Dynamic Pressure Distribution due to Horizontal Acceleration in Spherical LNG Tank with Cylindrical Central Part

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Abstract. Spherical LNG tanks having many advantages such as structural safety are used as a cargo containment system of LNG carriers. However, it is practically difficult to fabricate perfectly spherical tanks of different sizes in the yard. The most effective method of manufacturing LNG tanks of various capacities is to insert a cylindrical part at the center of existing spherical tanks. While a simplified high-precision analysis method for the initial design of the spherical tanks has been developed for both static and dynamic loads, in the case of spherical tanks with a cylindrical central part, the analysis method available only considers static loads. The purpose of the present study is to derive the dynamic pressure distribution due to horizontal acceleration, which is essential for developing an analysis method that considers dynamic loads as well.

1. Introduction
Because of the many advantages of spherical LNG tanks, especially when it comes to structural safety, these tanks are used as the cargo containment system in LNG carriers, even though their material cost is higher than that of membrane-type LNG tanks. To satisfy the demands of ship owners, the ship-builder must be able to design and construct spherical LNG tanks of various sizes. However, it is practically difficult to fabricate perfectly spherical tanks of different sizes in the yard because the number of fabrication shops required for making spherical LNG tanks is proportional to the radius of the tanks to be constructed. Thus, an efficient method of increasing the cargo capacity is to extend conventional spherical LNG tanks by inserting a cylindrical shell structure within them.

There have been several studies on spherical tanks, including studies on their initial design using finite element analysis [1], initial design using the simplified analysis method [2], fatigue strength and crack propagation behaviour [3, 4], and sloshing characteristics [5]. In a study on spherical tanks with a cylindrical central part, Shin and Ko developed and verified a simplified analysis method that considers static loads such as the overpressure, tank weight, and liquid cargo weight [6].

Because a spherical tank is a rotating body not only on the vertical axis but also on the horizontal axis, the pressure distribution due to horizontal acceleration is also axially symmetric. In contrast, a spherical tank with a cylindrical central part is a rotating body on the vertical axis but not on the horizontal axis. Further, no guidelines or classification rules exist for such tanks. Hence, in the present study, we determined the mathematical expressions for the pressure distribution caused by horizontal acceleration, as these are essential for the design and structural analysis of LNG carriers with a spherical cargo tank with a cylindrical central part.
2. Pressure distribution due to horizontal acceleration

The equations for the pressure distribution are determined for the case where the tank is subjected to horizontal acceleration and gravitational acceleration simultaneously. The pressures corresponding to horizontal acceleration alone were then obtained by subtracting gravitational acceleration from the calculated pressures.

Figure 1 show the direction of the resultant acceleration when the horizontal acceleration and the gravitational acceleration act simultaneously. Here, $a_H$ is the dimensionless horizontal acceleration obtained by dividing the horizontal acceleration by the gravitational acceleration, $g$, and $a_R$ is the dimensionless resultant acceleration obtained by dividing the resultant acceleration by $g$. The dimensionless resultant acceleration acts in the axial direction, which is $\beta^\circ$ off the vertical axis, as can be seen from the figure. This yields equations (1)-(3).

\[
\cos \beta = \frac{1}{a_R}, \quad \sin \beta = \frac{a_H}{a_R}, \quad a_R = \sqrt{a_H^2 + 1}
\]  

The slope of the tangent at point $T$ is $\tan \beta$, and the value of $c$ (the length of $SO$) is $R \sec \beta + D/2$. Thus, the value of $a$ (the length of $SU$) can be expressed as shown in equation (4).

\[
a = R(\sec \beta - 1) = R\left(\sqrt{a_H^2 + 1} - 1\right)
\]  

![Figure 1. Resultant acceleration.](image)

![Figure 2. Water head in each part.](image)
2.1. Pressure distribution in upper hemisphere

In order to calculate the water head in the upper hemisphere, it is convenient to introduce the distance, \( R \), from the central point of the cross-section to a point on the tank boundary, as shown in figure 2(a). \( R \) is a function of \( \mu \) based on point \( O \).

The water head at locations \( U \), \( U \), and \( U \) can be obtained from equations (5)-(7), respectively.

\[
\begin{align*}
    h_{U, \theta = 0} &= \left( R + \frac{D}{2} + a \right) \cos \beta - R(\mu) \cos(\mu - \beta) \\
    h_{U, \theta = \pi / 2} &= \left( R + \frac{D}{2} + a \right) \cos \beta - R(\mu) \cos(\mu + \beta) \\
    h_{U, \theta = \pi} &= \left( R + \frac{D}{2} + a \right) \cos \beta - R(\mu) \cos(\mu - \beta)
\end{align*}
\]

By substituting equations (1) and (2) in equations (5)-(7), we get equations (8)-(10).

\[
\begin{align*}
    h_{U, \theta = 0} &= \frac{1}{aR} (R + a - R \cos \varphi - aH \sin \varphi) \\
    h_{U, \theta = \pi / 2} &= \frac{1}{aR} (R + a - R \cos \varphi) \\
    h_{U, \theta = \pi} &= \frac{1}{aR} (R + a - R \cos \varphi + aH \sin \varphi)
\end{align*}
\]

Referring to equations (8)-(10), the water head in the upper hemisphere can be expressed through a single equation, which is given by equation (11), for the range \( 0 \leq \theta \leq \pi \).

\[
h_{U} = \frac{1}{aR} (R + a - R \cos \varphi - aH \sin \varphi \cos \theta)
\]

Since the resultant acceleration is dimensionless, a pressure due to the resultant acceleration can be calculated by multiplying \( \rho g a \) and the water head at the location in question. Therefore, the pressure due to the resultant acceleration in the upper hemisphere can be expressed as follows:

\[
P_U = \rho g (R + a - R \cos \varphi - aH \sin \varphi \cos \theta)
\]

By subtracting gravitational acceleration \( \rho g R (1 - \cos \varphi) \) from equation (12), the pressure corresponding to horizontal acceleration alone is as follows:

\[
P_{HU} = \rho g (a - aH \sin \varphi \cos \theta)
\]

2.2. Pressure distribution in cylindrical part

Figure 2(b) shows the points and line segments that are used for determining the water head in the cylindrical central part. The length of \( J_1C_1, SC_2 \) and \( J_2C_3 \) is \( R + a + D \), while the length of \( J_2E_2 \) and \( J_2E_2 \) is \( R \tan \beta \). That is, the length of \( E_1C_1 \) is \( R + a + D - R \tan \beta \), and the length of \( E_2C_3 \) is \( R + a + D + R \tan \beta \).

The water head at locations \( C_1, C_2, \) and \( C_3 \) can be obtained from equations (14)-(16), respectively.

\[
\begin{align*}
    h_{C, \theta = 0} &= (R + a + z - R \tan \beta) \cos \beta \\
    h_{C, \theta = \pi / 2} &= (R + a + z) \cos \beta \\
    h_{C, \theta = \pi} &= (R + a + z + R \tan \beta) \cos \beta
\end{align*}
\]

By substituting equations (1) and (2) in equations (14)-(16), we get equations (17)-(19), respectively.

\[
\begin{align*}
    h_{C, \theta = 0} &= \frac{1}{aR} (R + a + z - aH R) \\
    h_{C, \theta = \pi / 2} &= \frac{1}{aR} (R + a + z) \\
    h_{C, \theta = \pi} &= \frac{1}{aR} (R + a + z + aH R)
\end{align*}
\]
With reference to equations (17)-(19), the water head in the cylindrical part can be expressed using a single equation, which is given by equation (20), for the range $0 \leq \theta \leq \pi$.

$$h_C = \frac{1}{a_R} (R + a + z - a_H R \cos \theta)$$ (20)

If equation (20) is multiplied by $\rho g a_R$, it yields the pressure due to the resultant acceleration in the cylindrical part:

$$P_C = \rho g (R + a + z - a_H R \cos \theta)$$ (21)

By subtracting gravitational acceleration $\rho g (R + z)$ from equation (21), the pressure corresponding to horizontal acceleration alone is as follows:

$$P_{HC} = \rho g (a - a_H R \cos \theta)$$ (22)

### 2.3. Pressure distribution in lower hemisphere

Next, let us determine the water head at locations $L_1$, $L_2$, and $L_3$ in figure 2(c).

The procedure for determining the water head in the lower hemisphere is nearly the same as that in the case of the upper hemisphere. On the basis of a comparison of figures 2(a) and 2(c), $R (\mu) \cos (\mu - \beta)$ is subtracted from the expression for the water head at a location where $\theta$ is zero, whereas $R f (\mu) \cos (\mu + \beta)$ is added in the case of the lower hemisphere. Furthermore, $R f (\mu) \mu \cos \beta$ is subtracted from the expression for the water head in the upper hemisphere at a location where $\theta$ is $\pi/2$ whereas the value is added in the case of the lower hemisphere. Finally, $R f (\mu) \mu \cos (\mu - \beta)$ is subtracted from the expression for the water head at a location where $\theta$ is $\pi$ in the upper hemisphere whereas $R f (\mu) \mu \cos (\mu + \beta)$ is added in the case of the lower hemisphere. Therefore, water head at locations $L_1$, $L_2$, and $L_3$ can be obtained from equations (23)-(25), respectively.

$$h_{L_1, \theta=0} = \left( R + \frac{D}{2} + a \right) \cos \beta + R f (\mu) \cos (\mu + \beta)$$ (23)

$$h_{L_2, \theta=\frac{\pi}{2}} = \left( R + \frac{D}{2} + a \right) \cos \beta + R f (\mu) \cos \beta$$ (24)

$$h_{L_3, \theta=\pi} = \left( R + \frac{D}{2} + a \right) \cos \beta + R f (\mu) \cos (\mu - \beta)$$ (25)

If equations (23)-(25) are arranged using the same method as in Section A, they can be expressed as shown in equations (26)-(28), respectively.

$$h_{L_1, \theta=0} = \frac{1}{a_R} (R + a + D + R \cos \varphi - a_H R \sin \varphi)$$ (26)

$$h_{L_2, \theta=\frac{\pi}{2}} = \frac{1}{a_R} (R + a + D + R \cos \varphi)$$ (27)

$$h_{L_3, \theta=\pi} = \frac{1}{a_R} (R + a + D + R \cos \varphi + a_H R \sin \varphi)$$ (28)

With reference to equations (26)-(28), the water head in the lower hemisphere can be expressed through a single equation, which is given by equation (29), for the range $0 \leq \theta \leq \pi$.

$$h_L = \frac{1}{a_R} (R + a + D + R \cos \varphi - a_H R \sin \varphi \cos \theta)$$ (29)

Multiplying equation (29) with $\rho g a_R$ yields the pressure due to the resultant acceleration in the lower hemisphere:

$$P_L = \rho g (R + a + D + R \cos \varphi - a_H R \sin \varphi \cos \theta)$$ (30)

By subtracting gravitational acceleration $\rho g (R + R \cos \varphi + D)$ from equation (30), the pressure corresponding to horizontal acceleration alone is as follows:

$$P_{HL} = \rho g (a - a_H R \sin \varphi \cos \theta)$$ (31)
3. Conclusions
In this study, the equation for the distribution of the pressure resulting from horizontal acceleration was derived for spherical tanks with a cylindrical central part for use as a cargo containment system of LNG carriers, as shipbuilders in the world have no prior experience with the design and construction of such tanks.

For spherical LNG tanks with a cylindrical central part, the pressure distributions in the upper and lower hemispheres are similar to those in a perfect spherical tank if the magnitude of the horizontal acceleration is the same, and the maximum pressure occur at the cylindrical central part of the tank.

The equation for the pressure distribution in the cylindrical central part was derived clearly. This equation should be useful to designers for structural analysis of spherical LNG tanks with cylindrical extension.

For the future work, by using the equations derived in this study, a simplified analysis method suitable for dynamic load conditions will be developed and combined with a previously developed simplified analysis method for static loads [6] to develop a comprehensive simplified analysis method that allows for precise initial estimates and is more efficient than time-consuming finite element analyses.

4. References
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