Optimization method for multi-access capacity of wind power based on copula

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Abstract. With the increasing penetration of distributed wind power, the problem of wind power curtailment is becoming more and more serious due to the unreasonable installed capacity. A method is proposed to optimize the wind power capacity allocation for regional power grids. The main factors that affect wind power accommodation are analyzed according to the wind speed probability density function and the wind power output model. The correlation model of different wind farms is established by the Frank-Copula function. Aiming at reducing the total wind power curtailment, a distributed wind power installed capacity optimization method is proposed with the correlation model. The optimal capacity allocation of the system is calculated by genetic algorithm and the effectiveness of the proposed method is verified by the case of IEEE RTS-79.

1. Introduction
At present, Chinese renewable energy utilization is dominated by wind power. Wind resource characteristics cause the intermittent, volatility, uncertainty of wind power output [1]. When the installed capacity of wind power is small, its output power can be completely accommodated by the power grid. With the increasing scale of installed wind power, more peak regulation capacity is demanded in power grid. The problem of wind power accommodation has become increasingly serious. And the wind power curtailment occurred inevitably. In the period of wind farm design, how to determine the total installed capacity of distributed wind power in a regional grid has become an urgent problem [2-3].

Many scholars have studied the correlation of wind power. In the literature [4], a method based on Copula function is proposed for building the joint probability distribution of multi-wind farm power output. Jian J B studied the wind speed correlation of distributed wind farms at different locations based on Copula function and mean-variance model [5]. Pan X established a wind farm model based on the mixed Copula function [6]. Based on the Coupla theory, Zhang Y established the correlation between the two wind farms [7]. In the literature [8], a dynamic backtracking framework based on the extended Kalman filter is applied to predict the wind generation and the dynamic spatial correlations for the wind farms. Zhou H took into account the inherent time correlation of the target wind speed and its spatial correlation with reference wind farm [9].

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In the analysis of wind power capacity, in the literature [10], based on the non-sequential Monte Carlo simulation method, the probability of peak regulation shortage of wind power access grid are calculated. Fan P established an analytical model for wind power accommodation capacity with the goal of minimum operating cost [11]. In the literature [12], the optimal wind capacity allocation target is set by using voltage stability as a constraint. Alismail F considered two factors of wind power uncertainty and the availability of conventional generation units, wind power installed capacity optimization model was established [13]. In the literature [14], the maximum access capacity of wind farm in the region grid is calculated, based on the particle swarm optimization. Ye C took the peaking capacity of the power grid as one of the constraints and evaluated the wind power capacity of the power grid [15]. In the literature [16], the optimal wind power acceptance power of the grid is analyzed from the perspective of energy consumption.

Although many scholars have optimized the wind power access capacity through many different constraints, including economy, power grid peaking capability, voltage stability, power supply reliability, etc. They did not consider the impact of the correlation of multiple wind farms in the regional power grid on wind power access capacity.

Based on the correlation of distributed wind power output, this paper analyzes the relationship between different wind power installed capacity allocation and wind curtailment ratio. By using genetic algorithm, the optimal allocation of distributed wind power installed capacity in regional power grid is realized. The proposed method can accurately reflect the correlation of wind power, help planners understand the wind power accommodation capacity, and provides technical support for capacity allocation optimization of large scale distributed wind power accessing regional power grid.

2. Models
The randomness and uncontrollability of wind power is one of the main factors affecting its accommodation capacity. The wind power output depends mainly on the wind speed at the high point of the hub.

2.1. Wind power model

2.1.1. Wind speed model
The wind speed is generally in the Weibull distribution, and the probability density function is shown in equation (1).

\[
f(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp \left[ - \left( \frac{v}{c} \right)^k \right]
\]

where \( v \) is wind speed, \( c \) is scale parameter, \( k \) is a shape parameter. In the calculation analysis, the Weibull distribution parameter of the wind speed is known, and the wind speed sequence can be obtained by the inverse function of the cumulative probability function.

2.1.2. Wind power output model
The wind power output model is generally described by the wind turbine power characteristic curve, as shown by the piecewise function of equation (2).

\[
P(v) = \begin{cases} 
0 & (v \leq v_{c1}) \\
\frac{P_r}{v_r^3 - v_{c1}^3} \left( v^3 - v_{c1}^3 \right) & v_{c1} \leq v \leq v_r \\
P_r & v_r \leq v \leq v_{c2}
\end{cases}
\]

Where \( v_{c1} \) is cut into the wind speed, \( v_{c2} \) is cut out the wind speed, \( v_r \) is rated wind speed, \( P_r \) is the rated power, \( P(v) \) is the corresponding wind turbine output power when the wind speed is \( v \).

2.2. Factors affecting wind power accommodation capacity
On the grid side, the main factors affecting wind power accommodation capacity are as follows.
2.2.1. *Grid power balance constraints* The load and generator output in the power grid is a dynamic balancing process. The active power of the generator should be equal to the active power of the system load. The power balance constraint is as shown in equation (3):

\[ \sum_{i=1}^{n} P_{Gi} - \sum_{i=1}^{n} P_{Li} = 0 \]  

Where \( P_{Gi} \) is the active output of the \( i \)th generator; \( P_{Li} \) is the \( i \)th load active power.

2.2.2. *The thermal power unit peak regulation capacity constraint* When the output of the load and the wind power fluctuates, the output change of the conventional thermal power unit is:

\[ \Delta P_{Gi} = P_{Li} - P_{wi} - P_{Gi(i-1)} \]  

Where \( \Delta P_{Gi} \) is the magnitude of the change in the power output of the unit during a certain period of time; \( P_{Li} \) is the active power of system load at the \( i \)th moment; \( P_{wi} \) is the active power of wind power at the \( i \)th moment; \( P_{Gi(i-1)} \) is the active power output of the thermal power unit at the \((i-1)\)th moment.

Due to the limitation of the output power regulation rate (climbing rate) of the conventional thermal power unit, when the fluctuation of the wind output power causes the power variation of the conventional unit to be greater than the maximum climbing rate, the wind curtailment phenomenon will occur.

3. Theory

Wind power output curve is correlated and nonlinear. Copula theory provides an effective method for wind power correlation analysis.

3.1. *Copula theory*

3.1.1. *Copula function* The joint distribution function of an N-dimensional variable can be described by the edge distribution of the N variables and a Copula function, as shown in equation (5):

\[ G(x_1, x_2, ..., x_n) = C[G_1(x_1), G_2(x_2), ..., G_n(x_n)] \]  

Where \( G(x_1, x_2, ..., x_n) \) is the joint distribution function of the variable; \( G_i(x_i) \) is the edge distribution function of the variable; \( C(G_1, G_2, ..., G_n) \) is Copula function.

Typical Copula functions include normal Copula, t-Copula, Frank-Copula, Clayton-Copula, Gumbel-Copula functions, etc., where Frank-Copula, Clayton-Copula, and Gumbel-Copula function forms are as shown in equations (6) to (8):

\[ C(u,v) = -\frac{1}{\alpha} \ln \left[ 1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1} \right] \]  

(6)

\[ C(u,v) = \max [(u^{-\alpha} + v^{-\alpha} - 1), 0] \]  

(7)

\[ C(u,v) = -\frac{1}{\alpha} \ln \left[ 1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1} \right] \]  

(8)

3.1.2. *Kernel density estimation* In the Copula correlation analysis, the sample data needs to be subjected to kernel density estimation to obtain the edge probability density distribution. The kernel density is estimated based on the distribution characteristics of the data. \( t(x) \) is the kernel density estimation function value, the influence of these points on \( t(x) \) can be judged according to the distance between \( x \) and each point in \( x \) neighbourhood. \( X_1, X_2, ..., X_n \) are discrete samples from the measured data, and the density function estimate at \( x \) is as shown in equation (9):

\[ t(x) = \frac{1}{nh} \sum_{i=1}^{n} K \left( \frac{x - X_i}{h} \right) \]  

(9)
Where \( n \) is the number of samples; \( h \) is the bandwidth; \( K(u) \) is the kernel function that satisfies equation (10):

\[
\begin{align*}
\int K(u) du &= 1 \\
\int uK(u) du &= 0 \\
\int u^2K(u) du &> 0
\end{align*}
\]

3.1.3. Correlation coefficient

The correlation coefficient represents the degree of correlation between variables, including Pearson linear correlation coefficient \( \rho \), Kendall rank correlation coefficient \( \tau \), Spearman rank correlation coefficient \( \rho \) and so on. The linear correlation coefficient describes the linear relationship between the two variables, while the Kendall rank correlation coefficient and the Spearman rank correlation coefficient can describe the nonlinear relationship between the two variables. In the Copula correlation modeling, the Copula function type is selected according to the correlation coefficient.

3.2. Wind power correlation

The correlation analysis of wind power is essentially the correlation analysis of wind speed in different places. Through the analysis of wind speed correlation, the purpose of distributed wind power correlation analysis can be achieved. Based on the above Copula theoretical analysis method, the wind correlation analysis implementation steps are as follows.

1) Data preparation and pre-processing. The measured wind speed data of distributed wind power in different places are selected. The data record length is generally one year or more. The measured data can be used for subsequent calculation and analysis after pre-processing. Data preprocessing mainly includes abnormal data identification and elimination, missing point data reconstruction, data time benchmarking, etc. These three aspects of processing are also the main factors affecting the accuracy of subsequent calculations.

2) Wind speed kernel density estimation. For the pre-processed wind speed data, the kernel density is estimated by using equation (9). The result should correspond with the Weibull distribution model of wind speed proposed by equation (1).

3) Kendall correlation analysis. The wind speed is nonlinearly correlated. The Kendall rank correlation coefficient is used as the basis for selecting the Copula function. Let the two wind speed variables be \( v_1 \) and \( v_2 \) respectively, and the Kendall rank correlation coefficient \( \tau \) be calculated as equation (11):

\[
\tau = P[(v_1 - v_2)(t_1 - t_2) > 0] - P[(v_1 - v_2)(t_1 - t_2) < 0]
\]

Where \( P[(v_1 - v_2)(t_1 - t_2) > 0] \) is the probability of harmony between the two wind speed variables, \( P[(v_1 - v_2)(t_1 - t_2) > 0] \) is opposite; \( t_1, t_2 \) is the probability of distribution of wind speed variables.

4. Solution methodology

The wind power accommodation capacity is affected by many factors. In this case, the peak regulation capacity of the power grid is the key constraint factor. In order to reduce the total wind power curtailment, the distributed wind power is divided into regional power grids. By analyzing the relationship between installed capacity, installed proportion and wind power curtailment, and then the configuration optimization under certain installed capacity is realized.

According to the wind speed data, select the time length \( t \) (\( t \leq \) data recording time length) and the interval period \( T \) (\( T \geq \) data sampling point interval). \( N = t/T \) is the number of data sampling points. \( X \) is the installed capacity and \( K \) is the installed proportion. \( R \) is the ratio of total wind power curtailment to total wind power, as shown in equation (12):

\[
R(X, K) = \sum_i^NP_{wi} / \sum_i^NP_{wi}
\]

(12)
Where $P_{Wsi}$ is the wind power curtailment at the $i_{th}$ sampling time, $P_{Wt}$ is the theoretical power generation at the $i_{th}$ sampling time. $P_{Wi}$ is the sum of the theoretical wind power of the two regions. $P_{Wsi}$ is determined by equation (13):

$$P_{Wsi} = P_{Wt} - (P_{L} - P_{Gi})$$

At the $i_{th}$ sampling moment, the system load $P_{Li}$ is a constant value; The conventional thermal power unit output $P_{Gi}$ is constrained by the unit capacity $P_{GN}$, the minimum output rate $k_{Gpmin}$ and the maximum grade rate $k_{Gpmax}$, as shown in equation (14):

$$\begin{align*}
   k_{Gpmin} \cdot P_{GN} &\leq P_{Gi} \leq P_{GN} \\
   \Delta P_{Gi} - P_{Gi(i-1)} &\leq T \cdot k_{Gpmax} \cdot P_{GN}
\end{align*}$$

Where $P_{Gi(i-1)}$ is the output of the thermal power unit at the $(i-1)_{th}$ sampling time.

The problem can be solved by genetic algorithm. Under the condition that the installed capacity $X$ is constant, $R(X, K_j)$ is the fitness of the individual, and the minimum value corresponds to the optimal solution.

5. Test network
The validation example uses IEEE RTS-79. In this example, the installed capacity of thermal power for this regional power grid is 3310 MW. The maximum load of the system of this example is 2850 MW. The annual load sequence is shown in Figure 1.

![Annual load sequence](image1.png)

**Figure 1.** Annual load sequence.

![Annual wind speed sequence](image2.png)

**Figure 2.** Annual wind speed sequence.
This paper selects the actual wind speed data of two adjacent wind farms in Hebei Province, use hub height wind speed measurement and the measured height is 90 meters. Wind speed data are generated by Weibull distribution. The parameters of Weibull distribution are determined by the characteristics of actual data. The \(k\) of site A is 1.58 and the \(c\) is 3.14. The \(k\) of site B is 1.39 and the \(c\) is 3.2. The method proposed in this paper assumes that the wind turbine is always perfectly windward, and the wind power optimal access capacity is calculated on the premise. Therefore, the wind direction information is not considered, and the wind speed information is mainly collected. The annual wind speed sequence of two places is shown in Figure 2.

6. Results and discussions

![Permeability](image1)

(a) Permeability of different installed capacity

![Curtailment rate](image2)

(b) Curtailment rate of different installed capacity

![Optimal ratio](image3)

(c) Optimal ratio of different installed capacity

Figure 3. Optimization results of wind power installed capacity.
According to the result of Kendall coefficient, Frank-Copula function is selected for correlation modeling. The parameter $\alpha=12.654$ in formula (6) is obtained from the maximum likelihood estimation. Using the genetic algorithm, the reproduction algebra is $2^{00}$, the population number is 50, the mutation probability is 0.01, the crossing probability is 0.25, the optimization results of the installed capacity in two places. The range of installed capacity is $300 \sim 800\text{MW}$. The results are shown in Figure 3.

From Figure 3, it can be seen that when the total capacity of wind power installed at two places is $400\text{MW}$, the wind power permeability is 11.39%, and the ratio of wind power curtailment is less than 5%. The optimal capacity allocation ratio of site A and site B is 1.20, site A installed 218MW, and site B installed 182MW, and the system can better accommodate wind energy. With the increasing permeability of wind power in the system, the ratio of wind power curtailment also increases. When the wind power permeability is above 20%, the ratio of wind power curtailment is 8.78%, which exceeds the 8.5% of the national average wind power curtailment level in 2014. It shows that the accommodation ability of the system is insufficient, and the amount of wind power generation is wasted seriously.

7. Conclusions

Based on Frank-Copula theory, a correlation analysis model of wind power output is established in this paper. According to the wind resources of different locations, the proportion of installed capacity in different locations is optimized and the wind power curtailment is decreased under the condition of certain total installed capacity of wind power.

In the wind power operation, the failure rate of the turbine can not be ignored. In the subsequent work, the Monte Carlo method can be used to introduce the factor into the optimization allocation of the wind power installed capacity of the regional power grid.

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