The Symmetry energy of nuclear matter under a strong magnetic field

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Abstract. In this work we study the effects caused by strong magnetic fields on the symmetry energy and its slope. We compare two models under three intensities of magnetic field, we investigate the behaviour for the conditions with and without anomalous magnetic moment. It is discussed the possible effects on the structure of the pasta phase.

1. Introduction

Asymmetric nuclear matter \cite{1} is an important issue for both theoretical and experimental studies, specially for those interested in the description of stellar matter of compact stars. With the advent of new radioactive beams which will be operating in the near future, this area appears promising.

The symmetry energy is a quantity related to the neutron-proton asymmetry in the equation of state of nuclear matter, it is an important subject both for nuclear physics and astrophysics and is extremely important to the understanding of neutron star structure and composition \cite{2}. In particular, the symmetry energy slope defines nuclear matter properties, such as, neutron skin, neutron dripline, binding energy and core-crust transition density \cite{3}. We want to study the effect of the magnetic field on the symmetry energy.

We consider a system formed by protons and neutrons, interacting via the exchange of $\sigma$-$\omega$-$\rho$ mesons, in the presence of an uniform magnetic field $B$ along the z-axis. Three possible values of magnetic fields, $B = 0, 10^{17}$ and $10^{18}$G are taken. We will also study the effect of the anomalous magnetic moment for protons and neutrons. The NL3 and FSU parameterizations are used to describe the equations of state in the relativistic nuclear mean field formalism. These models present very different density dependence of the symmetry energy. At saturation, the slope $L$ is, respectively, 118 and 60 $MeV$.

2. Formalism

The Lagrangian used is given by:

$$\mathcal{L} = \bar{\psi}_b[i\gamma_\mu \partial^\mu + \eta_b \gamma_\mu A^\mu - g_\rho \gamma_b \gamma_\mu \partial^\mu - m_\sigma^b - g_\omega \gamma_\mu \omega^\mu - k_b \sigma_\mu F^{\mu\nu}] \psi_b$$
$$+ \frac{1}{2} (\partial_\mu \sigma \partial^{\mu} \sigma - m_\sigma^2 \sigma^2) + \frac{1}{3} b m_n (g_\sigma \sigma)^3 - \frac{1}{4} c (g_\sigma \sigma)^4 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu}$$

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The Lagrangian presents an isodoublet nucleon field ($\psi$) interacting via the exchange of the scalar meson ($\sigma$), the vector meson ($\omega^\mu$), the isovector meson ($\rho^\mu$) and the photon ($A^\mu$), where

\[ m_b^* = (m_b - g_b \sigma), \quad \Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu. \]

The nucleon anomalous magnetic moments are introduced via the coupling of the baryons to the $\sigma$ scalar meson ($\sigma$) interacting via the exchange of the $m$ meson with $\rho$ and $\pi$ mesons. The nucleon anomalous magnetic moments are included so we can compare the two models employed [6, 7].

Using the Lagrangian (1) and the mean-field approximation, the meson field expressions follow from the Euler-Lagrange equations:

\[ m_n^2 \sigma_0 = -cg_4 \sigma_0^3 - bm_n g_0^2 \sigma_0^2 + g_n n^s \]
\[ m_n^* \omega_0 = -\frac{\xi}{6} g_0^2 \omega_0^3 + g_n n_b \]
\[ m_n^* \rho_{03} = \tau_{03} n_3, \]

with $m_n^* = m_n^2 + 2\Lambda \omega g_0^2 \omega_0^2$, $m_n^s = m_n^2 + 2\Lambda \omega g_0^2 \omega_0^2$, $n^s = n_p^s + n_b^s$, $n_b = n_p + n_n$ and $n_3 = n_p - n_n$. Where:

\[ n_p = \frac{g_p B m_p^*}{2\pi^2} \sum_{\nu} \sum_s \left[ E^p_{F,S} \right] \ln \left( \frac{p_{F,P,S}^p + E_F^p}{m_n^2} \right) \]
\[ n_n = \frac{m_n^s}{4\pi^2} \sum_s \left[ (p_{F,S}^n)^3 \right] \left( 1 - \frac{1}{2} s \mu_N \kappa B \left( m_n^2 + E_F^p \left( \arcsin \left( \frac{m_n}{E_F^p} \right) - \frac{\pi}{2} \right) \right) \right). \]

**Table 1.** Parameter sets for the two models discussed in the text. The meson masses $m_{\sigma}$, $m_\omega$ and $m_\rho$ are given in MeV. The nucleon mass is 939 MeV in both models.

| Model | $m_\sigma$ | $m_\omega$ | $m_\rho$ | $g_\sigma$ | $g_\omega$ | $g_\rho$ | $c$ | $b$ | $\xi$ | $\Lambda_\omega$ |
|-------|------------|------------|----------|------------|------------|---------|----|----|-----|-------------|
| NL3   | 508.194    | 782.501    | 763.000  | 10.217     | 12.868     | 8.948   | -0.002651 | 0.002055 | 0.00 | 0.00         |
| FSU   | 491.5      | 782.5      | 763.0    | 10.592     | 14.302     | 11.767  | 0.0396    | 0.000756 | 0.06 | 0.03         |

The energy spectra for protons and neutrons are given by [8]:

\[ E_{F,S}^p = \sqrt{p_z^2 + \left( \frac{m_p^2 + 2\nu q_p B - \mu_N \kappa B}{2} \right)^2 + \frac{1}{2} g_{p^0}}, \] \[ E_{F,S}^n = \sqrt{p_z^2 + \left( \frac{m_n^2 + p_{\perp}^2 - \mu_N \kappa B}{2} \right)^2 + \frac{1}{2} g_{n^0}}, \]
where $\nu = n + \frac{1}{2} - \text{sgn}(q_\nu)\frac{2}{3} = 0, 1, 2, \ldots$ are the Landau levels for the fermions with electric charge $q$, $s$ is the quantum number of spin $s$ assume values $+1$ for spin up and $-1$ for spin down cases.

Defining $m_n = m^* - s\mu_N k_n B$, the Fermi momenta $p_{F,n,s}^n$ of the neutrons and $p_{F,p,s}^p$ of the protons and their relationship with the Fermi energies of protons $E_F^n$ and neutrons $E_F^p$ can be written as:

$$p_{F,n,s}^n = E_F^n - \sqrt{m_n^* - 2\nu q_n B - s\mu_N k_p B}^2, \quad p_{F,p,s}^p = E_F^p - m_n^*.$$  \hspace{1cm} (5)

The summation over the Landau level $\nu$ is made until $\nu_{\text{max}}$, which is the largest value of $\nu$ for which the square of Fermi momenta of the particle is still positive and corresponds to the closest integer, from below to:

$$\nu_{\text{max}} = \left[ \frac{(E_F^n + s\mu_N k_p B)^2 - m_n^*}{2|q_n B} \right].$$  \hspace{1cm} (6)

The chemical potentials of protons and neutrons are:

$$\mu_p = E_F^p + \frac{1}{2}g_\rho \rho^0, \quad \mu_n = E_F^n + \frac{1}{2}g_\rho \rho^0.$$  \hspace{1cm} (7)

The total energy density of the neutron star matter is given by:

$$\varepsilon_t = \sum_{b=p,n} \varepsilon_b + \frac{1}{2}m_\sigma \sigma_0^2 + \frac{1}{2}m_\omega \omega_0^2 + \frac{1}{2}m_\rho \rho_0^2$$

$$+ \frac{1}{8} \xi (g_\omega \omega_0)^4 + \frac{1}{3} b m_n (g_\sigma \sigma_0)^3 + \frac{1}{4!} c (g_\sigma \sigma_0)^4 + 3 \Lambda \omega (g_\omega \omega_0)^2 (g_\rho \rho_0)^2,$$  \hspace{1cm} (8)

where the expressions for the proton and neutron energy density are, respectively,

$$\varepsilon_p = \sum_{n=0}^{n_{\text{max}}} \sum_s \frac{|Q_n| B}{4\pi^2} \left[ p_{F,n,s}^p E_F + (\sqrt{m_p^2 + 2Q_p B n - s\mu_N k_p B})^2 \right]$$

$$\times \ln \left[ \frac{p_{F,n,s}^p + E_F}{(\sqrt{m_p^2 + 2Q_p B n - s\mu_N k_p B})} \right],$$

$$\varepsilon_n = \frac{1}{4\pi^2} \sum_s \frac{1}{2} E_F^n p_{F,n,s}^n - \frac{2}{3} s \mu_N k_n BE_F^n (\text{arc} \sin \frac{m}{E_F^n} - \frac{\pi}{2})$$

$$- \left( \frac{1}{3} s \mu_N k_n B + \frac{1}{4} m \right) [m p_{F,n,s}^n E_F^n + m^3 \ln \left( \frac{E_F^n + p_{F,n,s}^n}{m} \right)],$$

and the pressure for the baryons is $P_b = n_b \mu_b - \varepsilon_b$ (b=p,n).

The symmetry energy and its density slope are, respectively, defined as:

$$a_{\text{sym}} = \frac{1}{2} \frac{\partial^2 E/A}{\partial t^2}, \quad L = 3n_0 \frac{\partial a_{\text{sym}}}{\partial n},$$  \hspace{1cm} (10)

where $n_0 = 0.153 \text{ fm}^{-3}$ is the saturation density of nuclear matter and $t = \frac{n_n - n_p}{n_p + n_n}$ is the asymmetry parameter.
3. Results and discussions

In figure (1) we show how the behaviour of the symmetry energy, given in (10), varies with the baryonic density when different intensities of magnetic fields are applied. \( B = 0 \) \( G \) is used as a field of reference.

On the left side, the upper and lower panels represent the NL3 and FSU models for \( B = 10^{17} \) \( G \). The effect of Landau levels is seen, but due to the relatively low magnetic field, the curve of the symmetry energy is very similar to that of the \( B = 0 \) \( G \) field. The irregularities are due to the filling of Landau levels, which have small energy differences between them for fields of this intensities.

![Graphs showing symmetry energy vs baryonic density for NL3 and FSU models](image)

Figure 1. Symmetry energy versus baryonic density for NL3 and FSU models, with and without anomalous magnetic moment. The left panels presents \( B = 0 \) and \( 10^{17} \) \( G \) and the right panels presents \( B = 0 \) and \( 10^{18} \) \( G \) values for the magnetic field.

On the right side the curves for \( B = 10^{18} \) \( G \) field are shown. As seen from equation (6) a smaller number of Landau levels is filled, due to the higher value of magnetic field. The jumps obtained are larger than those arising with \( B = 10^{17} \) \( G \). This behaviour will affect the chemical composition of stellar matter. In particular, we get regions where the symmetry energy becomes larger implying larger proton fractions in \( \beta \)-equilibrium matter. The opposite occurs in neighbouring regions. The magnitude \( B = 10^{18} \) \( G \) may be larger than expected at the inner crust of a compact star, but it would be interesting to investigate the effect of \( B \) on the pasta phase. Since the effects depend on the filling of the Landau levels and, therefore, on the density, the different pasta phase geometries may be affected in different ways.
Comparing results for FSU and NL3 there are different trends due to different behavior of the symmetry energy with density in both models. For NL3, the symmetry energy for $B = 10^{17} G$ has a more linear dependence on the density which follows the $B = 0 G$ line tendency. This also happens for the $B = 10^{18} G$ scenario, but because of the smaller number of Landau levels the symmetry energy curve presents larger oscillations around the $B = 0 G$ line. The FSU model shows a similar behavior, both for the $B = 10^{17} G$ and $B = 10^{18} G$ curves, but the first one demonstrate a small discrepancy around the saturation density.

The inclusion of the anomalous magnetic moment has almost no effect for $B = 10^{17} G$. However, for a larger field such as $B = 10^{18} G$ a clear effect is observed due to the spin polarization of neutrons and protons, on top of the Landau level quantization effect on protons. The main result is an increase of the number and a decrease of the magnitude of the discontinuities around the average $B = 0$ result.

The slope of the symmetry energy is defining how strong is the distillation effect and the magnitude of the neutron drip in the inner crust pasta phase. Figure (1) has shown that the slope of the symmetry energy is also affected by the magnetic field and, therefore, we will next calculate and discuss this quantity.

In figure (2) the slope of the symmetry energy, eq. (10), is presented for both models, without (left panel) and with (right panel) the inclusion of the anomalous magnetic moment. We again
use the $B = 0 \, G$ as reference. As expected, the Landau levels generated by the magnetic fields have a strong effect on the slope, with regions where it is clearly larger, followed by regions where it is smaller, becoming even negative. In the regions where the slope of the symmetry energy becomes larger the isospin distillation effect will be stronger, and as a result, more isospin symmetric paste clusters are obtained. A second effect will be a denser neutron background gas. The opposite will occur in the neighbouring regions with a smaller slope $L$. These results show that the properties of the clusters will be affected by the magnetic field, and, as a result all transport properties that depend on the structure of this phase.

The inclusion of the anomalous magnetic moment will bring extra complexity to the problem with a less systematic behaviour due to the spin polarization effects.

4. Summary
In the present work, we have studied the behavior of the symmetry energy and its slope. In particular, we have compared these quantities obtained for two different models, NL3 and FSU, for three values of magnetic field, $B = 0$, $10^{17}$, $10^{18} \, G$ and under two different conditions, with and without the inclusion of the anomalous magnetic moment.

It was shown that the appearance of Landau levels due to the magnetic field will affect the symmetry energy, giving rise to discontinuities: the symmetry energy may become larger or smaller than the $B = 0$ reference for given densities which are determined with the filling of Landau levels or spin levels. It is expected that the observed behaviour will affect the structure of the inner crust of a neutron star, namely the pasta phase. Not only the fraction of protons in $\beta$-equilibrium matter will change but also the properties of the clusters and the background neutron gas. The effects are larger the more intense the magnetic field. The inclusion of the anomalous magnetic moment will only be noticed for quite high fields and will make the behaviour even more complex.

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References
[1] Ban S., Meng J., Satula W., Wyss R., Phys. Lett. B 633, 231-236 (2006).
[2] Horowitz C.J., Piekarewicz J., Phys. Rev. C 64, 062802 (2001)
[3] Chang Xu, Bao-An Li, Lie-Wen Chen, Phys. Rev. C 82, 054607 (2010)
[4] Rabhi A., Providência C., and Da Providência J., J. Phys. G: Nucl. Part. Phys. 37 075102 (2010).
[5] Broderick A., Prakash M., Lattimer M., ApJ 537, 351-367 (2000).
[6] Fattoyev F. J., Horowitz C. J., Piekarewicz J., and Shen G., Phys. Rev. C 82, 055803 (2010).
[7] Horowitz C.J., Piekarewicz J., Phys. Rev. Lett. 86, 25 (2001).
[8] Rabhi A., Providencia C., and Da Providência J., J. Phys. G: Nucl. Part. Phys. 35 125201 (2008).