Doubly charmed tetraquarks in a diquark-antidiquark model

Xiaojun Yan, Bin Zhong and Ruilin Zhu
Department of Physics and Institute of Theoretical Physics,
Nanjing Normal University, Nanjing, Jiangsu 210023, China

We study the spectra of the doubly charmed tetraquark states in a diquark-antidiquark model. The doubly charmed tetraquark states form an antitriplet and a sextet configurations according to flavor SU(3) symmetry. For the tetraquark state $[qq][cc]$, we show the mass for both bound and excited states. The two-body decays of tetraquark states $T^{cc}[0^+]$ and $T^{cc}[1^{-}]$ to charmed mesons have also been studied. In the end, the doubly charmed tetraquarks decays to a charmed baryon and a light baryon have been studied in the SU(3) flavor symmetry.

Keywords: Exotic states, tetraquark, diquark

I. INTRODUCTION

Since last decade, Belle, BABAR, BESIII, LHCb and other experimental data have indicated a considerable number of exotic hadronic resonances with charm or beauty, the so-called $XYZ$ and $P_c$ particles including tetraquark states and pentaquark states [1–17]. All of these exotic states have many unexpected properties, such as masses, decay widths and cross-sections, which are hard to be explained in the conventional quark model. The nature of these states is one of the most interesting subjects in hadron physics.

The study of heavy flavor tetraquark states consisting of two quarks and two antiquarks has a long history. Hidden charm and bottom tetraquark states have been investigated in the past decades [18–23]. The recent reviews of the exotic states can be found in Refs. [21, 22]. Currently, the exact inner physical picture of these exotic states have not been reached an agreement in both the theoretical and experimental communities. The possible explanations of the $XYZ$ resonances can be classified into hadronic molecules, compact tetraquarks, diquark-antidiquark states, conventional quarkonium, triangle anomaly, and kinematics effects.

In Ref. [19], the multiquark resonances are reviewed and a coherent description of the so-called X and Z resonances is presented. The first suggestion to use light diquarks to explain the exotic states is from Jaffe and Wilczek [22]. Maiani et al. then introduced heavy-light diquarks to explain the charmonium-like states in Ref. [24]. The evidence that in a tetraquark system the two quarks arrange their color in a diquark before interacting with the antiquarks has been found on the lattice [23].

Based on the diquark-antidiquark model, tetraquark states $[cq'][ar{c}ar{q}]$ can form an octet representation and a singlet representation in flavor SU(3) symmetry which may give an interpretation to such $XYZ$ states [24]. A tetraquark state can also be represented with four quark configuration $[qq'][ar{c}ar{c}]$ (two charm antiquarks $c's$ and two light quarks with up, down and strange quark $u$, $d$, and $s$), which is called as doubly charmed $(C=2)$ tetraquark $T^{cc}$. From a theoretical point of view, $T^{cc}$ may be a more interesting hadronic state because the two heavy charm quark are more likely to form a lower energy diquark and two light quarks will rotate round the charmed diquark. Besides, its flavor quantum numbers can give the possibility to clearly distinguish these tetraquarks from conventional quarkonia.

The doubly bottomed tetraquark states with four quark configuration $[bb][qq]$ (two bottom quarks and two light antiquarks) have been studied in Refs. [25–27]. In Ref. [28], a doubly bottomed tetraquark with spin-parity $J^P = 1^+$ and mass $m = 10389 MeV$ was predicted, which is around 215 MeV below $BB^*$ threshold. In Ref. [29], a doubly bottomed tetraquark around 135 MeV below $BB^*$ threshold was predicted. The Lattice calculations were performed in Refs. [30, 31], where the doubly bottomed tetraquark below $BB^*$ threshold were also predicted. In Ref. [32], the doubly bottomed and charmed tetraquarks were discussed in the heavy quark symmetry.

In this paper, we discuss the possibility of the doubly charmed tetraquark states, and attempt to investigate the mass spectrum and decay widths of doubly charmed tetraquarks in a diquark-antidiquark model. The doubly charmed tetraquark states can form an antitriplet configuration and a sextet configuration according to flavor SU(3) symmetry. We hope the doubly charmed tetraquark states $T^{cc}$ can be discovered at BES III, Belle II, LHCb and other experiments with their large data samples of high luminosity.

The paper is organized as the following. In Sec. II, we classify the doubly charmed tetraquark states into anti-triplet and sextet representations in flavor SU(3) symmetry. In Sec. III, the spectra of the doubly charmed tetraquark states are calculated from a diquark-antidiquark model. In Sec. IV, we study the two-body charmed mesons decays of doubly charmed tetraquarks. We summarize and conclude in the end.
II. DOUBLY CHARMED TETRAQUARKS IN FLAVOR SU(3) SYMMETRY

Considering the flavor SU(3) symmetry, the doubly charmed tetraquarks $T^{cc} \sim [qq'][cc]$ with two light quarks, $q$ and $q'$, can be conventionally classified into two groups. The three light quarks, $(u, d, s)$ form a triplet representation and the charm quark $c$ is a singlet. The doubly charmed tetraquarks $T^{cc} \sim [qq'][cc]$ can have the following irreducible representations

$$3 \otimes 3 = \bar{3} \otimes 6.$$  

We label $\bar{3}$ representation by $T^{cc}_{[i,j]}$. Here the flavor components are antisymmetric under the exchange of $i$ and $j$, and thus $T^{cc}_{[i,j]}$ is traceless as $T^{cc}_{[i,i]} = 0$. The components can be given explicitly as

$$T^{cc}_{[1,2]} = \frac{1}{\sqrt{2}} T^{cc}_{us}(\bar{3}), \quad T^{cc}_{[1,3]} = \frac{1}{\sqrt{2}} T^{cc}_{us}(3),$$

$$T^{cc}_{[2,3]} = \frac{1}{\sqrt{2}} T^{cc}_{ds}(\bar{3}).$$  

We label $6$ representation by $T^{cc}_{[i,j]}$. Here the flavor components are symmetric under the exchange of $i$ and $j$. In this case, the components are given by

$$T^{cc}_{[1,2]} = \frac{1}{\sqrt{2}} T^{cc}_{us}(6), \quad T^{cc}_{[1,3]} = \frac{1}{\sqrt{2}} T^{cc}_{us}(6),$$

$$T^{cc}_{[2,3]} = \frac{1}{\sqrt{2}} T^{cc}_{ds}(6), \quad T^{cc}_{[1,1]} = T^{cc}_{uu}(6),$$

$$T^{cc}_{[2,2]} = T^{cc}_{dd}(6), \quad T^{cc}_{[3,3]} = T^{cc}_{ss}(6).$$

To summarize, the flavor components of tetraquarks in flavor SU(3) symmetry can be explicitly obtained as below

$$T^{cc}_{us}(\bar{3}) = \frac{1}{\sqrt{2}} (ud - du) \bar{c}c, \quad T^{cc}_{us}(3) = \frac{1}{\sqrt{2}} (us - su) \bar{c}c,$$

$$T^{cc}_{ds}(\bar{3}) = \frac{1}{\sqrt{2}} (ds - sd) \bar{c}c, \quad T^{cc}_{ds}(6) = \frac{1}{\sqrt{2}} (ud + du) \bar{c}c,$$

$$T^{cc}_{us}(6) = \frac{1}{\sqrt{2}} (us + su) \bar{c}c, \quad T^{cc}_{dd}(6) = \frac{1}{\sqrt{2}} (ds + sd) \bar{c}c,$$

$$T^{cc}_{uu}(6) = \frac{1}{\sqrt{2}} uu \bar{c}c, \quad T^{cc}_{dd}(6) = \frac{1}{\sqrt{2}} dd \bar{c}c,$$

$$T^{cc}_{ss}(6) = ss \bar{c}c.$$  

The orbitally excited tetraquark states also form antitriplet and sextet representations. Considering the first orbital excitation with $L = 1$, the orbitally excited tetraquark states will have the spin-parity $1^-$. For the neutral tetraquarks, $T^{cc}_{uu}$ can have the definite charge-parity, thus their $J^{PCC}$ quantum numbers can be $1^{−−}$ or $1^{−+}$.

III. DOUBLY CHARMED TETRAQUARKS SPECTRA

Because the heavy charm quarks interact with each other at the momentum scale $m_c v$ with the heavy charm quark relative velocity $v$, and this scale $m_c v$ is a large quantity compared with the typical hadron scale, the two heavy charm quarks will have a small distance between each other and are easily to form an attractive diquark. Considering a diquark-antidiquark picture $\delta \delta'$ with $\delta = [qq']$ and $\delta' = [cc]$, the effective Hamiltonian includes three kinds of interactions: spin-spin interactions of quarks in the diquark, antidiquark and between them; spin-orbital interactions; orbit-orbit interactions. The effective Hamiltonian then can be written as [23]:

$$H = m_\delta + m_{\delta'} + H^S_{SS} + H^P_{SS} + H^R_{SS} + H_{SL} + H_{LL},$$

where $m_\delta$ and $m_{\delta'}$ are the constituent masses of the diquark $[qq']$ and the antidiquark $[cc]$, respectively. $H^S_{SS}$ and $H^R_{SS}$ denote the spin-spin interaction inside the diquark and antidiquark, respectively. $H^R_{SS}$ denotes the spin-spin interaction of quarks between diquark and antidiquark. $H_{SL}$ and $H_{LL}$ represent the spin-orbital and purely orbital interactions.

The explicit form of each Hamiltonian is written as

$$H^S_{SS} = 2(\kappa_{qq'})\bar{3}(S_q \cdot S_q'),$$

$$H^R_{SS} = 2(\kappa_{cc})\bar{3}(S_c \cdot S_c'),$$

$$H^P_{SS} = 4\kappa_{qq'}(S_{qq'} \cdot S_c) + 4\kappa_{cc}(S_{qq} \cdot S_{cc}),$$

$$H_{SL} = 2A_{S}(S_{qq} \cdot L) + 2A_{S}(S_{cc} \cdot L),$$

$$H_{LL} = B_{\delta \delta'}\frac{L(L+1)}{2}.$$  

where $S_q(q')$ and $S_c(c')$ are the spin operators of light and heavy quarks, respectively. $S_\delta$ and $S_{\delta'}$ denote the spin operators of the diquark and antidiquark, respectively. $L$ is the orbital angular momentum operator. The other parameters are all coefficients. $A_{S(\delta \delta')}$ and $B_{\delta \delta'}$ are the spin-orbit and orbit-orbit couplings, respectively. $(\kappa_{qq'})\bar{3}$ and $(\kappa_{cc})\bar{3}$ are the spin-spin couplings for diquark in color antitriplet $\bar{3}$. $\kappa_{qq'}$ and $\kappa_{cc}$ are the spin-spin couplings for a quark-antiquark pair.

The orbital angular momentum of ground states of tetraquark is zero. In this case, there are two possible tetraquark configurations with the spin-parity $J^P = 0^+$.
The mass matrix is given as

\[ M = \begin{pmatrix} m_\delta + m_{\delta'} & \frac{1}{2} \left( \langle \kappa_{qq'} \rangle_\delta + \langle \kappa_{\bar{q}\bar{q'}} \rangle_\delta \right) \\ \frac{1}{2} \left( \langle \kappa_{qq'} \rangle_\delta + \langle \kappa_{\bar{q}\bar{q'}} \rangle_\delta \right) & m_{\bar{q}\bar{q'}} \end{pmatrix}, \]

where \( |S_\delta, S_{\bar{q}\bar{q}}, S_J \rangle \) denotes the doubly charmed tetraquark; the \( S_\delta \) and \( S_{\bar{q}\bar{q}} \) represent the spin of diquark \([qq']\) and antidiquark \([\bar{q}\bar{q'}]\), respectively; the \( S_J \) represents the total angular momentum of the tetraquark.

The corresponding splitting mass matrix for the \( J^P = 0^+ \) tetraquarks is

\[ \Delta M(0^+) = \begin{pmatrix} -\frac{3}{2} \langle \kappa_{qq'} \rangle_\delta + \langle \kappa_{\bar{q}\bar{q'}} \rangle_\delta & 0 \\ 0 & h_1 \end{pmatrix}, \]

where

\[ h_1 = \frac{1}{2} \left( \langle \kappa_{qq'} \rangle_\delta + \langle \kappa_{\bar{q}\bar{q'}} \rangle_\delta - 4\kappa_{q\bar{q}} - 4\kappa_{q'\bar{q}'} \right). \]

The mass matrix is given as

\[ M(J^P) = m_\delta + m_{\delta'} + \Delta M(J^P). \]

The above mass matrix is naturally diagonalized, and one can easily obtain two different eigenvalues.

The corresponding tetraquark configuration for \( J^P = 2^+ \) is

\[ |1_\delta, 1_{\bar{q}\bar{q}}, 2_J \rangle = (|\bar{q}\bar{q'}\rangle |\bar{q}\bar{q'}\rangle |\bar{q}\bar{q'}\rangle |\bar{q}\bar{q'}\rangle), \]

with the mass

\[ M(2^+) = m_\delta + m_{\delta'} + \frac{1}{2} \left( \langle \kappa_{qq'} \rangle_\delta + \langle \kappa_{\bar{q}\bar{q'}} \rangle_\delta \right) + \frac{1}{2} \left( 2\kappa_{q\bar{q}} + 2\kappa_{q'\bar{q}'} \right). \]

The possible configurations for the tetraquark with \( J^P = 1^+ \) are

\[ |0_\delta, 1_{\bar{q}\bar{q}}, 1_J \rangle = \frac{1}{\sqrt{2}} \left( |\bar{q}q\rangle |\bar{q}q\rangle |\bar{q}q\rangle |\bar{q}q\rangle \right) \]

The mass splitting matrix \( \Delta M \) for \( J^P = 1^+ \) can be obtained with the base vectors defined in Eq. (13).

The heavy quark is highly static in the rest frame of the hadron. In the paper, we adopt the heavy quark mass as \( m_c = 1.670 \text{GeV} \) [24, 37].

Different diquark masses will obviously affect the tetraquark’s mass. In Refs. [34, 35], the diquark masses effects are studied. The scalar and axial-vector diquark masses are assumed to be equal in order to limit the number of parameters. The diquark mass is chosen to be \( m_{f}\) = \( \xi (m_c + m_g) \) where \( fg \in \{gg, qs, ss\} \) is the flavor content of the diquark. Naturally, the diquark mass parameter \( \xi \) is assumed to be \( \xi \in [0, 1] \) [33]. In order to consider the effects from the diquark mass, we denote the running diquark mass \( m_{f}\) and we have \( m_{f}\) = \( m_\delta + \Delta \delta \). Thus we have \( \Delta \delta = \xi (m_f + m_g) - m_\delta \).
For the spin-spin couplings, the strange quark is treated differently from the up and down quarks. The couplings are chosen as: \((\kappa_{qq})_{3} = 103\text{MeV}, (\kappa_{qq})_{3} = 64\text{MeV}, (\kappa_{cc})_{3} = 30\text{MeV}, (\kappa_{cc})_{10} = 70\text{MeV}\) and \((\kappa_{sd})_{10} = 72\text{MeV}\) where \(g\) also stands for \(u\) or \(d\). The relation \(\kappa_{ij} = \frac{1}{2}(\kappa_{ij})_{10}\) is satisfied for the quark-antiquark state, which is extracted from one gluon exchange model. The spin-orbit coupling \(A_{3}\) is estimated as 30MeV, and the orbit-orbit coupling \(B_{3}\) is estimated as 278 MeV.

The wave function of a tetraquark consists of four parts: space-coordinate, color, flavor, and spin subspaces,

\[
\Psi(q, q', c, c) = \psi(x_1, x_2, x_3, x_4) \otimes \chi_c(c_1, c_2, c_3, c_4) \\
\otimes \chi_f(f_1, f_2, f_3, f_4) \otimes \chi_s(s_1, s_2, s_3, s_4),
\]

(15)

where \(\psi(x_i)\), \(\chi_c(c_i)\), \(\chi_f(f_i)\), and \(\chi_s(s_i)\) denote the space, color, flavor, and spin wave functions, respectively. The sub-labels 1, 2, 3, 4 denote \(q, q', c, c\), respectively. The diquark is attractive only in the triplet representation in color space, thus the color wave function is antisymmetric. The antidiquark \([\bar{c}c]\) is symmetric in flavor space, thus the spin wave function of the antidiquark \([\bar{c}c]\) has to be symmetric with \(S_D = 1\) when we do not consider the orbital excitation in the inner diquark system.

First we focus on the tetraquarks with \(L = 0\), and then the space wave function is symmetric. If the spin wave function of the diquark system \([qq']\) is antisymmetric, i.e. \(S_3 = 0\), the flavor function should be also antisymmetric. In this case, the charmed tetraquarks can be decomposed into the 3 representation, with the spin-parity \(J^P = 1^+\). Inputting the parameter masses and using the fixed diquark mass with \(\Delta \delta = 0\), their masses are determined to be

\[
m(T_{uc}^{cc}(6)) = 3.60\text{GeV}, \quad J^P = 1^+,
\]

(16)

\[
m(T_{uc}^{cc}(3)) = m(T_{ds}^{cc}(3)) = 3.85\text{GeV}, \quad J^P = 1^+.
\]

(17)

If using the running diquark mass with \(\Delta \delta \neq 0\), the tetraquark mass \(m(T_{qq}^{cc})\) will become to be \(m(T_{qq}^{cc}) + \Delta \delta\). In Eqs. \(^{(16)}\), \(^{(17)}\), we have assumed \(\Delta \delta = 0\).

If the spin wave function of the diquark system \([qq']\) is symmetric, i.e. \(S_3 = 1\), the flavor function should be also symmetric. In this case, the charmed tetraquarks can be decomposed into the 6 representation, with the spin-parity \(J^P = 0^+, 1^+, 2^+\). Their masses are determined to be

\[
m(T_{uc}^{cc}(6)) = \begin{cases} 3.94\text{GeV}, & J^P = 0^+, \\ 3.97\text{GeV}, & J^P = 1^+, \\ 4.04\text{GeV}, & J^P = 2^+, \end{cases}
\]

(18)

Other tetraquarks’ masses can be obtained by

\[
m(T_{ud}^{cc}(6)) = \begin{cases} 4.11\text{GeV}, & J^P = 0^+, \\ 4.15\text{GeV}, & J^P = 1^+, \\ 4.22\text{GeV}, & J^P = 2^+, \end{cases}
\]

(19)

\[
m(T_{sd}^{cc}(6)) = \begin{cases} 4.30\text{GeV}, & J^P = 0^+, \\ 4.34\text{GeV}, & J^P = 1^+, \\ 4.41\text{GeV}, & J^P = 2^+. \end{cases}
\]

(20)

From the above calculation, a doubly charmed tetraquark \(T_{ud}^{cc}(3)\) with spin-parity \(J^P = 1^+\) and mass \(M = 3.60\text{GeV}\) is predicted in a diquark-antidiquark model, which is around 140MeV below the \(D\bar{D}\) threshold and 270MeV below the \(D\bar{D}^*(D\bar{D})\) threshold. Other doubly charmed tetraquark states are predicted above the \(D\bar{D}\) threshold, which will have large decay widths compared with the tetraquark \(T_{ud}^{cc}(3)\) with spin-parity \(J^P = 1^+\) and mass 3.60GeV.

For the orbitally excited tetraquarks states with \(L = 1\), here we just consider the tetraquarks with spin-parity \(J^P = 1^-\). The masses of the tetraquarks with the spin-parity \(J^P = 1^-\) in 3 representation are

\[
m(T_{ud}^{cc}(3)) = 3.82\text{GeV}, \quad J^P = 1^-.
\]

(21)

\[
m(T_{ds}^{cc}(3)) = 4.07\text{GeV}, \quad J^P = 1^-.
\]

(22)

The masses of the tetraquarks with the spin-parity \(J^P = 1^-\) in 6 representation are

\[
m(T_{ud}^{cc}(6)) = \begin{cases} 4.21\text{GeV}, & J^P = 1^-, \\ 4.19\text{GeV}, & J^P = 1^-, \\ 4.14\text{GeV}, & J^P = 1^-, \end{cases}
\]

(23)

\[
m(T_{sd}^{cc}(6)) = \begin{cases} 4.39\text{GeV}, & J^P = 1^-, \\ 4.36\text{GeV}, & J^P = 1^-, \\ 4.31\text{GeV}, & J^P = 1^-, \end{cases}
\]

(24)

\[
m(T_{us}^{cc}(6)) = \begin{cases} 4.58\text{GeV}, & J^P = 1^-, \\ 4.56\text{GeV}, & J^P = 1^-, \\ 4.51\text{GeV}, & J^P = 1^-, \end{cases}
\]

(25)

IV. DOUBLY CHARMED TETRAQUARKS DECAYS TO TWO CHARMED MESONS

When the doubly charmed tetraquarks lie above the \(D\bar{D}\) threshold, they can decay into two charmed mesons. For the tetraquarks with positive parity, the two-body decay amplitudes can be written as:

\[
\mathcal{M}(T^{cc}[0^+] \rightarrow D\bar{D}) = F_{D\bar{D}} f_{T^{cc}},
\]

(26)

where \(f_{T^{cc}}\) is the decay constant of the tetraquark, \(F_{D\bar{D}}\) denotes the effective coupling to diquark-antidiquark pair.
For the tetraquarks with $J^P = 1^-$, the two body decay amplitudes are written as:

$$\mathcal{M}(T^{cc}[1^-] \rightarrow \bar{D} \bar{D}) = \varepsilon_{T^{cc}} \cdot (P_{\bar{D}} - P'_{\bar{D}}) \frac{F_{\bar{D}} f_{T^{cc}}}{3\sqrt{2} m_{T^{cc}}},$$

(27)

$$\mathcal{M}(T^{cc}[1^-] \rightarrow \bar{D} \bar{D}^*) = \varepsilon_{T^{cc}}^\mu \varepsilon_{T^{cc}}^\nu \frac{F_{\bar{D}} f_{T^{cc}}}{3\sqrt{2} m_{T^{cc}}} \epsilon_{\mu \nu \rho \sigma} \frac{\epsilon_{\rho \sigma}}{2},$$

(28)

$$\mathcal{M}(T^{cc}[1^-] \rightarrow \bar{D}^*(P) \bar{D}^*(P')) = \varepsilon_{T^{cc}}^\mu \varepsilon_{T^{cc}}^\nu \frac{F_{\bar{D}} f_{T^{cc}}}{3\sqrt{2} m_{T^{cc}}} (g^{\mu \nu} (-P_T - P_{\bar{D}}))^\nu + g^{\mu \nu} P^\mu_{\bar{D}} + g^{\mu \nu} (P^\nu_{\bar{D}} - P_{\bar{D}})\epsilon_{\mu \nu \rho \sigma} \frac{\epsilon_{\rho \sigma}}{2},$$

(29)

The decay width of $T^{cc} \rightarrow \bar{D}(\bar{D}^*) \bar{D}(\bar{D}^*)$ can be written as:

$$\Gamma(T^{cc} \rightarrow \bar{D}(\bar{D}^*) + \bar{D}(\bar{D}^*)) = \frac{|p|}{8\pi m_{T^{cc}}^2} |\mathcal{M}|^2,$$

(30)

where

$$|p| = \frac{\sqrt{(m_{T^{cc}}^2 - (m_1 - m_2)^2) (m_{T^{cc}}^2 - (m_1 + m_2)^2)}}{2m_{T^{cc}}}.$$

which is the momentum modulus of final charmed meson in the tetraquark rest frame. $m_1$ and $m_2$ are the final charmed meson’s masses, respectively.

![FIG. 1: Feynman Diagrams for doubly charmed tetraquarks decay to two charmed mesons.](image)

The decay ratio for the tetraquark with positive parity is

$$\frac{\Gamma(T^{cc}[0^+] \rightarrow \bar{D} \bar{D})}{F_{\bar{D}}^2 |f_{T^{cc}}|^2 |p|^2} = \frac{1}{8\pi m_{T^{cc}}^2}.$$

The similar ratios for the tetraquarks with $J^P = 1^-$ are

$$\frac{\Gamma(T^{cc}[1^+] \rightarrow \bar{D} \bar{D})}{F_{\bar{D}}^2 |f_{T^{cc}}|^2 |p|^2} = \frac{1}{6\pi m_{T^{cc}}^2},$$

$$\frac{\Gamma(T^{cc}[1^-] \rightarrow \bar{D} \bar{D}^*)}{F_{\bar{D}}^2 |f_{T^{cc}}|^2 |p|^2} = \frac{1}{12\pi m_{T^{cc}}^2},$$

$$\frac{\Gamma(T^{cc}[1^-] \rightarrow \bar{D}^* \bar{D}^*)}{F_{\bar{D}}^2 |f_{T^{cc}}|^2 |p|^2} = \frac{m_{T^{cc}}^2 - 4\pi m_{T^{cc}}^2 |p|^2 + 4\pi^2 |p|^4}{2\pi m_{T^{cc}}^2 (2m_{T^{cc}}^2 - 4|p|^2)^2}.$$
where the representation $H^i(3)$ will vanish in the SU(3) flavor symmetry.

Singly charmed baryons with two light quarks can form an anti-triplet or sextet, which can be classified into two matrices

$$T^c_3 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & \Lambda^+_c & \Xi^+_c \\ -\Lambda^+_c & 0 & \Xi^0_c \\ -\Xi^+_c & -\Xi^0_c & 0 \end{pmatrix},$$

$$T^c_6 = \begin{pmatrix} \Sigma^+_c & \frac{1}{\sqrt{2}} \Sigma^+_c & \frac{1}{\sqrt{2}} \Sigma^0_c \\ \frac{1}{\sqrt{2}} \Sigma^+_c & \Sigma^0_c & \Xi^0_c \\ \frac{1}{\sqrt{2}} \Sigma^0_c & \frac{1}{\sqrt{2}} \Xi^0_c & O^0_c \end{pmatrix}. \tag{35}$$

The light baryons made of three light quarks with spin-parity $J^P = 1^+$ can form an octet, which has the expression

$$T_8 = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 \\ \frac{1}{\sqrt{2}} \Sigma^0 - \frac{1}{\sqrt{6}} \Lambda^0 \\ \Xi^0 - \sqrt{\frac{2}{3}} \Lambda^0 \end{pmatrix} \tag{36}$$

The decay amplitudes of Cabibbo-favored channels $A(T^{cc} \rightarrow T^c + T_8) = A(T^{cc} + T_8 | H_{eff} | T^{cc})$ can be written as $V_{cs}V^*_{ud}A(T^{cc} \rightarrow T^c + T_8)$.

For the decays $T^{cc}(3) \rightarrow T^c_3 + T_8$, the effective Hamiltonian can be written as

$$A(T^{cc}(3) \rightarrow T^c_3 + T_8) = a_6 T_{ij}^{cc}(3) T_{3,j}^{cc} \varepsilon_{ikn} (T_{ik}^{cc})^n H_{m}^{ijl} \tag{6}$$

$$+ a_6 T_{ij}^{cc}(3) T_{3,j}^{cc} \varepsilon_{ikm} (T_{ik}^{cc})^m H_{jn}^{jl} \tag{6}$$

$$+ b_6 T_{ij}^{cc}(3) T_{3,j}^{cc} \varepsilon_{ikn} (T_{ik}^{cc})^n H_{jn}^{jl} \tag{6}$$

$$+ a_15 T_{ij}^{cc}(3) T_{3,j}^{cc} \varepsilon_{ikm} (T_{ik}^{cc})^m H_{jn}^{jl} \tag{15}$$

$$+ a_15 T_{ij}^{cc}(3) T_{3,j}^{cc} \varepsilon_{ikn} (T_{ik}^{cc})^n H_{jn}^{jl} \tag{15}.$$ \tag{37}

Similarly, the effective Hamiltonian have the same formulae for the decays $T^{cc}(6) \rightarrow T^c_3 + T_8$ by the replacement of $T^{cc}(3) \rightarrow T^{cc}(ij)(6)$. The term proportional to $b_6$ will vanish for both the $T^{cc}(3) \rightarrow T^c_6 + T_8$ and $T^{cc}(6) \rightarrow T^c_6 + T_8$ decays. The Cabibbo-allowed amplitudes for doubly charmed tetraquarks decays to a charged baryon and a light baryon are given in Tabs. IV, III and V. For convenience, we give the ratio of the decay widths of the doubly charmed tetraquarks to a charged anti-baryon and a light baryon in the flavor SU(3) symmetry in Tabs. IV and V.

Whatever the doubly charmed tetraquarks decay to two charged mesons or decay to a charged baryon and a light baryon, BESIII, BelleII, and LHCb are excellent experiment platforms to search for it. BESIII has accumulated large $e^+e^-$ collision data samples at an

| TABLE I: Decay amplitudes of $T^{cc}(3) \rightarrow T^c_3 + T_8$ decays. |
|--------------------------------------|
| Decay channel | amplitude |
| $T^{cc}_{ud}(3) \rightarrow \Xi_n^0$ | $\frac{1}{2}(a_{15} - a_6 - 2b_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Xi_c^0$ | $-\frac{1}{2}(a_{15} + a_6 + 2b_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Xi_n^0 n$ | $\frac{1}{2}(a_{15} - 2a_6 - a_6 - 2b_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Xi_c^0 n$ | $\frac{1}{2}(a_{15} + a_6 + a_6 + 2b_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Lambda_c^n n$ | $-\frac{1}{2}(a_{15} + a_6 - a_6 - 2b_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Lambda_c^0 n$ | $\frac{1}{2}(a_{15} + a_6 + a_6 + 2b_6)$ |

| TABLE II: Decay amplitudes of $T^{cc}(3) \rightarrow T^c_6 + T_8$ decays. |
|--------------------------------------|
| Decay channel | amplitude |
| $T^{cc}_{ud}(3) \rightarrow \Sigma_n^0$ | $\frac{1}{2}(a_{15} - a_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Sigma_c^0$ | $-\frac{1}{2}(a_{15} - a_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Sigma_n^0 n$ | $-\frac{1}{2}(a_{15} + a_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Sigma_c^0 n$ | $\frac{1}{2}(a_{15} + a_6 + a_6 + 2b_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Omega_c^0$ | $\frac{1}{2}(a_{15} - a_6)$ |
| $T^{cc}_{ud}(3) \rightarrow \Omega_c^0 n$ | $\frac{1}{2}(a_{15} + a_6 + a_6 + 2b_6)$ |

| TABLE III: Decay amplitudes of $T^{cc}(6) \rightarrow T^c_3 + T_8$ decays. |
|--------------------------------------|
| Decay channel | amplitude |
| $T^{cc}_{ud}(6) \rightarrow \Xi_n^0$ | $-\frac{1}{2}(a_{15} - a_6 + 2b_6)$ |
| $T^{cc}_{ud}(6) \rightarrow \Xi_c^0$ | $-\frac{1}{2}(a_{15} + a_6 - 2b_6)$ |
| $T^{cc}_{ud}(6) \rightarrow \Xi_n^0 n$ | $\frac{1}{2}(a_{15} + a_6 + a_6 + 2b_6)$ |
| $T^{cc}_{ud}(6) \rightarrow \Xi_c^0 n$ | $\frac{1}{2}(a_{15} - a_6 + 2b_6)$ |
| $T^{cc}_{ud}(6) \rightarrow \Lambda_c^0 n$ | $\frac{1}{2}(a_{15} + a_6 + a_6 + 2b_6)$ |

FIG. 2: Feynman Diagrams for doubly charmed tetraquarks decays to baryon and anti-baryon. The two circles “⊕ ⊕” denote the four fermion weak effective vertex.
TABLE IV: Decay amplitudes of \( T^{cc}(6) \rightarrow \bar{T}_a^c + T_8 \) decays.

| Decay channel | amplitude |
|---------------|-----------|
| \( T_{uu}(6) \rightarrow \Sigma_{cc}^- n \) | \(-\frac{1}{\sqrt{2}}(a_{15} - a_6)\) |
| \( T_{uc}(6) \rightarrow \Sigma_{cc}^- n \) | \(-\frac{1}{\sqrt{2}}(a_{15} + a_6)\) |
| \( T_{du}(6) \rightarrow \Sigma_{cc}^- n \) | \(-\frac{1}{\sqrt{2}}(a_{15} + a_6)\) |
| \( T_{dd}(6) \rightarrow \Sigma_{cc}^- n \) | \(-\frac{1}{\sqrt{2}}(a_{15} + a_6)\) |
| \( T_{us}(6) \rightarrow \Sigma_{cc}^- n \) | \(\frac{1}{2}(a_{15} + a_6)\) |
| \( T_{us}(6) \rightarrow \Xi_{cc}^0 n \) | \(-\frac{1}{\sqrt{3}}(a_{15} + a'_6 - a_6)\) |
| \( T_{us}(6) \rightarrow \Omega_{cc}^- n \) | \(\frac{1}{2}(a_{15} + a'_6 - a_6)\) |
| \( T_{us}(6) \rightarrow \Sigma_{cc}^- p^+ \) | \(-\frac{1}{\sqrt{2}}(a_{15} + a'_6 - a_6)\) |
| \( T_{us}(6) \rightarrow \Xi_{cc}^0 p^+ \) | \(-\frac{1}{\sqrt{2}}(a_{15} + a'_6 - a_6)\) |
| \( T_{us}(6) \rightarrow \Omega_{cc}^- p^+ \) | \(-\frac{1}{\sqrt{2}}(a_{15} + a'_6 - a_6)\) |

TABLE V: SU(3) relations for decay widths for the \( \bar{3} \) doubly charmed tetraquark to a charmed anti-baryon and a light baryon. \( R \) denotes the ratio of two decay widths.

| \( \Gamma(channel_1)/\Gamma(channel_2) \) | \( R \) |
|-------------------------------|------|
| \( \Gamma(T_{uu}(6) \rightarrow \bar{\Sigma}_c^- + \Lambda) \) | 2 |
| \( \Gamma(T_{uc}(6) \rightarrow \bar{\Sigma}_c^- + \Sigma) \) | 2 |
| \( \Gamma(T_{du}(6) \rightarrow \bar{\Sigma}_c^- + \Lambda) \) | 2 |
| \( \Gamma(T_{dd}(6) \rightarrow \bar{\Sigma}_c^- + \Sigma) \) | 2 |
| \( \Gamma(T_{us}(6) \rightarrow \bar{\Sigma}_c^- + \Lambda) \) | 2 |
| \( \Gamma(T_{us}(6) \rightarrow \bar{\Sigma}_c^- + \Sigma) \) | 2 |

energy range from 3.8 GeV to 4.6 GeV and will continue taking data at open-charm energy region \([59]\). It would be an interesting research at those energy points with the world’s top integrated luminosity just as papers \([60][63]\) et al. The two charmed mesons channels \( T^{cc} \rightarrow D(D^*) + \bar{D}(\bar{D}^*) \) can be studied through the whole energy region from 3.8 GeV to 4.6 GeV. A charmed baryon and a light baryon decay channels showed in Tabs. 4, 11, 13 and 14 can also be studied carefully at the same energy region.

VI. CONCLUSION

We have considered the possibility for the doubly charmed tetraquark states with the quark configuration \([qq')[\bar{cc}]\). These states are straightforward consequences of the constituent diquark-antidiquark model. The doubly charmed tetraquarks form the antitriplet and sextet configuration in the flavor SU(3) symmetry. The mass spectrum and their spin-parities of tetraquark states have been investigated. We found that a doubly charmed tetraquark \( T_{uu}(\bar{3}) \) with spin-parity \( J^P = 1^+ \) is around 140 MeV below the \( DD \) threshold and 270 MeV below the \( D\bar{D}^*(D^*\bar{D}) \) threshold. For \( T^{cc}[0^+] \) and \( T^{cc}[1^-] \) tetraquark states, the decay modes of the two-body charmed mesons have also been presented. Furthermore, the doubly charmed tetraquarks decays to a charmed baryon and a light baryon have been studied in the SU(3) flavor symmetry. The search for such kind of states can be carried out at BESIII, BelleII, LHCb and other experiments with their large data samples of high luminosity. These Cabibbo-favored channels shall provide a window to discover the possible doubly charmed tetraquark states.
Acknowledgments

This work was supported in part by Natural Science Foundation of Jiangsu under Grant No. BK20171471, by

the National Natural Science Foundation of China under Grant No. U1732105, and by the Research Foundation for Advanced Talents of Nanjing Normal University under Grant No. 2014102XGQ0085.
[54] M. Oettel, G. Hellstern, R. Alkofer and H. Reinhardt, Phys. Rev. C 58, 2459 (1998) [nucl-th/9805054].
[55] D. Nicmorus, G. Eichmann, A. Krassnigg and R. Alkofer, Phys. Rev. D 80, 054028 (2009) [arXiv:0812.1665 [hep-ph]].
[56] G. Eichmann, H. Sanchis-Alepuz, R. Williams, R. Alkofer, and C. S. Fischer, Prog. Part. Nucl. Phys. 91, 1 (2016) [arXiv:1606.09602 [hep-ph]].
[57] W. Wang and R. L. Zhu, Phys. Rev. D 96(1), 014024 (2017) [arXiv:1704.00179 [hep-ph]].
[58] W. Wang, Z. P. Xing and J. Xu, Eur. Phys. J. C 77, no. 11, 800 (2017) [arXiv:1707.06570 [hep-ph]].
[59] M. Ablikim et al. [BESIII Collaboration], Chin. Phys. C 40(6), 063001 (2016).
[60] M. Ablikim et al. [BESIII Collaboration], Chin. Phys. C 39(4), 041001 (2015).
[61] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 94, 032009 (2016).
[62] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 93, 011102(R) (2016).
[63] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. D 96, 032004 (2017).