Hyperbolic Conservation Laws on Manifolds: 
An error estimate for finite volume schemes

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The mathematical theory of nonlinear hyperbolic conservation laws posed on manifolds was initiated by Ben-Artzi and LeFloch, together with several collaborators. For these equations, a suitable generalization of Kruzkov’s theory is now established for a large class of manifolds, and provides the existence and uniqueness of an entropy solution to the Cauchy problem, as well as the initial and boundary value problem. The convergence of finite volume schemes with monotone flux is also proven.

In the present work we consider the case where the conservation law is posed on a compact Riemannian manifold, and we establish an $L^1$-error estimate for a class of finite volume schemes which converge to the entropy solution of the Cauchy problem. Our proof relies on suitable generalization of a method due to Cockburn, Coquel, and LeFloch in the Euclidian setting. We need to revisit Kuznetzov’s approximation theory: our analysis has to take into account geometric terms and overcome several technical difficulties (doubling variable technique, discrete entropy dissipation estimates).

References

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