Charge extraction from a black hole

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Abstract

Following up on earlier work about across-horizon quantum scattering and information transfer outwards across the black-hole horizon, we show that it is also possible to extract electric charge from a static nonrotating black hole. This result can be extended to other types of charge with a gauge-boson coupling and to nonstatic rotating black holes.

Keywords: general relativity, black holes, electroweak standard model, scattering
I. INTRODUCTION

In a recent article \cite{1}, we discussed Coulomb scattering of two electrically charged elementary particles, with one of these particles inside the Schwarzschild black-hole horizon and the other outside. We, then, proposed a Gedankenexperiment which uses this quantum scattering process to transfer information from inside the black-hole horizon to outside.

Now, the question arises if it is, in principle, possible to extract an electric charge from a static nonrotating black hole by the exchange of a charged vector boson? In the present note, we will give an affirmative answer to this question, where we will use the results from Ref. \cite{1} and those of an earlier preprint version \cite{2}.

II. SCATTERING SET-UP

Consider the following elastic scattering process from the standard model of elementary particles in Minkowski spacetime:

\[ e^- + \nu_e \rightarrow e^- + \nu_e , \]  

(1)

as discussed in, e.g., Sec. 8.5 of Ref. \cite{3} and Sec. 12.6.2 of Ref. \cite{4}. The relevant position-space Feynman diagrams at tree level are given in Fig. 1.

The corresponding process in the black-hole context is given by the set-up of Fig. 2, which needs to be compared to Figs. 2 and 3 of Ref. \cite{1}. For the set-up of Fig. 2 in this note, we also assume that the initial inside-horizon electron (\(e^-\)) was produced by pair creation and that the corresponding initial inside-horizon positron (\(e^+\)) does not participate in the scattering with the initial outside-horizon neutrino (\(\nu_e\)). The total electric charge of this initial inside-horizon positron-electron pair is zero. Further discussion of the near-horizon region of a Schwarzschild black hole and the local Minkowski coordinates (\(T, X, Y, Z\)) can be found in Sec. 2 of Ref. \cite{1}.

III. CHARGE EXTRACTION

The position-space Feynman diagram on the left of Fig. 2 allows for electric charge extraction, as long as the initial electron is inside the event horizon and the final electron outside the event horizon with an appropriate outgoing momentum (similar to the recoil electron in elastic \(e^- \mu^-\) scattering as discussed in Ref. \cite{1}). Observe that the electric-charge-extraction process from Fig. 2 does not change the total lepton number inside the black-hole horizon.

From now on, we will use the conventions of Sec. 20.2 in Ref. \cite{5}. The tree-level W-exchange diagram of Fig. 2 corresponds to the following momentum-space probability amplitude:

\[ \mathcal{M}_W(e^- \nu_e \rightarrow e^- \nu_e) = -\frac{g^2}{2} \frac{1}{q^2 - m^2_W + i\varepsilon} \left[ \bar{u}^r(k') \gamma^\mu u^r(p) \right] \left[ \bar{u}^r'(p') \gamma^\mu u^s(k) \right] , \]  

(2)

with \(q \equiv k - p' = k' - p\) for momenta \(p/p'\) and \(k/k'\) of the initial/final neutrino and electron, respectively. On the left-hand side of (2), we have suppressed the spin indices \(r, r'\) of the neutrino and \(s, s'\) of the electron.
FIG. 1. Position-space Feynman diagrams for $e^- \nu_e$ scattering at tree level in Minkowski spacetime. The arrows in the diagrams show the flow of lepton number. The double lines indicate that electric charge is transported.

FIG. 2. Across-horizon $e^- \nu_e$ scattering (Fig. 1), in a local inertial coordinate system near the Schwarzschild black-hole horizon, allows for electric-charge extraction from a static nonrotating black hole (by the charged-current position-space Feynman diagram on the left). The projected black-hole horizon is indicated by the dashed line with the black-hole center to its left. The positron inside the black-hole horizon does not participate in the scattering.

Next, specialize to spin-up fermions, zero neutrino mass, and the following momenta:

\[
k^\mu = \left( \sqrt{k^2 + m_e^2}, k, 0, 0 \right), \tag{3a}
\]

\[
p^\mu = \left( p, -\sqrt{1/3} p, \sqrt{2/3} p, 0 \right), \tag{3b}
\]

\[
k'^\mu = k'^\mu, \tag{3c}
\]

\[
p'^\mu = p'^\mu, \tag{3d}
\]

with $k > 0$ and $p > 0$. Then, we have

\[
\mathcal{M}_W(e^- \nu_e \rightarrow e^- \nu_e) = \frac{2 g^2}{\sqrt{3}} \frac{p k}{m_e^2 - \sqrt{4/3} p k - 2p \sqrt{k^2 + m_e^2 - m_W^2 + i\varepsilon}} \sim \frac{g^2}{2} \left(1 - \sqrt{3}\right) \neq 0, \tag{4}
\]

in the ultrarelativistic limit of $k$ and $p$ [specifically, $\min(k, p) \gg m_W \gg m_e$]. For the calculation of the position-space amplitude shown on the left of Fig. 2, we must fold the above momentum-space amplitude with appropriate wave packages, as discussed in Appendix C of an earlier version of our article [2].

The nonvanishing $e^- \nu_e$ scattering amplitude (4) implies, according to the argument of Secs. 2 and 3 in Ref. [1] and App. C in Ref. [2], that we can extract electric charge from a static nonrotating black hole by repeating the scattering process of Fig. 2 a large number...
FIG. 3. Position-space Feynman diagram (in a near-horizon local inertial coordinate system) for electric-charge reduction of a static nonrotating charged black hole. Shown is a black hole with an initial negative electric charge $Q = -e$, which is changed to $Q = 0$ by an infalling positron (electric charge $e > 0$ and lepton number $-1$), while the corresponding electron (electric charge $-e < 0$ and lepton number $1$) escapes to spatial infinity. The double line stands for an electron and the single line for a quark (up or down quark with electric charge $2e/3$ or $-e/3$, each quark having baryon number $1/3$), where the arrows indicate the flow of negative electric charge. The projected black-hole horizon is shown by the dashed line with the black-hole center to its left.

of times ($N \gg 1$). Only a few events ($n \ll N$) extract electric charge, as both Feynman diagrams of Fig. 2 contribute to the process and even the charged-current diagram may not give an appropriate outgoing electron (cf. App. B in Ref. [1]).

IV. DISCUSSION

It is, of course, possible to change the electric charge of a static nonrotating black hole by other processes than the one considered up till now. Different from the electric-charge-extraction process (left-diagram of Fig. 2) is, for example, the electric-charge-reduction process as illustrated by Fig. 3. Observe that the electric-charge-reduction process from Fig. 3 changes the total lepton number inside the black-hole horizon. Note also that, for both processes, the backreaction on the metric has been neglected in Figs. 2 and 3.

Returning to our electric-charge-extraction process (Fig. 2), several remarks are in order. First, the very process of charge extraction considered in Sec. III does not imply that causality is violated, as there is no real (on-shell) particle that moves superluminally.

Second, we have thus seen that a quantum scattering process not only allows for the extraction of information from a static nonrotating black hole but also for the extraction of electric charge. In this respect, we note that electric charge is not just a measure of how strongly two electrons scatter with each other, but, rather, a measure of how the corresponding photon propagates (cf. Sec. III.7, pp. 204–205 in Ref. [6]). This becomes clear if we examine the structure of radiative corrections contributing to charge renormalization. It turns out that charge renormalization depends only on the photon field. Thus, we can say that the electric charge belongs to the gauge field (determining how the photon propagates), rather than to the matter field.

Third, our result for electric-charge extraction from a static nonrotating black hole can be extended to other types of charge involving a gauge boson and to nonstatic rotating black holes.
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