Constraining supersymmetric models from $B_d - \bar{B}_d$ mixing and the $B_d \to J/\psi K_S$ asymmetry

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Abstract

We analyze the chargino contributions to $B_d - \bar{B}_d$ mixing and CP asymmetry of the $B_d \to J/\psi K_S$ decay, in the framework of the mass insertion approximation. We derive model independent bounds on the relevant mass insertions. Moreover, we study these contributions in supersymmetric models with minimal flavor violation, Hermitian flavor structure, and small CP violating phases and universal strength Yukawa couplings. We show that in supersymmetric models with large flavor mixing, the observed values of $\sin 2\beta$ may be entirely due to the chargino–up–squark loops.

1 Introduction

Since its discovery in 1964 in the $K$-meson decays, the origin of CP violation remains an open question in particle physics. In standard model (SM), the phase of the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix provides an explanation of the CP violating effect in these decays. Although the SM is able to account for the observed CP violation in the kaon system and the recent measurement of the (time-dependent) CP asymmetry in $B_d \to J/\psi K_S$ decays, new CP violating sources are necessarily required to describe the observed baryon asymmetry [1]. Moreover, it is expected that with $B$ factories, the $B$–system will represent an ideal framework for crucial tests of the CP violation in SM and probing new physics effects at low energy.

It is a common feature for any new physics beyond the SM to possess additional CP violating phases beside the $\delta_{CKM}$ phase. In supersymmetric (SUSY) models, the soft SUSY breaking terms contain several parameters that may be complex, as may also the
SUSY preserving $\mu$ parameter. These new phases have significant implications for the electric dipole moment (EDM) of electron, neutron, and mercury atom [2]. It was shown that the EDM can be suppressed in SUSY models with small CP phases [3, 4] or in SUSY models with flavor off–diagonal CP violation [3, 5].

The idea of having small CP phases ($\lesssim 10^{-2}$) as an approximate CP–symmetry at low energy, could be an interesting possibility if supported by a mechanism of CP–symmetry restoration at high energy scale. However, this mechanism might also imply that the $\delta_{CKM}$ phase is small [6]. The large asymmetry of the $B$-meson decay $a_{J/\psi K_S}$ observed by BaBar and Belle [7] are in agreement with SM predictions for large $\delta_{CKM}$, and thus the idea of small phases might be disfavored.

However, in Ref.[8] it was shown that in the framework of the minimal supersymmetric standard model (MSSM) with non–universal soft terms and large flavor mixing in the Yukawa, supersymmetry can give the leading contribution to $a_{J/\psi K_S}$ with simultaneous account for the experimental results in $K$–system. Thus the supersymmetric models with small CP violating phases at high energy scale are still phenomenologically viable. An alternative possibility for suppressing the EDMs is that SUSY CP phases has a flavor off–diagonal character as in the SM [5, 9]. Such models would allow for phases of order $\mathcal{O}(1)$ which may have significant effect in $B$-physics [5].

A useful tool for analyzing SUSY contributions to flavour changing neutral current processes (FCNC) is provided, as known, by the mass insertion method [10]. One chooses a basis for the fermion and sfermion states where all the couplings of these particles to neutral gauginos are flavour diagonal, leaving all the sources of FC inside the off–diagonal terms of sfermion mass matrix. These terms are denoted by $(\Delta^{q}_{AB})^{ij}$, where as usual $A, B = (L, R)$ and $i, j = 1, 3$ indicate chiral and flavour indices respectively and $q = u, d$. The sfermion propagator is then expanded as a series of $(\delta^{q}_{AB})^{ij} = (\Delta^{q}_{AB})^{ij}/\tilde{m}^2$, where $\tilde{m}^2$ is an average sfermion mass. This method allows to parametrize, in a model independent way, the main sources of flavour violations in SUSY models. In this framework, the gluino and chargino contributions to the $K$–system have been analyzed in references [10] and [11] respectively. These analyses showed that the bounds on imaginary parts of mass insertions, coming from gluino exchanges to $\varepsilon_K$ and $\varepsilon'/\varepsilon$, are very severe [10], while the corresponding ones from chargino exchanges are less constrained [11]. In particular, in order to saturate $\varepsilon_K$ from the gluino contributions one should have [10] $\sqrt{|\text{Im}(\delta^{d}_{12})_{LL}^2|} \sim 10^{-3}$ or $\sqrt{|\text{Im}(\delta^{d}_{12})_{LR}^2|} \sim 10^{-4}$, and $\sqrt{|\text{Im}(\delta^{d}_{12})_{LR}^2|} \sim 10^{-5}$ from $\varepsilon'/\varepsilon$, while chargino contributions require [11] $\text{Im}(\delta^{d}_{12})_{LL}^2 \sim 10^{-2}$, for average squark masses of order of 500 GeV and gluino masses of the same order.

Recently, in the framework of mass insertion approximation, gluino contributions to the $B_d - \bar{B}_d$ mixing and CP asymmetry in the decay $B_d \to J/\psi K_S$ have been analyzed by including next-to-leading order (NLO) QCD corrections [12] (see also Ref.[13]). However, an analogous study for chargino contributions to these processes is still missing. This kind of analysis would be interesting for the following reasons. First, it would provide a new
set of upper bounds on the mass insertion parameters, namely \((\delta^u_{ij})_{AB}\) in the up-squark sector, which are complementary to the ones obtained from gluino exchanges (which only constrain \((\delta^d_{ij})_{AB}\)). Second, upper bounds on \((\delta^u_{ij})_{AB}\) would be very useful in order to perform easy tests on SUSY models which receive from chargino exchanges the main contributions to \(B_d - \bar{B}_d\) and CP asymmetry. Indeed, in many SUSY scenarios the gluino exchanges are always sub-leading.

In this paper we focus on the dominant chargino contributions to the \(B_d - \bar{B}_d\) mixing and CP asymmetry \(a_{J/\psi K_S}\). We use the mass insertion method and derive the corresponding bounds on the relevant mass insertion parameters. We perform this analysis at the NLO accuracy in QCD by using the results available in Ref.[12]. As an application of our analysis, we also provide a comparative study for supersymmetric models with minimal flavor violation, Hermitian flavor structure with small CP violating phases and universal strength of Yukawa couplings. We show that in all these scenarios, by comparing \((\delta^u_{ij})_{AB}\) and \((\delta^d_{ij})_{AB}\) with their corresponding upper bounds, the chargino contributions are dominant over the gluino ones.

The paper is organized as follows. In section 2 we present the supersymmetric contributions to \(B_d - \bar{B}_d\) mixing and CP asymmetry \(a_{J/\psi K_S}\). We start with a brief review on gluino contributions and then we present our results for the chargino ones, both in the mass insertion approach. In section 3 we derive model independent bounds on the relevant mass insertions involved in the \(B_d - \bar{B}_d\) mixing and \(a_{J/\psi K_S}\). In section 4 we generalize these results by including the case of a light stop–right. Section 5 is devoted to the study of the supersymmetric contribution to \(a_{J/\psi K_S}\) in three different supersymmetric models. We show that the observed values of \(\sin 2\beta\) may be entirely due to the chargino—up–squark loops in some classes of these models. Our conclusions are presented in section 6.

2 Supersymmetric contributions to \(\Delta B = 2\) transitions

We start this section by summarizing the main results on \(B_d - \bar{B}_d\) mixing and CP asymmetry \(a_{J/\psi K_S}\), then we will consider the relevant SUSY contributions to the effective Hamiltonian for \(\Delta B = 2\) transitions, given by the chargino and gluino box diagram exchanges.

In the \(B_d\) and \(\bar{B}_d\) system, the flavour eigenstates are given by \(B_d = (bd)\) and \(\bar{B}_d = (\bar{b}\bar{d})\). It is customary to denote the corresponding mass eigenstates by \(B_H = pB_d + q\bar{B}_d\) and \(B_L = p\bar{B}_d - qB_d\) where indices H and L refer to heavy and light mass eigenstates respectively, and \(p = (1 + \bar{\varepsilon}_B)/\sqrt{2(1 + |\bar{\varepsilon}_B|)}\), \(q = (1 - \bar{\varepsilon}_B)/\sqrt{2(1 + |\bar{\varepsilon}_B|)}\) where \(\bar{\varepsilon}_B\) is the corresponding CP violating parameter in the \(B_d - \bar{B}_d\) system, analogous of \(\bar{\varepsilon}\) in the kaon system [14]. Then the strength of \(B_d - \bar{B}_d\) mixing is described by the mass difference

\[\Delta M_{B_d} = M_{B_H} - M_{B_L}.\] (1)
whose present experimental value is $\Delta M_{B_d} = 0.484 \pm 0.010$ (ps)$^{-1}$ [14].

The CP asymmetry of the $B_d$ and $\bar{B}_d$ meson decay to the CP eigenstate $\psi K_S$ is given by

$$a_{\psi K_S}(t) = \frac{\Gamma(B^0_d(t) \to \psi K_S) - \Gamma(\bar{B}^0_d(t) \to \psi K_S)}{\Gamma(B^0_d(t) \to \psi K_S) + \Gamma(\bar{B}^0_d(t) \to \psi K_S)} = -a_{\psi K_S} \sin(\Delta m_{B_d} t).$$

(2)

The most recent measurements of this asymmetry are given by [7]

$$a_{\psi K_S} = 0.59 \pm 0.14 \pm 0.05 \quad \text{(BaBar)},$$

$$a_{\psi K_S} = 0.99 \pm 0.14 \pm 0.06 \quad \text{(Belle)}.$$  

(3)

where the second and third numbers correspond to statistic and systematic errors respectively, and so the present world average is given by $a_{\psi K_S} = 0.79 \pm 12$. These results show that there is a large CP asymmetry in the $B$ meson system. This implies that either the CP is not an approximate symmetry in nature and that the CKM mechanism is the dominant source of CP violation [5] or CP is an approximate symmetry with large flavour structure beyond the standard CKM matrix [8]. Generally, $\Delta M_{B_d}$ and $a_{\psi K_S}$ can be calculated via

$$\Delta M_{B_d} = 2|\langle B^0_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}^0_d \rangle|,$$

$$a_{\psi K_S} = \sin 2\beta_{\text{eff}}, \quad \beta_{\text{eff}} = \frac{1}{2} \arg\langle B^0_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}^0_d \rangle.$$  

(4)

(5)

where $H^{\Delta B=2}_{\text{eff}}$ is the effective Hamiltonian responsible for the $\Delta B = 2$ transitions. In the framework of the standard model (SM), $a_{\psi K_S}$ can be easily related to one of the inner angles of the unitarity triangles and parametrized by the $V_{CKM}$ elements as follows

$$a_{\psi K_S}^{\text{SM}} = \sin 2\beta, \quad \beta = \arg \left( -\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*} \right).$$

(6)

In supersymmetric theories the effective Hamiltonian for $\Delta B = 2$ transitions, can be generated, in addition to the $W$ box diagrams of SM, through other box diagrams mediated by charged Higgs, neutralino, photino, gluino, and chargino exchanges. The Higgs contributions are suppressed by the quark masses and can be neglected. The neutralino and photino exchange diagrams are also very suppressed compared to the gluino and chargino ones, due to their electroweak neutral couplings to fermion and sfermions. Thus, the dominant SUSY contributions to the off diagonal entry in the $B$-meson mass matrix, $\mathcal{M}_{12}(B_d) = \langle B^0_d|H^{\Delta B=2}_{\text{eff}}|\bar{B}^0_d \rangle$, is given by

$$\mathcal{M}_{12}(B_d) = \mathcal{M}_{12}^{\text{SM}}(B_d) + \mathcal{M}_{12}^{\tilde{g}}(B_d) + \mathcal{M}_{12}^{\tilde{\chi}^+}(B_d).$$

(7)

where $\mathcal{M}_{12}^{\text{SM}}(B_d)$, $\mathcal{M}_{12}^{\tilde{g}}(B_d)$, and $\mathcal{M}_{12}^{\tilde{\chi}^+}$ indicate the SM, gluino, and chargino contributions respectively. The SM contribution is known at NLO accuracy in QCD [14] (as well as the leading SUSY contributions [12]) and it is given by

$$\mathcal{M}_{12}^{\text{SM}}(B_d) = \frac{G_F^2}{12\pi^2} \eta_B \hat{B}_{B_d} f^2_{B_d} M_{B_d} M_{W} (V_{td}V_{tb}^*)^2 S_0(x_t),$$

(8)
where $f_{B_d}$ is the B meson decay constant, $\hat{B}_{B_d}$ is the renormalization group invariant $B$ parameter (for its definition and numerical value, see Ref. [14] and reference therein) and $\eta = 0.55 \pm 0.01$. The function $S_0(x_t)$, connected to the $\Delta B = 2$ box diagram with $W$ exchange, is given by

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1 - x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1 - x_t)^3},$$

(9)

where $x_t = M_t^2/M_W^2$.

The effect of supersymmetry can be simply described by a dimensionless parameter $r_d^2$ and a phase $2\theta_d$ defined as follows

$$r_d^2e^{2i\theta_d} = \frac{M_{12}(B_d)}{M_{12}^{SM}(B_d)},$$

(10)

where $\Delta M_{B_d} = 2|M_{12}^{SM}(B_d)|r_d^2$. Thus, in the presence of SUSY contributions, the CP asymmetry $B_d \rightarrow \psi K_S$ is modified, and now we have

$$a_{\psi K_S} = \sin 2\beta_{\text{eff}} = \sin(2\beta + 2\theta_d).$$

(11)

Therefore, the measurement of $a_{\psi K_S}$ would not determine $\sin 2\beta$ but rather $\sin 2\beta_{\text{eff}}$, where

$$2\theta_d = \arg \left(1 + \frac{M_{12}^{\text{SUSY}}(B_d)}{M_{12}^{SM}(B_d)}\right),$$

(12)

and $M_{12}^{\text{SUSY}}(B_d) = M_{12}^{\tilde{g}}(B_d) + M_{12}^{\tilde{χ}^+}(B_d)$.

### 2.1 Gluino contributions

The most general effective Hamiltonian for $\Delta B = 2$ processes, induced by gluino and chargino exchanges through $\Delta B = 2$ box diagrams, can be expressed as

$$H_{\text{eff}}^{\Delta B=2} = \sum_{i=1}^{5} C_i(\mu)Q_i(\mu) + \sum_{i=1}^{3} \tilde{C}_i(\mu)\tilde{Q}_i(\mu) + \text{h.c.},$$

(13)

where $C_i(\mu)$, $\tilde{C}_i(\mu)$ and $Q_i(\mu)$, $\tilde{Q}_i(\mu)$ are the Wilson coefficients and operators respectively renormalized at the scale $\mu$, with

$$Q_1 = \bar{d}_L^c \gamma_\mu b_L^c \bar{d}_L^c \gamma_\mu b_L^c, \quad Q_2 = \bar{d}_R^c b_L^c \bar{d}_R^c b_L^c, \quad Q_3 = \bar{d}_R^c b_L^c \bar{d}_R^c b_L^c,$$

$$Q_4 = \bar{d}_R^c b_L^c \bar{d}_R^c b_L^c, \quad Q_5 = \bar{d}_R^c b_L^c \bar{d}_R^c b_L^c.$$  

(14)
will show in the next section, in order to connect the Wilson coefficients at SUSY scale with the corresponding ones at low energy scale $\mu \simeq \mathcal{O}(m_b)$, the RGE equations for QCD corrections must be solved. In the case of the gluino exchange all the above operators give significant contributions and the corresponding Wilson coefficients are given by [10], [12]

$$
C_1(M_S) = -\frac{\alpha_s^2}{216m_q^2} \left( 24xf_6(x) + 66\tilde{f}_6(x) \right) (\delta_{13}^d)^2_{LL},
$$

$$
C_2(M_S) = -\frac{\alpha_s^2}{216m_q^2} 204xf_6(x)(\delta_{13}^d)^2_{RL},
$$

$$
C_3(M_S) = \frac{\alpha_s^2}{216m_q^2} 36xf_6(x)(\delta_{13}^d)^2_{RL},
$$

$$
C_4(M_S) = -\frac{\alpha_s^2}{216m_q^2} \left[ (504xf_6(x) - 72\tilde{f}_6(x)) (\delta_{13}^d)^{LL}(\delta_{13}^d)^{RR} - 132\tilde{f}_6(x)(\delta_{13}^d)^{LR}(\delta_{13}^d)^{RL} \right],
$$

$$
C_5(M_S) = -\frac{\alpha_s^2}{216m_q^2} \left[ (24xf_6(x) + 120\tilde{f}_6(x)) (\delta_{13}^d)^{LL}(\delta_{13}^d)^{RR} - 180\tilde{f}_6(x)(\delta_{13}^d)^{LR}(\delta_{13}^d)^{RL} \right],
$$

where $x = m_{\tilde{g}}^2/m_q^2$ and $\tilde{m}$ is an average squark mass. The expression for the functions $f_6(x)$ and $\tilde{f}_6(x)$ can be found in Ref.[12]. The Wilson coefficients $C_{1-3}$ are simply obtained by interchanging $L \leftrightarrow R$ in the mass insertions appearing in $C_{1-3}$.

2.2 Chargino contributions

Here we present our results for the chargino contributions to the effective Hamiltonian in Eq.(13) in the mass insertion approximation. The leading diagrams are illustrated in Fig. 1, where the cross in the middle of the squark propagator represents a single mass insertion. As we will explain in more details below, the dominant chargino exchange, can significantly affect the operators $Q_1$ and $Q_3$ only. We remind here that in the case of the $K-\bar{K}$ mixing, the relevant chargino exchange affects only the operator $Q_1$ [11], as in the SM.

In the framework of mass insertion approximation, one chooses a basis (super-CKM basis) where the couplings of the fermions and sfermions to neutral gauginos are flavour diagonal. In this basis, the interacting Lagrangian involving charginos is given by

$$
\mathcal{L}_{q\tilde{q}\tilde{\chi}^\pm} = -g \sum_k \sum_{a,b} \left( V_{k1} K^*_{ba} \bar{d}_L^{\tilde{K}} (\tilde{\chi}^+)^a \tilde{u}_L^b - U_{k2} (Y^\text{diag}_{d} K^+)_{ab} \bar{d}_R^{\tilde{K}} (\tilde{\chi}^+)^* \tilde{u}_L^b \right)
$$

$$
- V_{k2} (K Y^\text{diag}_{u} )_{ab} \bar{d}_L^{\tilde{K}} (\tilde{\chi}^+)^a \tilde{u}_R^b \right),
$$

where $Y^\text{diag}_{u,d}$ are the diagonal Yukawa matrices, and $K$ is the usual CKM matrix. The indices $a,b$ and $k$ label flavour and chargino mass eigenstates respectively, and $V, U$ are
the chargino mixing matrices defined by

$$U^* M_{\tilde{\chi}^+} V^{-1} = \text{diag}(m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2}), \text{ and } M_{\tilde{\chi}^+} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{pmatrix}.$$  \hspace{1cm} (17)

As one can see from Eq.(16), the Higgsino couplings are suppressed by Yukawas of the light quarks, and therefore they are negligible, except for the stop–bottom interaction which is directly enhanced by the top Yukawa ($Y_t$). The other vertex involving the down and stop could also be enhanced by $Y_t$, but one should pay the price of a $\lambda^3$ suppression, where $\lambda$ is the Cabibbo mixing. Since in our analysis we adopt the approximation of retaining only terms proportional to order $\lambda$, we will neglect the effect of this vertex. Moreover, we also set to zero the Higgsino contributions proportional to Yukawa couplings of light quarks with the exception of the bottom Yukawa $Y_b$, since its effect could be enhanced by large tan $\beta$. In this respect, it is clear that the chargino contribution to the Wilson coefficients $C_4$ and $C_5$ is negligible. Furthermore, due to the colour structure of chargino box diagrams there is no contribution to $C_2$ or $\tilde{C}_2$. However, as we will show in the next section, chargino contributions to $C_2$ or $\tilde{C}_2$ will be always induced at low energy by QCD corrections through the mixing with $C_3$.

Now we calculate the relevant Wilson coefficients $C_{Y,q}(M_S)$ at SUSY scale $M_S$, by using the mass insertion approximation. As mentioned in this case the flavor mixing is displayed by the non–diagonal entries of the sfermion mass matrices. Denoting by $\Delta^q_{AB}$ the off–diagonal terms in the sfermion ($\tilde{q} = \tilde{u}, \tilde{d}$) mass matrices for the up, down respectively, where $A, B$ indicate chirality couplings to fermions $A, B = (L, R)$, the A–B squark propagator can be expanded as

$$\langle \tilde{q}^a_A \tilde{q}^b_B \rangle = i \left( k^2 1 - \tilde{m}^2 1 - \Delta^q_{AB} \right)^{-1} \approx \frac{i \delta_{ab}}{k^2 - \tilde{m}^2} + \frac{i (\Delta^q_{AB})_{ab}}{(k^2 - \tilde{m}^2)^2} + O(\Delta^2),$$  \hspace{1cm} (18)

where $q = u, d$ selects up, down sector respectively, $a, b = (1, 2, 3)$ are flavour indices, $1$ is the unit matrix, and $\tilde{m}$ is the average squark mass. As we will see in the following,
it is convenient to parametrize this expansion in terms of the dimensionless quantity 
$$(\delta_{AB})_{ab} \equiv (\Delta_{AB})_{ab}/\tilde{m}^2$$. At the first order in the mass insertion approximation, we find
for the Wilson coefficients $C_{1,3}(M_S)$ the following result

$$C_1(M_S) = \frac{g^4}{768\pi^2} \sum_{i,j} \left\{ |V_{i1}|^2 |V_{j1}|^2 \left( (\delta_{LL})_{31}^2 + 2\lambda(\delta_{LL})_{31}(\delta_{LL})_{32} \right) \right.$$ 
$$- 2Y_i |V_{i1}|^2 V_{j1} V_{j2}^* \left( (\delta_{LL})_{31}^2 (\delta_{RL})_{31} + \lambda(\delta_{LL})_{32} (\delta_{RL})_{31} + \lambda(\delta_{LL})_{31} (\delta_{RL})_{32} \right)$$
$$+ Y_i^2 V_{i1} V_{j1} V_{j2}^* \left( (\delta_{LL})_{31}^2 + 2\lambda(\delta_{LL})_{31} (\delta_{RL})_{32} \right) \right\} L_2(x_i, x_j),$$

$$C_3(M_S) = \frac{g^4 Y_b^2}{192\pi^2\tilde{m}^2} \sum_{i,j} U_{i2} U_{j2} V_{j1} V_{i1} \left( (\delta_{LL})_{31}^2 + 2\lambda(\delta_{LL})_{31} (\delta_{RL})_{32} \right) L_0(x_i, x_j),$$

where $x_i = m^2_{\chi_i}/\tilde{m}^2$, and the functions $L_0(x, y)$ and $L_2(x, y)$ are given by

$$L_0(x, y) = \sqrt{xy} \left( \frac{x h_0(x) - y h_0(y)}{x - y} \right), \quad h_0(x) = -11 + 7x - 2x^2 \frac{6 \ln x}{(1 - x)^4}$$

$$L_2(x, y) = \frac{x h_2(x) - y h_2(y)}{x - y}, \quad h_2(x) = 2 + 5x - x^2 \frac{6 \ln x}{(1 - x)^4}$$

As in the gluino case, the corresponding results for $\tilde{C}_1$ and $\tilde{C}_1$ coefficients are simply obtained by interchanging $L \leftrightarrow R$ in the mass insertions appearing in the expressions for $C_{1,3}$.

### 3 Constraints from $\Delta M_{B_d}$ and $\sin 2\beta$

In this section we present our numerical results for the bounds on $(\delta_{AB})_{ij}$ which come from $\Delta M_{B_d}$ and CP violating parameter $\sin 2\beta$. We start with the chargino contributions which is found to be the dominant SUSY source in various models [5, 8, 15]. We also provide analytical expressions for $\Delta M_{B_d}$ and $\sin 2\beta$ as functions of the mass insertions in the Wilson coefficients $C_i(M_S)$ of Eqs. (19), (20). In our calculation we take into account NLO QCD corrections in both Wilson coefficients and hadronic matrix elements given in [12].

In order to connect $C_i(M_S)$ at SUSY scale $M_S$ with the corresponding low energy ones $C_i(\mu)$ (where $\mu \simeq \mathcal{O}(m_b)$), one has to solve the renormalization group equations (RGE) for the Wilson coefficients corresponding to the effective Hamiltonian in (13). Then, $C_i(\mu)$ will be related to $C_i(M_S)$ by [12]

$$C_r(\mu) = \sum_i \sum_s \left( b_i^{(r,s)} + \eta c_i^{(r,s)} \right) \eta^{a_i} C_s(M_S),$$

where $M_S > m_\chi$ and $\eta = \alpha_S(M_S)/\alpha_S(\mu)$. The values of the coefficients $b_i^{(r,s)}$, $c_i^{(r,s)}$, and $a_i$ appearing in (22) can be found in Ref.[12]. In our analysis the SUSY scale, where SUSY
particles are simultaneously integrated out, is identified with the average squark mass $\bar{m}$. By using the NLO results of [12], we obtain, for the relevant chargino contributions

$$C_1(\mu) = x_1(\mu) C_1(M_S), \quad C_2(\mu) = x_2(\mu) C_3(M_S), \quad C_3(\mu) = x_3(\mu) C_3(M_S),$$

(23)

while for the other coefficients $C_i(\mu) = 0, \ (i = 4, 5)$. Numerical values for $x_i(\mu)$, evaluated at $\mu = m_b$, are shown in table (1) for some representative values of $M_S$. Notice that the coefficients $b_i^{(23)}$ and $c_i^{(23)}$ are different from zero and so the contribution to $C_2(\mu)$ is radiatively generated at NLO by the off-diagonal mixing with $C_3(M_S)$. For the coefficients $\tilde{C}_{1-3}$ hold the same results as in Eq.(23) and in table (1), since the corresponding $\tilde{b}_i^{(r,s)}$ and $c_i^{(r,s)}$ coefficients in Eq.(22) are the same as the ones for the evolutions of $C_{1-3}$ [12].

The off diagonal matrix elements of the operators $Q_i$ are given by [12]

$$\langle B_d|Q_1|\tilde{B}_d\rangle = \frac{1}{3} m_{B_d} f_{B_d}^2 B_1(\mu),$$

$$\langle B_d|Q_2|\tilde{B}_d\rangle = -\frac{5}{24} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d} f_{B_d}^2 B_2(\mu),$$

$$\langle B_d|Q_3|\tilde{B}_d\rangle = \frac{1}{24} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d} f_{B_d}^2 B_3(\mu),$$

$$\langle B_d|Q_4|\tilde{B}_d\rangle = \frac{1}{4} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d} f_{B_d}^2 B_4(\mu),$$

$$\langle B_d|Q_5|\tilde{B}_d\rangle = \frac{1}{12} \left( \frac{m_{B_d}}{m_b(\mu) + m_d(\mu)} \right)^2 m_{B_d} f_{B_d}^2 B_4(\mu).$$

(24)

The value of $B_1$ has been extensively studied on the lattice [16], but for the other $B_i$ parameters, they have been recently calculated on the lattice by the collaboration in Ref. [17]. In our analysis we will use the following central values reported in [12], namely $B_1(\mu) = 0.87, \ B_2(\mu) = 0.82, \ B_3(\mu) = 1.02, \ B_4(\mu) = 1.16, \ B_5 = 1.91$. The same

| $\mu$ | $x_1(\mu)$ | $x_2(\mu)$ | $x_3(\mu)$ |
|-------|-------------|-------------|-------------|
| 200   | 0.844       | -0.327      | 0.571       |
| 400   | 0.827       | -0.367      | 0.536       |
| 600   | 0.817       | -0.389      | 0.518       |
| 800   | 0.810       | -0.404      | 0.506       |

Table 1: Numerical values for the coefficients $x_i$ (with $i = 1, 2, 3$) in Eq.(23) for some representative values of SUSY scale $M_S$, and evaluated at the low energy scale $\mu = m_b$.

results of Eq.(24) are also valid for the corresponding operators $\tilde{Q}_i$, with same values for the $B_i$ parameters, since strong interactions preserve parity.
Now we start our analysis, by discussing first the dominant chargino contribution to $\Delta M_{B_d}$. Using Eqs.(4), (19)–(20), and (23)–(24), we obtain for $\Delta M_{B_d}$ the following result

$$\Delta M_{B_d} = \frac{g^4 m_{B_d} f_{B_d}^2}{(4\pi)^2 m^2} |R + \bar{R}|$$  \hspace{1cm} (25)

$$R = \left( (\delta_{LL}^u)_{31}^2 + 2\lambda (\delta_{LL}^u)_{31}(\delta_{LL}^u)_{32} \right) \times \left( 2A_1 x_1(\mu) B_1(\mu) + A_4 X(\mu) (x_3(\mu) B_3(\mu) - 5x_2(\mu) B_2(\mu)) \right) + \left( (\delta_{LL}^u)_{31}(\delta_{RR}^u)_{31} + \lambda (\delta_{LL}^u)_{31}(\delta_{RL}^u)_{32} + (\delta_{LL}^u)_{32}(\delta_{RL}^u)_{31} \right) 2A_2 x_1(\mu) B_1(\mu) + \left( (\delta_{RL}^u)_{31}^2 + 2\lambda (\delta_{RR}^u)_{31}(\delta_{RL}^u)_{32} \right) 2A_3 x_1(\mu) B_1(\mu)$$  \hspace{1cm} (26)

where $\bar{R}$, which parametrizes the contributions of $\bar{Q}_{1-3}$ operators, is obtained from $R$ by exchanging $L \leftrightarrow R$ in the mass insertions, $X(\mu) = (m_{B_d}/(m_b(\mu) + m_d(\mu)))^2$, and the expressions of $A_i$ are given by

$$A_1 = \sum_{i,j} |V_{ii}|^2 |V_{jj}|^2 L_2(x_i, x_j), \hspace{1cm} A_2 = Y_t \sum_{i,j} |V_{ii}|^2 V_{ij}^* V_{j2}^* L_2(x_i, x_j),$$

$$A_3 = Y_t^2 \sum_{i,j} V_{ii}^* V_{i2}^* V_{j1}^* V_{j2}^* L_2(x_i, x_j), \hspace{1cm} A_4 = Y_b^2 \sum_{i,j} U_{i2}^* U_{j2}^* V_{i1}^* V_{j1}^* L_0(x_i, x_j)$$  \hspace{1cm} (27)

where the definition of the quantities appearing in (27) can be found in section [2]. Notice that the renormalization scheme dependence in Eq.(25) (for $\mu$ varying in the range $\mu \simeq (m_b/2, 2m_b)$), is strongly reduced due to the NLO QCD accuracy.

As customary in this kind of analysis [10], in order to find conservative upper bounds on mass insertions, the SM contribution to $\Delta M_{B_d}$ is set to zero. Moreover, since we are analyzing $\Delta M_{B_d}$ which is a CP conserving quantity, we keep the squark mass matrices real. Upper bounds are then obtained by requiring that the contribution of the real part of each independent combination of mass insertions in Eq.(25) does not exceed the experimental central value $\Delta M_{B_d} < 0.484(\text{ps})^{-1}$.

These constraints depend on the relevant MSSM low energy parameters, in particular, by $\tilde{m}$, $M_2$, $\mu$ and $\tan\beta$. Notice that with respect to the gluino mediated FCNC processes, which are parametrized by $\tilde{m}$, $M_3$, the chargino mediated ones contains two free parameters more.

In tables (2) and (3), we present our results for upper bounds on mass insertions coming from $\Delta M_{B_d}$, given for some representative values of $\tilde{m}$ and $M_2$ and for fixed values of $\mu = 200$ GeV and $\tan\beta = 5$. In table (2) we provide constraints on $\sqrt{|\text{Re}[(\delta_{LL}^u)_{31}]^2|}$ for several combinations of $\tilde{m}$ and $M_2$. We find that these bounds are almost insensitive to $\mu$ and $\tan\beta$ in the ranges of $200 - 500$ GeV and 3–40 respectively. This can be simply understood by noticing that the contributions to $\Delta B = 2$ transitions mediated by LL

*With abuse of notation, we used here the same symbol $\mu$ for the renormalization scale of Wilson coefficients and the Higgs mixing parameter of MSSM.
interactions are mainly given by the weak gaugino component of chargino. Therefore, the corresponding bounds are more sensitive to \( M_2 \) instead of \( \mu \) and \( \tan \beta \), since these last two parameters contribute to the Higgsino components of chargino. The only term in Eq.(25) which is quite sensitive to \( \tan \beta \) is \( A_4 \), because it is proportional to the bottom Yukawa coupling squared. However, \( (\delta^u_{LL})_{31} \), in addition to \( A_4 \), receives contributions also from the \( A_1 \) term. This term is larger than \( A_4 \) and almost insensitive to \( \tan \beta \), leaving the bounds on \( (\delta^u_{LL})_{31} \) almost independent from \( \tan \beta \).

| \( M_2 \) \( \setminus \) \( m \) | 300 | 500 | 700 | 900 |
|---|---|---|---|---|
| 150 | \( 1.3 \times 10^{-1} \) | \( 1.7 \times 10^{-1} \) | \( 2.2 \times 10^{-1} \) | \( 2.8 \times 10^{-1} \) |
| 250 | \( 1.9 \times 10^{-1} \) | \( 2.3 \times 10^{-1} \) | \( 2.7 \times 10^{-1} \) | \( 3.2 \times 10^{-1} \) |
| 350 | \( 2.7 \times 10^{-1} \) | \( 2.8 \times 10^{-1} \) | \( 3.3 \times 10^{-1} \) | \( 3.7 \times 10^{-1} \) |
| 450 | \( 3.6 \times 10^{-1} \) | \( 3.6 \times 10^{-1} \) | \( 3.9 \times 10^{-1} \) | \( 4.3 \times 10^{-1} \) |

Table 2: Upper bounds on \( \sqrt{|\text{Re}[(\delta^u_{LL})_{31}]^2|} \) from \( \Delta M_{B_d} \) (assuming zero CKM and SUSY phases), for \( \mu = 200 \) GeV and \( \tan \beta = 5 \), and for some values of \( \tilde{m} \) and \( M_2 \) (in GeV).

\[
\sqrt{|\text{Re}[(\delta^u_{LL})_{31}]^2|} \quad \sqrt{|\text{Re}[(\delta^u_{RL})_{31}]^2|} \quad \sqrt{|\text{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]|}
\]

\begin{array}{|c|c|c|c|}
\hline
\text{m} & \sqrt{|\text{Re}[(\delta^u_{LL})_{31}]^2|} & \sqrt{|\text{Re}[(\delta^u_{RL})_{31}]^2|} & \sqrt{|\text{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]|} \\
\hline
200 & 1.4 \times 10^{-1} & 4.7 \times 10^{-1} & 2.1 \times 10^{-1} \\
400 & 1.8 \times 10^{-1} & 9.0 \times 10^{-1} & 2.7 \times 10^{-1} \\
600 & 2.2 \times 10^{-1} & 1.5 & 3.4 \times 10^{-1} \\
800 & 2.7 \times 10^{-1} & 2.3 & 4.1 \times 10^{-1} \\
\hline
\end{array}

Table 3: Upper bounds on mass insertions as in table (2), for \( M_2 = \mu = 200 \) GeV and \( \tan \beta = 5 \).

\[
\sqrt{|\text{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]|} \quad \sqrt{|\text{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]|} \quad \sqrt{|\text{Re}[(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]|}
\]

\begin{array}{|c|c|c|c|}
\hline
\text{m} & \sqrt{|\text{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]|} & \sqrt{|\text{Re}[(\delta^u_{LL})_{31}(\delta^u_{RL})_{32}]|} & \sqrt{|\text{Re}[(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}]|} \\
\hline
200 & 1.8 \times 10^{-1} & 4.0 \times 10^{-1} & 7.1 \times 10^{-1} \\
400 & 3.0 \times 10^{-1} & 6.3 \times 10^{-1} & 1.3 \\
600 & 4.5 \times 10^{-1} & 9.5 \times 10^{-1} & 2.3 \\
800 & 6.3 \times 10^{-1} & 1.3 & 3.5 \\
\hline
\end{array}

Table 4: Upper bounds on mass insertions as in table (2), for \( M_2 = \mu = 200 \) GeV and \( \tan \beta = 5 \).

In tables (3) and (4) we give our results for the real parts of the other mass insertions (and as well as for \( \sqrt{|\text{Re}[(\delta^u_{LL})_{31}]^2|} \)) which are less constrained, for several values of \( \tilde{m} \)
Table 5: Upper bounds on $\sqrt{|\text{Im}[(\delta_{LL}^{u})_{31}^2]|}$ from $\sin 2\beta = 0.79$ (assuming a zero CKM phase), for $\mu = 200$ GeV and $\tan \beta = 5$, and for some values of $\tilde{m}$ and $M_2$ (in GeV).

| $M_2 \ \backslash \ m$ | 300 | 500 | 700 | 900 |
|------------------------|-----|-----|-----|-----|
| 150                    | $1.5 \times 10^{-1}$  | $2.0 \times 10^{-1}$  | $2.6 \times 10^{-1}$  | $3.1 \times 10^{-1}$  |
| 250                    | $2.2 \times 10^{-1}$  | $2.6 \times 10^{-1}$  | $3.1 \times 10^{-1}$  | $3.6 \times 10^{-1}$  |
| 350                    | $3.0 \times 10^{-1}$  | $3.3 \times 10^{-1}$  | $3.7 \times 10^{-1}$  | $4.2 \times 10^{-1}$  |
| 450                    | $4.0 \times 10^{-1}$  | $4.1 \times 10^{-1}$  | $4.4 \times 10^{-1}$  | $4.8 \times 10^{-1}$  |

and evaluated at $M_2 = \mu = 200$ GeV and $\tan \beta = 5$. For larger values of $\mu$ and $M_2$, these bounds become clearly less stringent due to the decoupling. Notice that they are also quite insensitive to $\tan \beta$, since no mass insertion in Eq.(25) receives leading contributions from bottom Yukawa couplings. It is also worth mentioning that the bounds on the mass insertion $(\delta_{LL}^{u})_{32} (\delta_{RL}^{u})_{31}$ are identically to the bounds of $(\delta_{LL}^{u})_{31} (\delta_{RL}^{u})_{32}$, since they have the same coefficients in $C_1^X$ as can be seen from Eq.(26). Therefore, here we just present the bounds of one of them. Moreover, due to the results in Eqs.(25)–(26) and the strategy adopted in setting constraints, the upper bounds for the other mass insertions combinations, where $L \leftrightarrow R$, turn out to be exactly the same as the corresponding ones in tables (2)–(4), and therefore we do not show them in our analysis.

In analogy to the procedure used for obtaining bounds from $\Delta M_{B_d}$, the imaginary parts will be constrained by switching off the SM CKM phase and imposing that the contribution of the SUSY phases to $\sin 2\beta$ does not exceed its experimental central value $(\sin 2\beta)^{\text{exp}} = 0.79$. In particular we obtain

$$(\tan 2\beta)^{\text{exp}} < \left( \frac{g^4 m_{B_d} f_{B_d}}{(48\pi)^2 \tilde{m}^2 \Delta M_{B_d} \text{Im}[R]} \right)$$

where $R$ is defined in Eq.(26).

In tables (5)–(7), we present our numerical results for the bounds on imaginary parts of mass insertions. Clearly, due to the procedure used in our analysis, these bounds turn out to be just proportional to the corresponding ones in tables (2)–(4), and therefore the same considerations about $\mu$ and $\tan \beta$ dependence hold for these bounds as well.

Next we consider the upper bounds on the relevant mass insertions in the down–squark sector, mediated by gluino exchange. In Ref.[12] the maximum allowed values for the real and imaginary parts of the mass insertions $(\delta_{LL}^{d})_{13}$ and $(\delta_{LR}^{d})_{13}$ are given by taking into account the NLO QCD corrections. However, in that analysis the SM contributions to $\Delta M_{B_d}$ and $\sin 2\beta$ are assumed not vanishing. In order to compare our bounds on up–squark mass insertions with the corresponding ones in the down-squark sector, we should use for these last ones the same strategy adopted above. Therefore, in order to find conservative upper bounds on down-squark mass insertions, we will impose that the pure
The upper bounds on the real parts of relevant combinations of mass insertions \((\delta_d^A B^B)_{13}\) (with \(A, B = (L, R)\)) from the gluino contribution to \(\Delta M_{Bd}\) are presented in table (8). In table (9) we show the corresponding bounds for the imaginary parts obtained from the gluino contribution to CP asymmetry \(a_{J/\psi K_S}\), again assuming zero SM contribution. The upper bounds on the other mass insertions in which \(L \leftrightarrow R\) are not shown here, since, as for the chargino case, they turn to be exactly the same as the corresponding ones in tables (8) and (9).

### Table 6: Upper bounds on mass insertions as in table (5), for \(M_2 = \mu = 200\) GeV and \(\tan\beta = 5\).

| \(m\) | \(\sqrt{\text{Im}[(\delta^u_{LL})_{31}]}\) | \(\sqrt{\text{Im}[(\delta^u_{RL})_{31}]}\) | \(\sqrt{\text{Im}[(\delta^u_{LL})_{31} (\delta^u_{RL})_{32}]}\) |
|---|---|---|---|
| 200 | \(1.6 \times 10^{-1}\) | \(5.4 \times 10^{-1}\) | \(2.4 \times 10^{-1}\) |
| 400 | \(2.0 \times 10^{-1}\) | \(1.0\) | \(3.0 \times 10^{-1}\) |
| 600 | \(2.5 \times 10^{-1}\) | \(1.7\) | \(3.8 \times 10^{-1}\) |
| 800 | \(3.1 \times 10^{-1}\) | \(2.7\) | \(4.6 \times 10^{-1}\) |

### Table 7: Upper bounds on mass insertions as in table (5), for \(M_2 = \mu = 200\) GeV and \(\tan\beta = 5\).

| \(m\) | \(\sqrt{\text{Im}[(\delta^u_{LL})_{31} (\delta^u_{RL})_{31}]}\) | \(\sqrt{\text{Im}[(\delta^u_{LL})_{31} (\delta^u_{RL})_{32}]}\) | \(\sqrt{\text{Im}[(\delta^u_{RL})_{31} (\delta^u_{RL})_{32}]}\) |
|---|---|---|---|
| 200 | \(2.1 \times 10^{-1}\) | \(4.5 \times 10^{-1}\) | \(8.0 \times 10^{-1}\) |
| 400 | \(3.4 \times 10^{-1}\) | \(7.2 \times 10^{-1}\) | \(1.5\) |
| 600 | \(5.1 \times 10^{-1}\) | \(1.1\) | \(2.5\) |
| 800 | \(7.2 \times 10^{-1}\) | \(1.5\) | \(4.0\) |

The upper bounds on the real parts of relevant combinations of mass insertions \((\delta_d^{d,k})_{13}\) (with \(A, B = (L, R)\)) from the gluino contribution to \(\Delta M_{Bd}\) are presented in table (8). In table (9) we show the corresponding bounds for the imaginary parts obtained from the gluino contribution to CP asymmetry \(a_{J/\psi K_S}\), again assuming zero SM contribution. The upper bounds on the other mass insertions in which \(L \leftrightarrow R\) are not shown here, since, as for the chargino case, they turn to be exactly the same as the corresponding ones in tables (8) and (9).

### 4 Light stop scenario

In this section we will provide analytical and numerical results for the bounds on mass insertions, in the particular case in which one of the eigenvalues of the up–squark mass matrix is much lighter than the other (almost degenerates) ones. This scenario appears in the specific model that we will analyze in section (5.3), where the mass of the stop–right \((m^2_{t_R})\) is lighter than the other diagonal terms in the up–squark mass matrix. Then the analytical results for the Wilson coefficients provided in section (3) will be generalized by
Table 8: Upper bounds on real parts of combinations of mass insertions $\delta_{AB}^d$, with $(A, B) = L, R$, from gluino contributions to $\Delta M_{B_u}$ (assuming zero SM contribution), evaluated at $\tilde{m} = 400$ GeV and for some values of gluino mass $M_3$ (in GeV).

| $M_3$ | $\sqrt{\text{Re} \left[ (\delta_{LL}^d)^2 \right]}$ | $\sqrt{\text{Re} \left[ (\delta_{RL}^d)^2 \right]}$ | $\sqrt{\text{Re} \left[ (\delta_{LL}^d)_{31}(\delta_{RR}^d)_{31} \right]}$ | $\sqrt{\text{Re} \left[ (\delta_{LR}^d)_{31}(\delta_{RL}^d)_{31} \right]}$ |
|---|---|---|---|---|
| 200 | $4.6 \times 10^{-2}$ | $2.2 \times 10^{-2}$ | $8.4 \times 10^{-3}$ | $1.1 \times 10^{-2}$ |
| 400 | $1.0 \times 10^{-1}$ | $2.4 \times 10^{-2}$ | $9.6 \times 10^{-3}$ | $1.9 \times 10^{-2}$ |
| 600 | $4.8 \times 10^{-1}$ | $2.9 \times 10^{-2}$ | $1.2 \times 10^{-2}$ | $3.0 \times 10^{-2}$ |
| 800 | $2.4 \times 10^{-1}$ | $3.4 \times 10^{-2}$ | $1.4 \times 10^{-2}$ | $4.4 \times 10^{-2}$ |

Table 9: Upper bounds on imaginary parts of combinations mass insertions $\delta_{AB}^d$, with $(A, B) = L, R$, from gluino contributions to $\sin 2\beta$ (assuming zero SM contribution), evaluated at $\tilde{m} = 400$ GeV and for some values of gluino mass $M_3$ (in GeV).

| $M_3$ | $\sqrt{\text{Im} \left[ (\delta_{LL}^d)^2 \right]}$ | $\sqrt{\text{Im} \left[ (\delta_{RL}^d)^2 \right]}$ | $\sqrt{\text{Im} \left[ (\delta_{LL}^d)_{31}(\delta_{RR}^d)_{31} \right]}$ | $\sqrt{\text{Im} \left[ (\delta_{LR}^d)_{31}(\delta_{RL}^d)_{31} \right]}$ |
|---|---|---|---|---|
| 200 | $5.2 \times 10^{-2}$ | $2.5 \times 10^{-2}$ | $9.6 \times 10^{-3}$ | $1.2 \times 10^{-2}$ |
| 400 | $1.2 \times 10^{-1}$ | $2.7 \times 10^{-2}$ | $1.1 \times 10^{-2}$ | $2.2 \times 10^{-2}$ |
| 600 | $5.5 \times 10^{-1}$ | $3.3 \times 10^{-2}$ | $1.3 \times 10^{-2}$ | $3.4 \times 10^{-2}$ |
| 800 | $2.8 \times 10^{-1}$ | $3.9 \times 10^{-2}$ | $1.6 \times 10^{-2}$ | $5.0 \times 10^{-2}$ |

including this effect.\(^\dagger\) In our case, this modification will affect only the expression for the Wilson coefficient $C_1^\chi(M_S)$ in Eq.\(\text{(19)}\), since the stop–right does not contribute to $C_3^\chi(M_S)$ at $\mathcal{O}(\lambda)$ order, as it can be seen from Eq.\(\text{(20)}\).

By taking different the mass of the stop–right from the average squark mass, we obtain the following result\(^\ddagger\)

$$ C_1^\chi(M_S) = \frac{g^4}{768\pi^2\tilde{m}^2} \sum_{i,j} \left[ |V_{i1}|^2 |V_{j1}|^2 \left( (\delta_{LL}^u)^2 + 2\lambda (\delta_{LL}^u)_{31}(\delta_{RR}^u)_{31} \right) L_2(x_i, x_j) \right. $$

$$ \left. - 2Y_t |V_{i1}|^2 V_{j1}^* V_{j2}^* \left( (\delta_{LL}^u)_{31}(\delta_{RR}^u)_{31} + \lambda (\delta_{LL}^u)_{32}(\delta_{RR}^u)_{32} + (\delta_{RR}^u)_{31}(\delta_{LL}^u)_{32} \right) R_2(x_i, x_j, z) \right] $$

$$ + Y_t^2 V_{i1}^* V_{j1}^* V_{j2}^* \left( (\delta_{RL}^u)^2 + 2\lambda (\delta_{RL}^u)_{31}(\delta_{RL}^u)_{32} \right) \tilde{R}_2(x_i, x_j, z), \quad (29) $$

where $x_i = m_{\tilde{\chi}_i^0}^2 / \tilde{m}^2$, $z = m_{\tilde{t}_R}^2 / \tilde{m}^2$ and the functions $R_2(x, y, z)$ and $\tilde{R}_2(x, y, z)$ are given

\(^\dagger\)We do not consider here the contributions of a light right-stop to the $\tilde{Q}_i$ operators, since in this case the effect of two mass insertions $\Delta_{RR}^d$ can invalidate the MIA method, being no heavy squarks running in the loop.

\(^\ddagger\)We have used the same method introduced in Ref.\([18]\), but our results are presented in a different way.
by

\[ R_2(x, y, z) = \frac{1}{x - y} (H_2(x, z) - H_2(y, z)), \quad \tilde{R}_2(x, y, z) = \frac{1}{x - y} (\tilde{H}_2(x, z) - \tilde{H}_2(y, z)) \]

\[ H_2(x, z) = \frac{3}{D_2(x, z)} \left\{ (-1 + x)(x - z)(-1 + z)(-1 - x - z + 3xz) + 6x^2(-1 + z)^3 \log(x) - 6(-1 + x)^3z^2 \log(z) \right\} \]

\[ \tilde{H}_2(x, z) = \frac{-6}{D_2(x, z)} \left\{ (-1 + x)(x - z)(-1 + z)(x + (-2 + x)z) + 6x^2(-1 + z)^3 \log(x) - 6(-1 + x)^2z(-2x + z + z^2) \log(z) \right\} \]

(30)

where \( D_2(x, z) = (-1 + x)^3(x - z)(-1 + z)^3 \) and \( \tilde{D}_2(x, z) = (-1 + x)^2(x - z)^2(-1 + z)^3 \). Notice that in the limit \( z \to 1 \), both the functions \( R_2(x, y, z) \) and \( \tilde{R}_2(x, y, z) \) tend to \( L_2(x, y) \), recovering the result in Eq.(19). Analogously, the expressions for \( A_2 \) and \( A_3 \) entering in Eq.(26) must be substituted by

\[ A_2 = Y_t \sum_{i,j} |V_{ti}|^2 V_{ji}^* R_2(x_i, x_j, z), \quad A_3 = Y_t^2 \sum_{i,j} V_{ti}^* V_{ji}^* V_{ji}^* V_{ji}^* \tilde{R}_2(x_i, x_j, z) \]

(31)

while \( A_1 \) and \( A_4 \) remain the same. In tables (10) and (11) we show our results, analogous to the ones in tables (3)-(6), for the bounds on real and imaginary parts on mass insertions respectively, by taking into account a light stop–right mass. We considered two representative cases of \( \tilde{m}_{t_R} \), for the bounds on mass insertions containing LL interactions. From these results we could see that the effect of taking \( \tilde{m}_{t_R} < \tilde{m} \) is sizable. In particular, on the bounds of the mass insertions \( (\delta_{RL}^u)_{31}(\delta_{RL}^u)_{3i}, (i = 1, 2) \) which are the most sensitive to a light stop–right.

From the results in tables (10) and (11), it is remarkable to notice that, in the limit of very heavy squark masses but with fixed right stop and chargino masses, the bounds on \( (\delta_{RL}^u)_{31}(\delta_{RL}^u)_{3i} \) tend to constant values. This is indeed an interesting property which shows a particular non–decoupling effect of supersymmetry when two light right–stop run inside the diagrams in Fig. (1). This feature is related to the infrared singularity of the loop function \( \tilde{R}_2(x, x, z) \) in the limit \( z \to 0 \). In particular, we find that \( \lim_{z \to 0} \tilde{R}_2(x, x, z) = f(x)/x \), where \( x = m^2/\tilde{m}^2 \), and \( f(x) \) is a non-singular and non-null function in \( x = 0 \). Then, in the limit \( \tilde{m} >> m_\chi \) the rescaling factor \( 1/\tilde{m}^2 \) in \( C^2_1 \) will be canceled by the \( 1/x \) dependence in the loop function and replaced by \( 1/m_\chi^2 \) times a constant factor.

This is a quite interesting result, since it shows that in the case of light right stop and charginos masses, in comparison to the other squark masses, the SUSY contribution (mediated by charginos) to the \( \Delta B = 2 \) processes might not decouple and could be sizable, provided that the mass insertions \( (\delta_{RL}^u)_{3i} \) are large enough. This effect could be achieved, for instance, in supersymmetric models with non–universal soft breaking terms.
Table 10: Upper bounds on real parts of mass insertions as in tables (3)–(4), for some values of \( \tilde{m} \) and \( \tilde{m}_{tr} \) (in GeV). In the fourth column the first number and the one in parenthesis correspond to \( i = 1 \) and \( i = 2 \) respectively. Upper bounds on mass insertions involving only LL interactions are the same as in tables (3)–(4).

| \( \tilde{m} \) | \( \tilde{m}_{tr} \) | \( \sqrt{|\text{Re}[(\delta_{RL}^{u})_{31}^2]|} \) | \( \sqrt{|\text{Re}[(\delta_{LL}^{u})_{31}^2(\delta_{RL}^{u})_{31}]|} \) | \( \sqrt{|\text{Re}[(\delta_{RL}^{u})_{31}(\delta_{RL}^{u})_{32}]|} \) |
|---|---|---|---|---|
| 400 | 100 | 1.9\times10^{-1} | 1.6(3.3)\times10^{-1} | 2.8\times10^{-1} |
| 600 | 100 | 1.8\times10^{-1} | 1.9(4.0)\times10^{-1} | 2.6\times10^{-1} |
| 800 | 100 | 1.8\times10^{-1} | 2.3(4.9)\times10^{-1} | 2.6\times10^{-1} |
| 400 | 200 | 3.5\times10^{-1} | 2.0(4.2)\times10^{-1} | 5.2\times10^{-1} |
| 600 | 200 | 3.3\times10^{-1} | 2.3(5.0)\times10^{-1} | 4.9\times10^{-1} |
| 800 | 200 | 3.2\times10^{-1} | 2.8(5.9)\times10^{-1} | 4.8\times10^{-1} |

Table 11: Upper bounds on imaginary parts of mass insertions as in tables (5)–(6), for some values of \( \tilde{m} \) and \( \tilde{m}_{tr} \) (in GeV). In the fourth column the first number and the one in parenthesis correspond to \( i = 1 \) and \( i = 2 \) respectively. Upper bounds on mass insertions involving only LL interactions are the same as in tables (5)–(6).

| \( \tilde{m} \) | \( \tilde{m}_{tr} \) | \( \sqrt{|\text{Im}[(\delta_{RL}^{u})_{31}^2]|} \) | \( \sqrt{|\text{Im}[(\delta_{LL}^{u})_{31}(\delta_{RL}^{u})_{31}]|} \) | \( \sqrt{|\text{Im}[(\delta_{RL}^{u})_{31}(\delta_{RL}^{u})_{32}]|} \) |
|---|---|---|---|---|
| 400 | 100 | 2.1\times10^{-1} | 1.8(3.7)\times10^{-1} | 3.1\times10^{-1} |
| 600 | 100 | 2.0\times10^{-1} | 2.2(4.6)\times10^{-1} | 3.0\times10^{-1} |
| 800 | 100 | 2.0\times10^{-1} | 2.6(5.5)\times10^{-1} | 3.0\times10^{-1} |
| 400 | 200 | 4.0\times10^{-1} | 2.2(4.8)\times10^{-1} | 6.0\times10^{-1} |
| 600 | 200 | 3.7\times10^{-1} | 2.7(5.6)\times10^{-1} | 5.6\times10^{-1} |
| 800 | 200 | 3.6\times10^{-1} | 3.1(6.7)\times10^{-1} | 5.4\times10^{-1} |

5 Specific supersymmetric models

In this section we focus on three specific supersymmetric models and study the impact of the constraints derived in previous sections on their predictions. We discuss first SUSY models with minimal flavor violation, then we study the ones with Hermitian flavor structure, and finally we consider a SUSY model with small CP violating phases with universal strength of Yukawa couplings.

5.1 SUSY models with minimal flavor violation

In supersymmetric models with minimal flavor violation (MFV) the CKM matrix is the only source of flavor violation. In the framework of MSSM (with \( R \) parity conserved) we consider a minimal model, like in the supergravity scenario, where the soft SUSY breaking
terms is assumed to be universal at grand unification scale, i.e., the soft scalar masses, gaugino masses and trilinear and bilinear couplings are given by
\[ m_i^2 = m_0^2, \quad M_a = m_{1/2} e^{-i\alpha M}, \quad A_\alpha = A_0 e^{-i\alpha A}, \quad B = B_0 e^{-i\alpha B}. \] (32)

As mentioned in the introduction, only two of the above phases are independent, and can be chosen as
\[ \phi_A = \arg(A^*M), \quad \phi_B = \arg(B^*M). \] (33)

The main constraints on \( \phi_A \) and \( \phi_B \) are due to the EDM of the electron, neutron and mercury atom. The present experimental bound on EDMs implies that \( \phi_{A,B} \) should be \( \lesssim 10^{-2} \) unless the SUSY masses are unnaturally heavy[3].

In these scenarios, where SUSY phases \( \phi_{A,B} \) are constrained to be very small by EDMs bounds, the supersymmetric contributions to CP violating phenomena in \( K \) and \( B \) mesons do not generate any sizable deviation from the SM prediction. We have to mention that the universal structure for the soft breaking terms, specially the universality of the trilinear couplings, is a very strong assumption. Indeed, in the light of recent works on SUSY breaking in string theories, the soft breaking sector at GUT scale is generally found to be non–universal [19]. Notice that, even if we start with universal soft breaking terms at GUT scale, some off diagonal terms in the squark mass matrices are induced at electroweak (EW) scale by Yukawas interactions through the renormalization group equation (RGE) evolution. Therefore these off–diagonal entries are suppressed by the smallness of the CKM angles and/or the smallness of the Yukawa couplings.

It is important to stress that even though one ignores the bounds from the EDMs and allows larger values (of order \( \mathcal{O}(1) \)) for the SUSY phases \( \phi_{A,B} \), this class of models with MFV can not generate any large contribution to \( \varepsilon_K \) and \( \varepsilon'/\varepsilon \). Therefore, the Yukawa couplings remain the main source of CP violation [20].

Here we also found that, within MFV scenarios, the SUSY contributions to \( \Delta M_{B_d} \) and \( a_{J/\psi K_S} \) are negligible. In fact, due to the universality assumption of soft SUSY breaking terms, it turns out that the gluino and chargino contributions are quite suppressed. For instance, for \( m_0 \sim m_{1/2} \sim A_0 \sim 200 \) GeV and \( \phi_{A,B} \sim \pi/2 \) (which corresponds to \( \tilde{m}^2 \) and \( m_\tilde{g} \) at SUSY scale of order 500 GeV) we find the following values of the relevant mass insertions: \( \text{Im}(\delta_{13}^d)_{LL} \sim \text{Re}(\delta_{13}^d)_{LL} \sim 10^{-4} \) and \( \text{Im}(\delta_{13}^d)_{LR} \sim \text{Re}(\delta_{13}^d)_{LR} \sim 10^{-6} \), which are clearly much smaller than the corresponding bounds mentioned in the previous section.\(^5\)

Therefore, we conclude that SUSY models with MFV do not give any genuine contribution to the CP violating and flavor changing processes in \( K \) and \( B \) systems and this scenario can not be distinguished from the SM model one.

\(^5\)In our analysis we have taken into account the effect of the CP violating phases in the RGE evolution.
5.2 SUSY models with Hermitian flavor structure

As discussed in the introduction, a possible solution for suppressing the EDMs in SUSY model is to have Hermitian flavor structures [5]. In this class of models, the flavor blind quantities, such as the $\mu$–terms and gaugino masses, are real while the Yukawa couplings and $A$-terms are Hermitian, i.e $Y^\dagger_{u,d} = Y_{u,d}$ and $A^\dagger_{u,d} = A_{u,d}$. It has been shown that these models are free from the EDM constraints and the off–diagonal phases lead to significant contribution to the observed CP violation in the kaon system, in particular to $\varepsilon'/\varepsilon$ [5].

Let us consider, for instance, the case of Hermitian and hierarchical quark mass matrices with three zeros [21]

$$M_i = \begin{pmatrix}
0 & a_i e^{i\alpha_i} & 0 \\
0 & b_i e^{i\beta_i} & 0 \\
0 & b_i e^{-i\beta_i} & B_i
\end{pmatrix}; \quad i = u, d \tag{34}
$$

with $A_i = (m_c, m_s)$, $B_i = (m_t - m_u, m_b - m_d)$, $a_i = (\sqrt{m_u m_c}, \sqrt{m_d m_s})$, and $b_i = (\sqrt{m_u m_t}, \sqrt{m_d m_b})$. The phases $\alpha_i$ and $\beta_i$ satisfy: $\alpha_d - \alpha_u = \pi/2$ and $\beta_d - \beta_u = \pi/2$.

These matrices reproduce the correct values for quark masses and CKM matrix. We also assume the following Hermitian $A$–terms:

$$A_d = A_u = \begin{pmatrix}
A_{11} & A_{12} e^{i\varphi_{12}} & A_{13} e^{i\varphi_{13}} \\
A_{12} e^{-i\varphi_{12}} & A_{22} & A_{23} e^{i\varphi_{23}} \\
A_{13} e^{-i\varphi_{13}} & A_{23} e^{-i\varphi_{23}} & A_{33}
\end{pmatrix}. \tag{35}
$$

Notice that, the scenario with non–degenerate $A$-terms is an interesting possibility for enhancing the SUSY contributions to $\varepsilon_K$ and $\varepsilon'/\varepsilon$ [9] and it is also well motivated by many string inspired models. In this case, the mass insertions are given by

$$(\delta^q_{ij})_{LL} = \frac{1}{m_q^2} \left( V^q Q^2 V^q \right)_{ij} \tag{36}$$

$$-(\delta^q_{ij})_{LR} = \frac{1}{m_q^2} \left[ (V^q Y^A_q V^q)_{ij} v_{1(2)} - \mu Y^q_i \delta_{ij} v_{2(1)} \right], \tag{37}
$$

where $q \equiv u, d$ and $(Y^A_q)^{ij} = Y^q_i A^q_{ij}$. Since the Yukawa are Hermitian matrices, they are diagonalized by only one unitary transformation.

In this class of models, we find that in most of the parameter space the chargino gives the dominant contribution to $B_d - \bar{B}_d$ mixing and CP asymmetry $a_{J/\psi K_S}$, while the gluino one is sub-leading. As we emphasized above, in order to have a significant gluino contribution for $\tilde{m} \sim m_q \sim 500$ GeV (i.e., $m_0 \sim M_{1/2} \sim 200$ at GUT scale), the real and imaginary parts of mass insertion $(\delta^d_{13})_{LL}$ or $(\delta^d_{13})_{LR}$ should be of order $10^{-1}$ and $10^{-2}$ respectively. However, with the above hierarchical Yukawas we find that these mass insertions are two orders of magnitude below the required values so that the gluino contributions are very small.
Concerning the chargino amplitude to the CP asymmetry \( a_{J/\psi K_S} \), we find that the mass insertions \( (\delta^{u}_{31})_{RL} \) and \( (\delta^{u}_{31})_{LL} \) give the leading contribution to \( a_{J/\psi K_S} \). However, for the representative case of \( m_0 = m_{1/2} = 200 \) and \( \phi_{ij} \simeq \pi/2 \) the values of these mass insertions are given by

\[
\sqrt{\left|\text{Im}[(\delta^{u}_{LL})_{31}]^2\right|} = 6 \times 10^{-4}, \tag{38}
\]
\[
\sqrt{\left|\text{Im}[(\delta^{u}_{LL})_{31}]^2\right|} = 4 \times 10^{-3}, \tag{39}
\]
\[
\sqrt{\left|\text{Im}[(\delta^{u}_{LL})(\delta^{u}_{RL})_{32}]\right|} = 1 \times 10^{-4}. \tag{40}
\]

These results show that, also for this class of models, SUSY contributions cannot give sizable effects to \( a_{J/\psi K_S} \). As expected, with hierarchical Yukawa couplings (where the mixing between different generations is very small), the SUSY contributions to the \( B - \bar{B} \) mixing and the CP asymmetry of \( B_d \to J/\psi K_S \) are sub-dominant and the SM should give the dominant contribution.

### 5.3 SUSY model with universal strength of Yukawa couplings

Supersymmetric models with small CP violating phases is a possible solution for suppressing the EDMs. In Ref.[8] it was shown that, among this class of models, the ones with universal strength of Yukawa couplings naturally provide very small CP violating phases. However due to the large mixing between different generations, it was found that the \( LL \) mass insertions can give sizable effects to \( \varepsilon_K \) and \( \varepsilon'/\varepsilon \) by means of gluino and chargino exchanges respectively. Furthermore, it was also emphasized that in these models the SM contribution to the CP asymmetry \( a_{J/\psi K_S} \) might be negligible, leaving the dominant SUSY effect (due to the chargino exchange) to account for the experimental results.

Here we will discuss the different contributions to \( B_d - \bar{B}_d \) and \( a_{J/\psi K_S} \) in terms of mass insertions and compare the predictions of this model with the corresponding ones of Hermitian flavor structure discussed in the previous subsection. In the framework of the universal strength Yukawa couplings, the quark Yukawa couplings can be written as

\[
U_{ij} = \frac{\lambda_u}{3} \exp \left[ i\Phi^u_{ij} \right] \quad \text{and} \quad D_{ij} = \frac{\lambda_d}{3} \exp \left[ i\Phi^d_{ij} \right], \tag{41}
\]

where \( \lambda_u,d \) are overall real constants, and \( \Phi^{u,d} \) are pure phase matrices which are constrained to be very small by the hierarchy of the quark masses [8]. The values of these parameters, that lead to the correct quark spectrum and mixing, can be found in Ref.[8]. As explained in that paper, an important feature of this model is the presence of a large mixing between the first and third generation. As we will show in the following, this property will account for large SUSY contributions to \( a_{J/\psi K_S} \).

In the framework of universal strength Yukawa couplings Eq.(41), due to the large generation mixing, the EDMs impose severe constraints on the parameter space and force
the trilinear couplings to take particular patterns as the factorizable matrix form \cite{8}, \emph{i.e.},

\[
A = m_0 \begin{pmatrix}
a & a & a \\
b & b & b \\
c & c & c
\end{pmatrix}.
\]

In order to satisfy the bound of the mercury EDM, the phases of the entries \(a, b\) and \(c\) should be of order \(10^{-2} - 10^{-1}\) \cite{8}. As an illustrative example, we consider \(m_0 = m_{1/2} = 200\) GeV, \(\phi_a = \phi_c = 0, \phi_b = 0.1\) and \(a = -1, b = -2, c = -3\). In this case one finds that at low energy the average squark mass is of order 500 GeV, however one of the stop masses (\(\tilde{t}_R\)) is much lighter, \(m_{\tilde{t}_R} \simeq 200\) GeV. The gaugino mass \(M_2\) is of order 170 GeV and, from the EW breaking condition, \(|\mu|\) turns out to be of order of 400 GeV. In this case, the relevant mass insertions for the gluino contribution are given by

\[
(\delta^d_{LL})_{31} \simeq -0.001 + 0.02 i, \quad (\delta^d_{RL})_{31} \simeq 0.00002 + 0.0009 i.
\]

Regarding the other mass insertions (LR and RR), they are much smaller (\(\lesssim \mathcal{O}(10^{-6})\)), and so we do not show them. It is clear that, with these values for the down–squark mass insertions, the gluino contribution to \(a_{J/\psi K_S}\) is negligible (of the order of 3\%).

On the contrary, the relevant up–squark mass insertions for the chargino contribution are given by

\[
(\delta^u_{LL})_{31} \simeq 0.001 + 0.05 i, \quad (\delta^u_{RL})_{31} \simeq -0.0004 + 0.13 i,
\]

\[
(\delta^u_{LL})_{32} \simeq -0.008 - 0.11 i, \quad (\delta^u_{RL})_{32} \simeq 0.01 - 0.28 i.
\]

Comparing these results with the ones in tables (10) and (11), we see that for this model, the chargino contribution to the imaginary parts \((\delta^u_{RL})_{31}\) and \((\delta^u_{RL})_{32}\) is of the same order of the corresponding upper bounds. Notice that these imaginary parts are of the same order, so that they might coherently contribute to give a sizable effect on \(a_{J/\psi K_S}\). In particular, by using the exact 1-loop calculation, we find that the chargino contribution leads to \(\sin(2\theta_d) \sim 0.75\). Moreover, as a check on our computations, we have compared our results from MIA approximation with the corresponding ones obtained by using the full calculation \cite{8}. In this case we find that, by taking into account the effect of a light stop, the MIA predictions are quite compatible with the results of the full computation.

6 Conclusions

In this paper we have studied the chargino contributions to \(B_d - \bar{B}_d\) mixing and CP asymmetry \(a_{J/\psi K_S}\) in the mass insertion approximation. In our analysis we have taken into account the NLO QCD corrections to the effective Hamiltonian for \(\Delta B = 2\) transitions \(H_{\text{eff}}^{\Delta B=2}\). We provided analytical results for the chargino contribution to \(H_{\text{eff}}^{\Delta B=2}\) in the framework of mass insertion method, and given the expressions for the \(B_d - \bar{B}_d\) and CP
asymmetry $a_{J/\psi K_S}$ at NLO in QCD, as a function of mass insertions in the up–squark sector. We have also provided model independent upper bounds on mass insertions by requiring that the pure chargino contribution does not exceed the experimental values of $B – \bar{B}$ mixing and CP asymmetry $a_{J/\psi K_S}$. Since in many SUSY models the chargino contribution gives the dominant effect to $B – \bar{B}$ mixing and CP asymmetry $a_{J/\psi K_S}$, our results are particularly useful for a ready check of the viability of these models. Moreover we generalized our results by including the case of a light right-stop scenario. In this case we found the interesting property that the bounds on mass insertions combinations $(\delta^u_{RL})_{31}(\delta^u_{RL})_{3i}$ are not sensitive to the common squark mass when this is very large in comparison to the chargino and stop–right ones.

Finally, we applied these results to a general class of SUSY models which are particularly suitable to solve the SUSY CP problem, namely the SUSY models with minimal flavour violations, hermitian flavour structure, and small CP violating phases with universal strength Yukawa couplings. We have shown that in SUSY models with minimal flavor violation and with Hermitian (and hierarchical) Yukawa couplings and $A$–terms, the SUSY contributions to the $B – \bar{B}$ mixing and the CP asymmetry $a_{\psi K_S}$ are very small and the SM contribution in these classes of models should give the dominant effect. On the contrary, in the case of SUSY scenarios with large mixing between different generations in the soft terms, the SUSY contributions become significant and can even be the dominant source for saturating the experimental value of $a_{\psi K_S}$. Among this class of models, we have investigated a SUSY model with universal strength of Yukawa couplings. In this case, we have found that the chargino exchange provides the leading contribution to $a_{\psi K_S}$ through the mass insertions $(\delta^u_{LL})_{31}(\delta^u_{RL})_{3i}$, $i = 1, 2$ and $(\delta^u_{RL})_{31}(\delta^u_{RL})_{32}$.

Acknowledgements

We acknowledge the kind hospitality of the CERN Theory Division where part of this work has been done. The work of S.K. was supported by PPARC. E.G. would like to thank Katri Huitu for useful discussions.

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