High precision speed measurement by using interferometric techniques

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Abstract: In this work we present the experimental realization of speed measurement by the use of a two wave interferometer and digital signal processing techniques. We built an automated Michelson interferometer and using an He-Ne laser and with the use of the Fast Fourier Transform (FFT) and computer algorithms we derived a method for finding the speed of displacement. We report uncertainties in the order of 2 -3 μm/s. with the use of this procedure. This brings the potential of another physical variable measurement like distance or pressure by this indirect measurement method. This approach is compared with an ultrasonic Logger Pro ® speed measurement system, and the results are compared between systems.

1. Introduction
High precision measurement techniques are a very important subject in development of Physics. It impacts also in the applied, industrial and commercial fields around the world. It is required to have standards that allow keeping congruent measurements in different parts of the world. With this idea in mind, many different techniques and devices are used and designed in order to achieve better measurements for specific purposes [1],[2]. In this work we use a Michelson interferometer and a a data acquisition system to store and process the interference signal in order to measure the velocity of a device which moves on a linear path. This device is coupled to an arm of the interferometer. By the use of digital signal processing algorithms we find indirectly the instantaneous velocity and acceleration of the linear displacement by the analysis of the frequency of the acquired signal. In the next section we describe the theory that helps to sustain our hypothesis, followed by the experimental setup, the results which includes the error analysis.

2. Theoretical Background
The normalized intensity equation at an interferometer photodetector using single-mode light with wavelength λ is [4]:
The previous expression is also known as the interference equation, where the propagation number $k = \frac{2\pi}{\lambda}$, and $OPD$ is the optical path difference. $I_0$ refers to the sum of the intensity of each arm of interferometer. Michelson interferometer experimental setup diagram is presented in figure 1.

**Figure 1.** Block diagram of experimental setup using a Michelson interferometer (a) Electromechanical displacement device used (b).

This interferometer provides an optical path difference $OPD = 2(d_1 - d_2)$, where $d_1$ and $d_2$ are the respective lengths of each arm of the interferometer. The previous expression (1) represents the interaction of two beams. The second part of the equation with the cosine function is also known as the interference term. When $OPD$ is an even multiple of $\lambda/2$, the intensity $I$ is maximum, while being odd multiple of $\lambda/4$, intensity is minimum.

By moving an arm of the interferometer with constant speed $v$, the interference terms becomes:

$$I = \frac{I_0}{2} + I_0 \cos(\frac{k OPD}{2}). \quad (1)$$

$$\cos(k \text{ OPD}(t)) = \cos\left(\frac{2\pi}{\lambda} 2(d_1 - d_2 + vt)\right) = \cos\left(\frac{2\pi}{\lambda} \left(\frac{2vt}{\lambda} + \phi_0\right)\right), \quad (2)$$

where $\phi_0$ is considered an initial phase. By considering the following analogy:

$$\cos(2\pi f t + \phi) = \cos\left(\frac{2\pi}{\lambda} \left(\frac{2vt}{\lambda} + \phi_0\right)\right). \quad (3)$$

Term to term, it is possible to see the following relationship between frequency and speed:

$$f = \frac{2v}{\lambda}. \quad (4)$$

The main objective is to take several measurements of an interference signal. With the aid of digital signal processing techniques, the acquired data will be filtered and processed with fast Fourier transform (FFT) [5],[6], and the data collected from those processes will be error analyzed [7],[8]. All the data samples are processed thru a window of 64 samples using a computer algorithm with FFT. By this way we compute the speed and record it, sample by sample. The process is repeated, and the operating voltage of the displacement device is changed to achieve a different speed. With the recorded data, we use an error analysis method [8] to verify the uncertainty of our measures. This process has the following steps.

- Compute the probability distribution function of the multiple speeds using histograms.
Calculate the i-th probability by computing the following quotient
\[ P_i = \frac{N_i}{N_t}, \]  
where \( N_i \) is the number of speeds that belong to the i-th bin and \( N_t \) is the total number of occurrences.

Calculate the expected speed \( \langle v \rangle \) for each measure with the following expression
\[ \langle v \rangle = \sum_{i=1}^{N} v_i P_i \]  
with \( v_i \) representing all possible speed values during the experiment.

With the expected value, standard deviation is computed by
\[ \sigma_x = \left( \sum_{i=1}^{n} P_i (v_i - \langle v \rangle)^2 \right)^{\frac{1}{2}} \]  
and the associated uncertainty of measurement is calculated with the following expression.
\[ \sigma_x^* = \frac{\sigma_x}{\sqrt{N_t}}. \]  

In the next section the experimental setup and frequency analysis is presented.

3. **Experimental Setup**

For the electromechanical part, the displacement device is a motor driven worm gear where a mirror is attached. It has a small step size about 0.5 mm/rev. The motor is controlled with an Arduino microcontroller with a pulse width modulation driver (L298 IC). Coherent light source is provided by an He-Ne laser with \( \lambda = 632.8 \) nm as specified by manufacturer.

For the data acquisition system a photodetector (Thorlabs FDS100) was used with a soundcard of a Netbook. The control system used a different network in order to reduce the amount of electrical noise. Figures 2 and 3 present an example of signal recorded with sampling frequency \( F_s = 44.1 \) kSamples/s. For different speeds the control system decreased motor voltage in steps of 0.5 volts, starting from 12 V to 9.5 V.

![Figure 2](image2.png)  
**Figure 2.** Example of acquired data in one second Voltage used for the motor is 12 V.  

![Figure 3](image3.png)  
**Figure 3.** Zoom of a small amount of time of the signal. It can be shown that frequency is not constant because of different speed of the displacement device.

To calculate speed from the recorded data, the following algorithm is presented:
In an empty data window with $N$ blank spaces is generated. A single data (from left to right) is introduced in the last position of the window. A Hamming window [6] is used to multiply the window point to point to improve frequency response. A FFT with $M$ points is computed and the maximum amplitude in the magnitude spectrum is taken. Corresponding frequency at this point is located with a resolution of $Fs/2M$. Speed is computed from expression in (4) and is recorded in another data set. Data in window is shifted one position to the left and the procedure is repeated from the second step of this algorithm.

Figures 4 shows an example of this procedure using $N = 64$ and $M = 16384$. Figure 5 shows calculated speed and acceleration computed with this method. Simultaneous to this experiment, there was the need to compare with another measurement device. This device was an ultrasonic speed measurement device Logger Pro®.

4. Results

These experiments were error analyzed using expressions (5) to (8) and the corresponding results (averaged) of the speed with associated uncertainty for proposed and ultrasonic method are presented in table 1.

Table 1. Speed measurement using frequency and ultrasonic method.

| Volts | Speed (mm/s) | Logger Pro (mm/s) |
|-------|--------------|-------------------|
| 12.0  | 2.150±0.002  | 2.10±0.10         |
| 11.5  | 2.050±0.002  | 2.06±0.08         |
| 11.0  | 2.000±0.002  | 1.97±0.08         |
| 10.5  | 1.900±0.002  | 1.92±0.10         |
| 10.0  | 1.835±0.003  | 1.90±0.12         |
| 9.5   | 1.770±0.003  | 1.80±0.11         |
The results presented shows three orders of magnitude in associated uncertainty. It can be shown that proposed method has three orders of magnitude of improvement compared to ultrasonic method.

5. Conclusions
This work presents an approach of measuring speed using frequency analysis. The method uses dynamic frequency analysis of an interference pattern from a Michelson interferometer attached to a displacement device. Error analysis shows that precision of this method is better than conventional methods using ultrasound. This approach needs to be improved and the potential of use includes the measurement of other physical variables like distance, pressure, optical properties, refraction index or temperature variations. More work is in progress to achieve a better measure system and test with other optical sources.

References
[1]. Demarest. Frank C , 1988. “High-resolution, high-speed, low data age uncertainty, heterodyne displacement measuring interferometer electronics”, Meas Sci and Tech Vol 9 Number 7 pp. 1024, IOP Publishing
[2]. Scalise L, Yanguang Yu ; Giuliani, G ; Plantier, Guy ; Bosch, T. 2004. “Self-mixing laser diode velocimetry: application to vibration and velocity measurement", IEEE Trans on Instr and Meas Vol 53, Issue: 1, pp. 223 -232,
[3]. Yanguang Yu ; Wollongong; Jiangtao Xi ; Chicharo, JF 2007 “Improving the Performance in an Optical feedback Self-mixing Interferometry System using Digital Signal Pre-processing”, IEEE Int Symp on Intell Sign Proc,. WISP 2007. pp. 1-6.
[4]. Hetch, E., 2000 “Óptics”; Ed. Addison Wesley.
[5]. Rao KR, Kin DN, Hwang J J; 2010. “Fast Fourier Transform: Algorithms and Applications”; Ed. Springer
[6]. Mitra S K; 2001 “Digital Signal Processing: A computer-based approach”; 2nd Edition; McGraw-Hill.;
[7]. Quarteroni A; 2006 “Scientific Computing with MATLAB and Octave”, 2nd Edition; Springer;.
[8]. Taylor J R; 1997 “An Introduction to Error Analysis”, 2ª Edition; University Science Books.

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