The Chemical Evolution of Iron-Peak Elements with Hypernovae

J.J. Grimmett,1⋆ Amanda I. Karakas,1 Alexander Heger,1,2 Bernhard Müller1

1 Monash Centre for Astrophysics, School of Physics and Astronomy, 19 Rainforest Walk, Monash University, VIC 3800, Australia
2 Tsung-Dao Lee Institute, Shanghai 200240, P. R. China

Accepted XXX. Received YYY; in original form ZZZ

ABSTRACT
We calculate the mean evolution of the iron-peak abundance ratios [(Cr,Mn,Co,Zn)/Fe] in the Galaxy, using modern supernova and hypernova chemical yields and a Galactic Chemical Evolution code that assumes homogeneous chemical evolution. We investigate a range of hypernova occurrence rates and are able to produce a chemical composition that is a reasonable fit to the observed values in metal-poor stars. This requires a hypernova occurrence rate that is large (50%) in the early Universe, decreasing throughout evolution to a value that is within present day observational constraints (≲ 1%). A large hypernova occurrence rate is beneficial to matching the high [Zn/Fe] observed in the most metal-poor stars, although including hypernovae with progenitor mass \( \geq 60 \text{M}_\odot \) is detrimental to matching the observed [(Mn,Co)/Fe] evolution at low [Fe/H]. A significant contribution from HNe seems to be critical for producing supersolar [(Co,Zn)/Fe] at low metallicity, though more work will need to be done in order to match the most extreme values. We also emphasise the need to update models for the enrichment sources at higher metallicity, as the satisfactory recovery of the solar values of [(Cr,Mn,Co,Zn)/Fe] still presents a challenge.

Key words: supernovae: general – Galaxy: abundances, evolution

1 INTRODUCTION
The chemical abundances in the upper layers of low-mass (\( \lesssim 0.8 \text{M}_\odot \)) stars remain relatively stable throughout their evolution. With the exception of perhaps the CNO elements, the abundances observed in the photosphere of low-mass stars reflect the chemical composition of the gas from the time and place that the star formed. Assuming that this gas is chemically enriched, a large part of this enrichment will have been from massive stars (\( \gtrsim 10 \text{M}_\odot \)) ending their lives as core-collapse supernovae. Ergo, a key metric in assessing our understanding of galactic evolution, stellar evolution, and the relationship between the two is given by how well we are able to reproduce the chemical abundances observed in the photospheres of low-mass, long-lived stars with theoretical models for the chemical end products of shorter lived, high-mass stars (e.g., Nomoto et al. 2013; Frebel & Norris 2015, for recent reviews). Although we can predict the ejected abundances for individual supernova models to compare with individual or small subsets of stellar observations, it is difficult to establish a "big picture" understanding in this manner. Galactic Chemical Evolution (GCE) models provide a useful framework to collectively integrate many different stellar models into a representation of the Galactic population. This allows a broad comparison to both the chemical abundances observed in large collections of stellar observations, and in chronological trends (e.g., Timmes et al. 1995; Kobayashi et al. 2006, 2011; Romano & Starkenburg 2013; Côté et al. 2016; Andrews et al. 2017). The evolutionary scales for galaxies and massive stars are both spatially and temporally disparate, which poses a computational challenge when considering the symbiotic evolution of the system. Furthermore, both galactic and stellar evolution have many unknowns and hurdles to overcome even as separate studies, and GCE is not simply a matter of combining two completely understood phenomena. Nevertheless, the scientific benefits of pursuing this goal are many, and several groups have made steady progress over the past few decades (van den Bergh 1957; Schmidt 1959, 1963; Truran & Cameron 1971; Talbot & Arnett 1971; Searle & Sargent 1972; Pagel & Patchett 1975; Tinsley 1980; Matteucci & Greggio 1986; Kobayashi et al. 2000; Kawata & Gibson 2003; Nomoto et al. 2006; Kobayashi et al. 2011).

* E-mail: james.grimmett@monash.edu

© xxxx The Authors

A persistent problem is the abundances of the iron-
peak elements and in particular \((\text{Cr}, \text{Mn}, \text{Co}, \text{Zn})/\text{Fe}\) observed in metal-poor stars, which have not been accurately reproduced by GCE models to date. This disparity reflects a significant shortcoming in our understanding of the chemical evolution in the Galaxy. In general, for decreasing \([\text{Fe}]/\text{H}\), and particularly for \([\text{Fe}]/\text{H}\) \(\lesssim -3\), the ratios \([\text{Cr}, \text{Mn}]/\text{Fe}\) are observed to decrease while \([\text{Co}, \text{Zn}]/\text{Fe}\) are seen to increase (McWilliam et al. 1995; Ryan et al. 1996; Cayrel et al. 2004; Bonifacio et al. 2009; Yong et al. 2013; Roederer et al. 2014; Reggiani et al. 2017). With the exception of perhaps \([\text{Mn}/\text{Fe}]\), these chemical ratios cannot be convincingly reconciled with the supernova yields available (Chieffi & Limongi 2002; Nomoto et al. 2006; Joggerst et al. 2010; Heger & Woosley 2010; Grimmett et al. 2018).

Stars with \([\text{Fe}]/\text{H} \lesssim -3\) are so iron-poor that they have most likely formed at a very early time in the Universe, when only very little metal enrichment had occurred. Though it is also possible that these stars may have formed in a poorly mixed region of the Galaxy at a later time, the general consensus is that these stars have been enriched by only one, or potentially a few, of the most massive and shortest lived first stars, i.e. those which formed from the primordial Universe (Audouze & Silk 1995; Argast et al. 2000, 2002). Whereas it has been shown that the iron-peak elemental abundances converge to the solar value in up-to-date GCE calculations, there is no convincing fit for the abundances of these elements in the most metal-poor stars (Kobayashi et al. 2006, 2011). This suggests that the supernovae (SNe) of the first stars must be in some way unusual.

Lacking robust multidimensional, neutrino-driven models for core-collapse supernovae of massive stars (see, e.g., Müller 2016), and also in the interest of computational cost, one-dimensional, parametrised models are commonly used to estimate the chemical yields of supernovae across a wide range of progenitors (Chieffi & Limongi 2002; Nomoto et al. 2006; Heger & Woosley 2010; Fryer et al. 2018). Physically motivated modifications to the explosion models, such as small variations to the amount of mass in the core which is destined to be trapped inside the compact remnant, and also the possibility that some amount of inner material may fail to be successfully ejected and instead fall back to accrete onto the compact remnant after some turbulent mixing, have allowed for some improvement in the right direction, in terms of reproducing the observed Fe-peak abundances in metal-poor stars (Nakamura et al. 1999; Umeda & Nomoto 2002; Heger & Woosley 2010). The most promising advance, however, has been made by considering unusually large explosion energies in models. By and large, most supernovae are estimated to have explosion energies of order \(10^{51}\) erg (e.g., Kasen & Woosley 2009). Beginning with the observation of the unusually energetic SN 1997bw, however, and of several similarly energetic supernovae since, it has been realised that at least some fraction of supernovae explode with energies an order of magnitude larger then usual, i.e. \(10^{52}\) erg, and have been commonly referred to as hypernovae (HNe) (Galama et al. 1998; Iwamoto et al. 1998; Matheson et al. 2003; Woosley & Bloom 2006). The exact occurrence rate of HNe is not yet well constrained, although recent estimates place it at less than 1 percent of the currently observed SN rate, with speculation that it may have been \(> 10\) percent in the early universe (Podsiadlowski et al. 2004; Woosley & Bloom 2006; Arcavi et al. 2010; Smith et al. 2011; Smidt et al. 2014). By modifying existing one-dimensional, parametrised supernova models to match the large explosion energy observed in these energetic events, it has been shown that spherical representations of hypernovae will typically heat larger regions of the stellar envelope to temperatures required for complete silicon burning, which results in more Fe, Co, Zn relative to Cr, Mn.

A growing library of hypernova observations have revealed a strong connection between hypernovae and gamma-ray bursts (GRBs) which, alongside other emerging evidence such as broad line features in hypernova spectra, indicate that hypernovae may be intrinsically aspherical, and possibly driven by jets (Woosley & Bloom 2006; Maeda et al. 2008; Wang & Wheeler 2008; Tanaka et al. 2017). Preliminary studies into the chemical yields to be expected from highly aspherical and energetic explosions indicate that they may also provide a favourable match to the abundances observed in stars with \([\text{Fe}]/\text{H} \lesssim -3\) (Maeda & Nomoto 2003; Pruet et al. 2003, 2004; Tominaga et al. 2007; Tominaga 2009). Though, models of this type are still in active development and will be refined as more observational constraints become available. Jet-driven supernovae may also be a site of r-process nucleosynthesis (Winteler et al. 2012; Nakamura et al. 2015; Nishimura et al. 2017; Halevi & Móst 2018).

Several earlier studies have made GCE calculations including the chemical yields from HNe, with promising results for the evolution of the iron-peak elements in relation to the observed trends in the Galaxy (Kobayashi et al. 2006, 2011; Komiyama 2011; Tsujimoto & Nishimura 2018; Hirai et al. 2018). These studies, however, have used SN and HN models which are now outdated and/or implement HN occurrence rates (e.g., 50% through all time) which cannot be reconciled with the observed rate. Meanwhile, many advances have been made in our understanding of stellar evolution and nuclear physics, and updated supernova and hypernova yields have become available (Rauscher et al. 2002; Heger & Woosley 2010; Côté et al. 2016; Grimmett et al. 2018). Additionally, we now have some broad observational constraints on the HN occurrence rate (Podsiadlowski et al. 2004; Woosley & Bloom 2006; Arcavi et al. 2010; Smith et al. 2011; Smidt et al. 2014). In this work, we seek to understand the range of chemical evolution results that are possible with the most modern and comprehensive chemical yield sets for SNe and HNe, and with realistic HN rates. For this purpose, we use a GCE model that assumes homogeneous chemical evolution throughout the galaxy i.e a one-zone model. One-zone models have proven to be effective and accurate for modelling the chemical evolution of well-mixed regions of the Galaxy, e.g., the thin disk, and are computationally cheap (Matteucci & Greggio 1986; Timmes et al. 1995; Kobayashi et al. 2000, 2006; Nomoto et al. 2006; Romano & Starkenburg 2013; Andrews et al. 2017). They are therefore an excellent tool for calculating the mean trends of chemical evolution in galaxies and assessing the viability of supernova yield
set, which we refer to as the Ertl set hereafter, some models fail to explode and instead collapse to black holes, ejecting only the fraction of the envelope that was driven away by stellar winds during evolution.

SNe Ia are represented by the W7 model from Nomoto et al. (1984), with chemical abundances taken from the yields table provided by Kobayashi et al. (2006).

### 2.2 Galactic Chemical Evolution Model

We use a basic one-zone chemical evolution model developed by the authors. The code solves a set of equations that represent a simplified model for an evolving galaxy, as introduced by Tinsley (1980). We emulate the formulation of these equations as described by Kobayashi et al. (1998); Kobayashi et al. (2000, 2006), with minor changes to accommodate our implementation of HNe, described below.

Similar to Kobayashi et al. (2006), we assume two varieties of core-collapse supernovae (CCSNe), those of typical supernova explosion energy of order $10^{51}\text{ erg}$, and those of hypernova explosion energy, of order $10^{52}\text{ erg}$ (specific values listed in Section 2.1). Rather than a constant HN rate, however, we set the fraction of massive stars exploding as hypernovae, $\epsilon_{\text{hn}}$, with the following metallicity ($Z$) dependent prescription:

$$\epsilon_{\text{hn}} = \max\left(\epsilon_{\text{hn},0} \exp\left(-\frac{Z}{0.001}\right), 0.001\right).$$  \hfill (1)

where $\epsilon_{\text{hn},0}$ is the HN fraction at $t = 0\text{yr}$ (i.e. the beginning of the Universe). The HN rate in the early universe is not well constrained but is predicted to be much larger than the observed rate today (Woosley & Bloom 2006; Smidt et al. 2014). The current HN rate is loosely constrained to be $\lesssim 1\%$ of the current SN rate (Podsiadlowski et al. 2004). This prescription allows us to explore a range of HN rates in the early universe, while ensuring that the rate drops rapidly to conform to the lower rate at higher metallicities. We implement $\epsilon_{\text{hn}}$ as a weighting factor to combine our SN and HN yields. Additionally, to allow for more massive HN progenitors, we extend the initial mass function (IMF) from a minimum mass of $0.05\,M_{\odot}$ to a maximum of $100\,M_{\odot}$.

We do not consider the nucleosynthetic output from low and intermediate-mass stars ($< 8\,M_{\odot}$), as the major contribution from this stellar group are intermediate mass elements, particularly the CNO and s-process elements (see, e.g., Kobayashi et al. 2011; Karakas & Lattanzio 2014), whereas our investigation is focused on the evolution of the Fe-peak elements. The large amount of gas that low and intermediate-mass stars return to the interstellar medium (ISM) via winds, however, is important to consider when following the evolution of gas in the Galaxy. For this purpose, we adopt the remnant mass ($m_{\text{rem}}$) prescription given by Iben & Tutukov (1984); Pagel (2009) for stars with mass $< 10\,M_{\odot}$.

---

**Table 1.** The hypernova models that we use in the GCE calculation.

| mass ($M_\odot$) | 15 | 20 | 30 | 40 | 60 | 80 |
|-----------------|----|----|----|----|----|----|
| explosion energy ($10^{51}\text{ erg}$) | 7.0 | 5.5 | 9.5 | 25 | 60 | 60 |

**Table 2.** The supernova models that we use in the GCE calculation.

| mass ($M_\odot$) | 13, 15, 20, 30 |
|-----------------|---------------|
| metallicity     | 0.00 | $1.53 \times 10^{-6}$ | $1.53 \times 10^{-5}$ |
|                 | $4.84 \times 10^{-3}$ | $1.53 \times 10^{-4}$ | $4.84 \times 10^{-4}$ |
|                 | $1.53 \times 10^{-3}$ | $2.43 \times 10^{-3}$ | $3.84 \times 10^{-3}$ |
|                 | $6.09 \times 10^{-3}$ | $9.66 \times 10^{-3}$ | $1.53 \times 10^{-2}$ |
|                 | $1.93 \times 10^{-2}$ | $2.43 \times 10^{-2}$ | $3.05 \times 10^{-2}$ |

Calculations. Given the uncertainties that are inherent to GCE modelling (homogenous models in particular), and the broad observational constraints for HN occurrence rates, we explore our results over a wide parameter space, to determine both the strengths and shortcomings of our current SN, HN, and thermonuclear Type Ia supernova (SN Ia) models. We aim to determine where we currently stand to evaluate the progenitor models for their "explodability" and are also models which produce relatively energy relation found by Nomoto et al. (2003) using light are selected to be in approximate agreement with the mass-

---

**MNRA 000, 1–18 (xxxxx)**
Here, $m$ is the main sequence mass of the star, and we leave the chemical composition of the gas unchanged between star formation and wind ejection in this mass range.

3 RESULTS

We first set about reproducing the results of Kobayashi et al. (2006), with results shown in Section A1.

3.1 GCE Results
Figure 1. The observed values of [(Cr,Mn,Co,Zn)/Fe] in the set of Milky Way stars that we have selected for comparison to our GCE results. The observational sources include metal-poor giant stars in the halo (Cayrel et al. 2004, black plus signs), metal-poor turnoff stars (Bonifacio et al. 2009, magenta triangles), metal-poor halo stars (Reggiani et al. 2017, gold stars), F and G dwarf stars in the solar neighbourhood (Bensby et al. 2014, grey hexagons), FGK stars in the solar neighbourhood (Adibekyan et al. 2012, green crosses). Where available, these abundances have been extracted from the STELLAB library (Ritter & Côté 2016).

Figure 2. IMF-weighted Fe mass in ejecta (top row), and [(Cr,Mn,Co,Zn)/Fe] IMF-weighted yields (bottom row) for individual model sets. (a,b); IMF-weighted HN yields, as a function of maximum HN progenitor mass, the SN Ia yields are plot as stars. (c,d); IMF-weighted minimum fallback SN yields as a function of [Fe/H]. (e,f); IMF-weighted SN yields from the Ertl set as a function of [Fe/H]. Models with primordial composition are plot at [Fe/H] = -4.5.
Figure 3. The HN fraction ($\epsilon_{hn}$) as a function of [Fe/H], for each value of initial HN fraction ($\epsilon_{hn,0}$). The solid lines represent the models with the minimum fallback SN (+HN/SN Ia) yields, and the dashed lines represent the models with the Ertl SN set (+HN/SN Ia). Here the HN mass upper limit is set as $30 \, M_\odot$. For different values of HN mass upper limit, the changes in $\epsilon_{hn}$ evolution are negligible, and the evolution shown here can be taken as representative for all values of HN mass upper limit.

Figure 4. The results of our GCE calculation for [Cr/Fe] as a function of [Fe/H] (blue lines). The solid blue lines represent the models with the minimum fallback SN (+HN/SN Ia) yields, and the dashed blue lines represent the models with the Ertl SN set (+HN/SN Ia). The HN mass upper limit in each model increases down the rows (i.e. HN mass upper limit by row, top to bottom: $30 \, M_\odot$, $40 \, M_\odot$, $60 \, M_\odot$, and $80 \, M_\odot$), and the initial HN fraction increases across the columns (i.e. initial HN rate by column, left to right: 5%, 10%, 25%, and 50%). The grey symbols are the observed values of [Cr/Fe] in stars, see Figure 1 for more detailed description of the observations.
Figure 5. Same as Figure 4, but for the evolution of $[\text{Mn}/\text{Fe}]$. 
Figure 6. Same as Figure 4, but for the evolution of [Co/Fe].
Figure 7. Same as Figure 4, but for the evolution of [Zn/Fe].
In Figures 4 through 7 we show the results for the evolution of \([\text{[Cr,Mn,Co,Zn]}/\text{Fe}]\) as a function of \([\text{Fe/H}]\). For each GCE model we vary SN/HN contributions, as described in Section 2.1. We compare our results to several recent sets of observed stellar abundances, which can be more easily differentiated in Figure 1, for reference. Due to the intrinsic uncertainties associated with basic one-zone GCE models (Section 1), our main goal is not to provide a perfect fit to the observed trends in chemical abundances. While general improvements to the absolute fit are significant and are discussed below, our central focus is to achieve a more comprehensive understanding of the scope of possible results for various sets of SN yields and HN contribution across a realistic parameter space. In the following, we discuss our results for each of \([\text{[Cr,Mn,Co,Zn]}/\text{Fe}]\) in turn.

### 3.2 Chromium

\([\text{Cr/Fe}]\) is produced at a value close to zero in SNe of all \([\text{Fe/H}]\) (Figure 2). This is because Cr and Fe are synthesised in similar regimes of temperature and neutron excess during explosive silicon burning, and therefore, it would be rare to enhance or inhibit the creation of one without likewise affecting the other. This is in contrast to the decreasing trend of \([\text{Cr/Fe}]\) at low metallicity. It has been found that at one pathway to lower \([\text{Cr/Fe}]\) is by increased explosion energy in core collapse models, where Fe production (as \(^{56}\text{Ni}\)) is increased due to a larger volume of the envelope undergoing explosive burning (Nakamura et al. 2001; Umeda & Nomoto 2005; Nomoto et al. 2006; Grimmett et al. 2018). This is the case in our HN models, which produce slightly lower \([\text{Cr/Fe}]\) than the SN models. The largest deviation in \([\text{Cr/Fe}]\) from zero occurs in the lower mass (\(\leq 30\,M_\odot\)) HN models, in which synthesis of Cr is suppressed as a result of extra heating from a reverse shock due to the specifics of the progenitor structure in this mass range (Grimmett et al. 2018). We have specifically selected HN models where deviation in \([\text{Cr/Fe}]\) is the largest to allow us to better investigate the full range of possible \([\text{Cr/Fe}]\) evolution histories. The W7 SN Ia model also produces a relatively low \([\text{Cr/Fe}]\) \(\sim -0.2\).

The similarity in \([\text{Cr/Fe}]\) between sources is reflected in our models of the chemical evolution of the galaxy (Figure 4). In general, \([\text{Cr/Fe}]\) is almost constant with \([\text{Fe/H}]\). At \([\text{Fe/H}] > -1\) when SNe Ia begin to contribute, there is a slight decrease in \([\text{Cr/Fe}]\) in each GCE model. Due to the similarity in \([\text{Cr/Fe}]\) between most SNe and HNe, the chemical evolution shows almost no change between models with different remnant mass prescriptions for SNe, upper mass limit for HNe, or HN fraction. The only (minor) exception to this result is for the models in which the contribution from the lower mass HNe (with low \([\text{Cr/Fe}]\)) is maximised. The results of these models are shown in the upper right corner of Figure 4, particularly in panels (c) and (d), which have a HN upper mass limit of 30\,M_\odot and the largest HN fraction. These models are still not able to match the low \([\text{Cr/Fe}]\) \(\sim -0.4\) observed in the most metal poor stars, but they are the only models which show signs of a decreased \([\text{Cr/Fe}]\) value for low \([\text{Fe/H}]\).

The evolution of the models with the Ertl SN set maintain slightly lower \([\text{Cr/Fe}]\) relative to the models with the minimum fallback prescription SNe, as the Ertl SN set contribute less ejecta overall, due to the stars which fail to explode, so the HN yields dominate the \([\text{Cr/Fe}]\) value. We are able to achieve \([\text{Cr/Fe}] < 0\) at low metallicity, which is required for matching the observed values in stars with \([\text{Fe/H}] \lesssim -3\).

### 3.3 Manganese

\([\text{Mn/Fe}]\) is produced by SNe in a ratio which is increasing with progenitor metallicity (Figure 2). This is because Mn has only one stable isotope, \(^{55}\text{Mn}\), which is neutron-rich. Therefore, \(^{55}\text{Mn}\) is produced more abundantly in environments with a supply of excess neutrons. An increasingly neutron-rich environment is provided by SN progenitors with greater metallicity. In our SN models, \([\text{Mn/Fe}]\) is further decreased by the enhanced Fe production that accompanies the large volume of explosively burned envelope during highly energetic explosions. The W7 SN Ia model produces the largest \([\text{Mn/Fe}]\) of our enrichment sources, at approximately the solar value.

The trend of increasing \([\text{Mn/Fe}]\) produced by higher metallicity SN models is clearly evident in all of our chemical evolution calculations (Figure 5). The decreasing contribution from HNe with larger \([\text{Fe/H}]\) serves to reinforce the positive relation between \([\text{Mn/Fe}]\) and \([\text{Fe/H}]\), as does the increasing contribution from SNe Ia at \([\text{Fe/H}] > -1\). Although the \([\text{Mn/Fe}]\) ratios from both SN sets are very similar, the models comprised of the SNe from the Ertl set are most strongly influenced by the HN and SN Ia contributions. This is because the SNe in the Ertl set collectively eject less mass than the minimum fallback SN set, and therefore the HN and SN Ia abundances are able to more strongly dominate the interstellar medium (ISM). This can be seen in the more extreme values of low \([\text{Mn/Fe}]\) at the lowest \([\text{Fe/H}]\), and high values of \([\text{Mn/Fe}]\) toward solar \([\text{Fe/H}]\) in the evolution of the models with the Ertl SN set relative to the models with the minimum fallback SN set. The HN sources provide the lowest value of \([\text{Mn/Fe}]\), and lower still from the most massive HNe, so the effect of both a larger HN fraction, and a larger HN upper mass limit is to decrease the \([\text{Mn/Fe}]\) value in the evolution. The best fit is provided by GCE models with a limited HN contribution, either by low HN fraction, low HN upper mass limit, or a combination of both (e.g., \(M_{\text{HNe}} < 40\,M_\odot\) or \(\epsilon_{\text{max}} \lesssim 0.1\)), hence the lower right hand corner of the grid of results in Figure 5 show the least favourable results. The increasing \([\text{Mn/Fe}]\) in the ejecta of SN progenitors with increasing metallicity, and the large \([\text{Mn/Fe}]\) contribution from SNe Ia ensures a fairly robust fit to the solar value of \([\text{Mn/Fe}]\) across all GCE models. The models with the minimum fallback SN set typically provide a better fit to the observed increasing trend in \([\text{Mn/Fe}]\) with \([\text{Fe/H}]\), although the flatter trend in \([\text{Mn/Fe}]\) in the models with the Ertl SN set is likely attributed to the contribution from only zero metallicity HNe. It is possible that there may be a trend of higher \([\text{Mn/Fe}]\) with progenitor \([\text{Fe/H}]\) in HNe, as there is with SNe, due to larger neutron excesses. This is the case in the HN models of Kobayashi et al. (2006).
3.4 Cobalt

Similar to $[\text{Mn/Fe}]$, $[\text{Co/Fe}]$ is produced in an increasing value by SN progenitors of larger $[\text{Fe/H}]$ (Figure 2). Unlike $[\text{Cr/Fe}]$ and $[\text{Mn/Fe}]$, there are significant differences between the values of $[\text{Co/Fe}]$ produced by the Ertl set and minimum fallback prescription SNe. In Figure 2 we saw that for lower metallicities, the IMF-weighted Ertl SN set produce $[\text{Co/Fe}]$ in a larger ratio than in the minimum fallback models, though for higher metallicity the values converge. $[\text{Co/Fe}]$ is typically produced at a value $[\text{Co/Fe}] \gtrsim 0.0$ by lower mass HNe, and in decreasing values for increasing progenitor mass, hence Figure 2 shows a decreasing IMF-weighted $[\text{Co/Fe}]$ value for increasing HN upper mass limit. The W7 SN Ia model produces $[\text{Co/Fe}] \approx -0.25$.

The impact of the difference in $[\text{Co/Fe}]$ between the SN model sets can immediately be seen in our results for the chemical evolution, shown in Figure 6. The models with the minimum fallback SN set typically have lower $[\text{Co/Fe}]$ values, though the results converge for higher HN fraction, higher upper mass limit for HNe, or a combination of both. In this regime, the HN yields tend to dominate the overall abundance ratios, and the differences between the SN model sets have less impact. We find that a larger HN fraction will typically increase the overall value of $[\text{Co/Fe}]$. This effect is more pronounced for the models with the minimum fallback SN set, for two main reasons: (i) the HNe and Ertl SN set $[\text{Co/Fe}]$ yields are similar, so an increased HN fraction has little effect on the average between the two. On the other hand, there is a large difference between the $[\text{Co/Fe}]$ yields from the HNe and the minimum fallback SN set, so a larger HN fraction has a stronger effect on the final value in this case; and (ii) the Ertl SN set eject less mass collectively, so the averaged $[\text{Co/Fe}]$ value is likely already dominated by HNe even for low HN fraction, and increasing HN fraction makes little difference.

For $[\text{Fe/H}] \gtrsim -1$, there is an increasing trend in the value of $[\text{Co/Fe}]$ for the models with the minimum fallback SN set, and a decreasing trend in the same value for the models with the Ertl SN set. This can be explained as follows: At $[\text{Fe/H}] \sim -1$ in each model, where HN contribution is rapidly decreasing, the $[\text{Co/Fe}]$ value at this point in the evolution is essentially a weighted average between the SN and SN Ia models. For the models with the Ertl SN set, the increasing SN Ia contribution mostly dominates the overall $[\text{Co/Fe}]$ value, due to the smaller mass of SN ejecta from these models, and the $[\text{Co/Fe}]$ value in the ISM trends toward the value ejected from SNe Ia, $[\text{Co/Fe}] \approx -0.25$. For the models with the minimum fallback SN set, the SNe have a stronger contribution due to larger IMF-weighted ejecta mass, and the final value resulting from the combined SN and SN Ia ejecta is reflected in the ISM as $[\text{Co/Fe}] \approx 0$.

Overall, we find the GCE models with the Ertl SN set produce a more robust fit to the observed abundances of $[\text{Co/Fe}]$ for $[\text{Fe/H}] < -1$, whereas models with the minimum fallback SNe provide a better fit for $[\text{Fe/H}] \gtrsim -1$. Neither of the models can convincingly reproduce the large $[\text{Co/Fe}]$ observed in the lowest metallicity stars. The GCE models with the Ertl SN set produce a better fit when higher mass HNe do not contribute (e.g., $M_{\text{hn}} \leq 40 M_{\odot}$), though all of these models underproduce $[\text{Co/Fe}]$ at solar metallicity. For the GCE models with the minimum fallback SNe, the best fit is produced with $40 M_{\odot} \leq M_{\text{hn}} \leq 60 M_{\odot}$ and $\epsilon_{\text{hn}} \geq 0.25$, though the $[\text{Co/Fe}]$ at the lowest metallicities remains too low.

3.5 Zinc

There is a significant difference in the IMF-weighted $[\text{Zn/Fe}]$ yields between the SNe with different remnant mass prescriptions (Figure 2). On average, the Ertl SN set produces larger $[\text{Zn/Fe}]$ relative to the minimum fallback SNe for $[\text{Fe/H}] \lesssim -1$, whereas the minimum fallback SN set produces larger $[\text{Zn/Fe}]$ relative to the Ertl SN set for $[\text{Fe/H}] \gtrsim -1$. The reason for the difference in $[\text{Zn/Fe}]$ between the two SN sets is not simple to explain, and is essentially due to which particular stars are able to explode at each given metallicity depending on the progenitor structure, see Ertl et al. (2016); Côté et al. (2016) for a more in-depth discussion on the explodability of models. Figure 2 also shows that the IMF-weighted HN yields consistently produce $[\text{Zn/Fe}] \geq 0$, and $[\text{Zn/Fe}] \geq 0.1$ for HN upper mass limit $\gtrsim 40 M_{\odot}$. The W7 SN Ia model produces very low $[\text{Zn/Fe}] \approx -1.5$.

The results for the evolution of $[\text{Zn/Fe}]$ are shown Figure 7. We see that typically, the GCE models with the Ertl SN set produce a larger $[\text{Zn/Fe}]$ value for $[\text{Fe/H}] \lesssim -1$, reflecting the high $[\text{Zn/Fe}]$ from both the low $[\text{Fe/H}]$ Ertl SN set, and dominant contribution from HN sources. Each of the models with different SN sets converge for high HN fraction, higher HN upper mass limit, or a combination of both, as the HN yields begin to dominate the chemical abundances in this regime (lower right corner of Figure 7). Variation of the HN upper mass limit and HN fraction has less effect in the models with the Ertl SN set source, relative to the effect on those with the minimum fallback SNe. This is partly because the $[\text{Zn/Fe}]$ yields from the Ertl SN set and HNe are similar, meaning that the variation of the HN contribution has little effect on the averaged value of $[\text{Zn/Fe}]$. Additionally, Ertl SN set eject less mass than the minimum fallback SNe (Figure 2), so the abundance ratio of $[\text{Zn/Fe}]$ is strongly weighted toward the HN yields, even for a low HN rate. Broadly speaking, each of the models with the Ertl SN set evolve with fairly constant $[\text{Zn/Fe}] \sim 0.2$ until $[\text{Fe/H}] \gtrsim -1$. At this point the contributions from SNe Ia and higher metallicity SNe commence, and the value of $[\text{Zn/Fe}]$ trends toward $\approx -0.5$ for each model.

Both a higher HN fraction and a higher HN upper mass limit have a strong effect on the $[\text{Zn/Fe}]$ evolution in the models which contain the minimum fallback SNe. When the HN contribution is minimal (upper left corner of Figure 7), we find $[\text{Zn/Fe}] \approx -0.2$ for $[\text{Fe/H}] \lesssim -1$. When the HN contribution is increased, $[\text{Zn/Fe}]$ is as high as $\sim 0.3$. This large value for $[\text{Zn/Fe}]$ requires either $\epsilon_{\text{hn}} \geq 0.1$ and HN upper mass limit $\gtrsim 60 M_{\odot}$, or $\epsilon_{\text{hn}} \geq 0.25$ and HN upper mass limit $\gtrsim 40 M_{\odot}$. Whereas the SN Ia contribution to $[\text{Zn/Fe}]$ at $[\text{Fe/H}] \gtrsim -1$ dominates the ISM abundances in the models with the Ertl SN set, the minimum fallback SNe...
have more massive ejecta and therefore a stronger contribution to the [Zn/Fe] value. The ISM [Zn/Fe] abundance in these models trends toward $\gtrsim -0.5$.

We are not able to match the high [Zn/Fe] $\gtrsim 0.5$ at the lowest metallicities with any of our models, nor are we able to match the mean solar value of [Zn/Fe]. Whereas all of our models with the Ertl SN set are able to produce a more robust fit to the observed values for [Fe/H] $\leq -1$, the value of [Zn/Fe] toward solar metallicity is far too low. Although the models with the minimum fallback SNe produce [Zn/Fe] that is too low overall, these models do provide a better fit to the trend of [Zn/Fe] versus [Fe/H]. In particular, the [Zn/Fe] increase for low metallicity and plateau for [Fe/H] $\gtrsim -1$. The models with the minimum fallback SNe, with HN upper mass limit $\geq 40 \, M_\odot$ and initial HN fraction $\epsilon_{\text{hn}} \geq 0.25$ provide the best fit, though the absolute value is too low, most notably at solar metallicity.

### 3.6 Early Evolution and Extremely Metal Poor Stars

With the exception of perhaps [Mn/Fe], we have not been able to match the observed abundances in stars with [Fe/H] $\lesssim -3$. It is possible, however, that the fit could be improved by varying some parameters that we have not investigated here, such as the SFR or IMF. There are indications that the IMF of the first stars may have been top-heavy (Bromm et al. 1999; Abel et al. 2002; Susa et al. 2014; Hirano et al. 2014; Hosokawa et al. 2016). It also has been predicted that the most metal-poor stars may have been enriched by the chemical yields from a single, or perhaps just a few, SN/HN events, whereas the one-zone GCE model assumes well-mixed material (Audouze & Silk 1995; Argast et al. 2000, 2002). In Figures 8 through 10, we show the yields of [(Cr,Mn,Co,Zn)/Fe] from each individual SN and HN model available in the sets that we selected from, including models that we did not use. In these figures it can be seen that we are in fact unable to match all of the observed abundances of [(Cr,Co,Zn)/Fe] with any single HN or SN model, and in turn, nor would any combination of the SN/HN models that we have not already investigated be capable of doing so.

On the other hand, when considering the possibility that some low metallicity stars may have been enriched by only one SN/HN, the lack of variation in [Cr/Fe] across all of our HN/SN models is consistent with the small scatter in the observed value (Cayrel et al. 2004). Furthermore, the range of [Mn/Fe] values produced by our zero-metallicity models is consistent with the larger scatter observed in [Mn/Fe]. The high energy ($\gtrsim 30 \times 10^{51} \, \text{erg}$) explosions of the $40 \, M_\odot$ model can potentially provide a reasonable fit to [(Mn,Co,Zn)/Fe], though the [Cr/Fe] becomes too large in these models.
Figure 8. The values of \([\text{Cr,Mn,Co,Zn}/\text{Fe}]\) in the ejecta of the hypernova models (Grimmett et al. 2018). The grey regions indicate models which undergo significant fallback and eject negligible amounts of iron-peak elements.

Figure 9. The values of \([\text{Cr,Mn,Co,Zn}/\text{Fe}]\) in the ejecta of the supernova models with the minimum fallback prescription.
Figure 10. The values of $[(\text{Cr,Mn,Co,Zn})/\text{Fe}]$ in the ejecta of the supernova models with the Ertl et al. (2016) remnant mass prescription. The grey regions indicate models which fail to explode and therefore eject negligible amounts of iron-peak elements.
4 DISCUSSION

We have calculated the mean evolution of the iron-peak ratios $[\text{Cr}, \text{Mn}, \text{Co}, \text{Zn}] / \text{Fe}$ in the Galaxy, using a GCE model that (i) applies the instantaneous mixing approximation, (ii) includes the time delay between formation and death of a star as a function of stellar mass and metallicity, and (iii) allows for inflow of material from a galactic halo. We include the chemical yields from SNe, HNe, and SNe Ia. Whereas this type of GCE model is not novel, the SN and HN yields that we include are the most modern and comprehensive sets available (Côté et al. 2016; Grimmett et al. 2018). Along with an improved understanding of the relationship between nucleosynthesis and explosion energy, there have been several developments in nuclear and stellar physics, and in the general implementation of this knowledge in the presupernova and supernova modelling (e.g., Rauscher et al. 2002; Heger & Woosley 2010). It is useful to periodically collate new nucleosynthetic results with updated GCE models, in order to establish an understanding of where we stand in explaining Galactic chemical trends. We have explored the effect of altering the HN contribution by occurrence rate and maximum progenitor mass, and also investigated SN models with two different remnant mass prescriptions. By exploring our results across a wide parameter space, our aim was to gain a thorough understanding of the strengths and shortcomings of current SN/HN models to explain the chemical evolution in the Galaxy, and in particular, to better understand the role of HNe in this process.

For the evolution of $[\text{Cr}/\text{Fe}]$ and $[\text{Mn}/\text{Fe}]$, we have found that a reasonable fit to the observed relation between $([\text{Cr}, \text{Mn}]/\text{Fe})$ and $[\text{Fe}/\text{H}]$ can be made with almost any combination of SN/HN models and HN rate. This statement is especially true for $[\text{Cr}/\text{Fe}]$, which varies only slightly with different SN sets and HN contributions. On one hand, the small variation in $[\text{Cr}/\text{Fe}]$ that is produced in our models is consistent with the particularly small scatter of observed values of $[\text{Cr}/\text{Fe}]$, as noted by Cayrel et al. (2004). On the other hand, if our models are accurate, then the flatt trend in $[\text{Cr}/\text{Fe}]$ that we see in our GCE results would support the suggestion that the observational values of $[\text{Cr}/\text{Fe}]$ reported for metal-poor stars suffer from metallicity dependent corrections (Cayrel et al. 2004; Lai et al. 2008). Bergemann & Cescutti (2010) find that neglect of non-local thermodynamic equilibrium effects (NLTE) in the analysis of Cr spectral lines result in an underestimate of the $[\text{Cr}/\text{Fe}]$ value in the most metal-poor stars. In any case, the consistency of the result for $[\text{Cr}/\text{Fe}]$ across our range of model parameters should be kept in mind when using $[\text{Cr}/\text{Fe}]$ as a diagnostic for the quality of any given GCE model. The evolution of $[\text{Mn}/\text{Fe}]$ shows a larger variation, depending on the SN/HN model parameters. Across the entire parameter space, however, we are still able to achieve a reasonably good fit to the observed trend of low $[\text{Mn}/\text{Fe}]$ at early times, increasing to the solar value. The robustness of the fit can be attributed to the low $[\text{Mn}/\text{Fe}]$ ejected from HN models, which are the dominant enrichment source at early times, shifting to the larger $[\text{Mn}/\text{Fe}]$ in the SN Ia ejecta, which becomes the dominant contribution for increasing metallicity. This is the typical $[\text{Mn}/\text{Fe}]$ evolution that has also been achieved in earlier studies (e.g., Kobayashi et al. 2006). We find that if the HN contribution becomes too large, i.e. when $M_{\text{HN}, u} \geq 60 \, M_\odot$ and $\eta_{\text{HN}} \geq 0.25$, the $[\text{Mn}/\text{Fe}]$ is driven too low to be consistent with the observed values at lower metallicity, though we still recover the solar value at later times in each of these models. This effect is particularly pronounced for the models with the Ertl SN set, where the HN ejecta is able to more strongly dominate. We have not been able to achieve the super-solar values of $[\text{Mn}/\text{Fe}]$ observed in stars with $[\text{Fe}/\text{H}] \gtrsim 0$. This result is unavoidable, as none of our individual SN or HN models produce super-solar $[\text{Mn}/\text{Fe}]$ (Figure 2, and Section 3.6). Seitenzahl et al. (2013) report that higher mass white dwarf (WD) progenitors (i.e. near Chandrasekhar mass) are required to produce super-solar $[\text{Mn}/\text{Fe}]$, regardless of the exact pathway to explosion. The progenitor systems of SNe Ia are yet to be identified, although observational and theoretical work in the near future will help to provide further constraints on the progenitor properties (see Maoz et al. (2014) for a review). Our results provide additional motivation to consider higher mass WD progenitors.

Both $[\text{Co}/\text{Fe}]$ and $[\text{Zn}/\text{Fe}]$ prove to be far more useful quantities for evaluating our models and associated parameter space, as the evolution of these element ratios show a large variation between GCE models with both different SN sets, and HN contributions. We find a reasonable fit to the observed evolution when implementing HN rates that are not irreconcilable with the observed rates. Furthermore, our models are able to produce both $[\text{Co}/\text{Fe}]$ and $[\text{Zn}/\text{Fe}]$ simultaneously in supersolar values at low metallicity. This is required to make progress in explaining the high observed values of $([\text{Co}, \text{Zn}]/\text{Fe})$ in stars with $[\text{Fe}/\text{H}] \lesssim -3$, which have proven to be difficult to reconcile (Timmes et al. 1995; Kobayashi et al. 2006, 2011; Hirai et al. 2018). We find that a strong contribution from HNe at low metallicity (e.g., $\gtrsim 25\%$ for $[\text{Fe}/\text{H}] \lesssim -2$) is favourable to producing large $([\text{Co}, \text{Zn}]/\text{Fe})$ at low metallicity, though even the largest values that we achieve are still too low to match the $[\text{Co}/\text{Fe}]$ observed in stars with $[\text{Fe}/\text{H}] < -3$. We also find, however, that if some fraction of SNe fail to explode as determined by the progenitor compactness criterion (i.e. the Ertl SN set), then the increasing contribution from SNe Ia dominates the ISM abundance, and the evolution of $[\text{Zn}/\text{Fe}]$ in our models becomes far too low near solar $[\text{Fe}/\text{H}]$ as a result. Although we have found that the models that include the Ertl SN set typically produce a more robust fit to the high $([\text{Co}, \text{Zn}]/\text{Fe})$ observed at low metallicity across the entire parameter space, this is more likely attributed to the smaller IMF-weighted ejecta from these SNe, which allows the HN contribution to more easily dominate. Moreover, the models with the Ertl SN set poorly fit the solar values of $([\text{Co}, \text{Zn}]/\text{Fe})$. The results of recent 3D supernova simulations performed by Burrows et al. (2019) indicate that there is no correlation between progenitor compactness and explodability, though, it certainly appears that not all progenitors in the core-collapse mass range will successfully explode. It may be useful to investigate other prescriptions for model explodability in future GCE studies.

Altogether, we find that the most favourable results
are achieved with a maximum HN mass of $40 \, M_\odot$, an initial HN occurrence rate of 50%, and the minimum fallback SN set i.e. the solid line in panel (h), Figures 4 through 7. In the GCE models with the minimum fallback SNe, we find that a combination of both a large initial HN fraction and more massive HN progenitors can be problematic, most notably for the fit to the observed trend in $[\text{Mn/Fe}]$, which is reasonably well fit with most other combinations of parameters. Furthermore, this combination of parameters is also detrimental for producing the slope in the observed values of $[(\text{Co,Zn})/\text{Fe}]$.

Ongoing investigations into the chemical yields from aspherical HN models will provide a critical step forward in understanding the role of HNe in GCE. Evidence has emerged that at least some HNe are accompanied by gamma-ray bursts (GRBs) (Galama et al. 1998; van Paradijs et al. 2000; Stanek et al. 2003; Malesani et al. 2004; Woosley & Bloom 2006). The two most popular models to explain the connection between the collapse of a massive star and a GRB, the collapsar model of MacFadyen & Woosley (1999); MacFadyen et al. (2001); Woosley & Heger (2003); Barkov & Komissarov (2008) and the magnetar model (e.g., LeBlanc & Wilson 1970; Wheeler et al. 2000; Akiyama et al. 2003; Burrows et al. 2007; Komissarov & Barkov 2007; Obergaulinger & Aloy 2017), would also necessitate intrinsically aspherical HNe. A key ingredient in both of these models is a rapidly rotating progenitor, so HNe from these sources may also provide a natural explanation for larger HN rates in the past, as rapid rotation in stars is predicted to be more common at low metallicity (Woosley & Bloom 2006; Woosley & Heger 2006; Stacy et al. 2011; Stacy & Bromm 2013). Preliminary models for these types of jet-powered explosions indicate that they may be able to provide ejecta with chemical abundances favourable to matching the large $[(\text{Co,Zn})/\text{Fe}]$ observed in metal-poor stars, and possess a total explosion energy which is of order $10^{52}$ erg (Maeda & Nomoto 2003; Tominaga et al. 2007; Tominaga 2009; Nishimura et al. 2017). Some recent first-order estimates also indicate that jetted explosions may help to explain the evolution of Zn abundances in the galaxy (Tsujimoto & Nishimura 2018), however more robust and realistic models will be required to confirm this.

Along with the need for more realistic HN models, there are several other sources of chemical enrichment and factors which could affect the GCE that we have investigated here. As our intention was to investigate the SN/HN contribution to GCE with modern nucleosynthetic results, we opted to keep other contributions to a minimum to avoid obscuring the effect of HN contribution. However, the following may have important implications in GCE calculations and are worth further consideration:

- The least massive ($\sim 9 \, M_\odot$) stars to undergo core-collapse are thought to explode as electron-capture SNe (ECSNe) after forming degenerate O-Ne-Mg cores (Nomoto 1984, 1987; Poelarends et al. 2008). These stars may make significant contributions to Galactic abundances of neutron-rich elements including Zn (Wanajo et al. 2013; Wanajo et al. 2018; Hirai et al. 2018). Although we do include CCSN yields down to $10 \, M_\odot$, the unique yields provided by ECSNe may be significant in GCE.
- There are indications that the IMF of the first stars may have been top-heavy, and that stars might have formed with mass of the order $100 \, M_\odot$ (Bromm et al. 1999; Abel et al. 2002; Susa et al. 2014; Hirano et al. 2014; Hosokawa et al. 2016). If they exist, some fraction of these stars are believed to explode as energetic pair-instability SNe (PISNe), with unique nucleosynthetic yields (Heger & Woosley 2002; Umeda & Nomoto 2002). These stars are so short-lived, however, that their contribution to GCE may be obscured (Komiya 2011).
- In agreement with previous studies, we have found that the evolution of Galactic chemical abundances towards solar metallicity depends critically on the SN Ia rate and chemical yields. There is still no consensus on the progenitor systems and explosion mechanisms for the thermonuclear events observed as SNe Ia. Likewise, there is still uncertainty around the nucleosynthetic end products of SNe Ia. This is an active area of research and future constraints on SN Ia contribution to GCE will be very important to our understanding of the Galactic chemical abundances (Kobayashi et al. 1998; Kobayashi et al. 2000; Matteucci & Recchi 2001; Kobayashi & Nomoto 2009; Seitenzahl et al. 2013; Seitenzahl & Townsley 2017).
- HN chemical yields are sensitive to the particular explosion energy chosen for each model (Grimmett et al. 2018). We have opted to select HN models that provide the most favourable fit to the abundances observed in EMP stars, within reasonable agreement to the theoretical mass-energy relation (Nomoto et al. 2003). Our aim was to test the limits of the most modern SN/HN models, although we certainly could have achieved less favourable results with even a small variation in the explosion energy of our chosen HN models. In finding the best fit that we can achieve with currently available HN models, we hope that that this can be built upon in the future with improved approximations for the explosion mechanism in hypernova modelling.
- Here we only use HN models from metal-free progenitors. The chemical end products for higher metallicity HNe are likely to diverge from the zero metallicity models, as we see for our SN models in Figure 2, and in the HN models of Kobayashi et al. (2006). The changes in SN chemical yield with metallicity, however, seem to be most significant towards solar $[\text{Fe/H}]$, and our HN rate quickly decreases as a function of metallicity, so the effect is likely minimal.

5 CONCLUSION

Using a modern and comprehensive set of SN and HN yields, we have found that we are able to achieve a reasonable fit to the observed Galactic trends in $[(\text{Co,Mn,Co,Zn})/\text{Fe}]$, with a hypernova rate that is within existing observational constraints. Our results indicate that the hypernova contribution to chemical enrichment is made by HNe with an upper limit to the progenitor mass of $40 \, M_\odot$, and an initial HN occurrence rate of 50%, decreasing to $\lesssim 1\%$ at present day. This result is indicative of a moderate contribution from HNe to the chemical enrichment of the Universe, but the specific constraints may change when aspherical HN models and
nucleosynthetic results become available. If some SNe fail to explode, as determined by the progenitor compactness criterion, then some additional source of enrichment will be required to reproduce the solar value of \( [\text{Co}, \text{Zn}] / \text{Fe} \). Complementary to earlier investigations into the role of HNe in Galactic chemical enrichment, our results demonstrate the crucial contribution that HNe provide to understanding the observed Fe-peak ratios in the Galaxy.

On the other hand, our aim was also to determine the areas where there is still significant discrepancy between GCE modelling and observational data. Our findings particularly make apparent the need for advancements in our understanding of (i) \( [\text{Zn}] / \text{Fe} \) enrichment near solar metallicity, and (ii) in \( [\text{Co}, \text{Zn}] / \text{Fe} \) at the lowest metallicities. In both cases we consistently achieve values which are too low to be in agreement with observations. We suggest that (i) may be improved with modern SN Ia chemical yields (which may also provide the supersolar \( [\text{Mn}] / \text{Fe} \) observed in stars with high metallicity) and/or the contribution from ECSNe, and (ii) indicates the need for developments in our understanding of the HN explosion mechanism and resultant nucleosynthesis, particularly with regards to the asphericity which seems to be intrinsic to HNe.

ACKNOWLEDGEMENTS

We thank Chiaki Kobayashi for her helpful discussions and willingness to assist us with the development of our GCE code. This is sincerely appreciated. This work was supported by the Australian Research Council through ARC Future Fellowship FT160100035 (BM) and Future Fellowship FT120100363 (AH). AIK acknowledges financial support from the Australian Research Council (DP170100521). AH has been supported, in part, by a grant from Science and Technology Foundation of Shanghai Municipality (Grants No.16DZ2260200) and National Natural Science Foundation of China (Grants No.11655002). This material is based upon work supported by the National Science Foundation under Grant No. PHY-1430152 (JINA Centre for the Evolution of the Elements). This research was undertaken with the assistance of resources from the National Computational Infrastructure (NCI), which is supported by the Australian Government and was supported by resources provided by the Pawsey Supercomputing Centre with funding from the Australian Government and the Government of Western Australia.

REFERENCES

Abel T., Bryan G. L., Norman M. L., 2002, *Science*, 295, 93
Adibekyan V. Z., Sousa S. G., Santos N. C., Delgado Mena E., González Hernández J. I., Israelian G., Mayor M., Khachaturyan G., 2012, *A&A*, 545, A32
Akiyama S., Wheeler J. C., Meier D. L., Lichtenstadt I., 2003, *ApJ*, 584, 954
Anders E., Grevesse N., 1989, *Geochimica et Cosmochimica Acta*, 53, 197
Andrews B. H., Weinberg D. H., Schönrich R., Johnson J. A., 2017, *ApJ*, 835, 224
Arcavi I., et al., 2010, *ApJ*, 721, 777
Argast D., Smailand M., Gerhard O. E., Thielemann F.-K., 2000, *A&A*, 356, 873
Argast D., Smailand M., Thielemann F. K., Gerhard O. E., 2002, *A&A*, 388, 842
Asplund M., Grevesse N., Sauval A. J., Scott P., 2009, *ARA&A*, 47, 481
Audouze J., Silk J., 1995, *ApJ*, 451, L49
Barkov M. V., Komissarov S. V., 2008. pp 608–611 (https://aip.scitation.org/doi/pdf/10.1063/1.3076747), doi:10.1063/1.3076747
Benz B., Perets H. B., 2005, *ApJ*, 618, 437
Benitez J., Della Valle M., 2006, *ApJ*, 641, 357
Bergerman M., Cecolini G., 2010, *A&A*, 522, A9
Bennicchi A., et al., 2009, *A&A*, 501, 519
Bromm V., Coppi P. S., Larson R. B., 1999, *ApJ*, 527, L5
Burrows A., Dessart L., Livne E., Ott C. D., Murphy J., 2007, *ApJ*, 664, 416
Burrows A., Radice D., Vartanyan D., Nagakura H., Skinner M. A., Dolenje J., 2019, arXiv e-prints, p. arXiv:1909.04152
Cayrel R., et al., 2004, *A&A*, 416, 1117
Chieffi A., Limongi M., 2002, *ApJ*, 577, 281
Côté B., West C., Heger A., Ritter C., O’Shea B. W., Herwig F., Travaglio C., Bisterzo S., 2016, *MNRAS*, 463, 3755
Ertl T., Janka H.-T., Woosley S. E., Sukhbold T., Ugliano M., 2016, *ApJ*, 818, 124
Freyal A., Norris J. E., 2015, *ARA&A*, 53, 631
Fryer C. L., Andrews S., Even W., Heger A., Sali-Harb S., 2018, *ApJ*, 856, 63
Galama T. J., et al., 1998, *Nature*, 395, 670
Grimmett J. J., Heger A., Karakas A. I., Müller B., 2018, *MNRAS*, 479, 495
Halevi G., Mósta P., 2018, *MNRAS*, 477, 2366
Heger A., Woosley S. E., 2002, *ApJ*, 567, 532
Heger A., Woosley S. E., 2010, *ApJ*, 724, 341
Hirai Y., Saitoh T. R., Ishimaru Y., Wanajo S., 2018, *ApJ*, 855, 63
Hirano S., Hosokawa T., Yoshida N., Umeda H., Omukai K., Chacko G., Yorke H. W., 2014, *ApJ*, 781, 60
Hosokawa T., Hirano S., Umeda H., Omukai K., Yoshida N., 2016, *ApJ*, 824, 119
Iben Jr. I., Tutukov A. V., 1984, *ApJS*, 54, 335
Iwamoto K., et al., 1998, *Nature*, 395, 672
Joggerst C. C., Almgren A., Bell J., Heger A., Whalen D., Woosley S. E., 2010, *ApJ*, 709, 11
Karakas A. I., Lattanzio J. C., 2014, *Publications of the Astronomical Society of Australia*, 31, e030
Kasen D., Woosley S. E., 2009, *ApJ*, 703, 2205
Kawata D., Gibson B. K., 2003, *MNRAS*, 340, 908
Kobayashi C., Nomoto K., 2009, *ApJ*, 707, 1466
Kobayashi C., Tsujimoto T., Nomoto K., Hachisu I., Kato M., 1998, *ApJ*, 503, L155
Kobayashi C., Tsujimoto T., Nomoto K., 2000, *ApJ*, 539, 26
Kobayashi C., Umeda H., Nomoto K., Tominaga N., Ishikawa G., Yorke H. W., 2006, *ApJ*, 653, 1145
Kobayashi C., Karakas A. I., Umeda H., 2011, *MNRAS*, 414, 3231
Komissarov S. S., Barkov M. V., 2007, *MNRAS*, 382, 1029
Komiyama Y., 2011, *ApJ*, 736, 73
Lai D. K., Bolte M., Johnson J. A., Lucatello S., Heger A., Woosley S. E., 2008, *ApJ*, 681, 1524
LeBlanc J. M., Wilson J. R., 1970, *ApJ*, 161, 541
MacFadyen A. I., Woosley S. E., 1999, *ApJ*, 524, 262
MacFadyen A. I., Woosley S. E., Heger A., 2001, *ApJ*, 550, 410
Maeda K., Nomoto K., 2002, *ApJ*, 588, 842
Maeda K., et al., 2008, *Science*, 319, 1220
Malesani D., et al., 2004, *ApJ*, 584, 432
Maoz D., Mannucci F., Nelemans G., 2014, *ARA&A*, 52, 107
Matheson T. J., et al., 2003, *ApJ*, 599, 394

*MNRA000, 1–18 (xxxxx)*

_Hypernovae and GCE_ 17
APPENDIX A: APPENDICES

A1 Reproducing the Results of Kobayashi et al. (2006)

In Figures A1 to A4, we show that we are able to reproduce the results of Kobayashi et al. (2000, 2006) with our implementation of the chemical evolution equations, using the chemical yields as provided by Kobayashi et al. (2006). Small differences between our results may be due to different treatments of the IMF discretisation, or in the interpolation between metallicities for SN/HN models.

To reproduce these results we implement an IMF with upper limit $m_u = 50 M_\odot$ and the solar values from Anders & Grevesse (1993) as is used by Kobayashi et al. (2006). For the remainder of our calculations, however, we use an upper limit $m_u = 50 M_\odot$ and the solar values provided by Asplund et al. (2009).

This paper has been typeset from a TeX/LATEX file prepared by the author.
Figure A2. Results for the evolution of \([\text{Fe/H}]\) from our model (solid) and the model of Kobayashi et al. (2006) (dashed).

Figure A3. Results for the evolution of the SFR from our model (solid) and the model of Kobayashi et al. (2006) (dashed).

Figure A4. The IMF weighted yields of \([\text{Cr}, \text{Mn}, \text{Co}, \text{Zn}]/\text{Fe}\) from SNe+HNe in our implementation (solid) and the implementation of Kobayashi et al. (2006) (dashed).