Injector-coupled transverse instabilities in a multi-element premixed combustor

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Abstract
Combustion instabilities in a high-pressure, multi-element combustor are studied in order to understand the relationship between the chamber and injector dynamics. A linear array of seven injectors supplies premixed natural gas and air into a rectangular combustion chamber designed to promote high-frequency, transverse thermoacoustic instabilities. The effect of equivalence ratio on the combustion dynamics was investigated for two injector lengths, 62.5 and 125 mm. For all operating conditions, the 125 mm injectors promote high-amplitude instabilities of the fundamental transverse (1T) mode, which has a frequency of 1750–1850 Hz. Reducing the injector length significantly lowers the instability amplitudes for all operating conditions and, for lower equivalence ratio cases, excites an additional mode near 1550 Hz. The delineating feature controlling the growth of the instabilities in each injector configuration is the coupling with axial pressure fluctuations in the injectors that occur in response to the transverse modes in the chamber.

Keywords
Combustion instabilities, self-excited, high-frequency, transverse, multi-element, high-pressure, gas turbine

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1. Introduction
Modern premixed combustion systems operate at continuously higher turbine inlet temperatures and power densities in order to increase the overall cycle efficiency while also meeting stringent NOx emission standards. At these conditions, the combustor is prone to a broader range of thermoacoustic instabilities than previous designs, which presents additional barriers to engine development. Modes oriented in the transverse direction relative to the mean flow through the engine are being observed more often, along with longitudinal modes that have been the focus of extensive characterizations. These additional modes introduce multi-dimensional acoustic effects that generally occur at higher frequencies than those caused by longitudinal modes in gas turbine combustors. While high-frequency (HF) dynamics have been widely studied in rocket engines, research on transverse instabilities and the mechanisms that cause them in gas turbine applications is relatively recent.

Multiple studies have been conducted to investigate the response of a flame and flow-field subjected to transverse acoustic waves. In the presence of transverse forcing, the flame response has been shown to depend on the location of the flame relative to the acoustic mode shape. Within the framework of standing pressure waves, a flame located at a pressure node has an asymmetric pressure field on either side of the flame, resulting in the formation of helical vortex structures. Whereas, a flame located at a pressure antinode has a symmetric pressure field across the injector outlet, creating symmetric ring vortex shedding. In addition to the location of the flame relative to the transverse pressure field, the size of the flame relative to the transverse mode and the confinement of the flame in the combustion chamber can also alter flame–acoustic interactions. When a flame is acoustically compact, the variation in pressure can be assumed to be constant over the width of the flame. In this case, with a flame located at a pressure node, the asymmetry of the heat...
release results in a weak contribution to the global, unsteady heat release. However, a flame located at a pressure anti-node has symmetric heat release, which can cause global heat release fluctuations.\textsuperscript{13,14}

Azimuthal, tangential, and radial modes can all be classified as transverse instabilities. Azimuthal modes are a spin-type instability, which have a continuous mode shape, whereas tangential modes are bound by two end points that generally occur at the combustor walls. Azimuthal modes are common in annular combustors, and they have been accurately described with an acoustically compact flame assumption.\textsuperscript{15} In can combustors, both azimuthal and tangential modes can occur, and depending on the given geometry, these transverse modes can be either compact or non-compact. In both systems, radial modes occur at higher frequencies and are seldom encountered.\textsuperscript{16}

Multiple pathways exist for transverse pressure oscillations to result in unsteady heat release, and these mechanisms can depend on whether the flame is compact or non-compact.\textsuperscript{17} A flame in a transverse pressure field can be directly excited by acoustic disturbances through mechanisms such as flame displacement and flame wrinkling. A model for these mechanisms has been developed using a single-element swirl burner at T.U. Munich in which the flame is acoustically non-compact.\textsuperscript{18} In the presence of transverse instabilities, the flame location is shifted by the acoustic velocity. This results in the flame being displaced in phase with the maximum transverse pressure, driving the instability.\textsuperscript{19,20} Transverse instabilities also cause local density fluctuations that can modulate the heat release via compression and expansion of the reaction zone, which again occurs in phase with the acoustic pressure. The local flame wrinkling and displacement resulting from transverse forcing of the shear layer can also change the heat release by modifying the flame surface area, but these smaller scale fluctuations do not necessarily occur in phase with the acoustic pressure.\textsuperscript{21,22} The local heat release profile, which directly impacts the local Rayleigh integral, determines which of these mechanisms play a dominant role in the driving of transverse instabilities.\textsuperscript{23–25}

Transverse to longitudinal coupling is another pathway that has been widely studied. In this mechanism, the fluctuating transverse pressure field in the combustion chamber causes axial pressure disturbances at the end of the injector. If the injector is located at a pressure anti-node, this results in variations in the mass flow rate and consequent heat release fluctuations at the injector exit. The transverse pressure oscillations in the chamber are often referred to as the “clock” that sets the frequency at which the instability occurs because the induced injector dynamics lead to heat release fluctuations.\textsuperscript{12,26} Large eddy simulations of an annular combustor by Staffelbach et al.\textsuperscript{27} show that two counter-rotating pressure waves cause mass flow rate fluctuations in the individual injectors, driving instability. Similar results have been obtained in laboratory-scale experiments, where stronger heat release fluctuations have been observed near pressure anti-nodes than near pressure nodes, indicating axial dynamics in the injector are primarily responsible for the instability.\textsuperscript{28–32} These studies show that the dominant transverse instability coupling mechanism can change depending on the configuration. For this reason, it is important to study the effect of transverse instabilities on single flame burners as well as multiple injector configurations, which more closely resemble fielded combustor designs.

Combustion processes in engines have much higher levels of turbulence and power density than laboratory-scale experiments, which are typically performed at or near atmospheric pressure.\textsuperscript{33} Lammel et al.\textsuperscript{34} state that a specific thermal power density of at least 10 MW/m\textsuperscript{2}-bar is typical of lean premixed combustors for power generation. At engine-relevant conditions, the range of length and time scales of the flow and chemical processes broadens, which allows for additional pathways through which the flame can be modulated by acoustics. Therefore, it is important to study how the transverse coupling mechanisms and flame response change as operation approaches realistic engine conditions. One of the few laboratory-scale experiments investigating HF instabilities in a gas turbine configuration at elevated pressure was performed by Buschhagen et al.\textsuperscript{35,36} This work characterized self-excited, transverse instabilities in a single-element, premixed jet flame that was acoustically non-compact. Changes to the overall flame shape and jet hydrodynamics were shown to influence the ability of flame-vortex coupling to drive transverse instabilities.

The objective of this paper is to investigate the dominant coupling mechanisms for transverse instabilities in a multi-element combustor operating at elevated pressure. Multi-element combustor designs support the study of inter-element interactions and their impact on transverse instabilities with acoustically compact flames. This allows for comparison with single-element experiments, which typically have non-compact flames with respect to the transverse acoustic wavelength and intra-element dynamics that dominate the flame–acoustic interactions.\textsuperscript{35} For this purpose, a canonical experiment was developed and tested up to 825 kPa with injection Reynolds number of $O(10^5)$. The resulting specific power density was 25–33 MW/m\textsuperscript{2}-bar (based on the combustor cross-sectional area), which is similar to the maximum value of 23 MW/m\textsuperscript{2}-bar reported by Lammel et al.\textsuperscript{34} Variation of the injector length resulted in multiple self-excited instability
regimes that are studied using high-speed CH$^*$ chemiluminescence and pressure measurements.

2. Experiment description

2.1. Combustor design

The VIPER–M (Versatile Intermediate Power Experimental Rig–Multi-element) experiment has a rectangular combustion chamber fed by a linear array of seven injector elements and operates with premixed preheated, non-vitiated air and natural gas. The design maintains similarities with several other rocket and gas turbine combustion experiments studied at Purdue University. VIPER–M was designed to promote self-excited, transverse instabilities while operating at a mean chamber pressure up to 825 kPa with an inlet air temperature up to 510 K. The multi-element configuration supports jet-stabilized combustion with flames anchored at the injector-exit plane (\(y = 0\)), as labeled in Figure 1. This injector configuration is similar to the FLOX$^*$ combustion experiments, which have been widely studied.

The combustion chamber has cross-section dimensions of 238 mm and 30 mm in the \(x\)- and \(z\)-directions, respectively. In the flow (\(y\)) direction, the cross-sectional area is constant for the first 115 mm and is followed by a 92 mm long converging section where the chamber aspect ratio is held constant. The length of the chamber was designed to offset the expected longitudinal and transverse mode frequencies. An odd number of injectors was chosen in order to locate the center injector at a pressure node of the fundamental transverse (1T) mode. The seven identical injectors, which are numbered in Figure 1, are cylindrical, with a 17 mm diameter. The distance between the centerlines of adjacent injectors is 34 mm, and the depth of the combustion chamber (\(z\)-direction) is 30 mm. This ensures that the confinement of the flames is similar in both spanwise (\(x\)- and \(z\)-) directions since the inter-element separation is comparable to the chamber depth.

The injectors were designed for a nominal operating condition with an inlet gas temperature (\(T_{inj}\)) of 500 K, an equivalence ratio (\(\phi\)) of 0.70, and a mean chamber pressure (\(p_c\)) of 800 kPa. The total reactant mass flow rate (\(\dot{m}\)) is 0.89 kg/s and is distributed equally between the injectors. The resulting Mach number in the injectors is 0.24 with a Reynolds number of \(4 \times 10^5\). This corresponds to a jet velocity of 108 m/s at the exit of the injectors, which is within the range of values used in similar jet-stabilized combustion experiments.

Based on the combustion product gas properties for these inlet conditions and the chamber width, the expected fundamental transverse mode frequency is 1860 Hz. In the first injector configuration, the injector length (\(L\)) was designed to support a half-wave pressure mode shape in the axial direction at the 1T frequency. This resulted in an injector length of 125 mm.

The presence of the expected transverse mode shape for this design condition was verified by solving the inhomogeneous Helmholtz equation in the frequency domain using COMSOL Multiphysics$^*$. The temperature in the

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**Figure 1.** Schematic diagram of VIPER–M combustor. FOV: field-of-view; HF: high-frequency; LF: low-frequency.
injected (T_{inj}) was set to the inlet gas temperature, and
the temperature in the chamber was assumed to be uni-
form at the adiabatic flame temperature (T_{ad}) with a step
change at the injector-exit plane. The small pressure drop
at the exit of the injectors was neglected, so the pressure
was 800 kPa throughout the combustor. A rigid wall
condition was applied to all boundaries. The computed
mode shapes are shown in Figure 2. For the 1T
(1860 Hz) mode in Figure 2(a), a pressure anti-node
occurs at the entrance and exit of all the injectors
except the center. This promotes a pressure-coupled
response at the entrance of the combustion chamber.
The second transverse (2T) mode, occurring at 3640 Hz
as depicted in Figure 2(b), shows this injector length also
supports harmonics of the fundamental transverse mode.

In order to decouple the injector acoustic response
from the fundamental transverse mode of the chamber,
a second injector configuration was designed with shorter
injectors, each 62.5 mm in length. Halving the length of
the injectors changes the supported pressure mode shape
in the axial direction in the injectors from a half-wave to
a quarter-wave at the combustion chamber 1T frequen-
cy. A schematic representation of the expected 1T mode
shape at 1860 Hz for this configuration is illustrated in
Figure 2(c). In contrast, the mode shape computed using
the COMSOL® model with the shortened injector
length, as shown in Figure 2(d), occurs at a frequency
of 1550 Hz and does not present the expected pressure
nodes at the injector-exit plane. The simplistic assump-
tions in the model, including uniform chamber tempera-
ture, a lack of mean flow, and a lack of heat release
model, may explain the differences between the expected and computed mode shape. However,
the results do show that shortening the injector length
affects the pressure field in the combustor. The exper-
imentally measured pressure mode shapes will be dis-
cussed in detail in subsequent sections. The 125 and
62.5 mm injector length configurations are referred to
as Configuration A and Configuration B, respectively,
for the remainder of the paper. As seen in Figure 1, an
injector block is installed for Configuration A, but it can
be removed to achieve the shorter injector length of
Configuration B.

Premixed operation is achieved by mixing preheated
air and natural gas upstream of the injector inlets in an
“external premixer” not pictured in Figure 1. In order
to isolate the mixing process from the chamber dynam-
es, each injector is supplied via nine, 2.44 mm diameter
choked orifices. The orifices were sized to establish a
pressure ratio between the external premixer and the
combustor of ∼2.5 so that they remain choked even in
the presence of high-amplitude instabilities. The
choked flow also provides a well-defined inlet acoustic
boundary condition for the injectors and ensures that
the mass flow rate into each injector is equal and
steady. The average injector pressure drop was 2%–
4% for all tests. The choked combustor throat at the
end of the converging section in the chamber allows for
system pressurization and provides a well-defined exit
acoustic boundary condition. All metal surfaces
exposed to the combustion products are thermal barri-
er coated to reduce heat transfer to the chamber walls.
The heat-sink stainless steel combustor design is pro-
vided with no water or film cooling, which limits a
typical test duration to 1–5 s. The application of ther-
mal barrier coating and the absence of active cooling,
which is typically present in gas turbine combustors,
limits potential loss mechanisms that could damp the
growth of combustion instabilities.

2.2. Operation and diagnostics

The flow rates of all reactants entering the combustor
are metered using sonic nozzles with electronically reg-
ulated upstream pressure. Low-frequency (LF) pres-
sure transducers and K-type thermocouples, sampled
at 500 Hz, are used to monitor the flow rates and
combustor operating parameters. Locations of these
sensors are highlighted in Figure 1. The relative uncer-
tainty of the calculated mass flow rates, computed
using the Kline–McClintock method of uncertainty
propagation, is no greater than 0.8% with a 95% con-
fidence interval.43 This includes the precision and bias
uncertainty of the measured temperature and pressure as well as the uncertainty of the sonic nozzle throat diameters and discharge coefficients. A typical test lasts approximately 1.0 s, during which the reactant mass flow rates and temperatures are held constant. Ignition of the main chamber reactants is achieved using a spark-ignited, hydrogen–oxygen torch located on a chamber end wall, as shown in Figure 1. The torch igniter mass flow rate is 1.25% of the total main chamber propellant mass flow rate. It is extinguished 0.5 s into the test, which allows the main chamber to achieve a robust state of operation. Combustion is terminated by replacing the fuel flow with an inert gas purge. The resulting test duration is sufficient for the chamber to reach limit-cycle behavior.

Water-cooled, HF pressure transducers (Kulite WCT312M-17/35BARA) are located at various positions throughout the combustion chamber as well as in the center and outermost injector elements. The corresponding designations for each transducer used in the forthcoming analysis are provided in Figure 1. An additional transducer not shown in Figure 1 is located in the external premixer. All HF transducers are installed in a recessed cavity with a port resonance greater than 10 kHz and sampled at 180 kHz.

OH* and CH* are commonly used as qualitative markers of global heat release in combustion diagnostics. For the purposes of this work, the change in chemiluminescence intensity is assumed to be proportional to the heat release rate from the flame. Line-of-sight integrated CH* chemiluminescence images are recorded at 75 kHz through a window in one side of the combustion chamber. The field-of-view (FOV), indicated in Figure 1, is 82 mm in the x-direction and 77 mm in the y-direction. The spatial resolution of the collected images is 0.249 mm/pixel. The emission signal of the A2Δ → X2Π transition is isolated from background light by using a filter centered at 434 nm with a 17 nm bandwidth (Semrock 434/17 Brightline Bandpass). The signal is then captured through a 200 mm focal length, f/4 lens (Nikon AF Micro-Nikkor), amplified by a Lambert HiCATT 25 intensifier with a 1:1 relay lens, and recorded with a Phantom v2512 high-speed CMOS camera.

### Table 1. Summary of test conditions.

| Case | L (mm) | \( \dot{m}_{\text{air}} \) (kg/s) | \( \phi \) | \( p_c \) (kPa) | \( T_{\text{inj}} \) (K) | \( T_{\text{ad}} \) (K) | \( p \) (MW/m²-bar) | Dominant mode \( f \) (Hz) | \( p_{\text{p-p}}/p_c \) (%) |
|------|--------|-------------------------------|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| A-1  | 125.0  | 0.85                          | 0.74   | 720            | 487            | 2048          | 30.0           | 1830           | 124.2          |
| B-1  | 62.5   | 0.86                          | 0.70   | 736            | 502            | 1989          | 27.8           | 1555           | 7.0            |
| B-2  | 62.5   | 0.85                          | 0.93   | 812            | 457            | 2290          | 31.6           | 1835           | 5.0            |

### 3. Results and discussion

The directional sensitivities of combustion instability to equivalence ratio from lean blow-out limit to stoichiometric composition were tested for the two injector configurations at air preheat temperatures of 450–505 K. Three representative cases that present discriminating dynamics are discussed in detail: one for Configuration A and two for Configuration B. The flow conditions for each case are provided in Table 1.

#### 3.1. Overview of operating conditions

Figure 3(a) depicts the HF pressure measured at the chamber end wall transducer (C-1) for the duration of Cases A-1 and B-2. The HF pressure is nondimensionalized by the mean chamber pressure (\( p_c \)) determined from the LF instrumentation. Stationary pressure amplitudes are achieved within 30 ms of ignition for both tests, but, as shown in Figure 3(b), Case A-1 undergoes a period of linear growth to a high-amplitude limit-cycle while the pressure fluctuations stay at low amplitudes for the duration of Case B-2. In order to quantify the instability amplitude, the probability density function (PDF) of the instantaneous pressure fluctuation magnitudes was calculated for a 0.2 s period of the test as denoted by the dashed vertical lines in Figure 3(a). This period begins 0.5 s after ignition, at which point the torch igniter is off. A second-order Butterworth high-pass filter with a cutoff frequency of 60 Hz was applied to the pressure data before generating the PDFs. The pressure fluctuation distribution for the two cases is shown in Figure 3(c). The distribution for Case A-1 is multimodal and left-skewed, which is characteristic of steep-fronted pressure waves because more samples are recorded in the expansion phase of the cycle compared to the compression phase. The histogram also shows two peaks in the positive part of the distribution, which suggests multiple dynamic processes occur with different amplitudes. For Case B-2, the distribution appears Gaussian, so further analysis is required to distinguish coherent, periodic pressure oscillations from broadband combustion noise.

The vertical lines in Figure 3(c) correspond to the 1% and 99% cumulative bin edges. These were used to determine the peak-to-peak pressure fluctuation...
amplitude \( \left( \frac{p'_0}{\nu} \right) \) for the Configuration B data. For Configuration A, in order to accurately capture the amplitude of the steep-fronted waves, the difference between the peak pressure and minimum pressure for each acoustic cycle in the 0.2 s time period was calculated. These values were then averaged to determine the \( \frac{p'_0}{\nu} \). The uncertainty of the resulting pressure fluctuation amplitudes for Configuration A is less than 5% with a 95% confidence interval. This accounts for the precision and bias uncertainty of the measured pressure, but it does not include possible amplification or attenuation of the measured peak pressure due to the installation of the pressure transducer in a recessed cavity port. As reported in Table 1, for Case A-1, the resulting \( \frac{p'_0}{\nu} \) is 124%, while for Case B-2, the \( \frac{p'_0}{\nu} \) is 5%.

The change in pressure fluctuation amplitude with a change in injector length is consistent for all of the tested conditions, as shown in Figure 4. The range of peak-to-peak pressure fluctuation amplitudes for Configuration A is 115%–130% of the mean chamber pressure, whereas the range for Configuration B is 4%–9%. No trend in instability level as a function of equivalence ratio is observed for either injector configuration. In contrast, for a combustor with a single, non-compact flame with an injector design similar to the current experiment, Buschhagen et al. showed that the dominant excited mode, either axial or transverse, and the amplitude of the instabilities were sensitive to the operating condition. The change in the local interaction between the intrinsic jet hydrodynamics and the flame resulted in this sensitivity. In the present multi-element configuration, the lack of influence of the operating condition on the combustor stability indicates that, in addition to the local dynamics, a system-wide response related to the injector length must play a role in the different instability amplitudes observed in the two configurations.

The time-averaged \( \text{CH}^* \) chemiluminescence images for Case A-1 and Case B-2 are shown in Figure 5, with dimensions normalized by the injector diameter \((D)\). The mean flame structure of the visible injector elements reveals significant differences between the two instability regimes. For Case B-2, which exhibits low instability amplitudes, distinct flames are present, extending beyond the FOV. The flames are stabilized in the jet shear layers with axially distributed heat release and minimal interaction between neighboring injector elements. Conversely, for Case A-1, the region of strongest heat release is much closer to the injector-exit plane. The high pressure fluctuation amplitudes in this case result in significant flame–flame interactions, leading to intense heat release in the recirculation zones between the injector jets.
3.2. Configuration A dynamics

Results from a representative test condition for Configuration A illustrate the dynamics that lead to high-amplitude instabilities with the 125 mm injector length. Figure 6(a) shows the HF pressure trace for a short time period at four spanwise locations in the combustion chamber, as labeled in Figure 1. At each location, the pressure is nondimensionalized by the average HF pressure ($\bar{p}$) for the 0.2 s period of the test shown in Figure 3(a). The signals were high-pass filtered at 60 Hz, and then the corresponding power spectral density (PSD) was calculated using the fast Fourier transform. The resulting power spectra are presented in Figure 6(b). The highest energy peaks in the PSD at the chamber end wall locations (C-1 and C-4) occur at 1830 Hz, which matches the expected 1T frequency in the combustion chamber. The steep-fronted pressure waves in the pressure trace and the high-amplitude, narrow peaks on the spectra at multiple harmonic frequencies are evidence of highly nonlinear dynamics in the chamber. It should be noted that the HF pressure transducer in the external premixer shows no frequency content at the 1T mode, indicating the injector inlets remain choked even in the presence of the large amplitude pressure fluctuations. While the amplitude of instabilities observed in this case significantly exceeds typical values in gas turbine combustors, it highlights the potential for the growth of instabilities through flame-acoustic coupling mechanisms at high thermal power densities and in the presence of limited loss mechanisms.

The pressure time history at two locations in one of the combustor side injectors (Injector 1 in Figure 1) shows the injector behavior in the presence of the transverse modes in the chamber. The pressures in the chamber (C-1) near the injector exit, slightly downstream of the injector axial center (P-1), and just downstream of C-4) and are 180° out of phase. Additionally, at the chamber center (C-2), the largest magnitude in the PSD is observed at 3670 Hz, which corresponds to the 2T mode. There is only a weak peak at 1830 Hz in the PSD, indicating the chamber center is at a pressure node for the 1T mode. Tracking the highest pressure at the chamber center (C-2) and off-center (C-3) locations shows that the pressure peaks traverse the chamber and reflect off of the chamber end walls. The pressure peaks at the chamber end walls are also followed a quarter cycle later by a lower amplitude, secondary peak. The secondary peaks indicate that dynamic processes occur at additional frequencies, so the numerous high energy frequencies in the PSDs are likely a result of not only the nonlinear nature of the instability, but also the presence of multiple harmonics of the 1T mode in the combustor.

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the injector inlet (P-3) are plotted in Figure 7. The pressure peaks in the injector appear to follow the primary and secondary pressure peaks in the chamber. Unlike in the combustion chamber, where two peaks in pressure occur at the transducer locations near the chamber center for each peak at a chamber end wall, there is only a single pressure peak per acoustic cycle at both locations in the injector. This indicates that the pressure waves originate in the combustion chamber and travel up into the injectors but do not reflect strongly off of the injector inlets. This is likely a result of the pressure waves being damped by the high-speed, under-expanded jets exiting the choked orifices at the injector inlets. The average measured static pressure at P-3 is 100 kPa less than the mean chamber pressure, which is evidence of the presence of high-speed flow. Therefore, while both the combustion chamber width and injector length correspond to half waves of the 1T mode frequency, their dynamics at this frequency are different.

Singular spectrum analysis (SSA) was employed to understand the instability characteristics present in the combustion chamber and injectors at specific, dominant frequencies. SSA is an energy-based decomposition technique that can isolate fluctuating, periodic components from a short, noisy signal. The original signal can then be reconstructed with all (or some fraction) of the decomposed modes while maintaining the phase relationship between various signals. The phase difference between similar frequency modes from two different signals was calculated from the difference between the arguments of the Hilbert transform. The reconstructed pressure signals and phase differences make it possible to infer the combustor mode shape at a given frequency. For Case A-1, the two highest energy SSA mode pairs are 1830 and 3655 Hz, which correspond to the 1T and 2T frequencies. The results are shown in Figure 8, with the signals colored based on the transducer locations depicted on a schematic of the combustor.

The reconstructed pressure signal for one cycle of the 1830 Hz mode from SSA is shown in Figure 8(a) for several transducer locations. The C-2 and P-5 signals are located at a pressure node for this mode and, therefore, do not have a corresponding SSA mode at this frequency. The phase differences at the other locations relative to the C-1 pressure transducer are shown in Figure 8(b). The longer time scale on this plot shows that the phase differences remain constant for the entire period of the test being analyzed. In the combustion chamber, the end wall reconstructed pressure signals, C-1 and C-4, are of equal magnitude and 180° out of phase, which is expected for the fundamental transverse mode with pressure anti-nodes located at the end walls. The pressure signal at C-3 is also 180° out of phase with the pressure at C-1 and has slightly lower amplitude, which is characteristic of a standing 1T mode in the combustion chamber.

In the injectors, a pressure anti-node is expected at both the injector inlet and exit since the injector length was scaled to support a half-wave pressure mode at the 1T frequency (see Figure 2(a)). The P-3 transducer, which is located 12 mm downstream of the Injector 1 inlet, is 125° out of phase with the C-1 transducer, lagging it by 235°. Mean flow effects cannot be neglected near the injector inlet because of the high-speed flow exiting the choked orifices. The phase difference is consistent with a wave traveling upstream into the injector but being slowed down by increasing mean flow approaching the injector inlet. The pressure at P-1, which is slightly downstream of the axial center of the injector, supports this observation. The P-1 transducer is 80° out of phase with C-1, which corresponds to the expected phase lag for a traveling wave based on the speed of sound in the injectors. If a standing wave was present, this phase difference would be near 0°. While the injectors show a strong response at the 1T frequency, it appears to result from traveling pressure waves and not the expected standing wave mode.

Injector Configuration A is also able to support multiple harmonic frequencies of the fundamental transverse mode. Figure 8(c) shows the 3655 Hz mode pressure data for the same period of time as the reconstructed 1T signals. In contrast to the 1T frequency, SSA extracts modes at the 2T frequency along the transverse centerline of the combustor because it is no longer a pressure node. As shown in Figure 2(b), pressure nodes are now expected to occur in Injectors 2 and 6 and in the chamber immediately downstream of these injectors. The reconstructed signals for the 3655 Hz mode generally have a p' amplitude that is about half that of the reconstructed 1T pressure. While the 2T mode adds less to the overall instability amplitude than the 1T mode, its contribution is still significant.
The phase differences relative to the C-1 and C-2 transducers at the 2T frequency are plotted in Figure 8(d). The chamber end wall pressure signals have a negligible phase difference because they are located at anti-nodes with two nodes in between. The two nodes cause the pressure at the center of the chamber (C-2) to be $180^\circ$ out of phase with the chamber end walls. The pressure amplitude at C-3 is very low relative to the other signals, confirming that a pressure node occurs downstream of Injector 6. Since the C-3 transducer is located on the left edge of Injector 6, it should be in phase with the pressure at C-2 for the 2T mode. The phase at C-3 relative to C-1 is close to $180^\circ$, but there is more variation compared to the other locations in the chamber, which is likely due to the close proximity to a pressure node. These phase relationships indicate that the high-amplitude dynamics of the 1T mode cause additional harmonics to be excited.

The injectors also respond to the harmonics occurring in the combustion chamber. For the 2T mode, the pressure near the axial center of Injectors 1 and 4, at P-1 and P-5 respectively, is almost entirely out of phase with the pressure in the chamber downstream of these injectors. This shows that a pressure anti-node is now present at the middle of all of the injectors, except Injectors 2 and 6. It is important to note that the magnitude of the phase difference is plotted. The reconstructed pressure signals at P-1 and P-5 show that the pressure in the outer injectors is out of phase with the pressure in the center injector, as expected from Figure 2(b). For the 2T mode, the pressure near the outer injector inlets should be in phase with the pressure at the chamber end walls. The increased mean flow velocity near the injector inlets shifts the phase at P-3 to $90^\circ$ out of phase with C-1. Although the phase near the injector inlets is difficult to interpret due to the mean flow effects, the pressure in the middle of the injectors closely resembles standing wave behavior for the 2T mode, as opposed to the 1T mode, which appears as a traveling wave in the injectors.

Reconstructing the pressure signal with a combination of the SSA modes at the first four transverse frequencies clarifies the effect of exciting multiple harmonics in the combustion chamber. As seen in Figure 9, the summation of the signals from the first four transverse modes nearly replicates the primary peaks as well as the secondary peaks observed in the original pressure data in Figure 6(a). The secondary peaks are a result of the combustor and the injectors responding to the harmonics of the fundamental frequency, causing additional fluctuations in pressure to occur. Scaling the injector length to correspond to a half-wave mode at the 1T frequency enables these multiple harmonics in the chamber to be sustained and grow to very high amplitudes. Pressure waves originating in the chamber and propagating up into the outer injectors are able to reflect off of the injector inlets and return to the combustion chamber in phase with the subsequent pressure wave. Although it has been shown that there is only weak reflection off of the injector inlets in this configuration, the nonlinear pressure fluctuations are high enough to temporarily stop the flow from exiting the injectors, even with a constant

Figure 8. Reconstructed 1830 Hz mode: (a) pressure fluctuation time series and (b) phase differences. Reconstructed 3655 Hz mode: (c) pressure fluctuation time series and (d) phase differences. Transducer locations are shown for reference.
mass flow rate entering the injectors. This leads to a pressure rise in the injectors, corresponding to an accumulation of reactants in the injectors, which then arrives in the combustion chamber in phase with the local maximum pressure. The strong response of the outer injectors to the 1T mode dynamics enables multiple harmonics to be excited and causes the instabilities to grow to very large amplitudes.

The CH* chemiluminescence images highlight the response of the flames to the chamber and injector dynamics. Figure 10(a) shows a series of instantaneous emission images for two segments of a single cycle of the 1T mode. Although the flow downstream of Injectors 4, 5, and part of 6 is visible in the FOV, it is difficult to distinguish between the combustion products originating from the different injectors. The chemiluminescence signal is dominated by a region of high intensity near the injector-exit plane, outlined by the dashed rectangles in Figure 10(a). In the same way that multiple pressure modes combine to form a steep-fronted wave, the unsteady heat release driven by the pressure harmonics coalesces into a single, compact reaction zone. This region traverses the width of the combustor at the speed of sound. For the first half of a cycle of the 1T mode, the area of maximum heat release is shown to move in the negative x-direction, as seen in the left column of images in Figure 10(a). In the second half of the 1T cycle, portrayed by the images in the right column, after the pressure is reflected off of the chamber end wall, the heat release moves in the positive x-direction. Relatively little combustion is observed in the region after the heat release wave passes the injectors, as evidenced by the low levels of chemiluminescence intensity.

The CH* intensity in a 12 × 12 pixel box around the C-2 pressure transducer port, denoted by the white square in Figure 10(a), is averaged and plotted along with the instantaneous pressure in Figure 10(b). For reference, the dashed vertical lines are located at the times of the images shown in Figure 10(a). The strongest peaks in both the pressure and intensity occur at the 2T frequency, but unlike the pressure signal, no secondary peaks in heat release are observed. The maximum heat release lags the maximum pressure by less than a quarter cycle of the 1T mode, as indicated by the upper x-axis in Figure 10(b). The steep-fronted pressure waves induce strong transverse velocity fluctuations, which compress the reaction zones and significantly distort the flame surface area. This leads to an increase in heat release following a short time delay.

The almost complete lack of heat release following the intense combustion shows a buildup of reactants that are rapidly consumed when the next pressure wave arrives. The response of the injectors to the strong transverse pressure waves in the combustion chamber makes it possible for this mechanism to occur. This is consistent with other experiments studying transverse instabilities with acoustically compact flames, which show the axial acoustics induced in the injectors as a result of transverse pressure fluctuations in the chamber are a primary driving mechanism. Any change to the flame dynamics with the varying equivalence ratio and the corresponding changes to the local transverse excitation methods, such as flame displacement, are small relative to the wavelength of the 1T mode and do not appear to have a strong influence on the overall dynamics. This explains the consistent instability levels across all operating conditions, as shown in Figure 4. The following results from Configuration B confirm that the injector coupling is the delineating feature leading to much higher levels of instability for Configuration A.

### 3.3. Configuration B dynamics

Comparing two different cases for the 62.5 mm injector configuration elucidates the dynamics that cause all of the conditions in Configuration B to have significantly lower instability amplitudes compared to Configuration A. Additionally, while changing the operating condition did not affect the range of stability amplitudes in either Configuration A or B, it did result in two distinct pressure modes being excited for Configuration B. A summary of the two test conditions is presented in Table 1. The nondimensionalized HF pressure at both the chamber left wall (C-1) and chamber right wall (C-4) is plotted for Case B-1 and Case B-2 in Figure 11(a). The signals are plotted beginning at the same time post-ignition in each case. Unlike Configuration A, which exhibited a steep-fronted pressure response with a clear frequency, periodic content is not immediately apparent.
in either of the raw pressure time series from Configuration B. There is also no clear phase relationship between the pressures at the chamber end walls.

For each case, the PSD of the pressure at the chamber left wall was calculated for a portion of the test equivalent to the analysis for Configuration A. As shown in Figure 11(b), the PSDs reveal narrowband frequency content is present in the noisy pressure signal. The sharpest peaks for Case B-1 and Case B-2 occur at two different frequencies, 1555 and 1835 Hz, respectively. Compared to the spectrum for Case A-1, which had sharp peaks at only the 1T mode and its harmonics, the dynamics in Configuration B are much less organized, with spectral power distributed across multiple frequencies. There is a peak around 400 Hz, which is a bulk mode of the combustor. At this frequency, the pressure oscillations occur in phase throughout the combustion chamber and in all of the injectors. This mode does not originate from the upstream supply system, which is evidenced by the absence of distinct frequency content in the external premixer at this frequency. The weak peak for Case B-2 around 5.3 kHz may correspond to the second harmonic of the injector, assuming it is a quarter-wave mode. Additionally, there is strong, broadband frequency content between 7 and 8 kHz, but this is not observed in the chemiluminescence signal. In the raw pressure data, especially for Case B-2, these higher frequencies are the easiest to discern. While many distinct dynamic processes are occurring, the peaks at the frequencies of 1555 and 1835 Hz differentiate the two cases.

For all of the tests performed in Configuration B, the PSDs show modes around both the 1550 Hz

**Figure 10.** (a) Instantaneous CH* chemiluminescence images for a different period of the 1T mode cycle in each column. (b) Corresponding time series from simultaneous CH* and HF pressure measurements at the C-2 transducer location plotted relative to the 1T mode phase (ψ) based on the C-1 transducer.

**Figure 11.** Chamber high-frequency pressure (a) time series segment and (b) PSD for Case B-1 and Case B-2.
frequency and the expected 1T frequency. These two modes appear simultaneously but with varying levels of power relative to each other. The power of each of these modes was determined by integrating the strongest peaks near 1550 and 1800 Hz in each PSD across the full width at half maximum. The results are shown in Figure 12, where the relative 1T mode power is the power of the 1T mode normalized by the total power of the two modes. If the relative 1T mode power was greater than 50%, the peak near the expected 1T frequency was determined to be the dominant mode frequency. Whereas, if the relative 1T mode power was less than 50%, the peak near 1550 Hz was set as the dominant mode frequency. This parameter is plotted as a function of the adiabatic flame temperature because the speed of sound and, thus, the expected frequency are directly dependent on it. The adiabatic flame temperature also accounts for differences in the inlet air temperature, equivalence ratio, and chamber pressure between tests.

A clear trend in the preferred mode as a function of the operating condition is seen in Figure 12. At higher equivalence ratios, where the adiabatic flame temperature is higher, the 1T mode frequency is favored, whereas at leaner conditions, the lower frequency 1550 Hz mode is preferred. There is also a slight increase in frequency for each mode as the adiabatic flame temperature rises, but the change between the 1550 and 1800 Hz modes is too large to be attributed to the differences in the speed of sound. The overlap between dominant modes for $T_{ad}$ between 2000 and 2200 K is due to both modes being present with often similar amplitudes. The relative mode power for each mode is generally higher for flame temperatures away from this region. The two cases being discussed in detail, as labeled in Figure 12, have comparatively high relative mode powers for the respective modes. These tests are analyzed further to illustrate the differences in the mode dynamics and the reason the 1550 Hz mode strengthens at lower power conditions.

Due to the large amount of noise in the pressure signals for Configuration B, multivariate singular spectrum analysis (m-SSA) was used to extract the highest energy modes from the HF pressure signals for each case. m-SSA is similar to SSA, except the signals from all measurement locations are processed together, resulting in a single set of data-adaptive eigenvectors that can be used to reconstruct the individual signals. The highest energy m-SSA mode pair for Case B-1 has a frequency of 1570 Hz, while the mode frequency for Case B-2 is 1840 Hz. The reconstructed pressure signals in each outer injector as well as at the chamber end walls are shown in Figure 13(a) for Case B-1 and in Figure 13(d) for Case B-2 for a 50 ms portion of each test. For both modes, the $p'$ amplitudes vary over the duration of the test, with maximum values slightly larger than 10 kPa in the injectors. This is significantly lower than the 1T mode amplitudes in Case A-1, which were as high as 200 kPa. Additionally, the pressure fluctuation magnitudes for the Configuration A results were relatively constant over the test duration, whereas the change in pressure amplitude for Configuration B indicates non-stationary behavior of the two dominant modes. Figure 13(b) and (e) shows a segment of the reconstructed pressure data from a period in each case when the pressure fluctuation amplitudes are highest. In both cases, the pressure magnitude in the injectors is double the chamber.

The phase relationships relative to the chamber end wall pressure measurements are plotted in Figure 13(c) and (f) for Case B-1 and Case B-2, respectively. Even though the pressure fluctuation amplitudes are modulated over the duration of the test, the calculated phase relationships are mostly constant. This highlights the ability of m-SSA to extract oscillatory behavior from noisy, non-stationary time series. For both cases, the chamber end wall pressures are 180° out of phase, as expected for a 1T mode. However, there is a clear shift in the phase relationship between the outer injectors and the chamber pressure when comparing the two cases. For Case B-1, which has a dominant 1570 Hz mode, the pressure in the left and right injectors is 125° and 110° out of phase with the pressure in the chamber downstream of these injectors. For Case B-2, where the dominant mode is 1840 Hz, this phase difference is increased to 155° on both sides of the combustor.

In Configuration B, the expected 1T mode frequency remains fixed around 1800 Hz because the combustion chamber dimensions were not changed from Configuration A. A pressure anti-node is also still expected to be present at the injector inlet since this remains an acoustically reflective boundary. However,
as depicted in Figure 2(c), because the injector length was halved, a pressure node is expected at the injector exit in contrast to a pressure anti-node in Configuration A. For Case B-2, which has a dominant mode frequency at the 1T frequency, the 155° phase difference between the pressure in the injector and in the chamber is close to the 180° phase difference that is expected if a pressure node occurs at the injector exit. However, with only one transducer in the injector, it cannot be definitely proven that a standing quarter-wave mode exists in that volume. Additionally, the effect of increased mean flow velocity near the choked injector inlet that was discussed for the Configuration A results adds uncertainty to the calculated phase. Although, this should not affect the change in the phase relationships for Case B-1 relative to Case B-2, which provides strong evidence that the overall combustor mode shape is different in each case.

The 1570 Hz mode frequency does not correspond to any length scale in the combustion chamber based on the combustion gas properties for Case B-1, so an additional part of the combustor must couple with the chamber to effectively lengthen the mode shape and reduce the observed frequency. If, similar to Configuration A, pressure waves originate in the combustor and propagate into the outer injectors, the phase lag between the pressure at the chamber end walls and at the injector inlets should be slightly greater than 90°. This is due to a combination of the injector length, which is a quarter of the 1T wavelength, and the increasing effect of mean flow approaching the injector inlet. For Case B-2, the measured phase lag is too large to correspond to a traveling wave, which supports the presence of standing quarter-wave modes in the outer injectors accompanied by the expected 1T mode in the combustion chamber. Conversely, Case B-1, which has a dominant frequency lower than expected for the 1T mode, has lower phase lags between the chamber and the outer injectors. The measured phase differences of 125° and 110° on either side of the combustor resemble the expected phase lags for traveling pressure waves in the injectors.

If pressure waves from the combustion chamber travel into the injectors, the overall length of the mode shape would be increased, reducing the mode frequency. The calculated frequency for the half-wave mode of a one-dimensional system that is a combination of the length of a single injector and the width of the chamber closely matches the dominant frequency of 1570 Hz in Case B-1. For this mode, the pressure node would no longer be located at the injector exit, but rather in the combustion chamber between anti-nodes at the outer injector inlet and the opposite chamber end wall. This 1570 Hz mode is best described as a system 1T mode, which has a pressure distribution similar to that shown in Figure 2(d). This provides a possible explanation for the change in frequency and injector phase lag between Case B-1 and Case B-2.

The shift in the dominant mode as a function of the adiabatic flame temperature provides further credence

Figure 13. Case B-1 reconstructed 1570 Hz mode: (a) extended pressure fluctuation time series, (b) segment of time series, and (c) phase differences. Case B-2 reconstructed 1840 Hz mode: (d) extended pressure fluctuation time series, (e) segment of time series, and (f) phase differences. Transducer locations are shown for reference.
to the hypothesis that a change in mode shape causes a shift in the observed combustion instability frequency. As shown in Figure 12, the higher flame temperature and, therefore, higher power density conditions favor the chamber 1T mode over the system 1T mode. As the equivalence ratio is increased, the combustion product gas properties change relative to the inlet gas properties, so the impedance difference between the fluid in the injectors and in the chamber is increased. For Case B-1, the ratio of the specific acoustic impedance, which is a product of the gas density and speed of sound, in the injector to the chamber is 1.65, while for Case B-2, the ratio of specific acoustic impedance is 2.02. As this ratio increases, the exit of the injector begins to approach a pressure release boundary condition. This supports a quarter-wave mode in the injector with a pressure anti-node at the inlet and a pressure node at the exit. Likewise, it becomes more difficult for pressure waves in the chamber to propagate into the injectors. This is consistent with the expected chamber 1T mode being favored at the higher power conditions, where there is a larger difference in acoustic impedance between the injectors and combustion chamber.

Phase conditioning the instantaneous CH\* chemiluminescence images shows the effect of the two modes on the flame structure. The images were sorted into eight bins based on the pressure at the chamber left wall (C-1) and then averaged. The results are shown in Figure 14(a) for Case B-1 and Figure 14(b) for Case B-2. A phase angle of 0° corresponds to minimum pressure at the left wall, whereas a phase angle of 180° corresponds to maximum pressure at the left wall. The center injector (Injector 4) is located at a pressure node and, therefore, a transverse velocity anti-node of the 1T mode. For a phase angle of 0°, it follows that the ρ′ is negative on the left side of Injector 4 but positive on the right side. As expected from previous experimental results, the out-of-phase pressure and transverse velocity fluctuations cause asymmetric vortex shedding downstream of the center injector. Similar flow structures occur downstream of Injector 5 even though it is not located at a pressure node because transverse velocity oscillations are still present. While the vortex shedding patterns between the two flames are similar, there are key differences to note from the phase-conditioned images. The regions of high heat release due to vortex roll-up for Case B-2 are more well-defined and compact than those in Case B-1. The spatial frequency of axially periodic shear layer structures is also higher in Case B-2 than in Case B-1. Given that the convective flow conditions are nearly identical for these two cases, this change in unsteady flow structure is attributed to the difference between the injector-coupled mode structures that drive these hydrodynamic instabilities.

The fluctuating pressure across each injector can be assumed to be constant since the flames in this configuration are compact relative to the wavelength of the 1T mode. Based on this assumption, the observed asymmetric heat release patterns occurring downstream of the center injectors are unable to contribute much energy to the unstable mode because the fluctuating heat release on either side of the injectors cancels out. This would not necessarily be the case for a non-compact flame, where the small changes in the shear layer dynamics seen in Figure 14 could have a more pronounced effect on the global, unsteady heat release. The outer injectors are also unable to add much energy to the unstable mode, as evidenced by the overall low instability amplitudes in this injector configuration. In Case B-1, a significant amount of the mode energy travels up into the injectors, reducing the mode frequency, while Case B-2 supports a quarter-wave mode in the injector with a pressure node at the exit. However, without a pressure anti-node at the injector exit in either case, neither is able to sustain a pressure-coupled response and drive high-amplitude instabilities.

Following the explanation for Configuration A, pressure waves that propagate into the injectors reflect off of the injector inlet, but now since the injector length is halved, the reflected waves return to the injector exit completely out of phase with the pressure oscillations in the combustion chamber. Any direct excitation of the flames via the weak transverse velocity fluctuations in the combustion chamber are effectively damped by the acoustic response of the injectors, as opposed to the injectors in Configuration A, which amplify the instabilities. The injectors in Configuration B behave similar to quarter-wave resonators, which have been used as passive control mechanisms for combustion instabilities in large-scale combustors. The extreme change in instability behavior between Configurations A and B along with the two different instability modes observed in Configuration B show that the coupling between the transverse modes in the chamber and the induced axial pressure fluctuations in the injectors is the primary mechanism that drives combustion instabilities for this multi-element combustor experiment.

4. Conclusions

Self-excited combustion instabilities in a high-pressure, multi-element combustor were investigated using HF pressure measurements and CH\* chemiluminescence imaging. Two injector lengths were tested, resulting in two distinct instability regimes. Configuration A, with 125 mm long injectors, presented high levels of instability that were independent of the operating condition. Phase relationships determined from SSA of the pressure time history showed the fundamental transverse...
mode and multiple harmonics were excited in the combustion chamber, and the superposition of these standing wave modes formed steep-fronted pressure waves. The chamber dynamics induced axial pressure waves in the injectors, originating in the combustion chamber and traveling into the injectors. This caused a modulation in the reactant flow exiting the injectors, leading to a localized region of intense heat release that traversed the width of the combustor following the peaks in the chamber pressure. The high levels of instability illustrate the potential for combustion dynamics to be driven in combustors operating at high power densities without many sources of damping.

The 62.5 mm long injectors in Configuration B significantly attenuated the pressure amplitudes in the combustor. This configuration also presented competing dynamics between the expected 1T mode and a lower frequency mode around 1550 Hz, where the 1T mode was favored at higher power density conditions. Comparing the relative pressure phase relationships from m-SSA for the two modes showed a change in the injector response depending on the dominant mode, resulting in two different frequencies being excited. In both cases though, the shortened injectors damped the instabilities in comparison to Configuration A. The stark change in instability behavior for similar operating conditions resulting from altering the injector length shows that the delineating feature between the two configurations is the ability of the injectors to support a pressure-coupled response with the combustion chamber. The importance of these global dynamics in the present experiment, as opposed to the local mechanisms of transverse excitation which seemed to play a secondary role, demonstrates that the mechanisms that define the stability of acoustically non-compact flames do not necessarily determine the transverse instability characteristics of a system with compact flames.

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Figure 14. Phase-conditioned CH* chemiluminescence images using dominant mode frequency for (a) Case B-1 (1570 Hz) and (b) Case B-2 (1840 Hz).
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