An analysis to student error of algebraic problem solving based on polya and newman theory

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Abstract. This study aims to reveal student algebraic problem-solving errors based on Polya and Newman hierarchical indicator. The subject was a study of 30 students of grade VII in one of the junior high school located in the Timor Tengah Utara, NTT, Indonesian, 2018/2019 period. Process of collecting data used tests and interviews. The result of the student problem-solving ability test was grouped into three parts, namely 5 of the student in the low category, 20 of the student in the medium category, and 5 of students in the high category. The result of the analysis based on Polya indicator indicated that a group of low students made an error on all indicators. The medium of category students made an error in carrying out the plan and looking back. For those in the high category, they make an error in looking back indicator. While the result of the analysis was based on Newman hierarchical indicator that a group of low students makes an error ranging from comprehension to encoding. Group medium student makes an error in the process skill and encoding process. For those in the high category, they tend to make an error at the encoding indicator.

1. Introduction
Mathematics is a science that has an important role in the development of science, technology, and also daily day life [1]. Line with this statement, [2] says that mathematical was one way to try to understand, interpret, and describe our world. One of the groups of mathematics that are taught in school is algebraic. The basic concept of algebra is an important foundation as an initial understanding for the student when learning an advanced algebraic concept because the algebraic concept was used as a mathematical problem-solving tool [3]. A problem can be modeled mathematically through algebra [4] because systematically, problems in daily life can be symbolized algebraically.

The algebraic concept was used as one of the material domain in the assessment of Trend in the International Mathematics and Science Study (TIMSS). The International Association for the Evaluation of Educational Achievement (IEA) report the result of the TIMSS in 2011 that Indonesia ranked 38 out of 42 countries [5], and in 2015, Indonesia was ranked 45 out of 50 participating countries [6]. Other international institutes that assess student mathematical abilities are the Program for International Student Assessment (PISA). The PISA report released by the Organization for Economic Cooperation and Development (OECD) show that in 2012, Indonesia ranked 64th out of 65 participating countries [7], and in 2015, Indonesia ranked 69th out of 76 participating countries [8].

In addition to the result of TIMSS and PISA report, the education and culture ministry center report on the result of the national examination shows that the national average of mathematics in
2017 is 50.31; and in 2018 amounting to 43.34. Whereas specifically in the Timor Tengah Utara District (NTT-Timor), the average math score of students in 2017 is 51.97 [9]; and the average math score in 2018 decreased to 42.59 [10].

Result of the TIMSS, PISA and national examination show the mathematics abilities from Indonesian students at the international and national level are not as expected. Indonesian students didn’t have the readiness to face challenges in real life, because the assessment carried out by TIMSS and PISA is oriented towards the future of students. [11] said that the assessment carried out by PISA aims to measure the readiness of the young generation at the end of their school in facing the challenges of temporary life. Whereas [12] said that TIMSS made students think about the importance of success in school and success in their future careers.

Therefore, students in schools must be accustomed to practicing solve a mathematics problem. Based on this, the National Council of Teachers of Mathematics (NCTM) emphasizes that learning programs must allow students to 1) construct new mathematics knowledge through problem-solving, 2) solve problems in mathematics and in other contexts, 3) apply and adjust various strategies that are appropriate for solving problem, and 4) monitor and reflect on the processes of mathematics problem solving [13].

Problem-solving is part of life skills which consist of analysis, interpretation, prediction, evaluation, and reflection [14]. Therefore, an evaluation of this capability is needed. According to [15] that the basic principles of problem-solving, i.e. 1) understand the problem, 2) devise a plan, 3) carry out the plan, and 4) look back. In addition to these basic principles, there are other steps to solve a problem is using the Newman hierarchy [16], namely 1) reading the problem, 2) comprehend what is read, 3) carry out a mental transformation from the words of the question to the selection of an appropriate mathematical strategy, 4) Apply the process skills demanded by the selected strategy; and 5) encode the answer in an acceptable written form. Furthermore, Newman recommended the questions that are used as a guide in classifying student errors, namely 1) please read the question for me (reading), 2) tell me what question is asking you to do (comprehension), 3) tell me a method you can use to find, and answer to the question (transformation), 4) show me how you worked out the answer to the question (Process skills), and 5) Now write down your answer to the question (encoding).

Based on Polya's basic principles and Newman's hierarchy, the researcher concluded that Polya's steps were used to uncover students' written problem-solving abilities, while Newman's steps were used to reveal students' oral problem-solving abilities. Based on that, this study aims to uncover student errors in solving mathematical problems using Polya's and Newman's hierarchical theory.

2. Method

The research approach used is a qualitative approach. While the research method used is descriptive qualitative. The subjects of this study were VII grade students in one of the Public Schools located in Kefamenanu City, Timor Tengah Utara Regency, NTT-Indonesia, 2018/2019 Academic Year totaling 30 people. The 30 students were included in the test to find out how far their algebraic problem-solving abilities. The test results of students' algebraic problem-solving abilities are grouped into high, medium, and low groups.

Data collection tools in the form of tests and interviews. The test is carried out with the aim to uncover students' algebraic problem-solving abilities use the Polya indicator. Whereas the aim interviews to uncover students' errors in solving algebraic problems. The form of the interview used is a semi-structured, which was developed based on Newman's hierarchical theory.

The analysis technique to validate the accuracy of the results and interpretation of the data is the triangulation technique. Triangulation is the process of strengthening evidence from different data types [17]. Strengthening the evidence intended in this study is to check the accuracy of the results and interpretation of data to the same source with different techniques, namely through the results of student work and interviews. Triangulation activities are carried out through stages of data reduction, data display, and conclusion.
3. Result and Discussion
The test result of the algebraic problem-solving ability of 30 students was analyzed descriptively and the obtained average of 22.30, a standard deviation of 4.69, a maximum value of 34, and a minimum value of 10 of the ideal maximum standard was 36. Based on the average and standard deviation then grouped the values of the student in the high, medium, and low categories. The number of students who get grades in the high category is 5 people, the value in the medium category is 20 people, and the value in the low category is 5 people.

Question test used was 4 numbers, but what was analyzed in this study were questions number 2 and number 4, with the reason that the research subject made more error on both of these questions. Analysis of student algebra problem-solving based on Polya and Newman’s theory is described in the following sections.

3.1. Analysis of Student Errors Based on Polya’ Indicator
The student answer was analyzed using quantitative descriptive based on the problem-solving ability indicator proposed by Polya, with the results as presented in Table 1.

| Number | Indicator 1 | Indicator 2 | Indicator 3 | Indicator 4 |
|--------|-------------|-------------|-------------|-------------|
| 2      | 13.33%      | 13.33%      | 43.33%      | 86.67%      |
| 4      | 30.00%      | 36.67%      | 53.33%      | 76.67%      |
| Average| 21.67%      | 25.00%      | 48.33%      | 81.67%      |

Table 1 shows the percentage of student errors based on problem-solving indicators of the two test questions analyzed in this study. Indicators of problem-solving according to Polya are conditional, and the results of the analysis show that the higher the indicator level, the greater the percentage of student errors. This is seen from the average percentage of errors in Table 1 which respectively are 21.67%, 25.00%, 48.33%, and 81.67%. Almost all students make errors in solving algebraic problems. This is the same as the results of research conducted by [18] that the average percentage of student problem-solving abilities is less than 50% and in the unfavorable category. The following describes the answers of some students to question number 2.

![Figure 1. Student Answer RN on Question 2.](image)

RN is one of the students who scored in the low category. In general, the answer in figure 1 shows that RN does not have the ability to think algebraically. The error made by RN student is (1). do not understand the problem (the problem is known and asked in the question), (2). do not understand the devise a plan (concepts/formula/rule used to solve the problem), (3). are unable to carry out the plan. In this section, the RN students are not able to add and or multiply the algebraic form. Even RN ignore the main concept of algebraic form in question involving variables. (4). unable to look back on the process and results. RN can give a conclusion to the answer but these conclusions are based on an error made on indicators 1, 2, and 3. So, RN has not been able to do the problem-solving process. [19] said that problem-solving is a process starting from when students are faced with a problem until the problem is resolved according to the four indicators of problem-solving.
JA is also one of the students who scored in the low category. The results of the answers show that JA does not have the ability to think algebraically. There are two planes, but JA only determines the circumference of the first plane. JA used the thinking concept of real numbers, not the concept of algebraic form. The errors made by JA are (1). don't understand the problem, (2). unable the devise a plan (concept/formula/rules used to solv...
without mathematical proof. It shows that VN student makes an error in the look back of the process and result. Indicator look back process and the result is an indicator that is deemed difficult for students. [20] said that one of the steps of problem-solving who difficult for students is the step of look back on the process and the results. At this step, the problem solver must be able to show the ability to think metacognitive in looking back at the plan or after carrying out the plan and linking it to the problem being asked.

3.2. Analysis of Student Errors Based on Newman's Indicator

Student errors in this section were uncovered through interviews. The percentage of students who make problem-solving error based on the Newman indicator is shown in Table 2 below.

| Number | 1     | 2     | 3     | 4     | 5     |
|--------|-------|-------|-------|-------|-------|
| 2      | 0.00% | 23.33%| 40.00%| 70.00%| 90.00%|
| 4      | 0.00% | 30.00%| 46.67%| 66.67%| 83.33%|
| Average| 0.00% | 26.67%| 43.33%| 68.33%| 86.67%|

Table 2 shows the percentage of student errors based on Newman's theory. Error in problem-solving students based on Newman is also conditional that the higher the level of the indicator, the greater the percentage of student error. This is seen from the average percentage of errors in Table 2 which respectively are 0.00%, 26.67%, 43.33%, 68.33%, and 86.67%. The following is described as a copy of student error obtained by interviewing researchers with several students on question number 2.

Copies of RN student error:

Errors made by RN are do not comprehend what is read (comprehension). When the RN was asked about what known and asked in question number 2, RN only reread the question without an explicit description of the answer to the question. This error consequence in indicators further. RN has not been able to choose the right strategy to answer the given question (transformation). When asked about this indicator, RN answered that question number 2 was solved by the formula of plane but did not describe what formula and how the formula meant. The researcher tried to direct the resolution strategy that the circumference of each plane was determined by adding the lengths of all edge of each plane, but RN had not been able to apply the requested process skills through the chosen strategy (process skill). RN has not been able to add algebraic form because the answer given is the result of integers. This error answer indicates that RN has not been able to encode the answer in an acceptable written form (Encoding).

Copies of JA student error:

JA has not fulfilled the indicator to comprehend what is read. When RN answers questions about known and asked elements in question number 2, only reread the whole question. The researcher explained the elements that were known and asked, but then the follow JA was unable to choose the right strategy to answer question number 2. Related to JA error on this indicator, the researcher directed the student to determine the circumference of each plane by adding the length of each edge of each plane. JA calculates according to the direction of the researcher, but JA still makes errors in this section. JA add the length of the edge of the first plane with the addition concept integers, not the concept of addition algebraic form, even from incomplete processes, JA says that the circumference of the first plane is longer than the circumference of the second plane, although not calculated the circumference of the second plane. This shows that JA has not been able to apply process skills, and has not been able to encode the answers in an acceptable written form.
Copies of MP student error:
MP is able to understand the questions that are read, able to describe the elements that are known and asked, able to choose the right strategy and be able to apply strategies in the process of solving the algebraic form. MP is able to add the algebraic form correctly, but after getting the result of the number of algebraic forms which are the circumference of two planes, MP is unable to compare it to decide which plane has a longer circumference between them.

Copies of VN student error:
VN is able to read and understand the questions that are read, able to choose and apply the right strategies in the process of solution algebraic forms. Errors made by VN are only technical errors in the calculation of algebraic forms, but when directed by the researcher, VN quickly and accurately to correct the errors that.

Copies of problem-solving based on Newman's theory show that students who problem-solving in the low category to make more fundamental errors than students who problem-solving in the medium category, or in the high category. This is in accordance with the result of the study [21] that students with low achievement make errors to comprehend what is read, and transformation error than high achieving students. Their finding is to show that students with low achievement may have been trapped in the early stage of the modeling process, so they did not continue at the next stage.

The following is a summary of the errors problem-solving of the student based Polya and Newman indicator.

Table 3. Category Error problem-solving of students based on low, medium and high groups.

| Student Group | Errors                               | Polya’s Indicator | Newman’s Indicator                  |
|---------------|--------------------------------------|-------------------|-------------------------------------|
| Low           | Understand the problem, devise a plan, carry out the plan, look back the process and result. | Comprehension, transformation, process skill, and encoding. |                                   |
| Medium        | Carry out the plan, as well as look back the process and result. | Process skill, and encoding. |                                   |
| High          | Look back the process and result.     | Encoding.         |                                     |

4. Conclusion.
Based on the result and discussion, it was concluded that:
1) Algebraic problem-solving errors based on the Polya’s indicator are as follow:
   a) Students whose algebraic problem-solving ability in the low category often make fundamental and causal errors. The fundamental error made is did not understanding the algebraic problem, being unable to devise a plan algebraic problem, being unable to carry out the plan algebraic problem, and not being able to look back at the result and process.
   b) Students who have the algebraic problem-solving ability in the medium category, often to make an error in carrying out the plan algebraic problem, and error look back on the result and process.
   c) Students who have the algebraic problem-solving ability in the high category, make causal errors in the calculation process, and error look back the result and process.
2) Algebraic problem-solving errors based on Newman's indicator are as follow:
   a) Students whose algebraic problem-solving abilities in the low category often to make comprehension error, transformation error, process skill error, and encoding error.
   b) Students whose algebraic problem-solving ability in the medium category, often to make errors in the process skill, and coding.
   c) Students whose algebraic problem-solving ability in the high category often make errors encoding. The errors made by students in this category are mostly caused by carelessness.
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