Efficient microwave-induced optical frequency conversion

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(March 3, 2022)

Frequency conversion process is studied in a medium of atoms with a Λ configuration of levels, where transition between two lower states is driven by a microwave field. In this system, conversion efficiency can be very high by virtue of the effect of electromagnetically induced transparency (EIT). Depending on intensity of the microwave field, two regimes of EIT are realized: "dark-state" EIT for the weak field, and Autler-Townes-type EIT for the strong one. We study both cases via analytical and numerical solution and find optimum conditions for the conversion.

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Frequency conversion is a useful technique for generation of coherent tunable radiation [1]. Efficient conversion of a continuous-wave (c.w.) radiation at relatively low pump intensities requires high nonlinear optical susceptibility of an (atomic) medium, which can be achieved by tuning to resonances. However, this will also increase the medium absorption and refraction seriously limiting the conversion efficiency. It was recently proposed [2] and demonstrated in many experiments that this problem can be overcome if one uses the effect of electromagnetically induced transparency (EIT) [3]. For example, the UV radiation has been generated by use of dc electric-field coupling in atomic hydrogen [4]. Radiation fields have been used to produce transparency in experiments on red to blue frequency conversion with molecular sodium [5], and on enhanced four-wave mixing with doped crystals [6]. Recently, blue to UV [7] and UV to VUV [8] conversion in atomic Pb vapor have been reached with almost unity photon-conversion efficiency.

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EIT is due to quantum interference in multilevel quantum systems (atoms, molecules, dopants in solids) induced by applied electromagnetic radiation. There are two basic mechanisms responsible for EIT. The first one occurs at large strength of one, ”coupling”, electromagnetic field which mixes and splits quantum states (Autler-Townes effect). When another, weaker ”probe” field is tuned in between the two mixed states, it experiences no absorption not only because of the splitting but also due to interference between excitation paths to two mixed states. This mechanism works well even in the case when the states mixed by the coupling field decay spontaneously. The second mechanism takes place at comparable intensities of applied fields. In this case, the cancellation of absorption and refraction can be explained by creation of a coherent superposition of atomic states (“dark” state) not excited by the radiation, and by preparation of atoms in this superposition (which is termed coherent population trapping - CPT) \[9\]. The dark state should be stable in order to allow for the population trapping. Therefore, the dark state should be a superposition of the metastable atomic states.

In the present paper we consider the frequency conversion in a scheme where both mechanisms of EIT are possible. This is a three-level Λ system (Fig. 1), where |1\rangle − |3\rangle and |2\rangle − |3\rangle are the dipole-allowed optical transitions, and the microwave (m.w.) transition |1\rangle − |2\rangle is a magnetic-dipole one. Such systems can be realized, e.g., on D-lines in alkali atoms, and may also be found in some molecules and doped crystals as well. Experiments on the absorption reduction induced by the m.w. field have recently been performed with similar systems in solids \[10\] \[12\]. The scheme is interesting, above all, because it allows easy control of the frequency conversion process by intensity of the microwave field. Possible applications of the present system include generation of the optical field which is phase- and amplitude-correlated to the input field, optical phase conjugation \[13\], generation of squeezed light \[14\], as well as quantum noise suppression and quantum correlation \[15\] \[16\].

When one of the optical fields (let say, \(\omega_{31}\)) and the m.w. field \(\omega_m\) are applied to the Λ atom, they induce an optical susceptibility on transition |2\rangle − |3\rangle. This leads to the generation of the optical field with frequency \(\omega_{32}\). In general, this field as well as the field
ω_{31} will be quickly absorbed if they are tuned close to the resonance. However, the absorption can be substantially reduced for particular values of the microwave intensity. For the weak m.w. field, optical waves create the dark state which is only slightly disturbed. The strong m.w. field mixes and splits both ground states |1⟩ and |2⟩ so that relatively weak optical fields experience EIT of the Autler-Townes type. Here we consider both cases, treating an interaction of the e.m. radiation with atoms as well as the propagation of radiation through the medium in exact manner.

The optical waves propagation along the z-axis in the medium is described by the Maxwell equations. In the slowly varying amplitude and phase approximation, and in continuous-wave limit these equations can be reduced to the following form \[17,18\]:

\[
\frac{dg_n}{d\zeta} = -\text{Im}(\bar{\sigma}_3n), \quad (1a)
\]

\[
\frac{d\varphi_n}{d\zeta} = -\frac{1}{g_n} \text{Re}(\bar{\sigma}_3n), \quad (1b)
\]

where \(g_n = d_{3n}E_n/2\hbar\gamma_{31}\) are the dimensionless optical field amplitudes (Rabi frequencies), \(E_n\) and \(\varphi_n\) \((n = 1, 2)\) are the optical amplitudes and phases, respectively; \(\gamma_{31}\) is the spontaneous decay rate in a channel \(|3⟩ \rightarrow |1⟩\), and \(d_{3n} = ⟨3|\hat{d}|n⟩\) are the matrix elements of the electric-dipole moment operator \(\hat{d}\) in the basis of atomic states \(|l⟩, l = 1, 2, 3\). The dimensionless optical length \(\zeta\) is expressed in terms of an absorption cross-section for the optical field \(\zeta = (2\pi\omega_{31}d_{31}^2/c\hbar\gamma_{31})Nz = \left(3\pi c^2/2\omega_{31}^2\right)Nz\), \(N\) is the density of active atoms.

The medium optical polarization components (the right-hand side of Eqs. (1)) are determined by the (steady-state) density matrix elements \(\sigma_{3n}\) averaged over the atomic velocities with the distribution \(w(v_z)\), where \(v_z\) is the z-projection of the atom velocity:

\[
\bar{\sigma}_{3n} = \int_{-\infty}^{+\infty} dv_z w(v_z) \sigma_{3n}(v_z), \quad \text{with} \quad \sigma_{3n}(v_z) = \rho_{3n}(v_z) \exp \left[i(\omega_{3n}t - k_{3n}z + \chi_{3n})\right], \quad \text{where} \quad \rho_{3n} \equiv ⟨3|\hat{\rho}|n⟩, \quad \hat{\rho} \text{ is the atomic density matrix.}
\]

The phase \(\chi_{3n}\) is the sum of the e.m. field phase \(\varphi_{3n}\) and the phase \(\vartheta_{3n}\) of the atomic dipole moment \(d_{3n} = |d_{3n}|e^{i\vartheta_{3n}}: \chi_{3n} = \varphi_{3n} + \vartheta_{3n}\). Similar quantities are determined for the microwave transition: Rabi frequency \(g_m = \mu H/2\hbar\gamma_{31}\) with the m.w. field amplitude \(H\) and phase \(\varphi_m\), and matrix element \(\mu \equiv ⟨1|\hat{\mu}|2⟩\) of the magnetic-dipole moment \(\hat{\mu}\).
Presence of the field on $|1\rangle - |2\rangle$ transition and/or both optical fields means, corresponding to the Maxwell equations, that the m.w. wave should also change along the propagation path. The EIT-assisted generation of a microwave radiation has recently been observed in atomic Cs vapor [19]. We, however, will not consider this effect here since the changes are of the order of $\Delta g_m^2 \approx (\omega_m/\omega_{31}) g_1^2 \approx (10^{-8} \div 10^{-5}) g_1^2$ at the most [20] which is negligible in the present context. Moreover, the propagation direction of the m.w. wave (traveling or standing one in a m.w. cavity) can be chosen perpendicular to the $z$–axis.

Let us now consider the case when all three e.m. fields are in resonance with corresponding transitions. This situation can be studied analytically if we additionally suppose equal spontaneous relaxation rates $\gamma_{31} = \gamma_{32} \equiv \gamma$, zero relaxation rate of the coherence between states $|1\rangle$ and $|2\rangle$: $\Gamma = 0$, and zero atomic velocity $v_z = 0$. Solution of the density-matrix equations for this case is given in the earlier work of one of us [17]. Nevertheless, we display it here again since it is important for further consideration:

$$
\text{Im}(\sigma_{31}) = -\frac{g_2g_mg_0^2(g_0^2 - 2g_m^2)}{2L} \sin \Phi + \frac{g_m^2g_1(g_1^2 - g_2^2 + 2g_2^2\sin^2 \Phi)}{L},
$$

$$
\text{Im}(\sigma_{32}) = \frac{g_1g_mg_0^2(g_0^2 - 2g_m^2)}{2L} \sin \Phi - \frac{g_m^2g_2(g_1^2 - g_2^2 - 2g_1^2\sin^2 \Phi)}{L},
$$

$$
\text{Re}(\sigma_{31}) = \frac{g_2g_m(g_1^2 - g_2^2)(g_0^2 - 2g_m^2)}{2L} \cos \Phi + \frac{g_m^2g_1g_2^2}{L} \sin 2\Phi,
$$

$$
\text{Re}(\sigma_{32}) = -\frac{g_1g_m(g_1^2 - g_2^2)(g_0^2 - 2g_m^2)}{2L} \cos \Phi - \frac{g_m^2g_1^2g_2}{L} \sin 2\Phi,
$$

and the excited state population is given by

$$
\rho_{33} = \frac{g_m^2 \left[ (g_1^2 - g_2^2)^2 + 4g_1^2g_2^2\sin^2 \Phi \right]}{L},
$$

with

$$
L = \frac{1}{2}g_0^6 + g_m^2 \left[ 3 \left( g_1^2 - g_2^2 \right) - 2g_0^4 + 2g_0^2 + 12g_1^2g_2^2\sin^2 \Phi \right] + 2g_0^2g_m^4,
$$

$$
g_0^2 = g_1^2 + g_2^2,
$$

and the relative phase $\Phi$ is determined as
\[
\Phi = (\chi_{31} - \chi_{32}) - \chi_{12},
\] (5)

where \(\chi_{12} = \varphi_m + \vartheta_{12}\) (\(\mu = |\mu| e^{i\vartheta_{12}}\)) is the phase of the m.w. transition.

One sees from Eqs. (2-4), that the medium is absolutely transparent and not refractive for

\[
\Phi = \pi n, \ n = 0, 1, 2, ...
\] (6)

and

\[
g_1 = g_2. \tag{7}
\]

These are exactly the conditions for the dark state in closed \(\Lambda\) system [17,21–23].

For arbitrary optical field amplitudes and phases, however, the refraction and absorption (or amplification) of individual frequency components can be substantial. Here we are interested in generation of the optical field \(\omega_{32}\) with lowest possible losses of the total e.m. energy. The change of the total energy flow is proportional to intensity \(I = I_{31} + I_{32}\) of the optical waves. The intensity is expressed in terms of Rabi frequency as

\[
I_n = (c/8\pi) E_n^2 = (2\hbar \omega_{3n}^3/3\pi c^2) g_n^2 \gamma, \ so \ that \ \frac{dI}{dz} \sim \frac{dg_1^2}{d\zeta} + \frac{dg_2^2}{d\zeta} = -2 (g_1 \text{Im} (\sigma_{31}) + g_2 \text{Im} (\sigma_{32})) = -2 \rho_{33}, \tag{8}
\]

where the last equality follows from the steady-state density matrix equations [17]. Thus, we arrive at almost obvious conclusion that the dissipation of the e.m. energy is small when the excited state population is small: \(\rho_{33} \ll 1\). Analysis of the expression (4) shows that, for arbitrary \(g_1\), \(g_2\) and \(\Phi\), this is the case for two ranges of the m.w. Rabi frequency: \(g_m \ll 1\), \(g_0\) and \(g_m \gg 1\), \(g_0\). These values correspond to EIT of the CPT-type and the Autler-Townes-type, respectively.

The change of the fields can be calculated analytically in present situation [24]. An interesting feature of the resonant case is that the phase equation can be solved for arbitrary values of \(g_m\). The propagation equation for the relative phase is as follows (if we neglect the change of the m.w. field phase \(\varphi_m\) and the atomic dipole phases \(\vartheta_{ns}\) along the propagation path):
\[
\frac{d\Phi}{d\zeta} = -\left( \frac{1}{g_1} \text{Re}(\sigma_{31}) - \frac{1}{g_2} \text{Re}(\sigma_{32}) \right).
\]

One can obtain from Eqs. (2) and (3) that
\[
\frac{1}{g_1} \text{Re}(\sigma_{31}) - \frac{1}{g_2} \text{Re}(\sigma_{32}) = (\cos \Phi / \sin \Phi) \left( g_2 \text{Im}(\sigma_{31}) + g_1 \text{Im}(\sigma_{32}) \right) / g_1 g_2
\]
so that
\[
\frac{d\Phi}{d\zeta} = -\cos \Phi \sin \Phi \left( g_2 \text{Im}(\sigma_{31}) + g_1 \text{Im}(\sigma_{32}) \right) / g_1 g_2,
\]
which can immediately be integrated to give the constant of motion:
\[
g_1 g_2 \cos \Phi = \Pi.
\]

The constant \(\Pi\) is determined from the boundary conditions at the \(\zeta = 0\). In particular, when one optical field is generated, \(g_2(\zeta = 0) = 0\), we have constant value of \(\cos \Phi\):
\[
\cos \Phi(\zeta) = 0.
\]

We now consider both EIT cases separately. For a weak m.w. field, \(g_m \ll 1\), the density matrix elements to the second order in \(g_m\) are:
\[
\text{Im}(\sigma_{31}) = -\frac{g_2 g_m}{g_0^2} \sin \Phi + \frac{2 g_m^2 g_1 \left( g_1^2 - g_2^2 + 2 g_2^2 \sin^2 \Phi \right)}{g_0^2},
\]
\[
\text{Im}(\sigma_{32}) = \frac{g_1 g_m}{g_0^2} \sin \Phi - \frac{2 g_m g_2 \left( g_1^2 - g_2^2 - 2 g_1^2 \sin^2 \Phi \right)}{g_0^2}.
\]

For generation of the field \(\omega_{32}\) the dissipation of total optical energy is proportional to (taking into account Eq. (10)):
\[
\frac{dg_0^2}{d\zeta} = -\frac{4g_m^2}{g_0^2},
\]
which has a solution
\[
g_0^4 = g_0^4(\zeta = 0) - 8g_m^2 \zeta.
\]

At sufficiently small optical length \(\zeta \ll 1/8g_m^2\), the total intensity decays linearly: \(g_0^2 = g_0^2(\zeta = 0) - (4g_m^2/g_0^2(\zeta = 0)) \zeta\). If we neglect this slow decay (which would simply correspond to neglect of terms of the second order in \(g_m\)), we obtain the following amplitude equations
\[
\frac{dg_1}{d\zeta} = -\frac{g_2 g_m}{g_0^2},
\]

\[
g_2 = g_0^2 - g_1^2
\]

which can be easily solved:

\[
g_1^2 = g_0^2 \cos^2 \left( \frac{g_m}{g_0} \zeta \right), \tag{13a}
\]

\[
g_2^2 = g_0^2 \sin^2 \left( \frac{g_m}{g_0} \zeta \right). \tag{13b}
\]

The solution indicates that the e.m. energy is transferred back and forth between two optical waves as the optical length increases. The period of these oscillations is \( \zeta_s = \pi g_0^2 / g_m \), which is much smaller than the characteristic length of the total energy dissipation: \( \zeta_{\text{diss}} \approx g_0^2(\zeta = 0)/4g_m^2 \), cf. Eq. (12). Therefore, very efficient conversion takes place at

\[
\zeta_{\text{max}} = \pi g_0^2(\zeta = 0)/2g_m. \tag{14}
\]

The loss of the optical intensity is \( \Delta I / I = 2\pi g_m \ll 1 \) at this point.

The reason for such an efficient process is a preparation of the medium in almost dark state. If the e.m. field between the states \( |1\rangle \) and \( |2\rangle \) is not applied then the dark state in \( \Lambda \) system takes the form [9]:

\[
|D\rangle = \frac{g_2}{g_0} |1\rangle - \exp \left( i\Phi \right) \frac{g_1}{g_0} |2\rangle. \tag{15}
\]

The population of the dark state can be expressed in terms of the ground-state density matrix elements:

\[
\rho_{DD} = \frac{g_2^2}{g_0^2} \rho_{11} + \frac{g_1^2}{g_0^2} \rho_{22} - \frac{2g_1 g_2}{g_0^2} \Re \left( \sigma_{21} \exp \left( -i\Phi \right) \right),
\]

which are, to the first order in \( g_m \),

\[
\Im(\sigma_{21}) = \frac{g_1 g_2}{g_0^2} \sin \Phi - \frac{g_m (g_1^2 - g_2^2)}{g_0^4},
\]

\[
\Re(\sigma_{21}) = -\frac{g_1 g_2}{g_0^2} \cos \Phi + \frac{g_m}{g_0^2} (1 + \sin \Phi) \cos \Phi,
\]

\[
\rho_{11} = \frac{g_2^2}{g_0^2} + \frac{2g_m g_1 g_2}{g_0^3} \sin \Phi,
\]

\[
\rho_{22} = \frac{g_1^2}{g_0^2} + \frac{2g_m g_1 g_2}{g_0^3} \sin \Phi.
\]
Thus, the population of the dark superposition is $\rho_{DD} = 1 - (2g_m g_1 g_2 / g_0^4) (1 + \sin \Phi) \cos^2 \Phi \approx 1$. It is interesting that a large lower-level coherence is not established in advance (since $g_2(\zeta = 0) = 0$). However, as soon as $g_2$ is generated, the coherence emerges, and the medium is prepared in the nonabsorbing state.

Even in real situation, when both the relaxation rate $\Gamma$ of the coherence between states $|1\rangle$ and $|2\rangle$ and the Doppler broadening are present, the parameters of the process are in fairly good agreement with calculations presented above. In Fig. 2 the spatial dependence of the field intensities and the phase $\Phi$ are plotted for the case when medium is a vapor of Na atoms, excited on $D_1$-line, at temperature $T = 440 K$ (this gives the saturated vapor density of $N = 4.42 \cdot 10^{11} cm^{-3}$ and corresponds to a most probable velocity of atoms of $v_p = 5.64 \cdot 10^4 cm/sec$, $\Gamma = 10^{-4}\gamma$ (1 kHz), input Rabi frequencies $g_{31}(\zeta = 0) = 2.0$, $g_m = 0.02$ (corresponding to intensities of $I_{31} = 12.6 mW/cm^2$ and $I_m = 1.26 \mu W/cm^2$). We see that dynamics of the intensities and the phase does not change qualitatively as compared to the case of negligible decay of the dark state. The behavior of the phase $\Phi$ in Fig. 2(b) follows the law $\cos \Phi(\zeta) = 0$ obtained analytically. The jumps in the phase occur at points where the intensity of the field being absorbed approaches zero, according to Eq. (1 (b)). The $\omega_{32}$ wave is generated and reaches its maximum at the length $\zeta = 340$ (this corresponds to the real length of the gas cell of $z = 1.9 cm$). This value is quite close to that calculated from analytical results, Eq. (14), $\zeta_{max} = 314$. The maximum intensity of the $\omega_{32}$ wave is $I_{32}/I_{31}(\zeta = 0) = 0.952$ which is slightly below the value 0.966 calculated from Eq. (12) because of an additional dissipation due to decay of the dark state with the rate $\Gamma$.

Obviously, this rate must be sufficiently small in order to allow for the population trapping in $|D\rangle$, namely it must be much smaller than the optical pumping rate into the dark state:

$$\frac{\Gamma}{\gamma} \ll \frac{g_0^2}{1 + \Delta^2}.$$  

Here, detuning $\Delta$ includes the Doppler shift: $\gamma\Delta = \Delta_{31} - k_{31}v_z \approx \Delta_{32} - k_{32}v_z$, where $\Delta_{3n} = \omega_{3n} - (E_3 - E_n) / \hbar$ are the laser frequency detunings from transitions $|n\rangle - |3\rangle$, ($n = 1, 2$), $E_n$ is the eigenenergy of the atomic state $|n\rangle$. For the resonance $\Delta_{31} = \Delta_{32} = 0$ and large
Doppler broadening \( k_{31} v_p \gg \gamma \), condition (16) reduces to
\[
g_0^2 \gg \frac{\Gamma}{\gamma} \left( \frac{k_{31} v_p}{\gamma} \right)^2.
\]
(17)
The better this condition is satisfied, the higher the efficiency of frequency conversion is.
The rate \( \Gamma \) is determined by m.w. field fluctuations, atomic collisions and other random phase disturbing processes. For parameters of the proposed here experiment, \( \Gamma \) can be very small. Recently, the rate \( \Gamma < 50 \, Hz \) has been observed in experiment [25].

Inasmuch as CPT is a basis for the considered above scheme, the generation occurs under quite restrictive conditions on e.m. wave frequencies. It is well known that CPT takes place when the optical frequencies are in the narrow range ("black line") around two-photon resonance [9]:
\[
\Delta_{32} = \Delta_{31}.
\]
(18)
Considering that the wave \( E_2 \) is always generated at frequency \( \omega_{32} = \omega_{31} - \omega_m \) (simply due to photon energy conservation), the condition (18) tells us that the generation takes place only if the m.w. field frequency is in the narrow range around resonance with transition \(|1\rangle - |2\rangle\): \( \omega_m = (E_2 - E_1)/\hbar \). Figure 3 demonstrates this fact. The width of the generation peak (width of the black line) is determined by the pumping rate into the dark state [9]. In the presence of large Doppler broadening this width is of the order of \( \delta \omega_m \approx g_0^2 / (k_{31} v_p/\gamma)^2 \gamma \), which is a few kHz for parameters of Fig. 3.

It is interesting that, at the same time, the large Doppler broadening allows for a broad tuning of the generated wave. In Fig. 4 we have plotted dependence of the generated intensity on detuning \( \Delta_{31} \) (for fixed \( \omega_m = (E_2 - E_1)/\hbar \)) at the optical length \( \zeta = 340 \). One can see that conversion efficiency remains fairly large for detunings of the order of the Doppler broadening (few GHz for Na vapor). This is because the CPT survives even at large common detunings \( \Delta_{32} = \Delta_{31} \) as long as the condition (16) is satisfied.

The second mechanism of EIT allowing efficient frequency conversion in the \( \Lambda \) medium takes place at strong m.w. fields, \( g_m \gg 1, g_0 \). In this case, the absorption coefficients to the second order in \( (1/g_m) \) are:
\[ \text{Im}(\sigma_{31}) = \frac{g_2}{2g_m} \sin \Phi + \frac{g_1 \left( g_2^2 - g_2^2 + 2g_2 \sin^2 \Phi \right)}{2g_m g_0^2}, \]  
\[ \text{Im}(\sigma_{32}) = -\frac{g_1}{2g_m} \sin \Phi - \frac{g_2 \left( g_2^2 - g_2^2 - 2g_1 \sin^2 \Phi \right)}{2g_m g_0^2}. \]

The energy dissipation is determined by the equation

\[ \frac{dg_0^2}{d\zeta} = -\frac{g_0^2}{g_m^2}, \]

with a solution

\[ g_0^2 = g_0^2(\zeta = 0) \exp \left(-g_m^{-2} \zeta \right). \]  

Again, if we neglect the slow total energy dissipation (i.e., we neglect terms of the second order in \( g_m \) in Eq. (19)), we obtain the solution of amplitude equations, very similar to the CPT case:

\[ g_1^2 = g_0^2 \cos^2 \left( \frac{1}{2g_m} \zeta \right), \]  
\[ g_2^2 = g_0^2 \sin^2 \left( \frac{1}{2g_m} \zeta \right). \]

Here, the period of intensity oscillations is \( \zeta_\pi = 2\pi g_m \), which is again much smaller than the characteristic length of the total energy dissipation: \( \zeta_{\text{diss}} \approx g_m^2 \). Maximum energy transfer to the \( \omega_{32} \) field occurs at

\[ \zeta_{\text{max}} = \pi g_m. \]

The loss of the optical intensity is \( \Delta I / I = 1 - \exp \left(-\pi / g_m\right) \ll 1 \) at this point.

Numerical calculations of the optical waves propagation give the results which are in very good agreement with analytical ones, and which are qualitatively very similar to the CPT-case in Fig. 2. The physical mechanism is, however, different. As we have discussed above, CPT does not work at strong m.w. fields except under the specific conditions (6) and (7). This can be proved by considering the density matrix elements. It turns out that for this case the ground state populations are equal to \( \rho_{11} = \rho_{22} = 0.5 \) up to the second order.
in \((1/g_m)\), \(\text{Im}(\sigma_{21}) = O(1/g_m^2)\) and \(\text{Re}(\sigma_{21}) = -(g_1g_2/g_0^2)\cos \Phi + O(1/g_m^2)\). Therefore, the atomic population is, for arbitrary \(g_1, g_2\) and \(\Phi\), not all pumped into the dark state:

\[\rho_{DD} = \frac{1}{2} + \left(2g_1^2g_2/g_0^2\right)\cos^2 \Phi < 1.\]

Under the condition \((10)\) taking place at the generation, the ground-state coherence \(\sigma_{21}\) is negligibly small and \(\rho_{DD} = 1/2\).

Thus, a very weak optical absorption at strong m.w. field is due to the Autler-Townes effect, or, in other terms, due to the capture of almost all atomic population by strong m.w. field in two-level system \(|1\rangle - |2\rangle\). This mechanism requires large m.w. intensities, but it has some advantages over the CPT-case. First of all, it is more robust. For example, the relaxation \(\Gamma\) does not play so important role, and the generation range of m.w. frequency \(\omega_m\) is much broader (it is of order of \(g_m\)) as compared to the case of weak m.w. field. Similar to the case with a weak m.w. field, there is a possibility to tune the generated radiation over the Doppler contour, and here the tuning is not as sensitive to the value of \(\Gamma\) as in former case. Note that here the optical length scales are determined only by \(g_m\) and do not depend on the input intensity, \(g_0^2(\zeta = 0)\). Therefore, the maximum conversion takes place at the same length for different input intensities. This is especially important for experiments with nonuniform light beams, e.g., with the Gaussian intensity profile. Another important advantage is that this mechanism can be applied not only to \(\Lambda\)-system, but also to a \(V\)-scheme with one ground and two excited states. We believe that the optical frequency conversion may be observed experimentally, for example, in a \(V\)-system of \(Pr^{3+} : YAlO_3\) solid where the reduced absorption was recently demonstrated \([12]\).

Finally, Fig. 5 represents the dependence of generated wave on Rabi frequency \(g_m\) of the m.w. field at fixed optical length \(\zeta = 340\). This figure clearly demonstrates two ranges of \(g_m\) where EIT and, correspondingly, efficient frequency conversion occur.

In summary, we have described a scheme for efficient optical conversion based on EIT in atomic \(\Lambda\) system where the interaction loop is closed by a microwave field. Depending on the m.w. intensity, two mechanisms of EIT work in this scheme: CPT and Autler-Townes effects. Intensity of the m.w. field plays also a role of controlling parameter in the conversion process - it determines optical length scales of the process, cf. Eqs. \((14)\), \((22)\), as
well as the degree of the total energy dissipation, cf. Eqs. (12), (20). An optimal choice is always possible, which should allow an experimental realization of the proposed scheme in different systems. Since the EIT-assisted frequency conversion combines large nonlinearity with substantially reduced spontaneous emission noise, one may expect that the generated signal will be fluctuation-correlated with the pump wave [3]. Such correlations persist even on a quantum level [16]. Therefore, the present scheme can be used for generation of two phase-correlated optical waves, which would be an alternative to conventional methods using electro- or acousto-optical modulators, or direct current modulation in laser diodes. Another possible application may be a generation of squeezed light [14].

I. ACKNOWLEDGMENTS

We are very grateful to Prof. L. Windholz for his continuous interest to this work and useful discussions. D.V. Kosachiov thanks the members of the Institut für Experimental-physik, TU Graz, for hospitality and support. This study was supported by the Austrian Science Foundation under project No. P 12894-PHY.

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Figure captions

Fig. 1. A system with two metastable states $|1\rangle$ and $|2\rangle$. $\omega_{31}$ and $\omega_{32}$ are the optical frequencies, $\omega_m$ is the microwave frequency.

Fig. 2. Spatial variations of: (a) optical field intensities $I_{31}$ (solid curve) and $I_{32}$ (dashed curve) in units of input intensity $I_0 \equiv I_{31}(\zeta = 0)$, (b) the relative phase $\Phi$, in a vapor of $^{23}\text{Na}$ atoms interacting with radiation in a $\Lambda$ configuration of levels $3^2S_{1/2}(F = 1) - 3^2S_{1/2}(F = 2) - 3^2P_{1/2}$. Vapor temperature $T = 440\ \text{K}$, $\Gamma = 10^{-4}\gamma$, detunings $\Delta_{31} = \Delta_{32} = 0$, Rabi frequencies of input fields $g_{31}(\zeta = 0) = 2.0$, $g_m = 0.02$.

Fig. 3. Generation of the $E_2$ wave (in units of input intensity $I_0 \equiv I_{31}(\zeta = 0)$) as a function of the microwave frequency $\omega_m$ (in units of the excited state relaxation rate $\gamma$). Other parameters are the same as in Fig. 2.

Fig. 4. Dependence of the generated intensity $I_{32}/I_0$ on detuning $\Delta_{31}$ (in units of the excited state relaxation rate $\gamma$) for fixed $\omega_m = (E_2 - E_1)/\hbar$, at the optical length $\zeta = 340$. Other parameters are the same as in Fig. 2.

Fig. 5. Dependence of the generated intensity $I_{32}/I_0$ on the Rabi frequency of microwave field $g_m$ at the optical length $\zeta = 340$. Other parameters are the same as in Fig. 2. Inset shows the range of small $g_m$. 
Fig. 1
Fig. 2

(a) Intensity $I_{3n}/I_0$

(b) Phase $\Phi$ ($\pi$ rad)
Fig. 3
Fig. 4

Intensity $I_{32} / I_0$ vs. detuning $\Delta_1$
Fig. 5

Intensity $I_{32} / I_0$

m.w. Rabi frequency $g_m$

The figure shows the intensity ratio $I_{32} / I_0$ as a function of the m.w. Rabi frequency $g_m$. The main graph displays a monotonic increase, while the inset highlights a sharp peak and subsequent decay.