EPR, Bell, and quantum locality

Robert B. Griffiths∗
Department of Physics, Carnegie-Mellon University,
Pittsburgh, PA 15213, USA

Version of 22 July 2011

Abstract

Maudlin has claimed that no local theory can reproduce the predictions of standard quantum mechanics that violate Bell’s inequality for Bohm’s version (two spin-half particles in a singlet state) of the Einstein-Podolsky-Rosen problem. It is argued that, on the contrary, standard quantum mechanics itself is a counterexample to Maudlin’s claim, because it is local in the appropriate sense (measurements at one place do not influence what occurs elsewhere there) when formulated using consistent principles in place of the inconsistent appeals to “measurement” found in current textbooks. This argument sheds light on the claim of Blaylock that counterfactual definiteness is an essential ingredient in derivations of Bell’s inequality.

Contents

I Introduction 1
II Quantum statics 3
III Quantum dynamics 6
IV Measurements 8
V EPR Correlations 9
   V A Entangled states .............................................. 9
   V B Dynamics ..................................................... 10
   V C Measurements ............................................... 11
VI Discussion 13
   VI A Locality ..................................................... 13
   VI B Counterfactuals ............................................. 14
VII Conclusion 15

I Introduction

Blaylock has argued that derivations of Bell’s inequality involve an implicit assumption of counterfactual definiteness as well as locality, and consequently the fact that quantum theory violates Bell’s inequality is consistent with the quantum world being local (as well as realistic and deterministic).1 In response Maudlin2 asserted that Bell’s result does not require counterfactual definiteness, and the fact that quantum theory violates it means that the quantum world is nonlocal: no local theory can reproduce The Predictions of quantum mechanics for the results of experiments done very far apart.

In this article we shall show that this last assertion is mistaken, by exhibiting a local theory for the situation of interest: measurements of various components of spin angular momentum on two spin-half

∗Electronic mail: rgrif@andrew.cmu.edu
particles prepared in a singlet state. (Blaylock considers polarization measurements on two photons, but these measurements amount to the same situation for the issues under discussion.) Because Maudlin’s argument is clearly written and relatively short, it is possible to make a useful comparison between it and the sort of reasoning needed to yield consistent results in the quantum domain. The connection with counterfactual definiteness will also be discussed.

The reader is probably aware that Bell’s inequality — see Ref. 1 for extensive references — puts constraints on certain correlation functions relating properties of separated systems, or measurements of these properties. Quantum mechanics predicts violations of these constraints, and measurements confirm the predictions of quantum theory. Consequently, any derivation of Bell’s inequality must violate one or more of the fundamental principles of quantum mechanics; or to put it another way, when viewed from a quantum perspective such a derivation contains one or more mistakes. On this point Blaylock, Maudlin, and the present author are in full agreement. The issue is: where and what are the mistakes? Finding mistakes is sometimes difficult. Many of us have experienced the frustration of dealing with a student asking us to find the error in a lengthy argument which he thinks is valid, but which leads to a result that we know is wrong. Various derivations of Bell’s inequality are complicated and involve probabilistic reasoning that is not straightforward. And sometimes additional assumptions, such as the existence of agents with free choice, move the discussion out of the realm of simple physics. Understanding these arguments, much less finding mistakes in them, can be an arduous task. We are fortunate that the argument for nonlocality in Ref. 2 is clear, compact, well written, and free choice plays no essential role.

The present paper is not intended to discuss all the assumptions that underlie various derivations of Bell’s inequality, or their possible validity in domains unrelated to quantum theory. Instead, the primary issue addressed is: at what point do Bell’s arguments, in particular as presented in Ref. 2, conflict with quantum theory?

Maudlin is not alone in maintaining that violations of Bell’s inequality imply that quantum mechanics is nonlocal. For an extended, though by no means complete, list of publications that have maintained this position, together with a significant number that have expressed disagreement, see the bibliography in Ref. 5, which contains a more detailed analysis of the Bell inequality from the perspective summarized in the present paper, along with a proof of an appropriate form of Einstein locality.

Issues of the sort under discussion cannot be resolved within the framework of standard quantum mechanics, when that is understood to be what is found in contemporary textbooks. Although textbooks provide students with many effective and efficient techniques for doing calculations, they do not contain an adequate presentation of the fundamental principles of the theory needed to understand why these techniques lead to reasonable answers, and what are their limits of applicability. Students become skilled at calculating the right answer, but are left confused or uncertain as to why, or whether, the answer really is right. Inquiries from the perplexed are met with Mermin’s well-known dictum: Shut up and calculate! In particular, attempts to make measurements part of the foundation of quantum mechanics are unsatisfactory because of the infamous measurement problem: the inability of such an approach to incorporate a physical measuring apparatus in fully quantum mechanical terms. The difficulties were pointed out by Wigner, and remain unsolved.

Thus in order to address the question of whether quantum theory is local, a consistent formulation of the principles that lie behind, and can be used to justify, the textbook calculations is needed: a formulation that, as Bell emphasized, is not based upon measurements, but instead allows us to understand, in quantum-mechanical terms, what goes on in real measuring processes. Such a formulation, in which probabilities (but not measurements) are fundamental, exists, and deserves to be better known. We shall refer to it as the histories approach; alternative names are consistent histories and decoherent histories. For further references and details see Ref. 13; for short introductions see Refs. 14 and 15. It yields what Maudlin calls “The Predictions” for Bohm’s version of the Einstein-Podolsky-Rosen (EPR) problem. It is explicitly local, as discussed below, and was applied by the author to the EPR problem in Ref. 18.

Because many readers will not be familiar with the histories approach (it is not mentioned in Refs. 1 and 2), some of its fundamental features are presented in a compact form in Sec. II, which deals with quantum statics, and in Sec. III, devoted to quantum dynamics. The histories approach is not based upon the concept of measurement, and readers who try to understand it in terms of measurements are likely to become confused. It is better to start with a mental picture of a classical system inside a closed box in which the dynamics is intrinsically random or stochastic. Things actually happen inside the box, but what happens in the future (or in the past) cannot be inferred with certainty from the state of affairs at a given point in time. As with any classical picture of the world, this one can only be a first step on the way to a fully consistent quantum description, but it is less likely to mislead than trying to think about things in
terms of measurements associated with mysterious wave function collapses, as in current textbooks.

Section IV discusses how genuine measurements, understood as real physical processes that go on in the real (quantum) world, can be understood from the perspective of fundamental quantum theory. The tools will then be in hand for a consistent analysis in Sec. V of the Bohm version of the EPR situation using two spin-half particles in a singlet state, both with and without measurements, demonstrating the locality of quantum theory.

Maudlin’s argument for nonlocality\(^2\) is examined in Sec. VI A in order to locate how and where it diverges from the analysis in Sec. V. Counterfactuals in the quantum domain are discussed in Sec. VI B, with the conclusion that their misuse can plausibly be part of one route leading to Bell’s inequality, as Blaylock indicates in Ref. 1, though perhaps it is not the only route. The paper ends with a brief summary and some comments in Sec. VII.

II Quantum statics

The most fundamental difference between classical and quantum mechanics is that the former makes use of a phase space whose individual points represent possible states of a physical system, with subsets of points representing physical properties. Whereas, as explained by von Neumann,\(^1^9\) quantum mechanics uses a complex Hilbert space, and physical properties correspond to subspaces, with a one-dimensional subspace (ray) the quantum analog of a single point in phase space. A finite-dimensional Hilbert space is adequate for purposes of the following discussion. In classical mechanics the logical negation of the property \(P\) that corresponds to a set of points \(\mathcal{P}\) in the phase space, which is to say the property “not \(P\)”, is represented by the set-theoretic complement \(\mathcal{P}^c\) of the set \(\mathcal{P}\). In the quantum case negation\(^{19}\) corresponds to the orthogonal complement \(\mathcal{P}^\perp\) of the subspace \(\mathcal{P}\): the collection of all vectors in the Hilbert space that are orthogonal to every vector in the subspace. Equivalently, if \(P\) is the projector onto \(\mathcal{P}\), its negation is represented by the projector \(I - P\), where \(I\) is the identity operator on the Hilbert space.

Figure 1: Schematic diagram of a two-dimensional Hilbert space using the real plane, with a basis consisting of the orthogonal kets \(|1\rangle\) and \(|2\rangle\), and various rays (one-dimensional subspaces) as discussed in the text.

This leads to a profound difference between classical and quantum properties, which is best illustrated by considering a two-dimensional Hilbert space representing possible properties of a spin-half particle, shown schematically in Fig. 1. (The figure shows a real, rather than a complex Hilbert space, but it is adequate for present purposes.) The line \(\mathcal{P}\) through the origin represents the physical property \(S_z = +1/2\) in units of \(\hbar\). The line \(\mathcal{P}^\perp\) perpendicular to this line is its orthogonal complement and represents the physical property \(S_z = -1/2\). The key point is that there are many other lines through the origin, such as \(Q\) corresponding (schematically) to the property \(S_x = 1/2\), which are neither the same as the property \(S_z = +1/2\) or its negation, \(S_z = -1/2\). This is fundamentally different from classical phase space where for a given property \(P\), any point in the phase space is either inside the set \(\mathcal{P}\), which means the system this property \(P\), or in its complement \(\mathcal{P}^c\), which means the system does not have property \(P\). In quantum mechanics we say
that two properties $P$ and $Q$, such as $S_z = +1/2$ and $S_z = 1/2$, are incompatible when the projectors onto the corresponding subspaces do not commute, $PQ \neq QP$. Whenever noncommuting operators appear in quantum mechanics we have a situation that lacks a precise analog in classical physics.

A consequence of adopting the von Neumann approach to quantum properties, which underlies the presentations in all contemporary textbooks, is that quantum reasoning must follow different rules from classical reasoning. For a classical phase space the conjunction of two properties $P$ and $Q$, $P \land Q$ or “$P$ AND $Q$,” is always defined: it is the property corresponding to the intersection $\mathcal{P} \cap \mathcal{Q}$ of the corresponding sets of points. But in the quantum case let $P$ be the property $S_z = +1/2$ and $Q$ be the incompatible property $S_z = -1/2$. Can we make any sense of the conjunction “$P$ AND $Q$”? Does it correspond to any ray in the Hilbert space? The answer is that it does not, or at least there is no plausible way to make such an association. Every ray in Hilbert space can be interpreted as the property that $S_w = +1/2$ for some direction in space $w$, so there is nothing left over to represent a conjunction of the sort in which we are interested. There is no room for it in Hilbert space. What shall we do?

Von Neumann was not unaware of this problem, and he and Birkhoff made a proposal that if $P$ and $Q$ refer to incompatible properties, “$P$ AND $Q$” should be the property corresponding to the intersection $\mathcal{P} \cap \mathcal{Q}$ of the corresponding subspaces of Hilbert space. As applied to the situation at hand, the intersection of the $S_z = +1/2$ and $S_z = -1/2$ rays is the subspace consisting of nothing but the zero vector, the origin in Fig. 1. The orthogonal complement of the zero vector is the entire Hilbert space, so the zero vector represents a proposition which is always false; it is the counterpart of the empty set for a classical phase space. This is an interesting proposal; let us pursue it for a bit and see where it leads, using a notation in which single letters
\[ z^+ = |z^+\rangle\langle z^+|, \quad x^- = |x^-\rangle\langle x^-|, \]
and so forth correspond to projectors onto the rays $S_z = +1/2$, $S_z = -1/2$, and so forth. Then the usual logical rules, with $\land$ and $\lor$ meaning AND and OR, lead to the expression
\[ (z^+ \land x^-) \lor (z^+ \land x^+) = z^+ \land (x^- \lor x^+) = z^+, \]
where we have used the fact that $x^- \lor x^+$ is always true, because it is the negation of the always false property $x^+ \land x^-$. But there is a serious problem with Eq. (2). The proposition or property on the left side is always false, as it is the disjunction of two always-false properties. So if we believe the equation, the property $z^+$, the final term on the right, is always false. We could just as well have used $z^-$ in place of $z^+$, and drawn the conclusion that $z^-$ is always false. Hence $z^+ \lor z^-$ is always false. But this contradicts the fact—see previous paragraph—that $z^+ \lor z^-$ is always true! We have arrived at a logical contradiction. How could two of the great mathematicians of the 20th century have made such a blunder? The answer is that they did not. They were quite aware that this problem was going to arise, and that the ordinary rules of propositional logic would have to be modified. So they proposed discarding the distributive rule, which justifies the first equality in Eq. (2), to produce what has come to be known as quantum logic.

Does quantum logic provide a consistent underpinning of the calculational techniques found in quantum textbooks? Not likely. Although quantum logic has been extensively studied as a subdiscipline of quantum foundations, so far as I am aware it has made no contribution to resolving the conceptual difficulties besetting quantum theory. It seems not to be used in quantum information theory, which is currently the “cutting edge” of developments in quantum theory, that is, in extending its concepts. This situation may simply reflect the fact that professional physicists are not smart enough to really understand quantum mechanics, and the problems of quantum foundations will have to remain unresolved until robots surpass our intelligence and can learn to think using the new rules. (But if they do come to understand quantum mechanics, will they be smart enough, or even sufficiently motivated, to explain it to us?)

But surely the textbooks must do something with incompatible properties, and if Birkhoff’s and von Neumann’s reasoning has not been adopted, what takes its place? One can discern two strategies. The first is based on measurements. There is no way to simultaneously measure $S_z$ and $S_x$ for a spin-half particle. On this everyone agrees, but we are back to the unsolved measurement problem. The second is to invoke the uncertainty principle. There are elegant mathematical formulations of various uncertainty principles to be found inside and outside textbooks. But what do they mean? What it means in practice for the student learning the subject is that certain things are best not discussed, or if they are, the discussion should be accompanied by an appropriate amount of vagueness and arm waving. If one is discussing $S_z$, then it is probably best to leave $S_x$ out of the discussion, though one is somewhat uncertain as to why certain things are uncertain.
The histories approach uses a precise principle in place of arm waving. Incompatible properties cannot be combined in a meaningful quantum description of the world, even if the individual properties by themselves make perfectly good sense. The statement \( S_z = +1/2 \) AND \( S_x = -1/2 \) makes no sense. It is very important to keep in mind the distinction between “makes no sense” or “meaningless,” and “always false.” A statement that is meaningful and false — see our discussion of the Birkhoff and von Neumann approach — is one whose negation is meaningful and true. By contrast, the negation of a meaningless statement is equally meaningless. In ordinary logic the statement \( P \land \lnot Q \), which is to say “\( P \) AND \( \lnot Q \),” is meaningless, because it has not been formed according to the syntactical rules appropriate to the language. One should think of \( \{ S_z = +1/2 \) AND \( S_x = -1/2 \} \) as similar — it is meaningless in the sense that quantum mechanics cannot assign it a meaning.

Similarly, \( \{ S_z = +1/2 \) OR \( S_x = -1/2 \} \) is not a meaningful statement about quantum properties, so it makes no sense to ask, “Is the spin-half particle in the state \( S_z = 1/2 \) or the state \( S_z = 1/2 \) or possibly both?” Students are taught that there is no way of measuring both \( S_z \) and \( S_x \) simultaneously, and this statement is correct. Unfortunately, they are not taught that the reason such measurements are impossible is that there is nothing there to be measured: there is no physical property in the quantum world that such a measurement might reveal. It is not the limited ability of experimental physicists that is at issue; in fact, one sign of a good experimental physicist is his inability to measure what is not there.

In the histories approach the rule for not combining incompatible properties is part of a more general single framework rule. A framework is a projective decomposition of the identity: a collection of projectors \( \{ P_\mu \} \), where \( \mu \) is a label not an exponent, which sum to the identity operator:

\[
I = \sum_\mu P_\mu, \quad P_\mu = (P_\mu)^\dagger = (P_\mu)^2, \quad P_\mu P_\nu = \delta_{\mu\nu} P_\mu.
\] (3)

Each projector corresponds to a physical property. The significance of \( P_\mu P_\nu = 0 \) when \( \mu \neq \nu \) is that these properties are mutually exclusive: if one is true the other is necessarily false. The collection of such properties form a quantum sample space: not only are they mutually exclusive, but together they exhaust all possibilities — that is the significance of \( \sum_\mu P_\mu = I \). The events, using the terminology of probability theory, associated with this sample space are projectors formed by taking sums of elements in the collection \( \{ P_\mu \} \), including \( I \) and the 0 projector; together they comprise a Boolean algebra under (operator) products and taking the complement \( I - P \) of \( P \).

Two such frameworks are compatible if the projectors in the first commute with all the projectors in the second; otherwise they are incompatible. Two compatible frameworks have a common refinement: a single framework whose Boolean algebra includes all the projectors of both of the original frameworks. Two incompatible frameworks have no common refinement. The single framework rule asserts that all reasoning about a quantum system must be carried out in a single framework in the sense that results from two frameworks cannot be combined. Given two compatible frameworks the single framework rule is easily satisfied: carry out the reasoning using a common refinement. Consequently, what the single framework rule prohibits is combining, or attempting to combine, results from incompatible frameworks.

It is important to understand just what the single framework rule allows and what it prohibits. Physicists are free to employ whatever framework they wish when describing a quantum system: formulas can be written down and probabilities calculated using the \( S_x \) or \( \{ x^+, x^- \} \), or the \( S_z \) framework for a spin-half particle. Call this freedom the principle of Liberty. There is no law of nature, or of quantum theory, that says that one of these is the “right” framework; from the perspective of fundamental quantum theory they are equally correct. Call this the principle of Equality. But Liberty and Equality are accompanied by a third principle: Incompatibility. Results in two incompatible frameworks cannot be combined in a meaningful way. What these principles amount to in practice will become apparent when we consider various examples.

Note that when a physicist chooses to use a particular framework rather than some other to describe a quantum system, this is not at all a matter of somehow “influencing” the system in question. Instead, such choices are somewhat like the choices a photographer makes in photographing Mount Rainier from, say, the north rather than from the south. Such a choice determines what kind of information is present, and consequently the utility or the use which can be made of the resulting photograph. But it has no influence on the mountain itself. Choosing a framework is thus very different from carrying out a measurement on a system, because a measurement can have a rather drastic effect upon a microscopic quantum system.
III Quantum dynamics

Von Neumann$^{19}$ taught us that quantum dynamics involves two distinct processes: a unitary or deterministic time evolution, and then a separate stochastic or probabilistic time evolution associated in some way with measurements. Although this is what students learn in textbooks, both students and teachers (and, one suspects, textbook writers — see Ref. 21) find it unaesthetic. One can sympathize with the efforts of Everett and his successors$^{22,23}$ to reduce all of quantum dynamics to a unitary form (for a closed system), even though the “many worlds” (or “many minds”) approach has its own share of obscurities and difficulties.$^{24–26}$ Blaylock$^1$ is in favor of the many-worlds interpretation, and Maudlin$^2$ is not.

The histories approach used in this paper is the exact opposite of Everett’s in the sense that quantum time evolution is always stochastic, or more precisely, should always be put in a stochastic or probabilistic framework. Classical (Hamiltonian) dynamics is deterministic: there is a unique map carrying each point in the phase space at a give time to another specific point at any particular later (or earlier) time. But there is no reason why the quantum world should necessarily possess the same sort of deterministic time development. And there is good experimental support, as in the random time of decay of unstable particles, for the idea that quantum dynamics has an intrinsically probabilistic character. But what about the wave function that satisfies the Schrödinger’s (time-dependent and deterministic) equation? Solutions to Schrödinger’s equation are to be used for calculating probabilities. This is what Born$^{27}$ taught us and what is done in textbooks. Formulating probabilistic ideas using histories, as in Ref. 13, Chap. 8, results in a consistent approach that removes (or at least “tames”) quantum paradoxes.

A consistent discussion of a stochastic process, whether classical or quantum, begins by introducing a sample space of mutually exclusive possibilities, which we shall refer to as histories. These can be represented, just as quantum properties at a single time, by subspaces on a history Hilbert space consisting of a tensor product of copies of the Hilbert space describing a quantum system at a single time, and with the projectors on the history space playing a role analogous to those corresponding to properties at a single time. (For details, see Ref. 13, Chap. 8.) Once again, there are many possibilities for such sample spaces, and the single framework rule says that the physicist is welcome to use any one he pleases (Liberty and Equality), as long as he does not put together meaningless combinations of incompatible sample spaces (Incompatibility). The simplest sorts of histories consist of just a sequence of quantum properties at a succession of different times, and the set of properties considered at one time may be incompatible with the set considered at a different time. Thus there is nothing inherently wrong with saying that at successive times $t_1 < t_2 < t_3$ a spin-half particle has $S_z = +1/2$ and then $S_z = -1/2$, and then $S_z = -1/2$, or perhaps $S_y = -1/2$ at the third time. These properties need not, and generally do not, map into one another under the unitary time evolution of a closed system generated by Schrödinger’s equation.

For this paper it will suffice to consider histories for a sequence of times $t_0 < t_1 < t_2 < \cdots < t_f$ which can be represented in the form

$$Y^\alpha = \Psi_0 \odot P_{t_1}^{\alpha_1} \odot P_{t_2}^{\alpha_2} \odot \cdots \odot P_{t_f}^{\alpha_f}, \tag{4}$$

where $\Psi_0 = \langle \Psi_0 | \Psi_0 \rangle$ projects onto the initial state $| \Psi_0 \rangle$ at $t_0$, so all histories begin with this initial state, and $\{P_{t_j}^{\alpha_j}\}$ denotes a (projective) decomposition of the identity $I$ at time $t_j$, see Eq. (3). The time is labeled by a subscript, and decompositions at different times need have no relation with one another. The superscript on the history $Y^\alpha$ denotes a string of labels:

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_f). \tag{5}$$

The $\odot$ symbols in Eq. (4) can be interpreted as variants of the usual $\otimes$ symbol denoting a tensor product, but for present purposes we can think of them as simply separating possible events at one time from events at the next time.

Probabilities can be assigned to histories if such a family represents events in a closed quantum system, in the following way. Let $T(t', t)$ be the unitary time development operator; for example, if the system Hamiltonian $H$ is independent of time, then $T(t', t) = \exp[-i(t' - t)H/\hbar]$. For history $\alpha$ define the corresponding chain ket:

$$| \alpha \rangle = P_{t_f}^{\alpha_f} T(t_f, t_{f-1}) \cdots P_{t_1}^{\alpha_1} T(t_1, t_0) | \Psi_0 \rangle. \tag{6}$$

Then provided the consistency condition

$$\langle \alpha | \beta \rangle = 0 \text{ whenever } \alpha \neq \beta \tag{7}$$

is satisfied, the probability of history $\alpha$ is given by

$$\Pr(\alpha) = | \langle \alpha | \alpha \rangle |, \tag{8}$$
assuming that $|\Psi_0\rangle$ in Eq. (6) is a normalized state. Here $\alpha \neq \beta$ means that for at least one $j$ it is the case that $\alpha_j \neq \beta_j$.

For a history family involving a large number of times Eq. (7) is a fairly stringent constraint. However, for families that involve only two times, thus one time $t_1$ following the initial time $t_0$, it is always satisfied because of the assumed orthogonality of the projectors at time $t_1$. In this case

$$\Pr(\alpha_1) = \langle \Psi_0 | T(t_0, t_1) P_1^{\alpha_1} T(t_1, t_0) | \Psi_0 \rangle$$

(9)

is simply the usual Born rule, so Eq. (8) represents a generalization of the Born rule to histories involving three or more times, for which one has to pay attention to Eq. (7). Note that the probability assignment in Eq. (8), of which Eq. (9) is a special case, makes no reference to measurements, and in this sense is distinct from what is found in textbooks. However, by using it as a fundamental axiom one can, as for example in Sec. IV, justify all the usual textbook calculations.

What is usually found in textbooks is not Eq. (9) but the equivalent expression

$$\Pr(\alpha_1) = \langle \Psi(t_1) | P_1^{\alpha_1} | \Psi(t_1) \rangle = |\langle \Phi_1^{\alpha_1} | \Psi(t_1) \rangle|^2,$$

(10)

where the second equality applies when $P_1^{\alpha_1}$ is a rank one projector onto the pure state $|\Phi_1^{\alpha_1}\rangle$. Here

$$|\Psi(t)\rangle := T(t, t_0) |\Psi_0\rangle$$

(11)

is obtained by integrating Schrödinger’s equation starting with $|\Psi_0\rangle$, and is often referred to as the wave function. However, this $|\Psi(t)\rangle$ is best viewed as a calculational tool, rather than, as in Everett’s interpretation, the actual state of the universe, or that part of the universe that constitutes the closed quantum system under discussion. There are two ways to see this. One is that in general the corresponding projector $\Psi(t) = |\Psi(t)\rangle \langle \Psi(t) |$ does not commute with the projector $P_1^{\alpha_1}$, which in the family of histories Eq. (4) represents a particular physical property at time $t_1$. As one cannot in the histories approach ascribe two incompatible properties to a single quantum system at the same instant of time, $|\Psi(t)\rangle$ cannot represent a physical property when we use the family Eq. (4).

The second way to see that $|\Psi(t_1)\rangle$ in Eq. (10) is nothing but a calculational tool is to calculate the same probability by another method which makes no mention of it:

$$\Pr(\alpha_1) = |\langle \Phi_1^{\alpha_1}(t_0) | \Psi_0 \rangle|^2,$$

(12)

where

$$|\Phi_1^{\alpha_1}(t)\rangle = T(t, t_1) |\Phi_1^{\alpha_1}\rangle$$

(13)

is again obtained by integrating Schrödinger’s equation (in the opposite time direction) to obtain the $|\Phi_1^{\alpha_1}(t_0)\rangle$ used in Eq. (12). Because the same probability can be obtained using either $|\Psi(t_1)\rangle$ or $|\Phi_1^{\alpha_1}(t_0)\rangle$, they can be regarded as alternative calculational tools, or pre-probabilities in the notation of Ref. 13, Sec. 9.4. A pre-probability is used to calculate probabilities of physical properties, and is thus if anything less real than the probabilities assigned to real properties. Hence “the” wave function plays a very secondary role in the histories approach, quite different from Everett or many worlds.

To be sure, there are special unitary families of histories in which “the” wave function is closely associated with physical properties; an example is given below. Before discussing it, let us note that just as there are incompatible quantum properties, there are also incompatible histories, and the latter must be treated in much the same way as the former. A collection of histories associated with a suitable sample space, for example, the $\{Y^\alpha\}$ defined in Eq. (4), is referred to as a framework or consistent family of histories provided the consistency conditions, Eq. (7), for the family under consideration, are satisfied. Two such families may be either compatible or incompatible. They are compatible if there is a common refinement, a family of more detailed histories which includes both families we started with, and also satisfies the consistency conditions. Otherwise they are incompatible, and the single framework rule says they are not to be combined, or the combination is meaningless (quantum theory can assign it no meaning). Once again, the pillars of good quantum reasoning are Liberty, Equality, and Incompatibility; see the discussion at the end of Sec. II. In particular, Equality means that from the point of view of fundamental quantum mechanics there is no reason to prefer one consistent family to another, there is no law of nature that singles out “the right family.”

\footnote{The terms “framework” and “consistent family” are used both for the sample space and the corresponding event algebra it generates. If the distinction is important, we can refer to a “consistent sample space of histories.”}
How this works can be illustrated by the case of Schrödinger’s poor cat whose sorry history is no doubt familiar to the reader. Under unitary time evolution the cat ends up in a state which can be described as a superposition of dead and alive, which Schrödinger and his successors found perplexing. So how does the histories approach deal with it? First, there must be a choice of a framework, which is to say to say a series of possibilities at the different times of interest. Let the initial state $|\Psi_0\rangle$ include the cat, the apparatus, and whatever else is needed to treat this as a closed quantum system. One possible family of histories is that in which at each time $t_j$, $P_j^0 = |\Psi(t_j)\rangle\langle\Psi(t_j)|$ is a projector onto the subspace containing “the” wave function as defined by Eq. (11); $P_j^1 = I - P_j^0$ is its negation. This family is unitary in the notation in Ref. 13, Sec. 8.7; it is consistent, and the physicist who wants to use it is at liberty to do so. But the historian will remind him that referring to “Schrödinger cat” states at later times constitutes something of a misnomer because the corresponding projector $P_j^0$ will (at least in general) not commute with projectors for physical properties of anything one would want to call a cat. If the physicist — says the historian — wants to discuss what is happening to something that can properly be called a cat, then it is necessary to adopt a different framework, one containing projectors referring to states in which the cat is dead or alive as mutually exclusive possibilities. Such frameworks exist, are just as fundamental (Equality) as the unitary framework, and of much greater utility to the physicist interested in biology or some other aspect of the ordinary macroscopic world.

### IV Measurements

One of the unfortunate features of contemporary quantum mechanics textbooks is their inability to make much sense of experiments of the sort that are done all the time in the laboratory by competent experimental physicists. Such measurements are typically designed to reveal properties possessed by systems before the measurement takes place. What does this electrical pulse mean? It means that the proton knocked out of the target by the high energy electron has ionized the gas near a pair of wires in the detector, etc. By the time they give talks reporting their results experimental physicists seem to have forgotten (fortunately) what they learned from textbooks about measurements magically producing reality out of nothing. When the quantum analysis is done properly using histories, one finds that the experimentalists are correct. Once one allows them liberty their discussions make perfectly good sense.

A simple schematic example of an idealized measuring process can be used to illustrate the essential points. We assume the Hilbert spaces of the system to be measured and the measuring apparatus are $\mathcal{H}_s$ and $\mathcal{H}_M$, and that at the earliest time, $t_0$, the two are in a product state $|s\rangle \otimes |M_0\rangle$. The unitary time development operator $T(t',t)$ is the identity between $t_0$ and $t_1$, but in the interval from $t_1$ to $t_2$ the system interacts with the apparatus as indicated here, with time steps in the order $t_0 \rightarrow t_1 \rightarrow t_2$:

$$|\Psi\rangle = |s\rangle \otimes |M_0\rangle \rightarrow |s\rangle \otimes |M_0\rangle \rightarrow |s\rangle \otimes |M_0\rangle$$

The $\{|s\rangle\}$ form an orthonormal basis of $\mathcal{H}_s$, assumed to be a Hilbert space of finite dimension, and the $\{|M\rangle\}$ are orthogonal to each other, and the $\{|s\rangle\}$ are normalized states in $\mathcal{H}_s$, but not necessarily orthogonal. We do not have to assume that $|\bar{s}\rangle = |s\rangle$.

Assume an initial state

$$|\Psi_0\rangle = |s\rangle \otimes |M_0\rangle = \left( \sum_j c_j |s_j\rangle \right) \otimes |M_0\rangle$$

at $t_0$, and a family of histories [see Eq. (4)],

$$\Psi_0 \otimes \{s\} \otimes \{M_k\}.$$  

As in Eq. (1) the letters denote projectors: $s_j = |s_j\rangle\langle s_j|$, etc. Each history begins with the initial state $|\Psi_0\rangle$ at $t_0$. At time $t_1$ the particle is in one of the states $|s\rangle$, and at $t_2$ the apparatus pointer is in the position indicated by the projector $M_k$. Notice that these histories contain no reference to the measuring apparatus at time $t_1$: think of $s_j$ as the same as $s_j \otimes I_M$. Nor is there any reference to the state of the particle at time $t_2$. Given the dynamics in Eq. (14) it is straightforward to show that the consistency conditions, Eq. (7), are satisfied and the probabilities are, in an obvious notation where subscripts denote the time,

$$\text{Pr}(s_1,M_k) = |c_j|^2 \delta_{jk}.$$
From these one can conclude by summing over \( j \) that the probability that the pointer is in state \( M^k \) at \( t_2 \) is equal to \( |c_j|^2 \), the same result as obtained by the usual textbook rules. But Eq. (17) also yields a conditional probability:

\[
\Pr(s_j^1 | M^k) = \delta_{jk}.
\]

That is, if the pointer is in position \( M^k \) at \( t_2 \), we can be sure that the system was in the state \( s^k \) at the earlier time \( t_1 \). The apparatus was doing what it was designed to do.

Notice that this analysis makes no assumption about the nature of the \( \{|s_j\}\) states that appear at time \( t_2 \) in the dynamics given in Eq. (14). They are irrelevant if one regards, as is typically the case, the purpose of the measurement to be that of determining some property of the measured system before interaction with the apparatus, which may well have altered the property in question, or even destroyed the system (as, for example, when a photon is absorbed). However, we can also consider the special case of a nondestructive measurement for which \( |s_j\rangle = |s^j\rangle \) for every \( j \). In this case we may replace the projectors at the final time \( t_2 \) in Eq. (16) with \( \{s^j \otimes M^k\} \), and after a short calculation derive the result

\[
\Pr(s_j^1, M^k, s_j^2) = |c_j|^2 \delta_{jk} \delta_{kl}.
\]

Thus given that the pointer is in the state \( M^k \) at \( t_2 \) we can deduce the state of the system \( s \) both before and after the measurement.

It is important to observe that the results given here, which agree with the results of textbook calculations and the belief of experimentalists that they can design good measurements, are not based on a “measurement postulate” of the sort made by von Neumann and later textbook writers. They are a consequence of Eq. (8) in a situation in which the consistency conditions, Eq. (7), apply. These are fundamental postulates that apply to any closed quantum system, which is to say to any situation in which the dynamics is determined by solving Schrödinger’s equation, not just cases in which a measurement is taking place.

But it is also worth noting the sense in which both von Neumann and the textbooks are to some degree correct. From the histories perspective they can be understood as using a family of histories of the form, focusing for simplicity on the nondestructive case,

\[
\Psi_0 \otimes \{\Psi_1, I - \Psi_1\} \otimes \{s^j \otimes M^k\},
\]

where \( \Psi_1 \) is the projector onto the unitarily-evolved state \( |\Psi_1\rangle = T(t_1, t_0) |\Psi_0\rangle \). It is easily checked that this family is consistent, and physicists are at Liberty to employ it. It provides just as good a description of nature as Eq. (16). However, the two families are incompatible if at least two of the \( c_j \) in Eq. (15) are nonzero; in particular \( \Psi_1 \) will (in general) not commute with \( s^j \). This fact means that using the family Eq. (20) precludes saying anything meaningful about the properties of \( s \) before the measurement takes place. So the reticence of textbook authors has some justification: their approach is unable to connect measurement outcomes with earlier properties, because to do so a framework is needed in which the relevant properties make sense. The problem is that the very possibility of alternative descriptions, the principles of Liberty and Equality, are absent from textbook treatments.

To summarize, in order to discuss measurements as measurements in the typical sense in which experimental physicists think of them, it is necessary to introduce frameworks or families in which both the measurement outcome and the relevant properties of the system before the measurement takes place are made part of a consistent family. Textbooks in effect adopt a framework in which outcomes make sense, because otherwise their discussions would never get off the ground: simple use of unitary time development leads to macroscopic quantum superpositions and the measurement problem. But then there is no good reason why, having taken one Liberty, a second, namely frameworks in which properties make sense before the measurement interaction, should be denied. And this second Liberty is used all the time by the experimentalists; it is past time for theoreticians and textbook writers to catch up. There is more to be said about wave function collapse, but we defer this to an appropriate point in the following discussion.

V EPR Correlations

VA Entangled states

Let us now turn to the EPR problem and its connection with locality, focusing on the standard example due to Bohm of two spin-half particles \( a \) and \( b \) in the state described by the singlet wave function

\[
|\psi_0\rangle = \left( |z_a^+ z_b^- \rangle - |z_a^- z_b^+ \rangle \right)/\sqrt{2},
\]

(21)
where \(|z^+_a z^+_b\) = \(|z^+_a\rangle \otimes \langle z^+_b|\) is an eigenstate of both \(S_{az}\) for the \(a\) particle, eigenvalue \(+1/2\), and of \(S_{bz}\) for the \(b\) particle, eigenvalue \(-1/2\); the state \(|z^-_a z^-_b\rangle\) has a similar interpretation with eigenvalues of the opposite sign. By contrast, the superposition represented by \(|\psi_0\rangle\) is not an eigenstate of either \(S_{az}\) or \(S_{bz}\). Indeed, the corresponding projector \(\psi_0 = |\psi_0\rangle \langle \psi_0|\) is incompatible with the property \(S_{aw} = +1/2\) of particle \(a\), where \(w\) is any direction in space, and likewise with \(S_{bw} = +1/2\). Thus it makes no sense to say that a quantum system which possesses the property \(\psi_0\) has any nontrivial property corresponding to particle \(a\) or particle \(b\), that is, to some subspace of the corresponding Hilbert space. (The trivial properties are the identity and zero projectors, which are, respectively, always true and always false.)

Before going further it is worth remarking that there is no notion of nonlocality intrinsic to the singlet state \(|\psi_0\rangle\) as such. After all, it is the spin state of a hydrogen atom or of positronium in its ground state, where one does not usually think of the particles as in distinct locations. Entanglement as such is a quantum concept distinct from any notion of nonlocality. To tie \(|\psi_0\rangle\) to nonlocality, one has to to suppose that the two particles are in different locations. Of course, just this sort of thing is frequently achieved in the laboratory nowadays, and, when it is, such states can with some justification be called nonlocal.

Nonlocality of this sort is, however, not what Bell and his successors have had in mind when speaking of nonlocality in quantum mechanics, for one does not need carefully constructed inequalities to demonstrate that entangled states are present in the quantum Hilbert space. The issue is not one of static nonlocality, but instead whether quantum theory allows or demands certain nonlocal dynamical effects. As Maudlin puts it, p. 123 of Ref. 2, the locality assumption which he thinks quantum mechanics violates is that “the measurement on one particle does not change the state of the other [particle].”

V B Dynamics

We begin our study by considering the dynamics of the spin degrees of freedom of particles \(a\) and \(b\), making the usual assumption that the particles are not interacting with each other and no magnetic fields are present, and hence the time development operator

\[
T(t', t) = T_a(t', t) \otimes T_b(t', t) = I_a \otimes I_b
\]

is trivial. Later we will explore the effects of measurements, but first let us examine what happens in their absence.

Consider the family of histories based on three times \(t_0 < t_1 < t_2\),

\[
\psi_0 \circ \{z^+_a, z^-_a\} \otimes \{z^+_b, z^-_b\} \circ \{z^+_a, z^-_a\} \otimes \{z^+_b, z^-_b\},
\]

(23)

which is to be interpreted as follows. All the histories begin at time \(t_0\) with the initial state \(|\psi_0\rangle\). At time \(t_1\) there are four possible properties or “events”: \(z^+_a z^+_b = z^+_a \otimes z^+_b\), which means \(S_{az} = +1/2\), \(S_{bz} = +1/2\), \(z^+_a z^-_b\), \(z^-_a z^+_b\), and \(z^-_a z^-_b\). The same four possibilities occur at the later time \(t_2\). Consistency can be checked and probabilities assigned to the \(4 \times 4 = 16\) histories as explained in Sec. III. Only two histories have nonzero probabilities:

\[
\Pr(z^+_a, z^-_b; z^+_a, z^-_b) = \Pr(z^-_a, z^+_b; z^-_a, z^+_b) = 1/2.
\]

(24)

Here semicolons rather than commas are used to separate events at different times, indicated by subscripts, to improve legibility. Both semicolons and commas should be read as “AND.”

Summing over the values at \(t_1\) yields the marginal probabilities

\[
\Pr(z^+_a, z^-_b) = \Pr(z^-_a, z^+_b) = 1/2
\]

(25)

at time \(t_2\). These can be obtained from \(|\psi_0\rangle\) by use of the Born rule (see Sec. III). There is no need to refer to measurements. Properly built apparatus, as discussed in Sec. V C below, reveals what is there before the measurement takes place. But Eq. (24) tells us more than Eq. (25). For example, it implies that the values of \(S_{az}\) are identical at times \(t_1\) and \(t_2\), a result that is physically reasonable but not implied by the Born rule.

Liberty allows many other families besides Eq. (23). If \(z\) is replaced everywhere in Eq. (23) by \(w\), where \(w\) could be \(x\) or \(y\) or any other direction in space, it follows from the spherical symmetry of \(|\psi_0\rangle\) that the family

\[
\psi_0 \circ \{w^+_a, w^-_a\} \otimes \{w^+_b, w^-_b\} \circ \{w^+_a, w^-_a\} \otimes \{w^+_b, w^-_b\},
\]

(26)
is consistent, yielding the same probabilities in Eq. (24) and marginals in Eq. (25) if \( w \) is replaced by \( z \) in these expressions. Perhaps of greater interest is the family

\[
\psi_0 \odot \{z_a^+, z_a^-\} \odot \{w_b^+, w_b^-\} \odot \{z_a^+, z_a^-\} \odot \{w_b^+, w_b^-\},
\]

(27)

with \( S_z \) for particle \( a \), but \( S_w \), with \( w \) an arbitrary but specific direction, for particle \( b \). The family is consistent, but now four histories have nonzero probability:

\[
\begin{align*}
\Pr(z_{a1}^+, w_{b1}^+; z_{a2}^-, w_{b2}^-) &= \Pr(z_{a1}^-, w_{b1}^-; z_{a2}^-, w_{b2}^-) \\
&= (1/2) \cos^2(\theta/2), \\
\Pr(z_{a1}^+, w_{b1}^+; z_{a2}^+, w_{b2}^+) &= \Pr(z_{a1}^-, w_{b1}^-; z_{a2}^-, w_{b2}^-) \\
&= (1/2) \sin^2(\theta/2).
\end{align*}
\]

(28)

Note that whatever value \( S_{bw} \) has at time \( t_1 \), it has exactly the same value (with probability 1) at time \( t_2 \), just as one would have expected for a particle in the absence of any interaction with the rest of the world. Again the marginals at time \( t_2 \), \( \Pr(z_{a2}^+, w_{b2}^-) \) and the like, have values given by the Born rule. And note that the three consistent families in Eqs. (23), (26), and (27) are all mutually incompatible, except for the special case in which \( w \) is equal to \( z \) or to \(-z\); there is no meaningful way to combine the corresponding probabilities.

V C Measurements

Consider a measurement of \( S_z \) on particle \( a \). In any fundamental quantum analysis the measuring apparatus must also be included as part of the total system, and described in fully quantum mechanical terms. Thus we use a Hilbert space \( \mathcal{H}_a \otimes \mathcal{H}_M \otimes \mathcal{H}_b \), and assume that for \( t \) and \( t' \) in the range of interest the total unitary time development operator

\[
T(t', t) = T_{aM}(t', t) \otimes T_b(t', t)
\]

(29)

factors into a piece \( T_{aM} \) in which particle \( a \) interacts with the apparatus, and a piece \( T_b(t', t) = I_b \): nothing is happening to particle \( b \). Further assume that the action of \( T_{aM}(t', t) \) is given for the range \( t_0 \to t_1 \to t_2 \to t_3 \) by

\[
\begin{align*}
|z_a^+\rangle \otimes |M_0\rangle &\to |z_a^+\rangle \otimes |M_0\rangle \\
&\to |z_a^+\rangle \otimes |M_0\rangle \to |z_a^+\rangle \otimes |M^+\rangle, \\
|z_a^-\rangle \otimes |M_0\rangle &\to |z_a^-\rangle \otimes |M_0\rangle \\
&\to |z_a^-\rangle \otimes |M_0\rangle \to |z_a^-\rangle \otimes |M^-\rangle,
\end{align*}
\]

(30)

where the arrows indicate how the ket to the left evolves unitarily to the ket on the right during the corresponding time interval. The final states \( |z_a^+\rangle \) and \( |z_a^-\rangle \) at \( t_3 \) are arbitrary and could be omitted from the discussion (see remarks in Sec. III); the reader who wants to remove the bars is welcome to do so.

Consider the family of histories at times \( t_0 < t_1 < t_2 < t_3 \)

\[
\Psi_0 \odot \{z_a^+, z_a^-\} \odot \{z_b^+, z_b^-\} \odot \{z_a^+, z_a^-\} \odot \{z_b^+, z_b^-\} \\
\odot \{M^+, M^-\} \odot \{z_a^+, z_a^-\},
\]

(31)

where \( \Psi_0 \) is the projector on the initial state \( |\Psi_0\rangle \otimes |M_0\rangle \) of the system at \( t_0 \): a singlet spin state together with an apparatus ready to measure \( S_{az} \). As in the family in Eq. (24), there are just two histories with nonzero probabilities:

\[
\begin{align*}
\Pr(z_{a1}^+, z_{a2}^+, z_{a3}^-, z_{a3}^+; M_{a3}^+, M_{a3}^-) &= \\
\Pr(z_{a1}^+, z_{a2}^+, z_{a3}^-, z_{a3}^+; M_{a3}^-, M_{a3}^+) = 1/2.
\end{align*}
\]

(32)

From these we can calculate marginal and conditional probabilities using the usual rules of ordinary probability theory. In particular,

\[
\Pr(z_{a}^+ | M_{a}^+) = \Pr(z_{a}^- | M_{a}^+) = 1,
\]

(33)
with \( j = 1 \) or \( 2 \) and \( k = 1, 2, \) or \( 3 \). That is, from the measurement outcome \( M^+ \) at time \( t_4 \) one can infer that particle \( a \) at earlier times (but later than \( t_0 \)) had \( S_3 = +1/2 \), while particle \( b \) had \( S_3 = -1/2 \) at times \( t_1 \) and \( t_2 \), and continued to possess this value at time \( t_3 \). All these results are reasonable in light of the earlier analysis of Eq. (23) and the probabilities in Eq. (24). Properly designed measurements reveal what is there to be measured; they do not somehow create reality out of a vacuum.

All these results are inaccessible using the calculational rules of standard textbooks, except for the result \( \Pr(z_{3a}^+ \mid M_3^+) = 1 \). One of the standard rules, which students find rather odd and ad hoc, states that if the outcome of a measurement is, say, \( M^+ \) at \( t_3 \), then one should “collapse” (first arrow) and renormalize (second arrow) the singlet state wave function:

\[
|\psi_0\rangle \rightarrow z_a^+|\psi_0\rangle = (1/\sqrt{2})|z_a^+z_b^-\rangle \rightarrow |\psi^+\rangle := |z_a^+z_b^-\rangle,
\]

and then calculate

\[
\Pr(z_{3a}^+ \mid M_3^+) = \langle \psi^+ | z_b^- \rangle |\psi^+\rangle = 1.
\]

This is a perfectly good calculational rule, and it can be justified, at least as used in the present context, on the basis of fundamental quantum principles, though we shall not take the time to do so here. Like most calculational rules it allows students to obtain the right answer without having to think about what they are doing, which is in effect calculating a conditional probability. The disadvantage is that they may mistakenly come to believe that wave function collapse is a physical process rather than a calculational tool. That there is no nonlocal physical effect associated with it can be seen by noting that the same “collapse” approach also yields the right answer, Eq. (33) for \( S_b \) at the earlier times \( t_1 \) and \( t_2 \) before the measurement of \( S_{az} \) occurs. And for those interested in relativistic quantum mechanics we remark that at time \( t_1 \) particle \( b \) can be in the backward light cone of the spacetime region, with \( t \) somewhere between \( t_2 \) and \( t_3 \), in which the measuring device interacts with particle \( a \). Backward as well as superluminal causation readily emerges from quantum theory when calculational methods are confused with physical causes.

Rather than Eq. (31) one can analyze the family

\[
\Psi_0 \odot \{z^+_a, z^-_a\} \odot \{w^+_b, w^-_b\} \odot
\{z^+_a, z^-_a\} \odot \{w^+_b, w^-_b\} \odot \{M^+, M^-\} \odot \{w^+_b, w^-_b\},
\]

where the focus is now on \( S_{bw} \) in place of \( S_{az} \), with \( w \) an arbitrary direction in space making an angle \( \theta \) with the positive \( z \) axis, as in Eq. (27). The resulting conditional probabilities include

\[
\Pr(z_{aj}^+ \mid M_3^+) = 1, \quad \Pr(w_{bk}^+ \mid M_3^+) = \sin^2(\theta/2),
\]

\[
\Pr(w_{bk}^- \mid w_{bk}^-) = 1,
\]

for \( j = 1, 2 \) and \( k = 1, 2, 3 \). These make perfectly good physical sense. In particular Eq. (38) tells us that nothing at all is happening to \( S_{bw} \) at any time during the interval between \( t_2 \) and \( t_3 \) when particle \( a \) is interacting with the measuring apparatus. Local measurements properly analyzed have no nonlocal effects.

One can also introduce a second measuring apparatus that measures \( S_{bw} \) for particle \( b \), with \( T_{bk}(t', t) \) for each time interval in \( t_0 \rightarrow t_1 \rightarrow t_2 \rightarrow t_3 \) equal to the identity, except \( t_2 \rightarrow t_3 \) given by [compare Eq. (30)]

\[
|w^+_b\rangle \otimes |N_0\rangle \rightarrow |\bar{w}^+_b\rangle \otimes |N^+\rangle, \quad |w^-_b\rangle \otimes |N_0\rangle \rightarrow |\bar{w}^-_b\rangle \otimes |N^-\rangle.
\]

Readers should have no difficulty checking the consistency and working out the probabilities associated with the family

\[
\Psi_0 \odot \{z^+_a, z^-_a\} \odot \{w^+_b, w^-_b\} \odot
\{z^+_a, z^-_a\} \odot \{w^+_b, w^-_b\} \odot \{M^+, M^-\} \odot \{N^+, N^-\},
\]

where \( \Psi_0 \) is now the projector on \( |\Psi_0\rangle = |\psi_0\rangle \otimes |M_0\rangle \otimes |N_0\rangle \), and deriving conditional probabilities such as

\[
\Pr(z_{aj}^+ \mid M_3^+) = \Pr(z_{aj}^- \mid M_3^-) = 1,
\]

\[
\Pr(w_{bk}^+ \mid N_3^+) = \Pr(w_{bk}^+ \mid N_3^-) = 1,
\]

\[
\Pr(N^+_3 \mid M_3^+) = \Pr(N^-_3 \mid M_3^-) = \sin^2(\theta/2),
\]

with \( j = 1, 2 \). Textbook rules yield Eq. (43), minus any insights obtained by relating the measurement outcomes to the particle properties they were designed to measure.

---

\(^3\)Strictly speaking, Eq. (34) should be applied only in cases in which the bars are absent in the final \( a \) states in the dynamics in Eq. (30). However, if we are only using the collapsed wave function to calculate properties of particle \( b \) this makes no difference.
VI Discussion

VI A Locality

What has been shown is that “The Predictions,” as Maudlin calls them, of quantum mechanics, in particular Eq. (43), are produced by a theory in which local measurements have no nonlocal effects: the measurement of $S_{az}$ that occurs between $t_2$ and $t_3$ has no effect on $S_{bw}$, see Eq. (38). This demonstrates that there is something amiss with the version of Bell’s argument found in Ref. 2, and because it is compact, we can hope to locate where the reasoning departs from the consistent principles of quantum mechanics discussed in Secs. II–IV.

My summary of Maudlin’s argument in Sec. II C of Ref. 2, with the notation changed to make it consistent with that in Sec. V, is as follows:

M1 Measurements of a given spin component on particles $a$ and $b$ will always yield opposite outcomes: $+1/2$ for one particle is perfectly correlated with $-1/2$ for the other.

M2 Assume locality: A measurement carried out on particle $a$ cannot affect the physical state of particle $b$.

M3 This means that particle $b$ must already have been disposed to yield the opposite result even before particle $a$ was measured.

M4 Particle $b$ must have been so disposed even when the total quantum mechanical state of the system was the singlet state $|\psi_0\rangle$, and at all times since its creation.

M5 Therefore the complete physical description of particle $b$ must determine how it is disposed to yield a particular outcome for each possible spin measurement, because M1 holds for any spin component.

M6 Hence a local theory must not only be deterministic but also a hidden variable theory (in the sense of M5).

M7 Any local theory that predicts the EPR correlations must also respect certain constraints on the correlations it predicts (Bell’s inequality).

M8 Quantum mechanics violates these constraints and thus the locality assumption in M2.

Let us compare these assertions with the analysis in Sec. V. Evidently M1 corresponds to the anticorrelation expressed in Eq. (25) in terms of the particles themselves, or in Eq. (43) for measurement outcomes, assuming $\theta = 0$, that is, measurements of the same spin component. Note that although these formulas were obtained for $S_{az}$ and $S_{bz}$, the results are equally valid if $z$ is replaced, for both particles, with an arbitrary direction $w$. As for M2, dynamical locality is ensured by the fact that the time development operator is a tensor product of the form $T_a \otimes T_b$ for the particles alone, or $T_{aM} \otimes T_{bN}$ when measuring apparatuses are included. Furthermore, direct calculation, see Eq. (38) and the comments following it, shows that the spin of particle $b$ is unaffected by making a measurement on particle $a$. And M3, the disposition of the $b$ particle to yield the opposite result even before the $a$ particle was measured, is evident from the fact that Eqs. (33) or (37) hold for all applicable values of $t_j$ and $t_k$: $t_1$ and $t_2$ before the measurement of particle $a$ and $t_3$ after the measurement. This disposition resides in the simple fact that the particle actually had at the earlier time the property which a measurement would later reveal. Thus far everything is fine.

But with M4 we run into difficulties. If by “state of the system” we are to understand $|\psi_0\rangle$ as the physical property possessed by the system, then, as noted in Sec. II, there is no meaningful way to ascribe a value to any component of the spin angular momentum of particle $b$ at this time, because the two projectors do not commute. Ignoring this is to ignore a crucial difference, one might say the crucial difference, between the quantum and the classical world: quantum incompatibility. One way out of the difficulty is to consider $|\psi_0\rangle$ a pre-probability rather than a property; see the discussion following Eq. (13), and in Ref. 13, Sec. 9.4. But it is not clear that a reference to $|\psi_0\rangle$ is really essential for the later stages of Maudlin’s argument, so perhaps we can replace M4 by a modified form consistent with the framework Eq. (31):

M4’ Particle $b$ was disposed to yield the opposite result of the measurement of particle $a$ at all times beginning shortly after the interaction of the two particles produced the singlet state.

---

Naturally, one has to use a different piece of apparatus to measure different components of the spin angular momentum.
VI B Counterfactuals

Blaylock\textsuperscript{1} claims that a key component in Bell’s inequality, or at least an argument leading to it, is counterfactual definiteness: the assumption that a measurement that was not performed had a single definite result. He concludes that therefore the violation of Bell’s inequality by quantum mechanics does not by itself imply nonlocality. Does our analysis throw some light on this matter?

It is helpful to explore the issue of counterfactuals in quantum mechanics starting with a simpler example than the one considered in Sec. V, namely a measurement of a component of spin angular momentum in a situation in which the apparatus can be adjusted to measure either $S_z$ or $S_y$. For example, for a particle traveling along the $y$ axis one could imagine rotating the Stern-Gerlach magnet so that the field gradient is parallel to $z$ or to $x$. Naturally, only one component can be measured in a given run; suppose that in a particular case it was $S_z$. Then the following counterfactual question seems sensible: What would have been the result if the apparatus had been set up to measure $S_x$ instead? And we can ask: is the answer to this question different for the (real) quantum world than it would be for a (hypothetical) classical world?

It is possible to set up a model in which the measurement axis is determined by the outcome of “flipping a quantum coin,” and address the situation inside the total closed system (coin plus apparatus plus particle being measured) using the appropriate quantum analysis to calculate the probabilities for a single consistent family; see Ref. 13, Chap. 19. There is no problem finding a decomposition of the identity which includes the apparatus pointer positions for both outcomes of the quantum coin flip. But to describe this as an authentic measurement in which the pointer position is correlated with a state before the measurement we need to include the corresponding particle property at a time before the measurement took place. In the case of the $S_z$ measurement the corresponding particle projectors will be $z^+$ and $z^-$, whereas for the counterfactual $S_x$ measurements we would like to use $x^+$ and $x^-$. However, the $z$ projectors do not commute with the $x$ projectors, so they cannot both be placed in the same consistent family. Note that the issue raised here has no direct connection with locality; we are considering only a single measurement on a single particle at a single location.

Valid counterfactual reasoning in the quantum domain ought to follow the same standards as ordinary reasoning, which is to say it must be restricted to a single consistent family. Thus the incompatibility at the microscopic level discussed in the previous paragraph can render a quantum counterfactual argument invalid even if the corresponding classical argument is correct, or at least plausible. There is never a problem if we limit the discussion to measurement outcomes and exclude all talk, even implicit, of particle properties preceding the measurement. However, Bell inequality derivations always make some assumption about what
goes on in the world prior to the measurements themselves, and Maudlin’s argument for nonlocality is filled with references to particle properties, so in either case counterfactual parts of the argument, if present, could well be in conflict with quantum theory. Even in the classical domain, counterfactual arguments can make an implicit reference to a preceding state of affairs, which may make it difficult to analyze their structure; see Ref. 13, Sec. 19.3. So is is plausible that something like that will also be present when these arguments are applied to quantum situations.

In fact it is possible to arrive at Maudlin’s M5, which as we pointed is in conflict with quantum theory as consistently interpreted using histories, by employing the following counterfactual argument:

We know that if $S_z$ is measured for particle $a$ and the outcome is $S_{az} = +1/2$, then $S_{bw}$ was $-1/2$ even before a measurement which confirmed that it had this value. If on the other hand $S_{aw}$, with $w$ some other direction in space, had been measured with, say, the outcome $+1/2$, then by the same line of reasoning $S_{bw}$ would have had the value $-1/2$, which would have been confirmed by a later measurement had it measured $S_{bw}$. So the $b$ particle must have had a definite value of $S_{bw}$ for every possible $w$ before any measurement took place, namely, the opposite of the outcome of the $S_{aw}$ measurement which did not but could have taken place.

This argument is of the counterfactual type because it starts with the assumption that $S_{az}$ was measured in the actual world and $S_{aw}$ in the imaginary or counterfactual world. In it one sees a connection with what Blaylock considers suspicious about derivations of Bell’s inequality: counterfactual definiteness, the assumption that unperformed measurements have definite outcomes. The historian will point to a failure to follow the single framework rule as the most likely error on the route from seemingly reasonable assumptions to a conclusion that is obviously wrong, at least if Hilbert space quantum mechanics is accepted as the physicist’s fundamental description of the world; see the discussion of M5 in Sec. VI A above. One cannot be sure that Maudlin was using this kind of argument to arrive at what I have called M5, although the following quotation from Ref. 2, p. 123, the second sentence of which is to a degree counterfactual (and contains a whiff of free choice), suggests it may not have been all that far from his thinking:

“The only way to give a local physical account of the EPR correlations is for each of the particles to be initially disposed to yield a particular outcome for each possible spin measurement. For if either particle is not so disposed and if we happen to measure the spin in the relevant direction (as we might), then there could be no guarantee that the outcomes of the two measurements will be anticorrelated.”

In summary it seems plausible that flawed (from the quantum perspective) counterfactual reasoning provides at least one possible route for deriving Bell’s inequality, with its conclusions inapplicable to the real (quantum) world, and in this respect the analysis given in this paper supports Blaylock’s conclusions. For more on the subject of the correct use of counterfactuals in the quantum domain, see Ref. 13, Chap. 19.

VII Conclusion

What we have shown by means of a counterexample is that quantum violations of Bell’s inequality are perfectly consistent with quantum mechanics being a local theory, in the sense that measurements near one point in space do not immediately affect what goes on elsewhere. So the claim by Maudlin that no local theory can reproduce the predictions of quantum mechanics is incorrect, at least for the situation under discussion: two spin-half particles initially in a singlet state. Much more can be said about quantum locality, and Ref. 5 gives a more extensive treatment of the topic using the histories formulation in a consistent manner to sort out the issues. There the suggestion is made that Bell’s inequality is best thought of as being appropriate to the domain of classical physics but not quantum physics: it has a perfectly good derivation, at least for all practical purposes, in the case of golf balls, and one can see explicitly how this derivation breaks down as the total angular momentum quantum number decreases from around 10 to 1/2. Thus there is nothing wrong with Maudlin’s reasoning, except that it is does not apply to the quantum world. Here new rules of reasoning are needed, as Birkhoff and von Neumann realized, even though their particular proposal has not turned out to be a fruitful approach for understanding quantum mechanics.

The extent to which counterfactual reasoning of a sort inconsistent with quantum principles is a necessary part of derivations of Bell’s inequality is less clear, though this is certainly one route for getting there, as Blaylock pointed out. In particular, it is plausible that Maudlin’s version of the Bell argument makes use of some form of counterfactual reasoning, and in this sense the present article supports the conclusion of
Blaylock that counterfactual reasoning, rather than an assumption of locality, is why derivations of Bell’s inequality lead to conclusions inconsistent with quantum theory and experiment.

An important lesson to be drawn from all of this is the need for a clear presentation of consistent principles of quantum reasoning in textbooks and courses. When teaching courses on quantum information I always stress the fact that there are no nonlocal influences in quantum theory, and point out that this principle is useful to keep in mind when analyzing quantum circuits. Unfortunately, physics students trained in traditional quantum courses have difficulty replacing, or at least augmenting, the calculational rules they learned by rote with a consistent probabilistic analysis of what is going on. They may already have learned that the superluminal influences reflected in violations of Bell’s inequality cannot be used to transmit information. But they also need to hear a simple explanation for why this is so: such influences do not exist. They are nothing but fudge factors needed to correct a mistaken use of classical reasoning in the quantum domain.

Acknowledgments

I have benefitted from correspondence with G. Blaylock and T. Maudlin, as well as comments by anonymous referees. Support for this research has come from the National Science Foundation through Grant 0757251.

References

[1] Guy Blaylock, “The EPR paradox, Bell’s inequality, and the question of locality,” Am. J. Phys. 78, 111–120 (2010).
[2] Tim Maudlin, “What Bell proved: A reply to Blaylock,” Am. J. Phys. 78, 121–125 (2010).
[3] Henry P. Stapp, “Nonlocal character of quantum theory,” Am. J. Phys. 65, 300–304 (1997).
[4] Bernard d’Espagnat, *On Physics and Philosophy* (Princeton University Press, Princeton, NJ, 2006).
[5] Robert B. Griffiths, “Quantum locality,” Found. Phys. 41, 705–733 (2011).
[6] N. David Mermin, “What’s wrong with this pillow?,” Phys. Today 42 (4), 9 (1989).
[7] Eugene P. Wigner, “The problem of measurement,” Am. J. Phys. 31, 6–15 (1963).
[8] Peter Mittelstaedt, *The Interpretation of Quantum Mechanics and the Measurement Process* (Cambridge University Press, Cambridge, U.K., 1998).
[9] Paul Busch and Abner Shimony, “Insolubility of the quantum measurement problem for unsharp observables,” Stud. Hist. Phil. Mod. Phys. 27, 397–404 (1996).
[10] J. S. Bell, “Against measurement,” in *Sixty-Two Years of Uncertainty*, edited by Arthur I. Miller (Plenum Press, New York, 1990).
[11] Robert B. Griffiths, “Consistent histories and the interpretation of quantum mechanics,” J. Stat. Phys. 36, 219–272 (1984).
[12] Murray Gell-Mann and James B. Hartle, “Quantum mechanics in the light of quantum cosmology,” in *Complexity, Entropy and the Physics of Information*, edited by W. H. Zurek (Addison-Wesley, Redwood City, CA, 1990).
[13] Robert B. Griffiths, *Consistent Quantum Theory* (Cambridge University Press, Cambridge, U.K., 2002).
[14] Robert B. Griffiths, “Consistent resolution of some relativistic quantum paradoxes,” Phys. Rev. A 66, 062101-1–21 (2002).
[15] Pierre C. Hohenberg, “An introduction to consistent quantum theory,” Rev. Mod. Phys. 82, 2835–2844 (2010).
[16] David Bohm, *Quantum Theory* (Prentice Hall, Englewood Cliffs, NJ, 1951), Chap. 22.
[17] A. Einstein, B. Podolsky, and N. Rosen, “Can quantum-mechanical description of physical reality be considered complete?,” Phys. Rev. **47**, 777–780 (1935).

[18] Robert B. Griffiths, “Correlations in separated quantum systems: A consistent history analysis of the EPR problem,” Am. J. Phys. **55**, 11–17 (1987).

[19] John von Neumann, *Mathematical Foundations of Quantum Mechanics* (Princeton University Press, Princeton, 1955). Translation of *Mathematische Grundlagen der Quantenmechanik* (Springer-Verlag, Berlin, 1932).

[20] G. Birkhoff and J. von Neumann, “The logic of quantum mechanics,” Ann. Math. **37**, 823–843 (1936).

[21] F. Laloë, “Do we really understand quantum mechanics? Strange correlations, paradoxes, and theorems,” Am. J. Phys. **69**, 655–701 (2001).

[22] Hugh Everett III, “Relative state' formulation of quantum mechanics,” Rev. Mod. Phys. **29**, 454–462 (1957).

[23] *The Many-Worlds Interpretation of Quantum Mechanics*, edited by Bryce S. DeWitt and Neill Graham (Princeton University Press, Princeton, NJ, 1973).

[24] Jeffrey A. Barrett, *The Quantum Mechanics of Minds and Worlds* (Oxford University Press, Oxford, 1999).

[25] Lev Vaidman, “The many-worlds interpretation of quantum mechanics,” in *Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, ¡plato.stanford.edu/entries/qm-manyworlds¿.

[26] Jeffrey A. Barrett, “Everett’s relative-state formulation of quantum mechanics,” in *Stanford Encyclopedia of Philosophy*, edited by Edward N. Zalta, ¡plato.stanford.edu/entries/qm-everett¿.

[27] Max Born, “Zur Quantenmechanik der Stoßvorgänge,” (“The quantum mechanics of scattering”) Z. Phys. **37**, 863–867 (1926).