Non-Poisson queueing model’s identification  
(Case study: AKAP and AKDP bus on the West Lines bus service of Tirtonadi Surakarta)

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Abstract. Transportation is an important factor to improve the economic level of an area. If the transportation does great, the economy will grow up. In case of that, Tirtonadi Surakarta Bus Station always try to provide optimum services to avoid long queue. Queue system on the west lanes nonpatas bus service of Tirtonadi Surakarta Bus Station (Solo-Yogyakarta, Solo-Semarang, Solo-Purwokerto, Solo-Jakarta, and Local Route) will be analyzed using queueing theory. The main goal of this project is to identify the distribution of the model of Non-Poisson and calculate the size of system performance. Primary data analysis is made up of equilibrium sample test (steady state) and tested the distribution of the arrivals number and the bus service’s time. Based on the analysis of queue process, there are non-Poisson queue models estimated with Triangular, Beta, Weibull, and the models are (M/TRIA/1) : (GD/∞/∞), (M/BETA/1) : (GD/∞/∞), (M/M/1) : (GD/∞/∞), (BETA/G/1) : (GD/∞/∞), and (M/WEIB/1) : (GD/∞/∞). The size of system performance shows that line A (Solo-Yogyakarta) and line B (Solo-Semarang) have a higher level of service rush than other service lines.

1. Introduction
Queuing phenomenon cannot be separated from daily life, including in the transportation sector. Transportation is one of the important facilities for the region to advanced and developing. The existence of transportation is expected to eliminate isolation and member of stimulant towards development in all sectors of life, such as trade, industry, and other sectors. If the transportation does great, the velocity of money in an area will grow faster.

Tirtonadi Bus Terminal in Surakarta is considered to have a strategic role because it has the midpoint from west to east and otherwise. The non-patas bus routes cause a lot of long lines in the service lane. A service facility is called good if it provides fast service and does not allow customers to wait too long in order to reduce the decrease in customer trust toward service. Therefore, to find out the effectiveness of non-patas bus service on the western route of Tirtonadi Surakarta Bus Station, the queue analysis is conducted. Afterwards, by doing the research it will give a conclusion which can be used to provide input that can help to describe the queue problems in the service lane so that the terminal as a service provider can provide excellent services and encourage the economic sector of Surakarta City to grow properly.

2. Literature Review
2.1. Measurement of steady state
Steady state is a condition when the properties of a system do not change within time (constant). According to [1], for example, \( \lambda \) is the average of customer arrival to service place per time unit. \( \mu \) is the average of customer served per unit of time. \( c \) is the number of service facilities (server), then \( \rho \) is defined as the ratio between the average of customer arrivals (\( \lambda \)) with the average of customer served per time unit (\( \mu \)) where the steady-state assumption is fulfilled if \( \lambda < \mu \) or can be written as follows:

\[
\rho = \frac{\lambda}{c \mu} \text{ with } \rho < 1
\]

2.2. Poisson process dan exponential distribution

According to [2], the queue process generally is assumed as the time between arrivals and service times following the Exponential distribution, or equal to the numbers of arrival and the number of services following the Poisson distribution. The number of cases stated \{N (t), t ≥ 0\} will be stated as a summation process if N (t) shows the numbers of arrivals numbers that occur until time t, with N (0) = 0 and will be stated as a Poisson process if it accordance three assumptions such as regularity, homogeneity in time, and independence.

2.3 Queue system models

2.3.1. (M/M/1) : (GD/∞/∞)

According to [1], this service model is a single service model with no limits of the system capacity or the calling source capacity where the numbers of arrivals per time unit following the Poisson distribution, service time with exponential distribution. There is a formula for obtaining all basic measures of service model performance (M/M/1):(GD/∞/∞):

1. The number of customers expected in the queue:

\[
L_q = L_s - \frac{\lambda}{\mu} = \frac{\rho^2}{1 - \rho}
\]

2. The number of customers estimated in the system:

\[
L_s = E(n) = \frac{\rho}{1 - \rho}
\]

3. The waiting time expected in the queue:

\[
W_q = \frac{L_q}{\lambda} = \frac{\rho}{\mu(1 - \rho)}
\]

4. The waiting time expected in the system:

\[
W_s = \frac{L_s}{\lambda} = \frac{1}{\mu(1 - \rho)}
\]

2.3.2. (M/G/1) : (GD/∞/∞)

According to [3], this model also called the Pollaczek-Khintchine (P-K), described through a single service with a situation based on these following three assumptions:

a. Arrivals distribution following the Poisson process with an average level \( \lambda \)

b. General service time distribution with expectations (average) \( E[t] = 1/\mu \) and variance \( \text{var}[t] \)

c. The steady state status is stated with \( \rho = \frac{\lambda}{\mu} < 1 \)

There is a formula for obtaining all basic measures of service model performance (M/G/1):(GD/∞/∞):

1. The number of customers expected in the queue:

\[
L_q = L_s - \frac{\lambda}{\mu}
\]

2. The number of customers estimated in the system:
3. The waiting time expected in the queue:
   \[ W_q = \frac{L_q}{\lambda} \]

4. The waiting time expected in the system:
   \[ W_s = \frac{L_s}{\lambda} \]

2.3.3. \((G/G/c) : (GD/\infty/\infty)\)
According to [2], the queue model \((G / G / c): (GD / \infty / \infty)\) is a queue model with a general pattern of arrival distribution (General), a general pattern of arrival distribution, with \(c\) service facilities as many as service. The queue discipline used in this model is a general FIFO (First In First Out), the maximum capacity allowed in the system is \(\infty\).

1. The number of customers expected in the queue:
   \[ L_q = \frac{\mu}{\lambda} \left[ \frac{1}{(c-1)(1-\rho)^2} \right] \rho_0 \]
   \[ L_q = \frac{\mu}{\lambda} \left[ \frac{1}{c!} \right] \rho_0 \]
   \[ L_q = L_q \frac{\mu^2 \nu t + \nu(t) \lambda^2}{2} \]

   With: \(\nu(t) = \left( \frac{1}{\mu^2} \right)^2\) and \(\nu(t) = \left( \frac{1}{\lambda^2} \right)^2\)

2. The number of customers estimated in the system:
   \[ L_s = L_q + \frac{\lambda}{\mu} \]

3. The waiting time expected in the queue:
   \[ W_q = \frac{L_q}{\lambda} \]

4. The waiting time expected in the system:
   \[ W_s = \frac{L_s}{\lambda} \]

2.4. Triangular Distribution
According to [4], Triangular distribution is a continuous distribution with three parameters: minimum value \(a\), maximum value \(b\), and value that is most likely to occur \(m\) with \(a \leq m \leq b\). The Triangular distribution is represented by \(Triangular (a, m, b)\). The probability of the Triangular distribution density function is as follows:

\[
f(x) = \begin{cases} 
\frac{2(x-a)}{(m-a)(b-a)}, & \text{for } a \leq x < m \\
\frac{2(b-x)}{(b-m)(b-a)}, & \text{for } m \leq x \leq b \\
0, & \text{others}
\end{cases}
\]

The average of the Triangular distribution can be found by the following formula:
\[ \mathbb{E}(x) = \frac{a + m + b}{3} \]
Triangular distribution variant:
\[
\text{Var}(X) = \frac{(a^2 + m^2 + b^2 - ma - ab - mb)}{18}
\]

2.5. Beta Distribution
According to [5], Beta distribution is a continuous but flexible distribution that is limited to vulnerabilities limitation used for probability models. The function of Beta distribution density with the parameters \(\alpha\) and \(\beta\):

\[
f(x) = \begin{cases} 
\frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1}, & 0 < x < 1 \\
0, & \text{others}
\end{cases}
\]

with the parameters \(\alpha > 0\) and \(\beta > 0\). While the mean and variance in the Beta distribution are:

\[
\mu = \frac{\alpha}{\alpha + \beta} \quad \sigma^2 = \frac{\alpha \beta}{(\alpha + \beta + 1)(\alpha + \beta)}
\]

2.6. Weibull distribution
According to [5], continuous random variable \(X\) has a Weibull distribution, with the parameters \(\alpha\) and \(\beta\) in the density function of the Weibull distribution as follows:

\[
f(x, \alpha, \beta) = \begin{cases} 
\alpha \beta x^{\beta-1} e^{-x^\beta}, & x > 0 \\
0, & \text{others}
\end{cases}
\]

with the parameters \(\alpha > 0\) and \(\beta > 0\). While the mean and variance in the Weibull distribution are:

\[
\mu = \frac{\alpha}{\beta} \Gamma(1 + \frac{1}{\beta}) \\
\sigma^2 = \left(\frac{1}{\beta}\right)^2 \Gamma(1 + \frac{2}{\beta}) - \left(\frac{1}{\beta}\right)^2 \Gamma^2(1 + \frac{1}{\beta})
\]

3. Research methodology
The research was conducted in the west lane of Tirtornadi Surakarta Bus Station on December 18\textsuperscript{th} - 23\textsuperscript{th}, 2017. The study started from 8 am to 5 pm every day. The software used was Xnote Stopwatch, Microsoft Excel 2016, SPSS Statistics 16.0, POM QM, and Rockwell Automation Arena v14. The data used were the number of bus arrivals and bus service time data.

3.1. Step analysis
Steps in the implementation of research and data analysis were as follows:

1. The data obtained must be accordant the steady-state conditions. If the data has not fit these conditions, it should be followed up by changing the appropriate time interval.
2. Carried out test distribution for data on the number of customers and data on the number of services by using the Kolmogorov-Smirnov test.
3. Determined the initial queue model.
4. Carried out the distribution of suitability test with Chi-Square for arrivals and services with General distribution. Data were obtained from Beta, Weibull, or Triangular distribution at output Arena.
5. Determined performance measures in the queue system, i.e. \(L_q\), \(L_s\), \(W_s\), \(W_q\). Made results and discussion obtained from system performance measures. It could be obtained an optimal model.
4. Result and discussion

4.1. General overview of the Tirtonadi Surakarta bus station queue system

Tirtonadi Surakarta Bus Station was divided into 3 parts, there were the arrival, service and departure terminals. The west lane terminal served buses with western majors, wherein this study there were 5 bus routes in the west lane, code A (Solo-Yogyakarta), code B (Solo-Semarang), code C (Solo-Purwokerto, Cilacap), code D (Solo-Jakarta, Bandung), and E code (Local Route). All buses entering the terminal and using the services were considered as customers, while the terminals were considered as servers. The time interval used in this analysis was 30 minutes, except the B code with a time interval of 15 minutes.

4.2 Descriptive analysis of Tirtonadi Surakarta bus station’s queue

Table 1. The number of non-patas bus in the west lane

| Day       | Line | A  | B  | C  | D  | E  |
|-----------|------|----|----|----|----|----|
| Monday    |      | 32 | 84 | 20 | 47 | 58 |
| Tuesday   |      | 29 | 76 | 20 | 35 | 60 |
| Wednesday |      | 29 | 85 | 19 | 39 | 66 |
| Thursday  |      | 33 | 78 | 19 | 45 | 58 |
| Friday    |      | 38 | 80 | 20 | 29 | 61 |
| Saturday  |      | 41 | 82 | 24 | 41 | 50 |

4.3 Measurement of steady state

From the research data, it was obtained the value on utility of service facilities for the five lanes as follows:

Table 2. Measurement of steady state

| Data       | Line | \( \rho \) |
|------------|------|-------------|
| Numbers of | A    | 0.81941     |
| Arrivals   | B    | 0.95551     |
| Time       | C    | 0.37926     |
| services   | D    | 0.20300     |
|            | E    | 0.31066     |

Value of utility level is less than one, which implies that the steady-state condition is accordant, meaning that average rate of vehicle arrival does not exceed the average service rate. So that the vehicle service system in the five lanes is good.

4.4 Distribution test

Hypothesis

H0: the number of arrival data followed Poisson Distribution and the time services data followed Exponential Distribution

H1: the number of arrival data did not follow Poisson Distribution and the time services data not followed Exponential Distribution

Test Criteria

H0 rejected if \( D \geq D_{tab}(1 - \alpha) \)

From table 3, giving the result that the queue model of non-patas bus in the west lane of Tirtonadi Terminal in Surakarta are code A (M/G/1) : (GD/∞/∞), code B (M/G/1) : (GD/∞/∞), code C (M/M/1) : (GD/∞/∞), code D (G/G/1) : (GD/∞/∞), and code E (M/G/1) : (GD/∞/∞).
### Table 3. Distribution test

| Line | Data                      | $D_{\text{manual}}$ | $D_{\text{table}}$ | Result     |
|------|---------------------------|---------------------|---------------------|------------|
| A    | Numbers of arrivals       | 0.11703             | 0.13087             | $H_0$ accepted |
|      | Time services             | 0.17976             | 0.13401             | $H_0$ rejected |
| B    | Numbers of arrivals       | 0.07419             | 0.09254             | $H_0$ accepted |
|      | Time services             | 0.21698             | 0.09254             | $H_0$ rejected |
| C    | Numbers of arrivals       | 0.10277             | 0.13087             | $H_0$ accepted |
|      | Time services             | 0.18872             | 0.16492             | $H_0$ rejected |
| D    | Numbers of arrivals       | 0.18810             | 0.13087             | $H_0$ rejected |
|      | Time services             | 0.15647             | 0.15399             | $H_0$ rejected |
| E    | Numbers of arrivals       | 0.10850             | 0.13087             | $H_0$ accepted |
|      | Time services             | 0.19825             | 0.13533             | $H_0$ rejected |

4.5 Queue system model

To know the real distribution of the number of bus arrivals and bus service time distributed by General distribution, so it was conducted the Chi-Square distribution test based on output Arena.

### Table 4. Distribution test

| Line | Data                      | Output Distribution | $p$-value | Result     |
|------|---------------------------|---------------------|-----------|------------|
| A    | Numbers of arrivals       | Poisson             | 0.117     | $H_0$ accepted |
|      | Time services             | Triangular          | 0.101     | $H_0$ accepted |
| B    | Numbers of arrivals       | Poisson             | 0.074     | $H_0$ accepted |
|      | Time services             | Beta                | 0.208     | $H_0$ accepted |
| C    | Numbers of arrivals       | Poisson             | 0.102     | $H_0$ accepted |
|      | Time services             | Exponential         | 0.096     | $H_0$ accepted |
| D    | Numbers of arrivals       | Beta                | 0.352     | $H_0$ accepted |
|      | Time services             | Weibull             | <0.005    | $H_0$ rejected |
| E    | Numbers of arrivals       | Poisson             | 0.109     | $H_0$ accepted |
|      | Time services             | Weibull             | 0.441     | $H_0$ accepted |

Based on table 4, giving the result that the final model of queue modeling non-patas bus in the west lane of Tirtonadi Terminal in Surakarta are code A (M/TRIA/1) : (GD/∞/∞), code B (M/BETA/1) : (GD/∞/∞), code C (M/M/1) : (GD/∞/∞), code D (BETA/G/1) : (GD/∞/∞), and code E (M/WEIB/1) : (GD/∞/∞).

4.6 System performance measures

Based on the output obtained by using POM QM software, it gave the measures of system queue performance of non-patas bus in the western lane of Tirtonadi Terminal Surakarta as follows:

### Table 5. Measures of service performance

| Line | $c$ | $\lambda$ | $\mu$ | $L_q$ | $L_s$ | $W_s$ | $W_q$ | $\rho$ |
|------|-----|-----------|-------|-------|-------|-------|-------|-------|
| A    | 1   | 1.87037   | 2.28258 | 3.71801 | 4.53742 | 2.42595 | 1.98785 | 0.81941 |
| B    | 1   | 2.22685   | 2.33052 | 20.5247 | 21.4802 | 9.6460 | 9.21692 | 0.95552 |
| C    | 1   | 1.12037   | 2.95412 | 0.23172 | 0.61097 | 0.54533 | 0.20682 | 0.37926 |
| D    | 1   | 2.18519   | 10.7647 | 0.05170 | 0.25470 | 0.11656 | 0.02366 | 0.20300 |
| E    | 1   | 3.26852   | 10.5211 | 0.14001 | 0.45067 | 0.13788 | 0.04283 | 0.31066 |
5. Conclusion
Based on this research, the final model for queue AKAP and AKDP bus on the west lane of Tirtonadi Bus Terminal in Surakarta with 1 server service facility, the discipline queue of FIFO (first in first out) with maximum capacity permitted in the system and the size of the unlimited calling source is:
1. Solo – Yogyakarta line with code A have Queue model (M/Tria/1):(GD/$\infty$/∞), number of arrival distribution is Poisson and time service distribution is Triangular.
2. Solo – Semarang line with code B have Queue model (M/Beta/1):(GD/$\infty$/∞), number of arrival distribution is Poisson and time service distribution is Beta.
3. Solo – Purwokerto line with code C have Queue model (M/M/1):(GD/$\infty$/∞), number of arrival distribution is Poisson and time service distribution is Exponential.
4. Solo – Jakarta line with code D have Queue model (M/Tria/1):(GD/$\infty$/∞), number of arrival distribution is Beta and time service distribution is General.
5. Village line with code E have Queue model (M/Weib/1):(GD/$\infty$/∞), number of arrival distribution is Poisson and time service distribution is Weibull.

From the results of the calculation of system performance measures, it can be concluded that the average server utilization ($\rho$) for bus A (Solo-Yogyakarta) and B (Solo-Semarang) has a higher level of service bustle than the other service lines.

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