Fisher Exponent from Pseudo-\(\varepsilon\) Expansion

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Abstract

Critical exponent \(\eta\) for three-dimensional systems with \(n\)-vector order parameter is evaluated in the frame of pseudo-\(\varepsilon\) expansion approach. Pseudo-\(\varepsilon\) expansion (\(\tau\)-series) for \(\eta\) found up to \(\tau^7\) term for \(n = 0, 1, 2, 3\) and within \(\tau^6\) order for general \(n\) is shown to have a structure rather favorable for getting numerical estimates. Use of Padé approximants and direct summation of \(\tau\)-series result in iteration procedures rapidly converging to the asymptotic values that are very close to most reliable numerical estimates of \(\eta\) known today. The origin of this fortune is discussed and shown to lie in general properties of the pseudo-\(\varepsilon\) expansion machinery interfering with some peculiarities of the renormalization group expansion of \(\eta\).

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I. INTRODUCTION

Field-theoretical renormalization group (RG) approach proved to be highly efficient when used to evaluate universal parameters characterizing the behavior of various systems near Curie point. It yields high-precision numerical estimates for critical exponents, renormalized coupling constants, universal ratios, etc., provided lengthy enough RG expansions are employed and proper resummation of these diverging series is made (see, e.g. Refs.1–10). Numerical estimates for critical exponents of the Ising, XY, Heisenberg and some other models obtained within field-theoretical RG machinery are referred today as canonical numbers11–14 and widely used in course of comparison of the theory with computer and physical experiments including advanced measurements performed in space15.

RG expansions being power series in renormalized quartic coupling constant $g$ or in $\epsilon = 4 - D$ have coefficients that grow factorially with their number $k$. To struggle this divergency the Borel transformation is usually employed which turns divergent series into expansions having non-zero radius of convergence. Resummation methods based on Borel transformation work very efficiently when original series are alternating and their coefficients demonstrate regular behavior, i.e. being monotonically decreasing functions of $k$ for moderate $k$, they monotonically grow up under $k \to \infty$. Fortunately, RG expansions for the $\beta$-function and ”big” critical exponents ($\gamma$, $\nu$ and some others) are precisely such regular series both in three and two1,2,16,17 dimensions. This is one of the main reasons why field-theoretical RG approach turned out to be so effective numerically in the phase transition problem.

However, there is a critical exponent for which RG series are not so ”friendly”. We mean the Fisher exponent $\eta$. Let us look at the perturbative expansion of $\eta$ for the three-dimensional Ising model and at the corresponding $\epsilon$-expansion that are known today in seven-loop1,18 and five-loop3 approximations respectively. They are as follows:

$$
\eta = 0.0109739369g^2 + 0.0009142223g^3 + 0.0017962229g^4 - 0.000653698g^5 + 0.00138781g^6 - 0.0016977g^7.
$$

(1)

$$
\eta = 0.01852\epsilon^2 + 0.01869\epsilon^3 - 0.00833\epsilon^4 + 0.02566\epsilon^5.
$$

(2)
The coefficients of above series as seen to be quite irregular, both in sign and modulo. That is why the resummation of such series by canonical (Pade-Borel, conform-Borel, etc.) methods is much less effective than in the case of Wilson fixed point location $g^*$ and big critical exponents. As a results, one usually prefers to evaluate the exponent $\eta$ via scaling relations instead of dealing with corresponding RG series (see, e. g. comprehensive review\cite{14}).

In such a situation it is reasonable to address some alternative technique which is able to turn original RG expansions into more appropriate ones. Here we do not mean avoiding of factorial growth of the coefficients since in series (1), (2) they are small or, at least, not too large. Instead, we are looking for a tool which would convert RG expansion for $\eta$ into the series regular in sign along with making higher-order coefficients to monotonically decrease with growing $k$.

Below, it will be shown that the pseudo-$\epsilon$ expansion can play a role of such a tool. This approach invented by B. Nickel many years ago (see Ref. 19 in the paper of Le Guillou and Zinn-Justin\cite{2}) exploits the idea that Wilson fixed point location in three dimensions may be found iteratively by means of introducing fictitious small parameter $\tau$ into linear term of perturbative series for $\beta$-function. Pseudo-$\epsilon$ expansion proved to be very efficient when used to estimate critical exponents and other universal quantities characterizing critical behavior of three-dimensional systems\cite{2,7,19,20,23}. Even in two dimensions, where RG series are shorter and more strongly divergent, it leads to good or satisfactory numerical results\cite{2,17,21,22,24}. As we will see, for the exponent $\eta$ the pseudo-$\epsilon$ expansion turns out to be highly effective as well: it generates iteration procedures rapidly converging to the asymptotic values that are in good agreement with the numbers extracted from alternative field-theoretical and lattice calculations.

It is worthy to note that ability of pseudo-$\epsilon$ expansion approach to accelerate RG iterations and to smooth oscillations of numerical estimates as functions of $k$ was discovered just after beginning of its application\cite{2}. It was observed also that in many cases pseudo-$\epsilon$ expansions do not require advanced resummation procedures; as a rule, use of Padé approximants or even direct summation are sufficient to lead to proper numerical results. In our case, however, pseudo-$\epsilon$ expansion demonstrates an extra advantage – it turns the series with a structure rather unfavorable from the computational point of view into those quite suitable for numerical estimates.
II. PSEUDO-\(\epsilon\) EXPANSIONS FOR \(n = 0, 1, 2, 3\)

Critical thermodynamics of three-dimensional systems with \(O(n)\)-symmetric vector order parameters is described by Euclidean field theory with the Hamiltonian:

\[
H = \int d^3x \left[ \frac{1}{2}(m_0^2 \varphi_\alpha^2 + (\nabla \varphi_\alpha)^2) + \frac{\lambda}{24}(\varphi_\alpha^2)^2 \right],
\]

where bare mass squared \(m_0^2\) is proportional to the deviation from mean field transition temperature. Perturbative expansions for the \(\beta\)-function and critical exponents of this model were calculated in the six-loop approximation within the massive theory\(^1\).\(^2\)\(^5\). Later, RG series for critical exponents were extended up to seven-loop order by Murray and Nickel in their unpublished work\(^18\); seven-loop terms were reported in the paper of Guida and Zinn-Justin\(^7\).

We derive the pseudo-\(\epsilon\) expansion for critical exponent \(\eta\) starting from RG series mentioned. To do this one has to substitute the \(\tau\)-series for the Wilson fixed point location \(g^*\) into perturbative expansion for the Fisher exponent and reexpand it in \(\tau\). Pseudo-\(\epsilon\) expansion of \(g^*\) for general \(n\) is known up to \(\tau^6\) term (six-loop order)\(^23\). At first glance, with this expansion in hand \(\tau\)-series for critical exponents may be found within the same \(\tau^6\) approximation. It is really so for all critical exponents but the Fisher one. Since the first non-zero term in RG expansion of \(\eta\) is proportional to \(g^2\) the length of \(\tau\)-series for \(g^*\) turns out to be sufficient to find \(\tau^7\) term. Seven-loop contribution in RG expansion of \(\eta\) was calculated for concrete values of \(n\) most interesting from the physical point of view\(^18\). That is why here we present pseudo-\(\epsilon\) expansions of \(\eta\) for \(n = 0, 1, 2, 3\) only, leaving six-loop (\(\tau^6\)) series at generic \(n\) for Section IV. Seven-loop \(\tau\)-series obtained are as follows:

\[
\eta = 0.0092592593\tau^2 + 0.0089160938\tau^3 + 0.004342287\tau^4 + 0.002834158\tau^5 + 0.0009392592\tau^6 + 0.0017464\tau^7, \quad n = 0
\]

\[
\eta = 0.0109739369\tau^2 + 0.0101871237\tau^3 + 0.005044182\tau^4 + 0.003205816\tau^5 + 0.00145159\tau^6 + 0.0016264\tau^7, \quad n = 1
\]

\[
\eta = 0.0118518519\tau^2 + 0.0105390747\tau^3 + 0.005188190\tau^4 + 0.003229563\tau^5 + 0.00145159\tau^6 + 0.0016264\tau^7, \quad n = 2
\]

\[
\eta = 0.0127525925\tau^2 + 0.0105390747\tau^3 + 0.005188190\tau^4 + 0.003229563\tau^5 + 0.00145159\tau^6 + 0.0016264\tau^7, \quad n = 3
\]

\[
\eta = 0.0136535925\tau^2 + 0.0105390747\tau^3 + 0.005188190\tau^4 + 0.003229563\tau^5 + 0.00145159\tau^6 + 0.0016264\tau^7, \quad n = 4
\]
\[
\eta = 0.0122436486 \tau^2 + 0.0104041740 \tau^3 + 0.005026652 \tau^4 \\
+ 0.003060806 \tau^5 + 0.0014632 \tau^6 + 0.0014657 \tau^7, \quad n = 3
\]  
(7)

Series (4)-(7) are seen to have much more regular structure than original RG expansions. Their coefficients possess the same sign and monotonically decrease with increasing \( k \), apart from those of seven-loop (\( \tau^7 \)) terms. These coefficients being small are nevertheless some bigger than their six-loop (\( \tau^6 \)) counterparts signalizing that \( \tau \)-series remain divergent. Despite of this, expansions (4)-(7) turn out to be quite suitable for getting numerical estimates.

III. NUMERICAL RESULTS: FAST CONVERGENCE TO ACCURATE ASYMPTOTES.

Numerical values of \( \eta \) are extracted from the series (4)-(7) by means of Padé approximants [L/M] and by direct summation (DS). Padé triangles for Ising and Heisenberg models are presented in Tables I and II as typical examples. Note that symbol [L/M] denotes here Padé approximants constructed for \( \eta/\tau^2 \), i. e. with insignificant factor \( \tau^2 \) having physical value \( \tau = 1 \) ignored.

As seen from Tables I and II the estimates of \( \eta \) given by highest-order near diagonal approximants [3/2] and [2/3] are very close to the numbers resulting from resummed 3D RG expansions; they differ from each other by 0.001 (3 per cent) or less. Moreover, the convergence of pseudo-\( \epsilon \) expansion estimates to the asymptotic values turns out to be fast what also may be seen from both tables. Similar behavior of estimates is observed in the case of direct summation of the series (4)-(7).

This is clearly demonstrated by Table III where Padé and DS estimates of \( \eta \) as functions of \( k \) are collected along with the values the field-theoretical and lattice calculations yield. One can see that for all four values of \( n \) both iteration schemes lead to the numbers which agree well with other high-precision estimates. As seen from Table III the deviation of pseudo-\( \epsilon \) expansion estimates from the alternative values is much smaller than characteristic difference between these values themselves. DS estimates reach their asymptotes monotonically what is a direct consequence of the \( \tau \)-series structure. On the contrary, the behavior of Padé estimates turns out to be oscillatory, i. e. typical for this and other, more sophisticated resummation procedures (see, e. g. classical papers). Corresponding oscillation, however, are weak what is known to be specific for the pseudo-\( \epsilon \) expansion technique.
Keeping in mind optimistic results just obtained, the question arises: will numerically favorable structure of pseudo-$\epsilon$ expansion for Fisher exponent demonstrated at $0 \leq n \leq 3$ persist for larger $n$? In other words, whether numerical power of the pseudo-$\epsilon$ expansion is its generic property or it manifests itself only for moderate $n$? To answer this questions we are in a position to study $\tau$-series for $\eta$ at arbitrary $n$.

IV. LARGE $n$ AND ROOTS OF FORTUNE.

Perturbative RG expansion of $\eta$ for general $n$ are known today within six-loop approximation\textsuperscript{4}. This enables us to derive corresponding pseudo-$\epsilon$ expansion ranging up to $\tau^6$ term. Straightforward calculation leads to the following $\tau$-series:

\[
\eta = \tau^2 \left( \frac{0.5925925926 + 0.2962962963n}{(n+8)^2} \right)
+ \tau^3 \left( \frac{36.52032036 + 26.24895084n + 4.043763332n^2 + 0.024684014n^3}{(n+8)^3} \right)
+ \tau^4 \left( \frac{1138.304360 + 1139.876143n + 362.9490746n^2}{(n+8)^4} \right)
+ \tau^5 \left( \frac{39.3100643n^3 + 0.2496327902n^4 - 0.0042985626n^5}{(n+8)^5} \right)
+ \tau^6 \left( \frac{47549.2884 + 57808.8268n + 26407.6964n^2 + 5708.39224n^3}{(n+8)^6} \right)
+ \tau^7 \left( \frac{519.915765n^4 + 6.06899481n^5 - 0.3213367385n^6 - 0.006550922n^7}{(n+8)^7} \right)
+ \tau^8 \left( \frac{1008457.5 + 1750566.0n + 1226035.8n^2 + 440973.33n^3}{(n+8)^8} \right)
+ \tau^9 \left( \frac{83719.223n^4 + 7199.2401n^5 + 92.760879n^6}{(n+8)^9} \right) - \tau^{10} \left( \frac{10.569441n^7 - 0.41561284n^8 - 0.00554892n^9}{(n+8)^{10}} \right).
\]

Analyzing this series under various $n$ lying between 4 and 64 we find that:

i) series (8) have positive and monotonically decreasing coefficients up to $n = 24$;

ii) coefficients of $\tau^5$ and $\tau^6$ terms change their signs at $n = 40$ and $n = 24$ respectively while other coefficients remain positive and monotonically decreasing;

iii) up to $n = 64$ coefficients of $\tau^5$ and $\tau^6$ terms persist to be tiny ($\approx 0.0007$ and smaller), so these terms do not influence appreciably upon numerical estimates the series (8) yields.

Hence, the structure favorable for getting numerical estimates is a generic property of the pseudo-$\epsilon$ expansion for Fisher exponent. Comparison of the values of $\eta$ resulting from
expansions (4)-(8) with each other and with their counterparts obtained within other approaches confirms this conclusion. These values are collected in Table IV, along with the numbers given by the $1/n$-expansion:

$$\eta = \frac{8}{3\pi^2} - \frac{512}{27\pi^4} \frac{1}{n^2} - \frac{1.881234507}{n^3}.$$  

(9)

All the data presented are seen to be in a good agreement at any $n$.

Why the pseudo-$\epsilon$ expansion technique turns out to be so efficient in particular case considered? We have an explanation of this fact. The point is that mechanism of pseudo-$\epsilon$ expansion is organized in such a way that it suppresses the divergency of RG expansions for critical exponents provided these expansions are alternating. The mechanism of suppression works well for alternate series because in course of transformation of RG series into pseudo-$\epsilon$ expansions multiple mutual subtractions ("destructive interference") of the terms of original series take place. If, however, we apply this technique to series with positive coefficients the subtraction is changed by summation ("constructive interference") what makes relevant terms in pseudo-$\epsilon$ expansion larger than that of RG series.

This can be demonstrated explicitly. Let the pseudo-$\epsilon$ expansion for renormalized quartic coupling constant at criticality (Wilson fixed point location) $g^*$ be:

$$g^* = \tau + A\tau^2 + B\tau^3 + C\tau^4 + D\tau^5 + ..., \quad (10)$$

while RG series for some critical exponent $\psi$ have a form:

$$\psi = p_0 - p_1 g + p_2 g^2 - p_3 g^3 + p_4 g^4 - p_5 g^5 + .... \quad (11)$$

Typically, all $p_i$ are positive, i. e. the series (11) is alternating. To obtain $\tau$-series for $\psi$, we have to substitute expansion (10) into (11). It yields:

$$\psi = p_0 - p_1 \tau + (-Ap_1 + p_2)\tau^2 + (-Bp_1 + 2Ap_2 - p_3)\tau^3 + [-Cp_1 + (A + 2B)p_2 - 3Ap_3 + p_4]\tau^4 + ... \quad (12)$$

If coefficients of pseudo-$\epsilon$ expansion (10) for $g^*$ are positive, what is really the case for several lower-order terms, coefficients of the series (11) interfere within (12) destructively. It is clearly seen from the structure of series (12). The character of interference, however, depends crucially on signs of coefficients of initial RG expansion. Indeed, if we changed signs of odd terms in (11), i. e. made all terms in (11) positive, destructive interference would turn into constructive what again is clearly seen from (12).
What happens in the case of Fisher exponent? Since lower-order terms in RG expansions for $\eta$ have the same sign (see, e. g. (1)) pseudo-$\epsilon$ expansion machine grows them up. On the contrary, for the higher-order terms this machine works as suppressive because starting from $g^4$ term series (1) becomes (looks as) alternating. As a result, pseudo-$\epsilon$ expansion technique transforms RG series with small and irregular coefficients into $\tau$-series which possesses larger lower-order coefficients and decreasing with $k$ higher-order ones, i. e. demonstrates behavior similar to that of converging series. Fig.1 illustrates such metamorphosis for the Ising ($n = 1$) and Heisenberg ($n = 3$) models.

In fact, pseudo-$\epsilon$ expansion does not generate convergent expansions. Instead, it replaces one diverging series by another, less strongly divergent. The resulting expansions, however, have much more favorable structure from the numerical point of view. In our case pseudo-$\epsilon$ expansions actually do not require resummation, even in its simplest – Padé – form: as seen from Tables III and IV the highest-order ($\tau^7$) estimates given by direct summation and found by means of Padé analysis differ from each other by 3 percents or less. This difference is much smaller than individual and overall error bars characteristic for data provided by high-precision field-theoretical and lattice calculations. Thus, as was argued earlier, the pseudo-$\epsilon$ expansion approach may be considered as a resummation method. This method, however, is somewhat specific – it does not turn divergent series into convergent but makes them very convenient for practical use.

V. CONCLUSION

To summarize, we have calculated pseudo-$\epsilon$ expansions of the Fisher critical exponent up to $\tau^7$ terms for $n = 0, 1, 2, 3$ and within six-loop ($\tau^6$) approximation for general $n$. These expansions have been found to possess a structure that is rather favorable for getting numerical estimates. Having processed $\tau$-series obtained by means of Padé approximants and performed their direct summation we’ve obtained numerical estimates of $\eta$ that are as accurate as those extracted from advanced field-theoretical and lattice calculations. The structure of $\tau$-series for $\eta$ persists to be favorable within the wide range of $n$ signaling that it is a generic property of the pseudo-$\epsilon$ expansion for $\eta$. We have found arguments shedding light on the roots of such fortune. They lie in the general properties of the pseudo-$\epsilon$ expansion
machinery interfering with some peculiarities of the RG expansion for $\eta$.

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TABLE I: Padé triangle for pseudo-ε expansion of critical exponent \( \eta \) of 3D Ising model. Approximants are constructed for \( \eta/\tau^2 \), i.e. with factor \( \eta/\tau^2 \) omitted. Approximants [0/1], [0/3], [0/5] and [4/1] have poles close to 1; their locations are shown as subscripts. Canonical values of \( \eta \) resulting from resummed RG series and ε-expansion are 0.0335 ± 0.0025 and 0.0365 ± 0.0050 respectively.

| \( M \setminus L \) | 0   | 1   | 2   | 3   | 4   | 5   |
|-------------------|-----|-----|-----|-----|-----|-----|
| 0                 | 0.0110 | 0.0212 | 0.0262 | 0.0294 | 0.0307 | 0.0324 |
| 1                 | 0.1531_{1.08} | 0.0312 | 0.0350 | 0.0316 | 0.0256_{0.75} |
| 2                 | 0.0232 | 0.0339 | 0.0327 | 0.0333 |
| 3                 | 0.0467_{1.25} | 0.0322 | 0.0332 |
| 4                 | 0.0258 | 0.0342 |
| 5                 | 0.0570_{1.13} |

TABLE II: The same as Table I but for \( n = 3 \) (Heisenberg model). Approximant [4/1] has a pole practically equal to 1; corresponding estimate does not exist. Resummed 3D RG series give 0.0355 ± 0.0025\( ^{7} \) and 0.0350 ± 0.0008\( ^{26} \) while the ε-expansion yields \( \eta = 0.0355^{7} \).

| \( M \setminus L \) | 0   | 1   | 2   | 3   | 4   | 5   |
|-------------------|-----|-----|-----|-----|-----|-----|
| 0                 | 0.0122 | 0.0226 | 0.0277 | 0.0307 | 0.0322 | 0.0337 |
| 1                 | 0.0815_{1.18} | 0.0324 | 0.0355 | 0.0335 | - |
| 2                 | 0.0265 | 0.0346 | 0.0341 | 0.0347 |
| 3                 | 0.0414 | 0.0339 | 0.0344 |
| 4                 | 0.0304 | 0.0354 |
| 5                 | 0.0432_{1.25} |
TABLE III: Convergence of two iteration schemes generated by the pseudo-\( \epsilon \) expansion of critical exponent \( \eta \) for the polymer (SAW), Ising, XY and Heisenberg models; \( k \) is the order of approximation (number of loops). Upper lines contain estimates obtained with a help of Padé approximants, lower lines correspond to direct summation. Padé estimates are those given by diagonal approximants \([1/1], [2/2]\) for \( \eta/\tau^2 \) or by near diagonal ones; in the latter case Pade estimates being the averages over two working approximants. Since approximant \([0/1]\) has pole close to 1 its counterpart \([1/0]\) is used for final estimates; they are marked with asterisks. High-precision values of Fisher exponent resulting from 6-loop RG series, 5-loop \( \epsilon \)-expansion and lattice calculations (LC) are presented for comparison.

| \( k \) | 2  | 3  | 4  | 5  | 6  | 7  | 3D RG | \( \epsilon \)-exp. | LC |
|-------|----|----|----|----|----|----|-------|----------------|----|
| \( n = 0 \) | Padé | 0.0093 | 0.0182* | 0.0266 | 0.0300 | 0.0280 | 0.0285 | 0.0284 \( ^7 \) | 0.0315 \( ^7 \) |
|       | DS  | 0.0093 | 0.0182 | 0.0225 | 0.0254 | 0.0263 | 0.0280 | 0.0284 \( ^7 \) | 0.0315 \( ^7 \) |
| \( n = 1 \) | Padé | 0.0110 | 0.0212* | 0.0312 | 0.0344 | 0.0327 | 0.0332 | 0.0335 \( ^7 \) | 0.0365 \( ^7 \) | 0.0360 \( ^{27-29} \) |
|       | DS  | 0.0110 | 0.0212 | 0.0262 | 0.0294 | 0.0307 | 0.0324 | 0.0335 \( ^7 \) | 0.0364 \( ^{11} \) |
| \( n = 2 \) | Padé | 0.0119 | 0.0224* | 0.0326 | 0.0356 | 0.0343 | 0.0348 | 0.0354 \( ^7 \) | 0.0370 \( ^7 \) | 0.0380 \( ^{30} \) |
|       | DS  | 0.0119 | 0.0224 | 0.0276 | 0.0308 | 0.0323 | 0.0339 | 0.0349 \( ^{26} \) |            |    |
| \( n = 3 \) | Padé | 0.0122 | 0.0226* | 0.0324 | 0.0351 | 0.0341 | 0.0346 | 0.0355 \( ^7 \) | 0.0355 \( ^7 \) | 0.0375 \( ^{31} \) |
|       | DS  | 0.0122 | 0.0226 | 0.0277 | 0.0307 | 0.0322 | 0.0337 | 0.0350 \( ^{26} \) |            |    |
TABLE IV: Numerical values of the Fisher exponent for various $n$ obtained from seven-loop ($n = 0, 1, 2, 3$) and six-loop ($n \geq 4$) pseudo-$\epsilon$ expansions processed with a help of Padé approximants and by direct summation (DS). Padé estimates are those given by diagonal approximants $[2/2]$ for $\eta/\tau^2$ or by near diagonal ones (for $n = 0, 1, 2, 3$); in the latter case Padé estimates being the averages over working approximants $[3/2]$ and $[2/3]$. At $n = 64$ and 48 approximant $[2/2]$ has pole close to 1; the values reported (marked with asterisks) are averages over numbers given by approximants $[3/1]$ and $[1/3]$. The values of $\eta$ resulting from 6-loop RG series in three dimensions, obtained within the $\epsilon$-expansion and $(1/n)$-expansion approaches are presented for comparison.

| $n$ | Padé | DS | 3D RG$^4$ | 3D RG$^7$ | $\epsilon$-exp$^7$ | $(1/n)$-exp. |
|-----|------|----|-----------|-----------|----------------|--------------|
| 0   | 0.0285 | 0.0280 | 0.0284 | 0.0315 |
| 1   | 0.0332 | 0.0324 | 0.038 | 0.0335 | 0.0365 |
| 2   | 0.0348 | 0.0339 | 0.039 | 0.0354 | 0.0370 |
| 3   | 0.0346 | 0.0337 | 0.038 | 0.0355 | 0.0355 |
| 4   | 0.0329 | 0.0313 | 0.036 | 0.0350 | 0.033 | 0.0260 |
| 8   | 0.0260 | 0.0252 | 0.027 | |
| 16  | 0.0164 | 0.0163 | 0.017 | |
| 24  | 0.01173 | 0.01168 | 0.012 | |
| 32  | 0.00915 | 0.00902 | 0.009 | |
| 40  | 0.00761 | 0.00732 | |
| 48  | 0.00627* | 0.00616 | |
| 64  | 0.00490* | 0.00466 | |
FIG. 1: (Color online) Fairy metamorphosis of perturbative RG series for Fisher exponent at $n = 1$ and $n = 3$ under the action of pseudo-$\epsilon$ expansion machinery. Left histograms depict weight of different terms in RG series at the Wilson fixed point, right histograms show analogous distributions for pseudo-$\epsilon$ expansions (5) and (7), $k$ being an order of a term.