Supplementary Information –
Conditional control of the quantum states of remote atomic memories for quantum networking

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Supporting documentation is provided for our manuscript Ref. [1].

I. DECOHERENCE

Figure 1 shows the variation of the conditional probabilities of detecting one ($p_{c2}$) and two ($p_{c22}$) photons in fields $2_L$ and $2_R$, once the two ensembles are ready to fire, as functions of the number $N$ of trials that occur between the two field-1 detections. This figure was obtained from the same raw data as Fig. 2 of Ref. [1]. Fields $2_L$ and $2_R$ have then orthogonal polarizations, and are combined at a beam splitter as shown in Fig. 1b of Ref. [1]. In order to obtain the quantities in Fig. 1, we divided the number of coincidences in field 2 that followed two temporally separated detections in field 1 by the number of times the two ensembles were prepared with that specific time separation. The solid lines are fittings considering an exponential decay of the conditional probability $p_c$ for the second photon from either of the two ensembles, once a detection has occurred in field 1. We assumed the same $p_c$ and decay time for the two systems. Note that the two systems were actually set up to have similar $p_c$ and similar Raman linewidths for transitions between the hyperfine ground states [2], which should correspond to the system coherence time. The expressions used to fit were then

$$p_{c22}(N) = \frac{p_c^2}{2}e^{-N/N_c},$$  \hfill (1)

$$p_{c2}(N) = \frac{p_c}{2}e^{-N/N_c} - p_{c22}(N).$$  \hfill (2)

FIG. 1: Conditional probabilities $p_{c2}$ and $p_{c22}$ of measuring one (red circles and green triangles) and two (black squares) photons in field 2, respectively, once the two ensembles are ready to fire. The red and black curves are fits using Eqs. (1) and (2), as discussed in the text. The green line is 0.95× the red line, as the field 2 level measured by $D_{\text{2b}}$ is always 5% lower than for $D_{\text{2a}}$, indicating a possible difference of detection efficiency of this magnitude.

We assume above that $p_{c22}$ always involves one photon coming from an excitation stored during $N$ trials. In this way, we are neglecting the two-photon component of field 2, as well as diverse sources of background. For $p_{c2}$, the first two terms take into account that the conditioned detection event can be originated from either an ensemble that has just been excited, or a stored (for $N$ trials) excitation. The third term subtracts the probability of having a joint detection...
on field 2 (\(p_2^c\) gives the probability of detecting an event on one detector and zero on the other). From the fitting, we then obtain \(p_c = 0.091\) and \(N_c = 18\), corresponding to a coherence time \(\tau_c = N_c \times 0.525\mu s = 9.5\mu s\). Keeping in mind the simplicity of the above expressions, which do not take into account any background in field 2 or its two-photon component, the inferred conditional probability \(p_c\) is then consistent with the independently measured values of about 0.085 for each ensemble.

From the above discussion, it is then straightforward to obtain an expression taking into account decoherence for the measured \(p_{1122}\), presented in Fig. 2 of Ref. [1]. Note first that, in the ideal case of very long coherence time (\(N_c \to \infty\)), \(p_{1122}\) can be obtained from the \(p_{11}\) expression presented in the Methods section of Ref. [1] by multiplying it by the conditional probability \(p_{22}^c(0)\) of obtaining a pair of photons with effectively zero delay (\(N/N_c \to 0\)) between them:

\[
p_{1122}^{\text{ideal}} = \frac{(2N-1)p_1^2p_2^c}{2}, \text{ when } p_1 \ll 1.
\]

Considering the measured value of \(p_2^c\) and the value of \(p_c\) obtained from the fits of Fig. 1, we then obtain the green line plotted in Fig. 2. In order to introduce decoherence in this analysis, each term of the \(p_{11}\) expression deduced in the Methods section should be multiplied by the proper \(p_{22}^c(N)\) as defined in Eq. 1:

\[
p_{1122}(N) = p_1 \left( p_1 p_{22}^c(0) + 2 \left[ (1-p_1)p_1 p_{22}^c(1) + (1-p_1)^2 p_1 p_{22}^c(2) + \cdots + (1-p_1)^{N-1} p_1 p_{22}^c(N-1) \right] \right).
\]

A plot of this expression, considering the \(p_c\) and \(N_c\) obtained from the fits in Fig. 1, is shown as the red curve in Fig. 2. The quite reasonable agreement with the experimental data (filled squares) indicates then that the experimentally observed increase in \(p_{1122}\) can be understood by the increase in \(p_{11}\) provided by the circuit combined with the decoherence of the stored collective excitation.

![FIG. 2: Probability \(p_{1122}\) of coincidence detection as functions of the number \(N\) of trials waited between the independent preparations of the two ensembles (L, R) with 1 excitation each. Filled squares give the measured joint probability \(p_{1122}\) of preparing the two ensembles and detecting a pair of photons, one in each output of the beam splitter, in fields \(2_L, 2_R\). Error bars indicate statistical errors. The polarizations for fields \(2_L, 2_R\) were set to be orthogonal. These experimental results were also presented in Fig. 2 of Ref. [1]. The green curve gives the theoretically expected \(p_{1122}^{\text{ideal}}\) for the ideal case of very long coherence time, as given by Eq. 3. The red curve gives the theoretical \(p_{1122}\) for the case of a finite coherence time, Eq. 4.](image)

Note finally that \(p_{1122}\) times the number of trials per second gives the rate of conditional joint detections in fields \(2_L, 2_R\). In this way, as shown in Fig. 2 we can see that a larger coherence time could still enhance this detection rate by up to a factor of 1.6 for \(N = 23\) (enhance the factor \(F_{1122}\) from 28 to 45). An increase on the conditional probability \(p_c\) would also greatly improve this rate, since it scales with \(p_2^c\). As discussed in the text, we infer that the probability \(q_c\) of extracting the photon from the ensemble is about 34% for our experimental conditions. Thus, considering the same amount of losses on the field 2 pathways and the same detection efficiencies, we infer that ideally, if one achieved \(q_c = 1\), \(p_c\) can still be increased by up to a factor of 3, which would increase the joint detections rate by 9. This indicates a good prospect for further optimizations of our system in the future.
II. VISIBILITY AS A FUNCTION OF $w$

In the case of two indistinguishable single-photon wavepackets combined at a 50/50 beam splitter (BS), no coincidences should be observed at the two outputs of the BS [3]. However, if the combined fields are not perfectly overlapping single photons, coincident detections at the output of the BS can occur due to the two-photon component in each input port. In this section, we evaluate the loss of visibility (as defined in Ref. [1]) due to this effect with a simple model.

Suppose that the two ensembles are prepared each with stored excitation, as heralded by a detection in field 1 for both ensembles. Let’s denote $P_n$ the probability of finding $n$ photons in field 2, and assume, for simplicity, the various $P_n$ are the same for both ensembles. In each field (before the BS), the two-photon suppression is characterized by the parameter $w$ [4]:

$$w = \frac{2P_2}{P_1^2},$$

so that the two-photon component can be written as:

$$P_2 = \frac{wP_1^2}{2}.\quad (6)$$

Let us now combine the two fields at the BS. The probability to have one photon at each output of the BS, when the two wavepackets do not overlap (e.g., if they have orthogonal polarizations) is given by:

$$p_\perp = \frac{P_2}{2} + \frac{P_2}{2} + \frac{wP_1^2}{4} + \frac{wP_1^2}{4} + \frac{P_1^2}{2},\quad (7)$$

where the two first terms corresponds to the terms with two photons in one input mode of the BS, and the third term to the case with one photon in each input mode. The factor $1/2$ corresponds to the 50% chance that the photons split at the BS. In this simplified calculation, we neglect the case where we have two photons in one input and one in the other one, whose probability is on the order of $P_3^1$.

If the two fields overlap perfectly at the BS (parallel polarizations), the term with one photon in each input does not lead to coincidences, and the probability to have one photon in each output is then:

$$p_\parallel = \frac{wP_1^2}{4} + \frac{wP_1^2}{4}.\quad (8)$$

Taking Eqs. (7) and (8) into account, we find that the visibility can be written as:

$$V = \frac{p_\perp - p_\parallel}{p_\perp} = \frac{1}{1 + w}.\quad (9)$$

In our case, we have $g_{12} \approx 23$ for the two ensembles, from which we estimate $w \approx 0.17$ [5]. This leads to a maximal visibility of $V_{\text{max}} = 0.85$ for a perfect overlap $\xi = 1.0$ between the fields. From our measured visibility of 0.77, we then estimate an overlap $\xi = 0.90$.

III. JOINT-DETECTION LEVELS FOR EVENTS IN DIFFERENT TRIALS

In Fig. 3b, we show how the conditional probability of detecting two photons, when the $(L, R)$ systems are ready, decreases as a function of the delay between the two detections for the situation where the fields $2_L, 2_R$ are combined with the same (red) or orthogonal (black) polarizations. It corresponds then to Fig. 3 of Ref. [1]. The time $t_d$ of the detections is obtained from the recording of events in our acquisition card, and it refers to a fixed reference that marks the beginning of the 525 ns repetition periods. From this list of detection times, we obtain the relative delay $\tau$ when the two detections occur in the same trial.

We also show in Fig. 3, the cases where the two detections in $D_{2a}$ and $D_{2b}$ occur in different trials, with the event in one detector occurring when the two ensembles are ready and the event in the other detector occurring the next time the ensembles are ready. Figures 3a and 3c give the cases in which one detector or the other registers an event first. The fact that the signal level is similar in both cases, with different polarizations, indicates that there is no large misalignment when the half-wave plate is turned to switch between the two polarization configurations. Even though, if we integrate the curves in Figs. 3a and 3c, the value obtained for orthogonal polarization is about 0.08 lower than...
the one obtained for parallel polarization. We confirmed this value by calculating it also from other different-trials peaks (for detection events separated by up to 5 ready signals). The curves with different polarizations were taking at alternate cycles of half-hour data taking, just turning a single half-wave plate between them. In this way, we believe the decrease for orthogonal polarizations comes just from a small misalignment in the fiber input introduced by this operation.

The level of the peak in (b) obtained with orthogonal polarizations should be half that observed in (a),(c) for the case where pure single photons arrive at the beam splitter. The experimental observed ratio \( r \) is found to be \( r = 0.60 \pm 0.05 \). This ratio can be explained by the two-photon component of our generated state. As previously done in section II, let’s denote \( P_1 \) and \( P_2 \) the probabilities of finding respectively one or two photons in each field 2. For the sake of simplicity, these probabilities are taken equal for the two ensembles. Including two-photon events, the probability for coincidence in the two detectors is given for the center peak (Fig. 2b) by:

\[
\frac{1}{2}P_1^2 + P_2
\]  

(10)

The first term takes into account the two cases where the single photons are both reflected or transmitted at the beam splitter. The second one corresponds to the case where two photons arriving at one input of the beam splitter are split into the two arms. Higher-order cases, which involve for instance two photons in each input, or two photon in one input and one in the other, are neglected.

FIG. 3: Conditional joint-detection probability \( p_{22}^c(\tau) \) of registering two events in \( D_2a \) and \( D_2b \), once a ready signal is generated as a result of the two ensembles being ready to fire, as a function of the time difference \( \tau \) between the two detections. (b) The two detections occur within the same trial. (a) Detector \( D_2a \) fires first and \( D_2b \) fires after the next ready signal. (c) Same as (a), but with the detector order inverted. The red circles (black squares) provide the results for field 2 from the two ensembles having orthogonal (parallel) polarizations.

In a similar way, the probability for the others peaks, where detections occur in different trials, can be written as

\[
(P_1 + 2P_2)^2
\]  

(11)

This expression corresponds basically to the mean photon number going to one detector times the mean photon number going to the other one.
With \( P_2 = wP_1^2/2 \), and by neglecting higher-order terms, the ratio becomes:

\[
\tau = \frac{1}{2} \frac{1 + w}{1 + 2wP_1}
\]

With \( P_1 = 0.085 \) and \( w = 0.17 \), this expression gives then an expected value \( r = 57\% \), which is consistent with the observed one.

IV. VISIBILITY AS A FUNCTION OF INTEGRATION WINDOWS

In Fig. 4 we show the results of the measurement of \( V \) for different integration windows around \( \tau = 0 \). We see that for an integration window around the center, from \( \tau = -6 \) ns to \( \tau = 6 \) ns, the visibility is \( 80 \pm 10\% \), while the integration using the whole window gives \( V = 77 \pm 6\% \), indicating that the suppression is roughly uniform for all \( \tau \), which is also consistent with having close to transform-limited wavepackets for both fields \( 2_L, 2_R \).

![Fig. 4: Visibility \( V \) of the two-photon coincidence suppression as a function of the integration window for the time difference between the two field-2 detections. The integration is made from \(-\tau_w\) to \(\tau_w\).](image)

V. TIME WINDOWS

The electronic time windows used for field 1 and field 2 detections were 80 ns and 90 ns long, respectively, positioned around the center of the respective wavepackets. In order to analyze the conditional field-2 wavepackets overlap in Fig. 3 of Ref. [1], and also in the above Figs. 3 and 4, we introduced in the analysis an additional time window, only 44 ns long around the conditional field 2. As can be seen in Fig. 4 of Ref. [1], this corresponds to consider the whole conditional field-2 wavepackets.

[1] Felinto, D. et al. manuscript submitted to Nature Physics (2006).
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