Understanding the outbreak of COVID-19 in Ecuador

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Abstract. In this study is presented a mathematical approach that can be used to estimate the variability of the growth rate coefficient ($\lambda$), the total number of cases, and the midpoint of maximum infection due to the COVID-19 pandemic. The different parameters are quantified using one-year data set reported for Ecuador (from March 2020 to February 2021) and the (discrete or differential) logistic model. In particular, the results evidence that the most critical months of the pandemic in Ecuador were March and April 2020. In the following months, the outbreak continues with low growth rate values but in a variable way, which can be attributed to state health policies and the social behavior of the population. The estimated number of confirmed cases is around 409 K agrees with the data reported at the end of May 2021, validating the proposed mathematical approach.

1. Introduction

The COVID-19 pandemic has accelerated a global crisis with more than 170 M confirmed cases and approximately 3.5 M confirmed deaths in more than 210 countries as of May 28, 2021. In December 2019, a cluster of cases associated with a new respiratory syndrome called coronaviruses diseases 2019 (COVID-19) caused by a novel coronavirus (SARS-CoV-2) was reported in China and spread rapidly around the world [1-3]. With person-to-person transmission confirmed in different countries outside China, COVID-19 was declared a Public Health Emergency of International Concern by the World Health Organization (WHO) on January 30, 2020 [3, 4]. On March 11, 2020, COVID-19 has declared a pandemic [5] because Europe reported more confirmed cases and deaths than other continents combined.

With over 400 K cases, Ecuador has been one of the most affected countries worldwide, accounting for population size. Ecuador has become the epicenter of the COVID-19 pandemic for Latin America during March and April 2020 due to the collapse of public health services, centralized control of information, limited decision-making, low laboratory capability, and inequalities in access to healthcare. As of February 26, 2021, Ecuador had a high mortality rate (882 pmp) and a positivity rate (28%) higher than Chile, Uruguay, and Colombia [6]. Although the government has implemented social distancing, case testing, and random quarantine for protecting the population against COVID-19, these directives appear to be unsuccessful in controlling the disease until today. Otherwise, considering the virus mutation and the varying intensities of the outbreak around the country, the Ecuador case represents a significant challenge for understanding the effect of COVID-19.
From the point of view of modeling, the infection growth rate, turning point, duration of outbreaks, and final size of COVID-19 are common questions. To address those questions, different approaches have been proposed, including models based on logistic function [7], improved SIR and SEIR models [8], hierarchical polynomial regression models [9], statistical models including machine learning techniques [10], time-varying Markov process [11], Gaussian mixture model [12], among others. However, above mentioned models should be used with caution, especially, if these models are used for long-term forecasting.

To date, a one-year data analysis of COVID-19 in Ecuador using the discrete logistic equation and ordinary differential equation (ODE) logistic model, has not been reported yet. In this communication, such an approach is presented. The proposed method allows estimating, particularly, the infection growth rate, turning point, and the number of cases. In addition, this model can be extended to other countries or locations in order to take immediate corrective measures.

2. Mathematical Approach

We use the Ecuadorian data (one year) from March 1, 2020, to February 28, 2021, to model the spread of the COVID-19 in Ecuador, published by worldometer

(https://www.worldometers.info/coronavirus/country/ecuador/). Furthermore, the following approach is based on our previous work [13].

At the early stage, the epidemic can be described by an exponential behavior as follow:

\[ N(t) = N_0 e^{\lambda t} \]  \hspace{1cm} (1)

where \( N_0 \) is the initial number of the infected people, \( t \) is the time since the first reported cases (day), and \( \lambda \) is the growth rate of the epidemic (day\(^{-1}\)). This model can be applied as long as there is no change in the curve. If a change in the curve is observed, the epidemic can be analyzed using the discrete logistic equation:

\[ N(t) = \frac{N_{max}}{1 + e^{-\lambda (t-t_0)}} \]  \hspace{1cm} (2)

which also allows estimating the maximum number of infected people (\( N_{max} \)) and the point of maximum growth (\( t_0 \)). However, the logistic equation written in the form of ODE can complement the understanding of the epidemic:

\[ \frac{dN(t)}{dt} = \lambda N(t) \left( 1 - \frac{N(t)}{N_{max}} \right) \]  \hspace{1cm} (3)

where the \( N(t) \) denotes the total number of confirmed cases as a function of time, and the respective solution is easily found as follow:

\[ N(t) = \frac{N_0 N_{max} e^{\lambda t}}{N_{max} + N_0 (e^{\lambda t} - 1)} \]  \hspace{1cm} (4)
note that Equation 2 and Equation 4 can be used to study the curve of the total case, but the $t_n$ parameter cannot be estimated from Equation 4. Solving the difference logistic equation 
\[
\frac{dN(t)}{dt} = N_{i+1} + N_t = K(t),
\]
where $K(t)$ are the daily reported cases, the following expression arises:

\[
K(t) = \lambda N(t) \left(1 - \frac{N(t)}{N_{\text{max}}}ight)
\]  

which shows a simple relation between $K(t)$ and $N(t)$. Interestingly, the growth rate can be easily found from Eq. (5):

\[
\lambda = \frac{K(t)}{N(t) \left(1 - \frac{N(t)}{N_{\text{max}}}ight)}
\]  

Equation 6 shows the growth rate variability as a function of total confirmed cases ($N(t)$) or time ($t$).

3. Results and Discussions

Figure 1. a) The number of infected people per day and b) the total number of confirmed cases reported on 02/28/2021. The reported data are represented in gray and the 7-point average data is in orange.
The outbreak of COVID-19 in Ecuador is shown in Figure 1. The number of infected people per day is presented in gray and the 7-point average data is in orange (Figure 1a). After twelve months, the number of daily cases continues to increase, suggesting not early saturation with current public health policies. Build on the same data, the total number of confirmed cases increases with time (Figure 1b). The COVID-19 data from Ecuador contrast, for instance, with those reported by China where saturation of infected people was achieved in approximately three months.

However, the spread of COVID-19 follows almost the same scenario in all Latin American countries, which means an exponential increase in the number of cases, and the pandemic has not yet reached its peak. The latter suggests an immediate intervention of the state and local authorities in the adoption of restrictive rules until the population has access to the vaccine.

Figure 2. The total number of cases as a function of time (orange) compared the different models: exponential model (blue), discrete logistic equation (green), and solved ODE logistic model (Eq. 4, black).

With this in mind, it is necessary to find the numerical values of the parameters, say, the growth rate ($\lambda$), the point of maximum growth ($t_p$), and the maximum number of infected people ($N_{max}$) in order to explain the outbreak of COVID-19 and predict its consequences. Figure 2 shows the curve of the total case compared to the exponential model (Equation 1, blue), discrete logistic equation (Equation 2, green), and solution of the ODE logistic model (Equation 4, black). As the exponential model can only be used at the beginning of the pandemic, data from the first 30 days were used. A complete data set was used in the other models.
Table 1. Estimated parameters using the different models of the present work. The $\lambda$ represents the estimated growth rate, $N_{max}$ is the maximum number of infected people, and $t_o$ denotes the sigmoid midpoint.

| Parameter | Estimated | Standard Error | $R^2$ |
|-----------|-----------|----------------|-------|
| $\lambda$ | 0.185     | 0.001          | 0.978 |
| $t_0$     | 225.563   | 2.231          |       |
| $N_{max}$ | 294730    | 3597.460       |       |

Equation 2

| $\lambda$ | 0.014     | 0.001          | 0.997 |
| $t_0$     | 254730    | 3597.460       |       |
| $N_{max}$ | 294730    | 3597.460       |       |

Equation 4

| $\lambda$ | 0.056     | 0.001          | 0.945 |
| $N_{max}$ | 203776    | 2922.990       |       |

Equation 5

| $\lambda$ | 0.011     | 0.001          | 0.902 |
| $N_{max}$ | 409156    | 18249.270      |       |

The estimated value of the constants and corresponding statistical parameters (standard error and $R^2$) are reported in Table 1. As observed, the growth rate coefficient estimated from the exponential model ($\lambda = 0.185$) is around 13.2 times and 3.3 times higher than those found by Equation 2 ($\lambda = 0.014$) and Equation 4 ($\lambda = 0.056$), respectively. The high growth rate can be attributed to the fact that at the beginning of the pandemic nothing was clear and health policies were not implemented promptly even though Ecuador had two months to prepare for the COVID-19 pandemic.

As public health strategies were implemented, i.e., mitigation (social distancing, case testing, and symptomatic case isolation) and suppression (case isolation, quarantine, treating COVID-19 patients, and viral testing), the growth rate decreases as illustrated in Figure 3. However, the growth rate estimated by Equation 4 (Figure 3c) is four times higher than that estimated by Equation 2 (Figure 3b), suggesting a chaotic/random behavior as a function of time (discussed below). The approximation of the available data is good enough ($R^2 > 0.9$), suggesting the approach proposed is sustainable for the Ecuador case [14].

Figure 3. Matrix representation of the growth rate coefficient ($\lambda$) from (a) exponential model, (b) discrete logistic equation, and (c) solved ODE logistic model. Red and green markers represent the infected and non-infected people by COVID-19.
The turning point \( t_0 = 225.56 \) can be estimated from Equation 2, which means that the maximum growth rate was reached. The latter is not observed since the data does not follow a sigmoidal trend (Figure 1b). To explain this outcome, we hypothesize that the outbreak of COVID-19 in Ecuador is characterized by a multi-stage behavior where the first, second, or more waves overlap, giving an average value of the turning point which could increase if more data points are added.

On the other hand, the maximum number of infected people can be estimated as either Equation 2 \( N_{\text{max}} = 294730 \) or Equation 4 \( N_{\text{max}} = 203776 \) but the resulting values were exceeded at the end of February 2021. For this reason, a more accurate value of the maximum number of infected people can be obtained from Equation 5. Thus, Figure 4 displays the relationship between daily reported cases \( (R) \) and total cases \( (N) \). The parabolic approximation (blue curve) arising from Equation 5, shows a reasonable agreement with the data reported \( (R^2 = 0.9) \), giving a \( N_{\text{max}} = 409156 \) (Table 1).

In fact, this result agrees with the data reported at the end of May 2021 \( N_{\text{max}} = 426037 \). As observed, the scatter of data points with respect to the parabolic curve is not small, indicating that it is necessary to consider the variability of the growth rate coefficient as a time function, particularly, if new daily cases are reported. As stated, the coefficient variability can be determined from Equation 6, expressed as a function of total cases (Figure 5) or time (Figure 6).

![Figure 4](image)

**Figure 4.** The daily cases as a function of the total confirmed cases. The orange markers show the data and the blue curve is the regression according to Eq. (5).
Figure 5. The growth rate coefficient as a function of the total confirmed cases, computed using Eq. (6) and considering the data in a one-year window in orange and a close picture in blue.

Figure 6. The growth rate coefficient of Eq. (6) is represented as a surface-color plot vs 30-days and 12-months. (a) the growth rate in a one-year window and (b) the growth rate from May 1, 2020, to February 28, 2021.

Figure 5 shows the variability of the coefficient $\lambda$. In particular, the data in orange show that the growth rate coefficient obtained the highest values in the first $10^3$ total confirmed cases ($0.2 < \lambda < 0.06$). After $20^3$ and $30^3$ total confirmed cases, the value of the coefficient $\lambda$ decreases to $\sim 0.05$ and $\sim 0.01$, respectively. The variability of the coefficient $\lambda$ is confirmed in the data reported in blue from $40^3$ to $300^3$ total cases with $0.02 < \lambda < 0.005$. While the confinement declared on March 16, 2020, had a positive effect on reducing the $\lambda$ values, the high number of infected people in the early days of the pandemic collapsed Ecuador's weak health service.

This assertion can be easily observed in Figure 6 where the coefficient $\lambda$ is reported in time. March and April 2020 were the most critical months in Ecuador, particularly, the maximum values of the coefficient $\lambda$ are observed in the third week (Figure 6a). In the remaining months (from May 2020 to February 2021), a variability of the coefficient $\lambda$ is also appreciated which can be attributed to the variable number of examined people per
day or social behavior. However, its values are not as high as in the first month because the population adopted for personal care such as the use of face masks, sanitizers, and social distancing (Figure 6b).

4. Conclusions

In summary, we have presented a mathematical approach based on the logistic model to explore the first year of COVID-19 in Ecuador. In particular, the predicted numerical value of the total number of infected people was found to be in good agreement with the real data reported at the end of May 2021. Thus, it was also evidenced that the most critical months were March and April 2020 with a growth rate coefficient of about 0.2 day\(^{-1}\). This simple approach can be calibrated by adding the daily data of infected people, most importantly, it can be used in other Latin American countries and localities to take immediate action against the COVID-19 disease.

5. References

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