A Laplace Sum-Rules Analysis of Heavy Pseudoscalar (J^{PC} = 0^{-+}) Hybrids

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Abstract

We use QCD sum-rules to predict ground state masses for pseudoscalar (J^{PC} = 0^{-+}) charmonium and bottomonium hybrids. We find that the inclusion of a six-dimensional gluon condensate contribution is needed to stabilize the analyses. For the charmonium hybrid, we find a mass of (3.82 \pm 0.13) GeV; for the bottomonium hybrid, we find a mass of (10.64 \pm 0.19) GeV. We comment on possible phenomenological implications concerning the Y(3940).

Keywords: hadron spectroscopy, Laplace sum-rules, XYZ resonances, heavy hybrids, pseudoscalar

Over the past decade, more than a dozen new charmonium-like states, the XYZ resonances, have been discovered (see [1] for a recent review). Few of these states can be easily accommodated with a conventional charmonium meson interpretation [2]. There are discrepancies between observations and calculations regarding masses and widths; certain J^{PC} sectors seem to be overpopulated—the vector (1^{--}) states, in particular; and electrically charged resonances have perhaps been seen. Naturally, there has been considerable speculation that some of these new states may lie outside of the constituent quark model.

Motivated by these findings, we use QCD sum-rules to predict ground state masses of both pseudoscalar (0^{-+}) charmonium and bottomonium hybrids. The first application of sum-rules to heavy hybrids was done in [3-5]. Therein, a variety of J^{PC} quantum numbers were investigated; however, in some cases (such as 1^{--}), the sum-rules were unstable and the resulting mass estimates were deemed unreliable. Recently, some of these sum-rules have been re-analysed [6-8]. In the vector analysis of [6], it was seen that the inclusion of a six-dimensional gluon condensate contribution stabilized the sum-rules and led to a reliable ground state mass prediction. Therefore, in an effort to update the original pseudoscalar work in a similar fashion, we also include a six-dimensional gluon condensate contribution.

As in [3], we define a current

\[ j_\mu = \frac{g}{2} \bar{Q} \gamma_\mu J^{a\mu^A} Q, \quad \bar{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\lambda} G^{a\rho\lambda} \]

where \( Q \) represents a heavy quark (charm or bottom) operator, and we define a corresponding two-point function

\[ \Pi_{\mu\nu}(q) = i \int d^4x e^{iqx} \langle 0 | T j_\mu(x) j_\nu(0) | 0 \rangle \]

\[ = \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) \Pi_{VV}(q^2) + \frac{q_\mu q_\nu}{q^2} \Pi_{SS}(q^2). \]

(The S stands for scalar; the V for vector.) Then, the longitudinal projection of \( \Pi_{\mu\nu} \)

\[ \Pi_{SS}(q^2) = \frac{d^4 q^\nu}{q^2} \Pi_{\mu\nu}(q) \]

serves as a probe of heavy, pseudoscalar hybrids.

As in [6], it was seen that the inclusion of a six-dimensional gluon condensate contribution stabilized the sum-rules and led to a reliable ground state mass prediction. Therefore, in an effort to update the original pseudoscalar work in a similar fashion, we also include a six-dimensional gluon condensate contribution.
The numerical values in (5) and (6) are extracted from heavy quark systems [9]. At leading-order in $\alpha$, the relevant Feynman diagrams are shown in Figure 1. Divergences are handled using dimensional regularization. The Wilson coefficients of both gluon condensate terms are calculated using fixed-point gauge methods (see [11, 12], for example). We use the program TARCER [13] to apply the two-loop integral recurrence relations of [14, 15] which significantly reduces the number of integral formulae needed to obtain exact results. The required two-loop integrals are in [16]; the required one-loop integrals are in [17]. Expanding in $\epsilon$ and omitting polynomials in $q^2$ as they ultimately provide no contribution to the sum-rules, we find

$$
\Pi_{(S)}^{\text{pert}}(q^2) = \frac{am^2}{\pi^3} \left( \frac{1}{30}(z - 1) \left( 4z^2 - 21z + 10 \right) \right.
\times 3 F_2(1, 1, 1; 3/2, 3; z)
\left. + \frac{1}{270}( 8z^3 + 8z^2 + 29z - 10) \right. 
\times 3 F_2(1, 1, 2; 5/2, 4; z),
$$

(8)

$$
\Pi_{(S)}^{GG}(q^2) = \frac{m^2}{18\pi^3} (2z + 1) \left( 2z - 2z + 1 \right) \times 3 F_1(1, 1, 5/2; z)
\times \left( \alpha G^2 \right),
$$

(9)

and

$$
\Pi_{(S)}^{GG}(q^2) = \frac{1}{384\pi^3(z - 1)^2} \left( 2z^2 - 2z + 1 \right)
\times 3 F_1(1, 1, 5/2; z)
\times \left( g^2 G^2 \right)
\times \left( z - 20z - 7 \right),
$$

(10)

where $z = \frac{q^2}{4m^2}$ and where $\,_{p}F_{q}(\cdots ; z)$ are generalized hypergeometric functions (see [18], for example). For $q = p - 1$, the function $\,_{p}F_{q}(\cdots ; z)$ has a branch cut discontinuity in the complex $z$-plane extending from $z = 1$ to $z \rightarrow \infty$; as such, it is readily seen from (8)–(10) that $\Pi_{(S)}^{QCD}$ has a branch cut extending from $q^2 = 4m^2$ to $q^2 \rightarrow \infty$ as expected.

Due to its analytic structure and asymptotic behaviour, $\Pi_{(S)}$ satisfies the following dispersion relation:

$$
\Pi_{(S)}(Q^2) = \frac{Q^2}{\pi} \int_{4m^2}^{\infty} \frac{\text{Im} \Pi_{(S)}(t)}{t^2(t + Q^2)} dt + \Pi_{(S)}(0)
+ \frac{Q^2}{2} \Pi_{(S)}'(0) + \frac{1}{6} Q^6 \Pi_{(S)}'''(0)
$$

(11)

where $Q^2 = -q^2$. To eliminate unwanted polynomials in $q^2$ and to suppress the high-energy (continuum) contributions from Im$\Pi_{(S)}$ to the integral on the right-hand side of (11), we apply the Borel transform

$$
\hat{\mathcal{B}} \equiv \lim_{N \rightarrow \infty} \frac{(-Q^2)^N}{N!} \left( \frac{d}{dQ^2} \right)^N
$$

(12)

to obtain

$$
\frac{1}{\tau} \hat{\mathcal{B}} \left[ (-Q^2)^k \Pi_{(S)}(Q^2) \right] = \frac{1}{\pi} \int_{4m^2}^{\infty} \hat{\mathcal{B}} e^{-\tau t} \text{Im} \Pi_{(S)}(t) dt.
$$

(13)

On the left-hand side of (13), we use (7)–(10) to approximate $\Pi_{(S)}$; on the right-hand side, we employ a single narrow resonance plus continuum model

$$
\text{Im} \Pi_{(S)}(t) = f^2 \delta(t - M^2) + \theta(t - s_0) \text{Im} \Pi_{(S)}^{QCD}(t)
$$

(14)

where $s_0$ is the continuum threshold parameter and $f$ represents a hadronic coupling. Moving the threshold contributions from the right- to the left-hand side of (13), we arrive at the Laplace sum-rules [13, 20]

$$
\mathcal{L}_\zeta(s_0) \equiv \frac{1}{\tau} \hat{\mathcal{B}} \left[ (-Q^2)^k \Pi_{(S)}^{QCD}(Q^2) \right]
$$

$$
= \frac{1}{\pi} \int_{s_0}^{\infty} \hat{\mathcal{B}} e^{-\tau t} \text{Im} \Pi_{(S)}^{QCD}(t) dt
$$

(15)

(15)

By exploiting a well-known relationship between the Borel transform and the inverse Laplace transform [21], we can simplify the remaining Borel transformed term.

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1 All Feynman diagrams are drawn using JaxoDraw [19].
in (15) (see [22], for example). Doing so gives

\[
\mathcal{L}_0(\tau, s_0) = \frac{4m^2}{\pi} \int_{s_0/4m^2}^{\infty} e^{-4m^2x^2} \left[ \text{Im} \Pi^{\text{pert}}_{(S)} (4m^2x) + \text{Im} \Pi^{\text{GG}}_{(S)} (4m^2x) \right] dx
\]

\[
+ \lim_{q \to 0^+} \left[ \frac{4m^2}{\pi} \int_{1+q}^{s_0/4m^2} e^{-4m^2x} \text{Im} \Pi^{\text{GG}}_{(S)} (4m^2x) dx \right]
\]

and

\[
\mathcal{L}_1(\tau, s_0) = -\frac{\partial}{\partial \tau} \mathcal{L}_0(\tau, s_0)
\]

where, for \( z > 1 \),

\[
\text{Im} \Pi^{\text{pert}}_{(S)} (q^2) = \frac{am^6}{120m^2z^2} \left[ \sqrt{z-1} \sqrt{z} \right.
\]

\[
\times \left( 30 - 115z + 166z^2 + 8z^3 + 16z^4 \right)
\]

\[- 15 \left( -2 + 9z - 16z^2 + 16z^3 \right) \log \left( \sqrt{z-1} + \sqrt{z} \right) \right],
\]

(18)

\[
\text{Im} \Pi^{\text{GG}}_{(S)} (q^2) = \frac{m^2}{12} (2z+1) \frac{\sqrt{z-1}}{\sqrt{z}} \langle \alpha G^2 \rangle,
\]

(19)

and

\[
\text{Im} \Pi^{\text{GG}}_{(S)} (q^2) = \frac{1}{256\pi^2(z-1)^2} \frac{\sqrt{z-1}}{\sqrt{z}}
\]

\[
\times (2z^2 - 2z + 1)(g^3G^2).
\]

(20)

We note that the portion of (16) that has a double pole at \( z = 1 \) ultimately contributes to both (16) and (17). hence, \( \text{Im} \Pi^{\text{GG}}_{(S)} \) alone is insufficient to formulate the sum-rules. A full calculation of \( \Pi^{\text{GG}}_{(S)} \) is required.

In (16) and (17), the parameters \( m \) and \( \alpha \) are \( \overline{\text{MS}} \)-scheme running quantities evaluated at a scale \( \mu \). For charm quarks,

\[
\alpha(\mu) = \frac{\alpha(M_c)}{1 + \frac{25am(M_c)}{12\pi} \log \left( \frac{\mu^2}{M_c^2} \right)}, \quad \alpha(M_c) = 0.33
\]

(21)

\[
m_c(\mu) = \bar{m}_c \left( \frac{\alpha(\mu)}{\alpha(\bar{m}_c)} \right)^{12/25}, \quad \bar{m}_c = (1.28 \pm 0.02) \text{ GeV}.
\]

(22)

For bottom quarks

\[
\alpha(\mu) = \frac{\alpha(M_b)}{1 + \frac{23am(M_b)}{12\pi} \log \left( \frac{\mu^2}{M_b^2} \right)}, \quad \alpha(M_b) = 0.118
\]

(23)

\[
m_b(\mu) = \bar{m}_b \left( \frac{\alpha(\mu)}{\alpha(\bar{m}_b)} \right)^{12/23}, \quad \bar{m}_b = (4.17 \pm 0.02) \text{ GeV}.
\]

(24)

In (21)–(24), the quark mass parameters are from [23–25], the \( \tau \) and \( Z \) masses are from [26], and \( \alpha(M_\tau) \) and \( \alpha(M_Z) \) are from [22]. Furthermore, renormalization-group improvement \( \mu = \frac{2}{\sqrt{\alpha}} \) [28].

From (13), it follows that

\[
\sqrt{\frac{\mathcal{L}_1(\tau, s_0)}{\mathcal{L}_0(\tau, s_0)}} = M^2.
\]

(25)

The range of acceptable \( \tau \)-values is determined by demanding that, on the left-hand side of (25), the continuum contributes less than 30% and the condensates contribute less than 10%. Then, \( s_0 \) and \( M \) are chosen to optimize (25) over this range. See [27] for further details. In the charmonium hybrid case, we find \( s_0 = 23 \text{ GeV}^2 \) and \( M = (3.82 \pm 0.13) \) GeV; in the bottomonium hybrid case, we find \( s_0 = 140 \text{ GeV}^2 \) and \( M = (10.64 \pm 0.19) \) GeV. The uncertainties in these mass predictions are dominated by the uncertainties in the quark mass parameters [23] and [24] and the uncertainties in the six-dimensional gluon condensate [6]. Plots of the left-hand side of (25) for various values of \( s_0 \) are shown in Figures [2] and [3] for charmonium and bottomonium hybrids respectively. Inclusion of a six-dimensional gluon condensate contribution does indeed seem to stabilize the sum-rule analysis.

![Figure 2: The ratio \( L^{\text{QCD}}_{0}(\tau, s_0)/L^{\text{QCD}}_{1}(\tau, s_0) \) for hybrid charmonium is shown as a function of the Borel scale \( 1/\tau \) for the optimized value \( s_0 = 23 \text{ GeV}^2 \) (solid curve). For comparison the ratio is also shown for \( s_0 = 28 \text{ GeV}^2 \) (upper dotted curve) and \( s_0 = 19 \text{ GeV}^2 \) (lower dotted curve). The uppermost dashed curve represents the \( s_0 \to \infty \) limit corresponding to the bound \( M < 3.96 \text{ GeV} \). Central values of the QCD parameters have been used.](image)
The $Y(3940)$ (likely the same resonance as the $X(3915)$) seen by both the Belle \cite{29} and BaBar \cite{30} collaborations is a $C = +$ isosinglet with a mass of 3.915 GeV. Unfortunately, its parity is unknown and its spin is uncertain: either $J = 0$ or $J = 2$. As such, the $Y(3940)$ could be a pseudoscalar, but additional work is needed to establish its $J^P$ assignment. In \cite{29}, this particle was already touted as a hybrid candidate based on both its production mechanism and its observed decay mode. The $Y(3940)$ is seen in $B$ decays which are thought to be particularly well-suited to hybrid production \cite{31}. Also, the $Y(3940)$ has not been seen to decay through kinematically allowed $DD^{(*)}$ modes, an observation consistent with a flux tube model-inspired selection rule which suppresses hybrid decays to pairs of $S$-wave mesons \cite{32,33}. If the $Y(3940)$ is ultimately shown to have $J^{PC} = 0^+$, then our mass prediction of $\langle 3.82 \pm 0.13 \rangle$ GeV would provide additional support in favour of a charmonium hybrid interpretation.

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\end{itemize}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{The ratio $L_{QCD}(\tau, s_0)/L_{QCD}(\tau, s_0)$ for hybrid bottomonium is shown as a function of the Borel scale $1/\tau$ for the optimized value $s_0 = 140$ GeV$^2$ (solid curve). For comparison the ratio is also shown for $s_0 = 155$ GeV$^2$ (upper dotted curve) and $s_0 = 116$ GeV$^2$ (lower dotted curve). The uppermost dashed curve represents the $s_0 \to \infty$ limit corresponding to the bound $M < 10.84$ GeV. Central values of the QCD parameters have been used.}
\end{figure}