Direct observation of irrotational flow and evidence of superfluidity in a rotating Bose-Einstein condensate

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We observed the expansion of vortex-free Bose-condensates after their sudden release from a slowly rotating anisotropic trap. Our results show clear experimental evidence of the irrotational flow expected for a superfluid. The expansion from a rotating trap has strong features associated with the superfluid nature of a Bose-condensate, namely that the condensate cannot at any point be cylindrically symmetric with respect to the axis of rotation since such a wavefunction cannot possess angular momentum. Consequently, an initially rotating condensate expands in a distinctly different way to one released from a static trap. We report measurements of this phenomenon in absorption images of the condensate taken along the direction of the rotation axis.

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In their recent theoretical paper, Edwards et al. calculated the time evolution of the expansion of a slowly rotating BEC. At higher rotation rates, greater than about half the radial trap frequency, the superfluid nature of the condensate has been demonstrated strikingly in the formation of quantised vortices. However they point out that the superfluidity of the condensate, together with the conservation of angular moment, also leads to clearly observable consequences at lower rotation rates, under conditions where no vortices are present. We report here the experimental verification of these predictions by time-of-flight measurements on a condensate released from a rotating anisotropic trap. These data give further evidence for the superfluidity of Bose-condensed gases. Other recent experiments on this important aspect of Bose-condensation include the observation of quantized vortices and the quenching of the moment of inertia in the scissors mode.

The solutions of the hydrodynamic equations for superfluids (that also describe the scissors mode) show that a condensate always has a moment inertia less than that of a rigid body of the same mass distribution. For the special case of cylindrical symmetry, the superfluid has zero moment of inertia about the symmetry axis. Thus a condensate with some angular momentum can never have a circular shape when viewed along the rotation axis, since this would imply the unphysical situation of infinite angular velocity. An initially rotating condensate therefore expands in a different way to one released from static trap. We observed this phenomenon in absorption images of the condensate taken along the direction of the rotational axis - the projection of the cloud in this direction never becomes circular (an aspect ratio of unity). The evolution of the condensate density distribution is calculated using the same hydrodynamic equations as in , and agrees well with our data. We give a brief review of the underlying theoretical aspects before describing the experimental procedure and results.

In our experiment we have an anisotropic harmonic potential with three angular frequencies \( \omega_x < \omega_y < \omega_z \). The potential rotates about the z-axis with angular frequency \( \Omega \). It is convenient to define two new parameters: the trap deformation, \( \lambda = (\omega_x^2 - \omega_y^2)/(\omega_x^2 + \omega_y^2) \) and the mean of the frequencies in the plane of rotation defined as \( \omega_m^2 = (\omega_x^2 + \omega_y^2)/2 \). In the hydrodynamic limit, a condensate rotating in this trap displays a quadrupolar flow pattern, as described in fig.1, and has a wavefunction which looks like:

\[
\Psi(r) = \sqrt{\rho(r)}e^{i \frac{2\pi}{\omega_y} xy},
\]

where \( \rho(r) \) is the condensate density given in Eq.3 below. The quadrupole frequency, \( \nu \), can be found from the solutions of the following cubic equation:

\[
\tilde{\nu}^3 + (1 - 2\lambda^2)\tilde{\nu} + \lambda\Omega = 0,
\]

where we introduced the dimensionless quantities \( \tilde{\nu} = \nu/\omega_m \) and \( \Omega/\omega_m \).

In a rotating trap, the condensate experiences effectively different trapping frequencies due to its quadrupolar motion and the condensate density can be written as:

\[
\rho(r) = \frac{\mu}{g} \left[ 1 - \frac{m}{2}(\tilde{\omega}_x^2 x^2 + \tilde{\omega}_y^2 y^2 + \tilde{\omega}_z^2 z^2) \right].
\]

The modified frequencies \( \tilde{\omega}_x \) and \( \tilde{\omega}_y \) are given by:

\[
\tilde{\omega}_x^2 = \omega_x^2 + \nu^2 - 2\nu\Omega \\
\tilde{\omega}_y^2 = \omega_y^2 - \nu^2 - 2\nu\Omega
\]

where \( \nu \) is found from Eq.3.

Hence, the aspect ratio of the condensate in the trap is

\[
\frac{R_y}{R_x} = \frac{\tilde{\omega}_y}{\tilde{\omega}_x} = \sqrt{\frac{\Omega + \nu}{\Omega - \nu}}.
\]
where \(R_x, R_y\) are the sizes of the condensate in the x and y-direction, respectively. The effective chemical potential and condensate sizes in the trap can also be calculated from Eqs. 4.

To calculate the expansion of a condensate when released from the anisotropic harmonic potential, we use the following ansatz of spheroidal form for the condensate density \(\rho(r, t)\) and the velocity field, \(\mathbf{v}(r, t)\), (as in [1]):

\[
\rho(r, t) = \rho_0(t) - \rho_2(t)x^2 - \rho_3(t)y^2 - \rho_4(t)z^2 - \rho_{xy}(t)xyz
\]

\[
\mathbf{v}(r, t) = \frac{1}{2} \nabla \left( v_x(t)x^2 + v_y(t)y^2 + v_z(t)z^2 + v_{xy}(t)xy \right)
\]

Inserting this ansatz into the hydrodynamic equations of superfluids yields a set of nine coupled differential equations for the expansion parameters, which we integrate numerically. The initial conditions for the density are found from Eq. 3. Note that the cross-term \(\rho_{xy}(0)\) is assumed to be zero at the instant when the condensate is released from the trap. During the evolution in time of flight, this term grows and can become comparable in size to \(\rho_x\), so that the elliptical condensate rotates with respect to its initial release angle. All components of the velocity field are initially zero, except for the cross term, which is the quadrupole velocity field of the rotating condensate, given by \(v_{xy} = 2 \nu\) (see Eq. 1). The behavior of the condensate is thus completely determined by the above initial conditions and the nine coupled differential equations.

The angle of the condensate and its sizes can be found by diagonalising the quadratic form defined in Eq. 4. One finds, in time of flight, that at first the condensate expands along its smaller axes until the aspect ratio has reached a critical value close to spherical. The condensate has a quadrupolar flow pattern and an associated angular momentum. Conservation of this angular momentum does not allow the condensate to become spherical, as a spherical quadrupolar flow pattern has no angular momentum. Thus, when approaching this critical value in the aspect ratio, the condensate rotates quickly and continues to expand along its original long axis. The rapid increase in angular velocity close to the critical aspect ratio arises from the decrease of the moment of inertia when the aspect ratio approaches spherical. However, energy conservation limits how fast the condensate can rotate and thus the aspect ratio cannot become smaller than a certain value. This behaviour is in contrast to that of a thermal gas which can become spherical and preserve angular momentum, since there are no constraints of irrotationality. Thus it is essentially the superfluid nature of the Bose-condensate which prevents it from becoming spherical.

To observe this behaviour we evaporatively cooled \(^{87}\text{Rb}\) atoms in a TOP trap to a temperature of \(0.5 T/T_c\), where \(T_c\) is the critical temperature for condensation.

This produced Bose-condensates of \(1.5 \times 10^4\) atoms, in a trapping potential with frequencies \(\omega_x/2\pi = \omega_y/2\pi = 124\) Hz and \(\omega_z/2\pi = 350\) Hz. We then made the trap elliptical with \(\omega_y/\omega_x = 1.4\) (corresponding to \(\lambda = 0.32\)) by changing the ratio of the two TOP-field components to \(B_x/B_y = 4.2\). The eccentric trap was rotated by modulating the high frequency sinusoidal TOP signal (7 kHz frequency) at the low trap rotation frequency \(\Omega\), as described in [1]. The condensate was spun up in 500 ms during which time the eccentricity was ramped up from zero to its final value. Then the condensate was left in the rotating trap for another 500 ms before it was released at a well defined point in the trap rotation, from which the angles of the expanding cloud were measured. Figure 2 shows typical absorption images of the condensate, taken along the axis of rotation, after different expansion times. For these pictures the trap was rotating at 28 Hz and at the instant of release the long axis of the cloud was horizontal. The angle and aspect ratio of the cloud was obtained from a 2D parabolic fit to the density distribution. Figure 2a shows the minimum aspect ratio of 1.31, reached after ~ 4 ms of expansion. The angle increases steadily to an asymptotic value of ~ 55 degrees in fig. 3.

It was necessary to go to a high trap eccentricity so that a clear deformation of the cloud could be observed. We investigated the evolution of the condensate in time of flight at two different trap rotation frequencies \(\Omega/2\pi = 20\) and 28 Hz. In both cases the condensate was released from a trap of \(\omega_x/2\pi = 60\) Hz, \(\omega_y/2\pi = 1.4 \times 60\) Hz and \(\omega_z/2\pi = 206\) Hz. Fig. 3 shows the calculated change of angle of a condensate in time of flight after being released from a trap rotating at \(\Omega/2\pi = 20\) Hz (solid line) and \(\Omega/2\pi = 28\) Hz (dotted line), with the experimental data points superimposed. In both cases the angle evolves in a similar manner and reaches 45 degrees after about 6 ms. After 18 ms the angle assumes an asymptotic value between 55 and 60 degrees.

Figure 4 shows the theoretical prediction for the evolution of the condensate’s aspect ratio in time of flight, with experimentally measured values superimposed, for \(\Omega/2\pi = 28\) Hz (upper curve), for \(\Omega/2\pi = 20\) Hz (middle curve) and for release from a nonrotating trap (lower curve). The data clearly demonstrate how the aspect ratio for an initially rotating condensate decreases up to a critical point, which is reached after approximately 4 ms. From that point on it does not continue to expand along its minor axis but the aspect ratio increases again because the condensate cannot become circular under these conditions. However, the condensate released from a static trap has no velocity field which prevents it from becoming circular and the aspect ratio decreases steadily from \(\omega_y/\omega_x\) to a final value of less than one. Every experimental point displayed is the average of several measurements. For each time of flight we had to refocus our imaging system as the atoms move out of focus under
gravity when released from the trap. However incorrect focusing would only result in a more circular image and the minimum value for the measurement of the aspect ratio of rotating condensates was never consistent with unity. There is remarkable agreement between the experimental data and the theoretical predictions. However, we observed a small deviation of the experimental data from the predicted values for the higher rotation frequency (Ω/2π = 28 Hz, shown in the upper curve). We do not know the origin of that deviation.

Our results show that an expanding vortex-free Bose-condensate with some angular momentum refuses to become circular about the axis of rotation, as predicted by Edwards et al \[1\]. This provides direct evidence that Bose-condensed gases flow irrotationally as a consequence of their superfluidity. These measurements complement the observations of the scissors mode \[6\] where the irrotational flow was deduced from measurements of the oscillation frequencies of a trapped condensate - a superfluid has less moment of inertia than a classical object, of the same size, undergoing rigid body motion hence the superfluid oscillates faster. Other measurements of the flow of a Bose-condensed gas would be of great interest, for example flow through a narrow tube \[10\], or array of holes, analogous to the superleak experiments with liquid helium.

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[1] M. Edwards, C.W. Clark, P. Pedri, L. Pitaevskii, and S. Stringari, cond-mat/0106080
[2] K.W. Madison, F. Chevy, W. Wohlleben, and J. Dalibard, Phys. Rev. Lett. 84, 806 (2000); J.R. Abo-Shaeer, C. Raman, J.M. Vogels and W. Ketterle, Science Express (online), March 2001
[3] J. Arlt, O.M. Maragò, E. Hodby, S. A. Hopkins, G. Hechenblaikner, S. Webster and C. J. Foot, J. Phys. B, 32, 5861 (1999)
[4] M.R. Matthews, B.P. Anderson, P.C. Haljan, D.S. Hall, C.E. Wieman, and E.A. Cornell, Phys. Rev. Lett. 83, 2498 (1999)
[5] C. Raman, M. Köhl, R. Onofrio, D.S. Durfee, C.E. Kuklewicz, Z. Hadzibabic, and W. Ketterle, Phys. Rev. Lett. 83, 2592 (1999)
[6] O.M. Maragò, S.A. Hopkins, J. Arlt, E. Hodby, G. Hechenblaikner and C.J. Foot, Phys. Rev. Lett. 84, 2056 (2000)
[7] S. Stringari, Phys. Rev. Lett. 77, 2360 (1996)
[8] F.Zambelli and S.Stringari, Phys. Rev. A, 63, 033602 (2001)
[9] James Anglin, private communication 1999; Gerald Hechenblaikner, first year report, Clarendon Laboratory (1999); A. Recati, F. Zambelli, and S. Stringari, Phys. Rev. Lett 86, 377 (2001)
[10] B. Jackson, J. F. McCann, and C. S. Adams, J. Phys. B. 31, 4489 (1998).
FIG. 2. Typical images of the condensate at different times after release from a trap rotating at 28Hz - (a) 1.09 ms (b) 3.97 ms (c) 12.00 ms (d) 16.06 ms. At the instant of release the long axis of the cloud was horizontal.

FIG. 3. The angle of the condensate plotted against the time of flight. The open circles and the filled squares denote the measured angles after release from a trap rotating at $\Omega/2\pi = 20$ and $\Omega/2\pi = 28$ Hz, respectively. The solid and the dotted line are the respective theoretical calculations.

FIG. 4. The aspect ratio of a condensate in time of flight. Initially rotating condensates (upper and middle theoretical curves) exhibit a strong backbending effect after 4 ms and the condensate never becomes circular. However, after release from the non-rotating trap, the aspect ratio decreases steadily and inverts (lower curve); it is unity at about 6 ms. Upper curve and filled squares, $\Omega/2\pi = 28$ Hz. Middle curve and open circles, $\Omega/2\pi = 20$ Hz. Lower curve and filled triangles, Static trap.