Dilaton driven Hawking radiation in AdS$_2$ black hole

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Abstract

A recent study shows that Hawking radiation of a massless scalar field does not appear on the two-dimensional AdS$_2$ black hole background. We shall study this issue by calculating absorption and reflection coefficients under dilaton coupling with the matter field. If the scalar field does not couple to the dilaton, then it is fully absorbed into the black hole without any outgoing mode. On the other hand, once it couples to the dilaton field, the outgoing mode of the massless scalar field exists, and the nontrivial Hawking radiation appears. Finally, we comment on this dilaton dependence of Hawking radiation in connection with a three-dimensional black hole.

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Recently, anti-de Sitter (AdS) spacetimes have been studied in connection with various aspects of AdS black hole physics, for example, in the calculation of black hole entropies \[1\] in terms of lower-dimensional AdS gravity theories, the three-dimensional Bañados-Teitelboim-Zanelli (BTZ) black hole \[2\] and AdS\(_2\) black holes \[3\].

On the other hand, greybody factor \[4,5\] of the massless scalar field on this BTZ background has been studied by Birmingham, Sachs, and Sen (BSS) \[6\] and they obtained the absorption cross section related to the Hawking radiation \[7\] by carefully considering the matching procedure between the bulk and the AdS boundary. In two-dimensional case, it has been shown that the absorption coefficient is one and there are no reflection modes \[8\], which means that there does not exist any massless radiation on this two-dimensional AdS black hole background. This fact seems to be consistent with the argument in Ref. \[9\] since expectation value of the energy-momentum tensor around the two-dimensional AdS black hole is zero.

In this paper, we would like to study this issue whether the Hawking radiation appears on the two-dimensional AdS black hole background or not in terms of a scattering analysis of the massless scalar field as a test field. We shall assume that the classical metric-dilaton background and then the scalar wave equation will be solved on this background for two cases. For the first case, the free scalar field equation is solved, and by carefully matching this solution with the boundary solution we obtain the expected null radiation. Once the boundary condition that the outgoing modes are absent is imposed at the horizon, there are no more outgoing modes at the bulk and the boundary. On the other hand, for the second case of the minimal coupling with the dilaton background, which is just a Jackiw-Teitelboim (JT) model \[10\], the scalar field equation is exactly solved in the bulk and the boundary. By imposing appropriate boundary conditions, we obtain at last the nontrivial Hawking radiation. In this latter case, the vanishing limit of the dilaton field does not exist in this calculation, and the two cases are distinct in the scattering analysis.

Let us now assume the following metric-dilaton background as
\[ds^2 = -\left(-M + \frac{r^2}{\ell^2}\right)dt^2 + \left(-M + \frac{r^2}{\ell^2}\right)^{-1}dr^2,\]

\[\psi = \gamma \ln \frac{r}{\ell} + \psi_0,\]

where the metric (1) describes AdS black hole with the horizon \(r_H = \sqrt{M\ell}\) which is asymptotically AdS\(_2\) spacetime, and its curvature is a constant, \(R = -\frac{2}{\ell^2}\). One can assert that the Hawking temperature as

\[T_H = \frac{r_H}{2\pi\ell^2},\]

which may be given by the conventional procedure to avoid the conical singularity of the metric in Euclidean formalism. At first glance, this temperature, however, does not seem to depend on the dilaton field. The purpose of the present paper in some sense is to study the dilaton dependence of the black hole temperature in terms of the scattering analysis.

We are now going to study the scattering amplitudes of a test field on the AdS black hole background in order to calculate the Hawking radiation and temperature from the dynamical process. Let us now consider the following scalar field \(f\) obeying

\[\Box f + \nabla_\mu \psi \nabla^\mu f = 0.\]

For \(\gamma = 0\), the matter field does not couple to the metric, whereas it couples explicitly with the dilaton field for \(\gamma = 1\). The Hawking radiation for the dilaton coupled scalar field on the CGHS black hole background has been studied in Ref. [14]. Here, we consider only two cases in order to compare the behavior of scattering of the scalar field on the AdS black hole background, and study the dilaton dependence of Hawking radiation. In fact, for the case of \(\gamma = 0\) corresponding to the constant dilaton background, the AdS\(_2\) model [9] has been constructed by using the Callan-Giddings-Harvey-Strominger (CGHS) model [12] (especially, for the Russo-Susskind-Thorlacius model [13], the AdS geometry was discussed in Ref. [11]). For the other case of \(\gamma = 1\), it has been well appreciated as a two-dimensional AdS gravity [15–17]. Let us study Hawking radiation process on these backgrounds by calculating the absorption and reflection coefficients, and obtain the desirable Hawking temperatures.
After separation of variables with \( f(t,r) = R(\gamma)(r)e^{-i\omega t} \), the spatial equation of motion yields

\[
(r^2 - r_H^2)\partial_r^2 R(\gamma)(r) + \frac{1}{r} \left[ 2r^2 + \gamma(r^2 - r_H^2) \right] \partial_r R(\gamma)(r) + \frac{\omega^2 \ell^4}{(r^2 - r_H^2)} R(\gamma)(r) = 0. \tag{5}
\]

Hereafter, let us treat two cases separately since they are drastically different.

For the first case of \( \gamma = 0 \), we shall calculate the scattering amplitude of the massless scalar field on the AdS black hole background, and then infer the Hawking radiation and temperature. By performing change of the variable as \( z = \frac{r - r_H}{r + r_H} \), \( 0 \leq z \leq 1 \), the equation of motion Eq. (5) can be simply written in the form of

\[
z (1 - z) \partial_z^2 R(0)(z) + (1 - z) \partial_z R(0)(z) + \frac{\omega^2 \ell^4}{4r_H^2} (\frac{1}{z} - 1) R(0)(z) = 0. \tag{6}
\]

By defining \( R(0)(z) = z^\alpha g(z) \), the wave equation is given as

\[
z (1 - z) \partial_z^2 g(z) + (1 - z)(2\alpha + 1) \partial_z g(z) + \left[ \frac{1}{z} \left( \alpha^2 + \frac{\omega^2 \ell^4}{4r_H^2} \right) - \left( \alpha^2 + \frac{\omega^2 \ell^4}{4r_H^2} \right) \right] g(z) = 0, \tag{7}
\]

and the nonsingular solution is finally obtained with \( \alpha^2 = -\frac{\omega^2 \ell^4}{4r_H^2} \),

\[
R(\gamma)(r) = C_{\text{in}} e^{-i\frac{\omega^2 r}{r_H}} + C_{\text{out}} e^{i\frac{\omega^2 r}{r_H}}. \tag{8}
\]

Note that this is an exact solution defined over the bulk. Further, in the far region, it is also simply written as

\[
R^{\text{far}}(\gamma)(r) = C_{\text{in}} e^{i\frac{\omega^2 r}{r}} + C_{\text{out}} e^{-i\frac{\omega^2 r}{r}}. \tag{9}
\]

At this stage, we should carefully consider the boundary of this AdS black hole because it has nontrivial background geometry in contrast to the asymptotically flat black hole. The background geometry of the usual black hole at the asymptotically far region is happened to be that of the massless limit of the black hole geometry. So, in that case, the far region limit means the massless limit of the black hole geometry, however, in our model this is not the case. Therefore, we should take the boundary metric by defining \( M = 0 \) in Eq. (I). Then the equation of motion at the boundary is given by
\[ r^2 \partial^2_r R_{(0)}(r) + 2r \partial_r R_{(0)}(r) + \frac{\omega^2 \ell^4}{r^2} R_{(0)}(r) = 0, \tag{10} \]

and its solution is easily obtained as

\[ R_{(0)}^{\text{boundary}}(r) = A_{\text{in}} e^{i \omega \frac{\ell^2}{r}} + A_{\text{out}} e^{-i \omega \frac{\ell^2}{r}}. \tag{11} \]

It is interesting to note that the asymptotic solution (9) is compatible with the boundary solution (11) if we identify \( C_{\text{in}} = A_{\text{in}} \) and \( C_{\text{out}} = A_{\text{out}} \).

By using the expression of the flux expressed as

\[ F_{(\gamma)} = \frac{2\pi}{t} \left( \frac{r^2 - r_H^2}{\ell^2} \right) \left[ R^*_{(\gamma)}(r) \partial_r R_{(\gamma)}(r) - R(r)_{(\gamma)} \partial_r R^*_{(\gamma)}(r) \right], \tag{12} \]

we can straightforwardly define the absorption coefficient (\( A \)) and reflection coefficient (\( R \)) as

\[ A = \left| \frac{F_{(\gamma)}^{\text{in}}(r = r_H)}{F_{(\gamma)}^{\text{in}}(r = \infty)} \right|, \quad R = \left| \frac{F_{(\gamma)}^{\text{out}}(r = \infty)}{F_{(\gamma)}^{\text{in}}(r = \infty)} \right|, \tag{13} \]

where \( F_{(\gamma)}^{\text{in}}(r = r_H; \infty) \) and \( F_{(\gamma)}^{\text{out}}(r = r_H; \infty) \) are ingoing and outgoing fluxes at the horizon and boundary, respectively. At the horizon, we impose the boundary condition as \( C_{\text{out}} = 0 \) \([18]\), then the outgoing mode does not appear at the bulk and the boundary. Therefore, the reflection coefficient is zero, \( R = 0 \). Even though one chooses the other boundary condition as \( A_{\text{out}} = 0 \) in Ref. [18], in this case also the reflection coefficient is zero as it should be. By the use of the following relation which relates the reflection coefficient and Hawking thermal radiation \([18]\),

\[ < 0 | \mathcal{N} | 0 > = \frac{R}{1 - R} = \frac{1}{e^{\frac{i \omega H}{T_H}} - 1}, \tag{14} \]

we can assume the vanishing Hawking temperature,

\[ T_H = 0 \tag{15} \]

for the finite mode of the scalar field, \( \omega > 0 \) since \( R = 0 \). This fact is compatible with the result of Ref. [9] where the Hawking radiation has been studied in terms of the energy-momentum tensor of massless scalar field. Therefore, for \( \gamma = 0 \) case, there is no reflection.
mode in AdS$_{2}$ black hole background and there does not exist massless radiation in our scattering analysis.

Let us now study the second case of $\gamma = 1$ with the dilaton coupling of Eq. (4). In this case, we perform a change of variable to solve the field equation exactly as $z = \frac{r^2-r_+^2}{r^2}$ ($0 \leq z \leq 1$). So the spatial(radial) equation of motion (5) is given by

$$z(1-z)\partial_z^2 R_{(1)}(z) + (1-z)\partial_z R_{(1)}(z) + \frac{\omega^2 \ell^4}{4r_H^2 z} R_{(1)}(z) = 0,$$

(16)

and it is rewritten as

$$z(1-z)\partial_z^2 g(z) + (1+2\kappa)(1-z)\partial_z g(z) + \left[ \frac{1}{z} \left( \kappa^2 + \frac{\omega^2 \ell^4}{4r_H^2} \right) - \kappa^2 \right] g(z) = 0,$$

(17)

where $R_{(1)}(z) = z^\kappa g(z)$. The equation of motion (17) becomes

$$z(1-z)\partial_z^2 g(z) + (1+2\kappa)(1-z)\partial_z g(z) - \kappa^2 g(z) = 0$$

(18)

after choosing the constant $\kappa$ as $\kappa^2 = -\frac{\omega^2 \ell^4}{4r_H^2}$, and the standard solution is given by the linear combination of two hypergeometric functions $F(a, b, c; z)$ and $z^{1-c}F(a+1-c, b+1-c, 2-c; z)$ where

$$a = \kappa, \quad b = \kappa, \quad c = 1+2\kappa.$$  

(19)

Then, the bulk solution can be neatly written as

$$R_{(1)}(r) = z^{-\kappa} C_{\text{in}} F(-\kappa, -\kappa, 1-2\kappa; z) + z^{\kappa} C_{\text{out}} F(\kappa, \kappa, 1+2\kappa; z).$$

(20)

Note that it is symmetric under interchange of the sign of $\kappa$, and we simply take the plus sign of $\kappa$.

In the near horizon limit ($z \to 0$), the solution is reduced to

$$R_{\text{near}}^{(1)}(r) = C_{\text{in}} e^{\frac{i\omega\ell^2}{2r_H} \ln \left( \frac{r^2-r_+^2}{r^2} \right)} + C_{\text{out}} e^{\frac{i\omega\ell^2}{2r_H} \ln \left( \frac{r^2-r_+^2}{r^2} \right)}.$$  

(21)

On the other hand, the large-$r$ behavior of the bulk solution (20) follows from the $z \to 1-z$ transformation law for the hypergeometric functions [19]:

6
\[ F(a, b, a + b + m; z) = \frac{\Gamma(m)\Gamma(a + b + m)}{\Gamma(a + m)\Gamma(b + m)} \sum_{n=0}^{m-1} \frac{(a)(b)n}{n!(1-m)n}(1-z)^n \]
\[ - \frac{\Gamma(a + b + m)}{\Gamma(a)\Gamma(b)} (z-1)^m \sum_{n=0}^{\infty} \frac{(a+b+m)n}{n!(n+m)!}(1-z)^n \]
\[ \times [\ln(1-z) - \psi(n+1) - \psi(n+m+1) + \psi(a+n+m) + \psi(b+n+m)]. \] (22)

Note that in our case \( m \) is unity in Eq. (22). Using this relation (22), we can obtain the far region solution from Eq. (20),

\[ R_{\text{far}}^{(1)}(r) = C_{\text{in}} \frac{\Gamma(1-2\kappa)}{\Gamma(1-\kappa)\Gamma(1-\kappa)} \left[ 1 + \kappa \left( \frac{r_H}{r} \right)^2 + \kappa^2 \left( \frac{r_H}{r} \right)^2 \left( 2 \ln \left( \frac{r_H}{r} \right) + \zeta_{\kappa}^{(0)} \right) \right] \]
\[ + C_{\text{out}} \frac{\Gamma(1+2\kappa)}{\Gamma(1+\kappa)\Gamma(1+\kappa)} \left[ 1 - \kappa \left( \frac{r_H}{r} \right)^2 + \kappa^2 \left( \frac{r_H}{r} \right)^2 \left( 2 \ln \left( \frac{r_H}{r} \right) + \zeta_{\kappa}^{(0)} \right) \right], \] (23)

where \( \zeta_{\pm \kappa}^{(0)} = 2\psi(1 \pm \kappa) - \psi(2) - \psi(1) \) and \( \psi(z) \) is a digamma function.

As was emphasized for the case of \( \gamma = 0 \), at the boundary \( (z = 1) \), we have to solve the wave equation in order to match their coefficients with those of Eq. (23). So the boundary wave equation

\[ r^2 \partial_r^2 R_{(1)}(r) + 3r \partial_r R_{(1)}(r) + \frac{\omega^2 \ell^4}{r^2} R_{(1)}(r) = 0, \] (24)

yields the following boundary solution

\[ R_{\text{boundary}}^{(1)}(r) = \frac{1}{r} \left[ \alpha K_1 \left( i\omega \frac{\ell^2}{r} \right) + \beta I_1 \left( i\omega \frac{\ell^2}{r} \right) \right], \] (25)

where \( K \) and \( I \) are modified Bessel functions, and \( \alpha \) and \( \beta \) are arbitrary constants. These functions are expanded as a well-known form in Ref. [19], which is given by

\[ I_\nu(z) = \left( \frac{1}{2} \right)^\nu \sum_{k=0}^{\infty} \frac{\left( \frac{1}{2}z \right)^{2k}}{k! \Gamma(\nu + k + 1)}, \]
\[ K_\nu(z) = \frac{1}{2} \left( \frac{1}{2} \right)^{-\nu} \sum_{k=0}^{\nu-1} \frac{(\nu - k - 1)!}{k!} \left( -\frac{z^2}{4} \right)^k \]
\[ + (-1)^{\nu+1} \ln \left( \frac{1}{2} \right) I_\nu(z) \]
\[ + (-1)^{\nu} \frac{1}{2} \left( \frac{1}{2} \right)^\nu \sum_{k=0}^{\infty} (\psi(k + 1) + \psi(\nu + k + 1)) \frac{\left( \frac{1}{2}z \right)^{2k}}{k!(\nu + k)!}. \] (26)

7
Considering the most leading term which corresponds to \( k = 0 \) in Eq. (26) for the limit of \( r \to \infty \), we obtain the following boundary solution given by

\[
R_{\text{boundary}}^{(1)}(r) = a + \frac{b}{r^2},
\]

(27)

where \( a \) and \( b \) are arbitrary constants. We can rewrite this solution in terms of the ingoing and outgoing modes by redefining the constants as follows,

\[
a = A_{\text{in}} + A_{\text{out}},
\]

\[
b = i\frac{r_H^2}{\pi}(A_{\text{in}} - A_{\text{out}}).
\]

(28)

So we have the mode decomposed boundary solution by

\[
R_{\text{boundary}}^{(1)}(r) = A_{\text{in}} \left(1 + i\frac{r_H^2}{\pi r^2}\right) + A_{\text{out}} \left(1 - i\frac{r_H^2}{\pi r^2}\right).
\]

(29)

By matching the two solutions Eqs. (23) and (29), the boundary coefficient \( A_{\text{in}} \) and \( A_{\text{out}} \) are determined as follows,

\[
A_{\text{in}} = \frac{1}{2}C_{\text{out}} \frac{\Gamma(1 + 2\kappa)}{\Gamma(1 + \kappa)\Gamma(1 + \kappa)} [1 + i\pi \kappa] + \frac{1}{2}C_{\text{in}} \frac{\Gamma(1 - 2\kappa)}{\Gamma(1 - \kappa)\Gamma(1 - \kappa)} [1 - i\pi \kappa],
\]

\[
A_{\text{out}} = \frac{1}{2}C_{\text{out}} \frac{\Gamma(1 + 2\kappa)}{\Gamma(1 + \kappa)\Gamma(1 + \kappa)} [1 - i\pi \kappa] + \frac{1}{2}C_{\text{in}} \frac{\Gamma(1 - 2\kappa)}{\Gamma(1 - \kappa)\Gamma(1 - \kappa)} [1 + i\pi \kappa].
\]

(30)

Similarly to the previous case of \( \gamma = 0 \), we choose the boundary condition of \( C_{\text{out}} = 0 \) at the horizon \([8]\), and in the limit of small \( \omega \), the absorption and reflection coefficients are easily calculated as

\[
A = \left| \frac{F_{\text{in}}^{(1)}(r = r_H)}{F_{\text{in}}^{(1)}(r = \infty)} \right| = \frac{8\omega \ell^2 r_H}{(2r_H + \pi \omega \ell^2)^2},
\]

\[
R = \left| \frac{F_{\text{out}}^{(1)}(r = \infty)}{F_{\text{in}}^{(1)}(r = \infty)} \right| = \left| \frac{A_{\text{out}}}{A_{\text{in}}} \right|^2 = \left( \frac{2r_H - \pi \omega \ell^2}{2r_H + \pi \omega \ell^2} \right)^2.
\]

(31)

We now evaluate Hawking temperature by using the relation of reflection coefficient and the number operator,
\[ <0|N|0> = \frac{R}{1-R} = \frac{1}{e^{\frac{R}{T_H}} - 1} \]  

where \[ T_H = \frac{\omega}{\ln\left(1 + \frac{8\pi\omega^2 r_H}{(2\pi H - \pi\omega^2)^2}\right)}. \]  

Note that if we take the \( \omega \to 0 \) limit, i.e., the energy quanta of test field are so small, we obtain the desirable Hawking temperature

\[ T_H \approx \frac{r_H}{2\pi\ell^2}, \]  

which is interestingly coincident with the statistical Hawking temperature Eq. (3). As for the second kind of boundary configuration, we can choose \( A_{out} = 0 \) as in [18]. So, after performing matching procedure, we can calculate the ratio of coefficients between ingoing and outgoing modes in the near horizon region, \( \left|\frac{C_{in}}{C_{out}}\right|^2 = \left(\frac{2\pi H + \pi\omega^2}{2\pi H - \pi\omega^2}\right)^2 \). We note that it is just the inverse of reflection coefficient in the first kind of boundary condition and the Hawking emission rate is given as \( <0|N|0> = \left(\left|\frac{C_{in}}{C_{out}}\right|^2 - 1\right)^{-1} \), which is in fact the same with that of Eq. (32).

Note that we had an ambiguity in the boundary solution Eq. (29) in AdS\(_2\) spacetime [6], and the Hawking temperature may not be uniquely determined. This phenomenon has already appeared in defining the surface gravity in the AdS black hole. There is no preferred normalization of timelike Killing vector in the asymptotic region and the surface gravity depends upon this normalization [20]. So we may not obtain the usual expression of the Hawking temperature in our calculation too. However, in our boundary solution, we fixed this ambiguity and the well-known expression of the Hawking temperature Eq. (3) is obtained.

We have shown that the nontrivial dilaton coupling for the second case of \( \gamma = 1 \) plays an important role in the Hawking radiation process on the two-dimensional AdS black hole, which is contrasted with the first case of \( \gamma = 0 \). This fact can be traced back, and the
origin may be found from the three-dimensional BTZ black hole. Let us consider the three-
dimensional model as

\[ S = \frac{1}{2\pi} \int d^3x \sqrt{-g} \left[ R + \frac{2}{\ell^2} \right] - \frac{1}{4\pi} \int d^3x \sqrt{-g} (\nabla f)^2 \]  

(35)

which yields a well-known BTZ black hole solution for \( f = 0 \) \[3\]. The absorption coefficient
and the cross section of the scalar field \( f \) around the BTZ black hole background have been
calculated by BSS in Ref. \[6\]. Therefore if we take \( S_1 \) compactification along the angular axis
as \((ds)^2 = \left( g_{\alpha\beta}^{(2)} + e^{2\psi} A_\alpha A_\beta \right) dx^\alpha dx^\beta + 2e^{2\psi} A_\alpha dx^\alpha dx_3 + e^{2\psi} dx_3^2\) where \( \alpha, \beta = 0, 1, \) and \( x_3 \)
is angular coordinate in three dimensions, the two-dimensional AdS black hole configuration
of Eqs. \[(1)\] and \[(2)\] are obtained for the vanishing Kaluza-Klein charge \[3\]. In this s-wave
reduction, the equation of motion for the massless field around this AdS\(_2\) black hole becomes
exactly Eq. \[(1)\] only for \( \gamma = 1 \). Therefore, the nonvanishing Hawking radiation in our model
is due to the dilaton related to the transverse radius of the three dimensions.

On the other hand, the triviality of the Hawking radiation for \( \gamma = 0 \) is apparently due
to the fact that the test field does not see the dilaton. In this case, there may exist another
explanation for this triviality of the Hawking radiation. The constant \( M \) in Eq. \[(1)\] in
the constant dilaton background can not be interpreted as a black hole mass as far as we
take the background metric as \( ds^2 = -r^2 dt^2 + \frac{\ell^2}{r^2} dr^2 \) since the ADM mass is proportional
to the derivative of the dilaton field. To make this explicit, the quasilocal mass for the
asymptotically nonflat solution is given by \( Q_\xi = -\sqrt{-M + \frac{r^2}{\ell^2}} \epsilon(r) \) where \( Q_\xi \) is a conserved
charge along the timelike Killing vector \( \xi \) and \( \epsilon(r) \) is a local energy density defined by
\( \epsilon(r) = -2\sqrt{-M + \frac{r^2}{\ell^2}} \partial_r e\psi - \epsilon_0(r) \) \[21\]. The background energy is chosen as \( \epsilon_0(r) = -\frac{2r}{\ell^2} \) to be satisfied with the condition of \( \epsilon(r) = 0 \) for \( M = 0 \). Then the ADM mass for the large-\( r \)
is given by exactly \( Q_\xi(r \to \infty) = \frac{M}{\ell} \) for \( \gamma = 1 \), while it becomes zero for the constant
dilaton background case of \( \gamma = 0 \). Therefore, in two dimensions, the nontrivial AdS black
hole solution seems to be accompanied with the nontrivial dilaton solution. This implies
that for the case of \( \gamma = 0 \), the parameter \( M \) in the metric Eq. \[(1)\] may be removed by
using some coordinate transformations, which is in fact given by \( y^\pm = \frac{2M}{\sqrt{M} \, \ell} \tanh \frac{M r^\pm}{2\ell_{\text{eff}}} \) where
\(y^\pm\) describes our vacuum geometry (\(M = 0\)) of \(ds^2 = -\frac{4r^2}{(y^+ - y^-)^2}dy^+dy^-\) in the conformal coordinate while \(\sigma^\pm = t \pm r^*\) does the metric Eq. (1). The tortoise coordinate \(r^*\) is defined by \(r^* = \int \frac{dr}{\sqrt{-M + r^2}} = \frac{\ell^2}{2r_H} \ln \left(\frac{r - r_H}{r + r_H}\right)\). Therefore, one can think that the two-dimensional AdS black hole and vacuum in some sense belong to the equivalent class if the dilaton is not involved where the global difference between them is rigorously studied very recently in Ref. [22]. Note that for the dilaton coupled case of \(\gamma = 1\), the above coordinate transformation is also possible, however, the dilaton field in the new coordinate contains the information of the AdS black hole. Therefore the parameter \(M\) turns out to be a coordinate artifact for \(\gamma = 0\), and it seems to be meaningless to interpret it as a black hole mass in our calculation.

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