Twenty Years of SUGRA

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Abstract. A brief review is given of the developments of mSUGRA and its extensions since the formulation of these models in 1982. Future directions and prospects are also discussed.

1 Introduction

Supersymmetry provides a technical solution to the so called gauge hierarchy problem in the form of a no renormalization theorem[4] which makes it an attractive candidate for model building. The main hurdle in the development of realistic models in the early days was the difficulty of breaking supersymmetry[2] in a phenomenologically viable manner[3]. The resolution of this problem arises in supergravity framework. In this paper we briefly summarize the developments over the last 20 years since the formulation of the minimal supergravity grand unified model[4,5] (mSUGRA) and its extensions including nonuniversalities to which we give the generic name: SUGRA models. The formulations of SUGRA models are based on techniques of applied supergravity where one couples gauge fields with matter fields \( z_i \) and then couples the combined system to \( N = 1 \) supergravity[6,5]. The coupled theory depends on three arbitrary functions: the gauge kinetic energy function \( f_{\alpha \beta}(z_i, z_i^\dagger) \), the Kahler potential \( K(z_i, z_i^\dagger) \), and the superpotential \( W(z_i) \). This allows one to construct grand unified models based on supergravity[7]. The central assumption of SUGRA models is that supersymmetry is broken in a so called hidden sector and the breaking is communicated by gravitational interactions to the visible sector where quarks, leptons and the Higgs fields reside[4]. The theory intrinsically contains large mass scales which include the Planck scale and the grand unification scale. It is then shown that the low energy theory that results after integration over the Planck scale and the GUT scale is free of both these high scales, i.e., the Planck scale and the GUT scale. The absence of the Planck scale from low energy theory was shown in the work of Chamseddine et al and Barbieri et al in Ref.[4] while the cancellation of the grand unification scale was shown in the work of Chamseddine et al and Hall et al in Ref.[4]. For the case of minimal supergravity unification which uses the flat Kahler potential \( (K = \sum z_i z_i^\dagger) \) the low energy theory results in just four soft breaking parameters which are the universal scalar mass \( m_0 \), the universal gaugino mass \( m_{\tilde{g}} \), the universal trilinear coupling \( A_0 \) and the universal bilinear couplings \( B_0 \). In addition, the low energy theory contains a Higgs...
mixing parameter $\mu_0$ which can arise in a variety of ways but its size is typically of the soft breaking scale. The universality of the soft parameters holds at unification scale and below this scale the soft parameters and the parameter $\mu$ evolve according to the renormalization group equations governing the gauge and Yukawa couplings and the soft parameters. In the development of mSUGRA models it was assumed that SUSY breaks in the hidden sector via some scalar fields developing vacuum expectation values. At the more fundamental level the breaking may arise via gaugino condensation with $\langle \lambda \gamma^0 \lambda \rangle \neq 0$. However, this phenomenon requires non-perturbative effects to occur and generally it is difficult to obtain explicit models where a satisfactory solution is achieved. Returning to SUGRA models, they have played a dominant role in the development of SUSY phenomenology and some of the early works are contained in Ref.\[14].

A remarkable aspect of SUGRA models is that they lead to the breaking of the electroweak symmetry, which is something that is rather ad hoc in the standard model. Further, an attractive mechanism for this is via renormalization group effects using renormalization group evolution. The radiative electroweak symmetry breaking solutions must be subject to the constraints of color and charge conservation. Under these constraints one minimizes the effective potential in the vicinity of the electroweak scale which leads to constraints on the vacuum expectation value of the Higgs fields. In the minimal supersymmetric standard model (MSSM) one has two Higgs fields, $H_i$ ($i=1,2$), which leads to two constraints which arise from the extremization conditions corresponding to these fields. One of these constraints can be used to eliminate $\mu$ (the value of $\mu_0$ at the electroweak scale) except for its sign while the second one allows one to eliminate the soft parameter $B_0$ in favor of $\tan \beta = \langle H_2 \rangle / \langle H_1 \rangle$, where $H_2$ gives mass to the up quark and $H_1$ gives mass to the down quark and the lepton. Thus the low energy theory can be described by only four parameters, i.e. the parameters $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$, and sign ($\mu$). This is to be contrasted with the large number of soft parameters one can add in MSSM consistent with the cancellation of the quadratic divergences. Historically local supersymmetry first arose in the form of supergravity in superspace called gauge supersymmetry. This formulation was a direct extension of Einstein gravity to superspace. The formulation of local supersymmetry directly in ordinary space was given in Refs.\[20,21].

The outline of the rest of the paper is as follows: In section 2 we discuss some of the signatures of SUGRA models which include a discussion of the phenomenon of scaling. We discuss the trileptonic signal and also emphasize the importance of the process $B_{0,d}^0 \rightarrow \mu^+ \mu^-$ as an important signal of SUSY and SUGRA for the case of large $\tan \beta$. In Sec.3 we discuss the current situation regarding the implications of the Brookhaven experiment on $a_\mu = (g_\mu - 2)/2$ for SUGRA models. It is argued that a SUSY contribution to $a_\mu$ of size $O(10^{-9})$ or larger implies upper limits for sparticle masses which should be accessible at the Large Hadron Collider (LHC). In Sec.4 we discuss the implications of CP phases arising from the soft breaking parameters becoming complex. In this section we
also discuss the constraints on CP phases from the electric dipole moment limits that exist for the electron, the neutron and for the atomic edm's such as $Hg^{199}$. Implications of large CP phases on low energy phenomena are also discussed. Of special interest are the phenomena involving the mixings of CP even and CP odd Higgs states induced by the CP phases via loop corrections to the Higgs masses. These mixings can lead to interesting effects in SUSY phenomena at colliders. In Sec.4 we also discuss the effects of CP phases on supersymmetric corrections to the $b$ quark mass and for the $\tau$ lepton mass, and the modifications of the Higgs decays $h \rightarrow b\bar{b}$, $\tau\bar{\tau}$, $c\bar{c}$ due to CP phases. An accurate measurements of these decays may reveal the effects of supersymmetry and of CP phases. In Sec.5 we discuss the hyperbolic branch (HB) of the radiative breaking in SUGRA models. This branch is characterized by the property that the soft parameters $m_0$ and $m_{1/2}$ lie on the surface of a hyperbola for fixed $\mu$ and thus can get large while $\mu$ remains fixed and small. It is also discussed in Sec.5 that the focus point (FP) region is a part of the hyperbolic branch corresponding to relatively low values of $m_{1/2}$. The hyperbolic branch naturally leads to heavy squarks and gluinos but it is shown in Sec.6 that this branch may still allow for sufficient relic density to be consistent with the current astrophysical data. In Sec.7 the current status of SUGRA GUTs is discussed. Also discussed is the relationship of SUGRA to strings. Conclusions are given in Sec.8. In this paper we do not discuss other developments such as the no scale supergravity\textsuperscript{[22]} and the gauge mediated breaking of supersymmetry\textsuperscript{[23]}. Indeed the literature following the development of SUGRA models is enormous and it is physically not possible to do justice to reviewing it in a conference talk. Thus although the bibliography looks extensive there is no claim it is complete as it is only a fraction of the existing literature. Further, we limit ourselves in this talk to only a few topics of current interest and many other interesting topics within SUGRA are not discussed.

2 SUGRA Signatures

An important hint for supersymmetry comes from the LEP data in terms of precise values for the gauge couplings constants and the fact that unification occurs within SUSY/SUGRA unified models\textsuperscript{[24,25,26]}. We note that the threshold corrections from the sparticle masses enter crucially in the gauge coupling unification and provide an indirect support for SUGRA. The analyses involve evolution of gauge and Yukawa couplings from the GUT scale to low energy using one and two loop renormalization group equations of the gauge and Yukawa couplings\textsuperscript{[10,27]} and of the soft parameters\textsuperscript{[11,28]}. Further, more accurate determinations of the sparticle masses at low energy require that the minimization of the effective potential include loop corrections\textsuperscript{[29,30,31]}. In the Higgs sector such loop corrections turn out to be crucial. Thus in mSUGRA one has the tree relation that the lightest Higgs mass should lie below the Z boson mass\textsuperscript{[5]}. However, this relation is modified by loop corrections which lift the lightest Higgs mass significantly above $M_Z$\textsuperscript{[32]}. The parameter space of SUGRA model is limited by experimental and theoretical constraints. One of these is the constraint of the
flavor changing neutral current process \( b \rightarrow s + \gamma \) which limits the parameter space depending on the \( \mu \) sign\(^{33}\).

An interesting phenomenon exhibited by renormalization group analyses is that of scaling\(^{33}\). It arises in regions of the parameter space where \( \mu \gg M_Z \) (and \( m_0, \tilde{m}_g \leq 1 \) TeV) and leads to interesting relation on the gaugino masses of the type \( m_{\tilde{\chi}^0_1} \simeq 0.5 m_{\tilde{\chi}^0_2}, m_{\tilde{\chi}^0_2} \simeq m_{\tilde{\chi}^0_3}/3, m_{\tilde{\chi}^0_0} \simeq m_{\tilde{\chi}^+_{1}}, m_{\tilde{\chi}_0} \simeq m_{\tilde{\chi}_3} \simeq \mu \) as well as interesting relations among the other sparticle masses. Many of the mSUGRA mass relations can be appropriately expressed as sum rules which can be put to test when one has a precise measurement of the sparticle masses\(^{34,35}\). One consequence of the RG analysis is that the lightest neutralino turns out to be the lightest supersymmetric particle (lsp) over most of the parameter space of the model. Further, in the region where scaling holds one finds that the lightest neutralino is in fact mostly a Bino\(^{34,36}\). This result has important implications for supersymmetric dark matter in SUGRA models with R parity conservation.

Further, the R parity conservation constraint leads to interesting missing energy signals in sparticles decays. One such signal is the trileptonic signal. It was noted early in the investigation of SUGRA models that the \( W^\pm \) decay into a chargino and the second lightest neutralino, i.e., \( \chi^+_1 + \chi^0_2 \) with the subsequent decays of the \( \chi^+_1 \) and \( \chi^0_2 \) can lead to a clean trileptonic signal\(^{14}\) and further work was carried out in Ref.\(^{37}\). In Ref.\(^{38}\) it was observed that the decays from an off-shell \( W \) can extend very significantly the potential for the discovery of the chargino \( \chi^+_1 \). A recent analysis of this process can be found in Ref.\(^{39}\). Another process which has been the subject of recent studies is the decay \( B^0_s \rightarrow \mu^+\mu^- \).

In the standard model the branching ratio for this process is rather small, i.e., \( Br(B^0_s \rightarrow \mu^+\mu^-) = (2.1 \pm 1.4) \times 10^{-9} \) (for \( V_{ts} = 0.04 \pm 0.002 \)) and beyond the reach of experiment in the near future. Thus the current experimental limit for this process is \( Br(B^0_s \rightarrow \mu^+\mu^-) < 2.6 \times 10^{-6} \) and one estimates that a branching ratio down to the level of \( 10^{-8} \) may be achievable at RUNII of the Tevatron. However, a check on the standard model branching ratio \( Br(B^0_s \rightarrow \mu^+\mu^-) \) would not be feasible even with this enhanced sensitivity. In supersymmetry the so called counterterm diagram is proportional to \( \tan^6 \beta \) for large \( \tan \beta \)\(^{40}\). Detailed numerical analyses show that indeed one expects a big enhancement of order \( 10^2 \) in some parts of the parameter space in SUGRA models\(^{11,12,13,14,15,16}\). Thus the process \( Br(B^0_s \rightarrow \mu^+\mu^-) \) is a strong indicator of supersymmetry and in fact an observation of this process will be a pointer to the existence of sparticles even before the sparticles are seen. There are a variety of other signals which have been discussed in the literature and the reader is directed to Ref.\(^{47}\) for a more comprehensive survey, to Refs.\(^{39,48,49}\) for more recent constraints on mSUGRA, to Refs.\(^{50,51,52}\) for more recent surveys, and to Refs.\(^{53,54}\) for the discovery potential of SUSY/SUGRA at ATLAS and CMS.

### 3 \( g_\mu - 2 \) and SUGRA

\( g_\mu - 2 \) is one of the most sensitively determined quantities in all of physics. The precision of this determination has further increased due to the recent
The standard model correction consists of several parts: the QED correction, the hadronic correction and the electroweak correction. Of these the QED correction and the electroweak correction are reliably determined and the largest source of error in the theoretical analysis arises from the hadronic correction. The hadronic correction consists of the leading order (LO) and the nonleading order hadronic (NLO) contributions and the light-by-light hadronic contribution. The NLO correction is well understood. However, the LO correction has been the subject of much scrutiny. Similarly, the light-by-light hadronic correction has seen a flip in its sign and is still a subject of some debate. A very recent estimate of the difference between experiment and theory gives \( \Delta a_{\mu} = (a_{\mu}^{\exp} - a_{\mu}^{SM}) \sim (33 \pm 10) \times 10^{-10} \) which is about a 3σ effect. However, this difference is likely to change with more accurate determinations of the LO hadronic correction, and with more data expected from Brookhaven. Now it was predicted already nearly twenty years ago that the supersymmetric electroweak correction should be of the same size as the standard model electroweak correction and that any experiment that tests the standard model electroweak correction will in fact also test the supersymmetric electroweak correction. Further, it was later pointed out that the sign of susy correction to \( g_{\mu} - 2 \) is the sign of \( \mu \) in a large part of the parameter space. An important issue concerns if extra dimensions might provide a strong background to the supersymmetric effects. That is to say if the corrections to \( g_{\mu} - 2 \) from the exchange of Kaluza-Klein states might produce a large enough correction which might mask the supersymmetric effect. This analysis was carried out in Ref. The analysis showed that using the current lower limits on the size of extra dimensions one finds that the corrections from the extra dimensions do not produce a serious background to the supersymmetric correction. Thus after the results of the Brookhaven experiment in the year 2001 which showed a 2.6σ effect there was a lot of theoretical activity to understand the implications of the results in the context of supersymmetry. One of the major consequences that emerged from these analyses was the result that the Brookhaven result implied the existence of upper limits on sparticle masses which appeared to lie within reach of the LHC. The Brookhaven result of 2002 is essentially consistent with its previous determination. However, in the meantime the theoretical evaluations of \( \Delta a_{\mu}^{SM} \) have changed due to reevaluations of the leading order hadronic correction and the light-by-light hadronic correction. While the new evaluations still indicate a significant effect, the situation is in a state of flux since the issue of hadronic corrections to \( a_{\mu}^{SM} \) is not fully settled and there is more data to come from Brookhaven.

4 CP Phases

The minimal supergravity model mSUGRA can be extended to have two phases which can be taken to be the phase of \( \mu_0 \) and the phase of the trilinear coupling \( A_0 \). The main problem encountered with the inclusion of phases is that there are severe experimental constraints on them, e.g. from the electron edm.
from the neutron edm\cite{71}. Theoretically there are many contributions to the electric dipole moment of an elementary particle. Thus while for the electron the contribution to the electric dipole moment arises from the electric dipole operator, for the quarks it arises from the electric dipole operator, from the chromo electric dipole operator and from the dimension six operator \cite{72}. These contributions are computed at the weak scale and evolved down to the low scales where the experimental measurements of these are given. In extracting contributions from the chromo electric dipole operator and from the dimension six operator one uses the so called naive dimensional analysis\cite{73}. Now typically phases O(1) tend to give edm contributions which are already in contradiction with current experiment\cite{70,71}. There are a variety of ways that have been discussed in the literature for controlling these edms. These include fine tuning to make the phases small\cite{74}, suppression of the edms by heavy masses\cite{75}, suppression of phases in a class of Left-Right symmetric models\cite{76}, and suppression by the cancellation mechanism\cite{77,78}. Additional ways for the suppression include putting the phases in the third generation\cite{79}. Further, the experimental limits on the atomic edms arising from Schiff moment and specifically the experimental atomic edm of $Hg^{199}$\cite{80} also impose important constraints on model building\cite{81,82}. Since a broad class of SUGRA and string models constrain soft breaking parameters with large phases, the cancellation mechanism is specially suited for these scenarios. We explain, therefore, in some detail how the cancellation mechanism by which the edms are reduced works. Consider, for example, the electric dipole moment of the electron which receives contributions only from the electric dipole operator. However, the supersymmetric contribution to this operator includes contributions from the exchange of two charginos and four neutralinos. In certain regions of the parameter space these contributions have opposite signs and naturally cancel reducing the edm below the experimental limit. For the quarks the situation is more complex. As mentioned above the quark edm receives contributions from the electric dipole, the chromoelectric dipole and the purely gluonic dimension six operator. Here the electric dipole operator and the chromoelectric dipole operators receive contributions from the exchange of the gluino, the charginos, and the neutralinos and thus there are even more possibilities for cancellations. Similarly for the dimension six operators one has contributions arising from the exchange of stops and sbottoms. In addition to the cancellations that can occur within each individual operator from gluino, chargino and neutralino exchanges, one has an additional possibility for cancellations for the quark case not available for the electron electric dipole moment, i.e., one may have cross cancellations among the electric dipole, the chromoelectric dipole, and the purely gluonic dimension six operators.

Such cancellations are further facilitated in the nonuniversal SUGRA model (nSUGRA). In nSUGRA one may give nonuniversal gaugino masses as well as nonuniversalities in the Higgs sector and in the third generation sector consistent with FCNC. We focus here on the gaugino sector. In this case one may have independent gaugino masses for the $SU(3)$, $SU(2)$ and $U(1)$ sector gaugino masses $\tilde{m}_i$ so that $m_i = |\tilde{m}_i| e^{i\xi_i}$, $(i=1,2,3)$. We note in passing that the physical quanti-
ties such as the edm's depend only on certain combinations of phases which have been classified in Ref.[83]. Even so the appearance of a larger number of phases allows a larger region of the parameter space for the cancellation mechanism to operate. The central point of the cancellation mechanism and of other mechanisms is that they allow the phases to be large and one can still satisfy the edm constraints. Now if the phases are large they will affect a variety of low energy phenomena. These include effects on Higgs phenomenology[84,85], on sparticle phenomenology[86,87], on flavor and B physics[88,89], on $B_{s,d} \to \mu^+\mu^-$[40], on $g_\mu - 2$[40], and on proton decay[91]. CP phases also affect loop corrections to the $b$ quark mass and the $\tau$ lepton mass[92]. It is known that the SUSY effects can produce large corrections to the $b$ quark mass for large $\tan \beta$[93]. It was found in Ref.[92] that CP effects on these can also be large. Similarly supersymmetric effects produce important corrections to the Higgs decays to $b\bar{b}$, $\tau\bar{\tau}$ and $c\bar{c}$[94]. Here also one finds that CP phases can produce large corrections to these decays of the Higgs bosons[95].

5 Hyperbolic Branch/Focus Point (HB/FP)

It is now known that there are two branches to the radiative breaking of the electroweak symmetry, an ellipsoidal branch and a hyperbolic branch[96]. These arise due to two solutions of $\mu$ using radiative symmetry breaking equation that determines $\mu$, i.e., $C_1 m_{1/2}^0 + C_3 m_{1/2}^2 + C_2 A_0^2 + \Delta \mu_{loop}^2 = \mu^2 + \frac{1}{2} M_Z^2$, where $m_{1/2} = m_{1/2}^{\prime} + \frac{1}{2} A_0 C_1/C_3$, and $C_1$ etc are determined purely in terms of gauge and Yukawa couplings and $\Delta \mu^2$ are the loop corrections. These loop corrections play an important role in the analysis. For small to moderate values of $\tan \beta$ the loop corrections are relatively small. Also from the renormalization group analysis one finds that the co-efficients $C_2^\prime$, $C_3$ are positive. In these cases the scale dependence of $C_1$ is relatively small and one finds $C_1 > 0$ for a range of scales $Q$ where the radiative electroweak symmetry breaking is realized. In this case one finds that the soft parameters for fixed $\mu$ lie on the surface of an ellipsoid. Now for larger values of $\tan \beta$, i.e., typically $\tan \beta > 5$ one finds that the loop corrections to $\mu$ become large. Further, for this case one also finds a rather significant variation in this correction with the scale $Q$ and also a significant variation of $C_1$ with the scale. The implications of this scale dependence can be seen by choosing a scale $Q_0$ at which the loop corrections to $\mu$ are minimized. At this scale one then finds that sign($C_1(Q_0)$)=-1. One immediately sees that the implication of this result is to change the nature of the radiative symmetry breaking equation above from an ellipsoidal to a hyperbolic constraint. The choice of $Q_0$ in the discussion above is for illustration purposes only and the phenomenon discussed above would occur for any $Q_0$ in the region of the electroweak symmetry breaking. Now the parameter $\mu$ can also be regarded as the fine tuning parameter[96] of the theory. Though there are by now several different criteria of what constitutes fine tuning[97] the parameter $\mu$ provides the simplest criterion. This parameter is especially suitable for interpreting the implications of the hyperbolic branch vs the ellipsoidal branch. Thus for fixed $\mu$ and hence for a fixed fine tuning, one
finds that the ellipsoidal branch of radiative breaking puts upper bounds on $m_0$ and $m_{1/2}$ while the hyperbolic branch does not. In the latter case one finds that $m_0$ and $m_{1/2}$ could lie in the several TeV region consistent with a small $\mu$.

If indeed the hyperbolic branch is realized masses of some of the supersymmetric particles, specifically the squarks, the gluino, the heavier neutralinos, the heavier chargino, and the heavy Higgs, could be very large. In this scenario the lightest particles will be the light Higgs and $\chi_{1,2,3}^{0,\pm}$. Typically for large $m_0$ and $m_{1/2}$ the pattern of masses for $\chi_{1,2}^{0,\pm}$ is given by $m_{\chi_1^0} < m_{\chi_1^\pm} < m_{\chi_2^0}$ at the tree level but loop corrections here may be significant\[88,89\]. In this scenario the mass differences $\Delta M^\pm = m_{\chi_1^\pm} - m_{\chi_1^0}$ and $\Delta M^0 = m_{\chi_2^0} - m_{\chi_1^0}$ are typically $\mathcal{O}(10)$ GeV. Thus the usual strategies for identification of supersymmetry in this region does not work and one must follow other strategies for identification of such particles\[100,101,102\]. However, it was argued in Ref.\[103\] that observation of supersymmetric dark matter is still possible even in this region. This comes about because as $m_0$ and $m_{1/2}$ get large for fixed $\mu$, $\chi_{1,2,3}^{0,\pm}$ move from being mostly gaugino like to mostly higgsino like. We will discuss more on this in the section on dark matter below. A part of the hyperbolic region corresponds to the so called focus point region (FP)\[104\]. As in the hyperbolic branch (HB) the focus point region also corresponds to a small $\mu$. However, the focus point case corresponds to that part of the hyperbolic branch for which $m_{1/2}$ is relatively small. As a result $m_0$ is also constrained to get not too large. Still values of $m_0$ in several TeV region can be gotten in this part of the hyperbolic branch. Thus the focus point region is contained in the hyperbolic branch and corresponds to the low end of the $m_{1/2}$ region on this branch (see also Refs.\[105,103\] in this context). Of course if the $g_\mu - 2$ experimental difference at the level currently seen continues to persist then a significant part of HB/FP region will be eliminated.

6 Dark Matter in SUGRA

Soon after the formulation of mSUGRA it was realized that the lightest neutralino with R parity conservation could be a candidate for dark matter\[106,107\]. While this was originally just a possibility a concrete realization of this possibility occurs when one carries out renormalization group analyses on sparticle masses in mSUGRA models and one does indeed produce the light neutralino as the lsp over a significant part of the parameters space. Further, as pointed out in Sec.2 in SUGRA models theoretical analyses show that for regions of the parameter space where $\mu \gg M_Z$ ($m_0, m_{\tilde{g}} \lesssim 1$ TeV), the lightest neutralino is a Bino\[34,36\]. Of course, as discussed above there are other regions (HB/FP) where the lightest neutralino would be mostly a higgsino. We discuss now some salient features of the analyses of supersymmetric dark matter. First one needs to check if the density of relic neutralinos falls within current limits given by the astrophysical observations. The quantity of interest is $\Omega_{CDM} h^2$ where $\Omega_{CDM} = \rho_{CDM} / \rho_c$ where $\rho_{CDM}$ is the mass density of cold dark matter in the universe and $\rho_c$ is the relic density needed to close the universe, and $h$ is the Hubble parameter in units of $100$ km/s/Mpc. The most recent data from the Wilkinson Microwave
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Anisotropy Probe indicates the result $\Omega_{CDM} h^2 = 0.1126^{+0.008}_{-0.009}$. Before we discuss the implications of this highly accurate determination for supersymmetry, we continue first with our general line of discussion. The analysis of the relic density is quite intricate in that the annihilation of the relic neutralinos can result in many final states, such as $f\bar{f}, WW, ZZ, Zh$ etc with the number of final states included depending on the mass of the relic neutralino. The analysis of the relic density involves the thermal averaged quantity $<\sigma_{eff}v>$ where $\sigma_{eff}$ is the neutralino annihilation cross section and $v$ is the relative velocity of the annihilating neutralinos.

In general thermal averaging will involve integrating over the Breit-Wigner poles which is somewhat of a delicate procedure. It turns out that the analysis of relic density is also significantly affected by the phenomenon of coannihilation. The quantity of interest in these analyses is the number density $n = \sum n_a$ where the sum runs over all the particle types that coannihilate and $n$ obeys the Boltzmann equation $\frac{dn}{dt} = -3Hn - <\sigma_{eff}v>(n^2 - n_0^2)$ where $H$ is the Hubble parameter and $n_0$ stands for the equilibrium number density while $\sigma_{eff} = \sum \sigma_{ab}r_{ab}$ where $\sigma_{ab}$ is the annihilation cross section of particles $a$ and $b$ and $r_{ab} = n_{0a}/n_0$ with $n_{0a}$ the density of particles of species $a$ at equilibrium. In the coannihilation process after the freeze out the next to the lowest supersymmetric particles (nlsps) decay to the lsp and thus $n$ becomes the number density of the lsp. The importance of coannihilation arises from the fact that it extends considerably the allowed region of the parameter space where the relic density constraints can be satisfied. Thus without coannihilation the allowed range of the neutralino mass where the relic density constraint can be satisfied extends typically to around 150-200 GeV in mSUGRA. However, with the inclusion of coannihilation the allowed range of the neutralino mass can extend up to around 700 GeV. In this case the processes that enter in the relic density analyses are $\tilde{\chi}_a^0 \tilde{\chi}_b^0 \rightarrow \ell^a \gamma, \ell^a Z, \ell^a h$, $\tilde{\tau}_R^a \tilde{\tau}_R^b \rightarrow \ell^a \ell^b$, and $\tilde{\ell}^a_{R\ell} \tilde{\ell}^a_{R\ell} \rightarrow \ell^a \ell^b \gamma \gamma, \gamma Z, ZZ, W^+ W^-, hh$. The most important coannihilation channel here turns out to be the one involving the staus.

Theoretical analyses over more than a decade and a half have investigated both the indirect and the direct detection of dark matter and a number of phenomena have been studied. These include the effects of nonuniversalities in the higgs sector and in the third generation sector and the gaugino sector can enter via non-singlet representations in the decomposition of the neutralino mass matrix. Thus, for example, for SU(5) the gaugino mass matrix transforms in general like the symmetric product $24 \times 24$ which in its decomposition contains the SU(5) representations $1,24, 75, 200$. The assumption of only the singlet
leads to universal gaugino masses while a non-vanishing contribution from the non-singlet parts will lead to nonuniversalities. A similar situation occurs for the case of SO(10) where the gaugino mass matrix transforms like the symmetric product $45 \times 45$ and contains the representations $1, 54, 210$ and $770$. The gaugino mass nonuniversalities affect very significantly the allowed range of the neutralino mass over which the relic density constraints can be satisfied. Further, the direct detection rates are also affected. Similarly, the effect of CP violation on dark matter analyses turn out to be important. The situation regarding the imposition of the Yukawa unification constraint and specifically the $b-\tau$ unification constraint is interesting. It is well known that $b-\tau$ unification requires a negative contribution to the $b$ quark mass. Now roughly the correction to the $b$ quark mass depends directly on the sign of $\mu$ and thus a negative contribution to the $b$ quark mass indicates that a negative sign of $\mu$ is preferable. However, the BNL experiment appears to indicate that the sign of $\mu$ is positive. Thus these two results appear to be in conflict. However, a closer scrutiny reveals that the sign of the $b$ quark correction depends on the sign $\mu m_{\tilde{g}}$ while the sign of $g_{\mu} - 2$ correction is controlled by the the chargino exchange and hence depends on $\mu$ and the $SU(2)$ gaugino mass $\tilde{m}_2$. Thus one obvious solution presents itself, i.e., that the signs of $m_{\tilde{g}}$ and $\tilde{m}_2$ are opposite. Specifically, choosing a negative sign for $\mu m_{\tilde{g}}$ resolves this conflict. The opposite correlation of $m_{\tilde{g}}$ and $\tilde{m}_2$ arises naturally if the gauginos belong to the 24plet representation for the $SU(5)$ case and to the 54plet representation for the $SO(10)$ case.

We turn now to a discussion of dark matter on the hyperbolic branch. It is quite interesting that on the hyperbolic branch we can satisfy the relic density constraints even though much of the sparticle spectrum is rather heavy. The satisfaction of the relic density constraints here arises once again due in part to the inclusion of coannihilation which arise because of the near degeneracy of $\chi_1^0$, $\chi_2^0$ and $\chi_1^\pm$. The coannihilation involving these particles lead to processes of the type $\chi_1^+ \chi_1^- \rightarrow u_i \bar{u}_i, d_i \bar{d}_i, W^+ W^-$ and $\chi_1^0 \chi_1^\pm \rightarrow u_i \bar{d}_i, \bar{e}_i \nu_i, AW^+, ZW^+, W^+ h$. The channel that dominates the coannihilation is the one that involves the particle which has the smallest mass difference with the lsp which in this case is between $\chi_1^+$ and $\chi_1^0$. As pointed out earlier the neutralino in the hyperbolic region is mostly a higgsino and this structure tends to enhance the neutralino-proton cross section. We return now to the constraint of the WMAP constraint. There have been several analyses recently to explore the implications of this constraint. One important result that emerges is that the new data limits more severely the parameters space of models. Quite interestingly HB/FP region is consistent with the WMAP constraint. Further, the WMAP constraint produces neutralino-proton cross sections that lie within range of the current and future dark matter experiments.
7 SUGRA, GUTS and Strings

Supergravity grand unification much like SUSY GUTS generates baryon and lepton number violation whose nature and strength is controlled by the nature of the grand unification group. Theories based on SU(5), SO(10), E(6) gauge groups generate below the grand unification scale baryon and lepton number violating dimension five operators with chiral structures LLLL and RRRR [144,145]. When dressed with the full set of chargino, gluino and neutralino exchange diagrams the dimension five operators produce dimension six operators with chiral structures LLLL, LLRR, RRLL and RRRR which can decay the proton [144,145] and a similar situation exists also in string models [146]. A detailed analysis of proton decay, however, is rather intricate and depends on both the high and the low energy structures of the theory. Thus the lifetime of the proton can be significantly affected by the soft breaking sector of SUGRA GUTS and by the Higgs triplet structure [147] and specifically by the textures in the Higgs triplet sector [145]. The most recent limits on the proton lifetime appear to disfavor the minimal SU(5) model [149] (see, however, Ref. [150]). Regarding SO(10) there are a whole variety of possible SO(10) models and so there are no necessarily definitive SO(10) predictions since the proton decay modes are highly model dependent [151,152,153,154]. An important issue concerns the role of large representations such as 120 and 126. The appearance of such representations can significantly affect analyses of proton decay [154] and of neutrino masses [155].

We turn now to a brief discussion of the connection of SUGRA models and strings. Since SUGRA models are derived from models involving supergravity and supergravity may be viewed as a low energy limit of string theory below the Planck scale, it is natural to imagine SUGRA arising as a low energy limit of a string model. There are two elements involved in such a connection. First one must try to deduce a realistic model with a standard model gauge group from string theory and efforts have been in this direction from the very beginning [156,157]. Second one must try to obtain a realistic breaking of supersymmetry from strings and there has some been progress also along these lines [13,158] specifically using dualities [159,160]. More recently the constraints of modular invariance on soft breaking have been investigated to make contact with low energy phenomenology [161]. An interesting issue concerns the constraints needed in modular invariant theories to derive universality of soft parameters and in Ref. [162] some dynamical constraints to achieve universality of soft parameters were identified. Thus while we do not yet have a fully realistic string model it is interesting that one can still make tentative contact between supergravity based models and string theory.

8 Conclusion

The advent of SUGRA models in 1982 spurred an activity in supersymmetry phenomenology that still continues. Historically it was only within the framework of SUGRA models that a phenomenologically consistent spontaneous breaking of
supersymmetry was first achieved. The basic concept of supersymmetry breaking in one sector and its communication to the physical sector also introduced first in SUGRA finds applications in string based scenarios. Further, SUGRA models with R parity predict the existence of cold dark matter (CDM), something that appears desirable from astrophysical considerations. The literature on mSUGRA, its extensions and their implications is enormous and a comprehensive review of the developments is obviously outside the scope of a conference talk. Thus we have focussed on a few topics of current interest. One of these topics concerns the difference \( \Delta a_\mu = (a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}}) \). There are several estimates of this quantity which differ mainly due to the different estimates of the leading order hadronic correction. According to the recent analysis of Ref. [59] this difference is \((33 \pm 10) \times 10^{-10}\) which amounts to about a \(3\sigma\) deviation between experiment and theory. An effect of this size is expected within SUGRA models since it was noted early on [65] that the size of the supersymmetric electroweak correction could be as large or larger than the standard model electroweak correction. However, the theoretical evaluations of the hadronic error are still in a state of flux and \(\Delta a_\mu\) is likely to shift before it settles down. However, we note that if a value of \(\Delta a_\mu\) persists at a perceptible level, i.e., \(\sim 10^{-9}\) then the sparticle mass limits lie within reach of the LHC and the direct observation of new physics is implied. Thus most of the sparticles (\(\tilde{g}, \tilde{q}, \tilde{W}, \chi^0, \ldots\)) should become visible at the LHC. Another interesting aspect of SUGRA models is that renormalization group analyses show that in a large part of the parameter space the sign of \(a_{\mu}^{SU3Y}\) is correlated with the sign of \(\mu\). Thus the current analyses on \(\Delta a_\mu\) imply a positivity of the \(\mu\) sign. A positive \(\mu\) is very desirable for the satisfaction of the \(b \rightarrow s + \gamma\) constraint and also for the observation of supersymmetric dark matter.

mSUGRA is consistent with the flavor changing neutral current constraints. However, it is possible to extend mSUGRA to include non-universalities by the assumption of a non-flat Kahler potential and a non-flat gauge kinetic energy function. These extensions allow one to include nonuniversalities in the Higgs sector, in the third generation sector and in the gaugino masses consistent with the FCNC constraints. Further, SUGRA models allow the soft breaking parameters to become complex in general. Thus mSUGRA allows up to two phases in the soft breaking sector while more phases can appear when one includes nonuniversalities. However, the inclusion of phases requires strong consistency checks with the current very sensitive limits on the electron and the neutron edms. Additionally, the atomic edms generated via the Schiff moments also constrain phases. This is the case specifically for the atomic edm of \(Hg^{199}\). Typically phases O(1) will violate these constraints unless a mechanism is invoked for their suppression. Several mechanisms for such suppressions have been discussed in the literature. One mechanism which leads to a natural suppression in certain regions of the parameter space of SUGRA models is the cancellation mechanism and there are many works exploiting this technique to allow for large phases. However, whatever mechanism is employed for the suppression of the edms, the presence of large phases typically has large effects on supersymmetry phenomenology. One
of the most dramatic effects occurs in the Higgs sector where the Higgs mass eigenstates are no longer CP even and CP odd states but rather admixtures of CP even and CP odd states. This mixing will lead to rather dramatic effects in SUSY phenomena at \( e^+e^- \) colliders and elsewhere. Further, the inclusion of phases produces important effects on SUSY corrections to the Higgs mass, on SUSY corrections to the b quark and \( \tau \) lepton masses and on Higgs decays to \( b\bar{b} \), \( \tau\bar{\tau} \) and \( c\bar{c} \). Thus, for example, the decay branching ratio of the Higgs to \( b\bar{b} \) will carry signatures of both supersymmetry and CP phases. Another process affected strongly by phases is the decay \( B^{0}_{s,d} \to \mu^+\mu^- \). The branching ratio for this process in the standard model is too small to be accessible to experiment in RUNII of the Tevatron. However, in SUGRA models the branching ratio for this process can be enhanced by as much as a factor of \( 10^2 \) for large \( \tan\beta \). Further, the inclusion of CP phases can produce additional enhancements which can be as large as another factor of \( 10^2 \). These enhancements put the the \( B^{0}_{s,d} \to \mu^+\mu^- \) branching ratio within reach of the Tevatron. Thus the observation of this process will be a strong hint for supersymmetry pointing to the existence of sparticles even before the sparticles are directly observed.

The existence of supersymmetric dark matter is an important prediction of SUGRA models with R parity invariance. Detailed analyses of the density of the relic neutralinos indicate that the predictions of mSUGRA and its extensions allow for consistency with the most recent determinations of \( \Omega_{\text{CDM}}h^2 \) from the WMAP data. Further, the relic density limits from WMAP more sharply constrain the sparticle spectrum and define more sharply the allowed ranges of the spin independent and spin dependent neutralino-proton cross sections. In this talk we have also reviewed the hyperbolic branch of the radiative breaking of the electroweak symmetry. A part of this branch allows for large values of \( m_0 \) and \( m_{1/2} \) for a fixed value of \( \mu \) and puts most of the sparticle in this region in the several TeV region. However, quite interestingly this region still produces a relic density consistent with the WMAP constraints and leads to scalar and spin dependent neutralino-proton cross sections which appear to be within reach of the future dark matter experiments such as GENIUS and ZEPLIN. Proton decay in SUGRA GUTs depends on two elements, on the sparticle spectrum and on the GUT group. Unlike the sparticle masses which in mSUGRA are essentially independent of the GUT structure as pointed out in section 1, proton decay hinges critically on the GUT structure. Thus this sector of the theory is more model dependent. While models do exist where one can make consistent the GUT theory with the current proton decay limits, there is not a uniqueness in fixing the GUT structure. Fortunately, the low energy predictions of SUGRA models are independent of the dimension 5 operators and thus the SUGRA predictions are not affected by issues related to the proton lifetime. Finally, we note that SUGRA models have gravity as an intrinsic piece of their fabric and have good chance of making contact with string theory. Thus more effort is needed to derive SUGRA, mSUGRA and other competing models from a top down approach. More than 20 Years after its invention SUGRA is still a leading candidate for new physics beyond the SM. Experiment is awaited to check the predictions of this model in
the laboratory.

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