Leptogenesis and low-energy phases

Sacha Davidson* and Alejandro Ibarra†

* Dept of Physics, University of Durham, Durham, DH1 3LE, UK
† Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, UK

Abstract

In supersymmetric models, the CP asymmetry produced in the decay of the lightest right-handed neutrino, \( \equiv \epsilon \), can be written as a function of weak scale parameters. We introduce a way of separating \( \epsilon \) into contributions from the various weak-scale phases, and study the contribution of potentially measurable neutrino phases to leptogenesis. We find that the Majorana phase \( \phi' \), which could have observable effects on neutrinoless double beta decay, is important for \( \epsilon \) unless there are cancellations among phases. If the phase \( \delta \) can be measured at a neutrino factory, then it contributes significantly to \( \epsilon \) over much of parameter space.

1 Introduction

After the discovery of neutrino oscillations[1,2], leptogenesis [3] stands as one of the most appealing explanations for the observed Baryon Asymmetry of the Universe (BAU) [4]. One of the crucial ingredients [5] for this mechanism is CP violation in the leptonic sector. However, there is no indication for it so far, although it could perhaps be seen at a neutrino factory or in neutrinoless double beta decay. It is therefore interesting to investigate whether there is any relation between the CP violation required for leptogenesis and the phases that could be measured at low energies in the neutrino sector.

The leptogenesis scenario relies on the seesaw model [6] for neutrino masses, that is usually analyzed in terms of high-energy parameters, not accessible to experiments. So, the resulting predictions are (texture) model-dependent. The above question has been addressed in such an approach [7–12]. Instead, we parametrize the seesaw in terms of weak scale variables [13]. This gives us a model-independent formulation of leptogenesis in terms of low energy inputs, in which we can study the above question.

The aim of this paper is to quantify, in a model independent way, the relation of the CP violation required for leptogenesis to the measurable low-energy phases. We express the CP asymmetry of leptogenesis as a function of real parameters and phases at the weak scale, and then introduce a definition of “phase overlap” between the leptogenesis phase and the individual low energy phases. This definition is not the
only possible one, but has linearity properties and is calculable. It is motivated by the notion of vector space, spanned by low energy phases (“basis vectors”), in which the CP asymmetry of leptogenesis is a “vector”. The relative importance of a low energy phase for leptogenesis would then be the “inner product” of the leptogenesis “vector” with the relevant “basis vector”. We will not be able to construct such a vector space, but it is a useful analogy to keep in mind.

The paper is organized as follows: the next section introduces CP violation in the leptonic sector. Section 3 contains the basic concepts of the supersymmetric see-saw and the generation of the BAU by leptogenesis, from a top-down point of view. In Section 4 we review the procedure to reformulate the see-saw mechanism from a bottom-up perspective. This will allow us to study leptogenesis in terms of low energy data, opening the possibility of relating, in a straight-forward way, the Baryon Asymmetry of the Universe with the CP violation measurable at neutrino factories. In section 5 we develop the general formalism to study quantitatively the above-mentioned relation. In Section 6 and 7 we show the results of our analysis, first for a particular case, and then for a more general case. In section 8 we present a self-contained summary and conclusions. Finally, we include an appendix with the procedure to evaluate numerically the contributions from the low-energy phases to leptogenesis.

2 Flavour and CP violation in the leptonic sector

In the last few years, the Superkamiokande collaboration [2] has provided compelling evidence that neutrinos have mass and oscillate. More recently, the SNO collaboration [14] has confirmed the oscillation hypothesis, and the first neutral current data [15] seem to favour the large angle MSW (LAMSW) solution to the solar neutrino problem [16]. These results, combined with those from a series of other experiments [17], have allowed to measure fairly well the mass splittings and mixing angles relevant for solar and atmospheric neutrino oscillations. In addition to this, other experiments have provided bounds on neutrino parameters from electron antineutrino disappearance (CHOOZ)[18], the non-observation of neutrinoless double beta decay [19], the shape of the tritium beta decay spectrum [20], and different cosmological and astrophysical considerations. However, no evidence has been found so far for CP violation in the leptonic sector.

The search for leptonic CP violation is theoretically motivated by several facts. First, the discovery of CP violation in the leptonic sector could shed some light on the mechanism that generates neutrino masses and perhaps hint at some underlying structure. Secondly, the observation of CP violation in the quark sector, (in the neutral kaon system, $\epsilon'/\epsilon$, and in $B \rightarrow \psi K_s$), encourages the search for CP violation in the neutrino sector. If there exists a symmetry relating quarks and leptons, these experimental results would point to CP violation also in the leptonic sector. Furthermore, particular models would give definite predictions that could be contrasted in the future. Lastly, and most importantly for the purposes of this paper, CP violation in the leptonic sector could be related to the observed Baryon Asymmetry of the Universe. This is possible in the context of the see-saw mechanism.
On the experimental side, the leptonic version of the CKM phase can be detected by comparing transition probabilities for neutrinos and antineutrinos:

\[ A = \frac{P(\nu_\alpha \rightarrow \nu_\beta) - P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)}{P(\nu_\alpha \rightarrow \nu_\beta) + P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)}. \] (1)

Unfortunately, it is not possible to measure such an asymmetry with the natural sources of neutrinos, i.e. the Sun and pions decaying in the atmosphere, since the “beam” cannot be switched from \( \nu \) to \( \bar{\nu} \). Hence, a lot of effort is being bestowed on the design of a neutrino factory [21,22]: an intense muon source to produce a high-intensity neutrino beam. In the muon storage ring, muons decay to produce muon neutrinos and electron antineutrinos. Whereas a muon neutrino would produce a muon in the detector, the oscillation of an electron antineutrino to a muon antineutrino would produce an antimuon. This antimuon (a “wrong sign” muon) would be a clear signature for oscillation, and \( P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) \) could be determined. One of the advantages of a neutrino factory is that the muons in the storage ring can be replaced by antimuons. This makes possible the measurement of \( P(\nu_e \rightarrow \nu_\mu) \) and hence the CP asymmetry. In practice, detecting CP violation in the neutrino sector is not an easy task [23,22], since the beam has to go through the Earth, that is CP asymmetric. In consequence, the matter effects on the oscillation pattern can obscure the CP violation intrinsic to neutrinos.

If neutrinos have Majorana masses, as predicted by the seesaw mechanism, there are also “Majorana” phases, in addition to the “Dirac” phase that could be detected at a neutrino factory. Neutrinoless double beta decay could be sensitive to these phases (see however [24]). This lepton number violating, but CP conserving, process probes the Majorana neutrino mass matrix element between \( \nu_e \) and \( \nu_\mu \), which depends on the masses and mixing angles, and also on the Majorana phases. Neutrinoless double beta decay is not observed at the moment. However, experiments which should see a signal, for the currently favored masses and mixing angles (LMA), are being discussed [25].

The see-saw mechanism [6] consists on adding three right-handed neutrinos to the Standard Model (SM) particle content, singlets with respect to the SM gauge group, and coupled to the Higgs doublet through a Yukawa coupling. Then, a Majorana mass term for the right-handed neutrinos is not forbidden by the gauge symmetry, and can be naturally much larger than the scale of electroweak symmetry breaking. These simple assumptions are enough to produce neutrino masses naturally small \(^1\). Furthermore, if CP is violated in the leptonic sector, the decay of the right-handed neutrinos in the early Universe produces a lepton asymmetry [3,28] that will be eventually reprocessed into a baryon asymmetry by sphalerons [29]. This leptogenesis scenario will be discussed in more detail in section 3.

In supersymmetric models, the seesaw mechanism can induce flavour violating processes involving charged leptons that could be observed in the future [26]. The neutrino Yukawa couplings generate off-diagonal elements in the slepton mass matrix, via renormalization group running. These flavour violating mass terms contribute inside loops

\(^1\) Nevertheless, this minimal model has a serious hierarchy problem: the right-handed neutrinos produce a (large) quadratically divergent radiative correction to the Higgs mass. Therefore, in what follows, we will restrict ourselves to the supersymmetric version of the see-saw mechanism.
to processes such as $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$. This has been extensively studied from various theoretical [30,12] and phenomenological [31,32,27] perspectives. The current experimental bound [33] on $\mu \to e\gamma$ imposes some restrictions on the parameter space of the SUSY seesaw. It is anticipated that the sensitivity to $\tau \to \mu\gamma$ and $\mu \to e\gamma$ could improve by as much as three orders of magnitude [34] in forthcoming years. This would provide interesting information about the flavour structure of the SUSY seesaw, irrespective of whether lepton flavour violation is observed or not.

In this paper, we will concentrate on the possible relation of the CP asymmetry in the leptonic sector with the Baryon Asymmetry of the Universe, in the framework of the supersymmetric leptogenesis. We suppose that the BAU is generated in the out-of-equilibrium decay of the lightest right-handed neutrino. The CP violation that gives rise to the BAU is not straightforwardly related to the CP violation that could be observed at low energy. This has been carefully and elegantly discussed, using Jarlskog invariants in [7]. These authors have also studied the relation of the leptogenesis phase to low energy phases for specific high scale models and various solar solutions [8]. They have an analytic approximation similar to ours, and our results seem to agree where they overlap. Correlations between leptogenesis and low energy parameters have been studied in left-right symmetric models where the Yukawa couplings are small [10], and in various Grand Unified Theories [11]. In certain classes of top-down models, it has been found [12] that the leptogenesis phase is unrelated to the MNS phases; these textures correspond to the third case we study, in section 7.2, so the conclusion [12] that $\delta$ makes little contribution to leptogenesis agrees with our result. The goal of this paper is to investigate the interplay between the CP violation at very high energies and at low energies, in a model independent way. We will also comment on the prospects to observe CP violation at a neutrino factory or in neutrinoless double beta decay, in view of the measured BAU, and inversely, what could be inferred about the BAU if CP violation is observed at low energy.

3 The see-saw mechanism and leptogenesis: the top-down approach

The supersymmetric version of the see-saw mechanism has a leptonic superpotential that reads

$$W_{lep} = e^c_R Y_e L \cdot H_d + \nu^c_R Y_\nu L \cdot H_u - \frac{1}{2} \nu^c_R M \nu^c_R,$$  \hspace{1cm} (2)

where $L_i$ and $e_Ri$ ($i = e, \mu, \tau$) are the left-handed lepton doublet and the right-handed charged-lepton singlet, respectively, and $H_d$ ($H_u$) is the hypercharge $-1/2$ ($+1/2$) Higgs doublet. $Y_e$ and $Y_\nu$ are the Yukawa couplings that give masses to the charged leptons and generate the neutrino Dirac mass, and $M$ is a $3 \times 3$ Majorana mass matrix that does not break the SM gauge symmetry. We do not make any assumptions about the structure of the matrices in eq.(2), but consider the most general case. Then, it can be proved that the number of independent physical parameters is 21: 15 real parameters and 6 phases [36].
It is natural to assume that the overall scale of the neutrino mass matrix, denoted by $M$, is much larger than the electroweak scale or any soft mass. Therefore, at low energies the right-handed neutrinos are decoupled and the corresponding effective Lagrangian reads

$$\delta L_{lep} = e_R^T Y_e L \cdot H_d - \frac{1}{2} (Y_{\nu} L \cdot H_u)^T M^{-1} (Y_{\nu} L \cdot H_u) + h.c..$$

(3)

So, after the electroweak symmetry breaking, the left-handed neutrinos acquire a Majorana mass, given by

$$M_{\nu} = m_D^T M^{-1} m_D,$$

(4)

suppressed with respect to the typical fermion masses by the inverse power of the large scale $M$.

We will find convenient to work in the flavour basis where the charged-lepton Yukawa matrix, $Y_e$, and the gauge interactions are flavour-diagonal. In this basis, the neutrino mass matrix, $M_{\nu}$, can be diagonalized by the MNS [37] matrix $U$, defined by

$$U^T M_{\nu} U = \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \equiv D_{M_{\nu}},$$

(5)

where $U$ is a unitary matrix that relates flavour to mass eigenstates

$$\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= U
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix},$$

(6)

and the $m_{\nu_i}$ can be chosen real and positive. Also, we label the masses in such a way that $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. We will assume throughout the paper that the light neutrinos have a hierarchical spectrum. We do not consider the inverse hierarchy, which may be more difficult to match with the neutrinos detected from SN1987A [39]; we anticipate that the contribution of $\delta$ to the leptogenesis phase could be suppressed by powers of $m_{\nu_1}$ in this case. The CP asymmetry required for leptogenesis is suppressed for degenerate light neutrinos [38], so we neglect this possibility.

$U$ can be written as

$$U = V \cdot \text{diag}(e^{-i\phi/2}, e^{-i\phi'/2}, 1),$$

(7)

where $\phi$ and $\phi'$ are CP violating phases (if different from 0 or $\pi$) and $V$ has the ordinary form of the CKM matrix

$$V = \begin{pmatrix}
c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\
-c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\delta} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\delta} & s_{23}c_{13} \\
s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}.$$

(8)

It is interesting to note that the neutrino mass matrix, $M_{\nu}$, depends on 9 parameters: 6 real parameters and 3 phases. Comparing with the complete theory, we discover that some information has been “lost” in the decoupling process, to be precise, 6 real parameters and three phases. We will return to this important issue later on.

Another remarkable feature of the see-saw mechanism is that it provides a natural framework to generate the baryon asymmetry of the Universe, defined by $\eta_B = (n_B -
\( n_B/s \), where \( s \) is the entropy density. This quantity is strongly constrained by Big Bang Nucleosynthesis to lie in the range \( \eta_B \simeq (0.3 - 0.9) \times 10^{-10} \), to successfully reproduce the observed abundances of the light nuclei D, \(^3\)He, \(^4\)He and \(^7\)Li [40]. As was shown by Sakharov, generating a baryon asymmetry requires baryon number violation, \( C \) and CP violation, and a deviation from thermal equilibrium. These three conditions are fulfilled in the out-of-equilibrium decay of the right-handed neutrinos and sneutrinos in the early Universe. For conciseness, and since we are concerned only with supersymmetric leptogenesis, in what follows we will use right-handed neutrinos, and the shorthand notation \( \nu_R \), to refer both to right-handed neutrinos and right-handed sneutrinos.

Let us briefly review the mechanism of generation of the BAU through leptogenesis [3,28]. At the end of inflation, a certain number density of right-handed neutrinos, \( n_{\nu_R} \), is produced, that depends on the cosmological scenario. These right-handed neutrinos decay, with a decay rate that reads, at tree level,

\[
\Gamma_{D_i} = \Gamma(\nu_{R_i} \to \ell_i H_u) + \Gamma(\nu_{R_i} \to \tilde{L}_i \tilde{h}_u) = \frac{1}{8\pi}(Y_\nu Y_\nu^\dagger)_{ii} M_i. \tag{9}
\]

The out of equilibrium decay of a right-handed neutrino \( \nu_{R_i} \) creates a lepton asymmetry given by

\[
\eta_L = \frac{n_\ell - n_{\bar{\ell}}}{s} = \frac{n_{\nu_R} + n_{\bar{\nu}_R}}{s} \epsilon_i \kappa, \tag{10}
\]

The value of \( (n_{\nu_R} + n_{\bar{\nu}_R})/s \) depends on the particular mechanism to generate the right-handed (s)neutrinos. On the other hand, the CP-violating parameter

\[
\epsilon_i = \frac{\Gamma_{D_i} - \bar{\Gamma}_{D_i}}{\Gamma_{D_i} + \bar{\Gamma}_{D_i}}, \tag{11}
\]

where \( \bar{\Gamma}_{D_i} \) is the CP conjugated version of \( \Gamma_{D_i} \), is determined by the particle physics model that gives the masses and couplings of the \( \nu_R \). Finally, \( \kappa \) is the fraction of the produced asymmetry that survives washout by lepton number violating interactions after \( \nu_R \) decay. To ensure \( \kappa \sim 1 \), lepton number violating interactions (decays, inverse decays and scatterings) must be out of equilibrium when the right-handed neutrinos decay. In the case of the lightest right-handed neutrino \( \nu_{R_1} \), this corresponds approximately to \( \Gamma_{D_1} < H|_{T=M_1} \), where \( H \) is the Hubble parameter at the temperature \( T \), and can be expressed in terms of an effective light neutrino mass [28,41], \( \tilde{m}_1 \), as

\[
\tilde{m}_1 = \frac{8\pi\langle H^0_u\rangle^2}{M_1^2} \Gamma_{D_1} = (Y_\nu Y_\nu^\dagger)_{11} \frac{\langle H^0_u\rangle^2}{M_1} \lesssim 5 \times 10^{-3} \text{eV}. \tag{12}
\]

This requirement has been carefully studied [28,41,50]; the precise numerical bound on \( \tilde{m}_1 \) depends on \( M_1 \), and can be found in [41].

The last step is the transformation of the lepton asymmetry into a baryon asymmetry by non-perturbative B+L violating (sphaleron) processes [29], giving

\[
\eta_B = \frac{C}{C - 1} \eta_L, \tag{13}
\]

where \( C \) is a number \( \mathcal{O}(1) \), that in the Minimal Supersymmetric Standard Model takes the value \( C = 8/23 \).
In this paper, we assume that a sufficient number of $\nu_R$ were produced — thermally, or in the decay of the inflaton, or as a scalar condensate of $\tilde{\nu}_R$s, or by some other mechanism. We will concentrate on the step of leptogenesis that is most directly related to neutrino physics, namely the generation of a CP asymmetry, $\epsilon$, in the decay of the right-handed neutrinos. It is convenient to work in a basis of right-handed neutrinos where $\mathcal{M}$ is diagonal

$$\mathcal{M} = \text{diag}(M_1, M_2, M_3),$$

with $M_i$ real and $0 \leq M_1 < M_2 < M_3$. In this basis, the CP asymmetry produced in the decay of $\nu_{R_i}$ reads

$$\epsilon_i \simeq -\frac{1}{8\pi} \left[ \frac{1}{|Y_{\nu} Y_{\nu}^\dagger|_{ii}} \sum_j \text{Im} \left\{ [Y_{\nu} Y_{\nu}^\dagger]^2_{ij} \right\} f \left( \frac{M_j^2}{M_i^2} \right) \right],$$

where [43]

$$f(x) = \sqrt{x} \left( \frac{2}{x-1} + \ln \left[ \frac{1+x}{x} \right] \right).$$

Here, we will assume that the masses of the right-handed neutrinos are hierarchical. We also assume that the lepton asymmetry is generated in the decay of the lightest right-handed neutrino. This second assumption is critical; if the asymmetry was generated by the decay of $\nu_{R_2}$ or $\nu_{R_3}$, it would depend on a different combination of phases. This assumption is also dubious if the $\nu_R$ are produced thermally, because the $\nu_{R_1}$ mass, $M_1$, is severely constrained in SUSY models. To get a large enough baryon asymmetry, $M_1 > 10^8$ GeV is required [38,42], but $M_1$ must be less than or of order the reheat temperature $T_{reh}$. To avoid overproducing gravitons in the early Universe, $T_{reh}$ is required to be $\lesssim 10^9 - 10^{10}$ GeV [44].

With these approximations, the CP asymmetry is

$$\epsilon_1 \simeq -\frac{3}{8\pi} \left[ \frac{1}{|Y_{\nu} Y_{\nu}^\dagger|_{11}} \sum_j \text{Im} \left\{ [Y_{\nu} Y_{\nu}^\dagger]^2_{1j} \right\} \left( \frac{M_j}{M_1} \right) \right],$$

$$= -\frac{3}{8\pi} \left( \frac{M_1}{|H_u|^2 \left[ Y_{\nu} Y_{\nu}^\dagger \right]_{11}} \right) \text{Im} \left\{ [Y_{\nu} \mathcal{M}_{\nu} \nu_{\nu}^T]_{11} \right\}.$$  

The CP asymmetry depends on quantities that appear in the superpotential of the complete theory, eq.(2), and that are not directly measurable with experiments. However, these quantities can be related to neutrino and sneutrino parameters, as we will discuss in the next section. One of the goals of this paper is to implement in an explicit way these constraints on the CP asymmetry, eq.(18).

4 The see-saw mechanism and leptogenesis: the bottom-up approach

Our starting point will be the procedure presented in [13]. In the basis defined in section 3, where the charged lepton mass matrix and the right-handed Majorana mass
matrix are diagonal, the neutrino Yukawa coupling must be necessarily non-diagonal. However, it can be diagonalized by two unitary transformations:

\[ Y_\nu = V_R^T D_Y V_L. \]  

(19)

It is clear that the CP asymmetry depends just on \( V_R \) and \( D_M \). These quantities are related to the physics of the right-handed neutrinos and are not directly testable by experiments, since they are related to very high energy physics. However, there is a reminiscence of \( V_R \) and \( D_M \) in the low energy neutrino mass matrix that can be exploited to obtain information about the high-energy physics from the neutrino data. Substituting eq.(19) in the see-saw formula, \( M_\nu = m_D^T M^{-1} m_D \), one obtains

\[ D_Y^{-1} V_L^T \frac{M_\nu}{\langle \tilde{H}_0 \rangle^2} V_L^T D_Y - V_R^T D_M^{-1} V_R \equiv M^{-1}. \]  

(20)

From this equation we can solve for \( V_R \) and \( D_M \) in terms of \( M_\nu \), \( D_Y \) and \( V_L \). \( M_\nu \) is constrained by neutrino experiments, whereas \( D_Y = \text{diag}(y_1, y_2, y_3) \) and \( V_L \) are unknown parameters at this stage. We choose a parametrization of the unitary matrix \( V_L \) such that

\[ V_L = \begin{pmatrix} 
-c_{23}^L s_{12}^L e^{i\varphi_{12}} - s_{23}^L c_{13}^L e^{i(\varphi_{13} - \varphi_{23})} & c_{23}^L c_{13}^L e^{-i\varphi_{12}} & s_{23}^L c_{13}^L e^{-i\varphi_{13}} \\
 s_{23}^L s_{12}^L e^{i(\varphi_{12} + \varphi_{23})} - c_{23}^L c_{13}^L e^{i\varphi_{13}} & -c_{23}^L s_{12}^L e^{-i(\varphi_{12} + \varphi_{23})} & -s_{23}^L c_{13}^L e^{-i\varphi_{23}} \\
 s_{12}^L c_{12}^L e^{i\varphi_{12}} - c_{12}^L s_{12}^L e^{-i\varphi_{12}} & -s_{12}^L c_{12}^L e^{-i(\varphi_{12} + \varphi_{13})} & c_{12}^L c_{12}^L e^{i\varphi_{13}} 
\end{pmatrix}, \]  

(21)

where \( c_{ij}^L = \cos \theta_{ij}^L \) and \( s_{ij}^L = \sin \theta_{ij}^L \), being \( \theta_{ij}^L \) the angles in the \( V_L \) matrix.

In certain scenarios, the parameters \( D_Y \) and \( V_L \) can be constrained experimentally. For example, in a scenario of minimal SUGRA, with just the MSSM+3\( \nu_R \)S below the GUT scale, \( D_Y \) and \( V_L \) can in principle be extracted from the radiative corrections to the left-handed slepton mass matrix, since the corresponding RGE depends on the combination \( Y_\nu^T Y_\nu = V_L^T D_Y^2 V_L \). To be more precise, at low energies the left-handed slepton mass matrix reads, in the leading-log approximation

\[ (m_{\tilde{\ell}_i \tilde{\nu}_j}^2)_{ij} \simeq (\text{diagonal part})_{\tilde{\ell}_i \tilde{\nu}_j} + \frac{1}{8\pi^2} (3m_0^2 + A_0^2) Y_{\nu_{i\tilde{k}}} Y_{\nu_{k\tilde{j}}} \log \frac{M_k}{M_{\text{GUT}}}. \]  

(22)

The off-diagonal terms in \( m_{\tilde{\ell}_i \tilde{\nu}_j}^2 \) manifest themselves in processes like \( \mu \rightarrow e\gamma \) or \( \tau \rightarrow \mu\gamma \), that could be observed in the near future. In addition to this, at tree level the three sneutrino masses are degenerate. However, radiative corrections induce a non-universality among the masses that could perhaps be measured experimentally. All these measurements could be used to disentangle some information about the neutrino Yukawa matrix and the right handed masses from radiative corrections. See [47] for a recent analysis of \( \ell_j \rightarrow \ell_i \gamma \) in this approach.

For leptogenesis we are particularly interested in the phases of \( Y_\nu \), that are in turn related to the phases in the left-handed slepton mass matrix. It is then an important
issue to measure the phases in $m_{\tilde{\nu}}^2$. The electric dipole moments (EDMs) of the electron and muon are CP violating observables that could provide information about these phases. However, this CP violation is flavour conserving and could come from another flavour conserving sector of the theory, like the charginos and neutralinos, instead of the Yukawa couplings. Furthermore, the EDMs care about the relative phases between the charged leptons and the sleptons, of which the see-saw only induces one. So, the contribution induced by the see-saw would be suppressed by small angles and Yukawa couplings [48]. Therefore, to constrain the phases in the Yukawa couplings with the EDMs, one has to make certain assumptions about the soft SUSY breaking lagrangian. A more detailed discussion of obtaining information about the complete theory from low energy data can be found in [13].

The $V_L, D_Y$ low-energy parametrization has several advantages. If we treat the 9 parameters of the neutrino mass matrix as “known”, there are 9 remaining unknown variables in the seesaw: three phases and six real numbers. Possible parametrizations of these unknowns are $D_Y$ and $V_L$, $D_M$ and the orthogonal complex matrix $R$ [27], or as in [47]. The angles and phases of $V_L$ are related in a simple way to the lepton flavour violating slepton mass matrix entries. These off-diagonal (in the charged lepton mass eigenstate basis) entries are currently constrained and could possibly be determined by radiative lepton decays $\ell_j \to \ell_i \gamma$. The eigenvalues of $D_Y$ are more difficult to determine experimentally. However, we do measure the Yukawa matrix eigenvalues for the quarks and charged leptons, so we can make theoretical guesses of the $Y_\nu$ eigenvalues with more confidence than e.g. guessing the $\nu_R$ Majorana masses.

It is convenient for our leptogenesis analysis to parametrize the sneutrino mass matrix with $D_Y$ and $V_L$. It would be more correct to express the lepton asymmetry in terms of the magnitude and phases of slepton mass matrix elements [23]. Alternatively, there is an intermediate parametrization, which can be useful for analytic estimates. The parameters we use, $V_L$ and $D_Y$, determine $Y_\nu^T Y_\nu$ rather than $Y_{\nu_{\ell\ell}}^T Y_{\nu_{\ell\ell}} \log \frac{M_k}{M_{GUT}}$, which is the expression that appears at leading log. It is easy, though, to relate $V_L$ and $D_Y$ to $Y_{\nu_{\ell\ell}}^T Y_{\nu_{\ell\ell}} \log \frac{M_k}{M_{GUT}}$. Noting that

$$Y_{\nu_{\ell\ell}}^T \log \frac{M_k}{M_{GUT}} Y_{\nu_{\ell\ell}} = (\tilde{Y}_\nu^T \tilde{Y}_\nu)_{ij} = (\tilde{V}_L^+ D_Y^2 \tilde{V}_L)_{ij}$$

$$\frac{M_{\nu_{\ell\ell}}}{H_u^0} = \tilde{Y}_{\nu_{\ell\ell}}^T \frac{1}{M_k} \tilde{Y}_{\nu_{\ell\ell}},$$

where $\tilde{Y}_{\ell\ell} = Y_{\ell\ell} \sqrt{\log \frac{M_k}{M_{GUT}}}$ and $\tilde{M}_k = M_k \log \frac{M_k}{M_{GUT}}$, it is possible to rewrite eq.(20) but using tilded parameters. So, one could parametrize the see-saw mechanism with the neutrino mass matrix, $M_\nu$, and $\tilde{V}_L, \tilde{D}_Y$, that are directly related to the leading-log approximate solution of the left-handed slepton RGEs. Also, from the definitions, it is straightforward to relate $V_L$ and $D_Y$ with their tilded-counterparts. However, since SUSY has not yet been discovered, we use $V_L$ and $D_Y$, with the knowledge that we can calculate $[m_{\tilde{\nu}}^2]$ from these parameters. This choice will be important when we discuss phase overlaps.

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3We will follow this approach in a subsequent publication [49].
We turn now to expressing the CP asymmetry in terms of neutrino masses, the MNS matrix, and other unknown parameters encoded in $D_Y$ and $V_L$. We can make an analytic approximation indicating the dependence of the CP asymmetry $\epsilon$ on our low energy parameters. To derive these estimates, we first assume $M_3 \gg M_1$ and $y_1 \ll y_2, y_3$. Then we assume that $[M^{-1} M^{-1}]_{11}$ is the largest element of $M^{-1} \bar{M}^{-1}$, in the basis where $Y_\nu$ is diagonal. As we will see, this is usually reasonable.

If a matrix $\Lambda$ has a zero eigenvalue, then the remaining two eigenvalues are

$$\lambda_1, \lambda_2 = \frac{1}{2} \left\{ \text{Tr} \Lambda \pm \sqrt{(\text{Tr} \Lambda)^2 - 4 (\Lambda_{11} \text{tr} \Lambda + \det \Lambda - |\Lambda_{12}|^2 - |\Lambda_{13}|^2)} \right\} , \quad (24)$$

where $\text{Tr}$ is the trace of the 3-d matrix, and $\text{tr}$ and $\det$ are defined on the 2-3 subspace. In the limit where $M_3 \to \infty$, this formula can be applied to the hermitian matrix $M^{-1} \bar{M}^{-1}$:

$$M^{-1} \bar{M}^{-1} = D^{-1}_Y Y \nu V_L^\dagger \bar{D}^{-2}_Y \bar{M} \nu V_L \bar{D}^{-1}_Y \equiv \frac{\Lambda}{y^4_1} . \quad (25)$$

To obtain simple expressions, we would like to expand eq. (24) in small dimensionless parameters. So to avoid confusion, we scale a factor $y^4_1$ out of $M^{-1} \bar{M}^{-1}$.

The largest eigenvalue of $\Lambda$ will be of order $m^2_\nu$, as can be seen by defining $\eta \equiv y_1 D^{-1}_Y Y \nu = \text{diag} \{ \eta_1, \eta_2, \eta_3 \}$ and $\Delta = V_L^* \bar{M} \nu V_L^\dagger$.

$$\Delta = V_L^* \bar{M} \nu V_L^\dagger \equiv W^* D \bar{M} \nu W^\dagger , \quad (26)$$

which gives

$$\Lambda = \frac{\eta \Delta^\dagger \eta^2 \Delta \eta}{\langle H^0 \rangle^4} . \quad (27)$$

We take $y_3 = 1$. The matrix $W = V_L U$ is the rotation from the basis where the $\nu_L$ masses are diagonal to the basis where the neutrino Yukawa matrix $Y^\dagger_\nu Y_\nu$ is diagonal.

The dominant contributions to the matrix elements of $\Lambda$ can be calculated as an expansion in $\eta_2$ and $\eta_3$. Only $\Lambda_{11}$ is zeroth order in $\eta_2$ and $\eta_3$, so generically $\Lambda_{11} \gg \Lambda_{22}, \Lambda_{33}$. Then from eq. (24), the lightest RH neutrino has mass

$$|M_1|^2 \simeq \frac{y^4_1}{\Lambda_{11}} , \quad (28)$$

and the associated eigenvector will be

$$\vec{v}_1 \simeq \begin{pmatrix} \Lambda_{11} \\ \Lambda_{21} \\ \Lambda_{31} \end{pmatrix} \times \frac{1}{\Lambda_{11}} = \begin{pmatrix} \Delta^*_1 \\ \eta_2 \Delta^*_{12} \\ \eta_3 \Delta^*_{13} \end{pmatrix} \times \frac{1}{\Delta^*_1} . \quad (29)$$

We can use this eigenvector to evaluate eq. (18), and find

$$\epsilon \simeq -\frac{3y^2_1 \Lambda^2_{11}}{8\pi [\Lambda D_Y^\dagger \Lambda]_{11}} \text{Im} \left\{ \frac{[\Lambda D_Y \Delta^\dagger D_Y \Lambda^\dagger]_{11}}{[\Lambda \eta \Delta^\dagger \eta \Lambda^\dagger]_{11}} \right\} = -\frac{3y^2_1}{8\pi \sum_j |W_{1j}|^2 m^2_{\nu_j}} \text{Im} \left\{ \sum_k W^2_{1k} m^3_{\nu_k} \right\} \sum_n W^2_{1n} m^2_{\nu_n} , \quad (30)$$

10
where we have dropped terms of order $\eta_2$ and $\eta_3$, and recall that $W$ is the rotation from the basis where the $\nu_L$ masses are diagonal to the basis where $Y^\dagger \nu Y$ is diagonal.

It is important to notice that the CP asymmetry depends only on the first row of the matrix $W$, that in turn depends only on the first row of $V_L$. In the parametrization that we have chosen for $V_L$, eq.(21), the first row depends on two angles and two phases. Therefore, at the end of the day, the CP asymmetry depends on the neutrino mass matrix and five unknown parameters: $y_1$, two angles and two phases. Note that for generic $\Delta$, the order of magnitude of $\epsilon$ is fixed by $y_1^2$. For the GUT-inspired value $y_1 \approx m_u/m_t \sim 10^{-4}$, $\epsilon \sim 10^{-9}$ unless there is some amplification in $\text{Im}\{[\Delta^\dagger \Delta]_{11} \Delta^*_{11}\}/(|[\Delta^\dagger \Delta]_{11}|^2)$.

5 Phases for leptogenesis

From the previous discussion, we find that in the parametrization we have chosen, the CP asymmetry depends on five phases, namely the phases in the MNS matrix, $\delta$, $\phi$ and $\phi'$, and the phases in the first row of the $V_L$ matrix, $\varphi_{12}$ and $\varphi_{13}$. In this section we would like to study the relative importance of these phases on the CP asymmetry, and whether any of them could be considered as the “leptogenesis phase”, i.e. the phase that is fully responsible of the CP asymmetry.

To this end, we first introduce a definition of “overlap” between the “leptogenesis phase” and the low energy phases. At the end of the section, we will discuss issues raised by our definition.

We define the contribution to the CP asymmetry from a phase $\alpha$ (this is not quite what we call a phase overlap) such that the total CP asymmetry is the sum of the different contributions:

$$\epsilon = \epsilon_\delta + \epsilon_\phi + \epsilon_{\phi'} + \epsilon_{\varphi_{12}} + \epsilon_{\varphi_{13}} \ .$$

(31)

To obtain a decomposition of the CP asymmetry in this way, and give a more precise definition of the different contributions, we Fourier expand the CP asymmetry:

$$\epsilon = \sum_{j,k,l,m,n} A_{ijklmn} \sin(j\delta + k\phi + l\phi' + m\varphi_{12} + n\varphi_{13}) \ .$$

(32)

This summation can be split in

$$\epsilon = \sum_{\alpha} C_\alpha + \sum_{\alpha<\beta} C_{\alpha\beta} + \sum_{\alpha<\beta<\gamma} C_{\alpha\beta\gamma} + \sum_{\alpha<\beta<\gamma<\rho} C_{\alpha\beta\gamma\rho} + \sum_{\alpha<\beta<\gamma<\rho<\sigma} C_{\alpha\beta\gamma\rho\sigma} \ ,$$

(33)

with $\{\alpha, \beta, \gamma, \rho, \sigma\}$ elements of the ordered set $\{\delta, \phi, \phi', \varphi_{12}, \varphi_{13}\}$. Note that the subindices of the C’s are ordered, so $\delta < \phi < \phi' < \varphi_{12} < \varphi_{13}$. (to avoid double counting, only $C_{\delta\phi}$ exists, and $C_{\phi\delta}$ does not.) Some of the terms in the summation are:

$$C_\delta = \sum_{j\neq 0} A_{j0000} \sin(j\delta)$$

(34)

$$C_{\delta\phi} = \sum_{j\neq 0, k\neq 0} A_{j0k00} \sin(j\delta + k\phi)$$

(35)
\[ C_{\delta\phi'} = \sum_{j \neq 0, k \neq 0, l \neq 0} A_{jkl00} \sin(j\delta + k\phi + l\phi') \] (36)

and so on.

We can now rewrite the summation eq.(33) in a way that resembles eq.(31). It is clear that \( C_\delta \) is a contribution from \( \delta \) to the CP asymmetry, so it must be one of the terms in \( \epsilon_\delta \). On the other hand, \( C_{\delta\phi} \) is a contribution from \( \delta \), but also from \( \phi \), and it is not possible to conclude whether it is a contribution from \( \delta \) or from \( \phi \). So, we will say that \( C_{\delta\phi} \) contributes in \( C_{\delta\phi}/2 \) to \( \epsilon_\delta \) and in \( C_{\delta\phi}/2 \) to \( \epsilon_\phi \). This rationale can be applied to the rest of the terms in the expansion eq.(33) to finally obtain

\[ \epsilon_\delta = C_\delta + \frac{1}{2} \sum_\beta C_{\delta\beta} + \frac{1}{3} \sum_{\beta<\gamma} C_{\delta\beta\gamma} + \frac{1}{4} \sum_{\beta<\gamma<\rho} C_{\delta\beta\gamma\rho} + \frac{1}{5} \sum_{\beta<\gamma<\rho<\sigma} C_{\delta\beta\gamma\rho\sigma} , \quad (37) \]

and similarly for \( \epsilon_\phi \), \( \epsilon_{\phi'} \), \( \epsilon_{\phi_{12}} \) and \( \epsilon_{\phi_{13}} \). It can be checked that with this decomposition, eq.(31) holds.

In this analysis, we are only concerned with the relative contributions of the different phases to the CP asymmetry, and not with the overall magnitude of the CP asymmetry itself (that is essentially determined by the unknown parameter \( y_1 \)). So, we normalize the different contributions to 1, and define “phase overlap” as the normalized contribution from a phase to the CP asymmetry. This quantity measures the relative importance of that phase for the CP asymmetry compared to the rest of the phases. For instance, the “overlap” of \( \delta \) with the leptogenesis phase is:

\[ O_\delta = \frac{|\epsilon_\delta|}{\sqrt{\sum_\alpha \epsilon_\alpha^2}} , \quad (38) \]

which satisfies \( \sum O_\alpha^2 = 1 \) and \( 0 \leq O_\alpha \leq 1 \). Had we chosen a linear normalization, i.e. \( O_\alpha = \epsilon_\alpha / \epsilon \), in some regions of the parameter space \( |O_\alpha| \) could be larger than one, due to cancellations among the different \( \epsilon_\alpha \)’s. So, we prefer to use a quadratic normalization, that satisfies \( 0 \leq O_\alpha \leq 1 \) to better represent the fractional contribution of the phase \( \alpha \) to \( \epsilon \).

The overlap defined in eq.(38) measures the importance of \( \delta \) for \( \epsilon \), provided that the phases in the expansion are independent and “orthogonal”. In any parametrization of the seesaw, six phases are required, so independence is automatic. The importance of “orthogonality” can be understood by analogy with linear algebra, where a vector can be uniquely decomposed in its components along a given orthonormal basis. Similarly, the phases of our parametrization must be “orthogonal”, as well as independent, to have a unique definition of the fraction of \( \epsilon \) due to \( \delta \). However, a mathematical definition of “orthogonality” is difficult, perhaps impossible, because we do not have an inner product between phases. So we opt for a physical notion: we assume as “orthogonal” the so-called “physical” phases, the phases that could be measured at low energy — in practice, or in principle in the best of all physicists worlds. Notice that the choice of low energy phases is important — had we parametrized with the phases of \( U \) and \( W \), then from eq. (30), \( \epsilon \) depends only on the phases of \( W_{1i} \) and is independent of the MNS phases. Similarly, if the seesaw is parametrized using \( U \) and the complex orthogonal
matrix $R$. The MNS matrix cancels out of the equation for $\epsilon$, and the $\delta$ dependence of $\epsilon$ is buried in $R$.

The measurable phases of the slepton sector are those of the slepton mass matrices, so we should expand $\epsilon$ on $\delta$, $\phi'$, $\phi$, and the phases of $[m_{L}^{2}]$. However, we find it convenient to parametrize the seesaw in terms of $V_{L}$ and $D_{Y}$, rather than $[m_{L}^{2}]$, as discussed in section 4. The $[m_{L}^{2}]$ are therefore functions of the real angles of $V_{L}$, as well as the phases of $V_{L}$, so $\varphi_{12}$ and $\varphi_{13}$ are not quite the correct physical phases. We expect this choice to have little effect on the relative importance of $\delta$ and $\phi'$ for leptogenesis. $V_{L}$ is closely related to the slepton mass matrix, and in the limit that the real angles in $V_{L}$ are small, the phases of $[m_{L}^{2}]$ are those of $V_{L}$ (or $\tilde{V}_{L}$) in leading log.

Finally, this definition of overlap is statistical. $O_{\delta}$ gives the probable importance of $\delta$ for $\epsilon$, assuming all phases are unknown and $O(1)$. But if, for instance, $\delta = 0$, $O_{\delta}$ will be non-zero, because parts of the crossed terms $\{C_{\delta a}, C_{\delta a, b}, \ldots\}$ are included in $O_{\delta}$, and these terms could be non-zero due to the other phases.

To avoid possible confusion, observe that our notion of “leptogenesis phase” differs from the one introduced in [42], who write $\epsilon = \epsilon_{\text{max}} \delta_{\text{eff}}$ where $\epsilon_{\text{max}}$ is the upper bound on $\epsilon$. The asymmetry is the imaginary part of a complex number $\epsilon \equiv \text{Im}\{|\epsilon_{c}| \epsilon^{i\rho}\} = |\epsilon_{c}| \sin \rho$, so we interpret the “leptogenesis phase” to be $\rho$.

The definition, eq. (38), of the fraction of $\epsilon$ that is due to $\delta$, depends on eight unknowns: three real angles and the five phases. We assume that the real angles could be measured, so we present results for different fixed values of the angles. We take random values of the phases, linearly distributed between 0 and $2\pi$, and make scatter plots of the overlaps $\{O_{\alpha}\}$. If most of the points are distributed at $|O_{\alpha}|^{2} > .3$, we conclude that the phase $\alpha$ contributes significantly to $\epsilon$ ($\alpha$ here represents any phase among $\{\delta, \phi', \phi, \phi_{12}, \phi_{13}\}$). Notice however that this is a statistical statement, based on choosing all the phases randomly and large.

6 The case $V_{L} = 1$

In this section we particularize the previous study to the case $V_{L} = 1$. In this case, there is no flavour or CP violation induced radiatively by right-handed neutrinos in the slepton mass matrices. Since we have fixed the $V_{L}$ matrix, the number of unknown parameters is reduced, and the CP asymmetry depends just on the neutrino mass matrix and $y_{1}$, the lightest eigenvalue of $Y_{l}^{\dagger}Y_{\nu}$.

When $V_{L} = 1$, only the phases in the MNS matrix ($\delta$, $\phi$ and $\phi'$) are relevant, hence the analysis of the previous section is greatly simplified. The CP asymmetry can be written as the sum of the contributions from the phases $\delta$, $\phi$ and $\phi'$,

$$\epsilon = \epsilon_{\delta} + \epsilon_{\phi} + \epsilon_{\phi'} .$$

As in the previous section, we Fourier expand $\epsilon$, yielding

$$\epsilon = \sum_{j, k, l} A_{jkl} \sin(j \delta + k \phi + l \phi') = C_{\delta} + C_{\phi} + C_{\phi'} + C_{\delta \phi} + C_{\delta \phi'} + C_{\phi \phi'} + C_{\delta \phi \phi'} ,$$

13
with $C_\alpha$, $C_{\alpha\beta}$ and $C_{\delta\phi\phi'}$ as in eqs.(34)-(36). Then, the contribution from the phases $\delta$, $\phi$ and $\phi'$ to the CP asymmetry read

$$
\epsilon_\delta = C_{\delta} + \frac{1}{2}(C_{\delta\phi} + C_{\delta\phi'}) + \frac{1}{3}C_{\delta\phi\phi'}
$$

$$
\epsilon_\phi = C_{\phi} + \frac{1}{2}(C_{\phi\phi} + C_{\phi\phi'}) + \frac{1}{3}C_{\phi\phi\phi'}
$$

$$
\epsilon_{\phi'} = C_{\phi'} + \frac{1}{2}(C_{\phi'\phi} + C_{\phi'\phi'}) + \frac{1}{3}C_{\phi'\phi\phi'}
$$

(41)

As before, and since we are only concerned with the relative contributions from the different phases to $\epsilon$ and not with the overall magnitude, we define the “phase overlaps” as:

$$
O_\alpha = \frac{|\epsilon_\alpha|}{\sqrt{\epsilon_\delta^2 + \epsilon_\phi^2 + \epsilon_{\phi'}^2}}
$$

(42)

where $\alpha = \delta, \phi, \phi'$. With these definitions, the following identity holds:

$$
(O_\delta)^2 + (O_\phi)^2 + (O_{\phi'})^2 = 1
$$

(43)

In Figure 1 we show the numerical results for different CHOOZ angles. We show the results for the LAMSW solution to the solar neutrino problem and the mass hierarchy $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$. Each point corresponds to a random value of the phases $\delta$, $\phi$ and $\phi'$ between 0 and $2\pi$. In view of eq.(43), we find convenient to present the results on a triangular plot, where the distance to the sides of the triangle corresponds to the “phase overlaps” squared, defined in eq.(42) (see upper left plot).

When the CHOOZ angle is close to the experimental bound (upper right plot) over most of the parameter space the relevant phases are $\delta$ and $\phi'$, and their contributions are approximately equal. In this case, the phase $\phi$ is essentially irrelevant, except for a few points that correspond to $2\delta - \phi' \simeq 0, \pi$. On the other hand, when the CHOOZ angle is moderately small (lower left plot), we find points scattered over the whole triangle: the three phases are relevant in this case. One can also see from the figure that the points seem to follow a circular pattern. We will come back to this issue later on. Finally, when the CHOOZ angle is very small (lower right plot), the relevant phases are $\phi$ and $\phi'$, except for the points for which $\phi - \phi' \simeq 0, \pi$, where the phase $\delta$ becomes relevant. A neutrino factory is expected to be sensitive to sin $\theta_{13} \simeq 10^{-3}$ [22] and to be able to see CP violation for phases of order 1 if $\sin\theta_{13} \gtrsim .01$ [23].

These plots can be understood analytically using the approximation for the CP asymmetry, eq.(30). When $V_L = 1$, the CP asymmetry has a fairly simple expression in terms of low energy neutrino data and $y_1$, the lightest eigenvalue of $Y_L^\dagger Y_{\nu}$:

$$
\epsilon \simeq \frac{3y^2_1}{8\pi D} \text{Im} \left\{ \frac{m^3_{\nu_1} c^2_{13} c^2_{12}}{m_{\nu_1} c^2_{13} c^2_{12}} e^{-i\phi} + m^3_{\nu_2} c^2_{13} s^2_{12} e^{-i\phi'} + m^3_{\nu_3} s^2_{13} e^{-2i\delta} \right\},
$$

(44)

where $D = m^2_{\nu_1} c^2_{13} c^2_{12} + m^2_{\nu_2} c^2_{13} s^2_{12} + m^2_{\nu_3} s^2_{13}$. For the mass hierarchy and the ranges of CHOOZ angles that we are using, it turns out that $m_{\nu_3} \gg m_{\nu_1}, m_{\nu_2}$, so we can
Figure 1: "Phase overlaps" for the case $V_L = 1$, i.e. $\mathbf{Y}_\nu^\dagger \mathbf{Y}_\nu$ diagonal. The upper left plot indicates the meaning of the distances to the different sides of the triangle. The rest of the triangles show density plots of the "phase overlaps" for random values of the phases and different CHOOZ angles: 0.1 (upper right), 0.01 (lower left) and 0.001 (lower right). The darkest regions correspond to the largest density of points.
expand the denominator in $\epsilon$. Approximating $\theta_{12}, \theta_{23} \sim \pi/4$, the result is:

$$\epsilon \simeq -\frac{3y_1^2}{4\pi} \left\{ \left( \frac{m_{\nu_3}}{m_{\nu_2}} \right)^3 2s_{13}^2 \sin(2\delta - \phi') - \frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi - \phi') \right\}. \quad (45)$$

When the CHOOZ angle is much larger than $\sqrt{m_{\nu_1}m_{\nu_2}/m_{\nu_3}}$ the first term in eq.(45) dominates, unless $2\delta - \phi'$ is close to 0 or $\pi$. This condition is satisfied in particular when the lightest neutrino is very light, which is an interesting physical possibility. For the mass hierarchy that we have chosen, the condition above reads $s_{13} \gg 0.01$, which is satisfied when the CHOOZ angle is close to the present experimental limit. Recall that $\theta_{13} \sim 0.1$ is required to detect $\delta$ at a neutrino factory, so this limit would hold if CP violation is observed at the neutrino factory. The CP asymmetry in this case can be approximated by

$$\epsilon \simeq -\frac{3y_1^2}{2\pi} \left( \frac{m_{\nu_1}}{m_{\nu_2}} \right)^3 s_{13}^2 \sin(2\delta - \phi'),$$

that does not depend on $\phi$; only on $\delta$ and $\phi'$. Furthermore, the dependence is such that one cannot conclude whether the "leptogenesis phase" is $\delta$ or $\phi'$. Instead, in this limit, the "leptogenesis phase" is the combination $2\delta - \phi'$. Comparing eq.(46) with the Fourier expansion, eq.(40), it follows that for most points, $\epsilon \simeq C_{\delta\phi'}$. Hence $\epsilon_\phi \simeq 0, \epsilon_\delta \simeq \epsilon_\phi' \simeq C_{\delta\phi'}/2$. ($\epsilon_\delta \simeq \epsilon_\phi'$ is a consequence of the fact that $\epsilon$ depends on a combination of $\delta$ and $\phi'$.) Consequently, most points in the scatter plot, Fig. 1, upper right, are concentrated in the middle of the side corresponding to $O_\phi = 0$.

On the other hand, when the CHOOZ angle is very small, it is the second term in eq.(45) the one that dominates, as long as $(\phi - \phi')$ is different from 0 or $\pi$. In this limit,

$$\epsilon \simeq -\frac{3y_1^2}{4\pi} \frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi - \phi'). \quad (47)$$

The CP asymmetry only depends on $\phi$ and $\phi'$, and the "leptogenesis phase" is $\phi - \phi'$. As before, comparing eq.(47) with the expansion eq.(40), we conclude that for most points, $\epsilon \simeq C_{\phi\phi'}$. Hence, $\epsilon_\delta \simeq 0, \epsilon_\phi \simeq \epsilon_\phi' \simeq C_{\phi\phi'}/2$. In consequence, most points in Fig. 1, lower right, are concentrated in the middle of the side corresponding to $O_\delta = 0$.

Finally, for values of the CHOOZ angle $\sim \sqrt{m_{\nu_1}m_{\nu_2}/m_{\nu_3}}$, both terms in eq.(45) have to be taken into account. In this case, we cannot say that there is a single "leptogenesis phase": both $\delta - 2\phi$ and $\phi - \phi'$ are "leptogenesis phases". Concerning the contributions from the phases $\delta, \phi$ and $\phi'$ to the CP asymmetry, it is apparent from eq.(45) that in a generic point the three contributions are going to be comparable. To be precise:

$$\epsilon_\delta = \frac{1}{2} C_{\delta\phi'}, \quad \epsilon_\phi = \frac{1}{2} C_{\phi\phi'}, \quad \epsilon_\phi' = \frac{1}{2} (C_{\delta\phi'} + C_{\phi\phi'}) \quad (48)$$
where,
\[ C_{\delta\phi'} \simeq \frac{-3y_1^2}{2\pi} \left( \frac{m_{\nu_2}}{m_{\nu_1}} \right)^3 s_{13}^2 \sin(2\delta - \phi') \]
\[ C_{\phi\phi'} \simeq \frac{3y_1^2}{4\pi} \frac{m_{\nu_1}}{m_{\nu_2}} \sin(\phi - \phi') . \]

From these formulas, it is possible to understand the circular pattern that appears in Fig. 1, lower left, changing from triangular coordinates to Cartesian coordinates. We define the Cartesian axes setting the origin at the lower left vertex of the triangle, and we denote as \( x \) (\( y \)) the horizontal (vertical) axis. The change of variables read,
\[ x = \frac{2}{\sqrt{3}} \left[ \frac{O_2^2}{2} + O^2 \right] \]
\[ y = O_2^2 \]
and using that \( \epsilon_{\phi'} \simeq \epsilon_\delta + \epsilon_\phi \), we obtain, after some algebra, \((x - \frac{1}{\sqrt{3}})^2 + (y - \frac{1}{3})^2 \simeq \frac{1}{9}\)
which is the equation of a circle centered in the barycentre of the triangle, with radius \( 1/3 \).

7 The general case

In the general case the number of unknown parameters involved is rather large (five phases, two angles in the \( V_L \) matrix and the CHOOZ angle), so the analysis is much more intricate since many different limits arise. However, we will see that only a few limits are distinct and physically interesting; the rest correspond to small regions in the parameter space that could arise in particular models, but that we will not consider, following the same bottom-up spirit as in the rest of the paper.

The different limits stem from the possible ways to expand the denominator in our approximate expression for the CP asymmetry
\[ \epsilon \simeq -\frac{3y_1^2}{8\pi \sum_n |W_{1n}|^2 m_{\nu_n}^2} \text{Im} \left\{ \sum_i W_{1i}^2 m_{\nu_i}^3 \sum_j \frac{W_{1j} W_{nj}}{m_{\nu_j}} \right\} . \]

The relevant elements in \( W \) for the calculation are
\[ W_{11} \simeq e^{-i\phi/2} \left[ \frac{1}{\sqrt{2}} c_{12}^L c_{13}^L + e^{i\varphi_{12}} s_{12}^L c_{13}^L \left( \frac{1}{2} + \frac{1}{2} e^{i\delta} s_{13} \right) - e^{i\varphi_{13}} s_{13}^L \left( \frac{1}{2} - \frac{1}{2} e^{i\delta} s_{13} \right) \right] \]
\[ W_{12} \simeq e^{-i\phi'/2} \left[ \frac{1}{\sqrt{2}} c_{12}^L c_{13}^L - e^{i\varphi_{12}} s_{12}^L c_{13}^L \left( \frac{1}{2} - \frac{1}{2} e^{i\delta} s_{13} \right) + e^{i\varphi_{13}} s_{13}^L \left( \frac{1}{2} + \frac{1}{2} e^{i\delta} s_{13} \right) \right] \]
\[ W_{13} \simeq e^{-i\delta} c_{12}^L c_{13}^L s_{13} - \frac{1}{\sqrt{2}} e^{i\varphi_{12}} s_{12}^L c_{13}^L - \frac{1}{\sqrt{2}} e^{i\varphi_{13}} s_{13}^L \]

where we have approximated \( \cos \theta_{13} \simeq 1 \) and we have assumed maximal solar and atmospheric mixings. It is apparent from these equations that different limits are going
to arise depending on the mixing angles in $V_L$ and the CHOOZ angle. We find then an interesting interplay between leptogenesis and lepton flavour violation, induced by radiative corrections through $Y_\nu^\dagger Y_\nu = V_L^\dagger D_2^L V_L$. However, from the parametrization we have chosen for $V_L$, eq.(21), one realizes that the off-diagonal elements of $Y_\nu^\dagger Y_\nu$ depend also on $\theta_{23}^L$, that does not play any role in the CP asymmetry generated in the decay of the lightest right-handed neutrino $^4$.

We obtain simple analytic expressions when the denominator of eq. (50) can be expanded in small parameters. In the whole of section 7, we assume $|W_{12}| > |W_{11}| \sqrt{m_{\nu_1}/m_{\nu_2}}$, so we can neglect $m_{\nu_1}$ terms in the denominator. When $|W_{13}|^2 < |W_{12}|^2 m_{\nu_2}^2/m_{\nu_3}^2$, which will be the case if $\theta_{12}^L$, $\theta_{13}^L$, $\theta_{13} < .1$, $\epsilon$ can be approximated by

$$\epsilon \simeq \frac{3y_i^2}{8\pi|W_{12}|^2} \text{Im} \left[ \frac{m_{\nu_3}}{m_{\nu_2}} \frac{3}{W_{13}^2} \left( \frac{W_{12}^2}{W_{12}^2} - \frac{W_{11}^2}{W_{12}^2} \right) \right]. \quad (52)$$

On the other hand, when the mixing in $V_L$ is large, in the sense that $\sin \theta_{12}^L$ or $\sin \theta_{13}^L$ is larger than $\sim 0.1$, then $|W_{13}|^2 > |W_{12}|^2 m_{\nu_2}^2/m_{\nu_3}^2$, and the CP asymmetry reads

$$\epsilon \simeq - \frac{3y_i^2}{8\pi|W_{13}|^2} \left( \frac{m_{\nu_2}}{m_{\nu_3}} \right) \text{Im} \left[ \frac{W_{12}^2}{W_{13}^2} \right]. \quad (53)$$

There is also an intermediate case, between these limits, where $|W_{13}|^2 m_{\nu_2}^2/m_{\nu_3}^2 < |W_{12}|^2 < |W_{13}|^2 m_{\nu_2}^2/m_{\nu_3}^2$. We do not discuss this, because $m_{\nu_3}/m_{\nu_2} \sim 10$ for the hierarchical LMA solution we consider.

Let us analyze the two cases separately.

### 7.1 $|W_{11}| \sim |W_{12}| \gg |W_{13}| m_{\nu_3}/m_{\nu_2}$

The analysis for this case is parallel to the one we performed in the previous section for the case $V_L = 1$, where $|U_{11}| \sim |U_{12}| \gg |U_{13}| m_{\nu_2}/m_{\nu_3}$. Using that $m_{\nu_2} \gg m_{\nu_1}$, $m_{\nu_3}$, $s_i s_j$, where $s_i$ is any of $s_{13}$, $s_{12}^L$, $s_{13}^L$, we can expand eq.(52), keeping the leading order terms in the expansion. The result is:

$$\epsilon \simeq - \frac{3y_i^2}{4\pi} \left\{ \left( \frac{m_{\nu_3}}{m_{\nu_2}} \right)^3 \left[ 2s_{13}^L \sin(2\phi - \phi') - (s_{12}^L)^2 \sin(2\varphi_{12} + \phi') - (s_{13}^L)^2 \sin(2\varphi_{13} + \phi') \right. \\
- 2\sqrt{2} s_{13} s_{12}^L \sin(\delta - \varphi_{12} - \phi') - 2\sqrt{2} s_{13} s_{12}^L \sin(\delta - \varphi_{13} - \phi') - 2s_{12}^L s_{13}^L \sin(\varphi_{12} + \varphi_{13} + \phi') \right] \\
- \left( \frac{m_{\nu_1}}{m_{\nu_2}} \right) \sin(\phi - \phi') \right\}. \quad (54)$$

Obviously, in the limit $s_{12}^L, s_{13}^L \to 0$ we recover eq.(45). In the case $V_L = 1$ we found different limits, depending on the value of the CHOOZ angle. Now, the role of the CHOOZ

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$^4$It is important, though, for the computation of the CP asymmetry generated in the decay of the heavier right-handed neutrinos, that could be relevant, or even dominant, in some scenarios (particularly in scenarios with non-thermal creation of right handed neutrinos). Research along this lines would be certainly interesting, since in this case lepton flavour violation could be intimately related with leptogenesis.
angle is played by the angles $\theta_{12}^L$, $\theta_{13}^L$ and the CHOOZ angle itself, and the results are different depending on the values of these angles compared with $\sqrt{m_{\nu_1} m_{\nu_2} / m_{\nu_3}}$.

- When any of the angles $s_{13}$, $s_{12}^L$ or $s_{13}^L$ is much larger than $\sqrt{m_{\nu_1} m_{\nu_2} / m_{\nu_3}}$, the term proportional to $(m_{\nu_3} / m_{\nu_2})^3$ in eq.(54) dominates:

$$\epsilon \simeq -\frac{3y_1^2}{4\pi} \left( \frac{m_{\nu_3}}{m_{\nu_2}} \right)^3 \left[ 2s_{13}^2 \sin(2\delta - \phi') - (s_{12}^L)^2 \sin(2\varphi_{12} + \phi') - (s_{13}^L)^2 \sin(2\varphi_{13} + \phi') 
- 2\sqrt{2}s_{13} s_{12}^L \sin(\delta - \varphi_{12} - \phi') - 2\sqrt{2}s_{13} s_{12}^L \sin(\delta - \varphi_{13} - \phi') - 2s_{12}^L s_{13}^L \sin(\varphi_{12} + \varphi_{13} + \phi') \right] .$$

We recall here that this limit corresponds to the case where the lightest neutrino mass is very small. On the other hand, for the mass hierarchy that we are using as reference to present our numerical results, $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$, this limit arises when any of the angles is much larger than $\sim 0.01$, in particular, when the CHOOZ angle is close to the experimental bound and the relevant angles in $V_L$ are comparable to or smaller than the CHOOZ angle.

When the three angles are comparable in size, we see that there are three "leptogenesis phases": $2\delta - \phi'$, $2\varphi_{12} + \phi'$ and $2\varphi_{13} + \phi'$ (the arguments of the sines in the last three terms of eq.(55) are combinations of these). Notice that in this limit $\phi'$ is an important phase for leptogenesis, although it cannot be regarded as the "leptogenesis phase", since the actual "leptogenesis phases" are combinations of $\phi'$ with other phases. However, an indication for a non-vanishing $\phi'$, coming for example from experiments on neutrinoless double beta decay, would provide an indication for leptogenesis.

If there are two angles that are comparable, while the third is much smaller than the others, then there are two "leptogenesis phases". To understand better the results for this limit, we analyze in some detail the case $s_{13} \simeq s_{12}^L \gg s_{13}^L$. If $s_{13}^L$ is also small, this case would produce small rates for $\mu \rightarrow e\gamma$, as can be checked from eq.(21). The results for the other possibilities, $s_{13} \simeq s_{13}^L \gg s_{12}^L$ and $s_{12} \simeq s_{13}^L \gg s_{13}$, can be easily deduced from this analysis, making the appropriate substitutions. We have computed numerically the different contributions to the CP asymmetry for the choice of angles $s_{13} = s_{12}^L = 0.03$, $s_{13}^L = 0$, the mass hierarchy $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$ and assigning random values, between 0 and 2$\pi$, to the phases. We obtain that for most of the parameter space, the only non-vanishing contributions to the CP asymmetry are $\epsilon_\delta$, $\epsilon_{\varphi_{12}}$ and $\epsilon_{\phi'}$ ($\varphi_{13}$ does not play any role, because we have set $s_{13}^L$ to 0). Since there are essentially only three contributions involved, a convenient way of presenting the results is using a triangular plot. In Fig.2, left, we explain how to interpret the distances to the different sides of the triangle, whereas in Fig.2, right, we show the numerical results of the calculation. We find that in general the three contributions are comparable, although the contribution from $\phi'$ is slightly larger than the other two. This can be understood from the analytical approximation, eq.(55), setting $s_{13}^L = 0$. The different contributions to the CP asymmetry are:

$$\epsilon_\delta = \frac{1}{2} C_{\delta \phi'} + \frac{1}{3} C_{\delta \varphi_{12}}$$

$$\epsilon_{\phi'} = \frac{1}{2} (C_{\phi' \varphi_{12}} + C_{\phi' \varphi_{12}}) + \frac{1}{3} C_{\delta \phi' \varphi_{12}}$$

(56)
Figure 2: The same as Fig.1, for the case where $s_{13} = s_{L12}^L = 0.03$, $s_{L13}^L = 0$. The left plot indicates how to interpret the distances to the different sides of the triangle, and the right plot shows a density plot of the “phase overlaps” for random values of the phases. The darkest regions correspond to the largest density of points.

$$
\epsilon_{\varphi_{12}} = \frac{1}{2} C_{\varphi'_{12}} + \frac{1}{3} C_{\delta \varphi'_{12}},
$$

where,

$$
C_{\delta \varphi'} \simeq -\frac{3y_1^2}{2\pi} \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}^2 \sin(2\delta - \varphi')
$$

$$
C_{\varphi'_{12}} \simeq \frac{3y_1^2}{4\pi} \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 (s_{12}^L)^2 \sin(2\varphi_{12} + \varphi')
$$

$$
C_{\delta \varphi'_{12}} \simeq -\frac{3y_1^2}{\sqrt{2}\pi} \left(\frac{m_{\nu_3}}{m_{\nu_2}}\right)^3 s_{13}s_{12}^L \sin(\delta - \varphi_{12} - \varphi')
$$

that are in general comparable.

Finally, if one of the angles dominates over the others, the conclusions are very similar as for the case $V_L = 1$, where the CP asymmetry received contributions from $\varphi'$ and $\delta$. Here, the role of $\delta$ is played by the phase corresponding to the angle that dominates ($\delta$ for $s_{13}$, $\varphi_{12}$ for $s_{L12}^L$, and $\varphi_{13}$ for $s_{L13}^L$). In this case, $\epsilon \simeq C_{x\varphi'}$, where $x$ is the relevant angle among $\delta$, $\varphi_{12}$ and $\varphi_{13}$. On the other hand, the normalized contributions are $O_x \simeq O_{\varphi'} \simeq 1/\sqrt{2}$, while they are vanishing for the rest of the phases. For example, if $s_{12}^L \gg s_{13}, s_{L13}^L$, then $\epsilon \simeq C_{\varphi'_{12}}$ and $O_{\varphi_{12}} \simeq O_{\varphi'} \simeq 1/\sqrt{2}$. 

20
When all the angles \((s_{13}, s_{12}^L, s_{13}^L)\) are much smaller than \(\sqrt{m_\nu_1 m_\nu_2} / m_\nu_3\), the term proportional to \(m_\nu_1 / m_\nu_2\) dominates in eq.(54) and the CP asymmetry reads

\[
\epsilon \simeq \frac{3y_1^2}{4\pi} \left( \frac{m_\nu_1}{m_\nu_2} \right) \sin(\phi - \phi').
\]  

In this limit, the results are identical as in the corresponding limit in the case \(V_L = 1\), and there is a single “leptogenesis phase”, \(\phi - \phi'\). So, the normalized contributions to the CP asymmetry from \(\phi\) and \(\phi'\) are equal to \(1/\sqrt{2}\), while the contributions from the rest of the phases vanish. The numerical analysis yield a plot that is very similar to Fig.1, lower right, where the role of \(O_\delta\) is played by either \(O_{\varphi_{12}}, O_{\varphi_{13}}\) or \(O_\delta\).

Lastly, in the situations where the two terms in eq.(54) are comparable, the analysis is very involved, since in principle there are four independent “leptogenesis phases”, namely, \(2\delta - \phi', 2\varphi_{12} - \phi', 2\varphi_{13} - \phi'\) and \(\phi - \phi'\). So, the CP asymmetry receives contributions from the five phases, and in general they are comparable in size. Hence, it is very difficult to extract any general conclusion for this case.

### 7.2 \(|W_{12}|^2 m_\nu_2 / m_\nu_3 < |W_{13}|^2\)

For simplicity, and since the number of phases and angles involved is rather large, we will set one of the angles in \(V_L\) equal to zero, say \(s_{13}^L = 0\), so the phase \(\varphi_{13}\) becomes irrelevant. Since \(s_{12}^L\) and \(s_{13}^L\) appear in a similar way in the formulas, one can qualitatively derive the result when \(s_{13}^L\) is different from zero. With this choice, we are left with only two angles, the CHOOZ angle, \(s_{13}\), and one angle in \(V_L\), \(s_{12}^L\). The limit we are studying in this section requires \(s_{12}^L\) larger than \(~0.1\). Then, using the experimental bound on the CHOOZ angle and that our phases are generically of order 1, the denominator can be expanded as

\[
\frac{1}{|W_{13}|^2} \simeq \left( \frac{2}{s_{12}^L} \right)^2 \left( 1 + 2\sqrt{\frac{s_{12}^L}{s_{12}}} s_{13} \cos(\delta + \varphi_{12}) \right).
\]  

Hence, the CP asymmetry can be approximated by

\[
\epsilon \simeq -\frac{3y_1^2 (c_{12}^L)^3}{4\pi (s_{12}^L)^5} \left\{ \left( \frac{s_{12}^L}{c_{12}^L} \right)^2 \left[ -2\sin(2\varphi_{12} + \phi') + 2\sqrt{2} \left( \frac{s_{12}^L}{c_{12}^L} \right) \sin(\varphi_{12} + \phi') - \left( \frac{s_{12}^L}{c_{12}^L} \right)^2 \sin \phi' \right] \right.
\]
\[+ s_{13} \left[ 2\sqrt{2}[\sin(\delta - \varphi_{12} - \phi') - 3\sin(\delta + 3\varphi_{12} + \phi')] \right.
\[+ 4 \left( \frac{s_{12}^L}{c_{12}^L} \right)[\sin(\delta - \phi') - 3\sin(\delta + 2\varphi_{12} + \phi')] + \sqrt{2} \left( \frac{s_{12}^L}{c_{12}^L} \right)^2 [2\sin(\delta - \varphi_{12} - \phi')
\[+ \sin(\delta + \varphi_{12} - \phi') - 3\sin(\delta + \varphi_{12} + \phi')] - 2 \left( \frac{s_{12}^L}{c_{12}^L} \right)^3 \sin(\delta - \phi') \left. \right]\}.
\]

This expression is rather cumbersome and it is difficult to extract information from it. It is not possible in general to identify the “leptogenesis phase”, although it is
clear that leptogenesis depends mainly on $\phi'$ and $\varphi_{12}$, whereas the dependence on $\delta$ is weaker.

In Fig. 3 we show the numerical results for this case. As usual, we show a triangle with the meaning of the distances to the different sides (upper left plot), and density plots of the “phase overlaps” for the mass hierarchy $m_{\nu_1} : m_{\nu_2} : m_{\nu_3} = 10^{-2} : 0.1 : 1$, taking random values for the phases between 0 and $2\pi$. In the upper right (lower left) plot we show the results for $\tan \theta_{12}^L = 0.5$ (1) and $s_{13} = 0.1$. In both plots, the points are concentrated close to the base of the triangle (that corresponds to $O_3$ small), due to the small value of the CHOOZ angle. In the plot corresponding to $\tan \theta_{12}^L = 0.5$ the points are concentrated around the center of the base, whereas for $\tan \theta_{12}^L = 1$, they are spread all over the base. This can be understood from the dependence of $\epsilon$ on $\cot \theta_{12}^L$. For $\tan \theta_{12}^L = 0.5$, the terms with both $\varphi_{12}$ and $\phi'$ are the dominant ones, so $C_{\phi'\varphi_{12}}$ is the largest contribution to the CP asymmetry. On the other hand, when $\tan \theta_{12}^L = 1$ these terms are comparable to the one with $\sin \phi'$, so $\epsilon$ is dominated by $C_{\phi'\varphi_{12}}$ and $C_{\phi'}$. Depending on the value of $\phi'$ the points spread along the basis of the triangle. In the lower right plot we show the numerical results for $\tan \theta_{12}^L = 0.5$ and $s_{13} = 0.01$. The plot is similar to the one with $s_{13} = 0.1$ but with an even smaller value of $O_3$.

8 Summary and Discussion

If neutrino masses are due to the seesaw mechanism, then the heavy right-handed neutrinos can generate a lepton asymmetry in the early Universe when they decay out of equilibrium, if they have CP violating couplings. Such complex couplings in the high energy parameters could induce three phases in the light neutrino sector, called $\phi$, $\phi'$ and $\delta$ (these are the phases that appear in the MNS matrix; see eqs. (7) and (8)).

Upcoming experiments may be sensitive to two of these phases: the Dirac phase $\delta$ could be measured at a neutrino factory, whereas the Majorana phase $\phi'$ might have some observable effects in neutrinoless double beta decay. In this paper, we are interested in the relative importance of the phases $\phi'$ and $\delta$ for leptogenesis.

To address this issue, we use a parametrization of the seesaw in terms of weak scale variables: the light neutrino masses, the MNS matrix, the eigenvalues of the neutrino Yukawa matrix, and a unitary matrix $V_L$. We assume a hierarchical light neutrino spectrum, with the lightest neutrino mass of order $m_{\nu_3}/100$, and an MNS matrix that corresponds to the LAMSW solution to the solar neutrino problem. The matrix $V_L$ is related to the off-diagonal (lepton flavour violating) elements of the slepton mass matrix, and contains three phases, two of which ($\varphi_{12}, \varphi_{13}$) are relevant for our calculation. It is important to use a parametrization in terms of “physical” weak scale phases; this is discussed in section 5.

In the parameter space we are interested in, we find a simple analytic approximation for the lepton asymmetry $\epsilon_1$, produced in the decay of the lightest right-handed neutrino $\nu_{R1}$:

$$\epsilon_1 \simeq \frac{3y_1^2}{8\pi} \sum_j |W_{1j}|^2 \left\{ \frac{\text{Im} \left\{ \sum_k W_{1k}^2 m_{\nu_k} \right\}}{\sum_k W_{1k}^2 m_{\nu_k}} \right\} \left( \sum_n W_{1n}^2 m_{\nu_n} \right)$$

(see eq. (30)). In this equation, $y_1$ is the smallest eigenvalue of the neutrino Yukawa
Figure 3: The same as fig.1 for different situations where $|W_{12}|^2 m_{\nu_2}/m_{\nu_3} < |W_{13}|^2$. The upper left plot indicates the meaning of the distances to the different sides of the triangle. The rest of the triangles show density plots of the “phase overlaps” for random values of the phases, $s_{13}^L = 0$ and $\tan \theta_{12}^L = 0.5$, $s_{13} = 0.1$ (upper right), $\tan \theta_{12}^L = 1$, $s_{13} = 0.1$ (lower left), and $\tan \theta_{12}^L = 0.5$, $s_{13} = 0.01$ (lower right). The darkest regions correspond to the largest density of points. (See sect. 7.2 for details.)
matrix, and $W$ is the unitary transformation from the basis where the $\nu_L$ mass matrix is diagonal to the basis where $Y^\dagger \nu Y$ is diagonal: $W_{1n} = [V_L]_{1m}[U_{mn}$ where $V_L$ and $U$ are defined in eqs. (7), (8) and (21).

We are interested in the relative importance of the phases $\phi'$ and $\delta$ for $\epsilon$. That is, we do not discuss whether we get $\epsilon$ large enough, which is essentially controlled by real parameters, such as $y_1$. We assume that the observed baryon asymmetry is produced in the out of equilibrium decay of $\nu_{R1}$ and study how important could be $\delta$ or $\phi'$ for the CP asymmetry. In other words, if we suppose that $\epsilon$ is of the correct size, what fraction of $\epsilon$ is due to $\delta$ or $\phi'$?

We Fourier expand $\epsilon$ on the five relevant phases of our low-energy parametrization

$$\epsilon = \sum_{j,k,l,m,n} A_{jklmn} \sin(j\delta + k\phi + l\phi' + m\varphi_{12} + n\varphi_{13})$$

and then divide the sum into five components, one due to each phase. In $\epsilon_\delta$, which is the component due to $\delta$, we put all the terms from the Fourier expansion that are $\propto \sin(j\delta)$. We define $C_{\delta\alpha}$ to be the sum of all the terms $\propto \sin(j\delta + n\alpha)$, and divide it equally between $\epsilon_\delta$ and $\epsilon_\alpha$ — that is, we add $\frac{1}{5} \sum_\alpha C_{\delta\alpha}$ to $\epsilon_\delta$. We also add the terms $\propto \sin(j\delta + n\alpha + m\beta)$, multiplied by 1/3, and so on ($\alpha$ and $\beta$ are one of $\{\phi, \phi', \varphi_{12}, \varphi_{13}\}$). This procedure is described in more detail in section 5, and the formula for $\epsilon_\delta$ can be found in eq.(37). Then we define a normalized “fraction of $\epsilon$ due to $\delta$”, which we call the overlap between the leptogenesis phase and $\delta$, as

$$O_\delta = \frac{|\epsilon_\delta|}{\sqrt{\sum_\alpha \epsilon^2_\alpha}}.$$  

The magnitude of $O_\delta$ or $O_{\phi'}$ depends on five phases and three unknown real parameters: the CHOOZ angle, $\theta_{13}$, and two angles from $V_L$, that is related to radiative decays. In the numerical calculation, we fix the hierarchy of light neutrino masses, the angles and assign random values (linearly distributed) to the phases between 0 and $2\pi$. The numerical results are shown in density plots in the $O_\delta - O_{\phi'}$ space.

We present results for two representative cases. In section 6 we discuss the $V_L = 1$ case, where the three relevant phases are $\delta$, $\phi'$ and $\phi$ (the phases of the MNS matrix), and in Section 7 we allow for two non-zero angles in $V_L$. For the sake of clarity in the presentation, in Section 7 we analyze simplified scenarios to reduce the number of phases involved to three. Since the definition of overlap satisfies the identity $\sum O^2_\alpha = 1$, a convenient way of showing the results is by using a triangular plot, where the distance to each side of the triangle corresponds to $O^2_\alpha$.

The $V_L = 1$ model should be a good approximation when the angles in $V_L$ are smaller than the CHOOZ angle. It can be checked from eqs.(51) and (61) that the Dirac phases $\delta$, $\varphi_{12}$ and $\varphi_{13}$ appear in $\epsilon$ multiplied by the sine of a real angle ($\theta_{13}$, $\theta_{12}^L$ and $\theta_{13}^L$, respectively). If $\theta_{12}^L$ and $\theta_{13}^L$ are much smaller than the CHOOZ angle $\theta_{13}$, then $\varphi_{12}$ and $\varphi_{13}$ are less important for $\epsilon$ than $\delta$.

In Section 7 we analyze the situation where there are angles in $V_L$ larger than the CHOOZ angle. The angles of $V_L$ are related to the branching ratios for $\ell_j \to \ell_i \gamma$, as discussed after eq. (22) : $BR(\ell_j \to \ell_i \gamma) \propto y^2_k |[V_L]_{kj}|^2 |[V_L]_{ki}|^2$. Current limits on
\( \tau \to \mu \gamma, \ \tau \to e \gamma, \ \text{and} \ \mu \to e \gamma \) are satisfied if all angles \( \theta^L_{ij} \lesssim .1 \). However, present bounds and anticipated improvements on all three branching ratios can be satisfied if e.g. \( \theta^L_{i3}, \theta^L_{23} \approx 0 \) and \( \theta^L_{12} \sim 1 \). So it is phenomenologically possible to have at least one large angle. The associated phase could then be important for leptogenesis; this possibility is studied in that section.

The approximate expression for \( \epsilon \) given in eq.(61), shows that for generic \( W_{ij} \), \( \epsilon \propto \text{Im}(W_{ij}W_{13})^2 \). That is, the terms proportional to \( m_{\nu_1} \) can be neglected when \( W_{12} > W_{11}\sqrt{m_{\nu_1}/m_{\nu_2}} \) and \( W_{13} > W_{11}\sqrt{m_{\nu_1}m_{\nu_2}^2/m_{\nu_3}^2} \), as discussed in section 7. Recall that \( W_{13} \) contains terms \( \sim \sin \theta_{13}, \sin \theta_{12}, \sin \theta_{13} \). We set \( \sin \theta_{13} \) to zero, to ensure that \( \ell_j \to \ell_i \gamma \) constraints are satisfied, and because the functional dependence of \( \epsilon \) on \( \sin \theta_{12} \) and \( \sin \theta_{13} \) is similar. So the case studied in that section has three phases: \( \delta, \phi' \) and \( \varphi_{12} \).

The CP asymmetry \( \epsilon \) usually depends on the interference between at least two phases. Our results can be divided into three representative cases, according to the neutrino masses and low energy mixing angles. The phases relevant to leptogenesis can be 1) the light majorana phases \( \phi \) and \( \phi' \), or 2) the neutrino factory phase \( \delta \) and \( \phi' \), or 3) \( \phi' \) and phase(s) from the slepton mass matrix. The first two cases were discussed in section 6, and the third in section 7. We outline here the observational consequences of each case.

The majorana phases \( \phi \) and \( \phi' \) are the relevant phases for leptogenesis (equivalently \( O_\phi \) and \( O_{\phi'} \) are large) when

\[
\theta_{13}, \ \theta^L_{ij} \lesssim \sqrt{\frac{m_{\nu_2}m_{\nu_1}}{m_{\nu_3}^3}}, \ \ (O_\phi, O_{\phi'} \ \text{large}) \tag{64}
\]

where \( j = 1, 2 \). For \( m_{\nu_1} \sim .01m_{\nu_3} \), \( \) these conditions imply a CHOOZ angle \( \theta_{13} < .01 \), \( BR(\tau \to e \gamma) \lesssim 10^{-10} \) and \( BR(\mu \to e \gamma) \lesssim 10^{-4}BR(\tau \to \mu \gamma) \). So if this scenario is realised, then with forseeable sensitivities, \( \tau \to \mu \gamma \) and \( \mu \to e \gamma \) could be observed, but \( \theta_{13} \) and \( \tau \to e \gamma \) will not be seen. If \( m_{\nu_1} \) was measured in 0\( \nu_2 \beta \) decay, then sufficient conditions (eqn 64) could be determined: knowing \( m_{\nu_1} \) fixes how small \( \theta_{13} \) and \( \theta^L_{ij} \) must be for their associated phases to be irrelevant.

Notice that \( \tau \to \mu \gamma \) has little impact on the importance of \( \delta \) for leptogenesis. This is because it is mostly related to \( \theta^L_{23} \), the angle from \( V_L \) which does not affect \( \epsilon \) in our parametrisation.

A second possibility, where \( \delta \) would be important for leptogenesis, arises if

\[
\theta^L_{ij}, \sqrt{\frac{m_{\nu_2}^2m_{\nu_1}}{m_{\nu_3}^3}} \ll \theta_{13} \ \ , \ \ (O_\delta, O_{\phi'} \ \text{large}) \tag{65}
\]

\( ^5 \text{this is because } \nu_3 \approx (\nu_e + \nu_\mu)/\sqrt{2} \)

\( ^6 \text{We use } m_{\nu_1} = .01m_{\nu_3} \text{ to estimate numerical upper bounds on the angles. The parameter space } \text{where this scenario obtains shrinks down as } m_{\nu_1} \text{ decreases.} \)

\( ^7 \text{In this discussion, we approximate (1) } BR(\ell_j \to \ell_i \gamma) \approx 10^{-7}y^2_\ell V_{Lki}V_{Lkj}^* (\frac{\tan \beta}{10})^2 \left( \frac{300 \text{GeV}}{m_{\nu_{mass}}} \right)^4, \text{ so these estimates are for flavour only and should be taken with large crystals of salt.} \)
This in the interesting case for the neutrino factory; it would arise for \( \theta_{13} > 0.1 \sqrt{\frac{m_{\nu_3}}{0.01m_{\nu_3}}} \) and \( \theta_{12}^L < \theta_{13} \). For \( m_{\nu_1} \sim 0.01m_{\nu_2} \), it corresponds to a CHOOZ angle accessible to the neutrino factory, \( \tau \rightarrow e\gamma \) below anticipated sensitivities \( \text{BR}(\tau \rightarrow e\gamma) < 10^{-9} \), and \( \text{BR}(\mu \rightarrow e\gamma) \lesssim \theta_{13}^2 \frac{1}{2} \text{BR}(\tau \rightarrow \mu\gamma) \). Case 2 (eq 65) corresponds to \( O_3 \) large, and therefore \( \tau \rightarrow e\gamma \) unobservably small.

A final possibility is that the phases of the slepton mass matrix are more important for leptogenesis than \( \delta \). This corresponds to

\[
\theta_{13}, \sqrt{\frac{m_{\nu_2}^2 m_{\nu_1}}{m_{\nu_3}^3}} < \theta_{1j}^L \ , \ (O_{\phi_{1j}}, O_{\phi'} \text{ large}) \quad (66)
\]

An example of this case would be \( \theta_{13} \sim 0.01 \)—so detectable at the neutrino factory, and \( \sin \theta_{13}^L \sim 1/\sqrt{5} \)—which would imply \( BR(\tau \rightarrow e\gamma) \sim 10^{-8} \). This example is plotted in figure 3 (if \{\theta_{13}, \phi_{13}\} \text{ and } \{\theta_{12}^L, \phi_{12}^L\} \text{ are interchanged}). If \( \tau \rightarrow e\gamma \) is observed, then we are in case 3, \( O_{\phi_{1j}} > O_3 \) and the slepton phases are probably more important for leptogenesis than \( \delta \). Notice however, that lepton flavour violating processes could be small even in this case, where \( \delta \) is not important for leptogenesis. The 1-2 angle of \( V_L \) can be large, without inducing observable \( \ell_j \rightarrow \ell_i\gamma \) rates, because it induces lepton flavour violating slepton masses proportional to \( y_{12}'^2 \). Such simultaneously small lepton flavour violation and \( O_3 \) could be disfavoured by model-building, because it requires \( \theta_{13}^L \ll \theta_{12}^L \).

To summarise these three cases: we find that the neutrino factory phase \( \delta \) can be important for leptogenesis. When the CHOOZ angle is large enough to detect CP violation at neutrino factories (which requires \( \theta_{13} \gtrsim 0.1 \)), and the lepton flavour violating branching ratios are vanishingly small \( ( \theta_{12}^L, \theta_{13}^L \lesssim \theta_{13} ) \), then \( \delta \) contributes significantly to the leptogenesis phase \( (|O_4|^2 \gtrsim 0.3) \). This can be seen from our low-energy approximation to \( \epsilon \), eq. (61): if \( W_{13} \) is not too small, \( \epsilon \) depends on the phase difference between \( W_{12} \) and \( W_{13} \), and if \( \theta_{13} \gtrsim \theta_{12}^L, \theta_{13}^L \), the phase of \( W_{13} \) is \( \delta \). Although \( \delta \) appears always suppressed by the CHOOZ angle, which is small, it plays an important role, because \( W_{13} \) is multiplied by the largest neutrino mass \( m_{\nu_3} \). The phase \( \delta \) is unlikely to be important for leptogenesis if either the CHOOZ angle is small \( (\theta_{13} < 0.01 \text{—see fig 1}) \), or if the \( \tau \rightarrow e\gamma \) or \( \mu \rightarrow e\gamma \) branching ratios are large \( (\theta_{12}^L, \theta_{13}^L > \theta_{13} \text{—see fig 3}) \).

We find that the Majorana phase \( \phi' \) is (almost) always important for leptogenesis. For instance, the fraction of \( \epsilon \) that is due to \( \phi', O_{\phi'} \), is significant in all the cases we have studied. Algebraically, the reason is that the main contribution to \( \epsilon \) in eq.(61) is generically proportional to \( W_{12} \), that is proportional to \( e^{i\phi'} \), unless some cancellations occur. The contribution from any of the other phases can be suppressed by sending a small parameter to zero. The Majorana phase of \( m_{\nu_1}, \phi \), becomes unimportant as \( m_{\nu_1} \rightarrow 0 \), and the three Dirac phases \( \delta, \phi_{12} \text{ and } \phi_{13} \) multiply angles which are positively small \( (s_{13}) \) or believed to be small \( (s_{12}^L, s_{13}^L) \). The contribution of \( \phi' \), on the other hand, is consistently significant.

It is interesting to study how closely related are the Majorana phases of the light neutrinos to the Majorana phases of the heavy right-handed neutrinos. It is well known that when neutrinos have Majorana masses, there is CP violation even in the
two generation model. This suggests that the Majorana phases of the $\nu_R$ sector could be more important for leptogenesis than the Dirac phase, because they can contribute to the CP asymmetry $\epsilon$ suppressed by mixing between only two of the $\nu_R$, rather than mixing among the three $\nu_R$, as required for the Dirac phase. However, there is no symmetry-based distinction between Majorana phases and Dirac phases. The high scale Majorana phases are functions of all the weak scale parameters—the real ones as well as all the Dirac and Majorana phases. So the reason $\phi'$ is important is not that low energy Majorana phases determine the high scale Majorana phases. One can check from the formulae in section 4 that $\phi'$ is usually significant because it multiples a not-very-small mass, rather than a (possibly) small mixing angle.

We do not find a simple correlation between the sign of low energy phases and the sign of the CP asymmetry $\epsilon$. Such a correlation would be interesting, and exists in certain models [45]. However, in our bottom-up approach, $\epsilon$ is usually proportional to phase differences ($\epsilon \sim \sin(j\alpha + n\beta)$), as can be seen from eq. (61) and the various limiting cases discussed in sections 6 and 7.

In summary, we have studied the relative importance of low energy phases for leptogenesis. Using a parametrization of the seesaw mechanism in terms of weak scale variables, we express the CP asymmetry produced in the decay of the lightest $\nu_R$ as a function of the “neutrino factory phase” $\delta$, the “neutrinoless double beta decay phase” $\phi'$, and three other “physical” weak scale phases. We introduce a way of splitting $\epsilon$ into contributions due to the different phases, $\delta$, $\phi'$, etc. We assume that $\epsilon$ is big enough to be responsible for the observed baryon asymmetry, and compare the relative size of the different contributions. We find that $\phi'$ is generically important for leptogenesis. The importance of $\delta$ depends on the mixing angles of the slepton sector. If these are smaller than the CHOOZ angle, then $\delta$ makes a significant contribution to leptogenesis.

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Addendum

After this work was completed, a related analysis appeared [51].

Appendix

In the appendix we explain the numerical procedure that we have followed to compute the contributions to the CP asymmetry from the different phases. For the sake of clarity we will only present the procedure we followed for the case $V_L = 1$, where only the phases $\delta$, $\phi$ and $\phi'$ were relevant. The extension to the general case is straight-forward.

Our starting point to compute the contributions was the Fourier expansion of the
CP asymmetry

\[ \epsilon = \sum_{j,k,l} A_{jkl} \sin(j\delta + k\phi + l\phi') = C_\delta + C_\phi + C_{\phi'} + C_{\delta\phi} + C_{\delta\phi'} + C_{\phi\phi'} \]  

(67)

with \( C_\alpha, C_{\alpha\beta} \) and \( C_{\delta\phi\phi'} \) as in eqs.(34)-(36). From the periodicity of \( \epsilon \) it is apparent that

\[ C_\delta = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\phi' \int_{-\pi}^{\pi} d\phi \epsilon \]

\[ C_\phi = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\delta \int_{-\pi}^{\pi} d\phi' \epsilon \]

\[ C_{\phi'} = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\delta \int_{-\pi}^{\pi} d\phi \epsilon \]

(68)

\[ C_{\delta\phi} = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\phi' \epsilon - (C_\delta + C_{\phi'}) \]

\[ C_{\delta\phi'} = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\phi \epsilon - (C_\delta + C_\phi) \]

\[ C_{\phi\phi'} = \frac{1}{(2\pi)^2} \int_{-\pi}^{\pi} d\delta \epsilon - (C_\phi + C_{\phi'}) \]

(69)

\[ C_{\delta\phi\phi'} = \epsilon - (C_\delta + C_\phi + C_{\phi'} + C_{\delta\phi} + C_{\delta\phi'} + C_{\phi\phi'}) \]

(70)

These integrals can be computed numerically, thus giving the different contributions to the CP asymmetry. This avoids difficulties with the points where the approximation eq. (30) breaks down. However, it is also interesting to solve this integrals analytically, to cross-check the results we obtained in Section 6. The results for the double integrals are

\[ C_\delta \approx 0 \quad C_\phi \approx 0 \quad C_{\phi'} \approx 0. \]

(71)

On the other hand, the results for the single integrals is more involved and depends on the particular point of the parameter space. These integrals can be computed using the residue theorem; the number of poles inside the unit circle depends on the values of the phases and other neutrino parameters, especially on the CHOOZ angle, hence the dependence of the result on the chosen parameters. However, some care must be exercised in using the residue theorem, because there can be poles in eq. (30) at points where the approximation breaks down. Such poles must be neglected.

The results for the single integrals are different depending on the CHOOZ angle. We consider three possibilities:

- When the CHOOZ angle is close to the experimental upper limit, or to be precise, when

\[ |m_{\nu_1} c_{12}^2 c_{13}^2 e^{i\phi} + m_{\nu_3} s_{13}^2 e^{2i\delta}| < m_{\nu_2} c_{13}^2 s_{12}^2 \]

\[ |m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'} + m_{\nu_3} c_{13}^2 e^{2i\delta}| > m_{\nu_1} c_{13}^2 c_{12}^2 \]

\[ |m_{\nu_1} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}| < m_{\nu_3} s_{13}^2 \]

(72)
the single integrals read

\[
C_{\delta \phi} \simeq 0
\]
\[
C_{\delta \phi'} \simeq -\frac{3y_1^2}{8\pi D} \text{Im} \left\{ \frac{m_{\nu_2}^3 c_{13}^2 s_{12}^2 e^{i\phi} + m_{\nu_1}^3 s_{13}^2 e^{2i\delta}}{m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi} + m_{\nu_1} s_{13}^2 e^{2i\delta}} \right\} \simeq -\frac{3y_1^2}{2\pi} \left( \frac{m_{\nu_3}}{m_{\nu_2}} \right)^3 s_{13}^2 \sin(2\delta - \phi')
\]
\[
C_{\phi \phi'} \simeq 0
\]

where \( D \) was defined after eq.(44). This result, coincide with eq.(46), that was obtained using a completely different method.

- When the CHOOZ angle is very small, or when the conditions

\[
|m_{\nu_1} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_3} s_{13}^2 e^{2i\delta}| < m_{\nu_2} c_{13}^2 s_{12}^2
\]
\[
|m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'} + m_{\nu_3} s_{13}^2 e^{2i\delta}| < m_{\nu_1} c_{13}^2 c_{12}^2
\]
\[
|m_{\nu_2} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}| > m_{\nu_3} s_{13}^2
\]

are fulfilled, the results for the single integrals are

\[
C_{\delta \phi} \simeq 0
\]
\[
C_{\delta \phi'} \simeq 0
\]
\[
C_{\phi \phi'} \simeq -\frac{3y_1^2}{8\pi D} \text{Im} \left\{ \frac{m_{\nu_1} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}}{m_{\nu_1} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}} \right\} \simeq \frac{3y_1^2}{4\pi} \frac{m_{\nu_3}}{m_{\nu_2}} \sin(\phi - \phi')
\]

This result is identical to the result obtained using series expansions in Section 6, eq.(47).

- For intermediate values of the CHOOZ angle, it is usually the case that

\[
|m_{\nu_1} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_3} s_{13}^2 e^{2i\delta}| < m_{\nu_2} c_{13}^2 s_{12}^2
\]
\[
|m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'} + m_{\nu_3} s_{13}^2 e^{2i\delta}| > m_{\nu_1} c_{13}^2 c_{12}^2
\]
\[
|m_{\nu_1} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}| > m_{\nu_3} s_{13}^2
\]

so the single integrals are

\[
C_{\delta \phi} \simeq 0
\]
\[
C_{\delta \phi'} \simeq -\frac{3y_1^2}{8\pi D} \text{Im} \left\{ \frac{m_{\nu_3} c_{13}^2 s_{12}^2 e^{i\phi} + m_{\nu_2} s_{13}^2 e^{2i\delta}}{m_{\nu_3} c_{13}^2 s_{12}^2 e^{i\phi} + m_{\nu_2} s_{13}^2 e^{2i\delta}} \right\} \simeq -\frac{3y_1^2}{2\pi} \left( \frac{m_{\nu_3}}{m_{\nu_2}} \right)^3 s_{13}^2 \sin(2\delta - \phi')
\]
\[
C_{\phi \phi'} \simeq -\frac{3y_1^2}{8\pi D} \text{Im} \left\{ \frac{m_{\nu_3} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}}{m_{\nu_3} c_{13}^2 c_{12}^2 e^{i\phi} + m_{\nu_2} c_{13}^2 s_{12}^2 e^{i\phi'}} \right\} \simeq \frac{3y_1^2}{4\pi} \frac{m_{\nu_3}}{m_{\nu_2}} \sin(\phi - \phi')
\]

that are identical to eq.(49).

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