Parton Distribution Functions

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• introduction and overview
• LHC benchmark cross sections: how do the various sets compare?
• issues and outlook

in collaboration with Alan Martin, Robert Thorne and Graeme Watt
1

introduction and overview
parton distribution functions

\[ f_i/A(x, Q^2) \]

- introduced by Feynman (1969) in the *parton model*, to explain Bjorken scaling in deep inelastic scattering data; interpretation as probability distributions

- according to the QCD *factorisation theorem* for inclusive hard scattering processes, universal distributions containing long-distance structure of hadrons; related to parton model distributions at leading order, but with logarithmic scaling violations (DGLAP)

- key ingredients for Tevatron and LHC phenomenology
for example, in Deep Inelastic Scattering

\[
\frac{1}{x} F_{2p}^{l}(x, Q^2) = x \sum_q e_q^2 \int_{x}^{1} \frac{dy}{y} q(y, Q^2) \left\{ \delta(1 - \frac{x}{y}) + \frac{\alpha_s(Q^2)}{2\pi} C_q(x/y) \right\} \\
+ x \sum_q e_q^2 \frac{\alpha_s(Q^2)}{2\pi} \int_{x}^{1} \frac{dy}{y} g(y, Q^2) C_g(x/y) + \mathcal{O}(\alpha_s^2) \\
+ \mathcal{O}(1/Q^2) \text{ (higher twist, mass corrections)}
\]

where the scale dependence of the parton distributions is calculable in QCD perturbation theory

\[
\mu^2 \frac{\partial}{\partial \mu^2} f_i(x, \mu^2) = \frac{\alpha_s(\mu^2)}{2\pi} \sum_j \int_{x}^{1} \frac{dy}{y} f_j(y, \mu^2) P_{ij}(x/y, \alpha_s(\mu^2))
\]

… and \( f_i(x, \mu_0^2) \) determined from

- lattice QCD (in principle)
- fits to data (in practice)
how pdfs are obtained*

• choose a factorisation scheme (e.g. $\overline{\text{MS}}$), an order in perturbation theory (LO, NLO, NNLO) and a ‘starting scale’ $Q_0$ where pQCD applies (e.g. 1-2 GeV)

• parametrise the quark and gluon distributions at $Q_0$, e.g.

$$f_i(x, Q_0^2) = A_i x^{a_i} [1 + b_i \sqrt{x} + c_i x] (1 - x)^{d_i}$$

• solve DGLAP equations to obtain the pdfs at any $x$ and scale $Q > Q_0$; fit data for parameters $\{A_i, a_i, \ldots, \alpha_S\}$

• approximate the exact solutions (e.g. interpolation grids, expansions in polynomials etc) for ease of use; thus the output ‘global fits’ are available ‘off the shelf”, e.g.

```
SUBROUTINE PDF (X, Q, U, UBAR, D, DBAR, ..., BBAR, GLU)
        input | output
```

*traditional method
the pdf industry

• many groups now extracting pdfs from ‘global’ data analyses (MSTW, CTEQ, NNPDF, …)

• broad agreement, but differences due to
  — choice of data sets (including cuts and corrections)
  — treatment of data errors
  — treatment of heavy quarks (s,c,b)
  — order of perturbation theory
  — parameterisation at $Q_0$
  — theoretical assumptions (if any) about:
    • flavour symmetries
    • $x \to 0, 1$ behaviour
    • …
## Examples of Data Sets Used in Fits

| Data set                              | $N_{\text{pts.}}$ |
|---------------------------------------|-------------------|
| H1 MB 99 $e^+p$ NC                   | 8                 |
| H1 MB 97 $e^+p$ NC                   | 64                |
| H1 low $Q^2$ 96–97 $e^+p$ NC          | 80                |
| H1 high $Q^2$ 98–99 $e^-p$ NC         | 126               |
| H1 high $Q^2$ 99–00 $e^+p$ NC         | 147               |
| ZEUS SVX 95 $e^+p$ NC                 | 30                |
| ZEUS 96–97 $e^+p$ NC                  | 144               |
| ZEUS 98–99 $e^-p$ NC                  | 92                |
| ZEUS 99–00 $e^+p$ NC                  | 90                |
| H1 99–00 $e^+p$ CC                    | 28                |
| ZEUS 99–00 $e^+p$ CC                  | 30                |
| H1/ZEUS $e^\pm p$ $F_2^{\text{charm}}$| 83                |
| H1 99–00 $e^+p$ incl. jets            | 30                |
| ZEUS 96–97 $e^+p$ incl. jets          | 30                |
| ZEUS 98–00 $e^\pm p$ incl. jets       | 30                |
| DØ II $p\bar{p}$ incl. jets          | 110               |
| CDF II $p\bar{p}$ incl. jets          | 76                |
| CDF II $W \rightarrow l\nu$ asym.     | 22                |
| DØ II $W \rightarrow l\nu$ asym.     | 10                |
| DØ II $Z$ rap.                        | 28                |
| CDF II $Z$ rap.                       | 29                |

### Data Sets

| Data set                              | $N_{\text{pts.}}$ |
|---------------------------------------|-------------------|
| BCDMS $\mu p$ $F_2$                   | 163               |
| BCDMS $\mu d$ $F_2$                   | 151               |
| NMC $\mu p$ $F_2$                     | 123               |
| NMC $\mu d$ $F_2$                     | 123               |
| NMC $\mu n/\mu p$                    | 148               |
| E665 $\mu p$ $F_2$                    | 53                |
| E665 $\mu d$ $F_2$                    | 53                |
| SLAC $ep$ $F_2$                       | 37                |
| SLAC $ed$ $F_2$                       | 38                |
| NMC/BCDMS/SLAC $F_L$                  | 31                |
| E866/NuSea $pp$ DY                    | 184               |
| E866/NuSea $pd/pp$ DY                 | 15                |
| NuTeV $\nu N$ $F_2$                   | 53                |
| CHORUS $\nu N$ $F_2$                  | 42                |
| NuTeV $\nu N$ $xF_3$                  | 45                |
| CHORUS $\nu N$ $xF_3$                 | 33                |
| CCFR $\nu N \rightarrow \mu \mu X$   | 86                |
| NuTeV $\nu N \rightarrow \mu \mu X$ | 84                |

### All Data Sets

| Data Set | $N_{\text{pts.}}$ |
|----------|--------------------|
| All data sets | 2743 |

*Red font = new wrt MRST2006 fit*
recent global or quasi-global pdf fits

| pdfs   | authors                                                                 | arXiv                          |
|--------|--------------------------------------------------------------------------|--------------------------------|
| ABKM   | S. Alekhin, J. Blümlein, S. Klein, S. Moch, and others                   | 0908.3128, 0908.2766, …         |
| CTEQ   | H.-L. Lai, M. Guzzi, J. Huston, Z. Li, P. Nadolsky, J. Pumplin, C.-P. Yuan, and others | 1007.2241, 1004.4624, 0910.4183, 0904.2424, 0802.0007, … |
| GJR    | M. Glück, P. Jimenez-Delgado, E. Reya, and others                        | 0909.1711, 0810.4274, …         |
| HERAPDF| H1 and ZEUS collaborations                                                | 1006.4471, 0906.1108, …         |
| MSTW   | A.D. Martin, W.J. Stirling, R.S. Thorne, G. Watt                         | 1006.2753, 0905.3531, 0901.0002, … |
| NNPDF  | R. Ball, L. Del Debbio, S. Forte, A. Guffanti, J. Latorre, J. Rojo, M. Ubiali, and others | 1005.0397, 1002.4407, 0912.2276, 0906.1958, … |
|                  | MSTW08 | CTEQ6.6* | NNPDF2.0 | HERAPDF1.0 | ABKM09 | GJR08 |
|------------------|--------|----------|----------|------------|--------|-------|
| HERA DIS         | ✓      | ✓        | ✓*       | ✓*         | ✓      | ✓     |
| F-T DIS          | ✓      | ✓        | ✓        | ✓          | ✓      | ✓     |
| F-T DY           | ✓      | ✓        | ✓        | ✓          | ✓      | ✓     |
| TEV W,Z          | ✓      | ✓+       | ✓        | ✓          | ✓      | ✓     |
| TEV jets         | ✓      | ✓+       | ✓        | ✓          | ✓      | ✓     |
| GM-VFNS          | ✓      | ✓        | ✓        | ✓          | ✓      | ✓     |
| NNLO             | ✓      | ✓        | ✓        | ✓          | ✓      | ✓     |

* Run 1 only
* includes new combined H1-ZEUS data → 1 – 2.5% increase in quarks at low x (depending on procedure), similar effect on $\alpha_S(M_Z^2)$ if free and somewhat less on gluon; more stable at NNLO (MSTW prelim.)

* New (July 2010) CT10 includes new combined H1-ZEUS data + Run 2 jet data + extended gluon parametrisation + … → more like MSTW08
impact of Tevatron jet data on fits

- a distinguishing feature of pdf sets is whether they use (MRST/MSTW, CTEQ, NNPDF, GJR,...) or do not use (HERAPDF, ABKM, ...) Tevatron jet data in the fit: the impact is on the high-x gluon
  
  (Note: Run II data requires slightly softer gluon than Run I data)

- the (still) missing ingredient is the full NNLO pQCD correction to the cross section, but not expected to have much impact in practice [Kidonakis, Owens (2001)]
D0 collaboration: arXiv:1002.4594
LO vs NLO vs NNLO?

in the MSTW2008 fit

\[ \chi^2_{\text{global}} / \text{dof} = \]
- 3066/2598 (LO)
- 2543/2699 (NLO)
- 2480/2615 (NNLO)

LO evolution too slow at small x; NNLO fit marginally better than NLO

**Note:**
- an important ingredient missing in the full NNLO global pdf fit is the NNLO correction to the Tevatron high \( E_T \) jet cross section
- LO can be improved (e.g. LO*) for MCs by adding K-factors, relaxing momentum conservation, etc.
pdf uncertainties

• most groups produce ‘pdfs with errors’

• typically, 20-40 ‘error’ sets based on a ‘best fit’ set to reflect $1\sigma$ variation of all the parameters* $\{A_i, a_i, \ldots, \alpha_S\}$ inherent in the fit

• these reflect the uncertainties on the data used in the global fit (e.g. $\delta F_2 \approx 3\% \rightarrow \delta u \approx 3\%$)

• however, there are also systematic pdf uncertainties reflecting theoretical assumptions/prejudices in the way the global fit is set up and performed (see earlier slide)

* e.g. $f_i(x, Q_0^2) = A_i x^{a_i} [1 + b_i \sqrt{x} + c_i x] (1 - x)^{d_i}$
determination of best fit and uncertainties

- **MSTW08** — 20 eigenvectors. Due to slight incompatibility of different sets (and perhaps to some extent parametrisation inflexibility) use ‘dynamical tolerance’ with inflated $\Delta \chi^2$ of 5 - 20 for eigenvectors

- **CTEQ6.6** — 22 eigenvectors. Inflated $\Delta \chi^2=50$ for 1 sigma for eigenvectors (no normalization uncertainties in CTEQ6.6, cf. CT10)

- **HERAPDF2.0** — 9 eigenvectors, use $\Delta \chi^2=20$. Additional model and parametrisation uncertainties

- **ABKM09** — 21 parton parameters, use $\Delta \chi^2=1$

- **GJR08** — 12 parton parameters. Use $\Delta \chi^2=20$. Impose strong theory (‘dynamical parton’) constraint on input form of pdfs.

**Note:** **NNPDF2.0** create many replicas of data and obtain PDF replicas in each case by fitting to training set and comparing to validation set $\Rightarrow$ uncertainty determined by spread of replicas. Direct relationship to $\chi^2$ in global fit not trivial.
determination of best fit and uncertainties contd.

• **NNPDF** and **MSTW** (due to extra parameters) have more complicated shape for gluon at smaller x and bigger small-x uncertainty

• choice of parametrisation leads to bigger very high-x gluon uncertainty for **CTEQ**

• different theory assumptions in strange quark pdf leads to vastly different uncertainties — **MSTW** small, **NNPDF** large; feeds into other ‘light’ quarks

• perhaps surprisingly all get rather similar uncertainties for pdfs and predicted cross sections — see later
example: MSTW2008(NLO) vs. CTEQ6.6

Note:

CTEQ error bands comparable with MSTW 90\% cl set (different definition of tolerance)

CTEQ light quarks and gluons slightly larger at small $x$ because of imposition of positivity on gluon at $Q_0^2$

CTEQ gluons slightly larger at large $x$ - only Run 1 jet data in fit

→ implications for ‘precision’ LHC cross sections (later)
pdfs and $\alpha_S(M_Z^2)$

- MSTW08, ABKM09 and GJR08: $\alpha_S(M_Z^2)$ values and uncertainty determined by global fit
- NNLO value about $0.003 - 0.004$ lower than NLO value, e.g. for MSTW08
  \[ \alpha_{S,\text{NLO}}^{MS}(M_Z^2) = 0.1202 \pm 0.012 \]
  \[ \alpha_{S,\text{NNLO}}^{MS}(M_Z^2) = 0.1171 \pm 0.014 \]
- CTEQ, NNPDF, HERAPDF choose standard values and uncertainties
- world average (PDG 2009)
  \[ \alpha_{S}^{MS}(M_Z^2) = 0.1184 \pm 0.0007 \]
- note that the pdfs and $\alpha_S$ are correlated!
- e.g. gluon $- \alpha_S$ anticorrelation at small $x$ and quark $- \alpha_S$ anticorrelation at large $x$
\( \alpha_S \) - pdf correlations

- Care needed when assessing impact of varying \( \alpha_S \) on cross sections \( \sim (\alpha_S)^n \)

Higgs \( (M_H = 120 \text{ GeV}) \) with MSTW 2008 NNLO PDFs

MSTW: arXiv:0905.3531
pdf + $\alpha_S$ uncertainties in jet cross sections

Inclusive jet cross sections with MSTW 2008 NLO PDFs

Tevatron, $\sqrt{s} = 1.96$ TeV

0.1 < $y$ < 0.7

Percentage uncertainty (68% C.L.)

$p_T$ (GeV)

fastNLO with $\mu_R = \mu_F = p_T$

$k_T$ algorithm with $D = 0.7$

LHC, $\sqrt{s} = 14$ TeV

0.0 < $y$ < 0.8

Percentage uncertainty (68% C.L.)

$p_T$ (GeV)

PDF only

PDF + $\alpha_S$
LHC benchmark cross sections
precision phenomenology at LHC

- **LO** for generic PS Monte Carlos

- **NLO** for NLO-MCs and many parton-level signal and background processes

- **NNLO** for a limited number of ‘precision observables’ (W, Z, DY, H, …)

  + E/W corrections, resummed HO terms etc…

\[ \delta \sigma_{th} = \delta \sigma_{pdf} \oplus \delta \sigma_{HO} \oplus \delta \sigma_{param} \oplus \ldots \]
parton luminosity functions

- a quick and easy way to assess the mass, collider energy and pdf dependence of production cross sections

\[ \hat{\sigma}_{ab \rightarrow X} = C_X \delta(\hat{s} - M_X^2) \]
\[ \sigma_X = \int_0^1 dx_a dx_b \, f_a(x_a, M_X^2) f_b(x_b, M_X^2) \, C_X \, \delta(x_a x_b - \tau) \]
\[ \equiv C_X \left[ \frac{1}{s} \frac{\partial L_{ab}}{\partial \tau} \right] \quad (\tau = M_X^2/s) \]
\[ \frac{\partial L_{ab}}{\partial \tau} = \int_0^1 dx_a dx_b \, f_a(x_a, M_X^2) f_b(x_b, M_X^2) \, \delta(x_a x_b - \tau) \]

- i.e. all the mass and energy dependence is contained in the \( X \)-independent parton luminosity function in [ ]
- useful combinations are \( ab = gg, \sum_q q\bar{q}, \ldots \)
- and also useful for assessing the uncertainty on cross sections due to uncertainties in the pdfs
more such luminosity plots available at [www.hep.phy.cam.ac.uk/~wjs/plots/plots.html](http://www.hep.phy.cam.ac.uk/~wjs/plots/plots.html)
ratios of parton luminosities at 7, 14 TeV LHC and Tevatron
pdfs at LHC – the issues

• high precision cross section predictions require accurate knowledge of pdfs: \( \delta \sigma_{th} = \delta \sigma_{pdf} + \ldots \)

→ how do the different pdf sets compare?

• can we learn more about pdfs from LHC measurements, e.g.
  - high-\(E_T\) jets → gluon?
  - \(W^+, W^-, Z^0\) → quarks?
  - very forward Drell-Yan (e.g. LHCb) → small \(x\)?
  - …
parton luminosity comparisons

Luminosity and cross section plots from Graeme Watt (MSTW, in preparation), available at projects.hepforge.org/mstwpdf/pdf4lhc

- positivity constraint on input gluon
- Run 1 vs. Run 2 Tevatron jet data
- No Tevatron jet data or FT-DIS data in fit
- ZM-VFNS
- momentum sum rule
restricted parametrisation

no Tevatron jet data in fit

gg luminosity at LHC ($\sqrt{s} = 7$ TeV)

Ratio to MSTW 2008 NLO (68\% C.L.)

- MSTW08
- CTEQ6.6
- ABKM09
- GJR08

$\sqrt{S} / s$
remarkably similar considering the different definitions of pdf uncertainties used by the 3 groups!
NLO and NNLO parton luminosity comparisons

NLO and NNLO parton luminosity comparisons.
W, Z
benchmark W,Z cross sections

NLO $W^\pm \rightarrow l^\mp \nu$ at the LHC ($\sqrt{s} = 7$ TeV)

NLO $W^+/W^-$ ratio at the LHC ($\sqrt{s} = 7$ TeV)

NLO $Z^0 \rightarrow l^+l^-$ at the LHC ($\sqrt{s} = 7$ TeV)

NLO W/Z ratio at the LHC ($\sqrt{s} = 7$ TeV)

differences probably due to sea quark flavour structure
predictions for $\sigma(W,Z) @$ Tevatron, LHC: NLO vs. NNLO
at LHC, ~30% of W and Z total cross sections involves s,c,b quarks

impact of sea quarks on the NLO W charge asymmetry ratio at 7 TeV:

| pdfs                          | R(W^+/W^-) |
|-------------------------------|------------|
| {udg} only                    | 1.53       |
| {udscbg} = MSTW08             | 1.42 ± 0.02|
| {udscbg}_{sea} only           | 0.99       |
| {udscbg}_{sym,sea} only       | 1.00       |
CTEQ6.6 vs. CT10, CT10W (NLO)

* CT10W: attempt to include recent D0 lepton asymmetry data in global fit → slightly different d/u
care needed with definition of ‘total cross section’ in these comparisons

All the results are in agreement with the Standard Model expectations.
using the $W^{\pm}$ charge asymmetry at the LHC

• at the Tevatron $\sigma(W^+) = \sigma(W^-)$, whereas at LHC $\sigma(W^+) \sim (1.4 - 1.3) \sigma(W^-)$

• can use this asymmetry to calibrate backgrounds to new physics, since typically $\sigma_{NP}(X \rightarrow W^+ + \ldots) = \sigma_{NP}(X \rightarrow W^- + \ldots)$

• example:

$$gg \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow W^{\pm}(\rightarrow l^{\pm} + \nu) + 4\text{jets}$$

in this case

$$\sigma_{\text{signal}}(W^+ + 4\text{jets}) = \sigma_{\text{signal}}(W^- + 4\text{jets})$$

whereas...

$$\sigma_{\text{QCD bkgd}}(W^+ + 4\text{jets}) \neq \sigma_{\text{QCD bkgd}}(W^- + 4\text{jets})$$

which can in principle help distinguish signal and background
$R^\pm$ larger at 7 TeV LHC

$R^\pm$ increases with jet $p_T^{\text{min}}$

| $n$ | $\sqrt{s} = 7$ TeV | $\sqrt{s} = 14$ TeV |
|-----|-----------------|-----------------|
| 0   | $1.52 \pm 0.01$ (scl) $\pm 0.02$ (pdf) | $1.31 \pm 0.01$ (scl) $\pm 0.01$ (pdf) |
| 1   | $1.45 \pm 0.01$ (scl) $\pm 0.01$ (pdf) | $1.27 \pm 0.01$ (scl) $\pm 0.01$ (pdf) |
| 2   | $1.56 \pm 0.02$ (scl) $\pm 0.02$ (pdf) | $1.33 \pm 0.02$ (scl) $\pm 0.01$ (pdf) |
| 3   | $1.72 \pm 0.03$ (scl) $\pm 0.03$ (pdf) | $1.45 \pm 0.03$ (scl) $\pm 0.02$ (pdf) |
| 4   | $1.87 \pm 0.04$ (scl) $\pm 0.03$ (pdf) | $1.55 \pm 0.04$ (scl) $\pm 0.02$ (pdf) |
Higgs
• only scale variation uncertainty shown

• central values calculated for a *fixed* set pdfs with a *fixed* value of $\alpha_S(M_Z)$
benchmark Higgs cross sections

\[ \text{NLO } gg \rightarrow H \text{ at the LHC (}\sqrt{s} = 7 \text{ TeV) for } M_H = 120 \text{ GeV} \]

... differences from both pdfs AND \( \alpha_s \)!
NLO $gg \rightarrow H$ at the LHC ($\sqrt{s} = 7$ TeV) for $M_H = 240$ GeV

Vertical error bars
Inner: PDF only
Outer: PDF+$\alpha_S$

68% C.L. PDF
- MSTW08
- CTEQ6.6
- NNPDF2.0
- HERAPDF1.0
- ABKM09
- GJR08
Central predictions use the values of $\alpha_s(M_Z)$ favoured by each PDF group, i.e. 0.1202 for MSTW08, 0.1180 for CTEQ6.6 and 0.1190 for NNPDF2.0. For MSTW08, $\alpha_s(M_Z)$ was determined simultaneously with the PDFs in the global fit. The experimental uncertainties on $\alpha_s(M_Z)$ are $+0.0012/-0.0015$ at 68% C.L The uncertainties on $\alpha_s(M_Z)$ for CTEQ6.6 and NNPDF2.0 are taken to be $\pm0.0012$ at 68% C.L. The combined PDF+$\alpha_s$ uncertainty is calculated following the prescription recommended by each group, i.e. $\alpha_s$ uncertainties are simply added in quadrature for CTEQ6.6, while for NNPDF2.0 the exact prescription is used as explained in arXiv:1004.0962.

How to define an overall ‘best theory prediction’?! See LHC Higgs Cross Section Working Group meeting, 5-6 July, higgs2010.to.infn.it

Note: (i) for MSTW08, uncertainty band similar at NNLO (ii) everything here is at fixed scale $\mu=M_H$!
benchmark top cross sections

NLO $t\bar{t}$ cross sections at the LHC ($\sqrt{s} = 7$ TeV)

68% C.L. PDF
- MSTW08
- CTEQ6.6
- NNPDF2.0
- HERAPDF1.0
- ABKM09
- GJR08

Vertical error bars
Inner: PDF only
Outer: PDF+$\alpha_s$
issues and outlook
issues and outlook

• continuing convergence between the various pdf sets

• outstanding issues include:
  — inclusion of combined HERA data (not yet in all fits)
  — difficulty of reconciling Run II Tevatron W asymmetry data
  — proper assessment of uncertainties due to treatment of heavy quark flavours (GM-VFNS optimal but not uniquely defined)
  — beyond NNLO? e.g. influence of $[\alpha_S \ln(1/x)]^n$ contributions
  — ‘QED pdfs’ (MSTW in preparation, cf. MRST 2004)

• much discussion (e.g. PDF4LHC workshops) among the pdf groups about how to define a ‘overall best’ theory prediction and uncertainty (be careful with ‘averaging’ and ‘envelopes’!)

• eagerly awaiting precision cross sections at 7 TeV!
Lepton asymmetry and CDF data

Recent progress in NNLO QCD calculations

Massimiliano Grazzini (INFN, Firenze)

HO10 CERN Theory Institute, 30 June 2010
Lepton asymmetry and new DØ data

Recent progress in NNLO QCD calculations

Massimiliano Grazzini (INFN, Firenze)

HO10 CERN Theory Institute, 30 June 2010
The Ring of Brodgar, Orkney Islands, Scotland
extra slides
... the same at 90% cl
gg luminosity at LHC ($\sqrt{s} = 7$ TeV)
... the same at 90\% cl
heavy quarks: charm, bottom, …

considered sufficiently massive to allow pQCD treatment: \( g \rightarrow Q\overline{Q} \)

distinguish two regimes:
(i) \( Q^2 \sim m_H^2 \) include full \( m_H \) dependence to get correct threshold behaviour
(ii) \( Q^2 \gg m_H^2 \) treat as ~massless partons to resum \( \alpha_S^{n\log^n(Q^2/m_H^2)} \) via DGLAP

**FFNS:** OK for (i) only  \hspace{1cm} **ZM-VFNS:** OK for (ii) only

consistent **GM (=general mass)-VFNS** now available (e.g. ACOT(\( \chi \)), RT, BMSN,…) which interpolates smoothly between the two regimes

Aivazis, Collins, Olness, Tung; Roberts, Thorne; Buza, Matiounine, Smith, Migneron, van Neerven, …

**Note:**
(i) the definition of these is tricky and non-unique (ambiguity in assignment of \( O(m_H^2/Q^2) \) contributions), and the implementation of improved treatment (e.g. in going from MRST2004→MRST2006 or CTEQ 6.1→6.5) can have a big effect on light partons
(ii) the **true** uncertainty on e.g. LHC predictions coming from ambiguities in the heavy quark treatment has yet to be quantified
charm and bottom structure functions

• MSTW 2008 uses *fixed* values of $m_c = 1.4$ GeV and $m_b = 4.75$ GeV in a GM-VFNS

• currently studying the sensitivity of the fit to these values, and impact on LHC cross sections
extrapolation uncertainties

theoretical insight for $x \to 0$:

$f \sim A x$

$f \sim A x^\delta$, $A > 0$

$f \sim A x^\delta$

no theoretical insight:

$f \sim ???$

…with only sum rules providing a constraint

Examples:

(i) the MSTW negative small-$x$ gluon at $Q_0$

(ii) the NNPDF ‘parameter free’ pdfs at small and large $x$
summary of DIS data

Note: must impose cuts on DIS data to ensure validity of leading-twist DGLAP formalism in analyses to determine pdfs, typically:

\[ Q^2 > 2 - 4 \text{ GeV}^2 \]

\[ W^2 = \frac{(1-x)}{x} Q^2 > 10 - 15 \text{ GeV}^2 \]
H1 95-97 incl. jet and dijet data, $\chi^2 = 13/32$ pts.

MSTW NLO PDF fit (preliminary. 17/10/2007)

- only in NLO fit (no NNLO correction yet)
improved LO pdfs

• conventional wisdom is to match pQCD order of pdfs with that of MEs

• but, in practice,
  — $\sigma_{\text{LO}} = \text{PDFs}(\text{LO}) \otimes \text{ME}(\text{LO})$ can be different from $\sigma_{\text{NLO}} = \text{PDFs}(\text{NLO}) \otimes \text{ME}(\text{NLO})$, in both shape and normalisation
  — LO pdfs have very poor $\chi^2$ in (LO) global fit (no surprise: NLO corrections at large and small $x$ are significant and preferred by the data)

• momentum conservation limits how much additional glue can be added to LO partons to compensate for missing NLO pQCD corrections (e.g. to get correct evolution rate of small-$x$ quarks)

• therefore relax momentum conservation and redo LO fit; study the impact of this on $\chi^2$, partons and cross sections

• e.g. Thorne & Shertsnev 2007: LO* partons
  — $\chi^2$: 3066/2235 $\rightarrow$ 2691/2235, momentum conservation: 100% $\rightarrow$ 113%
transverse momentum distribution in H → τ τ production at LHC

comparison of gluons at high Q^2
pdf uncertainty on $d\sigma(DY)/dM_{dy}$
$d\sigma(W^+)/dy$, $d\sigma(W^-)/dy$, $d\sigma(Z)/dy$

at LHC using MSTW2008NLO (68%cl)

$M = 8$ GeV

16 GeV

24 GeV

pdf uncertainty on $R_{WZ} = d\sigma(W)/d\sigma(Z)$, $R_{W-} = d\sigma(W^+)/d\sigma(W^-)$

$A_{ll} = (d\sigma(W^+)-d\sigma(W^-))/(d\sigma(W^+)+d\sigma(W^-))$

at LHC using MSTW2008NLO (68%cl)
pdf uncertainty on $\sigma(gg \to H)$

$\delta \sigma / \sigma (\%)$

$M_H \text{ (GeV)}$

$y_H$

→ typically 2-3% pdf uncertainty, except near edges of phase space
comparison of gluons extracted from LO, NLO, NNLO global fits

- large positive $P_{qg}$ contributions at small $x$ lead to smaller gluons at higher order

- clear instability at small $x, Q^2$, and this is reflected in predictions for $F_L$ (see later)
**sea quarks**

• the sea presumably arises when ‘primordial‘ valence quarks emit gluons which in turn split into quark-antiquark pairs, with suppressed splitting into heavier quark pairs

• so we naively expect

\[ u \approx d > s > c > \ldots \]

• but why such a big d-u asymmetry? Meson cloud, Pauli exclusion, …?

The ratio of Drell-Yan cross sections for \( pp, pn \rightarrow \mu^+\mu^- + X \) provides a measure of the difference between the \( u \) and \( d \) sea quark distributions

\[ \frac{\bar{d}(x, Q^2) - \bar{u}(x, Q^2)}{\bar{d}(x, Q^2) + \bar{u}(x, Q^2)} \]

antidown - antiup at \( Q^2 = 10 \text{ GeV}^2 \)

MSTW 2008 NLO (68% C.L.)
earliest pdf fits had SU(3) symmetry: \( s(x, Q_0^2) = \bar{s}(x, Q_0^2) = u(x, Q_0^2) = d(x, Q_0^2) \)

later relaxed to include (constant) strange suppression (cf. fragmentation):

\[
s(x, Q_0^2) = \bar{s}(x, Q_0^2) = \frac{\kappa}{2} \left[ u(x, Q_0^2) + d(x, Q_0^2) \right]
\]

with \( \kappa = 0.4 - 0.5 \)

nowadays, dimuon production in \( \nu N \) DIS (CCFR, NuTeV) allows ‘direct’ determination:

\[
\frac{d\sigma}{dxdy} (\nu_\mu (\bar{\nu}_\mu) N \rightarrow \mu^+ \mu^- X) = B_c N A \frac{d\sigma}{dxdy} (\nu_\mu \bar{s} (\bar{\nu}_\mu \bar{s}) \rightarrow c\mu^- (\bar{c}\mu^+) X)
\]

in the range \( 0.01 < x < 0.4 \)

data seem to slightly prefer \( s(x, Q_0^2) - \bar{s}(x, Q_0^2) \neq 0 \)

theoretical explanation?!
\[ \text{NuTeV} \frac{100\pi}{G_F M_N E_\nu} \frac{d\sigma}{dy}(\nu_{\mu} N \rightarrow \mu^+ \mu^- X) \text{ in GeV}^2 \]

MSTW 2008 NNLO PDF fit, \( \chi^2 = 13 \) for 21 DOF

- \( E_\nu = 88.29 \text{ GeV} \)
  - \( y = 0.324 \)

- \( E_\nu = 88.29 \text{ GeV} \)
  - \( y = 0.558 \)

- \( E_\nu = 88.29 \text{ GeV} \)
  - \( y = 0.771 \)

- \( E_\nu = 174.29 \text{ GeV} \)
  - \( y = 0.324 \)

- \( E_\nu = 174.29 \text{ GeV} \)
  - \( y = 0.558 \)

- \( E_\nu = 174.29 \text{ GeV} \)
  - \( y = 0.771 \)

- \( E_\nu = 247.00 \text{ GeV} \)
  - \( y = 0.324 \)

- \( E_\nu = 247.00 \text{ GeV} \)
  - \( y = 0.558 \)

- \( E_\nu = 247.00 \text{ GeV} \)
  - \( y = 0.771 \)
\[ \kappa(x, Q_0^2) = \frac{s(x, Q_0^2) + \bar{s}(x, Q_0^2)}{u(x, Q_0^2) + d(x, Q_0^2)} \]

\[ \int_0^1 dx \left[ x s(x, Q_0^2) / \int_0^1 dx \left[ x u(x, Q_0^2) + x d(x, Q_0^2) \right] \right] = 0.36 \pm 0.03 \]

\[ s_v \equiv s - \bar{s} \]

\[ \int_0^1 dx s_v(x, Q_0^2) = 0.0023 \pm 0.0015 \]

\[ x_0 = 0.0186 \]
strange quark in NNPDF

Note:

**MSTW:** assume u, d, s quarks have same $x^\delta$ behaviour as $x \to 0$

**NuTeV** $\sin^2 \theta_W$ anomaly largely removed
NLO

| MSTW (this work) | $\alpha_S(M_Z^2)$ (expt. unc. only) |
|------------------|------------------------------------|
| MSTW (this work) | 0.1202 $^{+0.0012}_{-0.0015}$     |
| CTEQ [2]         | 0.1170 $^{ \pm 0.0047}$            |
| H1 [23]          | 0.1150 $^{ \pm 0.0017}$            |
| ZEUS [48]        | 0.1183 $^{ \pm 0.0028}$            |
| Alekhin [57]     | 0.1171 $^{ \pm 0.0015}$            |
| BBG [58]         | 0.1148 $^{ \pm 0.0019}$            |
| GJR [59]         | 0.1145 $^{ \pm 0.0018}$            |

NNLO

| MSTW (this work) | $\alpha_S(M_Z^2)$ (expt. unc. only) |
|------------------|------------------------------------|
| MSTW (this work) | 0.1171 $^{+0.0014}_{-0.0014}$     |
| AMP [60]         | 0.1128 $^{ \pm 0.0015}$            |
| BBG [58]         | 0.1134 $^{+0.0019}_{-0.0021}$     |
| ABKM [61]        | 0.1129 $^{ \pm 0.0014}$            |
| JR [62]          | 0.1158 $^{ \pm 0.0035}$            |

- reasonable consistency between different analyses
- MSTW values slightly higher because of smaller low-x gluon needed for high-$p_T$ Tevatron jet fit

MSTW (2009):
full global NLO and NNLO fit

CTEQ (2008):
full global NLO fit

H1 (2001):
H1 + BCDMS

ZEUS (2005):
ZEUS inc. DIS-JET + photoprodn.

BBG = Blumlein, Bottcher, Guffanti (2006):
non-singlet DIS analysis

AMP = Alekhin, Melnikov, Petriello (2006):
DIS + DY

GJR = Gluck, Jimenez-Delgado, Reya (2008):
DIS + DY + Tevatron jet

JR = Jimenez-Delgado, Reya (2009):
DIS + DY

ABKM = Alekhin, Blumlein, Klein, Moch (2009):
DIS + DY

PDG(2008): $\alpha_S(M_Z^2) = 0.1176 \pm 0.002$
CT09G fit to Run I and Run II jet data simultaneously, find much harder gluon (with more flexible parameterisation)

… harder than valence quarks?!
• an independent measurement of the small-\(x\) gluon

• a test of the assumptions in the DGLAP LT pQCD analysis of small-\(x\) \(F_2\)

• higher-order \(\ln(1/x)\) and higher-twist contributions could be important
$F_L(x, Q^2)$

- $H1$ (Prelim.)
- $E_p = 460, 575, 920$ GeV

- Red: MSTW NLO
- Blue: MSTW NNLO
- Magenta: WT NLO + NLL(1/x)
MSTW2008 NLO
LHC (14 TeV)
LHCb

Unique features

• pseudo-rapidity range 1.9 - 4.9
  — 1.9 - 2.5 complementary to ATLAS/CMS
  — > 2.5 unique to LHCb

• beam defocused at LHCb: 1 year of running = 2 fb⁻¹

• trigger on low momentum muons: \( p > 8 \text{ GeV}, p_T > 1 \text{ GeV} \)

  access to unique range of \((x,Q^2)\)
→ detect forward, low $p_T$ muons from $q \bar{q} \rightarrow \mu^+ \mu^-$