Quantum Computing for Neutrino-nucleus Scattering with NISQ Devices

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Neutrino-nucleus scattering

- Accelerator Neutrino Experiments, e.g. DUNE

- Simulate scattering cross sections to predict detector efficiency and backgrounds
Simulate response function and cross sections

- Dynamical linear response function

\[ S(\omega, \hat{O}) = \sum_{\nu} |\langle \phi_{\nu} | \hat{O} | \phi_0 \rangle|^2 \delta(E_{\nu} - E_0 - \omega) = \int dt \langle \phi_0 | \hat{O}^\dagger e^{-i(\hat{H}-E_0-\omega)t} \hat{O} | \phi_0 \rangle \]

Nuclei: \( \hat{H} | \phi_{\nu} \rangle = E_{\nu} | \phi_{\nu} \rangle \)  
ground state: |\phi_0\rangle

- \( S(\omega, \hat{O}) \rightarrow \) inclusive cross sections

- Sample the final nuclei state |\phi_{\nu}\rangle \rightarrow \) semi-exclusive cross sections

- Quantum advantage: bigger nuclei, wide range of kinematics
Starting point: pionless effective field theory

\[ H = 2DtA - t \sum_{f=1}^{N_f} \sum_{f',i,j}^{M} \left[ c_{i,f}^\dagger c_{j,f} + c_{i,f}^\dagger c_{j,f}^\dagger \right] \]

Kinetic energy

\[ + \frac{1}{2} C_0 \sum_{f \neq f'}^{N_f} \sum_{i=1}^{M} n_{i,f} n_{i,f'} \]

Attractive 2-body contact interaction \((C_0 < 0)\)

\[ + \frac{D_0}{6} \sum_{f \neq f' \neq f''}^{N_f} \sum_{i=1}^{M} n_{i,f} n_{i,f'} n_{i,f''} \]

Repulsive 3-body interaction \((D_0 > 0)\) to avoid collapse into deeply bound state

- Approximately reproduce binding of 3 and 4 nucleons
  
  [Phys. Lett. B 772 839-848 (2017), PRL 124 143402 (2020)]

- Simple model for initial study and quantum resource estimation
  
  – Future: need interactions involving virtual pions for accurate prediction
Dynamic linear response quantum algorithm

\[ S(\omega, \hat{O}) = \sum_{\nu} |\langle \phi_{\nu} | \hat{O} | \phi_{0} \rangle|^2 \delta(E_{\nu} - E_{0} - \omega) \]

\[ = \sum_{\nu} |\langle \phi_{\nu} | \psi_{\hat{O}} \rangle|^2 \delta(E_{\nu} - E_{0} - \omega) \langle \phi_{0} | \hat{O}^\dagger \hat{O} | \phi_{0} \rangle \]

Prob. of \( |\psi_{\hat{O}} \rangle \) in eigenbasis \( |\phi_{\nu} \rangle \) \( \rightarrow \) QPE

Ground state meas.

1. Qubit encoding: represent the system by qubits

2. State preparation: \( |\psi_{\hat{O}} \rangle \)

3. Quantum phase estimation of \( |\psi_{\hat{O}} \rangle \) with \( \hat{U} = e^{i(\hat{H} - E_{0})} \)

4. Measure ancilla qubits: probability distribution \( \rightarrow S(\omega, \hat{O}) \)
(nuclei state by measuring the encoding qubits)

Complexity

\[ |\psi_{\hat{O}} \rangle = \frac{\hat{O} |\phi_{0} \rangle}{\sqrt{\langle \phi_{0} | \hat{O}^\dagger \hat{O} | \phi_{0} \rangle}} \]
Qubit encoding efficiency

- Nucleons (fermions) $\rightarrow$ qubits

- General mapping: Jordan-Wigner, Bravyi-Kitaev [1], etc.

- Special case of fixed nucleons: lattice-location encoding
  nucleon 1: $|1\rangle_{N1} = |0\rangle_{q0}|0\rangle_{q1}, |2\rangle_{N1} = |0\rangle_{q0}|1\rangle_{q1}, \ldots$
  nucleon 2: $|1\rangle_{N2} = |0\rangle_{q2}|0\rangle_{q3}, |2\rangle_{N2} = |0\rangle_{q2}|1\rangle_{q3}, \ldots$
  ...

- Efficiency: $A$ nucleons on a lattice with $M$ sites and $N_f$ fermion mode per site

  JW, BK : $N_f \times M$ qubits

  Lattice-location : $A \log_2 M$ qubits

\[
H = 2DtA - t \sum_{f=1}^{N_f} \sum_{i,j}^{M} \left[ c_{i,f}^\dagger c_{j,f} + c_{i,f}^\dagger c_{j,f} \right] \\
+ \frac{1}{2} C_0 \sum_{f \neq f'}^{N_f} \sum_{i=1}^{M} n_{i,f} n_{i,f'} \\
+ \frac{D_0}{6} \sum_{f \neq f' \neq f''}^{N_f} \sum_{i=1}^{M} n_{i,f} n_{i,f'} n_{i,f''},
\]

[1]: Ann. Phys. 298, 210 (2002)
Quantum phase estimation

- QFT and Control-$U$ circuits
  \( \hat{U} = e^{i(\hat{H} - E_0)} \): system propagator

- QFT: gate cost = $O(N^2)$
  $N$: number of ancilla qubits

- $U$ circuits: Trotter decompositions
  - $U_1(\tau) = e^{-i\tau K} e^{-i\tau V}$
  - $U_2^{K+V}(\tau) = e^{-i\tau K/2} e^{-i\tau V} e^{-i\tau K/2}$
  - $U_2^{V+K}(\tau) = e^{-i\tau V/2} e^{-i\tau K} e^{-i\tau V/2}$

- Control-$U$ circuits: replace gates by their controlled version

**Superposition**

\[
\begin{align*}
|0\rangle & \rightarrow H |0\rangle \rightarrow \cdots \\
|0\rangle & \rightarrow H |0\rangle \rightarrow \cdots \\
|0\rangle & \rightarrow H |0\rangle \rightarrow \cdots \\
|\psi\rangle & \rightarrow C - U^0 |\psi\rangle \rightarrow C - U^1 |\psi\rangle \rightarrow \cdots \rightarrow C - U^{N-1}
\end{align*}
\]

**Controlled $U$ Operations**

\[
\begin{align*}
H = 2D t A - t \sum_{f=1}^{N_f} \sum_{i=1}^{M} \left[ c_{i,f}^j c_{j,f} + c_{i,f}^j c_{j,f} \right]
+ \frac{1}{2} C_0 \sum_{f \neq f'} \sum_{i=1}^{N_f} n_{i,f} n_{i,f'}
+ \frac{D_0}{6} \sum_{f \neq f' \neq f''} \sum_{i=1}^{N_f} n_{i,f} n_{i,f'} n_{i,f''}
\end{align*}
\]

- $K$: kinetic energy
- $V$: potential energy
- Diagonal in qubit basis after JW
Gate counts of quantum phase estimation

- Gate counts based on 2 gates
  - CNOT: control-not, two-qubit gate
  - $R_Z$: rotation-Z, single-qubit gate

- Quadratic decomposition: favorable

- Gate counts $\rightarrow \sim 10^{10}$
  - Final 99% fidelity: $1 - e^{\frac{\ln 0.99}{10^{10}}}$
    $\rightarrow \sim 10^{-12}$ gate error rate
  - Need error-corrected qubits for full linear response algorithm simulating realistic model
NISQ implementation of modified linear response algorithm

1. Qubit encoding: small # of nucleons
   - Lattice-location encoding

2. State preparation: \( |\psi_\hat{\theta}\rangle = \frac{\hat{\theta}|\phi_0\rangle}{\sqrt{\langle \phi_0 | \hat{\theta}^\dagger \hat{\theta} | \phi_0 \rangle}} \)
   - Approximated low-energy state \( |\tilde{\phi}_0\rangle \) by a variational ansatz

3. Quantum phase estimation of \( |\psi_\hat{\theta}\rangle \) with \( \hat{U} = e^{i(\hat{H} - E_0)} \)
   - Time evolution by \( \hat{U}(t) = e^{i(\hat{H} - E_0)t} \) on a pretrained initial state

4. Measure ancilla qubits: probability distribution \( \rightarrow S(\omega, \hat{O}) \)
   - Directly measure \( S(\omega, \hat{O}) = \int dt \langle \phi_0 | \hat{O}^\dagger e^{-i (\hat{H} - E_0 - \omega)t} \hat{O} | \phi_0 \rangle \) (no ancilla qubits)
4-qubit proof-of-principle experiment

• Triton toy model:
  – 3 nucleons with one chosen to be static on a 2 by 2 lattice
  – 2 effective nucleons ($A = 2$, $N_f = 2$, $M = 4$)
  – Two-nucleon dynamics incorporates important information about nuclear response ([arXiv:1909.06400](https://arxiv.org/abs/1909.06400))

• Lattice-location encoding: $A \log M = 4$ qubits
  – In comparison, JW needs $N_f M = 8$ qubits

\[
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+ \frac{1}{2} C_0 \sum_{f\neq f'}^{N_f} \sum_{i=1}^{M} n_{i,f} n_{i,f'} \\
+ \frac{D_0}{6} \sum_{f\neq f'\neq f''}^{N_f} \sum_{i=1}^{M} n_{i,f} n_{i,f'} n_{i,f''}
\]

IBMQ Poughkeepsie

\[
C_0 = U, D_0 = -4U
\]
State preparation with a variational ansatz

• 2-parameter variational ansatz $|\phi(\vec{\theta})\rangle$

• Trained by a noiseless simulator to minimized the energy $E(\vec{\theta}) = \langle \phi(\vec{\theta})|H|\phi(\vec{\theta})\rangle$

• Optimized state: $|\tilde{\phi}_0\rangle = \hat{O}|\phi_0\rangle$ (low-energy state)

• Run the pretrained circuit on the IBM QPU

• QPU shows a promising result with error mitigation (readout error mitigation and noise extrapolation)
Time evolution with 1 Trotter step

- 1st order Trotter’s step: $U(\tau) = e^{-i\tau K} e^{-i\tau V}$

- Initial state: pretrained state $|\tilde{\phi}_0\rangle$

- 3-body contact with: $C_3(\tau) = |\langle 0000|U(\tau)\tilde{\phi}_0\rangle|^2$
  $|0000\rangle$: all nucleons at site 1

\[
H = 8t + \frac{U}{2} - 2t \sum_{k=1}^{4} X_k - \frac{U}{4} (Z_1 Z_4 + Z_2 Z_3) - \frac{U}{4} \sum_{i<j<k} Z_i Z_j Z_k
\]
Result of 1-Trotter-step time evolution

- Expt. result: 3-week-window collection
- Output: considerable change from run to run
- Error is noticeable for a single Trotter’s step → cannot do multiple Trotter’s steps
- Error mitigation is insufficient to bring down the error
Promising result and further studies needed

1. Qubit encoding: small # of nucleons
   - Lattice-location encoding

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   - Approximated low-energy state $|\tilde{\phi}_0\rangle$ by a variational ansatz

3. Quantum phase estimation of $|\psi_\tilde{\phi}\rangle$ with $\hat{U} = e^{i(\hat{H} - E_0)}$
   - Time evolution by $\hat{U}(t) = e^{i(\hat{H} - E_0)t}$ on a pretrained initial state

4. Measure ancilla qubits: probability distribution $\rightarrow S(\omega, \hat{O})$
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Further studies on error mitigation, hardware improvement
Overview

• Quantum algorithm for dynamic linear response $S(\omega, \hat{D})$
  – Inclusive/exclusive cross sections of neutrino-nucleus scattering
  – Components: state preparation and quantum phase estimation
  – Full scale studies with realistic model: potentially an important application of error-corrected quantum computer

• NISQ implementation
  – Components: ground state preparation and time evolution
  – Promising result with today hardware
  – Linear response of simple models: near-term applications with error mitigation strategies implemented and hardware improvement