Constraints on the Time Variation of the Fine Structure Constant by the 5-Year WMAP Data

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The constraints on the time variation of the fine structure constant relative to its present value, \(\Delta \alpha/\alpha \equiv (\alpha_{\text{rec}} - \alpha_{\text{now}})/\alpha_{\text{now}}\), are obtained from the analysis of the 5-year WMAP cosmic microwave background data. As a result of Markov-Chain Monte-Carlo analysis, it is found that, contrary to the analysis based on the previous WMAP data, the mean value of \(\Delta \alpha/\alpha = -0.0009\) does not change significantly whether we use the Hubble Space Telescope (HST) measurement of the Hubble parameter as a prior or not. The resultant 95% confidence ranges of \(\Delta \alpha/\alpha\) are \(-0.028 < \Delta \alpha/\alpha < 0.026\) with HST prior and \(-0.050 < \Delta \alpha/\alpha < 0.042\) without HST prior.

The variation of the fundamental physical constants is a long-standing issue. Dirac first considered such a possibility\(^{(1,2)}\) and proposed that the Newton constant should be inversely proportional to time. While his claim is not compatible with the current observations, recent unification theories such as superstring theories naturally predict the variation of the fundamental constants.\(^{(3)}\) Because of these theoretical motivations, it is important to measure their possible time variation observationally.

Among various fundamental constants, the time variation of the fine structure constant \(\alpha\) has been most extensively discussed in observational contexts. We briefly summarize those terrestrial and celestial limits on \(\alpha\) as follows:\(^{(4)}\)

- The atomic clocks constrain the current value of the temporal derivative of the fine structure constant as \(\dot{\alpha}/\alpha = (-3.3 \pm 3.0) \times 10^{-16} \text{ yr}^{-1}\)\(^{(5-8)}\).
- \(\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17} \text{ yr}^{-1}\) from the measurement of the frequency ratio of aluminum and mercury single-ion optical clocks.\(^{(9)}\)
- \(\Delta \alpha/\alpha = (-0.8 \pm 1.0) \times 10^{-8}\) or \(\Delta \alpha/\alpha = (0.88 \pm 0.07) \times 10^{-7}\) from the Oklo natural reactor in Gabon (\(z \sim 0.1\))\(^{(10)}\).
- \(\Delta \alpha/\alpha = (-0.57 \pm 0.11) \times 10^{-5}\) (\(z \sim 0.2 - 4.2\))\(^{(11)}\) and \(\Delta \alpha/\alpha = (-0.64 \pm 0.36) \times 10^{-5}\) (\(z \sim 0.4 - 2.3\))\(^{(12,13)}\) from spectra of quasars, the former of which is from the Keck/HIRES instrument, and the latter from the Ultraviolet and Visual Echelle Spectrograph (UVES) instrument. In Ref. 14, \(\Delta \alpha/\alpha = (-0.12 \pm 1.79) \times 10^{-6}\) (\(z = 1.15\)) and \(\Delta \alpha/\alpha = (-5.66 \pm 2.67) \times 10^{-6}\) (\(z = 1.84\)) from the Single Ion Differential \(\alpha\) Measurement (SIDAM) method are also presented.
\( -5.0 \times 10^{-2} < \Delta \alpha / \alpha < 1.0 \times 10^{-2} \) (95\% C.L.) from big bang nucleosynthesis (BBN, \( z \sim 10^9 - 10^{10} \))

- \( -0.048 < \Delta \alpha / \alpha < 0.032, 16 \), \( -0.06 < \Delta \alpha / \alpha < 0.01, 17 \) or \( -0.039 < \Delta \alpha / \alpha < 0.010, 18 \) (95\% C.L.) from the cosmic microwave background (CMB, \( z \sim 10^3 \)), the former two of which are based on the analysis of the 1-year WMAP data and the last one on the 3-year WMAP data.

We also note that the seasonal variation effect on \( \alpha \) has also been discussed in Ref. 19).

In this paper, we focus on the CMB constraint on \( \alpha \) from 5-year WMAP data, finding new limits on its value at the recombination epoch. While the other physical constants may vary in time simultaneously, they are so model-dependent that we consider only the variation of \( \alpha \) here. An example of such a class of models can be found in Refs. 21 and 22.

Both CMB and BBN\(^{23}\) are useful for obtaining the constraints of the variation of \( \alpha \) over a cosmological time scale. Although BBN is superior in the sense that it can probe a longer timespan, it has a drawback that the effect of \( \alpha \) in Helium abundance \( Y_p \) is model-dependent so that we cannot obtain a robust result from BBN analysis. On the other hand, the physics of the CMB is much simpler and well understood with high-precision data, so we can obtain a meaningful limit on the variation of \( \alpha \) from the CMB data.

As is well known, changing the value of the fine structure constant affects the CMB power spectrum mainly through the change of the recombination time.\(^{24, 25}\) Hence, it probes the value of \( \alpha \) in this particular epoch. Let us summarize the main part of the recombination process and see how \( \alpha \) appears.

Following the treatments of Refs. 26 and 27, which are implemented in the RECFAST code,\(^{28}\) the recombination process is approximated by the evolutions of three variables: the proton fraction \( x_p = n_p/n_H \), the fraction of the singly ionized Helium \( x_{He\text{II}} = n_{He\text{II}}/n_H \), and the matter temperature \( T_M \). Here, \( n_H \) is defined as the total Hydrogen number density. Their evolution equations read

\[
\frac{dx_p}{dz} = \frac{C_H}{H(z)(1+z)} \left[ \alpha_{H} x_e x_p n_H - \beta_{H}(1-x_e) \exp \left( -\frac{h\nu_H}{k_B T_M} \right) \right],
\]

\[
\frac{dx_{He\text{II}}}{dz} = \frac{C_{He}}{H(z)(1+z)} \left[ \alpha_{He} x_{He\text{II}} x_e n_H - \beta_{He}(f_{He} - x_{He\text{II}}) \exp \left( -\frac{h\nu_{He\text{II}}}{k_B T_M} \right) \right],
\]

\[
\frac{dT_M}{dz} = \frac{8 \sigma_{T} a_R T_R^4}{3 H(z)(1+z) m_e (1 + f_{He} + x_e)} \frac{x_e}{(T_M - T_R)} + \frac{2 T_M}{1 + z}.
\]

In the above equations, \( H(z) \) is the Hubble expansion rate at the redshift \( z \), \( \sigma_T = 2 \alpha^2 \hbar^2 / (3 \pi m_e^2 c^2) \) is the Thomson scattering cross section, \( a_R = k_B^4/(120 \pi c^3 \hbar^3) \) is the blackbody constant,\(^{a)*} \) \( \nu_H = c/(121.5682 \text{ nm}) \) is the Ly-\( \alpha \) frequency, \( \nu_{He\text{II}} = c/(60.1404 \text{ nm}) \) is the \( He\text{II} \) frequency, \( x_e = n_e/n_H = x_p + x_{He\text{II}} \) is the free electron fraction, \( T_R \) is the radiation temperature, and \( f_{He} = Y_p/(4(1 - Y_p)) \) is the number ratio of Helium to Hydrogen, where \( Y_p \) is the primordial Helium mass fraction, which we consider as 0.24. \( C_H (C_{He}) \) is the so-called Peebles reduction

\(^{a)*} \) \( h \) is the Planck constant, \( k_B \) is the Boltzmann constant, and \( c \) is the speed of light.
factor
\[
C_H = \frac{1 + K_H A_H n_H (1 - x_p)}{1 + K_H (A_H + \beta_H) n_H (1 - x_p)},
\]
(4)
\[
C_{He} = \frac{1 + K_{He} A_{He} n_{He} (f_{He} - x_{HeII}) \exp (-h \nu_{ps}/k_B T_M)}{1 + K_{He} (A_{He} + \beta_{He}) n_{He} (f_{He} - x_{HeII}) \exp (-h \nu_{ps}/k_B T_M)},
\]
(5)

where \( A_H \) is H 2s-1s two-photon decay rate, \( A_{He} \) is HeI 2s-1s two-photon decay rate, \( \nu_{ps} \equiv \nu_{He21p} - \nu_{He21s} \), \( K_H = c^3/(8 \pi \nu_{H^2 p}^3 H) \), and \( K_{He} = c^3/(8 \pi \nu_{He21p}^3 H) \). \( \alpha_H \) and \( \alpha_{He} \) are the case B recombination coefficients and they are well fitted by
\[
\alpha_H = 10^{-19} F \frac{a t^b}{1 + c t^d} \left[ \text{m}^3 \text{s}^{-1} \right],
\]
(6)
\[
\alpha_{He} = q \sqrt{T_M/T_2} \left( 1 + \frac{T_M}{T_2} \right)^{1-p} \left( 1 + \frac{T_M}{T_1} \right)^{1+p} \left[ \text{m}^3 \text{s}^{-1} \right],
\]
(7)

with \( t = T_M/10^4 \left[ \text{K} \right], \ a = 4.309, \ b = -0.6166, \ c = 0.6703, \ d = 0.5300, \ F = 1.14, \ q = 10^{-16.744}, \ \nu_p = 0.711, \ T_1 = 10^5.114 \left[ \text{K} \right], \ \text{and} \ T_2 = 3 \left[ \text{K} \right]. \) Finally, \( \beta_H \) and \( \beta_{He} \) are the photoionization coefficients
\[
\beta_H = \alpha_H \left( \frac{2 \pi m_e k_B T_M}{h^2} \right)^{3/2} \exp \left( -\frac{h \nu_{H^2 s}}{k_B T_M} \right),
\]
(8)
\[
\beta_{He} = \alpha_{He} \left( \frac{2 \pi m_e k_B T_M}{h^2} \right)^{3/2} \exp \left( -\frac{h \nu_{He21 s}}{k_B T_M} \right).
\]
(9)

Now, we show how the above quantities depend on \( \alpha \). Since binding energies scale as \( \alpha^2 \), the frequencies \( \nu \) also scale as \( \alpha^2 \), and \( K_H(K_{He}) \propto \nu_{H^2 p}^{-3}(\nu_{He21p}^{-3}) \propto \alpha^{-6} \). According to Ref. 25), two-photon decay rates \( A_H \) and \( A_{He} \) scale as \( \alpha^8 \), and the recombination coefficients \( \alpha_H \) and \( \alpha_{He} \) scale as \( \alpha^{2(1+\xi)} \), where we adopt \( \xi = 0.7 \) following Ref. 25).

As already pointed out in Refs. 16, 24), and 25), the larger value of \( \alpha \) at the recombination epoch results in the higher redshift of the last scattering surface. Thus, increasing \( \alpha \) results in three characteristic signatures in the angular power spectrum of the temperature anisotropy, namely, shift of the peaks to higher multipoles, increase in the height of the peaks due to the enhanced early integrated Sachs Wolfe effect, and decrease in the small-scale diffusion damping effect. These features can be seen in Fig. 1.

We constrain the variation of \( \alpha \) using three types of CMB anisotropy spectra, namely, angular power spectrum of temperature anisotropy, \( C_{TT}^\ell \), that of E-mode polarization, \( C_{EE}^\ell \), and cross correlation of temperature and E-mode polarization \( C_{TE}^\ell \) of the 5-year WMAP data.\(^{20–31} \) For this purpose, we have modified the CAMB code\(^{32,33} \) including the RECFAST code to calculate the theoretical anisotropy spectra for different values of \( \alpha \) at recombination and we performed the parameter estimation using Markov-Chain Monte-Carlo (MCMC) techniques implemented in the CosmoMC code.\(^{34,35} \)
Fig. 1. (color online) CMB temperature anisotropy spectra for no change of $\alpha$ (solid red curve), an increase in $\alpha$ by 10% (dashed yellow curve), a decrease in $\alpha$ by 10% (dotted blue curve).

We have run the CosmoMC code on four Markov chains. To check the convergence, we used the “variance of chain means” / “mean of chain variances” $R$ statistic and adopted the condition $R - 1 < 0.03$.

First, we consider the modified version of the flat $\Lambda$CDM model, that is, as for cosmological parameters, we take $(\Omega_B h^2, \Omega_{DM} h^2, H_0, n_s, A_s, \tau, \Delta \alpha / \alpha)$, where $\Omega_B h^2$ is the normalized baryon density, $\Omega_{DM} h^2$ is the normalized cold-dark-matter density, $H_0 \equiv 100h [\text{km sec}^{-1} \text{Mpc}^{-1}]$ is the Hubble constant, $n_s$ is the spectral index of the primordial curvature perturbation, $A_s$ is its amplitude, and $\Delta \alpha / \alpha \equiv (\alpha_{\text{rec}} - \alpha_{\text{now}}) / \alpha_{\text{now}}$ is the variation of the fine structure constant at recombination relative to its present value. We have also analyzed the standard flat $\Lambda$CDM model without $\Delta \alpha / \alpha$ and compared the other parameter values between these two models.

The results obtained from MCMC calculations are given in Table I, and Fig. 2. In Table I, we present the mean values and the 68% confidence intervals of the cosmological parameters and Fig. 2 shows the one-dimensional marginalized posterior distributions of the parameters. From these results, it can be seen that the effect of the additional parameter $\Delta \alpha / \alpha$ is only to increase the errors of the other parameters, and the mean values of the other parameters in modified flat $\Lambda$CDM are practically the same as in the standard flat $\Lambda$CDM. The marginalized distributions of $H_0$ and $\Omega_B$ in Fig. 2 suggest the degeneracy of these parameters with $\Delta \alpha / \alpha$.

Actually, in the above calculations, we have incorporated the result of the Hubble Key Project of the Hubble Space Telescope (HST) on the Hubble parameter $H_0$,

$^{36}$ that is, we have imposed a prior that $H_0$ is a Gaussian with the mean 72 [km sec$^{-1}$ Mpc$^{-1}$] and the variance 8 [km sec$^{-1}$ Mpc$^{-1}$]. If we do not use the
Table I. MCMC results on the mean values and 68% confidence intervals of cosmological parameters for the two models.

| Parameter | modified flat $\Lambda$CDM | standard flat $\Lambda$CDM |
|-----------|-----------------------------|-----------------------------|
| $100\Omega_B h^2$ | $2.27^{+0.000}_{-0.10}$ | $2.27^{+0.000}_{-0.05}$ |
| $\Omega_{DM} h^2$ | $0.109^{+0.006}_{-0.006}$ | $0.109^{+0.006}_{-0.006}$ |
| $\tau$ | $0.087^{+0.007}_{-0.008}$ | $0.0876^{+0.008}_{-0.008}$ |
| $n_s$ | $0.966^{+0.014}_{-0.015}$ | $0.964^{+0.014}_{-0.014}$ |
| $\log(10^{10}A_s)$ | $3.06^{+0.04}_{-0.04}$ | $3.06^{+0.04}_{-0.04}$ |
| $H_0$ | $71.9^{+7.1}_{-7.1}$ | $72.1^{+2.6}_{-2.6}$ |
| $\Delta \alpha/\alpha$ | $-0.000894^{+0.0148}_{-0.0148}$ | $-$ |

Fig. 2. (color online) One-dimensional marginalized posterior distributions for the parameters of the modified flat $\Lambda$CDM model (solid red curve), and for the standard flat $\Lambda$CDM model (dotted blue curve).

HST prior, we can only obtain weaker constraints on the parameter values because of projection degeneracy. To check the effect of the HST prior, we show two-dimensional marginalized distributions with and without HST prior in Fig. 3 and one-dimensional distributions in Fig. 4. It is confirmed that the HST prior is very important to realistically constrain the time variation of $\alpha$.

The 95% confidence interval and the mean value of $\Delta \alpha/\alpha$ from 5-year WMAP data with HST prior are

$$-0.028 < \Delta \alpha/\alpha < 0.026 \quad \text{and} \quad \Delta \alpha/\alpha = -0.000894,$$

respectively. Without the HST prior, they read

$$-0.050 < \Delta \alpha/\alpha < 0.042 \quad \text{and} \quad \Delta \alpha/\alpha = -0.00181,$$
Fig. 3. (color online) Two-dimensional marginalized posterior distributions for $\Delta \alpha/\alpha - \Omega_B h^2$ and $\Delta \alpha/\alpha - H_0$. Red contours (inner two contours) are with HST prior, and blue contours (outer two contours) are without it. For each contour pair, the inner one represents the 68% bound and the outer one 95%.

Fig. 4. (color online) One-dimensional marginalized posterior distributions for the results with HST prior (solid red curve), and without it (dotted blue curve).

respectively. Previous results from 1-year WMAP data are $-0.048 < \Delta \alpha/\alpha < 0.032$ and $-0.107 < \Delta \alpha/\alpha < 0.043$, with and without HST prior, respectively, so our results from 5-year WMAP data are about 30% tighter than those from the 1-year WMAP data. We also note that for the 1-year data, the mean value of $\Delta \alpha/\alpha$ was found to be $\Delta \alpha/\alpha = -0.04$ without the HST prior although it was practically equal
to 0 with it. For the 5-year data, we have found that the mean value remains practically intact whether we use the HST prior or not. This may be interpreted as an indication that the observational cosmology has made a step forward to the concordance at an even higher level.

Next, we take \((\Omega_B h^2, \Omega_{DM} h^2, H_0, n_s, A_s, \tau, w, \Delta \alpha/\alpha)\), where \(w\) is the dark energy equation of state. In addition to the 5-year WMAP data, we use the HST and Supernova Legacy Survey\(^\text{37}\) priors here. Although the possibility of the degeneracy of \(w\) with \(\Delta \alpha/\alpha\) was pointed out some time ago,\(^\text{38}\) we can conclude that such a degeneracy is very weak now, for we have both temperature and polarization data enough to constrain \(w\) and \(\Delta \alpha/\alpha\) simultaneously (see Fig. 5). The 95% confidence interval and the mean value of \(\Delta \alpha/\alpha\) in this case are

\[-0.033 < \Delta \alpha/\alpha < 0.032 \quad \text{and} \quad \Delta \alpha/\alpha = -0.00186.\]  \(\text{(12)}\)

The interval is slightly larger than that of the model with the cosmological constant.

In summary, in terms of the MCMC analysis using CosmoMC code, we have updated constraints on the time variation of the fine structure constant \(\alpha\) based on 5-year WMAP data. We obtained tighter constraints compared with previous results from 1-year WMAP data owing to the inclusion of the polarization data and the decrease in the statistical errors. Compared with the result based on the 3-year WMAP data,\(^\text{18}\) where no comparison between the cases with and without HST prior has been made, the resultant limit is almost of the same order of magnitude but the mean value of ours is closer to 0. We have verified that the null result is favored
concerning the variation of $\alpha$, and the addition of this new parameter $\Delta\alpha/\alpha$ does not essentially affect the determinations of the other standard parameters contrary to the case of the analysis based on the 1-year WMAP data.\textsuperscript{16)

We have also studied the possibility of the degeneracy between $w$ and $\Delta\alpha/\alpha$, finding the slightly relaxed limit on the latter parameter due to the addition of $w$, but no drastic degeneracy. Also in this case, we cannot find any evidence of time varying $\alpha$ in the 5-year WMAP CMB data.

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**Note added:** After we finished our analysis, we became aware of a paper by Scoccola, Landau, and Vucetich, who also analyzed constraints on the time variation of $\alpha$ using 5-year WMAP data. Their main focus, however, is dependence on the details of the recombination scenario, which they have shown to be weak. It appears that they did not incorporate HST prior in contrast to our analysis. Hence, our paper is complementary to theirs.