We show that the $AdS_3 \times S^3$ and $AdS_5 \times S^5$ superstring theories in the Pohlmeyer-reduced form reveal hidden $\mathcal{N} = (4,4)$ and $\mathcal{N} = (8,8)$ worldsheet supersymmetries. The characteristic feature of these transformations is the presence of non-local terms.

1 Pohlmeyer Reduction (PR): what and why

This sort of reduction was suggested by Pohlmeyer in 1984 [1] in the case of 2d bosonic sigma models (for further developments see, e.g., [2, 3]).

The basic idea of the PR was to reformulate the given 2d sigma model in terms of currents, rather than the target space coordinates, thus performing a non-local change of variables. The following equivalences with the well known 2d integrable models were revealed in this way:

$$O(3)/O(2) \text{ sigma model } \iff \text{sine-Gordon (one field)},$$
$$O(4)/O(3) \text{ sigma model } \iff \text{complex sine-Gordon (two fields)}.$$

The PR procedure preserves 2d lorentz symmetry and integrability. It deals with the minimal set of fields, one field being always eliminated by fixing the classical 2d conformal invariance of the original sigma model.

The above equivalences are broken at the quantum level, where classical 2d conformal invariance is plagued by UV-divergences. Nevertheless, one could expect that the similar equivalences are still held at the quantum level in theories like $AdS_5 \times S^5$ superstring, which retains 2d conformal invariance upon quantization.

This was the main motivation for Grigoriev and Tseytlin to apply PR to the $AdS_n \times S^n$ type superstrings in the sigma-model formulation [4, 5] (see also [6]). The eventual ambitious goal of this new approach to superstrings is to exactly solve the $AdS_5 \times S^5$ theory at the full quantum level.
2 Sketch of Pohlmeyer reduction in superstrings

2.1 AdS × S superstrings as supercoset sigma models

Superstring theories in a Green-Schwarz type formulation with manifest space-time supersymmetry are naturally described as WZW-type sigma models with a supercoset target space [7]. Of special interest, in view of the renowned AdS/CFT correspondence, are superstrings on the super AdS\(_n\) × S\(_n\) backgrounds, including the maximally supersymmetric D = 10 type IIB super AdS\(_5\) × S\(_5\) background [8].

The relevant supercosets are minimal superextensions of the following bosonic cosets:

\[
\begin{align*}
\text{AdS}_2 \times S^2 &= \frac{SU(1,1) \times SU(2)}{U(1) \times U(1)}, \\
\text{AdS}_3 \times S^3 &= \frac{SU(1,1) \times SU(1,1) \times SU(2) \times SU(2)}{SU(1,1) \times SU(2)}, \\
\text{AdS}_5 \times S^5 &= \frac{SU(2,2) \times SU(4)}{SO(1,4) \times SO(5)},
\end{align*}
\]

and they are

\[
\begin{align*}
\text{AdS}_2 \times S^2 : \quad \hat{F} &= \frac{PSU(1,1|2)}{U(1) \times U(1)}, \\
\text{AdS}_3 \times S^3 : \quad \hat{F} &= \frac{PSU(1,1|2) \times PSU(1,1|2)}{SU(1,1) \times SU(2)}, \\
\text{AdS}_5 \times S^5 : \quad \hat{F} &= \frac{PSU(2,2|4)}{SO(1,4) \times SO(5)}.
\end{align*}
\]

2.2 Z\(_4\) grading

A notable common feature of all cases \(n = 2, 3, 5\) listed above is that the superalgebra \(\hat{f}\) of the underlying supergroup \(\hat{F}\) admits a \(Z_4\) grading:

\[
\hat{f} = \hat{f}_0 \oplus \hat{f}_1 \oplus \hat{f}_2 \oplus \hat{f}_3, \quad [\hat{f}_i, \hat{f}_j] \subset \hat{f}_{i+j \mod 4}.
\]

Here \(\hat{f}_0\) is the algebra of the bosonic group \(\hat{G}\), \(\hat{f}_2\) is its orthogonal complement to the full bosonic subalgebra of \(\hat{f}\) and \(\hat{f}_{1,3}\) are fermionic.

Respectively, the currents \(J_\pm = F^{-1} \partial_\pm F\), where \(F \in \hat{F}\), can be decomposed as

\[
J_\pm = A_\pm + P_\pm + Q_{1\pm} + Q_{2\pm}, \quad A \in \hat{f}_0, \quad Q_1 \in \hat{f}_1, \quad P \in \hat{f}_2, \quad Q_2 \in \hat{f}_3.
\]

The very important consequence of the \(Z_4\) grading property is that the WZW 3-form is exact and so the WZW term in the Green-Schwarz (GS) action can be represented in a form bilinear in fermionic currents [9, 10, 11]:

\[
L_{GS} = \frac{1}{2} \text{Str} \left( \gamma^{ab} P_a P_b + \epsilon^{ab} Q_{a1} Q_{b2} \right), \quad \gamma^{ab} = \sqrt{-g} g^{ab}, \quad a, b = +, -.
\]
In such a form, the GS action is quite similar to the actions of the bosonic sigma models (which are also bilinear in currents) and so it is ideally suitable for performing the Pohlmeyer reduction by analogy with the bosonic case. A new feature is the local fermionic kappa symmetry of this GS action.

2.3 Basic steps of PR procedure

The PR procedure for $AdS_n \times S^n$ superstrings can be summarized as follows [4, 5]:

- Choose the conformal gauge $\gamma^{ab} = \eta^{ab}$;
- Take into account the Virasoro constraints $\text{STr}(P_\pm P_\pm) = 0$;
- Properly fix the bosonic gauge $G$ symmetry of the action, making use of the Virasoro constraints and the residual (anti)holomorphic conformal invariance:

$$P_+ = \mu T, \quad P_- = \mu g^{-1}Tg, \quad \text{where } g(\sigma) \in G,$$
$$T \sim \text{diag}(1, \ldots, 1, -1, \ldots, 1), \quad \mu \text{ is a constant of mass dimension};$$

The matrix $T$ makes it possible to split the full superalgebra $\hat{f}$ as

$$\hat{f} = \hat{f}^\| \oplus \hat{f}^\perp, \quad P^\| \zeta^\| = \zeta^\|, \quad P^\| \chi^\perp = 0,$$
$$\zeta^\| \in \hat{f}^\|, \quad \chi^\perp \in \hat{f}^\perp, \quad P^\| = -[T, [T, \cdot]], \quad [\hat{f}^\perp, \hat{f}^\perp] \subset \hat{f}^\perp, \quad [\hat{f}^\|, \hat{f}^\perp] \subset \hat{f}^\perp;$$

- Fully fix the fermionic kappa-symmetry, so that $Q_{1-} = Q_{2+} = 0$ and the minimal set of fermionic fields remains at the end:

$$\Psi_R = \frac{1}{\sqrt{\mu}} Q^\|_{1+} \in \hat{f}^\|_1, \quad \Psi_L = \frac{1}{\sqrt{\mu}} (gQ_{2-}g^{-1})^\| \in \hat{f}^\|_3.$$

This generalized PR applied to the equations of motion associated with the original GS Lagrangian (2.1) finally results in the following equations of motion for the reduced fields $g$, $\Psi_L$, $\Psi_R$:

$$D_-(g^{-1}D_+g) - F_{+-} = \mu^2 [T, g^{-1}Tg] + \mu [\Psi_R, g^{-1}\Psi_Lg],$$
$$D_-\Psi_R = \mu [T, g^{-1}\Psi_Lg], \quad D_+\Psi_L = \mu [T, g\Psi_Rg^{-1}],$$  \hspace{1cm} (2.2)

with $D_\pm = \partial_\pm + [A_\pm, \cdot]$ and $F_{+-} = \partial_+ A_- - \partial_- A_+ + [A_+, A_-].$

The 2d gauge fields $A_\pm$ take values in the algebra $\mathfrak{h}$ of subgroup $H$ of group $G$, defined by the condition $[T, h] = 0, \quad h \in \mathfrak{h}$, so that

$$g = \hat{f}_0 = \mathfrak{m} \oplus \mathfrak{h}, \quad \mathfrak{m} := \hat{f}_0^\|, \quad \mathfrak{h} := \hat{f}_0^\perp.$$
In the $n = 2$ case, with $G = U(1) \times U(1)$, the subgroup $H$ is empty and $A_\pm = 0$. In the $n = 3$ case, with $G = SU(1,1) \times SU(2)$, we have $H = U(1) \times U(1)$, and in the $n = 5$ case, with $G = SO(1,4) \times SO(5)$, we have $H = SO(4) \times SO(4) \sim [SU(2)]^4$.

The reduced equations (2.2) are covariant under the $H \times H$-valued gauge transformations

$$
g \rightarrow hg\tilde{h}^{-1}, \quad \Psi_L \rightarrow h\Psi_L\tilde{h}^{-1}, \quad \Psi_R \rightarrow \tilde{h}\Psi_R\tilde{h}^{-1},$$

$$A_+ \rightarrow h(A_+ + \partial_+)h^{-1}, \quad A_- \rightarrow \tilde{h}(A_- + \partial_-)\tilde{h}^{-1}$$

and are derivable from the action

$$S_{tot} = S_{gWZW} + \mu^2 \int d^2\sigma \text{Str}(g^{-1}TgT)$$

$$+ \int d^2\sigma \text{Str}(\Psi_L T \partial_+ \Psi_L + \Psi_R T \partial_- \Psi_R) + \mu \text{Str}(g\Psi_R g^{-1} \Psi_L),$$

(2.4)

where $S_{gWZW}$ is the action of the gauged $G/H$ WZW (gWZW) model [12, 13, 14]:

$$S_{gWZW} = S_{WZW}(g) + S_{gauge}(g, A_\pm),$$

$$S_{gauge} = \int d^2\sigma \text{Str}(A_+ \partial_- gg^{-1} - A_- g^{-1} \partial_+ g - g^{-1} A_+ g A_- + A_+ A_-).$$

The action (2.4) is invariant under the $H$-valued gauge transformations

$$g \rightarrow hg\tilde{h}^{-1}, \quad \Psi_{L,R} \rightarrow h\Psi_{L,R}\tilde{h}^{-1}, \quad A_\pm \rightarrow h(A_\pm + \partial_\pm)\tilde{h}^{-1},$$

which form a diagonal $h = \tilde{h}$ in the “on-shell” gauge group $H \times H$, eq. (2.3).

As a consequence of this Lagrangian formulation of the PR superstring equations there also appear additional algebraic constraints as equations of motion for the fields $A_\pm$:

a) $(g^{-1}D_+ g)_h = 2(T\Psi^2_R)_h, \quad$ b) $(gD_- g^{-1})_h = 2(T\Psi^2_L)_h.$

These equations can be interpreted as fixing of a certain gauge with respect to the extended on-shell gauge group.

### 2.4 Worldsheet supersymmetry?

The PR $AdS_n \times S^n$ superstring equations and the relevant off-shell actions are fermionic extensions of the bosonic gWZW systems. The numbers of off-shell bosonic and fermionic degrees of freedom are as follows

$$n = 2 : \quad 2 \text{ bosons}, \quad 4 \text{ fermions},$$

$$n = 3 : \quad 4 \text{ bosons}, \quad 8 \text{ fermions},$$

$$n = 5 : \quad 8 \text{ bosons}, \quad 16 \text{ fermions}.$$

They nicely match each other to suggest the presence of worldsheet supersymmetry ($\# \text{ bosons} = \frac{1}{2}\# \text{ fermions on shell}$).
Indeed, in the $n = 2$ case the action does possess $N = (2, 2)$ supersymmetry [4, 5]

\[
\delta_{\epsilon L} g = g[T, [\Psi_R, \epsilon_L]] , \quad \delta_{\epsilon L} \Psi_R = [(g^{-1}D_+g)^\parallel, \epsilon_L] , \quad \delta_{\epsilon L} \Psi_L = \mu[T, g\epsilon_Lg^{-1}] , \quad \delta_{\epsilon L} A_+ = 0 , \quad \delta_{\epsilon L} A_- = \mu[(g^{-1}\Psi_Lg)^\perp, \epsilon_L] ,
\]

where $\epsilon_L \in \tilde{f}_1^+$ (and analogously for the right chiral supersymmetry, with the parameter $\epsilon_R \in \tilde{f}_3^+$). Formally, these transformations could be applied also in the cases $n = 3$ and $n = 5$. However, the corresponding actions are invariant only under too strong condition $[\epsilon_L, h] = 0$, which can be fulfilled only for $n = 2$.

Before our paper [15], no way was known how to evade this obstruction for $n = 3$ and $n = 5$.

3 Worldsheets supersymmetry for $n = 3$ and $n = 5$

3.1 Modified PR superstring action

First, we use the Polyakov-Wiegmann representation for the gauge fields $A_\pm$ [16, 17]:

\[
A_+ = -\partial_+ uu^{-1} , \quad A_- = -\partial_- \bar{u}\bar{u}^{-1} ,
\]

where $u$ and $\bar{u}$ take values in $H$. The $H \times H$ gauge transformation laws of $A_\pm$ amount to the following gauge transformations of $u$ and $\bar{u}$:

\[
u \to hu , \quad \bar{u} \to \bar{h}\bar{u} .
\]

The above definition is not changed under the additional Kac-Moody transformations of $u, \bar{u}$

\[
u \to u \omega(\sigma^-) , \quad \bar{u} \to \bar{u} \bar{\omega}(\sigma^+) .
\]

Second, we propose to modify the PR superstring action (2.4) as

\[
S_{\text{tot}} \to S'_{\text{tot}} = S_{\text{tot}} + S_a , \quad S_a = S^{(H)}_{\text{WZW}}(B) , \quad B := u^{-1}\bar{u} .
\]

The field $B$ is manifestly invariant under the diagonal $h = \bar{h}$ subgroup of the $H \times H$ gauge transformations (2.3), so $S'_{\text{tot}}$ is gauge-invariant like $S_{\text{tot}}$. An important point is that the equations of motion (2.2) for the basic PR fields $g, \Psi_{L,R}$ are not changed upon the above modification, it affects only equations of motion for the gauge fields $u, \bar{u}$.

3.2 Off-shell worldsheet supersymmetry

The modified action is invariant, modulo a total derivative, under the following modified supersymmetry transformations

\[
\delta_{\epsilon L} g = g[T, [\Psi_R, \epsilon_L]] , \quad \delta_{\epsilon L} \Psi_R = [(g^{-1}D_+g)^\parallel, \epsilon_L] , \quad \delta_{\epsilon L} \Psi_L = \mu[T, g\epsilon_Lg^{-1}] , \quad \delta_{\epsilon L} A_+ = 0 , \quad \delta_{\epsilon L} A_- = \mu[(g^{-1}\Psi_Lg)^\perp, \epsilon_L] ,
\]

(3.2)
with \( \tilde{e}_L = \bar{u}e_L\bar{u}^{-1} \). The invariance is secured due to this dressing of the infinitesimal parameters and non-trivial transformation law of the addition \( S_a \). In checking the invariance, the crucial role is played by the relation \( D_-\tilde{e}_L = 0 \), no need in the constraint \([\epsilon_L, h_\epsilon] = 0\) arises.

The matrix \( \epsilon_L \in \hat{f}_1^+ \) encompasses \( 2(n - 1) \) independent parameters for the \( \text{AdS}_n \times S^n \) model. The transformations of the right chiral supersymmetry can be written in a symmetric way through the matrix parameter \( \tilde{e}_R = \epsilon_R u^{-1} \) with the same number of independent entries.

Thus we have \( \mathcal{N} = (2, 2) \), \( \mathcal{N} = (4, 4) \) and \( \mathcal{N} = (8, 8) \) chiral worldsheet supersymmetries in the \( n = 2, n = 3 \) and \( n = 5 \) cases.

Transformations written in terms of the “prepotentials” \( u \) and \( \bar{u} \) are nonlocal,

\[
D_- (\delta \bar{u} u^{-1}) = \mu [\tilde{\epsilon}_L, (g^{-1} \Psi_L g)^\dagger] \Rightarrow \bar{u}^{-1} \delta \bar{u} = \mu (\partial_-)^{-1} (\bar{u}^{-1} [\tilde{\epsilon}_L, (g^{-1} \Psi_L g)^\dagger] \bar{u}) .
\]

### 3.3 Supercurrent and closure

The characteristic feature of supersymmetric systems is the existence of conserved supercurrent by which the corresponding Noether supercharges can be constructed.

We vary \( S'_{\text{tot}} \) (3.1) with respect to the supergroup variations, in which the substitution \( \epsilon_L \to \epsilon_L (\sigma^+, \sigma^-) \) was made. Then the components of the supercurrent can be found from

\[
\delta S'_{\text{tot}} = \int d^2 \sigma \text{STr}(\partial_+ \epsilon_L J_- + \partial_- \epsilon_L J_+) ,
\]

whence

\[
\begin{align*}
J_+ &= \bar{u}^{-1} [(g^{-1} D_+ g)^\|, [T, \Psi_R]]u + \mu [\partial_-^{-1} (\bar{u}^{-1} (g^{-1} \Psi_L g)^\dagger \bar{u}), \tilde{O}_+] ,
J_- &= -\mu \bar{u}^{-1} (g^{-1} \Psi_L g)^\dagger \bar{u} ,
\end{align*}
\]

with \( \tilde{O}_+ = \bar{u}^{-1} (g^{-1} \partial_+ g + g^{-1} A_+ g - 2 T \Psi_R^2 \tilde{A}_+ h) \bar{u} \), \( \tilde{A}_+ = -\partial_+ \bar{u} u^{-1} \). The supercurrent (3.4) obeys the standard conservation law

\[
\partial_+ J_- + \partial_- J_+ = 0 .
\]

It is interesting to find the on-shell closure of the supersymmetry transformations. It is convenient to single out independent \( \mathcal{N} = (2, 2) \) subalgebras and to study the closure inside each subalgebra. For \( n = 2 \) there is only one such subalgebra, while in the cases \( n = 3 \) and \( n = 5 \) there are two and four ones, respectively.

For the left-chiral supersymmetries the Lie brackets of supersymmetry transformations have the unique form for all involved fields:

\[
\begin{align*}
(\delta_1 \delta_2 - \delta_2 \delta_1) g &= -2a^+ \partial_+ g - 2a^+ A_+ g + g (2a^+ A_+ + \dot{Q}) , \\
(\delta_1 \delta_2 - \delta_2 \delta_1) \Psi_R &= -2a^+ \partial_+ \Psi_R + [\Psi_R, 2a^+ A_+ + \dot{Q}] , \\
(\delta_1 \delta_2 - \delta_2 \delta_1) \Psi_L &= -2a^+ \partial_+ \Psi_L - [2a^+ A_+, \Psi_L] .
\end{align*}
\]
Here $Q$ is a field-dependent $h$-valued matrix. The structure of the closure is nicely transparent: it is a sum of the $2d\,\sigma^+$ translations (with the bracket parameter $a^+$) and compensating $H \times H$ gauge transformations. While checking the closure, we used some on-shell gauge and $\Psi_{L,R}$ field equations.

It still remains to calculate the on-shell Lie brackets between different pairs of $\mathcal{N} = (2,2)$ transformations and between the transformations with the (anti)holomorphic parameters within each pair.

4 Summary and outlook

We have shown that the actions of the Pohlmeyer-reduced $AdS_3 \times S^3$ and $AdS_5 \times S^5$ superstrings possess non-locally realized chiral worldsheet $\mathcal{N} = (4,4)$ and $\mathcal{N} = (8,8)$ supersymmetries. These systems can thus be treated as a new type of superextensions of the gauged mass-deformed gWZW models.

Though we started from a modified gWZW action, it follows from our consideration that the original Grigoriev-Tseytlin action is also invariant under non-local supersymmetries, namely, under

$$\delta_{\epsilon_L} g = g([T, [\Psi_R, \tilde{\epsilon}_L]] + \delta \tilde{h}), \quad \delta_{\epsilon_L} \Psi_R = [(g^{-1} D_+ g)^\parallel, \tilde{\epsilon}_L] + [\Psi_R, \delta \tilde{h}],$$

$$\delta_{\epsilon_L} \Psi_L = \mu[T, g \tilde{\epsilon}_L g^{-1}], \quad \delta A_\pm = 0, \quad \text{where } \delta \tilde{h} = \mu(D_-)^{-1}[\tilde{\epsilon}_L, (g^{-1} \Psi_L g)^\perp].$$

This agrees with the independent analysis of Hollowood & Miramontes [18] (see also [19]). They showed that the on-shell worldsheet Poincaré supersymmetry of the PR $AdS_5 \times S^5$ superstring actually extends to the symmetry $psu(2|2) \oplus psu(2|2)$. It is an open question whether the latter can be somehow continued off shell.

The PR form of $AdS_5 \times S^5$ superstring is UV finite at one loop and there are strong indications that this extends to all loops [20]. This property is related to the hidden $\mathcal{N} = (8,8)$ supersymmetry [21]. It is likely that in the full quantum theory this supersymmetry is deformed into a sort of quantum supergroup.

To analyze all these issues in full generality, it seems very important to work out the appropriate off-shell superfield formalism. Could Harmonic Superspace [22] be helpful for this purpose?

Acknowledgements

E. Ivanov thanks the Organizers of the QTS-7 Conference for inviting him to participate and for the warm hospitality in Prague. A partial support from the RFBR grants Nr. 09-02-01209, Nr. 11-02-90445 and a grant of the Heisenberg-Landau Program is cordially acknowledged.
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