Modal analysis and analysis of motor support based on workbench test bench design of motor dynamic shock absorber

YueMin Wang\textsuperscript{1,a}, Ke Xu\textsuperscript{2,b}, Qing Zhang\textsuperscript{1,c*}

\textsuperscript{1}School of Mechanical Engineering, Nanjing University of Science and Technology, Nanjing, China
\textsuperscript{2}215 Institute, Nanjing Chenguang Group, Nanjing, China
\textsuperscript{a}email: w13842384025@outlook.com, \textsuperscript{b}email: 1060992532@qq.com
\textsuperscript{c*}corresponding author: \textsuperscript{c}email: dazhang@njust.edu.cn

Abstract: This article takes the test bench support (hereinafter referred to as the support) as the research object, analyzes its natural frequency, establishes a three-dimensional model of the support through creo, and uses ansys workbench for modal analysis. The motor is an adjustable-speed motor with a maximum speed of 20000r/s and is fixed on the bracket. The rotation excitation frequency of the motor and the natural frequency of the bracket will form resonance in the first 10 steps, so a damping shock absorber is designed to transfer the vibration on the motor to the vibration reduction device, so as to achieve the purpose of eliminating vibration.

1. Introduction
Mechanical vibration is ubiquitous in engineering practice. The so-called mechanical vibration is the reciprocating motion of an object in a balanced position. The failure of mechanical equipment and the damage of parts are mostly related to vibration\textsuperscript{[1]}.

The modal analysis of the support can determine the dynamic parameters of the support, thereby determining the vibration form of the support when the motor is working. In the literature \textsuperscript{[2]}, through the modal analysis, the support will resonate at the low-order natural frequency, so the support The structure of the seat has been optimized to improve the stiffness and increase the natural frequency of the support to prevent the occurrence of resonance. Literature \textsuperscript{[3]} carried out a modal analysis and harmonious response analysis on the machine tool table, and carried out a topology optimization on the machine tool table, Reduce the quality and improve the rigidity. Literature \textsuperscript{[4]} analyzed the test bench supports under different working conditions to ensure the stability, safety and practicability of the test bench supports during testing. In this paper, by designing a vibration damping device on the motor, the vibration of the main system is transferred to the vibration damping device, thereby eliminating vibration, avoiding the occurrence of resonance phenomenon, and preventing damage to the mechanism.

2. Stent for modal analysis
2.1. Establishment of simulation model
The bracket is made of cast iron (HT200). Check the mechanical design manual\textsuperscript{[5]}. The density is 7150kg/m\textsuperscript{3}, the elastic modulus is 157Gpa, and the Poisson’s ratio is 0.24. The three-dimensional
Before importing the model, simplify the model and delete the bolt holes to ensure the successful meshing. After the model is simplified, it is imported into Ansys workbench for modal analysis. The model is meshed and the cell size is 10mm. The number of units is 141848, the nodes are 840960, and the grid is automatically divided. The model after grid division is shown in Figure 1. The bracket is composed of two parts, a platform and a frame, which are connected by welding, so binding constraints are adopted. The four rectangular tubes of the bracket are placed on the ground, so fixed supports are adopted.

![Figure 1 Grid division of the bracket](image)

## 2.2. Analysis of finite element simulation results

Through finite element calculation, take the first 12-order natural frequency parameters, and the modal parameters are shown in Table 1.

| Step | Frequency /HZ |
|------|---------------|
| 1    | 23.613        |
| 2    | 28.725        |
| 3    | 36.96         |
| 4    | 77.498        |
| 5    | 99.408        |
| 6    | 113.69        |
| 7    | 131.03        |
| 8    | 211.96        |
| 9    | 266.69        |
| 10   | 290.91        |
| 11   | 347.23        |
| 12   | 361.57        |

The motor is a speed-regulating motor, the maximum speed is 20000r/min, then the rotation excitation frequency of the motor is 333HZ, when the motor reaches the maximum speed, when the
natural frequency of the bracket is less than 333 hz, resonance may occur. It can be seen from Table 1 that there are 5 below 100 hz, 2 from 100 hz to 200 hz, and 3 from 200 hz to 300 hz. The natural frequency of the bracket is too densely distributed in the low frequency band and is less than 333 hz from the first to the tenth order. The motor will generate excitation in the low frequency band when it is started and closed, which will easily cause resonance, which will affect the entire test organization. Therefore, it is necessary to design a vibration damping device to eliminate vibration.

3. Design analysis of shock absorber

The motor is fixed on the bracket. When the motor is running, it will generate rotational excitation, and the generated vibration is unavoidable. To solve the vibration problem and prevent damage to the mechanism, two methods are usually adopted in engineering. The first is vibration isolation and vibration isolation. Usually, springs, rubber and other vibration isolation materials are installed at the bottom of the machine, which is equivalent to separating the vibration of the machine from the base surface. Since the motor in the test bench is hoisted on a bracket, it is not appropriate to add vibration isolation materials, and vibration isolation does not eliminate vibration, it can only isolate vibration, so vibration isolation is not adopted. The second is to add a shock absorber to the motor to absorb vibration, so as to achieve the purpose of eliminating vibration. There are two types of dynamic shock absorbers, namely, undamped dynamic shock absorbers and damped dynamic shock absorbers. This article introduces the design principles of the two types of shock absorbers, and finally chooses the damped dynamic shock absorber to eliminate the work of the motor when the rotation is excited.

3.1. Design analysis of undamped dynamic shock absorber

The equivalent mass of the motor is \( m_1 \), the equivalent stiffness is \( k_1 \), the undamped dynamic shock absorber is composed of mass \( m_2 \) and spring \( k_2 \), and the structure diagram of the undamped dynamic shock absorber is shown in Figure 2, which is composed of the motor system and the undamped dynamic shock absorber system. The combined system lists the vibration differential equations:

\[
\begin{align*}
    m_1 \ddot{x}_1 + (k_1 + k_2) \dot{x}_1 - k_2 x_2 &= F_1 \sin \omega t \\
    m_2 \ddot{x}_2 - k_2 x_1 + k_2 x_2 &= 0
\end{align*}
\]

(1)

Written in matrix form:

\[
\begin{pmatrix}
    m_1 & 0 \\
    0 & m_2
\end{pmatrix}
\begin{pmatrix}
    \ddot{x}_1 \\
    \ddot{x}_2
\end{pmatrix}
+ \begin{pmatrix}
    k_1 + k_2 & -k_2 \\
    -k_2 & k_2
\end{pmatrix}
\begin{pmatrix}
    \dot{x}_1 \\
    \dot{x}_2
\end{pmatrix}
= \begin{pmatrix}
    F_1 \sin \omega t \\
    0
\end{pmatrix}
\]

(2)

\[\begin{array}{c}
\end{array}\]

![Figure 2 Schematic diagram of undamped dynamic shock absorber](image)

\( x_1 = X_1 \sin \omega t \), \( x_2 = X_2 \sin \omega t \) Substituting into equation (1), get:
Introduce symbols:

- $\omega_1 = \sqrt{\frac{k_1}{m_1}}$ Natural frequency of motor system
- $\omega_2 = \sqrt{\frac{k_2}{m_2}}$ Natural frequency of shock absorber
- $x_1 = \frac{F_1}{k_1}$ Static deformation of the motor system
- $\mu = \frac{m_2}{m_1}$ The ratio of the mass of the shock absorber to the mass of the motor.

Substituting the symbol into equation (3), get:

$$X_1 = \frac{[1 - (\omega / \omega_2)^2]x_1}{[1 + \mu(\omega_2 / \omega_1)^2 - (\omega / \omega_1)^2][1 - (\omega / \omega_2)^2] - \mu(\omega_2 / \omega_1)^2}$$

$$X_2 = \frac{x_2}{[1 + \mu(\omega_2 / \omega_1)^2 - (\omega / \omega_1)^2][1 - (\omega / \omega_2)^2] - \mu(\omega_2 / \omega_1)^2}$$

According to equation (4), when the amplitude of the motor system $X_1$ is equal to 0, that is, when the excitation frequency $\omega$ is equal to the frequency $\omega_2$ of the shock absorber, the vibration is eliminated and the motor system remains unchanged. The amplitude $X_2$ of the shock absorber is:

$$X_2 = -\frac{F_1}{k_2} = -\frac{F_1 \sin \omega t}{k_2}$$

The force in the spring of the undamped dynamic shock absorber is:

$$k_2 x_2 = -F_1 \sin \omega t$$

The force in the spring of the undamped dynamic shock absorber just offsets the rotational excitation, so that the system vibration is transferred to the shock absorber. Figure 3 shows the frequency response curve of the motor system.

According to equation (4), when the amplitude of the motor system $X_1$ is equal to 0, that is, when the excitation frequency $\omega$ is equal to the frequency $\omega_2$ of the shock absorber, the vibration is eliminated and the motor system remains unchanged. The amplitude $X_2$ of the shock absorber is:

$$X_2 = -\frac{F_1}{k_2} = -\frac{F_1 \sin \omega t}{k_2}$$

The force in the spring of the undamped dynamic shock absorber is:

$$k_2 x_2 = -F_1 \sin \omega t$$

The force in the spring of the undamped dynamic shock absorber just offsets the rotational excitation, so that the system vibration is transferred to the shock absorber. Figure 3 shows the frequency response curve of the motor system.
freedom after the addition of the damping system, and the original system becomes two resonance frequency points. The probability of causing resonance increases and only works well in the shaded part. The adjustable speed range is too small for the speed-regulating motor. The undamped shock absorber is designed for a fixed operating frequency and is not suitable for variable frequency excitation. Therefore, it cannot meet the requirements of use, so it cannot be used to absorb vibration.

3.2. Design analysis of damped dynamic shock absorber

The design of the damped dynamic shock absorber can make the excitation frequency of the equipment change within a wider frequency and can eliminate vibration. As shown in Figure 4.

![Figure 4 Schematic diagram of damped dynamic shock absorber](image)

The main system consists of equivalent mass $m_1$ and equivalent stiffness $k_1$ groups Composition, mass $m_2$, spring $k_2$ and damping $c$ form a damping shock absorber system. Then the vibration differential equation of the system is:

$$m_1 \ddot{x}_1 + c \dot{x}_1 - c \dot{x}_2 + (k_1 + k_2)x_1 - k_2 x_2 = F_1 \sin \omega t$$

$$m_2 \ddot{x}_2 - c \dot{x}_1 + c \dot{x}_2 - k_2 x_1 + k_2 x_2 = 0$$

(5)

Considering the motor rotation excitation, using the complex expression method, the excitation at the right end of equation (5) is $F_1 e^{i\omega t}$.

$$x_1 = X_1 e^{i(\omega t - \phi)}$$

$$x_2 = X_2 e^{i(\omega t - \phi)}$$

Substituting into equation (5), get:

$$X_1 = F_1 \frac{(k_2 - \omega^2 m_2)^2 + \omega^2 c^2}{\sqrt{\left[(k_1 - \omega^2 m_1)(k_2 - \omega^2 m_2) - \omega^2 k_2 m_2 \right]^2 + \omega^2 c^2 (k_1 - \omega^2 m_1 - \omega^2 m_2)^2}}$$

$$X_2 = F_1 \sqrt{\frac{k_2^2 + \omega^2 c^2}{\left[(k_1 - \omega^2 m_1)(k_2 - \omega^2 m_2) - \omega^2 k_2 m_2 \right]^2 + \omega^2 c^2 (k_1 - \omega^2 m_1 - \omega^2 m_2)^2}}$$

(6)

Introduce symbols:

$$\omega_1 = \sqrt{\frac{k_1}{m_1}}$$  \hspace{1cm} \text{Natural frequency of motor system} \hspace{1cm} \omega_2 = \sqrt{\frac{k_2}{m_2}}$$  \hspace{1cm} \text{Natural frequency of shock}
absorber

\[ x_r = \frac{F_r}{k_i} \]  
Static deformation of the motor system

\[ \mu = \frac{m_2}{m_1} \]  
The ratio of the mass of the shock absorber to the mass of the motor

\[ \alpha = \frac{\omega_2}{\omega_1} \]  
The ratio of the frequency of the shock absorber to the frequency of the motor system

\[ \lambda = \frac{\omega}{\omega_1} \]  
Ratio of excitation frequency to motor system frequency

\[ \xi = \frac{c}{2m_2\omega_1} \]  
Damping ratio

Substitute the symbol into:

\[
X_{1x} = \left[ \frac{(\alpha^2 - \lambda^2)^2 + (2\xi\lambda)^2}{\left(1 - \lambda^2(\alpha^2 - \lambda^2) - \mu(\alpha^2 - \lambda^2)^2\right) + (2\xi\lambda)^2\left(1 - \lambda^2 - \mu\lambda^2\right)^2} \right]^{1/2} \]  
(7)

According to (7), the amplitude \( X_{1x} \) is a function of \( \alpha, \mu, \lambda, \xi \), and the amplitude frequency response curve of the motor system is shown in Figure 5.

When \( \xi = 0 \), there are two resonance frequencies. When \( \xi = \infty \), there is one resonance frequency. Draw two curves \( \xi = 0.10 \) and \( \xi = 0.32 \) between \( \xi = 0 \) and \( \xi = \infty \). The amplitude is obviously smaller and Regardless of the value of \( \xi \), all the curves pass through \( S \) and \( T \). Therefore, the design of the shock absorber should make \( X_{1x} \) below the amplitude of \( S \) and \( T \). Choose the appropriate \( \xi \) and \( \alpha \) to be effective in a wide frequency range. Reduce vibration. In order to find the abscissas \( \lambda_s \) and \( \lambda_t \) corresponding to \( S \), \( T \), the response \( X_{1x} \) when \( \xi = 0 \) and \( \xi = \infty \) are equal, namely:

\[
\lambda^4 - \frac{2(1 + \alpha^2 + \mu\alpha^2)}{2 + \mu} \lambda^2 + \frac{2\alpha^2}{2 + \mu} = 0 \]  
(8)

Figure 5 The amplitude frequency response curve of the motor system

Because the curve passes through points \( S \) and \( T \), the response of points \( S \) and \( T \) has nothing to do with damping \( c \). Therefore, the infinite damping response equation is used to find \( \lambda_s \) and \( \lambda_t \), namely:
Let \( \frac{X_{1s}}{x_s} \) and \( \frac{X_{1t}}{x_t} \) be equal to:

\[
\frac{X_{1s}}{x_s} = \frac{1}{1 - \lambda_s^2 - \mu \lambda_s^2}
\]
\[
\frac{X_{1t}}{x_t} = -\frac{1}{1 - \lambda_t^2 - \mu \lambda_t^2}
\]  

(9)

(10)

The sum of the two roots is equal to the negative coefficient of the middle term, namely:

\[
\lambda_s^2 + \lambda_t^2 = \frac{2}{1 + \mu}
\]  

(11)

Solve for: \( \alpha = \omega_s = \frac{1}{1 + \mu} = \frac{1}{1 + \frac{m_2}{m_1}} \)

Substituting \( \alpha = \frac{1}{1 + \mu} \) into equation (8) is: \( \lambda^4 - \frac{2}{2 + \mu} \lambda^2 + \frac{2}{(2 + \mu)(1 + \mu)^2} = 0 \)

Inferred: \( \lambda_{S,T}^2 = \frac{1}{1 + \mu} (1 + \frac{\mu}{\sqrt{2 + \mu}}) \)

Therefore \( \frac{X_{1s}}{x_s} = \frac{2 + \mu}{\mu} = \frac{X_{1t}}{x_t} \)

Knowing the maximum amplitude of the motor, \( \mu \) can be obtained, and the shock absorber mass \( m_2 \) is determined by \( \mu, \alpha \) is obtained, and \( \omega_s \) is obtained, and the spring coefficient \( k_2 \) is obtained. Finally, a suitable \( \xi \) must be selected to determine the damping \( c \) and \( \xi \) as The optimal value needs to meet two conditions, one is that \( S \) and \( T \) have the same amplitude, and the other is that the points \( S \) and \( T \) have horizontal tangents, so let the slopes of points \( S \) and \( T \) be zero to get the optimal value:

\[
\xi = \frac{c}{2m_2 \omega_s} = \frac{3\mu}{\sqrt{8(1 + \mu)^3}}
\]

The three coefficients of \( m_2, k_2, c \) are determined, and a damping shock absorber can be designed according to this to eliminate vibration.

4. Conclusion
Motor vibration is a very common problem. Vibration will cause damage to the transmission device, inaccurate measurement data, and noise will also cause harm to the human body. This article starts with the analysis of the motor support of the test bench, analyzes the frequency of resonance through the natural frequency of the support, and compares the design methods of undamped shock absorbers and damped shock absorbers, and finally chooses a damped dynamic shock absorber to absorb vibration. To eliminate the influence of motor vibration on the motor support of the test bench. It also provides a calculation method for vibration reduction for the design of the transmission mechanism in the future, which has certain reference value.

References
[1] Zhang Yimin. Mechanical vibration [M]. Beijing: Tsinghua University Press, 2013.8:1-7
[2] Huang Xiaotian, Li Ming, etc. Modal analysis and improvement of the motor support of the test bench based on the finite element method[J]. New technology and new process, 2018.11

[3] Zhang Xifeng, Gao Dongqiang, etc. Analysis and optimization of dynamic characteristics of machine tool table [J]. Process and Inspection, 2014

[4] Gao Zhipeng, Zhao Jian, etc. Vibration test bench design and analysis of dynamic characteristics[J]. Mechanical Engineer. 2016.8

[5] Huang Datong, Xie Liyang. Modern Mechanical Design Manual, Volume 3 [M]. Beijing: Chemical Industry Press, 2011.1: 111-135, 93~129