Deconfinement and chiral-symmetry restoration in finite temperature QCD

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Abstract

QCD sum rules are based on the Operator Product Expansion of current correlators, and on QCD-hadron duality. An extension of this program to finite temperature is discussed. This allows for a study of deconfinement and chiral-symmetry restoration. In addition, it is possible to relate certain hadronic matrix elements to expectation values of quark and gluon field operators by using thermal Finite Energy Sum Rules. In this way one can determine the temperature behaviour of hadron masses and couplings, as well as form factors. An attempt is made to clarify some misconceptions in the existing literature on QCD sum rules at finite temperature.

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1 Outline

This report is an expanded version of a talk given at CAM-94, the joint meeting of the Canadian Association of Physicists, the American Physical Society, and the Mexican Physical Society. I discuss here an extension of the QCD sum rule program to finite temperature. At $T = 0$, QCD sum rules offer a very successful quantum field theory framework to extract information on hadronic physics from QCD analytically. This great success, unfortunately, does not translate immediately into the finite temperature domain, where there are some serious unresolved problems with Laplace transform sum rules. In spite of this, some reasonable progress has been made through lowest moment Finite Energy Sum Rules (FESR). As with any new field, some degree of confusion is to be expected as our ideas take shape, and we gain a better understanding of the subject. It is important, though, not to persist on wrong notions, nor advocate results which have been shown to be in contradiction with more fundamental facts. I shall attempt to clarify here some misconceptions present in the existing literature on this subject. The purpose is not to antagonize, but rather to point out the problems and promote discussion that might help to solve them. I may group these misconceptions into two categories: (a) conceptual, and (b) specific results of applications. Among the first is the statement (without proof) that the notion of QCD-hadron duality, one of the two pillars of QCD sum rules, should abruptly disappear as soon as the temperature is turned on (by no matter what small amount). If correct, this proposition would invalidate a smooth extension of the QCD sum rule program to $T \neq 0$. I shall argue against this scenario. Second, and in connection with results of applications, there exist quite a few determinations of hadron masses based on Laplace transforms which ignore the underlying problems with these sum rules. Chief among these problems is the fact that they are inconsistent with the well known (and well established) temperature behaviour of the quark condensate and the gluon condensate. Not surprisingly, the predictions from this approach are in serious contradiction with the $T$-dependence of hadron masses obtained in other independent frameworks.

The outline of this report is as follows. In Section 2 I introduce briefly the key ideas behind QCD sum rules at $T = 0$, and argue for a smooth extension of this program to finite temperature. I discuss supportive evidence for the validity of both the Operator Product Expansion, and the notion of QCD-hadron duality at $T \neq 0$. Section 3 deals with chiral-symmetry restoration, and in Section 4 I review proposals for phenomenological order parameters to characterize deconfinement. In Section 5 I show how these two phase transitions have been related using a lowest moment FESR-QCD Sum Rule. Section 6 summarizes recent results for the temperature behaviour of the electromagnetic form factor of the pion in the space-like region, once again through a lowest moment FESR. Finally, in Section 7 I concentrate on the problems with Laplace transform QCD sum rules, as well as with higher moment FESR, and make critical comments on the existing
literature. Except for a major part of Section 7, the ideas and results presented here have already been published in the literature, and the reader may trace them through the list of references.

## 2 QCD sum rules

QCD sum rules at $T = 0$ are based on the Operator Product Expansion (OPE) of current correlators at short distances, suitably extended to include non-perturbative effects. The latter are parametrized in terms of a set of vacuum expectation values of the quark and gluon fields entering the basic QCD Lagrangian. Contact with the hadronic world of large distances is achieved by invoking the notion of QCD-hadron duality. This leads to relationships between fundamental QCD parameters ($\Lambda_{QCD}$, quark masses, vacuum condensates, etc.) and low energy parameters (hadron masses, widths, couplings, form factors, etc.). The values of the vacuum condensates in the OPE cannot be calculated analytically from first principles, as this would be tantamount to solving QCD exactly. Instead, they are extracted from certain channels where experimental information is available, e.g. $e^+e^-$ annihilation, and $\tau$ decays. It is also possible, in principle, to estimate them numerically from lattice QCD. To be more specific, let us consider the two-point function

$$\Pi(q) = i \int d^4x \exp(\imath qx) \langle 0 | T(J(x), J^\dagger(0)) | 0 \rangle ,$$

(2.1)

where $J(x)$ is a local current built from the quark and/or gluon fields entering the QCD Lagrangian, and having specific quantum numbers. In the sequel I concentrate on light quark flavours. The OPE of $\Pi(q)$ is formally written as

$$\Pi(q) = C_I < I > + \sum_r C_r(q) < O_r > ,$$

(2.2)

where the Wilson coefficients $C_r(q)$ depend on the Lorentz indices and quantum numbers of the external current $J(x)$, and also of the local gauge-invariant operators $O_r$ built from the quark and gluon fields of QCD. The unit operator $I$ in Eq.(2.2) represents the purely perturbative piece. The OPE is assumed valid, even in the presence of non-perturbative effects, for $q^2 < 0$ (spacelike), and $|q^2| \gg \Lambda^2_{QCD}$. In principle, all Wilson coefficients are calculable in perturbative QCD to any desired order in the strong coupling constant. In the sequel we shall work at leading (one loop) order for simplicity. The non-perturbative effects are then buried in the vacuum condensates. Since these have dimensions, the associated Wilson coefficients fall off as inverse powers of $Q^2 = -q^2$. For instance, if the current $J(x)$ in Eq.(2.1) is identified with the axial-vector current $A_\mu(x) = :\bar{u}(x)\gamma_\mu\gamma_5d(x) :$, then with

$$\Pi_{\mu\nu}(q) = -g_{\mu\nu}\Pi_1(q^2) + q_\mu q_\nu \Pi_0(q^2) ,$$

(2.3)
one easily finds \[1\]

\[
4\pi^2 \Pi_0(q) = - \ln \frac{Q^2}{\mu^2} + \frac{C_4 < O_4 >}{Q^4} + \frac{C_6 < O_6 >}{Q^6} + \cdots ,
\]

(2.4)

where \(\mu\) is a renormalization scale, and e.g. the leading vacuum condensate is given by

\[
C_4 < O_4 > = \frac{\pi}{3} < \alpha_s G^2 > - 8\pi^2 \tilde{m}_q < \bar{q}q > ,
\]

(2.5)

with \(\tilde{m}_q = (m_u + m_d)/2\), and \(< \bar{q}q > = < \bar{u}u > \simeq < \bar{d}d >\). In Eq.(2.4) a term proportional to \(m_q^2/Q^2\) has been neglected. The function \(\Pi_0(q)\), Eq. (2.4), satisfies a dispersion relation

\[
\Pi_0(Q^2) = \frac{1}{\pi} \int_0^\infty ds \ \frac{\text{Im} \ \Pi_0(s)}{s + Q^2} ,
\]

(2.6)

defined in this case up to one subtraction constant, which can be disposed of by e.g. taking the first derivative with respect to \(Q^2\) in Eq.(2.6). The notion of QCD-hadron duality is implemented by calculating the left hand side of Eq.(2.6) in QCD through the OPE, and parametrizing the spectral function entering the right hand side in terms of hadronic resonances, followed by a hadronic continuum modelled by perturbative QCD. In this fashion one relates fundamental QCD parameters, such as quark masses, renormalization scales, vacuum condensates, etc., to hadronic parameters such as particle masses, widths, couplings, etc.. The convergence of the Hilbert transform, Eq.(2.6), may be improved by considering other integral kernels. This leads to other versions of QCD sum rules, such as the Laplace transform, Finite Energy Sum Rules (FESR), etc.. For instance, the Laplace transform QCD sum rule for the axial-axial correlator is \[1\]

\[
\mathcal{L} \ \Pi_0(Q^2) = \Pi_0(M^2) = \int_0^\infty ds \ \exp(-s/M^2) \ \frac{1}{\pi} \text{Im} \Pi_0(s) ,
\]

(2.7)

where \(M^2\) is the Laplace parameter which plays the role of \(Q^2\) as the short distance expansion parameter (for light quark correlators). Performing the Laplace transform of Eq.(2.4) one finds \[1\]

\[
\int_0^\infty ds \ \exp(-s/M^2) \ \frac{1}{\pi} \text{Im} \Pi_0(s) = \frac{M^2}{4\pi^2} \left( 1 + \frac{C_4 < O_4 >}{M^4} + \frac{1}{2!} \frac{C_6 < O_6 >}{M^6} + \cdots \right) .
\]

(2.8)

One should notice from Eq.(2.8) that in Laplace transform QCD sum rules all condensates are involved. Their numerical importance, though, is suppressed by inverse powers of \(M^2\), as well as by factorial coefficients.

Either by using Cauchy theorem, or by expanding Eq.(2.7) in \(M^2\), the Laplace transform QCD sum rule Eq.(2.7) is equivalent to the infinite number of FESR

\[
(-)^{N-1} C_{2N} < O_{2N} > = 4\pi^2 \int_0^{s_0} ds \ s^{N-1} \ \frac{1}{\pi} \text{Im} \Pi_0(s)|_{\text{RES}} - \frac{s_0^N}{N} ,
\]

(2.9)
where $N=1,2,\ldots$, and $\text{Im} \Pi_0(s)|_{\text{RES}}$ stands for the purely resonant contribution to the hadronic spectral function (i.e. without the continuum). For $N = 1$, $C_2 < O_2 >$ is nothing but a short-hand notation for the perturbative quark mass insertion ($C_2 < O_2 > \propto m^2_q$).

A practical advantage of these FESR is that now the vacuum condensates of different dimensionality are effectively decoupled. This becomes particularly important if the objective is to extract the values of these condensates from a knowledge of the hadronic spectral function. In fact, this is the correct procedure followed in [2] to determine the condensates from data on $e^+e^-$ annihilation and $\tau$-decays. The decoupling of the different $C_N < O_N >$ is also important to study the self-consistency of the Laplace transform QCD sum rules at $T \neq 0$, as will be discussed in Section 7.

An extension of this QCD sum rule program to finite temperature was proposed some time ago in [3]. This proposal entails the assumptions that (a) the OPE continues to be valid, except that now the vacuum condensates will develop an (a-priori) unknown temperature dependence, and (b) the notion of QCD-hadron duality also remains valid. I shall discuss below some evidence in support of these two assumptions. Notice that in analogy with the situation at $T = 0$, the thermal behaviour of the vacuum condensates is not calculable analytically from first principles. Some model or approximation must be invoked, e.g. the dilute pion gas approximation, lattice QCD, etc.. The quark, the gluon, and the four-quark condensates at $T \neq 0$ have thus been estimated in the literature [4]-[7]. At finite temperature, the basic object to be considered is the retarded (advanced) two-point function after appropriate Gibbs averaging

$$\Pi(q,T) = i \int d^4x \exp(iqx) \theta(x_0) \langle [J(x),J^\dagger(0)] \rangle ,$$

(2.10)

where

$$\langle A \cdot B \rangle = \sum_n \exp(-E_n/T) \langle n|A \cdot B|n\rangle /Tr(\exp(-H/T)) ,$$

(2.11)

and $|n\rangle$ is a complete set of eigenstates of the (QCD) Hamiltonian. The OPE of $\Pi(q,T)$ is now written as

$$\Pi(q,T) = C_I \langle I \rangle + \sum_r C_r(q) \langle O_r \rangle ,$$

(2.12)

It must be stressed that the states $|n\rangle$ entering Eq.(2.11) can be any complete set of states, e.g. hadronic states, quark-gluon basis, etc.. The hadronic (mostly pion) basis has been advocated in [3], while the quark-gluon basis was first used in [3]. These two approaches are quite complementary, rather than in conflict, as the information they provide is somewhat different. The pion basis is well suited to determine the temperature dependence of vacuum condensates at low $T$. It does not make use of QCD-hadron duality, and thus has little relationship to the QCD sum rule program. On the other hand, use of the quark-gluon basis allows for a smooth extension of that program to finite temperature. As it continues to rely on both the OPE and QCD-hadron duality, this approach provides information on thermal Green functions provided the temperature
dependence of the condensates is known. Since the latter can be obtained e.g. from using
the pion basis in Eq.(2.11), the two choices of the complete set \( |n> \) complement each
other. However, it must be kept in mind that the choice of the pion basis, being a form
of the virial expansion, is restricted to low temperatures. This is not necessarily the case
for the quark-gluon basis approach. In fact, given an expression for the condensates in
some framework, accurate enough for all \( T \) up to the critical temperature, the QCD sum
rules will provide the \( T \)-dependence of hadronic matrix elements in the same temperature
range.

The validity of the OPE (at any temperature) beyond perturbation theory cannot be
proved from first principles, since one does not know how to solve QCD exactly. However,
at \( T = 0 \) one can solve exactly other field theories which bear some resemblance to
QCD, thus providing evidence in support of this assumption. For instance, a study of
the (exactly solvable) \( O(N) \) sigma model, in the large \( N \) limit, and of the Schwinger
model, both in two dimensions, shows that the short distance approximation to exact
Green functions agrees with the result from the OPE [9]. An extension of this analysis
to finite temperature [10] shows the same agreement, and thus supports the assumption
of the validity of the OPE in this regime. I briefly summarize the results of [10]. Let
me consider first the \( O(N) \) sigma model in 1+1 dimensions which is characterized by the
Lagrangian

\[
\mathcal{L} = \frac{1}{2} \left[ \partial_\mu \sigma^a(x) \right] \left[ \partial_\mu \sigma^a(x) \right],
\]  

where \( a = 1,...N \), and \( \sigma^a \sigma^a = N/f \), with \( f \) being the coupling constant. In the large \( N \)
limit this model can be solved exactly (for details see [9]), it is known to be asymptotically
free, and in spite of the absence of mass parameters in Eq.(2.13), it exhibits dynamical
mass generation. In addition, in this model there are vacuum condensates, e.g. to leading
order in \( 1/N \) : \( \langle 0 | \alpha | 0 \rangle = \sqrt{N} m^2 \), whereas all other condensates factorize, viz. \( \langle 0 | \alpha^k | 0 \rangle =\)
\( (\sqrt{N} m^2)^k \). The \( \alpha \) field is: \( \alpha = f(\partial_\mu \sigma^a)^2 / \sqrt{N} \), and we are interested in the Green
function associated with the propagation of quanta of this \( \alpha \) field. We have calculated
this Green function at finite temperature [10]. Its imaginary part can be integrated
analytically in closed form and is

\[
\text{Im} \Gamma(\omega, q = 0, T) = \frac{1}{2\omega^2} \left[ 1 + 3n_B(\omega/2T) \right]
\] 

\[
+ \frac{1}{2} \left[ \frac{2}{\sqrt{N}} \frac{<\alpha>}{\omega^4} + \frac{6}{N} \frac{<\alpha^2>}{\omega^6} + \cdots \right]
\]  

(2.14)

where the first term above corresponds to the perturbative contribution, the second to
the non-perturbative, and \( n_B \) is the thermal Bose factor. Equation (2.14) is valid in the
time-like region; the space-like region counterpart vanishes in 2 dimensions. Since the
model is exactly solvable, the thermal behaviour of the vacuum condensates can also be
calculated, viz.

\[
<\alpha> = <\alpha> \left[ 1 + 3n_B(\omega/2T) \right]
\]  

(2.15)

5
In this case the vacuum condensates contribute to the imaginary part, and as Eq.\((2.14)\) shows, the thermal dependence of the perturbative piece cannot be absorbed into the condensates. Hence, no confusion should arise between perturbative and non-perturbative contributions.

Next, I consider the Schwinger model in 1+1 dimensions, with the Lagrangian

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \gamma_\mu D_\mu \psi
\]

where \(D_\mu = i \partial_\mu + e A_\mu\). At \(T=0\) this model has been solved exactly, and in the framework of the OPE \([9]\). The short distance expansion of the exact solution coincides with that from the OPE. Here, we are interested in the two-point functions

\[
\Pi_{++}(x) = \langle 0 | T \{ j^+(x) j^+(0) \} | 0 \rangle
\]

\[
\Pi_{+-}(x) = \langle 0 | T \{ j^+(x) j^-(0) \} | 0 \rangle
\]

where the scalar currents are: \(j^+ = \bar{\psi}_R \psi_L\), \(j^- = \bar{\psi}_L \psi_R\), with \(\psi_{L,R} = (1 \pm \gamma_5)\psi/2\). The function \(\Pi_{++}(Q)\) vanishes identically in perturbation theory, and the leading non-perturbative contribution involves a four-fermion vacuum condensate. We have calculated the thermal behaviour of these current correlators \([10]\) and obtain, e.g. for their imaginary parts in the time-like region (again, there is no space-like contribution in 2 dimensions)

\[
\text{Im } \Pi_{++}(\omega, q = 0, T) = 0
\]

\[
\text{Im } \Pi_{+-}(\omega, q = 0, T) = \frac{1}{4} [1 - 2n_F(\omega/2T)]
\]

Hence, the choice of the fermion basis in the Gibbs average of current correlators does not imply confusing these fermions with condensates, as argued in \([9]\). As Eqs.\((2.19)-(2.20)\) indicate, (perturbative) fermion loop terms and (non-perturbative) vacuum condensates develop their own temperature dependence, which in this particular example happen to be different.

Concerning the notion of QCD-hadron duality, it has been suggested recently \([8]\) that it is not applicable at finite temperature. If correct, this would require a singular dynamical mechanism of a discontinuous nature in order to invalidate the inter-relationship between QCD and hadronic parameters effected by duality. No such mechanism has been proposed in \([8]\). That this inter-relationship would abruptly disappear by raising the temperature from \(T = 0\) to some arbitrary small value, say a nano-Kelvin, seems quite unlikely, especially in the absence of a concrete mechanism to achieve it. According to the QCD sum rule philosophy, at \(T = 0\) one calculates the theoretical left hand side of Eq.\((2.6)\) through the OPE Eq.\((2.2)\), i.e. one uses quark-gluon degrees of freedom, and duality relates this QCD part to a weighted average of the hadronic spectral function. The latter arises from using hadronic degrees of freedom. At very low temperatures the hadronic
spectrum is expected to change very little, and the external current will still convert into quark-antiquark pairs. The temperature dependence of the quark and gluon condensates is known, and at very low $T$ they also hardly change. Hence, it is only reasonable to assume that nothing drastic will happen to duality. There is a sort of temperature inertia affecting both QCD and hadronic physics at very low $T$. At finite temperature, though, there are some new effects coming into play, e.g. there are contributions to the QCD and hadronic spectral functions in the space-like region (as opposed to only the time-like region at $T = 0$), and the heat bath can support condensates with non-trivial quantum numbers. However, these additional contributions vanish smoothly as $T$ approaches zero, i.e. they do not introduce any discontinuous behaviour that would abruptly invalidate the notion of QCD-hadron duality. At moderate temperatures the hadronic spectrum is expected to suffer some rearrangement, in pace with changes in the condensates and the increasing importance of the new analytic structure in the complex energy plane. By retaining the notion of QCD-hadron duality one is able to relate quantitatively the temperature dependence of hadronic parameters with that of QCD parameters, as will be discussed in Sections 5-6.

3 Chiral-Symmetry and its restoration

I begin by discussing some of the symmetries of the QCD Lagrangian

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}(i\gamma^\mu D_\mu - M) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu},$$

where

$$F^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu + ig_0[G^\mu, G^\nu],$$

with

$$\begin{align*}
G^\mu &= \frac{1}{2} \lambda^a G_a^\mu \\
D^\mu &= \partial^\mu + ig_0 G^\mu \quad (a = 1, \cdots 8)
\end{align*}$$

and $M$ the quark mass matrix. For the purposes of this talk I consider only two quark flavours (up and down). Among the various symmetries of $\mathcal{L}_{\text{QCD}}$ one finds a (global) $SU(2)_V$ and an $SU(2)_R \otimes SU(2)_L$ symmetry. These Lagrangian symmetries are explicitly broken by the quark masses. In fact, the vector (I-spin) Noether current $V_\mu^i = \bar{\psi} \gamma_\mu \tau^i \psi$, and the axial-vector current $A_\mu^i = \bar{\psi} \gamma_\mu \gamma_5 \tau^i \psi$ have divergences

$$\partial^\mu V_\mu^i = i(m_d - m_u)\bar{d}u,$$

$$\partial^\mu A_\mu^i = i(m_d + m_u)\bar{d}\gamma_5 u.$$
In the limit $m_u = m_d = 0$, $SU(2)_V$, and $SU(2)_R \otimes SU(2)_L$ become exact Lagrangian symmetries. However, this is not what is usually meant by chiral-symmetry restoration, which refers to the symmetry of the vacuum. Given a Lagrangian symmetry one must investigate how it is realized in the states, starting with the vacuum. According to whether the Noether charges $Q^i = \int d^3x \ J^i_{0}(\vec{x}, t)$ annihilate the vacuum or not, one has a Wigner-Weyl or a Nambu-Goldstone realization of the Lagrangian symmetry. In the former case particles are classified according to the irreducible representations of the symmetry group, as in e.g. $SU(2)_V$. The Nambu-Goldstone realization (spontaneous symmetry breaking) corresponds to a hidden symmetry, as the vacuum does not share the symmetry of the Lagrangian. This is the case for $SU(2)_R \otimes SU(2)_L$ since e.g. there are no (quasi) degenerate parity doublets in the particle spectrum. No particle classification is possible in this phase as the vacuum is contaminated by an arbitrary number of massless (Nambu-Goldstone) bosons carrying non-trivial quantum numbers. In the case of $SU(2)_R \otimes SU(2)_L$ the three emerging Nambu-Goldstone bosons are readily identified with the pion ($\pi^\pm, \pi^0$), which decays to the (hadronic) vacuum through the axial-vector current, i.e.

$$\langle 0|A^i_{\mu}(0)|\pi^j(p)\rangle = i \ f_\pi \ p_\mu \ \delta^{ij}, \quad (3.6)$$

with $f_\pi = 93.2$ MeV. In the limit $m_u = m_d = 0$ the axial-vector current is strictly conserved and, hence, $f_\pi \ \mu_\pi^2 = 0$. In the Nambu-Goldstone phase

$$\mu_\pi^2 \propto (m_u + m_d) \to 0, \quad (3.7)$$

$$f_\pi^2 \propto \langle 0|\bar{u}u + \bar{d}d|0\rangle \neq 0, \quad (3.8)$$

with the proportionality constants such that $2f_\pi^2\mu_\pi^2 = (m_u + m_d)\langle 0|\bar{u}u + \bar{d}d|0\rangle$. A phase transition from a Nambu-Goldstone to a Wigner-Weyl mode (chiral-symmetry restoration) is characterized by the vanishing of the order parameter $f_\pi$, or alternatively $\langle 0|\bar{q}q|0\rangle$. Clearly, this can only happen at finite temperature, and if the phase transition does take place this should happen regardless of whether the quark masses are zero or not. The numerical value of the critical temperature, though, is expected to depend on this fact.

The temperature behaviour of $f_\pi(T)$ at low $T$ ($T \ll \mu_\pi$) has been investigated in chiral perturbation theory with the result \cite{[4]}

$$f_\pi(T) = f_\pi(0) \left[1 - \frac{T^2}{8f_\pi^2(0)} + O(T^4)\right], \quad (3.9)$$

in the chiral limit for three flavours. Chiral-symmetry breaking corrections to Eq.(3.9) have been also calculated \cite{[5]}, together with corrections due to massive states \cite{[5]}. The low temperature expansion of $\langle \bar{q}q\rangle_T$ has been carried out up to order $O(T^6)$ in \cite{[5]}. As the authors of \cite{[4]},\cite{[5]} have pointed out, this low temperature expansion should not be extrapolated to the critical temperature, as it is not valid there. For instance, if the phase transition is of second order then the critical exponent should be $\frac{1}{2}$, rather than 2 as one
would naively obtain from Eq.(3.9). If one is interested in the behaviour of \( f_\pi(T) \) near \( T = T_c \), other methods should be used, e.g. the composite operator formalism of \[3\], which reproduces Eq.(3.9) at low \( T \), but gives instead \( f_\pi(T) \propto (1 - T/T_c)^{1/2} \) as \( T \to T_c \). This feature will become particularly important in Section 5.

## 4 Quark Deconfinement

At zero temperature the shape of a typical hadronic spectral function consists of some delta functions plus resonances with increasing widths, followed by a smooth continuum starting at some threshold energy \( E_0 \). It is known from fits to actual data that for \( E > E_0 \) the hadronic spectral functions are well approximated by perturbative QCD; \( E_0 \) is thus called the asymptotic freedom (A.F.) threshold. This picture is well supported by all existing experimental data. Under the assumption that quark deconfinement does take place at some critical temperature \( T_d \), one would expect that by increasing \( T \) from \( T = 0 \) the resonance peaks in the spectral function should become broader. At \( T = T_d \) the resonance widths would then become infinite, signalling quark deconfinement. This resonance melting with increasing temperature would be accompanied by a shift of the asymptotic freedom threshold \( s_0 \) towards threshold. In this picture the resonance width, and/or the asymptotic freedom threshold provide a suitable order parameter.

In the complex \( s \)-plane, bound states (e.g. the pion) correspond to poles of the S-matrix lying on the real axis, and resonances correspond to poles located in the second Riemann sheet, their distance to the real axis being measured by the width \( \Gamma_R \). At finite temperature the form of the Green function will be

\[
G(E,T) \propto \frac{1}{E - M_R(T) + \frac{i}{2} \Gamma_R(T)}.
\] (4.1)

In the picture described above, an increase in the temperature will shift all poles farther away from the real axis as \( \Gamma_R(T) \) increases. While the mass \( M_R \) may depend on \( T \), it is not a relevant order parameter. With the second Riemann sheet poles infinitely far from the real axis, and the A.F. threshold correspondingly close to the origin, the spectral function is then described entirely by the QCD continuum extrapolated down to the kinematic threshold. One may interpret such a phase as a deconfined phase where all hadrons originally contributing to the spectral function have melted into quarks or quark-antiquark pairs. As an example, let us consider the following ansatz for the rho-meson width

\[
\Gamma_\rho(T) = \frac{\Gamma_\rho(0)}{(1 - T/T_d)^\alpha},
\] (4.2)
where $T_d$ is the critical temperature for deconfinement, and $\alpha$ a positive number. The behaviour of the hadronic spectral function in the vector channel, with $M_\rho(T) = M_\rho(T = 0)$, $\Gamma_\rho$ as in Eq.(4.2) with $\alpha = 0.5$, is illustrated in Fig. 1 for two values of the temperature: $T = 0$ (solid curve), and $T = T_d/2$ (dashed curve).

In the case of stable hadrons, e.g. the pion or the nucleon, for which $\Gamma(T = 0) = 0$, we would expect an imaginary part in their propagators to develop for $T \neq 0$. In this case, these hadronic thermal widths should be interpreted as damping coefficients of wave packets propagating through a dispersive medium (heat bath). Their growth with increasing temperature implies that, with hadrons corresponding to excitations in this medium, these excitations become less and less important. Combined with the notion of QCD-hadron duality, this may be interpreted as the melting of hadrons into their constituents, thus signalling a deconfinement phase transition.

I wish to stress that the relevant order parameter for deconfinement should be the width and not the mass. Claims have been made occasionally in the literature that the mass should vanish at the critical temperature. While this may happen in some cases, it is not a necessary condition for deconfinement. In fact, let us consider a stable hadron, i.e. one with zero width at zero temperature, such as the pion or the nucleon. If with increasing temperature the mass goes to zero, and nothing happens to the width, then at the critical temperature one would still see a peak in the hadronic spectral function at zero energy. The hadron has not disappeared from the spectrum, and hence has not melted! The only way a particle can melt is by having a temperature dependent width such that $\Gamma_R \to \infty$ as $T \to T_c$. What happens to the mass (defined as the position of the pole on the real axis) is irrelevant to this argument. Once the resonance becomes infinitely broad, the spectral function becomes smooth and should be well approximated by the quark degrees of freedom, i.e. by perturbative QCD. Since this proposal was first made in [11] independent supportive theoretical evidence has become available. In [14] it has been shown in the framework of the virial expansion that: (a) at low temperatures ($T < 50$ MeV) and in the chiral limit ($\mu_\pi = 0$) the mass of the nucleon $M_N(T) \simeq M_N(0)$, and its width $\Gamma_N(T) \simeq \Gamma_N(0) \equiv 0$; (b) At higher temperatures, and now away from the chiral limit, both $M_N(T)$ and $\mu_\pi(T)$ increase slightly (by 4% and 1%, respectively, up to $T \lesssim 160$ MeV), while $\Gamma_N(T)$ and $\Gamma_\pi(T)$ increase substantially, e.g. at $T = 160$ MeV the ratio of width to mass (imaginary to real part of the propagator) is about 43% for the pion and 20% for the nucleon. Since the pion and nucleon widths are strictly zero at $T = 0$, this is quite a dramatic effect. Additional evidence for the approximate constancy of $M_N$ and $\mu_\pi$ follows from the sigma model [15], which also gives increases over $M_N(0)$ and $\mu_\pi(0)$ at the level of a few percent. Further support comes from a recent calculation of $\mu_\pi(T)$ in the composite operator formalism [14], showing approximate constancy of the pion mass over a wide range of temperatures, with a tendency to increase near the critical temperature. QCD sum rules give a similar $T$-dependence of the nucleon mass [17]. The
temperature behaviour of the pion and nucleon widths have also been calculated recently in the framework of the sigma model [18]. The results are in qualitative agreement with [14], and are shown in Figs. 2-3 for three different values of the $\sigma$-meson mass: $M_\sigma = 400$ MeV (a), 600 MeV (b), 800 MeV (c). Concerning the rho-meson mass, it has been shown [19] that to first order in the virial expansion, unitarity of the $\pi - \pi$ scattering amplitude requires that if $\Gamma_\rho(T)$ increases with $T$, then $M_\rho(T)$ must also increase with $T$. I shall come back to this point in Section 7.

5 Relationship between deconfinement and chiral-symmetry restoration

Let us consider the retarded two-point function Eq.(2.10) involving the axial-vector current $A_\mu(x) = :\bar{u}(x)\gamma_\mu\gamma_5 d(x):$. As discussed in Section 2, the QCD sum rule program consists in calculating the two-point function in the deep euclidean region in perturbative QCD, adding non-perturbative effects parametrized in terms of vacuum expectation values of the quark and gluon fields appearing in the QCD Lagrangian, and relating the result to the hadronic spectral function by means of a dispersion relation.

In order to find the perturbative QCD behaviour of Eq.(2.10) we assume a dilute quark gas at temperature $T$ with zero chemical potential. The virtual quanta associated to the local current $A_\mu(x)$ will convert into $q\bar{q}$ pairs for $q^2 = \omega^2 - q^2 > 4m_q^2$, while for space-like momenta ($\omega^2 - q^2) < 0$ there is an additional cut in the complex $\omega$-plane centered around $\omega = 0$ [3]. These two distinct processes contribute to the spectral function as follows

$$\frac{1}{\pi} \text{Im} \Pi_{\mu\nu}^+(\omega, q) = \sum_q \int \text{LIPS}(\omega, q, E_1, p_1, E_2, p_2) \times \langle 0 | A_\mu | q\bar{q} \rangle \langle q\bar{q} | A_\nu | 0 \rangle \times [1 - n_F(E_1) - n_F(E_2)],$$  

(5.1)

for $q^2 \geq 0$, and

$$\frac{1}{\pi} \text{Im} \Pi_{\mu\nu}^-(\omega, q) = \sum_q \sum_{\bar{q}} \int \text{LIPS}(\omega, q, E_1, p_1, -E_2, -p_2) \times \langle q | A_\mu | \bar{q} \rangle \langle \bar{q} | A_\nu | q \rangle \times [n_F(E_1) - n_F(E_2)]$$  

(5.2)
for $q^2 < 0$. In the above equations $n_F(E)$ is the Fermi-Dirac distribution function, and the phase space is

$$LIPS(\omega, q, E_1, p_1, E_2, p_2) = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \times \delta(\omega - E_1 - E_2)\delta^{(3)}(q - p_1 - p_2).$$

(5.3)

It must be stressed that the above two contributions to the spectral function, arising from different physical mechanisms, are unrelated. The first contribution ($q^2 \geq 0$) leads to the usual right- and left-hand cuts in the complex energy plane, and does not vanish at $T = 0$. The second piece ($q^2 < 0$) comes from a new cut centered around $\omega = 0$, vanishes at $T = 0$, and is unrelated to duality.

On the hadronic side, at low temperatures $T < \mu_K$, only pions from the gas will contribute to the spectral function. In addition to the (time-like) pion pole contribution to $\text{Im}\Pi_{\mu\nu}^{(+)}$, there is a (space-like) piece in the hadronic spectral function $\text{Im}\Pi_{\mu\nu}^{(-)}$ from the center cut in the complex $\omega$-plane. The latter involves an integral of $\langle \pi | A_\mu | 2\pi \rangle \langle 2\pi | A_\nu | \pi \rangle$ weighted by Bose factors. However, since $A_\mu \propto \varphi (\varphi + \partial_\mu \varphi)$ this term appears at order $T^4/f_\pi^2$ in $\text{Im}\Pi(s)$. In addition, the numerical coefficient of this term is very small. Hence, a parametrization in terms of the pion pole plus a continuum modelled by perturbative QCD should be a good approximation to the hadronic spectral function.

Using all this information in the FESR Eq.(2.9), suitably extended to $T \neq 0$, choosing $N = 0$, and neglecting $C_2 < O_2 >$ (which is proportional to the quark mass squared) one obtains the following finite temperature FESR

$$8\pi^2 f^2_\pi(T) = \frac{1}{2} \int_{4m^2_q}^{s_0(T)} dz^2 v(z) [3 - v^2(z)] \tanh \left( \frac{z}{4T} \right)$$

$$+ \int_{4m^2_q}^{\infty} dz^2 v(z) [3 - v^2(z)] n_F \left( \frac{z}{2T} \right),$$

(5.4)

where $v(z) = (1 - 4m^2_q/z^2)^{1/2}$. Equation (5.4) is an eigenvalue equation fixing $s_0(T)$ once $f_\pi(T)$ is known independently. The advantage of our choice of Green function should be evident: at low temperatures ($T < \mu_K$) apart from $f_\pi(T)$ there are no other unknown $T$-dependent hadronic parameters such as masses and widths. For instance, QCD sum rules for the correlator of two vector currents at $T \neq 0$ [3], [12], involve $M_\rho(T)$ and $\Gamma_\rho(T)$ which, unlike $f_\pi(T)$, are a priori unknown and model-dependent. I shall come back to this point in Section 7.

In the chiral limit the FESR Eq.(5.4) takes the simple form

$$8\pi^2 f^2_\pi(T) - \frac{4\pi^2}{3} T^2 = \int_0^{s_0(T)} ds \tanh \left( \frac{\sqrt{s}}{4T} \right),$$

(5.5)
where \( s_0(T = 0) = 8\pi^2 f_π^2(0) \). In [13] the FESR Eqs.(5.4)-(5.5) were solved using the low temperature expansion for \( f_π(T) \) from [4], i.e. Eq.(3.9). The result is that \( s_0(T) \) vanishes at a critical temperature \( T_d \), but \( T_d < T_c \) (\( T_c \) being the critical temperature for chiral-symmetry restoration). However, one should keep in mind that Eq.(3.9) is not valid in the vicinity of \( T_c \). As shown in [6], if one uses the expression for \( f_π(T) \) from the composite operator formalism, \( s_0(T) \) vanishes at practically the same temperature as \( f_π(T) \), i.e. \( T_d \approx T_c \). This is illustrated in Fig.4 (reproduced from [6]), where \((s_0(T)/s_0(0))^{1/2} \approx f_π(T)/f_π(0)\), except very close to the critical temperature. Given the uncertainties of the method, this minor difference can be safely ignored.

Independent confirmation of this result may be obtained by using the first Weinberg sum rule at \( T \neq 0 \), which in the chiral limit is given by

\[
\frac{1}{\pi} \int_0^\infty ds [\text{Im} \Pi_V(s,T) - \text{Im} \Pi_A(S,T)] = f_π^2(T) \quad (5.6)
\]

This sum rule was studied in [13] using Eq.(3.9), the result being essentially the same as with the FESR Eq.(5.5). However, the authors of [6] obtained \( T_d/T_c \approx 0.99 \) using the more accurate expression for \( f_π(T) \) from the composite operator formalism.

In summary, Finite Energy QCD sum rules at \( T \neq 0 \) lead to the prediction that \( s_0(T) \) vanishes at some critical temperature, which is essentially the same as that for chiral-symmetry restoration: \( T_d \approx T_c \), provided one uses an expression for \( f_π(T) \) valid for all \( T \), such as e.g. the one in [6]. According to the interpretation of \( s_0(T) \) as a relevant order parameter for quark deconfinement (see Section 4), one can conclude that the QCD-FESR provide evidence for the existence of this phase transition.

### 6 Pion form factor at finite temperature

Independent phenomenological evidence for the deconfinement phase transition in QCD may be obtained e.g. by studying the thermal behaviour of the electromagnetic form factor of the pion, \( F_π \). In this case one would expect the size of the pion to increase with increasing temperature. At the critical temperature the pion radius should presumably diverge, indicating quark-gluon deconfinement. In this Section I discuss a recent determination [20] of the \( T \)-dependence of \( F_π \) in the space-like region using a Finite Energy QCD Sum Rule (FESR). The pion form factor at \( T = 0 \) has been extensively studied in the past with FESR, as well as with Laplace transform QCD sum rules [21]. In order to establish some notation, as well as the \( T = 0 \) normalization, I briefly describe the method.
at $T = 0$ before introducing thermal corrections.

The appropriate object to study is the three-point function

$$
\Pi_{\mu\nu\lambda}(p, p', q) = i^2 \int d^4 x \, d^4 y \, e^{i(p' x - q y)} < 0 | T(A^\dagger_{\mu}(x) \, V_\lambda(y) \, A_\mu(0)) | 0 > ,
$$

(6.1)

where $A_\mu(x) =: \bar{u}(x) \gamma_\mu \gamma_5 d(x)$ is the axial-vector current, $V_\lambda$ is the electromagnetic current, and $q = p' - p$ the momentum transfer. On general analyticity grounds, the three-point function Eq.(6.1) satisfies the double dispersion relation

$$
\Pi_{\mu\nu\lambda}(p^2, p'^2, Q^2) = \frac{1}{\pi^2} \int_0^\infty ds \int_0^\infty ds' \frac{\rho_{\mu\nu\lambda}(s, s', Q^2)}{(s + p^2)(s' + p'^2)},
$$

(6.2)

defined up to subtractions, which are disposed of by Laplace improving the Hilbert transform, or by considering FESR. The correlator Eq.(6.1) involves quite a few structure functions, associated with all the Lorentz structures that can be formed with the available four-momenta. In principle, it should not matter which particular structure one chooses to project the pion form factor. Following \[21\] in choosing the combination $P_\mu P_\nu P_\lambda$, where $P = p + p'$, the hadronic spectral function in the chiral-limit reads

$$
\rho(s, s', Q^2)|_{HAD} = \frac{1}{2} f_\pi^2 F_\pi(Q^2) \delta(s) \delta(s') + \rho(s, s', Q^2)|_{QCD}[1 - \theta(s_0 - s - s')],
$$

(6.3)

where $s_0$ signals the onset of the continuum, $f_\pi \simeq 93$ MeV, and

$$
\rho(s, s', Q^2)|_{QCD} = \frac{3}{16 \pi^2} \frac{Q^4}{\lambda^{7/2}} \left[ 3 \lambda (x + Q^2)(x + 2Q^2) - \lambda^2 - 5Q^2(x + Q^2)^3 \right],
$$

(6.4)

to one-loop order (and in the chiral-limit), with

$$
\lambda = y^2 + Q^2(2x + Q^2),
$$

(6.5)

and $x = s + s'$, $y = s - s'$. Since one is interested in writing the lowest moment FESR for $F_\pi$, i.e.

$$
F_\pi(Q^2) = \frac{1}{f_\pi^2} \int_0^{s_0} \int_{-x}^x dy \, \rho(x, y, Q^2)|_{QCD},
$$

(6.6)

rather than a Laplace transform QCD sum rule, the non-perturbative power corrections entering the OPE are of no concern here (they contribute to higher moment FESR). The integration region in Eq.(6.6) has been chosen to be a triangle in the $(s,s')$ plane, with base and height equal to $s_0$. Other choices of the integration region, e.g. a square region of side $s_1 \simeq s_0/\sqrt{2}$, give similar results. The solution to the FESR Eq.(6.6) is

$$
F_\pi(Q^2) = \frac{1}{16 \pi^2 f_\pi^2 (1 + Q^2/2s_0^2)^2},
$$

(6.7)
Although not evident from Eq.(6.7), it is important to realize that this analysis is only valid in the region $Q^2 \geq 1 \text{ GeV}^2$, where one expects a reasonable convergence of the OPE. This limitation is of no relevance if one is only interested in the thermal behaviour of the ratio $F_\pi(Q^2, T)/F_\pi(Q^2, 0)$. In any case, as shown in [21], Eq.(6.7) provides a reasonable fit to the experimental data in the region $Q^2 \simeq 1 - 4 \text{ GeV}^2$, if $s_0 \simeq 1 \text{ GeV}^2$.

The spectral function Eq.(6.4) at finite temperature was calculated in [20] using the Dolan-Jackiw formalism. After substitution in Eq.(6.6), the result can be expressed as

$$F_\pi(Q^2, T) = \frac{1}{f_\pi^2(T)} \int_0^{s_0(T)} dx \int_{-x}^x dy \rho(x, y, Q^2)|_{QCD} F(x, y, Q^2, T),$$

(6.8)

with

$$F(x, y, Q^2, T) = 1 - n_1 - n_2 - n_3 + n_1 n_2 + n_1 n_3 + n_2 n_3,$$

(6.9)

$$n_1 = n_2 \equiv n_F \left( \frac{1}{2T} \sqrt{\frac{x+y}{2}} \right),$$

(6.10)

$$n_3 \equiv n_F \left( \frac{Q^2 + (x-y)/2}{2T \sqrt{\frac{x+y}{2}}} \right),$$

(6.11)

and $n_F$ is the Fermi thermal factor. In the equations above a frame was chosen such that $p_\mu = (\omega, 0)$, and $p'_\mu = (\omega', p')$, in which case

$$\omega = \sqrt{\frac{x+y}{2}}, \quad \omega' = \frac{x + Q^2}{2\sqrt{\frac{x+y}{2}}}. $$

(6.12)

It has been explicitly checked that the ratio

$$R(T) \equiv \frac{F_\pi(Q^2, T)}{F_\pi(Q^2, 0)}$$

(6.13)

is essentially insensitive to other choices of frames. For instance, one may choose $p_\mu = (\omega, p)$, and $p'_\mu = (\omega', -p)$, which leads to different arguments in the thermal factors, but roughly the same ratio $R(T)$. The temperature dependence of the continuum threshold, $s_0(T)$, is given by [3] (see discussion in Section 5)

$$\sqrt{\frac{s_0(T)}{s_0(0)}} \sim \frac{f_\pi(T)}{f_\pi(0)},$$

(6.14)

The ratio Eq.(6.13) is shown in Fig.5 for $Q^2 = 1 \text{ GeV}^2$. This result for $R(T)$ is in nice agreement with the expectation that as the temperature increases, $F_\pi$ should decrease...
and eventually vanish at the critical temperature for deconfinement $T_d$.

Although the OPE breaks down at small values of $Q^2$, one may still extrapolate the ratio Eq.(6.13) into this region just to study the qualitative temperature behaviour of the electromagnetic radius ratio $<r^2_\pi>_T / <r^2_\pi>_0$. Doing this, one finds that this ratio increases monotonically with $T$, doubling at $T/T_d \sim 0.8$, and diverging at the critical temperature. This divergence of $<r^2_\pi>_T$ may be interpreted as a signal for quark deconfinement. In fact, the behaviour of $<r^2_\pi>_T$ can be traced back to the temperature behaviour of the asymptotic freedom threshold $s_0(T)$. As $s_0(T)$ decreases with increasing $T$, a signature of quark deconfinement, the root-mean-square radius of the pion increases. This result is in qualitative agreement with the one obtained in the framework of the Nambu-Jona Lasinio model [22].

7 Laplace sum rules: do they fail?

There are a few papers [23]-[27] devoted to Laplace transform QCD sum rules at finite temperature. I shall concentrate mostly in the vector-vector correlator, e.g. the $\rho$-meson channel. All of these analyses suffer from the following serious drawbacks.

(i) A single Laplace sum rule involves three unknowns: the mass and the width of the $\rho$-meson, $M_\rho(T)$ and $\Gamma_\rho(T)$, and the continuum threshold $s_0(T)$. I am assuming that the temperature dependence of the vacuum condensates is a known input. No trick of magic allows a determination of three unknown quantities from a single equation. In principle, at least, one could take derivatives with respect to the Laplace parameter $M^2$ in order to end up with three equations. However, this does not work in practice because of (ii) below.

(ii) The Laplace sum rule is inconsistent with the known temperature dependence of the gluon and the quark condensates, as will be shown below. This inconsistency translates into a breakdown of the FESR beyond the lowest moment.
A zero-width approximation has been used. While it is true that at very low temperatures this approximation does not differ much from a finite-width parametrization, at intermediate temperatures this is not the case. Since on physical grounds we expect the width to increase with increasing $T$ (see Section 4), the zero-width approximation is in principle self-contradictory.

Since the Laplace sum rule is equivalent to an infinite number of FESR, it is simpler to break it up into a series of FESR in order to show (ii) above. However, this is not strictly necessary; the same conclusion follows from the Laplace sum rule itself, but the procedure is a bit more involved. The first three FESR of the type Eq. (2.9) at finite temperature are (notice the change of normalization, as I am now discussing the neutral $\rho$-meson channel)

$$\int_0^{s_0(T)} ds \left( \frac{1}{\pi} Im\Pi_0(s,T)|_{\text{RES}} = \frac{1}{8\pi^2} \int_0^{s_0(T)} ds \tanh\left(\frac{\sqrt{s}}{4T}\right) + \frac{T^2}{18}, \right. \tag{7.1}$$

$$C_4 << O_4 >> = \int_0^{s_0(T)} ds \ tanh\left(\frac{\sqrt{s}}{4T}\right) - 8\pi^2 \int_0^{s_0(T)} ds \ s \left( \frac{1}{\pi} Im\Pi_0(s,T)|_{\text{RES}} \right), \tag{7.2}$$

$$C_6 << O_6 >> = - \int_0^{s_0(T)} ds \ s^2 \tanh\left(\frac{\sqrt{s}}{4T}\right) + 8\pi^2 \int_0^{s_0(T)} ds \ s^2 \left( \frac{1}{\pi} Im\Pi_0(s,T)|_{\text{RES}} \right), \tag{7.3}$$

where $C_N << O_N >> = C_N < O_N >(T)$, the term $C_2 << O_2 >>$ has been neglected (it is proportional to $m_q^2$), and the hadronic resonant spectral function is given by

$$\frac{1}{\pi} Im\Pi_0(s,T)|_{\text{RES}} = \frac{1}{48\pi^2} \frac{M^4_\rho(T) (1 + \Gamma^2_\rho(T)/M^2_\rho(T))}{(s - M^2_\rho(T))^2 + M^2_\rho(T)\Gamma^2_\rho(T)}. \tag{7.4}$$

We now have three equations to determine the three unknowns $s_0(T)$, $M_\rho(T)$, and $\Gamma_\rho(T)$, provided the temperature dependence of the condensates is used as an input. The gluon condensate in the chiral limit, and at low $T$, is given by [3]

$$<< \alpha_s G^2 >> = << \alpha_s G^2 > - \frac{4\pi^2}{1215} \frac{T^8}{f^4_\pi} (\ln \frac{\Lambda_p}{T} - \frac{1}{4}) \tag{7.5},$$

where $\Lambda_p = 275 \pm 65 MeV$. The above result is valid up to $T \simeq 100 - 120 MeV$. Beyond these temperatures the contribution of massive states, as well as chiral symmetry breaking corrections, become gradually more important. Numerically, though, with $<< \alpha_s G^2 >> \simeq 0.1 GeV^4$ [4] these corrections are below the 10% level at $T \simeq 180 MeV$ [28]. In view of this I shall take the gluon condensate to be temperature-independent. Concerning the dimension $d = 6$ four-quark condensate, I assume it is proportional to $<< \bar{q}q >>^2$, and use $<< \bar{q}q >>$ from the analysis of [3]. At dimension $d = 4$ and $d = 6$ the heat bath can sustain non-scalar condensates [27]. Numerically, though, they play a
negligible role in this analysis (I have used the estimates of [27]).

In trying to solve Eqs. (7.1)-(7.3) I found no meaningful global solution for all three unknowns: \(s_0(T), M_\rho(T), \Gamma_\rho(T)\). As I show next, the reason for the absence of a solution to all three FESR is that they are inconsistent with the assumed temperature dependence of the condensates.

Let us turn the argument around, and assume a temperature functional form for \(M_\rho(T)\) and \(\Gamma_\rho(T)\), and use the three FESR to determine \(s_0(T), C_4 << O_4 >>, \) and \(C_6 << O_6 >>.\) For definiteness I shall assume that \(\Gamma_\rho(T)\) is given by Eq.(4.2). Variations of this functional form do not change the conclusions. For \(M_\rho(T)\) I consider two possibilities: (a): \(M_\rho(T) = M_\rho(0),\) and (b): \(M_\rho(T)\) as determined using unitarity of the \(\pi - \pi\) scattering amplitude, and to lowest order in the virial expansion [19]. The latter analysis leads to a monotonically increasing \(M_\rho(T)\). Case (a) has already been discussed in [27]; the solutions of the first two FESR for \(s_0(T)\) and \(C_4 << O_4 >>,\) respectively, are shown in Fig.6. Although \(s_0(T)\) does decrease with \(T\) as expected, the thermal dependence of the gluon condensate is definitely wrong. Also, solving the third FESR Eq.(7.3) gives a quark condensate that decreases with \(T\) even faster than the gluon condensate, again in contradiction with expectations. Next, case (b) is qualitatively similar to case (a), as may be appreciated from Fig.7. In this figure, the function \(M_\rho(T)\) is taken from [19]. Hence, the conclusion from this analysis is that while the first FESR, Eq.(7.1), does lead to a reasonable result for \(s_0(T)\), the higher moment FESR are in contradiction with the known \(T\)-dependence of the vacuum condensates. Therefore, the FESR program breaks down beyond the lowest moment, and so does the Laplace transform. As mentioned before, this analysis can be carried out directly with Laplace sum rules, without reference to the FESR, but the conclusions are the same.

There is a third possibility, which I find quite attractive (although it does not solve the above inconsistency problem). This is to consider the ratio \(s_0(T)/s_0(0)\) as a universal function, i.e. the same for any channel. Clearly, the value of \(s_0(0)\) will depend on the particular channel under consideration, as it is obviously different in the vector and axial-vector channels, and the nucleon channel, etc.. However, the ratio \(s_0(T)/s_0(0)\) may be thought of as a phenomenological order parameter for deconfinement, in which case it is reasonable to assume it is channel-independent. In this case, one can use e.g. the first FESR Eq.(7.1) to predict \(M_\rho(T)\), and the second FESR Eq.(7.2) to predict \(C_4 << O_4 >>.\) The results of this exercise are shown in Fig.8, where I have used \(s_0(T)/s_0(0)\) from [8] (see Fig.4). Interestingly enough, the temperature dependence of the \(\rho\)-meson mass turns out to be in good agreement with the determination using unitarity of the \(\pi - \pi\) scattering to lowest order in the virial expansion [19]. This supports the validity of the lowest moment FESR Eq.(7.1), but unfortunately it does not solve the problem with the higher moment FESR or with the Laplace transform, as \(C_4 << O_4 >>\) comes out wrong. Whether
additional mechanisms could be brought into play in order to rescue the QCD sum rule program remains an open problem.

Concerning the existing analyses of the vector channel based on Laplace sum rules \[23, 24, 26, 27\], they all predict a decrease of the $\rho$-meson mass with increasing $T$. This is in contradiction with the result of \[19\], which is quite general. In the light of the above discussion, it is possible to understand why these analyses predict the wrong $T$-dependence of $M_{\rho}$. Laplace sum rules are inconsistent with the known thermal behaviour of the quark and gluon condensates. There is also a Laplace sum rule determination of the nucleon mass at finite temperature \[25\] which gives again a monotonically decreasing $M_N(T)$. This is in contradiction with a lowest moment FESR analysis \[17\] which predicts an almost constant nucleon mass. It is also inconsistent with \[14\]. The reason behind the discrepancy is the same as in the vector channel. I should add that in \[25\] it is claimed that the nucleon mass should decrease, since it is proportional to the quark condensate $<\bar{q}q>$. This argument is falacious because the contribution from the hadronic cut in the complex energy plane centered around the origin (the so called scattering term) prevents the nucleon mass from decreasing \[17\].

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