Derivations of Atomic Ionization Effects Induced by Neutrino Magnetic Moments

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Abstract

A recent paper [M.B. Voloshin, Phys. Rev. Lett. 105, 201801 (2010)] pointed out that our earlier derivations of atomic ionization cross-section due to neutrino magnetic moments (arXiv:1001.2074v2) involved unjustified assumptions. We confirm and elaborate on this comment with these notes. We caution that the results of the sum-rule approach in this paper contradict the expected behaviour in atomic transitions.

Advances in low energy detectors make it relevant to evaluate atomic effects induced by possible neutrino electromagnetic interactions. A recent paper (Ref. [1]) observed that there are unjustified assumptions implicit in our previous derivations of the atomic ionization (AI) cross-section induced by neutrino magnetic moment ($\mu_\nu$) (Ref. [2]). This comment is correct.

We use the pre-defined notations in Ref. [2] and work with positive $q^2$ for clarity. The $d\sigma/dT$ formula in Eq.10 is due to integration of Eq.8 over $d\Omega$ which, implicitly, is integration over $q^2$. However, the $q^2 \to 0$ limits have been taken in the assignments of the form factors $F_a(q^2, T)$ and $q^2 F_b(q^2, T)$. There is an unjustified assumption in Eq.10 that the form factors are constant within the integration range of $q^2$ from 0 to $\sim 4E_\nu^2$ (where $E_\nu \sim$ few MeV for reactor neutrinos). Consequently, Eq.10 as well as the results that follow are invalid.

The $q^2$-dependent components of Eq.8 in the laboratory frame can be written as:

$$\frac{d^2 \sigma}{dT d\Omega} \propto \left[ ME_\nu \left( \frac{E_\nu}{T} - 1 \right) F_a(q^2, T) + \frac{1}{4} q^2 F_b(q^2, T) \right] ,$$

where $F_a(0, T) \propto \sigma_{\gamma}(T)$.

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There are no experimental constraints on $F_a(q^2,T)$ and $F_b(q^2,T)$. It is natural to expect the electron mass scale ($m_e$) plays an important role. In the case where the form factors are exclusively defined by $m_e$, such that they are suppressed at $q^2 > 2m_eT$ ($\sim (0.1 \text{ MeV})^2$), the AI effects would be small compared to the free-electron cross-section. If, however, a higher mass scale like that of the atomic mass may have even a minor role to play in the process, the form factors can be finite up to $q^2 \sim E_\nu^2$. Large AI contributions are possible in this scenario and the discussions of Ref. [2] would still hold.

It is instructive to note how the equivalent photon approximation approach of Ref. [2] does produce valid results in two similar, but non-identical, problems.

1. $\mu_\nu$-induced deuteron disintegration with solar neutrinos [3] – the form factors are defined by the nucleon mass scale ($\sim \text{GeV}$) and so can be taken as constant within the range of integration up to $q^2 \sim (10 \text{ MeV})^2$, so that Eq.10 remains valid.

2. Charge-induced AI processes with relativistic minimum ionizing particles [4] – the kinematics involves an additional $(1/q^2)$ weight factor. The integral is dominated by contributions at $q^2 \to 0$ and insensitive to the behaviour of the form factors at large $q^2$. It is adequate to describe them by the physical photoelectric cross-section at $q^2 = 0$.

Ref. [1] adopted a sum-rule approach to arrive at an inclusive cross-section of $\mu_\nu$-induced scattering with atomic electrons. This is given in Eq.13 as:

$$\frac{d\sigma}{dT} \propto (1 - \delta) \frac{Z}{T}, \text{ where } \delta = \frac{T^2 \sigma_\gamma(T)}{4\pi^2 \alpha Z} \text{ and } 0 < \delta < 10^{-3}.\]

In atomic transitions where binding energies ($\Delta_b$) are involved, the cross-sections are expected to have a sharp increase across the transition edge from $T < \Delta_b$ to $T > \Delta_b$. The sum-rule results, however, represent a continuous cross-section, smoothed to $< 10^{-3}$. In addition, the contribution of photoelectric cross-section $\sigma_\gamma$ to the inclusive process is negative, which implies the total cross-section actually decreases across the transition edge. Both features contradict the expected behaviour. The results should therefore be taken with caution. We note that an alternative derivation using Hartree-Fock techniques results in a cross-section resembling that for free electrons scattering modified by step-functions [5].

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References

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