On parton distributions beyond the leading order

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Abstract

The importance of properly taking into account the factorization scheme dependence of parton distribution functions is emphasized. A serious error in the usual handling of this topic is pointed out and the correct procedure for transforming parton distribution functions from one factorisation scheme to another recalled. It is shown that the conventional MS and DIS definitions thereof are ill-defined due to the lack of distinction between the factorisation scheme dependence of parton distribution functions and renormalisation scheme dependence of the strong coupling constant $\alpha_s$. A novel definition of parton distribution functions is suggested and its role in the construction of consistent next-to-leading order event generators briefly outlined.

1 Introduction

During recent years significant progress in the determination of parton distribution functions (p.d.f.) in the nucleon has been achieved, basically as a result of new data [1, 2], combined with more sophisticated and reliable theoretical analyses [3, 4, 5]. In the CTEQ Collaboration [4] a number of theorists, phenomenologists and experimentalists have combined their efforts in order to deal properly with all experimental and theoretical subtleties of quantitative QCD analysis of vast amount of data from various experiments and processes. In [4, 5] p.d.f. are determined with high accuracy, unheard of just a few years ago. In such circumstances a careful reanalysis of various theoretical uncertainties is clearly needed. Although most of such uncertainties are discussed in sufficient detail in review papers like [3, 7], there is one which has not been so far satisfactorily covered in either these or in any other paper I am aware of. It concerns the factorization scheme (FS) dependence of finite order QCD predictions in processes involving hadrons in the initial state. The treatment of this ambiguity presented in [4, 5, 8] is incomplete and moreover contains an error in the very central point of the factorization mechanism. As the FS dependence of p.d.f. has so far obtained much less attention than it probably deserves, I discuss in this note several of its aspects, drawing on analogy with the much more publicised case of the renormalization scheme (RS) ambiguity of the running coupling constant (couplant) $\alpha_s$. I think that much of the confusion and
misunderstanding that surrounds this topic stems from the failure to distinguish these related but in principle separate uncertainties.

The organization of this paper is as follows. In Section 2 the notation is introduced and some basic facts about the RS dependence of the couplant are recalled. The crucial point, i.e. the dependence of p.d.f. on the choice of the FS is discussed in Section 3, followed in Section 4 by a few critical remarks on the currently used p.d.f.. In Section 5 the merits of the so-called “zero” FS are discussed, and in particular it is shown how it can be used for the construction of consistent next-to-leading order (NLO) event generators. The results are summarized and conclusions drawn in Section 6.

2 Remarks on renormalization scheme dependence

Before coming to the ambiguity in the definition of p.d.f. let me recall a few basic facts about RS dependence of the renormalized couplant $a(\mu) \equiv g^2(\mu)/4\pi^2$. In massless QCD (to which I restrict my attention) it obeys the equation

$$\frac{da(\mu)}{d\ln \mu} = -ba^2 \left( 1 + c_1a(\mu) + c_2a^2(\mu) + \cdots \right),$$

(1)

where the coefficients $b, c_1$, are fixed by the number of quark flavours, while all the higher ones are essentially free, defining the so called renormalization convention (RC), $\text{RC} = \{c_i, i \geq 2\}$. In the simplest case this couplant enters the perturbation expansion of a physical quantity $R$, depending on a single external momentum $Q$, in the form

$$R(Q) = a(\mu) \left( 1 + r_1(\mu/Q, c_1)a(\mu) + r_2(\mu/Q, c_1, c_2)a^2(\mu) + \cdots \right).$$

(2)

Although not written out explicitly, also the couplant $a(\mu)$ (when (1) is considered to the $k$-th order) depends on all $c_i, i \leq k-2$. Moreover, both the couplant and the coefficients $r_k$ depend also on the specification which of the infinite number of solutions to (1) we have in mind. Each of these solutions can be labelled, for instance, by the familiar $\Lambda$ parameter. Combining this last information with that on $c_i$ defines what is usually called the renormalization scheme. Only if this RS is fixed does the specification of the scale $\mu$ (together with $c_i, i \geq 2$) uniquely determine both the couplant and the coefficients $r_k$. Although I prefer the terminology advocated in [1], where the the term “RS” is reserved for a unique specification of both $a$ and $r_k$ (for detailed discussion of this point, see [2]), I adopt in the following the more conventional notation in order to stay in close contact with [3 4]. In this notation $\mu$ is set equal to some “natural” scale in the problem and the variation of the RS is parametrized by means of the corresponding $\Lambda_{\text{RS}}$ and the coefficients $c_i, i \geq 2$. Considering now (1,2) to the NLO, we are left with only one degree of freedom, corresponding to the variation of $\Lambda_{\text{RS}}$. To make the following considerations as transparent as possible, let me set $c_1 = 0$ in the rest of this note. The internal consistency of perturbation theory implies, using (1), the following

\footnote{No essential conclusion obtained in the following does, however, depend on this purely technical simplification.}
relation between $\mu$, $a$ and $r_1$:

$$r_1(\mu/Q, \text{RS}) = b \ln \left( \frac{\mu}{\Lambda_{\text{RS}}} \right) - \rho = \frac{1}{a} - \rho \quad \Rightarrow \quad a = \frac{1}{r_1 + \rho}$$  \tag{3}

where $\rho$

$$\rho = b \ln \left( \frac{Q}{\Lambda_{\text{RS}}} \right) - r_1(\mu = Q, \text{RS})$$  \tag{4}

is the renormalization group (RG) invariant [8], which contains the $Q$-dependence of $R(Q)$. As the variations of $\mu$ and the RS are actually two sides of the same coin, it is redundant to vary both of them. Without the loss of generality we can fix $\mu$ (for instance by setting it equal to some external momentum, like $\mu = Q$) in (3) and elsewhere and vary the RS only. On the other hand, as the “natural” scale in the problem is usually not so unambiguously defined after all, many authors vary both the scale $\mu$ and the RS. Although unnecessary complication, this procedure is certainly legal. One has, however, to keep in mind that without the specification of the RS the choice of $\mu$ doesn’t fix either the couplant $a(\mu)$ or the coefficients $r_k$. In different RS’s the same $\mu$ implies different $a(\mu/\Lambda_{\text{RS}})$ and $r_k$ and thus the choice of the RS is as important as that of $\mu$. With this in mind let me continue to label the RS by means of $\Lambda_{\text{RS}}$, but keep $\mu$ still as a free parameter. Eq. (3) furthermore suggests that instead of $\Lambda_{\text{RS}}$, the value of $r_1$ can equally well serve the purpose of labelling the various RS. Substituting (3) into (2) and truncating it to the NLO we get

$$R_{\text{NLO}}(\rho, r_1) = \frac{2r_1 + \rho}{(r_1 + \rho)^2} = \frac{1}{\rho} \left( \frac{1 + 2r_1/\rho}{(1 + r_1/\rho)^2} \right) \tag{5}$$

as an explicit function of $r_1$ and $\rho$. The obvious consequence of the nontrivial dependence of $R_{\text{NLO}}$ on the RS is that it would be a profound mistake to “transform” the couplant from one RS into another by means of equating the NLO (in fact any finite order) approximation to (2) in two different RS, say RS(1), RS(2), i.e. by solving the equation

$$a(\text{RS}(1))(1 + r_1(\text{RS}(1))a(\text{RS}(1))) = a(\text{RS}(2))(1 + r_1(\text{RS}(2))a(\text{RS}(2))) \tag{6}$$

Assuming (3) in RS(1) and expressing $r_1(\text{RS}(2))$ in terms of $a^{(1)} = a(\text{RS}(1)), a^{(2)} = a(\text{RS}(2))$ we get from (3)

$$r_1(\text{RS}(2)) = \frac{2a^{(1)}/a^{(2)} - 1}{a^{(2)}} - \rho \frac{a^{(1)}}{a^{(2)}}$$  \tag{7}

which yields the correct relation (3) between $r_1(\text{RS}(2))$ and $a^{(2)}$ only for the trivial case $a^{(1)} = a^{(2)}$. In other words imposing the relation (3) leads to inconsistency for any nontrivial RG transformation! In the particular case when RS(2) is defined by means of the effective charges approach of [10], corresponding to $r_1^{\text{ECH}} = 0$, (3) implies

$$a^{\text{ECH}} = \frac{1}{\rho} \left( \frac{1 + 2r_1^{(1)}/\rho}{(1 + r_1^{(1)}/\rho)^2} \right) \tag{8}$$

while the correct relation reads $a^{\text{ECH}} = 1/\rho$. 

3
3 Parton distributions in general factorization scheme

In processes involving hadrons in the initial state, there is, beside the mentioned RS dependence of the couplant, another kind of ambiguity, concerning the definition of p.d.f. beyond the leading order [11]. Again, to simplify the discussion as much as possible, I restrict the discussion to the nonsinglet (NS) quark distribution functions as revealed in the lepton-nucleon deep inelastic scattering (DIS). Contrary to the RS dependence of the couplant [9, 12, 13], this latter ambiguity has so far received much less attention [14]. This is somewhat surprising, taking into account that its consequences may actually be even more important than those discussed in Section 2.

In DIS the factorization theorem [11] implies that the generic NS structure function $F_{NS}(x, Q^2)$ can be written as a convolution (I drop the label “NS” in the following)

$$F(x, Q^2) = \int_x^1 \frac{dy}{y} q(y, M) C \left( \frac{Q}{M}, x, a(\mu) \right)$$

(9) of the perturbatively uncalculable quark distribution function $q(x, M)$, defined at the factorization scale $M$, and obeying the evolution equation

$$\frac{dq(x, M)}{d \ln M} = \int_x^1 \frac{dy}{y} \left[ a(M) P^{(0)} \left( \frac{x}{y} \right) + a^2(M) P^{(1)} \left( \frac{x}{y} \right) + \cdots \right]$$

(10) and the hard scattering cross-section $C(Q/M, z, a(\mu))$ admitting perturbation expansion in powers of the couplant at the hard scattering scale $\mu$, generally different from $M$

$$C(Q/M, z, a(\mu)) = \delta(1 - z) + a(\mu) C^{(1)}(Q/M, z) + \cdots$$

(11) While $P^{(0)}(z)$ is unique, both $P^{(1)}$ in (10) and $C^{(1)}$ in (11) are ambiguous, but internal consistency of the factorization procedure links the variation of the NLO hard scattering cross-section $C^{(1)}$ with that of the NLO branching function $P^{(1)}$ [11]:

$$C^{(1)}(Q/M, z, \text{RS}) = P^{(0)}(z) \ln(Q/M) + P^{(1)}(z)/b + \kappa(z, \text{RS})$$

(12) where the FS-invariant function $\kappa(z, \text{RS})$ still, however, depends on the RS of the couplant $a(M)$. Similar consistency conditions do exist at each order of perturbation theory. In the rest of this note I stay within the NLO approximation. The dependence of $C^{(1)}$ on the RS of the couplant appears as a consequence of the fact, that the r.h.s. of (10) is given as an expansion in powers of this couplant. I shall return to this point in the next section. For fixed RS of the couplant $a(M)$, $P^{(1)}$, or via (12), $C^{(1)}$, defines at the NLO the FS of p.d.f. : FS:=\{P^{(1)}(z)\}. In the above relations the factorization scale $M$ is again, as in the case of $\mu$, kept as a free parameter.

The first point I want to emphasize is that the renormalization of the couplant, to the NLO order fully described by variations of $\Lambda_{\text{RS}}$ in both the couplant $a(M)$ and the coefficients

\footnote{If considered to all orders of $a(\mu)$, $C(Q/M, z, a(\mu))$ doesn’t actually depend on $\mu$ [11]. Contrary to the $M$-dependence of $q(N, M)$ and $C^{(1)}(N, M)$, which is the basic feature of the factorization theorem and holds to all orders, the dependence of $C(Q/M, z)$ on $\mu$ is merely a consequence of truncating expansion (11) to a finite order.}
$r_k$, is independent of the FS of p.d.f. as specified by the NLO branching function $P^{(1)}$ in the evolution equation (1)! Any combination of the RS=$\{\Lambda_{RS}\}$ and FS=$\{P^{(1)}\}$ represents in principle equally legal definition of p.d.f.. Moreover, as $P^{(1)}$ is a function of $z$, it represents in fact an infinite number of of degrees of freedom, and thus its variations can be expected to be at least as important as that of the factorization scale $M$. Also this point will be elucidated in the next section.

Secondly, let me recall the obvious fact that although the physical observable $F(x, Q^2)$ is, when (10,12) are taken to all orders, independent of the parameters describing the renormalization and factorization schemes, any finite order approximation to these expansions inevitably leads to nontrivial dependence of $F(x, Q^2)$ on $M$, RS and FS. I mention it here as it is related to the basic question I want to address in this section and which concerns the way the p.d.f. transform when the FS=$\{P^{(1)}\}$ is varied. As all the considerations are much more transparent in terms of conventional moments, defined, for a generic function $f(x)$, as

$$f(N) = \int_0^1 x^{N-1} f(x) dx$$

(13)

let me rewrite (9-12) in terms of them, explicitly writing out the dependence on both the RS of the couplant $a(M)$ and the FS of the p.d.f. ($d_N$, $d_N^{(1)}$, $\kappa(N)$ are the moments of $P^{(0)}(z)$, $P^{(1)}(z)$, $\kappa(z)$ respectively):

$$\frac{dq(N, M, RS,FS)}{d\ln M} = q(N, M, RS,FS)\gamma_N; \quad \gamma_N \equiv d_N a(M, RS) + d_N^{(1)}(FS)a^2(M, RS).$$

(14)

This is easily solved:

$$q(N, M, RS,FS) = A_N(a(M, RS))^{-d_N/b} \exp\left[-a(M, RS)d_N^{(1)}(FS)/b\right],$$

(15)

where the constants $A_N$, introduced in (1), are independent of $M$ as well as $d_N^{(1)}$. For moments of the structure function (9) we get, explicitly writing out the dependence of the NLO approximation to $F(N, Q^2)$ on $M$ and FS=$\{d_N^{(1)}\}$:

$$F(N, Q^2, M, RS, d_N^{(1)}) = q(N, M, RS, d_N^{(1)}) \left(1 + a(\mu, RS)C^{(1)}(Q/M, N, RS, d_N^{(1)})\right),$$

(16)

with the following consistency condition, implied by (12),

$$C^{(1)}(Q/M, N, RS, d_N^{(1)}) = d_N \ln \frac{Q}{M} + \frac{d_N^{(1)}}{b} + \kappa(N, RS).$$

(17)

For each moment $N$, the expression (13) is a function of $M$ which, however, still depends on two further parameters, one specifying the RS of the couplant and the other ($d_N^{(1)}$) the FS of p.d.f.. If we now want to transform $q(N, M, RS,FS)$ from one FS=$\{d_N^{(1)}\}$ into another, specified by FS=$\{d_{N'}^{(1)}\}$, we again cannot do so by imposing the relation (the RS-dependence is suppressed in the rest of this section)

$$q(N, M, d_N^{(1)}) \left(1 + a(M)C^{(1)}(Q/M, N, d_N^{(1)})\right) = q(N, M, d_{N'}^{(1)}) \left(1 + a(M)C^{(1)}(Q/M, N, d_{N'}^{(1)})\right)$$

(18)
As emphasized above, \( F(N, Q^2, M, d_N^{(1)}) \) does nontrivially depend on \( M, d_N^{(1)} \), as well as the RS and thus postulating equation like (18) would violate this basic feature of finite order approximations. Formally this is clear from inserting (15) into (18) and solving the resulting equation. Similarly to (7), we get the following relation between \( C_N^{(1)} \equiv C^{(1)}(Q/M, N, d_N^{(1)}) \) and \( d_N^{(1)} \):

\[
d_N^{(1)} = d_N^{(1)} - \frac{b}{a(M)} \ln \left( \frac{1 + a(M)C_N^{(1)}}{1 + a(M)f_N^{(1)}} \right)
\]

which reduces to the correct one, as given in (12), only for the trivial case \( d_N^{(1)} = d_N^{(1)} \). For any other case the equation (19) is incompatible with the requirement of consistency (12) and thereby wrong. I discuss this point in detail as in many papers, including [3, 6, 7], the equation (converted into moments and restricted to the NS channel) used to transform the quark distribution function between the so-called DIS and \( \overline{\text{MS}} \) “schemes” (more on them in the next section)

\[
q_{\text{DIS}}(N, M) = q_{\overline{\text{MS}}}(N, M) \left( 1 + a(M)C_{\overline{\text{MS}}}^{(1)}(N, M) \right)
\]

is \( C_{\text{DIS}}^{(1)} = 0 \) by definition) precisely of the incorrect form (18)!

The only theoretically consistent way of transforming \( q(N, M, \text{FS}) \) from one \( \text{FS} = \{d_N^{(1)}\} \) into another is given explicitly in (15) with, as emphasized, the constants \( A_N \) held fixed. In [14] I have discussed the whole procedure, based on the use of Jacobi polynomials [15, 16], in \( x \)-space. Although currently other, superior, methods of solving the evolution equations are available [3], the fact that Jacobi polynomials are constructed from conventional moments (15) for which we know how the FS transformations operate, makes them invaluable in this kind of considerations.

Finally a remark. The constants \( A_N \) represent the most natural way of parametrizing the uncalculable nonperturbative properties of the nucleon. They are not related to any particular “initial” \( M_0 \), nor to any \( \text{FS} = \{d_N^{(1)}\} \), but determine the asymptotic behaviour of \( q(N, M, \text{RS}, d_N^{(1)}) \) as \( M \to \infty \), which is unique.

4 Remarks on current phenomenology

In the preceding Section I have discussed the central question of the FS dependence of the p.d.f.. Let me now turn to the current phenomenology of DIS, related to this subject.

The first remark concerns the meaning of the words “\( \overline{\text{MS}} \)” and “DIS”, when used in the connection with the p.d.f. at the NLO. The \( \overline{\text{MS}} \) p.d.f. are defined in [3] as those “which appear in the equation such as (9) with hard scattering part \( C^{(1)} \) calculated with the \( \overline{\text{MS}} \) subtraction prescription.” Although correct, this definition is obviously incomplete, as it specifies merely the RS of the couplant but tell us nothing about the FS=\{\( P^{(1)}(z) \)\} to be used in (10)!. Recall that the term \( \ln 4\pi - \gamma_E \) defining the \( \overline{\text{MS}} \) “subtraction scheme”, is an artifact of extending the definition of the couplant into 4-\( \epsilon \) dimensions. As any RS of the couplant may be combined with any FS of p.d.f., there is an infinite set of \( \overline{\text{MS}} \)-like p.d.f., sharing the same definition of the couplant, and therefore the right to be called “\( \overline{\text{MS}} \)”,
but arbitrarily differing in \( P^{(1)} \). The one usually understood in the literature under the label “MS” corresponds to the FS used in [17, 18], which should be called “MS” as it sets (within the dimensional regularization), the finite parts of the counterterms, renormalizing the appropriate composite operators, to zero. Although the name for a FS is basically a matter of semantics, the current use of the term “MS” is misleading as it fails to specify the FS of the p.d.f. used.

Ambiguity of a different kind is associated with the use of the label “DIS”. This scheme is supposed to be defined by the condition:

\[
C^{(1)}_{\text{DIS}}(Q = M, z, RS, FS) = 0 \Rightarrow P^{(1)}_{\text{DIS}}(z, RS) = -b\kappa(z, RS)
\]

which, however, is again not unique, due to the fact that the FS invariant \( \kappa(z, RS) \) still depends on the RS of the couplant. In fact it is the combination

\[
\varepsilon(z) \equiv \kappa(z, RS) + P^{(0)}(z)\ln(Q/\Lambda_{RS})
\]

which is independent of \( M, FS=\{P^{(1)}(z)\} \) as well as the RS of the couplant [14]. Consequently in a given RS of the couplant \( a(M, RS) \) we find

\[
P^{(1)}_{\text{DIS}}(z, RS) = -b\varepsilon(z) + bP^{(0)}(z)\ln\frac{Q}{\Lambda_{RS}}.
\]

For example we find

\[
P^{(1)}_{\text{DIS}}(z, \overline{\text{MS}}) = P^{(1)}_{\text{DIS}}(z, \text{MS}) + bP^{(0)}(z)\ln\frac{\Lambda_{\text{MS}}}{\Lambda_{\overline{\text{MS}}}}
\]

In general, there is again an infinite set of “DIS”-like p.d.f., which share the property \( C^{(1)} = 0 \), but differ in the NLO branching function \( P^{(1)}(z) \) and the couplant, thereby leading to different numerical predictions when inserted into (9). P.d.f. bearing the name “DIS”, like those of [19], or some of [4], tacitly assume \( \overline{\text{MS}} \) as the RS of the couplant. This, however, is not a must and thus for an unambiguous specification of the “DIS”-like FS the RS of the couplant should always be specified.

The second remark concerns the practical aspect of exploiting the vast freedom in the definition of p.d.f. at the NLO. As already mentioned, little phenomenological attention has so far been payed to the FS dependence of p.d.f.. This may be due in part to the failure to appreciate the independence of these two renormalization procedures. It is also true that to apply, for instance, the idea of “optimization” [8] to the FC dependence of p.d.f. in \( x \)-space is technically much more involved. In [14] I have, however, argued that at least the FS defined by setting \( P^{(1)} = 0 \) (at the NLO; at higher orders it would generalize by setting all higher order AP branching functions to zero) should seriously be considered. In this “zero” FS the full NLO correction is put into the hard scattering cross-section \( C^{(1)} \), thereby representing in some sense the opposite of the DIS FS, which sets \( C^{(1)}(z) = 0 \). Moreover, it turns out [14] that when the moments of p.d.f. are considered, this FS is very close to that obtained via the Principle of Minimum Sensitivity of [8] (for \( c_1 = 0 \) they even coincide). Although this results doesn’t automatically imply the same close relation for the p.d.f. themselves, it seems reasonable to add this FS to the list of those used in phenomenological applications.
5 “Zero” FS and NLO event generators

There is, in fact, another reason why this FS could be of considerable interest. Recall, that all currently used event generators, like HERWIG, PYTHIA, JETSET, LEPTO etc., are based on essentially leading-log parton showers. Although they are sometimes combined with NLO hard scattering cross-sections, the overall description remains only LO. The simple picture of LO parton showers becomes much more complicated when one attempts to generalize them to the NLO. To get a consistent NLO description of any hard scattering process in the “zero” FS we, however, need merely the LO parton showers as $P^{(1)}(z) = 0$ in this FS!

As the example of moments of structure functions shows, this choice may be quite reasonable and should definitely be tried. Although simple at first glance, one has to be careful in taking for the NLO cross-section that corresponding to this “zero” FS. From (12) we easily find its form:

$$C_{zero}^{(1)}(Q/M, z) = P^{(0)}(z) \ln(Q/M) + \kappa(z, RS)$$

Using the results of [18] on $P^{(1)}$ and $C^{(1)}$, $\kappa(z, RS)$ can straightforwardly be evaluated in the $\overline{\text{MS}}$ RS. Transformation to any other RS is then trivial.

Let me stress that this procedure is not equivalent to the so-called matching of parton showers to fixed order matrix elements, as recently implemented in LEPTO event generator [20]. There, the exact $O(\alpha_s)$ matrix element is matched to the parton shower at some particular value of incoming parton virtuality $t_m$ (see Fig.1) in the sense that below $t_m$ only parton showers are used while above $t_m$ the matrix element takes fully over. In the case of “zero” FS, the situation is different and the NLO cross-section $d\sigma^{NLO}(Q/M, z, t)/dt$ which, when integrated over $t$, yields [25] contributes at any virtuality, even below that given by the factorization scale $t = M^2$, as only the pole term $1/t$ plus some finite part is factorized into the parton p.d.f.. As a result, the NLO hard scattering cross-section $d\sigma^{NLO}(Q/M, z, t)/dt$ becomes a discontinuous function of $t$ at $t = M^2$, this discontinuity being cancelled by a similar discontinuity of the parton shower contribution, which is restricted by definition to the domain $t \leq M^2$!

The result of factorization, i.e. the separation of the full NLO cross-section, containing all infrared and parallel singularities, into a part included in the quark distribution function and the remaining, finite, hard scattering cross-section, is represented in Fig.2b by the dashed and dotted curves, discontinuous at the factorization scale $\tau = M^2$. For $\tau > M^2$ the hard scattering cross-section $d\sigma^{NLO}(z, Q, t)$ coincides with the full result, but for $\tau < M^2$ its definition is ambiguous as it depends on how much of the finite part will accompany the singular pole term into the definition of the quark distribution function. Fig.2b corresponds to the case that only the pole term $A/\tau$ is subtracted. Recall that, for instance, the term $P^{(0)}(z) \ln(Q/M)$ appearing in (12) is essentially the integral of the pole term $A/\tau$ from $M^2$ to the upper kinematically allowed value of $\tau$, proportional to $Q^2$.
6 Summary and conclusions

In this note I have discussed several aspects of factorization scheme dependence of parton distribution functions. I have emphasized potential importance of proper treatment of this ambiguity for the reliability of theoretical analyses of ever better data. Special attention has been payed to the correct transformation of p.d.f. between different factorization schemes and the ambiguities in the meaning of some of the currently most popular definition of p.d.f. have been brought to light. Finally the so-called “zero” FS has been proposed and shown to be potentially useful in the construction of NLO event generators.

Acknowledgment

I am grateful to P. Kolář for careful reading of the manuscript and many stimulating discussions.

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Figure captions

Fig.1: Feynman diagram describing the process $e^- + q \rightarrow e^- + q + g$ with parallel singularity in the $t$-channel.

Fig.2a: A typical shape of $d\sigma(Q, z, t)/dt$ as a function of $\tau = -t$ for fixed $x, Q^2$. In this example the pole term (dashed curve) is added to a linearly rising finite part (dotted line) to give the full NLO contribution (solid curve).

Fig.2b: Separation of the full NLO contribution (solid curve) into the part absorbed in the quark distribution function (dashed curve) and the finite NLO hard scattering cross-section (dotted curve). The discontinuities of the last two curves at $\tau = M^2$ cancel in the sum.
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