No-Lose ‘Theorem’ for Parity Violating
Nucleon-Nucleon Scattering Experiments

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Abstract

A purely left-chiral model of the weak interactions is used to show that the total parity-violating asymmetry in quark-quark scattering must grow with increasing energy. In the absence of other new physics, non-observation of a large asymmetry can therefore be used to infer an upper bound on the mass scale for new right-chiral weak vector bosons. Applying this idea to actual nucleon-nucleon scattering requires more involved calculations, as the dominant contribution appears to come from a component of diquark-quark scattering related to, but not identical to, wavefunction-mixing. Earlier criticism of this model by Simonius and Unger is refuted and a new calculation is proposed as an additional check on the result. Finally, we argue that the so-called ‘spin crisis’ does not affect our conclusions.

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1 Hadronic Scattering in a Purely Left-Handed World

Consider a world in which the weak interaction is described by a purely left-chiral SU(2) theory, spontaneously broken in the usual way. To one-loop order, there are then finite weak corrections to the left-chiral quark-gluon vertex function which do not exist for the corresponding right-chiral vertex. These (finite after wavefunction renormalization) vertex corrections have the structure of form-factors and so imply that the left-chiral quark-gluon form-factor falls more rapidly with increasing (spacelike) squared four-momentum of the gluon than does the right-chiral one.

The right-chiral quark-gluon vertex is completely unaffected in this regard because the right-chiral quarks do not couple at all to this weak interaction. The difference of these vertex strengths then induces a parity violation when, say, a beam of right-chiral quarks impinging on a target of left-chiral quark is compared with the same beam impinging on a target of right-chiral quarks. The strong interactions (gluon exchanges) are equal, in the two cases, as are the weak interactions at tree level, but the latter differ at one-loop because of the vertex correction to the left-chiral quark-gluon coupling. Note that in this simplified model, there are no other one-loop graphs (box or crossed-box), as in the actual case, again because the right-chiral quarks do not participate at all in this weak interaction.

Thus, the gluon-mediated scattering of left-chiral quarks on right-chiral quarks falls off more rapidly with four-momentum transfer than does the scattering of right-chiral quarks on right-chiral quarks. This parity-violating asymmetry (PVA) in quark-quark scattering, defined here as the difference between the right-chiral on right-chiral total cross-section and the left-chiral on right-chiral one divided by their sum, develops and grows as larger and larger four-momentum transfers contribute to the total cross-sections as the phase space expands with increasing energy. The statement holds for ‘elastic’ quark-quark scattering below the weak vector boson production threshold.

Above that threshold, the opening channels restore total cross-section strength and damp out the PVA because, even in this simplified model, bremsstrahlung of weak-bosons will compensate for the form-factor reduction of elastic quark-quark scattering. However, if the mass of the weak vector boson is large, it should be feasible, in principle, (unlike the corresponding electromagnetic case) to experimentally separate out the (weak-)elastic events from all others.

The below threshold growth of the PVA can only be ameliorated by the slow, logarithmic change with scale of the coupling constants, unless there are new right-chiral weak interactions which appear at some (higher) mass scale. These new interactions weaken the right-chiral quark-gluon vertex in a corresponding fashion, stopping the growth in the difference between the two cross-sections. As the average total cross-section continues to grow, the fractional PVA decreases. Thus we expect two possibilities for PVA measurements at medium and high energies: Either the Standard Model is correct and the PVA increases with energy, becoming less difficult to measure; or the PVA decreases and experiments can only put an upper bound on its value at higher energies. In the latter case, however, we can infer an upper bound on the mass scale of the vector bosons for a new right-chiral weak interaction. This is the no-lose ‘theorem’ referred to in the title.

Our explicit calculations [1] of the vertex function in the pure SU(2)\textsubscript{L} case show decrements in the vertex strength of only a tiny fraction of a percent of the dimensionless weak coupling constant at four-momentum transfers of order 1 GeV, but this grows by more than an order of magnitude by 10 GeV. It grows to 30% by the TeV region. Thus, the only practical questions are whether the PVA can be measured at all at lower energies, and whether the theoretical calculation of the effect in the Standard Model is consistent with those results. We believe the answer is positive in both regards [3], but this contention relies heavily on the results from an experiment [4] at ANL at 6 GeV/c and a more recent result [4] from LAMPF at even lower energies.
2 A Brief Digression on Low-Energy Measurements and Theories

Several theoretical and experimental attempts have been made in the past to measure and predict the PVA in nucleon-nucleon scattering, now defined as the difference between the total cross-sections for positive and negative helicity (right- and left-chiral) nucleons on unpolarized targets divided by their sum. Early low energy experiments were performed at energies of 15 and 45 MeV, finding negative PVA’s of order $10^{-7}$, to no one’s great surprise. The theoretical efforts naturally approach the problem from the point of view of meson-exchange forces and do produce the correct sign and order of magnitude. The main problem was to obtain independent estimates of the parity-violating meson-nucleon vertex function strengths. Some are available through current algebra and bounds may be obtained from limits on parity-violating admixtures in nuclear states. A brave attempt was made to calculate these quantities using QCD. Very recently, low energy polarized neutron scattering at LAMPF has provided a new way to extract this information at zero energy. These experiments are especially interesting as parity-violating effects above the 10% level have been discovered, compromising the old arguments that such effects must always be very small.

However, a common feature of the calculations emerged as they were extended to intermediate energies: As more partial waves or channels (Δ intermediate states, and multiple meson exchanges for example) are added, the PVA tends to become positive (the cross-over energy has been the subject of experimental efforts at TRIUMF) and an envelope of the collection of curves appears, which continues growing with energy even though each new individual contribution eventually falls off. For recent sophisticated efforts that include references to earlier work, see [9]. The impetus for these extensions was the ‘high’ energy result at ANL, which seemed incomprehensibly large, at $2 \times 10^{-6}$, compared to the model results, despite their clear inadequacy at such a relatively large energy.

The envelope feature, however, was reminiscent to me of features I had seen in other attempts to describe high energy phenomena in terms of mesons and baryons, phenomena which were more readily explained (i.e., in terms of fewer independent amplitudes) by using quarks and gluons. At the instigation of Darragh Nagle, I therefore set out with Dean Preston, to calculate the relevant components of the weak Hamiltonian for quarks in detail, and to apply it to nucleon-nucleon scattering.

3 The QCD Calculation

Returning now to the real world, we find many dilution factors for the PVA signal: Due to confinement, one must use proton beams and nucleon targets, not quarks, with the attendant averaging over different chirality quark beams and targets. Of course, since the quarks are not directly visible, for a differential cross-section, a high $Q^2$ jet would have to be observed to ascertain that a large momentum transfer did indeed occur in the scattering event. Box and crossed-box graphs appear. The mixing of the $U(1)$ factor with both chiralities of couplings to include electromagnetism in the standard electroweak model also produces some cancelling right-chiral interaction corrections. However, it is at least very unlikely (and was indeed found not to be the case in the restricted energy range calculations of Ref. [10]) for there to be a strong cancellation in both proton-proton and proton-neutron scattering, (where the beam proton is polarized).

From a high energy, quark point of view, the strong amplitude, $A$, is schematically

$$A \sim \alpha_s/q^2, \quad (1)$$

where $\alpha_s$ is the strong coupling, and $q^2$ is the four-momentum transfer, and the weak parity-violating amplitude, $B$, is similarly

$$B \sim G_F. \quad (2)$$
Then, since $\alpha_s \sim 1$ and $< q^2 > \sim 0.1 \text{ GeV}^2/c^2$,

\[ PVA \equiv \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \]

\[ \sim \frac{|A^* B|}{|A|^2} \]

\[ \sim G_F < q^2 > /\alpha_s \]

\[ \sim 10^{-6}, \quad (3) \]

where $\sigma_{\pm}$ are the total cross sections for positive and negative helicity nucleons on unpolarized targets. Note the general feature that the PVA is given by the overlap of a strong and weak amplitude, divided by the total strong interaction cross-section.

The result in Eqn. (3) suggests the ANL result is of normal size. However, when averaged over realistic quark distributions in nucleons and when all the 2's and $\pi$'s are included, this result is reduced by about one order of magnitude. More detailed calculations require studying strong and weak amplitudes involving box and crossed-box graphs of gluons and weak vector bosons as well as the vertex corrections referred to in the Introduction. We have calculated all of the terms in the non-strange part of the weak Hamiltonian, to all orders in perturbation theory using the leading logarithm approximation of the renormalization group [2].

The result is essentially unchanged for quark-quark scattering, where the two quarks involved come from the different nucleons. However, there is another component present, namely when the two quarks involved come from the same nucleon. At first sight, this appears to be a quark level description of wavefunction mixing, as was studied in a Regge model by Soffer and Preparata [11]. It was immediately recognised that their conclusions must be wrong [12], however, since they obtained the ANL size and sign for the PVA, but their effect is the only one that survives to zero energy, thus contradicting the experimental results at low energy (barring very rapidly varying amplitudes below 10 MeV).

There is a more subtle error in that approach also. It is due to the fact that the quark-quark scattering in one nucleon can be influenced by the proximity of the other nucleon in the overall process. To see this, think of the quarks in a nucleon as wavepackets continually rescattering on each other due to the attractive strong interaction. A parity-violating wavefunction mixing occurs when the rescattering of this pair of quarks (diquark) occurs due to the action of the weak Hamiltonian rather than the strong. (Note that only the isospin-one, spin-one, vector diquark can contribute.) However, if there is another nucleon nearby, a non-negligible probability develops for one of its quarks to scatter on one of the quarks in the diquark just before the weak scattering occurs. (Or just after, but the other time-ordering is intuitively easier to understand.) This injects a four-momentum which raises the intermediate state diquark (before the weak scattering) to larger mass scales. As such, this effect includes all relevant parity-violating mixing between the nucleon and higher mass baryonic states. It is not constrained by (low energy) nuclear data on (diagonal) parity violating components of the nucleon itself. The probability of the weak scattering has not been otherwise significantly affected because the ‘other’ quark of the pair was already ‘focused’ to rescatter with this boosted quark, due to the strong interactions that shaped their wavefunctions. In nuclear physics terms, this is a combination of distorted wave (initial and final) and three-body effects. Aside from the change in the flux relevant to the scattering process, the rapid growth of the weak scattering strength with energy means that the parity-violating state mixing can be markedly increased.

Preston and I did find [10, 13] a PVA growing with energy due to this quark-diquark scattering effect. Moreover, for reasonable parameter values, the size of the effect is consistent with the ANL result. Normalizing to that result and correcting for growth in the strong cross-section at lower energies, we
also predicted a value consistent with the smaller, but still positive, PVA found at lower energy in the LAMPF experiment. For completeness, we should note that in both the quark-quark and the quark-diquark scattering cases, a single term of the effective weak Hamiltonian dominates, but its strong interaction enhanced strength is uncertain by an estimated overall factor of four. This is why we view the energy dependence of our result as more reliable than the absolute scale.

4 Response to Criticism

Simonius and Unger (SU) have taken exception to our calculation. They argue that we must model both the strong and the weak amplitude for the scattering processes, just as is done in the low energy meson-baryon descriptions, in order to make a meaningful prediction of the PVA. In a sense, we agree with them but argue that we have done a much better job of this than they have.

We take QCD with conventional parameter values to represent the strong interaction. For the weak interaction, our approximation requires using a quark-diquark amplitude wherein the diquark undergoes a weak scattering and one of the quarks in it exchanges a gluon with the intruding quark. The leading term in the QCD amplitude which overlaps this, in our approximation, involves a similar structure, with the internal diquark weak scattering replaced by a gluon exchange to represent the strong interaction. This last is required because our diquark representation has been simplified to that of two quarks, each carrying precisely half of the diquark momentum. (The diquark momentum fraction distribution is taken as the complement to the quark momentum fraction distribution derived experimentally from deep inelastic lepton scattering.) As a result, without this strong scattering of the two quarks, there would be an unrealistically poor overlap with the final state distribution of the two quarks from the weak scattering, which spreads their strength widely over momentum space. In a sense, we are modeling the quark wavefunction in the diquark by its perturbative part. The actual amplitude is stronger, but falls off more rapidly with momentum transfer until reaching the perturbative level so that, if anything, we should have an underestimate of the overall size of the effect.

In addition, however, we use another approximation involving SU(3) group characters to reduce a twelve $\gamma$-matrix trace to a product of two traces each involving six $\gamma$-matrices. As a result, only one of several strong-weak overlap amplitudes survives the color and Dirac tracings. SU calculate the corresponding strong-strong overlap, still with only one term, and find a large and rapidly growing (with energy) contribution to the total strong interaction cross-section. From this their criticism devolves.

However, it is clear on several counts that the SU calculation is meaningless. Where we take the measured total cross section for the strong interaction, SU take only one quark-diquark graph to represent QCD. It is straightforward to see that this is not consistent, as there is no reason to assume that the other QCD graphs vanish. In fact, this graph represents neither a complete set nor even a gauge invariant subset of graphs, and hence, the procedure is not sensible. This is fortunate, since if their calculation were correct, they would have shown that the quark-diquark contribution to nucleon-nucleon scattering proves QCD is inconsistent with data!

That there is some difference is apparent from the fact that SU find 8 barns for the nucleon-nucleon total cross section at a total center-of-mass energy squared ($S$) of 13 GeV^2. If their calculation were valid, QCD itself would have proven false! But the SU calculation cannot be correct since it is well known that the contribution of such graphs in a renormalizable theory to the total cross-section cannot grow faster than $ln^2(S)$, and their cross-section grows much faster. In fact, our initial results grow only as $ln(S)$. Furthermore, we improved our calculation by including the logarithmic variation of the strong coupling with momentum transfer scale (which, to carry out the resulting integrations, required forcing an equality between the renormalization scale and effective gluon mass, or infrared cutoff). This causes the PVA to fall asymptotically as $ln(ln(S))/ln(S)$ despite the nonrenormalizable
weak vertex (which, by the way, limits the applicability of our results to $\sqrt{s} \lesssim 1$ TeV, where the effect of the W-boson propagator should become apparent), and so the SU result should fall even faster. We re-emphasize that their result does not fall, but rather increases rapidly with $S$.

We are unable to trace the source of the error, since they present only numerical results. Note, however, that their problem is reminiscent of similar ones in QED where gauge invariance has not been properly implemented. There, as here, a single graph at a given order can be larger than the sum, showing that there, as here, arbitrarily picking out one graph is completely unjustified. Our effectively single graph result for the weak PVA numerator came from examining all graphs to this order, and finding that, in that particular case, the rest were negligible, or vanished. It is clear this would not be the case for the QCD denominator.

Finally, we note that SU implicitly propose summation of the large coupling constant, divergent perturbation series for QCD. No such calculation would be credible. Even the operator-product type of analysis for summing leading logarithms to calculate strong interaction enhancements to the weak amplitude is subject to serious criticism, although its employment is standardly accepted. Rather, it is our experience that the leading term in a QCD calculation, with normal parameter values, gives a good representation of the physics in any given process, up to an overall strength factor. This is why we used the experimental value for the total strong-interaction cross-section, checking only that the leading graphs for this are consistent the experimental results for the (normally accepted) parameter values that we used in the rest of the calculation of the PVA.

5 A Proposal for an ‘Improved’ Calculation

Our calculation can, of course, be improved. We have recently realized that, at the cost of going to a level of complexity involving traces of products of eight $\gamma$-matrices, we can redo the quark-diquark scattering calculation in a different way. It requires modeling the diquark wavefunction, and we were initially reluctant to add such an ansatz. It will, however, allow us to calculate to one lower order in the strong interaction, since we can now use this wavefunction to provide for the overlap with the weak scattering amplitude. Thus, the strong interaction will appear only in the same order in the numerator and (conceptually) in the denominator of our PVA estimate. As we just argued in the previous section, a lower order calculation in QCD should be more reliable (at least, in that regard). Furthermore, such a calculation will provide a qualitative check on our result. From the structure of the integrals, we can see no reason for a markedly different result.

6 Effect of the QCD Spin Crisis

Our model is based on the heretofore conventional wisdom that all of the nucleon spin is carried by the valence quarks [13]. If the sea and gluons are highly polarized, then graphs for the weak amplitude which we have ignored, such as polarized gluon-gluon scattering, could become important. We would find this hard to credit except for one consideration: the two-phase vacuum model of confinement involves chromo-electric and -magnetic fields. These could carry significant spin, polarizing the sea quarks to produce a precise cancellation for an “empty” perturbative vacuum bubble. Introduction of polarized valence quarks would certainly disturb this cancellation, and it is precisely at small Bjorken $x$ where one would expect the largest effect. We speculate that this is related to high-$p_T$ polarization phenomena [16], becoming significant when the $p_T$ is large enough that the hard scattering involved occurred in one polarization region. However, this speculation and the effect of these considerations on the PVA require considerable additional effort before any conclusions can be drawn. Conversely,
the measured high energy PVA may be an important constraint for interpreting the results of such a theoretical study.

The situation may not be of too great concern, however, since our PVA contribution comes dominantly from harder scattering of quarks at higher Bjorken $x$. As Kunz, Mulders and Pollock have shown [17], the valence quark spin distribution is not significantly different from expectation. Indeed, they find that the entire measured result is not unreasonable, since any low $p_T$ quark model will evolve spin from the valence constituent quarks to the sea and gluons as $Q^2$ increases. Therefore, we do not expect this interesting development to invalidate our calculation or conclusions.

7 Conclusion

In summary, we have presented a simple model which, as for deep inelastic structure functions or Drell-Yan lepton-pair production, cannot supply an accurate prediction of the PVA at a given energy, but which should be valid for the (strong) energy dependence of the PVA at high energies. Due to approximations made in evaluating it, the model is not applicable above 1 TeV. Amazingly, it is consistent with data between 6 and 1.5 GeV/c, when the variation of the total nucleon-nucleon cross section between those beam momenta is taken crudely into account.

We predict that an experiment at Brookhaven should expect to find a PVA $\sim 10^{-5}$ and one at Fermilab, almost $10^{-4}$. Naturally, the prudent experimenter will design for an order of magnitude better sensitivity than these predictions, if possible. More importantly, perhaps, we have shown that, if PVA’s at these levels are not observed, one may interpret such a result as evidence for the existence of new, right-chiral weak interactions. Calculations are in progress to provide a firm numerical link between the value of (or upper bound on) the PVA at a given energy and the upper bound on the mass scale for the new weak vector boson. A confirmation of the PVA at somewhat lower energies is also needed.

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