Conformal symmetry: towards the link between the Fermi and the Planck scales

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based of works with Andrey Shkerin
Outline

- Introduction and motivations, naturalness?
- Semiclassical relation between the Fermi and Planck scales?
- Conclusions
Introduction and motivations, naturalness?
Triumph of the SM in particle physics

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- The Standard Model is now complete: the last particle - Higgs boson, predicted by the SM, has been found.
- No significant deviations from the SM have been observed.
- With experimental values of the masses of the top quark and of the Higgs boson the SM is a self-consistent effective field theory all the way up to the quantum gravity Planck scale $M_P$. 
Marginal evidence (less than $2\sigma$) for the SM vacuum metastability given uncertainties in relation between Monte-Carlo top mass and the top quark Yukawa coupling.

Vacuum is unstable at $\sim 1.5\sigma$.

\[ M_t = 173.21 \pm 0.87 \text{ GeV} \]
\[ M_H = 125.7 \pm 0.4 \text{ GeV} \]

\[ M_t = 172.25 \pm 0.63 \text{ GeV} \]
\[ M_H = 125.09 \pm 0.24 \text{ GeV} \]
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**unnatural**

[uhn-nach-er-uh l, -nach-ruh l]

*adjective*

1. contrary to the laws or course of nature.
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This is unfair to “unnatural” SM as it describes the Nature better than “natural” theories...
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$$\delta m_H^2 = \ldots SM \ldots + \ldots New \ldots \sim 0$$

right above EW scale
The original source of the naturalness requirement: hierarchy problem in Grand Unified theories
Extra GUT particles beyond the SM – leptoquarks (vector and scalar) must be very heavy, $M_X > 10^{15}$ GeV

- this is required by the gauge coupling unification
- this is needed for stability of matter, proton lifetime $\tau_p > 10^{34}$ years

Hierarchy: \[
\left( \frac{M_X}{M_W} \right)^2 \approx 10^{28}
\]
Two faces of hierarchy

- Ad hoc tuning between the parameters (masses and couplings of different multiplets) at the tree level with an accuracy of 26 orders of magnitude

- Stability of the Higgs mass against radiative corrections Gildener, ’76

\[ \delta m_H^2 \simeq \alpha_{GUT}^n M_X^2 \]

Tuning is needed up to 14th order of perturbation theory!
Proposed solutions

Stability of EW scale – requirement of “naturalness”: absence of quadratic divergencies in the Higgs mass

- Low energy SUSY: compensation of bosonic loops by fermionic loops
- Composite Higgs boson - new strong interactions
- Large extra dimensions

All require new physics right above the Fermi scale, which was expected to show up at the LHC
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An attempt towards this direction:
M.S., Shkerin, arXiv:1803.08907 + arXiv:1804.06376
Example of “complete” theory: the \( \nu \)MSM

\[ \nu \text{MSM} \equiv \text{Neutrino minimal Standard Model} \]

\[ \equiv \text{Minimal low scale see-saw model with 3 singlet fermions} \]

**Role of** \( N_2, \ N_3 \) with mass in 100 MeV – GeV region: “give” masses to neutrinos and produce baryon asymmetry of the Universe.

**Role of** \( N_1 \) with mass in keV region: dark matter.

**Role** of the Higgs boson: break the symmetry and inflate the Universe.
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Very small numbers in quantum physics: quantum tunnelling

- 1928, Gamow theory of alpha decay,

\[ ^{238}U \rightarrow ^{234}Th + \alpha, \Gamma = E_{\text{bound}}e^{-S} \ll E_{\text{bound}} \]

- 1951, Townes, Ammonia Maser,

\[ \omega = \omega_0 e^{-S} \ll \omega_0 \]

- Perhaps, \( m_H^2 = M_P^2 e^{-S} \) with \( S = 72 \)? Peculiar gravitational instanton, Shkerin, MS, arXiv:1803.08907 + arXiv:1804.06376
Semiclassical relation between the Fermi and Planck scales?
Higgs-Planck hierarchy: the ratio of the two scales is exponentially small,

\[ \frac{m_H}{M_P} \sim 10^{-16} \sim e^{-S}, \quad S \approx 36 \]
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Requirements to the theory

- Perturbatively \( m_H = 0 \), symmetry protection (?)
- Existence of (semi) classical configurations leading to \( \langle H \rangle \neq 0, \quad \langle H \rangle \ll M_P \)
If the mass of the Higgs boson is put to zero in the SM, the classical Lagrangian has a wider symmetry: it is scale and conformally invariant: 

**Dilatations** - global scale transformations ($\sigma = \text{const}$)

$$\Psi(x) \rightarrow \sigma^n \Psi(\sigma x),$$

$n = 1$ for scalars and vectors and $n = 3/2$ for fermions.

It is tempting to use this symmetry for solution of the hierarchy problem

**Bardeen '95**: why the Higgs boson mass is so small in comparison with the Planck scale?

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But what about quantum corrections?
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No go theorem?
The statement "any cutoff procedure necessarily involves a large mass" is not true. Counter-example: dimensional regularisation of \( \text{t’Hooft and Veltman} \) does not involve any large mass, just a normalisation point \( \mu \) which is not send to infinity. Still, the scale invariance is anomalous due to “dimensional transmutation”: the renormalisation-group running of the parameters leads to a non-vanishing trace of the energy-momentum tensor, which enters the divergence of the scale current. However, the physical quantities depend on the renormalisation scale only \text{logarithmically}. Any quadratically divergent contributions to the Higgs boson mass are purely technical and are introduced by explicitly breaking the conformal invariance by regulators.
Radiative symmetry breaking

Lagrangian is invariant at the classical level, and scale symmetry is broken by quantum corrections (conformal anomaly) a’la Coleman-Weinberg: Linde ’76; Weinberg ’76; Buchmuller, Dragon ’88; Hempfling ’96; Meissner, Nicolai ’06; Foot et al ’07, ’11; Iso, et al ’09; Boyle et al ’11; Wetterich ’11, Salvio, Strumia ’14; Lindner et al, ’14, ’15, ’17

Does not work for the SM:

- If the top quark mass $m_t \lesssim 172$ GeV, then the minimum of the effective potential is generated at $\langle H \rangle \simeq 100$ MeV due to chiral symmetry breaking in QCD
- If the top quark mass $m_t \gtrsim 172$ GeV, then an extra minimum of the effective potential is generated at $\langle H \rangle \gtrsim M_P$ due to top quark loops
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Gravity + conformally invariant SM is an ideal playground for looking at non-perturbative generation of the weak scale.
Scalar theory plus gravity with non-minimal coupling, $\xi > 0$:

$$\frac{\mathcal{L}_{\varphi,g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi \varphi^2)R + \frac{1}{2}(\partial \varphi)^2 + V(\varphi)$$

Scale invariant “matter” with

$$V(\varphi) = \frac{\lambda}{4} \varphi^4$$

We want to compute the Higgs vev:

$$\langle \varphi \rangle \sim \int \mathcal{D}\varphi \mathcal{D}g_{\mu\nu} \varphi e^{-S_E}.$$  

$S_E$ is the euclidean action of the model.
Remarks:

Euclidean path integral for gravity may not be well defined due to the problem with the conformal factor of the metric.

We will ignore this problem and follow the crowd: Hawking; Coleman, de Luccia; Veneziano; ..., Isidori, Rychkov, Strumia, Tetradis; ... Branchina, Messina, Sher;...

Small $\varphi \ll M_P$ - gravity is irrelevant – no contribution to the vev of the Higgs from scalar loops.

**Challenge:** account for contributions with $\varphi \gg M_P$.

Theory for large $h$:

$$\mathcal{L} = -\frac{1}{2} \xi \varphi^2 R + \frac{1}{2} (\partial \varphi)^2 + \frac{\lambda}{4} \varphi^4$$
Scale-invariant

Planck scale is dynamical, $\propto \sqrt{\xi \phi}$

**Conjecture:** contribution of large Higgs fields $\phi > M_P/\sqrt{\xi}$ to path integral is better to be found in the Einstein frame. Conformal transformation:

$$\hat{g}_{\mu\nu} = \Omega^2 g_{\mu\nu}, \quad \Omega^2 = 1 + \frac{\xi \phi^2}{M_P^2}$$
Redefinition of the Higgs field to make canonical kinetic term

\[
\frac{d\chi}{d\varphi} = \sqrt{\frac{\Omega^2 + 6\xi \varphi^2}{\Omega^4}} \Rightarrow \begin{cases} 
\varphi \simeq \chi & \text{for } \varphi < \frac{M_P}{\sqrt{\xi}} \\
\varphi \simeq \frac{M_P}{\sqrt{\xi}} \exp \left( \frac{\chi}{\sqrt{6}M_P} \right) & \text{for } \varphi > \frac{M_P}{\sqrt{\xi}}
\end{cases}
\]

Resulting kinetic part of the action

\[
S_{kin} = \int d^4x \sqrt{-\hat{g}} \left\{ - \frac{M_P^2}{2} \hat{R} + \frac{\partial_\mu \chi \partial^\mu \chi}{2} \right\}
\]

Most important:

\[
\langle \varphi(x) \rangle \sim \int \mathcal{D}\mathcal{A} \mathcal{D}\varphi(x) \mathcal{D}g_{\mu\nu} \varphi(x) e^{-S_E} \Rightarrow \int \mathcal{D}\mathcal{A} \mathcal{D}\chi \mathcal{D}\hat{g}_{\mu\nu} e^{\frac{\chi(x)}{\sqrt{6}M_P}} e^{-S_E}
\]

Modification of the action and equations of motion!
Equations of motion for $\chi$ contain a source term $\delta(x) \implies$ new classical solutions.

Similar to

- computation of $\int dx x^N e^{-x^2}$ for large $N$, 
- to computation of multi-particle production, 
- to proof of confinement in 3D Georgi-Glashow model, $\langle \exp(\int A_\mu dv^\mu) \rangle$ in Polyakov '76.

Schematically, modification of the right-hand side for scalar field equation:

$$\Box \chi + \ldots = \delta(x)/\sqrt{6}M_P$$
Path integral:

$$\int_{\varphi \gtrsim M_P/\sqrt{\xi}} D\varphi \varphi e^{-S_E} \rightarrow M_P \int_{\chi \gtrsim M_P \log(1/\sqrt{\xi})} D\chi J e^{-W},$$

where $W = \chi(0)/\sqrt{6}M_P + S_E$ and $J$ is the corresponding Jacobian.

**Conjecture:** Higgs vev

$$\langle \varphi \rangle \approx M_P e^{-\bar{W}},$$

is much smaller than $M_P$ because the action $\bar{W}$ on the saddle point

$$\bar{W} \gg 1.$$ 

Condensed matter analogue: exponentially small mass gap in superconductors.
Field equations in the Einstein frame for maximally $O(4)$ symmetric metric $d\tilde{s}^2 = g^2(\rho) d\rho^2 + \rho^2 d\Omega_3^2$ in the large $\chi$ regime:

$$\partial_\rho \left( \frac{\rho^3 \ddot{\chi}}{g a_{SI}} \right) = -\frac{1}{M_P} \delta(\rho), \quad g^2 = 1 - \frac{\rho^2 \ddot{\chi}^2}{6 a_{SI} M_P^2}, \quad a_{SI} = \frac{1}{1/\xi + 6}$$

Vacuum boundary conditions at infinity $\rho \to \infty$:

$g^2(\rho) \to 1, \quad h(\rho) \to 0$. The asymptotic behaviour of the scalar field $\chi$ at $\rho \to 0$: $\chi = -M_P \sqrt{6 a_{SI}} \log \rho M_P + c$ with $c$ a constant used to match with the asymptotics at large $\rho$.

Coincides with Hawking-Turok instanton, but the constant in front of $\log$ is fixed by the source term.
**blue:** configuration obeying the boundary condition imposed by the source. **green** is the one with the large euclidean action \( \bar{W} = 40 \). **red:** the bounce. **c:** fall-off at infinity, \( \chi = c \rho^{-2}, \rho \rightarrow \infty \).
blue: configuration obeying the boundary condition imposed by the source. green is the one with the large euclidean action $\bar{W} = 40$. red: the bounce. $c$: fall-off at infinity, $\chi = c\rho^{-2}$, $\rho \to \infty$.

The action is too small! Semiclassical approximation does not work, and $\langle \varphi \rangle \sim M_P$ is expected.
The appearance of a scale small compared with $M_P$ is not generic and requires a specific theory in UV, well along with our conjecture. Modifications of the theory in the UV:

- Include higher dimensional operators, for example

$$
\mathcal{O}_4 = \sqrt{g} \delta \frac{(\partial \phi)^4}{(M_P \Omega)^4}, \quad \Omega^2 = 1 + \frac{\xi \phi^2}{M_P^2}
$$

$\Omega^2$ is needed to keep the scale symmetry at large $h$. Leads to finite values of the Higgs field in the centre of the instanton!

- Action of the instanton depends on $a_{SI}$, $S \propto \sqrt{a_{SI}}$. The parameter $a_{SI}$ is a combination of a non-minimal coupling to gravity and scalar kinetic term. They can be changed for large $h$ to make the action large, without affecting low energy physics.
A theory that works

\[ \frac{\mathcal{L}_{h, g}}{\sqrt{g}} = -\frac{1}{2}(M_P^2 + \xi h^2)R + \frac{1}{2}G(h/M_P)(\partial h)^2 \]
\[ + \, \delta \xi^2 \frac{(\partial h)^4}{(M_P \Omega)^4} + \frac{\lambda}{4} h^4 , \]

where \( G(0) = 1, \ G(\infty) = \kappa = \text{const.} \). Then the asymptotic value of \( a_{SI} \) in the large-\( \chi \) regime modifies to

\[ a_{SI} \rightarrow a_{HE} = \frac{1}{\kappa/\xi + 6} , \ \rho \rightarrow 0 , \ h \gtrsim M_P , \]

Must have \( \kappa > -\frac{6}{\xi} \) (absence of ghosts). Taking \( \kappa = -\frac{6}{\xi} + \epsilon, \ \epsilon > 0, \ \epsilon \ll 1 \) can make the action large.
The instanton value of the functional $W$ plotted against the coefficient $a_{HE}$ and with the different choices of the parameter $\delta$.

**Intriguing fact:** large $a_{HE}$ and small $\delta$ correspond to an approximate Weyl symmetry.

**Conclusions:** the mechanism may be viable.
Remark: other known classical solutions

- Bounce: an indication of the vacuum instability Coleman, de Luccia
- Hawking-Moss instanton: dominates transitions between vacua at high temperature
- Gravitational instantons (Taub-NUT, Eguchi-Hanson, Gibbons-Hawking): pure gravity, no relation to scalar field
- Giddings-Strominger instanton: gravity + axion field: wormholes
- Hawking-Turok instanton: creation of Universe from nothing?

None works for the Higgs vev generation
Conclusions
Very small $m_H/M_P$ ratio is (perhaps) telling us that

- There are no new particles with masses between the Fermi and Planck scale
- The smallness of the Fermi scale is a semiclassical non-perturbative UV effect associated with gravity and new type of instantons
- The asymptotic theory of the SM at large scalar fields is nearly Weyl invariant

Open problems

- Unfortunately, we can make no prediction of the ratio $m_H/M_P$, as this depends on details of UV theory.
- We cannot estimate the contribution of the effects other than perturbative and semiclassical.
Remark

Everything can be generalised to a completely scale-invariant theory with spontaneous breaking of scale invariance.

Gravity part:

$$\mathcal{L}_G = - (\xi \chi \chi^2 + 2\xi_h \varphi^\dagger \varphi) \frac{R}{2},$$

The vev of extra field – dilaton – gives rise to the Planck scale.

For analysis and results see M.S., Shkerin, arXiv:1804.06376.