Analysis of the cylinder’s movement characteristics after entering water based on CFD

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Abstract. It’s a variable speed motion after the cylinder vertical entry the water. Using dynamic mesh is mostly unstructured grid, and the calculation results are not ideal and consume huge computing resources. CFD method is used to calculate the resistance of the cylinder at different velocities. Cubic spline interpolation method is used to obtain the resistance at fixed speeds. The finite difference method is used to solve the motion equation, and the acceleration, velocity, displacement and other physical quantities are obtained after the cylinder enters the water.

1. Introduction
Buoys, beacons and other objects are dropped into the water by helicopters, and their velocity variation in the water is non-uniform. At present, the two-dimensional problem can be directly calculated by dynamic mesh method, and even simulating its rotation process. However, the calculation using unstructured grid, the credibility of the results is doubtful. For the three-dimensional case, the dynamic mesh method consumes more computer resources, and time-consuming, and the results are not necessarily right.

This study focuses on the movement of the cylinder vertical entry into the water, without considering the rotation of the cylinder. In order to taking advantages of CFD, using CFD method to calculate the cylinder resistance at fixed speed, and then using interpolation method to calculate the resistance at different speeds.

2. Principle of curve interpolation
Curve interpolation usually has exponential function interpolation, Lagrange interpolation, Cubic spline interpolation, B spline curve, NURBS curve and so on.

2.1. Cubic spline curve interpolation
There, \( a = x_0 < x_1 < \cdots < x_n = b \), if function \( y(x) \) satisfying two condition:

(a) \( y(x) \) is Cubic Polynomial on each interval \([x_{i-1}, x_i], i = 1, 2, \cdots, n;\)

(b) \( y(x) \) is twice continuously differentiable function on interval \([a, b]\).

Then, \( y(x) \) is Cubic spline on interval \([a, b]\). There, \( x_i (i = 0, 1, \cdots, n) \) is the node. The interpolation points are shown in Table 1.
Table 1. Interpolation points

| $x$ | $x_0$ | $x_i$ | $\ldots$ | $x_n$ |
|-----|------|------|--------|------|
| $y$ | $y_0$ | $y_i$ | $\ldots$ | $y_n$ |

Cubic spline interpolation function:

$$
\begin{align*}
    s(x) &= \frac{h_k}{h_k^3} + \frac{2(x-x_k)}{h_k^3}(x-x_{k+1})^2 y_k + \frac{h_k}{h_k^3} - \frac{2(x-x_{k+1})}{h_k^3}(x-x_k)^2 y_{k+1} \\
    &+ \frac{(x-x_k)(x-x_{k+1})^2}{h_k^2} m_k + \frac{(x-x_{k+1})(x-x_k)^2}{h_k^2} m_{k+1} \\
\end{align*}
$$

(1)

In equality (1),

$$
    h_k = x_{k+1} - x_k \quad (k = 0, 1, \ldots, n - 1)
$$

(2)

$$
    \lambda_k m_{k-1} + 2m_k + \mu_k m_{k+1} = g_k \quad (k = 1, 2, \ldots, n - 1)
$$

(3)

Where,

$$
\lambda_k = \frac{h_k}{h_k + h_{k-1}}, \mu_k = \frac{h_{k-1}}{h_k + h_{k-1}}, g_k = 3(\mu_k \frac{y_{k+1} - y_k}{h_k} + \lambda_k \frac{y_k - y_{k-1}}{h_{k-1}})(k = 1, 2, \ldots, n - 1)
$$

3. Basic theory of fluid mechanics

3.1. Control equation

The continuity equation and momentum equation for incompressible viscous fluid are:

$$
\frac{\partial \bar{u}_i}{\partial x_i} = 0
$$

(4)

$$
\rho \frac{\partial \bar{u}_i}{\partial t} + \rho \bar{u}_j \frac{\partial \bar{u}_i}{\partial x_j} = \rho F_i - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} (\mu \frac{\partial \bar{u}_i}{\partial x_j} - \rho \bar{u}_i \bar{u}_j)
$$

(5)

Where, $\rho$ is density, $\mu$ is Viscosity Coefficient, $\bar{p}$ is average pressure, $F_i$ is external force, $\bar{u}_i$ is mean velocity, $u_i$ is fluctuating velocity, $-\rho \bar{u}_i \bar{u}_j$ is Reynolds stress.

3.2. Turbulence model

In order to make the equation closed, a new turbulent model equation must be introduced to link the fluctuating values with the time average in the stress terms. There, we chose the $RNGk - \varepsilon$ equations. The transport equations of turbulent kinetic energy and turbulent fluctuation intensity in the $RNGk - \varepsilon$ equation are:
\[
\frac{\partial}{\partial t} (\rho k) + \frac{\partial}{\partial x_j} (\rho k u_i) = \frac{\partial}{\partial x_j} [\alpha_k \mu_{eff} \frac{\partial k}{\partial x_j}] + G_k - \rho \varepsilon + S_k
\]  

(6)

\[
\frac{\partial}{\partial t} (\rho \varepsilon) + \frac{\partial}{\partial x_j} (\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} [\alpha_k \mu_{eff} \frac{\partial \varepsilon}{\partial x_j}] + G_k \varepsilon k - C_{2\varepsilon} \rho \varepsilon^2 \varepsilon - R_e + S_{\varepsilon}
\]  

(7)

Where, \( \mu_{eff} = \mu + \mu_t \), \( \mu_t = \rho C \mu \frac{k^2}{\varepsilon} \), \( G_k = -\rho u_t \frac{\partial u_j}{\partial x_i} \), \( R_e = \frac{C_{\rho} \eta^3 (1 - \frac{\eta}{\eta_0})}{1 + \beta \eta^3} \frac{\varepsilon^2}{k} \), \( \eta = \frac{S_k}{\varepsilon} \), \( S = \sqrt{2S_x S_y} \), \( S_x = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \), \( S_k \) and \( S_{\varepsilon} \) are user-defined source terms.

Constant, \( G_{1\varepsilon} = 1.42 \), \( G_{2\varepsilon} = 1.68 \), \( C_\mu = 0.0845 \), \( \sigma_k = 1.0 \), \( \sigma_{\varepsilon} = 1.3 \), \( \eta_0 = 4.38 \), \( \beta = 0.012 \)

4. CFD Simulation

The dimension of Cylinder is 200 mm, the distance is 1000 mm (Fig. 1).

![Figure 1. Cylinder](image)

In order to obtain high-quality meshes, structured grids are used for the entire computational domain. The governing equations and turbulence models are discretized by the finite volume method and solved by the coupled implicit solver.

The pressure - velocity coupling iterative algorithm using SIMPLExC algorithm, pressure by the standard discretization, momentum, turbulent kinetic energy and turbulent dissipation rate by two order upwind scheme, relaxation factors are the default values. Wall function method is adopted in the near wall region.

Finally, the resistance values of the rotating body at different speeds are shown in tab. 2.

| V(m/s) | 0   | 0.5 | 1   | 2   | 3   | 4   | 5   |
|-------|-----|-----|-----|-----|-----|-----|-----|
| R(N)  | 0   | 2.3 | 9.5 | 36.3| 80.5| 140.9| 217.3|

5. Calculation and analysis

When the cylinder moves in water, its force analysis can be seen in Figure 2. The gyroscopic motion and current action of cylinder after being thrown into the water by a flying vehicle are not considered.
The moment of motion of cylinder in water, its equation of motion is:

\[ G - V - F = ma \]  
(8)

The resistance \( F \) varies with the speed, so the differential equations cannot be solved by theoretical methods, only numerical methods can be used. Changing equation (8) to the following form:

\[ G - V - F = m \frac{V_{t+\Delta t} - V_t}{\Delta t} \]  
(9)

\( V_{t=0} \) has been known, we can use cubic spline curve interpolation to calculate \( V_{t+\Delta t} \). Taking \( V_{t=0} = 0 \) as an example, numerical calculation when \( \Delta t \) equal to 0.05 s, 0.1 s, 0.2 s and 0.5 s (Table 3.).

**Table 3.** Velocity, Resistance etc. at different \( \Delta t \) (t=5s)

| V(m/s) | R(N)  | a(m/s^2) | s(m)  | \( \Delta t \) (s) |
|--------|-------|----------|-------|---------------------|
| 1.60   | 23.37 | 0.02     | 5.89  | 0.01                |
| 1.60   | 23.39 | 0.02     | 5.90  | 0.05                |
| 1.60   | 23.42 | 0.02     | 5.93  | 0.1                 |
| 1.60   | 23.48 | 0.02     | 5.97  | 0.2                 |
| 1.61   | 23.67 | 0.01     | 6.10  | 0.5                 |

In order to ensure the efficiency of calculation and the accuracy of the result, \( \Delta t = 0.05 \) s, and the calculation results are shown in Table 4.

In table 4, the acceleration is 0 when the time is 10s, and the speed is stable at 1.62 m/s. That means the cylinder’s external force is 0 when the time after 10s. So, the cylinder will maintain uniform rectilinear motion.

Using this method, the time, velocity, acceleration and displacement of the cylinder can be calculated very quickly after it enter the water.
Table 4. Velocity, Resistance etc. at different time ($\Delta t = 0.05$ s)

| T(s) | V(m/s) | R(N) | a(m/s²) | s(m) |
|------|--------|------|---------|------|
| 1    | 0.72   | 4.00 | 0.68    | 0.39 |
| 2    | 1.23   | 13.81| 0.34    | 1.41 |
| 3    | 1.47   | 19.62| 0.15    | 2.79 |
| 4    | 1.56   | 22.33| 0.06    | 4.32 |
| 5    | 1.60   | 23.39| 0.02    | 5.90 |
| 6    | 1.61   | 23.78| 0.01    | 7.51 |
| 7    | 1.62   | 23.92| 0.003   | 9.13 |
| 8    | 1.62   | 23.97| 0.001   | 10.75|
| 9    | 1.62   | 23.99| 0       | 12.37|
| 10   | 1.62   | 24.00| 0       | 13.99|
| 11   | 1.62   | 24.00| 0       | 15.62|

6. Conclusion
Using CFD, cubic spline interpolation and finite difference method can effectively calculate the speed, acceleration and displacement etc. of the cylinder when it enter into water. Comparing with the dynamic mesh method, this method with less resource intensive, fast calculating. The method is suitable for analyzing the motion performance and trajectory analysis of heavy objects after entering water.

References
[1] Seddon C, Moatamedi M. Review of water entry with applications to aerospace structures [J]. International Journal of Impact Engineering, 2016, 32(7):1045―1067.
[2] Gu H B, Qian L, Causon D M, et al. Numerical simulation of water impact of solid bodies with vertical and oblique entries [J]. Ocean engineering, 2013, 75(5):128-137.
[3] Tanvir Mehedi Sayeed, Heather Peng, Brian Veitch. Experimental investigation of slamming loads on a wedge [R]. The International Conference on Marine Technology. 2010:107-112.
[4] Tanvir Mehedi Sayeed, Heather Peng, Brian Veitch. Experimental investigation of slamming loads on a wedge [R]. The International Conference on Marine Technology. 2010:107-112.
[5] Tveitnes T, Fairlie-Clarke A. C, Varyani K. An experimental investigation into the constant velocity water entry of wedge-shaped sections [J]. Ocean Engineering. 2008, 35:1463-1478.
[6] Wang Yonghu, Shi Xiuhua, Review on research and development of water-entry impact problem. Explosion and Shock Waves, 2008, 28(3): 276-282.
[7] Arai M, Cheng L Y, Inoue Y. A computing method for the analysis of water impact of arbitrary shaped bodies[J]. Journal of the Society of Naval Architects of Japan, 1995, 176: 233-240.
[8] Hadzica I, Hennig J, PERICM, et al. Computation of flow-induced motion of floating bodies [J]. Applied Mathematical Modelling, 2005, 29(12): 1196-1210.
[9] Qiu Haiqiang, Yuan Xulong, Wang Yadong, et al. Simulation on impact load and cavity shape in high speed vertical water entry for an axisymmetric body [J]. Torpedo Technology, 2013, 21(3): 161―164.
[10] Seddon C M, Moatamedi M. Review of water entry with applications to aerospace structures [J]. International Journal of Impact Engineering, 2006, 32:1045-1067.
[11] Carcaterraa A, Ciappi E. Hydrodynamic shock of elastic structures impacting on the water: theory and experiments [J]. Journal of Sound and Vibration. 2004, 271: 411-439.
[12] Zhao R, Faltinsen O M. Water-entry of two-dimensional bodies[J]. Journal of Fluid Mechanics, 1993, 246: 593-612.