In the brane-world scenario, our universe is understood as a three dimensional hypersurface embedded in a higher dimensional space-time. The fluctuations of the brane along the extra dimensions are seen from the four-dimensional point of view as new fields whose properties are determined by the geometry of the extra space. We show that such branon fields can be massive, stable and weakly interacting, and accordingly they are natural candidates to explain the universe missing mass problem. We also consider the possibility of producing branons non-thermally and their relevance in the cosmic coincidence problem. Finally we show that some of the branon distinctive signals could be detected in future colliders and in direct or indirect dark matter searches.

1 Introduction

The construction of extra-dimensional models has been revived in recent years within the so called brane-world scenario. The main assumption of this scenario is that by some (unknown) mechanism, matter fields are constrained to live in a three-dimensional hypersurface (brane) embedded in the higher dimensional (bulk) space. Only gravity is able to propagate in the bulk space, but the fundamental scale of gravity in $D$ dimensions $M_D$ can be much lower than the Planck scale, the volume of the extra dimensions being responsible for the actual value of the Newton constant in four dimensions. The fact that rigid objects are incompatible with General Relativity implies that the brane-world must be dynamical and can move and fluctuate along the extra dimensions. Branons are precisely the fields parametrizing the position of the brane in the extra coordinates. Thus, in four dimensions branons could be detected through their contribution to the induced space-time metric.

2 Branon dark matter

Let us consider our four-dimensional space-time $M_4$ to be embedded in a $D$-dimensional bulk space whose coordinates will be denoted by $(x^\mu, y^m)$, where $x^\mu$, with $\mu = 0, 1, 2, 3$, correspond to
the ordinary four dimensional space-time and $y^m$, with $m = 4, 5, \ldots, D - 1$, are coordinates of the compact extra space of typical size $R_B$. For simplicity we will assume that the bulk metric tensor takes the following form:

$$ds^2 = ˜g_{\mu\nu}(x)W(y)dx^\mu dx^\nu - g^\prime_{mn}(y)dy^m dy^n$$  \hspace{1cm} (1)

where the warp factor is normalized as $W(0) = 1$. The position of the brane in the bulk can be parametrized as $Y^M = (x^\mu, Y^m(x))$, and we assume for simplicity that the ground state of the brane corresponds to $Y^m(x) = 0$.

In the simplest case in which the metric is not warped along the extra dimensions, i.e. $W(y) = 1$, the transverse brane fluctuations are massless and they can be parametrized by the Goldstone boson fields $\pi^\alpha(x), \alpha = 4, 5, \ldots, D - 1$, associated to the spontaneous breaking of the extra-space translational symmetry. In that case we can choose the $y$ coordinates so that the branon fields are proportional to the extra-space coordinates: $\pi^\alpha(x) = f^2 \delta^\alpha_m Y^m(x)$, where the proportionality constant is related to the brane tension $\tau = f^4$.

In the general case, the curvature generated by the warp factor explicitly breaks the translational invariance in the extra space. Therefore branons acquire a mass matrix which is given precisely by the bulk Riemann tensor evaluated at the brane position: $M^{\alpha\beta}_2 = ˜g^{\mu\nu}R_{\mu\alpha\nu\beta}|_{y=0}$.

The dynamics of branons can be obtained from the Nambu-Goto action. In addition, it is also possible to get their couplings to the ordinary particles just by replacing the space-time by the induced metric in the Standard Model (SM) action. Thus we get up to quadratic terms in the branon fields:

$$S_{Br} = \int d^4 x \sqrt{g} \left[ \frac{1}{2} \left( ˜g^{\mu\nu} \partial_\mu \pi^\alpha \partial_\nu \pi^\alpha - M^{\alpha\beta}_2 \pi^\alpha \pi^\beta \right) + \frac{1}{8 f^4} \left( 4 \partial_\mu \pi^\alpha \partial_\nu \pi^\alpha - M^{\alpha\beta}_2 \pi^\alpha \pi^\beta \pi^\mu \pi^\nu \right) T^{\mu\nu}_{SM} \right]$$  \hspace{1cm} (2)

Figure 1: Relic abundance in the $f - M$ plane for a model with one branon of mass: $M$. The two lines on the left correspond to the $\Omega_{Br} h^2 = 0.0076$ and $\Omega_{Br} h^2 = 0.129 - 0.095$ curves for hot-warm relics, whereas the right line corresponds to the latter limits for cold relics (see [5] for details). The lower area is excluded by single-photon processes at LEP-II [4] together with monojet signal at Tevatron-I [6]. The astrophysical constraints are less restrictive and they mainly come from supernova cooling by branon emission [5].

We can see that branons interact with the SM particles through their energy-momentum tensor. The couplings are controlled by the brane tension scale $f$. For large $f$, branons are therefore weakly interacting particles. The sign of the branon fields is determined by the orientation of the brane submanifold in the bulk space. Under a parity transformation on the brane ($x^i \rightarrow -x^i$), the orientation of the brane changes sign provided the ordinary space has an
odd number of dimensions, whereas it remains unchanged for even spatial dimensions. In the case in which we are interested with three ordinary spatial dimensions, branons are therefore pseudoscalar particles. Parity on the brane then requires that branons always couple to SM particles by pairs, which ensures that they are stable particles. This fact means that branons are natural dark matter candidates.

When the branon annihilation rate, $\Gamma = n_{eq}(\sigma_AV)$, equals the universe expansion rate $H$, the branon abundance freezes out relative to the entropy density. This happens at the so called freeze-out temperature $T_f$. We have computed this relic branon abundance in two cases: either relativistic branons at freeze-out (hot-warm) or non-relativistic (cold), and assuming that the evolution of the universe is standard for $T < f$ (see Fig. 1).

3 Non-thermal branon production and the cosmic coincidence problem

So far we have considered only the thermal production of branons. However, if the maximum temperature reached in the universe is smaller than the branon freeze-out temperature, but larger than the explicit symmetry breaking scale, then branons can be considered as massless particles decoupled from the rest of matter and radiation. In such a case there is no reason to expect that initially the brane was located at the potential minimum $Y_0 = 0$, but in general we will have $Y_0 \simeq R_B$, i.e. $\pi_0 \simeq f^2R_B$. As the universe expands, the brane can start oscillating around the minimum, and the energy density of the oscillation can be seen as cold dark matter from the brane point of view. This is completely analogous to the misalignment mechanism for axion production. In the case in which $H(T) > \Gamma(T)$, with $\Gamma(T)$ the total branon annihilation rate, the amplitude of the oscillations is only damped by the Hubble expansion, but not by particle production. The corresponding present energy density would be given by:

$$\Omega_{Br}h^2 \simeq \frac{6.5 \cdot 10^{-20}N}{GeV^{5/2}}f^4 R_B^2 M^{1/2},$$  \hspace{1cm} (3)

where $N$ is the number of branon species. It is interesting to estimate typical values for $\Omega_{Br}h^2$ generated by this mechanism. Thus consider the simplest non-trivial model in six dimensions in which the bulk space only contains a (negative) cosmological constant $\Lambda_6$ (AdS$_6$ soliton). The solutions of Einstein equations in the case in which the extra space has azimuthal symmetry and the metric depends only on the radial coordinate $\rho$ with a periodic angular coordinate $\theta \in [0, 2\pi)$, is given by:

$$ds^2 = M^2(\rho)\eta_{\mu\nu}dx^\mu dx^\nu - d\rho^2 - L^2(\rho)d\theta^2,$$ \hspace{1cm} (4)

where, with $k = \sqrt{-5\Lambda_6/(8M_6^4)}$:

$$M(\rho) = \cosh^{2/5}(k\rho); \quad L(\rho) = \frac{\sinh(k\rho)}{k \cosh^{3/5}(k\rho)}.$$ \hspace{1cm} (5)

Notice that we have assumed that the presence of the brane has no effect on the bulk metric. However, even if we include the jump conditions at the brane position, it can be seen that the only consequence would be the introduction of a deficit angle in the $\theta$ coordinate, which is related to the brane tension. In addition, in order to compactify the extra dimensions, it has been shown that it is possible to truncate the extra space by introducing a 4-brane at a finite distance $\rho = R_B$ with an anisotropic energy-momentum tensor. The corresponding branon mass is given by $M^2 = 8k^2/5 = -\Lambda_6/M_6^2$.

Let us assume that there is only one fundamental scale in the theory which is close to the electroweak scale, i.e. we will have $f \sim M_6 \sim 1$ TeV, and also assume that the order of
magnitude of the bulk cosmological constant is fixed by bulk loop effects i.e. \( \Lambda_6 \sim R_B^6 \). In this case, the branon mass is \( M \sim 10^{-33} \text{ eV} \), and, in order to recover the usual four dimensional Planck scale, the size of the extra dimension should be \( R_B^{-1} \sim 10^{-3} \text{ eV} \). Substituting these values into Eq. (3), we get \( \Omega_B h^2 \simeq 0.1 \), in agreement with observations. Six dimensional models like the one above have been studied also in the context of the dark energy problem. It has been shown that integrating the volume of the extra space, the natural value for the brane cosmological constant would be \( \Lambda_4 \sim R_B^{-4} \sim (10^{-3} \text{eV})^4 \), also in agreement with observations. Since the own brane tension does not contribute to the brane cosmological constant in the \( D = 6 \) case, it has been suggested that the amount of fine tuning needed to solve the dark energy problem would be reduced in these models. In addition, as shown above the correct value of the dark matter energy density can also be obtained without including additional mass scales in the theory. Therefore, we see that in 6D brane world models the two fine-tuning problems, i.e. the gauge hierarchy and the cosmic coincidence can be related to a single one, namely, the existence of large extra dimensions.\(^7\)

4 Branon searches

If branons make up the galactic halo, they could be detected by direct search experiments from the energy transfer in elastic collisions with nuclei of a suitable target. For the allowed parameter region in Fig. 1, branons cannot be detected by present experiments such as DAMA, ZEPLIN 1 or EDELWEISS. However, they could be observed by future detectors such as CRESST II, CDMS or GENIUS\(^5\).

Branons could also be detected indirectly: their annihilations in the galactic halo can give rise to pairs of photons or \( e^+e^- \) which could be detected by \( \gamma \)-ray telescopes such as MAGIC or GLAST or antimatter detectors (see\(^5\) for an estimation of positron and photon fluxes from branon annihilation in AMS). Annihilation of branons trapped in the center of the sun or the earth can give rise to high-energy neutrinos which could be detectable by high-energy neutrino telescopes such as AMANDA, IceCube or ANTARES. These searches complement those in high-energy particle colliders (both in \( e^+e^- \) and hadron colliders) in which real (see Fig. 1) and virtual branon effects could be measured\(^5\).

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