Scaling behavior of \((N_{\text{ch}})^{-1}dN_{\text{ch}}/d\eta\) at \(\sqrt{s_{\text{NN}}} = 130\ \text{GeV}\) by PHOBOS Collaboration and its analyses in terms of stochastic approach

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Abstract

Recently interesting data on \(dN_{\text{ch}}/d\eta\) in Au-Au collisions \((\eta = -\ln \tan(\theta/2))\) with the centrality cuts have been reported by PHOBOS Collaboration. Their data are usually divided by the number of participants (nucleons) in collisions. Instead of this way, using the total multiplicity \(N_{\text{ch}} = \int (dN_{\text{ch}}/d\eta) d\eta\), we find that there is scaling phenomenon among \((N_{\text{ch}})^{-1}dN_{\text{ch}}/d\eta = dn/d\eta\) with different centrality cuts at \(\sqrt{s_{\text{NN}}} = 130\ \text{GeV}\). To explain this scaling behavior of \(dn/d\eta\), we consider the stochastic approach called the Ornstein-Uhlenbeck process with two sources. Moreover, comparisons of \(dn/d\eta\) at \(\sqrt{s_{\text{NN}}} = 130\ \text{GeV}\) with that at \(\sqrt{s_{\text{NN}}} = 200\ \text{GeV}\) have been made. A possible detection method of the quark-gluon plasma (QGP) thorough \(dn/d\eta\) is presented.

1 Introduction

Recently PHOBOS Collaboration has published an interesting data on \(dN_{\text{ch}}/d\eta\) \((\eta = -\ln \tan(\theta/2))\) in Au-Au collisions at \(\sqrt{s_{\text{NN}}} = 130\ \text{GeV}\) [1]. The authors of ref. [1] have calculated the following quantity,

\[
y = \frac{1}{2} \ln \frac{E + p_z}{E - p_z} = \tanh^{-1} \left( \frac{p_z}{E} \right) \approx -\ln \tan(\theta/2) \equiv \eta.
\]

\[
\eta = \frac{1}{2} \ln \frac{p + p_z}{p - p_z}, \quad \text{and} \quad \frac{dn}{d\eta} = \frac{p}{E} \frac{dn}{dy}.
\]
\[
\frac{1}{\langle N_{\text{part}} \rangle / 2} \frac{dN_{\text{ch}}}{d\eta} = f(N_{\text{part}}, N_{\text{coll}}, \eta),
\]

where \(N_{\text{part}}\) and \(N_{\text{coll}}\) mean the number of participants (nucleons) and number of collision particles in Au-Au collisions. It depends on the centrality cuts. The function \(f(N_{\text{part}}, N_{\text{coll}}, \eta = 0)\) is an increasing function, as \(N_{\text{part}}\) increases.

In this paper, instead of eq. (1), we consider the following physical quantity,

\[
\frac{1}{N_{\text{ch}}} \frac{dN_{\text{ch}}}{d\eta} = \frac{dn}{d\eta},
\]

where \(N_{\text{ch}} = f(dN_{\text{ch}}/d\eta)\). and \(f(dn/d\eta)d\eta = 1\). In Fig. 1, two sets of \(dn/d\eta\) are shown. They suggest us that there is scaling among \(dn/d\eta\)’s with different centrality cuts. Thus \(dn/d\eta\) is named a kind of the probability density, because \(dn/d\eta = f(\eta)\), where \(dn/d\eta\) is function of \(\eta\) only. This fact probably implies that the stochastic approach is available in analyses of \(dn/d\eta\).

Contents of the present paper are as follows. In the second paragraph, the Fokker-Planck equation is considered. In the third one, concrete analyses are presented. In the final one, concluding remarks are given.

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2 Multiplicity distributions in high energy collisions, i.e., \(P(n, \langle n \rangle)\)’s are the probability distributions, which are function of \(n\) and \(\langle n \rangle\). It is known that the KNO scaling functions,

\[
\lim_{n, \langle n \rangle \to \infty} \langle n \rangle P(n) = \psi(z = n/\langle n \rangle)
\]

are described by solutions of various Fokker-Planck equations [2–4].

3 Dokshitzer has calculated the generalized gamma distribution in QCD [5]

\[
P(z = n/\langle n \rangle) \approx \frac{2\mu^2 (Dz)^{3\mu/2}}{z^{1+\gamma}} \exp \left[ -(Dz)^\mu \right],
\]

where \(z, \mu, D, \gamma\) are KNO scaling variable, \(1 - \gamma = 1/\mu\), a parameter, the anomalous dimension in QCD, respectively. This is a steady solution of the following Fokker-Planck Equation

\[
\frac{\partial P}{\partial t} = -\frac{\partial}{\partial z} \left[ \left( d + \frac{1}{2} Q \right) z + bz^{1+\gamma} \right] P + \frac{1}{2} Q^2 \frac{\partial^2}{\partial z^2} [z^2 P].
\]
Fig. 1. Two sets of $dn/d\eta$ with different centrality cuts [1]. Data with 15-25% are used in both Figures, for the sake of comparisons.

2 Stochastic approach to $dn/d\eta$

It is well known that the rapidity ($y \approx \eta$) is a kind of the velocity. Moreover, in collisions leading particles in the beam and target nuclei, i.e., nucleons in gold at RHIC experiments, collide each other, lose their energies and emit various particles. In other words, there are fluctuations in the rapidity space or the pseudo-rapidity ($\eta$). Thus we would like to assume that a damping law governs the rapidity space

$$\frac{dX}{dt} = -\gamma X + f_W(t),$$  \hfill (3)

where $X = y$ or $\eta$, $\gamma$ and $f_W(t)$ are the frictional coefficient and the white noise, respectively. From eq. (3), we can derive the following Fokker-Planck equation,

$$\frac{\partial P(x, t)}{\partial t} = \gamma \left[ \frac{\partial}{\partial x} + \frac{1}{2} \frac{\sigma^2}{\gamma} \frac{\partial^2}{\partial x^2} \right] P(x, t),$$  \hfill (4)

where $\sigma$ and $P(x, t)$ are the dispersion and the probability density, respectively. Equation (4) is called the Ornstein-Uhlenbeck (O-U) process [6,7]. To look for the solutions of eq. (4), we have to assume the source functions at “time” $t = 0$. The typical solution with $\delta(\eta - \eta_0) = P(\eta, t = 0)$ is given as
\[ P(\eta|\eta_0, t) = \frac{1}{\sqrt{2\pi V^2(t)}} \exp \left[ -\frac{(\eta - \eta_0 e^{-\gamma t})^2}{2V^2(t)} \right], \tag{5} \]

where \( V^2(t) = \left( \sigma^2 / 2\gamma \right) (1 - e^{-2\gamma t}) \) and \( \eta_0 \) is the initial rapidity.

In Fig. 2(a), we depicted a simplified picture of heavy-ion collisions. According to Fig. 2(a), a model of two sources \( 0.5 \times \delta(\eta - \eta_{\text{max}}) \) and \( 0.5 \times \delta(\eta + \eta_{\text{max}}) \) at \( t = 0 \) seems to be reasonable. As our present model is very simple, \( 0.5 \times N_{\text{ch}} \) particles have been produced at \( \eta_{\text{max}} \) and the same \( 0.5 \times N_{\text{ch}} \) particles at \( -\eta_{\text{max}} \) at \( t = 0 \). In this case we have the following expression which is the sum of two solutions,

\[ P(\eta|\eta_{\text{max}}, t) = \frac{1}{\sqrt{8\pi V^2(t)}} \left\{ \exp \left[ -\frac{(\eta + \eta_{\text{max}} e^{-\gamma t})^2}{2V^2(t)} \right] + \exp \left[ -\frac{(\eta - \eta_{\text{max}} e^{-\gamma t})^2}{2V^2(t)} \right] \right\}. \tag{6} \]

The evolution of eq. (6) is shown in Fig. 2(b).

Fig. 2. (a) Simplified picture for A-A collisions. The black circles mean nucleons. (b) Evolution of eq. (6) with two sources at \( \eta_{\text{max}} \) and \( -\eta_{\text{max}} \).

3 Analyses of \( dn/d\eta \) by means of eq. (6)

By making use of eq. (6), we can analyse \( dn/d\eta \) shown in Fig. 1. Our results are shown in Fig. 3 and Table 1. In Fig. 4, we examine whether or not the dispersion \( V^2(t) \) and \( p = 1 - e^{-2\gamma t} \) depend on the centrality cuts. As is seen in Figs. 3 and 4, the scaling behavior among \( dn/d\eta \)'s at \( \sqrt{s_{\text{NN}}} = 130 \) GeV is explained by eq. (6) with small changes in the dispersion \( V^2(t) \). It can be said that the scaling behavior is explained by the O-U process with two sources at the beam (\( y_B \) or \( \eta_{\text{max}} \)) and target (\( y_T \) or \( -\eta_{\text{max}} \)) rapidities. Of course, it is obvious that the single source cannot explain it.
Fig. 3. Analyses of $d\eta/d\eta$ by eq. (6). See Table 1.

4 Concluding remarks

First of all, it can be stressed that there is the scaling among $d\eta/d\eta$'s with various centrality cuts at $\sqrt{s_{NN}} = 130$ GeV.

Second the scaling behavior of $d\eta/d\eta$ is described by the solutions of the Fokker-Planck equation, i.e., eq. (6). This suggests that $d\eta/d\eta$ with the centrality cut 0-6% do not show singular /or particular phenomenon relating to signatures of the Quark-Gluon Plasma (QGP). Of course, we should pay our attention that we are handling the averaged quantity in statistics. At present, however, it is difficult to conclude that the QGP is created, and the signature

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4 To estimate the “thermalization time” of the QGP, Hwa has considered the Fokker-Planck equation for the motion of the quarks and gluons in nuclei [8]. See also ref. [9] in which the Wiener process is considered for the quarks and gluons.
Table 1
Estimated parameters in our analyses by eq. (6) with two sources. Evolution of eq. (6) is stopped at minimum $\chi^2$'s. $\delta p = 0.006 \sim 0.004$.

| Fig. 3 | (a) | (b) | (c) | (d) | (e) | (f) |
|--------|-----|-----|-----|-----|-----|-----|
| centrality (%) | 45-55 | 35-45 | 25-35 | 15-25 | 6-15 | 0-6 |
| $p$ | $0.872\pm\delta p$ | $0.875\pm\delta p$ | $0.878\pm\delta p$ | $0.882\pm\delta p$ | $0.886\pm\delta p$ | $0.888\pm\delta p$ |
| $V^2(t)$ | $3.83\pm0.27$ | $3.61\pm0.21$ | $3.23\pm0.16$ | $3.00\pm0.13$ | $2.72\pm0.10$ | $2.47\pm0.08$ |
| $\langle N_{\text{part}} \rangle$ | — | 93 | 135 | 197 | 270 | 340 |
| $N_{\text{ch}}$ | $662\pm10$ | $1056\pm16$ | $1582\pm23$ | $2270\pm34$ | $3199\pm49$ | $4070\pm63$ |
| $\chi^2/\text{n.d.f.}$ | $8.61/51$ | $7.63/51$ | $5.88/51$ | $5.35/51$ | $3.57/51$ | $3.82/51$ |

![Fig. 4](image-url) Dispersal $V^2(t)$ and $p$ of Fig. 3 and Table 1.

From the QGP are washed out by the strong interactions between hadrons, if the QGP is created.

Here we should carefully observe Fig. 1. As is seen in Fig. 1, there are small differences in $dn/d\eta$'s over the range $|\eta| \lesssim 2$ with centrality cut 0-6% and others. To explore the differences more carefully, we need $dn/d\eta$ with smaller centrality cuts, 0-3% $\sim$ 0-5%.

Moreover, we can add the following fact. Very recently, PHOBOS Collaboration has reported the data on $dN_{\text{ch}}/d\eta$ with centrality cut 0-6% at $\sqrt{s_{NN}} = 200$ GeV [10]. They are compared with $dn/d\eta$ at $\sqrt{s_{NN}} = 130$ GeV in Fig. 5. Roughly speaking, the scaling on $dn/d\eta$ holds between $\sqrt{s_{NN}} = 130$ GeV and 200 GeV\(^5\). The distribution of $dN_{\text{ch}}/d\eta$ or $dn/d\eta$ with the centrality cut 0-6% does not show the particular behavior relating to the QGP in the sense of

\(^5\) Using the Bjorken’s picture [11] for the calculation of energy density near $|\Delta \eta| \leq 0.5$ with the geometrical picture of the gold ($R_c \approx 6-7$ fm, $c\tau_0 \approx 1-2$ fm, $V \approx \pi R_T^2(c\tau_0) \approx 300$ fm\(^3\)), we obtain the following values.
Fig. 5. Comparisons of $d n/d \eta$ with the centrality cut 0-6% at $\sqrt{s_{NN}} = 130$ GeV and 200 GeV. Solid line is obtained for latter energy: $p = 0.879 \pm 0.007$, $V^2(t) = 2.67 \pm 0.24$, $\chi^2/n.d.f = 0.63/22$.

average. To investigate the particular phenomena like the turbulence and/or deflagration in $dN_{\text{ch}}/d\eta$, we need to analyse the single event with smaller centrality cut than 0-6%.

Moreover, analyses of event-by-event, for example the intermittency and the wavelet, seem to be necessary for $dN_{\text{ch}}/d\eta$ with smaller centrality cut [12–18].

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$\varepsilon \sim \frac{3}{2} \left. \frac{1}{V} \frac{dN_{\text{ch}}}{d\eta} E_T \right|_{\Delta \eta \leq 0.5} \sim 1 \text{ GeV/fm}^3 \ (130 \text{ GeV}),$

$\varepsilon \sim 1.2 \text{ GeV/fm}^3 \ (200 \text{ GeV}).$
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