite{7}. SUSY models with large $\tan\beta$ (say $\gtrsim 10$) are favored by this excess. It was shown for mSUGRA in \cite{3} that with a good fit to $(g - 2)_\mu$ the branching ratio (Br) of $B^0_{d,s} \to l^+l^-$ can be enhanced by 100 times and within good reach at Tevatron Run II. Over the last two months the theoretical prediction of $a_\mu$ in the standard model (SM) has undergone a significant revision due to the change in sign of the light by light hadronic correction to $a_\mu$, which leads to

$$\Delta a_\mu = 26(16) \times 10^{-10}$$

(1)
corresponding to a 1.6$\sigma$ deviation from SM \cite{8}. In this letter we will examine the CP violating effects of the process in the light of this new result of $\Delta a_\mu$.

While there is no direct CP violation for this process, there might be CP violation induced by mixing of $B^0$ and $\bar{B}^0$ in the process

$$B^0 \to \bar{B}^0 \to f \quad \text{vs.} \quad \bar{B}^0 \to B^0 \to \bar{f}.$$  

(2)

One can define the CP violating observable as

$$A_{CP} = \frac{\int_0^\infty dt \sum_{i=1,2} \Gamma(B^0_{\text{phys}}(t) \to f_i) - \int_0^\infty dt \sum_{i=1,2} \Gamma(\bar{B}^0_{\text{phys}}(t) \to \bar{f}_i)}{\int_0^\infty dt \sum_{i=1,2} \Gamma(B^0_{\text{phys}}(t) \to f_i) + \int_0^\infty dt \sum_{i=1,2} \Gamma(\bar{B}^0_{\text{phys}}(t) \to \bar{f}_i)}.$$  

(3)
where \( f_{1,2} = \bar{l}_{L,R} l_{L,R} \) with \( l_{L,R} \) being the helicity eigenstate of eigenvalue \(-1(+1)\), \( \tilde{f} \) is the CP conjugated state of \( f \), i.e. \( \tilde{f}_{1,2} = l_{R,L} \bar{l}_{R,L} \). In the approximation \( |\frac{q}{p}| = 1 \) it is

\[
A_{CP} = -\frac{2Im(\xi)X_q}{(1 + |\xi|^2)(1 + X_q^2)}, \quad q = d, s,
\]

where \( X_q = \frac{\Delta m_q}{M_{\pm}}(q = d, s \text{ for } B^0_\mu \text{ and } B^0_\tau \text{ respectively}) \), \( p \) and \( q \) are parameters diagonalising the B meson mass matrix: \( |B^0_{L,H}| = p|B^0| \pm q|\bar{B}^0| \) (|\( p \|^2 + |q \|^2 = 1 \), and \( \xi \) is

\[
\xi = \frac{C_{Q_i}1\sqrt{1 - 4\hat{m}^2} + (C_{Q_4} + 2\hat{m}C_{10})}{C_{Q_1}1\sqrt{1 - 4\hat{m}^2} - (C_{Q_4} + 2\hat{m}C_{10})}
\]

with \( \hat{m}_i = m_i / m_{B_0} \). In Eq. (4) \( C_{Q_i} \)'s are Wilson coefficients accounting for neutral Higgs contributions, and \( C_{10} \) is the Wilson coefficient for the axial vector operator (for details, see refs. [1, 2, 4]). In deriving Eq. (4) we have used \( q = -\frac{M^{*}_{\mu}}{|M_{12}|} = \frac{\tilde{\Delta}_\mu}{A} \) which is the result for \( B^0 - \bar{B}^0 \) mixing in the SM. This is a good approximation in the MSSM if one limits himself to the regions with large \( \tan \beta \) (say, larger than 10 but smaller than 60), not too light charged Higgs boson (say, larger than 250 Gev), and heavy sparticles, and in the scenarios of the minimal flavor violation (MFV) [1] without new CP violating phases there is also no correction to the result [2, 10]. In MFV models with new CP violating phases, e.g., in the SUGRA with nonuniversal gaugino masses which we consider in the latter, a rough estimate gives that the correction to the SM value of \( q/p \) is below 20% in the parameter space we used in the latter. The detailed analysis of the effects on \( B - \bar{B} \) mixing of new CP violating phases in such models will be given in the forthcoming paper [11]. It is easy to see from Eq. (4) that the kinetic upper bound of \( A_{CP} \) is \( X_q/(1 + X_q^2) \) which is about 48% for \( B^0_\mu \) and 6.3% for \( B^0_\tau \). The SM predictions for these observables are of order \( 10^{-3} \) for \( B^0_\mu (B^0_\tau) \) if corrections to \( |\frac{\xi}{p}| = 1 \) included [2].

In our approximation they are zero in SM. We shall concentrate on the decay modes \( B^0_\mu \rightarrow l^+l^- \) and it is straightforward to apply to \( B^0_\tau \rightarrow l^+l^- \).

Before presenting numerical results, we can make an estimate on the magnitude of the CP asymmetry in the process. For the case of SUSY contributions dominated (e.g. \( Br \) enhanced by 100 times larger) the main contributions to \( C_{Q_i} \)'s are from the FCNC self-energy type diagram with neutral Higgs bosons coupled to the external bottom quark [2, 3]. So one may get \( m^2_{H_0} C_{Q_1} \approx -m^2_{A_0} C_{Q_2} \) (the light neutral Higgs contribution is not important in general by observing that the light neutral Higgs should decouple to resemble the SM Higgs if the SUSY spectrum is a little of bit high or \( \tan \beta \) is large). Consequently for \( l = e, \mu \) we have \( |\xi| \approx |(m^2_{H_0} - m^2_{A_0})/(m^2_{H_0} + m^2_{A_0})| \). This is a small number in most regions of the parameter space of MSSM, since in the case of large \( m_{A_0} \) (\( m^2_{A_0} \gg m^2_\tau \)) \( m_{H_0} \) and \( m_{A_0} \) are aligned with just a few percents discrepancies at most. This estimate is polluted in two ways. One is that penguin and box diagrams give a deviation from the relation \( m^2_{H_0} C_{Q_1} = -m^2_{A_0} C_{Q_2} \) which should be less than 5% for \( \tan \beta \) larger than 20. Another one is given by the appearance of \( 2\hat{m}_E C_{10} \).

\footnote{Note that the explicit expressions of the Wilson coefficients \( C_{Q_i} \)'s are the same in SUSY models with and without CP violating phases. In the SUSY models with CP violating phases the coefficients become complex since the new CP violating phases enter into squark and charged mass matrices [10].}

\footnote{MFV means the models in which the CKM matrix remains the unique source of flavor violation. The models have further be subdivided into the so-called MFV models and the generalized MFV models (GMFV) in ref. [1].}
in Eq. (3) which is about $-0.18$ and can give about 10%\% deviation. Interferences among all of these can give the CP asymmetry for $B^0_d \to \mu^+\mu^-$ of 15% if the phase of $\xi$ is large ($\sim \pi/2$).

Obviously this estimation is not valid for $l = \tau$ since the factor $\sqrt{1 - 4m^2} \approx 0.74$ is now important. CP asymmetry can then be increased by about 10% due to this factor.

For muon $g - 2$, it has been shown that in most of the parameter space in SUSY models the chargino exchange dominates the contribution\cite{13} which is proportional to $\sum_j \tan \beta/m_{\tilde{\chi}^+_j} Re(V_j^* U_{j2})$ (where $V$, $U$ are the matrices diagonalising the chargino mass matrix and the loop function is not shown) in the case of large $\tan \beta$. SUGRA models with nonuniversal gaugino masses have been reexamined \cite{14} in the light of the muon $g - 2$ constraint. It was shown that 60 -- 90\% of the regions of the parameter space are excluded by it. They are mainly regions with low $\tan \beta$, as expected. In the following we will consider scenarios with large $\tan \beta(> 10)$ and discuss the BNL constraint on the SUGRA model with nonuniversal gaugino masses. The values of CP violating phases in the model are obtained by satisfying the constraints from the electric dipole moments (EDMs) of the electron and of the neutron based on the cancellation mechanism\cite{15, 16}. In the SUGRA model with nonuniversal gaugino masses \cite{13, 17}, compared with mSUGRA \cite{16} there are two more real parameters (say, $|M_1|$ and $|M_2|$, where $M_1$ and $M_2$ are gauge masses corresponding to $U(1)$ and $SU(3)$ respectively), and two more independent phases arising from complex gaugino masses, which make the cancellations among various SUSY contributions to EDMs easier than in mSUGRA. Therefore relatively large values of the phase of $\mu$ are allowed. It was shown in \cite{16} that in the SUGRA model $\phi_\mu$ is bounded to be about 0.5 at most (as in \cite{16} we work in the convention that $B\mu$ and $M_2$ are real and the remaining phases of $A_0$, $M_3$ and $M_3$ and $M_\tau$ are then considered physical) while the phases of gaugino masses can be of order one and give interesting implications on CP violation of B decays.\cite{16} Accordingly, it is expected that in the SUGRA model the correlation between $Br(B_{d,s}^0 \to l^+l^-)$ and $a^{\mu}_{SUSY}$ which was shown for CP conserving mSUGRA in \cite{16} will be altered just a little by the presence of $\phi_\mu$ in the chargino mass matrix and the qualitative properties should be the same, that is, for a reasonable value of $a^{\mu}_{SUSY}$ we can get a quite large enhancement of $Br(B_{d,s}^0 \to l^+l^-)$ over its SM value. Our numerical calculations confirmed the expectation.

As well known, inclusive $b \to s\gamma$ is also sensitive to $\tan \beta$ in SUSY models and the CP asymmetry in $b \to s\gamma$ can be large (say 10%) in models beyond SM \cite{19, 20}, which is well under the probe at B factories. So while studying the CP violation of $B_{d,s}^0 \to l^+l^-$ we should also consider $b \to s\gamma$ and we would like to know what implications on $B_{d,s}^0 \to l^+l^-$ we can get if the CP asymmetry in $b \to s\gamma$ is measured to a good precision. The CP asymmetry in $b \to s\gamma$ is

$$a_{CP}^{b\to s\gamma}(\delta) = \frac{\Gamma(\bar{B} \to X_s\gamma) - \Gamma(B \to X_s\gamma)}{\Gamma(\bar{B} \to X_s\gamma) + \Gamma(B \to X_s\gamma)}\bigg|_{E_{\gamma} > (1 - \delta)E_{\gamma}^{max}}$$

$$= \frac{a_s(m_b)}{|C_7|^2} \left\{ \left[ \frac{40}{81} - \frac{8z}{9} [v(z) + b(z, \delta)] \right] \text{Im}(C_2C_7^*) - \frac{4}{9} \text{Im}(C_6C_7^*) + \frac{8z}{27} b(z, \delta) \text{Im}(C_2C_8^*) \right\} \ (6)$$

\footnote{In mSUGRA there in general are only four real parameters (the universal scalar and gaugino masses $M_0$ and $M_{1/2}$, the trilinear term $A_0$ and $\tan \beta$) and two CP violating phases (the phase of the Higgsino mass parameter $\mu$ ($\phi_\mu$) and the phase of $A_0$).}

\footnote{Essentially the effects of the phases of gaugino masses on $C_8$’s (and consequently CP violation in B decays) come from the running of $A_1$ since $A_1$ at the EW scale mainly depends on $M_3$ at the high energy (GUT) scale through RGE effects. For details, see ref. \cite{16} \cite{18}.}
Now the result with complete data has appeared\cite{24}. It is shown that there is a narrow region in which the $\sigma$ result (1) with a 100% CL. For the lightest Higgs boson mass $m_h$ we set $m_h \geq 112$ GeV.\footnote{In the autumn of year 2000 a 2$\sigma$ excess of Higgs boson signal was reported at energy of about 115 GeV.}

Figure 1: (a) $a_{CP}(b \rightarrow s\gamma)$ versus $A_{CP}(B^0_{d,s} \rightarrow \mu^+\mu^-)$ in the parameter space $150 < |M_1| < 350, 150 < M_2 < 300, 150 < |M_3| < 500, |A_0| = 300, M_0 = 250$ and $10 < \tan\beta < 50$; (b) $a_{CP}(b \rightarrow s\gamma)$ versus $a^{SUSY}_{\mu}$.

where $\sqrt{z} = m_c/m_b = 0.29$, $v(z)$ and $b(z, \delta)$ can be found in\cite{19}. Among all of the terms the important are interferences between $C_7$ and $C_2$ and between $C_8$ and $C_7$ (in the following we denote this CP asymmetry as $a_{CP}$ to avoid the confusion with that of $B^0_{d,s} \rightarrow l^+l^-$). It is the imaginary part of $C_7$ that plays the main role. Because $Im(C_7)$ for which SUSY contributions are responsible linearly depends on $\tan\beta$ in the large $\tan\beta$ case, we expect that if $a_{CP}$ is observed to be large (note that in this case the 2$\sigma$ Br bound, $2.0 \times 10^{-4} < Br(b \rightarrow s\gamma) < 4.5 \times 10^{-4}$\cite{21}, can be satisfied through the cancellation between the SM and SUSY contributions), $a^{SUSY}_{\mu}$ must also be large. $A_{CP}$, in this case, should also be significant by observing that SUSY contributions to $Q_{di}$’s share the vertex similar to that contributing to $C_7$. However, large enough $A_{CP}$ does not necessarily imply large enough $a_{CP}$. This is because $C_{Qi}$’s (depending on $\tan^3\beta$) can easily dominate the process $B^0_{d,s} \rightarrow l^+l^-$ while $\tan\beta$ is not large enough to make $a_{CP}$ significant. Until now the result on $a_{CP}$ is still preliminary: $-0.27 < A(b \rightarrow s\gamma) < 0.10$ at 90% CL\cite{22}.

Our results are shown in Figs. 1 and 2. In Fig.1(a) we give a plot for the correlation between the CP asymmetries of $B^0_d \rightarrow \mu^+\mu^-$ and $b \rightarrow s\gamma$ (for $\delta = 0.9$) with points scanned in the region as shown in the figure caption (in which the long-lived chargino mass bound is satisfied and the neutral LSP constraint included). Dimensionful parameters are with unit GeV. Besides the $b \rightarrow s\gamma$ Br bound, experimental constraints also considered include the muon $g - 2$ bound (the result \cite{1} with a 1$\sigma$ error corridor), $10 \times 10^{-10} < \Delta a_{\mu}(= a^{SUSY}_{\mu}) < 42 \times 10^{-10}$, the Br bound of $B^0_s \rightarrow \mu^+\mu^-$, $Br(B^0_s \rightarrow \mu^+\mu^-) < 2.0 \times 10^{-6}$ (90% CL)\cite{23} (the bound from $B^0_l \rightarrow \mu^+\mu^-$ is weaker and not relevant). For the lightest Higgs boson mass $m_{h_0}$ we set $m_{h_0} \geq 112$ GeV.\footnote{In the autumn of year 2000 a 2$\sigma$ excess of Higgs boson signal was reported at energy of about 115 GeV. Now the result with complete data has appeared\cite{24}. It is shown that there is a narrow region in which the $m_{h_0}$ and $m_{A_0}$ are allowed to both reach 89 GeV. Beyond this region $m_{h_0}$ and $m_{A_0}$ are bounded from below.
In calculating the Br and CP asymmetries we have included the higher loop corrections due to
the large bottom Yukawa coupling in the large tan β case\(^{25}\). The plot is shown for
\(Br(B^0_d \to \mu^+\mu^-)\) enhanced by at least 5 times larger than that of SM, i.e., \(\gtrsim 7.5 \times 10^{-10}\).
One may see that two asymmetries do not exhibit a strong correlation. Fig.1(b) is devoted to the correlation of \(a_{CP}\) and \(a^{SUSY}_\mu\) without the requirement \(Br(B^0_d \to \mu^+\mu^-) \gtrsim 7.5 \times 10^{-10}\).
One may find that 5% \(a_{CP}\) requires more than a 2\(\sigma\) deviation of \(a^{SUSY}_\mu\). Back to Fig.1(a), one may find that for \(a_{CP}\) of about 2% \(A_{CP}\) can still reach 10%. This means that one may use \(A_{CP}(B^0_d \to \mu^+\mu^-)\) to determine CP phases if \(a_{CP}\) is just about 2% which is hard to give an answer on new physics because of the about 1% SM prediction\(^{19}\). It’s also clear that for \(a_{CP}\) just above 2% \(A_{CP}\) would be inevitably large (\(\gtrsim 10\%\)). This is exactly what we expected. Unfortunately, the Br of this process is too small. Even in Tevatron Run IIb experiments with 20 fb\(^{-1}\) integrated luminosity collected, only several tens events can be got for Br enhanced by 100 times. One should wait LHCb experiments to get an answer on it. We have also calculated CP violation in \(B^0_s \to \mu^+\mu^-\) in the same region as that shown in the caption of Fig. 1 and the result is that the CP asymmetry can reach about 1.5%.

Problems will be overcome if tau events are identified since \(B \to \tau^+\tau^-\) has the Br over 160 times larger than that of \(B \to \mu^+\mu^-\). For a quite large part of points shown in Fig.1(a), corresponding \(A_{CP}(B^0_d \to \tau^+\tau^-)\) can reach 25%. If 900 events collected, for example, for which with the 20 fb\(^{-1}\) integrated luminosity the Br only needs to be enhanced by about 30 times of the SM prediction, then even all the errors including those of b-tagging, tau-tagging and the statistical altogether are of about 8% which is quite large, one may still determine the CP asymmetry, \(A_{CP}\), to a 3\(\sigma\) level. Since tau decays before it reaches detector, the helicity eigenstates can be measured according to the final lepton energy. The CP asymmetries defined referring to definite helicity, i.e. \(A_{CP}^i\)'s (\(i=1,2\))\(^{12}\), can then be useful. Because \(|\xi|\) is in general a number less than one, \(A_{CP}^1(B^0_d \to \tau^+\tau^-)\) is small because of \(X_d \approx 0.74\), while the magnitude of \(A_{CP}^2(B^0_d \to \tau^+\tau^-)\) is larger than \(A_{CP}\) and its kinematic maximum is 63%\(^{12}\). Basically \(A_{CP}\) and \(A_{CP}^2\) can give independent implications on the CP violating phases. However at the Tevatron only for the case of \(Br(B^0_d \to \tau^+\tau^-)\) enhanced by two orders of magnitude over its SM value it is possible to probe \(A_{CP}^2\) to a good precision. The reason is that \(Br(B^0_d \to \tau^+\tau^-)\) is proportional to \(|C_{Q1}\sqrt{1-4m^2_\tau}+(C_{Q2}+2m_\tau C_{10})|^2\) which is suppressed by an order of magnitude compared with the total Br because of the cancellation between \(C_{Q1}\) and \(C_{Q2}\), as we noted.

In the above analysis we assume that the Br of the process is increased a lot compared with that in the SM. This possibility can be probed by searching for \(B^0_s \to \mu^+\mu^-\) at Tevatron Run II. In fact there are large regions of the parameter space, e.g., for tan β of 10 – 20, in which the Br can not be increased too much. In particular, there is a possibility that the Br is increased at most by just about 60% compared with that of SM which is in the uncertain region of the SM prediction. For example, for \(B^0_d \to \tau^+\tau^-\) if enhanced by 60% then the Br is \((4.9 \pm 1.6) \times 10^{-8}\). It still has overlap with the SM prediction: \((3.1 \pm 1.0) \times 10^{-8}\). An especially interesting question then is whether one can still learn something about new physics from its CP violating effects if the Br of the process should be confirmed to be in the uncertain region of the SM prediction. To answer this question, we plot the correlations of \(A_{CP}(B^0_d \to \tau^+\tau^-)\) with \(a^{SUSY}_\mu\) in Fig.2(a),

by 112 – 113GeV. To stress the importance of our study we therefore leave that narrow region and make a conservative study by assuming that the Higgs boson mass should not be less than 112 GeV.

\(^3\)We also calculate the correlation of \(a^{SUSY}_\mu\) with \(A_{CP}\). The result is that for a 1\(\sigma\) deviation of \(a^{SUSY}_\mu\), \(A_{CP}\) can reach 10%. 

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setting the calculated center value of the Br to satisfy $Br(B^0_d \rightarrow \tau^+\tau^-) < 5.0 \times 10^{-8}$, scanning the parameter space as shown in Fig.1. The answer is obviously yes. CP asymmetry can reach 20% allowed by the muon $g-2$ constraint within 2σ deviations, which could be probed at the LHCb. We would like to note that $A_{CP}(B^0_d \rightarrow \mu^+\mu^-)$ in this case may not be reduced by about 10% compared with $A_{CP}(B^0_d \rightarrow \tau^+\tau^-)$ and can also reach 20%. Fig2(b) is devoted to the correlation between $A_{CP}(B^0_d \rightarrow \mu^+\mu^-)$ and $a_{CP}(b \rightarrow s \gamma)$. We may observe that $a_{CP}$ can reach only 2% while $A_{CP}$ can be as large as 20%. In this case $Br(B^0_d \rightarrow \tau^+\tau^-)$ is generally of the same order of the total $Br(B^0_d \rightarrow \tau^+\tau^-)$. Consequently $A^2_{CP}$ and $A_{CP}$ would exhibit a strong correlation.

We have done analyses in the SUGRA model with non-universal gaugino masses. For MSSM which has many more CP violating phases, the qualitative features of our results are also valid at least in the scenario of minimal flavor violation (MFV). As shown in ref. [27], the $a_{SUSY}^{\mu}$ dependence on phases from both the chargino and neutralino exchanges consists only of three combinations which depend on the phases of $\mu, A_{\mu}$ and gaugino masses. Compared with the SUGRA model we used above, there is one more phase, the phase of $A_{\mu}$, which enters $a_{SUSY}^{\mu}$ through the neutralino exchange. Because the regions of the parameter space we considered in the letter, as noted before, are the regions where the chargino exchange contribution dominates, it makes no significant effects on our result. Although the presence of phases of $A_{u,d,e}$ makes larger $\phi_{\mu}$ possible which may reduce $a_{SUSY}^{\mu}$, it is still hard for $\phi_{\mu}$ to reach one in the large tan $\beta$ case so that the effects of the phases of $A_{u,d,e}$ on $a_{SUSY}^{\mu}$ are small and consequently negligible. It is obvious that these more phases have no significant effects on the CP asymmetries in $b \rightarrow s \gamma$ and $B^0_{d,s} \rightarrow l^+l^-$ since the main contributions to the processes $b \rightarrow s \gamma$ and $B^0_{d,s} \rightarrow l^+l^-$ come from the third generation quarks and their superpartners. Therefore, we conclude that

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*If relaxing to the scenario of non MFV, e.g., there are non-diagonal trilinear terms, then the non-diagonal trilinear terms will make significant effects on the CP asymmetry in $b \rightarrow s \gamma$.*

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Figure 2: (a) $a_{SUSY}^{\mu}$ versus $A_{CP}(B^0_d \rightarrow \tau^+\tau^-)$ with $Br(B^0_d \rightarrow \tau^+\tau^-) < 5.0 \times 10^{-8}$; (b) $a_{CP}(b \rightarrow s \gamma)$ versus $A_{CP}(B^0_d \rightarrow \mu^+\mu^-)$. 

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*If relaxing to the scenario of non MFV, e.g., there are non-diagonal trilinear terms, then the non-diagonal trilinear terms will make significant effects on the CP asymmetry in $b \rightarrow s \gamma$. [28].

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qualitatively our results should be valid in quite large regions of the parameter space in MSSM.

We stress that CP asymmetry of $B^0_{d,s} \to l^+l^-$ is theoretically very clean because the decay constant, which is the only source of hadronic uncertainty for the process, is cancelled out for the asymmetry. The above analysis have shown that CP violation of $B^0_d \to \tau^+\tau^-$ is highly interesting for finding new physics and could be observed at Run II of the Tevatron if tau events can be identified. Based on the expectation we encourage experimentalists to do a good job on tau-tagging.

In the analysis so far we didn’t include the effects from the mixing of CP-odd and CP-even neutral Higgs bosons in SUSY models[24]. This effect is expected to increase the mass difference between $A^0$ and $H^0$ in the presence of CP violating phases. Roughly $(m_{H^0}^2 - m_{A^0}^2)/(m_{H^0}^2 + m_{A^0}^2)$ can be increased up to 10% for $m_{H^\pm}$ of 200−300 GeV, which implies larger CP asymmetry in the $B^0_{d,s} \to l^+l^-$ process.

In conclusion, we have shown the correlation of the CP asymmetry in $B^0_{d,s} \to l^+l^-$ with the muon anomalous magnetic moment $a_\mu$ and the CP asymmetry in $b \to s\gamma$ in SUSY models. Besides the branching ratio, the CP asymmetry $A_{CP}$ of the $B^0_{d,s} \to l^+l^-$ ($l=\mu, \tau$) process is also highly interesting to shed light on physics beyond SM. Compared with the branching ratio, theoretically CP asymmetry of $B^0_{d,s} \to l^+l^-$ is of more advantages because the common uncertain decay constant cancels out in the definitions of the CP violating observables. CP asymmetry of $B^0_d \to l^+l^-$ can be as large as several tens percents while the constraints from the muon anomalous magnetic moment and $b \to s\gamma$ are satisfied. If large CP violating effects were to be found out in the $b \to s\gamma$ process, CP asymmetry of $B^0_d \to l^+l^-$ would be inevitably large. On the other hand for smaller than 2% CP asymmetry of $b \to s\gamma$, $B^0_d \to l^+l^-$ can still have CP violating effects of 10%(20%) for $l = \mu(\tau)$. If tau events identified with 6% tagging error, one can measure $A_{CP}(B^0_d \to \tau^+\tau^-)$ to a 3$\sigma$ level at Run II of Tevatron with $Br(B^0_d \to \tau^+\tau^-)$ enhanced by a factor of about 30 compared to that of SM. It is interesting when the nature prefers a scenario for which new physics only increases the branching ratio a little so that it is still in the uncertain region of the SM prediction. CP asymmetry of $B^0_d \to l^+l^-$ in this scenario can still reach 20% allowed by the muon $g-2$ constraint within 2$\sigma$ deviations. Therefore it can provide a powerful way to probe new physics beyond the SM, while CP violation of $b \to s\gamma$ can only be about 2% at most which is not large enough to probe new physics since the prediction for CP violation of $b \to s\gamma$ in SM is of order 1% in magnitude[19].

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