A Decision Making in Selection of Bricks Using Multiple Attribute Decision Making Methods

Satish Kumar Jain

Abstract — All building materials such as brick, cement, paint, lime, steel, glass, etc. of various brands with small variation in their specifications and cost are available in the markets of construction. It becomes very difficult for contractors, engineers, and owners to make right choice of materials logically to maintain good quality and minimum cost of the work. Improper choice may result in either bad quality or higher cost. Multiple Attribute Decision Making Methods are very helpful in selection of any material. These methods have been used largely in various fields of engineering for deciding best of available options. This paper presents an overview of Simple Additive Weighting Method (SAW), Weighted Product Method (WPM) and Analytical Hierarchy Process (AHP) methods which can be simply and successfully used for selection of bricks.

Key Words — Building materials, Decision Making Methods, Brick, Performance Index, SAW, WPM, AHP.

I. INTRODUCTION

A large variety of materials is present in the market of civil engineering, mechanical engineering, irrigation, medical etc. with different advantages and characteristics of each material. Selection of required beneficial material is difficult and challenging for a selector because attributes of available alternatives are found conflicting due to day-by-day emerging new technologies in field of manufacturing. It becomes multiple decision making problem when requires selection of material from more than two material with more than two attributes.

In civil engineering, construction field is very vast and important poor or rich, all require home, infrastructural development is necessary for the development of nation. Cement, steel, stone, brick, glass, wood, tiles and many more materials are required for all types of construction works and all these materials of many verities are available with small variation in their attributes. Selection of best material for use in construction or manufacturing is a problem because selection criteria seem very close or conflicting. It needs some easy, scientific, and logical approaches which ease the selection procedure of any material that will be optimal for superior and economical construction.

Numerous methods for making the best selection have been proposed in the past by many researchers. Jee and Kang [2] presented technique for order preference by similarity to ideal solution (TOPSIS) method for material selection. Karsak [3] demonstrated the distance based fuzzy multiple criteria decision making method for the selection of flexible manufacturing system. The fuzzy approach presented by Karsak and Kuzgunkaya [4] is difficult due to involvement of mathematical equations, fuzzy distribution, weights representation etc. [5]. Yardakul [6] used AHP as strategic decision making tool in selection of machine tools. Many studies report various approaches proposed by researchers for selection of ideal flexible manufacturing system [7]- [11]. A PROMETHEE approach has been used in many fields for selection of optimal objective [12]. Albayrakoglu [13] justified a new manufacturing technology by proposing a strategic approach using AHP. AHP has been also implemented in a tractor plant [14].

The literature related to MADM does not show any application of these methods for the selection of materials used in the field of civil construction. There is large scope for use of these approaches in all subjects of civil engineering e.g., building materials, irrigation, water resources, structural design etc. Hence an attempt has been made in this study to implement these methods in selection of bricks from available alternatives with multiple attributes. The assigned values of attributes are for demonstration purpose only.

II. MADM METHODS AND METHODOLOGY

Simple Additive Weighting (SAW), Weighted Product Method (WPM) and Analytical Hierarchy Process (AHP), these three methods have been used in this study for to understand the applicability of MADM methods in selecting best brick from available alternatives bricks with multiple attributes.

A. Simple Additive Weighting (SAW) Method

SAW method is very simple and widely used in decision making problem and it is also known as weighted sum method [15]. After normalizing the data of decision table, SAW method can be used for any number of attributes of any type.

Assessment of the weights for each attribute is carried out according to the method proposed by Edwards et al. [16]. 10 points are assigned to the attribute of least importance. Then more than 10 points are assigned to the next least important attribute and so on. Relative importance should reflect in point assignment. Final weights are calculated by normalizing the sum of total to one.

In this method, assessment of each alternative is made regarding all attributes and overall performance index (Pi) of an alternative is calculated by (1):

\[ Pi = \sum_{j=1}^{M} w_j (rij) \text{normal} \]  

(1)
where \((rij)_{\text{normal}}\) is the normalized value of \(rij\) and \(Pi\) is the overall performance index of the alternative \(Ai\). The \(Pi\) values of all alternatives are arranged in descending order and first choice is considered with highest value of \(Pi\). Second, third and fourth etc. are corresponding to descending values of \(Pi\).

**B. Weighted Product Method (WPM)**

The assessment of relative importance of attribute and calculation of weights is similar to the SW method as discussed above. In this method each normalized value of attributes is raised to power of the relative weight of corresponding attributes as in (2)

\[
Pi = \prod_{j=1}^{M}((rij)_{\text{normal}})^{wj}
\]  

The final overall performance index (\(Pi\)) of an alternative is calculated by multiplying the performance of each attribute of that alternative. The composite \(Pi\) values of all alternatives are arranged in descending order. The alternative with highest \(Pi\) value is reported as first choice when subjective measures are considered, even when subjective values, the objective valued or ranked between 0 and 1 as given in Table I [1]. The scale range may vary 1-10, 0-50, 0-100, 1-1000 etc. for obtaining performance selection index. The alternative having highest value of selection index is considered the top choice for the purpose.

**C. Analytical Hierarchy Process (AHP) Method**

AHP method is very suitable to handle with objective as well as subjective attributes, even when subjective attributes playing an important role in decision.

**D. Value of Attribute**

The attribute value \(Ri\) of alternatives may be found either from estimation or available data. The attribute values may be objective or subjective data. The subjective measures are valued or ranked between 0-1 as given in Table I [1]. At the same time, along with subjective values, the objective values may have different units; hence normalization of attribute values is desirable to same scale as of the subjective values.

**TABLE I: VALUE OF ATTRIBUTE**

| Subjective measure of attribute | Assigned value |
|---------------------------------|---------------|
| Exceptionally low               | 0.0           |
| Extremely low                   | 0.1           |
| Very low                        | 0.2           |
| Low                             | 0.3           |
| Below average                   | 0.4           |
| Average                         | 0.5           |
| Above average                   | 0.6           |
| High                            | 0.7           |
| Very high                       | 0.8           |
| Extremely high                  | 0.9           |
| Exceptionally high              | 1.0           |

The normalized value \(Ri\) can be determined by \(Rii/ Riu\), in the case of beneficial attribute i.e., the higher value of the attribute is desirable, while for non beneficial attribute, i.e., the lower value of attribute is desirable by \(Rii/ Rii\).

Where \(Rii\) is lower attribute value, \(Riu\) is highest attribute value and \(Rii\) is lower attribute value.

The relative importance is also assigned to an attribute \((rij)\) for given problem, on a scale between 0-1. If relative importance value is assigned for \(i\)th attribute as 0.4 and compared with \(j\)th attribute, then relative importance value of \(j\)th attribute will be 0.6 \((rij = 1 - rji)\).

Table II suggests the six point relative importance values to be assigned for the attributes [1]. The scale range may vary 1-10, 0-50, 0-100, 1-1000 etc. for obtaining performance selection index. The alternative having highest value of selection index is considered the top choice for the purpose.

**E. Relative Importance between Attributes in AHP**

Satty [17] proposed a method of assigning relative importance values between two attributes \(rij\) as

1. Procedure is that a pair-wise comparison matrix is constructed on the basis of a relative importance scale. Value 1 is always assigned to the attribute, which is compared to own, hence all diagonal values remain 1 in the pair-wise comparison matrix.

**TABLE II: RELATIVE IMPORTANCE OF ATTRIBUTES**

| Class description | Relative importance |
|-------------------|---------------------|
| \(rij\) \(rij = 1 - rji\) |                      |
| Two attributes are equally important | 0.5 0.5 |
| One attribute is slightly more important over the other | 0.6 0.4 |
| One attribute is strongly more important over the other | 0.7 0.3 |
| One attribute is very strongly more important over the other | 0.8 0.2 |
| One attribute is extremely important over the other | 0.9 0.1 |
| One attribute is exceptionally more important over the other | 1.0 0.0 |

Off-diagonal values in the pair-wise comparison matrix are assigned 3, 5, 7 or 9 on the basis of judgements such as moderately important, strongly important, very strongly important or absolutely important respectively and 2, 4, 6, and 8 for compromise between previous values. A pair-wise comparison of attribute \(i\) with \(j\), when total number of attributes are \(M\), then \(A1_{MM}\) matrix is formed, in which \(rij\) shows the comparative importance of attribute \(i\) over attribute \(j\). In matrix \(A1\), \(rij = 1\) and \(rji = 1/ rij\), when \(i = j\).

2. Now the consistency in the judgement is checked. The relative normalized weight \((wj)\) of each attribute is found by calculating first geometric mean of \(i\)th row and then normalizing the geometric means of rows in the matrix \(A1\) as expressed by (3) and (4)

\[
\text{GMj} = \left[\prod_{i=1}^{M} rij\right]^{1/M}
\]  

\[
wj = \frac{\text{GMj}}{\sum_{j=1}^{M} \text{GMj}}
\]

Matrix of all attributes such as \([w1, w2, w3, \ldots,]^{T}\) is known as matrix \(A2\). This method is easy for finding
relative normalized weights and maximum Eigen value and to minimize the judgement inconsistency. 

Matrices A3 and A4 are found as:

\[ A3 = A1 \times A2 \] and \[ A4 = A3/A2 \]

1. Find Eigen value \( \lambda_{\text{max}} \), which is the average of A4. 
2. Find Consistency Index CI as:

\[ CI = (\lambda_{\text{max}} - M)/(M - 1) \]

Step 2. The units of all three attributes are different hence the values are normalized to bring them on same scale between 0-1. Normalization is carried out for beneficial attributes by dividing all the by highest value and for non-beneficial attributes all values are divided by lowest value as discussed above in methodology. Normalized attribute values are shown in Table V.

A. Simple Additive Weighting (SAW) Method

Step 3. Calculation of weight for each attribute is carried out by assigning 10 points to least important attribute porosity (P), 20 points are assigned to next least important attribute cost (C) and 40 points are assigned to crushing strength (CS). Now these points are divided by sum of all these points to obtain the relative weight of each attribute as discussed in methodology part of this method above. Calculation of relative weights is shown in Table VI.

Step 4. Weights wp, wc, and wcs are now operated on normalized data of attributes for different alternatives of bricks as shown in Table VII as explained in methodology part to obtain the performance index of SAW method. The values. The values of performance index (Pi) are arranged in descending order and ranked I-VII as shown in Table VII.

TABLE III: RANDOM INDEX (RI) VALUES

| Attributes | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|------------|----|----|----|----|----|----|----|----|
| RI         | 0.52 | 0.89 | 1.11 | 1.62 | 1.35 | 1.35 | 1.45 | 1.49 |

III. EXAMPLE

Here an example is taken to implement MADM as Simple Additive Weighted (SAW), Weighted Product Method (WPM) and Analytical Hierarchy Process (AHP) methods to check their performance or applicability in selection of bricks. There are 7 alternatives of bricks and three attributes as shown in Table IV. All attributes are quantitative data.

TABLE IV: QUANTITATIVE DATA OF ATTRIBUTES OF THE EXAMPLE CONSIDERED

| Alternative Bricks | Crushing Strength kg/cm\(^2\) (CS) | Porosity % (P) | Cost per Brick Rs. (C) |
|--------------------|-----------------------------------|-----------------|------------------------|
| 1                  | 110                               | 8               | 4.00                   |
| 2                  | 109                               | 9               | 3.50                   |
| 3                  | 90                                | 12              | 2.50                   |
| 4                  | 112                               | 14              | 4.50                   |
| 5                  | 95                                | 13              | 3.00                   |
| 6                  | 100                               | 11              | 3.25                   |
| 7                  | 108                               | 9               | 4.00                   |

Applicability of these three methods is demonstrated in following steps.

Step 1. Three quantitative attributes namely crushing strength (CS), Porosity (P) and Cost (C) of all 7 alternatives are considered in the decision making problem. Crushing strength is beneficial attribute i.e., higher values are desired for good quality of work and porosity and cost are non-beneficial attributes i.e., their lower values are desired for good quality and economy of work, respectively.

TABLE V: NORMALIZED DATA

| Alternative Bricks | Crushing Strength kg/cm\(^2\) (CS) | Porosity % (P) | Cost per Brick Rs. (C) |
|--------------------|-----------------------------------|-----------------|------------------------|
| 1                  | 0.9821 (110/112)                  | 1.0000 (8/8)    | 0.6250 (2.5/3.5)       |
| 2                  | 0.9375 (105/112)                  | 0.8888 (80/80)  | 0.7142 (2.5/3.5)       |
| 3                  | 0.8305 (90/112)                   | 0.6666 (6/12)   | 1.0000 (2.5/2.5)       |
| 4                  | 1.0000 (112/112)                  | 0.5714 (8/14)   | 0.5555 (2.5/4.5)       |
| 5                  | 0.8482                            | 0.6153          | 0.8333                 |
| 6                  | 0.8928                            | 0.7272          | 0.7692                 |
| 7                  | 0.9642                            | 0.8888          | 0.6250                 |

TABLE VI: CALCULATION OF WEIGHTS

| Attribute in ascending order of importance | Assigned points |
|-------------------------------------------|-----------------|
| Porosity (P)                              | 10              |
| Cost (C)                                  | 20              |
| Crushing strength (CS)                    | 40              |
| Total                                     | 70              |

TABLE VII: DETERMINATION OF BRICK PERFORMANCE INDEX (Pi)

| Alternative Bricks | CS | P | C | Performance index (Pi) | Rank |
|--------------------|----|---|---|------------------------|------|
| 1                  | 0.5611 | 0.1428 | 0.1714 | 0.8853 | I |
| 2                  | 0.5356 | 0.1269 | 0.2040 | 0.8666 | II |
| 3                  | 0.4511 | 0.0951 | 0.2857 | 0.8400 | IV |
| 4                  | 0.5714 | 0.0815 | 0.1587 | 0.8116 | VI |
| 5                  | 0.4846 | 0.8786 | 0.2380 | 0.8105 | VII |
| 6                  | 0.5101 | 0.1038 | 0.2197 | 0.8337 | V |
| 7                  | 0.5504 | 0.1269 | 0.1785 | 0.8564 | III |

Ranks of all alternatives are arranged in descending order according to the descending values of Pi 1-2-7-3-6-4-5. This order indicates that the brick designated as one is the best choice while the brick designated as fifth brick is the last choice.
B. Weighted Product Method (WPM)

Steps 1-3 explained in SAW method are same in this method.

Step 4. Weights wp, wc, and wcs are now operated on normalized data of attributes for different alternatives of bricks shown in Table V, as discussed in methodology part of this method to obtain overall performance index in this method as shown in Table VIII.

TABLE VIII: DETERMINATION OF BRICK PERFORMANCE INDEX (PI)

| Alternative Bricks | CS   | P    | C    | Selection Index (SI) | Rank |
|--------------------|------|------|------|----------------------|------|
| 1                  | 0.9897 | 1.0000 | 0.8743 | 0.8653              | I    |
| 2                  | 0.9637 | 0.9833 | 0.9083 | 0.8608              | II   |
| 3                  | 0.8824 | 0.9437 | 1.0000 | 0.8328              | IV   |
| 4                  | 1.0000 | 0.9231 | 0.8453 | 0.7804              | III  |
| 5                  | 0.9102 | 0.9329 | 0.9492 | 0.8061              | VI   |
| 6                  | 0.9372 | 0.9555 | 0.9277 | 0.8308              | V    |
| 7                  | 0.9793 | 0.9833 | 0.8743 | 0.8420              | III  |

Above values of composite performance index of all alternatives are arranged in descending order. The highest value of Pi is ranked first, and the lowest value of Pi is ranked last as 1-2-7-3-6-5-4. These ranks indicate that brick designated as one is the first choice and the brick designated as 4 is the last or seventh choice.

C. Analytical Hierarchy Process (AHP)

Steps 1 and discussed above are same in this method.

Step 3. Pair-wise comparison matrix is formed by assigning the relative importance of attributes (r_{ij}) as explained in methodology part of this method. Crushing strength (CS) is considered more important than porosity (P) in bricks selection, hence relative importance value of 3.5 is assigned to CS over P (i.e., r_{12} = 3.5) and a relative importance value of 1/3.5 is assigned to P over CS (i.e., r_{21} = 1/3). Porosity P is considered less important than cost (C) hence relative important value of 1/2 is assigned to P over C (i.e., r_{23} = 1/2) and relative importance value of 2 is assigned to C over P (i.e., r_{32} = 2). Similarly, relative importance values are assigned among other attributes as shown in pair-wise comparison matrix. Actually, the assignment of relative importance very much depends on the experience and requirement of the expert. In this decision making problem, porosity has been considered less important to cost but some other expert may consider porosity is more important over cost, so results will be different. Now the relative importance weights for each attribute are calculated as explained in methodology part of this method.

Step 4. Now Matrix A2, A3, and A4 are found as

Eigen value \( \lambda_{max} \) (average of matrix A4) is found 3.0018. CI is calculated as 0.00093. Taking RI = 0.52 from Table III for three attributes. Now, CR = 0.0017 which is very less than the permissible CR value of 0.1. Thus, good consistency exists in assigning the relative importance values among the attributes.

Step 5. The relative normalized weights (wcs = 0.5569, wp = wp = 0.1514 and wc = 0.2916) in matrix A2 are operated by multiplying these weights to corresponding normalized attributes of all alternatives in Table II. Performance index (Pi) of all alternatives is calculated as shown in Table IX.

TABLE IX: DETERMINATION OF BRICK PERFORMANCE INDEX (PI)

| Alternative Bricks | CS   | P    | C    | Performance index (SI) | Rank |
|--------------------|------|------|------|------------------------|------|
| 1                  | 0.5469 | 0.1514 | 0.1822 | 0.8805                | I    |
| 2                  | 0.5220 | 0.1345 | 0.2082 | 0.8649                | II   |
| 3                  | 0.4474 | 0.1009 | 0.2916 | 0.8399                | IV   |
| 4                  | 0.5569 | 0.0865 | 0.1619 | 0.8053                | VII  |
| 5                  | 0.4723 | 0.0931 | 0.2429 | 0.8085                | VI   |
| 6                  | 0.4972 | 0.1100 | 0.2242 | 0.8315                | V    |
| 7                  | 0.5369 | 0.1345 | 0.1822 | 0.8537                | III  |

The values of Pi of all alternatives are arranged in descending order and rank I is assigned to highest value of Pi, while rank VII is assigned to lowest value of Pi as 1-2-7-3-6-5-4. Thus, the brick designated as 1 is the first choice and brick designated as 4 is the last or seventh choice.

IV. DISCUSSION

SAW, WPM and AHP methods demonstrated for selection of bricks are general methods and can be used for selection of any material of civil engineering. These methods can be used for any number of alternatives with any number of attributes of subjective or objective nature. Ranks obtained by these methods are very much depend on the assignment of relative important points to the attributes by the decision makers, hence, different users may obtain different ranks by the same methods. But these methods are logical, simple and easy in use for reasonable selection. A person having practical experience about the material which is to be selected can avail advantage of these methods by assigning relative importance value wisely.

V. CONCLUSIONS

In present period of fast changing technologies and soon implementation of these new technologies in field of manufacturing causes emergence of lot of varieties of same product or material used in field of construction with conflicting criteria of selection. The contractors, engineers or architect engaged in work of construction face problem in selection of required material from the available many alternatives with close variation in their specifications/attributes to achieve highest quality and economy in work. Literature shows that MADM methods have been used in selection of machine, tools or any other material used in mechanical engineering. But the use of these methods has not been found for the selection of any civil engineering or construction material. Hence SAW, WPM and AHP.
methods have been demonstrated successfully in this study for making selection from 7 alternatives of bricks. WPM and AHP methods give same ranks for all 7 bricks while SAW method gave 1-5 same ranks as given by AHP and WPM.

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