Parameters of Sand-Tyre Chips Mixture for Hardening Soil Small Constitutive Model

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Abstract. Hardening Soil model with the small strain extension (HSS) is lately one of the most popular constitutive models to describe soil behaviour. It is versatile – includes the phenomena of shear strength, stress history, dilatancy, volumetric and shear hardening, hyperbolic stress-strain relationship in axial compression, stiffness dependency on stress and its degradation with strain, as well as the regain of the high stiffness after sharp loading reversals. Even though the model is advanced and complex, accordingly to its authors, it is relatively easy to calibrate based on results of standard tests and empirical formulas. In this paper an attempt was undertaken to estimate the parameters of untypical anthropogenic soils – mixtures of sand and scrap tyre rubber in order to build a database for future numerical analyses. A literature review was conducted and, eventually, the material parameters were determined based on results of a series of laboratory tests (cyclic and monotonic triaxial with bender elements, direct shear) published by researchers of Wollongong University of Australia.

1. Introduction

Design of any geotechnical structure with the use of a numerical analysis requires a thoughtful choice of the constitutive model and its parameters’ values. The choice of the model depends on the geotechnical category of the structure and character of the service loads. Wherever the displacements play an important role, the simplest and most commonly used linear elastic – perfectly plastic model with the Coulomb-Mohr failure surface becomes unreliable and should be replaced with an advanced constitutive model taking into account the non-linear dependencies between stress and strain in the pre-failure state. Unless the project is calculated by a professional experienced in development of her/his own constitutive models, a geotechnical engineer will rather choose a model that is available in a commercial software. A good example is the Hardening Soil Small (HSS) model offered in many popular finite element (FEM) codes, like Plaxis [4] or Z_Soil.PC [5]. The HSS model is actually the Hardening Soil model developed by Schantz et al. [1] and extended by Benz and co-workers [2][3] to improve the simulation of soil stiffness within the small strain zone (shear strains $\gamma$ from $10^{-6}$ to $10^{-3}$). It is versatile – includes the phenomena of shear strength (Coulomb-Mohr failure criterion), stress history, dilatancy, volumetric and shear hardening, hyperbolic stress-strain relationship in axial compression, stiffness dependency on stress and its degradation with strain, as well as the regain of the high stiffness after sharp loading reversals. As claimed by the authors of the model, it is relatively easy to implement as the parameters’ values can be determined from standard testing and empirical formulas [6].

The same requirements in terms of the choice of a constitutive model apply to anthropogenic soils that are to be used in geoengeering works. Recently, this role is very often taken by industrial wastes,
like carbonaceous shales from coal mining, fly ashes and slags from coal and lignite combustion, etc. Since the 90-ties of the XX century this group includes also tyre derived aggregates (TDA), which are made by shredding scrap car tyres to various fractions. They are used alone or mixed with natural soils in road embankments, backfills of retaining walls, drainage, thermal insulation or vibration damping layers around foundations. As the solution is relatively new, the database of the geotechnical parameters’ values of TDA obtained in the controlled stress and strain conditions is still not large – especially if models more advanced than elastic-perfectly plastic are concerned. One of the reasons why rubber is added to natural soils (mostly sands) is to reduce the influence of cyclic loading on structures – in such a case it is important to use a model capable of good simulation of hysteretic elastic behaviour. As long as the cyclic mobility is not crucial, this can be provided by the HSS model [5]. It shall be however mentioned that if the rubber is to be used as vibration isolation layer – other constitutive models, more advanced in terms of dynamic soil behaviour (liquefaction), shall be considered.

2. Hardening Soil Small model parameters
In the HSS model a multi-surface yield criterion is used to calculate plastic strains (figure 1). Hardening is isotropic and depends on plastic shear and volumetric strain (cap mechanism). A non-associated flow rule is assumed for the frictional hardening and an associated flow rule – for the cap hardening. The stiffness degradation with strain is described with the use of hyperbolic Hardin-Drnevich relation.

To analyse a numerical problem with the use of HSS model, in the version applied in Z_Soil.PC program [5], it is necessary to estimate the values of the following 17 parameters:

1) $E_0^{ref}$ [kPa] – the initial tangent slope of the axial strain–stress intensity ($\varepsilon_1 - q$) curve at the reference minor principal stress $\sigma_3^{ref}$ (figure 2a),
2) $\gamma_0$ [-] – the characteristic shear strain level $\gamma_s$ at which the ratio of the current and initial shear modulus $G/G_0 = 0.722$ (figure 2b),
3) $E_0^{ref}$ [kPa] – the unloading/reloading stiffness at engineering strains ($\varepsilon_1 \approx 10^{-3}$) at the reference minor principal stress $\sigma_3^{ref}$ (figure 2a),
4) $E_50^{ref}$ [kPa] – the secant stiffness at 50% of the ultimate deviatoric stress $q_f$ at the reference minor principal stress $\sigma_3^{ref}$,  
5) $m$ [-] – the exponent in equation (1) defining the dependence of stiffness moduli ($E = E_w$ or $E_50$) on stress (effective reference minor stress $\sigma_3^{ref}$, minor stress $\sigma_3 -$ not smaller than the limit minor stress $\sigma_L$ – by default: $\sigma_L = 10$ kPa), ‘cohesion’ $c'$ and internal angle of friction $\varphi$. 

![Figure 1. Multisurface yield criterion in HSS model](image)
\[
E = E^{ref} \left( \frac{\sigma'_3 + c' \cdot \cot \varphi}{\sigma'^{ref} + c' \cdot \cot \varphi} \right)^m
\]

1) \(W\) [MJ] – the Poisson’s ratio of minor and major strain \(\varepsilon_3/\varepsilon_1\) in an unloading-reloading cycle (elastic deformations),
2) \(R_f\) [-] – used to compute the hardening parameter \(\gamma_{PS}\) with the use of the asymptotic deviatoric stress \(q_s\) defining the hyperbolic function \(f_2\) (by default: \(R_f = 0.9\) (see figure 1 and figure 2a),
3) \(c'\) [kPa] – ‘effective cohesion’ – the intercept of the Mohr-Coulomb line at null stress condition,
4) \(\varphi'\) [°] – ‘effective angle of friction’ – the slope of the Mohr-Coulomb yield criterion,
5) \(y\) [°] – ‘dilatancy angle’ – the maximal slope of axial strain–volumetric strain \((\varepsilon_1 - \varepsilon_{vol})\) curve,
6) \(\nu_{ur}\) [-] – the Poisson’s ratio of minor and major strain \(\varepsilon_3/\varepsilon_1\) in an unloading-reloading cycle (elastic deformations),
7) \(R_f\) [-] – used to compute the hardening parameter \(\gamma_{PS}\) with the use of the asymptotic deviatoric stress \(q_s\) defining the hyperbolic function \(f_2\) (by default: \(R_f = 0.9\) (see figure 1 and figure 2a),
8) \(c'\) [kPa] – ‘effective cohesion’ – the intercept of the Mohr-Coulomb line at null stress condition,
9) \(\varphi'\) [°] – ‘effective angle of friction’ – the slope of the Mohr-Coulomb yield criterion,
10) \(y\) [°] – ‘dilatancy angle’ – the maximal slope of axial strain–volumetric strain \((\varepsilon_1 - \varepsilon_{vol})\) curve,
11) \(M\) [-] – controls Rowe’s dilatancy law in the contractancy domain (by default: \(D = 0.25\)),
12) \(e_{max}\) [-] – the maximal void ratio observed at the ultimate state (the cut-off limit),
13) \(f_t\) [kPa] – the maximal tensile strength of the material (by default: \(f_t = 0\) kPa),
14) \(D\) [-] – controls Rowe’s dilatancy law in the contractancy domain (by default: \(D = 0.25\)),
15) \(H\) [kPa] – defines the shape of the elliptical cap yield surface (see Figure 1),
16) OCR [-] or \(q_{POP}\) [kPa] – overconsolidation ratio or the maximum preoverburden pressure – sets the initial position of stress with respect to the cap surface – used to compute the hardening parameter \(\gamma_{PS}\) and preconsolidation pressure \(p_{c0}\),
17) \(K_{0SR}\) [-] – sets a historical position of the stress point \(\sigma^{SR}_3 (K_{0SR} = \sigma^{SR}_3 / \sigma^{SR}_v)\) with respect to the initial stress configuration for an overconsolidated material – used to compute the hardening parameter \(\gamma_{PS}\) and preconsolidation pressure \(p_{c0}\).

Figure 2. Definitions of: a) stiffness moduli, \(R_f\) and b) \(\gamma_{0.7}\) in HSS model

The HSS model parameters can be estimated based on laboratory and/or field test results. As far as anthropogenic soils are concerned the list gets practically limited to the laboratory tests, so the complete material investigation should include at least:

- 3 CID triaxial tests\(^2\) (consolidated isotropically with drained shearing) at three different effective confining stresses \(\sigma''_3\) with one unloading-reloading loop, bender element tests and local

\(^1\) Parameters \(M\) and \(H\) can be assessed based on oedometric modulus \(E_{oed}^{ref}\) obtained for primary loading (Normal Consolidation Line) at the reference stress \(\sigma_{oed}^{ref}\) and on the coefficient of earth pressure at rest \(K_0^{NC}\)

\(^2\) Optionally, to speed up the estimation of the parameters, instead of 3 CID tests – 1 CID and 2 CIU triaxial tests (consolidated isotropically with undrained shearing) could be conducted, unless there is not enough evidence to assume the values of the parameters: \(\gamma_{0.7}\) and \(m\).
measurement of strains to identify: $E_{oed}^{ref}, \gamma_{oed}^{ref}, E_{at}^{ref}, \nu_{at}, E_{oed}^{ref}, m, c', \varphi', \psi, R_s, D, e_{max}$ (the latter on a dense specimen)

- 1 OED oedometric test at $\sigma_{oed}^{ref}$ vertical stress to identify: $E_{oed}^{ref}, OCR$ and preconsolidation pressure $p_o$,

- If assumed that the material tested behaves in a manner similar to natural soils – some parameters’ values may be assumed: e.g. $D = 0.25$, $f_r = 0$ kPa (otherwise an isotropic extension test would be needed), for normally consolidated conditions $K_0^{SR}$ can be calculated using the Jaky’s formula:

$$K_0^{SR} = K_0^{NC} = 1 - \sin{\varphi'}$$

(otherwise a $K_0$ consolidation triaxial test or an oedometric test with measurement of horizontal stress is necessary).

Note, that even though the triaxial and oedometric tests are treated as ‘standard’ laboratory tests, it is not always easy to find a laboratory, whose apparatuses enable testing specimens of size large enough to be representative for anthropogenic soils. Also, bender elements and local strain transducers are still not typical accessories of triaxial apparatuses available in commercial laboratories. Theoretically a solution could be field tests (e.g. seismic cone penetration SCPT and/or dilatometer SDMT) conducted on trial embankments or in large size test chambers, however their interpretation would be reliable only if large set of stress/strain controlled test data was already available for their calibration [12].

The laboratory testing campaign can be a time and cost consuming procedure, but is necessary to identify the material parameters within the desired range of stress/strain and in the conditions as close as possible to the individual case. On the other hand, if the numerical model is to be used at the initial design stage or where the choice of the material is still under question – it is reasonable to estimate the parameters based on a literature review. Such an approach has been applied here – this paper presents estimation of the HSS model parameters for mixtures of sand and various amounts of scrap tyre rubber based on laboratory tests results published by other researchers. Some of the obtained parameter values were used in the numerical analysis of a flexible PVC pipe backfilled with a mixture of sand and 40% (by weight) of tyre chips to check the potential of such a solution; its results were presented at WMCAUS 2020 [11].

3. Material properties

The physical and mechanical properties of rubber and sand-rubber mixtures have been the subject of many studies, but most publications focus on only one or two selected aspects, giving too little data to evaluate all the parameters of the HSS model. One of the exceptions is the series of laboratory experiments: monotonic and cyclic triaxial, measurements of shear wave velocity and direct shear tests conducted by Dr M. S. Mashiri and her research group from the Wollongong University in Australia. The published results of their work [7][8][9][10] have been used here for estimation of the HSS model parameters.

The natural soil used in the mixtures was a poorly graded (coefficient of uniformity $C_u = 1.58$) beach sand with the maximum size of the smallest 10%, 30%, 50% and 60% of the sample: $D_{90} = 0.24$ mm, $D_{50} = 0.30$ mm, $D_{20} = 0.35$ mm and $D_{10} = 0.38$ mm, respectively. It was mixed with scrap tyre chips (TCh) in various proportions: $\zeta = 0, 10\%, 20\%, 30\%$ or $40\%$ (by weight). The chips were rectangular pieces of uniform thickness of about 6 mm, with no metal reinforcement, and the maximum width and length not exceeding 8 and 22 mm, respectively. The specific gravities of the sand and TCh were: 2.62 and 1.12, respectively. The relative density ($D_r$) of all the specimens was 50%.

Below presented is estimation of the HSS model parameters of the sand-tyre chips mixtures (STCh) based on the mentioned publications with all the obtained values and explanation shown in table 1. They have been grouped (just like it is used in the Z_Soil.PC program) into:
• physical parameters: unit bulk weight \( \gamma \) and initial void ratio \( e_0 \);
• parameters describing stiffness: \( E_0^{ref}, \gamma_0, \gamma, E_{ref}^{st}, E_{so}^{ref}, \nu_{ur}, m \) and \( \sigma_t \);
• parameters describing shear mechanism: \( c', \varphi', \psi, R_s, e_{max}, f_i \) and \( D \);
• parameter describing volumetric (cap) mechanism: \( E_{cap}^{ref} \);
• initial state variables representing the soil history: OCR [-] or \( q^{ref} \) and \( K_0^{scr} \).

Some of the values were obtained directly from the published texts or tables and some had to be calculated based on the values read from the printed graphs, which obviously may be burdened with some error resulting from the quality of the print. In table 1 light-grey colour has been used to mark the intermediate parameters used for further calculations of the HSS model parameters listed above.

3.1. Physical parameters

The specific unit weights \( \gamma_s \) of the sand and tyre chips are equal to \( \gamma_s^{so} = 26.2 \, \text{kN/m}^3 \) and \( \gamma_s^{TCh} = 11.2 \, \text{kN/m}^3 \) [9]. The values of \( \gamma_s \) for sand-rubber mixtures with various rubber contents (\( \xi \)) were calculated based on the formula:

\[
\gamma_s^{\xi} = \frac{1}{\frac{\xi}{\gamma_s^{TCh}} + \frac{1 - \xi}{\gamma_s^{so}}}
\]

(3)

The initial void ratio \( e_0 \) was calculated based on the values of the minimum and maximum void ratios (see [9]) and assumption that all the specimens were tested at \( D_s = 50\% \). Then, assumed that all the specimens were initially dry (water content \( w = 0\% \)) the bulk unit weights \( \gamma \) could be calculated from:

\[
\gamma^{\xi} = \gamma_s^{\xi} / (1 + e_0^{\xi})
\]

(4)

3.2. Stiffness parameters

The maximum (initial) shear stiffness moduli \( G_0 \) of the STCh mixtures were evaluated based on the results of bender element tests conducted by Mashiri et al. [7] on triaxial specimens with diameter equal to 50 mm and height equal to 100 mm. The specimens were saturated and isotropically consolidated to the effective confining stresses: 23, 46, 69 and 138 kPa. The shear moduli \( G_0 \) were calculated based on the formula:

\[
G_0 = q \cdot V_s^2
\]

(5)

where \( \rho \) is the specimen’s density at the end of the consolidation stage and \( V_s \) is the shear wave velocity obtained by using the \( \pi \)-point identification method [13] to determine the shear wave travel time. Eventually the values of \( G_0 \), for the reference stress \( \sigma_3^{ref} = 69 \, \text{kPa} \) (\( G_0^{ref} \)) are shown in table 1. In the very small strain zone, based on the theory of elasticity, the Young modulus \( E_0^{ref} \) can be calculated based on the shear modulus \( G_0^{ref} \) and Poisson’s ratio \( \nu_{ur} \), using the formula:

\[
E_0 = 2G_0(1 + \nu_{ur})
\]

(6)

As the Poisson ratio \( \nu_{ur} \) was not determined in [7], the values of \( \nu_{ur} \) for the effective stress 69 kPa and rubber contents 0%, 10% and 40% were taken from [9], where they were back-calculated from the values of the initial tangent bulk and shear moduli \( (K_i \) and \( G_i \), respectively) obtained from the shearing \( (q - e_0) \) and compressibility \( (p^*-e_0) \) characteristics in monotonic triaxial shearing, where \( e_0 = 2/3(e_1 - e_3) = 2/3\gamma \)
and $\varepsilon_p = \varepsilon_{vol} = \varepsilon_1 + 2\varepsilon_3$. For the intermediate rubber contents of 20% and 30% the $\nu_{ur}$ values were assumed as equal to 0.38.

The shear strain level $\gamma_{0.7}$ at which the ratio of the current and initial shear modulus $G/G_0 = 0.722$ could be read from figure 3 [7] or calculated based on the transformed formula given there:

$$G_s = \frac{G_0}{1 + (\gamma/\gamma_r)[1 + a \cdot \exp(-b \cdot \gamma/\gamma_r)]}$$  \hspace{1cm} (7)

where $a$ and $b$ are the soil constants, $\gamma$ is the shear strain (here $\gamma_{0.7}$) and $\gamma_r$ is the reference shear strain. The $\gamma_r$ is the ratio of the maximum shear stress at failure to the maximum shear modulus $G_0$ and was evaluated in [7] based on monotonic triaxial tests [8] – its values are presented in table 1. The parameters $a$ and $b$ were obtained by statistical curve fitting and can be calculated by means of the formulas:

$$a = -3.63 \ln \left( \frac{G_0}{p_a} \right) + 26.65$$ \hspace{1cm} (8)

$$b = -0.40 \ln \left( \frac{G_0}{p_a} \right) + 2.44$$ \hspace{1cm} (9)

where $p_a$ is the atmospheric pressure (equal to 102.3 kPa).

None of the analysed papers presented results of monotonic triaxial tests with an unloading-reloading loop, necessary for direct determination of $E_{ur}^{ref}$ value, however, based on instruction [5] this value can be assumed as the stiffness at engineering axial strains $\varepsilon \approx 10^{-3}$. In the analysed case, first, the $G_{ur}^{ref}$ was calculated based on equation (7) for shear strain $\gamma_{ur} = \frac{2}{3}(1 - \nu_{ur}) \varepsilon$ and then $E_{ur}^{ref}$ was calculated from equation (6), where instead of $E_{0} - E_{ur}$ was used.

The $E_{50}^{ref}$ modulus at $\sigma_3^{ref} = 69$ kPa for clean sand (0% rubber) was calculated based on the shearing characteristic curve ($q - \varepsilon_q$) and strain path ($\varepsilon_p - \varepsilon_q$) obtained from the triaxial monotonic test presented in [9] (figure 4 a, b). For other rubber contents available were only results for confining stress 138 kPa (figure 4 c, d). From these graphs the 50% $q_i$ values were determined and based on the corresponding $\varepsilon_p$ and $\varepsilon_q$ the axial strain $\varepsilon_1$ was obtained, which was then used to calculate the stiffness modulus $E_{50}$. The $E_{50}$, $E_{ur}$ and $E_0$ should be given for the same reference stress 69 kPa, so the values of $E_{50}^{ref}$ were calculated with equation (1) based on $E_{50}^{138\text{ kPa}}$, $m$, $c'$ and $\varphi$. 

![Figure 3. Shear modulus degradation with strain – based on [7]](image-url)
The procedure of determination of the exponent $m$ in equation (1) suggested in [5] requires plotting a rectilinear trend line $y = ax + b$, where $y = \ln E_{50}, x = \ln((\sigma_3^i + c \cdot \cot \phi)/(\sigma_3^{ref} + c \cdot \cot \phi))$ and $a = m$ – see figure 5. The values of $E_{50}$ for confining stresses $\sigma_3 = 23, 46, 69$ and 138 kPa were obtained based on figure 4. As there are available only results for clean sand and sand with 35% rubber content – the $m$ values for all the sand-rubber mixtures were assumed equal to 0.39 (the value obtained for $\xi = 35\%$).

Figure 4. Graphs used for determination of $E_{50}^{ref}$ and $\psi$ – based on [9]

The minimal limiting minor stress $\sigma_L$ was assumed equal 10 kPa in order to avoid zero stiffness when the mean effective stress $p'$ is close to 0.

3.3. Shear mechanism parameters
The Coulomb-Mohr shear strength parameters $c'$ and $\phi'$ for $\xi = 0\%$ and 40% were obtained based on peak strength values determined in monotonic triaxial tests by Mashiri [9]. The values for 10% and 30% rubber contents were calculated based on the results of direct shear tests presented by Vinod et al. [10]. Unfortunately no results of shear strength tests were found for sand-rubber mixtures with 20% rubber content, so the values were assumed as equal to the average of the ones for 10 and 30%.

Figure 5. Determination of parameter $m$
The dilatancy angle $\psi$ can be obtained using formula:

$$\psi = -\arcsin \frac{d}{2 - d}$$  \hspace{1cm} (10)

where $d$ is the maximum inclination of $\varepsilon_p - \varepsilon_l$ curve. The strain paths in the available publications were presented in the $\varepsilon_p - \varepsilon_q$ space, so the $d$ values had to be calculated using formula:

$$d = \frac{d^\ast}{1 + d^\ast/3}$$  \hspace{1cm} (11)

where $d^\ast$ is the maximum inclination of $\varepsilon_p - \varepsilon_q$ strain path. The $d^\ast$ values were obtained from the monotonic drained triaxial tests carried out by Mashiri [9] at 138 kPa confining stress (figure 4d). It can be noticed that $\psi$ value decreases almost linearly with the content of rubber. Based on [8] it shall be noted that the $d^\ast$ value decreases with increase of the mean effective stress $p^\prime$, however this phenomenon is not simulated in HSS model.

The maximum void ratio $e_{max}$, determined based on ASTM D 4253-16 [15], was read from [7].

The failure ratio value $R_f$ can be calculated based on $(\varepsilon_1/q - \varepsilon_1)$ graph – being a result of monotonic triaxial test. This kind of a graph was not available in none of the analysed papers, thus the $R_f$ value was assumed as being equal to 0.9 (a default value in Z_Soil.PC).

The tensile strength of all the materials $f_t$ was assumed as equal to zero. The default value of $D = 0.25$ was assumed as well.

3.4. Initial state and volumetric cap mechanism parameters

None of the available research included results of oedometric tests on the analysed sand-tyre chips mixtures, thus, accordingly to the instructions in [5], the oedometric modulus $E_{oed}^{ref}$ for the reference stress $\sigma^{\prime}o^{ref} = 138$ kPa was calculated based on formula:

$$E_{oed}^{138 \text{kPa}} = E_{50}^{138 \text{kPa}}(K_0^{NC})^m$$  \hspace{1cm} (12)

where the coefficient of lateral stress at rest $K_0^{NC}$ was assumed for normally consolidated soil ($OCR = 1$) based on the Yaky’s formula (2).

The minimum preconsolidation stress $p_{o,\min}$ was assumed as equal to about 10 kPa to avoid numerical errors at the finite elements close to the simulated terrain surface.

4. Summary and conclusions

When estimating parameters for a constitutive model based on several literature sources special attention must be paid to verify whether the described materials are exactly the same. This process becomes easier if the needed parameters constitute the direct purpose of the available research works.

In case of the HSS model parameters for the selected sand-tyre chips mixtures the data from 4 publications by the same team of authors were gathered. They referred to the physical properties of the materials (grading, densities, maximum and minimum void ratios) and various aspects of their behaviour – with much attention paid to the shear strength, dilatancy, stiffness and its degradation with strain. This made a valuable source of information, even though some data were scattered across various papers.
| Parameter symbol | unit | 0% | 10% | 20% | 30% | 40% | source, comments |
|------------------|------|----|-----|-----|-----|-----|-----------------|
| Physical parameters |      |    |     |     |     |     |                 |
| $\gamma$ | kN/m$^3$ | 26.2 | 23.1 | 20.7 | 18.7 | 17.1 | tab. 3.1 in [9]; eq. (3) |
| $e_0$ | - | 0.66 | 0.55 | 0.43 | 0.36 | 0.37 | based on tab 1. ($e_{max}$, $e_{min}$) in [9]; for $D_r = 50\%$ |
| $\gamma$ | kN/m$^3$ | 15.8 | 14.9 | 14.5 | 13.7 | 12.5 | 0% and 100% from [7]; others based on eq. (3) and (4) |
| Stiffness parameters |      |    |     |     |     |     |                 |
| $\nu_{ur}$ |      | - | 0.37 | 0.39 | 0.38 | 0.38 | 0.37 | table 5.2 in [9] |
| $G_{ur}^{69 \text{kPa}}$ | kPa | 74 553 | 56 566 | 53 271 | 39 103 | 26 429 | table 3 in [7] |
| $E_{ur}^{69 \text{kPa}}$ | kPa | 204 275 | 157 253 | 147 028 | 107 924 | 72 415 | eq. (6) |
| $G_{ur}^{0.722 \text{G}_0}$ | kPa | 53 827 | 40 841 | 38 462 | 28 232 | 19 082 |                 |
| $G_{ur}^{69 \text{kPa}}$ | kPa | 55 954 | 46 688 | 47 457 | 39 042 | 28 324 |                 |
| $E_{ur}^{69 \text{kPa}}$ | kPa | 32 813 | 30 067 | 17 293 | 10 115 | 5 790 | fig. 4.3 & 4.4 in [9] |
| $G_{ur}^{50 \text{kPa}}$ | kPa | 20 671 | 17 293 | 13 785 | 8 160 | 4 693 | 0%: fig. 4.5 & 4.6 in [9]; other: calc. using eq. (1) |
| $\sigma_L$ | kPa | 10 | 10 | 10 | 10 | 10 | assumed |
| Shear mechanism parameters |      |    |     |     |     |     |                 |
| $\varphi$ | deg | 39.7 | 35.5 | 37.1 | 38.7 | 35.9 | 0% & 40%: tab. 4.1. in [9]; 10% & 30%: fig. 6 in [10]; 20% - mean |
| $c'$ | kPa | 3.3 | 8 | 14 | 20 | 20.1 |                 |
| $d$ | | 0.642 | 0.547 | 0.416 | 0.316 | 0.223 | Figure 4d + eq. (11) (at 138 kPa) |
| $\psi$ | deg | 28.2 | 22.1 | 15.2 | 10.8 | 7.2 | eq. (10) |
| $R_f$ | - | 0.9 | 0.9 | 0.9 | 0.9 | 0.9 | default value |
| $e_{max}$ | - | 0.77 | 0.66 | 0.52 | 0.45 | 0.49 | table 1 in [7] |
| $f_r$ | kPa | 0 | 0 | 0 | 0 | 0 | assumed |
| $D$ | - | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | assumed |
| Stress history and volumetric cap mechanism parameters |      |    |     |     |     |     |                 |
| $K_{NC}^{e_{max}}$ | - | 0.36 | 0.42 | 0.40 | 0.37 | 0.41 | Yaky’s formula (2) |
| $OCR$ | - | 1 | 1 | 1 | 1 | 1 | assumed |
| $E_{oed}^{38 \text{kPa}}$ | kPa | 24 673 | 24 985 | 12 059 | 6 898 | 4 103 | eq. (12) |
| $p_{oed}^{min}$ | kPa | 10 | 10 | 10 | 10 | 10 | assumed |

* assumed based on results for other rubber contents
The only problem with determination of the HSS model parameters was related to the fact that the data were sometimes presented by the authors in other layout than the one required in the FEM program (Z_Soil.PC) (eg. shear modulus $G$ instead of Young modulus $E$) or at other confining stress value, which required some additional calculations.

Obviously, sand-rubber mixtures are not natural soils for which some suggested parameter values or value ranges are commonly available (see eg. [5]) based on the extensive experimental and theoretical knowledge. Some relations, however, are true also for this anthropogenic soils – a good example being the relation between the stiffness moduli: $E_{ref} > E_{ext} > E_{int}$. In the cases, where parameter values could not be obtained, some assumptions had to be done ($R_t$, $f_s$, $D$, $E_{mod}$), which need later verification.

Just like for any other materials, the best method to achieve reliable numerical simulations of sand-rubber layers in geoeengineering problems (within the possibilities of the chosen constitutive model) is to conduct own experimental campaign in order to evaluate the values of all the necessary model parameters. However, for an initial numerical analysis of a problem, in which STCh mixtures are to be used, the parameter values of HSS model listed in table 1 shall prove helpful.

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