Radiation induced zero-resistance states: a dressed electronic structure effect

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Recent results on magnetoresistance in a two dimensional electron gas under crossed magnetic and microwave fields show a new class of oscillations, suggesting a new kind of zero-resistance states. A complete understanding of the effect is still lacking. We consider the problem from the point of view of the electronic structure dressed by photons due to a plane linearly polarized ac field. The dramatic changes in the dressed electronic structure lead to a reinterpretation of the new magnetoresistance oscillations as a persistent-current like effect, induced by the radiation field.

PACS numbers: 73.21.-b, 78.67.-n, 73.43.-f

A new class of low temperature non-equilibrium zero-resistance states (ZRS) have recently been identified by Mani and coworkers in irradiated quantum Hall systems based on GaAs/AlGaAs heterostructures. The effect was confirmed in ultra high mobility GaAs/AlGaAs quantum wells by Zudov et al., who cited the phenomenon as evidence for a new dissipation less effect in 2D electronic transport. Such ZRS are induced in the two dimensional electron gas (2DEG) by electromagnetic-wave excitation with the ac electric field parallel to 2DEG, in a weak, static, perpendicular magnetic field. Oscillations of the resistance induced by microwave excitation, in low mobility specimens, had been reported previously. There is strong experimental evidence that the ZRS coincide with a gap in the electronic spectrum, previously. There is strong experimental evidence that the ZRS coincide with a gap in the electronic spectrum, previously.

Mani and coworkers in irradiated quantum Hall systems

3, 4. There is strong experimental evidence that the ZRS coincide with a gap in the electronic spectrum, although the positions of the extrema remains controversial. Mani and coworkers find resistance minima/(maxima) at \( \omega/\omega_e = \epsilon = j + 1/4(j + 3/4) \), where \( \omega \) is the ac field frequency, \( \omega_e \) the cyclotron frequency and \( j = 1, 2, ..., \) is the difference between the indexes of the participating Landau levels (LLs). Zudov et al. report different periodicities for the maxima and minima, with maxima at \( \epsilon = j \) and minima on the high field side of \( \epsilon = j + 1/2 \). Interest in this novel ZRS has already produced an extensive list of preliminary results, which aim to establish a theory for understanding this remarkable effect.

We show that a full understanding, however, has to consider the effect from the point of view of the LLs dressed by photons due to the coupling to ac fields.

The experimental parameters reveal a rich physical scenario. In summary, the quantum Hall effects are observed at high magnetic fields \( (B > 0.4 \, \text{T}) \), for low magnetic fields \( (B < 0.4 \, \text{T}) \) new oscillations in the magnetoresistance are observed under radiation. Indeed, the resistance vanishes, in a given data collection at \( B \approx 0.2 \, \text{T} \), for a radiation frequency of \( \nu \approx 100 \, \text{GHz} \). At \( B \approx 0.2 \, \text{T} \) the LL separation is \( \hbar \omega_c \approx 0.35 \, \text{meV} \), while the microwave photons have energy of the same order, namely \( \hbar \nu \approx 0.4 \, \text{meV} \). At the measurement temperature \( T = 1.5 \, \text{K} \), LLs are still resolved, since Shubnikov-de Haas oscillations are still seen below \( B = 0.2 \, \text{T} \) in absence of radiation. On the other hand, these new oscillations are observable down to \( B \approx 0.02 \, \text{T} \), corresponding to a magnetic length \( l_c \approx 0.18 \mu \text{m} \) and a classical cyclotron radius up to \( R_c \approx 4.5 \mu \text{m} \), still small compared to a mean free path of \( l_0 \approx 140 \mu \text{m} \). Besides that, the used microwaves have frequencies down to \( \nu = 30 \, \text{GHz} \), corresponding to a wavelength up to \( \lambda = 10 \, \text{mm} \), approximately a factor of 2 or 3 larger than the linear dimensions of the sample, \( w \). Indeed, oscillations have been reported at frequencies down to \( 3 \, \text{GHz} \) \( (\lambda = 10 \, \text{cm}) \). A ratio \( \lambda/w \approx 10 \) validates a dipole approximation for the radiation-sample coupling. More important is that the estimated power level is of \( \leq 1 \, \text{mW} \), over a cross sectional area of \( \leq 135 \, \text{mm}^2 \) in the vicinity of the sample. This represents a field intensity of \( I = 7.4 \, \text{W/m}^2 \), which can be related to the associated electric field by \( E = E_{rms}^2/(c \mu_0) \), \( E_{rms} \approx 50 \, \text{V/m} \). Considering the classical cyclotron radius as the relevant length scale and a frequency of 100 GHz, this leads to a ratio \( eR_cE_{rms}/(\hbar \nu) \approx 0.35 \) at \( B = 0.2 \, \text{T} \). Such ratio, between the energy gained from the ac field over a distance corresponding to the cyclotron radius and the photon energy, represents already a field intensity that can not be considered perturbatively. We address this problem within a tight-binding approach, considering non-perturbatively the two main ingredients of the problem: Landau quantization and dressing of the electronic states by means of a coupling with the ac fields. The dressed electronic structure shows non-trivial features which are clear signatures of the newly observed resistance oscillations.

The present model is a finite tight-binding lattice coupled non-perturbatively to an ac field by means of the Floquet method. The time-independent infinite matrix Hamiltonian obtained from transforming the time-dependent Schrödinger equation, describes entirely these processes without any further \textit{ad-hoc} hypothesis. There-
fore, the effects of an intense ac field on the electronic spectra have to be described by very large truncated matrix Hamiltonian. Such numerical endeavour is possible by means of a renormalization procedure, providing the spectral modulation as function of field intensity, as well as the intensity hierarchy of the quasi-energy spectra related to different photon replicas. An ac field will be considered parallel to one of the square sides. Hence, the model for the bare electronic system coupled to an ac field is described by the Hamiltonian $H = H_0 + H_{int}$, where

$$H_0 = \sum_{l_1,l_2} \varepsilon_{l_1,l_2} \sigma_{l_1,l_2} \sigma_{l_1,l_2}^\dagger + \frac{V}{2} \sum_{l_1,l_2} [\sigma_{l_1,l_2} \sigma_{l_1+1,l_2}^\dagger + \sigma_{l_1,l_2+1} \sigma_{l_1,l_2}^\dagger] + \sigma_{l_1+1,l_2} \sigma_{l_1,l_2+1}^\dagger + e^{i2\pi\alpha l_1} (\sigma_{l_1,l_2} \sigma_{l_1+1,l_2} + \sigma_{l_1,l_2+1} \sigma_{l_1+1,l_2}^\dagger)$$

(1)

and

$$H_{int} = eaF \cos \omega t \sum_{l_1,l_2} \sigma_{l_1,l_2} l_1 \sigma_{l_1,l_2}^\dagger,$$

(2)

Here $\sigma_{l_1,l_2} = |l_1,l_2 >, \sigma_{l_1,l_2}^\dagger = < l_1,l_2 |$, where $(l_1,l_2)$ are the $(x,y)$ coordinates of the sites. The phase factor $\alpha$ is defined as $\alpha = \Phi/\Phi_e$, where $\Phi_e = h/e$ is the magnetic flux quantum, and $\Phi = a^2 B$ is the magnetic flux per unit cell of the square lattice. The atomic energy will be taken constant, $\varepsilon_{l_1,l_2} = 4|V|$, for all sites. The hopping parameter can emulate the electronic effective-mass for the GaAs bottom of the conduction band, $m^* = 0.067 m_0$. Since $V = -h^2/(2m^*a^2)$, $V = -0.142$ eV for a lattice parameter of $a = 20 \text{Å}$. The ac field is defined by its frequency and amplitude, $\omega$ and $F$, respectively. The treatment of the time-dependent problem is based on Floquet states $(l_1,l_2, m >$ where $m$ is the photon index. We follow the procedure developed by Shirley, which consists in a transformation of the time-dependent Hamiltonian into a time-independent infinite matrix. The elements of this infinite matrix are

$$\left[ (\mathcal{E} - m\hbar \omega - \varepsilon_{l_1,l_2}) \delta_{l_1,l_2} - \frac{V}{2} \left\{ \delta_{l_1,l_1+1} + \delta_{l_1,l_1+1} \right\} \right] \delta_{m,m'}$$

(3)

where $F_1 = \frac{1}{2} ea F$. The energy eigenvalues, $\mathcal{E} - m\hbar \omega$, are quasi-energies of a system dressed by photons, shifted by multiples of the photon energy, usually called as the $m$-th “photon replica” of the system, which are coupled by the ac field. Diagonalization of a truncated Floquet matrix involves dimensions given by $L^2(2M + 1)$. $L$ is the lateral size of the square lattice in number of atomic sites, while $M$ is the maximum photon index. Since the ac field couples a Floquet state defined by $m$ photons to states with $m - 1$ or $m + 1$ photons, multiple photon processes become relevant with increasing field intensity. As a consequence, $M$, which determines how many “photon replicas” are taken into account, increases with field intensity. A truncated Floquet matrix is a tridiagonal block matrix which contain $L \times L$ diagonal blocks given by $\mathcal{E}M = (\mathcal{E} - m\hbar \omega)I + H_0$ representing a photon replica with the matrix elements given by the left hand side of Eq.(3). The coupling of system with the intense ac electric field is represented by the off-diagonal blocks $\mathcal{F}$, which are diagonal block matrices, with the elements given by $\mathcal{F} = F_1 \delta_{l_1,l_1} \delta_{l_2,l_2}$. The dimension of the problem can be reduced to $L^2$ by means of a renormalization procedure, based on the definition of the associated Green’s function, $G$, where $FG = 1$.

A detailed discussion of this method is given in a previous work. The final result of this renormalization of the Floquet matrix is the dressed Green’s function for one of the photon replicas, say $M = 0$, and a quasi-density of Floquet states, $\rho(\mathcal{E} + i\eta)$ can then be obtained:

$$\rho(\mathcal{E} + i\eta) = -\frac{1}{\pi} \text{Im} \left[ \text{Tr} G_{MM} \right].$$

(4)

The trace of the Green’s operator is taken over the atomic sites basis.

A finite square lattice in the presence of a perpendicular magnetic field shows a rich “quantum dot-like” spectrum, Fig.1(a), with a low magnetic flux region dominated by finite sample size quantization (cyclotron radius large compared with the linear dimensions of the sample), as well as bulk LLs and edge states, well defined for higher magnetic fluxes. We consider a $L = 10a$ square lattice, a size limited by computational costs, focusing on the states collapsing into LLs for $\Phi/\Phi_e > 0.1$. In Fig.1(a) the lowest two LLs are well defined, with a ladder of edge states between them. The advantage of finite lattices is that the magnetic flux can be continuously varied, as in the experimental measurements, since commensurability effects are absent. On the other hand, the unavoidable presence of edge states could hinder the interpretation of the ac fields on the bulk LLs. Nevertheless, ac field effects on bulk LLs and edge states can be distinguished, as shown below.

The spectrum of the system depicted in Fig.1(a), modified by an ac field, is shown in Fig.1(b), for a photon energy $h\omega = 10 \text{meV}$, which is lower than the quantum-dot-like states separation at very low magnetic
fluxes and much lower than the LL separation at values of $\Phi/\Phi_0$ where the LLs start to be well defined. In Fig.1(b) the field intensity is $eaF = 5$ meV, where $a \approx l_c$ at $\Phi/\Phi_0 \approx 0.1$ [25]. This represents already a non-perturbative field intensity $eaF/\hbar \omega = 0.5$. A dramatic change in the quasi density of states can be observed, with a coupling between different photon replicas leading to a flattening of the states, opening of gaps in the lower part of the spectrum and a coupling between edge and bulk states induced by the ac field. In the energy scale of the figure $E/\hbar \nu = 1$ is the separation between successive photon replicas, which are successively less intense with increasing the photon index $m$. At higher magnetic fluxes photon replicas of the lowest bulk LL can be clearly followed. Increasing the field intensity leads to the formation of higher order photon replicas of the lowest LLs, as well as new periodic structure (as a function of magnetic flux) in the quasi-energy spectrum (not shown here).

$\Phi_c/\Phi_0 = 0.5$ (not shown here), $\Phi_c$ is the field intensity in absence of an ac field. Black(white) stands for highest(lowest) density of states. Top: spectrum for a square lattice with $L = 10a$ (see text) in absence of an ac field. Bottom: spectrum for the same system with an ac field with $h \nu = 10$ meV and $eaF = 5$ meV.

![Figure 1](image1.png)

**FIG. 1:** Spectra of the density of states as a function of magnetic flux. Black (white) stands for highest (lowest) density of states. Top: spectrum for a square lattice with $L = 10a$ (see text) in absence of an ac field. Bottom: spectrum for the same system with an ac field with $h \nu = 10$ meV and $eaF = 5$ meV.

These results are intrinsically interesting, but we should focus on the experimental conditions [1, 2], i.e., frequencies of the order of the LL separation. In Fig.2(a) we choose $h \nu = 150$ meV, namely the LL separation for $\Phi/\Phi_0 \approx 0.1$, a flux for which the lowest LLs are already well defined. The main feature for the present discussion is the avoided crossing between the second LL and the first photon replica of the lowest LL (plus one photon). This avoided crossing at $\Phi/\Phi_0 \approx 0.1$ can be distinguished from other features of the spectrum in spite of the rather complex quasi density of states due to the ac filed. This anticrossing at $E/\hbar \nu \approx 1.5$ is better observed in a zoom of the density of states shown in Fig.2(b). An avoided crossing can also be seen between the lowest LL and a first photon replica of the second LL (minus 1 photon in this case). A crucial point is that the field intensity for the case illustrated in Fig.2 is $eaF = 30$ meV, corresponding to $eaF/\hbar \omega = 0.2$. Indeed the avoided crossings are already defined for $eaF/\hbar \omega = 0.05$ (not shown here), a value one order of magnitude lower than the estimative for the actual experimental conditions discussed in the introduction.

Having in mind such anticrossings in the density of states, induced by the ac field, one can figure out a complete picture of the phenomenon in the sketch depicted in Fig.3. Here we represent photon replicas of LLs in absence of ac field coupling. The energies are given by $E_{m,n} = e_n \pm m \hbar \nu$, where $e_n = (n + 1/2)\hbar \omega_c$ are the LL energies, while $\pm m$ indicates the replicas obtained by adding/subtracting $m$ photons. Only 5 LLs are show for sake of clarity. On the other hand, only $m = 0, 1$ are considered, since only the coupling between “nearest-neighbor replicas” should be important at the field intensities considered, although higher order effects are expected for even higher field intensities. Notice that the crossings between $\Delta m = \pm 1$ occur only at $\omega/\omega_c = j$ (indicated by dashed vertical lines) The crossing of these LLs become anticrossings by turning on the ac field and the anticrossings would lead the a modulated spectrum with the periodicity given by $\omega/\omega_c = j$. The numerical calculation illustrated in Fig.2 corresponds to the last crossing in Fig.3 at $\omega_c = 1.0$. Oscillatory behavior of $dE/d\Phi$ are therefore expected only for lower magnetic fluxes. Oscillations at higher magnetic fluxes would be higher order effects ($\Delta m = \pm 2$). Such oscillating spectrum resembles the spectrum of a quantum ring pierced by a magnetic flux [27], which reveals a persistent current. The present results indicate that the recently observed ac field induced oscillations in the magnetoresistance are *ac field induced persistent current-like effects*. A connection between static magnetic flux and intense ac field effects has been recently suggested for quantum ring structures [28].

The richness of the ac field induced features on the spectrum of a 2D system threated by a perpendicular magnetic field deserves further investigations. One of these features, anticrossing between LL photon replicas, leads to a simple interpretation of the very inter-
FIG. 2: Top: spectrum for a square lattice with \( L = 10a \) (see text) with an ac field with \( h\nu = 150 \) meV and \( eaF = 30 \) meV. Bottom: zoom of the same density of states spectrum shown in the top panel in the range of one of the avoided crossings at \( E/h\nu \approx 1.5 \) and \( \Phi/\Phi_c \approx 0.1 \).

FIG. 3: \( E_{n,m} = (n+1/2)\hbar\omega_c \pm m h\nu \) (see text), as a function of the cyclotron frequency, \( \omega_c \). Only \( m = 0, \pm 1 \) photon replicas are considered. Vertical dashed lines represents \( \omega/\omega_c = j \). The lowest LL of each photon replica is highlighted as a guide for the eyes.

The discrepancy in the oscillation phase, as well as the huge activation energies need further investigations within the framework proposed in the present work. The authors thank R. G. Mani for fruitful discussions and critical reading of the manuscript. P. A. S. acknowledges financial support from the Brazilian agency FAPESP.

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