HFLAV $\tau$ branching fractions fit and measurements of $|V_{us}|$ with $\tau$ lepton data

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Abstract

We report the status of the Heavy Flavour Averaging Group (HFLAV) averages of the $\tau$ lepton measurements. We then update the latest published HFLAV global fit of the $\tau$ lepton branching fractions (Spring 2017) with recent results by BaBar. We use the fit results to update the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{us}|$ measurements with the $\tau$ branching fractions. We combine the direct $\tau$ branching fraction measurements with indirect predictions using kaon branching fractions measurements to improve the determination of $|V_{us}|$ using $\tau$ branching fractions. The $|V_{us}|$ determinations based on the inclusive branching fraction of $\tau$ to strange final states are about $3\sigma$ lower than the $|V_{us}|$ determination from the CKM matrix unitarity.

Contents

1 Introduction 2
2 New $\tau$ branching fraction measurements 2
3 $|V_{us}|$ determination including the 2018 BaBar results 2
4 $\tau$ branching fraction predictions from kaon measurements 3
5 Consistency of $|V_{us}|$ with the CKM matrix unitarity 3
6 Conclusions 4
References 4
1 Introduction

The $\tau$ subgroup of the Heavy Flavour Averaging Group (HFLAV) provides a global fit of the $\tau$ branching fractions, the lepton universality tests and the $|V_{us}|$ determination based on $\tau$ measurements. The latest published report for the $\tau$ lepton is labelled “Spring 2017” [1]. A version of the HFLAV $\tau$ branching fractions fit with unitarity constraint is published on the Review of Particle Physics [2] (RPP). There are additional minor differences between the two fits [1,3]. The fit results are used to test lepton universality and to compute $|V_{us}|$ [1].

The HFLAV-Tau group collects and combines also a list of upper limits set by searches of lepton-flavour-violating $\tau$ decays [1].

In the following, we update the HFLAV-Tau global fit input with two $BA\bar{B}AR$ measurements that became public in 2018 [4,5] and we update the $|V_{us}|$ determinations based on $\tau$ data. The new results have a negligible effect on the lepton universality tests.

Finally, we add to the fit input measurements of three $\tau$ branching fractions that are indirectly determined using measurements of kaon branching fractions [6], in order to improve the precision on $|V_{us}|$.

2 New $\tau$ branching fraction measurements

Since the last HFLAV report, $BA\bar{B}AR$ published [4] a measurement of

$$B(\tau^- \to K^- K^0 \nu_\tau) = (14.78 \pm 0.22 \pm 0.40) \cdot 10^{-4}$$

and presented [5] preliminary measurements of

$$B(\tau^- \to K^- \nu_\tau) = (7.174 \pm 0.033 \pm 0.213) \cdot 10^{-3},$$
$$B(\tau^- \to K^- \pi^0 \nu_\tau) = (5.054 \pm 0.021 \pm 0.148) \cdot 10^{-3},$$
$$B(\tau^- \to K^- 2\pi^0 \nu_\tau \text{ (ex. } K^0) = (6.151 \pm 0.117 \pm 0.338) \cdot 10^{-4},$$
$$B(\tau^- \to K^- 3\pi^0 \nu_\tau \text{ (ex. } K^0, \eta) = (1.246 \pm 0.164 \pm 0.238) \cdot 10^{-4},$$
$$B(\tau^- \to \pi^- 3\pi^0 \nu_\tau \text{ (ex. } K^0, \eta) = (1.168 \pm 0.006 \pm 0.038) \cdot 10^{-2},$$
$$B(\tau^- \to \pi^- 4\pi^0 \nu_\tau \text{ (ex. } K^0, \eta) = (9.020 \pm 0.400 \pm 0.652) \cdot 10^{-4}.$$ .

3 $|V_{us}|$ determination including the 2018 $BA\bar{B}AR$ results

We add the measurements listed in the previous section to the HFLAV-Tau global fit, removing a former $BA\bar{B}AR$ measurement of $B(\tau^- \to K^- \pi^0 \nu_\tau)$ [7] that has been superseded [5]. The new measurements of the branching fractions $\tau$ decaying to a kaon and 0, 1, 2, 3 $\pi^0$’s improve the experimental resolution on several modes that most contribute to the uncertainty on $|V_{us}|$.

We compute $|V_{us}|_{\tau_s}$ using the total branching fraction of the $\tau$ to strange final states following Ref. [8]:

$$|V_{us}|_{\tau_s} = \sqrt{R_s/\left[ \frac{R_{VA}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]} = 0.2195 \pm 0.0019,$$
where $|V_{ud}| = 0.97420 \pm 0.00021$ [9], $R_s$ and $R_{VA}$ are the $\tau$ hadronic partial widths to strange and to non-strange hadronic final states ($\Gamma_s$ and $\Gamma_{\text{had}}$) divided by the universality-improved branching fraction $B(\tau \rightarrow e\nu\bar{\nu}) = B_{e}^{\text{uni}} = (17.814 \pm 0.022)\%$ [13], and the SU(3)-breaking term $\delta R_{\text{theory}} = 0.242 \pm 0.033$ is computed using inputs from Ref. [8] and $m_s = (95.00 \pm 6.70)\text{ MeV}$ [2] (the uncertainties on $m_s$ have been symmetrized).

We compute also

$$|V_{us}|_{\tau K/\pi} = |V_{ud}| \frac{f_{\pi \pm}^2 - m_{\pi}^2}{f_{K \pm}^2 - m_{K}^2} \sqrt{\frac{B(\tau^{-} \rightarrow K^{-}\nu_{\tau}) R_{\tau K/\pi}}{B(\tau^{-} \rightarrow \pi^{-}\nu_{\tau}) R_{\tau K/\pi} R_{\tau K/\tau\pi}}} = 0.2236 \pm 0.0016\, ,$$

where $f_{K \pm}/f_{\pi \pm} = 1.193 \pm 0.003$ from the FLAG 2016 Lattice averages with $N_f = 2 + 1 + 1$ [10,13] (the same value persists in the FLAG 2017 web update). The radiative correction terms are $R_{\tau K} = 1 + (0.90 \pm 0.22)\%$, $R_{\tau/\pi} = 1 + (0.16 \pm 0.14)\%$ [14,17], $R_{\tau K/\tau\pi} = 1 + (-0.69 \pm 0.17)\%$ [18,20]. The third value differs from the one quoted in the Spring 2017 HFLAV-Tau report [1], which incorrectly included a strong isospin-breaking correction that is not needed when using $f_{K \pm}/f_{\pi \pm}$ rather than its isospin-limit variant. The other parameters are taken from the Review of Particle Physics (RPP) 2018 [2].

Averaging the two above $|V_{us}|$ determinations, we obtain $|V_{us}|_{\tau} = 0.2220 \pm 0.0014$.

### 4 $\tau$ branching fraction predictions from kaon measurements

Assuming the validity of the Standard Model (SM), three $\tau$ branching fractions have been computed using the precisely measured $K_{\ell 2}$ and $K_{\ell 3}$ branching fractions and the measured $\tau^{-} \rightarrow (K\pi)^{-}\nu_{\tau}$ spectra [6]:

$$B(\tau^{-} \rightarrow K^{-}\nu_{\tau}) = (0.713 \pm 0.003)\%\, ,$$
$$B(\tau^{-} \rightarrow K^{-}\pi^{0}\nu_{\tau}) = (0.471 \pm 0.018)\%\, ,$$
$$B(\tau^{-} \rightarrow K^{0}\pi^{-}\nu_{\tau}) = (0.857 \pm 0.030)\%\, .$$

The uncertainties on the last two results are fully correlated. It has been observed [6,18] that all the above indirect values are higher than the corresponding directly measured $\tau$ branching fractions. If the indirect values replace the direct ones, $|V_{us}| = 0.2207 \pm 0.027$ [6].

We add the kaon-indirect determinations of the three above $\tau$ branching fractions to the data set used in the previous section in order to obtain improved calculations of $|V_{us}|_{\tau s} = 0.2202 \pm 0.0018$, $|V_{us}|_{\tau K/\pi} = 0.22546 \pm 0.00097$ and their average $|V_{us}|_{\tau} = 0.22439 \pm 0.00088$.

### 5 Consistency of $|V_{us}|$ with the CKM matrix unitarity

Assuming the CKM matrix unitarity,

$$|V_{us}|_{\text{uni}} = \sqrt{1 - |V_{ud}|^2 - |V_{ub}|^2} = 0.22565 \pm 0.00089\, ,$$

using $|V_{ud}| = 0.97420 \pm 0.00021$ [9] and $|V_{ub}| = (0.3940 \pm 0.0360) \cdot 10^{-2}$ [2]. Table 1 summarizes the residuals, expressed as numbers of standard deviations, of the above mentioned $|V_{us}|$ determinations with respect to the $|V_{us}|_{\text{uni}}$ computation from the CKM matrix unitarity. $|V_{us}|_{\text{uni}}$ computed with the $\tau$-inclusive method is significantly lower, but the significance of the discrepancy is mildly reduced alongside a mild progress in the experimental resolution.
Table 1: Deviations of $|V_{us}|$ computed with $\tau$ data with respect to $|V_{us}|$ obtained with CKM unitarity. The second and third row use the $|V_{us}|$ determinations performed in this paper.

|                          | $\Delta |V_{us}|_{\tau s} / \sigma$ | $\Delta |V_{us}|_{\tau K/\pi} / \sigma$ | $\Delta |V_{us}| / \sigma$ |
|--------------------------|----------------------------------|-------------------------------------|----------------------|
| HFLAV Spring 2017        | −3.0                             | −1.0                                | −2.3                 |
| HFLAV + BaBar 2018       | −2.9                             | −1.1                                | −2.3                 |
| HFLAV + BaBar 2018 + kaon predictions | −2.7                             | −0.1                                | −0.9                 |

6 Conclusions

Figure 1 reports the $|V_{us}|_{\tau s}$ determinations described above, a determination of $|V_{us}|_{\tau s}$ obtained replacing some $\tau$ branching fractions measurements with the indirect predictions based on kaon branching fractions [6], and other more complex determinations that use the $\tau$ spectral functions [21] and Lattice QCD techniques [22]. Updates on the last two determinations have been presented at the Tau 2018 workshop [23]. The last four determinations use an older and in some cases partial set of experimental $\tau$ branching fractions measurements.

The $\tau$ based $|V_{us}|$ determinations use the $|V_{ud}|$ measurements as input. The dependence on $|V_{ud}|$ is however very small, and there is just a small correlation between $|V_{us}|$ and $|V_{ud}|$ when doing a simultaneous fit. Figure 2 shows the results of a $|V_{ud}|$-$|V_{us}|$ simultaneous fit on the $\tau$ measurements corresponding to the HFLAV Spring 2017 fit and the BaBar 2018 results. The fit results are:

$$|V_{ud}| = 0.97420 \pm 0.00021$$
$$|V_{us}| = 0.2223 \pm 0.0014$$
$$|V_{ud}|$-$|V_{us}|$ correlation = 0.035

Tables 2 and 3 report the contributions to the $|V_{us}|_{\tau s}$ uncertainty before and after the BaBar 2018 results. The largest contributions come from the $\tau$ branching fractions to strange final states and from the theory. The BaBar 2018 measurements reduced significantly several large contributions. High multiplicity $\tau$ decays to strange final states dominate the $|V_{us}|_{\tau s}$ uncertainty. The Belle II super flavour factory will offer the opportunity to improve the experimental precision on the $\tau$ strange branching fractions. More precise $\tau$ branching fractions and spectral function measurements will help improving also the theory uncertainty.

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Figure 1: $|V_{us}|_{\tau s}$ determinations obtained in this document, from the top: $|V_{us}|_{\text{uni}}$, $|V_{us}|_{\tau s}$ with the HFLAV Spring 2017 fit, after adding the $\text{BABAR}$ 2018 data, after adding both the $\text{BABAR}$ 2018 and the kaon indirect predictions, from Ref. [6], from Ref. [21], and two determinations from Ref. [22].
Figure 2: Results of a $|V_{ud}|$-$|V_{us}|$ simultaneous fit. The bands describe the constraints corresponding to the $|V_{ud}|$ measurement, the $|V_{us}|_{\tau s}$ and the $|V_{us}|_{\tau K/\pi}$ determinations that use the $\tau$ measurements. The oblique line corresponds to the CKM matrix unitarity constraint. The ellipse corresponds to $1\sigma$ uncertainty on the $|V_{ud}|$ and $|V_{us}|$ fit results.

Table 2: Contributions to the $|V_{us}|_{\tau s}$ uncertainty in percent before the $BaBar$ 2018 results.

| Contribution | Uncertainty |
|--------------|-------------|
| $\pi^- K^0 \pi^0 \nu_\tau$ (ex. $K^0$) | 0.3963 |
| $K^- 2\pi^0 \nu_\tau$ (ex. $K^0$) | 0.3789 |
| $K^- 3\pi^0 \nu_\tau$ (ex. $K^0, \eta$) | 0.3714 |
| $K^0 h^- h^+ \nu_\tau$ | 0.3478 |
| $K^- \pi^0 \nu_\tau$ | 0.2561 |
| $K^- \pi^- \pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$) | 0.2456 |
| $\pi^- K^0 \nu_\tau$ | 0.2424 |
| $\pi^- K^0 \pi^0 \nu_\tau$ | 0.2219 |
| $K^- \nu_\tau$ | 0.1646 |
| $K^- \omega \nu_\tau$ | 0.1585 |
| $K^- \pi^- \pi^0 \nu_\tau$ (ex. $K^0, \omega$) | 0.1157 |
| $\pi^- K^0 \eta \nu_\tau$ | 0.0256 |
| $K^- \pi^0 \eta \nu_\tau$ | 0.0200 |
| $K^- \eta \nu_\tau$ | 0.0138 |
| $K^- \phi \nu_\tau$ ($\phi \to K^+ K^-$) | 0.0138 |
| $K^- \phi \nu_\tau$ ($\phi \to K^0 K^0$) | 0.0096 |
| $K^- 2\pi^- 2\pi^0 \nu_\tau$ (ex. $K^0$) | 0.0021 |
| $K^- 2\pi^- 2\pi^0 \pi^0 \nu_\tau$ (ex. $K^0$) | 0.0010 |
| $\tau \to$ non-strange | 0.0896 |
| $B_s^{univ}$ | 0.0045 |
| theory | 0.4861 |
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Table 3: Contributions to the $|V_{us}|_{\tau s}$ uncertainty in percent after the BABAR 2018 results.

| Contribution                                         | Contribution (ex. $K^0$) | 0.3931 |
|-------------------------------------------------------|---------------------------|--------|
| $\pi^- K^0 \pi^0 \nu_\tau$                           |                           |        |
| $K^0 h^- h^+ \nu_\tau$                                |                           | 0.3450 |
| $K^- \pi^- \pi^0 \nu_\tau$ (ex. $K^0, \omega, \eta$) |                           | 0.2436 |
| $\pi^- K^0 \nu_\tau$                                  |                           | 0.2372 |
| $\pi^- K^0 \pi^0 \nu_\tau$                           |                           | 0.2200 |
| $K^- \omega \nu_\tau$                                 |                           | 0.1572 |
| $K^- \pi^0 \nu_\tau$                                  |                           | 0.1554 |
| $K^- \nu_\tau$                                        |                           | 0.1459 |
| $K^- \pi^- \pi^+ \nu_\tau$ (ex. $K^0, \omega$)        |                           | 0.1147 |
| $K^- 2\pi^0 \nu_\tau$ (ex. $K^0$)                     |                           | 0.0460 |
| $K^- 3\pi^0 \nu_\tau$ (ex. $K^0, \eta$)              |                           | 0.0449 |
| $\pi^- K^0 \eta \nu_\tau$                            |                           | 0.0254 |
| $K^- \pi^0 \eta \nu_\tau$                            |                           | 0.0198 |
| $K^- \eta \nu_\tau$                                   |                           | 0.0137 |
| $K^- \phi \nu_\tau (\phi \rightarrow K^+ K^-)$       |                           | 0.0136 |
| $K^- \phi \nu_\tau (\phi \rightarrow K^0 K^0_{\ell})$|                           | 0.0095 |
| $K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. $K^0$)              |                           | 0.0021 |
| $K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. $K^0$)       |                           | 0.0010 |
| $\tau \rightarrow$ non-strange                        |                           | 0.0855 |
| $K^0_{\text{univ}}$                                    |                           | 0.0045 |
| theory                                                |                           | 0.4863 |