Robust Phase Estimation of a Hybrid Monte Carlo/Finite Memory Digital Phase-Locked Loop

Sang-Su LEE†, Nonmember, Sung-Hyun YOU†, Member, and Seok-Kyoon KIM††, Nonmember

SUMMARY Digital phase-locked loops (DPLLs) have been designed in a number of ways to correctly generate pulse signals in various systems. However, the existing DPLLs have poor acquisition performance or are prone to the divergence phenomenon when modeling and/or round-off errors exist and the noise statistics are incorrect. In this paper, we propose a novel DPLL whose phase estimator is designed in hybrid form that utilizes the advantages of Monte Carlo estimation, which is robust to nonlinear effects such as measurement quantization, and a finite memory estimator, which is robust against incorrect noise information and system model mismatch. The robustness of the proposed hybrid Monte Carlo/finite memory DPLL is demonstrated by comparing its phase estimation performance via a numerical example.

key words: digital phase-locked loop, Monte Carlo estimation, finite memory estimation

1. Introduction

Digital phase-locked loops (DPLLs) are responsible for synchronization, which plays an important role in signal transmission systems using a digital clock, and are widely used in electrical systems such as communication systems and control systems [1]. All-digital phase locked loops (ADPLLs), which is designed with fully digital loop control circuitry, have been attracting attention recently and applied to various radio frequency transceivers [2], [3]. It is necessary to estimate accurate phase for tracking input information immediately. Some progress has been made in the filter theory approach to estimate the unknown phase.

Kalman filter (KF) has been applied to DPLL based on the advantage of its fast operation speed in linear systems [4]. However, KF-based DPLL is only accurate when the noise is close to white Gaussian. Moreover, it is vulnerable to mismatch between the actual process and the system model. Particle filter (PF)-based DPLL has been designed to overcome the effects of any form of noise [5]. The PF performs a Monte Carlo (MC) estimation based on a defined number of particles [6]. However, a large number of particles must be defined to achieve sufficient reliability and it results in a long computation time. In addition, when the number of particles is insufficient, this may cause entire degeneracy called sample impoverishment [7]. Another fundamental disadvantage of KF and PF is that they have an infinite impulse response (IIR) structure, which cannot exclude the influence of an occasional incorrect measurement [8].

In this paper, we propose a hybrid MC/finite memory (FM) DPLL (MCFMDPLL) which has all of the advantages of both an MC estimator and an FM estimator to obtain accurate phase values in the ADPLL. The FM estimator estimates the state by using only the measurements during a given number of time steps, called the horizon size. Therefore, even if an incorrect measurement occurs temporarily, it can estimate more accurate value soon after. The Mahalanobis distance [9] between the actual and predicted output of the system is considered to determine the failure of the MC estimation due to an incorrect measurement. The FM estimator takes only the horizon size as a design parameter and does not require the noise statistic. In this paper, we derive the gain of the FM estimator by considering the unbiasedness condition with the given horizon size. The robustness of hybrid MCFMDPLL is verified via a numerical example.

This paper has three more sections organized as follows. In Sect. 2, we derive the algorithm and design method of the proposed hybrid MCFMDPLL. In Sect. 3, we verify the reliability of the proposed DPLL in various situations with a variety of disturbance types through simulations. Finally, Sect. 4 contains concluding remarks.

2. The Hybrid MCFMDPLL

At the k-th time step, the offset of positive going zero-crossing points $\phi_k$ and timing offset $\Delta \phi_k$ of a zero-crossing ADPLL can be defined as [4]

$$\phi_k = t_0 + k(T_1 - T_0),$$

$$\Delta \phi_k = T_1 - T_0,$$

where $t_0$ is the initial zero-crossing offset and $T_1$ and $T_0$ are the measured clock period and the nominal clock period, respectively. By defining the state $x_k = [\phi_k \Delta \phi_k]^T$, we can then represent the state-space model with the k-th measurement $y_k$ as follows:

$$x_{k+1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} w_{\phi_k} \\ w_{\Delta \phi_k} \end{bmatrix} = A x_k + w_k,$$

$$y_k = \begin{bmatrix} 1 & 0 \end{bmatrix} x_k + v_k = C x_k + v_k,$$

where $w_{\phi_k}$ and $w_{\Delta \phi_k}$ are process noise terms and $w_k$ and $v_k$ are measurement noise terms. The process noise terms $w_{\phi_k}$ and $w_{\Delta \phi_k}$ are assumed to be zero-mean Gaussian white noise processes, while the measurement noise terms $w_k$ and $v_k$ are assumed to be zero-mean Gaussian white noise processes as well.
where \( \mathbf{w}_k \) and \( \mathbf{v}_k \) are the process noise vector and measurement noise, respectively, and both are assumed to be zero-mean white Gaussian noise with covariances \( \mathbf{Q} = \text{diag}\{\sigma_q^2, \sigma_w^2\} \) and \( \mathbf{R} = \sigma_r^2 \).

The overall estimation procedure for the DPLL is shown in the flowchart in Fig. 1. First, in the initialization step, the parameter values for the MC estimation and the FM estimation are respectively initialized. Next, the prior step, the parameter values for the MC estimation and the covariance have the same value as before regeneration. In the MC estimation, if the threshold value is \( \sqrt{R} \), the confidence interval, the failure diagnosis algorithm shows that the MC estimation has failed. The threshold value is chosen as a chi-square distribution of the state vectors and the measurements in the MC estimation are loaded. In the MC estimation, if the measurement is significantly incorrect or invalid state vectors are selected during resampling [10], this can cause the divergence of the estimation error. The failure diagnosis algorithm operates to determine whether the MC estimator has diverged. The value to determine failure is the Mahalanobis distance [9] which is represented by

\[
\begin{align*}
d_{k} &= \frac{(\mathbf{y}_k^*-\hat{\mathbf{y}}_k^*)^2}{\mathbf{R}}.
\end{align*}
\]

where \( \mathbf{y}_k^* \) and \( \hat{\mathbf{y}}_k \) denote the actual measurement, the measurement noise covariance, and the predicted measurement, respectively. If the Mahalanobis distance is greater than a certain threshold value corresponding to a given confidence, the failure diagnosis algorithm shows that the MC estimation has failed. The threshold value is chosen as a chi-square value from the given confidence level and the chi-square table. If the threshold value is \( d_{th} \), the confidence interval, a new measurement range indicating that the MC estimation is normal is expressed as \( (\mathbf{y}_k^* - \sqrt{R} d_{th}, \mathbf{y}_k^* + \sqrt{R} d_{th}) \) by (5). In a case where failure is not detected, the MC estimation \( \hat{\mathbf{x}}_{MC} = \sum_{i=1}^{N_{MC}} \mathbf{w}_i \mathbf{x}_i \) becomes the final estimation result; otherwise, the following form of estimate \( \hat{\mathbf{x}}_{FM,k} \) can be represented as the final estimation result:

\[
\begin{align*}
\hat{\mathbf{x}}_{FM,k} &= \mathbf{H} \mathbf{Y}_{k-1},
\end{align*}
\]

where \( \mathbf{H} \in \mathbb{R}^{2 \times N}, \mathbf{Y}_{k-1} = [\mathbf{y}_{k-N} \mathbf{y}_{k-N+1} \cdots \mathbf{y}_{k-1}]^T \), and \( N \) are the gain matrix, an augmented measurement vector, and the size of the recent finite discrete time interval (also called the horizon size), respectively. According to system model Eqs. (3) and (4), the following relationships can be established:

\[
\begin{align*}
\mathbf{x}_{k-N} &= \mathbf{A}^{-N} \mathbf{x}_k - \mathbf{A}^{-N} \mathbf{F}_N \mathbf{W}_{k-1}, \\
\mathbf{y}_{k-1} &= \mathbf{C}_N \mathbf{x}_k + \mathbf{G}_N \mathbf{W}_{k-1} + \mathbf{v}_{k-1},
\end{align*}
\]

where

\[
\begin{align*}
\mathbf{C}_N &= \begin{bmatrix} \mathbf{C} \mathbf{A}^{-N} \\ \mathbf{C} \mathbf{A}^{-N} \\ \vdots \\ \mathbf{C} \mathbf{A}^{-1} \end{bmatrix}, \\
\mathbf{G}_N &= \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ \mathbf{C} & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{C} \mathbf{A}^{-N-2} & \mathbf{C} \mathbf{A}^{-N-3} & \cdots & \cdots & \mathbf{C} \mathbf{A}^{-N-2} & \cdots & \cdots & \mathbf{C} \mathbf{A}^{-N} \end{bmatrix}, \\
\mathbf{W}_{k-1} &= [\mathbf{w}_{k-N}^T \mathbf{w}_{k-N+1}^T \cdots \mathbf{w}_{k-2}^T \mathbf{w}_{k-1}^T]^T, \\
\mathbf{V}_{k-1} &= [\mathbf{v}_{k-N} \mathbf{v}_{k-N+1} \cdots \mathbf{v}_{k-1}]^T.
\end{align*}
\]

By substituting (8) into (6), the following relationship is obtained:

\[
\begin{align*}
\hat{\mathbf{x}}_k &= \mathbf{H} (\mathbf{C}_N \mathbf{x}_k + \mathbf{G}_N \mathbf{W}_{k-1} + \mathbf{v}_{k-1}).
\end{align*}
\]

Since the noise in all of the time steps is white-Gaussian, the following equation holds by taking the expectation:

\[
\begin{align*}
\mathbf{E} [\hat{\mathbf{x}}_k] &= \mathbf{H} \mathbf{C}_N \mathbf{x}_k.
\end{align*}
\]

In order to satisfy the unbiasedness condition, \( \mathbf{E} [\hat{\mathbf{x}}_k] = \mathbf{E} [\mathbf{x}_k] \), the following equation must be satisfied:

\[
\begin{align*}
\mathbf{H} \mathbf{C}_N &= \mathbf{I}.
\end{align*}
\]

Among the various solutions for \( \mathbf{H} \) available in (11), we can consider a minimal length solution. In other words, the gain matrix \( \mathbf{H} \) is derived using the following Lagrange function that solves the minimization problem with an equality constraint:

\[
\begin{align*}
\mathcal{L} &= \text{tr}(\mathbf{H} \mathbf{H}^T) + \Lambda (\mathbf{H} \mathbf{C}_N - \mathbf{I}),
\end{align*}
\]

where \( \Lambda \) is a Lagrange multiplier. The partial derivative of \( \mathcal{L} \) with respect to \( \mathbf{H} \) should be zero as follows:

\[
\begin{align*}
\frac{\partial \mathcal{L}}{\partial \mathbf{H}} &= 2 \mathbf{H} + \Lambda \mathbf{C}_N = 0.
\end{align*}
\]

Therefore, by (11) and (13), the solution of \( \mathbf{H} \) is given as

\[
\begin{align*}
\mathbf{H} &= (\mathbf{C}_N^T \mathbf{C}_N)^{-1} \mathbf{C}_N^T.
\end{align*}
\]

After FM estimation, the state vectors for the MC estimation are regenerated stochastically based on it. The mean of the regenerated state vectors is equal to the FM estimate, and the covariance has the same value as before regeneration. In the subsequent time steps, the MC estimation failure is diagnosed through the Mahalanobis distance between the distribution of the state vectors and the measurements in the MC estimation.
estimation procedure. If failure is detected, the aforementioned FM estimation and the state vectors are regenerated.

3. Numerical Example

In this section, we demonstrate the performance of the proposed hybrid MCFMDPLL in a numerical example. Environmental specifications are as follows. The nominal clock period and the measured clock period are set to \( T_0 = 0.0001 \) and \( T_1 = 1.17T_0 \), respectively. The process noise covariances are \( \sigma^2_0 = \sigma^2_{\Delta\phi} = T_0^2/12 \) and the measurement noise covariance is \( \sigma^2_v = T_0^2/32 \). The threshold of the Mahalanobis distance for the divergence of the MC estimation is set at 6.635 from the chi-square table to make the confidence level of the MC estimation 99%. In terms of designing the DPLL, the number of state vectors for the MC estimation is set at 200, and the horizon size \( N \) for finite memory estimation at 4. In addition, in the simulations, the estimators use \( Q = 0.01Q \) and \( R' = 100R \) for the process and measurement covariances, respectively, for considering incorrect noise information is used. The measurement quantization unit is set at 0.0001. The model mismatch can be expressed by adding a stochastic term in \( A \) as follows:

\[
A' = \begin{bmatrix} 1 & 1 + \delta_k \\ 0 & 1 \end{bmatrix},
\]

where \( \delta_k \) is regarded as a white Gaussian random variable with standard deviation \( \sigma_\delta = 0.1 \).

Figure 2 shows the estimating performance of the DPLLs in the presence of incorrect noise information, measurement quantization, and model mismatch. As shown in Table 1, the hybrid MCFMDPLL had 2.89 and 2.30 times fewer errors than the KF and PF-based DPLLs, respectively, under the same conditions. As a result, the proposed hybrid MCFMDPLL showed a more robust performance against inaccurate information than the existing DPLLs.

4. Conclusions

In this paper, we proposed the hybrid MCFMDPLL in which an MC estimation method estimates the state of the current time based on the state vectors at the previous time step. In order to prevent the divergence phenomenon, a criterion for judging the divergence of the MC estimation based on the Mahalanobis distance and chi-square values, the FM estimator, was derived to estimate the state based on finite time interval information when the MC estimator diverges. The FM estimator was designed by solving an optimization problem that takes into account the unbiasedness condition as an equality constraint by introducing a Lagrange function. From the simulation results, it is evident that the proposed hybrid MCFMDPLL is robust to temporary incorrect measurements or nonlinear effects while the conventional approach DPLLs diverge. We expect that the proposed estimation method is applicable to various fields.

Acknowledgments

This work was supported partially by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education (2018R1A6A1A03026005) and partially by “Human Resources program in Energy Technology” of the Korea Institute of Energy Technology Evaluation and Planning (KETEP) granted financial resource from the Ministry of Trade, Industry & Energy, Republic of Korea (No. 20174030201820).

References

[1] W.C. Lindsey and C.M. Chie, “A survey of digital phase-locked loops,” Proc. IEEE, vol.69, no.4, pp.410–431, 1981.
[2] R.B. Staszewski and P.T. Balsara, “All-digital PLL with ultra fast settling,” IEEE Trans. Circuits Syst. II, Exp. Briefs, vol.54, no.2, pp.181–185, 2007.
[3] J. Bae, S. Radhapuram, I. Jo, W. Wang, T. Kihara, and T. Matsuoka, “A design of 0.7-V 400-MHz all-digital phase-locked loop for implantable biomedical devices,” IEICE Trans. Electron., vol.E99-C, no.4, pp.431–439, 2016.
[4] Y. Qian, X. Cui, M. Lu, and Z. Feng, “Steady-state performance of Kalman filter for DPLL,” Tsinghua Science and Technology, vol.14, no.4, pp.470–473, 2009.
[5] J.H. Chung, S.H. You, J.M. Pak, J.H. Kim, M.T. Lim, and M.K. Song, “A novel particle filter-based digital phase-locked loop robust against quantization error,” International Journal of Control, Automation and Systems, vol.15, no.1, pp.457–461, 2017.
[6] A. Doucet, N. de Freitas, and N. Gordon, Sequential Monte Carlo Methods in Practice (Springer, New York, 2001).
[7] J.M. Pak, C.K. Ahn, Y.S. Shamliy, and M.T. Lim, “Improving reliability of particle filter-based localization in wireless sensor networks via hybrid particle/FIR filtering,” IEEE Trans. Ind. Informat., vol.11, no.5, pp.1089–1098, 2015.
[8] S.H. You, J.M. Pak, C.K. Ahn, P. Shi, and M.T. Lim, “Unbiased
finite-memory digital phase-locked loop," IEEE Trans. Circuits Syst. II, Exp. Briefs, vol.63, 798, 2016.

[9] J. Hardin and D.M. Rocke, “The distribution of robust distances,” Journal of Computational and Graphical Statistics, vol.14, no.4, pp.928–946, 2005.

[10] D. Simon, Optimal State Estimation, 1st ed., pp.466–468 (Wiley, US, 2006).

[11] Y.S. Shmaliy, S. Zhao, and C.K. Ahn, “Unbiased Finite Impulse Response Filtering: An Iterative Alternative to Kalman Filtering Ignoring Noise and Initial Conditions,” IEEE Control Syst. Mag., vol.37, no.5, pp.70–89, 2017.

[12] S. Zhao, Y.S. Shmaliy, P. Shi, and C.K. Ahn, “Fusion Kalman/UFIR Filter for State Estimation with Uncertain Parameters and Noise Statistics,” IEEE Trans. Ind. Electron., vol.64, no.4, pp.3075–3083, 2017.

[13] C.K. Ahn, P. Shi, and M.V. Basin, “Deadbeat Dissipative FIR Filtering,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol.63, no.8, pp.1210–1221, 2016.

[14] C.K. Ahn, S. Zhao, and Y.S. Shmaliy, “Frequency Efficient Receding Horizon $\mathcal{H}_\infty$ FIR Filtering in Discrete-Time State-Space,” IEEE Trans. Circuits Syst. I, Reg. Papers, vol.64, no.11, pp.2945–2953, 2017.