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Neutrinos in dense media: helicity- and pair correlations

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Abstract. Neutrinos propagating in media (matter and electromagnetic fields) undergo flavor and helicity oscillations, where helicity transitions are induced both by electromagnetic fields and matter currents. In addition, it has been shown that correlations between neutrinos and antineutrinos of opposite momentum can build up in anisotropic media. We discuss the neutrinos’ equations of motion in the mean-field approximation for homogeneous yet anisotropic media highlighting previous results on the physical implications and typical orders of magnitude of helicity- and pair correlations.

1. Introduction
Core-collapse supernovae are highly complex astrophysical systems. To our current understanding, explosions become possible because neutrinos, which are emitted out of equilibrium from the core, provide their energy to a shock front that finally disrupts the outer layers of the dying star. This explosion mechanism depends delicately on the details of the underlying physics since even small changes in the input parameters can turn an unsuccessful simulation of a supernova into a successful one [1].

Important aspects of neutrino propagation include their flavor content and energy spectrum which determine the efficiency of the energy transport towards the stalling shock wave. While the large matter density inside supernovae suppresses flavor oscillations, coherent $\nu-\bar{\nu}$-scattering leads to self-induced flavor conversions that might produce exponential growth of seemingly negligible terms and that make a detailed understanding of the propagation relevant [2, 3].

During propagation, neutrinos do not only undergo flavor oscillations. The helicity of neutrinos can be flipped when they cross anisotropic backgrounds converting a negative-helicity Dirac neutrino into a positive-helicity one, or turning a Majorana neutrino into an antineutrino. These helicity oscillations have been studied by, e.g., refs. [4, 5, 6]. Besides helicity flips, an anisotropic background also induces correlations between the vacuum and states with a neutrino/antineutrino pair. For these pair correlations that eventually lead to spontaneous pair creation, the formalism has been developed in refs. [7, 8] and complemented by ref. [9], while the physical implications have been pointed out in ref. [9].

In this proceeding, we summarize some of the main results of ref. [9], where besides re-deriving the equations of ref. [8], also the typical magnitudes of helicity and pair correlations were estimated and the possibility of adiabatic conversions were scrutinized. We also repeat the main arguments that lead to the interpretation of pair correlations as a coherence between the vacuum and a paired state.

The plan of this proceeding is as follows: In section 2, we give a short overview of the density matrix formalism that will be applied in section 3 to the problem of helicity-flips. This formalism will also be used to study pair correlations in section 4. We conclude in section 5.
2. Density matrices

The density matrix formalism for neutrino propagation was developed in ref. [10]. The evolution equation for the density matrix $\rho$ in a homogeneous background reads

$$i\dot{\rho} = [H, \rho] + \text{coll.},$$

where $H$ is the Hamiltonian matrix whose entries are derived from the Hamiltonian operator. The collision terms (coll.) are only relevant deep inside the supernova’s core and will be neglected in the following.

The entries of the density matrix for Dirac neutrinos are defined as

$$(2\pi)^3 \delta^3(p-k) \rho_{ij}(t,p) = \langle a_i^\dagger(t,p) a_j(t,k) \rangle,$$  

$$(2\pi)^3 \delta^3(p-k) \bar{\rho}_{ij}(t,-p) = \langle b_i^\dagger(t,-p) b_j(t,-k) \rangle,$$

where the ensemble average is taken over (anti)neutrino creation/annihilation operators $a^\dagger, a$ ($b^\dagger, b$) that correspond to mass eigenstates $i, j$. The vectors $p$ and $k$ denote 3-momenta. The diagonal $i = j$ elements of $\rho$ give the number densities of the mass eigenstates, and the off-diagonal elements describe the coherence between different mass eigenstates. We choose the mass basis in the following since this will simplify the description of helicity oscillations which are proportional to the neutrinos’ masses.

Majorana neutrinos can be described in a similar way as eqs. (2) and (3). For simplicity, we will, however, concentrate on Dirac neutrinos in the following.

3. Helicity oscillations

Neutrinos couple to matter through the weak interaction, which is chiral and, hence, sensitive to the helicity of a particle. This implies that if a Dirac neutrino, which is predominantly produced in a negative helicity state, undergoes a helicity flip, its coupling to matter will be suppressed by $m/2E$, where $m$ is its mass and $E$ the neutrino’s energy. This significantly reduces the neutrino’s opacity and its energy deposition at the shock wave. For Majorana neutrinos, a spin flip corresponds to converting a neutrino into its antiparticle, which affects nucleosynthesis through the proton-to-neutron ratio.

Helicity can be easily included in the density matrix formalism by extending $\rho$ and $H$ to larger matrices. The entries of the density matrix now read

$$(2\pi)^3 \delta^3(p-k) \rho_{ij,sh}(t,p) = \langle a_{ij}^\dagger(t,p) a_{is}(t,k) \rangle,$$

where $s, h \in [-,+]$ denote the helicity states. An analogous definition holds for antineutrinos. The Hamiltonian matrix is of the form

$$H^{\nu\nu} = \begin{pmatrix} H^{\nu\nu}_{++} & H^{\nu\nu}_{+\nu} \\ H^{\nu\nu}_{\nu+} & H^{\nu\nu}_{++} \end{pmatrix},$$

where each entry still contains a flavor structure that reduces to a number in the one-flavor case. The off-diagonal terms are of great importance since they are responsible for changes of the neutrinos’ helicities.

If neutrinos undergo a helicity flip, their angular momentum will change. We, therefore, need an anisotropic background that can take up this angular momentum. There are two vectors available in a supernova: background currents and magnetic fields. To estimate their relative importance, we have to compare the entries in the Hamiltonian matrix that lead to helicity flips,

$$H^{\nu\nu}_{++} = \frac{m}{2E} \beta V - \mu \cdot B.$$

Here, the first term corresponds to matter currents with a typical potential $V = G_F n_n / \sqrt{2}$ and velocity $\beta$ transversal to the neutrino’s direction of movement. The magnetic field $B$ couples to neutrinos via...
their magnetic moment \( \mu \). For typical values and standard model-like magnetic moments,

\[
\begin{align*}
H^\nu_+ &= 10^{-12} \text{eV} \left( \frac{m}{0.1 \text{eV}} \right) \left( \frac{60 \text{MeV}}{2E} \right) \left( \frac{\beta}{0.01} \right) \left( \frac{V}{0.1 \text{eV}} \right) \\
-10^{-16} \text{eV} \left( \frac{\mu}{3 \times 10^{-20} \mu_B} \right) \left( \frac{|B|}{10^{12} \text{G}} \right),
\end{align*}
\]

(7)

and the effect of the magnetic field can be neglected because it is four orders of magnitude smaller than the potential from matter currents.

To assess the importance of helicity oscillations, we also need to estimate the coherences \( \rho_{+-} \). Since neutrinos are emitted from the core that is in thermal equilibrium, a first estimate can be obtained by solving the evolution equation (1) in equilibrium to obtain

\[
\rho_{+-} = \frac{H^\nu_+}{H^\nu_- - H^\nu_+} (\rho_{--} - \rho_{++}) \approx \frac{m}{2E} \beta \approx 10^{-11},
\]

(8)

which is again suppressed by \( m/(2E) \) and, hence, a very small value compared to the density of negative helicity neutrinos \( \rho_{--} \sim O(0.1) \).

These coherences become large if the denominator of eq. (8) vanishes such that the energy levels of positive- and negative helicity states become quasi-degenerate. For Majorana neutrinos, such an avoided branch crossing has been studied in ref. [11], where it was found that for an electron fraction \( Y_e \approx 1/3 \) such an enhancement can be realized. However, this crossing has to be adiabatic for an efficient conversion, which requires some tuning [11].

4. Pair correlations
Traditionally, the entries of the density matrix [eq. (4)] involved either neutrino or antineutrino operators but not both. In ref. [7] however, the mathematical description of supernova neutrinos was extended by a new type of correlator \( \kappa \), which has been formerly used in nuclear physics. The homogeneous evolution equations in the mean-field limit and including the \( \kappa \)-correlators have then been derived in ref. [8]. The \( \kappa \)-matrices are

\[
(2\pi)^3 \delta^3(p + k) \kappa_{ij,s,h}(t, p) = \langle b_{j,h}(t, k) a_{i,s}(t, p) \rangle,
\]

(9)

\[
(2\pi)^3 \delta^3(p + k) \kappa^\dagger_{ij,s,h}(t, -p) = \langle a^\dagger_{j,h}(t, k) b^\dagger_{i,s}(t, p) \rangle.
\]

(10)

To incorporate these matrices in our eq. (1), we have to generalize the density matrix

\[
R = \begin{pmatrix}
\rho & \kappa \\
\kappa^\dagger & 1 - \beta
\end{pmatrix},
\]

(11)

such that the evolution equation reads

\[
i\dot{R} = [H, R],
\]

(12)

and the new Hamiltonian matrix can be decomposed as

\[
H = \begin{pmatrix}
H^\nu & H^{\bar{\nu}} \\
H^{\bar{\nu}} & H^\bar{\nu}
\end{pmatrix},
\]

(13)

where \( H^\nu \) and \( H^{\bar{\nu}} \) now mix neutrinos and antineutrinos. Flavor and helicity indices have been left implicit. Before studying the size of these new terms in the Hamiltonian and the density matrices, it is instructive to ask what physical interpretation adheres to the \( \kappa \)-matrices.
Physical interpretation. In ref. [9], we have shown that including non-zero $\kappa$-matrices corresponds to spontaneous creation of a back-to-back neutrino/antineutrino pair. To illustrate this point, we will in the following restrict ourselves to one generation of a massless neutrino and one momentum mode. Equation (12) then simplifies to a $2 \times 2$-system. With these approximations, (anti)particle number densities change according to

$$\dot{\rho} = -2\text{Im} \left( \kappa H^{\bar{\nu} \nu} \right),$$ \hspace{1cm} (14)

$$\dot{\bar{\rho}} = -2\text{Im} \left( \kappa H^{\nu \bar{\nu}} \right),$$ \hspace{1cm} (15)

such that the difference $\rho - \bar{\rho}$ is conserved, while the sum $(\rho + \bar{\rho})$ is not. For non-zero $\kappa$-matrices, neutrino and antineutrino densities increase and decrease simultaneously.

Pair creation can be more rigorously shown in the Schrödinger formalism. If we define a general, normalized state

$$|\psi(t)\rangle = A_{00}(t)|00\rangle + A_{11}(t)|11\rangle + A_{10}(t)|10\rangle + A_{01}(t)|01\rangle,$$ \hspace{1cm} (16)

as a superposition of the vacuum $|00\rangle$, the neutrino $|10\rangle$, antineutrino $|01\rangle$, and the paired state $|11\rangle$, the evolution separates into three equations [9]

$$i\partial_t \begin{pmatrix} A_{00} \\ A_{11} \end{pmatrix} = \begin{pmatrix} 0 & \beta V \\ \beta V & 2E \end{pmatrix} \begin{pmatrix} A_{00} \\ A_{11} \end{pmatrix},$$ \hspace{1cm} (17)

$$i\partial_t A_{10} = H^{\nu \bar{\nu}} A_{10},$$ \hspace{1cm} (18)

$$i\partial_t A_{01} = -H^{\nu \bar{\nu}} A_{01}.$$ \hspace{1cm} (19)

Here, an anisotropic background with velocity $\beta$ and potential $V$ induces transitions from the vacuum to the paired state, while the states that are filled with just one (anti)neutrino only develop a phase.

By defining $\rho = |A_{11}|^2 + |A_{10}|^2$, $\bar{\rho} = |A_{11}|^2 + |A_{01}|^2$, and $\kappa = A_{00}A_{11}$, one can show that these equations are completely equivalent to the evolution equations obtained from eq. (12). Especially, the definition of $\kappa = A_{00}A_{11}$ makes clear that non-zero $\kappa$ corresponds to a coherence between the vacuum and the paired state.

Explicitly, the evolution equation for $\kappa$ reads [8, 9]

$$i\dot{\kappa} = (H^{\nu \bar{\nu}} - H^{\bar{\nu} \nu})\kappa + H^{\nu \bar{\nu}}(1 - \rho - \bar{\rho})$$

$$\approx 2E\kappa + H^{\nu \bar{\nu}}(1 - \rho - \bar{\rho}),$$ \hspace{1cm} (20)

from which it can be seen that $\kappa$ oscillates with a very large frequency $2E \sim 10^{22}$ Hz for $E = 15$ MeV. This has been used by ref. [3] to invoke a new type of instability inside supernovae.

Order of magnitude. We perform a similar analysis as in sec. 3 to estimate the typical size of the particle/antiparticle correlators. Solving for $\kappa$ in equilibrium ($\dot{\kappa} = 0$) in eq. (20), we obtain

$$\kappa = -\frac{H^{\nu \bar{\nu}}}{H^{\nu \bar{\nu}} - H^{\bar{\nu} \nu}}(1 - \rho - \bar{\rho}) \sim \frac{H^{\nu \bar{\nu}}}{2E},$$ \hspace{1cm} (21)

where $H^{\nu \bar{\nu}}$ is a function that depends on an integral of $\kappa$ itself

$$H^{\nu \bar{\nu}} = -A\beta - 4\sqrt{2}G_F \hat{e} \cdot \int d^3q \text{Re}[\hat{e}\kappa].$$ \hspace{1cm} (22)

Here, $A$ is the potential that is generated by the anisotropy of neutrino and background matter currents. The vectors $\hat{e}$ correspond the spatial part of the polarization vectors of photons and project on a direction orthogonal to the neutrino momentum.
By solving for a self-consistent solution and truncating high-energy modes that go beyond our Fermi approximation, we obtain (see [9] for details)

\[ \kappa \approx G_F n \beta \frac{G_F}{2E} \approx 10^{-11}. \] (23)

Similar to helicity coherence, the equilibrium value is very small and negligible when compared to \( \rho = O(0.1) \). For the one-flavor, massless model, this estimate can also be obtained when one allows for small deviations from equilibrium [9].

Next, we can look at possible enhancements through adiabatic branch crossings. The Hamiltonian matrix reads explicitly

\[ H = \begin{pmatrix} E - V_\parallel & -V_\perp \\ -V_\perp & -E + V_\parallel \end{pmatrix}, \] (24)

where \( V_\perp \) (\( V_\parallel \)) is the potential induced by matter currents perpendicular (parallel) to the momentum direction of the neutrino. The resonance condition is fulfilled when \( V_\parallel / E \approx 1 \). For a typical supernova, however, \( V_\parallel / E \approx 10^{-9} \) such that a branch crossing cannot be obtained.

5. Outlook & conclusions

Self-induced flavor oscillations remain the most important non-trivial effect in supernova neutrino transport. We argued that the coherences associated with helicity flips are typically many orders of magnitude smaller than realistic neutrino number densities (\( \rho_+ \sim 10^{-11} \)) such that strong enhancements through MSW-like resonances or instabilities are needed to make helicity flips relevant. While the requirement of an avoided branch crossing might be obtained within realistic supernovae, the fulfillment of the second condition, that the crossing is adiabatic, is questionable.

For pair correlations \( \kappa \), the typical magnitude is very small as well (\( \kappa \sim 10^{-11} \)). Moreover, it is not possible to enhance the value of \( \kappa \) through a branch crossing since the densities that are required would be nine orders of magnitude above typical supernova densities. These results, however, have only been derived for the simple model of one momentum mode and one massless neutrino. Extending the analysis to a more realistic neutrino sector is a challenging task but might yield a much richer phenomenology. Similarly, it might be interesting to let go of the approximation of homogeneity. Unfortunately, this complicates the resulting equations enormously.

All in all, supernova neutrinos continue to be an interesting topic where even seemingly small effects as pair correlations might completely change our understanding of neutrino propagation in dense media.

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