Two-photon decays and photoproduction on electrons of $\eta(550)$, $\eta'(958)$, $\eta(1295)$, and $\eta(1475)$ mesons

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(Dated: January 31, 2013)

Electromagnetic interactions of the ground and first radial excited states of $\eta$ and $\eta'$ mesons in the framework of the extended $U(3) \times U(3)$ NJL model are considered. The radial excitations are described with the help of polynomial form factor of the second order over the inner quark momentum. The solution of the $U_A(1)$ problem by means of ’t Hooft interaction is taken into account. For diagonalization of the free Lagrangian the $4 \times 4$ mixing matrix is used. Two-photon decay widths of the ground $\eta$ and $\eta'$ meson are found to be in a satisfactory agreement with the experiment. Predictions for the two-photon decay widths of $\eta(1295)$ and $\eta(1475)$ are given. The probabilities of eta meson production by two-photon mechanism in $e^+e^-$ collisions and of their photoproduction on electrons are calculated.

PACS numbers: 12.39.Fe, 13.20.Jf, 13.66.Bc
Keywords: Nambu–Jona-Lasinio model, radially excited mesons, electron-positron annihilation into hadrons

I. INTRODUCTION

At the present time extensive investigations of meson production in electron-positron collisions of different energies are carried on at various experimental facilities like VEPP-2000 (Novosibirsk), DAΦNE (Frascati), BEPC-II (Beijing), KEK-B (KEK) and other. For theoretical description of these processes at low energies one can not use the perturbative QCD, therefore it is necessary to use phenomenological models: the vector meson dominance one, the ones based on the chiral symmetry etc.

In a set of recent papers we used the extended $SU(2) \times SU(2)$ Nambu–Jona-Lasinio (NJL) model with a polynomial form factor $[12]$ for description of electromagnetic interactions of $\pi$, $\rho$, and $\omega$ mesons and of their first radial excited states. In Ref. $[3]$ the two-photon decays of $\pi$ and $\pi'$ $\equiv \pi(1300)$ mesons and the processes of their production in $e^+e^-$ collisions were considered. In papers $[4,5]$ we computed the production cross sections of $\pi^0(\pi')\omega$ and $\pi^0(\pi')\gamma$ pairs at electron positron colliders. These results were in a satisfactory agreement as with the experimental data $[6,7]$ as well as with theoretical estimates $[8]$ obtained within the vector meson dominance model. Radiative decays of $\pi^0$, $\rho^0$, $\omega$, and their radial excited states were studied in Ref. $[10]$.

In this work we describe the two-photon decays of pseudoscalar isoscalar $\eta$, $\eta'(958)$, $\eta(1295)$, and $\eta(1475)$ mesons, the processes of their production in two-photon mechanism at electron-positron collisions, and in the Primakoff effect on an electron. The calculations are performed on the base of the $U(3) \times U(3)$ chiral NJL model with ’t Hooft interaction. The radial excitations of mesons are described again with the help of polynomial form factors for meson interactions with $u(d)$ and $s$ quarks $[11]$. Diagonalization of the free Lagrangian for the four eta-mesons with the transition to their physical states is performed by means of the $4 \times 4$ mixing matrix $[12,14]$.

II. LAGRANGIAN

We use a nonlocal separable four-quark interaction of a current-current form which admits nonlocal vertexes (form factors) in the quark currents, and a pure local six-quark ’t Hooft interaction $[15,16]$:

\[
\mathcal{L}(\bar{q}, q) = \int d^4x \bar{\tilde{q}}(x)(i\partial - m^0)q(x) + \mathcal{L}_{\text{int}}^{(4)} + \mathcal{L}_{\text{int}}^{(6)},
\]

\[
\mathcal{L}_{\text{int}}^{(4)} = \int d^4x \sum_{a=0}^8 \frac{G}{2} \left[ j_{S,4}^a(x)j_{S,4}^a(x) + j_{P,4}^a(x)j_{P,4}^a(x) \right],
\]

\[
\mathcal{L}_{\text{int}}^{(6)} = -K \left[ \det \left[ \bar{q}(1 + \gamma_5)q \right] + \det \left[ \bar{q}(1 - \gamma_5)q \right] \right].
\]

Here, $m^0$ is the current quark mass matrix ($m^0_u \approx m^0_d$) and $j_{S,P,4}^a(x)$ denotes the scalar (pseudoscalar) quark currents

\[
j_{S(P)}^a(x) = \int d^4x_1d^4x_2 \bar{q}(x_1)F_{S(P)}^a(x_1,x_2)q(x_2)
\]

where $F_{S(P)}^a(x_1,x_2)$ are the scalar (pseudoscalar) nonlocal quark vertex. The coupling constants $G = 3.14 \text{ GeV}^{-2}$ and $K = 6.1 \text{ GeV}^{-5}$ are fixed in the model $[13,14]$ from the pion mass and from the masses of $\eta$ and $\eta'$ mesons (taking into account the mixing of ground and excited states), respectively.

To describe the first radial excitations of mesons, we take the form factors in momentum space as follows (see...
where $\lambda^a$ are Gell–Mann matrices, $\lambda^0 = \sqrt{\frac{2}{3}}$, with 1 being the unit matrix. Here, we consider the form factors in the rest frame of mesons. The slope parameters $d_{u} = -1.78$ GeV$^{-2}$ and $d_{s} = -1.73$ GeV$^{-2}$) are defined from the condition that the tadpoles with the form factors for the corresponding quarks are equal to zero. That means that the excited states do not give contributions to the quark condensates. The coefficients $c_u = 1.5$, $c_s = 1.66$ are fitted from the masses of the radial excited physical states of eta-mesons.

Following [13, 16], after coupling of a pair of quarks in the 't Hooft interaction we get a modified four-quark interaction of the isoscalar pseudoscalar sector of the NJL model:

$$L_{\text{isos}} = \sum_{a,b=\pm} (\bar{q}_i i \gamma_5 \tau_a q) T^P_{ab}(\bar{q}_j i \gamma_5 \tau_a q),$$

where $T^P$ is a matrix with elements defined as follows

$$T^P_{88} = G_{u}^+/2, \quad T^P_{89} = G_{u}^-+/2, \quad T^P_{99} = G_{s}^+/2, \quad \tau_8 = \lambda_u = (\sqrt{2}\lambda_0 + \lambda_3)/\sqrt{3},$$

$$\tau_9 = \lambda_s = ( -\lambda_0 + \sqrt{2}\lambda_3 )/\sqrt{3}, \quad G_{u}^+ = G - 4K_{u}I_{1}(m_{u}), \quad G_{s}^+ = G,$$

$$G_{u}^- = 4\sqrt{2}K_{u}I_{1}(m_{u}).$$

Here $m_u$ and $m_s$ are the constituent quark masses and $I_{1}(m_{q})$ is the integral which for an arbitrary $n$ is defined as follows

$$I_{n}^{f_1\ldots f_s}(m_q) = -\frac{N_c}{(2\pi)^4} \int_{\Lambda_3} d^4k \frac{f_{a}(k)\ldots f_{a}(k)}{m^2 - k^2}.$$

The 3-dimensional cut-off $\Lambda_3 = 1.03$ GeV in is implemented to regularize the divergent integrals. Here $m_{u,s}$ are the constituent quark masses: $m_u = 280$ MeV and $m_s = 405$ MeV.

After bosonization we get the quark-meson interaction Lagrangian in the form

$$L_{\text{int}} = \bar{q}(p_{1}) \left[ (i\not{\tau} - m + i\gamma_5 \left( \lambda_u g_{\phi}^{\mu} \phi_{\mu} + \lambda_s g_{\phi}^{s} \phi_{s}^{s} \right) + \lambda_u g_{\phi}^{\mu} \phi_{\mu} + \lambda_s g_{\phi}^{s} \phi_{s}^{s} \right] q(p_{2}),$$

(6)

where $m = \text{diag}(m_{u}, m_{d}, m_{s})$, $p = p_{1} - p_{2}$.

$$g_{\phi}^{u} = [Z_{A1}^{u}]^{-1/2}, \quad g_{\phi}^{s} = [Z_{A2}^{s}]^{-1/2},$$

$$g_{u,s}^{u,s} = [\frac{1}{2G} - 8I_{1}^{f_1}(m_{u,s})],$$

(7)

where $\pi - a_{1}$ transitions accounted by factor

$$Z = 1 - \frac{6m_{c}^{2}}{M_{a_{1}}^{2}}$$

(8)

taken the same for ground $\phi_{u}$ and $\phi_{s}$ meson states. For the excited states these transitions can be omitted as discussed in Ref. [11].

From Eq. (4) taking into account renormalization of the kinetic terms in the one-loop approximation, one can get the free meson Lagrangian in the following form [12, 13]:

$$L^{(2)}(\phi) = \frac{1}{2} \sum_{i,j=1}^{2} \sum_{a,b=\pm}^{8} \phi_{i}K_{\phi,ij}(P)\phi_{j},$$

(9)

where

$$K_{\phi,11}(P) = P^2 - (m_{u} \pm m_{u})^2 - M_{\phi,1}^2,$$

$$K_{\phi,22}(P) = P^2 - (m_{u} \pm m_{u})^2 - M_{\phi,2}^2,$$

$$K_{\phi,33}(P) = P^2 - (m_{s} \pm m_{s})^2 - M_{\phi,3}^2,$$

$$K_{\phi,44}(P) = P^2 - (m_{s} \pm m_{s})^2 - M_{\phi,4}^2,$$

$$K_{\phi,12}(P) = K_{\phi,21}(P) = \Gamma_{\phi}(P^2 - (m_{u} \pm m_{u})^2),$$

$$K_{\phi,34}(P) = K_{\phi,43}(P) = \Gamma_{\phi}(P^2 - (m_{s} \pm m_{s})^2),$$

$$K_{\phi,13}(P) = K_{\phi,31}(P) = g_{\phi}^{u}g_{\phi}^{s}(P^2)_{88}^{1},$$

$$\Gamma_{\phi,u,s} = \frac{I_{u,s}^{f_1}(m_{u,s})}{\sqrt{I_{2}(m_{u,s})I_{1}^{f_1,u,s}(m_{u,s})}}.$$

The bare meson masses are

$$M_{\phi,1}^{2} = (g_{\phi}^{u})^{2} \left( \frac{1}{2} (P^2)^{-1/2}_{88} - 8I_{1}(m_{u}) \right),$$

$$M_{\phi,2}^{2} = (g_{\phi}^{u})^{2} \left( \frac{1}{2} (P^2)^{-1/2}_{99} - 8I_{1}(m_{u}) \right),$$

$$M_{\phi,3}^{2} = (g_{\phi}^{s})^{2} \left( \frac{1}{2G} - 8I_{1}^{f_1}(m_{u}) \right),$$

$$M_{\phi,4}^{2} = (g_{\phi}^{s})^{2} \left( \frac{1}{2G} - 8I_{1}^{f_1}(m_{s}) \right).$$

(11)

Transition from the bare states to the physical ones is performed with the help of $4 \times 4$ matrix $R$ which provides the diagonal form for the free Lagrangian. This matrix was found numerically in Refs. [12, 13], it is given in Table I.
Table I: The mixing coefficients for the isoscalar pseudoscalar meson states.

| $R_{i,j}$ | $\eta$ | $\eta'$ | $\eta''$ | $\eta'''$ |
|-----------|---------|---------|---------|---------|
| $\varphi^1_1$ | 0.71 | 0.62 | -0.32 | 0.56 |
| $\varphi^2_1$ | 0.11 | -0.87 | -0.48 | -0.54 |
| $\varphi^1_2$ | 0.62 | 0.19 | 0.56 | -0.67 |
| $\varphi^2_2$ | 0.06 | -0.66 | 0.30 | 0.82 |

Table II: Widths of $\eta$ meson two-photon decays.

| meson | $\eta$ | $\bar{\eta}$ | $\eta'$ | $\eta''$ |
|-------|--------|-------------|--------|--------|
| model [eV] | 520 | 93 | 4990 | 230 |
| exp. [eV] | 510 ± 26 | -4340 ± 140 | - |

III. DESCRIPTION OF RADIATIVE PROCESSES

A. Two-photon decays

Let us start with the 2-photon decays of the ground and excited states of $\eta$-mesons. To describe it we introduce the quark-photon interaction term $qQ\gamma^\mu A^\nu q$ into the interaction Lagrangian \cite{4}, with $Q = \mathrm{diag}(2/3, -1/3, -1/3)$. For the decay $\eta \to \gamma\gamma$ we get the amplitude

$$ T_{\eta\to\gamma\gamma} = \frac{\alpha}{g^2\pi} e^{\mu
u} [\gamma^\mu q_1^\nu \gamma^\nu q_2^\nu \varepsilon_1^\mu \varepsilon_2^\nu ] \left\{ R_{1,1} \frac{g_1^u}{m_u} 5I_3(m_u) + R_{2,1} \frac{g_2^u}{m_u} 5I_3^l(m_u) - R_{3,1} \frac{g_1^s}{m_s} \sqrt{2}I_3^s(m_s) - R_{4,1} \frac{g_2^s}{m_s} \sqrt{2}I_3^f(m_s) \right\}. $$

(12)

Analogously we got the amplitudes for two-photon decays of $\bar{\eta}$, $\eta'$, and $\eta''$ mesons. Our results for the widths of the four decays with comparison to the existing experimental data are given in Table I.

Consider now the processes of $\eta$-meson production in $e^+e^-$ collisions by two-photon mechanism:

$$ e^+ + e^- \to e^+ + e^- + \eta(\bar{\eta}, \eta', \eta''). $$

(13)

In the first approximation the total cross section \cite{17} reads

$$ \sigma_{\eta} = (4\alpha)^2 \ln^2 \frac{\sqrt{\gamma \Gamma(\eta \to 2\gamma)} Y \left( \frac{M^2(\eta)}{s} \right)}{2m_e M^3(\eta)}, $$

$$ Y(z) = (2 + z)^2 \ln \frac{1}{\sqrt{z}} - (3 + z)(1 - z). $$

(14)

The corresponding numerical results are given in Table II in comparison with the experimental data \cite{18}.

Table III: Total cross sections of $\eta$ meson production via the two-photon mechanism.

| meson | $\eta$ | $\bar{\eta}$ | $\eta'$ | $\eta''$ |
|-------|--------|-------------|--------|--------|
| model [nb] | 1.4 | 0.014 | 2.1 | 0.022 |
| exp. [nb] | 1.25 ± 0.13 | - | 1.8 ± 0.3 | - |

existing for the ground meson states at $\sqrt{s} = 29$ GeV total $e^+e^-$ energy in the center-of-mass.

In a similar manner we can describe the Primakoff process of $\eta$ meson production in photon–lepton collisions

$$ \gamma(k) + l(p) \to \eta_l(p_1) + l(p'), \quad l = e, \mu, $$

$$ p^2 = p'^2 = m_l^2, \quad k^2 = 0, \quad p_l^2 = M^2(\eta_l), $$

$$ s = 2kp > M^2(\eta_l) \gg m_l^2. $$

(15)

The total cross section of this process reads \cite{3}:

$$ \sigma^{l\to\eta_l} = \frac{\alpha \Gamma(\eta_l)}{M^3(\eta_l)} \left[ 1 + \left( \frac{M^2(\eta_l)}{s} \right)^2 \right] \times \left( \ln \frac{s^2}{m_l^2 M^2(\eta_l)} - 1 \right). $$

(16)

For electron-photon collisions at the center-of-mass energy $\sqrt{s} = 3$ GeV we get the following predictions for the total cross sections: $\sigma^{e\to\eta e} = 340$ pb, $\sigma^{e\to\eta' e} = 540$ pb, $\sigma^{e\to\eta'' e} = 3.7$ pb, $\sigma^{e\to\eta' e} = 5.7$ pb.

IV. CONCLUSIONS

The results of our calculations show that the application of the mixing matrix $R$ which diagonalizes the free meson Lagrangian leads to sufficiently good description radiative decays of the ground $\eta$ and $\eta'$ meson states. This allows us to hope to get reasonable predictions for their first radial excited states. A similar situation took place in the case of strong interactions of the ground and excited mesons \cite{2, 14}.

In the future we plan to consider also the processes $e^+e^- \to \eta_l \gamma$ taking into account the ground and the first radial excited vector mesons $\rho$, $\omega$ and $\phi$ in the intermediate state. Similar processes with pion production were considered recently in Ref. \cite{21}. In the same way one can consider processes $e^+e^- \to \eta_l \rho^0$ and $e^+e^- \to \eta_l \omega$. The analogous processes $e^+e^- \to \pi^+\pi^-\omega$ were studied in Ref. \cite{20}.

Acknowledgments

We are grateful to E.A. Kuraev for fruitful discussions and critical reading of the manuscript. This work was supported by RFBR grant 10-02-01295-a.
[1] M. K. Volkov and C. Weiss, Phys. Rev. D 56, 221 (1997).
[2] M. K. Volkov, D. Ebert and M. Nagy, Int. J. Mod. Phys. A 13, 5443 (1998).
[3] E. A. Kuraev and M. K. Volkov, Phys. Lett. B 682, 212 (2009). [arXiv:0901.0641 [hep-ph]].
[4] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Phys. Rev. C 83, 048201 (2011) [arXiv:1012.2459 [hep-ph]].
[5] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, arXiv:1106.2215 [hep-ph], to appear in Europ. Phys. J. A.
[6] CMD-2 Collab. (R. R. Akhmetshin et al.), Phys. Lett. B 562, 173 (2003).
[7] M. N. Achasov et al., Phys. Lett. B 486, 29 (2000).
[8] M. N. Achasov et al., Eur. Phys. J. C 12, 25 (2000).
[9] M. N. Achasov et al., Phys. Lett. B 559, 171 (2003).
[10] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Phys. Rev. C 82, 068201 (2010).
[11] M. K. Volkov, Phys. Atom. Nucl. 60, 1920 (1997).
[12] M. K. Volkov and V. L. Yudichev, Int. J. Mod. Phys. A 14, 4621 (1999).
[13] M. K. Volkov and V. L. Yudichev, Phys. Atom. Nucl. 63, 1835 (2000), [Yad. Fiz. 63N10, (2000)].
[14] M. K. Volkov and V. L. Yudichev, Phys. Part. Nucl. 31, 282 (2000), [Fiz. Elem. Chast. Atom. Yadra 31, 576 (2000)].
[15] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991).
[16] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[17] S. J. Brodsky, T. Kinoshita and H. Terazawa, Phys. Rev. Lett. 25, 972 (1970).
[18] N. A. Roe et al., Phys. Rev. D 41, 17 (1990).
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In a set of recent papers we used the extended $SU(2) \times SU(2)$ Nambu–Jona-Lasinio (NJL) model with a polynomial form factor for description of electromagnetic interactions of $\pi$, $\rho$, and $\omega$ mesons and of their first radial excited states. In Ref. [3] the two-photon decays of $\pi$ and $\pi' \equiv \pi(1300)$ mesons and the processes of their production in $e^+e^-$ collisions were considered. In papers [4, 5] we computed the production cross sections of $\pi_0(\pi')\omega$ and $\pi^0(\pi')\gamma$ pairs at electron positron colliders. These results were in a satisfactory agreement as with the experimental data as well as with theoretical estimates obtained within the vector meson dominance model. Radiative decays of $\pi^0$, $\rho^0$, $\omega$, and their radial excited states were studied in Ref. [10].

In this work we describe the two-photon decays of pseudoscalar isoscalar $\eta$, $\eta'(958)$, $\eta(1295)$, and $\eta(1475)$ mesons, the processes of their production in two-photon mechanism at electron-positron collisions, and in the Primakoff effect on an electron. The calculations are performed on the base of the $U(3) \times U(3)$ chiral NJL model with ’t Hooft interaction. The radial excitations of mesons are described again with the help of polynomial form factors for meson interactions with $u(d)$ and $s$ quarks [11]. Diagonalization of the free Lagrangian for the four eta-mesons with the transition to their physical states is performed by means of the $4 \times 4$ mixing matrix [12–14]. Note that we skip the $\eta(1405)$ state which is usually treated as a pseudoscalar glueball [15–17].

II. LAGRANGIAN

We use a nonlocal separable four-quark interaction of a current-current form which admits nonlocal vertexes (form factors) in the quark currents, and a pure local six-quark ’t Hooft interaction [18, 19]:

$$
\mathcal{L}(\bar{q}, q) = \int d^4x \bar{q}(x)(i\partial - m^0)q(x) + \mathcal{L}^{(4)}_{\text{int}} + \mathcal{L}^{(6)}_{\text{int}},
$$

$$
\mathcal{L}^{(4)}_{\text{int}} = \int d^4x \sum_{a=0}^{N} \sum_{i=1}^{2} G \left[ j^{a}_{S,i}(x)j^{a}_{S,i}(x) + j^{a}_{P,i}(x)j^{a}_{P,i}(x) \right],
$$

$$
\mathcal{L}^{(6)}_{\text{int}} = -K \left[ \det [\bar{q}(1 + \gamma_5)q] \right].
$$

Here, $m^0$ is the current quark mass matrix ($m^0_u \approx m^0_d$) and $j^{a}_{S(P),i}(x)$ denotes the scalar (pseudoscalar) quark currents

$$
j^{a}_{S(P),i}(x) = \int d^4x_1d^4x_2 \bar{q}(x_1)F^{a}_{S(P),i}(x; x_1, x_2)q(x_2)
$$

where $F^{a}_{S(P),i}(x; x_1, x_2)$ are the scalar (pseudoscalar) nonlocal quark vertex. The coupling constants $G = 3.14$ GeV$^{-2}$ and $K = 6.1$ GeV$^{-5}$ are fixed in the model [12–14] from the pion mass and from the masses of $\eta$ and $\eta'$ mesons (taking into account the mixing of ground and excited states), respectively.

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Following [18, 19], after coupling of a pair of quarks in the 't Hooft interaction we get a modified four-quark interaction of the isoscalar pseudoscalar sector of the NJL model:

$$L_{\text{isos}} = \sum_{a,b=8}^9 (\bar{q} i \gamma_5 \tau_a q) T^{P \mu}_{ab}(\bar{q} i \gamma_5 \tau_b q),$$

where $T^P$ is a matrix with elements defined as follows

$$T^{P}_{88} = G^+_u/2, \quad T^{P}_{89} = G^+_u/2,$$

$$T^{P}_{98} = G^+_s/2, \quad T^{P}_{99} = G^+_s/2,$$

$$\tau_8 = \lambda_u = (\sqrt{2} \lambda_0 + \lambda_8)/\sqrt{3},$$

$$\tau_9 = \lambda_s = (\lambda_0 + \sqrt{2} \lambda_8)/\sqrt{3},$$

$$G^+_u = G - 4K m_u I_1(m_a), \quad G^+_s = G,$$

$$G^+_{us} = 4\sqrt{2}K m_u I_1(m_a).$$

Here $m_u$ and $m_s$ are the constituent quark masses and $I_1(m_a)$ is the integral which for an arbitrary $n$ is defined as follows

$$I_n^{f_\ldots f_s}(m_a) = -\frac{N_c}{(2\pi)^3} \int d^4k f_a(k) \ldots f_a(k) \frac{1}{(m_a^2 - k^2)^n}.$$

The 3-dimensional cut-off $\Lambda_3 = 1.03$ GeV is implemented to regularize the divergent integrals. Here $m_{u,s}$ are the constituent quark masses: $m_u = 280$ MeV and $m_s = 405$ MeV.

After bosonization we get the quark-meson interaction Lagrangian in the form

$$\mathcal{L}_{\text{int}} = \bar{q}(p_1) \left[ (i \not{\nabla} - m) + i \gamma_5 \left( \lambda_u g_1^u \phi_1^u + \lambda_s g_1^s \phi_1^s \right) + \lambda_u g_2^u \phi_2^u f_2^u(p) + \lambda_s g_2^s \phi_2^s f_2^s(p) \right] q(p_2),$$

where $m = \text{diag}(m_u, m_d, m_s)$, $p = p_1 - p_2$.

$$g_1^u = [Z I^2_2]^{-1/2}, \quad g_1^s = [Z I^2_2]^{-1/2},$$

$$g_2^{u,s} = [4 I^2_2 f_{u,s}(m_{u,s})]^{-1/2},$$

where $\pi - a_1$ transitions accounted by factor

$$Z = 1 - \frac{6m^2}{M^2_{a_1}}$$

taken the same for ground $\phi_u$ and $\phi_s$ meson states. For the excited states these transitions can be omitted as discussed in Ref. [11].

Note that here we do not take into account transitions between pseudoscalar and axial-vector states, which were important for description of pions and kaons. This leads to a certain change with respect to the previous works [12–14].

From Eq. (4) taking into account renormalization of the kinetic terms in the one-loop approximation, one can get the free meson Lagrangian in the following form [12, 13]:

$$\mathcal{L}^{(2)}(\phi) = \frac{1}{2} \sum_{i,j=1}^9 \sum_{a,b=8}^9 \phi_i \mathcal{K}_{\phi,ij}(P) \phi_j,$$

where

$$\mathcal{K}_{\phi,11}(P) = P^2 - (m_u + m_u)^2 - M_{\phi,s}^2,$$

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$$\mathcal{K}_{\phi,44}(P) = P^2 - (m_s + m_s)^2 - M_{\phi,s}^2,$$

$$\mathcal{K}_{\phi,12}(P) = K_{\phi,21}(P) = \Gamma_{\eta_u}(P^2 - (m_u + m_u)^2),$$

$$\mathcal{K}_{\phi,34}(P) = K_{\phi,43}(P) = \Gamma_{\eta_u}(P^2 - (m_s + m_s)^2),$$

$$\mathcal{K}_{\phi,13}(P) = K_{\phi,31}(P) = g_{1}^u g_{1}^s [(T^P)^{-1}_{88}],$$

$$\Gamma_{\eta_u,s} = \sqrt{\frac{I_{2u,s}(m_{u,s})}{I_2(m_{u,s}) I_{1}^{f_{u,s}}(m_{u,s})}}.$$

The bare meson masses are

$$M_{\phi,s}^2 = (g_{1}^{u,s})^2 \left( \frac{1}{2} (T^P)^{-1}_{88} - 8 I_1(m_u) \right),$$

$$M_{\phi,s}^2 = (g_{1}^{u,s})^2 \left( \frac{1}{2} (T^P)^{-1}_{99} - 8 I_1(m_s) \right),$$

$$M_{\phi,s}^2 = (g_{2}^{u,s})^2 \left( \frac{1}{2G} - 8 I_1^{f_{u,s}}(m_{u,s}) \right).$$

Transition from the bare states to the physical ones is performed with the help of a $4 \times 4$ matrix $R$ which provides the diagonal form for the free Lagrangian. This
matrix was found numerically in [12, 13]; it is given in Table I. Note that this matrix is not unitary. This is due to the fact that it contains not only a rotation but also dilations. The dilations are related to two different renormalizations: one of them for the ground states and one for the first radial excited states. One can observe the same feature of non-unitary transformation in a more simple case SU(2) extended NJL model [1] where the mixing of π and π′ mesons was considered. It is worth to note that if we exclude from the consideration the excited states, we would get instead of the 4 × 4 matrix a simple 2 × 2 matrix corresponding to unitary orthogonal transformations with the standard singlet-octet mixing angle θ ≈ −19°, see Ref. [20]. The latter quantity is close to the values of the mixing angle obtained in numerous theoretical and experimental studies, see e.g. paper [21] and references therein.

III. DESCRIPTION OF RADIATIVE PROCESSES

A. Two-photon decays

Let us start with the 2-photon decays of the ground and excited states of η-mesons. To describe it we introduce the quark-photon interaction term $q\bar{Q}\gamma^\mu A^\mu q$ into the interaction Lagrangian [20], with $Q = \text{diag}(2/3, -1/3, -1/3)$. For the decay $\eta \rightarrow \gamma\gamma$ we get the amplitude

$$T_{\eta \rightarrow \gamma\gamma} = \frac{\alpha}{9\pi} \varepsilon_{\mu\nu\rho\sigma} q_1^\rho q_2^\sigma \varepsilon_1^\mu \varepsilon_2^\nu \left\{ R_{1,1} \frac{g_1^\rho}{m_u} 5I_3(m_u) + R_{2,1} \frac{g_2^\rho}{m_u} 5I_1^f(m_u) - R_{3,1} \frac{g_1^\rho}{m_s} \sqrt{2}I_3(m_s) - R_{4,1} \frac{g_2^\rho}{m_s} \sqrt{2}I_1^f(m_s) \right\}.$$ (12)

Analogously we got the amplitudes for two-photon decays of $\eta$, $\eta'$, and $\eta''$ mesons. Our results for the widths of the four decays with comparison to the existing experimental data are given in Table II.

Consider now the processes of η-meson production in $e^+e^-$ collisions by two-photon mechanism:

$$e^+ e^- \rightarrow e^+ e^- + \eta(\eta', \eta'')$$ (13)

Table I: The mixing coefficients for the isoscalar pseudoscalar meson states.

| $R_{i,j}$ | $\eta$ | $\hat{\eta}$ | $\eta'$ | $\hat{\eta}'$ |
|---------|--------|-------------|--------|-------------|
| $\varphi_1$ | 0.71 | 0.62 | -0.32 | 0.56 |
| $\varphi_2$ | 0.11 | -0.87 | -0.48 | -0.54 |
| $\varphi_3$ | 0.62 | 0.19 | 0.56 | -0.67 |
| $\varphi_4$ | 0.06 | -0.66 | 0.30 | 0.82 |

Table II: Widths of η meson two-photon decays.

| meson | $\eta$ | $\hat{\eta}$ | $\eta'$ | $\hat{\eta}'$ |
|-------|--------|-------------|--------|-------------|
| model [eV] | 520 | 93 | 4990 | 230 |
| exp. [eV] | 510 ± 26 | - | 4340 ± 140 | - |

Table III: Total cross sections of η meson production via the two-photon mechanism.

| meson | $\eta$ | $\hat{\eta}$ | $\eta'$ | $\hat{\eta}'$ |
|-------|--------|-------------|--------|-------------|
| model [nb] | 1.4 | 0.014 | 2.1 | 0.022 |
| exp. [nb] | 1.25 ± 0.13 | - | 1.8 ± 0.3 | - |

In the first approximation the total cross section reads

$$\sigma_{\eta \rightarrow \gamma\gamma} = (4\alpha)^2 \ln^2 \frac{\sqrt{s}}{2m_e} \frac{\Gamma(\eta \rightarrow 2\gamma)}{M^2(\eta)} Y \left( \frac{M^2(\eta)}{s} \right),$$

$$Y(z) = (2 + z)^2 \ln \frac{1}{\sqrt{z}} - (3 + z)(1 - z).$$ (14)

The corresponding numerical results are given in Table III in comparison with the experimental data [22] existing for the ground meson states at $\sqrt{s} = 29$ GeV total $e^+e^-$ energy in the center-of-mass.

In a similar manner we can describe the Primakoff process of η meson production in photon–lepton collisions

$$\gamma(k) + l(p) \rightarrow \eta_l(p_1) + l(p'), \quad l = e, \mu,

p^2 = p'^2 = m_l^2, \quad k^2 = 0, \quad p_1^2 = M^2(\eta_l),

s = 2kp > M^2(\eta_l) \gg m_l^2.$$ (15)

The total cross section of this process reads [2]:

$$\sigma_{\gamma l \rightarrow \eta l} = \frac{\alpha^2}{M^2(\eta_l)} \left[ 1 + \left( 1 - \frac{M^2(\eta_l)}{s} \right)^2 \right] \times \left( \ln \frac{s^2}{m_l^2 M^2(\eta_l)} - 1 \right).$$ (16)

For electron-photon collisions at the center-of-mass energy $\sqrt{s} = 3$ GeV we get the following predictions for the total cross sections: $\sigma_{\gamma e \rightarrow \eta e} = 340$ pb, $\sigma_{\gamma e \rightarrow \eta' e} = 540$ pb, $\sigma_{\gamma e \rightarrow \eta'' e} = 3.7$ pb, $\sigma_{\gamma e \rightarrow \eta'\eta'} = 5.7$ pb.

IV. CONCLUSIONS

The results of our calculations show that the application of the mixing matrix $R$ which diagonalizes the free meson Lagrangian leads to sufficiently good description radiative decays of the ground η and η' meson states. This allows us to hope to get reasonable predictions for
their first radial excited states. A similar situation took place in the case of strong interactions of the ground and excited mesons [2, 14].

The $\eta(1475)$ meson as a $q\bar{q}$ state was considered in Ref. [24], where a simple phenomenological model of mixing between only two $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components was used. In our approach we have mixing of four $q\bar{q}$ components: two of them for ground states and two for radial excited ones. However, we did not take into account mixing of $\eta$ mesons with pseudoscalar glueballs in particular with the $\eta(1405)$ state. Meanwhile we took into account the influence of the gluon anomaly (by means of the ’t Hooft interaction) onto the mixing of the $u\bar{u} + d\bar{d}$ and $s\bar{s}$ components in all four states being considered. Some other papers were devoted to studies of mixing between the ground $\eta(550)$ and $\eta'(958)$ mesons with a pseudoscalar glueball, see e.g. Refs. [17, 25, 26]. Note that the excited $\eta$ meson states were not taken into account there. Moreover, the mixing of the ground $\eta$ and $\eta'$ mesons with glueball was found to be not large. We argue that in future studies all these mixing effects should be considered together like it was done in ref. [27] for mixing of four isoscalar mesons and a glueball using $5 \times 5$ mixing matrix.

Acknowledgments

We are grateful to E.A. Kuraev for fruitful discussions and critical reading of the manuscript. This work was supported by RFBR grant 10-02-01295-a.

[1] M. K. Volkov and C. Weiss, Phys. Rev. D 56, 221 (1997).
[2] M. K. Volkov, D. Ebert and M. Nagy, Int. J. Mod. Phys. A 13, 5443 (1998).
[3] E. A. Kuraev and M. K. Volkov, Phys. Lett. B 682, 212 (2009). [arXiv:0908.1628 [hep-ph]].
[4] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Phys. Rev. C 83, 048201 (2011) [arXiv:1012.2455 [hep-ph]].
[5] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Eur. Phys. J. A 47, 103 (2011).
[6] CMD-2 Collab. (R. R. Akhmetshin et al.), Phys. Lett. B 562, 173 (2003).
[7] M. N. Achasov et al., Phys. Lett. B 486, 29 (2000).
[8] M. N. Achasov et al., Eur. Phys. J. C 12, 25 (2000).
[9] M. N. Achasov et al., Phys. Lett. B 559, 171 (2003).
[10] A. B. Arbuzov, E. A. Kuraev and M. K. Volkov, Phys. Rev. C 82, 068201 (2010).
[11] M. K. Volkov, Phys. Atom. Nucl. 60, 1920 (1997).
[12] M. K. Volkov and V. L. Yudichev, Int. J. Mod. Phys. A 14, 4621 (1999).
[13] M. K. Volkov and V. L. Yudichev, Phys. Atom. Nucl. 63, 1835 (2000). [Yad. Fiz. 63N10, (2000)].
[14] M. K. Volkov and V. L. Yudichev, Phys. Part. Nucl. 31, 282 (2000). [Fiz. Elem. Chast. Atom. Yadra 31, 576 (2000)].
[15] V. Crede, C. A. Meyer, Prog. Part. Nucl. Phys. 63, 74-116 (2009). [arXiv:0812.0600 [hep-ex]].
[16] V. Mathieu, Gluon Mass, Glueballs and Gluonic Mesons, [arXiv:1102.3875 [hep-ph]].
[17] H.-Y. Cheng, H.-n. Li, K.-F. Liu, Phys. Rev. D79, 014024 (2009).
[18] U. Vogl and W. Weise, Prog. Part. Nucl. Phys. 27, 195 (1991).
[19] S. P. Klevansky, Rev. Mod. Phys. 64, 649 (1992).
[20] M. K. Volkov, V. L. Yudichev, M. Nagy, Nuovo Cim. A112, 955 (1999).
[21] T. N. Pham, Phys. Lett. B694, 129 (2010).
[22] S. J. Brodsky, T. Kinoshita and H. Terazawa, Phys. Rev. Lett. 25, 972 (1970).
[23] N. A. Roe et al., Phys. Rev. D 41, 17 (1990).
[24] N. N. Achasov, G. N. Shestakov, Phys. Rev. D84, 034036 (2011).
[25] V. V. Anisovich, D. V. Bugg, D. I. Melikhov, V. A. Nikonov, Phys. Lett. B404, 166-172 (1997).
[26] S. Kiesewetter, V. Vento, Phys. Rev. D82, 034003 (2010). [arXiv:1003.5792 [hep-ph]].
[27] M. K. Volkov, V. L. Yudichev, Eur. Phys. J. A10, 223-235 (2001).