Inductive lattices of totally composition formations

Retículos inductivos de formaciones totalmente compositivas

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Abstract. Let \( \tau \) be a subgroup functor such that all subgroups of every finite group \( G \) contained in \( \tau(G) \) are subnormal in \( G \). In this paper, we give a simple proof of the fact that the lattice of all \( \tau \)-closed totally composition formations of finite groups is inductive.

Key words and phrases. Finite group, formation of groups, satellite of formation, \( \tau \)-closed totally composition formation, inductive lattice of formations.

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Resumen. Sea \( \tau \) un funtor de subgrupo de modo que todos los subgrupos de cualquier grupo finito \( G \) contenido en \( \tau(G) \) son subnormales en \( G \). En este artículo, damos una demostración simple de que el retículo de todas las formaciones de composición totalmente \( \tau \)-cerradas de los grupos finitos es inductivo.

Palabras y frases clave. Grupo finito, formación de grupos, satélite de formación, formación de composición totalmente \( \tau \)-cerrada, retículo inductivo de formaciones.

1. Introduction

All groups considered in this paper are finite. A class of groups is a collection of groups satisfying the property that if a group \( G \) belongs to the collection, then every group isomorphic to \( G \) is also in the collection.

If a class of groups is a formation, it is closed with respect to forming quotient groups and subdirect products. This notion introduced by Gaschütz [3] in 1963 immediately became an object of extensive investigations. Saturated formations are very important in group theory; composition formations form a
broader family of formations. By Baer’s theorem, composition formations are precisely solvably saturated formations [2, p. 373].

Skiba [10] introduced the concept of an inductive lattice of formations in order to adapt lattice-theoretical methods for the investigation of saturated formations. This concept plays an important role in the research of the lattices of formations and their law systems (see Chapter 4 of the book [10], Chapter 4 of the book [19]; and the papers [5, 6, 7, 8, 12, 13, 14, 18, 20, 21]).

Let \( \Theta \) be a complete lattice of formations. A satellite \( f \) is called \( \Theta \)-valued if all its values belong to \( \Theta \). We denote by \( \Theta^c \) the set of all formations having a composition \( \Theta \)-valued satellite. In [11, p. 901], it is shown that this set is a complete lattice of formations.

A complete lattice \( \Theta^c \) is called inductive if for any collection of formations \( \{ \mathfrak{F}_i = CLF(f_i) \mid i \in I \} \), where \( f_i \) is an integrated satellite of \( \mathfrak{F}_i \in \Theta^c \), the following equality holds:

\[
\bigvee_{\Theta^c} (\mathfrak{F}_i \mid i \in I) = CLF(\bigvee_{\Theta} (f_i \mid i \in I)).
\]

The inductance of a lattice \( \Theta^c \), in fact, means that a research of the operation \( \bigvee_{\Theta^c} \) on the set \( \Theta^c \) can be reduced to a research of the operation \( \bigvee_{\Theta} \) on the set \( \Theta \). Therefore, the inductance is one very useful property of the lattice \( \Theta^c \).

Vorob'ev [17] proved that the lattice of all totally saturated formations is inductive. Moreover, it is already known that the lattice of all multiply composition formations is inductive (see [16]). However, the following question was still open.

**Question.** Is the lattice of all totally composition formations inductive?

The aim of the present paper is to give a simple proof of the following theorem which gives a positive answer to this question.

**Theorem 1.1.** The lattice of all \( \tau \)-closed totally composition formations \( c^\tau_\infty \) is inductive.

2. Terminologies and notations

All unexplained notations and terminologies are standard. The reader is referred to [1, 2, 4, 11] if necessary.

2.1. Subgroup functor \( \tau \)

In various applications of the theory of classes of finite groups, it is often necessary to use formations closed with respect to some subgroup systems. Skiba [10] introduced the concept of a subgroup functor, which covers all the systems of subgroups under consideration.

In each group \( G \), we select a system of subgroups \( \tau(G) \). We say that \( \tau \) is a **subgroup functor** if (1) \( G \in \tau(G) \) for every group \( G \); (2) for every epimorphism
ϕ : A → B, and each H ∈ τ(A) and T ∈ τ(B), we have Hϕ ∈ τ(B) and Tϕ−1 ∈ τ(A).

If τ(G) = {G}, then the functor τ is called trivial. A formation  is called τ-closed if τ(G) ⊆  for every group G of (see [10]).

We consider only subgroup functors τ such that for every group G all subgroups of τ(G) are subnormal in G.

2.2. Composition formations

The set of all primes is denoted by . Let p ∈ , and G a group. Then the subgroup Cp(G) is the intersection of the centralizers of all the abelian p-chief factors of G, with Cp(G) = G if G has no abelian p-chief factors.

For every collection of groups , we write Com( ) to denote the class of all groups L such that L is isomorphic to some abelian composition factor of some group in . If is the set of one group G, then we write Com(G) instead of Com( ).

The symbol R(G) denotes the product of all solvable normal subgroups of G. We consider a function f of the form

\[
 f : \mathbb{P} \cup \{0\} \rightarrow \{\text{formations of groups}\}, \quad (*)
\]

and the class of groups

\[
 CLF(f) = \{G \mid G/R(G) \in f(0); G/C_p(G) \in f(p) \text{ for all } p \in \pi(\text{Com}(G))\}.
\]

If  is a formation such that  = CLF(f) for a function f of the form ( * ), then  is said to be composition (solvably saturated) formation, and f is said to be a composition satellite of  (see [4, p. 4]).

If the values of composition satellites of some formation are themselves composition formations, then this circumstance leads to the following natural definition. Every formation is 0-multiply composition: for n > 0, a formation  is called n-multiply composition if  = CLF(f), and all nonempty values of f are (n − 1)-multiply composition formations (see [11]).

A formation is called totally composition if it is n-multiply composition for all positive integers n.

2.3. Lattices of formations

A set of formations Θ is called a complete lattice of formations if the intersection of every set of formations in Θ belongs to Θ, and there is a formation  in Θ such that M ⊆  for every other formation M of Θ (see [10]).

Every complete lattice of formations is a complete lattice in the ordinary sense. Various collections of formations form complete lattices; for example, the set of all saturated formations [10, p. 151], and the set of all composition...
(solvably saturated) formations [9, p. 97] are complete lattices of formations. Moreover for all positive integers \( n \), the set of all \( n \)-multiply composition formations \( c_n \), and the set of all totally composition formations \( c_\infty = \bigcap_{n=1}^{\infty} c_n \) are complete lattices of formations (see [11, p. 904]).

A formation in \( \Theta \) is called a \( \Theta \)-formation. Let \( \Theta \) be a complete lattice of formations, and let \( \{ F_i \mid i \in I \} \) be an arbitrary collection of \( \Theta \)-formations. We denote
\[
\bigvee_\Theta (F_i \mid i \in I) = \Theta \text{form}(\bigcup_{i \in I} F_i).
\]
In particular, we write
\[
\bigvee_\tau (F_i \mid i \in I) = c_\tau \text{form}(\bigcup_{i \in I} F_i).
\]

If \( M, H \in \Theta \), then \( M \cap H \) is the greatest lower bound for \( \{ M, H \} \) in \( \Theta \); and \( M \lor \Theta H \) is the least upper bound for \( \{ M, H \} \) in \( \Theta \).

Let \( \{ f_i \mid i \in I \} \) be a collection of \( \Theta \)-valued functions of the form \((*)\). Then by \( \bigvee_\Theta (f_i \mid i \in I) \) we denote a function \( f \) such that
\[
f(a) = \Theta \text{form}(\bigcup_{i \in I} f_i(a))
\]
for all \( a \in \mathbb{P} \cup \{0\} \).

### 3. Preliminaries

Following the paper [11], we set for every collection of groups \( \mathfrak{X} \):
\[
\mathfrak{X}(C^p) = \begin{cases} 
\text{form}(G/C^p(G) \mid G \in \mathfrak{X}) & \text{if } p \in \pi(\text{Com}(\mathfrak{X})); \\
\emptyset & \text{if } p \in \mathbb{P} \setminus \pi(\text{Com}(\mathfrak{X})). 
\end{cases}
\]
We recall that the symbol \( \mathfrak{N}_p \) denotes the class of all \( p \)-groups. Let \( \mathfrak{F} = CLF(F) \), where \( F(0) = \mathfrak{F} \) and \( F(p) = \mathfrak{N}_p \mathfrak{F}(C^p) \) for all \( p \in \mathbb{P} \). Then the satellite \( F \) is called a canonical composition satellite of the formation \( \mathfrak{F} \). By [11, Remark 1], every composition formation possesses a canonical composition satellite.

**Lemma 3.1.** [11, Lemma 8] Let \( \Theta \) be a complete lattice of formations such that \( \Theta^c \subseteq \Theta \) and let the formation \( \mathfrak{N}_p \mathfrak{F} \) belongs to \( \Theta \) for each formation \( \mathfrak{F} \in \Theta \) and every prime \( p \). If \( \mathfrak{F} = CLF(F) \in \Theta^c \), then the satellite \( F \) is \( \Theta \)-valued.

**Lemma 3.2.** [16, Lemma 1] Let \( n \) be a positive integer. Then we have
\[
(c_{\tau n-1}^n)^c = c_n^\tau.
\]

**Corollary 3.3.** The following equality holds: \( (c_\infty^\tau)^c = c_\infty^\tau \).

**Proof.** The inclusion \( (c_\infty^\tau)^c \subseteq c_\infty^\tau \) is obvious. Let \( \mathfrak{F} \in c_\infty^\tau \) and \( F \) be a canonical composition satellite of \( \mathfrak{F} \). Then by Lemmas 3.1 and 3.2 for all \( a \in \mathbb{P} \cup \{0\} \) and each positive integer \( n \), the formation \( F(a) \) is \( \tau \)-closed \( n \)-multiply composition. Thus, the satellite \( F \) is \( c_\infty^\tau \)-valued. Consequently, \( \mathfrak{F} \in (c_\infty^\tau)^c \), and we have \( c_\infty^\tau \subseteq (c_\infty^\tau)^c \). \( \square \)
Lemma 3.4. [22, Lemma 2.1] Let $\mathcal{F} = \text{CLF}(F)$ be a $\tau$-closed $n$-multiply composition formation, where $n$ is a positive integer. Then the satellite $F$ is $c_\tau^n$-valued.

From Lemma 3.4 follows the corollary.

Corollary 3.5. Let $\mathcal{F} = \text{CLF}(F)$ be a $\tau$-closed totally $\omega$-composition formation. Then the satellite $F$ is $c_\tau^\infty$-valued.

Let $\{f_i | i \in I\}$ be a collection of composition satellites. Then by $\bigcap_{i \in I} f_i$, we denote the composition satellite $f$ such that $f(a) = \bigcap_{i \in I} f_i(a)$ for all $a \in \mathbb{P} \cup \{0\}$ (see [11]).

Lemma 3.6. [11, Lemma 2] Let $\mathcal{F} = \bigcap_{i \in I} F_i$, where $F_i = \text{CLF}(f_i)$. Then $\mathcal{F} = \text{CLF}(f)$, where $f = \bigcap_{i \in I} f_i$.

Let $\{f_i | i \in I\}$ be the collection of all composition $c_\tau^\infty$-valued satellites of a formation $\mathcal{F}$. Since the lattice $c_\tau^\infty$ is complete using Lemma 3.6, we conclude that $f = \bigcap_{i \in I} f_i$ is a composition $c_\tau^\infty$-valued satellite of $\mathcal{F}$. The satellite $f$ is called minimal.

Let $\Theta$ be a complete lattice of formations. Then $\Theta_{\text{form}}X$ is the intersection of all $\Theta$-formations containing a collection of groups $X$. Thus, $c_\tau^\infty_{\text{form}}X$ is the intersection of all $\tau$-closed totally composition formations containing a collection of groups $X$. The next lemma immediately follows from [11, Lemma 5] by Corollary 3.3, and gives a description of the minimal $c_\tau^\infty$-valued satellite of a formation $c_\tau^\infty_{\text{form}}X$.

Lemma 3.7. Let $X$ be a nonempty collection of groups, $\mathcal{F} = c_\tau^\infty_{\text{form}}X$, $\pi = \pi(\text{Com}(X))$, and let $f$ be the minimal $c_\tau^\infty$-valued composition satellite of $\mathcal{F}$. Then the following statements hold:

1) $f(0) = c_\tau^\infty_{\text{form}}(G/R(G) | G \in X)$;
2) $f(p) = c_\tau^\infty_{\text{form}}(G/C^p(G) | G \in X)$ for all $p \in \pi$;
3) $f(p) = \emptyset$ for all $p \in \mathbb{P} \setminus \pi$;
4) if $\mathcal{F} = \text{CLF}(h)$ and the satellite $h$ is $c_\tau^\infty$-valued, then for all $p \in \pi$ we have

$$f(p) = c_\tau^\infty_{\text{form}}(G | G \in h(p) \cap \mathcal{F} \text{ and } O_p(G) = 1),$$

and

$$f(0) = c_\tau^\infty_{\text{form}}(G | G \in h(0) \cap \mathcal{F} \text{ and } R(G) = 1).$$

By Lemma 3.7, it is easy to show the following assertion.

Corollary 3.8. Let $f_1$ and $f_2$ be the minimal composition $c_\tau^\infty$-valued satellites of formations $\mathcal{F}_1$ and $\mathcal{F}_2$ respectively. Then $\mathcal{F}_1 \subseteq \mathcal{F}_2$ if and only if $f_1 \leq f_2$.

If $\mathcal{F} = \text{CLF}(f)$ and $f(a) \subseteq \mathcal{F}$ for all $a \in \mathbb{P} \cup \{0\}$, then $f$ is called an integrated satellite of $\mathcal{F}$.
4. Inductance of the lattice $c_{\omega,\infty}$

**Proof of Theorem.** Let $\{\mathcal{F}_i \mid i \in I\}$ be a collection of $\tau$-closed totally composition formations, and $f_i$ be an integrated $c_{\omega,\infty}$-valued composition satellite of $\mathcal{F}_i$. Let

$$\mathcal{F} = CLF(f) = \vee_\omega(\mathcal{F}_i \mid i \in I), \quad \text{and} \quad \mathcal{M} = CLF(\vee_\omega(f_i \mid i \in I)).$$

We shall show that $\mathcal{F} = \mathcal{M}$ proceeding by induction on $i$.

**Step 1.** Let $i = 2$, $p \in \mathbb{P}$, and $h_j$ be the minimal $c_{\omega,\infty}$-valued composition satellite of the formation $\mathcal{F}_j = CLF(f_j)$, where $j = 1, 2$. Then by Corollary 3.5, we have

$$h_j(p) \subseteq f_j(p) \subseteq \Omega_p h_j(p) = F_j(p) \in c_{\omega,\infty},$$

where $F_j$ is the canonical $c_{\omega,\infty}$-valued composition satellite of the formation $\mathcal{F}_j$. Let $\mathcal{F} = CLF(F)$, where $F$ is the canonical $c_{\omega,\infty}$-valued composition satellite of the formation $\mathcal{F}$. Then by Lemma 3.7, we have

$$h(p) = c_{\omega,\infty}(\theta_1 \cup \theta_2)(C(p)) = c_{\omega,\infty}(\theta_1(C) \cup \theta_2(C)) =$$

$$c_{\omega,\infty}(\theta_1(p) \cup \theta_2(p)) \subseteq f(p) \subseteq$$

$$\Omega_p c_{\omega,\infty}(\theta_1(p) \cup \theta_2(p)) = \Omega_p h(p) = F(p).$$

Thus, we have $h(p) \subseteq f(p) \subseteq F(p)$ for all $p \in \mathbb{P}$; moreover, it holds $h(0) \subseteq f(0) \subseteq F(0)$. Hence, $h(a) \subseteq f(a) \subseteq F(a)$ for all $a \in \mathbb{P} \cup \{0\}$ implies $h \leq f \leq F$. Consequently, we have $\mathcal{F}_1 \vee_\omega \mathcal{F}_2 = CLF(f_1 \vee_\omega f_2)$.

**Step 2.** Let $i > 2$, and the assertion is true for $i = r - 1$ by induction. Then $\mathcal{F}_1 \vee_\omega \ldots \vee_\omega \mathcal{F}_{r-1} = CLF(f_1 \vee_\omega \ldots \vee_\omega f_{r-1})$. By Step 1, we have

$$\mathcal{F} = c_{\omega,\infty}(\theta_1 \vee_\omega \ldots \vee_\omega \mathcal{F}_{r-1} \cup \mathcal{F}_r) = CLF(f),$$

where

$$f(a) = c_{\omega,\infty}(f_1(a) \vee_\omega \ldots \vee_\omega f_{r-1}(a)) \cup f_r(a) =$$

$$f_1(a) \vee_\omega \ldots \vee_\omega f_r(a) = (f_1 \vee_\omega \ldots \vee_\omega f_r)(a)$$

for each $a \in \mathbb{P} \cup \{0\}$. Therefore, we have $f = f_1 \vee_\omega \ldots \vee_\omega f_r$. This proves the theorem.

Each complete sublattice of the inductive lattice is an inductive lattice. Thus, we have the following result.

**Corollary 4.1.** Let $\theta$ be a complete sublattice of the lattice $c_{\omega,\infty}$. Then $\theta$ is inductive.

If $\tau$ is trivial, we have the following result.

**Corollary 4.2.** The lattice of all totally composition formations is inductive.
Corollary 4.3. The lattice of all solvable totally composition formations is inductive.

By Lemma 3.6, we have the following corollary.

Corollary 4.4. Let \( \xi(x_1, \ldots, x_m) \) be a term of signature \( \{ \cap, \vee^\infty \} \), and let \( f_i \) be an integrated \( c_i^\infty \)-valued composition satellite of a formation \( \mathfrak{F}_i \), where \( i = 1, \ldots, m \). Then, we have \( \xi(\mathfrak{F}_1, \ldots, \mathfrak{F}_m) = CLF(\xi(f_1, \ldots, f_m)) \).

5. Some applications
Let \( A \) be a group, and \( p \) be a prime. We use \( Z_p \triangleq A \) to denote the regular wreath product of groups \( Z_p \) and \( A \) (see [2, p. 66]).

Lemma 5.1. Let \( \mathfrak{F}_i = c_i^\infty \) form \( (Z_p \triangleq A_i) \), where \( p \notin \pi(A_i) \) for \( i = 1, 2 \). Then \( f(p) = f_1(p) \cap f_2(p) \), where \( f_1 \) and \( f_2 \) are the minimal \( c_i^\infty \)-valued composition satellites of the formations \( \mathfrak{F}_i \) and \( \mathfrak{F} = \mathfrak{F}_1 \cap \mathfrak{F}_2 \), respectively.

Proof. See proof of [15, Lemma 3.1].

The following lemma is proved by direct calculation.

Lemma 5.2. Let \( f_i \) be the minimal \( c_i^\infty \)-valued composition satellite of a formation \( \mathfrak{F}_i \), where \( i \in I \). Then \( \vee^\infty (f_i \mid i \in I) \) is the minimal \( c_i^\infty \)-valued composition satellite of the formation \( \mathfrak{F} = \vee^\infty (\mathfrak{F}_i \mid i \in I) \).

Proposition 5.3. Let \( \mathfrak{F}_i = c_i^\infty \) form \( (Z_p \triangleq A_i) \) for \( p \notin \pi(A_i) \), where \( i = 1, \ldots, m \). Let \( f_i \) be the minimal \( c_i^\infty \)-valued composition satellite of \( \mathfrak{F}_i \) and \( f(p) = \xi(f_1, \ldots, f_m)(p) \), where \( \xi(x_1, \ldots, x_m) \) is a term of signature \( \{ \cap, \vee^\infty \} \). Then \( f \) is the minimal \( c_i^\infty \)-valued composition satellite of the formation \( \mathfrak{F} = \xi(\mathfrak{F}_1, \ldots, \mathfrak{F}_m) \).

Proof. Let \( h = \xi(f_1, \ldots, f_m) \). By Corollary 4.4, we have

\[
\xi(\mathfrak{F}_1, \ldots, \mathfrak{F}_m) = CLF(h).
\]

We shall show that \( h(p) = f(p) \) by induction on the number \( r \) of occurrences of the symbols in \( \{ \cap, \vee^\infty \} \) into \( \xi \).

The case \( r = 1 \) holds using Lemmas 5.1 and 5.2.

Let the term \( \xi \) have \( r > 1 \) occurrences of the symbols in \( \{ \cap, \vee^\infty \} \). Let \( \xi \) have the form \( \xi(x_1, \ldots, x_m) = \xi_1(x_{i_1}, \ldots, x_{i_a}) \Delta \xi_2(x_{j_1}, \ldots, x_{j_b}) \), where \( \{ x_{i_1}, \ldots, x_{i_a} \} \cup \{ x_{j_1}, \ldots, x_{j_b} \} = \{ x_1, \ldots, x_m \} \) and \( \Delta \in \{ \cap, \vee^\infty \} \). We suppose that the assertion is true for the terms \( \xi_1 \) and \( \xi_2 \). By induction, we have \( h_1(p) = \xi_1(f_{i_1}, \ldots, f_{i_a})(p) \) and \( h_2(p) = \xi_2(f_{j_1}, \ldots, f_{j_b})(p) \), where \( h_1 \) and \( h_2 \) are the minimal \( c_i^\infty \)-valued composition satellites of the formations \( \xi_1(\mathfrak{F}_{i_1}, \ldots, \mathfrak{F}_{i_a}) \) and \( \xi_2(\mathfrak{F}_{j_1}, \ldots, \mathfrak{F}_{j_b}) \), respectively. Thus, we have

\[
f(p) = h_1(p) \Delta h_2(p) = \]

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\begin{align*}
\xi_1(f_{i_1}(p), \ldots, f_{i_n}(p)) \triangledown \xi_2(f_{j_1}(p), \ldots, f_{j_m}(p)) &= \xi(f_1(p), \ldots, f_m(p)) = h(p),
\end{align*}

as claimed.

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