Interaction of metastable shock waves with perturbations

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Abstract. The shock wave metastable behavior in equilibrium media is detected in numerical experiments. This phenomenon is directly related to the ambiguous representation of a shock wave discontinuity when the shock belongs to the S-shaped fragment of the pressure-velocity Hugoniot. After interaction with weak perturbations the metastable shock wave recovers its original form and parameters (like stable shock). If the amplitude of the perturbations exceeds some critical value, the shock wave instability is developed. The instability is accompanied by formation of the transverse secondary waves switching local post-shock parameters between two Hugoniot segments separated from each other by the absolute instability region. The problem of the metastable shock wave interaction with local entropy perturbation passing through the shock front is studied in the framework of three-dimensional problem formulation. The shock wave transition to the limiting self-oscillating mode is described. Some features of this mode are completely determined by the shape of the Hugoniot and isentropes and does not depend on the shock wave structure. Namely, the shape of the shock front corrugations and flow structure evolution are analyzed. These features may be served as marker of the corresponding thermodynamic anomalies.

1. Introduction

The stability of shock waves in media with anomalous thermodynamic properties has remained the focus of research for several decades, beginning with the works of S P Dyakov [1] and V M Kontorovich [2], in which, within the framework of a linear theory, the criteria for the stability of shock waves for two-dimensional perturbations were first formulated. The results has been confirmed by another authors in the framework of different mathematical approaches [3–5] and extended to relativistic hydrodynamics [6–8].

According to the linear shock wave stability theory there are two cases of shock wave stability violation: neutral stability and instability. The condition of neutral stability of a shock wave is most often fulfilled for real media (see, for example, [9, 10]). The neutrally stable shock waves (observed in real gas, plasma etc) are characterized by the presence of a complicated pattern of secondary waves. The properties of such waves have been studied numerically [11,12]. Formation of the secondary waves during neutrally stable shock-vortex interaction was modeled in [13]. A structural mechanism of undamped secondary waves is proposed in [14]. In addition to the case of neutral stability, there are two instability conditions \( L < -1 \) and \( L > 1 + 2M \), where \( L \) is Dyakov’s parameter [1], \( M \) is the Mach number of the post-shock flow in the shock–attached frame. The sections of the Hugoniot curve on which the instability conditions are fulfilled...
are shown to be overlapped by sections with an ambiguous representation of the shock-wave discontinuity [15–17].

In the region of the ambiguous representation of the shock-wave discontinuity associated with the instability region $L < -1$, the viscous profile (solution in the form of traveling wave which describes the shock structure) of the shock wave does not exist, see, e.g., [18]. In this case, instead of a single shock wave, a composite compression wave is realized, which includes shock waves and isentropic compression waves (an example is the shock waves in hot nuclear matter accompanied by the quark-hadron phase transition in the framework of the hydrodynamic approximation [19, 20]). Numerical calculations showed that the numerical viscosity of the upwind shock-capturing numerical scheme is sufficient to provide the shock wave splitting [21].

Otherwise, under the condition of an ambiguous representation of the shock discontinuity associated with the absolute instability condition $L > 1 + 2M$, there exists a one-dimensional viscous shock wave profile; however, it turns out to be substantially unstable with respect to one-dimensional perturbations on the interval $L > 1 + 2M$ [22]. Numerical calculations of perturbed shock waves under condition of ambiguity of the shock-wave discontinuity representation in the model of a viscous heat-conductive gas are presented in [23]. The perturbations had the form of small amplitude compression and rarefaction waves with normal incidence direction. The calculations with model equation of state showed that in the case $L > 1 + 2M$, the shock waves split with formation one of the two possible wave-split configurations, first of which includes two diverging shock waves and the second one includes a diverging shock and a rarefaction wave. The choice of the resulting wave-split configuration is shown to depend on the perturbation. The shock waves satisfying instability criterion $L > 1 + 2M$ are not formed in any shock wave splitting processes in the associated region of ambiguous representation of a shock-wave discontinuity.

Shock waves with a shock state behind the instability segment either split or behave as a stable shock depending on the shape (compression or rarefaction) and the amplitude of the perturbation. Namely, the perturbations, which have the form of compression wave, stimulate splitting of the shock waves in the lower branch of the ambiguity segment. The resulting wave-split configuration includes two diverging shock waves. The perturbations, which have the form of rarefaction wave, stimulate splitting of the shock waves in the upper branch of the ambiguity segment. The resulting wave-split configuration is one including shock and rarefaction waves.

The numerical study of the shock waves in presence of multidimensional periodic perturbations has shown that a non-stationary mode of the shock wave propagation similar to case of cellular detonation is formed. In [23], the problem formulation was considered in which the multidimensional harmonic perturbation was introduced by the deviation of the shape of the shock wave. Thus, the perturbation was non-local and strong enough to initiate the unstable mode. In this paper, we consider the interaction of a shock wave with a compact perturbation under conditions where the final state belongs to the region of an ambiguous representation of the shock-wave discontinuity and at the same time is beyond the instability segment of the Hugoniot. This is necessary for understanding accompanying phenomena, such as specific secondary waves and possible transition to the self-oscillating mode in case of the symmetry conditions at the side boundaries. These phenomena may be served as marker of fulfillment of $L > 1 + 2M$ shock wave instability condition on the Hugoniot.

2. Equation of state
A model medium is considered which admits an S-shaped shock adiabat constructed in $p$–$u$ variables with the fulfillment of the shock wave instability criterion $L > 1 + 2M$. A modification of the equation of state proposed in [24] is used, which was previously used to study the behavior of shock waves when the instability criteria obtained in the framework of the linear shock wave stability theory were fulfilled on the shock adiabat (see [21,23]). Consider the basic parameters of the model equation of state. To calculate adiabatic flows, it is sufficient to have caloric equation
of state (EOS). Let us start from the EOS with conventional convex shape of isentropes, but with rapid diminishing of compressibility of the matter, at \( p \approx 1 \),

\[
V = V_0 - C \sqrt{\pi} \text{erf}(p),
\]

where

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt
\]

is the error function, \( C = \text{const}, \ p \) and \( V \) are dimensionless pressure and specific rest mass volume, accordingly. Corresponding internal energy, satisfying the condition \( e = 0 \) at \( p = 0 \), is given by \( e = C[1 - \exp(-p^2)] \). In accordance with [24], the anomalies of the shock curves, which lead to the shock wave instabilities, may be achieved by introducing the perturbation of this equation of the form \( C = 4 - \exp(-(V - 4)^2) \). The considered equation of state has segments in which the shock stability criteria are violated. Finally, the model equation of state reads

\[
e(V, p) = \left[1 - \exp(-p^2) + kp^2V\right]\{4 - \exp(-(V - 4)^2)\},
\]

where \( e \) is the internal energy, \( V = 1/\rho \), \( \rho \) is the density, \( p \) is the pressure. In particular case \( k = 0, \xi = 1 \), this equation becomes the model equation of state [24]. The introduced term \( kp^2V \) allows changing the shape of the shock adiabat, affecting the shock wave stability conditions.

The features of the behavior of shock waves studied in the work are due to the shape of the shock adiabats and Poisson adiabats, and the possible influence of the structural factor is limited (see below for an analysis of the interaction of the shock wave with incoming disturbances in the form of acoustic compression and rarefaction waves). For definiteness, we consider shock waves with a viscous structure and a non-negatively defined matrix of viscous coefficients (see [23]).

Numerical tests show that the change in viscosity does not significantly affect the cellular detonation structure described in [23]. This structure arises in the case of harmonic perturbation of the shape of the shock wave front under conditions of an ambiguous representation of the shock-wave discontinuity associated with the fulfillment of the condition \( L > 1 + 2M \). Corresponding initial-value problem was formulated in [23]. Comparison of the density fields for this problem at the same point in time and different viscosity is shown in figure 1.

In this paper we use dimensionless variables. The scales of the variables are defined as follows. The length scale is \( l_m = R \), where \( R \) is the radius of a sphere or a cylinder in which the density perturbation is defined; the time scale is given by \( t_m = l_m/u_m, \ u_m = (p_m/\rho_m)^{1/2} \). For the sake of correspondence to [24] the scales of pressure and density are defined as follows \( p_m = 10p_0 \) and \( \rho_m = 5.49\rho_0 \), where \( \rho_0 \) and \( p_0 \) correspond to the initial state of the model Hugoniot. Under this definition of scales the extension of the \( L > 1 + 2M \) instability segment on the model Hugoniot in pressures and specific volumes is of order of 1.

3. Interaction of plane metastable shock wave with acoustic waves coming to the shock front from the side of the compressed matter

The shock adiabat, which has an S-shape in the variables \( p-u \) (similar to that shown in figure 2), was considered, and the shock adiabat has two tangent points (E and F) with isentropes corresponding to a family of waves of the opposite directions. Figure 2 shows the structure independent mechanism of splitting of a metastable shock wave under the influence of the acoustic perturbations coming from the side of the compressed substance. Consider the case in which the metastable shock wave corresponds to a point located on the shock adiabat below point E (point A). If the amplitude of the compression wave arriving at the shock wave exceeds \( \Delta p_A \), the problem of the interaction of the compression wave with the original shock wave has a unique solution independent of structural factors.
Figure 1. Dependence of the numerical solution for cellular detonation-like structure on the model viscosity. Density for the same point in time is shown.

Figure 2. Interaction of metastable shocks with acoustic waves.

The solution is a wave configuration that includes two diverging shock waves separated by a contact discontinuity. From the construction in figure 2, where $A'G$ is the segment of the shock adiabat drawn from point $A'$, it follows that the pressure and velocity in the region between the shock waves correspond to the point $G$ on the initial shock adiabat. The similar result takes place for shock waves corresponding to the Hugoniot segment located above the upper instability boundary (point $F$). This case is considered by the example of a shock wave with a
final state at point B. If the amplitude of the perturbation in the form of a rarefaction wave exceeds $\Delta p_B$ (see figure 2), then the only solution, regardless of the structure of the shock wave, is the decay of a shock wave with the formation of a two-wave configuration: a shock wave with a final state at point D and a rarefaction wave represented in figure 2 by the segment B'D of the isentrope. If the post-shock states denoted by A and B approach corresponding instability boundaries (E and F), the critical amplitude of the perturbation wave goes to zero. Thus, the source of alternating acoustic perturbations behind the shock front can lead to a sequence of decays to the described configurations. In the two-dimensional case, the source of perturbations causing the “switching” of the parameters of the shock wave from the upper branch of the shock adiabat to the lower part and back are the secondary waves being a combination of acoustic and entropy-vortex waves propagating behind the shock front and consistent with the conditions at the shock-wave discontinuity.

4. Interaction of a metastable shock wave with density inhomogeneities

In this section, we consider the interaction of a metastable shock wave with medium density inhomogeneities in the framework of the formulation of a multidimensional problem. The shock wave parameters are as follows. Pre-shock density and pressure are $\rho_0 = 0.1821$ and $p_0 = 0.1$; the post-shock pressure is $p_1 = 2.5$. The parameters correspond to the segment DE of the Hugoniot. The shock wave with these parameters is stable with respect to small perturbations. This fact makes it possible to obtain it in the numerical solution. At the initial moment of time, a spherical density inhomogeneity is specified in the spherical region ahead of the shock wave front. On the side boundaries of the computational domain the conditions of symmetry are specified. The process of interaction is shown in figure 3, where the density distribution in the flowfield is given at successive points in time. The interaction generates a secondary wave propagating along the surface of the shock wave. This secondary wave is a “switching” wave, in which local parameters of the flow behind the shock front pass from the lower stable branch of the shock adiabat to the upper branch. The resulting perturbations of the shock wave front and the associated perturbations of the flow behind its front do not decay over time. The non-stationary (self-oscillatory) mode of shock wave propagation is established. The change of the mode of shock wave propagation induced by the interaction with the heterogeneity of the medium generates an entropy trace being the material boundary separating the particles of matter that crossed the front of the shock wave in its quasi-stationary and self-oscillatory mode (see figure 3, points in time $t = 6, 8, 10.5$ and $15$). The region between this boundary and the shock wave is characterized by a strong inhomogeneity of the density distribution. Namely, the particles of matter crossed the shock wave front sections with a higher post-shock pressure have a lower density, than one crossed the shock front at a lower shock wave intensity. This is a characteristic feature of the considered anomaly of the shock adiabat due to the positive slope of the Hugoniot on the $(p, V)$–plane, which is necessary for the instability condition $L > 1 + 2M$. Thus, a shock wave stable according to the linear stability theory turned out to be unstable with respect to finite perturbations.

The result of this instability is the transition of the flow to the self-oscillatory regime, in which local parameters behind the shock wave front undergo transitions between the lower and upper stable branches of the shock adiabat. In order to study the features of the secondary waves we consider a similar problem formulation without imposing periodicity conditions on the side boundaries. Without loss of generality, we consider the problem in two-dimensional formulation. Figure 4 shows the dependence of the reaction of the metastable shock wave on the strength of the perturbation. The density distribution after interaction of metastable shock wave with a cylindrical region with perturbed density is shown. The results of three calculations with different strength of the perturbation ($\rho/\rho_0 = 0.5; 0.9; 1.1$) are presented for the same point in time. It follows from the calculations that the shock wave in the second case ($\rho/\rho_0 = 0.9$)
behaves as stable one, while in the first and third cases the switching waves are formed. In contrast to the case shown in figure 3, the initial state of the shock wave corresponds to the
Figure 4. The response of the metastable shock wave to perturbations depends on the strength of the latter; density is shown for some point in time after passage of the metastable shock wave through the cylindrical region with perturbed density $\rho/\rho_0 = 0.5, 0.9$ and $1.1$ (the order of the pictures is from left to right): 1—the entropy trace of the perturbation; 2—the edge of the secondary waves, spreading from the place the interaction; 3—trailing secondary wave.

upper stable branch of the shock adiabat. Therefore, the leading secondary wave is a rarefaction wave. The parameters of the secondary wave (maximum and minimum pressure, the shape of the shock front perturbation) remain constant during its propagation along the shock-wave surface.

5. Properties of the switching waves
Considering a shock wave as a discontinuity surface at those points where this surface has no discontinuity, we can speak of local parameters behind the shock wave front, which must belong to Hugoniot. The change of the local post-shock parameters between the stable branches of the shock adiabat, separated by the instability region $L > 1 + 2M$ raises the question of the implementation of the parameters corresponding to unstable shock waves. A shock wave front section with switching waves is shown in figure 5 in terms of density and pressure distributions. Change of local post-shock parameters of the shock wave from the lower stable branch of the shock adiabat to the upper branch occurs abruptly as the nodal line of the three-wave configuration passes over the surface of the shock wave (point 1 in figure 5). This point (line) corresponds to the kink of the shock-wave surface. The reverse transition occurs in the vicinity...
of point 2, where the shape of the shock wave also has a local maximum curvature. With an increase in the grid resolution, the curvature in this area increases, and the corresponding curvilinear section tends to form a kink. Thus, in the limit of a small thickness of the shock wave structure, the linear instability conditions for the shock front are not realized. It should be noted that obtained in the calculations speed of propagation of the switching waves along the shock wave surface is subsonic.

6. Discussion
Due to the metastable behavior of the viscous shock wave in the lower region of the ambiguous representation (DE in figure 1), in the processes with a slow adiabatic change of parameters behind its front (for example, in the converging shock waves), the splitting of the shock wave have to occur when its parameters reach the lower instability boundary. The wave-split configuration includes two shock waves separated by a contact discontinuity. The same is valid for splitting of diverging shock waves in the upper region of ambiguous representation. The splitting is expected at point F. The wave-split configuration includes a shock wave and a rarefaction wave separated by a contact discontinuity. Example of shock wave metastability given above appears to be possible only in ambiguity region associated with \( L > 1 + 2M \) instability condition. In this case for each unstable shock wave satisfying \( L > 1 + 2M \) condition there exist a pair of corresponding wave-split configurations (RTS and STS, where S stands for shock wave, R denotes rarefaction
wave, T is contact discontinuity) with stable elements. Any configuration from the pair can be “switched” to its alternative by the splitting of the leading shock. The splitting can be induced locally (i.e., on the small segment of the shock-wave front) by interaction of the leading shock-wave with a finite perturbation. Then the splitting is spreading over the shock wave surface via the secondary switching waves. In the presence of boundaries the non-stationary mode of the flow can develop. It should be mentioned that there is non-complete analogy between the observed shock wave behavior and the behavior of detonation waves. In the framework of this analogy the lower stable branch of the Hugoniot corresponds to unreacted shocks, while the upper one corresponds to the reacted (with energy release) shock waves. Switching to the upper branch looks like transition to detonation. One can find some analogy between the described non-stationary mode and some modes of cellular detonation. However, there are important disagreements between these phenomena, which include questions of reversibility and characteristic length scale. These disagreements manifest themselves in the flow patterns. Currently in literature there are no examples of media in which the shock wave stability criterion $L < 1 + 2M$ can be violated. At the same time the necessary condition $L > 1$ is found to be fulfilled in many cases. It cannot be excluded that violation of this stability criterion will be detected in the future with respect to complex reactive and multiphase media. The examples of exotic shock wave behavior are markers of the $L < 1 + 2M$ instability condition.

7. Conclusion
Numerical experiments have been performed to simulate the interaction of metastable shock waves with density inhomogeneities. The metastable shock wave behavior is detected. This phenomenon is shown to be related to the ambiguous representation of a shock wave discontinuity when the shock wave belongs to the S-shaped in the $p–u$ plane Hugoniot fragment. After interaction with weak perturbations the metastable shock wave recovers its original parameters. If the amplitude of the perturbations exceeds some critical value, the shock wave instability is developed, which is accompanied by formation of the transverse secondary waves switching local post-shock parameters between two Hugoniot segments separated from each other by the absolute instability region. Transition to the limiting self-oscillating mode is observed in the calculations with symmetry conditions specified at side boundaries.

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