Observables for model-independent detections of $Z'$ boson at the ILC

V. Skalozub $^a$* and I. Kucher $^b$$^{†}$

$^a$ Dnipropetrovsk National University, 49010 Dnipropetrovsk, Ukraine
$^b$ Dnipropetrovsk National University, 49010 Dnipropetrovsk, Ukraine

Abstract

The integral observables for model-independent detections of Abelian $Z'$ gauge boson in $e^+e^- \rightarrow \mu^+\mu^- (\tau^+\tau^-)$ process with unpolarized beams at the ILC energies are proposed. They are based on the differential cross-section of deviations from the standard model predictions calculated with a low energy effective Lagrangian and taking into consideration the relations between the $Z'$ couplings to the fermions derived already. Due to these relations, the cross-section exhibits angular distribution giving a possibility for introducing one- or two parameter observables which effectively fit the mass $m_{Z'}$, the axial-vector $a^2_{Z'}$, and the product of vector couplings $v_{e}v_{\mu}(v_{e}v_{\tau})$. Determination of the basic $Z'$ model is discussed. Comparison with other results and approaches is given.

1 Introduction

Searching for new heavy particles beyond the energy scale of the standard model (SM) is one of the main goals of modern high energy physics. Nowadays it is established on the base of experimental data accumulated at hadron colliders such as Tevatron and the LHC. As it was planned beforehand, distinguishable important discoveries of these experiments will be further investigated in details at the ILC which will have energies of $\sim 500-1000$ GeV in the center-of-mass of beams but much better precision of measurements due to point-like structure of leptons and experiments with polarized initial and final fermions.

One of expected heavy particles beyond the SM is $Z'$ gauge boson which is related with an additional $U(1)$ group. It enters as a necessary element numerous GUT models like $SO(10)$, $E_6$ as well as superstrings, extra dimensions, etc.

*E-mail: skalozubv@daad-alumni.de
†E-mail: kucherimnaua@gmail.com
Detailed description of the $Z'$ is given in [1]-[4]. Searches for this particle have been established already within the LEP data in either model-dependent [5] or model-independent [8] approaches, and the Tevatron data [11], [12]. Modern model-dependent measurements constrain that the mass $m_{Z'}$ to be larger than $2.5 - 2.9$ TeV [13], [14]. So, at the ILC experiments the $Z'$ will be investigated as a virtual state.

At present about hundred $Z'$ models are discussed in the literature. In model-dependent searches established, only the most popular ones such as $LR$, $ALR$, $\chi$, $\psi$, $\eta$, B - L, $SSM$, have been investigated and the particle mass estimated. These models are also used as benchmarks in introducing effective observables for future experiments at the ILC [15], [16]. Analysis of the RS model of strong gravity in $e^+e^-$ annihilation into leptons see, for example, in [17]. Role beam polarizations in detecting of various anomalous particle couplings is discussed for many years in the literature. One of beginning papers is [18]. It worth also mentioning that most investigations devoted to model-dependent searches at the ILC deal with the polarized beams and corresponding observables are introduced.

On the other hand, recent studies of perspective variables for identification of the $Z'$ models [16], in particular, came to conclusion that, as complementary way, a model-independent approach is very desirable. An important feature of this method is that not only the $Z'$ mass but also the couplings to the SM fermions are unknown parameters which must be fitted in experiments. Estimations of couplings can be further used in specifying of the basic $Z'$ model. Usually, the couplings are considered as independent arbitrary numbers. However, this is not the case and they are correlated parameters, if some natural requirements, which this model has to satisfy, are assumed. For instance, in most cases we believe that the basic model is renormalizable one. Hence, correlations follow and the amount of free low energy parameters reduces. Moreover, the correlations between couplings influence kinematics of the processes that gives a possibility for introducing the specific observables which uniquely pick out the virtual state of interest – $Z'$ boson in our case. The noted additional requirement assumes searching for new particles within the class of renormalizable models. In other aspects the models are not specified. In what follows, we will say ”model-independent approach” in the case when either the mass or the couplings must be fitted. Clearly that different correlations fix coupling properties common to the specific classes of models. Such type analysis is in between the customary model-dependent method, when all the couplings are fixed and only the mass $m_{Z'}$ is free parameter, and model-independent searches assuming complete independence of couplings describing new physics.

In the present paper we search for the Abelian $Z'$ boson coming from the extended renormalizable model. We also assume that there is only one additional heavy particle relevant at considered energies. There are numerous models of such type. In particular, most of $E_6$ motivated models and mentioned above ones belong to this class. In general, the requirement of renormalizability admits
two sets of correlations between the low energy couplings [20]. First is used in what follows (Eq. (4)). Second corresponds to a massive neutral vector particle interacting with left-handed fermion species, only. It covers other class of extended models (see, for example, [22]). Searches for this type particle require other observables and separate analysis. For more details see Refs. [20], [21]. In what follows, we say $Z'$ boson for the Abelian one. We also assume, as usually, that the SM is the subgroup of the extended group and therefore no interactions of the type $ZZ^*W^+$ appear at a tree-level.

We apply the model-independent search for the $Z'$ by analyzing the deviations of the differential cross-sections for the annihilation process $e^+e^- \rightarrow \mu^+\mu^- (\tau^+\tau^-)$ from the SM predictions considered at center-of-mass energies 500 - 1000 GeV. We introduce new observables, $A(E, m_{Z'})$ (9), giving a possibility for estimating both the axial-vector coupling of the $Z'$ to the SM fermions $a_{Z'}$ and the mass $m_{Z'}$, and the observable $V(E, m_{Z'})$ (14), for fitting the products of vector couplings $v_{e\mu}, v_{e\tau}$ and the mass $m_{Z'}$. Our analysis is carried out within the effective low energy Lagrangian introduced in [6], [7] which describes interactions of the $Z'$ with the SM fermions. It was used already for the $Z'$ searches at the LEP experiments. Detailed description of it and the obtained results are presented in the review [8]. Here we apply this approach with modifications necessary in searching for the $Z'$ at the ILC. At giving energies and expected particle masses, distinguishable properties of the factors at couplings entering the cross-section are observed that gives a possibility for introducing noted observables. Their values can be used in subsequent determination of the basic $Z'$ model. Moreover, the ratio of $A(E, m_{Z'})$ (or $V(E, m_{Z'})$) taken at different energies depends on the $m_{Z'}$, only and may be used as new observables for model-independent estimation of it.

The paper is organized as follows. In next section we adduce necessary information on the approach used and write down the differential cross-section of deviations from the SM due to the $Z'$ contributions. In section 3 we analyze for different energies and mass $m_{Z'}$ the factors at the couplings entering this cross section. On the base of these considerations new integral observable dependent on the axial-vector coupling $a_{Z'}$ and mass $m_{Z'}$ is introduced. In section 4 the observable for model-independent estimate of $m_{Z'}$ is proposed. In section 5 the observable for detecting the product of the vector couplings and the mass is introduced. The last section is devoted to the discussion of the results obtained and comparison with other approaches. In Appendix 1 explicit expressions for the cross-sections are adduced. In Appendix 2 as application the model-independent discovery reach for the mass $m_{Z'}$ is estimated by using the ratio of observable $V(E, m_{Z'})$ taken at different energies. In Appendix 3 we calculate the discovery reach following from the observable $A(E, m_{Z'})$ with accounting for the values $a_{Z'}^2$ estimated from the data set of LEP experiments.
2 Cross-section for the $Z'$ detections

At low energies, $Z'$ boson can manifest itself as virtual intermediate state through the couplings to the SM fermions and scalars. Moreover, the $Z$ boson couplings are also modified due to a $Z-Z'$ mixing. As it is known (see reviews [2, 4, 8]), significant signals beyond the SM can be inspired by the couplings of renormalizable type. Such couplings can be described by adding new $\tilde{G}(Z')$-terms to the electroweak covariant derivatives $D^{ew}$ in the Lagrangian [6, 7]

$$L_f = i \sum_{f_L} \tilde{f}_L \gamma^\mu \left( \partial_\mu - \frac{ig}{2} \sigma_a W^a_\mu - \frac{ig'}{2} B_\mu Y_{f_L} - \frac{ig}{2} \tilde{B}_\mu \tilde{Y}_{f_L} \right) f_L$$

$$L_{\phi} = \left| \left( \partial_\mu - \frac{ig}{2} \sigma_a W^a_\mu - \frac{ig'}{2} B_\mu Y_{\phi} - \frac{ig}{2} \tilde{B}_\mu \tilde{Y}_{\phi} \right) \phi \right|^2,$$

where summation over all the SM left-handed fermion doublets, leptons and quarks, $f_L = (f_u)_L, (f_d)_L$, and the right-handed singlets, $f_R = (f_u)_R, (f_d)_R$, is understood. In these formulas, $g, g', \tilde{g}$ are the charges associated with the $SU(2)_L, U(1)_Y$, and the $Z'$ gauge groups, respectively, $\sigma_a$ are the Pauli matrices, $Q_f$ denotes the charge of $f$ in positron charge units, $Y_{\phi}$ is the $U(1)_Y$ hypercharge, and $Y_{f_L} = -1$ for leptons and $1/3$ for quarks. In case of Abelian $Z'$, the $\tilde{Y}_{f_L} = \tilde{Y}_{f_L} \text{diag}(1, 1)$ and $Y_{\phi} = \tilde{Y}_{\phi} \text{diag}(1, 1)$ are diagonal $2 \times 2$ matrices with corresponding coupling factors. These generators do not influence the $SU(2)_L$ symmetry.

The $Z-Z'$ mixing angle $\theta_0$ is determined by the coupling $\tilde{Y}_{\phi}$ as follows

$$\theta_0 = \frac{\tilde{g} \sin \theta_W \cos \theta_W}{\sqrt{4 \pi \alpha_{em}}} \frac{m_Z^2}{m_{Z'}^2} \tilde{Y}_{\phi} + O \left( \frac{m_Z^4}{m_{Z'}^4} \right),$$

where $\theta_W$ is the SM Weinberg angle, and $\alpha_{em}$ is the electromagnetic fine structure constant. Although the mixing angle is a small quantity of order $m_{Z'}^2$, it contributes to the $Z$-boson exchange amplitude and cannot be neglected in general. There are precision constraints on its value, coming, in particular, from the LEP1 experiments. It is one of the main parameters of the $Z'$ physics. It will be systematically accounted for in what follows.

Below, we will use the $Z'$ couplings to the vector and axial-vector fermion currents defined as

$$v_f = \frac{\tilde{g}}{2} \tilde{Y}_{L,f} + \tilde{Y}_{R,f}, \quad a_f = \frac{\tilde{g}}{2} \tilde{Y}_{R,f} - \tilde{Y}_{L,f}.$$

The Lagrangian (11) leads to the following interactions between the fermions and the $Z$ and $Z'$ mass eigenstates:

$$\mathcal{L}_{Zff} = \frac{1}{2} Z_\mu \tilde{f} \gamma^\mu \left[ (v_f^\text{SM} + \gamma^5 a_f^\text{SM}) \cos \theta_0 + (v_f + \gamma^5 a_f) \sin \theta_0 \right] f,$$
\[ \mathcal{L}_{Z'ff} = \frac{1}{2} Z'_\mu \bar{f} \gamma^\mu \left[ (v_f + \gamma^5 a_f) \cos \theta_0 - (v_{fZ}^{SM} + \gamma^5 a_{fZ}^{SM}) \sin \theta_0 \right] f, \]  

where \( f \) is an arbitrary SM fermion state; \( v_{fZ}^{SM}, a_{fZ}^{SM} \) are the SM couplings of the \( Z \)-boson.

As it occurs, if the extended model is renormalizable, the relations between couplings hold (see [19], [20], [8]):

\[ v_f - a_f = v_{f*} - a_{f*}, \quad a_f = T_{3f} \tilde{g} \tilde{Y}_f. \]  

Here \( f \) and \( f^* \) are the partners of the \( SU(2)_L \) fermion doublet (\( l^* = \nu_l, \nu^* = l, q_u^* = q_d \) and \( q_d^* = q_u \)), \( T_{3f} \) is the third component of weak isospin. They also can be derived by imposing the requirement of invariance of the SM Yukawa term with respect to the \( U(1) \) gauge transformations [9]. Therefore the relations (6) are independent of the number of scalar field doublets.

The couplings of the Abelian \( Z' \) to the axial-vector fermion current have a universal absolute value proportional to the \( Z' \) coupling to the scalar doublet. Then, the \( Z-Z' \) mixing angle (3) can be determined by the axial-vector coupling. As a result, the number of independent couplings is significantly reduced. Because of the universality we will omit the subscript \( f \) and write \( a \) instead of \( a_f \).

We assume no new light particles. The relations (6) could change essentially if the SM has to be modified at energies below the \( Z' \) mass. Thus, we suppose no supersymmetry below the \( Z' \) decoupling scale.

Although the relations (6) were derived for effective low-energy parameters, they nevertheless also hold at tree-level in a wide class of known models containing the Abelian \( Z' \) (see [8]). They also fulfill for the case of the Two-Higgs-Doublet SM. This is the reason to call them model-independent ones. The correlations (6) essentially influence the kinematics of scattering processes and give a possibility to uniquely detect the mass and couplings of the virtual \( Z' \) boson state.

Let us consider the process \( e^+e^- \rightarrow l^+l^- \) \((l = \mu, \tau)\) with the non-polarized initial and final fermions. Two classes of diagrams have to be taken into consideration. The first one includes the pure SM graphs. This part should be estimated as accurate as possible. The second class includes heavy \( Z' \) boson as the virtual state described by the effective Lagrangian (5) and the scalar particle contributions. We assume that \( Z' \) is decoupled and not excited inside loops at the ILC energies. The tree-level diagram \( e^+e^- \rightarrow Z' \rightarrow l^+l^- \) defines a leading contribution to the cross-section. It is enough to take into account this diagram to estimate the \( Z' \) signals. The cross-section includes the contribution of the interference of the SM amplitudes with the \( Z' \) exchange amplitude (having the order \( \sim a^2, v_{fa} \)) and the squared of the latter one (of the order \( \sim a^4, v_f^4 \)). Since the couplings of the \( Z' \) are small, the last contribution can be neglected at far from resonance energies. In our calculations, radiative corrections to the \( Z' \)-exchange diagram are incorporated in the improved Born approximation. This mainly influences the values of couplings at high energies and is sufficient for applied analysis.
Within these assumptions, the deviation of the differential cross-section for the process $e^+ e^- \rightarrow \mu^+ \mu^- (\tau^+ \tau^-)$ can be written in the form (see Eq. (27) in [8])

$$\Delta \sigma(z) = \frac{d\sigma}{dz} - \frac{d\sigma^{\text{SM}}}{dz} = f_{1\mu\mu}(z) \frac{a^2}{m_{Z'}^2} + f_{2\mu\mu}(z) \frac{v_{e\mu}}{m_{Z'}^2} + f_{3\mu\mu}(z) \frac{a v_{e}}{m_{Z'}^2} + f_{4\mu\mu}(z) \frac{a v_{\mu}}{m_{Z'}^2}. \quad (7)$$

Here, $z = \cos \theta$ is the cosine of scattering angle $\theta$. Eq. (7) is our definition of the $Z'$ signal.

This cross-section accounts for the relations (6) through the known dimensionless functions $f_i(z), i=1,\ldots,4$, since the coupling $\tilde{Y}_\phi$ (the mixing angle $\theta_0$) is substituted by the axial-vector coupling $a$ which is universal parameter. Usually, when a four-fermion effective Lagrangian is applied to describe physics beyond the SM, this dependence on the scalar field coupling is neglected at all [10], [16]. However, in our case, when we are interested in searching for signals of the $Z'$ boson on the base of the effective low-energy Lagrangian (1)–(2), these contributions to the cross-section are essential.

To introduce the observables of interest we have to investigate the $z$-dependence of the factors $f_i(z), i=1,\ldots,4$ for a number of proper energies and possible $Z'$ masses. In Appendix 1 we adduce this functions as well as the expression for the SM case in the chosen approximation.

### 3 Observable for estimation of $a^2$ and $m_{Z'}$

According to Eq.(7), the deviation of the differential cross-sections is described by four factors $f_i(z)$. Let us investigate their behavior assuming that couplings $a, v_f$ have the same order of magnitude. In this case the kinematics properties of $f_i(z)$ can be elucidated.

For definiteness, in Figs. 1, 2 we show the behavior for energy $E = 500$ GeV in the $e^+ e^-$ center-of-mass and the mass $m_{Z'} = 2500, 3000$ GeV. Here, some remark is needed. As it was reported in [13], [14], the lower bound on the mass $m_{Z'}$ obtained from the data on the Drell-Yan process at the LHC is $m_{Z'} \geq 2.5 - 2.9$ TeV. It has been estimated assuming the narrow resonances with the width $\Gamma_{Z'}$ of the order: $\Gamma_{Z'}/m_{Z'} \sim 0.01$. Similar assumptions were also used in [16] for the analysis of the process $e^+ e^- \rightarrow f \bar{f}$ at the ILC with the goal of introducing effective observables for the determination of the $Z'$ model. On the other hand, as it was argued in [11], the resonances with not small $\Gamma_{Z'}$ are not excluded. They could considerably decrease the value of the lower bound on the $m_{Z'}$. Below, to present the results we take the ratio $\Gamma_{Z'}/m_{Z'} \sim 0.1$ (the results for narrow resonances are similar).
Fig. 1 Behavior of factors $f_1(z)$, $f_2(z)$, $f_4(z)$ for $m_{Z'} = 2500$ GeV, width $\Gamma_{Z'} = 250$ GeV for $E = 500$ GeV

In the plots, the function $f_1(z)$ is presented as solid line, the $f_2(z)$ is shown as dot-dashed one and the functions $f_3, 4(z)$ are shown as dashing line. The $f_3, 4(z)$ coincide at high energies when one can neglect fermion masses. The function $f_1(z)$ has opposite signs for the forward and backward beams. This is in contrast to the factors $f_2(z)$ and $f_3, 4(z)$. The former is negative and the latter - positive.
one. Moreover, the factors $f_{3,4}(z)$ are suppressed by two orders of magnitude as compared to the $f_1(z)$ and $f_2(z)$. Shown angular dependence is typical and takes place in the wide mass interval and for other energies, for example, 1000 GeV. The mass interval $1.5 \leq m_{Z'} \leq 4$ TeV was investigated. This behavior makes reasonable introducing the integral observable which picks out the contribution coming from the first term in Eq. (7) and consequently the axial-vector coupling $a^2$.

Really, we can integrate $f_2(z)$ in the intervals $(-1 < z < -0.2)$ (where the function $f_1(z)$ is positive) and $(-0.2 < z < z^*)$ (where $f_1(z)$ is negative) and specify the limit $z^*$ in such a way that the difference of the integrals turns to zero:

$$(-0.2 \rightarrow z^*)(\int_{-1}^{z^*} - \int_{-0.2}^{z^*}) f_2^{\mu\mu}(z) dz = 0.$$  (8)

Since $f_2(z)$ is sign definite, this point always exists. At the same time, due to opposite signs of $f_1(z)$ in these intervals and sign definiteness of $f_{3,4}(z)$, the difference is mainly determined by the first term in (9). The partial cancelation of the contributions coming from $f_{3,4}(z)$ takes place. Although this is not very essential because of the significant suppression of these factors. As a result, the value of the universal coupling constant $a^2$ can be estimated with high accuracy. As explicit calculations showed, the upper limit of integration equals to $z^* = 0.489$ for a wide interval of both the mass $m_{Z'}$ and beam energies $E$. It is also important that the function $f_1(z)$ changes its sign at the point $z = -0.2$ for all energies and masses investigated.

On these grounds we introduce the observable for model-independent estimating of the $a^2$ and $m_{Z'}$:

$$A(E, m_{Z'}) = (\int_{-1}^{-0.2} - \int_{-0.2}^{z^*})(\frac{d\sigma}{dz} - \frac{d\sigma^{SM}}{dz}) dz.$$  (9)

Here, the lower and upper limits of integration are theoretical bounds. They can be substituted by other ones corresponding to actual set up of experiments. For example, for the lower limit $z_{lower} = -0.9$, that is close to the values of measured scattering angles, $10^\circ < \theta < 170^\circ$, planned for the ILC detectors [25], the upper limit is $z^* = 0.406$. To complete this section, we adduce the values of the observable (9) for the number of the mass and energy values.

In the tables, in first, second and third columns the energy, mass and width values (expressed in GeV) are given, correspondingly. In the fourth column the contribution coming from $f_1^{\mu\mu}(z)$ is adduced. In the fifth and sixth columns the values of the contributions coming from the factors $f_{3,4}^{\mu\mu}(z)$, $f_2^{\mu\mu}(z)$ Eq.(7) are shown.
From Tables 1, 2 it can be estimated as \( \kappa_a \).

Second case, both of the parameters can be found in two parameter fitting. Since simply estimated in a model-independent way within a one parameter fit. In the former case, for the given mass \( m \) for example, \([15], [16]\). First, the \( Z \) obtained at the LHC. Usually, two scenarios are discussed in the literature (see, \[Z\] ). First, the mass \( m \) and the integral \( |f| \) are also positive by construction. So, the positivity of the \( A(E, m_{Z'}) \) is determined by two couplings \( a^2 \) and \( m_{Z'} \).

Since the contribution of the factor \( f_2(z) \) is chosen to be zero, the \( A(E, m_{Z'}) \) is determined by two couplings \( a^2 \) and \( \mu \). The efficiency of the observable is determined from the relation:

\[
\kappa_A = \frac{|f^{\mu\mu}_{i}|}{|f^{\mu\mu}_{1}| + |f^{\mu\mu}_{3,4}|},
\]

Here the quantities \( |f^{\mu\mu}_{i}|, i = 1, 3, 4 \), mark the integrals

\[
\left( \int_{-0.2}^{z^*} - \int_{-1}^{0.2} \right) f^{\mu\mu}_{i}(z)dz > 0.
\]

From Tables 1, 2 it can be estimated as \( \kappa_A = 0.9896 \) for all the given energy and mass values.

The signature of the observable is also important. The coupling \( a^2 \) is positive and the integral \( |f^{\mu\mu}_{i}| \) is also positive by construction. So, the positivity of the \( A(E, m_{Z'}) \) is the distinguishable signal of the \( Z' \) boson.

This observable can be used in different ways dependently on the results obtained at the LHC. Usually, two scenarios are discussed in the literature (see, for example, [15], [16]). First, the \( Z' \) is detected and its mass is estimated but the model not. Second, neither the mass \( m_{Z'} \) nor the \( Z' \) model are determined. In the former case, for the given mass \( m_{Z'} \), the axial-vector coupling \( a^2 \) can be simply estimated in a model-independent way within a one parameter fit. In the second case, both of the parameters can be found in two parameter fitting. Since \( a^2 \) is universal it covers numerous \( Z' \) models.
4 Observable for estimation of $m_{Z'}$

Interesting application of the observable $A(E, m_{Z'})$ (9) is related with the model-independent determination of the mass $m_{Z'}$. Really, the observable (9) includes the factor $a^2$ which is canceled in the ratio $R^\text{experim}_A = \frac{A(E_1, m_{Z'})}{A(E_2, m_{Z'})}$. So that the behavior of it can be used in estimating $m_{Z'}$. We can consider two cross-sections with close energies $E_1$ and $E_2 = E_1 + \Delta E$ and write

$$R^\text{experim}_A = \frac{A(E_1, m_{Z'})}{A(E_2, m_{Z'})} = 1 - \frac{\partial \ln A(E_1, m_{Z'})}{\partial E_1} \Delta E. \quad (12)$$

As a theoretical curve $R^\text{theory}_A$ the function $f^{\mu\mu}_1$ from Eq.(7) has to be substituted in Eq.(12) instead of $A(E_1, m_{Z'})$. This is because contributions of all other form-factors are suppressed in the difference. As a result, we obtain the observable dependent on $m_{Z'}$, only. Hence, the value of the mass can be estimated by using a standard $\chi^2$ method. The value $\Delta E$ can be taken as the difference between the closer beam energies of experiments.

One may wonder how the mass can be estimated without any information about the $Z'$ boson, which will depend on the couplings. In particular, the measurement accuracy of the $Z'$ mass has to depend on the size of the new-physics signal determined by the coupling values. However, information on the coupling is completely removed from $R^\text{experim}_A$.

Nevertheless, in the developed approach this possibility is realized due to the following. First, the model-independent analysis is based on the cross-section of deviations $\Delta \sigma(z)$ (7), which tends to zero if couplings are very small. So that the deviation must be visible. Since the factors $f_i$ take into account the relations (6), they uniquely pick out the virtual $Z'$ boson state. Therefore, we expect that, if some deviation is generated by any other virtual particles, the estimated couplings $a$ and $v$ are to be zero and the mass very large. Second, the dependence on the mass is a propagator effect which uniquely exhibits itself though the function $f^{\mu\mu}_1$ after integration over the interval of $z$ where the contributions of the factors $f_2, f_3, f_4$ are canceled in the total. This integral is the same for various coupling values. We shall return to this problem in what follows.

The linear approximation used in the l.h.s. of Eq. (12) is sufficient for small $\Delta E$. It can be modified for relatively large $\Delta E$. In actual investigations, one can start from the mass estimates and then use the obtained results in determining of $a^2$ by means of a one parameter fit. This can be used as complementary analysis for the two parameter fitting mentioned above.

5 Observable for estimation of $\nu_\tau \nu_\mu$ ($\nu_\tau \nu_\mu$)

The behavior of factors shown in Fig. 1, Fig. 2 gives also a possibility of introducing the observable for model-independent determination of the product
\( v_e v_\mu (v_e v_\tau) \). As we see from the plots and Tables 1, 2, the contributions of the factors standing at \( av_\mu \) and \( av_v \) are suppressed and can be neglected. To exclude the contribution of the \( a^2 \)-dependent term we have to integrate the differential cross-section \( \Delta \sigma(z) \) over \( z \) in the interval \((-1 \leq z \leq z^v)\) and select the upper limit from the requirement

\[
\int_{-1}^{z^v} f_1^{\mu\mu}(z) \, dz = 0. \tag{13}
\]

Hence, we obtain the observable \( V_{e\mu}(E, m_{Z'}) \) for estimation of \( v_e v_\mu \) (or \( v_e v_\tau \)):

\[
V_{e\mu}(E, m_{Z'}) = \int_{-1}^{z^v} \left( \frac{d\sigma}{dz} - \frac{d\sigma^{\text{SM}}}{dz} \right) dz, \tag{14}
\]

where the limit \( z^v \) depends on the energy \( E \) and mass \( m_{Z'} \).

Let us adduce the values of \( z^v \) and \( V_{e\mu}(E, m_{Z'}) \) for the number of energy and mass values. In Table 3, the first three columns show the center-of-mass energy, mass and width, as in Tables 1, 2. In the fourth column the cosine of boundary angles is adduced. In the last two columns the corresponding values of \( V_{e\mu} \cdot m_{Z'}^2 \) and the contributions of the factor at the product \( av_\mu \) are presented. Of course, these limits can be substituted by other ones according to an experiment set up. As above, the contributions of the factors \( \sim av_e, av_\mu \) are negligibly small and can be omitted in the total.

The efficiency of the observable \( V(E, m_{Z'}) \) is determined analogously to the \( \kappa_A \) \(^{10}\) according to the condition

\[
\kappa_V = \frac{|f_2^{\mu\mu}|}{|f_2^{\mu\mu}| + |f_3^{\mu\mu}|}, \tag{15}
\]

where now the quantities \( |f_i^{\mu\mu}|, i = 2, 3, 4 \), mark the integrals over the interval \(-1 < z < z^v\). The efficiency is estimated as \( \kappa_V = 0.9891 \). Again we obtain very efficient observable.

Since the factor \( f_2(z) \) is negative, the sign of the observable \( V(E, m_{Z'}) \) depends on sign of the product \( v_e v_\mu \). If this value is positive, we have negatively defined observable. For this case, the negative sign is also the distinguishable signal of the virtual \( Z' \) boson.
As a result, we have obtained the two parameter observable for fitting of the product of vector couplings and the mass \( m_{Z'} \). In the case of family independence for vector couplings \( v_e = v_\mu = v_\tau = v \), as it is often assumed, \( v^2 \) can be determined in the model-independent way. The calculation procedures are quite similar to that described for the case of the \( a^2 \) coupling. Moreover, the positivity of \( A(E, m_{Z'}) \) and negativity of \( V(E, m_{Z'}) \) is the distinguishable signal of the \( Z' \) boson. Analogously to \( A(E, m_{Z'}) \) in sect. 4, the observable \( V(E, m_{Z'}) \) can be used for model-independent estimate of the \( Z' \) mass. To demonstrate this ability, in Appendix 2 we derive a discovery reach for \( m_{Z'} \), estimated for some values of \( \Delta E \).

The accuracy of possible estimates depends on both theoretical and experimental uncertainties. The former account for the accuracy of the cross-section calculation, which includes the SM terms and the additional terms coming from the low energy effective Lagrangian Eq.(1). The latter depend on the precision of measurements. The detailed analysis of that within the LEP1 and LEP2 experiments is provided in [8]. It also has relevance to considered case. Here we note that the deviation \( \Delta \sigma(z) \) (and therefore the introduced model independent observables) with high accuracy can be related with the standard variables - the deviation of the total cross-section \( \Delta \sigma^T \) and forward-backward asymmetry \( A^{FB} \). It looks as follows:

\[
\Delta \sigma(z) = (1 + z^2)\beta + z\eta + \delta(z),
\]

where \( \delta(z) \) is the difference between the exact and approximate cross-sections. The parameters \( \beta, \eta \) can be calculated as (see [8] for details)

\[
\Delta \sigma^T = \sigma^T - \sigma^{T,SM} = \frac{8\beta}{9} + \delta(-1),
\]

\[
\Delta \sigma^{FB} = \sigma^{FB} - \sigma^{FB,SM} = \eta + \delta(0),
\]

where \( \delta(-1) \), \( \delta(0) \) are the deviations at the specified \( z \). The forward-backward cross-section can be written in the form:

\[
\Delta \sigma^{FB} = \Delta \sigma^T A^{FB} + \sigma^{T,SM} \Delta A^{FB}.
\]

Through these relations the accuracy of measurements of the introduced observables \( A(E, m_{Z'}), V_{e\mu}(E, m_{Z'}) \) can be related with the accuracy of measurements of the total cross-section and the forward-backward asymmetry.

As it was estimated for LEP experiments (see [8]), the deviation \( \delta \) is much less than the systematic error, which includes also theoretical errors, and for the SM is of the order 2 \%. So that we assume that not larger values will be for the ILC. At considered energies, the contributions of the omitted terms \( \sim a^4, (v_e v_\mu)^2 \) are estimated as 0.1 \%. According to data in Tables 1-3, the neglected contributions coming from the factors \( f_3, f_4 \) are estimated as 1 - 1.5 \%. Hence, we estimate the theoretical errors as 3-4 \%. The accuracy of measurements of the leptonic
cross-sections is expected to be high. Thus, the couplings and the mass $m_{Z'}$ can be precisely measured either from the differential cross-sections or from data on the total cross-sections.

The derived values can be used further in determination of the basis renormalizable $Z'$ model. This procedure depends on the results obtained at the LHC. If the mass $m_{Z'}$ will be estimated, the couplings $a^2, v_ev_\mu$ can be determined in the one parameter fits.

6 Discussion

We have investigated the process $e^+e^- \rightarrow \mu^+\mu^- (\tau^+\tau^-)$ for unpolarized initial and final fermions at the center-of-mass energies 500 − 1000 GeV with the goal of introducing the integral observables for model-independent detections of the $Z'$ boson. In doing that the relations (6) have been used. The account of them considerably reduces the number of parameters which must be fitted in experiments. Moreover, the factors entering the differential cross-section (7) exhibit features giving a possibility for introducing the integral observables (9) and (14) dependent mainly on only one coupling $a^2$, or $v_ev_\mu (v_ev_\mu)$, correspondingly, and the mass $m_{Z'}$. So that all these parameters can be estimated within one- or two parameter fits. Remind that the coupling $a^2$ is universal according to the relations (6).

On the basis of these observables the model-independent estimate of the $Z'$ mass can be done. It may be of interest if the $Z'$ boson is heavy and could not be discovered at the LHC. At low energies, the data on the cross-sections at two different energies are needed. Then, the mass $m_{Z'}$ can be found from the observable $R_A^{\text{experim}}$ (12) related to the observable (9), or from the similar observable $R_{Ve\mu}^{\text{experim}}$ related to the $V_{e\mu}$ (14). To obtain the latter one we have to substitute the function $f^\mu\mu_1$ in the theoretical expression $R_A^{\text{theory}}$ by the $f^\mu\mu_2$ from (7) and make obvious modifications in sect. 4. The same can be done for the $\tau$-lepton final states. As a result, the mass $m_{Z'}$ can be fitted by using two different factor functions and therefore there are two ways of measuring this parameter.

It worth to mention that the observables (9), (14) are specialized mainly for detecting the couplings not the mass $m_{Z'}$. By construction, each of them is proportional to the constant determining the interaction strength. On the contrary, the dependence on the mass $m_{Z'}$ is the propagator effect which is described by smooth functions (see Appendix 1). That is why the model-independent determination of the $Z'$ mass can not be done with very good accuracy. This partially can be compensated by the number of different fits. In connection with application at the ILC, the main theoretical error of the observables $R_A$ and $R_V$ is related with sufficiently large interval, $\Delta E = 200 − 300$ GeV, between beam energies. For LEP experiment data, where $\Delta E \sim 10$ GeV, they are more reliable.

To have some ideas about the ability of the observables, in Appendix 2 we
estimate the discovery reach for $m_{Z'}$ followed from the observable $R_V$ for two values of $\Delta E = 300$ and $10$ GeV. The value $m_{Z'}^{DRV} = 1.5$ TeV obtained in the former case is approximately twice less than the lower bound on the mass reported in [13], [14]. In fact, this could be the consequence of the estimate roughness related with the linear approximation used. So, it may occur that two parameter fits including the couplings and the mass are more precise. On the contrary, for $\Delta E = 10$ GeV $m_{Z'}^{DRV} = 12$ TeV. All these require detailed analysis and comparisons of the results coming from the $R_A$ and $R_V$ observables. It will be done elsewhere separately.

In Appendix 3 we obtain the discovery reach for $Z'$ with taking into consideration the observable $A(E, m_{Z'})$ (9) and the axial-vector coupling $a^2$ (see Eq. (33)) estimated from the data set of LEP experiments and reported in [8]: $m_{Z'}^{ARA} = 4.4$ TeV. This value follows from the model-independent estimates obtained with accounting for the relations (6). It is not much larger than the low limits on the mass obtained in the model-depended searches for popular models: $m_{Z'} > 2.9 - 3$ TeV.

Next what can be verified on the base of the $V(E, m_{Z'})$ observable is family independence of $v_f$ couplings. Really, the ratio of the observables taken at a fixed energy

$$D_{v}^{\mu\tau} = \frac{V(E, m_{Z'})_{\mu}}{V(E, m_{Z'})_{\tau}} = \frac{v_{\mu}}{v_{\tau}}$$

depends on the coupling values and has to be unit in the case of the family independence. It can be simply checked.

The observable $D_{v}^{\mu\tau}$ can also be used for measuring the couplings $v_e, v_\mu, v_\tau$ in the leptonic processes. Usually it is believed [27], [28] that an additional information coming from hadronic processes is necessary. This speculation follows from the fact that in leptonic cross-sections the couplings enter as the products $v_e v_\mu = d_{e\mu}, v_e v_\tau = d_{e\tau}$. In the considered case, let us assume that the products $d_{e\mu}$ and $d_{e\tau}$ are measured. Then, the observable (20) equals to: $D_{v}^{\mu\tau} = d_{e\mu}/d_{e\tau}$. Hence

$$v_{\mu} = v_{\tau} D_{v}^{\mu\tau}, \quad v_e = \frac{d_{e\tau}}{v_{\tau}},$$

and we can express these couplings in terms of $v_\tau$. Combining this with the results on the Bhabha process $e^+ e^- \rightarrow e^+ e^-$, all the leptonic vector couplings can be measured.

It is essential that signature of the observables - positive sign of $A(E, m_{Z'})$ and negative sign of $V(E, m_{Z'})$ - is the signal of the Abelian $Z'$ boson.

The present approach can be used as an additional way for detecting at the ILC the $Z'$ boson as well as determining the model which it has to belong. Let us consider the case when the $Z'$ resonance state is observed at the LHC and we are interested in distinguishing between the models. This problem is reduced to distinguishing of model couplings. For example, in Ref. [28] the possibility of
separating the χ model coming from the $E_6$ symmetry breaking, LR-symmetric model (LR), Little Higgs model (LH), Simplest LH model (SLH) and KK excitations originating in theories of extra dimensions is discussed. Detailed analysis for the number of expected mass values is given within the sets of observables and beam polarizations $P_{e^+}, P_{e^-}$. This way is typical for model-dependent analysis.

From the point of view of the present approach accounting for the relations (6) or other ones corresponding to the chiral $Z'$ boson (see for details [8], [22]), select some classes of models. According this classification, the χ and LR models satisfy the relations (6) and can be analyzed with the observables considered. The LH and SLH models correspond to the effective theory which is not renormalizable. So, the couplings of these model do not fit these relations. The same concerns the KK model. The $Z'$ models investigated in [27] satisfy the relations (6) and can also be analyzed by means of the introduced observables.

Then, the found values of the couplings can be compared with the values for the specific renormalizable $Z'$ models. As a result, the number of the perspective candidates can be considerably reduced. This is very important because the identification reach for the $Z'$ models at the LHC is estimated as $m_{Z'} \leq 2 - 2.3$ TeV whereas the nowadays model-dependent lower bound is $\sim 2.5 - 2.9$ TeV [13], [14]. So, most probably, the basic model will not be identified at this collider at all. This problem must be attacked at the ILC.

Let us say a few words about the role of beam polarizations $P_{e^+}, P_{e^-}$. For the s-channel processes the cross-section reads (see, for example, eq.(3) in [28]):

$$\sigma_{P_{e^+}P_{e^-}} = (1 - P_{e^+}P_{e^-})[1 - P_{eff}A_{LR}] \sigma^{unpolarized},$$

(22)

where $A_{LR}$ is the left-right asymmetry and $P_{eff} = (P_{e^+} - P_{e^-})/(P_{e^+}P_{e^-} - 1)$ is the effective polarization. As we see, the cross-section $\sigma_{P_{e^+}P_{e^-}}$ is proportional to the unpolarized one. The polarization dependent factors modify the effective luminosity for the process. This does not change qualitatively the results discovered for unpolarized beams.

Now, let us compare the obtained results with the ones reported in the review [8], where the couplings $a^2, v_e^2$ and the mass $m_{Z'}$ were estimated within the data of the LEP1 and LEP2 experiments. The couplings have been estimated at 1 - 2 $\sigma$ CL. This, in particular, means that the $Z'$ boson is Abelian one belonging to the class covered by the relations (6). The mass was estimated to be $\sim 1.1 - 1.4$ TeV. This is in contrast to the results reported by the LEP Collaborations where no deviations from the SM at the 2 $\sigma$ CL have been determined. In fact, the main goal of that investigations was searching for the $Z'$ particle at energies $\sim 100 - 200$ GeV. So, the observables introduced were constructed with accounting for the contact four fermion couplings, as it was done by the LEP Collaborations. The energy of the beams 500 GeV was also considered. However, the simple and specific behavior of the factors $f_i(z)$ shown in Figs. 1, 2 was not determined. As a result, more complicated analysis was carried out and other observables
for model-independent fitting of the $a^2$ and $v_f$ were used. Their efficiency is sufficiently high. For instance, $\kappa_a^2 = 0.9587$ and $\kappa_v = 0.9533$ (see Eqs.(33), (29) in [8]). The systematic error of the calculations was estimated to be 5 - 10%. So, they also can be used in the analysis of experiments at the ILC and the results compared with obtained on the base of the observables $A(E, m_{Z'})$ and $V(E, m_{Z'})$.

As the present study shown, information on the differential cross-sections with unpolarized beams is sufficient for determining important characteristics of the virtual $Z'$ state. Of course, it is of interest to consider the case of polarized beams in more detail. It will be problem for the future.

To complete, we would like to note that the proposed observables are perspective for consistent analysis of future experiments at the ILC.

**Acknowledgements**

The authors are grateful to A.V. Gulov and A.A. Pankov for fruitful discussions and suggestions and A. A. Kozhushko for the help in preparation of the package for numeric calculations.

**Appendix 1**

In this appendix, we adduce the expression for the SM differential cross-section and the factors $f_i(z, E)$ entering Eq.(7) and calculated in the improved Born approximation. To realize that we have used the packages FeynArts [23], FormCalc and LoopTools [24] and Mathematica. The lepton masses are set to zero. For convenience, here we denote cosine of scattering angle as $x = z = \cos \theta$ and introduce the standard notations: $s_W = \sin \theta_W$, $c_W = \cos \theta_W$, where $\theta_W$ is the Weinberg angle, $\alpha$ is a fine structure constant.

The differential cross-section reads

$$\frac{\partial \sigma}{\partial x} = \sigma_{SM} + a^2 f_1(x) + v_e v_\mu f_2(x) + a v_e f_3(x) + a v_\rho f_4(x).$$

(23)

In contrast to Eq.(7) the factor $m_{Z'}^2$ is incorporated in the functions. The cross-section is measured in $GeV^{-2}$.

The SM part is expressed in terms of the resonant functions $f_Z$ and $f_{ZE}$:

$$\sigma_{SM} = \frac{1}{32 s_W^4 c_W} \left\{ (1 + x^2) \right\}$$

(24)

$$\times \left[ 4 s_W^4 c_W^4 / E^2 + f_{ZE}(1 - 4 s_W^2 + 8 s_W^4) + f_Z 2 s_W^2 c_W^2 (1 - 4 s_W^2) \right]$$

$$+ \ x \times \left[ 2 f_{ZE}(1 - 4 s_W^2) + f_Z 4 s_W^2 c_W^2 \right].$$
The factors are expressed in terms of the resonant functions $f_Z$, $f'_Z$, $f_{ZE}$, $f_{ZZ'}$:

\[
f_1(x) = \frac{-\alpha}{64s_W^4c_W^4m_{Z'}^4}(1 + x^2) \times \left[ f_{ZE}^4c_W^2s_{W,1}^2m_{Z',1}^2m_{Z',2}^2(1 - 4s_{W,1}^2 + 8s_{W,2}^2) - f_{Z'}^4c_W^2s_{W,1}^2m_{Z',2}^2(1 - 4s_{W,2}^2)^2 \right.
\]
\[
\left. - f_{Z}f_{Z'}^2s_{W,1}^2c_{W,1}^2(2m_{Z,1}^2) + f_{Z}f_{Z'}^2s_{W,1}^2c_{W,2}^2(2m_{Z,2}^2)(1 - 4s_{W,1}^2)^2 \right].
\]

\[
f_2(x) = \frac{-\alpha}{64s_W^4c_W^4m_{Z'}^4}(1 + x^2) \times \left[ f_{ZE}^4c_W^2s_{W,1}^2m_{Z',1}^2m_{Z',2}^2(1 - 4s_{W,1}^2) + f_{Z'}^4c_W^2s_{W,1}^2m_{Z',2}^2m_{Z',1}^2(1 - 4s_{W,2}^2) \right.
\]
\[
\left. + f_{Z}f_{Z'}^2s_{W,1}^2c_{W,1}^2m_{Z,1}^2m_{Z,2}^2(-1 + 8s_{W,2}^2 - 24s_{W,1}^2 + 32s_{W,1}^2) \right] - 4s_{W,1}^2c_{W,2}^2m_{Z,2}^2m_{Z,1}^2(1 - 4s_{W,1}^2)^2 \right].
\]

\[
f_3(x) = \frac{-\alpha}{64s_W^4c_W^4m_{Z'}^4}(1 + x^2) \times \left[ f_{Z}^4c_W^2s_{W,1}^2m_{Z',1}^2m_{Z',2}^2(1 - 4s_{W,1}^2) + f_{Z'}^4c_W^2s_{W,1}^2m_{Z',2}^2m_{Z',1}^2(1 - 4s_{W,2}^2) \right.
\]
\[
\left. - x \times 2f_{Z}f_{Z'}^2s_{W,1}^2c_{W,2}^2m_{Z,1}^4 \right].
\]

The resonant functions are:

\[
f_{Z} = \frac{(4E^2 - m_{Z,1}^2)}{(4E^2 - m_{Z,1}^2)^2 + m_{Z,1}^2\Gamma_{Z,1}^2},
\]
\[
f_{Z'} = \frac{(4E^2 - m_{Z,2}^2)}{(4E^2 - m_{Z,2}^2)^2 + m_{Z,2}^2\Gamma_{Z,2}^2},
\]
\[
f_{ZE} = \frac{E^2}{(4E^2 - m_{Z,1}^2)^2 + m_{Z,1}^2\Gamma_{Z,1}^2},
\]
\[
f_{ZZ'} = \frac{(4E^2 - m_{Z,1}^2)(4E^2 - m_{Z,2}^2) + m_{Z,1}^2\Gamma_{Z,1}^2m_{Z,2}^2\Gamma_{Z,2}^2}{(4E^2 - m_{Z,1}^2)^2 + m_{Z,1}^2\Gamma_{Z,1}^2},
\]

where $\Gamma_{Z,1}$, $\Gamma_{Z,2}$ are the widths of $Z$ and $Z'$ bosons.
Appendix 2

In this appendix, we calculate a model-independent discovery reach for the mass $m_{Z'}$ based on the observable $V_{e\mu}$ Eq.(14) and data given in Table 3 for energy 500 GeV. According to sect. 4, the observable reads

$$R_{V_{experim}}(m_{Z'}) = \frac{V(E_1, m_{Z'})}{V(E_2, m_{Z'})} = 1 - \frac{\partial \ln V(E_1, m_{Z'})}{\partial E_1} \Delta E.$$  (29)

As $\Delta E$ we first take the difference $\Delta E = 300$ GeV between the beam energies $E_1 = 500$ GeV and $E_2 = 800$ GeV planned for ILC experiments. The corresponding theoretical curve is the function $f^\mu\mu_2(E, z)$ Eq.(27).

Now, we calculate the necessary constituents for the analysis (for more details see, for example, Ref. [26] where the process $e^+e^- \rightarrow W^+W^-$ is investigated). These are the integral $I_{SM}^*$ of the SM cross-section (23) calculated over the interval $-1 \leq z \leq 0$. It gives a possibility for calculating in the SM the number $N_{SM}^* = I_{SM}^* L_{int}$ of the processes $e^+e^- \rightarrow \mu^+\mu^-$ at a given integral luminosity $L_{int} = 500 fb^{-1}$. Here, for simplicity, as the efficiency of the process reconstruction we take $\epsilon_{\mu^+\mu^-} = 0.95$. We also neglect the systematic errors which are to be much less than statistical ones. Since the beam energy is far from the resonance, we can put in Eq.(28) $\Gamma_{Z'} = 0$. The observable looks as follows

$$R_{f^\mu\mu_2}(m_{Z'}) = 1 - \frac{0.5675}{-1} \int_{-1}^{0.5675} \frac{df^\mu\mu_2(E, z)/dE}{f^\mu\mu_2(E, z)} dz |_{E=500 GeV} \Delta E.$$  (30)

To obtain the discovery reach we calculate the $\chi^2$ function

$$\chi^2 = \frac{(R_{f^\mu\mu_2}(m_{Z'}))^2}{(\delta R)^2_{SM}} \leq \chi^2_{min} + \chi^2_{CL},$$  (31)

where $(\delta R)_{SM}$ is the uncertainty of the observable $R_{f^\mu\mu_2}(m_{Z'})$ calculated for $N^*$, and find (at a chosen confidence level) the upper value of $m_{Z'}$ below which the observable is reliable. The value of $\chi^2_{min}$ depends on the value of $\Delta E$. In the considered case we get $\chi^2_{min} = 0.2334$. In our analysis we choose $\chi^2_{CL} = 5.99$ that corresponds to 2 $\sigma$ CL. Then accounting for that for the function $R_v = (\Delta \sigma(E_1)/\Delta \sigma(E_2)$ the dispersion is calculated as

$$\left(\frac{\delta R_v}{R_v}\right)^2 = \left(\frac{\delta \Delta \sigma(E_1)}{\Delta \sigma(E_1)}\right)^2 + \left(\frac{\delta \Delta \sigma(E_2)}{\Delta \sigma(E_2)}\right)^2$$  (32)

and that $I_{SM}^* = 1.6586 \cdot 10^{-10} GeV^{-2}$, we estimate the model-independent discovery reach $m^{DRV}_{Z'} = 1.5$ TeV. In the case of $\Delta E = 10$ GeV, $\chi^2_{min} = 269.84$ and $m^{DRV}_{Z'} = 12$ TeV.
Appendix 3

One of possibilities for determination of the $Z'$ discovery reach, $m_{Z'}^{DRA}$, is related with the observable $A(E, m_{Z'})$ \textsuperscript{[9]} and combining the results on estimating the coupling $a^2$ from the data set of the LEP experiments. This value has been obtained as (see review \textsuperscript{[8]} for details):

$$
\frac{a^2}{m_{Z'}^2} = 1.97 \times 10^{-2} \text{TeV}^{-2}.
$$

(33)

By using this value and the results of section 3 and Appendix 1 we can construct the $\chi^2$ function:

$$
\chi^2_A = \frac{(a^2 \int_{0.2}^{-0.2} f_{\mu\mu}^i(z)dz)^2}{(\delta A)^2} \leq \chi^2_{min} + \chi^2_{CL},
$$

(34)

where $(\delta A)$ is the uncertainty of the observable $A$, calculated for $N^*$, and find (at a chosen confidence level) the upper value of $m_{Z'}$ below which the observable is efficient. For this observable $\chi^2_{min} = 0$. Expressing this function in terms of a number of particles we get

$$
\chi^2_A = \frac{(a^2 \int_{0.2}^{-0.2} f_{\mu\mu}^i(z)dz)^2}{(\int_{-0.2}^{0.2} \sigma_{SM}(z)dz)^2} \frac{1}{N_{1SM} + N_{2SM}} \leq \chi^2_{CL},
$$

(35)

where $N_{1SM}, N_{2SM}$ are the number of muon pairs in the backward and forward bins calculated in the standard model at a given luminosity 500 $fb^{-1}$ and reconstruction efficiency $\epsilon_{\mu\mu} = 0.95$, $\sigma_{SM}(z)$ is the differential cross section for the process calculated in the SM \textsuperscript{(24)}:

$$
N_{1SM}^{SM} = \left| \int_{-0.2}^{-0.489} \sigma_{SM}(z)dz \times L_{int} \epsilon_{\mu^+\mu^-} \right|,
$$

$$
N_{2SM}^{SM} = \left| \int_{-0.2}^{-0.489} \sigma_{SM}(z)dz \times L_{int} \epsilon_{\mu^+\mu^-} \right|.
$$

(36)

Assuming $\chi^2_{CL} = 5.99$ we obtain $m_{Z'}^{DRA} = 4.4$ TeV.

References

[1] J.L. Hewett and T.G. Rizzo, Phys. Rep. 183, 193 (1989).
[2] A. Leike, Phys. Rep. 317, 143 (1999). arXiv:hep-ph/9805494

[3] T.G. Rizzo, arXiv:hep-ph/0610104

[4] P. Langacker, Rev. Mod. Phys., 81, 1199 (2009). arXiv: 0801.1345 [hep-ph]

[5] J. Erler, P. Langacker, S. Munir and E. Rojas, J. High Energy Phys. 0908, 017 (2009) arXiv: 0906.2435 [hep-ph]

[6] G. Degrassi and A. Sirlin, Phys. Rev. D, 40, 3066 (1989)

[7] M. Cvetič and B.W. Lynn, Phys. Rev. D, 35, 51 (1987)

[8] A.V. Gulov and V.V. Skalozub, Int. J. Mod. Phys. A, 25, 5787 (2010). arXiv:0905.2596v2 [hep-ph]

[9] A.V. Gulov and V.V. Skalozub, Int. J. Mod. Phys. A, 16, (2001) 179.

[10] A.A. Babich, et al., Eur. Phys. J. C, 29, 103 (2003). arXiv:hep-ph/0212163

[11] A. Gulov and A. Kozhushko, Int. J. Mod. Phys. A, 26, 4083 (2011). arXiv:1105.3025v1 [hep-ph]

[12] J. Erler, P. Langacker, S. Munir and E. Rojas, J. High Energy Phys. 1111, 076 (2011) arXiv: 1103.2659 [hep-ph]

[13] ATLAS Collaboration, ATLAS-CONF-2013

[14] CMS Collaboration, EX012061

[15] P. Osland, A.A. Pankov and A.V. Tsytrinov, Eur. Phys. J. C, 67, 191 (2010). arXiv:0912.2806 [hep-ph]

[16] T. Han, P. Langacker, Z. Liu and L.T. Wang, arXiv:1308.2738 [hep-ph]

[17] A.V. Kisselev, J. High Energy Phys. 0703, 006 (2007). arXiv:hep-ph/0610113

[18] V.V. Andreev, A.A. Pankov and N. Paver, Phys. Rev. D, 653, 2390 (1996). arXiv:hep-ph/9511300

[19] A.V. Gulov, V.V. Skalozub. Phys. Rev. D, 61, 055007 (2000)

[20] A.V. Gulov, V.V. Skalozub, Eur. Phys. J. C, 17, 685 (2000)

[21] A.V. Gulov, V.V. Skalozub, Phys. Atom. Nucl., 70, (2007) 1100.

[22] K.R. Lynch, S. Mrenna, M. Narain, E.H. Simmons, Phys. Rev. D, 63, (2001) 035006. arXiv:hep-ph/0007286v5
[23] T. Hahn, Comput. Phys. Commun., 140, 418 (2001). arXiv:hep-ph/0012260 v2

[24] T. Hahn and M. Perez-Victoria, Comput. Phys. Commun., 118, 153 (1999). arXiv:hep-ph/9807565 v2

[25] T. Bhenke, J.E. Brau, P.N. Burrows, J. Fuster, M. Peskin, et al., The International Linear Collider Technical Design Report - Vol. 4:Detectors. arXiv:1306.6329

[26] G. Moortgat-Pick, P. Osland, A.A. Pankov and A.V. Tsytrinov, Phys. Rev. D, 87 095017 (2013)

[27] J.A. Aguilar-Saavedra et al. [ECFA/DESY LC Physics Working Group], hep-ph/0106315

[28] S. Godfrey, P. Kalyniak, A. Tomkins, arXiv:hep-ph/0511335v1 29 Nov 2005