Double Higgs production at the 14 TeV LHC and the 100 TeV pp-collider

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We consider effective Higgs boson couplings, including both the CP-even and CP-odd couplings, that affect Higgs boson pair production in this study. Through the partial wave analysis, we find that the process $gg \to hh$ is dominated by the $s$-wave component even at a 100 TeV pp-collider. Making use of the $s$-wave kinematics, we propose a cut efficiency function to mimic the collider simulation and obtain the potential of measuring Higgs effective couplings at the 14 TeV LHC with an integrated luminosity of 3000 fb$^{-1}$ and at a 100 TeV pp-collider. Analytical expressions of the 2$\sigma$ exclusion limits at the LHC and the 5$\sigma$ discovery bands at the 100 TeV machine are given.

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I. INTRODUCTION

Double Higgs boson production is important to measure the trilinear Higgs coupling in order to determine the structure of the Higgs potential [1,11]. In addition to the trilinear Higgs coupling, the gluon-initiated process $gg \to hh$ also involves the coupling of Higgs boson to top quarks. Besides, in composite Higgs models [12,13] and Little Higgs models [14,15], the contact interactions $hhLtR + h.c.$ and $h(h)gg$ are naturally predicted. So far no new particle beyond the standard model (SM) is observed yet. It is natural to adopt the effective field theory (EFT) [16–18] approach to study the double Higgs production. In this paper, we extend the previous studies [19–24], which focus on the CP-even Higgs effective couplings, and include all the possible CP-odd Higgs effective couplings. The general effective Lagrangian of interest to us is [21–24]

$$\mathcal{L}_{\text{eff}} = -\frac{m_t}{\sqrt{2}}l(c_t + i\gamma_5)v + \frac{m_t}{2\sqrt{2}}l(c_t + i\gamma_5)v + \frac{\alpha_s}{12\pi}v(2c_2G_{\mu\nu}^A + \bar{c}_2G_{\mu\nu}^A)$$

where $m_t$ is the top quark mass, $v$ is the vacuum expectation value, $\alpha_s$ is the strong coupling constant and $G_{\mu\nu}^A(\equiv \partial_\mu G_\nu^A - \partial_\nu G_\mu^A - g_s F_{\mu\nu}^{ABC}G_A^BG_B^C)$ is the field strength of gluon and its dual is defined as $\tilde{G}_{\mu\nu} = \frac{i}{4} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$, with $\epsilon^{0123} = 1$. The terms of $c_t$, $c_2$, $c_3$, $c_{2g}$ and $c_{3g}$, describe the CP-even interactions while the terms of $\bar{c}_t$, $\bar{c}_2$, $\bar{c}_3$ and $\bar{c}_{2g}$ represent the CP-odd interactions. In the SM $c_t = 1$ and $c_{3g} = 1$ while all other coefficients vanish at the tree level. It is worth mentioning that the $c_2$ and $c_{2g}$ terms in the above interaction might be correlated in a given NP model. For example, $c_2 = c_{2g}$ when both terms arise from the same dimension-6 operator $\mathcal{O}_{HG} = H^H G_{\mu\nu}^A G_{\mu\nu}^A$, where $H$ denotes the Higgs doublet. Similarly, $\bar{c}_2 = -\bar{c}_{2g}$ when they are from the operator $\bar{\mathcal{O}}_{HG} = H^H G_{\mu\nu}^A G_{\mu\nu}^A$.

The remainder of this paper is organized as follows. In Sec. II, we present expressions of the single Higgs production amplitude under the Lagrangian in Eq. (1) and obtain constraints from current Higgs signal strength measurements and electric dipole moments (EDMs). In Sec. III A and III B, we give the expression of the amplitude in the double Higgs production and perform partial wave analysis to show this process is dominated by the $s$-wave component, respectively. We obtain a cut efficiency function based on the $s$-wave dominant feature of the amplitude in Sec. IV A.

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* The operator $\bar{\mathcal{O}}_{HG} = H^H G_{\mu\nu}^A G_{\mu\nu}^A$ can also induce a QCD $\theta$-term. However, it can be removed through invoking Peccei-Quinn mechanism.
Then we use the cut efficiency function to mimic the collider simulation to get the \(m_{hh}\) distribution in Sec. [IVB] and the cross section before and after the selection cuts at the HL-LHC and a 100 TeV pp-collider in Sec. [IVC]. The correlation and sensitivity of the Higgs effective couplings at the HL-LHC and a 100 TeV pp-collider is investigated in Sec. [IVD] and Sec. [IVE] respectively. Finally, we conclude in Sec. [V].

II. CONSTRAINTS FROM SINGLE HIGGS MEASUREMENTS AND ELECTRIC DIPOLE MOMENTS

The effective couplings \(c_t, \tilde{c}_t, c_g, \tilde{c}_g\) in Eq. (1) which are related to the double Higgs production also contribute to the single Higgs production and decay processes. Therefore, we consider the current constraints from the single Higgs measurements at the 7 TeV, 8 TeV and 13 TeV LHC as well as the low energy experiments.

The partonic amplitude of the single Higgs production \(g^{\ast\mu}(p_1)g^{b\nu}(p_2) \rightarrow h\) at the leading order (LO) is

\[
\mathcal{M}_h = \alpha_s \delta^{ab} \frac{1}{4\pi v} \left[(c_t F_\Delta + \frac{2}{3} c_g) A^{\mu\nu} - (\tilde{c}_t F^{(1)}_\Delta + \frac{2}{3} \tilde{c}_g) C^{\mu\nu} \epsilon_{\mu}(p_1) \epsilon_{\nu}(p_2)\right],
\]

where \(s = (p_1 + p_2)^2 = m_h^2, \alpha_s = g_s^2/(4\pi)\) and \(\text{Tr}(t^a t^b) = \delta^{ab}/2\). The form factors \(F_\Delta\) and \(F^{(1)}_\Delta\) can be expressed in terms of piecewise function \[26, 27\]

\[
F_\Delta = \tau_t[1 + (1 - \tau_t)f(\tau_t)], \quad F^{(1)}_\Delta = -\tau_t f(\tau_t),
\]

where \(\tau_t = 4m_t^2/m_h^2\) and

\[
f(\tau) = \begin{cases} \arcsin^2 \left(\frac{1}{\sqrt{\tau}}\right) & \tau \geq 1, \\ -\frac{1}{4} \left[\log \left(\frac{1 + \sqrt{1 - \tau}}{1 - \sqrt{1 - \tau}}\right) - i\pi\right]^2 & \tau < 1. \end{cases}
\]

In the large \(m_t\) limit \[28\], we have

\[
F_\Delta = \frac{2}{3} + \mathcal{O}(s/m_t^2), \quad F^{(1)}_\Delta = -1 + \mathcal{O}(s/m_t^2).
\]

The Lorentz structures \(A^{\mu\nu}\) and \(C^{\mu\nu}\) are defined as

\[
A^{\mu\nu} = g^{\mu\nu} - \frac{p_1^\mu p_2^\nu}{p_1 \cdot p_2}, \quad C^{\mu\nu} = \frac{p_{1\sigma} p_{2\nu} - p_{1\nu} p_{2\sigma}}{p_1 \cdot p_2} \epsilon^{\mu\nu\rho\sigma}.
\]

The single Higgs production cross section and partial decay width in the NP model, normalized to the SM values, are

\[
\kappa_s^2 = \frac{\sigma(gg \rightarrow h)}{\sigma(gg \rightarrow h)_{\text{SM}}} = \frac{|c_t F_\Delta + \frac{2}{3} c_g|^2 + |\tilde{c}_t F^{(1)}_\Delta + \frac{2}{3} \tilde{c}_g|^2}{|F_\Delta|^2},
\]

and

\[
\kappa_t^2 = \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{\text{SM}}} = \frac{|F_1(\tau_W) + \frac{4}{3} c_t F_\Delta|^2 + |4 \tilde{c}_t F^{(1)}_\Delta|^2}{|F_1(\tau_W) + \frac{4}{3} F_\Delta|^2},
\]

respectively known as the \(\kappa\)-framework \[29\], where the \(hWW\) coupling is assumed to be the SM value. The form factor \(F_1(\tau_W)\) is defined as \[20\]

\[
F_1(\tau_W) = -\frac{1}{2} |2 + 3\tau_W + 3\tau_W(2 - \tau_W)f(\tau_W)|, \quad \tau_W \equiv 4m_t^2/m_h^2.
\]

The current bound on \((\kappa_s, \kappa_t)\) from the combination of the ATLAS and CMS results at the 7 and 8 TeV LHC is shown in Refs. \[30, 31\]. Note that the 13 TeV LHC data does not give a stronger constraint \[32, 33\].

In Fig. [1] we display the allowed regions of the effective couplings by the current single Higgs measurements\(^{1}\) at the LHC at 95\% confidence level (CL), which are shown in blue bands. Only two effective couplings vary in each plot.

\(^{1}\) For the “single Higgs measurements”, we mean that there is only one Higgs boson that contributes to the process measured.
while other couplings are set to be the SM values. Scenario \((c_t, c_g)\) shows two isolated bands, since the constraint from \(\kappa_g^2\) is proportional to \(|c_t F_\Delta + \frac{g}{2} c_g|^2\), which can be simplified as \(|c_t + c_g|^2\) in the infinite \(m_t\) limit. Therefore, there are two degenerate regions satisfying the constraints, \(c_g + c_t \sim \pm 1\). Besides, the lower and upper limits of \(c_t\) come from the constraint of \(\kappa_g^2\). So \(c_t\) cannot be too small or large, otherwise \(\Gamma(h \to \gamma\gamma)\) will be too large. The constraint on scenario \((c_g, \tilde{c}_g)\) comes only from \(\kappa_g^2\), which is \(|c_g + 1|^2 + |\tilde{c}_g|^2\) in the infinite \(m_t\) limit. It’s obvious that in order to satisfy the single Higgs measurements, the allowed parameter space must be a ring. In scenario \((c_t, \tilde{c}_t)\), we have to consider both constraints on \(\kappa_g^2\) and \(\kappa_{g'}^2\). In the infinite \(m_t\) limit, \(\kappa_{g'}^2\) is approximated to be \(|c_t|^2 + \frac{9}{4}|\tilde{c}_t|^2\). The allowed region from this constraint is an elliptical ring. The constraint on \(\kappa_g^2\) will further reject the \(c_t \lesssim 0\) region. In scenario \((\tilde{c}_t, \tilde{c}_g)\), \(\kappa_g^2 \sim 1 + |\tilde{c}_g - \frac{2}{3} \tilde{c}_t|^2\) in the infinite \(m_t\) limit. The constraint on \(\kappa_{g'}^2\) will lead to \(\tilde{c}_g \sim \frac{2}{3} \tilde{c}_t\), and \(\kappa_g^2\) will give lower and upper limits on \(\tilde{c}_t\). The correlation in this scenario is different from scenario \((c_t, c_g)\) due to the relative minus sign between \(F_\Delta\) and \(F_{\Delta}^{(1)}\). Similar to scenario \((c_g, \tilde{c}_g)\) the allowed region is a ring in scenario \((c_g, \tilde{c}_t)\) as a fact of \(\kappa_{g'}^2 \sim |c_g + 1|^2 + \frac{9}{4}|\tilde{c}_t|^2\). In principle, \(\kappa_g^2\) will further give a constraint on \(\tilde{c}_t\). But it turns out that the constraint from \(\kappa_{g'}^2\) is not so strong such that the region allowed by \(\kappa_{g'}^2\) satisfies the \(\kappa_g^2\) constraint. The situation of scenario \((c_t, \tilde{c}_g)\) is also similar to scenario \((c_t, \tilde{c}_t)\). In the infinite \(m_t\) limit, we have \(\kappa_g^2 \sim |c_t|^2 + |\tilde{c}_g|^2\).

On the other hand, the individual constraints on \(c_t, \tilde{c}_t, c_g\) and \(\tilde{c}_g\) at 95% CL from the single Higgs measurements are

\[
c_t \in [0.839, 1.24], \quad \tilde{c}_t \in [-0.345, 0.186], \quad c_g \in [-2.134, -1.731] \cup [-0.161, 0.242], \quad \tilde{c}_g \in [-0.279, 0.517]. \tag{10}
\]

Apart from the LHC measurements, the CP-odd couplings are also constrained by the electric dipole moments of electron, neutron and mercury atom (Hg) [33, 36],

\[
|\tilde{c}_1| < 0.01, \quad |\tilde{c}_g| < 0.05. \tag{11}
\]
III. DOUBLE HIGGS PRODUCTION

A. Amplitude

In this section, we will discuss the amplitude of double Higgs production via gluon fusion $gg \to hh$. The LO partonic amplitude of $g^{a\mu}(p_1)g^{b\nu}(p_2) \to h(p_3)h(p_4)$ with the effective Lagrangian in Eq. 9 is

$$
\mathcal{M}_{hh} = \left[ c_1^2\mathcal{M}_{SM}^{(1)} + c_1^2\mathcal{M}_{SM}^{(2)} + c_4\mathcal{M}_{SM}^{(3)} + c_4\mathcal{M}_{SM}^{(4)} + c_4\mathcal{M}_{SM}^{(5)} + \bar{c}_2\mathcal{M}_{SM}^{(6)} + \bar{c}_2\mathcal{M}_{SM}^{(7)} \right] c_\mu(p_1)c_\nu(p_2)\delta^{ab},
$$

(12)

where $a$ and $b$ in the superscript denote the color index of gluons,

$$
\mathcal{M}_{SM}^{(1)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} (F_\square A^{\mu\nu} + G_\square B^{\mu\nu}),
\mathcal{M}_{SM}^{(2)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} \left( F_\square A^{\mu\nu} + G_\square B^{\mu\nu} \right),
\mathcal{M}_{SM}^{(3)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} \left( F_\square A^{\mu\nu} + G_\square B^{\mu\nu} \right),
\mathcal{M}_{SM}^{(4)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} \left( F_\square A^{\mu\nu} + G_\square B^{\mu\nu} \right),
\mathcal{M}_{SM}^{(5)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} \left( F_\square A^{\mu\nu} + G_\square B^{\mu\nu} \right),
\mathcal{M}_{SM}^{(6)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} \left( F_\square A^{\mu\nu} + G_\square B^{\mu\nu} \right),
\mathcal{M}_{SM}^{(7)} = -\frac{\alpha_s\hat{s}}{4\pi t^2} \left( F_\square A^{\mu\nu} + G_\square B^{\mu\nu} \right).
$$

(13)

Here the Mandelstam variables of the partonic process are defined as

$$
\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 - p_3)^2, \quad \hat{u} = (p_2 - p_3)^2.
$$

(14)

The Lorentz structures $A^{\mu\nu}$ and $C^{\mu\nu}$ are defined in Eqs. 9, while $B^{\mu\nu}$ is 11 [25]

$$
B^{\mu\nu} = g^{\mu\nu} + \frac{p_3^{\mu}p_3^{\nu}}{p_3^2} - \frac{2p_2 \cdot p_3 p_3^{\mu}}{p_2^2} - \frac{2p_1 \cdot p_3 p_3^{\mu}}{p_1^2} + \frac{2p_3^{\mu}p_3^{\nu}}{p_3^2},
$$

(15)

with $p_3^2 = (\hat{u} - m_h^2)/\hat{s}$. It can be easily verified that $A^{\mu\nu} A_{\mu\nu} = B^{\mu\nu} B_{\mu\nu} = C^{\mu\nu} C_{\mu\nu} = 2$ and $A^{\mu\nu} B_{\mu\nu} = B^{\mu\nu} C_{\mu\nu} = A^{\mu\nu} C_{\mu\nu} = 0$. Therefore we can expand the amplitude in terms of those tensor structures as follows

$$
\mathcal{M}_{hh} = -\frac{\alpha_s\hat{s}\delta^{ab}}{4\pi t^2} \epsilon_\mu(p_a)\epsilon_\nu(p_b) \left\{ c_1^2 F_\square + c_1^2 F_\square^{(1)} + \frac{3m_h^2}{s - m_h^2} c_3 h (c_1 F_\square + \frac{2}{3} c_2) + \frac{3m_h^2}{s - m_h^2} c_3 h (c_1 F_\square + \frac{2}{3} c_2) + \frac{3m_h^2}{s - m_h^2} c_3 h (c_1 F_\square + \frac{2}{3} c_2) + \frac{3m_h^2}{s - m_h^2} c_3 h (c_1 F_\square + \frac{2}{3} c_2) \right\}.
$$

(16)

The expressions of the form factors $F_\square, G_\square, F_\square^{(1)}, G_\square^{(1)}, F_\square^{(2)}, F_\square$ and $F_\square^{(4)}$ can be found in Appendix A. In the large $m_t$ limit [25], we have

$$
F_\square = -\frac{2}{3} + O(\hat{s}/m_t^2), \quad G_\square = O(\hat{s}/m_t^2), \quad F_\square^{(1)} = \frac{2}{3} + O(\hat{s}/m_t^2), \quad F_\square^{(2)} = 2 + O(\hat{s}/m_t^2), \quad G_\square^{(1)} = O(\hat{s}/m_t^2), \quad F_\square^{(4)} = -1 + O(\hat{s}/m_t^2),
$$

(17)

which can be obtained from the low energy theorem (LET) [37, 38].

Figure 2 shows the $\sqrt{s}$-dependence of each form factor, where we have chosen two specific values of $\theta$ which is defined as the scattering angle of the initial gluon and final Higgs boson. Numerically, the $F$ form factors are always larger than the $G$ form factors around the threshold region $\sqrt{s} \sim 400$ GeV, where the dominant cross section arises. Unlike the $G$ form factors, the $F$ form factors is insensitive to $\theta$. Thus the partonic cross section is dominated by the $s$-wave around the threshold region. To evaluate the $s$-wave and $d$-wave contributions at a large $\sqrt{s}$, it is necessary to perform a partial wave analysis.
FIG. 2: The form factors dependence on the partonic c.m. energy $\sqrt{s}$, where we have chosen the scattering angle $\theta = 0$ and $\pi/2$.

B. Partial wave analysis

The amplitude in the partial wave expansion is given by

$$M_{hh}(\hat{s}, \theta) = \sum_{\ell=0,2} (2\ell + 1) M_\ell(\hat{s}) P_\ell(\cos \theta), \quad (18)$$

where $P_\ell(x)$ are the Legendre polynomials satisfying the orthogonal relation $\int_{-1}^{1} dx P_\ell(x) P_{\ell'}(x) = \frac{2}{2\ell + 1} \delta_{\ell\ell'}$. The $s$-wave and $d$-wave components of the amplitude are proportional to $P_0(x)$ and $P_2(x)$ respectively, which are

$$M_0(\hat{s}) = \frac{1}{2} \int_{-1}^{1} d \cos \theta M_{hh}(\hat{s}, \theta) P_0(\cos \theta), \quad (19)$$

$$M_2(\hat{s}) = \frac{1}{2} \int_{-1}^{1} d \cos \theta M_{hh}(\hat{s}, \theta) P_2(\cos \theta). \quad (20)$$

Now the partonic level differential cross section with respect to $\cos \theta$ can be expanded into three terms

$$\frac{d\hat{\sigma}_{hh}(\hat{s})}{d \cos \theta} = \hat{\sigma}_0(\hat{s}) + \hat{\sigma}_2(\hat{s}) P_2(\cos \theta)^2 + \hat{\sigma}_{\text{int}}(\hat{s}) P_2(\cos \theta), \quad (21)$$

where the first and the second terms denote the $s$-wave and $d$-wave contributions, respectively. The third term, which arises from the interference of the $s$-wave and $d$-wave components of the amplitude, vanishes after integrating over $\cos \theta$. $\hat{\sigma}_0(\hat{s})$, $\hat{\sigma}_2(\hat{s})$ and $\hat{\sigma}_{\text{int}}(\hat{s})$ can be obtained numerically. The angular dependence of the form factors can be clearly revealed by choosing different combinations of $(c_t, \tilde{c}_t)$. In Fig. 3, we show the three terms in Eq. (21) with $\sqrt{s} = 400$ GeV, 1000 GeV for different $(c_t, \tilde{c}_t)$, where we fix other Higgs effective couplings to be the SM values and normalize the three terms to the total cross sections. Since the $s$-wave has no angular dependence, the distributions in Figs. 3(a) and (d) are flat. The $d$-wave and the interference contributions have nontrivial angular dependences, which are reflected in Figs. 3(b, e) and (c, f), respectively. From the distributions, the $d$-wave and the interference contributions are comparable for $\sqrt{s} = 400$ GeV and 1000 GeV. This is because the imaginary (real) parts of the form factors at $\sqrt{s} = 400$ (1000) GeV are small such that the interference contribution is suppressed. While increasing the $\sqrt{s}$ from 400 GeV to 1000 GeV, the fractions of the $d$-wave and the interference contributions grow almost one order of magnitude. However, their contributions are still overwhelmed by the $s$-wave. So the $d$-wave and the interference contributions to $\cos \theta$ distributions at the hadron level, such as transverse momentum or rapidity distributions, are small. Figure 4 shows the $s$ and $d$-wave contributions to the total cross sections after integrating over $\cos \theta$. It is clear that the $d$-wave contributions are at most of 10% of the total cross sections. As a result, the invariant mass distributions at the hadron level are dominated by the $s$-wave.

To be concrete, we show the $s$-wave and $d$-wave contributions to the total cross section at the hadron level,

$$\sigma_0 = \int_{\tau_0}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_g(\mu_F^2) f_g(\mu_F^2) \int_{-1}^{1} d \cos \theta \hat{\sigma}_0(\tau s) P_0(\cos \theta)^2, \quad (22)$$

$$\sigma_2 = \int_{\tau_0}^{1} d\tau \int_{\tau}^{1} \frac{dx}{x} f_g(\mu_F^2) f_g(\mu_F^2) \int_{-1}^{1} d \cos \theta \hat{\sigma}_2(\tau s) P_2(\cos \theta)^2, \quad (23)$$
where $\tau_0 = 4m_h^2/s$, $f_g$’s are the PDF functions of the initial gluons, and $\mu_F$ is the factorization scale. The hadronic cross sections can be expanded as follows,

$$
\frac{\sigma}{\sigma_{SM}^{hh}} = a_1 c_t^4 + a_2 c_t^3 + a_3 c_t^2 \tilde{c}_t^2 + a_4 c_t^2 + a_5 c_t \tilde{c}_t^2 + a_6 c_t^4 + a_7 \tilde{c}_t^2,
$$

(24)

$$
\frac{\sigma}{\sigma_{SM}^{hh}} = b_1 c_t^4 + b_2 c_t^3 + b_3 c_t^2 \tilde{c}_t^2 + b_4 c_t^2 + b_5 c_t \tilde{c}_t^2 + b_6 c_t^4 + b_7 \tilde{c}_t^2,
$$

(25)

where $\sigma_{SM}^{hh}$ denotes the hadronic cross section of $gg \rightarrow hh$ in the SM, which has been calculated at the LO [1, 28, 39], NLO [40–46], NLL [47], NNLO [48–54] and NNLL [55, 56]. The coefficients of the expansions are displayed in Table I. Total cross sections at the 14 TeV LHC and the 100 TeV pp-collider are dominated by the $s$-wave component. Besides, the fractions of the $s$-wave contribution at the 100 TeV pp-collider are smaller than the fractions at the 14 TeV LHC, while the fractions of the $d$-wave contribution at the 100 TeV pp-collider are larger than the fractions at the 14 TeV LHC.

From the above partial wave analysis, we draw a few conclusions, which do not rely on the Higgs effective couplings.

(1) The $d$-wave contributions to the $\cos \theta$ distributions at the hadron level, such as transverse momentum or rapidity distributions, are always small;
TABLE I: The coefficients $a_i$ and $b_i$ at $\sqrt{s} = 14$ TeV and 100 TeV.

| $\sqrt{s}$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
|------------|-------|-------|-------|-------|-------|-------|-------|
| 14 TeV     | 2.069 | -1.351| 13.858| 0.276 | -6.219| 0.706 | 0.861 |
| 100 TeV    | 1.891 | -1.108| 11.280| 0.208 | -4.795| 0.663 | 0.634 |

| $\sqrt{s}$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ | $b_5$ | $b_6$ | $b_7$ |
|------------|-------|-------|-------|-------|-------|-------|-------|
| 14 TeV     | 0.006 | 0     | 0.020 | 0     | -0.136| 0.013 | 0     |
| 100 TeV    | 0.009 | 0     | 0.027 | 0     | -0.137| 0.017 | 0     |

(2) The $d$-wave contributions to the invariant mass distributions at the hadron level are small;

(3) The $d$-wave contributions to the total cross sections are small.

Therefore, it is justified that the double Higgs production is dominated by the $s$-wave.

IV. COLLIDER SIMULATION

In Table II we collect references of the searches of double Higgs production at the 8 TeV and the 13 TeV LHC as well as the projected analyses at HL-LHC and 100 TeV pp-collider in the final states $bb\gamma\gamma$, $bbbb$, $b\tau^+\tau^-$, $W^+W^-$, $\gamma\gamma W^+W^-$ and $bbV(V = W, Z)$. Although $bbbb$ channel has the largest cross section, the QCD background is hard to control. On the other hand, the $bb\gamma\gamma$ channel, despite of its small decay branching ratio, exhibits clear collider signature. Therefore, it has been studied extensively in the literature \[3, 5, 20, 21, 23, 57\]. In this study, we focus our attention on the $bb\gamma\gamma$ channel and use the cut efficiency function $A(m_{hh})$ to mimic the detector effects in different NP scenarios\(^\dagger\). At the 14 TeV HL-LHC, we follow the analysis done by the ATLAS Collaboration \[58\] and adopt the cut efficiency function in Ref. \[57\]. At the 100 TeV pp-collider, we follow the projected analysis done by the 100 TeV group \[59\]. In the rest of this section, we will first derive cut efficiency function of $gg \rightarrow hh \rightarrow bb\gamma\gamma$ at the 10 TeV pp-collider, then we will discuss the correlations and sensitivities of Higgs effective couplings.

TABLE II: Searches of double Higgs production at the 8 TeV and 13 TeV by the ATLAS and CMS Collaborations, as well as the projected analysis at the 14 TeV High-Luminosity LHC (HL-LHC) and the 100 TeV pp-collider.

|                  | $bb\gamma\gamma$ | $bbbb$ | $b\tau^+\tau^-$ | $\gamma\gamma W^+W^-$ | $bbV$ |
|------------------|------------------|--------|-----------------|------------------------|-------|
| 8 TeV ATLAS      | \[60\][61]       |        |                 |                        | -     |
| 8 TeV CMS        | \[63\]           | \[64\] | \[65\][66]     |                        | -     |
| 13 TeV ATLAS     | \[67\]           | \[68\] |                 |                        | -     |
| 13 TeV CMS       | \[69\][70][71]   | \[72\] | \[73][74][75]  |                        | -     |
| 14 TeV HL-LHC ATLAS | \[58\] |              | \[76\][77]     |                        | -     |
| 14 TeV HL-LHC CMS | \[78\]           |        |                 |                        | -     |
| 100 TeV pp-collider Contino, et. al. | \[59\] | \[59\] | \[59\]         | \[59\] | \[59\] |

A. Cut efficiency function

The Born level differential cross section of double Higgs production can be written as

$$\frac{d^2\sigma}{d m_{hh} d\eta} = \frac{1}{2S} \int dx_1 dx_2 H(\hat{s}, \hat{\eta}, \mu_r, \{\theta_{NP}\}) f_{g/p}(x_1, \mu_f) f_{g/p}(x_2, \mu_f) \delta(x_1 x_2 S - \hat{s}) \det \left( \frac{\partial(\hat{s}, \hat{\eta})}{\partial(m_{hh}, \eta)} \right),$$

where $H$ is the hard scattering cross section depending on the center of mass energy (c.m.) square $\hat{s}$ and Higgs boson pseudo-rapidity $\hat{\eta}$ in the c.m. frame, $S$ is the collision energy of the hadron collider, $\mu_f$ is the factorization scale, $f_{i/j}$

\(^\dagger\) Hereafter, NP in this paper denotes those which can be described by the effective Lagrangian in Eq. \[1\].
is the parton distribution function (PDF) of parton \( i \) from \( j \), \( m_{hh} \) is the invariant mass of Higgs boson pair in the lab frame, \( \eta (\hat{\eta}) \) is the pseudo-rapidity of Higgs boson in the lab frame (c.m. frame). \( \{\theta_{NP}\} \) are the parameters from the new physics model. We do not write the parameters \( m_h^2 \) and \( m_t^2 \) explicitly, the azimuthal angle has been integrated out.

We know that \( \hat{s} = m_{hh}^2 \) at the Born level. The Jacobian determinant is \( 2m_{hh}|\partial \hat{\eta}/\partial \eta| \), and

\[
\frac{d^2\sigma}{dm_{hh}d\eta} = \frac{m_{hh}}{S} \int dx_1 dx_2 \left[ \frac{\partial \hat{\eta}}{\partial \eta} \right] H \left( m_{hh}, \hat{\eta} \left( m_{hh}, \eta, \frac{x_1}{x_2} \right), \mu_R \right) f_{g/p} (x_1, \mu_f) f_{g/p} (x_2, \mu_f) \delta \left( x_1 x_2 S - m_{hh}^2 \right)
\]

\[
= \int_{m_{hh}^2/S}^{1} \frac{m_{hh} dx_1}{x_1 S^2} H \left( m_{hh}, \hat{\eta} \left( m_{hh}, \eta, \frac{x_1^2 S}{m_{hh}^2} \right), \mu_R \right) f_{g/p} (x_1, \mu_f) f_{g/p} \left( \frac{m_{hh}^2}{x_1^2 S}, \mu_f \right) \left| \frac{\partial \hat{\eta}}{\partial \eta} \right|.
\]

(27)

The \( \eta \) and \( \hat{\eta} \) are related by

\[
\eta = -\frac{1}{2} \log \left[ \frac{\sqrt{\Delta^2 + 4x_1 x_2 \beta^2 \text{sech}^2 \hat{\eta}} - \Delta}{\sqrt{\Delta^2 + 4x_1 x_2 \beta^2 \text{sech}^2 \hat{\eta}} + \Delta} \right],
\]

(28)

where

\[
\beta \equiv \sqrt{1 - \frac{4m_h^2}{m_{hh}^2}}, \quad \Delta \equiv (x_1 - x_2) + (x_1 + x_2) \beta \tanh \hat{\eta}.
\]

(29)

Thus

\[
\frac{\partial \hat{\eta}}{\partial \eta} = \frac{\sqrt{\Delta^2 + 4x_1 x_2 \beta^2 \text{sech}^2 \hat{\eta}}}{(x_1 + x_2) \beta + (x_1 - x_2) \tanh \hat{\eta}}.
\]

(30)

For gluon-fusion initial state, the main contribution comes from the small-\( x \) region with \( x_1 \sim x_2 \). In that limit, it is a good approximation that \( \hat{\eta} = \eta \).

Owing to the scalar feature of Higgs boson, the kinematics of Higgs boson decay products is mainly controlled by the Higgs kinematics, e.g. \( p_T \) and \( \eta \) of the Higgs boson. Thus the cut efficiency depends on the \( p_T \) and \( \eta \) distributions of Higgs bosons. The transverse momentum of Higgs boson is

\[
p_T = \frac{\sqrt{s}}{2} \beta \text{sech} \hat{\eta} = \frac{m_{hh}}{2} \beta \text{sech} \left[ \hat{\eta} \left( m_{hh}, \eta_1, \frac{x_1^2 S}{m_{hh}^2} \right) \right].
\]

(31)

Denote \( \epsilon (p_T, \eta_1, \eta_2) \) to be the differential cut efficiency function. The pseudo-rapidity \( \eta_2 \) is determined by \( \hat{\eta}, m_{hh} \) and \( x_1 \). Therefore, \( \epsilon \) is a function of \( m_{hh}, x_1 \) and \( \eta \) (which is just \( \eta_1 \)).

The hard scattering function, \( H \), is generically \( \eta \)-dependent. Fortunately, for the SM-like double Higgs production induced by the effective Lagrangian given in Eq. 1, higher partial wave components are highly suppressed. Therefore, we can treat \( \{\theta_{NP}\} \) as \( \hat{\eta} \)-independent. Then the amplitude square will be \( \hat{\eta} \)-independent. Under such assumptions, the differential cross section can be factorized as following,

\[
\frac{d^2\sigma}{dm_{hh}d\eta} = \frac{m_{hh}}{S^2} H (m_{hh}, \mu_R) \int_{m_{hh}^2/S}^{1} \frac{dx_1}{x_1} f_{g/p} (x_1, \mu_f) f_{g/p} \left( \frac{m_{hh}^2}{x_1^2 S}, \mu_f \right) \left| \frac{\partial \hat{\eta}}{\partial \eta} \right|_{m_{hh}, \eta, x_1}.
\]

(32)

Integrating the pseudo-rapidity out, we have

\[
\frac{d\sigma}{dm_{hh}} = \frac{m_{hh}}{S^2} H (m_{hh}, \mu_R) \int_{m_{hh}^2/S}^{1} \frac{dx_1}{x_1} f_{g/p} \left( \frac{m_{hh}^2}{x_1^2 S}, \mu_f \right) f_{g/p} (x_1, \mu_f) \int d\eta \left| \frac{\partial \hat{\eta}}{\partial \eta} \right|_{m_{hh}, \eta, x_1} = \frac{m_{hh}}{S^2} H (m_{hh}, \mu_R) \Sigma (m_{hh}, S, \mu_f).
\]

(33)

We can also write down the differential cross section after kinematic cuts used by experimental groups,

\[
\frac{d\sigma_{\text{after cuts}}}{dm_{hh}} = \int d\hat{m}_{hh} \hat{m}_{hh} \frac{S}{2} H (\hat{m}_{hh}, \mu_R) \int_{\hat{m}_{hh}^2/S}^{1} \frac{dx_1}{x_1} f_{g/p} \left( \frac{\hat{m}_{hh}^2}{x_1^2 S}, \mu_f \right) f_{g/p} (x_1, \mu_f)

\times \int d\eta \left| \frac{\partial \hat{\eta}}{\partial \eta} \right|_{\hat{m}_{hh}, \eta, x_1} \epsilon (m_{hh}, \hat{m}_{hh}, x_1, \eta).
\]

(34)
where \( m_{hh} \) is the invariant mass of the Higgs-pair system measured in the experiment, \( \tilde{m}_{hh} \) is the real invariant mass of the Higgs-pair system of the same event, which is introduced to describe the finite energy smearing effect. For an ideal detector, we have

\[
\epsilon (m_{hh}, \tilde{m}_{hh}, x_1, \eta) = \epsilon (m_{hh}, x_1, \eta) \delta (\tilde{m}_{hh} - m_{hh}),
\]

which will be broken by finite energy smearing effect. Due to the cut effect, in general

\[
\epsilon (m_{hh}, \tilde{m}_{hh}, x_1, \eta) \neq \epsilon (m_{hh}, m_{hh}, x_1, \eta).
\]

To investigate the inclusive result, one can integrate \( m_{hh} \) and have

\[
\sigma_{\text{after cuts}} = \int dm_{hh} dm_{hh} \tilde{m}_{hh} S^2 H (\tilde{m}_{hh}, \mu_f) \int \frac{dx_1}{x_1} \frac{f_{g/p} \left( \frac{\tilde{m}_{hh}^2}{x_1 S}, \mu_f \right)}{f_{g/p} (x_1, \mu_f)} \nonumber \times \int d\eta \left[ \frac{\partial \eta}{\partial \tilde{m}_{hh}} \right] \epsilon (m_{hh}, \tilde{m}_{hh}, x_1, \eta).
\]

Define

\[
\Sigma (\tilde{m}_{hh}, S, \mu_f) = \int dm_{hh} \int \frac{dx_1}{x_1} \frac{f_{g/p} \left( \frac{\tilde{m}_{hh}^2}{x_1 S}, \mu_f \right)}{f_{g/p} (x_1, \mu_f)} \nonumber \times \int d\eta \left[ \frac{\partial \eta}{\partial \tilde{m}_{hh}} \right] \epsilon (m_{hh}, \tilde{m}_{hh}, x_1, \eta),
\]

we obtain

\[
\sigma_{\text{after cuts}} = \int dm_{hh} \tilde{m}_{hh} S^2 H (\tilde{m}_{hh}, \mu_f) \Sigma (\tilde{m}_{hh}, S, \mu_f).
\]

Then it is natural to define a differential cut efficiency as

\[
A (m_{hh}, S, \mu_f) = \frac{\Sigma (m_{hh}, S, \mu_f)}{\Sigma (m_{hh}, S, \mu_f)}.
\]

Such a differential cut acceptance function only depends on the collision energy and the detail of the PDF. When the new physics contribution is dominated by the \( s \)-wave, one can calculate the total cross section after cuts by a convolution of the differential cross section of \( \tilde{m}_{hh} \), and the differential cut acceptance function \( A(m_{hh}, S, \mu_f) \). Hence we obtain the \textit{master formula} for our study as following

\[
sigma_{\text{after cuts}} = \int dm_{hh} \tilde{m}_{hh} S^2 H (m_{hh}, \mu_f) \Sigma (m_{hh}, S, \mu_f) \nonumber = \int \frac{m_{hh} dm_{hh}}{S^2} H (m_{hh}, \mu_f) \Sigma (m_{hh}, S, \mu_f) \nonumber = \int \frac{m_{hh} dm_{hh}}{S^2} H (m_{hh}, \mu_f) \Sigma (m_{hh}, S, \mu_f) A (m_{hh}, S, \mu_f) \nonumber = \int dm_{hh} \frac{d\sigma}{dm_{hh}} A (m_{hh}, S, \mu_f).
\]

Equation (40) also tells us how to calculate the integral kernel \( \mathcal{A} \) practically. It can be calculated by generating \( s \)-wave events with fixed \( m_{hh} \) and counting the fraction of the events which pass the cuts. It is worth emphasizing that without the integration of \( m_{hh} \), the result is not exactly the differential distribution due to the finite invariant mass smearing effect. However, when the smearing effect is not too large, it is a good approximation to mimic the differential distribution after cut as following

\[
\left( \frac{d\sigma}{dm_{hh}} \right)_{\text{after cuts}} = \frac{d\sigma}{dm_{hh}} A (m_{hh}, S, \mu_f).
\]

As to be shown soon, this approximation works well for the Higgs boson pair production. Thus we will use this approximation to illustrate the differential cross section in our work.
At the 100 TeV pp-collider, we can also use this analytical function to include all the detector effects as we did for 14 TeV LHC in [57]. We follow the strategy in the 100 TeV report [59]. The main backgrounds consist of $b\bar{b}\gamma\gamma$, $b\bar{b}j\gamma$, $jj\gamma\gamma$, $b\bar{b}h(\gamma\gamma)$ and $t\bar{t}h(\gamma\gamma)$. The cuts used are

\begin{align*}
\gamma \text{ isolation } R &= 0.4, \text{ jets: anti-kt, parameter } R = 0.4, \Delta R_{bb} < 3.5, \Delta R_{\gamma\gamma} < 3.5,
p_T^{b_1} > 60 \text{ GeV}, p_T^{b_2} > 35 \text{ GeV}, |\eta^b| < 4.5, p_T^{\gamma_1} > 60 \text{ GeV}, p_T^{\gamma_2} > 35 \text{ GeV}, |\eta^\gamma| < 4.5, 
p_T(bb) > 100 \text{ GeV}, p_T(\gamma\gamma) > 100 \text{ GeV}, 100 \text{ GeV} < m_{bb} < 150 \text{ GeV}, 120.5 \text{ GeV} < m_{\gamma\gamma} < 129.5 \text{ GeV},
\end{align*}

where $b_1$ ($b_2$) and $\gamma_1$ ($\gamma_2$) represent the leading (subleading) $b$-jet and photon, respectively. The $b$-tagging probability and faking rates are

\begin{align*}
p_{b\rightarrow b} = 0.75, p_{c\rightarrow b} = 0.1, p_{j\rightarrow b} = 0.01.
\end{align*}

The light-jet-to-photon faking probability is parametrized via

\begin{align*}
p_{j\rightarrow \gamma} = \alpha \exp(-p_T/j/\beta), \alpha = 0.01, \beta = 30 \text{ GeV}.
\end{align*}

The photon identification efficiency is

\begin{align*}
\epsilon_\gamma(p_T) = \begin{cases} 
95\%, & \text{for } |\eta| < 1.5, \\
90\%, & \text{for } 1.4 < |\eta| < 4, \\
80\%, & \text{for } 4 < |\eta| < 6.
\end{cases}
\end{align*}

To get the cut efficiency function, we generate partonic level $pp \rightarrow hh \rightarrow b\bar{b}\gamma\gamma$ events with MadGraph5_aMC@NLO event generator [80] with CT14 PDF [81]. As we are interested only in the $s$-wave component, the default SM with trilinear Higgs coupling is enough. The events are generated with fixed $m_{hh}$ for each 10 GeV interval. The detector effects are mimicked with Gaussian smearing effects with the parameters given in [59].

![Graph](image.png)

**FIG. 5**: The cut efficiency function at the 100 TeV pp-collider.

We show the cut efficiency function for the 100 TeV pp-collider in Fig. 5. The structures in the figure can be easily understood as follows. For the “peak” structure, the boost factor of the Higgs boson is $\gamma \simeq 5$ around the crossing point. The angular distance between the Higgs decay products, i.e. $\Delta R \equiv \sqrt{(\Delta \eta)^2 + (\Delta \phi)^2}$ is approximated by $\Delta R \simeq 2/\gamma \simeq 0.4$. So crossing this point, the typical $\Delta R$ of the $bb$ system and the $\gamma\gamma$ system will become smaller. The signal events are likely to fail the $\Delta R$ cuts to yield a smaller cut efficiency. We would like to estimate the result analytically with some approximations. Let us define the 4-momenta of the partons (photons) in the Higgs rest frame.
with the z-direction defined by the Higgs 3-momentum in the lab frame. Then the exact result of the $\Delta R$ is

\[
\Delta R^2 = \arccos \left\{ \left[ 4r^2 \beta_T^2 + (1 - z^2) \left( \beta_T^2 - \cos^2 \phi \right) \sin^2 \phi \tanh^2 \eta \right] - z^2 \sech^2 \eta \right. \\
+ 2z \sin \phi \tan \eta \times \sqrt{(1 - z^2) \left( \beta_T^2 + \sech^2 \eta \right)} \\
\times \left[ (1 - z^2) \cos^2 \phi + \left( \sqrt{1 + 4r^2} \beta_T - \sqrt{1 - z^2} \sin \phi \tan \eta \right) + z\sqrt{\beta_T^2 + \sech^2 \eta} \right]^{2 - 1} \\
\times \left[ (1 - z^2) \cos^2 \phi + \left( \sqrt{1 + 4r^2} \beta_T + \sqrt{1 - z^2} \sin \phi \tan \eta \right) - z\sqrt{\beta_T^2 + \sech^2 \eta} \right]^{2 - 1} \\
+ \left. \arcsinh \left\{ \sinh \eta \left( \sqrt{1 + 4r^2} \beta_T + z\beta_T^2 + \sech^2 \eta \right) + \sin \phi \sech \eta \sqrt{1 - z^2} \right\} \right\}^2, \tag{47}
\]

where $\beta_T \equiv p_T/m_h$ is the ratio between the transverse momentum and the mass of the Higgs boson in the lab frame, $r = m/m_h$ is the mass ratio between the final state particle and the Higgs boson, which is 0 for photon and $m_b/m_h$ for bottom quark, $\eta$ is the pseudo-rapidity of the Higgs boson in the lab frame, $z \equiv \cos \theta$ is the cosine value of the polar angle of one parton in the Higgs rest frame, $\phi$ is the azimuthal angle of one parton in the Higgs rest frame. In the highly boost region, $\beta_T \gg 1$, and

\[
\Delta R = \arccos \left\{ 1 - \frac{2}{1 + (1 - z^2) \beta_T^2} \right\} \tag{48}
\]

is a good approximation for the massless final state particle. To pass the $\Delta R$ cuts, we need to solve this equation. The solutions are

\[
z_{\pm} = \pm \sqrt{\frac{1 - \beta_T^2 - (1 + \beta_T^{-2}) \cos \Delta R}{1 - \cos \Delta R}}. \tag{49}
\]

The region allowed by the cut is $[-1, z_-] \cup [z_+, 1]$. This is a hint that we can fit the high invariant mass tail with the function

\[
A(m_{hh}) = c_1 \left[ 1 - \frac{m_{hh}^2 (1 - \cos \Delta R) - 8 (m_h - \delta m)^2}{(1 - \cos \Delta R) (m_{hh}^2 - 4 (m_h - \delta m)^2)} \right]^{\gamma_c}, \tag{50}
\]

where $\Delta R = 0.4$ is the angular distance cut, the parameter $\gamma_c$ and $\delta m$ reflect the energy resolution effect and the invariant mass cut effect, $c$ is a normalization constant.

For massive final state particle, we have

\[
\Delta R = \arccos \left[ \frac{(1 - z^2 + r^2) \beta_T^2 - 1}{\sqrt{((1 - z^2 + r^2) \beta_T^2 - 1)^2 + 4 (1 - z^2) (1 + 4r^2) \beta_T^2}} \right]. \tag{51}
\]

The moving direction of a massive particle can be flipped by a Lorentz boost. For a very large $\beta_T > 1/r$, $z \to \pm 1$, we have $\Delta R = 0$. In this case, the region allowed by the cut is $[z_- - z_, 1] \cup [z_+, z_+ + 2]$ and could be $\emptyset$. However, because $m_b \ll m_h$, this will be only a tiny correction at very high $m_{hh}$ region ($p_{T,\text{cut}} > 3.3 \text{ TeV}$) and could be neglected.

The behavior of the cut efficiency function in the low invariant mass region can be understood as follows. The $p_T$ cuts on the Higgs bosons require the Higgs bosons have a large $p_T$, which means that the energy of the Higgs bosons
must be larger than $\sqrt{m_H^2 + p_{T,cut}^2} \approx 160$ GeV. This is the reason why the events with $m_{hh} < 320$ GeV have a very tiny (close to 0) cut acceptance. It is easy to know that the integration region of the polar angle in the c.m. frame is

\[
-\sqrt{1 - \frac{4(p_{T,cut}^h)^2}{m_{hh}^2 - 4m_h^2}}, \quad \sqrt{1 - \frac{4(p_{T,cut}^h)^2}{m_{hh}^2 - 4m_h^2}}.
\]

(52)

This is a hint that we could fit the low invariant mass region with

\[
A(m_{hh}) = a_1 \left[ 1 - \frac{4(p_{T,cut}^h)^2}{m_{hh}^2 - 4(m_h - \delta m_2)^2} \right]^{\beta_a} \left( \frac{2m_h}{m_{hh}} \right)^{\beta_h} \left[ 1 + a_2 \left( \frac{2m_h}{m_{hh}} \right) \log \left( \frac{2m_h}{m_{hh}} \right) \right],
\]

where $p_{T,cut}^h = 100$ GeV.

Finally, we obtain the analytic function $A(m_{hh}, s, \mu_F)$ at the 100 TeV $pp$-collider in the following form

\[
A(m_{hh}) = \begin{cases} 
  c_1 \left[ 1 - \frac{m_{hh}^2 (1 - \cos \Delta R_0) - 8 (m_h - \delta m_1)^2}{(1 - \cos \Delta R_0) \left( m_{hh}^2 - 4 (m_h - \delta m_1)^2 \right)} \right]^{\gamma_c}, & m_{hh} > M_{hh}^{(t)}, \\
  c_2 \left[ 1 - \frac{4(p_{T,cut}^h)^2}{m_{hh}^2 - 4(m_h - \delta m_2)^2} \right]^{\beta_a} \left( \frac{2m_h}{m_{hh}} \right)^{\beta_b} \left[ 1 + \beta_c \left( \frac{2m_h}{m_{hh}} \right) \log \left( \frac{2m_h}{m_{hh}} \right) \right], & 319.9 \text{ GeV} < m_{hh} < M_{hh}^{(t)}, \\
  0, & m_{hh} < 319.9 \text{ GeV},
\end{cases}
\]

(54)

where the fitting parameters $\delta m_1 = \delta m_2 = 0.15$ GeV, $\Delta R_0 = 0.4$, $c_1 = 40.30$, $\gamma_c = 0.938$, $c_2 = 8.269$, $\beta_a = 1.241$, $\beta_b = -0.565$, $\beta_c = -2.057$, and $M_{hh}^{(t)} = 1277.5$ GeV, in the low detector performance scenario.

For completeness, we also show the cut efficiency function at the HL-LHC below. The selection cuts used by the ATLAS Collaboration \[58\] are

\[
\begin{align*}
p_T^b &> 40 \text{ GeV}, \quad p_T^b > 25 \text{ GeV}, \quad |\eta^b| < 2.5, \\
p_T^\gamma &> 30 \text{ GeV}, \quad |\eta^\gamma| < 1.37 \text{ or } 1.52 < |\eta^\gamma| < 2.37, \\
\Delta R_0 &< \Delta R_{bb,\gamma\gamma} < 2.0, \quad \Delta R_{bb} > \Delta R_0, \quad \Delta R_0 = 0.4, \\
100 \text{ GeV} &< m_{bb} < 150 \text{ GeV}, \quad p_T^b > 110 \text{ GeV}, \\
123 \text{ GeV} &< m_{\gamma\gamma} < 128 \text{ GeV}, \quad p_T^\gamma > 110 \text{ GeV},
\end{align*}
\]

(55)

To mimic the detector effects, the final state parton momenta are smeared by a Gaussian distribution. The $b$-tagging efficiency is \[57, 82\]

\[
\epsilon_b (p_T, \eta) = 0.135 \tanh \left( \frac{p_T + 50}{75} \right) \tanh \left( \frac{450}{p_T + 80} \right) \times \left[ 3 + e^{-\left( |\eta| - \sqrt{\frac{p_T}{1000}} \right)^2 / 1.6} \right] e^{-|\eta|^2 p_T/1000}.
\]

(56)

and the photon energy resolution and identification efficiency are \[83\]

\[
\begin{align*}
\sigma (\text{GeV}) &= 0.3 \oplus 0.10 \times \sqrt{E(\text{GeV})} \oplus 0.010 \times E(\text{GeV}), \quad \text{for } |\eta| < 1.37, \\
\sigma (\text{GeV}) &= 0.3 \oplus 0.15 \times \sqrt{E(\text{GeV})} \oplus 0.015 \times E(\text{GeV}), \quad \text{for } 1.52 < |\eta| < 2.37,
\end{align*}
\]

and

\[
\epsilon_\gamma (p_T) = 0.76 - 1.98 \exp \left( -\frac{p_T}{16.1 \text{GeV}} \right),
\]

(58)

respectively.

After fitting the Monte Carlos simulation results with all the detector effects, we obtain the following cut efficiency...
function with $p_T^{\text{cut}} = 110$ GeV, which is slightly different from the function of the 100 TeV machine,

$\mathcal{A}(M_{hh}) = \begin{cases} 
  c_1 & \frac{1 - \frac{M_{hh}^2 (1 - \cos \Delta R_0) - 8 (m_H - \delta m_1)^2}{(1 - \cos \Delta R_0) \left( M_{hh}^2 - 4 (m_H - \delta m_1)^2 \right)}}{1 - \frac{4(p_T^{\text{cut}})^2}{M_{hh}^2 - 4 (m_H - \delta m_2)^2}} \beta_a \left( \frac{M_{hh}}{\sqrt{s}} \right) \beta_b \left[ 1 + \beta_c \left( \frac{M_{hh}}{\sqrt{s}} \right) \log \left( \frac{2M_{hh}}{\sqrt{s}} \right) \right], & M_{hh} > M_{hh}^{(t)}, \\
  c_2 & 0, & M_{hh} < 329.3 \text{ GeV}.
\end{cases}$

The fitting parameters are $c_1 = 1.1378$, $c_2 = 11.02$, $\delta m_1 = 50$ GeV, $\gamma_c = 1.675$, $\delta m_2 = 2.5$ GeV, $\beta_a = 1.13$, $\beta_b = 1.48$, $\beta_c = 4.88$, $\Delta R_0 = 0.4$ and $M_{hh}^{(t)} = 1260$ GeV [57].

B. The $m_{hh}$ distribution

Once knowing the cut efficiency function $\mathcal{A}(m_{hh})$, one can calculate numbers of events of Higgs boson pair production after a series of kinematic cuts listed in Eq. (43) or Eq. (55) using the master formula shown in Eq. (41). That requires knowledge of the inclusive $m_{hh}$ distribution before and after imposing experimental cuts.

Figure 6 displays the $m_{hh}$ distributions in the double Higgs production with CP-violating $htt$ and $h(h)gg$ couplings before and after the selection cuts at the 14 TeV LHC and at a future 100 TeV $pp$-collider, respectively. We derive the $m_{hh}$ distribution after cuts by convoluting the inclusive distribution with the cut efficiency function as stated in Eq. (42). Two combinations of Higgs effective couplings, $(c_t, \tilde{c}_t)$ and $(c_g, \tilde{c}_g)$, are considered. We fix all the other effective couplings as the SM values while varying the two effective couplings in each combination. We choose a few benchmark couplings listed as follows:

$$(c_t, \tilde{c}_t) = (1, 0), (0.8, 0.3), (0.5, 0.5), (0.2, 0.6),$$

$$(c_g, \tilde{c}_g) = (0, 0), (-2.0, 0), (-1, 1), (-0.3, 0.6),$$

which are well consistent with the measurements of single Higgs production at the LHC Run-I.

For the case of $(c_t, \tilde{c}_t)$, the invariant mass distribution of Higgs boson pairs peaks around 400 GeV in the SM, i.e. $(c_t, \tilde{c}_t) = (1, 0)$; see the black-solid curves in Figs. 6(a) and (b). Other values of $c_t$ and $\tilde{c}_t$ shift the peak to small $m_{hh}$ regions both at the 14 TeV and at the 100 TeV. It can be understood as follows. In the SM, a large cancellation between $F_\Box$ and $F_\Delta \times 3 m_h^2 / (\hat{s} - m_h^2)$ occurs near the threshold $m_{hh} \sim 2 m_h \approx 250$ GeV [37, 38]. However, the cancellation is spoiled when the $c_t$ coupling deviates sizably from the SM value $c_t = 1$. That shifts the peak position.

In addition, the contribution from $\tilde{c}_t (c_t F_\Box + F_\Delta (2) \times 3 m_h^2 / (\hat{s} - m_h^2))$ increases dramatically with $\tilde{c}_t$. Therefore, a large $\tilde{c}_t$, e.g. $(c_t, \tilde{c}_t) = (0.2, 0.6)$, distorts the smooth $m_{hh}$ distribution; see the blue curves in Figs. 6(a) and (b). We notice that the $m_{hh}$ distributions do not change much when increasing the collider energy from 14 TeV to 100 TeV. The $m_{hh}$ distributions in the small $m_{hh}$ region is sensitive to $c_t$ and $\tilde{c}_t$ before imposing any cuts. Different choices of $c_t$ and $\tilde{c}_t$ couplings yield distinct distributions. Unfortunately, the differences in low $m_{hh}$ region are washed out once imposing a hard $p_T$ cut on the Higgs boson in order to disentangle the signal out of huge SM background at the 14 TeV LHC and the 100 TeV $pp$-collider. Figures 6(c) and (f) show the $m_{hh}$ distributions after the selection cuts given in Eq. (55). After cuts all the curves are quite similar. If NP models only modify the $c_t$ and $\tilde{c}_t$ coupling, then it is difficult to discriminate the NP models through the $m_{hh}$ distributions.

We also show the $m_{hh}$ distributions for various combinations of $(c_g, \tilde{c}_g)$ in Fig. 6. The $c_g$ and $\tilde{c}_g$ couplings introduce a momentum dependence to the double Higgs production, and they are expected to play an important role in large $m_{hh}$ region. In the small $m_{hh}$ region, the invariant mass distributions are distorted at the 14 TeV and 100 TeV colliders, owing to the weaker cancellation when $c_g < 0$. In the high $m_{hh}$ regions, say $m_{hh} > 400$ GeV, the distributions are distinctly different, especially at the 100 TeV collider. See Figs. 6(c) and (d). It is because, unlike the $F$ form factors, the contributions from $h(h)gg$ interaction, which are proportional to $c_g$ and $\tilde{c}_g$, do not decrease in the large $m_{hh}$ region. More importantly, the differences of the $m_{hh}$ distributions remain even after imposing the selection cuts; see Figs. 6(g) and (h). As a consequence, it is possible to discriminate different NP models that modify $c_g$ and $\tilde{c}_g$ through the $m_{hh}$ distributions.

Next, we will discuss the correlation and sensitivity of the Higgs effective couplings in the scattering of $gg \rightarrow hh \rightarrow bb\gamma\gamma$ at the 14 TeV LHC and the 100 TeV $pp$-collider.
FIG. 6: $m_{hh}$ distributions in the double Higgs production with CP-violating $h(tt)$ and $h(h)gg$ couplings at the 14 TeV LHC and the 100 TeV pp-collider before (top row) and after the kinematic cuts (bottom row). The black (solid), green (dashed), red (dotdashed) and blue (dotted) lines in the upper panel correspond to $(c_t, c_h) = (1, 0), (0.8, 0.3), (0.5, 0.5), (0.2, 0.6)$, respectively. While the black (solid), green blue (dashed), red (dotdashed) and blue (dotted) lines in the lower panel correspond to $(c_b, c_h) = (0, 0), (-2, 0), (-1, 1), (-0.3, 0.6)$, respectively.

C. Signal strength and Higgs effective couplings

With the help of the cut efficiency function, we can easily obtain the total cross section of any NP described by the Higgs effective couplings after the selection cuts. Making use of the narrow width of the Higgs boson, the signal strength of the cross section $\sigma(pp \rightarrow hh \rightarrow b\bar{b}\gamma\gamma)$ of any $\mathrm{SM}$ can be defined as follows

$$\frac{\sigma(pp \rightarrow hh \rightarrow b\bar{b}\gamma\gamma)}{\sigma_{\mathrm{SM}}(pp \rightarrow hh \rightarrow b\bar{b}\gamma\gamma)} = \frac{\sigma_{hh}(pp \rightarrow hh)}{\sigma_{hh}^{\mathrm{SM}}(pp \rightarrow hh)} \times \frac{\mathrm{Br}(h \rightarrow b\bar{b})}{\mathrm{Br}(h \rightarrow b\bar{b})_{\mathrm{SM}}} \times \frac{\mathrm{Br}(h \rightarrow \gamma\gamma)}{\mathrm{Br}(h \rightarrow \gamma\gamma)_{\mathrm{SM}}} \equiv \mu_{hh} \times \mu_{b\bar{b}} \times \mu_{\gamma\gamma},$$

(61)

where $\mu_{hh,b\bar{b},\gamma\gamma}$ denote the signal strength of the cross section of double Higgs production, of the branching ratio of $h \rightarrow b\bar{b}$ decay, of the branching ratio of $h \rightarrow \gamma\gamma$ decay, defined as follows:

$$\mu_{hh} = \frac{\sigma_{hh}}{\sigma_{hh}^{\mathrm{SM}}}, \quad \mu_{b\bar{b}} = \frac{\mathrm{Br}(h \rightarrow b\bar{b})}{\mathrm{Br}(h \rightarrow b\bar{b})_{\mathrm{SM}}}, \quad \mu_{\gamma\gamma} = \frac{\mathrm{Br}(h \rightarrow \gamma\gamma)}{\mathrm{Br}(h \rightarrow \gamma\gamma)_{\mathrm{SM}}}.$$

(62)

The dependence of $\mu_{hh}$ on the effective couplings is

$$\mu_{hh} = A_1 c^2_{3h} c^2_t + A_2 c^2_{3h} c_t c_t + A_3 c^2_{3h} c^2_t + A_4 c_{3h} c^2_t c_t + A_5 c_{3h} c_t c^2_t + A_6 c_{3h} c_t c_t + A_7 c_{3h} c^2_t c_t + A_8 c_{3h} c_t c^2_t + A_9 c_{3h} c_t c_t + A_10 c_t^2 + A_{11} c_t^2 + A_{12} c_t^2 + A_{13} c_t^4 + A_{14} c_t^2 + A_{15} c_t^4 + A_{16} c^2_{3h} c^2_t + A_{17} c^2_{3h} c_t c_t + A_{18} c^2_{3h} c_t c_t + A_{19} c_{3h} c_t c_t c_t + A_{20} c_{3h} c_t c_t c_t + A_{21} c_{3h} c_t c_t c_t + A_{22} c^2_t + A_{23} c_t c_t c_t + A_{24} c^2_t + A_{25} c_t c_t c_t + A_{26} c_t c_t c_t + A_{27} c_t c_t c_t + A_{28} c_t^2 + A_{29} c_t^2 + A_{30} c_t c_t c_t + A_{31} c_t c_t c_t + A_{32} c_t c_t c_t + A_{33} c_t^2 + A_{34} c_t^2.$$

(63)

The product of $\mu_{b\bar{b}}$ and $\mu_{\gamma\gamma}$ is

$$\mu_{b\bar{b}} \times \mu_{\gamma\gamma} = \frac{\Gamma(h \rightarrow \gamma\gamma)}{\Gamma(h \rightarrow \gamma\gamma)_{\mathrm{SM}}} \left(\frac{\Gamma_{\mathrm{SM}}^{\mathrm{tot}}}{\Gamma_{\mathrm{tot}}}\right)^2 = \frac{\kappa^2_{\gamma}}{[1 + (\kappa^2_{\gamma} - 1)\mathrm{BR}^{\mathrm{SM}}_{\gamma} + (\kappa^2_{\gamma} - 1)\mathrm{BR}^{\mathrm{SM}}_{\gamma}]}$$

(64)

where we assume the Yukawa coupling of bottom quarks is not altered by NP effects. The $\kappa_{g}$ and $\kappa_{\gamma}$ couplings are defined in Eqs. [17] and [18]. The SM branching ratios are $\mathrm{BR}^{\mathrm{SM}}_{g} \equiv \mathrm{BR}(h \rightarrow gg)_{\mathrm{SM}} = 8.187\%$ and $\mathrm{BR}^{\mathrm{SM}}_{\gamma} \equiv \mathrm{BR}(h \rightarrow \gamma\gamma)_{\mathrm{SM}} = 0.227\%$. The SM branching ratios are $\mathrm{BR}^{\mathrm{SM}}_{g} = \mathrm{BR}(h \rightarrow gg)_{\mathrm{SM}} = 8.187\%$ and $\mathrm{BR}^{\mathrm{SM}}_{\gamma} = \mathrm{BR}(h \rightarrow \gamma\gamma)_{\mathrm{SM}} = 0.227\%$. 

[8]

[8]
TABLE III: The cross sections of $gg \rightarrow hh \rightarrow b\bar{b}\gamma\gamma$ in terms of the Higgs effective couplings at the 14 TeV LHC and 100 TeV $pp$-collider before (top panel) and after (bottom panel) the selection cuts.

| $\sqrt{s}$ | A1 | A2 | A3 | A4 | A5 | A6 | A7 | A8 | A9 | A10 | A11 | A12 |
|-----------|----|----|----|----|----|----|----|----|----|-----|-----|-----|
| 14 TeV    | 0.138 | 0.370 | 0.276 | 0.640 | -0.766 | 0.821 | 0.535 | -1.35 | -6.22 | 1.37 | -1.82 | 1.58 |
| 100 TeV   | 0.101 | 0.267 | 0.208 | 0.592 | -0.569 | 0.658 | 0.425 | -1.11 | -4.79 | 3.32 | -1.30 | 1.67 |
| $\sqrt{s}$ | A13 | A14 | A15 | A16 | A17 | A18 | A19 | A20 | A21 | A22 | A23 | A24 |
| 14 TeV    | 2.07 | 13.9 | 0.719 | 0.138 | -0.611 | 0.861 | 0.640 | 2.13 | -1.24 | 1.37 | 4.64 | 2.55 |
| 100 TeV   | 1.90 | 11.3 | 0.680 | 0.101 | -0.428 | 0.634 | 0.592 | 1.53 | -0.928 | 3.32 | 3.51 | 2.90 |
| $\sqrt{s}$ | A25 | A26 | A27 | A28 | A29 | A30 | A31 | A32 | A33 | A34 |
| 14 TeV    | 0.821 | 1.39 | 2.44 | -4.12 | 2.30 | -18.8 | 4.04 | -1.24 | 6.19 | -3.02 |
| 100 TeV   | 0.658 | 1.21 | 2.06 | -4.13 | 2.16 | -16.3 | 3.28 | -0.928 | 6.10 | -2.08 |

The values of the coefficients $A_i$’s are listed in Table III at the 14 TeV LHC and the 100 TeV $pp$-collider, before imposing any cuts (top panel) and after the series of cuts defined in Eq. 43 (bottom panel). The values of those coefficients at the 14 TeV LHC without any cut agree exactly with those values given in Ref. [23]. We notice that the imposition of any cuts (top panel) and after the series of cuts defined in Eq. 43 (bottom panel). The values of those coefficients are larger at the 100 TeV $pp$-collider than at the 14 TeV LHC. Those coefficients correspond to the couplings of $c_g^2$, $c_g c_t^2$, $c_g^2$ and $c_t^2$, which modify the $h(h)gg$ and $hhtt$ interactions and contribute significantly to the double Higgs production at the large $m_{hh}$ region.

Equipped with the inclusive $m_{hh}$ distributions and cut efficiency function, we are ready to explore the sensitivity of the HL-LHC and 100 TeV $pp$-collider on the Higgs effective couplings. The expected discovery significance and the exclusion limit can be evaluated with [80]

$$Z_0 = \sqrt{2 \left[ (n_s + n_b) \log \frac{n_s}{n_b} - n_s \right]},$$

$$Z_\mu = \sqrt{-2 \left( n_b \log \frac{n_s + n_b}{n_s} - n_s \right)},$$

respectively, where $n_s$ and $n_b$ denote the numbers of the signal and background events. The signal and background events in the SM at the 14 TeV HL-LHC with an integrated luminosity $\mathcal{L} = 3000$ fb$^{-1}$ [58] and the 100 TeV $pp$-collider with $\mathcal{L} = 30$ ab$^{-1}$ [59] are

14 TeV : $n_s = 8.4$, $n_b = 47$,
100 TeV : $n_s = 12061$, $n_b = 27118$.

D. Sensitivity to Higgs effective couplings at the HL-LHC

Figures 11 and 11 show the 2σ exclusion (red curves) and 5σ discovery (purple curves) contours for the double Higgs production at the 14 TeV LHC with an integrated luminosity $\mathcal{L} = 3000$ fb$^{-1}$, named as high luminosity LHC (HL-LHC). Throughout this study we vary only two effective couplings at a time. The blue regions denote the parameter space that is allowed by the current single Higgs measurements. The pair production of the SM Higgs bosons is expected to be observed at the HL-LHC at only 1.3σ confidence level [58]. Even though it is less promising
FIG. 7: Contours of 2σ exclusion (red curves) and 5σ discovery (purple curves) for different combinations of Higgs effective couplings at the HL-LHC. The blue regions represent the 95% CL constraints at the 7 and 8 TeV LHC.

to detect the double Higgs event, one can set an 2σ exclusion limit on the NP. On the other hand, if this process is discovered at the 5σ confidence level, it is a clear evidence of NP. We also show the 5σ discovery contours below.

In general, the shapes of the 2σ and 5σ boundary are similar. The large distortion occurs around the corners of the correlation contour of \(c_g\) and \(\tilde{c}_g\). The large \(c_g\) and \(\tilde{c}_g\) couplings could increase the total width of Higgs boson sizably\(^\S\); see Eq. (64). The enlarged width inevitably reduces the branching ratio of Higgs boson decaying into a pair of bottom quarks or photons and then reduces the discovery potential of Higgs pair events, especially in the region of \(c_g \gtrsim 2\) or \(|\tilde{c}_g| \gtrsim 2\). In order to compensate the reduction of branching ratio, the double Higgs production rate has to be dramatically enhanced to reach a 5σ discovery.

Figure 7 shows the sensitivity of the HL-LHC to a few combinations of effective couplings that can affect the single Higgs signal strength simultaneously. In the scenario \((c_t, c_g)\), one can use the double Higgs production to exclude the degenerate parameter space in the lower band allowed by the single Higgs measurements \(^5\), but only a portion of the upper band consisting of the SM is excluded; see the red curve. In the scenarios of \((c_g, \tilde{c}_g)\), \((\tilde{c}_t, c_g)\) and \((c_g, \tilde{c}_t)\), the parameter space away from the SM can be excluded; see Figs. 7(b), (d) and (e). That is mainly owing to the different correlations of effective couplings in single Higgs productions and double Higgs productions. For example, consider \((\tilde{c}_g, \tilde{c}_t)\). The double Higgs production rate is proportional to \((\tilde{c}_t + \frac{4}{3} \tilde{c}_g)^2\) while the single Higgs production rate proportional to \((-\tilde{c}_t + \frac{2}{3} \tilde{c}_g)^2\); see Eqs. (16) and (7). That yields the different slopes of the blue band and red (purple) curves. Unfortunately, the double Higgs process has less sensitivity to the parameter space in the scenarios of \((\tilde{c}_t, c_t)\) and \((c_t, \tilde{c}_g)\); see Figs. 7(c) and (f).

Not all the effective couplings affect the single Higgs production and Higgs boson decay. We separate the effective couplings into two categories: couplings sensitive to single Higgs production, say \(c_{3h}\) and \(\tilde{c}_{3h}\), and others effective couplings. Figure 8 shows the correlation among \(c_{3h} \tilde{c}_{3h}\) and others effective couplings. Plots in the first row in Fig. 8 show the correlations between \(c_{3h}\) and \(c_t, \tilde{c}_t, c_g, \tilde{c}_g\), respectively. The sign of \(c_{3h}\) coupling is important as it could alter the cancellation between the triangle diagram and the box diagram in the SM. A negative \(c_{3h}\) leads to an enhancement of the double Higgs production, easily yielding a 5σ discovery. On the other hand, the 2σ exclusion limit demands the \(c_{3h}\) being not too negatively large when \(c_t = 1\); see Figs. 8(b), (c) and (d). The tension is slightly alleviated in \((c_{3h}, c_t)\); it requires \(c_{3h} > -2\) if the double Higgs event is not observed at the HL-LHC; see Fig. 8(a). There is no stringent bound on \(c_{3h}\) from top, indicating that the double Higgs production is not sensitive to the quartic term in

\(^\S\) The current bound on the Higgs boson total width is about \(\Gamma_{tot} \lesssim 6 \Gamma_{SM}^{tot}\) \(^5\) \(^8\), which is still too weak to constrain the Higgs effective couplings.
the Higgs potential if the coefficient is positive. It has been pointed out in the comprehensive study in Ref. [23] which considers the CP-conserving operators. Our study shows the conclusion also holds for a CP-violating model.

Plots in the second (third) row of Fig. 8 show the correlations between $c_{2t}$, $\tilde{c}_{2t}$, $c_g$, $\tilde{c}_g$, respectively. If the NP model generates a sizable $c_{2t}$, then it is very promising to see its effects in the Higgs boson pair productions in both CP-conserving and CP-violating models; see the purple curves. Similar to the case of $c_{3h}$, the cancellation between $\Delta F$ and $\Box F$ also imposes a bound on $c_{2t}$ from bottom. Unlike the $c_{3h}$, the $c_t$ coupling is also bounded from top. If no deviation is observed in the double Higgs production, then one can impose a bound on $c_{2t}$ ($\tilde{c}_{2t}$), together with constraints obtained from the single Higgs production, as follows:

$$
(c_t, c_{2t}) : -0.469 \lesssim c_{2t} \lesssim 1.69,
$$

$$
(\tilde{c}_t, \tilde{c}_{2t}) : -0.339 \lesssim \tilde{c}_{2t} \lesssim 1.34,
$$

$$
(c_g, c_{2t}) : -0.429 \lesssim c_{2t} \lesssim 1.40,
$$

$$
(\tilde{c}_g, \tilde{c}_{2t}) : -0.339 \lesssim \tilde{c}_{2t} \lesssim 1.34,
$$

$$
(\tilde{c}_g, c_{2t}) : -0.339 \lesssim c_{2t} \lesssim 1.34,
$$

$$
(\tilde{c}_g, \tilde{c}_{2t}) : -0.519 \lesssim \tilde{c}_{2t} \lesssim 0.516.
$$

It is worth mentioning that the degenerate parameter spaces in $c_g$, i.e. the two blue bands in Figs. 8(c), (g) and (k), can be fully resolved at the HL-LHC.

Figure 9 shows the correlations among effective couplings ($c_{2t}$, $\tilde{c}_{2t}$ and $c_{3h}$) that do not affect the single Higgs production. The three couplings are completely free. They are constrained only by double Higgs production at the HL-LHC. If the NP effects are hidden in the three couplings, then one is not able to probe the NP effects no matter how accurately one measures the single Higgs boson production. The double Higgs production is sensitive to both magnitude and sign of the $c_{3h}$ coupling. If $c_{3h}$ is the only non-zero effective coupling, then null results of Higgs pair searches will require $c_{3h} > -1$. Including $c_{2t}$ completely relax the constraint on $c_{3h}$; see Fig. 9(a). It is owing to the
interference between $c_{3h}F_{\Delta}$ and $c_{2t}F_{\Delta}$ terms in Eq. \[(16).\] As a result, a large negative $c_{3h}$ is still allowed. The $\tilde{c}_{2t}$ coupling, which does not interfere with $c_{3h}$, has no strong impact on $c_{3h}$. The $c_{2t}$ and $\tilde{c}_{2t}$ do not interfere and result in the symmetric ellipse bound.

Finally, we list analytical expressions of all the $2\sigma$ exclusion limits below:

\[(c_t, c_g) : 2.25c_g^2 + c_g(0.508 - 1.15c_t)c_t + 1.64c_t^4 - 0.739c_t^3 + 0.0993c_t^2 - 1.00 < 1.28 ,
\]

\[(c_g, \tilde{c}_t) : 2.25c_g^2 - 0.645c_g + 2.25\tilde{c}_t^2 < 1.28 ,
\]

\[(c_t, \tilde{c}_t) : 1.64c_t^4 - 0.739c_t^3 + c_g^2(7.18\tilde{c}_t^2 + 0.0993) - 2.46c_t\tilde{c}_t^2 + 0.571\tilde{c}_t^4 + 0.257\tilde{c}_t^2 - 1.00 < 1.28 ,
\]

\[(c_t, \tilde{c}_g) : 2.25\tilde{c}_g^2 + 1.72\tilde{c}_g\tilde{c}_t + 0.571\tilde{c}_t^4 + 4.97\tilde{c}_t^2 < 1.28 ,
\]

\[(c_g, \tilde{c}_t) : 2.25c_g^2 + c_g(1.65c_t^2 - 0.645) + 0.571\tilde{c}_t^4 + 4.97\tilde{c}_t^2 < 1.28 ,
\]

\[(c_t, \tilde{c}_g) : 2.25c_g^2 + 1.64c_g^4 - 0.739c_g^3 + 0.0993c_g^2 - 1.00 < 1.28 ,
\]

\[(c_t, c_{3h}) : 0.0993c_{3h}c_g^2 - 0.739c_{3h}c_t^3 + 1.64c_t^4 - 1.00 < 1.28 ,
\]

\[(c_t, c_{3h}) : c_{3h}^2(0.257\tilde{c}_t^2 + 0.0993) + c_{3h}(-2.46\tilde{c}_t^2 - 0.739) + 0.571\tilde{c}_t^4 + 7.18\tilde{c}_t^2 + 0.639 < 1.28 ,
\]

\[(c_g, c_{3h}) : c_{3h}^2(0.0369c_g^2 + 0.0975c_g + 0.0993) + c_{3h}(0.406c_g^2 + 0.146c_g - 0.739) + 1.80c_g^2 - 0.888c_g + 0.639 < 1.28 ,
\]

\[(\tilde{c}_g, c_{3h}) : c_{3h}^2(0.0369\tilde{c}_g^2 + 0.0993) + c_{3h}(0.406\tilde{c}_g^2 - 0.739) + 1.80\tilde{c}_g^2 + 0.639 < 1.28 ,
\]

\[(c_t, c_{2t}) : 2.85c_{2t}^2 + c_{2t}(0.920 - 3.79c_t)c_t + 1.64c_t^4 - 0.739c_t^3 + 0.0993c_t^2 - 1.00 < 1.28 ,
\]

\[(c_{2t}, c_{2t}) : 2.85c_{2t}^2 + c_{2t}(1.91c_t^2 - 2.87) + 0.571c_t^4 + 4.97\tilde{c}_t^2 < 1.28 ,
\]

\[(c_g, c_{2t}) : 2.85c_{2t}^2 + c_{2t}(2.52c_g - 2.87) + 2.25c_g^2 - 0.645c_g < 1.28 ,
\]

\[(\tilde{c}_t, c_{2t}) : 2.85\tilde{c}_t^2 - 2.87c_{2t} + 2.25\tilde{c}_t^2 < 1.28 ,
\]

\[(c_t, c_{2t}) : 2.85c_{2t}^2 + 1.64c_t^4 - 0.739c_t^3 + 0.0993c_t^2 - 1.00 < 1.28 ,
\]

\[(c_t, c_{2t}) : 5.28c_{2t}^2 - 10.2c_{2t}\tilde{c}_t + 0.571c_t^4 + 4.97\tilde{c}_t^2 < 1.28 ,
\]

\[(c_g, \tilde{c}_{2t}) : 5.28\tilde{c}_{2t}^2 + 2.25\tilde{c}_g^2 - 0.645c_g < 1.28 ,
\]

\[(c_t, \tilde{c}_{2t}) : 2.85c_{2t}^2 - 2.07c_{2t}\tilde{c}_t + 2.25\tilde{c}_t^2 < 1.28 ,
\]

\[(c_{3h}, c_{2t}) : 2.85c_{2t}^2 + c_{2t}(0.920c_{3h} - 3.79) + 0.0993c_{3h} - 0.739c_{3h} - 0.639 < 1.28 ,
\]

\[(c_{3h}, \tilde{c}_{2t}) : 5.28c_{2t}^2 + 0.0993c_{3h}^2 - 0.739c_{3h} + 0.639 < 1.28 ,
\]

\[(c_{2t}, \tilde{c}_{2t}) : 2.85c_{2t}^2 - 2.87c_{2t} + 5.28c_{2t}^2 < 1.28 .
\]

Effective couplings violating the above inequalities can be excluded at the HL-LHC.

\section*{E. Sensitivity to Higgs effective couplings at a future 100 TeV pp-collider}

Now we study the potential of a future 100 TeV pp-collider on Higgs effective couplings. It is shown that increasing the collider energy improves the sensitivity significantly \cite{38,50}. Our simulation shows that the performance at the 100 TeV machine with an integrated luminosity of 10 fb$^{-1}$ is comparable to that at the HL-LHC. Moreover, the
hereafter. Figures 10-13 display the $5\sigma$ discovery of NP effects as follows: the EDM constraint.

The parameter space outside the EDM bound, then additional CP-violating interaction has to be included to respect production provides an alternative way to check $\tilde{\mu}$ luminosity enables us to discover NP effects in the double Higgs productions through $b\bar{b}\gamma\gamma$ channel.

$gg \rightarrow hh \rightarrow b\bar{b}\gamma\gamma$ process can be discovered with $\mathcal{L} = 256 \text{ fb}^{-1}$ at the 100 TeV $pp$-collider. Accumulating more luminosity enables us to discover NP effects in the double Higgs productions through $b\bar{b}\gamma\gamma$ channel.

As it is guaranteed to observe the Higgs pair signal in the SM at the 100 TeV machine, we focus on the NP searches hereafter. Figures 10-13 display the $5\sigma$ contours of discovering NP with an integrated luminosity of 30 ab$^{-1}$. The SM process $gg \rightarrow hh \rightarrow b\bar{b}\gamma\gamma$ is recognized as a background. The regions with the significance $Z_0 < 5$ are depicted with magenta curves. Outside of those magenta regions, the NP is expected to be observed. Again the constraints from the current single Higgs measurements are denoted in blue regions. We also include the EDM constraints on the CP-odd couplings $\tilde{c}_t$ and $\tilde{c}_g$; see the grey bands. The EDM constraints are very stringent on $\tilde{c}_t$ or $\tilde{c}_g$. The double Higgs production provides an alternative way to check $\tilde{c}_t$ and $\tilde{c}_g$. If the Higgs pair signal in the NP model is discovered in the parameter space outside the EDM bound, then additional CP-violating interaction has to be included to respect the EDM constraint.

We classify those figures into four categories according to the shapes of the boundary of $5\sigma$ discovery region. All the discovery regions in Fig. 10 are in a shape of ellipse; see the magenta curve. The parameter outside those ellipses can be discovered at more than $5\sigma$ confidence level. In the parameter space that is close to the SM, the modification of the decay branching ratios $\mu_{\gamma\gamma}$ and $\mu_{hh}$ can be ignored. We obtain analytic expressions corresponding to the $5\sigma$ discovery of NP effects as follows:

\begin{align*}
(c_g, \tilde{c}_g) & : 3.70c_g^2 - 0.290c_g + 3.70\tilde{c}_g^2 \geq 0.1 , \\
(\tilde{c}_g, \tilde{c}_t) & : 3.70\tilde{c}_g^2 + 1.40\tilde{c}_g\tilde{c}_t + 0.806\tilde{c}_t^2 + 4.54\tilde{c}_t^2 \geq 0.1 , \\
(c_g, \tilde{c}_t) & : 3.70c_g^2 + c_g(1.90c_t^2 - 0.290) + 0.806c_t^4 + 4.54c_t^2 \geq 0.1 , \\
(c_g, \tilde{c}_2t) & : 5.68\tilde{c}_2t^2 + 3.70c_g^2 - 0.290c_g \geq 0.1 , \\
(\tilde{c}_g, \tilde{c}_2t) & : 5.68\tilde{c}_2t^2 - 1.46\tilde{c}_2t\tilde{c}_g + 3.70\tilde{c}_g^2 \geq 0.1 , \\
(\tilde{c}_t, \tilde{c}_2t) & : 5.68\tilde{c}_2t^2 - 9.73\tilde{c}_2t\tilde{c}_t + 0.806\tilde{c}_t^4 + 4.54\tilde{c}_t^2 \geq 0.1 .
\end{align*}

The analytical expressions of one effective coupling can be derived from the above inequalities by setting the other coupling to be zero. The $5\sigma$ curve in $(\tilde{c}_t, \tilde{c}_2t)$ is stretched as a result of the significant interference effect between $\tilde{c}_t F_\Box^{(2)}$ and $\tilde{c}_2t F_\triangle^{(1)}$.

Figure 11 displays the correlation among effective couplings, of which the $5\sigma$ discovery boundary exhibits a ring type shape. Most of parameter space allowed by the single Higgs production can be covered by double Higgs production. The parameter outside the $5\sigma$ band produces more Higgs pair events, while the parameter inside the band reduces...
Couplings violating the above inequalities lead to a discovery of Higgs pair signal in the NP model. Figure 11 displays the $5\sigma$ discovery contours for different combinations of Higgs effective couplings at the 100 TeV $pp$-collider with the integrated luminosity $L = 30$ ab$^{-1}$. The blue regions represent the 95% CL constraints at the 7 and 8 TeV LHC.

Higgs pair events. The bands of discovery potential at a confidence level less than $5\sigma$ are listed as follows:

\[
\begin{align*}
(c_g, c_{2t}) : & \quad -0.1 < 3.33c_g^2 + c_{2t}(2.33c_g - 2.98) + 3.70c_{2t}^2 - 0.290c_g < 0.1, \\
(\tilde{c}_g, c_{2t}) : & \quad -0.1 < 3.33c_g^2 - 2.98c_{2t} + 3.70c_{2t}^2 < 0.1, \\
(\tilde{c}_t, c_{2t}) : & \quad -0.1 < 3.33c_{2t}^2 + 2.98c_{2t} + 3.70c_{2t}^2 < 0.1, \\
(c_t, \tilde{c}_t) : & \quad -0.1 < 5.68c_{2t}^2 + 1.58c_t^2 - 0.671c_t^3 + 0.0880c_t^2 - 1.00 < 0.1, \\
(c_t, c_g) : & \quad -0.1 < 3.70c_g^2 + c_g(0.457 - 0.746c_t)c_t + 1.58c_t^4 - 0.671c_t^3 + 0.0880c_t^2 - 1.00 < 0.1, \\
(c_t, \tilde{c}_g) : & \quad -0.1 < 1.58c_t^4 - 0.671c_t^3 + c_t^2(6.46c_t^2 + 0.0880) - 2.14c_t^2c_t^2 + 0.806c_t^2 + 0.222c_t^2 - 1.00 < 0.1, \\
(c_t, \tilde{c}_g) : & \quad -0.1 < 3.70c_g^2 + 1.58c_t^4 - 0.671c_t^3 + 0.0880c_t^2 - 1.00 < 0.1.
\end{align*}
\]

Couplings violating the above inequalities lead to a discovery of Higgs pair signal in the NP model.

Figure 12 displays the $5\sigma$ contour with a line shape. We notice that, owing to the insensitivity to $c_{3h}$, the $5\sigma$ discovery band in $(c_t, c_{3h})$ and $(c_{2t}, c_{3h})$ appears as a vertical line. The $5\sigma$ band in $(c_t, c_{2t})$ is determined by the

\[
\begin{align*}
(c_t, c_{3h}) : & \quad -0.1 < 1.58c_t^4 - 0.671c_t^3 + c_t^2(6.46c_t^2 + 0.0880) - 2.14c_t^2c_t^2 + 0.806c_t^2 + 0.222c_t^2 - 1.00 < 0.1, \\
(c_t, \tilde{c}_t) : & \quad -0.1 < 3.70c_g^2 + 1.58c_t^4 - 0.671c_t^3 + 0.0880c_t^2 - 1.00 < 0.1.
\end{align*}
\]

FIG. 11: $5\sigma$ discovery contours for different combinations of Higgs effective couplings at the 100 TeV $pp$-collider with the integrated luminosity $L = 30$ ab$^{-1}$. The blue regions represent the 95% CL constraints at the 7 and 8 TeV LHC.

FIG. 12: $5\sigma$ discovery contours for different combinations of Higgs effective couplings at the 100 TeV $pp$-collider with the integrated luminosity $L = 30$ ab$^{-1}$. The blue regions represent the 95% CL constraints at the 7 and 8 TeV LHC.
of Higgs pair searches also exclude a vast amount of parameter spaces. There are two islands in the parameter space of $(c_t, c_{3h})$, $(c_g, c_{3h})$, $(c_g, c_{2t})$ and $(c_t, c_{2t})$, which cannot be resolved by the single Higgs production. The double Higgs production could exclude the island that does not consist of the SM. We also presented the analytical expressions of those effective couplings at the 14 TeV HL-LHC and at the 100 TeV $pp$-collider. We followed the analysis in Refs.\[58, 59\] to derive the cuts and detector effects. Convoluting inclusive distribution of the invariant mass of Higgs pair with the cut efficiency function gives rise to the signal events after experimental cuts. We found that the double Higgs production can be discovered in the process $gg \to hh \to b\bar{b}\gamma\gamma$ at the 100 TeV $pp$-collider with an integrated luminosity of 30 ab$^{-1}$. The blue regions represent the 95\% CL constraints at the 7 and 8 TeV LHC.

\begin{equation}
(c_t, c_{3h}) : -0.1 < c\_{3h}^2(0.22c_t^2 + 0.0880) + c_{3h}(-2.14c_t^2 - 0.671) + 0.806c_t^4 + 6.46c_t^4 + 0.583 < 0.1 , \\
(c_g, c_{3h}) : -0.1 < c\_{3h}^2(0.0347c_g^2 + 0.0846c_g + 0.0880) + c_{3h}(0.465c_g^2 + 0.157c_g - 0.671) + 3.20c_g^2 - 0.531c_g + 0.583 < 0.1 , \\
(c_t, c_{2t}) : -0.1 < c\_{2t}(0.0347c_t^2 + 0.0880) + c_{2t}(0.465c_t^2 + 0.157c_t - 0.671) + 3.20c_t^2 + 0.583 < 0.1 , \\
(c_g, c_{2t}) : -0.1 < c\_{2t}(0.0347c_g^2 + 0.0880) + c_{2t}(0.465c_g^2 + 0.157c_g - 0.671) + 3.20c_g^2 - 0.531c_g + 0.583 < 0.1 .
\end{equation}

Finally, we plot in Fig.\[13\] the 5σ contour with an irregular shape. The bands of discovery potential at a confidence level less than 5σ are

\begin{equation}
(c_t, c_{3h}) : -0.1 < c\_{3h}^2F_{c_t} - 0.671c_{3h}c_t^2 + 1.58c_t^4 - 1.00 < 0.1 , \\
(c_g, c_{3h}) : -0.1 < 3.33c_{2t}^2 + c_{2t}(0.889c_{3h} - 3.87) + 0.0880c_{3h}^2 - 0.671c_{3h} + 0.583 < 0.1 , \\
(c_t, c_{2t}) : -0.1 < 3.33c_{2t}^2 + c_{2t}(0.889 - 3.87c_t)c_t + 1.58c_t^4 - 0.671c_t^4 + 0.0880c_t^2 - 1.00 < 0.1 .
\end{equation}

V. CONCLUSIONS

We considered effective Higgs boson couplings that affect the double Higgs production. For generality we included both CP-even and CP-odd effective couplings. Some of the effective couplings are loosely constrained by the single Higgs measurements at the 7 TeV and the 8 TeV LHC. The correlations of those effective couplings are different in single and double Higgs productions, therefore, one can probe those effective couplings by combining both the single and double Higgs productions. We examined the impact of the effective couplings on double Higgs production at the high luminosity LHC with an integrated luminosity of 3000 fb$^{-1}$ and at a future $pp$-collider operating at an energy of 100 TeV with an integrated luminosity of 30 ab$^{-1}$.

The amplitude of the double Higgs production depends on several form factors. From partial wave analysis, we found that the double Higgs production is still dominated by the s-wave component even at the 100 pp-collider. Making use of the s-wave dominant feature, we propose a universal cut efficiency function $A(m_{hh})$ to mimic the experimental cuts and detector effects. Convoluting inclusive distribution of the invariant mass of Higgs pair with the cut efficiency function gives rise to the signal events after experimental cuts. We followed the analysis in Refs.\[55, 59\] to derive the cut efficiency functions at the 14 TeV LHC and the 100 TeV $pp$-collider. Using the cut efficiency functions, we obtain the differential cross sections of $m_{hh}$ and total cross sections of $gg \to hh \to b\bar{b}\gamma\gamma$ after kinematics cuts. From there we obtained the potential of probing those effective couplings at the 14 TeV HL-LHC and at the 100 TeV $pp$-collider.

We varied two effective couplings at a time and fixed other couplings to be the SM values. With the tremendously high luminosity, the HL-LHC could cover a lot of parameter space, which could yield a 5σ discovery. Negative results of Higgs pair searches also exclude a vast amount of parameter spaces. There are two islands in the parameter space of $(c_t, c_g)$, $(c_g, c_{3h})$, $(c_g, c_{2t})$ and $(c_t, c_{2t})$, which cannot be resolved by the single Higgs production. The double Higgs production could exclude the island that does not consist of the SM. We also presented the analytical expressions of those 2σ exclusion limits in the parameter space.

We found that the double Higgs production can be discovered in the process $gg \to hh \to b\bar{b}\gamma\gamma$ at the 100 TeV $pp$-collider with an integrated luminosity of 256 fb$^{-1}$. We thus focused on searching for Higgs effective couplings at the 100 TeV machine with $L = 30$ ab$^{-1}$ and treat the SM double Higgs production as a background. Thanks to the
large center of mass energy, the 100 TeV $pp$-collider could cover almost entire parameter space of effective couplings, except $c_{3b}$ which is not sensitive to the Higgs pair production. Finally, we listed analytical expressions of the $5\sigma$ discovery bands which, together with the analytical expressions of the $2\sigma$ exclusion limits at the HL-LHC, is useful to probe new physics models.

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Appendix A: The expressions of form factors

In this appendix, we collect the explicit expressions of the form factors in the single and double Higgs productions,

\[
F_{\Box} = \frac{2m_t^2}{\hat{s}} \left\{ m_t^2(8m_h^2 - \hat{s} - 2m_h^2)(D_0^0 + D_0^u) + 4m_t^2(4m_t^2 - m_h^2)D_0^{tu} \right\} + 2m_t^2C_0^t + \frac{2}{\hat{s}}(m_h^2 - 4m_t^2)[(\hat{t} - m_h^2)C_0^t + (\hat{u} - m_h^2)C_0^u],
\]

\[G_{\Box} = \frac{m_t^2}{\hat{s}} \left\{ 2(8m_h^2 - \hat{s} - 2m_h^2)[m_t^2(D_0^0 + D_0^u + D_0^{tu}) - C_{0}^{sm}] - 2[\hat{s}C_0^u + (\hat{t} - m_h^2)C_0^t + (\hat{u} - m_h^2)C_0^u] \right\} + \frac{1}{\hat{s}^2} \left\{ \hat{s}u(8\hat{u}m_t^2 - \hat{u}^2 - m_h^4)D_0^u + \hat{s}t(8\hat{t}m_t^2 - \hat{t}^2 - m_h^4)D_0^t + (8m_h^2 + \hat{s} - m_h^2) \right\} \left\{ \hat{s}(\hat{s} - 2m_h^2)C_0^0 + \hat{s}(\hat{t} - 4m_h^2)C_{0}^{tu} + 2\hat{t}(m_h^2 - \hat{t})C_0^u + 2\hat{u}(m_h^2 - \hat{u})C_0^0 \right\},
\]

\[
F_{(1)} = \frac{2m_t^2}{\hat{s}^2} \left\{ m_h^2(2\hat{t}C_0^0 + 2\hat{u}C_0^u - \hat{t}\hat{u}D_0^{tu}) - 2m_h^4(C_0^0 + C_0^u) + m_h^2D_0^{tu} \right\} + \frac{1}{\hat{s}^2} \left\{ \hat{s}(\hat{s} - 2m_h^2)C_0^0 + \hat{s}(\hat{t} - 4m_h^2)C_{0}^{tu} + 2\hat{t}(m_h^2 - \hat{t})C_0^u + 2\hat{u}(m_h^2 - \hat{u})C_0^0 \right\},
\]

\[
G_{(1)} = \frac{m_t^2}{2\hat{s}} \left\{ \frac{2}{m_h^2 - \hat{t} - \hat{u}} \left\{ -\hat{s}u(2m_h^4 + \hat{t}^2 + \hat{u}^2)C_0^0 + 2(m_h^2 - \hat{t})(m_h^2 + \hat{t}^2)C_0^u + 2(m_h^2 - \hat{u})(m_h^2 + \hat{u}^2)C_0^u \right\} \right\} \left\{ \hat{s}(\hat{s} - 2m_h^2)C_0^0 + \hat{s}(\hat{t} - 4m_h^2)C_{0}^{tu} + 2\hat{t}(m_h^2 - \hat{t})C_0^u + \hat{s}(\hat{t} - 4m_h^2)C_{0}^{tu} \right\} \right\} \left\{ \hat{s}(\hat{s} - 2m_h^2)C_0^0 + \hat{s}(\hat{t} - 4m_h^2)C_{0}^{tu} + 2\hat{t}(m_h^2 - \hat{t})C_0^u + 2\hat{u}(m_h^2 - \hat{u})C_0^0 \right\},
\]

\[
F_{\Delta} = \frac{2m_t^2}{\hat{s}} \left\{ 2 + (4m_t^2 - \hat{s})C_0^0 \right\},
\]

\[
F_{\Delta(t)}^1 = 2m_t^2C_0^0.
\]

In the above we have the conventions \[55\]

\[
C_0 = C_0(0,0,\hat{s},m_h^2,m_t^2), \quad C_0^t = C_0(0,\hat{t},m_h^2,m_t^2,m_t^2), \quad C_0^u = C_0(0,\hat{u},m_h^2,m_t^2,m_t^2), \quad C_{0}^{sm} = C_0(m_h^2,\hat{s},m_t^2,m_t^2,m_t^2), \quad D_0^0 = D_0(m_h^2,0,0,m_h^2,\hat{t},\hat{u},m_h^2,m_t^2,m_t^2), \quad D_0^u = D_0(m_h^2,0,0,\hat{u},\hat{t},\hat{s},m_h^2,m_t^2,m_t^2), \quad D_0^{tu} = D_0(m_h^2,0,0,\hat{u},\hat{t},\hat{u},\hat{t},m_h^2,m_t^2,m_t^2),
\]

and the definitions of the scalar Passarino-Veltman functions are as follows \[80\]

\[
C_0(p_1^2,p_2^2,(p_1 + p_2)^2,m_1^2,m_2^2,m_3^2) = \frac{(2\pi \mu)^{4-D}}{i\pi^2} \int d^D q (q^2 - m_1^2)((q + p_1)^2 - m_2^2)((q + p_1 + p_2)^2 - m_3^2),
\]

\[
D_0(p_1^2,p_2^2,p_3^2,p_4^2,(p_1 + p_2)^2,(p_2 + p_3)^2,m_1^2,m_2^2,m_3^2,m_4^2) = \frac{(2\pi \mu)^{4-D}}{i\pi^2} \int d^D q (q^2 - m_1^2)((q + p_1)^2 - m_2^2)((q + p_1 + p_2)^2 - m_3^2)((q + p_1 + p_2 + p_3)^2 - m_4^2).
\]
where $\mu$ is the renormalization scale and $D$ is the space-time dimension.
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