New Physics / Resonances in Vector Boson Scattering at the LHC

Jürgen R. Reuter, DESY

based on work with A. Alboteanu, S. Brass, C. Fleper, W. Kilian, T. Ohl, M. Sekulla

w. EPJC [1807.02512] PRD93(16),3.036004 [1511.00022], PRD91(15) 096007 [1408.6207] JHEP 0811.010 [0806.4145]
Vector Boson Scattering after the Higgs Discovery

• Discovery of a light Higgs boson leaves still open questions:

1. **Nature of Electroweak Symmetry Breaking**
2. Higgs boson potential, all the way like the Standard Model!? 
3. Does “the Higgs” fulfill the US-fermion/Europe-boson rule? 
4. Is the 125 GeV state the only resonance in the system of EW vector bosons? 
5. How do EW vector bosons scatter? (true heart of weak interactions) 
6. Is there something related to the Little Hierarchy problem (strong or weak) 
7. Look for deviations in intricate cancellations of VBS amplitudes
Anatomy of Vector Boson Scattering (VBS)

\[ pp \rightarrow WWjj \rightarrow ℓννjj \]

- **Discovery for** \( W^+W^+jj \) (electroweak production)
  - ATLAS PRL 113(2014)14, 141803 [1405.6241] & 1611.02428; CMS PRL 114(2015), 051801 [1410.6315]

- **First limits on New Physics in** pure electroweak gauge/Goldstone sector
Anatomy of Vector Boson Scattering (VBS)

- Forward and high momentum jets
- Low central jet activity

VBS ZZjj Candidate Event from PLB 774 (2017) 682

shown by Kenneth Long, Seoul, ICHEP 2018
More channels came/coming up …

**Post-fit background normalisations**

\[ \mu_{\text{WZ-QCD}} = 0.60 \pm 0.25 \]
\[ \mu_{\text{tt\nu}} = 1.18 \pm 0.19 \]
\[ \mu_{\text{ZZ}} = 1.34 \pm 0.29 \]

**WZjj-EW measured signal strength:**

\[ \mu_{\text{EW}} = 1.77 \pm 0.41 \text{(stat.)} \pm 0.17 \text{(syst.)} = 1.77 \pm 0.45 \]

**Corresponding sign.:** \( 5.6\sigma \) (3.3\( \sigma \) expected)

**Corresponding fid. cross section:**

\[
\sigma_{\text{WZjj} \rightarrow l\nu l\nu jj} = 0.57 \pm 0.15 \text{ fb} \\
= 0.57 \pm 0.14 \text{ (stat.)} \pm 0.05 \text{ (sys.)} \pm 0.04 \text{ (th.) fb}
\]

**Observed (expected) of EW WZ 1.9\( \sigma \) (2.7\( \sigma \))**

**Observed (expected) of 5.5\( \sigma \) (5.7\( \sigma \))**

**Observed (expected) of 2.7\( \sigma \) (1.6\( \sigma \))**
Deviations as EFT — Dim 8 Operators

Motivated by SMEFT:

\[
\mathcal{L} = \mathcal{L}_{SM} + \sum_i \left[ \frac{c_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \frac{c_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{(7)} + \frac{c_i^{(8)}}{\Lambda^4} \mathcal{O}_i^{(8)} + \cdots \right]
\]

**Longitudinal operators**

\[
\begin{align*}
\mathcal{L}_{S,0} &= F_{S,0} \text{tr} \left[ (D_\mu H) \dagger (D_\nu H) \right] \text{tr} \left[ (D_\mu H) \dagger (D_\nu H) \right] \\
\mathcal{L}_{S,1} &= F_{S,1} \text{tr} \left[ (D_\mu H) \dagger (D_\mu H) \right] \text{tr} \left[ (D_\nu H) \dagger (D_\nu H) \right]
\end{align*}
\]

**Mixed operators**

\[
\begin{align*}
\mathcal{L}_{M,0} &= -g^2 F_{M,0} \text{tr} \left[ (D_\mu H) \dagger (D_\nu H) \right] \text{tr} \left[ W_{\nu \rho} W^{\nu \rho} \right] \\
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\mathcal{L}_{M,2} &= -g^2 F_{M,2} \text{tr} \left[ (D_\mu H) \dagger (D_\nu H) \right] \text{tr} \left[ B_{\nu \rho} B^{\nu \rho} \right] \\
\mathcal{L}_{M,3} &= -g^2 F_{M,3} \text{tr} \left[ (D_\mu H) \dagger (D_\nu H) \right] \text{tr} \left[ B_{\nu \rho} B^{\nu \mu} \right] \\
\mathcal{L}_{M,4} &= -g g' F_{M,4} \text{tr} \left[ (D_\mu H) \dagger W_{\nu \rho} (D_\mu H) B^{\nu \rho} \right] \\
\mathcal{L}_{M,5} &= -g g' F_{M,5} \text{tr} \left[ (D_\mu H) \dagger W_{\nu \rho} (D_\nu H) B^{\nu \mu} \right] \\
\mathcal{L}_{M,7} &= -g^2 F_{M,7} \text{tr} \left[ (D_\mu H) \dagger W_{\nu \rho} W^{\nu \mu} (D_\rho H) \right]
\end{align*}
\]

**Transversal operators**

\[
\begin{align*}
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Energy rise of operators lead to unitarity violation

Unitarity violation cancels between operators in UV-complete Theory

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Procedures to treat unitarity violations

Cut-off (a.k.a. “Event clipping”) \( \theta(\Lambda_C^2 - s) \)

- Unitarity bound (0th partial wave) at \( \Lambda_C \)
- No continuous transition beyond
- Effect on BDT training not clear
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Form factor

\[
\frac{1}{\left(1 + \frac{s}{\Lambda_{FF}}\right)^n}
\]

Applicable for arbitrary operators, tuning in 2 parameters: \( n \) damps unitarity violation, \( \Lambda_{FF} \)
highest value to satisfy 0th partial wave
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\( K/T\)-matrix saturation

\[ a = \frac{1}{\text{Re}(\frac{1}{a_0}) - i} \]

- Saturates amplitude [projection to unitarity circle], also for complex ampl., no additional parameters

Alboteanu/Kilian/JRR, 2008  Kilian/Ohl/JRR/Sekulla, 2014
VBS diboson spectra

General cuts: $M_{jj} > 500$ GeV; $\Delta \eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\Delta \eta_j| < 4.5$

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New Physics in VBS @ LHC

LHCP 2019, Puebla, 24.5.19
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**VBS diboson spectra**

Much more leeway for new physics in transversal gauge bosons and di-Higgs

**General cuts:** $M_{jj} > 500$ GeV; $\Delta \eta_{jj} > 2.4$; $p_T^j > 20$ GeV; $|\Delta \eta_j| < 4.5$

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(In)Validity of (In)Effective Field Theories

- Resonances in direct reach (not clear: strongly interacting models [e.g. $\sigma$ resonance])

- Estimate of operator coefficients (difficult for strongly coupled models)

\[ \mathcal{A}_{SM} \times \mathcal{A}_{\text{dim}-6} \gtrsim |\mathcal{A}_{\text{dim}-6}|^2 \]
\[ \mathcal{A}_{SM} \times \mathcal{A}_{\text{dim}-8} \gtrsim |\mathcal{A}_{\text{dim}-8}|^2 \]
\[ \mathcal{A}_{SM} \times \mathcal{A}_{\text{dim}-6} \gtrsim \mathcal{A}_{SM} \times \mathcal{A}_{\text{dim}-8} \]

- Partial wave unitarity: gives guidance on maximally possible event numbers

- Positivity constraints on operator coefficients

- Size of coefficients: dichotomy between validity and detectability

- EFT better/best[?] suited in intensity frontier [example: HEFT @ $\mathcal{O}(100 \text{ GeV})$]

- EFT borderline in VBS/energy frontier physics
Bumps vs. Tails
Simplified signal models: generic resonances

- Rise of amplitude: is Taylor expansion below a resonance
- Resonances might be in direct reach of LHC
- EFT framework EW-restored regime: $SU(2)_L \times SU(2)_R$, $SU(2)_L \times U(1)_Y$ gauged
- Include EFT operators in addition (more resonances, continuum contribution)
- Apply $T$-matrix unitarization beyond resonance ("UV-incomplete" model)

Spins 0, 2 considered, Spin 1 has (partially) different physics (mixing with $W/Z$)

|      | isoscalar | isotensor                          |
|------|-----------|------------------------------------|
| scalar | $\sigma^0$ | $\phi_t^-, \phi_t^-, \phi_t^0, \phi_t^+, \phi_t^{++}$, $\phi_v^-, \phi_v^0, \phi_v^+$, $\phi_s^0$ |
| tensor | $f^0$    | $(X_t^-, X_t^-, X_t^0, X_t^+, X_t^{++})$, $(X_v^-, X_v^0, X_v^+, X_v^{++})$, $(X_s^0)$ |

| $32\pi \Gamma / M^5$ |
|----------------------|
| $\sigma$ | $f$ | $X$ |
| $F_{S,0}$ | $\frac{1}{2}$ | 2 | 15 | 5 |
| $F_{S,1}$ | $-\frac{1}{2}$ | -5 | -35 |

Translation into Wilson coefficients below resonance
Comparison: Simplified Models & EFT

Black dashed line: saturation of $A_{22}(W^+W^+)/A_{00}(ZZ)$

- EFT fails at resonance
- aQGC describe rise of resonance
- Unitarization applied
- Tensor resonances better visible than scalars

$M_{jj} > 500 \text{ GeV}; \Delta \eta_{jj} > 2.4; p_T^j > 20 \text{ GeV}; |\Delta \eta_j| < 4.5$
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Triple [multiple] Vector Boson Production?

Evidence at ATLAS at $4\sigma$ level: smallest SM cross section

Relate to?

- Yes, same Feynman rule as in VBS, but …
- one external $W/Z/\gamma$ always far off-shell
- Unitarization: work in progress (needs $2 \rightarrow 3$ unitarizations, inelastic channels) [Kilian/Kreher/JRR, w.i.p.]
- Different Wilson coefficients dominate (particularly for resonances)
- Important physics (partially) independent from VBS (“different fiducial vol.”)
Conclusions / Summary

- Vector boson scattering a flagship measurement of Runs II/III (and FCC-hh !)
- EFT provides well-defined (and very limited) framework for SM deviations

- There is not really a true model-independent parameterization!
- Unitarization for theoretically sane description (allows reliable BDT analysis)
- $T$-matrix unitarization universal scheme for EFT and resonances
- Simplified models: generic electroweak resonances
- Limits from LHC still quite limited: $\Lambda_{\text{new physics}} \sim 700–800$ GeV
7th Workshop on Multi-Boson Interactions

August 26-28, 2019
Aristotle U., Thessaloniki

1. TU Dresden
2. BNL (Brookhaven Ntl. Lab)
3. DESY
4. U. of Wisconsin — Madison
5. KIT Karlsruhe
6. U. of Michigan — Ann Arbor
7. Aristotle U. Thessaloniki
BACKUP SLIDES
**Anatomy of Vector Boson Scattering (VBS)**

\[ pp \rightarrow WWjj \rightarrow \ell \nu \nu jj \]

Backgrounds:
- \( tt \rightarrow WbWb \)
- \( W + \) jets
- single top, misreconstructed jet
- \( WWjj \) QCD production
- \( ll + X + \text{Emiss} \) ("prompt")

Fiducial phase space volume:
- \( lljj \) tag
- \( m_{jj} > 500 \text{ GeV} \) ("jet recoil")
- \( |\Delta y_{jj}| > 2.4 \) ("rapidity distance")
- Cuts on \( E_{\text{j}}, p_{Tj} \)
- No mini jet vetoes

**CMS Preliminary**

May 2018

Production Cross Section, \( \sigma \) [pb]
Differential spectra in VBS

\[ pp \rightarrow e^+ \mu^+ \nu_e \nu_\mu jj \quad \sqrt{s} = 14 \text{ TeV} \quad \mathcal{L} = 1 \text{ ab}^{-1} \]

Simulations with WHIZARD [http://whizard.hepforge.org, Kilian/Ohl/JRR]
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\[ \mathcal{L}_{HD} = F_{HD} \text{tr} \left[ H^\dagger H - \frac{\nu^2}{4} \right] \cdot \text{tr} \left[ (D_\mu H)^\dagger D_\mu H \right] \]

\[ F_{HD} = 30 \text{ TeV}^{-2} \]
**Differential spectra in VBS**

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\[ F_{S,0} = 480 \text{ TeV}^{-4} \]

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How to get EFTs from New Physics

✦ Consider effects from heavy states by using (known) low-energy d.o.f.s

In addition to being a great convenience, effective field theory allows us to ask all the really scientific questions that we want to ask without committing ourselves to a picture of what happens at arbitrarily high energy.

H. Georgi, 1993
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✦ Integrating out heavy d.o.f.s marginalizes over details of short-distance interactions

✦ Toy Example: two interacting scalar fields $\varphi, \Phi$

Path integral

$$\mathcal{Z}[j, J] = \int D[\Phi] D[\varphi] \exp \left[ i \int dx \left( \frac{1}{2} (\partial \varphi)^2 - \frac{1}{2} \Phi (\square + M^2) \Phi - \lambda \varphi^2 \Phi - \ldots + J \Phi + j \varphi \right) \right]$$
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Completing the square (Gaussian integration)

$$\Phi' = \Phi + \frac{\lambda}{M^2} \left( 1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \quad \Rightarrow \quad \text{Diagram}$$
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Completing the square (Gaussian integration)

$$\Phi' = \Phi + \frac{\lambda}{M^2} \left( 1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2 \implies$$

In the Lagrangian remove the high-scale d.o.f.s:

$$\frac{1}{2} (\partial \Phi)^2 - \frac{1}{2} M^2 \Phi^2 - \lambda \varphi^2 \Phi = -\frac{1}{2} \Phi' (M^2 + \partial^2) \Phi' + \frac{\lambda^2}{2M^2} \varphi^2 \left( 1 + \frac{\partial^2}{M^2} \right)^{-1} \varphi^2$$

Irrelevant normalization of the path integral

Tower of higher and higher-dim. operators of light fields
1. **SM or weakly coupled physics (e.g. 2HDM):** amplitude remains close to origin

2. **Rising amplitude (at least one dim-8 operator):** rise beyond unitarity circle [unphys.], strongly interacting regime

3. **Inelastic channel opens (form-factor description):** new channels open out, multi-boson final states

4. **Saturation of amplitude:** maximal amplitude, strongly interacting continuum, K-/T-matrix unitarization

5. **New resonance:** amplitude turns over
Unitarity in vector boson scattering

**Optical Theorem** (Unitarity of the S(cattering) Matrix):

\[ \sigma_{\text{tot}} = \text{Im} \left[ \mathcal{M}_{ii}(t = 0) \right] / s \quad t = -s(1 - \cos \theta)/2 \]

Partial wave amplitudes:

\[ \mathcal{M}(s, t, u) = 32\pi \sum_\ell (2\ell + 1) A_\ell(s) P_\ell(\cos \theta) \] ("Power spectrum")
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SM longitudinal isospin eigenamplitudes \((A_{I,\text{spin}=J})\):
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exceeds unitarity bound \(|A_{IJ}| \lesssim \frac{1}{2}\) at:

- \(I = 0\): \(E \sim \sqrt{8\pi v} = 1.2\ \text{TeV}\)
- \(I = 1\): \(E \sim \sqrt{48\pi v} = 3.5\ \text{TeV}\)
- \(I = 2\): \(E \sim \sqrt{16\pi v} = 1.7\ \text{TeV}\)

Higgs exchange:

\[ A(s, t, u) = -\frac{M_H^2}{v^2} \frac{s}{s - M_H^2} \]

Unitarity:

\[ M_H \lesssim \sqrt{8\pi v} \sim 1.2\ \text{TeV} \]
Different unitarity projections

- **K-matrix**: Cayley transform of S-matrix
  - Heitler, 1941; Schwinger, 1949; Gupta, 1950

- Stereographic projection to Argand circle

\[
S = \frac{1 + iK/2}{1 - iK/2} \quad a_K(s) = \frac{a(s)}{1 - ia(s)}
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  Kilian/Ohl/JRR/Sekulla, 2014

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  Kilian/Ohl/JR/R/Sekulla, 2014

  Defined via  
  \[ |a - \frac{a_K}{2}| = \frac{a_K}{2} \implies a = \frac{1}{\text{Re} \left( \frac{1}{a_0} \right) - i} \]

  Identical to $K$ matrix for real amplitudes

  Points on Argand circle left invariant

  Does not rely on perturbation theory

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Complete LHC process at 14 TeV

$pp \rightarrow e^+ e^- \mu^+ \mu^- jj$ at 3 ab$^{-1}$

- $F_f = 17.4$ TeV$^{-1}$
- SM

$M_f = 1.0$ TeV

$M \left(e^+, e^-, \mu^+, \mu^- \right)$ [GeV]
Remark on alternative unitarizations

- Independent Amplitude Method (IAM) \cite{Truong, 1988; Dobado/Herrero/Truong, 1990}
- Padé Method \cite{Padé, 1890; Basdevant/Lee, 1970}
- N/D method \cite{Chew/Mandelstam, 1960}
- Focus on correct descriptions of certain explicit (known) resonance channels
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Unitarization of operators

- Clebsch-Gordan decomposition into spin–isospin eigenamplitudes
- Amplitudes should be modified only in \(s\)-channel configurations
  \[
  \mathcal{A}(I = 0) = 3\mathcal{A}(s, t, u) + \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)
  
  \mathcal{A}(I = 1) = \mathcal{A}(t, s, u) - \mathcal{A}(u, s, t)
  
  \mathcal{A}(I = 2) = \mathcal{A}(t, s, u) + \mathcal{A}(u, s, t)
  \]
- Evaluate modified Feynman rules off-shell
- Scale that is used for the diboson system in \(s\)-channel setups: \(\sqrt{\hat{s}} V V\)
Use spin-isospin eigenamplitudes exclusive in helicities:

\[ \mathcal{A}_0(s, t, u; \lambda) \]

Can be obtained by using Wigner’s \( d \)-functions [Wigner, 1931]

\[ \lambda = (\lambda_1, \lambda_2, \lambda_3, \lambda_4) \]
\[ \lambda = \lambda_1 - \lambda_2 \quad \lambda' = \lambda_3 - \lambda_4 \]

Extract all partial waves:

\[ A_{ij}(s; \lambda)/(g^4 s^2) = (c_0 F_{T_0} + c_1 F_{T_1} + c_2 F_{T_2}) \]

|   | 0     | 1     | 2     | \lambda |
|---|-------|-------|-------|---------|
| 0 | -6    | -2    | -3    | + + + + + |
|   | 0 0   | 0 0   | -2/3  | + - + -  |
|   | 0 0   | 0 0   | -2/3  | + - + -  |
|   | -22/3 | -14/3 | -11/3 | + + - -  |
| 1 | 0 0   | 0 0   | 0 0   | + + + + + |
|   | 0 0   | 0 0   | 2/5   | + - + -  |
|   | 0 0   | 0 0   | -2/5  | + - - +  |
|   | 0 0   | 2/5   | -1/3  | + + - -  |
| 2 | 0 -2  | -1    | 0 0   | + + + + + |
|   | 0 0   | 0 0   | -2/5  | + - - +  |
|   | 0 0   | 0 0   | -2/5  | + - - +  |
|   | -4/3  | -8/3  | -1/3  | + + - -  |

Braß/Fleper/Kilian/JRR/Sekulla, 1807.02512