Quantum States of Black Holes

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Abstract

I review the recent progress in providing a statistical foundation for black hole thermodynamics. In the context of string theory, one can now identify and count quantum states associated with black holes. One can also compute the analog of Hawking radiation (in a certain low energy regime) in a manifestly unitary way. Both extremal and nonextremal black holes are considered, including the Schwarzschild solution. Some implications of conjectured non-perturbative string “duality transformations” for the description of black hole states are also discussed.
1 Introduction

When I was a graduate student at the University of Chicago in the late 1970’s, Chandra often talked about the surprises he found in his study of black holes [1]: the separation of the Dirac equation in the Kerr metric, the equality of reflection and transmission coefficients for different types of perturbations, etc. Given his well known fascination with black holes, I am sure Chandra would be interested in the unexpected results about black holes described below, which have been discovered in the past year or two.

It is by now well known that black holes have thermodynamic properties (for a recent review see [2]). In particular, they have an entropy equal to one quarter of the area of their event horizon in Planck units. This is an enormous entropy. One way to see this is to consider thermal radiation (which, of course, has the highest entropy of ordinary matter) and ask what entropy it would have when it forms a black hole. In Planck units $G = c = \hbar = k = 1$, a ball of radiation at temperature $T$ and radius $R$ has mass $M \sim T^4 R^3$, and entropy $S \sim T^3 R^3$. The radiation will form a black hole when $R \sim M$ which implies $T \sim M^{-1/2}$ and hence $S \sim M^{3/2}$. In contrast, the entropy of the resulting black hole is $S_{bh} \sim M^2$. So any black hole with $M \gg 1$, i.e. mass much larger than the Planck mass, has an entropy much larger than the entropy of the thermal radiation that formed it. In terms of a more fundamental description, the entropy should be a measure of the number of underlying quantum states. The problem, which has puzzled physicists for more than twenty years, is to find a more fundamental theory which contains the quantum states predicted by black hole thermodynamics.

An answer has recently been provided in the context of string theory. This is a new physical theory which came into prominence in the mid 1980’s. This theory has been hailed as a “theory of everything” and scorned as a “typical end of the century phenomenon” [3]. I think the first view is much closer to the truth. String theory is a promising candidate for a consistent quantum theory of gravity and a unified theory of all forces and particles. It has not been proven that it can achieve these goals, but there is increasing evidence (especially over the past few years [4]) that it will.
To understand the basic idea behind the recent explanation of black hole entropy, one only needs three facts about string theory. The first is that when one quantizes a string in flat spacetime, there are an infinite tower of massive states. For every integer $N$ there are states with $M^2 \sim N/l_s^2$ where $l_s$ is a new length scale in the theory set by the string tension. These states are highly degenerate, and one can show that the number of string states at excitation level $N \gg 1$ is $e^{S_s}$ where

$$S_s \sim \sqrt{N}$$  \hfill (1.1)

i.e. the string entropy is proportional to the mass in string units. One can understand this in terms of a simple model of the string as a random walk with step size $l_s$. As a result of the string tension, the energy in the string after $n$ steps is proportional to its length: $E \sim n/l_s$. If one can move in $k$ possible directions at each step, the total number of configurations is $k^n$, so the entropy for large $n$ is proportional to $n$, i.e. proportional to the energy.

The second fact is that string interactions are governed by a string coupling constant $g$ (which is determined by a scalar field called the dilaton). Newton’s constant $G$ is related to $g$ and the string length $l_s$ by $G \sim g^2 l_s^2$ in four spacetime dimensions. We will sometimes use string units where $l_s = 1$, and sometimes use Planck units where $G = l_p^2 = 1$. It is important to distinguish them, especially when $g$ changes. Since $g$ is in fact determined by a dynamical field, one can imagine that it changes as a result of a physical process, e.g. a wave of dilaton passing by. However, it will often be convenient to assume the dilaton is constant and treat $g$ as just a parameter in the theory. In general, physical properties of a state can change when $g$ is varied. But we will see that in some cases, one can argue that certain properties remain unchanged.

The third fact is that the classical spacetime metric is well defined in string theory only when the curvature is less than the string scale $1/l_s^2$. This follows from the fact that fundamentally, the metric is unified with all the other modes of the string. This is easily seen in perturbation theory where

2This model is surprisingly accurate: The typical configuration of the string in a highly excited state is indeed a random walk.
the graviton is just one of the massless excitations of the string. When the curvature is small compared to $1/l_s^2$, one can integrate out the massive modes and obtain an effective low energy equation of motion which takes the form of Einstein’s equation with an infinite number of correction terms consisting of higher powers of the curvature multiplied by powers of $l_s$. When curvatures approach the string scale, this low energy approximation breaks down.

In the next section we give a general argument which shows that string theory can explain the entropy of black holes. This argument applies to essentially all black holes, and reproduces the correct dependence on the mass and charges, but is not precise enough to check the numerical coefficient. In section three we show that for certain black holes, even the numerical coefficient in the entropy can be computed and shown to agree. We also describe some recent calculations showing that the spectrum of Hawking radiation from black holes can be reproduced by string theory. In section four, we consider the effect of conjectured nonperturbative “duality” symmetries on the description of black hole states. Section five contains a discussion of some open issues.

2 Correspondence between black holes and strings

2.1 Schwarzschild Black Holes

A few years ago, Susskind [7] suggested that there should be a one to one correspondence between strings and black holes. The idea was the following: Consider a highly excited string state at level $N$ and zero string coupling. As we mentioned above, a typical configuration of such a string is a random walk with a length proportional to its energy $L \sim N^{1/2}l_s$ and hence a radius $R \sim N^{1/4}l_s$. Now imagine increasing the string coupling $g$ and recall that $G \sim g^2l_s^2$. Two effects take place. First, the gravitational attraction of one part of the string on the other causes the string size to decrease. Second, since $G$ increases, the gravitational field produced by the string becomes stronger and the effective Schwarzschild radius $GM$ increases in string units.
Clearly, for a sufficiently large value of the coupling, the string forms a black hole.

Conversely, suppose one starts with a black hole and decreases the string coupling. Then the Schwarzschild radius shrinks in string units and eventually becomes smaller than the string scale. At this point the metric is approximately flat except for a small region where it is no longer well defined. Susskind suggested that the black hole becomes an excited string state. Further evidence was presented in [8].

When I first heard this, I didn’t believe it. The first half of the argument sounded plausible enough, but the second half seemed to contradict the well known fact that the string entropy is proportional to the mass while the black hole entropy is proportional to the mass squared. It turns out that there is a simple resolution of this apparent contradiction [9]. Consider the familiar Schwarzschild black hole

$$ds^2 = -\left(1 - \frac{r_0}{r}\right)dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1}dr^2 + r^2d\Omega.$$  \hspace{1cm} (2.1)

The mass of the black hole is $M_{bh} = r_0/2G$. We want to equate this with the mass of a string state at excitation level $N$ which is $M_s^2 \sim N/l_s^2$ at zero string coupling. Now imagine increasing the string coupling, keeping the state fixed. Clearly, $M_s$ is constant in string units where $l_s$ is held fixed and Newton’s constant is changing. The analog of keeping the state fixed for the black hole is to keep the entropy fixed. Thus $M_{bh}$ is constant in Planck units where $G$ is fixed and the string length changes. (The black hole does not know about $g$ or $l_s$ separately, but only the combination $G \sim g^2l_s^2$.) In other words, the ratio $M_s/M_{bh}$ depends on $g$ and the masses cannot be equal for all values of the coupling. If we want to equate the masses, we have to decide at what value of the coupling they should be equal. Clearly the natural choice is to let $g$ be the value at which the string forms a black hole.

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3There is a small correction to $M_s$ due to gravitational binding energy, but this is negligible compared to the effect discussed here.

4For a system with a rapidly growing density of states, a narrow band of states can be labeled by its entropy. The argument below can be reinterpreted as saying that if you insist on keeping the entropy constant through the transition from strings to black holes, the mass changes by at most a factor of order unity.
or vice versa. If we start with a black hole and decrease the coupling the black hole description will remain valid until the horizon is of order the string scale. Setting the masses equal when $r_0 \sim l_s$ yields

$$M_{bh}^2 \sim \frac{l_s^2}{G^2} \sim \frac{N}{l_s^2} \quad (2.2)$$

The black hole entropy is then

$$S_{bh} \sim \frac{r_0^2}{G} \sim \frac{l_s^2}{G} \sim \sqrt{N} \quad (2.3)$$

So the Bekenstein-Hawking entropy is comparable to the string entropy! One cannot compare the numerical coefficient in the entropy formulas this way, since that would depend on exactly when the transition occurred. But this clearly shows that strings have enough states to reproduce the entropy of black holes. It follows from (2.3) that the transition from a string state to a black hole occurs when the string coupling is still rather small: $g \sim 1/N^{1/4} \ll 1$ for large $N$.

Let me emphasize that the fact that $r_0 \sim l_s$ does not mean that the black hole must be small. In fact, since the entropy is $S_{bh} \sim \sqrt{N}$, the Schwarzschild radius in Planck units is $r_0 \sim N^{1/4}$. It is probably more intuitive to describe the transition between string states and black holes in Planck units where $G$ is fixed. Suppose we start with a solar mass black hole. As we decrease $g$, the string length $l_s$ increases, i.e., the string tension decreases. Eventually, it is of order a kilometer and comparable to the horizon size. At this point, the black hole metric is no longer well defined in string theory, and the system is better described as a highly excited string state.

Starting at small coupling, the transition from a string to a black hole is analogous to a neutron star which accretes matter. In Planck units, the string tension increases like $g^2$. Thus for fixed excitation $N$, the mass of a string state increases as $g$ increases until it forms a black hole. For both the neutron star and the string, there is not a sudden change in mass when the black hole forms. However there is one important difference: Neutron stars have much less entropy than the resulting black hole, while strings do not.
The above counting of string states does not include the center of mass degree of freedom. This is appropriate since the entropy of a black hole does not include a contribution from its location in spacetime. It also does not include the possibility that a black hole is described by several strings at weak coupling. Since the string entropy is proportional to its energy, two strings with energy $E/2$ have the same entropy as one string with energy $E$. However, it turns out that the ratio of the total entropy of all multistring states to the entropy of a single string goes to one for large $E$ \[6\]. So the entropy can be approximated by a single string.

We have seen that as one increases $g$, a perturbative string state evolves into a black hole. In addition to the string-black hole transition, there seems to be an implicit quantum-classical transition as well. This can be made explicit as follows. Recall that for a given excitation $N$, the transition between the black hole and string regimes occurs when $gN^{1/4} \sim 1$. For $gN^{1/4} > 1$ the Schwarzschild radius is larger than the string scale and the black hole description is valid. If $gN^{1/4} < 1$, the effective Schwarzschild radius is less than the string scale and perturbative string description is valid. In either case, the classical limit is $g \to 0$, $N \to \infty$ with $gN^{1/4}$ fixed. In this limit, the Planck length $l_p \sim gl_s$ goes to zero, and the entropy $S \sim \sqrt{N}$ diverges, which agrees with the fact that a purely classical black hole indeed has infinite entropy. Quantum corrections to the black hole can be computed in terms of a string loop expansion. The classical limit on the perturbative string theory side does not yield a single classical string. Since string theory is a “second quantized” theory of strings, the states of the first quantized string describe the classical fields of the final theory. So the $g \to 0$ limit is essentially a classical field theory with an infinite number of fields. From this viewpoint, it is interesting to note that the black hole entropy is related to the number of fields in the theory with a given mass. (Since many of these fields have large spin, one is actually counting the number of components of these massive fields.) In the second quantized theory, the first quantized string states simply arise as states in the “one string sector” of the usual perturbative Fock space. In the following, we will often keep $N$ fixed and vary $g$. Thus, the black hole will be referred to as the “strong coupling regime”.

7
The agreement between black hole entropy and the number of string states extends to Schwarzschild black holes in higher dimensions. The easiest way to see this is to note that the entropy of a Schwarzschild black hole in any dimension can be expressed

$$S_{bh} \sim r_0 M_{bh} \quad (2.4)$$

Clearly, when $r_0 \sim l_s$, the entropy is proportional to the mass in string units, exactly like a free string.

### 2.2 A Charged Black Hole

The agreement between the black hole entropy and the counting of string states also extends to charged black holes. The simplest case to consider is a Kaluza-Klein black hole where the charge comes from momentum in an internal direction \[10\]. Given a five dimensional metric which is independent of $x_5$, define a four dimensional metric $g_{\mu\nu}$, gauge field $A_\mu$ and scalar $\chi$ by

$$ds^2 = e^{-4\chi/\sqrt{3}}(dx_5 + 2A_\mu dx^\mu)^2 + e^{2\chi/\sqrt{3}}g_{\mu\nu}dx^\mu dx^\nu \quad (2.5)$$

Then the five dimensional Einstein action is equivalent to

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [R - 2(\nabla \chi)^2 - e^{-2\sqrt{3}\chi}F_{\mu\nu}F^{\mu\nu}] \quad (2.6)$$

Charged black hole solutions can be obtained by starting with the product of the four dimensional Schwarzschild metric (2.1) and a line, boosting along the line, periodically identifying $x_5$ and reducing to four dimensions using (2.5). The result is

$$ds^2 = -\Delta^{-1/2} \left(1 - \frac{r_0}{r}\right) dt^2 + \Delta^{1/2} \left[ \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega \right] \quad (2.7)$$

$$A_t = -\frac{r_0 \sinh 2\gamma}{4r\Delta}, \quad e^{-4\chi/\sqrt{3}} = \Delta$$

where

$$\Delta = 1 + \frac{r_0 \sinh^2 \gamma}{r}$$
The ADM mass and electric charge are

\[
M_{bh} = \frac{r_0}{8G} (3 + \cosh 2\gamma) \quad (2.8)
\]

\[
Q = \frac{r_0}{4G} \sinh 2\gamma \quad (2.9)
\]

The black hole entropy is

\[
S_{bh} = \frac{\pi r_0^2}{G} \cosh \gamma \quad (2.10)
\]

We want to show that this entropy is reproduced by counting string states. The electric charge on the black hole is twice the total momentum in the internal direction. A string in five dimensions with momentum \( P \) in the fifth direction has entropy \( S_s \sim \sqrt{N} \) where now

\[
M_s^2 - P^2 \sim \frac{N}{l_s^2} \quad (2.11)
\]

and \( M_s \) is the four dimensional mass of the string. We want to match the black hole solution to the string solution so we set \( P = Q/2 \). We also set \( M_{bh} = M_s \) when the curvature is of order the string scale. It is the curvature of the original five dimensional solution which is important here so the matching occurs when \( r_0 \sim l_s \). The result is that

\[
\frac{N}{l_s^2} \sim M_{bh}^2 - \frac{Q^2}{4} \sim \frac{l_s^2}{G^2} [5 + 3 \cosh 2\gamma] \quad (2.12)
\]

Comparing with (2.10) we see that up to a (\( \gamma \) dependent) factor of order unity,

\[
S_{bh} \sim \sqrt{N} \sim S_s \quad (2.13)
\]

Thus the two entropies agree for all values of \( \gamma \), i.e. for all values of \( Q/M \).

### 2.3 Correspondence Principle

For the Kaluza-Klein black hole, we assumed there was an internal direction in spacetime. String theory actually predicts many internal directions.
(In the above examples, the extra dimensions were taken to be a trivial torus.) These give rise to a large number of different types of charges in the effective four dimensional theory. In addition to internal momentum, one has charges associated with strings winding around each compact direction. There are also higher dimensional extended objects called D-branes (which will be discussed in the next section) which look like localized charged particles when wrapped around the internal space. Black holes can carry any of these charges. In almost all cases, the size of the black hole in string units becomes smaller when $g$ is decreased.\footnote{The one exception is black holes carrying Kaluza-Klein monopole charge, or another magnetic charge dual to string winding number. Approaches to understanding the entropy of black holes with these charges have been discussed in \cite{11, 12, 13}.} The entropy of all of these black holes can be understood by matching onto a weak coupling description when the black hole is of order the string scale. However, there is one additional fact about string theory which must be taken into account. For many charged black holes, the dilaton field $\phi$ is not constant. In this case, strings couple to a metric $g^S_{\mu\nu}$ which is conformally related to the metric with the standard Einstein action $g^E_{\mu\nu}$. In $D$ spacetime dimensions, one has $g^S_{\mu\nu} = e^{4\phi/D-2} g^E_{\mu\nu}$. Both of these metrics play a role in our discussion. The spacetime metric ceases to be well defined when the curvature of the string metric is of order the string scale, while the black hole entropy is, of course, related to the area of the event horizon in the standard Einstein metric.

The general relation between black holes and strings can now be stated in terms of the following \textbf{correspondence principle}:

(i) When the curvature at the horizon of a black hole (in the string metric) becomes greater than the string scale, the typical black hole state becomes a typical state of strings and D-branes with the same charges and angular momentum.

(ii) The mass changes by at most a factor of order unity during the transition.

An appropriate measure of the curvature near the horizon are the curvature invariants, since these control the higher order string corrections to the field equations. (The physical tidal forces felt by an infalling observer can be much
larger than the curvature invariants would suggest. It has been shown that for a large class of charged black holes, this correspondence principle correctly reproduces the Bekenstein-Hawking entropy up to factors of order unity. The two examples discussed earlier are easily seen to be special cases of this principle.

As a final example, consider the following metric describing a five dimensional black string:

\[
\begin{align*}
\text{ds}^2 &= F \left[ - \left( 1 - \frac{r_0}{r} \right) dt^2 + dz^2 \right] + \left( 1 - \frac{r_0}{r} \right)^{-1} dr^2 + r^2 d\Omega \tag{2.14}
\end{align*}
\]

where

\[
F^{-1} = 1 + \frac{r_0 \sinh^2 \gamma_1}{r}. \tag{2.15}
\]

This is not the Einstein metric, but the string metric \( g^{S}_{\mu\nu} \). The dilaton is \( e^{2\phi} = F \). The extremal limit, \( r_0 \to 0, \gamma_1 \to \infty \) with \( r_0 \sinh^2 \gamma_1 \) fixed, describes the strong coupling geometry of an unexcited string wrapped around an internal direction. When this solution was discovered, it was suggested that the nonextremal black string should describe the strong coupling limit of the excited states of the wound string. We now see that this is indeed the case. In fact, (2.14) reduces to the same four dimensional black hole that we discussed earlier (2.7), and the counting of states for a string with nonzero winding number is again given by (2.11) with \( P \) replaced by the energy in the winding mode. So the number of string states with nonzero winding number agrees with the black string entropy.

3 Precise agreements between black holes and strings

3.1 Supersymmetric Black Holes

Using the correspondence principle to understand black hole entropy has the virtue that it can be applied to essentially any black hole, but it is not yet able to compare the precise coefficients in the entropy formulas. For a special class of black holes, as one decreases the string coupling, one has more control
over the transition to a perturbative string state and even the coefficients in the entropy formulas can be compared. These more precise calculations use the fact that string theory is supersymmetric. Its low energy limit is a supergravity theory that admits black hole solutions which are invariant under some supersymmetry transformations. These supersymmetric solutions are extremally charged black holes with mass and charge related by $M = cQ$ for some constant $c$. In the limit of weak coupling, the mass and charge of all perturbative string states satisfy inequalities of the form $M \geq cQ$. States with $M = cQ$ are called BPS states, and have the special property that their mass does not receive any quantum corrections.

One can thus start with a supersymmetric black hole, and decrease the string coupling. One then counts the number of BPS states at weak coupling with the same charge as the black hole. Notice that in this case, the issue of when to match the mass of the black hole to the perturbative string state never arises: In both regimes the mass is completely fixed by the charge. The remarkable result is that the number of BPS states at weak coupling turns out to be precisely the exponential of the Bekenstein-Hawking entropy of the black hole at strong coupling \[16\]. (For a comprehensive review, see \[17\].)

This sounds so easy, one might wonder why this calculation wasn’t done years ago. There are two main reasons. The first is that most supersymmetric black holes are not really black holes at all: They have zero horizon area. The problem is that supergravity theories have scalars which couple to the gauge fields. Nonextremal charged black hole solutions exist with familiar properties, but as one approaches the extremal limit, the scalars become large at the horizon causing it to shrink to zero size. To obtain a supersymmetric black hole with nonzero horizon area, one needs to include several charges to stabilize the scalars. This results in a second problem, since some of these charges are not carried by fundamental strings. Instead, they are carried by nonperturbative solitons. Thus, rather than simply counting states of a string at weak coupling, one must quantize the solitons and count bound states of the solitons and strings, which is much more difficult. These problems were

\[6\] This is only true for certain charges in extended supersymmetry which appear in the supersymmetry algebra.
recently solved when Polchinski discovered a new representation for these solitons called “D-branes” \[18\].

A D-brane has a mass proportional to $1/g$, so at weak coupling they are very massive (and hence nonperturbative). But since Newton’s constant $G \sim g^2$, the gravitational field produced by the D-brane, which is proportional to $GM$ goes to zero as $g \rightarrow 0$. Thus there exists a flat space description of these nonperturbative states. This is obtained by adding to one’s theory of closed strings, a set of open strings where the endpoints of the open strings are constrained to live on a particular surface. These surfaces can have any dimension, and can be viewed as generalizations of membranes. Since the endpoints of the open string satisfy Dirichlet boundary conditions in the directions normal to the surface, the surface is called a D-brane.

The excited states of a D-brane are described by quantizing the open strings. At low energies, only the massless modes contribute. The massless states of an open string include some scalars which describe the fluctuations of the brane in the surrounding spacetime, and a $U(1)$ gauge field on the brane. When two D-branes come together this is enhanced to a $U(2)$ gauge field. The extra massless states needed to change $U(1)^2$ into $U(2)$ arise from open strings that are stretched between the two D-branes. These states have a mass proportional to the separation of the branes. When this separation goes to zero, they become massless and combine with the $U(1)$ gauge fields on each brane to produce a $U(2)$ gauge field. Similarly, when $m$ D-branes come together one obtains a $U(m)$ gauge theory. In terms of the low energy gauge theory on the brane, the fact that $U(m)$ reduces to $U(1)^m$ when the branes are slightly separated is described by the usual Higgs mechanism.

There is a fascinating story which is emerging from the study of D-branes at very short distances. I do not have time to develop it in detail (and it is not required to understand the recent progress in black holes) but I cannot resist mentioning it. It appears that D-branes are able to probe distances much shorter than the string scale \[19\]. Since the graviton is a mode of a closed string, the usual metric description cannot apply at these short distances. Consider $p$-dimensional D-branes. When $m$ of them come together, their low-energy dynamics is governed by the reduction of a ten dimensional $U(m)$
Yang-Mills theory to \( p \) dimensions. Thus, in addition to the \( U(m) \) gauge field on the brane there are matter fields \( X_i, i = p + 1, \ldots, 9 \) which are \( m \times m \) hermitian matrices with a potential \( V \sim [X_i, X_j]^2 \). For the ground states, \( [X_i, X_j] = 0 \), and one can simultaneously diagonalize these matrices. When \( X_i \neq 0 \) the symmetry is broken to \( U(1)^m \) and the diagonal entries can be interpreted as the positions of the (now separated) branes. Therefore, there is a one-to-one relation between the position of the D-brane in spacetime and the moduli space of ground states of the gauge theory on the D-brane. For slowly moving D-branes there is a natural metric on this moduli space which controls the physics. In some cases, this metric turns out to be identical to the metric on spacetime measured at much larger distances. Perhaps the most intriguing observation is that the variables in the gauge theory which correspond to position in spacetime commute only for the ground states, but in general are noncommuting! This suggests that at very short distances, spacetime may be described by a form of noncommutative geometry \[20\].

Returning to our discussion of black holes, a single D-brane with \( p \) spatial dimensions carries a charge with respect to a \( p + 2 \) form field strength \( F \). In other words, the charge is defined by integrating \( \ast F \) over a sphere encircling the \( p + 1 \) dimensional world volume of the brane. In fact, a D-brane has the minimum possible mass for this type of charge and is (in a well-defined sense) a BPS state. Now suppose one compactifies spacetime on a \( p \) dimensional internal space, and wraps the D-brane around this space. From the standpoint of the reduced theory, the D-brane appears to be a point-like object carrying a charge associated with a usual two-form Maxwell field. Since a single D-brane carries only one type of charge, its strong coupling limit does not have nonzero horizon area. However, bound states of several different types of D-branes yield black holes with regular horizons.

It turns out that supersymmetric black holes with nonzero horizon area can only exist in four and five dimensions. In five dimensions one needs three different types of charges while in four dimensions, one needs four.\(^7\) The first

\(^7\) More precisely, this is true in \( N = 4 \) or \( N = 8 \) supergravity. In \( N = 2 \) supergravity, there are supersymmetric black holes with one charge and nonzero horizon area \[21\]. However, this theory does not arise when compactifying string theory on a torus. One needs more complicated internal spaces \[22\].
precise calculation of black hole entropy was performed by Strominger and Vafa [16] for an extreme nonrotating five dimensional black hole. This was quickly generalized to include rotation [23] and four dimensional extremal black holes [24, 25] (as well as small deviations from extremality which will be discussed shortly). Even the entropy of solutions which depend on arbitrary functions has been reproduced by counting states of D-branes [26].

As perhaps the most interesting example of the above results, we consider the familiar extreme Reissner-Nordström solution:

\[ ds^2 = - \left( 1 - \frac{GM}{r} \right)^2 dt^2 + \left( 1 - \frac{GM}{r} \right)^{-2} dr^2 + r^2 d\Omega . \]  

(3.1)

As explained above, the counting of states for this black hole is rather complicated since we need to consider four different charges. In other words, one views (3.1) as a composite of four different fundamental objects. There is a more general solution (given below) where these charges are all independent parameters, which reduces to (3.1) in a certain limit. Many of these charges are represented in weak coupling by D-branes wrapped around internal directions. There are, in fact, several different possible choices for the charges which all include (3.1) as a special case.

One way to obtain the Reissner-Nordström solution is the following [24]. One starts at weak coupling with ten dimensional flat spacetime and compactifies six dimensions on a torus, which is convenient to think of as the product of a four torus with volume \((2\pi)^4 V\) and two circles with radii \(R\) and \(\tilde{R}\). One then takes \(Q_6\) D-sixbranes wrapped around the six torus. One adds \(Q_2\) D-two branes wrapped around the two circles. One adds \(Q_5\) five branes wrapped around the four torus and \(R\). All of these branes lie over the same point in the three noncompact spatial directions, so this configuration describes a localized object in four spacetime dimensions. Notice that the intersection of these branes is the circle \(R\). When \(R\) is large the low energy excitations of these branes are described by open strings moving along this circle. It turns out that these strings have \(4Q_2Q_5Q_6\) massless bosonic degrees of freedom and an equal number of fermionic degrees of freedom. (This is

\(\text{In this case there is a small puzzle since although the horizon area is well defined, the curvature diverges there [27].}\)
obtained by analyzing the induced gauge theory on the branes.) Momentum along the circle is quantized in units of $1/R$. The number of states with right-moving momentum $n/R$ (and no left-moving momentum) is $e^S$ where $S$ is given by the general formula for a one dimensional gas $S = 2\pi \sqrt{cn/6}$. The constant $c$ receives a contribution of one for every bosonic field and one half for every fermionic field. In our case, we have $c = 6Q_2Q_5Q_6$ and hence

$$S = 2\pi \sqrt{Q_2Q_5Q_6n} \quad (3.2)$$

I should emphasize that this calculation is done in flat spacetime. There is no event horizon present. One is simply counting states of strings on D-branes. Notice that the entropy depends only on the integer charges, and not on the continuous parameters $V, R, \tilde{R}$.

The strong coupling description of this system is found by solving the ten dimensional supergravity equations with these charges. After reducing along the six torus, the four dimensional (Einstein) metric becomes\[29\]

$$ds^2 = -f^{-1/2}(r) \left(1 - \frac{r_0}{r}\right) dt^2 + f^{1/2}(r) \left[\left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega\right],$$

where

$$f(r) = \left(1 + \frac{r_0 \sinh^2 \alpha_2}{r}\right) \left(1 + \frac{r_0 \sinh^2 \alpha_5}{r}\right) \left(1 + \frac{r_0 \sinh^2 \alpha_6}{r}\right) \left(1 + \frac{r_0 \sinh^2 \alpha_p}{r}\right). \quad (3.3)$$

This metric is parameterized by the five independent quantities $\alpha_2, \alpha_5, \alpha_6, \alpha_p$ and $r_0$. They are related to the integer charges by

$$Q_2 = \frac{r_0 V}{g} \sinh 2\alpha_2,$$

$$Q_5 = r_0 \tilde{R} \sinh 2\alpha_5,$$

$$Q_6 = \frac{r_0}{g} \sinh 2\alpha_6,$$

$$n = \frac{r_0 V R^2 \tilde{R}}{g^2} \sinh 2\alpha_p, \quad (3.4)$$

\[9\text{We follow the discussion in [28].}\]
where we have set $l_s = 1$. The event horizon lies at $r = r_0$. The special case $\alpha_2 = \alpha_5 = \alpha_6 = \alpha_p$ corresponds to the Reissner-Nordström metric. Notice that if we set all charges except the momentum $n$ to zero, the metric (3.3) reduces to the Kaluza-Klein solution (2.7) as it should. Furthermore, since all charges enter (3.3) symmetrically, the four dimensional metric generated by any one of these charges is the Kaluza-Klein black hole. The precise relation between the four-dimensional Newton constant and the string coupling is $G = g^2/(8VR\tilde{R})$ in string units ($l_s = 1$). The ADM mass of the solution is

$$M = \frac{r_0 V R \tilde{R}}{g^2} (\cosh 2\alpha_2 + \cosh 2\alpha_5 + \cosh 2\alpha_6 + \cosh 2\alpha_p) \quad (3.5)$$

and the Bekenstein-Hawking entropy is

$$S_{bh} = \frac{A}{4G} = \frac{8\pi r_0^2 VR\tilde{R}}{g^2} \cosh \alpha_2 \cosh \alpha_5 \cosh \alpha_6 \cosh \alpha_p . \quad (3.6)$$

The extremal limit corresponds to $r_0 \to 0$, $\alpha_i \to \pm \infty$ with $Q_i$ fixed. In this limit, the entropy becomes

$$S_{bh} = 2\pi \sqrt{Q_2 Q_5 Q_6 n} \quad (3.7)$$

in precise agreement with the string result (3.2)! The Reissner-Nordström solution is clearly just a special case of this.

Even though we have not needed the correspondence principle to reproduce the black hole entropy, one can still use it to estimate when the black hole description breaks down, and must be replaced by the D-brane description. In string units, the area of the event horizon is approximately $g^2 \sqrt{Q_2 Q_5 Q_6 n}$. The curvature at the horizon will be of order the string scale when this is of order one. If we assume all the charges are comparable, the transition occurs when $gQ \sim 1$.

In the extremal limit, the ADM mass becomes

$$M = \frac{R \tilde{R}}{g} Q_2 + \frac{RV}{g^2} Q_5 + \frac{R \tilde{R} V}{g} Q_6 + \frac{n}{R} \quad (3.8)$$

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which is easily seen to be just the sum of the energy of each of the four con-
stituents making up the black hole.\footnote{The reason that the energy of the fivebranes goes like $1/g^2$ rather than $1/g$ is because
it is not a D-fivebrane but rather a solitonic fivebrane \cite{21}. Fortunately, the counting of states can still be carried out in this case. For an alternative description of the four
dimensional black hole without solitonic fivebranes, see \cite{30}.} In the case of the Reissner-Nordström solution when all the boost parameters are set equal, one can easily verify that each of these constituents contributes equally to the total mass. Notice that if one fixes the charges and size of the internal torus, the mass (in string units) changes with the string coupling. This is consistent with the BPS bound.

3.2 Near Extremal Black Holes

Since supersymmetry seemed to play an important role in the above discus-
sion, one might have thought that the entropy could be calculated precisely only in this case. This turns out to be incorrect. In fact, soon after the first precise calculation of black hole entropy, it was shown that the entropy of certain slightly nonextremal black holes can also be calculated exactly \cite{31, 32}. At present, this is well understood only in the regime where one of the constituents is much lighter than the rest. This case is clearly the simplest to consider since the maximum entropy is obtained by adding energy to the lightest degrees of freedom. For the case of Reissner-Nordström, where all the branes contribute equally to the mass, if one adds a small amount of energy, the excitation of all the branes will contribute equally to the entropy, and the counting is much more difficult. It is clear from (3.8) that when $R$ is large, the momentum modes are much lighter than the branes. So if one adds a small amount of excess energy, it simply goes into exciting left and right moving modes. Since the left and right moving modes are largely noninteracting at weak coupling, the entropy is additive and we obtain

\[ S = 2\pi \sqrt{Q_2 Q_5 Q_6} (\sqrt{n_L} + \sqrt{n_R}) \] (3.9)

where the left and right moving momenta are $n_L/R$ and $n_R/R$ respectively. This agrees precisely with the Bekenstein-Hawking entropy computed from
the near extremal hole solution (3.6). For other choices of the parameters $V, R, \tilde{R}$, one of the branes may be much lighter than the other constituents. In this case, the entropy can again be understood by a suitable counting argument. But when several branes are light (or all contribute equally, as in Reissner-Nordström), one does not yet have a precise counting of the entropy of the near extremal solution.

At weak coupling the interactions between the left and right moving modes are small but nonzero. Occasionally, a left-moving mode can combine with a right-moving mode to form a closed string which can leave the brane. This represents the decay of an excited configuration of D-branes and is the weak coupling analog of Hawking radiation. Given the remarkable agreement between the entropy of the black hole and the counting of states of the D-branes, the next step is clearly to ask how the radiation emitted by the D-brane compares to Hawking radiation. Since the entropy as a function of energy agrees in the two cases, it is not surprising that the radiation is approximately thermal with the same temperature in both cases. What is surprising is that the overall rate of radiation agrees [33]. What is even more remarkable is that the deviations from the black body spectrum also agree [34]. On the black hole side, these deviations arise since the radiation has to propagate through the curved spacetime outside the black hole. This produces potential barriers which give rise to frequency-dependent greybody factors. On the D-brane side there are deviations since the modes come from separate left and right moving sectors on the D-branes. The calculations of these deviations could not look more different. On the black hole side, one solves a wave equation in a black hole background. The solutions involve hypergeometric functions. On the D-brane side, one does a calculation in D-brane perturbation theory. Remarkably, the answers agree.

To be a little more precise, the calculations were first done for the (five dimensional) near-extremal black hole with $R \gg 1$. Since the black hole is near extremality, the temperature is very low and hence the wavelength of the radiation is much larger than the size of the black hole. One considers radiation by a minimally coupled scalar field. On the $D$-brane side, one starts with a thermal distribution of left and right moving modes with temperature
and $T_R$. The decay rate for left and right moving excitations, each with energy $\frac{\omega}{2}$, to produce an outgoing S-wave mode with energy $\omega$ is

$$\Gamma = \frac{g_{eff}^2 \omega}{(e^{\omega/2T_L} - 1)(e^{\omega/2T_R} - 1)(2\pi)^4} \, d^4 k$$  \hspace{1cm} (3.10)$$

The factors in the denominator are the usual thermal factors for the left and right moving modes and $g_{eff}$ is a frequency independent effective coupling constant. To compare with the black hole, one computes the average left and right moving energy, which determines $n_L$ and $n_R$. This fixes the total energy and momentum of the black hole. The Hawking temperature turns out to be related to $T_L$ and $T_R$ by

$$\frac{1}{T_R} + \frac{1}{T_L} = \frac{2}{T_H}$$  \hspace{1cm} (3.11)$$

The black hole decay rate is given by

$$\Gamma = \frac{\sigma_{abs}(\omega)}{(e^{\omega/T_H} - 1)} \, d^4 k \, (2\pi)^4$$  \hspace{1cm} (3.12)$$

where $\sigma_{abs}(\omega)$ is the greybody factor, which equals the classical absorption cross section. One calculates $\sigma_{abs}(\omega)$ by studying solutions to the wave equation in the black hole background. For the Schwarzschild and Kerr metrics, this was extensively studied more than twenty years ago. It has recently been shown that for any black hole, the limit of $\sigma_{abs}(\omega)$ as $\omega \to 0$ is the area of the event horizon. After a lengthy calculation in the metric analogous to (3.3) describing a five dimensional black hole with three charges, one finds that for $\omega \leq T_H$,

$$\sigma_{abs}(\omega) = \frac{g_{eff}^2 \omega (e^{\omega/T_H} - 1)}{(e^{\omega/2T_L} - 1)(e^{\omega/2T_R} - 1)}$$  \hspace{1cm} (3.13)$$

so the two rates (3.10) and (3.12) agree!

It is worth emphasizing that it is not just a few parameters which agree. Since the calculation is valid for $\omega \leq T_H$, the entire functional form is significant. It is as if the black hole knows that its states are described by an
effective 1 + 1 dimensional field theory with left and right moving modes. It appears that the black hole also knows that some of these modes are fermionic. In the weak-coupling description, an outgoing mode with angular momentum $\ell = 1$ arises from a left and right moving fermion on the D-brane. Remarkably, when the greybody factor is computed for this case, one again finds that it factors into left and right moving thermal factors, but now they take the form $(e^{\omega/2T} + 1)$ appropriate for fermions [38]! (The overall numerical coefficient has not yet been checked for this case). More generally, for $\ell$ odd, one obtains the fermionic factors, while for $\ell$ even, one obtains the bosonic factors as expected from the D-brane description.

It is not yet clear why a calculation of decay rates at weak coupling can be extrapolated without modification into the strong coupling regime. One possible explanation was given by Maldacena [39] who argued that at low energy, the relevant interactions did not receive quantum corrections due to a supersymmetric non-renormalization theorem (see also [40]).

Attempts to extend this calculation have met with mixed success. Agreement was found for four, as well as five, dimensional black holes [41], charged scalars [34, 41], and certain non-minimally coupled scalars [42, 43] but disagreement was found for higher energies [44, 45], other near-extremal regimes, e.g. $R \sim 1$ [46, 45], or other non-minimally-coupled scalars [47]. However, in most of the cases where disagreement was found, even the weak-coupling D-brane calculations are not yet completely understood.

These results have immediate implications for the well known black hole information puzzle. Hawking has argued that the radiation emitted from a black hole is independent of what falls in. Thus if an extreme black hole absorbs a small amount of matter and then radiates back to extremality, information will be lost, and unitarity will be violated. String theory provides a manifestly unitary description of a system of strings and D-branes with the same entropy and rate of radiation. If one throws a small amount of energy toward a system of D-branes, the branes become excited and then decay. The final system is in a pure state, with correlations between state of the D-branes and the radiation. If one traces over the D-brane states, the radiation is approximately thermal. One might imagine that the same thing happens
for black holes. But this does not avoid the difficulties of how information
gets out from inside the black hole. Suppose that one repeats this experiment
many times. When the entropy in the radiation becomes greater than the
entropy in the D-branes, it can no longer be the case that the radiation
is thermal after tracing out the D-brane states. There will be correlations
between the radiation emitted at early times and the radiation emitted at
late times [48]. Since this system of strings and D-branes becomes a black
hole when one increases the string coupling, it is very tempting to conclude
that even in the black hole regime, there will be correlations between the
radiation at early and late times and the evaporation process will be unitary.
In fact, the argument based on supersymmetric nonrenormalization theorems
[39] supports this view for very low energy quanta.

However, Hawking has stressed that the causal structure of the black hole
is very different from the flat space description, since there is no analog of
the event horizon. It is logically possible that the evaporation process is uni-
tary at weak coupling and fails to be unitary at strong coupling. Before one
can conclude that quantum mechanics is not violated in black hole evapora-
tion, one needs at least a convincing explanation of where Hawking’s original
arguments break down.

4 Duality

In the previous two sections, we have considered the transition from a weakly
coupled state of strings and D-branes to a black hole. We have seen that this
occurs when $gN^{1/4} \sim 1$ or $gQ \sim 1$ which implies that the fundamental
string coupling $g$ is still rather small. During the past few years there have
been a series of conjectures about the behavior of string theory when the
coupling becomes much greater than one. It was previously believed that
there were five fundamental string theories in ten dimensions which differed in
the amount of supersymmetry and type of gauge groups they contained. The
low energy limits of these theories were ten dimensional supergravity coupled
to matter. In addition there was an eleven dimensional supergravity theory
which did not seem to fit into string theory at all. It is now believed that
all of these theories are connected in the sense that the large coupling limit of one theory compactified on one type of internal space is equivalent to the weak coupling limit of another compactified on a possibly different internal space. This suggests that there is one universal theory with different weak coupling limits corresponding to each of the known theories. The conjectures relating these theories are known as S-duality. The evidence for them has been accumulating for the past two years and is now rather convincing [49]. This includes the fact that there are solitons in one theory with the same properties as the fundamental strings of the other, and the spectrum of BPS states in the two theories (which does not depend on the coupling) agrees.

I do not have time to discuss this exciting subject in detail, but let me mention two consequences for our discussion of black holes. For simplicity, I will not discuss compactification, but consider black holes directly in ten dimensions. Since all string theories include gravity, the Schwarzschild black hole is a solution to each theory. By taking different limits, one can represent its states in different ways. Let us consider the type IIB theory which has both fundamental strings and D-strings. The D-strings have a tension $1/(gl_s^2)$ and hence are very heavy when $g \ll 1$. But when $g \gg 1$, the D-strings become much lighter than the fundamental strings. This limit is described by another IIB string theory but now the role of the fundamental strings is played by the D-strings and the coupling constant is $\hat{g} = 1/g$. (In other words, the IIB theory is self-dual.) We saw in section 2 that when $g \ll 1$, the states of a Schwarzschild black hole could be represented by states of an ordinary string. We now see that in the limit $g \gg 1$, the same black hole can be described in terms of states of a weakly coupled D-string.

Thus one has the following picture as one increases $g$. One can start with an excited state of the fundamental string at level $N$ when $g = 0$. As we have seen, when $g \sim 1/N^{1/4}$ the Schwarzschild radius is of order the string scale and the state forms a black hole. If we continue to increase the coupling, the black hole remains unchanged until $g \sim N^{1/4}$ which is when the Schwarzschild radius is of order the length scale set by the D-string tension. Beyond this point, the black hole can be described by an excited state of a weakly coupled D-string at the same level $N$ as the initial fundamental
In fact, the low energy IIB string lagrangian has an $SL(2,\mathbb{Z})$ symmetry, under which the Einstein metric is invariant, but the fundamental string is mapped into a $(m, n)$ string which carries $m$ units of fundamental string charge and $n$ units of D-string charge. Starting with the Schwarzschild black hole, there are different weak coupling limits of this theory in which the states are excitations of the $(m, n)$ strings.

It is interesting to consider the black hole information puzzle from this standpoint. If we compactify five dimensions of the IIB string theory and wrap various D-branes around the internal five torus, one can obtain extreme and near extreme five dimensional black holes. We have seen that the spectrum of radiation at infinity remains unchanged as we increase the coupling and the description of the state changes from slightly excited D-branes to near extremal black holes. The D-brane radiation is known to be unitary, yet it has been argued that the black hole radiation will not be unitary. The duality conjectures imply that if we continue to increase $g$, the black hole can again be described by a weakly coupled dual string theory. The spectrum will remain the same, and the radiation will again be unitary. Of course the spacetime geometry in both limits $g \gg 1$ and $g \ll 1$ is flat, so there are qualitative differences from the black hole. But still it seems rather unlikely that a physical process would change from being unitary to nonunitary and back to unitary as a parameter is continuously increased.

As a second application of duality ideas to black holes, we consider the IIA string theory. This theory has a series of BPS states (bound states of D-zerobranes) with masses which are integer multiples of $1/(gl_s)$. This is similar to the spectrum of Kaluza-Klein states in a theory compactified on a circle of radius $R = gl_s$. Note that for weak coupling, the radius is very small, but at strong coupling, it becomes large. For this (and other) reasons, it is believed that the low energy limit of the strongly coupled ten dimensional IIA string theory is eleven dimensional supergravity compactified on a circle of radius $R = gl_s$. The eleven dimensional Planck length turns out to be $l_p = g^{1/3}l_s$. One can now trace out the following behavior of an excited IIA string state as the coupling is increased. One starts at zero coupling with a state at level $N$, string.
so \( M = \sqrt{N/l_s} \) and \( S \sim \sqrt{N} \). One now increases the string coupling keeping the state (i.e. the entropy) fixed. As before, when \( g \sim 1/N^{1/4} \), one forms a ten dimensional black hole with \( r_0 = N^{1/16} \) in ten dimensional Planck units (since \( S \sim r_0^8 \sim N^{1/2} \)). When \( g \sim 1 \), the length of the eleventh dimension becomes greater than the eleven dimensional Planck length, and so becomes physically meaningful. Since the ten dimensional IIA supergravity theory is just the dimensional reduction of eleven dimensional supergravity, the ten dimensional black hole then becomes an eleven dimensional black string, i.e. the solution becomes the product of Schwarzschild and a circle. As the coupling increases, the length of the eleventh dimension becomes larger. When it becomes of order \( r_0 \), the black string becomes unstable and probably forms an eleven dimensional black hole. This occurs when \( r_0 \sim R \), which implies \( R \sim S^{1/9} l_p \). Since \( S \sim N^{1/2} \), we have \( R \sim N^{1/18} l_p \) and hence \( g \sim N^{1/12} \). As \( g \) is increased further, corresponding to larger values of \( R \), the state remains an eleven dimensional black hole. Conversely, starting with an eleven dimensional Schwarzschild solution with one dimension periodically identified, one can follow the above description in reverse as one decreases \( g \): The eleven dimensional black hole transforms into an eleven dimensional black string, which then becomes a ten dimensional black hole, and finally a weakly coupled string in ten dimensions. This is one approach toward understanding the entropy of eleven dimensional black holes. Of course, it would be more satisfactory to have a direct explanation of the entropy of eleven dimensional black holes, which did not require a direction to be compactified. But this requires the full eleven dimensional quantum theory (called M theory) which is not yet well understood.

5 Discussion

Our understanding of black hole microstates provided by string theory is progressing rapidly. To illustrate this, let me briefly list some of the highlights from 1996.

\footnote{This is by no means an exhaustive list of all of the contributions to this subject, which would include well over a hundred papers.}
tween the entropy of an extremal five dimensional black hole and the counting of string states was performed [16]. In February, this was extended to near extremal black holes [31, 32], and extreme rotating black holes [23] (still in five dimensions). In March, the entropy of four dimensional black holes, both extremal [24, 25] and near extremal [28], was reproduced. In June, the rate of low energy radiation from a near extremal five dimensional black hole was shown to agree exactly with the rate from excited D-branes [33]. In August, this was extended to four dimensions [51]. In September it was shown that the deviations from the black body spectrum agree, both in five [34] and four [41] dimensions. Finally, in December the entropy of black holes far from extremality was understood, up to an overall coefficient, in terms of a correspondence principle [9].

Despite this enormous progress, our understanding is still far from complete. One outstanding issue is the resolution of the black hole information puzzle, which was discussed in the last two sections. Perhaps a more modest question is whether the entropy of black holes far from extremality can be reproduced exactly. When the supersymmetric black hole calculations were first carried out, there was a strong belief that supersymmetry was playing a key role, and it would be impossible to do the same thing for nonsupersymmetric configurations. But then it was found that the entropy continued to agree for near extremal black holes and also extremal black holes which are not supersymmetric, e.g., extreme rotating four dimensional black holes [28, 52]. I now believe that the entropy of all large black holes should be computable, including the overall coefficient. The correspondence principle certainly shows that the main contribution to the entropy can be understood without supersymmetry.

One might ask whether the correspondence principle could be extended to compare the precise coefficients in the expression for the entropy. At first sight this appears to be difficult since it requires a better understanding of the string state when it is at the string scale, and interactions become important. However, there are indications that things are simpler than they appear. One of these comes from studies of the near extremal threebrane. Using the correspondence principle, one can understand the entropy of all black
$p$-branes in terms of a gas of massless strings on the brane at weak coupling \[^9\]. However, for the threebrane, the entropy turns out to be independent of when the masses are set equal. If one takes the coupling all the way to zero and compares the entropy of a free gas with that of the black threebrane, one finds $S_{bh} = (3/4)S_{gas}$ \[^53\]. The correspondence principle predicts that the transition to the black $p$-brane occurs when the temperature of the gas is of order the string scale, so the interactions should be important. Since the potential energy is positive, this explains why $S_{bh} < S_{gas}$: By ignoring the interactions, the energy of each state of the gas has been underestimated, and hence the total number of states with a given energy has been overestimated. But it is not clear why the interactions should simply produce a factor of $3/4$.

We have seen that the transition from a perturbative string state to a black hole occurs when the string coupling is still rather small $g \sim 1/N^{1/4}$ or $g \sim 1/Q$ (for near extremal black holes). So one might wonder why string perturbation theory cannot be used to directly study properties of black holes. The reason is that the effective coupling constant, due to the long string or large number of D-branes, is really $gN^{1/4}$ or $gQ$, which is becoming of order one. Even though string perturbation theory is known to diverge \[^54\], there is an important difference between a coupling constant that is of order one, and one that is small. The perturbation series has the form $\sum C_n \tilde{g}^n$ where $C_n \sim (2n)!$ and $\tilde{g}$ is the effective coupling constant \[^25\]. This is an asymptotic series, and the best approximation to the exact answer is obtained by cutting the series off after $\tilde{g}^{-1/2}$ terms when the individual terms become greater than one. It turns out that the error one introduces this way is of order $e^{-1/\tilde{g}}$. (In ordinary field theory, $C_n \sim n!$, so cutting the series off after $1/\tilde{g}$ terms introduces an error $e^{-1/\tilde{g}^2}$.) Clearly, when $\tilde{g} \sim 1$, no useful information can be obtained from the perturbation series. Fortunately, for $\tilde{g} \gg 1$, one has an alternative description of the system in terms of a semi-classical black hole.

For the near extremal black holes discussed in section 3.2, the energy above extremality is independent of the string coupling $g$. So one can compare the entropy as a function of energy for the black holes and strings. Since they agree, one knows that the temperature of the two systems are equal.
This is not true for black holes far from extremality. We saw in section 2 that the mass of a state does not remain constant as one changes the string coupling $g$. In string units, the mass of a perturbative string state is approximately constant until it forms a black hole and then it decreases with $g$. In Planck units, the mass of a string state increases with $g$ until it forms a black hole and then it remains constant. This means that even though the entropies of black holes and strings agree, their temperatures do not. For a highly excited string the natural temperature is a constant of order the string scale (the Hagedorn temperature) rather than the Hawking temperature.

The understanding of black hole entropy provided by the correspondence principle leads to a simple picture of the evaporation of a Schwarzschild black hole, if the string coupling is small in nature. In most of our previous discussion, we imagined varying the string coupling $g$. Now we suppose that $g \ll 1$ is fixed, so the string length scale is much larger than the Planck length. A large black hole will Hawking evaporate until the curvature at the horizon reaches the string scale. At this point it turns into a highly excited string state with $N \sim 1/g^4$, since for this value of $N$, the string has a mass comparable to the black hole: $M_s^2 \sim N/l_s^2 \sim 1/(g^4 l_s^2) \sim l_s^2/G^2 \sim M_{bh}^2$. The entropy of the string is also comparable to the black hole entropy at this point, so the string can carry the remaining information in the black hole. The excited string will then continue to decay via string interactions. The temperature slowly increases as the black hole evaporates and reaches the string scale at the transition point. It then remains at this temperature as the excited string decays. Eventually one is left with an unexcited string, i.e., an elementary particle like a photon.

The history of particle physics is full of examples where objects which were thought to be elementary were later found to be composed of more fundamental entities. The fact that the entropy of black holes has now been reproduced by counting quantum states strongly suggests that we have finally identified the fundamental degrees of freedom, and there is not another level

\footnote{The mass is likely to decay exponentially if one assumes that each segment of the long string has the same probability to break off a small loop of string. Since $M \sim L$, $dM/dt \sim -M$. (I thank L. Susskind for pointing this out.)}
of structure waiting to be uncovered. However, as we saw in the last section, these fundamental degrees of freedom can take different forms in different weak coupling limits of the theory.

There remains the puzzling question of why the counting of string states turns out to reproduce the area of the event horizon of a black hole. This is undoubtably tied up with the fundamental question of what is the origin of spacetime geometry in string theory. One recent suggestion, which is motivated by the relation between position in spacetime and the moduli space of gauge theories on D-branes, starts with a quantum theory of $N \times N$ matrices in the limit of large $N$ [56]. The description of black holes in this context is beginning to be investigated [57, 58]. At the risk of sounding too conservative, I will state my belief that spacetime itself will ultimately be made up of strings. After all, perturbations in the spacetime metric are just one mode of the string. With the possible exception of zero-branes, it is difficult to see how charged objects like D-branes can produce a neutral object like Minkowski spacetime. I believe the entropy calculations are providing a glimpse into the quantum origin of spacetime. It is tempting to turn the current calculations around and use them to try to define a metric or area in spacetime in terms of the number of states in a Hilbert space. Following this approach may expand our glimpse until the full picture of quantum spacetime is finally revealed.

6 Acknowledgments

It is a pleasure to thank A. Ashtekar, T. Jacobson, B.-L. Hu, and S. Ross for raising questions which (it is hoped) improved the clarity of this presentation. I also wish to thank J. Polchinski, A. Strominger, and L. Susskind for discussions which improved my understanding of the results discussed here. This work was supported in part by the NSF under grant PHY95-07065.

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