The Super P-Brane Scan and S Duality

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(July, 1996)

Abstract

Taking into account the recent dualities we rederive the super p-brane scan. Our main results are the importance of the metric’s signature and the existence of an S self-dual super 5-brane at D=14 with signature (7,7) or (11,3).

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Recently, new developments in superstring theory have led to major advances in our understanding of the non-perturbative regime of the theory. Dualities taught us that there exists an underlying unique theory, it may allow different vacua. We have mainly two types of dualities S and T, sometimes, they can unify into U duality. It can happen that theory A at strong coupling is equivalent to theory B at weak coupling, they are said to be S dual, while, if theory A is compactified on a space of volume $\tau$ is equivalent to theory B compactified on a space of volume $\sim 1/\tau$, then they are called T dual. The S duality can be considered as a generalization of the Manton-Olive electromagnetic duality [1] where a theory at small coupling is mapped to its solitonic partner at strong coupling.

One of the striking consequences of these new achievements is the existence of the eleven dimensions M-theory [2], a quantizable version of the supermembrane and another more fundamental the F-theory [3]. The accumulating evidences of this F-theory indicate that it lives in 12 dimensions of signature (10,2). At first sight, it seems strange to have a physical theory in $D > 11$, since, a spinor in D dimensions has $2^{(D+2)/2}$ real components i.e in 12 dimensions, the spinor will have 128 real components whereas any physical theory (that does not couple to particles with helicity higher than two) have maximally 32 real components. Actually, this statement overlooked something very important since it ignores the signature of the underlying space-time. Supersymmetric theories are very sensitive to the dimension of the spinor, the nature of the spinorial representation of the lorentz group (being real or imaginary) which depends on the dimension and the signature of space-time and moreover the possible conditions that one can impose on his spinor and this is exactly what we are going to investigate in this note.

The construction of any supersymmetric theories is systematic and straightforward once the automorphism group of its corresponding Clifford algebra is given [4]. So the knowledge of Clifford algebra is vital. Such algebra are completely classified by mathematicians for any generic D dimensions with signature $(s,t)$ [4][5]. I have reproduced it in Table (I) as we will use it heavily.
Using Table I, we can generalize the results of [4] to any dimension D with signature (s,t) we have the following properties for our spinor

\[ s - t = \begin{align*}
2, & \text{ mod 8 complex,} \\
0, & \text{ mod 8 real,} \\
3, & \text{ mod 8 pseudoreal.}
\end{align*} \] (1)

One can read easily the dimension of the spinor from Table (I). Their automorphism group is given by

\[ s - t = \begin{align*}
2, & \text{ mod 8 SU(N) \times U(1),} \\
1, & \text{ mod 8 SO(N),} \\
0, & \text{ mod 8 SO(N)_+ \times SO(N)_-,} \\
3, & \text{ mod 8 USp(N),} \\
4, & \text{ mod 8 USp(N)_+ \times USp(N)_-.}
\end{align*} \] (4)

In this paper, we will concentrate on the Weyl-Majorana cases as they are the most promising ones in higher dimensions. It is well known that if we have a Weyl-Majorana on-shell (WM) spinor, it will reduce the number of components to the quarter. A Weyl-Majorana spinor is possible for D dimensions space-time of signature (s,t) if we can impose a Weyl condition [7]

\[ s - t = 0 \mod 4, \] (9)

while a Majorana condition is available for [4]
\[ s - t = 0, 1, 2 \mod 8. \]  \hspace{1cm} (10)

By combining the two conditions together, we have

\[ s - t = 0 \mod 8, \]  \hspace{1cm} (11)

another way to get this result is by combining a Majorana spinor with a pseudo-Majorana. One can check (11) explicitly by looking in Table(I) and search for the \( ^2R \) cases. I have produced all the allowed physical cases in details in Table (II).

- TABLE II -

Indeed in 12 dimensions a 32 real component spinor is possible for the following signatures (10,2) and (6,6). The (10,2) case has been noticed along time ago in [7] and recently in [3,8]. Generally, a WM spinor is possible for any even 2n dimensions with signature (n,n). Such cases are very interesting as it is conjectured that their supersymmetric self dual theories are the generators of all known - and still unknown - possible integrable models [9,10]. Fortunately for a Weyl-Majorana spinor, its automorphism group is well known and always of the type \( SO(2^{(D-2)/2})_+ \times SO(2^{(D-2)/2})_- \).

The first application of Table(II) can be the scan for possible (d-1)-superbranes [11]. The standard analysis goes as the following : An important key in constructing these Green-Schwarz type superbranes is the existence of the \( \kappa \) symmetry which means that half of the spinor degrees of freedom are redundant and may be eliminated by a physical gauge choice. Let \( M \) be the number of real components of the minimal spinor and \( N \) the number
of supersymmetries in D dimensions and let m and n be the corresponding quantities over
the d worldvolume dimensions, the number of bosonic effective degrees of freedom is

\[ N_B = D - d \]  

(12)

where we are working in the “static gauge choice”. the number of on-shell fermionic degrees
of freedom is

\[ N_F = \frac{1}{2} mn = \frac{1}{4} MN. \]

(13)

Worldvolume supersymmetry requires \( N_B = N_F \), hence

\[ D - d = \frac{1}{2} mn = \frac{1}{4} MN. \]

(14)

After substituting for \( m = 2^{(d-2)/2} \) and \( M = 2^{(D-2)/2} \) we have

\[ D - d = \frac{1}{2} 2^{(d-2)/2} n = \frac{1}{4} 2^{(D-2)/2} N. \]

(15)

We have shown the allowed values of p and n in the last column of Table (I I). We note
that \( D_{\text{max}} = 12 \) whereas \( d_{\text{max}} = 6 \). Hence, we have at max super 5-brane. From table(I I),
we have the possible situations

\[ d = 2 \quad D = 3, 4, 6, 10. \quad \text{string} \]  

(16)

\[ d = 3 \quad D = 4, 5, 7, 11. \quad \text{membrane} \]  

(17)

\[ d = 4 \quad D = 5, 6, 8, 12. \quad \text{3-brane} \]  

(18)

\[ d = 5 \quad D = 6, 7, 9. \quad \text{4-brane} \]  

(19)

\[ d = 6 \quad D = 8, 10. \quad \text{5-brane} \]  

(20)

It is conjectured that all super p-branes for \( D < 11 \) are solitonic solutions. But, for
\( D = 11 \) and 12, they are the low energy limit of the M and F theory respectively. From
(I6 I8), we see three distinct series, for any d=x, super (d-1)-brane exists for D=x+1, x+2,
x+4, x+8, we will call them the real, complex, quaternion and octonion series respectively.
Actually, we have taken the signature of the p-brane worldvolume’s metric to be that of the minimal spinor, if choose such metric to be (d-1,1) then m is minimal only for d = 2,3,9,10,11 as can be shown directly from Table (I). But this choice will affect only the number of supersymmetries on the super p-brane world volume and will not make any further changes to our results.

From the hierarchies of string/string, string/membrane ... dualities, one would like to make the final theory distinguishable. A possible way is to require it to be self dual with respect to the S duality. From [11], we find this possibility is only available for

\[ D = 2(d + 1), \]  

(21)

If we try to satisfy (21) for the real and complex series, d will lead to negative values of p whereas for the other series, we have

\[ d = 2 \quad D = 6 \quad \text{the quaternion series} \]

\[ d = 6 \quad D = 14 \quad \text{the octonion series} \]

(22)

(23)

So, the final theory may be a super 5-brane in 14 dimensions but its existence is a puzzle. If we accept that the final theory is too different then there should be a new “generalized” symmetry and there is no low energy limit’s traditional super-gravitational theory. This result suggests to replace (19) and (20) by

\[ d = 5 \quad D = 6, 7, 9, 13 \quad \text{4-brane} \]

\[ d = 6 \quad D = 7, 8, 10, 14 \quad \text{5-brane} \]

(24)

(25)

In order to mach these results with (14), there should be a generalization of \( \kappa \) symmetry, for D = 7 and 14 super 5-brane (as the minimal spinor in 14 dimensions with signature (7,7) or (11,3) has 64 real components) or we should include other fields so (14) will acquire new bosonic degrees of freedom as had been suggested in [13].

We can reach the conclusion that the 14 dimensional 5-brane theory is consistent and exists from another different route: Assuming that S duality is an exact symmetry of the
super p-branes then we can derive some important consequences about their quantization, at D dimensions, a (d-1) brane is dual to another (\(\tilde{d}-1\)) brane \[11\], for

\[
\tilde{d} = D - d - 2, \tag{26}
\]

whatever this condition seems trivial, we require for its validity the existence, a priori, of both the (d-1) and the (\(\tilde{d}-1\)) brane then we have only the following cases:

\[
\begin{align*}
D &= 14 & d &= 6 \text{ and } \tilde{d} = 6. \tag{27}
D &= 10 & d &= 2 \text{ and } \tilde{d} = 6. \tag{28}
D &= 6 & d &= 2 \text{ and } \tilde{d} = 2. \tag{29}
\end{align*}
\]

We require the existence of both the d and the \(\tilde{d}\) theory because S duality maps a (d-1) brane at weak coupling to a (\(\tilde{d}-1\)) brane at strong coupling and vice-versa. So, if one of these dual theories does not exist at this specific D, then our theory will be anomalous. Actually, this condition was too effective to rule out all the p branes except p=2 and 5 only. It indicates that superstring theory is consistent in 6 and 10 dimensions \[14\] whereas the super 5 brane is allowed only for 10 and 14 dimensions. We notice that these dimensions are exactly the ones where the gravitational anomaly exists (D=4k+2 for integer k). It remains to confirm this result on a stiff ground by an explicit anomaly calculation.

I am very grateful, especially, to Prof. P. Rotelli, and generally, to all the members of the physics department at Lecce university for their kind hospitality.
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TABLE I. The complete classification of real Clifford algebra in D dimensions with signature (s,t) where the real number of components (comp) < 64.

| s/t\(\text{^a}\) | 0   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  |
|------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| 0                | ±1  | R   | C   | H   | 2H  | H(2)| C(4)| R(8)| 2R(8)| R(16)| C(16)| X   | X   |
| 1                | R   | 2R  | R(2)| C(2)| H(2)| 2H(2)| H(4)| C(8)| R(16)| 2R(16)| R(32)| X   | X   |
| 2                | C   | R(2)| 2R(2)| R(4)| C(4)| H(4)| 2H(4)| H(8)| C(16)| R(32)| 2R(32)| X   | X   |
| 3                | H   | C(2)| R(4)| 2R(4)| R(8)| C(8)| H(8)| 2H(8)| X   | X   | X   | X   | X   |
| 4                | 2H  | H(2)| C(4)| R(8)| 2R(8)| R(16)| C(16)| X   | X   | X   | X   | X   | X   |
| 5                | H(2)| 2H(2)| H(4)| C(8)| R(16)| 2R(16)| R(32)| X   | X   | X   | X   | X   | X   |
| 6                | C(4)| H(4)| 2H(4)| H(8)| C(16)| R(32)| 2R(32)| X   | X   | X   | X   | X   | X   |
| 7                | R(8)| C(8)| H(8)| 2H(8)| X   | X   | X   | X   | X   | X   | X   | X   | X   |
| 8                | 2R(8)| R(16)| C(16)| X   | X   | X   | X   | X   | X   | X   | X   | X   | X   |
| 9                | R(16)| 2R(16)| R(32)| X   | X   | X   | X   | X   | X   | X   | X   | X   | X   |
| 10               | C(16)| R(32)| 2R(32)| X   | X   | X   | X   | X   | X   | X   | X   | X   | X   |
| 11               | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   |
| 12               | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   | X   |

\(\text{^a}(2A)\) means \(A \oplus A\)
TABLE II. The allowed minimal spinor (M) for D dimensions with signature (s,t) where the real number of components < 64. We present also the possible number of extended supersymmetry N and the allowed p-branes (p = d − 1) with their extended supersymmetry n.

| D | (s,t)  | M  | The automorphism group | N  | (p,n) |
|---|-------|----|------------------------|----|------|
| 1 | (1,0) | 1  | SO(1)                 | 1..8 |      |
| 2 | (1,1) | 1  | SO(1)⁺ × SO(1)⁻       | 1..8 | (1,2) |
| 3 | (2,1) | 2  | SO(2)                 | 1..8 |      |
| 4 | (2,2) | 2  | SO(2)⁺ × SO(2)⁻       | 1..8 | (1,4), (2,1) |
| 5 | (3,2) | 4  | SO(4)                 | 1..8 | (2,2), (3,1) |
| 6 | (3,3) | 4  | SO(4)⁺ × SO(4)⁻       | 1..8 | (1,8), (3,2) |
| 7 | (7,0), (4,3) | 8 | SO(8)                 | 1..4 | (2,4), (4,1) |
| 8 | (8,0), (4,4) | 8 | SO(8)⁺ × SO(8)⁻       | 1..4 | (3,4), (5,1) |
| 9 | (9,0), (8,1), (5,4) | 16 | SO(16)            | 1, 2 | (4,2) |
| 10 | (9,1), (5,5) | 16 | SO(16)⁺ × SO(16)⁻   | 1, 2 | (1,16), (5,2) |
| 11 | (10,1), (9,2), (6,5) | 32 | SO(32)              | 1   | (2,8) |
| 12 | (10,2), (6,6) | 32 | SO(32)⁺ × SO(32)⁻ | 1   | (3,8) |

*a* for space and *t* for time, obviously, the (t,s) case is also allowed.

*b* real components.

*c* again, we will restrict ourselves to the only physical cases i.e N≤8.