Research on calculation of machine-tool settings and flank modification for spiral bevel gear by duplex spread blade method based on 4-axis CNC milling machine

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Abstract
A method is proposed to manufacture spiral bevel gear by duplex spread blade method based on 4-axis CNC milling machine. Firstly, the geometry parameters and machine-tool settings are comprehensively considered. To meet with the tooth taper, a new way is raised to select reference point. Then machine-tool settings are calculated that only first-order mesh performance is guaranteed in the reference point. Therefore conjugate tooth surface is modified by a surface represented as second-order polynomial to optimize mesh performance, as a result a target tooth surface is established, and according to the sensitivity matrix the machine-tool settings are corrected. Finally, the proposed method is illustrated by a numerical example to reveal it’s effective and feasible.

Keywords: Spiral bevel gear, Duplex spread blade method, Reference point, Flank modification, Machine experiment

1. Introduction

Spiral bevel gear is widely used in aviation, aerospace, marine and machine tool for its advantages of driving smoothly, high transmission efficiency, excellent load capacity, etc. With the improvement of CNC, it is a trend that the machine of spiral bevel gear is converting to completing method from the traditional five-cut method, especially in the developed country of manufacturing. It is more efficient for completing method compared to five-cut method for the machine of pinion is one process for two sides simultaneously by completing, while separated by five-cut method, moreover which is more energy-saving and environmental protection by adopt power dry cutting. Obviously, completing method is an advanced method; however high-end six-axis milling machine is essential, such as Gleason phoenix series and Oerlikon C series.

Duplex helical method is firstly proposed by Gleason to machine spiral bevel gear, while cutting principle and machine setting are not fully revealed (Gleason works, 1971). Tsay et al., (1993) developed a mathematical model can be applied to simulate the tooth surface geometry machined by spread blade and duplex helical method. Gonzalez et al., (2011) presented conversion of the specific machine-tool settings of a given generator to neutral machine-tool settings by duplex helical method, and parabolic profile on the blades of the head cutter was applied to adjust the contact pattern. Zhang et al., (2015) revealed the generalized theory of helical method in which gear was non-generated and calculated the basic machine-tool settings by define three reference points, later a mathematical model with circular profile blade was established to adjust contact pattern (2016). Lv et al., (2018) calculated the machine settings of small
module hypoid gear by duplex helical method. All the above research, the calculation of machine settings is complex, and rigorous demands for related machine, such as helical motion, tilt and swivel.

For five-cut method, many exports and scholars have carried out a deep and systematical research. Litvin et al., (1981) suggested the Helixform and Formate methods for Hypoid gears, then realized (1988) the determination of setting of a tilted head-cutter for Hypoid and Spiral Bevel Gears, and proposed (1991) loyal synthesis to calculate machine-tool settings which realize the control of tooth contact and transmission error. Tsuji, I.et al (2011) presented manufacturing method of large-sized spiral bevel gears using multi-tasking machine tool with five axis control. Deng et al., (2014) presented a machining method for face milling using a disk cutter with a concave end by 5-axis CNC machine tool. Yang et al., (2015) proposed a pinion development approach in order to obtain excellent transmission performance of Hypoid and Spiral bevel gears. Jiang (2016) researched the machine setting parameters and cutting test by a new tilt milling machine. Cao (2018) proposed a new method for tooth contact analysis. While for the traditional five-cut method, the machine-tool settings of pinion are calculated based on the machine-tool settings of gear in the reference point by the local conjugate or local synthesis, and the machine settings are different for two sides. And the machine-tool settings are usually calculated in the reference point selected in the middle of tooth surface.

For flank modification, Wang and Fong (2006) proposed a methodology to synthesize the mating flanks of a face-milling pinion with a predetermined fourth-order motion curve. Fan et al., (2008) developed a new approach to correct tooth flank form error by using higher-order coefficients of the universal motions, besides the error surface is represented by polynomials in the paper. Shih (2010) proposed a flank modification methodology based on ease-off topography for face-hobbing. Artoni et al., (2013) presented a methodology to correct tooth deviate that gear deviations mapped into equivalent pinion deviations, and only pinion corrected. Mu and Fang (2017) proposed an ease-off flank modification curvature motion method to solve the problem of the design and fabrication of high contact ratio spiral bevel gear with seventh-order transmission error. Nie et al., (2018) proposed a tooth surface topography modification to improve the meshing performance of spiral bevel gears. And above literatures listed are all about five-cut method or completing, which provided valuable insights for this paper.

To solve the problem of complex calculating machine settings and the constraints of equipment by completing, this paper proposed process spiral bevel gear by duplex spread blade method. The geometry parameters and machine setting parameters are comprehensively considered. And a novel method to calculation reference point is proposed, which is different from traditional method selecting the middle of tooth surface as reference point. Then the geometry parameters are designed and machine-tool settings are calculated in the reference point, and the machine settings are the same to two sides for both gear and pinion. What’s more, the machine settings are without helical motion, tilt swivel. According to the machine-tool settings, ease-off topography is established by the original and conjugate pinion tooth surface to check the original parameters. Only the position of contact determined by machine-tool settings, in order to improve mesh performance, a second-order surface is built to modify the conjugate tooth surface. Finally, the machine setting parameters are corrected that guarantee mesh performance. The logic flow is showed as Fig.1.

![Fig.1 The logic flow of design.](image-url)
2. Determination parameters of spiral bevel gear

2.1 Determination geometry and tool parameters

A difference angle of tangent line between midpoints on either side of the tooth is formed comparing single side method with duplex method is deduced (Zeng 1989).

\[ \Delta \psi_i = \angle P_iO P_i - \angle P_i O' P_i = \frac{s}{R} \left(1 - \frac{R \sin \beta}{r_s}\right) \]

(1)

Where, \( P_i, P_i' \) represents the points in the middle of tooth flank line, \( R \) represents mean cone distance, \( O \) represents cross point of gear, \( O' \) represents the center of head cutter, \( s \) represents the tooth thickness.

Meanwhile, in order to reveal the internal relations between dedendum angle \( \theta_j \) and \( \Delta \psi_i \), two points are selected, and represented as

\[ \Delta \psi_2 = \tan \Delta \psi_2 = \frac{\Delta s}{\Delta L} = 2 \tan \theta_j \tan \alpha \cos \beta \]

(2)

Where, \( \alpha \) represents pressure angle, \( \beta \) represents spiral angle, \( \Delta s \) represents the distance along the tooth thickness, \( \Delta L \) represents the distance along the pitch line.

It is significant to minimize the difference value between \( \Delta \psi_2 \) and \( \Delta \psi_i \) to avoid excessive tooth taper which has a serious impact on gear strength and tool life. As a result the root line is tilted to obtain a suitable dedendum angle \( \theta_j \) which matches \( \Delta \psi_1 = \Delta \psi_2 \), and can be written as according to Eq. (1) and Eq. (2).

\[ \theta_j = \tan \theta_j = \frac{s}{2R \tan \alpha \cos \beta} \left(1 - \frac{R \sin \beta}{r_s}\right) \]

(3)

Above research is suited for both pinion and gear. Thus it is established to calculate the sum of dedendum angle \( \Sigma \theta_s \) as follow equation

\[ \Sigma \theta_s = \theta_{s1} + \theta_{s2} = \frac{s_1 + s_2}{2R \tan \alpha \cos \beta} \left(1 - \frac{R \sin \beta}{r_s}\right) \]

(4)

Where \( \theta_{s1}, \theta_{s2} \) is dedendum angle of pinion and gear respectively, \( s_1, s_2 \) is mean normal circular tooth thickness of pinion and gear respectively, and \( s_1, s_2 \) satisfy the equation \( z_0 (s_1 + s_2) = 2\pi R \), \( z_0 \) is equivalent number of teeth.

\[ \Sigma \theta_s = \frac{\pi}{z_0 \tan \alpha \cos \beta} \left(1 - \frac{R \sin \beta}{r_s}\right) \]

(5)

An assumption is established that sum of dedendum angle \( \Sigma \theta_s \) is satisfy with Eq. (5). Then an ideal cutter radius \( r_c \) can be deduced as

\[ r_c = \frac{R \sin \beta}{\left(\frac{\Sigma \theta_s z_0 \tan \alpha \cos \beta}{180}\right)} \]

(6)

Where, \( \Sigma \theta_s \) represents the sum of dedendum angle of standard taper.

The size of head cutter has been standardized, so a cutter radius \( r_c \) can be selected according to the ideal cutter radius \( r_c \). Once cutter radius \( r_c \) determined, the final sum of dedendum angle \( \Sigma \theta_s \) can be calculated based on Eq. (6).

Naturally, cut number \( N \) can be obtained as

\[ N = \frac{\Sigma \theta_s \sin \beta}{20} \]

(7)
It is different from the blade that can be grinded in any angle; the block is fixed when produced. Cut number may be different for couples. To reduce the quantity of head cutter, a series of cut number is set up. And then the cut number \( N_c \) closed to the calculated can be selected.

Once the cutter number \( N_c \) determined, the profile angle \( a \) is calculated as

\[
a_i = \pm a + \frac{N_c}{6}
\]  

(8)

Where, \( a \) is profile angle, upper sign is applied to inner cutter; lower sign is applied to outer cutter.

The dedendum angle \( \theta_d \) is modified to eliminate the effects caused by the different of cut number. The modification comes true by tilting root line about the mean point. The value of modification can be repent as

\[
\Delta \theta_d = \frac{10N_c}{\sin \beta} - \frac{\Sigma \theta}{2}
\]  

(9)

For the root tilted around the middle point, the tooth depth in heel and toe will change, \( \Delta h \) is the change of the depth and presented as

\[
\Delta h = \frac{b}{2}\tan \Delta \theta_d
\]  

(10)

Where, \( b \) is face width.

Then the geometry parameters can be obtained combining that of standard taper.

2.2 Determination machine setting parameters

The basic parameters (e.g., the tooth number, the pressure angle, the spiral angle) are determined as the couple designed. With the cut number changed, the mean cone distance will not be satisfied with Eq. (6). To solve the problem, a point \( M \) is selected as the reference point to calculate machine settings, and deduced as

\[
R' = \frac{r_0}{\sin \beta} \left[ 1 - \frac{z_a \tan \alpha N_c}{540 \tan \beta} \right]
\]  

(11)

It is important to note that the cut number should nearby avoiding the reference point far away from the midpoint of surface.

Pinion and gear are both generated; the only different is that the reference point is selected not at the mean cone distance. A simple model is established as Fig.2, a left hand pinion is illustrated to calculate the machine setting parameters. The tangent to the path trace at point \( M \) forms angle \( \beta \) with axis \( X \).

Fig.2  Installment of head cutter.
Radial distance \( S_r \) can be obtained as

\[
S_r = \sqrt{r_c^2 + (R')^2 - 2r_c R' \cos \beta}
\]

(12)

Whereas cradle angle \( q \) is obtained as

\[
q = \arctan \left( \frac{r_c \cos \beta}{R' - r_c \sin \beta} \right)
\]

(13)

The sliding base will be changed as root line tilted, and the variation is determined as

\[
X_s = R' \tan(\Delta \theta)
\]

(14)

And the other machine setting parameters are

\[
\begin{align*}
E_x &= 0 \\
X_e &= 0
\end{align*}
\]

(15)

Then the machine setting parameters are calculated.

3. Establishment of digital tooth surface

A universal model is established as Fig.3. \( S_{\text{set}}(x_{\text{set}}, y_{\text{set}}, z_{\text{set}}) \) is coordinate system of machine, \( S_i(x_i, y_i, z_i) \) is coordinate system of pinion, \( S_h(x_h, y_h, z_h) \) is coordinate system of head cutter, \( S_c(x_c, y_c, z_c) \) is coordinate system of cradle, \( S_{\text{set}}(x_{\text{set}}, y_{\text{set}}, z_{\text{set}}) \) is an auxiliary coordinate system. The machine-tool settings, for instance, radial distance is the distance between \( O_i \) and \( O_e \), and represents as \( S_{\text{set}} \), center roll position is the angle formed by \( S_{\text{set}} \) and \( x_{\text{set}} \), represented as \( q_i \), machine root angle \( \gamma_i \), Blank offset \( E_{\text{set}} \) and machine center to back \( X_{\text{set}} \), cutter radius \( R_c \), profile angle \( \alpha \), are showed in Fig.3. \( \phi_i \) represents the angle of cradle during the generate, \( \phi \) represents corresponding angle of pinion which meets with \( \phi = R_w \cdot \phi_i \). What needs illustration is that although the Blank offset \( E_{\text{set}} \) and machine center to back \( X_{\text{set}} \) are equal 0, they are shown in Fig.3, for the correction of tooth surface in the following.

Fig.3  Coordinate systems of machine and head cutter.
Equation of head cutter in $S_i$ can be expressed as

$$
t_c = r_c(s_p, \theta_p) = \begin{bmatrix} (R_p \pm s_p \sin \alpha_i \cos \theta_p) \\
(R_p \pm s_p \sin \alpha_i \sin \theta_p) \\
-s_p \cos \alpha_i \\
1 \end{bmatrix}
$$

(16)

Where, $s_p, \theta_p$ are parameters of tooth surface, as showed in Fig.3. $R_p$ is cutter radius measured in the plane of $z = 0$, and upper sign is applied to outer cutter, lower sign is applied to inner cutter.

Equation of pinion and unit normal vector in $S_i$ can be obtained by coordinate transformation and written as

$$
M_i(\psi_j, \phi_p) = M_i(\psi_j, \phi_p) \cdot r_p
$$

$$
n_i(s_p, \theta_p, \phi_p) = \frac{\partial r_i(s_p, \theta_p, \phi_p)}{\partial s_p} \times \frac{\partial r_i(s_p, \theta_p, \phi_p)}{\partial \theta_p}
$$

(17)

Where, $M_i(\psi_j, \phi_p)$ is transfer matrix from $S_i$ to $S_j$. $\psi_j$ represent machine-tool settings. And $M_i(\psi_j, \phi_p)$ can be detailed description as

$$
M_i = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
0 & a_{22} & a_{23} & a_{24} \\
0 & a_{32} & a_{33} & a_{34} \\
0 & 0 & 0 & a_{44}
\end{bmatrix}
$$

(18)

$$
a_{11} = \cos \phi_p \cos \gamma_i
$$

$$
a_{12} = -\sin \phi_p \cos \gamma_i
$$

$$
a_{13} = \sin \gamma_i
$$

$$
a_{14} = S_{r_i} \cos \gamma_i \cos (\phi_p + q_i) - X_{r_i} \sin \gamma_i
$$

$$
a_{21} = \cos \phi_p \sin \phi_p - \cos \phi_p \sin \phi_p \sin \gamma_i
$$

$$
a_{22} = \cos \phi_p \cos \phi_p + \sin \phi_p \sin \phi_p \sin \gamma_i
$$

$$
a_{23} = \cos \gamma_i \sin \phi_p
$$

$$
a_{24} = E_{r_i} \cos \phi_p - X_{r_i} \cos \gamma_i \sin \phi_p + S_{r_i} \cos \phi_p \sin (\phi_p + q_i) - S_{r_i} \sin \phi_p \sin \gamma_i \cos (\phi_p + q_i)
$$

$$
a_{31} = -\sin \phi_p \sin \phi_p - \cos \phi_p \cos \phi_p \sin \gamma_i
$$

$$
a_{32} = \sin \phi_p \cos \phi_p \sin \gamma_i - \cos \phi_p \sin \phi_p
$$

$$
a_{33} = \cos \gamma_i \cos \phi_p
$$

$$
a_{34} = -E_{r_i} \sin \phi_p - X_{r_i} \cos \gamma_i \cos \phi_p - S_{r_i} \sin \phi_p \sin (\phi_p + q_i) - S_{r_i} \cos \phi_p \sin \gamma_i \cos (\phi_p + q_i)
$$

$$
a_{41} = 1
$$

Meshing equation can be expressed as during the process of generating

$$
f(s_p, \theta_p, \phi_p) = n_i \cdot \frac{\partial r_p}{\partial \phi_p} = 0
$$

(19)

The theoretical tooth surface geometry can be numerically represented by the coordinates of a series of surface point defined in Fig.4. A grid of $n$ lines and $m$ columns is defined in the axial $XL - RL$ plane of pinion that $XL$ is coinciding with axis of pinion.
\[ XL = r_u(s_x, \theta_s, \phi_p) \]
\[ RL = \sqrt{r_u^2(s_x, \theta_s, \phi_p) + r_o^2(s_x, \theta_s, \phi_p)} \]

Where, \( r_u, r_p, r_o \) are the three components of \( r_e \).

![Fig.4 Definition of a tooth flank grid.](image)

The grid point can be calculated by solving the nonlinear equation made up of Eqs. (19)- (20). Reference point is chosen according to Eq. (11). There are three parameters \( s_x, \theta_s, \phi_p \) in the equations. As a result, a series of parameters about \( s_x, \theta_s, \phi_p \) can be obtained. Hence, the tooth surface equation and unit normal vector are obtained by Eq. (17). So is for gear. The models are built of pinion and gear in UG showed in Fig.5, simultaneously, an assemble model is set up.

![Fig.5 Model of gear, pinion and assemble.](image)

4. Flank modification of conjugate tooth surface

Only the pressure angle \( \alpha \) and spiral angle \( \beta \) are guaranteed in the reference point. In order to realize tooth surface active design and acquire a well performance, a method is put forward to modify the conjugate tooth surface by a second order surface that tooth lengthwise crowing, tooth profile crowing and tooth longitudinal twist are modified. The main steps are as follows:

(1) Deducing the conjugate tooth surface;
(2) Adjusting the coefficient to modify until a well mesh performance obtained.

4.1 Derivation of conjugate tooth surface

\( S_i(x_i, y_i, z_i) \) is mesh coordinate system which is rigidly attached to machine frame, as shown in Fig.6. Coordinate systems \( S(x, y, z) \) and \( S_i(x_i, y_i, z_i) \) are rigidly attached to the pinion and gear, respectively. An auxiliary coordinate system \( S_e(x_e, y_e, z_e) \) is established whose \( z_e \) is parallel to \( z \). \( \omega_l \) and \( \omega_g \) are angular velocity of pinion and gear. \( \Sigma \) is shaft angle formed by the rotation axes, \( E \) is blank offset the distance between the rotation axes. \( \phi_1, \phi_2 \) are the rotating angles that determine the meshing instantaneous position of pinion and gear, respectively.
Similarly, gear equation and unit normal vector can be represented as \( \mathbf{r}_s(\theta_s, \varphi_s) \) and \( \mathbf{n}_s(\theta_s, \varphi_s) \), where \( \theta_s, \varphi_s \) are surface parameters of gear.

![Fig.6 Mesh coordinate system.](image)

The gear equation \( \mathbf{r}_s(\theta_s, \varphi_s) \) and unit normal vector \( \mathbf{n}_s(\theta_s, \varphi_s) \) can be represented in \( S_s \) as

\[
\begin{align*}
\mathbf{r}_s &= M_{s2} \mathbf{r}_2 \\
\mathbf{n}_s &= L_{s2} \mathbf{n}_2
\end{align*}
\] (21)

Where,

\[
M_{s2} = \begin{bmatrix}
0 & -\sin \varphi_2 & -\cos \varphi_2 & 0 \\
0 & -\cos \varphi_2 & \sin \varphi_2 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\( L_{s2} \) is the \( 3 \times 3 \) submatrix \( M_{s2} \).

Meshing equation is met in contact point which expressed in \( S_s \) as follow

\[
(\theta_s, \varphi_s, \varphi_s) = \mathbf{n}_s \cdot \mathbf{v}_d^{(2)} = 0
\] (22)

The relative speed of meshing point can be expressed as

\[
\mathbf{v}_d^{(2)} = (\omega_d^{(2)} - \omega_d^{(1)}) \times \mathbf{r}_d - \mathbf{R}_j \times \omega_d^{(1)}
\] (23)

Where, \( \omega_d^{(1)} = [-1 \quad 0 \quad 0]^T \),

\[
\omega_d^{(2)} = \begin{bmatrix}
z_2 \\
0 \\
0
\end{bmatrix}
\]

\[
\mathbf{R}_j = \begin{bmatrix}
0 & -E & 0
\end{bmatrix}
\]

\( m_z = z_1 / z_2, z_1 \) is the number of pinion and \( z_2 \) is the number of gear.

According to Eqs. (21) - (23), \( \varphi_s \) can be solved, and expressed as

\[
\varphi_s = f(\theta_s, \varphi_s)
\] (24)
Substituting Eq. (24) in Eq. (21) the tooth surface \( r_i(\theta_1, \varphi_1) \) and vector \( n_i(\theta_1, \varphi_1) \) of gear represented in \( S_z \) can be obtained.

Then the corresponding points in pinion can be deduced as

\[
\begin{align*}
\mathbf{r}_i &= M_{\omega r} \mathbf{r}_i(\theta_1, \varphi_1) \\
\mathbf{n}_i &= L_{\omega r} \mathbf{n}_i(\theta_1, \varphi_1)
\end{align*}
\]  

(25)

Where

\[
M_{\omega r} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \varphi_i & -\sin \varphi_i & E \\ 0 & \sin \varphi_i & \cos \varphi_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\]

And, \( L_{\omega r} \) is the 3 \times 3 submatrix \( M_{\omega r} \).

4.2 Flank modification

A second order surface is developed to modify the conjugate tooth surface in order to regulate the curvature that aims at get a well performance which represents as

\[
\delta = a_t X^2 + a_r Y^2 + a_{XY} XY
\]

(26)

Where, \( X \) and \( Y \) are along tooth length and tooth profile, respectively. \( a_t \) represents the factor of tooth lengthwise crowning that modifies tooth lengthwise crowing, \( a_r \) represents the factor of tooth profile crowing that modifies tooth profile crowing, \( a_{XY} \) represents the factor of tooth longitudinal twist that modifies tooth longitudinal twist.

Equation (26) is appropriate for both convex side and concave side. Adjusting six coefficients, and then a different modification model can be built; here a sketch (only concave side) is listed in Fig.7.

![Modification of tooth surface.](image)

Fig.7 Modification of tooth surface.

The quantity of modification is calculated according to Eq. (26), represented as \( \delta \). The target tooth surface is established by Eq. (25) and Eq. (26), and represented as

\[
r_i(\theta_1, \varphi_1) = r_i(\theta_1, \varphi_1) + \delta \cdot n_i(\theta_1, \varphi_1)
\]

(27)

5. Calculate correction of machine-tool settings

The machine-tool settings are corrected to eliminate the deviations between target and original tooth surface in order to obtain a well mesh performance with the exist gear. The flank topographical correction method has been broadly developed and has been proven to be a reliable and effective method for reducing manufacturing errors.

First the deviations between target and original tooth surface can be expressed as follow
\[ r = r_p - r_p' (\theta_p, \varphi_p) \]  

Taking machine-tool settings into account, Eq. (28) can be represented as

\[ r = r_p - r_p' (\theta_p, \varphi_p, \psi_p) \]  

Then differential coefficient of Eq. (29) can be expressed

\[
\delta r = \left( \frac{\partial r}{\partial \delta s_p} \times \delta s_p + \frac{\partial r}{\partial \theta_p} \times \delta \theta_p + \frac{\partial r}{\partial \psi_1} \times \delta \psi_1 + ... + \frac{\partial r}{\partial \psi_i} \times \delta \psi_i \right) 
\]

(30)

Taking dot product of Eq. (30) with the vector \( n \),

\[
\delta r \cdot n = \left( \frac{\partial r}{\partial \psi_1} \cdot n_p \times \delta \psi_1 + \frac{\partial r}{\partial \psi_2} \times \delta \psi_2 + ... + \frac{\partial r}{\partial \psi_i} \times \delta \psi_i \right) \cdot n
\]

(31)

For vectors \( \frac{\partial r}{\partial \psi_i} \) and \( \frac{\partial r}{\partial \theta_p} \) are perpendicular to the surface normal \( n \), so it can be simplified as follows

\[
\delta r \cdot n = -\left( \frac{\partial r}{\partial \psi_1} \cdot n_p \times \delta \psi_1 + \frac{\partial r}{\partial \psi_2} \times \delta \psi_2 + ... + \frac{\partial r}{\partial \psi_i} \times \delta \psi_i \right)
\]

(32)

Where \( \delta r \cdot n \) denotes the deviation of tooth surface, \( \frac{\partial r}{\partial \psi_j} \) denotes the influence coefficient of machine-tool setting.

Using \( \Delta \delta \) represents the deviation of grid points, and then the sensitivity coefficient of parameter \( j \) is as follows

\[
\eta_j = \frac{\Delta \delta_j}{\Delta \psi_j}
\]

(33)

As mentioned, the tooth surface deviation of grid points which is made up of \( i \) \( (i = 2 \cdot m \cdot n) \) points can be written in matrix form as Eq. (34) and simplified as Eq. (35).

\[
\begin{bmatrix}
\Delta \delta_1 \\
\Delta \delta_2 \\
\Delta \delta_3 \\
... \\
\Delta \delta_j
\end{bmatrix} =
\begin{bmatrix}
\eta_{i_1} & \eta_{i_2} & \eta_{i_3} & ... & \eta_{i_k} \\
\eta_{i_2} & \eta_{i_3} & \eta_{i_4} & ... & \eta_{i_k} \\
\eta_{i_3} & \eta_{i_4} & \eta_{i_5} & ... & \eta_{i_k} \\
... & \eta_{i_2} & \eta_{i_3} & \eta_{i_4} & ... & \eta_{i_k} \\
\eta_{i_k} & \eta_{i_1} & \eta_{i_2} & \eta_{i_3} & ... & \eta_{i_{k-1}}
\end{bmatrix}
\begin{bmatrix}
\Delta \psi_1 \\
\Delta \psi_2 \\
\Delta \psi_3 \\
... \\
\Delta \psi_j
\end{bmatrix}
\]

(34)

\[
\begin{bmatrix}
\Delta \delta_1 \\
\Delta \delta_2 \\
\Delta \delta_3 \\
... \\
\Delta \delta_j
\end{bmatrix} =
\begin{bmatrix}
\eta_{i_1} \\
\eta_{i_2} \\
\eta_{i_3} \\
... \\
\eta_{i_j}
\end{bmatrix}
\begin{bmatrix}
\Delta \psi_1 \\
\Delta \psi_2 \\
\Delta \psi_3 \\
... \\
\Delta \psi_j
\end{bmatrix}
\]

(35)

Where, \( \eta \) represents the sensitivity coefficient of machine setting parameters \( j \) in point \( i \). \( \begin{bmatrix} \eta \end{bmatrix} \) represents the sensitivity matrix, \( \{ \Delta \delta \} \) represents the deviations between target and original gird points, \( \{ \Delta \psi \} \) represents the modification of machine-tool settings.

There are far more grid points than the machine-tool settings in Eq. (35), so it is over determined. The least squares method is used to solve the corrections.

\[
\begin{bmatrix} \Delta \psi \end{bmatrix} = \begin{bmatrix} \eta \end{bmatrix}^{-1} \begin{bmatrix} \eta \end{bmatrix}^{-1} \{ \Delta \delta \}
\]

(36)

The machine-tool settings will be corrected with the corresponding coefficients until the corrected tooth surface have a well mesh performance.
6. Numerical example and machine experiment
6.1 Numerical example

A couple that the basic parameters are given in Table 1 is selected as a number example to illustrate the proposed method. The machine-tool setting parameters are calculated according to the proposed method, and given in Table 2.

| Parameters                  | Pinion (Left hand) | Gear (Right hand) |
|-----------------------------|--------------------|-------------------|
| Number of teeth             | 18                 | 35                |
| Pitch cone angle/°           | 27.21611           | 62.78389          |
| Face cone angle/°            | 31.39276           | 65.58103          |
| Root cone angle/°            | 24.41897           | 58.60724          |
| Module/mm                   | 7.16               |                   |
| Face width/mm               | 53                 |                   |
| Shaft angle/°               | 90                 |                   |
| Mean spiral angle/°          | 35                 |                   |
| Pressure angle/°             | 20                 |                   |
| Whole depth/mm              | 13.66683           |                   |
| Cone distance of reference point/mm | 108.6804 |                   |

Table 2  Machine-tool setting parameters.

| Parameters                  | Pinion       | Gear          |
|-----------------------------|--------------|---------------|
| Radial setting/mm           | 103.0815     | 103.0815      |
| Initial cradle angle setting/° | -65.27172   | 65.27172      |
| Machine root angle/°        | 24.417       | 58.6          |
| Vertical offset, mm         | 0            | 0             |
| Machine center to back/mm   | 0            | 0             |
| Sliding base/mm             | 0.97         | 0.97          |
| Roll ratio                  | 2.186519     | 1.124495      |
| Cutter radius/mm            | 114.3        | 114.3         |
| Point width/mm              | 3.09         | 3.09          |
| Cut number                  | 12           | 12            |

The building of ease-off topography is presented in detail in the literature (Nie 2018). The ease-off topography between the original and conjugate tooth surface of pinion are established, as shown in Fig.8.

![Ease-off topography between original and conjugate tooth surface of pinion](image)

Fig.8  Ease-off topography between original and conjugate tooth surface of pinion.

As Fig.8 shows, for the concave side, the biggest indifference is 0.30064mm in the topland of heel, and as more offset reference point, the indifference becomes bigger. Then the contact pattern may be in the around of reference point. For the convex side, the biggest indifference is 0.3125mm in the root of heel. In order to display contact pattern more intuitive, a distribution of deviation in tooth surface is drawn as Fig.9. For elastic deformation is 0.00635mm, so
the gradient is selected to distinguish the deviations in different colors.

![Image](a) Concave of pinion  (b) Convex of pinion

Fig.9  Distribution of deviations between original and conjugate tooth surface of pinion.

As shown in Fig.9, for concave and convex side of pinion, the tooth contact patterns are nearly perpendicular to the root line and there is no edge contact. But unfortunately the tooth lengthwise crowing, tooth profile crowing and tooth longitudinal twist are undesirability that the tooth contact pattern will be too short. Therefore the polynomial coefficients that listed in Table.3 are applied to modify the conjugate tooth surface.

|                | $a_1$   | $a_2$   | $a_3$   |
|----------------|---------|---------|---------|
| Concave of pinion | 0.00027 | 0.00018 | 0.00015 |
| Convex of pinion  | 0.00032 | 0.00025 | -0.00013 |

The deviations between target and original tooth surface are as shown in Fig.10.

![Image](a) Concave of pinion  (b) Convex of pinion

Fig.10  Deviations between target and original tooth surface of pinion.

As Fig.10 illustrates, for the concave side, the biggest deviation is 0.081893mm that occurs in the topland of heel; for the convex side, the biggest deviation is 0.057434mm that occurs in the root of heel. Then the machine setting parameters are corrected by the method proposed in this paper. Need to supplement, if possible the tool parameters can also be corrected, while only machine setting parameters are corrected in this paper. The corrections are listed in Table.4.

| Parameters                      | Correction   |
|---------------------------------|--------------|
| radial setting/ mm              | -1.240238    |
| initial cradle angle setting/ ° | 0.085152     |
| machine root angle/ °           | -1.525668    |
| vertical offset/mm              | 0.846601     |
| machine center to back/ mm      | -0.012888    |
| sliding base/mm                 | 0.972381     |
| roll ratio                      | -0.015783    |

The deviations between corrected and original tooth surface are as shown in Fig.11.
As Fig. 11 illustrates, for the concave side, the biggest error between target and corrected tooth surface of pinion is 0.009098mm that occurs in the topland of heel, the biggest error decrease from 0.081893mm to 0.009098mm, and the errors mainly exists in the edge of the tooth surface; for the convex side, the biggest error is 0.009689mm that occurs in the root of toe, the biggest error decrease from 0.057434mm to 0.009689mm. Comparing to Fig.10, the errors decrease obviously, and the target tooth surfaces are approximate achieved.

The ease-off topography correspond to the corrected machine setting parameters are shown in Fig.12, and distributions of deviations are shown in Fig.13.

Referring to Fig.9 and Fig.13, the tooth contact patterns are becomes big along the tooth length, and the bias in is more obvious. Tooth contact analysis is carried out using the corrected machine-tool settings, the results are shown in Fig.14.
6.2 Machine experiment

Based on the approached method, an experiment is carried out by 4-axis CNC milling machine YK2260X. The pinion and gear are processed respectively both two sides. Instantaneous states during processing are shown in Fig.15.

![Gear process and Pinion process](image)

(a) Gear process  (b) Pinion process

Fig.15 Machine experiments.

Comparing the model (Fig.5) and product (Fig.15), they are in harmony with each other. Finally, rolling test is carried out after fillet. The scene of rolling test and contact patterns are showed in Fig.16. The transmission is steady without obvious vibration.

![Rolling test and results](image)

(a) Rolling test  (b) Convex contact pattern of gear  (c) Concave contact pattern of gear

Fig.16 Rolling test and results.

In order to make it is easy to understand the match of the actual tooth contact and the result of calculation. A comparison is carried out through the position of contact and the contact path. The rubbings of real contact pattern are made, and the position and contact path are mapped. The results are showed in Fig.17.

![Real and theory contact pattern comparison](image)

(a) Real contact pattern of gear convex  (b) Real contact pattern of gear concave  (c) Theory contact pattern of gear convex  (d) Theory contact pattern of gear concave

Fig.17 Real and theory contact pattern comparison.
As showed in Fig.17, (17a) is real contact pattern of gear convex. The angle between contact path and pitch line is 40.5°, and the distance between the middle point of contact and toe is 18.4mm, while the corresponding data showed in Fig.17(c) are 40.9° and 18.96mm. 17(b) is real contact pattern of gear concave. The angle between contact path and pitch line is 40.5°, and the distance between the middle point of contact and toe is 19.72mm, while the corresponding data showed in Fig.17(d) are 41° and 19.63mm.

From the comparison in the position and contact path, results of real contact pattern and TCA are matched. The difference may be caused by machine errors and installment errors during the process.

7. Conclusion

(1) In this paper, duplex spread blade method is proposed to machine spiral bevel gear. Geometry parameters and machine-tool setting parameters are comprehensively considered. The parameters are deduced in reference point which is calculated by a new way.

(2) A modification by second order surface is proposed to modification tooth lengthwise crowing, tooth profile crowing and tooth longitudinal twist for two sides simultaneously in order to get a well mesh performance.

(3) An experiment is carried out by a 4-axis CNC milling machine, results of real contact pattern and TCA are matched through the comparison in the position and contact path. It demonstrates that the proposed method to process spiral bevel gear by duplex spread blade is effective and feasible.

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