Algorithm for power systems mode optimization taking into account the frequency change in terms of probabilistic nature of initial information

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Abstract. The task of power systems mode optimization relates to the complex tasks of nonlinear mathematical programming. Despite the development over the past few decades many methods and algorithms for solving this problem, questions of their improvement taking into account the current operating conditions of energy systems remain an important task. This article proposes a new algorithm for the optimization of short-term modes of power systems, taking into account frequency changes in terms of the probabilistic nature of initial information. A distinctive feature of the algorithm is associated with the elimination of the need to choose the single slack bus with balancing power plant in calculations, which is typical for many existing methods. It is shown that taking into account frequency change in the optimization of power system mode in terms of probabilistic nature of initial information can introduce significant changes in the calculation results and lead to a corresponding increase in the resulting economic effect.

1. Introduction

Optimal planning of short-term modes of power systems is a complex nonlinear problem with many variables and various constraints in the form of equalities and inequalities. As the results of works carried out during decades of years by many scientists and specialists around the world, a set of methods and algorithms for solving this problem have been developed. These include classical methods [1-3], heuristic [4, 5] and artificial computational intelligence methods such as ant algorithms [6, 7], artificial neural networks [8, 9], fuzzy logic [10], cuckoo search [11], bee algorithms [7, 12, 17], particle swarm optimization [13], evolutionary algorithms [14, 15, 16], etc. Many of them are currently used to solve production problems. However, the need to take into account some additional factors poses the problem of improving these methods. One of the problems in optimization with sufficient accuracy is associated with the probabilistic nature and partial uncertainty of initial data on the electric network circuits and the loads in nodes, as well as the need to take into account the frequency changes in a power system. The above-mentioned methods of optimization provide, mainly, the use of deterministic initial information. For their application under conditions of probability or partial uncertainty, the problem can be reduced to a number of sequentially solved deterministic problems based on the application of appropriate procedures as in [18, 19, 20]. On the other hand, the calculation algorithms on these methods provide for the allocation of a single slack bus with a balancing power plant, which provides a power balance and thereby an allowable frequency in the power system. However, in cases...
of deviation of the actual load from the planned one, determined by forecasting, the power system mode obtained on the basis of such algorithms can have a significant error. This factor is more evident in the planning of power system mode with an increase in the number of power plants involved in frequency regulation, as well as in conditions of probability and partial uncertainty of initial information. Therefore, the development of algorithms for power systems mode optimization taking into account frequency changes, which provides eliminating the need to allocate a single slack bus with balancing power plants in calculations, is an important problem.

Below a new algorithm for power system mode optimization taking into account the frequency change in terms of probabilistic nature of initial information is proposed.

2. Methods
In tasks of optimal planning of power systems mode, probabilistic and partially uncertain initial data are often consumer loads and losses in electric networks, determined by the active resistances and conductivities of elements, which depend on the ambient temperature. In general cases, the probabilistic and partially uncertain nature of active resistances and conductivities of elements are taken into account by the introduction of appropriate amendments to their calculations taking into account the results of a forecast of weather conditions for the planned period. The issues of taking into account of probability and uncertainty of consumer loads in nodes of electric networks are solved in the process of optimization of short-term mode the power system.

As a result of optimal planning of power system short-term mode, the optimal load schedules of power plants involved in optimization are determined. Providing of the planned optimal mode of power system for each interval of the planning period is carried out by power plants carrying out schedules of their loads determined by the dispatcher. The deviation of a load of the power system from the planned one leads to the corresponding deviation of frequency in the power system. To maintain the frequency within acceptable limits, the total load deviation of the power system $\Delta P_L$ is covered by power plants that have a power reserve and are involved in frequency regulation. Distribution of total load deviation $\Delta P_L$ between such stations occurs according to the static characteristics of the turbine speed controllers in them.

To illustrate the provisions described above, Figure 1 shows the distribution of the total load $P_L$ and deviations from it $\Delta P_L$ between two blocks according to the static characteristics of the speed controllers of their turbines.

![Figure 1. Distribution of load between power blocks of power plants in accordance with the static characteristics of the turbine speed controllers.](image)
The total load for two stations $P_L$ is distributed between them at the rated frequency $f_L$ and the power of the power plants is obtained $P_1, P_2$. The increase of load $\Delta P_L$ (with the total load $P'_L = P_L + \Delta P_L$) is also distributed among the power plants in a similar way and their capacities become as

$$P'_1 = P_1 + \Delta P_1, \quad P'_2 = P_2 + \Delta P_2 \quad (1)$$

In this case, the frequency in the power system becomes $f'$. If the new frequency $f'$ is less than the permissible value, then the turbine speed regulators are triggered at the power plants, as a result of which their static characteristics rise up to $\Delta f$ relative to the initial positions. As a result, the nominal frequency will be restored at new power plant capacities.

Using the graphical method to determine the distribution of deviations of the total loads between power plants with a power reserve causes corresponding difficulties and inaccuracies in calculations. Therefore, it is advisable to use the following analytical expressions:

$$\Delta P_1 = k_1 \Delta f, \quad P_2 = k_2 \Delta f \quad (2)$$

$$\Delta P_1 + \Delta P_2 = \Delta F_L \quad (3)$$

$$(k_1 + k_2) \Delta f = \Delta P_L \quad (4)$$

$$\Delta f = \Delta P_L / (k_1 + k_2) \quad (5)$$

where $k_1, k_2$ are the static coefficients of the turbine speed controllers in power plants.

The proposed algorithm takes into account such distribution of deviations in power loads $\Delta P_L$ between power plants in process power system mode optimization in terms of probability of initial information about loads of consumers.

**The algorithm of optimization**

To illustrate the essence of the proposed algorithm, we consider a power system with thermal power plants (TPPs) involved in optimization.

The task of optimization of power system mode for any interval of the planning period is formed as follows:

minimize the objective function - the function of total fuel costs or equivalent fuel consumption in all $n$ TPPs

$$B = B_1(P_1) + B_2(P_2) + \ldots + B_n(P_n) \quad (6)$$

subject to constraints on the power balance in the power system

$$P_1 + P_2 + \ldots + P_n = P_L \quad (7)$$

and marginal capacity of power plants

$$P_{i\text{min}} \leq P_i \leq P_{i\text{max}}, \quad i = 1, 2, \ldots, n \quad (8)$$

The total load of the power system $P_L$ is probabilistic, i.e. its probable values of $P_{L1}, P_{L2}, \ldots, P_{Lm}$ are given with the corresponding probabilities of their appearance $P_{L1}, P_{L2}, \ldots, P_{Lm}$, where $P_{L1} + P_{L2} + \ldots + P_{Lm} = 1$.

We describe the proposed algorithm of power system mode optimization taking into account the frequency change in terms of probability of initial information:
1. The total loads of the power system $P_{L1}$, $P_{L2}$, ..., $P_{Lm}$ are optimally distributed between power plants, i.e. for different values of the total load the problem (6) - (8) is solved. At this stage of calculations the existing methods of power system mode optimization in terms of deterministic initial information, particularly above mentioned methods, can be used. In cases of discontinuity of energy characteristics of power plants can be used also the algorithm presented in [21]. The obtained optimal solutions with the corresponding probabilities $p_{L1}$, $p_{L2}$, ..., $p_{Lm}$ make conditionally optimal plans. Then, at these conditionally optimal plans, the values of the objective function, which forms the diagonal elements of the so-called “payment matrix” $B_{ii}$, $i = 1, 2, ..., m$ are calculated.

2. The values of the objective function for each of the conditionally optimal plans are calculated for all of the possible total loads of power systems. At this stage, the deviation in total load obtained due to the difference in total loads used for obtaining the corresponding conditionally optimal plan and other possible (probable) one are distributed between power plants involved in frequency control in accordance with the characteristics of the speed controllers of their turbines. In this case, the frequency deviation and corresponding changes of loads of power plants are determined by formulas (4) and (2), correspondingly. The values of object function at obtained here loads of power plants forms off-diagonal elements of the “payment matrix” $B_{ij}$, $i = 1, 2, ..., m; j = 1, 2, ..., m (i \neq j)$.

3. The optimal solution to the problem is selected from among the conditionally optimal plans as a result of a comparison of mathematical expectations of the objective function, defined as

$$MB_i = p_1 B_{i1} + p_2 B_{i2} + \ldots + p_m B_{im}. \quad (9)$$

The conditionally optimal plan corresponding to the minimum mathematical expectation of the objective function is adopted as the optimal solution to the problem.

3. Results and Discussion

The effectiveness of the described algorithm was investigated, in particular, an example of the problem of the optimal distribution of the probabilistic load of power system between four TPPs, taking into account the frequency change. The characteristics of the relative increment of the equivalent fuel consumption of TPPs are given in Table 1. This table also shows the minimum and maximum possible TPP loads, hourly equivalent fuel consumption at their minimum loads $B_{i0}$ (where t.f.e.- tons of fuel equivalent). The probabilistic load of the power system and the law of its distribution are given in table 2.

Power plants involved in frequency regulation are TPP-1 and TPP-3. The static coefficients of turbine speed regulators of them are $k_1 = 0.2$ and $k_3 = 0.15$, respectively.

**Table 1.** Relative increment characteristics of equivalent fuel consumption of TPPs

| TPP–1 | TPP–2 | TPP–3 | TPP–4 |
|-------|-------|-------|-------|
| $B_{i0} = 114.8$ t.f.e./h. | $B_{i0} = 32$ t.f.e./h. | $B_{i0} = 57$ t.f.e./h. | $B_{i0} = 97$ t.f.e./h. |
| $b_1$, t.f.e./MW.h | $P_1$ (MW) | $b_2$, t.f.e./MW.h | $P_2$ (MW) | $b_3$, t.f.e./MW.h | $P_3$ (MW) | $b_4$, t.f.e./MW.h | $P_4$ (MW) |
| 0.2705 | 400 | 0.34 | 70 | 0.314 | 160 | 0.22 | 320 |
| 0.2836 | 565 | 0.359 | 77 | 0.316 | 320 | 0.27 | 398 |
| 0.2967 | 583 | 0.366 | 92 | 0.317 | 384 | 0.295 | 438 |
| 0.3098 | 595 | 0.524 | 92 | 0.319 | 427 | 0.32 | 450 |
| 0.3229 | 647 | 0.533 | 100 | 0.334 | 428 | 0.345 | 495 |
Table 2. The probabilistic total load of power system and the law of its distribution

| $P_L$, MW | 1620 | 1660 | 1700 | 1740 | 1780 |
|-----------|------|------|------|------|------|
| $p$       | 0.1  | 0.2  | 0.4  | 0.2  | 0.1  |

This optimization problem is solved by the proposed algorithm. As a result, the conditionally optimal plans are obtained and payment matrix is formed. For comparison and analysis of the results, table 3 shows the 3rd conditionally optimal plan, i.e. the result of the optimal distribution of a total load of power system 1700 MW between TPPs, as well as possible options for its implementation with corresponding probabilities. Here, according to the proposed algorithm, the probable unbalances of the total load, defined as the difference between the total loads used to obtain the corresponding conditionally optimal plan and other possible one are covered by power plants TES-1 and TES-3 in accordance with the static characteristics of the turbine speed regulators in them.

Table 3. Possible options for implementation of the 3rd conditionally optimal plan with corresponding probabilities, obtained by proposed algorithm.

| Optimized parameters | Total loads of power system and probabilities of their occurrence |
|----------------------|---------------------------------------------------------------|
|                      | 0.1   | 0.2   | 0.4   | 0.2   | 0.1   |
|                      | 1620MW| 1660MW| 1700MW| 1740MW| 1780MW|
| $P_1$                | 659.92| 682.78| 705.64| 728.49| 751.35|
| $P_2$                | 70.00 | 70.00 | 70.00 | 70.00 | 70.00 |
| $P_3$                | 414.52| 431.67| 448.81| 465.95| 483.09|
| $P_4$                | 475.55| 475.55| 475.55| 475.55| 475.55|

Table 4 shows the mathematical expectations of the objective function, i.e. the total equivalent fuel consumption in TPPs corresponding to various conditionally optimal plans. According to these results, as the optimal one corresponding to the least mathematical expectation, we select the third conditionally optimal plan.

Table 4. The mathematical expectations of objective function.

| № of plans | $MB_i$ |
|------------|--------|
| 1          | 525.59 |
| 2          | 525.37 |
| 3          | 525.25 |
| 4          | 525.65 |
| 5          | 525.40 |

To compare the results obtained above based on the use of the proposed algorithm, optimization was also performed by the existing traditional algorithm with the selection of the single slack bus with bal-
ancing power plant TES-1, which controls the frequency, and the corresponding “payment matrix” is formed. Table 5 shows the 3rd conditionally optimal plan, i.e. the result of the optimal distribution of the third probabilistic load of 1700 MW between power plants, as well as possible options for implementing this plan with corresponding probabilities.

Table 5. Possible options for implementation of the 3rd conditionally optimal plan with corresponding probabilities obtained by existing algorithm

| Optimized parameters | Total loads of power system and probabilities of their occurrence |
|----------------------|---------------------------------------------------------------|
|                      | 0.1    | 0.2    | 0.4    | 0.2   | 0.1   |
|                      | 1620 MW | 1660 MW | 1700 MW | 1740 MW | 1780 MW |
| $P_1$                | 625.637 | 665.64 | 705.64 | 745.64 | 785.64 |
| $P_2$                | 70.00   | 70.00  | 70.00  | 70.00  | 70.00  |
| $P_3$                | 448.81  | 448.81 | 448.81 | 448.81 | 448.81 |
| $P_4$                | 475.55  | 475.55 | 475.55 | 475.55 | 475.55 |

Comparing the results obtained by the proposed algorithm, when for frequency regulation participate to power plants TPP-1 and TPP-3 (Table 3) and by existing algorithms, when the nominal frequency is provided by a single power plant in slack bus TPP-1 (table 5), we will sure that taking the frequency change into account makes a significant change in optimal total load distribution between power plants. If in the first case, i.e. when two power plants TES-1 and TES-3 participate in frequency regulation, the probable unbalances in a total load of the power system are distributed between them in accordance to the static coefficients of their turbine speed regulators, then in the second case, all these probable unbalances are covered by a single balancing plant. Moreover, if in the first case the mathematical expectation of the total hourly consumption of equivalent fuel is 525.25 t.f.e. / h, then in the second case 525.96 t.f.e/h. Thus, the hourly economic effect as a result of using the proposed algorithm is 0.71 t.f.e./h or 0.13%.

4. Conclusions
1. An algorithm for optimization of power systems mode taking into account frequency changes in terms of probabilistic nature of initial information, which differs by the exclusion of the need to allocate slack bus with balancing power plant in calculations, is proposed.
2. It has been revealed that taking into account frequency changes at the optimization of power systems mode in terms of probabilistic nature of initial information on consumer loads, in general cases, can significantly reduce the economic expenses related to total fuel consumption in thermal power plants.
3. The proposed algorithm of optimization can be used by power systems dispatching services for optimal planning of short-term modes of power systems with several stations involved in frequency regulation in terms of the probabilistic nature of initial information on loads of electric consumers.

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