Tachyon fields with effects of quantum matter in an Anti-de Sitter Universe

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Abstract

We consider an Anti-de Sitter universe filled by quantum conformal matter with the contribution from the usual tachyon and a perfect fluid. The model represents the combination of a trace-anomaly annihilated and a tachyon driven Anti-de Sitter universe. The influence exerted by the quantum effects and by the tachyon on the AdS space is studied. The radius corresponding to this universe is calculated and the effect of the tachyon potential is discussed, in particular, concerning to the possibility to get an accelerated scale factor for the proposed model (implying an accelerated expansion of the AdS type of universe). Fulfillment of the cosmological energy conditions in the model is also investigated.

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One of the most intriguing questions in today’s Physics concerns the nature of the dark energy present in the Universe we live in. The existence of such energy, with almost uniform density distribution and a substantial negative pressure, which completely dominates all other forms of matter, is inferred from recent astronomical observations [1]. In particular, according to recent astrophysical data analysis, this dark energy seems to behave like a cosmological constant, and it is responsible for the accelerating expansion of the observable universe. And there are reasons to believe that answering this question has much to do with the possibility to explain the physics of the very early Universe.

Models of dark energy are abundant. One of the proposed candidates for it is the phantom, thus called because it relies on a negative energy field. The peculiar properties of a phantom scalar (with negative kinetic energy) in a space with non-zero cosmological constant have been discussed in an interesting paper by Gibbons [2]. It has been indicated there, that phantom properties bear some similarity with quantum effects [3]. The interesting property of the investigation in [2] is that it is easily generalizable to other constant curvature spaces, as the Anti-de Sitter (AdS) space. There is presently some interest in such spaces, coming in particular from the AdS/CFT correspondence. According to it, the AdS space might have in fact cosmological relevance [4], e.g. by way of increasing the number of particles created on a given subspace [5]. It could also be used to study a cosmological AdS/CFT correspondence [6]: the study of a phantom field in AdS space may give us a hint about the origin of such field via the dual description. In the supergravity formulation, one may think of the phantom as of a special RG flow for scalars in gauged AdS supergravity. (Actually, such RG flow may correspond to an imaginary scalar.)

Another candidate for dark energy is the tachyon. This is an unstable field. The interest of models exhibiting a tachyon is motivated by its role in the Dirac-Born-Infeld (DBI) action as a description of the D-brane action [7, 8, 9]. In spite of the fact that the tachyon represents an unstable field, its role in cosmology is still considered useful as a source of dark matter [10] and, depending on the form of the associated potential [11, 12, 13, 14, 15], it can lead to a period of inflation. On the other hand, it is important to realize that a tachyon with negative kinetic energy (yet another type of phantom) can be introduced [16]. In that phantom/tachyon model \( w \) is naturally negative. In this case the late time de Sitter attractor solution is admissible, and this
is one of the main reasons why it can be considered as an interesting model for the dark energy [16]. Moreover, in order to understand the role of the tachyon in cosmology it is necessary to study its effects on other backgrounds, as in the case of an anti-de Sitter background.

In the present paper, we shall consider an AdS model filled with classical matter, a perfect fluid, and a phantom/tachyon scalar, taking also into account quantum contributions. The model can be viewed as some generalized phantom/tachyon cosmology. In our theory, quantum effects are described via the conformal anomaly, what is reminiscent of the well known anomaly-driven inflation [17]. Such quantum effects are typical for the vacuum energy (for a review, see [18]). In special, we investigate the analogies between our model formulated in AdS space and the corresponding one formulated in a true de Sitter (dS) universe [19]. By making use of the AdS/CFT correspondence, one can expect that the tachyon (phantom/tachyon) field may emerge out of some QFT instability in the dual description. It can originate as a result of some phase transition. We will also study how the energy conditions can indeed be fulfilled in a tachyonic AdS universe of this sort, and what is the effect of the tachyon on the scale factor and, correspondingly, on the accelerated inflation of such universe.

We start from an action for the tachyon, given by the following expression

\[
S_\phi = - \int d^4 \sqrt{-g} \left\{ V(\phi) \sqrt{1 + \lambda g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi} + U(\phi) \right\}.
\] (1)

This action describes an ordinary tachyon when \( \lambda = 1 \) and \( U(\phi) = 0 \), but if \( \lambda = -1 \) and \( U(\phi) = 0 \) it corresponds to a phantom/tachyon (see [16, 19]).

Let us now consider 4-dimensional Anti-de Sitter spacetime (AdS\(_4\)), with the metric chosen as [20]

\[
ds^2 = e^{-2\lambda \tilde{x}_3} (dt^2 - (dx^1)^2 - (dx^2)^2) - (d\tilde{x}^3)^2.
\] (2)

The simplest way to account for quantum effects (at least, for conformal matter) is to include the contributions coming from the conformal anomaly

\[
T = b \left( F + \frac{2}{3} \Box R \right) + b' G + b'' \Box R,
\] (3)

where \( F \) is the square of 4d Weyl tensor and \( G \) the Gauss-Bonnet invariant, which are given by

\[
F = \frac{1}{3} R^2 - 2 R_{ij} R^{ij} + R_{ijkl} R^{ijkl},
\]
\[ G = R^2 - 4R_{ij}R^{ij} + R_{ijkl}R^{ijkl}. \] (4)

In general, with \( N \) scalar, \( N_{1/2} \) spinor, \( N_1 \) vector fields, \( N_2 (= 0 \text{ or } 1) \) gravitons and \( N_{\text{HD}} \) higher derivative conformal scalars, \( b, b' \) and \( b'' \) turn out to be

\[
\begin{align*}
b &= \frac{N + 6N_{1/2} + 12N_1 + 611N_2 - 8N_{\text{HD}}}{120(4\pi)^2}, \\
b' &= -\frac{N + 11N_{1/2} + 62N_1 + 1411N_2 - 28N_{\text{HD}}}{360(4\pi)^2}, \\
b'' &= 0.
\end{align*}
\] (5)

The contributions of the conformal anomaly to \( \rho \) and \( p \) can be found in [21, 22], namely

\[
\begin{align*}
\rho_A &= -\frac{1}{a^4} \left[ b' \left( 6a^4H^4 + 12a^2H^2 \right) \\
   &\quad + \left( \frac{2}{3} b + b'' \right) \left\{ a^4 \left( -6HH_{,tt} - 18H^2H_{,t} + 3H_t^2 \right) + 6a^2H^2 \right\} \\
   &\quad - 2b + 6b' - 3b'' \right], \\
p_A &= b' \left\{ 6H^4 + 8H^2H_{,t} + \frac{1}{a^2} \left( 4H^2 + 8H_t \right) \right\} \\
   &\quad + \left( \frac{2}{3} b + b'' \right) \left\{ -2H_{,tt} - 12HH_{,tt} - 18H^2H_{,t} - 9H_t^2 \\
   &\quad + \frac{1}{a^2} \left( 2H^2 + 4H_t \right) \right\} - \frac{-2b + 6b' - 3b''}{3a^4}. \tag{7}
\end{align*}
\]

The “radius” of the Universe \( a \) and the Hubble parameter \( H \) may be taken as

\[ a \equiv Le^A, \quad H = \frac{1}{a} \frac{da}{dt} = \frac{dA}{dt}. \] (8)

Then, for such metric, the corresponding FRW equation has the following form

\[
\begin{align*}
H^2 &= \frac{-\kappa}{3} (\rho_\phi + \rho_A), \tag{9} \\
\frac{\ddot{a}}{a} &= \frac{\kappa}{3} \left\{ \frac{1}{2} (\rho_\phi + \rho_A) + \frac{3}{2} (p_\phi + p_A) \right\}. \tag{10}
\end{align*}
\]
Here $\rho_\phi$ and $p_\phi$ represent the tachyon energy density and pressure, respectively.

From another side, by choosing $\phi$ to be depending only on $x_3$, the tachyon action (1) takes the more simple form

$$
S_\phi = - \int d^4x a^{-3} \left\{ V(\phi) \sqrt{1 - \lambda \dot{\phi}^2} + U(\phi) \right\},
$$

(11)

and, after varying (11) with respect to $\phi$, the resulting equation of motion for the tachyon is

$$
\lambda \ddot{\phi} + \left( \frac{V'(\phi)}{V(\phi)} - 3 \lambda H \dot{\phi} \right) (1 - \lambda \dot{\phi}^2) + \frac{U'(\phi)}{V(\phi)} (1 - \lambda \dot{\phi}^2)^{3/2} = 0.
$$

(12)

In the same way, after varying (11) with respect to the metric $g_{\mu\nu}$, we obtain for the energy density $\rho_\phi$ and pressure $p_\phi$:

$$
\rho_\phi = \frac{V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} + U(\phi) \\
p_\phi = -V(\phi) \sqrt{1 - \lambda \dot{\phi}^2} - U(\phi).
$$

(13)

Assuming that the spacetime is Anti-deSitter for the scale factor we have $a = e^{-\frac{\phi}{2\Lambda}}$; thus, for the quantum energy density and pressure on gets [21, 22]

$$
\rho_A = -p_A = -\frac{6b'}{L^4}.
$$

(14)

According to this, the FRW equations (9) and (10) become

$$
\frac{1}{L^2} = \frac{\kappa}{3} \left[ \frac{V(\phi)}{2\sqrt{1 - \lambda \dot{\phi}^2}} - \frac{3}{2} V(\phi) \sqrt{1 - \lambda \dot{\phi}^2} - U(\phi) + 6b' \lambda^4 \right],
$$

(15)

$$
\frac{1}{L^2} = -\frac{\kappa}{3} \left[ \frac{V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} + U(\phi) - 6b' \lambda^4 \right].
$$

(16)

Combining now (15) and (16), we obtain for $\dot{\phi}$ a trivial solution $\dot{\phi}^2 = 0$. Thus $\phi$ should be a constant $\phi = \phi_o$, and therefore from eq. (16) it follows that

$$
L^{-2} = -\frac{\kappa}{3} \left( V(\phi_o) + U(\phi_o) - 6b' \lambda^4 \right).
$$

(17)
As we see, this solution differs from the one obtained in [19] only on its sign, but this is no trivial change since it will affect the solutions for the tachyon potentials $V(\phi)$ and $U(\phi)$ in our AdS spacetime. Indeed, from eq. (12) it follows that

$$V'(\phi_o) + U'(\phi_o) = 0.$$  \hspace{1cm} (18)

Such a solution means that $V(\phi_o) + U(\phi_o)$ has an extremum at $\phi = \phi_o$ or either that $V(\phi_o) + U(\phi_o)$ is a constant. Let us now look for a tachyon potential with $V(\phi)$ and $U(\phi)$ according to the proposal in [23] in the following form:

$$V(\phi) = V_o \left(1 + \frac{\phi}{\phi_o} \ln \left(1 + \frac{\phi}{\phi_o}\right)\right)$$  \hspace{1cm} (19)

$$U(\phi) = -V_o \frac{\phi}{\phi_o \ln \left(1 + \frac{\phi}{\phi_o}\right)}.$$  \hspace{1cm} (20)

It is easy to verify that $V(\phi) + U(\phi) = V_o = \text{const}$, for any value of $\phi$. On the other hand, as $\dot{\phi} = 0$, we get $\rho_{\phi} = -p_{\phi} = V(\phi_o) + U(\phi_o)$ and $w = -1$. From (17) the solution for $L^{-2}$ is found as

$$L^{-2} = \frac{1}{4b'\kappa} \pm \sqrt{\frac{1}{16b'^2\kappa^2} + \frac{U_o}{6b'}}.$$  \hspace{1cm} (21)

where $U_o \equiv V(\phi_o) + U(\phi_o)$. Since $b'$ is usually negative, we find real positive solutions only if $U_o < 0$. In fact we see that there is only one positive solution taking the plus sign. Thus, considering the case when $U_o$ is small enough, we find that

$$L^{-2} \sim \frac{-\kappa U_o}{3}.$$  \hspace{1cm} (22)

This solution corresponds to a universe which expands due only to the tachyon perturbed by quantum effects. Therefore, in this scenario the inflationary regime emerging purely from quantum effects [19] is not present in our AdS Universe as it occurs in the dS case [19].

On the other hand, taking into account that $\dot{\phi} = 0$, we see that our solutions satisfies

$$w = \frac{p_{\phi}}{\rho_{\phi}} = \frac{p_A}{\rho_A} = \frac{p_{\phi} + p_A}{\rho_{\phi} + \rho_A} = -1.$$  \hspace{1cm} (23)
Assuming $\dot{\phi} \neq 0$ in (37), the effective equation of state becomes

$$w = \frac{p_\phi}{\rho_\phi} = -1 + \frac{\lambda \dot{\phi}^2 \left( V(\phi) - \frac{\lambda \dot{\phi}^2 U(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} \right)}{V(\phi) + U(\phi) \sqrt{1 - \lambda \dot{\phi}^2}}.$$  \hfill (24)

In this case, considering the situation of a phantom/tachyon ($\lambda < 0$) for $V(\phi) > 0$ and $U(\phi) \geq 0$, one finds that $w \leq -1$.

Let us now look for solutions for the scale factor and the Hubble parameter. We are interesting in their dependence on the form chosen for the functions $V(\phi)$ and $U(\phi)$. From (10), (37) and (14), the solution for $H^2$ follows

$$H^2 = -\frac{\kappa}{3} \left[ \frac{V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} + U(\phi) - 6b'L^{-4} \right].$$  \hfill (25)

Then, for $\dot{\phi} = 0$, we have

$$H^2 = -\frac{\kappa}{3} \left[ V(\phi) + U(\phi) - 6b'L^{-4} \right] = -\frac{\kappa}{3} \left( V_o - 6b'L^{-4} \right).$$  \hfill (26)

This solution tells us that a non-imaginary scale factor is obtained only under the condition $V_o < 6b'L^{-4}$, and since $b'$ is negative, $V_o$ must be negative too.

Combining (9), (37) and (14), the following equation is obtained

$$\frac{\ddot{a}}{a} = \frac{\kappa}{3} \left[ \frac{V(\phi)}{2\sqrt{1 - \lambda \dot{\phi}^2}} \left( 3\lambda \dot{\phi}^2 - 2 \right) - U(\phi) + 6b'L^{-4} \right].$$  \hfill (27)

Now according to what was found before, we may consider the case when $\dot{\phi} = 0$; we get

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{3} \left[ V(\phi) + U(\phi) + 6b'L^{-4} \right].$$  \hfill (28)

Equation (28) may be rewritten as

$$\ddot{a} + \alpha_o a = 0,$$  \hfill (29)

where $\alpha_o = -\frac{\kappa}{3} \left[ V_o + 6b'L^{-4} \right]$.

Taking into account the fact that $V_o$ must be negative, it is possible to obtain a solution for $a$ depending on the sign of $\alpha_o$, as follows

$$a(x_3) = 2a_o \cosh (\alpha_o x_3).$$  \hfill (30)
This solution (30) tells us that from our model the AdS Universe is inflationary with an accelerating scale factor, since $\ddot{a} > 0$. Furthermore, we find that even when quantum effects are small, the AdS Universe may still remain inflationary accelerated, the inflation being induced only by the tachyon.

It is important to remark that the solutions obtained do not depend explicitly on the form of the functions $V(\phi)$ and $U(\phi)$ but only on the condition that $V(\phi) + U(\phi) = V_0 = \text{const}$, for any value of $\phi$, according to the form for the tachyon potential proposed in [23].

An important issue to be investigated concerns the energy conditions for this model, in other words, its consistency. In [24] the energy conditions were studied for an AdS universe with phantoms and compared with those for the corresponding model in a de Sitter universe. We want to know which of the energy conditions can be fulfilled in the present model. The standard ones in cosmology are the following:

1. Null Energy Condition (NEC):
   \[ \rho + p \geq 0. \] (31)

2. Weak Energy Condition (WEC):
   \[ \rho \geq 0 \text{ and } \rho + p \geq 0. \] (32)

3. Strong Energy Condition (SEC):
   \[ \rho + 3p \geq 0 \text{ and } \rho + p \geq 0. \] (33)

4. Dominant Energy Condition (DEC):
   \[ \rho \geq 0 \text{ and } \rho \pm p \geq 0. \] (34)

In analogy with what was obtained when coupling the above model with a usual phantom field [2] and the results for an AdS Universe with a phantom and quantum matter [24], it is sensible to rewrite eqs. (15) and (16) as

\[
\frac{1}{L^2} = \frac{\kappa}{3} \left[ \frac{V(\phi)}{2\sqrt{1 - \lambda \dot{\phi}^2}} - \frac{3}{2} V(\phi) \sqrt{1 - \lambda \dot{\phi}^2} - U(\phi) + \frac{6b'}{L^4} \right. \\
- \left. C^2 - \frac{\rho_m}{2} + \frac{3}{2} \rho_m \right], 
\] (35)

\[
\frac{1}{L^2} = -\frac{\kappa}{3} \left[ \frac{V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} + U(\phi) - \frac{6b'}{L^4} - \frac{C^2}{2} + \rho_m \right]. 
\] (36)
Now, solving eqs. (35) and (36) for $\rho_m$ and $p_m$, we obtain

$$\rho_m = -\frac{3}{\kappa L^2} - \frac{V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} - U(\phi) + 6b' L^{-4} + \frac{C^2}{2},$$ \hspace{1cm} (37)

$$p_m = \frac{3}{\kappa L^2} + V(\phi) \sqrt{1 - \lambda \dot{\phi}^2} + U(\phi) - 6b' L^{-4} + \frac{C^2}{2}. \hspace{1cm} (38)$$

Let us consider with care the implications of these equations, i.e. which are the restrictions on the anti-de Sitter cosmology which follow from the energy conditions (31-34).

First, by combining (37) and (38), we get

$$\rho_m + p_m = C^2 - \frac{\lambda \dot{\phi}^2 V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}}. \hspace{1cm} (39)$$

From (39) we see that the NEC is satisfied only if $V(\phi) > 0$ and $\lambda < 0$, i.e. for a phantom-like tachyon. Now let us introduce $\beta(\phi) \equiv \frac{C^2}{2} - \frac{V(\phi)}{\sqrt{1 - \lambda \dot{\phi}^2}} - U(\phi)$.

Then, eq. (37) is rewritten as follows

$$\rho_m = \frac{\beta(\phi)}{L^4} \left[ L^2 - \frac{3}{2\kappa \beta(\phi)} - \frac{1}{2} \sqrt{\left( \frac{3}{\kappa \beta(\phi)} \right)^2 - \frac{24b'}{\beta(\phi)}} \right]$$

$$\times \left[ L^2 - \frac{3}{2\kappa \beta(\phi)} + \frac{1}{2} \sqrt{\left( \frac{3}{\kappa \beta(\phi)} \right)^2 - \frac{24b'}{\beta(\phi)}} \right]. \hspace{1cm} (40)$$

We see that if $\beta(\phi) < 0$, $\rho_m < 0$, then the WEC or DEC is not satisfied in such case. If $\beta(\phi) > 0$, since $L^2 - \frac{3}{2\kappa \beta(\phi)} + \frac{1}{2} \sqrt{\left( \frac{3}{\kappa \beta(\phi)} \right)^2 - \frac{24b'}{\beta(\phi)}} > 0$, a non-trivial constraint on $L^2$ from WEC or DEC is obtained as follows:

$$L^2 > \frac{3}{2\kappa \beta(\phi)} + \sqrt{\left( \frac{3}{2\kappa \beta(\phi)} \right)^2 - \frac{6b'}{\beta(\phi)}}. \hspace{1cm} (41)$$

In addition, we have

$$\rho_m + 3p_m = \frac{\gamma(\phi)}{L^4} \left[ L^2 - \frac{3}{\kappa \gamma(\phi)} - \sqrt{\left( \frac{3}{\kappa \gamma(\phi)} \right)^2 + \frac{12b'}{\gamma(\phi)}} \right]$$
Here $\gamma(\phi) \equiv 2C^2 + \frac{2-3\lambda\dot{\phi}^2}{\sqrt{1-\lambda\dot{\phi}^2}}V(\phi) + 2U(\phi)$. Then if $V(\phi), U(\phi) > 0$ and $\lambda < 0$, we conclude that $\gamma(\phi) > 0$; on the other hand, if the quantity inside the square root is negative,

$$
\left(\frac{3}{\kappa\gamma(\phi)}\right)^2 + \frac{12b'}{\gamma} < 0,
$$

we obtain $\rho_m + 3p_m > 0$ and the SEC is satisfied.

On the other hand, if the quantity inside square root is positive, from the SEC we obtain a non-trivial constraint on $L^2$, namely

$$
L^2 < \frac{3}{\kappa\gamma(\phi)} - \left[\left(\frac{3}{\kappa\gamma(\phi)}\right)^2 + \frac{12b'}{\gamma}\right]^{1/2},
$$

(42)

For the DEC we also have

$$
\frac{\eta(\phi)}{L^4} \left[ L^2 - \frac{3}{\kappa\eta(\phi)} - \left( \frac{3}{\kappa\eta(\phi)} \right)^2 + \frac{12b'}{\eta(\phi)} \right] \times \left[ L^2 - \frac{3}{\kappa\eta(\phi)} + \left( \frac{3}{\kappa\eta(\phi)} \right)^2 + \frac{12b'}{\eta(\phi)} \right],
$$

(43)

where $\eta(\phi) = \frac{2-3\lambda\dot{\phi}^2}{\sqrt{1-\lambda\dot{\phi}^2}}V(\phi) + 2U(\phi)$. In this case $V(\phi), U(\phi) > 0$, then $\eta(\phi) > 0$ and from the DEC we get the following constraint on $L^2$

$$
\frac{3}{\kappa\eta(\phi)} - \sqrt{\left( \frac{3}{\kappa\eta(\phi)} \right)^2 + \frac{12b'}{\eta(\phi)}} < L^2 < \frac{3}{\kappa\eta(\phi)} + \sqrt{\left( \frac{3}{\kappa\eta(\phi)} \right)^2 + \frac{12b'}{\eta(\phi)}}.
$$

(44)

From the above analysis of the energy conditions, we are led to the conclusion that they can always be fulfilled, provided the constrains derived are imposed. In other words, these results lead to the possibility of the formation
of an AdS universe out of quantum matter effects and the presence of dark energy, which in our model is obtained from the contributions of the tachyon and phantom fields. As a proof of consistency of the derivations above, we may restrict the above analysis for the energy conditions (and the constraints derived for $L^2$) to the case when the contributions of the tachyon are absent. Then we immediately see that the results that we got in [24] for an anti-de Sitter Universe filled with quantum CFT with classical phantom matter and a perfect fluid are recovered.

To summarize, a number of interesting conclusions can be drawn from the study of the influence of tachyon/phantom and quantum effects in an AdS universe. In particular, the possibility to give sense to such a model, attending to the fact that the majority of the energy conditions can be preserved —within certain limits— when matter is composed of tachyon/phantom, perfect fluid and conformal matter.

In addition, it has been shown that our model represents a sort of inflationary accelerated anti-de Sitter universe and that it still may remain such, even if the quantum effects are not considered.

Furthermore, from our expressions we are able to recover (30), i.e. the situation when the tachyon is not present, in which case we again conclude that the anti-de Sitter Universe remains accelerated —knowing however that in this particular case it will not be stable [20, 25, 26].

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