Physics for the future colliders.

Fawzi Boudjema

Laboratoire de Physique Théorique LAPTH 1
Chemin de Bellevue, B.P. 110, F-74941 Annecy-le-Vieux, Cedex, France.

Abstract

Physics issues at the upcoming and planned colliders are discussed. We critically review the different arguments that suggest that New Physics is bound to materialise at the TeV scale and why we should keep an open minded approach. The complementarity of the LHC and a moderate energy $e^+e^-$ collider is stressed together with the need for higher energy machines.
1 Introduction: The particles of today

The extremely successful Standard Model, $\text{SM}$, is a theory that describes in a very neat, and in part economical, manner the interactions of all known fundamental spin-1 and spin-1/2 particles. However it also requires a spin-0 particle which has so far not been seen. Future experiments are needed either to unravel this missing particle or probe whether the scalar sector is in fact the tip of a beautiful iceberg full of much interesting physics to come. When trying to predict what physics is to be expected at high energies and which machines are best suitable for this physics one should ideally rid oneself of all theoretical prejudices even though some theoretical arguments backed by some very strong indirect experimental evidence are very tempting.

Take for instance the spin-1 sector. LEP has brought a beyond-doubt confirmation that all the forces of the theory are based on a local gauge symmetry, by the fact that all the couplings of the gauge bosons not only to fermions but among themselves are universal. Moreover the three independent gauge couplings for the $SU(2) \times U(1) \times SU(3)$ forces are now measured with such a precision that evolving them to high scale seems to suggest that they might unify at a scale of $\sim 2 \times 10^{16}\text{GeV}$, if one postulates the existence of supersymmetric particles at energies of the order 1TeV, or even below, see Figs 1. Since many of the problems that plague the Standard Model are solved in the more symmetric supersymmetric completion of the model, many view this unification as strong evidence for SUSY and that SUSY will be discovered at the LHC. To be cautious, one should also point out that, recently, one has found out[2] that models based on extra-dimensions, not necessarily supersymmetric, are also able to unify the three gauge couplings at scales closer to the TeV scale than the Planck scale, without guarantee that new particles be seen at the upcoming colliders. When talking about the forces, one should not forget
that the all pervading gravity still lacks a satisfactory quantum description. Although it is not included as part of the $\mathcal{SM}$, it does provide a scale, the Planck scale $M_P$, which conceptually poses a severe conflict with the scale of the electroweak theory.

**The spin-1/2.** With the relatively recent discovery of the top and the direct confirmation of the tau neutrino the matter content of the standard model is complete. The Lagrangian describing the interaction of the spin-1/2 and spin-1 of the theory, leaving aside the masses, is very simple and economical

$$\mathcal{L}_{1,1/2} = i \bar{f} \gamma^\mu f - \frac{1}{4} (F^\mu_{\nu})^2$$

with $D$ being the appropriate covariant derivative acting on the left handed doublets and the right-handed singlet. The latter do not mix in the absence of mass. Having defined the charges one only needs the three coupling constants.

Let us now turn to the spin-0 part. First, as mentioned earlier the Higgs, the physical spin-0 particle of the model, has not been confirmed. However a scalar doublet, with a non-zero expectation value is required in the $\mathcal{SM}$ to provide mass to all the particles in a gauge invariant way. It is here that the majority of the seemingly haphazard parameters of the $\mathcal{SM}$ are hidden. Moreover it is important that one checks the structure of this sector and the nature of the scalar potential. This is a large part of the physics of the future. It can be argued that it is not completely correct to claim that we have not seen any spin-0 in the $\mathcal{SM}$. Indeed, the Goldstone bosons $\varphi$ of the scalar doublet $\Phi$, are somehow the longitudinal modes of the $W$ and $Z$

$$\Phi = \left( \begin{array}{c} \varphi^+ \\ \frac{1}{\sqrt{2}} (v + H + i\varphi_3) \end{array} \right)$$

and therefore it is only $H$ that has not been seen yet.

The scalar potential is also mysterious

$$\mathcal{V}_{SSB} = \lambda \left[ \Phi^\dagger \Phi - \frac{v^2}{2} \right]^2 = \lambda \left[ \Phi^\dagger \Phi \right]^2 - \mu^2 \Phi^\dagger \Phi + \frac{\lambda v^4}{4}$$

Ideally one would like to explain how the minus sign for the “the mass of the doublet” emerges. The Higgs interaction, the masses and the mixing of the fermions and bosons are then contained in

$$\mathcal{L}_{0,1/2} = |D_\mu \Phi|^2 - V_{SSB} - (M_u^{ij} \bar{u}_R^i \Phi^j Q_L^j + M_d^{ij} \bar{d}_R^i \Phi^j Q_L^j + M_l^{ij} \bar{l}_R^i \Phi^j L_L^j + \text{h.c.})$$

Although the masses of the $W/Z$ are derived from gauge couplings, those of the fermions and the Higgs are Yukawa couplings. The matrices in family space involve a large number of parameters. Future probes of the flavour sector need to pin-down these matrices and check whether the hierarchical mass and mixing structures can be obtained in terms of very few parameters. For instance, texture-zero matrices of the form

$$M_u = \lambda_t \begin{pmatrix} 0 & 0 & c e^{3i\phi} \\ 0 & \lambda_c/\lambda_t & 0 \\ c e^{3i\phi} & 0 & 1 \end{pmatrix}, \quad M_d = \lambda_b \begin{pmatrix} 0 & a e^{3i\phi} & 0 \\ a e^{3i\phi} & e^2 & b e^2 \\ 0 & b e^2 & 1 \end{pmatrix}$$
with \( a, b, c \sim 1 \) successfully lead to \( V_{us} = \sqrt{m_d/m_s} \). Future precise measurements of \( V_{ij} \) will drastically discriminate between different ansätze. Recent evidence for neutrino masses, may also be a sign of a new mechanism of mass generation and hence of New Physics. The argument is theoretically biased, even though very appealing. It is biased because although it does not really explain why \( \lambda_e/\lambda_t \sim 10^{-6} \) it seeks to find a mechanism leading to a similar hierarchy factor within a generation. Since a right-handed neutrino has no electroweak quantum number and since neutrinos can get a Majorana mass, neutrino masses may be induced to be small through the see-saw mechanism. The latter gives the neutrino a tiny mass because the right-handed neutrinos are extremely heavy, with a mass scale related to the unification scale or some intermediate scale, and therefore New Physics: \( m_\nu = m_D^2/m_\nu_R \). With \( m_D \) the Dirac mass as generated for the quarks and charged leptons and hence with a value similar to those, while \( m_\nu_R \) is the extremely heavy right-handed neutrino mass.

While at it, note that the Higgs potential poses another irritation. Potentially it contains a huge vacuum energy density through the constant term in \( V_{SSB} \). This can be written as \( V_{H,cosmos} = \rho_{vac}^H = M_H^2/8 = \Lambda_H/8\pi G_{\text{Newton}} \). Even though recent results suggest \( \Lambda \neq 0 \), it rests that \( \rho_{vac} < 10^{-46}\text{GeV}^4 \), while for \( M_h > 100\text{GeV} \) \( \rho_{vac}^H > 10^8\text{GeV}^4 \). This problem is still there in broken SUSY.

Talking about the cosmos, and assuming that our cosmological models and other hypotheses are correct, then the matter content of the SM is not enough. There seems to be a large amount of dark matter in the universe which calls for New Physics and new particles.

2 Lessons from the past: Why there should be New Physics

Unitarity is a strong argument which is not based on some theoretical prejudice or bias. This is just a statement that one can not have a probability larger than one! This generally translates into the fact that cross sections do not grow indefinitely. A prime example is given by \( e^+e^- \to W^+W^- \) as measured at LEP2, Fig. 2. One sees that if the \( WWV \) coupling and the \( e\nu_\ell W \) couplings are not equal, the cross section can get catastrophically large. For this not to happen, either one has to enforce the gauge symmetry at all energies or one has to invoke New Physics to take place in case the \( WWV \) coupling deviates from its SM value. Therefore in a sense the trend observed at LEP2 was foreseen. This has happened time and again in the history of physics. Fermi had described beta-decay as a point-like interaction. This had to break down. The reaction \( \bar{\nu}_\mu \mu \to \bar{\nu}_e e \) calculated on the basis of the contact interaction grows as \( G_F^2E^2 \), \( E \) is the c.m energy. Unitarity would be violated at energies of the order \( E > G_F^{-1/2} \sim 300\text{GeV} \). However physics that restored this behaviour, namely the exchange of a \( W \) boson, appeared much earlier: 80\text{GeV}, while precision measurements hinted at a departure from a point-like structure even earlier.

Next we could consider \( \nu_\mu \bar{\nu}_\mu \to W^+W^- \) but with only the t-channel \( \mu \) exchange, no \( Z \) exchange. Unitarity in the \( J = 1 \) channel would have told us that unitarity would break at an energy \( E > \sqrt{3\pi/G_F} \sim 1\text{TeV} \). This is much higher than the LEP2 energy where the \( WWZ \) coupling has been measured at better than a few percents. In the situation
we are in today we can consider $W^+W^- \rightarrow W^+W^-$

\[
\begin{align*}
W^+ &\rightarrow \gamma, Z \rightarrow W^+ \\
W^- &\rightarrow \gamma, Z \rightarrow W^-
\end{align*}
\]

and take all the couplings to be gauge couplings. Adding all the diagrams the helicity amplitude when all $W$’s are longitudinal takes the form

\[\mathcal{M}_{LLLL} \sim \sqrt{2} G_F u\] (5)

$u$ is one of the Madelstam variables. This shows that the cross section will grow with energy. A partial wave analysis for the $J = 0$ channel shows that New Physics ought to be manifest at $\sqrt{s_{WW}} \geq 1.2$TeV. This kind of energies require post-LHC $pp$ machines and $e^+e^-$ (or even $\mu^+\mu^-$) facilities operating in the range of some 3TeV or so and with sufficient luminosity. This is not to say that a machine such as TESLA will not be sensitive to a strongly interacting regime of the weak interaction. In fact as we will see some useful constraints can be set for some scenarios. But this is not always guaranteed.

In the $SM$, the picture can be improved by the inclusion of the Higgs

\[
\begin{align*}
W^+ &\rightarrow H \rightarrow W^+ \\
W^- &\rightarrow H \rightarrow W^-
\end{align*}
\]

turning the amplitude, Eq. 5, into

\[
\mathcal{M}_{LLLL} \sim -\sqrt{2} G_F M_H^2 \left( \frac{s}{s - M_H^2} + \frac{t}{t - M_H^2} \right)
\] (6)

Figure 2: The small insert shows the latest data of the $W^+W^-$ cross section at LEP2[4]. The main figure shows the behaviour of the same cross section at much higher energies and the contribution of each channel.
but again at asymptotic energies the amplitude grows with the mass of the Higgs. This then puts a limit on the Higgs mass. Partial wave analysis requires the *perturbative limit*

\[ M_H \leq \frac{4\pi\sqrt{2}}{3G_F} \sim 700\text{GeV} \]  

within reach of the LHC.

### 3 Lessons from the present and near future

![Graph](image)

**Figure 3:** a) *Latest limit on the SM Higgs mass from the precision measurements.* The (yellow) shaded area gives the direct limit. b) *Expected discovery/exclusion mass limit on the Higgs mass at the Tevatron.*

Most people will ask why bother with this since LEP data indicates the presence of a Higgs and constrains its mass to be less than 170GeV at 95%, Fig. 3. A slightly higher limit is derived if one uses improved values for \(\alpha_{em}(M_Z)\). Moreover many see the excess at the end of LEP2 as a tantalising hint of a Higgs signal at \(M_h \simeq 115\text{GeV}\), that could be confirmed by the Tevatron.

Other arguments are often presented to back up the idea of a light Higgs. Requiring the Higgs Yukawa coupling, and hence its mass, to be perturbative up to a certain scale \(\Lambda\), where New Physics should show up, and that the vacuum be stable give the following constraint: *If* one requires that the theory be perturbative up to the unification scale and the vacuum be stable, then the Higgs mass is tightly constrained around 180GeV or so, see Fig. 4. A Higgs mass as suggested by the excess seen at LEP2 would not be compatible with a stable vacuum if \(\Lambda > 10^6\text{GeV}\). Such a light Higgs may then be a supersymmetric Higgs since the contribution of the SUSY spectrum will make the vacuum stable at scales compatible with the unification scale. Note on the other hand that a
Figure 4: Requiring the Higgs couplings to remain perturbative up to a certain scale $\Lambda$ gives the upper curve while imposing the vacuum to remain stable leads to the other curve, from [6].

Heavy Higgs ($M_H > 600\text{GeV}$) means Physics at the TeV scale. So again either a light Higgs which when combined with gauge coupling unification hints at new particles around the TeV scale or a heavy Higgs but again with some manifestation of New Physics at the TeV scale. In the first case one may be fortunate to discover, beside the light Higgs, other new particles at the upcoming colliders, while in the second case to reach the TeV scale might require a post-LHC hadron machine and/or a few TeV lepton collider.

Even if the Tevatron fails to see a Higgs, the LHC should have no problem discovering a Standard Model Higgs all the way from the direct LEP2 limit to 1TeV, see Fig. 5. Note however that the significance is lowest for $M_h \sim 115\text{GeV}$. Moreover if the Higgs

Figure 5: Statistical significance of a SM Higgs in ATLAS as a function of $m_H$. The various channels are shown, from [8].
(especially a light one) is slightly non standard, discovery may not be guaranteed. For instance invisible decays of a light Higgs can easily bring the significance below 5. While at it, let us stress that within the MSSM, an invisible Higgs invariably entails a rich production of neutralinos and charginos because these would be quite light. One could then make precision measurements on the masses of these to then explain why the Higgs has been missed. Recent analysis suggest that even at the LHC (and the Tevatron if it is produced) it may be possible to track down an invisible Higgs. So wait... Of course an invisible Higgs at the linear collider is no problem.

4 Three solutions to the hierarchy problem and the three main paths to the New Physics

The previous discussions should be convincing enough as to why New Physics should show up. Traditionally the main motivation for New Physics is related to the hierarchy problem and again stems from yet another frustration with the Higgs. Let us go once again through the spin content of the SM and the associated masses of the particles. Why should the mass of the photon remain exactly zero or why is it natural that those of the W/Z are ridiculously tiny compared to the what we think is the fundamental scale, \( \Lambda_P \sim 10^{19}\text{GeV} \)? This is because these particles are gauge bosons whose mass is protected by local gauge symmetry. For the spin-1/2, a massless electron is protected by a chiral symmetry. Though this is only a global symmetry it is still powerful enough to keep the correction to the electron mass tiny. Starting with a bare mass \( m^0_e \), at one loop one gets

\[
m_e = m^0_e \left(1 + \frac{3\alpha}{2\pi}\log(\Lambda^2/m^2_e)\right)
\]

The correction is only logarithmic, taking the cut-off \( \Lambda \) to be the Planck scale gives a 30% correction. Doing the same exercise for the scalar Higgs the correction for \( M^2_H \) is quadratic in \( \Lambda \). If this is the Planck scale, this calls for extraordinary fine-tuned adjustment close to 30 digits! Clearly this is unnatural and is related to the fact that there is no symmetry that protects a scalar mass. Hence the motivation for the New Physics based on the disparate scale of electroweak symmetry breaking, \( v \sim 246\text{GeV} \) and \( \Lambda_{\text{Planck}} = \Lambda_P \sim 10^{19}\text{GeV} \).

\[
M_H, v \ll \Lambda_{\text{Planck}}
\]

Notwithstanding that this is a numerical accident and also that \( \Lambda_{\text{Planck}} \) relates to a force which is not set on as firm foundation as the gauge theories of the SM, there are basically three main routes to solving the hierarchy problem. Each one tackles one side or one part of the above inequality.

i) \( M_H \) just not there! These models require no Higgs or rather no fundamental elementary scalar. The Higgs may be a composite particle that can be heavy. As we have seen earlier these models would suggest that the W interaction can become strongly interacting at the TeV.
ii) $\Lambda_P$ is not the fundamental scale. The fundamental scale is much lower and can be at the TeV scale. This is the very recent extra-dimension solution. It does not require a host of new particles. Although this brings the fundamental scale down to $v$, little has been convincingly done to dynamically induce $v$ from the fundamental scale. If this is not done, the fact that the two scales refer to quite different mechanisms, leaves a puzzle as to why they are so close to each other. For some recent attempts, see [12]

iii) $\ll$ is quite natural. This is the supersymmetric solution and seems to me more satisfying than the previous solution since it endows the scalar with a symmetry that protects its mass, by associating the scalars to fermions that have a protective mechanism. By doubling the known spectrum, the phenomenology is very rich.

4.1 The Higgsless models

First of all, it is worth stressing that one can have a perfectly gauge invariant theory without a Higgs. One only needs to make use of the Goldstones. One should also take into account the fact that the $\rho$ parameter being to a good approximation unity suggests a custodial symmetry. Then use (for notations and a mini review see [13])

$$\Sigma = \exp\left(i\omega^{\uparrow^i}\nu^i\right) \quad D_\mu \Sigma = \partial_\mu \Sigma + i\frac{1}{2} \left( g W_\mu \Sigma - g' B_\mu \Sigma \tau_3 \right)$$

The $W, Z$ masses are simply

$$L_M = \frac{v^2}{4} \text{Tr}(D^\mu \Sigma^\dagger D_\mu \Sigma)$$

which is the lowest order operator one can write.

The severe problem this model faces, is how to accommodate the LEP data that calls for a light Higgs. The answer is that those fits are only valid within the SM. Fitting the data with a larger Higgs mass calls for New Physics contributions to the LEP observables. In a more general context one has to consider the $S, T, U$ [14] (or $\varepsilon_1, 2, 3$ [15]) variables besides the Higgs mass. These can be approximated as

$$\varepsilon_1 = \Delta \rho = \alpha T = \frac{3 G_\mu M_Z^2}{8 \pi^2 \sqrt{2}} \left( \frac{m_t^2}{M_Z^2} - 2 s_W^2 \ln(M_h/M_Z) \right) + \varepsilon_{1\text{NP}}$$

$$\varepsilon_2 = -\frac{\alpha}{4 s_W^2} U = -\frac{G_\mu M_W^2}{2 \pi^2 \sqrt{2}} \left( \ln\left(\frac{m_t}{M_Z}\right) \right) + \varepsilon_{2\text{NP}}$$

$$\varepsilon_3 = \frac{\alpha}{4 s_W^2} S = \frac{G_\mu M_W^2}{12 \pi^2 \sqrt{2}} \left( - \ln\left(\frac{m_t^2}{M_Z^2}\right) + \ln(M_h/M_Z) \right) + \varepsilon_{3\text{NP}}$$

In models where the Higgs is absent, $M_h$ in the above should be interpreted as a cut-off at the TeV scale (1TeV to 4$\pi v \sim 3$TeV). The contribution which is most sensitive to the Higgs is $\varepsilon_3$. In $\varepsilon_1$ the Higgs dependence is somehow subleading compared to the quadratic top mass dependence. The $S$ variable has been a killer of naive technicolour. Precision measurements interpreted with a large Higgs mass need $S_{\text{new}} < 0$ to counterbalance the large Higgs mass/cut-off. Recent fits, allowing $T$ as free parameter, and taking the New
Physics at 3TeV give $10^{-3} < S_{\text{new}} < 10^{-1}$. Technicolour based on a naive rescaling of QCD, leads to $S > 0$. In an effective Lagrangian approach including next-to-leading operators to those contributing to the mass, one should expect the following operator, beside an operator that accounts for a slight breaking of the custodial symmetry and hence to $T$,\n
$$L_{10} = \frac{g}{16\pi^2} L_{10} \text{Tr}(B^{\mu\nu} \Sigma^\dagger W^{\mu\nu} W) \rightarrow L_{10} = -\pi S_{\text{New}}$$ \hspace{1cm} (12)\n
Limit from LEP/SLC thus suggest $L_{10} \sim \mathcal{O}(1)$. One can therefore fit the LEP data with a very heavy Higgs by including a “judicious” amount of $S$ and $T$. Some have argued that one should also consider a larger set of operators that involve the various fermion fields, and not just those bosonic that contribute only to the two variables $S$ and $T$. The fermionic operators give extremely constraining bounds and thus if all operators are of the same order, which could be argued is what is “natural”, then one can not so easily fit a large Higgs mass. However it may well be that the correct effective theory leads to negligible fermionic operators. It is not a very attractive possibility but one that can not be totally dismissed. Nonetheless, considering solely the bosonic operators, one can write others beside $L_{10}$. Although they do not contribute directly to the present precision measurements they do contribute to the tri-linear and quadri-linear vector boson couplings. For instance, $L_{9L} = -ig \frac{L_{9L}}{16\pi^2} \text{Tr}(W^{\mu\nu} D_\mu D_\nu \Sigma^\dagger) \text{ and } L_{11} = \frac{L_{11}}{16\pi^2} \left( \text{Tr}(D^\mu \Sigma^\dagger D_\mu \Sigma) \right)^2$ to cite only two (for more see [13]). Now these operators need to be probed at higher energies. In order that one learns more than what we have with the LEP data, these operators should be constrained better than $L_{10}$, i.e., the $L_i$ should be measured better than .1, ideally one should aim at the $10^{-2}$ level. This is hard since already $L_{9L} \sim .1$ implies measuring the $\Delta \kappa_\gamma$ in the $WW\gamma$ vertex at $\Delta \kappa_\gamma \sim 1.3 \times 10^{-4}$.

The figure shows how the tri-linear couplings would be measured at the colliders. $e^+e^-$ even at relatively modest energies does a good job, but one is still somehow below the sensitivity one has reached on $L_{10}$. Recent simulations for TESLA (500GeV) with improved luminosity (500fb) give $L_9 \sim .2$ while $pp \rightarrow WZ$ at the LHC give $L_{9L} \sim 1$ [13]. $WW$ scattering will constrain the quartic couplings $L_{1,2}$. For these one definitely needs an $e^+e^-$ in excess of a TeV, the precision on the couplings scales as $s^{-1/2}$ $T^{-1}$. A simulation for 1TeV $e^+e^-$ machine with a luminosity of 500fb$^{-1}$ give a limit $L_1 \sim 1.5$ while the LHC gives $\sim 5$ [13].

Once again if one takes the $L_{10}$ limit set by the present precision measurements as a yardstick, one would again come to the conclusion that the physics of a heavy Higgs needs a post-LHC and a post-LC500GeV. In fact if the latter can run as a Giga-Z [13] factory with polarised beams and with an improved measurement of the $W$ mass and the top mass ($\Delta m_t \sim 200$MeV is foreseen), then the $S, T, U$ variables will be incredibly constraining dwarfing the importance of the measurements of the other $L_i$’s from $W$ pair production. To be fair and stress the importance of future measurements, one should also add that it is also not excluded that $L_{10}$ be “naturally” much smaller than the other $L_i$’s. Indeed the couplings we have written invoke the custodial $SU(2)$ symmetry. One could impose an additional global symmetry $SU(2)_A$ [20] which would make $L_{10}$ vanish. For instance in some disguises of technicolour this would occur if the vector and axial vector mesons had the same mass. This is implemented in the extended BESS model [21] where $L_{10}$ is zero at the leading order. The model enjoys a decoupling property, where the
Figure 6: **Comparison of limits on the chiral Lagrangian parameters** $L_9$ **at the future colliders**, from [13].

Contributions to the $S, T, U$ variables become all of the same order and are subleading. Nonetheless to be consistent with a heavy Higgs, means that these contributions are not negligibly small and thus that the vector bosons are not asymptotically heavy. They should appear as resonances at the LHC or perhaps even at the LC.

### 4.2 Extra-dimensions

This scenario solves the hierarchy problem by suggesting that the natural fundamental scale is not the Planck scale with $\Lambda_P \sim 10^{19}$GeV but of the order a few TeV! This is based on the reasoning that if all matter and interactions were indeed occurring in a 4-dimensional world but that gravity were all pervading in a larger dimensional space, gravity will then look weaker to us while actually it is not [22]. As for the extra-dimensions one could think of them as being compactified with a “small” radius $R$. For distances $r$, with $r \gg R$ one would have the usual (4-dim) Newtonian law

$$F \propto \frac{1}{\Lambda_P^2} \frac{m_1 m_2}{r^2}$$

($\Lambda_P^2 = 4\pi G_N^{-1}$). However for $r \ll R$ one should feel the $4 + n$ law

$$F \propto \frac{1}{\Lambda_P^{2+n}} \frac{m_1 m_2}{r^{2+n}}.$$
By matching $\Lambda_P^2 = R^n \Lambda_F^{2+n}$. In the original ADD variant\cite{ADD}, it was required to have the fundamental scale $\Lambda_F \sim v \sim 1 TeV \sim 10^{-17} cm$. This already excludes $n = 1$ since we extract $R \sim 10^{15} cm$, i.e. distances where the $1/r^2$ law has been extremely well tested. $n = 2$ gives $R \sim mm$, it is still not totally excluded. $n = 5$ corresponds to $R \sim \mu m$. In this scenario the most striking signature is the production of gravitons, G, that could pop out of the detector into the extra world and act as missing energy. The most promising reactions are $e^+ e^- \rightarrow \gamma G$ and $pp \rightarrow g G$. Although gravitons still interact with a strength $1/\Lambda_P$, there is a huge density of them. The scaling law for the cross section can be explained easily. Take the simple case of an extra dimension with a simple geometry where the extra-dimension is compactified to a circle with radius $R$. The wave function can be factorised as

$$\Psi_G(x_4, y) = \sum_k \Psi_k(x_4) e^{iky/R}$$

From the point of view of the 4-dim world one has a tower of Kaluza-Klein states of mass $m_{G,k} \sim k/R$ and thus one can have for a total available energy $\sqrt{s}$ about $N = \sqrt{sR}$ states contribute to the cross section. For $n$ extra dimension one has $N = (\sqrt{sR})^n$, and therefore though the gravitational interaction is of order $1/\Lambda_P$, the cross section is $\sigma \sim N \times 1/\Lambda_P^2 \sim 1/\Lambda_P^2 \times (s/\Lambda_P^2)^{n/2}$. Limits one expects are given in the table below\cite{ADD}.

|       | n=4                  | n=6                  |
|-------|----------------------|----------------------|
|       | LEP                  | LHC                  | LC500                |
| $R/cm/\Lambda_F(\text{GeV})$ | $R/cm/\Lambda_F(\text{GeV})$ | $R/cm/\Lambda_F(\text{GeV})$ |
| LEP   | 1.9 $10^{-9}$ / 730  | 6.8 $10^{-12}$ / 530 |
| LHC   | 5.6 $10^{-11}$ / 7500| 2.7 $10^{-13}$ / 6000|
| LC500 | 1.2 $10^{-11}$ / 4500| 6.5 $10^{-13}$ / 3100|

Gravitons can also contribute indirectly leading to contact interactions but the limits are model dependent. What about the Higgs in these models. Here the Higgs mass is a free parameter, nonetheless though most studies concentrate on the $J = 2$ component, it is conceivable that the $J = 0$ mixes with the Higgs on the brane and leads to a decay of the Higgs into graviscalars\cite{ADD} and hence an invisible Higgs signature. This contribution can be large for a light Higgs. Both the LHC and the Tevatron could well miss the signal, but not the LC. One can of course also imagine that not only the gravitons but also the ordinary gauge bosons have siblings as Kaluza-Klein towers. For that one can assume the gauge bosons to propagate in more than 4-dim provided on takes for instance $R \sim 1 TeV \sim 10^{-17} cm$, $\Lambda_F \sim 10^{4} TeV$. The KK states of would be excited $W'/Z'$ could be looked for at the colliders. Moreover these states can mix with the ordinary vector bosons and indirectly contribute to the $\varepsilon_{1,2,3}$, Eq. \ref{epsilons} if not too heavy. In this case one could fit the LEP data with a heavier Higgs ($M_h < 500 \text{GeV}$)\cite{ADD} but would expect to see the additional KK gauge bosons at the LHC or LC500. It rests that, though the extra-dim scenario is in its infancy, it does not guarantee that one will see its manifestation. It may well happen that the Higgs (well at least that) is discovered at the LHC in accordance with the LEP limit and that it will not pose us a naturalness problem within this scheme...
4.3 SUSY

The SUSY solution is in my view a better solution to the hierarchy puzzle since it solves the problem through a symmetry which alas has to be broken. Let us start with the Higgs sector of the theory. A SUSY version of the SM requires, for anomaly cancellation, two Higgs doublets \((H_1, H_2)\). Proper symmetry breaking then gives 5 physical Higgses: 2 CP even \(h, H\), one CP odd \(A\) and the charged Higgs \(H^\pm\). But to do that SUSY must be broken. Indeed, before SUSY is broken the general supersymmetric potential is

\[
V = |\mu|^2 \left( |H_1|^2 + |H_2|^2 \right) + \frac{g^2 + g'^2}{8} \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{g^2}{2} |H_1^* H_2|^2 \geq 0 \quad (14)
\]

Note the appearance of the \(\mu\) term which is a SUSY conserving free parameter. But note also that the quartic couplings are gauge coupling. So one must add (soft) SUSY breaking parameters in such a way that one triggers electroweak symmetry breaking.

\[
V_H = (m_{11}^2 + |\mu|^2)|H_1|^2 + (m_{22}^2 + |\mu|^2)|H_2|^2 - m_{12}^2 \epsilon_{ij} \left( H_i^* H_j^2 + h.c. \right) + \frac{g^2 + g'^2}{8} \left( |H_1|^2 - |H_2|^2 \right)^2 + \frac{g^2}{2} |H_1^* H_2|^2 \quad (15)
\]

With appropriate soft-susy breaking terms electroweak symmetry breaking can be achieved. As this introduction to the Higgs potential already shows, understanding how supersymmetry is broken will be a prime motivation if supersymmetry is discovered. It will be crucial to reconstruct as much as possible the soft-susy parameters and test whether they satisfy some specific relations. A host of soft-susy breaking masses and mixing are also necessary in order to explain the splitting between the masses of the fermions and the sfermions. In all generality these masses and \(A\) terms should be matrices in flavour space. However because of the danger of potentially large FCNC one has to retort to the assumption of some alignment, \(i.e.\) diagonal matrices (the same basis as that of ordinary fermions). Most popular models even assume a common scalar mass for all and a common tri-linear \(A\) term at some unification scale. For each gaugino one must also attribute a soft susy breaking mass. It is also common to assume that the three gaugino masses also unify at the unification scale in the same way that within SUSY the three gauge couplings unify. All these assumptions need to be verified because they are probing theories and mechanisms at GUT scales or even string scales! One popular model with a minimum number of parameters is the mSUGRA\[27\] model where all scalar masses are equal at the unification scale as are all the gaugino masses. The SUSY spectrum it predicts at the weak scale is quite distinctive and is shown above, Fig. 7. One extremely appealing feature is that one can induce at the weak scale a “negative mass” for \(H_2 = H_u\)! This is however not always guaranteed and even if possible and it may be too fine-tuned (the \(\mu\) problem). Moreover it is not always assured that some other charged particles do not end up with an unwanted negative mass!. Still the reconstruction of the fundamental SUSY parameters will be a fascinating subject if SUSY is indeed discovered. Well at least one of the Higgses should be discovered.

At tree-level, and in the large \(M_A\) limit, one has \(m_h^2 = M_Z^2 \cos^2 2\beta\) which is excluded by LEP. The tree-level relation receives a large radiative correction due to the large top Yukawa coupling and the contribution of the stops, but one still has a definite upper limit
$m_h < 130\text{GeV}$\cite{28}. In models beyond the MSSM, this upper limit is relaxed. However insisting that all Yukawa couplings remain perturbative up to the Unification scale, then one finds that $m_h < 205\text{GeV}$\cite{23}. At LC500 such a light Higgs will be discovered within a day! Even if it decays invisibly. At the LHC, since the main signature is in $\gamma\gamma$ it is not so easy but all simulations show that one should not miss the Higgs and if lucky might even see some of the others, see Fig. 8.

The above picture, Fig. 8, is with the assumption that the other SUSY particles are too heavy to have an impact on the decay patterns and the production mechanisms. However if SUSY is correct it is very unlikely that the other particles are too heavy. One can for instance have light stops with large mixing that can reduce drastically the $gg \to h$ production\cite{30,31,32,33}. But the same scenario not only enhances the associated production signal but also guarantees discovery of $\tilde{t}_1$ and may even generate higgses through $\tilde{t}_2 \to \tilde{t}_1 h$\cite{33}. Another potential danger is the possibility of $h \to \tilde{\chi}_1^0 \tilde{\chi}_1^0$, that decays into invisible LSP’s\cite{34}. But again this means that there will be a nice study of the chargino neutralino system at the LHC and perhaps even at the Tevatron. Also in mSUGRA, it is very possible to produce the Higgs in the decay chain $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 h$\cite{34}. So light sparticles are often a blessing for the Higgs. The picture may be more complicated for decays of the heavier Higgses.

Although the latter may not be directly accessible at the LC500\cite{34} the bounty of $h$ in the very clean $e^+ e^-$ environment allows first class precision measurements on the Higgs properties. The spin and parity of the Higgs can be measured. Extraction of the light Higgs couplings to fermions (and $W$’s) at the 1% level (compared to 10 – 15% at the LHC) can distinguish between a SM Higgs and a SUSY Higgs and may either set a limit on $M_A$ ($M_A > 600\text{GeV}$) or constrain its mass if not too heavy $M_A < 500\text{GeV}$, practically\footnote{The $\gamma\gamma$ mode can extend the discovery potential of the heavier neutral Higgses. For a review on the $\gamma\gamma$ mode see \cite{36}. For a recent study on the detection of the heavier neutral Higgses see \cite{37}}.

![Figure 7: Evolution of the the SUSY masses in mSUGRA. From \cite{24}](image)
Figure 8: a Discovery potential of SUSY Higgses at the LHC. All other sparticles are assumed heavy, [8].

Figure 9: a) Precision with which the Higgs couplings $h\bar{t}t$ and $hWW$ couplings, normalised to those of the SM, can be measured at TESLA and LHC and how these parameters vary with $M_A$ in SUSY. b) Distinguishing a SUSY $h$ apart from a SM on the basis of a combination of Higgs branching ratios at TESLA with $1000\text{fb}^{-1}$. The sensitivity in $M_A/\tan\beta$ is shown at 68%, 90% and 95% level. From [35].
independently of $\tan \beta$, see Fig. 9.

Figure 10: Discovery potential of squarks and gluinos in CMS in the $m_0, M_{1/2}$ plane of mSUGRA for two values of $\tan \beta$. The $5\sigma$ contours corresponding to different final state signatures are shown. Isocurves for the masses are also shown. Note that constraints on the relic density, and the Higgs, are also shown. These would make these models not viable, however these can be circumvented in slightly more general that give the same LHC reach for gluinos and squarks, From [38].

Even if for $M_A > 600\text{GeV}$ one one would see only the lightest Higgs at both the LHC and LC500, squarks and gluinos with masses as high as $2 - 3\text{TeV}$ should be accessible at the LHC in a number of signatures, as Figure 10 shows. Apart from some quixotic scenarios, failing to see squarks and gluinos means that they have masses in excess of $3\text{TeV}$. This would cast some very serious doubt about SUSY as providing a neat answer to the hierarchy problem. Therefore if SUSY is correct and with such a large number of squarks produced one needs to precisely reconstruct the SUSY parameters. Until recently this has been done in the LHC simulations by assuming a specific model, most often mSUGRA. Recently many of the model assumptions have been dropped, however one still needs to rely on a specific decay chain to be able to do anything useful [39]. Take for instance the following chain, Fig 11.

Figure 11: A typical decay chain for the squark, from [40].

By measuring the different combinations of invariant mass distributions and studying their end-points it is possible to measure the SUSY masses with a precision close to a few
per-cent. In some cases only mass differences can be extracted, but even these can be very useful in distinguishing between models.

Nonetheless the conclusions would still be biased. For instance, one can not, in a model-independent way, claim that the slepton is the right-handed slepton, and it may be even not easy to ascertain its spin. Even more difficult, how do we know it is a supersymmetric process? A confirmation would be to measure the coupling $g_{\tilde{e}R\tilde{\chi}_0^1}$.

In this respect an $e^+e^-$ machine with polarised beams is a wonderful machine when it comes to providing model-independent precision measurements\[41\]. For instance, take the issue about the nature of the slepton in the process of pair production of a right-handed smuon which most probably will decay into the LSP neutralino and a muon. The signature is the same as that of $W$ pair production with the $W$’s decaying into muons and neutrinos and would constitute a formidable background. The use of polarisation becomes almost a must. First of all, $W$ pair production which is essentially an SU(2) weak process can be switched off by choosing right-handed electrons. Indeed, at high-energy one recovers the symmetric case where the $Z$ and $\gamma$ separate into the orthogonal $W^0$ and $B$ (hypercharge). The former not coupling to right-handed states. On the other hand the same argument shows that if only the hypercharge boson is exchanged and the fact that the hypercharge of the right-hand electron is twice that of the left-handed one, right smuon production will be four times larger than with left-handed $e^-$. Thus polarization achieves three things: tags the nature of the smuon (right-handed) independently of how it decays, increases the signal cross section and dramatically decreases the background. The smuon mass can be inferred either from a threshold scan which is independent of the decay or as is the case here, the measurement of the end-points of the muon energy which give both the smuon mass and the LSP mass. A combined fit, for the case above and for a modest luminosity $20 \, fb^{-1}$, gives these masses at the 1% level. One more thing, to confirm the scalar nature of the smuon one can look at its angular distribution which should show a $\sin^2\theta$ dependence. In the case of the right-handed selectron, this will not be the case since even with a right-handed electron on has to deal with a t-channel neutralino exchange. For the same reason as above only the bino component of the neutralino will be selected. If this component is not negligible one should observe a forward peak. This component is a function of the gaugino parameters $M_{1,2}$, the $\mu$ parameter and $tg\beta$. With the knowledge of $\chi_1^0$ one can measure how much of the LSP is bino. Similar beautiful experiments with chargino and neutralinos production can be conducted. They allow to check the gaugino unification condition and allow also to measure the couplings of the susy particle and verify them against the gauge couplings, see Figs. 12. Another important reason for extracting these model independent parameters, is that they allow to calculate the relic density. It could well be that the calculation turns out not to be compatible with the required value. This would urge us to review some of our assumptions concerning the nature of Dark Matter and/or the hypotheses that enter these calculations\[42\]. Of course to be sure to cover the parameter space as much as possible one needs to access as many states as possible, and this is guaranteed only by the highest energy linear collider.
Figure 12: Experimental confirmation at the LC of the GUT relation between the gaugino masses $M_1$ and $M_1$. b) Accuracy with which the SUSY gauge coupling can be measured at the LC, from [41].

5 Conclusions

The standard model has been extremely successful, but the jewel on the crown, the Higgs is still missing. The latest LEP data shows that this Higgs should be discovered at the LHC and extremely well studied at a moderate energy linear collider that could be built with existing technology (For a review of physics at the $e^+e^-$ see [43]). Though theoretically unnatural, a scenario with a heavy Higgs is still possible. However even this scenario predicts New Physics at the TeV scale, but unfortunately without a guarantee for direct observation at the LHC and a phase-I linear collider. These kind of scenarios require probably a next generation machines. The most motivated scenario is supersymmetry which has a good chance to be, even though partially, discovered at the LHC. However observation of supersymmetric particles is not enough. One needs to understand the breaking of SUSY by measuring as many of the parameters of the model as possible because this is a probe into the physics of a hidden sector at unification of even string scales. LHC could provide a few of these measurements but a polarised $e^+e^-$ machine is an ideal tool. In order to fully cover the parameter space one may need a second phase of a “leptonic” machine at a few TeV. Of course other scenarios we still have not thought of are possible. The recent hypothesis of extra-dimensions should teach us to be cautious and not see the future through too strait-laced arguments.

Acknowledgements

I would like to thank Emilian Dudas for a clarification about gauge unification at low scale, Geneviève Bélanger and Rohini Godbole for reading the manuscript.
References

[1] M. Peskin, in Proceedings the 1996 European School of High Energy Physics, Carry-le-Rouet, Sep. 96, SLAC-PUB-7479. hep-ph/9705479.

[2] K. R. Dienes, E. Dudas and T. Ghergetta, Nucl. Phys. B537 (1999) 47. hep-ph/9807011.

[3] M. Baillargeon, F. Boudjema, C. Hamzaoui and J. Lindig In Proceedings of the MRST’98 Conference, “Towards the Theory of Everything”, 80 (1998) Montréal. hep-ph/9809207.

[4] The LEP Electroweak Working Group, http://lepewwg.web.cern.ch/LEPWG/seminar/.

[5] M. Carena, et al., Report of the Tevatron Higgs Working Group, hep-ph/0010338.

[6] T. Hambye and K. Riesselman, Phys. Rev. D55 (1997) 7255.

[7] J. Ellis and D. Ross, hep-ph/0012067.

[8] For an update on the Higgs analysis in ATLAS see, the ATLAS Technical Design Report, the ATLAS Collaboration, 1999, http://atlasinfo.cern.ch/Atlas/TP/tp.html.

[9] G. Bélanger, F. Boudjema F. Donato, R. Godbole and S. Rosier-Lees Nucl.Phys. B581 (2000) 3. hep-ph/0002039.

[10] O. J. P. Eboli and D. Zeppenfeld, Phys. Lett. B495 (2000) 147 . hep-ph/0009158.

[11] S.P. Martin, J. D. Wells, Phys.Rev. D60 (1999) 035006. hep-ph/9903258.

[12] J.F. Gunion and B. Grzadkowski, Phys.Lett. B473 (2000). hep-ph/9910456.

[13] F. Boudjema, Invited talk at the Workshop on Physics and Experiments with Linear \(e^+e^−\) Colliders, Morioka, Japan, 1995, eds. A. Miyamoto et al., World Scientific, 1996, p. 199.

[14] M. Peskin and T. Takeuchi, Phys.Rev.Lett. 65 (1990) 964.

[15] G. Altarelli and R. Barbieri Phys.Lett. B253 (1991) 161.

[16] J.A. Bagger, A.F. Falk and M. Swartz, Phys.Rev.Lett. 84 (2000) 1385. hep-ph/9908327.

[17] R. Barbieri and A. Strumia, Phys.Lett. B462 (1999) 144. hep-ph/9905283.

[18] W. Kilian, talk at LCWS2000, http://conferences.fnal.gov/lcws2000 Boos et al., Phys.Rev. D57 (1998) 1553. hep-ph/9708310. ibid, Phys.Rev. D61 (2000) 077901. hep-ph/9908406.

[19] J. Erler, S.Heinemeyer, W.Hollik, G. Weiglein and P. Zerwas, Phys.Lett. B486 (2000) 125. hep-ph/0005024.

S.Heinemeyer and G. Weiglein hep-ph/0012364.
[20] T. Inami, C. S. Lim and A. Yamada, Mod. Phys. Lett. A7 (1992) 2789. See also, T. Inami, C. S. Lim in Proceedings of INS Workshop “Physics of $e^+e^-$, $e^-$ and $\gamma\gamma$ Collisions at Linear Accelerators, eds Z. Hioki, T. Ishii and R. Najima, p.229, INS-J-181, May 1995.

[21] See for instance, R. Casalbuoni et al., Phys. Lett. B435 (1998) 396. hep-ph/9805446.

[22] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429 (1998) 263. hep-ph/9803315.

[23] S. Cullen, M. Perelstein and M. Peskin, Phys. Rev. D62 (2000)055012. hep-ph/0001166.

[24] G.F. Giudice, R. Rattazzi and J.D. Wells, hep-ph/9906234.

[25] T.G. Rizzo and J. D. Wells, Phys. Rev. D61 (2000) 016007. hep-ph/9906234.

[26] G.L. Kane, C. Kolda, L. Roszkowski and J.D. Wells, Phys. Rev. D49 (1994) 6173.

[27] For a review see, H.P. Nilles, Phys. Rep. 110 (1984) 1.
R. Arnowitt, A. Chamseddine and P. Nath, Applied N=1 Supergravity, World Scientific, 1984.

[28] S. Heinemeyer, W. Hollik and G. Weiglein, Phys. Rev. D58 (1998) 091701. hep-ph/9803274.
ibid Phys.Lett. B440 (1998) 296, hep-ph/9807423. Eur. Phys. J. C9 (1999) 343, hep-ph/9812474.
Phys. Lett. B455 (1999) 179, hep-ph/9903404.
M. Carena et al., Nucl. Phys. B580 (2000) 29. hep-ph/0001002.
The limiting case of vanishing stop mixing and large $M_A$ and $\tan\beta$ has been considered by R. Hempfling and A. Hoang, Phys. Lett. B331 (1994) 99.

[29] M. Quiros and J.R. Espinosa. hep-ph/9809269.

[30] G. L Kane, G. D. Kribs, S.P Martin and J. D. Wells, Phys. Rev. D50 (1996) 213.

[31] B. Kileng, Z. Phys. C63 (1993) 87.
B. Kileng, P. Osland, P.N. Pandita, Z.Phys. C71 (1996)87. hep-ph/9506455.

[32] A. Djouadi, Phys.Lett. B435 (1998) 101, hep-ph/9806315.

[33] G. Bélanger, F. Boudjema and K. Sridhar, Nucl. Phys. B568 (2000) 3. hep-ph/9904348.

[34] G. Polesello, L. Poggioli, E. Richter-Was and J. Sderqvist, ATLAS Internal Note, PHYS-No-111, Oct. 1997.

[35] M. Battaglia and K. Desch, hep-ph/0101165.

[36] M. Baillargeon, G. Bélanger and F. Boudjema, in Proceedings of Two-photon Physics from $\DA\Phi\NE$ to LEP200 and Beyond, Paris, eds. F. Kapusta and J. Parisi, World Scientific, 1995 p. 267; hep-ph/9405355.

[37] M.M. Muhlleitner, M. Kramer, M. Spira and P.M. Zerwas, hep-ph/0101083.

[38] S. Abdullin and F. Charles, Nucl. Phys. B597 (1999) 60. hep-ph/9811402.

[39] For a nice recent review, see F. Paige, BNL-HET-98/1, hep-ph/9801254, to appear in TASI 97, Supersymmetry, Supergravity and Supercolliders.

[40] B.C. Allanach, C.G. Lester,, M.A. Parker and B.R. Webber, hep-ph/0007009.
For the extraction of the SUSY parameters see, T. Tsukamoto, K. Fujii, H. Murayama, M. Yamaguchi and Y. Okada. Phys. Rev. D51 (1995) 3153. M.M. Nojiri, K. Fujii and T. Tsukamoto, Phys. Rev. D54 (1996) 6756. For an extremely nice summary, see K. Fujii, in Physics and Experiments with Linear Colliders, Morioka, Japan, p. 283 Op. cit.

See for instance, K. Enqvist and J. McDonald, Phys. Lett. B440 (1998) 59, hep-ph/9807269. T. Moroi and L. Randall, Nucl. Phys. B570 (2000) 455, hep-ph/9906527. P. Gondolo, Phys.Lett. B494 (2000) 181, hep-ph/0002220. G.F. Giudice, E.W Kolb, A. Riotto, hep-ph/0005123.

Proc. of the Workshop on $e^+e^-$ Collisions at 500 GeV: The Physics Potential, ed. P. Zerwas, DESY-92-123A,B(1992); DESY-93-123C (1993); DESY-96-123D (1996). H. Murayama and M.E. Peskin, Ann. Rev. of Nucl. and Part. Sci. 46 (1996) 533. hep-ex/9606003. Also E. Accomondo et al., Phys. Rep. (1998). F. Boudjema, Pramana 51 (1998) 249. hep-ph/9809220. American Linear Collider Working Group (J. Bagger et al.), hep-ex/0007022.