Fully explorable horned particles hiding charge

E I Guendelman and M Vasihoun

Physics Department, Ben Gurion University, Beer Sheva, Israel
E-mail: guendel@bgu.ac.il and mahary@bgu.ac.il

Received 10 January 2012, in final form 10 March 2012
Published 11 April 2012
Online at stacks.iop.org/CQG/29/095004

Abstract
The charge-hiding effect by a horned particle, which was studied for the case where the gravity/gauge-field system is self-consistently interacting with a charged light-like brane (LLB) as a matter source, is now studied for the case of a time-like brane. From the demand that no surfaces of infinite coordinate time redshift (horizons) appear in the problem, we are now led to a completely explorable horned particle space for a traveller that goes through the horned particle (as was the case for the LLB). Now, in addition to this, the horned region is fully visible to a static external observer. This requires negative surface energy density for the shell sitting at the throat. We study a gauge-field subsystem that is of a special nonlinear form containing a square root of the Maxwell term and that previously has been shown to produce a QCD-like confining gauge-field dynamics in flat spacetime. The condition of finite energy of the system or asymptotic flatness on one side of the horned particle implies that the charged object sitting at the throat expels all the flux it produces into the other side of the horned particle, which turns out to be of a ‘tube-like’ nature. An outside observer in the asymptotically flat universe detects, therefore, an apparently neutral object. The hiding of the electric flux behind the tube-like region of a horned particle is the only possible way that a truly charged particle can still be of finite energy, in a theory that in flat space it describes confinement. This points to the physical relevance of such solutions, even though there is the need of negative energy density at the throat of the horned particle, which can be of quantum-mechanical origin.

PACS numbers: 11.25.−w, 04.50.+h

(Some figures may appear in colour only in the online journal)

1. Introduction
The charge-hiding effect by a horned particle, which are spaces containing a space where asymptotically $r \to \infty$ connected to a space where $r = \text{constant}$—this is called a ‘horned’
region because there is another coordinate (that replaces $r$ which is constant and therefore cannot be used as a coordinate) that runs along an infinite range\textsuperscript{1}—which was studied for the case where gravity/gauge-field system is self-consistently interacting with a charged light-like brane (LLB) as a matter source [1], is now studied for the case of a time-like brane. In this case we demand that no surfaces of infinite coordinate time redshift\textsuperscript{2} appear in the problem, leading therefore to a completely explorable horned particle space, according not only to the traveller that goes through the horned particle space (as was the case for the LLB), but also to a static external observer. This requires negative surface energy density for the shell sitting at the throat of the horned particle. We study a gauge-field subsystem that is of a special nonlinear form containing a square root of the Maxwell term and which has previously been shown to produce a QCD-like confining gauge-field dynamics in flat spacetime. The condition of finite energy of the system, or asymptotic flatness on one side of the horned particle, implies that the charged object sitting at the ‘throat’ expels all the flux it produces into the other side of the horned particle, which turns out to be of a ‘tube-like’ nature. An outside observer in the asymptotically flat universe detects, therefore, a neutral object. The hiding of the electric flux behind the horned region of the particle appears to be the only possible way that a truly charged particle can still be of finite energy, which points to the physical relevance of such solutions, even though there is the need of negative energy density at the throat, which can be of quantum-mechanical origin.

This effect is indeed the opposite to the famous Misner–Wheeler ‘charge without charge’ effect [7], one of the most interesting physical phenomena produced by wormholes. Misner and Wheeler realized that wormholes connecting two asymptotically flat spacetimes provide the possibility of existence of electromagnetically non-trivial solutions, where the lines of force of the electric field flow from one universe to the other without a source and give the impression of being positively charged in one universe and negatively charged in the other. Wormholes may be classified according to their traversability properties, which can be addressed according to whether a ‘traveller’ that attempts to cross from one side of the wormhole throat to the other side can do so in a finite proper time (the traveller’s proper time). In addition one may require that a static observer, which uses the coordinate time, will see the traveller go from one side of the throat and come back in a finite coordinate time. For a detailed account of the general theory of traversable wormholes according to this second, most stringent definition, we refer to Visser’s book [8] (see also [9, 10] and some more recent accounts [11–15]).

In contrast to the Misner and Wheeler effect, the charge-hiding effect by a horned particle means a genuinely charged matter source of gravity and electromagnetism may appear electrically neutral to an external observer. In previous publications, it has been shown that this phenomenon takes place in a gravity/gauge-field system self-consistently coupled to a charged LLB as a matter source, where the gauge-field subsystem is of a special nonlinear form containing a square root of the Maxwell term [1]. The latter has been previously shown [16–20] to produce a QCD-like confining (‘Cornell’ [23–25]) potential in flat spacetime. There the LLB at the ‘throat’ connects a ‘universe’ (where $r \to \infty$) with a ‘universe’ (where $r = \text{const}$) and is electrically charged; however, all of its flux flows into the ‘tube-like universe’

\textsuperscript{1} In [1], we called these spaces wormholes, but it has been pointed out to us that they are more correctly described as ‘horned particles’ as has been done for the spacetimes studied in [2, 3] that also have large tube-like structure. Somewhat similar are the so-called ‘gravitational bags’, where some extra dimensions grow very large at the center of the four-dimensional projected metric [4–6].

\textsuperscript{2} Such surface of infinite redshift is in fact a horizon at $r = r_h$, so the resulting object is similar to a black hole, but not exactly. In our previous studies of wormholes constructed this way, because there is no ‘interior’, i.e. $r > r_h$ everywhere in the previous studies, and since $r > r_h$, everywhere, there is no possibility of ‘collapse’ to $r = 0$. 

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Class. Quantum Grav. 29 (2012) 095004
E I Guendelman and M Vasihoun
only. No Coulomb field is produced in the ‘universe’ containing the \( r \to \infty \) region; therefore, the horned particle hides the charge from an external observer in the latter ‘universe’.

In the case where LLB are present as the source of the system, sitting at the throat, there is at the throat a surface of infinite redshift for the coordinate time; so, the horn is accessible only according to the weaker definition that the ‘traveller’ that attempts to cross from one side of the horn throat to the other side can do so in a finite proper time. If we want to assure the more stringent definition of traversability or accessibility, we must use time-like branes. This will be done here in the framework of the thin-wall approach to domain walls or thin shells coupled to gravity [27].

The gravity/gauge-field system with a square-root of the Maxwell term was recently studied in [28] (see the brief review in section 2), where the following interesting new features of the pertinent static spherically symmetric solutions have been found:

(i) appearance of a constant radial electric field (in addition to the Coulomb one) in charged black holes within the Reissner–Nordstrøm–de Sitter-type and/or Reissner–Nordstrøm–anti-de Sitter-type spacetimes, in particular, in electrically neutral black holes with the Schwarzschild–de Sitter and/or Schwarzschild–anti-de Sitter geometry;

(ii) novel mechanism of dynamical generation of a cosmological constant through the nonlinear gauge-field dynamics of the ‘square-root’ Maxwell term,

(iii) appearance of confining-type effective potential in charged test particle dynamics in the above black hole backgrounds.

In section 3 of this paper, we review the results of [1] concerning tube-type or Lévi-Civita–Bertotti–Robinson-type [29–31] solutions, with spacetime geometry of the form \( M_2 \times S^2 \) with \( M_2 \) being a two-dimensional anti-de Sitter, Minkowski or de Sitter space depending on the relative strength of the electric field w.r.t. coupling of the square-root Maxwell term.

In previous papers [32–42], an explicit reparametrization-invariant world-volume Lagrangian formulation of light-like \( p \)-branes was used to construct various types of wormhole, regular black hole and light-like braneworld solutions in \( D = 4 \) or higher dimensional asymptotically flat or asymptotically anti-de Sitter bulk spacetimes. In particular, in [40–42], it has been shown that LLB can trigger a series of transitions of spacetime regions, e.g., from ‘tube-like’ Lévi-Civita–Bertotti–Robinson spaces to non-compact Reissner–Nordstrøm or Reissner–Nordstrøm–de Sitter region or vice versa. Let us note that wormholes with ‘tube-like’ structure (and regular black holes with ‘tube-like’ core) have previously been obtained in [43–51].

Although LLB will not be used in this paper, one should point out nevertheless the essential role of the proper world-volume Lagrangian formulation of LLB, which manifests itself most clearly in the correct self-consistent construction [36, 39] of the simplest wormhole solution first proposed by Einstein and Rosen [52]—the Einstein–Rosen ‘bridge’ wormhole. Namely, in [36, 39], it has been shown that the Einstein–Rosen ‘bridge’ in its original formulation [52] naturally arises as the simplest particular case of static spherically symmetric wormhole solutions produced by LLB as gravitational sources, where the two identical ‘universes’ with Schwarzschild outer-region geometry are self-consistently glued together by an LLB occupying their common horizon—the wormhole ‘throat’. An understanding of this picture within the framework of the Kruskal–Szekeres manifold was subsequently provided in [55], which involves Rindler’s elliptic identification of the two antipodal future event horizons. The system resembles black hole in the sense that there is a surface of infinite redshift at \( r = r_h \) but unlike the standard black hole there is no \( r < r_h \), only two regions glued, and for both, \( r > r_h \).

At this point, let us strongly emphasize that the original notion of ‘Einstein–Rosen bridge’ in [52] is qualitatively different from the notion of ‘Einstein–Rosen bridge’ defined in several
popular textbooks (e.g., [53, 54]) using the Kruskal–Szekeres manifold, where the ‘bridge’ has dynamic spacetime geometry. Namely, the two regions in the Kruskal–Szekeres spacetime corresponding to the two copies of the outer Schwarzschild spacetime region \((r > 2m)\) (the building blocks of the original static Einstein–Rosen ‘bridge’) and labeled (I) and (III) in [53] are generally disconnected and share only a two-sphere (the angular part) as a common border \((U = 0, V = 0)\) in the Kruskal–Szekeres coordinates), whereas in the original Einstein–Rosen ‘bridge’ construction [52], the boundary between the two identical copies of the outer Schwarzschild spacetime region \((r > 2m)\) is a three-dimensional light-like hypersurface \((r = 2m)\).

In section 4, we consider the matching of an external solution, containing \(r \to \infty\) region, to a tube-like solution, where \(r = \text{const}\), through a time-like brane that will serve as a matter source of gravity and (nonlinear) electromagnetism. Then, in section 5, we recognize the interesting phenomenon that for asymptotic flatness (and therefore for the configuration to be recognized as a legitimate finite-energy excitation from flat space) the charged particle has to redirect all the flux it produces in the direction of the tube region. This new charge ‘confining’ phenomenon is entirely due to the presence of the ‘square-root’ Maxwell term, which assigns infinite energy to any configuration that does not hide the flux (i.e. does not send all the electric flux in the direction of the tube region).

2. Lagrangian formulation: spherically symmetric solutions

In [16–20], a flat spacetime model of a nonlinear gauge-field system with a square root of the Maxwell term was considered \((f \) is a positive constant that sets the scale for confinement effects in the theory):

\[
S = \int d^4x L(F^2), \quad L(F^2) = -\frac{1}{4} F^2 - \frac{f}{2} \sqrt{-g} F^2,
\]

\[F^2 \equiv F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \tag{1}\]

The equations of motion are

\[
\partial_\nu \left( \sqrt{-g} F^{\alpha\beta} - f \right) \frac{F^{\mu\nu}}{\sqrt{-g} F^{\alpha\beta}} = 0. \tag{2}\]

Then, assuming spherical symmetry and time independence, we find that, in addition to a Coulomb-like piece, a linear term proportional to \(f\) is obtained for \(A^0\), which is of the form of the well-known ‘Cornell’ potential [23–25] in quantum chromodynamics (QCD). Furthermore, these equations are consistent with the ’t Hooft criterion for perturbative confinement. In fact, in the infrared region, the above equation implies that

\[
F^{\mu\nu} = f \frac{F^{\mu\nu}}{\sqrt{-g} F^{\alpha\beta}} \tag{3}\]

plus negligible terms in the infrared sector. Interestingly enough, for a static source, this automatically implies that the chromoelectric field has a fixed amplitude. Confinement is obvious then, since in the presence of two external oppositely charged sources, by symmetry arguments, one can see that such a constant-amplitude chromoelectric field must be in the direction of the line joining the two charges. The potential that gives rise to this kind of field configuration is of course a linear potential. Also, for static-field configurations, the model (1) yields the following electric displacement field: \(\vec{D} = \vec{E} - \frac{f}{\sqrt{2}} \frac{\vec{F}}{|\vec{F}|}\). The pertinent energy density for the electrostatic case turns out to be \(\frac{1}{4} \vec{E} \cdot \vec{D}\), and for the case, \(\vec{E}\) and \(\vec{D}\) point
in the same direction, which is satisfied if \( E = |\vec{E}| > \frac{f}{\sqrt{2}} \), then \( \frac{1}{2} \vec{E}^2 = \frac{1}{2} \vec{D}^2 + \frac{f^2}{2} D + \frac{f^2}{4} \), so that it indeed contains a term linear w.r.t. \( D = |\vec{D}| \) as argued by 't Hooft [26]. The vacuum state is degenerated and is defined by the states with \( \vec{D} = 0 \); note that a charge source generates \( \vec{D} \), not \( \vec{E} \); furthermore, the states with \( \vec{D} = 0 \) are the solutions of the equations of motion, but not so the states with \( \vec{E} = 0 \); in fact, at such a point in field space, the equations of motion are not even well defined. However, for all the solutions studied in this paper, the excitations ‘over the vacuum’ will satisfy \( E = |\vec{E}| > \frac{f}{\sqrt{2}} \), while \( E = \frac{f}{\sqrt{2}} \) will correspond to the ‘vacuum configuration’ of the electrostatic theory, as it will be discussed. Similar connection between \( \vec{D} \) and \( \vec{E} \) has been considered as an example of a ‘classical model of confinement’ in [21] and analyzed generalizing the methods developed for the ‘leading logarithm model’ in [22].

The natural appearance of the ‘square-root’ Maxwell term in the effective gauge-field action (1) was further motivated by 't Hooft [26] who has proposed that such gauge-field actions are adequate for describing confinement (see especially equation (5.10) in [26]). He has in particular described a consistent quantum approach in which these kind of terms play the role of ‘infrared counter terms’. Also, it has been shown in first three [16–20] that the square root of the Maxwell term naturally arises as a result of spontaneous breakdown of scale symmetry of the original scale-invariant Maxwell theory with \( f \) appearing as an integration constant responsible for the latter spontaneous breakdown.

We will consider the simplest coupling to gravity of the nonlinear gauge-field system with a square-root of the Maxwell term. The relevant action is given by (we use units with the Newton constant \( G_N = 1 \))

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R(g)}{16\pi} - 2\Lambda + L(F^2) \right],
\]

\[
L(F^2) = -\frac{1}{4} F^2 \equiv \frac{f^2}{2} \sqrt{-g},
\]

\[
F^2 \equiv F_{\mu\nu} F^{\mu\nu} = g^{\mu\nu} g^{\rho\sigma} F_{\mu\rho} F_{\nu\sigma} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\]

Here, \( R(g) \) is the scalar curvature of the spacetime metric \( g_{\mu\nu} \) and \( g \equiv \det \| g_{\mu\nu} \| \), and \( f \) is a positive coupling constant.

Let us note that the Lagrangian \( L(F^2) \) in (4) contains both the usual Maxwell term as well as a non-analytic function of \( F^2 \), and thus is a non-standard form of nonlinear electrodynamics. In this way, it is significantly different from the original purely ‘square-root’ Lagrangian \( -\frac{f^2}{2} \sqrt{F^2} \) first proposed by Nielsen and Olesen [56] to describe string dynamics (see also [57, 58]). The Nielsen and Olesen action was designed to be applicable only to ‘magnetic-dominated’ configuration; for electric-dominated configuration, the square root becomes imaginary. In contrast, here we will be interested in the ‘electric’ sector of the theory and note that now for magnetic-dominated configurations the argument inside the square root becomes negative and the square root itself imaginary. This could be a real effect (that gauge fields must be electrically dominated), or since the action is not analytic, it is of course possible to construct a simple modification that allows magnetic-dominated configurations by taking the absolute value inside the square root. This will not affect the electrostatic sector of the theory, but it will be harder to motivate (for example from spontaneous symmetry breaking of scale invariance). That discussion would go well beyond the purpose of this paper, which is to see what a theory that provides confining theory in flat space can do in the presence of horned spacetimes.

As done in [26], we also mean to use Lagrangian (4) to describe a truncation of the non-Abelian theory, where for simplicity we take the gauge-field potential in a specific direction in color space (so commutator terms vanish). Let us also remark that one could start with the non-Abelian version of the gauge-field action in (4). Since we will be interested in static spherically

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\text{Class. Quantum Grav. 29 (2012) 095004}
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E I Guendelman and M Vasiouh

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symmetric solutions, the non-Abelian gauge theory effectively reduces to an Abelian one as pointed out in [16].

The corresponding equations of motion of (4) reads

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 8\pi T^{(F)}_{\mu\nu}, \]  

(5)

where

\[ T^{(F)}_{\mu\nu} = L(F^2) g_{\mu\nu} - 4L'(F^2) F_{\mu\kappa} F_{\nu\lambda} g^{\kappa\lambda}, \]  

(6)

and

\[ \partial_\nu (\sqrt{-g} L'(F^2) F_{\kappa\lambda} g^{\mu\kappa} g^{\nu\lambda}) = 0, \]  

(7)

where \( L'(F^2) \) denotes derivative w.r.t. \( F^2 \) of the function \( L(F^2) \) in (4).

A note concerning the vacuum of the theory is in order here. As opposed to the standard Maxwell theory, the vacuum of the theory is not obtained for the zero gauge-field strength \( F_{\mu\nu} = 0 \). Instead, the vacuum of the theory is obtained for a configuration such that \( L'(F^2) = 0 \); note that in such case \( T^{(F)}_{\mu\nu} \propto g_{\mu\nu} \), i.e. it gives a ‘cosmological constant-type contribution’, and also note that this is obtained for \( F^2 \neq 0 \), which agrees with the notion that the vacuum of confining theory contains gauge-field condensates.

In our preceding paper [28], we have shown that the gravity–gauge-field system (4) possesses static spherically symmetric solutions with a radial electric field containing both Coulomb and constant vacuum pieces:

\[ F_{0\eta} = \frac{\varepsilon F f}{\sqrt{2}} + \frac{Q}{r^2}, \quad \varepsilon_F = \text{sign}(Q), \]  

(8)

the sign of the second term, determined by \( \varepsilon_F \), which is the field strength \( F_{0\eta} \) divided by its absolute value, has the same sign as \( F_{0\eta} \) itself, and looking at small enough \( r \), we see that this sign is determined by \( Q \), since there the Coulomb part dominates. We see therefore that both contributions in (8) have the same sign, and therefore, \( |F_{0\eta}| > \frac{f}{\sqrt{2}} \), i.e. bigger than its vacuum value, and the spacetime metric that we have

\[ ds^2 = -A(r) dt^2 + \frac{dr^2}{A(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]  

(9)

\[ A(r) = 1 - \sqrt{8\pi} |Q| f - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda_{\text{eff}}}{3} r^2 \]  

(10)

is Reissner–Nordström–de-Sitter-type space, where \( \Lambda_{\text{eff}} = 2\pi f^2 + \Lambda \).

Appearance in (10) of a ‘leading’ constant term different from 1 resembles the effect on gravity produced by a spherically symmetric ‘hedgehog’ configuration of a nonlinear sigma-model scalar field with \( SO(3) \) symmetry, that is the field of a global monopole [59] (cf also [60]).

3. Generalized Lévi-Civitá–Bertotti–Robinson space-times

Here, we will look for the static solutions of Lévi-Civitá–Bertotti–Robinson type [29–31] for the system (5)–(7); this was studied in [1, 65], namely, with spacetime geometry of the form \( M_2 \times S^2 \), where \( M_2 \) is some two-dimensional manifold:

\[ ds^2 = -A(\eta) dt^2 + \frac{d\eta^2}{A(\eta)} + a^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad -\infty < \eta < \infty, \quad a = \text{const}, \]  

(11)

and the electromagnetic field is given by

\[ F_{\mu\nu} = 0 \quad \text{for} \quad \mu, \nu \neq 0, \eta, \quad F_{0\eta} = F_{0\eta}(\eta). \]  

(12)
The gauge-field equations of motion become
\[ \partial_\eta \left( F_{0\eta} - \frac{\varepsilon F}{\sqrt{2}} \right) = 0, \quad \varepsilon \equiv \text{sign}(F_{0\eta}), \]
yielding a constant vacuum electric field:
\[ F_{0\eta} = c_F = \text{arbitrary const}. \]

The (mixed) components of the energy–momentum tensor (6) read
\[ T^{(F)}_{0 \eta} = -\frac{1}{2} F_{0 \eta}^2, \quad T^{(F)}_{ij} = g_{ij} \left( \frac{1}{2} F_{0 \eta}^2 - \frac{f}{\sqrt{2}} |F_{0 \eta}| \right). \]

Taking into account (15), the Einstein equations (5) for \((ij)\), where \( R_{ij} = \frac{1}{a^2} R_{ij} \) because of the \( S^2 \) factor in (11), yield
\[ \frac{1}{a^2} = 4\pi c_F^2 + \Lambda. \]

The (00) Einstein equation (5) using the expression \( R^{(F)}_{00} = -\frac{1}{2} \partial_\eta^2 A \) ([61]; see also [62]) becomes
\[ \partial_\eta^2 A = 8\pi h(|c_F|), \quad h(|c_F|) \equiv c_F^2 - \sqrt{2} f |c_F| - \frac{\Lambda}{4\pi}. \]

In the particular case, where \( \Lambda = 0 \), studied in [1], we arrive at the following three distinct types of Lévi-Civita–Bertotti–Robinson solutions for gravity coupled to the non-Maxwell gauge-field system (4).

(i) \( \text{AdS}_2 \times S^2 \) with the strong constant vacuum electric field \(|F_{0\eta}| = |c_F| > \sqrt{2} f\), where \( \text{AdS}_2 \) is two-dimensional anti-de Sitter space with
\[ A(\eta) = 1 + 4\pi |c_F|(|c_F| - \sqrt{2} f)\eta^2 \]
in the metric (11).

(ii) \( M_2 \times S^2 \) with the constant vacuum electric field \(|F_{0\eta}| = |c_F| = \sqrt{2} f\), where \( M_2 \) is the flat two-dimensional space with
\[ A(\eta) = 1 \]
in the metric (11).

(iii) \( \text{dS}_2 \times S^2 \) with the weak constant vacuum electric field \(|F_{0\eta}| = |c_F| < \sqrt{2} f\), where \( \text{dS}_2 \) is two-dimensional de Sitter space with
\[ A(\eta) = 1 - 4\pi |c_F|(|\sqrt{2} f - |c_F|)\eta^2. \]

For the special value \(|c_F| = \frac{f}{\sqrt{2}}\), we recover the Nariai solution [63, 64] with \( A(\eta) = 1 - 2\pi f^2 \eta^2 \) and equality (up to signs) among energy density, radial and transverse pressures: \( \rho = -p_r = -p_\perp = \frac{f}{2} (T^{(F)})^\mu_\mu = \text{diag} - \rho, p_r, p_\perp, p_\perp) \).

In all three cases above, the size of the \( S^2 \) factor is given by (16). Solutions (19) and (20) are the new ones and are specifically due to the presence of the non-Maxwell square-root term in the gauge-field Lagrangian (4).

In this paper, we will consider the case \( \Lambda \neq 0 \) and demand that \( \Lambda_{\text{eff}} = 0 \). This leaves us only with solutions similar to (18) although with a different dependence on \(|c_F|\) (see equation (46)).
4. Matching through a regular (time-like) thin shell

Now, we want to discuss the matching of (9) to (11) at a spherically symmetric wall. The metric induced at the wall has to be well defined, and the coefficient of purely angular displacements \(d/\Omega^1\) in (9) and (11) has to agree at the position of the wall, to give the same value of \(ds^2\). Therefore, at the wall

\[ r = a. \]  

The equations of motion of a thin layer in GR have been obtained by Israel [27]; we now briefly review these results. To obtain those equations, it is useful to define a Gaussian normal coordinate system in a neighborhood of the wall as follows: denoting the \((2+1)\)-dimensional hypersurface \(\Sigma_1\) and introducing a coordinate system on \(\Sigma_1\), two are taken to be the angular variables \(\theta, \phi\), which are always well defined up to an overall rotation for a spherically symmetric configuration. For the other coordinate in the wall, one can use the proper time variable \(\tau\) that would be measured by an observer moving along with the wall. The fourth coordinate \(\xi\) is taken as the proper distance along the geodesics intersecting \(\Sigma_1\) orthogonally.

We adopt the convention that \(\xi\) is taken to be positive in the Reissner–Nordström–de-Sitter-type regime and negative in the generalized Lévi-Civitá–Bertotti–Robinson regime, and \(\xi = 0\) is of course the position of the wall. Thus, the full set of coordinates is given by \(x^\mu = (\tau, \theta, \phi, \xi)\);

\[ g^{\xi \xi} = g^{\xi i} = 1, \quad g^{\xi i} = 0. \]  

Also, we define \(n^\mu\) to be the normal to a \(\xi = \text{constant}\) hypersurface, which in Gaussian normal coordinates has the simple form \(n^\mu = (0, 0, 0, 1)\). We then define the extrinsic curvature corresponding to each \(\xi = \text{constant}\) hypersurface, which is a three-dimensional tensor whose components are defined by

\[ K_{ij} = \frac{\partial n_i}{\partial x^j} - \Gamma^\alpha_{ij} n_\alpha = -\Gamma^\xi_{ij} = \frac{1}{2} \frac{\partial}{\partial \xi} g_{ij}. \]  

As we can see, the extrinsic curvature gives the change of the metric in the direction perpendicular to the surface.

In terms of these variables, the Einstein’s equations take the form

\[ G^\xi_\xi = -\frac{1}{2} (R^{(3)} + \frac{1}{3} [\text{Tr} K^2 - \text{Tr}(K^2)] = 8\pi G T^\xi_\xi, \]

\[ G^\xi_i = K^n_{i\alpha} - (\text{Tr} K)_\alpha = 8\pi G T^\xi_i. \]  

\[ G^j_j = (K^j_j - \delta^j_i [\text{Tr} K]) \delta^i_j - (\text{Tr} K) K^j_j + \frac{1}{2} \delta^j_j [\text{Tr} K^2 + \text{Tr}(K^2)] = 8\pi G T^j_j, \]  

where the subscript vertical bar denotes the three-dimensional covariant derivative in the \((2+1)\)-dimensional space of coordinates \((\tau, \theta, \phi)\), and comma denotes an ordinary derivative. Also quantities like \(R^{(3)}\), \(G^j_j\), etc are to be evaluated as if they concerned with a purely three-dimensional metric \(g_{ij}\), without any reference as to how it is embedded in the higher four-dimensional space.

By definition, for a thin wall, the energy–momentum tensor \(T^{\mu \nu}\) has a delta-function singularity at the wall, so one can define a surface stress–energy tensor \(S^{\mu \nu}\):

\[ T^{\mu \nu} = S^{\mu \nu} \delta(\xi) + \text{regular terms}. \]  

When the energy–momentum tensor (26) is inserted into the field equations (24) and (25), we obtain that (24) are satisfied automatically, provided that they are satisfied for \(\xi \neq 0\) and provided that \(g_{ij}\) is continuous at \(\xi = 0\) (so that \(K_{ij}\) does not acquire a \(\delta\)-function singularity).
Equation (25) however, when integrated from $\xi = -\epsilon$ to $\xi = \epsilon$ ($\epsilon \rightarrow 0$ and $\epsilon > 0$), leads to the discontinuity condition

$$S_j^i = \frac{1}{8 \pi G} \left[ \gamma_j^i - \delta_j^i \text{Tr} \gamma \right]$$  \tag{27}$$
or equivalently

$$\gamma_j^i = -8 \pi G \left( S_j^i - \frac{1}{2} \delta_j^i \text{Tr} S \right),$$ \tag{28}$$
where

$$\gamma_{ij} = \lim_{\epsilon \to 0} [K_{ij}(\xi = +\epsilon) - K_{ij}(\xi = -\epsilon)]$$ \tag{29}$$
is therefore the jump of extrinsic curvature across $\Sigma$.

The local conservation of $T_{\mu\nu}$ and the demand of spherical symmetry give a surface stress–energy tensor of the form

$$S_{\mu\nu} = \sigma(\tau) U_\mu U_\nu - \omega(\tau) [h_{\mu\nu} + U_\mu U_\nu]$$ \tag{30}$$
where $h_{\mu\nu} = g_{\mu\nu} - n^\mu n^\nu$ is the metric projected onto the hypersurface of the wall, and $U^\mu = (1, 0, 0, 0)$ is the four velocity of the wall.

In (30), $\sigma$ has the interpretation of energy per unit surface, as detected by an observer at rest with respect to the wall, and $\omega$ has the interpretation of surface tension. For a given equation of state $p = p(\sigma)$ ($p = -\omega$), the local energy momentum at the wall gives $d(\sigma^2) = -pd(\sigma^2)$, but since we have $r = \text{const}$ at the matching point, we obtain $\sigma = \text{const}$ and from the generic equation of state $p = p(\sigma) = \text{const}$. For the shell at the junction $r = a$, the angular coordinates $(\theta, \phi)$ must be identified. Also the radial coordinate of the shell, as seen from the outside, is clearly $r = a$, but this smooth sewing so that the requirement that the induced metrics on the shell from the inside and outside giving the same result on the shell, and therefore being well defined, is not enough to determine $\eta$ in (11), which will have a non-trivial time dependence $\eta = \eta(t)$ to be determined by the Israel junction conditions.

In addition to this, the time-like brane can also have a delta function charge density $j^\mu = \delta^\mu_\xi q\delta(\xi)$ coming from the discontinuity in the gauge-field strength across the matching at $r = a$:

$$[F_{0\nu}]_{r=a} = q,$$ \tag{31}$$
which can also be defined in terms of the electric flux, or more clearly, by defining the electric displacement field $D_{0\nu}$ in the two regions, which in this case is significantly different from the electric field $F_{0\nu}$, due to the presence of the ‘square-root’ Maxwell term

$$D_{0\nu} - D_{0\nu} = q,$$ \tag{32}$$
where $D_{0\nu} = (1 - \frac{f}{\sqrt{2F_{0\nu}}}) F_{0\nu}$ and $D_{0\nu} = (1 - \frac{f}{\sqrt{2F_{0\nu}}}) F_{0\nu}$.

We now consider the matching through a thin spherical wall of the Reissner–Nordström–de-Sitter-type regime (denoted by ‘+’) to the generalized Lévi-Civita–Bertotti–Robinson regime (denoted by ‘−’). First, consider the discontinuity of $K_{\theta\theta}$. Using (21), (28) and (30), we obtain

$$K_{\theta\theta} = 4\pi \sigma a^2$$ \tag{33}$$
for the Lévi-Civita–Bertotti–Robinson space $g_{\theta\theta} = a^2$ = constant, so that $K_{\theta\theta} = 0$ according to (23). Therefore, $K_{\theta\theta} = -4\pi \sigma a^2$

$$\sqrt{A_0(a)} = -4\pi \sigma a,$$ \tag{34}$$
where $-A_0$ denotes the 0–0 component of the metric ‘outside’, on the $r > a$ region. So, if we are dealing with a standard static spacetime outside (e.g., in the case of Schwarzschild
space using only region I), the above equation implies $\sigma < 0$. That is, the matching of the tube spacetime with the ‘normal’ outside space will require negative energy densities. We could insist on $\sigma > 0$ but this requires the use of all regions of Kruskal space [43] (this allows the square root in (34) to be negative [43]), and therefore, the resulting wormhole is not traversable; we will not follow this approach in this paper.

The $\phi\phi$ component of equation (28) gives the same information due to the spherical symmetry of the problem. The additional information will come from the $\tau\tau$ component, which reads

$$K_{\tau\tau} - K_{\tau\tau}^+ = 4\pi (\sigma - 2\omega),$$

(35)

from $U^\mu n_\mu = 0$ we have $U^\mu n_{\mu;\nu} = -n_\mu U^\mu_{;\nu}$, and the $K_{\tau\tau}$ component then takes the form

$$K_{\tau\tau} = -n_\mu \left( \frac{dU^\mu}{d\tau} + \Gamma^\mu_{\alpha\beta} U^\alpha U^\beta \right).$$

(36)

Using this expression for the $\tau\tau$ component of the extrinsic curvature, its discontinuity equation then gives

$$- \frac{1}{\eta} \frac{d}{d\tau} \left( \sqrt{A_i(\eta)} + \dot{\eta}^2 \right) + \frac{1}{2\sqrt{A_0(a)}} \frac{\partial A_i(r)}{\partial r} \bigg|_{r=a} = 4\pi (\sigma - 2\omega),$$

(37)

where $-A_i(\eta)$ denotes the 0–0 component of the metric ‘inside’, on the tube-like compactified region.

Using (34),

$$- \frac{1}{\eta} \frac{d}{d\tau} \left( \sqrt{A_i(\eta)} + \dot{\eta}^2 \right) - \frac{1}{8\pi \sigma a} \frac{\partial A_i(r)}{\partial r} \bigg|_{r=a} = 4\pi (\sigma - 2\omega).$$

(38)

Multiplying by $\dot{\eta}$ and defining

$$\Delta \equiv \frac{1}{8\pi \sigma a} \frac{\partial A_i(r)}{\partial r} \bigg|_{r=a} + 4\pi (\sigma - 2\omega),$$

(39)

we can see that equation (38) becomes a total derivative of the proper time $\tau$

$$\frac{d}{d\tau} \left[ \sqrt{A_i(\eta)} + \dot{\eta}^2 + \Delta \eta \right] = 0$$

(40)

from which, upon integrating, we obtain

$$\sqrt{A_i(\eta)} + \dot{\eta}^2 + \Delta \eta = E = \text{const.}$$

(41)

Taking $\Delta$ to the right-hand side and then squaring, we finally obtain

$$\dot{\eta}^2 = (\Delta \eta - E)^2 - A_i(\eta),$$

(42)

where the general expression of $\Delta$, using the explicit form (10) for the uncompactified region, is

$$\Delta \equiv \frac{1}{4\pi \sigma a^4} \left( ma - Q^2 - \Lambda_{\text{eff}} a^4 \right) + 4\pi (\sigma - 2\omega).$$

(43)

5. Asymptotically flat, finite-energy solutions imply hiding the electric flux

Let us assume that our ground state is just flat space, and on top of that we would like to build finite-energy excitations. In the first place, this means that $\Lambda_{\text{eff}} = 0$, but still, this is not enough to ensure asymptotic flatness of finite energy. Indeed, if $Q \neq 0$, the leading behavior of the metric is not flat, but rather ‘hedgehog type’ [59, 60], which has an energy–momentum tensor that decreases only as the square of the radius for large distances, which of course are infinite-energy solutions.
This is of course consistent with the notion that in a confining theory an isolated charge has an associated infinite energy. Here, if we add the horned particle to the problem, the isolated charge can have finite energy, provided it sends all the electric flux it produces to the tube region, this requires the vanishing of the external Coulomb part of the electric field, or $Q = 0$.

When $Q = 0$, according to (8), the electric field outside has the magnitude $|F_0| = \frac{f}{\sqrt{2}}$. Therefore, in the $r > a$ region, the displacement field is equal to zero $D_0 = 0$. Thus, in the absence of Coulomb field, our choice $A_{\text{eff}} = 0$, and assuming the absence of magnetic field, the outer region then simply becomes a Schwarzschild solution with the vacuum electric field, which has a constant magnitude $\frac{f}{\sqrt{2}}$. But for this value of the field strength, we have that the gauge-field Lagrangian has a minimum, in fact, $L(F^2) = 0$; so, using only that $F^2 = f^2$, the equation of motion for the gauge field is satisfied automatically, that is to say, we now do not need the electric field to be radial, and the orientation is now completely arbitrary, once $F^2 = f^2$ is satisfied. In this disordered vacuum, where the electric field with a constant magnitude does not point in one fixed direction, a test charged particle will not be able to get energy from the electric field; instead, it will undergo a kind of Brownian motion, and therefore, no Schwinger pair-creation mechanism will take place.

From (43) and the above discussion, $\Delta$ takes the following simple form:

$$\Delta \to \frac{m}{4\pi \sigma a^3} + 4\pi (\sigma - 2\omega),$$

(44)

and from (17),

$$\partial_\eta^2 A_\eta(\eta) = 8\pi D(|c_F|)^2,$$

(45)

where $D(|c_F|)$ is the displacement field in the compactified region; this leads to a metric of the anti-de Sitter form for a two-dimensional space factor:

$$A_\eta(\eta) = 1 + 4\pi D^2 \eta^2.$$  

(46)

Equation (42) then becomes

$$\dot{\eta}^2 = E^2 - 1 + (\Delta^2 - 4\pi D^2)\eta^2 - 2\Delta E\eta,$$

(47)

which can be cast onto the form

$$\dot{\eta}^2 + (4\pi D^2 - \Delta^2)\left(\eta + \frac{\Delta E}{4\pi D^2 - \Delta^2}\right)^2 = \frac{4\pi D^2 E^2}{4\pi D^2 - \Delta^2} - 1.$$  

(48)

Going back to equation (32), the charge of the time-like shell, call it $K$, will be determined by the flux produced only by the tube region $D(|c_F|)$. Using $D_0 = 0$, we have $D_{\text{eff}} = \eta - q = -K/4\pi a^2$. Note that $D$ and therefore $q$ must be different from zero since equation (16), together with $A_{\text{eff}} = 0$, implies $|c_F| > f/\sqrt{2}$; otherwise, the radii $a$ will be ill-defined or infinite.

Using now the condition $A_{\text{eff}} = 0$, we obtain from (16)

$$\frac{1}{a^2} = 4\pi \left(|c_F| - \frac{f}{\sqrt{2}}\right) \left(|c_F| + \frac{f}{\sqrt{2}}\right).$$

(49)

Expressing $|c_F|$ in terms of $D$ and then $D$ in terms of $|K|$, and taking into account the signs of $c_F$ and $q$, we see that (49) provides a quadratic equation for the charge $|K|$ in terms of $a$ and $f$:

$$0 = \frac{|K|^2}{4\pi a^2} + \sqrt{2}f|K| - 1,$$

(50)

which has a positive solution for all possible values of those parameters, since the discriminant of such a quadratic equation is manifestly positive.

This completes the proof that there are indeed consistent, horned particle solutions where all the electric flux of the charged particle at the throat flows into the tube region; furthermore, these are the only finite-energy solutions.
Figure 1. An illustration of the possible solutions. (a) We have a stable solution with the ‘energy’ bounded $E^2 > 1 - \left(\frac{\Delta}{\sqrt{4\pi D}}\right)^2 \equiv E_{\text{min}}^2$. (b) An unstable solution is given. In this case, there is no bound on the ‘energy’ since $\frac{4\pi D^2}{4\pi D^2 - \Delta^2} - 1 < 0$ for all values of $E$.

6. Discussion and perspectives for future research

The charge-hiding effect by a wormhole, which was studied for the case where the gravity/gauge-field system is self-consistently interacting with a charged LLB as a matter source, is now studied for the case of a time-like brane. From the demand that no surfaces of infinite coordinate time redshift appear in the problem, we are now led to a horned-like
particle where the horn region of the particle is completely accessible from the outside region containing the $r \to \infty$ region and vice versa, according to not only the traveller that goes through the shell (as was the case for the LLB), but also to a static external observer. This requires negative surface energy density for the shell sitting at the throat. We study a gauge-field subsystem that is of a special nonlinear form containing a square-root of the Maxwell term and that previously has been shown to produce a QCD-like confining gauge-field dynamics in flat spacetime. The condition of finite energy of the system or asymptotic flatness on one side of the horned particle implies that the charged object sitting at the throat expels all the flux it produces into the other side of the horned particle, which turns out to be of a ‘tube-like’ nature. An outside observer in the asymptotically flat universe detects, therefore, an apparently neutral object. The hiding of the electric flux in the horn region is the only possible way that a truly charged particle can still be of finite energy, which points to the physical relevance of such solutions, even though there is the need of negative energy density at the throat, which can be of quantum-mechanical origin.

In addition to the ‘hiding’ effect, one can also study the ‘confinement’ [65], where instead of considering just the case of a matching of an external uncompactified region with a compactified tube region, we consider two asymptotically flat regions connected by a tube region, and at the two points where we match the corresponding uncompactified to the tube region we have a brane; the system will contain a brane plus associated ‘antibrane’ at the other matching point. In the case where LLB are used at the matching points, the branes are, classically at least, located at fixed coordinate locations [65]; for time-like branes, this will not be generically the case, so the brane could in principle collide with its antibrane. These possibilities will be studied in a future publication.

We have seen, for zero $Q$, that in the exterior region any configuration satisfying $\mathcal{L}'(F^2) = 0$ (or $D = 0$ in the electrostatic case) is a solution of the gauge-field equation, obtaining in the vacuum region a ‘disordered ferroelectric state’. This degeneracy of the outside state is good from the point of view of the high entropy content of the configuration, since it means that there is a great many ways, infinite indeed, to achieve this matching with zero Coulomb field outside. This subject and its presumably favorable consequences for the stability of this vacuum state with gauge-field condensation will be studied in future publications.

Going back to the hiding effect studied in this paper, it is interesting to consider now whether these solutions where the charged particle expels the flux exclusively in the direction of the tube region can take place in nature, or whether this is just a mathematical exercise. This
question naturally relates to whether the negative energy density at the throat is physically realizable. To start with, we know that negative energy densities can be achieved through quantum corrections, as it has been discussed in [66, 67]. These authors have however found, studying some field theory models, that to build up regions of negative energy density, one must 'pay' by building compensating regions with positive energy density elsewhere. More generally, this means that one cannot arbitrarily assign some negative energy density to some region of space and just blame quantum fluctuations for that. For explicit calculations showing that quantum fluctuations can be the origin of negative energy densities, see [68, 69].

It is interesting that the gauge-field model that produces confinement may give the possibility of obtaining negative energy densities. In the solutions studied so far in this paper, this has not been the case because the electric fields involved in all of the solutions considered in this paper are stronger than the vacuum value; recall for example that in the tube region $|c_F| > \frac{f}{\sqrt{2}}$. If we look at lower field strengths, if absolute values lower than the vacuum value could somehow be obtained (maybe as a result of quantum fluctuations) then negative energy densities could result. To see this, just consider the flat space situation and then recall that for static-field configurations, the model (1) yields the following electric displacement field:

$$\vec{D} = \vec{E} - \frac{\mu}{|\vec{E}|} \vec{E}.$$  

The pertinent energy density for the electrostatic case turns out to be $\frac{1}{2} \vec{E}^2$, and for the case, $\vec{E}$ and $\vec{D}$ point in opposite directions, which is satisfied if $E = |E| < \frac{f}{\sqrt{2}}$;

then, $\frac{1}{2} E^2 = \frac{1}{2} D^2 - \frac{\mu}{\sqrt{2}} D + \frac{\mu^2}{4}$, so that the term linear w.r.t. $D = |D|$ is negative now. For low values of $D$, this dominates over the quadratic contribution, and therefore we get an energy density lower than that of the vacuum state, which is the state with $E = |E| = \frac{f}{\sqrt{2}}$ (or $D = 0$). With the appropriate bare cosmological constant chosen here, the vacuum state vacuum energy density is zero; so, the low electric field configurations then produce a negative vacuum energy density. However, this way of achieving negative energy densities can be used, to achieve horned particle hiding charge remains an interesting subject for future research.

In the case of the charge hiding in a horned particle studied here, we have the special situation where the hiding of the electric flux behind the horn region, and no flux of $D$ going in the outside region, is the only possible way by which a truly charged particle (in a model with confining dynamics) can still be of finite energy while still remaining truly charged, which points to the physical relevance of such solutions, even though there is the need of negative energy density at the throat, which can be of quantum-mechanical origin or just classical (by means of low electric field strengths). One can then argue that a variational approach to the problem, based on minimization of energy, must produce, indeed select, the appropriate state that gives rise to the necessary negative energy density at the throat of the horned particle so that the flux lines can now be redirected into the horn region and make the finite-energy solution possible.

Acknowledgment

We would like to thank A Kaganovich, E Nissimov, S Pacheva for very useful conversations.

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