Comment on “Entry Distribution, Fission Barrier, and Formation Mechanism of $^{254}$No”

A recent paper [1] has reported the observation of the rotational band of $^{254}$No for spins up to $I=20$, showing that the compound nucleus was formed and survived fission decay at angular momenta $I \geq 20$. A microscopic description of $^{254}$No with the Gogny force predicts the observed survival of this nucleus against fission [2].

This finding may appear surprising at first, given the well known instability toward fission of these nuclei and the expected decrease in the fission barrier due to angular momentum. The question behind the surprise is why angular momentum, usually so effective in decreasing the fission barrier and in enhancing the fission to neutron emission branching ratio in lighter nuclei, appears here to be somewhat ineffective. The explanation of this puzzle is not only interesting for this case, but is also even more relevant for the resilience to angular momentum of superheavy nuclei.

To show the leading effects of angular momentum on the barrier height, we use perturbation theory to calculate the energy associated with the perturbation (rotational energy) using parameter values (moments of inertia, deformation, etc.) associated with no perturbation. This is the standard cranking approximation. Accordingly, the energy can be written as:

$$ E(\bar{\epsilon}) = E_0(\bar{\epsilon}) + \frac{I(I+1)\hbar^2}{2\mathcal{J}(\bar{\epsilon})} $$

where $\bar{\epsilon}$ is a generalized deformation vector, $E_0(\bar{\epsilon})$ and $\mathcal{J}(\bar{\epsilon})$ are the potential energy surface and moment of inertia at zero angular momentum.

As shown in Fig. 1, the decrease of the barrier height $\Delta B(I) = B_f(I) - B_f(0)$ due to angular momentum is

$$ \Delta B = \frac{h^2}{2} \left( \frac{1}{\mathcal{J}_g} - \frac{1}{\mathcal{J}_s} \right) I(I+1) $$

where $\mathcal{J}_g$ and $\mathcal{J}_s$ are the moments of inertia of the ground and saddle deformations. This decrease depends strictly on the values of the two moments of inertia at $I=0$ irrespective of their origin (liquid drop, shell effects, pairing, etc.). Higher order effects, such as changes in the ground and saddle deformation, and changes in the shell and pairing effects occur at higher angular momenta. The evidence for the goodness of the cranking approximation and thus for the lack of change of the shell effect in the ground state is evident in the fact that $^{254}$No (as well as most strongly deformed rare earth and actinide nuclei) is a good rigid rotor up to $I=20$. The moment of inertia changes by $\approx 10\%$ for $I=0$ to $I=20$.

In typical lighter nuclei, the saddle point is controlled by the liquid drop contributions and is found to be at large deformations. Therefore, $\mathcal{J}_g << \mathcal{J}_s$ and $\Delta B \approx \frac{h^2}{2} I(I+1)/2\mathcal{J}_g$. This produces the large effect of angular momentum on the fission barrier for lighter systems.

However, in trans-fermium nuclei, the ground state is already deformed at the values of $\epsilon$ typical of all actinides. The saddle occurs at a deformation only slightly greater, corresponding to the anti-shell immediately following the deformed minimum. We can rewrite Eq. (2) in terms of the fractional difference of the moments of inertia: small values of $\Delta \mathcal{J} = \mathcal{J}_s - \mathcal{J}_g$, obtaining

$$ \Delta B = \frac{h^2}{2} \left( \frac{1}{\mathcal{J}_g} - \frac{1}{\mathcal{J}_s} \right) I(I+1) \approx \frac{h^2}{2} \frac{\Delta \mathcal{J}}{\mathcal{J}_s} I(I+1) $$

where $\Delta \mathcal{J} = \mathcal{J}_s - \mathcal{J}_g$. Consequently, the decrease in barrier height is equal to the ground state rotational energy $E_{rot}^{gs} \approx 2.8$ MeV and $\Delta \mathcal{J}/\mathcal{J}_s \approx 0.40$ giving $\Delta B \approx 1.1$ MeV. These features are shown pictorially in Fig. 1. This estimate (1.1 MeV) of the change in fission barrier at $20\hbar$ is in excellent agreement with detailed calculations [2].

There is little doubt that the estimate described above explains the resiliency of the No barrier to angular momentum. The same arguments speak for a similar or even greater resilience in superheavy nuclei. Thus, we expect that we can safely graze in the pastures of the superheavy island of stability, without fear of (moderate) angular momentum values.

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