Shunt-capacitor-assisted synchronization of oscillations in intrinsic Josephson junctions stack

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Abstract

We show that shunt capacitor stabilizes synchronized oscillations in intrinsic Josephson junction stacks biased by DC current. This synchronization mechanism has an effect similar to the previously discussed radiative coupling between junctions, however, it is not defined by the geometry of the stack. It is particularly important in crystals with smaller number of junctions, where radiation coupling is week, and is comparable with the effect of strong super-radiation in crystal with many junctions. The shunt also helps to enter the phase-locked regime in the beginning of oscillations, after switching on the bias current. Shunt may be used to tune radiation power, which drops as shunt capacitance increases.
Recently THz radiation was obtained from mesa-type layered crystals with intrinsic Josephson junctions (IJJ). The number of junctions in that case was not very large, about 600, and power of radiation was enhanced by exciting resonance modes inherent to the crystal, which acts as a cavity. Part of energy stored in an excited mode leaked outside the crystal as radiation. The limitation of such a design is that the radiation frequency is fixed by the crystal resonances and cannot be continuously tuned. A general design for high power tunable source of THz radiation based on IJJ in layered superconductors was discussed in Ref. 2, see Fig. 1. The main idea behind this design is to get radiation from crystal boundaries from many synchronized IJJ (up to \( N = 10000 \)) biased with the DC current. The current induces DC voltage \( V \) between neighboring layers and thus produces Josephson oscillations with the frequency \( \omega_J = \frac{2eV}{\hbar} \) tunable by the DC current. It was proposed in Ref. 2 to use a crystal with dimensions \( L_y \gtrsim \frac{c}{\omega_J} \gg L_x \) to eliminate effect of resonance modes in the \( x \)-direction and use metallic screens at \( |z| > \frac{L_z}{2} \) to eliminate destructive interference of electromagnetic waves coming from the surfaces \( x = \pm \frac{L_x}{2} \). When all junctions are synchronized, the radiation power emitted from the crystal edge is proportional to \( N^2 \) and may reach 1 mW from crystal with \( L_y = 300 \mu m, L_x = 4 \mu m \) and \( L_z = 40 \mu m \).

It was shown in Ref. 2 that radiation by the junctions itself can synchronize oscillations. Without radiation, in-phase oscillations in different junctions may be unstable due to excitation of the Fiske modes in the layered crystal. Here we propose additional mechanism of synchronization of Josephson oscillations by means of external shunt capacitor, see Fig. 1. It works in a way similar to radiation from the crystal, but now all junctions contribute to the electric field inside the shunt capacitor. The effect of shunt stabilization of synchronized oscillations in an array of point-like Josephson junctions was discussed previously by Chernikov and Schmidt. Here we generalize their results for extended IJJ and find stabilization condition for such systems. We calculate stabilization effects of both, radiation and shunt, and compare them quantitatively. We show that stabilization effects of shunt with moderate capacitance and of radiation in the super-radiation regime (large number of junctions, of order \( 10^4 - 10^5 \)) are comparable, while shunt capacitor is much more effective in keeping oscillations in different junctions synchronized at smaller \( N \). We demonstrate also that increase of shunt capacitance results in suppression of radiation and thus radiation power may be tuned by shunt.

To account for the effect of external shunt on oscillations in the IJJ, we use the Lagrangian
FIG. 1: Stack of intrinsic Josephson junctions shunted by external capacitance. Light green plates are metallic screens, superconducting layers are shown by dark green.

The Lagrangian for the system with shunt shown in Fig. II is

$$\mathcal{L}\{\varphi_n\} = \frac{\Phi_0^2 s}{16\pi^3 \lambda_{ab}^2} \sum_n \int dr \left[ \frac{1}{2c_0^2} (1 - \alpha \nabla_{n}^2)^{-1} \dot{\varphi}_n^2 - \frac{1}{\lambda_{J}^2} (1 - \cos \varphi_n) - \frac{1}{2} Q_n^2 \right] - \int dr dz \frac{(\text{curl} A)^2}{8\pi} + \frac{\hbar^2}{8\varepsilon_2 C_s N^2} \dot{\varphi}^2. \quad (1)$$

Here $\varphi_n(r, t)$ is the gauge-invariant phase difference between the layers $n$ and $n + 1$, the coordinates inside layer are $r = x, y$, phase difference gradients are $\nabla \varphi_n = (\nabla_x \varphi_n, \nabla_y \varphi_n)$, the London penetration lengths are $\lambda_c$ and $\lambda_{ab}$ for currents between layers and inside layers, respectively, $\varepsilon_c$ and $\varepsilon_{ab}$ are high-frequency dielectric constant for electric fields perpendicular to layers (along the $z$-axis) and along layers, $\ell = \lambda_{ab}/s$, where $s$ is the interlayer distance, $c_0 = c/(\sqrt{\varepsilon_c \ell})$ and $\lambda_J = \gamma s$, where $\gamma = \lambda_c/\lambda_{ab}$ is the anisotropy ratio, and $\Phi_0 = \pi \hbar c/e$.

The first two terms account for the electro-chemical and the Josephson energies of the IJJ. The factor $(1 - \alpha \nabla_{n}^2)^{-1}$, with second discrete derivative $\nabla_{n}^2 A_n = A_{n+1} + A_{n-1} - 2A_n$ and $\alpha = e^{-1} s^{-2}(4\pi)^{-1} \partial \mu / \partial \rho$, originates from the relation between gauge invariant time derivative of the phase difference and difference in the chemical potentials, see Ref. 3:

$$\hbar \frac{\partial \varphi_n}{\partial t} = e(V_n - V_{n+1}) + \frac{\partial \mu}{\partial \rho_n} (\rho_n - \rho_{n+1}), \quad (2)$$
where $V_n, \rho_n = (E_{zn} - E_{z,n-1})/(4\pi s)$, and $\mu_n$ are the potential, the charge density, and the chemical potential in the layer $n$, while $E_n$ is the electric field in the junction $n$. Eq. (2) results in the relation

$$E_{zn}(r, t) = (1 - \alpha \nabla_{n}^2)^{-1}(B_c \ell \lambda/c) \phi_n(r, t), \quad B_c = \Phi_0/(2\pi \lambda_{ab} \lambda_c).$$

(3)

The next terms in the square brackets account for the kinetic energy of the intralayer currents. The intralayer current is $j_n = (c\Phi_0/8\pi^2 \lambda_{ab}^2) Q_n$, where $Q_n = -\nabla \phi_n - (2\pi/\Phi_0) A_n$, where we introduced the phase $\phi_n$ of the superconducting order parameter and the vector potential $A_n(r)$ in the layer $n$. The fourth term in the Lagrangian is the energy of magnetic field inside the crystal. The term with $\dot{Q}_n$ is omitted because its contribution is negligible at the low frequencies discussed here.

The dissipative function is $\mathcal{R} \{ \psi_n \} = \mathcal{R}_c \{ \phi_n \} + \mathcal{R}_{ab} \{ \phi_n \}$, where

$$\mathcal{R}_c \{ \phi_n \} = \frac{\Phi_0^2 s}{32\pi^3 \lambda_{ab}^2} \sum_n \int dr \frac{4\pi \sigma_c}{\varepsilon_c} \phi_n^2,$$

(5)

$$\mathcal{R}_{ab} \{ \phi_n \} = \frac{\Phi_0^2 s^3}{32\pi^3 \lambda_{ab}^2} \sum_n \int dr \frac{4\pi \sigma_{ab}}{\varepsilon_{ab}} \dot{Q}_n^2.$$

(6)

Here $\sigma_c$ and $\sigma_{ab}$ are the quasiparticle conductivities perpendicular and along the layers, respectively.

The Lagrangian and the dissipative function result in the equations of motion for the phases $\phi_n$ and the vector potential $A$

$$\frac{d}{dt} \frac{\delta \mathcal{L}}{\delta \phi_n} - \frac{\delta \mathcal{L}}{\delta \dot{\phi}_n} + \frac{\delta \mathcal{R}}{\delta \phi_n} = 0$$

(7)

and similar equation for $A$. We write them in the form

$$\frac{\partial^2}{\partial \tau^2} [\phi_n + \beta \phi] + (1 - \alpha \nabla_{n}^2) \left( \nu_c \frac{\partial \phi_n}{\partial \tau} + \sin \phi_n - \nabla_u h_{y,n} + \nabla_v h_{x,n} \right) = 0,$$

(8)

$$\left( \nabla_{n}^2 - \ell^{-2} T_{ab} \right) h_{y,n} + T_{ab} \nabla_u \phi_n = 0,$$

(9)

$$\left( \nabla_{n}^2 - \ell^{-2} T_{ab} \right) h_{x,n} - T_{ab} \nabla_v \phi_n = 0.$$

(10)

Here we use reduced coordinates $u = x/\lambda_J$ and $v = y/\lambda_J$, reduced time $\tau = t/\omega_p$ and frequency $\omega = \omega_J/\omega_p$ with $\omega_p = c/(\lambda_c \sqrt{\varepsilon_c})$ as well as reduced magnetic field $h = B/B_c$ and $\beta = NC_s/C_J$. 4
Further, $T_{ab} = 1 + \nu_{ab} \partial / \partial \tau$, and $C_J = \epsilon_c L_x L_y / (4 \pi s)$ is the junction capacitance. We introduce reduced dissipative parameters $\nu_{ab} = 4 \pi \sigma_{ab} / (\gamma^2 \epsilon_c \omega_p)$ and $\nu_c = 4 \pi \sigma_c / (\epsilon_c \omega_p)$. The shunt capacitor effectively enhances the capacitance of the Josephson junctions, but only for synchronized oscillations; this enhancement is proportional to the number of junctions.

The typical parameters of optimally doped BSCCO at low temperatures are $\epsilon_c = 12$, $s = 15.6$ Å, $\gamma = 500$, $\lambda_{ab} = 200$ nm, the critical current density $j_c = \Phi_0 c / (8 \pi^2 s \lambda_c^2) = 1700$ A/cm², $\ell = 130$, $\nu_{ab} = 0.2$ and $\nu_c = 0.002$. The parameter $\alpha \sim 0.1 - 1$. It was estimated in Ref. [2] that with these parameters crystal optimal for radiation should have sizes $L_z \approx 40 \mu$m, $L_x \approx 4 - 6 \mu$m, $L_y \approx 300 \mu$m.

The differential equations (10) should be completed by the boundary conditions at $x = \pm L_x / 2$ and $y = \pm L_y / 2$. Using Eqs. (10) and continuity of $B_x$ and $B_y$ for the time-independent part of the phase difference we obtain at these boundaries

$$\nabla_{x,y} \phi_n = \pm (2\pi s / \Phi_0) B_{y,x}. \quad (11)$$

The outside magnetic field is created by the bias DC current and by the induced alternating current. For $L_y \gg L_x$ we estimate $B_y(x = \pm L_x / 2, y = \pm L_y / 2) \approx \pm 2\pi I / (c L_y)$, while $B_x \lesssim B_y$. Here $I = j L_x L_y$ is the total interlayer bias current and $j$ is the bias current density. Hence, we estimate time-independent phase difference,

$$\varphi(y = L_y / 2) - \varphi(y = -L_y / 2) \lesssim (2\pi)^2 s j L_x L_y / (c \Phi_0) \approx 2\pi \sigma_c \omega_j L_x L_y / c^2. \quad (12)$$

Here we used the relation $j \approx \sigma_c E_z$ in the resistive state when voltage is present. The phase difference estimated here is very small for crystals with dimensions smaller than cm. Neglecting it we use approximation with $y$-independent phase difference.

For alternating part of the phase difference, we find boundary conditions by matching electromagnetic fields inside and outside the crystal. Outside fields, in half spaces $|x| > L_x / 2$, obey the Maxwell equations, which fix the ratio between electric and magnetic field. Inside the crystal, $B_n(r, t)$ given by Eq. (10) and the electric by Eq. (3). When $L_y \gg L_x, c / \omega_J$, the predominant radiation is along the $x$-axis. For weak radiation in the $y$-direction, we can use the boundary conditions $\nabla_y \varphi_n = 0$. Thus we can omit dependence of $\varphi_n$ on the $y$-coordinate also for alternating part of the phase differences. Then we obtain the boundary conditions at $x = \pm L_x / 2$:

$$\pm h_{y,n}(\omega) = \frac{i s \ell \omega}{2 \sqrt{\epsilon_c}} \sum_m \varphi_m(\omega) ([k_{\omega} J_0(k_{\omega} s |n - m|)] + i k_{\omega} N_0(k_{\omega} s |n - m|)), \quad (13)$$
where we use the Fourier transforms with respect to time, $k_\omega = \omega/c$, and $J_0(x)$ and $N_0(x)$ are the Bessel functions.

The following calculations are similar to those in Ref. 2 but accounting for shunt contribution. We consider high frequencies $\omega \gg 1$ and $N \gg \ell$ and neglect finite-size effects along the c-axis. The equation for $y$- and $n$-uniform phase is

$$\ddot{\varphi}(u) + \left(\beta/\bar{L}_x\right) \int_{-\bar{L}_x/2}^{\bar{L}_x/2} du \ddot{\varphi}(u) + \nu_c \dot{\varphi} + \sin \varphi - \ell^2 \nabla_u^2 \varphi = 0. \quad (14)$$

Here $\bar{L}_x = L_x/\lambda_J$. In the limit $\omega \gg 1$ we look for solution

$$\varphi(u, \tau) = \omega \tau + \eta(u, \tau), \quad \eta \ll 1. \quad (15)$$

The equation for $\eta$ is

$$\ddot{\eta}(u) + \left(\beta/\bar{L}_x\right) \int_{-\bar{L}_x}^{\bar{L}_x} du \ddot{\eta}(u) + \nu_c \dot{\eta} - \ell^2 \nabla_u^2 \eta = - \sin(\omega \tau), \quad (16)$$

which should be solved with the boundary conditions at $u = \pm \bar{L}_x/2$

$$\nabla_u \eta = \pm i \omega \zeta \eta, \quad (17)$$

$$\zeta = \frac{L_x}{2\ell \sqrt{|k_\omega|}} \left[|k_\omega| - ik_\omega L_\omega\right], \quad L_\omega \approx \frac{2}{\pi} \ln \left[\frac{5.03}{|k_\omega L_z|}\right].$$

The solution is

$$\eta = (1/2) \text{Im}[B + A \cos(\bar{k}_\omega u)], \quad \bar{k}_\omega = \omega/\ell \quad (18)$$

$$B = -[\omega^2(1 + \beta \xi) + i \omega \nu_c]^{-1},$$

$$A = i \zeta \left[|\bar{k}_\omega| \sin(\bar{k}_\omega \bar{L}_x/2) + i \zeta \omega \cos(\bar{k}_\omega \bar{L}_x/2)\right] \left[\omega(1 + \beta \xi) + i \nu_c\right]^{-1},$$

$$\xi = [1 + 2i \omega \zeta/(\bar{k}_\omega \bar{L}_x)]^{-1}.$$

where we approximate $\sin(\bar{k}_\omega \bar{L}_x/2) \approx \bar{k}_\omega \bar{L}_x/2$. The first term in $\eta$ is the amplitude of synchronized ($y$- and $n$-independent) Josephson oscillations. It drops as $\beta$ increases. The second term describes nonuniform electromagnetic wave inside the crystal. It is generated at the boundaries due to radiation field.

To analyze stability of synchronized Josephson oscillations we consider a small perturbation $\theta_n(u, \tau)$ to the solution uniform in $n$, $\varphi$,

$$\varphi_n(u, \tau) = \varphi(u, \tau) + \theta_n(u, \tau). \quad (19)$$
Equations for $\theta_n(u, \tau)$ are obtained by linearization of Eqs. (10) with respect to $\theta_n(u, \tau)$. The term $\cos[\eta(\tau)]\theta_n(u, \tau)$ in the linearized equation couples harmonics with small frequency $\Omega$ to high-frequency terms at $\Omega \pm \omega$. At $\omega \gg 1$ we can neglect coupling to the higher frequency harmonics $\Omega \pm m\omega$ with $m > 1$ and represent the phase perturbation as

$$\theta_n \approx \sum_q \left[ \bar{\theta}_q + \sum_{q\pm} \bar{\theta}_{q\pm} \exp(\pm i\omega \tau) \right] \cos(qn)e^{-i\Omega \tau},$$

where $q = \pi k/N, k = 1, 2, \ldots, N$. The complex eigenfrequencies $\Omega(q)$ are assumed to be small, $|\Omega| \ll \omega$. We will find them and also conditions when $\text{Im}[\Omega] < 0$ for all $q$ (stability condition). Substituting $\theta_n$ into linearized equations (8)-(10), excluding oscillating magnetic fields and separating the fast and slow parts, we obtain for $q \neq 0$ the coupled equations

$$\left[ \frac{\Omega^2}{1 + \alpha_q} + i\nu_c \Omega - \bar{C} \right] \bar{\theta}_q + G_q^{-2}\nabla_u^2 \bar{\theta}_q = \frac{\bar{\theta}_{q+} + \bar{\theta}_{q-}}{2},$$

$$\left[ \frac{(\Omega \pm \omega)^2}{1 + \alpha_q} + i\nu_c(\Omega \pm \omega) \right] \bar{\theta}_{q\pm} + G_{q\pm}^{-2}\nabla_u^2 \bar{\theta}_{q\pm} = \frac{\bar{\theta}_q}{2}.$$

$$\bar{C} = \langle \cos \eta \rangle_t \approx \text{Re}[\eta_v]/2 \approx -(1/2)\text{Re}\left[ \frac{1}{(1 + \beta \xi)\omega^2 + i\omega \nu_c} \right],$$

$$\alpha_q = 2\alpha(1 - \cos q), \quad G_{q,\pm}^2 \approx \frac{2(1 - \cos q)}{1 - i(\Omega \pm \omega)\nu_{ab} + \ell^{-2}}.$$

Here $G_q = G_{q,0}$. The boundary conditions for slow and fast components at $u = \pm \bar{L}_x/2$ and $q \gg \pi/N$ follow from Eq. (13).

Finally, we obtain Mathieu equation for slow-varying component with $q \neq 0$

$$\left( \frac{\Omega^2}{1 + \alpha_q} + i\nu_c \Omega + \Lambda - V(u) + G_q^{-2}\nabla_u^2 \right) \bar{\theta}_q = 0.$$

$$\Lambda = \text{Re}\left[ \frac{1}{2[\omega^2(1 + \beta \xi) + i\nu_c \omega]} \right] - \frac{1 + \alpha_q}{2[\omega^2 + \nu_c^2(1 + \alpha_q)^2]}.$$  

with potential $V(u) = V_1(u) + V_2(u)$,

$$V_1(u) = \frac{1}{2\omega^2} \text{Re}\left[ \frac{i\zeta \cos(\bar{\kappa}_\omega u)}{(1 + \beta \xi)[\bar{\kappa}_\omega \sin(\bar{\kappa}_\omega \bar{L}_x/2) + i\zeta \cos(\bar{\kappa}_\omega \bar{L}_x/2)]} \right],$$

$$V_2(u) = \frac{1}{2\omega^2} \text{Re}\left[ \frac{\kappa_+ \cos(p_+ u)}{(1 + \beta \xi)[p_+ \sin(p_+ \bar{L}_x/2) + \kappa_+ \cos(p_+ \bar{L}_x/2)]} \right].$$

Here $p_+ = \omega G_{q+}$ and $\kappa_+ \approx [(\Omega - \omega)^2 G_{q,\pm}^2]/[(1 + \alpha_q)\epsilon_c q \gamma]$. In the following we consider shunt with moderate capacitance $\beta \leq 1$. In the lowest order in $\bar{\kappa}_\omega \bar{L}_x = \omega \bar{L}_x/\ell \ll 1$ the part $V_1(u)$
reduces to a constant, $V_1(u) \approx \text{Re}[\mathcal{K}_\omega/(2\omega^2(1 + \beta \xi))]$,

$$K_\omega = \frac{\mathcal{L}_\omega(\mathcal{L}_\omega + a) + 1}{(\mathcal{L}_\omega + a)^2 + 1}, \quad a = \frac{\epsilon_c L_x}{L_z},$$  \hspace{1cm} (27)

Treating the coordinate-dependent part of $\bar{\theta}_q$ as a small perturbation, we find expression for $\Omega(q)$,

$$\Omega^2 + i\nu_c \Omega \approx \frac{1}{2\omega^2} \left\{ \text{Re} \left[ \frac{\beta \xi + K_\omega}{1 + \beta \xi} + \alpha_q - \frac{W_2(q)}{1 + \beta \xi} \right] \right\},$$  \hspace{1cm} (28)

$$W_2(q) = \frac{1}{p_+ \tilde{L}_x(p_+ / \kappa_+ + \cot(p_+ \tilde{L}_x/2))}.$$  \hspace{1cm} (29)

The first and the second terms in Eq. (28) represent the stabilization effects of shunt and radiation, and of the long-range interlayer capacitance, respectively. The last term, $W_2$, describes the effect of modes $\bar{\theta}_q^\pm$ induced inside the crystal due to radiation (parametric excitation of Fiske modes). This term leads to instability in the limit of zero dissipation and in the absence of other stabilizing terms. It is much smaller than unity. Capacitive shunt and radiation introduce the gap in the spectrum of weak distortions and are most effective in stabilization. Their contributions can both reach order one for $\epsilon_c L_x < L_z = N_s$ (in the super-radiation regime) and $NC_s \approx C_J$. As $C_J = 60$ cm for $s = 1200 \mu$m$^2$, it is easy to reach this condition. In order to achieve the maximal stabilization without sacrificing the radiation power, one needs to choose $\beta \approx 1$.

We conclude that shunt capacitor stabilizes synchronized oscillations in IJJ stack. The effect is particularly useful in crystals with the small junction number or at the initial stages of radiation. Shunt may also be used to tune the radiation power.

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