Deflection tomography of a complex flow field based on the visualization of projection array

Bin Zhang and Zhanli Miao

College of Electromechanical Engineering, Qingdao University of Science and Technology, Qingdao, Shandong 266061, PR China

E-mail: zb-sh@163.com

Abstract. Tomographic techniques are used for the investigation of complex flow fields by means of deflectometric methods. A new deflection tomographic setup for obtaining an array of multidirectional deflectograms is presented. Deflection projections in different angles of view can be captured synchronously in same optical path condition and arranged on the camera in two rows with three views in each row. Tikhonov regularization method is used to reconstruct temperature distribution from deflectometric projection data. The conjugate gradient method is used to compute the regularized solution for the least-square equations. The asymmetric flame temperature distribution in the horizontal section was reconstructed from limited view angle projections. The experimental results of reconstruction from real projection data were satisfactory when compared with the direct thermocouple measurements.

1. Introduction

Tomography is a technique for determining the local distribution of a parameter in a system from path integrated or line-of-sigh measurements. In fluid flow systems line-of-sight measurements are usually based on refraction of a light beam. Refractive techniques involve measurements of the optical phase by interferometry and holographic interferometry. The gradient in the refractive index is measured by deflectometry [1]. Compared with interferometric methods applied to flow visualization and measurement [2], the deflection technique has been shown to be simpler and easier to use, and has been successfully applied to the mapping of two-dimensional phase objects [3-5]. It should be noted that because of its large dynamic range and applicability to unfavourable circumstances, the moire deflectometric method is widely utilized for measuring complex flows, such as rocket jet plumes [6] and supersonic flows in wind tunnels [7].

Deflection tomography comprises two major steps in reconstructing three-dimensional refractive-index fields. First, multidirectional deflection projections have to be experimentally captured. Second, the data are numerically processed with computational tomographic algorithms for reconstruction from projections.

To date, various experimental configurations have been reported for obtaining field projections. Typical of these can be rotation of the object field or probing beam, and spatial or temporal cascade division and redirection of the probing beam [5]. These approaches exhibit both merits and shortcomings. The object or beam rotation scheme is simple, but it dose not allow truly instantaneous capture of a field. The beam division approach can provide only a limited number of projections as a result of the complexity that arises because the setup has to be duplicated to obtain each projection.

Computational reconstruction techniques can be classified into transform methods and series expansion methods [8]. Transform algorithms are direct and computationally efficient, but require
axisymmetric measured distribution or well-conditioned data that cover an angular range of 180° with a reasonable number of projections in the absence of an opaque object in the field. Series expansion methods, known as algebraic reconstruction technique (ART) [9], allow direct computational reconstruction from a limited data set, which may provide a restricted view, a limited number of projections, or incomplete projections as a result of the presence of an opaque object. Recently, a modified ART has been applied to moire deflection tomography [7, 8]. The algorithm was derived from the basic deflection formula and the deflection angles were used directly in iteration. Although the modified ART is proved to be more accurate than classic ART in simulated reconstruction using both limited and incomplete projection data, this algorithm has to be performed a large number of iterations.

In this paper, we introduce a new deflection tomographic setup that can obtain an array of multidirectional deflectograms. Deflection projections in different angles of view can be captured synchronously in same optical path condition. In reconstruction, the Tikhonov regularization method is used to calculate asymmetry temperature distributions. The technique is demonstrated by measuring a flame temperature field.

2. Deflection tomography apparatus

The optical configuration used in experiment is shown in figure 1. A helium-neon continuous wave laser (λ=632.8nm) is used as the light source. Two mirrors (M1, M2) and beam expanding optics (SF and L1) are used to produce a collimated beam. Mirrors M3 and M4 direct the ~40-mm diameter plane wave to a 50-50 beam splitter. The two resulting beams are directed onto separate diffraction gratings (DG1 and DG2). Each diffraction grating produces three beams of equal intensity. The resulting six probe beams intersect in the test volume with viewing directions of 0°, 26°, 52°, 88°, 114°, and 140°. After passing the apertures, the beams are directed onto an array of mirrors (two rows, each with three 30 × 30 mm mirrors). The mirror array is used to direct all the probe beams along the same optical axis. These parallel beams are then collected onto a 200-mm diameter parabolic mirror (PM), which was used to form a demagnified image of all views in front of lens L2. The probe beams are made parallel and imaged through two Ronchi gratings onto a CCD camera. The six deflectograms, where each deflectogram represents a tomographic viewing direction, are captured simultaneously on the camera with pixel resolution 600 (H) × 800 (W). The views are arranged on the camera in two rows with three views in each row and do not overlap.

3. Regularized reconstruction algorithm for deflection tomography
When a ray passes through a phase object, it is bent due to refraction associated with the gradients of the refractive-index within the object. In the coordinate system shown in figure 2, the ray deflection angle $\phi$ in the $y'$ direction is [2]

$$\phi \approx \int_{-\infty}^{\infty} \frac{\partial n(x, y)}{\partial y'} \, dx',$$

(1)

Where $n(x, y)$ is the refractive-index associated with the two-dimensional phase object at a point $(x, y)$ and $n_0$ is the refractive-index of the surroundings. In deriving equation (1) the in-plane deflection approximation and paraxial approximation have been used. The beam deflection angle obtained form deflectogram can be transformed into the path difference $p$ as follow

$$p(x', y') = \int_{-\infty}^{\infty} n_0 \cdot \phi(x', y') dy'.$$

(2)

A set of linear algebraic equations can be formed. That is

$$\mathbf{p} = \mathbf{An},$$

(3)

where $\mathbf{p}$ is the measurement vector, $\mathbf{A}$ is the basis function projection matrix, and $\mathbf{n}$ is the object vector.

The basic principle of Tikhonov regularization method [10] is to find a $\mathbf{n}$ that minimizes

$$J_\alpha(\mathbf{n}) = \|\mathbf{An} - \mathbf{p}\|^2 + \alpha \|\mathbf{Dn}\|^2,$$

(4)

where $\mathbf{D}$ is a regularization operator and $\alpha$ is a regularization parameter, which plays an important role in reconstruction. This regularization method is effective for the kind of reconstruction problems where the unknown quantity to be reconstructed is continuously distributed in space. An approximation for $\alpha$ is suggested by [10] as

$$\alpha \approx \frac{2\|\mathbf{p} - \mathbf{An}(0)\|^2}{\|\mathbf{Dn}(0)\|^2},$$

(5)

and $\mathbf{n}(0)$ is calculated by

$$\mathbf{n}(0) = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{p}.$$  

(6)

The solution of equation (3) under the minimization of equation (4) can be obtained from normalized equation (7)

$$(\mathbf{A}^T \mathbf{A} + \alpha \mathbf{D}^T \mathbf{D}) \mathbf{n}_\alpha = \mathbf{A}^T \mathbf{p}.$$  

(7)

A regularization operator for 2-D reconstruction [11] is used as follows

$$D_{0s} = n_0 - (1/8)(n_s + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8) \quad s = 1, 2, \cdots k.$$  

(8)

Three kinds of elements having different relationships with their neighbor elements are shown in figure 3. $D_{0s}$ in equation (8) is the element in the line matching $n_0$. 

![Figure 2. Schematic diagram of the coordinate system. is the ray deflection angle. O.R. represents a typical ray passing through the phase object P.O.](image-url)
A refractive index field is calculated with the conjugate gradient method. The detail steps of the method are as follows.

1. Initialize values.

\[ E = A^T p, \]
\[ F = A^T A + \alpha D^T D, \]
\[ P^{(1)} = Q^{(0)} = E - F n, \]
\[ \gamma = 1, \]

where \( \gamma \) is the iteration number.

2. Find values for a new iteration.

\[ \beta = (Q^{(r-1)}, Q^{(r-1)})/(P^{(r)}, FP^{(r)}), \]
\[ n^{(r)} = n^{(r-1)} + \beta P^{(r)}, \]
\[ Q^{(r)} = Q^{(r-1)} - \beta FP^{(r)}, \]
\[ \chi_{r+1} = (Q^{(r)}, Q^{(r)})/(Q^{(r-1)}, Q^{(r-1)}), \]
\[ P^{(r+1)} = Q^{(r)} + \chi_{r+1} P^{(r)} \]

3. Test the convergence.

If \( P^{(r+1)} / P^{(r)} \geq \varepsilon \), where \( \varepsilon \) is a precision parameter in estimation, return to step (2); otherwise, stop the iteration estimation.

4. Experimental results

The temperature field was generated by flames of three candles, which were fixed at each vertex of an equilateral triangle. The six deflectograms at view angles of 0°, 26°, 52°, 88°, 114°, and 140°, respectively, which were captured by the deflectometric system, are shown in figure 4. With the use of automatic deflectogram-processing software, the deflectograms were processed with a PC-based image-processing system. From the processed data, the refractive-index field was reconstructed by applying the Tikhonov regularization method. One 30×30 mm² square region, which was in the horizontal plane 1.0 cm above the candles, was selected to be reconstructed. The reconstructed area was performed over a 30×30 grid array. Each projection consisted of 150 rays. The temperature distribution was then calculated by using the Glastone-Dale formula with an ambient temperature of 30 °C. The cross-sectional temperature distribution is shown in figure 5. For cross-evaluation, the thermocouple measurements were also conducted. At the three points just above three candles, the construction result was in an average error of ~6% when compared with the thermocouple measurements.
5. Conclusions
A system for obtaining a large ensemble of scalar measurements using deflectometric tomography is presented. Six deflectograms, where each deflectogram represents a tomographic viewing direction, are captured simultaneously on the camera. The views are arranged on the camera in two rows with three views in each row and do not overlap. Tikhonov regularization method for deflection tomography is used to reconstruct the refractive-index field. A flame temperature distribution was reconstructed. In future research, some improvements to the current system will be performed, and some more complex flow fields, such as high temperature jet, will be measured with deflection tomography.

Acknowledgments
This work was supported in part by a grant from the Ph.D. Foundation of Department of Science and Technology of Shandong Province (No. 2008BS01004).

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