Comment on The Preprint

“Neutrino Flavor Evolution Near A Supernova’s Core”

(astro-ph 9405008) by J. Pantaleone

Yong-Zhong Qian and George M. Fuller¹

Institute for Nuclear Theory

University of Washington, Seattle, WA 98195

Abstract

The revised version of the widely circulated preprint “Neutrino Flavor Evolution Near A Supernova’s Core” by J. Pantaleone (astro-ph 9405008 on the Bulletin Board, Indiana University preprint IUHET-276) is wrong. It contains two errors which lead to incorrect conclusions regarding neutrino flavor transformation in the supernova environment. In this short note we discuss these errors.

¹ Permanent address: Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319.
The widely circulated preprint “Neutrino Flavor Evolution Near A Supernova’s Core” by J. Pantaleone [1] is wrong. We pointed out that the first version of this paper had a quantum mechanics error. In response the author has changed the entire second half of the paper. Unfortunately, this revised version still contains two errors which render its conclusions incorrect.

First, Pantaleone makes a conceptual error in his treatment of the neutrino density matrix. Individual neutrinos emitted from the neutrino sphere can be described as coherent states (kets). However, each emitted neutrino is related to every other emitted neutrino in an incoherent fashion. These states have random relative phases, as is characteristic of a thermal emission process. The total neutrino field is properly a mixed ensemble, not a coherent many-body state. The total neutrino density matrix is an incoherent sum over each single neutrino density matrix. The single neutrino density matrix can be written as

\[
|\psi(t)\rangle\langle\psi(t)| = |a_1(t)|^2|\nu_1(t)\rangle\langle\nu_1(t)| + |a_2(t)|^2|\nu_2(t)\rangle\langle\nu_2(t)| \\
+ a_1^*(t)a_2(t)|\nu_2(t)\rangle\langle\nu_1(t)| + a_1(t)a_2^*(t)|\nu_1(t)\rangle\langle\nu_2(t)|, \tag{1}
\]

where the single neutrino state is \(|\psi(t)\rangle = a_1(t)|\nu_1(t)\rangle + a_2(t)|\nu_2(t)\rangle\), and where \(|\nu_1(t)\rangle\) and \(|\nu_2(t)\rangle\) are the propagating physical mass eigenstates for the case of two-neutrino mixing. Here \(t\) represents any evolutionary parameter (e.g., density, radius, time, etc.) along the neutrino’s path. Note that the last two terms in Eq. (1) have coefficients \(a_1^*(t)a_2(t)\) and \(a_1(t)a_2^*(t)\). These are cross terms.

These cross terms contain phases which cause them to be rapidly varying with position above the neutrino sphere. Each cross term is proportional to a factor \(\sim \exp[i \int \omega_{12}(t)dt]\).
with $\omega_{12}$ the difference in the neutrino-flavor-oscillation frequencies of the two mass eigenstates $|\nu_1(t)\rangle$ and $|\nu_2(t)\rangle$. These oscillation frequencies are, in turn, dependent on density. Near the neutrino sphere it can be shown that $\omega_{12} \sim \sqrt{2}G_F n_e$, with $n_e = Y_e \rho N_A$ the net electron number density [2, 3].

At a point above the neutrino sphere one must average over the neutrino distribution functions in order to obtain an ensemble average for any physical quantity dependent on the local neutrino configurations. In any such ensemble average we will have to sum the individual neutrino contributions (Eq. [1]) over different neutrino paths from the neutrino sphere. Fig. 1 illustrates the arrangement of the neutrino sphere (radius $R_\nu$) and a point at radius $r$ above it. Three possible neutrino paths to the point at radius $r$ are shown. Each path with a different polar angle will have a different phase entering into the cross term coefficients of Eq. (1). The phase difference acquired from going through a region of density $\rho$ with a path length difference $\delta r$ is $\delta \phi \sim \sqrt{2}G_F Y_e \rho N_A \delta r \sim 19(Y_e/0.5)(\rho/10^{10} \text{ g cm}^{-3})(\delta r/1 \text{ cm}) \gg 1$. Clearly, the cross terms in Eq. (1) will average to zero in any ensemble average over neutrino distribution functions.

Pantaleone mistakenly retains these cross terms in his expressions for the ensemble-averaged neutrino density matrix elements (his Eqs. [13] and [14]). This introduces a spurious, and unphysical, "coherence" which leads Pantaleone to the erroneous conclusion that the cross terms dominate the neutrino flavor transformation phenomenon for cases intermediate between the adiabatic and non-adiabatic limits.

In a second error, Pantaleone incorrectly estimates the effect of the neutrino background on the adiabaticity of neutrino flavor evolution at resonance. The full flavor-basis
Hamiltonian for a neutrino propagating above the neutrino sphere is (cf. Ref. [3]),

\[
H = \frac{1}{2} \begin{pmatrix}
-\Delta \cos 2\theta + A + B & \Delta \sin 2\theta + B_{e\mu} \\
\Delta \sin 2\theta + B_{e\mu} & \Delta \cos 2\theta - A - B
\end{pmatrix},
\]

(2)

where \( \Delta \equiv \delta m^2 / 2E_\nu \) with \( \delta m^2 = m_2^2 - m_1^2 \) the difference of the squares of the vacuum mass eigenvalues and \( E_\nu \) is the neutrino energy. Note that Pantaleone’s \( \Delta \) [1] is our \( \delta m^2 \). Here \( \theta \) is the vacuum mixing angle in the unitary transformation between mass eigenstates and flavor eigenstates in vacuum. In this expression the contribution to the Hamiltonian from the net electron number density is \( A = \sqrt{2} G_F n_e \). The contributions to the Hamiltonian from neutrino-neutrino forward scattering on background neutrinos are \( B \) and \( B_{e\mu} \) (Eqs. [15a & b] in Ref. [3]). With these definitions the adiabaticity parameter at resonance is

\[
\gamma = \frac{(\Delta \sin 2\theta + B_{e\mu})^2}{\Delta \cos 2\theta} \left| \frac{d \ln(A + B)}{dr} \right|^{-1}_{\text{res}}.
\]

(3)

Resonance occurs when \( \Delta \cos 2\theta = A + B \).

We are interested in estimating \( \gamma \) at resonance for a given, fixed, value of \( \Delta \) and a given, fixed, value of \( \sin 2\theta \). The term \( B \) has the effect of shifting the resonance position. The terms \( B, B_{e\mu}, \) and \( |d \ln(A + B)/dr|^{-1}_{\text{res}} \) are to be evaluated at resonance. The essential nonlinearity of this problem demands that a self-consistent iteration be performed to obtain the resonance position and good estimates of \( B \) and \( B_{e\mu} \).

Instead of doing this, Pantaleone mistakenly takes \( B \) to produce a change in \( \Delta \). By doing this he estimates a \( \gamma \) which is irrelevant for the resonance position of a neutrino with the original values of parameters \( \Delta \) and \( \sin 2\theta \). This is evident in his Eq. (23) [1]. With these unphysical estimates for \( \gamma \) Pantaleone’s conclusions regarding the flavor conversion efficiencies for neutrinos with given \( \Delta \) and \( \sin 2\theta \) are wrong.
Furthermore, Pantaleone incorrectly interprets Fig. 2 of Ref. [2]. He states in Ref. [1] that “... when the neutrino background is neglected the flavor evolution is extremely adiabatic for almost all of the relevant parameter space, and also because neutrino masses are only probed if significant amounts of flavor conversion occur.” This is obviously false, as is revealed by a cursory estimate of flavor conversion efficiency along the $Y_e = 0.5$ line in Fig. 2 of Ref. [2] (it is only of order $\sim 30\%$ for $E_\nu = 25$ MeV neutrinos).

The conclusions of Pantaleone’s preprint regarding the effects of the neutrino background on neutrino flavor transformation are in error. His conclusion that “For $\Delta < 700$ eV$^2$ the connection between $r$-process nucleosynthesis and cosmologically relevant neutrino masses is enervated” is wrong. A consistent treatment of neutrino flavor evolution in supernovae, along with a proper ensemble average over neutrino background quantities is performed in Ref. [3]. There it is shown that the neutrino background produces only minor alterations in the results of Ref. [2].

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References

[1] J. Pantaleone, Indiana University Preprint No. IUHET-276 (1994) \texttt{astro-ph 9405008} on the bulletin board).

[2] Y.-Z. Qian, G. M. Fuller, G. J. Mathews, R. W. Mayle, J. R. Wilson, and S. E. Woosley, Phys. Rev. Lett. \textbf{71}, 1965 (1993).

[3] Y.-Z. Qian and G. M. Fuller, Institute for Nuclear Theory Preprint No. DOE/ER/40561-150-INT94-00-63 (1994).

Figure Caption

\textbf{Fig. 1} Illustration of the arrangement of the neutrino sphere (radius $R_{\nu}$) and a point at radius $r$ above it. Three possible neutrino paths to the point at radius $r$ are shown.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9406074v1
