Constraints on Torque-Reversing Accretion-Powered X-Ray Pulsars

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ABSTRACT

The observed abrupt torque reversals in X-ray pulsars, 4U 1626–67, GX 1+4, and OAO 1657–415, can be explained by a transition in accretion-flow rotation from Keplerian to sub-Keplerian, which takes place at a critical accretion rate, $\sim 10^{16}$–$10^{17}$ g s$^{-1}$. When a pulsar system spins up near equilibrium spin before the transition, the system goes into spin-down after transition to sub-Keplerian. If a system is well into the spin-up regime, the transition can cause a sharp decrease in spin-up rate but not a sudden spin-down. These observable types of abrupt torque change are distinguished from the smooth torque variation caused by a change of $\dot{M}$ in the Keplerian flow. The observed abrupt torque reversal is expected when the pulsar magnetic field $B_p \sim 5 \times 10^{-11} B^*_{12} r_{1}^{1/2} L_{36}^{1/2} P_{10}^{-1/2}$ G, where the magnetic pitch parameter $b_p$ is of the order of a few, $L_{36}$ is the X-ray luminosity in $10^{36}$ ergs s$^{-1}$, and $P_{10}$ is the pulsar spin period in 10 s. Observed quasi-periodic oscillation (QPO) periods tightly constrain the model. For 4U 1626–67, $M \approx 2.7 \times 10^{16}$ g s$^{-1}$ with $b_p^{1/2} B_p \approx 2 \times 10^{12}$ G. We estimate $M \approx 6 \times 10^{16}$ g s$^{-1}$ and $b_p^{1/2} B_p \approx 5 \times 10^{13}$ G for GX 1+4, and $M \approx 1 \times 10^{17}$ g s$^{-1}$ and $b_p^{1/2} B_p \approx 2 \times 10^{13}$ G for OAO 1657–415. Reliable detection of QPOs before and after torque reversal could directly test the model. We discuss some outstanding uncertainties and difficulties in the present model.

Subject headings: accretion, accretion disks — pulsars: general — stars: magnetic fields — X-rays: stars

1. INTRODUCTION

Sudden torque-reversal events in some accretion-powered X-ray pulsars, such as 4U 1626–67, GX 1+4, and OAO 1657–415, have recently been detected (e.g., Chakrabarty 1996; Chakrabarty et al. 1993, 1997a, 1997b). The spin-up and spin-down rates are puzzlingly similar, despite abrupt torque reversal. The torques remain largely steady before and after reversal, which plausibly indicates the existence of an ordered, stable accretion flow. These systems are distinguished from those showing random torque fluctuations seen in some wind-fed pulsar systems (e.g., Nagase 1989; Anzer & Börner 1995). In the torque-reversing systems, the mass accretion rate $\dot{M}$ appears gradually modulated with a small amplitude on a timescale $\gtrsim$ years, which is much longer than the typical reversal timescale $\gtrsim$ days. The recently reported flux and spectral changes around the time of the reversal in 4U 1626–67 show that $\dot{M}$ seems to change by about a few times 10% in the X-ray-emitting region close to the neutron star (Vaughan & Kitamoto 1998).

Recently, Yi, Wheeler, & Vishniac (1997) have suggested an explanation for the torque-reversal phenomenon. The reversal is triggered by transition of the accretion flow from Keplerian rotation to sub-Keplerian rotation. The transition occurs when $\dot{M}$ crosses the critical rate $\dot{M}_s \sim 10^{16}$–$10^{17}$ g s$^{-1}$. For magnetized pulsar systems, the inner region of accretion flow lies roughly at a radius $R_s \sim 5 \times 10^5 B^*_{12}^{1/2} M_{16}^{-7/2}$ cm, where $M_{16} = M/10^{16}$ g s$^{-1}$ and $B^*_{12}$ is the stellar field strength in units of $10^{12}$ G. This radius is similar to the size of a white dwarf, which strongly suggests that the transition could be similar to that of cataclysmic variables (e.g., Patterson & Raymond 1985; Narayan & Popham 1993).

There exist several outstanding uncertainties in the proposed explanation.

First, the details of the accretion-flow transition process remain unknown. In Yi et al. (1997), it was assumed that the accretion-flow transition occurs abruptly when the critical mass accretion rate is reached. Such an assumption is likely to be valid only when the accretion-flow transition occurs in a spatially limited region from which most of the spin-up/down torque arises (e.g., Wang 1995). The relevant timescale for the transition is likely to be the thermal timescale $t_{th} = (\alpha \Omega_s)^{-1}$ or the viscous-thermal timescale $t_{th} \sim (R/H)_{th}$, where $\alpha$ is the conventional $\alpha$-viscosity coefficient, $H$ is the thickness of the accretion disk at radius $R$ from the star, and $\Omega_s = (GM_*/R^3)^{1/2}$ is the Keplerian rotation rate for the stellar mass $M_*$. The timescales $t_{th} < t_{\alpha} \sim 10^3$ s for $\alpha \approx 0.3$, $R \sim 10^6$ cm, and $M_{16} = 1$ (Frank, King, & Raine 1992). It remains unclear whether the transition on these timescales can be sufficiently abrupt. The case of cataclysmic variables studied by Narayan & Popham (1993) was limited to the steady state. Moreover, the critical accretion rate separating the two accretion-flow states was not determined in sufficient detail to be directly relevant for the present case. For these reasons, it remains to be seen whether an abrupt transition indeed occurs in the pulsar systems.

Second, in the proposed explanation, the accretion-flow rotation needs to be substantially sub-Keplerian in the hot accretion-flow phase. After the transition, the rotation of the accretion flow becomes sub-Keplerian because of large internal pressure (see Narayan & Yi 1995). Significantly sub-Keplerian rotation is realized only when the accretion flow advects most of the accretion energy inward. Since the

An erratum to Yi et al. (1997) was published in ApJ Letters because of errors in the quoted parameters. The corrected parameters include substantially higher magnetic fields and mass accretion rates that are consistent with those in this work.
change of rotation induces an abrupt (≤ 1 day) decrease of torque exerted on the star by the accretion flow, the realization of the sudden change to the advective state is crucial for the success of the model. The self-consistent, magnetized, advection-dominated flows have not been calculated.

Third, Yi et al. (1997) concluded that the mass accretion rate changes little near the accretion-flow transitions in the three pulsar systems they considered. One of them, 4U 1626−67, has been detected with a very significant spectral transition, despite small changes in X-ray luminosities (Vaughan & Kitamoto 1998). It is unclear whether such spectral transitions also occur in other torque-reversing systems. Interestingly, the accretion-flow transitions in the cataclysmic variables show a similar phenomenon (Narayan & Popham 1993). Yi et al. (1997) suggested that the observable spectral transition is due to the appearance of the geometrically thick, hot accretion flow. The details of the spectral transition, however, have not been addressed.

Despite these major uncertainties, we attempt to derive useful information on the observed torque-reversing systems by combining the simple model with the observed quantities. The observed properties of the torque-reversal phenomenon are not likely to occur in all systems undergoing the accretion-flow transition. In fact, it turns out that only certain systems satisfying a tightly constrained set of parameters are able to cause torque reversals as a result of the accretion-flow transition. In this paper, we derive tight constraints on the pulsar system parameters that lead to the observed torque reversal. The estimated parameters are largely consistent with other available estimates. We also derive a simple criterion that identifies candidate pulsar systems by their observable parameters. Combining this with an additional constraint from the quasi-periodic oscillation (QPO), we suggest a possible test of the model. We also discuss different types of torque variation and how they differ observationally from one another. The observed reversal events are likely to occur in pulsar systems near spin equilibrium, with $M \sim M_c \sim$ a few times $10^{16}$ g s$^{-1}$. We take the neutron star moment of inertia $I_* = 10^{45}$ g cm$^2$, radius $R_* = 10^6$ cm, and $M_* = 1.4 M_\odot$. The magnetic field is assumed to be a dipole configuration (Frank et al. 1992). The angular velocity of the star $\Omega_* = 2\pi/P_*$. 

In § 2 we briefly review the torque-reversal mechanism proposed by Yi et al. (1997), derive basic equations, and discuss different types of time-varying torques. In § 3 the equations derived in § 2 are directly applied to the observed pulsar systems, and physical parameters are derived for individual systems. In § 4 general constraints on the torque-reversing pulsar systems are explicitly derived. In § 5 we briefly summarize and discuss some related outstanding issues.

2. ACCRETION-FLOW TRANSITION AND TORQUE REVERSAL

We briefly summarize the main ideas proposed in Yi et al. (1997). For an accretion flow with the Keplerian rotation, the interaction with the stellar magnetic field (e.g., Yi et al. 1997; Wang 1995 and references therein) gives the pitch of the azimuthally stretched magnetic field in a steady state. The basic phenomenological parameters are $\gamma \leq 1$, which measures the vertical velocity shear length scale and the dimensionless ratio $b_p = g/\gamma \sim O(1)$ (e.g., Wang 1995 and reference therein). The accretion flow is magnetically disrupted at $R = R_{in}$, determined by

$$\delta = \left(\frac{R_o}{R_c}\right)^{7/2} \left[ 1 - \left(\frac{R_o}{R_c}\right)^{3/2}\right]^{-1},$$

where the Keplerian corotation radius is given by $R_c = (GM_* P_0^2/4\pi^2)^{1/3}$.

$$\delta = \frac{2(2\pi)^{7/3} R_*^2 (\gamma/\gamma) B_*^2}{(GM_* R_s^3) L_x} \approx 2.1 \times 10^{-2} B_{eff,11}^2 P_0^{-7/3} L_{x,3}$$.  

where $N_0 = M(GM_* R_s)^{1/2}$ (Yi et al. 1997; Wang 1995). The spin equilibrium $N_0$ occurs when $R/R_c = x_{eq} = (\gamma/\gamma)^{1/3}$. The pulsar spin evolves as $P_* = -P_0 N_{2/3} I_*$, where $M$ varies linearly in time specified by $dM/dt$. For a Keplerian flow with gradually varying $M$, $R$, and $N$ vary according to equations (1) and (5). If a pulsar system evolves from a spin-up (spin-down) state and $M$ decreases (increases) gradually, it is possible that $R/R_c = x_{eq}$ is reached at a certain $M$ and then moves to $R/R_c > x_{eq}$, and hence spin-down (spin-up). As long as $M$ variation is smooth and gradual, the torque-reversal events is expected to be distinctively smooth, as shown in Figure 1 for 4U 1626−67.

When the decreasing $M$ reaches $M_c$, below which the accretion flow becomes sub-Keplerian (Yi et al. 1997), the accretion flow–magnetic field interaction would change on a timescale ≤ 1 day. For sub-Keplerian rotation, $\Omega/\Omega_k = A < 1$, the new corotation radius $R_c = A^{3/2} R_c$ and the new inner edge $R_{in}$ is determined by

$$\delta' = \left(\frac{R_o}{R_{in}}\right)^{7/2} \left[ 1 - \left(\frac{R_o}{R_{in}}\right)^{3/2}\right]^{-1},$$

where $\delta' = \delta A^{-7/3}$. The new torque on the star after the transition is

$$N' = \left(\frac{7}{6}\right) N_0 \frac{1 - (8/7) (R_{in}/R_c)^{3/2}}{1 - (R_{in}/R_c)^{3/2}},$$

where $N'_0 = A M(GM_* R_s)^{1/2}$, $N_0 = 0$ would still occur when $R_{in}/R_c = x_{eq}$. In a sub-Keplerian flow supported by internal pressure, $\Omega = A \Omega_k$ is largely determined by the ratio of magnetic to gas pressure in the accreted plasma. For equipartition between magnetic and gas pressures, with the pressure ratio, say, 0.05−1, we expect $A \approx 0.14−0.4$ for all $\alpha = 0.01−0.3$ (e.g., Narayan & Yi 1995).

For the Keplerian flow, the equilbrium spin period is $P_{eq} \approx 13 \left(b_p/10\right)^{3/7} B_{eff,12}^{-3/7} M_{16}^{-3/7}$ s. For the sub-Keplerian rotation, the new equilibrium period $P_{eq} = P_{eq}/A > P_{eq}$. A system in a spin-up state with a restricted period ratio $1 \leq P_{eq}/P_{eq} \leq A^{-1}$ before transition would evolve toward $P_{eq} > P_{eq}$ after transition, i.e., after a sudden torque reversal. Figure 1 shows a fit to the observed reversal event in 4U 1626−67 (see Yi et al. 1997). Vaughan & Kitamoto (1998) report that $M$ changes by ~20% around the observed reversal. The model in Figure 1 is consistent with this finding. On the other hand, if a system has a pretransition $P_{eq}/P_{eq} > A^{-1}$ that places it well into the spin-up regime, the transition could merely lead to a sudden decrease of spin-up rate but not to spin-down (i.e., $P_* > P_{eq}$), a behav-
ior most likely in systems with smaller $B_*$, Figure 1 shows a hypothetical pulsar similar to 4U 1626−67, except with a smaller $B_*$. Observations of this type of torque change in pulsar systems that have been recently displaced from their spin equilibrium (e.g., by abrupt $\dot{M}$ change) could support the present accretion-flow transition model.

QPO, if observed, could provide an additional constraint. Such a constraint would depend critically on the nature of the QPO mechanism. Adopting the widely used beat frequency model (e.g., Lamb et al. 1985), we get the QPO periods

$$P_{QPO} = P_*[(R_o/R_c)^{-3/2} - 1]^{-1},$$

$$P'_{QPO} = P_*[(R_o/R_c)^{-3/2} - 1]^{-1}$$

before and after transition, respectively. It remains unclear whether the conventional beat frequency model can explain the recently observed kHz QPOs in some X-ray pulsar systems. The QPO periods we use in the following section are much longer than the kHz QPO periods. There exists no indication that the long-period QPOs arise from other physical processes.

3. APPLICATION TO OBSERVED X-RAY PULSAR SYSTEMS

The expressions derived so far can be readily applied to the observed systems.

4U 1626−67 has $P_* \approx 7.660$ s at reversal, and the observed torques for the adopted $I_*$ are $N \approx 5.37 \times 10^{33}$ g cm$^2$ s$^{-2}$ and $N' \approx -4.51 \times 10^{33}$ g cm$^2$ s$^{-2}$, or the observed torque ratio $(N'/N)_{obs} \approx -0.840$ (Chakrabarty 1996;...
Chakrabarty et al. 1997a). Using the observed $P_{\text{QPO}} \approx 25\,\text{s}$ during spin-up (Shinoda et al. 1990), equation (6) gives $R_\ast/R_c \approx 0.84$, or $\delta \approx 2.3$, from equation (1). Using the observed $N$ and the derived $R_\ast/R_c$ in equation (3), we get $M \approx 2.7 \times 10^{16}$ g s$^{-1}$. Making use of the observed $N'$ and $\delta' = A^{-7/3}$, we solve equations (4) and (5) to obtain $A \approx 0.46$, which suggests that the accreted plasma has an equipartition-strength magnetic field. From equation (2), we then find $B_{\text{eff}} = b_{\text{j}}/2 \, B_\ast \approx 1.7 \times 10^{12}$ G. This field strength is smaller than the estimates $B_\ast \sim (6-8) 	imes 10^{12}$ G obtained by Pravdo et al. (1979) and Kii et al. (1986), based on the energy cutoff in the X-ray spectrum and the energy dependence of the pulse shape. Although uncertainties in the latter estimates of $B_\ast$ are not clear, a simple comparison suggests that $b_{\text{j}}/2$ is, at most, order unity (see Wang 1995). The observed $0.7-60$ keV flux $F_X \sim 2.4 \times 10^{-9}$ ergs s$^{-1}$ cm$^{-2}$ (Pravdo et al. 1979) gives the distance estimate $(L_X/4\pi F_X)^{1/2} \sim 4.2$ kpc, which is consistent with the previous estimates (e.g., Chakrabarty 1996). Equation (6) suggests a possible $P_{\text{QPO}} \sim 120\,\text{s}$ after transition. Detection of such a QPO could directly test the model.

GX 1+4 reversed torque from spin-down to spin-up around the spin period $P_\ast \approx 122.15$ s (Chakrabarty 1996; Chakrabarty et al. 1997b) with the measured torques $N \approx 3.77 \times 10^{34}$ g cm$^2$ s$^{-2}$ and $N' \approx 3.14 \times 10^{34}$ g cm$^2$ s$^{-2}$, which give $(N'/N)_{\text{obs}} \approx -0.833$. A QPO of period $P_{\text{QPO}} \sim 250\,\text{s}$ was reported during the 1976 spin-up (Doty, Hoffman, & Lewin 1981), but none were reported near the recent reversal episode. The applicability of this QPO is therefore questionable. Nevertheless, assuming that the 1976 spin-up state is similar to the recent spin-up, we can adopt this QPO to make estimates for GX 1+4. The spin period around 1976 is close to the recent spin period, and this may help to justify our use of the 1976 QPO data. Following the same procedure as for 4U 1626−67, we estimate that $R_\ast/R_c \sim 0.77$, $\delta \sim 1.2$, and $M \sim 2.9 \times 10^{16}$ g s$^{-1}$. We get $A \sim 0.12$, which again indicates a magnetic field strength near equipartition. Finally, from equation (2), $B_{\text{eff}} \sim 3 \times 10^{13}$ G, or $B_\ast \sim 10^{13}$ G for $b_\perp \sim 10^2$. The observed flux $F_X \sim 2 \times 10^{-16}$ ergs s$^{-1}$ cm$^{-2}$ in the range of 20−60 keV (Chakrabarty et al. 1997b) gives a distance estimate of $\sim 15$ kpc for the derived $M$, whereas the Doty et al. (1981) value of $F_X \sim 8 \times 10^{-9}$ ergs s$^{-1}$ cm$^{-2}$ in the 1.5−55 keV range gives a distance estimate of $\sim 2.4$ kpc. During the recent spin-down, $P_{\text{QPO}} \sim 6160$ s would be predicted. This prediction, however, is much less certain than that for 4U 1626−67, because of the lack of reliable $P_{\text{QPO}}$ measurements near the recent spin-up. We note that the recent GX 1+4 reversal is considerably more gradual than the 4U 1626−67 event. It is possible that the smooth transition type shown in Figure 1 could be relevant in this case (Yi et al. 1997).

OA 1657−415 has an observed pulse period $P_\ast \approx 37.665$ s at the time of reversal. Recent observations give $N \approx 4.40 \times 10^{34}$ g cm$^2$ s$^{-2}$ and $N' \approx -1.06 \times 10^{34}$ g cm$^2$ s$^{-2}$ (Chakrabarty et al. 1993) or $(N'/N)_{\text{obs}} \approx -0.241$. There exists no reported QPO for this system. If we take $A = 0.4$, based on our estimates in 4U 1626−67 and GX 1+4, the observed torque ratio $(N'/N)_{\text{obs}}$ corresponds to $\delta \sim 1.2$ or $R_\ast/R_c \sim 0.77$, as in GX 1+4, which gives $M \sim 1.5 \times 10^{17}$ g s$^{-1}$. Therefore, we get $B_{\text{eff}} \sim 2 \times 10^{13}$ G or $B_\ast \sim 10^{13}$ G for $b_{\perp} \sim 10$. For $P_\ast \sim 375$, $R_\ast/R_c \sim 0.77$, and $A = 0.4$, we expect $P_{\text{QPO}} \sim 75\,\text{s}$ and $P_{\text{QPO}} \sim 430\,\text{s}$ before and after reversal, respectively. For a detected flux of $\sim 10^{-9}$ ergs s$^{-1}$ cm$^{-2}$ (White & Pravdo 1979; Mereghetti et al. 1991) and the estimated $M \sim 1 \times 10^{17}$ g s$^{-1}$, the distance is $\sim 12$ kpc.

We caution that the predicted QPO frequencies may not be easily detectable. During spin-down, the accretion flow thickens considerably, and the pulsed polar emission may be significantly diluted by scattering (Yi et al. 1997). The long-period QPOs may appear as occasional flares, with repetition timescales of roughly a few times $10^3\,\text{s}$.

4. NECESSARY CONDITION FOR TORQUE REVERSAL IN SYSTEMS NEAR SPIN EQUILIBRIUM

The parameters derived above can be understood by a simple expression that constrains the pulsar parameters and essentially determines the pulsar magnetic field. We assume that the system reverses its torque from spin-up to spin-down. Just before reversal, we take $R_\ast/R_c = x_{\text{eq}} - \epsilon$, with $\epsilon < x_{\text{eq}}$. Expanding equations (1) and (3) to first order in $\epsilon/x_{\text{eq}}$ and combining, we get

$$N = \frac{7N_0}{4} \frac{\epsilon/x_{\text{eq}}}{1 - x_{\text{eq}}^2} = \frac{7}{2(7 - 4x_{\text{eq}}^2)} \left[ 1 - \frac{1 - x_{\text{eq}}^2}{x_{\text{eq}}^2} \right].$$

After transition to the sub-Kleptarian state, we take $R_\ast/R_c = x_{\text{eq}} + \epsilon'$, also with $\epsilon < x_{\text{eq}}$. We expand equations (4) and (5) to first order in $\epsilon'/x_{\text{eq}}$ and combine them to derive

$$N' = -\frac{7N_0}{4} \frac{\epsilon'}{1 - x_{\text{eq}}^2} = -\frac{7N_0}{2(7 - 4x_{\text{eq}}^2)} \left[ 1 - \frac{x_{\text{eq}}^2}{x_{\text{eq}}^2} \right].$$

The ratio of the sub-Kleptarian spin-down torque ($N'$) to the Kleptarian spin-up torque ($N$) is then

$$\frac{N'}{N} = -A^{4/3} \left[ 1 + \frac{g(x_{\text{eq}})(1 - x_{\text{eq}}^2)}{4(7 - 4x_{\text{eq}}^2)} (\delta' - \delta) \right] \frac{g(x_{\text{eq}})\delta' - 1}{1 - g(x_{\text{eq}})\delta'} ,$$

where $g(x_{\text{eq}}) = (1 - x_{\text{eq}}^2)/x_{\text{eq}}^2$. We have made use of the first-order expression

$$\frac{N_0'}{N_0} = A^{4/3} \left[ 1 + \frac{\epsilon' + \epsilon}{x_{\text{eq}}^2} \right] \frac{g(x_{\text{eq}})\delta' - 1}{1 - g(x_{\text{eq}})\delta'} .$$

For $x_{\text{eq}} = (\gamma/2)^{2/3}$, we get

$$\frac{N'}{N} = -A^{4/3} \left[ 1 + 6.10 \times 10^{-3}(\delta' - \delta) \right] \frac{0.171\delta' - 1}{1 - 0.171\delta} .$$

Given the observational constraint $N' \leq N$, equation (11) indicates that small values of $A \sim 0.1$ are unlikely, which is consistent with the result $A \geq 0.3$ in the previous section. This in turn suggests that the magnetic pressure−gas pressure ratio is roughly at the level of equipartition (see Narayan & Yi 1995). From equations (7) and (8), the first necessary condition for the torque reversal is $N'/N < 0$, or

$$A^{7/3} \frac{x_{\text{eq}}^2}{1 - x_{\text{eq}}^2} < \delta < \frac{x_{\text{eq}}^2}{1 - x_{\text{eq}}^2} ,$$

or, for $x_{\text{eq}} = (\gamma/2)^{2/3}$,

$$\delta_{\text{min}} = 5.86A^{7/3} < \delta < 5.86$$.
The second condition comes from the observational requirement, $|N'/N|_{\text{obs}} < 1$, which gives (see eq. [11])

$$\delta < 5.86 \frac{1 + A^{4/3}}{1 + A^{-1}}. \tag{14}$$

Finally, the observed continuous X-ray emission throughout the whole reversal process requires the accretion to be continuous. This translates into the condition $R_0/R_c < 1$ and $R_0/R_c < 1$, in order to avoid an angular momentum barrier at the corotation radius. By expressing $\epsilon'$ in terms of $\delta$, we get

$$\delta < 13.39 A^{7/3} = \delta_{\text{max}}. \tag{15}$$

For $A \leq 0.51$, $\delta_{\text{max}}$ replaces the upper bounds in equations (13) and (14). For $A \geq 0.51$, the upper bound on $\delta$ is given by equation (14), as shown in Figure 2. As $A \to 1$, the allowed parameter space shrinks rapidly, which simply indicates that the rotation gets closer to the Keplerian and that the spin-down becomes more difficult when the mass accretion rate is fixed. Therefore, the rotation needs to be substantially sub-Keplerian after transition in order for the reversal to occur. For most of the $A$ values, $\delta'$s range is limited to be within a factor of $\sim 2$. Therefore the condition for torque reversal becomes $\delta_{\text{min}} < \delta < \delta_{\text{max}}$, and the magnetic field should be close to

$$B_{\text{eff}} \approx 7 \times 10^{11} \delta_{1/2} L_{N,36}^{1/2} P_{*1.0}^{-0.7} \text{ G}, \tag{16}$$

where $\delta_{1/2} = 2.4 A^{7/6} \delta_{1/2} < 3.7 A^{7/6} = \delta_{\text{max}}$. If the accreted plasma is magnetized with near-equipartition between magnetic pressure and gas pressure, the sub-Keplerian flow with $A \sim 0.1$–0.4 (Narayan & Yi 1995) suggests that $\delta_{1/2}$ is essentially a constant of order unity (see Fig. 2). Therefore, equation (16) provides an estimate of the magnetic field within a factor of $\sim 2$. If the distance to the pulsar and an independent estimate on $B_*$ (say, through X-ray spectrum and emission) are known, this relation can constrain $b_* = (B_{\text{eff}}/B_*)^2$. The pulsar parameters derived in the previous section are consistent with equation (16). This can be checked as follows: Using equation (11), we derive

$$\delta = A^{4/3} \frac{1 - A^{-4/3} (N'/N)_{\text{obs}}}{0.1707 A^{-1} - (N'/N)_{\text{obs}}}. \tag{17}$$

For a given $A$, we confirm that the observed torque ratio gives $\delta's$ that are close enough to the derived values in the previous section. Therefore, we conclude that equation (16) can be reliably used to identify the characteristics of the accreting pulsar systems that should show torque reversals.

The condition for the torque reversal, $5.86 A^{7/3} < \delta < 13.39 A^{7/3}$ for $A \leq 0.51$, allows very tightly constrained physical parameters of pulsar systems. Based on the results in the previous section, our model suggests $A \approx 0.4$ and, therefore, $0.69 < \delta < 1.58$. Then, in equation (16), $0.8 < \delta_{1/2} < 1.3$ is determined accurately. If this is the case, we can derive $B_{\text{eff}} \approx 7 \times 10^{11} L_{N,36}^{1/2} P_{*1.0}^{-0.7} \text{ G}$ essentially without uncertainties in $\delta$. Since $\delta \sim A^{7/6}$, $B_{\text{eff}} \sim A^{7/6}$ has a crucial dependence on $A$.

5. DISCUSSION

We have examined the torque-reversing mechanism proposed by Yi et al. (1997) and shown various possible types of torque behavior in X-ray pulsar systems. Within the proposed mechanism, the physical parameters of the observed torque-reversing systems are tightly constrained. We have derived a general constraint on the torque reversal. If the accretion-flow transition is indeed responsible for the torque reversal, the observed pulsar systems must be near spin equilibrium, which immediately leads to a tight constraint on the pulsar magnetic fields. Combining with QPOs and X-ray luminosities, the torque-reversing pulsar systems could be unambiguously constrained. Detection of QPOs immediately before and after torque reversal remains an outstanding issue.

The current analysis and the study of Yi et al. (1997) suggest that the observed torque reversals occur when the accretion flow makes a transition at a critical rate $M_* \sim 10^{16} - 10^{17}$ g s$^{-1}$. This points to an interesting connection between X-ray pulsar systems and other compact accretion systems, such as cataclysmic variables and black hole transients. We speculate that the spectral transitions seen in the latter systems may be due to a common accretion–flow–transition mechanism (see Narayan & Yi 1995). The observed torque reversal could be a signature of a pulsar system near spin equilibrium, with $M$ gradually varying near $M_*$. The proposed mechanism tightly constrains the pulsar field strength. If a pulsar does not have an appropriate field strength (eq. [16]), it would not undergo a torque reversal even if $M$ passes through $M_*$, causing the transition. If all systems have a similar $M_*$ (Narayan & Yi 1995), the pulsar magnetic field strength can be determined despite a major uncertainty in the distance. It is possible that some systems with $P_*/P_{eq}$ abruptly change their torques without reversing sign at $M \sim M_*$. These types of sudden torque change are distinguishable from the smooth torque variation in a Keplerian accretion flow. If QPOs can be detected near torque reversal, the proposed model could be further tested in detail.

Although the spectral and flux changes are generally small in the torque-reversing systems (Chakrabarty 1996), which, at least in 4U 1626–67, indicates the small changes in $M$, some significant spectral and flux changes have been reported (Vaughan & Kitamoto 1998). The hardening of the
spectrum after the transition to spin-down could be attributed to the accretion-flow transition itself (Yi et al. 1997). It is, however, unclear whether the hot accretion flow with a large vertical thickness could directly affect the X-ray spectrum through scattering, since the expected scattering depth through the hot flow is at most a few times $10^{-2}$ for $M \sim 2 \times 10^{16}$ g s$^{-1}$ (Narayan & Yi 1995), appropriate for 4U 1626-67. The spectral transition issue needs further investigation in connection with other torque-reversing systems.

Gradual $M$ modulation on a timescale ranging from approximately a year to a few decades is required for the proposed torque reversal. Such a modulation can be driven by several known mechanisms: (i) In the case of GX 1+4, the binary orbital motion on a timescale of approximately a year is plausible (Chakrabarty 1996), which could cause $M$ variation through orbital modulation of mass transfer. In 4U 1626-67, an orbital timescale longer than a few years is unlikely (Rappaport et al. 1977; Joss et al. 1978; Shinoda et al. 1990). OAO 1657-415 has an orbital timescale of ~10 days (Chakrabarty et al. 1993). (ii) Several precession timescales involving the pulsar and the secondary remain viable (e.g., Thorne, Price, & MacDonald 1986). (iii) X-ray irradiation-induced, mass-flow oscillation (Meyer & Meyer-Hofmeister 1990) could provide long-timescale (greater than months) oscillations when certain conditions such as disk size and $z$ are met. Interestingly, in 4U 1626-67 the observed optical pulsation frequency is the same as that in X-rays, which has been attributed to the reprocessing of X-rays by the accretion disk (Ilovaisky, Motch, & Chevalier 1978; Chester 1979). (iv) The disk instability model (e.g., Smak 1984) has not been applied to neutron star systems, and long-term $M$ variation due to the disk instability needs further investigation.

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