Triminimal Parametrization of Quark Mixing Matrix

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Abstract

Starting from a new zeroth order basis for quark mixing (CKM) matrix based on the quark-lepton complementarity and the tri-bimaximal pattern of lepton mixing, we derive a triminimal parametrization of CKM matrix with three small angles and a CP-violating phase as its parameters. This new triminimal parametrization has the merits of fast convergence and simplicity in application. With the quark-lepton complementary relations, we derive relations between the two unified triminimal parametrizations for quark mixing obtained in this work and for lepton mixing obtained by Pakvasa-Rodejohann-Weiler. Parametrization deviating from quark-lepton complementarity is also discussed.

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Quarks are the fundamental constituents of matter. An appropriate parametrization of the quark mixing matrix is an important step to understand the mixing pattern and its underlying theory. The mixing matrix can be described by the Cabibbo\([1]\), Kobayashi and Maskawa\([2]\) (CKM) $V_{\text{CKM}}$ matrix. A commonly used form of $V_{\text{CKM}}$ for three generations of quarks is given by \([3, 4]\),

$$V_{\text{CKM}} = \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -s_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}, \quad (1)$$

where $s_{ij} = \sin \theta_{ij}$ and $c_{ij} = \cos \theta_{ij}$ are the rotation angles and $\delta$ is the CP violating phase. The above expression can be decomposed as

$$V_{\text{CKM}} = R_{23}(\theta_{23})U_{\delta}^{\dagger}R_{13}(\theta_{13})U_{\delta}R_{12}(\theta_{12}), \quad (2)$$

with

$$R_{23} = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}, \quad R_{13} = \begin{pmatrix}
c_{13} & 0 & s_{13} \\
0 & 1 & 0 \\
-s_{13} & 0 & c_{13}
\end{pmatrix}, \quad R_{12} = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}, \quad (3)$$

and $U_{\delta}$ is a diagonal phase matrix with the diagonal elements given by $\text{diag}(e^{i\delta/2}, 1, e^{-i\delta/2})$.

The elements in the CKM matrix have been determined to high precision with the allowed ranges of the magnitudes of its elements as \([4]\)

$$\begin{pmatrix}
0.97419 \pm 0.00022 & 0.2257 \pm 0.0010 & 0.00359 \pm 0.00016 \\
0.2256 \pm 0.0010 & 0.97334 \pm 0.00023 & 0.0415^{+0.0010}_{-0.0011} \\
0.00874^{+0.00026}_{-0.00037} & 0.0407 \pm 0.0010 & 0.999133^{+0.000044}_{-0.000043}
\end{pmatrix}. \quad (4)$$

With Eq. (1) and the data given in Eq. (4), we find that

$$\sin \theta_{12} = \frac{|V_{us}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} = 0.2257 \pm 0.0010,$$

$$\sin \theta_{23} = \frac{|V_{cb}|}{\sqrt{|V_{ud}|^2 + |V_{us}|^2}} = 0.0415^{+0.0010}_{-0.0011},$$

$$\sin \theta_{13} = |V_{ub}| = 0.00359 \pm 0.00016. \quad (5)$$

The CP violating phase has also been determined. A recent full fit from UTfit group gives $\delta = (66.7 \pm 6.4)^{\circ} \[5\].
The CKM matrix is close to the unit matrix with small deviations in the non-diagonal elements. This led Wolfenstien, some time ago, to parameterize the mixing matrix as

\[
V_{\text{CKM}} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^2 \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4).
\] (6)

This parametrization explicitly shows the deviations of the non-diagonal elements from the unit matrix in different powers of the parameter \( \lambda \) with \( \lambda = 0.2257^{+0.0009}_{-0.0009} \). Since \( A = 0.814^{+0.021}_{-0.022} \), \( \rho (1 - \lambda^2 / 2 + \ldots) = 0.135^{+0.031}_{-0.016} \), and \( \eta (1 - \lambda^2 / 2 + \ldots) = 0.349^{+0.015}_{-0.017} \) which are of order one, the hierarchy of the CKM elements in powers of \( \lambda \) in the expansion is evident.

The unit matrix as the zeroth leading order for expanding the CKM matrix is easy to use. However, we find that parameterizing the CKM matrix in ways leading to faster convergence also interesting which provides more information about mixing. One then asks the question that what physical reason(s) would help to make the zeroth leading order matrix. An appealing guideline in searching for such a method is the quark-lepton complementarity (QLC) \([7, 8]\), first introduced by Smirnov \([7]\) and then discussed in many other papers \([8]\), which relates quark and lepton mixing angles with

\[
\theta_{12}^Q + \theta_{12}^L = \frac{\pi}{4}, \quad \theta_{23}^Q + \theta_{23}^L = \frac{\pi}{4}, \quad \theta_{13}^Q \sim \theta_{13}^L \sim 0,
\] (7)

where the superscript \( Q \) indicates the mixing angles in the CKM matrix, and the superscript \( L \) indicates mixing angles in the lepton mixing matrix \( U_{\text{PMNS}} \), i.e, the Pontecorvo-Maki-Nakawaga-Sakata \([10]\) (PMNS) matrix. From now on we will use these notations to distinguish mixing angles in the CKM and PMNS matrices. The above relations enable one to determine the mixing angles use both quark and lepton mixing information, and therefore provide a unified understanding of quark mixing and lepton mixing phenomena.

Current experimental data show that the PMNS matrix is close to the tri-bimaximal pattern given by,

\[
U = \begin{pmatrix}
2/\sqrt{6} & 1/\sqrt{3} & 0 \\
-1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \\
1/\sqrt{6} & -1/\sqrt{3} & 1/\sqrt{2}
\end{pmatrix}.
\] (8)

The tri-bimaximal mixing has been studied extensively in literature \([11, 12]\). Parametrization of the PMNS matrix around the tri-bimaximal pattern has been treated in several ap-
proaches [13]. Recently, Pakvasa, Rodejohann, and Weiler (PRW) gave a new parametri-
zation of the PMNS matrix [14] with the matrix in Eq. (8) as the zeroth order in an expansion.
In their paper, the parameters chosen are not the traditional deviations within the elements
of the matrix, instead, they are the deviations from the tri-bimaximal angles. The method
pointed out a new way to parameterize the mixing matrix with all angles small, i.e. the
“triminimal” parametrization.

Enlightened by above idea, we propose to parameterize the CKM matrix with also three
small angles, which we refer as “triminimal” parametrization of the CKM matrix. In this
parametrization the zeroth leading order CKM matrix $V_b$ is inspired from QLC suggested in
Ref. [15],

$$\sin \theta_{12}^Q = \frac{\sqrt{2} - 1}{\sqrt{6}}, \quad \sin \theta_{23}^Q = 0, \quad \sin \theta_{13}^Q = 0. \quad (9)$$

which leads to the zeroth order basis as

$$V_b = \begin{pmatrix}
\frac{\sqrt{2}+1}{\sqrt{6}} & \frac{\sqrt{2}-1}{\sqrt{6}} & 0 \\
-\frac{\sqrt{2}-1}{\sqrt{6}} & \frac{\sqrt{2}+1}{\sqrt{6}} & 0 \\
0 & 0 & 1
\end{pmatrix}. \quad (10)$$

Although the new matrix in Eq. (10) is a little more complicated compared with the unit
matrix, it is closer to reality and deviations of the CKM matrix from the new matrix are
rather small. The expansion around $V_b$ converges very quickly.

We have motivated our choice of $V_b$ from QLC relations following Ref. [15]. If a different
principle is used, the form $V_b$ may be different. For example, it has been argued that the angle
$\theta_{12}^Q$ should satisfy [16] $\tan(2\theta_{12}) = 1/2$ which is numerically more close to the experimental
value than we choose. There are ambiguities in choosing the zeroth order matrix. Better
theoretical understanding is needed to determine the form of $V_b$. We believe that the QLC,
which leads to some unified understanding of mixing angles in lepton and quark sectors, is
a good starting point and will follow this rout to carry out our analysis. When using QLC
relations, one should also be careful about at which scale they hold. In this work we will
assume that QLC holds at a relatively low scale such that the renormalization group equation
(RGE) running effects down to SM scale can be neglected. If the QLC holds at a very high
energy, say grand unification scale, one should include possible large RGE running effects,
in particular possible large effects in neutrino sector [17]. This is an interesting possibility
which will be studied in a future work.
Using $\epsilon_{12}^Q$, $\epsilon_{23}^Q$, and $\epsilon_{13}^Q$ defined by

$$\theta_{12}^Q = \arcsin \sqrt{2} - \frac{1}{\sqrt{6}} + \epsilon_{12}^Q, \quad \theta_{23}^Q = \epsilon_{23}^Q, \quad \theta_{13}^Q = \epsilon_{13}^Q,$$

we obtain the triminimal form for the CKM matrix,

$$V_{\text{CKM}} = R_{23}(\theta_{23}^Q)U_{\delta}^\dagger R_{13}(\theta_{13}^Q)U_{\delta} R_{12}(\theta_{12}^Q)$$

$$= R_{23}(\epsilon_{23}^Q)U_{\delta}^\dagger R_{13}(\epsilon_{13}^Q)U_{\delta} R_{12}(\epsilon_{12}^Q)\left(\arcsin \frac{\sqrt{2} - 1}{\sqrt{6}}\right)$$

Comparing with the data in Eq. (11), we obtain

$$\epsilon_{12}^Q = 0.0577 \pm 0.0010, \quad \epsilon_{23}^Q = 0.0415^{+0.0010}_{-0.0011}, \quad \epsilon_{13}^Q = 0.00359 \pm 0.00016.$$

We see that the angles $\epsilon_{ij}^Q$ are indeed very small, much smaller than $\lambda$ in the Wolfenstein parametrization. Therefore using the parameters $\epsilon_{ij}^Q$, the expansion converges faster than the Wolfenstein parametrization.

To the second order expansion in $\epsilon_{ij}^Q$, $V_{\text{CKM}}$ is given by

$$V_{\text{CKM}} = V_6 + \epsilon_{12}^Q \left( \begin{array}{ccc} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 0 \end{array} \right) + \epsilon_{23}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) + \epsilon_{13}^Q \left( \begin{array}{ccc} 0 & 0 & e^{-i\delta} \\ 0 & 0 & 0 \\ 0 & 0 & e^{i\delta} \end{array} \right)$$

$$+ (\epsilon_{12}^Q)^2 \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) + (\epsilon_{23}^Q)^2 \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right) + (\epsilon_{13}^Q)^2 \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array} \right)$$

$$+ \epsilon_{12}^Q \epsilon_{23}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) + \epsilon_{12}^Q \epsilon_{13}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) + \epsilon_{23}^Q \epsilon_{13}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right)$$

$$+ \epsilon_{12}^Q \epsilon_{23}^Q \epsilon_{13}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) + \epsilon_{12}^Q \epsilon_{13}^Q \epsilon_{13}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right) + \epsilon_{23}^Q \epsilon_{13}^Q \epsilon_{13}^Q \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{array} \right)$$
Note that numerically we have, $\epsilon_{23}^Q \sim \epsilon_{12}^Q$, but $\epsilon_{13}^Q \sim (\epsilon_{12}^Q)^2$. This implies that $\epsilon_{12}^Q$, $\epsilon_{23}^Q$ are of order $O(\epsilon_{12}^Q)$, but $\epsilon_{13}^Q$ is of order $O((\epsilon_{12}^Q)^2)$. Therefore $\epsilon_{12}^Q \epsilon_{13}^Q$, $\epsilon_{23}^Q \epsilon_{13}^Q$ and $(\epsilon_{13}^Q)^2$ in Eq. (14) can be omitted if we make an approximation to order $O((\epsilon_{12}^Q)^2)$. Also note that the effect of CP violation exists in first power in $\epsilon_{13}^Q$, it is of order $O((\epsilon_{12}^Q)^2)$ in this parametrization, which is coordinated with the result in Ref. [15].

The rephasing-invariant Jarlskog parameter, $J \equiv \text{Im}(V_{us} V_{cb} V_{ub}^* V_{cs})$ [18], in our parametrization is given by a simple expression,

$$J = \left(\frac{1}{6} + \frac{2\sqrt{2}}{3} \epsilon_{12}^Q\right) \epsilon_{23}^Q \epsilon_{13}^Q \sin \delta. \quad (15)$$

Since $\frac{2\sqrt{2}}{3} \epsilon_{12}^Q \sim \frac{1}{3} \times \frac{1}{6}$, the $O((\epsilon_{ij}^Q)^3)$ term should be kept in the expression of $J$.

The QLC relations in Eq. (7) has important implications. If one imposes these empirical equations, the parameters in our parametrization and those proposed for lepton mixing in Ref. [14] are not independent, we find that

$$\epsilon_{12}^Q + \epsilon_{12}^L = 0,$$
$$\epsilon_{23}^Q + \epsilon_{23}^L = 0,$$
$$\epsilon_{13}^Q \sim \epsilon_{13}^L \sim 0. \quad (16)$$

The QLC leads to unified triminimal parametrizations for mixing in quark and lepton sectors.

Considerable experimental data on lepton mixing have been accumulated. To have some idea how good QLC relations provide guidance, in Table I we show the values of several quantities with their $1\sigma$($3\sigma$) errors, for QLC relations using global data fitting on CKM matrix elements in Eq. (4) [4], and on neutrino mixing angles obtained in [19], [20] and [21].

We see, from Table I that there are differences from different fits which reflects possible theoretical uncertainties in handling row data. However all fits obtain similar results. The best fit values of $\epsilon_{12,23}^L$ are minus relative to $\epsilon_{12,23}^Q$ which are in the right direction. But there are deviations. The values for $\epsilon_{12,23}^L$ are different from $-\epsilon_{12,23}^Q$. In particular the value for $\epsilon_{12}^L$ is substantially away from $\epsilon_{12}^Q$. The QLC relation for $\epsilon_{23}^{QL}$ is satisfied at $1\sigma$ level. The QLC relation for $\epsilon_{12}^{QL}$ is inconsistence at $1\sigma$ level. There is a tension with QLC predictions. However all QLC predictions are consistent at $3\sigma$ level. More experimental data are needed to test QLC. If the current trend persists, one should modify the QLC to some extent.

In most applications of QLC, $\epsilon_{13}^Q$ and $\epsilon_{13}^L$ do not have a specific relation, what is known is that they are both small. In this case the well determined $\epsilon_{13}^Q$ will not provide severe
constraint on $\epsilon_{13}^L$. If one requires $\epsilon_{13}^Q + \epsilon_{13}^L = 0$ or $\epsilon_{13}^L = \epsilon_{13}^Q$, then the value for $U_{e3}$ will be too small to be measured at near future experiments. However, this needs not be true as $\epsilon_{12}^Q$ and $\epsilon_{12}^L$ can be independent. At present it is not known whether the CP violating phases $\delta^Q$ and $\delta^L$ for CKM and PMNS matrices are related. Therefore knowing $\delta^Q$, one still does not know much about $\delta^L$. The determinations of $\epsilon_{13}^L$ and $\delta^L$ have to be left to future experiments or a underlying theory for CKM and PMNS matrices to decide. Also note that no information about possible Majorana phases in the PMNS matrix can be inferred from QLC in the present form.

We would like to stress that the triminimal parametrizations do not depend on QLC assumption since our parametrization and the parametrization in Ref. [14] are both general and can be independent of each other. If future experiments will rule out the QLC relations, one should not impose the relations in Eq. (7). The deviations from QLC relations, to the second order in $\epsilon_{ij}^{Q,L}$, can be written as

$$\sin(\theta_{12}^Q + \theta_{12}^L) = \frac{1}{\sqrt{2}} \left( \sin(\epsilon_{12}^Q + \epsilon_{12}^L) + \cos(\epsilon_{12}^Q + \epsilon_{12}^L) \right)$$

$$\approx \frac{1}{\sqrt{2}} \left( 1 + (\epsilon_{12}^Q + \epsilon_{12}^L) - \frac{1}{2} (\epsilon_{12}^Q + \epsilon_{12}^L)^2 \right),$$

$$\sin(\theta_{23}^Q + \theta_{23}^L) = \frac{1}{\sqrt{2}} \left( \sin(\epsilon_{23}^Q + \epsilon_{23}^L) + \cos(\epsilon_{23}^Q + \epsilon_{23}^L) \right)$$

$$\approx \frac{1}{\sqrt{2}} \left( 1 + (\epsilon_{23}^Q + \epsilon_{23}^L) - \frac{1}{2} (\epsilon_{23}^Q + \epsilon_{23}^L)^2 \right).$$

Clearly, the QLC is satisfied to zeroth order. That is to say, the QLC may be considered to be the lowest order approximation of the sum of the corresponding mixing angles of the CKM and PMNS matrices.

As pointed out earlier, there are evidences of violation of QLC since the best fit values $\epsilon_{12}^Q + \epsilon_{12}^L = (0.04, 0.03, 0.03)$ and $\epsilon_{23}^Q + \epsilon_{23}^L = (0.01, 0.01, 0.04)$ shown in Table I deviate from zero. One has to wait more data to make conclusions. In any case, parametrization deviating from QLC given above can provide a convenient way for experimental studies.

To conclude, we have proposed a new triminimal parametrization of the CKM matrix starting from a new zeroth order basis of quark mixing based on the quark-lepton complementarity and the tri-bimaximal pattern of lepton mixing. This new parametrization has three small angles and a CP-violating phase as its parameters. It not only has a unified form similar to the recently known PRW triminimal parametrization of lepton mixing [14],
but also has the merits of fast convergence and simplicity in application. With the quark-lepton complementary relations, we have derived relations between the two unified triminimal parametrizations for both quark mixing and lepton mixing. Parametrization deviating from quark-lepton complementarity is also discussed.

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