Numerical study of disturbances generated by localized impulse action in a flat-plate boundary layer at Mach 2.

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Abstract. The work is devoted to the numerical simulation of the excitation and development of localized disturbances in the supersonic boundary layer of a plate with a sharp leading edge at a Mach number of 2 in a two-dimensional formulation. Artificial disturbances are generated with the help of localized in time and space by changing the boundary conditions to the velocity component normal to the surface. The studies for two intensities of the impulse action were carried out. It was found that with an increase in the intensity of the action on the boundary layer, the amplitude of the generated disturbances increases. The development of disturbances of various amplitudes is similar. The ratio of the level of total temperature pulsations to the level of mass flow pulsations was estimated for the investigated disturbances. It was found that at small distances from the source of disturbances, this ratio increases downstream, at the same time in the far field, it reaches a constant value.

1. Introduction

One of the effective approaches to the study of the laminar-turbulent transition in boundary layers is the investigation of the development of disturbances with a wide spectrum (wave packets) artificially introduced into a boundary layer. This makes it possible to obtain data on the joint development of a wave packet and to study possible interactions. For the first time, experimental studies of the development of artificial wave packets in the boundary layer were carried out at low subsonic flow velocities in [1, 2]. Today, this approach is actively used to study the laminar-turbulent transition in high-speed boundary layers both in experiments [3-6] and in numerical modeling [7-10].

It is necessary to generate localized disturbances, the properties of which are close to disturbances in real flows for correct simulation of the development of disturbances in the boundary layer at all stages of the laminar-turbulent transition. This requires detailed knowledge about the influence of various parameters of generation of disturbances and it is required to compare the calculated disturbances with the properties of natural disturbances known from experimental studies.

In experimental studies [11-15] in supersonic boundary layers for a wide class of flows, it was found that the ratio of the level of total temperature fluctuation to the level of mass flow fluctuation remains constant at all stages of the laminar-turbulent transition. This value can be used to determine the proximity of the disturbances investigated in calculations to natural boundary layer disturbances in experiments.
In this work, we carry out numerical studies of the excitation and development of localized disturbances in the laminar boundary layer of the flat plate at Mach 2. The calculations are carried out in a two-dimensional formulation, which does not allow to fully model the development of disturbances in the supersonic boundary layer, however, it makes it possible to determine the regularities of the influence of the impact intensity, as well as to work out the methods comparing the calculated disturbances with the properties of natural pulsations. The ratio of the level of total temperature pulsations to the level of mass flow pulsations is estimated for the investigated disturbances. The paper considers artificial disturbances that are generated with the help of localized in time and space by changing the boundary conditions to the velocity component normal to the surface. Studies were carried out for two intensities of the action.

2. Calculation set-up
Direct numerical simulation of supersonic flow around the plate was carried out using the Navier-Stokes equations in the ANSYS Fluent software package. The problem was considered within the framework of a continuum model for an ideal compressible viscous heat-conducting gas. The gas was described as ideal, \( Cp = 1006.43 \, \text{J/(kg} \cdot \text{K)} \), thermal conductivity was taken according to the kinetic theory.

Numerical modeling was carried out for the following parameters of the oncoming flow: Mach number \( M = 2 \), flow temperature \( T_e = 164 \, \text{K} \), static pressure \( P_e = 5600 \, \text{Pa} \), which corresponds to the value of the unit Reynolds number \( \text{Re}_1 = 4.5 \times 10^6 \, \text{m}^{-1} \). Such parameters of the oncoming flow are close to the oncoming flow in the T-325 wind tunnel at the ITAM SB RAS.

The calculations were carried out in a two-dimensional setting. The rectangular computational domain had the following dimensions: vertically 105 mm, horizontally 155 mm. The plate surface was located at the lower boundary of the computational domain and started at a distance of 5 mm from the input boundary. The symmetry boundary condition was set on the free region in front of the plate. The condition of an adiabatic wall and no-slip condition were set on the plate surface. The left and upper boundaries of the computational domain corresponded to free flow conditions, and the right boundary had output conditions. In the longitudinal direction (\( x \)-coordinate), the computational domain was divided into cells with a size of 1 mm. In the direction perpendicular to the plate (\( y \)-coordinate), the computational domain was divided into two parts. The first part, closest to the plate, 5 mm high, was divided into 100 cells which clustered near the surface. The second, upper part, was divided into 30 cells. A structured rectangular grid was used.

The calculation was carried out in two stages: at the first stage, the stationary base flow was calculated. At the second stage a transition calculation with a time step of 1e– 9 sec was performed with the introduction of artificial disturbances. To generate artificial disturbances on the surface of the plate, a source region was selected, located at a distance of 30 mm from the leading edge (\( x = 30 \, \text{mm} \)) of the plate and having a length of 2 mm. In the source region, the boundary condition was set on the normal component of the velocity for a time of 50 \( \mu \text{s} \). After that, the initial no-slip boundary condition was returned, and the calculation was carried out further. In this paper, the intensity of the impact on the boundary layer is considered 25 and 50 m/s, which corresponds to approximately 5 and 10 % of the oncoming flow velocity, respectively. In total, for each calculation at the second stage 1020 thousand iterations were performed, that corresponds to 1020 \( \mu \text{s} \). In the process of non-stationary calculations, the instantaneous profiles of the longitudinal component of the velocity, density and temperature along the \( y \)-axis were recorded at various values of the longitudinal coordinate \( x = 40, 50, \ldots, 100 \, \text{mm} \).

3. Results
In experimental studies, the most detailed information on disturbances in the supersonic boundary layer can be obtained with the help of a hot-wire anemometer, which is usually sensitive to pulsations of the longitudinal component of the mass flow. Therefore, in this work, disturbances in the boundary
layer are analyzed from the mass flow fluctuations normalized to the mean local flow. The mass flow pulsation was calculated as follows:

\[
\rho U' = \frac{\rho U(t) - \rho U(0)}{\rho U(0)} \times 100\%,
\]

where \( \rho U(t) \) is the mass flow at each moment of time, \( \rho U(0) \) is the mass flow of base flow.

Figures 1 and 2 show a comparison of the calculated and experimental data on the profile of the mean flow and the rms level of mass flow rate pulsations. The experimental data were obtained in [4] in a T-325 wind tunnel of the ITAM SB RAS at a Mach number \( M = 2 \) in the boundary layer of the plate. The data for the value of the Reynolds number \( \text{Re}_x = \text{Re}_t * x = 0.36 \times 10^6 \) are presented. The mean flow is normalized to the external flow, and the rms level are normalized to the maximum value.

**Figure 1.** The calculated and experimental data on the profile of the mean mass flow.

**Figure 2.** The calculated and experimental data on the profile of the mass flow pulsations.

The results of calculations and experiments on the mean flow are in good agreement with each other at \( y > 0.45-0.5 \) mm, which corresponds to the supersonic region of the boundary layer. In the transonic part \( (y < 0.45-0.5 \) mm), a discrepancy is observed, which is associated with a feature of the hot-wire anemometer - the used hot-wire probe is calibrated for supersonic flows.

In the case of calculations, the maximum of pulsations is observed in the transonic part of the flow \( (M = 1.1+1.2) \). In the experiment, the maximum of pulsations is observed above the model. The discrepancies may be due to the fact that in the transonic region of the flow (Mach number is about 1), the hot-wire probe is less sensitive to mass flow pulsations.

Dependences of mass flow pulsations on time at different distances from the surface of the model and different values of the longitudinal coordinate \( x = 60 \) (figure 3), \( x = 100 \) (figure 4). The results for disturbance introduced by changing boundary condition on the velocity component normal to the plate surface 50 m/s are presented.
Figure 3. Mass flow pulsations in $x = 60$ mm at different values of the $y$ coordinate.

Figure 4. Mass flow pulsations in $x = 100$ mm at different values of the $y$ coordinate.

For small values of the longitudinal coordinate, the introduced disturbance is a localized flow defect. The largest amplitudes are observed inside the boundary layer. At the upper boundary of the boundary layer, the disturbance amplitude tends to zero. The largest amplitude of disturbances at various values of the longitudinal coordinate $x$ is in the boundary layer region, where the mean mass flow is about 0.5-0.6 of the oncoming flow. This is consistent with experimental observations.

Developing downstream, the initial localized disturbance decays. At the leading edge of the disturbance, with distance from the leading edge of the plate, the formation of wave structures is observed, which grow downstream. Note that similar results of the development of localized disturbances are observed at subsonic flow velocities [2].

The evolution downstream of mass flow pulsations are shown in figures 5 and 6. The results for various values of the longitudinal coordinate ($x = 40, 60, 80, 100$ mm) are presented. The pulsations at the layer where $\rho U/\rho U_\infty = 0.6$ and the disturbances amplitude is maximum are considered. Disturbance introduced by a change in the normal velocity component on the plate surface of 25 m/s is shown in figure 5. Figure 6 presents results for 50 m/s action.

Figure 5. Evolution of mass flow pulsations. Impulse action of 25 m/s.

Figure 6. Evolution of mass flow pulsations. Impulse action of 50 m/s.

The evolution of disturbances generated by the impulse action on the boundary layer of different intensities are similar. The propagation velocities are the same, the wave structures generated at the
leading front are similar. This indicates on the linear development of disturbances in these studies. The velocities of propagation of disturbances downstream, estimated from the maximum deviation, are approximately $0.6 U_\infty$, that coincides with the results of experimental studies on the development of disturbances from a pulsed discharge \[4, \ 5\].

In experimental studies in supersonic boundary layers using a hot-wire anemometer in the automatic scanning mode on the hot-wire probe overheating, it is possible to obtain data on the development of mass flow and total temperature pulsations. In [11-15] for a wide class of flows, it was found the ratio of the level of total temperature fluctuation to the level of mass flow fluctuation remains constant that at all stages of the laminar-turbulent transition and equal to approximately 0.1-0.2. This value can be used to determine the proximity of the disturbances investigated in calculations to natural boundary layer disturbances in experiments.

In this paper, the growth downstream of the ratio of root-mean-square level of the pulsations of total temperature to the level of the mass flow fluctuations is considered. The pulsation of the total temperature is determined as follows:

$$T_0' = \alpha T' + \beta u',$$

where $T'$, $u'$ – pulsations of temperature and velocity, $\alpha$ and $\beta$ are gas-dynamic functions, which are calculated as follows:

$$\alpha = \frac{1}{1 + \frac{\gamma-1}{2} M^2}, \quad \beta = (\gamma-1)\alpha M^2.$$

The results of the estimation of the ratio of root-mean-square level of the pulsations of total temperature to the level of the mass flow fluctuations are shown in figure 7. The ratio increases downstream up to $x = 80$ mm, reaching a constant value of 0.3.

![Figure 7](image)

**Figure 7.** An increase in the ratio of temperature pulsations to mass flow downstream.

Close to the source of disturbances ($x < 70$ mm), this ratio increases downstream. At $x > 80$ mm, the ratio practically does not change and is close to 0.3. As noted above, initially the disturbance is represented as a pulsed flow defect. While developing downstream, the initial defect decays, and a packet of growing waves is generated at its leading front. Apparently, active attenuation of the initial defect leads to the observed increase in the ratio of the total temperature pulsation level to the mass flow pulsation level. Subsequently, the growth of disturbances generated at the leading front occurs mainly.
4. Conclusion
Direct numerical simulation of the excitation and development of localized perturbations in the boundary layer of a plate with a sharp leading edge at Mach 2 has been carried out. It has been shown that the greatest amplitude of disturbances at different values of the longitudinal coordinate $x$ is observed in the boundary layer region, where the mean mass flow rate is 0.5-0.6 of the oncoming flow. This is consistent with experimental observations.

Initially the disturbance generated by a change in the normal velocity component on the plate surface is represented as a pulsed flow defect which decays downstream. At the leading edge of a localized disturbance, the formation of growing wave structures is observed. The development of disturbances generated by impulse action on the boundary layer of different intensities are of the same.

The velocities of propagation of disturbances downstream, estimated from the maximum deviation, are approximately $0.6 U_x$, that coincides with the results of experimental studies on the development of disturbances from a pulsed discharge [4, 5].

The ratio of the level of temperature pulsations to the level of mass flow pulsations increases downstream up to $x = 80$ mm, reaching a constant value of 0.3. In experimental studies, this ratio is constant and equal to 0.1-0.2. It can be concluded that the method of introducing disturbances into the boundary layer used in these calculations does not allow modeling disturbances in real flows. Further searches should be carried out for the optimal method of introducing artificial disturbances in the calculations.

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