Intermittency in an Optomechanical Cavity Near a Subcritical Hopf Bifurcation

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We experimentally study an optomechanical cavity consisting of an oscillating mechanical resonator embedded in a superconducting microwave transmission line cavity. Tunable optomechanical coupling between the mechanical resonator and the microwave cavity is introduced by positioning a niobium-coated single mode optical fiber above the mechanical resonator. The capacitance between the mechanical resonator and the coated fiber gives rise to optomechanical coupling, which can be controlled by varying the fiber-resonator distance. We study radiation pressure induced self-excited oscillations as a function of microwave driving parameters (frequency and power). Intermittency between limit cycle and steady state behaviors is observed with blue-detuned driving frequency. The experimental results are accounted for by a model that takes into account the Duffing-like nonlinearity of the microwave cavity. A stability analysis reveals a subcritical Hopf bifurcation near the region where intermittency is observed.

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The field of cavity optomechanics [10, 28, 41, 51] deals with a family of systems, each is composed of two coupled elements. The first one is a mechanical resonator, commonly having low damping rate, and the second one is an electromagnetic cavity, which is typically externally driven. Both radiation pressure [2, 13, 19, 27, 37, 57, 73, 78] and bolometric force [4, 39, 47, 50, 52, 54, 55, 56, 61, 67, 86, 88] can give rise to the coupling between the mechanical resonator and the cavity. In recent years a variety of cavity optomechanical systems have been constructed and studied [2, 14, 16, 19, 20, 27, 29, 31, 35, 44, 45, 55, 58, 60, 72, 75, 76, 79], and phenomena such as mode cooling [16, 40, 71, 75, 76], self-excited oscillations [1, 13, 15, 21, 31, 32, 52, 54, 60] and optically induced transparency [40, 59, 70, 82] have been investigated. In addition to applications in metrology [3, 18] and photonics [5, 34, 89], the appeal of optomechanics lies in the potential for observation of macroscopic quantum behavior in mechanical systems [6, 23, 26, 32, 41, 43, 48, 53, 58, 60, 62, 63, 65, 70, 77, 80, 81, 83, 84]. While much experimental and theoretical progress has been made in reaching the ground state and observing the linear dynamics of mechanical objects, it is becoming appreciated that nonlinearity allows the creation of non-classical mechanical states [49] and can be exploited for improving the efficiency of optomechanical cooling [56]. It is therefore important to study and shed light on the nonlinear dynamics of these devices.

In this work we experimentally study self-excited oscillations in an optomechanical cavity operating in the microwave band. We introduce a novel method for achieving strong and tunable optomechanical coupling, which is based on positioning a metallically coated optical fiber near the mechanical resonator. The microwave cavity, which is made of a superconducting aluminum microstrip, exhibits Kerr type nonlinearity [11, 22, 71, 85], which significantly affects the dynamics of the entire optomechanical system [56]. We study the dependence of the self-excited oscillations on the driving parameters of the cavity and found that a good agreement with theory can be obtained provided that cavity nonlinearity is taken into account [56]. We experimentally find that in a certain region of drive parameters the system exhibits random jumps between a limit-cycle (i.e. self-excited oscillations) and a steady-state. A theoretical stability analysis reveals that this observed intermittency behavior occurs near a subcritical [7, 33, 46, 53] Hopf bifurcation [48, 80].

The experimental setup is schematically depicted in Fig. 1. Magnetron DC sputtering is employed for coating a high resistivity silicon wafer with aluminum. The aluminum layer is annealed in situ at 400°C for 10 – 30 minutes to reduce internal stress in the layer [36]. A standard photo-lithography process is used to pattern the microwave microstrip cavity. At the open end of the microstrip a 100nm thick SiN membrane is fabricated [88]. The mechanical resonator is made by releasing a 100 × 100µm² trampoline supported by four beams using electron cyclotron resonance (ECR) dry etch. At the other end the cavity is weakly coupled to a feedline, which guides both the injected and reflected microwave signals. The results presented here are obtained with a device having a fundamental cavity resonance frequency $\omega_c/2\pi = 2.5465$ GHz, cavity linear damping rate $\gamma_c/2\pi = 420$ kHz, fundamental mechanical resonance frequency $\omega_m/2\pi = 12.1$ kHz and mechanical Q-factor $Q_m = 3700$.

As can be seen in Fig. 1 a single mode optical fiber coated with niobium is placed above the suspended trampoline. In the presence of the coated fiber two optomechanical cavities are formed, one in the microwave band and the other in the optical band [87]. The fact that both optomechanical cavities share the same mechanical resonator can be exploited for conversion between microwave and optical photons [11, 8, 17, 23, 88]. However, in the present work we employ the optical cavity and the
optical setup seen in Fig. 1 only for fiber positioning and for characterization of the mechanical resonator at high temperatures, whereas all low temperature measurements that are discussed below are done in the microwave band only.

We employ a telecom single mode optical fiber having a fiber Bragg grating (FBG) mirror and a focusing lens, made by melting the fiber tip. Magnetron DC sputtering is used for coating the fiber with niobium. To allow optical transmission, we etch the niobium coating using focused ion beam (FIB), exposing thus the core of the fiber at the tip. A cryogenic piezoelectric 3-axis positioning system having sub-nanometer resolution is employed for manipulating the position of the optical fiber.

Optomechanical coupling between the microwave cavity and the mechanical resonator is introduced due to the capacitance between the coated fiber and the suspended trampoline, which is given by approximately

\[ C_{SP} = 2\pi\epsilon_0 R_F \log(R_F/\Delta z) \]

where \( R_F = 350 \mu m \) is the radius of curvature of the melted fiber tip, \( \Delta z \) is the fiber-trampoline distance and \( \epsilon_0 \) is the vacuum permittivity. A hole of diameter \( d_H = 2.4 mm \) and depth \( h_H = 2.7 mm \) is drilled in the sample package, which is made of copper, above the trampoline in order to allow inserting the optical fiber, which has an outer diameter of \( d_F = 125 \mu m \). When the fiber is centered inside the hole, the fiber-package coaxial capacitance is given by

\[ C_k = 2\pi\epsilon_0 h_H/\log(d_H/d_F) \]

When radiation loss is disregarded, the effect of the coated fiber on microwave cavity modes can be accounted for by assuming that a termination having purely imaginary impedance given by \( Z_T = 1/i\omega C_k \), where \( C^{-1} = C_{SP}^{-1} + C_k^{-1} \), has been introduced between the microstrip end and ground. The frequencies \( f_a \) of the cavity modes can be found by solving \( \tan(\kappa l_M) = iZ_0/Z_T \), where the propagation constant \( \kappa \) is related to \( f_a \) by \( f_a = \kappa c'/2\pi \), \( c' \) is the propagation velocity in the microstrip, \( l_M \) is the length of the microstrip, and \( Z_0 = 48 \Omega \) is its characteristic impedance. Comparison between the measured and calculated values of the cavity fundamental mode frequency \( f_0 \) is shown in panel (d) of Fig. 1. The dependence of \( f_0 \) on fiber-trampoline distance \( \Delta z \) allows the extraction of optomechanical coupling coefficient \( \Omega \), which is found to be given by

\[ \Omega/\omega_{p0} = 55 \text{MHz} \mu m^{-1} \]

for our chosen operating point, where \( \omega_{p0} = \sqrt{\hbar/2m\omega_0} = 1.1 \times 10^{-5} \mu m \) is the mechanical zero point amplitude, and where \( m = 5.4 \times 10^{-12} \text{kg} \) is the effective mass of the mechanical mode.

The microwave cavity is excited by injecting a monochromatic pump signal having frequency \( f_p = \omega_p/2\pi \) and amplitude \( b_p \) into the feedline and monitoring the off-reflected signal using either a spectrum analyzer or a diode connected to an oscilloscope [see panel (a) of Fig. 1]. The amplitude \( b_p \) is related to the pump power \( P_p \) by \( P_p = \hbar\omega_0 |b_p|^2 / 2\gamma_01 \), where \( \gamma_01 \) represents the contribution to the total cavity linear damping rate \( \gamma_a \) due to cavity-feedline coupling. In the absence of any optomechanical coupling (i.e. when the fiber is positioned far from the trampoline) the cavity reflectivity exhibits bistability in a certain region in the plane of pump parameters (frequency \( f_p \) and amplitude \( b_p \)) originates by cavity Kerr nonlinearity. The border line of this region

FIG. 1: Experimental Setup (color online). The Microwave cavity is a microstrip made of aluminum over high resistivity silicon wafer coated with a 100 nm thick SiN layer. The mechanical resonator at the end of the microstrip is a suspended silicon wafer coated with a 100 nm thick SiN layer. The cavity is a microstrip made of aluminum over high resistivity silicon wafer coated with a 100 nm thick SiN layer. The mechanical resonator at the end of the microstrip is a suspended silicon wafer coated with a 100 nm thick SiN layer. The cavity is a microstrip made of aluminum over high resistivity silicon wafer coated with a 100 nm thick SiN layer. The mechanical resonator at the end of the microstrip is a suspended silicon wafer coated with a 100 nm thick SiN layer.
and frequency self-excited oscillations occur. The height of the peak at mechanical resonance frequency, for both forward and backward sweep analyzer, at frequency $f_c$ of microwave power, which is measured using a spectrum analyzer, is Boltzmann’s constant and $T$ is the temperature.

The stability map of the system is obtained using the numerical continuation package MATCONT (URL: http://www.matcont.ugent.be/). First, a steady state solution (i.e. solution to $\Theta_a = \Theta_b = 0$) is found for each operating point in the plane of pump parameters. Note that for the region seen in Fig. 2 the steady state is unique (since $b_p/b_c < 1$). Then MATCONT is employed to identify bifurcations. The solid black, dotted cyan and dotted green lines represent, respectively, supercritical Hopf, subcritical Hopf and limit point of cycle bifurcations. The following cavity parameters are employed for the bifurcation calculation $K_a/\omega_n = -1.44 \times 10^{-15}$ and $\gamma_{a3}/K_a = 0.04$.

FIG. 2: Self-Excited Oscillations (color online). Panels (a) and (b) show the reflected power at angular frequency $\omega_p - \omega_n$ for backward and forward frequency sweeps, respectively. The pump critical power is $P_c = -19.5$ dBm and the pump critical detuning is $\Delta_c = -2\pi \times 0.7$ MHz. Panel (c) shows the reflected power at angular frequency $\omega_p$. Note the pulling in the frequency response due to the cavity Kerr nonlinearity. The solid black, dotted cyan and dotted green lines represent, respectively, supercritical Hopf, subcritical Hopf and limit point of cycle bifurcations. The following cavity parameters are employed for the bifurcation calculation $K_a/\omega_n = -1.44 \times 10^{-15}$ and $\gamma_{a3}/K_a = 0.04$.

contains a cusp point, which is also known as the onset of bistability point [55]. The values of pump frequency and pump amplitude at that critical point are labeled by $f_c = \omega_c/2\pi$ and $b_c$, respectively. In what follows we employ normalized and dimensionless parameters for the pump detuning $\Delta/\Delta_c = (\omega_p - \omega_n) / |\omega_c - \omega_a|$ and for the pump amplitude $b_p/b_c$.

Panels (a) and (b) of Fig. 2 show the reflected microwave power, which is measured using a spectrum analyzer, at frequency $f_p - f_b$, where $f_b = \omega_b/2\pi$ is the mechanical resonance frequency, for both forward and backward sweeps of the pump frequency $f_p$. A strong peak is found at frequency $f_p - f_b$ as well as at other harmonics $f_p + n f_b$, where $n$ is integer, as can be seen in panel (a) of Fig. 3 in a certain region in the plane of normalized pump parameters $\Delta/\Delta_c$ and $b_p/b_c$, inside which self-excited oscillations occur. The height of the peak at frequency $f_p - f_b$ is plotted in panels (a) and (b) of Fig. 2. The results obtained with backward sweep [panel (a)] differ from those obtained with forward sweep [panel(b)], which indicates that the cavity response is hysteretic due to bistability. Panel (c) depicts the reflected power at $f_p$.

Cavity nonlinearity plays a crucial role in the observed behavior of the system. We employ the theoretical modeling of Refs. 12, 55 to account for cavity Kerr nonlinearity 22, 78. The equations of motion in the rotating frame of the cavity for the annihilation operators $A_a$ and $A_b$ of the cavity and mechanical resonator, respectively, are found to be given by $dA_a/dt + \Theta_a = F_a$ and $dA_b/dt + \Theta_b = F_b$ where

$$\Theta_a = [i \Delta_{a}^{\text{eff}} + \gamma_a + (i K_a + \gamma_{a3}) N_a] A_a + b_p$$

(1)

$$\Theta_b = (i \omega_b + \gamma_b) A_b + i \Omega N_a$$

(2)

and where $\Delta_{a}^{\text{eff}} = \omega_a - \omega_n + \Omega (A_b + A_b^{\dagger})$ is the effective cavity detuning, $\Omega$ is the optomechanical coupling coefficient, $K_a$ is the cavity Kerr nonlinearity constant, $\omega_a (\omega_b)$ is the cavity (mechanical) angular resonance frequency, $\gamma_a (\gamma_b)$ is the cavity (mechanical) linear damping rate, $\gamma_{a3}$ is the cavity nonlinear damping rate, $N_a = A_b^{\dagger} A_b$ is the cavity number operator and $b_p$ is the pump amplitude. The terms $F_a$ and $F_b$ represent white noise having (frequency independent) power spectrum given by

$$S_a = 2 \Gamma_a (1 - e^{-\beta \hbar \omega_n})^{-1}$$

and

$$S_b = 2 \gamma_b (1 - e^{-\beta \hbar \omega_n})^{-1},$$

respectively, where $\Gamma_a = \gamma_a + 2 \gamma_{a3} \langle N_a \rangle$, $\beta = 1/\hbar T$, $\hbar$ is Boltzmann’s constant and $T$ is the temperature.
dotted green lines in Fig. 2 represent, respectively, supercritical Hopf (labeled as $H_-$), subcritical Hopf (labeled as $H_+$) and limit point of cycle (LPC) bifurcations. In the numerical investigation noise is disregarded and the operators $A_a$ and $A_b$ are treated as c-numbers. The cavity parameters that are used for the numerical calculation are listed in the caption of Fig. 2. The bifurcation lines divide the region in the plane of pump parameters seen in Fig. 2 into three zones. In the zone between the $H_-$ and $H_+$ bifurcations only a single limit-cycle is found to be locally stable (though the existence of other locally stable solutions cannot be ruled out), in the zone between the $H_+$ and LPC bifurcations bistability of a limit-cycle and a steady state occurs, whereas elsewhere only a unique steady state is found.

Intermittency, i.e. random jumps between the limit-cycle and the steady state, is experimentally observed for $b_p/b_c > 0.2$. Fig. 3 shows frequency and time domain measurements taken with normalized pump amplitude given by $b_p/b_c = 0.58$. Panel (a) shows the frequency decomposition of the reflected signal as the pump frequency $f_p$ is scanned and the white line shows the reflected power at frequency $f_p$. Regular self-excited oscillations in the time domain can be seen in panel (c) for normalized detuning $\Delta/\Delta_c = 0.2$. At larger detuning $\Delta/\Delta_c = 1.5$ (in the bistable zone), however, random transitions between the limit cycle and the steady state occur, as can be seen in panel (b). The limit cycle and the steady state in the complex $A_a$ projection plane of phase space are plotted in panel (d). Note that the dynamics near the steady state remains relatively slow even when it becomes locally unstable (i.e. in the zone between the $H_-$ and $H_+$ bifurcations). Consequently, in the presence of noise even a locally unstable steady state can give rise to intermittency-like behavior provided that it is sufficiently close to the limit cycle. Indeed, intermittency is experimentally observed on both sides of the $H_+$ bifurcation.

In summary, we find that a subcritical Hopf bifurcation is the underlying mechanism that is responsible for the experimentally observed intermittency. While the current study is focused on the classical dynamics, future study will explore the possibility of exploiting dynamical bistability for the creation of macroscopic non-classical states of the optomechanical system 3.

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