Theory of exchange coupling in disordered magnetic multilayers

A.Yu. Zyuzin

A.F.Ioffe Physical-Technical Institute RAS, 194021 St.Petersburg, Russia

Abstract

We consider mechanism of exchange coupling based on interaction between electrons in nonmagnetic layer. Depending on ratio of inverse time of diffusion of electrons between ferromagnetic layers and ferromagnetic splitting of conducting electrons this mechanism describes transition from ferromagnetic to concollinear ordering of magnetizations of ferromagnetic layers.

I. INTRODUCTION AND MAIN RESULTS.

In metallic ferromagnet-nonferromagnet-ferromagnet multilayers (see fig. 1) magnetic structure oscillates between ferromagnetic and antiferromagnetic orientations of the ferromagnets’ magnetizations as a function of thickness of nonmagnetic metal $L$ with a period of order of the Fermi wave length. The explanation of this phenomenon is based on the fact that the interlayer coupling is due to Ruderman-Kittel interaction between electron spins in different ferromagnets.

Further investigations discovered structures with perpendicular orientations of the ferromagnets’ magnetizations (see for review [7]). Often phenomenological coupling between magnetizations of ferromagnetic layers can be represented as sum of bilinear and biquadratic contributions

$$E(\varphi) = J_1 \cos \varphi + J_2 \cos^2 \varphi$$

(1)
Here $\varphi$ is angle between directions of magnetizations of ferromagnetic films. Bilinear constant $J_1$ oscillates as function of interlayer distance $L$. In case of large positive biquadratic constant $J_2$ minimum of $E(\varphi)$ corresponds to $\varphi = \pi/2$. As explained by Slonzevskii large positive biquadratic coupling might be result of spatial fluctuations of bilinear coupling $J_1$ due to ferromagnet-nonferromagnet surface roughness [7].

In disordered system, when $L$ is larger than electron mean free path $l$, RKKY interaction $\langle J_1 \rangle$, averaged over realizations of scattering potential exponentially decreases [8]. At the same time fluctuations of local exchange become much larger than $\langle J_1 \rangle$ [9] giving rise to biquadratic contribution $J_2 \gg |\langle J_1 \rangle| [10]$.

Here we propose mechanism of coupling in disordered multilayers based on interaction between electrons in nonmagnetic layer. Spin fluctuations in system of interacting electrons give rise to contribution to thermodynamic potential [11], which depends on magnetic field or, in our case, on relative orientation of magnetizations in ferromagnetic layers. Here we show that in magnetic multilayer this mechanism describes transition between ferromagnetic and noncolinear ordering with increasing distance between ferromagnetic layers or value of ferromagnetic splitting of conducting electrons.

We assume that magnetic multilayer can be described by Hamiltonian

$$H = H_0 + \epsilon_{exc} \int d\mathbf{r} \Psi_\alpha^+(\mathbf{r}) \mathbf{n}(z) \sigma_{\alpha\beta} \Psi_\beta(\mathbf{r}) + H_{\text{int}}$$

(2)

Here $H_0$ is Hamiltonian of free electrons in random field. Second term describes exchange field in ferromagnetic layers. $\epsilon_{exc}$ is ferromagnetic splitting of conducting electrons. $\mathbf{n}(z)$ is unit vector of direction of magnetization of ferromagnetic layers. $\mathbf{n}(z) = \mathbf{n}_1$ at $z < -L/2$ and $\mathbf{n}(z) = \mathbf{n}_2$ at $z > L/2$ as it is shown on figure 1. $\Psi_\alpha^+(\mathbf{r})$ and $\Psi_\beta(\mathbf{r})$ are creation and annihilation operators, $\sigma_{\alpha\beta}$ are Pauli matrixes. Integration in second term is over ferromagnetic layers. The last term $H_{\text{int}}$ describes Coulomb interaction between electrons in nonmagnetic layer. We assume that interaction in ferromagnetic layers is taken into account self consistently in $\epsilon_{exc}$.

Details of calculation are given in the last part of the paper. Here we present the main
results. Characteristic energies in the problem are ferromagnetic splitting of conducting electrons $\epsilon_{exc}$ and Thouless energy $E_c \equiv D/L^2$. $D$ is diffusion constant of conduction electrons. We assume that it is the same in nonmagnetic and ferromagnetic layers.

In case of small thickness when $E_c > \epsilon_{exc}$ coupling between ferromagnetic layers has bilinear form and coupling energy per unit area is

$$E(\varphi) = -\frac{F}{8(4\pi L)^2} \frac{\epsilon_{exc}^2}{E_c} \cos \varphi$$  \hspace{1cm} (3)

Here $F$ is characteristic constant of interaction in diffusion channel $[11]$. It is positive for Coulomb repulsion between electrons. Let us note that in this regime coupling (3) does not depend on $L$. Minimum of (3) corresponds to ferromagnetic orientation of magnetizations in multilayer $\varphi = 0$. Note that result is obtained in limit when $L > l$, or $\epsilon_{exc}$ smaller that inverse mean free time $D/l^2$.

At larger distance $L$ when $E_c < \epsilon_{exc}$ coupling has biquadratic form and coupling energy per unit area is

$$E(\varphi) \simeq \frac{F}{(4\pi L)^2} E_c \cos^2 \varphi$$ \hspace{1cm} (4)

This quantity decreases as $L^{-4}$ with increasing distance. Minimum of coupling energy corresponds to noncollinear state $\varphi = \pi/2$.

Both expressions are given for the case of infinite thickness of ferromagnetic layers. Calculation show that in case $d > \sqrt{D/\epsilon_{exc}}$ coupling weakly depends on $d$.

Results (3) and (4) are also valid provided $L < \sqrt{D/T}$. In opposite case coupling energy decreases exponentially as $\exp \left( -\frac{8\pi T}{D} L \right)$.

Let us compare results (2) and (3) with biquadratic contribution due to mesoscopic fluctuations of RKKY interaction $[10]$, which is $J_2 \sim \frac{1}{L^2} \frac{E^2}{A d}$ at $E_c < \epsilon_{exc}$ and $J_2 \sim \frac{1}{L^2} \frac{\epsilon_{exc}^2}{A d E_c}$ at $E_c > \epsilon_{exc}$. Here $A$ is an intralayer ferromagnetic stiffness and thickness $d > \sqrt{D/\epsilon_{exc}}$.

The quantity $J_2$ decreases with $L$ much faster than (4). Also for $\epsilon_{exc}/Ad << 1$, $F \approx 1$ coupling energy given by expressions (3) and (4) is larger than biquadratic contribution due to mesoscopic fluctuations of RKKY in whole range of distances. In this case with increasing
distance $L >> l$ system is undergo transition between ferromagnetic and perpendicular \( \varphi = \pi/2 \) ordering. Such transition was observed in [12].

II. DERIVATION OF RESULTS.

Correction to thermodynamic potential which depends on \( \epsilon_{exc}n(z) \) is given by expression [11]

\[
\delta \Omega = \frac{F}{4} T \sum_{|\omega_n|<\omega,\tau} |\omega_n| \int \frac{d^2 q}{(2\pi)^2} \int_{|z|<L/2} dzD_{\beta}^{\alpha} (z, z, q, \omega_n)
\]  

\[d \] Here constant \( F \) describes screened Coulomb interaction in diffusion channel. \( \omega_n = 2\pi nT \) is Matsubara frequency. \( \tau \) is electron mean free time.

Diffusion ladder satisfies equation

\[
\left(-D \frac{d^2}{dz^2} + D q^2 + |\omega_n| \right) D_{\mu \eta}^{\alpha \beta} + i\epsilon_{exc}n(z) \left( \sigma_{\alpha \gamma} D_{\mu \eta}^{\gamma \beta} - D_{\mu \gamma}^{\alpha \beta} \sigma_{\gamma \eta} \right) \text{sign} \omega_n = \delta (z - z') \delta_{\alpha \beta} \delta_{\mu \eta}
\]

\[d \] It is convenient to present solution of equation (6) at \( |z| < L/2 \) in the form

\[
D_{\mu \eta}^{\alpha \beta} = A_{\mu \eta}^{\alpha \beta} \exp (-Qz) + U_{\alpha \gamma}^{+} C_{\mu \gamma}^{\beta} U_{\gamma \eta} \exp (Qz) + \frac{\exp (-Q |z - z'|)}{2DQ} \delta_{\alpha \beta} \delta_{\mu \eta}
\]

Here we introduce \( Q = \sqrt{q^2 + |\omega_n|} / D \). \( U \) is matrix of relative rotation of magnetizations of ferromagnetic layers. In case when direction of magnetization in ferromagnetic layer \( z < -L/2 \) is directed along \( z \) axes \( n(z) = (0,0,1) \) and at \( z > L/2 \) direction is \( n(z) = (\sin \varphi, 0, \cos \varphi) \), it is matrix of rotation along \( y \) axes \( U = \exp \left( \frac{i\varphi}{2} \sigma_y \right) \).

For simplicity we consider limit of semiinfinite ferromagnetic layers. More detail consideration shows that at \( d > \sqrt{D/\epsilon_{exc}} \) results weakly depend on thickness of ferromagnetic layers. It is convenient to introduce boundary conditions for diffusion ladder at ferromagnet-nonferromagnet interfaces taking into account that according to equation (6) in coordinate system where spins are directed along magnetization, components of ladder with antiparallel spins decreases as \( \exp (-Q_1 |z|) \) and \( \exp (-Q_1^* |z|) \) at \( |z| > L/2 \), where \( Q_1 = \sqrt{q^2 + |\omega + i\epsilon_{exc}|} / D \).
Components of ladder with parallel spins decreases as exp \((-Q |z|)\) at \(|z| > L/2\). At \(z = -L/2\) where \(n(z) = (0, 0, 1)\) boundary conditions are

\[
\left(\frac{d}{dz} - Q_1\right) P_{\alpha \gamma}^+ D^{\gamma \beta} P_{\gamma \eta}^- = \left(\frac{d}{dz} - Q_1^*\right) P_{\alpha \gamma}^- D^{\gamma \beta} P_{\gamma \eta}^+ = 0
\]

\[
\left(\frac{d}{dz} - Q\right) P_{\alpha \gamma}^\pm D^{\gamma \beta} P_{\gamma \eta}^\pm = 0
\]

(8)

Here we introduce projectors of spins on \(z\)-axes \(P_{\pm} = \frac{1 \pm \sigma_z}{2}\).

The same kind of boundary conditions can be introduced for rotated diffusion ladder \(U_{\alpha \gamma} D^{\gamma \beta} U_{\gamma \eta}^+\) at \(z = L/2\). Solving system of equations (6,8) we obtain

\[
\delta \Omega = -\frac{F_T}{2} \sum_{|\omega_n| < 1; \alpha, \beta} |\omega_n| \int \frac{d^2q}{(2\pi)^2} \frac{L}{DQ} \times \]

\[
\times \left[ (|\Lambda|^2 - (\text{Re} \Lambda)^2) \cos \varphi + \left(|\Lambda|^4 - \frac{1}{2}(\text{Re} \Lambda)^2 \right) \cos^2 \varphi - \frac{1}{2}(\text{Re} \Lambda) \left(1 - \frac{1}{2}|\Lambda|^2 - \frac{1}{2}|\Lambda|^4 \cos^2 \varphi \right) \frac{\sinh QL}{QL} \right] \left[ 1 - (\text{Re} \Lambda)^2 + 2(|\Lambda|^2 - (\text{Re} \Lambda)^2) \cos \varphi + (|\Lambda|^4 - (\text{Re} \Lambda)^2) \cos^2 \varphi \right]
\]

(9)

Here \(\Lambda = \frac{(Q_1 - Q)}{(Q_1 + Q)} \exp(-QL)\). Expression (9) contains divergent terms, which do not depend \(\varphi\) and must be subtracted.

In limit of large exchange splitting when \(|Q_1| > Q\) parameter \(\Lambda = \exp(-QL)\) is real. In this case energy is function of \(\cos^2 \varphi\). Subtracting in expression (9) terms which do no depend on angle we obtain

\[
\delta \Omega (\varphi) = \frac{F_T}{4} \sum_{|\omega_n| < 1; \alpha, \beta} |\omega_n| \int \frac{d^2q}{(2\pi)^2} \frac{L}{DQ} \frac{(1 - \Lambda \frac{\sinh QL}{QL})}{(1 - \Lambda^2 \cos^2 \varphi)} \frac{\Lambda^2 \cos^2 \varphi}{(1 - \Lambda^2 \cos^2 \varphi)} \]

(10)

Main contribution in expression (10) is from region where \(\Lambda < 1\), denominator therefore gives only small correction. Neglecting it we obtain expression (4).

In opposite limit of small exchange splitting \(\text{Re} \Lambda \sim \epsilon_{\text{ex}}^2\), \(|\Lambda| \sim \epsilon_{\text{ex}}\) and to the order \(\epsilon_{\text{ex}}^2\) coupling energy is proportional to \(\cos \varphi\).

\[
\delta \Omega (\varphi) = -\frac{F_T}{2} \sum_{|\omega_n| < 1; \alpha, \beta} |\omega_n| \int \frac{d^2q}{(2\pi)^2} \frac{L}{DQ} \frac{|\Lambda|^2}{\cos \varphi}
\]

(11)

Calculating (11) at zero temperature we obtain (3). Transition between limits (10) and (11) occurs at \(\epsilon_{\text{ex}} \sim D/L^2\).
III. ACKNOWLEDGEMENT

This work is supported by Russian Fund for Fundamental Research grant number 01-02-17794.
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