Effective field theory investigations of the XYZ puzzle

Jorge Segovia
Physik-Department, Technische Universität München,
James-Franck-Str. 1, 85748 Garching, Germany
E-mail: jorge.segovia@tum.de

Abstract. Quantum Chromodynamics, the theory of strong interactions, predicts several types of bound states. Among them are conventional mesons \((q\bar{q})\) and baryons \((qqq)\), which have been the only states observed in experiments for years. However, in the last decade, many states that do not fit this picture have been observed at \(B\)-factories (BaBar, Belle and CLEO), at \(\tau\)-charm facilities (CLEO-c, BESIII) and also at proton-proton colliders (CDF, D0, LHCB, ATLAS, CMS). There is growing evidence that at least some of the new charmonium- and bottomonium-like states, the so-called XYZ mesons, are new forms of matter such as quark-gluon hybrids, mesonic molecules or different arrangements of tetraquarks, pentaquarks... Effective Field Theories (EFTs) have been constructed for heavy-quark-antiquark bound states, but a general study of the XYZ mesons within the same framework has not yet been done. The scope of this conference proceedings is to discuss the possibilities we have in developing novel EFTs that, characterizing the conventional quarkonium states, facilitate also the systematic and model-independent description of the new exotic matter, in particular, the hybrid mesons.

1. Introduction
One of the basic properties of Quantum Chromodynamics (QCD) is its spectrum: the list of particles that are stable or at least sufficiently long-lived to be observed as resonances. The elementary constituents in QCD are quarks \((q)\), antiquarks \((\bar{q})\), and gluons \((g)\), and QCD requires them to be confined into color-singlet clusters called hadrons. The most stable hadrons are the clusters predicted by the quark model [1, 2], conventional mesons \((q\bar{q})\) and baryons \((qqq)\), which have been the only states observed in experiments for years [3]).

A decade ago, the Belle Collaboration discovered an unexpected enhancement at 3872 MeV in the \(\pi^+\pi^-J/\psi\) invariant mass spectrum while studying the reaction \(B^+ \rightarrow K^+\pi^+\pi^-J/\psi\) [4]. The \(X(3872)\) state was later studied by CDF, D0, and BaBar collaborations confirming that its quantum numbers, mass and decay patterns make it an unlikely conventional charmonium candidate. Therefore, the simple picture that had been so successful for 30 years was challenged leading to an explosion of related experimental activity. The number of new collectively denoted XYZ states has increased dramatically. Almost two dozen charmonium- and bottomonium-like XYZ states have forced an end to the era when heavy quarkonium was considered as a relatively well established heavy quark-antiquark bound system (see, e.g., reviews [5, 6] for more details).

The ultimate goal of theory is to describe the properties of the XYZ states from QCD’s first principles. However, since this task is quite challenging, a more modest goal to start with is the development of QCD motivated phenomenological models that specify the colored constituents, how they are clustered and the forces between them. For this reason, at the same time that...
many experimental measurements on XYZ physics were performed, theorists proposed different kind of exotic hadrons to classify them:

1) A Quarkonium hybrid, which has constituents $Q\bar{Q}g$, where $g$ is a constituent gluon. The $Q\bar{Q}$-pair is in a color-octet state because the color charge of a gluon is octet.

2) A Tetraquark, which consists on one heavy quark ($Q$), one heavy antiquark ($\bar{Q}$), one light quark ($q$) and one light antiquark ($\bar{q}$) whose different arrangements allowed by QCD give:
   2.1) A meson molecule [7], which consists of color-singlet ($Q\bar{q}$)$_1$ and ($\bar{Q}q$)$_1$ mesons bound by pseudoscalar-meson-exchange interactions.
   2.2) A diquarkonium [8], which consists of a color-antitriplet ($Qq$)$_{-3}$ diquark and a color-triplet ($\bar{Q}\bar{q}$)$_3$ anti-diquark bound by gluon exchanges.
   2.3) A hadro-quarkonium [9], which consists of a color-singlet ($q\bar{q}$)$_1$-pair bound through color van der Waals forces to a compact color-singlet ($Q\bar{Q}$)$_1$-pair.
   2.4) A quarkonium adjoint meson or Born-Oppenheimer tetraquark [10], which consists of two bounded color-octets, ($Q\bar{Q}$)$_8$ and ($q\bar{q}$)$_8$, with dynamics similar to that of hybrids.
   2.5) A compact tetraquark [11], in which the constituents do not form any distinguishable cluster but form as a whole a color singlet state: ($Q\bar{Q}q\bar{q}$)$_1$.

3) A Pentaquark and so on, which are the various multiquark configurations allowed by QCD that one can imagine and have not been already cited.

Thus far, the phenomenological descriptions specified above have produced compelling explanations for some of the XYZ states but not for all of them in an unified way. It would be desirable to have a single theoretical framework based firmly on QCD that describes all the XYZ mesons.

2. Nonrelativistic effective field theories for conventional

Heavy quarkonium systems are characterized by their nonrelativistic nature, i.e. the heavy quark bound-state velocity, $v$, satisfies $v \ll 1$. This is reasonably fulfilled in bottomonium ($v^2 \sim 0.1$) and to a certain extent in charmonium ($v^2 \sim 0.3$). Moreover, at least, three widely separated scales appear: the heavy quark mass $m$ (hard scale), the relative momentum of the bound state $p \sim mv$ (soft scale) and the binding energy $E \sim mv^2$ (ultrasoft scale). With $v \ll 1$, the following hierarchy of scales

$$m \gg p \sim 1/r \sim mv \gg E \sim mv^2$$  \hspace{1cm} (1)

is satisfied and this allows for a description in terms of EFTs of physical processes taking place at one of the lower scales. The integration out of modes associated with high-energy scales is performed as part of a matching procedure that enforces the equivalence between QCD and the EFT at a given order of the expansion in $v$. The final result is a factorization at the Lagrangian level between the high-energy modes, which are encoded in the matching coefficients, and the low-energy contributions carried by the dynamical degrees of freedom.

2.1. Physics at the scale $m$: NRQCD

The suitable EFT to describe heavy quarkonium annihilation and production, which take place at the scale $m$, is Nonrelativistic QCD (NRQCD) [12, 13]. It follows from QCD by integrating out the high energy modes of order $m$. As a consequence, the effective Lagrangian is organized as an expansion in $1/m$ and $\alpha_s(m)$:

$$\mathcal{L}_{\text{NRQCD}} = \sum_n \frac{c_n(\alpha_s(m), \mu)}{m^n} \times \mathcal{O}_n(\mu, mv, mv^2, \ldots),$$ \hspace{1cm} (2)

where $c_n$ are the Wilson coefficients that encode the contributions from the scale $m$, $\mu$ is the NRQCD factorization scale, and $\mathcal{O}_n$ are the low-energy operators constructed out of two or
four heavy-quark/antiquark fields plus gluons. The matrix elements of $O_n$ depend on the scales $\mu, mv, mv^2$ and $\Lambda_{\text{QCD}}$. Thus, the operators are counted in powers of $v$. The imaginary part of the coefficients of the four-fermion operators contains the information on heavy quarkonium annihilation and production.

The NRQCD heavy quarkonium Fock state is given by a series of terms, where the leading term is a $Q\bar{Q}$ in a color-singlet state, and the first correction, suppressed in $v$, comes from a $Q\bar{Q}$ in an octet state plus a gluon. Higher-order terms are subleading in increasing powers of $v$.

2.2. Physics at the scale $mv$: pNRQCD

The suitable EFT to describe heavy quarkonium formation, which takes place at the scale $mv$, is potential NRQCD (pNRQCD) [14, 15]. It follows from NRQCD by integrating out the modes of order $mv \sim 1/r$. The specific construction details of pNRQCD are slightly different depending upon the relative size between the soft, $mv$, and the confinement, $\Lambda_{\text{QCD}}$, scales. Two main situations are distinguished. When $mv \gg \Lambda_{\text{QCD}}$, we speak about weakly-coupled pNRQCD because the matching of NRQCD to pNRQCD may be performed in perturbation theory. When $mv \sim \Lambda_{\text{QCD}}$, we speak about strongly-coupled pNRQCD because the matching of NRQCD to pNRQCD is not possible in perturbation theory.

When the soft scale is perturbative, the effective pNRQCD Lagrangian is organized as an expansion in $1/m$ and $\alpha_s (m)$, inherited from NRQCD, but also as a multipole expansion in $r$:

$$L_{\text{pNRQCD}} = \int d^3 r \sum_n \sum_k \frac{c_n(\alpha_s (m), \mu)}{m^n} V_{n,k}(r, \mu', \mu) r^k \times O_k(\mu', mv^2, \ldots),$$

(3)

where $O_k$ are the operators of pNRQCD that depend on the scales $\mu', mv^2$ and $\Lambda_{\text{QCD}}$; the pNRQCD factorization scale is $\mu'$, and $V_{n,k}$ are the Wilson coefficients that encode the contributions from the scale $r$.

The degrees of freedom that make up the operators $O_k$ are $Q\bar{Q}$-pairs in a color-singlet or a color-octet state, and ultrasoft gluons. As one can see in Eq. (3), the Wilson coefficients $V_{n,k}$ depend on the relative distance $r$ between the quark and the antiquark. They are potential-like terms when $k = 0$ and are the $1/m^n$ potentials that enter in a Schrödinger-type equation. Non-potential interactions, $V_{n,k} \neq 0$, account for singlet to octet transitions via ultrasoft gluons and provide loop corrections to the leading potential picture. They are typically related to nonperturbative effects.

3. Non-relativistic effective field theories for exotics

The above EFTs have proven to be very successful for the description of conventional heavy quarkonium states which are below the open-flavour threshold ($D\bar{D}$ ($B\bar{B}$) in the charmonium (bottomonium) sector). However, no EFT description has yet been constructed for those states which close to or above threshold because the dynamical situation changes drastically [16, 17].

The threshold regions are actually the most interesting ones because many of the charmonium- and bottomonium-like XYZ states are located in such regions. Ab-initio lattice-regularized QCD computations are also in trouble when dealing with threshold regions. Moreover, calculations of excited states using this technique have been only recently pioneered and the full treatment of bottomonium on the lattice seems to be tricky. This all together explains why many of our expectations for the XYZ states still rely on phenomenological models.

Below we examine how things change when studying one particular case of the XYZ particles: the quark-gluon hybrids, and elucidate the future steps to be done towards an EFT description of them. It is important to keep in mind that the XYZ states belong to heavy quark sectors and thus the heavy quark mass is still an appropriate parameter from which begin a nonrelativistic expansion.
3.1. An example: hybrid states

Consider the case in which a system is made by a heavy quark, a heavy antiquark and gluonic excitations aiming to describe heavy quarkonium hybrids. In the static limit, at and above the AQCD threshold, a tower of hybrid static energies (i.e. of gluonic excitations) must be considered on top of the \(Q\bar{Q}\) static singlet energy [18, 19]. The spectrum has been thoroughly studied in lattice NRQCD [20] through the logarithm of large time generalized static Wilson loops divided by the interaction time:

\[
E_n^{(0)}(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle X_n, T/2|X_n, -T/2 \rangle ,
\]

where the NRQCD initial and final states can be constructed as follows

\[
|X_n\rangle = \chi(x_2)\phi(x_2, R)T^n P_n(R)\phi(R, x_1)\psi^\dagger(x_1)|\text{vac}\rangle .
\]

with \(\phi(x_2, x_1)\) a Wilson line from \(x_1\) to \(x_2\), and \(P_n\) is some gluonic operator that generates the desired quantum numbers \(n\) to calculate the static energy. A list of possible operators \(P_n\) can be found, e.g., in Refs. [15, 21].

Berwein et al., based on Refs. [15, 21, 22, 23], have set in Ref. [24] the first steps towards a pNRQCD characterization of the gluonic energies and thus of the hybrid potential. The matching between NRQCD and pNRQCD allows to establish that the spectrum of the hybrid static energies is described at short distances and in the leading multipole expansion of pNRQCD by the octet potential plus a mass scale called gluelump mass:

\[
E_n^{(0)}(r) = \lim_{T \to \infty} \frac{i}{T} \log \langle X_n, T/2|X_n, -T/2 \rangle = V_o(r) + \Lambda_H + O(r^2) .
\]

Equation (6) can be systematically improved by calculating higher orders in the multipole expansion. In particular, one can look at how the symmetry group of the hybrid system at short distances \(O(3) \otimes C\) is softly broken to the symmetry group that characterized the gluonic static energies in NRQCD \(D_{\infty h}\). We know that the leading correction coming from the multipole expansion is at \(O(r^2)\) and can be calculated in pNRQCD in terms of nonperturbative correlators to be eventually evaluated on the lattice or in QCD vacuum models. This correction has been treated phenomenologically in Ref. [24] and its computation will be one of our future goals.

Another of our goals will be going beyond the static limit and compute \(1/m^n\) terms with \(n = 1, 2, \ldots\). For instance, the NRQCD Hamiltonian for the one-quark-one-antiquark sector of the Fock space reads

\[
H_{\text{NRQCD}} = H^{(0)} + \frac{1}{m}H^{(1)} + \ldots ,
\]

\[
H^{(0)} = \int d^3x \frac{1}{2}(E^a \cdot E^a + B^a \cdot B^a) - \sum_{j=1}^{n_f} \int d^3x \bar{q}_j i\gamma D \cdot q_j ,
\]

\[
H^{(1)} = -\frac{1}{2} \int d^3x \psi^\dagger (D^2 + g_c F \sigma \cdot B) \psi + \frac{1}{2} \int d^3x \chi^\dagger (D^2 + g_c F \sigma \cdot B) \chi ,
\]

where we have shown only terms up to order \(1/m\) in the quark mass expansion, \(\psi (\chi^\dagger)\) is the Pauli spinor field that annihilates the heavy quark (antiquark), \(q_j\) is a massless quark of flavor \(j\), and \(c_F\) is a matching coefficient. We are not considering the light quarks as external dynamical sources but they can still appear in the form of sea quarks.

A partial computation of the \(1/m\) contributions has been performed in Ref. [24]: the kinetic part of \(H^{(1)}\) was included whereas its spin dependent term was ignored. This has produced spin-multiplets which compare nicely with lattice-regularized QCD results (see, for instance, Fig. 5 of...
Ref. [24]), providing the necessary support for continuing in this direction. It would be valuable to break the spin degeneracy and give a more detailed structure to the hybrid multiplets.

Finally, let us state here that our long term goal is to introduce an EFT description of heavy quarkonium hybrids without using the multipole expansion. At large distances, the dynamics of the system is nonperturbative but still reduces to a quantum mechanical problem with a number of potentials organized in powers of $1/m$ [25]. This would entail the definition of appropriate generalized Wilson loops that encode the dynamics of the nonperturbative matrix elements and obtaining in strongly-coupled pNRQCD the dynamical equations that couple them.

4. Conclusions

The search of exotic matter is a topic that has fascinated all generations of nuclear and particle physicists since the establishment of QCD as the theory of the strong interaction. In this respect, Europe is situated in a privileged position for the next decade(s) with two major experiments scheduled: LHCb@LHC and PANDA@FAIR. In support of the experimental effort, European institutions should also play a leading role in the theoretical counterpart. The work presented here aims to develop a state-of-the-art theoretical tool able to turn the description of exotic matter from a qualitative perspective into a quantitative one.

Acknowledgments

I would like to thank first Nora Brambilla and Antonio Vairo for insightful comments and careful reading of this manuscript; and second to some of my colleagues who attended the FAIRNESS2016 Workshop, M. Albaladejo, M. Berwein, S. Hwang, Z. Meisel, R. Navarro, C. Qi, U. Tamponi and J. Tarrañes-Castellà, for enjoyable physics discussions. I also want to express my gratitude to the organizers of the FAIRNESS2016 Workshop for enabling my participation, which proved very rewarding. I acknowledge financial support from the Alexander von Humboldt Foundation.

References

[1] Gell-Mann M 1964 Phys. Lett. 8 214–215
[2] Zweig G 1964 CERN Report No.8182/TH.401, CERN Report No.8419/TH.412
[3] Olive K et al. (Particle Data Group) 2014 Chin. Phys. C38 090001
[4] Choi S et al. (Belle Collaboration) 2003 Phys. Rev. Lett. 91 262001
[5] Brambilla N et al. 2011 Eur. Phys. J. C71 1534
[6] Brambilla N et al. 2014 Eur. Phys. J. C74 2981
[7] Tornqvist N A 1994 Z. Phys. C61 525–537
[8] Drenska N, Faccini R, Piccinini F, Polosa A, Renga F and Sabelli C 2010 Riv. Nuovo Cim. 33 633–712
[9] Duhynskiy S and Voloshin M B 2008 Phys. Lett. B666 344–346
[10] Braaten E 2013 Phys. Rev. Lett. 111 162003
[11] Vijande J, Weissman E, Valcarce A and Barnea N 2007 Phys. Rev. D76 094027
[12] Caswell W and Lepage G 1986 Phys. Lett. B167 437
[13] Bodwin G T, Braaten E and Lepage G P 1995 Phys. Rev. D51 1125–1171
[14] Pineda A and Soto J 1998 Nucl. Phys. Proc. Suppl. 64 428–432
[15] Brambilla N, Pineda A, Soto J and Vairo A 2000 Nucl. Phys. B566 275
[16] Vairo A 2007 Int. J. Mod. Phys. A22 5481–5491 [71(2006)]
[17] Brambilla N, Vairo A, Polosa A and Soto J 2008 Nucl. Phys. Proc. Suppl. 185 107–117
[18] Horn D and Mandula J 1978 Phys. Rev. D17 898
[19] Hasenfratz P, Horgan R R, Kuti J and Richard J M 1980 Phys. Lett. B95 299
[20] Juge K J, Kuti J and Morningstar C 2003 Phys. Rev. Lett. 90 161601
[21] Bali G S and Pineda A 2004 Phys. Rev. D69 094001
[22] Brambilla N, Pineda A, Soto J and Vairo A 2001 Phys. Rev. D63 014023
[23] Pineda A and Vairo A 2001 Phys. Rev. D63 054007 [Erratum: Phys. Rev. D64, 039902 (2001)]
[24] Berwein M, Brambilla N, Tarriss Castellà J and Vairo A 2015 Phys. Rev. D92 114019
[25] Brambilla N, Pineda A, Soto J and Vairo A 2005 Rev. Mod. Phys. 77 1423