Understanding Ghost Interference

Tabish Qureshi  
Center for Theoretical Physics, Jamia Milia Islamia, New Delhi 110025, India.

Pravabati Chingangbam  
Indian Institute of Astrophysics, Koramangala, Bangalore-560034, India.

Sheeba Shafaq  
Department of Physics, Jamia Milia Islamia, New Delhi 110025, India.

The ghost interference observed for entangled photons is theoretically analyzed using wave-packet dynamics. It is shown that ghost interference is a combined effect of virtual double-slit creation due to entanglement, and quantum erasure of which-path information for the interfering photon. For the case where the two photons are of different color, it is shown that fringe width of the interfering photon depends not only on its own wavelength, but also on the wavelength of the other photon which it is entangled with.

I. INTRODUCTION

Quantum entanglement is a concept which has intrigued people since the time Einstein, Podolsky and Rosen[1] first raised some objections against quantum mechanics, which led Schrödinger[2] to formalize the concept. Entangled quantum systems, even though far separated in space, show certain correlation in their measurement results. Einstein believed that such correlations implied a nonlocal action at a distance. Measurement on one system seems to affect a far away system which it is entangled to. Schrödinger went further to say, “It is rather discomforting that the theory should allow a system to be steered or piloted into one or the other type of state at the experimenter’s mercy in spite of his having no access to it.”[2]

As experimental techniques progressed, the effect of entanglement was experimentally demonstrated. One of the most dramatic demonstrations of nonlocality is in[3] the so-called ghost interference experiment by Strekalov et.al.[3] In the following we briefly describe the ghost interference experiment. Two entangled photons emerge from a spontaneous parametric down-conversion (SPDC) source S. Photon 1 and 2 are made to move in different directions, by a beam-splitter. A double-slit is kept in the path of photon 1, and there is a scanning detector D1 behind it (see Fig. 1(a)). No first order interference is observed for photon 1. This is surprising because normally one expects Young’s double-slit interference. The second surprise of the experiment is that when photon 2 is observed by detector D2, in coincidence with a fixed detector D1 detecting photon 1, photon 2 shows a double-slit interference pattern (see Fig. 1(b)). Note that photon 2 does not pass through any double-slit. The third surprise of the experiment is that the fringe width of the interference pattern follows a Young’s interference formula \( w = \frac{\lambda D}{d} \), where \( D \) is a very curious distance from double-slit, right through the SPDC source S to the detector D2. Note that photon 2 does not even travel that much of distance.

To explain the experimental results, the authors presented a geometrical model where the entangled photons, starting from a spatially extended source, travelled in exactly oppositely directed paths. The path length depends on the point of origin of the photon pair in the extended source. The difference in the lengths of two such paths passing through different slits was shown to lead to ghost interference[3]. The absence of first order interference behind the double-slit was attributed to “the considerably large angular propagation uncertainty of a single SPDC photon.”[3] The experiment attracted a lot of debate and

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*Electronic address: tabish@ctp-jamia.res.in
†Electronic address: prava@iap.res.in
‡Electronic address: shafaqsheeba1@gmail.com

FIG. 1: Schematic diagram of the two-slit ghost interference experiment. Entangled photons 1 and 2 emerge from the SPDC source S and travel in different directions along x-axis.
research attention.[4–9]

An explanation based on geometric trajectories is not very satisfying, and one would like to look for a more quantum argument. Also, from the explanation provided, the origin of ghost interference and the absence of first order interference seem to be not connected. In the following we will carry out an analysis based on time evolution of quantum wave-packets, and show that it gives us a much better understanding of ghost interference. In our analysis, we will show that the origin of ghost interference and the absence of first order interference, have a much deeper connection.

Recently a modified ghost interference experiment has been carried out using photon pairs generated via spontaneous four-wave mixing (SFWM).[10] This experiment is novel in the aspect that the correlated photons in a pair are of different color, with wavelengths $\lambda_1 = 1530$ nm and $\lambda_2 = 780$ nm. This phenomenon was called two-color ghost interference by the authors. We will analyze this experiment too, and present some interesting predictions for it.

II. WAVE-PACKET ANALYSIS

A. The entangled state

In order to theoretically analyze ghost interference, a major challenge is to formulate a well-behaved entangled state which captures the essence of entangled photons. The so-called EPR state is not well behaved in the sense that it is like a Dirac delta function. We propose, what we call, a generalized EPR state as follows:

$$\Psi(z_1, z_2) = C \int_{-\infty}^{\infty} dp e^{-p^2/4\hbar^2} e^{-i p z_2/\hbar} e^{i p z_1/\hbar} e^{-\left(\frac{(z_1 + z_2)^2}{4\Omega^2}\right)},$$

where $C$ is a normalization constant, and $\sigma, \Omega$ are certain parameters whose physical significance will become clear in the following. The state (1), unlike the EPR state, is well behaved and fully normalized. In the limit $\sigma, \Omega \to \infty$ the state (1) reduces to the EPR state.

The photons of the pair are assumed to be travelling in opposite directions along the x-axis, but the entanglement is in the z-direction. In our analysis we will ignore the dynamics along the x-axis as it does not affect entanglement. We just assume that during evolution for a time $t$, the photon travels a distance equal to $ct$. Integration over $p$ can be performed in (1) to obtain:

$$\Psi(z_1, z_2) = \sqrt{\frac{\sigma}{\pi \Omega}} e^{-\left(\frac{z_1 - z_2}{2\Omega}\right)^2} e^{-\left(\frac{z_1 + z_2}{2\Omega}\right)^2}. \tag{2}$$

The uncertainty in position and the wave-vector of the two photons, along the z-axis, is given by

$$\Delta z_1 = \Delta z_2 = \sqrt{\Omega^2 + 1/4\sigma^2},$$

$$\Delta k_{1z} = \Delta k_{2z} = \frac{1}{2} \sqrt{\frac{\sigma^2 + 1}{4\Omega^2}}. \tag{3}$$

For the above it is clear that $\Omega$ and $\sigma$ quantify the position and momentum spread of the photons in the z-direction.

B. Time evolution

We will first lay out our strategy for time evolution of a photon wave-packet. If the state at time $t = 0$ is $\psi(z, 0)$, the state at a later time is given by

$$\psi(z, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp(ik_2 z - i\omega(k_2) t) \tilde{\psi}(k_2, 0) dk_2, \tag{4}$$

where $\tilde{\psi}(k_2, 0)$ is the Fourier transform of $\psi(z, 0)$ with respect to $z$. Now photon is approximately travelling in the x-direction, but can slightly deviate in the z-direction (which allows it to pass through slits located at different z-positions), so that its true wave-vector will have a small component in the z-direction too. Thus

$$\omega(k_z) = c \sqrt{k_x^2 + k_z^2} \tag{5}$$

Since the photon is travelling along x-axis by and large, we can write $k_x \approx k_0$, where $k_0$ is the wavenumber of the photon associated with its wavelength, $k_0 = 2\pi/\lambda$. The dispersion along x-axis can then be approximated by

$$\omega(k_z) \approx c k_0 + ck_z^2/2k_0. \tag{6}$$

Using this, Eqn. (4) becomes

$$\psi(z, t) = \frac{e^{ict_0 t}}{2\pi} \int_{-\infty}^{\infty} \exp(ik_2 z - i\omega(k_2) t) \tilde{\psi}(k_2, 0) dk_2 \tag{7}$$

Coming back to our problem of entangled photons, we assume that after travelling for a time $t_0$, photon 1 reaches the double slit ($c t_0 = L_2$), and photon 2 travels a distance $L_2$ towards detector D2. Using the strategy outlined in the preceding discussion, we can write the state of the entangled photons after a time $t_0$ as follows:

$$\psi(z_1, z_2, t_0) = \frac{e^{i\omega k_0 t_0}}{4\pi^2} \int_{-\infty}^{\infty} dk_1 \exp(ik_1 z_1 - i\omega k_0 t_0) \tilde{\psi}(k_1, z_2, k_0),$$

$$\int_{-\infty}^{\infty} dk_2 \exp(ik_2 z_2 - i\omega k_2 t_0) \tilde{\psi}(k_1, k_2, 0), \tag{8}$$

where $\tilde{\psi}(k_1, k_2, 0)$ is the Fourier transform of (2) with respect to $z_1, z_2$.

C. Effect of double-slit

In order to see the effect of the double-slit on the entangled state, one would normally model a potential for the double-slit, and calculate the evolution of the state in that potential. That is not an easy task. We will follow an alternative strategy which captures the essence of
the effect of the double-slit on the state, without going into tedious calculation. When the state interacts with a single-slit, let us assume that what emerges from a single slit is a Gaussian wave-packet centered at the location of the slit, and whose width is related to the width of the slit. So, if the two slits are A and B, the packets which pass through will be, say, $|\phi_A(z_1)\rangle$ and $|\phi_B(z_1)\rangle$, respectively. Some part of the state of particle 1 will get blocked. Let us represent it by $\chi(z_1)$. These three states are obviously orthogonal, and the actual state of particle 1 can be expanded in this basis.

$$|\Psi(z_1, z_2, t_0)\rangle = |\phi_A\rangle|\phi_A\rangle|\Psi\rangle + |\phi_B\rangle|\phi_B\rangle|\Psi\rangle + |\chi\rangle|\chi\rangle|\Psi\rangle.$$  
(9)

The terms $|\phi_A\rangle|\Psi\rangle, |\phi_B\rangle|\Psi\rangle, |\chi\rangle|\Psi\rangle$ are states of particle 2 and can be explicitly calculated as

$$\psi_A(z_2) = |\langle\phi_A(z_1)|\Psi(z_1, z_2, t_0)\rangle|,$$
$$\psi_B(z_2) = |\langle\phi_B(z_1)|\Psi(z_1, z_2, t_0)\rangle|,$$
$$\psi_\chi(z_2) = |\langle\chi(z_1)|\Psi(z_1, z_2, t_0)\rangle|.$$  
(10)

So, the entangled state we get after particle 1 crosses the double-slit is:

$$|\Psi(z_1, z_2)\rangle = |\phi_A\rangle|\psi_A\rangle + |\phi_B\rangle|\psi_B\rangle + |\chi\rangle|\psi_\chi\rangle,$$  
(11)

where $|\phi_A\rangle$ and $|\phi_B\rangle$ are states of particle 1, and $|\psi_A\rangle$ and $|\psi_B\rangle$ are states of particle 2. The first two terms represent the amplitudes of particle 1 passing through the slits, and the last term represents the amplitude of it getting reflected or blocked. The linearity of the Schrödinger equation assures that the first two terms and the last term evolve independently. Since the experiment only looks for those photons 1 which have passed through the double-slit, we might as well throw away the last term. Doing this will not change anything except for renormalizing the part of the state that we retain.

In the following, we assume that $|\phi_A\rangle, |\phi_B\rangle$, are Gaussian wave-packets:

$$\phi_A(z_1) = \frac{1}{(\pi/2)^{1/4}}e^{-(z_1-z_0)^2/\epsilon^2},$$
$$\phi_B(z_1) = \frac{1}{(\pi/2)^{1/4}}e^{-(z_1+z_0)^2/\epsilon^2},$$  
(12)

where $z_0$ is the z-position of slit A and B, respectively, and $\epsilon$ their widths. Thus, the distance between the two slits is $2z_0 \equiv d$.

Using (10) and (8), wavefunctions for $|\psi_A\rangle, |\psi_B\rangle$ can be calculated, which, after normalization, have the form

$$\psi_A(z_2) = C_2e^{-\frac{(z_2-z'_0)^2}{\epsilon^2}}, \quad \psi_B(z_2) = C_2e^{-\frac{(z_2+z'_0)^2}{\epsilon^2}},$$  
(13)

where $C_2 = (2/\pi)^{1/4}(\sqrt{\Gamma_R} + \frac{i\Gamma_I}{\sqrt{\Gamma_R}})^{-1/2},$

$$z'_0 = \frac{z_0}{4\Gamma_R^2\sigma^2 + 1} + \frac{4\epsilon^2}{4\Gamma_R^2 - 1/\sigma^2}.$$  
(14)

and $\Gamma = \frac{e^2}{\pi} + \frac{1}{\sqrt{\pi}}\frac{\sqrt{\Gamma_R} + i\Gamma_I}{\sqrt{\Gamma_R}}.$

Here $\Gamma_R, \Gamma_I$ are the real and imaginary parts of $\Gamma$, respectively.

Thus, the state which emerges from the double slit, has the following form

$$\Psi(z_1, z_2) = C e^{-(z_1-z_0)^2/\epsilon^2} e^{-\frac{(z_2-z'_0)^2}{\epsilon^2}} + C e^{-(z_1+z_0)^2/\epsilon^2} e^{-\frac{(z_2+z'_0)^2}{\epsilon^2}},$$  
(15)

where $c = (1/\sqrt{\pi\epsilon})(\sqrt{\Gamma_R} + \frac{i\Gamma_I}{\sqrt{\Gamma_R}})^{-1/2}$. In obtaining this expression, we have dropped the phase factor in (8), as it is not important for our final analysis. Equation (15) represents two wave-packets of photon 1, of width $\epsilon$, and localized at $\pm z_0$, entangled with two wave-packets of photon 2, of width $\sqrt{\Gamma_R}/\sqrt{\pi}\epsilon$, localized at $\pm z'_0$.

Even at this early stage one can see that the amplitudes of photon 1 passing through slits A and B are correlated to spatially separated wave-packets of photon 2. Thus, in principle one can detect photon 2 and know which slit, A or B, photon 1 passed through. By Bohr’s principle of complementarity, if one knows which slit photon 1 passed through, it cannot show any interference. This is the fundamental reason for non-observance of first order interference in photon 1 in the ghost imaging experiment.

D. Ghost interference

The parts of the state for photon 2 are in the form of two spatially separated wave-packets. This suggests that photon 2 has passed through a virtual double-slit (of slit separation $2z'_0$), conditioned on photon 1 having passed through the real double-slit. As photon 2 evolves in time, the two wave-packets will overlap, and interference is a possibility. This is consistent with the ghost imaging observed for entangled photons. It must be emphasized here however, that although entanglement leads to ghost imaging, entanglement is not a requirement for it. Classical correlations in light are enough for demonstrating ghost imaging. This subject has been widely debated, and we can only refer the reader to two review articles and references therein.

Before reaching detector D2, particle 2 evolves for a further time $t$. The time evolution, transforms the state (15) to

$$\Psi(z_1, z_2, t) = C_t \exp \left[ \frac{-(z_1-z_0)^2}{\epsilon^2 + i\epsilon\chi_1\lambda_1/\pi} \right] \exp \left[ \frac{-(z_2-z'_0)^2}{\Gamma + i\epsilon\chi_2\lambda_2/\pi} \right]$$
$$+ C_t \exp \left[ \frac{-(z_1+z_0)^2}{\epsilon^2 + i\epsilon\chi_1\lambda_1/\pi} \right] \exp \left[ \frac{-(z_2+z'_0)^2}{\Gamma + i\epsilon\chi_2\lambda_2/\pi} \right],$$  
(16)
where
\[ C(t) = \frac{1}{\sqrt{\pi} \sqrt{\epsilon + i c t \lambda_1 / \gamma} \sqrt{1 + \epsilon} + (\Gamma_i + i c t \lambda_2 / \pi) / \sqrt{1 + \epsilon}}. \]  
(17)
If the correlation because of entanglement between the photons is good, one can make the following approximation: \( \Omega \gg \epsilon, \Omega \gg 1/\sigma \) and \( \Omega \gg 1 \). In this limit,
\[ \Gamma \approx \gamma^2 + 2i \hbar t_0 / \mu, \quad z_0' \approx z_0, \]  
(18)
where \( \gamma^2 = \epsilon^2 + 1/\sigma^2 \).

The wave-function (16) represents the combined state of the two photons when they reach detectors D1 and D2 respectively. The stage is now set to calculate the probability of coincident counting of D1 located at \( z_1 \) and D2 located at \( z_2 \).

If D1 and D2 are located at \( z_1 \) and \( z_2 \) respectively, the probability density of their coincident count is given by
\[ P(z_1, z_2) = |\Psi(z_1, z_2, t)|^2 \]
\[ = |C_t|^2 \left( \exp \left[ -\frac{2(z_1 - z_0)^2}{\epsilon^2 + \lambda_1^2 L_1^2 / \pi^2} - \frac{2(z_2 - z_0)^2}{\gamma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2} \right] + \exp \left[ -\frac{2(z_1 + z_0)^2}{\epsilon^2 + (\lambda_1 L_1)^2 / \pi^2} - \frac{2(z_2 + z_0)^2}{\gamma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2} \right] + \exp \left[ -\frac{2(z_1^2 + z_0^2)}{\epsilon^2 + (\lambda_1 L_1)^2 / \pi^2} - \frac{2(z_2^2 + z_0^2)}{\gamma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2} \right] \cdot 2 \cos (\theta_1 z_1 + \theta_2 z_2) \right), \]
(19)
where
\[ \theta_1 = \frac{2d \lambda_1 L_1 / \pi}{\epsilon^2 + \lambda_1^2 L_1^2 / \pi^2}, \quad \theta_2 = \frac{2 \pi d (\lambda_2 L + \lambda_1 L_2)}{\gamma^2/\sigma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2}, \]
(20)
\[ L = L_1 + L_2, \quad \text{and} \]
\[ C_t = \frac{1}{\sqrt{\pi} \sqrt{\epsilon + \lambda_1^2 L_1^2 / \pi^2} \sqrt{\gamma + \lambda_2 L + \lambda_1 L_2 / \pi \gamma}}. \]
(21)

**III. RESULTS**

**A. Ghost Interference**

Let us first analyze the original ghost interference experiment when the entangled photons have the same wave-length \( \lambda \) and detector D1 is fixed at \( z_1 = 0 \). In that case, (19) reduces to
\[ P(z_1, z_2) = |\Psi(z_1, z_2, t)|^2 \]
\[ = |C_t|^2 \exp \left[ -\frac{2z_0^2}{\epsilon^2 + \lambda_1^2 L_1^2 / \pi^2} \right] \cdot \exp \left[ -\frac{2(z_2^2 + z_0^2)}{\gamma^2 / \sigma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2} \right] \cdot 2 \cosh (4z_2 z_0 / \gamma_D^2) \right) \times \left( 1 + \frac{\cos (\theta_D z_2)}{\cosh (4z_2 z_0 / \gamma_D^2)} \right), \]
(22)
where
\[ \theta_D = \frac{2 \pi d \lambda D}{\gamma_D^2 / \sigma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2}, \]
(23)
and \( \gamma_D^2 = \gamma^2 + (\lambda D / \pi)^2 \). For \( \gamma^2 \gg \lambda D / \pi \), (25) represents an interference pattern for photon 2 with the fringe width given by
\[ w_2 \approx \frac{\lambda D}{d}. \]
(24)
This is ghost interference, and one should notice that the distance \( D \) appearing in the formula is the distance from the double-slit, right through the source to detector D2. This is exactly what was seen in the experiment.[4]

**B. Two-color ghost interference**

Now we analyze the two-color ghost interference experiment of Ding et al.[10] When the two photons have different wavelengths, and detector D1 is fixed at \( z_1 = 0 \), (19) reduces to
\[ P(z_1, z_2) = |C_t|^2 \exp \left[ -\frac{2z_0^2}{\epsilon^2 + \lambda_1^2 L_1^2 / \pi^2} \right] \cdot \exp \left[ -\frac{2(z_2^2 + z_0^2)}{\gamma_2^2 / \sigma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2} \right] \cdot 2 \cosh (4z_2 z_0 / \gamma_L^2) \times \left( 1 + \frac{\cos (\theta_L z_2)}{\cosh (4z_2 z_0 / \gamma_L^2)} \right), \]
(25)
where
\[ \theta_L = \frac{2 \pi d (\lambda_2 L + \lambda_1 L_2)}{\gamma_2^2 / \sigma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2}, \]
(26)
and \( \gamma_L^2 = \gamma^2 + (\lambda_2 L + \lambda_1 L_2)^2 / \pi^2 \). The expression (25) is an interference pattern for photons 2 which are detected in coincidence with a fixed D1 (see Fig. 2). For \( \gamma^2 \ll \lambda_2 L / \pi \), (25) represents an interference pattern for photon 2 with the fringe width given by
\[ w_2 \approx \frac{\lambda_2 (L_1 + L_2)}{d} + \frac{\lambda_1 L_2}{d}. \]
(27)
One can see that the fringe width of the ghost interference of photon 2 depends not only on its wavelength \( \lambda_2 \) but also depends on the wavelength \( \lambda_1 \) of the photon which it is entangled with. We believe this is a highly non-classical feature.

**C. Effect of converging lens**

In the two-color ghost interference experiment, Ding et al.[10] have used a converging lens between the source and detector D2. So the fringe-width formula given by (27) doesn’t directly apply. However, if one were to repeat this experiment without the converging lens, the validity of the formula (27) can be tested.
In the following we incorporate the effect of a converging lens in our theoretical analysis. This allows us to make contact with Ding et al.’s experimental results. In order to do that, we consider an experimental setup similar to that of Ding et al. [10] (see Fig. 3). A converging lens of focal length \( f \) is kept at distance \( f \) before the detector D2, where \( f \) is its focal length. The distance between the source and the lens is \( L_1 + L_2 - f \).

The effect of a converging lens on a general Gaussian wave-packet is such that in its subsequent dynamics, it narrows instead of spreading. In general, we can quantify the effect of the lens by a unitary transformation of the form [14]

\[
U_f \left( \frac{\pi}{2} \right)^{-1/4} \exp \left( \frac{-z_1^2}{\sigma^2 + i\Delta L} \right) = \frac{(\pi/2)^{-1/4}}{\sqrt{\sigma^2 + i\Delta L}} \exp \left( -\frac{z_1^2}{\sigma^2 + i\Delta L} \right), \tag{28}
\]

where \( L \) is the distance the wave-packet of an initial width \( \sigma \), traveled before passing through the lens, and \( \lambda = \Lambda\pi \) is the wavelength of the particle. This transformation respects the classical thin lens equation in the following way. If a Gaussian wave-packet of width \( \sigma \) starts from a distance \( L \) from the lens, it converges back until the imaginary terms in the exponent disappear. This happens at a distance \( u \) such that \( \frac{1}{u} = \frac{1}{f} - \frac{1}{L} \). The width of the wave-packet at that time is \( \frac{\sigma}{L-f} \).

In our calculation without a lens, particles 1 and 2 travel a distance \( L_2 \) so that photon 1 passes through the double-slit. When the photons emerge from the double-slit, the two-photon state is given by eqn. [15]. The situation in this case too remains the same, hence (15) still holds. However, after this, instead of travelling a distance \( L_2 \) to reach D2, photon 2 now travels a distance \( L_1 - f \) to reach the lens. The two-photon state at this time is given by

\[
\Psi(z_1, z_2, t) = c_1 e^{i\frac{z_1^2}{\lambda_1}} e^{i\frac{z_2^2}{\lambda_2}} + c_2 e^{i\frac{(z_1 + z_2)^2}{\lambda_1 + \lambda_2}} e^{i\frac{z_1^2}{\lambda_2}} + c_3 e^{i\frac{(z_1 + z_2)^2}{\lambda_1 + \lambda_2}} e^{i\frac{z_2^2}{\lambda_2}} \tag{29}
\]

where \( c_1 = \left( \sqrt{\pi} e^{-i\alpha L_1} \Gamma_1 e^{-i\gamma L_1} / \Gamma_1 \right)^{-1} \) and \( \Gamma \approx \gamma^2 + i(\Lambda_1 + \Lambda_2) L_2 \). Photon 2 wave-packet, when it reaches the lens, has apparently travelled a distance \( (1 + \alpha \Lambda_1 / \lambda_2) L_2 + L_1 \), and has an original width \( \gamma \).

We apply the lens transformation (28) on (29) and let the wave-packets evolve for a further distance \( \Delta \) to reach the respective detectors 1 and 2. The probability density of finding photon 2 at \( z_2 \), given that photon 1 is detected at \( z_1 = 0 \) is given by

\[
P(0, z_2) \approx 2|C_f|^2 e^{-\frac{z_2^2}{\Delta_2}} \exp \left( \frac{4z_2^2}{\Delta_2} \right) + \cos \left[ \frac{2\pi z_2 d}{\lambda_2 f \left( 1 + \alpha \Lambda_1 L_2 - L_1 \right)} \right], \tag{30}
\]

where \( \Delta_1 = \epsilon^2 + \frac{\lambda_1^2 L_1^2}{\epsilon^2} \), \( \Delta_2 = \left( \frac{\gamma f}{(1 + \alpha \Lambda_2) L_2 + L_1 - \Delta f} \right)^2 + \lambda_2^2 \left( \frac{2\alpha L_2 + 2\alpha L_1 - 3\Delta f}{\gamma f} \right)^2 \), \( \alpha = 1 + \lambda_1 / \lambda_2 \), and

\[
C_f = \frac{1}{\sqrt{\pi} e^{\frac{\gamma f}{\sqrt{\pi} L_2 - \Delta f}} \left( 1 + \alpha \Lambda_2 L_2 - L_1 \right)^{3/2}}. \tag{31}
\]

Expression (30) represents an interference pattern with a fringe-width given by

\[
w_f = \frac{\lambda_2 f \left( 1 + \alpha \Lambda_1 L_2 - L_1 \right)}{d}. \tag{31}
\]

Comparing the above with (27), we see that introducing a converging lens masks the dramatic effect of entanglement to large extent. A more simplistic analysis by Ding et al. gave \( w_f = \frac{\lambda_2^2 f}{d} \). [10]
IV. DISCUSSION

From the wave-packet analysis carried out here, the following inferences can be drawn. When one of the two entangled photons (photon 1) passes through a double-slit, the other one experiences a virtual double-slit or a ghost image of the real double-slit, by virtue of their entangled state. If one is only interested in those photon pairs whose photon 1 has passed through the double-slit, the resultant entangled state consists of two distinct wave-packets of photon 1, correlated with two distinct wave-packets of photon 2. One can, in principle, detect photon 2 without disturbing photon 1, and can figure out which of the two slits photon 1 went through. Bohr’s principle of complementarity implies that if we know which of the two slits photon 1 went through, it cannot show any interference. This is the real reason why first order interference behind the double-slit is absent in Strekalov et al.’s experiment. [3]

Since photon 2 experiences a virtual double-slit (conditioned on photon 1 passing through the real double-slit), it has the potential to show an interference pattern. However, by virtue of the entangled wave-packet state, photon 1 also carries information on which of the two virtual slits photon 2 passed through. Thus photon 2 should also not show any interference. However, in the experiment, detector D1 is fixed at z₁ = 0, far away from the double-slit. Photon 1 from either of the two slits is equally likely to reach the fixed detector D1. Thus by the act of fixing D1, we lose the knowledge of which slit photon 1 went through. The which-way information about photon 1, and consequently about photon 2, is erased. It is well known that if the which-way information in a two-slit experiment is erased, there are ways in which the interference pattern can be brought back. This phenomenon goes by the name of quantum erasure. [15] So, ghost interference is a result of formation of a virtual double-slit, by virtue of entanglement, and quantum erasure of which-way information for photon 2. A curious feature of this phenomenon is that the virtual double-slit seems to be located, not anywhere between the source and D2, as one might naively expect, but exactly at the location of the real double-slit! Photon 2 goes nowhere near that region. The distance D that appears in the formula for the fringe-width of ghost interference (24), is never travelled by photon 2.

As a corollary, if the detector D2 is fixed instead of D1, and D1 is scanned along z-axis, photon 1 will show an interference pattern, but only in coincidence with the fixed detector D2. The fringe-width in this case, will depend on the actual distance between the double-slit and D1, and not on D.

In the case of the two-color ghost interference, the situation is more interesting. The fringe-width of the ghost interference shown by photon 2 [27], depends not only on its own wavelength, but also on the wavelength of photon 1, which it is entangled with. This is a highly non-classical feature, arising from true entanglement. This effect is hidden in the case where λ₁ = λ₂. The two-color ghost interference experiment was carried out in the presence of a converging lens. Our analysis shows that the converging lens masks the dramatic feature seen in (27) to a large extent. Formula (27) can be verified if the two-color ghost interference experiment is carried out without the converging lens.

Acknowledgments

Tabish Qureshi thanks the organisers of the International Program on Quantum Information 2014, IOP, Bhubaneswar, for making possible a lively debate on various foundational issues.

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