Non-Anomalous Discrete $R$-symmetry Decrees Three Generations

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We show that more than two generations of quarks and leptons are required by anomaly free discrete $R$-symmetries larger than $R$-parity, provided that the supersymmetric Standard Model can be minimally embedded into grand unified theories.

INTRODUCTION

Approximately 70 years ago I. I. Rabi famously quipped “who ordered that?” in regards to the discovery of the second electron, i.e. the muon. Since that time the origin of multiple generations of quarks and leptons has been a mystery. A partial answer to this question can be found in the leptogenesis mechanism \cite{Fukugita:1986hr}. In leptogenesis, at least two generations of right-handed neutrinos are required for $CP$-violation \cite{Barr:1989ja}, an essential ingredient in baryogenesis. This solution, however, does not explain the existence of the third generation \cite{Fukugita:1986hr}.

In this letter, we show that more than two generations of quarks and leptons are necessary for an anomaly free discrete $R$-symmetry \cite{Ibe:2009pu}, $\mathbb{Z}_{N_R}$, of order $N > 2$ \cite{Ibe:2009pu}. An $R$-symmetry is important when considering model building and phenomenology with supersymmetry: generic \cite{Ibe:2009pu} and metastable supersymmetry breaking \cite{Ibe:2009pu}, proton decay \cite{Ibe:2009pu}, and the $\mu$ problem (see, for instance, \cite{Ibe:2009pu, Ibe:2009pu} and references therein) can all be solved by, or require, an $R$-symmetry. Furthermore, without a discrete $R$-symmetry, a constant term in the superpotential is allowed and expected to be of order the Planck scale. A large constant term in the superpotential necessitates Planck scale supersymmetry breaking to cancel the large cosmological constant. Therefore, low scale supersymmetric extensions of the Standard Model have various difficulties which well motivate an $R$-symmetry. By considering a minimal embedding into a grand unified theory (GUT), we show that this discrete $R$-symmetry requires (at least) three generations to be anomaly free.

ANOMALY FREE DISCRETE $R$-SYMMETRY

Now, let us consider the anomaly free conditions of a discrete $R$-symmetry. (Notice that we are assuming that a discrete $R$-symmetry stems from a gauged $R$-symmetry since no global symmetries are expected in a quantum theory of gravity \cite{Ibe:2009pu}.) In the following, we consider a class of GUT models where each generation of the quark and lepton supermultiplets are unified into a $10$ and a $5^*$ representation of $SU(5)_{\text{GUT}}$. Furthermore, we also assume that there are no additional light degrees of freedom charged under the supersymmetric Standard Model (SSM) gauge groups beyond the ones in the SSM. In particular, we expect that the colored Higgs associated with the SSM Higgs doublets have masses of order the GUT scale.

In this class of models, the anomaly free conditions from $SU(3)_c$ and $SU(2)_L$ gauge symmetries with the discrete $R$-symmetry give \cite{Ibe:2009pu, Ibe:2009pu},

\begin{align}
6 + n_g(3r_{10} + r_5 - 4) &= 0, \quad (1) \\
4 + n_g(3r_{10} + r_5 - 4) + (r_u + r_d - 2) &= 0, \quad (2)
\end{align}

respectively, where these equations are modulo $N$. Here, $n_g$ denotes the number of generations, $r_{10}, r_5, r_u, r_d$ are the $R$-charges of the superfields $10, 5^*, H_u, H_d$ respectively. The presence of Yukawa interactions constrains the $R$-charges

\begin{align}
2r_{10} + r_u &= 2, \quad (3) \\
r_{10} + r_5 + r_d &= 2, \quad (4)
\end{align}

modulo $N$. By combining Eqs. (1)–(4), the anomaly free conditions reduce to

\begin{align}
6 - 4n_g &= 0 \pmod{N}, \quad (5) \\
r_u + r_d &= 4 \pmod{N}. \quad (6)
\end{align}

The condition in Eq. (5) remarkably relates the number of generations of quarks and leptons to the order of the discrete $R$-symmetry. Interestingly, this condition shows that no discrete $R$-symmetry with $N > 2$ is allowed for $n_g = 1, 2$. In fact, three generations or more are needed to allow $R$-symmetries with $N \geq 3$ (see also Table I). Consequently, we find that anomaly free discrete $R$-symmetries with $N > 2$ require more than two generations of quarks and leptons, provided that the SSM is minimally embedded into GUT representations.

Another interesting feature of the above conditions is that the $R$-charges of $H_u$ and $H_d$ in Eq. (6) forbid the
so-called $\mu$-term when $N \geq 3$. Therefore, the $\mu$-term is automatically small in this class of models. The same arguments also apply to the infamous dimension five proton decay operator, $10 \ 10 \ 10^5 \ [14, \ 15]$, since the $R$-charge of this operator is

$$3 r_{10} + r_5 = 4 - (r_u + r_d) = 0 \quad (7)$$

modulo $N$. Thus, the dimension five proton decay operator is also automatically suppressed by the discrete $R$-symmetry.

One may wonder whether other anomaly free conditions such as $Z_{NR} U(1)_{Y}$, $Z_{NR}^2 U(1)_{Y}$, $Z_{NR}^3$, $Z_{NR} (\text{gravity})^2$, could give further constraints. The $Z_{NR}$ and $Z_{NR} (\text{gravity})^2$ anomaly conditions are, however, not useful for constraining the SSM $R$-symmetries since they depend on the $R$-charges of assignments of heavy fields not charged under the GUT gauge group. The $Z_{NR} U(1)^2_r$ and $Z_{NR}^2 U(1)_{Y}$ conditions are model dependent and require more careful treatment. For example, in grand unified theories with semi-simple gauge groups, these anomaly free conditions should be examined. On the other hand, if the GUT group is not semi-simple, these two conditions do not place any useful constraints since they depend on the $R$-charges of fields not charged under $SU(5)_{\text{GUT}}$. The condition $Z_{NR}^2 U(1)_{Y}$ is irrelevant for both cases. For semi-simple gauge groups, the $Z_{NR}^2 U(1)_{Y}$ condition is satisfied because of the tracelessness of $U(1)_{Y}$. This condition is irrelevant in models with non semi-simple gauge groups for the same reasons that $Z_{NR} U(1)^2_r$ was irrelevant.

With these points in mind, let us examine the anomaly free condition, $Z_{NR} U(1)^2_r$, assuming that the GUT gauge group is semi-simple. In this case, the anomaly free condition as a function of $n_g$ is given by

$$2 (-10 n_g + 3) = 0 \ (\text{mod } N) \quad (8)$$

Substituting $n_g = 3$ for each $N = 6, 3, 2, 1$, we find that the anomaly free condition for $Z_{NR} U(1)^2_r$ is also satisfied. For $n_g = 4$, the allowed $R$ symmetries are $N = 74, 37, 2$, which are not consistent with the other anomaly conditions for $n_g = 4$, with the exception of the (non-$R$) $Z_2$ (See Table I). This suggests that discrete $R$-symmetries are consistent with GUT models based on semi-simple gauge groups with three generations, although this is not a necessary condition.

### Table I

| $n_g$ | $Z_{NR}$ |
|-------|-----------|
| 1     | $N = 2, 1$ |
| 2     | $N = 2, 1$ |
| 3     | $N = 6, 3, 2, 1$ |
| 4     | $N = 10, 5, 2, 1$ |

### Table II

The $R$-charge assignments of the model based on $SU(5)_{\text{GUT}} \times U(3)_H$. The Higgs doublets are embedded into (anti-)fundamental representation of $SU(5)_{\text{GUT}}$ (i.e. $H_u \subset H(5^*)$ and $H_d \subset \bar{H}(5)$). We assign $U(1)_H$ charge $1/\sqrt{6}$ to the fundamental representation of $SU(3)_H$. Here, we assumed that the $R$-charge assignment of the right-handed neutrino $r_1 = 1$ so that the assignment is consistent with the see-saw mechanism (see Eqs. (10) and (11)).

| $U(3)_H$ | $Y(5)$ | $H(5^*)$ | $Q(5)$ | $Q(5^*)$ | $X(1)$ | $\bar{X}(1)$ | $\Phi(1)$ |
|----------|--------|---------|-------|---------|--------|-------------|---------|
| $Z_{NR}$ | $-1$   | $3$     | $4$   | $0$     | $0$    | $-2$        | $2$     |

### R-Invariant Grand Unified Theory

So far, we have not discussed the mechanism which gives mass to the colored Higgs, i.e. the infamous doublet-triplet splitting problem. It is, however, known to be difficult to realize doublet-triplet splitting naturally in GUT models with a simple gauge group and only the SSM matter content below the GUT scale. In addition to this naturalness problem, it was recently shown in Ref. 16 that a low scale SSM with discrete $R$-symmetries having $N > 2$ are not consistent with GUT models based on a simple gauge group.

In GUT models with a strongly coupled gauge theory, on the other hand, the doublet-triplet splitting can be naturally realized 18–23. In fact, an $R$-invariant GUT model of this class has been constructed in Ref. 20 based on a $SU(5)_{\text{GUT}} \times U(3)_H$ gauge symmetry (see also Ref. 13). In Table II we show the $Z_{NR}$-charge assignments of this GUT model which satisfies all the anomaly free conditions of the previous section. We also find that the $R$-charge assignment in the table satisfy the anomaly free conditions, $Z_{NR} SU(5)_{\text{GUT}}, Z_{NR} SU(3)_H$ and $Z_{NR}^2 U(1)^2_r$.

In the Higgs phase of this model, the GUT gauge group is broken by the vacuum expectation values of $Q$ and $\bar{Q}$ which leave $SU(3)_c$ and $U(1)_Y$ as the unbroken diagonal subgroups of $SU(5)_{\text{GUT}} \times U(3)_H$. The mass partner of the colored Higgs triplets are provided by the ef-

\[1\] The unification of gauge coupling constants is not disturbed by the addition of two chiral multiplets, one transforming as an octet of $SU(3)_c$ and the other as a triplet of $SU(2)_L$, with a mass below the GUT scale. Thus, the anomaly free conditions for the discrete $R$-symmetries can be altered without conflicting with unification. These light states allows us to avoid this no-go theorem for GUT models based on a simple gauge group. The detailed analysis of models with such incomplete GUT multiplets below the GUT scale which are consistent with gauge coupling unification will be given elsewhere.

\[5\] The anomaly free condition on $Z_{NR} U(1)^2_r$ need not be satisfied as explained in the previous section even if we require the exact SSM below the GUT scale.

\[6\] In the Higgs phase, $U(1)_H$ is necessary since the GUT gauge group is broken by the vacuum expectation values of $Q$ and $\bar{Q}$.
fective colored triplets $XQ$ and $X\bar{Q}$ with $R$-charge $-2$ and $2 \pmod{6}$, respectively, while no mass partners of the doublet Higgs are generated. As a result, this GUT model leads to the SSM with the $R$-charge assignments discussed in the previous section.

Finally, we comment on the gauge couplings. In the Higgs phase, the low energy coupling $\alpha_3$, is given by $1/\alpha_3d = 1/\alpha_5 + 1/\alpha_{3H}$ with a similar relationship between hypercharge, $\alpha_Y$, and $\alpha_{1H}$ [24]. Here $\alpha_5$, $\alpha_{3H}$, and $\alpha_{1H}$ are the gauge coupling constants of $SU(5)_{GUT}$, $SU(3)_{H}$, and $U(1)_{H}$ respectively. For a strongly coupled $SU(3)_{H}$ and $U(1)_{H}$, we see that at the GUT scale the approximate GUT relation, $\alpha_3 \simeq \alpha_2 \simeq \alpha_1$, still holds.

DISCUSSION

Any discrete $R$-symmetry with $N > 2$ should be spontaneously broken down to $R$-parity at some scale much lower than the GUT or Planck scale. Such spontaneous breaking of exact discrete symmetries could cause a cosmological domain wall problem [23]. One option for avoiding this domain wall problem is to assume that the spontaneous breaking of the discrete $R$-symmetry occurs well before inflation. This leads to constraints on the Hubble constant during inflation and the reheating temperature relative to the $R$-symmetry breaking scale. Another interesting possibility is a model where the vacuum expectation value of the inflaton breaks the discrete $R$-symmetry [26]. In this class of models, the flatness of the inflaton potential near the origin is naturally explained by the $R$-symmetry [27], while the domain wall problem is avoided because the radius of the coherent domain of the inflaton field is inflated to $e^{N_c}$ ($N_c \gtrsim 60$) times larger than the Hubble radius at the end of inflation.

In Ref. [29] (see also Ref. [31]), anomaly cancellation of discrete $R$-symmetries was also studied, but with the addition of the Green-Schwarz mechanism. Our approach is to take the minimal setup and constraints, and as we stated above, we do not include gravitational effects for the anomalies.

We also comment on the $R$-charge of the right-handed neutrinos, $r_1$, which are essential for the see-saw mechanism [28]. By including right-handed neutrinos, we obtain two additional conditions on the $R$-charges:

$$r_5 + r_4 + r_u = 2,$$
$$2r_1 = 2,$$

modulo $N$. The first condition is due to the Yukawa interactions of the right handed neutrinos, and the second condition is due to the Majorana masses of the right-handed neutrinos. By combining the conditions in Eqs. (1)–(4), (9) and (10), we find the $R$-charges in Table III. Thus, we find that the $Z_{6R}$ symmetry is consistent with the see-saw mechanism. However, we find that these charge assignments are not consistent with $SO(10)$ unification since $r_5 = r_4 = r_1$ is not satisfied, which was pointed out in Ref. [24].

We showed in the previous section that proton decay and the $\mu$ problem are also addressed by this discrete $R$ symmetry; the phenomenological aspects of a $Z_{6R}$ symmetry are rich. We now point out that this analysis is complicated by the addition of complete multiplets of $SU(5)$, even if the $(1)$ anomaly conditions are used. These additional multiplets affect the relationship between the number of generations and the size of the discrete $R$-symmetry. However, this analysis is model dependent and will be considered in [31].

To conclude, we have shown that a discrete $R$-symmetry (larger than $Z_2$) requires at least three generations of quarks and leptons to be anomaly free, assuming a minimal embedding in a GUT. This non-anomalous discrete $R$-symmetry is rich phenomenologically, and a useful ingredient in model building.

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