Different Realizations of Cooper-Frye Sampling with Conservation Laws

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Abstract. Approaches based on viscous hydrodynamics for the hot and dense stage and hadronic transport for the final dilute rescattering stage are successfully applied to the dynamic description of heavy ion reactions at high beam energies. One crucial step in such hybrid approaches is the so called particlization, the transition between the hydrodynamic description to microscopic degrees of freedom. For this purpose, individual particles are sampled on the Cooper-Frye hypersurface. In this work, 4 different realizations of sampling algorithms are compared, where three of them incorporate global conservation laws of quantum numbers in each event. The algorithms are compared within two types of scenarios: simple “box” hypersurface consisting of only one static cell and a typical particlization hypersurface for Au+Au collisions at $\sqrt{s_{\text{NN}}} = 200$ GeV. For all algorithms the mean multiplicities (or particle spectra) remain unaffected by global conservation laws in the case of large volumes. In contrast, the fluctuations of the particle numbers are affected considerably. The fluctuations of the newly developed SPREW algorithm based on exponential weights and the recently suggested SER algorithm based on ensemble rejection are smaller than without conservation laws and agree with the expectation from the canonical ensemble. The previously applied mode sampling algorithm produces dramatically larger fluctuations, than it is expected in the corresponding microcanonical ensemble, and therefore should be avoided in fluctuation studies. This study might be of interest for investigations of particle fluctuations and correlations, e.g. the suggested signatures for a phase transition or a critical endpoint, in hybrid approaches that are affected by global conservation laws.
1. Introduction

Bulk observables in heavy ion reactions at high beam energies carried out at the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) are successfully described by hybrid approaches (see reviews in [1, 2] and references therein). Event-by-event calculations based on relativistic viscous fluid dynamics starting from fluctuating initial conditions and coupled to hadronic transport approaches for the late dilute stages are the current state-of-the-art for a realistic dynamic description of these heavy ion reactions [3, 4, 5]. Nowadays, such calculations are performed in a multi-parameter space and by detailed comparisons to a plethora of observables quantitative constraints on the properties of hot and dense QCD matter are extracted by Bayesian techniques [6, 7, 8].

There are two crucial interfaces in such combined microscopic and macroscopic approaches: The construction of the initial state for hydrodynamics accompanied by rapid thermalization [9] and the particlization at the transition from the hydrodynamic description to individual particles. The first topic is under heavy investigation especially also in the context of the collective effects observed in small systems at RHIC and LHC [10, 11]. In this work, we concentrate on the second transition, where the sampling of particles on the hypersurface is usually performed according to the Cooper-Frye formula [12]. Physics-wise the transition from hydrodynamics to the non-equilibrium transport description should take place, when the degrees of freedom are hadrons and the Knudsen number grows too large for fluid dynamics to be applicable [13]. The chemical and kinetic freeze-out is then subsequently performed automatically in the transport approach [14, 15].

Usually the particles are sampled on the hypersurface according to grand-canonical distribution functions taking the flow velocity of the corresponding cell into account. In the grand-canonical ensemble temperature, volume and chemical potentials are fixed, but quantum numbers, energy and momentum are only conserved on the average over many events. Therefore in single events a discontinuity in the total energy, momentum and quantum numbers occurs at particlization. This contradicts the philosophy behind the hybrid approaches that particlization is just a smooth change of the formalism from hydrodynamics to transport, but not a physical transition. Additionally, at the first glance it seems simply unphysical to violate conservation laws. Nevertheless, the grand canonical sampling is justified in many cases, being a good and computationally fast approximation. Exploring the validity region of this approximation is one of the purposes of this article.

It is generally expected that the account of global conservation laws on an event-by-event basis does not change averages over many events significantly. Therefore, grand-canonical sampling should be a good approximation for bulk observables, such as transverse momentum and rapidity spectra. In the present manuscript it is verified that the relative difference for central Au+Au collisions at the highest RHIC energy, where hybrid approaches are often applied, does not exceed 2%. However, the error may become larger at lower collision energies (e.g. at which a hybrid approach [16] was recently applied), for smaller systems or for rare hadron species. On the other hand, in most cases there are many particle distributions sampled per hydro run, the so called “over-sampling” technique to increase the statistics [4, 16, 17, 18, 19, 20, 21, 22, 23]. With the oversampling factor $N_{\text{over}}$ the error decreases as $\sqrt{N_{\text{over}}}$, therefore for bulk observables one can increase the accuracy of the grand-canonical sampling approximation by increasing $N_{\text{over}}$.

Fluctuation and correlation observables are currently under intense experimental investigation, because they are associated with the possible existence of a first order phase transition between the hadron gas and the quark gluon plasma or a critical endpoint in the QCD phase diagram [24, 25]. Contrary to bulk observables, fluctuations and correlations cannot be reliably studied in a hybrid approach with grand-canonical sampling. Spurious event-by-event fluctuations of the energy, momentum and quantum numbers at participlization change fluctuations of multiplicities in an uncontrollable way [26]. Besides the fluctuations themselves, it may bias the selection of events according to centrality classes.

Therefore, obeying global event-by-event conservation laws at particlization is necessary to study multiplicity fluctuations with a hybrid approach. To study correlations, which typically make use of local variables, even this is not enough. Local conservation laws have to be ensured, as it was successfully tried for electric charge in [27].

Event-by-event conservation laws at particlization are important to study certain observables using hybrid approaches. This was the motivation in the original event-by-event hybrid approach based on the UrQMD transport model [28, 29], where one initial state is propagated through a hydrodynamic evolution and one final state is sampled for each event. Conservation laws (except momentum) are obeyed globally in this approach via the so-called “mode sampling” algorithm [29]. Recently it was observed that the multiplicity fluctuations produced by this algorithm depend on internal details [30], therefore, in this work a new algorithm to conserve global quantum numbers is suggested.

The new SPREW (Single Particle Rejection with
Exponential Weights) algorithm is proposed based on a typical grand-canonical sampling algorithm [31], enhanced by conservation laws. Exponential weights are introduced for each quantum number to favor configurations that fulfill the conservation laws. The conventional and the new algorithm are compared to the mode sampling algorithm and SER (Sequential Ensemble Rejection) algorithm. In Section 2 all realizations of particle sampling on the Cooper-Frye hypersurface are explained with more emphasis on the newly developed SPREW algorithm. As a first basic test in Section 3 all three algorithms are applied to a single static cell and the mean multiplicities as well as their fluctuations are compared to the thermal expectation. In Section 4 the different realizations are compared for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV and Section 5 summarizes the main findings.

2. The different sampling algorithms

The purpose of any particlization algorithm is to produce particles given a set of hydrodynamic variables on a predefined hypersurface: energy density $\epsilon(x)$, pressure $p(x)$, temperature $T(x)$, chemical potentials $\mu(x)$ and collective velocities $u^\mu(x)$. The hypersurface is numerically divided into many small pieces characterized by the local normal vectors $d\sigma_{\mu}(x)$. According to the Cooper-Frye formula, the average number of particles with 4-momentum $p^\mu$ from one cell is

$$dN = g \frac{p^\mu d\sigma_{\mu}}{(2\pi)^3} f(p^\mu u_{\mu}) d^3p,$$

where $f(p^\mu u_{\mu})$ is the distribution function and $g$ is the degeneracy of a hadron species. The average number of hadrons of species $i$ from a cell is

$$\pi_i = \frac{g_i u^\mu d\sigma_{\mu}}{2\pi^2h^3} \int f(p^\mu u_{\mu}) p^\mu dp,$$

which is obtained from Eq. (1) using its Lorentz-invariance. At this step in conventional sampling the grand-canonical ensemble is assumed, neglecting global conservation laws. This is a challenging assumption in heavy ion collisions, where net baryon number, electric charge, strangeness, energy or momentum are conserved in each event. Nevertheless, as discussed above, in many cases this assumption may be a good approximation.

For the grand-canonical ensemble one can express probabilities of occupation numbers $n$ via average $\pi_i$:

$$P(n) = \left\{ \begin{array}{ll} \frac{\pi^n}{n!} e^{-\pi}, & \text{Boltzmann stat.} \\ \pi^n (1 + \pi)^{-1-n}, & \text{Bose stat.} \\ \pi^n (1 - \pi)^{-1-n}, & \text{Fermi stat.} \end{array} \right.$$

(3)

Usually $P(n_i)$ for Boltzmann statistics is assumed even if Bose or Fermi statistics is used in Eq. (2). This simplifies sampling considerably, because the Poisson distribution is additive. This means that the total number of particles in a cell, as well as the overall number of particles are Poissonian variables. From this it also follows that, e.g., the total number of particles with charge $+1$ is also a Poissonian variable, as well as number of particles with charge $-1$. The total charge, or baryon number therefore has a Skellam distribution. The net strangeness, total energy and momentum are not fixed in the conventional sampling procedure, but distributed over a range of values as it has been discussed in [29]. Instead they should have values determined by the hypersurface ($X$ denotes the overall value of BSQ and $x_i$ are BSQ numbers of each particle, where BSQ stands for net baryon, strangeness and electric charge):

$$X_{tot} = \int_{\sigma} n_i x_i \, u^\mu \, d\sigma_{\mu},$$

(4)

where the integral runs over the hypersurface. The total energy and momentum should be fixed to

$$P_{tot}^\mu = (E_{tot}, \vec{p}_{tot})^\mu = \int_{\sigma} T^{\mu\nu} d\sigma_{\nu},$$

(5)

where the energy-momentum tensor $T^{\mu\nu}$ is provided by the hydrodynamics. The distribution to be sampled is therefore

$$w \sim \prod_{\text{particles}} g_j \frac{p_j^\mu d\sigma_{\mu}}{p_j^\mu} f(p_j^\mu u_{\mu}) d^3p_j \times \delta^{(4)} \left( \sum_j P_{j}^\mu - P_{tot}^\mu \right) \delta^{BSQ} \left( \sum_j x_j - X_{tot} \right).$$

(6)

Notice that in this distribution the momenta of the particles are not independent, which makes the distribution [6] extremely hard to sample exactly. Therefore the mode sampling algorithm and the newly developed SPREW algorithm attempt to sample it approximately, fulfilling the constraints given by the $\delta$-functions. In the following the algorithms themselves are described.

2.1. Conventional sampling

The assumptions of the conventional sampling algorithm are described above. It is realized in the following way:

(i) Average multiplicities for every hadron species in a cell are computed according to Eq. (2) and summed up to the total average multiplicity in a cell $\pi$.

(ii) The total number of particles in a cell is sampled $n \sim \text{Poisson}(\pi)$. 

(iii) For each particle the type is selected based on the probability $\frac{n_i}{n}$ (an equivalent alternative to get the multiplicities of hadron species is to sample a multinomial distribution)

(iv) The momentum is generated from the thermal distribution and boosted to the rest frame of the cell, which results in the proper distribution when the correct weighting factors are included.

(v) For every cell this is repeated.

2.2. SPREW sampling

The newly developed single particle rejection with exponential weights (SPREW) algorithm ensures the global conservation of quantum numbers in the particle sampling by introducing weights, that suppress configurations with unwanted quantum numbers. The general procedure is as for conventional sampling, however, the particle species sampled with probability $\frac{n_i}{n}$ can be either accepted or rejected based on the following scheme.

Firstly, the difference of the quantum number $X$ (representing the baryon number, electric charge or strangeness) of the so far produced particles to the value on the hypersurface is calculated

$$\Delta X = X_{\text{particles}} - X_{\text{surface}} \ .$$

For a particle $i$ with quantum numbers $x_i$, the particle is rejected with the probability $1 - e^{-|\Delta X|}$, if $\Delta X$ and $x_i$ have the same sign. This rejection is performed for every quantum number ($B$, $S$, $Q$), until one hadron is accepted. In the end, the quantum numbers are not necessarily reproduced exactly, therefore, the last few particles are adjusted by hand. If the total number of particles is too small, this is not possible, since it would produce too large bias.

The momenta of the particles are sampled in the same way as in the conventional method. After a particle ensemble has been obtained, all the energies and momenta of the individual hadrons $\vec{p}_i$ are adjusted to enforce energy and momentum conservation. Firstly, the momenta are centralized: $\vec{p}_i = \vec{p}_i - \frac{1}{N} \sum_i p_i$. Then all the particles are boosted to the center of mass frame (denoted by the primed quantities). The momenta are then rescaled with factor $(1 + a)$ such that

$$\sum_i \sqrt{(1 + a)^2 p_i^2 + m_i^2} = E'_{\text{hypersurface}}$$

and then boosted back. This enforces energy and momentum conservation and ensures that the on-shell condition $E^2 = p^2 + m^2$ is met for every particle. Given that the typical value of $a$ is very small ($|a| \sim 3\%$) in the case of Au-Au collisions, the SPREW sampling conserves energy and momentum without deformation of the momentum space distribution even though the energy-momentum conservation is not enforced at the point of sampling.

It is important to underline the difference between rescaling the energy and accounting for the energy conservation in the sampling directly. While the first does not affect the multiplicity distribution, the second implies the transition from canonical to microcanonical sampling leading to narrower multiplicity distributions.

2.3. Mode sampling

Another way of implementing global conservation laws has been applied within the UrQMD hybrid approach by the so called mode sampling procedure. Essentially, the sampling procedure is performed several times (different 'modes'), that are always terminated when the total energy is used for particle production. In the first mode only the particles containing positive strangeness are kept, while all others are discarded, the second one produces the corresponding number of particles with negative strangeness to ensure net strangeness conservation. The following two modes proceed in a similar way for net baryon number conservation and then the same procedure is pursued for charged particles that are non-strange mesons. Last, the energy is filled up with non-strange neutral mesons, which are mainly $\pi_0$'s. The mode sampling is described in more detail in \cite{28,29,30}.

2.4. SER sampling

One more algorithm of implementing a sampling respecting global conservation laws can be called sequential ensemble rejection (SER). This algorithm was already used and studied in \cite{30} under the name of “unbiased Becattini-Ferroni sampling”. The algorithm proceeds as follows:

(i) For the quantum number $X \in \{B,S,Q\}$ repeat the following. Sample the total number of $N_{X>0}$ and $N_{X<0}$ from a Poisson distribution. The multiplicities of particular hadron species with $X > 0$ and $X < 0$ are then sampled from multinomial distributions. If $N_{X>0} - N_{X<0} \neq X_{\text{surface}} - X_{\text{particles}}$, where $X_{\text{particles}}$ is the total of the previously sampled particles, then start from the very beginning.

(ii) Sample multiplicities of neutral mesons from the Poisson distributions.

(iii) If the total energy of the sampled particles $E_{\text{sampled}}$ deviates too much from the expected energy $E_{\text{surface}}$ then start from the beginning. The quantitative criterion was chosen as $|E_{\text{sampled}} - E_{\text{surface}}| / E_{\text{surface}} < 0.01$. This last step can be
omitted, then the energy conservation is not enforced. Without this last step the algorithm corresponds to a global canonical ensemble, with the last step it approximately corresponds to a global microcanononical ensemble.

3. Thermal fluctuations

In this Section a simple “box” test scenario is investigated, where the hypersurface is just one static cell with \( d\sigma_\mu = (V,0,0,0) \). The temperature in this box is assumed to be \( T = 150 \text{ MeV} \). All the chemical potentials are supposed to be \( \mu_B = \mu_S = \mu_I = 0 \).

The multiplicity distributions should be compared to the corresponding analytical expectations. The conventional sampling aims at producing the grand-canonical ensemble, where multiplicity distributions are simply Poissonian. The goal of the SPREW algorithm is to produce a canonical ensemble in global net baryon number \( B \), strangeness \( S \) and electric charge \( Q \). The SER algorithm and mode sampling additionally try to conserve the total energy, so they should be compared to the proper microcanononical ensemble. Finally, SER algorithm without the last step, which performs rejection by energy, is to be compared to the canonical ensemble in \( BSQ \).

The analytical canonical and microcanonical distributions are hard to obtain analytically in general, but they were computed in \(^{32}\) for the case of large volumes. In this case the (micro-)canonical multiplicity distributions approach a Gaussian distribution with the grand-canonical mean and a non-trivial variance, which is always smaller or equal than for the grand-canonical ensemble. To compute the analytical expectation Boltzmann-Maxwell distribution was used, Eqs. (28-33) from \(^{32}\) were applied for canonical ensemble and extended analogously to Eqs. (47-53) from the same article to add energy conservation.

Let us start by investigating a small box with a length of \( l = 5 \text{ fm} \) that contains a rather small number of particles. First, notice that theoretically the distributions of \( N_{\pi^+} \) and \( N_{\pi^-} \) should be identical in any ensemble for vanishing chemical potentials. Fig. [1](left) demonstrates that for mode sampling and SPREW these distributions do not coincide. The deviation is the largest for the mode sampling, which does not have enough energy left in the last modes and fails to produce enough \( \pi^- \) and especially \( \pi^0 \). Of course, this bias depends on the order of the modes, as it was already noticed in \(^{30}\). The SPREW algorithm leads to a smaller deviation, which we attribute not to the algorithm itself, but rather to the manual interventions in the final step to ensure exact conservation of quantum numbers. We have checked that for the SPREW sampling the results are insensitive to the order of the SPREW steps for the different quantum numbers in contrast to the mode sampling. The SER algorithm, which is not shown in Fig. [1] produces \( N_{\pi^+} \) and \( N_{\pi^-} \) distributions, which are identical to each other.

Despite the small volume, the multiplicity distributions are still rather close to Gaussian distributions. Therefore, the large volume analytical approximation should be applicable and the mean multiplicities should be close to the grand-canonical values for all algorithms. In Fig. [2] one can see that this is indeed the case for the SPREW algorithm (the SER also fits the expectation), but not for the mode sampling. The latter introduces a bias to the mean values in case of small particle number. Therefore, one has to be careful when applying hybrid approaches with conservation laws to small systems like p-p collisions.

The scaled variance \( \frac{\sigma^2}{\langle N \rangle^2} = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle^2} \) of the multiplicities is compared in Fig. [2](right). The SPREW and SER algorithm without total energy conservation match the approximate canonical expectation within statistical errors. The SER with total energy conservation matches the microcanonical expectation reasonably well. By varying the margin for the energy rejection the agreement can be increased even further. In contrast to previous algorithms, the mode sampling drastically deviates from the aimed microcanononical ensemble, producing even wider distributions than the grand-canonical one.

Figs. [3] and [4] show the same probability distributions and quantified mean values and standard deviations in a larger box with length \( l = 20 \text{ fm} \) that contains multiplicities similar to the ones in a Au+Au collision at the highest RHIC energies. The multiplicity distributions are very close to Gaussian distributions, so the large volume approximation should be accurate in this case. The expected mean values are reproduced nicely by all algorithms within 1% accuracy. The fluctuations behave similarly to the smaller box, namely for the mode sampling they overshoot the microcanonical and even the grand-canonical expectations dramatically.

In addition, we have checked that including Bose/Fermi distributions does not change the results for the conventional sampling algorithm by employing a completely independent implementation that has been used in \(^{5}\).

4. Au+Au collisions at \( \sqrt{s_{\text{NN}}} = 200 \text{ GeV} \)

After testing the different realizations of sampling particles with and without conservation laws in a box, let us now investigate the more realistic situation of heavy ion reactions. The transition from hydrodynamics to particles is usually performed on the
Cooper-Frye hypersurface at a constant temperature. This is a good approximation to the hypersurface where the expansion rate exceeds the scattering rate and the hydrodynamic evolution is not applicable anymore. To calculate the hydrodynamic evolution and the properties on the hypersurface the CLVisc code on GPUs \[33\] has been employed. Smooth Glauber initial conditions are propagated through an ideal hydrodynamic evolution to eliminate all additional sources of fluctuations. The hadronic rescattering has also been neglected for simplicity. The fluid densities are converted to single particles on the Cooper-Frye hypersurface at a freeze-out temperature of \( T = 137 \) MeV. The parameterization with a smooth crossover connecting lattice QCD EoS at high temperatures and a hadron resonance gas at low temperatures in chemical equilibrium (or partial chemical equilibrium if \( s95p-pce \) is applied) \[34\], is used in the current calculation. Particles are sampled on the hypersurface according to the algorithms described in Section 2.

Let us start with a discussion of all the conserved quantum numbers. Let us stress again, that in conventional sampling algorithms none of these is conserved in single events, only on the average the correct result is obtained. In Fig. 5 each of the panels contains the mean values and their standard deviations.
deviation. For the net baryon number, net strangeness and electric charge the average values are reproduced very well. For the energy and the momentum in $z$- and $x$- direction slight deviations can be observed. The mode sampling conserves all quantum numbers, but the rescaling of the momenta is not applied, which is reflected in small deviations for the total energy on the hypersurface as well. The newly developed SPREW algorithm nicely conserves all quantum numbers on an event-by-event basis. In the conventional algorithm the distributions around the mean are rather wide and allow for large fluctuations in single event particle samples. This has to be kept in mind, when calculating more involved particle correlation and fluctuation observables from hydrodynamic or hybrid approaches.

Here, we restrict ourselves to investigating the consequences of global conservation laws on basic bulk observables. In Fig. 6 it can be seen that the transverse momentum and rapidity spectra are not affected by conservation laws. All three algorithms yield exactly the same distribution for all particles in a central ($b = 0$ fm) Au+Au collision.

In Fig. 7 the relative multiplicities for the most abundant particle species are shown. All of them are reproduced by all three algorithms within a better accuracy than 0.5%. This is in agreement with our
Figure 5. Conserved quantities on the hypersurface in a central ($b = 0$ fm) Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV compared to the sampled particles for all three algorithms. The mean values of 10,000 events are given as numbers in the corresponding figures.

Figure 6. Transverse momentum (left) and rapidity (right) distribution for all particles in a central ($b = 0$ fm) Au+Au collision at $\sqrt{s_{NN}} = 200$ GeV for all three algorithms.
finding in the previous section, that the algorithms do not bias the particle multiplicities, if the abundances are large enough.

Fig. 8 shows the distributions of the different pion isospin states in more detail, analogously to the thermal box tests in Section 3 above. First of all, a slight difference in the mean values for $\pi^+)/-\pi^0$ is observed that is attributed to their different masses. Apart from the small difference in the mean values, the distributions for the conventional sampling algorithm are very similar for the three different types of pions. For the SPREW sampling, the distributions of charged pions are a little narrower than the original distribution, which can be attributed to charge conservation as discussed above. The mode sampling again leads to wider distributions and the width of the distribution is even more increased for neutral pions. This result confirms our findings that the SPREW sampling introduces less biases than the mode sampling, when enforcing global conservation laws.

One of the most direct consequences of different event-by-event fluctuations of multiplicities is expected for the division of events in centrality classes. If the fluctuations of the number of charged particles depend on the sampling algorithm that has an effect on the selection of events for certain centrality cuts. Especially, when extreme cuts for very central events are performed small differences in the fluctuations can lead to different resulting event samples for each centrality class. To quantify this effect, Fig. 9 shows the mean number of charged particles and their variance displayed by the shaded band as a function of impact parameter for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. Resonance decays have been taken into account. The bands and lines for the conventional and SPREW sampling overlap for all centralities, so the SPREW sampling will not affect the selection of centrality classes. The mode sampling on the other hand leads to wider distributions in line with our analysis above and might affect the resulting centrality class selection.

Besides the possible effects of event-by-event global conservation laws in heavy ion collisions, small systems might be affected much more strongly by conservation laws. Due to the measurement of collective effects and anisotropic flow coefficients in pp and pPb collisions at the LHC, hydrodynamic and
hybrid approaches are also applied to small systems. The effect of momentum conservation will strongly affect the particle correlations (at least on the 2-particle level), but for this purpose local conservation laws need to be implemented. We have nevertheless checked that global conservation laws do not affect nor introduce structures in the $\Delta \eta - \Delta \phi$ distribution in proton-proton collisions corresponding to a beam energy of $\sqrt{s} = 13$ TeV.

5. Conclusions and Outlook

In this article, we have studied the effects of global conservation laws on the particle sampling process on the Cooper-Frye hypersurface. This question is relevant for event-by-event hybrid approaches that are used as the ’standard model’ for the theoretical description of the dynamical evolution of heavy ion collisions at high beam energies. In addition to the conventional sampling without conservation of quantum numbers and the existing mode sampling and SER algorithms, a new SPREW algorithm using exponential weights to ensure global conservation of quantum numbers has been introduced.

The mode sampling, which is supposed to match the microcanonical ensemble, produces correct mean multiplicities only in case of the large volume. For a small volume mode sampling introduces species-dependent biases to means, especially the neutral mesons are affected. Fluctuations produced by modes sampling are always dramatically larger than the analytical expectation. The SPREW and SER algorithms correctly match the canonical ensemble, while SER with rejection by total energy approximately matches the microcanonical expectation. Although the SER algorithm seems to be the best one in terms of rigour, it is also the slowest one, especially with the energy rejection. The SPREW algorithm can be a fast alternative to SER to generate canonical ensemble.

Conserving quantum numbers on the Cooper-Frye transition between hydrodynamics and hadronic transport is potentially crucial for future studies of particle correlations and fluctuations both at high and at low beam energies. For example, any higher moment analysis with hybrid approaches at intermediate or lower beam energies needs to consider this effect. Therefore, the results of this study might have important consequences on the interpretation of experimental results related to the QCD critical point and first order phase transition.

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