Scaling of hadronic transverse momenta in a hydrodynamic treatment of relativistic heavy ion collisions

Dinesh Kumar Srivastava
Variable Energy Cyclotron Centre, 1/AF Bidhan Nagar, Kolkata 700 064, India
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The transverse momenta of hadrons in central nucleus-nucleus collisions are evaluated in a boost invariant hydrodynamics with transverse expansion. Quark gluon plasma is assumed to be formed in the initial state which expands and cools via a first order phase transition to a rich hadronic matter and ultimately undergoes a freeze-out. The average transverse momentum of pions, kaons, and protons is estimated for a wide range of multiplicity densities and transverse sizes of the system. For a given system it is found to scale with the square-root of the particle rapidity density per unit transverse area, and consistent with the corresponding values seen in $p\bar{p}$ experiments at 1800 GeV, suggesting a universal behaviour. The average transverse momentum shows only an approximate scaling with multiplicity density per nucleon which is at variance with the $p\bar{p}$ data.

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The last two decades have witnessed an active pursuit of relativistic heavy ion collisions in order to create a quark-gluon plasma, first ‘seen’ a few micro-seconds after the big-bang. The recently concluded experiments at the CERN SPS and the ongoing experiments at the Relativistic Heavy Ion Collider at Brookhaven do seem to suggest a very definite possibility of the confirmation of the quark-hadron phase transition.

It is known for quite some time[1] that the transverse momenta of particles produced in a high multiplicity environment in almost all models of particle production, roughly scale with the particle rapidity density $\langle dN/dy \rangle$ per unit transverse area ($\pi R_T^2$),

$$\nu = \frac{1}{\pi R_T^2} \frac{dN}{dy}, \quad (1)$$

so that,

$$\langle p_T \rangle \sim \nu^{1/2}. \quad (2)$$

This follows from dimensional arguments as $\nu$ is the only scale in the problem. This has been noted in minijet models[2], string models[3], and more recently suggested in partonic saturation descriptions like colour gluon condensate model for heavy ion collisions[4].

Let us approach this problem from another angle. Let us assume that in central nucleus-nucleus collisions at relativistic energies a thermally and chemically equilibrated quark gluon plasma is formed at an initial time $\tau_0$ at an initial temperature $T_0$. Once $\nu$ is known and an assumption is made about the applicability of Bjorken hydrodynamics[5] we get,

$$\frac{2\pi^4}{45\zeta(3)} \frac{1}{\pi R_T^2} \frac{dN}{dy} = 4aT_0^3\tau_0, \quad (3)$$

where $a = 42.25\pi^2/90$ for a plasma of massless quarks (u, d, and s) and gluons and we have put the number of flavours as $\approx 2.5$ to account for the mass of the strange quarks. Do the hadron spectra resulting from the hydrodynamic expansion of the plasma thus produced show some scaling behaviour?

Assuming that one has $\langle E \rangle = 3T$ for massless particles, we may take $\tau_0 = 1/3T_0$, from considerations of the uncertainty relation. Thus we see that the initial temperature scales with $\nu^{1/2}$. We may also add that it was suggested some-time ago that[6] the $S+Au$ and $Pb+Pb$ collisions at the CERN SPS present an interesting example as they lead to systems with different transverse sizes but similar initial temperatures due to the near equality of the scaling variable $\nu$. (We shall see that the differences for the hadronic spectra for the two cases are not large.)

In the present work we continue this argument and show that the average transverse momenta of the produced hadrons, estimated on the basis of a hydrodynamical expansion with transverse flow scale with $\nu^{1/2}$. The scaling is surprisingly similar to the corresponding behaviour for $p\bar{p}$ data at 1800 GeV, which is sought to be understood within a colour gluon condensate model[7]. It may be recalled that these data were used to provide a suggestion for baryon-flow and quark gluon plasma[8]. A straightforward explanation in terms of an increased minijet activity which leads to a larger multiplicity has also been offered for this[9].

The similarity of these behaviours is of interest as the hydrodynamics calculations start with a partonic plasma, which develops into a mixed phase and then into a hadronic phase. The spectra for particles having different masses are affected differently due to the celebrated radial flow velocity which develops during the expansion. More-over several details like the equation of state, the critical temperature, the freeze-out temperature, and a model of hadronization etc. affect the overall evolution of the system.

This would seem to suggest that the scaling of the particle transverse momenta may be ‘seeded’ in the initial conditions, and that this behaviour is broadly retained by the hydrodynamic expansion. We insist that radial flow is essential to obtain this result, as in absence of the
radial flow, the \( \langle p_T \rangle \) is uniquely determined by the freeze-out temperature and is independent of multiplicity \(^1\) and the size of the system. The method of solution of the hydrodynamic equations for the expansion and cooling of the quark-gluon plasma has been discussed extensively in literature. We only briefly mention the salient features. As discussed earlier, we assume the plasma to be in a thermally and chemically equilibrated state of quark gluon plasma, which expands, cools, undergoes a phase transition to a hot hadronic matter at \( T = T_C \) and when the hadronic density is too low to cause further scattering among the hadrons it undergoes a freeze-out. The plasma is assumed to undergo a longitudinally boost-invariant and an azimuthally symmetric transverse expansion.

The initial energy density profile is assumed to follow the so-called ‘wounded-nucleon’ distribution \(^2\). We further assume that the phase transition takes place at \( T = 180 \) MeV and the freeze-out takes place at 120 MeV. We use a hadronic equation of state consisting of all hadrons and resonances from the particle data table which have a mass less then 2.5 GeV \(^3\). In all the calculations reported here the initial time \( \tau_0 \) is taken as 1 fm/c. It is well known that as long as \( T^3 \tau_0 \) is kept fixed, a smaller initial time makes only a marginal difference to the flow. The relevant hydrodynamic equations are solved using the procedure \(^4\) discussed earlier and spectra for hadrons estimated \(^5\) using the Cooper-Fry formulation.

In Fig.1 we show our results for several systems \(^6\) having an identical particle rapidity density per unit transverse area \((\nu)\). We see that the average transverse momenta for pions, kaons, and protons for all the cases are similar, though they decrease slightly with the increasing transverse size. This decrease is about 4% for pions and about 7% for protons. Even though marginal, this is interesting as the radial flow introduces an additional scale in the problem, the size \( R_T \). At a time \( t \) the region beyond \( R_T = c_s t \) is affected by the transverse flow \(^7\). Thus a smaller system is more strongly affected by the flow, even if the initial temperature is similar and thus build-up \(^8\) of the radial flow velocity given by

\[
\nu_r(t_f) \propto \int_{t_i}^{t_f} \frac{P(t)}{\epsilon(t)} dt
\]

will be affected. In the above, \( P \) is the pressure and \( \epsilon \) is the energy density so that the quantity under the integral sign gives the speed of sound which passes though a minimum during the mixed phase.

A demonstration of this effect is seen in Fig.2, where we have plotted the emission of pions per unit transverse area for the S+Au (WA80 experiment \(^9\)) and Pb+Pb (WA98 experiment \(^10\)) systems which have the same \( \nu \) but different transverse sizes \((R_T)\). We add that the hydrodynamic treatment used here gives a perfect description to the hadronic spectra for the Pb+Pb system at SPS energy, which correspond to this value of \( \nu \) \(^11\).

We note that the inverse slope of the smaller system is marginally larger, as indeed Fig.1 implies. The slight difference in the radial flow which may develop in systems of varying sizes can be magnified by looking at the transverse momenta of heavier particles. This is seen in Fig.3 where we now plot the emission of protons per unit transverse area for the S+Au (NA35 \(^12\)) and Pb+Pb (NA44 \(^13\)) systems having the same \( \nu \). A much larger freeze-out temperature (with very different values for pions and protons) would be needed to understand these spectra if the flow is not included.

In Fig.4 we show the average transverse momenta of pions, kaons, and protons for central collision of lead nuclei for a range of multiplicity densities spanning the region likely to be covered at the RHIC and LHC. It is seen that the average transverse momentum rises with the square-root of the scaling variable \( \nu \) so that, the pion, kaon, and proton spectra determined from the hydrodynamics are well described by:

\[
\langle p_T \rangle = 0.326 + 0.052 \sqrt{\nu}; \quad \pi , \\
\langle p_T \rangle = 0.423 + 0.087 \sqrt{\nu}; \quad K , \\
\langle p_T \rangle = 0.530 + 0.137 \sqrt{\nu}; \quad p .
\]

We also find (see Fig.5) that the the hadronic-spectra scale as

\[
dN_h/d^2p_Tdy = \frac{\alpha}{(p_T)^2} F\left(\frac{p_T}{\langle p_T \rangle}\right) .
\]

The preliminary data for the transverse momenta are now available from the PHENIX experiment \(^14\) at \( \sqrt{s_{NN}} = 130 \) GeV. We see from the Fig.6 that these data conform to the hydrodynamic behaviour seen in this work.

In Fig.7 we compare the hydrodynamics results for Pb+Pb system with the experimental results for the \( pp \) collision at 1800 GeV \(^15\). Their agreement points to a universal scaling behaviour for the transverse momentum distributions. Considering that several models of particle production relate the increase in \( \langle p_T \rangle \) with the multiplicity to the initial conditions, we have plotted the \( \langle p_T \rangle \) of the hadrons against the initial temperature obtained and used in the hydrodynamic calculations here and we see (Fig.8) that the increase is directly related to initial temperature.

It has been suggested \(^16\) that hydrodynamic flow at the end of the three dimensional evolution of the plasma is uniquely determined by a dimensionless parameter \( \omega \):

\[
\omega = s_0 \tau_0 / s_c R_T
\]

where \( s_0 \) is initial entropy density, \( \tau_0 \) is initial time, \( s_c \) is the entropy density at the beginning of the phase-transition, and \( R_T \) is the transverse size. Recalling that \( s \sim T^3 \), one can easily prove that \( \omega = \tau_0 / R_T \), where \( \tau_0 \) is the proper time when the phase transition begins, provided the QGP cools according to a boost-invariant
longitudinal expansion. Obviously if $\omega$ is small, transverse flow effects should be small, as the QGP phase is over well before $R_T/c_s$, when the rare-fraction wave covers the distance $R_T$ and agitates the entire fluid. Using Eq. (3), we can also see that $\omega \sim (dN/dy)/A$. We now investigate this scaling in terms of a variable

$$\mu = \frac{1}{A} \frac{dN}{dy}$$

in the spirit of a similar study by Ruuskanen [24].

In Fig. 9 we have plotted the average transverse momenta of hadrons obtained for $S+S$, $Zr+Zr$ and $Pb+Pb$ systems for a large range of values of the scaling variable $\mu$. We see that the professed scaling is only approximately satisfied, and becomes worse with increasing mass of the hadron at large $\mu$. This could be due to the fact that for such values the development of the flow after the conversion to hadrons starts, can not be ignored. We also note that this scaling is not universal as the corresponding results for the $p\bar{p}$ data show a very different behaviour.

Before concluding, let us discuss the uncertainties in our estimates. The hydrodynamics calculations have several inputs which can slightly alter the extent of the deviations of the scaling behaviour observed here. We have used a freeze-out temperature of 120 MeV. Decreasing $T_F$ increases the $\langle p_T \rangle$ slightly, the largest increase being for protons. For $\nu = 5$, decreasing $T_F$ to 100 MeV increases the $\langle p_T \rangle$ for pions from 0.44 GeV/c to 0.46 GeV/c, while for protons the corresponding increase is from 0.83 GeV/c to 1.02 GeV/c. For $\nu = 50$, the $\langle p_T \rangle$ for pions increases from 0.70 GeV/c to 0.72 GeV/c when the freeze-out temperature is decreased to 100 MeV, while the increase for protons is from 1.5 GeV/c to 1.7 GeV/c. The critical temperature used affects the $\langle p_T \rangle$ too, in a very interesting manner. Increasing the $T_C$ increases the degrees of freedom in the hadronic matter (as higher resonances get excited) and the phase-transition is completed quickly. Thus the duration of the low-pressure gradient associated with the mixed phase is reduced and the $\langle p_T \rangle$ increases slightly, and the opposite happens when a lower value of $T_C$ is used. We have verified that the change in $\langle p_T \rangle$ for protons is less than 10% when $T_C$ is changed by $\pm 20$ MeV. For pions the results are much less altered. It is easy to argue that if a much simpler equation of state for hadrons is used (say having only pions) then the mixed phase will live much longer and $\langle p_T \rangle$ will decrease.

It is also useful to remember that the $p_T$ broadening and the so-called jet-quenching should also affect results discussed in Fig. 3, e.g. It is hard to decide the relative worth of hydrodynamic and such treatments till precise data at higher energies and up to large $p_T$ are available.

The discussion so far has involved central collisions, as it is not easy to extend the hydrodynamic treatment to non-central (non-zero impact parameter) collisions. A rough estimate could still be made if we ignore the azimuthal asymmetry of the region of overlap, approximating it to a circle having a radius $R \approx 1.2(N_{\text{part}}/2)^{1/3}$, where $N_{\text{part}}$ is the number of participants. This would then help assign a value of $\nu$ even to non-central collisions, as has been done in Ref. [4]. Recall that the NA49 data for $Pb + Pb$ collisions treated in this manner were found to similar to the E735 data for $p\bar{p}$ in the above work bolstering the arguments for a universal behaviour. We eagerly await the data from RHIC experiments to verify this aspect.

The similarity of the behaviour observed here for high multiplicity $p\bar{p}$ and $AA$ results raises one question, which is applicable to all the studies involving hydrodynamics; viz., are the conditions appropriate for the applicability of hydrodynamics realized in these situations? The similarity of the behaviour for the high multiplicity events in $p\bar{p}$ with those of $AA$ does suggest a plausible argument. It indicates that the the initial states of these systems are perhaps consisting of partons and the multiple collisions indeed drive the system to an equilibrium quickly. It is known from several studies that high multiplicity events in hadron-hadron collisions are also rich in multiple partonic collisions [1] and provide a basis for the parton cascade model [23].

In brief, we see that hadronic spectra generated from boost-invariant hydrodynamics with transverse expansion show a scaling behaviour in the particle rapidity density per unit transverse area. For smaller values of the scaling variable $\nu$, the results are quite close to values inferred from ISR data, which are sought to be understood in terms of a universal behaviour given by the colour gluon condensate model.

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FIG. 1. A model calculation of the average transverse momentum of hadrons for several systems which have identical particle rapidity density per unit transverse area.

FIG. 2. Emission of pions per unit transverse area for S+Au (WA80 [12]) and Pb+Pb (WA98 [14]) systems, which have the same multiplicity per unit transverse area but different transverse sizes.
FIG. 3. Emission of protons per unit transverse area for $S + Au$ and $Pb + Pb$ having same $v$.

FIG. 4. A model calculation of the average transverse momentum of hadrons for central collision of Pb nuclei as a function of the particle rapidity density per unit transverse area. The symbols represent the hydrodynamics results while the solid curves give the fits described in the text (Eq. 5).

FIG. 5. Scaling of hadronic spectra obtained from hydrodynamics (Eq. 6). The two curves differ by a factor which is equal to the ratio of the square of the average transverse momenta (see text).

FIG. 6. Hadronic spectra from PHENIX experiment [19].
FIG. 7. A comparison of scaling suggested by hydrodynamics (solid curves) and the corresponding data for $p\bar{p}$ scattering at 1800 GeV [20].

FIG. 8. The average transverse momenta attained as a function of the initial temperature.

FIG. 9. The average transverse momenta for pions, kaons, and protons for central collisions of $S + S$, $Zr + Zr$, and $Pb + Pb$ systems as a function of multiplicity per nucleon. The corresponding values for the $p\bar{p}$ data [20] are also given.