Empirical Regularities in Distributions of Individual Consumption Expenditure

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We empirically investigate distributions of individual consumption expenditure for four commodity categories conditional on fixed income levels. The data stems from the Family Expenditure Survey carried out annually in the United Kingdom. We use graphical techniques to test for normality and lognormality of these distributions. While mainstream economic theory does not predict any structure for these distributions, we find that in at least three commodity categories individual consumption expenditure conditional on a fixed income level is lognormally distributed.

*Keywords:* General Equilibrium Theory, econophysics, economic heterogeneity, lognormal distribution, consumption.

I. Introduction

Probabilistic concepts have been fundamental to economic theories of financial markets for almost a century \[1\] and these markets have received much attention from the econophysics community in recent years \[2\]. With a few exceptions (see \[3\]), this has not been the case for other markets, e.g. commodity markets although they might deserve broader attention from econophysicists and be a fruitful application area for methods of computational statistical physics as well.

The prevailing economic framework for describing markets for commodities is General Equilibrium Theory \[4\]. With its main results established about fifty years ago, it continues to be a fundamental paradigm in economic thought. Its starting point is a set $\mathcal{A}$ of agents and a space of individual consumption plans $R^L_+$. Each agent $a \in \mathcal{A}$ chooses a vector $q \in R^L_+$ (with coordinate $q_i$ denoting the quantity of commodity $i$ she wants to consume) as the maximal element with respect to an order relation over $R^L_+$, called her preference or taste, subject to the restriction that it must be affordable to her at the prevailing price system $p \in R^L_+$ (with $p_i$ denoting the price for commodity $i$). Choices of firms regarding supply of commodities are modelled in a similar manner. The main success of General Equilibrium Theory has been to prove the existence of a price system $p^*$ equilibrating aggregate demand and supply on the market for each commodity given any specifica-
tion of preferences and income on the set $A$ under very mild assumptions on the set $P$ of admissible individual preferences. Thus it has established a rigorous framework for understanding how a decentralized economic system where individuals and firms decide in a seemingly uncoordinated fashion about their individual demands and supplies can become self-coordinated by the price system. However, General Equilibrium Theory is an intrinsically static concept in many respects and therefore not capable of explaining some important issues. One major point is that it does not derive endogenously the shape and the dynamics of the distribution of agents’ characteristics, like preference, expectations and income. Because the space of admissible distributions of characteristics is not restricted in the model, General Equilibrium Theory has too little structure to produce empirically testable predictions on market outcomes. Already in 1974, a Markov Random Field model with a finite subset of $P$ as state space has been presented from which, leaving aside some technical difficulties, distributions of preferences can be derived. Unfortunately, this approach received little attention in the mainstream economics community despite the contention from social sciences that interaction between consumers is likely to be an important factor determining consumption decisions. As a result, to our knowledge no attempts have been made to derive empirically testable predictions from probabilistic models of preference dynamics. In this paper, we document empirical regularities in the distributions of individual cross-section consumption expenditure which might suggest that heterogeneity of individual consumption expenditure for certain groups of commodities is indeed governed by a common stochastic mechanism.

II. Data and methodology
Our expenditure data is provided by the Family Expenditure Survey carried out annually since 1957 in the United Kingdom. The survey is based on a representative sample of about 7000 households which amounts to 0.05% of all households in the United Kingdom. A household comprises one person living alone or a group of people living at the same address. Each household contributes information about its total income and its total expenditure for

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2The problem lies in the fact, that a distribution of preferences cannot be observed empirically because to do so we would need to know consumers’ consumption decisions for a large set of price systems $p \in \mathbb{R}^L_+$ (While relative prices do change over time, so do preferences; as a result, a ceteris paribus condition cannot be secured). On the other hand, the distribution of income is observable and substantial progress has been achieved in General Equilibrium Theory by taking it into account. However, this approach relies on ad-hoc assumptions on the distribution of preferences the validity of which cannot be tested within the prevailing theoretical framework.
goods and services in a time period of two weeks. Related types of goods and services are grouped into nine categories and expenditures are aggregated within each category. Information about expenditures is obtained partly by records kept by individual members, partly by interview in case of periodic expenditures. Details of income are obtained by interview. The periods for the record book and interviews are spread evenly over the year. Additionally, household characteristics like the number of household members, the age of the household head etc. are recorded. We confine our analysis to the categories Services, Fuel (comprising fuel, light and power), Food (comprising food and nonalcoholic beverages) and Travel (comprising transport and vehicles). For each of these categories, we investigate the distribution of expenditure within an annual sample. It is obvious that income is an important determinant of expenditure. However, we want to exclude the effect of income heterogeneity to focus solely on heterogeneity of tastes. Therefore we aim at estimating the distribution of consumption expenditure for a fixed value of income rather than in the whole sample. Clearly, we have to base our estimation on subsamples consisting of observations from narrow income intervals. Narrowing down these intervals is limited by the need to have a sufficient number of points in a subsample. Furthermore we include in one subsample only observations from households with a common number of persons, because consumption patterns presumably vary with the size of households. We choose from each annual sample four subsamples with a width of about 0.3% of the total income spectrum. Within each interval we estimated the income distribution using nonparametric techniques. Income tends to be spread evenly within these intervals with no notable regularities. To eliminate the effects of remaining income variance we corrected individual observations based on the slope of the Engel curve which regresses the dependence of consumption on income. With this slope being in most cases in the range between -0.1 and 0.3, we found that this procedure has negligible effect on the results except for some smoothing. Income and consumption expenditure do not correlate in any of the corrected subsample datasets. In summary, the stratification procedure resulted in 37 subsamples comprising 22 subsamples with one-person households and 15 with two-person households with the number of observations in each subsample between 300 and 700. In a first step, we used nonparametric density estimation techniques to get qualitative information on the shape of the density functions. In a second step, we used probability plotting to investigate the functional type of the

\[^3\]The shape and the origin of the income distribution constitute an extremely interesting research topic, but it is important to stress that regularities present in the distribution of income are not related to the regularities we aim at in this paper.
distributions. In probability plotting, the values of the empirical distribution function are transformed in such a way that they will follow a straight line if plotted against the observed realizations of the random variable $x$ (within sampling error) if the hypothesized distribution is the true underlying distribution. Assume the true distribution is $F$ with mean $\mu$ and variance $\sigma^2$. We write

$$ F(x) = G\left(\frac{x - \mu}{\sigma}\right) = G(z) $$

(1)

If we plot $z = G^{-1}(F(x)) = \frac{x - \mu}{\sigma}$ against $x$, the resulting plot will be a straight line. Probability plotting displays $z_i = G^{-1}(F_n(x(i)))$ on $x(i)$ with the empirical distribution function

$$ F_n(x(i)) = \frac{i - 0.5}{n} $$

(2)

and the ordered observations $x(1) \leq \ldots \leq x(n)$. If the hypothesized distribution is normal, [8] recommends that $F_n(x)$ be transformed by

$$ z = \text{sign}(F_n(x) - 0.5)(1, 238t(1 + 0.0262t)) $$

(3)

with

$$ t = \{ - \ln [4F_n(x)(1 - F_n(x))] \}^{1/2} $$

(4)

and plotted against $x$. If the hypothesized distribution is lognormal, [8] recommends that the same transformation be applied on $F_n(x)$ and $z$ to be plotted against $\ln x$.

III. Results

Nonparametric estimates show that for each category and in all subsamples the distribution of consumption expenditure is unimodal. The estimated density function oscillates in the tails due to limited sample size. The magnitude of these oscillations is similar as in nonparametric density estimates of Monte-Carlo generated samples from lognormal distributions presented in the literature [9]. For the good categories Services, Fuel and Travel, the nonparametric density estimates indicate that the distributions of expenditure are skewed to the right (see top of Figures 1, 2, 3 for representative plots). In the category Food, the estimated distributions appear to be slightly skewed to the right. With these preliminary findings, we tested the data of each subsample and for each category for normality and lognormality using probability plotting. In the categories Services, Fuel and Travel, the values obtained by formulae (3) and (4) follow a straight line in lognormal probability plots within sampling error indicating lognormality of the distributions (see bottom of Figures 1, 2, 3 for representative plots). In a few plots there
are outliers present in the upper and lower ends of the distribution which appear to result from contamination. Based on [10], where Monte-Carlo generated samples from mixtures of two lognormal distributions are displayed in probability plotting, we concluded that the weight of a possible contaminant distribution is less than 0.2. For the category Food, it is difficult to distinguish between normality and lognormality from probability plotting. In normal probability plotting, we obtain in most subsamples a slightly concave curve indicating that the distribution is skewed to the right [8] while obtaining a straight line in the remaining cases indicating normality. In the former instance, the sample points follow a slightly convex curve in lognormal probability plotting indicating a deviation from lognormality towards normality [8].

IV. Discussion
We see two potential explanations for the regularities found in this paper. First, the distributions might originate simply from fluctuations inherent in the process by which the data is obtained. The reported individual expenditures within a given category involve adding amounts from many instances of trading for goods and services of many types and brands. However, the fact that in at least three good categories we find lognormal distributions makes it unlikely that the regularities are attributable solely to random fluctuations. By the Central Limit Theorem a lognormal distribution of an observable would originate from a multiplicative process involving stochastically independent fluctuation on each stage, but we do not see how a multiplicative process could be involved in the process by which our data is obtained. Therefore we suggest that the observed regularities have a second - and deeper - origin in a stochastic process governing the heterogeneity of individual tastes [4]. Lognormal distributions are ubiquitous in natural sciences where their origin is some structure of the underlying system. The question of whether the regularities in consumption data have some deeper origin lying in the structure of socioeconomic systems is presumably worth exploring.

The authors thank J. Arns for extracting the data from the Family Expenditure Survey. We are indebted to W. Hildenbrand and D. Stauffer for many insightful discussions. S.P. gratefully acknowledges financial support from the Graduiertenförderung, University of Bonn.

4In economic theory, heterogeneity of tastes would be formalised as a distribution on an infinite dimensional space of functions representing preferences, as outlined in the introduction. However, the notion of preferences is a hypothetical concept and one might well doubt their existence if this concept does not provide testable predictions.
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Figure 1: Representative plots for the category Services; top: nonparametric density estimates for the subsamples: 1987, 1 person, income 40-70 (diamonds) and 1986, 2 persons, income 100-150 (+); bottom: lognormal probability plots for the same subsamples
Figure 2: Representative plots for the category Fuel; top: nonparametric density estimates for the subsamples: 1988, 1 person, income 70-100 (diamonds) and 1992, 2 persons, income 200-250 (+); bottom: lognormal probability plots for the same subsamples
Figure 3: Representative plots for the category Travel; top: nonparametric density estimates for the subsamples: 1988, 2 persons, income 150-200 (diamonds) and 1992, 2 persons, income 200-250 (+); bottom: lognormal probability plots for the same subsamples