Spin resolved Andreev reflection in ferromagnet-superconductor
junctions with Zeeman splitting

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Abstract

Andreev reflection in ferromagnet-superconductor junctions is derived in a
regime in which Zeeman splitting dominates the response of the superconduc-
tor to an applied magnetic field. Spin-up and spin-down Andreev reflections
are shown to be resolved as voltage is increased. In the metallic limit, the
transition from Andreev to tunnel conductivity in the spin-up channels has a
non trivial behavior when spin polarization is increased. The conductance is
asymmetric in a voltage reversal.

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The interplay between Andreev reflection and spin polarization has generated recently an important interest, both theoretical [1–5] and experimental [6–11]. The subgap conductance in normal metal-superconductor (NS) junctions originates from Andreev reflection [12]: a spin-$\sigma$ electron incoming from the N side is reflected as a hole in the spin-$(-\sigma)$ band while a spin-zero Cooper pair is transferred into the superconductor. Since the incoming electron and outgoing hole belong to opposite spin bands, Andreev reflection couples to a Fermi surface polarization in the N side of the junction. de Jong and Beenakker [1] showed theoretically that increasing the Fermi surface polarization in ferromagnet-superconductor (FS) junctions suppresses Andreev reflection because Andreev reflection is limited by the minority-spin channels. Their prediction was verified experimentally by Soulen et al. [10] and Upadhyay et al. [11], who used this effect to measure the Fermi surface polarization. On the other hand, Tedrow and Meservey [13] demonstrated that under specific conditions, a magnetic field can be used to tune a Zeeman splitting of the quasiparticle excitations in a superconductor [13], and used it to perform a spin resolved tunnel spectroscopy in FS junctions [13]. I show in the present Letter that Zeeman splitting can be used to resolve spin-up and spin-down Andreev reflections, with a different threshold voltage $eV_\pm = \Delta \mp \mu_B H$ for the transition from Andreev to tunnel conductivity in a magnetic field $H$. In NS junctions with Zeeman splitting, the spin-up and spin-down differential conductances have the same behavior at the Andreev reflection threshold voltages $V_\pm$. In FS junctions with Zeeman splitting, a non trivial behavior at the spin-up threshold voltage $V_+$ is predicted. In addition, the conductance is asymmetric in a voltage reversal. These behaviors can be probed experimentally.

Our modeling neglects disorder in the superconductor, as well as the proximity effect in the N side of the junction [8,14–20]. This approximation is justified in FS junctions, where superconducting correlations do not extend in the ferromagnet beyond the exchange length $\sqrt{\hbar D/J}$ of order 20 Å [7]. In NS junctions, we expect the qualitative physics arising from the coupling between Andreev reflection and Zeeman splitting to hold also in the presence of disorder. Let us first consider a NS junction with Zeeman splitting. The superconductor
is assumed to have a thin film geometry with the magnetic field applied parallel to the film. We assume a small orbital depairing parameter while the critical field for destroying superconductivity is set by Pauli paramagnetism [21], with large values of $H_{c2\parallel} \sim 5T$ for Al thin films [13]. The spin-orbit scattering length is supposed to be small compared to the superconductor coherence length $\xi$, as it is the case for light elements such as Al [13]. This insures that electrons in the superconductor have a well defined spin $\sigma$ at length $\xi$, and therefore a well defined Zeeman energy $-\mu_B H \sigma$ [13,22].

The coherence factors of spin-$\sigma$ electrons ($u_\sigma$) and holes in the spin-$(-\sigma)$ band ($v_{-\sigma}$) with an energy $\epsilon$ are

$$u_\sigma^2 = 1 - v_{-\sigma}^2 = \frac{1}{2} \left(1 + \frac{\sqrt{(\epsilon + \sigma \mu_B H)^2 - |\Delta|^2}}{\epsilon + \sigma \mu_B H}\right),$$

with therefore a coupling between Andreev reflection and Zeeman splitting. A step function variation of the superconducting gap at the interface is assumed: $\Delta(x) = \Delta \theta(x)$. We consider a $\delta$-function elastic interface scattering potential $V(x) = H_0 \delta(x)$, interpolating between a metallic contact if $H_0 = 0$ and a tunnel junction if $H_0 = \infty$ [23]. The interface barrier is normalized with respect to the Fermi velocity: $Z = m H_0 / (\hbar \sqrt{2m \mu})$, with $\mu = \hbar^2 k_F^2 / 2m$ the chemical potential [23]. The energy dependence of the transmitted quasiparticle wave vectors is irrelevant to the present calculation [24] and we consider identical Fermi wave vectors in the superconductor and the normal metal since this assumption does not change the qualitative physics. Given the coherence factors (1), the Andreev reflection transition probability of electrons with a spin-up and holes in the spin-down band with an energy $\epsilon$ is

$$A^{\epsilon\uparrow}(\epsilon) = A^{h\downarrow}(\epsilon) = A_{\text{BTK}}(\epsilon + \mu_B H),$$

with $A_{\text{BTK}}(\epsilon)$ the Blonder, Thinkham and Klapwijk (BTK) Andreev reflection coefficient [23]. Similarly in the spin-down sector,

$$A^{\epsilon\downarrow}(\epsilon) = A^{h\uparrow}(\epsilon) = A_{\text{BTK}}(\epsilon - \mu_B H).$$

Eqs. 2 and 3 are valid also if $\epsilon < 0$, in which case transmission of quasiparticles on negative energy branches should be considered. Noting $B_{\text{BTK}}$ the BTK backscattering coefficient [23], the zero-temperature differential conductance of spin-$\sigma$ carriers

3
\[
\frac{dI^\sigma}{dV}(eV, H) = \frac{e^2}{h} [1 + A_{\text{BTK}}(eV + \sigma \mu_B H) - B_{\text{BTK}}(eV + \sigma \mu_B H)]
\]

shows a Zeeman splitting for an arbitrary interface scattering in the sense that the magnetic field enters the conductivity via the combination \(eV + \sigma \mu_B H\) only. The tunnel spectrum in the limit \(Z \gg 1\) reproduces the Zeeman splitted density of states of the superconductor \(\rho_\sigma(\epsilon) = \rho_{\text{BCS}}(\epsilon + \sigma \mu_B H)\), with \(\rho_{\text{BCS}}\) the single-spin BCS density of states [13,22,23]. In the metallic limit \(Z = 0\) and below the spin-up threshold voltage \(eV_+ = \Delta - \mu_B H\), spin-up and spin-down transport originate from Andreev reflection, with a conductance of \(2e^2/h\) per spin channel (see Fig. [3]). Spin-up transport transits from Andreev reflection to tunneling at the spin-up threshold voltage, smaller than the spin-down threshold voltage \(eV_- = \Delta + \mu_B H\). In between \(V_+\) and \(V_-\) a plateau of \(3e^2/(2h)\) per spin channel develops in the conductance when \(H\) increases, corresponding to an Andreev reflection transport of spin-down carriers and a tunnel transport of spin-up carriers. Notice that the single-spin conductance in Eq. [4] is not symmetric in a voltage reversal. The total conductance of the NS junction is however symmetric in a voltage reversal because the two spin channels play a symmetric role in this junction.

We now extend our treatment to incorporate the effect of a spin polarization in the normal metal. We show a non trivial transition from Andreev to tunnel transport at the spin-up threshold voltage \(V_+\), as well as a conductance asymmetric in a voltage reversal. We denote by \(n\) and \(n'\) the quantum numbers associated to a quantized transverse motion in a clean FS point contact of cross sectional area \(a^2\). We assume a Stoner ferromagnet with an exchange field \(h_{\text{ex}}(x) = h_{\text{ex}}\theta(-x)\). The channel with transverse quantum numbers \((n, n')\) in the spin-\(\sigma\) band has a dispersion

\[
E_{n,n'}^{\sigma}(k^\sigma) = \frac{\hbar^2(k^\sigma)^2}{2m} - \sigma h_{\text{ex}} + \kappa(n^2 + n'^2),
\]

with the energy \(\kappa = (\hbar^2/2m)(\pi/a)^2\) inverse proportional to the junction area, and related to the number of spin-\(\sigma\) channels according to \(N^\sigma = \pi(\mu + \sigma h_{\text{ex}})/(4\kappa)\) [14]. The associated barrier parameter \(Z_{n,n'}^{F,\sigma}\) of spin-\(\sigma\) electrons in the channel \((n, n')\) is \(Z_{n,n'}^{F,\sigma} = \ldots\)
\[ (1 + \sigma \frac{h_{\text{ex}}}{\mu} - \frac{\kappa}{\mu}(n^2 + n'^2))^{-1/2} Z, \text{ with } Z = mH_0/(\hbar \sqrt{2m\mu}). \]

The transverse dimensions of the S side of the junction are assumed to be identical to the ones of the N side and the gap, the interface scattering and the exchange field are constant in the transverse direction, with therefore a conservation of the transverse quantum numbers across the interface \[24\]. The pairing Hamiltonian in the S side with a cross sectional area \(a^2\) is

\[
H^S = \sum_{n,n',k,\sigma} \left( \frac{\hbar^2 k^2}{2m} + \kappa(n^2 + n'^2) \right) c^n_{n,n',k,\sigma} c^{\dagger}_{n,n',k,\sigma} + \sum_{n,n',k} \left( \Delta c^n_{n,n',k,\uparrow} c^n_{n,n',k,\downarrow} + \text{h.c.} \right),
\]

with an associated barrier parameter \(Z^S_{n,n'} = \left( 1 - \frac{\kappa}{\mu}(n^2 + n'^2) \right)^{-1/2} Z\) different from \(Z^F_{n,n'}\) because of the Fermi wave vector mismatch between the ferromagnet and the superconductor \[5\]. The channels with a spin-up Fermi surface only have a real positive \(Z^F_{n,n'}^{\uparrow}\) and a pure imaginary \(Z^F_{n,n'}^{\downarrow}\). Physically, a spin-up electron incoming from the N side below the superconducting gap in such a channel is Andreev reflected into an evanescent hole state in the spin-down band, with a pure imaginary wave vector \(k^\downarrow\). The hole propagates in the ferromagnet over the length scale \(1/\text{Im}(k^\downarrow)\) before it is backscattered onto the interface and Andreev reflected as a spin-up electron, therefore not carrying current, as proposed by de Jong and Beenakker \[1\]. Incorporating this process under the form of a pure imaginary interface scattering allows to calculate transport above the superconducting gap. The matching of the wave functions between the F and S sides is solved similarly to Ref. \[5\], including the coherence factors in Eq. \[1\] and the barrier parameters \(Z^F_{n,n'}^{\sigma}\) and \(Z^S_{n,n'}\) \[26\]. The differential conductance spectra are shown on Fig. 2 in the metallic limit \(Z = 0\). At low voltage, the conductance shows a reduction of Andreev reflection by spin polarization \[1\]. The large voltage limiting value of the tunnel conductance per spin channel decreases from the Landauer value \(e^2/\hbar\) in the absence of spin polarization to \(e^2/(2\hbar)\) with a full polarization, because only the ferromagnet channels with a corresponding channel in the superconductor contribute to the tunnel conductance. The number of spin-down tunneling channels is \(N^\downarrow\), while spin-up tunneling is limited by the number of superconducting channels \(N^S = \pi\mu/4\kappa\). The total number of tunneling channels is \(\pi(2\mu - h_{\text{ex}})/4\kappa\), reduced by a factor of two when the exchange field \(h_{\text{ex}}\) increases from zero to \(\mu\).
Now the behavior of the differential conductance at the spin-up threshold voltage $eV_+ = \Delta - \mu_B H$ differs qualitatively in the weak and strong polarization regimes: the conductance decreases with voltage at $eV_+$ if spin polarization is weak while it increases if spin polarization is strong (see Fig. 2). With a weak polarization, most of the spin-up channels are Andreev reflected and the decrease in conductance at $eV_+$ can be understood qualitatively on the basis of the transition from Andreev to tunnel transport in the single channel BTK model \[23\]. If spin polarization is strong, a fraction $1 - (N_\downarrow/N_\uparrow)$ of the spin-up channels are not Andreev reflected if $V < V_+$. These channels however contribute to the tunnel current if $V > V_+$, with a spin-up tunnel conductance $\simeq (e^2/h)N^S$, larger than the Andreev conductance $\simeq (2e^2/h)N_\downarrow$ if $h_{\text{ex}} > \mu/2$. The conductance at the spin-down threshold voltage behaves similarly to the single channel BTK model because there is no suppression of Andreev reflection in the spin-down channels. As a result of the different behavior in the spin-up and spin-down channels, the conductance spectrum is asymmetric in a voltage reversal (see Fig. 2). Tedrow and Meservey used this asymmetry to probe spin polarization in the tunnel limit \[13\]. We predict an asymmetry in the metallic limit also, which can be used as a signature of the present effect in an experiment.

Finally, we have shown on Fig. 3 the behavior of the FS junction model with an interface scattering potential equal to the Fermi velocity ($Z = 1$). In this parameter range, and in the absence of spin polarization, two tunnel-like peaks coexist with a finite low-voltage conductance originating from Andreev reflection. The subgap conductance of the unpolarized junction is smaller than $2e^2/h$ because of the finite interface scattering. Increasing spin polarization results in a suppression of Andreev reflection by spin polarization and spin polarized tunneling (a spin-up peak at $eV_+$ with a stronger weight than the spin-down peak at $eV_-$). These two phenomena may therefore be observed simultaneously.

To conclude, we have shown that Zeeman splitting can be used to resolve the spin-up and spin-down Andreev reflections in NS and FS junctions. In metallic FS junctions, the spin-up tunnel current is larger than the spin-up Andreev reflection current if spin polarization is large, while the transition from Andreev to tunnel transport in the spin-down channels
has a BTK-type behavior. The different behavior in the spin-up and spin-down channels generates a conductance spectrum asymmetric in a voltage reversal.

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FIGURES

FIG. 1. Differential conductance of the NS junction in the metallic limit \( Z = 0 \), with a Zeeman splitting \( \mu_B H = 0 \) (\( \bigcirc \)), 0.2 (+), 0.4 (\( \square \)), 0.6 (\( \times \)) in units of the superconducting gap \( \Delta \). The conductance is normalized to the number of spin channels. The voltage is measured in units of the superconducting gap \( \Delta \). The conductance is symmetric in a voltage reversal. A plateau of \( 3e^2/(2h) \) per spin channel develops in the conductance when \( H \) increases.

FIG. 2. Differential conductance of the FS junction in the metallic limit \( Z = 0 \). The conductance is normalized to the number of spin channels. The voltage is measured in units of the superconducting gap \( \Delta \). The chemical potential is \( \mu = 10^4 \), and the Zeeman splitting is \( \mu_B H = 0.3 \) (in units of \( \Delta \)). The Fermi surface polarization \( P = (N^\uparrow - N^\downarrow)/(N^\uparrow + N^\downarrow) \) are \( P = 0 \) (\( \bigcirc \)), \( P = 0.21 \) (+), \( P = 0.42 \) (\( \square \)), \( P = 0.63 \) (\( \times \)), and \( P = 0.83 \) (\( \triangle \)). The conductance is asymmetric in a voltage reversal, and has a non trivial behavior at the spin-up threshold voltage \( eV_+ = \Delta - \mu_B H \).

FIG. 3. Differential conductance of the FS junction with \( Z = 1 \). The conductance is normalized to the number of spin channels. The voltage is measured in units of the superconducting gap \( \Delta \). The chemical potential is \( \mu = 10^4 \), and the Zeeman splitting is \( \mu_B H = 0.3 \) (in units of \( \Delta \)). The Fermi surface polarization \( P = (N^\uparrow - N^\downarrow)/(N^\uparrow + N^\downarrow) \) are \( P = 0 \) (\( \bigcirc \)), and \( P = 0.83 \) (\( \square \)). Spin polarization results simultaneously in a suppression of Andreev reflection and spin polarized tunneling.
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[25] We expect the same qualitative physics as in a contact with different transverse dimensions of the N and S sides. The asymmetric contact may be solved via a recursion transfert matrix method; see N. Kobayashi, S. Takahashi, and S. Maekawa, Report No cond-mat/9906406 and references therein.

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Differential conductance per spin channel

Voltage

NS junction, Z=0

H=0

H=0.2

H=0.4

H=0.6
Differential conductance per spin channel

FS junction, Z=0, H=0.3

P=0
P=0.21
P=0.42
P=0.63
P=0.83
Differential conductance per spin channel

Voltage

FS junction, Z=1, H=0.3

P=0

P=0.83