Nonlinear Dynamical Analysis of a Circular Truss Antenna in Thermal Environments

Ruiqin Wu, Wei Zhang
Beijing Key Laboratory on Nonlinear Vibrations and Strength of Mechanical Structures, College of Mechanical Engineering, Beijing University of Technology, Beijing, 100124, PR China
ruiqinwu@163.com

Abstract. This paper investigates the nonlinear dynamics of the circular truss antenna with considering the thermal effect. The first-order shear deformation shell theory and the Hamilton's principle are used to establish the mechanical model for the circular truss antenna system. The motion governing equation is derived for the equivalent circular cylindrical shell. For interest in 1:2:3 internal resonances, the continuous system of nonlinear partial governing differential equation of motion is truncated into a three-degree-of-freedom system of ordinary differential equation including quadratic and cubic nonlinearities. From the averaged equations obtained by the method of multiple scales, numerical simulations are presented to investigate the effects of the thermal excitation on the nonlinear responses of the truss antenna system. Bifurcation diagram, phase portraits, time histories, power spectra and max Lyapunov exponents are used to get the numerically results. It is found that there exist period doubling bifurcations, periodic motions and chaotic motions in the system. The numerical results show that the thermal excitation has significant influence on the nonlinear dynamical behaviors of the circular truss antenna system. The findings in this paper is also useful for the nonlinear vibration control of the shell structure.

1. Introduction
The truss antennas designed to be foldable and deployable structures with large apertures are widely used in many space missions, such as earth observation, land sensing and deep space exploration. The truss antenna system consists of a astrovilelcker, a mesh reflector, the solar wing and extended arms, in which the typical components of the mesh reflector are two lightweight paraboloidal meshes and a truss rim structure as show in Figure 1. Since most of components in the truss antenna system are lightweight and high-flexibility structures, the system has low stiffness. That is easy lead to large deformations and vibrations when the system under the influence of thermal excitation, which can adversely impact their performance and reliability. The study of dynamics for truss antennas would get away from those catastrophic vibrations. Thus, analyse the dynamics of the circular truss antenna system in thermal environments plays an important role in engineering and theories. In recent years, the stability of truss antennas has been increasingly attracting researchers. There were a number of literatures that have been conducted to study the dynamics of truss antenna systems.

Li et al. [1] presented a new parallel computation methodology based on the finite-element approach of absolute nodal coordinate formulation for the mesh reflector multi-body system, and used the numerical examples of the dynamic simulation for a complex Astro-Mesh reflector to validate the efficiency and accuracy of the new method. time-varying stiffness using the asymptotic perturbation method at the present of primary parametric resonance and 1/2 sub-harmonic resonance. In [2], the...
author discussed the deployment motions of the hoop truss deployable antenna using the kinematic, dynamic analysis and control methods and considered effects of the stiffness of torsional spring, damping in joints, gravity and the pretension forces in nets. Dai et al. [3] investigated the structural optimization for a new type of deployable antenna system with double-ring deployable truss. The optimized system results obtained by using genetic algorithms and gradient-based optimizers were experimentally tested with a 4.2-m scaled antenna system for dynamic properties. Morterolle et al. [4] discussed the dynamic behaviors of a large deployable space reflector using finite element method and considered the influence of suspension thread and gravity in the experimental model.

Figure 1   A truss rim structure of the deployable antenna is given.

Some researches focused on the cable net of the deployable antenna which is important for the precision of antenna. Zong et al. [5] utilized the analytical sensitivity analysis method to investigate the uncertainty carrying by parameters for the shape precision and cable tensions of a cable-network antenna. The result demonstrated that some factors such as slender front net cables, thick tension ties and high tension level can improve the performance of the cable-network antenna structures. Li et al. [6] proposed an effective form-finding method based on the iterative FDM and the minimum norm method to do the form-finding analysis for the mesh reflectors. They also investigated the influence of the mesh tension force on the reflector natural frequencies.

Thermal excitation must be considered when the structures worked in thermal environment since it can be induced deformations or vibrations of simple structures like beam, plates, shells and so on. In [7-8], the authors devoted to study thermally forced vibrations of beam and plate structures. In addition to the aforementioned literatures, Xue et al. [9] used the finite element method discussed the bending-torsion coupling vibration of large scale space structures induced by the thermal. Chen et al. [10] studied the nonlinear dynamic behaviors of a circular truss antenna by theoretical and numerical methodology. The results indicated that the thermal excitation has great impact on the nonlinear breathing vibrations of the antenna system.

As aforementioned, there are many investigations focusing on truss antennas, but not many researches are contributed on the dynamics of truss rims in circular antennas. This paper investigates nonlinear dynamics for a circular deployable truss antenna system by using the method of multiple scales [11-13]. The system is modelled by three-degree-of-freedom system of ordinary differential equation including quadratic and cubic nonlinearities using the first-order shear deformation shell theory and the Hamilton's principle. For interest in 1:2:3 internal resonances, from the averaged equations obtained by the method of multiple scales, numerical simulations are presented to investigate the effects of the thermal excitation on the nonlinear responses of the truss antenna system. Phase portraits, time histories, power spectra and max Lyapunov exponents are used to get the numerically results.

2. Equations of motion for circular truss antenna system
The truss antenna system studied in this paper is a deployable structure included two lightweight paraboloidal meshes and a truss rim structure. The truss rim structure is simplified be a continuum circular cylindrical shell by the principle of equivalent effect for theoretical analysis, as shown in Figure 2.

According to Reddy's first-order shear deformation theory (FSDT), the displacement field for the simplified shell is assumed to be

\[
\begin{align*}
    u(x, 0, z, t) &= u_0(x, 0, t) + z\phi_x(x, 0, t), \\
    v(x, 0, z, t) &= v_0(x, 0, t) + z\phi_y(x, 0, t), \\
    w(x, 0, z, t) &= w_0(x, 0, t),
\end{align*}
\]

Then we derived the strains-displacement relation based on the FSDT as following

\[
\begin{align*}
    \varepsilon_{xx} &= \frac{\partial u_0}{\partial x} + \frac{1}{2} \left( \frac{\partial w_0}{\partial x} \right)^2 + z \frac{\partial \phi_x}{\partial x}, \\
    \varepsilon_{yy} &= \frac{1}{R} \frac{\partial v_0}{\partial \theta} + \frac{1}{2} \left( \frac{\partial w_0}{\partial \theta} \right)^2 + \frac{w_0}{R} + z \frac{\partial \phi_y}{\partial \theta}, \\
    \gamma_{xz} &= \phi_x + \frac{\partial w_0}{\partial x},
\end{align*}
\]

From Figure 2 and based equations (1)-(2), the kinetic energy, potential energy and external virtual work are calculated, and substituted them into Hamilton's principle, we get the following equation

\[
\int_{\Omega} (\delta K - \delta U + \delta W) dt = 0,
\]

where

\[
\begin{align*}
    \delta K &= \int_{-h/2}^{h/2} \rho \left( \dot{u}\ddot{u} + \dot{v}\ddot{v} + \dot{w}\ddot{w} \right) dx dy dz, \\
    \delta W &= \int_{\Omega} F \dot{w}_0 dx dy - \int_{\Omega} \gamma_0 \dot{w}_0 dx dy, \\
    \delta U &= \int_{\Omega} \left( \sigma_{xx} \delta e_x + \sigma_{yy} \delta e_y + \tau_{xy} \delta e_{xy} + \tau_{xz} \delta e_{xz} + \tau_{yz} \delta e_{yz} \right) dx dy dz.
\end{align*}
\]

From the former equations, the nonlinear governing equations of motion for the circular cylindrical shell are derived as follows

\[
\begin{align*}
    \frac{\partial N_{xx}}{\partial x} + \frac{1}{r} \frac{\partial N_{yx}}{\partial \theta} &= I_0 \frac{\partial^2 \ddot{u}_0}{\partial t^2} + I_1 \frac{\partial^2 \ddot{\phi}_x}{\partial t^2}, \\
    \frac{1}{r} \frac{\partial N_{yx}}{\partial \theta} + \frac{\partial N_{yy}}{\partial x} + \frac{Q_{y0}}{R} &= I_0 \frac{\partial^2 \ddot{v}_0}{\partial t^2} + I_1 \frac{\partial^2 \ddot{\phi}_y}{\partial t^2},
\end{align*}
\]
\[ \frac{\partial}{\partial x} \left( N_{xx} \frac{\partial \omega_0}{\partial x} \right) + \frac{1}{r} \frac{\partial}{\partial x} \left( N_{s\theta} \frac{\partial \omega_0}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left( N_{s\theta} \frac{\partial \omega_0}{\partial \theta} \right) + \frac{1}{r^2} \frac{\partial Q_x}{\partial \theta} + \frac{1}{r} \frac{\partial Q_0}{\partial \theta} - \frac{N_{xx}}{R} + \mu \ddot{w}_0 = I_0 \ddot{w}_0, \]  
\[ \frac{\partial M_{xx}}{\partial x} + \frac{1}{r} \frac{\partial M_{s\theta}}{\partial \theta} - Q_x = I_1 \frac{\partial^2 \dot{u}_0}{\partial t^2} + I_2 \frac{\partial^2 \ddot{\theta}_0}{\partial t^2}, \]  
\[ \frac{1}{r} \frac{\partial M_{s\theta}}{\partial x} + \frac{1}{r} \frac{\partial M_{s\theta}}{\partial \theta} - Q_0 = I_1 \frac{\partial^2 \dot{u}_0}{\partial t^2} + I_2 \frac{\partial^2 \ddot{\theta}_0}{\partial t^2}, \]  
where
\[ \begin{bmatrix} N_{xx} \\ N_{s\theta} \\ M_{xx} \\ M_{s\theta} \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} \sigma_{xx} \\ \sigma_{s\theta} \\ \sigma_{xx} \\ \sigma_{s\theta} \end{bmatrix} dz, \quad \begin{bmatrix} Q_0 \\ Q_x \end{bmatrix} = K \int_{-h/2}^{h/2} \begin{bmatrix} \gamma_{0z} \\ \gamma_{xz} \end{bmatrix} dz. \]

The thermal stress resultants can be represented as follows
\[ \begin{bmatrix} N_{xx}^T \\ N_{s\theta}^T \\ M_{xx}^T \\ M_{s\theta}^T \end{bmatrix} = \int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} \\ Q_{12} \\ Q_{22} \\ Q_{66} \end{bmatrix} \Delta T dz, \quad \begin{bmatrix} Q_0 \\ Q_x \end{bmatrix} = K \int_{-h/2}^{h/2} \begin{bmatrix} \gamma_{0z} \\ \gamma_{xz} \end{bmatrix} dz. \]

\[ \Delta T = T_1 \cos \Omega t - T_0, \]  
where \( \Delta T \) is the temperature increment.

The boundary conditions of the shell are given as follows
\[ u_0 = v_0 = w_0 = \phi_x = \phi_y = 0, \quad (8a) \]
\[ N_{xx} = N_{s\theta} = M_{xx} = M_{s\theta} = 0. \quad (8b) \]

Next, we introduce the following non-dimensional variables and parameters in order to obtain the dimensionless equation
\[ \bar{u}_0 = \frac{u_0}{L}, \quad \bar{v}_0 = \frac{v_0}{E}, \quad \bar{w}_0 = \frac{w_0}{h}, \quad \bar{\phi}_x = \phi_x, \quad \bar{\phi}_y = \phi_y, \quad \bar{x} = \frac{x}{L}, \quad \bar{\Omega} = \frac{1}{\pi^2} \sqrt{\frac{LR\rho}{E}}, \quad \bar{t} = \frac{t}{\pi^2} \sqrt{\frac{E}{LR\rho}}. \]

\[ \bar{\mu} = \frac{(LR)^2}{E h^4} \sqrt{\frac{1}{p \rho}}, \quad \bar{\sigma}_{ij} = \frac{\sqrt{LR}}{E h^2} A_{ij}, \quad \bar{B}_{ij} = \frac{\sqrt{LR}}{E h^2} B_{ij}, \quad \bar{D}_{ij} = \frac{\sqrt{LR}}{E h^2} D_{ij}, \quad \bar{I}_i = \frac{1}{\sqrt{LR\rho}} I_i. \]

The mode function satisfies the boundary conditions of the shell can be assumed as follows
\[ u_0 = u(t) \phi_x(x) \sin \left( \frac{\theta}{2} \right), \quad v_0 = v(t) \phi_y(x) \sin \left( \frac{\theta}{2} \right), \quad \phi_x = \phi_x(t) \phi_x(x) \sin \left( \frac{\theta}{2} \right), \]
\[ w_0 = w_1(t) \phi_{w1}(\theta) + w_2(t) \phi_{w2}(\theta) + w_3(t) \phi_{w3}(\theta), \quad \phi_0 = \phi_0(t) \phi_0(x) \sin \left( \frac{\theta}{2} \right). \]

Transverse vibration is the main motion form in the system, for simplicity, we neglect all the inertia terms \( u_0, \ v_0, \ \phi_x, \ \phi_y \). Substituting equation (6)-(10) into equation (5), and applying the Galerkin procedure, the dimensionless governing equations of motion for the circular cylindrical shell are obtained as follows
\[ \ddot{w}_1 + \mu_1 \dot{w}_1 + \omega_1^2 w_1 + f_1 \cos(\Omega t)w_1 + \alpha_{11} w_1^2 + \alpha_{12} w_2^2 + \alpha_{13} w_3^2 + \alpha_{14} w_1^2 + \alpha_{15} w_2^2 + \alpha_{16} w_3^2 + \alpha_{17} w_1^2 + \alpha_{18} w_2^2 + \alpha_{19} w_3^2 + \alpha_{20} w_1^2 + \alpha_{21} w_2^2 + \alpha_{22} w_3^2 + \alpha_{23} w_1^2 + \alpha_{24} w_2^2 + \alpha_{25} w_3^2 + \alpha_{26} w_1 w_2 w_3 = F_1 \cos(\Omega t), \]
\[ \ddot{w}_2 + \mu_2 \dot{w}_2 + \alpha_{21} w_1^2 + \alpha_{22} w_2^2 + \alpha_{23} w_3^2 + \alpha_{24} w_1 w_2 w_3 = F_1 \cos(\Omega t). \]
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In order to utilize the method of multiple scales to analyse nonlinear responses of equation (11), the averaged equation. Based on the averaged equation, numerical approach is utilized to find the periodic, quasi-periodic and chaotic motions in the system. In the following analysis, we will use the method of multiple scales [11-13] to obtain the averaged equation. Based on the averaged equation, numerical approach is utilized to find the periodic, quasi-periodic and chaotic motions in the antenna system.

3. Perturbation analysis

In order to utilize the method of multiple scales to analyse nonlinear responses of equation (11), the scale transformations are introduced as

\[ \mu_i \rightarrow \varepsilon^2 \mu_i, \quad f_i \rightarrow \varepsilon^2 f_i, \quad F_i \rightarrow \varepsilon^2 F_i, \quad \alpha_{ui} \rightarrow \varepsilon \alpha_{ui}, \quad \beta_{ui} \rightarrow \varepsilon \beta_{ui}, \quad \gamma_{ui} \rightarrow \varepsilon \gamma_{ui}, \quad \alpha_{2j} \rightarrow \varepsilon^2 \alpha_{2j}, \]

\[ \beta_{2j} \rightarrow \varepsilon^2 \beta_{2j}, \quad \gamma_{2j} \rightarrow \varepsilon^2 \gamma_{2j}, \quad \gamma_{uj} \rightarrow \varepsilon^2 \gamma_{uj}, \quad i = 1,2,3, \quad j = 0,1,\ldots,6 \]

(12)

where \( \varepsilon \) is a small perturbation parameter.

Only considering the case of primary parametric resonance and 1:2:3 sub-harmonic resonance, there are the following relations

\[ \omega_1^2 = \Omega^2 - \varepsilon^2 \sigma_1, \quad \omega_2^2 = 4\Omega^2 - \varepsilon^2 \sigma_2, \quad \omega_3^2 = 9\Omega^2 - \varepsilon^2 \sigma_3, \quad \Omega = 1, \]

(13)

where \( \sigma_1, \sigma_2 \) and \( \sigma_3 \) are the three different detuning parameters.

Substituting equation (12) into equation (11), we obtain the following dimensionless three-degree-of-freedom nonlinear system under combined parametric and forcing excitations

\[ \ddot{w}_1 + \varepsilon^2 \mu_1 \dot{w}_1 + \omega_1^2 w_1 + \varepsilon^2 f_1 \cos(\Omega t) w_1 + \varepsilon \alpha_{11} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{12} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{13} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{14} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{15} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{16} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{17} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{18} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{19} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{20} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{21} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{22} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{23} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{24} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{25} \varepsilon^2 \omega_1^2 w_1 + \varepsilon \alpha_{26} \varepsilon^2 \omega_1^2 w_1 = \varepsilon^2 F_1 \cos(\Omega t), \]

(14a)

\[ \ddot{w}_2 + \varepsilon^2 \mu_2 \dot{w}_2 + \omega_2^2 w_2 + \varepsilon^2 f_2 \cos(\Omega t) w_2 + \varepsilon \alpha_{21} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{22} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{23} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{24} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{25} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{26} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{27} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{28} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{29} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{30} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{31} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{32} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{33} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{34} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{35} \varepsilon^2 \omega_2^2 w_2 + \varepsilon \alpha_{36} \varepsilon^2 \omega_2^2 w_2 = \varepsilon^2 F_2 \cos(\Omega t), \]

(14b)

\[ \ddot{w}_3 + \varepsilon^2 \mu_3 \dot{w}_3 + \omega_3^2 w_3 + \varepsilon^2 f_3 \cos(\Omega t) w_3 + \varepsilon \alpha_{31} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{32} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{33} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{34} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{35} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{36} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{37} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{38} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{39} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{40} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{41} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{42} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{43} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{44} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{45} \varepsilon^2 \omega_3^2 w_3 + \varepsilon \alpha_{46} \varepsilon^2 \omega_3^2 w_3 = \varepsilon^2 F_3 \cos(\Omega t), \]

(14c)

The above equations, which include the parametric and forcing excitations, describe the nonlinear oscillations of the circular cylindrical shell. In the following analysis, we will use the method of multiple scales to obtain the averaged equation. The method of multiple scales [11-13] is used to find the uniform solutions of equation (14) in the following form

\[ w_1 = w_{10}(T_0, T_1, T_2) + \varepsilon w_{11}(T_0, T_1, T_2) + \varepsilon^2 w_{12}(T_0, T_1, T_2) + \cdots, \]

(15a)

\[ w_2 = w_{20}(T_0, T_1, T_2) + \varepsilon w_{21}(T_0, T_1, T_2) + \varepsilon^2 w_{22}(T_0, T_1, T_2) + \cdots, \]

(15b)
\[ w_3 = w_{30}(T_0, T_1, T_2) + \varepsilon w_{31}(T_0, T_1, T_2) + \varepsilon^2 w_{32}(T_0, T_1, T_2) + \cdots, \]  
(15c)

where \( T_0, T_1, T_2 = t, \varepsilon t, t^2 \).

Then, the differential operators is obtained

\[ \frac{d}{dt} + \varepsilon \frac{d}{dT_0} + \varepsilon^2 \frac{d}{dT_1} + \cdots = D_0 + \varepsilon D_1 + \varepsilon^2 D_2 + \cdots, \]

(16a)

\[ \frac{d^2}{dt^2} + \varepsilon \left( \frac{d}{dT_0} + \varepsilon \frac{d}{dT_1} + \varepsilon^2 \frac{d}{dT_2} \right) + \cdots = D_0^2 + 2\varepsilon D_1 D_0 + \varepsilon^2 \left( D_1^2 + 2D_2 D_0 \right) + \cdots, \]

(16b)

where \( D_k = \frac{\partial}{\partial T_k}, k = 0, 1, 2 \).

Substituting equations (13), (15)-(16) into equation (14) and balancing the coefficients of like power of \( \varepsilon \) yield the following differential equations

order \( \varepsilon^0 \)

\[ D_0^2 w_{10} + w_{10} = 0, \quad D_0^2 w_{20} + 4w_{20} = 0, \quad D_0^2 w_{30} + 9w_{30} = 0, \quad (17) \]

order \( \varepsilon^1 \)

\[ D_0^2 w_{11} + w_{11} = -2D_0 D_1 w_{10} - \alpha_{11} w_{10} - \alpha_{12} w_{20} - \alpha_{13} w_{30} - \alpha_{17} w_{10} w_{20} - \alpha_{18} w_{20} w_{30} - \alpha_{19} w_{10} w_{30}, \]

(18a)

\[ D_0^2 w_{21} + 4w_{21} = -2D_0 D_1 w_{20} - \beta_{11} w_{10} - \beta_{12} w_{20} - \beta_{13} w_{30} - \beta_{17} w_{10} w_{20} - \beta_{18} w_{20} w_{30} - \beta_{19} w_{10} w_{30}, \]

(18b)

\[ D_0^2 w_{31} + 9w_{31} = -2D_0 D_1 w_{30} - \gamma_{11} w_{10} - \gamma_{12} w_{20} - \gamma_{13} w_{30} - \gamma_{17} w_{10} w_{20} - \gamma_{18} w_{20} w_{30} - \gamma_{19} w_{10} w_{30}, \]

(18c)

order \( \varepsilon^2 \)

\[ D_0^2 w_{12} + w_{12} = -2D_0 D_2 w_{10} - 2D_0 D_1 w_{11} + \sigma_{11} w_{10} - \mu_1 D_0 w_{20} - D_1^2 w_{10} - 2\alpha_{11} w_{11} w_{11} - 2\alpha_{12} w_{20} w_{10} - 2\alpha_{13} w_{30} - 2\alpha_{17} w_{10} w_{20} - 2\alpha_{18} w_{20} w_{30} - 2\alpha_{19} w_{10} w_{30}, \]

(19a)

\[ D_0^2 w_{22} + 4w_{22} = -2D_0 D_2 w_{20} - 2D_0 D_1 w_{21} + \sigma_{22} w_{20} - \mu_2 D_0 w_{30} - D_1^2 w_{20} - 2\beta_{11} w_{10} w_{11} - 2\beta_{12} w_{20} w_{20} - 2\beta_{13} w_{30} - 2\beta_{17} w_{10} w_{20} - 2\beta_{18} w_{20} w_{30} - 2\beta_{19} w_{10} w_{30}, \]

(19b)

\[ D_0^2 w_{32} + 9w_{32} = -2D_0 D_2 w_{30} - 2D_0 D_1 w_{31} + \sigma_{33} w_{30} - \mu_3 D_0 w_{40} - D_1^2 w_{30} - 2\gamma_{11} w_{10} w_{11} - 2\gamma_{12} w_{20} w_{20} - 2\gamma_{13} w_{30} - 2\gamma_{17} w_{10} w_{20} - 2\gamma_{18} w_{20} w_{30} - 2\gamma_{19} w_{10} w_{30}, \]

(19c)

The solutions of equation (17) in the complex form can be expressed as

\[ w_{10} = A_1(T_1, T_2)e^{iT_0} + cc, \quad w_{20} = A_2(T_1, T_2)e^{2iT_0} + cc, \quad w_{30} = A_3(T_1, T_2)e^{3iT_0} + cc. \]

(20)
Substituting equation (20) into equation (18) yields

\[ D_1 A_1 = \frac{i}{2} (\alpha_{17} A_7 + \alpha_{18} A_8), \quad D_1 A_2 = \frac{i}{2} (\beta_{17} A_7 + \beta_{19} A_9), \quad D_1 A_3 = \frac{i}{2} \gamma_{17} A_7. \]  

(21)

Eliminating the terms that produce secular terms from equation (21), the special solution is given

\[ w_{11} = \frac{1}{3} (\alpha_{11} A_1^2 + \alpha_{19} A_9 A_7 \theta^{215} + \frac{1}{5} (\alpha_{19} A_9 A_7 + \alpha_{12} A_2^2) \theta^{415} + \frac{1}{35} \alpha_{13} A_3^2 \theta^{615} - \alpha_{11} |A_1|^2 - \alpha_{12} |A_2|^2 - \alpha_{13} |A_3|^2 + cc, \]  

(22a)

\[ w_{21} = \frac{1}{3} (\beta_{17} A_7 + \beta_{18} A_8) \theta^{115} + \frac{1}{5} \beta_{17} A_7 A_8 \theta^{315} + \frac{1}{12} \beta_{12} A_2^2 \theta^{415} + \frac{1}{21} \beta_{18} A_8 A_9 \theta^{515} + \frac{1}{32} \beta_{13} A_3^2 \theta^{615} - \frac{1}{4} (\beta_{11} |A_1|^2 + \beta_{12} |A_2|^2 + \beta_{13} |A_3|^2) + cc, \]  

(22b)

\[ w_{31} = \frac{1}{8} (\gamma_{17} A_7 + \gamma_{18} A_8) \theta^{115} - \frac{1}{5} (\gamma_{11} A_1^2 + \gamma_{19} A_9 A_7 \theta^{215} + \frac{1}{7} (\gamma_{12} A_2^2 + \gamma_{19} A_9 A_7 \theta^{215} + \frac{1}{16} \gamma_{18} A_8 A_9 \theta^{515} + \frac{1}{27} \gamma_{13} A_3^2 \theta^{615} - \frac{1}{9} (\gamma_{11} A_1^2 + \gamma_{12} A_2^2 + \gamma_{13} A_3^2)^2 + cc. \]  

(22c)

Substituting equation (22) into equation (19) and eliminating the terms that produce secular terms, the averaged equation in the complex form is derived.

In order to transform the complex form of the averaged equation into the Cartesian form, let

\[ A_1 = x_1 + ix_2, \quad A_2 = x_3 + ix_4, \quad A_3 = x_5 + ix_6. \]  

(23)

Substituting equation (23) into the averaged equation in the complex form and separating the real and imaginary parts, we obtain the averaged equations as follows

\[ x_1' = -\frac{1}{2} \mu_1 x_1 + \frac{1}{2} \sigma_1 x_2 + a_1 x_3^2 + b_1 x_4^2 + a_2 x_5^2 + (a_7 x_4 + a_8 x_6) x_2^2 + (a_1 x_2 + a_3) x_5^2 - a_1 |x_1|^2 - a_2 |x_2|^2 - a_3 |x_3|^2 + cc, \]  

(24a)

\[ x_2' = -\frac{1}{2} \mu_2 x_2 + \frac{1}{2} \sigma_2 x_3 + a_1 x_2^2 + a_1 x_2 + a_2 x_4 + a_3 x_5 - a_4 x_3 + a_5 x_4 + a_6 x_5 x_1^2 - (a_7 x_4 + a_8 x_6) x_2^2 + (a_1 x_2 + a_3) x_5^2 - a_1 |x_1|^2 - a_2 |x_2|^2 - a_3 |x_3|^2 + cc, \]  

(24b)

\[ x_3' = -\frac{1}{2} \mu_3 x_3 + \frac{1}{4} \sigma_3 x_4 + b_1 x_2 + a_2 x_4^2 + b_2 x_4^2 + b_3 x_4^2 + (b_7 x_4 + b_8 x_6) x_2^2 + (b_4 x_2 + b_5 x_4 - b_6 x_6) x_5^2 - (b_7 x_4 + b_8 x_6) x_2^2 + (b_4 x_2 + b_5 x_4 - b_6 x_6) x_5^2 - b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 + (b_7 x_4 + b_8 x_6) x_2^2 + (b_4 x_2 + b_5 x_4 - b_6 x_6) x_5^2 + b_3 x_3^2 - b_5 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 - b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5, \]  

(24c)

\[ x_4' = -\frac{1}{2} \mu_4 x_4 + \frac{1}{4} \sigma_4 x_5 + b_1 x_2 + a_2 x_4^2 + b_2 x_4^2 + b_3 x_4^2 - (b_7 x_4 + b_8 x_6) x_2^2 + (b_4 x_2 + b_5 x_4 - b_6 x_6) x_5^2 - (b_7 x_4 + b_8 x_6) x_2^2 + (b_4 x_2 + b_5 x_4 - b_6 x_6) x_5^2 - b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 + (b_7 x_4 + b_8 x_6) x_2^2 + (b_4 x_2 + b_5 x_4 - b_6 x_6) x_5^2 + b_3 x_3^2 - b_5 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5 - b_2 x_2 x_3 x_5 + b_2 x_2 x_3 x_5, \]  

(24d)
+ (b_{17} x_1 + b_{18} x_3) x_3^2 - (b_{19} x_4 + b_{20} x_8) x_1 x_2 - b_3 x_5^2 - (b_{21} x_3 + b_{22} x_5) x_2 x_4 + (b_{23} x_3 + b_{24} x_5) x_2 x_6
+ b_{25} x_1 x_4 x_6 - b_{26} x_1 x_4 x_2 - (b_{27} x_3 + b_{28} x_5) x_4 x_6,
\quad \text{(24d)}

\begin{align}
x_2' &= -\frac{1}{2} \mu_3 x_5 + \frac{1}{6} \sigma_3 x_6 + c_1 x_2^3 + c_2 x_4^3 + (c_7 x_4 - c_6 x_6) x_2^2 + (c_4 x_2 + c_5 x_4 + c_6 x_6) x_2^3 \\
&\quad - (c_9 x_2 + c_{10} x_4 - c_{11} x_6) x_3^2 + (c_{12} x_2 + c_{13} x_4) x_4^2 + (c_{14} x_2 + c_{15} x_4 - c_{16} x_6) x_5^2 + c_3 x_6^3 \\
&\quad + (c_{17} x_3 + c_{20} x_5) x_1 x_2 + (c_{21} x_4 - c_{22} x_8) x_1 x_3 - (c_{17} x_2 + c_{18} x_4) x_2^2 - (c_{23} x_4 + c_{24} x_6) x_4 x_5 \\
&\quad + c_{25} x_2 x_3 x_5 + c_{26} x_2 x_4 x_6 + (c_{27} x_4 - c_{28} x_6) x_4 x_5,
\quad \text{(24e)}

x_5' &= -\frac{1}{2} \mu_3 x_6 - \frac{1}{6} \sigma_3 x_5 - \frac{1}{4} F_3 - c_1 x_1^3 - (c_7 x_3 - c_6 x_5) x_1^2 - (c_4 x_1 + c_5 x_4 - c_6 x_5) x_2^2 \\
&\quad + (c_9 x_1 + c_{10} x_4 + c_{11} x_6) x_3^2 - (c_{12} x_1 + c_{13} x_4) x_4^2 - (c_{14} x_1 + c_{15} x_4 - c_{16} x_6) x_5^2 - c_2 x_3^3 \\
&\quad + (c_{17} x_3 + c_{18} x_5) x_2^2 - (c_{19} x_4 + c_{20} x_6) x_1 x_2 - c_3 x_2^2 - (c_{21} x_3 - c_{22} x_5) x_2 x_4 \\
&\quad + c_{23} x_3 + c_{24} x_5 x_6 - c_{25} x_1 x_4 x_6 - c_{26} x_1 x_3 x_5 - (c_{27} x_3 - c_{28} x_5) x_4 x_6,
\quad \text{(24f)}
\end{align}

4. Numerical simulation of periodic and chaotic motions

In this section, the numerical results of the nonlinear responses of the circular cylindrical shell are presented under the thermal excitation. We choose the averaged Equation (24) to do numerical simulation.

The computer software *Matlab* and Runge-Kutta algorithm are utilized to explore the existence of the periodic and chaotic motions in the shell represent the truss rim structures of circular truss antenna. We plot the bifurcation diagrams, phase portraits, time histories, power spectra and max Lyapunov exponents. We choose the parameter $F_1$ that is closely related to the thermal excitation as a controlling parameter to investigate the influences of the change of the thermal excitation on the nonlinear dynamic response of the truss antenna system when the parameter $F_1$ varies from 0.2 to 1.8. Other parameters and the initial conditions in equation (24) are chosen as

![Figure 3 Bifurcation diagrams via the change of parameter $F_1$ are obtained.](image)
\( \mu_1 = 0.03, \mu_2 = 0.1, \mu_3 = 0.8, \sigma_1 = 0.2, \sigma_2 = 0.1, \sigma_3 = 0.3, F_2 = 0.4, F_3 = 0.2, a_2 = -0.06, \\
a_3 = -0.01, a_4 = 0.04, a_5 = 0.02, a_6 = -0.03, a_7 = -0.05, b_1 = -0.08, b_2 = -0.04, \\
b_3 = -0.02, b_4 = 0, b_5 = -0.03, b_6 = -0.07, b_7 = -0.03, c_1 = -0.05, c_2 = -0.02, c_3 = -0.03, \\
c_4 = 0.08, c_5 = 0.06, c_6 = -0.07, c_7 = 0.05, a_{13} = 0.04, a_{14} = 0.02, a_{15} = 0.01, a_{16} = -0.05, \\
b_{13} = -0.03, b_{14} = 0.05, b_{15} = -0.01, b_{16} = 0.01, c_{13} = 0.02, c_{14} = 0.01, c_{15} = 0.05, a_{23} = 0.03, \\
a_{24} = 0, a_{25} = -0.05, a_{26} = 0.03, a_{27} = -0.02, a_{28} = 0.01, b_{23} = -0.02, b_{24} = -0.01, b_{25} = 0, \\
b_{26} = -0.06, b_{27} = -0.05, b_{28} = 0.05, c_{23} = 0.05, c_{24} = 0.07, c_{25} = -0.03, c_{26} = 0, c_{27} = 0.04, \\
c_{28} = -0.06, x_{10} = -0.01, x_{20} = 0.002, x_{30} = 0.05, x_{40} = 0.08, x_{50} = 0.003, x_{60} = -0.07.

Figure 4 The period motion occurs when \( F_1 = 0.21 \).
The bifurcation diagrams are given in Figure 3, which $x$ axis represents the change of parameter $F_1$ and $y$ axis represents the displacement of the circular cylindrical shell. It is can be seen from Figure 3 that the dynamic behaviors of the shell begin to change from the periodic motion to the chaotic motion when the thermal excitation $F_1$ changes from 0.2 to 1.8. The change procedure of nonlinear dynamic responses of the circular cylindrical shell for the circular antenna system is investigated as shown in Figures 4-7.

Figure 5  The period-2 motion occurs when $F_1 = 0.51$.

Figure 4 show a typically periodic motion for $F_1 = 0.21$, other parameters are the same as Figure 3. Figure 4(a) and Figure 4(b) respectively show the 3-dimensional and 2-dimensinal phase portraits. Figure 4(c) is the time history of $w_1$. The frequency spectrum of $w_1$ is given in Figure 4(d). In Figure 4(e), the red dot means the Poincare map, and the maximum Lyapunov exponent is approaching to
zero in Figure 4(f). Figure 5 illustrate that the period-2 motion of the circular cylinder occurs when the parameter $F_1$ changes to $F_1 = 0.51$. Figure 6 demonstrates the existence of the period-4 motion for the circular cylindrical shell when $F_1 = 0.81$.

Figure 6  Period-4 motion occurs when $F_1 = 0.81$.

Figure 8 indicates that the chaotic motion for the shell occurs when $F_1 = 1.64$. In the aforementioned three cases, it is found that there exists large differences in the phase portrait, the frequency spectrum and Poincare map. The time history of $w_1$ in figure 7(c) seems like a random signal. The frequency spectrum of broadband in Figure 7(d) and a red area of Poincare map in Figure 7(e) clearly show the motion has countless frequencies. What's more, the maximum Lyapunov exponent in Figure 7(f) is bigger than zero. Those characteristics definitely confirm that the motion is chaotic motion.
Figure 7 The chaotic motion occurs when $F_1 = 1.64$.

5. Conclusions

In this paper, the governing equations of motion of the circular cylindrical shell for truss rim structures of circular antenna system are obtained by using the first-order shear deformation theory and applying the Galerkin procedure. The resulting dimensionless equation of motion for the circular cylindrical shell is presented with the three-degree-of-freedom system including parametric excitation, the quadratic and cubic nonlinearities. The method of multiple scales is applied to derive the steady state of modulation equations under the case of primary and internal resonance. For academic researching nonlinear phenomena, the phase portraits, the time histories, the frequency spectra, Poincare maps and the Lyapunov exponents are used to demonstrate that there exist periodic motions, and chaotic motions in system.

Based on the numerical results obtained above, it is found that there exists chaotic motion, period-1, period-2 and period-4 motion in the truss circular antenna system. It is also indicated that the motion in the truss circular antenna system is changed when the parameter $F_1$ take different values. It is demonstrated that the occurrence of the periodic-n and chaotic motions for truss antenna system
depends on the parameter $F_1$ under the certain conditions. Thus, in the situation investigated in this paper, the parameter $F_1$, or the thermal excitation is considered to be a controlling force can control the chaotic response in the truss antenna system to period motion. The results obtained here indicate that the thermal excitation has significant influence on nonlinear responses of truss rim structures in the truss circular antenna system. The findings of this paper is helpful for structurally designing the circular truss antenna and controlling its vibrations.

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