Abstract Tachyonic scalar field-driven late universe with dust matter content is considered. The cosmic expansion is modeled with power-law and phantom power-law expansion at late time, i.e. $z \lesssim 0.45$. WMAP7 and its combined data are used to constraint the model. The forms of potential and the field solution are different for quintessence and tachyonic cases. Power-law cosmology model (driven by either quintessence or tachyonic field) predicts unmatched equation of state parameter to the observational value, hence the power-law model is excluded for both quintessence and tachyonic field. In the opposite, the phantom power-law model predicts agreeing valued of equation of state parameter with the observational data for both quintessence and tachyonic cases, i.e. $w_{\phi,0} = -1.49^{+11.64}_{-4.08}$ (WMAP7+BAO+$H_0$) and $w_{\phi,0} = -1.51^{+3.89}_{-6.72}$ (WMAP7). The phantom-power law exponent $\beta$ must be less than about -6, so that the $-2 < w_{\phi,0} < -1$. The phantom power-law tachyonic potential is reconstructed. We found that dimensionless potential slope variable $\Gamma$ at present is about 1.5. The tachyonic potential reduced to $V = V_0 \phi^{-2}$ in the limit $\Omega_m,0 \to 0$.

Keywords power-law cosmology, tachyonic dark energy

1 Introduction

There have been clear evidences that the present universe is under accelerating expansion as observed in, e.g. the cosmic microwave background (CMB) (Masi et. al. 2002; Larson et. al. 2011; Komatsu et. al. 2011), large-scale structure surveys (Scranton et. al. 2003; Tegmark et. al. 2004), supernovae type Ia (SNia) (Perlmutter et. al. 1998, 1999; Riess et. al. 1998; Riess 1999; Goldhaber et. al. 2001; Tonry et. al. 2003; Riess et. al. 2007, 2004; Astier 2006; Amanullah 2010) and X-ray luminosity from galaxy clusters (Allen et. al. 2004; Rapetti et. al. 2005). One prime explanation is that the acceleration is an effect of a scalar field evolving under its potential to acquire negative pressure with $p < -\rho c^2/3$ giving repulsive gravity. Form of energy with this negative pressure range is generally called dark energy (Padmanabhan 2005; Copeland et. al. 2006; Padmanabhan 2006). Scalar field is responsible for symmetry breaking mechanisms and super-fast expansion in inflationary scenario, resolving horizon and flatness problems as well as explaining the origin of structures (Starobinsky 1980; Guth 1981; Sato 1981; Albrecht et. al. 1982; Linde 1982). Introducing a cosmological constant into the field equation is simplest way to have dark energy (Weinberg 1989; Ford 1987; Dolgov 1997), but it creates new problem on fine-tuning of energy density scales (Sahni et. al. 2000; Peebles et. al. 2003). For the cosmological constant to be viable, idea of varying cosmological constant needs to be installed (Sola et. al. 2005; Shapiro et. al. 2009). If dark energy is the scalar field, the field could have non-canonical kinetic part such as tachyon which is classified in a type of k-essence models (Armendariz-Picon et. al. 2000, 2001). The tachyon field is a negative mass mode of an unstable non-BPS D3-brane in string theory (Garousi 2000; Sen 2002a,b) or a massive scalar field on anti-D3
brane (Garousi et al. 2004). It was found that the tachyonic field potential must not be too steep, i.e. less steep than $V(\phi) \propto \phi^{-2}$ in order to account for the late acceleration (Padmanabhan 2002; Bagla et al. 2003; Kutasov 2003; Abram et al. 2003; Aguirregabiria et al. 2004; Copeland et al. 2005).

In this work, we considered dark energy in form of tachyonic scalar field in power-law cosmology of which the scale factor scaled as $a \propto t^n$ with $0 \leq \alpha \leq \infty$, corresponding to acceleration if $\alpha > 1$ and in addition we also consider phantom power-law cosmology with $a \propto (t_a - t)^\beta$. In cosmic history, there were epoch when radiation or dust is dominant component in the universe for which the scale factor evolves as power-law $a \propto t^{1/2}$ and $a \propto t^{2/3}$. A universe with mixed combination of different cosmic ingredients can be modeled using power-law expansion with some approximately constant $\alpha$ during a brief period of cosmic time. Adjustability of expansion rate is characterized by only one parameter, $\alpha$ which is used widely in astrophysical observations. There are also other situations that one can obtain the power-law solution. These are such as non-minimally coupled scalar-tensor theory in which the scalar field couples to the curvature contributing to energy density that cancels out the vacuum energy (Dolgov 1982; Ford 1987; Fujii et al. 1990; Dolgov 1997) and simple inflationary model in which the power-law cosmology can avoid flatness and horizon problems and can give simple spectrum (Lucchin et al. 1985). The power-law has proved to be a very good phenomenological description of the cosmic evolution, since it can describe radiation epoch, dark matter epoch, and dark energy epoch according to value of the exponent (Kolb 1989; Peebles 1993). Previously linear-coasting cosmology, $\alpha \approx 1$ was analyzed (Lohiya et al. 1996; Sethi et al. 1999; Dev et al. 2001, 2002) with motivation from SU(2) instanton cosmology (Allen 1999), higher order (Weyl) gravity (Manheim et al. 1990), or from scalar-tensor theories (Lohiya et al. 1999). However the universe expanding with $\alpha = 1$ (Melia et al. 2012) was not able to agree with observational constraint from Type Ia supernovae, Hubble rate data from cosmic chronometers and BAO (Blick et al. 2012) which indicates that $H'(z)=\text{const}$ and $q(z) = 0$ are not favored by the observations.

For a specific gravity or dark energy model, power-law cosmology is considered in $f(T)$ and $f(G)$ gravities (Rastkar et al. 2012; Setare et al. 2012) and in the case of which there is coupling between cosmic fluids (Cataldo et al. 2008). The power-law cosmology were also studied in context of scalar field cosmology (Gunjudpai et al. 2012; Gunjudpai 2013), phantom scalar field cosmology (Kaonikhom et al. 2011). There is also slightly different form of the power-law function which $\alpha$ can evolved with time so that it can parameterize cosmological observables (Wei 2001).

For the power-law to be valid throughout the cosmic evolution, it is not possible with constant exponent. For example, at big bang primordial nucleosynthesis (BBN), $\alpha$ is allowed to have maximum value at approximately 0.55 in order to be capable of light element abundances (Kaplinghat et al. 1999) (Kaplinghat et al. 2000). The value is about 1/2 at highly-radiation dominated era, about 2/3 at highly-dust dominated era and greater than one at present. Low value of $\alpha$ results in much younger cosmic age and does not give acceleration. On the other hand $\alpha \geq 1$ value is needed to solve age problem in the CDM model (Kolb 1989) without flatness and horizon problems. In universe dominated with cold dark matter and dark energy, considering that the power-law expansion happens long after matter-radiation equality era, $z \ll 3196$ (value from (Larson et al. 2011)), the BBN constraint can be relaxed and large $\alpha$ can be allowed. We consider power-law cosmology with a brief period of recent cosmic era when dark energy began to dominate, i.e. from $z \lesssim 0.45$ to present using results from WMAP7 (Larson et al. 2011) and WMAP7+BAO+$H_0$ combined datasets (Komatsu et al. 2011). There are tachyonic scalar field evolving under potential $V(\phi)$ and dust barotropic fluid (cold dark matter and baryonic matter) as two major ingredients. We aim to test whether the power-law cosmology is still valid in the scenario of tachyonic scalar field by looking at value of the equation of state predicted by the power-law tachyonic cosmology and that of varying dark energy equation of state direct-observational result. The WMAP7 and WMAP7+BAO+$H_0$ data used here are presented in Table 1. We also consider when the field is phantom, i.e. having negative kinetic term with phantom power-law expansion (Caldwell 2002; Caldwell et al. 2003), $a \propto (t_a - t)^\beta$, $\beta < 0$ from $z \lesssim 0.45$ till present. We determine tachyonic field equation of state parameter, $w_\phi$ and we perform parametric plot versus exponent $\beta$. We then analyze the result and conclude this work.

2 Background cosmology and observational data

We consider standard FLRW universe containing dust matter (cold dark matter and baryonic matter) with tachyonic field with Lagrangian,

$$L_{\text{tachyon}} = -V(\phi)\sqrt{1 - \partial_\mu \phi \partial^\mu \phi}$$

(1)
evolving under the background Friedmann equation,

\[ H^2 = \frac{8\pi G}{3} (\rho_\phi + \rho_m) - \frac{kc^2}{a^2} \]  

(2)

and acceleration rate,

\[ \dot{H} = \frac{\ddot{a}}{a} - H^2 = -\frac{4\pi G}{c^2} \left( \rho_\phi c^2 + \rho_m c^2 + p_m \right) + \frac{kc^2}{a^2} \]  

(3)

Tachyonic field energy density and pressure are

\[ \rho_\phi c^2 = \frac{V(\phi)}{\sqrt{1 - \epsilon \phi^2}} \]  

(4)

\[ p_\phi = -V(\phi) \sqrt{1 - \epsilon \phi^2} \]  

(5)

where \( \epsilon = \pm 1 \). The negative \( \epsilon \) represents the case when kinetic term of the tachyon is phantom. The tachyonic fluid equation reads

\[ \frac{\epsilon \dot{\phi}^2}{1 - \epsilon \phi^2} + 3H \epsilon \dot{\phi} + \frac{V'}{V} = 0 \]  

(6)

Using Eq. (4), (5) and (6) in the (3) (dust pressure is zero), we obtain

\[ \dot{H} = -\frac{4\pi G}{c^2} \left( \frac{V \epsilon \dot{\phi}^2}{\sqrt{1 - \epsilon \phi^2}} + \rho_m c^2 \right) + \frac{kc^2}{a^2} \]  

(7)

Using tachyonic density (4) in the Friedmann equation therefore

\[ \frac{V}{\sqrt{1 - \epsilon \phi^2}} = \frac{3}{8\pi G/c^2} \left( H^2 + \frac{kc^2}{a^2} \right) - \rho_m c^2 \]  

(8)

Substituting (8) into the equation (7), we obtain

\[ \dot{H} = -4\pi G \left[ \frac{3c^2 \dot{\phi}^2}{8\pi G} \left( H^2 + \frac{kc^2}{a^2} \right) - \rho_m \epsilon \dot{\phi}^2 + \rho_m \right] + \frac{kc^2}{a^2} \]  

(9)

which can be rewritten as

\[ \epsilon \dot{\phi}^2 = -\left[ \frac{2\dot{H} - (2kc^2/a^2) + 8\pi G \rho_m}{3H^2 + (3kc^2/a^2) - 8\pi G \rho_m} \right] \]  

(10)

and hence

\[ 1 - \epsilon \dot{\phi}^2 = \frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G \rho_m} \]  

(11)

We use the above expression in the equation (8), as a result we can get tachyonic potential

\[ V = \left[ \frac{3}{8\pi G/c^2} \left( H^2 + \frac{kc^2}{a^2} \right) - \rho_m c^2 \right] \times \sqrt{\frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G \rho_m}}. \]  

(12)

Tachyonic potential of the phantom-power law is in different form from the quintessential potential of the normal power-law cosmology. The tachyonic equation of state, \( w_\phi \) is, from (11),

\[ w_\phi = \frac{p}{\rho c^2} = -(1 - \epsilon \dot{\phi}^2) \]  

\[ = -\left[ \frac{3H^2 + 2\dot{H} + (kc^2/a^2)}{3H^2 + (3kc^2/a^2) - 8\pi G \rho_m} \right] \]  

(13)

This can be weighed with the dust-matter content to give effective equation of state, \( w_{\text{eff}} = \rho_\phi w_\phi / (\rho_\phi + \rho_m) \). With all information above, \( w_{\text{eff}} \) is expressed as

\[ w_{\text{eff}} = w_\phi \left[ 1 - \frac{8\pi G \rho_m / 3}{H^2 + (kc^2/a^2)} \right] \]  

(14)

We found that the equation (13) and (14) are the same for both quintessence scalar field (Gunjudpai 2013) and tachyonic field cases, albeit the \( \dot{\phi} \) and \( V(\phi) \) are expressed differently in both cases. That is for both quintessence and tachyonic cases, \( w_\phi \) does not depend on the scalar field model but depends on the form of expansion function. This is also true for \( w_{\text{eff},0} \). The equation of state is also independent of the sign of \( \epsilon \) which indicates negative kinetic energy. Using power-law expansion and phantom power-law expansion into (13), one can find the present value of the equation of state, \( w_{\phi,0} \). This value is a (phantom) power-law prediction of the \( w_{\phi,0} \). We can compare this predicted value to the \( w_{\phi,0} \) (of varying equation of state) obtained from CMB observation.

The derived data from WMAP7+BAO+H_\_0 and WMAP7 are presented in Table 1. We will set \( a_0 = 1 \) and consider flat universe \( k = 0 \) throughout (but kept \( k \) in the formulat for completeness). Dust density is defined as \( \rho_m = \Omega_m 0 a_0 c_0 \). Total dust fluid density at present is sum of that of all dust matter type \( \Omega_m = \Omega_{\text{CDM},0} + \Omega_m \). Present value of the critical density is \( \rho_c,0 = 3H_0^2 / 8\pi G \), and radiation density is negligible. We take the maximum likelihood value assuming spatially flat case. Although in deriving \( t_0 \), the \( \Lambda \)CDM model is assumed with the CMB data, however one can estimably use \( t_0 \) since \( \Delta \phi_G \) is very close to -1. In SI units, the reduced Planck mass squared is \( M_\text{P}^2 = \hbar c / 8 \pi G \). In this work, we also give correction to errors on future singularity time, \( t_s \) (phantom
power-law case) reported previously in (Kaeonikhom et. al. 2011) and improve values of \( w_{\phi,0} \) of the phantom power-law case in (Kaeonikhom et. al. 2011) and of the usual power-law case reported earlier (Gumjudpai 2013).

3 Power-law cosmology

Origin of power-law cosmology comes from a solution of the Friedmann equation with flat geometry and domination of dark energy, \( H^2 = 8\pi G \rho_{\phi}/3 \). For constant equation of state \( w_{\phi} \), the solution is well known as (see, for example, in page 150 of (Coles et. al. 2002))

\[
a = a_0 \left[ 1 + \frac{H(t_0)}{\alpha} (t - t_0) \right]^\alpha
\]

(15)

where \( \alpha = 2/[3(1 + w_{\phi})] \) is constant. For \(-1/3 > w_{\phi} > -1\), the solution takes power-law form,

\[
a(t) = a_0 \left( \frac{t}{t_0} \right)^{\alpha},
\]

(16)

Note that although the function is motivated by domination of constant \( w_{\phi} \) scalar field in the flat Friedmann equation which gives \( 1 < \alpha < \infty \), here we will consider the range \( 0 < \alpha < \infty \) (constant value of \( \alpha \)) and we will estimably use the power-law expansion in presence of barotropic dust fluid and varying \( w_{\phi} \) in a short range of redshift \( z \lesssim 0.45 \) to present. Later section on phantom-power law (Sec. 4) is based on the same estimation as well. In the power-law cosmology, the speed is \( \dot{a} = \alpha a/t \) and the acceleration is \( \ddot{a} = \alpha (\alpha - 1)a/t^2 \). The Hubble parameter is \( H(t) = \dot{a}/a = \alpha /t \) with \( H = -\alpha /t^2 \). The deceleration parameter in this scenario is \( q = -\ddot{a}/a^2 = (1/\alpha) - 1 \), that is \( \alpha = 1/(q+1) \). As \( \alpha \geq 0 \) is required in power-law cosmology, hence \( q \geq -1 \) and \( H_0 \geq 0 \). To convert into redshift \( z \), from \( 1 + z = a_0/a \) then \( 1 + z = (t_0/t)^{\alpha} \). Typically astrophysical tests for power-law cosmology indicating the value of \( \alpha \) are performed by observing \( H(z) \) data of SNIa or high-redshift objects such as distant globular clusters (Dev et. al. 2008; Sethi et. al. 2005; Kumar 2012). To indicate the value of \( \alpha \) one can also use gravitational lensing statistics (Dev et. al. 2002), compact-radio source (Jain et. al. 2003) or using X-ray gas mass fraction measurements of galaxy clusters (Zhu et. al. 2008) (Allen 2002, 2003). Study of angular size to \( z \) relation of a large sample of milliarcsecond compact radio sources in flat FLRW universe found that \( \alpha = 1.0 \pm 0.3 \) at 68 \% C.L. (Jain et. al. 2003). WMAP5 dataset gives \( \alpha = 1.01 \) for closed geometry (Gumjudpai et. al. 2012). Some procedures of measurement give large value of \( \alpha \) such as \( \alpha = 2.3^{+1.4}_{-0.7} \) (X-ray mass fraction data of galaxy clusters in flat geometry) (Zhu et. al. 2008) and \( \alpha = 1.6^{+0.10}_{-0.09} \) (joint test using Supernova Legacy Survey (SNLS) and \( H(z) \) data in flat geometry) (Dev et. al. 2008). Notice that assumption of non-zero spatial curvature \((\pm 1,0)\) is assumed in these results in evaluating of \( \alpha \) except in the WMAP5 of which the result puts also constraint on the spatial curvature. When \( \alpha \) is found with curvature-independent procedure (i.e. with neither SNIa nor cluster X-ray gas mass fraction) or in flat case, \( \alpha \) is near unity. For example, \( H(z) \) data gives \( \alpha = 1.07^{+0.11}_{-0.09} \) (Dev et. al. 2008) and \( \alpha = 1.11^{+0.21}_{-0.14} \) (Gumjudpai 2013; Kumar 2012).

Short review of recent \( \alpha \) values can be found in Ref. (Gumjudpai 2013). Here \( \alpha \) is calculated from value at present \( H_0, t_0 \) as \( \alpha = H_0 t_0 \). From (13) and (14), in case of power-law cosmology driven by tachyonic field, the equation of state of dark energy is

\[
w_{\phi} = -\frac{\frac{3\alpha^2}{\alpha^2} - \frac{2\alpha}{t_0^2} + \frac{k\alpha^2}{t_0^2} \left( \frac{t_0}{t} \right)^{2\alpha}}{\frac{3\alpha^2}{\alpha^2} + \frac{3k\alpha^2}{t_0^2} \left( \frac{t_0}{t} \right)^{2\alpha} - 8\pi G \rho_{m,0} \left( \frac{t_0}{t} \right)^{3\alpha}}
\]

(17)

and

\[
w_{\text{eff}} = w_{\phi} \left[ 1 - \frac{(8\pi G/3)\rho_{m,0}(t_0/t)^{3\alpha}}{(\alpha^2/t^2) + (k\alpha^2/t_0^2)\left( t_0/t \right)^{2\alpha}} \right]
\]

(18)

At present, \( t = t_0 \), \( w_{\phi,0} = -1 + 2/(3\alpha) \). In Table 2, values of equation of state parameters derived in the power-law cosmology (true for both tachyonic and quintessence) do not match observational data, i.e. \( w_{\phi,0} \) and \( w_{\text{eff,0}} \) found here are much greater than observational (spatially flat) WMAP derived results, for example WMAP7\(^1\): \( w_{\phi,0} = -1.12^{+0.42}_{-0.43} \), WMAP7+BAO+H0 combined\(^2\): \( w_{\phi,0} = -1.10^{+0.14}_{-0.14} \) (68 \% CL), WMAP7+BAO+H0+SN\(^3\): \( w_{\phi,0} = -1.34^{+0.36}_{-0.36} \) (68 \% CL) and WMAP7+BAO+H0+SN with time delay distance information correction\(^4\): \( w_{\phi,0} = -1.31^{+0.67}_{-0.38} \) (68 \% CL). We conclude that the power-law expansion universe with quintessential scalar field (Gumjudpai 2013) or tachyonic field is neither viable.

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\(^{1}\)flat geometry, constant \( w_{\phi,0} \) (Sec. 4.2.5 of Ref. (Larson et. al. 2011))

\(^{2}\)flat geometry, constant \( w_{\phi,0} \) (Sec. 5.1 of Ref. (Komatsu et. al. 2011))

\(^{3}\)flat geometry, time varying dark energy EoS, \( w_{\phi}(a) = w_0 + w_a(1 - a) \) with \( w_0 = -0.93 \pm 0.13 \), \( w_a = -0.41^{+0.72}_{-0.71} \) (Sec. 5.3 of Ref. (Komatsu et. al. 2011))

\(^{4}\)flat geometry, time varying dark energy EoS, \( w_{\phi}(a) = w_0 + w_a(1 - a) \) with \( w_0 = -0.93 \pm 0.12 \), \( w_a = -0.38^{+0.66}_{-0.65} \) (Sec. 5.3 of Ref. (Komatsu et. al. 2011))
Table 1  Combined WMAP7+BAO+$H_0$ and WMAP derived parameters (maximum likelihood) from Refs. (Larson et. al. 2011) and (Komatsu et. al. 2011). Here we also calculate (with error analysis) $\Omega_{m,0} = \Omega_{b,0} + \Omega_{CDM,0}$, critical density: $\rho_c,0 = 3H_0^2/8\pi G$ and matter density: $\rho_m,0 = \Omega_{m,0}\rho_c,0$. The space is flat and $a_0$ is set to unity.

| Parameter | WMAP7+BAO+$H_0$ | WMAP7 |
|-----------|----------------|--------|
| $t_0$     | $13.76 \pm 0.11$ Gyr or $(4.34 \pm 0.03) \times 10^{17}$ sec | $13.79 \pm 0.13$ Gyr or $(4.35 \pm 0.04) \times 10^{17}$ sec |
| $H_0$     | $70.4 \pm 1.4$ km/s/Mpc | $70.3 \pm 2.5$ km/s/Mpc |
|           | $(2.28 \pm 0.04) \times 10^{-18}$ sec$^{-1}$ | $(2.28 \pm 0.08) \times 10^{-18}$ sec$^{-1}$ |
| $\Omega_{b,0}$ | $0.0455 \pm 0.0016$ | $0.0451 \pm 0.0028$ |
| $\Omega_{CDM,0}$ | $0.226 \pm 0.015$ | $0.226 \pm 0.027$ |
| $\Omega_{m,0}$ | $0.271(5) \pm 0.015(1)$ | $0.271(1) \pm 0.027(1)$ |
| $\rho_{m,0}$ | $(2.52(49)^{+0.18(24)}_{-0.16(64)}) \times 10^{-27}$ kg/m$^3$ | $(2.52(12)^{+0.39(97)}_{-0.39(61)}) \times 10^{-27}$ kg/m$^3$ |
| $\rho_{c,0}$ | $(9.29(99)^{+0.32(92)}_{-0.32(45)}) \times 10^{-27}$ kg/m$^3$ | $(9.29(99)^{+0.66(41)}_{-0.64(12)}) \times 10^{-27}$ kg/m$^3$ |

Table 2  Power-law cosmology exponent and its prediction of equation of state parameters. The value does not match the WMAP7 results.

| Parameter | WMAP7+BAO+$H_0$ | WMAP7 |
|-----------|----------------|--------|
| $\alpha$  | $0.98(95) \pm 0.01(87)$ | $0.99(18) \pm 0.03(60)$ |
| $w_{\phi,0}$ (with power-law cosmology) | $-0.44(79)^{+0.01(66)}_{-0.01(54)}$ | $-0.44(98)^{+0.02(97)}_{-0.02(82)}$ |
| $w_{\text{eff},0}$ (with power-law cosmology) | $-0.32(63) \pm 0.01(25)$ | $-0.32(78) \pm 0.02(35)$ |

4 Phantom power-law cosmology

In this section, we can check if phantom power-law could be a valid solution for the tachyonic-driven universe. From Eq. (15), with constant $w_{\phi} < -1$, the solution becomes phantom power-law,

$$a(t) = a_0 \left( \frac{t_s - t}{t_s - t_0} \right)^\beta$$

with speed,

$$\dot{a} = -a_0 \beta \left( \frac{t_s - t}{t_s - t_0} \right)^{\beta - 1} \frac{a}{(t_s - t)}$$

and acceleration,

$$\ddot{a} = a_0 \beta^2 \left( \frac{t_s - t}{t_s - t_0} \right)^{\beta - 2} \frac{a}{(t_s - t)^2}$$

where $t_s \equiv t_0 + |\beta|/H(t_0)$ (Coles et. al. 2002) is future big-rip singularity time (Caldwell 2002; Caldwell et. al. 2003) and we use $\beta$ instead of $\alpha$ to distinct the two solutions. For both $\dot{a}$ and $\ddot{a}$ to be greater than zero, i.e. both expanding and accelerating, the condition $\beta < 0$ is needed. The Hubble parameter is therefore,

$$H = \frac{\dot{a}}{a} = -\frac{\beta}{t_s - t}$$

hence

$$\dot{H} = -\frac{\beta^2}{(t_s - t)^2}$$

At present, $\beta = H_0(t_0 - t_s)$. The deceleration parameter is $q \equiv -a\ddot{a}/\dot{a}^2 = (1/\beta) - 1$. The dust matter density,

$$\rho_m = \rho_{m,0} a_0^3/a^3$$

is then

$$\rho_m = \rho_{m,0} \left( \frac{t_s - t_0}{t_s - t} \right)^{3\beta}$$

Substituting these equations into (13) and (14), we obtain,

$$w_{\phi} = -\left[ \frac{\beta(3\beta - 2)}{(t_s - t_0)^2} + \frac{(kc^2)}{a_0^2} \left( \frac{t_s - t_0}{t_s - t} \right)^{2\beta} \right]$$

$$\frac{3\beta^2}{(t_s - t)^2} + \frac{3kc^2}{a_0^2} \left( \frac{t_s - t_0}{t_s - t} \right)^{2\beta} - 8\pi G \rho_{m,0} \left[ \frac{t_s - t_0}{t_s - t} \right]^{3\beta}$$

$$w_{\text{eff}} = w_{\phi} \left[ 1 - \left( \frac{8\pi G \rho_{m,0}}{3} \right) \left( \frac{t_s - t_0}{t_s - t} \right)^{3\beta} \right]$$

$$+ \left( \frac{\beta^2}{(t_s - t)^2} + \frac{(kc^2)}{a_0^2} \left( \frac{t_s - t_0}{t_s - t} \right)^{2\beta} \right)$$

(23)
To convert to redshift one can use $1 + z = a_0/a$ therefore $1 + z = [(t_0 - t)/(t_0 - t)]^\beta$ and $t_s - t = (t_0 - t_0)(1 + z)^{-1/\beta}$. At present, $t = t_0$, $w_{\text{eff},0} = -1 + 2/(3\beta)$. The big-rip time $t_s$ can be calculated from

$$t_s \approx t_0 - \frac{2}{3(1 + w_{\text{DE}})} \frac{1}{H_0 \sqrt{1 - \Omega_{m,0}}}$$ (24)

Here, $w_{\text{DE}}$ must be less than -1 and in deriving this above expression flat geometry and constant dark energy equation of state is assumed (Caldwell 2002; Caldwell et. al. 2003). We will estimably use $t_s$ from this formula. In finding error bar of $t_s$, we exploit better procedure than that performed earlier in (Kacenikhom et. al. 2011) by considering that the second order of error bar multiplications are too large to be neglected. We discuss this in the appendix. Results presented in Table 3 are $\beta$, $t_s$ and the equation of state. For phantom power-law cosmology driven by tachyonic field (also true for phantom quintessence), the resulting value is $w_{\phi,0} = -1.49^{+1.14}_{-1.08}$ (using WMAP7+BAO+H0) and $-1.51^{+3.13}_{-6.72}$ (using WMAP7). These do not much differ from results from WMAP7+BAO+H0+SN data (flat, varying dark energy EoS) which gives $w_{\phi,0} = -1.34^{+1.74}_{-0.36}$ (68 % CL) and WMAP7+BAO+H0+SN+time delay distance correction data (flat varying dark energy EoS) which gives $w_{\phi,0} = -1.31^{+1.67}_{-0.38}$ (68 % CL) (Komatsu et. al. 2011). Using observational data in Tables 1 and 3 we derive

$$w_{\phi,0} = \left[ 1 - \frac{2/(3\beta)}{1 - (16.60/3^2)} \right] (\text{WMAP7+BAO+H0})$$ (25)

$$w_{\phi,0} = \left[ 1 - \frac{2/(3\beta)}{1 - (11.47/3^2)} \right] (\text{WMAP7})$$ (26)

With these, we show parametric plots of the $w_{\phi,0}$ and $\beta$ in Fig. 1. The values measured for $\beta$ and $w_{\phi,0}$ are the purple cross (WMAP7+BAO+H0) and yellow spot (WMAP7). For $-\infty < \beta < -6$, $w_{\phi,0}$ lies in the range ($-1, -2$). Fig. 2 shows evolution of $w(z)$ in late phantom power-law universe from $0 < z < 0.45$, i.e. $t = 8.45$ Gyr (both datasets) till present era (this is to avoid singularity in $w_s$ at $z = 0.492$ (WMAP7+BAO+H0) and at $z = 0.484$ (WMAP7)). These are equivalent to the past 5.28 Gyr ago (WMAP7+BAO+H0) and the past 5.31 Gyr ago (WMAP7).

## 5 Tachyonic potential for phantom power-law cosmology

### 5.1 Tachyonic field dominant case

When the field is phantom ($\epsilon = -1$) and is the dominant component, the equation (10) for flat space hence

$$\dot{\phi}^2 = \frac{2\dot{H}}{3H^2} = -\frac{2}{3\beta}$$ (27)

Integrating from $t$ to $t_s$, and choosing positive solution,

$$\phi(t) = \sqrt{\frac{2}{3|\beta|(t_s - t)}}$$ (28)

Since $\beta < 0$ hence $-\beta = |\beta|$. From (12) the tachyonic potential is

$$V(\phi) = \frac{2c^2|\beta|}{\kappa \phi^2} \sqrt{1 + \frac{2}{3|\beta|}}$$ (29)

where $\kappa \equiv 8\pi G$. With parameters in Table 3, the potential is plotted in Fig. 3 which is no surprised as it was found earlier (Padmanabhan 2002) regardless of the expansion is either normal power-law or phantom power-law. The steepness of the potential is typically determined by a dimensionless variable

$$\Gamma = \frac{V''V}{V^2}$$ (30)

where $'$ denotes d/d$\phi$. For the potential (29), it is found that $\Gamma = 3/2$.

### 5.2 Using tachyonic field dominant solution to approximate $V(\phi)$ in mixed fluid universe

Considering equation (10) for flat space and $\epsilon = -1$ hence $\dot{\phi}^2 = (2H + 8\pi G\rho_m)/(3H^2 - 8\pi G\rho_m)$. We approximate that the dust term is much less contributive compared to the $\dot{H}$ and $H^2$ terms therefore,

$$\dot{\phi}^2 \approx \frac{2\dot{H}}{3H^2} = -\frac{2}{3\beta}$$, $\phi(t) \approx \sqrt{\frac{2}{3|\beta|(t_s - t)}}$ (31)

Now we will use this $\phi(t)$ solution found with tachyonic field dominant approximation to find the potential. This is not exact way of deriving the potential which has also contribution of baryonic matter density. However the approximation made here does not much alter the result and could be roughly acceptable. Let $B \equiv \sqrt{3|\beta|/2}$, hence $t_s - t = B\phi$. Using the equation
Table 3  Phantom power-law cosmology exponent and its prediction of equation of state parameters. The equation of state lies in acceptable range of values given by WMAP7 results. Large error bar of \(w_{\phi,0}\) is an effect of large error bar in \(t_s\).

| Parameter | WMAP7+BAO+Hubble | WMAP7 |
|-----------|------------------|--------|
| \(\beta\) | \(-7.81(08) +1.11(08) -4.56(01)\) | \(-6.50(02) +3.91(02) -5.09(96)\) |
| \(t_s\) | \(122.30(00) +162.83(07) -63.36(00)\) Gyr | \(104.21(54) +54.37(03) -70.79(90)\) Gyr |
| \(w_{\phi,0}\) (with phantom power-law cosmology) | \(-1.48(99) +11.64(46) -4.08(45)\) | \(-1.51(26) +3.89(23) -6.71(90)\) |
| \(w_{\phi,0}\) (with phantom power-law cosmology) | \(-1.08(54) -0.11(98)\) | \(-1.10(24) -0.37(12)\) |

(12) we find that

\[
V(\phi) \approx \left[ \frac{3c^2}{\kappa} \beta^3 - \rho_{m,0} c^2 \left( \frac{t_s - t_0}{B_{10}} \right)^{3\beta} \right] \times \left[ \frac{1 - 2/(3\beta)}{1 - \rho_{m,0}[\kappa/(3\beta^2)](B)^{2-3\beta}(t_s - t_0)^{3\beta}} \right]^{1/2}
\]

Note that the term \(1 - 2/(3\beta)\) is just \(-w_{\phi,0}\). We can rearrange the potential in form of cosmological observables \(H_0, \Omega_{m,0}\) and \(q\),

\[
V \approx \frac{c^2}{\kappa} \left[ \frac{2|\beta|}{\phi^2} - 3 \left( \frac{3}{2|\beta|} \right)^{3|\beta|} \Omega_{m,0} H_0^{2+3|\beta|} \phi^{3|\beta|} \right] \times \left[ \frac{1 + 2/(3|\beta|)}{1 - \left( \frac{3}{2} \right)^{1+3|\beta|} \Omega_{m,0} (H_0)^{2+3|\beta|} |\beta|^{-1-3|\beta|}} \right]^{1/2}
\]

where \(\beta = \beta(q) = (1 + q)^{-1}\). This is plotted in Fig. 4 where the field values at present and at \(z = 0.45\) are

\[
\phi|_{z=0} = 1.268 \times 10^{17} \text{ sec and} \\
\phi|_{z=0.45} = 7.803 \times 10^{16} \text{ sec (WAMP7 + BAO + Hubble)} \\
\phi|_{z=0} = 1.393 \times 10^{17} \text{ sec and} \\
\phi|_{z=0.45} = 8.555 \times 10^{16} \text{ sec (WAMP7)}
\]

It has been known that in order for the tachyonic potential to account for the late acceleration, it should not be steeper than the potential \(V \propto \phi^{-2}\) (Padmanabhan 2002; Bagla et. al. 2003). To check if our derived tachyonic potential could fit in this criteria, i.e. shallower than \(V \propto \phi^{-2}\), we use dimensionless variable, \(\Gamma\). For the potential \(V \propto \phi^{-2}\) in previous section, \(\Gamma = 3/2\). Hence in general the potential with \(\Gamma < 3/2\) satisfies this criteria. Considering the potential (33) we use both derived datasets to compute its dynamical slope \(\Gamma(\phi)\) which is in very complicated form. We plot this in Fig. 5. We found that using our data with the field value at present, for WMAP7+BAO+Hubble, \(\Gamma(\phi(z = 0)) = 1.500\) and for WMAP7, \(\Gamma(\phi(z = 0)) = 1.500\). Up to three decimal digits, these values are approximately the same as that of \(V \propto \phi^{-2}\). Note that the \(V \propto \phi^{-2}\) potential is found when the universe is filled with tachyon field as single component. Indeed in the limit \(\Omega_{m,0} \rightarrow 0\), our derived potential (33) becomes \(V \propto \phi^{-2}\). The other tachyonic potentials such as \(V = V_0/[\cosh(a\phi/2)]\) and \(V = V_0(1/b)^m\phi^2\) have \(V = 1 - \cosh^2(a\phi/2)\) and \(1 + (m\phi)^{-2}\) respectively. These examples are typical tachyonic potentials which also have dynamical slopes. In Fig. 5, \(\Gamma(\phi)\) diverges twice however, in the region we consider (\(z = 0.45 \rightarrow z = 0\)), the value of \(\Gamma\) stays approximately at 1.5.

6 Conclusion

In this work model of tachyonic-driven universe are investigated for normal power-law cosmology and phantom power-law cosmology. The universe is flat FLRW filled with tachyonic scalar field and dust. We consider late universe when dark energy has dominated, i.e. \(z < 0.45\). WMAP7 data and its derived data when combined with BAO and \(H(z)\) data are used in this study. We find exponents of power-law and phantom-power-law expansion and other cosmological observables. We improve data reported earlier in (Kaoenikhom et. al. 2011). We find that although the forms of potential and the field solution are different for quintessential scalar field (Gumjudpai 2013) (Kaoenikhom et. al. 2011) and tachyonic field, however the equation of state are identical for both quintessential scalar field and tachyonic field. This is to say that, for quintessence and tachyonic field, the equation of state does not depend on type of the scalar field but depends only on form of expansion function of the scale factor. The present value of dark energy equation of state predicted by quintessential and tachyonic normal power-law cosmology models...
**Fig. 1** Present value of phantom tachyonic dark energy equation of state plotted versus $\beta$. Their error bar results from the error bar in $\beta$. This is the same for quintessence case.

**Fig. 2** Phantom tachyonic (and quintessence) dark energy equation of state versus $z$. 
Fig. 3  Potential versus field using WMAP7+BAO+$H_0$, WMAP7 for the case of tachyonic field domination ($V \propto \phi^{-2}$).

Fig. 4  Approximated potential versus field using WMAP7+BAO+$H_0$, WMAP7 for the case of mixed tachyonic field with barotropic dust.
do not match both WMAP7 datasets. We conclude that the usual power-law cosmology model with either quintessence or with tachyonic field are excluded by these observational data. When considering the other case, the phantom power-law cosmology, the model predicts values of equation of state not much differ from observational results (for both quintessence and tachyonic cases), i.e. \( w_{\phi,0} = -1.49_{-4.68}^{+1.64} \) (phantom power-law using WMAP7+BAO+\( H_0 \)) and \( w_{\phi,0} = -1.51_{-3.89}^{+3.39} \) (phantom power-law using WMAP7) compared to \( w_{\phi,0} = -1.34_{-0.36}^{+1.74} \) (WMAP7+BAO+\( H_0 + \text{SN} \)) and \( w_{\phi,0} = -1.31_{-0.38}^{+1.67} \) (WMAP7+BAO+\( H_0 + \text{SN} + \text{time delay distance correction} \)) (Komatsu et. al. 2011). From parametric plot in Fig. 1, at \( \beta \lesssim -6 \), \( w_{\phi,0} \) is in the expected range (-2, -1). We reconstruct the tachyonic potential in this scenario and we find that the dimensionless slope variable \( \Gamma \) of our derived potential at present time is about 1.5. The phantom-power-law tachyonic potential found here reduced to \( V = V_0 \phi^{-2} \) in the limit \( \Omega_{m,0} \to 0 \).

**Appendix: Errors Analysis**

In calculating of the accumulated errors, we follow the procedure here. If \( f \) is valued of answer in the form

\[
 f = f(x_1, x_2, \ldots, x_n)
\]

and \( f_0 \) is the value when \( x_i \) is set to their measured values, then the value of \( f_i \) is defined as

\[
 f_i = f(x_1, \ldots, x_i + \sigma_i, \ldots, x_n)
\]

This value of \( f \) is the value with effect of error in variable \( x_i \), that is \( \sigma_i \). One can find square of the accumulated error from

\[
 \sigma_f^2 = \sum_{i} (f_i - f_0)^2
\]

Hence giving the error of \( f \) from accumulating effect from errors of \( x_i \).

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