High energy neutrino in a nuclear environment: mirror asymmetry of the shadowing effect\textsuperscript{1}

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Abstract

The parity non-conservation effect in diffractive charged current DIS is quantified in terms of color dipole sizes of left-handed and right-handed electroweak bosons. We identify the origin and estimate the strength of the left-right asymmetry effect and present comparison with experimental data on the parity-odd structure function \( \Delta x F_3 = x F_{3}^{pN} - x F_{3}^{\bar{p}N} \). We study the shadowing effect in absorption of left-handed and right-handed \( W \)-bosons by atomic nuclei. The target nucleus is found to be quite transparent for the charmed-strange Fock component of the light-cone \( W^+ \) in the helicity state \( \lambda = +1 \) and rather opaque for the \( cs \) dipole with \( \lambda = -1 \).

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In this paper we discuss the effects of mirror asymmetry in small-\(x\) charged current (CC) deep inelastic scattering (DIS). Since the physical picture of a process may change dramatically in different reference frames, one can gain deeper insight into the dynamics of a process by choosing a particular frame. In a frame where the parton picture of a nucleon is manifest the mirror asymmetry appears as a difference of relevant parton and anti-parton densities, in a dual frame the mirror asymmetry shows up as a difference of color dipole sizes of left-handed and right-handed \(W\)-bosons \((W_{L,R})\). Indeed, in the brick wall frame in the chiral limit the \(W_L\) interacts only with quarks while the \(W_R\) interacts only with anti-quarks. Hence, the representation for the standard parity-odd structure function \(F_3\) in terms of parton densities [1]

\[
F_3^{\nu p} = 2\{ (d - \bar{u}) + (s - \bar{c}) \} \quad (1) \\
F_3^{\nu n} = 2\{ (u - \bar{d}) + (s - \bar{c}) \} \quad (2) \\
F_3^{\bar{\nu} p} = 2\{ (u - \bar{d}) + (c - \bar{s}) \} \quad (3) \\
F_3^{\bar{\nu} n} = 2\{ (d - \bar{u}) + (c - \bar{s}) \} , \quad (4)
\]

The isospin symmetry of the nucleon sea implies that at small Bjorken \(x\) the structure function \(F_3\) is dominated by the charm-strange weak current and for an isoscalar nucleon target at \(x \to 0\)

\[
F_3^{\nu N} = 2(s - \bar{c}) \quad (5) \\
F_3^{\bar{\nu} N} = 2(c - \bar{s}) . \quad (6)
\]

The non-partonic sea contribution to \(F_3\) coming from the u-d current is proportional to the light quark mass splitting \(\sim \Delta m_{ud}/m_d\) and vanishes in the limit \(m_u = m_d\).

In the dipole/laboratory frame the space-time picture of the phenomenon is quite different [2, 3]. In the color dipole approach [4, 5] (for the review see [6]) the small-\(x\) DIS is treated in terms of the interaction of the quark-antiquark color dipole of size \(r\), that the virtual \(W\) transforms into, with the target nucleon. This interaction is described by the beam- and flavor-independent color dipole cross section \(\sigma(x, r)\). At small \(x\) the dipole size \(r\) is a conserved quantum number and the parity non-conservation effect can be quantified in terms of color
dipole sizes of left-handed and right-handed $W$-bosons. Hence, the representation of $F_3$ in terms of absorption cross sections for $W_L$- and $W_R$-bosons ($\sigma_{L,R}$)

$$F_3 \sim \sigma_L - \sigma_R. \quad (7)$$

These cross sections are calculated as a quantum mechanical expectation values of $\sigma(x,r)$. Once the light-cone wave function (LCWF) of a color dipole state is specified the evaluation of $\sigma_{L,R}$ becomes a routine quantum mechanical procedure.

Below we report our extension of the color dipole analysis onto the charged current DIS with particular emphasis on the left-right asymmetry of diffractive interactions of electroweak bosons of different helicity. We make use of the LCWF derived in [2, 3] and evaluate the structure functions $xF_3$ in the vacuum exchange dominated region of $x \lesssim 0.01$. We present comparison of our results with experimental data. In experiments with neutrino beams the nuclear targets are in use and the nucleon structure functions are extracted from nuclear data. In particular, from the differential cross section of $\nu Fe$-scattering. The latter is considered as the incoherent sum of $\nu N$ cross sections. However, at small $x$ the effect of non-additivity of nuclear cross sections plays important role. The interference of multiple scattering amplitudes leads to $\sigma^{\nu A} \neq A \sigma^{\nu N}$ and to the so called nuclear shadowing effect. The shadowing effect is found to be different for left-handed and right-handed $W$-bosons and mirror asymmetry is shown to be enhanced by the large thickness of a nucleus.

At small $x$ the contribution of diffractive excitation of open charm/strangeness to the absorption cross section for scalar, ($\lambda = 0$), left-handed, ($\lambda = -1$), and right-handed, ($\lambda = +1$), $W$-boson of virtuality $Q^2$, is given by the color dipole factorization formula [7, 8]

$$\sigma_\lambda(x, Q^2) = \int dzd^2r \sum_{\lambda_1, \lambda_2} |\Psi_{\lambda_1,\lambda_2}^\lambda(z, r)|^2 \sigma(x, r). \quad (8)$$

In Eq. (8) $\Psi_{\lambda_1,\lambda_2}^\lambda(z, r)$ is the LCWF of the $|c\bar{s}\rangle$ state with the $c$ quark carrying fraction $z$ of the $W^+$ light-cone momentum and $\bar{s}$ with momentum fraction $1 - z$ [2, 3]. The $c$- and $\bar{s}$-quark helicities are $\lambda_1 = \pm 1/2$ and $\lambda_2 = \pm 1/2$, respectively.

Only diagonal elements of the density matrix [2, 3]

$$\rho_{\lambda\lambda'} = \sum_{\lambda_1, \lambda_2} \Psi_{\lambda_1,\lambda_2}^\lambda \left(\Psi_{\lambda_1,\lambda_2}^{\lambda'}\right)^* \quad (9)$$
for \( \lambda = \lambda' = L, R \) enter Eq. (8):

\[
\rho_{RR}(z, r) = \left| \Psi_R^{+1/2,+1/2}(z, r) \right|^2 + \left| \Psi_R^{-1/2,+1/2}(z, r) \right|^2
= \frac{8\alpha_W N_c}{(2\pi)^2} (1 - z)^2 \left[ m^2 K_0^2(\varepsilon r) + \varepsilon^2 K_1^2(\varepsilon r) \right]
\]

(10)

and

\[
\rho_{LL}(z, r) = \left| \Psi_L^{-1/2,-1/2}(z, r) \right|^2 + \left| \Psi_L^{-1/2,+1/2}(z, r) \right|^2
= \frac{8\alpha_W N_c}{(2\pi)^2} z^2 \left[ \mu^2 K_0^2(\varepsilon r) + \varepsilon^2 K_1^2(\varepsilon r) \right],
\]

(11)

where

\[
\varepsilon^2 = z(1 - z)Q^2 + (1 - z)m^2 + \mu^2,
\]

(12)

\( K_\nu(x) \) is the modified Bessel function, \( \alpha_W = g^2/4\pi \) and

\[
\frac{G_F}{\sqrt{2}} = \frac{g^2}{m_W^2}.
\]

(13)

The c-quark and \( \bar{s} \)-quark masses are \( m \) and \( \mu \), respectively.

The left-handedness of weak currents leads to the striking momentum partition asymmetry of both \( \rho_{LL} \) and \( \rho_{RR} \) [2, 3]. The left-handed quark in the decay of left-handed \( W^+_L \) carries away most of the \( W^+_L \) light-cone momentum. The same does the right-handed antiquark in the light-cone decay of \( W^-_R \). This is the phenomenon of the same nature as spin-spin correlations in the neutron \( \beta \)-decay. The observable which is strongly affected by this left-right asymmetry is the parity-odd structure function of the neutrino-nucleon DIS named \( F_3 \). Its definition in terms of \( \sigma_R \) and \( \sigma_L \) of Eq. (8) is as follows:

\[
2xF_3(x, Q^2) = \frac{Q^2}{4\pi^2\alpha_W} \left[ \sigma_L(x, Q^2) - \sigma_R(x, Q^2) \right],
\]

(14)

To estimate consequences of the left-right asymmetry for \( F_3 \) at high \( Q^2 \), such that

\[
\frac{m^2}{Q^2} \ll 1, \quad \frac{\mu^2}{Q^2} \ll 1,
\]

(15)

one should take into account that the dipole cross-section \( \sigma(x, r) \) in Eq. (8) is related to the un-integrated gluon structure function \( F(x, \kappa^2) = \partial G(x, \kappa^2)/\partial \log \kappa^2 \), [9]:
\[
\sigma(x, r) = \frac{\pi^2}{N_c} r^2 \alpha_S(r^2) \int \frac{d\kappa^2 \kappa^2}{(\kappa^2 + \mu_G^2)^2} \frac{4[1 - J_0(\kappa r)]}{\kappa^2 r^2} \mathcal{F}(x_g, \kappa^2). \tag{16}
\]

In the Double Leading Logarithm Approximation (DLLA), i.e. for small dipoles,

\[
\sigma(x, r) \approx \frac{\pi^2}{N_c} r^2 \alpha_S(r^2) G(x_g, A/r^2), \tag{17}
\]

where \(\mu_G = 1/R_c\) is the inverse correlation radius of perturbative gluons and \(A \approx 10\) comes from properties of the Bessel function \(J_0(y)\). Because of scaling violation \(G(x, Q^2)\) rises with \(Q^2\), but the product \(\alpha_S(r^2)G(x, A/r^2)\) is approximately flat in \(r^2\). At large \(Q^2\) the leading contribution to \(\sigma\lambda(x, Q^2)\) comes from the P-wave term, \(\varepsilon^2 K_1(\varepsilon r)^2\), in Eqs. (10) and (11). The asymptotic behavior of the Bessel function, \(K_1(x) \approx \exp(-x)/\sqrt{2\pi/x}\) makes the \(r\)-integration rapidly convergent at \(\varepsilon r > 1\). Integration over \(r\) in Eq. (8) yields

\[
\sigma_L \propto \int_0^1 dz \frac{z^2}{\varepsilon^2} \alpha_S G \sim \frac{\alpha_S G}{Q^2} \log \frac{Q^2}{\mu^2}, \tag{18}
\]

and similarly

\[
\sigma_R \propto \int_0^1 dz \frac{(1 - z)^2}{\varepsilon^2} \alpha_S G \sim \frac{\alpha_S G}{Q^2} \log \frac{Q^2}{m^2}. \tag{19}
\]

It should be emphasized that we focus on the vacuum exchange contribution to \(x F_3\) corresponding to the excitation of the \(c\bar{s}\) state in the process the \(W^+\)-gluon fusion,

\[
W^+ g \rightarrow c\bar{s}. \tag{20}
\]

Therefore, the structure function \(x F_3\) differs from zero only due to the strong left-right asymmetry of the light-cone \(|c\bar{s}\rangle\) Fock state. One should bear in mind, however, that in experiment the smallest available values of \(x\) are in fact only moderately small and there is quite significant valence contribution to \(x F_3\). The valence term, \(xV\) is the same for both \(\nu N\) and \(\bar{\nu} N\) structure functions of an iso-scalar nucleon. The sea-quark term in the \(x F_3^{\nu N}\) which we are interested in is denoted by \(x S(x, Q^2)\) and has opposite sign for \(x F_3^{\bar{\nu} N}\). Indeed, the substitution \(m \leftrightarrow \mu\) in Eqs. (10) and (11) entails \(\sigma_L \leftrightarrow \sigma_R\). Therefore,

\[
x F_3^{\nu N} = x V + x S, \tag{21}
\]

and

\[
x F_3^{\bar{\nu} N} = x V - x S. \tag{22}
\]
Figure 1: $\Delta x F_3$ data as a function of $Q^2$ [11]. Shown by solid lines are the results of color dipole description.

One can combine the $\nu N$ and $\bar{\nu} N$ structure functions to isolate the Pomeron exchange term,

$$\Delta x F_3 = x F_{3}^{\nu N} - x F_{3}^{\bar{\nu} N} = 2xS.$$  \hspace{1cm} (23)

The extraction of $\Delta x F_3$ from CCFR $\nu_\mu Fe$ and $\bar{\nu}_\mu Fe$ differential cross section in a model-independent way has been reported in [11]. Figure 1 shows the extracted values of $\Delta x F_3$ as a function of $Q^2$ for two smallest values of $x$. The solid curves are calculated making use of Eqs. (8) and (16) with the differential gluon density function $\mathcal{F}(x g, \kappa^2)$ determined in [12].

The $\Delta x F_3$ slowly increases with growing $Q^2$ because of the logarithmic scaling violation in $\sigma_{L,R}$. Note that in the region of moderately small $x$, $x > 0.01$, the mass threshold effect suppresses $\Delta x F_3$ at $Q^2 \lesssim (m + \mu)^2$

The CCFR/NuTeV structure functions $x F_{3}^{\nu N}$ and $x F_{3}^{\bar{\nu} N}$ are extracted from the $\nu Fe$ and $\bar{\nu} Fe$ data [10, 11]. For the nuclear absorption cross section the color dipole factorization gives

$$\sigma^A_\lambda(x, Q^2) = \langle \Psi_\lambda | \sigma^A(x, r) | \Psi_\lambda \rangle$$

$$= \int dz d^2r \sum_{\lambda_1, \lambda_2} | \Psi^{\lambda_1, \lambda_2}_\lambda(z, r) |^2 \sigma^A(x, r),$$  \hspace{1cm} (24)
where \[13\]
\[\sigma^A(x, r) = 2 \int d^2 b \left\{ 1 - \exp \left[ -\frac{1}{2} \sigma(x, r) T(b) \right] \right\}. \tag{25}\]

Here \(T(b)\) is the optical thickness of a nucleus,
\[T(b) = \int_{-\infty}^{+\infty} dz n(z, \sqrt{z^2 + b^2}). \tag{26}\]

\(b\) is the impact parameter and \(n(r)\) is the nuclear matter density normalized as follows:
\[\int d^3 r n(r) = A. \tag{27}\]

It is assumed that \(A \gg 1\). One can expand the exponential in Eq. (25) to separate the impulse approximation term and the shadowing correction, \(\delta \sigma^A_\lambda\), in (24),
\[\sigma^A_\lambda = A \sigma_\lambda - \delta \sigma^A_\lambda. \tag{28}\]

To the lowest order in \(\sigma T\) the shadowing term reads
\[\delta \sigma^A_\lambda \simeq \frac{\pi}{4} \langle \sigma^2 \rangle S^2_A(k_L) \int db^2 T^2(b), \tag{29}\]
where
\[\langle \sigma^2 \rangle = \langle \Psi_\lambda \sigma(x, r)^2 | \Psi_\lambda \rangle\]
\[= S^2_A(k_L) \int dz d^2 r \sum_{\lambda_1, \lambda_2} |\Psi_{\lambda_1, \lambda_2}(z, r)|^2 \sigma^2(x, r). \tag{30}\]

The longitudinal nuclear form factor \(S_A(k_L)\) in Eq. (29) takes care about the coherency constraint,
\[l \gg R_A. \tag{31}\]

The approximation (29) represents the driving term of shadowing, the double-scattering term. It is reduced by the higher-order rescatterings by about 30% for iron and 50% for lead nuclei. This accuracy is quite sufficient for order-of-magnitude estimates. The numerical calculations presented below are done for the full Glauber series (25),
\[\delta \sigma^A_\lambda = \pi S^2_A(k_L) \sum_{n=2}^{\infty} \frac{(-1)^n \langle \sigma^n \rangle}{n!2^{n-1}} \int db^2 T^n(b), \tag{32}\]
where the effect of finite coherence length is modeled by the factor $S_A^2(k_L)$ in rhs. A consistent description of the latter effect in electro-production was obtained in Ref. [14] based on the light-cone path integral technique of Ref. [15].

To estimate the strength of the nuclear shadowing effect in $x F_3$ at high $Q^2$ such that

$$\frac{m^2}{Q^2} \ll 1, \frac{\mu^2}{Q^2} \ll 1$$

one can use the dipole cross section $\sigma(x, r)$ of Eq.(16) Let us estimate first the contribution to $\langle \sigma_3^2 \rangle$ coming from the P-wave term, $\varepsilon^2 K_1(\varepsilon r)^2$, in Eqs. (10) and (11). Integration over $r$ in Eq. (30) yields

$$\langle \sigma_3^2 \rangle \propto \int_0^1 dz \frac{z^2}{\varepsilon^4} \propto \frac{1}{Q^2 \mu^2}$$

and similarly

$$\langle \sigma_R^2 \rangle \propto \int_0^1 dz \frac{(1-z)^2}{\varepsilon^4} \propto \frac{1}{Q^2 m^2}.$$  

Obviously, the integral (34) is dominated by $z \gtrsim 1 - \mu^2/Q^2$ i.e., by $\varepsilon^2 \sim \mu^2$ and, consequently, by $r^2 \sim 1/\varepsilon^2 \sim 1/\mu^2$. A comparable contribution to (34) comes from the S-wave term $\propto \mu^2 K_0(\varepsilon r)^2$ in $\rho_{LL}$. In Eq. (35) the integral is dominated by $z \lessgtr m^2/Q^2$, corresponding to $\varepsilon^2 \sim m^2$. Therefore, $r^2 \sim 1/\varepsilon^2 \sim 1/m^2$. Thus, we conclude that the typical dipole sizes which dominate $\sigma_3$ and $\langle \sigma_3^2 \rangle$ are very different. In Ref. [2] basing on the color dipole approach we found the scaling cross sections $\sigma_L$ and $\sigma_R \propto 1/Q^2$ times the Leading-Log scaling violation factors $\propto \log Q^2/\mu^2$ and $\propto \log Q^2/m^2$, respectively (see Eqs.(18,19)). The scaling violations were found to be (logarithmically) dominated by

$$r^2 \sim 1/Q^2.$$  

On the contrary, the contribution of small-size dipoles, $\sim 1/Q^2$, to $\langle \sigma_3^2 \rangle$, defined in Eq. (30), proved to be negligible. At $\lambda = -1$ $\langle \sigma_3^2 \rangle$ is dominated by large hadronic size $c\bar{s}$-dipoles, $r \sim 1/\mu$. Consequently,

$$\delta \sigma_L^A \propto 1/\mu^2. \quad (37)$$

At $\lambda = +1$ a typical $c\bar{s}$-dipole is rather small, $r \sim 1/m$, and $\delta \sigma_R^A$ is small as well:

$$\delta \sigma_R^A \propto 1/m^2. \quad (38)$$
Thus, there is a sort of filtering phenomenon, the target nucleus absorbs the $c\bar{s}$ Fock component of $W^+$ with $\lambda = -1$, but is nearly transparent for $c\bar{s}$ states with opposite helicity, $\lambda = +1$.

From Eq.(28) it follows that the shadowing correction to nucleonic $\Delta xF_3$ extracted from nuclear data is

$$\delta(\Delta xF_3) = \frac{Q^2}{4\pi^2\alpha_W} \frac{1}{A} \left( \delta\sigma^A_L - \delta\sigma^A_R \right). \quad (39)$$

To give an idea of the magnitude of the shadowing effect we evaluate the ratio of the nuclear shadowing correction, $\delta(\Delta xF_3)$, to the nuclear structure function of the impulse approximation, $A \cdot \Delta xF_3$,

$$R = \frac{\delta(\Delta xF_3)}{A\Delta xF_3} = \frac{\delta\sigma^A_L - \delta\sigma^A_R}{A\sigma_L - A\sigma_R}. \quad (40)$$

Hence, the nuclear shadowing correction $R \cdot \Delta xF_3$ which should be added to $\Delta xF_3$ extracted from the $\nu F e$ data to get the "genuine" $\Delta xF_3$. This correction positive-valued and does increase $\Delta xF_3$ of the impulse approximation. We calculate $R$ as a function of $Q^2$ for several values of Bjorken $x$ in the kinematical range of CCFR/NuTeV experiment. Our results obtained for realistic nuclear densities of Ref.[16] are presented in Figure 2. Shown is the ratio
$R(Q^2)$ for different nuclear targets including $^{56}Fe$. At small $x$ and high $Q^2$ the shadowing correction scales, $\delta\sigma_{L,R} \propto 1/Q^2$. The absorption cross section $\sigma_{L,R}$ scales as well. The ratio $\delta\sigma_{L,R}/\sigma_{L,R}$ slowly decreases with growing $Q^2$ because of the logarithmic scaling violation in $\sigma_{L,R}$. Toward the region of $x > 0.01$, both the nuclear form factor and the mass threshold effect suppress $R$ at $Q^2 \lesssim (m + \mu)^2$ (see Fig.2).

Summarizing, we developed the light-cone color dipole description of the left-right asymmetry effect in charged current DIS at small Bjorken $x$. We evaluated the contribution of the diffractive excitation of charm-strange Fock states of the light-cone W-boson to the structure function $\Delta x F_3 = x F_3^\nu - x F_3^{\bar{\nu}}$ and compared our results with experimental data. Theory is in reasonable agreement with data. We presented the color dipole analysis of nuclear effects in charge current DIS. The emphasis was put on the pronounced effect of left-right asymmetry of shadowing in neutrino-nucleus DIS at small values of Bjorken $x$. We predicted strikingly different scaling behavior of nuclear shadowing for the left-handed and right-handed $W^\pm$. Large, about $20 - 25\%$, shadowing in the $Fe$ structure functions is predicted, which is important for a precise determination of the nucleon structure functions $xF_3$ and $\Delta x F_3$.

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