Comparison of priori and posteriori approach of multi-objective optimization for WEDM on Ti6Al4V alloy

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Abstract

Titanium alloys are known to have some excellent properties, such as good biocompatibility, good fatigue resistance and high strength to weight ratio. Due to these properties, Ti6Al4V alloy is widely used in the biomedical field, aerospace and automobile industries. In this article, pulse on-time (TON), pulse off time (TOFF), and servo voltage (SV) were selected as process parameters for wire electric discharge machining (WEDM) on Ti6Al4V alloy. The material removal rate (MRR) and surface roughness (SR) were determined as responses. MRR and SR have been equated by a central composite design (CCD: a response surface method technique). Then multi-objective Artificial Bee Colony optimization (MO-ABC) with Gray relational analysis (GRA) was selected as a priori approach for multi-objective optimization. Also, a multi-objective grasshopper optimization algorithm (MO-GOA) has been chosen as a posterior approach for optimization. These two algorithms have been tested on various iterations and populations. Based on the elapsed time, it has been found that the priori approach of multi-objective optimization is better than the posterior approach of multi-objective optimization. When comparing these algorithms based on the results, it is obtained that the posterior approach gives a better combination of multiple results. The major outcome of the research is that the priori method is quick, while the posterior approach produces many promising solutions.

1. Introduction

Titanium is the ninth most abundant material found in the Earth’s crust. Nevertheless, due to the complexity of its extraction process, it is one of the costly materials that limits its use [1]. Titanium undergoes an allotropic transformation at 882.5°C, which changes its phase from alpha to beta. The addition of certain elements to titanium metal that raises its transition temperature are called alpha stabilizers. They are Al, O, N, and C. The elements that lower the transition temperature of titanium metal are called beta stabilizers, and they are Mo, V, Nb, Cu and Si. The elements with minimal effect on the transition temperature of titanium metal are called neutral elements, and they are Sn & Zr. In Ti6Al4V alloy, Al is added as alpha stabilizer, while V is added as beta stabilizer. The alpha phase of substances is a closed-packed hexagonal system of an elementary crystal structure type. The alpha phase of titanium is highly corrosion resistant, while the beta phase of substances is a body-centered cubic crystal structure type & the beta phase of titanium is more ductile. The Ti6Al4V alloy has an alpha + beta phase, where the best combination of both is applied. Therefore, Ti6Al4V is a good ductile, strong, formable, and corrosion-resistant alloy [2–11].

Titanium has a high strength-to-weight ratio, even at elevated temperatures. Due to this, titanium is widely used in the aerospace and automobile sectors [12]. Titanium is a highly corrosion resistant metal used in chemical industries [13]. When titanium is implanted in the human body, an oxide film forms on the titanium metal (due to its goodness), and it prevents corrosion and helps in osteointegration. Therefore, titanium has good biocompatibility and is therefore used in making biomedical implants [14, 15]. However, in addition to its
good properties, titanium alloys have some difficulties in conventional machining due to other properties (such as high strength even at elevated temperatures, high chemical reaction propensity and low thermal conductivity) [16–19]. Therefore, for the machining of Ti6Al4V alloys, unconventional machining is one of the better choices [20]. In this study, WEDM is used as an unconventional machining for Ti6Al4V alloy. In WEDM, a wire is used to cut the workpiece by creating a spark between the wire and the workpiece [21–23]. This spark during machining depends on factors such as TON, Toff, SV, wire tension (WT), wire feed rate (WFR), etc TON, Toff, and SV are chosen as process parameters in this study.

Chopra et al investigated the power consumption on En31 steel during WEDM. They found that MRR and power consumption increase with increase in TON and decrease in voltage [24]. Ubale et al optimized WEDM on W-Cu metal matrix composites. They found a 24.11% improvement in MRR and a 27.45% improvement in SR due to the desirability task multi-objective optimization approach [25]. Siva Kumar et al processed titanium based human implant material via wire EDM. They found that good surface quality could be achieved due to WEDM [26]. Prabhud et al compared brass and molybdenum wire during wire-cut EDM using GRA with PCA. They found that molybdenum wire gives good surface quality. Whereas they found that maximum MRR and SR are achieved at high peak current [27]. Pranamik et al investigated the geometric error during WEDM on Ti alloy. They found that wire tension and flushing pressure are the most important factors for geometric error [28]. Singh et al studied the EDM process on Biomedical Materials. They have found that the EDM process makes the biomaterial more biocompatible [30].

Devarasiddappa et al performed WEDM on Ti6Al4V alloy. Four process parameters, TON, Toff, current (Ip), and wire-speed (WS), were chosen to analyze MRR and Sr Their research modeled the two responses and then selected a TLBO-based multi-objective optimization algorithm to simultaneously maximize MRR and minimize Sr [31].

Sharma et al proposed WEDM on aero-engine alloy A-706. They have selected Toff, TON, SV, and WFR as input parameters. While MRR is maximized and SR is minimized. They found that WEDM cut surfaces have high amounts of molten droplets, craters, and microscopic holes due to improper flushing and high energy levels. They also concluded that good surface morphology and lowest roughness values are observed on TON 105 μS, SV 60 V, and Toff 55 μS due to low discharge energy and better flushing action [32].

Srinivasrao G et al performed WEDM on alpha-beta Ti alloy. They considered five input process parameters: TON, Toff, Ip, SV, and WFR. At the same time, MRR and SR were chosen as responses. They found that the optimized MRR 2.78066 mm3 min−1 and SR 1.292 micro-meters achieve on the TON 108 μS, Toff 51 μS, Ip 12 A, Vs 51 V, and WFR at 15 m min−1 [33].

Singh et al examined WEDM for the surface roughness of AISI H13 using the ANOVA method. They have found that the SR is minimum when using a diffused wire. SR is max for plain brass wire and high pulse on time [34].

Saini et al optimized the WEDM parameters considering the MRR and SR as the responses. They have found that MRR and SR increase while pulse on time increases [35].

Goyal et al performed WEDM on H-13 die tool steel considering TON, TOFF, wire type and peak current as process parameters. They concluded from ANOVA that TON is the most important factor influencing MRR and SR [36].

Babu et al proposed a feed-forward neural network model using adaptive particle swarm optimization algorithms to optimize surface roughness during drilling. They found that adaptive PSO gives results very close to experimental [37].

Karboga et al introduced ABC optimization for real-world problems, and they proved that it has better performance than other types of algorithms [38]. Mirjalili et al proposed a new algorithm for adaptation from locust swarm navigation in nature. This algorithm can provide competitive results in Pareto optimal solutions related to other algorithms [39]. Therefore, the priori type artificial Bee colony optimization algorithm and the posterior type grasshopper optimization algorithm were compared in this study.

The novelty of this work is to explore the machinability aspect of WEDM on Ti6Al4V alloy and optimize the responses by comparing the primary and posterior approach type algorithms. The research gap of this work is to consider responses other than MRR and SR in order to understand the full machinability aspect of WEDM. Different responses can be kerf width, recast layer thickness, micro-hardness, residual stress, maximum peak-to-valley height, etc.

2. Experimental process

In this investigation, a sample length of 240 mm and 4.8 mm thickness of the Ti6Al4V sample was taken as the machined length for WEDM. Each sample length has been machined at different process parameter conditions.
In the present work the experiments were performed on the FANUC ROBOCUT ALPHA1iB wire cut electric discharge machine. It is shown in figure 1, and its manufacturer is FANUC Corporation Japan. In these experiments, a 0.25 mm hard brass wire has been selected as the wire material. At the same time, deionized water is used as the dielectric medium. The machine can bear a maximum workpiece weight of 1000kg and is 13KVA in power. This machine has an X-axis of 550 mm, a Y-axis of 370 mm, and a Z-axis of 310 mm.

WEDM has a lot of process parameters. Some are the following:

1. **Pulse ON Time**: It is the time period in which current flows during machining.
2. **Pulse OFF Time**: This is the time period in which no current is supplied between the electrodes.
3. **Servo Voltage**: This is the voltage maintaining the spark gap between the electrode and the workpiece.
4. **Wire Tension**: Wire tension is a load in the wire. Which straightens the wire by continuously feeding the wire between the wire guides.
5. **Peak Current**: Peak current is the amount of current flowing in the wire to cut the workpiece by spark generation.

Machining process parameters such as T\text{ON}, T\text{OFF} and SV significantly affect the quality of the products that manufactured by WEDM. Setting the right parameters can result in high MRR and low SR performance. These three parameters were selected on the basis of literature review and pilot experiment.

In this work, the surface roughness is measured by Surf Test SJ-210. During the measurement, the cut-off length was 0.8 mm, the sample length was 4.8 mm, and the measuring speed was 0.5 mm s\(^{-1}\). In this surface tester, a contact-type diamond probe is used as a sensor to measure surface roughness. In this experiment, the arithmetic mean height (Ra) indicates the mean absolute value along the sample length.

\[
Ra = \frac{1}{l_s} \int_0^{l_s} |Z(x)| \, dx
\]

Here, \(l_s\) is the sampling length, while \(Z(x)\) is the absolute roughness value from the mean.

Jewelry and medicine weighing scales were used to measure weight in this research. The Baijnath Premnath Atom MH 88 Palm Scale has an accuracy of 10 mg (0.01 g). Time was measured during the experiments with a stopwatch.
Here $W_i$ is the initial weight, $W_f$ is the final weight after the experiment, and $T$ is the Time during the experiment [40].

The central composite design was used to design the experiments. WEDM has a lot of input process parameters like $T_{ON}$, $T_{OFF}$, SV, Ip, WS, WT, etc. Nevertheless, according to some researchers, the most commonly affected process parameters are $T_{ON}$, $T_{OFF}$ and SV that affect WEDM on Ti6Al4V. In Table 1, five levels of $T_{ON}$, five levels of $T_{OFF}$, and five levels of SV were selected according to the feasibility of the machine by the operator’s experience and previous research.

The CCD for the three factors has $2^3$ factorial runs, $2^3$ star runs, and six replicas of the centre levels, i.e., 20 experiments are done. The readings of the responses measured at these settings of the experiments are shown in Table 2 below.

## 3. Optimization

### 3.1. Multi-objective artificial bee colony optimization

Basically artificial bees in an artificial bee colony are classified into three categories:

I. Employed bees.

II. Onlooker bees.

III. Scout bees.

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**Table 1. Levels of process parameters of WEDM.**

| S.No. | Parameters     | Units | Symbol | Range          |
|-------|----------------|-------|--------|----------------|
| 1     | Pulse ON Time  | μS    | $T_{ON}$ | 6.8,10,12,14  |
| 2     | Pulse OFF Time | μS    | $T_{OFF}$ | 20,22,25,27,30 |
| 3     | Servo Voltage  | V     | SV     | 30,35,40,45,50 |

**Table 2. Experimental chart for WEDM.**

| Run | Pulse on time | Pulse off time | Servo voltage | MRR (g/h) | SR (μm) |
|-----|---------------|----------------|---------------|-----------|---------|
| 1   | 12            | 22             | 45            | 3.102     | 4.803   |
| 2   | 12            | 27             | 45            | 3.342     | 4.901   |
| 3   | 10            | 25             | 40            | 2.664     | 4.401   |
| 4   | 12            | 22             | 35            | 3.048     | 4.855   |
| 5   | 12            | 27             | 35            | 3.276     | 4.998   |
| 6   | 10            | 25             | 30            | 2.166     | 5.141   |
| 7   | 10            | 20             | 40            | 2.238     | 4.601   |
| 8   | 6             | 25             | 40            | 2.526     | 2.801   |
| 9   | 10            | 25             | 40            | 2.910     | 4.398   |
| 10  | 10            | 25             | 40            | 2.616     | 4.451   |
| 11  | 10            | 25             | 40            | 2.568     | 3.981   |
| 12  | 8             | 22             | 35            | 2.286     | 3.952   |
| 13  | 10            | 30             | 40            | 3.144     | 4.612   |
| 14  | 10            | 25             | 40            | 2.964     | 3.950   |
| 15  | 10            | 25             | 50            | 2.286     | 4.957   |
| 16  | 8             | 27             | 35            | 2.706     | 3.654   |
| 17  | 8             | 22             | 45            | 1.902     | 3.801   |
| 18  | 8             | 27             | 45            | 2.604     | 3.481   |
| 19  | 14            | 25             | 40            | 3.810     | 5.201   |
| 20  | 10            | 25             | 40            | 2.988     | 4.105   |

\[
Material\ Removal\ Rate = \frac{W_i - W_f}{T}
\]
Generally, bees visit the food source and collect nectar from them. This principle is used in MO-ABC to optimize the responses. The employed bees first search for the food source and then evaluate its nectar. The number of all employed bees and onlooker bees equals the number of food sources. While the amount of nectar is proportional to the fitness value, the state of the food source is the state of the solution. When all employed bees complete the evaluation of nectar, each employed bee goes randomly to a neighbouring food source by equation (3) and reevaluates the nectar.

\[ x_{i,j} = x_i + \phi(x_i - x_j) \]  \hspace{1cm} (3)

Here, \( x_i \) & \( x_j \) are the \( i \)th & \( j \)th position of the bee, \( x_{i,j} \) is the position of the new bee, & \( \phi \) is the acceleration coefficient.

If this new position \( x_j \) of the food source has more nectar than the previous one, update it to be the new position of the \( i \)th bee. After a food source assessment of these employed bees, onlooker bees come into play. When the nectar of a food source is better, the employed bees begin to dance, and on the dance area, the onlooker bees update the position of their food source by equation (4).

\[ x_{i,j} = x_i + \varphi(x_i - x_j) \]  \hspace{1cm} (4)

Here, \( x_i \) is the \( i \)th position of the food source obtained by the Roulette Wheel Selection, & \( x_j \) is the \( j \)th neighbouring food source position of the bee, \( x_{i,j} \) is the position of the new bee, & \( \varphi \) is the acceleration coefficient. In roulette wheel selection, the probability of \( i \)th onlooker bee selection is \( p_i \) obtained by equation (5):

\[ p_i = \frac{\text{fitness}_i}{\sum_{m=1}^{n} \text{fitness}_m} \]  \hspace{1cm} (5)

Here, \( \text{fitness}_i \) is the fitness of \( i \)th bee, while \( n \) is the total no. of bees.

When all these employed and sighting bees move randomly near neighbouring food sources & the bees do not update for several cycles (if no better solution is found). So, the scout bees come into play, and this number of cycles is called the abandonment limit. The positions of the scout bees are updated randomly by a uniform distribution (equation (6)):

\[ x_i = x_i + \text{random number} \times (x_{\text{max}} - x_{\text{min}}) \]  \hspace{1cm} (6)

The best status of bees is updated. After that, the cycles continue and repeat. The iterations are stopped when the iteration reaches the maximum number of iterations defined in the initial step. Bees attain the best source of food and its best nectar \([38]\).

In this article, the priori type multi-objective artificial bee colony algorithm was used to optimize MRR and SR simultaneously. The pseudocode and flow chart for the multi-objective ABC algorithm is shown in figures 2 and 3, respectively. For this, the weightage for each response was determined by the GRA method, the details of which are as follows:

3.1.1. Gray relational analysis

In the priori approach to multi-objective optimization, the equations for all responses are reduced to a single equation by assigning different weights to those responses. The advantage of GRA is that the weightage of the responses is determined by its experimental results. So, for this priori approach of MO-ABC, the problem can be formulated as:

\[ \text{Fitness Function} = W_1 \times \left( \frac{1}{\text{MRR}} \right) + W_2 \times \left( \frac{\text{SR}}{\text{SR}_{\text{min}}} \right) \]  \hspace{1cm} (7)

Here, \( W_1 \) and \( W_2 \) are the weightings of MRR and SR, respectively. \( 1/\text{MRR} \) is taken to minimize the MRR (as we want to maximize the MRR). For this, \( \text{MRR}_{\text{max}} \) and \( \text{SR}_{\text{min}} \) are calculated as a single objective function taking \( W_1 = 1, W_2 = 0, \) and \( W_1 = 0, W_2 = 1, \) respectively. At the same time, MRR and SR are regression equations derived from DOE.

The GRA method can be understood through the following steps:

**Step I**

First, the responses are normalized by equation (8) (if one wants to maximize) and equation (9) (if one wants to minimize) \([41]\):

\[ Y_{n} = \frac{Z_{n} - \min (Z_{n}, n = 1, 2, 3...p, \sigma = 1, 2...q)}{\max (Z_{n}, n = 1, 2, 3...p, \sigma = 1, 2...q) - \min (Z_{n}, n = 1, 2, 3...p, \sigma = 1, 2...q)} \]  \hspace{1cm} (8)

\[ Y_{n} = \frac{Z_{n} - \max (Z_{n}, n = 1, 2, 3...p, \sigma = 1, 2...q)}{\max (Z_{n}, n = 1, 2, 3...p, \sigma = 1, 2...q) - \min (Z_{n}, n = 1, 2, 3...p, \sigma = 1, 2...q)} \]  \hspace{1cm} (9)
Here, $Y_{no}$ is the normalized value of the nth number of the experimental run for oth response.

$Z_{no}$ is the experimental value of the nth number of the experimental run for oth response.

**Step II**

After normalizing the responses, the Gray Relational coefficient (GRC) is calculated by equation (10), & shown in the following table 3:
Here,

\[
GRC(r) = \gamma (Y_r(r), Y_{ref}(r)) = \frac{\Delta_{\text{min}} + \xi \Delta_{\text{max}}}{\Delta_{\text{ref}}(r) + \xi \Delta_{\text{max}}}
\]

(10)

\[
\Delta_{\text{ref}}(r) = \text{coefficient of variation between } Y_r(r) \text{ and } Y_{ref}(r) = \| Y_r(r) - Y_{ref}(r) \|
\]

\[
\Delta_{\text{max}} = \text{maximum value of } \Delta_{\text{ref}}(r)
\]

\[
\Delta_{\text{min}} = \text{minimum value of } \Delta_{\text{ref}}(r)
\]
\[ \xi = \text{discrimination coefficient (0.5)} \]

\[ 0 < \xi < 1, [42] \text{.} \]

**Step III**

After the GRC is calculated, its average is calculated at each process parameter level. Then its maximum Range is calculated by equation (11), and the weight of each response is calculated by equation (12), as shown in table 4:

\[ R_{ot} = \max \{ A_{ot1}, A_{ot2}, \ldots, A_{otv} \} - \min \{ A_{ot1}, A_{ot2}, \ldots, A_{otv} \} \]

\[ w = \frac{\sum_{t=1}^{u} R_{ot}}{\sum_{o=1}^{q} \sum_{t=1}^{u} R_{ot}} \]

Here,

- \( R_{ot} = \text{Range of grey relational coefficient} \)
- \( \Lambda = \text{Average grey relational coefficient of each process parameter at the same level} \)
- \( \alpha = 1, 2, \ldots, q \text{ is number of objectives} \)
- \( t = 1, 2, \ldots, u \text{ is number of cutting parameters} \)
- \( v = 1, 2, \ldots, w \text{ is number of levels of cutting parameters} [43] \)

The weightage derived from GRA for MRR and SR is 0.517 and 0.483, respectively.
3.2. Multi-objective grasshopper optimization algorithm

Basically, in the Grasshopper Optimization Algorithm, the position of the grasshopper in the swarm represents a possible solution to the optimization. The position of the nth grasshopper can be represented as $Z_m$ and can be defined according to equation (13):

$$Z_n = S_n + G_n + A_n$$  \hspace{1cm} (13)

Here, $S_n$ is the social interaction, $G_n$ is the gravity force on the nth grasshopper, $A_n$ is the wind advection.

Here in this article, the posterior approach type GOA is used to optimize the MRR and Sr. First, the archive size, maximum number of iterations, population size, coverage parameters, attraction intensity ($F$), and attractive length scale ($L$) are defined. Then first, the position of the grasshopper is determined at random by the uniform distribution. Then its fitness is calculated. Then update the archive. If the non-dominated solution collection exceeds the maximum size, rank it, and truncate it to store the full extent. Then select any other mth grasshopper by roulette wheel selection. Then calculate the distance between random grasshoppers. Their social interaction is updated by equation (14).

$$S_n = \sum_{m=1}^{N} S(d_{mn})d_{mn}$$  \hspace{1cm} (14)

Here, $d_{mn}$ is the distance between mth and nth grasshopper.

$$S(R) = \text{function for social forces} = f^R(e^{-R}) - e^{-R}$$  \hspace{1cm} (15)

$f$ is attraction intensity, $L$ is attractive length scale.

Then the position of the grasshopper is updated by equation (16).

$$Y_n^d = c \left\{ \sum_{m=1}^{N} e^{\frac{1}{2} h d_{mn}^2 - \frac{1}{2} b d_{mn}^2 S((x_m^d - x_n^d)\frac{x_m - x_n}{d_{mn}})} \right\} + \hat{T}_d$$  \hspace{1cm} (16)

Here, $C$ is the coverage parameter that shows the forces of attraction and repulsion between the grasshoppers. So, it is updated by equation (17).

$$C = C_{max} - \text{iter} \left( \frac{C_{max} - C_{min}}{\text{iter}_{max}} \right)$$  \hspace{1cm} (17)

Here, iter is the current iteration, $\text{iter}_{max}$ is the maximum number of iterations, $C_{max}$ is the maximum coverage, $C_{min}$ is the minimum coverage.

Then after this collection of locust positions its fitness is calculated, and the process is repeated again and again until the stopping criterion is reached. Then the final solution is obtained [39]. The pseudocode and flowchart for the multi-objective GOA algorithm is shown in figures 4 and 5, respectively.

4. Results and discussions

4.1. DOE analysis of MRR

For the material removal rate, the model’s fitness is determined by the $R^2$ value, which is 0.9318 and is very close to 1. It, therefore, reflects the excellent fitness of the quadratic model for MRR. At the same time, the difference between Adjusted $R^2$ (0.8705) and Predicted-$R^2$ (0.7374) is less than 0.2 and very close. So, it reveals the excellent quality of prediction of the quadratic model of MRR. Statistical Analysis of MRR is shown in table 5.

In the analysis of variance for MRR, the model F-value is 15.19, and the corresponding p-value is < 0.0001, whereas for the lack of fit, the F-value is 0.5714, and the corresponding P-value is 0.7230, indicating good fitness of the quadratic model. The ANOVA suggests that the linear effect of $T_{ON}$ and $T_{off}$, and the quadratic effect of $T_{ON}$ and $SV$, significantly increase MRR. Whereas ANOVA for MRR is shown in table 6.

The perturbation Plot for MRR is shown in figure 6. The perturbation plot shows that the MRR increases significantly due to higher discharge energy with an increasing $T_{ON}$. Whereas the $T_{off}$ shows a slight increase in MRR because when the $T_{off}$ increases, the flushing time increases, and more material is removed [31].

Increasing the $SV$ increases the MRR, and further increasing the $SV$ decreases the MRR. So, machining can be done very quickly (no wire breaks) at some optimum value of $SV$ and increases the MRR due to good discharge energy [33].

The 3D plot for a $T_{ON}$ versus $T_{off}$ versus MRR shows that increasing $T_{ON}$ and $T_{off}$ together increases MRR. The 3D surface plot for MRR is shown in figure 7.

9
Figure 4. Flowchart of Multi-objective GOA.
Pseudocode for Multi Objective Grasshopper Optimization Algorithm

START

Define Input Parameters like number of variables, limits of variable (lower bound and upper bound), number of objectives, maximum number of iterations, population, Archive_Max_Size, Archive Position, Archive Fitness, Maximum number of iterations, Population Size, Coverage parameters (Cmax and Cmin), Intensity of Attraction (I), Attractive Length Scale(l);

Initialize Grasshopper Position;

For iter = 1: Maximum iteration
    For i = 1: Population Size
        Check whether Grasshopper position within limits or not otherwise limit them to lower bound & upper bound;
        Grasshopper Fitness Calculate;
    End

Update Archive Position & Archive Fitness;
If (Archive_member_no > Archive_Max_Size)
    Ranked the Archive;
    Chopped the Archive to Archive_Max_Size;
Else
    Ranked the Archive;
End

Ranked the Archive;
Select Target Grasshopper from Archive by Roulette Wheel Selection;
Update Coverage parameter by eq. 17;
    For i = 1: Population Size
        Choose random grasshoppers & calculate distance between them;
        Social interaction update by eq. 14;
        Grasshopper Position Updated by eq. 16;
    End

End

Archive of Grasshopper found;

End

Figure 5. Pseudocode for Multi-Objective GOA.

Table 5. Statistical Analysis of MRR.

| Parameters | Value | Parameters | Value |
|------------|-------|------------|-------|
| Std. Dev.  | 0.1678| $R^2$      | 0.9318|
| Mean       | 2.76  | Adjusted $R^2$ | 0.8705|
| C.V. %     | 6.09  | Predicted $R^2$ | 0.7374|
|            |       | Adeq Precision | 16.9878|
The regression equation for MRR is shown in equation (18).

\[
MRR = -5.73973 + 0.210901 \text{ Pulse ON Time} \\
+ 0.213828 \text{ Pulse OFF Time} \\
+ 0.251256 \text{ Servo Voltage} \\
- 0.017750 \text{ Pulse ON Time} \times \text{ Pulse OFF Time} \\
+ 0.007575 \text{ Pulse ON Time} \times \text{ Servo Voltage} \\
+ 0.003476 \text{ Pulse OFF Time} \times \text{ Servo Voltage} \\
+ 0.026483 \text{ Pulse ON Time} \\
- 0.005183 \text{ Servo Voltage}.
\]

(18)

4.2. DOE analysis of SR

The R² for SR is 0.9490, and the difference between adjusted R² and estimated R² is very close, i.e., it shows the excellent model prediction for Sr Statistical Analysis of SR is shown in table 7.

For SR, analysis of variance shows that the quadratic model with an F value of 20.68 and a P-value <0.0001 shows good fitness. At the same time, the p-value and F-value for lack of fit for SR are 0.7885 (>0.05) and 0.4664 (very small), respectively. It, therefore, indicates the excellent fitness of the regression model for SR ANOVA suggests that T_ON is the most critical parameter that affects Sr ANOVA for SR as shown in table 8.
The perturbation plot also reveals the same results as ANOVA for Sr. As the \( T_{\text{ON}} \) increases, the SR increases. Due to more discharge energy, more area is affected; hence the SR deteriorates. At the same time, Toff and SV have little effect on SR [33]. The SR decreases as the Toff increases due to the longer flushing time. When the pulse-off time increases rapidly, the SR increases slightly due to the accumulation of debris without flushing [31]. Similar results shown by the 3D plot for Sr. Perturbation Plot for SR is shown in figure 8. The 3D surface plot for SR is shown in figure 9.

| Table 7. Statistical analysis of Sr |
|-----------------------------------|
| Parameter | Value | Parameter | Value |
| Std. Dev. | 0.1956 | \( R^2 \) | 0.9490 |
| Mean | 4.35 | Adjusted \( R^2 \) | 0.9031 |
| C.V. % | 4.49 | Predicted \( R^2 \) | 0.8169 |
| | | Adeq Precision | 17.4231 |

| Table 8. ANOVA for Sr |
|-----------------------|
| Source | Sum of squares | df | Mean square | F-value | p-value |
| Model | 7.12 | 9 | 0.7912 | 20.68 | <0.0001 | significant |
| A-Pulse ON Time | 5.70 | 1 | 5.70 | 148.86 | <0.0001 |
| B-Pulse OFF Time | 0.0002 | 1 | 0.0002 | 0.0034 | 0.9426 |
| C-Servo Voltage | 0.0445 | 1 | 0.0445 | 1.16 | 0.3062 |
| AB | 0.0932 | 1 | 0.0932 | 2.44 | 0.1497 |
| AC | 0.0038 | 1 | 0.0038 | 0.1000 | 0.7583 |
| BC | 0.0004 | 1 | 0.0004 | 0.0102 | 0.9215 |
| \( A^2 \) | 0.1166 | 1 | 0.1166 | 3.05 | 0.1114 |
| \( B^2 \) | 0.1823 | 1 | 0.1823 | 4.76 | 0.0540 |
| \( C^2 \) | 0.9460 | 1 | 0.9460 | 24.72 | 0.0006 |
| Residual | 0.3827 | 10 | 0.0383 |
| Lack of Fit | 0.1218 | 5 | 0.0244 | 0.4669 | 0.7885 | not significant |
| Pure Error | 0.2609 | 5 | 0.0522 |
| Cor Total | 7.50 | 19 |
The regression equation for SR is shown in equation (19).

\[
SR = 26.34347 + 0.019807 \text{Pulse ON Time} - 0.862938 \text{Pulse OFF Time} - 0.639202 \text{Servo Voltage} + 0.021375 \text{Pulse ON Time} \times \text{Pulse OFF Time} + 0.002188 \text{Pulse ON Time} \times \text{Servo Voltage} - 0.002188 \text{Pulse OFF Time} \times \text{Servo Voltage} - 0.017022 \text{Pulse ON Time}^2 + 0.013456 \text{Pulse OFF Time}^2 + 0.007757 \text{Servo Voltage}^2.
\]  

4.3. Optimization results

The priori approach type MOABC setting parameters are below. First, for this priori approach, 100% weight was given to each response and the individual optimum of MRR and SR was obtained. Whereas this was followed by multi-objective optimization giving 51.7% wt for MRR and 48.3% wt for SR, which was calculated by GRA. Here, three parameters have been selected for the experiments. Whereas on a combination of different populations and iterations, optimization is done. MO-ABC setting parameters are shown in table 9.

The convergence curve is a curve that indicates how quickly the algorithm converges to the optimal value with a few iterations. The convergence curves for single objective MRR and SR are shown in figure 10:
For different populations and iterations, MRR and SR are chosen as responses to analyze multi-objective ABC as priori approaches & shown in the following table and figure 11. ABC shows that the maximum MRR is $4.0956 \text{ g h}^{-1}$ at $\text{TON} \ 14 \ \mu \text{s}$, $\text{Toff} \ 30 \ \mu \text{s}$, and $\text{SV} \ 44.728 \ \text{V}$. At the same time, the elapsed time is $2.939815$ seconds for 500 iterations and 50 populations.

ABC shows that the minimum SR is $2.561 \ \mu \text{m}$ at $\text{TON} \ 6 \ \mu \text{s}$, $\text{Toff} \ 28.2819 \ \mu \text{s}$, and $\text{SV} \ 41.2199 \ \text{V}$. At the same time, the elapsed time is $3.163739$ seconds for 500 iterations and 50 populations.

MOABC shows that the optimum SR is $2.615 \ \mu \text{m}$ & optimum MRR is $2.67 \text{ g h}^{-1}$, @ $\text{TON} \ 6 \ \mu \text{s}$, $\text{Toff} \ 30 \ \mu \text{s}$, and $\text{SV} \ 40.3620 \ \text{V}$. At the same time, the elapsed time is $3.080010$ seconds, for 500 iterations and 50 populations.

Priori approach ABC shows that the elapsed time increases as the number of calculations increases.

A Pareto-front diagram is a diagram that shows how two responses affect each other. The Pareto-front diagram shows that maximum MRR is achieved at high Sr Priori approach type ABC also confirms the DOE results, that the MRR is higher on high $\text{TON}$, high Toff, and medium voltage. In contrast, the SR is minimal at the low $\text{TON}$, moderate Toff, and SV. When MRR and SR are formulated as a single priority problem in MOABC, optimum MRR and SR are obtained at a low $\text{TON}$, Toff, and moderate SV.

Whereas the posterior approach type MOGOA gives the combination of results in an archive to optimize MRR and SR together.
When we compare these two (priori-based MOABC and posterior type MOGOA) algorithms, it is found that a posteriori approach is better than the priori approach type algorithm as it gives a more optimal set of solutions. In contrast, the priori approach type algorithm only provides a single optimal solution. At the same time, the posterior approach algorithm took more elapsed time than the priority approach algorithm [44, 45].

The setting parameters, Pareto front, for the posterior approach MOGOA, are shown below in table 11, and figure 12, respectively:

Posterior approach MOGOA gives several results. The optimum SR is 2.564 micro-meter & the optimum MRR is 3.0654 g h$^{-1}$, @ TON 6 μs, Toff 28.6607 μs, and SV 41.2163 V. At the same time, another optimum result is MRR 4.0914 g h$^{-1}$ & SR 5.931 μm @ TON 13.997 μs, Toff 29.6363 μs, and SV 44.2687 V. While the elapsed time is 42.990352 seconds for 200 iterations and 125 populations. The other results are shown in table 12.

In figure 12, the true Pareto-front is compared with the obtained Pareto-front. This indicates that the obtained Pareto-front is in a viable range. The MOGOA Pareto Front also shows that higher MRR achieves at higher Sr.
Table 11. MOGOA setting parameters.

| MOGOA setting parameters | Value   |
|--------------------------|---------|
| Number of variables      | 3       |
| number of objectives     | 2       |
| Archive_Max_Size         | 100     |
| Coverage parameters (Cmax and Cmin) | 1 to 0.00004 |
| Intensity of attraction (f) | 0.5     |
| Attractive length scale(l) | 1.5     |
| Population size          | 200     |
| Maximum iterations       | 1000 & 250 & 125 |

Table 12. Posteriori approach analysis for MOGOA.

| Archive Solutions No. | Optimized Parameters | Optimized Responses |
|-----------------------|----------------------|---------------------|
| Population 200, Iteration 1000, Elapsed Time 343.129155 Seconds | | |
| 01. 6 μs, 29.2895 μs, 40.8245 V | MRR 3.165 g h$^{-1}$, SR 2.580 μm |
| 02. 14 μs, 29.1108 μs, 44.0561 V | MRR 4.0857 g h$^{-1}$, SR 5.812 μm |
| 03. 12.4769 μs, 24.4981 μs, 40.8843 V | MRR 3.432 g h$^{-1}$, SR 4.803 μm |
| Population 200, Iteration 250, Elapsed Time 87.689611 Seconds | | |
| 04. 14 μs, 22.3893 μs, 41.2890 V | MRR 3.9474 g h$^{-1}$, SR 4.993 μm |
| 05. 6 μs, 30 μs, 39.3913 V | MRR 3.2826 g h$^{-1}$, SR 2.6391 μm |
| 06. 6 μs, 28.1858 μs, 41.2342 V | MRR 2.9964 g h$^{-1}$, SR 2.561 μm |
| Population 200, Iteration 125, Elapsed Time 42.99352 Seconds | | |
| 07. 6.0001 μs, 28.6607 μs, 41.2163 V | MRR 3.0654 g h$^{-1}$, SR 2.564 μm |
| 08. 13.9997 μs, 29.6363 μs, 44.2687 V | MRR 4.0914 g h$^{-1}$, SR 5.931 μm |
| 09. 6 μs, 29.9932 μs, 39.7702 V | MRR 3.4692 g h$^{-1}$, SR 4.825 μm |

5. Conclusion

The present research work is undertaken to correctly understand the difference between the primary approach of multi-purpose optimization and the posterior approach of multi-purpose optimization while performing WEDM on Ti6Al4V alloy. Mathematical modelling of MRR and SR is done using CCD. The ANOVA shows the critical factors influencing MRR and Sr. Whereas the 3D plot and perturbation plot show the effect of process parameters on MRR and Sr. Then fits the regression model for MRR and Sr. GRA is applied for weightage calculation for MRR and SR, and then MO-ABC is applied. The posterior type MOGOA is then implemented to optimize the responses. Although the priori approach type ABC with a different number of counts gives similar results and these results are different for the posterior type MOGOA for the same number of counts. As the No Free Lunch Theorem shows, one algorithm is better for one class of problems, while for another problem, the other algorithm is promising. So, in this paper, both types of approaches are compared for a class of problems and find that the priori method is quick while the posterior approach gives many promising solutions. MRR is maximized for manufacturing purposes, whereas SR is minimized for good quality. The following points are also concluded here:

1. The CCD shows that the quadratic model is fitted for MRR and SR, respectively.
2. ANOVA for MRR shows, that the linear effect of T$_{ON}$, Toff, and the quadratic effects of SV & T$_{ON}$ are the most critical factors affecting MRR.
3. The MRR increases with an increasing T$_{ON}$ and T$_{off}$.
4. According to ANOVA, the linear effect of T$_{ON}$ and the quadratic effect of SV is the most critical factor for Sr.
5. Increasing the T$_{ON}$ increases the Sr. Other parameters like T$_{off}$ and SV have little effect on Sr.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).
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