A new evaluation index of rockburst tendency considering tunnel excavation and fracture distribution

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Abstract. Rockburst risk and tendency analysis of fractured rock mass after excavation and unloading has been gaining importance. Within the framework of a constitutive model based on internal variable thermodynamics, the energy dissipation rate is proposed as a new evaluation index of rockburst tendency. Taking the numerical simulation of a traffic tunnel in western China as an example to carry out the rockburst tendency analysis, the models with parallel fractures of different dip angles were established in FLAC3D by Monte-Carlo method and MATLAB programming. Combined with the estimated in-situ stress of the deep-buried tunnel and the inversion results of constitutive parameters of rock mass, the influence of fracture dip angles on rockburst tendency of the surrounding rock is analyzed. The analysis results under different calculation conditions show that with the increase of fracture dip angle, the possible location of rockburst gradually transfers from the vault to the left arch waist. In addition, the comparison with the stress index verifies that the energy dissipation rate index can effectively evaluate the potential dangerous areas and rockburst positions after excavation and unloading of fractured rock mass, which has a reference value for the rockburst risk assessment in deep-buried tunnels.

1. Introduction
With the gradual completion of shallow rock mass engineering, deep rock mass excavation has become a new requirement and trend of underground engineering. A large number of deep-buried long tunnels have been built in China over recent years, many of which were experiencing the rockburst problem during construction[1, 2]. Deep-buried tunnel is characterized by high geostress conditions and strong excavation disturbance, and deep rock mass is in practice a discontinuity composed of rocks and fractures. After the tunnel excavation, the deterioration of rock mass properties caused by fractures of surrounding rock will be more obvious. Rockburst risk and tendency analysis of surrounding rock after excavation and unloading of fractured rock mass is of great importance to ensure the safety and stability of deep-buried tunnel.

Many scholars have studied the rockburst tendency on the basis of theories or tests and proposed some evaluation indices. The representative stress-based indices are mainly stress-strength ratios derived from the engineering data analysis. Kidybinski[3] proposed the ratio of the maximum shear stress to the uniaxial compressive strength of the rock. Barton et al.[4] obtained the ratio of the uniaxial compressive strength to the maximum principal stress of the rock. In essence, rockburst is a complex dynamic instability failure phenomenon caused by the sudden release of elastic strain energy stored in rock mass. Therefore, it has become a trend to reveal the failure mechanism of unloading rock mass from the perspective of energy. The most notable energy index is the strain energy storage index proposed by Kidybiński[5], which defines as the ratio of retained strain energy to dissipated strain energy.
Goodman[6] introduced the impact energy index, which is the ratio of the energy before the peak strength to that after the peak strength. Tang et al.[7] proposed the surplus energy index to analyze rockburst proneness. Gong et al.[8] made a review on rockburst tendency indices, recommending residual elastic energy (REE) to be the best index. Additionally, some statistical and intelligent approaches are being used in the field of rockburst tendency evaluation[9]. In this paper, within the framework of a constitutive model based on internal variable thermodynamics, the energy dissipation rate is proposed as the new evaluation index of rockburst tendency. Then, the numerical models of deep-buried tunnel excavation under different fracture conditions are established based on Monte-Carlo method and FLAC3D. In addition, the new evaluation index is verified by numerical simulations under different calculation conditions. Based on simulation results, the influence of fracture dip angles on the rockburst tendency of surrounding rock is analyzed.

2. Energy dissipation rate as the evaluation index of rockburst tendency

2.1. General form of constitutive equation based on internal variable thermodynamics

Rockburst is essentially a kind of dynamic instability, therefore, the evaluation index of rock mass stability can also be used to analyze the rockburst tendency. However, the traditional constitutive models cannot effectively establish the stability analysis method. In order to overcome the shortcomings of the traditional constitutive model, a constitutive model based on internal variable thermodynamics[10] is adopted to describe the internal structure changes of the system and the internal energy dissipation process of rock materials. This constitutive model is the basis of the research on the thermodynamic stability of rock materials and has been redeveloped in FLAC3D.

The thermodynamic force conjugate with viscoplastic internal variable $\dot{\lambda} (\dot{\lambda}_1, \dot{\lambda}_2)$ is:

$$f_{1\text{vp}}^p = P_1 = \sqrt{J_2}$$
$$f_{2\text{vp}}^p = P_2 = (1 + b \dot{\lambda}) (a \dot{\lambda}_1 + \sqrt{J_2})$$

The thermodynamic force conjugate with the internal variable $\dot{\chi}$ is:

$$f_{s} = b \lambda_2 (a \dot{\lambda}_1 + \sqrt{J_2})$$

where $a$ and $b$ are material parameters. $I_1$ is the first invariant of stress tensor. $J_2$ is the second invariant of stress bias.

The viscoplastic strain rate equation is:

$$\dot{\varepsilon}_{m\text{vp}} = a \{(1 + b \dot{\lambda}) \dot{\lambda}_2 + b \lambda_2 \dot{\lambda}_2\}$$
$$\dot{\varepsilon}_{s\text{vp}} = \left[\dot{\lambda}_1 + (1 + b \dot{\lambda}) \dot{\lambda}_2 + b \lambda_2 \dot{\lambda}_2\right] \frac{s_{ij}}{2 \sqrt{J_2}}$$

where $\varepsilon_{m\text{vp}}$ is viscoplastic volumetric strain and $\varepsilon_{s\text{vp}}$ is viscoplastic deviatoric strain.

The evolution equation of internal variables is:

$$\dot{\lambda}_1 = \frac{1}{\eta_{p1}} \left\{ f_{1\text{vp}}^p - h \dot{\lambda}_1 \right\}$$
$$\dot{\lambda}_2 = \kappa_{p2} \left\{ \frac{f_{2\text{vp}}^p - R}{R} \right\}^p$$
$$\dot{\chi} = \kappa_{p3} \exp(m \dot{\chi}) \left( \frac{f_{2\text{vp}}^p}{R} \right)^2 \text{sign} (\dot{\lambda}_1)$$

where $\eta_{p1}$, $\kappa_{p2}$ and $\kappa_{p3}$ are viscosity coefficients. $m$ is the equation parameter. $p$, $h$ and $R$ are material constants. The Macaulay bracket $< >$ indicates:

$$\langle x \rangle = \begin{cases} x & x > 0 \\ 0 & x \leq 0 \end{cases}$$

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sign(x) is a symbolic function as follows:

\[
\text{sign}(x) = \begin{cases} 
1, & x > 0 \\
0, & x = 0 \\
-1, & x < 0
\end{cases}
\]  

(6)

2.2. Energy dissipation rate

According to the Clausius-Duhem inequality in continuum mechanics which expresses the second law of thermodynamics:

\[
\theta \dot{\eta}^p = \frac{1}{V} \sum_{a} \dot{\xi} - \frac{\theta}{\theta} q_k \geq 0
\]  

(7)

where \( \theta \) is the temperature, and \( \dot{\eta}^p \) is the entropy production. \( V \) is the volume of the representative volume element (RVE). \( \dot{\xi} \) is the microscopic internal variable. \( f = (f_1, f_2, \ldots, f_n) \) is the conjugate thermodynamic force of the internal variable \( \dot{\xi} \). \( q_k \) is the component of the heat flow in the \( k \) direction.

The second law of thermodynamics requires that the equation (7) is not less than 0. For rock materials at room temperature, the influence of temperature \( \theta \) and heat flux \( q \) need not be considered, and the equation can be reduced as follows:

\[
\Phi = \theta \dot{\eta}^p = \frac{1}{V} \sum_{a} \dot{\xi} \geq 0
\]  

(8)

where \( \Phi \) is defined as the intrinsic energy dissipation rate.

Energy dissipation rate as a measure of the current thermodynamic state and equilibrium state of materials, can reveal the weak part of rock mass after excavation and unloading. The larger the value of \( \Phi \) is, the farther the rock mass deviates from the equilibrium state, and the internal structure of rock mass moves more violently and the system tends to be more unstable, that is, the greater the risk of rockburst is. Therefore, in the framework of the constitutive model based on internal variable thermodynamics, rockburst can be regarded as a kind of instability failure which occurs when the current thermodynamic state deviates far from the equilibrium state, and energy dissipation rate can be adopted as an evaluation index of rockburst tendency.

3. Numerical model of tunnel excavation considering fractures

3.1. Generation of parallel fractures with different dip angles

The simulation and generation of fractures can be attributed to the description of fracture geometry and location. The random variables used to describe fracture geometry and location obey certain probability distribution, thus, the Monte Carlo method can be used to simulate. Random variables that determine the distribution of fractures are shown in Table 1. Based on the Monte Carlo method, a random fracture generating program is developed in MATLAB. The parallel fractures with different dip angles are shown in Figure 1. The number of fractures is set to 50.

| Table 1. Statistical parameter values of random variables with different fracture dip angles. |
|---------------------------------------------------------------|
| **Random variables**  | **Probability distribution** | **Statistical parameter values** |
|----------------------|------------------------------|---------------------------------|
| Trace                | Lognormal distribution       | \( \mu = 15 \) m \( \sigma = 5 \) m |
| Dip angle            | Normal distribution          | \( \mu = 0^\circ, 15^\circ, 30^\circ, 45^\circ, 60^\circ, 75^\circ, 90^\circ \) \( \sigma = 0 \) |
| Width                | Lognormal distribution       | \( \mu = 0.05 \) m \( \sigma = 0 \) |
| Location             | Uniform distribution         | \( a = -20 \) m \( b = 20 \) m |

Note: \( \mu \) and \( \sigma \) are the variance and the standard deviation respectively. \( a \) and \( b \) are the two ends of the uniform distribution.
3.2. Numerical model

The numerical model is based on the engineering data of a deep-buried tunnel in western China. The tunnel section is circular and the excavation diameter is 9.13 m. The plane strain model is adopted and the dimension of the model is $100 \times 100 \times 8$ m ($100 \times 100$ m in transverse direction and 8 m in longitudinal direction) intended to reduce the boundary effect.

For the excavation through rock mass with fractures, four groups should be considered for mesh generation: the group of the surrounding rock, the group of the tunnel excavation section, the group of fractures in the surrounding rock and the group of fractures in the tunnel excavation section. Taking the tunnel with parallel fractures of $0^\circ$ dip angle for an example, the final model implemented in FLAC$^3$D is shown in Figure 2. Other models under several fracture dip angles are also realized in FLAC$^3$D, which are not shown due to space limitation.

![Figure 1](image1.png)  
**Figure 1.** Parallel fractures with different dip angles.

![Figure 2](image2.png)  
**Figure 2.** The tunnel model with parallel fractures ($0^\circ$ dip angle).
4. Influence of fracture on rockburst tendency

4.1. Numerical analysis conditions

According to the engineering data of a deep-buried tunnel in western China, the density of rock mass is 2500 kg/m$^3$, the burial depth is 800 m at the origin of the model, and the vertical stress and horizontal stress are 20 MPa and 30 MPa respectively (that is, lateral pressure coefficient is 1.5). It is assumed that the horizontal stress and vertical stress are linearly distributed along the buried depth, and the lateral pressure coefficient is a constant. On the above basis, the initial geostress field is generated. The constitutive model for surrounding rock and fractures adopted the previously proposed constitutive model based on the internal variable thermodynamics, which has been redeveloped in FLAC$^{3D}$. The physical parameters of surrounding rock and fractures for numerical calculation are shown in Table 2, in which the physical parameters of surrounding rock are derived from the physical parameters of granite gneiss around the tunnel by inversion analysis, and the fracture physical parameters refer to the parameters of the bedding fault zone in layer C3 on the left bank of Baihetan Hydropower Station. The undetermined parameters of fractures are empirically taken as 1/100 of the surrounding rock parameters.

Table 2. Statistical parameter values of random variables with different fracture dip angles.

| Parameter type                  | Parameter | Surrounding rock | Fracture |
|---------------------------------|-----------|------------------|----------|
| Parameters of stiffness         | $E$ (GPa) | 25               | 0.2      |
|                                 | $\nu$     | 0.26             | 0.35     |
| Parameters of strength          | $\sigma'$ (MPa) | 3.5 | 0.035 |
|                                 | $R$ (MPa)  | 7.5              | 0.075    |
| Parameters of viscoplastic equation | $a$     | 0                | 0        |
|                                 | $\eta_p$ (GPa·s) | 1.0×10$^{14}$ | 1.0×10$^{12}$ |
|                                 | $h$ (GPa)  | 1.5              | 0.015    |
|                                 | $\kappa$ (s$^{-1}$) | 1.0×10$^8$ | 1.0×10$^8$ |
|                                 | $p$        | 1.70             | 1.70     |
| Parameters of equation about internal variable $\chi$ | $\kappa_0$ (s$^{-1}$) | 0 | 0 |
|                                 | $b$        | 0                | 0        |
|                                 | $m$        | 1000             | 1000     |

The boundary condition is as follows. The vertical stress is applied on the top of the model to simulate the overlying rock mass, and the velocity constraint is applied on the other surfaces.

4.2. Analysis of rockburst tendency based on energy dissipation rate index

The contour of energy dissipation rate after tunnel excavation under different fracture dip angles is shown in Figure 3. It can be seen from the figure that the fracture dip angle has a significant impact on the distribution of energy dissipation rate of the surrounding rock. When the fracture dip angle is 0°, the position where the energy dissipation rate of the surrounding rock reaches the maximum is located at the vault. With the increase of fracture dip angle, the position of the maximum energy dissipation rate deflects to a certain extent. When the fracture dip angle is 90°, the position of the maximum energy dissipation rate is located at the left arch waist. Based on the analysis above, it can be concluded that with the increase of the fracture dip angle, the location where rockburst is most likely to occur gradually transfers from the tunnel crown to the left arch waist.
**Figure 3.** Contour of energy dissipation rate under different fracture dip angles.

4.3. Verification based on the stress criterion

Based on the viewpoint of traditional stress criterion, the potential locations where rockburst is likely to occur are generally zones with high stress concentration in the redistributed stress field. The higher the stress concentration is, the greater the risk of rockburst is. Thus, the stress index can be adopted to verify the validity of the proposed energy dissipation rate index through comparison. The contour of maximum principal compressive stress under different fracture dip angles is shown in Figure 4.

**Figure 4.** Contour of maximum principal compressive stress under different fracture dip angles.

As can be seen in Figure 4, the fracture dip angle has a significant impact on the distribution of maximum principal compressive stress of the surrounding rock. When the fracture dip angle is 0°, the position where the maximum principal compressive stress of the surrounding rock reaches the maximum is
located at the vault. With the increase of fracture dip angle, the position where maximum principal compressive stress reach a peak deflects to a certain extent. When the fracture dip angle is 90°, the position of the stress peak is located at the left arch waist. Furthermore, it can be seen that the distribution of energy dissipation rate is basically consistent with the stress index, which shows the proposed energy dissipation rate index can be effectively used for analyzing the rockburst tendency.

5. Conclusions
Rockburst is a bottle-neck factor affecting the stability of deep-buried tunnels and rockburst tendency analysis has been gaining importance. In this paper, a new evaluation index for rockburst tendency based on energy dissipation rate is proposed. Based on the data of trace width, location and dip angle of fractures, the fractures with different dip angles were generated by using Monte-Carlo method and MATLAB programming. Then, the tunnel models with parallel fractures of different dip angles were established in FLAC$^{3D}$. Moreover, the rockburst tendency analysis was carried out based on the energy dissipation rate index. The main conclusions draw from this paper are as follows:
(1) In the framework of the constitutive model based on internal variable thermodynamics, rockburst is considered as a kind of instability failure which occurs when the current thermodynamic state deviates far from the equilibrium state. The energy dissipation rate $\Phi$ can be used as the evaluation index of rockburst tendency to reveal the weak position of rock mass after excavation and unloading. The larger the value of $\Phi$ is, the greater the risk of rockburst is.
(2) Based on the FLAC$^{3D}$ simulation of tunnel excavation, the influence of the fracture dip angle on the rockburst tendency was analyzed. Analysis results show that the fracture dip angle has a significant impact on the location where rockburst is likely to occur. When the fracture dip angle is 0°, the rockburst is most likely to occur near the tunnel crown. With the increase of the fracture dip angle, the possible location of rockburst gradually transfers to the left arch waist.

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