Precision Theoretical Tests of Effective Lagrangians:
Pion Photoproduction from Threshold
Through the Delta Resonance

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Abstract

We shall review here our current knowledge and new frontiers of the pion photoproduction study, from threshold through the Delta resonance region, emphasizing the hadron physics program, of interest to a 250 MeV, high duty factor electron facility, such as the upgraded MAX-lab at the Lund University, Sweden, where it would be possible to produce tagged photons of high quality, with energy below approximately 200 MeV.

I. INTRODUCTION

We are happy to be able to share with you our enthusiasm for physics that can be done in the future with a high-quality real photon beam below 200 MeV, such as that planned for the new MAX-lab. At the outset, you may think that this is too low an energy region to be of much interest to hadron physics, in particular, there being excellent existing facilities in this energy region elsewhere. While it is true that photon facilities at places like Saskatoon, Mainz and others have had a go at this physics already, much still remain to be done at a high precision. The advent and success of chiral perturbation theory (χPT) in explaining

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the pion photoproduction very near threshold have raised our expectations of a QCD-based understanding of this region, where already a solid understanding has existed on the basis of effective Lagrangians motivated by chiral symmetry [2]. In particular, the roles of the matter fields, such as \( \rho \) and \( \omega \) mesons and the \( \Delta(1232) \) resonance, which we have recently stressed in our work [2], remain to be explored carefully, both theoretically and experimentally. We believe future facilities like the MAX-lab will have a crucial role in this. We also remain concerned at the future of facilities like SAL at Saskatoon, Canada, which is already doing excellent exploration of this physics, as is nicely represented in several contributions to this workshop. We refer to these talks, as well as the theoretical review of the \( \chi \)PT by J. Bijnens at this workshop for a nice background to the physics issues of interest here.

In order to make our discussion a bit more general, we want to ask here the question: What can we learn from a high-quality \( E_\gamma \leq 200 \text{ MeV} \) facility, in terms of precision theoretical tests of effective Lagrangians? This means probing \( \chi \)PT, QCD and/or whatever else you have for hadron structure. The facilities where we can do this include Mainz, SAL, MAX-Lab, TUNL, LEGS and others of this category. We shall also assume that polarized photons and polarized targets will be eventually available for experimental use. These tools add considerably to our arsenals in order to probe relatively sensitive small amplitudes.

A few short remarks on the history of this subject, which is indeed a very old one. The Delta resonance was discovered by Fermi, Steinberger and their collaborators in the late forties to early fifties. The first low-energy theorem (LET) was proposed by Kroll and Ruderman, and the first dispersion theory of pion photoproduction by Chew \textit{et al.} (CGLN), all in the fifties. The developments in the current algebra by Gell-Mann, Nambu, Weinberg, Adler, Fubini and others in the sixties led to the LET’s as we know them. In the seventies, \( \chi \)PT got developed, thanks largely due to the efforts by Pagels, Weinberg, Leutwyler and Gasser. We have stressed the roles of the \( \Delta, \rho, \omega \) in the famous \( E_{0^+} \) amplitude for neutral pion production, in the eighties [2]. Then came the discovery of Meißner and collaborators [1] of the vital role of pion loops in the \( E_{0^+} \) amplitude, later verified by the experiments [3].

On the nuclear physics side, the dominance of the Gamow-Teller operator in the charged
pion photoproduction near threshold led in the early seventies to selected nuclear excitation of the Gamow-Teller states \[4\] in these and related processes (electromagnetic excitation, weak processes such as \(\beta\) decay and \(\mu\)-capture). Finally, the prominent roles of the Gamow-Teller excitations in the \((p,n)\) and \((p,n)\) processes were found \[5\]. Recently, even the excitation of Gamow-Teller states in the neutrino reactions have become routine.

For the remainder of this paper, we shall cover the following topics. In section II, we discuss what is new in the \(\Delta(1232)\) electromagnetic excitation. In section III, we examine how the effective Lagrangian approach (ELA), so successful at the Delta peak, does near the pion photoproduction threshold, discussing the similarity and differences of \(\chi PT\) and the simpler ELA’s at low energy. In section IV, we examine the critical energy region of interest to the future MAX-Lab program. We conclude with a summary.

**II. WHAT IS NEW IN THE \(\Delta(1232)\) ELECTROMAGNETIC EXCITATION?**

Thanks to recent experimental studies using polarized photons at the Brookhaven LEGS \[6\] and the more recent ones at the Mainz Microtron \[7\], via the reaction \(\vec{\gamma}p \rightarrow p\pi^0\), we can infer \[8\], in the framework of the ELA, the following values of the resonant \(N \rightarrow \Delta\) helicity amplitudes:

\[
A_{1/2} = -127.8 \pm 1.2 \\
A_{3/2} = -252.4 \pm 1.3 ,
\]

in the units of \(10^{-3} \text{ GeV}^{-1/2}\). These come from the more precise Mainz results. Translated into the resonant magnetic dipole \((M1)\) and electric quadrupole \((E2)\) amplitudes, we get from (1), the values, in the same units,

\[
M1 = 282.5 \pm 1.3 \\
E2 = 9.00 \pm 0.66 ,
\]

thereby yielding an \(E2/M1\) ratio \((R_{EM})\)
Important points to note here are that the $M1$ amplitude is substantially larger than the quark model (QM) estimates, around 200; the $E2$ is also much larger than the QM values\(^\text{(3)}\). This is a critical discrepancy in the QM, origins of which are as yet unclear.

These important amplitudes have been “measured” on the lattice for the first time. the results are\(^\text{(4)}\):

\[
M1 = 231 \pm 41 \quad R_{EM} = (3 \pm 8)\% .
\]

The discrepancy of the $M1$ amplitude between (2) and (4) is there, but not as severe as that with the QM. The $R_{EM}$ is too noisy to be of decisive importance. Clearly, there is a lot of room for improvements in the lattice “measurement”.

III. PREDICTIONS FOR THE EFFECTIVE LAGRANGIAN APPROACHES FOR PION PHOTOPRODUCTION NEAR THRESHOLD

The chiral symmetry constraints put the conventional ELA with considerable predictive power in describing charged pion photoproduction near threshold. However, there is a large correction in the $E_{0^+}$ amplitude for the neutral pion photoproduction. Thus, for the reaction $\gamma p \rightarrow p\pi^0$, the old tree-level results yield, for s- and u- channel exchange\(^\text{[11]}\),

\[
E_{0^+} = -2.23 ,
\]

in the usual units of $10^{-3}/M_{\pi^+}$. We have shown\(^\text{[2]}\) that the t-channel vector meson exchanges and the $\Delta$ exchange change this number to

\[
E_{0^+} \approx -1.94 .
\]

The $\chi$PT result, first established by Bernard\( et al.\)^{[1]}\, gives a sizable one-pion loop correction to yield

\[
E_{0^+} \approx -1.00 .
\]
The current situation for the $\pi^0$ threshold multipoles is summarized in Table I. Clearly, the $\chi$PT results are the closest to the experiment in the $E_{0+}$ case, while our theoretical tree-level results fare as well in the cases of the multipoles $P_1$ and $P_{23}$. The $\chi$PT results for the s-wave multipole still suffer from considerable uncertainty. For the p-waves, clearly the matter fields play important roles (Table II).

IV. SIMILARITIES AND DIFFERENCES BETWEEN $\chi$PT AND SIMPLER EFFECTIVE LAGRANGIANS

Table I already gives us a strong indication of the similarities and differences between predictions of $\chi$PT and simpler ELA’s for $\pi^0$ near threshold. As matter fields become stronger in the p-wave multipoles, the disagreement between $\chi$PT and the ELA of the simpler sort becomes more acute. Thus, for $E_\gamma \geq 160$ MeV, these differences begin to stand out. Thus, the energy region $E_\gamma \sim 170$ to 200 MeV is quite crucial to sort out this difference between the two classes of theories which begin to show their dynamical difference. Experiments are needed to precisely map out this region and the MAX-Lab can be of vital help in this.

V. CONCLUSIONS

From this survey, the following points can be stressed:

(1) The $E_{0+}$ multipole, probed by the $\pi^0$ photoproduction near threshold, is an important test of $\chi$PT, where the conventional ELA’s at the tree level do emphasize some role of the matter fields.

(2) The multipoles like $M_{1+}$ favor the conventional ELA’s which provide efficient treatments of the $\Delta$ resonance. The $\chi$PT has some work to do to clean up this physics.

(3) The transition region between these two classes of theories for $E_\gamma$ is from 170 to 200 MeV. Considerable theoretical and experimental works are needed here to make the $\chi$PT tests a precise theoretical enterprise.
Thus, the MAX-Lab and labs like that have an important QCD mission in this energy domain.

Let us max the MAX! This is one of the best ways to celebrate at Lund the memory of the late Professor Janne Rydberg in the next century.

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TABLE I. Threshold $\pi^0$ multipoles in the $\chi$PT \[1\], in the simpler effective Lagrangian approach (DMW) at the tree level \[2\] and in the recent experiment of Fuchs \textit{et al.} \[3\]. The multipoles are in the usual units.

|     | $\chi$PT       | DMW          | Expt.          |
|-----|----------------|--------------|----------------|
| $E_{0+}$ | -1 to -1.5 | -2.15 ± 0.23 | -1.31 ± 0.08   |
| $P_1$   | 10.3          | 11.28 ± 0.14 | 10.02 ± 0.15   |
| $P_{23}$ | 11.25         | 11.85 ± 0.20 | 11.44 ± 0.08   |
TABLE II. Importance of matter fields in controlling the $\pi^0$ photoproduction multipoles at threshold. The contributions enumerated are evaluated from the fits of Davidson et al. (DMW).

|       | PV Born | Vector Mesons | Delta $g_1$ coupling | Delta $g_2$ coupling |
|-------|---------|---------------|----------------------|----------------------|
| $E_{0+}$ | -2.47   | +0.08         | -0.05                | +0.40                |
| $M_{1-}$ | -6.57   | +0.89         | +2.03                | +0.19                |
| $E_{1+}$ | +0.04   | -0.01         | -0.15                | +0.10                |
| $M_{1+}$ | +3.39   | +0.74         | +3.96                | -0.14                |
[1] V. Bernard, N. Kaiser and U. Meißner, Phys. Lett. B268, 219 (1991); see also Report no. KFA-IKP(Th)-1996-14 and references therein. R. Baur and R. Urech, Report no. ZU-TH 30/96. J. Bijnens, this workshop.

[2] R.M. Davidson, N.C. Mukhopadhyay and R.S. Wittman, Phys. Rev. Lett. 56, 804 (1986); Phys. Rev. D 43, 71 (1991). R.M. Davidson and N.C. Mukhopadhyay, Phys. Rev. Lett. 60, 748 (1988); 70, 3834 (1993), and in press.

[3] M. Fuchs et al., Phys. Lett. B368, 20 (1996)

[4] See N.C. Mukhopadhyay, Phys. Rep. 30C, 1 (1977) and references therein. See also R. Erarmzhyan, this workshop.

[5] By C. Goodman and others.

[6] M. Khandaker and A.M. Sandorfi, Phys. Rev. D 51, 3966 (1995) and reference therein.

[7] R. Beck et al., Phys. Rev. Lett. 78, 606 (1997).

[8] R.M. Davidson and N.C. Mukhopadhyay, Phys. Rev. Lett., in press.

[9] S. Capstick and G. Karl, Phys. Rev. D 41, 2767 (1990); S. Capstick, ibid. D 46, 2864 (1992).

[10] D.B. Leinweber, T. Draper and R.M. Woloshyn, Phys. Rev. D 43, 1654 (1991).

[11] P. de Baenst, Nucl. Phys. B24, 633 (1970). A. Vainstein and V. Zakharov, ibid., B36, 589 (1972).