Simulation of the optimal control of a reagent supply in the in-situ uranium leaching process

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Abstract. This article develops an optimization task model for a physicochemical process that takes place in a mineral-bearing formation at the in-situ leaching of mineral. As a result of research, a certain hydrodynamic pressure and velocity field are obtained in a reservoir under the action network of wells, distributive reagent concentrations, mineral in a solid and liquid phase. Furthermore, according to this technique, the intensity-reagent impacts are studied on an extraction degree and mineral extraction time.

1. Introduction
The mineral extraction process using an in-situ leaching method (ISL) involves pumping a solvent into a mineral-bearing formation through injection wells, filtering a solution through an ore body with the selective mineral dissolution and its transfer from a solid phase to a liquid, and pumping the mineral-containing solution through pumping wells [1].

As a reagent in the leaching of uranium, sulfuric acid solutions, carbonate containing ammonium salts, sodium, potassium, etc. are used. The reagent choice depends on the mineralogical ore-deposits properties. In Kazakhstan, a sulfuric acid leaching has become widespread.

2. Mathematical and numerical models
The main reaction for describing the uranium leaching process with a sulfuric acid solution in a schematic form is [1], [2].

\[ \text{UO}_3 + \text{H}_2\text{SO}_4 = \text{UO}_2\text{SO}_4 + \text{H}_2\text{O} \]

This is explained by the good solubility of hexavalent uranium oxides in the sulfuric acid solutions, while the minerals of tetravalent uranium and coffinite under conditions of in-situ leaching practically do not interact with the sulfuric acid solutions without the oxidizing agents' addition to the solutions.

In an operational blocks-development by an ISL method, a reservoir is in the saturated state, therefore, a solution can be taken incompressible. The filtration process of incompressible solution (liquid) in a reservoir containing a dissolved mineral is described by the law of a mass conservation and filtration (Darcy's law) [3].

\[ \text{div} \, \mathbf{v} = W, \quad (1) \]
\[ v = \frac{k}{\mu} \text{grad} p. \]  

(2)

Substituting an equation (2) into the equation (1) and replacing \( h = p / \rho g \), it is possible to get an equation for a hydrodynamic pressure \( h \) in a reservoir in a form:

\[
\frac{\partial}{\partial x} \left( K_{xx} \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_{yy} \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_{zz} \frac{\partial h}{\partial z} \right) = -W,
\]

(3)

where \( \rho \) – fluid density;

\( K_{xx}, K_{yy}, K_{zz} \) – filtration coefficients (hydraulic conductivities), respectively, along with directions \( x, y, z \);

\( W \) – sum of pumping and injection wells.

3. Discussion and results

Because of low uranium concentration in a reservoir, changes in the porosity and density of by-product due to the uranium dissolution with a reagent are neglected. The equations system describing mass transfer processes occurring during the uranium extraction by the in-situ leaching method is written as [4]:

\[
\frac{\partial \vec{C}_m}{\partial t} = -\gamma \varepsilon C_R^0 \bar{C}_m \bar{C}_r,
\]

(4)

\[
\frac{\partial \varepsilon \bar{C}_r}{\partial t} = \text{div} \left( \varepsilon D \text{grad} \bar{C}_r - V \bar{C}_r \right) - \nu_1 \gamma \varepsilon C_m^0 \bar{C}_m \bar{C}_r - \sum_d Q \delta(x_d, y_d, z) \bar{C}_r + \sum_p Q \delta(x - x_0 P, y - y_0 P, z) \bar{C}_p.
\]

(5)

\[
\frac{\partial \varepsilon \bar{C}_p}{\partial t} = \text{div} \left( \varepsilon D \text{grad} \bar{C}_p - V \bar{C}_p \right) + \nu_1 \gamma \varepsilon C_m^0 \bar{C}_m \bar{C}_r - \sum_p Q \delta(x - x_0 P, y - y_0 P, z) \bar{C}_p.
\]

(6)

where

\[ \bar{C}_m = \frac{C_m}{C_m^0}, \quad \bar{C}_r = \frac{C_r}{C_r^0}, \quad \bar{C}_p = \frac{C_p v_1}{C_r^0 v_2}; \]

here \( V \) – filtration rate;

\( \gamma \) – coefficient characterizing a reaction rate;

\( C_m \) – uranium concentration in a solid phase;

\( C_r \) – concentration, sulfuric acid in a solution;

\( C_p \) – productive component concentration (uranium) in a solution;

\( Q \) – well flow rates (\( Q_d < 0 \) for pumping wells, \( Q_p > 0 \) for injection wells);

\( \varepsilon \) – reservoir porosity,

\( v_1 = v_r R / v_m M \),

\( v_2 = v_p P / v_m M \),

\( C_m^0 \) – initial mineral content in a reservoir;

\( C_m^0 \) – initial mineral concentration in a solid phase;
$C_r^0$ – reagent concentration on a pumping well;
$D_{i,j}$ – hydro-dispersion coefficient, determined by the following way:

$$
D_{xx} = \frac{\alpha_1 u^2}{|V|} + \frac{\alpha_1 v^2}{|V|} + \frac{\alpha_1 w^2}{|V|} + D^*;
$$

$$
D_{yy} = \frac{\alpha_1 v^2}{|V|} + \frac{\alpha_1 u^2}{|V|} + \frac{\alpha_1 w^2}{|V|} + D^*;
$$

$$
D_{zz} = \frac{\alpha_1 w^2}{|V|} + \frac{\alpha_1 u^2}{|V|} + \frac{\alpha_1 v^2}{|V|} + D^*;
$$

where $\alpha_1, \alpha_1$ – longitudinal and transverse dispersion coefficients;
$D^*$ – molecular diffusion coefficient.

Equations (4) - (6) are solved under the following initial and boundary conditions: at an initial moment, it is known that the mineral distribution in a reservoir, the concentration of a solution and dissolved useful component are absent:

$$
C_m |_{t=0}= C_m^0, \quad C_r |_{t=0}= C_r^0, \quad C_p |_{t=0}= C_p^0, \quad C_m |_{S=0}, \quad C_r |_{S=0}, \quad C_p |_{S=0}.
$$

When in-situ leaching, the reservoir-formation degree strongly depends on the input reagent concentration and the network location of wells [5]. Therefore, solving the optimizing tasks the mineral leaching process is reduced to solving two problems, one of which is the optimal control task of the flow rate of reagent and the other is the optimal network location of wells.

During leaching, when a well’s location is already known, a process occurring in a reservoir can be controlled only by the input reagent concentration in an injection well. Therefore, in this paper, the reagent concentration $C_r^0$ in an injection well, a change in a mineral value in a liquid phase $C_p^0$ was investigated. In this case, a reagent concentration value set at an injection well should not exceed a certain maximum value. Accordingly, the optimal process control task is reduced to controlling a reagent supply rate in pressure well.

There are two restrictions on reagent concentration values input:
- Inequality limiting a reagent concentration at each injection well:

$$
0 \leq C_{rk}^0 (t) \leq C^0_{r, \text{max}};
$$

- Total concentration equality of reagent at all injection wells to a certain value at each moment in time $t$:

$$
\sum_k \int_0^T C_{rk}^0 (t) dt = C_R,
$$

where $k$ – number of injection wells.

An objective function for this task has a view:

$$
f = \sum_d \int_0^T C_p (x_d, y_d, t) dt \rightarrow \text{max},
$$

where $x_d, y_d$ are decision variables.
where \( d \) – number of pumped wells;
\( C_p \) – mineral concentration in a liquid phase.

An algorithm for solving a problem is as follows: an initial approximation \( C_T^0 \) is set, a direct problem is solved, which involves solving an equations system describing the mineral dissolution process from a reservoir when filtering a reagent-containing solution, in particular: an equation for the hydrodynamic pressure (3) is solved using an upper relaxation iteration method, a velocity field is determined from Darcy's law (1), the equations system for a mineral dissolution (4), the liquid-solution transfer (5) and dissolved mineral (6) is solved jointly by a Crank-Nicholson scheme. An objective function (9) is calculated from the found-values concentration of useful component.

Next, an inverse problem is solved, where the objective function gradient \( \nabla f \) is calculated using a software package. Using the results of direct and inverse problems, an optimization problem (7) - (9) is solved, by means of which the optimal reagent-supply intensity values are determined at an injection well.

In modeling the uranium leaching with a sulfuric acid solution, the linear and hexagonal distribution of wells were considered. Due to the fact that linear wells are often used at the fields, a test calculation was carried out for a field block 20 meters wide and 40 meters long, consisting of three wells: two injection wells and one pumping (figure 1 (a)).

**Figure 1.** Schematic drawing of a considered area: (a) linear well network location in the case of 3 wells: at edges - pumping, in the middle - pumping out; (b) hexagonal wells network location: at edges - pumping, in the middle - pumping.

Figure 2 shows the pressure and isoline distributions of reagent concentrations, mineral in a solid phase and useful component in a liquid phase, at \( t = 100 \) days (vertical section along a well). Figure 3 demonstrates the degree-dependence formation of a reservoir and mineral concentration in pumped well overtime for the hexagonal location cases of wells. The scheme shows that at 70% recovery, the mineral concentration in a pumped well reaches a maximum value, and with the further mineral extraction through mining, the mineral concentration decreases. Figure 4 (a) and (b) illustrate the optimal reagent-concentration values in injection wells when constraints (7) and (8), respectively.
Figure 2. Pressure distribution and concentration isolines: a) pressure distribution, b) reagent, c) mineral in a solid phase, d) useful component in a liquid phase, with t = 100 days (vertical section along a well).

Figure 3. Dependence degree of reservoir formation and mineral concentration in a pumping well over time for the hexagonal location cases of wells: 1 - development field degree; 2 - relative mineral concentrations in a pumped well.

4. Conclusion
In this paper, an optimization problem model was developed for a physicochemical process that takes place in a mineral-containing formation at in-situ leaching. As a result of numerical studies, the obtained hydrodynamic pressure and velocity field were obtained in a reservoir under the action network of wells, the concentrations distribution of reagent, mineral in a solid and liquid phase. Obtained the optimal concentration values of the reagent in injection wells. The intensity influences of reagent supply on the extraction degree and mineral extraction time during the mineral extraction were investigated by an in-situ leaching method.
Figure 4. Optimal values concentration of the reagent in injection wells under different restrictions.

A created model for determining optimal flow regimes for a uranium mining can also be used for other mineral deposits developed by the in-situ leaching method.

References

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