Quarkonium decay into photon plus graviton: a golden channel to discriminate General Relativity from Massive Gravity?

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Abstract

After the recent historical discovery of gravitational wave, it is curious to speculate upon the detection prospect of the quantum graviton in the terrestrial accelerator-based experiment. We carefully investigate the “golden” channels, $J/\psi(\Upsilon) \rightarrow \gamma + \text{graviton}$, which can be pursued at BESIII and Belle 2 experiments, by searching for single-photon plus missing energy events. Within the effective field theory (EFT) framework of General Relativity (GR) together with Nonrelativistic QCD (NRQCD), we are capable of making solid predictions for the corresponding decay rates. It is found that these extremely suppressed decays are completely swamped by the Standard Model background events $J/\psi(\Upsilon) \rightarrow \gamma + \bar{\nu}\nu$. Meanwhile, we also study these rare decay processes in the context of massive gravity, and find the respective decay rates in the limit of vanishing graviton mass drastically differ from their counterparts in GR. Counterintuitive as the failure of smoothly recovering GR results may look, our finding is reminiscent of the van Dam-Veltman-Zakharov (vDVZ) discontinuity widely known in classical gravity, which can be traced to the finite contribution of the helicity-zero graviton in the massless limit. Nevertheless, at this stage we are not certain about the fate of the discontinuity encountered in this work, whether it is merely a pathology or not. If it could be endowed with some physical significance, the future observation of these rare decay channels, would, in principle, shed important light on the nature of gravitation, whether the graviton is strictly massless, or bears a very small but nonzero mass.

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1. **Introduction.** One century after Einstein’s avantgarde prediction based on his newly-formulated General Relativity (GR) [1], the recent legendary discovery of gravitational wave from binary black hole mergers by LIGO Scientific Collaboration and Virgo Collaboration [2, 3], has marked a major milestone in humankind’s advance in fundamental physics. A natural question then arises: how and when will we experimentally establish the existence of the graviton, the quantum of gravitational field and the force carrier of gravitation? Due to the extreme weakness of gravitational coupling, many physicists believe that humankind will never be able to detect graviton.

Looking back in history, one sees that the forerunners of quantum electromagnetism are much luckier. The making of electromagnetic wave by Hertz in 1887 [4], was only a quarter of a century after Maxwell’s ground-breaking prediction for the existence of electromagnetic wave [5]. Miraculously, in the same year, Hertz also discovered the photoelectric effect [6], which, from the modern viewpoint, was the direct experimental evidence for the existence of photon, the force carrier of electromagnetism and the quantum of electromagnetic field.

The path for detecting the graviton is doomed to be much, much more twisty than that for discovering photon. One may appreciate the difficulty by looking at a simple example. In his authoritative monograph on gravitation [7], Weinberg estimated the rate of the 3d-state hydrogen atom transitioning into the 1s state via single-graviton emission to be about $2.5 \times 10^{-44} \text{ sec}^{-1}$, completely overwhelmed by the spontaneous photon emission rates of order $10^9 \text{ sec}^{-1}$. So there seems absolutely no chance to detect the graviton in atomic physics laboratories.

This paper reports an exploratory study of graviton-hunting in the terrestrial high-energy collision experiments. Concretely speaking, we make a comprehensive investigation on, arguably one of the best channels to discover graviton in the accelerator-based experiments, $J/\psi(\Upsilon) \rightarrow \gamma + G$ (Henceforth we will use $G$ to denote graviton). Since the graviton carries quantum number $J^{PC} = 2^{++}$, this decay process, circumventing the Landau-Yang theorem and allowed by $C$-invariance, is permissible. The merit of focusing on vector quarkonia decay is that they can be copiously produced in $e^+ e^-$ collision experiments with quite clean environment, and also bear very narrow width owing to the OZI-suppression. The experimental signature is also very simple: a single photon with energy exactly half of the quarkonium mass, plus invisible events. We note that CLEO [8, 9] and BaBar [10] have already placed the upper bounds for the decays $J/\psi(\Upsilon(1S)) \rightarrow \gamma + \text{invisible}$, but their motivation was to search for the hypothetic dark matter particle, light Higgs and axion. Here we extend their hunting candidates by adding the graviton. Roughly speaking, about $10^{10}$ $J/\psi$ samples are accumulated at BESIII, and about $10^9$ $\Upsilon(nS)$ ($n = 1, 2, 3$) samples will be accumulated in the forthcoming Belle 2 experiment, therefore we expect the upper bound on this rare decay process will continue to improve with respect to Refs. [8–10].

To reliably account for this process, we first need couple the Standard Model (SM) of particle physics, especially QCD, which is responsible for the binding of heavy quarkonium, with the gravitation theory, in a consistent manner. The old folklore was that we do not yet have consistent theory of quantum gravity, since General Relativity is a nonrenormalizable theory. Nevertheless, with the increasing popularity of the effective field theory (EFT) motif, the paradigm has gradually shifted to that, as long as treating GR as the low-energy EFT with UV cutoff around Planck mass, one is then capable of making consistent and controlled predictions, regardless of our ignorance of the hitherto unknown UV-completed quantum gravity [11, 12]. The underlying tenet is intimately analogous to the chiral effective field theory as the powerful low-energy EFT of QCD [13]. Among the beautiful applications

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of this quantum gravity EFT are the quantum corrections to Newton’s gravitational law between two masses \[11, 12, 14, 15\], quantum correction to the bending of light in the external gravitational field \[16, 18\].

Meanwhile, a model-independent description of the rare decay of vector quarkonium, a color-singlet meson formed by the heavy quark-antiquark pair (with the quark-model spectroscopic symbol \(^3S_1\)), necessitates a controlled way of tackling the nonperturbative aspects of strong interaction. Historically, heavy quarkonium decays have played a key role in establishing the asymptotic freedom of QCD \[13, 20\]. Due to the non-relativistic nature of the heavy quarks inside quarkonium (the characteristic velocity of heavy quark \(v \ll c\)), the decay rates were traditionally expressed as the squared bound-state wave function at the origin, which is sensitive to the long-distance binding mechanism, multiplying the short-distance quark-antiquark annihilation decay rates, which can be accessible to perturbative \(\alpha_s\) expansion. After the advent of the nonrelativistic QCD (NRQCD) EFT \[21\], this intuitive factorization picture has been put on a field-theoretical ground, and one is allowed to systematically conduct a double expansion in \(\alpha_s\) and \(v/c\) \[22\]. To date a vast number of quarkonium decay and production processes have been fruitfully tackled using this powerful EFT approach \[23\]. Within the EFT framework of GR together with NRQCD, we are capable of making a solid prediction for the corresponding decay rates, which, not surprisingly, are extremely small. Unfortunately, these extremely suppressed decays appear to be completely swamped by the SM background events \(J/\psi(\Upsilon) \rightarrow \gamma + \nu \bar{\nu}\), which themselves already correspond to rather rare decay modes.

The standard theory of gravitation, GR, as an inevitable consequence of the nontrivially self-interacting massless spin-2 gravitons, has passed many stringent experimental tests \[24\]. Nevertheless, from the observational side, it remains possible that the graviton might be massive \[25\]. A special class of modified gravity theories, massive gravity (MG) \[26\], has the potential to account for the accelerated expansion of Universe by endowing the graviton a Hubble-scale mass rather than introducing the dark energy, which makes it phenomenologically appealing \[27\]. To date, astrophysical bound on the graviton mass is \(m_G < 6 \times 10^{-32}\) eV, many orders of magnitude tighter than that on the photon mass \[24\].

Perhaps the most interesting outcome of this work arises from our comparative study of the rare decays \(J/\psi(\Upsilon) \rightarrow \gamma + G\) from both GR and MG. There we come across a striking finding: the decay rates in the limit of vanishing graviton mass drastically differ from their counterparts in GR! This discontinuity can be traced to the contribution from the scalar-polarized graviton, which is absent in GR. It is natural to interpret this discontinuity as a quantum realization of the van Dam-Veltman-Zakharov (vDVZ) discontinuity \[28, 29\], which is widely known in classical gravity. As elucidated by Vainshtein long ago, the vDVZ discontinuity was just an artifact of linearized gravity, which can be dissolved by including nonlinearities nonperturbatively \[30\]. Nevertheless, in our case, we are unable of envisaging the relevant mechanism to eliminate this discontinuity even if it is merely a pathology. On the other hand, this discontinuity may bear some physical significance. Consequently, if these rare decays were observed one day in distant future (in extremely unlikely situation), one would then be capable of pinpointing, in principle, whether the graviton is strictly massless or it has an very small yet nonzero mass. This criterion, if true, appears to be of fundamental impact on the theory of gravitation.

2. GR+SM as an EFT for quantum gravity. The SM of particle physics and GR for gravity have long been two eminent pillars in fundamental physics. In modern EFT paradigm, SM
and GR are not only not incompatible, but can be fruitfully combined into a predictive framework for quantum gravity, provided that the probed energy scale is far below the Planck mass. The action for the quantum gravity EFT can be built upon the principle of general coordinate invariance, and can be divided into gravity part and matter part:

\[ S = S_{\text{grav}} + S_{\text{matt}} = \int d^4x \sqrt{-g} (\mathcal{L}_{\text{grav}} + \mathcal{L}_{\text{SM}}). \]  

The pure gravity sector can be organized as the curvature (energy) expansion:

\[ \mathcal{L}_{\text{grav}} = -\Lambda - \frac{2}{\kappa^2} R + c_1 R^2 + c_2 R_{\mu\nu} R_{\mu\nu} + \cdots, \]  

where \( \kappa = \sqrt{32\pi G_N} \), with Newton’s constant \( G_N = 6.709 \times 10^{-39} \) GeV\(^{-2} \). \( g_{\mu\nu}(x) \) signifies the metric field, \( R_{\mu\nu} \) is Ricci tensor, and \( R \) is the Ricci scalar. The first term corresponds to the cosmological constant, completely immune to local accelerator experiments. The effects of quadratic curvature terms are so much suppressed that no meaningful constraints can be imposed on the dimensionless couplings \( c_i \) (\( i = 1, 2 \)): \( c_{1,2} < 10^{74} \) \[31\]. In this work, suffice it for us to stay with the Einstein-Hilbert action.

The matter action is comprised of all the SM fields that are minimally coupled with gravity:

\[ \mathcal{L}_{\text{SM}} = -\frac{1}{4} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} - \frac{1}{4} g^{\mu\alpha} g^{\nu\beta} G_{\mu\alpha} G_{\nu\beta} + \sum_f \bar{q}_f (i\gamma^\mu \gamma^a D_\mu - m_f) q_f + \cdots. \]  

For our purpose, we need only retain the matter contents of photon, gluons, and the quarks. \( F_{\mu\nu} \) and \( G^a_{\mu\nu} \) represent the field strengths for the photon and gluon, respectively. The \( f \)-flavor quark field is denoted by \( q_f \), with \( m_f, e_f \) its mass and electric charge. \( e_\alpha^\mu \) is the vierbein field. \( D_\mu = \partial_\mu - ie_f e A_\mu - ig_s G_\mu^a T^a + \frac{1}{2} \sigma^{ab} \omega_{\mu ab} \) is the covariant derivative acting on the quark fields, with \( \sigma^{ab} = \frac{1}{2} [\gamma^a, \gamma^b] \), and \( \omega^a_{\mu \nu} \) the spin connection. Apart from the ordinary \( SU(3)_c \times U(1)_{\text{em}} \) gauge couplings, the last term in \( D_\mu \) generates the spin-dependent gravitational interaction.

Since we are only interested in terrestrial accelerator experiment, it is legitimate to conduct the weak-field approximation, by decomposing \( g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu} \), and treating \( h_{\mu\nu} \) as a small spin-2 quantum fluctuation around flat spacetime background. The most-minus-signature \((+ - - -)\) is adopted for Minkowski metric \( \eta_{\mu\nu} \). Expanding \(\text{(3)}\) to linear order in \( h_{\mu\nu} \), one can schematically express the graviton-matter interactions as

\[ \mathcal{L}_{\text{int}} = -\frac{\kappa}{2} h_{\mu\nu} T^{\mu\nu} = \mathcal{L}_{ff\bar{g}} + \mathcal{L}_{ffg\bar{g}} + \mathcal{L}_{gg\bar{g}} + \mathcal{L}_{g\gamma\bar{g}} + \cdots, \]  

where \( T^{\mu\nu} \) is the Belinfante energy-momentum tensor of the SM.

3. Polarized decay rates and helicity amplitudes. Let us choose to work in the \( J/\psi \) rest frame. Suppose the spin projection of the \( J/\psi \) along the \( \hat{z} \) axis to be \( S_z \), the helicities carried by the outgoing photon and graviton to be \( \lambda_1, \lambda_2 \), respectively. Let \( \theta \) signify the polar angle between the direction of the photon 3-momentum and the \( \hat{z} \) axis. The differential polarized decay rate can be expressed as \[32, 33\]

\[ \frac{d\Gamma[J/\psi(S_z) \to \gamma(\lambda_1) + \mathcal{G}(\lambda_2)]}{d\cos\theta} = \frac{1}{32\pi M_{J/\psi}} \left| d^4_{S_z,\lambda_1 - \lambda_2}(\theta) \right|^2 |M_{\lambda_1,\lambda_2}|^2, \]  

where \( d^4_{S_z,\lambda_1 - \lambda_2}(\theta) \) is the polarization matrix.
where $\mathcal{M}_{\lambda_1, \lambda_2}$ characterizes the helicity amplitude which encodes nontrivial dynamics. The angular distribution is entailed in the Wigner rotation matrix $d^{ij}_{m, m'}(\theta)$. Note the angular momentum conservation constrains that $|\lambda_1 - \lambda_2| \leq 1$.

Integrating (5) over the polar angle, and averaging over three $J/\psi$ polarizations, one finds the integrated decay rate of $J/\psi$ into $\gamma(\lambda_1) + G(\lambda_2)$ reads

$$\Gamma\left[J/\psi \rightarrow \gamma(\lambda_1) + G(\lambda_2)\right] = \frac{1}{48\pi M_{J/\psi}} |\mathcal{M}_{\lambda_1, \lambda_2}|^2.$$  

Since this decay process is mediated by the strong, electromagnetic, gravitational interactions, the relation $M_{-\lambda_1, -\lambda_2} = M_{\lambda_1, \lambda_2}$ constrained by parity invariance, can be invoked to reduce the number of independent helicity amplitudes.

4. $J/\psi \rightarrow \gamma + \text{massless graviton}$. We are going to apply the NRQCD factorization recipe to the decay $J/\psi \rightarrow \gamma + G$, starting from SM+GR EFT. Intuitively, the $c$ and $\bar{c}$ have to get very close to each other in order to annihilate into a hard photon plus a hard graviton (here hard means that momentum is of order charm quark mass, $m_c$, or greater). The computation of this process is in spirit similar to, but more involved than, the electromagnetic quarkonium decay processes such as $J/\psi \rightarrow e^+e^-$ and $\eta_c \rightarrow \gamma\gamma$, with the complication that the graviton couples universally to all the matter fields via the energy-momentum tensor of SM, including quark, photon and gluon. The NRQCD short-distance coefficients (SDCs), which encompass hard quantum fluctuations emerging in the length scale $\lesssim \frac{1}{m_c}$, can be computed in perturbative QCD owing to the asymptotic freedom. The methodology of extracting SDCs is well known, and we refer the interested readers to Refs. [22, 34, 35] for encyclopedic introduction.

In accordance with the Feynman rules generated from the linearized gravitational interaction encoded in (4), four quark-level diagrams arise at the lowest order (LO) in $\alpha_s$, as is shown in Fig. 1. Let us first consider the static limit, that is, by neglecting the relative momentum between $c$ and $\bar{c}$. Under dimensional analysis, the LO helicity amplitudes for $J/\psi \rightarrow \gamma + G$ can be written as

$$\mathcal{M}_{\pm 1, \pm 2} = e_c e_{\bar{c}} \kappa A^{LO} \sqrt{M_{J/\psi}} \langle 0 | \chi^\dagger \sigma \cdot \epsilon^* \psi | J/\psi(\epsilon) \rangle,$$  

where $A^{LO}$ in (7) is expected to be a dimensionless $O(1)$ number. Here $\langle 0 | \chi^\dagger \sigma \cdot \epsilon^* \psi | J/\psi(\epsilon) \rangle$ represents the lowest-order $J/\psi$-to-vacuum NRQCD matrix element, where $\psi, \chi$ represent the quark and anti-quark Pauli spinor fields in NRQCD, $\epsilon$ represents the polarization vector of $J/\psi$. This nonperturbative matrix element is often approximated by $\sqrt{\frac{N_c}{2\pi}} R_{J/\psi}(0)$ ($N_c = 3$ is the number of colors in QCD), where $R_{J/\psi}(0)$ denotes the radial wave function at the origin for the $J/\psi$ in the quark potential model, characterizing the probability amplitude for $c$ and $\bar{c}$ coincide in space. Without causing confusion, we will use $R_{J/\psi}(0)$ and the NRQCD matrix element interchangeably.
As a matter of fact, $A^{LO}$ happens to vanish. This pattern is reminiscent of the exclusive strong decay process $η_b \to J/ψJ/ψ$, where the LO amplitude also vanishes accidently, but becomes nonzero once going to higher order $[36, 37]$. In a similar vein, we also proceed to the next-to-leading order (NLO) corrections, along the both directions in $v$ and $α_s$ expansion. Assuming $α_s ≈ v^2$, we ought to include both the order-$α_s$ and order-$v^2$ corrections coherently. We utilize the all-order-in-$v$ spin projectors developed in $[35]$ to facilitate the computation of the relativistic correction. For the calculation of the radiative correction, as is indicated in Fig. 2 we set the relative momentum between $c$ and $\bar{c}$ to zero prior to conducting loop integration, which amounts to directly extracting the order-$α_s$ SDC without concerning about Coulomb divergences $[38]$. Dimensional regularization is employed to cope with both UV and IR divergences in intermediate steps. The calculation is also expedited by using Mathematica packages FeynRules $[39]$, FeynArts $[40]$, FeynCalc $[41]$, $\text{FIRE} [43]$. Ultimately, the NLO helicity amplitudes read

\[ \mathcal{M}_{NLO}^{±1,±2} = \frac{e_c e_κ}{6\sqrt{π}} \sqrt{N_c M_{J/ψ}} R_{J/ψ}(0) \left( \langle v^2 \rangle_{J/ψ} + \frac{3C_F α_s}{4π} (1 - 4 \ln 2) \right), \] (8)

where the color Casimir $C_F = \frac{N_c^2 - 1}{2N_c}$, and the leading relativistic correction is encoded in $\langle v^2 \rangle_{J/ψ}$, the dimensionless ratio of the following NRQCD matrix elements:

\[ \langle v^2 \rangle_{J/ψ} = \frac{\langle 0 | \chi^4 \sigma \cdot ε^∗ (∑_{a=1}^{3} D_a)^2 | J/ψ(ε) \rangle}{m_c^2 \langle 0 | \chi^4 \sigma \cdot ε^∗ | J/ψ(ε) \rangle}, \] (9)

where $D$ denotes the spatial part of the color-covariant derivative.

Some remarks are in order. First, the cancelation of IR divergence in the $O(α_s)$ short-distance coefficient in $[8]$, serves a nontrivial validation of NRQCD approach in a novel setting involving quantum gravity. Second, both types of NLO corrections are indeed non-vanishing, yet suffer from severe destructive interference.

Substituting $\mathcal{M}_{NLO}^{\pm 1,\pm 2}$ in $[8]$ into $[8]$, taking both helicity configurations ($±1,±2$) into account, we then obtain the unpolarized decay rate:

\[ \Gamma[J/ψ \to γ + G] = \frac{4c^2 α G_N}{27} N_c |R_{J/ψ}(0)|^2 \left( \langle v^2 \rangle_{J/ψ} + \frac{3C_F α_s}{4π} (1 - 4 ln 2) \right)^2. \] (10)

FIG. 2: Representative Feynman diagrams for $c\bar{c}(S_1^{(1)}) \to γ + G$ in NLO in $α_s$. 

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It is amusing to see that the coupling constants entering Nature’s fundamental interactions, strong, electroweak, and gravitational, are all packed in a single formula.

5. $J/\psi \rightarrow \gamma + \text{massive graviton}$. Let us revisit the decay $J/\psi \rightarrow \gamma + \mathcal{G}$ in a special class of infraredly-modified gravitation theories, massive gravity. For the gravity sector, suffice it for us to add the Fierz-Pauli mass term to Einstein-Hilbert action:

$$\mathcal{L}_{\text{grav}} = \sqrt{-g} \left[-\frac{2}{\kappa^2} R + \frac{m_G^2}{2} (h^{\mu\nu} h_{\mu\nu} - h^{\mu}_{\mu} h^{\nu}_{\nu})\right],$$

where $m_G$ signifies the graviton mass. For simplicity, we assume the matter fields coupled with the gravity in the same manner as in (3). Since we are only concerned with linearized gravity, we do not bother to specify the self-interactions for gravitons, so the detailed textures of the influential massive gravity models, exemplified by the $\Lambda_5$ model [44] and the ghost-free de Rham-Gabadadze-Tolley (dRGT) model (also referred to as $\Lambda_3$ model) [45, 46], are largely irrelevant.

Summing (6) over three independent helicity configurations, the decay rate can be expressed as

$$\Gamma[J/\psi \rightarrow \gamma + \mathcal{G}] = \frac{1}{48\pi M_{J/\psi}} \left(2 |\mathcal{M}_{1,0}|^2 + 2 |\mathcal{M}_{1,1}|^2 + 2 |\mathcal{M}_{1,2}|^2\right),$$

where we have neglected the tiny graviton mass in phase space integral.

Contrary to the GR case, the amplitudes in massive gravity at LO in $\alpha_s$ and $v$, no longer vanish. In the $m_G \rightarrow 0$ limit, each helicity amplitude bears the following asymptotic behavior:

$$\mathcal{M}^{\text{LO}}_{\pm 1, \pm 2} \rightarrow \left(\frac{m_G}{M_{J/\psi}}\right)^2 \frac{e_c e \kappa}{2\sqrt{\pi}} \sqrt{N_c M_{J/\psi}} R_{J/\psi}(0),$$

$$\mathcal{M}^{\text{LO}}_{\pm 1, \pm 1} \rightarrow \left(\frac{m_G}{M_{J/\psi}}\right) \frac{e_c e \kappa}{8\sqrt{\pi}} \sqrt{N_c M_{J/\psi}} R_{J/\psi}(0),$$

$$\mathcal{M}^{\text{LO}}_{\pm 1, 0} \rightarrow \frac{e_c e \kappa}{2\sqrt{6\pi}} \sqrt{N_c M_{J/\psi}} R_{J/\psi}(0).$$

Since $m_G/M_{J/\psi} < 2 \times 10^{-41}$, the contributions from the helicity-±2 and helicity-±1 gravitons are utterly negligible, whereas the helicity-0 graviton (graviscalar) survives this limit. As is well known in classical gravity, by employing the St"ukelberg trick, one readily shows that the coupling of the graviscalar with the trace of the energy-momentum tensor generally survives the $m_G \rightarrow 0$ limit [26]. This is nothing but the very origin of the vDVZ discontinuity.

Therefore, the decay rate is saturated by the graviscalar’s contribution solely:

$$\Gamma[J/\psi \rightarrow \gamma + \mathcal{G}] = \frac{2e_c^2 \alpha G_N}{9} N_c |R_{J/\psi}(0)|^2.$$
The vDVZ discontinuity is commonly regarded as an artifact of linearized classical gravity, which is of no physical significance. Once the nonlinearities of gravity are taken into account nonperturbatively, this discontinuity should fade away, so that the principle of continuity in parameters is recovered [30]. But in our case, it is far from obvious to identify the relevant Vainshtein mechanism to remove the discontinuity. It is unclear to us how to analytically resum a class of real emission and loop diagrams incorporating nonlinear multi-graviton interactions, and ultimately yield a vanishing amplitude as in GR. Actually, in our case, QCD corrections appear much more important than the quantum gravitational corrections.

We note that, by treating massive gravity as the quantum EFT, both the Λ₅ and Λ₃ models bear very low UV cutoffs in the particle physics standard, say, many orders of magnitude smaller than the characteristic hadronic scale of order GeV. This seems to cast serious doubt on the reliability of our prediction for $J/\psi \rightarrow \gamma G$, consequently the discontinuity found in (14) might not even be physically relevant.

Nevertheless, if a weakly-coupled UV completion of massive gravity could be realized [48], our predictions based on the linearized gravity would essentially remain intact, since only very rudimentary knowledge about massive graviton is required in our calculation: it has five degrees of freedom rather than two. Frankly speaking, we are not certain about the ultimate fate of this discontinuity.

6. Numerical predictions. To make concrete predictions, we specify the input parameters as follows [24, 49]: $e_c = \frac{2}{3}$, $\alpha = 1/137$, $\alpha_s(M_{J/\psi}/2) = 0.30$, $\Gamma_{J/\psi} = 92.9$ keV, $|R_{J/\psi}(0)|^2 = 0.922 \text{ GeV}^3$, $\langle v^2 \rangle_{J/\psi} = 0.225$. Substituting these numbers into (10) and (14), we then predict

$$\text{Br}(J/\psi \rightarrow \gamma + G) = (2 \sim 8) \times 10^{-40}, \quad \text{GR} \quad (15a)$$
$$\text{Br}(J/\psi \rightarrow \gamma + G) = 1.4 \times 10^{-37}, \quad \text{MG} \quad (15b)$$

For the GR prediction, we have estimated the uncertainty by varying the renormalization scale in $\alpha_s$ with from 1 GeV to $M_{J/\psi}$. The relatively large error can be attributed to the delicate destructive interference between the order-$\alpha_s$ and order-$v^2$ corrections of comparable size. It is striking to note that the branching fraction in massive gravity is more than two order-of-magnitude greater than that in GR!

For completeness, we also make the predictions for $\Upsilon(1S) \rightarrow \gamma + G$. Substituting $e_b = -\frac{1}{3}$, taking $\alpha_s(M_T/2) = 0.21$, and $\Gamma_{\Upsilon(1S)} = 54.02$ keV, $|R_{\Upsilon(1S)}(0)|^2 = 6.43 \text{ GeV}^3$, $\langle v^2 \rangle_{\Upsilon} = -0.009 \frac{\text{GeV}^2}{\text{MeV}^2}$, we find

$$\text{Br}(\Upsilon(1S) \rightarrow \gamma + G) = (3 \sim 4) \times 10^{-39}, \quad \text{GR} \quad (16a)$$
$$\text{Br}(\Upsilon(1S) \rightarrow \gamma + G) = 4.1 \times 10^{-37}, \quad \text{MG} \quad (16b)$$

For the GR prediction, we have estimated the error by varying the renormalization scale from 1.5 GeV to $M_T$. The relatively small uncertainty is due to the much smaller relativistic correction compared with the radiative correction, so that the destructive interference does not play a significant role.

Needless to say, both GR and MG predictions in (15) and (16) are many orders of magnitude beyond the maximal sensitivity of the BESIII and Belle 2 experiments.

Meanwhile, the really striking message is the dramatic difference between the decay rates predicted by GR and MG, which may in principle offer an experimental criterion to judge whether graviton bears a nonzero mass.
7. Primary SM background. One must be concerned with the nuisance that \( J/\psi \) non-gravitational rare decays may dominate over our desired \( \gamma + \text{graviton} \) signals. Aside from the hypothetical Beyond-SM scenarios that entail dark matter, light Higgs boson and axion, the major SM background is from \( J/\psi \rightarrow \gamma + (Z^* \rightarrow \nu \bar{\nu}) \), where neutrinos simply escape the detector and manifest themselves as missing energy. C-parity invariance dictates that only the axial-vector part of \( Zc \) coupling contribute. At LO in \( \alpha_s \) and \( \nu \), the photon energy spectrum is predicted to be \([51, 52]\):

\[
\frac{d\Gamma[J/\psi \rightarrow \gamma (E_\gamma) + \nu \bar{\nu}]}{dE_\gamma} = N_\nu \frac{e_\nu^2 \alpha G_F^2}{9 \pi^3} N_c \left| R_{J/\psi}(0) \right|^2 E_\gamma \left( 1 - \frac{E_\gamma}{M_{J/\psi}} \right),
\]

where \( N_\nu = 3 \) counts the number of neutrino flavors, \( G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \) is the Fermi coupling constant. The \( Z^0 \) exchange has been mimicked by a contact interaction since \( M_{J/\psi} \ll M_Z \). The photon spectrum is a monotonically increasing function with the photon energy \( E_\gamma \). The integrated decay rate \( \Gamma[J/\psi \rightarrow \gamma \nu \bar{\nu}] = N_\nu \frac{e_\nu^2 \alpha G_F^2 M^2_{J/\psi}}{9 \pi^3} N_c \left| R_{J/\psi}(0) \right|^2 \), and the corresponding branching fraction is about \( 10^{-10} \).

Realistic electromagnetic calorimeters always have limited resolution, typically 2\% of the photon energy at BESIII and Belle. We can make a rough estimate for these SM background events that can fake the desired \( \gamma + \mathcal{G} \) signals. Multiplying \([17]\) at \( E_\gamma \text{Max} = \frac{M_{J/\psi}}{2} \) by an energy interval \( \Delta E_\gamma = 0.02 E_\gamma \text{Max} \), we found these “fake” signals bear a branching fraction about \( 3 \times 10^{-12} \). The observation prospect of these SM events at BESIII is already rather pessimistic, let alone our desired signals. Moreover, even if BESIII would operate an unlimited period of time, the intended \( \gamma + \mathcal{G} \) signals, unfortunately, seem to be completely overwhelmed by these SM background events.

One can also use \([17]\) to estimate the decay rate for \( \Upsilon(1S) \rightarrow \gamma + \nu \bar{\nu} \) by making straightforward substitutions. One finds the total branching fraction of this decay channel is about \( 3 \times 10^{-9} \), and the “fake” single photon plus invisible possess a branching fraction about \( 8 \times 10^{-11} \). It appears unlikely to observe these SM background events in the forthcoming Belle 2 experiment.

8. Summary. After decades of heroic efforts to search the gravitational wave, establishing the experimental evidence for graviton is to become the holy grail in fundamental physics. It is deserved to be ranked among the top formidable tasks in the history of physics. Conceivably, the humankind’s zest for unraveling the existence of graviton will last for centuries, if not forever, and the hope will never be extinguished.

In this work, we have investigated, arguably one of the cleanest channels to search for the graviton in terrestrial accelerator-based experiments such as BESIII and Belle 2: the decay process \( J/\psi(\Upsilon) \rightarrow \gamma + \mathcal{G} \), with clean signature of a photon with energy exactly half of the quarkonium plus invisible. Based on the EFT paradigm of SM+GR matched onto NRQCD, we are able to make a solid prediction for this rare decay channel. The extremely suppressed branching fraction, even many orders of magnitude smaller than the dominant SM background events \( J/\psi(\Upsilon) \rightarrow \gamma + \nu \bar{\nu} \), render the detection of these decay modes futile in foreseeable future. Nevertheless, unceasing experimental endeavour in pushing the upper bound is always rewarding.

From the theoretical angle, our exploration on the rare decays \( J/\psi(\Upsilon) \rightarrow \gamma + \mathcal{G} \) brings forth some notable novelties. First, it is an amazing fact that all the fundamental forces in Nature are intertwined in a single process. Second, our study has illuminated the strength
of EFT, in particular, we have verified the internal consistency for the marriage of the EFT of quantum gravity, with NRQCD, which describes the strong dynamics for heavy quark bound states.

Most importantly, we have come across one interesting discontinuity when studying $J/\psi(\Upsilon) \rightarrow \gamma + G$ in the context of massive gravity. When the graviton mass approaches zero, the resulting decay rate drastically differs from what is predicted by GR. Since this discontinuity solely stems from the helicity-zero graviton, it is natural to interpret this as the quantum counterpart of the vDVZ discontinuity. Nevertheless, we are not certain whether the discontinuity discovered here is merely a pathology or not. If it is just an artifact of linearized quantum gravity, we have not been able to envisage the concrete Vainshtein mechanism for its dissolution. If this discontinuity could be affiliated with some physical significance, the future measurement of these rare decays, in principle, would help us to discriminate whether graviton mass is mathematically zero or not. It is definitely worth further investigation to elucidate the fate of this discontinuity.

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