Abstract—This paper describes a statistically-principled semi-supervised method of automatic chord estimation (ACE) that can make effective use of any music signals regardless of the availability of chord annotations. The typical approach to ACE is to train a deep classification model (neural chord estimator) in a supervised manner by using only a limited amount of annotated music signals. In this discriminative approach, prior knowledge about chord label sequences (characteristics of model output) has scarcely been taken into account. In contrast, we propose a unified generative and discriminative approach in the framework of amortized variational inference. More specifically, we formulate a deep generative model that represents the complex generative process of chroma vectors (observed variables) from the discrete labels and continuous textures of chords (latent variables). Chord labels and textures are assumed to follow a Markov model favoring self-transitions and a standard Gaussian distribution, respectively. Given chroma vectors as observed data, the posterior distributions of latent chord labels and distribution, respectively. Given chroma vectors as observed variables, the model is trained in a semi-supervised manner by using deep classification and recognition models, respectively. These three models are combined to form a variational autoencoder and trained jointly in a semi-supervised manner. The experimental results show that the performance of the classification model can be improved by additionally using non-annotated music signals and/or by regularizing the classification model with the Markov model of chord labels and the generative model of chroma vectors even in the fully-supervised condition.

Index Terms—Automatic chord estimation, semi-supervised training, variational autoencoder.

I. INTRODUCTION

CHORD is a mid-level representation of polyphonic music that lies between the highly-abstracted musical intensions of humans and actual musical sounds. Unlike musical notes, chord sequences do not tell the actual pitches, but abstractly represent the harmonic content evolving over time. In lead sheets (a form of music notation consisting of melody, chords, and lyrics), only chord labels are shown to musicians to roughly suggest the intentions on how musical notes should be arranged and played in each bar.

Automatic chord estimation (ACE), which aims to automate the process of transcribing a music signal into a chord sequence, has been one of the fundamental research topics in the field of music information retrieval (MIR). The diversity and complexity of the acoustic characteristics of music signals make the ACE task very challenging. ACE studies have thus been focusing on data-driven methods based on statistical models [1], which are roughly categorized into generative and discriminative approaches.

Early studies took the generative approach based on a classical probabilistic latent variable model. For example, a hidden Markov model (HMM) is formulated to represent the stochastic generative process of chroma vectors (observed variables) from chord labels (latent variables) [2]. Given chroma vectors, the chord labels are inferred such that their likelihood is maximized. An advantage of this approach is that the prior distribution of chord labels can be explicitly taken into account. For example, musically meaningful chord sequences can be estimated by considering the temporal structures of chord labels. Another advantage is that in theory generative models can be trained in a semi-supervised or even unsupervised manner. To efficiently infer the optimal latent variables, however, one needs to pose some unrealistic assumptions, e.g., first-order Markovian dynamics of chord sequences and conditional independence of acoustic features for mathematical convenient. In addition, the expressive power of simple generative models are
insufficient for representing the complex relationships between acoustic features and chord labels.

The discriminative approach, in contrast, aims to directly convert acoustic features into chord labels. Recently, deep neural networks (DNNs) have often been used for estimating the posterior probabilities of chord labels. Because the generative process is not considered explicitly, the oversimplified assumptions posed by the generative approach can be avoided, and chord label inference is more straightforward. A problem here is that the classifier is usually trained for learning a frame-wise audio-to-label mapping without considering the characteristics of chord label sequences and the generative process of music. In addition, the model needs to be trained in a fully-supervised manner. Manual chord annotation is a labor-intensive task, and the performance of ACE heavily depends on the amount, diversity and quality of annotated music signals because of the nature of supervised training.

Conjecturing that the performance of ACE could be improved by integrating the complementary generative and discriminative approaches, we propose a semi-supervised neural chord estimation method based on a variational autoencoder (VAE) [5] (Fig. 1). Because chroma vectors can be represented more precisely by considering their continuous textures (deviations from basic chroma patterns) in addition to the underlying discrete chord labels, we formulate a DNN-based generative model $p_q$ that represents the generative process of a sequence of chroma vectors from that of chord labels and that of chord textures. To complete Bayesian formulation, we introduce prior distributions on the latent chord labels and textures. Specifically, the chord labels are assumed to follow a first-order Markov model favoring self transitions, and the chord textures, which are abstract features representing the fine structure of the chroma vectors, are assumed to follow a standard Gaussian distribution. In the framework of amortized variational inference (AVI) [6], DNN-based discriminative models $q_s$ and $q_p$ are then introduced as variational posterior distributions to approximate the posterior distribution of latent variables from an observed chroma sequence. The generative and discriminative models can be trained jointly in a semi-supervised manner by using music signals with and without chord annotations.

The main contribution of this paper is to draw the potential of the powerful deep discriminative model by integrating it into the principled statistical inference formalism of the generative approach to ACE. This is the first attempt to use a VAE for semi-supervised ACE. A key feature of our VAE is that the Markov model, which defines the prior probability of chord label sequences, works as a chord language model and prevents frequent frame-level transitions of chord labels in the joint training of the generative and discriminative models for given chroma vectors. We experimentally show the effectiveness of this regularization on semi-supervised training.

II. RELATED WORK

This section reviews related work on ACE based on generative and discriminative approaches and machine-learning strategies for integrating these approaches.

A. Generative approach

In the generative approach, a probabilistic generative model $p(X,S) = p(X|S)p(S)$ has often been formulated as an HMM, where $X$ and $S$ denote a sequence of acoustic features (e.g., chroma vectors [7]) and a sequence of chord labels [8], respectively. The emission probabilities of $X$ for each chord, $p(X|S)$, and the chord transition probabilities, $p(S)$, are typically given by a Gaussian mixture model (GMM) and a first- or higher-order Markov model (n-gram model), respectively. Given $X$ as observed data, the optimal chord label sequence maximizing the posterior probability $p(S|X)$ can be estimated efficiently by using the Viterbi algorithm.

Many HMM-based methods have been proposed for ACE. Lee and Slaney [4], for example, proposed joint training of multiple HMMs corresponding to different keys. Another research direction is to simultaneously deal with multiple kinds of musical elements in addition to chord labels by explicitly considering the relationships of those elements. Mauch and Dixon [9] proposed a dynamic Bayesian network (DBN) that represents the hierarchical structure over metrical positions, musical keys, bass notes, and chords, and formulates the generative process of treble and bass chroma vectors. Similarly, Ni et al. [10] proposed a harmony progression analyzer (HPA) based on a DBN whose latent states represent chords, inversions, and musical keys.

Some studies have focused on the temporal characteristics of frame-wise chord sequences. To represent repetitions of chord patterns, for example, multi-order HMMs [11] and duration-explicit HMMs [12] have been proposed, where the results of ACE were found to be insensitive to the frame-wise transition probabilities of chord labels [12]. This indicates that the main advantage of using a frame-wise chord transition model is just to emphasize the chord label continuity instead of serving as a musically-meaningful chord language model.

B. Discriminative approach

Recently, DNNs have intensively been used as a powerful discriminative model for directly estimating the posterior probability $p(S|X)$. Humphrey and Bello [13], for example, used a convolutional neural network (CNN) for learning an effective representation of raw audio spectrograms. A number of ACE methods using DNN classifiers have then been proposed for using low-level audio representations such as constant-Q transform (CQT) spectrograms [14]. In general, these DNN-based methods outperform HMM-based generative methods [15].

Because typical DNN-based methods estimate the posterior probabilities of chord labels at the frame level, some smoothing technique is often used for estimating temporally-coherent chord labels. An HMM [16] or a conditional random field (CRF) [15], for example, can be used for estimating the optimal path of chord labels from the estimated posterior probabilities. Recurrent neural networks (RNNs) have recently been used as a language model that represents the long-term dependency of chord labels [17], [18]. Note that RNN-based models are still incapable of learning the symbol-level syntactic structure from frame-level chord sequences [19]. To solve this problem, Korzeniowski and Widmer [20] formulated a
symbol-level chord transition model with a duration distribution. Chen and Su [21] proposed an extension of the transformer model [22] for joint chord segmentation and labeling in a context of multi-task learning.

Even the state-of-the-art neural chord estimators have difficulty in identifying infrequent chord types because the numbers of occurrences of chord types are highly imbalanced. In fact, the majority of chords belongs to major and minor triads. Another difficulty lies in distinguishing chords that include several tones in common (e.g., C9 and Csus2). This problem can be mitigated by reflecting the musical knowledge about the constituent tones and taxonomy of chord labels [23, 25] into objective functions and/or by using an even-chance training scheme [26]. However, dealing with a large chord vocabulary is still an open problem.

C. Unified generative and discriminative approach

Integration of deep generative and discriminative models has actively been explored in the context of unsupervised or semi-supervised training. One of the most popular strategies is to use a variational autoencoder (VAE) [5] that jointly optimizes a deep generative model $p(X|S)$ and a deep recognition model $p(S|X)$ (approximated by a variational posterior $q(S|X)$ introduced for deriving the lower bound of the marginal likelihood $p(X)$). Its extension, known as conditional VAE [27], can be used for semi-supervised training of latent representations disentangled from given labels (conditions) [28]. In the field of Automatic Speech Recognition (ASR), some studies tried to jointly train a speech-to-text model with a text-to-speech model to improve the performance of ASR by using both annotated and non-annotated speech signals [29].

In this paper we use a VAE with two different latent variables corresponding to chord labels (categorical variables) and chord textures (continuous variables). This model is similar to JointVAE [30] in a sense that both discrete and continuous representations are learned jointly. A key feature of our model is that a Markov prior favoring self transitions is put on categorical variables. This prior acts as a regularization term that can be mitigated by reflecting the musical knowledge about the chords without referring to their chord labels (Section III-D).

Our goal is to train a classification model $p(S|X)$ and use it for estimating chord labels behind unseen music signals. In an unsupervised condition, the classification model should be...
trained from $X$ without using the ground-truth data of $S$. In a supervised condition, the classification model can be trained by using paired data of $X$ and $S$. In a semi-supervised condition, some part of $S$ is given as ground-truth data.

### B. Generative model

We formulate a probabilistic hierarchical generative model (joint probability distribution) of chord labels $S$, chroma texture $Z$, and chroma vectors $X$ as follows:

$$p_{\theta, \phi}(X, S, Z) = p_\theta(X|S, Z)p_\phi(S)p(Z),$$

where $p_\theta(X|S, Z)$ is a likelihood function of $S$ and $Z$ for $X$, $p_\phi(S)$ is a prior distribution of $S$, $p(Z)$ is that of $Z$, and $\theta$ and $\phi$ are model parameters. While standard HMMs used for ACE are represented as $p_\theta(X, S) = p_\theta(X|S)p_\phi(S)$, we use both $S$ and $Z$ for precisely representing $X$, i.e., $p_\theta(X|S, Z)$ is a deep generative model represented as follows:

$$p_\theta(X|S, Z) = \prod_{n=1}^{N} \prod_{d=1}^{D} \text{Bernoulli}(x_{nd}|[\omega_\theta(S, Z)]_{nd}),$$

where $\omega_\theta(S, Z)$ is the $ND$-dimensional output of a DNN with parameters $\theta$ that takes $S$ and $Z$ as input and the notation $[A]_{ij}$ indicates the $ij$-th element of $A$. Although $x_{nd}$ should take a binary value in theory, it allow us to take a real value between 0 and 1 from a practical point of view.

Since chord labels $S$ have temporal continuity, i.e., change infrequently at the frame level, we use a first-order Markov model favoring self-transitions of chord labels as the chord label prior $p_\phi(S)$ as follows:

$$p_\phi(S) = p(s_1) \prod_{n=2}^{N} p(s_n|s_{n-1}) = \prod_{k=1}^{K} \phi_k s_k \prod_{n=2}^{N} \prod_{k'=1}^{K} \phi_{k'n-1} s_{n-1} s_k,$$

where $\phi_k$ is the initial probability of chord $k$ and $\phi_{k'n}$ is the transition probability from chord $k'$ to chord $k$. The self-transition probability $\phi_{kk}$ is expected to take a larger value. $\phi$ can be either learned automatically or given manually. In this paper, we use $\phi_{kk} = 0.9$. As shown in Section IV-B, if $p(S)$ is set to a uniform distribution, estimated chord labels lose the temporal continuity, resulting in the negative impact even in the supervised learning.

Since we have no strong belief about abstract chroma textures $Z$, the chroma texture prior $p(Z)$ is set to a standard Gaussian distribution as follows:

$$p(Z) = \prod_{n=1}^{N} \mathcal{N}(z_n|0_L, I_L),$$

where $0_L$ is the all-zero vector of size $L$ and $I_L$ is the identity matrix of size $L$.

### C. Classification and recognition models

Given chroma vectors $X$ as observed data, we aim to infer the chord labels $S$ and the chroma textures $Z$ from $X$ and estimate the model parameters $\theta$ and $\phi$ in the framework of maximum likelihood estimation. Because the DNN-based generative modeling makes the posterior distribution $p_{\theta, \phi}(S, Z|X) \propto p_{\theta, \phi}(X, S, Z)$ analytically intractable, we compute it approximately with an AVI technique. More specifically, we introduce a sufficiently-expressive variational distribution $q_{\alpha, \beta}(S, Z|X)$ parametrized by $\alpha$ and $\beta$ and optimize it such that the Kullback-Leibler (KL) divergence from $q_{\alpha, \beta}(S, Z|X)$ to $p_{\theta, \phi}(S, Z|X)$ is minimized. In this paper we assume that $S$ and $Z$ are conditionally independent as follows:

$$q_{\alpha, \beta}(S, Z|X) = q_\alpha(S|X)q_\beta(Z|X),$$

where $q_\alpha(S|X)$ and $q_\beta(Z|X)$ are classification and recognition models that infer $S$ and $Z$, respectively. In this paper, these models are implemented with DNNs parameterized by $\alpha$ and $\beta$ as follows:

$$q_\alpha(S|X) = \prod_{n=1}^{N} \text{Categorical}(s_n|[\pi_\alpha(X)]_n),$$

$$q_\beta(Z|X) = \prod_{n=1}^{N} \mathcal{N}(z_n|\mu_\beta(X), \sigma^2_\beta(X)), $$

where $\pi_\alpha(X)$ is the $NK$-dimensional output of the DNN with parameters $\alpha$, and $\mu_\beta(X)$ and $\sigma^2_\beta(X)$ are the $NL$-dimensional outputs of the DNN with parameters $\beta$. Similar to the deep generative model, the outputs of the DNNs represent the parameters of certain probabilistic distributions.

### D. Unsupervised training

Instead of directly maximizing the log-marginal likelihood $\log p_{\theta, \phi}(X)$ with respect to the model parameters $\theta$ and $\phi$, we maximize its variational lower bound $\mathcal{L}_X(\theta, \phi, \alpha, \beta)$ derived by introducing $q_{\alpha, \beta}(S, Z|X)$ as follows:

$$\log p_{\theta, \phi}(X) = \log \int p_{\theta, \phi}(X, S, Z) dS dZ$$

$$= \log \int q_{\alpha, \beta}(S, Z|X) \log p_{\theta, \phi}(X, S, Z) dS dZ$$

$$\geq \int q_{\alpha, \beta}(S, Z|X) \log \frac{p_{\theta, \phi}(X, S, Z)}{q_{\alpha, \beta}(S, Z|X)} dS dZ$$

$$\triangleq \mathcal{L}_X(\theta, \phi, \alpha, \beta),$$

where the equality holds, i.e., $\mathcal{L}_X(\theta, \phi, \alpha, \beta)$ is maximized, if and only if $q_{\alpha, \beta}(S, Z|X) = p_{\theta, \phi}(S, Z|X)$. Note that in reality this equality condition cannot be satisfied because $p_{\theta, \phi}(S, Z|X)$ is hard to compute. However, the gap between $\log p_{\theta, \phi}(X)$ and $\mathcal{L}_X(\theta, \phi, \alpha, \beta)$ in (8) is given as the KL divergence from $q_{\alpha, \beta}(S, Z|X)$ to $p_{\theta, \phi}(S, Z|X)$. Therefore, the minimization of the KL divergence is equivalent to the maximization of $\mathcal{L}_X(\theta, \phi, \alpha, \beta)$.
To approximately compute $\mathcal{L}_X(\theta, \phi, \alpha, \beta)$, we use Monte Carlo integration as follows:

$$
\mathcal{L}_X(\theta, \phi, \alpha, \beta) = \mathbb{E}_{q_\alpha(S|X)}[\log p_\theta(X|S, Z)]
+ \mathbb{E}_{q_\alpha(S|X)}[\log p(Z) - \log q_\beta(Z|X)]
+ \mathbb{E}_{q_\alpha(S|X)}[\log p_\theta(S) - \log q_\alpha(S|X)]
\approx \frac{1}{T} \sum_{i=1}^{T} \log p_\theta(X|S_i, Z_i) - KL(q_\beta(Z|X)||p(Z))
+ \text{Entropy}[q_\alpha(S|X)] + \mathbb{E}_{q_\alpha(S|X)}[\log p_\theta(S)],
$$

(9)

where $\{S_i, Z_i\}_{i=1}^{T}$ are $I$ samples drawn from $q_\alpha(S|X)q_\beta(Z|X)$. Following the typical VAE implementation, we set $I = 1$ and hence the index $i$ can be omitted. To make (9) partially differentiable with respect to the model parameters $\theta, \phi, \alpha, \beta$, we use reparametrization tricks [5, 43] for deterministically representing the categorical variables $S$ and the Gaussian variables $Z$ as follows:

$$
\epsilon_n^s \sim \text{Gumbel}(0_K, 1_K),
$$

(10)

$$
s_n = \text{softmax}(\log([\pi_n(X)]_n + \epsilon_n^s) / \tau),
$$

(11)

$$
\epsilon_n^z \sim \mathcal{N}(0, I_L),
$$

(12)

$$
z_n = [\mu_\beta(X)]_n + \epsilon_n^z \odot [\sigma_\beta(X)]_n,
$$

(13)

where (10) indicates the standard Gumbel distribution, $1_K$ is the all-one vector of size $K$, $\odot$ means the element-wise product, and $\tau > 0$ is a temperature parameter that controls the uniformness of $s_n$ ($\tau = 0.1$ in this paper).

We can now compute the four terms of (9) evaluating the fitness of $S$ and $Z$: a reconstruction term indicating the likelihood of $S$ and $Z$ for $X$ and three regularization terms making the posterior $q_\beta(Z|X)$ close to the prior $p(Z)$, increasing the entropy of $q_\alpha(S|X)$, and making $S$ temporally coherent, respectively (Fig. 2). More specifically, the first term is given by (2) and the second and third terms are given by

$$
\text{KL}(q_\beta(Z|X)||p(Z))
= \sum_{n=1}^{N} \sum_{k=1}^{K} \left( \frac{[\mu_\beta(X)]_n^2}{2} - \log([\sigma_\beta(X)]_n) \right),
$$

(14)

$$
\text{Entropy}[q_\alpha(S|X)]
= - \sum_{n=1}^{N} \sum_{k=1}^{K} [\pi_\alpha(X)]_n \log([\pi_\alpha(X)]_n).
$$

(15)

The last term $\mathbb{E}_{q_\alpha(S|X)}[\log p_\theta(S)]$ can be calculated efficiently by using a dynamic programming method similar to the forward algorithm of the HMM. Let $\gamma(s_n) = \mathbb{E}_{q_\alpha(S|X)}[\log p(s_{1:n})]$, be a forward message at frame $s$, which can be calculated recursively as follows:

$$
\gamma(s_1) = \log p(s_1),
$$

(16)

$$
\gamma(s_n) = \sum_{s_{n-1}} q_\alpha(s_{n-1}|X) \left( \gamma(s_{n-1}) + \log p(s_n|s_{n-1}) \right),
$$

(17)

$$
\mathbb{E}_{q_\alpha(S|X)}[\log p_\theta(S)] = \sum_{s_N} q_\alpha(s_N|X) \gamma(s_N).
$$

(18)

We now have a VAE (Fig. 3) that consists of:

- the deep classification model $q_\alpha(S|X)$ given by (6) with the reparametrization trick given by (10) and (11),
- the deep recognition model $q_\beta(Z|X)$ given by (7) with the reparametrization trick given by (12) and (13), and
- the deep generative model $p_\theta(X|S, Z)$ given by (2) with the prior distributions $p_\theta(S)$ and $p(Z)$ given by (3) and (4), respectively.

In the unsupervised condition, all models are jointly optimized in the framework of the VAE by using a variant of stochastic gradient descent such that (9) is maximized with respect to $\theta, \phi, \alpha, \beta$.

E. Supervised training

Under the supervised condition that chroma vectors $X$ and the corresponding chord labels $S$ are given as observed data, we aim to maximize a variational lower bound $\mathcal{L}_{X,S}(\theta, \beta)$ of the log-likelihood $\log p_\theta(S|X)$, which can be derived in a way similar to (8) as follows:

$$
\log p_\theta(X|S) = \log \int q_\beta(Z|X) \frac{p_\theta(X|Z, S)}{q_\beta(Z|X)} dZ
\geq \int q_\beta(Z|X) \log \frac{p_\theta(X|Z, S)}{q_\beta(Z|X)} dZ
\triangleq \mathcal{L}_{X,S}(\theta, \beta).
$$

(19)

Using Monte Carlo integration, $\mathcal{L}_{X,S}(\theta, \beta)$ can be approximately computed as follows:

$$
\mathcal{L}_{X,S}(\theta, \beta) \triangleq \mathbb{E}_{q_\beta(Z|X)}[\log p_\theta(X|S, Z) + \log p(Z) - \log q_\beta(Z|X)]
\approx \frac{1}{T} \sum_{i=1}^{T} \log p_\theta(X|S_i, Z_i) + KL(q_\beta(Z|X)||p(Z)),
$$

(20)

where $\{Z_i\}_{i=1}^{T}$ are $I$ samples drawn from $q_\beta(Z|X, S)$ using the reparametrization trick ($I = 1$ in this paper). Because the chord estimator $q_\alpha(S|X)$, which plays a central role in ACE, does not appear in (20), $q_\alpha(S|X)$ cannot be trained only by maximizing (20). As suggested in the semi-supervised learning of a VAE [27], one could thus define an alternative objective function $\mathcal{L}_{X,S}(\theta, \alpha, \beta)$ including $\alpha$ by adding a classification performance term to (20) as follows:

$$
\mathcal{L}_{X,S}(\theta, \alpha, \beta) \triangleq \mathcal{L}_{X,S}(\theta, \beta) + \log q_\alpha(S|X).
$$

(21)

A problem of this approach, however, is that the regularization terms [15] and [18] enhancing the entropy of $q_\alpha(S|X)$ (preventing the overfitting) and smoothing the output of $q_\alpha(S|X)$, respectively, are not taken into account.

To solve this problem, we propose a new objective function $\mathcal{L}_{X,S}(\theta, \phi, \alpha, \beta)$ by summing (9) and (21) as follows:

$$
\mathcal{L}_{X,S}(\theta, \phi, \alpha, \beta) = \mathcal{L}_X(\theta, \phi, \alpha, \beta) + \mathcal{L}_{X,S}(\theta, \alpha, \beta).
$$

(22)

The new objective function means that in our approach, the chroma vectors $X$ with the annotations $S$ are used twice as unsupervised and supervised training data as if they were not annotated (first term) and as they are (second term), respectively.
F. Semi-supervised training

Under the semi-supervised condition that partially-annotated chroma vectors are available, we define an objective function by summing the objective functions (9) and (22) corresponding to the unsupervised and supervised conditions, respectively, as follows:

\[
\mathcal{L}'_{X,S}(\theta, \phi, \alpha, \beta) = \sum_{X \text{ w/o } S} \mathcal{L}_X(\theta, \phi, \alpha, \beta) + \sum_{X \text{ with } S} \mathcal{L}_{X,S}(\theta, \phi, \alpha, \beta),
\]

(23)

Note that \(q_\alpha(S|X)\) can always be regularized regardless of the availability of annotations.

G. Training and prediction

The optimal model parameters \(\theta, \phi, \alpha, \) and \(\beta\) are estimated with a stochastic gradient descent algorithm (e.g., Adam [34]) such that an objective function (9), (22), or (23) is maximized. Under the semi-supervised condition, each mini-batch consists of non-annotated and annotated chroma vectors (50%–50% in this paper) randomly selected from the training dataset.

In the test phase, using the neural chord estimator \(q_\alpha(S|X)\), a sequence of the posterior probabilities of chord labels \(S\) are calculated from a sequence of chroma vectors \(X\) extracted from a target music signal. The optimal temporally-coherent path of chord labels is then estimated uniquely from the posterior sequence by using Viterbi algorithm with the transition probabilities \(\phi\).

IV. Evaluation

This section reports comparative experiments conducted for evaluating the effectiveness of additionally using non-annotated data for semi-supervised training and that of introducing the Markov prior on chord labels.

A. Experimental conditions

We explain ACE methods, datasets, and evaluation measures used for the comparative experiment.

1) Compared methods: As listed in Table I, we tested five methods that train a neural chord estimator \(q_\alpha(S|X)\) in different ways:

- **Encoder-Sup (baseline):** As in conventional studies on ACE, \(q_\alpha(S|X)\) is trained in a supervised manner by minimizing the cross-entropy loss for the ground-truth labels \(S\), i.e., maximizing the following objective function:

  \[
  \mathcal{L}_{X,S}(\alpha) = \log q_\alpha(S|X).
  \]
  (24)

- **VAE-Markov-Sup:** \(q_\alpha(S|X)\) is trained in a supervised manner by maximizing (22).

- **VAE-Markov-SemiSup (proposed):** \(q_\alpha(S|X)\) is trained in a semi-supervised manner by maximizing (23).

- **VAE-Uniform-Sup:** \(q_\alpha(S|X)\) is trained in the same way as VAE-Markov-Sup except that a uniform distribution is used as \(p(S)\) instead of the Markov prior given by (3).

| TABLE I |
|-----------------|-----------------|-----------------|
| Non-annotated   | Generative model | Markov prior |
| Encoder-Sup (baseline) | ✓               | ✓             |
| VAE-Uniform-Sup   | ✓               | ✓             |
| VAE-Markov-Sup     | ✓               | ✓             |
| VAE-Uniform-SemiSup | ✓           | ✓             |
| VAE-Markov-SemiSup | ✓               | ✓             |

| TABLE II |
|-----------------|-----------------|-----------------|
| Chord type      | Duration [h]    |                |
| map             | 53.09           |                |
| min             | 16.63           |                |
| aug             | 0.15            |                |
| dim             | 0.36            |                |
| sus4            | 1.63            |                |
| sus2            | 0.25            |                |
| 1               | 0.76            |                |
| 5               | 0.84            |                |

- **VAE-Uniform-SemiSup:** \(q_\alpha(S|X)\) is trained in the same way as VAE-Markov-SemiSup except that a uniform distribution is used as \(p(S)\) as in VAE-Uniform-Sup. In VAE-Uniform-Sup or VAE-Uniform-SemiSup, we use

  \[
  p(S) = \prod_{n=1}^{N} \text{Categorical} (s_n| [\frac{1}{K}, \cdots, \frac{1}{K}]),
  \]

  (25)

  and the regularization term given by (18), which is used in (22) or (23), is replaced with

  \[
  E_{q_\alpha(S|X)}[\log p(S)] = -\sum_{n=1}^{N} \sum_{k=1}^{K} \log p_k(X) \log K.
  \]

  (26)

For convenience, let ‘*’ denote the wild-card character, e.g., VAE-*-Sup means VAE-Markov-Sup or VAE-Uniform-Sup. Comparing Encoder-Sup with VAE-*-Sup, we evaluated the effectiveness of the VAE architecture in regularizing \(q_\alpha(S|X)\). Comparing VAE-*-Sup with VAE-*-SemiSup, we evaluated the effectiveness of the semi-supervised training. Comparing VAE-Markov-*- with VAE-Uniform-*-, we evaluated the effectiveness of the Markov prior on \(S\).

2) Network configurations: Each of the classifier \(q_\alpha(S|X)\), the recognizer \(q_\beta(Z|X)\), and the generator \(p_\theta(X|S, Z)\) was implemented with a three-layered BLSTM network [35] followed by layer normalization [36]. The final layer of \(q_\alpha(S|X)\) consisted of softmax functions that output the frame-wise posterior probabilities of \(K\) chord labels. The final layer of \(q_\beta(Z|X)\) consisted of linear units that output \(\mu_\beta(X)\) and \(\log \sigma_\beta^2(X)\). The final layer of \(p_\theta(X|S, Z)\) consisted of sigmoid functions that output \(\omega_\theta(S, Z)\). Each hidden layer of the BLSTM had 256 units. Because the architecture of \(q_\alpha(S|X)\) was similar to that of a state-of-the-art chord estimator [31], Encoder-Sup was considered as a reasonable baseline method.

To maximize the objective function of each method, we used Adam optimizer [34], where the learning rate was first set to 0.001 and then decreased exponentially by a scaling factor of 0.99 per epoch and the gradient clipping with norm 5 was used. The length of each sequence was fixed to 645 frames (1 minute in audio duration) for computational efficiency. The
parameter of the Markov prior $\phi$ was fixed to 0.9, and $\theta$, $\alpha$ and $\beta$ were iteratively updated for 300 epochs.

3) Datasets: We collected 1210 annotated songs consisting of 198 songs from Isophonics [32], 100 songs from RWC-MDB-P-2001 [37], annotated 186 songs from uspop2002 [38], and 726 songs from McGill Billboard dataset [39]. As shown in Table II the triad chord types of the annotated data are heavily biased, where the majority of chord annotations belong to the major and minor triads. In addition, we collected 700 non-annotated popular songs composed by Japanese and American artists. Each music audio signal sampled at 44.1kHz was analyzed with constant-Q transform (CQT) with a shifting interval of 4096 samples and a frequency resolution of 1 semitone per bin. The CQT spectrogram and its 1-, 2-, 3-, 4-octave-shifted versions were then stacked to yield a five-layered harmonic CQT (HCQT) representation, which was fed to a neural multipitch estimator [31] for computing chroma vectors $X$.

4) Evaluation procedure: To conduct five-fold cross validation, we divided the 1210 annotated songs into five subsets (242 songs each). In each fold, a subset was kept as test data and the remaining four subsets were used as training data, in which $I$ and $4-I$ subsets were treated as annotated and non-annotated songs, respectively ($I \in \{1, 2, 3, 4\}$). Not only the classifier $q_{q}(S|X)$ but also the recognizer $q_{r}(Z|X)$ and the generator $p_{h}(X|S, Z)$ were trained jointly by using the entire training data (VAE-***-SemiSup) or the annotated data (VAE-***-Sup). In Encoder-Sup, in contrast, only the classifier $q_{q}(S|X)$ was trained by using the annotated data.

VAE-***-SemiSup was further tested under an extended semi-supervised condition that annotated 976 songs (four subsets) and $M$ non-annotated songs ($M \in \{250, 500, 700\}$) were given as training data in each fold. Note that the performance was measured on the remaining 242 annotated songs, which were totally different from the non-annotated songs.

5) Evaluation measure: The chord estimation performance (accuracy) of each method was measured in terms of the frame-level match rate between estimated and ground-truth chord sequences. The weighed accuracy for each song was measured with mir_eval library [40] in terms of the majmin criterion considering only major and minor triads plus a no-chord label ($K = 25$) and the triads criterion with the vocabulary defined in Section III.A ($K = 97$). The overall accuracy was given as the average of the piece-wise accuracies weighed by the song lengths. To compensate for the imbalance of the ratios of the 12 chord roots in the training data, chroma vectors and chord labels were jointly pitch-rotated by a random number of semitones on each training iteration.

B. Experimental results

The accuracies of each method w.r.t. the amount of training data are shown in Fig. 3. Comparing Encoder-Sup with VAE-Markov-Sup and VAE-Uniform-Sup, we found that the VAE-based formulation given by (10) helped to improve the performance of the classifier $q_{q}(S|X)$ under the supervised condition, i.e., $q_{q}(S|X)$ can be regularized effectively by considering its entropy, the Markov or uniform prior of $S$, and the reconstruction quality of $X$ based on $S$ and $Z$. The superiority of VAE-Markov-Sup over VAE-Uniform-Sup and of VAE-Markov-SemiSup over VAE-Uniform-SemiSup revealed the effectiveness of the Markov prior in suppressing the frequent switching of chord labels, as shown in Fig. 4. The superiority of VAE-Markov-SemiSup over VAE-Markov-Sup and that of VAE-Uniform-SemiSup over VAE-Uniform-Sup indicated the effectiveness of the VAE-based semi-supervised learning. A reason of the relatively small improvements would be that the characteristics of test data were similar to those of training data because both data were taken from the mixed dataset (each of the four dataset was divided into five subsets). The proposed VAE-Markov-SemiSup achieved the best accuracy in each of the semi-supervised conditions.

We found that the Markov prior of $S$ played an important role, especially under the extended semi-supervised condition using the non-annotated data with different characteristics. As shown in the right half of Fig. 3 the performance of VAE-Uniform-SemiSup was significantly degraded once, then gradually improved, and finally saturated according to the increase of the external non-annotated data. In contrast, the performance of VAE-Markov-SemiSup tended to be improved constantly without clear drop. The differences between the two sides of Fig. 3 showed the impact of music styles used for the supervised and unsupervised training.

As shown in Fig. 4 some of the errors with relatively long durations made by Encoder-Sup were fixed by the VAE-based methods. The chord label sequence obtained by VAE-Uniform-
SemiSup, however, still included a number of incorrect short fragments, which were caused by the local fluctuations of the chroma vectors. A possible reason is that the chord estimator $q_\theta(S|X)$ could be encouraged to frequently vary over time such that the fine structure of chroma vectors could reconstructed precisely in the VAE framework. In contrast, the chord label sequence obtained by VAE-Markov-SemiSup included less frequent transitions and was much closer to the ground-truth label sequence. This comparison shows the regularization effect of the Markov prior reflecting the temporal continuity of frame-level chord labels.

C. Further observations

The effect of the proposed method (VAE-Markov-SemiSup) depends on the chord types. Fig. 5 shows the confusion matrices obtained by Encoder-Sup and VAE-Markov-SemiSup. While the accuracies on the maj, min, aug and dim types were improved, the accuracies on other uncommon chord types were significantly degraded with the semi-supervised training. Rare chords tended to be wrongly classified to the maj or min triads. Interestingly, the accuracies were not necessarily correlated to the amount of chord examples in the training data (Table 1).

The different behaviors between the popular and rare chord categories can also be observed from the outputs of the generative model $p_\theta(X|S,Z)$. As shown in Fig. 6, $p_\theta(X|S,Z)$ could output the reasonable distributions of chroma vectors when conditioned by the popular major, minor, augmented and diminished triads. In contrast, the distributions conditioned by the rare sus2, sus4 triads and the 5 (power) chords were partly corrupted. Fig. 3 and Fig. 6 indicate the relations between the performance of chord estimation and the conditional chroma vector generation: if the generator fails to learn the pitch distribution of a chord type, the classifier cannot be trained properly to recognize that type. Consequently, the generator tend to reconstruct the observed chroma vectors more well when conditioned by maj triads, thus encouraging the classifier to output maj triads. To sum up, the generative model tends to enhance the negative effect of the ambiguities in chroma vectors.

V. CONCLUSION

This paper described a statistical method that trains a neural chord estimator in a semi-supervised manner by constructing a VAE with discrete and continuous latent variables corresponding to chord labels and chroma textures. This is a new approach to ACE that unifies the generative and discriminative methods. Our method can incorporate a Markov prior on chord labels to encourage the temporal continuity of chord labels estimated with the chord estimator. The comparative experiment clearly showed the effectiveness of using the generative model of chroma vectors and the Markov prior on chord labels in appropriately regularizing the chord estimator, resulting in the performance improvement. We also qualitatively showed how the Markov prior affects the training process.

The experimental results also revealed the limitations of the proposed semi-supervised learning method. It tends to mistakenly classify rare chord types into popular chord types and is vulnerable to the ambiguity between several chord types in observed chroma vectors. This becomes more problematic when the ACE system needs to support a larger chord vocabulary including various seventh chords and chord inversions. Solutions to these problems are included in future work.

The success of the semi-supervised VAE for ACE indicates the effectiveness of unifying the deep generative and discriminative methods in automatic music transcription (AMT), which have been studied separately. Using the amortized variational inference (AVI) framework, we could able to explicitly introduce some prior knowledge on musical symbol sequences as a regularization term. Such knowledge is hard to automatically extract from training data in supervised discriminative methods. One way to improve the performance of ACE is to replace the frame-level Markov prior of chord labels with a beat- or symbol-level language model, which is considered to be effective for solving the ambiguity in acoustic features.

![Figure 4](image-url) An example of estimation results before post-filtering obtained by the supervised and semi-supervised methods. For readability, only the first 24 dimensions (bass and middle channels) of the chroma vectors are displayed.
We also plan to develop a comprehensive AMT system based on the AVI for a unified generative model that has mutually dependent musical elements such as keys, beats, and notes as its latent variables.

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