Estimation of the Mass Outflow Rate From Compact Objects

Sandip K. Chakrabarti
S.N. Bose National Centre for Basic Sciences, JD-Block, Sector III, Salt Lake, Calcutta 700091, India

ABSTRACT

Outflows are common in many astrophysical systems which contain black holes and neutron stars. Difference between stellar outflows and outflows from these systems is that the outflows in these systems have to form out of the inflowing material only. The inflowing material can form a hot and dense cloud surrounding the compact object either because of centrifugal barrier or a denser barrier due to pair plasma or pre-heating effects. This barrier behaves like a stellar surface as far as the mass loss is concerned. We estimate the outflow rate from such considerations. These estimated rates roughly match with the rates in real observations as well as those obtained from numerical experiments.

Subject headings: accretion, accretion disks – black hole physics – jets – outflows – shock waves – stars: neutron – stars: individual (SS 433)

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1. INTRODUCTION

Cosmic radio jets are believed to be originated from the centers of active galaxies which harbor black holes (e.g., Chakrabarti 1996a, hereafter C96a). Even in so called ‘micro-quasars’, such as GRS 1915+105 which are believed to have stellar mass black holes (Mirabel & Rodríguez, 1994), the outflows are common. The well collimated outflows in SS 433 is well known for almost two decades (Margon, 1984). Similarly, systems with neutron stars also show outflows as is believed to be the case in X-ray bursters (e.g. Titarchuk, 1994).

There are large number of works in the literature which attempt to explain the origin of these outflows. These works can be broadly divided into three sets. In one set, the jets are believed to come out due to hydrodynamic or magneto-hydrodynamic pressure effects and are treated separately from the disks (e.g., Blandford & Payne 1982; Fukue 1982; Chakrabarti 1986; Contopoulos 1995). In another set, efforts are made to correlate the disk structure with that of the outflow (e.g., Königl 1989; Chakrabarti & Bhaskaran 1992). In the third set, numerical simulations are carried out to actually see how matter is deflected from the equatorial plane towards the axis (e.g., Hawley, Smarr & Wilson 1985; Eggum, Coroniti & Katz, 1985; Molteni, Lanzafame & Chakrabarti 1994; Molteni, Ryu & Chakrabarti 1996). Nevertheless, the definitive understanding of the formation of outflows is still lacking, and more importantly, it has always been difficult to estimate the outflow rate from first principles. In the first set, the outflow is not self-consistently derived from the inflow. In the second set, only self-similar steady solutions are found and in the third set, either a Keplerian disk or a constant angular momentum disk was studied, neither being the best possible assumption. On the other hand, the mass outflow rate of the normal stars are calculated very accurately from the stellar luminosity. Theory of radiatively driven winds seems to be very well understood (e.g., Castor, Abott & Klein, 1975). Given that the black holes and the neutron stars are much simpler celestial objects, and the flow around them is sufficiently hot to be generally ionized, it should have been simpler to compute the outflow rate from an inflow rate than the methods employed in stellar physics.

Our approach to the mass outflow rate computation is somewhat different from that used in the literature so far. Though we consider simple minded equations to make our points, such as those applicable to conical inflows and outflows, we add a fundamental ingredient to the system, whose importance is being revealed only very recently in the literature. This is the quasi-spherical centrifugally supported dense atmosphere of typical size 5 to 10 Schwarzschild radius around a black hole and a neutron star. Whether a shock actually forms or not, this dense region exists, as long as the angular momentum of the flow close to the compact object is roughly constant and is generally away from a Keplerian distribution as is the case in reality (Chakrabarti, 1989 [hereafter C89]; Chakrabarti 1996b; hereafter C96b). This centrifugally supported region (which basically forms the boundary layer of black holes and weakly magnetized neutron stars) successfully replaced the so called ‘Compton cloud’ (Chakrabarti & Titarchuk 1995 [hereafter CT95]; Chakrabarti, Titarchuk, Kazanas & Ebisawa 1996; Chakrabarti, 1997 [hereafter C97]) in explaining hard and soft states of black hole candidates, and the converging flow property of this
region successfully produced the power-law spectral slope in the soft states of black hole candidates (CT95; Titarchuk, Mastichiadis & Kylafis 1997). The oscillation of this region successfully explains the general properties of the quasi-periodic oscillation (Molteni, Sponholz & Chakrabarti, 1996; Ryu, Chakrabarti & Molteni 1997) from black holes and neutron stars. It is therefore of curiosity if this region plays any major role in formation of outflows.

Several authors have also mentioned denser regions due to different physical effects. Chang & Ostriker, 1985 showed that pre-heating of the gas could produce standing shocks at a large distance. Kazanas & Ellison (1986) mentioned that pressure due to pair plasma could produce standing shocks at smaller distances around a black hole as well. Our computation is insensitive to the actual mechanism by which the boundary layer is produced. All we require is that the gas should be hot at the region where the compression takes place. Thus, since Comptonization processes cool this region (CT95) for larger accretion rates ($\dot{M} \gtrsim 0.1 \dot{M}_{Edd}$) our process is valid only for low-luminosity objects, consistent with current observations. Begelman & Rees (1984) talked about a so-called ‘cauldron’ model of compact objects where jets were assumed to emerge from a dense mixture of matter and radiation by boring de-Laval nozzle as in Blandford & Rees (1974) model. The difference between this model and the present one is that there very high accretion rate was required ($\dot{M}_m \sim 1000 \dot{M}_E$) while we consider thermally driven outflows of smaller accretion rate. Second, the size of the ‘cauldron’ was thousands of Schwarzschild radii (where gravity was so weak that channel has to have shape of a de Laval nozzle), while we have a CENBOL of about $10R_g$ (where the gravity plays an active role in creating the transonic wind) in our mind. Third, in the present case, matter is assumed to pass through a sonic point using the pre-determined funnel where rotating pre-jet matter is accelerated (Chakrabarti, 1984) and not through a ‘bored nozzle’ even though symbolically a quasi-spherical CENBOL is considered for mathematical convenience. Fourth, for the first time we compute the outflow rate completely analytically starting from the inflow rate alone. To our knowledge such a calculation has not been done in the literature at all.

Once the presence of our centrifugal pressure supported boundary layer (CENBOL) is accepted, the mechanism of the formation of the outflow becomes clearer. One basic criteria is that the outflowing winds should have positive Bernoulli constant (C89). Just as photons from the stellar surface deposit momentum on the outflowing wind and keeps the flow roughly isothermal (Tarafdar, 1988) at least up to the sonic point, one may assume that the outflowing wind close to the black hole is kept isothermal due to deposition of momentum from hard photons. In the case of the sun, it’s luminosity is only $10^{-5} L_{Edd}$ and the typical mass outflow rate from the solar surface is $10^{-14} M_\odot$ year$^{-1}$ (Priest, 1982). Proportionately, for a star with a Eddington luminosity, the outflow rate would be $10^{-9} M_\odot$ year$^{-1}$. This is roughly half as the Eddington rate for a stellar mass star. Thus if the flow is compressed and heated at the centrifugal barrier around a black hole, it would also radiate enough to keep the flow isothermal (at least up to the sonic point) if the efficiency were exactly identical. Physically, both requirements may be equally difficult to meet, but in reality with photons shining on outflows near a black hole with almost $4\pi$ solid angle
(from funnel wall) it is easier to maintain the isothermality in the slowly moving (subsonic) region in the present context. Another reason is this: the process of momentum deposition on electrons is more efficient near a black hole. The electron density \( n_e \) falls off as \( r^{-3/2} \) while the photon density \( n_\gamma \) falls off as \( r^{-2} \). Thus the ratio \( n_e/n_\gamma \propto r^{1/2} \) increases with the size of the region. Thus a compact object will have lesser number of electrons per photon and the momentum transfer is more efficient. In a simpler minded way the physics is scale-invariant, though. In solar physics, it is customary to chose a momentum deposition term which keeps the flow isothermal to be of the form (Kopp & Holzer, 1976),

\[
F_r = \int_{R_s}^r D dr
\]

where, \( D \) is the momentum deposition (localized around \( r_p \)) factor with a typical spatial dependence,

\[
D = D_0 e^{-\alpha(r/r_p-1)^2}
\]

Here, \( D_0, \alpha \) are constants and \( R_s \) is the location of the stellar surface. Since \( r \) and \( r_p \) comes in ratio, exactly same physical consideration would be applicable to black hole physics, with the same result provided \( D_0 \) is scaled with luminosity (However, as we showed above, \( D_0 \) goes up for a compact object.). However, as CT95 showed, high accretion rate (\( \dot{M} \gtrsim 0.3\dot{M}_{Edd} \)) will reduce the temperature of the CENBOL catastrophically, and therefore our assumption of isothermality of the outflow would severely breakdown at these high rates. It is to be noted that in the context of stellar physics, it is shown (Pauldrachi, Puls & Kudritzki, 1986) that the temperature stratification is the outflowing wind has little effect on the mass loss rate.

Having thus convinced that isothermality of the outflow, at least upto the sonic point, is easier to maintain near a black hole, we present in this paper a simple derivation of the ratio of the mass outflow rate and mass inflow rate assuming the flow is externally collimated. We find that the ratio is a function of the compression ratio of the gas at the boundary of the hot, dense, centrifugally supported region. We estimate that the outflow rate should generally be less than a few percent if the outflow is well collimated. Finally, in \( \S \)3, we draw our conclusions.

2. DERIVATION OF THE OUTFLOW RATE

Assume for the sake of argument that our system is made up of the infalling gas, the dense boundary layer of the compact object (CENBOL), and collimated outflowing wind. Figure 1 shows a schematic diagram of the system with the components marked. Matter near the equatorial plane is assumed to fall in conical shape onto the black hole or the neutron star. The sub-Keplerian, hot and dense, quasi-spherical region forms either due to centrifugal barrier or due to pair plasma pressure or pre-heating effects. The outflowing wind is assumed to be also conical in shape for simplicity and is flowing out along the axis. It is assumed that the wind is collimated by an external pressure. Both the inflow and the outflow are assumed to be thin enough so that the velocity and density variations across the flow could be ignored.
The accretion rate of the incoming accretion flow is given by,

$$\dot{M}_{\text{in}} = \Theta_{\text{in}} \rho \vartheta r^2.$$  

(1)

Here, $\Theta_{\text{in}}$ is the solid angle subtended by the inflow, $\rho$ and $\vartheta$ are the density and velocity respectively, and $r$ is the radial distance. For simplicity, we assume geometric units ($G = 1 = M_{\text{BH}} = c$; $G$ is the gravitational constant, $M_{\text{BH}}$ is the mass of the central black hole, and $c$ is the velocity of light) to measure all the quantities. In this unit, for a freely falling gas,

$$\vartheta(r) = \left[1 - \Gamma r^2\right]^{1/2}$$  

(2)

and

$$\rho(r) = \frac{\dot{M}_{\text{in}}}{\Theta_{\text{in}}} \left[1 - \Gamma/r^2\right]^{-1/2} r^{-3/2}$$  

(3)

Here, $\Gamma/r^2$ (with $\Gamma$ assumed to be a constant) is the outward force due to radiation.

We assume that the boundary of the denser cloud is at $r = r_s$ (typically a few Schwarzschild radii, see, Chakrabarti 1996b) where the inflow gas is compressed. The compression could be abrupt due to standing shock or gradual as in a shock-free flow with angular momentum. This details are irrelevant. At this barrier, then

$$\rho_+(r_s) = R \rho_-(r_s)$$  

(4a)

and

$$\vartheta_+(r_s) = R^{-1} \vartheta_-(r_s)$$  

(4b)

where, $R$ is the compression ratio. Exact value of the compression ratio is a function of the flow parameters, such as the specific energy and the angular momentum (e.g., C89; Chakrabarti 1990 [hereafter C90], Chakrabarti, 1996c). Here, the subscripts $-$ and $+$ denote the pre-shock and post-shock quantities respectively. At the shock surface, the total pressure (thermal pressure plus ram pressure) is balanced.

$$P_-(r_s) + \rho_-(r_s) \vartheta^2(r_s) = P_+(r_s) + \rho_+(r_s) \vartheta^2_+(r_s).$$  

(5)

Assuming that the thermal pressure of the pre-shock incoming flow is negligible compared to the ram pressure, using eqs. 4(a-b) we find,

$$P_+(r_s) = \frac{R - 1}{R} \rho_-(r_s) \vartheta^2(r_s).$$  

(6)

The isothermal sound speed in the post-shock region is then,

$$C_s^2 = \frac{P_+}{\rho_+} = \frac{(R - 1)(1 - \Gamma)}{R^2} \frac{1}{r_s} = \frac{(1 - \Gamma)}{f_0 r_s}$$  

(7)

where, $f_0 = R^2/(R - 1)$. An outflow which is generated from this dense region with very low flow velocity along the axis is necessarily subsonic in this region, however, at a large distance,
the outflow velocity is expected to be much higher compared to the sound speed, and therefore
the flow must be supersonic. In the subsonic region of the outflow, the pressure and density are
expected to be almost constant and thus it is customary to assume isothermality condition up to
the sonic point (Tarafdar, 1988). As argued in the introduction, in the case of black hole accretion
also, such an assumption may be justified. With isothermality assumption or a given temperature
distribution \(T \propto r^{-\beta}\) with \(\beta\) a constant) the result is derivable in analytical form. The sonic
point conditions are obtained from the radial momentum equation,
\[
\frac{\partial}{\partial r} \frac{d\vartheta}{dr} + \frac{1}{\rho} \frac{dP}{dr} + \frac{1-\Gamma}{r^2} = 0. \tag{8}
\]
and the continuity equation
\[
\frac{1}{r^2} \frac{d(\rho \vartheta r^2)}{dr} = 0 \tag{9}
\]
in the usual way, i.e., by eliminating \(d\rho/dr\),
\[
\frac{d\vartheta}{dr} = \frac{N}{D} \tag{10}
\]
where
\[
N = 2C_s^2 - \frac{1-\Gamma}{r^2}
\]
and
\[
D = \vartheta - \frac{C_s^2}{\vartheta}
\]
and putting \(N = 0\) and \(D = 0\) conditions. These conditions yield, at the sonic point \(r = r_c\), for an
isothermal flow,
\[
\vartheta(r_c) = C_s. \tag{11a}
\]
and
\[
r_c = \frac{1-\Gamma}{2C_s^2} = \frac{f_{or_s}}{2} \tag{11b}
\]
where, we have utilized eq. (7) to substitute for \(C_s\).

Since the sonic point of a hotter outflow is located closer to the black hole, clearly, the
condition of isothermality is best maintained if the temperature is high enough. However if the
temperature is too high, so that \(r_c < r_s\), then the flow has to bore a hole through the cloud just
as in the ‘cauldron’ model of Begelman & Rees (1984), although it is a different situation — here
the temperature is high, while in the ‘cauldron’ model the temperature was low. In reality, a
pre-defined funnel caused by centrifugal barrier does not require to bore any nozzle at all, but our
simple quasi-spherical calculation fails to describe this case properly. This is done in detail in Das
& Chakrabarti (submitted).

The constancy of the integral of the radial momentum equation (eq. 8) in an isothermal flow
gives:
\[
C_s^2 \ln \rho_+ - \frac{1-\Gamma}{r_s} = \frac{1}{2} C_s^2 + C_s^2 \ln \rho_c - \frac{1-\Gamma}{r_c} \tag{12}
\]
where, we have ignored the initial value of the outflowing radial velocity \( \vartheta(r_s) \) at the dense region boundary, and also used eq. (11a). We have also put \( \rho(r_c) = \rho_c \) and \( \rho(r_s) = \rho_+ \). Upon simplification, we obtain,

\[
\rho_c = \rho_+ \exp(-f)
\]

where,

\[
f = f_0 - \frac{3}{2}
\]

Thus, the outflow rate is given by,

\[
\dot{M}_{\text{out}} = \Theta_{\text{out}} \rho_c \vartheta_c r_c^2
\]

where, \( \Theta_{\text{out}} \) is the solid angle subtended by the outflowing cone. Upon substitution, one obtains,

\[
\frac{\dot{M}_{\text{out}}}{M_{\text{in}}} = R_{\text{in}} = \frac{\Theta_{\text{out}}}{\Theta_{\text{in}}} \frac{R}{4} \int_0^{3/2} \exp(-f)
\]

which, explicitly depends only on the compression ratio:

\[
\frac{\dot{M}_{\text{out}}}{M_{\text{in}}} = R_{\text{in}} = \frac{\Theta_{\text{out}}}{\Theta_{\text{in}}} \frac{R^2}{4(R - 1)} \left[ \frac{2}{3} \exp\left(1 - \frac{R^2}{R - 1}\right)\right]
\]

apart from the geometric factors. Notice that this simple result does not depend on the location of the sonic points or the the size of the dense cloud or the outward radiation force constant \( \Gamma \). This is because the Newtonian potential was used throughout and the radiation force was also assumed to be very simple minded (\( \Gamma/r^2 \)). Also, effects of centrifugal force was ignored. Similarly, the ratio is independent of the mass accretion rate which should be valid only for low luminosity objects. For high luminosity flows, Comptonization would cool the dense region completely (CT95) and the mass loss will be negligible. Pair plasma supported quasi-spherical shocks forms for low luminosity as well (Kazanas & Ellison, 1986). In reality there would be a dependence on these quantities when full general relativistic considerations of the rotating flows are made. Exact and detailed computations using both the transonic inflow and outflow (where the compression ratio \( R \) is also computed self-consistently) are in progress, and the results would be presented elsewhere (Das & Chakrabarti, submitted).

Figure 2 contains the basic results. The solid curve shows the ratio \( R_{\text{in}} \) as a function of the compression ratio \( R \) (plotted from 1 to 7), while the dashed curve shows the same quantity as a function of the polytropic constant \( n = (\gamma - 1)^{-1} \) (drawn from \( n = 3/2 \) to 3), \( \gamma \) being the adiabatic index. The solid curve is drawn for any generic compression ratio and the dashed curve is drawn assuming the strong shock limit only: \( R = (\gamma + 1)/(\gamma - 1) = 2n + 1 \). In both the curves, \( \Theta_{\text{out}} \sim \Theta_{\text{in}} \) has been assumed for simplicity. Note that if the compression does not take place (namely, if the denser region does not form), then there is no outflow in this model. Indeed for, \( R = 1 \), the ratio \( R_{\text{in}} \) is zero as expected. Thus the driving force of the outflow is primarily coming from the hot and compressed region.
In a relativistic inflow or for a radiation dominated inflow, \( n = 3 \) and \( \gamma = 4/3 \). In the strong shock limit, the compression ratio is \( R = 7 \) and the ratio of inflow and outflow rates becomes,

\[
R_{in} = 0.052 \frac{\Theta_{out}}{\Theta_{in}} \quad \text{(17a)}
\]

For the inflow of a mono-atomic ionized gas \( n = 3/2 \) and \( \gamma = 5/3 \). The compression ratio is \( R = 4 \), and the ratio in this case becomes,

\[
R_{in} = 0.266 \frac{\Theta_{out}}{\Theta_{in}} \quad \text{(17b)}
\]

Since \( f_0 \) is smaller for \( \gamma = 5/3 \) case, the density at the sonic point in the outflow is much higher (due to exponential dependence of density on \( f_0 \), see, eq. 7) which causes the higher outflow rate, even when the actual jump in density in the postshock region, the location of the sonic point and the velocity of the flow at the sonic point are much lower. It is to be noted that generally for \( \gamma > 1.5 \) shocks are not expected (C90), but the centrifugal barrier supported dense region would still exist. As is clear, the entire behavior of the outflow depends only on the compression ratio, \( R \) and the collimating property of the outflow \( \Theta_{out}/\Theta_{in} \).

Outflows are usually concentrated near the axis, while the inflow is near the equatorial plane. Assuming a half angle of 10° in each case, we obtain,

\[
\Theta_{in} = \frac{2\pi^2}{9}; \quad \Theta_{out} = \frac{\pi^3}{162}
\]

and

\[
\frac{\Theta_{out}}{\Theta_{in}} = \frac{\pi}{36}. \quad \text{(18)}
\]

The ratios of the rates for \( \gamma = 4/3 \) and \( \gamma = 5/3 \) are then

\[
R_{in} = 0.0045 \quad \text{(19a)}
\]

and

\[
R_{in} = 0.023 \quad \text{(19b)}
\]

respectively. Thus, in quasi-spherical systems, in the case of strong shock limit, the outflow rate is at the most a couple of percent of the inflow. If this assumption is dropped, then for a cold inflow, the rate could be higher by about fifty percent (see, Fig. 2).

It is to be noted that the above expression for the outflow rate is strictly valid if the flow could be kept isothermal at least up to the sonic point. In the event this assumption is dropped the expression for the outflow rate becomes dependent on several parameters. As an example, we consider a polytropic outflow of same index \( \gamma \) but of a different entropy function \( K \) (We assume the equation of state to be \( P = K\rho^\gamma \), with \( \gamma \neq 1 \)). The expression (11b) would be replaced by,

\[
r_c = \frac{f_0 r_s}{2\gamma} \quad \text{(20)}
\]
and eq. (12) would be replaced by,

\[ na_s^2 + \frac{1 - \Gamma}{r_s} = \left( \frac{1}{2} + n \right) a_s^2 - \frac{1 - \Gamma}{r_c} \]

(21)

where \( n = 1/(\gamma - 1) \) is the polytropic constant of the flow and \( a_+ = (\gamma P_+ / \rho_+)^{1/2} \) and \( a_c = (\gamma P_c / \rho_c)^{1/2} \) are the adiabatic sound speeds at the starting point and the sonic point of the outflow. It is easily shown that a power law temperature fall off of the outflow \( (T \propto r^{-\beta}) \) would yield

\[ R_{\dot{m}} = \frac{\Theta_{\text{out}}}{\Theta_{\text{in}}} \left( \frac{K_i}{K_o} \right)^n \left( \frac{f_0}{2\gamma} \right)^{3/2 - \beta} \]

(22)

where, \( K_i \) and \( K_o \) are the entropy functions of the inflow and the outflow. This derivation is strictly valid for a non-isothermal flow. Since \( K_i < K_o \), \( n > 3/2 \) and \( f_0 \), for \( \Theta_{\text{out}} \sim \Theta_{\text{in}} \), \( R_{\dot{m}} \ll 1 \) is guaranteed provided \( \beta > \frac{3}{2} \), i.e., if the temperature falls for sufficiently rapidly. For an isothermal flow \( \beta = 0 \) and the rate tends to be higher. Note that since \( n \sim \infty \) in this case, any small jump in entropy due to compression will off-balance the the effect of \( f_0^{-3/2} \) factor. Thus \( R_{\dot{m}} \) remains smaller than unity. The first factor decreases with entropy jump while the second factor increases with the compression ratio \( (R) \) when \( \beta < 3/2 \). Thus the solution is still expected to be similar to what is shown in Fig. 2. Numerical results of the transonic flow using non-isothermal equation of state are discussed in Das & Chakrabarti (submitted).

3. CONCLUDING REMARKS

Although the outflows are common in many astrophysical systems which include compact objects such as black holes and neutron stars, it had been difficult to compute the outflow rates since these objects do not have any intrinsic atmospheres and outflowing matter has to be originated from the inflow only. We showed in the present paper, that assuming the formation of a dense region around these objects (as provided by a centrifugal barrier, for instance), it is possible to obtain the outflow rate in a compact form with an assumption of isothermality of the outflow at least up to the sonic point and the ratio thus obtained seems to be quite reasonable. Computation of the outflow rate with a non-isothermal outflow explicitly depends on several flow parameters. Our primary goal in this paper was to obtain the rates, and not the process of collimation. Since observed jets are generally hollow (Begelman, Blandford & Rees, 1984) they must be externally supported (either by ambient medium pressure or by magnetic hoop stress). This is assumed here for simplicity. Our assumption of isothermality of the wind till the sonic point is based on ‘experience’ borrowed from stellar physics. Momentum deposition from the hot photons from the dense cloud, or magnetic heating may or may not isothermalize the expanding outflow, depending on accretion rates and covering factors. However, it is clear that since the solid angle at which photons shine on electrons is close to \( 4\pi \) (as in a narrow funnel wall), and since the number of electrons per photon is much smaller in a compact region, it may be easier to maintain the isothermality close to a black hole than near a stellar surface.
The centrifugal pressure supported region that may be present in presence of angular momentum was found to be very useful in explaining the soft and the hard states (CT95; C97), rough agreement with power-law slopes in soft states (CT95; Titarchuk, Kylafis & Mastichiadis, 1997) as well as the amplitude and frequency of Quasi-Periodic Oscillations (Molteni, Sponholz & Chakrabarti 1996; Ryu, Chakrabarti & Molteni 1997) in black hole candidates. Therefore, our reasonable estimate of the outflow rate from these considerations further supports the view that such regions may be common around compact objects. Particularly interesting is the fact that since the wind here is thermally driven, the outflow rate is higher for hotter gas, i.e., when Comptonization is unimportant, that is, for low accretion rate. It is obvious that the non-magnetized neutron stars should also have the same dense region we discussed here and all the considerations mentioned here would be equally applicable.

It is to be noted that although the existence of outflows are well known, their rates are not. The only definite candidate whose outflow rate is known with any certainty is probably SS433 whose mass outflow rate was estimated to be $\dot{M}_{\text{out}} \gtrsim 1.6 \times 10^{-6} f^{-1} n_{13}^{-1} D_5^2 M_\odot \text{ yr}^{-1}$ (Watson et al. 1986), where $f$ is the volume filling factor, $n_{13}$ is the electron density $n_e$ in units of $10^{13} \text{ cm}^{-3}$, $D_5$ is the distance of SS433 in units of 5kpc. Considering a central black hole of mass $10 M_\odot$, the Eddington rate is $\dot{M}_{\text{Ed}} \sim 0.2 \times 10^{-7} M_\odot \text{ yr}^{-1}$ and assuming an efficiency of conversion of rest mass into gravitational energy $\eta \sim 0.06$, the critical rate would be roughly $\dot{M}_{\text{crit}} = \dot{M}_{\text{Ed}}/\eta \sim 3.2 \times 10^{-7} M_\odot \text{ yr}^{-1}$. Thus, in order to produce the outflow rate mentioned above even with our highest possible estimated $R_{\dot{m}} \sim 0.4$ (see, Fig. 2), one must have $\dot{M}_{\text{in}} \sim 12.5 \dot{M}_{\text{crit}}$ which is very high indeed. One possible reason why the above rate might have been over-estimated would be that below $10^{12} \text{ cm}$ from the central mass (Watson et al. 1986), $n_{13} >> 1$ because of the existence of the dense region at the base of the outflow.

In numerical simulations the ratio of the outflow and inflow has been computed in several occasions (Eggum, Coroniti & Katz, 1985; Molteni, Lanzafame & Chakrabarti, 1994). Eggum et al. (1985) found the ratio to be $R_{\dot{m}} \sim 0.004$ for a radiation pressure dominated flow. This is generally comparable with what we found above (eq. 19a). In Molteni et al. (1994) the centrifugally driven outflowing wind generated a ratio of $R_{\dot{m}} \sim 0.1$. Here, the angular momentum was present in both inflow as well as outflow, and the shock was not very strong. Thus, the result is again comparable with what we find here.
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**FIGURE CAPTIONS**

**Fig. 1** Schematic diagram of inflow and outflow around a compact object. Hot, dense region around the object either due to centrifugal barrier or due to plasma pressure effect or pre-heating, acts like a 'stellar surface' from which the outflowing wind is developed.

**Fig. 2** Ratio $\dot{R}_{in}$ of the outflow rate and the inflow rate as a function of the compression ratio of the gas at the dense region boundary (solid curve). Also shown in dashed curve is its variation with the polytropic constant $n$ in the strong shock limit. Solid angles subtended by the inflow and the outflow are assumed to be comparable.
Centrifugal Barrier supported dense region

Quasi-spherical Outflow

Compact Object

R_s

Quasi-spherical Sub-Keplerian Inflow
