Autonomous navigation data integrity monitoring of satellite radio navigation systems based on residual method

A V Ivanov¹, D V Boykov¹, O V Trapeznikova², A P Pudovkin¹ and E V Trapeznikov²

¹ Tambov State Technical University, 106, Sovetskaya Str., Tambov, 392000, Russia
² Omsk State Technical University, 11, Mira av., Omsk 644050, Russia
E-mail: resbn@mail.ru, evtrapeznikov@yandex.ru

Abstract. Statistical computer modeling is implemented to analyze an autonomous system for navigation data integrity monitoring of satellite radio navigation systems using the measurement residual summation method in a sliding window. The analysis is made for integrated optimal information processing algorithms obtained by the methods of the optimal estimation Markov theory. Algorithms are generated for the vertical channel of the navigation complex, which includes an inertial navigation system, signal reception equipment from satellite radio navigation systems, and a barometric altimeter. The possibility of detecting unreliable data in using different measurement residuals, as well as depending on the size of the sliding window, is investigated in the modeling process.

1. Introduction

To determine the location coordinates and motion parameters of mobile ground objects, as well as the spatial orientation of their longitudinal and transverse axes, navigation complexes installed on them are used. The basis of all modern navigation systems for mobile ground objects (MGO) is the signal reception equipment (RE) of satellite radio navigation systems (SRNS). The rest of the navigation system may include a fairly large variety of different non-radio engineering instruments, such as the inertial navigation system (INS); barometric altimeter (BA); angular velocity sensor (AVS); accelerometer; magnetometer; speed sensor (odometer), and others. The use of the RE SRNS as a part of the navigation complex provides high accuracy in determining the coordinates of the object position, as well as the possibility of additional attitude angle detection and position rate of change. However, the use of the RE SRNS leads to two significant problems. The first problem is that at any given time abnormal radio signals at the RE SRNS input or even short-term no-signal states are possible. The reasons for this are different. They are receiving antenna shadowing, on-board navigation satellite equipment failure, influence of the radio signal propagation channel or the surrounding landscape. The second problem is related to the possibility of distortion of the navigation data transmitted to the user in the navigation message of the SRNS radio signal. Such distortions can be caused, for example, by intended interference simulating a radio signal from a navigation satellite.

Due to these problems, the use of satellite radio navigation systems as part of a navigation complex requires integrity. Integrity means a set of measures to control the condition of the
radio navigation field and timely transmission to users information on changes in the quality of their service. Information redundancy is necessary to ensure integrity. Depending on the type of redundancy, integrity systems can be divided into two large groups:

- integrity control by devices and systems not being a part of the on-board equipment of the object where the navigation complex is installed (external control). Earth-fixed stations with a multi-channel receiver and proper mathematical software, transmitting to users information about the "health" of satellites by the terrestrial radio channel along with differential corrections (GIC), or via geostationary satellites of wide area augmentation (WAAS) are used as such devices. Also, such devices and systems can include equipment on board a navigation satellite (NS) that monitors its performance in order to form a sign of serviceability or malfunction in the information transmitted to the user. If NS is determined to be faulty, the fault symptom appears in the non-operational information of navigation messages on all navigation satellites no later than 16 hours. In the radio signals of the GLONASS-M series satellites, the transmission discreteness of the corresponding symptom is not more than 4 s, which provides a failure notification time of up to 10 s;
- integrity monitoring with devices and systems on board of the object;
- autonomous integrity monitoring [1].

Implementation of this control is possible due to information redundancy on board of a mobile object, for example, simultaneous parallel reception of radio signals from more than four navigation satellites, or by introducing additional devices into the navigation complex, such as a barometric altimeter, which provides redundancy of information (measurement of the same or functionally related variables by devices or systems operating on different physical principles).

The advantage of autonomous integrity monitoring systems on board is primarily their efficiency, autonomy and independence from the data transmission channel.

To determine the integrity of navigation data transmitted by satellite radio navigation systems, it is more promising to use autonomous integrity control systems based on information redundancy that occurs during complex information processing [1]. In this case, the following methods are applied to control the integrity of navigation data: the maximum solution separation method [2]; the range comparison method [3]; the position comparison method [3]; the least-squares residual method [4], and filtering methods [5].

In the works [6–8], optimal algorithms for information processing are synthesized for navigation systems. Algorithms of the autonomous system for SRNS navigation data integrity is developed in combination with the synthesized algorithms. To create them, in addition to the INS and RE SRNS, a barometric altimeter is added to the navigation complex of a mobile ground object. The detection of unreliable SRNS data was performed by comparing two estimates with the specified threshold values. As estimates, a constant component of the relative altitude error measurement by a barometric altimeter and a constant component of the error in INS acceleration measurement are used. An important advantage of the developed algorithms in comparison with other algorithms is that the estimates for comparison are combined with estimates of the coordinates and parameters of the MGO motion as a result of solving the synthesis problem using the Markov optimal estimation theory [9,10].

In [11] for the same algorithms developed in [6–8], to control the integrity of navigation data of the SRNS a residual summation method of measurements in a sliding window was proposed. This method assumes simultaneous summation of two measurement residuals at each step of estimating the coordinates of the location and object motion parameters, namely: the residual signal of the barometric altimeter and the residual signal at the output of the RE SRNS. The modeling performed for this work has not determined the impact of each residual on SRNS uncertain data detection.
Object of the work: to apply statistical computer modeling methods to analyze the operation of an autonomous system of navigation data integrity monitoring of SRNS, using the residual summation method in a sliding window in order to determine the impact of individual residuals on the possibility of detecting unreliable SRNS data.

2. Problem statement
To analyze the operation of the autonomous system of navigation data integrity monitoring of the SRNS, based on the method of summing up the measurement residuals in the sliding window, we use integrated optimal algorithms for processing information from the works [6–8]. Integrated optimal information processing algorithms were synthesized for the navigation complex of a mobile ground object. The navigation complex included: INS; SRNS, and BA. For integrity control we use a vertical channel of information processing.

As a coordinate system, which determines the position coordinates and motion parameters of the MGO, we use a geocentric (spherical) coordinate system with a radius vector \( R \). The relative altitude of the MGO (altitude relative to the level \( R_0 \)) is measured by the BA. We suppose that allowance is made for BA systematic error in its initial alignment. The signal at the BA output at discrete time intervals \( t_k, k = 0, 1, 2, ... \) has the form [6–8]:

\[
H_{OTH}^{BA}(t_{k+1}) = H_{OTH}(t_{k+1}) + \Delta H(t_{k+1}) + u_{BA}(t_{k+1})
\]

where \( H_{OTH}(t_{k+1}) \) is the true value of the relative altitude of the object; \( \Delta H(t_{k+1}) \) and \( u_{BA}(t_{k+1}) \), respectively, are the constant error and fluctuation error described by the expressions:

\[
\Delta H(t_{k+1}) = \Delta H(t_k)
\]

\[
u_{BA}(t_{k+1}) = \varphi_{uu}(t_{k+1}, t_k) u_{BA}(t_k) + \gamma_{u}(t_{k+1}, t_k) n_{u}(t_k), u_{BA}(t_0) = u_{BA0}
\]

in which \( \varphi_{uu}(t_{k+1}, t_k) = \exp(-\gamma_{BA}T) \); \( \gamma_{u}(t_{k+1}, t_k) = \sigma_{BA} \left[ 1 - \varphi_{uu}^2(t_{k+1}, t_k) \right]^{0.5} \); \( \gamma_{BA} \) is the coefficient characterizing the width of the random error spectrum; and \( \sigma_{BA} \) is the mean square value of the random error; \( n_{u}(t_k) \) are independent samples of a Gaussian process that has zero mathematical expectation and unit variance.

Radio signals from navigation spacecrafts are received by the SRNS signal reception equipment. In the considered coordinate system, the output signals from the RE SRNS about the altitude of the object \( H^{SRNS}(t_{k+1}) \) relative to the center of the Earth at discrete time intervals \( t_k, k = 0, 1, 2, ... \) are represented as [6–8]:

\[
H^{SRNS}(t_{k+1}) = H_{OTH}(t_{k+1}) + R_0 + \sigma_z n_z(t_{k+1})
\]

where \( (H_{OTH}(t_{k+1}) + R_0) \) is the true value of the object altitude relative to the center of the Earth; \( \sigma_z \) is the mean square value of the random error in measuring the object altitude; \( n_z(t_{k+1}) \) are independent samples of the random Gaussian process. We assume that the random Gaussian process has zero mathematical expectation and unit variance.

The information distribution approach is used to describe the model of the MGO motion in the vertical plane [6, 7]. In applying this approach, information is redistributed between the observation and control vectors. In accordance with this approach, we replace the true value of the vertical acceleration of the object \( a_Z(t_{k+1}) \) with the value of the output signal of the inertial navigation system \( a_{ZINS}^{INS}(t_{k+1}) \) in the vertical plane, that is, we refer the measured INS value (output signal) \( a_{ZINS}^{INS}(t_{k+1}) \) to the control vector. The output signal of the INS at discrete time intervals \( t_k, k = 0, 1, 2, ... \) has the form
\[ a_Z^{INS}(t_{k+1}) = a_Z(t_{k+1}) + \Delta a_Z(t_{k+1}) + g + \sigma_a \left( \frac{2T}{\alpha_a} \right)^{0.5} n_aZ(t_{k+1}) \]  

(4)

where \( g \) is the constant representing the acceleration of gravity; \( \alpha_a \) is the coefficient characterizing the spectrum width of the fluctuation error of acceleration measurement; \( \sigma_a \) is the mean square value of the fluctuation error of acceleration measurement; \( T = (t_{k+1} - t_k) \) is the time-sampling interval; \( n_aZ(t_k) \) are mutually independent samples of stochastic Gaussian process that has a zero mathematical expectation and unit dispersion; \( \Delta a_Z(t_k) \) is slowly varying random error of acceleration measurement

\[ \Delta a_Z(t_{k+1}) = \Delta a_Z(t_k) \]  

(5)

The mathematical model of the MGO relative altitude change discrete time intervals is described by the expression:

\[ H_{OTH}(t_{k+1}) = H_{OTH}(t_k) + TV_Z(t_k) + 0.5T^2a_Z^{INS}(t_k) - 0.5T^2\Delta a_Z(t_k) - 0.5T^2g - 0.5T^2\sigma_a \left( \frac{2T}{\alpha_a} \right)^{0.5} n_aZ(t_k) \]  

(6)

\[ V_Z(t_{k+1}) = V_Z(t_k) + Ta_Z^{INS}(t_k) - T\Delta a_Z(t_k) - Tg - T\sigma_a \left( \frac{2T}{\alpha_a} \right)^{0.5} n_aZ(t_k) \]  

(7)

where \( V_Z(t_{k+1}) \) is the variable representing the relative altitude rate of change.

The state vector has the dimension (4x1) and includes the relative altitude of the MGO, relative altitude rate of change, constant component of the relative altitude measurement error, and slowly changing random acceleration measurement error \( X(t_k) = [H_{OTH}(t_k), V_Z(t_k), \Delta H(t_k), \Delta a_Z(t_k)]^T \). A mathematical model for the state vector change at discrete time intervals \( t_k, k = 0, 1, 2, ... \) in accordance with (2), (5) – (7) is described by a difference vector-matrix stochastic equation

\[ X(t_{k+1}) = F_xX(t_{k+1}, t_k)X(t_k) + \Psi(t_{k+1}, t_k)W(t_k)G_x(t_{k+1}, t_k)N_x(t_k) \]  

(8)

In equation (8): \( W(t_k) \) is the dimension control vector (2x1) given in the form \( W = [a_Z^{INS}, g]^T \); \( \Psi \) is the known size control matrix (4x2) with the following non-zero elements \( \psi_{11} = 0, 5T^2, \psi_{12} = 0, 5T^2, \psi_{21} = T, \psi_{22} = -T; N_x(t_k) = n_aZ(t_k) \); are mutually independent samples of the random Gaussian process that has zero expectation and unit variance; \( F_x \) is the transition matrix of dimension (4x4) with non-zero elements \( f_{xx11} = f_{xx22} = f_{xx33} = f_{xx44} = 1; f_{xx12} = T, f_{xx14} = -0.5T^2, f_{xx24} = -T; G_x \) is the known disturbance vector with the dimension (4x1) and non-zero elements, \( g_{x1} = -0.5T^2\sigma_a(2T/\alpha_a)^{0.5}, g_{x21} = -T\sigma_a(2T/\alpha_a)^{0.5} \).

The observation vector at discrete time intervals \( t_k, k = 0, 1, 2, ... \) is written as \( \Xi(t_k) = [\xi_1(t_k), \xi_2(t_k)]^T \). As the components of the observation vector the following measurements are used: measurements at the BA output \( \xi_1(t_k) = H_{BA}^{BA}(t_k) \) ; measurements at the RE output of SINS signals \( \xi_2(t_k) = H_{SRNS}^{SRNS}(t_k) \) . Measurements at the output of the BA and RE SRNS in accordance with (1), (3) are described by the expressions:

\[ \xi_1(t_k) = H_1(t_k)X(t_k) + u_{BA}(t_k); \]  

\[ \xi_2(t_k) = H_2(t_k)X(t_k) + V_2 + g_2(t_k)N_2(t_k) \]  

(9)

where \( H_1 \) is the known observation matrix with dimensions (1x4) that has non-zero elements \( h_{111} = h_{113} = 1; H_2 \) is the known observation matrix with dimensions (1x4) that has a non-zero element \( h_{213} = 1; V_2 = R_0; g_2 = \sigma_2; N_2(t_k) = n_z(t_k) \) are mutually independent samples of a random Gaussian process that has zero expectation and unit variance.
3. Integrated optimal algorithms for information processing

To obtain integrated optimal algorithms for information processing in the navigation complex of the MGO, the methods of the Markov optimal estimation theory are employed. In conformance with these methods, the estimation of the state vector at discrete time intervals $t_k, k = 0, 1, 2, \ldots$ is described by the expression [6, 7]

$$X^* (t_{k+1}) = F_{xx} (t_{k+1}, t_k) X^* (t_k) + \Psi (t_{k+1}, t_k) W (t_k) + K_1 (t_{k+1}) [\xi_1 (t_{k+1}) -$$

$$- \varphi_{uu} (t_{k+1}, t_k) \xi_1 (t_k) - H_1 (t_{k+1}) \Psi (t_{k+1}, t_k) W (t_k) + \varphi_{uu} (t_{k+1}, t_k) H_1 X^* (t_k) -$$

$$- H_1 (t_{k+1}) F_{xx} (t_{k+1}, t_k) X^* (t_k)] + K_2 (t_{k+1}) [\xi_2 (t_{k+1}) - H_2 (t_{k+1}) \Psi (t_{k+1}, t_k) *$$

$$* W (t_k) - V_2 - H_2 (t_{k+1}) F_{xx} (t_{k+1}, t_k) X^* (t_k)]$$

(10)

where $K_1 (t_{k+1})$ and $K_2 (t_{k+1})$ are the column-vectors with dimensions (4x1) of the optimal transmission coefficients matrix $K (t_{k+1}) = \begin{bmatrix} K_1 (t_{k+1}) ; K_2 (t_{k+1}) \end{bmatrix}$ defined by

$$K (t_{k+1}) = \begin{bmatrix} F_{xx} (t_{k+1}, t_k) P (t_k) F_{yx}^T (t_{k+1}, t_k) + B_{xy} \end{bmatrix}\ast$$

$$\ast \begin{bmatrix} B_{yy} + F_{yx} (t_{k+1}, t_k) P (t_k) F_{yx}^T (t_{k+1}, t_k) \end{bmatrix}^{-1};$$

$$P (t_{k+1}) = \begin{bmatrix} F_{xx} (t_{k+1}, t_k) P (t_k) F_{yx}^T (t_{k+1}, t_k) + B_{xy} \end{bmatrix}$$

$$- K (t_{k+1}) \begin{bmatrix} B_{xy} + F_{xx} (t_{k+1}, t_k) P (t_k) F_{yx}^T (t_{k+1}, t_k) \end{bmatrix}^T;$$

in which $P (t_{k+1})$ is the matrix of a posteriori variances of estimation errors with dimensions (4x4); $B_{xx} (t_{k+1}, t_k) = G_x (t_{k+1}, t_k) G_x^T (t_{k+1}, t_k); F_{yx} (t_{k+1}, t_k), B_{xy}, B_{yy}$ are block matrices in the form

$$F_{yx} (t_{k+1}, t_k) = \begin{bmatrix} H_1 (t_{k+1}) F_{xx} (t_{k+1}, t_k) - \varphi_{uu} (t_{k+1}, t_k) H_1 (t_k) \end{bmatrix} / H_2 (t_{k+1}) F_{xx} (t_{k+1}, t_k);$$

$$B_{xy} = \begin{bmatrix} G_x (t_{k+1}, t_k) G_x^T (t_{k+1}, t_k) H_1^T (t_{k+1}) G_x (t_{k+1}, t_k) G_x^T (t_{k+1}, t_k) H_1^T (t_{k+1}) ) \end{bmatrix};$$

$$B_{yy} = \begin{bmatrix} H_1 (t_{k+1}) G_x (t_{k+1}, t_k) G_x^T (t_{k+1}, t_k) H_1^T (t_{k+1}) - \varphi_{uu}^2 (t_{k+1}, t_k) \end{bmatrix} / H_2 (t_{k+1}) G_x (t_{k+1}, t_k) G_x^T (t_{k+1}, t_k) H_1^T (t_{k+1}) + \varphi_{uu}^2 (t_{k+1}, t_k) \end{bmatrix};$$

$$H_1 (t_{k+1}) G_x (t_{k+1}, t_k) G_x^T (t_{k+1}, t_k) H_1^T (t_{k+1}) + \varphi_{uu}^2 (t_{k+1}, t_k) \end{bmatrix}.$$

The expressions (10), (11) are integrated optimal algorithms, and they enable, in particular, estimating the relative altitude of the MGO $H_{OTH} (t_k)$ and the speed of its change $V_Z (t_k)$.  

4. Autonomous integrity control system based on the residual method

Two measurement residuals can be used to detect unreliable SRNS data in the developed integrated optimal algorithms (10) and (11), namely: the first is the residual (error) between the received at a time $t_i, i = 1, 2, \ldots, K$ actual signal value of the barometric altimeter and forecast of the signal value; the second is the residual (error) between the received at a time $t_i, i = 1, 2, \ldots, K$ real signal value at the output of the RE SRNS and the forecast of the signal value. To solve the problem of autonomous integrity control, we use the residual summation method in a sliding window. This method of detecting changes in the analyzed function is based on limiting the interval of discrete samples analysis. The analyzed process is the change in the radio signal of
the satellite radio navigation system $H^{SRNS}(t_k)$, \( k = 0, 1, 2, \ldots \), and the analyzed function is a function $J_{12}(t_k)$ defined by the expression

$$J_{12}(t_k) = S_{12}(t_k) - J_{12}(t_{k-N})$$

where $N = 1, 2, \ldots, k$ is the number, which selected value determines the size of the window; $S_{12}(t_k)$ and $S_{12}(t_{k-N})$ are the sums of errors between the actual signal value received at the estimated step and its forecast (the sums of measurement residuals), determined in accordance with the expressions

$$S_{12}(t_k) = \sum_{i=1}^{K} [G_1(t_i) + G_2(t_i)], \quad S_{12}(t_{k-N}) = \sum_{i=1}^{K-N} [G_1(t_i) + G_2(t_i)]$$

in which $G_1(t_i)$ is the error between the received at a time $t_i$, $i = 1, 2, \ldots, K$ actual signal value of the barometric altimeter and forecast of the signal value; $G_2(t_i)$ is the error between the received at a time $t_i$, $i = 1, 2, \ldots, K$ real signal value at the output of the RE SRNS and the forecast of the signal value. According to expression (10), the values of these errors are described by expressions:

$$G_1(t_i) = H_{OTH}^{BA}(t_i) - \varphi_{uu} \cdot H_{OTH}^{BA}(t_{i-1}) - 0.5 \cdot T^2 \cdot a_{INS} Z(t_{i-1}) + 0.5 \cdot T^2 \cdot g + \varphi_{uu} \cdot H_{OTH}^{BA}(t_{i-1}) - \Delta H^{BA}(t_{i-1}) - T \cdot V_Z^* (t_{i-1})$$

$$G_2(t_i) = H^{SRNS}(t_i) - 0.5 \cdot T^2 \cdot a_{INS} Z(t_{i-1}) + 0.5 \cdot T^2 \cdot g - R_0 - H_{OTH}^{BA}(t_{i-1}) - T \cdot V_Z^* (t_{i-1}) 0.5 \cdot T^2 \cdot \Delta a_Z (t_{i-1})$$

In contrast to [11], let us consider two functions (16) and (17) in addition:

$$J_1(t_k) = S_1(t_k) - S_1(t_{k-N})$$

$$J_2(t_k) = S_2(t_k) - S_2(t_{k-N})$$

in which

$$S_1(t_k) = \sum_{i=1}^{K} G_1(t_i), \quad S_1(t_{k-N}) = \sum_{i=1}^{K-N} G_1(t_i)$$

$$S_2(t_k) = \sum_{i=1}^{K} G_2(t_i), \quad S_2(t_{k-N}) = \sum_{i=1}^{K-N} G_2(t_i)$$

These two functions $J_1(t_k)$ and $J_2(t_k)$ enable creating two separate channels for detecting anomalies of the SRNS radio signal or transmitting unreliable data, which increases the probability of their detection.

Computer simulation methods were used to evaluate the detection of unreliable data with the proposed algorithms. For this purpose, the barometric altimeter output signal was simulated in accordance with expressions (1), (2), the RE SRNS output signal was simulated in accordance with expression (3), the INS output signal was simulated in accordance with expression (4), and the integrated information processing algorithms were simulated in accordance with expressions (10), (11).

The initial data for modeling the output signal of the BA $H_{OTH}^{BA}(t_{k+1})$ had the following values: $\Delta H(t_k) = 5m$, \( k = 0, 1, 2, \ldots \); $T = 0.02 s$; $\gamma_{BA} = 10 \text{ s}^{-1}$ and $\sigma_{BA} = 1 \text{ m}$. 
The initial data for modeling the output signal of the BA SRNS $H_{SRNS}^{(t_{k+1})}$ had the following values: $R_0 = 6371110 \text{ m}$; $H_{OTH}(t_k) = 1000 \text{ m}$, $k = 0, 1, 2, \ldots$; $\sigma_z = 3 \text{ m}$.

The initial data for modeling the output signal of the INS $a_{Z}^{INS}(t_{k+1})$ had the following values: $a_Z = 0$, $\alpha_a = 50 \text{ s}^{-1}$; $\sigma_a = 0.03 \text{ ms}^{-2}$; $\Delta a_Z(t_k) = 0.2 \text{ ms}^{-2}$, $k = 0, 1, 2, \ldots$.

In the simulation, the object was assumed to move at an altitude higher than the radius-vector of the geocentric coordinate system $R_0$ at 1000 m.

Three cases of changing signals at the RE SRNS output due to abnormal measurements or transmission of unreliable navigation data in the radio signals of satellite radio navigation systems were considered:

- sudden abrupt change in the signal $H_{SRNS}^{(t_{k+1})}$ at the output of the RE SRNS;
- linear change of the signal $H_{SRNS}^{(t_{k+1})}$ at the RE SRNS output;
- sharp change of the signal $H_{SRNS}^{(t_{k+1})}$ at the output of RE SRNS and fixing this value at a certain level.

Computer modeling of algorithms for autonomous system of navigation data integrity control was performed in the software package for solving technical computing problems MATLAB (Matrix Laboratory).

4.1. The analysis of the algorithm operation in case of a sudden abrupt change of the signal $H_{SRNS}^{(t_{k+1})}$ at the output of the RE SRNS

Figure 1 shows the results of computer modeling of functions $J_{12}(t_k)$, $J_1(t_k)$ and $J_2(t_k)$ described by expressions (12), (16) and (17) when the signal $H_{SRNS}^{(t_{k+1})}$ at the output of the RE SRNS changes at 200 second. Residuals $S_1(t_k)$, $S_2(t_k)$ and $S_{12}(t_k)$ were calculated by the formulas (18), (19), and (13), respectively.

The analysis of the graphs shows that when the window size is $N=1$, the most pronounced reaction to an abrupt jump of the signal $H_{SRNS}^{(t_{k+1})}$ at the output of the RE SRNS occurs.
for the functions $J_2(t_k)$ and $J_{12}(t_k)$, described by the expressions (17) and (12), respectively. For the function $J_1(t_k)$ described by expression (16), the reaction is weakly expressed.

Figure 2 and Figure 3 show the change of the functions $J_{12}(t_k)$, $J_1(t_k)$, and $J_2(t_k)$, defined by expressions (12), (16), (17), with the window size $N=10$ (Figure 2) and $N=30$ (Figure 3).

The analysis shows that the optimal window size is 10. Further enlargement of the window results in the blurred response.

4.2. Analysis of the algorithm operation in case of a linear change of the signal $H^{SRNS}(t_{k+1})$ at the output of the RE SRNS

For the linear change of the signal $H^{SRNS}(t_{k+1})$ at the output of RE SRNS, the results are shown in Figures 4, 5 and 6 (the window size is $N=1$, $N=10$, and $N=30$, respectively). The reaction when using all three variants of accumulated residuals is quite pronounced. As the window value increases, the response becomes weaker.

4.3. Analysis of the algorithm operation in case of a sharp change in the signal $H^{SRNS}(t_{k+1})$ at the output of RE SRNS and fixing its value

Figures 7, 8 and 9 present a simulation of a sudden change in the signal $H^{SRNS}(t_{k+1})$ at the output of the RE SRNS and fixing its value for the window sizes $N=1$, $N=10$, and $N=30$, respectively.

The response to the error is sufficiently expressed in all three variants of summing up the residuals, but when the window is enlarged, it blurs.

4.4. Analysis of the algorithm for all three variants of making an error

Figures 10, 11 and 12 show the simulation for all the above variants of the signal change $H^{SRNS}(t_{k+1})$ at the output of RE SRNS with the size of the windows $N=1$, $N=10$ and $N=30$, respectively.

The graphs in Figures 10, 11 and 12 show that when the window value increases, the response to changes in the signal $H^{SRNS}(t_{k+1})$ at the RE SRNS output becomes weaker.

Conclusion

The following conclusions can be drawn from the results of the simulation:

- the use of measurement residuals can detect a failure or transmission of unreliable navigation data in the radio signals of satellite radio navigation systems, which are manifested in changing a signal on the altitude of the MGO at the output of the SRNS signal reception equipment;
- the function $J_1(t_k)$ that uses residual measurements of the inertial navigation system has the least informative value for detecting a failure or unreliable transmission of navigation data to the SRNS;
- to analyze the detection of a failure or unreliable transmission of navigation data, it is advisable to use the function $J_2(t_k)$ with the signal measurement residuals of the satellite radio navigation system, since it has the same informativeness as the function $J_{12}(t_k)$ that uses measurement residuals of the inertial navigation system together with signal measurements residuals of a satellite radio navigation system, but its calculation requires almost half the computational effort;
- the window size and threshold value are configurable parameters of the algorithm for detecting a failure or transmitting unreliable navigation data.

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Figure 2.

Figure 3.
Figure 4.

Figure 5.
Figure 6.

Figure 7.
Figure 8.

Figure 9.
Figure 10.

Figure 11.
Figure 12.

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