Method of Locating the Disturbance Source of Forced Power Oscillation Based on Equivalent Electrical Distance

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Abstract. This paper proposes a novel method of locating the disturbance source of forced power oscillation based on equivalent electrical distance. Firstly, actual grid nodes are mapped onto complex plane on the basis of equivalent electrical distance theory. Secondly, PMU monitoring nodes are selected according to the defined index of coupling degree of generator and load. Lastly, the disturbance source is effectively located by resolving optimization problem based on Geiger theory on seismology and the half-plane method. This proposed method can be implemented by acquiring and analyzing waveforms of frequency variance of several monitoring nodes, and is of great practical value for actual grid. A simulation for IEEE 30-buses system verified the validity and accuracy of this locating method.

1. Introduction
The forced power oscillation is another classical theory differing from under-damping oscillation, which successfully explains the occurrence of low frequency oscillation when power system is sufficiently damped. This theory indicates that the power system will present a drastic power oscillation when subjected to continuous and periodical power disturbance, whose frequency value is close to the one of natural frequency of power system[1,2]. Cutting the disturbance source can make the forced oscillation decay fast, which poses as a direct and valid way to suppress the oscillation. Thus locating the disturbance source is important to the suppression of forced oscillation.

Previous work has made large amount of achievements. Ref[3,4] addresses the locating problem by studying system energy change properties during the power oscillation, which requires massive calculation. Ref[5,6] analyzes and compares the similarity of voltage waveforms of several monitoring nodes, and uses the propagation time difference of disturbance wave in transmission lines to locate the disturbance source. Ref[7] proposes an online perception and location method based on CELL and decision-making tree theory. Ref[8] utilizes graph theory and compares the secret key from partial data to the one from global data to get the location of disturbance source.

This paper proposes a novel method of locating the disturbance source of forced power oscillation based on equivalent electrical distance. Firstly, actual grid nodes are complex coordinated according to the equivalent electrical distance theory. Secondly, PMU monitoring nodes are selected through defining the index of coupling degree of generator and load. In the last place, the disturbance source is precisely located by resolving optimization problem by using seismology Geiger theory and the half-plane method. This proposed method is convenient to be implemented and has great practical value for power system.
2. Equivalent Electrical Distance

In modern power system, the grid structural properties and propagation principles of system faults are commonly studied from the perspective of network topology. However, conventional network topology cannot describe well the essential feature of power system, while the electrical view can truly reflect the coupling connection between grid nodes[9,10].

For any two grid nodes $i$ and $j$, the equivalent electrical distance can be described as the equivalent impedance $Z_{ij,eq}$ between the two nodes, which is numerically equal to the voltage $U_{ij}$ between nodes $i$ and $j$ when unit current flows into node $i$ and out of node $j$. The equation can be expressed as

$$Z_{ij,eq} = \frac{U_{ij}}{I_i} = \frac{U_{ij}}{I_j}$$

(1)

According to superposition theorem, equation (1) can be properly transformed. The illustration of superposition theorem is shown below.

$$Z_{ij,eq} = Z_{ii} I_j + Z_{ij} I_j = Z_{ij}$$

(2)

Figure.1 Illustration of superposition theorem

Considering the physical meaning of elements of power system impedance matrix: when node $i$ is injected with unit current while the other nodes are injected with no current, $Z_{ii}$ is equal to the voltage $U_i$ of node $i$, $Z_{ij}$ is equal to the voltage $U_j$ of node $j$, where $Z_{ii}$ is the element of impedance matrix in $i$th row and $i$th rank, $Z_{ij}$ is the element of impedance matrix in $i$th row and $j$th rank.

Based on the above, the equivalent impedance $Z_{ij,eq}$ can be written as follows:

$$Z_{ij,eq} = U_{ij}^{(1)} + U_{ij}^{(2)}$$

$$= (Z_{ii} - Z_{ij}) + (Z_{jj} - Z_{ij})$$

$$= Z_{ii} + Z_{jj} - 2Z_{ij}$$

(2)

3. Complex Coordinates of Grid Nodes

To reflect the electrical connection between actual grid nodes, grid nodes are mapped onto complex coordinate according to theory of equivalent electrical distance. Every actual grid nodes has its corresponding coordinate in the complex plane, hence grid nodes can be complex coordinated as the following steps:

1) Define the complex coordinate origin. Select the slack bus as the complex coordinate origin, and chose only one if there are more than one slack buses.

2) Obtain the impedance matrix of grid. Get the grid impedance matrix by reversing of the admittance matrix which can be easily get from the grid topology.

3) Calculate the equivalent impedance. On the basis of the obtained impedance matrix, calculate the equivalent impedance between every single node and coordinate origin node by using equation (2).

4) Get the complex coordinate of every node. Assume that the equivalent impedance between node $k$ and origin node $o$ is $Z_{ok} = R_{ok} + jX_{ok}$, then the complex coordinate of node $k$ is $(R_{ok}, X_{ok})$. 

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4. Selection for PMU Monitoring Nodes

The power disturbance in both generator and load can trigger forced power oscillation in power system, and the grid nodes which have close connection with generator and load will early and sensitively feel the oscillations. Therefore, those can be selected as PMU monitoring nodes.

For a \( n \)-nodes power system, assuming that \( \Omega_G \) is set of all generator nodes, \( \Omega_L \) is set of all load nodes, we can define the index of coupling degree of generator and load as followed:

\[
\ell_{c,i} = \frac{1}{\sum_{j\in \Omega_G \cup \Omega_L} |Z_{ij,eq}|}
\]

where \( \ell_{c,i} \) is the coupling degree between node \( i \) and generator and load nodes, \( Z_{ij,eq} \) is the equivalent impedance between node \( i \) and node \( j \). A bigger value of \( \ell_{c,i} \) means that node \( i \) has a more intense electrical connection with the generator and load nodes, and is more sensitive to the disturbance in generator and load. And vice versa.

To obtain the oscillation waveforms better, we should make a proper selection of the PMU monitoring nodes. By calculating the coupling degree \( \ell_{c,i} \) and sorting the index in descending order, we can choose several PMU monitoring nodes with bigger coupling degree, and assure that the selected node is not too close to another selected one.

5. Method of Locating the Disturbance Source Based on Equivalent Electrical Distance

According to the classic Geiger theory on seismology\[11\], assuming that seismic origin is \((x_0, y_0, z_0)\), seismic starting time is \(t_0\), the time seismic waves arrive in the \( n \) seismic observation stations is respectively \(t_1, t_2, \ldots, t_n\), the problem of locating the seismic origin can be transformed into resolving the minimal value of the objective function below:

\[
\phi(t_0, x_0, y_0, z_0) = \sum_{i=1}^{n} r_i^2
\]

where \( r_i \) is residual error of reaching time, and \( T_i \) the calculating time of the \( i \)th observation station. And we also have

\[
r_i = t_i - t_0 - T_i(x_0, y_0, z_0)
\]

Assuming that disturbance electromechanical wave spreads in a same velocity in every direction, the problem of locating the disturbance source of forced oscillation can be translated to resolving the optimization problem given as followed\[12\]:

\[
\phi(x_0, y_0) = \sum_{i=1}^{N} d_i^2
\]

where \( d_i = \sqrt{(x_i - x_0)^2 + (y_i - y_0)^2} \), representing the equivalent electric distance between node \( i \) and disturbance node.

In combination with the half-plane locating method, we can set constraint conditions for equation (6). Given that there are \( N \) PMU monitoring nodes in a \( n \)-nodes grid, re-number the monitoring nodes according to the electrical distance between disturbance node and monitoring nodes in ascending order. Therefore, the optimization problem can be written as
Complementally, we can judge the reaching time of disturbance electromechanical wave by acquiring and analyzing waveforms of frequency variance $\Delta f$ of monitoring nodes. The earlier signal $\Delta f$ first reaches to the peak, the closer the monitoring node is to the disturbance source[14].

Based on the above, the method of locating the disturbance source of forced power oscillation can be designed in following steps:

1) Get actual grid nodes mapped onto the complex plane;
2) Select proper PMU monitoring nodes through calculating the coupling degree of generator and load;
3) Analyze waveforms of frequency variance to judge the timing sequence of reaching PMU monitoring nodes;
4) Resolve the optimization problem (7), and get the complex coordinates of disturbance source.

6. Simulation Analysis

The IEEE 30-buses system is adopted to verify the validity of the locating method proposed in this paper. The topology of simulation system is shown in Figure 2, and bus 1 is the slack bus.

![Figure 2: Topology of IEEE 30-buses system](image)

In the first place, the actual grid nodes are mapped onto the complex plane. The corresponding complex coordinates are given in Table 1.

| node number | complex coordinate | node number | complex coordinate |
|-------------|--------------------|-------------|--------------------|
| 1           | (0,0)              | 16          | (0.0790,0.3084)    |
| 2           | (0.0164,0.0504)    | 17          | (0.0601,0.2687)    |
| 3           | (0.0270,0.0934)    | 18          | (0.1135,0.3641)    |
| 4           | (0.0269,0.0884)    | 19          | (0.1082,0.3610)    |
| 5           | (0.0484,0.1575)    | 20          | (0.0983,0.3431)    |
The distribution of the 30 nodes in complex plane can be showed in Figure 3.

Secondly, the PMU monitoring nodes are properly selected. The indexes of coupling degree of generator and load are calculated and sorted in descending order in Table 2. As we can see from Table 2 that nodes {6, 1, 9, 22, 2, 25} are the ones with biggest value of coupling degree of generator and load. Considering that node 2 is very electrically close both to node 1 and node 6, we can discard node 2 and select nodes {6, 1, 9, 22, 25} as PMU monitoring nodes.

| node number | coupling degree | node number | coupling degree |
|-------------|----------------|-------------|----------------|
| 6           | 0.1773         | 12          | 0.0185         |
| 1           | 0.1292         | 13          | 0.0170         |
| 9           | 0.1013         | 14          | 0.0159         |
| 22          | 0.0727         | 15          | 0.0150         |
| 2           | 0.0699         | 16          | 0.0141         |
| 25          | 0.0531         | 17          | 0.0134         |
| 3           | 0.0499         | 18          | 0.0127         |
| 27          | 0.0426         | 19          | 0.0121         |
| 4           | 0.0387         | 20          | 0.0115         |
| 28          | 0.0353         | 21          | 0.0110         |
| 5           | 0.0327         | 23          | 0.0105         |
| 7           | 0.0282         | 24          | 0.0101         |
| 8           | 0.0247         | 26          | 0.0089         |
| 10          | 0.0224         | 29          | 0.0081         |
In the following step, the IEEE 30-buses system is built in PSD-BPA. To stimulate the forced oscillation, a 0.91Hz periodical power disturbance with 100% amplitude of initial mechanical power is applied to the generator 11. The time domain simulation curves of frequency variance of monitoring nodes are showed in Figure 4. We can tell from the waveforms that the reaching time sequence of disturbance electromechanical wave in descending order: 9→6→22→1→25.

![Figure 4 The time domain simulation curves of frequency variance](image)

After all the steps above, we can lastly resolve the optimization problem given in equation (7). The complex coordinates of actual node 11 is (0.0339, 0.4305). The optimal solution can be calculated as (0.0361, 0.4495), which is very close to the actual node 11. Therefore, this case study demonstrates the validity of the proposed method.

7. Conclusion
A novel method of locating the disturbance source of forced power oscillation is proposed in this paper in terms of equivalent electrical distance theory. The method is effective and precise in locating the disturbance source. Calculating the complex coordinates and selecting PMU monitoring nodes can be implemented off-line, and resolving the optimization problem can be easily realized, which makes this method an efficient and practical way to locate the disturbance source of forced power oscillation.

References
[1] Tang Yong. Fundamental theory of forced power oscillation in power system[J]. Power System Technology, 2006, 30(10): 29-33.
[2] Wang Tieqiang, He Renmu, Wang Weiguo, et al. The mechanism study of low frequency oscillation in power system[J]. Proceedings of the CSEE, 2002, 22(2): 21-25.
[3] YuYiping, Min Yong, Chen Lei, et al. Disturbance source location using energy functions[J]. Automation of Electric Power Systems, 2010, 34(5): 1-6.
[4] Yang Dongjun, Ding Jianyong, Li Jisheng, et al. A disturbance source location method for forced power oscillations based on parameter identification[J]. Automation of Electric Power Systems, 2012, 36(2): 26-29.
[5] Dong Qing, Liang Jing, Yan Xiangwu, et al. Locating method of disturbance source of low frequency oscillation in large scale power grid[J]. Proceedings of the CSEE, 2012, 32(1): 78-83.
[6] Dong Qing, Zhang Ling, Yan Xiangwu, et al. Automatic locating source method of compelled resonance low frequency oscillation in power grid[J]. Proceedings of the CSEE, 2012, 32 (28): 68-75.

[7] Jiang Changjiang, Liu Junyong, Liu Youbo, et al. Online forced oscillation detection and identification based on wide area measurement system and CELL theory[J]. Electric Power Automation Equipment, 2015, 35(2): 125-131.

[8] Nudell T, Chakrabrtty A. Graph-theoretic methods for measurement-based input localization in large networked dynamic systems. IEEE Transactions on Control Systems Technology, 2015, 60(8): 2114-2128.

[9] Holmgren A. Using graph models to analyze the vulnerability of electric power networks[J]. Risk Analysis, 2006, 26(4): 955-969.

[10] Tan Yudong, Li Xinran, Cai Ye, et al. Critical node identification for complex power grid based on electrical distance[J]. Proceedings of the CSEE, 2014, 34(1): 146-152.

[11] Geiger L. Probability method for the determination of earthquake epicenters from arrival time only[J]. Bull.St.Louis.University, 1912, 8: 60-71.

[12] Robert M. Gardner, Joshua K. Wang, Yilu Liu. Power system event location analysis using wide-area measurements[J]. IEEE PSCE, 2006: 1-7.

[13] Robert M. Gardner, Jason N. Bank, Joshua K. Wang, et al. Non-parametric power system event location using wide-area measurements[J]. IEEE PSCE, 2006: 1668-1675.

[14] HU Nan, LI Xingyuan, YANG Yiqiang, et al. Research for space-time variational features of frequency for power system forced oscillations and disturbance source location[J]. Journal of Sichuan University(Engineering science edition), 2013, 45(6): 135-142.