Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks based on correlation functions with scattering interpolating operators both at the source and at the sink

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We present first results of a recently started lattice QCD investigation of antiheavy-antiheavy-light-light tetraquark systems including scattering interpolating operators in correlation functions both at the source and at the sink. In particular, we discuss the importance of such scattering interpolating operators for a precise computation of the low-lying energy levels. We focus on the $\bar{b}b\bar{u}d$ four-quark system with quantum numbers $I(J^P) = 0(1^+)$, which has a ground state below the lowest meson-meson threshold. We carry out a scattering analysis using Lüscher’s method to extrapolate the binding energy of the corresponding QCD-stable tetraquark to infinite spatial volume. Our calculation uses clover $u$, $d$ valence quarks and NRQCD $b$ valence quarks on gauge-link ensembles with HISQ sea quarks that were generated by the MILC collaboration.

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1. Introduction

We report on a recently started lattice QCD project in which we aim to study possibly existing heavy-heavy-light-light tetraquark resonances. In the following, we focus on the \( \bar{b}b\bar{u}d \) tetraquark with quantum numbers \( I(J^P) = 0(1^+) \), which is theoretically simpler compared to other tetraquark candidates because it is QCD-stable. This tetraquark is the counterpart of the \( \bar{c}c\bar{u}d \) tetraquark \( T_{cc} \) recently discovered by LHCb \[1, 2\].

In the past couple of years, several independent lattice QCD studies of \( \bar{b}bq\bar{q} \) and \( \bar{b}c\bar{q}q \) systems (\( q \) denotes a light \( u \), \( d \) or \( s \) quark) were published. These computations employed either exclusively local four-quark interpolating operators \[3–7\] or local and scattering four-quark interpolating operators, but the latter only at the sink \[8, 9\].

In the work presented here, we include scattering interpolating operators both at the source and at the sink. This allows a more precise determination of finite-volume energy levels not only for bound states, but also for scattering states. This is particularly important for \( \bar{Q}\bar{Q}qq \) systems, where bound states and scattering states are very close, or where bound states do not exist but resonances might exist. An example is a future full lattice QCD investigation of a possibly existing \( \bar{b}b\bar{u}d \) tetraquark resonance with \( I(J^P) = 0(1^-) \) proposed in Ref. \[10\].

2. Interpolating operators

To study the \( \bar{b}b\bar{u}d \) four-quark system with quantum numbers \( I(J^P) = 0(1^+) \), we use local interpolating operators

\[
O_1 = O_{BB^\dagger}(0) = \sum_x \bar{b}_y s(x) \bar{b}_y j_u(x) - (d \leftrightarrow u),
\]

\[
O_2 = O_{B^\dagger B}(0) = \epsilon_{jkl} \sum_x \bar{b}_j k d(x) \bar{b}_l u(x) - (d \leftrightarrow u),
\]

\[
O_3 = O_{Dd^\dagger}(0) = \sum_x \bar{b}^a \gamma_j c \bar{b}^b \bar{t} (x) d^a \gamma_5 u^b (x) - (d \leftrightarrow u),
\]

and scattering interpolating operators

\[
O_4 = O_{B(0)B^\dagger(0)} = \left( \sum_x \bar{b}_y s(x) \right) \left( \sum_y \bar{b}_y j_u(y) \right) - (d \leftrightarrow u),
\]

\[
O_5 = O_{B^\dagger(0)B(0)} = \epsilon_{jkl} \left( \sum_x \bar{b}_j k d(x) \right) \left( \sum_y \bar{b}_l u(y) \right) - (d \leftrightarrow u).
\]

Here, \( C \) denotes the charge conjugation matrix, and upper indices \( a \) and \( b \) are color indices. For more details we refer to our previous work \[8\].

3. Lattice setup

We use 2 + 1 + 1-flavor HISQ gauge-link ensembles generated by the MILC collaboration \[11\] as summarized in Table 1.
Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks

Marc Wagner

Table 1: Gauge-link ensembles (a: lattice spacing; \( N_s, N_t \): number of lattice sites in spatial and temporal direction; \( m_\pi^{(\text{sea})}, m_\pi^{(\text{val})} \): pion mass corresponding to light sea and light valence quarks; \( N_{\text{conf}} \): number of gauge-link configurations used for computations).

| Ensemble    | \( a \) [fm] | \( N_s \times N_t \) | \( m_\pi^{(\text{sea})} \) [MeV] | \( m_\pi^{(\text{val})} \) [MeV] | \( N_{\text{conf}} \) |
|-------------|---------------|---------------------|-------------------------------|-------------------------------|-----------------|
| a12m310     | 0.1207(11)    | 24^3 \times 64     | 305.3(4)                      | 310.2(2.8)                    | 1053            |
| a12m220     | 0.1202(12)    | 24^3 \times 64     | 218.1(4)                      | 225.0(2.3)                    | 1020            |
| a12m220     | 0.1184(10)    | 32^3 \times 64     | 216.9(2)                      | 227.9(1.9)                    | 1000            |
| a12m220L    | 0.1189(09)    | 40^3 \times 64     | 217.0(2)                      | 227.6(1.7)                    | 1030            |
| a09m310     | 0.0888(08)    | 32^3 \times 96     | 312.7(6)                      | 313.0(2.8)                    | 1166            |
| a09m220     | 0.0872(07)    | 48^3 \times 96     | 220.3(2)                      | 225.9(1.8)                    | 657             |

We use a mixed-action setup with Wilson-clover \( u \) and \( d \) valence quarks [12, 13]. For the \( b \) valence quarks we use lattice NRQCD [14]. Correlation functions are computed with point-to-all propagators if there is a local operator at the source. If there is a scattering operator at the source, we use stochastic timeslice-to-all propagators combined with the one-end trick (see e.g. Ref. [15]). Moreover, we use APE smearing for the gauge links and Gaussian smearing for the quark fields.

For the analysis of the correlation matrices we employ two independent methods: solving standard generalized eigenvalue problems (GEVP) as well as the Athens Model Independent Analysis Scheme (AMIAS) [16].

4. Effective masses from “exclusively local” versus “local and scattering” interpolating operators

Based on previous lattice QCD computations [3, 5, 7, 8] we expect the ground state around 100 MeV below the \( BB^* \) threshold (the lowest meson-meson threshold in this channel) representing the QCD-stable tetraquark. The first and second excitations in the finite spatial volume should be meson-meson scattering states resembling \( BB^* \) and \( B^*B^* \) close to the respective thresholds.

The left plot in Figure 1 shows effective masses from a GEVP using only local interpolating operators \( O_1, O_2 \) and \( O_3 \) (i.e. corresponding to a 3 \times 3 \) matrix). The plateaus exhibit a strong discrepancy with the expectation discussed in the previous paragraph. The right plot in Figure 1 shows effective masses from a GEVP using both local interpolating operators \( O_1, O_2 \) and \( O_3 \) as well as scattering interpolating operators \( O_4 \) and \( O_5 \) (i.e. corresponding to a 5 \times 5 \) matrix). These effective masses are consistent with the expectation. Thus, Figure 1 demonstrates that scattering operators are essential for a precise determination of scattering states.

5. Scattering analysis

To determine the mass of the \( \bar{b}\bar{b}ud \) tetraquark in infinite volume (for a given ensemble, i.e. at given \( m_\pi \) and nonzero \( a \)), we proceed as in our previous work [8]:

1. Compute the two lowest energy levels in the finite spatial volume (see section 4).
Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks

Marc Wagner

Figure 1: Effective energies from a GEVP for ensemble a09m310. (left) 3 × 3 matrix, only local interpolating operators $O_1$, $O_2$ and $O_3$. (right) 5 × 5 matrix, both local interpolating operators $O_1$, $O_2$ and $O_3$ and scattering interpolating operators $O_4$ and $O_5$.

(2) Compute the corresponding phase shifts $\delta_0(k_0)$, $\delta_0(k_1)$ using Lüscher’s finite-volume method [17].

(3) Parameterize $\delta_0(k_0)$, $\delta_0(k_1)$ using the effective-range expansion,

$$k \cot(\delta_0(k)) = \frac{1}{a_0} + \frac{r_0}{2} k^2$$

with fit parameters $a_0$ and $r_0$.

(4) The mass of the $\bar{b}b\bar{d}u$ tetraquark (and the energy of the first excitation) in infinite spatial volume corresponds to a pole in the scattering amplitude

$$T_0(k) = \frac{1}{\cot(\delta_0(k)) - i}.$$  

The position of the pole can be obtained via Eqs. (6) and (7).

The dark-gray data points in the left plot of Figure 2 represent lattice QCD finite-volume energy levels (ground state and first excitation) for three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but identical $a ≈ 0.12$ fm and $m_\pi ≈ 220$ MeV (ensembles a12m220S, a12m220, a12m220L). The orange curves correspond to the two lowest finite volume energy levels as functions of the spatial extent $L$, computed with Lüscher’s finite-volume method using the effective-range expansion (6). There are rather small differences between the finite-volume and infinite-volume energy levels. We attribute this to the large binding energy, $\Delta E_0 ≈ O(100$ MeV). Scattering analyses are, however, expected to be more important for smaller binding energies (e.g. for the $\bar{b}b\bar{s}u$ system with $J^P = 1^+$) and essential for tetraquark resonances (e.g. for the $\bar{b}b\bar{d}u$ system with $I(J^P) = 0(1^-)$ [10]).

The right plot of Figure 2 shows an extrapolation of the tetraquark binding energy in the light $u/d$ quark mass based on the six ensembles listed in Table 1. The preliminary result at the physical pion mass $m_{\pi,\text{phys}} = 135$ MeV is

$$\Delta E_0(m_{\pi,\text{phys}}) ≈ (-103 ± 8) \text{ MeV},$$
Lattice QCD study of antiheavy-antiheavy-light-light tetraquarks

Marc Wagner

Figure 2: (left) Lattice QCD finite volume energy levels (dark gray: ground state and first excitation) for three different volumes $V = L^3$ with $L/a = 24, 32, 40$, but identical $a \approx 0.12 \text{ fm}$ and $m_\pi \approx 220 \text{ MeV}$ (ensembles a12m220S, a12m220, a12m220L) together with a fit based on the effective range expansion (6). (right) Extrapolation of the tetraquark binding energy in the light $u/d$ quark mass to the physical point.

where only the statistical uncertainty is shown. This binding energy is slightly smaller than, but consistent with, previous lattice results [3, 5–8].

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References

[1] R. Aaij et al. [LHCb], Nature Phys. 18, no. 7, 751–754 (2022) [arXiv:2109.01038 [hep-ex]].
[2] R. Aaij et al. [LHCb], Nature Commun. 13, no. 1, 3351 (2022) [arXiv:2109.01056 [hep-ex]].

[3] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. Lett. 118, 142001 (2017) [arXiv:1607.05214 [hep-lat]].

[4] A. Francis, R. J. Hudspith, R. Lewis and K. Maltman, Phys. Rev. D 99, 054505 (2019) [arXiv:1810.10550 [hep-lat]].

[5] P. Junnarkar, N. Mathur and M. Padmanath, Phys. Rev. D 99, 034507 (2019) [arXiv:1810.12285 [hep-lat]].

[6] R. J. Hudspith, B. Colquhoun, A. Francis, R. Lewis and K. Maltman, Phys. Rev. D 102, 114506 (2020) [arXiv:2006.14294 [hep-lat]].

[7] P. Mohanta and S. Basak, Phys. Rev. D 102, 094516 (2020) [arXiv:2008.11146 [hep-lat]].

[8] L. Leskovec, S. Meinel, M. Pflaumer and M. Wagner, Phys. Rev. D 100, 014503 (2019) [arXiv:1904.04197 [hep-lat]].

[9] S. Meinel, M. Pflaumer and M. Wagner, Phys. Rev. D 106, 034507 (2022) [arXiv:2205.13982 [hep-lat]].

[10] J. Hoffmann, A. Zimermmane-Santos, M. Pflaumer, M. Wagner, poster at LATTICE2022.

[11] A. Bazavov et al. [MILC], Phys. Rev. D 87, 054505 (2013) [arXiv:1212.4768 [hep-lat]].

[12] T. Bhattacharya et al. [PNDME], Phys. Rev. D 92, 094511 (2015) [arXiv:1506.06411 [hep-lat]].

[13] R. Gupta, Y. C. Jang, B. Yoon, H. W. Lin, V. Cirigliano and T. Bhattacharya, Phys. Rev. D 98, 034503 (2018) [arXiv:1806.09006 [hep-lat]].

[14] R. J. Dowdall et al. [HPQCD], Phys. Rev. D 85, 054509 (2012) [arXiv:1110.6887 [hep-lat]].

[15] A. Abdel-Rehim, C. Alexandrou, J. Berlin, M. Dalla Brida, J. Finkenrath and M. Wagner, Comput. Phys. Commun. 220, 97-121 (2017) [arXiv:1701.07228 [hep-lat]].

[16] C. Alexandrou, T. Leontiou, C. N. Papanicolas and E. Stiliaris, Phys. Rev. D 91, 014506 (2015) [arXiv:1411.6765 [hep-lat]].

[17] M. Lüscher, Nucl. Phys. B 354, 531-578 (1991).