On the factor set of code loops *‡

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Abstract

A Code loop on a binary linear code that is doubly even with a factor set is shown to be a central loop, conjugacy closed loop, Burn loop and extra loop. General forms of the identities that define the factor set of a code are deduced.

1 Introduction

Let $L$ be a loop and $Z \leq Z(L)$. If $L/Z \cong C$ where $C$ is an abelian group then, $L \cong L(\phi)$ where $\phi : C \times C \rightarrow F$, $F$ been a field and $L(\phi) = F \times C$ with binary operation $*$ such that for all $(a, u), (b, v) \in L(\phi)$:

$$(a, u) * (b, v) = (a + b + \phi(u, v), u + v)$$

and $(L(\phi), *)$ is a loop. The fact that $(L(\phi), *)$ is a loop is shown in [13].

Let $V$ be a finite dimensional vector space over the field $F$ and $C \leq V$. With $|F| = 2$ such that $F = \mathbb{Z}_2 = \{0, 1\}$, $C$ is called a binary linear code and its elements (vectors) are called code words. For all $u, v \in C$, let $||u||$ denote the number of non-zero co-ordinates in $u$ (this is simply equal to the ’norm squared’ of $u$ if $V = \mathbb{R}^n$), let $u \cdot v$ denote the number of the corresponding co-ordinates of $u$ and $v$ that are non-zero (this is simply equal to the ’inner product’ of $u$ and $v$ if $V = \mathbb{R}^n$) and let $c(u, v, w)$ represent the number of corresponding co-ordinates of $u, v$ and $w$ that are non-zero.

$C$ is called a double even code if :

1. $u \cdot v$ is even
2. $||u||$ is divisible by 4.

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\( \phi \) is called a factor set on \( C \) if it satisfy the conditions:

\[
\phi(u, u) \equiv \frac{||u||}{4} \mod 2 \quad (2)
\]

\[
\phi(u, v) + \phi(v, u) \equiv \frac{u \cdot v}{2} \mod 2 \quad (3)
\]

\[
\phi(u, v) + \phi(u + v, w) + \phi(v, w) + \phi(u, v + w) \equiv c(u, v, w) \mod 2 \quad (4)
\]

As mentioned in [4], for all \( u \in C \), \( \phi(u, 0) = \phi(0, u) = 0 \).

For more on the varieties of loops and their general properties, readers can check [1], [2], [3], [7] and [12].

Code loops were first shown to be Moufang loops by Griess [8] and was later shown to be Moufang loops with unique non-identity square in [5]. They have also been of interest to Chein and Goodaire [6] and Vojtĕchovský [9], [10], [11] and [13]. Here we shall approach them in the style of [5].

A Code loop on a binary linear code that is doubly even with a factor set is shown to be a central loop, conjugacy closed loop, Burn loop and extra loop. General forms of the identities that define the factor set of a code are deduced.

In all, LB, RB, LC, RC, C, LCC, RCC, and CC represent left Bol, right Bol, left central, right central, central, left conjugacy closed, right conjugacy closed and conjugacy closed respectively.

## 2 Preliminaries

**Definition 2.1** Let \((G, \circ)\) be a loop. \( G \) is a left(right) Burn loop if and only if it is a LB(RB)-loop and a LCC(RCC)-loop. It is called a Burn loop if and only if it is both a Moufang loop and a CC-loop.

**Definition 2.2** Let \( \phi \) be the factor set of a binary linear code \( C \) that is doubly even. \( \phi \) is called a left weakly linear (lwl) factor set on \( C \) if and only if \( \phi(u + v, u) = \phi(u, u) + \phi(v, u) \) \( \forall u, v \in C \).

Similarly, \( \phi \) is called a right weakly linear (rwl) factor set on \( C \) if and only if \( \phi(u, u + v) = \phi(u, u) + \phi(u, v) \) \( \forall u, v \in C \).

\( \phi \) is called a weakly linear (wl) factor set on \( C \) if and only if \( \phi \) is both a lwl and rwl factor set on \( C \).

**Lemma 2.1** Let \( \phi \) be the factor set of a binary linear code \( C \) that is doubly even.

1. If \( \phi \) is lwl, then \( \phi(u + v, u) \equiv [\phi(u, u) + \frac{u \cdot v}{2} + \phi(u, v)] \mod 2 \).
2. If \( \phi \) is rwl, then \( \phi(u, u + v) \equiv [\phi(u, u) + \frac{u \cdot v}{2} + \phi(v, u)] \mod 2 \).

**Proof**

In a doubly even code \( C \) with factor set \( \phi : \phi(u, v) + \phi(v, u) \equiv \frac{u \cdot v}{2} \mod 2 \). Thus:

\[
\phi(u, v) \equiv \left(\frac{u \cdot v}{2} + \phi(v, u)\right) \mod 2 \quad (5)
\]
\[ \phi(v, u) \equiv \left( \frac{u \cdot v}{2} + \phi(u, v) \right) \mod 2 \quad (6) \]

1. Since \( \phi \) is lwl, then using (6), \( \phi(u + v, u) \equiv [\phi(u, u) + \frac{u \cdot v}{2} + \phi(u, v)] \mod 2 \).

2. Since \( \phi \) is rwl, then using (5), \( \phi(u, u + v) \equiv [\phi(u, u) + \frac{u \cdot v}{2} + \phi(v, u)] \mod 2 \).

**Corollary 2.1** Let \( \phi \) be the factor set of a binary linear code \( C \) that is doubly even. If \( \phi \) is wl, then \( \phi(u + v, u) + \phi(u, u + v) \equiv \frac{u \cdot v}{2} \mod 2 \).

**Proof**
This follows by Lemma 2.1.

**Corollary 2.2** Let \( \phi \) be the factor set of a binary linear code \( C \) that is doubly even. The difference between \( \phi \) been rwl and lwl is equal to half of the number of corresponding coordinates that are non-zero in any two code words.

**Proof**
This follows from the fact in Corollary 2.1 above. \( \phi(u + v, u) - \phi(u, u + v) \equiv \frac{u \cdot v}{2} \mod 2 \) and \( \phi(u, u + v) - \phi(u + v, u) \equiv \frac{u \cdot v}{2} \mod 2 \). Hence the proof.

**Lemma 2.2** Let \( \phi \) be the factor set of a binary linear code \( C \) that is doubly even. If \( \phi \) is both rwl and lwl, then : \( \phi(u, u + v) = \phi(u + v, v) \iff \phi(u, v) = \phi(v, u) \forall u, v \in C \).

**Proof**
With \( \phi \) been rwl and lwl ; \( \phi(u, u + v) = \phi(u + v, v) \iff \phi(u, u) + \phi(u, v) = \phi(u, u) + \phi(v, u) \iff \phi(u, v) = \phi(v, u) \).

**Corollary 2.3** Let \( \phi \) be the factor set of a binary linear code \( C \) that is doubly even. If \( \phi \) is wl then : \( \phi(u, u + v) = \phi(u + v, v) \iff \phi(u, v) = \phi(v, u) \forall u, v \in C \).

**Proof**
This follows from Lemma 2.2.
Main Results

3. Code Loops and Central Loops

**Theorem 3.1** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). \( L(\phi) \) is an LC-loop if and only if any of the following equivalent conditions is true for all \( u, v, w \in C \).

1. \( A_1(u, v) = \phi(u, u + v) + \phi(u, v) + \phi(u, u) = 0. \)
2. \( A_2(u, v, w) = \phi(u, v) + \phi(u, u + v) + \phi(u, u + v + w) = 0. \)
3. \( A_3(u, v, w) = \phi(u, u) + \phi(u, v + w) + \phi(u, u + v + w) = 0. \)

**Proof**

1. \( L(\phi) \) is a LC-loop \( \iff xx * yz = (x * xy)z \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi). \)
   \( (a, u)(a, u) * (b, v)(c, w) = [(a, u) * (a, u)(b, v)](c, w) \iff (b + c + \phi(u, u) + \phi(v, w), v + w) = (b + c + \phi(u, v) + \phi(u, u + v) + \phi(v, w), v + w) \iff A_1(u, v) = \phi(u, u + v) + \phi(u, v) + \phi(u, u) = 0. \)
2. \( L(\phi) \) is a LC-loop \( \iff (x * xy)z = x(x * yz) \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi). \)
   \( [(a, u) * (a, u)(b, v)](c, w) = (a, u)[(a, u) * (b, v)(c, w)] \iff [(a, u) * (a + b + \phi(u, v), u + v)](c, w) = (a, u)[(a, u) * (b + c + \phi(v, w), v + w)] \iff (b + c + \phi(u, v) + \phi(u, u + v) + \phi(v, w), v + w) = (b + c + \phi(v, w) + \phi(u, u + v) + \phi(u, u + v + w), v + w) \iff A_2(u, v, w) = \phi(u, v) + \phi(u, u + v) + \phi(u, v + w) + \phi(u, u + v + w) = 0. \)
3. \( L(\phi) \) is a LC-loop \( \iff (xx * yz) = (x * yz) \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi). \)
   \( [(a, u)(a, u) * (b, v)](c, w) = (a, u)[(a, u)(b, v)(c, w)] \iff [(a, u) * (b + c + \phi(u, v), v + w)] = (a, u)[b + c + \phi(u, v) + \phi(u, v, w), v + w] \iff A_3(u, v, w) = \phi(u, u) + \phi(u, v + w) + \phi(u, u + v + w) + \phi(u, u + v + w) = 0. \)

**Corollary 3.1** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). \( L(\phi) \) is an LC-loop if and only if \( \phi \) is a rwl factor set.

**Proof**

Following Theorem 3.1 \( A_1(u, v) = 0 \iff \phi(u, u + v) = \phi(u, u) + \phi(u, v) \forall u, v \in C. \)

**Corollary 3.2** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). If \( L(\phi) \) is an LC-loop:

1. \( \phi(u, u + v) \equiv [\phi(u, u) + \frac{u + v}{2} + \phi(v, u)] \mod 2. \)
2. The difference between \( \phi \) been rwl and lwl is \( \frac{u + v}{2} \mod 2 \forall u, v \in C. \)
Proof
These follows from Lemma 2.1 and Corollary 2.2

Theorem 3.2 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is an RC-loop if and only if any of the following equivalent conditions is true for all $u, v, w \in C$.

1. $B_1(u, w) = \phi(u, u) + \phi(w, u) + \phi(w + u, u) = 0$.
2. $B_2(u, w, v) = \phi(w, u) + \phi(w + u, u) + \phi(v + w, u) + \phi(v + w + u, u) = 0$.
3. $B_3(u, v, w) = \phi(u, u) + \phi(v + w, u) + \phi(v + w + u, u) = 0$.

Proof

1. $L(\phi)$ is a RC-loop $\iff yz \ast xx = y(zx \ast x) \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi)$. $B(v)(c, w) * (a, u) * (a, u) = (b, v)[(c, w)(a, u) * (a, u) * (a, u) = (b, v) * (c + a + \phi(w, u), w + u)] \iff (b + c + \phi(v, w) + \phi(v + w + u) + \phi(v + w + u, u) + \phi(v + w + u, u, v + w) = (b + c + \phi(w, u) + \phi(w + u, u) + \phi(u, v), v + w) \iff B_1(u, w) = \phi(u, u) + \phi(w, u) + \phi(w + u, u) = 0$.

2. $L(\phi)$ is a RC-loop $\iff (yz \ast x)x = y(zx \ast x) \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi)$. $B(v)(c, w) * (a, u) * (a, u) = (b, v)[(c, w)(a, u) * (a, u) * (a, u) = (b, v) * (c + a + \phi(w, u), w + u)] \iff (b + c + \phi(v, w) + \phi(v + w + u) + \phi(v + w + u, u, v + w) = (b + c + \phi(w, u) + \phi(w + u, u) + \phi(v, w), v + w) \iff B_2(u, w, v) = \phi(w, u) + \phi(w + u, u) + \phi(v + w, u) + \phi(v + w + u, u) = 0$.

3. $L(\phi)$ is a RC-loop $\iff (yz \ast x)zx = y(zx \ast x) \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi)$. $B(v)(c, w) * (a, u) * (a, u) = (b, v)[(c, w)(a, u) * (a, u) * (a, u) = (b, v) * (c + a + \phi(w, u), w + u)] \iff (b + c + \phi(v, w) + \phi(v + w, u) + \phi(v + w + u, u, v + w) = (b + c + \phi(w, u) + \phi(w + u, u) + \phi(v, w), v + w) \iff B_3(u, v, w) = \phi(u, u) + \phi(v + w, u) + \phi(v + w + u, u) = 0$.

Corollary 3.3 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is an RC-loop if and only if $\phi$ is a lwl factor set.

Proof
Following Theorem 3.2 $B_1(u, w) = 0 \iff \phi(u + w, u) = \phi(u, u) + \phi(w, u) \forall u, w \in C$.

Corollary 3.4 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. If $L(\phi)$ is a RC-loop ;

1. $\phi(u + v, u) \equiv \phi(u, u) + \frac{u + v}{2} \equiv \phi(u, v) \mod 2$.

2. The difference between $\phi$ been rl and lwl is $\frac{u + v}{2} \mod 2 \forall u, v \in C$.

Proof
These follows from Lemma 2.1 and Corollary 2.2.
Corollary 3.5 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is a C-loop if and only if any of the following equivalent conditions is true for all $u, v, w \in C$.

Proof

1. $A_1(u, v) = 0$ and $B_1(u, v) = 0$.
2. $A_2(u, v, w) = 0$ and $B_2(u, v, w) = 0$.
3. $A_3(u, v, w) = 0$ and $B_3(u, v, w) = 0$.

L(\phi) is a C-loop if and only if it is both a LC-loop and an RC-loop. The rest follow from Theorem 3.1 and Theorem 3.2.

Corollary 3.6 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is a C-loop if and only if $\phi$ is a wl factor set on $C$.

Proof

L(\phi) is a C-loop if and only if it is both a LC-loop and an RC-loop. The rest follow from Corollary 3.1 and Corollary 3.3.

Corollary 3.7 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. If $L(\phi)$ is a C-loop;

1. $\phi(u, u + v) + \phi(u + v, u) \equiv \frac{uv}{2} \mod 2$.
2. The difference between $\phi$ been rwl and lwl is $\frac{uv}{2} \mod 2$ $\forall u, v \in C$.
3. $\phi(u, u + v) = \phi(u + v, v) \iff \phi(u, v) = \phi(v, u) \forall u, v \in C$.

Proof

Since a C-loop is both an RC-loop and an LC-loop, (1) and (2) follow from Corollary 3.4 and Corollary 3.2, (3) follows from Corollary 3.6 and Lemma 2.3.

Theorem 3.3 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is a C-loop if and only if $D(u, v, w) = \phi(v, u) + \phi(u, w) + \phi(v + u, v) + \phi(u, u + w) = 0$.

Proof

$L(\phi)$ is a C-loop $\iff \ y(x * xz) = (yx * x)z \ \forall \ x = (a, u), \ y = (b, v), \ z = (c, w) \in L(\phi). (b, v)[(a, u) * (a, u)(c, w)] = [(b, v)(a, u) * (a, u)](c, w) \iff (b + c + \phi(u, w) + \phi(u, u + w) + \phi(v, w), v + w) = (b + c + \phi(v, u) + \phi(v + u, v) + \phi(v, w), v + w) \iff D(u, v, w) = \phi(v, u) + \phi(u, w) + \phi(v + u, v) + \phi(u, u + w) = 0$. 6
4 Code Loops and Conjugacy Close Loops

**Lemma 4.1** Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is a LCC-loop if and only if $\phi$ is a rwl factor set.

**Proof**
A code loop is a Moufang loop, hence an LB-loop. A LB-loop is an LC-loop if and only if it is an LCC-loop. Thus by Corollary 3.1, the result follows.

**Lemma 4.2** Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is a RCC-loop if and only if $\phi$ is a lwl factor set.

**Proof**
A code loop is a Moufang loop, hence an RB-loop. A RB-loop is an RC-loop if and only if it is an RCC-loop. Thus by Corollary 3.3, the result follows.

**Lemma 4.3** Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is a CC-loop if and only if $\phi$ is a wl factor set.

**Proof**
A code loop is a Moufang loop which is possible if and only if it is a C-loop. Hence, the result follows from Lemma 4.1 and Lemma 4.2.

5 Code Loops and Extra Loops

**Lemma 5.1** Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is an extra loop if and only if $\phi$ is a wl factor set.

**Proof**
A code loop is a Moufang loop. A Moufang loop is an extra loop if and only if it is a CC-loop. Hence, the claim follows from Lemma 4.3.

**Corollary 5.1** Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is an extra loop if and only if $L(\phi)$ is nuclear square.

**Proof**
A code loop is a Moufang loop. A Moufang loop is an extra loop if and only if it is nuclear square. Hence, the proof.

**Theorem 5.1** Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. $L(\phi)$ is an extra loop if and only if any of the following equivalent conditions is true.

1. $E_1(u, v, w) = \phi(u, v) + \phi(w, u) + \phi(u + v, w) + \phi(v + w, u) + \phi(v, w + u) + \phi(u, v + w) = 0.$
2. \( E_2(u, v, w) = \phi(u, u) + \phi(u, v) + \phi(v, u) + \phi(u, w) + \phi(v + u, w) + \phi(u + v, u + w) = 0. \)

3. \( E_3(u, v, w) = \phi(u, u) + \phi(v, u) + \phi(u, w) + \phi(v, u + w) + \phi(v + w, u) + \phi(v + u, w + u) = 0. \)

**Proof**

1. \( L(\phi) \) is an extra loop \( \iff \) \((xy \ast z)x = x(y \ast zx) \) \( \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi). \)

\[
[(a, u)(b, v) * (c, w)](a, u) = (a, u)[(b, v) * (c, w)](a, u) \iff (b + c + \phi(u, v) + \phi(u + v, w) + \phi(u + v + w, u), v + w) = (b + c + \phi(w, u) + \phi(v, w + u) + \phi(u, v + w + u), v + w) \iff E_1(u, v, w) = \phi(u, v) + \phi(w, u) + \phi(u + v, w) + \phi(v + w, u) + \phi(u + v + w, u) = 0.
\]

2. \( L(\phi) \) is an extra loop \( \iff \) \( xy \ast xz = x(yx \ast z) \) \( \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi). \)

\[
(a, u)(b, v) * (c, w) = (a, u)[(b, v) * (c, w)](a, u) \iff (b + c + \phi(u, v) + \phi(u + v, w) + \phi(u + v + w, u), v + w) = (b + c + \phi(v, u) + \phi(v + u, w) + \phi(v + u + w, u), v + w) \iff E_2(u, v, w) = \phi(u, u) + \phi(u, v) + \phi(v, u) + \phi(u, w) + \phi(v + u, w) + \phi(u + v, u + w) = 0.
\]

3. \( L(\phi) \) is an extra loop \( \iff \) \( yx \ast xz = (y \ast xz)x \) \( \forall x = (a, u), y = (b, v), z = (c, w) \in L(\phi). \)

\[
(b, v)(a, u) * (c, w)(a, u) = [(b, v) * (a, u)(c, w)](a, u) \iff (b + c + \phi(v, u) + \phi(w, u) + \phi(v + w + u, v + w) = (b + c + \phi(u, w) + \phi(v, u + w) + \phi(v + u + w, u), v + w) \iff E_3(u, v, w) = \phi(u, u) + \phi(v, u) + \phi(u, w) + \phi(w, u) + \phi(v, u + w) + \phi(v + w, u) + \phi(v + u, w + u) = 0.
\]

### 6 Burn Loops and Code Loops

**Lemma 6.1** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). \( L(\phi) \) is left Burn loop if and only if \( \phi \) is lwl.

**Proof**

A left Burn loop is a LB-loop that is also an LCC-loop. The result follows by Lemma 4.1

**Lemma 6.2** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). \( L(\phi) \) is right Burn loop if and only if \( \phi \) is lw.

**Proof**

A right Burn loop is a RB-loop that is also an RCC-loop. The result follows by Lemma 4.2

**Lemma 6.3** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). \( L(\phi) \) is Burn loop if and only if \( \phi \) is w.

**Proof**

A Burn loop is a Moufang loop that is also an CC-loop. The result follows by Lemma 4.3

**Corollary 6.1** Let \( L(\phi) \) be a code loop on a binary linear code \( C \) that is doubly even with factor set \( \phi \). The following statements about \( L(\phi) \) are equivalent.
1. $L(\phi)$ is an $LC(RC)$-loop.
2. $L(\phi)$ is an $LCC(RCC)$-loop.
3. $L(\phi)$ is a left(right) Burn loop.
4. $\phi$ is rwl(lwl).

Proof
This follows from Corollary 3.1, Corollary 3.3, Lemma 4.1, Lemma 4.2, Lemma 6.1 and Lemma 6.2.

Corollary 6.2 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. The following statements about $L(\phi)$ are equivalent.

1. $L(\phi)$ is a C-loop.
2. $L(\phi)$ is a CC-loop.
3. $L(\phi)$ is a Burn loop.
4. $L(\phi)$ is an extra loop.
5. $\phi$ is wl.

Proof
This follows from Corollary 3.6, Lemma 4.3, and Lemma 6.3 or just by Corollary 6.1.

Theorem 6.1 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. The following statements about $L(\phi)$ are true.

1. $L(\phi)$ is a C-loop.
2. $L(\phi)$ is a CC-loop.
3. $L(\phi)$ is a Burn loop.
4. $L(\phi)$ is an extra loop.

Proof
$L(\phi)$ is a Moufang loop, so it is alternative which is true if and only if it is left alternative(LA) and right alternative(RA). It is easy to show that $L(\phi)$ is LA if and only if $\phi$ is rwl and $L(\phi)$ is RA if and only if $\phi$ is lwl. So, $L(\phi)$ is alternative if and only if $\phi$ is wl. So following Corollary 6.2, the claims are true.

Corollary 6.3 Let $L(\phi)$ be a code loop on a binary linear code $C$ that is doubly even with factor set $\phi$. The following are true.

1. $\phi(u, v) + \phi(w, u) + \phi(u + v, w) + \phi(v, w + u) \equiv \frac{u(v+w)}{2} \mod 2$. 
2. $\phi(u + v, u + w) \equiv \left[ |u| + \frac{u \cdot w}{2} + \phi(u, w) + \phi(v + u, w) + \phi(u, v + w) \right] \mod 2$.

3. $\phi(u + v, u + w) \equiv \left[ |u| + \frac{u \cdot w}{2} + \phi(u, v) + \phi(v, u + w) + \phi(v, w + u) \right] \mod 2$.

4. $\phi(v, u) + \phi(w, u) + \phi(u + v, w) + \phi(v, w + u) \equiv \frac{u \cdot v + u \cdot (v + w)}{2} \mod 2$.

5. $\phi(w, u) + \phi(v, w) + \phi(u, v + w) + \phi(v, w + u) \equiv [c(u, v, w) + \frac{v \cdot (v + w)}{2}] \mod 2$.

6. $\phi(u + v, u) = \phi(u, u) + \phi(v, u)$ and $\phi(u, u + v) = \phi(u, u) + \phi(u, v)$.

**Proof**

All these are achieved by Theorem 5.1, equations (2), (3) and (4). We use Theorem 5.1 because by Theorem 6.1, $L(\phi)$ is an extra loop.

**Remark 6.1** The congruence equations in Corollary 6.3 generalise the congruence equations (2), (3) and (4).

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