Photon versus Hadron Interferometry∗

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Abstract

Photons and mesons are both bosons and therefore satisfy the same Bose-Einstein statistics. This leads to certain similarities in the corresponding Bose-Einstein correlations which underly photon and hadron intensity interferometry. However there are also important differences between the two effects and these will be analyzed in the following.

1 Introduction

Hanbury-Brown and Twiss (HBT) [1] developed in the mid fifties the method of photon intensity interferometry to be used as an alternative to the amplitude interferometry of Michelson. Initially this “alternative” was considered merely as a technical improvement of interest only for astronomy and it is therefore not surprising that Goldhaber, Goldhaber Lee and Pais (GGLP) [2] were not aware of the HBT experiment when they discovered in 1959-1960 that pairs of identical pions were bunched and interpreted this effect as due to Bose-Einstein correlations. This initial separation between the two developments is in part due to the fact that the techniques used in the original HBT experiment and in the GGLP experiment were very different: the HBT interferometry in astronomy consists in measurements of distance correlations (actually correlations of time arrivals) in order to determine (angular) diameters of stars, while in GGLP experiments one measures momentum correlations in order to derive radii and lifetimes of sources of elementary particles.

On the other hand for (some) people working in optics it did not take much time to realize the quantum statistical significance of the HBT experiment and it turned out that the apparently small step in the history of interferometry due to HBT represented a huge step in the history of physics, leading to the creation of quantum optics with all its theoretical and practical developments. The implications and the importance of the HBT and GGLP effects for particle physics were appreciated only much later. It is

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‡As far as we can gather the link between the two experiments is mentioned for the first time in ref. [3].
therefore very timely that a conference like the present one where astronomers, particle and nuclear physicists meet, is organized. At present the HBT/GGLP effect is an important tool in particle and nuclear physics, being the only direct experimental method known so far for the determination of space-time characteristics of particle sources. Moreover, the phenomenon of Bose-Einstein correlations (BEC) presents interesting and important theoretical problems in itself and it is thus understandable that in the last decenium of this century it has become an independent subject of research.

Although both the HBT effect in quantum optics and in astronomy use photons, quantum optics, being a microscopic discipline, is of course much more related to particle physics than to astronomy. Among other things, in quantum optics, too, one measures momenta, rather than distance correlations. On the other hand photon interferometry is not restricted only to astronomy and quantum optics, but finds applications also in particle and nuclear physics. As a matter of fact, photon interferometry in particles physics is from a certain point of view superior to hadron interferometry, because photons are weakly interacting particles, while hadrons interact strongly. This has two important consequences in photon BEC: (i) there is (up to higher order corrections) no final state interaction between photons, so that the BEC effect is “clean”; (ii) in a high energy reaction, hadrons are produced only at the end of the reaction (at freeze-out), while photons from the beginning, so that photons can provide unique information about the initial state. For the search of quark-gluon plasma this is essential, because if such a state of matter is formed, then this happens only in the early stages of the reaction. This is also important in lower energy heavy ion reactions where the dynamics of the reaction as well as its space-time geometry are studied in this way (cf. the talk by R. Barbera in these proceedings).

These advantages of photon interferometry have stimulated theoretical and experimental studies, despite the technical difficulties due to the small rates of photon production and the background due to $\pi^0$ decays.

Besides the difference in the coupling constant, photons and hadrons (for the sake of concreteness we shall refer in the following to pions) have also other distinguishing properties like spin, isospin, and mass which manifest themselves in the corresponding BEC and which sometimes are overlooked. This is the subject of this talk.

### 2 Comparison between photon and hadron BEC

Table 1 contains an enumeration of differences between photons and pions which appear relevant from the point of view of intensity interferometry. We will comment upon three topics in the following:\footnote{From 1990 meetings dedicated (almost) entirely to this subject were held, beginning with CAMP.} classical versus quantum fields, condensates, and the role of spin in photon BEC.

#### 2.1 Classical versus quantum fields; coherence and chaos

As is well known BEC are sensitive to the amount of coherence of the source and this makes intensity interferometry a useful tool in the determination of coherence, both for photons and for hadrons. While classical fields are always coherent, quantum fields may...
| Photons                  | Properties          | Hadrons (Pions)                      |
|-------------------------|---------------------|-------------------------------------|
| Trivial (electromagnetic)| Classical fields    | Remarkable (Higgs, sigma meson)     |
| Remarkable              | Quantum fields      | Trivial                             |
| Trivial                 | Chaos               | Remarkable                          |
| Lasers                  | Condensates         | Pion condensates                    |
| No                      | Final state         | Yes                                 |
| interactions            |                     |                                     |
| $m = 0$                 | Mass                | $m \neq 0$                          |
| Yes, if effective       | Multiparticle       | Yes, if energy                      |
| coupling is big         | production          | is big enough                       |
| enough (lasers)         |                     |                                     |
| $S = 1$                 | Spin                | $S = 0$                             |
| $I = 0$                 | Isospin             | $I = 1$                             |
| $1/3 \leq C_2 \leq 3$   | **Correlations**    | $2/3 \leq C_2 \leq 2$, (for charged pions) |
|                         |                     | $1/3 \leq C_2 \leq 3$ (for neutral pions) |
| Astronomy, gravitational waves, quantum optics, atomic physics, chemistry, biochemistry | **Applications** | Particle and nuclear physics, search for quark-gluon plasma, determination of the mass of the W |

Table 1
be coherent or chaotic. Electromagnetic fields which are at the basis of optical phenomena are “classical”, i.e. quantum phenomena do not play a role there (the Planck constant \( h \) does not appear in the Maxwell equations). Therefore the discovery of photons i.e. of quanta of light was so important, as it lead to the creation of modern quantum physics. Particle physics developed much later and it was quantum from the very beginning. Therefore the fact that the associated particle fields are quantized is from this point of view trivial. On the other hand in the seventies it became clear that the symmetries observed in particle physics are spontaneously broken. This fact, which was brilliantly confirmed by the discovery of intermediate bosons, lead via the Golstodone-Higgs-Kibble mechanism necessarily to classical fields. Hence in particle physics the existence of classical fields is far from trivial. With chaos the situation is rather inversed. Conventional optical sources are thermal and therefore chaotic. However in particle physics where the wave lengths of particles are of the order of the dimensions of the sources and the lifetime of sources may be small compared with the time necessary for equilibration, one would expect coherence as a rule and thermal equilibrium as exceptional.

2.2 Condensates

One of the most important effects of quantum optics which is based on coherence is the phenomenon of lasing. Lasers are Bose condensates and it has been speculated that such condensates, in particular pion condensates, may exist also in nuclei (cf. e.g. [5]) or be created in heavy ion reactions (cf. e.g. [6], [7]).

However there exist important differences between photon condensates i.e. lasers and pion condensates. Furthermore there are different theoretical approaches to the problem of pion condensates and some confusing statements as to how pion condensates are produced. In the following we shall discuss briefly these issues.

2.2.1 Lasers versus pion condensates; pasers?

BEC for inclusive processes, which constitute by far the most interesting and most studied reactions both with hadrons and photons have to be treated by quantum field theory, which is the appropriate formalism when the number of particles is not conserved. For certain purposes however, sometimes one is interested in considering events with a fixed number of particles. Thus the number of particles in a given event can help selecting central collisions with small impact parameter. Theoretically this situation can be handled within field theory, using the methods of quantum statistics [8]. On the other hand for the construction of event generators wave functions appear so far to be a convenient tool and therefore, and also for historical reasons, some theorists have continued to use the “traditional” method of wave function (wf), as introduced in the original GGLP paper. This implies the explicit symmetrization of the products of single particle wf, while in field theory the symmetrization (of amplitudes) is automatically achieved through the commutation relations of the field operators. When the multiplicities are large, the symmetrization of the wf becomes tedious. This led Zajc [9] to use numerical Monte Carlo techniques for estimating \( n \) particle symmetrized probabilities, which he then applied to calculate two-particle BEC. He was thus able also to study the question of the dependence of BEC parameters on the multiplicity \( n \). Using as input a second order BEC function parametrized in the form

\[
C_2 \sim 1 + \lambda \exp(-q^2 R^2),
\]  

(2.1)
where \( q \) is the momentum transfer and \( R \) the radius, Zajc found, and this was confirmed in \([8]\), that the “incoherence” parameter \( \lambda \) decreased with increasing \( n \).

However Zajc did not consider that this effect means that events with higher pion multiplicities are denser and more coherent. On the contrary he warned against such an interpretation and concluded that his results have to be used in order to eliminate the bias introduced by this effect into experimental observations. \[9\]

This warning apparently did not deter the authors of \([6]\) and \([10]\) to do just that. Ref.\([6]\) went even so far to derive the possible existence of pionic lasers (pasers) from considerations of this type.

Ref.\([6]\) starts by proposing an algorithm for symmetrizing the \( \psi \) which presents the advantages that it reduces very much the computing time when using numerical techniques, which is applicable also for Wigner type source functions and not only plane wave functions, and which for Gaussian sources provides even analytical results.

Subsequently in ref.\([11]\) wave packets were symmetrized and in special cases the matrix density at fixed and arbitrary \( n \) was derived in analytical form. This algorithm was then applied to calculate the influence of symmetrization on BEC and multiplicity distributions. As in \([9]\) it was found that the symmetrization produces an effective decrease of the radius of the source, a broadening of the multiplicity distribution \( P(n) \) and an increase of the mean multiplicity as compared to the non-symmetrized case. What is new in \([6]\) is (besides the algorithm) mainly the meaning the author attributes to these results.

In a concrete example Pratt considers a non-relativistic source distribution \( S \) in the absence of symmetrization effects:

\[
S(k, x) = \frac{1}{(2\pi R^2 m T)^{3/2}} \exp \left( -\frac{k_0 T}{T} - \frac{x^2}{2R^2} \right) \delta(x_0) \quad (2.2)
\]

where

\[
k_0/T = k^2 / 2\Delta^2
\]

Here \( T \) is an effective temperature, \( R \) an effective radius, \( m \) the pion mass, and \( \Delta \) a constant with dimensions of momentum.

Let \( \eta_0 \) and \( \eta \) be the number densities before and after symmetrization, respectively. In terms of \( S(k, x) \) we have

\[
\eta_0 = \int S(k, x) d^4k d^4x \quad (2.4)
\]

and a corresponding expression for \( \eta \) with \( S \) replaced by the source function after symmetrization.

Then one finds \([6]\) that \( \eta \) increases with \( \eta_0 \) and above a certain critical density \( \eta_0^{crit} \), \( \eta \) diverges. This is interpreted by Pratt as pasers.

The reader may be rightly puzzled by the fact that while \( \eta \) has a clear physical significance the number density \( \eta_0 \) and a fortiori its critical value have no physical significance, because in nature there does not exist a system of bosons the \( \psi \) of which is not symmetrized. Thus contrary to what is alluded to in ref.\([6]\), this paper does not does address really the question how a condensate is reached. Indeed, the physical factors which induce

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4In \([6]\) the clumping in phase space due to Bose symmetry was also illustrated;

5The same interpretation of the multiplicity dependence of BEC was given in \([8]\). In this reference the nature of the “fake” coherence induced by fixing the multiplicity is even clearer, as one studies there explicitely partial coherence in a consistent quantum statistical formalism.
condensation are, for systems in (local) thermal and chemical equilibrium, pressure and temperature and the symmetrization is contained automatically in the form of the distribution function

\[ f = \frac{1}{\exp[(E - \mu)/T] - 1} \]  

(2.5)

where \( E \) is the energy and \( \mu \) the chemical potential.

To realize what is going on it is useful to observe that the increase of \( \eta_0 \) can be achieved by decreasing \( R \) and/or \( T \). Thus \( \eta_0 \) can be substituted by one or both of these two physical quantities. Then the blow-up of the number density \( \eta \) can be thought of as occurring due to a decrease of \( T \) and/or \( R \). However this is nothing but the well known Bose-Einstein condensation phenomenon.

While from a purely mathematical point of view the condensation effect can be achieved also by starting with a non-symmetrized \( \psi \) and symmetrizing it afterwards “by hand”, the causal i.e. physical relationship is different: one starts with a bosonic i.e symmetrized system and obtains condensation by decreasing the temperature or by increasing the density of this bosonic system. To obtain a pion condensate e.g., the chemical potential has to equate the pion mass.

A scenario for such an effect in heavy ion reactions has been proposed in [7]. It is based on the decay of short lived resonances which leads to an accumulation of pions and takes place if the hadronic (dense) matter decouples from chemical equilibrium earlier than from thermal equilibrium. In [7] it was found that if a pionic Bose condensate is formed at any stage of the collision, it can be expected to survive until pions decouple from the dense matter, and thus it can affect the spectra and correlations of final state pions.

This effect was then studied quantitatively by solving the equations of relativistic hydrodynamics for a fluid which contains also a superfluid component, corresponding to the pion condensate. From the results obtained in this way we quote: in the single inclusive transverse momentum distribution the signature of a maximum velocity appears, which is specific for a superfluid system. The second order correlation function \( C_2 \) presents the typical features of a partially coherent system i.e. a lowering of the intercept and a double structure, which in principle could be quite dramatic (up to a given value of \( q, C_2 \) vanishes). These features are rather specific for a pion condensate and distinguish such a system from optical condensates.

To conclude the “paser” topic, one must correct another confusing interpretation which relates to the observation made also in [9] that the symmetrization produces a broadening of the multiplicity distribution (MD). In particular starting with a Poisson MD for the non-symmetrized \( \psi \) one ends up after symmetrization with a negative binomial. While Zajc correctly considers this as a simple consequence of Bose statistics, ref.[6] goes further and associates this with the so called pasing effect. That such an interpretation is incorrect is obvious from the fact that for true lasers the opposite effect takes place. Before “condensing” i.e. below threshold their MD is in general broad and of negative binomial form corresponding to a chaotic (thermal) distribution while above threshold the

\[ ^{6}\text{For lasers the determining dynamical factor is among other things the inversion of the occupation of atomic levels.} \]

\[ ^{7}\text{None of the “paser” papers [6] - [11] address the crucial issue of directional coherence which is an essential characteristic of optical lasers. This casts doubts whether the terminology of “paser” is appropriate. For a model of directional coherence, not necessarily related to pion condensates, cf.[12]; experimental hints of this effect have possibly been seen in [13].} \]
laser condensate is produced and as such corresponds to a coherent state and therefore is characterized by a Poisson MD.

2.3 Photon interferometry. Photon spin and bounds of BEC.

In this section we discuss the difference between BEC for photons and for pions. Certain erroneous results and statements in the recent literature will be corrected.

Following [14] and [15] we consider a heavy ion reaction where photons are produced through bremsstrahlung from protons in independent proton-neutron collisions\(^8\). The corresponding elementary dipole currents are

\[
j^\lambda(k) = \frac{ie}{mk^0} p\epsilon_\lambda(k) \tag{2.6}
\]

where \( p = p_i - p_f \) is the difference between the initial and the final momentum of the proton, \( \epsilon_\lambda \) is the vector of linear polarization and \( k \) the photon 4-momentum; \( e \) and \( m \) are the charge and mass of the proton respectively. The total current is written

\[
J^\lambda(k) = \sum_{n=1}^{N} e^{ikx_n} j^\lambda_n(k) \tag{2.7}
\]

For simplicity we will discuss in the following only the case of pure chaotic currents \(< J^\lambda(k) > = 0 \). The index \( n \) labels the independent nucleon collisions which take place at different space-time points \( x_n \). These points are assumed to be randomly distributed in the space-time volume of the source with a distribution function \( f(x) \) for each elementary collision. The current correlator then reads

\[
< J^\lambda_1(k_1) J^\lambda_2(k_2) > = < J^\lambda_1(k_1) J^\lambda_2(-k_2) > \equiv C^{\lambda_1\lambda_2}(k_1, k_2) \\
= \sum_{n,m=1}^{N} \int \prod_{l=1}^{N} dx_1 f(x_1) \exp(ik_1 x_n - ik_2 x_m) < j^\lambda_1(k_1) j^\lambda_2(-k_2) > \\
= \sum_{n=1}^{N} [\tilde{f}(k_1 - k_2) < j^\lambda_1(k_1) j^\lambda_2(-k_2) > - \tilde{f}(k_1) \tilde{f}(-k_2) < j^\lambda_1(k_1) > \\
< j^\lambda_2(-k_2) > ] + < J^\lambda_1(k_1) > < J^\lambda_2(-k_2) > \tag{2.8}
\]

Here \( \tilde{f}(k) \) is the Fourier transform of \( f(x) \) with the normalization \( \tilde{f}(k = 0) = 1 \). The function \( \tilde{f} \) has a maximum at \( k = 0 \) and becomes usually negligible for \( kR \gg 1 \) where \( R \) is the effective radius of the source.

We will limit further the discussion to the important case from the experimental point of view of unpolarized photons. The corresponding cross sections are obtained by summing over the the polarization indexes and the elementary currents \( j_n \). Thus the correlator defined above will be proportional to products of the form

\[
< J^\lambda_1(k_1) J^\lambda_2(-k_2) > = \epsilon_\lambda_1(k_1) \left( \sum_{n=1}^{N} < p^i_n p^j_n > \right) \epsilon_\lambda_2(k_2) \tag{2.9}
\]

Due to the axial symmetry around the beam direction one has for the momenta the tensor decomposition

\[
< p^i_n p^j_n > = \frac{1}{3} \sigma_n \delta^{ij} + \delta_n l^i l^j, \tag{2.10}
\]

\(^8\)Photon emission from proton-proton collisions is suppressed because it is of quadrupole form.
where \( l \) is the unit vector in the beam direction and \( \sigma_n, \delta_n \) are real positive constants. In [14] an isotropic distribution of the momenta was assumed. This corresponds to the particular case \( \delta_n = 0 \). The generalization to the form (2.11) is due to [15]. The summation over polarization indexes is performed by using the relations

\[
< (\epsilon^i \cdot \mathbf{p}_l) (\epsilon^j \cdot \mathbf{p}_{l'}) > = \frac{1}{3} (\epsilon^i \cdot \epsilon^j) \delta_{ll'}
\]  

(2.11)

and

\[
\sum_{\lambda=1}^{2} \epsilon^i_{\lambda}(k) \cdot \epsilon^j_{\lambda}(k) = \delta^{ij} - \mathbf{n}^i \mathbf{n}^j,
\]  

(2.12)

where \( \mathbf{n} = k/|k| \).

We write below the results for the second order correlation function

\[
C_2(k_1, k_2) = \frac{\rho_2(k_1, k_2)}{\rho_1(k_1) \rho_1(k_2)}
\]  

(2.13)

for two extreme cases: (1) Uncorrelated elementary currents (isotropy) \((\sigma \gg \delta)\)

\[
C_2(k_1, k_2; \sigma \neq 0, \delta = 0) = 1 + \frac{1}{4} \left[ 1 + (\mathbf{n}_1 \cdot \mathbf{n}_2)^2 \right] \left[ |\tilde{f}(k_1 - k_2)|^2 + |\tilde{f}(k_1 + k_2)|^2 \right],
\]  

(2.14)

leading to an intercept

\[
C_2(k, k) = \frac{3}{2} + \frac{1}{2} |\tilde{f}(2k)|^2
\]  

(2.15)

limited by the values \((3/2, 2)\). (2) Strong anisotropy \((\sigma \ll \delta)\):

\[
C_2(k_1, k_2; \sigma = 0, \delta \neq 0) = 1 + |\tilde{f}(k_1 - k_2)|^2 + |\tilde{f}(k_1 + k_2)|^2
\]  

(2.16)

with an intercept

\[
C_2(k, k) = 2 + |\tilde{f}(2k)|^2
\]  

(2.17)

limited this time by the values \((2, 3)\).

These results are remarkable among other things because they illustrate the specific effects of photon spin on BEC. Thus while for (pseudo-)scalar pions the intercept is a constant (2 for charged pions and 3 for neutral ones) even for unpolarized photons the intercept is a function of \( k \). One thus finds that, while for a system of charged pions (i.e. a mixture of 50% positive and 50% negative) the maximum value of the intercept \( \text{Max} C_2(k, k) \) is 1.5, for photons \( \text{Max} C_2(k, k) \) exceeds this value and this excess reflects the space-time properties of the source represented by \( \tilde{f}(k) \), the degree of (an)isotropy of the source represented by the quantities \( \sigma \) and \( \delta \), and the supplementary degree of freedom represented by the photon spin. The fact that the differences between charged pions and photons are enhanced for soft photons reminds us of a similar effect found with neutral pions (cf. ref. [13]). Neutral pions are in general more bunched than identically charged ones and this difference is more pronounced for soft pions. This similarity is not accidental, because photons as well as \( \pi^0 \) particles are neutral and this circumstance has quantum field theoretical implications which will be mentioned also below.

We see thus that in principle photon BEC can provide information both about the space-time form of the source and the dynamics.

These results on photon correlations refer to the case that the sources are “static” i.e. not expanding. Expanding sources were considered in [17] within a covariant formalism.
The results quoted above, in particular eqs. (2.14,2.15), which had been initially derived by Neuhauser, were challenged by Slotta and Heinz [18]. Among other things, these authors claim that for photon correlations due to a chaotic source “the only change relative to 2-pion interferometry is a statistical factor $\frac{1}{2}$ for the overall strength of the correlation which results from the experimental averaging over the photon spin”. In [18] an intercept $\frac{3}{2}$ is derived which is in contradiction with the results presented above and in particular with eq.(2.15) where besides the factor $\frac{3}{2}$ there appears also the $k$ dependent function $\frac{1}{2}|f(2k)|^2$.

We would like to point out here that the reason for the difference between the results of [14],[15] on the one hand and those of ref.[18] on the other is mainly due to the fact that in [18] a formalism was used which is less general than that used in [14] and [15] and which is inadequate for the present problem. This implies among other things that unpolarized photons cannot be treated in the naive way proposed in [18] and that the results of [14] and [15] are correct, while the results of [18] are not.

In [18] the following formula for the second order correlation function is used:

$$C(k_1, k_2) = 1 + \frac{\tilde{g}_{\mu\nu}(q, K)\tilde{g}^{\mu\nu}(-q, K)}{\tilde{g}_{\mu\nu}(0, k_1)\tilde{g}^{\mu\nu}(0, k_2)}$$

(2.18)

Here $\tilde{g}$ is the Fourier transform of a source function $(g(x, K)$ and $q = k_1 - k_2, K = k_1 + k_2$.

This formula is a particular case of a more general formula for the second order correlation function derived by Shuryak [19] using a model of uncorrelated sources, when emission of particles from the same space-time point is negligible.

As is clear from this derivation there exists also a third term, neglected in eq.(2.18), and which corresponds to the simultaneous emission of two particles from the same point (cf. [14]). While for massive particles this term is in general suppressed, this is not true for massless particles and in particular for soft photons. Indeed in [14] and [15] this additional term had not been neglected as it was done subsequently in [18] and therefore it is not surprising that ref.[18] could not recover the results of refs.[14] and [15]. The neglect of the term corresponding to emission of two particles from the same space-time point is not permitted in the present case. Emission of particles from the same space-time point corresponds in a first approximation to particle-antiparticle correlations and this type of effect leads also to the difference between BEC for identical charged pions and the BEC for neutral pions. This is so because neutral particles coincide with the corresponding antiparticles. (As a consequence of this circumstance e.g. while for charged pions the maximum of the intercept is 2, for neutral pions it is 3 (cf. [14] and Table 1). Photons being neutral particles, similar effects like those observed for $\pi^0$-s are expected and indeed found.

This misapplication of the current formalism invalidates completely the conclusions of ref.[18].

Intuitively the fact that for unpolarized photons Max$C_2(k, k)$ is 2 and not 1.5 as stated in [18], can be explained as follows: a system of unpolarized photons consists on the average of 50% photons with the same helicities and 50% photons with opposite helicities. The first ones contribute to the maximum intercept with a factor of 3 and the last ones with a factor of 1 (corresponding to unidentical particles).

For the sake of clarification it must be mentioned that ref.[18] contains also other incorrect statements. Thus the claim in [18] that the approach by Neuhauser “does not correctly take into account the constraints from current conservation” is completely unfounded as can be seen from eq.(2.11) which is an obvious consequence of current conservation. Last but not least the statement that because the tensor structure in eq.(20)
of ref. [17] is parametrized in terms of \(k_1\) and \(k_2\) separately “instead of only in terms of \(K\), leading to spurious terms in the tensor structure which eventually result in their spurious momentum-dependent prefactor”, has also to be qualified. Indeed the additional term, unduely neglected in [13], depends not only on \(K\) but also on \(k_1\) and \(k_2\) separately and this contradicts the entire argumentaton of [13] regarding the “spurious terms”.

The considerations presented above refer to the effects of photon spin on the upper bounds of the correlation function. Similar specific effects exist also for the lower bounds [16] (cf. Table 1): \(C_{2}^{−−}(k_1, k_2) \geq 2/3\) and \(C_{2}^{00}(k_1, k_2) \geq 1/3\). Here the indeces \(−−\) and \(00\) refer to charged and neutral pions (photons) respectively. These lower bounds have also lead to confusion in the literature and this issue was clarified and corrected in [21]. For further details of the topics discussed here cf. [21].

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