A Quantizable Model of Massive Gauge Vector Bosons without Higgs

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Abstract

We incorporate the parameters of the gauge group \( G \) into the gauge theory of interactions through a non-linear partial-trace \( \sigma \)-model Lagrangian on \( G/H \). The minimal coupling of the new (Goldstone-like) scalar bosons provides mass terms to those intermediate vector bosons associated with the quotient \( G/H \), without spoiling gauge invariance, remaining the \( H \)-vector potentials massless. The main virtue of a partial trace on \( G/H \), rather than on the entire \( G \), is that we can find an infinite-dimensional symmetry, with non-trivial Noether invariants, which ensures quantum integrability in a non-canonical quantization scheme. The present formalism is explicitly applied to the case \( G = SU(2) \times U(1) \), as a Higgs-less alternative to the Standard Model of electroweak interactions, although it can also be used in low-energy phenomenological models for strong interactions.

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1 Introduction

In the 1960’s the mechanism of spontaneously broken symmetry, usually referred to as the Higgs-Kibble mechanism [1], came into the particle physics scenario [2], imported from solid state physics (mainly in relation to Meissner effect), to match the masses of the intermediate vector bosons with renormalizability [3]. However, in spite of the wide acceptance today of the Standard Model of electroweak interactions as a whole and of its phenomenological accuracy (putting aside the existence of the Higgs particle), there exists a rather extended feeling that a deeper structure is underneath, owing specially to the artificiality of the mass generation mechanism.

In this paper we face the chief point of the mass generation mechanism aiming at outlining a conceptually and mathematically neat framework within which the fundamentals of the Standard Model can be reproduced. This framework is essentially based on the inclusion of the gauge-group parameters into the theory as scalar dynamical fields paralleling the standard Goldstone bosons. With a proper Lagrangian for these new fields of the \( \sigma \)-model type and appropriate rewriting of the traditional Minimal Coupling Prescription we arrive at a general Massive Gauge Theory explicitly exhibiting gauge symmetry. When applied to the electroweak symmetry the new prescription provides mass to the \( W^\pm \) and \( Z \) vector bosons without the need for the Higgs particle, leaving naturally the electromagnetic field massless. It might also be used to address

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low energy effective models for the strong interaction according to the schemes handled in Refs. [4].

The explicit use of the Goldstone bosons in the description of physical processes is by no means new in the literature. In fact, as a consequence of the widely named “Equivalence Theorem” [5, 6], according to which a very heavy Higgs particle can be eliminated from the broken symmetry programme in favour of non-linear $\sigma$-like Goldstone bosons, the actual computation of Feynman diagrams involving the longitudinal polarizations of the (massive) vector bosons in electroweak interactions can be resolved in terms of the corresponding diagrams among those scalar fields. But even more, the possibility of incorporating explicitly the Goldstone bosons into the theory as some sort of matter fields has also been considered in the framework of non-abelian (generalized) Stueckelberg theory without Higgs [7]. Unfortunately, the use of a non-linear $\sigma$-Lagrangian, as a trace over the whole gauge group, has led to an insoluble dichotomy unitarity-renormalizability [8, 9] (see also the review [10] and references therein).

In this paper we introduce a simple, though essential, modification to the non-abelian Stueckelberg model. We shall adopt a non-linear partial-trace $\sigma$-model Lagrangian on $G/H$ instead of on the whole $G$. The minimal coupling of the new (Goldstone-like) scalar bosons provides mass terms to those intermediate vector bosons associated with the quotient $G/H$, without spoiling gauge invariance, so that the $H$-vector potentials remain massless in a natural way. The advantage of considering a partial trace on $G/H$, rather than on the entire $G$, lies on the existence of an infinite-dimensional symmetry enlarging the gauge symmetry group, providing as many non-zero Noether invariants as field degrees of freedom in the solution manifold of the physical system. This ensures quantum integrability, at least under a non-canonical quantization scheme based on symmetry grounds, as has been widely demonstrated in those systems bearing enough symmetries as happens in, for instance, conformal field theories.

It is well known that the non-linear sigma model, in general, suffers from unavoidable renormalizability problems under the canonical quantization programme (see, for instance, [11]). In fact, the trouble that canonical quantization faces in dealing with systems bearing non-trivial topology could be traced back to the “tangent space” approximation imposed at the very beginning of the (canonical) quantization program [12]. Already in the simple case of “free” particles moving on spheres, a proper quantization requires the replacement of canonical commutators with the Lie-algebra commutators of the Euclidean group [12, 13]. Going further in this direction, we shall replace canonical commutators between coordinates and momenta with Lie-algebra commutators between group generators of the enlarged local symmetry. In fact, the new “canonical” structure of the solution manifold can be derived directly from the symmetry group as one of its canonical invariant forms (giving the symplectic potential).

The present paper is organized as follows. In Sec. 2 we briefly report on the partial-trace version of the non-Abelian Stueckelberg Model. In Sec. 3, a finite quantization of the $G = SU(2)$ case will be sketched, although our symmetry-based quantization proposal can be applied as well to any semi-simple gauge group. In Sec. 4 we consider the case of $G = SU(2) \times U(1)$, aiming at reformulating the fundamentals of the Standard Model for electroweak interactions, along with an additional simple symmetry mixing which generates the mass of the elementary charged fermions.
2 A revision of the non-Abelian Stueckelberg model

As already commented in the introduction, Stueckelberg’s original idea (versus Proca) for giving mass to $U(1)$ gauge vector bosons in a renormalizable way consists roughly speaking in giving dynamical content to the gauge group parameters $U(x) = e^{i\varphi(x)} \in U(1)(M)$, $x \in M$, through the kinematical $U(1) - \sigma$ model Lagrangian (density)

$$\mathcal{L}^{U(1)}_{\sigma} = \frac{1}{2} \partial_\mu U \partial^\mu U^\dagger = \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi = \theta_\mu \theta^\mu, \quad (1)$$

($\theta_\mu \equiv -i\partial_\mu U U^\dagger = \partial_\mu \varphi$) minimally coupled according to the standard prescription:

$$\tilde{\mathcal{L}}^{U(1)}_{\sigma} = \frac{1}{2} (D_\mu U)(D^\mu U)^\dagger = \frac{1}{2} (\theta_\mu - A_\mu)(\theta^\mu - A^\mu) \quad (2)$$

where $D_\mu = \partial_\mu - iA_\mu$ stands for the covariant derivative. Then, the complete Abelian Stueckelberg Lagrangian is:

$$\mathcal{L}^{U(1)}_{MA} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + m^2 \tilde{\mathcal{L}}^{U(1)}_{\sigma}. \quad (3)$$

Quite remarkably, this Lagrangian is gauge invariant, even though it explicitly contains mass terms for the gauge bosons and no symmetry breaking has taken place.

The natural non-Abelian extension of the Stueckelberg formalism for a general gauge group $G$ follows similar steps. Now $U(x) = e^{i\varphi^a(x)} T^a \in G(M)$, where $T^a$, $a = 1, \ldots, \text{dim}(G)$ are the Lie-algebra generators of $G$ with commutation relations $[T^a, T^b] = C^c_{ab} T^c$. We shall restrict ourselves to unitary groups and set the normalization $\text{Tr}(T^a T^b) = \delta^{ab}$. When referring to the canonical 1-form on $G$, we must distinguish between the left- and right-invariant ones: $\theta^L_\mu = -i\partial_\mu U U^\dagger$ and $\theta^R_\mu = -i\partial_\mu U U^\dagger$, respectively. The $G$-invariant $\sigma$-model Lagrangian now reads:

$$\mathcal{L}^{G}_{\sigma} = \frac{1}{2} \text{Tr}(\theta_\mu U \partial^\mu U^\dagger) = \frac{1}{2} \text{Tr}(\theta_\mu \theta^\mu) = \frac{1}{2} \text{Tr}(\theta^{L\mu} \theta^{L^\mu}) \equiv \frac{1}{2} \theta_{ab}(\varphi) \partial_\mu \varphi^a \partial^\mu \varphi^b \quad (4)$$

which is highly non-linear and chiral. The minimal coupling is formally analogous to the Abelian case, namely

$$\tilde{\mathcal{L}}^{G}_{\sigma} = \frac{1}{2} \text{Tr}((D_\mu U)(D^\mu U)^\dagger) = \frac{1}{2} \text{Tr}((\theta_\mu - A_\mu)(\theta^\mu - A^\mu)), \quad (5)$$

although $A_\mu$ must be understood as $A_\mu = A^a_\mu T^a$. This Lagrangian is invariant, in particular, under

$$U \rightarrow VU, A_\mu \rightarrow VA_\mu V^\dagger - i\partial_\mu V V^\dagger. \quad (6)$$

Adding the standard kinematical Lagrangian for Yang-Mills fields $\mathcal{L}^{G}_{YM} = -\frac{1}{4} \text{Tr}(F^{\mu\nu} F_{\mu\nu})$, with

$$F_{\mu\nu}(A) \equiv \partial_\mu A_\nu - \partial_\nu A_\mu + [A_\mu, A_\nu], \quad (7)$$

to (5), we arrive at the full Lagrangian for Massive Yang-Mills bosons

$$\mathcal{L}^{G}_{MYM} = \mathcal{L}^{G}_{YM} + m^2 \tilde{\mathcal{L}}^{G}_{\sigma}. \quad (8)$$

As already mentioned in the introduction, this model prevents the massive Yang-Mills theory from being both unitary and renormalizable, at least in the canonical quantization approach.
Our main proposal here in this paper lies on a revision of this model consisting in restricting
the whole trace on $G$ to a partial trace on a quotient manifold $G/H$. $H$ is the isotropy subgroup
of a given direction $\lambda = \lambda^a T_a$, in the Lie-algebra of $G$, under the adjoint action $\lambda \rightarrow V \lambda V^\dagger$, where $\lambda^a$ are real numbers subjected to $\text{Tr}(\lambda^2) = 1$. Let us define $\Lambda \equiv U \lambda U^\dagger$. The claimed $G/H - \sigma$ Lagrangian has the following expression:

$$L^{G/H}_{\sigma} = \frac{1}{2} \text{Tr}([\lambda^T U \partial_\mu U, \lambda]) = \frac{1}{2} \text{Tr}([\theta_\mu^L, \lambda]^2) = \frac{1}{2} \text{Tr}([\theta_\mu, \Lambda]^2) = \frac{1}{2} \text{Tr}((\partial_\mu \Lambda)^2).$$  \(9\)

The minimally coupled version

$$\tilde{L}^{G/H}_{\sigma} = \frac{1}{2} \text{Tr}([-iU^\dagger D_\mu U, \lambda])$$

is again gauge invariant under \(\tilde{G}\). As in \(\tilde{G}\), the partial-trace $(G/H)$ Massive Yang-Mills Lagrangian now follows:

$$L^{G/H}_{\text{MYM}} = L^{G}_{\text{YM}} + m^2 \tilde{L}^{G/H}_{\sigma}. \quad (11)$$

We should remark that the change of variables

$$\tilde{A}_\mu = U^\dagger (A_\mu - \theta_\mu) U = U^\dagger A_\mu U + iU^\dagger \partial_\mu U$$ \(12\)

and the fact that $F(A) = UF(\tilde{A})U^\dagger$ renders the Lagrangian \(11\) into the simple form

$$L^{G/H}_{\text{MYM}} = -\frac{1}{4} \text{Tr}(F^{\mu\nu}(\tilde{A}))^2 + \frac{1}{2} m^2 \text{Tr}([\tilde{A}_\mu, \lambda]^2). \quad (13)$$

This change of variables, formally mimicking the shift to the unitary gauge, turns the actual degrees of freedom of the theory apparent. On the other hand, it must be eventually completed with the change of variables $\phi = U^\dagger \psi$ when the fermionic matter field $\psi$ will be introduced.

For instance, for $G = SU(2)$ with the standard spherical basis $T_\pm, T_0$ of the Lie algebra, and taking $\lambda = \lambda^0 T_0$, that is $H = U(1)$, the mass term in \(13\) is written

$$\frac{1}{2} m^2 \text{Tr}([\tilde{A}_\mu, \lambda]^2) = m_W^2 \tilde{W}^+ \tilde{W}^-, \quad (14)$$

where, as usual, $\tilde{W}^\pm = \tilde{A}^1 \pm i \tilde{A}^2$ and $m_W = m^0$, and $\tilde{W}^0 = \tilde{A}^3$ remains massless.

## 3 Quantization

For linear systems, the quantum theory proceeds according to the usual canonical quantization
rules, which lead to commutators between basic operators realizing the Lie algebra of the
Heisenberg-Weyl group in the corresponding dimension. In field theories, this means postulating
equal-time commutation relations between fields and their time derivatives, or conjugate
momenta, $[\phi(x), \pi(y)] = i\delta(x - y)$. Going to nonlinear systems with non-flat phase space should
require a different approach. This is precisely the situation we are facing now, as a result of the
introduction of the group parameters as physical degrees of freedom, with a (curved) compact
target space $G/H$. 

Our strategy is to look for a replacement of the Heisenberg-Weyl group with a (more involved) symmetry group of the solution manifold, keeping the general idea of considering as basic conjugate operators those giving central terms under commutation. Therefore, we should be able to identify such symmetry group by analyzing the symplectic potential (or Liouville 1-form) in the solution manifold, which generalizes $p_idq^i$ from particle mechanics.\[^1\] Regarding the sigma sector, and from (9), this can be written:

$$\Theta^{G/H}_{\sigma} = \int_{\Sigma} \text{Tr}([\theta_\mu, \Lambda][-i\delta UU^\dagger, \Lambda])d\sigma^\mu = \int_{\Sigma} \text{Tr}(\{[\theta_\mu, \Lambda] + \Lambda \text{Tr}(\Lambda \theta_\mu)][-i\delta UU^\dagger, \Lambda])d\sigma^\mu \equiv \int_{\Sigma} \text{Tr}([\vartheta_\mu[-i\delta UU^\dagger, \Lambda])d\sigma^\mu,$$

where in the second step we have added a null contribution intended to make the change

$$\theta_\mu \rightarrow \vartheta_\mu \equiv [\theta_\mu, \Lambda] + \Lambda \text{Tr}(\Lambda \theta_\mu),$$

invertible. The key observation is that the group of transformations (with parameters $U'$ and $\vartheta'_\mu$)

$$U \rightarrow U'U$$

$$\vartheta_\mu \rightarrow U'\vartheta_\mu U'^\dagger + \vartheta'_\mu$$

renders (15) invariant up to a total differential. This can be easily checked, taking into account the identity $\delta \Lambda \equiv \delta(U\lambda U^\dagger) = [\delta UU^\dagger, U\lambda U^\dagger]$.

It should be noted that the choice of coordinates (16) also represents a deviation from the canonical prescription; it generalizes to field theory the selection of angular momentum $\vec{L}$ and position $\vec{q}$ as basic conjugate variables for the quantum mechanical motion on the sphere $S^2$.\[^3\] A similar change to (16) applies to the vector potentials,

$$A_\mu \equiv [A_\mu, \Lambda] + \Lambda \text{Tr}(\Lambda A_\mu),$$

which now acquire the same composition law as in the second line in (17), without being forced to be a “flat connection”. Once we combine (16) and (18) together with (12), we can define

$$\tilde{A}_\mu = U^\dagger(A_\mu - \vartheta_\mu)U = [\tilde{A}_\mu, \lambda] + \lambda \text{Tr}(\lambda \tilde{A}_\mu).$$

The complete symplectic potential for the Massive Yang-Mills theory is written in terms of the new $\tilde{A}_\mu$ as

$$\Theta^{G/H}_{M YM} = \int_{\Sigma} \text{Tr}(F^{\mu\nu}(\tilde{A})\delta \tilde{A}_\nu + m^2\tilde{A}_\mu[-i\delta UU^\dagger, \lambda])d\sigma^\mu.$$ 

This expression tells us directly the actual conjugate couples of basic “coordinates” and “momenta” to be quantized. It is apparent now that the components of $\tilde{A}_\mu$ perpendicular to $\Sigma$ and $\lambda$ (in space-time and group directions, respectively) have the gauge group parameters $U$ themselves as conjugate coordinates.

\[^1\]The symplectic potential can be obtained by integrating the Lagrangian Poincaré-Cartan form on a Cauchy hypersurface $\Sigma$. 

5
Let us go back to the original variables $A^a_\mu$ and write
\begin{equation}
\hat{G}_a(x) = \frac{\delta}{\delta \varphi^a(x)} + C^c_{ab} \varphi^b(x) \frac{\delta}{\delta \varphi^c(x)} + \ldots
\end{equation}
for the generators of gauge transformations, and denote $\hat{E}_\mu^a(x)$ for the generator of translations in $A^\mu_a(x)$ and $\hat{A}_\mu^a(x)$ for the generator of translations in $F^0_a(x) \equiv E^0_a(x)$, in much the same way the momentum $\hat{p}$ is the generator of translations in $q$ in standard Quantum Mechanics.

Choosing $\Sigma = \mathbb{R}^3$ in the time direction (i.e., $d\sigma_\mu \to d^3x$), we propose the following equal-time ($x^0 = y^0$) commutators:
\begin{align}
\left[ \hat{G}_a(x), \hat{G}_b(y) \right] &= iC^{\bar{c}}_{ab} \hat{G}_c(x) \delta(x - y), \\
\left[ \hat{A}_\mu^a(x), \hat{E}_\nu^b(y) \right] &= i\delta^a_{\nu} \delta^b_\mu \delta(x - y), \\
\left[ \hat{G}_a(x), \hat{A}^b_\nu(y) \right] &= iC^c_{a\nu} \hat{A}^c_\nu(x) \delta(x - y) - i\delta^b_\nu \partial^\nu \delta(x - y), \\
\left[ \hat{G}_a(x), \hat{E}_\mu^b(y) \right] &= iC^c_{ab} \hat{E}'_\mu^c(x) \delta(x - y) - im^2 \delta^b_\mu C^{cd}_a \lambda_c \delta(x - y),
\end{align}
as corresponds to the Lie-algebra of the symmetry group of the system (see Appendix). The commutator in the last line of (22) algebraically expresses the comment after (20) concerning the conjugated character of translations in $A_0$ and $U$.

A unitary, irreducible representation of this (infinite-dimensional) Lie algebra on wave functionals $\Psi(E^\mu)$ in the “electric-field representation” $E^\mu_a$, where $E^0_a \equiv m^2 \text{Tr}(T_a A)$, can be achieved as (the actual details will be given in [14]):
\begin{align}
\hat{E}_a^\mu \Psi(E) &= (E_a^\mu - m^2 \delta^\mu_0 \lambda_a) \Psi(E), \\
\hat{A}^a_\mu \Psi(E) &= i \frac{\delta}{\delta E^a_\mu} \Psi(E), \\
\hat{G}_a \Psi(E) &= \left( \vec{\nabla} \cdot \vec{E}_a + iC^c_{ab} (E^\mu_c - m^2 \delta^\mu_0 \lambda_c) \frac{\delta}{\delta E^c_\mu} \right) \Psi(E)
\end{align}
The last expression accounts for the non-Abelian “Gauss law” when the constraint condition $\hat{G}_a \Psi(E) = 0$ is required.

It should be stressed that the central term proportional to $\lambda_c$ in the last commutator of (22) could also be considered as a remnant of some sort of “symmetry breaking” in the sense that it can be hidden into a redefinition of $\hat{E}_a^0$:
\begin{equation}
\hat{E}_a^0 \to \hat{E}_a^0 \equiv \hat{E}_a^0 + m^2 \lambda_a,
\end{equation}
which now acquires a non-zero vacuum expectation value proportional to the mass $m^2 \lambda_a$, that is:
\begin{equation}
\langle 0 | \hat{E}_a^0 | 0 \rangle = 0 \longrightarrow \langle 0 | \hat{E}_a^0 | 0 \rangle = m^2 \lambda_a.
\end{equation}

4 The $SU(2) \times U(1)$ group and the Standard Model

In this section we shall denote by $B_\mu = B^a_\mu T_a, a = 1, \ldots , 4$, the $SU(2) \times U(1)$ Lie-algebra valued vector potential, keeping $A_\mu$ for the electromagnetic potential, as usual. The new generator $T_4 (\equiv \frac{1}{2} Y$, the halved hypercharge) corresponds to the direct factor $U(1)$.
The key point in this section consists in combining the construction above for \( G = SU(2) \) with the traditional Stueckelberg model for a selected \( H^\perp = U(1) \). However, this time we shall choose \( \lambda \) in the electric charge (mixed) direction

\[
\lambda \propto Q \equiv T_3 + T_4, \tag{26}
\]

according to the usual Gell-Mann-Nishijima relation, and we shall choose \( H^\perp \) in the orthogonal direction:

\[
Q^\perp = T_3 - T_4, \tag{27}
\]

in the sense that \( \text{Tr}(QQ^\perp) = 0 \). This way we shall provide mass to three vector bosons, say \( W_\mu^\pm \propto B_\mu^1 \pm iB_\mu^2 \) and \( Z_\mu \propto \text{Tr}(Q^\perp B_\mu) \), out of the original four vector potentials \( B_\mu^a, a = 1, \ldots, 4 \), leaving the electromagnetic potential \( A_\mu \) massless. In fact, the Standard Model Lagrangian for the Yang-Mills sector will be

\[
\mathcal{L}_{\text{SM}}^{\text{YM}} = \frac{1}{4} \text{Tr}(F_{\mu\nu})^2 + \frac{1}{2} m^2 \text{Tr}([\theta_\mu - B_\mu, UQU^\dagger])^2 + \frac{1}{2} m^2 \text{Tr}((\theta_\mu - B_\mu)UQ^\perp U^\dagger)^2)
\]

\[
= -\frac{1}{4} \text{Tr}(F_{\mu\nu})^2 + \frac{1}{2} m^2 \text{Tr}((\tilde{B}_\mu)Q)^2 + \frac{1}{2} m^2 \text{Tr}((\tilde{B}_\mu Q^\perp)^2) \tag{28}
\]

\[
= -\frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + m^2 W_\mu^\dagger W^\mu - \frac{1}{2} m^2 Z^\mu Z_\mu \tag{29}
\]

where \( \tilde{B}_\mu \) is related to \( B_\mu \) in a way similar to that of Eq. (12). This Lagrangian reproduces the Yang-Mills sector of the Standard Model for electroweak interactions when we introduce the usual coupling constants \( g, g', e \) according to \( \tilde{B}_\mu^3 \equiv gB_\mu^3, \tilde{B}_\mu^4 \equiv g'B_\mu^4, \tilde{Z}_\mu \equiv \frac{2g'}{e} Z_\mu \). Writing \( Z_\mu \) in terms of \( B_\mu^3 \) and \( B_\mu^4 \), we have:

\[
Z_\mu = \frac{e}{gg'} \tilde{Z}_\mu = \frac{e}{gg'}(gB_\mu^3 - g'B_\mu^4) \equiv \cos(\vartheta_W)B_\mu^3 - \sin(\vartheta_W)B_\mu^4, \tag{30}
\]

which, together with the orthogonal relation

\[
A_\mu \equiv \sin(\vartheta_W)B_\mu^3 + \cos(\vartheta_W)B_\mu^4, \tag{31}
\]

(the electromagnetic vector potential) defines the usual Weinberg rotation of angle \( \vartheta_W \).

### 4.1 Giving mass to fermionic matter

The introduction of mass for fermionic matter can be accomplished by a non-trivial mixing between space-time and internal symmetries. Although the general setting of this symmetry mixing is rather ambitious, here we shall consider the consequences of the simplest, nontrivial, mixing between the Poincaré group \( P \) and the electromagnetic gauge subgroup \( H = U(1)_Q \), which has been widely developed in \[15\] and references therein. A more general symmetry mixing scheme, involving conformal symmetry and larger internal symmetries, is under consideration \[16\].

To be precise, we propose a mass-generating mechanism associated with a non-trivial mixing of the Poincaré group and \( SU(2) \times U(1) \). This mixing takes place through a linear combination \( P'_0 \equiv P_0 + \kappa Q \) between the time translation generator \( P_0 \) and \( Q \), in much the same way the generator \( Q \) had to be found as a linear combination of \( T_3 \) and \( T_4 \). The spirit of the redefinition
$P'_0$ is the same as the shifting \([24]\), with $\lambda \propto Q$, ultimately responsible for the mass $m_W$. In fact, with the new mass operator
\[
M'^2 = P'^2 - \vec{P}'^2
\]
the mass shell condition for fermionic fields $\psi$ becomes
\[
M'^2\psi = (P'^2 - \vec{P}'^2)\psi = m'^2\psi \rightarrow M'^2\psi = (m'^2 + 2\kappa P_0 Q + \kappa^2 Q^2)\psi.
\] (33)

At the rest frame we have
\[
M'^2\psi = (m'^2 + 2\kappa m_0 Q + \kappa^2 Q^2)\psi.
\] (34)

Then, for “originally” massless particles ($m_0 = 0$),
\[
M'^2\psi = \kappa^2 Q^2\psi,
\] (35)
so that only charged particles acquire mass. This is in agreement with the fact that there is no elementary fermionic massive particles without electric charge.

A Symmetry group of massive Yang-Mills theories

In this appendix we simply provide the composition law of the infinite-dimensional symmetry group from which the physical operators and their commutators \([22]\) can be explicitly derived as right-invariant vector fields. The group law:

\[
\begin{align*}
U''(x) &= U'(x)U(x) \quad (\Rightarrow \theta''_\mu(x) = U'(x)\theta_\mu(x)U'^\dagger(x) + \theta'_\mu(x)), \\
A''_\mu(x) &= U'(x)A_\mu(x)U'^\dagger(x) + A'_\mu(x), \\
F''_{\mu\nu}(x) &= U'(x)F_{\mu\nu}(x)U'^\dagger(x) + F'_{\mu\nu}(x), \\
\zeta'' &= \zeta' \zeta \exp \left( i \int_\Sigma d\sigma^\mu(x)J_\mu(U', A', F', U, A, F) \right), \\
J_\mu &= J^{YM}_\mu + J^\sigma_\mu, \\
J^{YM}_{\mu} &= \frac{1}{2} \text{Tr} \left( (A'^\nu - \theta'^\nu)U'F_{\mu\nu}U'^\dagger - F'_{\mu\nu}U'(A' - \theta'^\nu)U'^\dagger \right), \\
J^\sigma_{\mu} &= m^2 \text{Tr} \left( \lambda(U'(A_\mu - \theta_\mu)U'^\dagger - (A_\mu - \theta_\mu)) \right),
\end{align*}
\] (36)

is a central extension by $U(1) \ni \zeta$ of the basic symmetry group containing gauge transformations $U$ and translations in $A$ and $F$. This central extension is given by a two-cocycle $\int_\Sigma d\sigma^\mu(x)J_\mu(U', A', F'; U, A, F)$ defined through a symplectic potential current $J_\mu$ made of two pieces: $J^{YM}_{\mu}$ accounting for the symplectic structure of the pure Yang-Mills theory and $J^\sigma_{\mu}$ concerning the sigma (massive) sector. The unitary, irreducible representations of this infinite-dimensional symmetry group will be explicitly given in \([14]\), inside a Group Approach to Quantization scheme.
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References

[1] P.W. Higgs, Phys. Rev. Lett. 13, 508 (1964); F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); G. S. Guralnik, C. R. Hagen, and T. W. Kibble, Phys. Rev. Lett. 13, 585 (1964).

[2] S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967).

[3] G. ’t Hooft, Nucl. Phys. 33, 173 (1971); 35, 167 (1971).

[4] M. Bando, T. Kugo and K. Yamawaki, Nucl. Phys. B259, 493 (1985); Phys. Rep. 164, 217 (1988).

[5] A.C. Longhitano, Nucl. Phys. B188, 118 (1981).

[6] W.A. Bardeen and K. Shizuya, Phys. Rev. D18, 1969 (1978).

[7] T. Kunimasa and T. Goto, Prog. Theor. Phys. 37, 452-464 (1967).

[8] R. Delbourgo and G. Thompson, Phys. Rev. Lett. 57, 2610-2612 (1986); J. Kubo, Phys. Rev. Lett. 58, 2000 (1987); P. Kosinski and L. Szymanowski, Phys. Rev. Lett. 58, 2001 (1987); R. Delbourgo, S. Twisk and G. Thompson, Int. J. Mod. Phys. A3, 435-449 (1988).

[9] T. Hurth, Helv. Phys. Acta 70, 406-416 (1997).

[10] H. Ruegg and M. Ruiz-Altaba, Int. J. Mod. Phys. A19, 3265-3348 (2004).

[11] S.V. Ketov, Quantum Non-linear Sigma-Models, Springer-Verlag Berlin Heidelberg (2000).

[12] C.J. Isham, Topological and global aspects of quantum theory, in Relativity, Groups and Topology II, Les Houches Summer School, R. Stora and B. S. DeWitt (Eds.) (1986)

[13] V. Aldaya, M. Calixto, J. Guerrero and F.F. Lopez-Ruiz, Group-quantization of nonlinear sigma models: particle on S^2 revisited, Rep. Math. Phys. (2009), to appear.

[14] V. Aldaya, M. Calixto and F.F. Lopez-Ruiz, Symmetry group of massive Yang-Mills theories and its quantization, In preparation.

[15] V. Aldaya and E. Sanchez-Sastre, J. Phys. A39, 1729 (2006).

[16] Under investigation