Determining the motion of the Solar system relative to the cosmic microwave background using Type Ia supernovae

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ABSTRACT
We estimate the Solar system motion relative to the cosmic microwave background using Type Ia supernovae (SNe) measurements. We take into account the correlations in the error bars of the SNe measurements arising from correlated peculiar velocities. Without accounting for correlations in the peculiar velocities, the SNe data we use appear to detect the peculiar velocity of the Solar system at about the 3.5σ level. However, when the correlations are correctly accounted for, the SNe data only detect the Solar system peculiar velocity at about the 2.5σ level. We forecast that the Solar system peculiar velocity will be detected at the 9σ level by GAIA and the 11σ level by the Large Synoptic Survey Telescope. For these surveys, we find the correlations are much less important as most of the signal comes from higher redshifts where the number density of SNe is insufficient for the correlations to be important.

Key words: Solar system: general – supernovae: general – cosmic microwave background.

1 INTRODUCTION
The cosmic microwave background (CMB) has a 3.4 mK dipole anisotropy (Hinshaw et al. 2007) which can naturally be explained as being due to the motion of the Solar system with respect to the CMB rest frame (Lynden-Bell, Lahav & Burstein 1989; Strauss et al. 1992; Erdogdu et al. 2006; Loeb & Narayan 2008). An interesting consistency check of this is to evaluate the Solar system motion from peculiar velocity (PV) surveys (see e.g. Dale & Giovanelli 2000).

Supernovae (SNe) luminosity measurements provide an accurate probe of PVs. Using observed correlations between SNe light curves, we can estimate the SNe absolute magnitudes and thus obtain accurate distance estimates to the SNe. Combined with spectroscopic measurements of the host galaxies’ redshifts, this can be used to estimate the PV of each SNe’s host galaxy. The motion of the Solar system will then show up as a dipole anisotropy in the SNe-derived PVs. It is interesting to compare the estimates of the Solar system motion from the SNe with those derived from the CMB. If they turn out to be inconsistent, then it may be an indication that there is a significantly large intrinsic temperature dipole on the CMB surface of last scattering (Turner 1991; Langlois & Piran 1996), which could be caused by a double inflation model (Langlois 1996) for example.

A number of studies have made this comparison (Riess, Press & Kirshner 1995; Bonvin, Durrer & Kunz 2006b; Jha, Riess & Kirshner 2007), and a simplifying assumption used in these studies was that the PVs of the individual SNe were uncorrelated with each other. However, as the PVs are caused by variations in the density field, neighbouring SNe will have correlated PVs (Wang, Spergel & Turner 1998; Sugiura, Sugiyama & Sasaki 1999; Bonvin, Durrer & Gasparini 2006a; Hui & Greene 2006; Gordon, Land & Slosar 2007). These correlations will increase the error bars on our PV estimate as each new SNe measurement does not represent a completely independent realization of the velocity field.

In this article, we include the correlations of the PVs when estimating the motion of the Solar system with respect to the cosmic rest frame. In Section 2, we give a simple example of the underestimation of the uncertainty that occurs when correlations between observations are not taken into account. In Section 3, we outline the formalism we use for the SNe correlations, and in Section 4 we apply the method to spectral adaptive light-curve template (SALT)-calibrated SNe data. In Section 5, we look at the implications for future surveys. A summary and discussion of the results is given in Section 6.

2 SIMPLE EXAMPLE OF CORRELATED ERRORS
In order to illustrate the effect of correlated errors, we consider a simple example (also discussed below equation 5 of Cooray & Caldwell 2006) where we analyse N data points (x_i) drawn from a multivariate Gaussian likelihood
\[ L \propto |C|^{-1/2} \exp\left(-\frac{1}{2}(x - \mu)^T C^{-1}(x - \mu)\right). \] (1)

The vector x is made up of the data points (x_i), and each element of the vector \( \mu \) is equal to a constant, \( \mu \). The covariance matrix (C) has diagonal terms which are \( \sigma^2 \) and the off-diagonal terms which are \( \rho \sigma^2 \). That is, each data point has correlation \( \rho \) with the other data

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If there are correlations present, then it follows from equation (4) to estimate the dark energy equation of state, low-redshift SNe are not accounted for when using them in a sample analogous underestimation of the error happens if the correlations in show that there is also an underestimation in error on the motion of $\alpha$.

Comparing equations (6) and (10), we see that if the data are correlated, one would overestimate the error if the data are anti-correlated, although we note that this case is restricted as for the covariance matrix to be positive definite, $\rho > -(N - 1)^{-1}$.

As was shown in a earlier studies (Hui & Greene 2006; Gordon, Land & Slosar 2007; Neill, Hudson & Conley 2007), an analogous underestimation of the error happens if the correlations in low-redshift SNe are not accounted for when using them in a sample to estimate the dark energy equation of state, $w$. In this article, we show that there is also an underestimation in error on the motion of our Solar system when the correlations in the SNe are not accounted for.

3 METHOD

The luminosity distance, $d_L$, to a SN at redshift $z$ is defined such that

$$F = \frac{L}{4\pi d_L^2},$$

where $F$ is the observed flux and $L$ is the SN's intrinsic luminosity. Astronomers use magnitudes, which are related to the luminosity distance (in megaparsec) by

$$m = M + 5 \log_{10} d_L^{obs} + 25, \quad (11)$$

where $m$ and $M$ are the apparent and absolute magnitudes, respectively. In the context of SNe, $M$ is a 'nuisance parameter' which is completely degenerate with $\log(H_0)$ and is marginalized over. For a Friedmann–Robertson–Walker universe, the predicted luminosity distance is given by

$$d_L(z) = (1 + z) \int_0^z \frac{dz'}{H(z')}$$

(12)

(taking $c = 1$), where $H$ is the Hubble parameter. In the limit of low redshift, this reduces to $d_L \approx z/H_0$.

Given very stringent limits on the curvature of the universe, we can safely work within the assumption of a flatness as the allowed curvature would not play any role at the scales of interest. In this case, the effect of a PV leads to a perturbation in the luminosity distance ($\delta d_L$) given by (Sasaki 1987; Sugita et al. 1999; Pyne & Birkinshaw 2004; Bonvin et al. 2006a; Hui & Greene 2006)

$$\frac{\delta d_L}{d_L} \approx \dot{r} \cdot \left[ v - \frac{(1 + z)^2}{H(z)} \left[ v - v_0 \right] \right].$$

(13)

where $r$ is the position of the SN, and $v_0$ and $v$ are the PVs of the observer and SN, respectively. In the limit of low redshift, $\delta d_L \approx \dot{r} \cdot \left[ v_0 - v \right]/H_0$. This demonstrates how a SNe survey that measures $m$ and $z$ can estimate the projected PV field. We now relate this to the cosmology.

The projected velocity correlation function, $\xi(r, r') = \langle (\mathbf{v}(r) \cdot \mathbf{F}) (\mathbf{v}(r') \cdot \mathbf{F}) \rangle$, must be rotationally invariant, and therefore it can be decomposed into a parallel and perpendicular components (Gorski 1988; Groth, Juszkiewicz & Ostrik 1989; Dodson 2003):

$$\xi(r, r') = \sin \theta \sin \theta' \zeta_{\parallel}(\Delta r, z, z') + \cos \theta \cos \theta' \zeta_{\perp}(\Delta r, z, z')$$

(14)

where $\Delta r = r - r'$, $\Delta r = |\Delta r|$, $\cos \theta \equiv \hat{r} \cdot \hat{r}$, and $\cos \theta' \equiv \hat{r} \cdot \hat{r}'$. In linear theory, these are given by (Gorski 1988; Groth et al. 1989; Dodson 2003)

$$\zeta_{\parallel, \perp}(z, z') = D(z) D'(z') \int_0^\infty \frac{dk}{2\pi} P(k) K_{\parallel, \perp}(kr),$$

(15)

where for an arbitrary variable $x$, $K_{\parallel}(x) \equiv j_0(x) - \frac{2 j_1(x)}{x}$, $K_{\perp}(x) \equiv j_1(x)/x$. $D(z)$ is the growth function, and derivatives are with respect to conformal time. $P(k)$ is the matter power spectrum which can be evaluated either numerically (e.g. Code for Anisotropies in the Microwave Background (CAMB); Lewis, Challinor & Lasenby 2000) or using analytical approximations (Eisenstein & Hu 1998).

The above estimate of $\xi(r, r')$ is based on linear theory. On scales smaller than about $10^{-1}$ Mpc, non-linear contributions dominate. These are usually modelled as an uncorrelated term which is independent of redshift, often set to $\sigma_n \sim 300$ km s$^{-1}$. Comparison with N-body simulations (Silberman et al. 2001) indicates that this is an effective way of accounting for the non-linearities. Other random errors that are usually considered are those from the light-curve.
fitting ($m_{\text{err}}$) and intrinsic magnitude scatter ($\sigma_{m}$). Just these three errors are usually included in the analysis of SNe.

The residual deviations of luminosity distance from the homogeneous expansion can be packed into a data vector

$$\left( \frac{\delta d_L}{d_L} \right)_i = \frac{d_L^{\text{true}}(i) - d_L(z(i))}{d_L(z(i))},$$

whose covariance matrix (from the correlated PVs) is given by

$$C_{\nu}(i, j) = \left( 1 - \frac{(1 + z)^2}{H d_L} \right) \left( 1 - \frac{(1 + z)^2}{H d_L} \right) \delta_{ij} \xi(r_i, r_j).$$

while the standard uncorrelated random errors are given by

$$\sigma(i)^2 = \left( \frac{\ln(10)}{5} \right)^2 \left( \sigma_{m, i}^2 + m_{\text{err}}(i)^2 \right) + \left( 1 - \frac{(1 + z)^2}{H d_L} \right)^2 \delta_{ij} \sigma_v^2.$$

Some example plots of $C_{\nu}$ were given in Gordon et al. (2007). The likelihood is then

$$\mathcal{L} = (2\pi)^{-N/2}|\Sigma|^{-1/2} \exp \left( -\frac{1}{2} \Delta^T \Sigma^{-1} \Delta \right)$$

where

$$\Sigma(i, j) = C_{\nu}(i, j) + \sigma(i)^2 \delta_{ij}$$

and

$$\Delta_i = \left( \frac{\delta d_L}{d_L} \right)_i - \left( 1 + \frac{z}{H d_L} \right) \hat{r}_i \cdot \mathbf{v}_0.$$

We now proceed to find constraints on the observer velocity $\mathbf{v}_0$. We assume a standard $\Lambda$ cold dark matter ($\Lambda$CDM) cosmology and impose big bang nucleosynthesis (BBN) prior $\Omega_0 h^2 \sim N(0.0214, 0.002)$ (Kirkman et al. 2003) and a Hubble Space Telescope (HST) prior $h \sim N(0.72, 0.08)$ (Freedman et al. 2001). These two priors remove models that are wildly at odds with standard cosmological probes, but do not unduly bias results towards standard cosmology. The likelihood has almost negligible dependence on $n_s$, and to keep it in a range consistent with CMB and large-scale structure estimates we give it a uniform prior $n \in [-0.9, 1.1]$.

We parametrize the Solar system PV as a magnitude ($v_0$) and direction in Galactic coordinates ($l, b$). The prior on ($l, b$) was assumed to be uniform on the sphere, i.e. flat on $l$ and cos ($b$). The prior on $v_0$ was set to be uniform.

We use a SALT (Guy et al. 2005) calibrated low-redshift SNe data set$^1$ with heliocentric redshifts in the range $cz \in [2278, 37163]$ km s$^{-1}$, ($z \in [0.0076, 0.124]$). A histogram of the 61 redshifts is shown in Fig. 1 and the sky positions of the data are shown in Fig. 2. We also used the higher redshift 71 SNe from the Supernovae Legacy Survey (SNLS) data set.$^2$

The SALT calibration involves the additional parameters ($\alpha, \beta$), which account for the shape/luminosity and colour/luminosity relations of SNe. These and the other parameters ($\Omega_m, \sigma_8, \sigma_m, M_{\text{corr}}$) are all given broad uniform priors. We use the standard Markov Chain Monte Carlo (MCMC) method to generate samples from the posterior distribution of the parameters (Lewis & Bridle 2002). Convergence was checked using multiple chains with different starting positions, and also the $R - 1$ statistic (Gelman & Rubin 1992). We also checked that the estimated posterior distributions reduced to the prior distributions when no data were used. The analysis was checked with two completely independent codes and MCMC chains.

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$^1$ Obtained from http://gold.astro.utoronto.ca/conley/bubble/

$^2$ Obtained from http://snls.in2p3.fr/conf/papers/cosmol1/
convert the number of standard deviations of the detection into upper bounds on the Bayesian odds ratio (Gordon & Trotta 2007). Without correlations the odds, from SNe data, of \( v_O \) being non-zero appear to be at best 119:1. While if the correlations are accounted for, then the odds are at best only 7:1. In Table 1, we also give an estimate for the local group motion which was obtained by subtracting the Solar system velocity relative to the local group (Yahil, Tammann & Sandage 1977).

5 FORECASTS

In this section, we forecast constraints on the motion of the Solar system from GAIA (the ‘super-Hipparcos’ satellite) and LSST. For GAIA, based on the simulations by Belokurov & Evans (2003), we generate a sample of 6317 SNe distributed over the full sky with \( z < 0.14 \). For LSST, we generate 30 000 SNe distributed over the full sky with \( z < 0.3 \) (Wang, Pinto & Zhan 2005). We weighted the distribution of SNe by \( \cos(b)z^2 \) to account for the volume in spherical coordinates, i.e. we keep the density constant with \( z \). As our fiducial model, we took \( \{v_O, l, b, \Omega_m, \Omega_b, h, n_s, w, \sigma_8, \sigma_s, \sigma_m, m_{obs}\} = \{369, 264, 48, 0.3, 0.041, 0.72, 0.96, -1, 0.85, 300, 0.1, 0.1\} \). We do not include the SALT calibration parameters \( (\alpha, \beta) \) in the forecast, but we have checked using forecasts for the data sets used in Section 3 that this does not have a significant effect. In order to use the Fisher matrix (see equation 9), we consider the function

\[
d = d_L^{obs} 10^{\theta/5},
\]

where \( d_L^{obs} \) is given by equation (11). The expectation value vector has each element given by

\[
\langle d \rangle = 10^{\theta/5} d_L^{eh} + P \cdot v_0 \left(1 + \frac{z^2}{H(z)} \right),
\]

with \( d_L^{eh} \) given by equation (12). The covariance matrix \( (C) \) is as before, equation (20), but with the extra factor \( (10^{\theta/5})^2 d_L(i) d_L(j) \).

Table 1. The mean and standard deviation for the estimate of the Solar system and local group velocity from current SNe data. The results for both the correlated and uncorrelated PVs are shown.

| l     | b     | \( v_O \) (km s\(^{-1}\)) |
|-------|-------|--------------------------|
| Solar system uncorrelated | 238 \(\pm 26\) & 45 \(\pm 14\) & 475 \(\pm 134\) |
| Solar system correlated   | 234 \(\pm 44\) & 39 \(\pm 21\) & 468 \(\pm 186\) |
| Local Group uncorrelated  | 260 \(\pm 14\) & 32 \(\pm 11\) & 697 \(\pm 137\) |
| Local Group correlated    | 257 \(\pm 24\) & 29 \(\pm 16\) & 690 \(\pm 201\) |
increasing as exactly compensated by the number of SN in a redshift slice, which weakening of the dipole signal, that drops as $1/z$ in independent measurement errors in equation (18). In this limit, the experiment) they become unimportant compared to the redshift-weighted GAIA and LSST can rely on the
tions. In other words, under the assumption of a volume-weighted
redshift distribution for the SNe, GAIA and LSST can rely on the
number of SNe for which the covariance will become important to
set. Equation (10) tells us that this is approximately equal to the
SN luminosity to its covariance with a typical SN in the data
ratio is therefore low and increasing at low redshift, tailing off to a
constant value at redshifts at which PVs become unimportant.

As a rough measure of whether the correlated error will be im-
portant, at a redshift $z$, we look at the ratio $N = \sigma^2 / C$, where $C$, is evaluated using equation (17) with two SN at redshift $z$ and 90° apart. This is effectively the ratio of the measurement error on the SN luminosity to its covariance with a typical SN in the data set. Equation (10) tells us that this is approximately equal to the number of SNe for which the covariance will become important to our error estimates, and we plot this in Fig. 6. As can be seen, this simple estimate is in agreement with the more complete analysis: current data are affected by correlated errors much more so than GAIA and LSST, which will be practically unaffected by correlations. In other words, under the assumption of a volume-weighted redshift distribution for the SNe, GAIA and LSST can rely on the

Note the reason for the slightly different function of the data compared to equation (21) is so that the data vector does not depend on any of the parameters.

In Table 2, we present our main forecast results. As can be seen, there is a dramatic improvement in the constraints compared to current data. Also, unlike current data, taking into account the correlations does not have a large effect. This is because most of the constraining power for GAIA and LSST comes from higher redshifts, where the PV errors are negligible compared to the other types of error. This can be understood as follows. From equation (17), we see that the error induced by peculiar coherent velocity flows drops as $1/z$, and thus for high enough redshift ($z > 0.015$ for a typical experiment) they become unimportant compared to the redshift-independent measurement errors in equation (18). In this limit, the weakening of the dipole signal, that drops as $1/z$ in equation (21), is exactly compensated by the number of SN in a redshift slice, which increases as $z^2$, for a volume-weighted survey. The signal-to-noise ratio is therefore low and increasing at low redshift, tailing off to a constant value at redshifts at which PVs become unimportant.

|                | $l$ | $b$ | $v_0$ (km s$^{-1}$) |
|----------------|-----|-----|---------------------|
| GAIA uncorrelated PVs | $8^\circ$ | $5^\circ$ | 36 |
| LSST uncorrelated PVs | $7^\circ$ | $5^\circ$ | 32 |
| GAIA correlated PVS | $10^\circ$ | $6^\circ$ | 42 |
| LSST correlated PVS | $8^\circ$ | $5^\circ$ | 34 |

6 DISCUSSION

To summarize, we have used SALT-calibrated SNe data to estimate the motion of the Solar system. As seen from Table 1, the error bars are underestimated by about 50 per cent if the correlations in the PV are not accounted for.

We now compare our findings to previous published results. In Bonvin et al. (2006b), they used 44 SALT-calibrated SNe. They only allowed {$l, b, v_0$} to vary and all the other parameters were fixed to standard values. They did not account for correlations in the PVs. They found $v_0 = 405 \pm 192$ km s$^{-1}$ which is compatible with our result.

In Jha et al. (2007), they used 69 SNe with $z \in (0.005, 0.025)$. They also only allowed {$l, b, v_0$} to vary. Additionally, they used MLCs2k2 to calibrate the data, rather than the SALT method. They also did not account for the correlations in the PVs. They evaluated the motion of the local group and found {$l, b, v_0$} = {258 ± 18, 51 ± 12.5, 541 ± 75 km s$^{-1}$}. Our local group velocity results, which are listed in Table 1, are compatible with those of Jha et al. (2007) but, even when we do not take into account the correlations in the PVs, our error on the magnitude of $v_0$ is about 80 per cent larger than that of Jha et al. (2007) study. This is due to several factors. They had a lower redshift limit than us: 0.005 versus 0.0076. One of the reasons we did not go to such a low redshift is that the PVs (including the motion of our Solar system) become of the order of the Hubble expansion. This means that the motion of our Solar system has a high signal (hence the low error bars obtained by Jha et al. 2007) but one can no longer use equation (13) to evaluate the effects of PV on the luminosity distance. It would be possible to use a higher order version of equation (13) but this was not done by Jha et al. (2007) and so their results will have an additional unreported systematic error due to the induced luminosity change being calculated incorrectly. Also, the extra very low redshift SNe that were used are dominated by SNe that are too close together to model the correlations in the PV using linear theory (equation 15). Overall, our results are the only ones that take into account the correlations in PVs and account for the uncertainties in the cosmological and calibration parameters.

We also made forecasts for the GAIA and LSST surveys, assuming volume weighting for the redshift distribution. We found that the error bars will be about four times smaller than those of current data, but still not competitive with those from the CMB by a factor of ~10 (assuming that the CMB dipole is due to our local motion). Also, for GAIA and especially LSST we found that correlations had little effect as most of the signal came from higher redshifts where the correlations are almost negligible for the sample sizes considered.

In future work, we plan to test the techniques we have used in this paper against simulated SNe surveys generated from N-body simulations. These will test the assumptions that go into our data modelling – most importantly the effect of non-linearities (Haugboelle et al. 2006).

In Watkins & Feldman (2007), it was shown that the large-scale properties (bulk flow and shear; Kaiser 1991; Jaffe & Kaiser 1995) of the PV field derived from a low-redshift sample of 73 SNe was consistent with the bulk flow and shear of the velocity field derived from the Spiral Field I-band (SFI), Nearby Early-type Galaxies Survey (ENEAR) and Surface Brightness Fluctuations (SBF) surveys. It would be interesting to combine all these surveys together (with SFI replaced by SFI++; Springob et al. 2007) to estimate the Solar...
system velocity with respect to the CMB and put robust limits on the intrinsic CMB dipole.

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