Solar Wind Turbulence Around Mars: Relation Between The Energy Cascade Rate And The Proton Cyclotron Waves Activity

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Key Points:
• We estimate, for the first time, the incompressible energy cascade rate obtained in the solar wind near Mars.
• We find that the nonlinear cascade of energy is slightly amplified when proton cyclotron waves are present in the plasma.
• These statistical results show the presence of Alfvénic and non-Alfvénic turbulent fluctuations in a magnetic dominant regime.

Abstract
The first estimation of the incompressible energy cascade rate at magnetohydrodynamic (MHD) scales is obtained in the plasma upstream of the Martian bow shock, using MAVEN observations and an exact relation derived for MHD turbulence. The energy cascade rate is computed for events with and without proton cyclotron wave (PCW) activity, for time intervals when MAVEN was in the solar wind with no magnetic connection to the bow shock. It is shown that the nonlinear cascade of energy at the MHD scales is slightly amplified when PCWs are present in the plasma. The analysis of the normalized cross helicity and residual energy for the turbulent fluctuations shows the presence of Alfvénic and non-Alfvénic fluctuations in a magnetic dominant regime for the majority of the cases.

Plain Language Summary
Throughout its radial expansion from the Sun, the solar wind develops a strongly turbulent regime, which can be characterized by in situ observations of proton density, velocity and magnetic field fluctuations. Turbulence serves as a reservoir of energy that cascades through the inertial range down to the smallest scales, where it is dissipated by kinetic effects. For the first time, we compute the energy cascade rate which is transferred through different scales in the inertial range. This energy rate is computed for cases with and without proton cyclotron waves activity, when MAVEN was in the solar wind. Our results show that the energy cascade rate is emphasized when waves are present in the plasma.

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1 Introduction

Turbulence is a unique phenomenon present in several space environments, like the solar corona (Hendrix & Van Hoven, 1996; Dmitruk et al., 2002), planetary environments (Sahraoui et al., 2020) or the solar wind (Bruno & Carbone, 2005; W. Matthaeus & Velli, 2011). In particular, solar wind turbulence is partially characterized by an inertial range, where energy is transferred without dissipation through different spatial and temporal scales (e.g., Frisch, 1995). Typically, in the largest magnetohydrodynamic (MHD) scales, the solar wind magnetic spectrum presents a $-5/3$ slope (Kolmogorov, 1941a,b; W. H. Matthaeus & Goldstein, 1982; Leamon et al., 1998; Chen, 2016), which is generally compatible with a constant energy cascade rate as a function of such scales (Sorriso-Valvo et al., 2007; Marino et al., 2008; Coburn et al., 2014; Coburn et al., 2015; Hadid et al., 2017). A constant energy cascade rate reflects a well accepted idea that large (MHD) scale turbulence serves as a reservoir of energy that cascades down to the smallest scales, where it can be dissipated by kinetic effects (e.g., Leamon et al., 1998; Sahraoui et al., 2009; Alexandrova et al., 2009; Andrés et al., 2014).

Assuming spatial homogeneity and full isotropy, an exact relation for fully developed incompressible MHD turbulence can be derived (Politano & Pouquet, 1998a,b). Among its potential applications (e.g., Weygand et al., 2007; W. H. Matthaeus et al., 1999; MacBride et al., 2008; Benzi et al., 1993; Grossmann et al., 1997; Andrés & Banerjee, 2019), the exact relation provides a precise computation of the amount of energy per unit time and volume (or heating rate) as a function of the velocity and magnetic correlation functions. The MHD exact relation and its connection with the nonlinear energy cascade rate has been numerically validated for both incompressible and compressible MHD turbulence (Andrés et al., 2018), and has been generalized to include sub-ion scale effects (Andrés et al., 2018, 2019; Hadid et al., 2018; Hellinger et al., 2018; Ferrand et al., 2019; Banerjee & Andrés, 2020). Estimations of the energy cascade rate in the inertial range of solar wind turbulence have been previously computed at 1 Astronomical Unit (AU) (see, Marino et al., 2008; Coburn et al., 2014; Coburn et al., 2015; Banerjee & Andrés, 2019; Hadid et al., 2017) and more recently at $\sim 0.2$ AU (Bandyopadhyay et al., 2020; Chen et al., 2020). In particular, Hadid et al. (2017) have investigated in detail the role of the compressible fluctuations (Banerjee & Galtier, 2013; Andrés & Sahraoui, 2017) in modifying the energy cascade rate with respect to the prediction of the incompressible MHD model, based in situ data from the THEMIS/ARTEMIS spacecraft in the fast and slow solar wind.

The induced magnetosphere of Mars is formed as a result of the interaction between the solar wind and the planet’s atmosphere, including its exosphere, ionosphere and the crustal magnetic fields (Acuña, 1998, 1999). This interaction starts upstream of the Martian bow shock, due to the lack of an intrinsic global planetary magnetic field and the presence of an extended hydrogen exosphere (e.g., Chaffin et al., 2015). The response of this atmospheric obstacle is significantly modified by time-dependent physical processes (e.g., Edberg et al., 2010; Jakosky et al., 2015b; Romanelli et al., 2018a), as a result of temporal variability of the planetary and solar wind properties over different timescales (e.g., Edberg et al., 2009; Modolo et al., 2012; Mau et al., 2014; Fang et al., 2015; Romanelli et al., 2018b, 2019). The seasonal variability of the Martian hydrogen exosphere has been identified by several spacecraft (Bhattacharyya et al., 2015; Chaffin et al., 2014; Clarke et al., 2014, 2017; Halekas et al., 2017; Halekas, 2017). In particular, the Martian exosphere is subject to several ionizing mechanisms giving rise to newborn planetary protons, allowing one to indirectly observe such seasonal dependence with plasma instruments (e.g., Yamauchi et al., 2015; Rahmati et al., 2017). For instance, higher pickup ions detection rates were observed when Mars is near perihelion (Yamauchi et al., 2015). Moreover, when available, the measurement of the resulting proton velocity distribution function at these altitudes is composed of a core of solar wind particles and a non thermal proton population due to the presence of newborn planetary ions (seen in the solar wind reference frame). Such particle velocity distribution function is highly unstable and can give rise to several ultra-low frequency plasma waves (C. S. Wu & Davidson, 1972; C. Wu & Hartle, 1974; Brinca, 1991; Gary, 1991; Mazelle & Neubauer, 1993; Cowee et al., 2012).

In addition to their capability to excite different plasma waves, the relative velocity between the newborn planetary proton reference frame (very close to the planetary and spacecraft rest
frames) and the solar wind is also responsible for a Doppler shift that defines the observed wave frequency near the local proton cyclotron frequency in the spacecraft reference frame (e.g., Russell et al., 1990; Brain, 2002; Mazelle et al., 2004; Romanelli et al., 2013, 2016; Ruhunusiri et al., 2015, 2016; Liu et al., 2020). Variability in the proton cyclotron waves (PCWs) occurrence rate has been observed based on Mars Global Surveyor magnetic field data (Romanelli et al., 2013; Bertucci et al., 2015) and more recently with MAVEN Magnetometer (MAG) observations (Romanelli et al., 2016; Jakosky et al., 2015a; Connerney et al., 2015). In particular, Romanelli et al. (2016) have analyzed MAG observations between October 2014 and March 2016. The authors reported that the PCWs occurrence rate upstream of the Martian bow shock varies with time and takes higher values near the Martian perihelion. Such long term trend was associated with higher hydrogen exospheric densities around that orbital position (derived from numerical simulations) and was also in agreement with the long term trend observed in the irradiances in the 121-122 nm range by MAVEN extreme ultraviolet monitor (EUV) measurements (Eparvier et al., 2015), which provide a proxy to study the temporal variability of the photoionization frequency of the neutral H exosphere.

Ruhunusiri et al. (2017) have characterized magnetic energy spectra in the Mars plasma environment using the MAVEN MAG observations, in the frequency range 0.005 Hz to 16 Hz. By computing the spectral indices for the magnetic energy, the authors showed a wide range of values in the upstream solar wind and the magnetosheath plasma. Also, they observed a seasonal variability of the spectral indices, indicative of a clear connection with the seasonal variability of the PCWs. Nevertheless, to the best of our knowledge, no estimation of the energy cascade rate has been reported yet in the Martian plasma environment. In the present Letter, we aim to extend the current state of knowledge of the solar wind turbulence upstream the Martian shock by computing for the first time the energy transfer rate using an exact relation for fully developed turbulence. Using both magnetic field and plasma moments observations at ~1.8 AU, we investigate how turbulence is affected not only by the heliocentric distance, but also for the presence of PCWs.

2 Incompressible MHD Turbulence

The three-dimensional (3D) incompressible MHD equations can be written as,

\[
\frac{\partial \mathbf{u}}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{u}_A \cdot \nabla \mathbf{u}_A - \frac{1}{\rho_0} \nabla (P + P_M) + f_k + \mathbf{d}_k, \tag{1}
\]

\[
\frac{\partial \mathbf{u}_A}{\partial t} = -\mathbf{u} \cdot \nabla \mathbf{u}_A + \mathbf{u}_A \cdot \nabla \mathbf{u} + f_m + \mathbf{d}_m, \tag{2}
\]

\[
\nabla \cdot \mathbf{u} = 0, \tag{3}
\]

\[
\nabla \cdot \mathbf{u}_A = 0 \tag{4}
\]

where we have defined the incompressible Alfvén velocity \( \mathbf{u}_A \equiv \mathbf{B}/\sqrt{\frac{\pi \rho_0}{\rho_0}} \) (where \( \rho_0 \) the mean mass density) and \( P_M \equiv \rho_0 u_A^2/2 \) is the magnetic pressure. Then, both field variables, \( \mathbf{u} \) and \( \mathbf{u}_A \), are expressed in speed units. Finally, \( f_{k,m} \) are respectively a mechanical and the curl of the electromotive large-scale forcings, and \( \mathbf{d}_{k,m} \) are respectively the small-scale kinetic and magnetic dissipation terms (Andrés, Mininni, et al., 2016; Banerjee & Kritsuk, 2018).

Using Eq. (1)-(4) and following the usual assumptions for fully developed homogeneous turbulence (i.e., infinite kinetic and magnetic Reynolds numbers and a steady state with a balance between forcing and dissipation (see, e.g. Andrés & Sahraoui, 2017), an exact relation for incompressible MHD turbulence can be obtained as (Politano & Pouquet, 1998a,b),

\[
-4 \varepsilon = \rho_0 \nabla \cdot \langle (\delta \mathbf{u} \cdot \delta \mathbf{u} + \delta \mathbf{u}_A \cdot \delta \mathbf{u}_A) \delta \mathbf{u} - (\delta \mathbf{u} \cdot \delta \mathbf{u}_A + \delta \mathbf{u}_A \cdot \delta \mathbf{u}) \delta \mathbf{u}_A \rangle, \tag{5}
\]

where \( \varepsilon \) is the total energy cascade rate per unit volume. Fields are evaluated at position \( \mathbf{x} \) or \( \mathbf{x}' = \mathbf{x} + \ell \); in the latter case a prime is added to the field. The angular bracket \( \langle \cdot \rangle \) denotes an ensemble average (Batchelor, 1953), which is taken here as time average assuming ergodicity. Finally, we have introduced the usual increments definition, i.e., \( \delta \alpha \equiv \alpha' - \alpha \). Here is we are interested in estimating \( \varepsilon \) from Eq. (5), which is fully defined by velocity and magnetic field increments (or fluctuations) that we can estimate from MAVEN observations.
3 Analysis and Results

3.1 MAVEN observations

The MAVEN Magnetometer (MAG) provides vector magnetic field measurements with a 32 Hz maximum sampling frequency and absolute vector accuracy of 0.05% (Connerney et al., 2015). MAVEN’s Solar Wind Ion Analyzer (SWIA) is an energy and angular ion spectrometer covering an energy range between 25 eV/q and 25 keV/q with a field of view of 360° × 90° (Halekas et al., 2015). In this study, we have analyzed the MAVEN MAG and SWIA data sets as follows. Magnetic field observations with 32 Hz cadence are analyzed to discriminate events in the pristine solar wind with PCWs and without wave activity. To estimate the energy cascade rate at MHD scales (i.e., frequencies below ~ 0.1 Hz) we averaged the magnetic field data over 4 s to match SWIA onboard moments cadence (Halekas et al., 2015).

As discussed in the Introduction, Romanelli et al. (2016) have found that the PCWs occurrence rate increases (up to ~ 50%) when Mars is close to the perihelion (1.38 AU) on December 12 2014 and remains relatively low and constant (~ 25%) after reaching the Martian Northern Spring Equinox-Southern Autumn Equinox (NSE-SAE). Also, the authors concluded that the increment in the PCWs occurrence rate cannot be the result of biases associated with MAVEN’s spatial coverage of the upstream region or of the differences in the spatial distribution of the crustal magnetic fields. Therefore, to investigate how PCWs activity may affect the nonlinear transfer of energy, we consider two data sets. Set A contains observations from December 1 2014 until January 31 2015; and set B from January 1, 2015 until February 29, 2016. Set A includes MAVEN observations around perihelion and a local maximum of PCWs activity, while set B includes the Martian Northern Summer Solstice-Southern Winter Solstice (NSS-SWS) that took place on January 3 2016 (and corresponds to a local minimum of waves activity).

3.2 Selection criteria

For sets A and B (~ 330 orbits per set), during time periods when MAVEN was traveling in the solar wind with no connection to the shock (Gruesbeck et al., 2018), we looked for intervals in which the number density fluctuation level was lower than 20% (to be as close as possible to the incompressibility condition). Moreover, in order to have reliable estimate of the energy cascade rate $\varepsilon$ (both its sign and its absolute value (Halekas et al., 2017)) we only consider the events in which the $\theta_{uB}$ (the angle between the magnetic and velocity field) was relatively stationary (Andrés et al., 2019). The long time intervals that fulfil these criteria were divided into a series of sample events with a duration of 30 minutes. This duration ensures having at least one correlation time of the turbulent fluctuations (Hadid et al., 2017; Marquette et al., 2018). Finally, for set A (set B) we considered only cases when PCWs activity was present (absent). By doing this, we can assess the effects that the PCWs may have on the solar wind turbulence. This selection eventually resulted in 184 and 208 events for sets A and B, respectively.

Figure 1 shows two examples of the typical events analyzed in the present Letter (panels (a)-(h) show an example from set A, and panels (i)-(p) from set B). Figure 1 (a)-(f) show the time series for the proton and Alfvén velocity field components in Mars-centered Solar Orbital (MSO) coordinate system (where the x-axis points from Mars to the Sun, z-axis is perpendicular to Mars’ orbital plane and is positive toward the ecliptic north; the y-axis completes the right-handed system). Figure 1 (g)-(h) show the angle between the magnetic and velocity field $\theta_{uB}$ and the density fluctuation level (i.e., $\Delta n/\langle n \rangle$), respectively. Both examples show approximately the same level of density fluctuations and the same $\theta_{uB}$ angle. Finally, the Supporting Information shows that both sets A and B have similar distributions for the density, velocity and magnetic fluctuation values.

3.3 PSD of the magnetic field fluctuations

To determine if a given time interval presents PCWs activity or not, we used a criterion similar to the one in Romanelli et al. (2016). An event is considered to present PCWs activity when the magnetic energy power spectral density (PSD) displays an increase in a frequency interval centered
around the local proton cyclotron frequency $f_{ci}$ when compared to two contiguous windows of width 0.2 $f_{ci}$. More precisely,

$$\max \{ \text{PSD}[B(f)]_{\Delta f}^{1.2 f_{ci}} \} > \max \{ \text{PSD}[B(f)]_{\Delta f}^{1.4 f_{ci}}, \max \{ \text{PSD}[B(f)]_{\Delta f}^{0.8 f_{ci}} \}$$

(6)

where $\max$ corresponds to the maximum value in the PSD in the corresponding window.

Figure 2 (a) and (b) show the PSD for all the events in sets A and B, respectively. For reference, we plot a straight line with Kolmogorov-like slope (i.e., -5/3) in both cases. As we expected, all events near the Martian perihelion (i.e., set A) show a clear peak in their PSD near the proton cyclotron frequency $f_{ci}$. Moreover, all the cases analyzed in the present Letter show a Kolmogorov-like slope in the MHD scales (see, Ruhunusiri et al., 2017). The inset in Figure 2 (a) and (b) show the MAVEN location (where $R_{MSO} = \sqrt{y_{MSO}^2 + z_{MSO}^2}$) for each event for sets A and B, respectively. Finally, the gray dashed line corresponds to best fit of the bow shock extract from Gruesbeck et al. (2018).

### 3.4 Energy cascade rates

To compute the right hand side of Eq. (5), we constructed temporal correlation functions of the different turbulent fields at different time lags $\tau$ in the interval [4,1800] s, which allows covering the MHD inertial range (Ruhunusiri et al., 2017; Hadid et al., 2017). More precisely, assuming the Taylor hypothesis (i.e., $\tau \equiv \ell/V$, where $V$ is the mean plasma flow speed), Eq. (5) can be expressed as a function of time lags $\tau$. Therefore, for each event in both sets, we compute $\varepsilon(\tau)$.

Figure 3 (a) and (b) show the absolute value of the energy cascade rate as a function of the time lag ($\tau$) for both sets. Figure 3 (c) shows the histogram for the (log) mean values $\log(\langle \varepsilon \rangle)_{\text{MHD}}$ in the MHD scales ($\tau = 5 \times 10^2 - 1.5 \times 10^3$ s). It is worth emphasizing that if $\varepsilon$ is changing significantly in amplitude and/or sign, then the resulting mean values would not be reliable (see, e.g., Hadid et al., 2018; Andrés et al., 2019). Therefore, as we mentioned before, we kept only the intervals for which the cascade rate shows a constant (negative or positive) sign for all the time lags in the MHD range. By doing so, the mean value of $\varepsilon$ for each event is robust and so is its absolute value (Coburn et al., 2015; Hadid et al., 2018).

The only limitation of analyzing the non-signed $\varepsilon$ is related to the direct vs. inverse nature of the energy cascade rate. This is because the convergence of the sign of $\varepsilon$ is more stringent than its absolute value (see, Coburn et al., 2015; Hadid et al., 2018), thus demanding a much larger statistical sample than the one considered in the present work. For both data set A and B, the cascade rate values are lower than the averaged value observed at 1 AU, $\varepsilon \sim 10^{-16} - 10^{-17}$ J m$^{-3}$ s$^{-1}$ (Hadid et al., 2018). Also, it is worth mentioning that the energy cascade rate slightly increases when PCWs are present in the solar wind, based on our statistical analysis.

### 3.5 Alfvénic fluctuations

The cross helicity $H_c = \langle \mathbf{u} \cdot \mathbf{u}_A \rangle$ and the total energy $E_T \equiv \langle \langle |\mathbf{u}|^2 \rangle + \langle |\mathbf{u}_A|^2 \rangle \rangle / 2$ (where $\mathbf{u}$ and $\mathbf{u}_A$ are the proton and Alfvén velocities fluctuations) are the two rugged invariant of the ideal incompressible MHD model (see Eqs. 1-4). The dimensionless measure of the normalized cross-helicity corresponds to $\sigma_c \equiv H_c/E_T$, with $-1 \leq \sigma_c \leq 1$. Usually, fluctuations with $|\sigma_c| \sim 1$ are described as being Alfvénic. Another related measurement to quantify the relative energy present in the kinetic and magnetic fluctuations is the normalized residual energy $\sigma_r \equiv \langle \langle |\mathbf{u}|^2 \rangle - \langle |\mathbf{u}_A|^2 \rangle \rangle / E_T$. This parameter also range between -1 and 1.

Figure 4 shows the scatter plot of $\sigma_r$ as a function of $\sigma_c$, for both sets A and B, respectively. The colorbar corresponds to the mean value of the energy cascade rate in the MHD scales $\langle \langle \varepsilon \rangle \rangle_{\text{MHD}}$. The statistical results show a wide variety of possible values of $\sigma_r$ and $\sigma_c$, independently of the presence of PCWs. However, for set B, the events gather around $|\sigma_c| \sim 0.75$ and $\sigma_r \sim -0.4$. 

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4 Discussions and Conclusions

In the present work, we analyzed two data sets by considering separately the cases with (set A) and without PCWs (set B). In agreement with previous studies, our findings are consistent with the seasonal variability of PCWs (Romanelli et al., 2013; Bertucci et al., 2013; Romanelli et al., 2016). We confirmed that such variability is not the result of biases associated with the spatial coverage of MAVEN or with changes in the background velocity or magnetic fields.

Our statistical results show slopes compatible with a Kolmogorov scaling in the largest MHD scales in both sets. Ruhunusiri et al. (2017) determined spectra of magnetic field fluctuations in order to characterize turbulence in the Mars plasma environment. Using 512 s sliding windows, the authors found that magnetic spectrum slopes present different values. In particular, they found that the slope is typically $\sim -1.2$ at the solar wind (in the MHD scales), which differs from the Kolmogorov spectrum. This discrepancy between the computed slopes could be due to several factors: i) we are including only the cases where the cascade rate and the angle $\theta_{EB}$ are approximately constant; ii) the sliding window size used in Ruhunusiri et al. (2017) may not include enough correlation times to yield reliable PSD slopes; and iii) we are separating between PCWs and no waves events, while Ruhunusiri et al. (2017) included all the available data. It is worth mentioning that Gurnett et al. (2010) showed that the magnetic field fluctuations have a Kolmogorov scaling using magnetic field values derived from electron cyclotron echoes from Mars Express observations. Also, the $f^{-5/3}$ spectrum for the magnetic energy is theoretically compatible with our constant energy cascade rate assumption (Andrés, Mininni, et al., 2016; Andrés, Galtier, & Sahraoui, 2016).

We found that the energy cascade rate at Mars ($\sim 1.8$ AU) decreases comparing with previous results at 1 AU and smaller distance from the Sun (see, W. Mattheeus & Velli, 2011; Bruno & Carbone, 2005; Hadid et al., 2017; Bandyopadhyay et al., 2020). In particular, the statistical results for the data set B (no presence of PCWs activity) show a decrease of $|\varepsilon|$ of at least 1 order of magnitude with respect to the value at 1 AU (i.e., $10^{-16} - 10^{-17}$ J m$^{-3}$ s$^{-1}$) (Hadid et al., 2018). However, for the data set A, we observe a slight increase in the transfer of energy when waves are present in the plasma. Our results suggest that PCWs at the sub ion scales may affect the turbulence properties at the MHD scales. In other words, while Eq. (5) is valid only in the MHD inertial range, our results suggest that the instabilities and consequent nonlinear waves at frequencies $\sim f_c$ may affect the largest MHD scales (Osman et al., 2013; Hadid et al., 2018). However, several theoretical papers have shown that newborn planetary ions are capable of providing the free energy for the presence of PCWs (e.g., Brinca, 1991), the PCWs observed upstream from the Martian bow shock are nonlinear and likely not saturated (Cowee et al., 2012). While a increase in $|\varepsilon|$ in correlation with PCWs activity has not been observed before, an analysis of the local velocity distribution functions is still needed to better characterize the growing stage of the observed PCWs and its connection with the reported results.

While both sets show similar values in the parameter space of number density, velocity and Alfvén velocity fields values, our results show a wide variability in the possible values of $\sigma_c$ and $\sigma_r$. In particular, the events in set B correspond to Alfvénic and magnetic dominant fluctuations ($|\sigma_c| \sim 0.75$ and $\sigma_r \sim -0.4$). Interestingly, these events correspond to the higher values of the cascade rate in the set B. Moreover, for both sets the events have mainly negative $\sigma_r$ values with a majority gathering around $\sigma_r \sim -0.25$ and $\sigma_r \sim -0.4$, respectively. This majority of events in the magnetic dominant regime is compatible with previous results between 1 and 8 AU (Roberts et al., 1990; Bruno et al., 2007; Halekas et al., 2017). In particular, Halekas et al. (2017) have investigated the spatial distributions of $\sigma_r$ and $\sigma_c$ using 30 minutes time intervals with a 45 s cadence. Separating observations into four subsets based on the $B_p$ sign and the time range (near perihelion or aphelion), the authors found that the temporal decrease in $\sigma_r$ appears to be equally present in all upstream regions sampled by MAVEN. Our results using 4 s or 45 s (not shown here) cadence exhibit a similar statistical trend. Therefore, the PCWs activity is not affecting significantly the mean value of the statistical distributions of $\sigma_c$ and $\sigma_r$. Slight differences with Halekas et al. (2017) are probably due to the considered selection criteria.

Finally, in this study we have not computed the compressible component of the energy cascade rate (Banerjee & Galtier, 2013; Andrés & Sahraoui, 2017). In particular, we expect to obtain an
strong increases in the nonlinear cascade rate of energy in the Martian magnetosheath, where compressibility plays a major role, like in the Earth’s magnetosheath (Hadid et al., 2018; Andrés et al., 2019). Furthermore, a possible seasonal variability of the incompressible and/or compressible energy cascade rate may be present in the Martian environment. These studies will be part of future works.

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Figure 1. Time series for two examples from sets A and B. In particular, the proton and Alfvén velocity field components (in MSO coordinate system), the angle between magnetic and velocity fields and the density fluctuation level, respectively.
Figure 2. Magnetic power spectra density for both sets A and B, respectively. Inset: MAVEN location of each event in MSO reference frame and the bow shock best fit.

Figure 3. Energy cascade rate (absolute value) as a function of the time lag for sets (a) A and (b) B, respectively. (c) Histogram of $\log(\langle |\varepsilon| \rangle_{\text{MHD}})$ for both sets.
Figure 4. Scatter plot of $\sigma_r$ as a function of $\sigma_r$ for both sets A and B, respectively. Color bars correspond to the mean cascade rate in the MHD scales.