Search for $T$ Violation in Charm Meson Decays

The FOCUS Collaboration

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Abstract

Using data from the FOCUS (E831) experiment, we have searched for $T$ violation in charm meson decays using the four-body decay channels $D^0 \to K^-K^+\pi^-\pi^+$, $D^+ \to K_S^0K^+\pi^-\pi^+$, and $D_s^+ \to K_S^0K^+\pi^-\pi^+$. The $T$ violation asymmetry is obtained using triple-product correlations and assuming the validity of the CPT theorem. We find the asymmetry values to be $A_{T\text{viol}}(D^0) = 0.010 \pm 0.057\text{(stat.)} \pm 0.037\text{(syst.)}$, $A_{T\text{viol}}(D^+) = 0.023 \pm 0.062\text{(stat.)} \pm 0.022\text{(syst.)}$, and $A_{T\text{viol}}(D_s^+) = -0.036 \pm 0.067\text{(stat.)} \pm 0.023\text{(syst.)}$. Each measurement is consistent with no $T$ violation. New measurements of the $CP$ asymmetries for some of these decay modes are also presented.

1. Introduction

The origin of $CP$ violation remains one of the most important open questions in particle physics. Within the Standard Model, $CP$ violation arises due to the presence of a phase in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix. Although the main focus has been on rate asymmetries, there is another type of $CP$ violating signal which could potentially reveal the presence of physics beyond the Standard Model. Triple-product correlations of the form

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1 See http://www-focus.fnal.gov/authors.html for additional author information.
$\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3)$, where each $\vec{v}_i$ is a spin or momentum, are odd under time reversal ($T$). By the CPT theorem, a nonzero value for these correlations would also be a signal of $CP$ violation. A nonzero triple-product correlation is evidenced by a nonzero value of the asymmetry [1]

$$A_T \equiv \frac{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) - \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)}{\Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) > 0) + \Gamma(\vec{v}_1 \cdot (\vec{v}_2 \times \vec{v}_3) < 0)} \quad (1)$$

where $\Gamma$ is the decay rate for the process. There is a well-known technical complication: strong phases can produce a nonzero value of $A_T$, even if the weak phases are zero, that is $CP$ and $T$ violation are not necessarily present. Thus, strictly speaking, the asymmetry $A_T$ is not in fact a $T$-violating effect. Nevertheless, one can still obtain a true $T$-violating signal by measuring a nonzero value of

$$A_{T\text{viol}} \equiv \frac{1}{2} (A_T - \overline{A_T}) \quad (2)$$

where $\overline{A_T}$ is the $T$-odd asymmetry measured in the $CP$-conjugate decay process [2].

This study was inspired by a paper of Ikaros Bigi [3]. In this paper Bigi suggested a search for $T$ violation by looking at the triple-product correlation (using the momenta of the final state particles) in the decay mode $D^0 \rightarrow K^-K^+\pi^-\pi^+$. Such a correlation must necessarily involve at least four final-state particles. This can be understood by considering the rest frame of the decaying particle and invoking momentum conservation. The number of independent three-momenta is one less than the number of final-state particles, so a triple product composed entirely of momenta requires four particles in the final state [4].

We calculate $A_{T\text{viol}}$ for the decay modes $D^0 \rightarrow K^-K^+\pi^-\pi^+$ and $D^+_{(s)} \rightarrow K^0_S K^+\pi^-\pi^+$ using data from the FOCUS experiment.

FOCUS is a charm photoproduction experiment [5] which collected data during the 1996–97 fixed target run at Fermilab. Electron and positron beams (with typically 300 GeV endpoint energy) obtained from the 800 GeV Tevatron proton beam produce, by means of bremsstrahlung, a photon beam which interacts with a segmented BeO target. The mean photon energy for triggered events is $\sim 180$ GeV. A system of three multicell threshold Čerenkov counters performs the charged particle identification, separating kaons from pions up to 60 GeV/c of momentum. Two systems of silicon microvertex detectors are used to track particles: the first system consists of 4 planes of microstrips interleaved with the experimental target [6] and the second system consists
of 12 planes of microstrips located downstream of the target. These detectors provide high resolution in the transverse plane (approximately 9 $\mu$m), allowing the identification and separation of the primary (production) and the charm secondary (decay) vertices. Charged particle momentum is determined by measuring deflections in two magnets of opposite polarity through five stations of multiwire proportional chambers.

2. Search for $T$ violation in the decay mode $D^0 \to K^- K^+ \pi^- \pi^+$

The decay mode $D^0 \to K^- K^+ \pi^- \pi^+$ is Cabibbo-suppressed and may be produced as a non-resonant final state or via two-body and three-body intermediate resonant states. In a previous paper we determined its resonant substructure and the branching ratio $\Gamma(D^0 \to K^- K^+ \pi^- \pi^+)/\Gamma(D^0 \to K^- \pi^- \pi^+ \pi^+)$ [7].

The final states are selected using a candidate driven vertex algorithm [5]. A secondary vertex is formed from the four candidate tracks. The momentum vector of the resultant $D^0$ candidate is used as a seed track to intersect the other reconstructed tracks and to search for a primary vertex. The confidence levels of both vertices are required to be greater than 1%. Once the production and decay vertices are determined, the distance $L$ between the vertices and its error $\sigma_L$ are computed. The quantity $L/\sigma_L$ is an unbiased measure of the significance of detachment between the primary and secondary vertices. This is the most important variable for separating charm events from non-charm prompt backgrounds. Signal quality is further enhanced by cutting on $Iso2$, which is the confidence level that other tracks in the event might be associated with the secondary vertex. We use $L/\sigma_L > 6$ and $Iso2 < 10\%$. We also require the $D^0$ momentum to be in the range 25–250 GeV/$c$ (a very loose cut) and the primary vertex to be formed with at least two reconstructed tracks in addition to the $D^0$ seed.

The Čerenkov identification cuts used in FOCUS are based on likelihood ratios between the various particle identification hypotheses. These likelihoods are computed for a given track from the observed firing response (on or off) of all the cells that are within the track’s ($\beta = 1$) Čerenkov cone for each of our three Čerenkov counters. The product of all firing probabilities for all the cells within the three Čerenkov cones produces a $\chi^2$-like variable $W_i = -2 \ln(\text{Likelihood})$ where $i$ ranges over the electron, pion, kaon and proton hypotheses [8]. All kaon tracks are required to have $\Delta_K = W_\pi - W_K$ (kaonicity) greater than 3; whereas all the pion tracks are required to be separated by less than 5 units from the best hypothesis, that is $picon = W_{\text{min}} - W_\pi$ (pion consistency) is greater than $-5$.

In addition to these cuts (also used in our previous analysis of this decay mode), we require a $D^*-\text{tag}$. The sign of the bachelor pion in the $D^{*-}$ decay
chain $D^{*+(-)} \to D^0(D^0)\pi^{+(-)}$ is used to identify the neutral $D$ as either a $D^0$ or a $D^0$. We require that the mass difference between the $D^0$ and the $D^*$ mass be within 4 MeV/$c^2$ of the nominal mass difference [9].

Using the set of selection cuts just described, we obtain the invariant mass distributions for $K^-K^+\pi^-\pi^+$ shown in Fig. 1, where the first plot is the total sample and the other two plots show the $D^0$ and $\bar{D}^0$ samples separately.

The mass plots are fit with a function that includes two Gaussians with the same mean but different sigmas to take into account different momentum resolutions in our spectrometer [5] and a second-order polynomial for the combinatorial background. A log-likelihood fit gives a signal of $828 \pm 46 K^-K^+\pi^-\pi^+$ events for the total sample, $362 \pm 31 D^0$ events, and $472 \pm 34 \bar{D}^0$ events. The fitted $D^0$ masses are in good agreement with the world average [9] and the widths are in good agreement with those of our Monte Carlo simulation.

From the $D^0$ sample we can form a $T$-odd correlation with the momenta:

$$C_T \equiv \vec{p}_{K^-} \cdot (\vec{p}_{\pi^+} \times \vec{p}_{\pi^-})$$

and from the $\bar{D}^0$ sample we form:

$$\bar{C}_T \equiv \vec{p}_{K^+} \cdot (\vec{p}_{\pi^-} \times \vec{p}_{\pi^+}).$$

As we have seen in the introduction, finding a distribution of $C_T$ different from $-\bar{C}_T$ establishes $CP$ violation [3].

Fig. 2 shows $D^0(\bar{D}^0)$ signals separated by the sign of $C_T(\bar{C}_T)$. A log-likelihood fit, with the same fit function described previously, gives the yields summarized in Table 1.

| Decay mode | Request | Events       |
|------------|---------|--------------|
| $D^0 \to K^-K^+\pi^-\pi^+$ | $C_T > 0$ | $174 \pm 21$ |
| $D^0 \to K^-K^+\pi^-\pi^+$ | $C_T < 0$  | $190 \pm 24$ |
| $\bar{D}^0 \to K^-K^+\pi^-\pi^+$ | $\bar{C}_T > 0$ | $255 \pm 24$ |
| $\bar{D}^0 \to K^-K^+\pi^-\pi^+$ | $\bar{C}_T < 0$ | $220 \pm 25$ |

Before forming the asymmetry $A_T(A_T)$ we have to correct for detection efficiencies, accounting for possible differences in spectrometer acceptance and Čerenkov identification efficiency for positive/negative kaons and pions.  

2 It is well-known that in fixed-target experiments there are production asymme-
is, however, a small effect. From the efficiency corrected yields we compute the asymmetry:

\[ A_T = \frac{\Gamma(C_T > 0) - \Gamma(C_T < 0)}{\Gamma(C_T > 0) + \Gamma(C_T < 0)} \]  

(5)

and

\[ \bar{A}_T = \frac{\Gamma(-C_T > 0) - \Gamma(-C_T < 0)}{\Gamma(-C_T > 0) + \Gamma(-C_T < 0)}. \]  

(6)

The resulting \( T \)-violation asymmetry \( A_{T\text{viol}} \) is:

\[ A_{T\text{viol}} = \frac{1}{2}(A_T - \bar{A}_T) = 0.010 \pm 0.057. \]  

(7)

Without the efficiency correction it would have been \( A_{T\text{viol}} = 0.014 \pm 0.057 \).

This determination has been tested by modifying each of the vertex and Čerenkov cuts individually. Although the statistics is limited, the \( T \)-violation asymmetry is stable versus several sets of cuts as shown in Fig. 3. All the measurements are consistent with 0 for the \( T \)-violation asymmetry.

3. Search for \( T \) violation in the decay mode \( D \to K_S^0 K^+ \pi^- \pi^+ \)

The decay channel \( D^+ \to K_S^0 K^+ \pi^- \pi^+ \) is Cabibbo-suppressed and like \( D^0 \to K^- K^+ \pi^- \pi^+ \), it may be produced as a non-resonant final state or via two-body and three-body intermediate resonant states. Its relative branching ratio \( \Gamma(D^+ \to K_S^0 K^+ \pi^- \pi^+)/\Gamma(D^+ \to K_S^0 \pi^+ \pi^- \pi^+) \) has been measured \([10]\). \( D^+ \to K_S^0 K^+ \pi^- \pi^+ \) is observed in the same histogram as \( D^+ \to K_S^0 \pi^+ \pi^- \pi^+ \) and we fit for both signals.

The final states are selected using a candidate driven vertex algorithm as described in the previous section. The \( K_S^0 \) is reconstructed using techniques described elsewhere \([11]\). The \( K_S^0 \) and the charged tracks are used to form a \( D \) candidate which is used as a seed track to intersect the other reconstructed tracks and to search for a primary vertex. The confidence levels of both vertices must be greater than 1%. We also use \( L/\sigma_L > 6 \) and \( \text{Iso2} < 1\% \) and require the primary vertex to be composed of at least two reconstructed tracks in addition to the \( D \) seed.

tries between charm and anticharm particles. As a result the \( D^0 \) momentum distribution is different from the \( \bar{D}^0 \) distribution.
Using these selection cuts, we obtain the invariant mass distributions for $K_S^0 K^+ \pi^- \pi^+$ shown in Fig. 4, where the top plot is the total sample and the bottom two plots show the $D$ and $\bar{D}$ samples separately.

The mass plots are fit with a function that includes a Gaussian for the $D^+$ and a Gaussian for the $D_s^+$ with the widths fixed to those given from our Monte Carlo simulations. We use a second-order polynomial for the combinatorial background in addition to two reflection peaks from $\Lambda_c^+ \rightarrow p K_S^0 \pi^- \pi^+$ and $D^+ \rightarrow K_S^0 \pi^+ \pi^- \pi^+$. The $\Lambda_c^+$ yield is fixed after first fitting the sample with the $K^+$ mass changed to the proton mass. The $D^+ \rightarrow K_S^0 \pi^+ \pi^- \pi^+$ yield is determined by using the Monte Carlo misidentification rate of a pion as a kaon and the yield of $D^+ \rightarrow K_S^0 K^+ \pi^- \pi^+$. A log-likelihood fit gives a signal of $523 \pm 32$ events for the $D^+$ and a signal of $508 \pm 34$ events for $D_s^+$. The $K_S^0 \pi^+ \pi^- \pi^+$ sample has $240 \pm 22$ $D^+$ and $270 \pm 25$ $D_s^+$ events, while the $K_S^0 K^- \pi^- \pi^+$ sample has $282 \pm 23$ $D^-$ and $239 \pm 24$ $D_s^-$ events. The fitted $D$ masses are in good agreement with the world average [9]. Also the excess of $D^-$ over $D^+$ events is consistent with more $D^0$ mesons than $D_0$ mesons being produced. These photoproduced excesses have been observed in previous higher statistics studies by FOCUS [12,13].

The mass plots shown in Fig. 5 are the $D^+(s)$ ($D^-(s)$) signals split by the sign of $C_T(\overline{C_T})$. A log-likelihood fit, with the same fit function described previously, gives the yields summarized in Table 2.

| Final State  | Request | $D^+$ Events | $D_s^+$ Events |
|-------------|---------|--------------|---------------|
| $K_S^0 K^+ \pi^- \pi^+$ | $C_T > 0$ | $122 \pm 16$ | $126 \pm 17$ |
| $K_S^0 K^+ \pi^- \pi^+$ | $C_T < 0$ | $118 \pm 16$ | $147 \pm 18$ |
| $K_S^0 K^- \pi^- \pi^+$ | $\overline{C_T} > 0$ | $145 \pm 16$ | $120 \pm 17$ |
| $K_S^0 K^- \pi^- \pi^+$ | $\overline{C_T} < 0$ | $137 \pm 16$ | $119 \pm 16$ |

After correcting for detection and reconstruction efficiencies as given by the Monte Carlo simulation, we form the asymmetry $A_T(\overline{A_T})$ as given by equation (5) and equation (6):

$$A_{T\text{viol}}(D^+) = \frac{1}{2}(A_T - \overline{A_T}) = 0.023 \pm 0.062$$ \hspace{1cm} (8)

$$A_{T\text{viol}}(D^+) = \frac{1}{2}(A_T - \overline{A_T}) = -0.036 \pm 0.067.$$ \hspace{1cm} (9)

Without the efficiency corrections the numbers are essentially the same. A
scan of $A_{T\text{viol}}$ under a variety of different selection criteria is presented in Fig. 6.

4. Systematic Uncertainties

Systematic uncertainties on the $T$-violation asymmetry measurement can come from different sources. We determine five independent contributions to the systematic uncertainty: the *split sample* component, the *fit variant* component, the component due to the particular choice of the vertex and Čerenkov cuts (discussed previously), the dilution due to an erroneous $D^*$ tag for the $D^0 \to K^- K^+ \pi^- \pi^+$ channel, and a component due to the limited statistics of the Monte Carlo.

The *split sample* component addresses the systematics introduced by a residual difference between data and Monte Carlo, due to a possible mismatch in the reproduction of the $D$ momentum and the changing experimental conditions during data collection. This component has been determined by splitting data into four independent subsamples, according to the $D$ momentum range (high and low momentum) and the configuration of the vertex detector, that is, before and after the insertion of an upstream silicon system. A technique, employed in FOCUS and in the predecessor experiment E687, modeled after the *S-factor method* from the Particle Data Group [9], is used to try to separate true systematic variations from statistical fluctuations. The $T$-violation asymmetry is evaluated for each of the $4 (= 2^2)$ statistically independent subsamples and a *scaled variance* $\tilde{\sigma}$ (that is the errors are boosted when $\chi^2/(N-1) > 1$) is calculated. The *split sample* variance $\sigma_{\text{split}}$ is defined as the difference between the reported statistical variance and the scaled variance, if the scaled variance exceeds the statistical variance [14].

Another possible source of systematic uncertainty is the *fit variant*. This component is computed by varying, in a reasonable manner, the fitting conditions on the whole data set. In our study of the $D^0$ mode, we fixed the widths of the Gaussians to the values obtained by the Monte Carlo simulation, we changed the background parametrization (varying the degree of the polynomial), we modified the fit function in order to take into account the reflection peak from $D^0 \to K^- \pi^+ \pi^- \pi^+$ [7], and we use one Gaussian instead of two. For all modes, the variation of the computed efficiencies due to the different resonant substructure simulated in the Monte Carlo has been taken into account. The $T$-violation values obtained by these variants are all *a priori* equally likely, therefore this uncertainty can be estimated by the *r.m.s.* of the measurements [14].

Analogously to the *fit variant*, the cut component is estimated using the standard deviation of the several sets of cuts shown in Fig. 3 and Fig. 6. Actually,
this is an overestimate of the cut component because the statistics of the cut samples are different.

An erroneous $D^*$ tag can obviously dilute the measured asymmetry $A_{T\text{viol}}$. We find a dilution $^3$ of $0.9846 \pm 0.0029$ for the $D^0$ sample and $0.9882 \pm 0.0025$ for the $\bar{D}^0$ events (the dilutions are slightly different because, as we have already seen, the $D^0$ and $\bar{D}^0$ momentum distributions are different). Then we computed the $A_{T\text{viol}}$ asymmetry taking into account this dilution and estimated the uncertainty by using the difference between this determination and the standard one.

Finally, there is a further contribution due to the limited statistics of the Monte Carlo simulation used to determine the efficiencies. Adding in quadrature all of these components, we obtain the final systematic errors which are summarized in Table 3.

Table 3
Contribution to the systematic uncertainties of the $T$-violation parameters for $D^0$, $D^+$, and $D_s^+$.

| Source                | $D^0$ Uncertainty | $D^+$ Uncertainty | $D_s^+$ Uncertainty |
|-----------------------|-------------------|-------------------|---------------------|
| Split sample          | 0.000             | 0.000             | 0.000               |
| Fit Variant           | 0.009             | 0.006             | 0.004               |
| Set of cuts           | 0.035             | 0.021             | 0.022               |
| $D^*$-tag dilution    | 0.002             | —                 | —                   |
| MC statistics         | 0.009             | 0.004             | 0.006               |
| Total systematic error| 0.037             | 0.022             | 0.023               |

5. Conclusions

Using data from the FOCUS (E831) experiment at Fermilab we have searched for $T$ violation in charm meson decays. It is a clean and alternative way to search for $CP$ violation. This is the first time such a measurement has been performed in the charm sector.

We determine the final values for the $T$-violation asymmetries to be

$$A_{T\text{viol}}(D^0) = 0.010 \pm 0.057(\text{stat.}) \pm 0.037(\text{syst.}),$$

$^3$ The dilution, measured by means of our Monte Carlo simulation, is defined as $D = \frac{R-W}{R+W}$. For the $D^0$ ($\bar{D}^0$) dilution, $R$ is the number of generated $D^0$ ($\bar{D}^0$) events reconstructed correctly as $D^0$ ($\bar{D}^0$) and $W$ is the number of generated $D^0$ ($\bar{D}^0$) events reconstructed as $\bar{D}^0$ ($D^0$).
$A_{T\text{viol}}(D^+) = 0.023 \pm 0.062(\text{stat.}) \pm 0.022(\text{syst.})$, and

$A_{T\text{viol}}(D^+_s) = -0.036 \pm 0.067(\text{stat.}) \pm 0.023(\text{syst.})$.

It is interesting to compare the $A_{T\text{viol}}$ measurements with the usual $CP$ asymmetry measurements. Following the procedure described in a previous paper [12] we determine $A_{CP}$ for $D^0 \rightarrow K^-K^+\pi^-\pi^+$ and $D^+ \rightarrow K^0_S K^+\pi^-\pi^+$, where we used $D^0 \rightarrow K^-\pi^+\pi^-\pi^+$ and $D^+ \rightarrow K^0_S \pi^+\pi^-\pi^+$ to account for differences at production level. Systematic errors are obtained from the same sources and in the same manner as for the $A_{T\text{viol}}$ measurement.

We measure:

$A_{CP}(D^0) = -0.082 \pm 0.056(\text{stat.}) \pm 0.047(\text{syst.})$ and

$A_{CP}(D^+) = -0.042 \pm 0.064(\text{stat.}) \pm 0.022(\text{syst.})$.

Both $A_{T\text{viol}}$ and $A_{CP}$ are consistent with zero. While our measurements are consistent with no $T$ violation, we encourage higher statistics experiments to repeat these measurements.

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Fig. 1. $K^-K^+\pi^-\pi^+$ invariant mass distributions for: (a) total sample $D^{*+(-)} \rightarrow D^0(D^{0})\pi^{+(-)}$, (b) $D^0$ sample, $D^{*+} \rightarrow D^0\pi^+$ and (c) $D^{*0}$ sample, $D^{*-} \rightarrow D^{0}\pi^-$. The fit (solid curve) is explained in the text.
Fig. 2. $K^−K^+\pi^−\pi^+$ invariant mass distributions for: (a) $D^0$ sample with $C_T > 0$, (b) $D^0$ sample with $C_T < 0$, (c) $\bar{D}^0$ sample with $C_T > 0$ and (d) $\bar{D}^0$ sample with $C_T < 0$. The fit (solid curve) is explained in the text.
Fig. 3. $T$-violating asymmetry $A_{T\text{viol}}$ versus several sets of cuts. We varied the confidence level of the secondary vertex from 1% to 5% (5 points), $Iso2$ from $10^{-6}$ to 1 (7 points), $L/\sigma_L$ from 5 to 15 (11 points), $\Delta_K$ from 1 to 5 (9 points), $picon$ from −6 to −2 (9 points). The dashed lines show the quoted $A_{T\text{viol}}$ asymmetry $\pm 1\sigma$. 
Fig. 4. Upper plot is an invariant mass plots for the state $K^0_S K^+\pi^-\pi^+$. Lower plots are invariant mass plots for $D^+ \rightarrow K^0_S K^+\pi^-\pi^+$ (left) and for $D^- \rightarrow K^0_S K^-\pi^+\pi^-$ (right). All plots have an $L/\sigma > 6$ cut. The broad light shaded region is a reflection from $\Lambda_c^+ \rightarrow p K^0_S \pi^-\pi^+$, while the darker shaded region above the $D^+$ mass results from $D^+ \rightarrow K^0_S \pi^+\pi^-\pi^+$ where one pion is misidentified as a kaon.
Fig. 5. Upper plots are invariant mass plots for $D^+ \rightarrow K^0_S K^+ \pi^+ \pi^-$ for $C_T > 0$ in Fig. 5a and for $C_T < 0$ in Fig. 5b. Lower plots are invariant mass plots for $D^- \rightarrow K^0_S K^- \pi^+ \pi^-$ for $C_T > 0$ in Fig. 5c and for $C_T < 0$ in Fig. 5d. All plots have an $L/\sigma > 6$ cut. The shaded regions shown in each plot are explained in the text and in Fig. 4.
Fig. 6. \(T\)-violating asymmetry \(A_{T\text{viol}}\) versus several sets of cuts. We varied the \(\sigma_L\) from 4 to 12 (8 points), \(\text{Iso2}\) from \(10^{-2}\) to \(10^{-6}\) (5 points), confidence level of the secondary vertex from 1\% to 5\% (5 points), \(\Delta_K\) from 2 to 5 (7 points). The dashed lines show the quoted \(A_{T\text{viol}}\) asymmetry \(\pm 1\sigma\).