Cold atoms interacting with highly twisted laser beams mimic the forces involved in Millikan’s experiment

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Received 20 December 2018, revised 26 December 2018
Accepted for publication 26 December 2018
Published 8 February 2019

Abstract
This paper considers the analogy between the force exerted on cold atoms when they interact with a highly twisted tightly focused laser beam and the forces exerted on a charged dielectric particle inside a uniform electric field when we perform the Millikan’s experiment. In the later case the particle experiences its weight, a force due to the electric field which is arranged in the opposite direction of the weight, and a drag force due to the air proportional and opposite to the velocity of the particle. The force due to the electric field is ‘quantised’ since the charge of the particle is always an integer multiple of the electron charge. In the case of the cold atoms the total force is made up by three terms. The second term is quantised since it is proportional to the beam helicity which is an integer number. The sign of this term is opposite to the first. The third term is a damping force force proportional and opposite to the velocity of the atom. We present numerical calculations with parameters taken from experimental data which show an impressive analogy between the involved forces.

Keywords: cold atoms, twisted beams, optical vortices

(Some figures may appear in colour only in the online journal)
direction. Being in the excited state the atom could emit a photon back to the beam (stimulated emission) and return to its ground state while its gross motion gets a momentum ‘kick’ equal to \( h k \) in the direction opposite to the beam’s propagation direction. It is obvious that such a cycle of photon absorption–emission does not change the atomic momentum. But this is not the whole story as the atom has another way to emit a photon while being in its excited state. This is the spontaneous emission which results from the fluctuations of the electromagnetic vacuum. The spontaneously emitted photon has a random direction, so once it is emitted the atom recoils in a random direction while the photon does not ‘return’ to the beam. As the spontaneous emission has a random direction it does not change, on average, the atomic momentum, as the simulated emission does. In low intensities the spontaneous emission dominates over the stimulated emission so in practice the atomic momentum changes only by the effect of stimulated photon absorptions [1]. The size of the momentum ‘kick’ that is given to an atom by a single photon absorption or emission is very small compared to the atomic momentum so at first sight it seems strange that we can change the atomic momentum by a considerable amount. But the presence of laser ensures a large number of photon absorption and spontaneous emission cycles and the atom gets a net and considerable momentum along the beam propagation direction. It is like to try pushing or stopping a big box by repeatedly hitting it with pebbles.

On average this exchange results to a net transfer of momentum from the light field to the atom and gives rise to the so called radiation scattering (or dissipative) force. This force depends on the rate \( \Gamma \) with which spontaneous emission occurs and on the momentum carried by the photons. But the mechanical effects of light on atoms proved to be even more interesting after the discovery of Laguerre–Gaussian (LG) laser beams [4]. The name of this beams is after the Laguerre Gaussian (LG) mechanical effects of light on atoms proved to be even more prominent near the focus plane of the LG light mode. Both the Gouy and the curvature phase terms have so far been ignored in the analysis, with the Gouy phase strongly dependent on the values of \( l \) and \( p \).

2. The scattering force on a two-level atom

The prototype physical system under consideration is an atom which is irradiated by a coherent LG laser beam. In this case and due to the sharply defined frequency of the beam the light can induce electron transitions only between two atom energy levels, namely a ground state [1] and an excited state [2]. These two states have an energy separation \( E_{12} = h \omega_0 \), where \( \omega_0 \) is the so called atomic transition frequency. The excited atomic state [2] is characterized by a spontaneous emission rate \( \Gamma \).

The LG beam has a frequency \( \omega_z \) and is considered as propagating along the \( z \)-direction and being polarised along the \( x \)-direction. The electric field of such a beam is given by [6]:

\[
E_{\ell p}(\mathbf{R}) = \frac{1}{2}E_0 f(r,z) \exp \left[ i \Theta_{\ell p}(r,z,\phi) \right] \exp \left( -i \omega_z t \right) i + C.C.,
\]

where the amplitude \( E_{\ell p}(r,z) \) is given by

\[
E_{\ell p}(r,z) = E_0 f(r,z),
\]

with

\[
f(r,z) = \frac{C_{\ell p}}{\sqrt{1 + z^2/z_R^2}} \left( \frac{r \sqrt{2}}{w(z)} \right)^l \left( \frac{2r^2}{w^2(z)} \right) \exp \left( - \frac{r^2}{w^2(z)} \right),
\]

and the phase \( \Theta_{\ell p}(r,z,\phi) \) is given by the relation

\[
\Theta_{\ell p}(r,z,\phi) = k z + l \phi - (2p + 1) \tan^{-1}(z/z_R) + \frac{kr^2 z}{2 \left( z^2 + z_R^2 \right)},
\]

In the above relations we have that \( w(z) = w_0 \sqrt{1 + z^2/z_R^2} \), where \( w_0 \) is the beam waist and \( z_R = \pi w_0^2/\lambda \) the Rayleigh range of the beam. The quantity \( E_0 \) is the electric field amplitude which corresponds to a Gaussian beam of the same power and beam waist while \( L_{\ell p}^{(l)} \) is the associated Laguerre polynomial while \( C_{\ell p} = \sqrt{\frac{p!}{\pi^{\ell/2} \Gamma(p+\ell + 1)}} \). The third and the fourth terms in the RHS of equation (4) are the Gouy and the curvature phase respectively.

The physical basis of the scattering force is the continuous exchange of photons between the laser beam and the atom and their removal from the total system beam+atom to the vacuum due to spontaneous emission. The scattering force for an atom moving with velocity \( \mathbf{v} \) is given by [6]:

\[
\text{Laser Phys.} \ 29 \ (2019) \ 035503 \ V \ E \ Lembessis
\begin{equation}
\langle F_{\text{disc}}(v) \rangle = \hbar \nabla \Theta_{\text{f}p} \frac{\Omega^2/4}{\Delta^2(v) + (l^2/4) + (l^2/2)}. \tag{5}
\end{equation}

In the above relation \( \Delta(v) = \omega_L - \omega_0 - (\nabla \Theta \cdot v) \) is the detuning, i.e. the difference between the laser frequency and the atomic transition frequency. The quantity \( \nabla \Theta \cdot v \) is the Doppler shift of the laser beam frequency as observed by the moving atom [10]. The quantity \( \Omega \) is the so called Rabi frequency, i.e. the frequency with which the transitions induced by the laser namely the stimulated photon emission and absorption. The Rabi frequency is given by \( \Omega = \Omega_0 f(r,z) \) and is associated with the laser power and intensity since \( \Omega_0 = \sqrt{I_0 \Gamma/2l} \), where \( I_0 \) is the saturation intensity for the corresponding atomic transition [11]. The formula in equation (5) has the following meaning: the force results from scattering of photons away from the beam so it depends on the momentum of each photon \( \hbar \nabla \Theta \) (if the light field is a plane wave then the phase \( \Theta \) for a plane is just \( kz \), thus \( \nabla \Theta = k \)) and on the rate \( l \) at which this scattering occurs. The spontaneous emissions have a statistical character which is reflected at the Lorentzian term \( \frac{\Omega^2/4}{\Delta^2(v) + (l^2/4) + (l^2/2)} \). This term reaches its maximum value when the Doppler shift compensates the detuning, i.e. when \( (\nabla \Theta \cdot v) = \omega_L - \omega_0 \) and thus the interaction between the beam and the atom is resonant.

If we consider that the beam is an LG beam with large winding number the dissipative force for an atom near the beam focus, \( z = 0 \), is given by:

\begin{equation}
\langle F_{\text{disc}}(v) \rangle = \hbar \Gamma \frac{\Omega^2/4}{\Delta^2(v) + (l^2/4) + (l^2/2)} \left[ k \left( 1 - \frac{(2p + |l| + 1)}{kzR} \right)^2 + \frac{\ell}{l} \right]. \tag{6}
\end{equation}

As we see the force is made up from two components one along the axial direction and one along the azimuthal direction. The later as has been shown is responsible for a torque along the axial direction on the atom [12]. We are going to focus on the axial component.

We consider the case where the radial index \( p = 0 \) and we assume that our atom is located at a a radial distance equal to \( r = w_0 \sqrt{|l|/2} \), i.e. at the radial distance at which the beam intensity has its maximum when \( p = 0 \) and thus atom-beam interaction is stronger. We also assume a very large value of the index \( l \) such that \( |l| \gg 1 \). In this way we can show that the dissipative force assumes the form:

\begin{equation}
\langle F_{\text{disc}}(v) \rangle |_{z} = \hbar \Gamma k \frac{\Omega^2/4}{\Delta^2(v) + (l^2/4) + (l^2/2)} \left( 1 - \frac{|l|}{2kzR} \right) \hat{z}. \tag{7}
\end{equation}

Now as we know the atomic velocity is far smaller than the speed of light so a power series expansion around \( v = 0 \) of the force gives us the result:

\begin{equation}
\langle F_{\text{disc}}(v) \rangle |_{z} \approx \langle F_{\text{disc}}(0) \rangle |_{z} + av + ..., \tag{8}
\end{equation}

where \( \langle F_{\text{disc}}(0) \rangle \) is the force for zero atomic velocity and the coefficient \( a \) is given by:

\begin{equation}
a = \hbar k \left( 1 - \frac{|l|}{2kzR} \right)^2 s \frac{\Delta(0) \Gamma}{(1 + s)^2 \Delta^2(0) + (l^2/4)} \tag{9}
\end{equation}

with \( s \) being the so called saturation parameter given by \( s = (\Omega^2/2)/(\Delta^2(0) + l^2/4) \). The coefficient \( a \) is negative when the detuning \( \Delta(0) \) is negative. In this case the last term in equation (8) is a damping term. After all the above analysis we can write, for negative detuning, the total force as:

\begin{equation}
\langle F_{\text{disc}}(v) \rangle |_{z} = \hbar \Gamma \frac{\Omega^2/4}{\Delta^2(0) + (l^2/4) + (l^2/2)} \left( 1 - \frac{|l|}{2kzR} \right) \hat{z} - |a| \hat{v}. \tag{10}
\end{equation}

As we clearly see the total force expression is made up from three terms: the first term \( F_{\text{d}} = \hbar \Gamma k \Omega^2/4 \Delta^2(0) + (l^2/4) + (l^2/2) \) is the well-known expression for the axial scattering force, the second term \( F_{\text{l}} = \hbar \Gamma |l|/2zR \Delta^2(0) + (l^2/4) + (l^2/2) \) is opposite to the first and is ‘quantised’ since \( |l| = 1, 2, ..., \) The second term of the force has a considerable amount for large values of the beam helicity \( l \) and for tight focussing i.e. for small values of beam waist \( w_0 \) and Rayleigh range \( z_0 \). The third term \( F_{\text{D}} = -|a| |v| \) is a damping force. This term is responsible for the optical molasses effect, a viscous type of motion of a sample of atoms when they are irradiated by counter-propagating laser beams [13].

Before we proceed to the next paragraph we must clarify a crucial point in our calculations. The above argument for a force term proportional to \(|l|\) is true provided that the coefficient \( \frac{\Omega^2/4}{\Delta^2(0) + (l^2/4) + (l^2/2)} \) is a quantity which does not depend on \( l \). At first site this seems not to be true since the Rabi frequency depends on the parameter \( f(r,z) \) given in equation (3). This quantity depends clearly on \( l \) because of the existence of the term \( C_{pl} \) and the structure of the LG mode. However we must recall that the LG beam has a maximum intensity at a radial distance equal to \( r = w_0 \sqrt{|l|/2} \). At such radial distances the mechanical effects of such beams on atoms become important. At this radial distance, and for large values of \( l \), it has been shown that the Rabi frequency, for \( p = 0 \), gets the value \( \Omega \approx \Omega_0 C_0 |l|/\sqrt{2\pi |l|} \) [6], where \( \Omega_0 \) is the Rabi frequency associated with \( E_0 \) in equation (2). By replacing this quantity into the term \( \frac{\Omega^2/4}{\Delta^2(0) + (l^2/4) + (l^2/2)} \) this term keeps a constant value which does not depend on \( l \) for large values of \( l \) and for Rabi frequency \( \Omega_0 \) and detuning values normally used in atom cooling experiments.

3. Connection with the Millikan experiment

As we know in the famous Millikan experiment a small charged dielectric object, like an oil droplet or a latex sphere, experiences three forces: its weight \( W = mg \), the electric force from a uniform electric field \( E \) given by \( F_{\text{el}} = Eq \) which is quantised since the charge of the droplet is an integer multiple of the electron charge \( q = ne \) and finally a drag force due to air which is opposite to the velocity of the charged particle \( F_{\text{d}} = -K_{\text{D}} v \). We consider that the weight direction is along the
positive z-axis direction. The experimental set up is arranged in such a way that the electric force is in the opposite direction of the weight. In figure 1 we show the involved forces in the two cases which we study in this paper. Thus we can say that the resultant force on the particle is given by:

\[ \mathbf{F}_{\text{res}} = (mg - n|e|E - K_d V) \hat{z}. \]  

(11)

If we compare the expressions in equations (11) and (10) we see a clear analogy. The force \( F_\text{d} \) is the analogue of the weight \( W \), the force \( F_1 \) is the analogue of \( F_\text{d} \) while in both expressions we have damping forces opposite to the atom’s and particle’s velocities respectively. The similarity of the two cases is even more striking. We consider a typical undergraduate Millikan experiment with latex spheres of density \( \rho = 1.05 \text{ g cm}^{-3} \) and diameter \( D = 1.070 \text{ \mu m} \) \cite{14}. The spheres are considered to be inside a uniform electric field created by two parallel plates at a distance \( d = 0.6 \text{ mm} \) apart and kept at a potential difference \( V = 50 \text{ V} \). The charge of the sphere in the experiment varied from 1 to 10 times the charge of electron. In this case the weight of the sphere is the order of magnitude of the weight, the electrostatic force, if we consider a sphere of mass \( m \) and charge \( n|e| \) inside an electric field \( E \) is

\[ F_\text{ed} = n|e|E. \]

But this analogy is even more striking. Now let us consider the strengths of the damping forces. The coefficient \( K_d \) of the drag force on the latex spheres is given by

\[ K_d = 6\pi\eta r/(1 + b/(pr)) \]

where \( \eta \) is the viscosity of the air, \( r \) the radius of the charged latex sphere, \( b \) is an experimentally determined constant and \( p \) the atmospheric pressure. For the parameters we used in our numerical work the value of the coefficient \( K_d \) is \( K_d = 1.6 \times 10^{-10} \text{ kg s}^{-1} \). In this case the latex spheres acquire a terminal velocity equal to \( v_t = 0.3 \text{ mm} \text{s}^{-1} \). This gives us an estimation of the order of magnitude of the damping force which is \( F_d = 5 \times 10^{-14} \text{ N} \). Comparing this force to the weight of the latex sphere we have \( F_d/W = 0.95 \).

In the case of the damping force acting on the atom we can achieve such a value for the ratio \( F_D/F_\text{ed} \) for an atomic speed of \( v = 40 \text{ m} \text{s}^{-1} \) which is a speed that can be achieved by cooling the atomic motion from an initial thermal speed (of the order \( 10^3 \text{ m} \text{s}^{-1} \)). As we know the lower speed that can be achieved with scattering forces acting on a Cs atom is \( 0.088 \text{ m} \text{s}^{-1} \). All the above numerical examples show clearly that the analogy between these physically different effects is complete and justified.

4. Conclusions

We presented the analogy which exists between the axial scattering force exerted on a two-level atom when it is irradiated by a highly twisted LG beam, and the resultant of the forces exerted on a dielectric charged particle inside a uniform electric field. We would like to emphasize the term ‘analogy’ because in the case of the Millikan experiment we have three distinct and fundamentally different forces namely the weight, the electrostatic force and the buoyant force, while in the case of the irradiation of the atom by a twisted beam the ‘quantized’ force \( F_\text{I} \) and the damping force \( F_\text{D} \) forces were just correction terms of the scattering force: \( F_\text{I} \) becomes important when the beam helicity is
large and tightly focussed, while the damping force $F_D$ appears as the atomic speed is such that Doppler shifts the laser beam frequency. The most provoking similarity is the ‘quantization’ in the electric force and the scattering force term $F_s$. We must point out that the term $F_s$ emerges when we use highly twisted and strongly focussed LG beams. If we had used a typical Gaussian laser beam this term would not have appeared. On the contrary the quantization of the electric forces is a result of a fundamental property of the electric charge. We must also point out that the additional term in the force component along the $z$-direction which will result in the force $F_s$ appears because we have kept in our workings the Gouy phase and the curvature phase terms respectively. The Gouy phase has been shown to originate from the in-plane confinement of the focussed beam [15], or as a geometrical quantum effect, a result of the uncertainty principle when the beam is transmitted through a space with a modified volume [16]. This phase is significant when the winding numbers are high and also for tight focussing. Tight focussing is a geometrical quantum effect, a result of the uncertainty principle.

An interesting point is whether we can isolate the force $F_s$ which, as we have shown, appears as a correction term to the radiation pressure force. As it can be easily shown the force expression in equation (7), in the saturation limit, can take the form 

$$F_{\text{diss, sat}}(v) = \frac{h \gamma}{2} \left(1 - \frac{\| \mathbf{P} \|}{2 \gamma} \right) \mathbf{z}.$$  

As it easily seen this force is proportional to $1/r$. The action of this force is equivalent to a force exerted by a tangential electric field of magnitude 

$$F_{\text{ind}} = \frac{h \gamma}{2} \frac{\partial^2 \mathbf{P}}{\partial r^2} \frac{1}{r},$$  

on a particle of charge $q = l|e|$. Such a force is exerted by an induced electric field on a charged particle. Indeed if we consider the case of a magnetic field of constant direction but of a time varying magnitude in a circular region of radius $r_0$ this, at distances $r > r_0$, creates an induced electric field $E_{\text{ind}} = \frac{2 \partial \mathbf{P}}{\partial r} \frac{1}{r}$. This simple reasoning shows that the azimuthal motion of an atom may mimic the motion of an electric charge in an induced electric field.

Finally an interesting point is the role played by the other characteristic number of the LG beam, namely the radial index $p$, which in the above calculations was taken as equal to zero in order to simplify the calculations. This index is also an integer and could, thus, contribute to the ‘quantized’ force $F_s$ even in the case where we have a beam with $l = 0$ but a non zero $p$. So far researchers have not payed much attention to this index and thus there are a few works concerning its physical significance [17, 18]. As it has been pointed out this number can be considered as a conjugate quantity of the spatial confinement of the beam [17]. The fact that this number appears in the expressions for the radiation pressure forces on atoms is a nice chance for observing its physical effects in an experiment. The cylindrical symmetry of our modes and their spatial confinement are at the heart of the results taken in this paper. In conclusion our analysis has revealed a simulation of electromagnetic and damping forces with cold atoms interacting with highly twisted laser beams.

Acknowledgments

This project was supported by King Saud University, Deanship of Scientific Research, College of Sciences Research Center.

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