Phase transition in the one-dimensional Kondo lattice model with attractive electron-electron interaction

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(July 15, 2021)

The one-dimensional Kondo lattice model with attractive interaction among the conduction electrons is analyzed in the case of half-filling. It is shown that there are three distinct phases depending on the coupling constants of the model. Two phases have a spin and charge gap. While one shows a clear separation of the spin and charge excitation spectrum the other phase may be characterized as a band insulator type where both excitations are due to two-particle states. The third phase is gapless in both channels and has quasi long-range order in the spin and charge density wave correlation. In this phase the spin and charge excitations have again a clearly separated spectrum.

For the analysis we discuss first two limiting cases. Then a density matrix renormalization group calculation on finite systems is applied to determine the phase diagram and the correlation functions in the gapped and gapless phase for general coupling constants.

I. INTRODUCTION

The Kondo lattice model (KLM) describes a many-body system of two distinct types of degrees of freedom, itinerant electrons and localized spins which are arranged on a regular lattice. This model can be considered as an extension of the single impurity Kondo model where electrons interact with a single localized spin. The Hamiltonian of the KLM consists of two parts, the kinetic energy of the itinerant conduction electrons and the local exchange interaction between electron spin $S$ and localized spin $S_t$, both spin $1/2$ degrees of freedom,

$$H_{\text{KLM}} = -t \sum_{\langle i,j \rangle} \sum_s (c_i^s c_j^s + c_j^s c_i^s) + J \sum_i S_i \cdot S_{ti}$$  \hspace{1cm} (1)

where the operator $c_i^s$ ($c_i^{\dagger}$) annihilates (creates) a conduction electron on site $i$ with spin $s$ ($=\uparrow, \downarrow$) ($S_i^\mu = (\hbar/2) \sum_{s,s'} c_i^{s \dagger} \sigma_{s,s'}^\mu c_{si}$) and the sum in the first term runs over all nearest neighbor bonds $\langle i,j \rangle$. Furthermore, $t$ denotes the hopping matrix element and $J$ is the antiferromagnetic exchange coupling $\mathcal{J}$.

In this model both the conduction electrons and the localized spins are each uncorrelated. Correlation only appears through the exchange interaction. It is important to realize that the understanding of the single impurity Kondo problem does not simply extend to the lattice case. Indeed the research during recent years has shown that complicated correlation effects occur within the lattice model yielding a variety of physical phenomena beyond the single impurity picture. Although the KLM is certainly of interest of its own as a generic model of strongly correlated electrons, studies of this model are also motivated by real materials such as the so-called heavy fermion compounds or the Kondo insulators. Both systems where correlation effects between itinerant and localized electrons clearly dominate the low-energy physics.

In this article we discuss the extension of the KLM obtained by including the direct interaction among the conduction electrons. We use the most simple form for the interaction, which acts only between electrons on the same site,

$$\mathcal{H}_{\text{int}} = U \sum_i c_i^{\uparrow\dagger} c_i^{\uparrow\dagger} c_i^{\downarrow\dagger} c_i^{\downarrow}.$$  \hspace{1cm} (2)

We call $\mathcal{H} = \mathcal{H}_{\text{KLM}} + \mathcal{H}_{\text{int}}$ Kondo-Hubbard model (KHM). This model was recently also considered in connection with the ferromagnetic ground state away from half filling. In the following we will focus on the one-dimensional (1D) system with a half-filled electron band.

The half-filled KLM is considered as a good starting point to understand the Kondo insulators, a class of materials which show a spin and a charge gap at low temperatures. In contrast to ordinary band insulators the two gaps are different, indicating a separation of the spin and charge degrees of freedom due to correlation effects. The half-filled KLM shows indeed this type of properties. The properties of the spins are dominated by short-ranged antiferromagnetic correlations, i.e. this state can be considered as a spin liquid. A particular feature of the 1D KLM is that the spin liquid state exists for all finite values of $J$. This could be shown numerically using exact diagonalization or, more recently, the density matrix renormalization group technique and the mapping to a non-linear sigma model. (Note that in higher dimensions a transition between the spin liquid and an antiferromagnetically ordered state is expected at a critical value of $J$. This has, however, not been established so far either by analytical or numerical methods.) For weak coupling, $J \ll t$, one finds that the spin gap ($\Delta_s$) and charge gap ($\Delta_c$) depend in a very different way on $J$,

$$\Delta_s \propto \exp(-1/\alpha \rho J)$$

$$\Delta_c \propto J$$  \hspace{1cm} (3)
where $\rho$ is the density of states at the Fermi level for the free electrons. The spin gap energy gives an energy scale formally related to the Kondo temperature $T_K$ of a single localized spin ($\alpha = 1$). For the lattice of localized spins $\alpha$ is enhanced by a value of about 1.4 obtained from numerical simulations. The similarity with the Kondo energy scale indicates that the ground state of the half-filled KLM corresponds to a singlet bound state like the Kondo singlet state. However, correlation effects among the localized spins tend to increase the binding energy through the formation of a collective singlet state. On the other hand, the charge gap has no counterpart in the single impurity case (where the system forms a Fermi liquid). The charge gap proportional to $J$ originates from the strong antiferromagnetic correlation of the localized spins. Although short ranged it provides a staggered background for the electron motion, which yields the features of a doubled unit cell as found in a spin density wave state. Thus the energy scale is set by the exchange coupling between the electrons and the localized spins. Also in the strong coupling limit ($J \gg t$) we find a clear distinction between the two excitations as we will consider below.

The spin liquid state of the KLM can be characterized as the formation of a collective Kondo singlet involving the conduction electrons and localized spins. The singlet formation is optimized if configurations of conduction electrons with doubly occupied and empty sites are suppressed. Both configurations “remove” the electron spin degree of freedom so that the exchange term cannot be active. Therefore strong charge fluctuations of the conduction electrons tend to weaken the spin liquid state. The inclusion of a repulsive interaction between the conduction electrons as in Eq. (2) with $U > 0$ leads to a suppression of the charge fluctuations. It is a well-known fact that the Hubbard model with repulsive interaction develops a charge gap at half-filling for any finite $U$ in one dimension, while the spin excitation remain gapless. Consequently, we expect that the spin liquid state is further stabilized by the repulsive interaction. This is clearly seen in numerical calculations where, in particular, in the limit of $J \ll t$ the spin gap is enhanced through an increase of the factor $\alpha$ in the exponent Eq. (3). While the spin gap goes to zero for $J \rightarrow 0$ the charge gap remains finite, if $U > 0$.

We now ask what will happen if the interaction among the conduction electrons is attractive. There is a relation between the positive and negative $U$ Hubbard model due to particle-hole symmetry at half filling. The change of sign for $U$ leads effectively to an exchange of the charge and spin degrees of freedom. Indeed the charge excitations can be described as isospins completely analogous to the spins, as we will see below. For $U < 0$ the spin sector of the conduction electrons has a gap while the charge (isospin) excitations are gapless. Obviously this spin gap weakens the spin liquid phase characterized by the formation of the singlets between localized and electron spins. Therefore a competition arises between the the spin and the charge fluctuations whose behavior is determined by the relative strength of the coupling constants $J$ and $U$.

It is the goal of this paper to investigate this competition for the case of $U < 0$. By the analysis of two limiting cases we will demonstrate that the attractive interaction leads to a phase transition where the character of the ground state and the excitations change qualitatively. The state which is in competition with the spin liquid phase may be characterized by the property that it has quasi-long-range order in the spin and charge sectors. The dominant correlations are that of an antiferromagnet for the localized spins and of a charge density wave for the conduction electrons, i.e. a spin-charge density wave (SCDW). Additionally we find that within the spin liquid a phase can occur where the spin and charge excitations have the same energy scale and behave similar to a band insulator. In the following we will analyze these properties first by characterizing the states in the limit $t \ll |U|, J$. In a second step we will investigate the phase transition and the phase diagram, $U$ versus $J$, by means of numerical simulation using the density matrix renormalization group method (DMRG).

II. PHASES IN TWO LIMITING CASES

In this section we show that there are two distinct phases for the KHM with attractive interaction. To this end it is helpful to consider two limiting cases which allow a simple analysis of the model. These limits are $t \ll J \ll -U$ and $t \ll -U < J$. At the very beginning let us start with $t = 0$, the atomic limit. Then the states can be represented most easily in a real space basis. For $U = 0$ the ground state $|\Psi_s\rangle$ is a product of onsite singlets, i.e. on each site we find one conduction electron which forms a spin singlet with the localized spin $s_i$. For $\alpha \gg U$, the states can be represented most easily in a real space basis. For $\alpha > 0$ the spin liquid a phase can occur where the spin and charge excitations have the same energy scale and behave similar to a band insulator.
Turning on $U$ now we can change the level scheme (Fig. 1). The relative position of the singlet and triplet states, $|\Psi_s\rangle$ and $|\Psi_t\rangle$, is unchanged. However, the charge (doublon-holon) excitation is shifted according to $3J/2 + U$. For positive $U$ no qualitative change occurs in the level scheme. Negative $U$, however, lead to a re-arrangement of the onsite energy levels for sufficiently large $|U|$. For $U = -J/2$ the charge excitation passes the triplet state and for $U = -3J/2$ the singlet state. This indicates that the attractive interaction between electrons yields a qualitative modification of the system. Now let us discuss the situation where $t$ is finite and the above mentioned degeneracies are lifted.

**A. The spin liquid state: $|U| \ll J$**

For small $t$ we can now use perturbation theory to describe the effect of hopping of the conduction electrons. A detailed discussion of this type of perturbation can be found in Ref. [3]. For the limit $|U| \ll J$ the singlet state $|\Psi_s\rangle$ remains the ground state. Its energy acquires a correction in second order due to the polarization of the onsite singlet, i.e. the neighbor electron and localized spin are involved in the formation of the singlet which becomes more extended.

The hopping term leads to the mobility of the spin triplet and the doublon-holon excitations. The former behaves as a single quasiparticle with an excitation energy

$$E_t(q) = J + \frac{4t^2}{3J + 2U} - \frac{4t^2}{J + 2U}(1 - \cos q) \quad(4)$$

where $q$ is the momentum (lattice constant $a = 1$). The excitation energy for the doublon-holon state has the form of two-particle excitation with an energy depending on two momenta $k$ and $q$.

Thus, the doublon-holon excitations form a continuum. The spin triplet excitation may be understood also as a bound state of a doublon and holon with their spins in the triplet configuration. In this sense the triplet excitation has the character of an “exciton” within the gap between the singlet ground state and doublon-holon continuum.

The discussion of the level scheme for $t = 0$ suggests that there is some change in the properties of the spin liquid phase as we turn $U$ from 0 towards negative values. There is a critical value $U = -J/2$ where the doublon-holon state falls below the triplet excitation (for $t = 0$). If $U$ is smaller than $-J/2$ the triplet excitation discussed above is absorbed into the continuum of the spin-triplet channel of the doublon-holon continuum. Therefore the lowest spin and charge excitations have both two-particle character like a particle-hole excitation. This state has still a finite gap and the structure of the excitations is essentially similar to that of a band insulator. The quasiparticles, the doublons and holons, are spin 1/2 fermions, composed of conduction electrons and localized spins.

At the second critical value of $U$ ($= -3J/2$) the holon-doublon state passes the singlet ground state. Here the character of the ground state has to change so that this critical value should belong to a phase transition. In the following we want to discuss the properties of the new state.

The perturbative treatment of the different states given above does not allow the evaluation of the transition points for finite $t$. Close to the transition points a more complicated perturbation for degenerate states would be necessary. This leads, however, already to a complicated many-body problem. Thus, we leave the discussion of the phase boundary lines to the numerical part of our paper.

**B. The SCDW state: $|U| \gg J$**

In the limit $|U| \gg J$ the ground state is highly degenerate if we assume $t = 0$. A complete basis of these degenerate states is given by all real-space configurations of holons and doublons, with an equal number of each (half filling). Both onsite singlets $|\Psi_s\rangle$ or triplets $|\Psi_t\rangle$ are now excited states which always should occur in pairs, due to the conservation of the electron number, with energies $E_{st} = -3J/2 - U$, $E_{dt} = -J/2 - U$ and $E_{tt} = J/2 - U$ for a singlet-singlet, singlet-triplet and a triplet-triplet pair. All these states are highly degenerate.
In order to lift the degeneracy of the ground state we include now the hopping process, assuming a small, but finite $t$. We can now generate an effective Hamiltonian in the Hilbert-subspace of doublons and holons only. At this point it is convenient to introduce the notion of isospin $I$ (I=1/2) for the charge degrees of freedom of the holon and doublon. We identify a holon with isospin up and a doublon with isospin down. Like the ordinary spin also the isospin transforms according to the SU(2)-symmetry group. Additionally we introduce a phase convention by dividing the lattice into two sublattices $A$ and $B$. Then the real space basis functions should be multiplied by $\prod_{i \in B} \exp(i \pi I_z)$.

$$I_i^+ = s_i c_{i\uparrow} c_{i\downarrow}^\dagger$$
$$I_i^- = s_i c_{i\downarrow} c_{i\uparrow}^\dagger$$
$$I_i^z = \frac{1}{2} (c_{i\uparrow}^\dagger c_{i\uparrow} + c_{i\downarrow}^\dagger c_{i\downarrow} - 1)$$

where $s_i = +1$ if $i \in A$ and $s_i = -1$ if $i \in B$. With this notation the effective Hamiltonian which lifts the degeneracy in lowest order perturbation theory has the form,

$$\mathcal{H}_{\text{eff}} = \sum_{\langle i,j \rangle} \left[ I_i \cdot I_j - \frac{1}{4} \right] [a S_{fi} \cdot S_{fj} + b].$$

where the two constants are

$$a = -\frac{2t^2}{U + \frac{t^2}{2}} + \frac{t^2}{2} + \frac{t^2}{U + \frac{3t^2}{2}}$$
$$b = \frac{9t^2/4}{U - \frac{t^2}{2}} - \frac{3t^2/2}{U + \frac{t^2}{2}} - \frac{t^2/4}{U + \frac{3t^2}{2}}$$

(derived in the Appendix). In this formulation the SO(4)-symmetry of both the spin and isospin is obvious (SO(4) = SU(2) × SU(2)). The spin degrees of freedom are only due to the localized spins while the conduction electron spin is completely quenched in the effective model. Thus the conduction electrons only appear as charge degrees of freedom, i.e. isospins. It is easy to see that the coupling constant $a$ is negative for $U < -3J/2$ and vanishes for $J \to 0$, because this leads to a complete decoupling of the localized spins from the conduction electrons. Note that the positive constant $b$ determining the spectrum of the conduction electrons remains finite for $J = 0$ so that the isospin degrees of freedom remain coupled. The electron spins appear only in states including onsite singlet or triplet components which are much higher in energy.

For this type of Hamiltonian the following two exact statements are known. (1) For a finite system we can show that the ground state is a singlet in both the spin and the isospin channel and it is non-degenerate. This can be proven using a generalization of Marshall’s theorem. (2) In the thermodynamic limit the excitations in both channels are either gapless or the ground state is degenerate with spontaneously broken parity. This can be demonstrated using a generalization of the Lieb-Schultz-Mattis theorem as given by Affleck and Lieb. There is no need to reproduce the proofs of these two statements here, since they are completely analogous to those applied to the Heisenberg spin system.

The latter statement proves that in the limit $|U| \gg J$ the phase is different from the spin liquid state. While the spin liquid phase has a unique ground state and a gap in all excitations, this phase has either a degenerate ground state or gapless excitations. In the case of gapless excitations we expect that the system has quasi-long-range order in both spin and isospin analogous to the 1D Heisenberg model. The dominant correlation of both degrees of freedom is “antiferromagnetic”, which corresponds to the usual charge density wave correlation for the isospin part. Due to the qualitative difference of the two ground states, we conclude that there must be a phase transition between the two limits, depending on $U$ and $J$. In the following we will use numerical methods to demonstrate that the state for $U \ll -J$ has actually gapless excitations.

### III. THE PHASE DIAGRAM BY DMRG

In order to determine the transition line in the phase diagram, $U$ versus $J$, and the character of the different phases, we calculate the excitation energies and the correlation functions of the ground state in the KHM numerically. For this purpose it is necessary to treat long enough systems such that the transition between different phases can be observed reliably. We use the density matrix renormalization group (DMRG) method which allows us to study long chains by iteratively enlarging the system size and obtain ground state wave functions with only small systematic errors.

We calculate the elementary excitations for various $U$ to identify the three phases discussed in Sec. II. The DMRG scheme is designed to obtain a very good approximation of the ground state of a model. For the evaluation of excited states and their energies we consider the model in those Hilbert-subspaces which contain these states as lowest energy states. The real ground state is found for the subspace with $S_{\text{tot}}^z = 0$ and $N_c = L$ ($N_c$: number of conduction electrons). The spin excited state is obtained for $S_{\text{tot}}^z = 1$ and $N_c = L$ and the charge excited state for $S_{\text{tot}}^z = 0$ and $N_c = L \pm 2$. In these sectors the lowest energy state is calculated for various system sizes up to $L = 48$ with the finite system algorithm using open boundary conditions. The extrapolation to the bulk limit is obtained from the scaling laws. For the gapped spin liquid phase the form $\Delta(L) = \Delta(\infty) + \beta L^{-2} + O(L^{-4})$ is assumed, since the lowest excited state generally corresponds to the bottom of an excitation spectrum which can be expanded in terms of the square of the momentum,
energies, exchange constant $J$ in units of $t$.

In the DMRG calculations are 1 in the DMRG calculations are $5 \times 10^{-6}$ for $U/J = -1.4$. Gap energies, exchange constant $J$, and Coulomb interaction $U$ are in units of $t$.

FIG. 2. Excitation energies of the half-filled Kondo lattice model with attractive interactions. Typical truncation errors in the DMRG calculations are $1 \times 10^{-6}$ for $U/J = -1.4$. Gap energies, exchange constant $J$, and Coulomb interaction $U$ are in units of $t$.

First let us discuss the results for the case of strong exchange coupling $J = 10.0t$. The data for the excitation gaps are shown in Fig. 2. It is not difficult to identify the three phases as we scan $U$ from 0 to $-2J$: the ordinary spin liquid phase $\Delta_c = \Delta_s > 0$ for $U/J > -0.6$, the spin liquid phase with identical spin and charge excitations $\Delta_c = \Delta_s > 0$ for $-0.8 > U/J > -1.4$, and the gapless phase (SCDW) $\Delta_c = \Delta_s = 0$ for $U/J < -1.8$.

In the intermediate regime located between $U/J = -0.8$ and $-1.2$, the difference of $\Delta_c$ and $\Delta_s$ vanishes in the bulk limit within the accuracy of the present calculation. Therefore this confirms the existence of the intermediate phase where both spin and charge excitations have the same energy properties and are essentially of the two particle type as anticipated in the previous section.

In addition the single quasiparticle excitation gap $\Delta_{qp}$ is shown in Fig. 2. The quasiparticles originate from doublons or holons and have therefore fermionic character. The $\Delta_{qp}$ is obtained from the difference of the lowest energy in the Hilbert spaces with $S^z_{\text{tot}} = 0$ with $N_c = L$ and $S^z_{\text{tot}} = \pm 1/2$ with $N_c = L \pm 1$. The $\Delta_{qp}$ is half of the $\Delta_c$ for $U/J > -1.4$. The reason is that the effective interaction between the quasiparticles is repulsive in that region. On the other hand, for $U/J < -1.8$ the $\Delta_{qp}$ increases with growing $-U/J$ while $\Delta_c$ is zero (within our accuracy). Hence, we may interpret this as a switching from a repulsive to an attractive quasiparticle interaction, indicated by the minimum of the $\Delta_{qp}$ around $U/J = -1.6$. Since the character of the excitation changes here we expect that this switching coincides with the phase transition to the gapless phase. Obviously, in the thermodynamic limit all gaps should disappear at the transition point. Despite careful scaling analysis it is difficult to determine the exact position of the transition from the present DMRG study because close to the transition point the truncation error in the density matrix becomes large and the convergence of the RG iteration is rather slow.

Next we turn to a weaker exchange coupling. The results for $J = 2.0t$ are shown in Fig. 3. In contrast to the strong coupling case we cannot find any indication of the intermediate regime with $\Delta_c = \Delta_s > 0$. This means that this regime is either absent or confined to a very small region close to the transition point which we locate at around $U/J \approx -2.0t$. For $U/J < -2.0t$ the two gaps essentially coincide and are practically zero. Corresponding to this change the quasiparticle gap shows a minimum around the critical value of $U$.

Similar calculations are carried out for $J/t = 4, 6, 8$ and the phase diagram is obtained. In Fig. 3 we show the three phases by different dots which are numerically determined. The crosses in Fig. 3 represent estimated minimum points in the quasiparticle gap where we expect the phase transition to be located. There are technical limitations on the details of the phase diagram, which do not allow us to determine clearly the extension of the intermediate phase ($\Delta_c = \Delta_s > 0$).

$k^2$. On the other hand, for the region which is supposed to possess the gapless phase the bulk limit value of the excitation energies is estimated simply using $1/L$ scaling and $1/\log L$ scaling.
A definite advantage of the DMRG scheme is that we can get very good approximations of the ground state wave functions for rather large systems. This allows the study of various correlation functions at least sufficiently far from the transition line of the phase diagram. Here we have analyzed the spin-spin and charge-charge correlation functions which show characteristic features of the phases. The correlation functions can be observed through the perturbing effect of the boundaries. This induces an oscillating disturbance into the wave function leading to a charge or spin density modulation analogous to the Friedel oscillations around an impurity. For example if \( \Delta_c \) is finite then the density-density correlation has an exponential form and the charge density Friedel oscillations induced by an impurity potential shows the same exponential decay. The length scale is related to the ratio between the charge velocity and the gap, \( v_c/\Delta_c \). On the other hand, if \( \Delta_c \) is zero, then a power law decay of the correlation function, similar to that of a Tomonaga-Luttinger liquid, is expected. This power law decay occurs naturally in the Friedel oscillations of both spin and charge.

In Fig. 4 we show the charge and spin density Friedel oscillations, \( \delta \rho(x) \) and \( \delta \sigma(x) \), in the intermediate phase with \( J/t = 10 \) and \( U/J = 1.4 \). The charge density oscillations are naturally induced by the open boundary conditions, while for generating the spin density oscillations we have to apply a local magnetic field, \( H_{\text{local}} = 2h(S^+_1 - s^+_1 - S^+_L + s^+_L) \), coupling to the spins at both ends of the finite system. Obviously both the charge and spin density oscillations decay exponentially and our analysis shows that their correlation length is essentially the same. This means that not only the excitation gaps but also the velocities of the excitations are identical as we expect for the excitations of the particle-hole type. This clearly indicates that the picture of this phase as a kind of band insulator is appropriate.
shows log-log plot of the oscillations for $U$ correspond to $J$ is 30 sites and lattice model with attractive interactions. The system size is $J = 10t$. The solid line and the broken line correspond to $U = -20t$ and $-14t$, respectively. The inset shows log-log plot of the oscillations for $U = -20t$.

**FIG. 6.** Charge density oscillations of the half-filled Kondo lattice model with attractive interactions. The system size is 30 sites and $J = 10t$. The solid line and the broken line correspond to $U = -20t$ and $-14t$, respectively. The inset shows log-log plot of the oscillations for $U = -20t$.

$J = 10.0t$
$U = -14.0t$
$U = -20.0t$

**FIG. 7.** Spin density oscillations of the half-filled Kondo lattice model with attractive interactions. The system size is 30 sites and $J = 10t$. The solid line and the broken line correspond to $U = -20t$ and $-14t$, respectively. The inset shows log-log plot of the oscillations for $U = -20t$.

$J = 10.0t$
$U = -14.0t$
$U = -20.0t$

**IV. CONCLUSION**

We have seen that the 1D half-filled Kondo-Hubbard lattice with attractive electron-electron interaction has three phases, two gapped phases (spin liquid and band insulator-like) with short range correlations and one gapless phase with quasi long-range ordered spin and charge density waves (SCDW). Note that for positive $U$ only the spin liquid exists. An indication of the difference between positive and negative $U$ can be found in the small-$J$ limit. One may be tempted to lift the large spin degeneracy found for $J = 0$ by perturbation theory, i.e., introducing the RKKY-interaction among the localized spins. In lowest order one separates the system into free conduction electrons and interacting localized spins. A simple calculation shows, however, that this perturbation concept fails for the 1D system, because the effective interaction between the spins diverges for the wave vector $q = 2k_F = \pi$ for all $U \geq 0$. This indicates that there is no separation between these degrees of freedoms and numerical calculations suggest that for any finite $J$ the ground state has spin liquid properties. In contrast, for negative $U$ the perturbation converges for all wave vectors and one can derive a sensible RKKY-model. Under this condition the localized spins and the conduction electrons can separately exhibit gapless phases. Note, however, that the electron spin excitation has a gap for negative $U$ and only the charge part is gapless with a tendency towards an enhancement of charge density wave correlations. The effective Hamiltonian (7) describes this kind of system in the extreme limit ($|U| \gg J$).

An interesting change occurs also for the nature of the excitations. If we scan from $U = 0$ to $U \gg J$ for fixed values of $t$ and $J$ then we find close to the transition ($U \approx -3J/2$) a region where the lowest spin and charge excitations are due to doublon-holon excitation. Therefore, the two excitations have the same spectrum as a band insulator. We would like to point out, however, that the fermions, the holons and doublons, building the excitations are quasiparticles which are composed of the itinerant electronic and the localized spin degree of freedom. For finite $t$ they are real dressed quasiparticles involving complex correlation effects. Thus it is not an entirely trivial feature of this strongly correlated system to mimic the properties of a simple band insulator. On the other hand, in the spin liquid phase for $|U| \ll J$ the spin and charge excitation have different nature. The spin triplet excitation has excitonic character, i.e., it is a bound state of a holon-doublon excitation.

Also in the gapless phase the spin and charge excitations are separated as is seen in the difference of the correlation functions. However, the origin of the spin-charge separation is different. The conduction electron and localized spins provide nearly independent degrees of freedom in close connection to the RKKY-model mentioned above.

The KHM with attractive interaction is an example of a strongly correlated electron system which possesses a number of phase of rather different character. These phases depend only on the relation of different coupling constants. Several of the mentioned features can be transfered to higher dimensional systems. The gapless ground state has very likely long-range order in both the
charge and the spin density wave correlation. However, it has to be noticed that the spin liquid phase for $U \geq 0$ might have antiferromagnetic long-range order, if $J$ is sufficiently small. This has not been proven so far. However, this long-range ordered state has a charge gap (gap of a spin density wave state) and the spin excitations are due to both the electron and the localized spin. This is in contrast to the SCW-state we discussed above. Thus in higher dimensions we may expect to see a richer phase diagram.

ACKNOWLEDGMENTS

We would like to thank to H. Tsunetsugu, K. Ueda and H. Schoeller for helpful discussions. This work is financially supported by the Swiss Nationalfonds. M.S. is grateful for the PROFIL-fellowship by the Swiss Nationalfonds.

APPENDIX A: EFFECTIVE HAMILTONIAN IN THE LIMIT $|U| \gg J \gg T$

The low-energy Hilbert subspace $\Lambda$ in the limit of large attractive interaction consists of all real-space configurations of holons and doublons which appear in equal number at half-filling. All other states containing on-site singlets or triplets are higher in energy by a multiple of order $|U|$. For convenience we introduced the notion of isospin $\tilde{I}$ as an additional local SU(2) degree of freedom besides the localized spins. The doublon and the holon are mapped to isospin up and down, respectively (Eq. (6)). Thus each site has four basis states: $|\tilde{I}^z, S^z\rangle = \{|+, \uparrow\rangle, |+,-\rangle, |-, \uparrow\rangle, |-, \downarrow\rangle\}$. For vanishing hopping $t$ the whole subspace $\Lambda$ is disconnected leading to complete degeneracy. We would now like to generate the effective Hamiltonian which lifts this degeneracy and determines the low-energy physics. We use second order perturbation theory in the hopping where the higher energy states outside of $\Lambda$ appear as intermediate states. In this way the effective Hamiltonian will only have nearest neighbor coupling and be of the generic form,

$$H_{\text{eff}} = \sum_i \left\{ a\tilde{I}_i \tilde{I}_{i+1}^z + \frac{b}{2} (\tilde{I}_i^+ \tilde{I}_{i+1}^- + \tilde{I}_i^- \tilde{I}_{i+1}^+) + c \right\} \times \left\{ a' S_i^z S_{i+1}^z + \frac{b'}{2} (S_i^+ S_{i+1}^- + S_i^- S_{i+1}^+) + c' \right\}. \tag{A1}$$

We consider now the coupling on a single bond, say between site 1 and 2, where we represent the local states by $|\tilde{I}_1^z; S_1^z\rangle, |\tilde{I}_2^z; S_2^z\rangle$. There are three different cases to take into account:

1) $|\tilde{I}, \tilde{I}; S, \pm S\rangle$: If the z-component of the isospins on a bond are the same, then the hopping is ineffective and this state is not connected to any other via this bond.

2) $|\tilde{I}, -\tilde{I}; S, S\rangle$: We consider the example $|+, -; \uparrow\rangle$ which leads to the matrix elements,

$$m_1 = \langle +, -; \uparrow\uparrow | H_{\text{eff}} | +, -; \uparrow\uparrow \rangle = (-\frac{a}{4} + c)(\frac{a'}{4} + c') \tag{A2}$$

where the intermediate states consist of two triplets ($E = -U + J/2$) or of one singlet and one triplet ($E = -U - J/2$).

3) $|\tilde{I}, -\tilde{I}; S, -S\rangle$: It is sufficient to consider the example $|+, -; \downarrow\rangle$,

$$m_2 = \langle +, -; \downarrow\downarrow | H_{\text{eff}} | +, -; \downarrow\downarrow \rangle = (-\frac{a}{4} + c)(\frac{a'}{4} + c') \tag{A3}$$

and

$$m_3 = \langle +, -; \downarrow\downarrow | H_{\text{eff}} | +, -; \downarrow\downarrow \rangle = (-\frac{a}{4} + c)(\frac{b'}{2}) \tag{A4}$$

Note that $m_1 - m_2 = m_3$. Now we can determine the coefficients in Eq. (7). Fixing $a = 1$ we find $b = -1, c = -1/4, a' = b'$ and

$$a' = 4(m_2 - m_3) < 0 \tag{A5}$$

$$c' = -(m_1 + m_2) > 0. \tag{A6}$$

Next, we rotate the isospin on every second site by $\pi$ around the z-axis, $I^{x,y} \rightarrow I^{x,y} = -I^{x,y}$ and $I^z \rightarrow \tilde{I}^z$ by applying the phase factor $\exp(i\pi I^z)$. With this phase convention on the basis states $b \rightarrow -b$ and the effective Hamiltonian reaches its apparently SU(2) rotational symmetric form in both spin and isospin space as given in Eq. (7).
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