Selection Rules and RR Couplings on Non-BPS Branes

Ehsan Hatefi

Abstract

We compute three and four point functions of the non-BPS scattering amplitudes, including a closed string Ramond-Ramond, gauge/scalar and tachyon in type IIA (IIB) superstring theories. We then discover a unique expansion for tachyon amplitudes in both non-BPS and D-brane anti D-brane formalisms. Based on remarks on Chan-Paton factors and arXiv:1304.3711, we propose selection rules for all non-BPS scattering amplitudes of type II superstring theory. These selection rules, rule out various non-BPS higher point correlation functions of the string theory.
1 Introduction

It is shown that $D_p$-branes have to be realised as sources for Ramond-Ramond field (RR) for all kinds of either non-BPS or stable BPS D-branes in string theory [1, 2]. Through RR couplings several important subjects have been addressed. Let us just introduce some of the most important ones. The phenomenon brane inside branes [3, 4, 5, 6], K-theory (of course with D-brane language) [7], Dielectric effect (which is known to be Myers effect [8]), all order higher derivative corrections to Myers effect for stable branes [9, 10, 11] and more importantly all corrections to non-BPS branes in [12] are found out.

To begin with non-BPS branes in IIA (IIB) theory, $D_p$-branes with $p$ (spatial dimension of branes) odd (even) should be considered. In [13] we obtained a conjecture for an infinite number of the higher derivative corrections to all orders in $\alpha'$ for both BPS and non-BPS amplitudes. In a very recent paper, [14] investigated in detail this conjecture at some fundamental levels that can be applied to fermionic amplitudes as well.

In the case of non-BPS branes and their effective actions, it is known that the spectrum of non-BPS branes includes all massless, some massive and tachyon strings [15]. Specifically diverse discussions have clarified that the effective theory of unstable branes must have just massless and tachyon states as one needs to integrate out all massive modes. Their effective actions indeed have two parts involving DBI and Wess-Zumino part.

There are some methods to actually obtain these effective actions, such as Boundary String Field theory (BSFT) and S-Matrix method. Apart from basic references for BSFT [16, 17], BSFT has already been explained in the introduction of [18]. On the other hand the Wess-Zumino part is given in [19] as follows

$$S_{WZ} = \mu_p \int_{\Sigma(p+1)} C \wedge \text{Str} \, e^{i2\pi\alpha'\mathcal{F}},$$

with super connection’s curvature as

$$i\mathcal{F} = \begin{pmatrix} iF - \beta'^2T^2 & \beta'DT \\ \beta'DT & iF - \beta'^2T^2 \end{pmatrix},$$

where $\beta'$ is a normalisation constant.

The second method to begin with non-BPS branes effective actions is S-matrix method. This approach may be used to discover new couplings with exact $\alpha'$ corrections without any on-shell ambiguity. If one works with this method then one is able to fix all the coefficients.
of the higher derivative corrections to all orders in $\alpha'$. Notice that the internal CP matrix of tachyons around unstable point of tachyon DBI has to be taken into account, otherwise the computations of the S-matrices in type II string theory do not make sense. Basically open string tachyons in zero picture come with $\sigma_1$ while in (-1) picture they carry $\sigma_2$ Pauli matrix. The correct form of tachyon DBI effective action including its internal CP matrices is introduced in [18, 19]

$$S_{DBI} \sim \int d^{p+1}\sigma \text{Str} \left( V(T^iT_i) \sqrt{1 + \frac{1}{2}[T^i, T^j][T^j, T^i]} \right) \times \sqrt{-\det(\eta_{ab} + 2\pi\alpha'F_{ab} + 2\pi\alpha'D_aT^i(Q^{-1})^{ij}D_bT^j)},$$

with

$$V(T^iT_i) = e^{-\pi T^iT^i/2}, \quad Q^{ij} = I\delta^{ij} - i[T^i, T^j],$$

where $i, j = 1, 2$, i.e., $T^1 = T\sigma_1$, $T^2 = T\sigma_2$ (more information can be found in the next section). Since the total super ghost charge must be (-2) for disk amplitudes, one realises that by expanding the square roots, one has to keep two tachyons in (-1) pictures and they have to be carried with $T^2$. Symmetric trace is also needed.

Surprisingly, this action around tachyon potential’s stable point, takes the usual tachyon DBI action [20, 21] with $T^4V(T^2)$ potential. Note that by sending tachyon to infinity, the term $T^4V(TT)$ will send to zero. This is also expected from condensing an unstable brane. It is worth mentioning that (2) makes consistent results with the computations of the amplitudes of one RR ($C$), one tachyon ($T$) and two scalar fields ($\phi\phi$) including all their infinite higher derivative corrections [19].

On the other hand based on direct S-matrix computations of (CTTA) amplitude in the world volume of brane -anti brane system, the correct form of D-brane anti D-brane ($D\bar{D}$) effective action (symmetrized trace effective action) with all infinite higher derivative corrections of two tachyons and two gauge fields is derived in [22]. More recently by making use of the direct computations, all order $\alpha'$ higher derivative corrections to two tachyons and two scalars in the world volume of brane -anti brane system are discovered in [23]. To look for some of the applications of the higher derivative corrections for BPS branes in M-theory [24] (considering the $N^3$ entropy growth of M5 branes and Myers effect) and [25] are suggested. For some other applications to all order corrections in gauge-gravity duality [26] is proposed. The importance of the higher derivative corrections for tachyon amplitudes just
around the unstable point of tachyon action is also explained in [19]. Subsequently, using direct conformal field theory calculations tachyon DBI supersymmetrized action ([12], [27], [28]) is obtained.

\[ L = -T_p V(T) \sqrt{-\det(\eta_{ab} + 2\alpha' F_{ab} - 2\pi \alpha' \bar{\Psi} \gamma_b \partial_a \Psi + \pi^2 \alpha'^2 \bar{\Psi} \gamma^\mu \partial_a \Psi \bar{\Psi} \gamma_\mu \partial_b \Psi + 2\pi \alpha' \partial_a T \partial_b T)} \]

In [12], we have explored all order \( \alpha' \) higher derivative corrections to two tachyons and two fermions as well. Employing unstable branes might help us to realise some properties of the superstring theory in time-dependent backgrounds [29, 30, 15, 31]. It is known that tachyons are the sources of the instabilities in flat space thus by studying them in an effective field theory, we may be able to get some fascinating results. Indeed tachyon Born-Infeld effective action [21] in string theory can explain some of the decay properties of non-BPS D_\( p \)-branes [30, 32]. If one works with this effective action then one may understand the evolution of these unstable branes in time-dependent backgrounds. One can also deal with the possible cosmological applications of the tachyonic DBI action. In order to start with the explanation of the inflation in string theory [33, 34, 35], we need to know the effective action of \( D\bar{D} \) system involving its all order \( \alpha' \) higher derivative corrections [22, 23]. Some other applications on unstable branes are considered in [36]. To work with spontaneous chiral symmetry breaking (SCSB), tachyons and their higher derivative corrections are used in diverse models such as holographic model of the QCD [37, 38, 39]. If we take the brane, anti brane system just as a background (to be dual to confined color theory) then one can introduce flavor branes [40] where by decreasing the number of flavor branes with respect to the number of color branes, the \( D\bar{D} \) system may be considered as a probe.

The outline of the paper is as follows.

In the next section in the presence of non-BPS branes and based on internal CP matrix of all strings, we are going to construct selection rules for non-BPS amplitudes. We observe that these selection rules must be used to rule out several non-BPS higher point correlation functions of type IIA (IIB) superstring theory without the need for knowing world-sheet integrals. More significantly the proposed selection rules show that there should not be any couplings for some of the open/closed strings in the effective actions of non-BPS branes. In section three by carrying out explicit calculations we obtain all order \( \alpha' \) higher derivative corrections to the amplitude of one RR, one tachyon and one scalar field \( < V_C V_T V_\phi > \) in the world volume of non-BPS branes.

In particular we discover a unique expansion for tachyon amplitudes. This expansion is
very useful because by applying it to string amplitudes, one finds out all the singularities of the non-BPS higher point correlation functions of the string theory without knowing the exact results of the world-sheet integrals. This idea clearly had been applied to the amplitude of one RR and four tachyons in the world volume of $D \overline{D}$ system but has not been announced yet [11]. Using selection rules and universal tachyon expansion in section four, we will deal with the amplitude of one RR, one scalar, one gauge field and one tachyon in the world volume of non-BPS branes.

Having used some of new Wess-Zumino terms, tachyonic DBI action, this universal tachyon expansion, the proposed selection rules and the derived corrections of one RR, one tachyon and one scalar field (results in section four), we are able to exactly produce infinite number of $(u' = u + 1/4)$-channel tachyon and $t$-channel massless scalar poles of $< V_C V_T V_A V_\phi >$ amplitude as well as their higher derivative corrections. If we apply the selection rules to the field theory of $< V_C V_T V_A V_\phi >$ then we understand neither there should be single / double $s'$, $(s' + u' + t)$ channel tachyon, gauge/ scalar poles nor infinite poles. Indeed this $< V_C V_T V_A V_\phi >$ amplitude is an exceptional S-matrix.

2 Selection rules for non-BPS amplitudes

Based on Chan-Paton factors and new proposal appeared in [12], we are going to propose the complete selection rules for non-BPS scattering amplitudes of type II superstring theory. Without any knowledge of mixture of the closed-open string theory correlators, these rules can be used to show that some of the non-BPS higher point correlation functions of the string theory will be ruled out.

As mentioned by [42], one can read off the internal Chan-Paton matrix of the non-BPS D-brane open strings from D-brane anti D-brane Chan-Paton matrix. The open strings of the brane anti brane system (two real tachyons, massless scalar/gauge fields) are given by the following CP matrices

\[
(a) : \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (b) : \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad (c) : \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \quad (d) : \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},
\]

\[(4)\]

(a),(b) matrices are related to scalar/gauge located either on brane or anti brane and (c), (d) matrices show tachyons stretched from brane to anti brane and vice versa. The projection operator $(-1)^F_L$ does not play any role on the world-sheet fields; however, it changes brane to anti brane. Hence the projection operator plays the role on CP matrix $\Lambda$ as follows
\[ \Lambda \rightarrow \sigma_1 \Lambda (\sigma_1)^{-1}, \]

Once the projection operator is applied, only the states carrying either \( I \) or \( \sigma_1 \) CP matrices will remain. Therefore it is easy to show that all massless strings should carry internal \( I \) CP matrix. In order to get consistent results with the non-BPS S-matrix elements in string theory, tachyon vertex operator in zero picture must carry \( \sigma_1 \) CP matrix [19]. Let us devote \( I \) CP matrix to massless strings and \( \sigma_1 \) CP matrix to tachyon vertex operator in the zero picture. Since the picture changing operator for a non-BPS brane carries \( \sigma_3 \) CP factor [43], one can apply it to all vertex operators in zero picture to obtain the CP factor of the strings in \((-1)\)-picture.

Applying the picture changing operator on massless vertex operators in zero picture, we show that their internal CP matrix in \((-1)\) picture is \( I \sigma_3 = \sigma_3 \) and particularly the CP matrix of tachyon in \((-1)\) picture becomes \( \sigma_1 \sigma_3 \sim \sigma_2 \). The amplitude of two fermions and one gauge field makes sense even in the presence of non-BPS branes. Given this fact, we are able to fix the CP matrices of the fermion fields in [12] as follows

\[ A \Psi^{-1/2} \Psi^{-1/2} \Lambda^{-1} \sim \text{Tr} (\sigma_3 I \sigma_3) \neq 0. \]

Ramond-Ramond vertex operator carries CP matrix like other strings. Since there is a non zero coupling between two tachyons and one RR in the world volume of brane anti brane systems [44, 22], it has been pointed out in [19] that the RR for brane anti brane systems in \((-1/2, -1/2)\) picture has to carry \( \sigma_3 \) CP matrix. It is also discussed in detail in [19] that due to [42] there is an extra coefficient in RR vertex operator in \((-1)\) picture for the non-BPS branes which means that RR in \((-1)\) picture carries \( \sigma_3 \sigma_1 \) for the non-BPS brane systems. Its CP factor in \((-2)\) picture has also been addressed in [19]. As the closed strings (NS-NS sector) in \((0, 0)\) picture come with \( I \) internal CP matrix the internal CP factors can be verified by the S-matrix element of one RR, one graviton and one tachyon. The field theory of non-BPS D-branes reveals that the amplitude must have tachyon pole.

In field theory, this pole is found by the Feynman amplitude for which tachyon propagates between \( CT \) and \( TTh \) where \( C \) and \( h \) denote RR and graviton appropriately; however, there is no such pole in the field theory of brane anti brane system (i.e., there is no \( CT \) coupling in \((D\bar{D})\) case). Therefore, the internal CP factor \((D\bar{D})\) system is zero but for the non-BPS case, the internal CP factor is non-zero. It is easy to see that for the RR
vertex operator in \((-1/2, -1/2)\) along with graviton in \((0, 0)\) and tachyon in \((-1)\) picture, the CP factor of the \(CTh\) amplitude for \(D\bar{D}\) case is \(\text{Tr}(\sigma_3 I\sigma_2) = 0\) while its CP matrix is \(\text{Tr}(\sigma_3\sigma_1 I\sigma_2) \neq 0\) for the non-BPS brane. Accordingly, when we choose the RR in \((-1/2, -1/2)\), graviton in \((-1, 0)\) and tachyon in \(0\) picture, then the CP factor for \(D\bar{D}\) case is \(\text{Tr}(\sigma_3\sigma_3\sigma_1) = 0\) while its CP matrix is non-zero \((\text{Tr}(\sigma_3\sigma_1\sigma_3\sigma_1) \neq 0)\) for non-BPS brane. We are going to generalise these selection rules.

Based on internal CP matrices discussed in [12], there is no coupling between two closed string Ramond-Ramond fields and one tachyon in the world volume of non-BPS branes, because

\[
A^{C^{-1}C^{-1}T^0} \sim \text{Tr}(\sigma_3\sigma_1\sigma_3\sigma_1) = 0,
\]  
(5)

Note that the correlation function of four spin operators and one fermion field in 10 dimensions is non zero; however, based on internal CP matrices in (5), there are no couplings between two RR’s and one tachyon in type IIA(B) superstring theory.

Now we can generalise these selection rules further. Since we know that the total super ghost charge must be \((-2)\) for disk level S-matrix and scalars, gauges in zero picture carry \((I)\) CP matrix, we conclude that the amplitude of two RR’s, an arbitrary number of gauges/scalars and odd-number of tachyons in type IIA(B) superstring theory is zero. Of course one can add some arbitrary number of (NS-NS) to these amplitudes as well for which the result remains zero again, i.e.,

\[
A^{C^{-1}C^{-1}A^0\phi^0...A^0\phi^0T^0...T^0} = 0.
\]

In addition to the above equation, we can see that many amplitudes have no contributions to superstring theory. The amplitudes of one RR, some arbitrary number of (NS-NS) closed string states and odd number of tachyons in the world volume of brane anti brane vanish. More significantly the CP factor of a RR, some arbitrary closed string (NS-NS) states and even number of tachyons in the world volume of non-BPS branes is also zero, thus there are no such couplings in type IIA(B) superstring theories.

Based on internal CP matrices of the open strings in [12], we show that the tree level S-matrix elements of two fermion fields and one tachyon in the world volume of non-BPS branes becomes zero result; however, the correlation function of this \((\bar{\psi}\psi T)\) amplitude
system is vertex operators as below:

\[
<: S_A(x_1) : S_B(x_2) := x_{12}^{-5/4}(C^{-1})_{AB}
\]

is non zero. We can easily generalise and postulate these internal CP matrices to all the vertex operators as below:

\[
\begin{align*}
V_T^{(0)}(x) & = \alpha^i k \psi(x)e^{\alpha^i k \cdot X(x)}\lambda \otimes \sigma_1, \\
V_T^{(-1)}(x) & = e^{-\phi(x)}e^{\alpha^i k \cdot X(x)}\lambda \otimes \sigma_2 \\
V_\phi^{(-1)}(x) & = e^{-\phi(x)}\xi\psi^i(x)e^{\alpha^i q x(X(x)}\lambda \otimes \sigma_3 \\
V_A^{(-1)}(x) & = e^{-\phi(x)}\xi^a \psi^a(x)e^{\alpha^i q x(X(x)}\lambda \otimes \sigma_3 \\
V_\phi^{(0)}(x) & = \xi_{ik}(\partial X(x) + i\alpha^l k.\psi^i(x))e^{\alpha^i k \cdot X(x)} \otimes I \\
V_A^{(0)}(x) & = \xi_{ia}(\partial X(x) + i\alpha^l k.\psi^a(x))e^{\alpha^i k \cdot X(x)} \otimes I \\
V_\phi^{(-1/2)}(x) & = \tilde{u}^A e^{-\phi(x)/2}S_A(x) e^{\alpha^i q x(X(x)}\lambda \otimes \sigma_3 \\
V_\phi^{(-1/2)}(x) & = u^B e^{-\phi(x)/2}S_B(x) e^{\alpha^i q x(X(x)}\lambda \otimes I \\
V_C^{(-1/2)}(z, \bar{z}) & = (P_H(n)M_p)_{\alpha \beta} e^{-\phi(z)/2}S_A(z)e^{\imath q \cdot p X(z)}e^{-\phi(\bar{z})/2}S_\beta(\bar{z})e^{\imath \bar{\imath} \cdot p D \cdot X(\bar{z})} \lambda \otimes \sigma_3 \sigma_1,
\end{align*}
\]

where the CP factor of RR for non-BPS branes is \(\sigma_3 \sigma_1\), meanwhile its CP factor for \(D \bar{D}\) system is \(\sigma_3\). Given [12], we are going to generalise non-BPS fermionic amplitudes further.

The amplitude of two fermions, an arbitrary number of scalar/gauge fields and odd number of tachyons in the world volume of non-BPS branes in superstring theory is given by the following correlation function:

\[
A_{\Psi^{1/2}, \Psi^{1/2}, T^{1/2}, T^{0}, \ldots, T^{0}, A_1^{0}, \phi_1^{0}, \ldots A_m^{0}, \phi_l^{0}} \sim \int dx_1 \cdots dx_{(2n+m+l+3)} \text{Tr} \left[V_\Psi^{(-1/2)}(x_1)V_\Psi^{(-1/2)}(x_2) \cdots V_T^{(-1)}(x_3)V_T^{0}(x_4) \cdots V_T^{0}(x_{2n+1})V_A^{0}(x_{m_1})V_\phi^{0}(x_{l_1}) \cdots V_A^{0}(x_{m_m})V_\phi^{0}(x_{l_l}) \right],
\]

We have shown in [19] that the correlator of two spin operators and some arbitrary number of fermion and/or currents in ten dimensions is non-zero. Thus one may guess that the above S-matrix element is non zero, but the internal CP factor of the above S-matrix is zero for all orderings of tachyons, gauges and scalars, i.e.,

\[
A_{\Psi^{1/2}, \Psi^{1/2}, T^{1/2}, T^{0}, \ldots, T^{0}, A_1^{0}, \phi_1^{0}, \ldots A_m^{0}, \phi_l^{0}} = 0.
\]
Hence we discovered that in the world volume of non-BPS branes (in both type IIA and IIB superstring theories) there are no couplings between odd number of tachyons, two fermion fields and an arbitrary number of scalar/gauge vertex operators.

It is worth emphasising that these selection rules are very important to make sense of non-zero higher point correlation functions. Because by making use of them, we could remove several higher non-BPS correlators of the string theory without any knowledge of the world sheet integrals of the S-matrices. Now let us just talk about some other non-BPS three and four point functions of $C-\phi-T$ and $C-\phi-A-T$ amplitudes which make sense just in the world volume of non-BPS branes.

3 The $T-\phi-C$ amplitude

In this section we are going to apply direct conformal field theory techniques [45] to obtain the three point amplitude between one RR, one tachyon and one scalar field in the world volume of non-BPS branes. The amplitude is given by the following correlation function

$$\mathcal{A}_{T,\phi,RR} \sim \int dx dy d^2 z \langle V_T^{(0)}(y)V_\phi^{(-1)}(x)V_{RR}^{(-1)}(z, \bar{z}) \rangle,$$

where tachyon, scalar field and RR vertex operators (including their CP factors) are given in (6) such that all open strings have to be located in the boundary of the disk and RR must be replaced in the middle of the disk.

$q, p, k_1$ are the momenta of scalar, RR and tachyon field which satisfy the following on-shell condition

$$q^2 = p^2 = 0, \quad k_1^2 = 1/4, \quad k_1 \xi_1 = q \xi_1 = 0,$$

where the definitions of projector operator and RR field strength are

$$P_- = \frac{1}{2}(1 - \gamma^{11}), \quad \mathbb{H}(n) = \frac{a_n}{n!} \mathbb{H}_{\mu_1...\mu_n} \gamma^{\mu_1} ... \gamma^{\mu_n},$$

In type IIA (type IIB) $n = 2, 4, a_n = i$ ($n = 1, 3, 5, a_n = 1$) and also the notation for the spinor is

$$(P_- \mathbb{H}(n))^{\alpha\beta} = C^{\alpha\delta}(P_- \mathbb{H}(n))_{\delta\beta}.$$

In order to make use of the holomorphic components of the world-sheet fields, one needs to work with doubling trick. Thus we need to apply the following change of variables

$$\tilde{X}^\mu(\bar{z}) \to D^\mu_{\nu}(\bar{z}) \tilde{X}^\nu(\bar{z}), \quad \tilde{\psi}_{\mu}(\bar{z}) \to D^\mu_{\nu}\psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \to \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \to M_\alpha^\beta S_\beta(\bar{z}).$$
with the following matrices
\[ D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \]
and
\[ M_p = \begin{cases} \frac{-i}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \cdots \gamma^{i_{p+1}} & \text{for } p \text{ even} \\ \frac{-1}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \cdots \gamma^{i_{p+1}} \gamma_{i_1 \cdots i_{p+1}} & \text{for } p \text{ odd} \end{cases} \]

Now we are allowed to use just holomorphic part of the propagators for \( X^\mu, \psi^\mu, \phi, \) as follows
\[
\langle X^\mu(z)X^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z-w),
\]
\[
\langle \psi^\mu(z)\psi^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} (z-w)^{-1},
\]
\[
\langle \phi(z)\phi(w) \rangle = -\log(z-w). \tag{9}
\]

The \( \sigma \)-factor of the above S-matrix element is \( \text{Tr} (\sigma_3 \sigma_1 \sigma_1 \sigma_3) = 2 \) and the CP factor is \( 2 \text{Tr} (\lambda_1 \lambda_2). \) If one considers the related vertices then the amplitude is given as follows
\[
\int dx_1 dx_2 dx_4 dx_5 \alpha'(ik_{1a}\xi_i)(x_{24}x_{25})^{-1/2}(x_{45})^{-1/4} I_1(P_-H_{(n)}M_p)^{\alpha\beta} \times \langle S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_1) : \psi^i(x_2) : \rangle,
\]
where \( x_4 = z = x + iy, x_5 = \bar{z} = x - iy \)
\[
I_1 = |x_{12}|^{\alpha' k_1 \cdot k_2} |x_{14}x_{15}|^{\frac{\alpha' k_1 \cdot p}{2}} |x_{24}x_{25}|^{\frac{\alpha' k_2 \cdot p}{2}} |x_{45}|^{\frac{\alpha' k_1 \cdot p}{2}}, \tag{10}
\]

Now using Wick-like rule one can derive the following correlator
\[
\langle S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_1) : \psi^i(x_2) : \rangle = 2^{-1} (x_{14}x_{15}x_{24}x_{25})^{-1/2}(x_{45})^{-1/4}(\Gamma^{ia} C^{-1})_{\alpha\beta}.
\]

By applying this correlator into the above amplitude, we can easily show that the integrand is \( SL(2, R) \) invariant. Let us do gauge fixing as \( (x_1, x_2, z, \bar{z}) = (x, -x, i, -i) \) so that now the integrand is proportional to
\[
4k_{1a}\xi_i \int_{-\infty}^{\infty} dx (2x)^{-2u-1/2}(1 + x^2)^{-1/2+2u} \left( \text{Tr} (P_- H_{(n)}M_p \Gamma^{ia}) \right),
\]
where \( u = -\frac{\alpha'}{2} (k_1 + k_2)^2 \) and particularly one has to apply the conservation of momentum along the world volume of brane \( (k_1^a + k_2^a + p^a = 0). \)

\(^2\text{From now on we set } \alpha' = 2.\)
The integral of the amplitude is
\[ A_{T,\phi,RR} = (\pi \beta' \mu_p') 2 \sqrt{\pi} \Gamma[-u + 1/4] \Gamma[3/4 - u] \Gamma[a]k_1 \xi \Gamma[\lambda_1 \lambda_2]. \] (11)

The amplitude is normalised by the coefficient of \((\pi \beta' \mu_p'/2)\) where \(\beta'\) and \(\mu_p\) are Wess-Zumino normalisation and Ramond-Ramond charge of branes appropriately. The trace is non zero for \(p + 1 = n\) and it is extracted as follows
\[ \text{Tr} \left( \frac{H(n)M_p}{n} (\xi,\gamma)(k_1,\gamma) \right) \delta_{p+1,n} = \pm \frac{32}{(p + 1)!} \epsilon^{a_0 \cdots a_{p-1} a} H_{i a_0 \cdots a_{p-1} k_1 a} \xi \delta_{p+1,n}, \]

Given the fact that we want to compare S-matrix elements of \(C T \phi\) with its field theory, we do not fix the overall coefficient of the amplitude. Note that the trace involving \(\gamma^{11}\) shows that the above results can be held for the following
\[ p > 3, H_n = *H_{10-n}, n \geq 5. \]

It has been discussed in detail in [19] that the momentum expansion of any amplitude including tachyons should be obtained either with \(k_i k_j \to 0\) or \((k_i + k_j)^2 \to 0\) where the latter appears just for massless channel pole. In the other words momentum expansion can be read by analysing massless or tachyon poles of the amplitude. Nevertheless in this amplitude for both \(u \to 0\) and \(u \to -1/4\) (mass of tachyon), we observe that there are no massless/tachyon poles in the Gamma functions inside the amplitude of (11). Thus we might wonder about the momentum expansion of this amplitude. It is argued in [19] that the momentum expansion of non-BPS branes makes sense just in the presence of the following constraint:
\[ u = -p^a p_a \to -\frac{1}{4}, \] (12)
while for D-brane anti D-brane system \(p^a p_a \to 0\) makes sense in the S-matrix elements of type II superstring theory [23].

This \(C \phi T\) amplitude makes sense just for non-BPS branes, thus if we apply the momentum conservation along the world volume of brane then one understands that the correct expansion is \(u \to -1/4\), or in terms of momenta the expansion is around \(k_1 \cdot k_2 \to 0\).

Having expanded the Gamma functions that appeared in the amplitude we obtain an infinite number of higher derivative couplings of one scalar, one tachyon and one \(C_p\) field. They are related to the higher derivative extensions of the following Wess-Zumino coupling:
\[ 2i \beta' \mu_p'(2\pi \alpha')^2 \int_{\Sigma_{p+1}} \left( \text{Tr} (\partial_i C_p \wedge DT \phi^i) \right), \] (13)
In the above coupling Taylor expansion for scalar field has been employed. The infinite higher derivative corrections of the above coupling must be obtained by considering the momentum expansion inside the S-matrix. The expansion is

\[ \sqrt{\frac{\pi}{\Gamma[u + 1/4]}} = \pi \sum_{m=-1}^{\infty} c_m (u + 1/4)^{m+1} \]

with the following coefficients

- \( c_{-1} = 1 \),
- \( c_0 = 2\ln(2) \),
- \( c_1 = \frac{\pi}{6}(\pi^2 + 12\ln(2)^2) \),
- \( c_2 = \frac{1}{3}(6\zeta(3) + \pi^2\ln(2) + 4\ln(2)^3) \),
- \( c_3 = \frac{1}{360}(1440\zeta(3)\ln(2) + 120\pi^2\ln(2)^2 + 19\pi^4 + 240\ln(2)^4) \).

Note that the above coefficients are different from the coefficients that appeared in the momentum expansion of the S-matrix element of one RR and two gauge fields (one tachyon and one gauge field). If we replace the derived expansion in the final result of the amplitude and compare it with field theory coupling (13) then we realise that the term including \( c_{-1} \) can be precisely produced by (13). On the other hand the higher derivative corrections of (13) can be read with similar way as follows

\[ \frac{2\beta' \mu_p^p}{p!} (2\pi \alpha')^2 c_{a_0...a_p} \partial_i C_{a_0...a_{p-1}} \wedge \text{Tr} \left( \sum_{m=-1}^{\infty} c_m (\alpha')^{m+1} D_{a_1} \cdots D_{a_{m+1}} D_{a_p} T D_{a_1} \cdots D_{a_{m+1}} \phi^i \right) \]

where in the above coupling all commutator terms should be ignored because we are looking for the infinite couplings of one RR, one scalar and one tachyon in the world volume of non-BPS branes.

4 The \( C - \phi - A - T \) amplitude

4.1 The \( C^{-1}\phi^{-1}A^0T^0 \) amplitude

In this section we are going to look for the closed form of the amplitude of one RR, one scalar, one gauge and one tachyon in the world volume of non-BPS branes in type IIA (IIB) superstring theory. This \( \langle V_C V_\phi V_A V_T \rangle \) amplitude is a four point (technically five point)
function. It is pointed out in [19] that the vertex operators for non-BPS D-branes carry internal Chan-Paton factor. We know that the S-matrix element of BPS states does not depend on their pictures but for non BPS branes the amplitude is just independent of the picture of the vertex operators if and only if we take into account the CP matrices inside the vertex operators. Thus \[ < V_C V_\phi V_A V_T > \]
is given by the following correlation function:

\[
A^{C\phi AT} \sim \int dx_1 dx_2 dx_3 dz d\bar{z} \left( V_\phi^{(-1)}(x_1) V_A^{(0)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) \right),
\]
such that all open strings have to be located in the boundary of the disk and RR must be replaced in the middle of the disk. \( k_1, k_2, p, k_3 \) are the momenta of scalar, gauge, RR and tachyon field which satisfy the following on-shell condition

\[
k_1^2 = k_2^2 = p^2 = 0, \quad k_3^2 = 1/4, \quad k_2 \cdot \xi_2 = k_2 \cdot \xi_1 = k_1 \cdot \xi_1 = k_3 \cdot \xi_1 = 0
\]

Obviously the definitions of the projector, the field strength of RR, D and \( M_p \) matrices are exactly the same definitions that appeared in the last section. We also use holomorphic correlators [9].

If we consider the vertex operators then the amplitude is given by

\[
A^{C\phi AT} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P - H_{(n)} M_p)^{\alpha\beta \xi_1 \xi_2 \alpha' k_3} \frac{1}{x_1 x_2 x_3 x_4 x_5}^{1/4}(x_4 x_5)^{-1/2} \\
\times (I_1 + I_2) \text{Tr} (\lambda_1 \lambda_2 \lambda_3) \text{Tr} (\sigma_3 \sigma_1 \sigma_3 I \sigma_1),
\]

where \( x_{ij} = x_i - x_j \) and by applying the Wick theorem one finds the correlators as follows

\[
I_1 = \langle : e^{\alpha' i k_1 X(x_1)} : \partial X^a(x_2) e^{\alpha' i k_2 X(x_2)} : e^{\alpha' i k_3 X(x_3)} : e^{\alpha' i p D X(x_4)} : e^{\alpha' i p D X(x_5)} : \\
\times \langle: S_\alpha(x_4) : S_\beta(x_5) : \psi^j(x_1) : \psi^c(x_3) : >
\]

\[
I_2 = \langle : e^{\alpha' i k_1 X(x_1)} : e^{\alpha' i k_2 X(x_2)} : e^{\alpha' i k_3 X(x_3)} : e^{\alpha' i p D X(x_4)} : e^{\alpha' i p D X(x_5)} : >
\]

\[
\alpha' i k_2 d : e^{\alpha' i k_1 X(x_1)} : S_\alpha(x_4) : S_\beta(x_5) : \psi^j(x_1) : \psi^d \psi^a(x_2) : \psi^c(x_3) : >,
\]

We need to take into account the Wick-theorem and [9] to be able to compute the correlators of \( X \). In order to obtain the correlation function including two spin operators,
one current and two fermion fields we use Wick-like rule [46, 47] and its generalisation [11]. First of all we need to find the following correlator

\[ I_3^{ci} = \langle S_{\alpha}(x_4) : S_{\beta}(x_5) : \psi^i(x_1) : \psi^c(x_3) : \rangle = 2^{-1}x_{45}^{-1/4}(x_{14}x_{15}x_{34}x_{35})^{-1/2}\left\{ (\Gamma^{ci}C^{-1})_{\alpha\beta} \right\}. \]

The computation of the correlation function of a current, two fermions and two spin operators is really tedious, however, concerning the generalisation of Wick-Like rule [11] now one can easily obtain it as below

\[ I_4^{cadi} = \langle S_{\alpha}(x_4) : S_{\beta}(x_5) : \psi^i(x_1) : \psi^d\psi^a(x_2) : \psi^c(x_3) : \rangle = \left\{ (\Gamma^{cadi}C^{-1})_{\alpha\beta} + \frac{\text{Re}[x_{24}x_{35}]}{x_{23}x_{45}}(2\eta^{dc}(\Gamma^{ci}C^{-1})_{\alpha\beta} - 2\eta^{ac}(\Gamma^{cadi}C^{-1})_{\alpha\beta}) \right\} 2^{-2}x_{45}^{3/4}(x_{14}x_{15}x_{34}x_{35})^{-1/2}(x_{24}x_{25})^{-1}. \] (19)

We substitute all the related spin correlators in (17) and derive the general form of the amplitude as follows

\[ \mathcal{A}^{\phi AT} \sim \int dx_1dx_2dx_3dx_4dx_5(P_{\mathcal{H}}(n)M_p)^{\alpha_3\alpha_4}\xi_{14}^{\alpha_3}\xi_{25}^{\alpha_4}(-2\alpha'ik_{3e})x_{45}^{-1/4}(x_{14}x_{15})^{-1/2} \times \left( a_1^{\alpha}I_3^{ci} + \alpha'ik_{2d}I_4^{cadi} \right) \text{Tr} (\lambda_1\lambda_2\lambda_3), \] (20)

such that

\[ I = |x_{12}|^{\alpha_1^{2}k_1,k_2}|x_{13}|^{\alpha_2^{2}k_1,k_3}|x_{14}x_{15}|^{\alpha_3^{2}k_1,p}|x_{23}|^{\alpha_1^{2}k_2,k_3}|x_{24}x_{25}|^{\alpha_2^{2}k_2,p}|x_{34}x_{35}|^{\alpha_3^{2}k_3,p}|x_{45}|^{\alpha_4^{2}p,D,p}, \]

\[ a_1^{\alpha} = ik_1^{\alpha} \left( \frac{x_{14}}{x_{12}x_{24}} + \frac{x_{15}}{x_{12}x_{25}} \right) + ik_2^{\alpha} \left( \frac{x_{24}}{x_{23}x_{24}} + \frac{x_{25}}{x_{23}x_{25}} \right). \] (21)

Now the amplitude is written such that it is explicitly SL(2,R) invariant. By gauge fixing the location of open strings as

\[ x_1 = 0, \quad x_2 = 1, \quad x_3 \to \infty, \] (22)

we reach to the following integrals on the upper half plane

\[ \int d^2z|1 - z|^a|z|^b(z - \bar{z})^c(z + \bar{z})^d, \] (23)

where \( a, b, c \) are related to the Mandelstam variables as below

\[ s = -\frac{\alpha'}{2}(k_1 + k_3)^2, \quad t = -\frac{\alpha'}{2}(k_1 + k_2)^2, \quad u = -\frac{\alpha'}{2}(k_2 + k_3)^2. \]
where for \( d = 0,1 \) and for \( d = 2 \) the results are derived accordingly in [48], [18]. Given the solutions for integrals [48], [18], one can write the final result of the amplitude (17) as
\[
\mathcal{A}^C_{\phi AT} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3,
\] (24)
where
\[
\begin{align*}
\mathcal{A}_1 & \sim 2 \xi_1 \xi_2 a k_3 c k_2 d \text{Tr} (P_- \mathcal{H} (n) M_p \Gamma^{c a d i}) L_1, \\
\mathcal{A}_2 & \sim 2 \left\{ \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . \xi_2 \gamma . \xi_1) (u + \frac{1}{4}) + 2 k_3 \xi_2 \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . k_2 \gamma . \xi_1) \right\} L_2, \\
\mathcal{A}_3 & \sim -2 \left\{ 2 t (k_3, \xi_2) \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . k_3 \gamma . \xi_1) + 2 (-u - \frac{1}{4}) k_1 \xi_2 \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . k_3 \gamma . \xi_1) \right\} L_3,
\end{align*}
\] (25)
where
\[
\begin{align*}
L_1 &= (2)^{-2(t+s+u)} \pi \frac{\Gamma(-u + \frac{1}{4}) \Gamma(-s + \frac{1}{4}) \Gamma(-t + \frac{1}{4}) \Gamma(-t - s - u + \frac{1}{2})}{\Gamma(-u - t + \frac{3}{4}) \Gamma(-t - s + \frac{3}{4}) \Gamma(-s - u + \frac{1}{2})}, \\
L_2 &= (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u - \frac{1}{4}) \Gamma(-s + \frac{3}{4}) \Gamma(-t + 1) \Gamma(-t - s - u)}{\Gamma(-u - t + \frac{5}{4}) \Gamma(-t - s + \frac{5}{4}) \Gamma(-s - u + \frac{1}{2})}, \\
L_3 &= (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u - \frac{1}{4}) \Gamma(-s + \frac{3}{4}) \Gamma(-t) \Gamma(-t - s - u)}{\Gamma(-u - t + \frac{3}{4}) \Gamma(-t - s + \frac{3}{4}) \Gamma(-s - u + \frac{1}{2})}.
\end{align*}
\]
The amplitude now satisfies Ward identity associated to the gauge field, which means that by replacing \( \xi_2 a \rightarrow k_2 a \) it gives zero result. One can write the amplitude by doing more simplifications. Using momentum conservation along the world volume of brane and making use of the various identities we obtain
\[
\mathcal{A}^C_{\phi AT} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3,
\] (26)
where
\[
\begin{align*}
\mathcal{A}_1 & \sim 2 \xi_1 \xi_2 a k_3 c k_2 d \text{Tr} (P_- \mathcal{H} (n) M_p \Gamma^{c a d i}) L_1, \\
\mathcal{A}_2 & \sim \left\{ - \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . \xi_2 \gamma . \xi_1) (u + \frac{1}{4}) - 2 k_3 \xi_2 \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . k_2 \gamma . \xi_1) \right\} L_3 (2t) \\
& \quad + \text{Tr} (P_- \mathcal{H} (n) M_p \gamma . k_3 \gamma . \xi_1) \left\{ 2 t (k_3, \xi_2) + 2 (-u - \frac{1}{4}) k_1 \xi_2 \right\} (-2 L_3).
\end{align*}
\] (27)
\( \mathcal{H} (n), M_p, \Gamma^{c a d i} \) are constructed out of the antisymmetric gamma matrices thus the amplitude is non zero for \( p = n + 1 \) and \( p + 1 = n \) cases. If we apply momentum conservation, we find
Regarding the facts that for non-BPS branes $p^a p_a \rightarrow \frac{1}{4}$ and the vertex of two scalars and one gauge field is non zero, we understand $t \rightarrow 0$. Applying the above constraints, we obtain the universal tachyon expansion. Indeed the following unique expansions should be held for all four point tachyon amplitudes:

$$t \rightarrow 0, \quad s \rightarrow -\frac{1}{4}, \quad u \rightarrow -\frac{1}{4},$$

as they have been argued in [19]. Notice that in terms of momenta the correct expansion is:

$$(k_1 + k_2)^2 \rightarrow 0, \quad k_1.k_3 \rightarrow 0, \quad k_2.k_3 \rightarrow 0,$$

Now if we apply the expansion (28) to the all Gamma functions of the amplitude then we reveal that $C\phi AT$ amplitude has to have infinite massless scalar and tachyon poles. Before proceeding more several remarks have to be made.

It is argued that all four point functions including one RR, tachyon, gauge and scalar vertex operators must have infinite $t, s', u', (t + s' + u')$ channel poles, where $s' = s + \frac{1}{4}, u' = u + \frac{1}{4}$.

The important point for the $C\phi AT$ amplitude is that it is an exceptional amplitude. Due to the following reasons neither does it have $s', (t + s' + u')$ tachyon, scalar nor infinite poles. Due to the kinematic reasons, neither there is a coupling between two tachyons, one gauge and one scalar nor a coupling between two tachyons and one scalar field (even these amplitudes have non-zero CP factors). In fact conformal field theory techniques tell us that the amplitude of $TT\phi$ is vanished. Thus there is no $s'$ channel pole for $C\phi AT$ amplitude.

By applying selections rules (based on string CP matrices), one understands that the amplitudes

$$<V_0 V_0 V_{\phi^{-1}} V_{\phi^{-1}}>, <V_A V_0 V_{A^{-1}} V_{\phi^{-1}}> .$$

do not make sense in type II superstring theory because $\text{Tr} (I\sigma_1\sigma_3\sigma_3) = 0$.

Therefore neither there are single poles nor infinite poles of $s', (s' + t + u')$, as the amplitude (25) does not carry the coefficients of $\Gamma(-s - \frac{1}{2}), \Gamma(-t - s - u - 1/2)$ in its final form. Let us now analyse all infinite $u'$ tachyon and massless $t$ channel scalar poles of the amplitude.
5 Infinite tachyon poles of the $C\phi AT$ amplitude for $n = p + 1$ case

Given the explained universal expansion for tachyons, one reveals that the first and second terms of (25) do not contribute to any poles and in fact they are just contact terms in which we are not interested in producing them (the method of producing all contact interactions in string theory has been explained in [19]).

On the other hand the third and fourth terms of (25) have to be added to give rise to all infinite $u'$ channel tachyon poles and essentially the last term of (25) has infinite $t$-channel scalar poles.

Let us write down all the infinite tachyon poles as below

$$4k_3\ell_2(k_{2a} + k_{3a})\xi_{11}\left(\text{Tr}(P_-(\tilde{H}_{(n)}M_p\Gamma^{ai}))\right)L_2,$$

One has to use momentum conservation along the brane’s world volume. There is also an identity for RR as

$$p_a\epsilon^{a_0...a_{p-1}a} = 0.$$

In order to be able to obtain all the infinite $u'$-channel tachyon poles one has to apply all the identities and we find all tachyon poles as follows

$$-4k_3\ell_2k_{1a}\xi_{1i}\left(\text{Tr}(P_-(\tilde{H}_{(n)}M_p\Gamma^{ai}))\right)L_2,$$

where the expansion of $L_2$ is

$$L_2 = -\pi^{3/2}\left(\frac{1}{u'}\sum_{n=-1}^{\infty} c_n(s' + t)^{n+1} + \sum_{p,n,m=0}^{\infty} h_{p,n,m}(u')^p(t s')^n(t + s')^m\right),$$

some of the coefficients are

$$c_{-1} = 1, c_0 = 0, c_1 = \frac{1}{6}\pi^2, h_{0,0,1} = \frac{1}{3}\pi^2, h_{1,0,1} = h_{0,0,2} = 6\zeta(3).$$

In this section by making use of the infinite corrections of one RR, one tachyon and one scalar field [15], we want to produce all the infinite tachyon poles and to actually show that those obtained corrections are exact and have no on-shell ambiguity. Let us write down infinite tachyon $u'$-channel poles (there is no $s'$ channel pole for this amplitude) as
\[-(4k_3 \xi_2)k_{1a} \xi_{1i} \frac{16}{(p)!} (\pi^{3/2}) (\mu'_p \beta' \pi^{1/2}) \sum_{n=-1}^{\infty} c_n \frac{1}{u'} (s' + t)^{n+1} \times H^i_{a0 \ldots a_{p-1}} \epsilon^{a_0 \ldots a_{p-1} a \beta} \Tr (\lambda_1 \lambda_2 \lambda_3) \].

The above amplitude is normalised by a coefficient of \((\mu'_p \beta' \pi^{1/2})\). The following field theory amplitude should be considered

\[ A = V^\alpha (C_p, \phi_1, T) G^\alpha\beta (T) V^\beta (T, T_3, A_2) \].

We also need to take into account several remarks.

Tachyon propagator \((G^\alpha\beta (T))\) is derived from the kinetic term of the tachyon. To obtain \(V^\beta (T, T_3, A_2)\) one has to extract the covariant derivative of tachyon \((D_a T = \partial_T - i[A^a, T])\) inside the non-Abelion kinetic term of tachyon. \(V^\alpha (C_p, \phi_1, T)\) is derived from the coupling \((13)\), such that

\[ V^\beta (T, T_3, A_2) = iT_p (2\pi \alpha') (k_3 - k) \xi_2 \Tr (\lambda_2 \lambda_3 \Lambda^\beta) = 2i T_p (2\pi \alpha') k_3 \xi_2 \Tr (\lambda_2 \lambda_3 \Lambda^\beta), \]

\[ V^\alpha (C_p, \phi_1, T) = 2\mu'_p \beta' (2\pi \alpha')^2 (p)! \epsilon^{a_0 \ldots a_p} H^i_{a0 \ldots a_{p-1}} k_{a_p} \xi_{1i} \Tr (\lambda_1 \lambda^a), \]

\[ G^\alpha\beta (T) = \frac{-i \delta^{\alpha\beta}}{(2\pi \alpha') T_p (k^2 + m^2)} = \frac{-i \delta^{\alpha\beta}}{(2\pi \alpha') T_p (u')} \].

In the vertex of \(V^\beta (T, T_3, A_2)\), we have used the momentum conservation on the right hand side of the Feynman rule \((k_3 + k_2 + k)^a = 0\), in which \(k\) is the momentum of tachyon propagator. If we replace the above vertices \((34)\) inside the field theory amplitude \((33)\), use the momentum conservation in left hand side of the Feynman rule, namely \(((k_1 + p)^a = k^a)\) and take into account the first coefficient of the expansion \(c_{-1} = 1 \) in \((32)\) then we are able to exactly produce the first simple tachyon \(u'\)-channel pole; however, as it is seen from \((32)\), the amplitude involves infinite \(u'\) channel tachyon poles. In order to deal with those infinite poles, one has to use some of the arguments that appeared in \([12]\). Basically both of the simple tachyon pole and \(V^\beta (T, T_3, A_2)\) do not receive any correction. Because the kinetic term of tachyon has been fixed in tachyon DBI action and therefore it has no correction. Hence in order to produce those infinite tachyon poles, one has to find out the infinite corrections to one RR \((p\text{-form})\), one tachyon and one on-shell scalar. In fact these corrections have been derived by performing the scattering amplitude of \(C \phi T\) in \((15)\).
Once we apply all the infinite higher derivative corrections of $\partial_i C_p \wedge DT\phi^i \ (15)$, we can find

$$V^\alpha(C_p, T, \phi_1) = \frac{2\mu_p^\prime \beta'(2\pi\alpha')^2 k_{a_p} \xi_{1i} \epsilon_{a_0 \cdots a_p} H_{a_0 \cdots a_{p-1}}}{(p)!} \sum_{m=-1}^\infty c_m (\alpha' k_1 \cdot k)^{m+1} \text{Tr} (\lambda_1 \Lambda^\alpha). \tag{35}$$

Now if we use the fact that $\sum_{m=-1}^\infty c_m (\alpha' k_1 \cdot k)^{m+1} = \sum_{m=-1}^\infty c_m (t + s + 1/4)$ and substitute (35) inside the field theory amplitude (33) (while keep the other vertices fixed) then we are exactly able to explore all the infinite $u'$ channel poles of (32).

This clearly shows that the obtained higher derivative corrections of (15) are exact and have no on-shell ambiguity. Note also that there are no residual contact interactions left over in producing an infinite number of the tachyon poles of the $C\phi AT$ S-Matrix.

### 6 Infinite scalar t-channel poles of the $C\phi AT$ amplitude for $n = p + 1$ case

The last term of (25) includes an infinite number of t-channel massless scalar poles as follows

$$- 4k_1 \xi_2 k_3 \xi_1 \left( \text{Tr} (P_{-H(n)} M_p \Gamma^{a_1}) \right) (-u - 1/4) L_3, \tag{36}$$

and the expansion of $(-u - 1/4) L_3$ is

$$(-u - 1/4) L_3 = -\pi^{3/2} \left( \frac{1}{6} \sum_{n=-1}^\infty c_n (u' + s')^{n+1} + \sum_{p,n,m=0}^\infty h_{p,n,m} t^p (u' s')^n (u' + s')^m \right), \tag{37}$$

where $c_n$ coefficients are precisely the coefficients that have been written down in the momentum expansion of the S-matrix element of $C\phi T$. Some other coefficients of $c_n, h_{p,n,m}$ can be summarised as

$$c_2 = 2\zeta(3), h_{2,0,0} = h_{0,1,0} = 2\zeta(3), h_{1,0,0} = \frac{1}{6} \pi^2, h_{1,0,2} = \frac{19}{60} \pi^4,$$

$$h_{0,0,1} = \frac{1}{3} \pi^2, h_{0,0,3} = e_{2,0,1} = \frac{19}{90} \pi^4, h_{1,1,0} = h_{0,1,1} = \frac{1}{30} \pi^4.$$

In order to follow this section two main goals are needed.
1) We would like to explicitly show that the infinite corrections of one RR, one tachyon and one scalar field that obtained in (15) are exact and do work.

2) We are going to show that there are infinite t-channel scalar poles which must be obtained in field theory by applying the correct higher derivative corrections. In particular we want to show that our expansion in (28) is unique because by using it we can precisely produce an infinite number of the scalar poles of the $C\phi AT$ string amplitude. If we insert the first term of the expansion of $(-u-1/4)L_3$ into the amplitude then we can just write down the infinite $t$-channel scalar poles of the amplitude (there is no $(s'+t+u')$ scalar pole for this amplitude) as below

$$-4k_1\xi_2k_3\xi_{11}\frac{16}{(p)!}(\pi^{3/2})(\mu'\beta'\pi^{1/2})\sum_{n=-1}^{\infty}c_n\frac{1}{t}(s'+u')^{n+1}$$

$$\times H_{a_0\ldots a_{p-1}}^{i}a^{ao\ldots ap-1a}\text{Tr}(\lambda_1\lambda_2\lambda_3).$$

The rule for the field theory amplitude for this case is

$$A = V^\alpha(C_p, T_3, \phi)G^{\alpha\beta}(\phi)V^\beta(\phi, \phi_1, A_2),$$

Now we are going to make some remarks about field theory analysis. Scalar propagator ($G^{\alpha\beta}(\phi)$) should be found from the kinetic term of the scalar. $V^\beta(\phi, \phi_1, A_2)$ must be derived by extracting the covariant derivative of the scalar field in the non-Abelian kinetic term of the scalars ($D_a\phi^i = \partial_a\phi^i - i[A^a, \phi^i]$). $V^\alpha(C_p, T, \phi)$ is related to the previous S-matrix , i.e., the coupling (13) and $V^\beta(\phi, \phi_1, A_2)$ is derived in [10]. Let us write them down in below :

$$V^\beta(\phi, \phi_1, A_2) = -2i(2\pi\alpha')^2T_p k_1\xi_2\xi_{11}\text{Tr}(\lambda_1\lambda_2\Lambda^\beta),$$

$$V^\alpha(C_p, \phi, T_3) = 2\mu'\beta'\frac{(2\pi\alpha')^2}{(p)!}a^{ao\ldots ap}H_{ap\ldots ao-1}^{i}\text{Tr}(\lambda_3\Lambda^\alpha),$$

$$G^{\alpha\beta}(\phi) = \frac{-i\delta^{\alpha\beta}\delta^{ij}}{(2\pi\alpha')^2T_p(k^2)} = \frac{-i\delta^{\alpha\beta}\delta^{ij}}{(2\pi\alpha')^2T_p(t)}.$$
field theory side; however, as it is seen from \((-u - 1/4)L_3\) expansion (37), the amplitude includes an infinite number of t-channel scalar poles.

In order to produce those singularities in field theory amplitude, one has to apply several points. Basically both simple t-channel scalar pole and \(V^\beta(\phi, \phi_1, A_2)\) do not receive any corrections because the kinetic term of scalars does not obtain any correction as it has been fixed in standard DBI action and also simple pole does not receive any correction. Thus in order to look for those infinite massless scalar poles one needs to look for infinite corrections to one RR (p-form), one on-shell tachyon and one off-shell scalar field. Surprisingly these corrections can be derived from the scattering amplitude of \(C\phi T\) in (15).

Hence, if we use all the infinite higher derivative corrections of \(\partial_i C_p \wedge DT\phi^i\) (15) then we can explore all order extensions of the \(V^\alpha(C_p, \phi, T_3)\) vertex operator as follows

\[
V^\alpha(C_p, \phi, T_3) = 2\mu_p^{\prime}\beta(2\pi\alpha')^2\epsilon_{a_0...a_p}H^i_{a_0...a_p}k_{3a_p} \sum_{m=-1}^\infty c_m(\alpha'k_3 \cdot k)^{m+1} \text{Tr} (\lambda_3 \Lambda^\alpha), (42)
\]

We use momentum conservation in world volume direction such that \(\sum_{m=-1}^\infty c_m(\alpha'k_3 \cdot k)^{m+1} = \sum_{m=-1}^\infty c_m(u + s + 1/2)\), substitute (42) inside the field theory amplitude (39) and simultaneously keep the form of the other vertices fixed. If we do so then we are able to find out:

\[
-4k_1.\xi_2k_{3a_p}\xi_1, 16\pi^2\mu_p^{\prime}\beta(2\pi\alpha')^2\epsilon_{a_0...a_p}H^i_{a_0...a_p}k_{3a_p} \sum_{n=-1}^\infty c_n(s + u + 1/2)^{n+1} \times \text{Tr} (\lambda_1 \lambda_2 \lambda_3).
\]

which is exactly all the infinite t-channel massless scalar poles of (38). Hence this again shows that the derived higher derivative corrections of (15) are exact and have no on-shell ambiguities. Notice that in exploring an infinite number of t-channel scalar poles of the \(C\phi AT\) S-Matrix there are no residual contact interactions left over. This can be understood by comparing field theory amplitude (43) with string theory S-matrix elements (38).

Finally note that the method of extracting all the contact terms in string theory has been extensively pointed out in [19, 13]; however, it is worth pointing out the following remarks.

In order to look for an infinite number of contact interactions of the \(C\phi AT\) (for \(p+1 = n\)}
the following coupling has to be considered
\[ \int_{\Sigma_{p+1}} \partial^i C_p \wedge DT \phi^i \]

particularly one has to extract the covariant derivative of tachyon and take into account the commutator inside the covariant derivative of tachyon \([A^a, T]\). More importantly one should add together all the contact interactions coming from the second parts of \(L_2\) and \((-u')L_3\) expansions in (31) and (37) accordingly.

7 Conclusions

In this paper based on internal CP matrix of the strings in the presence of non-BPS branes, we have constructed the selection rules for all non-BPS amplitudes. We observed that, these selection rules have the significant potential to rule out several non-BPS higher point correlation functions of type IIA (IIB) string theory without the need for taking integrals of the world-sheet. More significantly these selection rules showed us that there should not be any couplings for some of the non-BPS open/closed strings in their effective actions. Based on the suggested rules we realise that what kinds of couplings are not allowed in these theories. In section three by explicit calculations we discovered all order \(\alpha'\) corrections of one RR, one tachyon and one scalar field in the world volume of non-BPS branes.

In particular we obtained a unique expansion for tachyon amplitudes. This expansion is very useful because by applying it to the string amplitudes, one can find out all singularities of non-BPS higher point functions of the string theory without the need for knowing the complete results of the world-sheet integrals. This idea clearly has been applied to the amplitude of one RR and four tachyons in the world volume of \(D\bar{D}\) system but has not been publicized yet [41].

In section four using selection rules and universal tachyon expansion we dealt with the amplitude of one RR, one scalar, one gauge field and one tachyon in the world volume of non-BPS branes. By applying selection rules to the field theory amplitude we revealed that neither does the \(C\phi AT\) amplitude have single /double/ infinite \((s')\) tachyon nor \((s' + u' + t)\) poles. Thus it is certainly an exceptional S-matrix.

Essentially by making use of the selection rules, the universal tachyon expansion and the derived corrections of one RR, one tachyon and one scalar field (results in section four), we were able to produce an infinite number of \(u'\) tachyon and \(t\)-channel massless scalar
field poles of the $C\phi AT$ amplitude and their higher derivative corrections. It is of high
importance to mention that the Wess-Zumino effective actions of the unstable branes give
all order corrections of the related string amplitudes.

Acknowledgments
The author would like to thank J. Polchinski, A. Sen, R. Myers, K.S.Narain, N.Lambert,
F. Quevedo, N. Arkani-Hamed, G. Veneziano, G. Moore, A. Sagnotti and L. Alvarez-Gaume for
valuable comments. He also thanks E. Martinec and P. Horava for very useful discussions
during recent CMT workshop at ICTP.

References

[1] J. Polchinski, “Dirichlet Branes and Ramond-Ramond charges,” Phys. Rev. Lett. 75, 4724 (1995) [hep-th/9510017].
[2] E. Witten, “Bound states of strings and p-branes,” Nucl. Phys. B 460, 335 (1996) [hep-th/9510135].
[3] M. R. Douglas, “Branes within branes,” In *Cargese 1997, Strings, branes and dualities* 267-275 [hep-th/9512077].
[4] M. R. Douglas, “D-branes and matrix theory in curved space,” Nucl. Phys. Proc. Suppl. 68 (1998) 381 [hep-th/9707228]; E. Hatefi, “Three Point Tree Level Amplitude in Superstring Theory,” Nucl. Phys. Proc. Suppl. 216 (2011) 234 [arXiv:1102.5042 [hep-th]].
[5] M. Li, “Boundary states of D-branes and Dy strings,” Nucl. Phys. B 460, 351 (1996) [hep-th/9510161].
[6] M. B. Green, J. A. Harvey and G. W. Moore, “I-brane inflow and anomalous couplings on d-branes,” Class. Quant. Grav. 14 (1997) 47 [hep-th/9605033].
[7] E. Witten, “D-branes and K theory,” JHEP 9812, 019 (1998) [hep-th/9810188].
[8] R. C. Myers, “Dielectric branes,” JHEP 9912 (1999) 022
[9] E. Hatefi, “Shedding light on new Wess-Zumino couplings with their corrections to all orders in alpha-prime,” JHEP 1304, 070 (2013) [arXiv:1211.2413 [hep-th]].

[10] E. Hatefi and I. Y. Park, “More on closed string induced higher derivative interactions on D-branes,” Phys. Rev. D 85, 125039 (2012) [arXiv:1203.5553 [hep-th]].

[11] E. Hatefi, “On effective actions of BPS branes and their higher derivative corrections,” JHEP 1005, 080 (2010) [arXiv:1003.0314 [hep-th]].

[12] E. Hatefi, ‘All order $\alpha'$ higher derivative corrections to non-BPS branes of type IIB Super string theory,” JHEP 1307, 002 (2013) [arXiv:1304.3711 [hep-th]].

[13] E. Hatefi and I. Y. Park, “Universality in all-order $\alpha'$ corrections to BPS/non-BPS brane world volume theories,” Nucl. Phys. B 864, 640 (2012) [arXiv:1205.5079 [hep-th]].

[14] E. Hatefi, “Closed string Ramond-Ramond proposed higher derivative interactions on fermionic amplitudes in IIB,” arXiv:1302.5024 [hep-th], to appear in NPB.

[15] A. Sen, “Tachyon dynamics in open string theory,” Int. J. Mod. Phys. A 20, 5513 (2005) [hep-th/0410103].

[16] P. Kraus and F. Larsen, “Boundary string field theory of the D anti-D system,” Phys. Rev. D 63, 106004 (2001) [hep-th/0012198].

[17] T. Takayanagi, S. Terashima and T. Uesugi, “Brane - anti-brane action from boundary string field theory,” JHEP 0103, 019 (2001) [hep-th/0012210].

[18] M. R. Garousi and E. Hatefi, “More on WZ action of non-BPS branes,” JHEP 0903, 008 (2009) [arXiv:0812.4216 [hep-th]].

[19] E. Hatefi, “On higher derivative corrections to Wess-Zumino and Tachyonic actions in type II super string theory,” Phys. Rev. D 86, 046003 (2012) [arXiv:1203.1329 [hep-th]].

[20] A. Sen, “Supersymmetric world volume action for nonBPS D-branes,” JHEP 9910, 008 (1999) [hep-th/9909062].

[21] J. Kluson, “Proposal for nonBPS D-brane action,” Phys. Rev. D 62, 126003 (2000) [hep-th/0004106].
[22] M. R. Garousi and E. Hatefi, “On Wess-Zumino terms of Brane-Antibrane systems,” Nucl. Phys. B 800, 502 (2008) [arXiv:0710.5875 [hep-th]].

[23] E. Hatefi, “On D-brane anti D-brane effective actions and their corrections to all orders in alpha-prime,” JCAP 1309, 011 (2013) [arXiv:1211.5538 [hep-th]].

[24] E. Hatefi, A. J. Nurmagambetov and I. Y. Park, “$N^3$ entropy of $M5$ branes from dielectric effect,” Nucl. Phys. B 866, 58 (2013) [arXiv:1204.2711 [hep-th]]; E. Hatefi, A. J. Nurmagambetov and I. Y. Park, “Near-Extremal Black-Branes with $n*3$ Entropy Growth,” Int. J. Mod. Phys. A 27, 1250182 (2012) [arXiv:1204.6303 [hep-th]]; E. Hatefi, “SuperYang-Mills, Chern-Simons couplings and their all order $\alpha'$ corrections in IIB superstring theory,” [arXiv:1310.8308 [hep-th]].

[25] J. McOrist and S. Sethi, “M-theory and Type IIA Flux Compactifications,” JHEP 1212, 122 (2012) [arXiv:1208.0261 [hep-th]].

[26] E. Hatefi, A. J. Nurmagambetov and I. Y. Park, “ADM reduction of IIB on $H^{p,q}$ to dS braneworld,” JHEP 1304, 170 (2013) [arXiv:1210.3825 [hep-th]].

[27] M. Aganagic, C. Popescu and J. H. Schwarz, “Gauge invariant and gauge fixed D-brane actions,” Nucl. Phys. B 495, 99 (1997) [hep-th/9612080].

[28] M. Aganagic, J. Park, C. Popescu and J. H. Schwarz, “Dual D-brane actions,” Nucl. Phys. B 496, 215 (1997) [hep-th/9702133].

[29] M. Gutperle and A. Strominger, JHEP 0204, 018 (2002) [hep-th/0202210].

[30] A. Sen, “Tachyon matter,” JHEP 0207 (2002) 065 [hep-th/0203265]; A. Sen, “Rolling tachyon,” JHEP 0204, 048 (2002) [hep-th/0203211]; A. Sen, “Tachyon condensation on the brane anti-brane system,” JHEP 9808, 012 (1998) [hep-th/9805170]; A. Sen, “Time evolution in open string theory,” JHEP 0210, 003 (2002) [hep-th/0207105].

[31] N. D. Lambert, H. Liu and J. M. Maldacena, “Closed strings from decaying D-branes,” JHEP 0703, 014 (2007) [hep-th/0303139].

[32] A. Sen, “Field theory of tachyon matter,” Mod. Phys. Lett. A 17, 1797 (2002) [hep-th/0204143].
[33] G. R. Dvali and S. H. H. Tye, “Brane inflation,” Phys. Lett. B 450 (1999) 72 [hep-ph/9812483].

[34] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, “The Inflationary brane anti-brane universe,” JHEP 0107, 047 (2001) [hep-th/0105204].

[35] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, “Towards inflation in string theory,” JCAP 0310, 013 (2003) [hep-th/0308055].

[36] D. Choudhury, D. Ghoshal, D. P. Jatkar and S. Panda, “Hybrid inflation and brane - anti-brane system,” JCAP 0307, 009 (2003) [hep-th/0305104]; S. de Alwis, R. K. Gupta, E. Hatefi and F. Quevedo, “Stability, Tunneling and Flux Changing de Sitter Transitions in the Large Volume String Scenario,” arXiv:1308.1222 [hep-th], to appear in JHEP; M. Frau, L. Gallot, A. Lerda and P. Strigazzi, “Stable nonBPS D-branes in type I string theory,” Nucl. Phys. B 564, 60 (2000) [hep-th/9903123]; E. Dudas, J. Mourad and A. Sagnotti, “Charged and uncharged D-branes in various string theories,” Nucl. Phys. B 620, 109 (2002) [hep-th/0107081]; E. Eyras and S. Panda, “The Space-time life of a nonBPS D particle,” Nucl. Phys. B 584 (2000) 251 [hep-th/0003033]; E. Eyras and S. Panda, “NonBPS branes in a type I orbifold,” JHEP 0105, 056 (2001) [hep-th/0009224].

[37] R. Casero, E. Kiritsis and A. Paredes, “Chiral symmetry breaking as open string tachyon condensation,” Nucl. Phys. B 787, 98 (2007) [hep-th/0702155][HEP-TH]].

[38] A. Dhar and P. Nag, “Sakai-Sugimoto model, Tachyon Condensation and Chiral symmetry Breaking,” JHEP 0801, 055 (2008) [arXiv:0708.3233 [hep-th]].

[39] A. Dhar and P. Nag, “Tachyon condensation and quark mass in modified Sakai-Sugimoto model,” Phys. Rev. D 78, 066021 (2008) [arXiv:0804.4807 [hep-th]].

[40] T. Sakai and S. Sugimoto, “Low energy hadron physics in holographic QCD,” Prog. Theor. Phys. 113, 843 (2005) [hep-th/0412141].

[41] E. Hatefi, one RR and four tachyon couplings and all order D- brane anti D-brane effective actions[work in progress].
[42] A. Sen, “NonBPS states and Branes in string theory,” hep-th/9904207.

[43] P.-J. De Smet and J. Raeymaekers, “The Tachyon potential in Witten’s superstring field theory,” JHEP 0008 (2000) 020 [hep-th/0004112].

[44] C. Kennedy and A. Wilkins, “Ramond-Ramond couplings on Brane - anti-Brane systems,” Phys. Lett. B 464, 206 (1999) [hep-th/9905195].

[45] D. Friedan, E. J. Martinec and S. H. Shenker, “Conformal Invariance, Supersymmetry and String Theory,” Nucl. Phys. B 271 (1986) 93;

[46] H. Liu and J. Michelson, “-trek III: The search for Ramond-Ramond couplings,” Nucl. Phys. B 614, 330 (2001) arXiv:hep-th/0107172.

[47] V. A. Kostelecky, O. Lechtenfeld, W. Lerche, S. Samuel and S. Watamura, “Conformal Techniques, Bosonization and Tree Level String Amplitudes,” Nucl. Phys. B 288, 173 (1987).

[48] A. Fotopoulos, “On (alpha')**2 corrections to the D-brane action for non-geodesic world-volume embeddings,” JHEP 0109, 005 (2001) arXiv:hep-th/0104146.