On the Ejection of Dark Matter from Globular Clusters

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ABSTRACT
We propose a novel mechanism for the removal of a Dark Matter halo from a Globular Cluster: Through multi-body gravitational interactions, a Dark Matter particle can be accelerated above the escape speed of the cluster and be ejected. We find that this mechanism is not sufficient to eject a massive, extended Dark Matter halo by the present time. Combined with observations of isolated Globular Clusters that show no evidence of tidal stripping, these results suggest that Globular Clusters likely never possessed significant Dark Matter halos.

Key words: (cosmology:) dark matter – (Galaxy:) globular clusters: general

1 INTRODUCTION
In the ΛCDM paradigm, Dark Matter is the first matter constituent to collapse, forming Dark Matter halos which serve as the seeds for galaxy formation. Progressively larger structures are built through the mergers of halos. This hierarchical structure formation is predicted by theory and is seen in N-body simulations, as well as observed in the structure of galaxies and galaxy clusters.

One seeming exception to this scenario are Globular Clusters (GC). Peebles (1984) was the first to propose that GCs form in gas compressed by shocks Gunn (1980); Harris & Pudritz (1994). However, observations of many GCs reveal thin tidal tails which N-body simulations predict should not form if they possess halos. Moreover, recent studies of several GCs indicate that the ratio of the mass in Dark Matter to stars in several GCs $M_{DM}/M_\ast \lesssim 1$ Grillmair et al. (1995); Odenkirchen et al. (2003); Moore (1996); Shin et al. (2013); Conroy et al. (2011); Ibata et al. (2013) and is potentially $\lesssim 10^{-2}$ if the Dark Matter is a low mass ($m_\chi \sim 10$ GeV) weakly interacting particle Hurst et al. (2015).

It is now generally thought GCs formed in gas compressed by shocks Gunn (1980); Harris & Pudritz (1994). However, the formation scenarios of GCs remain controversial in part because of the complex abundance patterns measured in stars. These observations indicate that GCs must have been much more massive in the past in order to retain significant amounts of heavy elements that would have been ejected by supernovae Gratton et al. (2004, 2012); Conroy & Spergel (2011). As pointed out in Conroy et al. (2011) this formation scenario is further complicated by the existence of nuclear star clusters, which demonstrates that at least some GC-like systems form in Dark Matter halos (e.g. Taylor et al. (2015); Böker et al. (2004); Walcher et al. (2005, 2006)).

Though they seemingly do not possess Dark Matter halos today, GCs could have had them in the past and subsequently lost their Dark Matter. One mechanism invoked for the removal of the halo is tidal stripping by the galaxy Bromm & Clarke (2002); Mashchenko & Sills (2005). While the majority of the Galactic Globular Clusters (GGC) orbit within strong tidal fields, there does exist a population of isolated GCs with galactocentric distances $r_{GC} > 70$ kpc that should not have lost their halos through tidal interactions. Two such GCs are NGC 2419 ($r_{GC} = 89.9$ kpc) and MGC1, which at $\sim 200$ kpc from M31 is the most isolated cluster in the local group Harris (1996); Conroy et al. (2011); Mackey et al. (2010). Observations of both these cluster indicate that $M_{DM}/M_\ast \lesssim 1$ Conroy et al. (2011); Ibata et al. (2013).

There is an additional mechanism by which GCs could eject Dark Matter halos: through multi-body gravitational interactions. In a close encounter with a star, a Dark Matter particle can be accelerated above the escape speed of the GC and be ejected. In principle, Dark Matter can also evaporate by slowly building up speed through multiple interactions. However, this mechanism is not efficient in GCs because particles with velocities near the escape speed spend most of their time near the outskirts of the GC and therefore, rarely experience an encounter with a star Henon (1969).

In this paper we will investigate the escape rate of Dark Matter particles from a spherically symmetric stellar system in order to ascertain the viability of the ejection scenario. As the interaction is gravitational, we shall not trouble ourselves with the details of the Dark Matter particle. The only assumption we make of the Dark Matter particle is that its mass is significantly less than the mass of a typical star.

The remainder of the paper is organized as follows: in §2 we outline the calculation of the escape rate of Dark Matter particles from an isolated, spherical stellar system. Some details of the calculated are relegated to an appendix (§5). In §3 we present our results and in §4 we discuss our conclusions.
2 METHODS

Our calculation will follow the approach of a pair of classic papers by Hénon (Hénon 1960; Henon 1969). We begin with the assumption that the Dark Matter and stellar distributions are spherically symmetric and that the particle velocities are isotropic. Then, the number of Dark Matter particles in a phase space volume element \( d^3r d^3v \) is

\[
(4\pi)^2 r^2 v^2 f(r, v) dr dv,
\]

where \( f(r, v) \) is the Dark Matter distribution function. Similarly, if the stellar distribution function is \( g(r, v', m') \) then the number of stars in the volume element \( d^3r d^3v' dm' \) is

\[
(4\pi)^2 r^2 v'^2 g(r, v', m') dr dv' dm'.
\]

Consider a Dark Matter particle of mass \( m_\chi \) and coordinates \((r, v)\). According to Paper 1 the probability that a particle will experience an encounter that takes it from a velocity \( \vec{v} \rightarrow \vec{v} + \vec{e} \) is

\[
P = 8\pi G^2 \int_0^\infty m'^2 dm' \int_0^{\infty} g(r, v', m') v' dv',
\]

where integration should satisfy:

\[
-1 \leq \cos \delta \leq 1,
\]

\[
0 \leq e,
\]

\[
v^2 + e^2 + 2ve \cos \delta \leq v_c^2 + e^2 + 2ve \cos \delta,
\]

\[
v \cos \delta + \frac{e}{2} \leq v' < v_c.
\]

For a bound Dark Matter particle it must be the case that \( v < v_c \), then from (6)

\[
v_c^2 \leq v^2 + e^2 + 2ve \cos \delta \leq v_c^2 + e^2 + 2ve \cos \delta,
\]

therefore,

\[
v \cos \delta \geq -\frac{e}{2}.
\]

Hence, we can drop the absolute value in (4). Now, we make is that \( m_\chi \ll m' \) so

\[
v_0' = \frac{1}{e} |\vec{v} \cdot \vec{e} + \frac{e^2}{2}| = |v \cos \delta + \frac{e}{2}|.
\]

The particle will escape if

\[
|\vec{v} + \vec{e}| > v_c(r),
\]

where \( v_c(r) \) is the local escape velocity. In the remainder of the paper we will denote the local escape velocity simply as \( v_c \). Using the notation of Paper 2, let \( e, \delta, \varphi \) be a set of spherical coordinates for the kick velocity \( \vec{e} \). Then from Equation (5), the condition for escape is

\[
v^2 + e^2 + 2ve \cos \delta \geq v_c^2.
\]

Then we can write the probability that the Dark Matter particle will escape in a time \( dt \) as:

\[
Q = 8\pi G^2 \int_0^\infty m'^2 dm' \int_0^{\infty} g(r, v', m') v' dv' \int_0^{2\pi} \sin \delta d\delta \int e^{-3} de.
\]

To find the escape rate, we now integrate over the position and velocity of the Dark Matter particle. Let \( N_\chi \) be the number of Dark Matter particles in the cluster,

\[
N_\chi = \int_0^{\infty} 4\pi r^2 dr \int_0^{v_e} 4\pi v^2 f(r, v) dv \int_0^{\infty} N_\chi(m) dm,
\]

with \( f(r, v) \) normalized to 1 and \( N_\chi(m) = N_\chi \delta(m - m_\chi) \) assuming the halo is composed of a single Dark Matter constituent. Then the specific escape rate is

\[
\left| \frac{1}{N_\chi} \frac{\partial N_\chi}{\partial t} \right| = \int_0^{\infty} 4\pi r^2 dr \int_0^{v_e} 4\pi v^2 \frac{Q}{dr} f(r, v) dv = 256 \pi^2 G^2 \int_0^{\infty} r^2 dr \int_0^{v_e} v^2 f(r, v) dv \int_0^{\infty} m'^2 dm' \int_0^{\infty} g(r, v, m') v' dv' \int e^{-3} de \int d\cos \delta,
\]

with the limits in Equations(11)-(14) satisfied and where we have
taken the magnitude since $\frac{\partial N_\psi}{\partial r} \rho$ is negative. If the magnitude of the specific escape rate is greater than $\tau^{-1}$ with $\tau$ the age of the Universe, then a typical Dark Matter particle will have been ejected from the halo. It is therefore likely that the GC would have dissipated its halo by the present time via this mechanism. We normalized Equation (15) to $N_\psi$ rather than 1 to make this point explicit.

This expression looks quite intractable, but the integrals in $\epsilon$ and $\delta$ can in fact be calculated analytically Hennon (1969) (the details are in the appendix §5). With these integrals calculated, we must specify the stellar and Dark Matter distribution functions in order to proceed. We take for the stellar component a Plummer model

$$\rho_\star(r) = \frac{3M_\star}{4\pi r_0^2} \left( \frac{r_0^2}{r^2 + r_0^2} \right)^{3/2},$$

where $r_0$ is the half-mass radius of the GC. As there is little guidance on what the distribution function of Dark Matter in a GC might be, we will also use a Plummer model for the Dark Matter

$$\rho_{DM}(r) = \frac{3M_{DM}}{4\pi r_\chi} \left( \frac{r_\chi^2}{r^2 + r_\chi^2} \right)^{3/2},$$

where $r_\chi$ is the half-mass radius of the Dark Matter halo. We choose the Plummer model for the Dark Matter in part because it has some nice mathematical properties that make it a convenient choice (see § 5). Moreover, the Plummer model is reasonably realistic for GCs Hennon (1969) and is similar to the structures of simulated Dark Matter halos and elliptical galaxies. One shortcoming of the Plummer model is that it lacks mass segregation which is known to occur (e.g. Aarseth (1966)). This in turn implies that velocities are uncorrelated, but the error is small and there is no known analytical cluster model with mass segregation Hennon (1969).

Now the gravitational potential is

$$\phi(r) = -\frac{GM_\star}{\sqrt{r^2 + r_0^2}} + \frac{-GM_{DM}}{\sqrt{r^2 + r_\chi^2}}.$$

In general the half-mass radii of the 2 components need not be the same. If $r_\chi \neq r_0$ the analytic expressions needed to derive the distribution function become cumbersome and we treat this case numerically. Due to the assumption of isotropy, the distribution function depends only on the magnitude of the velocity, or equivalently the kinetic energy. Figure 1 shows the distribution function $f(\epsilon)$ as a function of the magnitude of the specific energy ($\epsilon = \frac{1}{2}(v^2 - v_\star^2)$) for a GC with $M_\star = 2 \times 10^6 M_\odot$, $r_0 = 10$ pc. The solid line is the standard Plummer model in the case that $r_\chi = r_0$. The dashed green line shows the numerical result for this case, which is in agreement with the analytic case. The dotted line shows the distribution function in the case that $r_\chi = r_0/10$ while the dot-dashed line shows the case where $r_\chi = 10r_0$. The inset is a zoom in of the latter case, which shows the feature at $\epsilon = 150$.

where $\phi_0 = \frac{GM_\star}{r_0}$ with $M = M_\star + M_{DM}$ the total mass of the cluster and $E = -\frac{\phi_0}{M_{DM}}$ its energy.

With the choice that $r_\chi = r_0$ we have that

$$v_c = \sqrt{\frac{2\phi_0}{r_0}},$$

where we have defined $\phi(r) = -\phi(r)$.

3 RESULTS

In Figure 2 we consider the result of integrating Equation (43) numerically for different values of the ratio $M_{DM}/M_\star$ and compare these results to the GGCs (as well as the cluster MGC1 located in M31). Contours of the specific escape rate for GCs with $r_0 = r_\chi$ are shown with solid black lines. The red star represents MGC1, an isolated cluster orbiting M31, while the blue diamonds represent the isolated population of GGCs ($r_{GC} > 70$ kpc). As noted in §1, most of the GGCs could have lost their Dark Matter halos through tidal interactions with the Galaxy. We shall therefore pay particular attention to the most isolated GGCs. GGCs that have been selected for further investigation in Figure 3 are marked with pink squares while the green triangles denote the remaining GGCs. The solid blue line is the location where the specific escape rate is 1/τ with $\tau = 13.8$ Gyr the approximate age of the Universe Planck Collaboration et al. (2015). GCs with escape rates comparable to or exceeding this limit should have ejected a significant portion of their Dark Matter halos. However, none of the clusters reach this limit regardless of the value

\[
f(r, v) = \frac{24\sqrt{2}}{7\pi^3 r_0^3 \phi_0^2} \left( \frac{v^2 - v_\star^2}{2} \right)^2.
\]
where the specific escape rate is

\[ \frac{\chi}{M} \]

triangles denote the remaining GGCs. The solid blue line is the location

of the isolated population of GGCs (MGC1, an isolated cluster orbiting M31, while the blue diamonds represent

none of the clusters reach this limit regardless of the value of

limit should have ejected a significant portion of their DM halos. However,

within the number of Dark Matter particles within the stellar content (say

halo, slightly less than 20% of the Dark Matter particles are within

experience close encounters with stars. In the case of an extended

summarized in Table 1 and span the full range of GGCs. Note

that decreasing \( r_x \) increases the escape rate. This result is perhaps

counter intuitive as a smaller halo should have a deeper potential well,

which is correspondingly more difficult to escape from. However,

in a smaller halo, the probability of encountering an encounter

is much higher, which explains the results. Of course, the opposite

is true for larger halos. Though they are easier to escape from, the

probability of encounter is decreased. Note, that for \( M_{DM}/M_* \approx 1 \)

the only halo which exceeds 1/\( \tau \) is that of Pal 1 in the case that

\( r_x = 10^{-1} \). However, the escape rate can be increased by an addi-

tional half dex for smaller values of the ratio \( M_{DM}/M_* \). Fig 3 then

indicates that clusters with \( M_{DM}/M_* \approx 10^{-2} \) and \( r_0 \) not more

than a few parsecs, could have ejected a small remnant halo after

the initial halo was tidally stripped. This also suggests that such

clusters could have significantly dispersed the inner regions of their

halos, even if their halos were larger.

In Figure 4 we consider the escape rates for the most isolated

clusters in the Milky Way (and M31) which are marked with blue

diamonds (and pink star) in Figure 2. The parameters for these clusters are

summarized in Table 2. Due to their large sizes (\( r_0 > 10 \) pc), these clusters all have escape rates far below 1/\( \tau \). This is

further evidence against the formation of GGCs in Dark Matter halos.
under the assumption that significant halos today (tidally stripped (if they ever possessed them). Observations of 2 of the population of isolated GCs which should not have had their halos possess halos today, it is possible that they did in the past. One of the Universe not dominated by Dark Matter. Though they do not have massive Dark Matter halos.\rem{theremovalofDarkMatterfromaGC:theejectionofDarkMatter}

Figure 4. Escape rates for the isolated GCs for different values of $r_g/r_0$ under the assumption that $M_{DM}/M_*$ = 1. The solid blue line denotes 1/$r$.

| GC     | $M_*(M_\odot)$ | $n_0$ (pc) | $r_{gc}$ (kpc) | $M_{DM}/M_*$ |
|--------|---------------|------------|----------------|--------------|
| AM 1   | $1.81 \times 10^4$ | 14.7   | 124.6         | —            |
| Eridanus | $2.30 \times 10^4$   | 12.1     | 95.0           | —            |
| Pal 3  | $6.38 \times 10^4$   | 17.5    | 95.7           | —            |
| Pal 4  | $5.41 \times 10^4$   | 16.1    | 111.2          | —            |
| NGC 2419 | $1.60 \times 10^6$  | 21.4    | 89.9           | $\leq 1$    |
| Pal 14 | $2.00 \times 10^3$   | 27.1    | 71.6           | —            |
| MGC 1  | $1 \times 10^6$      | 20      | 200            | $\leq 1$    |

Table 2. Parameters for the isolated GCs in Figure 4.

4 CONCLUSIONS

GCs are peculiar systems in that they are the largest structures in the Universe not dominated by Dark Matter. Though they do not possess halos today, it is possible that they did in the past. One viable mechanism by which GCs can lose Dark Matter halos is through tidal interactions with the Galaxy. However, there exists a population of isolated GCs which should not have had their halos tidally stripped (if they ever possessed them). Observations of 2 of these GCs (NGC 2419 & MGC1) indicate that they do not possess significant halos today ($M_{DM} \leq M_*$ see Table 2).

In this paper we have investigated an additional mechanism for the removal of Dark Matter from a GC: the ejection of Dark Matter by multi-body gravitational interactions. We have found that GCs could not have ejected a significant Dark Matter halo—with one exception. GCs that are sufficiently small could have ejected a small remnant halo after the majority of the halo was tidally stripped. Our results cast further doubt on the formation of GCs in extended, massive Dark Matter halos.

In the context of WIMP astronomy, GCs remain interesting targets. As the stellar density of a GC is extremely high ($10^4 - 10^6$ stars/pc$^3$), even a subdominant Dark Matter halo could have a density several orders of magnitude greater than that of the Solar neighborhood $\rho_{DM} \sim 0.4$ GeV/cm$^3$. Current limits on the mass of any hypothetical Dark Matter halo are of order the stellar mass of the cluster. Our results indicate that such a halo should persist to the present day.

5 APPENDIX

Keeping with the notation of Henon (1969) let

$$S = \int e^{-3} \, de \int d\cos \delta,$$

and let $C = \cos \delta$. From (13)

$$C \geq \frac{e^2 - v^2 - e^2}{2ve} = C_1,$$

from (14)

$$C \leq \frac{v' - e}{v} = C_2,$$

and from (11)

$$C_1 = -1 \leq C \leq 1 = C_4,$$

In order for $S$ to be non-zero we must have that $C_1 < C_4$. Now $C_1 < C_2$ trivially. $C_1 < C_4$ requires that,

$$e > v_e - v = e_1,$$

which is stronger than (12). $C_1 < C_2$ requires that,

$$e > \frac{v_2^2 - v_2^2}{2v_2} = e_2,$$

which is again stronger than (12). And $C_3 < C_2$ requires that,

$$e < 2(v' + v) = e_3,$$

which further restricts (12). $C_3$ will be the lower limit of the dC integral when $C_1 < C_3$ or when

$$e > v + v_e = e_4,$$

and $C_2$ will be the upper limit when $C_2 < C_4$ or when

$$e > 2(v' - v) = e_5.$$

Thus, in order to determine the limits of the integrals in $S$, we must consider the order of $e_1, e_2, e_3, e_4$, and $e_5$. Elementary calculations show that

$$v' \geq \frac{1}{2}(v_e - 3v) = v'_1 \Rightarrow e_1 \leq e_3,$$

$$v' \geq \frac{1}{2}(v_e - v) = v'_2 \Rightarrow e_2 \leq e_3,\ e_2 \leq e_4,\ e_4 \leq e_3,$$

$$v' \geq \frac{1}{2}(v_e + v) = v'_3 \Rightarrow e_2 \leq e_1,\ e_1 \leq e_5,\ e_2 \leq e_5,$$

and it is always true that $e_1 \leq e_4$ and $e_5 \leq e_3$. These relations divide the $v$-$v'$ plane into 5 regions A, B, C, D, and E (see Figure 5). In region A,

$$e_5 \leq e_1 \leq e_2 \leq e_4 \leq e_3.$$

Thus in region A we have,

$$S_A = \int e_2 e^{-3} \, de \int C_1 dC + \int e_4 e^{-3} \, de \int C_1 dC,$$

$$= \frac{2v_3^3}{3v_2^2(v_2^2 - v^2)^2} + \frac{1}{8v(v' + v)} = \frac{2v_e + v}{6v(v_e + v)^2}.$$

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In region B, 
\[ e_2 \leq e_1 \leq e_5 \leq e_4 \leq e_3, \]  
Hence,
\[ S_B = \int_{e_1}^{e_5} e^{-3} \, dc \int_{C_x}^{C_z} dc + \int_{e_1}^{e_4} e^{-3} \, dc \int_{C_x}^{C_z} dc + \int_{e_4}^{e_5} e^{-3} \, dc \int_{C_x}^{C_z} dc, \]
\[ = \frac{3v_0^2 - v^2}{3(v_e - v)^2} \frac{1}{4(v_e^2 - v^2)}. \]  

In region C, 
\[ e_2 \leq e_1 \leq e_4 \leq e_5 \leq e_3, \]  
Hence,
\[ S_C = \int_{e_1}^{e_5} e^{-3} \, dc \int_{C_x}^{C_z} dc + \int_{e_1}^{e_4} e^{-3} \, dc \int_{C_x}^{C_z} dc + \int_{e_4}^{e_5} e^{-3} \, dc \int_{C_x}^{C_z} dc, \]
\[ = \frac{3v_0^2 - v^2}{3(v_e - v)^2} \frac{1}{4(v_e^2 - v^2)}, \]  
\[ = S_B. \]

In region D, 
Here we can not simultaneously satisfy \( e > e_1, e > e_2 \), and \( e < e_3 \), thus region D is forbidden. In region E, 
\[ e_5 \leq e_3 \leq e_1 \leq e_4 \leq e_2, \]  
So region E is forbidden for the same reason as D. Then Equation (16) becomes,
\[ \left| \frac{1}{N_x} \frac{\partial N_x}{\partial t} \right| = 256\pi^2 G^2 \int_0^\infty r^2 \, dr \int_0^\infty m^2 \, dm \]
\[ \times \left\{ \int_0^{v_e} v^2 f(r, v) \, dv \int_{v_5}^{v_7} v' S_A \, dv' + \int_0^{v_e} v^2 f(r, v) \, dv \int_{v_5}^{v_7} v' S_B \, dv' \right\}. \]

Utilizing a Plummer model for the stellar and Dark Matter components as described in the text and defining the stellar mass spectrum \( N_\star(m) \, dm \) as the number of stars in the mass interval \( m \rightarrow m + dm \), we have that \( g(r, v', m') = f(r, v') N_\star(m') \). Then Equation (40) becomes
\[ \left| \frac{1}{N_x} \frac{\partial N_x}{\partial t} \right| = \frac{2304G^2}{49\pi^2 v_5^4 \phi_0} \int_0^{R_{in}} r^2 \, dr \int_0^\infty N_\star(m) m^2 \, dm \]
\[ \times \left\{ \int_0^{v_e} v^2 (v_5^2 - v^2)^2 \, dv \int_{v_5}^{v_7} v' S_A (v_5^2 - v^2)^{7/2} \, dv' + \int_0^{v_e/3} v^2 (v_5^2 - v^2)^2 \, dv \int_{v_5}^{v_7} v' S_B (v_5^2 - v^2)^{7/2} \, dv' \right\}. \]
where the virial radius $R_{\text{vir}}$ of the Dark Matter halo is chosen to be suitably large ($\sim 10r_{\odot}$) such that the integrals in Equation (41) are all converged.

We now define new variables:

$$x = v/v_e, \quad x' = v'/v_e.$$  \hspace{1cm} (42)

Then we can remove $v_e$ from the integrals over $v$ and $v'$ and perform those integrals separately from the radial integral. It is proven in Appendix II of Henon (1969) that the Plummer model is the only steady state distribution for which this separation is possible. Then Equation (41) becomes

$$\int \frac{1}{N_X} \frac{dx}{dt} = \frac{2304G^2}{49\sigma^2_{00}} \int_0^{R_{\text{vir}}} v_e^2 r^2 \, dr \int_0^\infty N_e(m') m^2 \, dm' \times \left\{ \int_0^1 x^2(1-x^2)^2 \, dx \int_{x'_A}^{x'_B} x'S_A(1-x^2)^{7/2} \, dx' + \int_0^{1/3} x^2(1-x^2)^2 \, dx \int_{x'_A}^{x'_B} x'S_A(1-x^2)^{7/2} \, dx' \right\},$$

where

$$x'_A = \frac{1}{2}(1-x), \quad x'_B = \frac{1}{2}(1+x), \quad (44)$$

and the $r$ in $S_r'$ denotes that it is now a function of $x$ and $x'$ with $v_e$ factored out.

Let us now choose a particular stellar mass spectrum. We begin with the Initial Mass Function (IMF) from Kroupa (2001). All of the GGCs should have ages of order $\sim 10$ Gyr, meaning that their Main Sequence (MS) turnoffs should be at approximately $1 M_\odot$. Therefore, in order to obtain a crude approximation of the present day stellar mass spectrum, we simply cut off the IMF at $1 M_\odot$. Note that this is highly conservative as stellar remnants such as Neutron Stars, White Dwarfs, and Black Holes as well as any stars still on the Giant and Horizontal Branches should contribute to the escape rate. Furthermore, higher mass stars are given more weight in the integral over mass in Equation (43).

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Figure 5. The integration regions over the kick velocity $\vec{\epsilon}$. The shading denotes the fact that regions D & E are forbidden because we cannot simultaneously satisfy all of the required inequalities in Equations (11-14).

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