A fully coupled flow-deformation model for cyclic elasto-plastic analysis of 
multiphase porous media

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ABSTRACT

A fully coupled flow-deformation model is presented for the nonlinear cyclic analysis of multiphase unsaturated soils. The coupling between fluid flow and deformation fields is established using the effective stress parameters. The hydraulic hysteresis is accounted for in the model through the effective stress parameters and the soil water characteristic curve. The volume change dependency of the effective stress parameters and the soil water characteristic curve is addressed in the formulation. The elastico-plastic behaviour due to cyclic loading is captured using the bounding surface plasticity. Numerical examples are presented, demonstrating the application of the proposed approach.

Keywords: Unsaturated Soils, Cyclic, Effective Stress, Hydraulic Hysteresis, Bounding Surface

1 INTRODUCTION

The description of the hydro-mechanical behaviour of unsaturated soils has been a key area of research in modern geomechanics. The plastic volume change in an unsaturated soil affects the soil water characteristics curve and results in a change in the degree of saturation. Wetting and drying cycles, on the other hand, increase the stiffness [1] and cause irreversible volumetric strain [2].

Over the past two decades, several coupled hydro-mechanical models based on the multiphase mixture theory have been proposed for the cyclic analysis of unsaturated soils. Among the notable contributions have included the works of Zienkiewicz et al. [3], Meroi et al. [4], Schrefler & Scotta [5], Muraleetharan & Wei [6], Khalili et al. [7], Oka & Kimoto [8], Uzuoka & Borja [9], Shahbodagh et al. [10], Oka et al. [11]. Two different approaches have mainly been adopted to enforce the coupling between fluid flow and deformation fields, i.e. the independent stress state variables approach [6, 13] and the effective stress approach [3-5, 10-12]. In the first approach two stress variables, i.e. net stress and suction, are adopted to cast the constitutive relationships of the soil. These models require a considerable amount of laboratory testing to identify the model parameters, rendering their practical application time consuming and cost prohibitive. In the effective stress based models, the degree of saturation was mainly adopted as the effective stress parameter, which is not supported by the experimental evidence [14]. They also ignored the hydraulic hysteresis effects which can significantly alter the response of an unsaturated soil in particular during dynamic loading that involves complex cycles of strain-induced wetting and drying.

This paper presents a fully coupled flow-deformation model for the nonlinear cyclic analysis of unsaturated soils. The coupling between fluid flow and deformation fields is established following the effective stress approach proposed by Khalili et al. [7]. The hydraulic hysteresis is accounted for in the model through the effective stress parameters and the soil water characteristic curve. The volume change dependency of the effective stress parameters and the soil water characteristic curve is addressed in the formulation. The elasto-plastic behaviour and mechanical hysteresis are captured using the bounding surface plasticity. Numerical results are then presented to demonstrate the application of the proposed model.

2 EFFECTIVE STRESS

The concept of effective stress as a powerful tool for the quantitative assessment of response in saturated and unsaturated soils plays a central role in the present formulation. For unsaturated soils, the effective stress tensor is expressed as

\[ \sigma'_{ij} = \sigma_{\text{net}}_{ij} - \chi \delta_{ij} \]  \hspace{1cm} (1)

where \( \sigma_{\text{net}}_{ij} = \sigma_{ij} + p_{ij} \delta_{ij} \) is the net stress tensor,
\( s = p_G - p_W \) is the suction, \( \chi \) is the effective stress parameter, \( \sigma_{ij} \) is the total stress tensor, \( p_G \) is the pore gas pressure, \( p_W \) is the pore water pressure, and \( \delta_{ij} \) is the Kronecker delta. Since elasto-plastic constitutive relations are highly nonlinear, they are generally expressed using incremental equations. The incremental form of the effective Cauchy stress equation is written as

\[
\dot{\sigma}_{ij} = \dot{\sigma}_{net,ij} - \psi \dot{s} \delta_{ij}
\]  

(2)

where \( \psi = \frac{\partial (\chi s)}{\partial s} \) is the incremental effective stress parameter, and superimposed dot denotes the time derivative with respect to the solid phase. Following the work of Khalili & Khabbaz [15] and Khalili et al. [16], the effective stress parameter is defined as

\[
\chi = \begin{cases} 
1 & \text{for } \frac{s}{s_e} \leq 1 \\
\left( \frac{s}{s_e} \right)^{-\Omega} & \text{for } \frac{s}{s_e} > 1 
\end{cases}
\]  

(3)

where \( \Omega \) is a material parameter with the best fit value of 0.55, and \( s_e \) is the suction value marking the transition between saturated and unsaturated states. For wetting process, \( s_e \) is equal to the air expulsion value, \( s_{ex} \), whereas for drying process, \( s_e \) is equal to the air entry value, \( s_{ae} \). \( s_e \) is a priori a function of the specific volume or volume change of the solid skeleton [17]. This leads to a shift to the right of the effective stress parameter curve and soil water characteristic curve with increasing density (see Figure 1). The effect of hydraulic hysteresis on the effective stress parameter, is taken into account using the following correlation for suction reversals [18]:

\[
\chi = \begin{cases} 
\frac{S_{ae}}{S_{ex}} - \Omega \left( \frac{S}{S_{ex}} \right)^{-\zeta} & \text{for wetting path reversal} \\
\frac{S_{rd}}{S_{ae}} - \Omega \left( \frac{S}{S_{rd}} \right)^{-\zeta} & \text{for drying path reversal} 
\end{cases}
\]  

(4)

where \( \zeta \) is the slope of the transition line between the main wetting and main drying paths in a \( \ln \chi - \ln s \) plane, and \( s_{ew} \) and \( s_{ed} \) are the points of suction reversal on the main wetting and main drying paths, respectively.

Fig. 1. (above) Evolution of the effective stress parameter \( \chi \) and (below) soil water characteristic curve including hydraulic hysteresis.

3 SOIL WATER CHARACTERISTIC CURVE (SWCC)

Another key aspect in the mechanics of unsaturated soils is the SWCC. In this formulation, the SWCC model proposed by Brooks and Corey [19], extended to include hydraulic hysteresis effect [7], is adopted as

\[
S_{eff} = \begin{cases} 
1 & \text{for } \frac{s}{s_e} \leq 1 \\
\left( \frac{S}{S_{rw}} \right)^{-\zeta} & \text{for } \frac{s}{s_e} > 1 
\end{cases}
\]  

(5)
For drying path reversal \[
S_{\text{eff}} = \begin{cases} 
\frac{S}{s} & \text{for } s_a \leq s \leq s_{ad} \\
\frac{S}{s} & \text{for } s_{ad} \leq s \leq s_a 
\end{cases}
\]
where \(s_p\) is the pore size distribution index, \(S_{\text{eff}} = (S_r - S_{\text{res}})/(1 - S_{\text{res}})\) is the effective degree of saturation, and \(S_{\text{res}}\) is the residual degree of saturation (see Figure 1).

4 CONSERVATION OF MASS

By ignoring mass exchanges among the phases, the balance of mass for the water phase \((W)\) and the gas \((G)\) phase can, respectively, be expressed by

\[
\begin{align*}
\frac{-1}{\rho_w} (\rho_w \dot{\omega}_{W})_j &= n_w c_w \frac{d}{dt} p_w + \varphi \dot{v}_r + n \frac{\partial S}{\partial \hat{s}} \\
\frac{-1}{\rho_g} (\rho_g \dot{\omega}_G)_j &= n_g c_g \frac{d}{dt} p_g + (1 - \varphi) \dot{v}_r - n \frac{\partial S}{\partial \hat{s}}
\end{align*}
\]

where \(\dot{\omega}\) / \(\partial t = \hat{\omega}/\hat{t} + (\nabla \cdot v)\) is the material time derivative with respect to the solid phase \((S)\), \(v_s\) is the velocity vector of the solid phase, \(\dot{\omega}_{\beta}\) is the relative velocity of the fluid phases with respect to the solid phase as

\[
\dot{\omega}_{\beta} = n^\beta (v_{\beta} - v_s)
\]

\(v_{\alpha}\) is the velocity vector of phase \(\alpha = S, W, G\), \(\dot{v}_r = v_{r,ij}\) is the volumetric strain rate, \(\rho_a\) is the intrinsic density, \(S_r\) is the degree of saturation, \(n\) is the porosity, and \(c_{\beta}\) is the compressibility coefficient for phase \(\beta\).

5 CONSERVATION OF MOMENTUM

The conservation of linear momentum is expressed by

\[
\sigma_{\alpha,j} - \rho_a n^\alpha b_i + \sum_{\beta \neq \alpha} h_{ij}^\beta = \rho_a n^\alpha a_{\alpha i}
\]

where \(\alpha, \gamma = S, W, G\), \(\sigma_{\alpha}^\alpha\) is the partial Cauchy stress, \(b_i\) is the body force per unit mass, \(h_{ij}^\beta (= -h_{ij}^\alpha)\) is the interaction force per unit volume exerted by phase \(\gamma\) on phase \(\alpha\), and \(a_{\alpha i} = d_{\alpha i} v_{\alpha i}/\partial t\) is the acceleration vector. Assuming that the relative acceleration \(\dot{\omega}_{\beta}\) is much smaller than the acceleration of the solid phase, the summation of the conservation of momentum equations of the three phases yields

\[
\sigma_{\alpha,j} + \rho b_i = \rho a_{\alpha i}
\]

where \(\rho\) is the density of the multiphase mixture. The assumption is valid for most of practical problems such as earthquake analysis of porous media, in particular for unsaturated soils due to their lower permeability compared to the saturated soils.

6 CONTINUITY EQUATIONS

From the incorporation of the momentum balance into the mass balance equations, the continuity equations for the fluid phases become

\[
\begin{align*}
\frac{1}{\rho_w} \left( k_w \left( p_{W,j} - \rho_w b_j + \rho_w a_{W,j} \right) \right)_j &= n_w c_w \frac{d}{dt} p_w + \varphi \dot{v}_r + n \frac{\partial S}{\partial \hat{s}} \\
\frac{1}{\rho_g} \left( k_g \left( p_{G,j} - \rho_g b_j + \rho_g a_{G,j} \right) \right)_j &= n_g c_g \frac{d}{dt} p_g + (1 - \varphi) \dot{v}_r - n \frac{\partial S}{\partial \hat{s}}
\end{align*}
\]

7 PLASTICITY MODEL

The elasto-plastic behaviour of the solid skeleton is captured through the bounding surface plasticity [29]. In this approach, plastic deformation occurs when the current stress point lies on or within the bounding surface. In the model, the total strain increment \(\delta \varepsilon\) can be decomposed into elastic \(\delta \varepsilon^e\) and plastic \(\delta \varepsilon^p\) parts as

\[
\delta \varepsilon = \delta \varepsilon^e + \delta \varepsilon^p
\]

The elastic response can be described using an incremental stress-strain relationship as

\[
\delta \sigma^e = D^e \delta \varepsilon^e
\]

where \(D^e\) is the elastic stiffness matrix. Due to highly nonlinear behaviour of soils, the total
stress-strain relationship of the soil skeleton is written in the incremental format as
\[ \delta \sigma' = D^{\sigma} \delta \varepsilon \] (16)
where \( D^{\sigma} \) is the constitutive tensor, and \( \delta \varepsilon \) is the soil skeleton strain tensor. The constitutive model adopted must be able to explain the deformation characteristics of the multiphase porous medium under cyclic loading conditions [8, 20-30].

7.1 Bounding surface

The bounding surface adopted in the present formulation is described by [24, 32]
\[ F(\bar{p}', \bar{q}, \bar{p}') = \left( \frac{\bar{q}}{M_{\text{cr}} \bar{p}'} \right)^N - \frac{\ln(\bar{p}' / \bar{p})}{\ln R} = 0 \] (17)
where the superimposed bar denotes stress conditions on the bounding surface, parameter \( \bar{p}' \) controls the size of the bounding surface and is a function of plastic volumetric strain \( \varepsilon_p' \) and suction \( s \), the material constant \( R \) represents the ratio between \( \bar{p}' \) and the value of \( \bar{p}' \) at the intercept of \( F \) with the CSL in the \( q - p' \) plane, the material constant \( N \) controls the curvature of the surface, and \( M_{\text{cr}} \) is the slope of the CSL in the \( q - p' \) plane.

7.2 Loading surface

The loading surface is assumed to be of the same shape and homologous to the bounding surfaces about the centre of homology. For first time loading, the centre of homology is located at the origin of stresses in \( q - p' \) plane. For cyclic loading, the centre of homology moves to the last point of stress reversal and the maximum loading surface through the point of stress reversal serves as a local bounding surface. To maintain similarity with the bounding surface, the loading surfaces undergo kinematic hardening during loading and unloading such that they are tangent to the maximum loading surface at the centre of homology. The image point for cyclic loading is located sequentially by projecting the stress point onto a series of intermediate image points on successive local bounding surfaces passing through each point of stress reversal [32]. The loading history of the soil is captured through the stress reversal points and the corresponding maximum loading surfaces. Within this context, the loading surface takes the form
\[ f(\hat{p}', \hat{q}, \hat{p}') = \left( \frac{\hat{q}}{M_{\text{cr}} \hat{p}'} \right)^N - \frac{\ln(\hat{p}' / \hat{p}')}{\ln R} = 0 \] (18)
where \( \hat{p}' = p' - \alpha_p \) is an isotropic hardening parameter controlling the size of the loading surface, \( \alpha = [\alpha_p, \alpha_q] \) is the kinematic hardening vector controlling the position of the loading surface, \( \hat{q} = q - \alpha_q \), and \( \hat{p}' = p' - \alpha_p \).

7.3 Plastic potential

The plastic potential defines the direction of the plastic strain increments. The plastic potential \( g \) is defined by integrating an extension of Rowe’s stress-dilatancy relationship with respect to \( p' \) and \( q \) as
\[ g(p', q, p_0) = \hat{t} \left( q + M_{\text{cr}} p' \ln \left( \frac{p'}{p_0} \right) \right) \text{ for } A = 1 \]
\[ g(p', q, p_0) = \hat{t} \left( q + \frac{AM_p p'}{A-1} \left( \frac{p'}{p_0} \right)^{A-1} - 1 \right) \text{ for } A \neq 1 \] (19-20)
where \( p_0 \) is the variable controlling the size of the plastic potential, \( \hat{t} \) is the loading direction multiplier with \( \hat{t} = +1 \) for compression and \( \hat{t} = -1 \) for extension, and \( A \) is a material constant dependent on the mechanism and amount of energy dissipation. The direction of plastic flow is defined as
\[ \mathbf{m} = \frac{\partial g}{\partial \sigma'} = \frac{\partial g}{\partial \sigma'} \] (21)

7.4 Hardening rule

Following the conventional approach in the bounding surface plasticity, the strain hardening modulus \( h \) can be divided into two components
\[ h = h_b + h_f \] (22)
where \( h_b \) is the strain modulus at stress point \( \bar{\sigma}' \) on the bounding surface and \( h_f \) is an arbitrary modulus defined as a function of the distance between \( \bar{\sigma}' \) and \( \sigma' \). \( h_b \) is determined by imposing the consistency condition at the bounding surface and incorporating the
hardening effects of plastic volumetric strain and matric suction leading to
\[ h_f = - \frac{\partial F}{\partial \bar{\sigma}_c} \left( \frac{\partial \bar{\sigma}_c^{ep}}{\partial \bar{\sigma}_p^{ep}} + \frac{\partial \bar{\sigma}_c^{ep}}{\partial \bar{v}_p} + \frac{\partial \bar{\sigma}_c^{ep}}{\partial \bar{\epsilon}_p} \right) \frac{m_p}{\partial F / \partial \bar{\sigma}} \] \hspace{1cm} (23)

The modulus \( h_f \) is defined such that it is zero on the bounding surface and infinity at the point of stress reversal. Following Khalili et al. [32], \( h_f \) is taken as
\[ h_f = i \left( \frac{\partial \bar{\sigma}_c^{ep}}{\partial \bar{\epsilon}_p} + \frac{\partial \bar{\sigma}_c^{ep}}{\partial \bar{v}_p} \right) \left[ \frac{\bar{p}_c}{\bar{p}_c^{ep}} - 1 \right] k_m (\eta_p - \eta) \] \hspace{1cm} (24)

where \( \eta_p = M_c (1 - 2(\nu - \nu_c)) \) is the slope of the peak strength line in the \( q - p' \) plane and \( k_m \) is a material parameter controlling the steepness of the response in the \( q - \epsilon_q \) plane.

7.5 Stress-strain relation
The elasto-plastic stress-strain relation for unsaturated soils is expressed as
\[ \partial \sigma' = D^{pp} \partial \epsilon \] \hspace{1cm} (25)

where
\[ D^{pp} = D' - D' m n' D' \frac{1}{h + n' D' m} \] \hspace{1cm} (26)
is the elasto-plastic stress-strain matrix of the soil.

8 NUMERICAL EXAMPLES
The finite element method is used for the spatial discretization of the equation of motion and the continuity equations, whereas the time integration is conducted using the Newmark technique. As the first numerical example, a boundary value problem consisting of an unsaturated porous medium of 10 m height and 10 m width with an initial suction of 20 kPa is considered. The mesh is composed of 400 mixed quadrilateral elements of dimensions 0.5m×0.5m, see Figure 2. The upper boundary is drained and partially subjected to a load of width \( B = 3.0 \) m with a uniform intensity \( f(t) \), while the remaining boundaries are impervious. Table 1 shows the material parameters adopted for the analysis. The parameters for the fully saturated state are the same as those used by Simon et al. [33]. For the case with hydraulic hysteresis effect, the air entry and air expulsion values of \( s_{ae} = 10 \) kPa, \( s_{ex} = 5 \) kPa, along with \( \xi = 0.04 \), and \( \zeta = 0.15 \) are used, whereas for the case without hysteresis effect \( s_{ae} = s_{ex} = 10 \) kPa are adopted. In the case with hydraulic hysteresis, wetting and drying occur along the scanning and main drying paths, respectively, whereas without hysteresis the wetting-drying cycles occur along the main drying path.

A harmonic load \( f(t) = \sigma_{\text{max}} \left[ 1 - \cos(\omega t) \right] \) with angular frequency \( \omega = 31.4 \) rad/sec and \( \sigma_{\text{max}} = 20 \) kPa is applied on top of the soil sample. A comparison of numerical results of the proposed hysteretic approach with those of the non-hysteretic approach is demonstrated in Figure 3.

Table 1. Material parameters considered for the 2D numerical analysis.

| Parameter                        | Value |
|----------------------------------|-------|
| Initial void ratio \( e_0 \)     | 0.5   |
| Lame’s constant \( kPa \)        | 833.3 |
| Lame’s constant \( kPa \)        | 1250.0|
| Permeability of liquid at \( S_r = 1 \) \( m/s \) | 0.01425|
| Permeability of gas at \( S_r = 0 \) \( m/s \) | 0.05   |
| Density of saturated mixture \( t/m^3 \) | 1.8   |
| Residual degree of saturation \( S_{res} \) | 0.2   |

Fig. 2. Finite element mesh, boundary conditions, and applied loads.

Fig. 3. Effect of hydraulic hysteresis on cyclic response of the unsaturated porous medium.
As seen in the figure, considering the hydraulic hysteresis induces markedly larger variations in suction level during dynamic loading.

The second example deals with a partially saturated embankment under cyclic loading. The embankment is founded on a 15 m layer of soil. The geometry of the embankment and the boundary conditions of the finite element model are shown in Figure 4. Material parameters of Kaolin clay shown in Table 2 are used for the analysis. A harmonic load $f(t)$ with frequency of 0.1 Hz and $\sigma_{\text{max}} = 200 \text{kPa}$ is applied to the top surface of the embankment. The initial suction value of $s = 200 \text{kPa}$ is assigned over the whole area. The initial void ratio is 0.8 and soil's permeability is set to $k = 4.8 \times 10^{-9} \text{m/s}$. Figure 5 illustrates the change in displacement versus time at point A located in the middle of the top surface of the embankment. It is seen that the maximum vertical deformation in the first cycle is greater than the second cycle. This is due to change in soil stiffness. In fact soil behaves stiffer as the number of cycles increase. Also the value of $k_0$ is different for the first time loading comparing to other unloading/reloadings which leads to lower $h_f$ values for the first loading.

![Figure 4: Finite element mesh and boundary conditions for road embankment.](image)

![Figure 5: Variations of displacement at point A versus time.](image)

### Table 2. Material properties of soft Kaolin clay

| Material parameters                  | Values |
|--------------------------------------|--------|
| Slope of recompression line ($K'$)  | 0.05   |
| Poisson’s ratio ($V$)                | 0.15   |
| $M_{cy}$                             | 1.05   |
| Slope of compression line ($\lambda$) | 0.14   |
| $\Gamma_0$                          | 2.676  |
| $N$                                  | 1.44   |
| $R$                                  | 2.9    |
| $k^*$                                | 2      |
| $A$                                  | 1.0    |

### 9 CONCLUSIONS

A fully coupled flow deformation model based on the multiphase mixture theory is presented for describing the nonlinear cyclic behaviour of unsaturated soils including hydraulic and mechanical hysteresis effects. The coupling between solid and fluid phases is established through the effective stress principle taking suction dependency and volume change of the effective stress parameters into account. The hydraulic hysteresis is accounted for through the effective stress parameter and the soil water characteristic curve. The volume change dependency of the effective stress parameters and the soil water characteristic curve is addressed in the formulation. The elastic-plastic behaviour due to cyclic loading is captured using the bounding surface plasticity. Numerical results are presented demonstrating that the hydraulic hysteresis markedly alter the response of unsaturated soils under cyclic loadings.

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