Design of fractional slot windings with coil span of two slots for use in six-phase synchronous machines

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Abstract: The use of a winding step of two slots attempts to strike a balance between traditional distributed windings and fractional slot concentrated windings. It promises less ohmic losses and space for the end windings than distributed windings and less harmonics than fractional slot concentrated windings. This paper gives an introduction into the design of windings with a winding step of two slots for six-phase machines. The design procedure relies on the star of slots. Whereas for a given combination of slots and poles, the winding of a three-phase machine is unique, there exist several winding designs for six-phase machines. They differ by the shape of the phase belts and by the phase shift between the two three-phase subsystems. The definition of two characteristic figures facilitates the comparison of these windings: one describes the content of sub-harmonics, the other is a measure for rotor eddy losses. The comparison leads to some recommended winding implementations.

1 Introduction

When dealing with six-phase synchronous machines, fault tolerance is often in the focus of attention. There are also other reasons for the use of such a design [1]: the lower current rating per phase for a given power is a decisive advantage in all low-voltage high-power applications. Another reason is less torque ripple in large variable speed drives with current source inverter. Finally, the lower space-harmonic content in the mmf is advantageous for smoothness and efficiency.

Yet, it is impossible to have all these benefits entirely at the same time. If the requirements on fault tolerance are high, the machine must have a single-layer fractional slot concentrated windings in order to decouple the phases. In this case the requirements on smoothness and efficiency are compromised. However, such a winding design entails a huge amount of harmonics in the armature field. If fault tolerance is not the main cause for the choice of a six-phase design a traditional dual-layer chorded winding can keep the harmonics very low. The coil pitch of such distributed windings is normally at least three slots which brings about a high winding resistance and occupies much space for the end windings. The use of windings with a coil span of two slots promises a better balance between ohmic losses and end windings space on the one hand and harmonic content on the other. This idea found an early three-phase implementation in [2] where the authors created such a winding by nesting two fractional slot concentrated windings into each other. A systematic elaboration of three-phase windings with a winding step of two slots takes place in [3, 4]. With respect to six-phase machines, the literature discusses some specific combinations of slots and poles: a design with 24 slots and 10 poles can suppress the first-order harmonic in the mmf [5]. The authors of [6] show for an 18 slots stator and a rotor with 8 or 10 poles how one three-phase winding can split into two three-phase subsystems. Reference [7] suggests a six-phase traction drive with 18 slots and 8 poles which is further analysed regarding demagnetisation behaviour in [8].

This paper gives a general insight into the design of six-phase windings with a coil span of two slots. It depicts that for a given combination of slots and poles, the winding is not unique. Several winding designs are possible which differ by the shape of the phase belts and by the phase-shift between the two three-phase subsystems. The paper illustrates the construction of these different designs and analyses their individual mmf harmonic content. The results for each winding are merged into two characteristic figures which facilitate the comparison of these windings: one describes the overall content of sub-harmonics, the other is a measure for rotor losses by eddy currents. The comparison leads to some recommended winding implementations.

2 Design of six-phase windings

2.1 Number of slots per pole per phase

The term ‘fractional slot winding’ refers to the fact that the number of slots per pole per phase \( q \) is not an integer but a fraction. Doubling the number of phases \( m \) compared to a three-phase system halves the number of slots per pole per phase, which depends also on the number of slots \( Q_s \) and the pole pair number \( p \):

\[
q = \frac{Q_s}{2mp} \Rightarrow q_{2m} = q_{m}/2
\] (1)

As six-phase machines are just a combination of two three-phase systems, possible combinations of the number of slots \( Q_s \) and the number of poles \( 2p \) are a subset of the combinations for three-phase windings. Among the numerous combinations which reference [4] lists for three-phase windings, there are only some which have a satisfactory high winding factor and these are within a range of \( 1/2 < q_3 < 1 \). Hence, interesting six-phase windings exhibit \( 1/4 < q_{2m} < 1/2 \). The number of stator slots \( Q_s \) must be a multiple of the number of phases, i.e. \( Q_s = 6, 12, 18, \ldots \) for \( m = 6 \) phases. Reasonable dual-layer fractional slot windings with even number of slots and a coil span of two slots require at least 18 slots. Table 1 lists suitable combinations of stator slots and rotor poles for six-phase windings and gives the corresponding number of slots per pole per phase \( q_{6m} \).

2.2 Winding design with star of slots

The star of slots is a collection of phasors which illustrates all possible phase shifts of the slot emfs. Each of these phasors is valid for \( t \) slots, so the star of slots consists of \( Q_s = Q_s/t \) basic phasors. The counter \( t \) is the greatest common divisor of the number of slots \( Q_s \) and the number of pole pairs \( p \). Neighbouring phasors in the star of slots include a phase angle \( \alpha_p = 2\pi/Q_s \). The electrical phase shift between neighbouring slot emfs is \( \alpha_e = \alpha_p \cdot p/t \). In order to label the
phases with slot numbers, start with slot 1. When incrementing the slot number, jump by the electrical angle \( \alpha_e \) to the next phasor. If you stop at slot number \( Q_h \), each phasor has its first slot number. If \( t > 1 \) you will get the further slot numbers for each phasor by adding \( Q_h, \ldots, (t-1)Q_h \) to the first slot number of each phasor. Fig. 1a shows an example for \( Q_h = 36 \) and \( p = 7 \). As \( t = \gcd(Q_h, p) = 1 \), the number of phasors \( Q_h \) equals to the number of slots \( Q_s \).

Adjacent phasors include an angle \( \alpha_e = 10° \), and the electrical angle between neighbouring emfs is \( \alpha_e = 70° \).

The star of slots, i.e. the phasors and their numbering, is identical for three and six phases. Only the assignment to the phases alters. One possibility is to split each of the original six-phase belts of a three-phase system into two equal portions. One portion belongs to the first three-phase system and the other to the second three-phase system. For example phase belt \( u+ \) divides into \( u1+ \) and \( u2+ \). Altogether there are 12 phase belts which implement two three-phase systems with an electrical phase shift of 30° between them. Fig. 1a shows these 12-phase belts for the example with 36 slots and 14 poles. The phase belts indicate the assignment of the first layer to the phases. The second layer of the winding scheme follows by shifting the first layer by two slots, and Fig. 1b demonstrates the final setup of the winding.

The solution with 12-phase belts mixes coils in the same slots which belong to different three-phase systems (Fig. 1b). This is critical with respect to fault tolerance as a failure in one three-phase system can easily affect the other three-phase system. So it may be a wish to locate the two three-phase systems in different slots. Note that a strict separation of phases like in fractional slot concentrated windings is not possible because with a coil span of two slots coils belonging to different three-phase systems will always touch each other in the end windings. Nevertheless, two three-phase windings in different slots will improve the reliability. In order to achieve this, change the placing of the slot numbers in the star of slots: place all odd slot numbers on an imaginary inner circle near to the phasors and all even slot numbers on an imaginary outer circle (see Fig. 1c). The first three-phase system belongs to the odd slot numbers and is defined by overlaying the inner circle with six-phase belts. The second three-phase system belongs to the even slot numbers and is defined by overlaying the outer circle with six-phase belts. A shift of the outer phase belts against the inner ones of half of the belt width implements a phase shift of 30° between the two three-phase systems. Altogether there are again 12-phase belts, but they have double width. This width is now the same like in three-phase windings. The description ‘2 × 6 phase belts’ refers to this way of winding design.

### Table 1  Number of slots per pole per phase for six-phase fractional slot windings with a coil span of two slots

| \( Q_s \) | 2p  | \( Q_h \sim \) |
|---|---|---|
| 18 | 8  | 3/8 |
| 24 | 10 | 3/10 |
| 30 | 14 | 2/7 |
| 36 | 16 | 5/14 |

2.3 Mmf harmonics

The setup of the winding dictates the shape of the mmf curve which emerges when feeding with two balanced sinusoidal three-phase currents. The harmonic analysis of this stepped mmf curve delivers the individual content of each harmonic \( v \).

Fig. 2 compares the mmf harmonics contents of the two six-phase solutions and the three-phase layout for the example with 36 slots and 14 poles. The first-order component which is distinct with three phases vanishes completely with both six-phase variants. However, with 12-phase belts, the content of the 5th remains unaffected, whereas \( 2 \times 6 \) phase belts increase the effect of the 5th.

All variants produce only harmonics with an uneven ordinal number \( v \). This is due to the uneven denominator in the number of slots per pole per phase. All variants of Table 1 where the denominator of \( q \) is even have a broader spectrum which contains also even ordinal numbers [9]. Among the harmonics with an ordinal number \( v > p \) the slot harmonics are dominant for all variants. They have an ordinal number \( v = k \cdot Q_h \pm p \) (wherein \( k \) is a positive integer).

### 3 Phase shift between three-phase systems

The phase shift of 30° between the two three-phase systems in the example above is an optimum. With respect to Table 1, it is
variants – 12-phase belts and 2 × 6 phase belts. If a phase shift of at
least 30° is not possible, a phase shift of 60° is an option which
avoids: \( v = r \alpha_p \). The ordinal number of the working harmonic is
now \( v = r \alpha_p \). The harmonic \( v = 1 \) refers to an mmf wave with two
poles along the circumference of the machine. It is only existent, if
the winding scheme repeats \( r \) times along the circumference: \( v = r \alpha_p \). This may cause
confusion which the following definition of ordinal numbers
should solve:

### Criteria for comparison of windings

The choice of a winding for a six-phase machine is ambiguous even if the combination of slots and poles is fixed. Is the variant with 12-phase belts always superior to the 2 × 6 phase belts variant with respect to harmonics content? What is the recommended phase shift when using the 2 × 6 phase belts for reliability reasons? There is a need for criteria in order to compare the alternatives. Therefore, this section introduces two characteristic figures, namely the sub-harmonic distortion and a rotor loss figure. Both of them utilise the mmf harmonic content.

There exist different ways in order to count the ordinal numbers
of mmf harmonics which occur in fractional slot windings. Reference [9] suggests to assign the ordinal number \( v^* = 1 \) to the harmonic with the longest wave length. A fractional slot winding produces harmonics with \( v^* = 1, 4, 7... \). Running into the positive direction and harmonics with \( v^* = 2, 5, 8... \) running into the negative direction. With this way of counting the harmonic with ordinal number \( v^* = 1 \) always exists. However, the ordinal number \( v^*_w \) of the working harmonic differs from \( p \), if the winding scheme repeats \( r \) times along the circumference: \( v^*_w = p/r \). This may cause confusion which the following definition of ordinal numbers avoids: \( v = r \alpha_p \). The ordinal number of the working harmonic is now \( v = r \alpha_p \). The harmonic \( v = 1 \) refers to an mmf wave with two poles along the circumference of the machine. It is only existent, if the winding scheme does not repeat (\( r = 1 \)).

### 3.2 Six-phase windings with 2 × 6 phase belts

The shift angle of the outer phase belts against the inner phase belts determines the phase shift between the two three-phase systems. However, possible shifts are linked to the angle \( \alpha_p \) between adjacent phasors. If the number of phasors in the star of slots \( Q_b \) is a multiple of 12, the phase shift can only be an odd multiple of \( \alpha_p \).

For a stator with 24 slots, the angle \( \alpha_p \) has the same value for both possible pole pair numbers \( p = 5 \) and \( p = 7 \): \( \alpha_p = 15° \). This means that neither a shift of 30° nor a shift of 60° between the three-phase systems is possible. The windings can implement only a shift of 15° or 45°.

If \( Q_b \) is a multiple of 6 but not of 12, this allows the phase shift to be a multiple of \( \alpha_p \).

If \( Q_b \) is not a multiple of 6, the phase shift can be chosen as a multiple of \( \alpha_p/2 \). Fig. 5 illustrates this case with an example showing 30 slots and 16 poles. The instruction to label the star of slots with slot numbers, until each phasor has three phasors meet directly the limiting line between two adjacent phasors. If the number of phasors in the star of slots \( Q_b \) is a multiple of 6 but not of 12, writing odd slot numbers on an inner and even slot numbers on an outer circle takes care of the separation of three-phase systems into different slots. The shift angle of the outer phase belts against the inner ones corresponds to the phase shift. So the design in Fig. 5 implements a phase shift \( (3/2) \alpha_p = 36° \).
length is high compared to the tooth pitch they penetrate into all parts of the stator and will cause additional iron losses and local saturation. Moreover, they can excite bending modes of the stator. Their interaction with the rotor brings torque ripple about. So the rotor sees \( v \) pole pairs of the harmonic \( v \) running over him. The frequency in the rotor caused by this harmonic is therefore,

\[
\omega_r = v \cdot \omega_{rel,v} = \begin{cases} 
\omega_i \times (1 + v/p) & \text{if } v \text{ and } v_p = p \\
\omega_i \times |v/p - 1| & \text{if } v \text{ and } v_p = p
\end{cases}
\] (4)

The sign indicates the running direction of the harmonic. Setting \( v = p \) in this equation delivers the mechanical rotor speed \( \omega_m \). From the rotor point of view, all other harmonics run with a relative speed \( \omega_{rel,v} = \omega_r - \omega_m \). Within one relative revolution, a point on the rotor sees \( v \) pole pairs of the harmonic \( v \) running over him. The frequency in the rotor caused by this harmonic is therefore,

\[
\omega_r = v \cdot \omega_{rel,v} = \begin{cases} 
\omega_i \times (1 + v/p) & \text{if } v \text{ and } v_p = p \\
\omega_i \times |v/p - 1| & \text{if } v \text{ and } v_p = p
\end{cases}
\] (4)

The associated definition of a relative rotor frequency \( f_{rel,r\nu} \) eliminates the stator frequency:

\[
f_{rel,r\nu} = \left| 1 + \frac{v}{p} \right| \left| \frac{v}{p} - 1 \right| \] (5)

A simple approach for eddy losses is \( P_{ed} = f^{1.5} B^2 \) [10]. Let us assume that the flux densities \( B \) rise linearly with the amplitudes of the mmfs \( \tilde{V}_v \). However, high-order harmonics are damped by the air gap. So we can create a rotor loss coefficient \( C_r \) which sums up the influence of the harmonics using normalised mmf amplitudes:

\[
C_r = \sum_{v} f_{rel,r\nu}^{1.5} \cdot \tilde{V}_{rel,v} \cdot e^{0.02v} \cdot \tilde{V}_{rel,v} = \frac{\tilde{V}_v}{\tilde{V}_p} \] (6)

The choice of the factor 0.025 in the exponent of the damping term is somewhat arbitrary. There are more sophisticated definitions of rotor loss coefficients [11]. However, these depend on additional geometrical parameters of the design. The approach presented here needs only the number of slots and poles and the winding arrangement.

5 Comparison results

Table 2 lists the results for the criteria introduced above if the machine is in normal operation with equal load on both three-phase systems. For each slot/pole-combination, the first row lists the sub-harmonic distortion and the second row contains the rotor loss coefficient. The smaller the result for each criterion, the better the winding.

For a given combination of slots and poles, \( C_r \) is always better for 12-phase belts than for \( 2 \times 6 \) phase belts. For a given number of slots, \( C_r \) worsens for the higher pole number. This effect is independent of the way the phase belts are defined.

With rising slot number, \( C_r \) diminishes if the design adopts the lowest suitable pole number.

The shd is always better for 12-phase belts than for \( 2 \times 6 \) phase belts. For the variant with 24 slots, 10 poles and 12 phase belts, the sub-harmonics disappear completely.

The results suggest the following recommendations for the choice of the shift in windings with \( 2 \times 6 \) phase belts: with 18 slots and 8 poles a shift of 20° or 40° has slight advantages compared to 60°, whereas with 10 poles a decision is difficult: for 60° the shd is a little bit better, but \( C_r \) has grown slightly. With 30 slots and 14 poles, a shift of 24° or 36° is advantageous. However, with 16 poles, there is no clear recommendation as for 60° the shd is better, but \( C_r \) is worse.

A complete comparison demands to consider also the backup situation when only one three-phase system is working and the other is off. Table 3 shows the results for this situation. The calculation of the characteristics used the mmf amplitude of the working harmonic \( \tilde{V}_p \) in normal operation as reference value. In nearly all cases, \( C_r \) is lower than the respective entry in Table 2. That means the rotor losses with one system at full load are less than with both systems at full load. However, \( C_r \) never reduces by half, so related to the output power the rotor losses rise, and the

4.2 Characteristic for rotor losses

All harmonics except the working harmonic travel along the circumference with speeds that differ from the rotor speed. Therefore, they cause eddy losses in the rotor. The angular speed of an individual harmonic \( v \) depends on the stator frequency \( \omega_p \):

\[
\omega_i = \pm \frac{\omega_p}{v}
\] (3)

Table 2: Sub-harmonic distortion and rotor loss coefficient for six-phase windings in normal operation

| \( Q_s \) | 12 phase belts | \( 2 \times 6 \) phase belts |
|-------|----------------|--------------------------|
|       | Shift ≥ 30°    | Shift ≥ 60°               | Shift ≥ 60°               |
| 18    | 0.133          | 0.267                    | 0.296                     |
|       | 1.403          | 1.467                    | 1.544                     |
| 10    | 0.209          | 0.358                    | 0.321                     |
|       | 1.697          | 1.754                    | 1.826                     |
| 24    | 0              | 0.254                    | –                         |
|       | 1.029          | 1.076                    | –                         |
| 14    | 0.184          | 0.355                    | –                         |
|       | 1.411          | 1.434                    | –                         |
| 30    | 0.140          | 0.242                    | 0.299                     |
|       | 0.892          | 0.977                    | 1.061                     |
| 16    | 0.187          | 0.341                    | 0.299                     |
|       | 1.016          | 1.082                    | 1.180                     |
| 14    | 0.121          | 0.211                    | –                         |
|       | 0.635          | 0.703                    | –                         |
| 22    | 0.195          | 0.461                    | –                         |
|       | 1.002          | 1.231                    | –                         |

Fig. 5 Star of slots with \( 2 \times 6 \) phase belts for a six-phase machine with 30 slots and 16 poles

Table 3: Sub-harmonic distortion and rotor loss coefficient for six-phase windings in normal and backup operation

| \( Q_s \) | 12 phase belts | \( 2 \times 6 \) phase belts |
|-------|----------------|--------------------------|
|       | Shift ≥ 30°    | Shift ≥ 60°               | Shift ≥ 60°               |
| 18    | 0.133          | 0.267                    | 0.296                     |
|       | 1.403          | 1.467                    | 1.544                     |
| 10    | 0.209          | 0.358                    | 0.321                     |
|       | 1.697          | 1.754                    | 1.826                     |
| 24    | 0              | 0.254                    | –                         |
|       | 1.029          | 1.076                    | –                         |
| 14    | 0.184          | 0.355                    | –                         |
|       | 1.411          | 1.434                    | –                         |
| 30    | 0.140          | 0.242                    | 0.299                     |
|       | 0.892          | 0.977                    | 1.061                     |
| 16    | 0.187          | 0.341                    | 0.299                     |
|       | 1.016          | 1.082                    | 1.180                     |
| 14    | 0.121          | 0.211                    | –                         |
|       | 0.635          | 0.703                    | –                         |
| 22    | 0.195          | 0.461                    | –                         |
|       | 1.002          | 1.231                    | –                         |
Table 3  Sub-harmonic distortion and rotor loss coefficient for six-phase windings if only one three-phase system is working

| Qs | 2p | 12 phase belts | 2 × 6 phase belts |
|----|----|----------------|------------------|
| 18 | 8  | 0.514          | 0.196            |
|    |    | 0.928          | 1.164            |
| 10 | 5  | 0.536          | 0.671            |
|    |    | 0.966          | 1.647            |
| 24 | 10 | 0.536          | 0.179            |
|    |    | 0.645          | 0.889            |
| 14 | 12 | 0.583          | 0.744            |
|    |    | 0.641          | 1.591            |
| 30 | 14 | 0.502          | 0.2              |
|    |    | 0.833          | 0.953            |
| 16 | 14 | 0.472          | 0.615            |
|    |    | 0.849          | 1.196            |
| 36 | 14 | 0.501          | 0.181            |
|    |    | 0.448          | 0.634            |
| 16/20 see Qs = 18 and 2p = 8 and 10 respectively |
| 22 | 16 | 0.578          | 0.835            |
|    |    | 0.521          | 1.552            |

efficiency will be worse. It still holds that $C_r$ is better for 12 phase belts than for $2 \times 6$ phase belts.

The shd gets high with 12-phase belts. An even higher shd occurs if the design with $2 \times 6$ phase belts adopts the higher pole number for a given number of slots. Windings with $2 \times 6$ phase belts which implement the lower pole number for a certain number of slots exhibit acceptable values for the shd.

If the operation with only one three-phase system is not an issue a design with 12-phase belts is ideal as it better diminishes sub-harmonics as well as rotor losses. In contrast machines intended to work with only one three-phase system once in a while should use $2 \times 6$ phase belts and a lower pole number for a given number of slots.

6 Conclusion

This paper describes in detail six-phase fractional slot windings with a winding step of two slots. It discusses possible combinations of slots and poles together with the corresponding number of slots per pole per phase. Based on the star of slots, it introduces two different ways for winding design. The first one uses 12 phase belts. The second one separates the two three-phase windings into different slots by using $2 \times 6$ phase belts. Both ways of winding design separate the six phases into two three-phase subsystems. The phase shift between these two subsystems cannot reach always the optimum value of $30^\circ$, and the paper discusses possible values for the phase shift. Two criteria are presented in order to compare the different windings: the sub-harmonic distortion and a simple rotor loss coefficient. A comparison of the considered windings which deals with normal operation of both three-phase systems and with the operation of only one three-phase system reveals their strengths and weaknesses. Windings with 12 phase belts are better suited for normal operation but are inferior with respect to reliability and produce more sub-harmonics content when operating with only one three-phase system. If fault tolerance is important, a design with $2 \times 6$ phase belts and a low pole number for a given number of slots together with a proper phase shift is the best choice.

7 References

[1] Levi, E.: ‘Multiphase electric machines for variable-speed applications’, IEEE Trans. Ind. Electron., 2008, 55, (5), pp. 1893–1909
[2] Dajaku, G., Gerling, D.: ‘A novel 24-slots/10-Poles winding topology for electrical machines’, IEEE Int. Elect. Machines & Drives Conf. (IEMDC), Niagara Falls, 2011, pp. 65–70
[3] Reddy, P.B., Huh, K.-K., El-Refaie, A.M.: ‘Generalized approach of stator shifting in interior permanent-magnet machines equipped with fractional-slot concentrated windings’, IEEE Trans. Ind. Electron., 2014, 61, (9), pp. 5035–5046
[4] Wang, K., Zhu, Z.Q., Ombach, G.: ‘Synthesis of high-performance fractional-slot permanent-magnet machines with coil-pitch of two slot-pitches’, IEEE Trans. Energy Convers., 2014, 29, (3), pp. 758–778
[5] Abdel-Khalik, A.S., Ahmed, S., Massoud, A.M.: ‘A six-phase 24-slot/10-pole permanent-magnet machine with low space harmonics for electric vehicle applications’, IEEE Trans. Magn., 2016, 52, (6), Article # 8700130
[6] Wang, J., Patel, V.L., Wang, W.: ‘Fractional-slot permanent magnet brushless machines with low space harmonic contents’, IEEE Trans. Magn., 2014, 50, (1), Article # 8200209
[7] Patel, V.L., Wang, J., Wang, W., et al.: ‘Analysis and design of 6-phase fractional slot per pole per phase permanent magnet machines with low space harmonics’, IEEE Int. Elect. Machines & Drives Conf. (IEMDC), Chicago, 2013, pp. 386–393
[8] Patel, V.L., Wang, J., Nair, S.S.: ‘Demagnetization assessment of fractional-slot and distributed wound 6-phase permanent magnet machines’, IEEE Trans. Magn., 2015, 51, (6), Article # 8105511
[9] Binder, A.: ‘Elektrische maschinen und antriebe’ (Springer-Verlag, Berlin/Heidelberg, 2012)
[10] Jassal, A., Polinder, H., Ferreira, J.A.: ‘Literature survey of eddy-current loss analysis in rotating electrical machines’, IET Electr. Power Appl., 2012, 6, (9), pp. 743–752
[11] Bianchi, N., Fornasier, E.: ‘Index of rotor losses in three-phase fractional-slot permanent magnet machines’, IET Electr. Power Appl., 2009, 3, (5), pp. 381–388