Secular Evolution of Disks

Xiaolei Zhang

*Raytheon ITSS/NASA GSFC, Code 685, Greenbelt, MD 20771*

**Abstract.** It is found that a previously thought-to-be well established result of density wave theory, that there is no interaction between a quasi-stationary spiral density wave and the basic state (i.e. the axisymmetric part) of the galactic disk, is in fact false. When solved as a nonlinear and globally self-consistent problem, the presence of non-axisymmetric patterns such as spirals or bars is shown to lead to significant interaction of the basic state and the density wave, with the net result being the simultaneous acquisition of a quasi-steady wave amplitude and the secular redistribution of disk matter. The secular operation of this dynamical mechanism leads to the evolution of the Hubble type of a galaxy from late to early.

1. Introduction

During the past decade, growing evidence has pointed to a trend of secular morphological evolution of galaxies over the cosmic time. A firm theoretical foundation needs to be established on the operations of the dominant mechanisms driving this secular evolution. In what follows, we present a dynamical mechanism responsible for the secular morphological evolution of galaxies which is *internal* to galaxies which possess large scale spiral or bar patterns. We demonstrate the validity and inevitability of the operation of this mechanism, the astrophysical consequences as well as the connection of the current work with previous work in this field.

2. Basic Results

It was first shown in Zhang (1996) that for a spontaneous spiral mode, the potential spiral lags the density spiral in phase inside corotation, and vice versa outside corotation. The phase shift between the potential and density spirals means that there is a torque exerted by the potential spiral on the density spiral, and a secular transfer of energy and angular momentum between the disk matter and the density wave at the quasi-steady state of the wave mode (Zhang 1998, 1999). The torque \( T(r) \) applied by the spiral potential on the density in an annulus of unit width can be written as

\[
T(r) = \frac{dL}{dt} = r \int_0^{2\pi} -\Sigma(r \times \nabla V)_z d\phi
\]
\[ r \int_0^{2\pi} -\Sigma_1 \frac{\partial V_1}{\partial \phi} d\phi = -\pi mr \Sigma_1(r) V_1(r) \cdot \sin(m \phi_0(r)), \]  

where \( \Sigma, V, \Sigma_1, V_1 \) are the surface density and potential, as well as the spiral perturbation density and potential in the annulus, respectively, \( L \) is the angular momentum of the disk matter in the annulus, \( \phi_0 \) is the potential-density phase shift and \( m \) is the number of spiral arms.

At the quasi-steady state, the energy and angular momentum transfer between the basic state matter and the spiral density wave is achieved through a temporary local gravitational instability at the spiral arms (Zhang 1996). The length scale of this instability at the solar neighborhood is about 1 kpc, which coincides with the length scale of the giant molecular and HI complexes near the Galactic spiral arm region. The presence of the instability condition at the spiral arms, coupled with the supersonic to subsonic transition of particle streaming velocity with respect to the spiral arm, indicate that the nature of the large-scale spiral pattern in galaxies is in fact spiral gravitational shocks.

3. Astrophysical Consequences

The wave-basic state interaction leads naturally to the damping of the growing wave mode and to the acquirement of quasi-steady state (Zhang 1998). A by-product of the wave-basic state interaction is that an average star inside corotation will tend to lose energy and angular momentum to the wave secularly and spiral inward. The rate of this orbital delay can be shown to be

\[ \frac{dr}{dt} = -\frac{1}{2} F^2 v_0 \tan i \sin(m \phi_0) \]  

where \( F \) is the fractional wave amplitude, and \( v_0 \) is the circular velocity of the star, and \( i \) is the pitch angle of the spiral. This evolution rate expression has been quantitatively confirmed in the N-body simulations (Zhang 1998). This orbital decay rate is about 2 kpc per \( 10^{10} \) years for our own Galaxy, which corresponds to a mass accretion rate of about \( 6 \times 10^9 M_\odot \) per \( 10^{10} \) years. A substantial fraction of the bulge can thus be built up in a Hubble time.

Another important consequence of spiral-induced wave-basic state interaction is the secular heating of the disk stars. Since a spiral density wave of pattern speed \( \Omega_p \) can only gain energy and angular momentum in proportion to \( \Omega_p \), the pattern speed of the wave, and a disk star which moves on a nearly circular orbit loses its orbital energy and angular momentum in proportion to \( \Omega \), the circular speed of the star, an average star cannot lose orbital energy entirely to the wave for galactic radii other than the corotation, and thus the excess energy is used for the secular heating of the disk stars. For our Galaxy, the diffusion coefficient due to the spiral-induced secular heating is estimated to be

\[ D = (\Omega - \Omega_p) F^2 v_c^2 \tan i \sin(m \phi_0) \approx 6.0(kms^{-1})^2 yr^{-1}, \]  

if using the same set of spiral parameters as used above for estimating Bulge building. This value of \( D \) fits very well the age-velocity dispersion relation for the solar neighborhood stars (Zhang 1999). The above expression for \( D \) can be shown to be approximately constant across the galactic radii (Zhang 1999),
which agrees with the known isothermal distribution of the stellar and gaseous mass. Similar energy injection into the interstellar medium can serve as the top-level source for the subsequent supersonic turbulence cascade.

In general, the radial mass accretion process causes the disk mass to be more and more centrally concentrated, and causes the morphological type of a galaxy to evolve from late to early along the Hubble sequence. Such morphological transformation is most pronounced in dense clusters, which is the well-known Butcher-Oemler effect. In the current scenario, the enhanced mass accretion for cluster galaxies is due to the large amplitude and open spiral patterns induced through tidal interactions among cluster members, since the effective evolution rate due to spiral structure is seen to be proportional to wave amplitude squared and the spiral pitch angle (equation 2, note that the phase shift $\phi_0$ itself is approximately proportional to spiral pitch angle).

4. Discussions

Zhang (1998) further demonstrated that the phase shift between the potential and density spirals is intimately related to the gradient of the so-called torque coupling integrals (which is the same thing as an angular momentum flux in the radial direction) defined in LBK, such that during the linear modal growth process, $T(r) = -dC_g/dr$ where $C_g$ is the gravitational torque coupling integral; and that at the quasi steady state of the wave mode, $T(r) = -(dC_a + dC_g)/dr = -(dC)/dr$ where $C_a$ is the advective torque coupling integral. Since $T(r) < 0$ inside corotation and $T(r) > 0$ outside corotation due to the sign of the phase shift, it follows that $dC/dr > 0$ inside corotation and $dC/dr < 0$ outside corotation, which means that the $C(r)$ function is of a characteristic bell shape with the peak of the bell at the corotation radius. This bell shaped angular momentum flux says that a spiral mode not only transports angular momentum outward from the innermost region to the outer part of the disk, as originally stated in LBK, it also picks up angular momentum from all galactic radii inside corotation, and dumps angular momentum unto all radii outside corotation en route of the outward angular momentum transport.

This bell-shaped torque couple demonstrated in Zhang (1998) turns out to be intimately related to the ability of the wave to spontaneously grow in the linear regime, and for the inevitability of the basic state evolution at the quasi-steady state of the wave mode, as we show below. Since $dC/dr = d(C_a + C_g)/dr = -dL/dt$ (i.e. the gradient of the radial angular momentum flux is the rate of angular momentum change in the local annulus, which is again a direct consequence of angular momentum conservation), we have in general

$$\frac{dC}{dr} = -\frac{d(L_{\text{basic state}} + L_{\text{wave}})}{dt} \quad (4)$$

based on the angular momentum conservation.

In the linear regime:

$$\frac{dL_{\text{basic state}}}{dt} = 0, \quad (5)$$

therefore

$$-\frac{dC}{dr} = \frac{dL_{\text{wave}}}{dt} = 2\gamma g L_{\text{wave}}, \quad (6)$$
where $\gamma_g$ is the amplitude growth rate of the wave mode.

At the quasi-steady state:

$$\frac{dL_{\text{wave}}}{dt} = 0,$$

therefore

$$-\frac{dC}{dr} = \frac{dL_{\text{basic state}}}{dt}.$$ 

(8)

We see from the above expressions that instead of the outward angular momentum flux $C$ (as originally thought by LBK) it is rather the gradient of this transport $dC/dr$ (which is itself proportional to the phase shift-induced torque $T(r)$) that is responsible for the homogeneous modal growth across the entire disk surface in the linear regime, and for the homogeneous evolution of the basic state at the quasi-steady state of the wave mode.

In the past discussion of secular evolution mechanisms emphasis has been placed on the accretion of gas under the influence of the central bar. However, the microscopic viscosity in the gas component is known to be inadequate to support a reasonable accretion rate even for proto-stellar accretion disks. Furthermore, since gravity cannot really distinguish whether the underlying matter is made of stars or gas, the two component are expected to play essentially identical roles in the spiral-or-bar-induced viscous accretion processes. Indeed, the two-component N-body simulations involving both the stars and gas have invariably found that the phase shift between the stellar and gaseous densities are very small (Carlberg & Freedman 1985; Zhang 1998). It is their common phase shift with respect to the spiral potential that caused the stars and gas to both drift towards the center (Zhang 1998).

5. Conclusions

We have shown that a globally self-consistent solution for a spontaneous spiral mode in the disk geometry can be obtained as a dynamical equilibrium state, with the growth tendency of the spiral mode balanced by the local dissipation in the basic state. The resulting secular energy and angular momentum exchange between the wave mode and the basic state is mediated by a temporary local gravitational instability at the spiral arms.

The closed form equations for the rate of energy and angular momentum exchange between the basic state and the wave mode have been quantitatively confirmed in N-body simulations, and these expressions can be used to compute the evolution rate of physical galaxies. The gravitational torque mechanism is expected to be operating in other types of astrophysical disks as well.

References

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