Control of bound-pair transport by periodic driving

K. Kudo,1 T. Boness,2 and T.S. Monteiro2

1Division of Advanced Sciences, Ochadai Academic Production,
Ochanomizu University, 2-1-1 Ohtsuka, Bunkyo-ku, Tokyo 112-8610, Japan
2Department of Physics and Astronomy, University College London,
Gower Street, London WC1E 6BT, United Kingdom
(Dated: October 19, 2009)

We investigate the effect of periodic driving by an external field on systems with attractive pairing interactions. These include spin systems (like the ferromagnetic XXZ model) as well as ultracold fermionic atoms described by the attractive Hubbard model. We show that a well-known phenomenon seen in periodically driven systems—the renormalization of the exchange coupling strength—acts selectively on bound-pairs of spins/atoms, relative to magnon/bare atom states. Thus one can control the direction and speed of transport of bound-pair relative to magnon/unpaired atom states, and thus coherently achieve spatial separation of these components. Applications to recent experiments on transport with fermionic atoms in optical lattices which consist of mixtures of bound-pairs and bare atoms are discussed.

PACS numbers: 75.10.Pq, 67.85.Hj, 05.60.Gg, 03.75.Lm

I. INTRODUCTION

The dynamics of quantum spin transport and in particular spin correlations are of much current interest in the field of quantum information [1]. Cold-atoms in optical lattices provide clean realizations of a range of many-body Hamiltonians, including well-known spin systems. There is also enormous interest in pairing phenomena, motivated by many ground-breaking experiments with ultracold fermionic atoms in optical lattices [2].

Neglecting particle interactions, many simple many-body Hamiltonians, such as Heisenberg, Hubbard or Bose-Hubbard, comprise a hopping term $J H h$ characterized by an exchange coupling strength $J$. If such a system is subjected to an additional spatially-linear, but tonian:

$$H(t) = -J H_h + B \sin \omega t \sum_{n=1}^{N} n \sigma_z^n / 2.$$  \hspace{1cm} (1)

For these, as well as other analogous driven systems, there are regimes where the exchange strength takes an effective, renormalized, value $J_{\text{eff}}$:

$$J_{\text{eff}} = J J_0(\frac{B}{\omega}),$$  \hspace{1cm} (2)

where $J_0$ denotes an ordinary Bessel function. The inter-site transport is completely suppressed at values of $\frac{B}{\omega} = 2.4, 5.52, ...$ corresponding to $J_0(\frac{B}{\omega}) = 0$. The oscillating potential in Eq. (1) can be implemented with ultracold atoms in shaken optical lattices; thus Eq. (2) was recently demonstrated experimentally by mapping $|J_{\text{eff}}|$ as a function of the ratio $\frac{B}{\omega}$.

In the high frequency regime, where $B, \omega \gg J$, the suppression is often termed Coherent Destruction of Tunneling (CDT) [3, 4]. The behavior of the underlying quantum spectrum and the application of CDT to controlling the superfluid-Mott Insulator transition in atomic systems were investigated theoretically in Ref. [5, 10]. However, this CDT is also closely related to a phenomenon termed Dynamic Localization (DL)—identified even earlier in 1986 [6]—which also impedes transport in periodically driven systems at $J_0(\frac{B}{\omega}) \approx 0$. It too has been theoretically investigated in atomic systems [11, 12]. The precise relationship between CDT and DL remains of much interest: a recent theoretical analysis is provided by Ref. [13, but in brief, the CDT mechanism is associated with high-frequency driving ($\omega \gg J$) and complete suppression of hopping. For CDT, particles remain completely frozen at their original sites even for a 2-site system: CDT was initially identified in driven double-well systems [3, 4, 8]. CDT can persist even in the presence of some inter-particle interactions [9, 10]. DL, on the other hand, entails a less complete suppression of transport: for $J_0(\frac{B}{\omega}) \approx 0$, the single-particle wavepacket position may oscillate, but the particle returns periodically to its original position. DL is associated with lower frequency driving, negligible particle interactions and the large $N$ limit [13].

Here we investigate for the first time periodic-driving for systems with a bound-pair component. In the absence of driving, transport for fermionic systems with attractive pairing interactions has been previously investigated in, e.g., ferromagnetic XXZ spin models [14] and even experimentally with ultracold atoms which are a mixture of bound-pairs and bare atoms, in a regime of the attractive Hubbard model [15]. Our main finding is that the DL regime (but not CDT) can provide a mechanism to spatially separate the paired and unpaired fraction in such spin or cold atoms systems. This is because the driving can generate a hitherto unnoticed mechanism of directed motion; in contrast, previous studies to date, such as Refs. [3, 4, 10], probed only $|J_{\text{eff}}|$, i.e., the presence or absence of diffusive expansion of an atomic cloud, not global transport.
We note that both the DL and CDT transport-suppression mechanisms are usually analyzed in terms of the stationary states (Floquet states) of the driven Hamiltonian: typically, the Floquet eigenstates (i.e., their quasienergies) become degenerate or approximately degenerate if \( J_\sigma(n) \approx 0 \). However, here, in the \( N \to \infty \) limit, we analyze the quantum transport without any reference to the detailed quantum structure, or quasienergy near-degeneracies; we obtain the renormalization Eq. (2) (as well as the ratchet mechanism) from a straightforward integration of the classical equations of motion of the driven Hamiltonian. This entails a somewhat modified perspective, since now the key division is not between high-frequency CDT freezing-of-motion and low-frequency DL wavepacket revival. It is rather between the \( N \to \infty \) limit, where both high and low frequency suppression of transport arises as a classical phenomenon (i.e., it is purely an effect in the group velocity), and the small \( N \) limit, where a quantal analysis of a few Floquet state degeneracies remains essential—and where only high frequency driving suppresses transport. The effect of inter-particle interactions seems difficult to treat classically. But, below we identify an example where we can include the effect of certain important interactions by considering two limiting effective Hamiltonians.

The paper is organized as follows: In Sec. II, we introduce our models. One is a Heisenberg XXZ spin chain; its eigenstates include both magnon-like spin-waves (analogous to Bloch waves) and bound-pair states. Explicit expressions for both are obtainable via the Bethe ansatz. The close correspondence with the two-particle attractive Hubbard model, which also includes both bare atom and bound-pair states, is explained. The Hubbard model is of particular interest as it is already realized in current cold atoms experiments. In Sec. III the quantum CDT mechanism, dominant for low \( N \) and \( \omega \gg J \) is introduced. Its effects are demonstrated for both unpaired and bound-pair states by exact numerical solution of the quantum Hamiltonian. In Sec. IV the low-frequency, high-\( N \) regime is investigated; control of wavepacket dynamics is shown for the one and two-particle regimes. The underlying mechanism of directed motion are obtained from the classical equations of motion. The behavior of both bound and unpaired cases are analyzed with two limiting classical Hamiltonians and are shown to agree with the full quantum solutions. Conclusions and outlook are given in Sec. V.

II. STATIC MODELS

In general, inter-particle interactions are important so the full many-body Hamiltonian takes the form \( H = H_h + (J \Delta) H_{\text{int}} \), where \( H_{\text{int}} \) is an interaction term characterized by an interaction strength \( U = J \Delta \). For instance, the well-known spin-1/2 Heisenberg XXZ ferromagnetic chain of length \( N \), is governed by the Hamiltonian:

\[
-H = - \sum_{n=1}^{N} \left( \frac{J}{2} (\sigma^+_{n} \sigma^-_{n+1} + \sigma^+_{n} \sigma^-_{n+1}) + \frac{J \Delta}{4} \sigma^z_n \sigma^z_{n+1} \right)
\]

where \( n \in [1 : N] \) indicates the \( n \)-th site, and \( \Delta \) denotes the anisotropy. For large \( \Delta \) (and low density of excitations), there are two dominant classes of quasi-particle states: magnon-like and bound-pair states.

The Hamiltonian Eq.(3) conserves the number of spin-flips; a single excitation represents a spin-wave, or magnon, which distributes a single spin-flip throughout the chain. The spin-wave eigenstates of Eq.(3) are

\[
| \kappa \rangle = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} e^{i \kappa n} | n \rangle \quad \text{(for periodic boundary conditions)}
\]

where \( | n \rangle \) denotes a state with the spin-flip at sites \( n \); they obey the dispersion relation \( E_{\kappa} - E_0 = - J \cos \kappa \), where \( E_0 \) is the ground state. Higher excited states correspond to multiple spin-waves which interact when they coincide through (i) an exclusion process (no two spin flips can simultaneously occupy the same site) and (ii) an effective attractive interaction induced by the \( (J \Delta) H_{\text{int}} = \sum_{n} \frac{J \Delta}{4} \sigma^z_n \sigma^z_{n+1} \) interaction term. For the long wavelength processes considered in our key results Figs.2 and 3 only (ii) is of significance.

Now we consider the two-particle case. For the two-excitation case, the eigenstates of Eq.(3) may be expressed, via the Bethe ansatz, as spin waves [14, 17]:

\[
| \kappa_1, \kappa_2 \rangle = A_{\kappa_1, \kappa_2} \sum_{n_1, n_2} a_{n_1, n_2}^{\kappa_1, \kappa_2} | n_1, n_2 \rangle,
\]

where \( A_{\kappa_1, \kappa_2} \) is a normalization constant and \( | n_1, n_2 \rangle \) indicates a spin-flip at sites \( n_1 \) and \( n_2 \). Further details are in Refs. [14, 17] but the eigenstates divide into two distinct classes: (1) Magnon-like scattering states, where the spins move separately, with dispersion relation \( E_{\kappa_1, \kappa_2} - E_0 = J (2 \Delta - \cos \kappa_1 - \cos \kappa_2) \) which is a straightforward extension of the singly-excited case and (2) Bound-pair states, for which the probability amplitudes decay exponentially with the separation of the flips. For \( \Delta > 0 \), and as \( N \to \infty \) they obey the dispersion relation, \( E_{\kappa_1, \kappa_2} - E_0 = J \Delta - \frac{J}{2} [1 + \cos(\kappa_1 + \kappa_2)] \). Note that the sum \( (\kappa_1 + \kappa_2) \) is always real. In Ref. [14] a perturbation expansion in spin coupling strength \( J \) is employed to produce an effective Hamiltonian when \( \Delta \gg 1 \). In this approximation the bound state amplitudes are

\[
a_{n_1, n_2}^{\kappa_1, \kappa_2} = \delta_{n_1, n_2-1} e^{i(\kappa_1 + \kappa_2)n_1}
\]

and thus the bound states become confined to the nearest neighbor (NN) subspace, \( \{ | n, n+1 \rangle \} \); their dispersion relation remains unchanged from the above. In “center of mass” coordinates \( 2 \kappa = \kappa_1 + \kappa_2 \), the bound states are given by \( | \kappa \rangle = A_{\kappa} \sum_{n=1}^{N} e^{i 2 \kappa n} | n, n+1 \rangle \) and are of analogous form to the single magnon solution. Namely, two initially neighboring spin-flips thus hop together but their
propagation speed is slower relative to a single spin-flip: transport occurs by a second order effective Hamiltonian and an effective coupling $J_{\text{eff}} = \frac{1}{4} < J$.

The transport is rather analogous to that seen in the attractive Hubbard model $-JH = J \sum_k n_k (\epsilon_{\sigma} c_{k\sigma}^\dagger c_{k\sigma} + \text{H.c.}) + J_\Delta \sum_k n_k n_{k+1}^\dagger$ characterized by tunneling amplitude $J$ and interaction $U = J\Delta$; for $U < 0$, local bound-pairs form which tunnel by a second-order process, with amplitude of order $\sim J^2/U \sim J/\Delta$ akin to the spin system. This situation was investigated experimentally using fermionic atoms for $U < 0$ in Ref. [13]; note of course that the atomic local bound-pairs (BPs) correspond to a pair of atoms of opposite spin occupying the same lattice site, while the spin BPs occupy adjacent sites.

III. QUANTUM DYNAMICS: HIGH FREQUENCY DRIVING AND COHERENT DESTRUCTION OF TUNNELING

We now consider the regime with $B, \omega \gg J$ which can lead to coherent destruction of tunneling. Using units of scaled time $t' = \omega t$, the quantum dynamics generated by Eq. (1) depends only on two global parameters $J' = J/\omega$ and $B' = B/\omega$ (for a given $\Delta$). The high-frequency driving condition for CDT thus becomes $J' \ll 1$. We follow the usual procedure for justifying CDT [8, 10]: for $B' \gg J'$, we initially neglect $H_k$ and set $H(t) = B' \sin t' \sum_{n=1}^{N} n \sigma_n^z/2$.

Then, the Schrödinger equation has the following time-dependent solutions:

$$|n_1, n_2, t\rangle = e^{i[B'(n_1+n_2) \cos t]} |n_1, n_2\rangle,$$

(6) to within a constant phase term $e^{iB' \varphi \cos t}$ where $\varphi = \sum_{n=1}^{N} n = N(N+1)/4$. The stationary states of periodically driven systems (Floquet states) have a Brillouin zone type structure [8, 10] and the above correspond to a family of solutions $|n_1, n_2, m, t\rangle = |n_1, n_2, t\rangle e^{i\omega t}$ which are periodic in time. The multi-photon band index $m$ is used to label states which differ only by the absorption or emission of $m$ quanta of energy $E_m = m\hbar \omega$. The next step is to treat the spin-exchange as a perturbation, in the extended Hilbert space where time appears as an extra coordinate. The matrix elements of $H_k$ are simply

$$\langle \langle n_1', n_2', m', t'|JH_k|n_1, n_2, m, t\rangle \rangle \approx \delta_{m'm}$$

$$J\mathcal{J}_0[B'(n_1' - n_1) + B'(n_2' - n_2)] |n_1', n_2'\rangle |H_k|n_1, n_2\rangle.$$  

(7)

Here $\langle \langle \cdot \cdot \cdot \rangle \rangle = \frac{1}{T} \int_0^T \langle \cdot \cdot \cdot \rangle$ denotes a scalar product in the extended space [9] and $\langle \cdot \cdot \rangle$ denotes the scalar product in position coordinates. It is easily seen that in general, the matrix elements $|n_1', n_2'| |H_k|n_1, n_2\rangle = \delta_{n_1', n_1+1} \delta_{n_2', n_2} + \delta_{n_1', n_1} \delta_{n_2', n_2+1} - \delta_{n_1', n_1+1} \delta_{n_2', n_2}$ for $B' < 0$. In other words, they involve hopping of single spins only. Thus, we have

$$J\mathcal{J}_0[B'(n_1' - n_1) + B'(n_2' - n_2)] = J\mathcal{J}_0(B')$$

(8)

as in Eq. (2). This also corresponds to the situation explored in the cold atoms experiments [7, 8]. A more careful analysis [9] reveals terms coupling off-diagonal in the photon band index $m$. If these off-diagonal terms are significant, the above analysis is no longer valid; however, it has been shown that provided the energy scale for the band separation far exceeds the intra-band energy scale (i.e., $\omega \gg J$), these couplings are negligible. Thus within the above framework, the Eq. (2) renormalization holds for high-frequency driving.

However, the bound pairs, for $\Delta \gg 1$, evolve under an effective second-order Hamiltonian $H_{\text{eff}}$ [14] which results in the two spins hopping together. When $n_2 = n_1+1$, for instance, the argument of the Bessel function becomes

$$B'(n_1' - n_1) + B'[n_1' + 1 - (n_1 + 1)] = 2B'(n_1' - n_1).$$

(9)

Thus, the transport is then determined by matrix elements $J\mathcal{J}_0(2B')/n, n+1 \mid H_{\text{eff}} \mid n, n+1 \pm 1$: note the doubling of the argument of the Bessel function. Since the Bessel is an oscillating function one can then, for example, choose a value of $B'$ for which $J_{\text{eff}} \approx J\mathcal{J}_0(B')$ for the magnon-like states is positive, while $J_{\text{eff}} \approx J\mathcal{J}_0(2B')$ for the bound-pairs is negative (or zero).

An arbitrary spin state, in general, has a projection on both the magnon-like and the bound-pair eigenstates. For $\Delta \gg 1$, however, one may prepare a good approximation to a pure bound-pair state by simply flipping two adjacent spins of a ferromagnetic chain in its ground state (all spins aligned). Conversely, two well-separated spins will approximate pure magnon-like states. In a corresponding atomic experiment with Fermionic atoms one may either consider unpaired single atoms or a pure bound-pair, as well as a superposition of these two extremes.
In Fig. 1 we demonstrate CDT for the two extremes (an initial state which is either a pure magnon state or a pure bound-pair for large $\Delta$). Driving with a field $B' = 5.52$ “freezes” two initially well-separated spin-flips at their original sites, since $J_0(B') = 0$. In the absence of driving, or at values of $B'$ with $J_0(B') \neq 0$, both spins rapidly diffuse along the chain. On the other hand, if the two-flips are initially adjacent, they remain frozen at their positions if $J_0(2B') = 0$ and delocalize otherwise.

Other than for these extremes, CDT (e.g., for initial states which are superpositions of magnon/bound-pairs) is less effective; while both the magnon-like scattering states and bound-pair states are eigenstates of the Hamiltonian Eq. (3), the respective subspaces are coupled by the driving: in general, for CDT, the driving strength $B \gg J\Delta$, where $J(\Delta - 2)$ is the energetic separation between the two subspaces.

IV. THE LARGE-\(N\) REGIME AND DYNAMIC LOCALIZATION

We now investigate the regime $N \to \infty$, usually associated with Dynamic Localization. We can work in the more favorable regime $J\Delta \gtrsim B$ where coupling between bound-pairs and unpaired states is suppressed. We evolve Eq. (11) for an open chain with $N = 100$ sites (for computational reasons, much larger $N$ is difficult). Since our analysis below does not introduce any high-frequency condition, we can also consider low-frequency driving, for which $B', J' \gg 1$.

We prepare a one-spin flip Gaussian wavepacket, at the center of the chain. Figure 2(a) shows that the spin-packet unexpectedly moves as a whole, and with little spreading, along the chain, but its direction of travel reverses at $B' \simeq 5.5$: for $B' = 5.3$ it moves upwards, for $B' = 5.7$ it moves downwards; for $B' = 5.5$ it exhibits behavior characteristic of Dynamic Localization (static, but with oscillations). The results for $B' = 6.3$ may seem at first sight even more puzzling: the wavepacket’s center of mass is static, but the excitation diffuses along the chain. In order to understand these results, one notes that Fig. 2(a) demonstrates a combination of DL type renormalization, as well as a (hitherto unnoticed) type of ratchet (meaning directed motion without bias).

We note that $N$ plays the role of an effective $\hbar$. Thus, in the presence of driving and in the $N \to \infty$ limit, one may map the system to an “image” classical Hamiltonian:

$$H(x, p) = -J' \cos p - B' x \sin t'$$

by mapping site to continuous position $n \to x$ and $\kappa \to p$. Integrating Hamilton’s classical equations of motion over one period, it is easy to see that the distance traveled per period $v = (x(t + T) - x(t))/(2\pi)$, where $T = 2\pi/\omega$, for a particle with position $x(t = 0) = x_0$ and momentum $p(t = 0) = p_0$ is simply:

$$v = J' J_0(B') \sin(p_0 + B').$$

In effect, this is the center of mass velocity of the wavepacket. It is independent of initial position $x_0$. The renormalization of the velocity already appears, but it is no longer a question of simply renormalizing $J'$ by $J_0(B')$ because there is also a $\sin(p_0 + B')$ factor which can also change sign or suppress transport. We find, however, that it does provide great advantages in reducing dispersive spreading.

An initial Gaussian wavepacket, initially localized over a finite number of sites $1/\delta_n$, corresponds to localization in momentum of $\sim \delta_n$ in the image phasespace. Thus the corresponding momentum distribution is $N(p) \sim \exp[-(p_0 - \langle p \rangle)^2/\delta_n^2]$. Low-energy spin wavepackets (and cold atom clouds) correspond to distributions well-localized about zero momentum, i.e., $\kappa \sim 0$ thus $\langle p_0 \rangle \sim 0$. The distribution thus samples velocities $J' J_0(B') \sin(B' \pm \delta_n)$. Hence provided $m\pi \lesssim |B' \pm \delta_n| \lesssim (m + 1)\pi$, all momenta $-\delta_n < p_0 < \delta_n$ correspond to the same direction of motion and there is little dispersion. This results in directed motion. In contrast, when $B' \sim m\pi$ with $m = 1, 2, \ldots$, trajectories with $p_0 < 0$
move in opposite direction to those with $p_0 > 0$ and the wavepacket, though static, spreads diffusively along the chain. In the presence of DL ($B' = 5.5$), $J_0(B') = 0$, so there is, of course no transport. In Fig.2(b) we see that a calculation of the velocity from the numerics [displacement of the quantum wavepacket per period obtained from a quantum solution of Eq. (11)] is in excellent agreement with Eq. (11).

Conversely, a single bound-pair state moves under a different image Hamiltonian $H = \frac{J_0(x)}{\Delta} \cos 2P - 2B' \sin t$, where $2P = p_1 + p_2$ and $X = (x_1 + x_2)/2$, indicate center of mass coordinates. Thus the bound-pair velocity:

$$v_{bs} = \frac{J'}{2\Delta} J_0(2B') \sin(p_0 + 2B').$$  \hfill (12)

For multiple excitations, we analyze the numerics by assuming that the magnon-like projection of the wavepackets obey Eq. (11), while the Hilbert space fraction of bound-pair states moves under Eq. (12). We thus assume that coupling between the two subspaces is negligible, a reasonable assumption for $2B' \ll J$. Figure 2(b) (broken line) shows the corresponding velocity. One sees that the bound-pair and magnon velocities may even have opposite signs for the same driving field. This enables us to steer the two components in opposite directions or to stop one and transport the other. These different situations are demonstrated in Fig.3 where the initial state is the product of two Gaussian wavepackets peaked on $\kappa \approx 0$ and at positions $n_1 = 45$ and $n_2 = 50$. The $\kappa \approx 0$ condition corresponds to the low-energy distribution—also typical of an ultracold gas (centered on $p_0 \approx 0$). There is no special significance in the initial chosen positions [neither Eq. (11) nor (12) depends on initial $x$] other than choosing an initial state with $n_1$ and $n_2$ close but not adjacent yields an appreciable projection in both bound-pair and magnon subspaces. Note also that the extent of the initial distribution should not exceed that of the chain/lattice, so that center of mass displacement is observable. Equivalently, in a Hubbard Hamiltonian, two delocalized atoms with distribution peaked one or two sites apart would yield an appreciable probability to both form a pair/remain unpaired.

The directed wavepacket motion is in sharp contrast to the usual motion seen in studies of CDT/DL transport; also to the undriven case: while magnon states diffuse faster than bound-pair components (by a factor of $2\Delta$) both ultimately simply delocalize along the length of the chain. Figure 3(e) illustrates the spin correlations for Fig.3(d) (for large $\Delta = 5$) where the bound-pair component is immobilized, while the component with separated spins is transmitted.

V. CONCLUSIONS AND OUTLOOK

In conclusion, we have investigated the transport in a system with attractive pairing interactions periodically driven by an external field. The effects of periodic driving act selectively on unpaired and bound-pair states, which enable us to control separately the relative direction and speed of each wavepacket of the two states.

One should also consider higher numbers of particles. For low-numbers of excitations (modest filling factors in the atomic case) the dominant processes are the above. Nevertheless, the effect of including higher numbers of particles remains an important question. In the case of high-frequency driving and CDT, this is less of a concern: e.g., in Ref. 11 it was shown that a 2-particle model adequately described the CDT behavior of a full many-body Hamiltonian. However, for low frequencies, this remains an open question. One of our key findings is the onset of center of mass motion in the DL regime, which would open new experimental possibilities, and is well within reach of current techniques. The experiments of Ref. 7 observed the effects of Eq. (2) in the range $J' \approx 3 \rightarrow 1/30$; for lower frequencies, the effect was lost due to inter-particle interactions. This range of $J'$ overlaps with our DL wave-packet splitting regime [e.g., Fig.2(d) corresponds to $J' = 2$ and $U/J = 5$]. In addition, a system corresponding to the attractive Hubbard model is somewhat more favorable than the bosonic atoms used in CDT experiments since there are *two* good DL limits provided by the fully paired and unpaired extremes, while for the bosonic systems, low-frequency DL occurs only in the limit of negligible interactions. A full numerical study of the DL regime (high $N$ and many particles) is numerically challenging, but future calculations will need to address this question more fully.

Within the regime of validity of the present work, even for not too large $J\Delta/B$ we see that the wavepacket splits
cleanly into two components, whose relative speed and direction may be controlled as in Eqs. (11) and (12). This shows that coupling between the magnon and bound-pair subspaces by the external driving is limited. In fact it is easy to show that, once the wavepackets separate, further interaction is negligible. If the separation is slow, collisions between the paired and unpaired particles may be important; but this itself exemplifies a topic of much current interest; for example recent experiments probed transport of impurity wavepackets accelerating through a static cold cloud by means of a linear potential \( V = gx \) due to gravity \([18]\). The study of dynamics of bound-pairs moving through a cloud due to the linear oscillating lattice potential \( V = F x \cos \omega t \) is also within current experimental capabilities and offers new possibilities for transport in the presence of interactions.

Acknowledgments

We acknowledge helpful discussions with S. Bose, C. Creffield and M. Oberthaler. This work is partly supported by KAKENHI(21740289).

[1] S. Bose, Phys. Rev. Lett. 91, 207901 (2003); L. Amico, R. Fazio, A. Osterloh and V. Vedral, Rev. Mod. Phys. 80, 517 (2008); T.S. Cubitt and J.I. Cirac, Phys. Rev. Lett. 100, 180406 (2008).
[2] See, e.g., C. Chin et al, Nature 443, 961 (2006) and references therein.
[3] D.H. Dunlap and V.M. Kenkre, Phys. Rev. B 34, 3625 (1986).
[4] F. Grossmann, P. Jung, T. Dittrich, and P. Hanggi, Z. Phys. B 84, 315 (1991); F. Grossmann, P. Jung, T. Dittrich, and P. Hanggi, Phys. Rev. Lett. 67, 516 (1991).
[5] M. Grifoni and P. Hanggi, Phys. Rep. 303, 229 (1998).
[6] M. Holthaus, Phys. Rev. Lett. 69, 351 (1992).
[7] H. Lignier, C. Sias, D. Ciampini, Y. Singh, A. Zenesini, O. Morsch, and E. Arimondo, Phys. Rev. Lett. 99, 220403 (2007).
[8] E. Kierig, U. Schnorrberger, A. Schietinger, J. Tomkovic, and M. K. Oberthaler, Phys. Rev. Lett. 100, 190405 (2008).
[9] A. Eckardt et al, Phys. Rev. Lett. 95, 260404 (2005).
[10] C. Creffield and T.S. Monteiro, Phys. Rev. Lett. 96, 210403 (2006).
[11] Q. Thommen, J-C Garreau, V Zehnle, Phys. Rev. A 65, 053406 (2002); A. R. Kolovsky and H. J. Korsch, Int. J. Mod. Phys. B 18, 1235 (2004).
[12] A. Eckardt et al, Phys. Rev. A. 79, 013611 (2009).
[13] Y. Kyanuma and K. Saito, Phys. Rev. A. 77, 010101(R) (2008).
[14] L.F. Santos and M.I. Dykman, Phys. Rev. B 68, 214410 (2003).
[15] N. Strohmaier et al, Phys. Rev. Lett. 99, 220601 (2007).
[16] H. Bethe, Z. Phys. 71, 205 (1931); Yu.N. Skryabin “Statistical Mechanics of Magnetically Ordered Systems”, Kluwer Academic (1988).
[17] T. Boness, K. Kudo, T.S. Monteiro, arXiv:0810.0454.
[18] S. Palzer et al, arXiv:0903.4823.