Supplementary Information for
“Exploring Quasiparticles in High-T\textsubscript{c} Cuprates Through Photoemission, Tunneling, and X-ray Scattering Experiments”

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SI-1. ARPES SPECTRA AND FERMI ARCS

In the section we discuss implications of our model for ARPES experiments. Previous works [1-4] have already pointed out that a finite quasiparticle lifetime provides a natural explanation for the ARPES spectra, including the emergence of Fermi arcs in underdoped samples. Here we review their arguments and relate them to the Green’s function formalism used in this paper. At low temperatures ARPES probes the spectral function, defined as the imaginary part of the diagonal elements of $G(q, \omega)$ [5]. For momenta on the Fermi surface, $\epsilon_k = \mu$, the (symmetrized) ARPES signal is then given by

$$I_{\text{ARPES}}(k, E) = \text{Im} \left[ \frac{E - i\Gamma}{(E - i\Gamma)^2 - (\Delta^2_k)} \right]$$

(S1)

In Fig. S1a, c we directly compare the imaginary part of $G(k, \omega)$ with the symmetrized spectrum observed in ARPES experiments and find a very good agreement. Eq. (S1) behaves differently depending on the ratio $\Gamma/\Delta_k$. For $\Gamma/\Delta_k < \sqrt{3}$, it has two maxima at

$$E = \pm \sqrt{2\Delta_k \sqrt{\Delta^2_k + \Gamma^2} - \Delta^2_k - \Gamma^2}.$$  

(S2)

In Fig. S1b, d we show that this expression qualitatively reproduces the evolution of the “Fermi arcs”, provided that $\Gamma$ is assumed to be temperature dependent. For $\Gamma/\Delta_k > \sqrt{3}$ the same curve has a single maximum at $E = 0$. As a consequence, “Fermi arcs” are expected to be observed in the vicinity of the nodes for all momenta satisfying $\Delta_k = \Delta_0 (\cos(k_x) - \cos(k_y))/2 < \Gamma/\sqrt{3}$. The growth of the Fermi arcs with increasing temperature [6] and underdoping [7] can be explained in terms of a growth of $\Gamma$, rather than a closing gap. Because ARPES directly probes the nodal quasiparticles, while STM is mostly sensitive to antinodal quasiparticles, a systematic comparison of these two methods on the same materials and temperatures will deliver valuable information about the anisotropy of the inelastic scattering, and help to understand its physical origin.

SI-2. THEORETICAL DESCRIPTION OF STM MEASUREMENTS

In this Appendix we present the derivation of Fig. (2), describing the Fourier transformed STM signal induced by a single time-independent impurity. As mentioned in the text, STM measures the differential conductance

$$g(r, V) \equiv \frac{dI(r, V)}{dV} = \text{Im} [G(r, r, V)] ,$$

(S3)
FIG. S1. **Energy gap on the Fermi surface as detected by ARPES.** a, Predicted spectral function, Eq.(S1), for different points on the Fermi surface. The angle \( \theta \) is measured with respect to the nodal points (\( \theta = 45 \) corresponds to the antinodes). The phenomenological parameters are \( \Gamma = 12\text{meV} \), and \( \Delta_0 = 40\text{meV} \), and \( \mu = 20\text{meV} \) (\( p = 0.11 \)). b, Position of the maximal intensity of the ARPES signal on the Fermi surface, Eq. (S2), as function of \( \sin(2\theta) \) for increasing values of \( \Gamma \), with \( \Delta_0 = 50\text{meV} \), and \( \mu = 30\text{meV} \) (\( p = 0.08 \)). c-d, Experimental measurements of underdoped samples of Bi2212 with respectively c \( T_c = 65K \) [4] (measured at \( T = 75K \)) and d \( T_c = 55K \) [7] (measured at \( T = 11K - 60K \)). Subplot c is reprinted by permission from Macmillan Publishers Ltd: Nature Physics Ref. [4], copyright (2012).

where \( G(r, r', V) \) is the dressed Green’s function including the effects of disorder. In the case of a time-independent scatterer at position \( r_0 \), first-order perturbation theory gives:

\[
G(r, r, V) = G(0, V) + \int dr_1 \int dr_2 G(r - r_1, V)T_0(r_1 - r_0, r_2 - r_0)G(r_2 - r, V)
\]  

(S4)

Here \( G(r - r', V) = G(r, r', V) \) is the translational-invariant bare Green’s function (1), which includes the effects of interactions. Introducing its Fourier transform \( G(k, V) = \)
\[ \int dr \ e^{i k r} \ G(r, V) \ \\
\text{we obtain} \ \\
G(r, r, V) = \sum_k G(k, V) + \sum_{k_1, k_2} G(k_1, V) e^{i (k_1 - k_2)(r - r_0)} T_0(k_1, k_2) G(k_2, V) \ , \tag{S5} \]

where \( T_0(k_1, k_2) = \int dr_1 \int dr_2 \ e^{i k_1 r_1 - i k_2 r_2} T_0(r_1, r_2) \).

For a local impurity \( T_0(k_1, k_2) = T_0(k_1) + T_0(k_2) \) and

\[ G(r, r, V) = \sum_k G(k, V) + \sum_{k_1, k_2} e^{i (k_1 - k_2)(r - r_0)} G(k_1, V) (T_0(k_1) + T_0(k_2)) G(k_2, V) \] \tag{S6}

If both the bare Green’s function and the impurity scattering are invariant under inversion symmetry \( k \rightarrow -k \), only the cosine component contributes to the integral and

\[ G(r, r, V) = \sum_k G(k, V) + \sum_{k_1, k_2} \cos((k_1 - k_2)(r - r_0)) G(k_1, V) (T_0(k_1) + T_0(k_2)) G(k_2, V) \ . \tag{S7} \]

The Fourier transformed STM signal is then

\[ g(q, V) = \int dr \ e^{i q r} \ \text{Im} \ [G(r, r, V)] \ \\
= \sum_k \text{Im} \ [G(k, V)] \ \delta_{q,0} + \int dr \ e^{i q r} \sum_{k_1, k_2} \cos((k_1 - k_2)(r - r_0)) \times \text{Im} \ [G(k_1, V) (T_0(k_1) + T_0(k_2)) G(k_2, V)] \] \tag{S8}

Using the identity \( \int dr \ e^{i q r} \cos(q'(r - r_0)) = e^{i q r_0} (\delta_{q + q'} + \delta_{q - q'}) / 2 \) we find

\[ g(q, V) = \sum_k \text{Im} \ [G(k, V)] \ \delta_{q,0} + \frac{1}{2} e^{i q r_0} \sum_{k_1 - k_2 = \pm q} \text{Im} \ [G(k_1, V) (T_0(k_1) + T_0(k_2)) G(k_2, V)] \] \tag{S9}

Due to the above-mentioned symmetry \( (k \leftrightarrow -k) \) the contributions from terms with \( k - k' = q \) and \( k - k' = -q \) are identical, and the finite-\( q \) components of Eq. (S10) are given by Eq. (2).

**SI-3. COHERENCE FACTORS OF NODAL AND ANTINODAL QUASIPARTICLES**

In the text we explained that the non-dispersive peaks observed in STM originate from enhanced scattering at the antinodes (see also Ref. 8). This observation is in contradiction with the well-known “octet model” 9, which predicts that antinodal quasiparticles should contribute only at a specific voltage, \( V \approx \Delta_0 \), due to energy conservation. Accordingly, for
small voltages $V \ll \Delta_0$ only nodal quasiparticles are expected to contribute. We now show that this picture is dramatically changed when the appropriate matrix elements ("coherence factors") are taken into account. For convenience, we define the integrand of Eq. (2) as

$$S^{(\alpha)}(k, k + q, V) = \text{Im} \left[ G(k, V) \left( T^{(\alpha)}_k + T^{(\alpha)}_{k+q} \right) G(k + q, V) \right].$$

(S11)

Eq. (S11) determines the contribution to the differential conductance originating from the scattering of a quasiparticle from momentum $k$ to momentum $k + q$. The experimental observable $g(q, V)$ is obtained by integrating $S$ over all momenta $k$.

Let us first consider modulations of the chemical potential ($\alpha = 1$), in the limiting case of zero voltage and $\Gamma \to 0$. In this limit Eq. (S11) simplifies to

$$S^{(1)}(k, q, 0) = \frac{((\epsilon_k - \mu) + (\epsilon_{k+q} - \mu))}{(\Delta_k^2 + (\epsilon_k - \mu)^2)(\Delta_{k+q}^2 + (\epsilon_{k+q} - \mu)^2)}.$$  

(S12)

The denominator of Eq. (S12) vanishes if both $k$ and $k + q$ correspond to the nodal points (where $\epsilon_k = \epsilon_{k+q} = \mu$ and $\Delta_k = 0$) in agreement with the octet picture. However, precisely at this point the numerator vanishes as well and the contribution to the differential conductance is zero. Because on the two sides of the nodal point Eq. (S12) has opposite sign (depending on whether $\epsilon_k$ is larger or smaller than $\mu$), the integral over $k$ gives an almost-vanishing contribution. In this case, the peak predicted by the octet model is completely washed out.

We now consider the role of the coherence factors (S11) in generating the non-dispersive peak at $q \approx (0.2, 0) \times 2\pi$. Fig. S2b presents a colorplot of $S^{(1)}(k, k + q, V)$, associated with the modulations of the chemical potential. In agreement with our previous argument, we find that the coherence factors change sign across the Fermi surface, and are strongly suppressed when the sum over all $k$ is taken into account. In contrast, the coherence factors due to modulations of the pairing gap (subplot c) do not change sign and therefore dominate the predicted STM signal at this wavevector. It is interesting to compare our results with the octet model. This model predicts contributions from quasiparticles with a specific momentum, given by the intersection between the Fermi surface and the line $k_x = q_x/2$. For $q = q^*$ this momentum is approximately half-way between the nodal and antinodal points. In contrast to the octet model, our approach shows that the STM signal is determined by quasiparticles with a broad range of momenta, close to the antinodal points (blue regions in Fig. S2b).
FIG. S2. Coherence factors \( S_{11} \) as function of momentum \( \vec{k} = (k_x, k_y) \) for Pb-Bi2201 (\( p=0.16 \)). 

(a) Band structure of this material. We consider the scattering at wavevector connecting two antinodes, \( q^* = (0.1924, 0) \times 2\pi \) (distance between the dotted lines). (a) The contributions from modulations of the chemical potential \( S^{(1)} \) change sign as a function of momentum and are strongly suppressed when the sum over \( k \) is taken into account. (b) The contributions from modulations of the pairing gap \( S^{(4)} \) do not change sign and are peaked in a finite region in momentum space around the antinodes.

To further highlight the predominance of antinodal scattering in STM signal we now consider a momentum-dependent quasiparticle lifetime of the form

\[ \Gamma_k = \Gamma_n + (\cos(k_x) - \cos(k_y)) (\Gamma_a - \Gamma_n), \]  

(S13)

where \( \Gamma_n \) and \( \Gamma_a \) are, respectively, the quasiparticle lifetime at the nodes and at the antinodes. The resulting predictions for the Fourier-transformed STM signal is shown in Fig. S3. We find that the predicted differential conductance is strongly dependent on \( \Gamma_a \) and almost insensitive to \( \Gamma_n \). This result has an intuitive explanation: modulations of the pairing gap mainly affect the scattering of antinodal quasiparticles and their effects are controlled by the lifetime of these quasiparticles. Because the results of our calculations are independent on the lifetime of the nodal quasiparticles, we are allowed to adopt the simplifying assumption of \( \Gamma_n = \Gamma_a \), leading to the constant \( \Gamma_k = \Gamma \) used in our analysis of the STM and REXS signals.
FIG. S3. **STM signal with a momentum-dependent quasiparticle lifetime.** Same as Fig. 5g, but with the angular dependent quasiparticle lifetime [S13]. The predicted differential conductance depends significantly only on the antinodal quasiparticle scattering.

**SI-4. EMERGENCE OF TWO ENERGY SCALES IN STM EXPERIMENTS**

STM measurements of Bi2212 show dispersive features at low voltages and sharp non-dispersive ones at high voltages. As noted in Refs. [10–12], the transition between these two regimes is interrupted by an intermediate voltage interval in which neither dispersive nor sharp non-dispersive peaks are observed. The size of this region increases with underdoping, giving the impression of two independent energy scales. In Fig. S4 it is shown that the transition between the different regimes corresponds approximately to $0.8(\Delta_0 - \Gamma)$ and $0.8(\Delta_0 + \Gamma)$. According to this interpretation, the second energy scale observed in STM measurements is related to the quasiparticle lifetime, rather than a distinct energy gap.

FIG. S4. **Interplay of dispersive and non-dispersive peaks as function of the inverse quasiparticle lifetime $\Gamma$.** Same as Fig. 1a for increasing values of the gap and inverse quasiparticle lifetime (corresponding to increasing underdoping). Sharp non-dispersive features are seen at voltages $V > 0.8(\Delta_0 + \Gamma_0)$ and dispersive features at $V < 0.8(\Delta_0 - \Gamma_0)$. 

SI-5. HOMOGENEOUS COMPONENT OF THE STM SIGNAL

In this section we consider the spatially-homogenous conductance \( dI/dV \). In actual experiments, this component can be measured by averaging the STM signal over a large area \( \Omega \):

\[
\frac{dI}{dV} = \frac{1}{\Omega} \int_{\Omega} dI(r,V) \equiv g(q = 0, V) = \sum_k \text{Im}[G(k, \omega)]
\]

(S14)

The theoretical predictions and experimental observation of this component are compared in Fig. S5a-b and show a good agreement, within the large error bars of the experimental observations. These error bars are due to the inhomogenous component due to the scat-

![Graphs showing theoretical and experimental data for homogeneous component of LDOS in Pb-Bi2201.](image)

FIG. S5. Homogeneous component of the LDOS in Pb-Bi2201. a-b, Theoretical calculation and experimental measurement of the homogeneous component of the differential conductance in four samples of Bi2201. The theoretical calculations were performed using the parameters of Table I. The large error bars in the experimental measurements are due to the inhomogenous component. c, Theoretical predictions of the normalized conductance (see text) for increasing values of \( \Gamma \) (all energies are given in meV). d, Experimental measurements for an overdoped sample of Pb-Bi2201 (OD15K), at different temperatures. Adapted from Ref. [13]. Each curve is shifted by 0.3 in the vertical direction for clarity.
tering from local impurities and can be reduced only by averaging over larger areas. At
positive voltages, the theoretical curves show a maximum in correspondence of the super-
conducting gap, $V_+ \approx \Delta_0$. At negative voltages the signal shows a broad maximum at the
doping-dependent voltage $V_- \approx -40\text{meV} - \mu$. This maximum, which is simply due to the
particular form of the band structure, could create the impression of gap that increases with
underdoping. For any given point in space, the actual STM spectrum is the sum of the
homogenous component (S14), peaked at $V_+$ and $V_-$ and an inhomogenous contribution (2),
whose actual maximum varies in space. The sum of these two terms is expected to give
rise to a “kink”, which was indeed universally observed in experiments (see for example
Ref. [14]).

An alternative method to extract the homogeneous component of the STM signal has been
proposed in Ref. [15]. Assuming that in the normal phase $g(x, V) = \text{const}$, the homogenous
component of the differential conductance can be obtained from $g(x, V, T)/g(x, V, T_{\text{norm}})$,
where $T_{\text{norm}} > T_c$ is an arbitrary temperature. The experimental signal is reproduced in
Fig. S5c for an overdoped sample of Pb-Bi$_2$2201 with $T_c = 15\text{K}$, using $T_{\text{norm}} = 17\text{K}$. The
position of the peaks coincides with our identification of the superconducting gap for this
sample, $\Delta_0 = 8\text{meV}$. As the temperature increases, the distance between the peaks does
not significantly vary, but their visibility rapidly diminishes. In Fig. S5c we reproduce this
result by assuming a linear dependence between $\Gamma$ and the temperature. (In our case, we
have established that, at $T = 6\text{K}$, the inverse quasiparticle lifetime $\Gamma = 6\text{meV}$, leading to
the simple relation $\Gamma/T \approx 1\text{meV/K}$, see also Appendix SI-9). Using this assumption and
normalizing the theoretical predictions with respect to the value at $\Gamma = 17\text{meV}$, we obtain
a good agreement between theory and experiment, as shown in Fig. S5c-d.

SI-6. ANALYSIS OF THE NON-DISPERSIVE PEAK AT $q_{\pi,\pi} = (0.5, 0.5) \times 2\pi$

Figure S6 shows a two-dimensional cut of the data at fixed voltage $V = 5\text{meV}$, for
wavevectors inside the first Brillouin zone. Comparing subplots a and c we find that the
theory quantitatively reproduces the experiment, with one important exception: the experi-
ment shows a broad peak around $q_{\pi,\pi} \equiv (\pm 0.5, \pm 0.5) \times 2\pi$, while the theoretical predictions
exactly vanishes there (due to the symmetry of the coherence factors appearing in Eq. (2)).
To identify the nature of the $q_{\pi,\pi}$ peak, we study its voltage dependence (Fig S7) and find
it to be anti-symmetric with respect to the $V \to -V$. As explained above, this behavior is characteristic of the scattering from local modulations of the chemical potential. Fig. S6 shows that, indeed, this type of perturbation leads to a $g$-map that is peaked around $q_{\pi,\pi}$. Our findings may also explain the experimental observations of Ref. [8], who showed that the peak at $q_{\pi,\pi}$ responds to magnetic field and temperature in the opposite way than the rest of the map, highlighting its different physical origin.

SI-7. EFFECTS OF THE BAND STRUCTURE ON THE REXS SIGNAL

In the main body of the article we found that the predicted width of the REXS peak in Y123 is larger than the one observed in experiments [16]. One interesting possibility is that this discrepancy is due to an enhancement of the CDW order caused by electron-electron interactions. This effect can be described using an random-phase approximation (RPA) [17, 18] and, in general, acts to sharpen the predicted peak. Here we follow a simpler interpretation and relate the observed discrepancy in the REXS signal to a deviation of the actual band structure from the phenomenological model obtained in Ref. [19]. Unlike the

FIG. S6. STM signal at $\vec{q} = (q_x, q_y) \times 2\pi$ and fixed voltage $V = 5\text{meV}$. a, Theoretical predictions, for local modulations of the gap, Eq. (2) with $T_k = T_k^{(4)}$. b, Same as before, for local modulations of the chemical potential, $T_k = T_k^{(1)}$. c, Experimental signal $g(q,V)$ for the an underdoped sample (UD32K). The intensity peak at $q_{\pi,\pi} = (0.5, 0.5) \times 2\pi$ observed in the experiment is due to modulations of the chemical potential.
FIG. S7. Voltage dependence of the experimental signal at different wavevectors. Same as Fig. 5a-c for the optimally doped sample OPT35K and $e^{i\phi(q)} = \rho(q,V=-10\text{meV})/|\rho(q,V=-10\text{meV})|$. The Fourier component at small wavevectors is predominantly symmetric (gap modulations), while the Fourier component around the $q_{\pi,\pi}$ peak is predominantly antisymmetric (charge modulations).

case of Bi2212, the band structure of Y123 is known less accurately, due to surface effects and to the presence of CuO chains [20]. In Fig. S8 we compare calculations for the REXS signal using two different phenomenological band structures with similar Fermi surfaces. The position of the REXS peak $q = 0.31$ is uniquely determined by the doping, and is largely model independent. In contrast, the widths of the predicted signals significantly differ between the two models and vary from $\xi = 0.1$ to $\xi = 0.07$. We note that the phenomenological model with a larger number of parameters ($N = 5$) displays a sharper peak and offers a better agreement with experiments. In general, a sharp peak in the REXS signal requires a nested band-structure, whose characterization involves many fitting parameters. Accordingly, we observe that the band structure of Ref. [21], obtained using one single fitting parameter, does not generate any significant REXS peak. To further explore this point we consider the effects of an additional momentum-dependent term in the band structure of Ref. [19], with approximately the same amplitude as the previous ones (see last column of Table S1). We find that this term leads to a further sharpening of the REXS peak and an excellent agreement with experiments. We therefore propose that REXS experiments
FIG. S8. **Wavevector dependence of the REXS signal of Y123 at \( \vec{q} = (q, 0) \times 2\pi \).** a, Fermi surface associated with three different phenomenological models, including \( N \) fitting parameter (see Table S1). The model \( N = 6 \) is obtained by adding an additional arbitrary term to the band structure of Ref.[19] (and adjusting the chemical potential to keep the density fixed). b, Predicted (continuous curves) and measured (symbols and dotted lines) REXS signal. The discrepancy between theory (see text) and experiment ([16]) is far below the combined uncertainty of both. The theoretical curves where obtained using the same parameters as in Fig. 4. To allow a comparison between the different models and experiments, we have renormalized each curve by subtracting the value at \( q = 0.22 \) and dividing by the maximal intensity.

can be used to probe the band structure of the antinodal regions of cuprates.
### TABLE S1. Phenomenological band structure of Bi2212 (used for calculations of Pb-Bi2201 as well) and Y123 (bonding (B) and antibonding (A)). The first four columns were obtained from least square fits of ARPES measurements and used without modifications in the present analysis. Ref. [20] further analyses the effects of intraplane couplings, which are neglected in the present analysis. The last column is obtained from the model of Ref. [19], by adding an additional momentum-dependent term to the band structure and correcting the chemical potential to conserve the area of the Fermi surface.

| Material     | Bi2212 | Y123(A) | Y123(B) | Y123(B) | Y123(B) |
|--------------|--------|---------|---------|---------|---------|
| Number of fitting parameters | N=5    | N=5     | N=5     | N=4     | N=6     |
| Reference    | [22]   | [19]    | [19]    | [20]    |         |
| 1            | 0.1305 | 0.4368  | 0.1756  | 0.0500  | 0.1456  |
| \(\frac{1}{2}(\cos k_x + \cos k_y)\) | -0.5951 | -1.0939 | -1.1259 | -0.4200 | -1.1259 |
| \(\cos k_x \cos k_y\) | 0.1636 | 0.5612  | 0.5540  | 0.1163  | 0.5540  |
| \(\frac{1}{2}(\cos 2k_x + \cos 2k_y)\) | -0.0519 | -0.0776 | -0.1774 | -0.0983 | -0.1774 |
| \(\frac{1}{2}(\cos 2k_x \cos k_y + \cos k_x \cos 2k_y)\) | -0.1117 | -0.1041 | -0.0701 | -0.0353 | -0.0701 |
| \(\cos 2k_x \cos 2k_y\) | 0.0510 | 0.0674  | 0.1286  | 0.0000  | 0.1286  |
| \(\cos 2k_x \cos k_x + \cos 2k_y \cos k_y\) | 0       | 0       | 0       | 0       | -0.1000 |

SI-8. REXS SIGNAL IN TWO DIMENSIONS

In Fig. S9 we plot the predicted REXS signal as a function of the two-dimensional wave-vector \(\vec{q}\). Each subplot corresponds to a different type of local scatterer: a, local modulations of the chemical potential; b, local modulations of the pairing gap; c, the sum of the previous two; and d, their difference. See Methods section for the definition of the corresponding scattering matrices \(T^{(a)}\). By comparing subplots a and d we observe that REXS experiments couple more strongly to the modulations of the chemical potential, than to modulations of the pairing gap. This effect is due to the integration over frequencies appearing in Eq. 4: as shown in Fig. 5, the differential conductance induced by a modulation of the pairing gap \(T^{(4)}\) is very small for any \(\omega > \Delta_0\), while the effects of modulation of the chemical potential survives far above \(\Delta_0\). Exploiting the results obtained from the analysis of the STM signal (Appendix SI-6), we conjecture that a sum of the contributions from \(T^{(1)}\) and
FIG. S9. Predicted REXS signal as a function of the two-dimensional wavevector $\mathbf{q} = (q_x, q_y) \times 2\pi$. The physical parameters refer to an underdoped sample of Bi2212 (see the inset of Fig. 4). Each subplot describes the effects of a different type of local modulation (see text).

$T^{(4)}$ should best reproduce the physical situation, as shown in Fig. 4c. In addition to the peaks at $q_a = (0.25, 0) \times 2\pi$ and $q_b = (0, 0.25) \times 2\pi$, we predict a pronounced peak at $q_a \pm q_b$, whose maximal intensity is larger than the one predicted for $q_a$ and $q_b$. In contrast, if the REXS intensity is dominated by local impurities with $d$-wave symmetry ($T(3)$), no peak is predicted in the $(q, q)$ direction.
Our analysis indicates that a finite quasiparticle lifetime is fundamental for understanding the single-particle properties of cuprates. Here we explore the possibility that $\Gamma$ may also play an important role in determining the critical temperature $T_c$. We observe that, in Pb-Bi2201, $T_c$ seems to correspond to the point where antinodal quasiparticles become over-damped, i.e. where their inverse lifetime equals to twice their gap: $\Gamma(T_c) = 2\Delta_0$. To obtain this result, we assume a linear dependence of $\Gamma$ on the temperature, $\Gamma(T) = \alpha T$, found in both theoretical calculations \cite{23,25} and experiments \cite{26}. Starting from the observed values of $\Delta_0$ and $\Gamma$ (see Table I, obtained from STM measurements at $T = 6K$) and requiring $\alpha T_c = 2\Delta_0$, we obtain $T_c = (26 \pm 2)K, (30 \pm 3)K, (31 \pm 3)K, (16 \pm 2)K$, consistent with the actual values $T_c = 25K, 32K, 35K, 15K$. In optimally-doped Bi2212 the gap is 1.5 times larger ($\Delta_0 \approx 30\text{meV}$) and the quasiparticle lifetime 2 times smaller (as can be inferred from the measured value $\Gamma \approx 1\text{meV}$ at $1.9K$ \cite{10}), leading to a critical temperature that is approximately 3 times larger, $T_c \approx 90K$. This phenomenological observation suggests that the critical temperature could be further increased by decreasing $\Gamma$. The opposite effect (i.e. a decrease of $T_c$ for increasing $\Gamma$) has been recently demonstrated in experiments \cite{27}.

**FIG. S10.** Evolution of the model parameters as function of doping and temperature. a, Evolution of the superconducting gap $\Delta_0$, inverse quasiparticle lifetime $\Gamma$, and critical temperature $T_c$ in 4 samples of Pb-Bi2201 (Table I. b, Same as a for 8 samples of Bi2212 analyzed by Dipasupil et al. \cite{28} and Alldredge et al. \cite{10}. c, Proposed phase diagram of Pb-Bi2201.
SI-10. NORMALIZATION OF THE STM DATA

One main technical difficulty in performing STM experiments is related to the unknown distance between the tip and the sample, which can vary from point to point. To overcome this problem, the experimental data is usually normalized at each point by the current at the maximal observed voltage \( I_{\text{max}}(r) = I(r, V_{\text{max}}) = \Delta V \sum_{0}^{V_{\text{max}}} dI(r, V)/dV \). Alternative normalization procedures include dividing by the current at the minimal voltage \( I_{\text{min}}(r) = I(r, V_{\text{min}}) = \Delta V \sum_{0}^{V_{\text{min}}} dI(r, V)/dV \), or by the difference \( I_{\text{max}}(r) - I_{\text{min}}(r) \). This third normalization was used in generating the plots of Fig. 5e-g because it does not introduce spurious asymmetries between positive and negative voltages. The same plots, but with different normalizations are shown Fig. S11. In subplots a and d we observe that the normalization procedure does not significantly affect the absolute value of the signal at small wavevectors (black and green curves), but radically changes the signal at large wavevectors (red and blue curves). These changes are mitigated by splitting the signal into its real and imaginary components (subplots b-c, e-f). In particular, we observe that the peak at \( V \approx 50\, \text{meV} \), observed in the absolute value of the large-wavector signal (curves red and blue of subplot d) is actually a local minimum, associated with a change in sign of the real signal at \( V \approx \Delta_0 = 20\, \text{meV} \).

FIG. S11. Effects of the different normalizations on the experimental signal. Same as Fig. 5e-g but with a different normalization (see text). a-c, The spectrum at each point in real space is divided by \( I_{\text{max}}(r) \). d-f, Each point is divided by \( I_{\text{min}}(r) \).
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