1. INTRODUCTION

1.1. Protoplanetary Dust Growth

The formation of planetesimals, the kilometer-sized solid bodies whose further growth is controlled by mutual gravitational attraction, is still enigmatic. Collisions among the dust aggregates are controlled by Brownian motion, drift motions with respect to the gas of the protoplanetary disk, and turbulence in the gas (Weidenschilling 1977a; Weidenschilling & Cuzzi 1993). Once in contact, two dust grains experience a mutual van der Waals force (Heim et al. 1999). From the theoretical and experimental standpoints, it is evident that the (sub)micrometer-sized protoplanetary dust grains initially undergo hit-and-stick collisions, which lead to the formation of fractal aggregates (Weidenschilling & Cuzzi 1993; Blum et al. 2000; Krause & Blum 2004). As the collision energy increases, due to increasing aggregate mass and collision velocity, dust aggregates undergo a restructuring phase, in which they acquire denser structures (Dominik & Tielens 1997; Blum & Wurm 2000; Wada et al. 2007, 2008). Laboratory experiments showed that collisions among the dust aggregates result in fragmentation, i.e., in mass loss, if the impact velocities exceed \( \sim 1 \text{ m s}^{-1} \) (Blum & Wurm 2008). Depending on the disk model, this means that the direct collisional growth process ends (at the latest) at aggregate sizes for which this velocity is exceeded. For a minimum-mass solar nebula model (Weidenschilling 1977b; Hayashi et al. 1985), this size is approximately 10 cm.

The further growth is still highly speculative. Wurm et al. (2001) and Blum (2004) proposed the accretion of collisional fragments by aerodynamic and electrostatic effects, respectively. Wurm et al. (2005) and Teiser & Wurm (2009) showed experimentally that a fraction of a dust projectile can stick to a solidified larger dust target even at very large velocities. None of these processes, however, seem to work globally and under all circumstances so that very specific conditions are required for the dust aggregates to grow at high impact velocities. There is clearly a lack of understanding the detailed physics involved in the collisions between macroscopic dust aggregates of arbitrary composition and porosity. Without better knowledge of the collisional physics of these bodies, any attempt to model the formation of planetesimals as an aggregation process will have to fail.

1.2. Previous Work

In the three previous papers of this series, we described the collisional physics of high-porosity protoplanetary dust aggregates up to the centimeter-sized regime. In Paper I (Blum et al. 2006), we introduced a method to experimentally produce monolithic dust aggregates with diameters of 2.5 cm. By choosing either monodisperse spherical monomer particles, quasi-monodisperse irregular particles, or polydisperse irregular grains, we produced dust aggregates with volume filling factors (i.e., packing densities) \( \phi = \rho / \rho_s \) of \( \phi = 0.15, \phi = 0.11 \), and \( \phi = 0.07 \), respectively (see Table 1 in Paper I for more details about the monomer-particle properties). Here, \( \rho \) and \( \rho_s \) are the aggregate and the monomer density. Static uniaxial compression of these dust samples revealed that the maximum compaction for these high-porosity dust aggregates is \( \phi_{\text{max}} = 0.20 \ldots 0.33 \), a value very close to the overall porosity found in comets.
The tensile strengths of our dust samples were determined to \( |T| = 200 \ldots 6300 \) Pa, depending on the monomer properties and the compaction. Also these values are close to those found for comets. Paper II (Langkowski et al. 2008) concentrated on low-velocity impacts into these high-porosity dust samples. We showed that sticking by penetration is the dominating process for impacts above a threshold velocity of \( \sim 1 \) m s\(^{-1}\) for projectiles in the millimeter-sized regime and flat dust targets. For shallow penetration, i.e., for impacts below the threshold velocity, the projectiles bounce off, leaving a well-defined crater. It is obvious that the collisions result in the compaction of the target. In Paper III (Weidling et al. 2009), we investigated the compaction for high-porosity millimeter-sized dust aggregates in bouncing collisions. Bouncing collisions among dust aggregates show considerable energy losses (Blum & Münch 1993) so that it collisions. bouncing collisions among dust aggregates show considerable energy losses (Blum & Münch 1993) so that it was natural to assume some degree of compaction. In Paper III, we found that—although a single collision leads only to very localized compaction of the dust aggregate—millimeter-sized dust aggregates in protoplanetary disks can reach volume filling factors of \( \phi \sim 0.35 \) within a few dozen years.

1.3. Objectives

All previous experiments (see Sections 1.1 and 1.2) were limited by the experimentally available dust-aggregate sizes and morphologies and the achievable collision velocities. In the astrophysical context, the need for numerical simulations of collisions between dust aggregates of arbitrary composition, size, and impact velocity arises from the fact that only a limited parameter space can be covered by experiments. The ongoing debate about threshold velocities for sticking, bouncing, compaction, and fragmentation as well as the fragment size distribution requires a thorough investigation of a wide range of collisions, varied over supposedly critical parameters, such as collision velocity, porosity, size, impact parameter, impact angle, and shape of the colliding dust aggregates. An extensive parameter study of that kind is not feasible under laboratory conditions for the parameter ranges in question. Therefore, we aim to calibrate a smooth particle hydrodynamics (SPH) code and validate this model thoroughly with a series of independent benchmark tests. Hence, the SPH code gains a deeper reliability and the conducted numerical simulations provide well-grounded insight into the physical behavior of dust aggregates.

2. SPH IN DUST COLLISIONS

SPH is a meshless Lagrangian particle method originally developed for astrophysical hydrodynamics applications. A detailed description of the original SPH method may, e.g., be found in Monaghan (2005). The SPH code we utilize for the simulations in this work and the underlying porosity model are introduced and described in full depth in R. J. Geretschauer et al. (2009, in preparation). In the 1990s, SPH has been extended to model the elastic and plastic behavior of solids, see, e.g., Libersky et al. (1993) and Randles & Libersky (1996). The continuous solid objects are discretized into interacting mass packages called particles, which form a natural frame of reference for any deformation and fragmentation that may occur.

The SPH code solves the equations of continuum mechanics in Lagrangian form, in particular the continuity equation

\[
\frac{d\rho}{dt} + \rho \frac{\partial v_\alpha}{\partial x_\alpha} = 0, \tag{1}
\]

and the equation of motion

\[
\frac{d v_\alpha}{dt} = \frac{1}{\rho} \frac{\partial \sigma_{\alpha\beta}}{\partial x_\beta}. \tag{2}
\]

Here, Einstein’s summing convention holds throughout and Greek indices denote spatial coordinates. The variables have their usual meanings, i.e., \( \rho \) denotes the density, \( v \) the velocity, and \( \sigma_{\alpha\beta} \) the stress tensor. The latter is defined according to

\[
\sigma_{\alpha\beta} = -p \delta_{\alpha\beta} + S_{\alpha\beta}, \tag{3}
\]

consisting of a pressure part with pressure \( p \) and a shear part given by the traceless deviatoric stress tensor \( S_{\alpha\beta} \).

The deviatoric stress is defined by the constitutive equations. To model elastic behavior according to Hooke’s law, we adopt the approach by Benz & Asphaug (1994) for the time evolution of the deviatoric stress,

\[
\frac{d S_{\alpha\beta}}{d t} = 2\mu \left( \epsilon_{\alpha\beta} - \frac{1}{d \delta_{\alpha\beta} \epsilon_{\gamma\gamma}} \right) + S_{\gamma\gamma} R_{\gamma\beta} + S_{\beta\gamma} R_{\gamma\alpha}, \tag{4}
\]

where \( \mu \) is the shear modulus and \( d \) denotes the dimension. The rotation rate tensor \( R_{\alpha\beta} \) reads

\[
R_{\alpha\beta} = \frac{1}{2} \left( \frac{\partial v_\alpha}{\partial x_\beta} - \frac{\partial v_\beta}{\partial x_\alpha} \right), \tag{5}
\]

and the strain rate tensor \( \dot{\epsilon}_{\alpha\beta} \) accordingly

\[
\dot{\epsilon}_{\alpha\beta} = \frac{1}{2} \left( \frac{\partial v_\alpha}{\partial x_\beta} + \frac{\partial v_\beta}{\partial x_\alpha} \right). \tag{6}
\]

This set of equations is closed by a suitable equation of state and describes the elastic behavior of a solid body.

In order to simulate also the plastic behavior of porous bodies, we adopt a modified version of the porosity model by Sirono (2004; Figure 1). According to this approach, plasticity is modeled within the equation of state, which is divided into three different regimes. In the first regime, plastic behavior is caused by compression that exceeds a critical limit, the compressive strength \( \Sigma(\rho) \), while in the second regime, tension exceeds the tensile-strength limit \( T(\rho) \). In between these limits, the third, the elastic regime of the material is described by a special version of the Murnaghan equation of state. Thus, the full equation of state reads

\[
p(\rho) = \begin{cases} 
\Sigma(\rho) & \rho > \rho_{c}^+ \\
K(\rho_{c}) (\rho/\rho_{c}) - 1 & \rho_{c}^- \leq \rho \leq \rho_{c}^+ \\
T(\rho) & \rho < \rho_{c}^- .
\end{cases} \tag{7}
\]

The quantity \( \rho_{c}^- \) denotes the reference density, which is the density of the material without any external stress. \( \rho_{c}^+ \) and \( \rho_{c}^- \) are limiting quantities, where the transition between the elastic and plastic regime for compression and tension, respectively, takes place. Once these limits are exceeded, the material leaves the elastic path which represents the path where energy is conserved, and loses internal energy by following the paths of the compressive and tensile strength (Figure 1).

3. TOWARD AN EQUATION OF STATE FOR DUST AGGREGATES

In this laboratory section, we will provide the macroscopic material parameters, which are necessary for the SPH model.
introduced in Section 2. We recapitulate the tensile-strength measurements of Blum & Schräpler (2004) and give an interpolation for different volume filling factors. The compressive strength for unidirectional (one-dimensional) compression was also measured by Blum & Schräpler (2004), while in this paper we will present measurements on omnidirectional (three-dimensional) compression. Moreover, we will introduce a simple impact experiment, which will be used for calibrating the SPH model: a millimeter-sized glass bead (or a glass bead analog) impacts into a well-defined 2.5 cm dust sample at a collision speed of 0.1–1 m s\(^{-1}\). The dust sample, consisting of 1.5 \(\mu\)m SiO\(_2\) monodisperse spheres, was formed by random ballistic deposition (RBD) and has therefore a volume filling factor of \(\phi_0 = 0.15\) (see Blum & Schräpler 2004 and references therein).

The deceleration curve, penetration depth, and impact duration of the glass bead are measured as well as the compression of the dust beneath the glass bead to compare these results with an impact computed by the SPH model.

### 3.1. Tensile Strength

In Blum & Schräpler (2004) and Paper I, we reported on measurements of the tensile strength of dust samples of various compositions (i.e., monomer size distribution, morphology, composition, volume filling factor). The best set of data was collected for the dust aggregates consisting of spherical 1.5 \(\mu\)m SiO\(_2\) monomers (see above). For packing densities of \(\phi = 0.15, \phi = 0.41, \phi = 0.54, \) and \(\phi = 0.66\), we found tensile strengths of \(|T| = 1000\) Pa, \(|T| = 2400\) Pa, \(|T| = 3700\) Pa, and \(|T| = 6300\) Pa, respectively. To a good approximation, these values can be expressed by a relation of the form

\[
T(\phi) = -\left(10^{2.8+1.48\phi}\right).
\] (8)

This expression will be used throughout this paper for the packing density dependence of the tensile strength.

### 3.2. Static Measurement of Compressive-Strength Curves

The compression curve of a given material tells us how the material behaves under an applied pressure \(\Sigma\) in changing its volume filling factor \(\phi\). If the material can be described by macroscopic parameters, the volume filling factor is representative for the material density and so the development of the compression curve \(\phi(\Sigma)\) (compare to Equation (7)) is essential to establish a collision model and learn about collisions of protoplanetary dust aggregates.

### Table 1

| Compression Type  | \(\phi_1\) | \(\phi_2\) | \(p_m\) (kPa) | \(\Delta\) (dex) |
|------------------|----------|----------|--------------|-------------|
| Unidirectional   | 0.15     | 0.33     | 5.6          | 0.33        |
| Omnidirectional  | 0.12     | 0.58     | 13.0         | 0.58        |

Measurements of the compression curve were already performed by Blum & Schräpler (2004) and in Paper I. We will again focus on the dust samples made of 1.5 \(\mu\)m SiO\(_2\) spheres, whose properties are compiled in Table 1 in Blum & Schräpler (2004). In the compression experiments of Blum & Schräpler (2004), a dust sample was fixed between two parallel glass plates, which were then pushed together with an increasing force. The measurements of the dust mass, dust volume, compression force, and, thus, pressure yield the compression curve \(\phi(\Sigma)\). The force was applied in one direction, which is therefore called unidirectional compression. The dust sample flattens in the direction of the force but, at the same time, also expands in the other directions. For dust samples made of 1.5 \(\mu\)m SiO\(_2\) spheres, this leads to an equilibrium filling factor of 0.33 for pressures exceeding 10\(^5\) Pa. This compression curve is only applicable to protoplanetary dust collisions, if the material compressed in the impact zone creeps sideways as it did in the static experiments. As we will show later by X-ray analysis of the compression next to an impact site, this is not the case.

Consequently, a second way to measure the compressive-strength curve is to fix the dust sample at the sides with closed walls. In this case, the pressure cannot be released and acts from all sides, thus omnidirectional compression. We performed experiments in which we cut a cylindrical section from an RBD dust sample with a thin-walled plastic tube of 7 mm diameter. This cylindrical dust sample of approximately 1 cm height was then put into a 7 mm borehole in an aluminum block. Carefully pushing a piston into this borehole leads to an omnidirectional pressure onto the dust sample (see inset in Figure 2). The setup was put onto a balance and the piston was loaded with weights of increasing mass. This weight force, divided by the piston area, yields the pressure \(\Sigma\), while the mass and height of the dust sample determine the volume filling factor \(\phi\) (Figure 2). Due to the fact that the dust sample is not a frictionless fluid, force chains inside the sample might locally reduce the pressure. Thus, the pressure for the idealized compression curve can be slightly lower.

The solid line in Figure 2 denotes an analytical approximation of the mean filling factor of nine individual experiments as a function of the applied pressure and the gray shaded area is the standard deviation of the measurements. The analytical function is based on a Fermi distribution with logarithmic pressure in the energy term

\[
\phi(\Sigma) = \phi_2 - \frac{\phi_2 - \phi_1}{\exp(\frac{\ln\Sigma - \ln p_m}{\Delta}) + 1}.
\] (9)

and is only valid for \(\phi \geq \phi_0\). For pressures below \(\Sigma(\phi_0)\), the dust aggregate behaves elastically. The parameters for the unidirectional and omnidirectional compression curve are given in Table 1. The bottom plot in Figure 2 gives the deviation between the analytical approximation and the data, which is within \(\phi_{\text{err}} = \pm 0.02\). Often, the inverse function \(\Sigma(\phi)\) is used (see Equation (7)), which is here

\[
\Sigma(\phi) = p_m \cdot \left(\frac{\phi_2 - \phi_1}{\phi_2 - \phi} - 1\right) \Delta \ln 10.
\] (10)
Compared with the unidirectional compression curve (dashed line) of Blum & Schr"apler (2004), the filling factor also starts off at the original dust-sample filling factor of $\phi_0 = 0.15$ (compare to Blum & Schr"apler 2004), but diverges from the unidirectional curve for pressures $p \gtrsim 10^5 \text{ Pa}$. For those pressures, the filling factor is systematically higher, meaning on the other hand that the same filling factor is much easier to achieve if the pressure acts from all sides. So far there was no equilibrium filling factor found like in the case of the unidirectional compression experiments. The filling factor still significantly increases for the highest applied pressure of $10^6 \text{ Pa}$. However, the analytical approximation indicates an equilibrium for $\phi_2 = 0.58$, which is not far from random close packing of monodisperse spheres ($\phi \sim 0.64$, see, e.g., Torquato et al. 2000), the maximal possible compression without breaking the dust grains.

The new compression curve is still a static measurement. It is applicable for omnidirectional static pressures like the hydrostatic equilibrium inside planetesimal bodies. It is questionable if this compression curve is valid for dynamic collisions but it is a second attempt to assume that surrounding material, which does not interact in a collision, acts as a confining wall to the active impact volume instead of creeping sideways.

3.3. Deceleration Experiments

3.3.1. Experimental Setup

The experimental setup consists of a vacuum chamber (gas pressure $\sim 0.5 \text{ mbar}$) in which a projectile is suspended on a thin fiber (Figure 3) with negligible mass to prevent rotation and lateral velocities. The distance between the suspended projectile and the surface of the dust sample determines the impact velocity $v_0$. The projectile consists of an elongated solidified epoxy droplet at the bottom and a cylindrical plastic tube at the top end. After the release of the projectile, it is accelerated by gravity and decelerated once it is in contact with the dust sample. The deceleration within the dust sample is observed by a high-speed camera (Figure 4). From the deceleration curve of the projectile, we can derive fundamental dynamic properties of the target dust aggregate.

The bottom shapes of the projectiles were spherical with radii of $R \approx 0.5 \text{ mm}$ and $R \approx 1.5 \text{ mm}$ and masses of $m \approx 1 \text{ mg}$ and $m \approx 30 \text{ mg}$, respectively (see Table 2). The effective densities of the projectiles of $\rho = 2400 \ldots 3100 \text{ kg m}^{-3}$ match those of the astronomically relevant silicates, while the combination of low-density epoxy and plastic tube made sure that the top of the projectile was always visible to the camera even if the intrusion depth was larger than the projectile diameter. The high-speed camera was operated at a frame rate of 12,000 frames per second with a resolution of $\sim 30 \mu\text{m pixel}^{-1}$ and the position of the upper edge of the projectile was measured with subpixel accuracy of $\sim 3 \mu\text{m}$. The first touch of the projectile with the surface of the dust sample marks the time $t = 0$ and can clearly be determined from the deviation of the trajectory compared to a free-falling projectile. After its deepest penetration, the projectile bounces back (by $\sim 100 \mu\text{m}$) and oscillates in the vertical direction, which we will not take into account in the further discussion.

3.3.2. Experimental Results

We performed 15 impacts of our projectiles into the porous dust samples, which are compiled in Table 2. The time-resolved deceleration data $h(t)$ were cleaned from gravitational influence by adding $\frac{1}{2}gt^2$ to the negative intrusions so that the gravity-independent deepest penetration depth $D$ and stopping time $T$ could be determined. The intrusion curves were normalized in space and time through $h'(t') = h(t)/D$ and $t' = t/T$ so that $h'(t' = 0) = 0$ (first contact) and $h'(t' = 1) = -1$ (deepest intrusion), and can then be well represented by a sine function

$$h'(t') = -\sin \left( t' \cdot \frac{\pi}{2} \right). \quad (11)$$

Alternatively, a fourth-order polynomial with only one free parameter was used for fitting the data, where the mean standard deviation between the fit and the $N$ data points $\sigma = \ldots$
\( \sqrt{\frac{1}{N} \sum_{i=1}^{N} (h'(t_i) - h_i)^2} \) amounts to only 2–4 \( \mu \text{m} \) in absolute units (compare to Table 2). Although the standard deviation for the sine function is rather of the order of 10 \( \mu \text{m} \), we take the sine function because it has no free parameter and the standard deviation is still less than the pixel size of 30 \( \mu \text{m} \). In few experiments, the 1 mm projectiles canted over before coming to rest. In these cases, the data were used as long as reliable and the remaining deceleration curve and, thus, the penetration depth was extrapolated.

Figure 5 shows all measured deceleration curves in absolute units. Different intrusion depths and stopping times can clearly be distinguished in this plot. The intrusion depths increase with increasing impact velocities (i.e., with the absolute values of the initial slopes of the curves), while the stopping times are rather constant for one projectile size (\( T \sim 6 \text{ ms} \) for 3 mm projectiles (nos. 1–9 in Figure 5) and \( T \sim 3 \text{ ms} \) for 1 mm projectiles (nos. 10–15 in Figure 5) and, thus, independent of the impact velocity \( v_0 \). A \( \chi^2 \) test yielded the best-fitting power-law relation between the penetration depth, impact velocity, and mass of the form

\[
D = \gamma D_0 \cdot m^{\alpha D} \cdot v_0^{\beta D}, \tag{12}
\]

with \( \alpha D = 0.23 \pm 0.13, \beta D = 0.89 \pm 0.34, \) and \( \gamma D = (3.86 \pm 0.11) \times 10^{-2} \text{ kg m}^{1/3} \text{s}^{-0.09} \) (Figure 6). The respective errors denote the 1\% uncertainties. A more intuitive relation would be \( D \propto m v_0 A^{-1} \), with \( A = \pi R^2 \) being the cross section of the projectile. This relation has a clear physical meaning as the penetration depth is determined by the quotient of the momentum \( m v_0 \) as driving force and the cross-sectional area \( A \) as resistive parameter. With \( \alpha T = 1/3 \) and \( \beta T = 1 \) being possible within the uncertainties, the linear relation \( D \propto m v_0 A^{-1} \) or \( \beta D = 1 \) in Equation (12) was unfortunately not feasible due to the too small variations in the effective projectile density \( \rho_0 \) (compare to Table 2).

For the stopping time, we found

\[
T = \gamma T \cdot m^{\alpha T} \cdot v_0^{\beta T}, \tag{13}
\]

with \( \alpha T = 0.23 \pm 0.08, \beta T = 0.01 \pm 0.23, \) and \( \gamma T = (6.77 \pm 0.20) \times 10^{-2} \text{ kg m}^{-0.23} \text{s}^{1.01} \).

After pulling the projectiles out of the dust sample, dust stuck to the surface with which it had been in contact before (Figure 7).

### Table 2

| Experiment Number | Projectile Diameter 2R (mm) | Projectile Mass m (mg) | Effective Density \( \rho \) (kg m\(^{-3}\)) | Impact Velocity \( v_0 \) (m s\(^{-1}\)) | Penetration Depth \( D \) (mm) | Stopping Time \( T \) (ms) | Standard Deviation \( \sigma \) for Polynomial (\( \mu \text{m} \)) | Standard Deviation \( \sigma \) for Sine (\( \mu \text{m} \)) |
|-------------------|-----------------------------|------------------------|----------------------------------|-----------------|------------------|-----------------|--------------------------|--------------------------|
| 1                 | 2.73                        | 25.7                   | 2412                             | 0.89            | 3.16             | 5.92            | 2.43                     | 21.48                    |
| 2                 | 2.94                        | 32.0                   | 2404                             | 0.85            | 3.08             | 5.93            | 2.99                     | 22.64                    |
| 3                 | 2.77                        | 26.4                   | 2372                             | 0.73            | 2.70             | 6.09            | 1.87                     | 17.29                    |
| 4                 | 2.94                        | 32.0                   | 2404                             | 0.17            | 0.70             | 6.18            | 4.63                     | 10.78                    |
| 5                 | 2.94                        | 32.0                   | 2404                             | 0.16            | 0.75             | 7.13            | 2.84                     | 7.69                     |
| 6                 | 2.94                        | 32.0                   | 2404                             | 0.20            | 0.80             | 5.92            | 2.96                     | 8.95                     |
| 7                 | 2.94                        | 32.0                   | 2404                             | 0.32            | 1.31             | 6.47            | 1.62                     | 2.53                     |
| 8                 | 2.77                        | 27.6                   | 2480                             | 0.50            | 2.03             | 6.38            | 1.82                     | 3.57                     |
| 9                 | 2.77                        | 26.4                   | 2372                             | 0.37            | 1.33             | 5.71            | 3.65                     | 6.26                     |
| 10                | 0.99                        | 1.5                    | 2854                             | 0.67            | 1.06             | 3.09            | 3.48                     | 27.77                    |
| 11                | 0.99                        | 1.5                    | 2854                             | 0.76            | 1.15             | 2.94            | 1.41                     | 23.24                    |
| 12                | 0.99                        | 1.5                    | 2854                             | 0.79            | 1.47             | 3.12            | 14.29                    | 5.62                     |
| 13                | 0.85                        | 1.0                    | 3109                             | 0.19            | 0.32             | 2.46            | 1.94                     | 3.79                     |
| 14                | 0.85                        | 1.0                    | 3109                             | 0.35            | 0.83             | 3.28            | 2.60                     | 11.15                    |
| 15                | 0.85                        | 1.0                    | 3109                             | 0.36            | 0.72             | 3.11            | 2.71                     | 3.73                     |

With the preliminary assumption that this is compacted dust and the layer where it broke off is the transition from compacted to non-compacted dust (transition in tensile strength), this gives an indication for the compressed volume which will be analyzed in detail in the forthcoming section.

### 3.4. Dynamic Compression Experiments

#### 3.4.1. Experimental Setup

In order to investigate in more detail the compression behavior of the dust aggregates by collisions, we performed impact experiments with subsequent X-ray microtomography (XRT) measurements to analyze the degree of compaction.

Under vacuum conditions, we dropped a single glass spherule with a diameter of \( \sim 1 \text{ mm} \) from a given height of \( \sim 75 \text{ mm} \) into an RBD dust sample within a plastic tube with 7 mm diameter. To ensure the sphere to preferably hit the center of the dust sample within the narrow plastic tube, the released projectile was guided by falling through a tube. Due to friction and collisions with the tube’s walls, the impact velocity of \( (0.8 \pm 0.1) \text{ m s}^{-1} \), that was independently measured by high-speed imaging in 10 drops, is much lower than expected from free fall. However, the velocity in the specific experiment was not measured and can well be in the lower range of the error. From comparison of the observed
Figure 6. Best relation for the penetration depths from a \(\chi^2\) test yields a dependence of \(D \propto m^{0.23}v_0^{0.89}\). The intuitive relation \(D \propto mvA^{-1}\) is possible within the uncertainties.

Figure 7. Dust sticks to the projectiles after pulling them out of the dust sample. This is an indication of compacted material under the projectile as will be confirmed in Section 3.4.

Figure 8. Setup of the X-ray micro-CT measurement: the sample is rotated between an X-ray source and a detector. A three-dimensional density reconstruction can be computed from the transmission images.

penetration depth (see Figure 9 in Section 3.4.2) with the results in Figure 6, we expect a velocity of \(v = 0.65\) m s\(^{-1}\), which we will use for the further study.

For analyzing the density distribution of the dust-sample cutout with the embedded glass sphere, the dust sample was scanned by an X-ray micro-computer-tomograph (Micro-CT SkyScan 1074) at the University of Osnabrück. The dust sample was positioned on top of a rotatable sample carrier between the X-ray source and the detector (CCD camera; Figure 8). While rotating stepwise around by 360°, 400 transmission images were captured. Based on this data set, a three-dimensional density reconstruction was calculated by the SkyScan Cone-Beam Reconstruction Software provided with the X-ray micro-CT instrument.

3.4.2. Experimental Results

In the following, we present the results of two impact experiments. Further experiments with differently sized spheres and different impact velocities are intended. To visualize the spatial density distribution of the observed dust sample with the impacted glass sphere, the three-dimensional reconstruction data were cylinder-symmetrically averaged with the vertical axis aligned with the sphere center. Figure 9 displays the mean volume filling factor as a function of height and radius, whereas the data are mirrored with respect to the vertical center line of the diagram. The color gradient from yellow to light blue underneath the impacted sphere (red color: saturated density values of the considerably denser glass spherule) clearly shows the densification of the porous dust sample with an initial volume filling factor of \(\phi_0 \approx 0.15\). The compressed area, emphasized by overplotted contour lines, is located almost cylindrically shaped beneath the sphere and extends only slightly over the lateral borders of the sphere. Thus, the assumption of an omnidirectional compression curve, made in Section 3.2, seems to be justified.

Analysis of the distribution of occurring volume filling factors related to their fraction of volume within an uncompressed dust sample provides a Gaussian-shaped distribution with a mean value of \(\phi \approx 0.15\) (Figure 10). Figure 11 (top) shows the volume fraction (normalized by the sphere volume) of volume filling factors, which we determined only regarding the compacted volume underneath the impacted sphere for the two impact experiments. In both curves, the most prominent volume filling factor is around \(\phi = 0.23\), indicated by the dashed and dotted lines. The decreasing left flank of the curves corresponds to the transition region between compressed and uncompressed dust materials (see right curve flank of Figure 10). The same data plotted in a cumulative way (Figure 11, bottom) represent the
amount of compacted volume in units of the sphere volume that complies with a volume filling factor greater than a certain value. According to the volume filling factor values at the boundary to the uncompressed dust, given by the minima of the left side of the curves in Figure 11 (top), we can conclude from the cumulative curves (Figure 11, bottom) that the compressed volume due to an impacting sphere of 1 mm size into a high-porosity dust sample \( \phi \approx 0.15 \) fills the volume of \( \sim 0.8–1.2 \) sphere volumes.

### 3.5. Requirements of a Dynamic Compressive-Strength Curve

As seen in the previous sections, we have abundant indirect information about the compression behavior of loose dust samples. However, the basic question how the dynamic compressive-strength curve, \( \phi(\Sigma) \), looks, remains unanswered. We approach this problem the following way: (1) for low compressions, \( \Sigma \to 0 \), the volume filling factor is given by the initial properties of the material, i.e., \( \phi \to \phi_1 \) (see Table 1). (2) The maximum compression for \( \Sigma \to \infty \) is given by the value \( \phi_2 \) in Table 1 for the omnidirectional case, because the XRT analysis shows no material creeping sideways as was the case for the unidirectional flow (see Blum & Schräpler 2004). (3) With these two limits in mind, we apply Equation (9) as an approximation to the functionality of the dynamic compressive strength, which leaves us with the two free parameters \( \Delta \) and \( p_m \). The maximum slope of the compression at \( \Sigma = p_m \) is given by \( d\phi/d \log \Sigma = (\phi_2 - \phi_1)/\Delta \). For the unidirectional and omnidirectional static curves, we get slope values of 0.55 and 0.79, respectively (see Table 1). These are in fact not so different so that we adopt for the dynamic case the slope of the omnidirectional compression. Thus, we assume \( \Delta = 0.58 \) dex for the dynamic case. A refined study that takes both, \( \Delta \) and \( p_m \), as free parameters will be conducted in R. J. Geretschauser et al. (2009, in preparation), but in this paper we only vary \( p_m \).

### 4. CALIBRATING THE SPH CODE

The laboratory experiments in the previous section provided the static omnidirectional compressive strength \( \Sigma \) and the tensile-strength relation \( T \) as most important ingredients for the Sirono porosity model implemented in the SPH code by R. J. Geretschauser et al. (2009, in preparation). However, as it was already pointed out in the laboratory section, the compressive-strength relation has to be considered dynamically. The only free parameter \( p_m \) (see Section 3.5) cannot be determined by experiments. Hence, it has to be constrained by a numerical parameter study. We will use the stopping time of the impacting glass bead as reference for this parameter.

In addition, a relation for the shear strength is also very hard to measure in the laboratory. Therefore, we suggest three simple relations depending on the dynamic compressive-strength and tensile-strength relations and use the qualitative comparison of the filling factor profile under the glass bead after impact to constrain this unknown quantity.

Finally, we utilize the remaining experimentally measured independent features of the experiments described in the laboratory section to validate our calibration.

#### 4.1. Benchmark Test—Setup

The given experimental setup (see Section 3.4) was modeled with high resolution in two dimensions. Initially, the SPH particles were put on a triangular grid. All simulations were performed with the influence of gravity taken into account.

The projectile was modeled with a circle of 1.1 mm in diameter consisting of 1519 SPH particles. Its material properties were simulated using the Murnaghan equation of state

\[
p = \left( \frac{K_0}{n} \right) \left[ \left( \frac{\rho}{\rho_0} \right)^n - 1 \right]
\]

with \( \rho_0 = 2540 \text{ kg m}^{-3} \) (total two-dimensional mass per unit length \( m_{2D} = 2.4 \times 10^{-3} \text{ kg m}^{-1} \)), \( K_0 = 5.0 \times 10^9 \text{ Pa} \), and \( n = 4 \). The density has been chosen such that it matches the experimental specifications. The other material parameters are similar to those of sandstone. They can be found together with the Murnaghan equation of state in Melosh (1989). The exact choice of the bulk modulus \( K_0 \) and the Murnaghan exponent \( n \) does not have significant effects. The glass bead was treated as fully elastic. The impact velocity was 0.65 m s\(^{-1}\).

The dust sample was modeled as a 8 \( \times \) 5 mm\(^2\) rectangle with 64,421 SPH particles. About 0.15 mm at the bottom and 0.56 mm at each side of the rectangle were used as reflecting boundary by setting their acceleration to zero at each time step. The porous material was simulated by using the modified version of the Sirono model presented in Section 2. The initial density was expressed via the filling factor \( \phi = \rho/\rho_s \) with \( \phi = 0.15 \) and \( \rho_s = 2000 \text{ kg m}^{-3} \). For the tensile strength, we used the semianalytical relation, derived in Section 3.1 (Equation (8)) that matches the findings of Blum & Schräpler (2004) and Paper I. The bulk modulus was modeled by a power law

\[
K(\rho) = K_0 \left( \frac{\rho}{\rho_i} \right)^4
\]

with \( K_0 = 300 \text{ kPa} \) and the initial density of the dust aggregate \( \rho_i = 300 \text{ kg m}^{-3} \). The bulk modulus \( K_0 = \rho_i c^2 \) for uncompressed material was determined by the measurement of the
sound speed, which is \( c = 30 \text{ m s}^{-1} \) (Blum & Wurm 2008; Paszun & Dominik 2008).

For the compressive strength, several different relations were tested. At first we adopted the relation from the uniaxial experiments by Blum & Schräpler (2004). Second, we used the omnidirectional compression curve presented in this paper. After it turned out that a modified relation for the dynamical compressive-strength curve had to be considered, the omnidirectional compression curve (Equation (10)) was shifted toward lower pressure regimes using the free parameter \( p_m \) (see Section 3.5).

Since no experimental data were available for the shear strength \( Y \), parameter studies were carried out with three different relations \( Y(|T|, \Sigma); Y = |T|, Y = \Sigma \) and, following Sirono (2004), \( Y = \sqrt{\Sigma|T|} \), which represents the geometric mean of both quantities.

Due to reasons of stability, the two materials in contact (solid projectile, dusty target) have to be separated by artificial viscosity. We use the approach by Monaghan & Gingold (1983) and apply an \( \alpha \)-viscosity of 1.0 to all particles of the sphere and all particles interacting with the sphere. All other dust-sample particles were simulated without artificial viscosity following Sirono (2004). All details regarding the SPH code can be found in R. J. Geretshauser et al. (2009, in preparation).

4.2. Calibration Procedure

With the aim of reproducing the experimental results presented in the laboratory section, an SPH simulation using the omnidirectional compressive strength curve (ODC) was conducted. In the resulting pressure regime, the ODC relation and the relation from Blum & Schräpler (2004) are almost identical. Therefore, they can be treated as one case.

The impact velocity of the 1.1 mm glass bead was 0.65 m s\(^{-1}\) and we will compare the results of the simulation with the vertical density profile along a line through the center of the sphere perpendicular to the bottom of the dust sample, which was measured with X-ray microtomography as described in Section 3.4. Figure 12 shows the results for three different shear strength models, which are compared to two density profiles as measured in the experiments (lines with blue and green crosses). The initial surface of the dust sample is at 0 mm.

For the original ODC relation \( \left( p_m = 13 \text{ kPa} \right) \), the simulations for all shear strength models resulted in a much too shallow intrusion depth and an insufficient maximum filling factor underneath the sphere. These findings indicated, that the compressive-strength curve had to be modified in order to reproduce the experimental data. Therefore, we performed a parameter study varying the parameter \( p_m \), i.e., shifting the compressive-strength curve to lower pressures for the different shear strength models. For the complete study, see R. J. Geretshauser et al. (2009, in preparation). Independent experiments (Paper III) also support a lower \( p_m \) which can quantitatively explain the amount of compression in bouncing collisions.

A significant increase of the intrusion depth was only observed in case of \( Y = \sqrt{\Sigma|T|} \) and \( Y = \Sigma \) (see Figure 12, top and center). In case of \( Y = |T| \), the intrusion depth hardly changed with decreasing \( p_m \) (Figure 12, bottom). Since the shear strength remained constant and changing \( p_m \) did not have a significant effect, it can be concluded that shearing plays an important role during the intrusion.

Compared to the other cases, the shear strength reaches its highest values in the \( Y = |T| \) case. Hence, the material can hardly be pushed away due to shear and has to be compressed. Therefore, the highest filling factors can be found in this case (see Figure 12, bottom). The \( Y = \Sigma \) model yields the lowest shear strength values. Hence, material is mostly sheared aside, less material is compressed and therefore this model leads to filling factors below the reference data (see Figure 12, center).

Figure 13 shows intrusion depth over stopping time regarding the shear strength models \( Y = \Sigma \) and \( Y = \sqrt{\Sigma|T|} \) for all \( p_m \). Since \( Y = |T| \) did only yield insufficient intrusion depths, this model was omitted here. A good time/depth match was achieved for \( p_m = 3.9 \text{ kPa} \) using \( Y = \Sigma \) and for \( p_m = 1.3 \text{ kPa} \) using \( Y = \sqrt{\Sigma|T|} \). However, the \( Y = \Sigma \) model cannot reproduce the high values in the vertical filling factor profile (Figure 12, center) whereas the \( Y = \sqrt{\Sigma|T|} \) model yields an almost perfect match (Figure 12, top). Therefore, the latter with \( p_m = 1.3 \text{ kPa} \) gives a good match in Figure 12 (top) as well as in Figure 13 and is therefore used for further simulations. A more detailed study on the determination on the \( p_m \) value can be found in R. J. Geretshauser et al. (2009, in preparation).
Hereby, we have determined parameters for all previously unknown material relations and thus have calibrated the SPH model with respect to the presented experiments. The resulting strength curves of compression (Equation (10)), $p_m = 1.3$ kPa), tension (Equation (8)) and shear ($Y = \sqrt{\Sigma T}$) are illustrated in Figure 14.

However, the fact that the filling factor does not rapidly drop to $\sim 0.15$ at a depth of 1.5 mm requires further investigation.

### 4.3. Reproducing Experimental Features

Since intrusion time and depth as well as the filling factor profile underneath the sphere have been used to determine $p_m$ and the correct shear strength model, further features have to be reproduced in order to validate the calibration.

One of these features is the cumulated volume over filling factor relation (Figure 15). While the filling factor profile only displays a cut through the compressed volume, this curve represents the total compressed volume with its filling factors. Both curves are not fully, but mostly independent from each other. The chosen model and $p_m$ value yield an almost perfect match for filling factors $> 0.22$. The deviation for lower filling factors is due to the larger amount of compressed volume. This effect was already seen in the filling factor profile and is also very prominent in the comparison of the spatially density distribution plots (compare Figures 9 and 16).

Another feature to be reproduced is the relation $D \propto mvA^{-1}$ found in a similar way in the drop experiments (compare to Figure 6). We performed a series of two-dimensional simulations with spheres of 1 mm and 3 mm diameter and evaluated the maximum intrusion depth with respect to the impact velocity $v$. The latter was varied from 0.1 m s$^{-1}$ to 1.0 m s$^{-1}$ in steps of 0.1 m s$^{-1}$.

Two-dimensional simulation and experiment cannot be compared directly due to the different geometry (the two-dimensional setup represents a cut through an infinitely long cylinder). The advantage of using the quantity $mvA^{-1}$ instead of the more accurate Equation (12) is given by the fact that the former can be “converted” into two-dimensional by the following correction:

\[
\frac{m_{3D}v}{A_{3D}} = \frac{4\pi r^3 \rho \cdot v}{\pi r^2} = \frac{8 \pi r^2 \rho \cdot v}{2r} = \frac{8 \pi r^2 \rho \cdot v}{2r} = \frac{8 \pi r^2 \rho \cdot v}{2r} = \frac{8 m_{2D}v}{A_{2D}}.
\]

In comparison with the experimental results, the data from the simulation match very well for $mvA^{-1} > 1.0 \text{ kg m}^{-1} \text{ s}^{-1}$ (Figure 17). For smaller values, the simulation yields a shallower
Figure 17. In the momentum–intrusion relation, the agreement between simulation and experimental results is very good for values of \(\frac{mv}{A} \gtrsim 1\) kg m\(^{-1}\) s\(^{-1}\).

Figure 18. Normalized deceleration curve compared to the results. The deceleration curve in the SPH simulation is slightly lower than the experimentally observed sine curve, but well within the errors. This effect will be analyzed in future work. However, the range of experimental data encompasses the simulation results.

Comparing the simulated and experimentally acquired normalized deceleration curves (Figure 18), the simulated data slightly deviate from the experimental mean but remain within standard derivation limits. The deviation could arise from the geometric difference of the two-dimensional and three-dimensional case and has to be investigated in future works.

5. APPLICATION OF SPH TO DUST COLLISIONS IN PPDs, CONCLUSIONS, AND OUTLOOK

In this section, we will present some preliminary applications of SPH simulations to dust collisions in protoplanetary disks. We will present two examples of previously unfeasible calculations of interparticle collisions among macroscopic dust aggregates and will qualitatively compare them to similar dust experiments performed in the laboratory. Then, we will speculate about how the SPH code should be used in research on protoplanetary growth. Finally, we will sketch future work in preparation.

5.1. Qualitative Comparison Between SPH Simulations and Laboratory Experiments

The strength of the SPH simulations—besides the well-known examples in hyper-velocity collisions—over laboratory experiments and molecular-dynamics simulations is that low-velocity collisions among arbitrary dust aggregates can be investigated. Here, we show two examples recently observed in the lab, which can so far not be described by any other model. Example 1 deals with the frequently observed bouncing collisions in aggregate–aggregate interactions. Example 2 describes the impact of a single dust aggregate onto a solid flat target, which shows the co-occurrence of (partial) sticking and fragmentation.

5.1.1. Example 1

Bouncing in collisions between dust aggregates has been observed in many laboratory experiments (Blum & Münch 1993; Langkowski et al. 2008, Paper III; D. Heißelmann et al. 2009, in preparation), although molecular-dynamics simulations always show a direct transition from sticking to fragmentation when the collision energy exceeds a threshold value (Dominik & Tielens 1997; Wada et al. 2007, 2008). Nature obviously chooses a wider bouncing transition between those two stages, at least for aggregates above a certain size. It turns out that the SPH method is capable of describing the bouncing phase quite well. We have run a three-dimensional SPH simulation of a low-velocity impact of a 1 mm (diameter) fluffy aggregate onto a flat target. Due to symmetry arguments, this is identical to a two-aggregate (central) collision with twice the collision velocity. In our case, the aggregate was composed of 33,377 SPH particles and had an initial volume filling factor of 0.15. All other material parameters were identical to those in the previous section, i.e., \(K_0 = 300\) kPa, \(p_m = 1.3\) kPa, and \(Y = \sqrt{\sum I}\). The impact velocity was 0.2 m s\(^{-1}\), matching exactly the situation in the aggregate-wall experiments performed in Paper III and also those in the aggregate–aggregate collisions investigated by D. Heißelmann et al. (2009, in preparation) with a collision speed of 0.4 m s\(^{-1}\). Figure 19 shows a sequence of snapshots with a cut through the center of the aggregate, indicating the internal compaction due to the impact. Our simulation can correctly predict the coefficient of restitution of \(\sim 0.2\) (Blum & Münch 1993; D. Heißelmann et al. 2009, in preparation), although details in the compaction behavior still deviate from the laboratory results, which might be caused by insufficient resolution in the SPH simulation.
5.1.2. Example 2

In the previous example, we have seen that bouncing marks the broad transition regime between sticking and fragmentation. However, in the case of the impact of a dust aggregate onto a solid target, laboratory experiments have shown that, for impact experiments above the fragmentation threshold, fragmentation is always accompanied by partial sticking of the aggregate to the target. This effect was first found by Wurm et al. (2005) for compacted dust aggregates and impact velocities above 25 m s⁻¹ and later confirmed in our laboratory for \( \phi = 0.35 \) aggregates and impact velocities above 1 m s⁻¹. Figure 20 shows an image sequence of an impact experiment with fragmentation and partial sticking. An average of 10% of the projectile mass sticks to an initially smooth target at normal impact, which is consistent with the low-velocity results of Wurm et al. (2005). The remainder of the projectile mass is fragmented into a power-law mass distribution (see Blum & Münch 1993). The fragments leave the target under extremely flat angles. Our SPH simulation (Figure 21, left) featuring the calibration parameters of Section 4.2 cannot reproduce the fragmentation behavior seen in the experiments. Here, the predominant part of the dust sample sticks to the target. Only a few bigger chunks and single SPH particles burst off. However, a simulation with the same setup, but using the shifted unidirectional compressive-strength relation (Section 3.2) and a shear strength that is equal to the tensile strength, matches the experimental observations at least qualitatively (Figure 21, right). From that we conclude that the SPH code is in principle capable of simulating fragmentation of highly porous aggregates, even without the damage model adopted in the original Sirono (2004) porosity model.

We conclude that the shear model \( Y = \Sigma^{0.5} \cdot |T|^{0.5} \) tested for the dynamic compression experiments (Section 4.1) is unable to explain the fragmentation findings which are rather dominated by shear and tension, whereas a shear model \( Y = \Sigma^{0} \cdot |T|^{1} \) shows qualitative agreement. The imperfect shear model can also be responsible for the narrow but deep compressed volume in Figure 16 compared to Figure 9. A future task will therefore be to refine the shear calibration in a way that we will use \( Y = C \cdot \Sigma^{\alpha} \cdot |T|^{1-\alpha} \) with the free parameters \( C \) and \( \alpha \). Comprising both experiments for calibration, we will be able to find a shear model that can reproduce both cases.

5.2. Use of the SPH Code in Research on Protoplanetary Growth

The above examples show that the SPH method is a powerful tool to investigate the outcomes of protoplanetary dust collisions. When properly calibrated with laboratory experiments, SPH calculations allow access to parameter-space regions that are unavailable to laboratory experiments. Whereas molecular-dynamics simulations can be used for studying collisions of very small dust aggregates, SPH is most useful for very large samples. Such samples, particularly those with fluffy compositions,
cannot be built or treated in laboratories, and the experimental study of collisions seems impossible.

A particularly interesting and still unsolved problem is the dichotomy in the collision behavior of pairs of dust aggregates with similar and different sizes, respectively. In Paper II, we found sticking by deep penetration for impacts of millimeter-sized dusty projectiles into flat, centimeter-sized dusty targets ("projectile–target" collisions) above $\sim 1$ m s$^{-1}$. Both dust aggregates, projectile and target, consisted of identical particles and had equal porosity. Using similar dust aggregates, but giving projectile and target comparable size ("projectile–projectile" collisions), Blum & Münch (1993) and D. Heißelmann et al. (2009, in preparation) found that collisions either lead to bouncing or to fragmentation. Bouncing instead of sticking was also observed in Paper II when the target aggregates were prepared such that the local radius of curvature corresponded to the projectile’s radius. To find out where the boundary between "projectile–target" and "projectile–projectile" collisions occurs, will be one of our future applications of our SPH code.

5.3. Future Work

We have only begun to explore the potentials of SPH simulations of collisions between protoplanetary dust aggregates. Before we can start to investigate the full parameter space in protoplanetary dust collisions, i.e., before we can begin to find out what the collisional outcome is for all combinations of aggregate size, porosity, collision velocity, impact angle, state of rotation, temperature and state of sintering, material, and size (distribution) of the constituent dust grains, etc., the material parameters of macroscopic dust aggregates have to be fully explored. This will be the next task in our investigation. To achieve this, we will perform more calibration experiments of the type described in this paper for dust aggregates of various compositions and porosities. In addition to that, other calibration experiments will be explored, like the ones described in Sections 5.1.1 and 5.1.2.

We thank M.-B. Kallenrode and the University of Osnabrück for providing access to the XRT setup and Jens Teiser for the first feasibility tests for the experiments. The SPH simulations were performed on the university and bwGrid clusters of the computing center (ZDV) of the University of Tübingen. This project was funded by the Deutsche Forschungsgemeinschaft within the Forschergruppe 759 “The Formation of Planets: The Critical First Growth Phase” under grants Bl 298/7-1, Bl 298/8-1, and Kl 650/8-1.

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