From the CMF to the IMF: Failure of the Core-Collapse Model

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ABSTRACT
Observations have indicated that the prestellar core mass function is similar to the IMF, except for an offset towards larger masses. This has led to the idea that there is a one-to-one relation between cores and stars, such that the whole stellar mass reservoir is contained in a gravitationally-bound prestellar core, as postulated by the core-collapse model, and assumed in recent theoretical models of the stellar IMF. We test the validity of this assumption by comparing the final mass of stars with the mass of their progenitor cores in a high-resolution star-formation simulation that generates a realistic IMF under physical conditions characteristic of observed molecular clouds. We find that the core mass function and the IMF are closely related in a statistical sense only; for any individual star there is only a weak correlation between the progenitor core mass and the final stellar mass. In particular, for high mass stars only a small fraction of the final stellar mass comes from the progenitor core, and even for low mass stars the fraction is highly variable, with a median fraction of only about 50%. We conclude that the core-collapse scenario and related models for the origin of the IMF are incomplete.

Key words: stars: formation – MHD – stars: luminosity function, mass function

1 INTRODUCTION
The origin of the stellar initial mass function (IMF) is still considered an open problem, partly due to a lack of observational constraints. While the overall shape of the IMF is well documented in stellar clusters, its possible dependence on environmental parameters is still unclear. The turbulent nature of star-forming gas compounds the problem, although it may also be viewed as a key to solve it (e.g. Padoan & Nordlund 2002; Hennebelle & Chabrier 2008; Hopkins 2012).

The similarities of the stellar IMF (Kroupa 2001; Chabrier 2005) with the prestellar core mass function (CMF) derived from simulations (e.g. Klessen 2001; Tilley & Pudritz 2004, 2007; Padoan et al. 2007; Schmidt et al. 2010) and observations (e.g. Motte et al. 1998; Alves et al. 2007; Enoch et al. 2007; Nutter & Ward-Thompson 2007; Könyves et al. 2010, 2015; Marsh et al. 2016; Könyves et al. 2020; Ladjelate et al. 2020) has led to the suggestion that prestellar cores are true progenitors of stars, meaning that the mass reservoir of a star is fully contained in the progenitor core. The final mass of a star, $M_{\text{star}}$, is then given by that of its progenitor core, $M_{\text{prog}}$, multiplied by an efficiency factor, $M_{\text{star}} = \epsilon_{\text{prog}} M_{\text{prog}}$, and the IMF is interpreted as a CMF offset in mass by the factor $\epsilon_{\text{prog}}$ (e.g. Alves et al. 2007; André et al. 2010, 2014).

The idea that in the prestellar phase the stellar mass reservoir is fully contained in a bound overdensity is a fundamental assumption in the IMF models of Hennebelle & Chabrier (2008) and Hopkins (2012). A bound overdensity collapses when it becomes gravitationally unstable, and some form of thermal, magnetic or kinetic support is needed before the start of the collapse, if the overdensity is to accumulate all the stellar mass reservoir. Given the very broad mass range of stars, and the relatively small temperature variations in the dense molecular gas, magnetic and/or kinetic energy support are needed, particularly in the case of the most massive stars that would require the most massive prestellar cores (McKee & Tan 2002, 2003). This scenario for the origin of the IMF and massive stars, often referred to as core collapse, has been called into question by results of numerical simulations and observations.

Simulations of star formation in turbulent clouds result in relatively long star-formation timescales: high-mass protostars acquire their mass on a timescale comparable to the dynamical timescale of the star-forming cloud (Bate 2009, 2012; Padoan et al. 2014, 2019), which is much longer than...
the characteristic free-fall time of prestellar cores. In the case of the formation of intermediate- and high-mass stars, using SPH simulations or tracer particles in grid simulations, it has been shown that the mass reservoir of sink particles extends far beyond the prestellar cores (Bonnell et al. 2004; Smith et al. 2009; Padoan et al. 2019). Both results seem to be at odds with the core-collapse idea.

Recent interferometric observations of regions of massive star formation have revealed a scarcity of massive prestellar cores that could serve as the mass reservoir for high-mass stars (e.g. Sanhueza et al. 2017, 2019; Li et al. 2019; Pillai et al. 2019; Kong 2019; Servajean et al. 2019), implying that such a mass reservoir is spread over larger scales. This is also suggested by the evidence of parsec-scale filamentary accretion onto massive protostellar cores (e.g. Peretto et al. 2013). Furthermore, while the prestellar CMFs in the Aquila and Orion regions are found to peak at a mass a few times larger than that of the stellar IMF, corresponding to a progenitor core efficiency $\epsilon_{\text{prog}} \approx 0.2 - 0.4$ (André et al. 2010; Könyves et al. 2015, 2020), the peak mass of the CMFs in Taurus and Ophiucus is very close to that of the stellar IMF, or even smaller if candidate cores are included (Marsh et al. 2016; Ladjelate et al. 2020). This would imply that $\epsilon_{\text{prog}} > 1$ in Taurus and Ophiucus, in contradiction with the core-collapse scenario. Protostellar jets and outflows remove a significant fraction of the accreting mass (e.g. Matzner & McKee 2000; Tanaka et al. 2017), so a core-collapse model requires $\epsilon_{\text{prog}} \lesssim 0.5$.

Some of the alternatives to the core-collapse models are the competitive accretion model (e.g. Zinnecker 1982; Bonnell et al. 2001a,b; Bonnell & Bate 2006) and the inertial-inflow model (Padoan et al. 2019). In the competitive accretion model, all stars have initially low mass, and most of their final mass is accreted after the initial collapse and may originate far away from the initial core. The accretion is due to the stellar gravity, so the accretion rate is expected to grow with the stellar mass. However, the Bondi-Hoyle accretion rate in turbulent clouds is too low for this model to explain the stellar IMF, unless the star-forming region has a very low virial parameter, with a high density and comparatively low velocity dispersion (Krumholz et al. 2005). Thus, although this model is quite different from the core-collapse model, it still requires that the feeding region of a star is gravitationally bound and essentially in free-fall collapse.

In the inertial-inflow model (Padoan et al. 2019), stars are fed by mass inflows that are not driven primarily by gravity, as they are an intrinsic feature of supersonic turbulence. The scenario is inspired by the IMF model of Padoan & Nordlund (2002), where prestellar cores are formed by shocks in converging flows. The characteristic time of the compression is the turnover time of the turbulence on a given scale, which is generally larger than the free-fall time in the postshock gas, so a prestellar core may collapse into a protostar well before the full stellar mass reservoir has reached the core (Padoan & Nordlund 2011). After the initial collapse, the star can continue to grow, as the converging flows that were feeding the prestellar core continue to feed the star, through the mediation of a disk. Thus, the stellar mass reservoir can extend over a turbulent and unbound region much larger than the prestellar core. Because converging flows occur spontaneously in supersonic turbulence and can assemble the stellar mass without relying on a global collapse or on the gravity of the growing star (Padoan et al. 2014, 2019), the inertial-inflow scenario is fundamentally different from competitive-accretion.

The main goal of this work is to test the core-collapse model, hence the validity of the IMF and massive-star formation theories based on that scenario. The most direct way to test the core-collapse model with numerical experiments is to use simulations where the formation and growth of stars is captured with accreting sink particles, hereafter called 'stars' or 'star particles'. The final mass of each star particle, $M_{\text{star}}$, can be compared with the mass of its progenitor core, $M_{\text{prog}}$, and the hypothesis of the core-collapse model, $M_{\text{star}} \approx \epsilon_{\text{prog}} M_{\text{prog}}$ (with $\epsilon_{\text{prog}} \lesssim 0.5$ and approximately independent of mass), can be tested directly.

The relation between core and star particle masses was already discussed in Smith et al. (2009) and Gong & Ostriker (2015). Both works found a reasonable correlation between the masses of cores and stars at early times, when the stars were still growing. At later times, the stellar masses were significantly larger than the core masses (see Figure 10 in Smith et al. (2009) and Figure 22 in Gong & Ostriker (2015)). These results are in contradiction with the core-collapse model, as they imply that $\epsilon_{\text{prog}} > 1$. However, both simulations have very low resolution, and therefore cannot resolve the peak of the IMF (Haugbølle et al. 2018). Although initial velocity perturbations are imposed (without any further driving), velocity fluctuations at core scales must be severely underestimated, with respect to the global rms Mach number, due to the small dynamic range of the simulations. The low resolution is also reflected in the values of the stellar masses. In Gong & Ostriker (2015), the resolved peak of the IMF is at approximately $2 \, M_\odot$, assuming reasonable parameters. At the end of the simulation, the stellar masses are approximately three times larger than the core masses, hence the stellar mass distribution (not shown in their paper), must peak at approximately $6 \, M_\odot$, that is more than 20 times larger than the stellar IMF peak. Furthermore, because a large fraction of stars are still accreting at the end of the simulations, it is unclear what the final correlation between stellar and core masses should be. Finally, both works neglect magnetic fields, which are expected to play an important role in the fragmentation process (e.g. Krumholz & Federrath 2019).

In this work, we address the relation between the masses of stars and their progenitor cores to test the core-collapse model, using one of the adaptive-mesh-refinement (AMR), magneto-hydrodynamic (MHD) simulations of Haugbølle et al. (2018), where it was demonstrated for the first time that isothermal, supersonic, MHD turbulence can explain the origin of the stellar IMF. The high dynamic range, the inclusion of magnetic fields, and the long integration time overcome the limitations of previous studies, and result in a

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1 The distinction between the prestellar CMF and the stellar IMF based on the difference between the turbulence turn-over time and the free-fall time was not mentioned in Padoan & Nordlund (2002), and the model has been presented and interpreted incorrectly as a core-collapse model. The implication of the model with respect to the distinction between the prestellar CMF and the stellar IMF was explained later in (Padoan & Nordlund 2011).
mass distribution of star particles that is consistent with the observed stellar IMF, from brown dwarfs to massive stars. By the end of this simulation, a large fraction of star particles has stopped accreting, so final masses are well defined. Stars stop accreting when their converging mass inflows are terminated, when pressure forces decouple the gas from the stars, or when they are dynamically ejected from multiple systems while still accreting.

The paper is structured as follows. § 2 briefly summarizes the simulation and the numerical methods, and § 3 describes the selection of bound progenitor cores around newly formed star particles from the simulation. § 4 describes the physical properties of the progenitor cores, including their masses, sizes and sonic Mach numbers, and verifies that the progenitors are supercritical with respect to their magnetic support. The final star particle masses are compared with their progenitor core masses in § 5, and tracer particles are used to identify the fractions of mass accreted from the progenitor and from farther away. The results are discussed in § 6 in the context of the core collapse model and the observations of the core mass function. § 7 summarizes the conclusions.

2 SIMULATION

The simulation used in this study is the high reference simulation from Haugbolle et al. (2018) (see their Table 1). Details of the numerical methods and the simulation setup are in that paper, and are only briefly summarized in the following.

The MHD code used for the simulation is a version, developed in Copenhagen, of the public AMR code RAMSES (Teyssier 2007). It includes random turbulence driving and a robust algorithm for star particles. Periodic boundaries and an isothermal equation of state are adopted for the simulation. The grid refinement is based on overdensity, first at ten times the mean density and then at each factor of four in density. With a root grid of 256$^3$ cells and six levels of refinement, the highest effective resolution is 16,384$^3$, corresponding to a smallest cell size of 50 AU for the assumed box size of 4 pc. The initial turbulent state is achieved by running the simulation without self-gravity for ~ 20 dynamical times, with a random solenoidal acceleration giving an rms sonic Mach number of approximately 10. Then self-gravity is switched on while maintaining the random driving, and the simulation is run for another ~ 2.6 Myr, with snapshots saved every 22 kyr.

The physical parameters of the simulation are as follows. The temperature is set at 10 K and the mean molecular weight $\mu = 2.37$, appropriate for cold molecular clouds, resulting in an isothermal sound speed of 0.18 kms$^{-1}$. Assuming a box size $L_{\text{box}} = 4$ pc, the total mass, mean density and mean magnetic field strength are $M_{\text{box}} = 3000 M_\odot$, $\bar{n} = 795$cm$^{-3}$, and $B_{\text{box}} = 7.2$µG. The corresponding freefall time and dynamical time are 1.18 Myr and 1.08 Myr, respectively.

Star particles are created at a local gravitational potential minimum when the gas density in the cell is above a critical density, set at $1.7 \times 10^4$ cm$^{-3}$. They are created without mass, but start accreting from their surroundings. The mass-loss from winds and outflows is not modelled in the current simulation; it is however accounted for through an accretion efficiency factor, $\epsilon_{\text{acc}} = 0.5$, meaning that only half of the accreting mass is added to the stellar mass. At the end of the simulation, 413 star particles have been created, with a mass distribution consistent with the observed stellar IMF (Haugbolle et al. (2018)).

We use $10^5$ tracer particles in the global simulation to trace the mass flow in the box. The tracer particles are passively advected with the fluid velocity. This may lead to a growing inaccuracy between the gas density field and the density field defined by the tracer particles (Genel et al. 2013). It is a secular effect that at the end of the simulation, after 2.6 Myr, can introduce a discrepancy between mass defined by tracers compared to defined by the gas mass of $\approx 10\%$ on the scale of 1 $M_\odot$. We remedy this effect by re-defining trace particle masses according to the density field, at the time of core selection, making the core masses defined by gas density or by tracers consistent. The typical mass accretion time scale for a low-mass core is 100 kyr, and the difference between using the gas density field and passive tracers is below the percent-level in the analysis below. Passive tracers are accreted probabilistically to star particles with a probability matching the fraction of gas accreted in the enclosing cells.

Figure 1 shows a snapshot of column density of the whole simulation box. The three zoom-in regions show examples of 0.5 pc subcubes that are used in the detection of the progenitor cores (see Section 3.3), drawn in the same color scale.

3 SELECTION OF PROGENITOR CORES

3.1 Clumpfind algorithm

Cores are selected from the simulation with the clumpfind algorithm introduced in Padoan et al. (2007), updated in this work to consider both thermal and kinetic energies. In the clumpfind algorithm, cores are defined as “gravitationally unstable connected overdensities that cannot be split into two or more overdensities of amplitude $\delta n/n > f$, all of which remain unstable”. Cores are considered unstable if their gravitational binding energy is larger than the sum of their kinetic and thermal energies (see § 3.2 for details). The algorithm scans the density field with discrete density levels, each of amplitude $f$ relative to the previous one. Only the connected regions above each density level that are found to be unstable are retained. After this selection, the unstable cores from all levels form a hierarchy tree. Only the final (unsplit) core of each branch is retained. Each core is assigned only the mass within the density isosurface that defines the core (below that density level the core would be merged with its next neighbor). This algorithm is different from the construction of a dendrogram (Rosolowsky et al. 2008), where a hierarchy tree of cores is first computed irrespective of the cores energy ratios, and unstable cores are later identified among the leaves of the dendrogram. In our algorithm, the condition for instability is imposed while building the tree, otherwise some large unstable cores would be incorrectly eliminated if they were split into smaller cores that were later rejected by being stable.

Once the physical size and mean density of the system
are chosen, the clumpfind algorithm depends only on three parameters: (1) the spacing of the discrete density levels, \( f \), (2) the minimum density above which cores are selected, \( n_{\text{min}} \), and (3) the minimum number of cells per core. In principle, there is no need to define a minimum density, but in practice it speeds up the algorithm. The parameter \( f \) may be chosen according to a physical model providing the value of the smallest density fluctuation that could collapse separately from its contracting background. In practice, the \( f \) parameter is set by looking for numerical convergence of the mass distribution with decreasing values of \( f \). The minimum number of cells is set to 4, to ensure that all the detected cores are at least minimally resolved in the simulation. The parameters of clumpfind and the effect of numerical resolution are discussed in Appendix A.

### 3.2 Stability condition

In the clumpfind algorithm, the gravitational binding energy is given by the formula for a uniform sphere:

\[
E_g = \frac{3GM_{\text{prog}}^2}{5R_{\text{prog}}},
\]

where \( G \) is the gravitational constant, \( M_{\text{prog}} \) is the mass of the core, \( R_{\text{prog}} \) is the radius of a sphere of an equivalent volume to that of the core. This proved to be a good approximation for the gravitational binding energy for the cores that we find, when compared to a pair-wise calculation of the potential energy between the cells of the core.

The kinetic energy is given by:

\[
E_k = \sum_{i=1}^{N} \frac{1}{2} m_i (v_i - \bar{v})^2,
\]

where \( N \) is the number of cells in the core, \( m_i \) and \( v_i \) are the mass and the velocity of each cell, and \( \bar{v} \) is the mass-weighted mean velocity of the core. The thermal energy is given by the formula for an ideal di-atomic gas:

\[
E_t = \frac{5}{2} N k T,
\]

where \( N \) is the number of hydrogen molecules, \( k \) is the Boltzmann constant and \( T \) is the temperature, set to 10K. The condition that cores are unstable is then based on the energy ratio,

\[
\frac{E_k + E_t}{E_g} \leq 1.
\]

### 3.3 Selection setup

We extract 1,024³ subcubes of 0.5 pc in size centred on each star particle, in the snapshot where the star particle first appears. This resolution corresponds to 8,192³ resolution for the full 4 pc simulation box and a physical scale of 100 AU per cell. Resolution has a large effect on the masses of the selected cores; lower resolutions result in higher core masses as well as increase the number of cores with multiple stars inside them (see Appendix A). The clumpfind algorithm should in principle be applied to the whole periodic box, as the cores could depend on how neighboring cores are selected, but this would be computationally infeasible at the highest resolution. However, most of the cores are well-defined within a 0.5 pc subcube. Due to their large size, for...
22 cores with masses higher than $5 \, M_\odot$ in the 0.5 pc subcube, we repeat the clumpfind search using 1 pc subcubes.

Ideally, we would use a snapshot at a time immediately before the star is created. In the absence of that, and in the spirit of characterizing the core condition at the time immediately before the star formation, we add twice the mass of the star particle back to the density cube to account for the gravitational energy of the gas that has already accreted onto the star particle. The factor of 2 is due to our star accretion efficiency, $\epsilon_{\text{acc}} = 0.5$, which is used to mimic the effect of protostellar jets and outflows. The mass of other stars present in the subcube is also added to the density cube, using a factor of 2 if they formed in the same snapshot as the central star or a factor of 1 if they had formed in a previous snapshot (we assume that the jet would have time to disperse the $\epsilon_{\text{acc}}$ fraction of the accreted mass away from the core in that case). The mass is added into the cell where each star is located. The details of how the mass is added do not affect the calculation of the gravitational binding energy, as only the total mass is needed in the uniform sphere approximation of Equation 1. Additionally, this method ensures that, if there is no extended material around the star, the single cell ‘core’ is not picked up, because the minimum number of cells per core is set to 4.

For the overdensity amplitude, $f$, we adopt a conservative value, $f = 2\%$. Given that the 0.5 pc subcubes are in overdense regions of the full box, we select the minimum number density level to be $n_{\text{min}} = 10 \, \bar{n} = 7950 \, \text{cm}^{-3}$. We have varied $f$ and $n_{\text{min}}$ using $f = 8\%$ and $n_{\text{min}} = 2 \, \bar{n}$ and verified it resulted in almost identical core masses.

The clumpfind algorithm is run for each subcube, to find all the unstable cores. Then a search is made for the core that contains the star particle (in the central cell of the subcube). This core is labelled as the progenitor core for that star particle. If no core containing the central cell is found, we search for the closest core inside a $21^3$ cube around the star, and record the distance to the star if a core is found. Otherwise, the distance is set to a high number to indicate that no match is found.

## 4 PROGENITOR CORES

The clumpfind algorithm outputs a list of cell indices that are part of each core, as well as the masses of the cores. Once the progenitor cores are identified, their cell indices are used to find any other star particles within the progenitor. As explained in Section 3.3, we include twice the masses of the star particles that formed in the same snapshot as the primary star into the density cube, whereas older star particles were already considered to be stars.

Out of a total of 413 star particles in the simulation, we are able to match 382 unambiguously with a progenitor core. Among the other 31 stars, predominantly of low mass, 22 of them are found within 1,000 AU of a core and 9 are more distant. Some of these stars are brown dwarfs that have collapsed quickly from a small core and have already accreted their parent core by the time of the snapshot we analyze. Others are stars that have already decoupled from the accreting gas due to pressure forces acting on the gas or dynamical encounters with other stars. A small fraction of them may be numerical artifacts, as discussed in Haugbølle et al. (2018). We exclude those 31 stars from the analysis.

We further divide our 382 unambiguously matched progenitors in three categories based on the stars within the progenitor: 1) a single star (the primary one, 312 cores); 2) multiple stars, but all formed in the same snapshot (32 cores); 3) one or more older stars in addition to the primary one (38 cores). The older stars may dominate the mass reservoir inside the core. To be conservative, we exclude category three from the analysis.

The final sample used in the paper is therefore 344 stars with an unambiguous progenitor detection and no older stars inside the progenitor core.

To study the mass distribution of the accreted gas (see Section 5), we extract the passive tracer particles that are within a progenitor core at the time of its identification. These tracer particles are used to calculate the fractions of the progenitor mass accreted by the primary star ($f_{\text{tr,prog}}$), accreted by any other stars ($f_{\text{tr,other}}$), and not accreted by any star ($f_{\text{tr,unacc}}$) at the end of the simulation. Furthermore, we calculate the fraction of the final stellar mass contained within the progenitor core: $f_{\text{tr,star}} = \epsilon_{\text{acc}, \text{tr,prog}} M_{\text{prog}} / M_{\text{star}}$. These fractions evolve while the stars accrete their mass, and some of them have not stopped accreting yet. Thus, we further check if the stars have stopped accreting at the end of the simulation by computing the final accretion rate as the mass increase between the last two snapshots, divided by the time interval between the snapshots. This is an accretion rate averaged over 22 kyr. If the stars need 10 Myr or longer to double their final mass given that accretion rate, we consider them as having finished accreting (242 stars out of 344). Finally, to characterize the size of the region from which a star accretes its mass, we calculate the infall radius, $R_{\text{inf}}$, as was done in Padoan et al. (2019). $R_{\text{inf}}$ is defined as the radius of the sphere that contains 95% of the tracer particles’ mass accreted by the star by the end of the simulation. We refer to it as the infall radius in reference to our scenario where growing stars are fed by inertial inflows, as extensively demonstrated in Padoan et al. (2019). The mass fractions and the inflow radius defined through the tracer particles are used in § 5, where we address the relationship between the stars and their progenitor cores.

Since the clumpfind algorithm does not consider the magnetic field, we check for magnetic support against gravitational collapse after the fact. We follow the same methodology as in Ntormousi & Hennebelle (2019). We calculate the magnetic critical mass (Mouschovias & Spitzer 1976),

$$M_{\text{crit}} = \frac{c_1}{3\pi} \left( \frac{5}{G} \right)^{\frac{1}{2}} \phi$$

(5)

where $c_1 = 1$ for a uniform sphere, $G$ is the gravitational constant and $\phi$ is the magnetic flux.

We measure $\phi$ for each core by selecting three planes going through the core centre perpendicular to each axis. We then flag the cells belonging to the core, and sum the flux along the perpendicular axis over all the cells. We then select the highest of the three fluxes to calculate the magnetic critical mass. The mass-to-flux ratio, normalized by the magnetic critical mass, is then simply given by:

$$\mu = M_{\text{prog}} / M_{\text{crit}}$$

(6)
where $M_{\text{prog}}$ is the mass of the core and $M_{\text{BE}}$ is the magnetic critical mass, assuming a uniform sphere. Figure 2 shows the normalized mass-to-flux ratio, $\mu$, as a function of core mass, $M_{\text{prog}}$. We find that all of our unstable cores are supercritical, which is to be expected, as we selected for unstable cores associated with recently formed star particles.

To further describe the sample of progenitor cores, Figure 3 shows their size (upper panel) and their rms sonic Mach number (lower panel) as a function of their mass (blue dots). Although we make no attempt to retrieve observational properties of the cores through synthetic observations, it is still instructive to compare these quantities with some observational values. For that purpose, Figure 3 also shows values of sizes, rms Mach numbers and masses of prestellar cores from a number of large observational surveys.

The cores from the simulation are studied at the time when they have just started to collapse in order to identify the maximum value of their mass in the prestellar phase. Thus, our cores are in the transition from prestellar to protostellar, and they could also be compared with very young protostellar cores whose central star is still a small fraction of the core mass. For simplicity, we consider only observed cores in the prestellar phase.

The gravitational stability of cores is often defined by reference to a critical Bonnor-Ebert sphere. The critical radius, $R_{\text{BE}}$, of a Bonnor-Ebert sphere with a temperature of 10 K is given by $R_{\text{BE}} = M_{\text{BE}}/20.2$, where $M_{\text{BE}}$ is the mass of the sphere in $M_\odot$, and the radius is in parsecs. This expression for the critical radius as a function of the mass is shown by the red dashed line in the upper panel of Figure 3. The progenitor cores from our simulation span a range of values between approximately 100 AU and 0.2 pc, with a median value of approximately 800 AU. They are all well below the critical Bonnor-Ebert radius, because they are selected as gravitationally unstable cores accounting also for their internal kinetic energy, as shown in Equation (4).

For comparison, we show the observational values of the prestellar cores in Aquila from the Herschel Gould Belt Survey (Könyves et al. 2015), in Orion from a sub-sample of a SCUBA survey that includes followup line observations (Kirk et al. 2017), in a number of infrared dark clouds from a large ALMA survey (Sanhueza et al. 2019), and in another infrared dark clouds observed with ALMA where core velocity dispersions are also published (Ohashi et al. 2016). There is a significant overlap between the observed prestellar cores and the cores from our simulation, except that core sizes below 1,000 AU are missing in the observational samples, due to their spatial-resolution limit². Furthermore, the observed

² Although the least massive of our cores have the shortest free-fall times (they must be very dense to be gravitationally unstable), and so they may be more appropriately compared with cores
core masses at any given radius tend to extend to larger values than in our sample, probably also as an effect of the limited spatial resolution (see discussion in § 9.2 of Padoan et al. 2019). Finally, for the Aquila and Orion cores we have selected all prestellar cores with a radius smaller than or equal to the critical Bonnor-Ebert radius for their mass, which explains the large number of cores near the critical radius line. Although such cores, and even those with half a critical Bonnor-Ebert mass, are usually considered prestellar in dust-continuum studies, a fraction of them are probably not gravitationally bound, due to their internal kinetic energy.

To illustrate the importance of internal motions in the progenitor cores, the lower panel of Figure 3 shows the one-dimensional velocity dispersion inside the cores, $\sigma_v$, in units of the isothermal speed of sound for a temperature of 10 K, $c_s \approx 0.18 \, \text{km s}^{-1}$, as a function of the core mass, $M_{\text{prog}}$. In the case of the observational samples, the sound speed value estimated in the corresponding papers is adopted. Most of the progenitor cores (blue dots) are within a factor of two from the equipartition between kinetic and thermal energy, marked by the horizontal dashed line. They show a trend of increasing rms Mach number with increasing core mass, and some of the least massive cores have an rms Mach number several times smaller than the value corresponding to the energy equipartition, or velocity dispersions as low as 1/5 of the sound speed.

Apart from the prestellar cores in Orion and in the infrared dark cloud already shown in the upper panel of Figure 3, we consider also the velocity dispersion of prestellar cores in Perseus from Kirk et al. (2007). The observed values overlap with those of the progenitor cores from our simulation. However, in the range of core masses between approximately 1 and 10 $M_\odot$, the observed cores extend to lower Mach number values than our progenitor cores. This partial discrepancy may have two origins. First, as mentioned above and extensively documented in § 9.2 of Padoan et al. (2019), because of the limited spatial resolution of the observational surveys core masses may be systematically overestimated by large factors. If the observed cores were split into their lower-mass components, they may overlap in both velocity dispersion and mass with our low-mass progenitor cores in Figure 3. The second reason for the partial discrepancy is that we have computed the core rms Mach number from the ratio of kinetic and thermal energies, meaning that the rms velocity is density weighted. In the observations, the linewidths may more closely correspond to a velocity dispersion weighted by the square of the density, in the case of the observational samples, the sound speed value estimated in the corresponding papers is adopted. Most of the progenitor cores (blue dots) are within a factor of two from the equipartition between kinetic and thermal energy, marked by the horizontal dashed line. They show a trend of increasing rms Mach number with increasing core mass, and some of the least massive cores have an rms Mach number several times smaller than the value corresponding to the energy equipartition, or velocity dispersions as low as 1/5 of the sound speed.

As mentioned above, we make no attempt here to derive observational core properties with synthetic observations, as this work is primarily focused on testing a fundamental theoretical assumption of star-formation models. This comparison with observed prestellar cores is shown to illustrate that the values of the physical parameters of the cores from our simulation are reasonable, which does not require that such values cover the full range of parameter space from the observations. All the physical parameters of the progenitor cores used in this study are listed in Table 1. The Table, as well as other supplemental material, can also be obtained from a dedicated public URL (http://www.erda.dk/vgrid/core-mass-function/).

5 PROGENITOR MASSES VERSUS STELLAR MASSES

5.1 Statistical comparison

The main goal of this work is to test the hypothesis of the core-collapse model, which we express as $M_{\text{star}} \approx \epsilon_{\text{prog}} M_{\text{prog}}$, with $\epsilon_{\text{prog}} \leq 0.5$ and approximately independent of mass, using previously defined quantities. Before comparing cores and stars one-to-one, a look at their mass distribution is already instructive. Figure 4 shows the mass distribution of the progenitors selected at the star-formation snapshots and that of the stars at the end of the simulation. Only the 344 prestellar progenitors, those without older star particles in them (see Sect. 4), are included in the figure. Both the progenitor CMF and the IMF distribution peak at $\sim 0.3 M_\odot$, which is problematic for the core-collapse model.
as it would imply \( \epsilon_{\text{prog}} \approx 1 \). Our resolution convergence test in Appendix A indicates that the mass peak of the CMF is converging towards \( M_{\text{conv}} \approx 0.22 M_\odot \).

The similarity of the mass distributions continues to the high-mass tail, as well. However, this should not be taken to mean that the high-mass progenitors are undergoing a monolithic collapse. In order to show this, we used the tracer particles (see §4) to study how the gas from the progenitor core is either accreted by the primary star \( (f_{\text{tr,prog}}, \text{blue}) \), by other stars \( (f_{\text{tr,other}}, \text{red}) \), or remains unaccreted \( (f_{\text{tr,unacc}}, \text{black}) \). These mass fractions are shown in Figure 5 as a bar chart for the 16 cores with masses larger than 5 \( M_\odot \), where the x-axis is ordered by growing progenitor mass. The mass fractions for two high-mass cores, 7.5 and 88 \( M_\odot \), are not shown as they are identified less than 100 kyr before the end of the simulation, and most of their mass is unaccreted. The bar charts show that in the majority of the massive progenitors, the primary star only accretes less than half of the progenitor’s mass, with the trend worsening towards higher progenitor masses.

This result is also illustrated by the fact that the massive cores show a lot of internal structure, and their time evolution always results in their fragmentation. This is exemplified for star 391 in Figure 6, where the lower panel shows that another star has formed 66 kyr later and the bound core around star 391 has shrunk to a mere 0.16 \( M_\odot \). The vast majority of the gas in its 88 \( M_\odot \) progenitor core is no longer bound to star 391. It is not even the locally dominant core, as the mass of the progenitor core of star 405 is 1.1 \( M_\odot \).

Thus, the statistical similarity between the high-mass tails of the CMF and the stellar IMF does not imply a monolithic collapse of the massive cores as further discussed in the next subsection.

**Table 1.** Stellar and progenitor parameters for a set of 10 stars and their progenitors. The full table is included as an electronic download. The columns are: 1 star index, 2 snapshot number when the new star was recorded, 3 final mass of the star, 4 mass of the progenitor, 5 radius of the progenitor (as an equivalent volume sphere), 6 radius within which the star accretes 95% of its mass, 7 one-dimensional sonic Mach number, 8 kinetic-to-gravitational energy ratio, 9 thermal-to-gravitational energy ratio, 10 inverse of normalized mass-to-flux ratio, 11 final accretion rate as a fraction of the final stellar mass, 12 fraction of progenitor mass which is accreted by other stars, 13 fraction of final stellar mass already present in the progenitor, and 14 fraction of progenitor mass which remains unaccreted by any star can be derived from \( f_{\text{tr,unacc}} = 1 - (f_{\text{tr,other}} + f_{\text{tr,prog}}) \).

![](https://example.com/bar_chart.png)

**Figure 5.** Bar chart of the fractions where the progenitor mass goes, for progenitors with \( M_{\text{prog}} > 5 M_\odot \). The total mass of the progenitor is stated on the x-axis, and the fractions are unaccreted mass \( (f_{\text{tr,unacc}}, \text{black}) \), mass accreted by other stars \( (f_{\text{tr,other}}, \text{red}) \), and mass accreted by the star whose progenitor the core is \( (f_{\text{tr,prog}}, \text{blue}) \). Two high-mass cores, 7.5 and 88 \( M_\odot \), are not shown as they are identified less than 100 kyr before the end of the simulation.

### 5.2 One-to-one comparison

The one-to-one relation between progenitor masses and final stellar masses is addressed by the scatter plots in Figure 7. The figure shows that, for a given core mass, there is a scatter of about two orders of magnitude in the resulting stellar masses (Pearson’s correlation coefficient, \( r \), is 0.51 for all stars). The top panel of Fig. 7 distinguishes the stars that have finished accreting (magenta open circles). Limiting the sample to these stars does not make the correlation signifi-
Figure 6. Upper panel: Column density map of the progenitor core of star 391, seen from x direction. Cyan contours are drawn where there is at least one cell belonging to the progenitor core on the line of sight. The star being formed is in the middle (white circle and number), but it is clear that there are other nodes forming in this high-mass ($88 M_\odot$) core, too. The white dashed box shows the approximate location of the lower panel. Lower panel: As above, but for star 405, 66 kyr (3 snapshots) later. Star 391 is clearly identifiable, but the bound core around it is now only 0.16 $M_\odot$. The progenitor core associated with star 405 is 1.1 $M_\odot$.

Figure 7. Scatter plots of progenitor mass, $M_{\text{prog}}$, and final stellar mass, $M_{\text{star}}$. Upper panel: Blue crosses are all the stars with detected progenitors, and magenta circles are the stars that have also stopped accreting by the end of the simulation (needing 10 Myr or longer to double their mass at the current accretion rate). The blue line is a power-law fit ($M_{\text{star}} \propto M_{\text{prog}}$) to all stars ($a = 0.52$), while the magenta dashed line is a power-law fit to the stars that have stopped accreting ($a = 0.60$). Pearson’s correlation coefficient, $r$, for the blue and the magenta dashed line is 0.51 and 0.57, respectively. The red line shows the relation $M_{\text{star}} = \epsilon_{\text{prog}} M_{\text{prog}}$, with $\epsilon_{\text{prog}} = \epsilon_{\text{acc}} = 0.5$, the value adopted in the simulation. Lower panel: Same as the upper panel, but distinguishing progenitors with only one star (blue crosses) and progenitors with multiple stars born at the same time within them (green triangle). The blue line is a power-law fit to the single stars, with $a = 0.53$ and $r = 0.53$. The correlation is even worse at lower resolution (see Appendix A).

The relatively weak correlation between core and stellar masses is in contradiction with the core-collapse model, because it shows that even if we know the mass of the progenitor core at the birth time of the star, we cannot predict the final stellar mass with any reasonable accuracy. Furthermore, a least-square fit to the data points gives a slope of $a = 0.52$ for all stars (blue line), and $a = 0.60$ for the stars that have finished accreting (magenta dashed line). This shows that even the average relation between core and stellar masses is inconsistent with the idea of a constant efficiency factor, $\epsilon_{\text{prog}}$, as a constant efficiency would imply a slope $a = 1$ (red solid line in Figure 7).
Some of the high-mass progenitors can be expected to fragment and contribute to several stars, as already mentioned in Sect. 5.1. In the bottom panel of Fig. 7 we distinguish between stars that formed alone or accompanied by other stars. Progenitors with multiple stars have an elevated progenitor mass on average at lower stellar masses. This is easily understood, as those progenitors are feeding two or more stars, while at higher masses, the progenitor alone is not enough for those massive stars to grow. If the star is the first one to form inside the core, even if it appears later in another progenitors, it is still counted as single (when we check for stars that appear later in other progenitors, we do not find a clear correlation with their own progenitor masses, so this does not bias the result). For single stars, the Pearson’s correlation coefficient and the slope are $r = 0.53$ and $a = 0.53$, comparable to that of all the stars that have finished accreting. In the case of the stars that grow beyond their progenitor core mass, more mass needs to come in from outside the progenitor. Converging flows may continue feeding mass into the core from beyond its gravitationally-bound limit, allowing the star to accrete much more mass than what was initially available from the core. This is the case in particular with high-mass stars, as further documented in the following, using tracer particles.

Figure 8 shows the histograms of mass ratios, $f_{\text{star,prog}}$ (solid lines) and $f_{\text{star,star}}$ (dashed lines). The first fraction, $f_{\text{star,prog}}$ is the fraction of the progenitor mass that is accreted onto the star; the second fraction, $f_{\text{star,star}}$, is the fraction of the final stellar mass that originates from the progenitor core, as explained in Section 4. Black lines are for all stars, while magenta lines are for stars that have stopped accreting, as explained previously. Vertical lines show the median values. The dots are the values of these fractions for the example progenitors depicted in Figure 9, black for high-mass and red for low-mass. Most stars have a progenitor mass fraction, $f_{\text{star,prog}}$, higher than 0.5, and the distribution of $f_{\text{star,prog}}$ has a strong peak at $f_{\text{star,prog}} = 0.9 - 1.0$. The median value is $\approx 0.85$, meaning that for half of the progenitors, 85% or more of the progenitor mass eventually accretes onto the star. However, the stellar mass fraction, $f_{\text{star,star}}$, has an almost uniform distribution (dashed lines), with a median value of $\approx 0.5$. Thus, many stars have to accrete mass from outside their initial progenitor.

The origin of the mass that feeds the growth of a star is further illustrated in Figure 9 and Figure 10. Fig. 9 shows two example progenitors, where $\sim 95\%$ of the final stellar mass is accreted from the entire 4 pc simulation box for the high-mass example, and close to 60% from the 0.5 pc subcube for the low-mass example. Fig. 10 shows scatter plots of the progenitor radius, $R_{\text{prog}}$ (upper panel), and the inflow radius, $R_{05}$ (lower panel), as a function of the final stellar mass, $M_{\text{star}}$, for all 344 stars. Values of $R_{05}$ above 1 pc may be affected by the finite size of the simulation box, 4 pc, centred on each star for this analysis. The box size limits the distance where tracer particles can originate from, and the random driving force, applied to scales between the box size and half the box size may also affect the particle trajectories at those scales. The red line is the same in both panels, $R_{05} = 0.05 \, \text{pc} \times (M_{\text{star}}/1 \, \text{M}_\odot)^{1/4}$, taken from Figure 23 of Padoan et al. (2019), where it was obtained as a fit to the inflow radius of the stars with the highest accretion rate and was shown to be a good approximation to the lower envelope of the $R_{05}$ versus $M_{\text{star}}$ plot. In Padoan et al. (2019) that plot covered stellar masses above 2-3 $\text{M}_\odot$. Here, the same function is found to be a good approximation to the lower envelope of the plot for all stellar masses, down to brown dwarfs.

The inset in the lower panel shows the ratio of $R_{05}/R_{\text{prog}}$ as a function of the final stellar mass for stars that have already finished accreting, with the dashed red line indicating a ratio of unity. The scatter plot covers rather uniformly a range of values of $R_{05}/R_{\text{prog}}$ between approximately 1 and $10^3$, with a median of 14. There is no correlation between the ratio and the final stellar mass, which indicates that even lower-mass stars are influenced by proportionally as large a region around them as the more massive stars. Of the five lowest ratio cases (with a ratio $< 1$), four are massive progenitors ($M_{\text{prog}} > 7 \, \text{M}_\odot$) that later undergo sub-fragmentation to form relatively low-mass stars (0.3, 0.4, 0.7 and 1.7 $\text{M}_\odot$), and the remaining one is a low-mass progenitor ($M_{\text{prog}} = 0.3 \, \text{M}_\odot$) that results in the formation of a brown dwarf ($0.05 \, \text{M}_\odot$).

Figure 11 shows the progenitor mass fraction, $f_{\text{star,prog}}$, and the stellar mass fraction, $f_{\text{star,star}}$, in a scatter plot. The ratio of the stellar mass and the progenitor mass is $M_{\text{star}}/M_{\text{prog}} \propto f_{\text{star,prog}}/f_{\text{star,star}}$, so the core-collapse model may appear to be satisfied along the diagonal of Figure 11, where $f_{\text{star,prog}} = f_{\text{star,star}}$, or, equivalently, the red line in Figure 7. However, for many stars where $M_{\text{star}} \sim c_{\text{prog}} M_{\text{prog}}$, this agreement is only due to the fact that both $f_{\text{star,prog}}$ and $f_{\text{star,star}}$ are $< 1$, so both mass fractions actually violate the physical assumption of the core collapse model (although not its mathematical expression based only on $M_{\text{star}}$ and $M_{\text{prog}}$). In other words, the core-collapse model is truly satisfied, within an error of a factor of two, only in the upper
6 DISCUSSION

6.1 The Core-Collapse Model

We have compared the final stellar masses with those of their prestellar cores. Accurate predictions of both masses are difficult to obtain, due to the complexity of the star-formation process. However, we argue that uncertainties related to such mass determinations are not critical to our approach to test the core-collapse model and do not affect our conclusions. The mass of a star at the end of the simulation is a good approximation to the final stellar mass, particularly for the stars that were tagged as finished accreting. Because the simulation does not resolve jets and outflows, only half of the accreting mass is assigned to the star particle, a typical value found in theoretical models (e.g. Matzner & McKee 2000). This simple approach to modeling the mass loss from protostellar jets and outflows introduces some uncertainty in the final stellar mass. However, this uncertainty does not affect the fundamental question addressed here, which is the origin of the stellar mass reservoir, irrespective of how much of that may be ejected during its formation.

The estimation of the prestellar core mass is particularly difficult, because they arise within dense filaments as a result of converging flows in the turbulent gas, so they rarely appear as isolated objects with simple morphology. Furthermore, a comparison of prestellar core masses from a simulation with those from observations would have to address a number of observational limitations that are briefly mentioned in § 6.2. This would require an analysis of the simulation based on synthetic observations, and a method of core selection following the observational procedures. We have not attempted that, as our main focus is to test a theoretical idea. In the theoretical models, the stellar progenitors are well-defined entities. In the IMF models of Hen-
nebelle & Chabrier (2008) and Hopkins (2012) the stellar-mass reservoir is a gravitationally bound, overdense region in the turbulent flow, consistent with the core-collapse scenario. Such mass reservoir may not correspond precisely to what the observers identify as prestellar cores (e.g. it may have to contract somewhat before reaching a characteristic core density), or with analytical models of isolated cores (e.g. McKee & Tan 2002, 2003), as it is selected through a statistical description of a complex turbulent flow. However, that complexity is fully accounted for in our simulation, and our progenitor cores are defined as bound regions, as in the models. So the progenitor masses we derive are relevant for testing the IMF models based on the idea of core-collapse, and our approach is not affected by the uncertainties related to a comparison with the observations.

We have found 1) a poor correlation between progenitor and stellar masses, 2) a mean dependence of stellar mass on core mass with a slope significantly shallower than unity, 3) a stellar mass reservoir that extends well beyond the limits of the gravitationally-bound prestellar core. These results are inconsistent with the main hypothesis of the core-collapse model, which is $M_{\text{star}} \approx \epsilon_{\text{prog}} M_{\text{prog}}$ (with $\epsilon_{\text{prog}} \lesssim 0.5$ and approximately independent of mass) for the simulation. Because most of the stellar mass comes from a larger and unbound region, rather than from a gravitationally-bound reservoir, the fundamental assumption of the IMF models of Hennebelle & Chabrier (2008) and Hopkins (2012) and their further developments (e.g. Hennebelle & Chabrier 2009; Hopkins 2013) is incorrect, and an understanding of the IMF based on those models is incomplete at best. The competitive-accretion model is also ruled out, because the Bondi-Hoyle accretion rate from a gravitationally unbound mass reservoir would be too small to explain the accretion rates.

A similar conclusion was reached in Padoan et al. (2019), based on a star-formation simulation of a 250 pc region of the interstellar medium, driven by supernovae. However, in that work only high-mass stars are resolved, so the core-collapse model, as well as the competitive-accretion model, are ruled out only for the origin of massive stars. Here, instead, we show for the first time that the lack of correlation between core and stellar masses applies to the whole IMF, although it may be more extreme for the most massive stars. To further confirm this, we show in Figure 12 a scatter plot of the stellar mass fraction, $f_{\text{tr,star}}$, as a function of the final stellar mass, $M_{\text{star}}$. The plot shows that the stellar mass fraction that comes from the progenitor has a rather
uniform distribution between approximately 0.05 and 1, irrespective of $M_{\text{star}}$. There is only a slight trend with mass. Even at the IMF peak, $\sim 0.2 M_\odot$, there are stars with values as low as $f_{\text{tr},\text{star}} = 0.1$. For low-mass stars ($M_{\text{star}} < 2 M_\odot$) with $f_{\text{tr},\text{prog}} > 0.5$, we find that the median value of $f_{\text{tr},\text{star}}$ is 0.48 (172 stars). For intermediate-mass stars ($2 < 5 M_\odot$), the median value of $f_{\text{tr},\text{star}}$ is only 0.14 (13 stars). Thus, the main assumption of the core collapse model is clearly violated for a majority of stars at all masses, not only for the most massive stars.

6.2 The Observed Core Mass Function

We have identified the progenitor cores from the 3D density and velocity data-cubes of the simulation, and with the knowledge of when and where the star particles are formed. Although this allows us to test the main hypothesis of the core-collapse model, it does not closely resemble the process of identification of prestellar cores in the observations. The observed prestellar CMFs are extracted from 2D intensity information, such as dust-emission or dust-extinction maps, which involves a degree of line-of-sight confusion (e.g. Juvela et al. 2019). Furthermore, the observed quantities must be converted into column density, which, in the case of sub-mm observations, depends on possible temperature and dust-opacity variations, leading to a significant uncertainty in the mass determination (e.g. Malinen et al. 2011; Roy et al. 2014; Pagani et al. 2015; Men’shchikov 2016). Resolution plays a role in deriving the CMF in our simulation, as shown in Appendix A: lowering the resolution by a factor of 8 results in an increase of the median mass by a factor of 3. A similar dependence of the CMF on resolution must also affect the observations.

When a prestellar core is identified in the observations, it is hard to know how close it is to produce a protostar. The core may further increase in mass, or perhaps fragment into smaller cores, before the start of the collapse. In our study, we have selected the progenitors at the (approximate) time of star-particle formation. If we search for cores over the whole simulation box, using all available snapshots, and independent of the star-formation time (see Appendix B), we recover prestellar cores with median masses about a factor of 3 higher than in the case of the progenitor cores at the time of star formation (Figure B1). We interpret this result as showing that, prior to their collapse, prestellar cores are more massive, and later fragment into smaller units by the time they collapse into protostars. A similar factor may apply to the determination of the observed CMF, as the masses of cores may change by the time they collapse, adding further uncertainty to the observed CMF.

It should also be stressed that CMFs are often derived from dust-continuum data without a knowledge of the internal velocity dispersion in the cores, so the internal kinetic energy is neglected in the selection of gravitationally-bound cores. Beside the neglect of the kinetic energy, the core thermal and gravitational energies derived from the observations have significant uncertainties. To account for such uncertainties, observers often assume that a core is gravitationally bound, and can be classified as prestellar, if its estimated mass is half of the critical Bonnor-Ebert mass corresponding to the core radius. Besides the uncertainties in the mass determination mentioned above, the peak of the derived CMF is sensitive to this definition of prestellar cores, and would shift to larger masses if only gravitationally unstable cores (more massive than the critical Bonnor-Ebert mass) were selected, and/or the core internal kinetic energy were included.

The issues we have raised with regards to the observed CMFs may explain the variation of the peak of the CMF from region to region, even within a homogeneous set of CMFs (same telescope, same data analysis, same research group). For example, the peak mass is found to be a few times larger than the IMF peak in Aquila and Orion (Könnyves et al. 2015, 2020), while it is very close to, or even slightly smaller than the IMF peak in Taurus and Ophiucus (Marsh et al. 2016; Ladjelate et al. 2020). Despite the observational uncertainties and the significant differences in the way cores are extracted from the simulation and the observations, we do find a statistical similarity between the CMF and the IMF, as in the observations. However, our results show that, rather than looking at the IMF as being born from the CMF through a constant efficiency factor, both the CMF and the IMF should be viewed as being formed and fed by the same inertial inflows that arise naturally in supersonic turbulence.

7 CONCLUSIONS

We have addressed the relation between the final mass of a star and that of its prestellar core to test the main hypothesis of the core-collapse model. Using a simulation that produces a realistic stellar IMF under realistic physical conditions found in molecular clouds, we have extracted, for each star particle, its gravitationally-bound progenitor core, at the time when the star is created. From a statistical analysis, a one-to-one comparison between progenitor and stellar masses, and a study of the mass flow based on tracer particles, we have reached the conclusions listed in the following.

(i) The progenitor CMF converges with resolution, with a peak moving from 0.66 $M_\odot$ to 0.28 $M_\odot$ using resolutions from 800 AU to 100 AU. The estimated converged position of the peak is 0.22 $M_\odot$, close to the IMF peak, which is contradictory to the core-collapse model.

(ii) The CMF derived from the simulation is very similar to the stellar IMF from the same simulation. Irrespective of this statistical similarity, we find no direct correlation between the progenitor core mass and the final stellar mass for individual stars, contrary to the hypothesis of the core-collapse model.

(iii) A significant fraction of the mass reservoir of stars is generally outside of the progenitor cores. This applies across the whole IMF, not just for massive stars. For stars less than 1 $M_\odot$, $\approx 50\%$ of the stellar mass originates outside the core. This increases to $\approx 90\%$ for intermediate-mass stars ($2 < M/M_\odot < 5$).

(iv) The inflow region that contains 95% of the mass reservoir of a star is generally much larger than the size of the progenitor core. The ratio between the inflow radius and the core radius has a median value of 14 and its largest values are $\sim 10^3$. This size ratio shows no significant correlation with the final stellar mass.

(v) The competitive-accretion model is also ruled out: the
region that amounts to the stellar mass reservoir is not gravitationally bound, hence the Bondi-Hoyle accretion rate from that region would be too small to explain the actual accretion rates. (vi) The similarity between observed CMFs and the stellar IMF is confirmed by the simulation. However, observed CMFs should in principle result in a larger core mass on average, compared to the cores selected in the numerical data, because of limitations in resolution and not having a priori knowledge of which cores will form stars in the future. Inclusion of cores that may not be gravitationally unstable in the observed CMFs may have the opposite effect.

The main conclusion of this work is that the results from one of the most realistic star-formation simulations to date, which resolves both the IMF and the CMF, rule out the core-collapse idea for all stellar masses. Because this idea is a fundamental assumption in some recent theoretical models of the stellar IMF, our work implies that attempts at understanding the stellar IMF based on this assumption are incomplete. Specifically, the similarities of the observed CMF with the stellar IMF should not be interpreted as a one-to-one relation between individual core masses and final stellar masses, as the two masses are poorly correlated even when prestellar cores are identified without all the uncertainties affecting the observations. Having now established that the stellar mass reservoir resides well beyond the limits of gravitationally-bound prestellar cores, future theoretical and observational work should address the role of inertial inflows in shaping the stellar IMF.

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DATA AVAILABILITY

Table 1, as well as other supplemental material, can also be obtained from a dedicated public URL (http://www.erdadk/vgrid/core-mass-function/).

REFERENCES

Alves J., Lombardi M., Lada C. J., 2007, A&A, 462, L17
André P., et al., 2010, A&A, 518, L102+
André P., Di Francesco J., Ward-Thompson D., Inutsuka S. I., Padgett R. E., Pineda J. E., 2014, in Beuther H., Klessen R. S., Dullemond C. P., Henning T., eds, Protostars and Planets VI, p. 27 (arXiv:1312.6232), doi:10.2458/azu_uapress_9780816531240-d0002
Bate M. R., 2009, MNRAS, 392, 590
Bate M. R., 2012, MNRAS, 419, 3115
Bonelli I. A., Bate M. R., 2006, MNRAS, 370, 488
Bonelli I. A., Bate M. R., Clarke C. J., Pringle J. E., 2001a, MNRAS, 323, 785
Bonelli I. A., Clarke C. J., Bate M. R., Pringle J. E., 2001b, MNRAS, 324, 573
Bonelli I. A., Vine S. G., Bate M. R., 2004, MNRAS, 349, 735
Chabrier G., 2005, in E. Corbelli, F. Palla, & H. Zinnecker ed., Astrophysics and Space Science Library Vol. 327, The Initial Mass Function 50 Years Later. pp 41–+ (arXiv:astro-ph/0409465)
Enoch M. L., Glenn J., Evans II N. J., Sargent A. I., Young K. E., Huard T. L., 2007, ApJ, 666, 982
Genel S., Vogelsberger M., Nelson D., Sijacki D., Springel V., Hernquist L., 2013, MNRAS, 435, 1426
Gong M., Ostriker E. C., 2015, ApJ, 806, 31
Haugbølle T., Padoan P., Nordlund Å., 2018, ApJ, 854, 35
Hennebelle P., Chabrier G., 2008, ApJ, 684, 395
Hennebelle P., Chabrier G., 2009, ApJ, 702, 1428
Hopkins P. F., 2012, MNRAS, 423, 2037
Hopkins P. F., 2013, MNRAS, 430, 1653
Juvela M., Padoan P., Ristorcelli I., Pelkonen V.-M., 2019, A&A, 629, A63
Kirk H., Johnstone D., Tafalla M., 2007, ApJ, 668, 1042
Kirk H., et al., 2017, ApJ, 846, 144
Klessen R. S., 2001, ApJ, 546, 837
Kong S., 2019, ApJ, 873, 31
Könyves V., et al., 2010, A&A, 518, L106+
Könyves V., et al., 2015, A&A, 584, A91
Könyves V., et al., 2020, A&A, 635, A34
Kroupa P., 2001, MNRAS, 322, 231
Krumholz M. R., Federrath C., 2019, Frontiers in Astronomy and Space Sciences, 6, 7
Krumholz M. R., McKee C. F., Klein R. I., 2005, Nature, 438, 332
Ladjelate B., et al., 2020, A&A, 638, A74
Li S., Zhang Q., Pillai T., Stephens I. W., Wang J., Li F., 2019, arXiv e-prints, p. arXiv:1909.08916
Malinen J., Juvela M., Collins D. C., Lunttila T., Padoan P., 2011, A&A, 530, A101
Marsh K. A., et al., 2016, MNRAS, 459, 342
Matzner C. D., McKee C. F., 2000, ApJ, 545, 364
McKee C. F., Tan J. C., 2002, Nature, 416, 59
McKee C. F., Tan J. C., 2003, ApJ, 585, 850
Men'shchikov A., 2016, A&A, 593, A71
Motte F., Andre P., Neri R., 1998, A&A, 336, 150
Mouschovias T. C., Spitzer L. J., 1976, ApJ, 210, 326
Ntormousi E., Hennebelle P., 2019, A&A, 625, A82
Nutter D., Ward-Thompson D., 2007, MNRAS, 374, 1413
Ohashi S., Sanhueza P., Chen H.-R. V., Zhang Q., Busquet G., Nakamura F., Palau A., Tatamatsu K., 2016, ApJ, 833, 209
Padoan P., Nordlund Å., 2002, ApJ, 576, 870
Padoan P., Nordlund Å., 2011, ApJ, 741, L22
Padoan P., Nordlund Å., Kritsuk A. G., Norman M. L., Li P. S., 2007, ApJ, 661, 972
Padoan P., Haugbølle T., Nordlund Å., 2014, ApJ, 797, 32
Padoan P., Pan L., Juvela M., Haugbølle T., Nordlund Å., 2019, arXiv e-prints, p. arXiv:1911.04465
Pagani L., Lefèvre C., Juvela M., Pelkonen V. M., Schuller F., 2015, A&A, 574, L5
Peretto N., et al., 2013, A&A, 555, A112
Pillai T., Kaufmann J., Zhang Q., Sanhueza P., Leurini S., Wang K., Sridharan T. K., König C., 2019, A&A, 622, A54
Rosolowsky E. W., Pineda J. E., Kaufmann J., Goodman A. A., 2008, ApJ, 679, 1338
Roy A., et al., 2014, A&A, 562, A138
Sanhueza P., Jackson J. M., Zhang Q., Guzmán A. E., Lu X., Stephens I. W., Wang K., Tatamatsu K., 2017, ApJ, 841, 97
Sanhueza P., et al., 2019, arXiv e-prints, p. arXiv:1909.07985

MNRAS 000, 1–16 (2020)
Schmidt W., Kern S. A. W., Federrath C., Klessen R. S., 2010, A&A, 516, A25
Servajean E., Garay G., Rathborne J., Contreras Y., Gomez L., 2019, ApJ, 878, 146
Smith R. J., Longmore S., Bonnell I., 2009, MNRAS, 400, 1775
Tanaka K. E. I., Tan J. C., Zhang Y., 2017, ApJ, 835, 32
Teyssier R., 2007, Geophysical and Astrophysical Fluid Dynamics, 101, 199
Tilley D. A., Pudritz R. E., 2004, MNRAS, 353, 769
Tilley D. A., Pudritz R. E., 2007, MNRAS, 382, 73
Zinnecker H., 1982, Annals of the New York Academy of Sciences, 395, 226

APPENDIX A: CLUMPFIND PARAMETERS

Figure A1 shows the histograms of core mass at different resolution, with the clumpfind algorithm run for the 0.5 pc subcubes around the star particles with $f = 2\%$. It is clear that the peak of the mass distribution moves to the lower masses with the increasing resolution, summarized in Table A1. This is because it is easier to distinguish the border of the individual cores at higher resolutions, rather than smear everything out and thus select whole clouds. By contrast, ratio, $f$, does not seem to have an effect in the highest resolution, 8192$^3$, run. The mass histograms are almost identical for $f = 2\%, 4\%, 8\%$. This is presumably because at 8192$^3$, the core boundary is already sharp enough that we do not need to have a finer density difference sampling to separate the cores.

Figure A2 shows how the peak of the progenitor CMF ($M_{\text{peak}}$, blue line), the fraction of the detected stars ($f_d$, red line) and the fraction of single star progenitors ($f_s$, dashed red line) depend on the resolution, as reported also in Table A1. As mentioned above, the peak of the CMF moves to lower masses with increasing resolution. The estimated value of the converged peak mass, $M_{\text{conv}} = 0.215 M_\odot$, is determined by the least-squares fitting minimum, when $M_{\text{peak}}(x) - M_{\text{conv}}$ is best approximated by a straight line in logarithm space, where $x$ is the resolution. Figure A2 also shows that the fraction of the single star progenitors, $f_s$, increases with the resolution, which is another reason to use the highest feasible resolution.

Figure A3 shows the one-to-one scatterplot of final stellar masses and the masses of their progenitors, like in bottom panel of Fig. 7, but the selection is done at 1024$^3$, the same as in Figure A1, and the fractions of detected progenitors, $f_d$, and of single-star progenitors out of the detected ones, $f_s$, used in Figure A2.

APPENDIX B: THE CMF OF THE WHOLE BOX

What does the CMF look like, if we do the detection on the whole 4 pc simulation box without apriori knowledge of the star particles? We add the stellar mass in (see Sect. 3.3) and do the bound core selection in each snapshot at 1024$^3$ resolution with the clumpfind algorithm run for the 0.5 pc subcubes around the star particles with $f = 2\%$. The core boundary is already sharp enough that we do not need to have a finer density difference sampling to separate the cores. By contrast, ratio, $f$, does not seem to have an effect in the highest resolution, 8192$^3$, run.

![Figure A1](image1.png)

**Figure A1.** Prestellar progenitor mass histograms for resolutions 1024$^3$, 2048$^3$, 4096$^3$ and 8192$^3$ for the whole box, using $f = 2\%$ and 0.5 pc subcubes around the star particles in the selection. Dashed lines are lognormal fits ($N \times \exp(-(\log_{10} M - \log_{10} M_{\text{peak}})^2/2\sigma^2)$ to masses lower than $2 M_\odot$, and the values are given in Table A1.

![Figure A2](image2.png)

**Figure A2.** The peak of the core mass function, the fraction of detected stars of all the 413 stars, and the fraction of single star progenitors of the detected stars, as a function of the full box resolution. The peak of the core mass function converges towards $M_{\text{conv}} = 0.215 M_\odot$, as shown by $M_{\text{peak}}(x) - M_{\text{conv}}$ almost forming a straight line in logarithm space. Selections of the cores are done using $f = 2\%$ and 0.5 pc subcubes around the star particles.

**Table A1.** Parameters of the lognormal fits to mass histograms in Figure A1, and the fractions of detected progenitors, $f_d$, and of single-star progenitors out of the detected ones, $f_s$, used in Figure A2.

| Resolution | $M_{\text{peak}} [M_\odot]$ | $\sigma$ | $f_d$ | $f_s$ |
|------------|-------------------------------|---------|-------|-------|
| 1024       | 0.66                          | 0.43    | 0.93  | 0.49  |
| 2048       | 0.46                          | 0.37    | 0.96  | 0.62  |
| 4096       | 0.34                          | 0.43    | 0.96  | 0.74  |
| 8192       | 0.28                          | 0.42    | 0.92  | 0.81  |
Figure A3. Same as the bottom panel in Fig. 7, but core selection is done at 1024$^3$ whole box resolution in 0.5 pc subcubes around the star particles. Progenitors are plotted with different symbols based on how many and how old stars they have inside them: progenitors with only one star (blue crosses), progenitors that have multiple stars born at the same time (green triangle), and progenitors that have already formed protostars in them (red dots). The red line is drawn at constant $\epsilon_{\text{acc}} = 50\%$ that was used in the simulation, if all progenitor mass would be accreted by the star, while black and blue lines are fits to all progenitors and the single star progenitors, respectively.

Resolution, using minimum density level 10 and $f = 2\%$. We then categorize them as prestellar cores without stars, prestellar cores with stars (one or more stars that formed in that snapshot) which would be our progenitor cores, and protostellar cores (one or more older stars), according to the star locations and ages.

Figure B1 shows the mass histograms of all prestellar cores detected in the 114 snapshots of the simulation, and in subcategories of prestellar cores without stars and progenitors where the star is currently forming. We can see from the median values that if we do not have the star information, the median mass of all prestellar cores is about three times higher than that of just the progenitors.

APPENDIX C: PROGENITOR COLUMN DENSITY MAPS

Column density maps for progenitors in Table 1, identified by the number in the upper left corner, with the letter based on the line-of-sight axis. Each map is based on the density subcube used in the identification of the progenitor, usually a 0.5 pc region (see the scale bar on the bottom left corner), centred on the newly-born star, with the time of the identification in the bottom right corner. The logarithmic colorbar is the same as in Figure 1, running from $N_{\text{H}_2} = 10^{21.5}$ cm$^{-3}$ to $N_{\text{H}_2} = 10^{24}$ cm$^{-3}$ before saturating. Cyan contours are drawn where there is at least one cell belonging to the progenitor core on the line of sight. Similar maps for all progenitors at even higher resolution are available at URL: http://www.erda.dk/vgrid/core-mass-function/.
