Finite Temperature Effects And Axion Cosmology

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Abstract

We investigate the impact of finite temperature effects on axions in the context of cosmology. The temperature dependence of the decay constant is modeled analogous to pions. For the two interesting cases considered here, we find that the temperature effects do lead to changes relevant for detailed and precise abundance and rate calculations. We also find that the axion decoupling temperature starts showing large deviations for larger values of the axion decay constant.
1 Introduction

Axions, the pseudo Goldstone bosons, associated with the Peccei-Quinn symmetry, $U(1)_{PQ}$, remain the most popular solution to the strong CP problem [1, 2, 3, 4]. Axions, QCD type or some other variant, appear in many extensions of the standard model. The mass and coupling of the axion is inversely proportional to the scale at which $U(1)_{PQ}$ is broken, denoted by $f_a$, and here after referred to as Peccei-Quinn (PQ) scale or axion decay constant interchangeably. This has important implications for axion cosmology because axions can only be produced thermally when the temperature of the universe drops below $f_a$. The PQ scale, $f_a$, is constrained by astrophysics, cosmology and laboratory experiments. For a review of latest bounds on $f_a$ refer to [5, 6], while a more comprehensive review/collection of (earlier) bounds on $f_a$ can be found in [7, 8, 9, 10, 11]. Further, axions are also one of the theoretically well motivated contenders for the cold dark matter in the universe. The DAMA collaboration has also explored the possibility of scalar and pseudo-scalar interpretation of their results [12]. More recently, results from the PVLAS experiment [13] indicate the existence of an axion like particle, though at the moment it is difficult to reconcile any standard axion like particle with these results. For a summary of some of the possible ways out to this difficulty refer to [14].

In this note we would like to study some cosmological implications/effects in the context of axions. To the best of our knowledge, all the existing studies have ignored the finite temperature effects to axion decay constant while discussing issues like relic axion abundance etc. We intend to explore the possible impact of such corrections on axion physics at different epochs of the early universe. Before proceeding with the finite temperature effects, we fix our notation. It is worthwhile to mention that axions share the basic properties with the pions, notably, the axion coupling to two gluons is given by the anomaly term (the two photon coupling has an analogous form)

\[ L = \frac{1}{f_a} \frac{\alpha_s}{8\pi} G^{b\mu\nu} \tilde{G}^b_{\mu\nu} a , \]

where $\alpha_s$ is the QCD coupling constant and $\tilde{G}^b_{\mu\nu}$ is the dual of the gluon field defined as

\[ \tilde{G}^b_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^b_{\rho\sigma} , \]

where $b$ is the color index. Let us rewrite the above as

\[ L = g_{agg} \epsilon_{\mu\nu\rho\sigma} G^{b\mu\nu} G^{b\rho\sigma} a , \]

where $g_{agg}$ is the axion coupling which is inversely proportional to $f_a$. The axions can be further divided into two categories - (a) Hadronic axions which couple to quarks only and (b) non-Hadronic axions having a coupling to the leptons. Mass of hadronic axions is given by

\[ m_a = \frac{z^{1/2} f_\pi m_\pi}{1 + z} \frac{f_\pi}{f_a} , \]

where $f_\pi = 93$ MeV is the pion decay constant, and $z = m_u/m_d$ is the mass ratio of up and down quarks. $z$ can lie within the interval $0.3 - 0.7$ [15]. Here we take $z = 0.56$ [16, 17].

Having noted that the axion coupling to massless gauge bosons is the same as pion-photon-photon coupling, we model the temperature effects in an analogous way as well. To this end, we first note
that the results on chiral dynamics and anomaly at finite temperature [18, 19] imply the following

\[ f_\pi(T) = \left(1 - \frac{1}{12} \frac{T^2}{f_\pi^2}\right) f_\pi, \]  

where \( f_\pi \equiv f_\pi(T = 0) \sim 93 \text{ MeV} \) is the zero temperature pion decay constant measured in the laboratory. It is important to keep in mind that such a result is valid only for \( T < f_\pi \). In the case of axions, it is natural to look at temperatures lower than the axion decay constant as it is only then that the axions can be thermally produced. Therefore, the above result for the pions can be directly extended to the axion case. The analysis in [19] however brings out a crucial point. The effective pion-photon-photon coupling is inversely proportional to the decay constant and one would have expected that the coupling changes the way dictated by the change in decay constant with the temperature, i.e., one would have expected the coupling to change as \( g_{\pi\gamma\gamma} \rightarrow g_{\pi\gamma\gamma} \left(1 - \frac{1}{12} \frac{T^2}{f_\pi^2}\right)^{-1} \) or that to \( \mathcal{O}(T^2/f_\pi^2) \), \( g_{\pi\gamma\gamma}(T) \sim 1/f_\pi(T) \). What is found instead is that to this order \( g_{\pi\gamma\gamma}(T) \sim f_\pi(T) \), i.e.,

\[ g_{\pi\gamma\gamma}(T) = \left(1 - \frac{1}{12} \frac{T^2}{f_\pi^2}\right) g_{\pi\gamma\gamma}. \]  

We therefore take this form for the axion-gluon-gluon coupling and work with this.\(^1\)

Further, following [18, 22], we have for the pion mass (again in the regime \( T < f_\pi \)),

\[ m_\pi(T) = m_\pi \left(1 + \frac{T^2}{24 f_\pi^2}\right). \]  

Note that since the limits on axion mass indicate a very small mass, we do not consider temperature dependent effects for axion mass.

We are now ready to explore two epochs in the history of the universe, particularly important and relevant for axion cosmology. First, we study axion thermalization and abundance in the very early universe including the temperature effects. The second case investigated is the axion hadron interactions in the post QCD era. In both these cases we find that the temperature effects - via axion or pion decay constant, pion mass or axion-gluon-gluon coupling, whichever is applicable at the relevant epoch - do lead to change in the results for certain values of the temperatures.

## 2 Axion thermalization and abundance

As mentioned earlier, axions can be thermally produced in the universe once the temperature falls below the axion decay constant. However, axions can also be produced via some non-thermal processes (see [23] and references therein). In [24], the authors identify the conditions under which there is significant production and/or thermalization of axions. We therefore closely follow [24] but now include temperature effects as mentioned before. The main thermalization processes are axion-gluon and axion-quark scatterings. From Eq.(5) and Eq.(6), we see that both \( f_a \) and

\(^1\)It is worthwhile to point out that the configuration considered in [19] does not correspond to the physical situation while the authors in [20] consider the decay of pseudoscalar to real photons and find difference with the results obtained in [19]. See [21] for a discussion about this difference and the resolution. However, we only use the results concerning the change in decay constant with temperature and therefore this issue does not affect us here.
increase with the inclusion of the temperature effects. This means that the axions can be thermally produced at a slightly lower temperature and the interaction rate also goes down a bit. The Boltzmann equation for the abundance in an expanding universe is given by

$$x \frac{dY}{dx} = \frac{\Gamma}{H}(Y^{eq} - Y),$$  \hspace{1cm} (8)$$

where $x = \frac{f_a}{T}$, $H$ is the Hubble expansion rate and $\Gamma$ is the thermally averaged rate of reaction of axion processes. In the radiation dominated era, the Friedmann equation yields

$$H = \left(\frac{4\pi^3 g_{eff}}{45}\right)^{1/2} \frac{T^2}{M_P},$$  \hspace{1cm} (9)$$

where $g_{eff}$ is the effective degrees of freedom at the temperature $T$. For a generic process $a + i \rightarrow j + k$ ($i, j, k \neq a$), thermally averaged rate is

$$\Gamma = \frac{1}{n_a} \int dp_a dp_i dp_j dp_k (2\pi)^4 \delta^4(p_a + p_i - p_k - p_k)|\mathcal{M}|^2 f_a f_i (1 + f_j)(1 + f_k),$$  \hspace{1cm} (10)$$

where $\tilde{dp} = \frac{dp}{(2\pi)^3 2E}$, $f$’s are the equilibrium distribution functions and $n_a$ is the equilibrium axion number density. Compared to the expression for the thermal averaged interaction rate in [24], we will have an extra temperature dependent factor stemming from the correction to the axion-gluon-gluon coupling and the rate now looks

$$\Gamma \simeq 7.1 \times 10^{-6} \frac{T^3}{f_a} \left(1 - \frac{T^2}{12f_a^2}\right)^2 = \Gamma_0 \left(1 - \frac{T^2}{12f_a^2}\right)^2.$$  \hspace{1cm} (11)$$

$Y^{eq}$ is the abundance when axions are in thermal equilibrium with other SM particles. In terms of variables $\eta = \frac{Y}{Y^{eq}}$ and $k = x \frac{f_a}{H}$, Eq. (8) becomes

$$x^2 \frac{d\eta}{dx} = k \left(1 - \frac{1}{12x^2}\right)^2 (1 - \eta).$$  \hspace{1cm} (12)$$

Retaining only the leading term, the solution is

$$\eta(x) = 1 + C \ exp \left[\frac{k}{x} - \frac{k}{18x^3}\right].$$  \hspace{1cm} (13)$$

where $C$ is a constant determined by boundary condition, $\eta = 0$ for $\frac{f_a(T)}{T} = 1$. This gives $\eta = 0$ at $x \sim 1.08$ instead of at $x = 1$ as the boundary condition and the constant $C$ is evaluated accordingly. Fig.1 compares the axion abundance with and without the temperature effects taken into consideration for $f_a = 1.2 \times 10^{12}$ GeV. We see that there is a perceptible change in the abundance for the temperature range $(6-10)\times10^{11}$ GeV. As a consequence of the shift in the $x$ value from unity to 1.08, the results in [24] shift by this factor.

3 Hadronic axion in the post QCD era

The hadronic axion has no lowest level coupling to the charged leptons and therefore the induced coupling is expected to be small enough to be neglected altogether. For updated bounds on hadronic
The axion-pion interaction is of the form \[ L_{\alpha \pi} = \frac{C_{\alpha \pi}}{f_\pi f_\alpha} \left( \pi^0 \pi^+ \partial_\mu \pi^- + \pi^0 \pi^- \partial_\mu \pi^+ - 2\pi^+ \pi^- \partial_\mu \pi^0 \right) \partial_\mu a. \] (14)

In hadronic axion models, the coupling constant is

\[ C_{\alpha \pi} = \frac{1 - z}{3(1 + z)}. \] (15)

The relevant processes are \( a\pi^\pm \to \pi^0 \pi^\pm \) and \( a\pi^0 \to \pi^+ \pi^- \). Following \[ 24, 26 \], we calculate the axion decoupling temperature in this context but now taking the finite temperature effects in the pion decay constant and mass into account given in Eq. (5) and Eq. (7) respectively (valid for \( T < f_\pi \)). The decoupling temperature is that where the Hubble expansion rate equals the thermally averaged interaction rate. In the Fig. 2 we plot the expansion rate of the universe \( (H) \) and the interaction rates \( (\Gamma) \) - with and without temperature effects put in. The points where
the $\Gamma$ curves cut the $H$ curve give the decoupling temperature. This is shown in the figure for $f_a = 10^7$ GeV. It is clear from the figure that the decoupling temperature is lowered once the finite temperature effects are included and the reason is easy to see. In the epoch of interest, the temperatures are much lower than the axion decay constant and therefore, we do not consider any temperature dependent effects in $f_a$, while $f_\pi$ decreases and $m_\pi$ increases with temperature. Both these effects lead to larger interaction rate and therefore, the axions decouple later in time or at lower temperatures. From the figure, it is also clear that the difference in the two cases is not very large. We also compare decoupling temperature of axions in the present calculation with the previous ones (without the temperature effects) for several representative values of $f_a$ in Table 1. We find that for higher values of $f_a$, the temperature effects start becoming significant. However, within the approximation adopted here, it is not possible to explore the range $f_a \geq 1.5 \times 10^7$ GeV as the temperatures exceed the pion decay constant, thus taking us away from the validity of the approximation employed. It is therefore important to investigate the temperature effects going beyond this approximation and exploring the consequences.

![Figure 2: Hadronic axion reaction rate with(solid) and without(dashed) temperature effect for $f_a = 10^7 GeV.$](image)

| $f_a$ (GeV) | $T_{D1}$ (MeV) | $T_{D2}$ (MeV) |
|------------|----------------|----------------|
| $3 \times 10^5$  | 26.43          | 26.43          |
| $1 \times 10^6$  | 35.34          | 35.34          |
| $3 \times 10^6$  | 49.84          | 49.5           |
| $1 \times 10^7$  | 81.04          | 79.12          |
| $1.2 \times 10^7$ | 87.61          | 85.48          |
| $1.3 \times 10^7$ | 90.1           | 87.9           |

Table 1: Decoupling temperature of axions $T_{D1}$ (without) and $T_{D2}$ (with) temperature effects for different values of $f_a$. 

4 Discussion

We have investigated the impact of temperature effects, though in a limited sense, on the axion cosmology. We have focused our attention on the temperature effects in the axion decay constant, pion decay constant and pion mass in two important epochs of the universe. The results show that there can be perceptible differences in the predictions of abundances and decoupling temperatures. However, we must remark that these effects have been considered in the approximation when the temperature of the universe is lower than the decay constant(s). In the case of axion thermalization in the early universe this has a physical meaning because it is only then that the axions can be thermally produced. In this case we see some change in the abundance of the axions with temperature. In the case of axion interacting with pions in the post QCD era, we find that temperature effects in the pion mass and decay constant lead to a lower decoupling temperature for the axions. We find that as the axion decay constant is increased the difference between the zero temperature and finite temperature cases starts becoming significant. However, we have worked in the limit when temperatures are smaller than $f_\pi$. This puts a natural limit to the region that we can explore. It is worthwhile to try to explore what happens when $T > f_\pi$. This will require going beyond the validity of the present calculation. We would also like to mention that a more detailed and consistent calculation requires computing the matrix elements including thermal loops also to the same order in temperature corrections and then studying the full impact.

In summary, we can say that we have studied the impact of temperature effects due to a very limited source(s) entering the axion cosmology. The results show perceptible departure from the zero temperature calculations. Let us just compare the situation with the case of neutrino decoupling. In that case, naively, the temperature effects shift the decoupling temperature by a seemingly small amount. However, for precision studies, such a shift makes a significant difference and hence turns out to be important. Our approach is also guided by this fact. See [30] (and references therein) for related discussion. Whether, finite temperature effects are actually significant and substantial can be decided by more detailed computations, taking into account all sources of corrections to this order. Also, in the case of axion decoupling in the post QCD era, it’ll be interesting to try to go beyond the approximation of $T < f_\pi$ and include finite temperature effects in the masses and all other parameters entering the expressions. This can have serious implications for axion cosmology and can lead to very different bounds/limits on various parameters, thereby possibly changing our current understanding.

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