Analogue gravitational lensing in optical Bose-Einstein condensates

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We consider acoustic propagation of phonons in the presence of a non-rotating vortex with radial flow in a Bose-Einstein condensate (BEC) of photons. Since the vortex can be used to simulate a static acoustic black hole, the phonon would experience a considerable spacetime curvature at appreciable distance from the vortex core. The trajectory of the phonons is bended after passing by the vortex, which can be used as a simulation of gravitational lensing for phonons in a photonic BEC.

I. INTRODUCTION

Analogue gravity has been considered and developed since 1923 [1,2]. However, it is the discovery of a formal analogy between the dynamics of light in a black hole spacetime and that of sound waves in a moving fluid [3] what set the dawn of modern gravitational analogues. The importance of this result relies on the recognition that Hawing radiation is not related to general relativity itself, but it is an effect that must be present whenever we consider quantum field theories on curved spacetimes. Since then, the field witnessed an impressive development, both theoretically and experimentally, expanding beyond sonic waves in classical fluids. Among these achievements, we can cite phonons in Bose-Einstein condensates [4–11], surface waves in water flows [12–15], slow light in optical systems [16–21] and magnons in magnetic systems [22,23]. These analogue systems have become an important testing ground for some aspects of general relativity, while providing insights for the quantum nature of gravity [24,25].

Among the phenomena that can be mimicked, we find gravitational lensing, which is a general name for light deflection by a gravitational field generated by some astrophysical source like stars, black holes, and galaxies [26]. The first observation of this effect was carried out in 1919, when Eddington’s team confirmed Einstein’s prediction of the deflection of light by the gravitational field produced by the sun [27]. Since then, many other theoretical and experimental developments have appeared [28].

Considering acoustic propagation around a vortex in a superfluid system, it was shown in Ref. [29,30] that the phonon trajectories are deflected by an angle determined by the vortex circulation. In this way, the vortex acts like a converging lens for the phonons trajectories. Light deflection was observed in Ref. [16], where a microstructured waveguide around a microsphere was employed in order to mimic a curved spacetime. Optical materials were employed in order to mimic equatorial Kerr-Newman null geodesics, thus allowing the study of light trajectories in a non-static spacetime [17].

Here, we consider optical Bose-Einstein condensates [28] in order to mimic the gravitational lensing phenomenon. When particles of integer spin accumulate in the ground state at high density and low temperature, a Bose-Einstein condensate is formed [31]. However, in the case of massless particles, this is more complicated since lowering the temperature of a gas of massless particles does not preserve the number of particles. In fact, the ground state, or vacuum, has no particles at all. This problem was circumvented by considering a dye-filled optical cavity [32], where a photonic BEC could be achieved [33]. The process works because the system effectively provides a mass to the photons that can then thermalize to the cavity ground state while conserving the number of particles. The physical process behind this is the multiple scattering of the trapped photons by dye molecules, ultimately leading to thermalization.

Specifically, we consider phonons propagating in a photonic Bose-Einstein condensate. We expect the radial vortex to have important influence on the trajectories of the phonon since it creates an effective curved spacetime. We show that the quasiclassical scattering process of phonons by a non-rotating radial vortex leads to a scattering angle, thus deviating the trajectories from a straight line. In other words, we show that the effect of the effective spacetime curvature on the phonon trajectories is the same of a converging lens, as predicted by the gravitational lensing effect.

The advantage of this approach is that photonic condensates are formed at room temperature, contrary to the very low temperatures that are employed in other sys-

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tems. Moreover, due to the simplicity of the experimental setup on which such condensates are implemented, a dye-filled pumped optical cavity, it offers a manifold of possibilities for analogue gravity.

In Sec. [11] we describe the system under consideration and how it can be employed to simulate a curved spacetime. Section [11II] is devoted to present our main result, the analogue gravitational lensing in a photonic condensate of light is in steady state, hence the dissipation and the pumping rate compensate each other, while the condensate density is steady state limit, the decay rate and the pumping rate given by $mc^2/\Gamma$. From here on, we assume that the condensate of light is in steady state, hence the dissipative term in the previous equation can be recast as a wave equation on a curved spacetime, $\chi t^c c \gamma_0 r/\Gamma$. From this construction in the following paragraphs. The idea here is to create a quantum fluid where sound (phonons) can travel. The above velocity profile leads to subsonic and supersonic regions in the fluid. The transition between them is the analogue event horizon, since sound cannot leave the supersonic region.

We are here interested in the dynamics of small perturbations around a stationary background, characterized by the particle density and velocity profile. Within the regime of validity of hydrodynamics, the perturbations for the density and for the phase obeys a set of two coupled differential equations, the Bogoliubov system. Such system can be transformed into a single differential equation for the phase perturbation $\theta$ [40, 43],

$$\left\{\nabla^2 - \left(\partial_t - \frac{c_0}{r} \mathbf{\nabla} \cdot \mathbf{\nabla}\right)^2\right\} \theta(r, t) = 0 ,$$

where we have used the dimensionless variables $r \rightarrow \xi r$ and $t \rightarrow \xi t/c$ for simplicity.

Now, in order to find a relation with curved spacetimes, we introduce an effective metric $g^{\mu\nu}$. By doing this, the previous equation can be recast as a wave equation on a curved spacetime,

$$\frac{1}{\sqrt{-g}} \partial_{\mu} (\sqrt{-g} g^{\mu\nu} \partial_{\nu}) \theta = 0 .$$

Here, the effective acoustic metric $g_{\mu\nu}$ is given by

$$ds^2 = -f(r)dr^2 + f^{-1}(r)dz^2 + r^2 d\phi^2 ,$$

with $f(r) = 1 - c_0^2/r^2$ being the warp factor. To get this form of the metric, we have applied the following coordinate transformation

$$d\tau = dt - \frac{c_0}{r(1 - \xi^2)} dr .$$

Therefore, we see that under appropriate conditions, when the wavelengths of the perturbations are sufficiently large, the dynamics of the phonons (collective perturbations) mimics the one of a massless field on a curved spacetime described by Eq. [5]. Here, we see that the effective acoustic metric for the radial vortex is similar to a
Schwarzschild black hole, with the parameter $c_0^2$ playing the role of the black-hole mass \cite{40}. While for the irrotational vortex, the azimuthal flow induces an effective acoustic metric that asymptotically approaches the massless spinning cosmic string at large distance \cite{29}. Furthermore, there is also another difference between these two kind of vortexes – as an analogue of Schwarzschild black-hole, the metric Eq. (5) can have an acoustic event horizon, but for the cosmic string metric induced by the irrotational vortex, no event horizon can be formed.

The event horizon of the metric in Eq. (5) appears when the radial flow velocity towards the center exceeds the speed of sound, specified by the metric singularity at

$$r_h = c_0. \quad (7)$$

Note that we have a black-hole solution in 2+1 dimensions, which is not present in general relativity. This is a consequence of the fact that Einstein’s equations do not play a role here in the analogue system. In order to get an idea of the curvature of our effective spacetime, we can compute the Kretschmann scalar, which is given by

$$K = R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} = \frac{44c_0^2}{r^8}, \quad (8)$$

with $R_{\alpha\beta\gamma\delta}$ being the Riemann curvature tensor constructed from the metric. $K$ is well behaved at $r_h$, showing that the singularity at $r_h$ is just a breakdown of the employed coordinate system, and not a true singularity. The genuine singularity of this acoustic spacetime lies at $r = 0$, which is at the center of the vortex.

In addition, the Ricci scalar of the metric is straightforward to compute, resulting in

$$R_c = g^\mu\nu R_{\mu\nu}^{\alpha} = \frac{2c_0^2}{r^4}. \quad (9)$$

Therefore, the particles living on this spacetime will experience a significant curvature at distances well outside the vortex core.

III. ANALOGUE GRAVITATIONAL LENSING

As shown in the previous section, the vortex creates an effective curved spacetime for the phonons. The metric in Eq. (5) has no explicit dependence on $t$ and $\phi$. This means that the associated spacetime is invariant under time translation and rotation along the direction defining the angle $\phi$ (remember that we only have 2+1 dimensions). Hence, there are two Killing vectors for this metric – one related to the time-translation invariance, $(k_1)^\mu = (1, 0, 0)$, and the other to the rotational symmetry, $(k_2)^\mu = (0, 0, 1)$. Note that, in the low-energy limit, the phonons follow a linear dispersion relation, thus traveling like massless particles on the effective spacetime created by the vortex. This is also clear from Eq. (4) and, therefore, we expect the phonons to follow null geodesics.

Along these null geodesics, we can define the conserved quantities associated with Killing vectors, $k_1$ and $k_2$, as

$$E = -(k_1)^\mu g_{\mu\nu} \frac{dx^\nu}{d\lambda} = \left(1 - \frac{c_0^2}{r^2}\right) \frac{dr}{d\lambda},$$

$$L = (k_2)^\mu g_{\mu\nu} \frac{dx^\nu}{d\lambda} = r^2 \frac{d\phi}{d\lambda}. \quad (10)$$

Here, $E$ is the energy and $L$ is the angular momentum. $\lambda$ denotes an affine parameter for the null geodesics.

By rewriting these equations, we can obtain the dynamics equations for $\tau, \phi$ with respect to the affine parameter $\lambda$,

$$\frac{d\tau}{d\lambda} = \frac{Er^2}{r^2 - c_0^2},$$

$$\frac{d\phi}{d\lambda} = \frac{L}{r^2}. \quad (11)$$

Since the phonons are travelling along null geodesics, this acoustic metric is determined by

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0. \quad (12)$$

Using the acoustic metric in Eq. (5), we get the equation

$$- \left(1 - \frac{c_0^2}{r^2}\right) \frac{d\tau}{d\lambda} r^2 + \left(1 - \frac{c_0^2}{r^2}\right)^{-1} \left(\frac{d\tau}{d\lambda} r^2 + r^2 \frac{d\phi}{d\lambda}\right)^2 = 0. \quad (13)$$

After substituting $d\tau/d\lambda$ and $d\phi/d\lambda$ into the above equation, we can have the radial motion equation

$$\left(\frac{dr}{d\lambda}\right)^2 = E^2 + \frac{c_0^2 L^2}{r^2} - L^2 \frac{r^2}{r^2}. \quad (14)$$

Hence, with Eqs. (11) and (13), the path of phonons can be completely specified. In addition, the radial motion equation can also be recast as an energy equation form

$$\frac{1}{2} \left(\frac{dr}{d\lambda}\right)^2 + V(r) = \frac{1}{2} E^2, \quad (15)$$

with an effective potential

$$V(r) = \frac{L^2}{2r^2} - \frac{c_0^2 L^2}{2r^4}. \quad (16)$$

The extreme value of the effective potential is determined by $dV(r)/dr = 0$, leading to

$$r_m = \sqrt{2}c_0. \quad (17)$$

As we can see, at $r_{max}$ the potential has a maximum value $V(r_m) = L^2/8c_0^2$, after which it decreases quadratically as the distance from the vortex core increases. If the energy of the incident phonon is lower than the potential barrier, $E^2 < 2V(r_m)$, the phonon can be deflected by the potential. In this manner, phonons with energy bigger than the barrier $E^2 > 2V(r_m)$ can travel through the barrier and be dragged down to the vortex.
center. From the critical value $E^2 = 2V(r_m)$, the corresponding critical impact parameter can be determined as $b_m = L/E = 2c_0$. At this point, the phonon will rotate around the vortex in an unstable circular orbit.

When a phonon is deflected by the vortex, the turning point is determined by $dr/d\varphi = 0$, which locates at

$$r_t = b \left[\frac{1}{2} + \sqrt{\frac{(b^2 - 4c_0^2)}{2b}}\right],$$

(18)

where we have used the impact factor $b = L/E$. If the phonon is directed towards the vortex with a large impact factor, so that $c_0/b$ would be a small parameter, we can expand $r_t$ in series of $c_0/b$ to second order,

$$r_t = |b| \left[1 - \frac{1}{2} \left(\frac{c_0}{b}\right)^2 + O\left(\frac{c_0}{b}\right)^4\right].$$

(19)

The general deflection angle of the phonon passing by the acoustic black hole is obtained by integrating

$$\Delta \varphi = 2 \int_{r_t}^{\infty} \frac{d\varphi}{dr} = 2 \int_{r_t}^{\infty} \frac{b}{\sqrt{r^4 + b^2(c_0^2 - r^2)^2}} dr.$$  

(20)

Since this integral is not easy to calculate directly, we expand it with the parameter $c_0/b$ and integrate to second order, obtaining the following relation for the deflection angle,

$$\Delta \varphi = \pi \text{sign}(b)(1 + \frac{3c_0^2}{4b^2}) + O\left(\frac{c_0}{b}\right)^4.$$  

(21)

In consequence, in the absence of the vortex, the phonon would follow a straight line with phase changed by $\pi$. The presence of vortex leads to an extra deflection angle. This result is similar for the deflection of the phonon in the acoustic vortex spacetime [29]. However, they have a different origin. In Eq. (21), the deflection angle is determined by the dissipation parameter $c_0$, while in the rotational case it is given by the vortex winding number [29]. In the radial vortex case, the deflections of phonons around the vortex resemble the light deflection around a black hole, where the deflection angle is determined by the dissipation parameter $c_0$. This means that the light deflection angle around the black hole is determined by the black-hole mass, so the effective mass of the radial vortex is also determined by $c_0$. In the case of the irrotational vortex, no horizon appears. This means that, when the azimuthal flow becomes supersonic, the phonons can also be deflected around irrotational vortex and the deflection angle is related to the winding number.

The phonon deflection phenomena induced by the vortex can also lead to the interesting convergence effect. Two initially parallel phonon beams, after travelling through opposite paths with respect to the vortex center with impact parameter $b$, will intersect at a distance $l = 2f$ beyond the vortex. From the deflection angle in Eq. (21), the focal length $f$ can be obtained as

$$f = \frac{2}{3\pi} \frac{b^3}{c_0} = \frac{2}{3\pi} \frac{b^3}{r_h}.$$  

(22)

Therefore, the dissipative vortex plays the role of an effective phonon lens.

Besides the phonon convergence effect, in a paradigmatic analogue of gravitational lens in general relativity, similar effects for the phonons can also be realized in the case of dissipative vortex photonic BECs. As demonstrated in Fig. 1 if the source and observer are far away from the vortex, where $r_b \ll d_L, d_s$, the phonon beams are travelling along straight lines and all the phonon deflections occur around the vortex. Given that the angles $\theta_s, \theta_E$ are all very small, we have

$$d_L \theta_E = (d_s - d_L) \theta_s = b.$$  

(23)

In addition, as we can see in Fig. 1 the deflection angle is also equal to

$$\Delta \phi = \theta_s + \theta_E,$$  

(24)

where $\Delta \phi = \frac{3\pi c_0^2}{4\lambda_c^2}$ is the deflection angle.

By combining Eqs. (23) and (24), we can obtain the Einstein angle as

$$\theta_E = \left(\frac{3\pi c_0^2(d_s - d_L)}{4d_s d_L^2}\right)^{1/3}.$$  

(25)

In the proposed scenario of photonic BECs, the vortex size is much smaller than the distance between the source, the vortex, and the observer, which leads the vortex to behave as a thin analogue gravitational lens for the phonons. From Eq. (25), we can see that given a certain $d_L$ satisfying $d_L \ll d_s$, the Einstein angle $\theta_E$ depends only on the dissipating parameter $c_0$. Comparing with the case of a Schwarzschild black hole, clearly the parameter $c_0^2$ plays the same role as its mass [29].

In the above discussion, the analysis of the phonon trajectory deflection was based on the semiclassical approximation, so the phonon wavelength should be bigger than the healing length. Hence, the maximum values of the deflection angle (up to second orders) induced by the radial vortex is determined by

$$\Delta \varphi = \frac{3\pi c_0^2}{4\lambda_c^2}.$$  

(26)
where \( \lambda \) is the wave length of the phonons. For the typical values of the photonic BEC, the effective mass is \( m = 6.7 \times 10^{-36} \text{kg} \), the dimensionless interaction strength is of the order \( = (mg)/\hbar^2 \approx 7 \times 10^{-4} \), while the particle density is of order \( n = 10^{12} \sim 10^{13} \text{m}^{-2} \). Restricting to the case in which the phonon wavelength is in the linear dispersion region with \( \lambda = 2\xi \), we get the deflection angle as

\[
d\phi = \frac{3\pi}{4} \left( \frac{c_0 \hbar}{mc\lambda} \right)^2 \approx 0.59^\circ, 
\]

while the focal length is approximated as

\[
f = \frac{2 \lambda^3 m^2 c^2}{3 \hbar^2} \approx 20.3 \mu\text{m}.
\]

In the above estimations, we have restored the physical quantities that we made dimensionless before and set the parameter \( c_0 = 1 \). For the dye-based experiments, the cavity size is about 1.46\(\mu\)m, which is too small for observing these effects. However, a bigger system, or stronger interaction, can be achieved with suitable improvements. According to our calculations, if the condensed particle density and the inter-particle interaction strength can be increased one order of magnitude, the deflection angle can preserve the same value, while the focal length can be reduced to around 2\(\mu\)m. Therefore, we expect that our proposal may become feasible in the lab in the near future for testing the discussed phonon deflection effects.

IV. DISCUSSION

By using a radial vortex in photonic BEC as a Schwarzschild black-hole analog, we have studied the phonon trajectory bending effects induced by the vortex. This is similar to the light bending effects around the black hole in general relativity. With the acoustic spacetime approach, the quasi-classical scattering process of phonons by the vortex leads to a scattering angle, which is quadratic in the dissipating parameter \( c_0 \). We expect that the initially parallel phonon beams, after scattering along opposite side of the vortex, shall converge at a distance beyond the vortex. This implies that the radial vortex can be considered as an effective “phonon lens”. In analog with the observation of light bending effects in general relativity, we have also discussed the possible “thin phonon lens” effects that can be induced by the vortex. Here, the “image photon source” location perceived by the observer is deviated from the actual source by an Einstein angle \( \theta_E \). In addition, in the 2 + 1 dimensional system, there will be two image sources for the observer.

Throughout this paper we have assumed the hydrodynamic approximation is hold. Consequently, in the whole process, the system should be kept in a steady state without small scaled perturbations below healing length scale, which is also consistent with the analogue gravity research requirement. For possible experimental implementations, the BEC of light in a semiconductor microcavity or dye-based setups might be used, in particular due to the fact that the semiconductor microcavity setup can have stronger interactions.

V. ACKNOWLEDGEMENTS

We acknowledge support from QMiCS (820505) and OpenSuperQ (820363) projects from EU Flagship on Quantum Technologies, National Natural Science Foundation of China grant (NSFC) (12075145), Shanghai Government grant STCSM (2019SHZDZX01-ZX04), Spanish Government grant PGC2018-095113-B-I00 (MCIU/AEI/FEDER, UE), Basque Government IT986-16, EU FET Open Quromorphic and EPIQUS projects. LCC acknowledges financial support from the Brazilian agencies CNPq (PQ Grant No. 305740/2016-4 and INCT-IQ 246569/2014-0) and FAPEG (PRONEX 201710267000503).

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