Cho and Pak Reply: A recent Comment \[1\] has criticized the logarithmic correction term of the effective action of QED in our Letter \[2\].

\[
\mathcal{L}_{\text{eff}} = -\frac{a^2 - b^2}{2e^2} \left( 1 - \frac{e^2}{12\pi^2} \ln \frac{m^2}{\mu^2} \right) 
- \frac{ab}{4\pi^2} \sum_{n=1}^{\infty} \frac{1}{n} \left\{ \coth \left( \frac{n\pi b}{a} \right) \left( \text{ci} \left( \frac{n\pi m^2}{a} \right) \cos \left( \frac{n\pi m^2}{a} \right) \right) + i \left( \frac{n\pi m^2}{a} \right) \sin \left( \frac{n\pi m^2}{a} \right) \right\} 
- \frac{1}{2} \left( \coth \left( \frac{n\pi a}{b} \right) \left( \exp \left( \frac{n\pi m^2}{b} \right) \text{Ei} \left( -\frac{n\pi m^2}{b} \right) \right) + \exp \left( -\frac{n\pi m^2}{b} \right) \text{Ei} \left( \frac{n\pi m^2}{b} - ie \right) \right\},
\]

(1)

where \( \mu \) is the subtraction parameter and

\[
a = \frac{e}{2} \sqrt{F^4 + (F\tilde{F})^2 + F^2}, \\
b = \frac{e}{2} \sqrt{F^4 + (F\tilde{F})^2 - F^2}.
\]

The Comment claims that “the logarithmic correction term found by Cho and Pak vanishes when the final result is written in terms of the finite, renormalized, physical electron charge”, asserting that “these terms do not appear if on-mass shell renormalization is used”.

We have no intention to dispute this claim, because this is a simple reiteration of what everybody knows. What we like to point out here is that this criticism is based on the confusion of the regularization with the renormalization. Our Letter \[2\] deals only with the regularized effective action, not the renormalized one. And the logarithmic term in Eq. (1) contains an important piece of information, the subtraction dependence of the regularized effective action. The renormalization (and the renormalization group invariance) of the effective action has already been fully discussed in a separate paper \[3\], which was quoted in Ref. [12] of \[2\]. Here we simply note that the logarithmic term does disappear if we use the mass-shell subtraction \( \mu = m \). This is exactly what the Comment asserts. This nullifies the criticism of the Comment even without the renormalization of the effective action.

We also stand by our remark in Ref. [9] of \[2\]: An honest regularization of the divergent integral expression of the QED effective action must produce both the imaginary part and the logarithmic term. Only after the renormalization does the logarithmic term disappear, as we discussed in \[3\]. Furthermore, we remark that the logarithmic term plays an important role in proving the fact that the QED effective action has no infra-red divergence in the massless limit when \( ab = 0 \). This is evident from Eqs. (10), (12), and (14) of \[2\].

One might wonder why we did not use the simple “mass shell renormalization” in our Letter \[2\]. The reason is that one can not use the mass shell renormalization in a massless theory, because it can not control the infra-red divergence properly. And if one wants to discuss the massless limit of QED, one must do the the subtraction dependent regularization first. This point becomes more important when one tries to calculate the effective action of QCD \[4\].

The renormalization of the effective action gives us an unexpected surprise. To renormalize the effective action, one need to define the running coupling \( \bar{e}(\bar{\mu}) \) by

\[
\frac{d^2 V_{\text{eff}}}{da^2} \bigg|_{a=\bar{\mu}^2} = \frac{1}{\bar{e}^2},
\]

(2)

where \( V_{\text{eff}} \) is the effective potential obtained from Eq. (1). A remarkable point here is that this definition produces the running coupling which is different from what one obtains from the perturbative calculation \[3\]. This is surprising, because in QCD the above definition and the perturbative calculation produce an identical result \[3\]. Only in QED do we have this discrepancy. A possible interpretation of the origin of this difference is discussed in \[3\].

Note Added: There is a typographical mistake in \[2\]. The RHS of Eq. (24) in \[2\] should have an overall minus sign.

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