Entanglement Transfer via XXZ Heisenberg chain with DM Interaction

Morteza Rafiee\(^1\) *, Morteza Soltani\(^2\)†, Hamidreza Mohammadi\(^3\)‡ and Hossein Mokhtari\(^4\)§

\(^1,4\)Department of Physics, University of Yazd, Yazd, Iran and
\(^2,3\)Department of Physics, University of Isfahan, Hezar Jarib Ave., Isfahan, Iran

Abstract

The role of spin-orbit interaction, arises from the Dzyaloshinski-Moriya anisotropic antisymmetric interaction, on the entanglement transfer via an antiferromagnetic XXZ Heisenberg chain is investigated. From symmetrical point of view, the XXZ Hamiltonian with Dzyaloshinski-Moriya interaction can be replaced by a modified XXZ Hamiltonian which is defined by a new exchange coupling constant and rotated Pauli operators. The modified coupling constant and the angle of rotations are depend on the strength of Dzyaloshinski-Moriya interaction. In this paper we study the dynamical behavior of the entanglement propagation through a system which is consist of a pair of maximally entangled spins coupled to one end of the chain. The calculations are performed for the ground state and the thermal state of the chain, separately. In both cases the presence of this anisotropic interaction make our channel more efficient, such that the speed of transmission and the amount of the entanglement are improved as this interaction is switched on. We show that for large values of the strength of this interaction a large family of XXZ chains becomes efficient quantum channels, for whole values of an isotropy parameter in the region \(-2 \leq \Delta \leq 2\).

PACS numbers:

* m.rafiee178@gmail.com
† msoltani@Phys.ui.ac.ir
‡ h.mohammadi@phys.ui.ac.ir
§ hmaftab@yahoo.com
I. INTRODUCTION

Recently transmitting a quantum state is a most important task in quantum information and computation processing [1]. Today this purpose could be achieved in two manner: i) using standard teleportation protocols and, ii) information transfer via quantum networks. Bayat et.al [2] have shown that the fidelity of transmission is the same for both cases. Additionally, the former case employs the flying qbits as quantum channel and hence the fidelity of transmission reduces due to incompleteness of stationary-to-flying qbits conversion process [3]. Thus the later case is superior to the former, particularly in solid state devices. Among the numerous quantum systems suitable for quantum networks implementation, the spin chains offer a great advantages. One of the most interesting art of the spin chains is their ability to use as quantum wires in the information transfer protocols over the short distances [4–11]. Tunable spin interaction in these systems plays the key role to motivate one for using this permanently potential in the quantum information transfer processing. The major works on spin chain are in the Ferromagnetic(FM) phase [7] and the effects of temperature [8] and decoherence [9] have been investigated for FM channels. There are lesser works on Antiferromagnetic(AFM) phase [12, 13], but as mentioned in Refs. [2, 11], AFM spin chain have higher ability in transfer of information and thus is a better and faster alternative. So we prefer to study AFM spin chain. Fortunately, Antiferromagnetic spin chains with short length(up to 10 spins) have been built experimentally [14]. However, much attention has been paid to the entanglement in spin systems with only spin-spin interaction and spin-orbit interaction has been leaved. The spin-orbit interaction produces an anisotropic part of exchange interaction between localized conduction band electrons in crystals with lack of inversion symmetry, including all low dimensional structures and also bulk semiconductors with zinc-blende and wurtzite type of crystal lattice [15]. The main part of the anisotropic interaction has the form of Dzyaloshinski-Moriya interaction (DMI) [16–18] which have explained the weak ferromagnetism of antiferromagnetic crystals ($\alpha - Fe_2O_3$, $MnCO_3$ and $CrF_3$). As was shown [19–21] in the two-qubit Heisenberg systems the (DMI) plays an important role. The (DMI) express by

$$\vec{D},[\vec{S}_1 \otimes \vec{S}_2].$$

This interaction arises from extending the Anderson’s theory of superexchange interaction by including the spin-orbit coupling effects [17]. In this paper we interested to consider the
model with DMI and also investigated the effects of this interaction on the information transfer processing.

Following the approach proposed by S. Bose, we place a spin encoding the state at one end of the chain (which is now equipped with DMI) and wait for specific amount of time to let this state propagate to the other end. Therefore, entanglement could be transferred from one end of the chain to the other. We will show that the XXZ chain with DMI can be reduced to the modified XXZ chain with new coupling constant and rotated Pauli operators. The modified coupling constant and the angle of rotations are depend on the strength of DMI. Consequently, the entanglement transfer protocol through the XXZ+DMI chains becomes the same as a protocol via a general form of XXZ chain with new definition of coupling constant and Pauli matrices, but it should be noted that at this case the initial state may be changed. Indeed, both modified coupling constant and the angle of rotations, which is referred by "phase factor" through the text, are the efficient parameters for entanglement transfer. For the sake of clarity, the effects of phase factor and coupling constant have been separated. Our calculation shows that this phase factor does not affect on the entanglement transfer when $\Delta = 0$ for the XXZ chain. Whereas, this phase factor has desirable effects on the parameters of entanglement transfer for the case of $\Delta < 0$ and undesirable effects when $\Delta > 0$. However, there is a contest between the phase factor and coupling constant. For large amounts of the strength of DMI($D$), the effects of coupling constant are more dominant than the effects of phase factor and has desirable effects. Also, in the whole range of $\Delta$, the effects of the modified coupling constants and the phase factor simultaneously investigated on our main goal, i.e, information or entanglement transmission. Furthermore, the other advantage of DMI is that the speed of information transmission increases as increasing the strength of this interaction. Also, at nonzero temperature this interaction (DMI) improves both $E_{\text{max}}$ and $t_{\text{opt}}$ as increasing the strength of DMI ($D$) for both positive and negative amounts of $\Delta$.

The paper is organized as follows. In Sec. II we introduce the XXZ Hamiltonian with DMI and discuss the way to change the form of Hamiltonian to the general XXZ Hamiltonian. The new set of eigenstates have been introduced in this Sec. In Sec. III we introduce our state transmission. In Sec. IV we show analytically the dependence of concurrence on $D$ and pase factor. In Sec. V we show our numerical calculation of entanglement transfer and corresponding times in both zero and nonzero temperature and calculated the speed of
information transfer while, we summarize our results in Sec. VI.

II. HAMILTONIAN AND MODEL

The Hamiltonian of the open XXZ chain in presence of spin-orbit interaction is defined as

\[
H_{ch} = \sum_{i=1}^{N_{ch}-1} \{ J [\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y] + \Delta \sigma_i^z \sigma_{i+1}^z \} + \sum_{i=1}^{N_{ch}-1} \{ \vec{D} \cdot (\vec{\sigma}_i \times \vec{\sigma}_{i+1}) \},
\]

(1)

Where \( N_{ch} \) is the number of spin in 1D chain, \( \vec{\sigma}_i = (\sigma_i^x, \sigma_i^y, \sigma_i^z) \) is the vector of pauli matrices, \( J \) is the exchange coupling, the parameter \( \Delta \) is the anisotropy exchange coupling in the z direction and \( \vec{D} \) is the Dzyaloshinski-Moriya vector. Different phases of chain is depending on different range of \( J \) and \( \Delta \). For the case of \( J < 0 \) the chain is called Ferromagnetic(FM) Heisenberg chain and the case \( J > 0 \) is Antiferromagnetic(AFM) Heisenberg chain. The AFM Heisenberg chain includes FM phase(\( \Delta J < 0 \)), AFM phase(\( 0 < \Delta J < 1 \)) and Néel phase(\( \Delta J > 1 \))[22]. If we take \( \vec{D} = D \hat{z} \), then the above Hamiltonian can be written as

\[
H_{ch} = \sum_{i=1}^{N_{ch}-1} \{ J [\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y] + \Delta \sigma_i^z \sigma_{i+1}^z \} + D \sum_{i=1}^{N_{ch}-1} \{ \sigma_i^y \sigma_{i+1}^y - \sigma_i^y \sigma_{i+1}^y \}.
\]

(2)

This Hamiltonian is invariant under z-axis rotation, i.e, \([H, S_z] = 0\), where \( S_z = \frac{1}{2} \sum_{i=1}^{N_{ch}} \sigma_i^z \). By this property the Hamiltonian can be express in the form of usual XXZ model without explicit DMI. For this purpose, the pauli operators in x-y directions are manipulated by the unitary transformation which is depend on the spin sites[23]

\[
U_{N_{ch}} = \exp \left( -i \frac{\phi}{2} \sum_{m=2}^{N_{ch}} (m-1) \phi \sigma_m^z \right),
\]

(3)

where \( \phi = tan^{-1}(D/J) \). Hence these spin coordinate transformations reads

\[
\begin{align*}
\tilde{\sigma}_i^x &= \sigma_i^x \cos \phi_i + \sigma_i^y \sin \phi_i, \\
\tilde{\sigma}_i^y &= -\sigma_i^x \sin \phi_i + \sigma_i^y \cos \phi_i, \\
\tilde{\sigma}_i^z &= \sigma_i^z,
\end{align*}
\]

(4)

where \( \phi_i = (i-1)\phi \). So, the modified Hamiltonian is express as

\[
\tilde{H}_{ch} = U_{N_{ch}} H_{ch} U_{N_{ch}}^\dagger = \tilde{J} \sum_{i=1}^{N_{ch}-1} \{ \tilde{\sigma}_i^x \tilde{\sigma}_{i+1}^x + \tilde{\sigma}_i^y \tilde{\sigma}_{i+1}^y \} + \Delta \sum_{i=1}^{N_{ch}-1} \sigma_i^z \sigma_{i+1}^z,
\]

(5)

\[
\tilde{J} = sgn(J) \sqrt{J^2 + D^2}.
\]
The eigenstates of the new Hamiltonian ($|\tilde{\psi}_n\rangle$) are related to the earlier one ($|\psi_n\rangle$) via $|\tilde{\psi}_n\rangle = U_{N_{ch}} |\psi_n\rangle$. As the model of the system is specified, we can investigate the information transmission processing in this system.

III. ENTANGLEMENT TRANSMISSION

The quantum information transmission of one part of a two-spin maximally entangled state ($0'0$) via XXZ+DMI spin chain is investigated while the spin chain is in its ground state ($|\psi_{gs}\rangle_{ch}$). At $t=0$ we interact the spin 0 with the first spin of the chain. We suppose that the chain is prepare in a unique grand state, initially. The preparation could be performed by applying a small magnetic field, if it is required. Furthermore, the interaction between spin 0 and first spin of the chain has the same form of the rest of interaction,

$$H_I = J(\sigma_0^x\sigma_1^x + \sigma_0^y\sigma_1^y + \Delta\sigma_0^z\sigma_1^z) + D(\sigma_0^x\sigma_1^y - \sigma_0^y\sigma_1^x).$$

(6)

Indeed, the system is consist of $0'0$ and $N_{ch}$ spins and hence the total length of the system is $N = N_{ch} + 2$. The initial state of the system is

$$|\psi(0)\rangle = |\psi^\rangle_{0'0} \otimes |\psi_{gs}\rangle_{ch},$$

(7)

where

$$|\psi^\rangle_{0'0} = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.$$  

(8)

This $|\psi(0)\rangle$ is used as a channel which transfer the entanglement. Therefore, the total Hamiltonian being

$$H = I_{0'} \otimes (H_{ch} + H_I),$$

(9)

By this Hamiltonian, the initial state evolves to the state $|\psi(t)\rangle = e^{-iHt}|\psi(0)\rangle$ and the two sites reduced density matrix of can be computed as $\rho_{mn}(t) = tr_{\bar{m}\bar{n}}\{ |\psi(t)\rangle\langle\psi(t)| \}$, where $tr_{\bar{m}\bar{n}}$ is the partial trace over the system except sites $m$ and $n$. The two sites reduced density matrix in computational basis ($|00\rangle, |01\rangle, |10\rangle, |11\rangle$) has the general form as

$$\rho_{mn}(t) = \begin{pmatrix}
a(t) & 0 & 0 & 0 \\
0 & x(t) & z(t) & 0 \\
0 & z^*(t) & y(t) & 0 \\
0 & 0 & 0 & b(t)
\end{pmatrix}.$$  

(10)
Although, the density matrix has been written in the Schrödinger picture, but \( \rho_{mn}(t) \) in terms of spin-spin correlation function could be expressed in the Heisenberg picture as follows [24]

\[
a(t) = 1 + \frac{1}{2} \langle \sigma^z_m(t) + \sigma^z_n(t) \rangle + \langle \sigma^z_m(t) \sigma^z_n(t) \rangle,
\]
\[
x(t) = 1 + \frac{1}{2} \langle \sigma^z_m(t) - \sigma^z_n(t) \rangle - \langle \sigma^z_m(t) \sigma^z_n(t) \rangle,
\]
\[
y(t) = 1 - \frac{1}{2} \langle \sigma^z_m(t) - \sigma^z_n(t) \rangle - \langle \sigma^z_m(t) \sigma^z_n(t) \rangle,
\]
\[
b(t) = 1 - \frac{1}{2} \langle \sigma^z_m(t) + \sigma^z_n(t) \rangle + \langle \sigma^z_m(t) \sigma^z_n(t) \rangle,
\]
\[
z(t) = \langle \sigma^z_m(t) \sigma^z_n(t) \rangle + \langle \sigma^y_m(t) \sigma^y_n(t) \rangle + i(\langle \sigma^x_m(t) \sigma^y_n(t) \rangle - \langle \sigma^y_m(t) \sigma^x_n(t) \rangle),
\]

(11)

these correlations are computed in terms of the initial state (Eq.7) and \( \sigma^\alpha_m(t) = e^{-iHt} \sigma^\alpha_m e^{-iHt} \) where \( \alpha = \{x, y, z\} \). Since the concurrence is directly defined in terms of the density matrix and so any minimization procedure is not necessary, the concurrence is used as a measure of entanglement for arbitrary mixed state of two qubits [25],

\[
C = \max \{0, 2\lambda_{\max} - \text{tr}\sqrt{R}\},
\]

(12)

\[
R = \rho \sigma^y \otimes \sigma^y \rho^* \sigma^y \otimes \sigma^y,
\]

(13)

where \( \lambda_{\max} \) is the largest eigenvalues of the matrix \( \sqrt{R} \). For our density matrix the concurrence results to be

\[
C = 2\max \{0, C^{(1)}, C^{(2)}\},
\]

(14)

where \( C^{(1)} = -\sqrt{xy} \) and \( C^{(2)} = |z| - \sqrt{ab} \). Because, \( C^{(1)} \) is always negative here the concurrence is [22]

\[
C = 2\max \{0, |z| - \sqrt{ab}\}.
\]

(15)

For our main goal the first site refers to 0’ and other site refers to the spin located at the end of the chain, say (j). So in terms of density matrix the subscript \( m \) is change to 0’ and \( n \) is change to j. The singlet fraction of the state, \( \rho_{0'j} \), as an indicator of the average fidelity of state transferring could be obtain as

\[
F = \langle \psi^- | \rho_{0'j} | \psi^- \rangle = \frac{1}{2}(x + y - 2z).
\]

(16)

With the aid of \( \tilde{H}_{ch} = U_{N_{ch}} H_{ch} U_{N_{ch}}^\dagger \) we have

\[
\langle \sigma^\alpha_m(t) \rangle = \langle e^{-iHt} \sigma^\alpha_m e^{iHt} \rangle
\]

\[
= \langle 0'0| \psi_{gs} | U_{N_{ch}+1} e^{-i\tilde{H}t} \tilde{\sigma}^\alpha_m e^{i\tilde{H}t} U_{N_{ch}+1}^\dagger | \psi_{gs} \rangle | 0'0 \rangle,
\]

(17)
with use of the $|\tilde{\psi}_n\rangle = U_{Nch}|\psi_n\rangle$, the expectation value can be express as

$$\langle \sigma_m^\alpha(t) \rangle = \langle 0'|\tilde{\psi}_{gs}\rangle_{ch} U_{Nch} U_{Nch+1} \tilde{\sigma}_m^\alpha(t) U_{Nch+1}^\dagger U_{Nch}^\dagger |\tilde{\psi}_{gs}\rangle_{ch} |0'\rangle,$$

where $U_{Nch} U_{Nch+1} = \exp \left(-i \sum_{m=1}^{N_{ch}} (2m-1) \phi \sigma_m^z \right)$ is phase factor which modify the states on the right hand of above equation. So, we can conclude that this model is similar to usual XXZ model with the new strength coupling in XY direction ($\tilde{J}$) and the new set of states which are multiplied by the phase factor. In the following, the effects of these parameters (phase factor and $\tilde{J}$) will investigated on entanglement transfer processing.

**IV. ANALYTICAL CALCULATION**

To more clarifying, the concurrence between the $0'$ site and the end of the chain with the length of($N_{ch} = 2$) have been calculated analytically in the appendix. In these calculations the explicit form of $\rho_{mn}(t) = tr_{mn}\{|\psi(t)\rangle\langle \psi(t)|\}$ was used and these results are quality compatible with the numerical results for higher N. From the relation(A5) the concurrence for the case of ($\Delta = 0$) is

$$C(t) = \frac{1}{8} \left( 4 | \cos (\xi t) - 1 | - | e^{2i\phi}(1 + \cos (\xi t)) \right)$$

$$+ i\sqrt{2} \sin (\xi t) | \times [ 1 + \cos (\xi t) + i\sqrt{2} e^{2i\phi} \sin (\xi t)] ,$$

where $\xi = 2\sqrt{2} \tilde{J}$ and we have the maximum entanglement($C=1$) for $\xi t = \pi$. As we can see, amounts of entanglement at the first peak ($E_{max}$) and corresponding time ($t = t_{opt}$) in this case are independent on the $\phi$.

Furthermore, for the case of $\Delta \neq 0$ the concurrence is obtained in Eq.(A10). In this case $E_{max}$ and $t_{opt}$ depend on $\phi$ as indicated in Fig. 1. In these figure, for clarifying the role of $\phi$, individually, we fixed the size of $\tilde{J}$. The results show that, in the case of $\Delta < 0$, increasing $\phi$ improves both $E_{max}$ and $t_{opt}$ and for the case $\Delta > 0$, the increase of $\phi$ has undesirable effects on both $E_{max}$ and $t_{opt}$. The calculation becomes more involved when $N$ exceeds 4, this prevents one from writing an analytical expression for the concurrence. So, we solve this problem numerically.
V. NUMERICAL CALCULATION

A. Entanglement at T=0

Whereas, in the XXZ spin chain the AFM Heisenberg chains ($J > 0$) is a better candidate for information transfer than FM chains ($J < 0$) [11], we confined our calculations to the case $J > 0$ and take $J = 1$ to simplify the calculation. The entanglement is calculated between $0'$ and spin located at the end of the chain which has the length of $N=8$. In Fig. 2(a), $E_{\max}$, as measured by concurrence, have been plotted in the domain of $(-2 \leq \Delta \leq 2)$ for different values of $D$ and Fig. 2(b) illustrates the behavior of $E_{\max}$ as a function of $\Delta$ for different values of $\phi$, where $\tilde{J}$ is fixed. For the special case $D=0$ (i.e, $\tilde{J} = J$ and $\phi = 0$) the results are the same as in Ref. [2], qualitatively. In this case $E_{\max}$ vanishes at the quantum phase transition (QPT) point $(\tilde{\Delta} = -1)$ [22]. In the presence of $D$, $\tilde{J} \neq J$ and hence the QPT point shifts to the $\tilde{\Delta}/J = -1$, e.g, for the case of $D = 1$ the QPT occurs at $\Delta = -\sqrt{2}$ [28]. Despite to the results of [2], at this modified transition point $E_{\max}$ does not vanish, this is due to the presence of the phase factor $\phi$ as indicated in Fig. 2(b). Furthermore, the amount of $\tilde{J}$ is so large for large values of $D$ and so the QPT point disappear in the frame of $-2 \leq \Delta \leq 2$. In this range of $\Delta$ and $D$, the amount of $E_{\max}$ is unsensible to the values of $\phi$ and hence $\tilde{J}$ plays the main role to quantifying $E_{\max}$. Since, $\tilde{\Delta}/J$ approaches zero as $D$ becomes large, the system treat as the case $\Delta = 0$, i.e, $E_{\max} \rightarrow E_{\max}(\Delta = 0)$. Fig. 3(a) reveals that the behavior of $t_{opt}$ is compatible with the result of Fig. 2(a). This figure shows that $t_{opt}$ decreases as increasing $D$, hence the presence of DMI enhances the speed of information transmission. Also, Figs. 2(b) and 3(b) show that the effect of $\phi$ on $E_{\max}$ and $t_{opt}$ is undesirable for the case of $\Delta > 0$. In contrast, for the case $\Delta < 0$, the maximum entanglement at the first peak and speed of transmission enhances with $\phi$. In summary, all of the chains with $-2 \leq \Delta \leq 2$ can be used as protocol for information transfer processing with the same cost.

Fig. 4 depicts the singlet fraction in terms of $\Delta$ for different values of $D$. The result of this picture confirms the above mentioned effects of DMI on entanglement transfer properties.

In order to better illustrate the effects of $D$ on the entanglement transfer properties, $E_{\max}$ is plotted in terms of $D$ and $\phi$ in Figs. 5(a)-5(d) for different values of $\Delta$ respectively. The dips appearing in Figs. 5(a) and 5(c) are due to the effects of the phase factor ($\phi$) which
is obvious from Figs. 5(b) and 5(d). The effects of DMI on speed of state transmission are plotted in Figs. 6(a) and 6(b), more obviously.

**B. Thermal Entanglement**

Since, preparing the system at $T = 0$ is far from access, the presence of thermal excitations is unavoidable. So, we consider the state of channel as a thermal state instead of ground state. The thermal state of channel at temperature $T$ is given by density matrix $\rho_{ch} = \frac{e^{-\beta H_{ch}}}{Z}$, where $\beta = \frac{1}{k_B T}$, $k_B$ is the Boltzman constant and $Z = tr(e^{-\beta H_{ch}})$ is the partition function. So the initial state of system is

$$\rho(0) = |\psi^-\rangle\langle \psi^-| \otimes \frac{e^{-\beta H_{ch}}}{Z}. \quad (20)$$

Employing the system parameters as before, we repeat the calculations for this new initial state and the results have been shown in Figs 7 and 8 for $\Delta = 1$ and $\Delta = -1$, respectively. Two considerations are in order at this stage. Firstly, at high temperatures, thermal fluctuations suppress the quantum correlations and hence the entanglement vanishes and also, $t_{opt}$ becomes so large. Secondly, the presence of DMI amplify the quantum correlations, so by increasing $D$ we can obtain nonzero entanglement at larger temperatures. For instance Fig 8(a) shows that for the case of $\Delta = -1$ the entanglement can be exist at higher temperatures in the presence of DMI while it is zero for all temperature in the absence of DMI. Also, the speed of transfer improves as $D$ increases.

**C. Information speed**

As mentioned before, the DMI imposes desirable effects on the speed of transmission. In order to better clarify, we compare $t_{opt}$ with $v^{-1}$ (which $v$ is the spin-wave velocity) which is obtained with the aid of field theoretic techniques \[27\]. Note that, the later get the qualitative behavior of correlations in the thermodynamic limit while the former is calculated for very short chain ($N = 8$). Also, in the field theoretic techniques the dynamical correlations are computed for the ground state of the system while in our problem it is not the case. Despite difference, we still can use some well-known results of the field theoretic techniques. For instance, in our modified XXZ model (XXZ model equipped with DMI) and for the range
$-1 < \Delta < 1$, the spin-spin correlation function in the asymptotic thermodynamical limit have the following form [27]

$$\langle \sigma_i^\mu(t)\sigma_{i+n}^\mu \rangle \sim (-1)^n [n^2 - v^2 t^2]^{-(1/2)} \eta_v,$$  

(21)

$$\eta_x = \eta_y = \eta_z^{-1} = 1 - \frac{\gamma}{\pi},$$

where

$$\Delta = \cos(\gamma),$$  

(22)

and the propagation velocity of excitation in the chain can be written as

$$v \sim \frac{\pi J \sin(\gamma)}{\gamma}.$$  

(23)

This velocity is proportional to the strength of DMI ($D$), which is included in $\tilde{J}$, and also it depends on the size of $\Delta$. According to the Ref. [27], $v$ refers to the velocity of propagation of the correlations which is related to the entanglement transmission speed. Fig. 9 shows $v^{-1}$, which is obtained from Eq. (23), and $t_{opt}$, which is calculated in previous section, in terms of $D$ for two different values of $\Delta$. As we can see both quantities have similar behavior qualitatively, such that they descent with increasing of $D$. As this figure illustrates, despite of the field theory calculation for spin-wave propagation time ($v^{-1}$), the curves of $t_{opt}$ cross each other, indeed at the cross point the effect of the phase factor ($\phi$) becomes considerable. As a consequence, for large values of $D$ the speed of information transmission raises up and ultimately reaches the asymptotic value. Faster dynamics in large values of $D$ stems to the entanglement enhancement of the channel in this region.

VI. CONCLUSIONS

In this paper we examined a XXZ Heisenberg chain equipped with the Dzyaloshinski-Moriya interaction (DMI) as a quantum channel for investigation of the entanglement transfer properties. We had shown that the presence of DMI enhances the amount of coupling constant, $J$, and also imposes a phase factor on the state of the channel. In order to clarifying the role of DMI, we trace the effects of the new coupling constant ($\tilde{J}$) and phase factor ($\phi$) for a wide range of the anisotropy parameter, $\Delta$, separately. Indeed, increasing $\tilde{J}$ with $D$ leads to increase the strength of spin-spin correlations and ultimately improves the amount of the entanglement. For the case $\Delta < 0$ the effects of $\phi$ on the entanglement properties of
transmission is more desirable. In contrast, for the case $\Delta > 0$ increasing $\phi$, individually, destroys the entanglement of the channel. The effects of $\tilde{J}$ dominate for large values of $D$ and hence the channel becomes more efficient for all values of anisotropy parameter in the region $-2 \leq \Delta \leq 2$. We calculated the entanglement properties for the ground state and the thermal state of the chain, separately. Our results show that the amount of entanglement and the speed of transmission increase as $D$ increases. Also, we show that the entanglement can be exist at higher temperatures as $D$ increases and hence the transmission channel could be work more efficiently at higher temperatures.

Appendix A: Analytical Calculation for $N=4$

In this appendix, we give analytical calculation for the XXX and XXZ chains with $N = 4$, separately. For XXX chain ($\Delta = 0$), the initial state $|\psi(0)\rangle$ in the basis of the Hamiltonian $H_3$ for $N = 3$ can be written as \[ |\psi(0)\rangle = \frac{e^{-i\phi}}{2} [ |0\rangle_{0'} \otimes (\sqrt{2} |\psi_2\rangle + \frac{1}{2} (\sqrt{2} + e^{2i\phi}) |\psi_5\rangle + \frac{1}{2} (-\sqrt{2} + e^{2i\phi}) |\psi_7\rangle ) + |1\rangle_{0'} \otimes ( \frac{1}{\sqrt{2}} |\psi_3\rangle + \frac{1}{2} (1 + \sqrt{2} e^{2i\phi}) |\psi_6\rangle + \frac{1}{2} (1 - \sqrt{2} e^{2i\phi}) |\psi_8\rangle ] , \] (A1)

where

\[
|\psi_2\rangle = \frac{1}{\sqrt{2}} (|011\rangle + |110\rangle), \\
|\psi_3\rangle = \frac{1}{\sqrt{2}} (|001\rangle + |100\rangle), \\
|\psi_5\rangle = \frac{1}{2} (|011\rangle - \sqrt{2}|101\rangle + |110\rangle), \\
|\psi_6\rangle = \frac{1}{2} (|001\rangle - \sqrt{2}|010\rangle + |100\rangle), \\
|\psi_7\rangle = \frac{1}{2} (|011\rangle + \sqrt{2}|101\rangle + |110\rangle), \\
|\psi_8\rangle = \frac{1}{2} (|001\rangle + \sqrt{2}|010\rangle + |100\rangle),
\] (A2)

are the relevant eigenstates of $H_3(\Delta = 0)$ and the corresponding eigenvalues are $E_2 = E_3 = 0$, $E_5 = E_6 = -\xi$, and $E_7 = E_8 = \xi$, here we define $\xi = 2\sqrt{2}\tilde{J}$. The state of system at later
times can be obtained as

\[
\psi(t) = \frac{e^{-i\phi}}{2} |0\rangle_\psi \otimes \left( \frac{e^{2i\phi}}{\sqrt{2}} |\psi_2\rangle + \frac{1}{2} (\sqrt{2} + e^{2i\phi}) e^{i\xi t} |\psi_5\rangle \right) \\
+ \frac{1}{2} (-\sqrt{2} + e^{2i\phi}) e^{-i\xi t} |\psi_7\rangle + |1\rangle_\psi \otimes \left( -\frac{1}{\sqrt{2}} |\psi_3\rangle + \frac{1}{2} (1 - \sqrt{2} e^{2i\phi}) e^{-i\xi t} |\psi_8\rangle \right) \\
+ \frac{1}{2} (1 + \sqrt{2} e^{2i\phi}) e^{i\xi t} |\psi_6\rangle + \frac{1}{2} (1 - \sqrt{2} e^{2i\phi}) e^{-i\xi t} |\psi_8\rangle).
\]

(A3)

The corresponding reduced density matrix, \( \rho_{02}(t) \) is in the form of Eq. (10) with the following components

\[
a = \frac{1}{16} \left| e^{2i\phi} (1 + \cos (\xi t)) + i\sqrt{2} \sin (\xi t) \right|^2,
\]

\[
b = \frac{1}{16} \left| e^{2i\phi} (-1 + \cos (\xi t)) + i\sqrt{2} \sin (\xi t) \right|^2,
\]

\[
y = \frac{1}{8} \left| \sqrt{2} e^{2i\phi} \cos (\xi t) + i \sin (\xi t) \right|^2,
\]

\[
z = \frac{1}{4} \left| -1 + \cos (\xi t) \right|^2.
\]

(A4)

Therefore the concurrence could be computed as

\[
C(t) = \frac{1}{8} \left| 4 - 1 + \cos (\xi t) \right| - \left| e^{2i\phi} (1 + \cos (\xi t)) \right|
\]

\[
+ i\sqrt{2} \sin (\xi t) \times \left| 1 + \cos (\xi t) \right|
\]

\[
+ i\sqrt{2} e^{2i\phi} \sin (\xi t) \right|).
\]

(A5)

Following the same procedure for XXZ chain (\( \Delta \neq 0 \)), the initial state \( |\psi(0)\rangle \) in the basis of corresponding Hamiltonian is

\[
|\psi(0)\rangle = \frac{e^{-i\phi}}{2} |0\rangle_\psi \otimes \left( \frac{e^{2i\phi}}{\sqrt{2}} |\psi_2\rangle + \frac{\alpha + e^{2i\phi}}{\sqrt{2 + \alpha^2}} |\psi_5\rangle + \frac{-\beta + e^{2i\phi}}{\sqrt{2 + \beta^2}} |\psi_7\rangle \right)
\]

\[
+ |1\rangle_\psi \otimes \left( -\frac{1}{\sqrt{2}} |\psi_3\rangle + \frac{1 + \alpha e^{2i\phi}}{\sqrt{2 + \alpha^2}} |\psi_6\rangle + \frac{1 - \beta e^{2i\phi}}{\sqrt{2 + \beta^2}} |\psi_8\rangle \right),
\]

(A6)
where $\alpha = \frac{\Delta + \sqrt{8J^2 + \Delta^2}}{2J}$, $\beta = \frac{\Delta - \sqrt{8J^2 + \Delta^2}}{2J}$ and

\begin{align*}
|\psi_2\rangle &= \frac{1}{\sqrt{2}}(-|011\rangle + |110\rangle), \\
|\psi_3\rangle &= \frac{1}{\sqrt{2}}(-|001\rangle + |100\rangle), \\
|\psi_5\rangle &= \frac{1}{\sqrt{2 + \alpha^2}}(|011\rangle - \alpha|101\rangle + |110\rangle), \\
|\psi_6\rangle &= \frac{1}{\sqrt{2 + \alpha^2}}(|001\rangle - \alpha|010\rangle + |100\rangle), \\
|\psi_7\rangle &= \frac{1}{\sqrt{2 + \beta^2}}(|011\rangle + \beta|101\rangle + |110\rangle), \\
|\psi_8\rangle &= \frac{1}{\sqrt{2 + \beta^2}}(|001\rangle + \beta|010\rangle + |100\rangle),
\end{align*}

(A7)

are the relevant eigenstates of $H_3(\Delta \neq 0)$ and the corresponding eigenvalues are $E_2 = E_3 = 0$, $E_5 = E_6 = -2\bar{J}\alpha$ and $E_7 = E_8 = 2\bar{J}\beta$. Therefore, the initial state, $|\psi(0)\rangle$ evolves to the state

\begin{align*}
|\psi(t)\rangle &= \frac{e^{-i\phi}}{2} [|0\rangle_0 \otimes (\frac{e^{2i\phi}}{\sqrt{2}})|\psi_2\rangle + \frac{\alpha + e^{2i\phi}}{\sqrt{2 + \alpha^2}}|\psi_5\rangle + \frac{(-\beta + e^{2i\phi})e^{-iE_5t}}{\sqrt{2 + \beta^2}}|\psi_7\rangle + |1\rangle_0 \otimes (\frac{-1}{\sqrt{2}})|\psi_3\rangle \\
&\quad + \frac{(1 + \alpha e^{2i\phi})e^{-iE_6t}}{\sqrt{2 + \alpha^2}}|\psi_6\rangle + \frac{(1 - \beta e^{2i\phi})e^{-iE_8t}}{\sqrt{2 + \beta^2}}|\psi_8\rangle].
\end{align*}

(A8)
The corresponding reduced density matrix, $\rho_{\nu 2}(t)$, is in the form of Eq. (10) with the following components

$$
a = \frac{1}{4} \left| \frac{e^{2i\tilde{J}\alpha}(\alpha + e^{2i\phi})}{\alpha^2 + 2} - \frac{1}{2} e^{2i\phi} + \frac{e^{-2i\tilde{J}\beta}(e^{2i\phi} - \beta)}{\beta^2 + 2} \right|^2,
$$

$$
x = \frac{1}{4} \left| -\frac{1}{2} e^{2i\phi} + \frac{(\alpha + e^{2i\phi})e^{2i\alpha\tilde{J}t}}{2 + \alpha^2} + \frac{(-\beta + e^{2i\phi})e^{-2i\beta\tilde{J}t}}{2 + \beta^2} \right|^2,
$$

$$
y = \frac{1}{4} \left| -\frac{1}{2} + \frac{1 + \alpha e^{2i\phi} e^{2i\alpha\tilde{J}t}}{2 + \alpha^2} + \frac{(1 - \beta e^{2i\phi})e^{-2i\beta\tilde{J}t}}{2 + \beta^2} \right|^2,
$$

$$
b = \frac{1}{4} \left| \frac{1}{2} + \frac{(1 + \alpha e^{2i\phi})e^{2i\alpha\tilde{J}t}}{2 + \alpha^2} + \frac{(1 - \beta e^{2i\phi})e^{-2i\beta\tilde{J}t}}{2 + \beta^2} \right|^2,
$$

$$
z = \frac{1}{4} \left( \frac{\beta(-\beta + e^{-2i\phi})e^{2i\tilde{J}\alpha\beta}}{\beta^2 + 2} - \frac{\alpha(\alpha + e^{-2i\phi})e^{-2i\tilde{J}\alpha}}{\alpha^2 + 2} \right) \times \left( \frac{(1 + \alpha e^{2i\phi})e^{2i\tilde{J}\alpha}}{\alpha^2 + 2} + \frac{1 - \beta e^{2i\phi}}{\beta^2 + 2} - \frac{1}{2} \right)
$$

$$
+ \left( \frac{\alpha + e^{-2i\phi}}{\alpha^2 + 2} + \frac{e^{-2i\tilde{J}\alpha}(\alpha + e^{-2i\phi})}{\alpha^2 + 2} + \frac{1}{2} e^{-2i\phi} + \frac{e^{2i\tilde{J}\alpha}(e^{2i\phi} - \alpha)}{\alpha^2 + 2} \right) \times \left( \frac{1}{2} e^{-2i\phi} + \frac{e^{2i\tilde{J}\alpha}(e^{2i\phi} - \alpha)}{\alpha^2 + 2} \right).
$$

and hence

$$
C(t) = \frac{1}{2} \left( \left| \frac{e^{2i\tilde{J}\alpha}(\alpha + e^{2i\phi} - \beta)}{\beta^2 + 2} \right|^2 - \frac{e^{-2i\tilde{J}\alpha}(\alpha + e^{-2i\phi})}{\alpha^2 + 2} \right) \times \left( \frac{e^{2i\tilde{J}\alpha}(\alpha + e^{2i\phi})}{\alpha^2 + 2} + \frac{e^{-2i\tilde{J}\beta}(1 - e^{2i\phi} - \beta)}{\beta^2 + 2} \right)
$$

$$
+ \left( \frac{e^{-2i\tilde{J}\beta}(1 - e^{2i\phi} - \beta)}{\beta^2 + 2} \right) \times \left( \frac{e^{2i\tilde{J}\alpha}(\alpha + e^{2i\phi})}{\alpha^2 + 2} + \frac{1}{2} e^{-2i\phi} + \frac{e^{2i\tilde{J}\alpha}(e^{2i\phi} - \alpha)}{\alpha^2 + 2} \right)
$$

$$
\times \left| \frac{e^{2i\tilde{J}\alpha}(e^{2i\phi} - \alpha)}{\alpha^2 + 2} + \frac{1}{2} e^{-2i\phi} + \frac{e^{2i\tilde{J}\beta}(e^{2i\phi} - \beta)}{\beta^2 + 2} \right| + \frac{1}{2} \right).$$
[1] C. H. Bennett and D. P. DiVincenzo, Nature (London) 404, 247 (2000).
[2] A. Bayat and S. Bose, Phys. Rev. A 81, 012304 (2010).
[3] D. P. DiVincenzo, Fortschr. Phys. 48, 771 (2000).
[4] S. Bose, Phys. Rev. Lett. 91, 207901 (2003).
[5] J. Eisert et. al., Phys. Rev. Lett. 93, 190402 (2004); M. Christandl et. al., Phys. Rev. Lett. 92, 187902 (2004).
[6] V. Giovannetti and D. Burgarth, Phys. Rev. Lett. 96, 030501 (2006); J. Fitzsimons and J. Twamley, Phys. Rev. Lett. 97, 090502 (2006); A. Kay, Phys. Rev. Lett. 98, 010501 (2007).
[7] T. J. Osborne and N. Linden, Phys. Rev. A 69, 052315 (2004); A. Lyakhov and C. Bruder, Phys. Rev. B 74, 235303 (2006);
[8] A. Bayat and V. Karimipour, Phys. Rev. A 71, 042330 (2005);
[9] D. Burgarth, S. Bose, Phys. Rev. A 73, 062321 (2006); L. Zhou, J. Lu, T. Shi and C. P. Sun, quant-ph/0608135
[10] D. Burgarth and S. Bose, Phys. Rev. A 71, 052315 (2005); M. Avellino, A. J. Fisher, S. Bose, Phys. Rev. A 74, 012321 (2006);
[11] A. Bayat and S. Bose, Advances in Mathematical Physics, 2010, 127182 (2010); A. Bayat, D. Burgarth, S. Mancini and S. Bose, Phys. Rev. A 77, 050306(R) (2008).
[12] M. Asoudeh and V. Karimipour, Phys. Rev. A 73, 062109 (2006)
[13] K. Eckert, O. R. Isart, and A. Sanpera, New J. Phys. 9, 155 (2007); J. Ren and S. Zhu, Phys. Rev. A 81, 014302 (2010).
[14] C. F. Hirjibehedin, C. P. Lutz and A. J. Heinrich, Science 312, 1021 (2006).
[15] K. V. Kavokin, Phys. Rev. B 64, 075305 (2001).
[16] I. Dzyaloshinski, J. Phys. Chem. Solids 4, 241 (1958).
[17] T. Moriya, Phys. Rev. 117, 635 (1960).
[18] T. Moriya, Phys. Rev. Lett. 4, 228 (1960).
[19] F. Kheirandish, S. J. Akhtarshenas, and H. Mohammadi, Phys. Rev. A 77, 042309 (2008).
[20] F. Kheirandish, S. J. Akhtarshenas, and H. Mohammadi, Eur. Phys. J. D 57, 1 (2010).
[21] X. Wang, Phys. Lett. A 281, 101 (2001)
[22] H. Mikeska and A. Kolezhuk, Lect. Notes Phys. 645, 1 (2004).
It is important to note that, in the presence of DM interaction the phase diagram of the chain may be changed \cite{26} and hence we must be care on employing the words such that phase transition and so on. Whereas, in this paper, we modeled the XXZ chain with DM interaction by modified with XXZ chain and hence we can still borrow this words from the phase transition terminology.

since the spin $0'$ does not coupled to the chain it remains untouched during the evolution of the system. Thus, unitary transformation affects only on the sites $(0, 1, 2)$. 

\[\text{References}\]

\cite{23} O. Derzhko and T. Verkholyak, e-print \texttt{arXiv:0712.2507}.

\cite{24} L. Amico, A. Osterloh, F. Plastina, G. Palma, and R. Fazio, Phys. Rev. A \textbf{69}, 022304 (2004).

\cite{25} W. K. Wootters, Phys. Rev. Lett. \textbf{80}, 2245 (1998).

\cite{26} R. Jafari and A. Langari \texttt{arXiv:0812.1862}.

\cite{27} K. Fabricius, U. Low, and J. Stolze, Phys. Rev. B \textbf{55}, 5833 (1997).

\cite{28} Since the spin $0'$ does not coupled to the chain it remains untouched during the evolution of the system. Thus, unitary transformation affects only on the sites $(0, 1, 2)$. 

\cite{29} 

16
FIG. 1: (Color online) $C(\rho y_4)$ vs. $\phi$ and $t$, for (a) $\Delta = 0.9$ and (b) $\Delta = -0.9$. Here the length of the chain is chosen to be $N = 4$.

FIG. 2: (Color online) $E_{\text{max}}$ as a function of $\Delta$, for (a) different values of $D$ and (b) corresponding $\phi$.
FIG. 3: (Color online) $t_{opt}$ vs. $\Delta$, for (a) different values of $D$ and (b) corresponding $\phi$.

FIG. 4: Singlet fraction $F$ vs. $\Delta$ at $t_{opt}$ for different values of $D$. 
FIG. 5: (Color online) First maximum entanglement vs. $D$ and corresponding $\phi$. Graphs (a) and (b) refer to the case $\Delta \geq 0$ and (c) and (d) refer to the case $\Delta \leq 0$.

FIG. 6: (Color online) $t_{opt}$ in terms of $D$, for different values of (a) $\Delta \geq 0$ and (b) $\Delta \leq 0$. 
FIG. 7: (Color online) (a) $E_{\text{max}}$ and (b) $t_{\text{opt}}$ in terms of inverse temperature for different values of $D$ at $\Delta = 1$.

FIG. 8: (Color online) $E_{\text{max}}$ and (b) $t_{\text{opt}}$ in terms of inverse temperature for different values of $D$ at $\Delta = -1$. 
FIG. 9: (Color online) $v^{-1}$ (for an infinite chain) and $t_{opt}$ (for the chain of length N=8) vs. D for the case of $\Delta = 0.6$ and $\Delta = -0.6$. 