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Attention based parameter estimation and states forecasting of COVID-19 pandemic using modified SIQRD Model

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A B S T R A C T

In this work, we propose a new mathematical modeling of the spread of COVID-19 infection in an arbitrary population, by modifying the SIQRD model as m-SIQRD model, while taking into consideration the eight governmental interventions such as cancellation of events, closure of public places etc., as well as the influence of the asymptomatic cases on the states of the model. We introduce robustness and improved accuracy in predictions of these models by utilizing a novel deep learning scheme. This scheme comprises of attention based architecture, alongside with Generative Adversarial Network (GAN) based data augmentation, for robust estimation of time varying parameters of m-SIQRD model. In this regard, we also utilized a novel feature extraction methodology by employing noise removal operation by Spline interpolation and Savitzky–Golay filter, followed by Principal Component Analysis (PCA). These parameters are later directed towards two main tasks: forecasting of states to the next 15 days, and estimation of best policy encodings to control the infected and deceased number within the framework of data driven synergetic control theory. We validated the superiority of the forecasting performance of the proposed scheme over countries of South Korea and Germany and compared this performance with 7 benchmark forecasting models. We also showed the potential of this scheme to determine best policy encodings in South Korea for 15 day forecast horizon.

1. Introduction

Owing to significant transmission and fatality rates [1] of the SARS-COV-2, the principle virus behind the current pandemic, COVID-19 has led to major economic, political and social impact globally. Even though the systematic vaccination schemes have covered a significant part of the globe, leading to overall damping of infected numbers, the current pandemic is still at large with persistence of the same challenges [2]. The dynamics of the loss caused by the virus fundamentally begin with spreading operation, which can comprise several spatial and temporal modes like transmission from local infected objects or people spatially, or transmission from asymptomatic cases temporally. Therefore, one of the challenges at the governmental level is to implement measures to control the spread through actions like contact tracing, social distancing, enforcement of mask wearing and quarantine, etc. The enforcement of these policies and vigorous vaccination implementations has helped control the intensity of the pandemic, but its status is still unclear and gets worsened due to the continuous emergence of new variants of the virus, as well as transmissibility from asymptomatic individuals.

On the other hand, such measures as contact tracing can not only prove to be costly but indirectly negatively impact the economical and social lives of citizens [3]. Therefore the optimal policies to be determined must be necessary to reduce the impact of a pandemic, while having minimal impact on lives of people. In this course, modeling of several states of the pandemic is essential to determine the current and future trends of the pandemic to determine appropriate policies accordingly.

Henceforth, one of the active research topics in this course is not only modeling of pandemic states but also the determination of optimal policies to effectively mitigate the spread of the virus amongst the population, as well as reduce the number of casualities. In literature, usually, the topics of modeling and inference of policies are considered as separate topics, with later sometimes loosely connected to the prior topic.

The topic of modeling of pandemic states is usually dealt as a forecasting problem or fitting of a mathematical model on a real world data. In consideration of a forecasting model, a black box model like

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optimal strategies for governmental intervention to be real-world data-limited to simulations. In this regard, we believe that the designing of and social distancing based strategies, but these works are largely control [42] have considered strategies like pharmacological, isolation, etc. [39]. But this consideration may not be practical, as the individual epidemiologically its the responsibility of governments to design suitable policies from one compartment to another. These compartment models are easily transformed into a non-linear differential system, that can be tackled with techniques from dynamical systems. The compartment models add a layer of interpretability to such models. In this respect, many models have been proposed like SIR [19], SEIR [20–22], SITR [23], SIQRD [24], SIHR [25], SEQIR [26] and SEIRQJD [27] to name a few. Of course, while complexification of these models either by the addition or introduction of more states, considering fractional order version of these models [28] strengthens the conformity of these models with real-life situations, but this increment of degrees of freedom of the system directly impacts the computations [29], existence of solutions in a certain domain, as well as chaotic characteristics of the differential system [30]. Furthermore, usually data available may not comprise the necessary information to derive all states of a complex model. Therefore, a mathematical model with enough complexity, not more, is a necessity in modeling of real life pandemic.

The parameters of these mathematical models capture the global behavior of the trajectories of epidemic numbers, which may be inferred or interpreted as the behavior of the population in terms of mobility [20], as well as transmissibility of virus [31] that may lead to the implementation of measures [21] like social distancing, quarantine management, mandatory mask wearing and closing of public and private places. In literature, these parameters are either assumed, or estimated through optimization techniques from the domain of nature-inspired algorithms [23], Bayesian algorithms [32] and gradient-based algorithms [33], in application of fitting the real world data over differential system [34], basis functions [35], spline fitting [36], Kalman filter [37], neural nets [38] and even recurrent nets [39]. Furthermore, stochastic [31] or smooth [36] perturbations in the parameters of these models have been studied that lead to the better fitting of the epidemic models over real-world data. Not only this but these parameter variations have been studied to be essential, as the fluctuations in infection-related parameters were found to be linked with the stability of infected curves [31].

In terms of deriving necessary policies, to control the pandemic via forecasting or mathematical models, is not found to be automated, but primarily based on modeled educated inferences. In this regard, the application of control theory has proven to be fruitful by introducing several control variables that model either individual or collective behavior like a population’s conscious desire to use masks or antibiotics, etc. [39]. But this consideration may not be practical, as the individual behavior of the population cannot be directly controlled, but realistically its the responsibility of governments to design suitable policies in order the mitigate and control the pandemic. In other works on Lyapunov functions [40], backstepping control [41] and sliding mode control [42] have considered strategies like pharmacological, isolation, and social distancing based strategies, but these works are largely limited to simulations. In this regard, we believe that the designing of optimal strategies for governmental intervention to be real-world data-driven. In another direction, governmental policies have been used to further reparametrize mathematical models [24] as means to improve predictions the mathematical models. These mathematical models have also been frequently hybridized in deep learning schemes [7,43] to improve the robustness and accuracy of state prediction. Furthermore, these mathematical models may additionally be combined with data like mobility statistics or positive test rates, and have been reported to produce better predictive capabilities, over changing population behaviors and implementation of policies.

In this paper, our focus is towards mathematical models, more precisely SIQRD model [24,44] which comprises five states of the COVID-19 pandemic. This model has been previously studied to model the effect of different scenarios like information communication in dynamics of an arbitrary disease outbreak [45]. This model has also been previously studied to identify dynamics and forecasting of the spread of the Ebola virus [46,47] by the addition of a new state termed ‘Exposed’ between susceptible and infected states. This state referred to susceptible people during of period of being infected, while corresponding symptoms of the disease have not become visible yet. In COVID-19 context, this model has been largely employed to study the effects of different scenarios like demotivation to follow governmental guidelines [44], vaccine allocation [48] and political decisions [24] onto states of COVID-19 pandemic.

It has been noted in the literature that while the mathematical models perform well for short-term data, their long-term performance significantly deteriorates [49]. Whilst some studies have considered increasing the accuracy and robustness of the mathematical models over variable parameters and noisy data, by complimenting them with deep learning models [16–18], there is limited work on the systematic relationship of these parameters identified with optimal policies that require implementation by government. As already discussed, the work on deriving optimal policies is also limited due to negligible real world data consideration. Even if we extend our focus beyond differential models [16–18], we can identify another major limitation that the other category mathematical models either realize a single or a few set of specific policies into identifying control of pandemic dynamics. While a more realistic world scenario is design of much broader set of governmental policies that not only mitigate the effect of pandemic, but also supporting several other private and public sectors.

On the other hand, inference of certain policies from the trajectory of states or estimated parameters may be prone to a lack of robustness and human error. Furthermore, in many of the present approaches, the modifications of base SIR model are performed by the addition of states over real-world considerations without regarding the feedback effect on already present states, i.e. infected, susceptible and removed.

In another direction, in consideration of deep learning application to mathematical models, the resultant hybrid models have been mostly tested upon the city and provincial datasets, where the population is only a small fraction of the whole country. It is plausible that the errors associated with these models might become significant in consideration of the whole country. Not only this but these models have been used in forecasting selective states, not all states like deceased, infected and recovered numbers, etc., which are granted with much lesser magnitudes than other states like susceptible numbers. This consideration might deteriorate the overall accuracy of models upon all states. In the same context, there is a very sparse consideration of feature extraction to improve the accuracy of models. One study [7] employed principal component analysis (PCA) as a mean for data compression and feature extraction from states, but prior denoising of states was not considered, which is important since PCA is sensitive to noise. Furthermore, the context of data augmentation to improve the generalizability of such models is absent.

In direction of control theory application to stabilize the pandemic dynamics, while works on control theory branches like optimal control [39,50] have research upon, it has been observed that application of synergetic control theory [51] has been absent. This particular
technique will surely be one of the tools we bring into live in our current research work.

Attention-based models have proven to be state-of-the-art models in natural language processing and computer vision tasks, where fundamentally these give priority to a certain part of the input sequences, on contrary to the recurrent model where they give priority to every part of the sequence. From a forecasting perspective, they have been employed to predict COVID-19 states [52], but their application has been absent in the context of mathematical models.

Based on these findings, we essentially propose a novel scheme centered around a mathematical model and deep learning model to allow forecasting of states, as well as determination of necessary policies to control the spread and causalities caused by the pandemic. In this regard, we proposed a new mathematical model, named 'm-SQIRD' model that can model states like susceptible, infected, recovered, deceased and quarantine states of the pandemic. It is in fact a modification of the SIQRD model already proposed in the literature, and we demonstrate the superiority of the proposed variation in several contexts. We associate a deep learning model, which is centered upon an attention model in combination with a CNN-GRU model, to allow estimation of time-varying parameters of the m-SIQRD model. We introduce a robust to noise feature extraction method from state data, comprising techniques like PCA, cubic spline interpolation, and Savitzky–Golay filter. Furthermore, we improve the generalizability and regularization of the deep learning model through data augmentation by Generative Adversarial Networks (GAN) models. Finally, we re-utilize the information from estimated parameters to allow the determination of optimal policies to control the infected and deceased numbers in the future, using deep learning and synergetic control theory [51]. Our proposed approach of determining optimal policies is greatly flexible and can accommodate arbitrary number of policies. The novelties in our approach lie in the utility of new models in each application, methodology to estimate optimal parameters using control theory, and consideration of dataset with policy information. Not only this, but we also compared the forecasting performance of our scheme with 7 other state-of-the-art forecasting approaches, and showed that our model dominates in overall performance. Furthermore, we also provide novel proposals to improve this scheme and extend this work towards general time series related problems.

In summary, the contributions of our work are the following:

- proposed a novel deep learning scheme to utilize the modified SIQRD model in order to achieve objectives of state forecasting to next 15 days, as well as determination of optimal policies to control the infected and deceased numbers.
- proposed a novel feature extraction scheme for the proposed scheme that is robust to noise in data.
- proposed a novel parameter extraction scheme for the proposed scheme as an alternative to normalization and standardization, by decomposing the original parameters into pseudo-parameters and parameters.
- validated the forecasting performance of proposed scheme over South Korean and German datasets
- compared the performance of proposed scheme with 7 state of the art forecasting models.
- determined the optimal policies for the case of South Korea in the forecast horizon, in order to control the infected and deceased numbers.

The structure of this paper is as follows. Section 2 gives an overview of the datasets utilized, which is central to investigations of the performance of the proposed scheme. Section 3 first provides an introduction to prior and proposed SIQRD model. This is followed by the context to the deep learning models employed, dataset pre-processing pipeline, the architecture of the proposed scheme, and evaluation metrics employed in the paper to compare performance. Section 4 demonstrates and provides a performance comparison between these two models, in the domain of curve fitting, relationship to governmental policies, and their capacity as features to a deep learning model. Furthermore, this section also provides performance and validation of forecasting of proposed scheme on two country dataset, alongside with evaluation of optimal policies for case of Korea. Section 5 provides a drawback of the proposed scheme, and introduces their solution, as well as a sketch of further modifications to allow the proposed scheme applicable to general time series data. Finally, the conclusion is discussed in Section 6.

2. Dataset description

The dataset considered for application of the proposed scheme was availed from [53], which a metadata source on COVID-19 maintained by SNU ARIC. While there is a variety of COVID-19-related information provided regarding multiple countries, we shall focus on date indexed data on cumulative confirmed cases, cumulative deaths, cumulative recovered, and daily quarantined numbers, which are encoded as variable ‘MVAR1’, ‘MVAR2’, ‘MVAR3’, ‘MVAR4’ respectively in the dataset. Furthermore, there is also information on the date-indexed

| Table 1 |
| Description of policy related variables in the utilized dataset. |

| Policy # | Variable | Description | Encoding description |
|----------|----------|-------------|----------------------|
| 1        | OVAR1    | School closing | 0 (no action), 1 (recommend closing), 2 (require closing partially), 3 (require closing fully) |
| 2        | OVAR3    | Workplace closing | 0 (no action), 1 (recommend closing), 2 (require closing partially), 3 (require closing fully) |
| 3        | OVAR5    | Public event cancelling | 0 (no action), 1 (recommend closing), 2 (require cancelling) |
| 4        | OVAR7    | Gathering restriction | 0 (no action), 1 (restriction on gathering above 1000), 2 (restriction on gathering between 101-1000), 3 (restriction on gathering between 11-100), 4 (restrictions on gathering on 10 people or less) |
| 5        | OVAR10   | Inter-city travel restrictions | 0 (no action), 1 (recommend to not travel), 2 (restrict travel) |
| 6        | OVAR15   | International travel restrictions | 0 (no action), 1 (screening arrivals), 2 (quarantine arrivals), 3 (partial ban arrival), 4 (fully ban arrival) |
| 7        | OVAR24   | Contact tracing | 0 (no action), 1 (limited contact tracing), 2 (comprehensive contact tracing) |
| 8        | OVAR40   | Mask wearing | 0 (no action), 1 (recommend), 2 (required partially in some public places), 3 (required fully in all public places), 4 (required fully outside home) |
3. Proposed scheme

The proposed scheme revolves around a proposed modification of SIQRD model (as ‘m-SIQRD’ model), that is complemented with deep learning, more specifically through attention-based deep learning architecture to estimate the parameters of m-SIQRD model. This estimation is further proceeded with forecasting of states of m-SIQRD model, as well as determination of optimal policies to control the curves of infected and deceased numbers. The logical flow of the proposed scheme is represented in Fig. 1. In this respect, prior 3 days state data is given as an input to this scheme to allow forecasting to next 15 days, with the determination of optimal policies to be implemented at that time to control the number of the deceased and infected population.

In simplification, the proposed scheme comprises of m-SIQRD model and the deep learning framework, which is described in next of subsections.

3.1. The m-SIQRD model

3.1.1. Brief introduction to SIQRD model

Starting from the basic SIR (Suspected–Infected–Removed) model, in an attempt to mathematically describe the spread of infectious diseases [19], more states and parameters have been added, in order to capture few essential aspects of an ongoing pandemic caused by governmental interventions like quarantine measures [24], vaccination efforts [44,55] and even particular states like “Removed” are decomposed into several decompositions like deceased and recovered. In this pursuit, several compartment topologies have been proposed, but the main consideration of this paper is the SIQRD model, which is originally represented as a compartment model, as shown in Fig. 2(a).

In this model, $S$, $I$, and $R$ represent the number of susceptible, infected, and recovered individuals, respectively, and $Q$ represents the number of deceased individuals. The parameters $\beta$, $\gamma_1$, and $\gamma_2$ represent the infection rate, recovery rate, and death rate, respectively. The model is defined by the following system of differential equations:

$$\dot{S} = -\beta SI$$  \hspace{1cm} (1)
$$\dot{I} = \beta SI - \gamma_1 I - \gamma_2 I$$ \hspace{1cm} (2)
$$\dot{Q} = \gamma_1 I - \alpha_1 Q - \delta Q$$ \hspace{1cm} (3)
$$\dot{R} = \gamma_2 I + \alpha_2 Q$$ \hspace{1cm} (4)

These equations describe the dynamics of the model, where $\dot{S}$, $\dot{I}$, $\dot{Q}$, and $\dot{R}$ represent the time derivatives of the susceptible, infected, deceased, and recovered populations, respectively.
The description of each state is described below:

- **S**: Daily number of susceptible part of population, that are possibly at risk of getting infected
- **I**: Daily number of infected part of population
- **Q**: Daily number of quarantined individuals in accordance with contact tracing or governmental policy operation
- **R**: Daily number of recovered individuals, who were previously infected, after being treated from COVID-19 infection
- **D**: Daily number of deceased individuals

The corresponding parameters \(a, \beta, \gamma, \delta\) signify quarantine rate, infection rate, the recovery rate of infected population that is not tested, the recovery rate of infected population that is tested positive and death rate respectively. The sum of the states of the SIQRD model leads to the total population ‘\(N\)’, which is assumed to be constant over the trajectory of states.

### 3.1.2. Modifications of the SIQRD model

We perform a major modification to the prior SIQRD model by first realizing that the SIQRD model takes into account the human intervention to control the trajectory of the pandemic, and in this respect, a fundamental term can be realized, which is \((N - S)\). This term objectively implies how much susceptible population is close to the total population. In the worst case of a pandemic, this term should be greater due to the dominance of the infected population, hence the objective of human intervention is to reduce the term \(N - S\), and the infected population variation should be proportional to this term. Certainly, a pandemic has ended when this term is minimum for the largest amount of time. We may further remark from this that in a controlled pandemic environment, the term \(\int_{0}^{\infty} (N - S) dt\) should be minimized for some large \(\varepsilon\), which may perhaps invoke the application of the calculus of variations. But for our proposed model, we rely on the interpretable importance of the term \((N - S)\) on variations of infected and susceptible numbers, and we term it as ‘pandemic loss’.

We may realize another important aspect of authority-controlled pandemic environments that since enforcement of quarantine measures has drastic effect in reduction in causalities [56], hence the variations in deaths number should be dominated by the quarantine numbers. Henceforth, the quarantine trajectory requires careful modification to allow the modified SIQRD to approximate well on the real-world data. The first thing to conceptualize is that in the case of governmental interventions, even if infected and quarantined numbers become zero, the quarantine measures are not immediately lifted, but instead there is latency in the process, leading room for some residual quarantine number variations. Additionally, the variation in quarantine numbers is proportional to infected numbers, since the detection of greater infected numbers would force the government to strengthen quarantine measures [57], while at the same, this variation is controlled by the magnitude of the already quarantined number. To this end, in a consideration of government monitored epidemic system, where quarantine numbers form a monotonic one-to-one mapping to infected numbers, the deceased number variations should be controlled by quarantine number, in proportion to the quarantine rate. In this course, fraction of quarantine numbers may become recovered, a fraction of which may become susceptible again.

While quarantine numbers play a dominant role in controlling the number of deaths, fundamentally deaths occur as a fraction of infected individuals. Here we propose the scaling of this fraction of infected individuals by deceased numbers by considering some facts. The deceased numbers are usually much stationery in variations, in governmental intervention controlled and hospitalization resourceful parts of the country, even though the infected numbers are prone to drastic fluctuations. In addition, the amplitude and variations associated with death numbers is much lower than infected numbers.

Therefore, this scaling by deceased numbers allows regularization of variation of deceased numbers from a rather unrealistic spontaneous transformation from infected to a deceased individual.

Historically, overshoots in infected cases are unavoidable, even with the implementation of rigorous prevention measures. These phases are usually termed as ‘\(n\)-th waves’ [58] and they occur mainly due to non-deterministic transmission from asymptomatic cases, alongside with evolution of variants of SARS-CoV-2 virus [59], we may conclude that the change in infected population is not only proportional to the current infected number but also the time, in the neighborhood of critical time gap, beyond which the transmission of asymptomatic cases reaches significant levels and the infected number starts overshooting. Due to the random nature of the occurrence of this phenomenon, there is a lack of synchronicity in implemented measures to deal with the corresponding situation, leading to an inverse effect in the number of recovered individuals. Furthermore, an increase in the number of infected persons, leads to an overburden of testing and treatment facilities, thus providing less room for the influx of patients due to treatment progress of already filled positions, leading to an overall decrease in the number of recovered cases in general.

In above contexts, we propose modification of SIQRD model with a new topology defined by a differential system as presented by Eqs. (6) to (10).

\[
\begin{align*}
S &= -\zeta (N - S) + \gamma R \\
I &= \zeta (N - S) - \beta_1 (t_o - t) I - \beta_2 I - K - \delta I D \\
Q &= \beta_2 I + K - \alpha_1 Q - \alpha_2 Q \\
R &= \beta_1 (t_o - t) I + \alpha_1 Q - \gamma R \\
D &= \delta I D + \alpha_2 Q
\end{align*}
\]

Where \(\zeta\) is the pandemic loss coefficient, \(\beta_1\) and \(\beta_2\) are the primary and secondary infection rate respectively, \(\alpha_1\) and \(\alpha_2\) are the primary and secondary quarantine rate respectively, and \(K\) is the residual quarantine constant. \(\delta\) is the death rate, while \(t_o\) signify time gap associated to an overshoot behavior in infected numbers. The corresponding compartment model of the m-SIQRD model is provided in Fig. 2(b).

It is clear from equation system (6) to (10), that the sum of derivative of all states of m-SIQRD model is equal to zero, in the same way as the prior SIQRD model. This would imply that net sum of all states of particular state is a constant. Additionally, it can be verified that if we represent the m-SIQRD model as \(u(t) = f(u(t))\), for \(u(t) \in \mathbb{R}^3\), then Eq. (6) to (10) would have unique solutions in some neighborhood of initial time \(t_o\). Here \(u(t)\) would encode the states of m-SIQRD model. This is possible since the \(f(u)\) is Lipschitz continuous in \(u\), satisfying the condition for Picard–Lindelöf theorem [60], within the context of m-SIQRD differential system (6) to (10).

Further empirical analysis on comparison of proposed m-SIQRD model and prior model is described in section ‘Results and Discussion’.

### 3.2. Deep learning framework

The second part of the proposed scheme is the deep learning framework, and is composed of three abstract deep learning models, whose individual purpose is described below:

- **Approximator Model**: This model is composed of a attention based architecture followed by CNN-GRU architecture to approximate the time-varying parameters of SIQRD model given 3 day state data.
- **Generator Model**: This model is used to augment the training dataset for the approximator model, to improve its generalizability, robustness and accuracy.
3.2.1. Deep learning units

To construct the approximator, generator, and inference models, the following basic deep learning units were utilized for their construction, and are briefly described below. Further theoretical and mathematical details of these units can be discovered in [12,61–64]. The architectures of these units and their intrinsic mathematical operations are highlighted in Fig. 3.

3.2.1.1. Multi-layer perceptron (MLP) unit. MLP is an orchestration of fundamental units, comprising a linear mapping followed by a non-linear mapping, in a topology of a feedforward network [61]. The linear mapping is matrix multiplication, followed by the application of choice non-linear activation functions like ‘ReLU’, ‘Tanh’ or ‘Sigmoid’ to name a few. While MLPs are known to outperform traditional machine learning methods in the most task, they also have the capacity to be included in every complex deep neural network due to their versatile nature.

3.2.1.2. Multi-headed attention unit. Transformers, and their variants, are a state of the art of deep learning family, that were originally proposed to solve NLP tasks, but quickly evolved to solve general sequential learning problems. The original transformer model [12], originally proposed in 2017, had encoder–decoder abstractions, composed of self-attention modules where scores were awarded for relative importance to each part of a target sequence, using parameters ‘queries’, ‘keys’, and ‘values’. But these computations could be parallelized by decomposing these parameters into independently learned linear projections, to be fed into independent attention mechanisms, named ‘Attention Heads’, that can produce individual outputs, which can be combined later by concatenation, followed by a linear transformation by MLP to produce the final output. This would allow learning different behaviors and dependencies at various levels within a sequence, for example local vs...
global scale. The union of these attention heads forms a multi-head attentions unit, and is depicted in Fig. 3(b).

3.2.1.3. Gated Recurrent Unit (GRU). GRU was proposed in order to mitigate the drawbacks of RNN, i.e. they experience the gradient vanishing [62] and exploding problems [65]. They are endowed with a similar structure to that of rather more popular architecture, LSTM, but comprise of lesser number of operations and parameters. While generally LSTM surpass GRU in terms of accuracy, GRU is usually faster in training time than LSTM, up to 30 percent [65], due to the fact that they have much less trainable parameters. Furthermore, they are also known to surpass in accuracy scores [65] upon small datasets, as compared to LSTM.

3.2.1.4. Convolution unit. These units are integral parts of convolutional neural networks (CNN), besides pooling and MLP units, and provide a methodology to extract local features from the data [66], that have lesser dimensionality as compared to original data. This is achieved by the means of convolution with a kernel tensor, and provides a contrasting operation to standard tensor multiplication.

3.2.1.5. Deconvolution unit. Also referred to as “Transposed Convolution Units”, these units utilize transposed convolution operation, that is in a vague sense an inverse operation to convolution operation, such that where convolution units reduce the feature size, the deconvolution units tend to upsample the corresponding feature size [63]. These units tend to improve the signal-to-noise of resultant output from lesser amount of information, which fundamentally dictates its point of usage.

3.2.2. Dataset processing

For the processing of available datasets, to be later used by scheme models, first the state data for S, I, Q, R and D is determined from variables ‘MVAR1’, ‘MVAR2’, ‘MVAR3’ and ‘MVAR4’ in the dataset, by converting the cumulative numbers to daily numbers.

These state trajectories are further partitioned into sets of 3 days, in order to produce samples of dataset alongside with associated parameters. The reason for 3 day partition is that for the determination of 6 parameters of m-SIQRD model, at least 3 days state data is required, since the differential system is defined by Eqs. (6), (7), (8), (9) and (10) can be discretized into matrix form, and for the condition of the determined system to be met, the corresponding matrix form should be adjoined at least two times, and the corresponding two-time difference operator would require 3 days state data. Furthermore, to capture the persistence of behavior of these states, not only the number of days, associated with consistent parameters should be minimum, but also the SIQRD model should fit over the whole dataset with associated parameters. In this case, the case of rolling window-based data partitioning strategy would violate the last aspect of the proposed scheme.

3.2.2.1. Feature extraction. We employ a novel feature extraction scheme, where in order to extract robust to noise and less correlated features, first each state data is interpolated to 60 samples via spline interpolation [67]. This is followed by application of Savitzky–Golay filter [67] to improve signal to noise ratio and smoothens the data. This step is also necessary for later application of principle component analysis (PCA), since this technique is sensitive to noise [68].

Each resultant state data is wrapped into 2D matrix form, on which PCA is performed to estimate principal components. The trajectory of each state, over different time intervals, can be thought of as a feature. These features may be correlated, assumed that in these time windows the parameters are fundamentally constant. These intrinsic features are then projected into a new coordinate system through principal component analysis, to determine uncorrelated features with an independent amount of information. These features are then concatenated to maximize the amount of information. Next we select the first PCA component as our feature, which has the maximum variance amongst all PCA components. Consequently, the feature has less dimensionality, i.e. one dimension, as compared to original states input with 5 dimensions.

3.2.2.2. Parameter extraction.

In a similar manner to feature extraction, the noise is reduced from state data by a sequence of spline interpolation followed by a Savitzky–Golay filter. The spline interpolation permits later calculation of derivatives present in the m-SIQRD model. Thereafter, the parameters are estimated from the corresponding state’s data by converting the equation system defined by (6), (7), (8), (9) and (10) into a squared L2 norm error term, with 6 unknown variables as parameters, as defined by Eq. (11).

\[
\theta (\xi, \alpha, \beta, \gamma, \delta, K) = \| S + \xi (N-S) - y \| _2^2 + \\
\| I - \xi (N-S) + \beta (I-\xi) + \gamma + \delta I D \| _2^2 + \\
\| Q - \beta I - K - \gamma Q + \alpha Q \| _2^2 + \\
\| R - \beta I (I-\xi - \gamma) I - \alpha Q + \gamma R \| _2^2 + \\
\| D - \delta I D - \alpha Q \| _2^2.
\]

Determination of parameters of m-SIQRD is equivalent to least squares minimization of residuals [69], which is equivalent to finding parameters \( \xi, \alpha, \beta, \gamma, \delta, \text{ and } K \), that minimize \( \theta \) from Eq. (11). To this end, interior point based minimization [70] is used to minimize this objective function, in order to determine the corresponding parameters.

3.2.2.3. Pseudo-parameter extraction. The magnitudes of parameters of m-SIQRD model may not be uniform, especially due to parameters \( \xi, \alpha, \beta, \gamma, \delta, \text{ and } K \), since their influence in Eqs. (7), (8), and (9) permit them to occupy greater magnitudes as compared to the rest of the parameters. In this context, a deep learning model would not prove robust in approximating the parameters, when the training proceeds with mean squared \( L_2 \) norm minimization, as this tends to minimize the error by homogeneously spreading it out over dimensions of the output [71], and thus the output predictions uniformly converge towards reference along the output dimensions. Furthermore, the same model has to perform additional learning of proper input representations to be mapped towards parameters, which leads to additional complexity and less room for convergence of parameters of the last layer to weights with high variance, to be able to approximate parameters with high variance as well. Techniques like data normalization and standardization [72] may transform the parameters to have equal contributions in magnitudes, which can be later inverse scaled, but such strategies have known to work well on inputs, while our consideration is output. Furthermore, \( L_1 \) norm-based training may also help with this situation, due to its sparsity-promoting nature [71], but this may lead to thresholding of fewer magnitude parameters to zero.

We propose a novel solution to this problem, by first decomposing the standard dataset consisting of feature-parameters tuple, into features-pseudo-parameters-parameters tuple. The pseudo-parameters act as an intermediate target for deep learning model, which is already well homogeneous, and can easily be learned by the original model. These parameters act as a ‘deformed’ version of original parameters and would comprise less variance. Then we introduce an auxiliary deep learning model, to map these pseudo-parameters towards original parameters, hence the auxiliary neural network trains its weights to be able to approximate the high variance inherent to these parameters. In this respect, we propose a process of extraction of pseudo-parameters from original parameters, as highlighted in Fig. 4(c).

In this scheme, the parameters are interpolated by spline interpolation, followed by a small Gaussian noise addition and Savitzky–Golay interpolation. This process reduces the standard deviation of absolute amplitudes, without significantly disturbing the characteristic shape of the original parameters. This later proceeds with downsampling the interpolated sequences to 6 samples. Lastly, the hyperbolic tangent operation is applied on a scaled version of this signal to normalize the data between –1 and 1. Scaling controls the amount of homogeneity of absolute amplitudes in the resultant operation.
3.2.3. Generator model

Generator Model comprises two generative adversarial networks (GAN), which function to generate new dataset on parameters and states independently, and thus generator models intend to augment the data to improve robustness to input variability, as well as generalizability [73] of the approximator model. Originally proposed in [74] as a generative framework, GANs have found applications in unsupervised, supervised, semi-supervised, and reinforcement learning problems. They are fundamentally a scheme to train an original deep learning model, namely ‘Generator’, by adding an auxiliary neural network ‘Discriminator’ that trains while monitoring the loss of Generator. Henceforth, the Generator is able to generate variations of data, that have similar morphology to original dataset.

In our application of GAN, we more specifically employ a variation of deep convolutional generative neural networks (DCGAN) [75] that utilize convolutional and deconvolutional layers in the Discriminator and Generator model respectively. In the generation of new parameters and states, ‘parameter GAN’ and ‘state GAN’ are constructed respectively. While the parameter GAN utilizes a standard DCGAN architecture, the state GAN has a similar architecture DCGAN structure, except the convolutional layers in the discriminator of parameter GAN are replaced with MLP. The reason for this is that for the training of state GAN on our chosen datasets, the discriminator model was overfitting on real data, leading to the underfitting of the generator. Thus, the Generator is able to generate variations of data, that have similar morphology to original dataset.

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The approximator model is an abstraction centered around the attention model, which intends to map features towards pseudo-parameters. This attention model begins with an encoder block comprising mainly of a multi-headed attention layer, followed by 1D-CNN layers that capture further features from the encoding of the relative importance of input features. This block is connected with another block of the same structure through residual connections [77], as a mean to increase the accuracy performance of the model with the increasing complexity of the architecture. The resultant sequence is mapped towards pseudo-parameters through a single layer MLP, based on the assertion in Section 3.2.2 that since parameters, or even their deformations as pseudo-parameters encode information of the global trajectory of states, therefore, two-way (past to present and future to future) sequence to sequence regression, forms an optimal choice for last recurrent structure to finally map towards pseudo-parameters.

For mapping of pseudo-parameters towards parameters, a CNN-GRU architecture is employed where CNN extracts the distinct features that are invariant to noise presence, while the GRU intends to exploit the temporal dependencies of those features to map towards parameters. In the end, an MLP is applied to apply corrections in predictions from GRU layers. As pointed out in Section 3.2.1.3 that GRU proves to be a better alternative to rather more popular LSTM, in the case of smaller datasets,
while providing more computational efficiency. Therefore, for the training of the 1DCNN-GRU model, the original dataset is utilized, instead of the augmented one, which not only provides a reduction of training time to almost 50%, but also leads to better accuracy as compared to the alternative CNN-LSTM, in our evaluations. These evaluations are not further elaborated in this paper, since they are outside the scope of objectives of this paper. In the training of both constituents of approximator model, mean square error (MSE) was considered, which when originally considered for the case of a singular deep learning model to directly map features towards parameters would have given rise to problems, as already discussed in Section 3.2.2.3.

After discovery of time varying parameters from the approximator model, leads to determination of state trajectories of the m-SIQRD model over the next 15 days. However, the varying time parameters may lead to addition of high frequency noise into the trajectories, which is compensated by LOWESS smoothing [78]. This is also depicted in Fig. 1, as part of state forecasting pipeline.

### 3.2.5. Inference model

One of the important objectives of governmental intervention in a pandemic system is to control its spread, which directly implies implementing policies that lead to overall decreasing infected numbers and causalities. The decrease in infected number may slow down due to transmission of the virus from asymptomatic cases, inefficient vaccination schemes and mutation of associated virus [59]. In all variations, the decisions made at a particular time, directly and indirectly, control the unobservable and intractable causes, in terms of time and intensity of the effect, and therefore, the purpose of abstraction of inference model to tackle this situation.

Under the objective of this model, to map parameters towards optimal policies that control the number of infected and deceased numbers, the way it progresses is by mapping controlled parameters towards policies. In fact, these critically controlled parameters are the estimated parameters, if the future trend of infected and deceased numbers is determined to be of a generally decreasing nature. If this condition is not met, then we employ a control theoretical technique.
to vary critical parameters such that the new set of parameters, termed as ‘controlled parameters’, would lead to decreasing infected and deceased numbers. To realize this concept of decreasing nature of infected and deceased states, we employ a neural network to approximate the mapping between restricted parameter set and function metricizing ‘decreasant’ nature, instead of manually determining this function from the forecasting scheme, since it will allow making our predictions more robust and independent of any small errors that are possible in deceased numbers forecast, that may change the outlook of decreasant function.

We shall first discuss the control theoretical determination of critical parameters, in case the estimate parameters are identified to lead towards non-decreasing infected and deceased numbers. In this respect, we more specifically employ synergetic control theory [51] such that parameters and a variable that quantifies the increasing or decreasant nature of critical states, i.e. infected or deceased numbers. So, we introduce a decreasant function associated with critical state vector \( \mathbf{x}(t) \), and defined by following relation, which informs the probability of \( \mathbf{x}(t) \) as being overall decreasant over the trajectory.

In Eq. (17), the parameters \( \zeta', \beta'_I, d', \nu', \) and \( K' \) are the candidate critical parameters \( \delta \) of m-SIQRD model, which are subset of total parameters availed from the approximator model. These parameters relate to original parameters as \( p' = p + \delta p \), where \( p \in \delta \). \( \delta p \) are the variations in parameters, that are equivalent to control input, that leads to decreasing nature of infected and deceased states in future. While, \( \mu \) and \( k \) are chosen appropriately as per the expectation from epidemic and goal of governmental intervention to how much control the number of deceased and infected numbers.

Since the parameters of m-SIQRD model control the trajectories of states of pandemic, therefore a proper mapping must exist between parameters and a variable that quantifies the increasing or decreasant nature of critical states, i.e. infected or deceased numbers. So, we introduce a decreasant function \( D \) associated with critical state vector \( \mathbf{x}(t) \), and defined by following relation, which informs the probability of \( \mathbf{x}(t) \) as being overall decreasant over the trajectory.

The definition of \( \mathbb{D}(\mathbf{x}(t)) \) defined by Eq. (19) imposes a strict binarization towards decreasing behavior of trajectories to observe if they
are significantly decreasing globally. The mapping between $\mathcal{D}(x(t))$ and restricted parameter set can be easily be approximated by a neural network, due to their universal approximation capability [79], and a MLP is employed in this context as the appropriate mapper.

Utilizing the non-decreasant definition, we will construct two MLP models to approximate the mapping between restricted parameter set, and the non-decreasant function associated to both infected and deceased numbers respectively. The output of this MLP model are probabilities, due to presence of sigmoid activation function, which is later binarized to either 0 and 1, depending whether the received probability is beyond 70% or not.

For determination of critical parameters that need to be varied for the decreasant function to saturate towards 1, we utilize the model-agnostic interpretations of the MLP model, and determine the most influential parameters that impact the output of MLP models associated to infected and deceaseded numbers, which would be characterized as critical parameters. Since the critical states are two, therefore the maximum number of critical parameters would be two out of five candidate critical parameters. For this purpose, we utilize local interpretable model-agnostic explanations (LIME) [80] as interpretable models to interpret the predictions of both MLPs. These models are local surrogate models that train to explain individual predictions of each MLP model.

Fig. 7. (a) SHAP plots of influence of parameters of prior SIQRD model upon buffer model that maps parameters to 8 governmental policies (b) SHAP plots of influence of parameters of m-SIQRD model upon buffer model that maps parameters to 8 governmental policies.
4. Results and discussion

The following section experimentally elucidates upon the ideas presented in previous sections by first asserting upon the advantage of proposed m-SIQRD model over prior model. Then we explore the performance of proposed scheme into tasks of forecasting of states and determination of optimal policies to control the infected and deceased numbers.

4.1. Comparison between prior and proposed SIQRD models

In this section, we provide three comparisons to illuminate how the m-SIQRD model surpasses the SIQRD model in several important aspects, which essentially strengthen the criteria for utilization of the new model for the proposed scheme.

4.1.1. Curve fitting capability

In order to proceed with current and later subsections, raw state data utilized for the case of Korea and Germany are plotted in Fig. 5. In order to present empirical evidence of how the proposed m-SIQRD model performs better than the conventional SIQRD model, these models are fitted to pandemic states of Korea for a duration of first 90 days. The 90 days are partitioned into 3-day subsets upon which a model is fitted through the estimation of parameters. The MAE error trends are presented in Fig. 6. Further details of this dataset are provided in the later section. While it is apparent that the topology of connections between compartments of the m-SIQRD model is somewhat same as the SIQRD, as shown in Fig. 2, redefining many aspects of the model has allowed the mean error of fitting of the m-SIQRD model to be 2-fold better both in mean and standard deviation of the absolute error distribution.

4.1.2. Parameter relationship with policies

Since parameters encode relatively global behavior of the states of the pandemic, and are impacted by policies implemented in that timeline, therefore it requires systematic evidence to verify the hypothesis that whether variations in parameters are related to variations in policies. To this end, we introduce using Shapley Additive exPlanations (SHAP) plots of a buffer regressor model that maps parameters to policies, to verify the hypothesis. The buffer regressor model is only responsible to provide a function to map between parameters and policies. The main verification comes from SHAP-based plots on this function to depict the contribution of parameters towards output policies. Hence the SHAP values plot depicts how the policies map to the policies through the function learned by the regressor model.

SHAP is a model interpretability technique, based on cooperative game theory, to not only determine important features for the machine learning model but also explain how the features impact individual predictions of a black box model locally or globally [81]. They are widely employed as SHAP beeswarm plots, where the corresponding
SHAP values are grouped by the features on the $y$-axis and plotted as spots color-coded by the value of the parameters. The order of these groups (associated with each feature) is ordered by the mean SHAP values. The $x$-axis represents the SHAP values which provide a metric on the impact on output. In this way, a reader can observe both feature importance as well as feature impact on the model's output. For a demonstration of our comparison, we will consider SHAP plots over the whole dataset, upon which the buffer model is trained, to determine global explanations.

The regressor model considered is a multi-layer perceptron (MLP) with a 32 node hidden layer, and ReLU as activation function. Furthermore, both parameter and policy data were normalized during the training phase of the regressor model. Consequentially, the corresponding SHAP beeswarm plots to the cases of prior SIQRD model and m-SIQRD model are plotted in Fig. 7(a) and (b) respectively, in consideration of the Korean dataset. For all 8 policies, it is clear that for the case of SIQRD model, majority of the SHAP values for each parameter are centered around the origin (with zero SHAP value), with
their spread heavily concentrated around the origin, while only a few outliers deflecting away from the origin, showing an overall diminished impact on the regressor model's output. On the other hand, the same is not the case for the m-SIQRD model, where the spread of SHAP values is dense away from origin with a larger spread, showing that the regressor model was better in learning the variations in parameters in response to variations in policies. Hence we conclude that the parameters of the m-SIQRD model excel in encoding the policy information implemented at the time points of the trajectories.

4.1.3. Potential as features

We first introduce a notion of utilization of parameters of ODE, that can fit onto a time series data to be utilized as features for a deep learning model upon the same time series data. Surely these parameters capture the concise global representation of trajectories, so they qualify as trait of features for a deep learning model as being populated with information with lesser dimensions [82]. But there is even more important aspect of features as being least correlated
among themselves [82], an aspect which we exploit in comparison of parameters of prior and m-SIQRD model.

In order to show comparison between parameters of both mathematical models, in term of independency among themselves, we plot Pearson correlation coefficient based matrix for both cases, and is presented in Fig. 8. The first observation from the figure is that parameters are majorly non-correlated for both model cases. While overall the difference of correlation nature of parameters is not drastic, the range of values correlation matrix (except at diagonals) for prior SIQRD model are $-0.076 \pm 0.347$ and for m-SIQRD model are $0.072 \pm 0.157$. Hence, the parameters of m-SIQRD model are lesser correlated among themselves as compared the prior SIQRD model, and show more potential to be considered as features for a deep learning model, a theme which is later explored in this paper.

### 4.2. Forecasting of states

This section discusses the results constituent to the proposed scheme, for forecasting of 5 states of m-SIQRD model for next 15 days, given past 3 days data. The architectures of corresponding generation, approximation and inference models employed in this scheme, are presented in Figs. 9, 10 and 11 respectively.

#### 4.2.1. Data augmentation

The pipeline for forecasting states primarily begins with the approximation model, whose training is dependent upon the generation model, which is responsible for data augmentation. Henceforth, the training progress of the generation model is demonstrated for the case of the Korean dataset, where training loss curves of state-GAN and parameter-GAN are shown in Fig. 12(c) and (d) respectively. It is evident that for both state-GAN and parameter-GAN cases, the loss of Generator is decreasing, while loss of Discriminator is moving in opposite direction. We may remark that loss of state-GAN is relatively more prone to fluctuations, which are most likely caused by the dropout layer of discriminator. In this respect, reduction of drop out, followed by further fine tuning of Generator’s hyperparameters of transposed convolution layers may lead to stability.
Fig. 12. (a) Training and validation loss of CNN-GRU Model (b) Training and validation loss of Attention Model (c) Generator and Discriminator loss of state-GAN (d) Generator and Discriminator loss of parameter-GAN (e) Training loss of MLP models associated to deceased and infected states (f) Training loss of D-CNN model.

Fig. 13. (a) Comparison of reference real parameters and synthetic parameters of m-SIQRD model generated by parameter-GAN (b) Comparison of reference real states of and synthetic states of m-SIQRD model generated by parameter-GAN.
The results of generation of synthetic parameters and states from these models is depicted in Fig. 13(a) and (b) respectively over Korean dataset, where they are further utilized to reproduce further parameters and states data respectively, for increment in cardinality of the dataset. It is apparent from Fig. 13(a) and (b) that the morphology of synthetic data is preserved with slight amplitude differences.

### 4.2.2. Parameter estimation

As per the pipeline of the proposed scheme of states forecasting, the determination of time-varying parameters is the most pivotal part of robust estimation of future states, through virtue of m-SIQRD model. Since this is the objective of the approximation model, therefore, the training progress of constituents of the approximation model i.e. CNN-GRU model and Attention model are shown in Fig. 12(a) and (b) respectively. For training progress, the dataset utilized was processed, in accordance with pipeline mention in Fig. 4(a), (b) and (c) for achievement of features, pseudo-parameters and parameters respectively. It can be observed from Fig. 14 that the proposed features, extracted from dataset, are much less correlated as compared to the original states of m-SIQRD model, extracted from dataset, which was already claimed in Section 3.2.2.1.

Based on the scheme described in Fig. 1(a), the parameters estimated over each prediction of states through integration of SIQRD model, are collected and concatenated to form a time series, which are the time varying parameters. The results of the trained approximation model based estimation of time-varying parameters for next 15 days, from prior 3 day states data are shown in Fig. 15(a) and (b) for the case of Korean and German datasets respectively.

### 4.2.3. Simulation of future trajectories

After determination of time-varying parameters through the approximation model, they are utilized in the integration of m-SIQRD model, with initial condition as the 3rd day of prior 3 day data (as per the scheme in Fig. 10(a)) to forecast the states for next 15 days. The corresponding results for the case of Korean and German datasets are depicted in Figs. 16 and 17 respectively. The corresponding MAE and RMSE scores are tabulated in Table 2. Overall the MAE and RMSE performance for forecasting of Korean dataset states is 68.09 and 84.71 respectively. On the other hand, the MAE and RMSE scores for forecasting of German datasets states are 556.77 and 606.15 respectively. The margin between MAE and RMSE squares are not significant to conclude that scheme performance shows robustness in predictions. It can be remarked that the proposed scheme performs well in the forecasting of different amplitude range states, that can be as much as 80 million and as low as 2, like susceptible, infected, quarantined, recovered and deceased number.

### 4.2.4. Validation

The overall forecasting performance, as mentioned in previous subsection, is very good for the utilized dataset, and we validate this by comparing it with 7 state of art forecasting models, which are summarized in Table 3, with further information on utilized hyperparameters also provided.

Consequently, the overall MAE performances of these models are provided in Table 4, for forecasting on the same dataset is documented in Table 4, where it is clear that proposed scheme excels overall. However, to provide resolution into the difference of performances into individual states, three validation models were selected, i.e. Temporal Fusion Transformer (TFT), Temporal Convolution Transformer (TCT) and Transformer. The comparison of forecasting of these models with the proposed approach is done upon three selected states, namely susceptible, infected and deceased number, which fall in large, medium and small amplitude ranges. The performance is metricized through SMAPE (in units of percentage), and the results are shown in Fig. 18(a) and (b) for the case of Korean and German datasets respectively. The summary of these results is that the proposed approach has consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performance over other methods, in forecasting of large (Susceptible) and medium (Quarantined) range states. The performance over small range (Deceased) state for proposed scheme is in median consistent good performanc...
Fig. 15. (a) Estimation of time varying parameters by approximation model for next 15 days for Korean dataset (b) Estimation of time varying parameters by approximation model for next 15 days for German dataset.

Fig. 16. Proposed scheme based forecasting of m-SIQRD states (a) S (b) I (c) Q, (d) R and (e) D for Korean dataset for next 15 days using proposed scheme.

in Fig. 1(c). As imminent from the pipeline discussed in Section 3.2.5, first the MLP and D-CNN models are trained upon the Korean dataset, and their loss curves are provided in Fig. 12(e) and (f).

After training of MLP model, local interpretations of the contribution of parameters into predictions of MLP model are made using LIME. In this respect, these parameters are the arithmetic mean of estimated time varying parameters, availed from Korean dataset in Section 4.2.2. The corresponding LIME based plots, depicting influence of parameters in mapping of MLP model towards $\mathcal{D}$ function as decreasant probability, are shown in Fig. 19. Fig. 19 (a) and (b) are respective LIME plots for individual MLP models associated to infected and deceased states. The MLP models, associated to deceased and infected states, estimated the decreasant probability to be $4.4 \times 10^{-1}$ and $1.4 \times 10^{-18}$ respectively, depicting that infected and deceased numbers are overall not dominantly decreasing in a strict sense. This can be observed from Fig. 16(b) and (e) that the trends are prone to positive and negative fluctuations, and there is no consistent sharp decreasing behavior, the aspect which is captured by the $\mathcal{D}$ function.

From Fig. 19(a) and (b), we determine that the parameters $a$ and $K$ are contributing most towards the non-decreasant behavior of the infected and deceased states respectively, henceforth these are the critical parameters. Therefore, these parameters are required to be varied appropriately, as discussed in Section 3.2.5, to lead towards controlled parameters, that encodes the information of the policies to be implemented. The decoding of these policies is achieved from the D-CNN model.

In order to determine the variations $\delta a$ and $\delta K$ of identified critical parameters, we optimize the synergetic control objective function $\theta'$ as per Eq. (20), using interior point based minimization. In this case, $k$ and $\mu$ were set to 10 and 0.1 respectively.

Consequently, the critical parameters are controlled through updates $\beta' = \beta + \delta \beta$, and $\zeta' = \zeta + \delta \zeta$. The comparison of critical and controlled critical parameters is shown in Fig. 20. These control critical parameters are substituted in original parameters to form the complete set of controlled parameters, which are then mapped by the D-CNN model the policy encoding vector. The resultant encoding and...
Fig. 17. Proposed scheme based forecasting of m-SIQRD states (a) S (b) I (c) Q, (d) R and (e) D for German dataset for next 15 days using proposed scheme.

Fig. 18. (a) Comparison of forecasting curves by TCN (M-1), Transformer (M-5), TFT (M-6) and Proposed method (M-8), followed by documentation of corresponding SMAPE (%) score over Korean dataset(b) Comparison of forecasting curves by TCN (M-1), Transformer (M-5), TFT (M-6) and Proposed method (M-8), followed by documentation of corresponding SMAPE(%) score over German dataset.
Table 3
Methods utilized in comparison of forecasting performance to next 15 days, alongside with their corresponding hyperparameters and reference.

| Method name | Description | Nature | Hyperparameters | Ref |
|-------------|-------------|--------|-----------------|-----|
| M-1         | Temporal Convolutional Network | Forecasting | epochs = 200, input length = 30, output length = 1, dropout = 0.1, kernel size = 20, filter number = 58, min-max scaling = True | [11] |
| M-2         | Exponential Smoothing | Forecasting | seasonality mode = additive, min-max scaling = True | [10] |
| M-3         | Fb-Prophet Model | Forecasting | seasonality mode = additive, min-max scaling = True | [8] |
| M-4         | Neural-Prophet Model | Forecasting | seasonality mode = additive, min-max scaling = True | [9] |
| M-5         | Transformer | Forecasting | epochs = 200, input length = 30, output length = 1, attention head number = 16, number of encoders = 2, number of decoders = 2, MLP neurons = 128, dropout rate = 0.1, activation function = ReLU, min-max scaling = True | [12] |
| M-6         | Temporal Fusion Transformer | Forecasting | epochs = 200, input length = 30, output length = 1, hidden layer neurons = 10, LSTM layer number = 1, Attention head number = 1, dropout rate = 0.1, min-max scaling = True | [13] |
| M-7         | Bi-LSTM | Forecasting | Bi-LSTM cell number = 64, Bi-LSTM layers number = 1, Batch Normalization = True, dropout rate = 0.5, epochs = 200, min-max scaling = True | [7] |
| M-8         | Proposed Methodology | Mathematical Modeling | – | – |

Table 4
Comparison of overall MAE values of forecasting for all states of m-SIQRD model for next 15 days over Korean and German dataset through methods described in Table 2.

| Dataset | M-1 | M-2 | M-3 | M-4 | M-5 | M-6 | M-7 | M-8 |
|---------|-----|-----|-----|-----|-----|-----|-----|-----|
| S.Korea | 2468.39 | 651.08 | 421.60 | 366.12 | 336.73 | 645.78 | 220.69 | 68.09 |
| Germany | 2772.49 | 3854.37 | 47017.52 | 31759.03 | 4368.03 | 2636.28 | 4128.97 | 556.77 |

Fig. 19. (a) LIME plot over predictions of MLP model associated with deceased state over mean of estimated time varying parameters (b) LIME plot over predictions of MLP model associated with infected state over mean of estimated time varying parameters.

its decoding as policy action is tabulated in Table 5. The consequent trajectory of infected and deceased numbers, recognized as optimal trajectories, are plotted and compared with original numbers in Fig. 21, with in the 15-day forecasting horizon, by integrating the m-SIQRD model with a critical parameter set. It is clear that optimal trajectories much dominant decreasing characteristics, and this would be implied by the estimated best policy encodings, tabulated in Table 4. We may remark that the proposed scheme determines the public gathering and events as the most critical factors in controlling the infected and deceased numbers, and recommends imposing most restricted action upon them, while other factors like mask wearing, quarantine of arriving passengers etc. play a secondary role in reducing the infected and deceased numbers, as depicted by expected trajectory comparison in Fig. 21.

5. Future works

The proposed scheme is an orchestration of several data-driven modules, each with its own responsibility, but all in all, the proposed scheme is centered upon an ODE model (m-SIQRD model in our case) to achieve two fundamental objectives. i.e. forecasting and inference. This scheme extracts information from the ODE model like the future trajectory of states, and infers upon the environment of the states to allow its control, through the application of deep learning. While
in our case, the COVID-19 pandemic is considered, where we demonstrated its overall better performance as compared to state-of-the-art deep learning forecasting models over whole country data sets, where states can cover magnitudes up to 80 million, and as low as 50. We also demonstrated its ability for automated determination of best policy encodings to control the infected and deceased numbers in the future. However, the proposed approach has one drawback. In the proposed scheme, there are four deep learning models that orchestrate to provide two main functions, i.e. forecasting and inference, while other two models (from the generation model) are only employed for data augmentation. But certainly, the computational complexity of these models is a concern. In this models, the most space and time complexity revolves around the two models of approximation model abstraction, and we can significantly mitigate these complexity costs, by replacing both models with a single transformer, alongside with low-rank approximation \[ L_1 \] norm minimization during training, that can significantly reduce both memory and computation costs, while maintaining accuracy.

We also propose the tendency of the proposed scheme to be generalizable as well as more efficient, to be applicable to general time series related problem through the following modifications:

- Discovery of robust governing non-linear ODE system for a general time series data through Sparse Identification of Nonlinear Dynamical systems (SINDy) \[ 84 \].
- Regularization of loss during training of approximation model through physics of the trajectory of states of time series from the discovered ODE.
- The variable parameter estimation can be made more robust by directly estimating time-varying parameters of model for next 15 days, instead of current piecewise estimation scheme as depicted in Fig. 1(a).

6. Conclusion

In this paper, we first proposed a novel modification of SIQRD model as m-SIQRD model by taking into account how governmental interventions and asymptomatic individuals affect the dynamic of some states. We empirically showed that not only this new model has much better fitting capabilities over real pandemic states, but the determined parameters of this model are very sensitive to the governmental policies implemented to control the pandemic in the corresponding time. Thus these new set of parameters greatly encode the information of the then implemented policies. Then, we proposed a novel deep learning scheme over a proposed modification of SIQRD model (m-SIQRD) to accomplish two main objectives. The first objective concerned with forecasting of states of m-SIQRD model to next 15 days. The second objective was to determine the best policy encodings, as what sequence of governmental actions would allow required decrease of infected and
deceased numbers over time. We validated the forecasting performance of the proposed scheme from dataset of two countries, i.e. South Korea and Germany with 7 state of the art forecasting models proposed in COVID-19 forecasting literature. We showed that our model’s performance to be very competitive with overall MAE and RMSE score of 68.09 and 84.71 for South Korea case, and 556.77 and 606.15 for Germany case respectively. These scores greatly exceed the scores of other 7 benchmark forecasting models. We also utilized our scheme to determine the best policy encodings for the control of pandemic in South Korea, within the 15 days forecast horizon. Not only this, but we also proposed an improvement and extension of proposed scheme to general time series related problems, showing a great potential for proposed scheme beyond epidemic dynamics.

CRediT authorship contribution statement

Junaid Iqbal Khan: Conceptualization, Methodology, Software, Writing. Farman Ullah: Conceptualization, Writing – review & editing. Sungchung Lee: Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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