Two-photon total annihilation of molecular positronium

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Abstract – The rate for complete two-photon annihilation of molecular positronium Ps2 is reported. This decay channel involves a four-body collision among the fermions forming Ps2, and two photons of 1.022 MeV, each, as the final state. The quantum electrodynamics result for the rate of this process is found to be $\Gamma_{Ps2\rightarrow\gamma\gamma} = 9.0 \times 10^{-12} \text{s}^{-1}$. This decay channel completes the most comprehensive decay chart for Ps2 up to date.

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Introduction. – Positronium or Ps is the bound state of an electron and its antiparticle, the positron, forming a metastable hydrogen-like atom [1]. In the 1940s, Wheeler speculated that two Ps atoms may form molecular positronium Ps2, in analogy with two hydrogen atoms that can combine to form molecular hydrogen [2]. In the same decade, calculations of the binding energy of Ps2 were carried out, and it turned out to be 0.4 eV [3], supporting Wheeler’s prediction. More recently, in 2007, Cassidy and Mills reported the first observation of molecular positronium [4].

Molecular positronium can decay to different final states or channels. The characterization of the decay channels is essential in order to estimate its lifetime. Moreover, the complete characterization of the Ps2 decay channels and their partial widths could lead to the design of efficient detection schemes for this molecule. For a bound state, the total annihilation rate $\Gamma$ is determined as the sum of partial annihilation rates associated with each allowed decay channel $\Gamma_i$, i.e., $\Gamma = \sum_{i=1}^{N} \Gamma_i$, where $N$ represents the total number of decay channels. Each of the $\Gamma_i$ has to be computed by including all the topologically distinct Feynman diagrams associated with such channels, and in some cases, it can be important to include radiative corrections [5]. Frolov has reported the most complete chart of decay channels as well as partial annihilation rates up to date [6], including all the main decay channels, going from zero photon decay up to the 5-photon decay channel. However, a higher-order decay channel of Ps2 involving two-photons as the final state has not been considered in any estimation of the Ps2 lifetime, and apparently never previously contemplated as a possible decay channel.

The present study reports the calculation of the two-photon complete annihilation rate of Ps2, in which two electrons and two positrons annihilate simultaneously, producing two photons of 1.022 MeV energy each. The calculations have been carried out by using standard techniques of quantum field theory, such as the Feynman rules and trace technology [7]. This decay channel completes the decay chart of Ps2, previously reported in part by Frolov [6], besides the six-photon and seven-photon decay channels. While this decay is rare, it is worth mentioning that it provides a unique experimental signature of the presence of molecular positronium.

Two-photon annihilation of Ps2. – The annihilation of Ps2 into two photons (denoted here as $Ps2\rightarrow\gamma\gamma$) is governed by eight topologically distinct Feynman diagrams. Four of them are shown in fig. 1. The rest of the diagrams emerge as cross-terms of the ones shown in fig. 1, i.e., in which the momenta of the outgoing photons are interchanged. Figure 1 shows that the decay channel at hand is a four-body event, where the energy-momentum vectors of the incoming fermions are labelled as $p_1$, $p_2$, $p_3$, and $p_4$, whereas the energy-momenta of the outgoing photons are labelled as $k_1$ and $k_2$. Here, the energy-momentum vectors are represented as $(E, \vec{p})$, and natural units ($\hbar = c = 1$, with $\alpha \simeq 1/137$, being the fine-structure constant) are assumed.

The momenta of the electrons and positrons in Ps2 are very low in comparison with their rest mass energy. Hence the binding energy of the Ps2 molecule is negligible in

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The quantity $|\Psi_{Ps2}(0,0,0,0)|^2$ represents the probability of finding the four fermions at the same point in position space. Some details about its calculation are given below. Equation (1) can be viewed as an extension of the previous generalization of Kryuchkov [8] where a three-body initial state was taken into account for the single-photon decay of $Ps^-$. $M$ represents the transition matrix associated with the decay channel, and therefore $|M|^2$ represents the probability for such a transition. It is obtained by averaging the squared modulus of the total amplitude $\mathcal{A}$ over the spin states of the incoming particles $(e^{-}(p_1, s_1), e^{+}(p_2, s_2), e^{+}(p_4, s_4), e^{-}(p_3, s_3))$, here $s_i$ represents the spin of each particle and by summing over the polarizations of the outgoing particles $(e(k_1), e(k_2))$, here $e(k_i)$ denotes the polarization of each photon), i.e.,

$$|\mathcal{M}|^2 = \sum_{\epsilon(k_1)\epsilon(k_2)} \sum_{s_1s_2s_3s_4} \frac{1}{24} \sum_{k_1k_2} \sum_{k_3k_4} |\mathcal{A}|^2. \quad (2)$$

The amplitude $\mathcal{A}$ associated with the decay channel $Ps_2 \rightarrow \gamma\gamma$ contains eight terms, each of them associated with Feynman diagrams that contributes to the process (see fig. 1). The amplitude is given by

$$\mathcal{A} = e^4 \left[ v(p_4, s_4)\gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + m_e}{(p_3 - k_1)^2 - m_e^2} \gamma^\nu \epsilon_\nu(k_1) u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\nu \epsilon_\nu(k_1) \frac{p_3 - \not{k}_1 + m_e}{(p_4 - k_1)^2 - m_e^2} \gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\nu \epsilon_\nu(k_1) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu \epsilon_\nu(k_1) u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\nu \epsilon_\nu(k_1) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\nu \epsilon_\nu(k_1) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu \epsilon_\nu(k_1) u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\nu \epsilon_\nu(k_1) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right. \left. + v(p_4, s_4)\gamma^\nu \epsilon_\nu(k_1) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\lambda \epsilon_\lambda(k_2) \frac{p_3 - \not{k}_1 + \not{k}_2 + m_e}{(p_4 - k_1 - k_2)^2 - m_e^2} \gamma^\nu u(p_3, s_3) \right. \times \left. \frac{g_{\mu\nu}}{(p_1 + p_2)^2} v(p_2, s_2)\gamma^\mu u(p_1, s_1) \right]. \quad (3)$$

Here the Feynman gauge has been employed as well as the slashed notation, i.e., $\not{p}^\mu = \gamma^\mu p^\mu$. The $\gamma$ matrices are the Dirac matrices as defined in ref. [7]. Once the amplitude $\mathcal{A}$ of the process is known, the transition probability associated with the decay channel at hand can be found by

\[ \Gamma_{Ps2\rightarrow\gamma\gamma} = \frac{|\Psi_{Ps2}(0,0,0,0)|^2}{4} \int \frac{d^3\vec{k}_1}{(2\pi)^3 2|k_1|} \frac{d^3\vec{k}_2}{(2\pi)^3 2|k_2|} \left( \prod_{i=1,4} (2E_i) \right) |\mathcal{A}|^2. \quad (1) \]
Two-photon total annihilation of molecular positronium

After inserting the probability to find the four fermions at the same point $|\Psi_{PS_2}(0,0,0,0)|^2$, using the relation between atomic units and natural units, and taking into account eq. (4), we find $\Gamma_{PS_2\rightarrow\gamma\gamma} = 9.0 \times 10^{-12} \text{s}^{-1}$. This decay rate is smaller than the alternative decay channels explored thus far, and which have been previously reported by Frolov [6]. Table 1 shows a comparison between the rate for the two-photon decay and all the decay channels previously reported. Table 1 implies that the rate reported here, although smaller than the rest, is still comparable with the zero-photon decay channel. It is related with the number of vertices in each decay channel. The zero-photon decay involves three vertices, whereas the two-photon decay channels require four vertices. This difference implies that $|\mathcal{M}|^2$ has an extra factor of $\alpha$ for the case of two-photon decay, in comparison with the zero-photon decay.

**Conclusions.** – The two-photon annihilation rate of $PS_2$ has been calculated using a non-relativistic reduction of quantum electrodynamic methods. This annihilation process refers to the simultaneous decay of two electrons and two positrons into two photons, providing a rare but unambiguously unique signature of the presence of the $PS_2$ molecule. All the Feynman diagrams contributing to such process have been taken into account for the calculation of the transition probability. The wave function for the ground state of $PS_2$ has been calculated by employing correlated Gaussian basis functions in combination with hyperspherical coordinates [11]. The annihilation rate for this process turns out to be $\Gamma_{PS_2\rightarrow\gamma\gamma} = 9.0 \times 10^{-12} \text{s}^{-1}$. While this value is smaller than that of other decay channels of $PS_2$, it is nevertheless in the same range as the rate associated with the zero-photon decay [6].

The observation of the event studied here will be very challenging due to its very long lifetime. However, from a fundamental point of view, the two-photon annihilation of $PS_2$ constitutes a way to sample the $PS_2$ wave function, from a four-body perspective, yielding crucial information about the nature of the bound state. Finally, we point out that in some astrophysical regions such as near the galactic center where a high density of positrons and electrons is available, this event may be observed, due to its unique emission signature of two photons with energies equal to 1.022 MeV. This region of the gamma-ray spectra remains largely unexplored to date, although the International Gamma-Ray Astrophysics Laboratory (INTEGRAL) telescope has the capability for it. Indeed this telescope has found the signatures of two-photon annihilation in Ps [12].

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