The decay $\tau \rightarrow \pi\omega\nu$ in the extended NJL model

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Abstract

The decay width $\tau \rightarrow \pi\omega\nu$ in the framework of the extended NJL model is calculated. The contributions of the intermediate vector mesons $\rho(770)$ and $\rho'(1450)$ are taken into account. The computed partial width and the spectral function of the decay $\tau \rightarrow \pi\omega\nu$ are in satisfactory agreement with experimental data.

Keywords: tau decays, chiral symmetry, Nambu-Jona-Lasinio model, radial excited mesons

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1 Introduction

The process of the decay $\tau \rightarrow \pi\omega\nu$ is intensively investigating from experimental [1,2] as well as from theoretical [3–5] points of view. In these works phenomenological models with intermediate vector mesons $\rho(770)$, $\rho(1450)$ and $\rho(1700)$ were used. In all these models arbitrary parameters was introduced and adjusted to fit experimental data.

On the other hand the similar processes $e^-e^+\rightarrow \pi^0\gamma, \pi^0\rho^0, \pi^0\omega, \pi^+\pi^-$ in the extended Nambu-Jona-Lasinio (NJL) model [13–16] were described [6–9].

Let us note that NJL [16–21] model allows us to describe the number of tau lepton decays [10–12]. In this work the investigation of the $\tau$ decays is continued and the decay $\tau \rightarrow \pi\omega\nu$ is calculated in the framework of the extended NJL model which takes into account intermediate $\rho(770)$ and $\rho(1450)$ mesons.

2 The decay $\tau \rightarrow \pi\omega\nu$

The amplitude of the decay is described by the Feynman diagrams given in Figs. 1 and 2. These diagrams are similar to diagrams used in Ref. [6] to describe the process $e^-e^+\rightarrow \omega+\pi^0$.

The Lagrangian of quark-meson interactions in the framework of the extended NJL model was given in Refs. [6,7,15]. Therefore, in present work we give only the expression for the amplitude describing the decay $\tau \rightarrow \pi\omega\nu$:

$$T = G_F|V_{ud}|\bar{\nu}(1 - \gamma^5)\gamma^\mu \tau(T_{W\rho} + T_{\rho'})\epsilon_{\mu\nu\rho\sigma}p_\omega^\nu p_\pi^\sigma,$$

where $G_F = 1.16637 \cdot 10^{-11}$ MeV$^{-2}$ is the Fermi coupling constant; $|V_{ud}| = 0.97428$ is the cosine of the Cabibbo angle, $p_\omega$ and $p_\pi$ are the $\omega$ and $\pi$ meson momenta.

The $T_{W\rho}$ term corresponds to the contribution given by the contact diagram and the diagram with an intermediate $\rho(770)$ meson. Using the factor for $W-\rho$ transition, we can get the expression that coincides with

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one given by the vector meson dominance model:

\[
T_{W\rho} = \frac{C_{\rho}}{g_{\rho_1}} \frac{1 - i\Gamma_{\rho}/m_{\rho}}{m_{\rho}^2 - p^2 - im_{\rho}\Gamma_{\rho}} m_{\rho}^2,
\]

where \( m_{\rho} = 775.49 \) MeV is the mass of \( \rho(770) \) meson and \( \Gamma(m_{\rho}^2) = 149.1 \) MeV is its total decay width. The contribution of the amplitude with an intermediate \( \rho(1450) \) meson reads

\[
T_{\rho'} = C_{\rho'} C_{W\rho'} \frac{p^2}{m_{\rho'}^2 - p^2 - i\sqrt{p^2}\Gamma_{\rho'}(p^2)},
\]

where \( C_{W\rho'} \) corresponds to the \( W - \rho' \) transition, it was defined in [12]. The vertex constants \( C_{\rho} \) and \( C_{\rho'} \) are defined from the extended NJL model Lagrangian [6]:

\[
\frac{C_{\rho}}{g_{\pi_1}} = \left( g_{p_2} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \right)^2 I_3 + \left( g_{p_2} \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \right)^2 I_{3f}^f + 2g_{p_1}g_{p_2} \frac{\sin(\beta + \beta_0) \sin(\beta - \beta_0)}{\sin(2\beta_0) \sin(\beta_0)} I_3^f
\]

\[
\frac{C_{\rho'}}{g_{\pi_1}} = g_{p_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} I_3 + g_{p_2} \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} I_{3f}^f + g_{p_1} \frac{\sin(\beta + \beta_0)}{\sin(2\beta_0)} g_{p_2} \frac{\cos(\beta + \beta_0)}{\sin(2\beta_0)} I_3^f + g_{p_2} \frac{\cos(\beta - \beta_0)}{\sin(2\beta_0)} g_{p_1} \frac{\sin(\beta - \beta_0)}{\sin(2\beta_0)} I_3^f,
\]

where \( g_{p_1} = 6.14, g_{p_2} = 10.56, g_{\pi_1} = g_{p_1}/\sqrt{6}, \beta_0 = 61.44^\circ, \beta = 79.85^\circ \). The definitions of integrals \( I_3, I_{3f}^f, I_3^f \)

\footnote{In this work we use re-calculated values for the set of parameters of the extended NJL model which are a bit different from the ones used in the previous works.}
Using the same set of parameters and the approach to loop integral computation, we re-calculated the energy dependence of the $e^+ + e^- \rightarrow \omega + \pi^0$ process cross-section. Here we get a better description of the experimental data with respect to our earlier result \( [6] \) obtained within the same model (but with another treatment of the loop integrals and other parameter values). It allows us to get a better agreement with experimental data for $e^+ e^- \rightarrow \pi \omega$ cross-section, see Fig. 4.

Figure 3: Comparison of CLEO \( [1] \) spectral function (dots) with the NJL predictions. Solid, dotted and dashed lines are the total, $T_{W \rho}$ and $T_{\rho'}$ contributions, respectively.

Use this formulas we get values for the branching of the $\tau \rightarrow \pi \omega \nu$:

$$Br^{NJL} = 1.85\%$$

(6)

In experimental work \( [1] \) the results of the fit CVC model is given. With the help of the formula given in \( [1] \) we can get a prediction for the spectral density, see Fig. 3. The position of the peak in the NJL prediction for the spectral density differs from the one seen in the data. First of all, this is because we took the standard value of the $m_{\rho'} = 1465$ MeV value, while the fit of the CLEO data gives about 1520 MeV. Moreover, we did not take into account the contribution of the $\rho''$ intermediate state. The latter is important for the spectral function at large $q$ values, while it is very much suppressed in the decay spectrum.

From our model one can get parameters for the density and the form factor:

$$g_{\rho \omega \pi} = \frac{3g_{\rho}^2}{8\pi^2 F_{\pi}} = 15.4 \text{ GeV}^{-1},$$

(7)

$$A_1 = \frac{C_{\rho}C_{W\rho'}}{C_{\rho}} = -0.13.$$

(8)

The comparison of experimental and theoretical values for these parameters is presented in Table 1.

3 Conclusions

The presented calculation show that the extended NJL model allows to describe the branching of the decay $\tau \rightarrow \pi \omega \nu$ in a satisfactory agreement with experimental data without introduction of any additional arbitrary parameters. This fact distinguishes our model from phenomenological approaches used earlier in Refs. \( [3-5] \).

\( ^2 \)In present work integrals $I_3, I_4, I_4'$ was calculated in the $p^2$ approximation as it was done in more recent works \( [7, 22] \) with 3-dimensional cut-off.
Table 1: Experimental and theoretical data.

| Theory       | $g_{\rho\omega\pi}$, 1/GeV | $A_1$ | Br($\tau \to \pi\omega\nu$), % |
|--------------|----------------------------|-------|-------------------------------|
| CLEO [1]     | 16.10 ± 0.06               | −0.23 ± 0.02 | 1.95 ± 0.08                  |
| ALEPH [2]    | −                         | −     | 1.91 ± 0.13                   |
| SND-2011 [24]| 15.75 ± 0.45               | −0.29 ± 0.09 | −                            |

Figure 4: Comparison of experimental data of SND-2 (squares [23] and dots [24]) for $e^+e^- \to \pi\omega$ with the NJL prediction (solid line).

We note that CLEO experimental [1] values for energy range from 1.4 to 1.5 GeV don’t coincide with number of other experiments [24] and NJL prediction. It may affect to values for $\rho'$ mass and $\rho''$ contribution given by fits [1].

In future works we are going to describe within the same model the tau lepton decays with the creation of $\eta, \eta'$ mesons.

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References

[1] K. W. Edwards et al. [CLEO Collaboration], Phys. Rev. D 61, 072003 (2000) [hep-ex/9908024].
[2] D. Buskulic et al. [ALEPH Collaboration], Z. Phys. C 74, 263 (1997).
[3] G. Lopez Castro and D. A. Lopez Falcon, Phys. Rev. D 54, 4400 (1996) [hep-ph/9607409].
[4] A. Flores-Tlalpa and G. Lopez-Castro, Phys. Rev. D 77, 113011 (2008) [arXiv:0709.4039 [hep-ph]].
[5] Z. -H. Guo, Phys. Rev. D 78, 033004 (2008) [arXiv:0806.4322 [hep-ph]].
[6] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Phys. Rev. C 83, 048201 (2011) [arXiv:1012.2455 [hep-ph]].
[7] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Eur. Phys. J. A 47, 103 (2011).
[8] M. K. Volkov and D. G. Kostunin, arXiv:1204.1455 [hep-ph].
[9] A. I. Ahmadov, E. A. Kuraev and M. K. Volkov, arXiv:1111.2124 [hep-ph].
[10] Y. P. Ivanov, A. A. Osipov and M. K. Volkov, Z. Phys. C 49, 563 (1991).
[11] M. K. Volkov, Y. P. Ivanov and A. A. Osipov, Sov. J. Nucl. Phys. 52, 82 (1990) [Yad. Fiz. 52, 129 (1990)].
[12] M. K. Volkov and D. G. Kostunin, arXiv:1202.0506 [hep-ph].
[13] M. K. Volkov and C. Weiss, Phys. Rev. D 56, 221 (1997) [hep-ph/9608347].
[14] M. K. Volkov, Phys. Atom. Nucl. 60, 997 (1997) [Yad. Fiz. 60, 1115 (1997)].
[15] M. K. Volkov, D. Ebert and M. Nagy, Int. J. Mod. Phys. A 13, 5443 (1998) [hep-ph/9705334].
[16] M. K. Volkov and A. E. Radzhabov, Phys. Usp. 49, 551 (2006).
[17] M. K. Volkov, Annals Phys. 157, 282 (1984).
[18] M. K. Volkov, Sov. J. Part. Nucl. 17, 186 (1986) [Fiz. Elem. Chast. Atom. Yadra 17, 433 (1986)].
[19] D. Ebert and H. Reinhardt, Nucl. Phys. B 271, 188 (1986).
[20] M. K. Volkov, Phys. Part. Nucl. 24, 35 (1993).
[21] D. Ebert, H. Reinhardt and M. K. Volkov, Prog. Part. Nucl. Phys. 33, 1 (1994).
[22] A. B. Arbuzov and M. K. Volkov, Phys. Rev. C 84, 058201 (2011) [arXiv:1108.0050 [hep-ph]].
[23] M. N. Achasov et al., Phys. Lett. B 486, 29 (2000) [arXiv:hep-ex/0005032].
[24] M. N. Achasov, A. Y. Baryyakov, K. I. Beloborodov, A. V. Berdyugin, D. E. Berkaev, A. G. Bogdanchikov, A. A. Botov and D. A. Bukin et al., JETP Lett. 94, 2 (2012).