Performance analysis of ERS using R134a - An experimental investigation (Part 1) – Ejector design and performance analysis

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Abstract. The present study proposed a mathematical model for calculating the entrainment ratio and area-ratio for the single-phase ejector working double choking conditions. The model contains gas dynamic equations and is used to calculate performance of ejector for R134a and R141b refrigerants. The model is also validated with the experimental results of the previous researcher and found results are in good coherence with average deviation of -0.287% and 2.48% for area-ratio and entrainment ratio respectively.

Keyword: Mathematical model, ejector, performance.

1. Introduction
Refrigeration systems using these days are either consumes large amount of electric energy (i.e., in case vapor compression refrigeration system) or very bulky (i.e., in case of vapor absorption refrigeration system). The demand of refrigeration system working by consuming heat energy is very high and researchers are focusing their attention to design and develop heat assisted refrigeration systems. The ejector refrigeration system (ERS) is an attractive system to meet the current requirements. ERS consumes low temperature heat which makes it suitable to operate on solar energy or industrial waste heat. Another advantage of the ERS is its simplicity in design, low initial investment and light in weight. The schematic diagram of ERS is shown in figure 1. The liquid refrigerant condensed by the condenser is pumped by a liquid pump to the generator where the liquid refrigerant gets converted into the high-pressure vapor refrigerant by absorbing latent heat. This high-pressure vapor refrigerant is used as motive fluid by the ejector to suck the low-pressure vapor refrigerant from the evaporator and keeps the evaporator at low pressure. This low-pressure refrigerant is also known as secondary refrigerant. Both primary and secondary refrigerant mixes with each and moves together to the condenser at an intermediate pressure where the mixed refrigerant is condensed by releasing its latent heat in the condenser.

The main and principal component of the system is ejector. An ejector can be defined as a compressing unit which utilizes the fluid energy of the high pressure, high temperature incoming fluid
also known as primary fluid. When this motive fluid passes through the convergent-divergent primary nozzle, it expands and accelerates to a very high velocity normally above the speed of sound and thus creates a very low-pressure environment in the suction chamber which is responsible for the generation of a suction effect. This suction force entrains and draws the low-pressure vapor refrigerants from the evaporator. While passing through the constant area section of the ejector, these fluids mix with each other and when this mixed fluid moves through the diffuser section then the pressure is again raised on the compensation of the loss of kinetic energy.

A large amount of literature is available on the performance analysis of the ERS. But most of the available studies are theoretical and a few researchers have analyzed the ERS for various ejector geometry and various operating conditions. Huang et al. [1] has developed a mathematical model using R141b for analyzing the single-phase ejector using gas dynamics equations. They have studied the behavior of the ejector but not analyzed the complete system in detail. They used eleven different ejector geometries for validating the results of the developed mathematical model. Chou et al. [2] had also developed a mathematical model to study the ejector’s behavior but instead of focusing on performance analysis, they have analyzed the choking condition’s influence on the various section of the ejector by varying the condenser pressure. They use three different refrigerants i.e., R113, R141b and steam in their analysis. Kumar and Sachdeva [3] proposed a novel 1-D mathematical model for predicting the and refining the geometry of a single-phase ejector. In addition to the gas dynamics equation, they also use some new concepts in their models i.e., Prandtl’s mixing length, Kelvin-Helmholtz instability, Prandtl-Meyer expansion wave and Baroclinic effect. By considering real gas equations Tyagi and Murty [4] had developed a mathematical model using R11 and R113. The generator temperature is kept low i.e., in between 70-90°C. While keeping the condenser temperature constant at 30°C and evaporator temperature at 0°C, they found 0.43 and 0.38 COP with R11 and R113 respectively.

Boumaraf and Lallemand [5] formed a mathematical model to simulate the performance of ERS, which separately included the efficiencies of each and every component. They used R141b and R600a as the working fluids in their simulation for optimized and off operating conditions. They established their results with the previously available experimental and theoretical 1-D models [1]. COP of the system was found to increase with the decrease in source temperature but it decreased the critical condenser pressure of the system. Zhu and Li [6] studied and proposed a novel model for calculating performance of an ejector for both dry and wet vapor as working fluid at the critical conditions. Instead of utilizing simple thermodynamic relations, they used real velocity approach and integrated the velocity function for finding mass flow rates of primary and secondary fluid. Assuming the mixing layer in between the primary and secondary fluid in choking conditions, all the equations were derived accordingly.

Elahkdar et al. [7] developed a one-dimensional mathematical model to understand the frictional losses between the primary and secondary fluid flow during mixing and its effect on the performance. They reported the error difference of 4.27% in the entrainment ratio obtained from their model and the actual experimental results of Huang et al. [1]. The concept of Fanno flow to introduce frictional
compressible flow in the mixing chamber of the ejector was introduced in the mathematical model by Satheesh and Ooi [8]. Moreover, the average heat capacity ratio was calculated at different sections in the ejector rather than assuming a constant heat capacity ratio like other researchers. An average error of only 4% in the entrainment ratio was projected with that of the experimental results of Huang et al. [1].

2. Mathematical model

A one-dimensional mathematical model has been developed and proposed to evaluate performance of single-phase ejector for known dimensions and operating conditions using gas dynamics compressible flow equations.

**Assumptions:** Following assumptions have been made for the analysis:

- Primary/motive nozzle is working without shock wave and flow exit is above Mach one.
- To increase the model efficiency appropriate coefficient are selected on account of friction and mixing losses.
- The heat transfer rate in between the ejector wall and the working fluid is assumed to be zero i.e., an adiabatic flow.
- An assumption is taken into considered in the 1-D model that both fluids start mixing with each other at hypothetical throat.
- Both fluid mixes with each other in the constant area section.
- The secondary fluid is assumed to be choked at inlet face of the constant area section.
- The velocity of the flow while exiting from the diffuser is assumed to be zero.
- The refrigerant inside the ejector is assumed to be dry vapors.

![Single phase Ejector geometry with sections](image)

**Figure 2** Single phase Ejector geometry with sections

2.1 Governing gas dynamic equations

The refrigeration systems are normally designed for a specific application that contains a certain evaporator temperature. Consequently, the ejector must be designed for desired refrigeration effect and evaporator temperature. Furthermore, for obtaining desired cooling effect and evaporator temperature, the entrained secondary fluid mass and the suction pressure must be maintained constant by the designed ejector. Here in this case, it is assumed that the mass flow
rate of primary fluid provided by the generator is fixed at a specified saturated generator pressure and has some velocity at input to the primary nozzle of the ejector. Hence, this velocity can be calculated by the formula below:

\[
m_p = \rho_{p,1} \times A_{p,1} \times V_{p,1}
\]

where \(m\) and \(\rho\) are the primary fluid mass flow rate and the density respectively. The generator pressure \((P_{gn})\) and temperature \((T_{gn})\) is used to calculate the stagnation temperature \((T_{p,1})\) and pressure \((P_{p,1})\) of the primary fluid at inlet of the primary nozzle isentropic relations [9, 10]:

\[
T_{p,1} = T_{gn} + \frac{V_{p,1}^2}{2 \times c_{p,gn}}
\]

\[
\frac{P_{p,1}}{P_{gn}} = \left(\frac{T_{p,1}}{T_{gn}}\right)^{\frac{\gamma_{gn}}{\gamma_{gn}-1}}
\]

The mass flow rate of the primary/motive fluid calculated using isentropic gas dynamics relation [10, 11]:

\[
m_p = A_{p,2} \times P_{p,1} \times \left(\frac{\gamma_{p,1}}{R \times T_{p,1}}\right) \times \left(\frac{2}{\gamma_{p,1}+1}\right)^{\frac{1}{2}} \times \left(\eta_p\right)^{\frac{1}{2}}
\]

Frictional losses produced into the primary nozzle is taken to consideration increase the model efficiency i.e., isentropic efficiency of the primary nozzle \((\eta_p)\). It is assumed that the hypothetical throat of the secondary fluid exists at the inlet section of the CAS at section 4-4. The pressure of the secondary fluid \((P_{s,4})\) is evaluated by the relation as under at section 4-4 [9, 10]:

\[
\frac{P_{s,4}}{P_{ep}} = \left(\frac{2}{\gamma_{ep}+1}\right)^{\frac{\gamma_{ep}}{\gamma_{ep}-1}}
\]

The assumption of equal pressure of both primary and secondary fluid in the present model is considered [11].

\[
P_{s,3} = P_{p,3}
\]

It is also found numerically that difference in the velocities of the both i.e., primary and secondary fluid is very large. So, both fluids may not mix with each other until the difference in their velocities may not reduce considerably.

At section 3-3 i.e., at exit plane of the primary nozzle the Mach number \((M_{p,3})\) of the primary/motive fluid has been calculated by the following relation [9, 10]:

\[
\frac{P_{p,1}}{P_{p,3}} = \left\{1 + \left[\left(\frac{\gamma_{p,1}-1}{2}\right) \times \left(M_{p,3}\right)^2\right]\right\} \times \left(\frac{\gamma_{p,1}}{\gamma_{p,1}-1}\right)
\]

The area of cross-sectional of the motive nozzle at its exit \((A_{p,3})\) is evaluated as below [10, 11]:

\[
\frac{A_{p,3}}{A_{p,2}} = \frac{1}{M_{p,3}} \times \left\{\left[\frac{2}{\gamma_{p,2}+1}\right] \times \left(1 + \left[\left(\frac{\gamma_{p,2}-1}{2}\right) \times \left(M_{p,3}\right)^2\right]\right]\right\} \times \left(\frac{\gamma_{p,2}+1}{\gamma_{p,2}-1}\right)
\]

The “Prandtl-Meyer expansion wave” has been used for evaluating divergence angle of the motive nozzle as follows [12]:

\[
\theta_p = \frac{1}{2} \left[\nu(M_{p,3}) - \nu(M_{p,2})\right]
\]

Here \(\theta_p\) is the nozzle divergence angle and \(\nu(M_{p,2})\) and \(\nu(M_{p,3})\) are the Prandtl-Meyer functions [12].
\[ v(M_{p,3}) = \left( \frac{y_{p,3}+1}{y_{p,3}-1} \right)^{\frac{1}{2}} \times \tan^{-1} \left( \frac{y_{p,3}-1}{y_{p,3}+1} \right)^{\frac{1}{2}} - \tan^{-1} \left( M_{p,3}^2 - 1 \right)^{\frac{1}{2}} \] (10)

\[ v(M_{p,2}) = \left( \frac{y_{p,2}+1}{y_{p,2}-1} \right)^{\frac{1}{2}} \times \tan^{-1} \left( \frac{y_{p,2}-1}{y_{p,2}+1} \right)^{\frac{1}{2}} - \tan^{-1} \left( M_{p,2}^2 - 1 \right)^{\frac{1}{2}} \] (11)

The Mach number \((M_{p,\cdot})\) of the motive flow is evaluated by [9, 10, 11]:

\[ \frac{p_{p,4}}{p_{p,3}} = \left[ \frac{1 + ((y_{p,3}-1)/2)M_{p,3}^2}{1 + ((y_{p,3}-1)/2)M_{p,4}^2} \right]^{\frac{y_{p,3}}{y_{p,3}-1}} \] (12)

At hypothetic throat, both fluids are assumed to be at same pressure [1].

\[ p_{p,4} = p_{s,4} \] (13)

Cross-sectional area at section 4-4 for the primary fluid can be calculated by relation

\[ \frac{A_{p,4}}{A_{p,3}} = \left( \frac{\phi_{p}/M_{p,4}}{1/M_{p,3}} \right) \times \left( \frac{2/(y_{p,3}+1)}{1/(y_{p,3}+1)} \right) \left( \frac{(1+((y_{p,3}-1)/2)M_{p,3}^2)}{1+((y_{p,3}-1)/2)M_{p,4}^2} \right) \] (14)

Secondary fluid mass flow rate can be calculated as:

\[ \omega \times \dot{m}_p = \dot{m}_s = \dot{m}_{ep} \times A_{s,4} \times \left( \frac{\gamma_{ep}}{R \times T_{ep}} \right) \times \left( \frac{2}{\gamma_{ep}+1} \right)^{\frac{1}{2}} \left( \eta_s^2 \right) \] (15)

The total area of cross-section at section 4-4 \((A_i)\) is the sum of the area of the primary and secondary fluid:

\[ A_4 = A_{p,4} + A_{s,4} \] (16)

The secondary fluid flow’s area \((A_{s,i})\) is calculated by relations:

\[ \frac{p_{s,4}}{p_{s,3}} = \left( \frac{1 + \frac{y_{s,3}-1}{y_{s,3}}M_{s,3}^2}{1 + \frac{y_{s,3}-1}{2}M_{s,3}^2} \right)^{\frac{y_{s,3}}{y_{s,3}-1}} \] (17)

\[ \frac{A_{s,4}}{A_{s,3}} = \left( \frac{1 + \frac{y_{s,3}-1}{y_{s,3}}M_{s,3}^2}{1 + \frac{y_{s,3}-1}{2}M_{s,3}^2} \right)^{\frac{y_{s,3}+1}{2(y_{s,3}-1)}} \] (18)

Both fluids mix with each other in between section 4-4 to 5-5. Momentum and energy balance equations are used to calculate velocity and the temperature [1]:

\[ \varphi_m (\dot{m}_p V_{p,4} + \dot{m}_s V_{s,4}) = (\dot{m}_p + \dot{m}_s) V_5 \] (19)

\[ \dot{m}_p \left( C_{p,4} T_{p,4} + \frac{V_{p,4}}{2} \right) + \dot{m}_s \left( C_{p,s,4} T_{s,4} + \frac{V_{s,4}}{2} \right) = (\dot{m}_p + \dot{m}_s) \left( C_{p_s} T_5 + \frac{V_5}{2} \right) \] (20)

A normal shock is assumed at section 6-6 i.e., at the inlet face of the diffuser. And by using the following relations, pressure and temperature after the diffuser can be calculated:

\[ M_6 = \frac{M_5^{y_6} \left( \frac{2}{y_5} \right)^{\frac{y_5}{y_6-1}}}{y_6-1} \] (21)
\[
\frac{P_6}{P_5} = \frac{M_6 \sqrt{1+M_5^2(\gamma_5-1)/2}}{M_6 \sqrt{1+M_5^2(\gamma_5-1)/2}}
\]  
(22)

\[
\frac{T_6}{T_5} = \left[1 + \frac{2\gamma_5}{\gamma_5+1} (M_5^2 - 1)\right] \frac{2+(\gamma_5-1)M_5^2}{(\gamma_5+1)M_5^2}
\]  
(23)

The flow is assumed to be stagnant at the exit of the diffuser and the flow properties is calculated by the following relations:

\[
\frac{P_7}{P_6} = \left(1 + \frac{\gamma_6-1}{2} M_6^2\right) \frac{\gamma_6}{\gamma_6-1}
\]  
(24)

\[
\frac{A_7}{A_{s,final}} = \exp\left\{\frac{(P_7 - P_6)(1-M_6^2)}{\rho_6 M_6^2 \gamma_6 R T_6}\right\}
\]  
(25)

3. Results and numerical procedure

The temperature \((T_p)\), pressure \((P_p)\) and the mass flow rate \((m_p)\) of the primary fluid at the inlet of primary nozzle; pressure \((P_s)\), temperature \((T_s)\) of the secondary fluid in the evaporator and critical condenser pressure \((P_c)\) are the inputs to the mathematical model. The numerical procedure has been carried out in Engineer Equation Solver (EES) [13].

The model is designed to calculate the entrainment ratio for the desired input conditions. Additionally, it also calculates the area-ratio of the single-phase ejector. The present model is used to calculate the entrainment ratio and the area-ratio for R141b and R134a refrigerants.

3.1 Model Validation

The results of the proposed mathematical model are first validated by comparing the model’s results with the results obtained by Huang et al. [1]. Figure 3 shows the deviation of the results of the area-ratio obtained by the present model while comparing with Huang et al. [1]. The average deviation of -0.285% is found in area-ratio while comparing with the experimental results of the Huang et al. [1]. Similarly, figure 4 shows the comparison for the entrainment ratio between the present model and the experimental results of the Huang et al. [1]. Graphs show that the results obtained by the proposed model ae in good coherence with experimental results of Huang et al. [1].

![Graph showing average deviation is -0.287%](image)

**Average deviation is -0.287%**
Figure 3 Area ratio’s comparison between results obtained by proposed model and that of Huang et al. [1]

![Comparison between model results and experimental results of Huang et al. [1]](image)

Average Deviation is 2.48%

Figure 4 Entrainment ratio’s comparison between the results obtained using proposed model and that of Huang et al. [1]

Table 1 Geometries of ejector used in the experiments

| Sr. No | Ejector | Primary nozzle geometry | Secondary nozzle geometry | Area Ratio = (A₄/A₃) |
|--------|---------|-------------------------|---------------------------|----------------------|
|        | Inlet Area (A₃₁) (mm²) | Throat Area (A₃₂) (mm²) | Exit Area (A₃₃) (mm²) | CAS Area (A₄) (mm²) | CAS’s Length (L) (mm) | (NXP) (mm) |
| 1      | I       | 31.7                    | 3.14                      | 10.3                 | 31.7                 | 80            | 2.54 | 10.08 |
| 2      | II      | 31.7                    | 3.14                      | 10.3                 | 31.7                 | 120           | 2.54 | 10.08 |
| 3      | III     | 31.7                    | 4.91                      | 15.9                 | 31.7                 | 80            | 2.54 | 6.45  |
| 4      | IV      | 31.7                    | 4.91                      | 15.9                 | 31.7                 | 120           | 2.54 | 6.45  |
| 5      | V       | 31.7                    | 7.07                      | 20.6                 | 67.2                 | 80            | 2.54 | 9.51  |
| 6      | VI      | 31.7                    | 7.07                      | 20.6                 | 67.2                 | 120           | 2.54 | 9.51  |

Table 1 shows the ejector geometry calculated by the proposed model for R134a refrigerant. The input parameters for the model are throat area of the primary nozzle, temperature and pressure of generator, condenser and evaporator. These dimensions are calculated for 84°C generator temperature, 30°C condenser temperature and 10°C evaporator temperature.

4. Conclusion

The paper proposed a mathematical model of single-phase ejector to evaluate the performance analysis and a few ejector geometries. The results obtained by the model for R141b refrigerant is validated by comparing them with the experimental results of Huang et al. [1] and the results found are in good coherence. The proposed model is also used to calculate the performance and geometry of the single-
phase ejector using R134a refrigerant. All dimensions and performance parameters are discussed in the next research paper of the sequence.

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