Primordial black holes and oscillating gravitational waves in slow-roll and slow-climb inflation with an intermediate non-inflationary phase

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Abstract

We propose a new single field inflation model in which the usual slow-roll inflation is joined to a new period of slow-climb and slow-roll inflation through a short intermediate non-inflationary phase. We then show that primordial curvature perturbations can be enhanced at small scales, a sizable amount of primordial black holes (PBHs) can be produced which make up most of dark matter, and the gravitational waves (GWs) induced by scalar metric perturbations that accompany with the formation of PBHs can be detectable by future GW experiments. Remarkably, the GW spectrum displays a unique oscillating character in the ultraviolet regions which originates from the short non-inflationary phase. A detection of such oscillations in the GW spectrum may suggest the existence of such a non-inflationary phase in the whole inflation, thus providing us a chance to reveal an interesting period in the evolution of the early universe and distinguish our model from others.

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I. INTRODUCTION

Black holes (BHs), whose merger and shadow have been, respectively, listened by LIGO-Virgo observation [1, 2] and seen through Event Horizon Telescope [3, 4], are among the most remarkable and fascinating objects in nature. They can be generated from the stellar evolution with masses around $10 M_\odot$, as well as from the collapse of over-dense regions induced by large enough curvature perturbations generated during inflation, long before ordinary stellar BHs were formed. The latter kind of the BHs is dubbed primordial black holes (PBHs) [5–7] and their masses can span many orders of magnitude contingent upon the formation mechanism.

PBHs have been proposed as a promising candidate for the origin of BHs detected by the LIGO-Virgo Collaboration [8–11] since many observed binary BHs have masses around $30 M_\odot$ which are too heavy for BHs formed by the stellar evolution. For PBHs to be capable of accounting for the LIGO-Virgo gravitational wave (GW) events, it is required that they comprise $\mathcal{O}(1)\%$ of the total dark matter (DM) [10]. Actually, PBHs can make up all DM if their masses are around around $10^{-16} - 10^{-14} M_\odot$ and around $10^{-13} - 10^{-11} M_\odot$ [12–15]. Furthermore, PBHs with a mass around $\mathcal{O}(10^{-5}) M_\odot$ can be used to explain the ultrashort-timescale microlensing events in the OGLE data [16, 17]. Recently, the clustering of orbits of Kuiper belt objects has given some hints that there is a novel object (“Planet 9”) with a mass $5 - 10 M_\oplus$ in the outer Solar System [18–20]. It has been suggested that Planet 9 might be a PBH [21, 22]. As PBHs seem to hold the key to explaining the above mentioned astrophysical and cosmological observations, they are becoming increasingly attractive in physics, astronomy and cosmology.

The production of a sizable amount of PBHs requires that the power spectrum of primordial curvature perturbations $P_R$ produced during the inflationary era is amplified about seven orders of magnitude compared with that, $P_R \sim \mathcal{O}(10^{-9})$, on the cosmic microwave background (CMB) scales. Since in the slow-roll inflation model the power spectrum of primordial curvature perturbations is inversely proportional to one of the slow-roll parameters, a natural mechanism to enhance the curvature perturbations is to flatten the potential to realize a period of ultra-slow-roll inflation, i.e., the so-called inflection-point inflation [23–28]. The ultra-slow-roll inflation can also be obtained through slowing down the motion of the inflaton by increasing friction [29] or introducing a non-canonical kinetic term [30]. In ad-
dition, it has been found that the primordial curvature perturbations can also be enhanced through some other mechanisms, such as making the sound speed to approach to zero during some stages of inflation [31, 32], the parametric resonance arising from the oscillating sound speed squared [33] or the oscillating potential [34], and so on. Associated with the formation of PBHs, the enhanced curvature perturbations, when they reenter the horizon, may also induce very large scalar metric fluctuations, which would generate a significant stochastic GW background [35–45]. These scalar induced GWs (SIGWs) are very important since they might be detected by future GW projects.

In the standard inflation theory, the cosmic inflation is followed by a so-called reheating phase, in which the inflaton rolls rapidly through the minimum of its potential and oscillates around it to thermalize the Universe. As we have already mentioned, one way to amplify the curvature perturbations at small scales so that a sizable amount of PBHs will be generated is to slow down the rolling of the inflaton during the late stage of inflation either by flattening the potential or increasing the friction. Here, we propose a new inflation model which also naturally achieves the goal of enhancing curvature perturbations at small scales. Our basic idea is that the inflaton, after the usual slow-roll phase and passing across the minimum of its potential, undergoes a new period of slow-climb and slow-roll as it climbs up a flattened potential, which is then followed by the standard reheating. In our model, the usual slow-roll inflation is joined to a new slow-climb and slow-roll inflation with a short non-inflationary phase in between, i.e, altogether, there exist three inflationary phases and one non-inflationary phase in the whole inflation era (See Fig. 1 for a graphic description). Since the slow-climb phase is a non-attractor inflation due to the rapid change of the velocity of the inflaton, the curvature perturbations can be enhanced as so to produce a sizable amount of the PBHs. In the following, we construct a concrete simple single field model and study its consequences. As we will show later, the generated PBHs can make up most of DM and the SIGWs can be detectable by future GW projects. Remarkably, as a result of the short non-inflationary phase in the whole inflation era, the spectrum of the SIGWs displays an oscillating behavior in the ultraviolet regions which is testable by future GW experiments.
II. MODEL

To realize a non-attractor inflation to enhance the curvature perturbations after the inflaton rolls through the minimum of its potential for the first time, we consider the following potential

$$V(\phi) = \Lambda^4 \left\{ \left(1 + \frac{\phi^2}{M_1^2}\right)^{\frac{q}{2}} - 1 \right\} \Theta(\phi) + \xi \tanh^2 \left(\frac{\phi}{M_2}\right) \Theta(-\phi)$$

(1)

with \(q = 2/5\). Here, \(\Lambda, M_1\) and \(M_2\) are the parameters with the dimension of mass, \(\xi\) is a dimensionless parameter, and \(\Theta\) is the Heaviside theta function. Let us note that the \(\phi > 0\) part of this potential (1) is of the monodromy type [46, 47] while the other part is similar to that of an \(\alpha\)-attractor T-model [48]. To ensure that the potential (1) has a continuous second derivative with respect to \(\phi\) at \(\phi = 0\), parameter \(\xi\) is determined to be \(\xi = q(M_2/M_1)^2/2\). So, parameters \(M_1\) and \(M_2\) control the inflationary dynamics. The potential is plotted in Fig. 1.

A successful inflation must satisfy the CMB observations. The Planck results [49] have constrained the amplitude of the power spectrum, the scalar spectral index and the tensor-to-scalar ratio at the pivot scale \(k^* = 0.05\) Mpc\(^{-1}\) to be \(\ln(10^{10}P_R) = 3.044 \pm 0.014 \) (68% C.L.), \(n_s = 0.9649 \pm 0.0042 \) (68% C.L.), and \(r < 0.07 \) (95% C.L.), respectively. To obtain the allowed region of \(M_1\) (or \(M_2\)) constrained by \(n_s\), we need to give a concrete value to \(M_2\) (or \(M_1\)). Thus, there is only one free parameter in our model since \(\Lambda\) is determined only by \(P_R\) after \(M_1\) and \(M_2\) are given. Since parameter \(M_2\) is decisive to enhancing the curvature perturbations, we take \(M_2\) as the free parameter. In our analysis, we choose \(M_2 = 0.009M_p\), where \(M_p\) is the reduced Planck mass and \(M_p = 1\) is set in the numerical calculations. After setting the \(e\)-folding number from the time when \(k_s\) exits the horizon to the end of the whole inflation as \(N_s = 60\), we find that \(M_1\) is constrained to \(0.00179670 < M_1/M_p < 0.00179694\) by \(n_s\). We adopt \(M_1 = 0.00179687M_p\) as an example in the our analysis. In this case, we find that \(n_s \simeq 0.9639\) and \(r \simeq 0.049\), which are compatible with the current CMB observations. Using \(P_R \simeq 2.1 \times 10^{-9}\) at the scale \(k_s\), we obtain \(\Lambda \simeq 0.0028\).

Now, we discuss the inflationary dynamics of our model. Initially, the inflaton rolls down its potential from \(\phi > 0\). We set \(t_i = 0\) as the time when \(k_s\) exits the horizon. Figure 2 shows the evolutions of slow-roll parameters \(\epsilon = -\dot{H}/H^2\) and \(\eta = \ddot{\phi}/(H\dot{\phi})\) with respect to

\(t\).
the rescaled time $\Lambda^2 t$, where $H = \frac{\dot{a}}{a}$ with $a$ being the scale factor and a overdot denoting the derivative with respect to the cosmic time $t$. The whole evolution can be divided into four stages, which are described in Fig. 1 and Fig. 2 with four different color regions. The first stage is the usual slow-roll inflation in which the inflaton rolls slowly down the monodromy type potential. This stage ends at the time $t = t_1 \simeq 13.757 \Lambda^{-2}$ ($\epsilon = 1$ when $t = t_1$) and generates approximately $\Delta N \simeq 34.42$ e-folds for model parameters given the above. Then, the inflaton rolls rapidly through the minimum of its potential and climbs up the potential. The velocity of the inflaton decreases quickly during the climb-up. When the slow-roll parameter $\epsilon$ is very small, a new inflation stage begins. Taking the time $t_2 \simeq 13.865 \Lambda^{-2}$ at which $\epsilon = 0.01$ as the onset time of this new slow-climb inflation, the Universe undergoes a short non-inflationary phase in the period of $t_1 < t < t_2$, generating just $\Delta N \simeq 0.15$ e-folds. The new slow-climb inflation which follows the non-inflationary phase is a non-attractor inflation since the velocity of the inflaton changes rapidly, which can be seen from the right panel of Fig. 2. This non-attractor inflation goes on until $t = t_3 \simeq 15.782 \Lambda^{-2}$ ($\eta = 1$ when $t = t_3$), generating about $\Delta N \simeq 3.13$ e-folds. The last stage ($t > t_3$) is a period of the usual slow-roll inflation again, which generates about $\Delta N \simeq 22.3$ e-folds. After this stage, the Universe enters the standard reheating phase in which the inflaton oscillates rapidly around the minimum of the potential to thermalize the Universe.
To produce PBHs, the curvature perturbations at the small scales must be enhanced during inflation. The curvature perturbation $\mathcal{R}_k$ in the momentum space satisfies the equation of motion

$$\ddot{\mathcal{R}}_k + (3 + 2\epsilon + 2\eta) H \dot{\mathcal{R}}_k + \frac{k^2}{a^2} \mathcal{R}_k = 0 .$$  \hspace{1cm} (2)

During the usual slow-roll inflation, the third term can be neglected at the super Hubble horizon scales. So, the general solution of this equation contains a constant term and a decaying term, which results in a nearly scale-invariant power spectrum. While in the non-attractor inflation phase since the coefficient of the second term in Eq. (2) becomes negative, the curvature perturbations will be increased. For the modes which have left the horizon before several $e$-folds at the end of the first slow-roll inflation, their growth during the non-attractor inflation can not overtake the decay, and thus they remain nearly constant, which gives a scale-invariant power spectrum at the large scales to satisfy the CMB observations. For the modes leaving the horizon during the last several $e$-folds of the first slow-roll inflation and the slow-climb inflation, the non-attractor inflation will make them grow significantly. So, the power spectrum at these scales will be enhanced.

Figure 3 shows the resulting power spectrum $\mathcal{P}_\mathcal{R}$. One can see that the power spectrum remains nearly scale invariant on the large scales, which is compatible with the Planck 2018 results. At the small scales, the power spectrum is enhanced by the non-attractor inflation. The enhanced power spectrum displays an oscillating behavior, which is a result of the existence of the non-inflationary stage. During this stage, the second term in Eq. (2)
FIG. 3: The power spectrum $\mathcal{P}_\mathcal{R}$ (red line). The green-shaded region is excluded by the current CMB observations [49].

becomes dominant and its coefficient turns negative in the latter part of this stage since $\eta$ is much less than 0, which results in some modes to grow in a short time interval. As a consequence, the phase difference between the real and imaginary parts of the modes whose wave lengths are in the horizon but not too small changes from $\pi/2$ to near 0 after they undergo this short non-inflationary phase. This can be seen from Fig. 4. Accordingly, $|\mathcal{R}_k(t)|^2$ will evolve into an oscillating era, which leads to the oscillating structure in the power spectrum. This is a unique character for the model proposed in this paper. The detailed discussion for the growth of curvature perturbations will be given elsewhere.

III. PRIMORDIAL BLACK HOLES

When the enhanced curvature perturbations reenter the horizon, they may cause overdense regions to collapse into BHs. PBHs have been proposed as a component of DM. The current fraction of PBHs against the total DM can be expressed as

$$\frac{\Omega_{\text{PBH}}}{\Omega_{\text{DM}}} = \int \frac{dM}{M} f_{\text{PBH}}(M),$$

where $M$ is the PBH mass, which is relative to the horizon mass at the horizon entry of the perturbations through the relation $M(k) = \gamma \frac{4\pi}{\kappa^2 H} |_{k=aH} \simeq$
FIG. 4: The time evolutions of \( k^3 \text{Re}[\mathcal{R}_k(t)]^2/(2\pi^2) \) (red line) and \( k^3 \text{Im}[\mathcal{R}_k(t)]^2/(2\pi^2) \) (blue line) for \( k = 1.08 \times 10^{15}\text{Mpc}^{-1} \). \( \text{Re}[\mathcal{R}_k(t)] \) and \( \text{Im}[\mathcal{R}_k(t)] \) denote the real part and imaginary part of \( \mathcal{R}_k(t) \), respectively. The light-yellow region represents the non-inflationary stage.

\[
M_\odot \left( \frac{\gamma}{0.2} \right) \left( \frac{g_*}{10.75} \right)^{-\frac{1}{3}} \left( \frac{k}{1.9 \times 10^6 \text{Mpc}^{-1}} \right)^{-2}, \quad \text{and}
\]

\[
f_{\text{PBH}}(M) \equiv \frac{1}{\Omega_{\text{DM}}} \frac{d\Omega_{\text{PBH}}}{d\ln M} \approx \beta(M) \frac{\gamma}{1.84 \times 10^{-8}} \left( \frac{10.75}{g_*} \right)^{\frac{1}{2}} \left( \frac{0.12}{\Omega_{\text{DM}} h^2} \right) \left( \frac{M}{M_\odot} \right)^{-\frac{1}{2}}. \quad (4)
\]

Here, \( \Omega_{\text{DM}} \) is the current density parameter of DM, which is limited to be \( \Omega_{\text{DM}} h^2 \approx 0.12 \) by the Planck 2018 observations \cite{53} and \( g_* \) is the number of effectively relativistic degrees of freedom at the PBH formation. Assuming that the PBHs are formed deep in the radiation-dominated era, one has \( g_* = 106.75 \). Parameter \( \gamma \) is the ratio of the PBH mass to the horizon mass and represents the efficiency of collapse, which, in our analysis, is set to be \( \gamma \approx (1/\sqrt{3})^3 \) estimated by the simple analytical calculation \cite{7}.

Function \( \beta(M) \) in Eq. (4) is the production rate of PBHs with mass \( M(k) \). Based on the Press-Schechter theory, one has \( \beta(M) = \int_{\delta_c}^\infty \frac{d\delta}{\sqrt{2\pi} \sigma^2(M)} e^{-\frac{\delta^2}{2\sigma^2(M)}} = \frac{1}{2} \text{erfc}\left( \frac{\delta_c}{\sqrt{2\sigma^2(M)}} \right) \) if the probability distribution function of perturbations is assumed to be Gaussian \cite{50}. Here, \( \text{erfc} \) denotes the complementary error function and \( \delta_c \) is the threshold of the density perturbations for the PBH formation. We adopt \( \delta_c = 0.412 \) in our subsequent calculations since recent studies \cite{51, 52} have shown that \( \delta_c \approx 0.4 \sim 0.5 \). The variance \( \sigma^2(M) \) represents the coarse-grained density contrast with the smoothing scale \( k \), which is defined as
The predicted mass spectrum of PBHs, $f_{PBH}(M)$, is plotted in Fig. 5, in which the current observational constraints on the PBH abundance are also shown. We find that the mass spectrum of PBHs has a sharp peak at $1.12 \times 10^{-15} M_\odot$, and its height is about 2.1. These PBHs can make up most of DM since $\frac{\Omega_{PBH}}{\Omega_{DM}} \simeq 0.91$.

**IV. SCALAR INDUCED GRAVITATIONAL WAVES**

When the enhanced curvature perturbations reenter the horizon, the PBHs will be formed in the over-dense regions, and at the same time very large scalar metric perturbations will be induced. These large metric perturbations become a significant GW source and emit abundant GW signals via the second-order effect to form a stochastic GW background. It has been found that, during the radiation-dominated era, the energy density of SIGWs per logarithmic interval of $k$ can be evaluated, at conformal time $\tau_c$ which represents the time...
when $\Omega_{GW}$ stops to grow, as [35]

$$
\Omega_{GW}(\tau_c, k) = \frac{1}{12} \int_0^\infty dv \int_{1-v}^{1+v} du \left( 4v^2 - (1 + v^2 - u^2)^2 \right)^2 \mathcal{P}_R(ku) \mathcal{P}_R(kv) \left( \frac{3}{4u^3v^3} \right)^2 (u^2 + v^2 - 3)^2 \right]

\begin{align*}
\left\{ [-4uv + (u^2 + v^2 - 3) \ln \frac{3 - (u + v)^2}{3 - (u - v)^2}]^2 + \pi^2 (u^2 + v^2 - 3)^2 \Theta(v + u - \sqrt{3}) \right\}.
\end{align*}

The current energy spectra of SIGWs can be expressed as [40]

$$
\Omega_{GW,0} h^2 = 0.83 \left( \frac{g_*}{10.75} \right)^{-1/3} \Omega_{r,0} h^2 \Omega_{GW}(\tau_c, k),
$$

where $\Omega_{r,0} h^2 \simeq 4.2 \times 10^{-5}$ is the current density parameter of radiation. The current frequency $f$ of SIGWs, which relates to the comoving wave number $k$, can be obtained through $f = 1.546 \times 10^{-15} \frac{k}{1 \text{ Mpc}^{-1}} \text{Hz}$.

We show the predicted current energy spectrum of SIGWs associated with the production of PBHs in Fig. 6. One can see that the GW signals could be probed by future GW projects, i.e., LISA [58], DECIGO [59] and BBO [60]. It is interesting to note that the GW power spectrum displays a special oscillating structure in the ultraviolet regions, which is a unique feature that distinguishes our model from others. Since these oscillations originate from
the short non-inflationary stage, if they are found in the future GW experiments, we may conclude that there exists a non-inflationary phase in the whole inflation era in the early evolution of our Universe.

V. CONCLUSION

In this paper, we propose a simple single field inflation model, which can enhance naturally the curvature perturbations so that a sizable amount of PBHs can be produced. In our model, the whole inflation contains four stages: two slow-roll inflation phases, one slow-climb inflation phase and a short non-inflationary one. Since the slow-climb phase is a non-attractor inflation, the curvature perturbations can be enhanced during this stage to satisfy the condition for a production of a sizable amount of PBHs. We find that the produced PBHs can make up most of DM, and SIGWs accompanying with the formation of PBHs can be detectable by future GW experiments, i.e. LISA, BBO, and DECIGO. Remarkably, the spectrum of SIGWs shows a unique oscillating character in the ultraviolet regions, which is a result of the short non-inflationary phase in our model. A detection of such oscillations in the GW spectrum may suggest the existence of such a non-inflation phase in the whole inflation, thus providing us a chance to reveal an interesting period in the evolution of the early universe and distinguish our model from other inflation models.

Acknowledgments

This work was supported in part by the NSFC under Grants No. 11775077, No. 11435006, No. 11690034, and No. 11805063, and by the Science and Technology Innovation Plan of Hunan province under Grant No. 2017XK2019.

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