Spectral synthesis provides 2-D videos on a 1-D screen with 360°-visibility and mirror-immunity

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1. Introduction

Light sources whose spectrum can be adjusted for specific wavelengths at specific intensities are needed for applications ranging from microscopy and endoscopy [1], to colorimetry and color imaging [2-4], to stage lighting [5] and hyperspectral imaging [6-8]. Such tunable light sources are typically based on light emitting diodes (LED)[9] or spatial light modulators (SLM) to generate light via spectral synthesis.

SLM-based tunable light sources synthesize light usually by subtracting multiple parts from a single spectrum: A white slit image, mostly from a xenon [2,4] or mercury arc lamp, is dispersed by prisms [2,5] or gratings [4,5,10] onto a digital mirror device (DMD) or liquid crystal device (LCD) [2] panel, which masks out parts of the spectrum. Each column of the SLM represents a specific peak wavelength, whose intensity is regulated either by the number of 'on' pixels in that column [2,10], or through pulse width modulation [5,11]. Thus, a spectrum, or rows of spectra, are synthesized. They are combined into a projected line [11], on a diffusing plate [2], in an integrating cavity [5], in an optical fiber, or in a liquid light guide [4]. For calibration and further use of the light engine, the spectral power distribution (SPD) of the light is measured with a camera [4] or a spectroradiometer [2,12]. An optional feedback loop optimizes the spectral output [2,4,12]. Essentially, such SLM-based light engines are conversions of a spectroscopic setup.

This conventional approach to spectral synthesis is intuitive and effective. Conversely, there are two drawbacks: A) A complex apparatus is needed for three major steps: 1) dispersion of white light, 2) SLM-based modification, 3) recombination. B) Because the SLM is used to encode wavelengths as two-dimensional (2-D) patterns, its potential to encode 2-D images as wavelengths [13] has not been realized with such light engines.

Correspondingly, I propose an SLM-based light engine that has two advantages: A) The setup is simpler, yet as effective. It synthesizes light by combining single parts from multiple spectra. This Superposition of Newtonian Spectra (SNS) takes only two steps: 1) SLM-based modification of white light, 2) dispersion. B) With the light engine, one can watch 2-D, mirror-immune videos from all around a 1-D projection screen, in a viewing method called Projected-Image Circumlineascopy (PICS).

In Section 2, I introduce SNS spectral synthesis. Building on SNS, I present PICS. Section 3 describes a setup for both SNS
and PICS. Section 4 contains the experimental results. In Section 5, I compare SNS to other methods of spectral synthesis, and discuss PICS image properties. Finally, I conceptually unify spectral synthesis with spectral encoding. Section 6 summarizes the concepts and findings, and suggests future research.

2. From SNS essentials to PICS pictures

A. Spectral Synthesis based on a Superposition of Newtonian Spectra (SNS)

![Fig. 1. In SNS, a grayscale pattern is dispersed to synthesize light at \( L_e \).](image)

1. The essence of SNS

    Light of a desired wavelength composition can be synthesized by superposing (at a linear locus \( L_e \)) different color stripes from multiple Newtonian spectra (see Fig. 1). We obtain a single Newtonian spectrum by projecting a white slit image from a broadband source through a dispersive element (e.g., a prism or grating) [14]. Different color stripes in the spectrum represent different peak wavelengths, from \( \lambda_b \) to \( \lambda_r \). For multiple Newtonian spectra, we project multiple white lines through the same dispersive element for synthesis (DE\(_n\)).

    For spectral synthesis, consider the geometry of the spectra. Suppose we project a white line through a DE\(_n\), whose distance from the projector is \( d_p \) as shown in Fig. 2(a). Then, the rays for \( \lambda_b \) and \( \lambda_r \) emerge from DE\(_n\) under a dispersion angle \( \delta_s \). (Consider \( \delta_s \) positive if the ray for \( \lambda_b \) can be made to coincide with the ray for \( \lambda_r \) by a clockwise rotation of less than 90° about the vertex.) At a distance \( d_s \) from DE\(_n\), the rays for \( \lambda_b \) and \( \lambda_r \) create two monochromatic images whose mutual displacement, the so-called dispersive displacement for synthesis, is

    \[
    s = 2\tan(0.5\delta_s)d_s \tag{1}
    \]

    (If a ray passes a series of dispersive elements, we apply Eq. (1) sequentially.)

![Fig. 2. Setup and wavelength calibration for SNS. (a)-(b) Top view. (b)-(d) Line AD is dispersed to obtain a calibration spectrum at \( L_e \).

A white line (in the \( y \)-direction) of width \( \Delta x \) at a projection distance \( D_P = d_P + d_s \) is dispersed by DE\(_n\) (in the \( x \)-direction) into a Newtonian spectrum of width

    \[
    w_s = |s| + \Delta x \tag{2}
    \]

    The linear locus of spectral synthesis \( L_s \) is the place where two Newtonian spectra of lines AB and CD kiss each other to synthesize light from the ends of the spectrum (see Fig. 1). The kissing condition is that the distance between lines AB and CD equals the dispersive spread [15], i.e., the absolute value of the dispersive displacement for synthesis:

    \[
    |AC| = |BD| = |s| \tag{3}
    \]

    Under this condition, the spectra of lines AB and CD barely overlap in a narrow zone of width \( \Delta x \), supplying peak wavelengths \( \lambda_b \) and \( \lambda_r \) to \( L_s \) (in the \( y \)-direction). Then, intermediate spectra from lines between AB and CD contribute intermediate wavelengths to \( L_s \). With lines AB and CD in place, we insert a grayscale pattern that resembles the desired slit spectrum, as in Fig. 1.

2. Calibrating the grayscale pattern

    Having established lines AB and CD, we use a white line AD for wavelength calibration, and a white rectangle ABCD to calibrate the intensity of the grayscale pattern.

    For the wavelength scale, we assign to each pattern coordinate \( x \) the peak wavelength \( \lambda_{LS}(x) \) obtained at \( L_s \):

    \[
    x = x(\lambda_{LS}) \tag{4}
    \]

    Projected through DE\(_n\), line AD (\( y = kx \), \( k \) any constant, see Fig. 2(b)) yields a calibration spectrum at \( L_s \), as in Fig. 2(d). Its wavelengths \( \lambda_{LS} = \lambda_{LS}(y) \) imply \( \lambda_{LS} = \lambda_{LS}(x) \).

    For the intensity \( I(x) \) of the grayscale pattern, we relate the desired spectral intensity distribution \( I(\lambda_{LS}) \) to the spectrum of synthesized white light \( I_{LS} = I_{LS}(\lambda_{LS}) \):

    \[
    I(x) = \mu I(\lambda_{LS}) = \frac{I(x)(\lambda_{LS})}{I_{LS}(\lambda_{LS})} \tag{5}
    \]

    The weighting function \( \mu \) holds \( I(x) \) below or at the intensity of white light, \( I_{max} \), and compensates for the fact that each pixel contributes a certain range of wavelengths. Projected through DE\(_n\), rectangle ABCD of intensity \( R(x) = I_{max} \) yields \( I_{LS} \) at \( L_s \).
3. Calculating SNS spectral bandwidth

With wavelengths \( \lambda_{LS} \), the bandwidths \( \Delta \lambda_{LS}(x) \) follow from Dispersion Diagrams [15], as in Fig. 3.

Fig. 3. Deriving SNS spectral bandwidth from Dispersion Diagrams. Dispersion vector \( s = s \hat{x}_S \); unit vector \( \hat{x}_S \) pointing to the right on \( S1 \), see Fig. 2(a). If we assume equal dispersion for all coordinates \( x \), the curvature of the dispersed wavelength distribution in Fig. 3(b) mirrors the curve for \( \lambda_{LS} = \lambda_{LS}(x) \).

To derive the wavelength distribution of the synthesized light, we shear the grayscale pattern from Fig. 3(a) according to dispersion vector \( \lambda = \frac{s}{N} \) and bandwidth \( \lambda_{LS}(x) \) being roughly constant across \( w \).

If, to simplify, we assume linear dispersion

\[
\lambda_{LS}(x) = \lambda_{LS}(0) + (\lambda_{LS}(0) - \lambda_{LS}(x)) \frac{x}{s} \quad ,
\]

then, each pixel yields a spectral bandwidth

\[
\Delta \lambda_{LS}(x) = \frac{\lambda_{LS}(0) - \lambda_{LS}(x)}{N-1} \left( 1 + \frac{w}{\Delta x} \right) \quad .
\]

B. SNS for Projected-Image Circumlinescopy (PICS)

With SNS, we obtain a spectral intensity distribution \( I_0(\lambda_{LS}, y) \) at \( L_S \), by projecting a grayscale pattern of intensity \( I = I(x,y) \) through \( DE_S \). In plain words, the slit pattern of the synthesized line of light is always a rainbow-colored version of the grayscale image.

This allows us to insert grayscale text, photos or videos between lines AB and CD, and see corresponding spectral images by looking at the line of light at \( L_S \) through a dispersive element for analysis (DE\( _S \)).

To visualize the transformation from grayscale to spectral image, see the Dispersion Diagram in Fig. 4. DE\( _S \) disperses the grayscale image in the \( x_S \)-direction, each of the monochromatic constituents supplying a different image stripe to \( L_S \). DE\( _S \) rearranges these stripes according to wavelength to form the spectral image.

3.1 setup for both SNS and PICS

An SLM-projector (SONY LCD VPL – CX 70; UHP mercury arc lamp, 2,000 ANSI lumens, 1024 x 768 pixels), connected to a PC, and an amici prism (\( \delta_s = 4.3^\circ \)) as DE\( _S \) (unless otherwise specified), were used.

As in Figure 2(a), an image of white lines AB and CD in a black presentation slide was projected at a distance \( D_r = 2m \) onto two 0.6m x 0.8m cardboards as \( S1 \). With DE\( _S \) at \( d_s = 1.93m \), this yielded two spectra, each of width \( w_s = 0.15m \), cf. Figs. 7(a)-(b). The image width was adjusted until Eq. (9) was fulfilled. The purple light at \( L_S \) exited the slit between the cardboards.

The light of the calibration spectrum from line AD went at an angle \( \theta = 0^\circ + l/3^\circ \) through a transmission grating \( G \) (grating constant \( 1/g = 1000/mm \)) at a distance \( d = 3cm \) behind the slit and onto a 2-D translucent screen \( S2 \) at a distance \( d_s = 27cm \) behind the grating, as in Figs. 2(b)-(e). From the pixel positions in a digital photograph of diffraction orders \( m=0 \) and \( m=1 \), \( \lambda_{LS} = \lambda_{LS}(y) \) was calculated with the grating formula

\[
g(\sin \theta + \sin \theta_m) = m \lambda \quad ,
\]

\( \theta_m \) being the diffraction angle.

As a translucent 1-D projection screen for the synthesized light, an uncooked capellini (diameter \( w = 0.9mm \), length \( l = 26cm \)) was placed at the slit, and \( S1 \) was removed.

For SNS spectral synthesis, grayscale patterns spanned from line AB to CD, cf. Fig. 7(a). The capellini was photographed from the left with a Nikon D80 through a diffraction grating at a distance \( d_A = 0.4m \).

For PICS, grayscale text (cf. Fig. 9(a)), photos (cf. Fig. 8(a), Fig. 11(a)) or videos (played via VLC media player) between lines AB and CD were optimized in contrast and brightness. The capellini was viewed through one or two amici prisms (dispersion angle \( \delta_A = 4.3^\circ \)) or a transmission grating (\( 1/g = 1000/mm \)) as DE\( _A \) within a radius \( d_s \leq 2m \) around the capellini, see Fig. 5.
4. Experimental Results

A. SNS Spectral Synthesis

Peak wavelengths from $\lambda_B = 431$nm to $\lambda_R = 642$ nm were identified and translated into pattern coordinates $x(x_{LS}) = 169.9 - ([\lambda_{LS} - 433.2]/0.007)^{0.5}$ using a quadratic fit, see Fig. 6. With a pixel width $\Delta x = 1$mm$\pm$0.1mm, bandwidths ranged from $\Delta \lambda_{LS}(431$nm) $\approx 0.5$nm to $\Delta \lambda_{LS}(642$nm) $\approx 5$nm, according to Eq. (6). The grayscale patterns synthesized corresponding spectra, see Fig. 7.

**Fig. 6.** The fit to the data points constitutes a wavelength calibration.

B. PICS Spectral Images

**Fig. 8.** Example images from PICS. (a) Original grayscale image of ‘Lena’. (b) Spectral image of a capellini. (c) High-resolution spectral image of a capellini.

1. Overall appearance

For some views through $DE_A$, see Figs. 8(b)-(c), 9(b)-(d), and 11(c)-(d). The virtual spectral images appeared like semitransparent banners attached to the capellini. They turned about the capellini to face a circumambulating viewer constantly. While their height looked the same as the height of the light on the capellini, their width increased with the distance $d_i$ from $DE_A$ to the capellini. $DE_A$ with stronger dispersion produced wider images. Their position, size and sharpness did not notably change for a changed distance $d_i$ between viewer and $DE_A$. If the capellini was viewed obliquely from above or below, the left and right sides of the spectral image were tilted at the same angle as the capellini was due to perspective, see Fig. 11(c). The height was foreshortened so if the capellini was seen from directly above, the image reduced to a line.

2. Image proportions

**Fig. 9.** Different spectral image proportions for different $DE_A$.

With an amici prism as $DE_A$, $d_A = 1$m was good for viewing, yet correct image proportions arose at $d_A = 2$m. With the grating as $DE_A$, $d_A = 0.5$m was suitable. At smaller distances, the image was too narrow and blurry; at larger distances, the spectral image was too wide and faint. The spectral image had different proportions for different $DE_A$. Generally, the width-to-height ratio $w/A/h_s$ of the spectral image was different from the width-to-height ratio $w/s/h_s$ of the grayscale image, see Fig. 9. Specifically, through an amici prism at $d_A = 0.95$m, the text “Hello world!” of Fig. 9(a) had $w/A/h_s = (0.5+/-.01)w/s/h_s$, see Fig. 9(c). Through a series of amici prisms at distances $d_{i1} = 0.95$m and $d_{i2} = 0.82$m, the text had $w/A/h_s = (0.9+/-.2)w/s/h_s$, see Fig. 9(d). The letters were evenly spaced in both cases. Through the grating at $d_A = 0.4$m, the text had $w/A/h_s = 0.6(+/-0.01)w/s/h_s$, being squashed toward blue letters, see Fig. 9(b).
3. Image transformations

Shifting the capellini to the left of \( L_S \) (in the negative \( x_S \)-direction) removed the right part of the image by shifting the colors. Shifting in the opposite direction effected the opposite. Shifting the capellini from \( L_S \) toward \( D_E_S \) removed parts from both sides of the image. Shifting the capellini away from \( D_E_S \) reduced the color range, but left the image intact. In both cases, the image got out of focus, yet did not notably change within a range of about 0.5m. Thus, with a series of capellini within that range, a single grayscale image yielded multiple, almost identical and sharp spectral images at once.

Flipping the orientation of dispersion of \( D_E_A \) caused the spectral image and its colours to flip, as for \( w_4 \) in Figs. 12(b)-(c). Flipping the orientation of dispersion of \( D_E_S \) caused the spectral image to flip geometrically, but the orientation of the color spectrum remained, cf. Fig. 11. Combining both procedures flipped the colors, but not the orientation of the spectral image, cf. Figs. 8(b)-(c).

Fig. 10. Mirror Immunity with PICS. (a) A regular 2-D object in front of an upright mirror \( M \) is not mirror-immune because its mirror image has reverse orientation. (b) An upright 1-D screen is mirror-immune because its mirror image is the same. (c) Hence, even the 2-D spectral image of the 1-D screen is mirror-immune.

In a mirror parallel to the capellini, the spectral image was not a mirror image, but had the same orientation as the spectral image of the capellini before the mirror, as illustrated in Fig. 10.

4. Multiple-screen, multiple-image PICS

With grayscale images beside the original one, as in Fig. 11(a), the spectral images of multiple capellini beside \( L_S \) were arranged in space as the capellini themselves. They overlapped for some viewing positions, as in Figs. 11(b)-(d). The color result depended on the spatial sequence in which the capellini were viewed. Some capellini produced a spectrally shifted ‘doppelgänger’ of the spectral image of a neighboring capellini. This happened when the capellini, and the projected grayscale images, were less than a spectrum width \( w_S \) apart, as in Fig. 11.

Fig. 11. Multiple-screen, multiple-image PICS. (a) Arrangement of 4 grayscale portraits (Newton, Fresnel, Goethe and Huygens) on a presentation slide. (b) The slide was projected across 4 capellini through a single amici prism with horizontal dispersion as \( D_E_S \). Photos (c) and (d) were directly taken through a transmission grating at positions 1 and 2. Like rotatable banners attached to the capellini, the spectral portraits always face the viewer. The amici prism at the projector (in photo (d) a white spot) can be flipped horizontally to flip each portrait horizontally.

5. Image resolution

Image resolution was enhanced with \( L_S \) further from the projector as the grayscale image could be enlarged relative to the presentation slide. Additionally, a second amici prism at the projector allowed the grayscale image to take up almost twice as many pixels horizontally. Thus, horizontal image resolution improved, as in Figs. 8(b)-(c).

Image resolution depended on the orientation of dispersion, apparently because the screen was not perfectly one-dimensional. To investigate how the width \( w \) of the projection screen at \( L_S \) affects PICS image resolution, a translucent paper was placed at a distance \( d_s = 1.5m \) from the amici prism to scatter the whole dispersed grayscale image. It was viewed through an amici prism as \( D_E_A \) from behind the paper while the effective screen width was reduced with a slit aperture directly at the paper (see Fig. 12).

Generally, a \( D_E_A \) with parallel orientation of dispersion (relative to \( D_E_S \)) made the image even blurrier, while an antiparallel \( D_E_A \) reduced the blurriness. Specifically, if the spectral images were viewed at a distance \( d_A = 1.5m \) behind the translucent paper, the difference was extreme: On the one hand, for an antiparallel \( D_E_A \), the spectral image was sharp for an arbitrarily wide screen, see Fig. 12(b). On the other hand, for a parallel \( D_E_A \), the spectral image was only sharp for an extremely narrow screen, see Fig. 12(c). The same was true for an equivalent setup with gratings. Synthesized on a single blond human hair of length \( l \approx 0.5m \),
spectral images were sharpest, yet faint, and movements in a video were easily recognizable.

![Fig. 12. Screen width and image resolution.](image)

(a) The width of a 2-D translucent screen is successively reduced from $w_1 = 17\text{cm}$, $w_2 = 8\text{cm}$, $w_3 = 2\text{cm}$ to $w_4 = 0.5\text{cm}$ using a slit aperture while the grayscale image of ‘Lena’ is projected through DE$_A$ from behind. (b) DE$_A$ compensates the dispersion of DE$_\text{S}$. (c) DE$_\text{S}$ enhances the dispersion of DE$_A$. Uncannily, ‘Lena’ looks left in the blurry spectral image at $w_1$ and $w_2$, but looks right in the sharp image at $w_3$, appearing Janus-faced at $w_4$.

5. Discussion and Analysis

A. SNS versus other methods of spectral synthesis

Superposing single parts from multiple spectra is as effective in spectral synthesis as modifying a single spectrum, but less efficient: In SNS, only $w/2w_s \leq 1$% of the dispersed light is used. An analogous approach has been used to synthesize infrared spectra, but not visible light [10].

The SNS setup in Fig. 2(a) resembles the variable spectrum generator (VSG), but the VSG has a linear variable filter for wavelength selection and a cylindrical lens for optical compression [16]. Conveniently, DE$_A$ fulfills both functions.

Wavelength calibration as in Figs. 2(b)-(e) is simple, but presupposes negligible distortion of line AD by DE$_s$, negligible divergence of the dispersed rays from L$_s$, and a distortion-free photograph of the diffraction image. Hence, the error in Fig. 6 was +/-4nm. Although intensity calibration with rectangle ABCD is valid, Eq. (5) requires perfect darkness for ‘off’ pixels, which is hard to fulfill in practice. Conventional calibration methods are more accurate [2,10].

Whereas the VSG yields bandwidths around 2nm, the SNS light engine can produce much smaller bandwidths thanks to its flexible setup geometry.

As with the VSG, the grayscale pattern intensity may take on 256 values. $I_x(x)$ can be further regulated via the number of ‘on’ pixels in a given SLM column. This enables the SNS light engine to synthesize spectra with the same quality as the more complex spectral integrator [2].

B. Properties of PICS Images

1. Geometry and colors of the virtual spectral image

How can we quantify the position, size, orientation, and colors of a virtual spectral image?

Fig. 13. The virtual spectral image inspected through DE$_A$ (top view) is geometrically analogous to a real image projected through it.

Through an amici prism, the spectral image appears centered at the same position as the line of light itself, see Fig. 13. After all, the prism displaces the monochromatic constituents only in the $x_\text{T}$-direction. Through a grating, the spectral image appears to the left and right of the line of light (at $m=-1$ and $m=+1$). The spectral image has the same distance to the grating as the line of light, measured from the point where the central ray (for $\lambda_G = 0.5(\lambda_R + \lambda_B)$) to the viewer passes the grating [17]. In this sense, the spectral image is at the same distance $d_\text{s}$ from DE$_\text{A}$ as the line of light, both for the amici prism, and the grating.

As already Newton found in his Experiment XI in Book I, Part II of his Opticks [18], equal but opposite dispersions by DE$_s$ and DE$_A$ reproduce the original image. This allows us to express, analogous to Eq. (1), the dispersive displacement for analysis for a DE$_\text{A}$ with dispersion angle $\delta_\text{A}$ (see Fig. 13) as

$$a = 2\tan(0.5 \delta_\text{A})d_\text{A},$$

(10)

While $s$ determines the coloration of image stripes (cf. Fig. 4), $a$ dictates the orientation of the spectral image via the order of the color stripes. Analogous to Eq. (2), $a$ also determines the width of the spectral image

$$w_\text{s} = |a| + w,$$

(11)

$w$ being the width of the 1-D screen, which is assumed to be cylindrical, here.

Because dispersion in the $x_\text{A}$-direction leaves the view unchanged in the $y$-direction, the height $h_\text{A}$ of the virtual spectral image equals the height $h_\text{s}$ of the synthesized line of light, or the height of the projected grayscale image:

$$h_\text{s} = h_\text{A}.$$

(12)

Equations (10)-(12) hold for a prism with negligible magnification, and for a grating. Further, with these equations, we predict the spectral image to have constant position, size, orientation and colors even if $d_\text{s}$ varies, as long as $d_\text{A}$ is constant. Suppose DE$_\text{A}$ is not moved relative to the line of light. Then, for an amici prism, $d_\text{A}$ is always constant. In contrast, for a grating, Eq. (9) implies that $d_\text{A}$ varies according to the viewer’s movements [17]; except along a single moving direction, given by the central ray diffracted to the viewer.

The image transformations described in Section 4 can be understood with Fig. 4: Shifting the 1-D screen along the $x_\text{A}$-axis corresponds to shifting the $\lambda_\text{y}$-plane. Shifting the 1-D screen toward or away from DE$_A$ corresponds to varying the displacement between the monochromatic constituents according to $s$. 

2. Calculating PICS image proportions

As we saw in the experiment, the spectral image has a different width-to-height ratio than the grayscale image, except at a specified distance $d_\text{A}$, depending on DE$_\text{A}$.
Let us assume the grayscale image extends from line AB to CD, so \( w \cdot \left\lfloor \frac{1}{2} \right\rfloor + \Delta x = 1 \), as in Eq. (2). Then, based on Eqs. (11)-(12), the width-to-height ratio \( w_s/h_s \) of the spectral image relates to that \( w_h/h_h \) of the grayscale image as follows:

\[
\frac{w_s}{h_s} = \frac{\left\lfloor \frac{1}{2} \right\rfloor + 0.5 w_s}{\left\lfloor \frac{1}{2} \right\rfloor + 0.5 \Delta x} \cdot \frac{h_h}{h_s}.
\]

(13)

Thus, to obtain a spectral image with a desired width-to-height ratio, we may adapt the height \( h_s \) of the grayscale image according to the viewer (see Fig. 13) - thus, at \( L \) and \( S \), all grayscale pixels translate into spectral pixels. In (b), (c) and (e), additional dispersion of DEa shears the spectral stack according to dispersion vector \( s \). Thus, at \( L_s \), all grayscale pixels translate into spectral pixels. This asymmetry in the graphs in Fig. 15 depicts our observation that image resolution depends on the relative dispersive displacement for analysis, \( a/s \).

(a) At \( a/s = 0 \), all spectral pixels are superposed. (b) At \( a/s = +1 \), all spectral pixels lie next to each other. (c) At \( a/s = -1 \), all spectral pixels partially overlap. In (d), where \( a/s = 0 \), and in (e), where \( a/s = 0.5 \), the dispersion of DEa is twice as strong as in (a) and (c), allowing more pixels to be represented at the linear locus, yielding higher pixel resolution compared to (b). (f) The spectral pixel width \( \Delta x_h \), and the FWHM of a spectral pixel, \( \Delta x_h \), depend on the width \( w \) of the line of light at the linear locus, on the grayscale pixel width \( \Delta x \), and the relative dispersive displacement for analysis, \( a/s \).

Finally, let us define spectral image resolution as the number of spectral pixels that would fit within the width of the spectral image, namely as

\[
R_s = \frac{w_s}{\Delta x}.
\]

(19)

This is in analogy to the grayscale image resolution, \( R_g = w_s/\Delta x \). The asymmetry in the graphs in Fig. 15 depicts our observation that image resolution depends on the orientations of dispersion of DEa and DEa. Only for vanishing \( w/\Delta x \), which means a perfectly 1-D line of light, the asymmetry vanishes.

\[①\]

3. Calculating PICS image resolution

To grasp PICS image resolution intuitively, cut a picture into narrow vertical stripes of width \( w \) and stack them. This is analogous to Fig. 12(c) at \( w = w_s \). If you then spread the stripes out in the original direction, the original image appears: analogous to Fig. 12(b) at \( w_s \). If, instead, you spread the stripes out in the opposite direction, a mirror image, less sharp, emerges: analogous to Fig. 12(e) at \( w_s \). This analogy also explains why ‘Lena’ turns her face in Fig. 12(e) as the wide screen is narrowed.

To describe PICS image resolution quantitatively, we refer to Dispersion Diagrams [15], as in Fig. 14. Let us discuss the straightforward case of amici prisms as DEa, and DEsa. As shown in Figs. 14(a) and (d), DEsa shears the wavelength distribution of the spectral pixels that would - based on our resolution criterion - fit within the width of the spectral image, namely as in Fig. 14(a) and (d), the wavelength distribution of a grayscale image, dispersed by DEsa according to dispersion vector \( a \), contains a spectral stack of width \( w \) at the linear locus (accentuated) that represents all grayscale pixels as spectral pixels. In (b), (c) and (e), additional dispersion of DEsa shears the spectral stack according to dispersion vector \( s \), the spread-out spectral pixels forming a spectral image whose image resolution depends on the relative dispersive displacement for analysis, \( a/s \).

Fig. 14. Deriving PICS spectral image resolution from Dispersion Diagrams. In (a) and (d), the wavelength distribution of a grayscale image, dispersed by DEsa according to dispersion vector \( s \), contains a spectral stack of width \( w \) at the linear locus (accentuated) that represents all grayscale pixels as spectral pixels. In (b), (c) and (e), additional dispersion of DEsa shears the spectral stack according to dispersion vector \( a \), the spread-out spectral pixels forming a spectral image whose image resolution depends on the relative dispersive displacement for analysis, \( a/s \).

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\[
R_s = \frac{w_s}{\Delta x}.
\]

(19)
projection to object tracking [19]. Imagine a cinema where
4. Suggested applications of PICS

and contain cylindrical, and contain orthogonal projection onto the x_A-axis. Accordingly, we introduce \( s_A = s \cdot x_A \). Generalizing Eq. (16), we get

\[
\Delta x_A = \left( \frac{a}{s} \right) \Delta x + \left( \frac{a + s_A}{s} \right) w .
\]

Generalizing Eqs. (16) and (17), we obtain

\[
\Delta x_H = \left( \frac{a}{s} \right) \Delta x , \text{ for } \frac{a}{s_A} \leq -0.5
\]

and

\[
\Delta x_H = (\Delta x - w) \left( \frac{a}{s} \right) + w \left( \frac{a + s_A}{s} \right) , \text{ for } \frac{a}{s_A} \geq -0.5 .
\]

4. Suggested applications of PICS

360°-visibility offers applications from text display to image projection to object tracking [19]. Imagine a cinema where viewers with diffraction glasses sit around a translucent 1-D screen. (A reflective metal rod of diameter \( w = 2\)mm offers ca. 310°-visibility, as I found in preliminary experiments.) A grating constant that is inversely proportional to \( d_x \) enables correct image proportions. Alternatively, a cylindrical transmission grating around the 1-D screen could display correctly-proportioned spectral images to anyone around it.

Being semitransparent, virtual images may be superposed onto an object or image, whether for geometric comparison or color experiments. Further, three virtual spectral images of a single 1-D screen may compose a real-color image. To obtain the RGB image components, their grayscale versions are inserted in the corresponding intervals (600-700nm, 500-600nm, or 400nm-500nm, respectively; cf. [20]) between AB and CD. However, their superposition requires a specially designed DE_A.

The 1-D screen takes up little space and material. Besides, its light does not disturb a disinterested individual. Moreover, the relevant light beam is narrow, allowing projection in confined or crowded spaces, even for large images. This solves the problem stated in a recently published paper [21].

Being metameric, the synthesized line of light does not betray the image to viewers without or beyond DE_A. This is valuable in police interrogation, medical communication, advertising, and beyond.

Mirror immunity is intrinsic to a 1-D screen. Spectral images may be multiplied or delivered elsewhere via mirrors parallel to the 1-D screen, without ever changing image orientation.

C. Unifying spectral synthesis and spectral encoding

For PICS, 2-D images are spectrally encoded in 1D. Spectral encoding, whereby locations are translated into wavelengths of light, has already been applied to 2-D image acquisition via 0-D or 1-D apertures, whether in spectrally-encoded endoscopy (SEE) [22], wavelength-multiplexed microscopy [23], or modern pseudoscopy [24]. Spectral encoding was also proposed for the transmission of a 2-D image via an optical fiber [25-27]. Still, it has not yet been applied to video projection.

Until now, SLM-based light engines were thought to encode wavelengths as patterns on the SLM. Looking back on SNS and PICS, we may now state the reverse: SLM-based light engines encode images as wavelengths. This makes any of these light engines suitable for PICS.

A precursor to SNS and PICS is Newton’s Experiment I in Book I, Part II of his Opticks [18]. Focused on proving his theory, he did not see the practical value of the experiment, however.

6. Conclusion

By projecting a grayscale image through a prism or grating, we obtain light at a linear locus L_0 whose spectrum is a rainbow-colored version of the grayscale image.

This provides i) a method for synthesizing light with a desired spectral power distribution, called Superposition of Newtonian Spectra (SNS); and ii) a method for viewing 2-D images that are spectrally encoded on a 1-D projection screen, called Projected-image circumlineascopy (PICS).

For SNS, an SLM-projector and a prism or grating were used. This setup is the simplest among equally effective SLM-based tunable light sources: the trade-off being considerable light loss.
For PICS, grayscale text, photos and videos from an SLM-projector were dispersed by a prism across an upright capellini. If the capellini was viewed through another prism or grating, rainbow-colored versions of the grayscale images appeared. Floating in midair, the semitransparent images were correctly oriented for any azimuthal viewing angle, even if the capellini was reflected in an upright mirror. Real-color, mirror-immune, surround-view images are achievable with PICS by superposing three virtual spectral images of a single 1-D screen, yet a three-component viewing device needs to be designed for the RGB mixture. An advanced version of PICS is conceivable where a 0-D point of light is dispersed into 2 dimensions, yielding a 2-D spectral image that has correct image proportions and constant apparent size, for any viewing position.

Geometric optics were used in tandem with Dispersion Diagrams to visualize the transformation from grayscale to spectral image, and to derive formulas for SNS spectral resolution, PICS image proportions, and PICS image resolution. Further research could investigate PICS image transformations for variations of the setup, for example with a diagonal 1-D screen. Beyond that, a systematic treatment of generic virtual images in spatial relation to the viewer would be useful.

Besides the SNS tunable source, all SLM-based light engines are limited to abstract patterns, but may as well be concrete images.

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