Stokes Vector of Photon in the Decays \(B^0 \rightarrow \rho^0 \gamma\) and \(B^0 \rightarrow K^* \gamma\)

L. M. Sehgal and J. van Leusen
Institute of Theoretical Physics, RWTH Aachen,
D-52056 Aachen, Germany

Abstract

We consider a model for the decay \(B^0 \rightarrow \rho^0 \gamma\) in which the short-distance amplitude determined by the Hamiltonian describing \(b \rightarrow d \gamma\) is combined with a typical long-distance contribution \(B^0 \rightarrow D^0 \rightarrow \rho^0 \gamma\). The latter possesses a significant dynamical phase which induces a \(CP\)-violating asymmetry \(A_{CP}\), as well as an important modification of the Stokes vector of the photon. The components \(S_1\) and \(S_3\) of the Stokes vector can be measured in the decay \(B^0 \rightarrow \rho^0 \gamma^* \rightarrow \pi^+\pi^-e^+e^-\) where they produce a characteristic effect in the angular distribution \(d\Gamma/d\phi\), \(\phi\) being the angle between the \(\pi^+\pi^-\) and \(e^+e^-\) planes. A similar analysis is carried out for the decays \(B^0 \rightarrow K^* \gamma\) and \(B^0 \rightarrow K^* \gamma^* \rightarrow \pi^+K^-e^+e^-\).

1 Introduction

We study in this paper a long-distance contribution to the decay \(B^0 \rightarrow \rho^0 \gamma\), which has the interesting feature of possessing a large dynamical phase. When added to the short-distance amplitude determined by the \(b \rightarrow d \gamma\) penguin operator, this produces an asymmetry \(A_{CP}\) between \(\Gamma(B^0 \rightarrow \rho^0 \gamma)\) and \(\Gamma(B^0 \rightarrow \rho^0 \gamma)\). In addition, the presence of the long-distance component affects the polarization state (Stokes vector) of the photon. This effect can be measured in the decay \(B^0 \rightarrow \rho^0 e^+e^- \rightarrow \pi^+\pi^-e^+e^-\). An analogous effect on the Stokes vector occurs in the decay \(B^0 \rightarrow K^\ast \gamma\). The Stokes vector turns out to be very sensitive to the proposed long-distance contribution and thus may give more insight into the structure of the radiative decay amplitude.

The main contribution to the amplitude of the decay \(B^0 \rightarrow \rho^0 \gamma\) is believed to come from the effective Hamiltonian

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* c_7 O_7.
\]

\(G_F\) is Fermi’s constant, \(V_{ij}\) the CKM matrix elements, \(c_7\) the Wilson coefficient and \(O_7\) is the electromagnetic penguin operator

\[
O_7 = \frac{e}{8\pi^2} \bar{q} \gamma^\mu m_b (1 + \gamma_5) b F^{\mu\nu}.
\]

The corresponding amplitude contains a parity conserving (magnetic) term and a parity violating (electric) term and can be written as [1]:

\[
A(B^0 \rightarrow \rho^0 \gamma) = \frac{e G_F}{\sqrt{2}} \left( \epsilon_{\mu \rho \sigma} q_1^{\mu} q_2^{\rho} \epsilon_1^{\sigma} M_{SD} + i \epsilon_1^{\mu} \epsilon_2^{\nu} (g_{\mu \nu} p - p_{\mu} q_{1\nu}) E_{SD} \right).
\]
where \( p \) is the momentum of the \( B^0 \) meson, \( q_1 \) is the momentum of the photon and \( \epsilon_1 \) its polarization vector, \( q_2 \) is the momentum of the \( \rho^0 \) meson and \( \epsilon_2 \) its polarization vector. (The subscript \( SD \) denotes short-distance.)

Using the identity \( \sigma_{\mu\nu} = \frac{i}{2} \epsilon_{\mu\alpha\beta\gamma} \sigma^{\alpha\beta} \gamma_5 \) it immediately follows that

\[
E_{SD} = M_{SD}.
\]

Since only one weak phase is involved, this amplitude on its own produces no \( CP \) violation. The branching ratio is:

\[
Br(B^0 \to \rho^0 \gamma) = \frac{G_F^2 \alpha}{16 \pi} m_{B^0}^3 \left( 1 - \frac{m_{\rho}^2}{m_{B^0}^2} \right) \left( |E_{SD}|^2 + |M_{SD}|^2 \right),
\]

where \( M_{SD} = -V_{tb} V_{td}^* \frac{m_b}{2\pi^2} T_{1}^{B^- \to \rho^-}(0) \)

and \( T_{1}^{B^- \to \rho^-}(0) \) is the form factor of the \( B^- \to \rho^- \) transition due to the tensor current \( [6] \).

The decay \( B^0 \to \rho^0 \gamma \) possesses another observable: the Stokes vector, specifying the polarization state of the photon. Conforming to the notation of Ref. \( [2] \), we rewrite the decay amplitude \( [3] \) in the general form

\[
A(B^0 \to \rho^0 \gamma) = \frac{\epsilon G_F}{\sqrt{2}} \left( \epsilon_{\mu\rho\sigma} q_1^\mu q_1^\rho q_2^\sigma M + \epsilon_1^\mu \epsilon_2^\nu \left( g_{\mu\nu} p \cdot q_1 - p \mu q_1 \nu \right) E \right),
\]

where \( M = M_{SD} + \ldots \) and \( E = i(E_{SD} + \ldots) \), the dots denoting further interaction terms to be introduced later. The polarization of the photon is defined by the density matrix \( \rho \):

\[
\rho = \left( \begin{array}{cc} |E|^2 & E^* M \\ E M^* & |M|^2 \end{array} \right) = \frac{1}{2} \left( |E|^2 + |M|^2 \right) \left( 1 + \vec{S} \cdot \vec{\tau} \right),
\]

where \( \vec{\tau} \) are the Pauli matrices and \( \vec{S} \) is the Stokes vector. The components of the Stokes vector are according to Eq. \( [3] \):

\[
S_1 = \frac{2 \text{Re}(E^* M)}{|E|^2 + |M|^2}, \\
S_2 = \frac{2 \text{Im}(E^* M)}{|E|^2 + |M|^2}, \\
S_3 = \frac{|E|^2 - |M|^2}{|E|^2 + |M|^2}.
\]

The component \( S_2 \) describes the circular polarisation of the photon which has been discussed by Grinstein and Pirjol \( [3] \). More interesting from our point of view are the components \( S_1 \) and \( S_3 \) which can be measured indirectly by studying the Dalitz pair process \( B^0 \to \rho^0 \gamma \to \rho^0 e^+ e^- \).
If the short-distance amplitude is the only contribution there is no $CP$ asymmetry ($A_{CP} = 0$) and the photon is purely left-handed polarized, the Stokes vector reducing to the trivial form ($S_{1,3} = 0$, $S_2 = -1$).

In the next Section, we introduce an extra contribution to the $B^0 \to \rho^0 \gamma$ amplitude that carries not only a different weak phase but a non-trivial dynamical (strong) phase, thereby generating non-vanishing values for the observables $A_{CP}$, $S_1$ and $S_3$.

2 A long-distance contribution to $B^0 \to \rho^0 \gamma$

We consider the long-distance contribution to the decay $B^0 \to \rho^0 \gamma$ depicted by the triangle graphs in Fig. (1) with $D^+ D^-$ mesons as intermediate states\(^1\). The amplitude is calculated by analogy to the pion-loop model used for the decay $K_S \to \gamma \gamma^*$, discussed in detail in Ref. [4]. In such a model, based on minimal electromagnetic coupling, both real and imaginary parts of the amplitude are finite and calculable. Using dimensional regularization the gauge-invariant amplitude for the three graphs (triangle + crossed + sea-gull) is purely electric:

\[
A_{LD} = \frac{e G_F}{\sqrt{2}} i \epsilon_1^{\mu} \epsilon_2^{\nu} (g_{\mu\nu} p \cdot q_1 - p_{\mu} q_{1\nu}) E_{LD},
\]

where

\[
E_{LD} = -V_{cb} V_{cd}^* \frac{2 f_{\rho D^+ D^-} \epsilon_{B^0 D^+ D^-}}{(4\pi)^2 m_{D^+}^2} F(m_{B^0}^2, m_{\rho^0}^2)
\]

and [7]

\[
F(m_{B^0}^2, m_{\rho^0}^2) = -\frac{1}{2(a-b)} + \frac{1}{(a-b)^2} \left[ \frac{f_a - f_b}{2} + b(g_a - g_b) \right],
\]

\[
f_a = -\left( \ln \left( \sqrt{a} + \sqrt{a-1} \right) - i \frac{\pi}{2} \right)^2,
\]

\[
g_a = \sqrt{a-1} \left( \ln \left( \sqrt{a} + \sqrt{a-1} \right) - i \frac{\pi}{2} \right),
\]

\[
f_b = \arcsin^2(\sqrt{b}),
\]

\[
g_b = \sqrt{1-b} \arcsin(\sqrt{b}),
\]

\[
a = \frac{m_{B^0}^2}{4 m_{D^+}^2},
\]

\[
b = \frac{m_{\rho^0}^2}{4 m_{D^+}^2}.
\]

\(^1\)Similar graphs have been considered in connection with the long-range contribution to $B^0 \to \gamma \gamma$; see, for example, Ref. [3]. It may be mentioned that long-distance effects of operators containing $\bar{c}c$ currents in charmless $B$-decays have also been discussed under the appellation *charming penguins* [5]. We are not aware of a discussion of the radiative decays $B^0 \to \rho^0 \gamma (K^0 \to \rho^0 \gamma)$ along these lines.
In this model there are only two parameters left: the coupling constants $g_{B^0 D^- D^-}$ and $f_{D^+ D^-}$. The coupling $g_{B^0 D^- D^-}$ is determined by data $(Br(B^0 \rightarrow D^+ D^-) = 2.46 \times 10^{-4})$. For $f_{D^+ D^-}$ we use the vector dominance hypothesis, which implies $f_{D^+ D^-} = \frac{1}{2} f_{D^0 D^-}$. Using the empirical value $f_{D^0 D^-}^2 / 4\pi \approx 2.5$, we thus have

$$g_{B^0 D^- D^-} = \frac{4}{G_F |V_{cb} V_{cd}^*|} \frac{2\pi m_{B^0}}{\sqrt{Br(B^0 \rightarrow D^+ D^-)}} \sqrt{\frac{1 - 4m_{D^+}^2}{m_{B^0}^2}} \approx 2.5 \pi,$$

$$f_{D^+ D^-} = \frac{1}{2} f_{D^0 D^-} \approx \sqrt{2.5 \pi}.$$

A comparison of the penguin amplitude with the above long-distance contribution reveals several interesting features.

(a) The two amplitudes have different CKM factors, hence different weak phases. In addition, the long-distance part has a large absorptive part, producing a significant strong phase:

$$\delta_{\text{dyn}} = \arctan \left( \frac{\text{Im} \left[ F(m_{B^0}^2, m_{D^+}^2) \right]}{\text{Re} \left[ F(m_{B^0}^2, m_{D^+}^2) \right]} \right) \approx 97^\circ.$$

This opens the way to a non-zero $CP$-violating asymmetry $A_{CP}$.

(b) The long-distance component is quite sizeable in comparison to the short-distance amplitude. Taking the estimates in Eqs. (13) and (14) at face value,

$$\frac{\Gamma_{\text{LD}}}{\Gamma_{\text{SD}}} \approx 30\%.$$

(c) The long-distance amplitude generated by the $D^+ D^-$ intermediate state is purely electric, in contrast to the equality of $E_{\text{SD}}$ and $M_{\text{SD}}$ (Eq. (11)). This implies that the Stokes vector component $S_3$ will be non-zero. The existence of the strong phase $\delta_{\text{dyn}}$ also means that the component $S_1$ will be different from zero. Thus, we can expect non-trivial effects associated with $S_{1,3} \neq 0$ in the Dalitz pair reaction $B^0 \rightarrow \rho^0 \gamma^* \rightarrow \rho^0 e^+ e^-$. The amplitude $A_{\text{LD}}$ given by Eqs. (11) - (12), is based on minimal electromagnetic coupling, and serves as a convenient reference value for the long-range contribution to $B^0 \rightarrow \rho^0 \gamma$, possessing finite real and imaginary parts. The composite nature of the $D$-meson implies that there will be other intermediate states such as $DD^*$, $D^*D^*$ etc., as well as possible form factor effects in the $D\bar{D}$ contribution. Data on $B^0$ decays suggest that the $DD^*$, $D^*D^*$ final states are dominantly $CP = +1$, the same as for $D^+ D^-$, implying that the effect of these intermediate states on $B^0 \rightarrow \rho^0 \gamma$ is mainly in the electric amplitude $E$. In what follows we simulate the total long-distance amplitude by using an expression of the form $A_{\text{LD}} = \xi A_{\text{LD}}(D^+ D^-)$, allowing the parameter $\xi$ to vary in the range $-1 \leq \xi \leq 1$.

Inserting this parametrization in the Stokes vector in Eq. (9) (neglecting the small $W$-exchange effects for clarity) the results are:

$$S_1 = \frac{-2\xi |E_{\text{SD}}||E_{\text{LD}}| \sin(\delta_{\text{dyn}} - \beta)}{|E|^2 + |M|^2},$$
\[ S_2 = \frac{-2|E_{SD}|^2 - 2\xi|E_{SD}|E_{LD}| \cos(\delta_{dy} - \beta)}{|\mathcal{E}|^2 + |\mathcal{M}|^2}, \]
\[ S_3 = \frac{\xi^2|E_{LD}|^2 + 2\xi|E_{SD}|E_{LD}| \cos(\delta_{dy} - \beta)}{|\mathcal{E}|^2 + |\mathcal{M}|^2}. \]

And the CP asymmetry is \( A_{CP} = (\Gamma(B^0) - \Gamma(B^0))/(\Gamma(B^0) + \Gamma(B^0)) \)
\[ A_{CP} = \frac{4\xi|E_{SD}|E_{LD}| \sin(\delta_{dy} \sin \beta)}{|\mathcal{E}|^2 + |\mathcal{M}|^2 + |\mathcal{E}|^2 + |\mathcal{M}|^2}. \]

The results are shown in Fig. 2 for the CP asymmetry and in Fig. 3 for the Stokes vector, respectively. The central values used for the weak CKM phases are \( \beta = 23^\circ \) and \( \gamma = 59^\circ \).

The CP asymmetry in Fig. 2 ranges from \(-22\%\) for \( \xi = -1 \) to 26\% for \( \xi = 1 \) and vanishes at \( \xi = 0 \). Even for \( |\xi| \approx 0.3 \) which corresponds to a long-distance contribution of 3\% in the decay rate the asymmetry is still large: \( |A_{CP}| \approx 10\% \).

The effects of the long distance contribution to the Stokes vector are also large. The component \( S_1 \) (solid line) has a value around 0.7 for \( \xi = -1 \) and around -0.5 for \( \xi = 1 \) to be compared to \( S_1 = 0 \) at \( \xi = 0 \). The component \( S_3 \) (dashed line) is small for \( \xi < 0 \) but approaches 0.35 for \( \xi = 1 \). The component \( S_2 \) (dotted line) is plotted for completeness.

The Stokes vector components \( S_1 \) and \( S_3 \) are observable in the decay \( B^0 \to \rho^0 \gamma^* \to \pi^+ \pi^- e^+ e^- \) according to 2
\[ \frac{d\Gamma}{ds_1 d\phi} \sim 1 - (\Sigma_1(s_1) \sin 2\phi + \Sigma_3(s_1) \cos 2\phi), \]

where \( \Sigma_1(0) \) and \( \Sigma_3(0) \) are proportional to \( S_1 \) and \( S_3 \), respectively. \( \phi \) is the angle between the dipion and the dilepton plane.

To determine the magnitude of \( \Sigma_1 \) and \( \Sigma_3 \) we follow reference [10]. There, the decay \( B^0 \to \pi^+ \pi^- e^+ e^- \) is constructed under the assumption that the pion pair is produced at the \( \rho^0 \) resonance in a narrow width approximation. Additional to the short-distance contribution due to \( \mathcal{O}_7 \) in the Hamiltonian (Eq. 11) the operators \( \mathcal{O}_9 \) and \( \mathcal{O}_{10} \) have to be included. They dominate the decay rate in the region of higher dilepton mass \( s_1 \). We use in our calculation the Wilson coefficients \( c_7 = -0.315, c_9 = 4.224 \) and \( c_{10} = -4.642 \).

The resulting differential decay rate is written in a compact form in Eq. (3.7) of [10]. The effects of the long-distance contributions to this decay are incorporated by modifying the form factors \( g_+(s_l) \) and \( g_-(s_l) \) (\( q^2 \equiv s_l \)) in Eqs. (2.16) and (2.18) of [10]:

\[ g_+(s_l) \to g_+(s_l) - \xi g_{LD}(s_l), \]
\[ g_-(s_l) \to g_-(s_l) + \xi g_{LD}(s_l), \]

where
\[ g_{LD}(0) = \frac{V_{td}^* V_{us}^* f_{\rho^0 \pi^+ \pi^-} - g_{\rho^0 \rho^0} f_{\rho^0 \pi^+ \pi^-}}{V_{ub}^* V_{ts}^* 4\sqrt{\lambda} m_{D^*}^2} F(m_{B^0}^2, m_{\rho^0}^2), \]

\[ \text{For the form factors, we have used the parametrization in Table IV of [10]. However, the normalization has been updated to take account of the value } g_+(0)|_{B^0 \to \rho^-} = -T_{1}^{B^0 \to \rho^-}(0) = -0.27 \text{ instead of the value } -0.18 \text{ used in [10]. Note that } g_+(0) \equiv g_+(0)|_{B^0 \to \rho^-} = -\frac{1}{\sqrt{2}} g_+(0)|_{B^0 \to \rho^-}. \]
and
\[
\frac{g_{LD}(s_l)}{g_{LD}(0)} = G(m_{B^0}^2, s_l) F_{em}^D(s_l). \tag{23}
\]

The function \(G(m_{B^0}^2, s_l)\) describes the effects due to the triangle graph in Fig. 1 (assuming, for simplicity, scalar external particles):

\[
G(m_{B^0}^2, s_l) = \left[ 1 - \frac{f_c \theta(c - 1) + f'_c \theta(1 - c)}{f_a} \right] \left( 1 - \frac{s_l}{m_{B^0}^2} \right)^{-1}, \tag{24}
\]

\[
f_c = -\left( \ln\left(\sqrt{c} + \sqrt{c - 1}\right) - \frac{\pi}{2}\right)^2,
\]

\[
f'_c = \arcsin^2(\sqrt{c}),
\]

\[
c = \frac{s_l}{4m_{D^+}^2},
\]

and \(f_a\) as in Eq. 12. The factor \(F_{em}^D(s_l)\) is the electromagnetic form factor of the \(D\) meson in vector dominance approximation:

\[
F_{em}^D(s_l) = \frac{3}{2} \frac{1}{2 - s_l/m_{\rho^0}^2 - \frac{i}{m_{\rho^0}^2} \left( 1 - \frac{m_{D^+}^2}{s_l} \right)^2 \theta(s_l - 4m_{\pi}^2)} \tag{25}
\]

\[
- \frac{1}{2} \frac{1}{2 - s_l/m_{\rho^0}^2 - \frac{i}{m_{\rho^0}^2} \theta(s_l - 9m_{\pi}^2)}
\]

Integrating all variables but \(\phi\) and \(s_l\) in the differential decay rate yields Eq. 19. The results for \(\Sigma_1(s_l)\) and \(\Sigma_3(s_l)\) are shown in Fig. 11 and Fig. 15, respectively. Comparing the Stokes parameters \(S_1\) to \(\Sigma_1(0)\) and \(S_3\) to \(\Sigma_3(0)\) shows that they differ only by a factor of roughly 2. Both, \(\Sigma_1(s_l)\) and \(\Sigma_3(s_l)\) can be dominated in the region \(s_l < 2 GeV^2\) by the proposed long-distance effect, depending on the choice of the parameter \(\xi\). The branching ratio for \(B^0 \to \rho^0 \gamma^* \to \pi^+\pi^-e^+e^-\) in the region \(s_l < 2 GeV^2\) is found to be \((1.9, 1.3, 1.8) \times 10^{-8}\) for \(\xi = (\frac{\pi}{2}, 0, -\frac{\pi}{2})\). For \(s_l > 2 GeV^2\) the branching ratio is \(2.7 \times 10^{-8}\), almost independent of \(\xi\).

3 Remarks on Stokes Parameter for \(B^0 \to K^+\gamma\)

Using a description for the short-distance amplitude in the decay \(B^0 \to K^+\gamma\) similar to that of Eq. 3, it immediately follows, as in pure short-distance \(B^0 \to \rho^0 \gamma\), that \(A_{CP} = 0, S_{1,2} = 0, S_2 = -1\). To embed the long-distance model in the decay \(B^0 \to K^+\gamma\) we used the same definitions as in \(B^0 \to \rho^0 \gamma\) and made the following modifications, assuming \(SU(3)\)-symmetry: The vertices \(B^0 \to D^+D^-\) and \(\rho^0 D^+D^-\) change to \(B^0 \to D^+D^-\) and \(K^+D^+D^-\), respectively. In the CKM matrix elements exchange \(d \leftrightarrow s\) and align the form factors to the values found in the tables of \(B \to K^\pm\gamma\) decays.

The coupling \(g_{B^0D^+D^-}\) can be calculated from the branching ratio \(Br(B^0 \to D^+D^-) = 9.6 \times 10^{-3}\) \([11]\), and \(SU(3)\)-symmetry gives \(f_{K^+D^+D^-} = \sqrt{2} f_{\rho^0 D^+D^-}\).

Since there is no relative weak phase (up to order \(\lambda^4\)) in \(B^0 \to K^+\gamma\) even after including the proposed long-distance contribution, the \(CP\)-asymmetry is \(A_{CP} = 0\) for all values of \(\xi\). However,
due to the absorptive part of the triangle graph a strong (dynamical) phase is still present. In fact, it is essentially as large as in $B^0 \to \rho^0 \gamma$: $\delta_{\text{dyn}} \approx 97^\circ$. The presence of this phase can be seen in the Stokes vector components which are shown in Fig. (6). For $\xi \neq 0$ the components $S_1$ and $S_3$ display a significant deviation from zero.

The Stokes vector components $S_1$ and $S_3$ can be detected in $B^0 \to K^* \gamma^* \to \pi^+ K^- e^+ e^- \ [10]$ in the differential decay rate:

$$\frac{d\Gamma}{ds_l d\phi} \sim 1 - (\Sigma_1(s_l) \sin 2\phi + \Sigma_3(s_l) \cos 2\phi), \quad (26)$$

which is derived in a way analogous to that in the decay $B^0 \to \pi^+ \pi^- e^+ e^-$ described before. The results for $\Sigma_1$ and $\Sigma_3$ are shown in Figs. (7) and (8), respectively. Again, in the lower region of $s_l$ the long-distance contribution can play an important role depending on the parameter $\xi$. The branching ratio in the domain $s_l < 2 GeV^2$ is $(0.8, 0.6, 0.9) \times 10^{-6}$ for $\xi = (+\frac{1}{2}, 0, -\frac{1}{2})$, while for $s_l > 2 GeV^2$, the corresponding value is $0.9 \times 10^{-6}$, essentially independent of $\xi$.

\section{4 Summary}

We have examined a long-distance contribution to the decay $B^0 \to \rho^0 \gamma$, which induces a non-zero $CP$-violating asymmetry, $A_{CP} \neq 0$. At the same time, the Stokes parameters $S_1$, $S_3$ of the photon acquire non-zero values that can be detected in the correlation of the $\pi^+ \pi^-$ and $e^+ e^-$ planes in the decay $B^0 \to \rho^0 \gamma^* \to \pi^+ \pi^- e^+ e^-$. The same long-distance mechanism has been examined in the case of $B^0 \to K^* \gamma$. Although $A_{CP}$ remains zero in this case, significant effects due to the Stokes parameters $S_1$, $S_3$ are predicted in the correlation of the hadron and lepton planes in the Dalitz pair process $B^0 \to K^* \gamma^* \to \pi^+ K^- e^+ e^-$.  

\textbf{References}

[1] M. Beyer, D. Melikhov, N. Nikitin and B. Stech, Phys. Rev. D \textbf{64}, 094006 (2001).

[2] L. M. Sehgal and J. van Leusen, Phys. Rev. Lett. \textbf{83}, 4933 (1999).

[3] B. Grinstein, D. Pirjol, Phys. Rev. D \textbf{62}, 093002 (2000); A. Ali, A. Y. Parkhomenko Eur. Phys. J. C \textbf{23}, 89 (2002).

[4] D. Choudhury and J. Ellis, Phys. Lett. B \textbf{433}, 102 (1998); W. Liu, B. Zhang and H. Zheng, Phys. Lett. B \textbf{461}, 295 (1999).

[5] M. Ciuchini, R. Contino, E. Franco, G. Martinelli and L. Silvestrini, Nucl. Phys. B \textbf{512} 3 (1998); \textit{ibid.} B \textbf{531} 656 (1998); A. J. Buras and L. Silvestrini, Nucl. Phys. B \textbf{569} 3 (2000).

[6] M. Wirbel, B. Stech and C. Bauer, Z. Phys. C \textbf{29}, 637 (1985); D. Melikhov, Phys. Rev. D \textbf{53}, 2460 (1996); D \textbf{56}, 7089 (1997); M. Beyer, D. Melikhov, Phys. Lett. B \textbf{452}, 121 (1999); D. Melikhov, B. Stech, Phys. Rev. D \textbf{62}, 014066 (2000).
Figure 1: Proposed long-distance contribution for $\overline{B^0} \to \rho^0 \gamma$: triangle, crossed and sea-gull graph

[7] J. Pestieau, C. Smith and S. Trine, Int. J. Mod. Phys. A 17, 1355 (2002); C. Smith, Ph. D. dissertation, Appendix A, Université Catholique de Louvain, Louvain-la-Neuve, May 2002; see also L. M. Sehgal, Phys. Rev. D 7, 3303 (1973).

[8] T. Browder, talk at Lepton & Photon 2003, Batavia, Illinois, 2003.

[9] Babar Collaboration, B. Aubert et al., Phys. Rev. Lett. 91, 131801 (2003).

[10] F. Krüger, L. M. Sehgal, N. Sinha and R. Sinha, Phys. Rev. D 61, 114028 (2000); ibid. 63, 019901(E) (2000).

[11] M. Neubert and B. Stech, in “Heavy Flavours”, 2nd Edition, edited by A. J. Buras and M. Lindner (World Scientific, Singapore); hep-ph/9705292
Figure 2: The $CP$ asymmetry as a function of the scale parameter $\xi$ of the long distance contribution in $B \to \rho^0 \gamma$. 
Figure 3: The Stokes vector $\vec{S}$ as a function of the scale parameter $\xi$ of the long distance contribution in (a) $B^0 \to \rho^0 \gamma$, (b) $B^0 \to \rho^0 \gamma$ and (c) an untagged mixture of $B^0/B^0 \to \rho^0 \gamma$. The solid line describes the $S_1$ component, the dotted line the $S_2$ component and the dashed line the $S_3$ component.
Figure 4: The component $\Sigma_1$ as a function of the scale parameter $\xi$ of the long distance contribution and the dilepton energy $s_l$ in $B^0 \to \pi^+\pi^-e^+e^-$. 

Figure 5: The component $\Sigma_3$ as a function of the scale parameter $\xi$ of the long distance contribution and the dilepton energy $s_l$ in $B^0 \to \pi^+\pi^-e^+e^-$. 

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Figure 6: The Stokesvector $\vec{S}$ as a function of the scale parameter $\xi$ of the long distance contribution in $B^0 \to K^*\gamma$. The solid line describes the $S_1$ component, the dotted line the $S_2$ component and the dashed line the $S_3$ component.

Figure 7: The component $\Sigma_1$ as a function of the scale parameter $\xi$ of the long distance contribution and the dilepton energy $s_l$ in $B^0 \to \pi^+K^-e^+e^-$. 
Figure 8: The component $\Sigma_3$ as a function of the scale parameter $\xi$ of the long distance contribution and the dilepton energy $s_l$ in $B^0 \to \pi^+ K^- e^+ e^-$. 