QFT holography near the horizon of Schwarzschild-like spacetimes.

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Abstract: It is argued that free QFT can be defined on the event horizon of a Schwarzschild-like spacetime and that that theory is unitarily and algebraically equivalent to QFT in the bulk (near the horizon). Under that unitary equivalence the bulk hidden \(SL(2,\mathbb{R})\) symmetry found in a previous work becomes manifest on the event horizon, it being induced by a group of horizon diffeomorphisms. The class of generators of that group can be enlarged to include a full Virasoro algebra of fields defined on the event horizon. These generators have a quantum representation in QFT on the event horizon and thus in the bulk.

1. Introduction. A number of papers has been concerned with the issue of the statistical origin of black-hole entropy. Holographic principle [1, 2, 3] arose by the idea that gravity near the horizon should be described by a low dimensional theory with a higher dimensional group of symmetry. The correspondence between quantum field theories of different dimensions was conjectured by Maldacena [4] using the machinery of string theory: There is a correspondence between quantum field theory in a, asymptotically \(AdS, d+1\) dimensional spacetime (the “bulk”) and a conformal theory in a \(d\) dimensional manifold (the (conformal) “boundary” at spacelike infinity). Witten [5] described the above correspondence in terms of relations of observables of the two theories. Rehren proved rigorously some holographic results for free quantum fields in a \(AdS\) background, establishing a correspondence between bulk observables and boundary observables without employing string machinery [6, 7]. To explain the correspondence one should notice that the conformal group which acts in the \(d\)-dimensional \(AdS_{d+1}\) boundary can be realized as the group of the isometries of the \(AdS_{d+1}\) bulk. In this way, the bulk-boundary correspondence has a geometric nature. The boundary of a Schwarzschild spacetime (dropping the boundary at infinity) is the event horizon of the black hole. The \(AdS\) correspondence has been used directly in Minkowski spacetime for massless particle in [8] with the help of the optical metric. Is there any bulk-boundary correspondence in a manifold containing a Schwarzschild-like black hole? Two-dimensional Rindler spacetime embedded in Minkowski spacetime approximates the
nontrivial part of the spacetime structure near a bifurcate horizon as that of a Schwarzschild black hole embedded in Kruskal spacetime. In that context, we have argued in a recent work [9] that free quantum field theory in two-dimensional Rindler space presents a “hidden” $SL(2, \mathbb{R})$ symmetry: The theory turns out to be invariant under a unitary representation of $SL(2, \mathbb{R})$ but such a quantum symmetry cannot be induced by the geometric background. $SL(2, \mathbb{R})$ is the group of symmetry of the zero-dimensional conformal field theory in the sense of [10], so, as for the case of AdS spacetime, it suggests the existence of a possible correspondence between quantum field theory in Rindler space and a conformal field theory defined on its event horizon. In this letter we illustrate the basic results that can be found in a forthcoming technical paper [11] where we have shown that it is possible to build up the wanted correspondence of a free quantum theory defined in the bulk and a quantum field theory defined on the event horizon of a two dimensional Rindler space. Other involved results are that the $SL(2, \mathbb{R})$ symmetry reveals a clear geometric meaning if it is examined on the horizon and, in that context, a whole Virasoro algebra of symmetries arises.

Some overlap with our results is present in the literature. Guido, Longo, Roberts and Verch [12] discussed in some detail the extent to which an algebraic QFT on a spacetime with a bifurcate Killing horizon induces a conformal QFT on that bifurcate Killing horizon. Along a similar theme, Schroer and Wiesbrock [13] have studied the relationship between QFTs on horizons and QFTs on the ambient spacetime. They even use the term “hidden symmetry” a sense similar as we do here and we done in [9]. In related follow-up works by Schroer [14] and by Schroer and Fassarella [15] the relation to holography and diffeomorphism covariance is also discussed.

2. Hidden $SL(2, \mathbb{R})$ symmetry. Consider a general Schwarzschild-like metric (namely a static black hole metric with bifurcate event horizon), $ds_S^2 = -A(r)dt^2 + A^{-1}(r)dr^2 + r^2d\Omega^2$, $\Omega$ denoting angular coordinates. Near the horizon ($r = r_h$), the nonangular part of the metric reduces to the metric of a two-dimensional Rindler wedge $\mathbb{R}$, $ds_R^2 = -\kappa^2 y^2 dt^2 + dy^2$ with $A'(r_h) = 2\kappa$, and $\kappa y^2 = 4(r - r_h)$. Also dropping the angular coordinates, let us consider a free Klein-Gordon scalar field $\phi$ with motion equation $-\partial_t^2 \phi + \kappa^2 (y \partial_y y \partial_y - y^2 m^2) \phi = 0$. To built up the one-particle Hilbert space referred to the quantization with respect to the Rindler Killing time $t$, any real solution $\psi$ of the K-G equation must be decomposed in $\partial_t$-stationary modes as follows

$$\psi(t, y) = \int_{0}^{+\infty} \sum_{\alpha} \Phi_{E}^{(\alpha)}(t, y) \psi_{+}^{(\alpha)}(E) dE + c.c. \quad (1)$$

$E \in [0, +\infty) = \mathbb{R}^+$ is an element of the spectrum of the Rindler Hamiltonian $H$ associated with $\partial_t$ evolution. Concerning the index $\alpha$ we distinguish between two cases: if $m > 0$ there is a unique mode $\Phi_{E}^{(\alpha)} = \Phi_{E}$ whose expression is $\sqrt{2E \sinh(\pi E/\kappa)} / \sqrt{2\pi^2 \kappa E} e^{-iEt} K_{iE/\kappa}(my)$. If $m = 0$ there are two values of $\alpha$, corresponding to ingoing and outgoing modes, $\Phi_{E}^{(\alpha)}$ whose expression are $e^{-iEt(\pm \ln(\kappa y)/\kappa)} / \sqrt{4\pi E}$. If $m > 0$ there is no energy degeneration and the one-particle Hilbert space \( \mathcal{H} \) generated by the positive frequency part of the decomposition above is isomorphic to $L^2(\mathbb{R}^+, dE)$. In the other case ($m = 0$), twofold degeneracy implies that $\mathcal{H} \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$. Quantum field operators, acting in the symmetrized Fock space
$F(\mathcal{H})$ and referred to the Rindler vacuum $|0\rangle$ — that is $|0\rangle_{in} \otimes |0\rangle_{out}$ if $m = 0$ — read

$$\hat{\phi}(t,y) = \int_{0}^{\infty} \sum_{\alpha} \Phi_{E}^{(\alpha)}(t,y)a_{E\alpha} + \overline{\Phi_{E}^{(\alpha)}}(t,y)a_{E\alpha}^{\dagger} dE. \quad (2)$$

As usual, the causal propagator $\Delta$ satisfies $[\hat{\phi}(x), \hat{\phi}(x')] = -i\Delta(x,x').$

In [9] we have found that, if $m > 0,$ $\mathcal{H}$ is irreducible under a unitary representation of $SL(2, \mathbb{R})$ generated by (self-adjoint extensions) of the operators $iH, iD, iC$ (which enjoy the commutation relations of the Lie algebra of $SL(2, \mathbb{R})),$ with

$$H := E, \quad D := -i\left(\frac{1}{2} + E \frac{d}{dE}\right), \quad C := -\frac{d}{dE} E \frac{d}{dE} + \frac{(k - \frac{1}{2})^{2}}{E}. \quad (3)$$

$k$ can arbitrarily be fixed in $\{1/2, 1, 3/2, \ldots\}.$ See [11] for details on domains an all that. If $m = 0$ and so $\mathcal{H} \cong L^{2}(\mathbb{R}^{+}, dE) \oplus L^{2}(\mathbb{R}^{+}, dE),$ an analogue representation exists in each space $L^{2}(\mathbb{R}^{+}, dE).$ Making use of Heisenberg representation it is simply proven that the algebra generated by $H, D, C,$ with depending-on-time coefficients, is made of constant of motions [9, 11, 10]. Thus $SL(2, \mathbb{R})$ is a symmetry of the one-particle system (that can straightforwardly be extended to the free quantum field in Fock space). The crucial point is that the found symmetry is hidden: it cannot be induced by the background geometry since the Killing fields of Rindler spacetime enjoy a different Lie algebra from that of $H, D, C.$

3. Fields on the horizon. Let us to investigate the nature of the found symmetry exactly on the event horizon assuming $\mathbb{R}$ to be naturally embedded in a Minkowski spacetime. In particular we want to investigate its geometrical nature, if any, on the event horizon.
(Rindler) light coordinates \( u = t - \log(\kappa y)/\kappa \), \( v = t + \log(\kappa y)/\kappa \) (where \( u, v \in \mathbb{R} \)) cover the (open) Rindler space \( \mathbb{R} \). Separately, \( v \) is well defined on the future horizon \( F \), \( u \to +\infty \), and \( u \) is well defined on the past horizon \( P \), \( v \to -\infty \) (see figure). Take the wavefunction in (1) and consider the limit on the future horizon \( u \to +\infty \). That is equivalent to restrict the wavefunction on the event horizon when it is considered as a wavefunction in Minkowski spacetime, obtaining

\[
\psi(v) = \int \frac{e^{-iEv}}{\sqrt{4\pi E}} e^{i\rho_{m,\kappa}(E)} \tilde{\psi}_+(E) \, dE + \text{c.c.} \tag{4}
\]

\( e^{i\rho_{m,\kappa}(E)} \) is a pure phase (see [11] for details). In coordinate \( u \in \mathbb{R} \), the restriction of \( \psi \) to \( P \) is similar with the \( v \) replaced for \( u \) and \( \rho_{m,\kappa}(E) \) replaced by \( -\rho_{m,\kappa}(E) \). If \( m = 0 \) the restrictions to \( F \) and \( P \) read respectively

\[
\psi(v) = \int_{\mathbb{R}^+} e^{-iEv} \tilde{\psi}_{(0)}(E) \, dE + \text{c.c.}, \quad \psi(u) = \int_{\mathbb{R}^+} \frac{e^{-iEu}}{\sqrt{4\pi E}} \tilde{\psi}_{(0)}(E) \, dE + \text{c.c.} \tag{5}
\]

Discarding the phase it is possible to consider the following real “field on the future Horizon”:

\[
\varphi(v) = \int_{\mathbb{R}^+} \frac{e^{-iEv}}{\sqrt{4\pi E}} \tilde{\varphi}_+(E) \, dE + \int_{\mathbb{R}^+} \frac{e^{+iEv}}{\sqrt{4\pi E}} \tilde{\varphi}_+(E) \, dE \tag{6}
\]

as the basic object in defining a quantum field theory on the future event horizon. The same can be done for the past event horizon. The one-particle Hilbert space \( \mathcal{H}_F \) is defined as the space generated by positive frequency parts \( \tilde{\psi}_+(E) \) and turns out to be isomorphic to \( L^2(\mathbb{R}^+, dE) \) once again. The field operator reads, on the symmetrized Fock space \( \mathcal{F}(\mathcal{H}_F) \) with vacuum \( |0\rangle_F \),

\[
\hat{\phi}_F(v) = \int_0^\infty \frac{e^{-iEv}}{\sqrt{4\pi E}} a_E + \frac{e^{iEv}}{\sqrt{4\pi E}} a_E^\dagger \, dE \tag{7}
\]

The causal propagator \( \Delta_F \) is defined by imposing \( [\hat{\phi}(v), \hat{\phi}(v')] = -i\Delta_F(v, v') \) and it takes the form \((1/4)\text{sign}(v - v')\). In spite of the absence of any motion equation the essential features of free quantum field theory are preserved by that definition as proven in [11]. Degeneracy of the metric on the horizon prevents from smearing field operators by functions due to the ill-definiteness of the induced volume measure. However, employing the symplectic approach [16], a well-defined smearing-procedure is that of field operators and exact 1-forms \( \eta = df \) where \( f = f(v) \) vanishes fast as \( v \to \pm\infty \). The integration of forms does not need any measure. In other words for a real exact 1-form \( \eta \) as said above

\[
\hat{\phi}_F(\eta) = \int_0^\infty \frac{dE}{\sqrt{4\pi E}} \left( \int_{\mathbb{R}} e^{-iEv} \eta(v) \right) a_E + \left( \int_{\mathbb{R}} e^{iEv} \eta(v) \right) a_E^\dagger \tag{8}
\]

is well defined and diffeomorphism invariant. In a suitable domain the map \( \eta(v) \mapsto \Delta_F(\eta) = \frac{1}{4} \int_{\mathbb{R}} \text{sign}(v - v') \eta(v') = \psi_\eta(v) \) defines a one-to-one correspondence between exact one-forms and
horizon wavefunctions of the form $H$ and $\eta = 2d\psi$. Finally, similarly to usual quantum field theory [16], it holds

$$[\hat{\phi}_F(\eta), \hat{\phi}_F(\eta')] = -i\Delta_F(\eta, \eta') = \int_F \psi_{\eta'}d\psi_\eta - \psi_\eta d\psi_{\eta'}.$$ 

The last term define a diffeomorphism-invariant symplectic form on horizon wavefunctions.

4. Unitary and algebraic holography theorems. It is possible to prove the existence of a unitary equivalence between the theory in the bulk and that on the horizon in the sense we are going to describe. Consider the case $m > 0$ and the future horizon $F$.

**Theorem 1.** There is a unitary map $U_F : \mathcal{F}(H) \to \mathcal{F}(H_F)$ such that $U_F|0\rangle_F = |0\rangle_F$ and $U_F^{-1}\hat{\phi}_F(\eta)U_F = \hat{\phi}(f)$ for any smooth compactly supported function $f$ used to smear the bulk field, $\eta = 2d(\Delta(f)|_F)$. (See figure.) Details on the construction of $U_F$ are supplied in [11], here we give only the main idea. Take $f$ as said and consider the associated bulk wavefunction $\psi_f = \Delta(f)$, restrict $\psi_f$ to $F$ obtaining a horizon wavefunction as in [11] with positive frequency part $e^{im\omega}(E)\tilde{\psi}_{f+}(E)$. Then define a horizon wavefunction $\varphi_f$ as in [11] with $\tilde{\varphi}_+$ replaced by $\tilde{\psi}_{f+}$. It is clear that the map $\psi_f \mapsto \varphi_f$ corresponds to a unitary operator from $H$ to $H_F$. That is, by definition $U_F|\mathcal{H}$. Imposing $U_F|0\rangle_F = |0\rangle_F$, by taking tensor products of $U_F|\mathcal{H}$, this map extends to a unitary map $U_F : \mathcal{F}(H) \to \mathcal{F}(H_F)$. Finally, by direct inspection one finds that, if $\eta = 2d\varphi_f$, one also has $U_F^{-1}\hat{\phi}_F(\eta)U_F = \hat{\phi}(f)$.

The same procedure can be used to define an analogous unitary operator referred to $P$. If $m = 0$ two unitary operators arises. One is $V_F : \mathcal{F}(H_{in}) \to \mathcal{F}(H_F)$ such that $V_F|0\rangle_{in} = |0\rangle_F$ and $V_F^{-1}\hat{\phi}_F(\eta_f)V_F = \hat{\phi}_{in}(f)$. $H_{in}$ is the bulk Hilbert space associated with the ingoing modes and $\hat{\phi}_{in}(f)$ is the part of bulk field operator built up using only ingoing modes. The other unitary operator $V_P : \mathcal{F}(H_{out}) \to \mathcal{F}(H_P)$ plays an analogous rôle with $in$ replaced for $out$ and $F$ replaced for $P$ everywhere. (More generally $V_P \otimes V_F : \mathcal{F}(H) \to \mathcal{F}(H_P) \otimes \mathcal{F}(H_F)$ define a unitary operator which transforms the vacuum states into vacuum states and field operators into field operators.) As a consequence of the cited theorem, e.g. if $m > 0$, one has the invariance of vacuum expectation values:

$$F\langle 0|\hat{\phi}_F(\eta_1)\cdots\hat{\phi}_F(\eta_n)|0\rangle_F = \langle 0|\hat{\phi}(f_1)\cdots\hat{\phi}(f_n)|0\rangle_F.$$ 

Similarly to the extent in the bulk case, one focuses on the algebra $A_F$ of linear combinations of product of field operators $\phi_F(\omega)$ varying $\omega$ in the space of allowed complex 1-forms. We assume that $A_F$ also contain the unit operator $I$. The Hermitian elements of $A_F$ are the natural observables associated with the horizon field. From an abstract point of view the found algebra is a unital $*$-algebra of formal operators $\phi_F(\eta)$ with the additional properties $[\hat{\phi}_F(\eta), \hat{\phi}_F(\eta')] = -i\Delta_F(\eta, \eta')$, $\hat{\phi}_F(\eta)^* = \hat{\phi}_F(\overline{\eta})$ and linearity in the form $\eta$. $A_F$ can be studied no matter any operator representation in any Fock space. Operator representations are obtained via GNS

\footnote{The analogous algebra of operators in the bulk fulfill the further requirement $\phi(f) = 0$ if (and only if) $f = Kg$, $K$ being the Klein-Gordon operator. No analogous requirement makes sense for $A_F$ since there is no equation of motion on the horizon.}
Theorem 2. There is a unique injective unital $*$-algebras homomorphism $\chi_F : \mathcal{A}_R \to \mathcal{A}_F$ such that $\chi_F(\phi(f)) = \phi_F(\eta_f)$, where $\eta = 2d(\Delta(f)|_F)$. Moreover in GNS representations in the respectively associated Fock spaces $\mathcal{F}(H), \mathcal{F}(H_F)$ built up over $|0\rangle$ and $|0\rangle_F$ respectively, $\chi_F$ has a unitary implementation and reduces to $U_F$.

Notice that, in particular $\chi_F$ preserves the causal propagator, in the sense that it must be $-i\Delta(f, g) = [\phi(f), \phi(g)]I = [\phi(f), \phi(g)]\chi_F(I) = \chi_F([\phi(f), \phi(g)]I) = [\chi(\phi(f)), \chi(\phi(g))]$

Analogous algebraic homomorphism theorems can be given for $P$. In the case of the massless case $[11]$. Let $\varphi$ be the horizon wavefunction (see $[11]$). Let $\varphi = \varphi_+(E)$ be the positive frequency part of $\varphi$. The wavefunction $\varphi_g$ associated with $U_g \varphi_+$ reads

$$\varphi_g(v) = \varphi \left( \frac{av + b}{cv + d} \right) - \frac{b}{d}, \quad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

The term $-\varphi(b/d)$ assures that $\varphi_g$ vanishes as $v \to \pm \infty$. Notice that the added term disappears when referring to $d\varphi$ rather than $\varphi$. The group of diffeomorphisms of $F$, i.e. the real line$^2$,

$$v \mapsto \frac{av + b}{cv + d}, \quad g = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{R})$$

can be obtained by composition of one-parameter subgroups associated with the following three vector fields on $\mathfrak{F}$: $\partial_v, v\partial_v, v^2\partial_v$. It is simply proven that the Lie brackets of those fields is a realization of the Lie algebra of $SL(2, \mathbb{R})$. Moreover, it turns out that $[11]$:

$^2$Actually one has to consider the projective line $\mathbb{P} \cup \{\infty\}$. 

Theorem 3. If $k = 1$ in $[8]$, the action of every $U_g$ on a state $\varphi_+ = \varphi_+(E)$ is essentially equivalent to the action of a corresponding $F$-diffeomorphism on the associated (by $[6]$) horizon wavefunction $\varphi$. More precisely, take a matrix $g \in SL(2, \mathbb{R})$ and $\varphi = \varphi(v)$ in a suitable space of horizon wavefunction (see $[11]$). Let $\varphi_+ = \varphi_+(E)$ be the positive frequency part of $\varphi$. The wavefunction $\varphi_g$ associated with $U_g \varphi_+$ reads

$$\varphi_g(v) = \varphi \left( \frac{av + b}{cv + d} \right) - \frac{b}{d}, \quad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \quad (9)$$

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Theorem 4 (a) If $k = 1$ in (3), the unitary one-parameter group generated by $i\mathcal{H}_F$ is associated, through Theorem 3, to the one-group of $F$-diffeomorphisms generated by $\partial_\nu$. (b) the unitary one-parameter group generated by $iD_F$ is associated to the one-parameter group of $F$-diffeomorphisms generated by $v \partial_\nu$ and (c) the unitary one-parameter group generated by $iC_F$ is associated to the one-group of $F$-diffeomorphisms generated by $v^2 \partial_\nu$.

6. Appearance of Virasoro algebra. The bulk $SL(2, \mathbb{R})$-symmetry is manifest when examined on the event horizon, in the sense that it is induced by the geometry. The Lie algebra generated by vector fields $\partial_\nu, v \partial_\nu, v^2 \partial_\nu$ play a crucial rôle in proving this fact. That algebra can be extended to include all the class of fields defined on the event horizon $\mathcal{L}_n$ with $\mathcal{L}_n = v^{n+1} \partial_\nu$. It is interesting to notice that these fields enjoy Virasoro commutation relations without central charge, $\{\mathcal{L}_n, \mathcal{L}_m\} = (n-m)\mathcal{L}_{n+m}$. A natural question arises:

Is it possible to give a quantum representation of these generators in the sense of Theorem 4?

At least formally, the answer is positive. Indeed, by employing Theorem 4 one finds out that the infinitesimal action of the one parameter group of diffeomorphisms generated by $\mathcal{L}_n$ on a horizon wavefunction $\varphi = \varphi(v)$ is equivalent to the action of an anti-Hermitean operator $L_n$ on the positive frequency part $\tilde{\varphi}_+ = \tilde{\varphi}_+(E)$. $L_n$ is defined as, respectively for $n \geq -1$ and $n < -1$,

$$
(L_n \tilde{\varphi}_+)(E) := i^{n+2} \sqrt{E} \frac{d^{n+1}}{dE^{n+1}} \sqrt{E \tilde{\varphi}_+(E)},
$$

$$
(L_n \tilde{\varphi}_+)(E) := -i^{-(n+2)} \sqrt{E} \int_0^E \int_0^{E_1} \cdots \int_0^{E_{-(n+1)}} \sqrt{E_{-(n+1)} \tilde{\varphi}_+(E_{-(n+1)})}.
$$

Those operators are at least anti-Hermitean on suitable domains and enjoy Virasoro commutation rules $[L_n, L_m] = (n-m) L_{n+m}$.

7. Four dimensional case. Up to now we have investigated only the two dimensional spacetimes, but it is possible to extend our results to a four dimensional case which better approximates the Schwarzschild extent. For this purpose consider the near-horizon approximation of a Schwarzschild-like spacetime without discarding the angular variables $\theta, \phi$, so that $ds^2 = -\kappa^2 y^2 dt^2 + dy^2 + y^2 d\Omega^2$. Every field takes an angular part described by the usual spherical harmonics $Y^l_m(\theta, \phi)$. QFT in the bulk involves the one-particle Hilbert space $\oplus_{l=0}^\infty \mathbb{C}^{2l+1} \otimes \mathcal{K}_l$ with $\mathcal{K}_l \cong L^2(\mathbb{R}^+, dE)$ if $l > 0$, $\mathbb{C}^{2l+1}$ being the space at fixed total angular momentum $l$ and $\mathcal{K}_0 \cong L^2(\mathbb{R}^+, dE)$ in the massive case but $\mathcal{K}_0 \cong L^2(\mathbb{R}^+, dE) \oplus L^2(\mathbb{R}^+, dE)$ in the massless case. For wavefunctions with components in a fixed space $\mathbb{C}^{2l+1} \otimes L^2(\mathbb{R}^+, dE)$ Klein-Gordon equation reduces to the two-dimensional one with a positive contribution to the mass depending on $l$. Quantum field theory can be constructed on the future horizon $F \cong \mathbb{S}^2 \times \mathbb{R}$. The appropriate causal propagator reads

$$
\Delta_F(x, x') = (1/4) \text{sign}(v - v') \delta(\theta - \theta') \delta(\phi - \phi') \sqrt{g_{\mathbb{S}^2}(\theta, \phi)}.
$$

The horizon field operator $\hat{\Phi}_F$ has to be smeared with 3-forms as $df(v, \theta, \phi) \wedge d\theta \wedge d\phi$ and Theorems 1 and 2, at least in the massive case, can be restated as they stand for the two-dimensional
case. Theorems 3 and 4 hold true at fixed angular variables.

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