ABSTRACT

Pumps are critical infrastructure in the Oil and Gas industry, and have been widely used in pipeline transportations of petroleum products. The electrical energy needed by a pump to meet the minimum pipeline operational requirement plays an important role in the overall cost and evaluation of pumping systems performance, which has become an important issue in pump energy management and pump station designs. This paper provides a quantitative and analytical method using Bernoulli’s equation for studying energy dependence between two pumps (Booster and Mainline pumps) in series within a pump station as a function of pump’s head, flow-rate, and density. Using actual parameters from a pump station, the derived equations are validated on four different products. The densities of products are 1000 kg/cm$^3$, 835 kg/cm$^3$, 800 kg/cm$^3$ and 660 kg/cm$^3$ for Water, Automotive Gas Oil (AGO), Dual Purpose Kerosene (DPK), and Premium Motor Spirit (PMS) respectively. The results show that the energy requirement of the Booster pump is determined by the energy demand of the Mainline pump as a function of flowrate, density and pump’s head. The study is essential for developing energy saving strategy in pipeline operations and in electrical consideration when selecting the right electric motors for pumps in pump station design.

Keywords: Pumps; Pump Station; Pipelines; Bernoulli’s equation; Petroleum products.
1. INTRODUCTION

The efficient use of electrical energy is a global problem especially in industries where huge amount of energy is required to drive machineries. The cost of using energy has a significant impact on business operators and overall production cost. Pumps are among the highest energy consumers in industries, accounting for 20 to 25% of global energy consumption [1,2,3]. One of the major contributing factors for energy consumption in pumps at industrial scale is that pumps are usually coupled to a constant speed, 3-phase, induction motors, which work under varying pump load conditions. This considerably increases the energy demand of the motors [3] (Sarbu, 2016). Pump energy consumption has been an important subject in pump performance and energy efficiency. According to Kaya et al. [4] a good pump design and selection can increase efficiency and save energy consumption by about 30 to 35%. Studies have been conducted on how to optimize the energy used by pumps. An excellent design and optimization of large capacity centrifugal pumps have shown to reduce energy consumption [5,4,6,7]. Improvement in impeller design has increased pump efficiency and reduced energy consumption of an electric motor [2]. The use of variable speed drive (VSD) can optimize the performance of a pump and save significant amounts of energy [8,1,2] Sarbu 2016. Energy efficiency in single and multi-pumps have been studied with very impressive results [1]. The experimental study indicates that two 0.75 KW pumps in parallel save about 29.8% more than a single 1.5 KW pump. Energy issues and optimization in multi pumps have been extensively studied both experimentally and analytically. Sustainability in energy providing to save water in three-pump (multi-pump) systems have been studied using Power Field Oriented Control (P-FOC) to control the pumps [9]. Using particle swarm optimization (PSO) model, the power consumption and reliability of pumps in parallel was studied by controlling the operation points of the pumps [10].

Energy consumption in pumps has been extensively studied. Optimization based study of energy consumption in pipeline operations using genetic algorithm was discussed [11]. In their study, the goal was to determine the optimal set of parameters such as the suction pressure, discharge pressure and station temperature that would provide a minimum energy consumption. Energy savings strategy such as the replacement of low efficiency pumps can be proposed by measuring pumping parameters and using the data to calculate pump and electric motors efficiencies [12,4,11].

This paper studies the effect of pumping parameters on energy consumption in a series cascaded pumps, the case study of Booster and Mainline pumps. The aim is to explore analytical method using fluid mechanics to provide a quantitative tool for studying energy consumptions in a series cascaded pump (Booster and Mainline pumps) that are electrically driven. The effect of flow rate, fluid density, pump discharge flow control valve, and pump’s head on electrical current demands of the two pumps have been studied in details. The power/current relationship between Booster pump and the Mainline has been established with approximate constant of proportionality based on some selected products densities. The equations derived in this paper can be used in programs that monitors energy consumptions in pumping systems. This paper is organized as follows: Section 1 provides a background studies on the energy problems in pumping systems and related works on energy optimization. In section 2, the underlying theoretical framework and mathematical formulation of the fluid equations is established. Section 3 discusses the methods and mathematical modeling of the two pumps in series. The result is presented and explained in section 4. Finally, in the conclusion, the main results of the work are highlighted.

2. MATHEMATICAL FORMULATION

2.1 Pump Energy and Efficiency

Pump energy and efficiency are essential part of design considerations in pumps and/or motor selections in designs. Energy consumption in pumps and pumping systems are important issues in optimization and energy savings as a result of increasing energy demands of pumping systems [13,3] (Sarbu, 2016). Many strategies, quantitative analysis and attempts have been made to develop ways that can help to reduce the energy consumption and increase the efficiency of pumping systems [13,4] Wu et al, 2014. When pumping equipment is sized correctly, it allows pumps to attain their optimal operating points during operations. Electric motors should match the power and speed requirements of their pumps to reduce inefficiencies and minimize energy wastage in pumping systems [13,5,12,6]. There are many factors that can significantly affect the
efficiency of pumps and pumping systems. Faulty bearing, shaft vibration due to wide sleeves bearing clearance, worn-out shaft, incorrectly sized motors or drivers, and imbalanced impellers. These problems increase the amount of energy consumption by the pumping systems and cost of production in general.

The efficiency of a pump can be written as [4]:

$$\eta_p = \frac{P_{\text{fluid}}}{P_{\text{shaft}}}$$  \hspace{1cm} (1)$$

Equation (1) is a metric that quantifies the performance of a mechanical pump. Similarly, the efficiency of a three (3) phase electric motor can be written as:

$$\eta_m = \frac{P_{\text{shaft}}}{P_{3\phi}}$$  \hspace{1cm} (2)$$

It is not enough to only improve the efficiency of the pump. The efficiency of the electric motor plays a part in the overall pump system efficiency. Hence, the overall efficiency of a pump-motor assembly is given as (Marchi et al. 2012) [4]:

$$\eta_{\text{overall}} = \frac{P_{\text{fluid}}}{P_{\text{shaft}}} \times \frac{P_{\text{shaft}}}{P_{3\phi}} = \frac{P_{\text{fluid}}}{P_{3\phi}}$$  \hspace{1cm} (3)$$

The power and energy requirements of a pump depend on the overall efficiency of the pumping system in equation (3). The mechanical power requirement of a pump is given as (Marchi et al., 2012) [14]:

$$P = \frac{QH\rho g}{\eta}$$  \hspace{1cm} (4)$$

Where P is the power, H is the pump’s head, Q is the flow-rate, ρ is the density of the fluid, and η is the efficiency.

### 2.2 Electrical Energy Consumption

The electrical energy drawn by an electric motor to drive the pump is usually a three (3) phase supply. Power in 3-phase circuit can be classified into Apparent, Active and Reactive [4,15,16,17]. In an electric motor circuit, both real (and active) and apparent power can pass through the circuit.

The equations for power in the 3-phase circuit can be expressed as:

Reactive, \[ Q = \sum_{n=1}^{\infty} \sqrt{3} V_n l_n \sin(\phi_n) \] \hspace{1cm} (5)$$

Active, \[ P = \sum_{n=1}^{\infty} \sqrt{3} V_n l_n \cos(\phi_n) \] \hspace{1cm} (6)$$

Apparent, \[ Q = \sum_{n=1}^{\infty} V_n l_n \] \hspace{1cm} (7)$$

Where n represents the n\textsuperscript{th} Harmonics. Not all the electrical power drawn by the electric motor is useful. A fraction of the network’s power is dissipated in form of heat due to winding resistance, R, while the remaining power does the work and drive the motor’s shaft. The power consumed or used by an electric motor depends on the power factor as shown in equation (6). The ratio of mechanical power (P\textsubscript{m}) to electrical power (P\textsubscript{e}) is the efficiency of the motor, which is an important performance indicator in electrical machines. Fig.1 shows stages of electrical energy conversion in 3-phase circuit.

The efficiency, \[ \eta = \frac{P_m}{P_e} \] \hspace{1cm} (8)$$

Substituting equation (6) into (8) yields, \[ \eta = \frac{P_m}{\sum_{n=1}^{\infty} \sqrt{3} V_n l_n \cos(\phi_n)} \] \hspace{1cm} (9)$$

where \( P_m \) the shaft is output power, \( P_e \) is the input electrical power and \( \eta \) is the motor’s efficiency.

### 2.3 Bernoulli’s Equation

Bernoulli theory was developed as a powerful mathematical model for solving problems involving fluid motion under the influence of gravitational force. Bernoulli’s equation has become one of the most important theories in fluid dynamics with wide variety of applications in many areas of science and engineering [18]. The formulation in its original form is only applicable to viscous fluid and it is non-Galilean invariant [19,20,21,22].

Bernoulli’s equation in head form, \[ P_i + \frac{1}{2} \rho u_i^2 + \rho g z_i = \text{Constant} \] \hspace{1cm} (10)$$
Bernoulli’s equation in energy form,
\[
\frac{P_i}{\rho g} + \frac{1}{2}u_i^2 + z_i = \text{Constant}
\]  
(11)

Both equation (10) and (11) are balanced equations.

Bernoulli’s equation has played many roles in solving practical engineering problems in fluid related processes that include energy and losses. Phase composition of gas condensate in pipeline has been studied using Bernoulli’s theory [23]. Oil leakages and emission during pipeline operations have been studied and quantitatively estimated by applying Bernoulli’s theorem [24]. The method enables the leaked volume in a pipeline to be determined with great accuracy.

3. METHODS AND MODELING

Consider two pumps that are mechanically connected in series. In such arrangement, the total head is the algebraic sum of the individual heads of the pumps. Fig. 2 shows a simplified version of a pump station indicating major pumping equipment and storage facilities. Typically, for long distance depot to depot refined petroleum products distribution, the distance between storage tank A and storage tank B is in kilometers.

Fig. 2. A typical pump station for transportation of refined petroleum products through single multi-product pipeline showing cascaded (series) pumps
Consider Fig.3 and applying Bernoulli’s Equation in head form to points a and b.

\[
P_a \frac{U_a^2}{2g} + H_1 = P_b \frac{U_b^2}{2g} + z_1 \quad (12)
\]

where \( P_a \) is the pressure at point a, \( P_b \) is the pressure at point b, \( H_1 \) is the dynamic height of the fluid in the storage tank, \( U \) is the velocity of the fluid through the pipeline and \( z \) is the distance from the datum to the suction (or discharge) of the pump.

Assuming the pump is not in operation (OFF state), the velocity at point b can be estimated as:

\[
U_b = \sqrt{2g(H_1 - z_1)}
\] (13)

The quantity of flow through a restriction with an area, \( A \), is given as:

\[
Q = AU_b = A\sqrt{2g(H_1 - z_1)}
\] (14)

Equation (12) can be re-written as:

\[
P_a \frac{U_a^2}{2g} + z_1 + h_{p(a)} = P_b \frac{U_b^2}{2g} + z_2
\] (15)

\[
\text{Where}
\]

\[
P_b = P_1, \quad U_b^2 = U_1^2
\]

\[
P_a \frac{U_a^2}{2g} + z_1 + h_{p(a)} = P_2 \frac{U_2^2}{2g} + z_2
\] (16)

\[
h_{p(b)} = \frac{1}{\rho g}(P_2 - P_1) + \frac{1}{2g}(U_2^2 - U_1^2) + (z_2 - z_1)
\] (17)

Similarly, the Bernoulli’s equation for the mainline pump with head, \( h_{p(m)} \), is expressed as:

\[
P_a \frac{U_a^2}{2g} + z_2 + h_{p(m)} = P_3 \frac{U_3^2}{2g} + z_3
\] (18)

\[
h_{p(m)} = \frac{1}{\rho g}(P_3 - P_2) + \frac{1}{2g}(U_3^2 - U_2^2) + (z_3 - z_2)
\] (19)

For two pumps in series, the total head is the algebraic sum of their individual heads. Adding equation (17) into equation (19) yields,

\[
h_{p(b)} + h_{p(m)} = \frac{1}{\rho g}(P_2 - P_1) + \frac{1}{2g}(U_2^2 - U_1^2) + (z_2 - z_1)
\] (20)

\[
h_{p(b)} + h_{p(m)} = \frac{1}{\rho g}(P_4 - P_3) + \frac{1}{2g}(U_2^2 - U_1^2) + (z_3 - z_2)
\] (21)

Equation (21) can be recast as,

\[
P_a \frac{U_a^2}{2g} + z_1 + h_{p(b)} + h_{p(m)} = P_4 \frac{U_4^2}{2g} + z_3
\] (22)

From equation (16),

\[
U_1 = \sqrt{\frac{2}{\rho} \left( P_2 - P_1 \right) + 2g(z_2 - z_1) + U_2^2 - 2gh_{p(b)}}
\] (23)

Similarly, from equation (18),

\[
U_3 = \sqrt{\frac{2}{\rho} \left( P_4 - P_3 \right) + 2g(z_3 - z_2) + U_2^2 + 2gh_{p(m)}}
\] (24)

By assuming that the rate of fluid flow per area into the Booster pump equals the rate of fluid flow
per area being discharged by the mainline pump, then the continuity equation states:

\[ AU_1 = AU_3 \] (25)

Substituting equation (23) and equation (24) into equation (25) yields,

\[
A \frac{2}{\rho} \left( (p_2 - p_1) + 2g(z_2 - z_1) + u_2^2 - 2gh_{p(b)} \right) = A \frac{2}{\rho} \left( (p_2 - p_4) + 2g(z_2 - z_3) + u_2^2 + 2gh_{p(m)} \right) \] (25)

\[
(P_2 - P_1) + \rho g (z_2 - z_1) - \rho g h_{p(b)} = (P_2 - P_4) + \rho g (z_2 - z_3) + \rho g h_{p(m)} \] (26)

\[ h_{p(b)} + h_{p(m)} = \frac{1}{\rho g} (P_4 - P_1) + (z_3 - z_1) \] (27)

Pumps head is related to power, \( P \), developed by a pump is written as:

\[ P = \frac{Q \rho g H}{\eta} \] (28)

From equation (27), we can write the total head in the form,

\[ H = h_{p(b)} + h_{p(m)} \] (29)

\[ H = \frac{1}{\rho g} (P_4 - P_1) + \frac{1}{2g} (U_2^2 - U_1^2) + (z_3 - z_1) \] (30)

Substituting equation (30) into equation (28), we obtained the total power requirement of the series pumps as;

\[ P_T = \frac{Q \rho g}{\eta} \left[ \frac{1}{\rho g} (P_4 - P_1) + \frac{1}{2g} (U_2^2 - U_1^2) + (z_3 - z_1) \right] \] (31)

\[ P_T = \frac{1}{\eta} \left[ Q (P_4 - P_1) + \frac{Q \rho}{2} (U_2^2 - U_1^2) + Q \rho g (z_3 - z_1) \right] \] (32)

From equation (27), the power requirement is expressed as (If \( U_3 = U_1 \)):

\[ \frac{P_T}{\eta} = \frac{1}{\eta} \left[ Q (P_4 - P_1) + Q \rho g (z_3 - z_1) \right] \] (33)

Where

\( \eta \) is the pump’s efficiency, \( Q \) is the flow rate, \( \rho \) is the fluid density and \( H \) is the pump’s head.

The three (3)-phase electrical power in an electric motor circuit driving the pump can be expressed as:

\[ P_{in} = P_{rotor} + 3I_a^2 R_a \] (34)

\[ P_{rotor} = P_{in} - 3I_a^2 R_a \] (35)

The losses \( (3I_a^2 R_a) \) can be accounted in the motor’s efficiency as;

\[ P_{rotor} = \eta P_{in} \] (36)

\[ P_{in} = \sqrt{3} V_L I_L \cos \theta \] (37)

\[ P_{rotor} = \eta \sqrt{3} V_L I_L \cos \theta \] (38)

where \( V_L \) and \( I_L \) are the line voltage and current respectively.

Neglecting mechanical losses and assuming all the rotor power is transferred to the pump’s shaft, then

\[ P_{rotor} = P_1 = P_2 \] (39)

Substituting equation (38) into equation (28), the Booster pump power requirement can be written as:

\[ \eta_m^B \sqrt{3} V_L^B I_L^B \cos \theta = \frac{Q \rho g H^B}{\eta_{p}^B} \] (40)

The Booster pump electric motor current can be expressed as;

\[ I_{L}^{(B)} = \frac{Q^{(B)} \rho g H^{(B)}}{\eta_{p}^{(B)} \eta_{m}^{(B)} \sqrt{3} V_{L}^{(B)} \cos \theta} \] (41)

The Mainline pump electric motor current can be expressed as;

\[ I_{L}^{(m)} = \frac{Q^{(m)} \rho g H^{(m)}}{\eta_{p}^{(m)} \eta_{m}^{(m)} \sqrt{3} V_{L}^{(m)} \cos \theta} \] (42)
In series pumps configuration, the Booster pump electric motor current varies proportionally to the mainline electric motor current. Thus,

\[ I_L^{(B)} \propto I_L^{(m)} \]  

(43)

\[ I_L^{(B)} = K I_L^{(m)} \]  

(44)

Substituting equations (41) and (42) into equation (44) yields,

\[ K = \frac{\eta_p^{(m)}}{\eta_m^{(m)}} \frac{V_L^{(m)} Q^{(B)}}{V_L^{(m)} Q^{(m)}} \]  

(45)

equation (45) estimate the factor of proportionality between the Mainline electric motor current and that of the Booster pump.

4. RESULT AND DISCUSSION

In this section, the derived equations are validated using data in Table 1. The pump heads were estimated but can vary arbitrarily to observe the effect of heads on pump’s energy consumption. Using MATLAB, plots of various characteristics of the two pumps in series are presented.

Substituting the parameters in Table 1 into equations (41), (42) and (45) yields,

\[ I_{L_{(B)}} = \frac{Q_{(B)} \rho g H^{(B)}}{\eta_p^{(B)} \eta_m^{(B)} V_L^{(B)} \cos \theta} \]  

(46)

\[ I_{L_{(m)}} = \frac{Q_{(m)} \rho g H^{(m)}}{\eta_p^{(m)} \eta_m^{(m)} V_L^{(m)} \cos \theta} \]  

(47)

\[ K = 7.95181 \times \frac{Q_{(B)} H^{(B)}}{Q_{(m)} H^{(m)}} \]  

(48)

4.1 Effect of Pump’s Head and Flow Rate on Electric Motor Current at Different Densities

Using the parameters in Table 1, the approximate equations relating heads and flow rate to electrical current at different densities for the two pumps are derived and presented in Table 2. The electric current of the Booster pump’s motor varies with flow rate and head by a factor of 0.02505, 0.02067, 0.020048 and 0.01654 for Water, AGO, DPK and PMS respectively. Similarly, the current of the Mainline pump’s motor varies with flow rate and pump’s head by a factor of 0.00315176, 0.0026, 0.00252, and 0.00208 for Water, AGO, DPK and PMS.

| Parameter                     | Booster pump | Mainline pump |
|-------------------------------|--------------|---------------|
| Pump efficiency               | \( \eta_p^{(B)} \) = 0.85 | \( \eta_p^{(m)} \) = 0.85 |
| Motor efficiency              | \( \eta_m^{(B)} \) = 0.80 | \( \eta_m^{(m)} \) = 0.80 |
| Electric motor power factor   | \( \cos \theta \) = 0.8 | \( \cos \theta \) = 0.8 |
| Supply voltage (V)            | \( V_L^{(B)} \) = 415 | \( V_L^{(m)} \) = 3300 |
| Head (m)                      | \( H^{(B)} \) = 30-50 | \( H^{(m)} \) = 370-500 |
| Flow rate m3/hr.              | \( Q^{(B)} \) = 120 | \( Q^{(m)} \) = 200 |
| The constant, g               | 9.8          | 9.8           |

Table 2. Approximate current of mainline and booster electric motors at different densities from equation (46) and (47)

| Product | Density (Specific gravity) | Equation |
|---------|----------------------------|----------|
| Water   | 1                          | \[ I_{L_{(B)}} = 0.02505Q^{(B)} H^{(B)} \] \[ I_{L_{(m)}} = 0.00315176 Q^{(m)} H^{(m)} \] |
| AGO     | 0.825                      | \[ I_{L_{(B)}} = 0.02067Q^{(B)} H^{(B)} \] \[ I_{L_{(m)}} = 0.0026Q^{(m)} H^{(m)} \] |
| DPK     | 0.800                      | \[ I_{L_{(B)}} = 0.02048Q^{(B)} H^{(B)} \] \[ I_{L_{(m)}} = 0.00252Q^{(m)} H^{(m)} \] |
| PMS     | 0.660                      | \[ I_{L_{(B)}} = 0.01654Q^{(B)} H^{(B)} \] \[ I_{L_{(m)}} = 0.00208Q^{(m)} H^{(m)} \] |
Fig. 4 shows the electric current consumption of a Booster pump’s motor at different flow rates and product densities when the pump’s head (in meters) is fixed. First, as the graph indicates, density plays a role in energy consumption of a pump. Products with higher densities require higher electrical energy to transport. The plot in Fig. 4 shows that an increase in flow-rate (Q) results in a corresponding increase in current ($I_L^{(B)}$) or energy demand of the Booster pump. The difference in magnitude is accounted for by the density of the fluid being pumped.

In the characteristics plot shown in Fig. 5, the electric current, $I_L^{(B)}$, varies proportionally with pump’s head at different densities. There is a linear relationship between motor current and pump’s head. Changes in static head affects power consumption of the pump and energy demand of the electric motor.

The Mainline pump exhibits similar characteristics as the Booster pump. Increase in flow-rate (Q) also leads to an increase in energy demand of the motor in the Mainline pump. Similarly, an increase in head (H) affects the energy ($I_L^{(M)}$) demand of the pump as shown in Fig. 6.

In both cases, the Booster and Mainline pumps’ energy demand are being affected by the nature (density) of the fluid. Since Water is heavier than AGO, DKP and PMS, it consumes more energy at same head and flow rate.

Using the parameters from Table 1, the value of the constant of proportionality, K, from equation (48) is calculated:

$$K = 7.95181 \times \frac{Q^{(B)}H^{(B)}}{Q^{(m)}H^{(m)}} = 0.38685 \quad (49)$$

$$I_L^{(B)} = 0.38685 \times I_L^{(m)} \quad (50)$$

By substituting the values of $I_L^{(m)}$ from Table 2 into equation (5), the dependence of Booster pump current, $I_L^{(B)}$, on the Mainline pump’s head and flow rate can be obtained as tabulated in Table 3.

Using the value of the constant, K, the mathematical relationships between the Mainline pump’s head and flow rate is presented in Table 3. These relates how current of the Booster pump is being affected by the variation in the Mainline pump flow rate and head. Fig. 8 shows a graph of $I_L^{(B)}$ and $I_L^{(m)}$ which varies by a factor of 0.38685 proportionally.

To observe how the flow rate and static head of the Mainline pump automatically affects the electric current of the Booster pump, a plot of $I_L^{(B)}$ and $H^{(m)}$ at constant $Q^{(m)}$ shows that the Booster pump motor current varies proportionally with the Mainline static head, $H^{(m)}$ as shown in Fig. 9.

Also, a graph of $I_L^{(B)}$ and $Q^{(m)}$ at constant $H^{(m)}$ in fig.10 shows that an increase in flow rate of the Mainline pump increases the Booster pump
motor current proportionally. This implies that changes in the demand of the Mainline pump's static head or flow rate automatically affect the energy consumption of the Booster pump since the two pumps are connected in series [26,27].

4.1 Effect of pump’s head and flow rate on the power consumption of the pump

Using the parameters in Table 1, the power demand of the Booster and Mainline pumps can be estimated. By substituting the values of $I_L^{(B)}$ and $I_L^{(M)}$ from table 2 into equation (39), the power consumption of the Mainline pump, $P_m$, and the Booster pump, $P_B$, can be written as:

$$P_B = \eta_m \sqrt{3} V_L^{(B)} I_L^{(B)} \cos \theta = 460.0327 I_L^{(B)} \quad (51)$$

$$P_m = \eta_m \sqrt{3} V_L^{(m)} I_L^{(m)} \cos \theta = 3658.0913 I_L^{(m)} \quad (52)$$

The power requirement can be expressed in terms of flow-rate, Q, density and pump's head, H as shown in Table 4.

Using electrical power equation in three phase circuit and pump flow equation, the effect of pump’s head and flow rate on energy consumption can be observed as presented in table 4. As shown previously, pump’s head and flow rate variation significantly affect the power consumption of a pump. Figs. 11,12,13 and 14 show the characteristics of Booster and Mainline pump power variation with head and flow-rate.

The above analytical results agree with the experimental results obtained by Oshurbekov et al. [1]. Fig. 15 shows the plot of mechanical power as a function of flowrate. The relations between power and flow rate is linear and positive. As flow rate is increasing, the pump demands more power to meet the change in flowrates.

4.2 Total Electrical Power Consumption of the Mainline Pump

In this section, the total power of the Mainline pump as a function of fluid density and flow rate is presented to study how these parameters affect total power requirement. From equation (32) and equation (33),

$$P_1 = \frac{1}{\eta} \left\{ \frac{Q}{2} \left( \rho \Delta P + \rho g z_f - z_i \right) \right\}$$

$$P_2 = \frac{1}{\eta} \left\{ \frac{Q}{2} \left( \rho \Delta P + \rho g z_f - z_i \right) \right\}$$

In actual pump station, flow control valve is commonly used at the discharge of the Mainline pump to regulate flow-rate, mainline pressures and power consumption of the pump. For Newtonian fluids, the square of the quantity of flow, Q, through the flow control valve is proportional to the pressure differentials between the upstream and downstream of the valve [28-30]. Where specific gravity, $\rho$, pressure drop, $\Delta P$, and valve sizing coefficient, $C_v$. $\theta \propto \sqrt{\Delta P}$

$$C_v = \frac{Q}{\sqrt{\Delta P}} \quad (54)$$

$$C_v = \frac{Q}{\sqrt{\Delta P}} = 156.38$$

In Metric unit,

$$K_v = C_v \cdot 0.865 = 135.37$$

From Fig. 2, pressure drop across the flow control valve is given as;

$$P_4 = P_h = \left( \frac{Q}{C_v} \right)^2 \rho \quad (55)$$

$$P_1 = \left( \frac{Q}{C_v} \right)^2 \rho + P_h \quad (56)$$

Substituting equation (56) into equations (32) and (33) yields,

$$P_1 = \frac{1}{\eta} Q \left\{ \left( \frac{Q}{C_v} \right)^2 \rho + P_h - P_f + \frac{Q \rho}{2} \left( U_f^2 - U_i^2 \right) \right\} \quad (57)$$

$$P_2 = \frac{1}{\eta} Q \left\{ \left( \frac{Q}{135.27} \right)^2 \rho + P_h - P_f + \frac{Q \rho}{2} \left( U_f^2 - U_i^2 \right) \right\} + Q \rho g (z_f - z_i) \quad (58)$$

$$P_3 = \frac{1}{\eta} Q \left\{ \left( \frac{Q}{135.27} \right)^2 \rho + P_h - P_f \right\} + Q \rho g (z_f - z_i) \quad (59)$$

$$P_4 = \frac{1}{\eta} Q \left\{ \left( \frac{Q}{135.27} \right)^2 \rho + P_h - P_f \right\} + Q \rho g (z_f - z_i) \quad (60)$$
Fig. 5. Effect of head (H) on Booster pump electric motor current ($I_{b}$) at flow rate of 120 m3/hr

![Graph showing the effect of head (H) on Booster pump electric motor current ($I_{b}$) at flow rate of 120 m3/hr.]

Fig. 6. Effect of flow rate (Q) on the mainline electric motor current ($I_{m}$) at the head of 370 m

![Graph showing the effect of flow rate (Q) on the mainline electric motor current ($I_{m}$) at the head of 370 m.]

Fig. 7. Effect of heads (H) on mainline electric motor current (I) at flow rate of 200 m3/hr

![Graph showing the effect of heads (H) on mainline electric motor current (I) at flow rate of 200 m3/hr.]
Table 3. Variation of Booster pump motor current with Mainline pump’s head and flow rate

| Product                   | Density (Specific gravity) | Equation                             |
|---------------------------|----------------------------|--------------------------------------|
| Water                     | 1                          | $I_{B}^{(B)} = 0.001219242Q^{(m)}H^{(m)}$ |
| Automotive Gas Oil        | 0.825                      | $I_{B}^{(R)} = 0.0010058Q^{(m)}H^{(m)}$ |
| Dual Purpose Kerosene     | 0.800                      | $I_{B}^{(B)} = 0.00097485Q^{(m)}H^{(m)}$ |
| Premium Motor Spirit      | 0.660                      | $I_{B}^{(B)} = 0.00080464Q^{(m)}H^{(m)}$ |

Fig. 8. Variation of booster pump motor current with mainline pump motor current

Fig. 9. Effect of mainline pump head on the booster pump current at Q=200 m$^3$/hrs
Table 4. Estimate of pump power equation at different densities

| Product                  | Density (Specific gravity) | Equation                  |
|--------------------------|----------------------------|---------------------------|
| Water                    | 1                          | \[ P_B = 11.5238Q^{(B)}H^{(B)} \]  
|                          |                            | \[ P_m = 11.5294Q^{(m)}H^{(m)} \]  |
| Automotive Gas Oil       | 0.825                      | \[ P_B = 9.5089Q^{(B)}H^{(B)} \]  
|                          |                            | \[ P_m = 9.5110Q^{(m)}H^{(m)} \]  |
| Dual Purpose Kerosene    | 0.800                      | \[ P_B = 9.2227Q^{(B)}H^{(B)} \]  
|                          |                            | \[ P_m = 9.2184Q^{(m)}H^{(m)} \]  |
| Premium Motor Spirit     | 0.660                      | \[ P_B = 7.6089Q^{(B)}H^{(B)} \]  
|                          |                            | \[ P_m = 7.6088Q^{(m)}H^{(m)} \]  |

Fig. 10. Effect of mainline pump flow rate on the booster pump motor current at H=370 m

Fig. 11. Effect of flow rate on the power consumption of booster pump electric motor at pump head of 30 m
Fig. 12. Effect of flow rate on the power consumption of main line pump electric motor at pump head of 370 m

Fig. 13. Effect of pumps head on the power consumption of booster pump electric motor at flow rate of 120 m³/hr

Fig. 14. Effect of pump's head on the power consumption of main pump electric motor at the flow rate of 200 m³/hr
Table 5. Valve resizing parameters

| Parameter | Value         |
|-----------|---------------|
| $\rho$    | 1             |
| $\Delta P$ | 150 PSI     |
| $Q$       | 435 m$^3$/hrs., 1915.25 gpm |

Table 6. Pressure and pump parameters

| Parameter | Value         |
|-----------|---------------|
| $\eta_m^{(m)}$ | 0.8         |
| $P_6$     | 20 Bar        |
| $P_1$     | 5 Bar         |
| $(z_3 - z_1)$ | 0.5 m      |

Fig. 15. Experimental results of pump mechanical power (W) and flow rate (m$^3$/h$^{-1}$) [1]

Fig. 16. Effect of mainline flow rate on the total power requirement/consumption at z=0.5 m
The elevation, \((z_3 - z_1)\), above the datum level is selected arbitrary to fit the approximate pump station parameters. Using Table 16, the total power demand is plotted from equation (60) as shown in Fig.16.

The total power requirement at different densities is shown in Fig. 16. As described by equations (54) and (56), increase in flowrate causes an increase in energy demand of the two pumps in series. The power requirement is higher during pumping products with higher densities and vice versa [31-33]. This validates the fact that products viscosities significantly affect power consumption of a pump and energy requirement of pumping systems [34-37]. As previously explained, variation in pump’s head affects the energy consumption of the pumping systems.

5. CONCLUSION

In this paper, the power/current relationships between two pumps (Mainline and Booster pumps) in series as a function of flowrate, fluid density and pump’s head has been derived. We analytically determined the effect of pump’s head, density and flow rate on the energy consumption of the two pumps. The results show that the Booster pump energy consumption depends solely on the Mainline energy demands based on pump configuration of Fig. 2. Increase in current demand of the Main line electric motor is accompanied by a corresponding increase in the booster pump electrical power consumption.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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