Angular cross-correlation of galaxies: a probe of gravitational lensing by large-scale structure

R. Moessner and Bhuvnesh Jain

Max Planck Institut für Astrophysik, Karl-Schwarzschild-Straße 1, 85740 Garching, Germany
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218, USA

Accepted 1997 December 10. Received 1997 November 24; in original form 1997 September 29

ABSTRACT

The angular cross-correlation between two galaxy samples separated in redshift is shown to be a useful measure of weak lensing by large-scale structure. Angular correlations in faint galaxies arise as a result of spatial clustering of the galaxies as well as gravitational lensing by dark matter along the line of sight. The lensing contribution to the two-point autocorrelation function is typically small compared with the gravitational clustering. However, the cross-correlation between two galaxy samples is almost unaffected by gravitational clustering provided that their redshift distributions do not overlap. The cross-correlation is then induced by magnification bias resulting from lensing by large-scale structure. We compute the expected amplitude of the cross-correlation for popular theoretical models of structure formation. For two populations with mean redshifts of \( z = 0.3 \) and 1, we find a cross-correlation signal of \( \sim 1 \) per cent on arcmin scales and \( \sim 3 \) per cent on scales of a few arcsec. The dependence on the cosmological parameters \( \Omega_m \) and \( \Lambda \), the dark matter power spectrum and the bias factor of the foreground galaxy population is explored.

Key words: galaxies: clusters: general – cosmology: observations – gravitational lensing – large-scale structure of Universe.

1 INTRODUCTION

The angular autocorrelation function of galaxies has been widely used to characterize the large-scale structure of the Universe (e.g. Peebles 1980). The observed galaxy distribution is well described by a power law \( w(y) \propto y^{-0.8} \), with slope \( \gamma \simeq 0.8 \).

Gravitational lensing by large-scale structure along the line of sight can alter the amplitude of \( w(y) \) (Gunn 1967). Lensing increases the area of a given patch on the sky, thus diluting the number density. On the other hand, galaxies too faint to be included in a sample of given limiting magnitude are brightened as a result of lensing and may therefore be included in the sample. The net effect, known as magnification bias, can go either way: it can lead to enhancement or suppression of the observed number density of galaxies, depending on the slope of the number–magnitude relation. Variations in the number density that are correlated over some angular separation alter \( w(y) \). Following Kaiser (1992) and Villumsen (1996), Moessner, Jain & Villumsen (1997; henceforth MJV) have considered the effect of non-linear gravitational evolution and magnification bias on \( w(y) \). MJV found that for faint samples, with mean redshift \( z \approx 1 \), lensing contributes 5–20 per cent of the signal, depending on the cosmological model and angular scale. As the lensing contribution is small even for distant galaxies, it is difficult to interpret a measurement, especially because it requires knowledge of the biasing of high-redshift galaxies relative to the mass.

In this paper we explore a different statistic, the cross-correlation function of two different galaxy samples, in order to isolate the effect of magnification bias. Consider two galaxy samples with non-overlapping redshift distributions. If the minimal distance between the two samples is several 100 Mpc, the effects of gravitational clustering are negligible. The cross-correlation function in such a case is affected entirely by magnification bias, and the dominant term is provided by the lensing effect of dark matter associated with the foreground galaxy population. Cross-correlating observables affected by gravitational lensing, such as
Angular cross-correlation of galaxies

The goal of this paper is to compute the cross-correlation \( \omega(\theta) \) arising from these two effects.

For simplicity we assume a linear bias model where galaxies trace the underlying dark matter fluctuations
\[
\delta_i(x) = \bar{b}_i \delta(x).
\] (4)

The fluctuations on the sky resulting from intrinsic clustering are a projection of the density fluctuations along the line of sight, weighted with the bias factor and the radial distribution \( W(\chi) \) of the galaxies, i.e.
\[
\delta_n^2(\phi) = b \int_0^\infty \! d\chi W(\chi) \delta[r(\chi) \varnothing, a].
\] (5)

The metric, the comoving radial coordinate \( \chi \) and the comoving angular diameter distance \( r(\chi) \) are introduced in the Appendix. We assume for simplicity a constant bias factor independent of scale and redshift for each galaxy population.

To determine the fluctuations resulting from magnification bias, consider the logarithmic slope \( s \) of the number counts of galaxies \( N_s(m) \) in a sample with limiting magnitude \( m \) (see MJV for details):
\[
\frac{d \log N_s(m)}{dm}.
\] (6)

Magnification by amount \( \mu \) changes the number counts to (e.g. Broadhurst, Taylor & Peacock 1995)
\[
N^*(m) = N_s(m) \mu^{2.5s-1}.
\] (7)

In the weak lensing limit, the magnification is \( \mu = 1 + 2\kappa \), where the convergence \( \kappa \) is a weighted projection of the density field along the line of sight (see equation 9 below). As \( \kappa \ll 1 \) for weak lensing, equation (7) for the number counts reduces to
\[
N^*(m) = N_s(m)[1 + 5(s - 0.4) \kappa].
\] (8)

Using \( g(\chi) \) to denote the radial weight function (e.g. Jain & Seljak 1997), the convergence \( \kappa \) is
\[
\kappa_s(\phi) = \frac{3}{2} \Omega_m \int_0^\infty \! d\chi \frac{g(\chi)}{a} \delta[r(\chi) \varnothing, a].
\] (9)

The expression for \( g(\chi) \) in terms of \( r(\chi) \) is given in the Appendix.

Finally, using the above relations we can rewrite equation (3) for \( \delta_n(\phi) \) as
\[
\delta n(\phi) = \int_0^\infty \! d\chi f_s(\chi) \delta[r(\chi) \varnothing, a],
\] (10)

with
\[
f_s(\chi) = b \Omega_m(2.5s - 1) \frac{g(\chi)}{a}.
\] (11)

Inserting the above two equations into equation (1), using the small-angle approximation \( \theta \ll 1 \), and assuming that the radial weight functions \( f_s(\chi) \) vary slowly compared with the scale of density perturbations of interest gives (Villumsen 1996)
The power spectrum of dark matter fluctuations $P(\chi, k)$ is defined by

$$\langle \delta(k) \delta(k') \rangle = (2\pi)^3 P(\chi, k) \delta(k - k').$$

The angular cross-correlation function is composed of four terms. In the case of $\langle z_2 \rangle > \langle z_1 \rangle$, these terms are

$$\omega(\theta) = \langle \delta n(\hat{\phi}) \delta n(\hat{\phi}') \rangle + \langle \delta n(\hat{\phi}) \delta n(\hat{\phi}^*) \rangle + \langle \delta n(\hat{\phi}^*) \delta n(\hat{\phi}') \rangle.$$ (14)

The first term results from the intrinsic clustering of the galaxies of the two samples where their redshift distributions overlap,

$$\omega_{g_0}(\theta) = b_3 3 \Omega_{m} (2.5 \Omega_{s} - 1) 4 \pi^2 \int_{0}^{\chi_{\max}} \text{d} \chi W_{1}(\chi) W_{2}(\chi)$$

$$\times \int_{0}^{\chi_{\max}} \text{d} k P(\chi, k) J_{0}[k] \theta.$$ (15)

Ideally we would like this term to be zero, in order to distinguish the contribution resulting from lensing more clearly. This could be achieved by obtaining photometric redshifts for the galaxies, and selecting two galaxy populations that do not overlap in their redshift distributions. The cross-correlation of two such samples minimizes the contribution from intrinsic clustering, which removes uncertainties in the predictions caused by the unknown physical evolution of the galaxies.

The second term in equation (14) results from the lensing of the background galaxies by the dark matter in front of it, which is traced by the foreground galaxies. The correlation thus induced between galaxies in the two samples is given by

$$\omega_{g_0}(\theta) = b_3 3 \Omega_{m} (2.5 \Omega_{s} - 1) 4 \pi^2 \int_{0}^{\chi_{\max}} \text{d} \chi W_{1}(\chi) W_{2}(\chi)$$

$$\times \int_{0}^{\chi_{\max}} \text{d} k P(\chi, k) J_{0}[k] \theta.$$ (16)

The third term results from dark matter in front of both of the galaxy samples causing the lensing. The fourth is the result of dark matter traced by the background galaxies lensing the foreground galaxies. It is non-zero only if there is an overlap in the redshift distributions of the two samples. For all cases of interest these two terms are negligible compared with the second term, $\omega_{g_0}$.

### 2.1 Dependence on the cosmological model

Equation (15) shows how $\omega(\theta)$ depends linearly on $\Omega_{m}$, aside from the dependences on $\Omega_{s}$ and $\Omega_{l}$ contained in the line-of-sight integral. These arise from two sources: (i) the distance factors contained in $J_{0}$, $g(\chi)$ and $W(\chi)$, and (ii) the growth and amplitude of the power spectrum. In the linear regime, $P(\chi, k)$ depends on the linear growing mode of density perturbations $D(\chi)$ and the normalization $\sigma_{8}$, which in turn can depend on the cosmological parameters as

$$P(\chi, k) = (\sigma_{8} D(\chi))^2.$$ (17)

The linear growing mode is well approximated by (Carroll, Press & Turner 1992)

$$D(\chi) = \frac{1}{2} a \Omega_{m} \Omega_{l}^{-\beta} - \lambda(a)$$

$$+ [1 + \Omega_{m}(a)/2] [1 + \lambda(a)/70]^{-1},$$ (18)

where we have defined, following Mo, Jing & Börner (1996), the time-dependent fractions of density in matter and vacuum energy, $\Omega_{m}$ and $\Omega_{l}$:

$$\Omega_{m} = \frac{\Omega_{m}}{a + \Omega_{m}(1-a) + \Omega_{s}(a^3_a - a)}$$ (19)

and

$$\lambda(a) = \frac{a^3 \Omega_{s}}{a + \Omega_{m}(1-a) + \Omega_{s}(a^3_a - a)}.$$ (20)

Moreover, the spatial geometries differ in different models, leading to a dependence of the angular distance $r(\chi)$ on $\Omega_{m}$ and $\Omega_{l}$ according to equation (A2).

The redshift distribution of galaxies can be modelled by

$$n(z) = \frac{\beta z^2}{z_{\text{c}}^2} \Gamma \left[ \frac{4}{3} / \beta \right] - (z/z_{\text{c}})^{\gamma}$$ (21)

for $\beta = 2.5$, which agrees reasonably well with the redshift distribution estimated for the Hubble Deep Field from photometric redshifts (Mobasher et al. 1996). The mean redshift is then given by

$$\langle z \rangle = \frac{\Gamma (4/3) / \Gamma (3/\beta)}{z_{\text{c}}}.$$ (22)

Three of the four different cosmological models we consider are a flat universe with $\Omega_{m} = 1$, an open model with $\Omega_{m} = 0.3$ and a flat $\Lambda$-dominated model with $\Omega_{m} = 0.3$ and $\Omega_{s} = 0.7$. These three models are normalized to cluster abundances (White, Efstathiou & Frenk 1993; Viana & Liddle 1996; Eke, Cole & Frenk 1996; Pen 1996), $\sigma_{8} \approx 0.6 \Omega_{m}^{-0.6}$. (For $\Omega_{m} = 0.3$, we use $\sigma_{8} = 1$, which is close to the results of the approximate formula given above.) Our fourth model is a flat universe, $\Omega_{m} = 1$, with a high normalization of $\sigma_{8} = 1$.

The non-linear power spectrum is obtained from the linear one through the fitting formulae of Jain, Mo & White (1995) for $\Omega_{m} = 1$, and from those of Peacock & Dodds (1996) for the open and $\Lambda$-dominated models. These fitting formulae are based on the idea of relating the non-linear power spectrum at scale $k$ to the linear power spectrum at a larger scale $k_s$, where the relation $k(k_s)$ depends on the power spectrum itself (Hamilton et al. 1991). They have been calibrated from and tested extensively against N-body
simulations. For the linear power spectrum we take a CDM-like spectrum with shape parameter $\Gamma = 0.25$, which provides a good fit to observations. The transfer function for the initially scale-invariant power spectrum is taken from Bardeen et al. (1992).

3 CROSS-CORRELATION FUNCTION FOR CDM-LIKE POWER SPECTRA

For samples not overlapping in their redshift distributions, the cross-correlation $\omega_{gl}$ induced by lensing dominates the contribution to the cross-correlation function. The signal caused by intrinsic clustering is negligibly small, although if the redshift distributions overlap significantly it would swamp the lensing signal. In this section we present results on the cross-correlation of two galaxy samples with non-overlapping redshift distributions.

We computed $\omega_{gl}(\theta = 1$ arcmin) for two populations with mean redshifts of 0.3 and 1.5, using the redshift distribution $n(z)$ of equation (21). We found that the results differ by 5–10 per cent from those obtained using a delta-function redshift distribution. We therefore will use the simpler form of a delta-function distribution for computing $\omega_{gl}$. For calculating contributions resulting from intrinsic clustering, however, it is necessary to use the full $n(z)$ of equation (21). We do this in Section 4 below to estimate the relative error caused by mis-identifying background galaxies as foreground ones.

In Fig. 1 we plot $\omega_{gl}$ as a function of $\theta$ for a foreground galaxy sample at $z_1 = 0.3$ and a background sample at $z_2 = 1.5$, for the four cosmological models described in Section 2.1. We choose a bias of $b = 1/\sigma_8$, which is in agreement with large-scale galaxy clustering data; $\omega_{gl}$ is proportional to $b$. We assume a number-count slope of $s = 0.2$ for the background sample. For $s < 0.4$ the induced correlations are negative because $\omega_{gl} \propto (s - 0.4)$. This linear relation also makes it simple to scale our results to other values of $s$. A sample with a slope close to 0.2 may be obtained by defining colour-selected subsamples, using the fact that the number-count slope is a decreasing function of $V - I$ colour (Villumsen, Freudling & da Costa 1997). The price of selecting a subsample is a smaller number of galaxies and therefore larger Poisson errors, so in practice a careful cut suited to the available data would need to be made.

Fig. 1 shows that the typical cross-correlation signal expected on subarcmin scales is a few per cent. On angular scales larger than 1 arcmin, the signal drops to less than 1 per cent. For fixed $\sigma_8$, it is largest for the Einstein–de Sitter model and smallest for the cosmological constant model, at least on arcmin scales or smaller. If $\sigma_8$ is determined from cluster abundances, however, the cross-correlation on small scales is largest for the open model.

The dependence of the cross-correlation on the redshift of the background sample is shown in Fig. 2. We have plotted the cross-correlation function $\omega_{gl}(\theta = 1$ arcmin) for a foreground galaxy sample at $z_1 = 0.3$ as a function of the redshift $z_2$ of the background sample. Fig. 3 is the same as Fig. 2, but for $\theta = 0.2$ arcmin. These figures show the slow increase in the amplitude of the signal with $\langle z_2 \rangle$ above a redshift of 1. There is no significant variation in the shape of the curves among the four cosmological models. If the amplitudes were normalized to the same value at $z_2 \approx 1$, therefore, there would be very little difference between the curves at higher $z_2$.

The difference between the predictions of the four models shown in Figs 1–3 can be qualitatively understood as follows. The dominant dependence arises because of the factor of $\Omega_m$ outside the integral in equation (16) for $\omega_{gl}$.

Figure 1. The cross-correlation function $\omega_{gl}$ as a function of $\theta$ is shown for the four cosmological models. As the number count slope $s = 0.2$, magnification bias induces an anticorrelation between the foreground and background samples, hence the sign of $\omega_{gl}$ is negative. The differences between the four models are discussed in the text.
Taking the normalization of the power spectrum into account, this is reduced to $W_0$ for the cluster-abundance normalization of $\sigma_8$ and the bias relation $b = 1/\sigma_8$. This is because the expression for $\omega_{\gamma\gamma}$ depends explicitly on the factor $b\Omega_m \sigma_8^4$.

The line-of-sight integral in equation (16) further weakens the dependence on $\Omega_m$. In a low-$\Omega_m$ universe, the growth of perturbations is slowed down at late times. Hence, normalizing to present-day cluster abundances leads to a higher normalization at earlier times compared with the $\Omega_m = 1$ models. This in turn means that non-linear effects, which are significant on angular scales of 1 arcmin or less, become important earlier on and lead to a larger enhancement resulting from non-linear clustering by today. The non-linear enhancement is reinforced by a geometrical effect: for lower $\Omega_m$, and even more so for larger $\Omega$, (for given $\Omega_m$),

Figure 2. Cross-correlation function as a function of $z_1$ is shown for $\theta = 0.1$ arcmin for the four cosmological models. The negative correlations induced by magnification bias become stronger with $z_1$, but do not change much beyond a redshift of 1.

Figure 3. Cross-correlation function as a function of $z_1$ is shown for $\theta = 0.2$ arcmin for the four cosmological models. All parameters except $\theta$ are as in the previous figure.
the physical distance to a given redshift is larger. This leads to a larger lensing path-length and thus a further increase in the lensing signal.

The combination of all these effects is shown in Fig. 1. On scales well below an arcmin, the signal for the open model becomes comparable to that in the Einstein–de Sitter model. This is a result of the dominance of non-linear enhancement on these scales. The curve for the open model is also distinctly steeper than for the others. For \( \theta > 2 \) arcmin, however, there is no significant variation in the shape of the curves. For the \( \Lambda \)-dominated model, the effect of the growing mode is not as strong as for the open model. Thus on small scales, even though the geometric effect gives a stronger enhancement than for the open model, the net amplitude of \( \omega_g \) is smaller than for the open model.

Finally, note that \( \omega_g \) is proportional to this bias factor of the foreground sample. If this bias factor is larger or smaller than the value of \( b_{1/8} \) that we assumed to fit the large-scale structure data, \( \omega_g \) will be correspondingly altered.

### 4 Estimate of Errors

We consider two sources of error involved in an observational determination of the cross-correlation from two galaxy samples. The first potential error arises when a background galaxy is mis-identified as a foreground galaxy. In this case, the autocorrelation of the background galaxy sample will erroneously enter into the measured cross-correlation. Say \( \epsilon \) per cent of the background galaxy redshifts are sufficiently mis-estimated that they are taken to be part of the foreground sample. The observed cross-correlation is then given by

\[
\omega_m = \omega_g + \epsilon \frac{N_2}{N_1} \omega_{gg}(z_2),
\]

where the second term is the error caused by assigning galaxies to the wrong sample. It is proportional to the fraction of galaxies that is mis-identified and to \( \omega_{gg}(z_2) \), the autocorrelation function evaluated at the mean redshift of the background sample, for the redshift distribution of equation (21). We have calculated the autocorrelation at 1 arcmin and scaled it to 0.1 arcmin, assuming a power-law slope of \( -0.8 \); \( N_1 \) and \( N_2 \) are the numbers of galaxies in the foreground and background samples.

In Fig. 4 we plot the cross-correlation \( \omega_g(\theta=0.1 \text{ arcmin}) \) (lower set of matching curves) together with the measured cross-correlation function \( \omega_m \) (upper set of matching curves). Two values of \( \epsilon \), 1 and 5 per cent, are used (for samples of equal size). The results show that for \( z_2 > 1 \) the error is small for \( \epsilon = 1 \) per cent but not when \( \epsilon = 5 \) per cent. This sets an approximate standard required for using photometric redshifts or other possible methods to select the two galaxy samples.

A second source of error is the statistical uncertainty in estimating the angular correlations. Using a Poisson distribution to estimate the error in \( \omega \) provides a rough guide for the number of galaxies required to estimate the cross-correlation signal. The standard deviation \( \delta \omega(\theta) \) in the estimate of \( \omega(\theta) \) for a random distribution of objects is given by (Peebles 1980)

\[
\delta \omega(\theta)^2 = \frac{1}{N_1 N_2} \frac{\Omega}{\delta \Omega},
\]

where \( \Omega \) is the solid angle subtended by the survey area and \( \delta \Omega \) is the fraction in the bin used for angle \( \theta \). Note that \( \delta \omega^2 \) is just the inverse of the number of pairs in a given bin in \( \theta \).

For a sample with about \( 10^3 \) galaxies per 0.01 degree\(^2\) [e.g., as in each field of Woods & Fahlman (1997), whose sample reaches limiting magnitudes of \( V \sim 25 \), \( R \sim 25 \), \( I \sim 24 \)], the above estimate gives \( \delta \omega \approx 4 \times 10^{-1} \) if 10 bins in \( \theta \) are used. Thus in excess of about \( 10^3 \) galaxies each in the

---

**Figure 4.** True cross-correlation function \( \omega_g \) (lower set of matching curves) and measured cross-correlation function \( \omega_m \) (upper set of matching curves) when \( \epsilon \) per cent of the background galaxies are mis-identified as foreground ones, as a function of \( z_2 \), for \( \theta = 0.1 \) arcmin, \( z_1 = 0.3 \) and the four cosmological models.
foreground and background sample would be required for a detection of a \( \gtrsim 1 \) per cent cross-correlation signal with a high level of significance.

5 CONCLUSIONS

We have presented results for the cross-correlation of two galaxy samples with different redshift distributions. The signal is dominated by the effect of magnification bias resulting from weak lensing. To ensure that the contribution from gravitational clustering of the galaxies is negligible, we have assumed that the redshift distributions of the two samples do not overlap. With the use of photometric redshifts (e.g. Connolly et al. 1995; Sawicki, Lin & Yee 1997), it is possible to obtain deep galaxy samples that can be separated into subsamples with the desired redshift distributions. If only limiting magnitudes are used to create two subsamples, there will be a significant overlap and the interpretation of the signal is not as clear. Theoretical predictions can, however, be made for the expected signal assuming a redshift distribution.

The results shown in Figs 1–3 demonstrate that most models predict a signal of 1–4 per cent for the cross-correlation function. These numbers apply for a background sample with a mean redshift \( z \approx 1 \) and a number count slope of 0.2, on angular scales from a few arcsec to 1 arcmin. As argued in Section 4, the measurement of such a signal appears feasible in the near future.

The cross-correlation function is a measure of the projected dark matter power spectrum. For a given spectrum, the variation with angle on small scales is largest for open cosmological models and thus provides a probe of \( W \). Otherwise, for a given cosmology, it can constrain the bias factor of galaxies. Ideally, by combining the cross-correlation with other lensing measurements that are independent of bias, such as the ellipticity autocorrelation function, constraints on the cosmological model as well as on the biasing of galaxies at intermediate redshifts can be obtained.

ACKNOWLEDGMENTS

We are grateful to Simon White for many helpful suggestions. We would like to thank Matthias Bartelmann, Andrew Connolly, Peter Schneider, Alex Szalay and Jens Villumsen for stimulating discussions. The paper also benefited from helpful comments by the referee, Andrew Taylor.

REFERENCES

Bardeen J. M., Bond J. R., Kaiser N., Szalay A. S., 1986, ApJ, 304, 15
Bartelmann M., 1995, A&A, 298, 661
Benitez N., Martínez-Gonzales E. A., 1996, astro-ph/9609183
Bernardeau F., van Waerbeke L., Mellier Y., 1997, A&A, 322, 1
Blandford R., Saust A., Brainard T., Villumsen J., 1991, MNRAS, 251, 600
Broadhurst T., Taylor A., Peacock J., 1995, ApJ, 438, 49
Carroll J., Press W., Turner E., 1992, ARA&A, 30, 499
Connolly A. J., Scabia I., Szalay A. S., Koo D. C., Munn J. A., 1995, AJ, 110, 2655
Dolag K., Bartelmann M., 1997, astro-ph/9704217
Eke V. R., Cole S., Frenk C. S., 1996, MNRAS, 282, 263
Gunn J. E., 1987, ApJ, 147, 1
Hamilton A. J. S., Kumar P., Lu E., Mather A. A., 1991, ApJ, 374, L1
Jain B., Seljak U., 1997, ApJ, 484, 560
Jain B., M o H., White S. D. M., 1995, MNRAS, 276, L25
Kaiser N., 1992, ApJ, 388, 272
Kaiser N., 1996, astro-ph/9610120
Milgalda-Escude J., 1991, ApJ, 380, 1
Mo H., Jing Y.-P., Börner G., 1996, astro-ph/9607143
Mobasher B., Rowan-Robinson M., Georgakakis A., Eaton N., 1996, MNRAS, 282, 7
Moessner R., Jain B., Villumsen J., 1997, astro-ph/9708271
Peacock J., Doods A., 1996, MNRAS, 282, 119
Peebles P. J. E., 1980, The Large-Scale Structure of the Universe. Princeton Univ. Press, Princeton NJ
Pen U., 1996, astro-ph/9610147
Sanz J. L., Martínez-González E. A., Benítez N., 1996, astro-ph/9706278
Sawicki M. J., Lin H., Yee H. K. C., 1997, AJ, 113, 1
Schneider P., 1997, astro-ph/9708269
Viana P. T. P., Liddle A. R., 1996, MNRAS, 281, 323
Villumsen J., 1996, MNRAS, 281, 369
Villumsen J., Freudling W., da Costa L., 1997, ApJ, 481, 578
White S. D. M., Efstathiou G., Frenk C. S., 1993, MNRAS, 262, 1023
Woods D., Fakhlan G. G., 1997, astro-ph/9707127

APPENDIX A: NOTATION

Following the notation of Jain & Seljak (1997), we introduce the unperturbed metric

\[ ds^2 = a^2(\tau) \left[-dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)\right], \]  

(A1)

with \( \tau \) being conformal time and \( \chi \) the radial comoving distance. \( \chi_0 \) is used to denote the distance to the horizon. The comoving angular diameter distance \( r(\chi) \) is

\[ r(\chi) = \sin \chi, \quad K > 0, \]

\[ r(\chi) = r_{\text{cutoff}}, \quad K = 0, \]

\[ r(\chi) = (1 - K)^{-1/2} \sinh(1 - K)^{1/2} \chi, \quad K < 0, \]

(A2)

where \( K \) is the spatial curvature given by \( K = -H^2(1 - \Omega_m - \Omega_\Lambda) \), with \( H_0 \) being the Hubble parameter today.

With \( W(\chi) \) denoting the radial distribution of galaxies in the sample, the radial weight function \( g(\chi) \) is given by

\[ g(\chi) = r(\chi) \left(\int_{r(\chi')}^{r(\chi)} d\chi' W(\chi')\right) \]

(A3)

For a delta-function distribution of galaxies, \( W(\chi') = \delta(\chi' - \chi_0) \), and \( g(\chi) \) reduces to

\[ g(\chi) = r(\chi) r(\chi_0 - \chi_0) / r(\chi_0). \]

© 1998 RAS, MNRAS 294, L18–L24