Abstract

This article presents the $A$-dependence of the differential cross section for the coherent electroproduction of vector mesons on nuclei in forward direction, at fixed values of longitudinal momentum transfer $q_L$. It is shown that such cross section has complicated behavior over the atomic mass number $A$ with local minimums and maximums. It is also shown that a ratio of the real to the imaginary parts of the forward coherent amplitude on nuclei $\alpha_A = \Re f_A / \Im f_A$ has breaking points at some values of $A$. Comparison of the behaviors of the normalized cross section $\left( \frac{d\sigma}{d\Omega} \right)_A / \left( \frac{d\sigma}{d\Omega} \right)_N$ and $\alpha_A$ over $A$ shows that the location of minimums of the cross section are very close to the breaking points of $\alpha_A$. 

Normally the behavior of the cross section for the reactions on nuclear targets can be presented in the form $\sigma \sim A^\alpha$, where $A$ is the atomic mass number of the nuclear target. For many reactions $\alpha$ is a constant or a weakly changing function of $A$. The experimental data on coherent photoproduction of $\rho^0$ mesons on nuclear targets in forward direction, in the region of moderate energies of photons [1], indicates that the ratio $\left( \frac{1}{A} \frac{d\sigma}{d\Omega} \right)_A / \left( \frac{1}{Be} \frac{d\sigma}{d\Omega} \right)$ as a function of $A$ increases in the domain of light nuclei ($A \sim 40 \div 60$) and then decreases. The position of the maximum depends from the value of the photon energy $\nu$. When the energy of the photon increases, the position of the maximum shifts to the larger values of $A$. This behavior is governed mainly by the longitudinal momentum transfer, $q_L$. For the photoproduction process $q_L = M_V^2 / 2\nu$, where $M_V$ is the vector meson mass. The value of $q_L$ quickly decreases with the increase of the photon energy. In the case of vector mesons electroproduction $q_L$ depends from $\nu$ and the photon virtuality $Q^2$. Contrary to the real photoproduction case, for values of $Q^2 > M_V^2$, $q_L$ may differ from zero in a wide enough range of $\nu$.

Calculations performed for the case of coherent electroproduction of vector mesons on nuclei indicate that the $A$-dependence of the coherent differential cross sections have complicated behavior with minimums and maximums.

We study also a ratio of the real to the imaginary parts of forward amplitude on nucleus $\alpha_A = \Re f_A / \Im f_A$ as a function of $A$. It is shown that $\alpha_A$ has breaking points, i.e. jumps from $-\infty$ to $\infty$ at some values of $A$. It happens when imaginary part of nuclear amplitude crosses zero. As a result of the performed calculations it was shown that the minimums of cross sections in scale of the atomic mass number $A$ are in the close neighbourhood with the breaking points of $\alpha_A$. With the increase of the photon virtuality $Q^2$, the location of the minimums moves to the location of breaking points, which means, that they have a common nature.

Within the Glauber multiple scattering theory [2] the coherent forward amplitude for the

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2Inverse statement is not true, the corresponding minimum in cross section does not exist for every breaking point
The process of diffractive electroproduction of the vector meson on nucleus can be written as:

\[ f_{A} = 2\pi A f_{N} \int_{0}^{\infty} b db \int_{-\infty}^{\infty} dz \exp\left\{ -A \frac{\sigma_{\text{tot}}^{VN}}{2} \int_{z}^{\infty} \rho(b, y) dy \right\} \rho(b, z) e^{i q_{L} z}, \]

where \( f_{N} = i \Im m f_{N} (1 - i \alpha_{N}) \) \( (\alpha_{N} = \Re f_{N}/\Im m f_{N}) \) is the forward amplitude for vector meson electroproduction by an individual nucleon, \( b \) and \( z \) are the impact parameter and longitudinal coordinate of the point in nucleus where vector meson production took place, \( \sigma_{\text{tot}}^{VN} \) is the vector meson - nucleon total cross section. Longitudinal momentum transfer in the individual collision at forward direction inside the nucleus is equal

\[ q_{L} = \frac{Q^{2} + M_{V}^{2}}{2\nu}, \]

where \( \nu \) and \( q^{2} = -Q^{2} \) are the energy and the square of the four momentum of virtual photon. In the literature there are usually discussions about the two extreme cases of this variable, so called low and high energy limits, i.e. \( q_{L} \to \infty \) and \( q_{L} \to 0 \) (see, for instance, Ref. [4]). We consider intermediate case of different from zero and infinity values of \( q_{L} \). As an example the results of calculations performed at values of variables \( Q^{2} \) and \( \nu \) equal to 4 GeV\(^2\) and 10 GeV, are presented. According to the Eq. 2, it corresponds to \( q_{L} \approx 0.23 \text{GeV} \). Its inverse value \( l_{c} \), which is called the coherence length is equal to \( l_{c} \approx 0.87 \text{fm} \). We show that in this region there is nontrivial dependence of the coherent differential cross sections from \( A \), because of the typical values of \( q_{L} r_{A} \sim 3 \div 10 \) (\( r_{A} \) - nuclear radius), which means that nuclear amplitude strongly oscillates (see Eq. 1). The forward differential cross section on nucleus \( \left( \frac{d\sigma}{d\Omega} \right)_{A} \) is obtained by taking the absolute magnitude squared of the amplitude given in Eq. 1.

\[ \frac{d\sigma}{d\Omega} = \left| f_{A} \right|^{2}. \]

The ratio of the real to the imaginary parts of forward amplitude for electroproduction of vector mesons on the nucleus is

\[ \alpha_{A} = \frac{\Re f_{A}}{\Im m f_{A}} = \]

\[ \frac{\int d^{2}b \int_{-\infty}^{\infty} dz \exp\left\{ -A \frac{\sigma_{\text{tot}}^{VN}}{2} \int_{z}^{\infty} \rho(b, y) dy \right\} \rho(b, z) \sin(q_{L} z)}{\int d^{2}b \int_{-\infty}^{\infty} dz \exp\left\{ -A \frac{\sigma_{\text{tot}}^{VN}}{2} \int_{z}^{\infty} \rho(b, y) dy \right\} \rho(b, z) \cos(q_{L} z)}. \]
Calculations are performed for the case of $\rho^0$-mesons production, and for $\sigma_{tot}^{VN}$ the value of 25mb is used. Also another values for $\sigma_{tot}^{VN}$ were used to simulate the case of $\phi$ meson production, as well as to simulate possible $Q^2$ dependence of cross section due to color transparency effect. Calculations are done for the nuclear density function in the form of the Woods-Saxon distribution:

$$\rho(r) = \rho_0/(1 + \exp((r - r_A)/a)),$$

(6)

where parameters are $a = 0.545\,fm$ and $r_A = 1.14A^{1/3}\,fm$, the values of $\rho_0$ are determined from the normalization condition $\int d^3r \rho(r) = 1$. To study the $A$ - dependence one needs to include in consideration large range of nuclei, from light to heavy. It is clear, that the description of whole region by one set of nuclear parameters presented above, leads to the qualitative representation only. On Fig. 1 we present the ratio of the real to the imaginary parts of nuclear amplitudes, at $\nu = 10\,GeV$ and $Q^2 = 4\,GeV^2$. Dashed, solid and dotted curves correspond to the values of $\alpha_N = -0.2, 0$ and 0.2, respectively.

![Figure 1: A-dependence of the $\alpha_A$, the ratio of the real to the imaginary parts of nuclear amplitudes, at $\nu = 10\,GeV$ and $Q^2 = 4\,GeV^2$. Dashed, solid and dotted curves correspond to the values of $\alpha_N = -0.2, 0$ and 0.2, respectively.](image)

parts of the amplitude, $\alpha_A$ as a function of $A$ (see Eq. 5). As was mentioned above, $Q^2$ and $\nu$ were fixed at the values of $4\,GeV^2$ and $10\,GeV$, respectively. One can see from Fig. 1 that changing of $\alpha_N$ do not change essentially the shape of curve, but slightly shifts it from the central position. Taking this into account we will use for the calculations the value of $\alpha_N = 0$ only. On Fig. 2 the ratio of the coherent differential cross sections in forward direction, as a function of atomic mass number $A$ is calculated following to Eq. 4. Positions of breaking point coincide with the minimum of cross section, which indicates on

$^3$For the chosen kinematics the most realistic value of $\alpha_N$ is $\alpha_N = -0.2$ (see e.g. 5).
Figure 2: Ratio of the coherent differential cross sections in forward direction, as a function of atomic mass number $A$ calculated at the values of $\nu = 10\text{GeV}$, $Q^2 = 4\text{GeV}^2$ and $\sigma_{tot}^{VN} = 25\text{mb}$. Vertical lines correspond to the $\alpha_A$ breaking point positions.

their common nature. Calculations performed with higher values of $Q^2$ show that existing minimum moves to the smaller values of $A$ and becomes more deeper, also, at higher values of $A$, the second minimum arises. On Fig. 3 the ratio of the coherent differential cross sections in forward direction \(\frac{d\sigma}{d\Omega}_A / \frac{d\sigma}{d\Omega}_N\), as a function of atomic mass number $A$ is calculated for different values of $\sigma_{tot}^{VN}$. Used values of $\sigma_{tot}^{VN} = 15\text{mb}$ and $25\text{mb}$ are close to the accepted values for $\phi$ - and $\rho$ - meson - nucleon total cross sections, respectively.\(^4\) One can see, that the nuclear cross sections within the local minimums are most sensitive to the value of $\sigma_{tot}^{VN}$. It can be used for precise measurement of the vector meson - nucleon total cross sections. It is interesting to note, that position of minimum does not depend from the value of cross section. Although Glauber multiple scattering theory does not take into account the color transparency effect, some influence of this effect can be realized by means of variation of the average values of $\sigma_{tot}^{VN}$.

Our conclusions are as follows:

• The $A$-dependence of the differential cross sections for the coherent electroproduction of vector mesons on nuclei in forward direction is studied. It is shown that they are complicated functions of $A$ with minimums and maximums.

• The ratio of the real to the imaginary parts of the forward amplitude on nucleus $\alpha_A = \text{Re}f_A / \text{Im}f_A$ has breaking points at some values of $A$.

\(^4\)In order to make the strong statement concerning the correspondance to the $\phi$ - meson case we have to change also the definition of $q_L$ (see Eq. 2), where we use for all cases the $\rho$ -meson mass.
Figure 3: Ratio of the coherent differential cross sections in forward direction, as a function of atomic mass number $A$ calculated at the values of $\nu = 10\text{GeV}$ and $Q^2 = 4\text{GeV}^2$. Solid, dotted, dashed and dash-dotted curves correspond to the $\sigma_{tot}^{VN}$ values equal to 15, 20, 25 and 30mb, respectively.

- Comparison of the $A$ dependence of differential cross sections with $\alpha_A$ shows that minimums of cross sections are in the close neighbourhood with the breaking points of $\alpha_A$. With the increasing of photon virtuality $Q^2$, minimums move to the breaking points.
- Nuclear cross sections are very sensitive to the values of vector meson-nucleon total cross sections and can be used for their precise determination.
- One should note that the approach used in this paper can only claim for qualitative description of the vector mesons coherent electroproduction in forward direction. We intend to include into consideration in further papers also the effects connected with the various nuclear density distributions; other vector mesons production; as well as to involve the color transparency effect.

References

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