The dynamic programming algorithm for management of real-time aircraft assignments

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Abstract. The schedule is the basis of any airline. In this paper, we provide one of the problems of schedule theory. This problem of optimal management of flight schedule is to minimize flight departure delays and to improve flight on-time performance. This paper provides a conceptual and mathematical formulation of optimizing an operating flight schedule problem. The problem solution is to manage the assignments of specific air-crafts on flights considering the operational information about the implementation of the planned flight schedule. This problem is NP-hard and can not be solved exactly for any real dimensions. We propose the parametric effective algorithm for its approximate solution and provide the statistical data for testing.

1. Introduction. Statement of the conceptual problem

The planning of airline operations is a traditional field where optimization techniques can be used. The time schedule is the basis of any airline’s activities, solving the problem of optimum schedule management. Therefore, the authors present the most important problem. Traditionally, the problem of schedule management is divided into several subtasks, which can be tackled either separately or jointly [1]:

- scheduling;
- fleet assignment over the areas of operation;
- development of aircraft routing taking into account the necessary maintenance;
- crew scheduling.

The first attempt to solve the problem of optimal schedule management was undertaken in 1989 by Abara for American Airlines, Inc. Subsequently, similar problems were solved for such airlines as United Airlines and Delta. It is quite obvious that the tasks listed above are closely interrelated. All their actual formal representations belong to the class of intractable mixed-programming problems. The approaches used to search for approximate solutions rely on classical schemes such as Lagrangian relaxation methods, column generation and Benders decomposition techniques. Also, well-known computational tools of combinatorial optimization and cut-off methods as well as constraint-based programming approaches are used [2], [3].

In the present paper, we address the original conceptual statement of a special applied problem concerning the optimum operational control of aircraft fleet assignments aimed at the maximization of on-time
performance of flight arrivals to be achieved through minimization of propagated delays. The basic material is supplemented with demonstrating the solution of a model example based on the use of an efficient algorithm for optimizing the schedules of a parallel system with delays at the start of service.

On-time performance (or punctuality) is one of the key indicators characterizing the work of airlines in delivering passengers, baggage and cargoes in accordance with the transportation agreement. This is a percentage ratio of the number of flights accomplished on time (with an acceptable delay) and the total number of completed flights. In the Russian Federation, the rules for recording on-time performance are defined in Order No. 6 of the Civil Aviation Ministry of the USSR dated 10.01.1990, "On approval and implementation of the Guidelines for provision and keeping a record of on-time performance of flights of civil aviation aircrafts". The practical value of the applied problem under consideration is in the positive influence of on-time performance on both the passenger loyalty and flight servicing costs. Possible ways to manage both the costs and passenger loyalty through the prism of on-time performance are listed below.

- Reducing the number of delays is normalized by the reduction of direct expenditures for servicing the flight and passengers.
- Airlines operating flights with high on-time performance may receive additional discounts on ground handling from airports.
- Reducing the number of delays allows the minimization of the number of passenger disruptions, which results in a positive effect on a company’s image.

In terms of on-time performance management, the most urgent task is the operational management of an airline’s schedule. Operational management involves adjusting the assignments of particular numbers of the aircraft tail for flights with an optimality criterion based on the information on the current status of flight operations and the estimated delay time. While solving the task of aircraft assignment for flights, it is also important to take into account the fact that a delay in one flight may entail a delay in the departure of the next flight if there is not enough time to properly service the aircraft between the two flights. The accumulation of delays can ultimately lead to the violation of flight schedules, disruption of transfer passengers, and violation of the crew’s work schedule (especially at basic airports). Lan, Clark and Barnhart [1] have proposed the following classification of delays:

- propagated delays, or delays arising due to the delay of the aircraft at the previous stage of flight;
- non-propagated delays, or delays not related to the violation of the schedule at the previous stages of the aircraft’s flight.

The basis for calculating the delay propagation time is the concept of the minimum aircraft handling time at the airport of operation (Minimum Turn Time, or MTT). MTT is the minimum time required to prepare an aircraft for departure of the next flight; it is determined for a particular type of aircraft. As a rule, the scheduled time interval between flights exceeds MTT in order to provide a reserve of time in the case of delays. The idea underlying the task in question is to develop an optimum aircraft traffic schedules ensuring the necessary time reserve to compensate for the delays at the previous flight stages.

2. Formal statement of the problem of Airline Schedule Management

A problem of schedule synthesis of parallel-sequential support facilities (hereinafter - PSF) is, in respect to industrial controls, the most general and current problem of scheduling theory. A general statement of the problem, connected subtasks and a description of some solution algorithms can be found in [4]. In this particular case, we take an airline fleet as an example of PSF. In this instance one of the major problems is the enhancement of aircraft assignment and airline flight schedule. We introduce the following notation:

\[ l, - \text{ airport number } l \in L, \overline{L} = \sup L ; \]
$i_j$ - flight number $i \in I_j$, $\bigcup_{l \in I_j} I_l = I$, $I_l \cap I_{l'} = \emptyset$, $\forall l, l' \in L, I = \sup I$;

$s_j$ - aircraft type $s \in S$, $\overline{S} = \sup S$; aircraft type is a category unifying particular aircraft rating according to its technical and economic characteristics (flight distance, number of passengers and etc.). The coding of aircraft types is also recommended by IATA.

$j_i$ - aircraft tail number $j \in J_s$, $\bigcup_{s \in S} J_s = J$, $J_s \cap J_{s'} = \emptyset$, $\forall s, s' \in S, J = \sup J$. It is important to note that the aircraft tail number is unique and is given by a manufacturer upon production.

$t_{i,j}$ - service time required for aircraft preparation $j$ and flight time $i$, $T = \|t_{i,j}\|$, $i \in I_j$, $\forall l \in L$, $j \in J_s$, $s \in S$.

Equation (1) means that only one aircraft is allocated on flight $i$:

$$\sum_{j \in J} x_{i,j} = 1, \forall l \in L,$$  (2)

Equations (2) mean that only one aircraft is allocated on flight $i$:

Let us determine $\tau_{i,j}$ - estimated delay time of an aircraft $j$ at this stage (flight $i$) is a recursive function of delays of preceding flights of this aircraft:
\[ \hat{\tau}_{i,j} = \tau_{i,j} + y_{i,j} \geq 0, \quad i \in I, \quad \forall l \in L, \quad j \in J, \quad \forall s \in S, \tag{5} \]
\[ y_{i,j} \geq 0, \quad i \in I, \quad \forall l \in L, \quad j \in J, \quad \forall s \in S. \tag{6} \]

The conditions (5) and (6) neutralize possible negative delays through variable equalizers \( y_{i,j} \geq 0 \); thereafter \( \hat{\tau}_{i,j} \geq 0 \) response variable problems are corrected delays of departure of an aircraft \( j \) flight \( i \).

\[ \sum_{i \in I, j \in J} \hat{\tau}_{i,j} x_{i,j} + \sum_{i \in I, j \in J} t_{i,j} x_{i,j} \leq \lambda, \quad \forall l \in L, \quad j \in J, \quad s \in S, \tag{7} \]
\[ \lambda \rightarrow \min. \tag{8} \]

3. Implementation and testing of the algorithm

Due to recursions availability Dynamic Programming (DP) is almost single computational method applicable straight to solution of the problem (1)-(8). However, its direct application is not effective. At attempt of exact solution of (1)-(8) DP leads to full enumeration of any available possibilities. It is easy to calculate the number of such possibilities \( N \). For instance, if \( k \) is a stage number and if we set \( \bar{b}_j = 0 \), and \( \bar{b}_j = \sup I \), then, as it is shown below, we consider geometrical progression of possibilities number by DP steps, \( N = \left( \frac{J^{T+1} - J}{J - 1} \right)/2 \). For this reason, the application of the DP method for solution of the problem (1)-(8) is time-consuming and leads to excess of exponential, which makes this method inapplicable with real dimensions.

We use a general DP scheme for building an effective approximate algorithm with recursive elimination of the most inadequate possibilities on a number of DP steps (stages). This approach was tested by the authors earlier when solving the problem of schedule optimization of unrelated parallel instruments with delays of inception of service [5], and it showed good accuracy and speed results in practice.

Algorithm \( A_p \)

1. Input of initial values \( (\tau_{i,j}^0, t_{i,j}) \), \( j \in J, s \in S \), \( i \in I, l \in L \) and arguments \( k \) and \( N' \). Set \( \phi_{0,j}(\tau_{0,j}, t_{i,j}, x_{i,j}) = 0 \) and determine introductory number of stage \( \eta := 0 \).
2. Increase the number of stage \( \eta := \eta + 1 \).
3. Check the number of the stage. If \( \eta > \bar{T} \), go to step 7; if not, then go to the next step.
4. At the stage \( \eta \) we determine the order of succession of further stages (reallocating the flights list), calculate the variables of delays \( \hat{\tau}_{n,j} \), and generate all possible assignments and calculate \( f_{n,j}(\hat{\tau}_{n,j}, t_{n,j}, x_{n,j}) \) and schedule length \( \phi_{n,j}(\hat{\tau}_{n,j}, t_{n,j}, x_{n,j}) \).
5. Check \( N' \) number of cases \( \phi_{n,j}(\hat{\tau}_{n,j}, t_{n,j}, x_{n,j}) \) at the stage \( \eta \). If \( \eta < k \), i.e. \( N' \leq N' \), then go to step 2. Otherwise, go to the next step.
6. Eliminate \( J^{T-1}(J - 1) \) all generated in step 4 possibilities with maximum values of the schedule.
7. Choose the possibilities with the shortest schedule. Get summarized scheduling with reverse trace of DP.

Let us note the algorithm $A_p$ in relation to an evaluation of values of delays $\tilde{t}_{i,j}$. At each step of the algorithm, it is required to evaluate the values $\tilde{t}_{i,j}$ for those aircrafts that have not yet arrived to the airport of departure. Evaluation of the values $\tilde{t}_{i,j}$ is performed by formulating the "string" of flights. For each flight in the string delay, calculation is operated taking into account delays of preceding flights.

The "string" is understood to be a sequence of flights operated sequentially by the same aircraft. The flights in the string are connected with account of maintenance standards and turnaround time as well as on condition that the airport of entry of preceding flight aligns with the airport of departure of the next flight.

The formulating of the "strings" is possible using the solution of subtasks of search for the shortest routes in a graph of potential connections between the airports. In general, it is required to perform formulating of the "strings" at every stage because they can change gradually depending on preceding local flights assignments [6]. The described problem is not considered in this paper.

The software implementation of the parametric algorithm $A_p$ has allowed us to perform the calculation for the test dataset.

Let us suppose that there is a schedule of flights of some airline performing flights between 3 airports: DME (Moscow), OVB (Novosibirsk), AER (Sochi).

In total, the schedule contains 11 flights. The aircraft fleet consists of 3 aircrafts: Boeing 738 (2 aircrafts), Airbus 320 (1 aircraft).

All the initial data are given in Table 1. Specified data here are the identification number for each flight, the departure and arrival airports, the planned aircraft type, and the aircraft tail number currently assigned for the flight.

At the initial stage, we arrange the flights according to the times of the initial delays (the input schedule $\|\tilde{t}_{i,j}^0\|$), taking into account the location of the aircraft at the scheduling time. Then, we calculate the time of performance accomplishment for the aircraft $j$ for each flight at each stage. At the next step, we discard the least-priority assignment option and continue the calculation for the next set of flight assignment options 2 for one of the available aircrafts. Further, similarly to the described approach, we perform step-by-step calculation, cutting off the least-priority options for the distribution of flights over aircrafts. For the test dataset, 2 schedules were found and their estimated delays were 209 and 249 min.
Table 1. Test data

| Flight number | Airport of departure | Airport of arrival | Aircraft type | Flight time and ground handling time (min) | Aircraft tail number | Estimated time of departure (UTC) |
|---------------|----------------------|--------------------|---------------|-------------------------------------------|----------------------|---------------------------------|
| 1             | DME                  | AAQ                | 738           | 185                                       | 1                    | 9:52                            |
| 2             | DME                  | ROV                | 319           | 165                                       | 2                    | 10:05                           |
| 3             | DME                  | KGD                | 319           | 175                                       | 3                    | 10:50                           |
| 4             | AAQ                  | DME                | 738           | 185                                       | 1                    | 12:55                           |
| 5             | ROV                  | DME                | 319           | 245                                       | 2                    | 13:00                           |
| 6             | KGD                  | DME                | 319           | 215                                       | 3                    | 13:50                           |
| 7             | DME                  | KZN                | 319           | 135                                       | 3                    | 16:50                           |
| 8             | DME                  | LED                | 319           | 145                                       | 2                    | 17:25                           |
| 9             | DME                  | OVB                | 738           | 305                                       | 1                    | 18:25                           |
| 10            | KZN                  | DME                | 319           | 145                                       | 3                    | 19:10                           |
| 11            | LED                  | DME                | 319           | 145                                       | 2                    | 19:55                           |

Table 2 gives the calculation results for these data.

Table 2. Testing results

| Flight number | Airport of departure | Airport of arrival | j=1 (738) | j=1 (738) | j=1 (738) |
|---------------|----------------------|--------------------|-----------|-----------|-----------|
| 1             | DME                  | AAQ                | 1         | 0         | 0         |
| 2             | DME                  | ROV                | 0         | 1         | 0         |
| 3             | DME                  | KGD                | 0         | 0         | 1         |
| 4             | AAQ                  | DME                | 1         | 0         | 0         |
| 5             | ROV                  | DME                | 0         | 1         | 0         |
| 6             | KGD                  | DME                | 0         | 0         | 1         |
| 7             | DME                  | KZN                | 0         | 1         | 0         |
| 8             | DME                  | LED                | 0         | 1         | 0         |
| 9             | DME                  | OVB                | 1         | 0         | 0         |
| 10            | KZN                  | DME                | 0         | 0         | 1         |
| 11            | LED                  | DME                | 0         | 1         | 0         |

4. Conclusions

The posed problem is of high practical significance since the use of decision-making automation and optimization methods contributes to a significant increase in an airline’s efficiency. The effect due to the good solution of the problem of optimum aircraft assignments is estimated to ensure a 1-3% increase in an airline's revenue, which is quite substantial in absolute terms [7].

Despite the fact that a profound pool of approaches and methodologies for solving problems of optimizing aviation operations has been developed, permanent changes in the airline industry lead to the formulation of new problems that in turn require adaptation of the existing and the development of new approaches to their solution. Competition of both foreign and domestic airlines is permanently increasing, and the costs for ground handling and aviation fuel are growing. All this leads to the fact that the air companies are not armed with serious software products allowing these air companies to optimize their activities and lost revenue.
Because of the complexity of the problem of managing an airline’s schedule and the fact that on-time performance is the most important indicator of an airline’s activities, it can be concluded that the solution to the original applied problem has good prospects for introducing it into airline planning practices in air companies of all sizes.

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