MECHANISM DESIGN FOR MULTI-PARTY MACHINE LEARNING

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ABSTRACT

In a multi-party machine learning system, different parties cooperate on optimizing towards better models by sharing data in a privacy-preserving way. A major challenge in the learning is the incentive issue. For example, if there is competition among the parties, one may strategically hide his data to prevent other parties from getting better models.

In this paper, we study the problem through the lens of mechanism design. Compared with the standard mechanism design setting, our setting has several fundamental differences. First, each agent’s valuation has externalities that depend on others’ true types. We call this setting mechanism design with type-imposed externalities. Second, each agent can only misreport a lower type, but not the other way round. We show that some results (e.g., the truthfulness of the VCG mechanism) in the standard mechanism design setting fail to hold.

We provide the optimal truthful mechanism in the quasi-monotone utility setting. We also provide necessary and sufficient conditions for truthful mechanisms in the most general case. Finally, we show the existence of such mechanisms are highly affected by the market growth rate and give empirical analysis.

1 Introduction

Due to limited computational resources and data size, a single data owner may not be able to train a model with very high quality. In multi-party machine learning, a group of parties cooperates on optimizing towards better models. This concept has attracted much attention recently [18, 31, 32]. The advantage of this approach is that, it can make use of the distributed datasets and computational power to learn a powerful model that anyone in the group cannot achieve alone.

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To make multi-party machine learning practical, a large body of works focus on preserving data privacy in the learning process. Some techniques such as differential privacy-like techniques [1], homomorphic encryption [34, 33] aim to learn the model on the encrypted training datasets instead of the data itself. The federated learning approach, which does not even require the parties to hand over their data, but only collects an update from each data owner, has been proposed recently [31, 22, 23].

However, even the techniques of the secure distributed learning guarantee good performance in theory, the multi-party machine learning can still be fragile when putting into practice, as participants may have incentives to misbehave. The incentive issues may result in a significant reduction in the effectiveness of multi-party learning. Hence, one must tackle the incentive issues in order to make the multi-party learning work well in real applications. However, the importance of agent’s incentives in the multi-party machine learning setting has largely been ignored in most previous studies, where all the parties usually share the same global model with the best performance regardless of their contributions. This allocation works well when there are no conflicts of interest among the parties. For example, an app developer wants to use the users’ usage data to improve the user experience. All users are happy to contribute their data since they can all benefit from such improvements [23].

When the parties are competing with one another, they may be reluctant to participate in the learning process since their competitors can also benefit from their contributions. Consider the case where companies from the same industry are trying to adopt federated learning to level up the industry’s service qualities. Improving other companies’ services can possibly harm their own market share, especially when there are several monopolists that own most of the data.

Such a cooperative and competitive relation poses an interesting challenge that prevents the multi-party learning approach from being applied to a wider range of environments. In this paper, we view this problem from the multi-agent system perspective, and address the incentive issues mentioned above with the mechanism design theory.

1.1 Mechanism Design with Type-Imposed Externality

Although we apply techniques from mechanism design to analyze the problem, the setting we consider is, however, fundamentally different from the standard mechanism design, where each agent’s valuation only depends on his type and the outcome of the mechanism. In our setting, the agent’s valuation also depends on other agents’ true types and we call our setting mechanism design with type-imposed externalities. Recall in the competing companies example, the valuation (or market share) of a company depends on the models used by other companies to serve their customers and these companies may have incentives to hide some of their data (to prevent others from getting better models) and use the model trained with all their data instead.

Note that the externalities in our setting is also different from standard externalities in the literature, as the externalities in our setting are imposed by other agents’ types, but not their allocations. Therefore, some classic and well-known results in the standard mechanism design setting may no longer hold.

Another key difference is that each agent cannot “make up” a dataset that is of higher quality than his actual one. We incorporate this feature into the action space of agents. The setting that agents
can never over-report is widespread in practice. One straightforward example is that the sports competitions where athletes can show lower performance than their actual abilities but not over-perform. The restriction on the action space poses more constraints on the agent behaviors, and allows more flexibility in the design space.

We first formulate the problem mathematically, and then apply techniques from the mechanism design theory to analyze it. Our model is more general than the standard mechanism design framework, and is also able to describe other similar problems involving both cooperation and competition.

We make the following contributions in this paper:

- We model and formulate the mechanism design problem with type-imposed externalities, and identify the differences between our setting and the standard mechanism design setting. We show that some classic results, including the Myerson-Satterthwaite Theorem, do not hold in our setting.

- We propose a implementable learning protocol for the multi-party learning to accompany our mechanism.

- For the quasi-monotone externalities setting, we provide the revenue-optimal and truthful mechanism. For the general valuation functions, we provide both the necessary and the sufficient conditions for all truthful and individually rational mechanisms.

- We analyze the influence of the data size disparity among agents and the market size on mechanisms. When the disparity is huge and the market grows slowly, there may not exist a mechanism that achieves all the desirable properties we focus on.

1.2 Related Work

There is a series of works that focus on mechanism design with a restricted action space [4, 3, 5, 6, 2]. The discrete-bid ascending auctions [11, 9, 2] belong to action-bounded mechanisms, where all bidders’ action spaces are the same bid level set. Several works restrict the number of actions, such as bounded communications [5]. However, all the restricted action spaces are discrete in previous works while the restricted action spaces in our model can be continuous, and are related to agents’ types, i.e., agents can never report types exceeding their actual types.

The external effect on agents’ valuations aims to model competitions and cooperations among agents. A vast literature have studied mechanisms with externalities[19, 13, 20]. For example, Je-hiel et al. [19] propose an optimal auction for selling one item when potential buyers have negative externalities in the full information setting. Deng and Pekec [13] follow their work and exploit auctions with negative externalities for multiple items. Our setting is different from theirs, as external effects are related to other agents’ actual types.

The learned model can be copied and distributed to as many agents as possible, so the supply is unlimited. A line of literature focuses on selling items in unlimited supply such as digital goods [17, 16, 14]. However, the seller sells the same item to buyers while in our setting we can allocate models with different qualities to different agents.
Redko and Laclau [30] study the optimal strategies of agents for collaborative machine learning problems where they can choose to share their datasets or keep them private. Both their work and ours capture the cooperation and competition among the agents, but they only consider the case where agents reveal their total datasets while agents can choose to contribute only a fraction in our setting. Kang et al. [21] study the incentive design problem for federated learning, but all their results are about a non-competitive environment, which may not hold in real-world applications.

Another closely related topic is the incentives in machine learning problems, such as strategyproof classification [25, 24], incentive compatible linear regression [8, 10, 12, 29]. The main objective of these mechanisms is to get an unbiased learning model with strategic agents and all agents receive the results from the same model. We also consider the collaborative learning setting, but different models are distributed to different agents.

There is also a line of works on distributed algorithmic mechanism design [28, 15, 27]. They focus on distributed mechanisms under the condition that the untrusted center cannot be used. We also analyze mechanisms that are distributed, but the center is trusted in our setting.

2 Preliminaries

In this section, we introduce the general concepts of mechanism design and formulate the multi-party machine learning as a mechanism design problem. A multi-party learning consists of a central platform and several companies or data owners (called agents hereafter). The agents serve their customers with their models trained using their private data. Each agent can choose whether to enter the platform. If an agent does not participate, then he trains his model with only his own data. The platform requires all the participating agents to contribute their data in a privacy-preserving way and trains a model for each participant using a (weighted) combination of all the contributions. Then the platform returns the trained models to the agents.

We assume that all agents, no matter whether they enter the platform or not, use the same model structure. Therefore, each participating agent may be able to train a better model by making use of his private data and the model allocated to him. One important problem in this process is the incentive issue. For example, if the participants have conflicts of interest, then they may only want to make use of others’ contributions but are not willing to contribute with all their own data. To align their incentives, we allow the platform to charge the participants according to some predefined rules after the models are returned to them. In this paper, we analyze this problem from the angle of mechanism design.

Our goal is to design mechanisms that encourage all agents to join the multi-party learning as well as to contribute their all data.

2.1 Valid Data Size (Type)

Suppose there are \( n \) agents, denoted by \( N = (1, 2, \ldots, n) \), and each of them has a private dataset \( D_i \) where \( D_i \cap D_j = \emptyset, \forall i \neq j \). We assume that a model is fully characterized by its quality (e.g., the prediction accuracy), denoted by \( Q \), and the quality only depends on the data used to train it. For simplicity, we measure the contribution of a dataset to a trained model by its valid dataset size. Thus we have the following assumption:
**Assumption 1.** The model quality $Q$ is bounded and monotone increasing with respect to the valid data size $s \geq 0$ of the training data, that is

1. $Q(0) = 0$ and $Q(s) \leq 1$, $\forall s$;
2. $Q(s') > Q(s)$, $\forall s' > s$.

The valid data size of every contributor’s data is validated by the platform in a secure protocol (see Section 2.2). Let $t_i \in \mathbb{R}_+$ be the valid data size of agent $i$’s private dataset $D_i$. We call $t_i$ the agent’s type. The agent can only falsify his type by using a dataset of lower quality (for example, using a subset of $D_i$, or adding fake data), which decreases the contribution to the trained model as well as the size of valid data. As a result, the agent with type $t_i$ cannot contribute to the platform with a dataset higher than his type:

**Assumption 2.** Each agent $i$ can only report a type lower than his actual type $t_i$, i.e., the action space of agent $i$ is $[0, t_i]$.

### 2.2 Learning Protocol

In this section, we describe the learning protocol that could enable the implementation of our mechanism. We assume that the platform also has its own dataset, which we call the validation dataset. Analogy to the standard mechanism design setting, where each agent submits his type to the mechanism, we also require the agents to report their valid data size $t_i$. This could be done by asking each agent to submit the best model that he can possibly obtain by using his own dataset. Then the platform computes the model quality $q_i$ using its own validation dataset and get the agent’s valid data size $t_i$ by $t_i = Q^{-1}(q_i)$. The agent type $t_i$ will be used in the training process (e.g., aggregate weighted model updates), as well as to determine the the final allocation, which is a model with quality $x_i$.

After a mechanism is announced to the agents, the platform should guarantee to deliver to each agent the model determined by the mechanism. However, it is possible that an agent reports $t_i$ in the beginning but only contribute $t'_i < t_i$ in the actual training process. In the extreme case where all agents contribute nothing to the training process, the platform will fail to allocate a model to each agent with the quality determined by the mechanism. To address this issue, the platform can train $n$ additional models simultaneously, with the $i$-th model trained only using the data from agent $i$. During the training process, the platform can apply secure multi-party computation techniques, such as homomorphic encryption [34, 33], to prevent the agent from knowing which model is sent to him to compute the update. And after the training, the platform can compute the quality $t'_i$ of the $i$-th model again using the validation dataset. If the qualities $t'_i$ and $t_i$ match, we know with high probability that the dataset contributed by the agent is consistent with the type he reports. Otherwise, the platform can just exclude the agent and start over the training process again.

The above protocol only ensures that the type reported by each agent is the same as the type he participates in the actual training process with. To incentivize all agents to participate and contribute all their data, we still need to design a mechanism with desirable properties, to which we devote the rest of the paper.
2.3 Mechanism

Let \( t = (t_1, t_2, \ldots, t_n) \) be the type profile of all agents, and \( t_{-i} = (t_1, \ldots, t_{i-1}, t_{i+1}, \ldots, t_n) \) be the profile of all agents without \( i \). Given the type profile of all agents, a mechanism specifies a numerical allocation and payment for each agent, where the allocation is a model in the multi-party learning. Formally, we have:

**Definition 1 (Mechanism).** A mechanism \( M = (x, p) \) is a tuple, where

- \( x = (x_1, x_2, \ldots, x_n) \), where \( x_i : \mathbb{R}^n_+ \to \mathbb{R} \) is the allocation function for agent \( i \), which takes the agents’ reported types as input and decides the model quality for agent \( i \) as output;
- \( p = (p_1, p_2, \ldots, p_n) \), where \( p_i : \mathbb{R}^n_+ \to \mathbb{R} \) is the payment function for agent \( i \), which takes the agents’ reported types as input and specifies how much agent \( i \) should pay to the mechanism.

In a competitive environment, each agent will try their best to compete in the market. It is possible that a strategic agent hides some of his data and does not use the model he receives from the platform. Thus the final model quality depends on both the allocation and his actual type. We use valuation function \( v_i(x(t'), t) \) to measure the profit that agent \( i \) can make in the market.

**Definition 2 (Valuation with Type-Imposed Externalities).** A valuation \( v_i(x(t'), t) \) has type-imposed externalities, if it depends not only on the allocation outcome \( x(t') \) where \( t' \) is the reported type profile, but also on the actual type profile \( t \).

We assume agent \( i \) uses the following model to serve his customers:

\[
q_i = \max\{x_i(t'), Q(t_i)\},
\]

where \( Q(t_i) \) is the model trained with his own data. The valuation of agent \( i \) depends on the final model qualities of all agents due to their competition. Hence \( v_i \) can also be expressed as \( v_i(q_1, \ldots, q_n) \).

We make the following assumption on agent \( i \)’s valuation:

**Assumption 3.** Agent \( i \)’s valuation is monotone increasing with respect to true type \( t_i \) when the outcome \( x \) is fixed.

\[
v_i(x, t_i, t_{-i}) \geq v_i(x, \hat{t}_i, t_{-i}), \forall x, \forall t_i \geq \hat{t}_i, \forall t_{-i}, \forall i.
\]

This is because possessing more valid data will not lower one’s valuation. Otherwise, an agent is always able to discard part of his dataset to make his true type \( t_i' \). Suppose that each agent \( i \)’s utility \( u_i(t, t') \) has the form:

\[
u_i(t, t') = v_i(x(t'), t) - p_i(t'),
\]

where \( t \) and \( t' \) are actual types and reported types of all agents respectively. As mentioned above, an agent may lie about his actual type in order to benefit from the mechanism. To prevent the agents from lying, the mechanism should incentivize truthful reports.
Definition 3 (Incentive Compatibility (IC)). A mechanism is said to be incentive compatible, or truthful, if reporting truthfully is always the best response for each agent when the other agents report truthfully:

\[ u_i(x(t_i, t_{-i}), t) \geq u_i(x(t'_i, t_{-i}), t), \forall t_i \geq t'_i, \forall t_{-i}, \forall i. \]

For ease of presentation, we say agent \( i \) reports \( \emptyset \) if he chooses not to participate (so we have \( x_i(\emptyset, t_{-i}) = 0 \) and \( p_i(\emptyset, t_{-i}) = 0 \)). To encourage the agents to participate in the mechanism, the following property should be satisfied:

Definition 4 (Individual Rationality (IR)). A mechanism is said to be individually rational, if no agent loses by participation when the other agents report truthfully:

\[ u_i(x(t_i, t_{-i}), t) \geq u_i(x(\emptyset, t_{-i}), t), \forall t_i, t_{-i}, \forall i. \]

The revenue and welfare of a mechanism are defined to be all the payments collected from the agents and all the valuations of the agents.

Definition 5. The revenue and welfare of a mechanism \((x, p)\) are:

\[
\text{REV}(x, p) = \sum_{i=1}^{n} p_i(t'), \\
\text{WEL}(x, p) = \sum_{i=1}^{n} v_i(x, t).
\]

We say that a mechanism is efficient if

\[ x = \arg\max_x \sum_i v_i(x, t). \]

A mechanism is weakly budget-balance if it never loses money.

Definition 6 (Weak Budget Balance). A mechanism is weakly budget-balance if:

\[ \text{REV}(x, p) \geq 0, \forall t. \]

Definition 7 (Desirable Mechanism). We say a mechanism is desirable if it is simultaneously IC, IR, efficient and weakly budget-balance.

3 Comparison with Standard Mechanism Design

As discussed in previous sections, our setting includes external effects imposed by other agents’ true types. In the standard mechanism design setting, the utility function of an agent only depends on his own true type and his allocation. Even though some existing works study external effect, they only consider externalities imposed by others’ allocations. Such externalities are known to the mechanism designer, as allocations are determined by the mechanism. But in our setting, the mechanism designer may never know the actual externalities that are put on each agent.
At first glance, such a problem seems intractable as some important information about the externalities is missing. However, our setting has another key difference from the standard mechanism design setting, which enables our solution to the problem. In our setting, each agent has a limited action space and cannot report a type higher than his actual type while both over-reporting and under-reporting are allowed in the standard setting. Our setting puts a stronger restriction on agents’ behaviors, thus has a larger design space.

Due substantial differences, some classic results may no longer hold in our setting. In this section, we show that the well-known VCG mechanism does not guarantee the IC property and the Myerson-Satterthwaite Theorem fails to hold in our setting.

3.1 The VCG Mechanism Is Not IC

The VCG mechanism chooses an allocation function that maximizes the social welfare and charges each agent for the harm he causes to others due to his participation.

**Definition 8** (Vickrey-Clarke-Groves (VCG) mechanism). *VCG mechanism* $(x, p)$ is a mechanism where

$$x(t') = \arg \max_x \sum_{i=1}^{n} v_i(x(x(t'), t'),$$

$$p_i(t') = \sum_{j \neq i} v_j(x(\emptyset, t'_{-i}), t') - \sum_{j \neq i} v_j(x(t'), t').$$

It is known that the VCG mechanism is IR, IC and efficient. However, the VCG mechanism can no longer guarantee all these three properties in our setting. The following example shows that the VCG mechanism violates the constraint of IC.

**Example 1.** Let $q_i = \max\{x_i, Q(t_i)\}$ be the model that agent $i$ serves the his customers with and assume $Q(s) = s$. We assume agents are in a fixed market where $\sum_i^n v_i = 1$, and the values of agents are proportional to their model qualities in the market, thus we have:

$$q_i = \max\{x_i, Q(t_i)\} = \max\{x_i, t_i\},$$

$$v_i = \frac{q_i}{\sum_j q_j}.$$

We consider two agents whose true types are $t_1$ and $t_2$ respectively and $t_1 = 10t_2$. Because the whole market is fixed, any arbitrary allocation function is efficient. Suppose the two agents report $t'_1$ and $t'_2$, the efficient allocation is

$$x_1 = t'_1 + t'_2, x_2 = t'_1 + t'_2.$$

That is, we give the best model to both of them.
Consider the case when both the two agents truthfully report, according to the payment rules of the VCG mechanism, we have

\[ x_1 = x_2 = t_1 + t_2, \quad q_1 = q_2 = t_1 + t_2; \]
\[ p_1 = \frac{t_2}{t_1 + t_2} - \frac{1}{2}, \]
\[ u_1 = v_1 - p_1 = \frac{1}{2} \left( -\frac{t_2}{t_1 + t_2} + \frac{1}{2} \right) = \frac{t_1}{t_1 + t_2} = \frac{10}{11}. \]

If agent 1 reports \( t_1' = 3t_2 \), then by the VCG mechanism,

\[ x_1 = x_2 = t_1' + t_2, \quad q_1 = \max \{ t_1' + t_2, t_1 \} = t_1, \quad q_2 = x_2 = t_1' + t_2; \]
\[ p_1 = \frac{t_2}{t_1' + t_2} - \frac{1}{2}, \]
\[ u_1 = v_1 - p_1 = \frac{t_1}{t_1' + t_2} - \left( \frac{t_2}{t_1' + t_2} - \frac{1}{2} \right) = \frac{27}{28} > \frac{10}{11}. \]

So untruthfully reporting would bring more utility to agent 1, which violates the IC constraint. This example shows that the VCG would not work when there exists competition among agents.

The reason why the VCG mechanism does not work in our setting lies in the fact that each agent’s utility depends on the actual types of other agents. We also find that when such externalities are removed, the VCG mechanism guarantees both IR and IC again.

3.2 The Myerson-Satterthwaite Theorem Does Not Hold

The Myerson-Satterwaite Theorem is an important impossibility theorem which showing that no desirable mechanism exists in the standard mechanism design setting.

**Theorem 1** (Myerson and Satterthwaite [26]). In the standard mechanism design setting, no mechanism is simultaneously IC, IR, efficient and weakly budget-balance.

In our multi-party learning setting, mechanisms with all these properties can be found. We omit the proof, as the optimal truthful mechanism in Section 4 serves as a counter example.

4 Quasi-monotone Externality Setting

As described in Section 3, each agent’s utility may also depend on the models that other agents actually use. Such externalities lead to interesting and complicated interactions between the agents. For example, by contributing more data, one may improve the others’ model quality, and end up harming his own market share.

In this section, we study the setting where agents have quasi-monotone externalities.

**Definition 9** (Quasi-Monotone Valuation). Let \( q_i \) be the final selected model quality of the agent and \( q_{-i} \) be the profile of model qualities of all the agents except \( i \). A valuation function is quasi-monotone if it is in the form:

\[ v_i(q_i, q_{-i}) = F_i(q_i) + \theta_i(q_{-i}), \]

where \( F_i \) is a monotone function and \( \theta_i \) is an arbitrary function.
Example 2. Let’s consider a special quasi-monotone valuation: the linear externality setting, where the valuation for each agent is defined as $v_i = \sum_j \alpha_{ij} q_j$ with $q_j$ being the model that agent $j$ uses. The externality coefficient $\alpha_{ij}$ means the influence of agent $j$ to agent $i$ and captures either the competitive or cooperative relations among agents. If the increase of agent $j$’s model quality imposes a negative effect on agent $i$’s utility (e.g., major opponents in the market), $\alpha_{ij}$ would be negative. Also $\alpha_{ij}$ could be positive if agent $i$ and agent $j$ are collaborators. Additionally, $\alpha_{ii}$ should always be positive, naturally.

In the linear externality setting, the efficient allocation is straightforward. For each agent $i$, if $\sum_j \alpha_{ji} \geq 0$, the platform gives agent $i$ the training model with best possible quality. Otherwise, agent $i$ are not allocated any model if $\sum_j \alpha_{ji} < 0$.

We introduce a payment function called maximal exploitation payment, and show that the mechanism with efficient allocation and the maximal exploitation payment guarantees individually rationality, truthfulness, efficiency and revenue optimum.

Definition 10 (Maximal Exploitation Payment (MEP)). For a given allocation function $x$, suppose the agent $i$ reports a type $t'_i$ and the other agents report $t'_{-i}$, the maximal exploitation payment is to charge $i$

$$p_i(t'_i, t'_{-i}) = v_i(x(t'_i, t'_{-i}), t'_i, t'_{-i}) - v_i(x(\emptyset, t'_{-i}), t'_i, t'_{-i}).$$

Theorem 2. Under the quasi-monotone valuation setting, any mechanism with MEP is the mechanism with the maximal revenue among all IR mechanisms, and it is IC.

Proof. Intuitively, the MEP rule charges agent $i$ the profit he gets from an model that the mechanism allocates to him. If the mechanism charges higher than the MEP, an agent would have negative utility after taking part in. The IR constraint would then be violated. So it’s easy to see that the MEP is the maximal payment among all IR mechanisms.

Then we prove that this payment rule also guarantees the IC condition. It suffices to show that if an agent hides some data, no matter which model he chooses to use, he would never get more utility than that of truthful reporting. We suppose that agent $i$’s type is $t'_i$ and he untruthfully reports $t'_i$.

Suppose that the agent $i$ truthfully reports the type $t'_i = t_i$, since the payment function is defined to charge this agent until he reaches the valuation when he does not take part in the mechanism, the utility of this honest agent would be

$$u_i^0(t'_i) = F_i(Q(t'_i)) + \theta_i(q_{-i}(\emptyset, t'_{-i})).$$

If the agent does not report truthfully, we suppose that the agent reports $t'_i$ where $t'_i \leq t_i$. According to the MEP, the payment function for agent $i$ would be

$$p_i(t'_i, t'_{-i}) = F_i(q_i(t'_i, t'_{-i})) + \theta_i(q_{-i}(t'_i, t'_{-i})) - F_i(Q(t'_i)) - \theta_i(q_{-i}(\emptyset, t'_{-i})).$$

It can be seen that the mechanism would never give an agent a worse model than the model trained by its reported data, otherwise the agents would surely select their private data to train models. Hence it is without loss of generality to assume that the allocation $x_i(t'_i, t'_{-i}) \geq Q(t'_i), \forall t'_i, t'_{-i}, \forall i$. Thus we have $q_{-i}(t'_i, t'_{-i}) = x_{-i}(t'_i, t'_{-i})$. We discuss the utility of agent $i$ by two cases of choosing models.
Case 1: the agent chooses the allocation \( x_i \). Since agent \( i \) selects the allocated model, we have \( q_i = x_i(t_i', t_{i-1}') \). Then the utility of agent \( i \) would be

\[
\begin{align*}
    u^1_i &= v_i(t_i', t_{i-1}') - p_i(t_i', t_{i-1}') \\
    &= F_i(x_i(t_i', t_{i-1}')) + \theta_i(x_{i-}(t_i', t_{i-1}')) - p_i(t_i', t_{i-1}') \\
    &= F_i(x_i(t_i', t_{i-1}')) + \theta_i(x_{i-}(t_i', t_{i-1}')) + F_i(Q(t_i')) \\
    &\quad - \theta_i(x_{i-}((\emptyset, t_{i-}'))) - F_i(x_i(t_i', t_{i-1}')) - \theta_i(x_{i-}(t_i', t_{i-}')) \\
    &= F_i(Q(t_i')) + \theta_i(x_{i-}((\emptyset, t_{i-}'))) - F_i(x_i(t_i', t_{i-1}')).
\end{align*}
\]

Because both \( F_i \) and \( Q \) are monotone increasing functions and \( t_i \geq t_i' \), we have \( u^1_i \leq F_i(Q(t_i)) + \theta_i(x_{i-}((\emptyset, t_{i-}'))) = u^0_i \).

Case 2: the agent chooses \( Q(t_i) \). Since agent \( i \) selects the model trained by his private data, we have \( q_i = Q(t_i) \). The final utility of agent \( i \) would be

\[
\begin{align*}
    u^2_i &= v_i(t_i', t_{i-1}') - p_i(t_i', t_{i-1}') \\
    &= F_i(Q(t_i)) + \theta_i(x_{i-}(t_i', t_{i-}')) - p_i(t_i', t_{i-}').
\end{align*}
\]

Subtract the original utility from the both sides, then we have

\[
\begin{align*}
    u^2_i - u^0_i &= F_i(Q(t_i)) + \theta_i(x_{i-}((\emptyset, t_{i-}'))) - F_i(Q(t_i)) - \theta_i(x_{i-}((\emptyset, t_{i-}'))) \\
    &= F_i(Q(t_i')) - F_i(x_i(t_i', t_{i-}')).
\end{align*}
\]

Because \( x_i(t_i', t_{i-1}') \geq Q(t_i') \), \( \forall t_i', t_{i-1}', \forall i \) and because \( F_i \) is a monotonically increasing function, we can get \( u^2_i - u^0_i \leq 0 \). Therefore \( \max\{u^1_i, u^2_i\} \leq u^0_i \), lying would not bring more benefits to any agent, and the mechanism is IC.

\[\square\]

5 General Externality Setting

In this section, we consider the general externality setting where the valuation of agent \( i \) can be any function of allocation outcome and types of all agents. The limitation on the reporting space and the type-imposed value functions make the IC and IR mechanisms hard to characterize. It is possible that given a allocation rule, there exist several mechanisms with different payments that satisfy both IC and IR constraints. To understand what makes a mechanism IC and IR, we analyze some properties of truthful mechanisms in this section.

For ease of presentation, we assume that the functions \( v(\cdot), x(\cdot) \) and \( p(\cdot) \) are differentiable.

**Theorem 3** (Necessary Condition). If a mechanism \((x, p)\) is both IR and IC, for all possible valuation functions satisfying Assumption 3, then the payment function satisfies \( \forall t_i \geq t_i', \forall t_{i-}, \forall i, \)

\[
\begin{align*}
    p_i(0, t_{i-}) &\leq v_i(x(0, t_{i-}), 0, t_{i-}) - v_i(x(\emptyset, t_{i-}), 0, t_{i-}), \\
    p_i(t_i, t_{i-}) - p_i(t_i', t_{i-}) &\leq \int_{t'_i}^{t_i} \partial v_i(x(s', t_{i-}), s, t_{i-}) \left| \frac{\partial s'}{\partial s} \right| ds'.
\end{align*}
\]

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where we view $v_i(x(t_i', t_i), s, t_i)$ as a function of $t_i$, $t_i'$ and $t_i$ for simplicity and

$$\frac{\partial v_i(x(t_i', t_i), s, t_i)}{\partial t_i'} = \sum_{j=1}^{n} \frac{\partial v_i(x(t_i', t_i), s, t_i)}{\partial x_j(t_i', t_i)} \frac{\partial x_j(t_i', t_i)}{\partial t_i'}.$$  

Proof. We first prove that Equation (1) holds. Observe that

$$u_i(x(t_i, t_i'), t_i, t_i) - u_i(x(t_i', t_i'), t_i', t_i')$$

$$\geq [v_i(x(t_i, t_i'), t_i, t_i) - p_i(t_i, t_i')] - [v_i(x(t_i', t_i), t_i', t_i') - p_i(t_i', t_i')]$$

$$\geq [v_i(x(t_i, t_i'), t_i, t_i) - p_i(t_i, t_i')] - [v_i(x(t_i', t_i'), t_i', t_i') - p_i(t_i', t_i')]$$

$$= u_i(x(t_i, t_i'), t_i, t_i) - u_i(x(t_i', t_i'), t_i, t_i')$$

$$\geq 0,$$  \hspace{1cm} (3)

where the first inequality is because of Assumption 3, and the last inequality is because of the DSIC property.

Let $t_i' = 0$ in Equation (3). We have

$$u_i(x(t_i, t_i'), t_i, t_i) \geq u_i(x(0, t_i'), 0, t_i).$$

The IR property further requires that $u_i(x(0, t_i'), 0, t_i) \geq u_i(x(0, t_i'), 0, t_i')$, which Equation (1) follows.

To show Equation (2) must hold, we rewrite Equation (3):

$$p_i(t_i, t_i') - p_i(t_i', t_i') \leq v_i(x(t_i, t_i'), t_i, t_i) - v_i(x(t_i', t_i'), t_i, t_i')$$

$$= \int_{t_i'}^{t_i} \frac{dv_i(x(s', t_i'), s, t_i)}{ds'} ds'.$$  \hspace{1cm} (4)

Fixing $t_i$ and $t_i'$, the total derivative of the valuation function $v_i(x(s', t_i'), s, t_i)$ is:

$$\frac{dv_i(x(s', t_i'), s, t_i)}{ds'} = \frac{\partial v_i(x(s', t_i'), s, t_i)}{\partial s'} ds' + \frac{\partial v_i(x(s', t_i'), s, t_i)}{\partial s} ds.$$

View $s$ as a function of $s'$ and let $s(s') = s'$:

$$\frac{dv_i(x(s', t_i'), s(s'), t_i)}{ds'} = \frac{\partial v_i(x(s', t_i'), s, t_i)}{\partial s'} \bigg|_{s=s'} + \frac{\partial v_i(x(s', t_i'), s(s'), t_i)}{\partial s} \frac{ds(s')}{ds'} ds'.$$

Plug into Equation (4), and we obtain:

$$p_i(t_i, t_i') - p_i(t_i', t_i') \leq \int_{t_i'}^{t_i} \frac{\partial v_i(x(s', t_i'), s, t_i)}{\partial s'} \bigg|_{s=s'} + \int_{t_i'}^{t_i} \frac{\partial v_i(x(s', t_i'), s(s'), t_i)}{\partial s} \frac{ds(s')}{ds'} ds'.$$

Since the above inequality holds for any valuation function with $v_i(x(t_i, t_i), \forall x, \forall t_i, \forall t_i \geq t_i'$, we have:

$$p_i(t_i, t_i') - p_i(t_i', t_i') \leq \int_{t_i'}^{t_i} \frac{\partial v_i(x(s', t_i'), s, t_i)}{\partial s'} \bigg|_{s=s'} ds'.$$

\qed
Theorem 3 describes what the payment \( p \) is like in all IC and IR mechanisms. In fact, the conditions in Theorem 3 are also crucial in making a mechanism truthful. However, to ensure IC and IR property, we still need to restrict the allocation function \( x \).

**Theorem 4 (Sufficient Condition).** A mechanism \((x, p)\) satisfies both IR and IC, for all possible valuation functions satisfying Assumption 3, if for each agent \( i \), for all \( t_i \geq t'_i \), and all \( t_{-i} \). Equations (1) and the following two hold

\[
t'_i = \arg \min_{t_i, t_{-i} > t'_i} \frac{\partial v_i(x(t'_i, t_{-i}), t_i, t_{-i})}{\partial t'_i}
\]

\[
p_i(t_i, t_{-i}) - p_i(t'_i, t_{-i}) \leq \int_{t'_i}^{t_i} \frac{\partial v_i(x(s', t_{-i}), s, t_{-i})}{\partial s'} ds' - \int_{t'_i}^{t_i} \frac{\partial v_i(x(\emptyset, t_{-i}), s, t_{-i})}{\partial s} ds.
\]

**Proof.** Equation (5) indicates that the function \( \frac{\partial v_i(x(t'_i, t_{-i}'), t_i, t_{-i})}{\partial t'_i} \) is minimized at \( t'_i \):

\[
\frac{\partial v_i(x(t'_i, t_{-i}'), s, t_{-i})}{\partial t'_i} \bigg|_{s = t'_i} \leq \frac{\partial v_i(x(t'_i, t_{-i}'), t_i, t_{-i})}{\partial t'_i}.
\]

Therefore, we have

\[
u_i(x(t_i, t_{-i}'), t_i, t_{-i}) - u_i(x(t'_i, t_{-i}'), t_i, t_{-i}) \]
\[
= \int_{t'_i}^{t_i} \frac{\partial v_i(x(s', t_{-i}'), t_i, t_{-i})}{\partial s'} ds' - p_i(t_i, t_{-i}') + p_i(t'_i, t_{-i}')
\]
\[
\geq \int_{t'_i}^{t_i} \frac{\partial v_i(x(s', t_{-i}'), s, t_{-i})}{\partial s'} ds' - p_i(t_i, t_{-i}') + p_i(t'_i, t_{-i}')
\]
\[
\geq \int_{t'_i}^{t_i} \frac{\partial v_i(x(\emptyset, t_{-i}'), s, t_{-i})}{\partial s} ds,
\]

where the two inequalities are due to Equation (7) and (6), respectively. Since \( v_i(x, t_i, t_{-i}) \geq v_i(x, t'_i, t_{-i}), \forall x, \forall t_{-i}, \forall t_i \geq t'_i \) indicates \( \frac{\partial v_i(x(\emptyset, t_{-i}'), s, t_{-i})}{\partial s} \geq 0 \), the above inequality shows that the mechanism guarantees the DSIC property.

To prove that the mechanism is IR, we first observe that

\[
[u_i(x(t_i, t_{-i}'), t_i, t_{-i}) - v_i(x(\emptyset, t_{-i}'), t_i, t_{-i})] - [u_i(x(t'_i, t_{-i}'), t_i, t_{-i}) - v_i(\emptyset, x(t_{-i}), t'_i, t_{-i})]
\]
\[
= u_i(x(t_i, t_{-i}'), t_i, t_{-i}) - u_i(x(t'_i, t_{-i}'), t_i, t_{-i}) - \int_{t'_i}^{t_i} \frac{\partial v_i(x(\emptyset, t_{-i}'), s, t_{-i})}{\partial s} ds
\]
\[
\geq u_i(x(t_i, t_{-i}'), t_i, t_{-i}) - u_i(x(t'_i, t_{-i}'), t_i, t_{-i}) - \int_{t'_i}^{t_i} \frac{\partial v_i(x(\emptyset, t_{-i}'), s, t_{-i})}{\partial s} ds
\]
\[
\geq 0,
\]
where the two inequalities are Assumption 3 and Equation (8). Letting \( t'_i = 0 \) using Equation (2), we get:

\[
\begin{align*}
&u_i(x(t_i, t'_{-i}), t_i, t_{-i}) - v_i(x(\emptyset, t'_{-i}), t_i, t_{-i}) \\
&\geq u_i(x(0, t'_{-i}), 0, t_{-i}) - v_i(x(\emptyset, t'_{-i}), 0, t_{-i}) \\
&= v_i(x(0, t'_{-i}), 0, t_{-i}) - p_i(0, t'_{-i}) - v_i(x(\emptyset, t'_{-i}), 0, t_{-i}) \\
&\geq 0.
\end{align*}
\]

\[\square\]

6 Market Growth Rate

In this section, we will analyze a factor, the market growth rate, for the existence of the desirable mechanism. Expanding the market size would reduce competition among the agents, meaning that the damage to an agent’s existing market caused by joining the mechanism is more likely to be covered by the market growth. Thus our intuition is that if the market grows quickly, a desirable mechanism is more likely to exist.

As mentioned above, each agent’s valuation is the profit made from the market, so formally we define the market size to be the sum of the valuations of all the agents. Let \( M(q) \) be the agents’ total valuations where \( q = (q_1, q_2, \ldots, q_n) \) is the set of actual model qualities they use. We have:

\[
M(q) = \sum_{i=1}^{n} v_i(x, t).
\]

In general, the multi-party learning process improves all agents’ models. So we do not consider the case where the market shrinks due to the agents’ participation, and assume that the market is growing.

**Assumption 4 (Growing Market).** \( q \succeq q' \) implies \( M(q) \geq M(q') \).

A special case of the growing market is the non-competitive market where agent’s values are not affected by others’ model qualities, formally:

**Definition 11 (Non-competitive Market).** A market is non-competitive if and only if \( \frac{\partial v_i(q)}{\partial q_j} \geq 0, \forall i, j \).

**Theorem 5.** In a non-competitive market, there always exists a desirable mechanism, that gives the best possible model to each agent and charges nothing.

**Proof.** Suppose that the platform uses the mechanism mentioned in the theorem. Then for each agent, contributing with more data increases all participants’ model qualities. By definition, in a non-competitive market, improving others’ models does not decrease one’s profit. Therefore, the optimal strategy for each participant is to contribute with all his valid data, making the mechanism truthful. Also because of the definition, entering the platform always weakly increases one’s model quality. Thus the mechanism is IR. With the IC and IR properties, it is easy to see that the mechanism is also efficient and weakly budget-balance. \[\square\]
However, when the competition exists in a growing market, it is not easy to determine whether a desirable mechanism exists. Since when the market is growing, the efficient mechanism both redistributes existing markets and enlarges the market size by giving the best learned model. We will give the empirical analysis of the influence of competition and the market growth rate on the existence of desirable mechanisms.

7 Experiments

We design experiments to demonstrate the performance of our mechanism for practical use. We first show the mechanism with maximal exploitation payments (MEP) can guarantee good quality of trained models and high revenues under the linear externality cases. Then we conduct simulations to show the relation of the market growth of competitive market to the existence of desirable mechanisms.

7.1 The MEP Mechanism

We consider the valuation with linear externalities setting where \( \alpha_{ij} \)'s (defined in Example ??) are generated uniformly in \([-1, 1]\). Each agent’s type is drawn uniformly from \([0, 1]\) independently and the \( Q(t) = \frac{1-e^{-t}}{1+e^{-t}} \). The performance of a mechanism is measured by the platform’s revenue and its best quality of trained model under the mechanism. All the values of each instance are averaged over 50 samples. We both show the performance changes as the number of agents increases and as the agents’ type changes.

When the number of agents becomes larger, the platform can obtain more revenues and train better models (see Figure 1). Particularly, the model quality is close to be optimal when the number of agents over 12. An interesting phenomenon is that the revenue may surpass the social welfare. This is because the average external effect of other agents on one agent \( i \) tends to be negative when agent \( i \) does not join in the mechanism, thus the second term in the MEP payment is averagely negative and revenue is larger than the welfare.

To see the influence of type on performance, we fix one agent’s type to be 1 and set the other agent’s type from 0 to 2. It can be seen in Figure 2 that the welfare and opponent agent’s utility (ut\(_{i,2}\)) increase as the opponent’s type increases but the platform’s revenue and the utility of the static agent (ut\(_{i,1}\)) are almost not affected by the type. So we draw the conclusion that the most efficient way for the platform to earn more revenue is to attract more small companies to join the mechanism, since in the figure 1 the revenue obviously increases as the number of agents increases.

7.2 Existence for Desirable Mechanisms

We assume all the agents’ type lies in \([0, D]\), and the type space can be discretized into intervals of length \( \epsilon \), which is also the minimal size of the data. Thus each agent’s type is a multiple of \( \epsilon \). The data disparity is defined as the ratio of the largest possible data size to the smallest possible data size, denoted as, \( D/\epsilon \). We measure the condition for existence of desirable mechanisms by the maximal data disparity when the market growth rate is given.
Figure 1: Performance of MEP under different numbers of agents

Figure 2: Performance of MEP under different types

To describe the market growth, we use the following form of valuation function and model quality function:

\[ Q(t) = t \text{ and } v_i(q) = \left( \sum_{j=1}^{n} Q(q_j) \right)^{\alpha} \cdot Q(q_i), \forall i, \]

where the parameter \( \alpha \) indicates the market growth rate. As we can see, when \( \alpha < -1 \), the market is not a growing market. When \( \alpha \geq 0 \), the market becomes non-competitive, therefore by Theorem 5, a desirable mechanism trivially exists. Thus we consider the competitive growing market case where \(-1 \leq \alpha < 0\).

We provide an algorithm to find the desirable mechanisms. Since the utility function is general, all the points in the action space would influence the properties and existence of the mechanism, thus the input space of the algorithm is exponential. The algorithm can be seen in Appendix A.1. We enumerate the value of \( \alpha \) and run the algorithm to figure out the boundary of \( D/\epsilon \) under different
α in a market with 2 agents. It turns out that when \( \alpha \) is near \(-0.67\), the boundary of disparity grows very fast. When \( \alpha \) is close to \(-0.66\), the disparity boundary is over 10000 and is beyond our computing capability, so we just enumerate \( \alpha \) from \(-1\) to \(-0.668\) with step length 0.002.

Figure 3 demonstrates the maximal data disparity given the existence of a desirable mechanism under different market growth rates. The y-axis is the data disparity \( D/\epsilon \) and the x-axis shows the market growth rate \( \alpha \). For every fixed \( \alpha \), there does not exist any desirable mechanism when the biggest size of the agent’s dataset is larger than \( D \) if the smallest size of dataset is \( \epsilon \). It can be seen an obvious trend that when \( \alpha \) becomes larger, the constraint on data size disparity would become looser. In another word, if a desirable mechanism exists, a market with a larger \( \alpha \) can tolerate a larger data size disparity. A desirable mechanism is more likely to exist in a market that grows faster. When the market is not growing, there would not be such a desirable mechanism at all. On the other hand, if the market grows so fast such that there does not exist any competition between the agents, the mechanism always exists.

Appendix

A Algorithm

A.1 Finding a Desirable Mechanism

We provide an algorithm, that given the agents’ valuations, computes whether the mechanism that is simultaneously truthful, individually rational, efficient and weakly budget-balance exists, and outputs the one that optimizes revenue, if any.

Since each agent can only under-report, according to the IR property, we must have:

\[
 u_i(x(t_i, t_{-i}), t) \geq u_i(x(\emptyset, t_{-i}), t), \forall t, \forall i.
\]
Equivalently, we get $\forall t, \forall i,$
\[
  u_i(x(\emptyset, t_i), t) \leq v_i(x(t_i, t_{i-}), t) - p_i(t_i, t_{i-}),
\]
\[
p_i(t_i, t_{i-}) \leq v_i(x(t_i, t_{i-}), t) - u_i(x(\emptyset, t_{i-}), t),
\]
\[
p_i(t) \leq v_i(x(t_i, t_{i-}), t) - u_i(x(\emptyset, t_{i-}), t).
\]

For simplicity, we define the upper bound of $p(t')$ as
\[
  \overline{p(t)} \triangleq \{ v_i(x(t_i, t_{i-}), t) - u_i(x(\emptyset, t_{i-}), t) \}.
\]

The IC property requires that $\forall t_i \geq t_{i}', \forall t_{i-}, \forall i,$
\[
  u_i(x(t_i, t_{i-}), t) \geq u_i(x(t_{i}', t_{i-}), t).
\]

A little rearrangement gives:
\[
  p_i(t_i, t_{i-}) - p_i(t_{i}', t_{i-})
\]
\[
  \leq v_i(x(t_i, t_{i-}), t) - v_i(x(t_{i}', t_{i-}), t)
\]
\[
  \triangleq \text{Gap}_i(t_{i}', t_i, t_{i-}).
\]

Note that the inequality correlations between the payments form a system of difference constraints. The form of update of the payments is almost identical to that of the shortest path problem. Therefore, we make use of this observation to design the algorithm.

We assume that all the value functions are common knowledge, the efficient allocation is then determined because the mechanism always chooses the one that maximizes the social welfare. Thus it suffices to figure out whether there is a payment rule $p(t')$ which makes the mechanism IR, IC and weakly budget-balance. Since the valid data size for each agent is bounded in practice, we assume the mechanism only decides the payment functions on the data range $[0, D]$, and discretize the type space into intervals of length $\epsilon$, which is also the minimal size of the data. Thus each agent’s type is a multiple of $\epsilon$. Note that since the utility function is general, all the points in the action space would influence the properties and existence of the mechanism, thus it is necessary to enumerate all the points in the space. The exponential value function space, i.e., the exponential input space, determines that the complexity of our algorithm is exponential in $D$.

We give the following algorithmic characterization for the existence of a desirable mechanism.

The following theorem proves the correctness of Algorithm 1.

**Theorem 6.** Taking agents’ valuation functions as input, Algorithm 1 outputs the answer of the decision problem of whether there exists a mechanism that guarantees IR, IC, efficiency and weak budget balance simultaneously, and specifies the payments that achieve maximal revenue if the answer is yes.

**Proof.** Suppose that there is a larger payment for agent $i$ such that $p_i(t') > p_{i_{\text{max}}}(t')$ where $t'$ is the profile of reported types. In the process of our algorithm, the $p_{i_{\text{max}}}(t')$ is the minimal path length from $VB_{t_{i-}}$ to $V_{t_1} t_{i-}$, denoted by $(VB_{t_{i-}}, V_{t_1} t_{i-}, V_{t_2} t_{i-}, \cdots, V_{t_{ik}} = t_{i-})$. By the definition of edge
Adding these inequalities together, we get

\[
p_i(t_{i1}, t_{-i}) \leq p_i(t_{i1}, t_{-i}) ,
\]
\[
p_i(t_{i2}, t_{-i}) - p_i(t_{i1}, t_{-i}) \leq \text{Gap}_i(t_{i1}, t_{i2}, t_{-i}) ,
\]
\[
\vdots
\]
\[
p_i(t_{ik}, t_{-i}) - p_i(t_{i(k-1)}, t_{-i}) \leq \text{Gap}_i(t_{i1}, t_{i2}, t_{-i}).
\]

Adding these inequalities together, we get

\[
p_i(t') \leq p_i(t_{i1}, t_{-i}) + \sum_{j=1}^{k-1} \text{Gap}_i(t_{ij}, t_{i(j+1)}, t_{-i}) = p_i^{\text{max}}(t').
\]

If \( p_i(t') < p_i^{\text{max}}(t') \) holds, this would violate at least 1 of the \( k \) inequalities above. If the first inequality is violated, the mechanism would not be IR, by the definition of \( p_i(t_{i1}, t_{-i}) \). If any other inequality is violated, the mechanism would not be IC, by the definition of \( \text{Gap}_i(t_{ij}, t_{i(j+1)}, t_{-i}) \).

On the other hand, if we select \( p_i^{\text{max}}(t') \) to be payment of agent \( i \), all the inequalities should be satisfied, otherwise the shortest path would be updated to a smaller length.

Therefore the \( p_i^{\text{max}}(t') \) must be the maximum payment for agent \( i \). If the maximal payment sum up to less than 0, there would obviously be no mechanism that is IR, IC and weakly budget-balanced under the efficient allocation function.
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