Photon structure functions with heavy particle mass effects

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Abstract

In the framework of the perturbative QCD we investigate heavy particle mass effects on the unpolarized and polarized photon structure functions, $F^2_\gamma$ and $g^\gamma_1$, respectively. We present our basic formalism to treat heavy particle mass effects to NLO in perturbative QCD. We also study heavy quark effects on the QCD sum rule for the first moment of $g^\gamma_1$, which is related to axial anomaly. The photon structure function in supersymmetric QCD is also briefly discussed.

Keywords: QCD, photon structure function, heavy particle mass effects, sum rule, $g^\gamma_1$, axial anomaly

1. Introduction

In this talk I would like to address the question about how to incorporate the heavy particle mass effects into the photon structure functions. This talk is based on the works done in collaboration with Yoshio Kitadono, Ryo Sahara, Ken Sasaki, Takahiro Ueda and Yutaka Yoshida.

Now let me make some remarks on why the photon structure is so interesting. First of all, it provides a good probe to study the QCD dynamics in perturbation theory. The unpolarized virtual photon structure function $F^2_\gamma$ was studied up to the next-to-next-to-leading order (NNLO) and the polarized virtual photon structure function $g^\gamma_1$ was investigated to next-to-leading order (NLO).

The QCD sum rule for the first moment of $g^\gamma_1$ has attracted much attention in the literature since it is related to the axial anomaly. In this talk we investigate the heavy particle mass effects on the photon structure functions including the polarized structure function.

Here we investigate two-photon processes (Figure 1) with the kinematical region where the mass squared of the probe photon ($Q^2$) is much larger than that of the target photon ($P^2$) which is in turn much bigger than the $\Lambda^2_{\text{QCD}}$, the QCD scale parameter squared. The advantage for studying the virtual photon target is that we can calculate whole structure functions up to next-leading-order (NLO), in contrast to the real photon target where there remain uncalculable non-perturbative pieces.

2. QCD calculation of photon structure functions

The structure function $F^2_\gamma(x, Q^2)$ for the real photon target ($P^2 = 0$) was first studied in the parton model \cite{1,2} and then studied to the leading order (LO) QCD based on the operator product expansion (OPE) in ref.\cite{3}. The next-to-leading order QCD computation of $F^2_\gamma(x, Q^2)$ was performed in the OPE and Renormalization Group (RG) method by Bardeen and Buras \cite{4} and in the QCD-improved parton model by Glück and Reya \cite{5} and Fontannaz and Pilon \cite{6} and by other people quoted in ref.\cite{7}.

For a virtual photon target ($\Lambda^2_{\text{QCD}} \ll P^2 \ll Q^2$) the unpolarized photon structure function $F^2_\gamma(x, Q^2, P^2)$ to LO in QCD was studied in ref.\cite{8} and to NLO in ref.\cite{9}.

The master formula for the $n$-th moment to NLO is...
given by
\[
\int_0^1 dx x^{n-2} F'_2(x, Q^2, P^2) = \frac{\alpha_s}{4\pi} \frac{1}{2\beta_0} \times \left[ \sum_{i=+,-,NS} L^0_i \left( \frac{4\pi}{\alpha_s(Q^2)} \left( 1 - \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\frac{n}{2}\beta_0 + 1} \right) + \sum_{i=+,-,NS} \mathcal{A}^0_i \left( 1 - \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\frac{n}{2}\beta_0 + 1} + \sum_{i=+,-,NS} B^0_i \left( 1 - \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\frac{n}{2}\beta_0 + 1} + C^n + O(\alpha_s) \right],
\]
where \( L^0_i, \mathcal{A}^0_i, B^0_i \) and \( C^n \) are computed from the one- and two-loop anomalous dimensions together with one-loop coefficient functions. All of them are shown to be renormalization-scheme independent. \( \alpha_s(Q^2) \) is the QCD running coupling constant, \( \beta_0 \) is the one-loop beta function, and \( \lambda^0_0 (i = +, -, NS) \) denote the eigenvalues of one-loop anomalous dimensions \( \lambda^0_{ij} (i, j = \psi, G) \).

### 3. Heavy quark mass effects

We compute the deviation arising from heavy quark mass effects on the photon matrix elements of twist-2 quark and gluon operators and the corresponding coefficient functions to NLO QCD \([10]\).

The moment of \( F'_2 \) can be decomposed as
\[
M'_2(n, Q^2, P^2, m^2) \equiv \int_0^1 dx x^{n-2} F'_2(x, Q^2, P^2, m^2)
= M'_2(n, Q^2, P^2, m^2 = 0) + \Delta M'_2(n, Q^2, P^2, m^2),
\]
where the additional moment due to mass effects reads
\[
\Delta M'_2(n, Q^2, P^2, m^2) = \int_0^1 dx x^{n-2} \Delta F'_2(x, Q^2, P^2, m^2)
= \frac{\alpha_s}{4\pi} \frac{1}{2\beta_0} \left[ \sum_{i=+,-,NS} \Delta \mathcal{A}^0_i \left( 1 - \frac{\alpha_s(Q^2)}{\alpha_s(P^2)} \right)^{\frac{n}{2}\beta_0 + 1} + \Delta C^n \right] + O(\alpha_s),
\]
define the deviations of the quantities turn out to be
\[
\Delta \mathcal{A}^0_{NS} = -12\beta_0 e_H^2 (e_H^2 - \langle e_H^2 \rangle_n) (\Delta \lambda^0_{NS}/n_f),
\Delta \mathcal{A}^0_i = -12\beta_0 e_i^2 (e_i^2 - \langle e_i^2 \rangle_n) (\Delta \lambda^0_i (n_f),
\Delta B^0_{NS} = \Delta B^0_i = 0, \quad \Delta C^n = 12\beta_0 e_i^2 (\Delta \lambda^0_i / n_f) - \lambda^0_i - \lambda^0_i,
\]
where \( e_H \) is the heavy-quark charge and the average charge squared is defined as \( \langle e^2 \rangle_n = \sum_n e_i^2 / n_f \) with \( n_f \) being the number of active flavors and the deviation of operator matrix element \( \Delta \lambda^0_{NS}/n_f \) is
\[
\Delta \lambda^0_{NS}/n_f = 2 \left[ \frac{n^2 + n + 2}{n(n+1)(n+2)} \ln \frac{m_2}{P^2} + \frac{1}{n} - \frac{1}{m^2} \right].
\]

The Figure 2 shows the theoretical evaluation with charm quark mass effects compared to the PLUTO’s experimental data \([11]\) for the effective structure function \( F^{\gamma}_{eff}(x, Q^2, P^2) = F'_2(x, Q^2, P^2) + (3/2) F'_2(x, Q^2, P^2) \), where \( F'_2 \) is the longitudinal structure function. The theoretical prediction (red curve) with the charm quark mass effects shows the trend of reducing the massless QCD calculation (purple curve) and becomes consistent with the experimental data. Also shown is the theoretical curve (blue curve) with the resummation prescription given in \([12]\).

![Figure 2](image-url)

Figure 2: \( F^{\gamma}_{eff} \) to NLO in QCD with charm quark mass effects compared to PLUTO data for \( n_f = 4, Q^2 = 5 GeV^2, P^2 = 0.35 GeV^2, m_c = 1.3 GeV \) \([13]\).

### 4. Polarized photon structure function

We now turn to the polarized photon structure function \( g'_{1} \). Let us first consider massless quark contributions to the photon structure functions.

For a real photon target \((P^2 = 0)\), Bass, Brodsky and Schmidt have shown that the 1st moment of \( g'_{1}(x, Q^2) \) vanishes to all orders of \( \alpha_s(Q^2) \) in QCD \([13]\):
\[
\int_0^1 dx g'_{1}(x, Q^2) = 0.
\]

Now the question is what about the \( n = 1 \) moment of the virtual photon case. By replacing the \( x^{n-2} F'_2(x, Q^2, P^2) \)
with $x^{-1}g_1^0(x, Q^2, P^2)$ in \([1]\) and taking $n \to 1$ limit, the first three terms vanish. Denoting $e_i$, the $i$-th quark charge and $n_f$, the number of active flavors, we have

$$\int_0^1 dx g_1^0(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} e_i^4 + {\cal O}(\alpha^2_e) \ . \ (5)$$

We can now go to $\cal O(\alpha\alpha_e)$ corrections which turn out to be \([7]\):

$$\int_0^1 dx g_1^0(x, Q^2, P^2) = -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} e_i^4 \left(1 - \frac{\alpha_s(Q^2)}{\pi}\right) - \frac{2}{\beta_0} \left(\sum_{i=1}^{n_f} e_i^4 \right)^2 \left(\frac{\alpha_s(P^2)}{\pi} - \frac{\alpha_s(Q^2)}{\pi}\right) + {\cal O}(\alpha_e^2) . \ (6)$$

This result coincides with the one obtained by Narison, Shore and Veneziano \([13]\), apart from the overall sign for the definition of $g_1^f$. We then extended this NLO result to the next-to-next-to-leading order (NNLO) calculation, which is $\cal O(\alpha_s^2)$, where we need the 3-loop anomalous dimensions as well as 2-loop coefficient functions \([15]\).

The same formalism can be applied to the heavy quark mass effects as in the case of unpolarized structure functions. Of course we have to replace the one- and two-loop anomalous dimensions as well as the one-loop coefficient functions by the polarized counterparts. In addition we put the operator matrix elements with the photon states. Namely

$$\Delta \tilde{\Lambda}_W^N \left(\frac{1}{n_f}\right) = \left[1 - \frac{n - 1}{n(n + 1)} \ln \frac{m^2}{\mu^2} + \frac{1}{n} + \frac{1}{n^2}\right] - \frac{4}{(n + 1)^2} \left[\frac{n - 1}{n(n + 1)} \sum_{j=1}^{n_f} \frac{1}{j}\right] . \ (7)$$

Let us first consider the heavy quark mass effects on the first moment of the virtual photon structure function $g_1^f(x, Q^2, P^2)$. To the lowest order the first moment is given by

$$\int_0^1 dx g_1^f(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} C_{\alpha e}^{n=1} + {\cal O}(\alpha^2_e). \ (8)$$

For the massless case we have

$$C_{\alpha e}^{n=1} = 12\beta_0 \langle e_i^4 \rangle_n (\beta_0^N + \Delta \tilde{\Lambda}_W^N)_{n=1} = -24\beta_0 \langle e_i^4 \rangle_n , \ (9)$$

where $\langle e_i^4 \rangle_n = \sum_{j=1}^{n_f} e_i^4 / n_f$. Now in the presence of the heavy quark, the $n_f$-th flavor, with the charge $e_{n_f} = e_H$,

$$\Delta C_{\alpha e}^{n=1} = 24\beta_0 e_H^4 . \ (10)$$

Hence we find that the heavy quark mass effects lead to

$$\int_0^1 dx g_1^f(x, Q^2, P^2) = \frac{\alpha}{4\pi} \frac{1}{2\beta_0} (C_{\alpha e}^{n=1} + \Delta C_{\alpha e}^{n=1})$$

$$= -\frac{3\alpha}{\pi} \sum_{i=1}^{n_f} e_i^4 + {\cal O}(\alpha^2_e) \ . \ (11)$$

Thus the heavy quark decouples from the sum rule. This argument can be extended to the order $\cal O(\alpha_s\alpha_e)$ and the sum rule becomes eq.(5) where $n_f$ is replaced by $n_f - 1$.

We now investigate the heavy quark mass effects on the virtual photon structure function $g_1^f(x, Q^2, P^2)$ in the case of $Q^2 = 20\text{GeV}^2$ and $P^2 = 0.35\text{GeV}^2$, where $n_f = 4$ and the heavy quark is the charm quark with mass squared $m_c^2 = 2.25\text{GeV}^2$.

![Figure 3: $g_1^f(x, Q^2, P^2)$ in units of $3n_f/\alpha^2_e N \ln Q^2/P^2$ with and without the charm quark mass effects for $n_f = 4$, $Q^2 = 20\text{GeV}^2$, $P^2 = 0.35\text{GeV}^2$](figure3)

![Figure 4: $g_1^f(x, Q^2, P^2)$ in units of $3n_f/\alpha^2_e N \ln Q^2/P^2$ with and without the bottom quark mass effects for $n_f = 5$, $Q^2 = 120\text{GeV}^2$, $P^2 = 3.7\text{GeV}^2$](figure4)
Note that the polarized structure function for the massive case (red curve) is suppressed in the large $x$ region compared to the massless case (blue curve) and becomes less negative for small $x$ region when the charm quark mass effect is taken into account. Varying the values of $Q^2$ and $P^2$ for the kinematical region where the charm quark can be regarded as the heavy quark we get the similar behaviors.

We have also shown in Figure 4, the structure function for the the case of $Q^2 = 120\text{GeV}^2$ and $P^2 = 3.7\text{GeV}^2$, where $n_f = 5$ and the heavy quark is the bottom quark with mass squared $m_b^2 = 20.25\text{GeV}^2$. We note that the deviation from massless case is almost negligible, since the charge factor is much smaller compared to the charm case i.e. $e_b^2 = (1/16)e_c^2$.

We have also applied our formalism for the photon structure functions in supersymmetric QCD (SQCD) where there exist squarks and gluinos in addition to the quarks and gluons. The squark contribution in the parton model was studied in [17]. For the SQCD radiative corrections, we based our argument on the parton distribution functions (PDF’s) which satisfy the DGLAP-type evolution equation. The heavy mass effects can be incorporated by the boundary conditions where we require the PDF for each particle to vanish at $Q^2 = m_i^2$ ($i=$gluino, squark, heavy quark). By solving the coupled boundary conditions we can determine the initial conditions and then we get the solution for the moment of the each PDF. By performing the inverse Mellin transform we finally get structure functions. In Figure 5 we have plotted $F_2^g(x,Q^2,P^2)$ as a function of $x$. There we have also taken into account the threshold effects $0 \leq x \leq x_{\text{max}}$ with $x_{\text{max}} = 1/(1 + P^2/Q^2 + 4m_x^2/Q^2)$.

We note that at small $x$ there is no significant difference between massless and massive QCD, while there exists a large difference between massless and massive SQCD. At large $x$, the significant mass-effects exist both for non-SUSY and SUSY QCD. The SQCD case is seen to be much suppressed at large $x$ compared to the QCD.

5. Conclusion

In this talk we discussed the heavy particle mass effects for the virtual photon structure functions which are calculable by perturbative QCD (SQCD). Here we are particularly interested in evaluation of unpolarized and polarized structure functions, $F_2^g$ and $g_1^\gamma$, with the heavy particle mass effects. We first discussed our basic framework of studying heavy quark mass effects for $F_2^g$. We then present the analysis of the polarized photon structure function $g_1^\gamma$ in two aspects; QCD sum rule of the first moment which is related to the axial anomaly as well as a the $x$ dependence of the structure function. We have shown that the heavy particle decouples from the QCD sum rule. The analysis shows that the large $x$ region of $g_1^\gamma(x, Q^2, P^2)$ is suppressed in the presence of the heavy quark effects. We also investigated the photon structure functions in supersymmetric QCD. Experimental confrontation in the future is anticipated.

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References

[1] T. F. Walsh and P. M. Zerwas, Phys. Lett. B44 (1973) 195.
[2] R. L. Kingsley, Nucl. Phys. B60 (1973) 45.
[3] E. Witten, Nucl. Phys. B120 (1977) 189.
[4] W. A. Bardeen and A. J. Buras, Phys. Rev. D20 (1979) 166.
[5] M. Glück and E. Reya, Phys. Rev. D28 (1983) 2749.
[6] M. Fontannaz and E. Pilon, Phys. Rev. D45 (1992) 382.
[7] K. Sasaki and T. Uematsu, Phys. Rev. D59 (1999) 114011.
[8] T. Uematsu and T. F. Walsh, Phys. Lett. B101 (1981) 263.
[9] T. Uematsu and T. F. Walsh, Nucl. Phys. B199 (1982) 93.
[10] Y. Kitadono, K. Sasaki, T. Ueda and T. Uematsu, Prog. Theor. Phys. 121 (2009) 054019; Phys. Rev. D81 (2010) 074029.
[11] Ch. Berger et al., Phys. Lett. B142 (1984) 119.
[12] Y. Kitadono, R. Sahara, T. Ueda and T. Uematsu, Eur. Phys. J. C70 (2010) 999.
[13] S. D. Bass, S. J. Brodsky and I. Schmidt, Phys. Lett. B437 (1998) 417.
[14] S. Narison, G. M. Shore and G. Veneziano, Nucl. Phys. B391 (1993) 69.
[15] K. Sasaki, T. Ueda and T. Uematsu, Phys. Rev. D73 (2006) 094024.
[16] T. Uematsu, in preparation.
[17] Y. Kitadono, Y. Yoshida, R. Sahara and T. Uematsu, Phys. Rev. D84 (2011) 074031.
[18] R. Sahara, T. Uematsu and Y. Kitadono, Phys. Lett. B707 (2012) 517.