Design of Attitude Controller of Two-Dimensional Correction Projectile Based on UAS-SMC Method

Zhong Yang\textsuperscript{1,a,*}, Wang Liangming\textsuperscript{1,b} and Fu Jian\textsuperscript{1}

\textsuperscript{1}School of Energy and Power Engineering, Nanjing University of Science and Technology, Nanjing, China

\textsuperscript{*}E-mail: afishing0508@163.com; \textsuperscript{b}lmwang802@163.com

Abstract. For the spin stabilized two-dimensional trajectory correction projectiles with nutation and precession phenomenon, an attitude controller with sliding mode control method and unidiational auxiliary surfaces (UAS-SMC) is proposed in this paper. And the underactuated control allocation in two-dimensional trajectory correction projectile with UAS-SMC controller is also discussed here. Simulation results show that the proposed controller can effectively control the attitude of the projectile and has strong robustness, which provides reference for engineering applications.

1. Introduction

For the low-speed fin-stabilized projectiles, the relationship between the direction of the control moment and the direction of the projectile swing is simple and clear. For high-elastic bombs, the gyro effect causes the two-dimensional trajectory correction projectiles to produce nutation and precession under the action of control moments, which leading to the control relationship becomes very complicated. At the same time, the addition of correction components to the head of the projectile will cause mass eccentricity, installation errors, and complex aerodynamic characteristics, which cause model uncertainty and external disturbance. Therefore, the robustness of the controller algorithm is extremely high.

In practical engineering applications, traditional sliding mode control methods have unacceptable chattering phenomena. This chattering phenomenon may provoke vibrations in the unmodeled dynamics of the system, thereby reducing the performance of the control system. In order to solve this problem, experts and scholars from various countries have proposed control methods such as boundary layer sliding mode\cite{1}, high-order sliding mode\cite{2}, and double sliding mode\cite{3}, but these control methods require more or less different costs for the system.

Aiming at this problem, this paper proposes a new sliding mode control method, UAS-SMC by modifying the control structure of the traditional sliding mode by modifying the control system robustness and constraint ability. Under the premise of retaining the strong robustness of the traditional sliding mode, the method has no chattering of the controller, and the approaching speed near the switching surface is strengthened. No differential information is needed, and there is a positive invariant set in the system state space. The UAS-SMC improves the control performance while taking into account the application requirements in engineering practice, without the additional cost of the controller. Therefore, it has broad application prospects in practical engineering practice.
The nonlinear attitude model [4-5] of trajectory correction projectiles is given as follows:

\[
\mathbf{\dot{\Omega}} = \mathbf{f}_x + \mathbf{g}_x \mathbf{\omega} \\
\mathbf{\dot{\omega}} = \mathbf{f}_y + \mathbf{g}_y \mathbf{M} + \mathbf{d}(t)
\]  

(1)

where

\[
\mathbf{f}_x = \begin{bmatrix}
-\frac{1}{mv \cos \beta} (F_x \sin \alpha + F_y \cos \alpha) + \frac{g}{v \cos \beta} (\sin \beta \sin \alpha + \cos \beta \cos \alpha) \\
\frac{1}{mv} (-F_x \cos \alpha \sin \beta + F_y \sin \alpha \sin \beta + F_z \cos \beta) + \frac{g \sin \beta}{v} (\cos \alpha \sin \beta - \sin \alpha \cos \beta)
\end{bmatrix},
\]

\[
\mathbf{g}_x = \begin{bmatrix}
\tan \beta (\cos \alpha - \cos \alpha \tan \beta) & 1 \\
0 & (\cos \alpha + \sin \alpha \tan \beta)
\end{bmatrix},
\]

\[
\mathbf{f}_y = \begin{bmatrix}
\frac{M_z - (I_z \omega_z + I_y \omega_y) \omega_z}{I_y} + \omega_x \omega_z \tan \beta \\
\frac{M_z - (I_z \omega_z + I_y \omega_y) \omega_y}{I_y} - \omega_x^2 \tan \beta
\end{bmatrix},
\]

\[
\mathbf{g}_y = \begin{bmatrix}
1 / I_x & 0 \\
0 & 1 / I_z
\end{bmatrix},
\]

\[
\mathbf{M}_e = \begin{bmatrix}
\mathbf{M}_x \\
\mathbf{M}_y \\
\mathbf{M}_z
\end{bmatrix},
\]

\[
\mathbf{\Omega} = \begin{bmatrix}
\alpha \\
\beta
\end{bmatrix}^T \text{ are the attitude angles, } \mathbf{\omega} = \begin{bmatrix}
\omega_x \\
\omega_y \\
\omega_z
\end{bmatrix}^T \text{ are the angular rates, } \mathbf{d}(t) = \begin{bmatrix}
d_1 \\
d_2
\end{bmatrix}^T \text{ are the unknown torque disturbances in attitude motion of the projectile. Attitude tracking command signal } \mathbf{\Omega} = \begin{bmatrix}
\alpha \\
\beta
\end{bmatrix}^T .
\]

In order to design the UAS-SMC controller for trajectory correction projectiles (1), the following reasonable assumptions are needed.

a) Assume that the attitude tracking command signals \( \mathbf{\Omega} = [\alpha, \beta]^T \) are always continuous.

b) Assuming that matrix \( \mathbf{g}_x, \mathbf{g}_y \), always reversible during system motion.

c) Assume that all elements \( \mathbf{g}_x, \mathbf{f}_x, \mathbf{g}_y \) in the matrix are continuous functions.

3. Design of Control System Based on UAS-SMC

The tracking errors of two-dimension trajectory correction projectile are defined as

\[
\mathbf{e}_1 = \mathbf{\Omega} - \mathbf{\Omega} \\
\mathbf{e}_2 = \mathbf{\omega} - \mathbf{\omega}_1
\]  

(2)

where \( \mathbf{e}_1 = \begin{bmatrix} e_{11}, e_{12} \end{bmatrix}^T \in \mathbb{R}^2 \) is the attitude angle error, \( \mathbf{e}_2 = \begin{bmatrix} e_{21}, e_{22} \end{bmatrix}^T \in \mathbb{R}^2 \) is the angular rate error, \( \mathbf{\omega}_1 \) is the virtual control law of the design. Then equation (1) can be converted into the form of error equation:

\[
\mathbf{\ddot{\mathbf{e}}}_1 = \mathbf{f}_x + \mathbf{g}_x (\mathbf{\mathbf{e}}_1 + \mathbf{\mathbf{v}}_1) - \mathbf{\ddot{\mathbf{\mathbf{e}}}}_1, \mathbf{\ddot{\mathbf{e}}}_2 = \mathbf{f}_y + \mathbf{g}_y \mathbf{M} + \mathbf{d}(t) - \mathbf{\ddot{\mathbf{e}}}_2
\]  

(3)

The virtual control law is designed for the attitude loop error in equation (3). Switching surface is

\[
s_{11} = e_{11} + \xi_{11} \int e_{11} = 0, s_{12} = e_{12} + \xi_{12} \int e_{12} = 0
\]  

(4)

where \( \xi_{11} = \text{diag} [\xi_{111}, \xi_{112}], \xi_{12} = \text{diag} [\xi_{121}, \xi_{122}], s_{11} = [s_{111}, s_{112}]^T, s_{12} = [s_{121}, s_{122}]^T, \xi_{111}, \xi_{122} > 0, i \in \{1, 2\} .
\]

The auxiliary surface \( h_i \) is designed as follows with two auxiliary parameters \( \alpha_{101}, \alpha_{3n2} \) and two switching surface parameters \( \xi_{111}, \xi_{122} \):

\[
h_{ii} = \alpha_{101} e_{1i} + \alpha_{3n2} \int e_{1i} + m_{ii}
\]  

(5)

where \( \alpha_{hi} \neq 0 \) and
\[
\begin{align*}
\alpha_{t1} &= \begin{cases} 1 & s_{11} < 0, s_{12} < 0 \\
\omega_{111} & s_{11} < 0, s_{12} = 0 \\
\alpha_{212} & s_{11} \geq 0, s_{12} < 0 \\
-1 & s_{11} \geq 0, s_{12} \geq 0
\end{cases}, \\
\alpha_{t2} &= \begin{cases} \omega_{m2} & s_{11} < 0, s_{12} < 0 \\
\omega_{112} & s_{11} < 0, s_{12} = 0 \\
\omega_{222} & s_{11} \geq 0, s_{12} < 0 \\
-\omega_{212} & s_{11} \geq 0, s_{12} \geq 0
\end{cases}, \\
\alpha_{t3} &= \begin{cases} \omega_{m3} & s_{11} < 0, s_{12} < 0 \\
\omega_{113} & s_{11} < 0, s_{12} = 0 \\
\omega_{223} & s_{11} \geq 0, s_{12} < 0 \\
-\omega_{213} & s_{11} \geq 0, s_{12} \geq 0
\end{cases}.
\end{align*}
\]

The attitude control loop virtual control law is shown below.

\[v_i = g^{-1}(-f_i + \Omega_i - \Omega_i^{-1} \Omega_{ii} e_i + \Omega_{ii}^{-1} N_i)\]   \hspace{1cm} (6)

where \(N_i = [N_{1i}, N_{2i}]^T\) is the design approach law and satisfies \(N_{1i} > 0, N_{2i} > 0\). The compacted form for the Unidirectional auxiliary surface is as follows.

\[h_i = \Omega_i e_i + \Omega_{ii}^{-1} N_i \]   \hspace{1cm} (7)

where \(h_i = [h_{1i}, h_{2i}]^T\Omega_{ii} = \text{diag} (\omega_{11i}, \omega_{12i})\), \(\Omega_{ii} = \text{diag} (\omega_{11i}, \omega_{12i})\), \(m_i = [m_{1i}, m_{2i}]^T\). From equation (5), it is noted that \(\omega_{ja} \neq 0\), and the matrix \(\Omega_{ii}\) is a reversible matrix.

It can be known from equation (5), equation (6) and equation (7), there is the following relationship.

\[h_i = N_i + \Omega_{ii} g, e_i\] \hspace{1cm} (8)

The controller is designed for the angular rate loop error equation in equation (3). Switching surface is

\[S_{21} = e_1 + \xi_{21} \int e_1 \, dt = 0, S_{22} = e_2 + \xi_{22} \int e_2 \, dt = 0\]   \hspace{1cm} (9)

where \(\xi_{21} = \text{diag} (\xi_{211}, \xi_{212})\), \(\xi_{22} = \text{diag} (\xi_{221}, \xi_{222})\), \(S_{21} = [s_{211}, s_{212}]^T\), \(S_{22} = [s_{221}, s_{222}]^T\), \(\xi_{21i} > 0, i \in \{1, 2\}\)

In the same way, design auxiliary parameters \(\omega_{20i}, \omega_{21i}, \xi_{21i}, \xi_{22i}\) can complete the design of the auxiliary surface.

\[h_{2i} = \omega_{20i} e_{2i} + \omega_{21i} \int e_{2i} \, dt + m_{2i}\] \hspace{1cm} (10)

where

\[\omega_{20i} = \begin{cases} 1 & s_{21i} < 0, s_{22i} < 0 \\
\omega_{201} & s_{21i} < 0, s_{22i} = 0 \\
\omega_{221} & s_{21i} \geq 0, s_{22i} < 0 \\
-1 & s_{21i} \geq 0, s_{22i} \geq 0
\end{cases}, \quad \omega_{21i} = \begin{cases} \omega_{20i} & s_{21i} < 0, s_{22i} < 0 \\
\omega_{212} & s_{21i} < 0, s_{22i} = 0 \\
\omega_{222} & s_{21i} \geq 0, s_{22i} < 0 \\
-\omega_{212} & s_{21i} \geq 0, s_{22i} \geq 0
\end{cases}, \quad \omega_{22i} = \begin{cases} \omega_{20i} & s_{21i} < 0, s_{22i} < 0 \\
\omega_{212} & s_{21i} < 0, s_{22i} = 0 \\
\omega_{222} & s_{21i} \geq 0, s_{22i} < 0 \\
-\omega_{212} & s_{21i} \geq 0, s_{22i} \geq 0
\end{cases}.
\]

Angular rate loop controller is

\[M_i = g^{-1}(-f_i + \dot{\theta} - \Omega_{ii} \Omega_{ii}^{-1} e_i + \Omega_{ii}^{-1} N_i)\] \hspace{1cm} (11)

where \(N_i = [N_{1i}, N_{2i}]^T\) is the reaching law and satisfies \(N_{1i} > 0\). The linear equation of the Unidirectional auxiliary surface is

\[H_{2i} = \omega_{20i} e_{2i} + \omega_{21i} \int e_{2i} \, dt + m_{2i}\] \hspace{1cm} (12)
where \( k \in \{0, 1, 2, 3\} \), \( \omega_{2ki} \neq 0 \) and \( m_{2i} > 0 \) are real numbers. The current unidirectional auxiliary sliding surface linear equation is

\[
h_2 = \Omega_{2i} e_i + \Omega_{2i} \int e_i + m_2
\]

where \( h_2 = [h_{21}, h_{22}]^T \), \( \Omega_{21} = \text{diag}\{\omega_{211}, \omega_{221}\} \), \( \Omega_{22} = \text{diag}\{\omega_{212}, \omega_{222}\} \), \( m_2 = [m_{21}, m_{22}]^T \). It can be seen by the equation (13) that \( \omega_{2ki} \neq 0 \) and \( \Omega_{2i} \) is a reversible matrix.

Theorem 1 [6] Considering system (1), the closed-loop system is stable under control (11) if the switching surfaces \( S_{1i} = 0, S_{12} = 0, S_{21} = 0, S_{22} = 0 \) are stable and the approaching laws \( N_1 = [N_{11}, N_{12}]^T, N_2 = [N_{21}, N_{22}]^T \) satisfy \( N_{11} > 0, N_{12} > 0, N_{21} > 0, N_{22} > 0 \).

4. Control Allocation
As shown in Figure 1, the projectile correction component is mounted on the head of the projectile, consisting of four fixed rudders, two of which are in the same direction (control rudder) and two in opposite directions (differential rudder). The control rudder can generate control moment to change the attitude of projectile, and the differential rudder provides a torque moment which can change the direction of control aerodynamics. By the control rudder and differential rudder, two-dimensional trajectory correction projectiles can achieve the purpose of trajectory correction.

![Figure 1. Schematic diagram of two-dimensional trajectory correction projectiles.](image)

It is known from equation (1) that the desired control torque generated by the controller is \( \mathbf{M}_c = [M_{cy}, M_{cz}]^T \) and meet the following relationship.

\[
M_{cy} = \frac{1}{2} \rho v^2 s \sin \gamma_t \delta_e \cos \gamma_t, \quad M_{cz} = \frac{1}{2} \rho v^2 s \sin \gamma_t \delta_e \sin \gamma_t
\]

where \( \gamma_t \) is the roll angle of the correct component. The roll angle is expressed as

\[
\gamma_t = \begin{cases} 
0 & M_{cy} \leq 0, M_{cz} > 0 \\
\arctan \left( \frac{M_{cy}}{M_{cz}} \right) & M_{cy} \leq 0, M_{cz} < 0 \\
\pi - \arctan \left( \frac{M_{cy}}{M_{cz}} \right) & M_{cy} \geq 0, M_{cz} < 0 \\
2\pi - \arctan \left( \frac{M_{cy}}{M_{cz}} \right) & M_{cy} \geq 0, M_{cz} > 0 \\
\frac{\pi}{2} & M_{cy} < 0, M_{cz} = 0 \\
3\pi / 2 & M_{cy} > 0, M_{cz} = 0
\end{cases}
\]

where \( \arctan(\cdot) \) represents the arctangent function.

5. Simulation and Analysis of Trajectory Correction Projectile Controller
The actual control moment \( \mathbf{M}_{\text{control}} \) is shown below.
\[ M_{\text{real}} = k_i M_c \]  \hspace{1cm} (16)

where \( k_i = \sqrt{\frac{M_x^2 + M_y^2}{0.5\rho v_f^2 S l C_{\delta_f}}}. \) First, substituting the desired control torque into equation (16), then substituting \( M_{\text{real}} \) for \( M_c \) with equation (1). Finally, the closed loop control equation under the actual control torque is obtained. The specific simulation results are as follows.

**Figure 2.** Dynamic inverse controller without control allocation.

**Figure 3.** Dynamic inverse controller with control allocation.

**Figure 4.** UAS-SMC controller without control allocation.
Simulation results of attitude control system based on Dynamic inverse controller and UAS-SMC controller are shown in Figure 2 to Figure 6. As can be seen from Figure 2 and Figure 3, the open loop control based on the inverse system is less effective. Moreover, the control effect of controlling the magnitude and direction of the moment is better than the control effect of controlling only the moment direction. However, if the magnitude and direction of the control moment are to be achieved, it is necessary to achieve this through the equivalent periodic force control. This control method needs very large moment which is very difficult for the motor. Therefore, this paper adopts a control method that the direction of control moment can be variable and the magnitude is not variable. Figure 4 shows the UAS-SMC controller without control allocation, and it works well under ideal conditions. But in practice, you need to consider control allocation. Figure 5 gives a result of UAS-SMC controller with control allocation, and it can effectively control the attitude of the projectile. Figure 6 shows the result of UAS-SMC controller with disturbances of 5% aerodynamics error, and it proved strong robustness and the control effect can meet the system requirements.

6. Conclusions
Aiming at the nonlinear attitude control system of ballistic correction projectile, a backstepping unidirectional sliding mode control method is developed, and a nonlinear attitude control system is designed. Finally, verify its control effect through simulation. The simulation results show that open loop control based on inverse system is less effective, and the control effect of controlling the
magnitude and direction of the moment is better than the control effect of controlling only the moment direction. In fact, in order to meet the low-cost design requirements in the design, it is recommended to use a control method with variable torque direction and variable magnitude. Results of UAS-SMC controller with control allocation and aerodynamic disturbance show that the method has strong robustness and the control effect can meet the system requirements.

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