Lepton Flavor Violation at a Future LC

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Abstract.
We investigate the prospects for detection of lepton flavour violation (LFV) in sparticle
production and decays at a Linear Collider (LC). We study the Constrained Minimal
Supersymmetric extension of the Standard Model (CMSSM), focusing on the subset of the
supersymmetric parameter space that also leads to cosmologically interesting values of the relic
neutralino LSP density. Emphasis is given to the complementarity between the LC and the
LHC signals.

1. Introduction
Data from both atmospheric [1], solar [2] and long-baseline reactor [3] and accelerator [4, 5]
neutrino experiments have by now confirmed the existence of neutrino oscillations with near-
maximal $\nu_{\mu} - \nu_\tau$ mixing and large $\nu_e \rightarrow \nu_\mu$ one. These observations would also imply violation
of the corresponding charged-lepton numbers, which in supersymmetric theories might be
significant and observable in low-energy experiments [6, 7]. This may be enhanced sufficiently
to become observable in a class of theories predicting new physics at the TeV scale accessible to
colliders, particularly in supersymmetric theories. In this presentation, we review the detection
channels for lepton flavour violation at a Linear Collider [8] and compare with a previous work
[9] where we found that a signal for $\tau$ flavour-violating $\chi_2$ decays may be observable at the
LHC. In our sampling of the parameter space, we focus on regions that satisfy not only all
phenomenological constraints, but also the cosmological relic density considerations.

2. LFV in Slepton Production
2.1. CMSSM with LFV
In the unrotated charged-lepton flavour basis $\tilde{\ell} = (\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}^*_R, \tilde{\mu}^*_R, \tilde{\tau}^*_R)$, the charged slepton
mass matrix is:

$$
M_{\tilde{\ell}}^2 = \begin{pmatrix}
M_{L L}^2 & M_{L R}^2 \\
M_{R L}^2 & M_{R R}^2
\end{pmatrix}
$$

(1)

where

$$
M_{L L}^2 = m_{\ell}^2 + M_{L}^2 - \frac{1}{2}(2m_W^2 - m_Z^2) \cos 2\beta \, I,
$$

$$
M_{R R}^2 = m_{\ell}^2 + M_{R}^2 - (m_Z^2 - m_W^2) \cos 2\beta \, I,
$$

$$
M_{L R}^2 = (A - \mu \tan \beta) \, m_{\ell},
$$

$$
M_{R L}^2 = (M_{L R}^2)^\dagger.
$$

(2)
Here we parametrize trilinear soft supersymmetry-breaking terms as $A_{ij}^e (\lambda_e)_{ij}$, where the $\lambda_e$ are the respective Yukawa couplings. For universal soft terms at some high input scale, one has

$$M_L^2 = M_R^2 = m_0^2 \, I, \quad A_{ij}^e = A_0 \delta_{ij},$$

whereas flavour-mixing entries may be parametrized by:

$$\delta_{ij}^{XX} = (M_{XX}^2)^{ij} / (M_{XX}^2)^{ii} \quad (X = L, R).$$

The evaluation of the LFV observables is done by performing the diagonalization of the slepton mass matrices (see [10], for instance), inserting the full rotation matrices in the lepton-slepton-gaugino vertices and summing over all the mass eigenstates of the exchanged particles.

### 2.2. LFV at the LHC

At the LHC sleptons may appear in cascade decays of heavy squarks or gluinos, we pay particular attention to regions that lead to large values of $\Gamma(\chi_2 \rightarrow \chi + \tau^\pm + \mu^\mp)$ via the on-shell slepton production mechanism: $BR(\chi_2 \rightarrow \chi\tau^\pm + \mu^\mp) = \sum_{i=1}^3 \left[BR(\chi_2 \rightarrow \tilde{\ell}_i \mu)BR(\tilde{\ell}_i \rightarrow \tau\chi) + BR(\chi_2 \rightarrow \tilde{\ell}_i \tau)BR(\tilde{\ell}_i \rightarrow \mu\chi)\right]$. The simulations performed in [9] indicate that this BR must be above 0.1 to allow the identification of LFV events. Furthermore, these ratios can only be achieved if the sleptons mass matrix contain large RR entries in a basis where the charged sleptons are diagonal.

The inclusion of a see-saw mechanism to explain neutrino masses may lead to LL mixings with interesting predictions for radiative decays like $\tau \rightarrow \mu \gamma$. However, the required LL mixing values to observe stau flavour oscillation will imply a large violation of this radiative decay. For instance, the cases presented in [9] show that in order to achieve a branching ratio for $\tilde{\chi}_2 \rightarrow \tilde{\chi}_1 \tau^\pm \mu^\mp$ that is of interest for the LHC, we need $\delta_{RR} \sim 0.15$ for $\delta_{LL} = 0$ or $\delta_{LL} \sim 0.35$ for $\delta_{RR} = 0$. While the bound on $BR(\tau \rightarrow \mu \gamma)$ is very restrictive on the size of $\delta_{LL}$, imposing a maximum value $\sim 0.03$ on it. These large mixings are difficult to generate in models with universal soft terms at hight energy scales, however they may be present in non-minimal GUT models as shown in ref.[11].

### 2.3. LFV at the LC

Charged-lepton flavour violation at a LC may occur either directly in slepton pair production or indirectly via slepton production in cascade decays [12]. Processes leading to lepton production in the decays of a pair of sleptons include

$$e^+ e^- \rightarrow \tilde{\ell}_i^- \tilde{\ell}_j^+ \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \chi_1^0,$$

$$e^+ e^- \rightarrow \nu_i^+ \nu_j^- \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \chi_2^+ + \chi_2^- \chi_1^-,$$

for which representative Feynman diagrams are shown in Fig. 1. Slepton production may also result from the the cascade decays of the heavier gauginos, e.g., via the processes

$$e^+ e^- \rightarrow \tilde{\chi}_2^+ \chi_1^- \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^+ \chi_1^0,$$

$$e^+ e^- \rightarrow \tilde{\chi}_2^0 \chi_1^- \rightarrow \tau^\pm \mu^\mp \tilde{\chi}_1^0 \chi_1^0.$$
3. LFV observation at the LC.
We take regions of SUSY parameter space relevant from the cosmological point of view, in which sizable $\tilde{\tau} - \chi$ coannihilations or direct-channel $H/A$ resonances lead to values of $\Omega_\chi h^2$ consistent with WMAP. As a representative of the coannihilation case, we chose $\tan \beta = 35$, a value that leads to a relatively heavy sparticle spectrum compatible with WMAP. At $\tan \beta = 45$ we found points in the resonance funnel, which have larger scalar masses since $m_0$ can take higher values than in the coannihilation area. We need a choice of flavour mixing parameters for a more detailed discussion of LFV cross sections. We assume the same values of the mixing parameters for both choices of $\tan \beta$, namely:

\[
\begin{align*}
(\delta_{LL})_{13} &= 0.02, & (\delta_{LL})_{23} &= 0.02, \\
(\delta_{RR})_{13} &= 0.04, & (\delta_{RR})_{23} &= 0.15.
\end{align*}
\]

For completeness, we also introduce a small mixing between the first and second generation:

\[
(\delta_{LL/RR})_{12} = 0.2 \cdot (\delta_{LL/RR})_{13}.
\]

The LFV decays into $\tau - \mu$ pairs are not heavily dependent on this parameter while, with this choice, the bound on $\text{BR}(\mu \to e\gamma)$ does not over-constrain the parameter space.

Cross sections for $e^+e^- \to \tau^+\tau^- + 2\chi^0$ above 1 fb can be reached in the areas of parameters shown in Fig. 2 for energies below those indicated. The large areas between the thick solid red, black and blue lines, on the one hand, and the shaded regions that are excluded by the indicated present experimental constraints, on the other hand, demonstrate that there are ample opportunities for LFV discovery and measurement at the LC. These opportunities are exemplified by the benchmark points marked by crosses in Fig. 2 and studied with more detail in ref. [8].

In Fig. 3 we present the maximum values of the cross sections in the areas allowed by all the current constraints. The shaded areas show the possible ranges of $\sigma(e^+e^- \to \tau^\pm \mu^\mp + 2\chi^0)$, and we note that points along the WMAP strips (solid lines) generally have high values of the cross sections. The dashed lines show the possible values of $\sigma(e^+e^- \to \tau^\pm e^\mp + 2\chi^0)$ along the WMAP strips, and we see that these cross sections may be of the same order of magnitude as those for $\mu - \tau$ pairs.

4. Conclusions
The LC enhances significantly the prospects of detecting LFV for heavy sparticle spectra, where flavour-violating rare decays and conversions are significantly suppressed. This allows probing an entirely different range of the flavour-violating parameters. Unlike the LHC of ref. [9], at the LC we can have significant LFV within the CMSSM via $LL$ mixing.
Figure 2. The solid red (black) (blue) lines are contours where $\sigma(e^+e^-\rightarrow \tau^+\mu^- + 2\chi^0) = 1$ fb at $\sqrt{s} = 500, 1000, 2000$ GeV, assuming the left mixing parameters (7) in the upper panels and the right mixing (8) in the lower panels. The left panels are for for $\tan\beta = 35$ and the right panels for $\tan\beta = 45$, assuming $A_0 = 0$ in both cases. Each panel also shows the areas excluded by current bounds on $BR(\mu \rightarrow e\gamma)$ (thin solid line), $BR(\tau \rightarrow e\gamma)$ (thin dash line), $BR(\tau \rightarrow \mu\gamma)$ (thin dot-dash line). The areas excluded by $BR(b \rightarrow s\gamma)$ and the LEP Higgs search are also displayed, and the green area denotes the WMAP favored region. We see that there are ample opportunities for LFV discovery and measurement at the LC, and the benchmark points chosen for further studies are indicated by crosses.

The expectations for models of massive neutrinos, in which quantum corrections provide a significant source of LFV in the $LL$ channel, provide a potential link between LC observables and neutrino mass and mixing parameters. The cross sections expected in non-minimal extensions of the theory might not only enhance channels that in the current scheme are more suppressed, but could also enable a comparison of the allowed range of mixing parameters in different models.

Overall, it seems that the LC provides an optimal environment for the study of LFV, whereas the LHC is limited to specific channels that have significant backgrounds. The fact that the LC opens up additional possibilities may prove significant for making the link between observable cross sections and flavour model building.

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Figure 3. The shaded areas show the possible ranges of $\sigma(e^+e^- \rightarrow \tau^\pm \mu^\mp + 2\chi^0)$ for $m_0 < 1000\text{ GeV}$, $\tan\beta = 35$ (left panels) and $\tan\beta = 45$ (right panels), assuming $A_0 = 0$ in both cases, with $\sqrt{s}$ fixed to 500 GeV (grey), 1000 GeV (green) and 2000 GeV (orange). In the upper panels, we assume the left mixing parameters (7), whereas in the lower panels we assume the right mixing (8). The solid lines present the possible values for models along the center of the WMAP strips, with the lines corresponding to $\sqrt{s}=500$, 1000, 2000 GeV being progressively thicker. The corresponding predictions for $\sigma(e^+e^- \rightarrow \tau^\pm e^\mp + 2\chi^0)$ are shown by the dashed lines.

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