Transient and harmonic analyses of South Memnon Colossus including soil-structure interaction

Taize Yu1, *, Xiaokun Chen2, Siyu Liu3, Yijie Ling4
1 International division, ShenZhen Senior High School, ShenZhen, 518040, China
2 College of civil science and engineering, YangZhou University, YangZhou, JiangSu 225009, China
3 School of mechanics and construction engineering, JiNan University, GuangZhou 510632, China
4 Guangdong Country Garden School, ShunDe 528000, China
* Corresponding author’s e-mail: yutz@cn-school.com

Abstract. In this work, a 3D model of an ancient statue -- the South Memnon Colossus, was built, and its dynamic response to sinusoidal earthquake motion was investigated. In the transient analysis, parametric studies were conducted to better understand the impact of ground motion characteristics (frequency, amplitude, and direction) on the structure's seismic response. The comparison was made of the dynamic response with and without soil-structure interaction effects. The analyses were conducted in MSUP (linear) mode of a commercial finite element code ANSYS. The solution was obtained in the frequency domain and showed increasing deformation as the load frequency approaches the natural frequency of the structure (resonance).

1. Introduction

Best-known for 'Vocal Memnon', credited for 'singing' at dawn, the Colossi of Memnon are a pair of giant statues made of stone, which are located in the Theban Necropolis in Luxor, Upper Egypt. Situated right at the Mediterranean seismic zone[1], where frequently experienced earthquakes and floods, two statues have been severely damaged. Since both statues could topple at any moment during an earthquake in the future, remedial measures are proposed to maintain these two invaluable assets, including strategies fastening blocks together and adding substance to fulfill fractures. Non-invasive techniques such as simulations and computational modeling are required at this point to give a reasonable prediction on current statues’ responses toward different earthquake magnitude.

A known computational model on seismic response of the South Memnon Colossus was by Verdel[2-5] and co-workers by using Finite Element (FE) analyses. Verdel’s team carried out a 2D discontinuous modeling by dividing the structure into four blocks and using the distinct element method to handle the interaction between adjacent blocks. A shortcoming of Verdel’s model is its accuracy due to its coarse volume definition. Thus, finer 3D models were conducted to improve accuracy.
2. Methodology

2.1. Physical property of material

The Colossi were made up of sandstone which is assumed to be isotropic linear elastic[2] with a Young's modulus of $E = 20 \text{ GPa}$, a Poisson's ratio $\nu = 0.2$, and a mass density of $\rho = 1800 \text{ kg/m}^3$.

The soil layer was made up of silt and hard limestone base. In Borja's research[2], they divided the soil into six sub-layers featured with fine mesh. To simplify the problem and shorten the time spending, it is assumed that all the layers for each material was the same, which account for only two layers underground. The first layer was silt that had the property of Young's modulus $E = 5 \text{ Mpa}$, and Poisson's ratio $\nu = 0.33$, a total mass density $\rho = 1400 \text{ kg/m}^3$. The limestone layer was much stiffer, with Young's modulus $E = 12 \text{ Gpa}$, an elastic Poisson's ratio of $\nu = 0.21$, and a total mass density of $\rho = 2400\text{kg/m}^3$.

2.2. The Modeling

Two models were built in order to make the comparison. The statue only model was made up merely of sandstone. Based on the model of Verdel, parts of the statue were removed since the volume of the Verdel's model was an overestimation. The right arm was removed to simulate the reality. With the meshing size of 1000 mm, the statue calculation has its reality and accuracy.

The soil-statue combined model was divided into three parts. With the upper statue remaining the same, the lower parts were composed of a 6-meter-thick silt layer and a 5-meter-thick limestone layer. The contact between the silt layer and the limestone layer were considered as bonded. According to the site condition, the statue was inserted into the silt layer. The contact was set bonded when considering the horizontal contact between the statue and the silt layer. As for the vertical contacts, it was simplified as frictionless in order to avoid nonlinear problems. Due to the limitation of the software, the soil foundation was built as playground-shape with the meshing size of 2500mm. Commonly, the vertical boundary of the soil foundation was set up as frictionless and the bottom of the soil foundation was set
up as fixed in order to mimic the interaction in the soil foundation. The isometric model of the statue and the statue-soil combined structure is shown as Figure.1.

2.3. The Modeling

To identify the natural mode of vibration[6], modal analyses, which are often used in vibration engineering field, would be applied first. Each mode in modal analyses refers to a natural vibration characteristic of the mechanical structure, and each mode has its own specific natural frequency, damping ratio and mode shape. Through modal analyses, it was able to distinguish the structure’s main properties under a range of frequencies that would readily affect the structure, and thus be able to predict the structure’s vibration response under various external and internal vibration within particular frequency band. The goal of modal analyses, in this case, was to identify the modal parameters of the system and provide the basis for further transient analysis and harmonic response analysis, since it is one of the prerequisites of base-excitation.

In total, two models were built in order to compare the difference between the structure only condition and the structure-soil combined condition. Therefore, two modal analyses were conducted separately. Since the modal analyses aim to find the natural frequencies, which were the inherent properties of the structure, no load would be applied in modal analyses process. Correspondingly, boundary condition (fixed support in both cases) were applied at the bottom of the statue or soil foundation since natural frequencies vary under different boundary conditions.

2.4. Transient analyses

Transient analyses are the responses of the structure given a time domain. Certain time steps were chosen in these analyses. There was 1 time step which lasts for 2s and 0.1s for the each sub-steps. Thus, there were total 20 intervals in the time domain that need to be calculated by the program.

The parametric comparison would be applied in the transient analyses in order to know whether there was a linear or nonlinear relationship between the parameter and the result. Since the sinusoidal waves were used to simulate the earthquake motion, there were three parameters in the transient analyses; amplitude, frequency, and direction of the sinusoidal wave can be controlled separately. The phase angle was not included since it was the option in the harmonic response analyses. By controlling one parameter each time, reasonable relationships were expected in order to predict results that have certain reference value in the future application.

The fundamental sinusoidal wave was represented as a displacement function: \( u = A \sin(\pi t) \) and the original frequency\((\omega/2\pi)\) was set at 0.5, along with the original amplitude\( (A)\) of 1. For \( f = 1\text{Hz} \) and \( 2\text{Hz} \), the expression would be \( A \sin(2\pi t) \) and \( A \sin(4\pi t) \). Amplitude would be selected from 1mm, 10mm, and 100mm. The direction would be varied from the x, z axis or both combined.

There are two solution methods in the transient system: full method and mode superposition(or MSUP) method. The full method provided a more precise calculation. It could support all types of loads and constraints as well as the non-linear problems. However, this method needed much more time to calculate. Mode superposition method needed less time than that of full method, but it did not support non-zero forced displacement or any non-linear problems. To compare the difference between these two methods, both of them were used.

The parametric methods above would be applied to both the statue model and the statue-soil model, respectively. Then, three comparison would be made: the statue with itself, the statue-soil with itself, and the comparison between the statue and the statue-soil combination. Total deformation and equivalent stress would be taken into consideration during the comparison. Note that in the statue-soil model, only the statue part would take into account in the comparison for the consistency.

2.5. Harmonic Response

Harmonic response analyses, a method used to determine steady-state responses of a linear structure under the effect of time dependent load source that vibrates in a regular sinusoidal fashion, which is
Instead of considering the transient vibration when excitation was applied, merely the steady state forced vibration of the structure was calculated during the harmonic response analyses. The purpose of harmonic response analyses is to calculate the response value (total deformation and equivalent stress) of the structure at several frequencies. These results enabled us to predict the continuous dynamic characteristics of the structure.

The harmonic response analyses were conducted to the statue only model during the process. Since the range of the harmonic response was 1.5 times smaller than the range of the modal analysis, the range of the harmonic was set 15Hz-35Hz accordingly, with 30 solution intervals. The base excitation displacement was exerted with the magnitude of 1mm and phrase angle of 0. After conducting the solution, the deformation and equivalent stress of the frequency domain were maintained and evaluated. Plus, the frequency was once set precisely close to the first natural frequency to observe how the structure would response.

2.6. Transient (Full)
Consider the non-linearity, full system was used to analyse the statue only model with the friction existed between the statue and the its base. Elastic support with the stiffness of $\frac{3}{100} \text{mm}N$ was added on a frictional surface with frictional coefficient of 0.3, forced displacement at the bottom as the excitation with wave $\sin(\pi t) \sin(2\pi t) 10\sin(\pi t)$. The analysis settings contained a total time domain of 2 seconds, only 1 time steps with initial and maximum sub-steps of 200, minimum sub-steps of 20.

3. Results and Conclusion

3.1. Modal analyses

Figure 2. Six natural modes of the statue vibration
For the statue only condition, the first six modes are 16.851Hz, 17.644Hz, 42.495Hz, 44.429Hz, 47.833Hz, 63.856Hz respectively, which is shown in Figure 2.

As for the statue-soil combined condition, the first six modes are 4.1258E-004Hz, 4.6297E-004Hz, 1.01E-003Hz, 1.116E-003Hz, 1.1949E-003Hz, and 1.2104E-003Hz respectively.

The first 6 natural modes of the statue are represented in Figure 2.

3.2. Transient analyses
Through processes of analyses as describe in previous sections, total deformation and equivalent stress that the statue endured under varied frequencies, amplitudes, and directions in each case with and without considering the existence of soil beneath the colossus were obtained.

Table 1. Varied frequencies in statue only model

| Variations | 1  | 2  | 3  | 4  |
|------------|----|----|----|----|
| Frequency (Hz) | 0.5 | 1  | 2  | 105.88 |
| Total deformation (mm) | 1.0151 | 1.0981 | 1.2749 | 1.8323 |
| Von Mises stress (Mpa) | 0.2666 | 0.5445 | 1.2276 | 2.9066 |

Table 2. Varied amplitudes in statue only model

| Variations | 1  | 2  | 3  |
|------------|----|----|----|
| Amplitude | 1  | 10 | 100 |
| Total deformation (mm) | 1.0151 | 10.1510 | 101.5100 |
| Von Mises stress (Mpa) | 0.2666 | 2.6657 | 26.657 |

Table 3. Varied directions in statue only model

| Variations | 1  | 2  | 3  |
|------------|----|----|----|
| Direction | NS | EW | NSEW |
| Total deformation (mm) | 1.0151 | 1.0229 | 1.444 |
| Von Mises stress (Mpa) | 0.2666 | 0.4663 | 0.2370 |

Figure 3(a). Response of the statue when frequency varies (b) Response of the statue-soil structure when frequency varies
Figure 4(a). Response of the statue when amplitude varies (b) Response of the statue-soil structure when amplitude varies

Figure 5(a). Response of the statue when direction varies (b) Response of the statue-soil structure when direction varies

Table 1 and Figure 3(a) illustrated the maximum deformation and equivalent stress trend when changing the frequency of sinusoidal function of displacement applied at the base of the statue. At this point, 0.5Hz, 1Hz, 2Hz, and 16.851Hz (the first natural frequency) in particular were chosen as the frequency of the function and process the result obtained into a broken line graph so that the tendency of the quantity can be evidently shown. The tendency turns out that, during the domain of 0.5Hz-2Hz, as the function’s frequency increases, the maximum total deformation and maximum equivalent stress of the colossus raised in a linear-like trend. Though maximum total deformation and maximum equivalent stress obtained from 16.851Hz did not conform with the slope before, the colossus would not experience extremities as the frequency of sinusoidal function approach the natural frequency of statue.

Besides, the amplitude of the sinusoidal function were changed. As shown in Table 2 and Figure 4(a), the maximum magnitude of stress and deformation on the colossus were strictly proportional to the amplitude of the seismic function.

To better simulate the actual earthquake, the seismic wave upon the statue were also applied in varied orientation, including north-south(NS) direction, east-west(EW) direction and northwest-southeast(NW-SE) direction. In NS and EW, the two total deformation are nearly the same, while the total deformation of NW-SE direction is around \( \sqrt{2} \), which is within the anticipation. When it comes to stress, the EW direction is around twice the NW-SE direction. The results were shown in Table 3 and Figure 5(a).

| \( \omega \) | 1  | 2  | 3  | 4  | 5  | 6  |
|-------------|----|----|----|----|----|----|
| Frequency(Hz) | 0.25 | 0.5 | 0.75 | 1  | 2  | 8  |
| Total deformation(mm) | 0.23802 | 0.23802 | 0.23702 | 0.22637 | 0.22738 | 0.22638 |
| Von Mises stress(Mpa) | 7.5839E-10 | 7.5843E-10 | 7.5847E-10 | 7.2133E-10 | 7.2132E-10 | 7.2133E-10 |

Table 4. Varied frequencies in statue-soil model
Table 5. Varied amplitudes in statue-soil model

| Amplitudes in soil-statue structure | 1       | 2       | 3       |
|------------------------------------|---------|---------|---------|
| ω                                 | 3.1416  | 3.1416  | 3.1416  |
| Amplitude                          | 1       | 10      | 100     |
| Total deformation (mm)             | 0.2380  | 2.3802  | 23.8020 |
| Von Mises stress (Mpa)             | 0.0000  | 7.58E-009 | 7.58E-008 |

Table 6. Varied directions in statue-soil model

| Varied directions in soil-statue structure | 1       | 2       | 3       |
|-------------------------------------------|---------|---------|---------|
| ω                                         | 3.1416  | 3.1416  | 3.1416  |
| Direction                                 | NS      | EW      | NSEW    |
| Total deformation (mm)                    | 0.2380  | 4.7122  | 4.7006  |
| Von Mises stress (Mpa)                    | 7.5843E-010 | 1.84E-08 | 1.79E-08 |

Same as the previous analyses, this time the acquired soil-statue results were very different. The total deformation represented a linear decrement (0.5Hz-1Hz) followed by a horizontal line (1Hz-2Hz) which the deformation was almost identical, just as shown in Table.2(a) and Figure.4(a). Unconvinced of the strange trend, more frequencies were taken into account: 0.25Hz, 0.75Hz, 8Hz. It turned out that, from 0.25Hz to 0.75Hz, the total deformation remained a horizontal line, then with a descending line connecting 0.75Hz and 1Hz, the slope from 1Hz to 8Hz was also 0. In the soil-statue case, the equivalent stress generally became smaller as the frequency became larger, specifically in a periodic declination. The corresponding value of stresses when frequency equaled to 1Hz and 2Hz were also almost identical while a big drop from 0.5Hz to 1Hz, which indicates the declination period. However, since the equivalent stress was very close to 0 in all three cases (frequency, amplitude and direction), the trending analyses seem relatively redundant. As shown in Table.4 and Figure.3(b).

When different amplitude was used as shown in Table.5 and Figure.4(b), an obvious verdict that the maximum magnitude of stress and deformation on the colossus were linearly proportional to the amplitude of the seismic function could be made.

For different seismic wave orientation, once again, three directions were measured: EW direction, NS direction, and NW-SE direction. The maximal deformation on the structure occurred when the seismic wave was applied at EW direction while the most moderate in NS way. Note that when applying the EW direction function, the total deformation was surprisingly greater than 1, which was out of the anticipation earlier. The results were shown in Table.6 and Figure.5(b).

3.3. Harmonic Response

Harmonic response analyses were applied to the statue only model. The model had a first-order modal frequency at 16.851Hz which was analyzed previously in the modal analyses process, and 30 frequencies were selected with equal interval from 15Hz to 35Hz, covering the first two modal frequencies. It was expected to see the resonance effect that when the frequency approached the natural frequencies. When the excitation frequency was 16.851Hz, which was the first modal frequency, its maximum total deformation was approximately 58864mm, not realistic at all.
Figure 6. The responses of the statue within frequency domain

As the Figure.6 illustrates, when the frequency was 17.667Hz, close to the second modal frequency (17.644Hz), the maximum of directional deformation approached to 118.3mm. However, when the frequency increased, its directional deformation dropped to an ordinary level. Conclusions were drawn from the results that resonance (applying frequency close to modal frequency) could result in a significant level of deformation and could lead to serious damage to the structure.

3.4 Transient (Full)

The results were illustrated by those figures. The total deformation curve of the full analysis was really similar to that of MSUP system(Figure.8). To avoid the divergence of the result, force convergence was used to check if the simulation was converge among the time domain.(Figure.7) It could be concluded from the green line which was converged sub-step and the absence of the bisection that the simulation was converged. It was observed that when the wave comes to $10 \sin(\pi t)$ and $\sin(2\pi t)$, the statue started sliding on the base, as in Figure.9.

Figure 7. Solution out: force convergence

Figure 8(a). Deformation under FULL analysis model (b) Deformation under MSUP analysis model
4. Conclusion

Compared with the data obtained with and without the soil foundation, several conclusions were conducted:

1. Total deformation and equivalent stress values of the statue under identical displacement function at the base were dramatically smaller when considering the soil as a part of the FE model. The deformation with soil was approximately 1/5 of the value from the model without soil; Thanks to the contribution of soil, the stress in soil-statue model could be reduced to nearly 0 (in all three cases: frequency, amplitude and direction).

2. All results indicated that the amplitude of the seismic wave was linearly proportionate with the total deformation and equivalent stress, strictly.

3. The deformation and stress in statue-soil combined model were proportionate with each other in a particular direction, while no obvious relationship was observed between those quantities in statue only model. In both cases, the minimum stress occurred at NS direction while maximum equivalent stress appeared in the EW direction. Besides, it was out of the anticipation that when soil was considered, the deformation in EW direction was even 4.7 times larger than the statue only model.

Further refinement needs to be done. First, the response of the structure may be influenced by the shape and the size of the soil foundation since a different soil foundation was applied and the results varied. Furthermore, a more realistic seismic motion could be applied along with a more proper step time and boundary conditions. The more precise meshing and time steps could be executed on an advanced computer. Besides, the methods to distinguish under what deformation the statue would topple could be further developed. Finally, the repairing methods according to the collected response, damper for example, would be proposed and analyzed[7].

Acknowledgments

Taize Yu and Xiaokun Chen contributed equally to this work and they are considered co-first authors.

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