Polarized deep inelastic scattering (DIS) of leptons on nucleons has helped to establish the modern understanding of nucleon spin structure function (SSF). The Bjorken sum rule obtained from the non-interacting partonic description of QCD. For a full understanding, the interactions among nucleon constituents must also be probed by looking into contributions beyond leading order, which grow in importance at lower energies. Of particular interest are the so called ‘higher twist’ terms, which measure correlations among partons (quarks and gluons), and offer unique perspectives on the phenomenon of confinement inside nucleons and other hadrons.

Access to these subleading interactions is possible with transverse polarized inelastic scattering. In this configuration, the target spin is held orthogonal to the incident lepton spin, and the $g_2$ spin structure function dominates compared to $g_1$. QCD matrix elements that arise from multi-parton correlations can be extracted directly from moments of $g_2$. However, this approach has been limited due to the technical challenges involved in aligning proton spins at 90° relative to the beam. As we detail below, we have carried out a complete measurement of
the longitudinal and transverse inclusive SSF’s $g_1$ and $g_2$ of the proton and deuteron, with the goal of extracting matrix elements related to quark-gluon, $qg$, interactions.

The Operator Product Expansion (OPE) [3] relates the moments of the SSF’s to reduced quark and gluon matrix elements representing the multi-parton correlations that lead to confinement. This perturbative approach to QCD is formulated in terms of the Cornwall-Norton (CN) moments [6]:

$$
\Gamma_{1,2}^{(n)}(Q^2) = \int_0^1 dx \, x^{n-1} g_{1,2}(x, Q^2)
$$

where $x = Q^2/2M \nu$ is the Bjorken scaling variable, $Q^2$ is the four-momentum transfer squared of the scattering process, $M$ the nucleon mass, and $\nu$ the energy transfer. Conventionally, the index $n = 1$ is not displayed.

In the OPE, the CN moments are expanded in a power series in $1/Q^{\tau - 2}$ of increasing ‘twist’ $\tau$, which is defined as the mass dimension minus the spin $n$ of the corresponding QCD operator [3]. The leading twist-2 terms map to the successful predictions of the parton model, such as the $Q^2$-independence of the structure functions up to logarithmic corrections. Higher twist (HT) corrections arise from the non-perturbative multiparton interactions, whose contributions at low energy increase as $1/Q^2$, reflecting confinement. Specifically, twist-3 represents $qgq$ correlations, so a non-zero result at a given $Q^2$ for the term representing twist-3 signals a departure from the non-interacting partonic regime.

Studies of higher twist effects [6,11] have typically focused on the CN moments $I(Q^2) = \int_0^1 dx \, x^2 \left( 2g_1 + 3g_2 \right)$ and $\Gamma_1(Q^2)$, in order to extract the twist-3 and twist-4 matrix elements. However, as recently stressed in [6,12], this approach is only appropriate when terms of purely kinematic origin, known as ‘target mass corrections’ (TMC) [13,18] for their connection to the finite nucleon mass, can be neglected. These terms, of order $O(M^2/Q^2)$, are formally related to twist-2 operators and must be cleanly separated from the desired dynamical higher twists. $I(Q^2)$ is often labeled ‘$d_2$’ in the literature, although, as discussed below, it is not equivalent to the HT matrix element indicated by the same symbol.

As emphasized in [12], dynamical HT can be extracted to order $O(M^2 / Q^2)$ by using Nachtmann moments [16,19], which depend on the scaling variable $x = 2x/(1 + \sqrt{1 + (2xM^2)/Q^2})$. This variable extends scaling as $x$ approaches 1, for data measured at low $Q^2$.

The Nachtmann moments are defined as

$$
M_1^n(Q^2) = \frac{1}{2} \tilde{a}_{n-1} \equiv \frac{1}{2} d_n E_n^m = \int_0^1 dx \, x^{n+1} 2^n \frac{2^n x^n}{x^2} [ \frac{x}{\xi} - \left( \frac{n^2 (n+2)^2}{(n+1)^2 \xi^2} \right) g_1 - 4n \frac{M^2 x^2}{Q^2} g_2 ]
$$

and

$$
M_2^n(Q^2) = \frac{1}{2} \tilde{d}_{n-1} \equiv \frac{1}{2} d_n E_2^m = \int_0^1 dx \, x^{n+1} 2^n \frac{2^n x^n}{x^2} [ \frac{x}{\xi} g_1 + \left( \frac{n}{n-1} \xi^2 - \frac{n}{n+1} \frac{M^2 x^2}{Q^2} \right) g_2 ]
$$

Here, the $a_n$ ($d_n$) represent twist-2 (-3) matrix elements, while the $E_2^n$ are the corresponding Wilson coefficients, which contain logarithmic QCD corrections. For convenience, these corrections are absorbed into the definition of the effective matrix elements $\tilde{a}_n$ and $\tilde{d}_n$. The index runs over $n = 1, 3, \ldots$ for $M_1$, and $n = 3, 5, \ldots$ for $M_2$. When $Q^2 \gg M^2$, the Nachtmann moments simplify:

$$
M_1^n(Q^2) \to \Gamma_1(Q^2), \quad 2M_2^n(Q^2) \to I(Q^2)
$$

Consequently, deviations from unity of the ratio of Nachtmann to CN moments give a quantitative measure of the TMCs at finite $Q^2$:

$$
R_1(Q^2) = \frac{M_1^n(Q^2)}{\Gamma_1(Q^2)}, \quad R_2(Q^2) = \frac{2M_2^n(Q^2)}{I(Q^2)}
$$

The leading twist-2 contribution to $g_2$ can be determined from $g_1$ via the Wandzura-Wilczek relation [27]:

$$
g_2^{tw}(x, Q^2) = -g_1(x, Q^2) + \int_x^1 y^{-1} g_1(y, Q^2) dy
$$

So any twist-3 effects arise entirely from $g_2 = g_2 - g_2^{tw}$.

**Experiment E01-006** was conducted in Hall C of the Thomas Jefferson National Accelerator Facility by the Resonance Spin Structure (RSS) collaboration. We measured the parallel and perpendicular double spin asymmetries $A_{||}$ and $A_{\perp}$ in the scattering of 100 nA polarized 5.755 GeV electrons on polarized protons and deuterons. Scattered electrons were detected at an angle of 13.15° using the Hall C High Momentum Spectrometer. The kinematic coverage in invariant mass was $1.090 < W < 1.910$ GeV, corresponding to $x_0 = 0.316 < x < x_{\text{max}} = 0.823$, at an average four-momentum transfer of $(Q^2) = 1.28 \pm 0.21$ GeV$^2$. Systematic uncertainties are detailed in Table I with more details in [20] and [21].

The SSFs were extracted from the measured asymmetries using a fit to the ratio of longitudinal to transverse cross sections $R$ [22] and the unpolarized structure function $F_1$ [23], based primarily on data measured previously in Hall C. The standard formulas and procedure used to obtain $A_{||}$ and $A_{\perp}$ and the SSFs from the data are detailed in [1] and [20]. Simultaneous determination of both SSFs

|     | $\Gamma_1^n$ | $\Gamma_1^1$ | $d_2^n$ | $d_2^1$ |
|-----|------------|------------|--------|--------|
| $f_1$ | 4.9 | 5.1 | 4.9 | 5.1 |
| RC | 3.2 | 4.0 | 9.5 | 4.5 |
| $F_1$ | 3.0 | 4.0 | 3.0 | 4.0 |
| $R$ | 0.9 | 1.8 | 1.2 | 2.8 |
| $P_3, P_4$ | 1.6 | 1.6 | 5.2 | 3.1 |
| $Q^2$ dependence | 0.33 | 0.1 | 4.3 | 0.3 |
| Total | 6.8 | 8.0 | 13.0 | 8.9 |

**TABLE I.** Measured integral systematic uncertainties (in %) arising from the target dilution factor $f$, radiative corrections RC, the $F_1$ and $R$ data fits, beam and target polarizations $P_3$ and $P_4$, and the evolution of our data fit to constant $Q^2$. 
from the measured resonance region labeled with ‘m’), the well known x assumed for the electromagnetic form factors. Relative uncertainties of 5, 1, 14, and 2.5% have been calculated using the form factor parameterizations of [23].

allowed us to evaluate the moments without model input for g2, as was necessary in some previous analyses [9]. The moments reported herein are evaluated at ⟨Q2⟩ = 1.28 GeV2, and have been decomposed into contributions from the measured resonance region x0 < x < xthr (labeled with ’m’), the well known x = 1 elastic (‘el’) contribution, and the unmeasured (‘u’) portion below x0. We note that the small difference between our experimental xmax and the nucleon inelastic threshold xthr has negligible impact on the integrated results. We have used fits [24] to our data to evaluate the moments in the measured region. The nucleon elastic contribution was calculated using the form factor parameterizations of [23]. Relative uncertainties of 5, 1, 14, and 2.5% have been assumed for the electromagnetic form factors GF,E,M and GF,N,M respectively. For the deuteron, x = 1 represents quasileastic scattering, the strength of which we estimate by combining the nucleon elastic contributions using the D-state correction discussed below. The deuteron nuclear elastic contribution is negligible here. We have also evaluated the neutron and non-singlet (proton - neutron) moments by using the relation GF = ΓN = ΓD − ΓP, where the singlet ΓΣ = ΓΣ/D, and γD ≈ 0.926 ± 0.016 [23] is the D-state correction to the deuteron wave function. The uncertainty arising from this approach is estimated [24] to be O(1%). The singlet and non-singlet results assume negligible heavy quark contributions.

Table III provides numerical values for the moments. The ratio R2 differs significantly from unity, indicating large unwanted twist-2 kinematical contributions to I(Q2). The full results for the matrix element d2 indicate clear twist-3 at more than 6σ for the proton and 3σ for the neutron. These non-vanishing results unambiguously indicate the presence of qgq correlations. Their magnitudes can be used in comparing with nucleon models.

The values of I(Q2) and d2 for x < x0 were estimated to be zero, with a systematic error described below. In this unmeasured region, ξ ≈ x and the CN and Nachtmann truncated moments converge, so there is little difference between I(Q2) and d2. I(Q2) is expected to be small in this unmeasured region, because of the decreasing importance of higher twists at low x, and the strong suppression due to the x2 weighting of the integral. This assumed low x behavior is supported by our data. Figure 1(left) displays the x−dependence of the measured g2 structure function. It is clear that g2 is nearly constant and consistent with zero within errors for the proton, near the low end of our measured range x0 < x ≲ 0.37. The deuteron data shows a similar behavior, different from zero only at the one σ (statistical) level. Neutron data [25] at x ≈ 0.2 and similar Q2 are also consistent with γ2 ≃ 0. We take the error on γ2, evaluated at x0, δγ2, as a conservative upper limit for the integral of I(Q2) in the unmeasured low x region. For the proton, we determined this upper limit by assuming a constant extrapolation of the value of δγ2(x0) down to x = 0. For the deuteron, we evaluated both a linear and constant extrapolation, averaged both assumptions and added quadratically one-half their difference as ‘model’ error to the δγ2 fit error to obtain the value in Table III.

A divergence of g2 as x → 0 could invalidate this assumption. Such a possible divergence for x ≲ 0.001 was discussed in [29]. Normalizing the low x dependence of γ2 given in [29] to our δγ2 estimates, we find the additional contribution to be negligible.

It is instructive to compare our twist-3 results to previous measurements. SLAC E155 [1] reported an evaluation of I(Q2) at ⟨Q2⟩ = 5 GeV2. We have corrected those results for TMCS [12], to obtain 0.0028 ± 0.0015 and 0.0072 ± 0.0044 for the proton and neutron, respectively. For direct comparison, we performed a pQCD evolution [30] from RSS to the SLAC kinematics. At LO, we find d2 n = 0.0021 ± 0.0006 and d2 p = 0.0031 ± 0.0038, which are consistent with the E155 results. The elastic contribution [23] at these kinematics is smaller than the uncertainties and is not included in the results of this paragraph. NLO corrections [31] to our data have been calculated and will be discussed in a future publication.

Twist-3 effects also manifest in the first moment of g2. The Burkhardt-Cottingham (BC) sum rule [32] predicts that Γ2 vanishes for all Q2. This sum rule can be derived from the unsubtracted dispersion relation for the virtual Compton scattering amplitudes, in an analogous fashion to the more famous GDH and Bjorken sum rules. It provides a remarkably clean tool to investigate nucleon

FIG. 1. Left-top: RSS proton g2 data [20], along with the RSS fit [23]. The shaded curve is g2 new evaluated from the RSS fit. Left-bottom: RSS deuteron g2 data, and similar curves as above. Right-top: The open circle is the measured RSS proton Γ2(Q2) data. The full circle is ∆Γ2(Q2). The inner (outer) error bars represent statistical (total) uncertainty. The shaded curve is the elastic contribution to Γ2(Q2). Right-bottom: Same as above, but for deuteron.
structure, since it is free from both QCD radiative corrections and TMCs \[32\]. Initial measurements by the E155 collaboration \[1\] found a 2\(\pi\) inconsistency with the proton BC sum rule at large \(Q^2\), while the same group found that the deuteron sum rule to hold. Later measurements \[3, 7\] at lower momentum transfer found agreement with both the neutron and \(^3\)He BC sum rules.

The leading twist \(g_2^{\pi\pi}\) satisfies the BC sum rule exactly, so any violation must arise from higher twist effects. For convenience we split the full integral as:

\[
\Gamma_2 = \Gamma_2^{\pi\pi} + \Gamma_2^{\text{el}} + \Gamma_2^{\text{cl}}
\]

where the overbar signifies removal of the leading twist contribution, and we have made use of the fact that \(\Gamma_2^{\pi\pi} = 0\) by definition. The elastic contribution is well known, as displayed in Fig. 1, and we can evaluate \(\Gamma_2^{\text{el}}\) directly from our own data. \(\Gamma_2^{\text{cl}}\) is pure higher twist, and cannot be directly determined, due to the lack of low \(x\) \(g_2\) data. However, the difference,

\[
\Delta\Gamma_2 \equiv \Gamma_2 - \Gamma_2^{\text{el}}
\]

depends only on measured data. A significant non-zero result for this quantity would indicate a higher twist contribution to the integral in the region \(x < x_0\), assuming of course, that the BC sum rule holds. We find that the proton results are consistent with vanishing HT effects at low \(x\). On the other hand, the singlet results indicate the need for significant HT contributions to \(\Gamma_2\) at low \(x\), if the BC sum rule is to be satisfied. This could indicate an isospin dependence of the BC sum rule, or alternatively, a modification to the sum rule due to nuclear effects.

Nachtmann moments are also useful for obtaining leading twist matrix elements free of TMCs. The \(a_0\) non-singlet matrix element corresponds to the lowest moment \(\Gamma_1^{I-n}\) of the isovector nucleon \(g_1\), which is related to the nucleon axial charge \(g_A\) via the Bjorken sum rule \[34\]. This relation is a direct consequence of QCD, and experimental tests \[1, 2\] of the sum rule have played a critical role in confirming QCD as the correct theory of the strong interaction. The target mass corrections to \(\Gamma_1\) given by the ratio \(R\) are small at our kinematics, due to a combination of dominant contributions from the region below our lowest measured \(x_0\), and the nearly identical values of \(x\) and \(\xi\) in that region. It is interesting to note that \(R \approx 93\%\) for all targets, when the elastic contribution is included. The \(M_1\) non-singlet result including the elastic part \((0.161 \pm 0.019)\) is in excellent agreement with theory: Ref. \[35\] predicts 0.212, which reduces to 0.155 after application of the next to next-to-leading order corrections \[36\] appropriate at our \(Q^2\).

In summary, we have measured the spin structure function moments of the proton, deuteron and neutron in the resonance region. The kinematic weighting of higher moments suppresses contributions from the low \(x\) region, thus minimizing the systematic uncertainty of this unmeasured piece of the integral. Our analysis indicates a large TMC contamination of the CN moment \(I(Q^2)\), which has often been used to determine the magnitude of higher twist effects. The alternate Nachtmann moment \(d_2\) minimizes TMCs and allows a clean extraction of the twist-3 reduced matrix element. The previously undetermined \(d_2\) provides unambiguous evidence of dynamical

| Table II: RSS Moments evaluated at \(\langle Q^2 \rangle = 1.28 \text{ GeV}^2\). The ratio \(R_1 = M_1^I / \Gamma_1\) and \(R_2 = d_2 / \Gamma_1\). | Proton | Deuteron | Neutron | Non-Singlet |
|---|---|---|---|---|
| Value | Stat | Syst | Value | Stat | Syst | Value | Total | Value | Total |
| \(0 < x < x_0\) | \(0.0676 \cdots 0.0060\) | \(0.0274 \cdots 0.0104\) | \(-0.0381 \pm 0.0132\) | \(0.1057 \pm 0.0178\) |
| \(x_0 < x < x_{\text{th}}\) | \(0.0683 \cdots 0.0060\) | \(0.0280 \cdots 0.0099\) | \(-0.0381 \pm 0.0127\) | \(0.1065 \pm 0.0176\) |
| \(x = 1\) (elastic) | \(0.0330 \pm 0.0022\) | \(0.0190 \pm 0.0035\) | \(0.0 \pm 0.0016\) | \(0.0 \pm 0.0021\) |
| \(0 < x < 1\) (Total) | \(0.1293 \pm 0.0073\) | \(0.0870 \pm 0.0010\) | \(-0.0318 \pm 0.0019\) | \(0.0518 \pm 0.0031\) |
| \(0 < x < 1\) | \(0.933 \pm 0.003\) | \(0.932 \pm 0.012\) | \(0.936 \pm 0.037\) | \(0.934 \pm 0.008\) |
|        | Proton |         | Deuteron |         | Neutron |         | Non-Singlet |         |
|--------|--------|---------|----------|---------|---------|---------|------------|---------|
|        | Value  | Stat    | Syst     | Value   | Stat    | Syst    | Value Total | Syst    |
|        |        |         |          |        |         |          |            |         |
| \( \Gamma_2 \) | -0.0132 | 0.0004  |          | -0.0219 | 0.0009  | 0.0009  | 0.0105 | 0.0005  | -0.0027 | 0.0007 |
| \( \Gamma_3 \) | -0.0138 | 0.0007  | 0.0009   | -0.0107 | 0.0012  | 0.0009  | 0.0222 | 0.0020  | -0.0160 | 0.0028 |
| \( \Gamma_4 \) | 0.0126  | 0.0008  | 0.0020   | 0.0129  | 0.0016  | 0.0018  | 0.0013 | 0.0034  | 0.0113  | 0.0051 |
| \( \Delta \Gamma_2 \) | 0.0006  | 0.0008  | 0.0021   | -0.0090 | 0.0016  | 0.0020  | -0.0092 | 0.0035  | 0.0086  | 0.0051 |

Table III. The higher twist \( \Delta \Gamma_2 = \Gamma_2 - \Gamma_2 \) evaluated at \((Q^2) = 1.28 \text{ GeV}^2\).

Twist-3 effects, which arise from quark-gluon interactions. The first moment of \( g_2 \) indicates an isospin dependence of HT effects in the region of small Bjorken scaling variable, assuming that the BC sum rule holds. After nearly thirty years of spin-dependent investigations, the \( g_2^p \) structure function remains largely unknown. But the data reported here and future analyses will shed light on this fundamental nucleon observable.

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