Precision charmonium and D physics from lattice QCD and determination of the charm quark mass

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QCD is key part of SM but quark confinement tricky

Lattice QCD = full QCD effects

**RECIPE**

- Generate sets of gluon fields for Monte Carlo integrn of Path Integral (inc effect of sea quarks)
- Calculate averaged “hadron correlators” from valence q props.

\[ \langle 0 | M^\dagger(0)M(t) | 0 \rangle \]

- Fit for masses and simple matrix elements
- Fix \( m_q \) and determine \( a \) to get physical results
HPQCD Priority PRECISION lattice QCD i.e. ~ 1%

- Allows non-trivial tests of QCD i.e. better than models.
- Allows accurate determn of SM parameters (inc CKM)
- Provides the underpinning for other calcs.

Possible for ‘gold-plated quantities’ i.e. stable hadron masses and weak/em decay rates to single hadron states

Statistical errors must be very good to test systematics.

Systematics from:

Expect an error budget ....

- disc. errors (need several $a$ values)
- extrapoln to physical u/d masses $m_s/10 < m_u/d < m_s/2$
- finite volume
- errors in fixing QCD parameters. Use:

$\Upsilon(2S - 1S), m_\pi, m_K, m_{\eta_c}, m_\gamma$
2007 HPQCD/MILC/FNAL results

Analysis on MILC configs that include u,d, s improved staggered sea quarks - numerically fast

Recent highlight - very accurate charm physics
Charm quarks in lattice QCD - heavy or light?

Advantages of relativistic light quarks:

• \( E_{\text{sim}} = m \)
• PCAC relation (if enough chiral symmetry) gives \( Z = 1 \)
• same action as for u, d, s, so cancellation in ratios

Key issue is discretisation errors:

\[
m = m_{a=0}(1 + A(m_c a)^2 + B(m_c a)^4 + \ldots)
\]

\( m_c a \approx 0.4, (m_c a)^2 \approx 0.2, \alpha_s(m_c a)^2 \approx 0.06, (m_c a)^4 \approx 0.04 \)

for \( a \approx 0.1 \text{ fm} \)

Need to remove all of these errors for precision results

This is done in the Highly Improved Staggered Quark formalism, further improving Improved Staggered Quarks
Very precise D/Ds masses obtained

NO free parameters

charmonium masses, HISQ on fine MILC

D/Ds masses vs expt.

Fix $m_c$

lattice errors 6 MeV - $a^2$ extrap /error in $a$ and em corrns

A key test of disc. errors since charmonium and D have different dynamics → stringent test of QCD.

E. Follana et al, 0706.1726[hep-lat]
Decay constants of $D/D_s/K/\pi$ to 2%.

$\text{Br}(H \rightarrow \mu\nu) \propto V_{ab}^2 f_H^2$

$f_H m_H = \langle 0 | \bar{\psi} \gamma_0 \gamma_5 \psi | H \rangle$

$f$ is a property of the meson calculable in lattice QCD

Value can be extracted from expt if $V_{ab}$ known

E. Follana et al, 0706.1726[hep-lat]
2008 Improved accuracy from CLEO-c
Leptonic rate $\rightarrow$ decay constant using $V_{cs} = V_{ud}$, $V_{cd} = V_{us}$

$\begin{align*}
\hat{f}_{D} & \quad \hat{f}_{Ds} \\
\text{agree} & \quad 206(9) \quad 268(9) \\
\text{(exptl)} & \quad 3\sigma \\
207(4) & \quad 241(3) \\
\end{align*}$

$\text{Belle} \quad \text{EPS2007}$
$\text{BaBar} \quad \text{hep-ex/0607094}$
$\text{CLEO-c, 0806.2112, ICHEP08}$

$\text{HPQCD HISQ u,d,s sea} \quad \text{0706.1726[hep-lat]}$
$\text{FNAL/MILC u,d,s sea} \quad \text{LAT08 prelim.}$
$\text{ETMC u,d sea} \quad \text{LAT08 prelim.}$

$\text{First disagreement between lattice and expt. New physics?}$
Further checks of lattice QCD calcns important ...

1. Further masses of hadrons containing charm

Mass splitting V-PS accurately calculable

For staggered quarks there are different ‘tastes’

No dependence on $m_u/d$

Good agreement for all with expt. as $a \rightarrow 0$

New prelim. results on $a=0.06$fm lattices

Hyperfine splittings

$D_s^* - D_s$

$ψ - η_c$
2. Further decay constants of hadrons containing charm and strange

\[ \Gamma_{e^+e^-} = \frac{4\pi}{3} \alpha_{QED}^2 e^2 Q f_V^2 m_V \]

\[ f_V m_V = \langle 0 | J | V \rangle \]

Good agreement with expt for all tastes as \( a \to 0 \)

Need to complete with conserved vector current
3. Compare charmonium correlators to perturbation theory - allows accurate determn $m_c, \alpha_s$

Small $t$ correlators perturbative - take $t$ moments

$$G_n = \sum_t (t/a)^n G(t)$$

$$\rightarrow \frac{\partial^n}{\partial E^n} \Pi(E = 0)$$

$$G_n = \frac{g_n(\alpha_{\overline{MS}}(\mu), \mu/m_c)}{(am_c(\mu))^{n-4}}$$

J. Kühn talk

QCD/Lattice

continuum pert. th.

(4-loop for low $n$)

I. Allison et al, 0805.2999[hep-lat]

HPQCD + Karlsruhe/Brookhaven

+ new results here
Gives 1% accurate value for $m_c$

Best lattice result from pseudoscalar

$m_c(3\text{GeV}) = 0.986(10)\text{GeV}$ \quad $m_c(m_c) = 1.267(9)\text{GeV}$

Contnm uses vector, $R(e^+ e^-) = 0.986(13)\text{ GeV}$

PRELIMINARY

4 different currents agree

Full error budget – biggest is determinn of a

$\mu = 3\text{GeV}$
\[ \alpha_s \text{ determination} \]

\[ \alpha_{\text{MSB}}(M_Z, n_f = 5) \]

\[ \alpha_{\text{MS}}(M_Z) = 0.1183(7) \]

Reduction moments have less a dependence

New superflourine results

agrees with determn from Wilson loops (2008)

\[ \alpha_{\text{MS}}(M_Z) = 0.1174(12) \]

Give \( m_c \)

Davies et al, 0807.1687

PRELIMINARY
Conclusions

- We now have lattice results in charm physics with accuracy (2%) similar to that for light hadrons.

- $D_s$ decay constant is the *only* result (from $\sim 15$ quantities) that disagrees with experiment.

- Further tests this year confirm confidence in the lattice calculation must take this seriously.

Future:

- Need significantly improved experimental error on $f_{D_s}$ - currently $3x$ lattice error.

- Further lattice calculations in other formalisms needed.

- Similarly accurate semileptonic form factors for $D/D_s/K$ need to be calculated.
## Error budgets

| Source | $f_K/f_\pi$ | $f_K$ | $f_\pi$ | $f_{Ds}/f_D$ | $f_{Ds}$ | $f_D$ | $\Delta_s/\Delta_d$ |
|--------|-------------|-------|---------|--------------|----------|-------|---------------------|
| $r_1$ uncertainty | 0.3 | 1.1 | 1.4 | 0.4 | 1.0 | 1.4 | 0.7 |
| $a^2$ extrapol. | 0.2 | 0.2 | 0.2 | 0.4 | 0.5 | 0.6 | 0.5 |
| finite vol. | 0.4 | 0.4 | 0.8 | 0.3 | 0.1 | 0.3 | 0.1 |
| $m_{u/d}$ extrapol. | 0.2 | 0.3 | 0.4 | 0.2 | 0.3 | 0.4 | 0.2 |
| stat. errors | 0.2 | 0.4 | 0.5 | 0.5 | 0.6 | 0.7 | 0.6 |
| $m_s$ evoln. | 0.1 | 0.1 | 0.1 | 0.3 | 0.3 | 0.3 | 0.5 |
| $m_d$, QED etc | 0.0 | 0.0 | 0.0 | 0.1 | 0.0 | 0.1 | 0.5 |
| Total % | 0.6 | 1.3 | 1.7 | 0.9 | 1.3 | 1.8 | 1.2 |

### $m_c(\mu)$

| Source | $R_6$ | $R_8$ | $R_4$ | $R_6/R_8$ |
|--------|-------|-------|-------|----------|
| $a^2$ extrapolation | 0.3% | 0.3% | 0.4% | 0.2% |
| perturbation theory | 0.4 | 0.3 | 0.6 | 0.6 |
| $\alpha_{\overline{MS}}$ uncertainty | 0.3 | 0.4 | 0.0 | 0.0 |
| $m_c(\mu)$ uncertainty | 0.0 | 0.0 | 0.1 | 0.1 |
| gluon condensate | 0.3 | 0.0 | 0.4 | 0.7 |
| statistical errors | 0.1 | 0.0 | 0.2 | 0.1 |
| $m_{0c}$ errors from $r_1/a$ | 0.5 | 0.5 | 0.3 | 0.4 |
| $m_{0c}$ errors from $r_1$ | 0.6 | 0.6 | 0.1 | 0.1 |
| $m_{u/d/s}$ extrapolation | 0.2 | 0.2 | 0.1 | 0.2 |
| finite volume | 0.1 | 0.1 | 0.0 | 0.3 |
| $\mu \to M_Z$ evolution | 0.0 | 0.0 | 0.1 | 0.1 |
| Total | 1.0% | 1.0% | 1.0% | 1.1% |

Update of:

I.Allison et al,
0805.2999[hep-lat]
HPQCD + Karlsruhe/
Brookhaven