Multi-view Registration Based on Weighted Low Rank and Sparse Matrix Decomposition of Motions

Congcong Jin\textsuperscript{a,b}, Jihua Zhu\textsuperscript{b}, Yaochen Li\textsuperscript{b}, Shanmin Pang\textsuperscript{b}, Lei Chen\textsuperscript{c}, Jun Wang\textsuperscript{d}

\textsuperscript{a}State Key Laboratory of Rail Transit Engineering Informatization (FSDI), Xi’an, 710043
\textsuperscript{b}School of Software, Xi’an Jiaotong University, Xi’an, 710049
\textsuperscript{c}School of Computer Science, Nanjing University of Posts and Telecommunications, Nanjing, 210003
\textsuperscript{d}School of Digital Media, Jiangnan University, Wuxi, 214122

Abstract

Recently, the low rank and sparse (LRS) matrix decomposition has been introduced as an effective mean to solve the multi-view registration. However, this method presents two notable disadvantages: the registration result is quite sensitive to the sparsity of the LRS matrix; besides, the decomposition process treats each block element equally in spite of their reliability. Therefore, this paper firstly proposes a matrix completion method based on the overlap percentage of scan pairs. By completing the LRS matrix with reliable block elements as much as possible, more synchronization constraints of relative motions can be utilized for registration. Furthermore, it is observed that the reliability of each element in the LRS matrix can be weighed by the relationship between its corresponding model and data shapes. Therefore, a weight matrix is designed to measure the contribution of each element to decomposition and accordingly, the decomposition result is closer to the ground truth than before. Benefited from the more informative LRS matrix as well as the weight matrix, experimental results conducted on several public datasets demonstrate the superiority of the proposed approach over other methods on both accuracy and robustness.

Keywords: multi-view registration, low rank and sparse matrix, weighted decomposition, matrix completion, relative motions, global motions

1. Introduction

Due to its wide applications in robot mapping [1, 2], 3D model reconstruction [3], object recognition [4, 5] and etc, point scan registration has attracted
broad interests among researchers. The aim of registration is to calculate the optimal transformation for two or more point scans and bring them into one coordinate system to recover the original scene of a 3D object. Based on the number of scans to be registered, this problem can be classified into two categories: pair-wise registration and multi-view registration.

The most popular method for pair-wise registration is the iterative closest point (ICP) algorithm proposed by Besl et al. [6], which iteratively builds up correspondences and calculates the rigid transformation by minimizing the residual error. Apart from the low efficiency, a significant disadvantage of this method is that it can only apply to two absolutely overlapping scans. Afterward, Fitzgibbon et al. [7] employed the Levenberg-Marquardt algorithm to improve the low efficiency of ICP; besides, Jost et al. [8] proposed a coarse-to-fine multi-resolution technique with the neighbor search algorithm to speed it up. To deal with the partially overlapping problem, an original idea was to reject outliers with distances greater than a preset threshold. Further, Chetverikov et al. [9] proposed the trimmed ICP (TrICP) algorithm, which introduced an overlap percentage into the original ICP algorithm. But it is ineffective. After that, Phillips et al. [10] proposed the fractional TrICP (FTrICP) algorithm to simultaneously compute the overlap percentage and transformation for partially overlapping scans.

To boost the accuracy of the aforementioned approaches, some probabilistic methods [11, 12, 13, 14] were also proposed. Much precise these methods may be, the huge computational resources they require pose a great challenge to most application areas. To address the local minimum problem existing in many algorithms, invariant features were introduced into ICP algorithm by Lee et al. [15]. Moreover, the genetic algorithm [16, 17] and particle filter [18] were also presented to determine the optimal rigid transformation. As for the low robustness, some methods have also been investigated in recent years. For example, Zhu et al. [19] proposed an ICP variant based on the ratio of bidirectional distances to assign a probability for each correspondence. [20] introduced correntropy into ICP and solve this algorithm by maximizing the correntropy. Although these algorithms can obtain optimal transformation in most situations, the robustness should be further improved.

Many researchers explore the thought of pair-wise algorithms and extend them into multi-view registration. The most original approach was proposed by Chen et al. [21], which repeatedly registers two scans into one model until all range scans are integrated into the whole model. The most significant problem in this method is error accumulation. To address this issue, Bergevin et al. [22] proposed to simultaneously register all scans through the ICP algorithm. Since this approach
establishes correspondences for every point in each scan, it is time-consuming. Afterwards, the multi-z-buffer technique [23] was applied into multi-view registration. To further reduce the accumulative error, some methods [24, 25] have proposed to optimize this problem over the graph of adjacent scans and transfer the registration error between coordinate systems. However, this approach cannot decrease the error essentially since it does not update the correspondences during registration.

In recent years, some novel methods have been introduced to deal with multi-view registration. For example, Mateo et al. [26] presented the Bayesian framework to deal with missing data of pair-wise correspondences, which then be solved by the Expectation-Maximization algorithm. Related works can also be found in [27]. Besides, [28] employs an ICP and Generalized Procrustes Analysis [29] combined strategy to obtain the registration result. To further explore the redundant information of non-adjacent point scans, Godvin et al. [30, 31] introduced the motion averaging algorithm [30] to refine global motions iteratively by Lie-algebra averaging, which then be extended by Li et al. [32] to achieve more accurate and efficient registration. Besides, Guo et al. [33] proposed a weighted motion averaging algorithm to improve the registration accuracy. In [34], Arrigoni et al. cast the multi-view problem into the low-rank and sparse matrix decomposition frame. By decomposing the relative motion stacked matrix, noise and outliers can be discarded accordingly, and the set of global motions can be recovered by any column of the decomposed matrix.

In this paper, we extend the method proposed in [34] to achieve more robust and accurate registration. The innovations of this paper can be illustrated as follows: it is acknowledged that the more elements the decomposed matrix contains, the more precise the decomposition consequence should be. Therefore, a novel matrix completion method based on overlap percentage is proposed. Apart from that, the weights of each relative motion are considered according to the relationship between the model and data shapes. Consequently, a weight matrix is designed to indicate relative motions reliability. Finally, a refined L1-ALM [35] algorithm is applied to decompose the matrix and obtain precise global motions. To demonstrate the efficacy of the proposed method, experiments are conducted on several public datasets.

The rest of this paper is organized as follows: section [2] briefly introduces the framework of LRS matrix based method [34]. Then in section [3], the proposed method is described in detail. Following that is section [4] in which experiments are conducted covering different aspects of the proposed method. At last, some conclusions are drawn in section [5].
2. The framework of LRS matrix based method

To achieve accurate multi-view registration, [34] proposes to construct a low rank and sparse matrix stacked with pair-wise registration. By decomposing the sparse matrix, global motions can be recovered accordingly after excluding noise and outliers. This approach can be illustrated in Fig. 1. For a set of given point scans, the first scan is chosen as the reference scan. During registration, two kinds of motions are involved, e.g., global motion $M_i$ (denoting the motion from $i$th scan to the reference scan) and relative motion $M_{ij}$ (denoting the motion from $j$th scan to $i$th scan). The transformation between global and relative motions can be expressed as:

$$M_{ij} = M_i^{-1}M_j$$ (1)

To fully utilize the relationship between global and relative motions, two block matrices are defined:

$$M = [M_1 \ M_2 \ \cdots \ M_N]$$ (2)

and

$$M^{-b} = \begin{pmatrix}
M_1^{-1} \\
M_2^{-1} \\
\vdots \\
M_N^{-1}
\end{pmatrix}$$ (3)

where $N$ is the number of scans to be registered. For further exploration, block matrix $X$ is defined to take advantage of the synchronization constraints:

$$X = \begin{bmatrix}
I & M_{12} & \ldots & M_{1N} \\
M_{21} & I & \ldots & M_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
M_{N1} & M_{N2} & \ldots & I
\end{bmatrix}$$ (4)

where $I$ is the identity matrix. Accordingly, $X$ can also be expressed as $X = M^{-b}M$. In ideal situations, $X$ has the same rank as $M_{ij}$, e.g., 4.

To construct matrix $X$, relative motions (denoted as blocks in Fig. 1) should be first provided. [34] uses the traditional ICP algorithm to obtain initial pair-wise registration results. According to the structure of $X$, global motions can
Figure 1: The framework of the LRS method, where the grey blocks represent missing relative motions, the green blocks on the diagonal are identity matrices, the golden blocks denote relative motions that their overlap percentages are smaller than a predefined threshold while blue blocks indicate relative motions with overlap percentages greater than the threshold, the red blocks mean outliers.

be acquired according to its any complete column. However, the problem is that accurate relative motions can not be acquired in most situations due to noise and outliers. Considering the characteristics of $X$, matrix decomposition approach is applied to recover its low rank part:

$$X = \hat{X} + E$$  \hspace{1cm} (5)

where $\hat{X}$ contains the low-rank structure of $X$ while $E$ represents a sparse matrix containing outliers. In practical scenarios, $X$ does not have all entries since a point scan usually does not overlap with all the others. Therefore, Eq. (5) can be solved by minimizing the following objective function:

$$\min_{\hat{X}} \|P_W(X - \hat{X})\|_1 + \lambda \|\hat{X}\|_*$$  \hspace{1cm} \text{s.t.} \quad \text{rank}(\hat{X}) \leq 4, X = \hat{X} + E$$  \hspace{1cm} (6)

where $P(\cdot)$ represents the projection of $X - \hat{X}$ onto $W$ and $W$ is an indicator matrix denoting whether the corresponding element is observed, $\|\|_*$ is the nuclear norm.

Once the accurate low rank part $\hat{X}$ is obtained, precise global motions can be accordingly acquired based on the aforementioned theories.
3. The proposed approach

While the method proposed in [34] has satisfactory registration result in ideal situations, two significant defects cannot be neglected. Firstly, the decomposition method it uses cannot achieve matrix completion significantly, thus the sparsity property poses a great challenge to the result. Besides, it does not take the reliability of each relative motion into consideration, which may include much noise into registration. Aiming at the above problems, we improve the original LRS decomposition based registration method from these two perspectives.

3.1. The construction of $X$

As Eq. (4) demonstrates, to decompose $X$, relative motions should be provided first. Considering the partially overlapping problem, the TrICP algorithm is selected to generate relative motions for some scan pairs. Therefore, the following TrICP objective function is computed based on an initial transformation:

$$
\min_{\mathbf{R}, \mathbf{t}} \frac{1}{|D_\xi|} \xi^{1+\lambda} \sum_{\mathbf{d}_i \in D_\xi} \left\| \mathbf{R}\mathbf{d}_i + \mathbf{t} - \mathbf{m}_{c(i)} \right\|_2^2 
$$

s.t. $\mathbf{R}^T \mathbf{R} = I_{3 \times 3}$, $\det(\mathbf{R}) = 1$

(7)

where $\xi$ is the overlap percentage parameter, $D_\xi$ represents the overlapping part of data shape $D$ to model shape $M$, $\mathbf{m}_{c(i)}$ denotes the correspondence of $\mathbf{d}_i$ and $\lambda$ is a preset parameter.

As explained before, the reliability of refined relative motions should also be considered, since two scans with pretty low overlap percentage are inclined to introduce outliers into $X$. Therefore, only the scan pairs that satisfy $\xi_{ij} > \xi_{thr}$ will be chosen for multi-view registration, where $\xi_{thr}$ is a predefined threshold. To estimate the overlap percentage between any arbitrary scan pair, the method proposed in [32] is adopted. For the $i$th point scan, it firstly searches correspondences from all the other scans. The largest correspondence distance that minimizes objective function (7) is selected as a threshold. Then for the other $j$th scan with $n_j$ points, suppose there are $n_j'$ points that are deemed as inliers (points in the overlap area), which have shorter distances than the selected threshold. Then the overlap percentage of the $j$th scan to $i$th scan can be computed as:

$$
\xi_{ij} = \frac{n_j'}{n_j}
$$

(8)
Furtherly, for the scan pair whose overlap percentage satisfies $\xi_{ij} > \xi_{thr}$, we can assume that the computed relative motion is relatively precise. What’s more, for any pair of precise relative motion, the following formula can be inferred based on its property:

$$M_{ji} = M_{ij}^{-1}$$

(9)

Accordingly, the motion characteristic based matrix completion method is introduced into this algorithm. For the reliable motion $M_{ij}$, its inverse motion $M_{ji}$ can be obtained based on Eq. (9) instead of being computed via TrICP algorithm. This strategy can not only obtain more precise result since more elements are involved in multi-view registration, but can greatly boost the efficiency. Therefore, we propose that relative motions whose overlap percentage satisfy $\xi_{ij} > \xi'_{thr}$, their inverse motions can be obtained according to Eq. (9), where $\xi'_{thr}$ is another threshold smaller than $\xi_{thr}$. This process can be illustrated in Fig. 1(b). For the computed reliable relative motions in Fig. 1(a), denoted as blue blocks, their dual motions can be accordingly computed. Based on the proposed matrix completion, the sparsity of $X$ can be reduced significantly.

3.2. Designing of the weight matrix $W$

Relative motions refined by TrICP algorithm may be affected by noise in many scenarios. To reduce its interference, a weight matrix $W$ is introduced into registration. It is understandable that the weight is closely related to the reliability of relative motions. A significant assumption in TrICP is that the correspondences between a scan pair can be measured by Euclidean distance. Besides, the minimized objective function, as well as the overlap percentage, is also based on this assumption. Therefore, the weight of relative motions relies heavily on the reliability of correspondences. However, in most cases, the value of problem (7) is heavily affected by the density of scans. For explanation simplicity, two variables are defined:

$$Me = \frac{1}{N_m} \left( \sum_{i=1}^{N_m} d_{\tilde{m}_i \tilde{m}_{c(i)}}^2 \right)$$

(10)

$$De = \frac{1}{|D_\xi|} \left( \sum_{i=1}^{|D_\xi|} d_{d_{\tilde{m}_i \tilde{m}_{c(i)}}}^2 \right)$$

where $d_{\tilde{m}_i \tilde{m}_{c(i)}}$ represents the distance between each $\tilde{m}_i$ in the model shape and its nearest neighbor $\tilde{m}_{c(i)}$ in the model shape itself, while $d_{d_{\tilde{m}_i \tilde{m}_{c(i)}}}$ represents the
distance between each $\vec{d}_i$ in the data shape and its correspondence $\vec{m}_{c(i)}$ in the model shape. Then the following observation can be captured: for a dense model shape, the value of $d_{\vec{m}_i,\vec{m}_{c(i)}}$ should be small. Meanwhile, the value of $d_{\vec{d}_i,\vec{m}_{c(i)}}$ should be small, too. However, for a sparse model shape, the situation is just the opposite. Another significant phenomenon is that with the increase of the sparsity, the increase of $Me$ is lower than that of $De$, since $d_{\vec{m}_i,\vec{m}_{c(i)}}$ is usually shorter than $d_{\vec{d}_i,\vec{m}_{c(i)}}$. Therefore, we propose the following formula to measure the contribution of each relative motion:

$$A_{ij} = \frac{Me}{De}$$  \hspace{1cm} (11)

However, $A$ is just the contribution matrix and should be normalized for computation convenience. Therefore, the following weight matrix can be obtained:

$$A_{ij} = \frac{A_{ij}}{\max(A)}$$ \hspace{1cm} (12)

where $\max(A)$ is the maximum value in $A$. It should also be noticed that this is the weight of each relative motion. To extend it into each element of $M_{ij}$, the following expanding is adopted:

$$(W_{ij})_{4\times4} = A_{ij} \otimes 1_{4\times4}$$ \hspace{1cm} (13)

where $\otimes$ is the Kronecker product and $1_{4\times4}$ is a 4*4 matrix filled by ones.

3.3. The decomposition process

To recover the refined global motions $\{I, M_2, \ldots, M_N\}$ from the initial input, the decomposition strategy is adopted to constrain the rank of $X$ while excluding outliers. In this paper, we adopt the $L_1$-norm based Augmented Lagrange Multiplier (ALM) method to solve objective function (6). Since any matrix $\hat{X}$ of fixed rank $r$ admits the factorization of $\hat{X} = UV$, where $U \in \mathbb{R}^{m \times r}$ and $V \in \mathbb{R}^{r \times n}$, recovering $\hat{X}$ can be translated into solving $U$ and $V$. To shrink the solution space, the matrix $U$ is constrained to be column-orthogonal, i.e., $U^T U = I_r$. Accordingly, problem (6) can be translated as:

$$\min_{U,V,E} \|W \odot E\|_1 + \lambda \|V\|_*$$

s.t. \hspace{0.5cm} $\text{rank}(V) \leq 4, X = UV + E, U^T U = I_r$ \hspace{1cm} (14)

where $E$ represents outliers contained in $X$, as shown in Fig. [c].
Benefited from the ALM algorithm, the corresponding augmented Lagrange function can be derived as:

\[
f(U, V, E, L, \mu) = \|W \odot E\|_1 + \lambda\|V\|_* + \langle L, X - UV - E \rangle + \frac{\mu}{2} \|X - UV - E\|^2_F
\] (15)

where \(L\) is the Lagrange multiplier and \(\mu\) is the penalty parameter. \(\langle A, B \rangle\) is the inner product of two matrices equivalent to the trace of \(A^T B\). To solve this problem, the Gauss-Seidel Iteration as adopted in [35] is applied to this question. The following three parameters are updated in each iteration.

### 3.3.1. Solving \(U\) via Orthogonal Procrustes

Based on the known \(E\) and \(V\), the following objective function is minimized to update \(U\):

\[
\min_U \frac{\mu}{2} \left\| (X - E + \frac{1}{\mu}L) - UV \right\|^2_F
\]

s.t. \(X = UV + E, U^T U = I_r\) (16)

The above orthogonal procrustes problem can be solved by the SVD method of \((X - E + \frac{1}{\mu}L)V^T\):

\[
[U_1 \quad S_1 \quad V_1] = \text{svd}((X - E + \frac{1}{\mu}L)V^T)
\] (17)

Consequently, \(U\) can be derived as:

\[
U = U_1 V_1^T
\] (18)

### 3.3.2. Solving \(V\) via Singular Value Decomposition

Based on the \(E\) and \(U\), the updating of \(V\) can be specified via the following minimization:

\[
\min_V \{\lambda\|V\|_* + \langle L, X - E - UV \rangle + \frac{\mu}{2} \|X - E - UV\|^2_F\}
\] (19)

Since \(U^T U = I_r\), the above objective function can be reformulated as:

\[
\min_V \{\lambda\|V\|_* + \frac{\mu}{2} \left\| V - U^T (X - E + \frac{1}{\mu}L) \right\|^2_F\}
\] (20)
**Algorithm 1:** Decomposition process using $L_1 - ALM$

| Input | $X$, $W$, $L$, $\mu$, and $\rho = 1.05$ |
|-------|---------------------------------|
| Output | Accurate $U$, $V$, $E$ and $L$, $\mu$ |

▷ Outer Iteration

while not converged do

▷ Inner Iteration

while not converged do

Update $U$ according to Eq. (17) and (18);
Update $V$ according to Eq. (22) and (23);
Update $E$ according to Eq. (25);
end

Update $L$ via $L = L + \mu(M - E - UV)$;
Update $\mu$ via $\mu = \min \{\rho\mu, 1e20\}$.
end

To solve problem (20), the soft-thresholding (shrinkage) operator is adopted:

$$ S_\varepsilon [q] = \max(|q| - \varepsilon, 0) \text{sgn}(q) $$

where $\text{sgn}(q)$ is the sign function. Then the singular values of $U^T(X - E + \frac{1}{\mu}L)$ is computed as:

$$ [U_2 \quad S_2 \quad V_2] = \text{svd}(U^T(X - E + \frac{1}{\mu}L)) $$

Finally, the shrinkage operator is applied to the singular values and the optimal $V$ can be updated as:

$$ V = U_2 S_{\frac{1}{\mu}} [S_2] V_2^T $$

### 3.3.3 Solving $E$ via Absolute Value Shrinkage

To update $E$, the following objective function is derived:

$$ \min_E \|W \odot E\|_1 + \frac{\mu}{2} \left\| X - E - (UV - \frac{1}{\mu}L) \right\|_F^2 $$

By applying the Absolute Value Shrinkage, this problem can be solved as:

$$ E = S_{\frac{1}{\mu}} [X - UV + \frac{1}{\mu} L] $$
After obtaining the precise $U$ and $V$, $\hat{X}$ can be accordingly computed as $\hat{X} = UV$. The whole process of obtaining precise $U$ and $V$ can be summarized in Algorithm 1.

### 3.4. Implementation details

In practical scenarios, the translation vector $\vec{t}_{ij}$ may pose significant effect since its elements have no boundaries. Accordingly, $\vec{t}_{ij}$ should be normalized to exclude the influence of peak values before decomposition. Therefore, the following normalization strategy is adopted:

$$\vec{t}_{ij} = \frac{\vec{t}_{ij}}{\max(\vec{t}_{ij})}$$  \hspace{1cm} (26)

where $\max(\vec{t}_{ij})$ represents the maximal value of $\vec{t}_{ij}$. After the accurate relative motions are obtained, it is restored to the original scale.

After decomposition, each element in $\hat{X}$ may not possess the essential properties to register multiple scans. To solve this issue, the 3*3 rotation matrix $R_{ij}$ in each block $X_{ij}$ should be firstly orthogonized:

$$[U_3 \quad S_3 \quad V_3] = \text{svd}(R_{ij})$$  \hspace{1cm} (27)

Accordingly, the following equation can be obtained:

$$R_{ij} = U_3 Q V_3^T$$  \hspace{1cm} (28)

where $Q$ is a square diagonal matrix with the elements of $\{1, 1, \det(U_3 V_3^T)\}$ on the main diagonal.

The introduced method for registration can be concluded as: for a given set of initial global motions, relative motions can be acquired based on the transformation between relative and global motions. With the help of TrICP algorithm, relative motions can be accordingly refined to form the matrix $X$. For providing more information for decomposition, matrix completion method is adopted to exclude the interference of outliers. For consideration of noise, a weight matrix is proposed to project each relative motion. In summarization of the proposed algorithm, Algorithm 2 gives a concise illustration:

### 4. Experiments

To testify its superior performances of the proposed approach, we conduct the following experiments from two perspectives: firstly, we validate the efficacy of
Algorithm 2: The proposed approach

**Input**: Point scans \( \{S_1, S_2, ..., S_N\} \) and initial global motions \( \{I, \hat{M}_1, ..., \hat{M}_N\} \)

**Output**: Accurate global motions \( \{I, M_1, ..., M_N\} \)

\( \triangleright \) **Outer iteration**

\( \text{while not converged do} \)

\( \triangleright \) **Inner iteration**

Estimate the overlap percentage between scan pairs;
Select scan pairs that satisfy \( \xi_{ij} > \xi_{thr} \);

\( \triangleright \) **for Each selected scan pair do**

Update their relative motions according to TrICP algorithm;
Compute their weights to decomposition based on Eq. (11), (12) and (13);

end

Update \( X \) and \( A \) based on Eq. (4) and (13);
Complete missing elements in \( X \) if \( \xi_{ij} > \xi_{thr}' \);
Decompose block matrix \( X \) according to Algorithm 1;
Refine global motions based on Eq. (1) and (4);

end

the two contributions in this paper separately, i.e., the matrix completion (MC) strategy and the weighted LRS decomposition method; then we conduct several comparison experiments to demonstrate the superiority of the proposed method from efficiency, accuracy and robustness. For comparing accuracy, the objective function proposed in [36] is adopted. Its value (ObjV) is computed by averaging the sum of TrICP algorithm, in which each time a scan registers to all the other scans until all the scans being taken as a data shape. All the following experiments were implemented in MATLAB on a four-core 3.6GHz computer with 8GB of memory. Besides, the results were averaged over 20 MC trials.

4.1. Validation

To validate the improvements of the proposed algorithm, four methods are compared in terms of accuracy and efficiency, i.e., the original LRS matrix based method, LRS matrix based method with the proposed matrix completion (LRS with MC), weighted LRS matrix based method and the proposed approach (weighted decomposition with matrix completion).
For each dataset, the same initial parameters ($R$ and $\vec{t}$) are provided for each method. The value of ObjV and runtime for each method is recorded in Tab. 1. Besides, the most accurate and efficient values are demonstrated in bold formulations.

**Table 1: Validation of accuracy and runtime.**

| Datasets | LRS | LRS with MC | Weighted LRS | Ours |
|----------|-----|-------------|--------------|------|
|          | ObjV | T(s)        | ObjV         | T(s)  |
|          | 0.842 | 21.374     | 0.837        | 15.705 |
| Bunny    | 0.826 | 18.852     | 0.821        | 26.496 |
|          | 0.552 | 33.824     | 0.541        | 14.083 |
| Dragon   | 0.528 | 20.852     | 0.527        | 23.205 |
|          | 0.179 | 81.203     | 0.168        | 66.285 |
| Happy    | 0.165 | 66.323     | 0.163        | 67.416 |

As shown in Tab. 1, the original LRS method performs the least accurate in the four competed approaches. Its performance is boosted a little after the matrix completion strategy is adopted. Furthermore, the accuracy is improved much more after adopting the weighted decomposition method than using the matrix completion method. This illustrates that the weighted decomposition method contributes more to accuracy than the matrix completion method. However, the performance of adopting the matrix completion or weighted decomposition method exclusively is not as better as that of the proposed method, which combines the matrix completion and weighted decomposition methods together.

Besides, our proposed method performs well as for efficiency, while the original LRS method performs the poorest in the four approaches. This is because the proposed MC method has greatly reduced the amount of relative motions to be refined by TrICP algorithm. Both the LRS matrix based method with MC and the weighted LRS algorithms are more efficient than the proposed method, since the matrix completion and the weighted decomposition together will consume more time than each of these two strategies. But the efficiency of the proposed algorithm is not slower too much.

**4.2. Comparison**

In this section, three algorithms, i.e., the motion averaged based ICP (MAICP) algorithm [31], the coarse to fine (CFTICP) algorithm [36] and the LRS based matrix decomposition method [34], were chosen to compare with the proposed one in terms of accuracy, efficiency as well as robustness. It should be noted that
the TrICP algorithm was selected to substitute for the original ICP algorithm in
MAICP approach.

4.2.1. Accuracy and efficiency
In this section, the Stanford repository [37] and UWA 3D Modeling Dataset [38]
were selected to testify the performances of all competed approaches. To com-
pare the accuracy, the objective function value (ObjV) introduced in [36] was
adopted. All competed approaches were testified under the same initial trans-
formation for one dataset. The results of accuracy and efficiency were recorded
in Tab. 2. For simplicity, we illustrated the cross-section results of the Stanford
repository dataset, which is shown in Fig. 2.

| Datasets | MAICP [31] | CFTrICP [36] | LRS [34] | Ours |
|----------|------------|--------------|----------|------|
|          | ObjV      | T(s)         | ObjV     | T(s) | ObjV | T(s)         | ObjV | T(s)         |
| Bunny    | 0.867     | 9.535        | 0.919    | 55.066 | 0.951 | 31.131        | 0.832 | 22.888       |
| Happy    | 0.273     | 72.250       | 0.442    | 243.029 | 0.249 | 64.654 | 0.168 | 42.233       |
| Dragon   | 3.464     | 17.662       | 2.159    | 77.258 | 2.730 | 34.087 | 0.533 | 16.803       |
| Trex     | 0.495     | 20.704       | 0.352    | 27.839 | 0.314 | 57.042 | 0.309 | 17.960       |
| Chicken  | 0.720     | 9.010        | 0.473    | 16.261 | 0.460 | 65.697 | 0.446 | 15.412       |
| Parasa   | 0.546     | 11.913       | 0.425    | 80.241 | 0.404 | 32.256 | 0.386 | 11.221       |
| Chef     | 0.394     | 54.199       | 0.306    | 111.855 | 0.274 | 323.031 | 0.259 | 71.595       |

As shown in Fig. 2, all competed approaches achieve satisfactory results on
the Bunny dataset. However, only the LRS matrix based method and the pro-
posed one perform well on Happy Buddha dataset. As for the Dragon dataset, all
methods fail except the proposed approach. In terms of accuracy, Tab. 2 made
a strong argument. It is obvious that the MAICP performs the poorest among
the four competed approaches. Experiments demonstrated that it performed well
when the initial transformation was close to the ground truth. However, its ac-
curacy decayed quickly once an inaccurate transformation was provided as initial
parameters. The CFTrICP algorithm achieved comparatively precise registration
since it registered one scan to all the other scans each time, which could provide
more correspondence information and large overlap percentage could be utilized
to compute pretty precise transformation. As for the LRS matrix based method,
Figure 2: Cross-section comparison for different approaches. (a) Reconstructed 3D models. (b) MAICP. (c) CFTrICP. (d) LRS. (e) Ours

it obtained more accurate results than the aforementioned two methods. The high accuracy of LRS benefits from its proposed framework, which can utilize more motion constraints while exclude the influence of outliers. For the proposed approach, besides adopting the framework of LRS, it also takes the weights of motions into consideration and provides more relative motions for decomposition, thus can achieve the most accurate registration among the four approaches.

As for efficiency, both the MAICP and the proposed algorithms achieved much faster registration than the other two methods. The proposed algorithm adopts the motion property based matrix completion method, thus can avoid much relative motion computation. As for the CFTrICP algorithm, it needs to build the $k-d$ tree for every iteration, which consumes much time. To make the original LRS matrix based method accurate, more relative motions should be computed. Therefore, it is much time-consuming. Based on the above analysis, the proposed approach is the most superior one among all the competed approaches.

4.2.2. Robustness

To further testify its robustness objectively, the following experiments were designed. In this part, the Dragon dataset was selected for registration. Before registration, the initial rotation matrix $R_0$ obtained beforehand was imposed by different turbulences $R_r$, where the Euler angles were randomly drawn from the uniform distribution $[-0.02, 0.02]$ degree, $[-0.04, 0.04]$ degree, $[-0.06, 0.06]$ degree.
gree, [−0.08, 0.08] degree and [−0.10, 0.10] degree. Then, all the competed approaches took the rigid transformation $R_r \cdot R_0, t_0$ as initial parameter. To compare their performances on each dataset, Tab. 3 recorded the objective function value and the corresponding runtime.

| Terms         | MAICP [31] | CFTrICP [36] | LRS [34] | Ours          |
|---------------|------------|--------------|----------|---------------|
| [−0.02, 0.02] | 0.627      | **14.226**   | 0.539    | 52.075        | 0.566         | 24.751        | 0.528         | **14.252**    |
| [−0.04, 0.04] | 2.569      | 53.074       | 0.944    | 68.703        | 2.015         | 111.700       | **0.532**     | **18.497**    |
| [−0.06, 0.06] | 3.643      | 130.872      | 1.072    | 72.837        | 3.124         | 176.212       | **0.540**     | **22.197**    |
| [−0.08, 0.08] | 3.687      | 133.437      | 1.646    | 78.721        | 3.268         | 196.932       | **0.554**     | **24.751**    |
| [−0.10, 0.10] | 3.821      | 137.394      | 2.484    | 79.210        | 3.356         | 218.032       | **0.607**     | **27.646**    |

As shown in Tab. 3, all the competed approaches can achieve satisfactory registration under a small turbulence. This means that all the four competed approaches can obtain precise registration under an accurate initial parameter. However, the objective function value of MAICP algorithm increased dramatically with the raise of noise. Besides, it cannot achieve good registration even under a not too large turbulence. The same situation happened on the LRS matrix based method. This is because both the MAICP algorithm and the LRS based method rely heavily on their initial transformations. That is to say, both of these two methods ask the TrICP algorithm to provide relative motions. However, the TrICP algorithm performs well when the initial transformation is precise. Once a set of pretty coarse transformation is provided, the TrICP algorithm based MAICP and LRS methods will perform poorly since there is no strategy to check whether the relative motions are precise or not. Besides, their efficiency is much lower than the other two methods since the objective function converges slowly and may even not converge in many situations. On the other hand, the CFTrICP algorithm performs much better than the aforementioned two approaches, since it registers one scan to all the other scans and accordingly has more information to utilize. The most robust approach is the proposed one in this paper, which can still achieve precise and efficient registration under the biggest turbulence. It is obvious that the weighting strategy introduced in this paper takes into the reliability of each relative motion into account, thus can avoid much noise. According to Tab. 3, it is
evident that the proposed approach is the most robust one among all the competed approaches.

5. Conclusions

For the low rank and sparse matrix based multi-view registration problem, this paper proposes two schemas to overcome its defects. First, an overlap percentage based matrix completion method is introduced into this approach. Accordingly, more synchronization constraints can be leveraged, which significantly alleviates the sparsity problem of the original LRS matrix based method. Second, a weighting matrix based on the relationship of model and data shapes is proposed to exclude the interference of noise for registration. Consequently, each relative motion has a corresponding weight in decomposition, which can result in more accurate registration. By restraining the rank of relative motion stacked matrix, refined global motions can be accordingly obtained via matrix decomposition method. Owing to the more informative block matrix and the weighting decomposition strategy, extensive experiments tested on public datasets testified its superior performances in accuracy and robustness.

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