Premium rate determination of crop insurance product based on rainfall index consideration

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Abstract. Crop insurance product based on rainfall index is an alternative product to minimize the risk in the agricultural sector. Rainfall index is supposed to represent crop failure in a specific area. Rainfall index is composed of parameters, trigger, and exit. The trigger is a benchmark value of rainfall that causes the partial risk and gives the right to the policyholder to begin partial benefit payment. The exit is the benchmark value of rainfall that consists of total crop failure and assigns the right to the policyholder to submit full benefit claim. The trigger and the exit in a rainfall index provide the value of the premium. A case study is carried out with cumulative rainfall data from June to August in the Dramaga area of Bogor from 1984 until 2017 to ensure paddy. The two-component Gamma Mixture distribution model is applied to determine the rate of premium that must be paid by the policyholder on the specified rainfall index.

1. Introduction

The agricultural sector is faced by the unpredict rainfall, which causes rainfall risk, particularly in the rainfed area. The rainfall risk is related to a decrease in productivity of agricultural commodity. Rainfall that decreases sharply for a long time causes drought and rainfall that increases sharply occurs to flood. Both drought and flood decline productivity of commodity and threaten the farmer’s income. The worst effect is the farmer cannot plant commodity in the next cropping season. To minimize the risk of drought, the crop insurance product is created.

Based on [5] and [1], crop insurance product based on rainfall index is an alternative product to assure the risk of partial crop failure or the threat of total crop failure. It ensures rainfall index which is used as a proxy for loss in a specific region (village or regency) rather than upon the individual loss. Rainfall is being underlying of the index because of an objective measurement which exhibits a strong correlation with failure crop. Rainfall index has the trigger and the exit as benchmark value of rainfall to explain some crop failure conditions. In addition, the insurer promises to pay a benefit to the policyholder based on actual rainfall which it is measured over a pre-specified period of time at a particular rainfall station.

To obtain protection, policyholder buys a policy that is called premium. The premium cannot be set by the insurer based on known cost. Premium is predicted by the mainly focussed framework on actuarial control cycle. Premium relies on historical data. The premium rate should cover all claims. The premium rate is determined by the expected benefit claim to cover the expected loss. The aim of this research is to determine the premium rate of crop insurance product based on rainfall index consideration.
2. Methods

2.1. Rainfall modelling

The rainfall modelling is important to choose an approached distribution, which is good to represent the historical rainfall data. The selected distribution is used to predict the probability of loss. The two-component Gamma Mixture distribution model is presented. The method of parameter estimation and the goodness-of-fit test are espoused.

Based on [4], the two-component Gamma Mixture distribution \( (p_1, p_2, \alpha_1, \alpha_2, \lambda_1, \lambda_2) \) is primarily used to model distribution from two populations. \( \alpha_i \) denotes the shape parameter of \( i \)-th component, and \( 1/\lambda_i \) denotes the scale parameter of \( i \)-th component. Cumulative distribution function \( (F_R) \) and probability density function \( (f_R) \) of two-component Gamma Mixture distribution model are as follows

\[
F_R(r) = p_1 \frac{1}{\Gamma(\alpha_1)} \gamma(\alpha_1, \lambda_1 r) + p_2 \frac{1}{\Gamma(\alpha_2)} \gamma(\alpha_2, \lambda_2 r),
\]

\[
f_R(r) = p_1 \frac{\lambda_1^{\alpha_1}}{\Gamma(\alpha_1)} r^{\alpha_1-1} \exp(-\lambda_1 r) + p_2 \frac{\lambda_2^{\alpha_2}}{\Gamma(\alpha_2)} r^{\alpha_2-1} \exp(-\lambda_2 r),
\]

with \( p_1 \) and \( p_2 \) are mixture proportions, \( p_1 + p_2 = 1 \), \( p_1 > 0 \), \( p_2 > 0 \), \( \alpha_1 > 0 \), \( \alpha_2 > 0 \), \( \lambda_1 > 0 \), \( \lambda_2 > 0 \), \( r \geq 0 \), \( \Gamma(\alpha) = \int_0^\alpha \exp(-r) r^{\alpha-1} dr \), and \( \gamma(\alpha, \lambda r) = \int_0^\lambda u^{\alpha-1} \exp(-u) du \).

Log-likelihood function for two-component Gamma Mixture distribution is given by

\[
\ln L(r; \theta) = \ln \prod_{i=1}^n \left( p_1 \frac{\lambda_1^{\alpha_1}}{\Gamma(\alpha_1)} r_i^{\alpha_1-1} \exp(-\lambda_1 r_i) + p_2 \frac{\lambda_2^{\alpha_2}}{\Gamma(\alpha_2)} r_i^{\alpha_2-1} \exp(-\lambda_2 r_i) \right).
\]

From equation 3, it can be easily seen that the results from deriving log-likelihood equation for each parameter and making them equal to zero have no closed form solution. Consequently, gammamixEM() function in mix tools package of R program is used to solve it.

The goodness-of-fit test is used for checking the validity of two-component Gamma Mixture distribution. Empirical Distribution Function (EDF) provides a discrepancy measurement between the empirical and theoretical distribution. The most well-known EDF test is the Kolmogorov-Smirnov test (K-S) and is applied here. Based on [3], Empirical distribution \( F_N(x) \) is given by

\[
F_N(x) = \frac{n_i}{n},
\]

with \( n_i \) is the size of data which every observation is sorted and it is less than the \( i \)-th observation, \( n \) is the sample size. Let \( D^+ \) and \( D^- \) are given by

\[
D^+ = \max_{1 \leq i \leq n} \left\{ \frac{1}{n} - F(x(i)) \right\} \quad \text{and} \quad D^- = \max_{1 \leq i \leq n} \left\{ F(x(i)) - \frac{i-1}{n} \right\},
\]

where \( x(i) \) denotes the \( i \) th ordered statistic of the sample and \( F \) denotes the cumulative distribution of theoretical distribution. Then, the statistic value \( (D) \) is given by \( \max(D^+, D^-) \). Beside it, the critical value for this test is \( 1.36/\sqrt{n} \) with a significant level at five per cent. The \( D \) value, which is less than the critical value statistic, provides the theoretical distribution, is good to represent the empirical distribution.

2.2. Insurance modelling

Based on [5], crop insurance product based on rainfall index, the developed model from [1], ensures rainfall index in a specific region. Rainfall index has the trigger and the exit. The exit is the benchmark value of rainfall that consists of total crop failure and gives the right to the policyholder to submit full benefit claim. The trigger is a benchmark value of rainfall that causes the partial risk and assigns the right to the policyholder to begin partial benefit payment. The exit and the trigger can be calculated by equation (6) and equation (7) as follow

\[
K = F_R(E) = P(R \leq E),
\]

\[
E = F_R^{-1}(0.5).
\]
\[ L = F_R(T) = P(R \leq T), \]  

(7)

with \( F_R \) denotes cumulative distribution function of rainfall data and \( K \) and \( L \) denote constants in per cent, which represent the willingness of the insurer to pay full benefit claim and partial benefit claim, respectively.

In other words, the trigger and the exit establish the range of values which benefit claim can be classified into three conditions by appropriate actual rainfall (expressed in millimeters). Benefit claim is a random variable and is defined by \( X \) (shown in per cent of benefit). Then, the benefit is denoted by \( b \) (expressed in rupiah). Let see figure 1.

Based on [5], three conditions in figure 1 can be modeled and explained as

- **Condition 1** \((x = b)\). The full benefit can be claimed if the actual rainfall is under the exit. This condition is composed when the total crop failure occurs. The probability is \( P(R \leq E) \) if this condition is encountered.

- **Condition 2** \((x = \frac{b(T-r)}{T-E})\). Partial benefit can be claimed if the actual rainfall is under the trigger until the agreed exit is reached. This condition is composed when the agriculture begins to alleviate the productivity of commodity. The probability is \( P(E < r < T) \) if this condition is encountered.

- **Condition 3** \((x = 0)\). If the actual rainfall is upper than the trigger so policyholder cannot claim benefit. The probability is \( P(R \geq T) \) if this condition is encountered.

The premium method can be found in [2]. \( P \) is a premium. So, the formula of the premium is \( P = (1 + \theta)E[X] + S \) where \( \theta \) is premium loading or adding cost, which depends on \( E[X] \) and \( S \) represents an administration fee. Expected benefit claim of crop insurance product based on rainfall index is showed by the formula of equation (8)

\[
E[X] = b \ P^*(R \leq E) + \int_{E}^{T} \left( \frac{b(T-r)}{T-E} \right) f_R(r) dr + 0 \ P^*(R \geq T). \]

(8)

\( E[X] \) represents the average of benefits claims that will be collected by policyholders. \( E[X] \) means rate premium. The research will be focused on determining \( E[X] \) of the product.

**Figure 1.** Benefit claim model.
3. Case study

3.1. Data

The rainfall data were used to represent total accumulated rainfall during the paddy cropping season data which it was recorded in Dramaga station Bogor. The data were collected from January 1984 to December 2017 and were taken from BMKG. Paddy cropping season was assumed from June until August. So, those were 34 observations in the data set.

3.2. The result of parameter estimation and Kolmogorov-Smirnov test

Data set is modeled by two-component Gamma Mixture distribution. The approach is assumed that five parameters of the two-component Gamma Mixture distribution are unknown. From the numerical illustration, MLE is implemented, the under-performance of gammamixEM() produces the estimated five parameters which are reported in table 1.

| No | Distribution                          | Estimated Parameter       | K-S Statistic Value |
|----|---------------------------------------|---------------------------|---------------------|
| 1  | Two-component Gamma Mixture            | \( \hat{\alpha}_1 = 4.85, \hat{\alpha}_2 = 18.44 \) | 0.06                |
|    |                                       | \( \hat{\lambda}_1 = 68.29, \hat{\lambda}_2 = 43.04 \) | (0.24)              |
|    |                                       | \( \hat{p}_1 = 0.27, \hat{p}_2 = 0.73 \) |                     |

Figure 2 and figure 3 indicate that the presence of distribution is good to represent the distribution of rainfall data. To strengthen, it can be observed that the Kolmogorov-Smirnov statistic value is smaller than the critical value (0.24) with a significant level at five per cent. It means that two-component Gamma Mixture distribution is good to be used as a distribution model of rainfall data.

3.3. The result of premium rate determination

To determine the premium rate in this paper, the case study has some assumptions. They are: 1) the risk of loss is only caused by drought, 2) the high rainfall, pest attacks, etc. do not reduce agricultural production in an area, 3) the insurance period provides from June to August. The rainfall distribution is
as same as the past rainfall distribution, 4) the crop insurance based on rainfall index requires the policyholder to pay premium at the pre-period and to claim benefit at the end of the period if rainfall index is fulfilled, and 5) full benefit is as same as production cost, 6 million IDR.

Two-component Gamma Mixture distribution is used to calculate the trigger value, the exit value, and the rate premium. Two cases are done. Afterwards, the results of the two cases can be seen in table 2.

| No | Case | Exit (mm) | Trigger (mm) | Rate Premium (IDR) |
|----|------|-----------|--------------|-------------------|
| 1  | $K = 5\%, L = 50\%$ | 196.75 | 698.13 | 1 452 638 |
| 2  | $K = 4\%, L = 20\%$ | 181.45 | 402.23 | 728 527 |

From table 2, case I explains that the insurer is willing to assure rainfall index in 5% of full benefit claim and 45% of partial benefit ($K = 5\%, L = 50\%$). The case I am illustrated because there are two seasons in Indonesia. Each of which has a 50% chance, so the insurer is willing to assure benefit claim up to 50%. For case II, the insurer is ready to ensure rainfall index in 4% of full benefit claim and 20% of partial benefit ($K = 4\%, L = 20\%$). Case II is illustrated by premium in 12% of the full advantage. Let see in figure 4 and 5. If the exit is changed from 0 to 698.13 mm, then the rate premium does not increase linearly from 0 to 3 million IDR. If the exit is changed from 0 to 402.23 mm, then the rate premium does not increase proportionately from 0 to 1.2 million IDR.

4. Conclusion
Rainfall index has the trigger and the exit which give premium value for this crop insurance product based on rainfall index. The trigger and the exit can be calculated by selected distribution. The two-component Gamma Mixture distribution model is good to represent the rainfall distribution of total accumulated rainfall for the paddy cropping season in Dramaga Bogor. The premium rate corresponds
with the potential risk of loss that is made. Premium will be costly if the potential risk of loss is high. Vice versa, the premium will be cheap if the potential risk of loss is low.

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