Trilinear Neutral Gauge Boson Couplings in Effective Theories

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We list all the lowest dimension effective operators inducing off-shell trilinear neutral gauge boson couplings $ZZ\gamma$, $Z\gamma\gamma$, and $ZZZ$ within the effective Lagrangian approach, both in the linear and nonlinear realizations of the SU(2)$_L \times$ U(1)$_Y$ gauge symmetry. In the linear scenario we find that these couplings can be generated only by dimension eight operators necessarily including the Higgs boson field, whereas in the nonlinear case they are induced by dimension six operators. We consider the impact of these couplings on some precision measurements such as the magnetic and electric dipole moments of fermions, as well as the $Z$ boson rare decay $Z \to \nu\nu\gamma$. If the underlying new physics is of a decoupling nature, it is not expected that trilinear neutral gauge boson couplings may affect considerably any of these observables. On the contrary, it is just in the nonlinear scenario where these couplings have the more promising prospects of being perceptible through high precision experiments.

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I. INTRODUCTION

The present agreement between experimental data and the standard model (SM) suggests that the energy scale $\Lambda$ associated with any new physics should be large compared with the electroweak scale $v = (\sqrt{2}G_F)^{1/2} = 246$ GeV. To infer the existence of new particles as heavy as $\Lambda$ through their virtual effects, effective Lagrangian (EL) techniques have been extensively used to study quantities which are forbidden or highly suppressed within the SM [1–3]. Among these quantities, self-couplings of electroweak gauge bosons constitute a sensitive probe of nonstandard interactions [4]. Experimental bounds on possible anomalous $W^+W^-Z(\gamma)$ couplings have reached an accuracy of the few percent level in both hadronic and leptonic colliders [5,6], but the situation looks less promising for anomalous $ZZZ$, $ZZ\gamma$, and $Z\gamma\gamma$ couplings [7]. Unlike $W^+W^-Z(\gamma)$ couplings, trilinear neutral gauge boson couplings (TNGBC) vanish when the three bosons are real. Another interesting peculiarity of TNGBC is that they must be induced by loop effects in any renormalizable theory since they cannot possess a renormalizable structure. In the SM, TNGBC are generated at one-loop level by fermion triangles [8], being very suppressed even in the presence of a fourth fermion family [9]. There follows that it is convenient to carry out a model independent study of TNGBC using the EL method to parametrize any anomalous contribution. Within this approach, there are two well motivated schemes to parametrize virtual effects of physics beyond the Fermi scale via effective operators involving only SM fields, namely the linear and the nonlinear realizations.

In the linear realization or decoupling scenario it is assumed that the light spectrum of particles, which fill out multiplets of the electroweak SU(2)$_L \times$ U(1)$_Y$ gauge group, includes at least the physical Higgs boson of the SM. Because of the decoupling theorem, virtual effects of heavy physics cannot affect low energy processes dramatically. Nonetheless, any new effect, in spite of its smallness, may have significant effects on the couplings which are absent or highly suppressed within the SM. Starting from the SM fields and assuming lepton and baryon number conservation, there is no way to construct any odd dimension operator respecting the linearly realized SU(2)$_L \times$ U(1)$_Y$ symmetry.

1Throughout this work we consider the general case of off-shell bosons, unless stated otherwise, but they will be denoted by $V$ rather than $V^*$. 
As for dimension six, operators of this class were comprehensively studied in [10]. It was shown that there are 84 independent dimension six operators.

In the case of the nonlinear realization or nondecoupling scenario, the parametrization of new physics effects arises when it is assumed that the Higgs bosons are very heavy or do not exist at all. The scalar sector is comprised only by Goldstone bosons, which transform nonlinearly under the SU(2)\textsubscript{L} × U(1)\textsubscript{Y} group. It is also possible to introduce light scalar fields in this parametrization, but they cannot be recognized as Higgs bosons since such fields do not couple to the remaining light particles as dictated by the Higgs mechanism [13]. Since the low energy theory is nonrenormalizable under Dyson prescription, heavy physics does not decouple from the low energy processes. We may think of this scenario as the one in which the EL parametrizes unknown physics which would not obey the Higgs mechanism. In this case, the most important operators are the ones which induce the masses of the W and Z gauge bosons, prescribing also the general structure of the W\textsuperscript{+}W\textsuperscript{−}Z(γ) couplings [14]. These operators have dimension two and four.

At the lowest order, anomalous W\textsuperscript{+}W\textsuperscript{−}Z(γ) couplings are induced by dimension six operators in the decoupling scenario. In the nonlinear scheme, they receive contributions from dimension four operators. In contrast, at the lowest order, TNGBC are induced by dimension eight operators in the linear realization and by dimension six operators in the nonlinear one. In the latter case there are also some dimension four operators which give rise to the ZZZ coupling, but they are proportional to the scalar part of the Z boson (\(\partial_\mu Z^\mu\)). It can be shown that such operators may be eliminated by means of a transformation which leaves invariant the S-matrix [15]. Consequently, any anomalous contribution to TNGBC is expected to be more suppressed than those inducing nonstandard W\textsuperscript{+}W\textsuperscript{−}Z(γ) couplings. It must be stressed, however, that any potential effect must be carefully examined as it may constitute a clear evidence of new physics.

The structure of TNGBC has already been studied in the context of effective theories, initially at the level of vertex functions [14]. However, in this approach it was considered the case where two particles are real and just one is virtual. It is only very recently that the analysis of the off-shell vertices has been done under the U(1)\textsubscript{em} gauge invariant framework, including the study of the respective EL. By invoking Bose symmetry, Lorentz covariance, and electromagnetic gauge invariance, the most general structures inducing TNGBC with three off-shell neutral bosons were constructed [15]. As was shown in [15], the U(1)\textsubscript{em} gauge invariant framework is equivalent to the nonlinearly realized SU(2)\textsubscript{L} × U(1)\textsubscript{Y} invariant case. Such an equivalence is explicit in the unitary gauge. The choice of using either framework is only a matter of convenience. In particular, the nonlinear scheme is convenient in working out loop calculations, as the presence of Goldstone bosons allows to quantize the theory with the aid of a renormalizable R\textsubscript{ξ} gauge.

It is clear that a comprehensive study of TNGBC must include both the linear and the nonlinear schemes. To our knowledge the former has never been studied before. One of the aims of the present paper is to present a complete list of the effective operators which induce TNGBC at the lowest order in both realizations of the SU(2)\textsubscript{L} × U(1)\textsubscript{Y} gauge symmetry. Not all the operators that can be constructed respecting the Lorentz and electroweak symmetries are independent since a certain class of general transformations allows to rule out some of them without affecting the S-matrix elements [16]. In the course of our classification we have found operators with terms containing higher derivatives which resemble the covariant structure of the equations of motion; there are also operators with terms which are proportional to the scalar part of the Z boson (\(\partial_\mu Z^\mu\)). It has been shown in [16] that both types of structures can be eliminated in favor of other operators already present in the effective Lagrangian. Such a procedure is only valid at first order in the unknown effective parameters of the theory as any effective Lagrangian is assumed to describe the effects of well-behaved new physics just in this approximation. Consequently, after performing the required transformation, the equations of motions can be used to eliminate any redundant structure, expressing the respective operator in terms of other ones. This whole procedure does not affect the S-matrix elements. In order to present all the independent operators, we will classify them according to the following criterion: those which can not be reduced using the equations of motion will be referred to as irreducible, the remaining ones will be referred to as reducible.

After classifying the operators, our paper will be concerned with the sensitivity of some precision experiments to new physics effects arising from TNGBC. Although persuasive theoretical arguments indicate that trilinear gauge boson couplings are not expected to be larger than the one percent [19,20], the Large Hadron Collider (LHC) as well as the planned Next Linear Collider (NLC) are expected to constrain them at a level of 10\textsuperscript{−4}–10\textsuperscript{−6} [4,21]. As long as TNGBC are concerned, the size of their effects will be suppressed by powers of \((\nu/\Lambda)^4\) and \((\nu/\Lambda)^6\) in the linear and the nonlinear scenarios, respectively. We will examine whether some high precision measurements may lead to any reasonable bound on these couplings. The anomalous W\textsuperscript{+}W\textsuperscript{−}Z(γ) couplings have been constrained from a global analysis of the LEP/SLC observables at the Z pole [2]. To draw any inference about the size of TNGBC we will consider the muon g − 2 value, the known limit on the electric dipole moment (EDM) of the electron, and the current limit on the rare decay \(Z \rightarrow \nu\bar{\nu}\gamma\).

Our paper is organized as follows. All the lowest dimension operators that generate TNGBC in the linear scheme are
presented in Sec. II, following the classification criterion already explained. Besides, the respective Lagrangians and vertex functions are shown explicitly. In Sec. III, a similar analysis within the nonlinear scenario is presented. Sec. IV is devoted to examine the possibility of obtaining constraints on the couplings out of high precision experiments. Finally, the paper is closed with some concluding remarks in Sec. V.

II. THE DECOUPLING SCENARIO

This section focuses on the itemization of all the lowest dimension operators that generate at least one of the couplings $\mathcal{Z}Z$, $\mathcal{Z}\gamma$ or $Z\gamma\gamma$ within the linear realization of the SU(2)$_L \times$ U(1)$_Y$ electroweak group. To construct a basis of independent operators with a given dimension, we must consider some aspects concerning the independence of the $S$-matrix under a wide class of transformations which leave it invariant [8]. For instance, it was shown in [8] that some operators, which consist of a piece containing higher derivatives, can be eliminated in favor of others by using a specific transformation, leaving unchanged the $S$-matrix elements at any order of perturbation theory. Another situation arises when an operator is proportional to the scalar part of the $Z$ boson. While the latter kind of structures give vanishing contributions when the $Z$ boson is on mass shell or is virtual but couples to light fermions, the situation is not the same in the case of the top quark. In this respect, this kind of operators can also be eliminated by performing a transformation which does not alter the $S$-matrix elements [3]. It must be noted that both transformations are equivalent to applying the equations of motion. Beside these considerations, we have made a systematic use of integration by parts to rule out any operator related to others through a surface term. Consequently, we will catalog the operators inducing TNGBC as reducible or irreducible.

Any SU(2)$_L \times$ U(1)$_Y$ invariant involving only bosonic fields can be constructed out of the covariant structures $B_{\mu\nu}$, $W_{\mu\nu} = \frac{1}{2} \sigma^i W^i_{\mu\nu}$, $\Phi$, and $D_\mu Z$, where the covariant derivative is defined as $D_\mu = \partial_\mu - ig W_\mu - ig' B_\mu$, and $\Phi$ is the Higgs doublet. Using these basic structures, we can built the following SU(2)$_L \times$ U(1)$_Y$ invariant and Lorentz covariant structures of dimension two through five

\begin{align}
B_{\mu\nu}, \Phi \Phi, \Phi^\dagger D_\mu \Phi, \Phi^\dagger W_{\mu\nu} \Phi, B_{\mu\nu} B^{\lambda\rho}, \text{Tr}[W_{\mu\nu} W^{\lambda\rho}], \Phi^\dagger (D_{\mu} D_{\nu} + D_{\nu} D_{\mu}) \Phi, \Phi^\dagger W_{\mu\nu} D_{\lambda} \Phi.
\end{align}

Note that another set of SU(2)$_L \times$ U(1)$_Y$ invariant and Lorentz covariant structures can be generated by operating with the ordinary derivative on these expressions. Any nonrenormalizable bosonic operator can be built by choosing the appropriate combinations of these structures to form Lorentz scalars. The ordinary derivative can act on the last expressions in several ways, but the contractions $\partial^{\mu} B_{\mu\nu}$ and $\partial^{\mu} (\Phi^\dagger D_\mu \Phi)$, being proportional to the scalar part of the $Z$ boson, are special because in both cases we can use the equations of motion to eliminate the resulting operator.

Let us now discuss the general Lorentz structure of TNGBC. The lowest dimension operators which can be assembled out of the basic structures have dimension six [10]. It is easy to see that no dimension six operator induce TNGBC, which unavoidably leads to search for eight dimension operators. In principle, the combination which can give rise to TNGBC may involve the 4-vectors $A_\mu$ and $Z_\mu$, together with the antisymmetric tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$. Owing to U(1)$_{\text{em}}$ gauge symmetry, the electromagnetic field can only appear as $A_\mu$, through the respective covariant derivative, which operates on charged fields only. Therefore, the photon must appear in any term through the tensor field $F_{\mu\nu}$. Due to the antisymmetry of the $F_{\mu\nu}$ and $Z_{\mu\nu}$ tensors, it is not possible to generate TNGBC using only these structures: it would be necessary to have a t our disposal three antisymmetric tensors. Therefore, the photon must appear in any term through the tensor field $F_{\mu\nu}$.

To construct the $\mathcal{Z}Z$, $\mathcal{Z}\gamma$, and $Z\gamma\gamma$ vertices, we must use at least a $Z$ boson in the $Z_\mu$ form, which is allowed because this field couples to neutral fields. The 4-vector $Z_\mu$ is contained in the covariant derivative, which in the bosonic sector operates only on the Higgs doublet. As a consequence, the Higgs mechanism plays a special role in this type of couplings. In particular, the Higgs presence increases the dimension at which the operators can be generated in comparison to the nonlinear case, where this field is absent. The $Z$ boson may appear through the combinations $Z_\lambda Z_{\mu\nu}$, $Z_\lambda Z_{\mu\nu}$, $Z_\lambda Z_{\mu\nu}$, and $Z_\mu$. The building blocks necessary to construct these couplings are $\Phi^\dagger D_\mu \Phi$, $\Phi^\dagger (D_{\mu} D_{\nu} + D_{\nu} D_{\mu}) \Phi$, and $\Phi^\dagger W_{\mu\nu} D_{\lambda} \Phi$, which, after spontaneous symmetry breaking (SSB), induce the structures $Z_{\mu\nu}$, $Z_{\mu\nu}$, and $Z_{\mu\nu}$ respectively. The irreducible operators may contribute to a given physical process through the specific structure of TNGBC, while the reducible ones may contribute to it via contact diagrams in which an internal line associated with either a $Z$ boson or a photon has been amputated, for instance when the equations of motion are used to replace the term $\partial_\mu B^{\mu\nu}$ with the respective current. Therefore, the reducible operators deserve a more careful study than the reducible ones. We will present thus the Lagrangians and vertex functions in the irreducible case, whereas in the reducible case we will list only the respective operators and the Lagrangian prescribing the off-shell electromagnetic properties of the $Z$ boson. In the next section we will enumerate the operators of dimension eight that generate TNGBC.
A. Irreducible operators

We begin by classifying those operators which cannot be eliminated using the equations of motion. We will categorize them according to $CP$ symmetry.

1. CP-odd operators

The operators we are interested in have the form $O_i \partial^\mu O_j$, where $O_i$ is any of the $SU(2)_L \times U(1)_Y$ invariant expressions shown in (1). Given these operators it is immediate to construct the new ones $(\partial^\mu O_i)O_j$, which also belong to the irreducible group, but they are not independent at all since they are related to the original operators through a surface term. Bearing this in mind, we obtain the following four independent CP-odd operators of dimension eight

\[ O_{W1} = i2\bar{\Phi}(\Phi^\dagger D_\mu \Phi)\text{Tr}[W^{\mu\nu}W_{\lambda\nu}] + h.c., \]
\[ O_{W1B} = i(\Phi^\dagger W_{\mu\nu}\Phi)\partial^\lambda B^{\mu\nu} + h.c., \]
\[ O_{W2} = i(\Phi^\dagger W_{\mu\nu}\Phi)\partial^\lambda B^{\mu\nu} + h.c., \]
\[ O_{B1} = i(\Phi^\dagger D_\mu \Phi)B_{\lambda\nu}\partial^\lambda B^{\mu\nu} + h.c. \]

Notice that the operator $O_{B1}$ contains three $SU(2)_L \times U(1)_Y$ invariant structures which can be contracted with the ordinary derivative in three different ways, leading to the same number of operators. One of them, namely $i\partial^\lambda(\Phi^\dagger D_\mu \Phi)B_{\lambda\nu}B^{\mu\nu}$, is irreducible, but can be expressed by means of integration by parts in terms of $O_{B1}$ and the reducible operator $i(\Phi^\dagger D_\mu \Phi)(\partial^\lambda B_{\lambda\nu})B^{\mu\nu}$, which will be considered later.

2. CP-odd structure of the $ZZZ$, $ZZ\gamma$, and $Z\gamma\gamma$ couplings

The 4-vector $Z_\mu$ arises from the term $(\Phi^\dagger D_\mu \Phi)$ after SSB, whereas the antisymmetric field tensors $F_{\mu\nu}$ and $Z_{\mu\nu}$ appear through the relations

\[ B_{\mu\nu} = c_w F_{\mu\nu} - s_w Z_{\mu\nu}, \]
\[ W^3_{\mu\nu} = s_w F_{\mu\nu} + c_w Z_{\mu\nu} + ig(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu), \]

where $s_w(c_w) = \sin\theta_w(\cos\theta_w)$, with $\theta_w$ the weak mixing angle. After the decomposition of these operators in terms of the mass eigenstate fields, we are left with several Lorentz structures corresponding to TNGBC, though not all of them are independent. Some of them are identical, which is manifest after a subtle manipulation of their Lorentz indices, whereas other ones are related through a surface term. Consequently, the $ZZZ$, $ZZ\gamma$ and $Z\gamma\gamma$ couplings can be described by the following independent Lorentz structures

\[ L^{CP-odd}_{L-ZZZ} = f^{ZZZ}_L Z_\lambda Z_{\mu\nu}\partial^\lambda Z^{\mu\nu} + f^{ZZZ}_L Z_{\mu\nu}Z^{\lambda\nu}\partial^\lambda Z^{\mu}, \]
\[ L^{CP-odd}_{L-ZZ\gamma} = f^{ZZ\gamma}_L Z^\mu F_{\lambda\nu}\partial^\lambda Z_\mu + f^{ZZ\gamma}_L Z_{\mu\nu}\partial^\lambda F^{\mu\nu} + f^{ZZ\gamma}_L Z_{\mu\nu}\partial^\lambda Z^{\mu\nu}, \]
\[ L^{CP-odd}_{L-Z\gamma\gamma} = f^{Z\gamma\gamma}_L F^{\mu\nu} F_{\lambda\nu}\partial^\lambda Z_\mu + f^{Z\gamma\gamma}_L Z_{\mu\nu}\partial^\lambda F^{\mu\nu}, \]

where $L$ is a subscript standing for the linear scheme. The coefficients $f^{ZZZ}_L$ are defined by

\[ f^{ZZZ}_L = \frac{s^2_w}{4gm^2_Z} \left[ 2(c_w\epsilon_{WB1} + s_w\epsilon_{BB1}) + c_w\epsilon_{WB2} \right], \]
\[ f^{ZZZ}_L = \frac{c^3_w}{gm^2_Z} f_{WW1}, \]
with dual, namely $\tilde{O}$ associated with each operator in the EL scheme. The vertex functions are presented in Appendix I.

Operators of this kind can be obtained from the $CP$-odd ones by replacing each strength tensor with its respective dual, namely $\mathbf{W}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\lambda\rho} \mathbf{W}^{\lambda\rho}$, and a similar expression for $\mathbf{B}_{\mu\nu}$. There is a couple of independent $CP$-even operators associated with each one of the $CP$-odd operators $\mathcal{O}_{WW1}$, $\mathcal{O}_{WB2}$, and $\mathcal{O}_{BB1}$. Note that in these operators both $\mathbf{W}$ tensors are contracted via only one of their indices, leading to two independent combinations of the dual tensor. On the other hand, in $\mathcal{O}_{WB1}$ the $\mathbf{W}$ and $B$ tensors appear contracted by both indices. Since the two possible combinations of dual tensors are equivalent, just one $CP$-even operator can be constructed from $\mathcal{O}_{WB1}$. In this way, there are seven independent $CP$-even operators

$$\mathcal{O}_{WW1} = i2\partial^\lambda(\Phi^\dagger D_\mu \Phi) \text{Tr} \left[ \mathbf{W}^{\mu\nu} \mathbf{W}_{\lambda\nu} \right] + h.c.,$$

$$\mathcal{O}_{WB1} = i(\Phi^\dagger \mathbf{W}_{\mu\nu} D_\lambda \Phi) \partial^\lambda \mathbf{B}^{\mu\nu} + h.c.,$$

$$\mathcal{O}_{WB2} = i(\Phi^\dagger \mathbf{W}_{\mu\nu} D_\lambda \Phi) \partial^\mu \mathbf{B}^{\lambda\nu} + h.c.,$$

$$\mathcal{O}_{BB1} = i(\Phi^\dagger D_\mu \Phi) \mathbf{B}_{\lambda\nu} \partial^\lambda \mathbf{B}^{\mu\nu} + h.c.,$$

$$\mathcal{O}_{BB2} = i(\Phi^\dagger D_\mu \Phi) \mathbf{B}_{\lambda\nu} \partial^\mu \mathbf{B}^{\lambda\nu} + h.c.,$$

We can make the ordinary derivative operate on the remaining $SU(2)_L \times U(1)_Y$ invariant terms out of which the previous operators are constructed. The resulting operators are also of the irreducible kind, but they are not independent since, as explained in the $CP$-odd case, all of them are related to the first ones through a surface term.

4. $CP$-even structure of the $ZZZ$, $ZZ\gamma$, and $Z\gamma\gamma$ couplings

After a careful analysis of the Lorentz structure induced by the $CP$-even operators, we find that the $ZZZ$, $ZZ\gamma$, and $Z\gamma\gamma$ couplings are characterized, respectively, by two, five, and three independent Lorentz structures

$$\mathcal{L}^{CP\text{-even}}_{L-ZZZ} = g^{ZZZ}_{L1} Z_\lambda Z_{\mu\nu} \partial^\lambda Z^{\mu\nu} + g^{ZZZ}_{L2} Z_\lambda Z_{\mu\nu} \partial^\mu \bar{Z}^{\lambda\nu} + g^{ZZZ}_{L3} Z_\lambda Z_{\mu\nu} \partial^\nu \bar{Z}^{\lambda\mu},$$

$$\mathcal{L}^{CP\text{-even}}_{L-ZZ\gamma} = g^{ZZ\gamma}_{L1} F_{\mu\nu} Z^{\lambda\nu} \partial_\lambda Z^{\mu} + g^{ZZ\gamma}_{L2} \bar{Z}_{\mu\nu} F^{\lambda\nu} \partial_\lambda Z^{\mu} + g^{ZZ\gamma}_{L3} \bar{Z}_{\mu\nu} F^{\lambda\nu} \partial_\mu \bar{Z}^{\lambda\nu} + g^{ZZ\gamma}_{L4} Z^{\lambda\nu} F^{\mu\nu} \partial_\mu \bar{Z}^{\lambda\nu} + g^{ZZ\gamma}_{L5} Z^{\lambda\nu} F^{\mu\nu} \partial_\nu \bar{Z}^{\lambda\mu} + g^{ZZ\gamma}_{L6} Z^{\lambda\nu} F^{\mu\nu} \partial_\nu \bar{Z}^{\lambda\mu}. $$
where the coefficients are

$$g_{L1}^{ZZZ} = \frac{s_{2w}}{4g_{Z}^{2}} \left[ \frac{2c_{w}^{2}}{s_{w}} (\epsilon_{W}W_{1} + \epsilon_{\bar{W}}\bar{W}_{1}) + 2(c_{w}^{2} - s_{w}^{2}) \right],$$

$$g_{L2}^{ZZZ} = \frac{s_{2w}}{4g_{Z}^{2}} \left[ c_{w}^{2} + 2s_{w}^{2} \right],$$

$$g_{L3}^{ZZZ} = -\frac{2c_{w}}{2g_{Z}^{2}} \left[ c_{w}^{2} + 2s_{w}^{2} \right],$$

$$g_{L4}^{ZZZ} = \frac{c_{w}^{2}}{2g_{Z}^{2}} \left[ s_{w}^{2} \right],$$

$$g_{L5}^{ZZZ} = \frac{c_{w}^{2}}{2g_{Z}^{2}} \left[ 2s_{w} \right],$$

$$g_{L1}^{Z\gamma\gamma} = -\frac{c_{w}^{2}}{2g_{Z}^{2}} \left[ s_{w}^{2} \right],$$

$$g_{L2}^{Z\gamma\gamma} = \frac{s_{2w}}{4g_{Z}^{2}} \left[ c_{w}^{2} + 2s_{w}^{2} \right],$$

$$g_{L3}^{Z\gamma\gamma} = -\frac{s_{2w}}{4g_{Z}^{2}} \left[ c_{w}^{2} + 2s_{w}^{2} \right].$$

The vertex functions are also presented in Appendix I.

**B. Reducible operators**

The operators belonging to the reducible class are proportional to the SU(2)$_{L} \times U(1)$_Y invariants $\partial^{\mu}(\Phi^{I}D_{\mu}\Phi)$ and $\partial^{\mu}B_{\mu \nu}$. While those operators containing the term $\partial^{\mu}(\Phi^{I}D_{\mu}\Phi)$ are proportional to the scalar part of the Z boson, those proportional to the $\partial^{\mu}B_{\mu \nu}$ have the peculiarity that they generate the Lorentz structures required to define the off-shell electromagnetic properties of the Z boson, namely the transition magnetic (electric) dipole and quadrupole moments. All of these operators can be reduced to others by using the equations of motion. To define these structures, it will be necessary to include some operators of dimension ten, but as they can always be expressed in terms of other operators we will content ourselves with list them. We will also present the Lagrangian prescribing the off-shell electromagnetic properties of the Z boson. The operators will be classified according to these properties.
1. Operators that generate the off-shell electromagnetic properties of the Z boson

All these operators are proportional to the SU(2)$_L$ × U(1)$_Y$ invariant $\partial^\mu B_{\mu\nu}$. There are four operators of this class: one couple of CP-odd ones and another couple of CP-even ones

\[
O_{WB3} = i(\Phi\dagger W^\mu{}_{\nu\rho\sigma})D_\rho B_{\sigma\nu} + h.c.,
\]

\[
O_{BB3} = i(\Phi\dagger D_\rho B_{\sigma\nu})D_\rho B_{\sigma\nu} + h.c.,
\]

\[
O_{\tilde{WB}3} = i(\tilde{\Phi}\dagger \tilde{W}^\mu{}_{\nu\rho\sigma})D_\rho B_{\sigma\nu} + h.c.,
\]

\[
O_{\tilde{BB}3} = i(\tilde{\Phi}\dagger D_\rho B_{\sigma\nu})D_\rho B_{\sigma\nu} + h.c.
\]

To define the off-shell electromagnetic properties of the Z boson, it is necessary to include the following operators of dimension ten

\[
O_{WB}^{10} = i(\Phi\dagger W_{\mu\nu}D_\lambda B_{\lambda\nu})\partial_{\mu}\partial^\lambda\partial^\rho B_{\rho\nu} + h.c.,
\]

\[
O_{BB}^{10} = i(\Phi\dagger D_{\lambda\nu}B^\mu\rho\sigma)\partial_{\mu}\partial^\lambda\partial^\rho B_{\rho\nu} + h.c.,
\]

\[
O_{\tilde{WB}}^{10} = i(\tilde{\Phi}\dagger \tilde{W}_{\mu\nu}D_\lambda B_{\lambda\nu})\partial_{\mu}\partial^\lambda\partial^\rho B_{\rho\nu} + h.c.,
\]

\[
O_{\tilde{BB}}^{10} = i(\tilde{\Phi}\dagger D_{\lambda\nu}B^\mu\rho\sigma)\partial_{\mu}\partial^\lambda\partial^\rho B_{\rho\nu} + h.c.
\]

We have excluded any redundant operator, as the ones related through a surface term. The operator $O_{DB} = \Phi\dagger (D_{\mu}D_{\nu} + D_{\nu}D_{\mu})\Phi\partial^\mu\partial^\lambda B_{\lambda\nu}$, which does not contribute to the electromagnetic properties of the Z boson, can be eliminated by using the equations of motion. The Lorentz structures defining the off-shell electromagnetic properties of the Z boson can be conveniently parametrized by the following Lagrangian

\[
\mathcal{L}_{ZZ} = -e \left[ \left( h_1^F F_{\mu\nu} + h_3^F \tilde{F}^\mu_{\nu\rho\sigma} \right) Z_{\mu} \frac{\partial^\lambda Z_{\lambda\nu}}{m_Z^2} + \left( h_2^F F_{\mu\nu} + h_4^F \tilde{F}^\mu_{\nu\rho\sigma} \right) Z_{\mu} \frac{\partial^\lambda Z_{\lambda\nu}}{m_Z^2} \right],
\]

where the coefficients are

\[
h_1^Z = -\frac{s_{2w}}{8s_w^2\epsilon_0} \epsilon_8,
\]

\[
h_3^Z = -\frac{s_{2w}}{8s_w\epsilon_0^2} \epsilon_8,
\]

\[
h_2^Z = -\frac{s_{2w}}{8w^2\epsilon_0} \epsilon_{10},
\]

\[
h_4^Z = -\frac{s_{2w}}{8w^6\epsilon_0} \epsilon_{10}.
\]

The parameters $\epsilon_8$ and $\epsilon_{10}$ depend on the coefficients of the operators of dimension eight and ten, and are given by

\[
\epsilon_8 = s_w^2\epsilon_{WB3} + 2c_w^2\epsilon_{BB3},
\]

\[
\tilde{\epsilon}_8 = s_w^2\epsilon_{\tilde{WB}3} + 2c_w^2\epsilon_{\tilde{BB}3},
\]

\[
\epsilon_{10} = s_w^2\epsilon_{WB}^{10} + 2c_w^2\epsilon_{BB}^{10},
\]

\[
\tilde{\epsilon}_{10} = s_w^2\epsilon_{\tilde{WB}}^{10} + 2c_w^2\epsilon_{\tilde{BB}}^{10}.
\]

where $\epsilon_{i}^{10} = (m_Z/\Lambda)^6\alpha_i$ are the coefficients of the dimension ten operators. The transition moments are defined as
\[\mu_Z = -\frac{e}{\sqrt{2m_Z}} \frac{E^2}{m_Z^2} (h_1^Z - h_2^Z),\]  
(37a)

\[Q_Z = -\frac{2\sqrt{10}e}{m_Z^2} h_1^Z,\]  
(37b)

\[d_Z = -\frac{e}{\sqrt{2m_Z}} \frac{E^2}{m_Z^2} (h_2^Z - h_3^Z),\]  
(37c)

\[Q_Z^m = -\frac{2\sqrt{10}e}{m_Z^2} h_3^Z,\]  
(37d)

where \(\mu_Z\) (\(d_Z\)) is the off-shell magnetic (electric) dipole moment and \(Q_Z^m (Q_Z)\) is the magnetic (electric) quadrupole moment of the Z boson.

2. Operators proportional to the scalar part of the Z boson

These operators are characterized by the SU(2)_L \times U(1)_Y invariant \(\partial_\mu (\Phi^\dagger D^\mu \Phi)\). There are three CP-odd operators of this type

\[O_{WW2} = i 2 \partial_\lambda (\Phi^\dagger D^\lambda \Phi) \text{Tr} [W^\mu \nu W^{\mu\nu}] + h.c.,\]  
(38)

\[O_{BB2} = i \partial_\lambda (\Phi^\dagger D^\lambda \Phi) B_\mu B^{\mu\nu} + h.c.,\]  
(39)

\[O_{D\Phi} = i \Phi^\dagger (D_\mu D_\nu + D_\nu D_\mu) \Phi \Phi^\dagger (\Phi^\dagger D^\nu \Phi) + h.c.\]  
(40)

The last operator generates only the \(ZZZ\) coupling, which can be expressed by integration by parts as a coupling proportional to the scalar part of the Z boson. As for CP-even operators, there are only a pair of this kind

\[O_{W\bar{W}2} = i 2 \partial_\lambda (\Phi^\dagger D^\lambda \Phi) \text{Tr} [W^\mu \nu \tilde{W}^{\mu\nu}] + h.c.,\]  
(41)

\[O_{B\bar{B}2} = i \partial_\lambda (\Phi^\dagger D^\lambda \Phi) B_\mu \bar{B}^{\mu\nu} + h.c.\]  
(42)

We have ignored any operator which can be expressed as a linear combination of those given above.

III. THE NONDECOUPLING SCENARIO

We will proceed now to consider the possibility that new physics effects do not decouple from low energy physics. In this situation, the relevant SU(2)_L \times U(1)_Y invariant structures are the same as in the linear case, with the Higgs doublet being replaced by the following unitary matrix

\[U = \exp(\frac{2i\sigma^i \phi^i}{v}),\]  
(43)

where the \(\phi^i\) scalars would become Goldstone bosons. The covariant derivative in the nonlinear realization of the SU(2)_L \times U(1)_Y group is defined as \(D_\mu U = \partial_\mu U + iqW_\mu U - iqU B_\mu\), where the Abelian field is now defined as \(B_\mu = (\sigma^3/2)B_\mu\). The basic structures out of which TNGBC can be constructed are the SU(2)_L \times U(1)_Y invariants \(\text{Tr} [\sigma^3 U \dagger D_\mu U], \text{Tr} [U \dagger (D_\mu D_\nu + D_\nu D_\mu) U],\) and \(\text{Tr} [U \dagger W^\mu \nu D_\lambda U],\) which in mass units have dimension one, two, and three. Like their linear counterparts, these invariants are essential to construct any TNGBC because they induce the Lorentz structures \(Z_\mu, Z_\nu Z_\mu,\) and \(Z_\lambda Z_\mu Z_\nu (F_\mu \nu).\) Since these structures have a lower dimension than their analogous in the linear case, in the nonlinear scenario not only it is possible to construct dimension six operators inducing TNGBC, but it is also possible a larger number of independent operators. As we will show below, there exist some operators of dimension four which induce the \(ZZZ\) coupling, though not the \(ZZ\gamma\) and \(Z\gamma\gamma\) ones. Nevertheless, such operators are proportional to the scalar part of the Z boson and belong to the reducible group. We will use the same criterion used in the linear case to classify all of the independent operators. We will refrain from any technical detail already explained in the linear case if it is not relevant for the present discussion.
A. Irreducible operators

These operators are proportional to the SU(2) \(_L\) \(\times\) U(1)\(_Y\) invariant structures \(\text{Tr} \left[ \sigma^3 U^d \mu U \right]\) and \(\text{Tr} \left[ U^\dagger W_{\mu \nu} D_\lambda U \right]\). We will classify them according to \(CP\) symmetry.

1. \(CP\)-odd operators

The dimension six operators resembling those of the linear scenario are the following

\[
\mathcal{L}_{WW1} = 2i \frac{\lambda_{WW1}}{\Lambda^2} \partial^\lambda \text{Tr} \left[ \sigma^3 U^d \mu U \right] \text{Tr} \left[ W^{\mu \nu} W_{\lambda \nu} \right] + h.c.,
\]

\[
\mathcal{L}_{WB1} = i \frac{\lambda_{WB1}}{\Lambda^2} \text{Tr} \left[ U^\dagger W_{\mu \nu} D_\lambda U \right] \partial^\lambda B^{\mu \nu} + h.c.,
\]

\[
\mathcal{L}_{WB2} = i \frac{\lambda_{WB2}}{\Lambda^2} \text{Tr} \left[ U^\dagger W_{\mu \nu} D_\lambda U \right] \partial^\mu B^{\lambda \nu} + h.c.,
\]

\[
\mathcal{L}_{BB1} = i \frac{\lambda_{BB1}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^d \mu U \right] B_{\lambda \nu} \partial^\lambda B^{\mu \nu} + h.c.,
\]

where we are using the symbol \(\Lambda\), introduced in the linear case, to denote the new physics scale. As the structure \(\text{Tr} \left[ \sigma^3 U^d \mu U \right]\) has dimension one, we can construct three new independent operators of dimension six which have no dimension eight counterpart in the linear realization. They are given by

\[
\mathcal{L}_{DD} = i \frac{\lambda_{DD}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^d \mu U \right] \partial^\mu \text{Tr} \left[ \sigma^3 U^d \mu U \right] + h.c.,
\]

\[
\mathcal{L}_{DB1} = \frac{\lambda_{DB1}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^d \mu U \right] \partial_\lambda \text{Tr} \left[ \sigma^3 U^d \mu U \right] \partial^\lambda B^{\mu \nu} + h.c.,
\]

\[
\mathcal{L}_{DB2} = \frac{\lambda_{DB2}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^d \mu U \right] \partial_\nu \text{Tr} \left[ \sigma^3 U^d \mu U \right] \partial^\lambda B^{\mu \nu} + h.c.
\]

Note that in the linear scheme the operator corresponding to \(\mathcal{L}_{DD}\) have dimension twelve, whereas those related to \(\mathcal{L}_{DB1}\) and \(\mathcal{L}_{DB2}\) are of dimension ten. These operators have the peculiarity that they induce TNGBC exclusively, i.e. there are no interactions containing a charged \(W\) boson, which can be seen by noting that the structure \(\text{Tr} \left[ \sigma^3 U^d \mu U \right]\) is proportional to the \(Z_\mu\) boson in the unitary gauge. While the first one of these operators induces only the \(ZZZ\) coupling, the remaining ones generate both the \(ZZZ\) and \(Z\gamma\) couplings. There is no \(Z\gamma\) coupling arising from these kind of operators, which implies that the Lorentz structure of it is the same in both the linear and the nonlinear realizations of the electroweak group, at least at this order.

2. \(CP\)-odd structure of the \(ZZZ\), \(ZZ\gamma\), and \(Z\gamma\) couplings.

After decomposing the nonlinear \(CP\)-odd operators in terms of the physical fields, we have found that the \(ZZZ\) coupling can be described by five independent Lorentz structures, and so is the \(Z\gamma\) vertex. On the other hand, the \(Z\gamma\) coupling becomes changed, as compared to its counterpart in the linear case, in its coefficients but not in its Lorentz structure. We thus have

\[
\mathcal{L}_{NL-ZZZ}^{CP-odd} = \mathcal{L}_{L-ZZZ}^{CP-odd} + f_{NL}^{ZZZ} Z_\mu \Box Z_\nu \partial^\mu Z^\nu + f_{NL}^{ZZZ} Z_\mu \partial_\lambda Z_\nu \partial^\lambda Z^{\mu \nu} + f_{NL}^{ZZZ} Z_\mu \partial_\nu Z_\lambda \partial^\lambda Z^{\mu \nu},
\]

\[
\mathcal{L}_{NL-ZZ\gamma}^{CP-odd} = \mathcal{L}_{L-ZZ\gamma}^{CP-odd} + f_{NL}^{ZZ\gamma} Z_\mu \partial_\lambda Z_\nu \partial^\lambda F^{\mu \nu} + f_{NL}^{ZZ\gamma} Z_\mu \partial_\nu Z_\lambda \partial^\lambda F^{\mu \nu},
\]

\[
\mathcal{L}_{NL-Z\gamma\gamma}^{CP-odd} = \mathcal{L}_{L-Z\gamma\gamma}^{CP-odd},
\]

where the respective coefficients are obtained from those of the linear scenario (Sec. [I]) through the relation

\[
f_{NL}^{VVV} = \left( \frac{\Lambda}{m_Z} \right)^2 f_{LL}^{VVV},
\]
with the remaining coefficients being given by

\[
\begin{align*}
\tilde f_{NL3}^{ZZZ} & = \frac{2g^3}{c_w^3 m_Z^2} \epsilon_{DD}, \quad (55a) \\
\tilde f_{NL4}^{ZZZ} & = \frac{2g^2 s_w}{c_w^2 m_Z^2} \epsilon_{DB1}, \quad (55b) \\
\tilde f_{NL5}^{ZZZ} & = \frac{2g^2 s_w}{c_w m_Z^2} \epsilon_{DB2}, \quad (55c) \\
\tilde f_{NL4}^{ZZ} & = \frac{2g^2}{c_w m_Z^2} \epsilon_{DB1}, \quad (56a) \\
\tilde f_{NL5}^{ZZ} & = \frac{2g^2}{c_w m_Z^2} \epsilon_{DB2}. \quad (56b)
\end{align*}
\]

We have also introduced the definition \( \epsilon_i = (m_Z/\Lambda)^2 \lambda_i \), where \( \lambda_i \) represents the coefficient associated with each operator of the nonlinear scenario. All the vertex functions are given in Appendix II.

3. CP-even operators

There are eight operators belonging to this class. Seven of them can be easily obtained from their linear counterparts whereas a new one is obtained from the CP-odd operator \( \mathcal{L}_{DB1} \) when the tensor \( B_{\mu\nu} \) is replaced by its dual. The CP-odd operator which is equivalent to \( \mathcal{L}_{DB2} \) is not independent as it generates TNGBC with a Lorentz structure already induced by the operators resembling those of the linear case. In this way, we are left with the following independent CP-even operators

\[
\begin{align*}
\tilde \mathcal{L}_{WW1} & = 2i \frac{W_1}{\Lambda^2} \partial^\lambda \text{Tr} \left[ \sigma^3 U^\dagger D_{\mu} U \right] \text{Tr} \left[ \tilde W^{\mu\nu} W_{\lambda\nu} \right] + h.c., \quad (57) \\
\tilde \mathcal{L}_{WW1} & = 2i \frac{W_1}{\Lambda^2} \partial^\lambda \text{Tr} \left[ \sigma^3 U^\dagger D_{\mu} U \right] \text{Tr} \left[ W^{\mu\nu} \tilde W_{\lambda\nu} \right] + h.c., \quad (58) \\
\tilde \mathcal{L}_{WB1} & = i \frac{W_B}{\Lambda^2} \text{Tr} \left[ U^\dagger W_{\mu\nu} D_{\lambda} U \right] \partial^\lambda \tilde B^{\mu\nu} + h.c., \quad (59) \\
\tilde \mathcal{L}_{WB2} & = i \frac{W_B}{\Lambda^2} \text{Tr} \left[ U^\dagger W_{\mu\nu} D_{\lambda} U \right] \partial^\mu \tilde B^{\lambda\nu} + h.c., \quad (60) \\
\tilde \mathcal{L}_{WB2} & = i \frac{W_B}{\Lambda^2} \text{Tr} \left[ U^\dagger W_{\mu\nu} D_{\lambda} U \right] \partial^\mu \tilde B^{\lambda\nu} + h.c., \quad (61) \\
\tilde \mathcal{L}_{BB1} & = i \frac{B_{\mu\nu}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^\dagger D_{\mu} U \right] B_{\lambda\nu} \partial^\lambda B^{\mu\nu} + h.c., \quad (62) \\
\tilde \mathcal{L}_{BB1} & = i \frac{B_{\mu\nu}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^\dagger D_{\mu} U \right] B_{\lambda\nu} \partial^\lambda B^{\mu\nu} + h.c., \quad (63) \\
\tilde \mathcal{L}_{BB1} & = i \frac{B_{\mu\nu}}{\Lambda^2} \text{Tr} \left[ \sigma^3 U^\dagger D_{\mu} U \right] \partial_{\lambda} \text{Tr} \left[ \sigma^3 U^\dagger D_{\nu} U \right] \partial^\lambda \tilde B^{\mu\nu} + h.c. \quad (64)
\end{align*}
\]

4. CP-even structure of the ZZZ, ZZ\(\gamma\), and \(Z\gamma\gamma\) couplings

As far as their Lorentz structure is concerned, both the ZZZ and the ZZ\(\gamma\) couplings differ from their analogues in the linear realization. They receive a new contribution arising from the operator \( \tilde \mathcal{L}_{DB1} \). The ZZZ coupling in turn is characterized by three independent Lorentz structures
The ones of dimension six are given by counterpart by replacing $\Phi^\dagger$ electromagnetic properties of the $Z$ which, however, does not contribute to the electromagnetic properties of the $Z$. The Lorentz structure of the $Z\gamma\gamma$ vertex coincides with one of its linear counterpart. As for the coefficients appearing in the last equations, they are given in terms of the linear ones by means of a relation similar to (64). The remaining coefficients are

$$g_{NL3}^{ZZZ} = \frac{2g^2 s_w}{c_w m_Z^2} \mu_1$$

The respective vertex function are presented in Appendix II.

B. Reducible operators

As was the case in the linear scenario, we can classify the reducible operators in those contributing to the off-shell electromagnetic properties of the $Z$ boson, and those which are proportional to the scalar part of the $Z$ boson.

1. Operators that generate the off-shell electromagnetic properties of the $Z$ boson

These operators are proportional to the SU(2)$_L$ × U(1)$_Y$ invariant $\partial_\mu B^{\mu\nu}$, and are obtained from their linear counterpart by replacing $\Phi^\dagger D_\mu \Phi$ with $\text{Tr} [\sigma^3 U^\dagger D_\mu U]$. This give rise to dimension six and dimension eight operators. The ones of dimension six are given by

$$\mathcal{L}_{WB3} = i \frac{\lambda_{WB3}}{\Lambda^2} \text{Tr} [U^\dagger W^{\mu\nu} D_\mu U] \partial^\lambda B_{\lambda\nu} + \text{h.c.},$$

$$\mathcal{L}_{BB3} = i \frac{\lambda_{BB3}}{\Lambda^2} \text{Tr} [U^\dagger D_\mu U] B^{\mu\nu} \partial^\lambda B_{\lambda\nu} + \text{h.c.},$$

$$\mathcal{L}_{WB3} = i \frac{\lambda_{WB3}}{\Lambda^2} \text{Tr} [U^\dagger \tilde{W}^{\mu\nu} D_\mu U] \partial^\lambda B_{\lambda\nu} + \text{h.c.},$$

$$\mathcal{L}_{BB3} = i \frac{\lambda_{BB3}}{\Lambda^2} \text{Tr} [\sigma^3 U^\dagger D_\mu U] \tilde{B}^{\mu\nu} \partial^\lambda B_{\lambda\nu} + \text{h.c.}.$$  

Just as in the linear realization, there is another $CP$-odd dimension six operator being given by

$$\mathcal{L}_{DB} = \frac{\lambda_{DB}}{\Lambda^2} \text{Tr} [U^\dagger (D_\mu D_\nu + D_\nu D_\mu) U] \partial^\mu \partial^\nu \partial^\lambda B_{\lambda\nu},$$

which, however, does not contributes to the electromagnetic properties of the $Z$ boson. The operators of dimension eight, necessary to an adequate definition of the electric and magnetic transition dipole and quadrupole moments, are given by

$$\mathcal{L}_{WB}^8 = i \frac{\lambda_{WB}}{\Lambda^4} \text{Tr} [U^\dagger W^{\mu\nu} D_\lambda U] \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \text{h.c.},$$

$$\mathcal{L}_{BB}^8 = i \frac{\lambda_{BB}}{\Lambda^4} \text{Tr} [\sigma^3 U^\dagger D_\lambda U] \partial_{\mu\nu} \partial_\mu \partial^\lambda D_{\rho\nu} + \text{h.c.},$$

$$\mathcal{L}_{WB}^8 = i \frac{\lambda_{WB}}{\Lambda^4} \text{Tr} [U^\dagger \tilde{W}^{\mu\nu} D_\lambda U] \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \text{h.c.},$$

$$\mathcal{L}_{BB}^8 = i \frac{\lambda_{BB}}{\Lambda^4} \text{Tr} [\sigma^3 U^\dagger D_\lambda U] \tilde{B}^{\mu\nu} \partial_\mu \partial^\lambda \partial^\rho B_{\rho\nu} + \text{h.c.}.$$  

They induce the off-shell electromagnetic properties of the $Z$ boson through the Lagrangian given in Sec. II. The coefficients $h_{1,3}^Z$ and $h_{2,4}^Z$ are obtained from those of the linear scenario after multiplying them by the factor $(\Lambda/m_Z)^2$ and $(\Lambda/m_Z)^4$, respectively.
These operators are proportional to the SU(2)$_L \times$ U(1)$_Y$ invariant $\partial_\mu \text{Tr} \left[ \sigma^3 U^\dagger D^\mu U \right]$. As previously mentioned, there are a pair of dimension four CP-odd operators which generate just the ZZZ vertex. They are given by

\begin{align}
\mathcal{L}^4_1 &= i \lambda_1 \text{Tr} \left[ \sigma^3 U^\dagger D_\nu U \right] \text{Tr} \left[ \sigma^3 U^\dagger D^\nu U \right] \partial_\mu \text{Tr} \left[ \sigma^3 U^\dagger D^\mu U \right] + h.c., \\
\mathcal{L}^4_2 &= i \lambda_2 \text{Tr} \left[ \sigma^3 U^\dagger D^\nu U \right] \partial^\mu \text{Tr} \left[ U^\dagger (D_\mu D_\nu + D_\nu D_\mu) U \right] + h.c.
\end{align}

(78)  
(79)

The linear counterpart of the operator $\mathcal{L}^4_1$ has dimension ten, while the one associated with $\mathcal{L}^4_2$ has dimension eight, as described in Sec. I. The remaining operators have dimension six, and are obtained from those given in the linear case by the replacement of $\Phi^\dagger D_\mu \Phi$ by $\text{Tr} \left[ \sigma^3 U^\dagger D_\mu U \right]$. There are four operators of this type: one pair of CP-odd ones as well as one pair of CP-even ones.

\begin{align}
\mathcal{L}_{WW2} &= 2i \frac{\lambda_{WW2}}{\Lambda^2} \partial_\lambda \text{Tr} \left[ \sigma^3 U^\dagger D^\lambda U \right] \text{Tr} \left[ W_{\mu\nu} W^{\mu\nu} \right] + h.c., \\
\mathcal{L}_{BB2} &= i \frac{\lambda_{BB2}}{\Lambda^2} \partial_\lambda \text{Tr} \left[ U^\dagger D^\lambda U \right] B_{\mu\nu} B^{\mu\nu} + h.c., \\
\mathcal{L}_{WB2} &= 2i \frac{\lambda_{WB2}}{\Lambda^2} \partial_\lambda \text{Tr} \left[ \sigma^3 U^\dagger D^\lambda U \right] \text{Tr} \left[ W_{\mu\nu} \tilde{W}^{\mu\nu} \right] + h.c., \\
\mathcal{L}_{B\tilde{B}2} &= i \frac{\lambda_{B\tilde{B}2}}{\Lambda^2} \partial_\lambda \text{Tr} \left[ U^\dagger D^\lambda U \right] B_{\mu\nu} \tilde{B}^{\mu\nu} + h.c.
\end{align}

(80)  
(81)  
(82)  
(83)

IV. CONSTRAINTS FROM PRECISION MEASUREMENTS

Once a complete treatment of the effective operators inducing TNGBC has been presented within both the linear and the nonlinear realizations of the SU(2)$_L \times$ U(1)$_Y$ gauge symmetry, our major concern lies in how to get bounds on the respective coefficients of these operators from current phenomenology. In this respect, considerable work exists in the literature where bounds on anomalous trilinear gauge boson couplings $W^+W^-\gamma$ have been analyzed. To this purpose, measurements on some observables have been extensively used, such as the magnetic and the electric dipole moments of elementary fermions, the $Z \rightarrow bb$ branching fraction, as well as the processes $e^+e^- \rightarrow WW$ and $W^* \rightarrow W\gamma \hspace{1cm} (84)$. As for TNGBC, bounds on these couplings have been obtained through the processes $e^+e^- \rightarrow Z\gamma(Z)$ and $q\bar{q} \rightarrow Z\gamma(Z)$, although such studies involve only those operators in which two gauge bosons are on-shell. To obtain bounds on our operators, we will follow a similar approach as that in previous works. We will also consider the rare decay $Z \rightarrow \nu\bar{\nu}\gamma$, which is affected at tree level by TNGBC through the $ZZ\gamma$ vertex. Since its SM contribution is insignificant, this process might offer an invaluable mode to unravel any latent new physics effect.

A. Decoupling scenario

We will start by examining the situation in the decoupling scheme of EL. Rather than performing any explicit calculation, it is worth to begin with estimating on a general basis the expectations we should have as regards to the size of TNGBC. It has been pointed out that persuasive theoretical arguments indicate that one loop generated anomalous trilinear gauge boson couplings are unlikely expected to be above the one percent level (85). Indeed, the fact that TNGBC are induced at one loop level suggests that they are of order $(g/4\pi)^2$ in a wide class of models. It has also been conjectured that even in theories with underlying strong dynamics, trilinear gauge couplings are expected to have a sizeable enhancement. In the SM, the $ZZ\gamma(Z)$ couplings are severely constrained, even in the presence of a fourth fermion family they are highly suppressed and thus out of the range of detectability (86)(87). Regarding the bounds arising from phenomenological grounds, we would like to begin by examining in a qualitative way whether the current measurements on the magnetic and electric dipole moments of elementary fermions can give any useful bound on TNGBC.

The effective operators presented so far not only induce TNGBC but also anomalous $W^+W^-\gamma$ couplings. An exhaustive analysis on phenomenological constraints would require to compute every contribution to the observable under study, including also the ones coming from all of the lower dimension operators inducing vertices which also affect the process. For the sake of simplicity, a crude estimate can be obtained if just some operators are considered at a time. In the specific case of the magnetic moment of leptons, which receives contributions from CP-even
operators exclusively, a profuse work has been devoted to study comprehensively the contributions from the lowest order effective operators respecting the SU(2)L × U(1)Y gauge invariance, linearly and nonlinearly realized, which induce nonstandard anomalous couplings. In this respect, there are one loop generated operators of dimension six which induce $W^+ W^- \gamma$ couplings, but not TNGBC. These operators contribute to the magnetic moment of leptons via their insertion in the loop diagram depicted in Fig. 1 [23]. Secondly, some dimension six operators directly induce the magnetic moment term at tree level, though they are generated at the one loop level. Finally, the redefinition of the gauge fields, necessary to an adequate definition of the quadratic part of the theory, also affects the anomalous magnetic moment value. To obtain bounds on the coefficients of the CP conserving operators, the full contribution to the anomalous magnetic moment of the muon was computed [24]. In this respect, the strategy which has been found to be the most suitable for estimating the size of loops involving an effective vertex is that of dimensional regularization, together with the minimal subtraction (MS) renormalization scheme. According to it and retaining only the leading logarithmic dependence on the new physics scale $\Lambda$, it was found that the contribution from dimension six operators inducing the $W^+ W^- \gamma$ vertex is given by

$$\delta a_\mu = \eta_0 \left( \frac{m_\mu}{\Lambda} \right)^2 O(\log^2 \frac{m_\mu^2}{m_W^2} \alpha_L),$$

(84)

where $\eta_0$ is a factor dependent on the particular graph, and $\alpha_L$ is directly related to the operator coefficients. Numerically, one obtains from this equation $|\delta a_\mu|/10^{-3} = \alpha_L \left( 1 + \log \Lambda / \Lambda^2 \right)$. If it is taken the accepted lowest value of 1 TeV for the new physics scale $\Lambda$, we are left with the unpromising result that the operator coefficient should be of order $O(1)$ to have any chance of being detected. But this result is far beyond the estimate of $\alpha_L$ being of order $(g/4\pi)^2$. Indeed, only the direct contribution is expected to give a measurable contribution to the magnetic moment of the muon. In view of this result, it is natural to think that we should not expect a better situation for TNGBC since they are generated by higher order operators. We note that dimension eight operators are suppressed by the factor $(v/\Lambda)^2$, with $v = 246$ GeV the vacuum expectation value, with respect to dimension six operators. A rough estimate is obtained if we multiply [23] with the suppression factor and evaluate again at $\Lambda = 1$ TeV. We obtain the discouraging result that $\alpha_L$ should be of order $O(100)$, which is very unlikely to occur, to allow any TNGBC to be experimentally detected. By way of illustration, we have explicitly computed the contribution to the muon anomalous magnetic moment which is obtained by introducing in the one loop diagram of Fig. 1 the effective $ZZ\gamma$ vertex whose Feynman rule is given by equations (24a) and (103). After isolating the divergent part, the application of the MS scheme gives

$$\delta a_{\mu_{L1}}^{L1} = \frac{\ell_\omega (4s_\omega^2 - 1)g}{256\pi^2} \left( \frac{m_\mu}{\Lambda} \right)^2 \left( \frac{m_\mu}{m_Z} \right)^2 \left( \log \left( \frac{\Lambda}{m_Z} \right)^2 + \frac{3}{4} \right) \bar{\epsilon}_{L1},$$

(85)

where $\bar{\epsilon}_{L1}$ is the factor multiplying $s_\omega/(gm_Z^2)$ in the coefficient $g_L^{ZZ\gamma}$ which is defined in (24). In fact, if this equation is numerically evaluated we find that the actual bound is looser than the rough estimate. We thus see that there is few hopes that a reasonable bound on CP-even TNGBC could be obtained from precision measurements on the magnetic moment of the muon. Although we have examined the situation of only one vertex, the same result is expected for the remaining ones. In fact, as shown in the appendices, the Lorentz structure which parametrizes TNGBC does not differ essentially for each case. The best result would be obtained if all contributions add up coherently, though there is no evident reason to expect it.

A similar analysis can be done for the $CP$-odd operators which contribute to the electric dipole moment of fermions. In this case a strong bound, from precision measurements on the electric dipole moment of neutron, exists on dimension six operators inducing anomalous $W^+ W^- \gamma$ couplings [23]. The respective operator coefficients are constrained to lie below the $10^{-3}$ level. Since our $CP$-odd operators, which also induce anomalous $W^+ W^- \gamma$ couplings, are of dimension eight in the decoupling scenario we could not expect to get a better bound for their coefficients. Once more, a rough estimate would be obtained by dividing the bound on dimension six operators by the suppression factor $(v/\Lambda)^2$.

Now let us focus on the rare $Z$ boson decay $Z \to \nu \tau \gamma$, which has been studied within both the SM realm and the EL approach [3, 4]. It was shown that the SM contribution turns out to be negligible small, with a branching ratio of order $10^{-10}$ [4]. In the EL approach, this process arises at tree level, as depicted in Fig. 2. In addition it has also the advantage of receiving contributions from TNGBC only through the $ZZ\gamma$ vertex, as depicted in Fig. 2a. Although there are also lower dimension effective operators contributing to $Z \to \nu \tau \gamma$ through the Feynman diagrams of Fig. 2b and 2c, we will not include those contributions in here since they are not associated with TNGBC. Furthermore, we are only interested on estimating the best possible bound on TNGBC.

The measurement of energetic single-photons at LEP arising from the decay $Z \to \nu \tau \gamma$ has been used to put a direct limit on the magnetic moment of the $\tau$ neutrino [26]. For the purpose of the present analysis, the search for the energetic single-photons events on the data collected by the L3 collaboration may be translated into bounds on
TNGBC. In order to reduce backgrounds, the L3 collaboration required the photon energy to be greater than one half the $e^+e^-$ beam energy. It was obtained a limit on the branching ratio for $Z \to \nu\bar{\nu}\gamma$ of one part in a million when the photon energy is above 30 GeV \[9\]. Expressing the invariant amplitude $M$ in terms of the variables $x = 2k_1.p_1/m_Z^2$ and $y = 2k_2.p_2/m_Z^2$, the $Z(k_2) \to A(k_1)\nu(p_1)\bar{\nu}(p_2)$ decay width is given by

$$
\Gamma(Z \to \bar{\nu}\nu\gamma) = \frac{m_Z^3}{256\pi^3} \int_0^1 dx \int_0^{1-x} dy |M|^2.
$$

We have not imposed any energy cutoff since we want to estimate the TNGBC bounds in a conservative way. From the Feynman rules for the $ZZ\gamma$ vertex in Appendix I is obtained

$$
|\mathcal{M}|^2 = \frac{1}{32} \left((x^2 + y^2)(1-x-y) - 4xy\right)\left(\alpha^2 + \bar{\alpha}^2\right),
$$

$$
\alpha \equiv \alpha_L = \alpha_{L1} + \alpha_{L2} - \alpha_{L3},
$$

$$
\bar{\alpha} \equiv \bar{\alpha}_L = \bar{\alpha}_{L1} - \bar{\alpha}_{L2} - \bar{\alpha}_{L3} + \bar{\alpha}_{L4} + \bar{\alpha}_{L5}.
$$

As natural, there is no interference between $CP$ violating and $CP$ conserving couplings. The coefficients $\alpha_{Li}$ ($\bar{\alpha}_{Li}$), which in turn are related to the $CP$-odd ($CP$-even) operator coefficients, can be extracted from \[(1)\] and \[(2)\], being given by

$$
\alpha_{Li} = \left(\frac{g m_Z^2}{c_w}\right) \bar{f}_{Li}^{ZZ\gamma},
$$

$$
\bar{\alpha}_{Li} = \left(\frac{g m_Z^2}{c_w}\right) g_{Li}^{ZZ\gamma}.
$$

After performing the integration in equation \[(86)\] we have

$$
BR(Z \to \bar{\nu}\nu\gamma) = 2.912 \times 10^{-5} \left(\alpha_L^2 + \bar{\alpha}_L^2\right).
$$

taking the value $\Lambda = 1$ TeV and considering the L3 bound on the respective branching fraction, we obtain again the result that the size of the $ZZ\gamma$ coupling should be beyond any reasonable expectation to become perceptible through the process $Z \to \nu\bar{\nu}\gamma$. Stated in other words, we may not expect moderate bounds from this process. The reason of such a discouraging result is the natural suppression of dimension eight operators. Our viewpoint would be more pessimistic if we consider that in this calculation only those contributions arising from effective operators inducing the $ZZ\gamma$ coupling have been included. However, there is no compelling reason to disregard any other new physics contributions, such as the ones coming from the Feynman diagrams shown in the figures 2a and 2b \[3\]. In view of our results, it is conceivable to state that any TNGBC associated with underlying physics respecting linearly the SU(2)$_L \times U(1)_Y$ symmetry would not be measurable through the processes investigated in this work. However, we cannot discard the case in which certain TNGBC is given by a sum of loops whose contributions add up coherently to give a large value.

### B. Nondecoupling scenario

We will turn to analyze the situation in the nonlinear scenario, where TNGBC are generated by dimension six operators. Therefore, we might expect a better situation than that in the decoupling scenario. We will see that the discussion for the linear scenario can be easily translated to comprise the nonlinear case. To begin with, we can see from the Feynman rules in appendix II that in the nonlinear scenario the $CP$-even $ZZ\gamma$ vertex is parametrized by one extra Lorentz structure in addition to those parametrizing this vertex in the decoupling case. The result given in \[(83)\] for the linear scenario can be directly used if we consider the substitution rule applicable to the operator coefficients, that is $g_{N Li}^{ZZ\gamma} = (\Lambda/m_Z)^2 g_{Li}^{ZZ\gamma}$. We then have that the leading term obtained by including in the loop graph of Fig. \[3\] the $ZZ\gamma$ vertex associated with $g_{N Li}$ is
\[ \delta a_{\mu}^{NL} = \frac{t_w (4s_w^2 - 1) g^2 (\frac{m_\mu}{v})^2}{256\pi^2} \left( \log \left( \frac{v}{m_Z} \right)^2 + \frac{3}{4} \right) \tilde{\epsilon}_{NL1}, \]

where we have employed the conservative value \( \Lambda \to v \). Numerically one obtains \( \delta a_{\mu}^{NL} = -0.767 \times 10^{-9} \tilde{\epsilon}_{NL1} \). On the other hand, the data collected through the BNL E281 experiment together with the SM predictions put a bound on any new physics contribution to \( \alpha_{\mu} \) of \(-7.1 \times 10^{-9} < \delta a_{\mu} < 22.1 \times 10^{-9} \) at 95 \%CL \[27\]. As a consequence, probing \( \delta a_{\mu}^{NL} \) at the \( \pm 10^{-9} \) level provides a sensitivity to \( \tilde{\epsilon}_{NL} \) of about \( O(1) \) at most, which translates into a loose bound for the operator coefficients \( \lambda_i \). This situation is not better than the result obtained in \[24\] for the dimension four operators inducing \( W^+W^-\gamma \) couplings within the nonlinear scheme. Moreover, as there are other sources of new physics which can affect the anomalous magnetic moment, it is hard to think that any TNGBC could be competitive in this process, even in the nonlinear scenario.

Regarding the rare decay \( Z \to \nu\gamma \), after the inclusion of all the contributions arising from the \( ZZ\gamma \) vertex we have that \[27\] remains valid, though \( (88a) \) and \( (88b) \) now read

\[ \alpha \equiv \alpha_{NL} = \alpha_{NL1} + \alpha_{NL2} - 2\alpha_{NL3} + 2\alpha_{NL4} + \alpha_{NL5}, \]

\[ \alpha \equiv \tilde{\alpha}_{NL} = \tilde{\alpha}_{NL1} - \tilde{\alpha}_{NL2} - \tilde{\alpha}_{NL3} + \tilde{\alpha}_{NL4} + \tilde{\alpha}_{NL5} + 2\tilde{\alpha}_{NL6}. \]

The new coefficients \( \alpha_{NLi} \) and \( \tilde{\alpha}_{NLi} \) are obtained, with the adequate subscript substitutions, via the relations \( (89a) \) and \( (89b) \), which also hold for the nonlinear scenario. The coefficients \( J_{NLi}^{ZZ\gamma} \) have been given in previous sections. We will only concentrate in the \( CP \)-conserving term, which has been widely studied in the literature. Equation \( (10) \) and the L3 limit for the respective branching ratio give the bound \( |\tilde{\alpha}_{NL}| < 1.8 \times 10^{-11} \) if \( \alpha_{NLi} = 0 \). This is a more promising result than that previously found in the linear scenario. In fact, there exist a direct relation between the bound just obtained within the nonlinear scenario and the ones presented elsewhere under the parametrization derived in \[13\]. It will be shown below that \( \alpha_{NL} = 2h^Z_{10}/c_{\omega}s_w \) and \( \tilde{\alpha}_{NL} = 2h^Z_{30}/c_{\omega}s_w \) correspond to the low energy limit of the form factors \( h^Z_{10} \) used extensively to study the \( ZZ\gamma \) vertex in the case in which one \( Z \) boson and the photon are on-shell \[28\]. Our bound translates thus into

\[ |h^Z_{20}| < 0.38 \]

if \( h^Z_{10} = 0 \), which agrees with previous bounds \[25\]. Of course, the same result applies to \( h^Z_{30} \) when \( h^Z_{30} = 0 \). In this analysis, we have considered that the SM contribution to the rare decay \( Z \to \nu\gamma \) is negligible, what is a good approximation since it was found that the branching ratio is of order \( 10^{-10} \) \[14\]. We have also neglected the contributions coming from the operators which give rise to the effective vertices shown in Fig. 2b and 2c. This is the most optimum scenario indeed. It is likely that any TNGBC may be screened by any other sources of new physics arising from lower dimension operators. Therefore, a more comprehensive analysis must be done to disentangle any new physics contributing to the processes \( e^+e^- (qq) \to ZZ\gamma \) \[28\].

C. Connection with results derived within the \( U(1)_{em} \) formalism

In the last subsections, the EL parametrization we elaborated earlier was used to examine the impact of TNGBC on some loop induced processes. It was also examined whether it is possible to obtain any reasonable bound from the current limit on the branching ratio of the rare decay \( Z \to \nu\gamma \). These vertices were studied for the first time long ago, when only one particle was allowed to be off-shell \[14\]. Following this approach, it has been customary to parametrize any new physics effects inducing TNGBC by certain structures derived out of \( U(1)_{em} \) gauge invariance, Lorentz covariance, as well as Bose symmetry, which corresponds to the so called \( U(1)_{em} \) framework \[21\]. The coefficients of such Lorentz structures are taken to be form factors which actually comprise all our ignorance on the underlying dynamics inducing TNGBC. In general, these form factors depend on the squared momenta of the participating particles. Furthermore, as this dependence is unknown since the form factors are determined by the up to now unknown physics, it is necessary to make some assumptions to describe their behavior. This scheme has proved to be useful to constrain the low energy values of the form factors through \( Z\gamma \) production in \( e^+e^- \) and \( qq \) collisions at LEP, the Tevatron, and the future LHC \[21\].

The latter formalism is to be contrasted with the EL method followed in this work, which in turn is well suited for studying new physics effects in a model independent way and no form factors nor extra assumptions on the unknown physics are required, but all our ignorance of the new physics lies in dimensionless (or dimensionful) coefficients associated with each effective operator, which in turn only depend on the new physics energy scale. Another peculiarity
of the EL formalism is that we are allowed to know what operators the new physics comes from, in contrast to the form factor scheme where we only know that the form factors themselves are generated at a given order in the U(1)_{em} effective Lagrangian. To establish a direct connection between these two different formalisms is not an immediate nor an easy task. In a previous work \[13\] both approaches, within the U(1)_{em} gauge invariant scheme, were considered and their relation was established. It was shown how the form factors are related to the coefficients associated with the effective operators arising from the U(1)_{em} framework. At this point, it is natural to ask whether there is a direct connection between our own results, when it is considered the case of only one off-shell particle, and those derived from the form factor parametrization. We will show that in the case where the form factors are given their low energy values $h^{\alpha}_{10}$, there is a simple connection indeed.

To show the relation between our results and previous ones, we will consider only the $Z\Z\gamma$ coupling, in the specific case where both the initial Z boson and the photon are on-shell since it is the only coupling involved in the rare process $Z \to \nu\bar{\nu}\gamma$. The most general structure for the $Z\Z\gamma$ vertex respecting Lorentz covariance, U(1)_{em} gauge invariance and Bose symmetry is given by

$$
\Gamma_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = \frac{ie (k_2^2 - m_Z^2)}{m_Z^2} \left( h^Z_1 (k^{\alpha_1} g^{\alpha_2} - k^{\alpha_2} g^{\alpha_1}) + \frac{h^Z_2}{m_Z^2} k^{\alpha_1}_2 (k_2 \cdot k g^{\alpha_2\alpha} - k^{\alpha_2} k^2_2) + h^{Z\epsilon}_{3} \epsilon^{\alpha_1\alpha_2\alpha\mu} k^\mu + \frac{h^{Z\epsilon}_4}{m_Z^2} \epsilon^{\alpha_1\alpha_2\alpha\mu\nu} k^{\mu}_1 k^{\nu}_2 \right)
$$

(94)

where all the momenta are taken as incoming. Any term proportional to $k^{\alpha}$ and $k^{\alpha}_1$ has been omitted, the same is true for those proportional to $k^{2\alpha}$ because it is also assumed that the virtual Z boson couples to light fermions, as actually happens in the decay $Z \to \ell\ell\gamma$. In this parametrization, the $CP$-conserving terms $h^Z_{1,2}$ as well as the $CP$-violating ones $h^Z_{3,4}$ are taken as form factors which depend on the dynamics of the underlying new physics, in general they are unknown functions of the squared momenta of the neutral bosons, namely $k^2$, $k^2_1$, and $k^2_2$. Within the U(1)_{em} formalism, as far as the form factors $h^Z_{3,4}$ are concerned, they receive contributions from dimension six operators, whereas the ones $h^Z_{1,2}$ can be induced by dimension eight or higher operators. Based on unitarity requirement, some authors have extensively used the approximation $h^Z_2 = h^Z_0/(1 + s/<\Lambda^2>)^n$, with $n$ an integer, $h^Z_0$ the form factor low energy value, and $s$ the squared momentum of the virtual Z boson \[28\]. If the energy scale $\Lambda$ associated with the new physics inducing TNGBC is larger than the energy scale involved in the process, i.e. the squared momentum of the virtual particle, it is a good approximation to use the low energy values of the form factors. After the replacement $h^Z_2 \to h^Z_0$ is done in \[14\], we are left with the expression for the $Z\Z\gamma$ vertex which must coincide with the one obtained from our results in the nonlinear scenario.

Considering the assumptions just described, we can obtain from the appendices the expression for the $Z\Z\gamma$ vertex arising from the lower dimension operators within either the linear scenario or the nonlinear one. The $CP$-odd part is given by

$$
\Gamma^{Z\Z\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = g_1 \left( k^2_2 - m_Z^2 \right) \epsilon_{\alpha_1\alpha_2\alpha\mu} k^\mu + g_2 \left[ k_{\alpha_1} \epsilon_{\alpha_2\alpha\mu\nu} k^\mu_1 k^\nu_2 - k_{\alpha_2} \epsilon_{\alpha_1\alpha\mu\nu} k^\mu_1 k^\nu_2 \right],
$$

(95)

while the $CP$-even part is

$$
\Gamma^{Z\Z\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = f_1 \left( k^2_2 - m_Z^2 \right) (k_{\alpha_2} g_{\alpha_1\alpha} - k_{\alpha_1} g_{\alpha_2\alpha})
$$

$$
+ f_2 \left[ k_{\alpha_1} (k_1 \cdot k g_{\alpha_2\alpha}) + k_{\alpha_2} (k_1 \cdot k g_{\alpha_1\alpha}) \right],
$$

(96)

where the factors $f_1$ and $g_1$ are related to the coefficients $f^{Z\Z\gamma}_1$ and $g^{Z\Z\gamma}_1$, respectively. At first sight it seems there is no direct coincidence of the terms which multiply $f_2$ and $g_2$ with those multiplying $h^Z_2$ and $h^Z_0$ in \[14\]. However, after a judicious manipulation and with the aid of Shouten’s identity, it can be shown that this is the case indeed. We thus obtain a simple expression for the $Z\Z\gamma$ vertex

$$
\Gamma^{Z\Z\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = \left( k^2_2 - m_Z^2 \right) \left[ g^{Z\Z\gamma}_2 (k_{\alpha_2} g_{\alpha_1\alpha} - k_{\alpha_1} g_{\alpha_2\alpha}) + f^{Z\Z\gamma}_2 \epsilon_{\alpha_1\alpha_2\alpha\mu} k^\mu \right],
$$

(97)

which do show an obvious connection with \[14\]. Instead of giving explicit expressions for $f^{Z\Z\gamma}_1$ and $g^{Z\Z\gamma}_1$, we will establish the connection of $h^Z_{10}$ and $h^Z_{20}$ with the coefficients $\alpha$ and $\tilde{\alpha}$ appearing in \[88\] for the linear scenario and \[92\] for the nonlinear case. We thus have

$$
h^Z_{10} = \frac{c_w s_w \alpha \mu}{2 g^2},
$$

(98a)
\[ h_{30}^Z = \frac{c_w s_w \tilde{\alpha}_L}{2g^2}, \]  

the same relation holds for these coefficients in the nonlinear scenario. Explicit expressions from \( f^{ZZ\gamma} \) and \( g^{ZZ\gamma} \) can be easily extracted from (89a) and (89b).

Finally, a few comments are in order. Although the \( ZZ\gamma \) has the same Lorentz structure in both realizations of the \( SU(2)_L \times U(1)_Y \) gauge symmetry, the main difference is that the operators inducing these structures are of dimension six in the nonlinear scenario, whereas in the linear case they are induced by dimension eight operators. As a result, though the bounds found for the coefficients \( h_{10}^Z \) and \( h_{30}^Z \), apply in both scenarios, if they were translated into the operator coefficients \( \alpha_i \) and \( \lambda_i \), looser bounds would be obtained in the linear scenario. Regarding the remaining TNGBC, a similar analysis following the lines sketched above was done for the \( ZZZ \) and \( Z\gamma\gamma \) couplings. It was found that our results agree with those previously presented. Another interesting point to be noted is that, since the context of the effective Lagrangian approach, both in the linear and the nonlinear realizations of the \( SU(2) \) symmetry (endowing the gauge bosons with mass), the possibility of measuring their effects still remains. The EL approach indicates that, owing to the suppression of the operators inducing TNGBC, it is difficult that the effects arising from them may compete with those coming from other sources of new physics induced by lower dimension operators. However, it may happen that some fortuitous fact, such as some resonant effect, could give rise to large TNGBC in a particular model. In this context, it would be useful a study in a model dependent way to have more evidences which could lead us to a deeper understanding of TNGBC.

V. CONCLUSIONS

In this work we have presented an analysis of trilinear neutral gauge boson couplings, \( ZZZ \), \( ZZ\gamma \) and \( Z\gamma\gamma \), under the context of the effective Lagrangian approach, both in the linear and the nonlinear realizations of the \( SU(2)_L \times U(1)_Y \) gauge symmetry. Particular emphasis has been given to the linear scenario since current literature lacks of an analysis in this line. The most general case with three off-shell bosons is considered. In the linear scenario these couplings receive contributions from dimension eight operators, whereas in the nonlinear scenario they are induced by dimension six operators. For completeness, we have included the Lorentz structure which parametrizes these vertices, ready to be used in any future calculation. Based on general considerations and actual calculations, we conclude that, if the until now unknown physics underlying the SM is of a decoupling nature, it is not expected that TNGBC could have a considerable impact either through their virtual effects or via direct production. In contrast, if new physics effects arise from a strong coupling regime at higher energies which is responsible for the breaking of the \( SU(2)_L \times U(1)_Y \) symmetry (endowing the gauge bosons with mass), the possibility of measuring their effects still remains. The EL approach indicates that, owing to the suppression of the operators inducing TNGBC, it is difficult that the effects arising from them may compete with those coming from other sources of new physics induced by lower dimension operators. However, it may happen that some fortuitous fact, such as some resonant effect, could give rise to large TNGBC in a particular model. In this context, it would be useful a study in a model dependent way to have more evidences which could lead us to a deeper understanding of TNGBC.

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APPENDIX I: TNGBC VERTEX FUNCTIONS IN THE LINEAR SCENARIO

In this appendix the vertex functions for the \( ZZZ \), \( ZZ\gamma \), and \( Z\gamma\gamma \) couplings within the linear scenario are presented. We consider the most general case where the three bosons are virtual. The particle momenta are denoted as described below and will be taken as incoming everywhere. The \( CP \)-odd vertex functions are given by

\[ Z_{\alpha_1}(k_1)Z_{\alpha_2}(k_2)Z_{\alpha_3}(k_3) \text{ vertex:} \]

\[ \Gamma_{\alpha_1\alpha_2\alpha_3}^{L=ZZZ}(k_1, k_2, k_3) = \sum_{i=1}^{2} f_{Li}^{ZZZ} \Gamma_{\alpha_1\alpha_2\alpha_3}^{Li=ZZZ}(k_1, k_2, k_3), \]

\[ \Gamma_{\alpha_1\alpha_2\alpha_3}^{Li=ZZZ}(k_1, k_2, k_3) \]

\[ = 2k_{1\alpha_1}(k_{2\alpha_2}k_{3\alpha_3} - k_{2\alpha_3}k_{3\alpha_2}) + 2k_{2\alpha_2}(k_{1\alpha_1}k_{3\alpha_3} - k_{1\alpha_3}k_{3\alpha_2}) + 2k_{3\alpha_3}(k_{1\alpha_1}k_{2\alpha_2} - k_{1\alpha_2}k_{2\alpha_3}), \]

\[ \Gamma_{\alpha_1\alpha_2\alpha_3}^{Lz=ZZZ}(k_1, k_2, k_3) \]

\[ = k_{1\alpha_1}(k_{2\alpha_2}k_{3\alpha_3} - k_{2\alpha_3}k_{3\alpha_2}) + k_{1\alpha_2}(k_{2\alpha_3}k_{3\alpha_1} - k_{2\alpha_1}k_{3\alpha_3}) + k_{1\alpha_3}(k_{2\alpha_1}k_{3\alpha_2} - k_{2\alpha_2}k_{3\alpha_1}) \]

\[ + k_{2\alpha_2}(k_{1\alpha_3}k_{3\alpha_1} - k_{1\alpha_1}k_{3\alpha_3}) + k_{2\alpha_3}(k_{1\alpha_1}k_{3\alpha_2} - k_{1\alpha_2}k_{3\alpha_1}) + k_{3\alpha_3}(k_{1\alpha_1}k_{2\alpha_2} - k_{1\alpha_2}k_{2\alpha_3}) \]

\[ + k_{3\alpha_1}(k_{1\alpha_2}k_{2\alpha_3} - k_{1\alpha_3}k_{2\alpha_2}) + k_{3\alpha_2}(k_{1\alpha_1}k_{2\alpha_3} - k_{1\alpha_3}k_{2\alpha_1}). \]

17
$Z_{\alpha_1}(k_1) Z_{\alpha_2}(k_2) A_{\alpha}(k)$ vertex:

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = \frac{3}{2} \sum_{i=1}^{3} g_{L_1}^{ZZ\gamma} \Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k), \quad (100a)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = k_{1a}(k_2 \cdot g_{a_1 a} - k_{2a} g_{a_1 a} + k_{1a} g_{a_2 a} - k_{2a} g_{a_2 a}), \quad (100b)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = k_{2a}(k_1 \cdot g_{a_2 a} - k_{1a} g_{a_2 a} + k_{2a} g_{a_1 a} - k_{1a} g_{a_1 a}), \quad (100c)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = k_{1a}(k_2 \cdot g_{a_1 a} - k_{2a} g_{a_1 a} + k_{1a} g_{a_2 a} - k_{2a} g_{a_2 a}). \quad (100d)$$

$A_{\alpha}(k_1) A_{\alpha_2}(k_2) Z_{\alpha}(k)$ vertex:

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = \frac{3}{2} \sum_{i=1}^{3} g_{L_1}^{ZZ\gamma} \Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k), \quad (101a)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = k_{1a} k_{2a} g_{a_1 a}(k_3 - k_1) + k_{2a} k_{1a} g_{a_2 a}(k_3 - k_2). \quad (101b)$$

Following the same conventions, the $CP$-even vertex functions are given by

$ZZ\gamma$ vertex:

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k_3) = \frac{3}{2} \sum_{i=1}^{3} g_{L_1}^{ZZ\gamma} \Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k_3), \quad (102a)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k_3) = k_{1a} k_{2a} k_{3a} g_{a_1 a}(k_1 - k_2) g_{a_2 a}(k_2 - k_3) + k_{2a} k_{1a} k_{3a} g_{a_2 a}(k_1 - k_2) g_{a_1 a}(k_3 - k_1) + k_{3a} k_{1a} k_{2a} g_{a_2 a}(k_1 - k_3) g_{a_1 a}(k_2 - k_1) + k_{3a} k_{2a} k_{1a} g_{a_2 a}(k_1 - k_3) g_{a_1 a}(k_2 - k_1) + k_{1a} k_{2a} k_{3a} g_{a_1 a}(k_1 - k_2) g_{a_2 a}(k_2 - k_3) + k_{3a} k_{1a} k_{2a} g_{a_2 a}(k_1 - k_3) g_{a_1 a}(k_2 - k_1). \quad (102b)$$

$ZZ\gamma$ vertex:

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k_3) = \frac{3}{2} \sum_{i=1}^{3} g_{L_1}^{ZZ\gamma} \Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k_3), \quad (103a)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k_3) = k_{1a} k_{2a} k_{3a} g_{a_1 a}(k_1 - k_2) g_{a_2 a}(k_2 - k_3) + k_{2a} k_{3a} k_{1a} g_{a_2 a}(k_1 - k_2) g_{a_1 a}(k_3 - k_1) + k_{3a} k_{1a} k_{2a} g_{a_2 a}(k_1 - k_3) g_{a_1 a}(k_2 - k_1) + k_{3a} k_{2a} k_{1a} g_{a_2 a}(k_1 - k_3) g_{a_1 a}(k_2 - k_1) + k_{1a} k_{2a} k_{3a} g_{a_1 a}(k_1 - k_2) g_{a_2 a}(k_2 - k_3) + k_{3a} k_{1a} k_{2a} g_{a_2 a}(k_1 - k_3) g_{a_1 a}(k_2 - k_1). \quad (103b)$$

$Z\gamma\gamma$ vertex:

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = \frac{3}{2} \sum_{i=1}^{3} g_{L_1}^{ZZ\gamma} \Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k), \quad (104a)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = k_{1a} k_{2a} (g_{a_1 a}(k_1 - k_2) + g_{a_2 a}(k_1 - k_2) k_{1a}), \quad (104b)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = -k_{a} g_{a_1 a_2 a}(k_1 - k_2). \quad (104c)$$

$$\Gamma_{a_1 a_2 a}^{L-Z\gamma}(k_1, k_2, k) = k_{1a} k_{2a} (g_{a_1 a}(k_1 - k_2) + g_{a_2 a}(k_1 - k_2) k_{1a}) - k_{a} g_{a_1 a_2 a}(k_1 - k_2). \quad (104d)$$

Note that the vertex functions vanish for three on-shell particles. They are also symmetric under the interchange of identical particles, in perfect agreement with Bose symmetry.
APPENDIX II: TNGBC VERTEX FUNCTIONS IN THE NONLINEAR SCENARIO

We are using the same conventions used in the linear case. The CP-odd vertex functions can be written in terms of those of the linear scenario plus some new terms

ZZZ vertex:

\[ \Gamma^{NL-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) = \Gamma^{L-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) + \sum_{i=3}^{5} f^{ZZZ}_{NLi} \Gamma^{NL_i-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3), \]  

\[ \Gamma^{NL3-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) = k_1^2(k_3g_{\alpha_2\alpha_3} + g_{\alpha_1\alpha_2\alpha_3}) + k_2^2(k_3g_{\alpha_2\alpha_3} + g_{\alpha_1\alpha_2\alpha_3}) + k_3^2(k_2g_{\alpha_2\alpha_3} + g_{\alpha_1\alpha_2\alpha_3}), \]

\[ \Gamma^{NL4-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) = -k_1 \cdot k_2(k_3g_{\alpha_2\alpha_3} + g_{\alpha_1\alpha_2\alpha_3}) + k_3g_{\alpha_2\alpha_3} + k_3g_{\alpha_1\alpha_2\alpha_3} \]

\[ \Gamma^{NL5-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) = k_2g_{\alpha_1\alpha_2\alpha_3} - k_1 \cdot k_3g_{\alpha_2\alpha_3} + k_3g_{\alpha_1\alpha_2\alpha_3} + k_3g_{\alpha_1\alpha_2\alpha_3} \]

ZZ\gamma vertex:

\[ \Gamma^{NL-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = \Gamma^{L-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) + \sum_{i=3}^{5} f^{ZZ\gamma}_{NLi} \Gamma^{NL_i-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k), \]

\[ \Gamma^{NL4-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = (k_1 - k_2) \cdot k(k_3g_{\alpha_2\alpha_3} - g_{\alpha_1\alpha_2\alpha_3}), \]

\[ \Gamma^{NL5-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = g_{\alpha_1\alpha_2\alpha_3} - k_1 \cdot k_2g_{\alpha_2\alpha_3} + g_{\alpha_1\alpha_2\alpha_3} - k_2 \cdot k_3g_{\alpha_2\alpha_3} \]

The respective CP-even vertex functions can also been written in terms of those of the linear case.

ZZZ vertex:

\[ \bar{\Gamma}^{NL-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) = \bar{\Gamma}^{L-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) + g^{ZZZ}_{NL3} \bar{\Gamma}^{NL3-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3), \]

\[ \bar{\Gamma}^{NL3-ZZZ}_{\alpha_1\alpha_2\alpha_3}(k_1, k_2, k_3) = k_1 \cdot (k_2 - k_3)\epsilon_{\alpha_1\alpha_2\alpha_3\mu}k_1^\mu + k_2 \cdot (k_3 - k_1)\epsilon_{\alpha_1\alpha_2\alpha_3\mu}k_2^\mu + k_3 \cdot (k_1 - k_2)\epsilon_{\alpha_1\alpha_2\alpha_3\mu}k_3^\mu. \]

ZZ\gamma vertex:

\[ \bar{\Gamma}^{NL-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = \bar{\Gamma}^{L-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) + g^{ZZ\gamma}_{NL6} \bar{\Gamma}^{NL6-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k), \]

\[ \bar{\Gamma}^{NL6-ZZ\gamma}_{\alpha_1\alpha_2\alpha}(k_1, k_2, k) = k \cdot (k_1 - k_2)\epsilon_{\alpha_1\alpha_2\alpha\mu}k_1^\mu. \]

Finally, the Z\gamma\gamma vertex has the same Lorentz structure in both the linear and the nonlinear scenarios.

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FIG. 1. Contribution from TNGBC to the anomalous magnetic moment of fermions in the effective Lagrangian approach.
FIG. 2. Feynman diagrams contributing to the decay $Z \to \nu \bar{\nu} \gamma$ in the effective Lagrangian approach.