Flavors and Phases in Unparticle Physics

Chuan-Hung Chen\textsuperscript{1,2}\textsuperscript{*} and Chao-Qiang Geng\textsuperscript{3}\textsuperscript{†}

\textsuperscript{1}Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan
\textsuperscript{2}National Center for Theoretical Sciences, Hsinchu 300, Taiwan
\textsuperscript{3}Department of Physics, National Tsing-Hua University, Hsinchu 300, Taiwan

(Dated: February 1, 2008)

Abstract

Inspired by the recent Georgi’s unparticle proposal, we study the flavor structures of the standard model (SM) particles when they couple to unparticles. At a very high energy scale, we introduce $BZ$ charges for the SM particles, which are universal for each generation and allow $BZ$ fields to distinguish flavor generations. At the $\Lambda_{U}$ scale, $BZ$ operators and charges are matched onto unparticle operators and charges, respectively. In this scenario, we find that tree flavor changing neutral currents (FCNCs) can be induced by the rediagonalizations of the SM fermions. As an illustration, we employ the Fritzsch ansatz to the SM fermion mass matrices and we find that the FCNC effects could be simplified to be associated with the mass ratios denoted by $\sqrt{m_i m_j/m_3^2}$, where $m_3$ is the mass of the heaviest particle in each type of fermion generations and $i, j$ are the flavor indices. In addition, we show that there is no new CP violating phase for FCNCs in down type quarks beside the unique one in the CKM matrix. We use $\bar{B}_q \rightarrow \ell^+ \ell^-$ as examples to display the new FCNC effects. In particular, we demonstrate that the direct CP asymmetries in the decays can be $O(10\%)$ due to the peculiar CP conserving phase in the unparticle propagator.

\textsuperscript{*} Email: physchen@mail.ncku.edu.tw
\textsuperscript{†} Email: geng@phys.nthu.edu.tw
In the standard model (SM), it is known that flavor changing processes at tree level can only be generated for charged currents mediated the W gauge boson in the quark sector. These charged currents will induce flavor changing neutral currents (FCNCs) via quantum loops. Consequently, the most impressive features of flavor physics are the Glashow-Iliopoulos-Maiani (GIM) mechanism and the large top quark mass. For instance, the former makes the $P^0 - \bar{P}^0$ ($P = K$ and $D$) mixings and rare $P$ decays naturally small while the latter leads to large $B_q - \bar{B}_q$ mixings ($q = d, s$) as well as the time-dependence CP asymmetry for the decay of $B_d \to J/\Psi K_S$. Among these effects, the most important measured quantities are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, coming from the unitary matrices which diagonalize the left-handed up and down-type quark matrices. Although there are no disagreements between the SM and experiments, which might give us some clue as to what may lie beyond the SM, it is important to keep searching for any discrepancies. In particular, the next generation of flavor factories such as SuperKEKB and LHCb with design luminosities of $5 \times 10^{35}$ and $5 \times 10^{32} \text{cm}^{-2}\text{s}^{-1}$, respectively, may provide some hints for new flavor effects. Thus, theoretically it should be interesting to explore the possible new phenomena related to flavor physics.

Recently, Georgi has proposed that an invisible sector dictated by the scale invariant may weakly couple to the particles of the SM. Since the scale invariant stuff cannot have a definite mass unless it is zero, it should be made of unparticles as the SM particles have definite masses. In terms of the two-point function with the scale invariance, it is found that the unparticle with the scaling dimension $d_U$ behaves like a non-integral number $d_U$ of invisible particles. Consequently, the unparticle physics phenomenology has been extensively explored in Refs. Some illustrative examples such as $t \to u + U$ and $e^+ e^- \to \mu^+ \mu^-$ have been given to display the unparticle properties. It is also suggested that the unparticle production in high energy colliders might be detected by searching for the missing energy and momentum distributions. Nevertheless, flavor factories with high luminosities mentioned above should also provide good environments to search for unparticles via their virtual effects.

Besides the Lorentz structure, so far there is no rule to govern the interactions between the SM particles and unparticles. The flavor physics associated with unparticles is quite arbitrary, i.e., the couplings could be flavor conserving or changing. Moreover, there is no
any correlation in the transitions among three generations for flavor changing processes. In this note, we will study the possible flavor structures for the SM particles when they couple to unparticles. Since the gauge structure of unparticles involves more theoretical uncertainties, we only pay attention to the interactions with the charged fermion sectors. We will not discuss the neutrino sector because the nature of neutrino flavors is still unclear.

We start from the scheme proposed in Ref. [8]. For the system with the scale invariance there exist so-called Banks-Zaks ($BZ$) fields that have a nontrivial infrared fixed point at a very high energy scale [49]. Above the electroweak scale, since all SM particles are massless, we cannot tell the differences between down-type quarks or up-type quarks. In the SM, we have $SU(3)_D \times SU(3)_U \times SU(3)_Q$ flavor symmetries [50], where $D$ and $U$ denote the singlet states for down and up-type quarks, respectively, while $Q$ stands for the quark doublet. Therefore, if $BZ$ fields are flavor blind, plausibly new flavor mixing effects cannot be generated for vector and axial-vector currents after the electroweak symmetry breaking (EWSB). It is worthy to mention that scalar-type couplings, illustrated by $\bar{d}d\mathcal{O}$ and $\bar{d}\gamma_5d\mathcal{O}$ in weak eigenstates, could basically produce FCNCs after the spontaneous symmetry breaking. For example, $\bar{d}_i \left( V_D^R V_D^{D\dagger} \right)_{ij} P_L d_j \mathcal{O}$ would be induced for the coupling of $\bar{d} P_L d \mathcal{O}$ after the EWSB, where $V_D^D_{R,L}$ are unitary matrices to diagonalize the down-type quark Yukawa matrix. Note that since $\bar{d} \Gamma d$ ($\Gamma = 1, \gamma_5$) have to be $SU(2)_L$ singlets, the $d$-quark has to be either left-handed or right-handed before the EWSB as it should be. For the convention of $V_L^U = 1$, $V_D^D$ is just the CKM matrix. Immediately, we suffer a serious problem from the $K^0 - \bar{K}^0$ mixing due to the coupling for $\bar{d}s\mathcal{O}$ being associated with $(V_D^R_{12} - \lambda)$ where $\lambda$ is the Wolfenstein parameter [51]. To avoid the large FCNC problem, one can set the Yukawa matrix be hermitian so that $V_D^D = V_L^D$. As a result, the FCNCs at tree level via scalar-type interactions are removed. In any event, despite the property of Yukawa matrices, to get natural small FCNCs at tree level for scalar and vector-type interactions, we need some internal degrees of freedom for fermions that could differentiate flavors by the scale invariant stuff.

In order to reveal the new flavor mixing effects due to the involvement of unparticles, we assume that the SM particles carry some kind of $BZ$ charges so that $BZ$ fields could distinguish flavor species. In terms of the prescription in Ref. [8], the interactions between
$BZ$ and SM fields are given by

$$\frac{g_{BZ}}{M_{U}^{k}}\bar{F}Q^{BZ}_{\Gamma}FO_{BZ}$$

(1)

where $M_{U}$ is the high energy mass scale of the messenger, $g_{BZ}$ is a free parameter, $F^{T} = (f_{1}, f_{2}, f_{3})$ denote the 3-generation of fermions in the SM, $\text{dia}\bar{Q}^{BZ} = (Q_{1}^{BZ}, Q_{2}^{BZ}, Q_{3}^{BZ})$ are the corresponding $BZ$ charges, $\Gamma$ is the possible Dirac matrix and $O_{BZ}$ is the operator composed of $BZ$ fields. We note that although $Q_{i}^{BZ}$ are different for each generation, the interactions are still flavor conserved. To simplify our discussion, we regard that all fermions in each generation have the same $BZ$ charge at the high energy scale and we assume that the interactions with the $BZ$ fields are invariant under parity. Subsequently, with the dimensional transmutation at the $\Lambda_{U}$ scale, the $BZ$ operators in Eq. (1) will match onto unparticle operators. The effective interactions are obtained to be

$$C_{U}^{F} \frac{\Lambda_{U}^{d_{BZ}}}{M_{U}^{d_{U}}} \bar{F}Q^{U}_{\Gamma}FO_{U},$$

(2)

where $C_{U}^{F}$ is a Wilson-like coefficient function and $d_{BZ(U)}$ is the scaling dimension of the $BZ$ (unparticle) operator. Here the unparticle operators have been set to be hermitian [9]. In principle, $Q^{U}$ could be related to $Q^{BZ}$ by a complicated matching procedure. However, at the current stage, it is impossible to give any explicit calculations for the matching. With the property of the diagonal $Q^{BZ}$ matrix, we know that $Q^{U}$ should be also a diagonal one, parametrized by $\text{dia}\bar{Q}^{U} = (Q_{1}^{U}, Q_{2}^{U}, Q_{3}^{U})$. Hence, below the $\Lambda_{U}$ scale, $Q^{U}$ could be regarded as unparticle charges carried by the SM fermions to distinguish the flavors by the unparticle stuff.

When the energy scale goes down below the EWSB scale, described by the vacuum expectation value (VEV) of the Higgs field $\langle H \rangle = v/\sqrt{2}$, the flavor symmetry will be broken by the Yukawa terms and the charged fermions become massive. Afterward, the weak eigenstates of the fermions appearing in Eq. (2) need to be transformed to the physical eigenstates by proper unitary transformations. Hence, Eq. (2) is found to be

$$\mathcal{L}_{U} = \frac{C_{S}^{F}}{\Lambda_{U}^{d_{U}-1}} \left( FV_{R}^{F}Q^{U}_{\Gamma}V_{L}^{F}P_{L}F + h.c. \right) O_{U}$$

$$+ \frac{1}{\Lambda_{U}^{d_{U}-1}} \left( FV_{L}^{F}Q^{U}_{\Gamma}V_{L}^{F}P_{L}F + L \rightarrow R \right) \left( C_{V}^{F}O_{U}^{\mu} + \frac{C_{V}^{F}}{\Lambda_{U}} \partial^{\mu}O_{U} \right) + \ldots,$$

(3)

where we have redefined the coefficient functions to be dimensionless free parameters denoted by $C_{S}^{F}, C_{V}^{F}$ and $C_{V,S}^{F}$, respectively. In Eq. (3), the power of $\Lambda_{U}$ is taken to fit the dimension
of the effective Lagrangian in four-dimension spacetime and the explicit terms represent the main FCNC effects. Note that we have separated the interactions in terms of the fermion chirality. In addition, $V_{R,L}^F$ are the unitary matrices to diagonalize the Yukawa matrix of $F$-type fermions, where $F$ could be up and down-type quarks and charged leptons. According to Eq. (3), in general we have two types of sources for new FCNCs, i.e., $V_R^F V_L^{F\dagger}$ and $V_{L(R)}^F Q^H V_{L(R)}^{F\dagger}$. As known, the determination of flavor mixing matrices $V_{L,R}^F$ is governed by the detailed patterns of the mass matrices. For convenience, we just focus on the quark sector. It has been known that the CKM matrix, defined by $V^{U} = V^{U}_{LL} V^{D \dagger}_{LR}$, is approximately an unity matrix. This indicates that the quark mass matrices are very likely aligned and have the relationship of $M_D = M_U + \Delta(\lambda^2)$ with $M_{U(D)} = m_{U(D)}/m_{t(b)}$ [53, 54, 55]. In Ref. [55], it showed that the Fritzsch quark mass matrices, given by [52, 54]

$$M_F = R_F \bar{M}_F H_F$$

with

$$\bar{M}_F = \begin{pmatrix} 0 & A_F & 0 \\ A_F & 0 & B_F \\ 0 & B_F & C_F \end{pmatrix}$$

(4)

where $R_F$ and $H_F$ are diagonal phase matrices, could lead to reasonable structures for the mixing angles and CP violating phase in the CKM matrix just in terms of the quark masses. From the hierarchy $m_u(d) \ll m_c(s) \ll m_t(b)$, it is found that the interesting equalities [55]

$$\sqrt{m_d/m_s} - \sqrt{m_u/m_c} \approx V_{us}$$

$$\sqrt{m_s/m_b} - \sqrt{m_c/m_t} \approx V_{cb}$$

(5)

are satisfied. Although the extensions of the Fritzsch ansatz could have more degrees of freedom to fit the experimental data [56], however, since our goal of this study is to explore the flavor structure affected by unparticles, we will take the simplest version of the Fritzsch ansatz in Eq. (4) as our working base. In addition, we have checked that due to the character of mass hierarchy, the extensions of Eq. (4) do not change our following results.

Since the SM has been extended to include new flavor interactions, we have to be careful to use the phase convention because the rotated phases will flow to Eq. (3). To avoid the phase ambiguity, we should start from the flavor basis in Eqs. (2) and (4). At first, we rotate away $R_U$ and $H_U$ from $M_U$ by redefining the phases of the up-type quarks. In order to make the weak charged currents to be invariant under this transformation, left-handed down quarks should make the transformation $d_L \rightarrow H_U d_L$ simultaneously. Then, the interactions
in Eq. (2) for up and down quarks to the scalar unparticle become

$$\bar{u}_R Q^U [R_U H^U_L^\dagger] u_L \mathcal{O}_U, \quad \bar{d}_R Q^D H^D_L^\dagger d_L \mathcal{O}_D.$$  

Due to $Q^U$, $R_F$ and $H_F$ being all diagonal matrices, the phase redefinition will not influence the vector current interactions. Since $\mathcal{M}$ is a real and symmetric matrix, it can be diagonalized by an orthogonal matrix $O$ such that $\mathcal{M}^{\text{dia}} = O \mathcal{M} O^T$. Accordingly, we obtain

$$V^U_L = V^U_R = O_U, \quad V^D_L = O_D H_D H^U_U, \quad V^D_R = O_D R_D$$

and $V_{CKM} = O_U H_U H^D_D O_D^T$. Then, the flavor structures could be expressed by

$$\bar{F} [O_F Q^U R_F H^U_F O^T_F] P_L F + \text{h.c.},$$

$$\bar{F} [O_F Q^D O^T_F] \gamma_{\mu} P_L F + (P_R \rightarrow P_L).$$

(6)

We note that the phases in $R_F$ and $H_F$ appear only in the scalar-type interactions. With $tr \mathcal{M}_F, tr \mathcal{M}_F^2$ and $det \mathcal{M}_F$ and the convention of $\text{dia} \mathcal{M}_F^{\text{dia}} = (m_1, -m_2, m_3)$ where $m_{1,2,3}$ denote $m_{u,c,t}[d,s,b]$ and $F = U[D]$, we find that $A_F \approx \sqrt{m_1 m_2}$, $B_F \approx \sqrt{m_2 m_3}$ and $C \approx m_3$. As a result, the orthogonal matrix could be obtained as [55]

$$O_F \approx \begin{pmatrix} 1 - m_1/2m_2 & \sqrt{m_1/m_2} & -\sqrt{m_1/m_3} \\ -\sqrt{m_1/m_2} & 1 - m_1/2m_2 - m_2/2m_3 & -\sqrt{m_2/m_3} \\ \sqrt{m_1/m_3}m_2/m_3 & \sqrt{m_2/m_3} & 1 - m_2/2m_3 \end{pmatrix}. \quad (7)$$

Since the CKM matrix expressed by $V_{CKM} = O_U H_U H^D_D O_D^T$ in general has six phases, we can redefine the phases in up and down-type quarks again so that $V_{CKM} = X O_U H_U H^D_D O_D^T Y^\dagger$ satisfies one single CKM phase convention [3]. With the new phases in $\text{dia} X = e^{i(\alpha - \beta)}(-i, 1, 1)$, $\text{dia} Y = e^{i\alpha}(-1, 1, 1)$ and $\text{dia} H_U H^D_D = e^{i\beta}(-i, 1, 1)$, Eq. (6) becomes

$$\bar{F} Z_F \left[ O_F Q^U R_F H^U_F O^T_F \right] Z_F^\dagger P_L F + \text{h.c.},$$

$$\bar{F} Z_F \left[ O_F Q^D O^T_F \right] Z_F^\dagger \gamma_{\mu} F,$$

(8)

where $\text{dia} Z_U = (-i, 1, 1)$, $\text{dia} Z_D = (-1, 1, 1)$ and the vector-type interactions are parity conserved. We note that the flavor structures in Eq. (8) have some restrictions on $Q^U$. To see the problem clearly, we decompose the flavor changing effects to be

$$(O_F Q^U O^T_F)_{ij} = Q_1 [\delta_{ij} + (r_{21} - 1) O_{Fi2} O_{Fj2} + (r_{31} - 1) O_{Fi3} O_{Fj3}]$$
with \( r_{ij} = Q_i^U / Q_j^U \). Since all phase matrices are in diagonal forms, an analysis on \( O_F Q^U O_F^T \) will not lose the generality. Using the elements in Eq. (7), the possible flavor changing effects are explicitly given by

\[
\begin{align*}
(O_F Q^U O_F^T)_{12} &= Q_1^U (r_{21} - 1) \sqrt{\frac{m_1}{m_2}} + (r_{31} - 1) \sqrt{\frac{m_1 m_2}{m_3}}, \\
(O_F Q^U O_F^T)_{13} &= Q_1^U (r_{21} - r_{31}) \sqrt{\frac{m_1}{m_3}}, \\
(O_F Q^U O_F^T)_{23} &= Q_1^U (r_{21} - r_{31}) \sqrt{\frac{m_2}{m_3}}.
\end{align*}
\]  

(9)

Phenomenologically, \( (O_F Q^U O_F^T)_{12} \) and \( (O_D Q^U O_D^T)_{12,13,23} \) are dictated by \( D^0 - \bar{D}^0, K^0 - \bar{K}^0, B_d - \bar{B}^d \) and \( B_s - \bar{B}^s \) mixings, respectively. From Eq. (9), one can easily see that if \( r_{21} - 1 \sim O(\lambda) \), a strict constraint on \( Q_1^U \) is inevitable due to \( \sqrt{m_d/m_s} \sim \lambda \). If \( r_{21} \) and \( r_{31} \) are in the same order of magnitude, it will make the FCNC effects involving the third generation be less interesting. Motivated by the successful SM results for pseudoscalar meson oscillations in the down-type quark systems, where the related CKM matrix elements for \( \Delta m_K, \Delta m_B \) and \( \Delta m_{B_s} \) have the ratios \( \lambda^3 : \lambda : 1 \), we find that \( | r_{21} - 1 | \sim O(\lambda^2) \) in \( (O_D Q^U O_D^T)_{12} \) should be satisfied, i.e., \( Q_1^U \sim Q_2^U + O(\lambda^2) \). In addition, the sign and the specific magnitude should be chosen to somewhat cancel out the second term of the first line in Eq. (9). With this scheme, we then have

\[
(r_{21} - 1) \sqrt{\frac{m_d m_b}{m_s^2}} - \sqrt{\frac{m_d}{m_b}} \sim O \left( \sqrt{\frac{m_d m_b}{m_s^2}} \right),
\]  

(10)

which is needed for the phenomenological reason.

With the experimental data and Fritzsch ansatz, we have obtained the FCNC effects from the couplings of quarks and unparticles. According to the results in Eq. (8), we highlight some interesting characters as follows:

- If the phase matrices \( R_F \) and \( H_F \) are independent, from Eq. (8) we find that only scalar-type FCNCs could have different couplings for different chiralities. Even there are some new CP violating phases in \( \mathcal{N}_F \equiv Z_F \left[ O_F Q^U R_F H_F^T O_F^T \right] Z_F^\dagger \), due to \( \mathcal{N}_F^\dagger = \mathcal{N}_F^* \), we see that the scalar-type interactions are in fact associated with \( \tilde{F} (Re \mathcal{N}_F - iIm \mathcal{N}_F \gamma_5) F \). Thus, there are no new physical CP violating effects unless the processes involve \( Re \mathcal{N}_F \cdot Im \mathcal{N}_F \). It is also true for cases with the vector current couplings.

- If \( R_F = H_F \), from Eq. (11) we can easily find that the corresponding mass matrices are hermitian. The FCNC effects are all related to \( Z_F \left[ O_F Q^U O_F^T \right] Z_F^\dagger \) which is also hermitian.
As a result, in terms of the quark currents, the couplings of fermions and unparticles are parity-even and no new CP phase is induced for down type quarks in this case. It should be worthy to mention that the hermitian mass matrices could be naturally realized in gauge models such as left-right symmetric models \[57\]. The hermiticity could help us to solve the CP problem in models with supersymmetry (SUSY) \[58\] and it has an important implication on CP violation in Hyperon decays \[59\].

- From Eq. (9), we find that $\Delta m_{B_s}/\Delta m_{B_d} \approx m_d/m_s \sim \lambda^2$, which is consistent with the experimental data \[3\].
- Since the masses of the charged leptons also have the mass hierarchy $m_e \ll m_\mu \ll m_\tau$, if we take the same phase convention, Eqs. (8) and (9) should be straightforwardly extended to the charged lepton sector.

To illustrate the peculiar phenomena in Eq. (8) associated with unparticles, we take $\bar{B}_q \rightarrow \ell^+\ell^-$ as examples. For simplicity, we adopt scalar-type interactions as the representative. The effective interactions are

$$L_U = \frac{1}{\Lambda_{d_U}^2} \bar{q} (N_{qb} P_L + N_{qb}^* P_R) b + \frac{1}{\Lambda_{d_U}^2} \bar{\ell} (N_{\ell\ell} P_L + N_{\ell\ell}^* P_R) \ell$$

with

$$N_{qb} = \sqrt{\frac{m_d}{m_b}} (\bar{Q}_d e^{i\theta_3} - \bar{Q}_2 e^{i\theta_2}),$$

$$N_{\ell\ell} = \bar{Q}_\ell e^{i\theta_\ell}.$$

Since the coefficient functions are always associated with $U$-charges, we define $Q_1^U = C^D_S Q_1^U$ and $Q_\ell^U = C^L_S Q_\ell^U$. With the propagator of the scalar unparticle operator proposed in Refs. \[8, 9\], given by

$$\int e^{iqx} \langle 0 | T \mathcal{O}_U(x) \mathcal{O}_U(0) | 0 \rangle = i \frac{A_{d_U}}{2 \sin \frac{d_U \pi}{2} (q^2)^{2-d_U}},$$

$$A_{d_U} = \frac{16 \pi^{5/2}}{(2\pi)^{2d_U} \Gamma(2d_U - 1) \Gamma(d_U - 1) \Gamma(d_U + 1/2)},$$

$$\phi_{d_U} = (d_U - 2)\pi,$$

the decay amplitudes for $\bar{B}_q \rightarrow \ell^+\ell^-$ by due to unparticles are expressed by

$$A(\bar{B}_q \rightarrow \ell^+\ell^-) = i \frac{f_{B_q}}{m_{B_q}} \left( \frac{m_{B_q}^2}{N_{d_U}^2} \right)^{d_U - 1} A_{d_U} \frac{1}{2 \sin \frac{d_U \pi}{2}} e^{-i\phi_{d_U}}$$

$$\times \text{Im} N_{qb} [\text{Re} N_{\ell\ell} \bar{\ell} \ell - i \text{Im} N_{\ell\ell} \bar{\ell} \gamma_5 \ell].$$
We note that $\phi_q$ is a CP conserving phase. Combining with the SM contributions, the corresponding branching ratios (BRs) are

$$\frac{1}{m_q^2} B_q \to \ell^+ \ell^- = \kappa_{B_q} \left[ \left| \Sigma_{SM} e^{i\beta_q} + \Sigma_{ul} e^{-i\phi_u} \right|^2 + \left| \Sigma_{ul} \right|^2 \right], \quad (11)$$

where the angle $\beta_q$ is from $V_{tq} = |V_{tq}| e^{-i\beta_q}$ with

$$\beta_{d(s)} = \beta(0),$$

$$\kappa_{B_q} = \frac{1}{m_q^2} \frac{\alpha_{em}^2 B(B^+ \to \tau^+\nu_\tau)}{\pi^2 \sin^2 \theta_W} \frac{m_{B_q} f_{B_q}^2 \tau_{B_q}}{m_{B^+} f_{B^+}^2 \tau_{B^+}},$$

$$\Sigma_{SM} = \frac{|V_{tb} V_{ts}^*|}{|V_{ub}|} Y(m_t^2/m_W^2),$$

$$\Sigma_{ul} = \frac{8\pi \sin^2 \theta_W}{g^2 \alpha_{em} |V_{ub}|} \frac{m_W^2}{m_t m_q} \frac{A_{dt}}{2 \sin d_{ut} \pi} \frac{m_{B_q}^2}{\Lambda_{ul}^2} d_{ut}^{-1} \text{Im} N_{q_b} I \text{m} N_{\ell\ell},$$

$$\Sigma_{ul}^* = \Sigma_{ul}^* \text{Re} N_{\ell\ell}/\text{Im} N_{\ell\ell}.$$

Due to $m_W \ll m_t$, the function of $Y(m_t^2/m_W^2)$ can be simplified to $Y(x_t) = 0.315 x_t^{0.78}$ [60]. Here, we have used the measured $B^- \to \tau \bar{\nu}_\tau$ decay to remove the uncertainties from $f_B$ and $|V_{tq}|$. Besides the BRs, from Eq. (11) we can also study the direct CP asymmetries (CPAs) in the two-body exclusive $B$ decays, defined by

$$A_{CP}(B_q \to \ell^+ \ell^-) = \frac{\mathcal{B}(\bar{B}_q \to \ell^+ \ell^-) - \mathcal{B}(B_q \to \ell^+ \ell^-)}{\mathcal{B}(\bar{B}_q \to \ell^+ \ell^-) + \mathcal{B}(B_q \to \ell^+ \ell^-)}. \quad (12)$$

It is known that in a process the direct CPA needs CP conserving and unrotated CP violating phases simultaneously. Since the unparticle stuff provides a CP-conserved phase, if the process carries a physical CP violating phase, a nonvanishing CPA is expected. In $B_q \to \ell^+ \ell^-$, the new free parameters are $d_{ut}$, $\Lambda_{ul}$ and $N_{q_b}$ ($N_{\ell\ell}$), which can be constrained by $\Delta m_{B_q}$ ($\Delta a_\ell$) of the $B_q - \bar{B}_q$ mixings (lepton anomalous magnetic dipole moments). Explicitly, we find that

$$\Delta m_{B_q} = 2 \text{Re} \langle B_q | H_{ul} (|\Delta B| = 2) | \bar{B}_q \rangle$$

$$= \frac{1}{6} \frac{f_{B_q}^2}{m_{B_q}^2} \frac{A_{dt}}{2 \sin d_{ut} \pi} \left( \frac{m_{B_q}^2}{\Lambda_{ul}^2} \right)^{d_{ut}^{-1}} \cos \phi_u \left[ (\text{Re} N_{q_b})^2 + 6 (\text{Im} N_{q_b})^2 \right],$$

$$\Delta a_\ell = \frac{1}{4\pi} \frac{A_{dt}}{2 \sin d_{ut} \pi} \left( \frac{m_{B_q}^2}{\Lambda_{ul}^2} \right)^{d_{ut}^{-1}}$$

$$\times \left[ \text{Re} (N_{\ell\ell})^2 \frac{\Gamma(2 - d_{ut}) \Gamma(2d_{ut} - 1)}{\Gamma(d_{ut} + 1)} + |N_{\ell\ell}|^2 \frac{\Gamma(3 - d_{ut}) \Gamma(2d_{ut} - 1)}{\Gamma(d_{ut} + 2)} \right]. \quad (13)$$
To estimate the numerical values, we take $|V_{ub}| = 4.3 \times 10^{-3}$, $V_{td} = 7.4 \times 10^{-3}e^{-i\beta}$ with $\beta = 25^\circ$, $V_{ts} = -0.041$, $m_{B_d(s)} = 5.28$ (5.37) GeV, $f_{B_d(s)} = 0.2$ (0.22) GeV and $\sin^2 \theta_W \approx 0.234$. For the mixing parameters of $\Delta m_{B_d}$ and $\Delta m_{B_s}$, measured to be $(3.337\pm 0.033) \times 10^{-13}$ GeV and $(11.69 \pm 0.08) \times 10^{-12}$ GeV [61], respectively, we will use their central values as the inputs to constrain $N_{d(s)b}$. For $\Delta a_\ell$, we will concentrate on the muon one with $\ell = \mu$. The difference between the experimental value and the SM prediction on the muon $g - 2$ is given by $\Delta a_\mu = a_\mu^{\exp} - a_\mu^{\SM} = (22 \pm 10) \times 10^{-10}$ [3]. We will take the upper limit to bound the free parameter $N_{\ell\ell}$. For simplicity, we set $\Lambda_U = 1$ TeV, $1 < d_\ell < 2$, $ReN_{q\bar{q}} \sim ImN_{q\bar{q}}$ and $ReN_{\ell\ell} \sim ImN_{\ell\ell}$. To see the effects of the scalar unparticle on the muon $g - 2$ and $B_q - \bar{B}_q$ mixings, we first show the results in Fig. 1, where the solid, dotted, dashed and dash-dotted lines stand for $d_\ell = 1.2$, 1.4, 1.6 and 1.8, respectively. We note that $ImN_{db}$ is treated as a free parameter due to $N_{sb} = \sqrt{m_d/m_s}N_{db}$. From the figures, we find that the muon $g - 2$ and $B_q - \bar{B}_q$ mixings are very sensitive to the scale dimension $d_\ell$. The smaller $d_\ell$ it is, the stronger constraint on $ImN_{\mu\mu(db)}$ we get. Furthermore, with the inputs and the allowed values of $ImN_{\ell}$ and $ImN_{db}$, the BRs for $B_d \rightarrow \mu^+\mu^-$ [solid] and $B_s \rightarrow \mu^+\mu^-$ [dashed] and CPA for $B_d \rightarrow \mu^+\mu^-$ as functions of $d_\ell$ are presented in Fig. 2. Due to the current
upper limit on $\mathcal{B}(B_s \rightarrow \mu^+\mu^-)$, $d_U$ should be less than 1.66. The flat curves in Fig. 2(a) correspond to the SM predictions. Amazingly, from Fig. 2(b) we see that an unusual direct CPA of $O(10\%)$ is generated in $B_d \rightarrow \mu^+\mu^-$. Besides the necessary weak CP violating phase $\beta$ existed in the SM, the result mainly depends on the CP conserving phase carried by the unparticle in its propagator. This is a unique phenomenon since it is supposed to be vanishing small even some new CP violating phases are introduced. In other words, if a signal of the CPA in $B_d \rightarrow \mu^+\mu^-$ is observed, it must be the unparticle effect. Similar results are also expected in the dielectron and ditau modes. However, there is no direct CPA for $B_s \rightarrow \ell^+\ell^-$ decays due to $\beta_s = 0$ in the SM.

In summary, we have studied the flavor structures of the SM fermions when they couple to the scale invariant stuff. In order to get naturally suppressed FCNC effects at tree level and more correlative transitions among three generations, we have introduced the $BZ$ charges that are universal in each generation but generation un-blind. The $BZ$ charges could be regarded as the internal degrees of freedom carried by the fermions for which the $BZ$-fields can distinguish the flavor generations. By the dimensional transmutation, the $BZ$ charges are matched onto the unparticle charges when the $BZ$ operators onto the unparticle operators. After the EWSB, the FCNCs are induced by the rediagonalizations.
of the fermion mass matrices. To demonstrate the FCNC effects, we have adopted the simplest Fritzsch ansatz for quarks. Consequently, we have found that the FCNC effects are associated with the square roots of the mass ratios, \( \sqrt{m_i m_j / m_3^2} \). In addition, although the couplings of the FCNCs could be complex, there is no more new CP violating phase available because the matrices \( O_F Q^{ij} R_F H_F^1 O_F^T \) responsible to the FCNC effects are symmetric. Moreover, we have used \( \bar{B}_q \to \ell^+ \ell^- \) decays to illustrate the influence of unparticles. In particular, with the peculiar CP conserving phases carried by unparticles, a unique phenomenon is generated in the direct CPAs of \( B_d \to \ell^+ \ell^- \). If any CP violating signal is found in these decays, it must indicate the existence of unparticles.

**Acknowledgments**

We would like to thank Prof. Ling-Fong Li for useful discussions. This work is supported in part by the National Science Council of R.O.C. under Grant #s: NSC-95-2112-M-006-013-MY2 and NSC-95-2112-M-007-059-MY3.

[1] S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D2, 1285 (1970).

[2] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[3] Particle Data Group, W.M. Yao et al., J. Phys. G: Nucl. Part. Phys. 33, 1 (2006).

[4] A. G. Akeroyd et al. [SuperKEKB Physics Working Group], arXiv:hep-ex/0406071.

[5] L. Camilleri [LHCb Collaboration], CERN-LHCB-2007-096; H. Dijkstra, arXiv:0708.2665 [hep-ex].

[6] C. H. Chen and C. Q. Geng, Phys. Rev. D66, 014007 (2002); Phys. Rev. D66, 094018 (2002); Phys. Rev. D71, 054012 (2005); Phys. Rev. D72, 037701 (2005); Mod. Phys. Lett. A21, 1137 (2006); Phys. Rev. D71, 077501 (2005); Phys. Rev. D71, 115004 (2005); Phys. Rev. D74, 035010 (2006); JHEP 0610, 053 (2006); Phys. Lett. B645, 189 (2007).

[7] C. H. Chen, Phys. Lett. B579, 371 (2004); C. H. Chen, C. Q. Geng and A. K. Giri, Phys. Lett. B621, 253 (2005); C. H. Chen and H. Hatanaka, Phys. Rev. D73, 075003 (2006); C. H. Chen, C. Q. Geng and T. C. Yuan, Phys. Rev. D75, 077301 (2007); C. H. Chen, C. Q. Geng and S. H. Nam, Phys. Rev. Lett. 99, 019101 (2007); C. H. Chen, C. Q. Geng and C. W. Kao,
[8] H. Georgi, Phys. Rev. Lett. 98, 221601, (2007) [arXiv:hep-ph/0703260].
[9] H. Georgi, Phys. Lett. B650, 275 (2007) [arXiv:0704.2457 [hep-ph]].
[10] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. Lett. 99, 051803 (2007) [arXiv:0704.2588 [hep-ph]]; [arXiv:0706.3155 [hep-ph]].
[11] M. X. Luo and G. H. Zhu, [arXiv:0704.3532 [hep-ph]]; M. X. Luo, W. Wu and G. H. Zhu, [arXiv:0708.0671 [hep-ph]].
[12] C. H. Chen and C. Q. Geng, [arXiv:0705.0689 [hep-ph]]; Phys. Rev. D76, 036007 (2007) [arXiv:0706.0850 [hep-ph]].
[13] G. J. Ding and M. L. Yan, [arXiv:0705.0794 [hep-ph]]; [arXiv:0706.0325 [hep-ph]].
[14] Y. Liao, [arXiv:0705.0837 [hep-ph]]; [arXiv:0708.3327 [hep-ph]]; Y. Liao and J. Y. Liu, [arXiv:0706.1284 [hep-ph]].
[15] T. M. Aliev, A. S. Cornell and N. Gaur, [arXiv:0705.1326 [hep-ph]]; JHEP 07, 072 (2007) [arXiv:0705.4542 [hep-ph]].
[16] S. Catterall and F. Sannino, Phys. Rev. D76, 034504 (2007) [arXiv:0705.1664 [hep-lat]].
[17] X. Q. Li and Z. T. Wei, Phys. Lett. B651, 380 (2007) [arXiv:0705.1821 [hep-ph]]; [arXiv:0707.2285 [hep-ph]].
[18] C. D. Lu, W. Wang and Y. M. Wang, [arXiv:0705.2909 [hep-ph]].
[19] M. A. Stephanov, [arXiv:0705.3049 [hep-ph]].
[20] P. J. Fox, A. Rajaraman and Y. Shirman, [arXiv:0705.3092 [hep-ph]].
[21] N. Greiner, [arXiv:0705.3518 [hep-ph]].
[22] H. Davoudiasl, [arXiv:0705.3636 [hep-ph]].
[23] D. Choudhury, D. K. Ghosh and Mamta, [arXiv:0705.3637 [hep-ph]].
[24] S. L. Chen and X. G. He, [arXiv:0705.3946 [hep-ph]]; S. L. Chen, X. G. He and H. C. Tsai, [arXiv:0707.0187 [hep-ph]].
[25] P. Mathews and V. Ravindran, [arXiv:0705.4599 [hep-ph]].
[26] S. Zhou, [arXiv:0706.0302 [hep-ph]].
[27] R. Foadi, M. T. Frandsen, T. A. Ryttov and F. Sannino, [arXiv:0706.1696 [hep-ph]].
[28] M. Bander, J. L. Feng, A. Rajaraman and Y. Shirman, [arXiv:0706.2677 [hep-ph]].
[29] T. G. Rizzo, [arXiv:0706.3025 [hep-ph]].
[30] H. Goldberg and P. Nath, [arXiv:0706.3898 [hep-ph]].
[31] R. Zwicky, arXiv:0707.0677 [hep-ph].
[32] T. Kikuchi and N. Okada, arXiv:0707.0893 [hep-ph].
[33] R. Mohanta and A. K. Giri, arXiv:0707.1234 [hep-ph]; arXiv:0707.3308 [hep-ph].
[34] C. S. Huang and X. H. Wu, arXiv:0707.1268 [hep-ph].
[35] N. V. Krasnikov, arXiv:0707.1419 [hep-ph].
[36] A. Lenz, arXiv:0707.1535 [hep-ph].
[37] D. Choudhury and D. K. Ghosh, arXiv:0707.2074 [hep-ph].
[38] H. Zhang, C. S. Li and Z. Li, arXiv:0707.2132 [hep-ph].
[39] Y. Nakayama, arXiv:0707.2451 [hep-ph].
[40] N. G. Deshpande, X. G. He and J. Jiang, arXiv:0707.2959 [hep-ph]; N. G. Deshpande, S. D. H. Hsu and J. Jiang, arXiv:0708.2735 [hep-ph].
[41] A. Delgado, J. R. Espinosa and M. Quiros, arXiv:0707.4309 [hep-ph].
[42] M. Neubert, arXiv:0708.0036 [hep-ph].
[43] S. Hannestad, G. Raffelt and Y. Y. Wong, arXiv:0708.1404 [hep-ph].
[44] P. K. Das, arXiv:0708.2812 [hep-ph].
[45] G. Battacharyya, D. Choudhury and D. K. Ghosh, arXiv:0708.2835 [hep-ph].
[46] D. Majumdar, arXiv:0708.3485 [hep-ph].
[47] A. T. Alan and N. K. Pak, arXiv:0708.3802 [hep-ph].
[48] A. Freitas and D. Wyler, arXiv:0708.4339 [hep-ph].
[49] T. Banks and A. Zaks, Nucl. Phys. B196, 189 (1982).
[50] A.V. Manohar and M.B. Wise, Phys. Rev. D74, 035009 (2006).
[51] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983).
[52] H. Fritzsch, Phys. Lett. B70, 4361 (1977).
[53] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985); Z. Phys. C29, 491 (1985).
[54] P.H. Frampton and C. Jarlskog, Phys. Lett. B154, 421 (1985).
[55] T.P. Cheng and L.F. Li, Phys. Rev. Lett. 55, 2249 (1985); Phys. Rev. D34, 219 (1986).
[56] K. Matsuda and H. Nishiura, Phys. Rev. D74, 033014 (2006).
[57] R.N. Mohapatra and G. Senjanovic, Phys. Lett. B79, 283 (1978); R.N. Mohapatra and A. Rasin, Phys. Rev. Lett. 76, 3490 (1996); Phys. Rev. D54, 5835 (1996); K.S. Babu, B. Dutta and R. N. Mohapatra, Phys. Rev. D61, 091701 (2000).
[58] S. Abel et al., Phys. Lett. B504, 241 (2001); S. Abel, S. Khalil, and O. Lebedev, Phys. Rev.
Lett. \textbf{89}, 121601 (2002).

[59] C.H. Chen, Phys. Lett. B\textbf{521}, 315 (2001).

[60] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. \textbf{68}, 1125 (1996).

[61] A. Abulencia \textit{et al.} (CDF Collaboration), Phys. Rev. Lett. \textbf{97}, 062003 (2006) [arXiv:hep-ex/0606027]; V.M. Abazov \textit{et al.} (D0 Collaboration), Phys. Rev. Lett. \textbf{97}, 021802 (2006) [arXiv:hep-ex/0603029].