OPTIMAL ORDERING POLICY FOR INVENTORY MECHANISM WITH A STOCHASTIC SHORT-TERM PRICE DISCOUNT

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Abstract. This paper considers an inventory mechanism in which the supplier may provide a short-term price discount to the retailer at a future time with some uncertainty. To maximize the retailer’s profit in this setting, we establish an optimal replenishment and stocking strategy model. Based on the retailer’s inventory cost-benefit analysis, we present a closed-form solution for the inventory model and provide an optimal ordering policy to the retailer. Numerical experiments and numerical sensitivity are given to provide some high insights to the inventory model.

1. Introduction. In modern consumer products market, enterprises adopt various marketing strategies to enhance their market share and competitiveness, and among them, the price discount may be a popular one [14, 31, 29]. Generally, the price discount can motivate retailers to make larger orders, which can not only help suppliers to reduce storage and clear stock to relieve their capital pressure, but also decrease their order processing cost [5, 6]. However, suppliers do not always have the incentive to offer the price discount to the retailer since it would decrease their selling profit. Typically, price discount occurs under some situations, e.g., the enterprise has a large number of backlogs at the end of the year, faces a tight budget or a special promoting opportunity. Therefore, the granting of the price discount is temporary and/or conditional, and research on the inventory system with discount is usually established in an EOQ environment [31].

The broad inventory research on price discount can be divided into two categories. The first one is on recurrent discounts which generally operates in an infinite horizon setting [9], the second one is on one-time special purchase opportunity which often occurs in the EOQ environment [31]. We begin with the latter.

For one-time discount, there are three streams of research in the literature: instantaneous discounts in the present or at a future time [31], discounts over an interval of time [10], and discounts in the form of an extended credit period [8]. For
the first stream of research, Ardalan [1, 2] analyzes the discounted inventory model which allows an arbitrary level of inventory when the special opportunity occurs. Lev and Weiss [13] consider the discounted inventory model in a finite horizon setting. Yang et al. [28] study a one-time discount inventory model with a capacity constraint. Tersine and Schwarzkopf [25] examine the discounted inventory model for a nonrestrictive time duration in an infinite time horizon setting. Mousavi et al. [15, 16, 17, 18] consider the multi-item multi-period discounted inventory control problem in a fuzzy environment. Aull-Hyde [4], Cardenas-Barron et al. [5], Kindi and Sarker [12], and Taleizadeh et al. [23], among others, investigate the inventory system with shortages allowed.

For the second stream of research, Ardalan [1] considers a one-time discount inventory model where the demand is sensitive to the discounted price. Chang et al. [6], Taleizadeh et al. [24], and Kevin et al. [11] consider the discounted inventory model with deteriorating or imperfect items. Sari et al. [19] consider the inventory model with several discount offers and different discounts at different times. Chu et al. [7] consider the inventory model in which the discounted purchased quantity can be selected from a restricted set of values. Sarker and Kindi [20] examine the inventory model with multiple types of one-interval discounts.

For the research on the inventory with discounts in the form of an extended credit period, Zhang and Tang [30] consider the setting of the inventory model from the perspective of the supplier. Arcelus et al. [3] study one-time price incentives for a model of perishable items. By taking the supplier capital constraint into consideration, Wang et al. [26] establish a conditional and partial trade credit model for the two-echelon supply chain with a stable market demand.

The discussion above assumes that a one-time discount occurs with certainty. Nevertheless, due to the variation of the demand and unforeseen incidents in market, even the supplier is not entirely confident whether the discount could be offered for a given future time. This means that the occurrence of the event is uncertain and the research on the inventory systems with probabilistic discount is significant in inventory control and management. For this, Shaposhnik, et al. [21] consider the inventory system with a probabilistic one-time discount. Note that when the discount takes place, it usually lasts for a short period. For instance, GREE Ltd. China has been launching promotions on a fixed number of discounted air-conditioning deals around Labor Day long weekend in recent years. Besides, many enterprises opt to hold a three-to-five day clearance sale at the end of the year. This means that the inventory model with a stochastic short-term price discount is more significant in practice and this constitutes the issue considered in this paper. More precisely, in this paper, we consider the inventory mechanism in which the supplier may provide a short-term price discount to the retailer at a future time with some uncertainty. For this setting, to maximize the retailer’s profit, we establish an optimal replenishment and stocking strategy model in this paper. Based on the retailer’s cost-benefit analysis of inventory, we propose a closed form solution for the model and provide an optimal replenishment policy to the retailer.

The remainder of the paper is organized as follows. Section 2 presents the assumptions made on the inventory model and notations used in the subsequent analysis. An optimization model of the problem is also established in this section. Section 3 provides three heuristic inventory policies based on the EOQ ordering policy. On account of the retailer’s cost-benefit analysis of inventory, we derive a global optimal solution in closed-form for the model and present an optimal replenishment policy.
for retailers in Section 4. Numerical experiments on sensitivity analysis are given in Section 5 to provide high insights to the model.

2. Assumptions, notations and problem formulation. First, we present the elementary assumptions on the concerned inventory model:

(1) time horizon is infinite;
(2) the demand rate is constant;
(3) shortage is not allowed;
(4) the lead time of an order is zero;
(5) retailer’s inventory storage space and the availability of capital is unlimited;

Further more, we assume that a short-term price discount may take place at a future time, and if it takes place at that time, then it will last for a short period.

Based on the assumption, the running pattern of the concerned inventory system is as follows: The supplier may provide a price discount to the retailer at a future time with uncertainty. If the discount takes place at that time, then it will last for a short period and the retailer has an opportunity to make a special order with a lower purchase price during that period. For this system, to minimize the retailer’s inventory cost and/or to maximize retailer’s inventory benefit, we should find an optimal replenishment and stocking strategy by taking the possible short-term price discount into considerations. To this end, we need the following notations.

| Symbol | Description                          | Symbol | Description                          |
|--------|--------------------------------------|--------|--------------------------------------|
| λ      | retailer’s market demand rate         | ts     | the start time of possible discount   |
| K      | fixed ordering cost                   | te     | the end time of possible discount     |
| c      | retailer’s unit purchase price        | t_r    | the special ordering time             |
| b      | retailer’s unit selling price         | q_s    | remaining inventory at ts             |
| h      | retailer’s inventory holding cost     | q_e    | remaining inventory at te             |
| p      | probability that the price discount takes place | q_r    | remaining inventory at t_r          |
| γ      | discount rate                         | Q_0    | order size before ts                  |
|        |                                       | Q_d    | special order size                    |

For the inventory holding cost for each item, we assume that it depends on the retailer’s purchase price. Based on this, the retailer’s holding cost is γh for each discounted item in unit time. For the retailer’s economic order quantity $Q_{EOQ}$, it holds that $Q_{EOQ} = \sqrt{2AK/h}$ [31].

Without loss of generality, we assume that the stock level is zero at the beginning of the planning horizon. Further, we assume that the retailer can only make at most one special order during the discount period and $t_e - t_s \leq Q_{EOQ}^{EOQ}/\lambda$.

For convenience, if price discount takes place, we use $T_1$ to denote the time period from the beginning to the end of the special replenishment cycle, and use $F_1(Q_0, Q_d, t_r)$ to denote the increased profit under an adjusted replenishment policy relative to the EOQ order policy in the same period; and if price discount does not takes place, we use $T_2$ to denote the time period from the beginning to the first reorder point after $t_s$, and use $F_2(Q_0)$ to denote the increased profit under an adjusted replenishment policy relative to the EOQ order policy in the same time horizon. According to the assumptions and the principle of inventory cost minimization, and considering the randomness of the price discount, we can formulate the
optimal replenishment and stocking strategy model as the following optimization problem \[22, 27\]

\[
\max_{Q_0, Q_d, t_r} E = pF_1(Q_0, Q_d, t_r) + (1 - p)F_2(Q_0)
\]

(2.1)

In the following two sections, we will first present three heuristic solution methods for the problem based on the EOQ ordering policy, and then present a closed form solution method for the model.

3. **Heuristic solution methods upon EOQ model.** According to the assumptions on the inventory model and the running pattern of the model, to maximize the retailer’s profit, i.e., to solve problem (2.1), we should determine the order size \(Q_0\) prior to the event, the special order time \(t_r\) and the special order size \(Q_d\) when the event takes place. For these quantities, we have the following conclusions.

**Theorem 3.1.** [21] For any optimal ordering policy of problem (2.1), orders made before \(t_s\) are placed only upon stock depletion and are equal.

Since the discount will last for a short period if the event takes place and the lead time of an order is zero, the following conclusion on special order time is evident.

**Theorem 3.2.** For any optimal ordering policy, if the remaining inventory at any time in time horizon \([t_s, t_e]\) is nonzero, then the optimal special ordering time is \(t_e\) as long as the price discount takes place.

If a special order is made when the remaining stock is zero, then we have the following conclusion.

**Theorem 3.3.** For any optimal ordering policy of problem (2.1), if the special order is made when the remaining inventory is zero, then the optimal special order size is

\[
Q_d^* = \frac{(1 - \gamma)c}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\lambda}{h}}.
\]

**Proof.** To show the conclusion, we consider the increased profit caused by the special order relative to the EOQ ordering policy.

Since the remaining inventory at the beginning of the special replenishment cycle is zero, the retailer’s profit derived from the special replenishment cycle is

\[
f^D = (b - \gamma c)Q_d - K - \frac{\gamma hQ_d^2}{2\lambda},
\]

where the first term refers to the selling profit, the second term refers to the fixed ordering cost, and the last term refers to the inventory holding cost.

Clearly, the length of the special replenishment cycle is \(\frac{Q_d}{\lambda}\) and the retailer’s profit in this time horizon under the normal ordering policy, i.e., the EOQ ordering policy is [31]

\[
f^N = (b - c)Q_d - \frac{Q_d}{\lambda} \sqrt{2\lambda K h}.
\]

Therefore, relative to the EOQ ordering policy, the increased profit of inventory brought by the special order is

\[
F(Q_d) = f^D - f^N = (c - \gamma c)Q_d - K - \frac{\gamma hQ_d^2}{2\lambda} + Q_d\sqrt{\frac{2\lambda K}{\lambda}}.
\]

Obviously, the function is concave, and hence its maximum is reached at the zero point of its derivative.
The desired result follows.

For the inventory mechanism of concern, since the demand rate is constant, $Q_{EOQ}$ is retailer’s optimal order size without regard to the price discount. In consideration of the possibility of discount in $[t_s, t_e]$, the retailer should adjust the EOQ order policy to better enjoy the discount if the event takes place. Specifically, to maximize the retailer’s profit, the retailer should determine the order size before the event and determine the special ordering time and order size when the event takes place. To this end, we break the discussion into two scenarios according to whether the retailer’s EOQ ordering point falls in the discount period $[t_s, t_e]$ or not.

**Scenario 1.** The EOQ ordering point falls in the discount period $[t_s, t_e]$. Denote the EOQ reordering point in time horizon $[t_s, t_e]$ by $t_0$. Surely, if the discount takes place, the retailer should make a special order at time $t_0$ with size $Q_d^* = \left(1 - \gamma\right)\frac{c\lambda}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\lambda}{h}}$ according to Theorem 3.3, and make an EOQ order size otherwise. The retailer’s optimal ordering policy with and without price discount is illustrated in Figure 3.1.

**Scenario 2.** The EOQ ordering point does not fall in the discount period $[t_s, t_e]$. Since price discount can increase the retailer’s benefit, to better enjoy the possible price discount when the event takes place, the retailer needs to adjust the EOQ orders prior to the event to make the remaining inventory in time horizon $[t_s, t_e]$ as low as possible. However, this adjustment will lead to an increase in the inventory holding cost in time horizon $[0, t_s]$ as shown in the following conclusion.

**Lemma 3.1.** [31] The inventory cost in the unit time is a convex function of $Q$ and arrives its minimum at $Q_{EOQ}$, or more precisely, the inventory cost in the unit time is decreasing w.r.t. $Q \in (0, Q_{EOQ}]$ and is increasing w.r.t. $Q \in [Q_{EOQ}, \infty)$. From the conclusion, we need to make a tradeoff between enjoying the benefit of the possible price discount and bearing the increase of inventory holding cost lead by the adjustment of the orders before $t_s$. In view of this, we now present three heuristic ordering policies for problem (2.1) in Scenario 2 based on the EOQ policy provided that a special order is made as long as the event takes place. To proceed, we denote the order times before $t_s$ under the EOQ model by $m$, that is, $m = \left\lceil \frac{\lambda t_s}{Q_{EOQ}} \right\rceil$.

**Policy 1.** Make a slight increase to the EOQ order size before $t_s$ so that $q_s = 0$ and the order times before $t_s$ is $m - 1$, i.e., $\left\lceil \frac{\lambda t_s}{Q_{EOQ}} \right\rceil$. According to Theorem 3.1, the...
optimal order size before \( t_s \) is \( Q_s = \frac{M}{m-1} \). Further, if the discount takes place, then a special order is made at \( t_s \) with order size \( Q_d' \) given in Theorem 3.3; otherwise, the system reverts to the EOQ model from \( t_s \). We denote the ordering policy by \( \pi_s \) which is illustrated in Figure 3.2.

For ordering policy \( \pi_s \), if the price discount takes place, then \( T_1 = [0, t_s + \frac{Q_d'}{\lambda}] \) and the retailer’s profit in this period is

\[
f^{\pi_s} = (b - c) \lambda t_s + (b - \gamma c) Q_d' - mK - \frac{h\lambda t_s^2}{2(m-1)} - \frac{\gamma h (Q_d')^2}{2\lambda}.
\]

Considering the retailer’s profit in \( T_1 \) under the normal ordering policy, i.e., EOQ ordering policy

\[
f^N = (b - c) \lambda (t_s + \frac{Q_d}{\lambda}) - \sqrt{2\lambda K h (t_s + \frac{Q_d}{\lambda})},
\]

we can obtain the retailer’s increased profit relative to the EOQ ordering policy when the discount takes place,

\[
F_1 = f^{\pi_s} - f^N = \left(1 - \gamma\right)cQ_d' - mK - \frac{h\lambda t_s^2}{2(m-1)} - \frac{\gamma h (Q_d')^2}{2\lambda} + \sqrt{2\lambda K h (t_s + \frac{Q_d'}{\lambda})}.
\]

On the other hand, if price discount does not takes place, the retailer’s increased profit relative to the EOQ ordering policy in time horizon \( [0, t_s] \) is

\[
F_2 = -(m - 1)K - \frac{h\lambda t_s^2}{2(m-1)} + \sqrt{2\lambda K h t_s}.
\]

Taking the probability of the discount into consideration, we can obtain the retailer’s expected increased profit relative to the EOQ ordering policy

\[
E_{\pi_s} = p F_1 + (1 - p) F_2 = \left[\left(1 - \gamma\right)cQ_d' - \frac{\gamma h (Q_d')^2}{2\lambda} + Q_d' \sqrt{\frac{2K h}{\lambda} - K}\right] - (m - 1)K - \frac{h\lambda t_s^2}{2(m-1)} + \sqrt{2\lambda K h t_s}.
\]

**Policy 2.** Slightly reduce the EOQ order size before \( t_s \) to make \( q_e = 0 \) and retain the order times in time horizon \([0, t_s]\) as \( m \). Under the adjusted ordering policy, the order size before times \( t_s \) is \( Q_e = \frac{\lambda t_e}{m} \). Further, if the discount takes place, then a special order is made at \( t_e \) with order size \( Q_d' \) given in Theorem 3.3; otherwise,
the system reverts to the EOQ model since $t_e$. Denote the ordering policy by $\pi_e$ which is illustrated in Figure 3.3.

For the adjusted ordering policy, if the price discount takes place, then $T_1 = [0, t_e + \frac{Q_e^*}{\lambda}]$ and the retailer’s profit in this time horizon is

$$f^{\pi_e} = (b - c)\lambda t_e + (b - \gamma_c)Q_d^* - (m + 1)K - \frac{h\lambda t_e^2}{2m} - \frac{\gamma h (Q^*_d)^2}{2\lambda},$$

where the first two items are the selling profit, the third item is the fixed ordering cost and the last two items are the inventory holding cost.

It is readily to compute the retailer’s profit in time horizon $T_1$ under the EOQ ordering policy

$$f^N = (b - c)\lambda(t_e + \frac{Q_d^*}{\lambda}) - \sqrt{2\lambda K h t_e}(t_e + \frac{Q_d^*}{\lambda}).$$

Hence, for ordering policy $\pi_e$, if the price discount takes place, then the retailer’s increased profit relative to the EOQ policy is

$$F_1 = f^{\pi_e} - f^N = (1 - \gamma_c)Q_d^* - (m + 1)K - \frac{h\lambda t_e^2}{2m} - \frac{\gamma h (Q_d^*)^2}{2\lambda} + \sqrt{2\lambda K h t_e}(t_e + \frac{Q_d^*}{\lambda}),$$

and if the price discount does not take place, then the retailer’s increased profit relative to the EOQ policy is

$$F_2 = -mK - \frac{h\lambda t_e^2}{2m} + \sqrt{2\lambda K h t_e}.$$

Therefore, under the adjusted ordering policy, the retailer’s expected increased profit relative to the EOQ ordering policy is

$$E_{\pi_e} = pF_1 + (1 - p)F_2 = p[(1 - \gamma_c)Q_d^* - \frac{\gamma h (Q_d^*)^2}{2\lambda} + Q_d^*\sqrt{\frac{2\lambda K h}{\lambda} - K}] - mK - \frac{h\lambda t_e^2}{2m} + \sqrt{2\lambda K h t_e}.$$

**Policy 3.** Slightly reduce the EOQ order size before $t_s$ to make $q_e < mQ_{EOQ} - \lambda t_e$ and retain the ordering times before $t_e$ as $m$. The ordering policy is denoted by $\pi$ and it is illustrated in Figure 3.4.

For the adjusted policy, it is easy to see that the order size $Q_0$ before $t_s$ satisfies that $Q_e < Q_0 < Q_{EOQ}$.

For this adjusted policy, if the price discount takes place, then the retailer’s profit in $T_1$ is

$$f^\pi = (b - c)\lambda(t_e + \frac{q_e}{\lambda}) + (b - \gamma_c)Q_d - (m + 1)K - m\frac{hQ_0^2}{2\lambda} - (\gamma hQ_d \frac{q_e}{\lambda} + \frac{\gamma hQ_d^2}{2\lambda}).$$
While the retailer’s profit in this time horizon under the EOQ ordering policy is
\[ f^N = (b - c)\lambda(t_e + \frac{Q_d + q_e}{\lambda}) - \sqrt{2\lambda Kh}(t_e + \frac{Q_d + q_e}{\lambda}). \]

Thus, if the price discount takes place, then the retailer’s increased profit relative to the EOQ ordering policy is
\[ F_1 = \hat{F} - f^N = (1 - \gamma)cQ_d - (m + 1)K - m\frac{hQ_d^2}{2\lambda} - \frac{2hQ_d q_e}{\lambda} + \sqrt{2\lambda Kh}(t_e + \frac{2e^*Q_d}{\lambda}). \]

On the other hand, if the price discount does not take place, then the retailer’s increased profit relative to the EOQ ordering policy in interval \( T_2 \) is
\[ F_2 = -mK - m\frac{hQ_d^2}{2\lambda} + \sqrt{2\lambda Kh}(t_e + \frac{q_e}{\lambda}). \]

Therefore, for the adjusted ordering policy, the retailer’s expected increased profit relative to the EOQ ordering policy is
\[
E_{\bar{\pi}}(Q_0, Q_d) = pF_1 + (1 - p)F_2
\]
\[
= p\left[(1 - \gamma)cQ_d - (m + 1)K - m\frac{hQ_d^2}{2\lambda} - \frac{2hQ_d q_e}{\lambda} + \sqrt{2\lambda Kh}(t_e + \frac{2e^*Q_d}{\lambda})\right] - mK - m\frac{hQ_d^2}{2\lambda} + \sqrt{2\lambda Kh}(t_e + \frac{q_e}{\lambda})
\]
\[
= p\left[-\frac{\gamma hQ_d^2}{2\lambda} + ((1 - \gamma)c + \sqrt{\frac{2Kh}{\lambda} - \frac{\gamma h q_e}{\lambda}})Q_d - K\right]
\]
\[
- mK - m\frac{hQ_d^2}{2\lambda} + \sqrt{2\lambda Kh}(t_e + \frac{q_e}{\lambda}).
\]

To maximize function \( E_{\bar{\pi}} \) w.r.t. \( Q_0 \) and \( Q_d \), by the fact that
\[
\max E_{\bar{\pi}}(Q_0, Q_d) = \max Q_0 \max Q_d E_{\bar{\pi}}(Q_0, Q_d)
\]
we may first maximize the function w.r.t. \( Q_d \) and then w.r.t. \( Q_0 \).

It is easy to see that the function is concave in \( Q_d \). Thus, differentiating the function w.r.t. \( Q_d \) and setting the derivative to zero yield the function’s maximizer w.r.t. \( Q_d \),
\[
Q_d' = \frac{(1 - \gamma)c\lambda}{\gamma h} + \frac{1}{\gamma}\sqrt{\frac{2\lambda K}{h}} - q_e.
\]
Setting \( M = \frac{(1-\gamma)cM}{\delta h} + \frac{1}{\gamma} \sqrt{\frac{2K\lambda}{h}} \) and substituting \( M, Q_d^* \) and \( q_e = mQ_0 - \lambda t_e \) into equation (3.1) yield
\[
E_{\pi}(Q_0) = E_{\pi}(Q_0, Q_d^* )
\]
\[
= p \left[ - \frac{\gamma h(Q_0^2)}{2\lambda} + \frac{\gamma h}{\lambda}(Q_d^*)^2 - K \right] - mK - m \frac{hQ_0^2}{2\lambda} + \sqrt{2\lambda Kh(t_e + \frac{q_e}{\gamma})}
\]
\[
= p \frac{\gamma h(Q_0^2)}{2\lambda} - pK - mK - m \frac{hQ_0^2}{2\lambda} + mQ_0 \sqrt{2\lambda Kh}
\]
\[
= p \frac{\gamma h(M-q_e)}{2\lambda} - pK - mK - m \frac{hQ_0^2}{2\lambda} + mQ_0 \sqrt{2\lambda Kh}
\]
\[
= m \frac{hQ_0^2}{2\lambda} (mp\gamma - 1) - mQ_0 \left( p\gamma h(t_e + \frac{M}{h}) \right) - \sqrt{2\lambda Kh}
\]
\[
+ p\gamma h\lambda^2 + mp\gamma h\lambda K + p\gamma h M t_e - (p+m)K.
\]

If \( mp\gamma - 1 < 0 \), then the function is concave in \( Q_0 \) and its maximum arrives at the zero point of the function’s derivative:
\[
Q_0 = \frac{p\gamma h(Mt_e + M)}{h(mp\gamma - 1)}.
\]  \hspace{1cm} (3.3)

Considering the constraint \( Q_e \leq Q_0 \leq Q_{EQQ} \), we conclude that the maximizer of the function is \( Q_0^* = \max(Q_0, Q_e, Q_{EQQ}) \). If \( mp\gamma - 1 \geq 0 \), then \( E_{\pi}(Q_0) \) is convex in \( Q_0 \) and it reaches the maximum on the domain boundary, i.e., the function arrives its maximum at \( Q_e \) or \( Q_{EQQ} \).

4. **Solution method for the model.** In this section, we first show that for the case that the EOQ reordering point does not fall in time horizon \([t_s, t_e]\), the optimal ordering policy with a special order will be one of three heuristic ordering policies given in Section 3, and then present a closed-form solution to problem (2.1).

**Theorem 4.1.** If the replenishment point of the EOQ model does not fall in the price discount period \([t_s, t_e]\), then for any optimal strategy with a special order, the optimal order size in time horizon \([0, t_s]\) satisfies that \( Q_0^* \in \{ Q_s \} \cup [Q_e, Q_{EQQ}] \) and the optimal special order time is
\[
t_e = \begin{cases} 
t_s, & \text{if } Q_0^* = Q_s; \\
\hat{t}_e, & \text{if } Q_0^* \in [Q_e, Q_{EQQ}].
\end{cases}
\]

To show the conclusion, we need the following two conclusions.

**Lemma 4.1.** Let \( Q_1, Q_2 \) be such that \( 0 < Q_1 < Q_2 \leq Q_{EQQ} \), (or, \( Q_1 > Q_2 \geq Q_{EQQ} \)) and \( m_1, m_2 \) be two positive integers such that \( m_1 \geq m_2 \). Define the following two ordering policies on the basic EOQ inventory model:
\( \pi_1 \): make \( m_1 \) orders with size \( Q_1 \) from the beginning and revert to the EOQ ordering policy after these orders are depleted;
\( \pi_2 \): make \( m_2 \) orders with size \( Q_2 \) from the beginning and revert to the EOQ ordering policy after these orders are depleted.

Then Policy \( \pi_2 \) is superior to Policy \( \pi_1 \).

**Proof.** We only show the conclusion for the case that \( Q_1 < Q_2 \leq Q_{EQQ} \), as for the case that \( Q_1 > Q_2 \geq Q_{EQQ} \) can similarly be proved.

To show the conclusion, it suffices to show that the conclusion holds for the case that \( m = n = 1 \), as for case of \( n \geq m \geq 1 \), the conclusion follows by using the conclusion for the case that \( m = n = 1 \) and Lemma 3.1.
According to the assumption, the order size in time horizon \([0, \frac{Q_1}{\lambda}]\) is \(Q_1\) under Policy \(\pi_1\), the order size in time horizon \([0, \frac{Q_2}{\lambda}]\) is \(Q_2\) under Policy \(\pi_2\), and these two inventory systems would respectively revert to the classical EOQ model after time \(\frac{Q_1}{\lambda}\) and \(\frac{Q_2}{\lambda}\). Thus, the operating cost in inventory during time horizon \([0, \frac{Q_1}{\lambda}]\) under Policies \(\pi_1\) and \(\pi_2\) are respectively

\[
f_1 = K + \frac{Q_1^2 h}{2 \lambda} + \sqrt{2 \lambda h K} \frac{Q_2 - Q_1}{\lambda}, \quad f_2 = K + \frac{Q_2^2 h}{2 \lambda}.
\]

Then

\[
f_1 - f_2 = \frac{Q_1^2 h}{2 \lambda} + \sqrt{2 \lambda h K} \frac{Q_2 - Q_1}{\lambda} - \frac{Q_2^2 h}{2 \lambda} = \sqrt{2 \lambda h K} \frac{Q_2 - Q_1}{\lambda} + \frac{h}{2 \lambda} (Q_1^2 - Q_2^2) = \frac{h (Q_2 - Q_1)}{\lambda} \left( \frac{2 \lambda K}{h} - \frac{Q_2^2 - Q_1^2}{\lambda^2} \right).
\]

Using the fact that \(Q_1 < Q_2 \leq Q_{EOQ}^2 = \frac{2 \lambda K}{h}\), we conclude that \(f_1 > f_2\) and the desired result follows.

From Lemma 4.1, we can readily obtain the following conclusion.

**Lemma 4.2.** Let \(Q_1, Q_2\) be such that \(Q_1 > Q_2 \geq Q_{EOQ}^2\), and \(m\) be a positive integer. Define the following two ordering policies on the basic EOQ inventory model:

- **Policy \(\pi_1\):** make \(m\) orders with size \(Q_1\) from the beginning, and return to EOQ ordering policy after these orders are depleted;
- **Policy \(\pi_2\):** make \(m\) orders with size \(Q_2\) from the beginning, and return to EOQ ordering policy after these orders are depleted.

Then Policy \(\pi_2\) is superior to Policy \(\pi_1\).

**Proof of Theorem 4.1.** To show the conclusion, it suffices to show that for any optimal ordering policy, if it does not satisfy the condition, then the ordering policy can be adjusted with the condition being satisfied and an increased profit.

Let \(\pi\) be an optimal ordering policy which does not satisfy the condition. According to Theorem 3.1, all orders before \(t_e\), denoted by \(Q_{\pi}^*\), are equal. For Policy \(\pi\), denote the special order size by \(Q_{\pi}^*\), which is made at \(t_0^*\) \(\in [t_s, t_e]\).

From the discussion in Section 3, we know that \(Q_e \leq Q_{EOQ}^2 \leq Q_s\), where \(Q_e, Q_s\) are respectively defined in Policies 1 and 2. Now, we break the discussion into three cases according to the relationship of \(Q_0^*\) with \(Q_e, Q_s\) and \(Q_{EOQ}^2\).

**Case 1.** \(Q_0^* < Q_e\). Make the following adjustment to Policy \(\pi\) to obtain ordering Policy \(\pi'\): let the order size before \(t_s\) be \(Q_e\), and let the special order be made at \(t_e\) with size \(Q_{\pi'}^*\) if the discount takes place.

Now, we make a cost-benefit analysis of inventory for Policies \(\pi\) and \(\pi'\). First, since the remaining stock is zero at the special ordering time under Policy \(\pi'\), thus the special order has no idle time in the stock. As the special order sizes under Policies \(\pi\) and \(\pi'\) are equal, thus the operating cost of inventory under Policy \(\pi'\) is not larger than that under Policy \(\pi\).

Second, since \(Q_0^* < Q_{\pi'}^* = Q_e \leq Q_{EOQ}^2\), we know that the order times with size \(Q_0^*\) under Policy \(\pi\) is not less than \(Q_{\pi'}^*\) under Policy \(\pi'\). By Lemma 4.1 and using the fact that the system would revert to the EOQ ordering policy after the special order is depleted, we conclude that the operating cost in inventory under Policy \(\pi'\) is less than that under Policy \(\pi\). This contradicts the optimality of Policy \(\pi\). Hence, the case does not exist.

**Case 2.** \(Q_{EOQ}^2 < Q_0^* < Q_s\). According the assumption on the inventory model and discussion for Policy 1 in Section 2, it holds that \(q_{\pi_{EOQ}} < q_{\pi}^*\), and the remaining stock
at any time in horizon \([t_s, t_e]\) under Policy \(\pi\) is nonzero. Due to the optimality of Policy \(\pi\) and by Theorem 3.2, the special order is made at \(t_e\). Now, we make an adjustment to Policy \(\pi\) to obtain Policy \(\pi'\): the order size before \(t_s\) is \(Q^\text{EOQ}_0\), and a special order is made at \(t_e\) with size \(Q^\pi_t\).

Similarly, we make a cost-benefit analysis of inventory for Policies \(\pi\) and \(\pi'\). First, since \(q^\pi_t < q^\pi_e\) and \(Q^\pi_t = Q^\pi_\pi\), the inventory cost caused by the special order under Policy \(\pi\) is higher than that under Policy \(\pi'\). Second, since \(Q^\pi_\pi = Q^\text{EOQ} < Q^\pi_t < Q^\pi_s\), the ordering times before \(t_s\) with size \(Q^\pi_t\) under Policy \(\pi\) and that with size \(Q^\pi_\pi\) under Policy \(\pi'\) are the same according to the argument for Policy 1. Then by Lemma 4.2 and using the fact that the inventory system would revert to the EOQ under Policy \(\pi\) after the special replenishment cycle under these two ordering policies, we conclude that Policy \(\pi'\) is superior to Policy \(\pi\). The contradiction shows that the case does not exist.

**Case 3.** \(Q^\pi_\pi > Q_s\). For this, we make an adjustment to Policy \(\pi'\) to obtain Policy \(\pi'\): the order size before \(t_s\) is \(Q_s\) and a special order is made at \(t_s\) with size \(Q^\pi_s\).

Now, we make a cost-benefit analysis of inventory for these two ordering policies. First, since the remaining stock at \(t_s\) under Policy \(\pi'\) is zero, the special order has no idle time in the stock. As the special order sizes under Policies \(\pi\) and \(\pi'\) are equal, thus the operating cost for the special order under Policy \(\pi'\) is not larger than that under Policy \(\pi\). Second, since \(Q^\text{EOQ} < Q^\pi' = Q_s < Q^\pi_\pi\), the running time horizon of orders with size \(Q^\pi_\pi\) under Policy \(\pi\) is longer than that of orders with size \(Q^\pi_t\) under Policy \(\pi'\). By Lemma 3.1, we conclude that Policy \(\pi'\) is superior Policy \(\pi\). The contradiction shows that this case does not exist.

The desired result follows by combining the discussions above.

From Theorem 4.1 and considering the EOQ ordering policy without special ordering when the event takes place, we can obtain all possible optimal ordering policies for the case that the replenishment point of the EOQ model does not fall in discount period \([t_s, t_e]\):

- **Policy \(\pi_s\):** \(Q_0 = Q_s = \frac{M_s}{m-1}, t_r = t_s\) and \(Q_d = \frac{(1-\gamma)c\lambda}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\Lambda}{h}}\);
- **Policy \(\pi_e\):** \(Q_0 = Q_s = \frac{M_s}{m}, t_r = t_e\) and \(Q_d = \frac{(1-\gamma)c\lambda}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\Lambda}{h}}\);
- **Policy \(\pi\):** \(Q_0 = \tilde{Q}_0 = \frac{\gamma h (M_s + M) - \sqrt{2MK\Lambda}}{h(m \gamma^2 - 1)}, t_r = t_e\) and \(Q_d = \frac{(1-\gamma)c\lambda}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\Lambda}{h}} - (mQ_0 - \lambda t_e)\);
- **Policy \(\pi_{\text{EOQ}}\):** \(Q_0 = Q^\text{EOQ}, t_r = t_e\) and \(Q_d = \frac{(1-\gamma)c\lambda}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\Lambda}{h}} - (mQ^\text{EOQ} - \lambda t_e)\);
- **Policy EOQ:** \(Q_0 = Q^\text{EOQ},\) make no special order when discount takes place.

Considering the ordering policy discussed in Section 3 for the scenario that the replenishment point of the EOQ model falls in the discount period \([t_s, t_e]\), we can present the following algorithm for problem (2.1).

**Algorithm 4.1.**

**Step 1.** Input value of parameters \(\lambda, K, b, c, p, \gamma, h, t_s, t_e\).

**Step 2.** Compute \([\frac{M_s}{m-1}], [\frac{M_s}{m}]\) and \(m = \lceil \frac{M_s}{Q^\text{EOQ}} \rceil\). If \(\lceil \frac{M_s}{Q^\text{EOQ}} \rceil < \lceil \frac{M_s}{Q^\text{EOQ}} \rceil\), then set \(Q^\pi_\pi = Q^\text{EOQ}\), make a special order at time \(t^*_s = mQ^\text{EOQ}_{\lambda}\) with size \(Q^\pi_{t^*_s} = \frac{(1-\gamma)c\lambda}{\gamma h} + \frac{1}{\gamma} \sqrt{\frac{2K\Lambda}{h}}\) if the price discount takes place, and make an EOQ order at \(t^*_s\) otherwise, terminate. Otherwise, go to Step 3.
Step 3. Compute the order sizes before $t_s$ under different ordering policies

$$Q_s^* = \begin{cases} \frac{M_s}{m - \gamma}, & \text{for Policy } \pi_s; \\ \frac{M_e}{m}, & \text{for Policy } \pi_e; \\ \sqrt{\frac{2\Delta K}{h}}, & \text{for Policy } \text{EOQ and } \pi_{\text{EOQ}}; \\ \frac{p GH(2M - M_e - \Delta K)}{h(\gamma - 1)}, & \text{for Policy } \bar{\pi}. \end{cases}$$

If $m p \gamma - 1 < 0$ and $Q_e < Q_0 < Q_{\text{EOQ}}$, then the candidate optimal policy set is $\{\pi_s, \bar{\pi}, \text{EOQ}\}$, and goto Step 4; otherwise, the candidate optimal policy set is $\{\pi_s, \pi_e, \pi_{\text{EOQ}}, \text{EOQ}\}$, goto Step 7.

Step 4. Compute the optimal special order size $Q_s^*$ under Policies $\pi_s$ and $\bar{\pi}$ by

$$Q_s^* = \frac{(1 - \gamma) c \lambda}{\gamma} + \frac{1}{\gamma} \sqrt{\frac{2K \lambda}{h}} - q_r,$$

where

$$q_r = \begin{cases} 0, & \text{for Policies } \pi_s; \\ mQ_0 - \lambda e, & \text{for Policy } \bar{\pi}. \end{cases}$$

Step 5. Compute the expected increased profit $E_{\pi_s}$ and $E_{\bar{\pi}}$ relative to the EOQ ordering policy.

Step 6. If $\max\{E_{\pi_s}, E_{\bar{\pi}}\} \leq 0$, then choose the EOQ policy, i.e., $Q_s^* = Q_{\text{EOQ}}$ and make no special order when discount takes place; otherwise, take the policy with the largest expected increased profit from ordering policy set $\{\pi_s, \bar{\pi}\}$, terminate.

Step 7. Compute special order size $Q_s^*$ under Policies $\pi_s$, $\pi_e$ and $\pi_{\text{EOQ}}$ by

$$Q_s^* = \frac{(1 - \gamma) c \lambda}{\gamma} + \frac{1}{\gamma} \sqrt{\frac{2K \lambda}{h}} - q_r,$$

where

$$q_r = \begin{cases} 0, & \text{for Policies } \pi_s \text{ and } \pi_e; \\ mQ_{\text{EOQ}} - \lambda e, & \text{for Policy } \pi_{\text{EOQ}}. \end{cases}$$

Step 8. Compute the expected increased profit $E_{\pi_s}$, $E_{\pi_e}$ and $E_{\text{EOQ}}$ relative to the EOQ policy given in Section 3.

Step 9. If $\max\{E_{\pi_s}, E_{\pi_e}, E_{\text{EOQ}}\} \leq 0$, then choose the EOQ policy, that is, take $Q_s^* = Q_{\text{EOQ}}$ and make no special order when discount takes place; otherwise, choose the one with the largest expected increased profit from ordering policy set $\{\pi_s, \pi_e, \pi_{\text{EOQ}}\}$, and terminate.

5. Numerical experiments and sensitivity analysis. In this section, we will make some numerical experiments for the proposed solution method to provide a high insight to the inventory model.

Example 5.1. Consider the inventory system with the following parameters: $\lambda = 5, K = 169, b = 8, c = 5, \gamma = 0.9, t_s = 60, \tau = 0.02, h = 0.1, p = 0.3, t_e = 70$.

For this inventory system, since $\lfloor \frac{M_s}{m - \gamma} \rfloor = \lfloor \frac{M_e}{m} \rfloor$, the EOQ ordering point does not belong to time horizon $[t_s, t_e]$. By Algorithm 4.1, the candidate optimal policy set is $\{\pi_s, \pi_e, \pi_{\text{EOQ}}, \text{EOQ}\}$, and each policy's expected increased profit relative to the EOQ order policy is presented in Table 5.1 from which we can see that the optimal ordering policy is $\pi_e$. Under this ordering policy, the retailer's expected increased profit relative to the EOQ ordering policy is 24.05.
Table 5.1: Numerical result for Example 5.1

| Policy | $Q_0$ | $q_r$ | $Q_d$  | $E$  |
|--------|-------|-------|--------|------|
| $\pi_{EOQ}$ | 130   | 40    | 132.22 | -3.50 |
| $\pi_s$     | 150   | 0     | 172.22 | 21.38 |
| $\pi_e$     | 116.67| 0     | 172.22 | 24.05 |

**Example 5.2.** For the inventory system considered in Example 5.1, set $p = 0.05$ and other parameters remain unchanged.

For this inventory system, by Algorithm 4.1, the candidate optimal policy set is $\{\pi_s, \bar{\pi}, \text{EOQ}\}$ and the expected increased profits relative to the EOQ ordering policy are presented in Table 5.2 which shows that the optimal ordering policy is $\bar{\pi}$, and under this ordering policy, the retailer’s expected increased profit relative to the EOQ ordering policy is 2.54.

Table 5.2: Numerical result for Example 5.2

| Policy | $Q_0$ | $q_r$ | $Q_d$  | $E$  |
|--------|-------|-------|--------|------|
| $\pi_s$     | 150   | 0     | 172.22 | 0.25 |
| $\bar{\pi}$ | 117.86| 3.58  | 168.64 | 2.54 |

From the numerical results for Examples 5.1 and 5.2, we can see that the discount probability $p$ significantly affects the retailer’s ordering policy. To better understand the effect of discount probability $p$ on the ordering policy, we conduct a sensitivity analysis of the model by varying parameter $p$ while keeping the other parameters fixed. The numerical results are presented in Table 5.3 and Figure 5.1.

Table 5.3: Impact of parameter $p$ on the retailer’s profit

| $p$  | $\pi_{EOQ}$ | $\pi_s$ | $\pi_e$ | $\bar{\pi}$ | $\text{EOQ}$ | ordering policy |
|------|-------------|---------|---------|-------------|---------------|----------------|
| 0.01 | -0.11       | -7.02   | -4.35   | -0.07       | 0             | $\text{EOQ}$   |
| 0.05 | -0.58       | -3.10   | -0.43   | 0.64        | 0             | $\bar{\pi}$    |
| 0.10 | -1.16       | 1.79    | 4.46    | /           | 0             | $\pi_e$        |
| 0.15 | -1.74       | 6.69    | 9.36    | /           | 0             | $\pi_e$        |
| 0.30 | -3.50       | 21.38   | 24.05   | /           | 0             | $\pi_e$        |
| 0.50 | -5.83       | 40.97   | 43.64   | /           | 0             | $\pi_e$        |
| 0.80 | -9.32       | 70.36   | 73.02   | /           | 0             | $\pi_e$        |
| 0.90 | -10.49      | 80.15   | 82.82   | /           | 0             | $\pi_e$        |
| 0.95 | -11.07      | 85.05   | 87.71   | /           | 0             | $\pi_e$        |

Figure 5.1. The expected increased profit as a function of parameter $p$
From Table 5.3 and Figure 5.1, we can see that policy \( \bar{\pi} \) is only relevant for a small range of discount probability \( p \). In detail, when \( p \) grows from zero to 0.085, the order size \( \bar{Q}_0 \) shifts from \( Q^{\text{EOQ}} \) to \( Q_e \), and it is no longer relevant thereafter. For the order policy, when \( p \) grows from 0 to a certain value in \((0, 1)\), the retailer's optimal policy becomes \( \bar{\pi} \) from the EOQ policy, and if \( p \) further grows to another certain value in \((0, 1)\), then the optimal policy becomes \( \pi_e \). This indicates that when \( p \) is relatively small, the retailer can gain more benefits by slightly adjusting the EOQ model. However, when \( p \) is relatively large, the retailer can gain more benefits by making a major adjustment to the EOQ model.

**Example 5.3.** For the inventory considered in Example 5.1, set \( \gamma = 0.98 \) and other parameters remain unchanged.

For this system, by Algorithm 4.1, the candidate optimal policy set is \( \{ \pi_s, \pi_e, \pi_{\text{EOQ}}, \text{EOQ} \} \) and its expected increased profit relative to the EOQ ordering policy are presented in Table 5.4 which indicates that the optimal ordering policy is EOQ.

| Policy | \( Q_0 \) | \( q_r \) | \( Q_d \) | \( E \) |
|--------|--------|--------|--------|------|
| \( \pi_{\text{EOQ}} \) | 130 | 40 | 97.76 | -22.60 |
| \( \pi_s \) | 150 | 0 | 137.76 | -2.91 |
| \( \pi_e \) | 116.67 | 0 | 137.76 | -0.24 |

Table 5.4: Numerical results for Example 5.3

To show the effect of the discount rate \( \gamma \) on the optimal ordering policy, we conduct a sensitivity analysis by varying discount rate \( \gamma \) while keeping the other parameters fixed on the example. The numerical results are presented in Table 5.5 and Figure 5.2 which show that as \( \gamma \) grows from a certain value in \((0, 1)\), the retailer’s expected increased profit under the candidate optimal policies decreases, and when the discount rate is close to 1, the retailer’s optimal policy becomes the EOQ policy.

| \( \gamma \) | \( \pi_{\text{EOQ}} \) | \( \pi_s \) | \( \pi_e \) | \( \bar{\pi} \) | \( \text{EOQ} \) |
|--------|--------|--------|--------|------|-----|
| 0.5    | 280.65 | 331.45 | 334.11 | /     | 0   |
| 0.6    | 161.48 | 205.80 | 208.47 | /     | 0   |
| 0.7    | 83.57  | 121.41 | 124.07 | /     | 0   |
| 0.8    | 31.44  | 62.80  | 65.47  | /     | 0   |
| 0.9    | -3.50  | 21.38  | 24.05  | /     | 0   |
| 0.95   | -16.21 | 5.43   | 8.09   | /     | 0   |
| 0.98   | -22.61 | -2.91  | -0.24  | /     | EOQ |
| 0.99   | -24.55 | -5.50  | -2.83  | /     | EOQ |

**Example 5.5.** For the inventory considered in Example 5.1, set \( \gamma = 0.98 \) and other parameters remain unchanged.

6. **Conclusion and extensions.** For the inventory mechanism with a statistic short-term discount, we established an optimal replenishment and stocking strategy model and propose a solution method for the model. The given numerical experiments provide a high insight to the inventory model.

In the model, shortage and backorder are not allowed. To make the model more practical, we can extend it to the case that shortage and backorder are allowed. This will be considered in the future research.
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