Interplay of magnetic field and geometry in magneto-transport of mesoscopic loops with Rashba and Dresselhaus spin-orbit interactions

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Electronic transport in closed loop structures is addressed within a tight-binding formalism and in the presence of both the Rashba and Dresselhaus spin-orbit interactions. It has been shown that any one of the spin-orbit (SO) fields can be estimated precisely if the other one is known, by observing either the transmission resonance or anti-resonance of unpolarized electrons. The result is obtained through an exact analytic calculation for a simple square loop, and through a numerically exact formulation for a circular ring. The sensitivity of the transport properties on the geometry of the interferometer is discussed in details.

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I. INTRODUCTION

Spintronics is a recent field of utmost research interest that include magnetic memory circuits, quantum computers, magnetic nano-structures and quasi one-dimensional semiconductor rings which have been acknowledged as ideal candidates for investigating the effects of quantum coherence in low-dimensions, and have been examined as the prospective quantum devices.1–4 A central mechanism governing the physics in the meso and nano-length scales are the Rashba and Dresselhaus spin-orbit interactions which result from a structural inversion asymmetry, and bulk inversion asymmetry, respectively. The effects are pronounced in quantum rings formed at the interface of two narrow gap semiconductors, as already discussed in the literature.5–8

Needless to say, an accurate estimation of the spin-orbit interaction (SOI) strengths is crucial in the field of spintronics. The Rashba spin-orbit interaction (RSOI) can be controlled by a gate voltage placed in the vicinity of the sample and hence, can be ‘measured’. Comparatively speaking, reports on the techniques of measurement of the Dresselhaus spin-orbit interaction (DSOI) are relatively few. Very recently we have put forward an idea of estimating the DSOI strength provided the RSOI is known by measuring a minimum in the Drude weight.9–11. A minimum in the Drude weight appears only when the strengths of the RSOI and DSOI are exactly identical.

From a closer look at the Rashba and Dresselhaus SO interactions it turns out that both the interactions are equivalent to the SU(2) gauge field which introduces a phase in the wave function. In this communication we describe the role of this phase and considering its effect on quantum interference we develop a simple idea about how one of the two SOI’s can be estimated while the other is known. This quantum interference effect in presence of SO interactions has not been described in our previous work and the present analysis may provide a basic understanding of designing switching devices for spintronic applications in near future. A simple version of a quantum ring, in the form of a loop with a rhombic geometry is considered for an analytical attack on the problem, while numerically exact results are provided for circular rings with and without disorder and with a magnetic flux threading these polygonal structures. We adopt a tight-binding formalism in contrast to a recently proposed scheme where a continuous version of the model is presented16 to consider the combined effect of the RSOI and the DSOI. When strengths of the RSOI and DSOI are equal, the end to end transmission across the rhombic loop is shown to be equal to unity and vanishes when the loop is threaded by a magnetic flux \( \phi \) equal to the half flux-quantum (\( \phi_0/2 \)). Thus, one can estimate the DSOI by observing the peak when \( \phi = 0 \) or dip when \( \phi = \phi_0/2 \) in the transmission (conductance) spectrum when the RSOI is known, and vice versa.

The idea is extended to the case of rings with circular geometry, where we have evaluated the transmission coefficient numerically. The essential differences in the cases of a rhombic loop and a circular ring are discussed to highlight the sensitivity of the results on the loop geometry. In what follows we describe the procedure and the results. In section II, we present the model and the method. In section III we discuss the sensitivity of the results on...
the geometry of the closed loop structures, and in section IV we draw our conclusions.

II. THE MODEL AND THE METHOD

The Hamiltonian: Let us consider the rhombic loop depicted in Fig. 1 which is threaded by a magnetic flux $\Phi$. Each side of the loop is of length $L$, and the loop contains $N$ number of equispaced atomic sites with ‘lattice constant’ $a$. Within a tight-binding framework the Hamiltonian for this network in the presence of the RSOI is assumed to be zero throughout the geometry.

$$H = H_0 - i\alpha H_R + i\beta H_D$$

where,

$$H_0 = -\sum_i \left( c_i^\dagger tc_{i+1}e^{i\phi} + c_{i+1}^\dagger tc_i e^{-i\phi} \right)$$

The Rashba and Dresselhaus spin-orbit parts of the Hamiltonian, viz, $H_R$ and $H_D$, are given by,

$$H_R = \sum_i \left( \sin \theta c_i^\dagger \sigma_x c_{i+1} - \cos \theta c_i^\dagger \sigma_y c_{i+1} \right) e^{i\phi} + h.c.$$  

$$H_D = \sum_i \left( -\cos \theta c_i^\dagger \sigma_y c_{i+1} + \sin \theta c_i^\dagger \sigma_y c_{i+1} \right) e^{i\phi} + h.c.$$  

where, $i$ refers to the atomic sites in the arms of the loop. $\phi = 2\pi \Phi / Na$, and $\phi_0 = \hbar c / e$, the fundamental flux-quantum. The other operators in Eq. 3 are as follows,

$$c_i = \begin{pmatrix} c_i^\uparrow \\ c_i^\downarrow \end{pmatrix} \quad \text{and} \quad t = t \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$ 

Here the site energy of an electron at any $i$-th site is assumed to be zero throughout the geometry. $t$ is the nearest-neighbor hopping integral. $\alpha$ and $\beta$ are the isotropic nearest-neighbor transfer integrals which measure the strengths of Rashba and Dresselhaus SOI, respectively. $\sigma_x, \sigma_y$ and $\sigma_z$ are the Pauli spin matrices. $c_{i,\sigma}^\dagger (c_{i,\sigma})$ is the creation (annihilation) operator of an electron at the site $i$ with spin $\sigma$ ($\uparrow, \downarrow$).

Eigenvalues and eigenfunctions of the Hamiltonian: These are obtained by adopting a k-space description of the Hamiltonian given in Eq. 3 viz, $H = \sum_k c_k^\dagger H_k c_k$. Using discrete Fourier transform $c_k = \frac{1}{\sqrt{N}} \sum_n c_n \exp(-ik_na)$, the Hamiltonian matrix reads,

$$H_k = \begin{pmatrix} \epsilon_k & \gamma_k + i\delta_k \\ \gamma_k - i\delta_k & \epsilon_k \end{pmatrix}$$

where,

$$\epsilon_k = -2t \cos(ka + \phi)$$

$$\gamma_k = 2(\alpha \sin \theta + \beta \cos \theta) \sin(ka + \phi)$$

$$\delta_k = 2(\alpha \cos \theta + \beta \sin \theta) \sin(ka + \phi)$$

While writing the above expressions, we have set the lattice constant $a = 1$. The energy eigenvalues are obtained from Eq. 4 and are given by, $E_{k\pm} = \epsilon_k \pm \sqrt{\gamma_k^2 + \delta_k^2}$.

Let us denote the left vertex of the loop in Fig. 1 as the ‘origin’ $(0, 0)$. A simple but lengthy algebra now allows one to write the wave function for an energy $E$ at a distance $L$ along any arm of the rhombic loop as,

$$|\Phi_E(L, a)\rangle = R_E(L, \alpha, \beta, \theta)|\Phi_E(0)\rangle$$

where, the elements of the matrix $R_E(L, \alpha, \beta, \theta)$ are,

$$R_E(L, \alpha, \beta, \theta)_{11} = \frac{1}{2} \left( e^{i(k_L + L\alpha)} + e^{i(-k_L - L\alpha)} \right)$$

$$R_E(L, \alpha, \beta, \theta)_{12} = \frac{1}{2} \left( e^{i(k_L + L\alpha + \nu_k \alpha)} - e^{i(-k_L - L\alpha + \nu_k \alpha)} \right)$$

$$R_E(L, \alpha, \beta, \theta)_{21} = \frac{1}{2} \left( e^{i(k_L - L\alpha + \nu_k \alpha)} - e^{i(-k_L + L\alpha - \nu_k \alpha)} \right)$$

$$R_E(L, \alpha, \beta, \theta)_{22} = \frac{1}{2} \left( e^{i(k_L - L\alpha)} + e^{i(-k_L - L\alpha)} \right)$$

Here, $\nu_k = \tan^{-1}(\delta k_\pm / \gamma k_\pm)$. $k_\pm$ are the wave vectors corresponding to the energy values $\epsilon_k \pm \sqrt{\gamma_k^2 + \delta_k^2}$, as already mentioned.

Transmission of unpolarized electrons: Let us now assume that the electrons enter the loop at the point $A$ through the source, and are drained out at $B$. In addition, without any loss of generality, and to get a relatively simple set of equations, we take our loop to be a square one with $\theta = \pi / 4$. For an electron traveling in the loop in the clockwise sense with a specified energy $E$, the wave vector $k_\pm$ are the solutions of the equation $E = E_{k\pm}$, and can be explicitly written as,

$$k_\pm a = \cos^{-1} \xi_\pm(E) - \phi$$

where,

$$\xi_\pm(E) = \frac{1}{\sqrt{1 + (\alpha+\beta)^2 t^2}} \left[ \frac{E}{2t} \sqrt{1 + \frac{(\alpha+\beta)^2 t^2}{E^2}} \right]$$

$$\pm \frac{\alpha + \beta}{t} \sqrt{1 - \frac{E^2}{4t^2(1 + \frac{(\alpha+\beta)^2}{t^2})}}$$

In a similar manner, we need to work out the wave vectors $k_\pm'$ for the electrons with the same energy $E$, and traveling in the counter-clockwise sense. The result is,

$$k_\pm' a = \cos^{-1} \xi_\pm'(E) - \phi$$

where,

$$\xi_\pm'(E) = \frac{1}{\sqrt{1 + (\alpha-\beta)^2 t^2}} \left[ \frac{E}{2t} \sqrt{1 + \frac{(\alpha-\beta)^2 t^2}{E^2}} \right]$$

$$\pm \frac{\alpha - \beta}{t} \sqrt{1 - \frac{E^2}{4t^2(1 + \frac{(\alpha-\beta)^2}{t^2})}}$$
The probability that the electrons travel in the clockwise or the counter-clockwise sense are assumed to be equal. The transmission amplitude is given by the matrix,

\[ \tau = \begin{pmatrix} \tau_{\uparrow\uparrow} & \tau_{\uparrow\downarrow} \\ \tau_{\downarrow\uparrow} & \tau_{\downarrow\downarrow} \end{pmatrix} \] (11)

which, for \( \theta = \pi/4 \), simplifies to,

\[ \tau = \frac{1}{2} \left[ \mathcal{R}_E(L, \alpha, \beta, -\pi/4) \mathcal{R}_E(L, \alpha, \beta, \pi/4) + \mathcal{R}_E(L, \alpha, \beta, \pi/4) \mathcal{R}_E(L, \alpha, \beta, -\pi/4) \right] \] (12)

The transmission amplitude is given by the matrix,

\[ \mathcal{R}_E \]

or the counter-clockwise sense are assumed to be equal.

The probability that the electrons travel in the clockwise is

\[ \tau = \frac{1}{2} \left[ \mathcal{R}_E(L, \alpha, \beta, -\pi/4) \mathcal{R}_E(L, \alpha, \beta, \pi/4) + \mathcal{R}_E(L, \alpha, \beta, \pi/4) \mathcal{R}_E(L, \alpha, \beta, -\pi/4) \right] \] (12)

\[ \mathcal{R}_E \] matrices can be cast in to the forms,

\[ \mathcal{R}_E(L, \alpha, \beta, \pi/4) = \frac{e^{i\phi L}}{2} \left( \begin{array}{c} A_1 \sigma_x + B_1 \sigma_y \\ B_1 \sigma_y + A_1 \sigma_x \end{array} \right) \] (13)

\[ \mathcal{R}_E(L, \alpha, \beta, -\pi/4) = \frac{e^{i\phi L}}{2} \left( \begin{array}{c} A_2 \sigma_x + B_2 \sigma_y \\ B_2 \sigma_y + A_2 \sigma_x \end{array} \right) \] (13)

with,

\[ A_1 = \exp(ik_L a) + \exp(ik_{-} La) \]
\[ A_2 = \exp(ik_L ' a) + \exp(ik_{-} ' La) \]
\[ B_1 = \exp(ik_L La) - \exp(ik_{-} La) \]
\[ B_2 = \exp(ik_L ' La) - \exp(ik_{-} ' La) \] (14)

The transmission amplitude given by Eq. (12) can now be explicitly written as,

\[ \tau = \frac{1}{8} \left[ 2A_1A_2 + \sqrt{2}(A_2B_1 + A_1B_2)\sigma_x \\ + \sqrt{2}(A_1B_2 - A_2B_1)\sigma_y \right] \cos 2\phi L \]
\[ + \frac{i}{4}(\sigma_x \sigma_y - \sigma_y \sigma_x)B_1B_2 \sin 2\phi L \] (15)

The coefficient of transmission for an incoming up-spin electron is \( T_\uparrow = |\tau_{\uparrow\uparrow} + \tau_{\uparrow\downarrow}|^2 \), and the transmission coefficient for an incoming down-spin electron is \( T_\downarrow = |\tau_{\downarrow\uparrow} + \tau_{\downarrow\downarrow}|^2 \), so that the final transmission coefficient for spin unpolarized electrons turns out to be,

\[ T = \frac{1}{2}(T_\uparrow + T_\downarrow) \]
\[ = \frac{1}{2} \left[ \frac{|A_1A_2|^2}{8} + \frac{|A_2B_1 + A_1B_2|^2}{16} + \frac{|A_1B_2 - A_2B_1|^2}{16} \right] \]
\[ = \cos^2 \left( \frac{(k_+ - k_-) La}{2} \right) \sin^2 \left( \frac{(k_+ + k_-) La}{2} \right) \]
\[ \times \cos^2 \left( \frac{(k_+ - k_-) La}{2} \right), \quad \text{for } \phi = 0 \]
\[ = 4\sin^2 \left( \frac{(k_+ - k_-) La}{2} \right) \sin^2 \left( \frac{(k_+ - k_-) La}{2} \right) \]
\[ \text{for } \phi = \pi/4L \] (16)

When \( \alpha = \beta \), from Eq. (11) we observe that \( \xi_+ = \xi_- \) or \( k'_+ = k'_- \) and it gives \( B_2 = 0 \). Therefore, for \( \phi = 0 \), the transmission coefficient \( T = 1 \), and, \( T = 0 \) for \( \phi = \pi/4L \). Thus we get perfect transmission for \( \phi = 0 \) while a vanishing of transmission coefficient for \( \phi = \pi/4L \).

### III. EFFECT OF LOOP GEOMETRY

Electronic transport turns out to be sensitive to the geometry of the mesoscopic loop. To this end, we have numerically calculated the two-terminal spin transport in a ring geometry threaded by a magnetic flux \( \Phi \) (Fig. 2). The role of \( \Phi \) will be discussed later. Here, for the time being, we ignore the flux. For a ring like structure, the azimuthal angle keeps on changing as one traverses the perimeter of the ring. This generates an effective site dependent hopping integral in the Hamiltonian. As a result, scattering takes place as the electron travels across the sites on the ring. The scattering becomes a maximum when \( \alpha = \beta \), and naturally, the two-terminal transport...
is expected to show up a minimum at $\alpha = \beta$. We use exact numerical methods. In Fig. 3 we show the variation of the two-terminal transmission coefficient for an ordered ($W = 0$, $W$ measures the strength of disorder) ring of 40 sites. Similar observations are presented in Fig. 6 for a 40-site ring with random diagonal disorder. The results in the latter case have been averaged over 60 disorder configurations. $\Phi$ is fixed at 0.5.

Before we end this section, it should be appreciated that, in an experiment the transmission minimum is not easy to locate, and hence an inaccuracy in the value of the SOI might be introduced. This difficulty may be circumvented if the transmission minimum becomes precisely equal to zero. This is easily achieved if the ring is threaded by a magnetic flux which is set equal to half the flux-quantum $\phi_0 = \hbar c/e$. In Figs. 5 and 6 the results are presented for a 40-site ordered ring and a randomly disordered ring of the same size. The flux threading the rings is set at $\Phi = \phi_0/2$. The two-terminal transmission coefficient exhibits clear zeros in both the cases as soon as the DSOI equals the strength of the RSOI. Once again, the transmission zero in this special situation is independent of the disorder configuration.

IV. CONCLUSION

In conclusion, we have presented exact analytical results to show that the spin-orbit interactions present in a mesoscopic sample can be measured by observing two-terminal transmission resonance in a simple square network. The transmission coefficient is shown to be exactly equal to unity when the Rashba and the Dresselhaus interactions become equal in strength. Thus, knowing the Rashba interaction, for example, a determination of the Dresselhaus term is possible. For a multi-site ring, the transmission maximum observed for the single square loop gets converted in to transmission minimum. This happens again when the Rashba and the Dresselhaus interactions are equal. With a magnetic flux equal to half the flux-quantum, the transmission minimum becomes an exact transmission zero, and hence facilitates a possible experimental measurement.

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