Interpreting $f_0(600)$ and $a_0(980)$ as $\bar{q}q$ states from an $N_f = 3$ sigma model with (axial-)vectors

Denis Parganlija

Institute for Theoretical Physics, Goethe University, Max-von-Laue-Str. 1, D-60438 Frankfurt am Main, Germany
E-mail: parganlija@th.physik.uni-frankfurt.de

Received 22 September 2011
Accepted for publication 16 November 2011
Published 28 September 2012
Online at stacks.iop.org/PhysScr/T150/014029

Abstract

We address the question of whether it is possible to interpret the low-lying scalar mesons $f_0(600)$ and $a_0(980)$ as $\bar{q}q$ states within a $U(3) \times U(3)$ linear sigma model containing vector and axial-vector degrees of freedom.

PACS numbers: 12.39.Fe, 12.40.Yx, 14.40.Be, 14.40.Df

1. Introduction

The structure of scalar mesons has been a matter of debate for many decades. Experimental data [1] suggest the existence of at least five $IJ^{PC} = 00^{+*}$ states in the region up to 1.75 GeV: $f_0(600)$, $f_0(980)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$. In the scalar sector, the existence of the scalar $K_0^*(1430)$ state has been confirmed, unlike the existence of the low-lying scalar kaon $K_0^*(800)$ (or $\kappa$). Kaons and other mesons containing strange quarks are expected to play an important role in vacuum phenomenology and also in the restoration of the $U(N_f)_L \times U(N_f)_R$ chiral symmetry [3], a feature of the quantum chromodynamics broken in vacuum spontaneously [4] by the quark condensate and explicitly by non-vanishing quark masses (here, $N_f$ is the number of quark flavours).

Meson phenomenology has been considered in various $\sigma$ model approaches (see [3] and references therein). In this paper, we present an extended linear sigma model (eLSM; [2, 5, 6]) containing scalar, pseudoscalar, vector and axial-vector mesons in both the non-strange and strange sectors. We have addressed the issue of the structure of scalar mesons $f_0(1370)$ and $f_0(1710)$ in [2]; the conclusion was that one could find a reasonable global fit of scalar, pseudoscalar, vector and axial-vector masses within a single model assuming that $a_0(1450)$ and $K_0^*(1430)$ are $\bar{q}q$ states. A consequence of the fit was the statement that $a_0(1450)$ and $f_0(1710)$ correspond to scalar $\bar{q}q$ states. However, the work [2] did not address a different possibility: whether such a fit could also be found assuming that $a_0(980)$ and $\kappa$, rather than $a_0(1450)$ and $K_0^*(1430)$, are $\bar{q}q$ states. If a reasonable fit exists in this case, then the results of [2] are inconclusive; however, if the opposite is true, then the suggestion is confirmed that scalars above 1 GeV favour being $\bar{q}q$ states.

This paper is organized as follows. In section 2 we present the Lagrangian, discuss the results in section 3 and provide the summary and outlook of future work in section 4.

2. The model

The Lagrangian of the extended linear sigma model with $U(3)_L \times U(3)_R$ symmetry reads [3, 5, 6]

$$L = \text{Tr}[(D^a \Phi)^\dagger (D^a \Phi)] - m_0^2 \text{Tr}(\Phi^\dagger \Phi)$$

$$- \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi \Phi^\dagger \Phi)$$

$$- \frac{1}{4} \text{Tr}[(L^{\mu \nu})^2 + (R^{\mu \nu})^2] + \text{Tr} \left[ \frac{m_1^2}{2} + \Delta \right] (L^{\mu})^2 + (R^{\mu})^2]$$

$$+ \text{Tr}[H(\Phi + \Phi^\dagger)]$$

$$+ \frac{i}{2} c_1 (\text{det} \Phi - \text{det} \Phi^\dagger)^2 + \frac{g_2}{2} \text{Tr}[L_{\mu \nu} [L^\mu, L^\nu]]$$

$$+ \text{Tr} [R_{\mu \nu} [R^\mu, R^\nu]]$$

$$+ \frac{h_1}{2} \text{Tr}(\Phi^\dagger \Phi) \text{Tr}[(L^{\mu})^2 + (R^{\mu})^2]$$

$$+ \frac{h_2}{2} \text{Tr}[(L^{\mu})^2 + (R^{\mu})^2] + 2 h_3 \text{Tr}(\Phi R_{\mu} \Phi^\dagger L^\mu),$$

(1)
\[ \Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} a_0^+ + i\sigma^+ & K_S^+ + iK^+ \\ a_0^- + i\sigma^- & K_S^- + iK^- \\ \sqrt{2} & \sqrt{2} \end{pmatrix} \]

is a matrix containing the scalar and pseudoscalar degrees of freedom, \( L^\mu = V^\mu + A^\mu \) and \( R^\mu = V^\mu - A^\mu \) are, respectively, the left-handed and right-handed matrices containing vector and axial-vector degrees of freedom with

\[ V^\mu = \frac{1}{\sqrt{2}} \begin{pmatrix} \lambda_{\pi\pi}^\rho \rho^+ & \rho^+ \\ \rho^- & \rho^- \end{pmatrix} \]

and \( \Delta = \text{diag}(\delta_N, \delta_S, \delta_S) \) describes explicit breaking of the chiral symmetry in the (axial-)vector channel. The explicit symmetry breaking in the (pseudo)scalar sector is described by \( \text{Tr}[H(\Phi + \Phi^\dagger)] \) with \( H = 1/2 \text{diag}(h_{ON}, h_{ON}, \sqrt{2}h_{ON}) \). \( h_{ON} \) const and \( h_{OS} \) const.

Also, \( D^\mu \Phi = \partial^\mu \Phi - i\mathcal{G}_1(L^\mu \Phi - \Phi R^\mu) \) is the covariant derivative; \( L^{\mu\nu} = \partial^\mu L^\nu - \partial^\nu L^\mu \), \( R^{\mu\nu} = \partial^\mu R^\nu - \partial^\nu R^\mu \) are, respectively, the left-handed and right-handed field strength tensors and the term \( c_1(\det \Phi - \det \Phi^\dagger)^2 \) describes the \( U(1)_A \) anomaly [7].

We assign the fields \( \pi \) and \( \eta_{K} \) to the pion and the \( SU(2) \) counterpart of the \( \eta \) meson, \( \eta_{K} \equiv (u\overline{u} + d\overline{d})/\sqrt{2} \). The fields \( a_0^\rho, \bar{a}_0^\rho, f_{IS}^1 \) and \( A_1^\rho \) are assigned to the \( \omega(782) \), \( \rho(770) \), \( f_1(1285) \) and \( a_1(1260) \) mesons, respectively [8]. We also assign the \( K \) fields to the kaons; \( \eta_S \) is the strange contribution to the \( \eta \) and \( \eta_S \) fields and the \( \bar{a}_0^\rho, f_{IS}^1, K^\rho \) and \( K_1^\rho \) fields correspond to the \( \phi(1020) \), \( f_1(1420) \), \( K^*(892) \) and \( K_1(1270) \) mesons, respectively.

The assignment of the scalar fields in our model to physical resonances is ambiguous. In this work, we assign the \( a_0^\rho \) field to \( a_0^\rho(980) \) and \( K \) to the physical \( K^\rho(800) \) state. We thus presuppose that these two states below \( 1 \text{ GeV} \) are \( \bar{q}q \) states (as all the fields present in our model are \( \bar{q}q \) states [8]) and describe in the following section whether a global fit of masses can be found under this assumption.

### 3. The global fit and two-pion decay of the sigma mesons

Results from our best fit are shown in table 1. We note that \( m_{\sigma}, m_{a_1}, m_{K_1}, m_{\omega}, m_{\sigma^0}, m_{\sigma^+}, m_{\sigma^0} \) deviate substantially from their respective experimental values [1]. This is in particular a problem for the rather sharp resonances \( \omega_S \equiv \varphi(1020) \) and \( f_1(1420) \equiv f_0(S) \).

In the scalar sector, if we set \( m_{\sigma_1} = 705 \text{ MeV} \) and \( m_{\sigma_2} = 1200 \text{ MeV} \), then we obtain \( \Gamma_{\sigma_1 \rightarrow \pi \pi} = 305 \text{ MeV} \) and \( \Gamma_{\sigma_2 \rightarrow \pi \pi} = 207 \text{ MeV} \) as well as \( \Gamma_{\sigma_1 \rightarrow K K} = 240 \text{ MeV} \). These results correspond very well to experiment [1, 9]. We thus assign \( \sigma_1 \) to \( f_0(600) \) and \( \sigma_2 \) to \( f_0(1370) \). Consequently, \( f_0(600) \) is interpreted as a predominantly non-strange \( \bar{q}q \) state, while \( f_0(1370) \) is interpreted as a predominantly \( \bar{s}s \) state. The results also suggest, however, that \( f_0(1370) \) should predominantly decay into kaons (as \( \Gamma_{\sigma_2 \rightarrow K K}/\Gamma_{\sigma_2 \rightarrow \pi \pi} = 1.15 \) — not surprising for an \( \bar{s}s \) state but clearly at odds with experimental data [1].

Additionally, the phenomenology in the vector and axial-vector channels is not well described. The decay width \( \Gamma_{\sigma_1(1260) \rightarrow \rho \pi} \) depends on the parameter \( g_2 \), fixed via \( \Gamma_{\rho \rightarrow \pi \pi} \) [8]. A calculation of the decay width \( \Gamma_{\sigma_1(1260) \rightarrow \rho \pi} \) then yields values of more than \( 10 \text{ GeV} \) if we set \( \Gamma_{\rho \rightarrow \pi \pi} = 149.1 \text{ MeV} \) (as suggested by the Particle Data Group [1]). Alternatively, if one forces \( \Gamma_{\sigma_1(1260) \rightarrow \rho \pi} < 600 \text{ MeV} \) to comply with the data, then \( \Gamma_{\rho \rightarrow \pi \pi} < 38 \text{ MeV} \) is
obtained—a value that is approximately 100 MeV less than the experimental result.

Then the fit results, and thus the assumption of scalar \( \bar{q}q \) states below 1 GeV, are problematic.

4. Summary and outlook

We have presented a \( U(3)_L \times U(3)_R \) linear sigma model with (axial-)vector mesons. A global fit of all masses (except the sigma masses) has been performed to determine model parameters. The fit included masses of scalar states \( \vec{a}_0 \) and \( K_S \), assigned, respectively, to \( a_0(980) \) and \( \kappa \). The fit does not yield particularly good mass values: several of the masses considered deviate by more than 100 MeV from the corresponding Particle Data Group value (see table 1). We find very good correspondence of \( \Gamma_{\sigma_1 \rightarrow \pi\pi} \) and \( \Gamma_{\sigma_2 \rightarrow K\bar{K}} \) to the experiment if one considers \( m_{\sigma_1} = 705 \) MeV and \( m_{\sigma_2} = 1200 \) MeV. We thus assign \( \sigma_1 \) to \( f_0(600) \) and \( \sigma_2 \) to \( f_0(1370) \). This implies that \( f_0(1370) \) decays predominantly into kaons—at odds with experiment [1]. Additionally, we cannot accommodate a correct (axial-)vector phenomenology into the fit: either \( a_1(1260) \) is too broad (\( \gtrsim 10 \) GeV) or the \( \rho \) meson is too narrow (\( \gtrsim 40 \) MeV). We thus conclude that the fit of [2], where the scalar states were assumed to be above (rather than, as in this work, below) 1 GeV, described meson phenomenology better than the fit presented here and that this work confirms the conclusion of [2] that scalar \( \bar{q}q \) states are favoured to have mass above 1 GeV.

Acknowledgments

I am grateful to Francesco Giacosa, Dirk Rischke, Péter Kovács and György Wolf for valuable discussions on my work.

References

[1] Nakamura K et al (Particle Data Group) 2010 J. Phys. G: Nucl. Part. Phys. 37 075021 and 2011 partial update for the 2012 edition
[2] Parganlija D 2011 Acta Phys. Polon. Suppl. 4 727 (arXiv:1105.3647)
[3] Lenaghan J T, Rischke D H and Schaffner-Bielich J 2000 Phys. Rev. D 62 085008 (arXiv:nucl-th/0004006)
[4] Goldstone J 1961 Nuovo Cimento 19 154
[5] Goldstone J, Salam A and Weinberg S 1962 Phys. Rev. 127 965
[6] Parganlija D, Giacosa F, Kovacs P and Wolf G 2011 Int. J. Mod. Phys. A 26 607 (arXiv:1009.2250)
[7] Klempt E, Metsch B C, Munz C R and Petry H R 1995 Phys. Rev. D 52 1383
[8] Dmitrasinovic V 1996 Phys. Rev. C 53 1383
[9] Ritter C, Metsch B C, Munz C R and Petry H R 1996 Phys. Lett. B 380 431 (arXiv:hep-ph/9601246)
[10] Dmitrasinovic V 1997 Phys. Rev. D 56 247
[11] Fariborz A H, Jora R and Schechter J 2008 Phys. Rev. D 77 094004 (arXiv:0801.2552)
[12] Parganlija D, Giacosa F and Rischke D H 2010 Phys. Rev. D 82 054024 (arXiv:1003.4934)
[13] Bugg D V 2007 Eur. Phys. J. C 52 55 (arXiv:0706.1341)