Dynamics of Heavy-Ion Collisions at Fermi Energies: Challenges and Opportunities

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ABSTRACT

The dynamics of heavy-ion reactions at Fermi energies is dominated by a dissipative mechanism modified by the concurrent emission of non-statistical nucleons, light particles, and nuclear clusters. Experimental observables are available to monitor the relaxation processes driving the evolution of an interacting nuclear system towards equilibrium. Isospin degrees of freedom provide interesting new access to fundamental information on the reaction mechanism and the effective in-medium nucleonic interactions.

1. Introduction

It is well known that heavy-ion reactions at bombarding energies near the interaction barrier are “fueled” by the stochastic exchange of nucleons between the reaction partners [Sch84]. In such dissipative collisions, the interacting projectile and target nuclei exchange independent nucleons, giving rise to one-body transport phenomena occurring on a time scale which is fast compared to that on which the collective degrees of freedom change (separation of time scales). At near-barrier energies, the concept of an underlying stochastic nucleon-exchange mechanism explains qualitatively multi-dimensional experimental correlations between the net loss in kinetic energy of the massive reaction partners, their individual intrinsic (“thermal”) excitation energies, their gains and losses in mass, charge, and spin. As illustrated by numerous examples at low energies [Sch84], the progress of a heavy-ion reaction can be mapped in terms of a number of experimental indicators (“clocks”, “thermometers”, “impact-parameter filters”). These indicators include the average fragment kinetic-energy loss, the average multiplicity of neutrons or of light particles evaporated sequentially from the hot primary projectile-like (PLF) and target-like (TLF) reaction fragments, the mass-to-charge (A/Z) asymmetry of PLF and TLF, among others. Experimental results [Sch84, Pla88] have clearly demonstrated that in general equilibrium is not reached in dissipative heavy-ion collisions at energies close to the barrier, except possibly for the most central collisions and the longest interaction times, or when the system is initially already close to such equilibrium. It seems unlikely that such equilibrium is reached for higher bombarding energies.

Heavy-ion reactions at Fermi bombarding energies [$\epsilon = E/A \approx (20 - 100) \text{ MeV}$] are expected to be much more complex than those near the barrier. Of practical
concern is that the energy available in the entrance channel equals several times what is required for the complete disassembly of a system into its constituents, opening up an enormous number of reaction and decay channels. Under such conditions, it is experimentally difficult to reconstruct the interesting primary reaction events (prior to evaporative decay). On a conceptual level, at Fermi energies, there is no separation of the respective time scales for intrinsic and collective degrees of freedom. Macroscopic motion occurs on a time scale comparable to estimated internal relaxation times ($t_{rel} \sim 10^{-22} \text{s} \approx 40 \text{ fm/c}$). This concurrence presents difficulties for the interpretation of reaction phenomena, for example, in terms of effective reaction products and the forces acting on their collective motion, or an interpretation in terms of statistical thermodynamics.

On the other hand, and precisely because the time scales for intrinsic nucleonic motion and collective variables merge at Fermi energies, the mean field cannot adjust adiabatically to the rapidly approaching or receding projectile and target nuclei. This situation may lead to sudden compression or dilution of nuclear matter, resisted by the density-and isospin-dependent in-medium interactions. In this fashion, diabatic nucleus-nucleus interactions arise, which can influence heavy-ion collision dynamics in a measurable way, and differently for neutrons and protons. Here, the reaction dynamics become dependent on the projectile-target A/Z asymmetry. There are iso-scalar and iso-vector parts of the nucleus-nucleus liquid-drop and proximity interactions [Pom80] implying different effects for different projectile-target A/Z asymmetries [Sob95, Col98]. Another, related consequence of these isospin-dependent effects is that the properties of the reaction products, e.g., their differential branching ratios for different A/Z ratios, emission angles, and energies, depend on the dynamics of a collision. This behavior of interacting nuclei establishes a connection between the isospin-dependent nuclear equation of state and observable reaction dynamics and opens up an interesting new access to a fundamental property of nuclear matter.

In the following, first examples of experimental data showing a dependence of dynamical reaction variables on projectile and target “isospin” will be illustrated and discussed, mostly in terms of the qualitative physics. Here and in the following, the term “isospin” is used in its colloquial meaning, referring to the third component of the corresponding isospin, or better, the A/Z or N/Z ratios, making no claim as to a conservation of isospin quantum numbers in heavy-ion reactions. Available experimental experience demonstrates that the reaction environment is more complicated than often assumed and that a host of simultaneously measured experimental parameters and sophisticated analytical procedures are needed to deduce a meaningful reaction scenario. Therefore, first the need for a consistent view of the reaction environment at bombarding energies in the Fermi energy domain is explained in Section II, followed in Section III by a discussion of newly observed isospin-dependent trends and some of the concepts useful for their interpretation. In particular, model predictions for the relevant isospin-dependent dynamics of heavy-ion reactions are confronted with experimental data.
2. Empirical Reaction Environment at Fermi Energies

2.1. General Reaction Dynamics

As has been stated in the Introduction, heavy-ion reactions at bombarding energies near the interaction barrier are driven by the stochastic exchange of nucleons between the reaction partners [Sch84]. In such dissipative collisions, projectile and target nuclei are believed to develop a matter bridge between them, a “neck,” and to form a transient di-nuclear system. Through this neck interface, the interacting fragments exchange independent nucleons (single particles), giving rise to one-body transport phenomena occurring on a time scale which is fast compared to that on which the collective degrees of freedom change (separation of time scales). This exchange process dissipates kinetic energy of relative motion, as the dinuclear complex rotates (“orbits”) about its center of gravity. The dinuclear system is mechanically unstable and holds together only temporarily by the action of adhesive liquid-drop and proximity forces, which are counterbalanced by repulsive Coulomb and centrifugal forces. For all but light reaction systems and a narrow range of impact parameters associated with central collisions, the repulsive forces are sufficiently strong to drive the di-nuclear system apart again, leading to projectile-like and target-like fragments, PLF and TLF, respectively. The primary fragments are typically highly excited and decay subsequently by evaporation or fission-like processes. Both, approach and re-separation are retarded by one-body frictional forces mediated by nucleon exchange. The experimentally observed reaction features are well reproduced, often quantitatively, by semi-classical nucleon-exchange models (NEM) developed by Randrup and collaborators [Ran76-82] and others [Bor90, Fel85], even though some of the simplifying assumptions made in these models appear difficult to justify.

In any case, at near-barrier energies, the concept of an underlying nucleon-exchange mechanism explains qualitatively multi-dimensional experimental correlations between the net loss in kinetic energy of relative motion of the massive reaction partners, their individual intrinsic (“thermal”) excitation energies, their gains and losses in mass, charge, and spin. Based on such reaction models, it is plausible that the energy loss is stochastically (anti-) correlated with impact parameter and associated interaction time. This means that all variables are characterized by stochastic probability functions, as they are determined by a stochastic nucleon exchange mechanism. Therefore, as demonstrated by numerous examples [Sch84], the progress of a heavy-ion reaction at low energies can be followed in terms of experimental indicators such as the average fragment kinetic-energy loss, the average multiplicity of evaporated light particles, and the mass-to-charge (A/Z) asymmetry of reaction products.

As already stated, heavy-ion reactions at Fermi bombarding energies are much more difficult to analyze and to understand than those at near-barrier energies. For example, at these energies it is conceivable that upon impact the system breaks up directly into many light particles and clusters. Even if such catastrophic events are
unlikely to occur, massive remnants of the primary PLFs and TLFs, or their collective energies, may not be identifiable any longer. It is then not obvious how to reconstruct important reaction parameters such as the amount of initial kinetic energy dissipated into intrinsic heat. On a conceptual level, at Fermi energies there is no separation of time scales between the motion along intrinsic and collective degrees of freedom. This concurrence presents difficulties for the interpretation of reaction phenomena in terms of effective system trajectories, reaction products, and forces acting on the collective motion, or even in terms of an interpretation in terms of statistical thermodynamics. In fact, classical concepts such as that of an average system trajectory may not be applicable any longer.

A meaningful experimental access to heavy-ion reactions at Fermi energies has been gained through so-called $4\pi$ experiments. In such measurements, all important reaction observables are sampled with a broad dynamic range and throughout the entire solid angle (in the corresponding center-of-mass reference frame). Typically, such coverage is achieved for light and intermediate-mass charged reaction products. Experimental detection includes less frequently also the emitted neutrons, and the massive PLFs, TLFs, and other heavy residues (HR), in addition to the lighter charged products. So far in a few cases only has it been possible to reconstruct primary reaction fragments. Approximately, this task was accomplished in the reaction $^{209}$Bi+$^{136}$Xe both at bombarding energies per nucleon of $\epsilon = E/A = 28$ [Bal93, 95, Lot93, Tok94-96] and 55 MeV [Sku97, 98]. Much of the overall reaction dynamics prevailing at Fermi energies can be gleaned from these two reactions. Other heavy and intermediate reaction systems such as $^{197}$Au+$^{86}$Kr [Sku96, Dje96] and $^{112}$Sn+$^{40,48}$Ca [Agn97, 98] at $\epsilon = 35$ MeV provide consistent and supplementary information.

In Fig. 1, experimental results are displayed in form of two-dimensional contour diagrams, for the angle-energy correlations (Wilczyński Plots) for the above heavy-ion reactions and the reaction $^{112}$Sn+$^{40}$Ca [Agn97, 98], all measured at Fermi-type bombarding energies. All but the latter reaction have been measured with $4\pi$ coverage. In this figure, the observed yield of PLFs is plotted vs. laboratory kinetic energy and angle. For all reactions, one observes consistent trends in the yields, which in each case form a ridge of elastically scattered projectiles at high (elastic-type) energies, extending up to the respective grazing angle [Sch84]. Starting from the grazing angle, the correlation ridge of reaction events indicates PLF emission at successively smaller (more forward) angles, as energy is dissipated. Eventually, the forward-going ridge escapes the angular acceptance of the detection system. However, the picture appears consistent with the trend continuing, until PLFs are emitted on the other (“negative”) side of the beam. For large enough angles, PLFs emitted at negative angles are again accepted by the detector, leading to the appearance of a backward-going ridge at lower energies, the reflection of the negative-angle ridge. In the plots included in Fig. 1, no correction has been made to the data for the decay of the primary PLFs. This is the main reason for the lack of definition of the yield ridges. Considering this circumstance, these data are reminiscent of the process of
“dissipative orbiting” of a dinuclear complex formed in low-energy heavy-ion reactions. The curves superimposed on the data in Fig. 1 represent calculations with the nucleon exchange model (NEM) mentioned earlier. The calculations shown have been corrected on average for the number of particles evaporated sequentially from the primary hot PLFs. The resulting agreement between data and calculations, while not quantitatively excellent, nevertheless supports a generally dissipative character of the reactions studied in the Fermi energy domain.

From this type of dissipative reaction, in a first step, two massive reaction fragments emerge, which preserve memory of projectile and target nucleus, in several respects. This feature is recovered for the two $^{209}$Bi+$^{136}$Xe reactions, for which a partial reconstruction of the primary PLF has been possible, adding the average of the combined charge of light charged particles evaporated from the primary PLF to the measured charge of the PLF evaporation residue. Figure 2 displays the so-called diffusion plot for the reaction $^{209}$Bi+$^{136}$Xe at $\epsilon = 28$ MeV. Here, the angle-integrated yield of reconstructed primary PLFs is plotted vs. the PLF atomic number ($Z$) and multiplicity ($m_n$) of evaporated neutrons. The multiplicity of neutrons is a measure of the dissipated kinetic energy. In this plot, the ordinate reflects essentially the number of neutrons evaporated from the PLF, measuring the internal excitation energy of the primary PLF. One observes that, for small and intermediate amounts of dissipated energy, the yield is centered at the atomic number ($Z = 54$) of the projectile. The distribution broadens considerably, as energy is dissipated into heat of the reaction products. The degree of broadening is quantitatively consistent with expectations based on the NEM, given the sizeable uncertainties inherent in the average reconstruction procedure. A discrepancy definitively exists for the higher neutron multiplicities, where an average shift of the PLF Z distribution to smaller atomic numbers is observed. This effect is understood to be due to the emission of intermediate-mass fragments (IMF) from the intermediate dinuclear system, which were not assignable to either PLF or TLF, as explained below. It is not due to a diffusive “drift” of the Z-distribution reflecting the underlying potential [Sku98]

The particles evaporated from PLF or TLF have been identified based on their characteristic emission patterns. Examples of such patterns are displayed in Fig. 3, showing the Galilei-invariant plots of the differential cross section for protons, tritons, Li, and C fragments, emitted in the reaction $^{209}$Bi+$^{136}$Xe for peripheral collisions with energy losses around $E_{loss} = 0.3$ GeV. The yield is presented as a two-dimensional contour diagram of the yield plotted vs. the particle velocities parallel and perpendicular to the beam. The origin represents the c.m. system. The geometry consistently seen in the invariant cross section diagrams, for the light charged particles (p,d,t, and $\alpha$-particles) and essentially all energy losses, is that of two intersecting semi-circles, one centered at the velocity of the PLF (arrows on right) and one at the velocity of the TLF reaction partner (arrows on left). The radius of each semi-circle corresponds to the most probable (Coulomb plus thermal) energy of the emitted particle. Each semi-circle is centered at the velocity of the emitter. These “Coulomb circles” are
indicative of an evaporation of light charged particles (LCP) from primary hot PLF and TLF, occurring after significant acceleration in the mutual Coulomb field has taken place already. Owing to these characteristic emission patterns, the origin of the evaporated particles can be assessed, not event by event but at least on average. The emission patterns, shown in the two bottom panels of Fig.3, for intermediate-mass (IMF) reaction products Li and C are distinctly different from those of the LCPs. They exhibit no Coulomb circles but are produced on average at rest in the cm system.

2.2. Non-Statistical Emission of Particles and Clusters

The data in the above three figures (Figs. 1 to 3) demonstrate the dominantly dissipative character of the reaction mechanism. However, in addition to the similarities between heavy-ion reaction mechanisms at low and Fermi energies, there are also important differences. As was noted in an early experiment [Que93] on the system $^{197}$Au+$^{208}$Pb at $\epsilon = 29$ MeV, measuring neutrons along with LCPs, there is a limit to the amount of entrance-channel kinetic energy which can be dissipated, converted into intrinsic heat, and finally emitted again in form of nucleons and light particles. For heavy systems and at bombarding energies around $\epsilon = 30$ MeV, typically only (50-60)% of the available energy is converted into heat. In particular in central collisions, the remaining energy can be consumed in the emission of energetic non-statistical light particles and IMFs.

Non-statistical particle emission has been studied extensively for fusion-type reactions [Hil88]. As sample illustration of non-statistical particle emission, experimental neutron energy spectra observed [Agn97, 98] for semi-peripheral $^{112}$Sn+$^{48}$Ca collisions at $\epsilon = 35$ MeV and three selected (out of 12 available) angles are depicted in Fig. 4, together with theoretical fits. These spectra are very similar to those measured [Que93], both for the heavy system $^{197}$Au+$^{208}$Pb reaction at 29 MeV and for light system $^{12}$C+$^{35}$Cl at $\epsilon = 43$ MeV [Lar 98]. In order to understand the data in either of these cases, a minimum of three separate moving neutron emitters has to be assumed. The spectra at backward angles (e.g., at $\Theta_{lab} = 104^\circ$ for $^{112}$Sn+$^{48}$Ca) are dominated by evaporation from a target-like reaction fragment, i.e., have a Maxwell-Boltzmann ("moving source") form, in the laboratory given by

$$\frac{dm}{d\Omega dE} \propto \sqrt{Ee^{\frac{E-2\sqrt{\epsilon}}{T}}}$$

for temperature T. Here, $\epsilon$ is the emitter energy per nucleon and $\Theta$ the particle detection angle. Transformed to forward angles (e.g., $\Theta_{lab} = 15^\circ$), this component is found in the data at very low laboratory energies. The remainder of this forward-angle spectrum is due to evaporation from a fast-moving PLF. The spectrum is well represented already by the two evaporative (PLF and TLF) components. However, at observation angles sideways to the beam, e.g., at $\Theta_{lab} = 55^\circ$ in Fig. 4, these two evaporation components are insufficient to explain the exponential high-energy tail of the neutron spectrum. This region is kinematically essentially inaccessible to
evaporation from PLF or TLF, and the neutron spectrum is much harder than the evaporation spectra observed for the same PLF/TLF events both at more forward or more backward angles. Clearly, a third, non-statistical neutron component is needed. The energy-angle emission pattern (12 angles) of this non-statistical component can be described well by a hypothetical third source, moving with about half the beam velocity and emitting neutrons isotropically in its own rest frame. This frame of reference is approximately identical with the NN cm frame. As suggested by fusion-like reaction studies, the corresponding non-statistical neutron energy spectrum is very similar to a Maxwell-Boltzmann distribution (cf. Equ. 1), in the rest frame of the emitter given by

\[ \frac{dN}{dE} \propto \sqrt{E}e^{E_0} \]

even though the emission is non-thermal. For the case shown in Fig. 4, the inverse exponential slope parameter is large, of the order of \( E_0 \approx 10 \text{ MeV} \). In comparison, the evaporation spectra are characteristic of a nuclear temperature of only \( T = 3.3 \text{ MeV} \) in the respective emitter frames. This discrepancy is due to the fact that non-statistical emission is initiated in collisions between a few nucleons only, much before thermalization, i.e., distribution of the kinetic energy over all degrees of freedom, is completed.

The obvious, two-component nature of the neutron spectra exhibited in Fig. 4 for neutrons emitted in peripheral collisions disappears for higher bombarding energies and/or more central collisions. Here the evaporation temperatures are not so different anymore from the inverse logarithmic slopes \( E_0 \) characterizing the spectra of non-statistical particles. Similar observations are made for the spectra of protons measured for reactions such as \( ^{112}\text{Sn}+^{40,48}\text{Ca} \). Because of the Coulomb barrier for emission, the proton energy spectra exhibit a higher effective energy threshold than that for neutrons, thus suppressing an important part of the proton evaporative spectra. On the other hand, because of their shift to high energies, non-statistical proton spectra are not significantly affected by Coulomb barriers. Non-statistical nucleon emission is weak for peripheral collisions, but its multiplicity increases with decreasing impact parameter. To put the importance of non-statistical nucleon emission in perspective, for \( ^{112}\text{Sn}+^{40,48}\text{Ca} \), this non-equilibrium process may consume up to 25% of the available energy.

IMF emission presents another modification of the dissipative reaction mechanism. Like emission of non-statistical light particles, this process occurs infrequently in peripheral collisions but becomes important for collisions that are more central. Although, the IMF data in the lower two panels of Fig. 3, taken by themselves, suggest a single source emitting at intermediate velocity, there is no doubt as to the presence of two massive fragments (PLF and TLF) from the same events. These projectile and target remnants survive violent central nuclear collision. Apparently, even at Fermi energies, nucleon exchanges between the reaction partners, as well as the frictional
effects caused by this transport process, remain important. This behavior appears to be universal for the Fermi energy domain. For example, in the reaction $^{197}$Au+$^{86}$Kr at 35 MeV, correlated PLF/TLF pairs, as well as TLF evaporation residues, have been measured [Sku96, Dje96] for energy losses corresponding to a range of peripheral to central collisions. Even for lighter reaction systems and higher energies, such as $^{12}$C+$^{35}$Cl at $\epsilon = 43$ MeV, and for events in which the entire system has vaporized, event reconstruction reveals [Lar98] that PLF and TLF survive the first dissipative reaction step. The general conclusions on the reaction mechanism, drawn already from studies of the reactions $^{197}$Au+$^{208}$Pb [Sch92, Que93] and $^{209}$Bi+$^{136}$Xe [Lot93, Sch92, 94, Bal95] have now been confirmed up to relatively high energies. Recently, the reaction $^{58}$Ni+$^{36}$Ar has been studied [Bor99] at $\epsilon = 95$ MeV, reporting the reconstruction properties of vaporized PLFs, i.e., for very high excitation energies or energy losses. Uncertainties as to the competition between dissipative and fusion-like mechanisms persist only on the level of the innermost 1% of the impact parameter range.

Experimentally, it is not yet clear at what reaction stage intermediate-mass fragments are emitted, e.g., whether at the beginning or at the end of the dissipative interaction between projectile-like and target-like fragments. Certainly, because of substantial masses and energies, IMF emission influences significantly the balance of mass, charge, and energy available to the rest of the reaction products. As an example, a reduction of the average PLF atomic number by IMF emission is seen in Fig. 3 for large neutron multiplicities, i.e., for central collisions and several IMFs. Properties of the IMFs emitted in the reaction $^{209}$Bi+$^{136}$Xe at $\epsilon = 28$ MeV are shown in Fig. 5. Here, the atomic-number ($Z$) distributions for IMFs emitted in the reaction $^{209}$Bi+$^{136}$Xe at $\epsilon = 28$ MeV are plotted on the left, for various numbers of IMFs measured in coincidence, i.e., different IMF multiplicities ($m_{IMF}$). These $Z$ distributions have a generally exponential character. It is an unexpected fact that the shapes of the $Z$ distributions are very similar and independent of the IMF multiplicity. The average charge of an IMF is $<Z_{IMF}> \approx 8$, corresponding to oxygen fragments. On the r.h.s. of this figure, the corresponding transverse-energy spectra are shown for the same events. These latter spectra look again very much like thermal Maxwell-Boltzmann distributions (see Equ. 2), albeit with very large inverse slope parameters ($E_o \approx 30$ MeV) and average IMF energies of $<E_{IMF}> \approx 60$ MeV. For the same events, the statistical neutron and LCP spectra show temperature parameters of only a few MeV ($T \leq 5.5$ MeV).

The IMF energy spectra are also seen to be independent of the IMF multiplicity. The resulting reducibility of the total kinetic energy spectrum of several IMFs, in terms of a fundamental single-particle energy spectrum, has given rise to a statistical interpretation [Mor97] of IMF emission from a nuclear system in equilibrium with an infinite heat bath at temperature $T$. While this schematic approach can account for some of the combinatorial aspects of multi-IMF emission, the model is inconsistent with fundamental properties of the reaction mechanism in general and with the IMF
emission process in particular. For an equilibrated (micro-canonical) nuclear source, emission of IMFs would compete with that of other particles, such as neutrons, protons, and other LCPs, dividing the same fixed total excitation energy according to the available numbers of microstates. The joint neutron/LCP multiplicity distributions $P(m_n, m_{LCP})$ plotted in Fig. 6 for different values of $m_{IMF}$ demonstrate that such statistical competition does in fact not exist. Apart from the distributions for a transitional region ($0 \leq m_{IMF} \leq 2$), the distributions are relatively well defined, with only small fluctuations, and approximately independent of $m_{IMF}$. This implies that the thermal energy of the system “saturates” at energies per nucleon of $\epsilon \approx (4 - 5)$ MeV, i.e., remains constant, while hundreds of MeV of entrance-channel kinetic energy are still available, and are eventually expended in, IMF emission from the system. A trend consistent with thermal IMF emission would have consisted in a significant shift away from the origin and a considerable broadening of the joint multiplicity distributions with increasing IMF multiplicity. However, there is no mystery as to where the energy comes from which is carried away by multiple IMFs: With increasing $m_{IMF}$, the velocity of the PLF remnant is seen to decrease [Tok96].

Certainly, the hypothetical IMF-emitting source is not in thermal equilibrium with the PLF and TLF in the same reaction event. Kinetic energy of relative motion is channeled more or less directly into dynamical emission of IMFs and other particles. The non-statistical processes of nucleon and IMF emission appear to have similarities but are not obviously reducible to one another. Both types of process lead to an emission of relatively hard Maxwell-Boltzmann-type distributions with large slope parameters and show some isotropy in a system of reference kinematically between those of PLF and TLF and close to the cm system. A possible scenario attributes both, nucleon and IMF emission, to slightly different stages of the collision impact process. Fast nucleons could be emitted in the first encounters of nucleons from the projectile with those of the target. Presumably, a breakup of the projectile into IMFs requires significant overlap, mixing, and compression of the matter density distributions of projectile and target. Thus, the question could be decided by experiments studying both types of non-statistical emission as functions of projectile-target A/Z asymmetry, impact parameter, and bombarding energy. First experiments [Agn97] of this kind have yielded indeed information about the time evolution of these processes, encouraging further, and more specific investigations.

2.3. The Charge Density Asymmetry Degree of Freedom

As stated previously, the charge density, or A/Z, asymmetry of reaction products has been found to be an effective indicator of the degree of (chemical) equilibrium reached in a heavy-ion reaction. At near-barrier energies, interacting heavy-ion systems evolve slowly from an initial state towards equilibrium [Sch84]. Chemical equilibrium can be defined in terms of a local minimum in the driving potential energy surface ($PES$) or by the bulk A/Z ratio of the composite system. More rigorously, the time-dependent free energy $F$ defines an evolution of the chemical equilibrium
along a system trajectory. Practically, however, only the asymptotic potentials, the binding, and the interaction energies of representative nuclei with well-defined shapes are accessible. Therefore, experiments have often been evaluated only with respect to some static $PES$ or its derivatives, the chemical potentials. To devise a more model-independent method to explore chemical equilibration processes represents one of the long-standing challenges in heavy-ion studies. This applies in particular to the fluctuations in these variables, which contain independent information on the reaction and its products. In the absence of such methods, ambiguities in the interpretation of experimental data caused by such over-simplifications should be kept in mind. The approximations made in such analysis are probably best justified for peripheral collisions, where distortions due to nuclear overlap are smallest.

Experimental results [Sch84, Pla88] have clearly demonstrated that, in general, equilibrium (in the above operational sense) is not reached in dissipative heavy-ion collisions at bombarding energies close to the barrier, except possibly for the most central collisions, the longest interaction times, or when the system is initially already close to such equilibrium. It is interesting that, at these lower energies, the rapidity $d < Z_{PLF} > /dE_{Loss}$ of the drift of the overall PLF-Z distribution clearly correlates with the gradient $dV/dZ$ of the static $PES(V)$ for nucleon exchange. Data are shown in Fig. 7, as obtained [Des88] for reactions involving a $^{238}$U target and several medium-weight projectiles at the same bombarding energy of $\epsilon = 8.5$ MeV. The correlation suggests that the propensity of $^{40}$Ca to transfer one of its protons to the $^{238}$U target is related to the initially steep gradient of the $PES$ in that direction. Possibly because of the large curvature ($d^2V/dZ^2$) of the $PES$, one observes net (average) transfers between projectile and target of never more than a few nucleons. It should be noted that otherwise successful NEM models have never been able to describe consistently the details of the average mass and charge drifts of the fragment distributions at near-barrier energies [Sch84].

Because of the merging interaction and relaxation time scales in the Fermi energy domain, one does not intuitively expect energetic collisions to drive a system closer to chemical equilibrium than at lower energies. In addition, because of the diminishing role of potential energies and Q-values for nucleon transfer at these energies, chemical equilibrium should be defined somewhat differently, i.e., more in terms of thermodynamic properties of free projectile and target nucleon gases. However, detailed experiments in this energy region studying $A/Z$ equilibration in dissipative reactions [Agn97,98] or fusion-like reactions [Yen94, Joh96,97] are rare. Experimental systematics similar to those of Fig. 7, or theoretical model predictions, are largely unavailable for these higher energies.

Interesting first information about $A/Z$ relaxation, and a host of information about other “isospin” effects in the Fermi energy domain, has been reported by Agnihotri et al. [Agn97, 98], who have measured PLFs in coincidence with neutrons, LCPs, and IMFs emitted from the two reactions $^{112}$Sn+$^{40}$Ca and $^{112}$Sn+$^{48}$Ca at $\epsilon = 35$ MeV. These two systems have been chosen because of their different initial conditions with
respect to the local minima of the corresponding static potential energy surfaces. The PES for fragmentations of the composite system $^{112}\text{Sn}^{+}40\text{Ca}$ is depicted in Fig. 8. It is defined as,

$$V(Z_1, N_1; Z, N) = V_{LD}(Z_1, N_1) + V_{LD}(Z - Z_1, N - N_1) + V_{Coul}(Z_1, Z_2) + V_{Nucl}(A_1, A_2)$$

(3)

Here, the different terms represent the liquid-drop binding energies of two nuclei, the Coulomb and the nuclear interactions, respectively. The potential is arbitrarily normalized.

As shown in this figure, the initial configuration (“injection point”) for the $^{112}\text{Sn}^{+}40\text{Ca}$ reaction is associated with a point on the steep wall of the PES, some 8-10 MeV above the local minimum. In contrast, the injection point for the $^{112}\text{Sn}^{+}48\text{Ca}$ reaction is very close to the minimum in the corresponding PES. Based on the low-energy systematics depicted in Fig. 7, one would expect very different neutron and proton pickup vs. stripping tendencies of the two projectiles $^{40}\text{Ca}$ and $^{48}\text{Ca}$. Only the reaction $^{112}\text{Sn}^{+}40\text{Ca}$ is expected to show an evolution towards chemical equilibrium at all and provide thus information about the degree to which this equilibrium is reached in a heavy-ion reaction at Fermi energies. The experiment sampled almost the entire reaction cross section. The measured cross sections are consistent with an exclusively dissipative reaction mechanism. However, the data allow for a 10% contribution of fusion-like events, which were not detected in the experiment.

Some of the neutron spectra measured [Agn98] in coincidence with PLFs for the reaction $^{112}\text{Sn}^{+}48\text{Ca}$ have been shown already in Fig. 4. The solid lines included in that figure indicate the contributions assigned to the PLF, TLF, and non-statistical (PRE) sources. Similar spectra were obtained for the associated reaction $^{112}\text{Sn}^{+}40\text{Ca}$, as well as for protons and LCPs in both cases. From the information contained in the measured post-evaporative properties of PLFs and the multiplicities and energies of the evaporated light particles, mostly neutrons, the properties of the primary fragments were reconstructed on average, as functions of energy loss or reconstructed fragment excitation energy. Corrections were made allowing for mass and energy loss due to pre-equilibrium neutron and proton emission, processes to be discussed further below. Iterative evaporation calculations were performed with the statistical-model code GEMINI [Cha88].

An important observation was made from the $^{112}\text{Sn}^{+}40,48\text{Ca}$ data with respect to thermal equilibrium, pertains to the sharing of the dissipated energy between the reaction partners. The data indicate that thermal equilibrium (equipartition) has not been reached, even for the largest excitation energies ($< E^* > \approx 400$ MeV) measured in the experiment. NEM calculations predict an almost equal sharing of this energy between PLF and TLF, in spite of their very different numbers of nucleons. This is a more extreme disequilibrium situation than observed in the data, which follow a gradual evolution towards equilibrium, with increasing excitation energy, i.e., with decreasing impact parameter. For the events accessible in the measurements, an
equilibrium excitation energy distribution is never reached. In this sense, the two reactions $^{112}\text{Sn}+^{40,48}\text{Ca}$ behave similarly at low and high energies.

Relaxation of the A/Z degree of freedom in the two $^{112}\text{Sn}+^{40,48}\text{Ca}$ reactions is illustrated in Fig. 9. Here, the neutron-to-proton multiplicity ratio (circles with error bars) is plotted vs. excitation energy. This ratio of multiplicities of evaporated particles, combined with measured properties of the secondary (post-evaporative) PLF, represents an observable sensitive to the A/Z ratio of the primary PLF. The sizable errors reflect overall systematic uncertainties in the method, e.g., pertaining to level density parameters, etc. The abscissa scale in Fig. 9 can be thought of as an effective impact-parameter or time scale. The curves included in this figure represent calculations assuming different primary A/Z (or N/Z) ratios and are meant to give a conservative estimate of the sensitivity of the data. The label “Equilibrium” refers to the respective local $PES$ minima.

It is obvious from the different behavior shown by the reactions $^{112}\text{Sn}+^{48}\text{Ca}$ and $^{112}\text{Sn}+^{40}\text{Ca}$ that the charge density asymmetry is a dynamical variable, evolving with impact parameter and interaction time. The large multiplicity ratios $M_n/M_p$ for $^{48}\text{Ca}$ at low excitations could be taken to reflect simply the large neutron excess of the projectile and, perhaps to a lesser extent, the efficiency of the Coulomb barrier in hindering the emission of protons. In such picture, higher excitation energies would simply reduce the Coulomb effect for proton emission. However, a similar hypothetical scenario for the reaction $^{112}\text{Sn}+^{40}\text{Ca}$ would predict a trend opposite to that actually observed. One concludes that the observed different evolution of the neutron/proton multiplicity ratios must reflect differences in the constitutions of the emitting PLFs, which change with dissipated energy or impact parameter.

From the experimental data, it appears that the mass-to-charge ratio of the $^{48}\text{Ca}$-induced reaction remains fixed, at A/Z = 2.4, solely because the initial projectile/target fragmentation is already at the minimum of the $PES$. This behavior could have been mistaken for a fast A/Z equilibration of the system. On the other hand, PLFs from the $^{40}\text{Ca}$-induced reaction, where the injection point is removed from the local $PES$ minimum, show an obvious evolution of their average A/Z ratio with increasing excitation energy (or decreasing impact parameter). The change, an increase from the initial value of $(A/Z)_0 = 2$ to $(A/Z)_\infty = 2.15$, is significant. Within the present experimental accuracy, one has to conclude that chemical equilibrium can possibly be reached in semi-central $^{112}\text{Sn}+^{40,48}\text{Ca}$ collisions at $\epsilon = 35$ MeV. However, a more accurate determination of cross sections and primary fragment A/Z ratios would be desirable, in order to improve the present, somewhat broad constraints on reaction models. Nevertheless, the above measurements suggest already that the mass-to-charge (A/Z) or neutron-to-proton (N/Z) ratios of the reaction products represent important variables providing independent information needed for an interpretation of the reaction dynamics in terms of non-equilibrium transport processes. There are indications [Ram00] that these variables remain powerful indicators of reaction dynamics at even higher energies.
Average A/Z ratios can be determined in a relative straightforward fashion from the isotopic distributions of light particles and clusters emitted in heavy-ion reactions. However, taken as isolated aspects, these observables may be ambiguous. For example, a neutron-rich flow of particles emitted from an equilibrated composite system could indicate nothing more interesting than the size of the available phase space, whereas in a direct breakup scenario, this fact may reflect initial-state correlations or incomplete mixing of projectile and target constituents. However, taken together with simultaneously measured other observables, the isotopic distributions of the final reaction products can play a crucial role in establishing the reaction mechanism, e.g., for its similarity to thermal (statistical) dynamics.

In this context, it should be pointed out that a determination of only the first moments of the multivariate probability distribution for system observables, a task that already exceeds the capacity of many contemporary experimental setups, provides an incomplete picture of heavy-ion reaction mechanisms. For example, statistical processes characterized by certain correlations between different moments of the probability distribution, e.g., as governed by a fluctuation-dissipation theorem [Sch84].

3. Isospin-Physics: The Next New Thing

In addition to its role as an indicator of proximity to chemical equilibrium, the A/Z degree of freedom may provide genuinely new information on effective nucleon-nucleon interaction forces in heavy-ion reactions at Fermi energies. Because here the time scales for intrinsic nucleonic motion and collective variables merge, the mean field cannot adjust adiabatically to the collective motion. The resulting sudden compression or dilution of nuclear matter, mainly in the overlap region, is resisted by diabatic, density-and momentum-dependent in-medium interactions. These regions of non-normal nuclear matter density (\(\rho\)) can develop mechanical and chemical instabilities, which influence reaction dynamics and the decay of the intermediate nuclear system.

In a heavy-ion collision, to first order, the average (mean-field) energy (\(\epsilon\)) of a nucleon in the nuclear medium changes by amounts that have to be provided by the relative motion of projectile and target nuclei. In this fashion, diabatic nucleus-nucleus interactions arise, which can influence the heavy-ion reaction dynamics in a measurable way. This effect has the interesting consequence that, in principle, information on the nuclear equation of state (EOS) may be obtained from dynamical observables. Furthermore, because of the nuclear symmetry energy, the interaction energy is different for neutrons and protons. Therefore, there are iso-scalar and iso-vector parts of the nucleus-nucleus liquid-drop and proximity interactions [Pom80] leading to effective interactions depending on the asymmetry in projectile-target A/Z values or isospins, I=(N-Z)/A. It is customary also to consider the relative neutron excess degree of freedom,
\[ \delta = \frac{(\rho_n - \rho_p)}{\rho} \]  

and to express the isospin-dependent nuclear equation of state \( \epsilon(\rho, \delta) \) in terms of this variable.

Different models for isospin-dependent interactions are reviewed in other sections of this book. For example, a model by Myers and Swiatecki [Mye98] considers a semi-classical Thomas-Fermi approach to the motion of neutrons and protons in finite nuclei, interacting via a short-range effective interaction of the Yukawa type. In this model, the isospin-dependent single-particle energy is given by a series expansion in the relative density \( \rho/\rho_o \),

\[ \epsilon(\rho, \delta) = \epsilon_F \left[ a(\delta) \left( \frac{\rho}{\rho_o} \right)^{2/3} - b(\rho) \frac{\rho}{\rho_o} + c(\delta) \left( \frac{\rho}{\rho_o} \right)^{5/3} \right] \]  

where \( \epsilon_F \) is the Fermi energy and \( a, b, \) and \( c \) are functions of the (local) neutron excess \( \delta \). This relation represents the equation of state (EOS) in the model, containing kinetic and (mean-field) potential energy parts. In other approaches, effective Skyrme forces are used to describe these interactions. The mean-field interaction potential applicable to heavy-ion collisions has the typical form of an expansion in the relative density \( \rho/\rho_o \),

\[ V^{n(p)}(\rho, \delta) = a(\delta) \frac{\rho}{\rho_o} + b(\delta) \left( \frac{\rho}{\rho_o} \right)^{\sigma} + V_{Coul}^p + V_{asym}^{n(p)}(\rho, \delta) \]  

In this expression [Li98], the coefficients \( a \) and \( b \) are functions of the (local) neutron excess \( \delta \), and \( \sigma \) is an exponent characteristic of the Skyrme type used for the effective force. The Coulomb interaction acts only on the protons in the system. The last term in Equ. (6) is the symmetry term, containing the dependence of the nuclear asymmetry energy on the matter density and the neutron excess. These are poorly known physical variables, which are of fundamental interest for several fields of science, for example for cosmology.

Of interest for heavy-ion reactions are also the chemical potentials, since they determine the net exchange of neutrons and protons between the reaction partners, an experimental observable. For moderately high nuclear temperatures, \( T \geq 4 \) MeV, these potentials can be expressed as

\[ \mu_{n(p)} \approx V^{n(p)} + T f(\rho_{n(p)}, T) \]  

in terms of the mean field and a density- and temperature-dependent function \( f \). In this fashion, a connection is drawn between the isospin-dependent equation of state (the mean nucleus-nucleus interaction) and the dynamic driving forces influencing nucleon exchange between the reaction partners in a heavy-ion collision. As a result, one expects the essentially binary reaction dynamics of heavy-ion reactions in the Fermi energy domain to be sensitive to these fundamental in-medium interactions revealed, for example, by the nature of PLF deflection functions, the relative
importance of fusion as compared to the dissipative reaction cross section. Furthermore, these interactions determine the moments of the probability distributions for the transfer of neutrons and protons between projectile and target, e.g., average and (co-)variances. Additional information is carried by fast particles promptly emitted during the nuclear interaction.

In the absence of residual nucleon-nucleon interactions, the above diabatic interactions are essentially conservative, returning compressional energy back into kinetic energy of collective expansion. However, in general, higher-order in-medium NN scattering exists, giving rise to dissipative nucleus-nucleus interaction forces. These latter dissipative forces are very different from the one-body, wall-and window-type [Sch84] dissipation prominent at near-barrier bombarding energies and should be detectable in heavy-ion reactions at Fermi energies. In addition to causing dissipative retardation of collective nucleus-nucleus motion, isospin-dependent in-medium NN scattering effects the prompt emission of scattered nucleons into the continuum. While multiple NN scattering quickly leads to a distribution of the initial kinetic energy onto many intrinsic degrees of freedom, at every stage of such an equilibration cascade, fast nucleons can be emitted from the interacting system as “non-statistical” (pre-equilibrium) particles. Again, the expected effects are different for neutrons and protons. Thus, the multiplicities, angular and energy distributions of these non-equilibrium particles contain independent information on the isospin-dependent in-medium nucleonic interactions. Together with the mean-field influences on the reaction dynamics, the isospin-dependent dissipation and particle-emission processes should fit into an internally consistent picture of the reaction mechanism.

Quantitative comparisons to data rely on an intermediate layer of model descriptions, namely the modeling (propagation) of the essential reaction dynamics. A number of very different theory “platforms” proposed in the literature have been utilized for such comparisons, from relatively simple trajectory type calculations, e.g., based on the nucleon exchange model NEM referred to earlier, to more detailed microscopic, multi-dimensional transport models. The latter models include various realizations of Boltzmann-Ühling-Ulenbeck-type (BUU, etc.) [Ber88, Bau93] approaches, classical or quantal molecular-dynamics models (QMD, AMD, FMD, etc.) [Aic91, Ono92, Fel90, 97]. In addition, models have been proposed [Bon85, Gro93] for the statistical decay of hot and rather diluted nuclear systems. To date, most work is based on NEM or BUU simulation calculations to collisions between heavy ions at Fermi energies.

As an example, calculations of PLF deflection functions with a BUU model [Sob95] for the $^{209}$Bi+$^{136}$Xe reaction are shown in Fig. 10. In this figure, the open and filled squares represent a stiff and a soft EOS, respectively, for symmetric nuclear matter. The diamonds are the results of a calculation assuming an isospin-soft EOS. The deflection functions are meant to illustrate expected sensitivities of the data, but no attempt has been made as yet to adjust parameters to fit experimental particular angle-energy data. Arguably, such a parameter adjustment should only be done to describe the entire set of observables simultaneously.
The trends illustrated in Fig. 10 are intuitively expected: The isospin-dependent interaction (open diamonds) is the most repulsive one, not leading to orbiting but to a bouncing-off behavior. Consequently, for this case, the cross section for fusion is smallest, which is best consistent with experiment, where there is no positive evidence for fusion at all [Bal93, 95]. All calculations predict a monotonic relation between impact parameter and PLF kinetic energy \((E/A)\) or energy loss. Experimentally, this latter behavior is confirmed qualitatively. However, experimentally a damping exceeding 60% of the initial kinetic energy, i.e., a slowing-down further to full damping \((E/A \approx 10 \text{ MeV})\), is unlikely for this system. Furthermore, orbiting is observed for the \(^{209}\text{Bi} + ^{136}\text{Xe}\) reaction from bombarding energies of \(E/A = 28 \text{ MeV}\) to \(E/A = 55 \text{ MeV}\) [see Fig. 1]. Therefore, the so-called balance energy must be significantly higher than 55 MeV per nucleon.

In other BUU-type isospin-dependent calculations [Col98] including density fluctuations in the nuclear interior, the different effects of isospin-dependent interactions on isobaric reaction pairs are emphasized. For example, for the n-rich light and more symmetric system \(^{64}\text{Ni} + ^{46}\text{Ar}\) at 30 MeV per nucleon, an isospin-soft EOS results in more attraction between projectile and target, and there is more dissipation, than for the case of an isospin-stiff EOS. In the former case, the system may fuse at a certain impact parameter, while the isospin-stiff EOS leads to more repulsion and binary dissipative collisions. For the \(N=Z\) isobaric analog reaction \(^{64}\text{Ge} + ^{46}\text{V}\), the behavior is inverted. A test of such interesting predictions has to await future experiments with exotic, secondary beams with a range of projectile \(A/Z\) ratios.

Fragmentation dynamics is expected to provide additional testing grounds for the exploration of isospin dependent interactions. Figure 11 provides a two-dimensional (x-z plane) overview over the time sequence of a simulated \(^{112}\text{Sn} + ^{40}\text{Ca}\) collision (left column) and of a \(^{112}\text{Sn} + ^{48}\text{Ca}\) collision (right column). The z direction is that of the beam. In both cases, an impact parameter of \(b = 7 \text{ fm}\) was chosen for the calculations, corresponding to a semi-peripheral collision. The nuclear EOS was chosen to be soft in the symmetry energy degree of freedom, as selected by the scaling function \(F_2\) [Li98].

As can be seen from the right column in this figure corresponding to the neutron-rich system \(^{112}\text{Sn} + ^{48}\text{Ca}\), projectile and target fuse and intermix their nucleons significantly during the first \((100-200) \text{ fm/c}\) of the interaction time. Later, the system stretches and forms a neck. This neck subsequently breaks and leaves one or two intermediate-mass clusters behind, in addition to a number of energetic light particles. A very different situation is predicted for the neutron-poor reaction system \(^{112}\text{Sn} + ^{40}\text{Ca}\) illustrated on the left of Fig. 11. Here, none or only very small nuclear clusters are produced in a similar collision. Such an inhibition of IMF emission for neutron-rich systems appears to be at variance with experiment [Dem96]. It should also be noted that the above BUU predictions for \(^{112}\text{Sn} + ^{40,48}\text{Ca}\) depend on the stiffness or softness of the EOS employed and on impact parameter. For example, calculations done with a stiff EOS predict no IMF production at all. Predicted IMF
emission probabilities depend significantly also on the number of test particles used in the calculations. In fact, IMF cluster emission is predicted to occur only in a narrow region of impact parameters around $b = 7$ fm, while experimentally, such clusters are observed for a large range of impact parameters. This is an striking puzzle, an inconsistency between model and experiment that demands closer scrutiny. In addition, it is a known fact that, the more test particles are used in the BUU calculations, the smoother the reaction features become, including a disappearance of the cluster emission process. However, one can argue that, in reality, density fluctuations would be present, having a similar effect as the artificial fluctuations introduced by poor sampling statistics in the calculations. This observation calls for a simultaneous measurement of observables that measure fluctuations independently, such as the widths of fragment A and Z distributions.

The above BUU calculations illustrate a dynamical process of nuclear cluster production in heavy-ion reactions, which would fit the general kinematics of IMF emission discussed previously (see Fig. 3). The calculations suggest origin and structure for an “intermediate, mid-rapidity source,” revealed in experiments. In the simulations, IMFs are formed from both projectile and target matter and appear relatively late in a collision, when the isospin asymmetry has largely relaxed. The neck matter is pulled out of both PLF and TLF, when the intermediate complex breaks apart. The average velocities of such IMFs are expected to be intermediate between those of PLF and TLF and probably somewhat closer to the velocity of the TLF or that of the cm system. In addition, the fact that IMF emission occurs late and after considerable intermixing of all nucleons in the model, the IMF clusters should reflect the A/Z ratio of the composite system, characteristic for overall isospin equilibration. The model makes definite predictions about the isospin relaxation as a function of impact parameter and bombarding energy, which can be subjected to equally unambiguous experimental test.

Unfortunately, experiments are extremely rare, where IMFs have been identified with respect to A and Z, and none is known yet that would allow one to perform a detailed, multi-faceted comparison to reaction models. Some summary information is available for the $^{112}$Sn+$^{40,48}$Ca reactions introduced already above, as well as for “incomplete fusion” reactions induced by 32A-MeV $^{14}$N projectiles on Sn and Au targets [Mur95].

In the former, $^{112}$Sn+$^{40,48}$Ca experiments, IMFs were detected in addition to PLFs and light particles. In a range of forward and sideways angles, the intermediate “neck-like” source is emphasized. It is kinematically not likely that the IMFs considered here have been evaporated statistically from excited primary PLF or TLF reaction fragments. However, because of poor statistics, the inclusive isotopic yields are averages over energy loss or impact parameter, emphasizing semi-central collisions.

Ratios $Y(^{48}$Ca)/$Y(^{40}$Ca) of isotopic IMF yields from the corresponding intermediate, neck-like source are available for the elements lithium through oxygen, as depicted in Fig. 12. It is obvious from these yield distributions that in the reaction $^{112}$Sn+$^{48}$Ca,
more neutron-rich, as well as fewer neutron-poor clusters are emitted than in the reaction induced by the neutron poorer $^{40}$Ca projectile. The enhancement is significant, as the yield ratios vary by up to factor of 3, in particular, when considering the fact that the N/Z ratios for the two composites differ by only 10%. The variation in yield is even larger than the difference in the N/Z ratios of the projectiles $^{40}$Ca and $^{48}$Ca, which amounts to only 40%.

Certainly, the results displayed in Fig. 12 are not indicative of overall, or even local, isospin equilibration. For these two reactions, it is known that the PLF/TLF isospin asymmetry relaxes with increasing excitation energy and decreasing impact parameter. As illustrated by Fig. 9, the PLF N/Z ratio either stays constant (for the $^{112}$Sn+$^{48}$Ca reaction) or increases (for the $^{112}$Sn+$^{40}$Ca reaction) with increasing energy dissipation. Therefore, with increasing energy dissipation, the PLF N/Z ratios of the two reactions approach each other. Large IMF emission probabilities sample presumably more dominantly the central, rather than peripheral reactions and should reflect the nearly equilibrated charge density in the overlap zone between PLF and TLF. If the BUU scenario discussed above were realistic, one would expect more mixing of PLF and TLF matter to have occurred in the formation of a neck than reflected in the IMF isotopic yield ratios of Fig.12.

A simple and convincing explanation of the isotopic IMF yields discussed above is currently not available. The BUU scenario implies several inconsistencies with the data, both concerning the range of impact parameters leading to IMF emission and the variation of isotopic IMF yields. If the data shown in Fig. 12 can be confirmed, they could indicate that significant spatial charge polarization occurs within projectile and target. This process would occur necessarily early on in a collision approach phase. It would be followed by an early IMF emission preserving this polarization. Such a process could perhaps also show up in the abundance and other properties of the light particles emitted non-statistically on a fast time scale.

The diagrams of Fig. 11 picturing the progress of $^{112}$Sn+$^{40,48}$Ca collisions in BUU simulation also illustrate properties of the non-statistical particle emission process. In fact, the calculations predict that the emission of such nucleons and other light particles occurs on a “mesoscopic” timescale, during the collision but not confined to the shortest times. Although the emission process commences in the approach phase, at about 50 fm/c, the bulk of the fast particles is emitted throughout the collision. The first of each of the 4 frames in Fig. 11 represents an early stage ($t = 50$ fm/c) of the collision. At this time, significant matter overlap between projectile and target has already developed, and one expects substantial mixing of projectile and target nucleons to have occurred. However, there is very little emission of fast particles. Only for times larger than a few hundred fm/c are there many fast (test) particles visible in the plots. Hence, in the model, particle emission takes place during the heavy-ion collision, but it occurs dominantly at its later, and not in its early stages. Since many particles are emitted late, one might also expect a relatively soft energy spectrum for these particles.
Figure 13 puts this observation on a quantitative basis. Here, the predicted ratio of neutron-to-proton multiplicities is plotted vs. elapsed interaction time, for an impact parameter of $b = 7$ fm. Two different parameterizations of the symmetry energy $V_{asym}$ are compared in the two panels. The parameterization on the left ($asym = 1$) is more repulsive for neutrons and more attractive for protons than the parameterization depicted on the right ($asym = 3$). The latter form has actually a negative curvature for neutrons. (These symmetry potentials correspond to the scaling functions $F_2$ and $F_3$, respectively, in [Li98]).

For both parameterizations shown in Fig. 13, the neutron-to-proton ratio is initially larger for the neutron-rich projectile $^{48}$Ca than for $^{40}$Ca. At later times, the ratio drops precipitously and takes on an asymptotically constant value. The time dependencies of the multiplicity ratios are very different for the two reactions and show characteristic sensitivity to the symmetry energy. The dependence on the projectile $N/Z$ ratio is particularly obvious for the parameterization on the right. In the latter case, the emission of fast neutrons builds up in time.

The examples of the two isospin parameterizations in Fig. 13 demonstrate a notable sensitivity of the asymptotic nucleon multiplicity ratios to the effective asymmetry potentials (and the associated EOS). However, appreciating that an average correlation between the energy of a non-statistical particle and its time of emission may exist, one can realize an even greater sensitivity to this symmetry energy. Typically, in an exciton model [Bla75] of non-statistical (pre-equilibrium) particle emission, early emission times lead to higher particle kinetic energies. This behavior is expected also for other nuclear models emphasizing nucleonic interactions such as QMD, but it is not necessarily the case in BUU-type approaches. In BUU transport models, fast, non-statistical nucleon emission can occur in at least two ways. The first, presumably less important mechanism is that of in-medium nucleon-nucleon scattering, which bears similarities to an exciton model. The second and more powerful mechanism has a collective origin: Nucleons can be ejected from the strong mean-field distortions caused by compressional effects generated in a heavy-ion collision.

If non-statistical nucleon emission is due to collective compressional effects, the particle spectra will be harder for delayed than for early emission, because of the time needed to produce a significant matter overlap between projectile and target. This effect should be strong for the parameterization of the symmetry energy illustrated on the r.h.s. of Fig. 14, since for this case, substantial compressional energy is built up, before neutrons are finally ejected from the mean field. This feature seems to be reflected in the energy spectra of non-statistical neutrons predicted by isospin-dependent BUU simulations. In this figure, the neutron yields are plotted vs. energy for two emission angles. Obviously, there is a very strong dependence of the particle spectra on the isospin-dependent EOS. If the strength of the effect is confirmed, the particle energy spectra could present the most sensitive indicators of the strength of the isospin-dependent mean field. The energy spectra on the right (for $asym = 3$), are much harder than typically observed experimentally (cf. Fig. 4). Even the spectra
for the parameterization illustrated on the left are somewhat harder than observed.
In addition to the shape of the energy spectra of the fast particles emitted, their angular distributions carry information about the emission process. Anisotropies in the angular distributions may result from various sources, including the velocity of the effective emitting source, the anisotropy of the nucleon-nucleon scattering process, as well as final-state interactions leading to attenuation and multiple scattering in the nuclear medium. All these effects need to be treated consistently within a single model, in order to derive meaningful information even on selected aspects of the effective nucleonic interactions, e.g., the EOS. However, it is encouraging already that detailed BUU calculations, e.g., by Li et al. [Li98], predict a net sensitivity of the kinetic-energy spectra of fast particles to the isospin-dependent EOS.

The validity of these general ideas can be subjected to first tests, utilizing the data on non-statistical nucleon emission in the same two reactions $^{112}\text{Sn} + ^{40,48}\text{Ca}$ discussed above. Some instructive data on neutron and proton spectra from these reactions are depicted in Fig. 15. Here, for a detection angle of $\Theta = 60^\circ$, spectra are shown, arranged from top to bottom according to increasing energy loss, $<E^* > \approx 100\text{ MeV} \rightarrow <E^* > \approx 400\text{ MeV}$, in the reactions. In the same direction, the collision impact parameter should decrease. The left column in Fig. 15 depicts neutron (circles) and proton (squares) data for $^{112}\text{Sn} + ^{40}\text{Ca}$, while the right column shows the same for $^{112}\text{Sn} + ^{48}\text{Ca}$. The neutron spectra show the two spectral components discussed already in the context of Fig. 4. At the present detection angle of $\Theta = 60^\circ$, the spectra are a superposition of a low-energy evaporative component, associated with the TLF, and a high-energy, non-statistical component. Because of Coulomb barrier and detection thresholds for protons, the low-energy evaporative component is suppressed and less obvious in the proton spectra. Within present experimental accuracy, the angular distribution of the non-statistical particles is isotropic in the cm system of the nucleon-nucleon system. Hence, one can fit these spectra at measured angles and determine the overall multiplicities, $M_n$ and $M_p$, by interpolation and extrapolation. To no great surprise, the neutron multiplicities are generally larger than proton multiplicities. However, there is a definite evolution with increasing energy loss of the relative yields of non-statistical neutrons and protons. In addition, there is a marked difference in the behavior of the two reaction systems.

As discussed previously (see Fig. 9), for the reaction $^{112}\text{Sn} + ^{40}\text{Ca}$, with increasing energy loss an evolution of the PLF/TLF isospin asymmetry degree of freedom towards equilibrium has been observed. Presumably, this evolution is associated with an increasing overlap of PLF and TLF matter distributions and the ensuing mixing of the nucleons of the two fragments. In agreement with this picture, proceeding from top to bottom, one observes a decrease of the non-statistical multiplicity ratio $R = (M_n/M_p)|_{\text{neq}}$ from $R \approx 2$ to $R \approx 1$.

The situation is very different for the reaction illustrated on the r.h.s. of Fig. 15 for the neutron-rich system $^{112}\text{Sn} + ^{48}\text{Ca}$. First, the multiplicity ratio $R$ for the non-statistical particles is very much larger than for the neutron-poorer system. With
increasing energy loss, one observes the same type of evolution as for the neutron-poor system. The decrease in $R$ is very steep, which requires a steep asymmetry potential, i.e., significant chemical-potential gradients. However, the multiplicity ratio never decreases below $R \approx 2.5$, although the bulk N/Z ratios of the two $^{112}\text{Sn}^{40,48}\text{Ca}$ systems are identical within 10%. This behavior indicates that even semi-central $^{112}\text{Sn}^{48}\text{Ca}$ collisions do not lead to chemical equilibrium characterized by significant mixing of projectile and target nucleons, before non-statistical particles are emitted. The same conditions must hold for the neutron-poor system where, however, the corresponding evidence is hidden.

These observations impose tight constraints on the various timescales for fusion and isospin equilibration. Apparently, fast nucleons are emitted from non-equilibrated and very neutron-rich subsystems. As a likely scenario, prompt emission of particles from the surface regions of the interacting nuclei during the early phases of a collision comes to mind. The experimental observations may also imply a more important role of in-medium nucleon-nucleon scattering than previously thought. This is a trend expected from the large average values and n-p asymmetries for the in-medium cross sections in low-density regions, regions that are also neutron rich. Alternatively, one also has to consider the possibility of breakup of PLF and TLF on impact. This scenario would be challenged by other experimental data, such as deflection functions, should it require a stiff EOS with a stiff asymmetry term. This is a trend expected from the large average values and n-p asymmetries for the in-medium cross sections in low-density regions, regions that are also neutron rich. Alternatively, one also has to consider the possibility of breakup of PLF and TLF on impact. This scenario would be challenged by other experimental data, such as deflection functions, should it require a stiff EOS with a stiff asymmetry term. Such a situation can be modeled and studied more appropriately in molecular-dynamics simulations.

The few, but rather clear-cut, experimental data for non-statistical nucleon emission discussed above show some generic similarities with the emission of IMFs in heavy-ion reactions at Fermi energies. Such a similarity is apparent also in other data [Mur95], where both, fast particles and energetic clusters (Fig. 5) have been measured. Both types of particles appear to be emitted from a hypothetical third source traveling at some speed intermediate between those of PLF and TLF. In BUU, such a source is modeled as the decay of a neck between the main fragments. Experimentally, this source is not in thermal, mechanical, or chemical equilibrium with the massive PLF and TLF nuclei. It appears to be neutron rich, or the underlying process that this source simulates seems to emit preferentially neutron-rich particle and cluster species. For fast nucleons, this experimental fact has been demonstrated above. The emission of light particles and clusters such as $^3,^4,^6\text{He}$ and $\text{Li}$ ions has been studied [Dem96] in the reactions $^{112,124}\text{Sn}^{124,136}\text{Xe}$ at $\epsilon = 55$ MeV. It was concluded from this study that particles at intermediate velocities are substantially more neutron rich than those evaporated from the heavy, Xe-like PLFs. This is consistent with the scenario for IMF and fast particle emission.

In view of this experience, it seems currently unlikely that isospin equilibrium is reached in the Fermi energy domain, where reactions still lead to massive remnants of projectile and target, even in central collisions [Lot93, Bal95, Lar98]. In fact, a dominantly non-statistical, non-equilibrium nature of reactions at Fermi bombarding energies is suggested by an observation of incomplete damping [Que93] of the kinetic
energy available in the entrance channel and by the dynamic emission patterns of intermediate-mass fragments [Tok95-96] in such reactions. A basically dissipative reaction environment is reflected also in the display of memory of the entrance-channel A/Z asymmetry observed [Tok94-96, Agni98,99, Dem96] for different types of products from the reactions $^{209}$Bi+$^{136}$Xe, $^{112}$Sn+$^{40,48}$Ca, and $^{120}$Sn+$^{136}$Xe. The lack of chemical equilibrium suggested at the present state of research would fit well into this scenario. Such a scenario is fundamentally dynamical, following a multi-dimensional path toward relaxation, which can be followed experimentally. Thus, the challenge to understand the motion along the isospin degrees of freedom also provides new experimental opportunities to monitor the time evolution of the microscopic reaction mechanism.

4. Conclusions

Accepting the dominance of dynamics in nuclear collisions at Fermi energies and a substantial absence of final nuclear configurations in mutual statistical equilibrium, the pursuit of fundamental scientific goals in nuclear mechanistic studies at Fermi energies continues to present a formidable task. These goals include gathering information on the effective isospin-dependent in-medium nucleonic interactions, i.e., the mean-field equation of state (EOS) of nuclear matter, the residual interactions leading to in-medium nucleon-nucleon scattering, and nucleonic correlations responsible for nucleonic aggregation and properties of intermediate-mass nuclear clusters.

Conventional strategies aiming at information on the nuclear EOS have consisted in searching for critical behavior of nuclear systems produced in central heavy-ion collisions, such as critical temperatures and cluster distributions reflecting a nuclear liquid-gas phase transition. While it may be premature to abandon altogether the hope of observing this statistical nuclear phase transition in heavy-ion reactions, a new access to the fundamental information on the nuclear EOS and in-medium effects has been found, provided by new observations associated with the isospin degrees of freedom.

To date, there have been very few experiments exploring these new degrees of freedom. Available data are not sufficient to develop a coherent reaction scenario with these new observables. However, it is clear that the initial projectile-target isospin (A/Z) asymmetry influences the reaction dynamics, as reflected consistently in the isotopic distributions of the various types of final products.

On the other hand, non-equilibrium emission of light particles and clusters are illustrative examples demonstrating interesting new isospin-dependent effects. For example, it has been shown that relative abundances and energy spectra of non-statistical (pre-equilibrium) nucleons and clusters contain information about the nuclear EOS, in particular on its dependence on the nuclear asymmetry energy. However, there is no consistent reaction scenario as yet that would explain consistently all observations. Some of the observed dependencies are expected in certain reaction models, others are not. Exploiting the additional isospin degree of freedom will
help to develop such an internally consistent, holistic reaction scenario explaining simultaneously a host of reaction features, which also incorporates important features of the nuclear equation of state. While there is no detailed roadmap for future research available that could guarantee success in this venture, new experimental and theoretical discoveries are pointing to new horizons.

Much of the experimental work on the reactions $^{112}\text{Sn} + ^{40,48}\text{Ca}$ reported here has been done by Dr. D.K. Agnihotri, whose contributions we gratefully acknowledge. One of us (WUS) has benefited from discussions with B.A. Li and M. DiToro, whose interest in our experiments has been encouraging.

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5. Figure Captions

Figure 1. Two-dimensional contour diagrams of the yield of charged products vs. lab energy and angle, for the reactions $^{112}\text{Sn}+^{48}\text{Ca}$ (top left), $^{197}\text{Au}+^{86}\text{Kr}$ (top right), and $^{209}\text{Bi}+^{136}\text{Xe}$ (bottom left and right), for the indicated bombarding energies. The solid dots and lines represent predictions by the NEM, corrected for sequential evaporation. (From [Agn98])

Figure 2: Contour plot of the correlation between PLF atomic number and the multiplicity of neutrons for the $^{209}\text{Bi}+^{136}\text{Xe}$ reaction. The PLF Z has been corrected for evaporated light charged particles. (From [Bal95])

Figure 3: Galilei-invariant cross section of protons (p), tritons (t), lithium (Li), and carbon (C) fragments emitted from the $^{209}\text{Bi}+^{136}\text{Xe}$ reaction. Contours of constant yields are plotted vs parallel and transverse particle velocity. The arrows indicate the velocities of PLF and TLF, respectively (From [Tok96]).

Figure 4: Neutron energy spectra for three lab angles for the reaction $^{112}\text{Sn}+^{48}\text{Ca}$ at $\epsilon = 35$ MeV. Solid lines indicate moving-source fits. (From [Agn98])

Figure 5: Atomic-number (left) and transverse-energy distributions for intermediate-mass fragments emitted in the $^{209}\text{Bi}+^{136}\text{Xe}$ reaction. The distributions are sorted for different IMF multiplicities and displaced along the ordinate (From [Tok98]).

Figure 6: Contour plots of the joint multiplicity distributions $P(m_n, m_{LCP})$ for different IMF multiplicities. The upper left distribution represents the unconditional multiplicity distribution.

Figure 7: Rate of initial drift in average PLF-Z value vs. potential gradient. (From [Des88]).

Figure 8: Static liquid-drop (plus shell corrections) potential energy surface for fragmentations of the composite system $^{152}\text{Yb}$. The spacing between the contour lines is 4 MeV and the “injection point” fragmentation $^{112}\text{Sn}+^{40}\text{Ca}$ is indicated by a cross.

Figure 9: Neutron-to-proton multiplicity ratio vs. total excitation energy, for the two reactions $^{112}\text{Sn}+^{48}\text{Ca}$ (left) and $^{112}\text{Sn}+^{48}\text{Ca}$ (right) at $\epsilon = 35$ MeV.
Figure 10: Theoretical correlations of angle/impact parameter (a) and angle/energy per nucleon (b), predicted by BUU for symmetric matter (squares) and an isospin-soft EOS (diamonds). (From [Sob94])

Figure 11: Two-dimensional time sequence of BUU simulations of $^{112}\text{Sn}^{+}40\text{Ca}$ (left) and $^{112}\text{Sn}^{+}48\text{Ca}$ (right) reactions at $\epsilon = 35$ MeV and $b = 7$ fm. An iso-soft EOS has been used.

Figure 12: Ratio of yields of IMF isotopes in the reactions $^{112}\text{Sn}^{+}48\text{Ca}$ and $^{112}\text{Sn}^{+}40\text{Ca}$.

Figure 13: BUU simulations of the multiplicity ratios of neutrons and protons emitted promptly in $^{112}\text{Sn}^{+}40,48\text{Ca}$ reactions. Iso-spin-soft (left) and stiff (right) EOS assumed in BUU.

Figure 14: Lab energy spectra of promptly emitted neutrons in the reaction $^{112}\text{Sn}^{+}48\text{Ca}$, as predicted with BUU with the same parameterizations as used for Fig. 13.

Figure 15: $^{112}\text{Sn}^{+}40,48\text{Ca}$ neutron and proton spectra measured at a lab angle of $\Theta = 60^\circ$. The spectra are arranged from top to bottom according to increasing energy loss. (From [Agn98])
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