Thermomagnetic instability of a rotating magnetized plasma disk

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ABSTRACT

We analyze the stability of a thin plasma disk which is rotating around a compact astrophysical object and is embedded in the strong magnetic field of such a source. The aim of this study is the determination of a new type of unstable modes, able to replace the magneto-rotational instability profile for low $\beta$ values and for sufficiently small scales of the perturbations, where it fails. In particular, we consider the magneto-hydrodynamical scheme including a non-zero Nernst coefficient, corresponding to first-order collisional effects. As a result, modes with imaginary frequency lead to an instability regime when the magnetic tension vanishes. Finally, we show that, even in the presence of resistive effects, it remains a good candidate to ensure the onset of a turbulent behavior in the absence of the magneto-rotational instability.

Key words: Accretion Disks; Plasmas.

1 INTRODUCTION

The description of the accretion process on a compact object constitutes one of the most relevant open questions in astrophysics, both for the understanding of crucial phenomena, like Gamma Ray Bursts \cite{Piran1999} and Active Galactic Nuclei \cite{Krolik1999}, as well as because the accreting plasma can trigger the formation of matter jets, widely observed in nature \cite{Fender2010}. The “standard model” for the formation and behavior of accretion disks is due to Shakura \cite{Shakura1973}; Shakura, Sunyaev \cite{Shakura1973} and relies on the possibility to transport angular momentum outwards by means of the shear viscosity emerging from the disk differential rotation. Indeed, such a viscous effect can not be justified by the microscopic properties of the plasma and it is commonly accepted to be associated with the turbulent regime arising from the linear instability of the plasma dynamics with respect to small perturbations. The natural scenario able to induce turbulence is recognized in the Magnetorotational Instability (MRI) \cite{Velikhov1959, Chandrasekhar1960} (for more recent developments, see \cite{Balbus1991}), which holds as far as an arbitrarily small magnetic field is introduced in the problem.

The scope of the present analysis (referred to as a thin rotating disk) is to determine a new type of instability active in that region of the parameters where the MRI contribution is significantly suppressed, \textit{i.e.}, for low $\beta$ values and at small spatial scales. We succeeded in this attempt by fixing a Thermomagnetic Instability (TMI) which is triggered by a non-zero Nernst coefficient (for a classical treatment of the thermomagnetic waves in plasma and TMI, see \cite{Tidman1974, Dolginov1974, Urpin1981}). Due to the microscopic value this term has in the quasi-ideal disk plasma, this instability will be relevant for very small scales where MRI is instead strongly suppressed. At small $\beta$ values, the only relevant dissipative effect corresponds to a non-zero plasma resistivity. We show how this additional contribution behaves as a damping of the instability growth rate, yet it is unable to suppress it by more than 50%. This analysis is able to provide an alternative and complementary paradigm to MRI.

2 BASIC FEATURES OF THE STANDARD ACCRETION MODEL

Let us now briefly remind the standard accretion disk paradigm. In the Shakura model, the local equilibrium configuration \cite{Bisnovatyi-Kogan2001} is mainly described by the hydrodynamical equations (for a critical approach to this, see \cite{Montani2012}). The radial force balance fixes the Keplerian nature of the disk angular frequency $\Omega$, while the vertical gravostatic equilibrium fixes the mass density profile and the half-depth of the disk $H$. Moreover, the continuity equation provides a constant value of the accretion rate $\dot{M}$ and the azimuthal force balance expresses the angular momentum transport across the disk.
in terms of the viscous stress tensor \( \tau = -3\eta \rho \Omega / 2 \), namely
\[ M(L - L^*) \sim -H \tau r^2, \]
where \( L = \Omega r^2 \) is the specific angular momentum (\( r \) being the disk radial coordinate), \( L^* \) is a given value of \( L \) on the inner boundary layer of the disk and \( \eta \) the shear viscosity coefficient. The viscous stress tensor corresponds to the relevant non-zero correlation function of the turbulent regime which, in the Shakura proposal, is responsible for the dissipative effects, i.e., \( \tau = -(\rho \nu_r v_\phi) \sim -\alpha \rho v_\phi^2 \), where \( \alpha \) is a parameter smaller than unity, \( \nu_r, \nu_\phi \) denotes the radial and toroidal components of the velocity field, \( \rho \) is the disk mass density and \( v_\phi \) the sound speed. Comparing the expressions of the stress tensor, one easily gets \( \eta \sim \alpha \nu_r v_\phi H \).

The puzzle of such a scheme (which successfully accounts for many observed accretion features in astrophysical systems) consists of the stability that the hydrodynamical equilibrium manifests under small perturbations, preserving the axial symmetry. Therefore, unless peculiar scenarios (like the resonance phenomenon) are advocated, the paradigm of a turbulent regime would fall down.

The solution to this problem is offered by including an even small magnetic field in the equilibrium configuration, thus generating the MRI mentioned above. This scenario is well-grounded for astrophysical settings, because many accreting compact sources are endowed with a relevant intrinsic magnetic field and, for many real systems, it can be properly approximated by a dipole-like configuration that (for a thin disk) results in essentially a vertical field. In the MRI analysis, the local stability of a magnetically confined plasma disk can be then studied in a simplified perturbative scheme, in which the background is characterized by a purely Keplerian rotating configuration of constant mass density (and pressure) and embedded in a constant vertical magnetic field. The perturbations are consequently taken as local and incompressible (the so-called Boussinesq approximation) and propagating along the magnetic field direction. Furthermore, the linearity of the fluctuation dynamics allows us to use a plane wave form of wave-vector \( k \) and frequency \( \omega \). It can be shown that the normal modes associated to this scheme are characterized by the following dispersion relation (for a review, see [Balbus, Hawley (1998)]
\[ \omega^2 - \omega^2 (\Omega^2 + 2(kv_A)^2) + (kv_A)^2 (kv_A)^2 - 3\Omega^2) = 0, \tag{1} \]
where \( v_A \) is the Alfvén speed associated to the background magnetic field. Eq. (1) leads to the following condition for rising unstable modes \( k < k_c \equiv \sqrt{3 \Omega/v_A} \). MRI is therefore ensured in the presence of sufficiently small wavenumber (large scales). In this respect, it is worth underlining the presence of a minimum for the values of \( k \) which corresponds to \( k_{min} = \pi / H \) (i.e., to the maximum disk scale). Thus, the MRI mechanism is not applicable if \( k_{min} > k_c \) (at sufficiently small scales) since all modes would result stable. The latter condition can be expressed in terms of the plasma \( \beta \) parameter as \( \beta < \pi^2/3 \) and, since a large class of real accretion systems are indeed characterized by \( \beta > \pi^2/3 \), the corresponding MRI is commonly considered as the privileged mechanism for rising instabilities within an axisymmetric rotating plasma configuration. However, the existence of plasma disks for which \( \beta \) is close to unity can not be excluded at all. In particular, in the case of very thin profile, MRI is not applicable as far as the magnetic field \( B \) induced by the central object is sufficiently strong (we recall that \( \beta \sim B^{-2} \)). Since the sound speed is related to the disk local background temperature by the relation \( v_s^2 = 5T/3m_i \) (\( m_i \) being the ion mass), for a sufficiently cold and magnetized plasma disk we are led to search for new kind of instabilities, able to trigger the turbulent behavior.

### 3. THERMOMAGNETIC DYNAMICAL PARADIGM

We analyze the equilibrium configuration in the presence of a magnetic field strong enough to break down the MRI paradigm. In this respect, we consider the collisional effect described by the Nernst coefficient \( \mathcal{N} \) [Lifshitz, Pitaevskii (1981)], which emerges when the temperature gradient does not vanish. Such a transport coefficient is determined by the first-order correction to the equilibrium distribution function when the Boltzmann equation is expanded in powers of the synchrotron frequency inverse. In this case, the dissipative energy density flux \( q \) is written as
\[ q = \mathcal{N} \tau B \times J, \tag{2} \]
where \( T, B \) and \( J \) are the temperature, the magnetic field and the current in the disk, respectively, and we use units such that \( K_B = 1 \). Correspondingly, the generalized Ohm law assumes the following form
\[ E + v \times B/c = \mathcal{N} B \times \nabla T, \tag{3} \]
where \( E \) and \( v \) denotes the electric and velocity field, respectively. The microscopic expression of the Nernst coefficient reads as
\[ \mathcal{N} = -v_{ie}/[\sqrt{2\pi} c m_e \omega_{ie}], \tag{4} \]
where \( v_{ie} \) denotes the ion-electron collision frequency, \( m_e \) the electron mass and \( \omega_{ie} \) the electron cyclotron frequency.

Our analysis is implemented ([r, \( \phi, z \) indicate cylindrical coordinates]) at a fixed distance \( r = r \) (in the following, we denote quantities evaluated at this radius with the bar symbol) from the central body of mass \( M \) and the accretion disk is assumed to be thin, i.e., the half-depth \( H(r) \) verifies the inequality \( H \ll r \). The local equilibrium configuration is described by a first-order perturbation of the usual magneto-hydrodynamics (MHD) equations and we denote the background quantities with \( (...) \) and the fluctuations with \( (...) \). A generic variable \( A \) is thus perturbed near \( r \) as \( A = A_0 + A_1 \), with \( A_1 \ll A_0 \). We underline that the effects associated with the vector \( q \) are neglected in the background dynamics. Since we are dealing with small scale perturbations, the local approximation results are well-grounded and the radial variation of the zeroth-order quantities can be effectively frozen to a given fiducial radius.

The background configuration is specialized for an adiabatic equation of state \( p_0 \sim \rho_0^{\gamma/3} \), \( \rho_0(r, z) \) and \( p_0(r, z) \) being the disk thermodynamical pressure and mass density, respectively. The zeroth-order radial momentum conservation (which reduces to the balance of the gravitational force with the centripetal one) locally fixes the Keplerian nature of the disk angular frequency \( \Omega(r) \) as \( \Omega_0 = \Omega_K = \sqrt{GM/r^3} \). The background vertical local equilibrium determines instead the gravostatic profile of decay for the mass density (and for the pressure) as the vertical coordinate increases. Indeed, the equilibrium between the pressure and the vertical gravitational force simply results, in to
the background profile [Bisnovatyi-Kogan, Lovelace (2001)]
\[ \rho_0(\mathbf{r}, z) = \rho_0^0(1 - z^2/H^2)^{3/2}, \]
where \( H = \sqrt{2p_0/\mu_0 \mu_B} \),
with \( \rho_0^0 \) being the density of the pressure
Correspondingly, the thermodynamics of the disk plasma is
B_0 \cdot \nabla \ldots \right) = 0 \) (for an approach to TMI built up on the opposite limit, when the vertical
gradient of the temperature dominates the equilibrium
configuration of the disk and when the energy transport is neglected by assuming that the energy is irradiated outwards
the disk, i.e., Eqs. (17) and (8) are disregarded in the
perturbation scheme, see [Liverts, Mond, Urpin (2010)]). In this
scheme, the magnetic induction equation, at the first order
boundary, stands as
\[ \partial_t B_{1r} = 0, \quad (5a) \]
\[ \partial_t B_{1\phi} + \frac{1}{\Omega} B_{1r} = 0, \quad (5b) \]
\[ \partial_t B_{1z} + B_{0z} (\nabla \cdot v_1 + c\nabla \nabla^2 T_1) = 0, \quad (5c) \]
where, for the axial symmetry, we have neglected the \( \phi \)
derivative. While the magnetic tension vanishes identically, the
magnetic pressure, at first order, reads as \( p_{1r} = B_0 \cdot B_1 / 4\pi \) and plays a significant role in determining the disk
stability properties.

4 THERMOMAGNETIC INSTABILITY

Addressing the perturbation theory, alternatively to MRI, we deal with a fluctuation dependence orthogonal to the
background magnetic field, i.e., \( B_0 \cdot \nabla \ldots = 0 \) (for an approach to TMI built up on the opposite limit, when the vertical
gradient of the temperature dominates the equilibrium
configuration of the disk and when the energy transport is neglected by assuming that the energy is irradiated outwards
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stability properties.

The structure of the plasma velocity field leads us to
write the Euler equations for the perturbative dynamics in the
following form
\[ \rho_0 \partial_t v_{1r} - 2\rho_0 \Omega K v_{1\phi} + \partial_r (p_1 + p_{1r}^m) = 0, \quad (6a) \]
\[ \rho_0 \partial_t v_{1\phi} + \partial_z (p_1 + p_{1r}^m) = 0, \quad (6b) \]
\[ 2\partial_t v_{1z} + \Omega K v_{1\phi} = 0. \quad (6c) \]
Correspondingly, the thermodynamics of the disk plasma is
described by the evolution of the perturbed mass density \( \rho_1 \) and
of the pressure \( p_1 \), i.e.,
\[ \partial_t \rho_1 + \rho_0 \nabla \cdot v_1 = 0, \quad (7a) \]
\[ \partial_t p_1 + \frac{5}{3} \rho_0 \nabla \cdot v_1 + \frac{c}{6\pi} N\bar{T}_0 B_0 \cdot \nabla^2 B_1 = 0, \quad (7b) \]

Eq. (17b) can be rewritten in terms of the temperature
\( T_1 \), by using \( \bar{T}_0 = (\bar{n}_0 + n_1) / (\bar{n}_0 + n_1 + 1) \) (where \( n \) denotes
the number density), as
\[ \frac{3\rho_0}{2} \partial_t T_1 + \rho_0 \nabla \cdot v_1 + \frac{c}{4\pi} N\bar{T}_0 B_0 \cdot \nabla^2 B_1 = 0, \quad (8) \]
Finally, combining Eq. (7b) and (8), we easily obtain
\[ \partial_t (p_1/\rho_0) + \nabla \cdot v_1 - \partial_t (T_1/T_0) = 0, \quad (9) \]
and, deriving Eq. (9) with respect to time using Eqs. (5) to
express \( \partial_t B_1 \), we get the following forth order equation for
\( T_1 \) and \( v_1 \)
\[ \frac{3}{2} \frac{\partial^2 T_1}{\bar{T}_0} - \frac{2c N\bar{T}_0}{\beta} (\nabla^2 (\nabla \cdot v_1) + c\nabla \nabla^4 T_1) + \partial_t \nabla \cdot (\partial_t v_1) = 0, \quad (10) \]
where \( \beta = 8\pi \rho_0 / B_0^2 \).

If we now derive with respect to time Eq. (6a) using
Eq. (12b) to eliminate \( \partial_t v_1 \), we get the relation
\[ \rho_0 \partial_t^2 v_{1r} + \rho_0 \Omega K v_{1\phi} + \partial_r (p_1 + p_{1r}^m) = 0, \quad (11) \]
while the time derivative of the thermomagnetic pressure is provided by Eq. (10) and the corresponding magnetic term
(\( \partial_t p_{1r}^m \) can be cast via Eqs. (5) as
\[ \partial_t p_{1r}^m = \frac{1}{4\pi} B_0 \cdot \partial_r B_1 = -\rho_0 \Omega K \nabla \cdot v_1 + c\nabla \nabla^4 T_1, \quad (12) \]
where the Alfvén velocity reads \( v_A = B_0^2 / 4\pi \rho_0 \).

Dealing with a vertical background magnetic field and
working in axial symmetry, the request of an orthogonal
perturbation dependence leads us to \( v_{1r} = v_{1r} (r, t) \) and
\( T_1 = T_1 (r, t) \), only. In this scheme, we obtain the relation
\( \nabla \cdot v_1 = (1/\rho) \partial_r (r v_{1r}) \approx \partial_r v_{1r} = \partial_r (\partial_r \xi_r) \), where
we have used the small scale nature of the perturbations and
\( \xi_r = (\xi_r, \xi_\phi, \xi_s) \) denotes the plasma shift vector. Recalling that
\( v_{1r}^2 = 5\rho_0 / 3\rho_0 \), in the adiabatic case, Eq. (11) rewrites
by means of \( \xi_r \) as
\[ \partial_t \xi_r + \partial_t v_1 \xi_r = \left[ \frac{3}{5} v_r^2 + \frac{3}{5} \right] \partial_t ^2 \xi_r + \left[ -c\nabla v_1 \cdot \frac{3}{5} v_r^2 \partial_r \partial_r \frac{\partial_t (T_1 / \bar{T}_0)}{T_0} \right] = 0, \quad (13) \]
This equation must be coupled with Eq. (10) expressed in
terms of \( \xi_r \). In view of the linear nature of such equations,
we are able to consider the plane wave representation:
\( \xi_r = \xi_r \exp[i(k_t r - \omega t)] \) and \( T_1 = T_1 \exp[i(k_t r - \omega t)] \), where \( \xi_r \) and
\( T_1 \) are constant values. Defining now \( \Omega_a \equiv k_t v_A \), \( \Omega_\Omega \equiv k_t \Omega \)
and \( \Omega_N \equiv -c\nabla k^2 T_0 \) (we recall that \( N < 0 \)), Eqs. (10) and (13)
rewrites as
\[ \frac{T_1^3}{T_0} + 4\Omega_N + 2\Omega A \frac{\omega}{3\Omega^2 \omega^2 + 4\Omega_N} \omega \kappa \xi_r = 0, \quad (14) \]
\[ \omega^3 - (\Omega_\Omega + 3\Omega_A^2) / 5 + \Omega_N^2 \omega^2 - \frac{\Omega^2 \Omega_N}{\kappa} \frac{T_1^3}{T_0} = 0, \quad (15) \]
respectively. Combining the expression above, we finally get the following dispersion relation
\[ \omega^4 + \left( \frac{4\Omega^2}{3\beta} - \Omega_\Omega \right) \omega^2 + \frac{4}{3} \Omega_A^2 \Omega_N \omega + \frac{4\Omega^2}{3\beta} \left[ \Omega^2 - \left( 1 + \frac{\beta}{3} \right) \Omega_A^2 \right] = 0, \quad (16) \]
where we have set \( \Omega^2 = \Omega_\Omega^2 + \Omega_A^2 + \Omega_N^2 \) and we have used the adiabatic relation \( \Omega_A^2 = 5\beta \Omega_\Omega^2 / 6 \). Let us now divide
Eq. (14) by \( \Omega^2 \) and define the dimensionless quantities
\( Z_{A,N} \equiv \Omega_A \Omega_N / \Omega \), and we = \( -\omega / \Omega \). Eq. (16) rewrites as
\[ y^4 + \left( 1 - \frac{4\Omega_A^2 / 3\beta} {3} \right) y^2 - \frac{4}{3} \Omega_A^2 \Omega_N y + \frac{4\Omega^2}{3\beta} \left[ -1 - \left( 1 + \frac{\beta}{3} \right) \Omega_A^2 \right] = 0. \quad (17) \]
In this scheme, unstable modes clearly correspond to solutions with $\text{Re}[y] > 0$.

In the general case, there is always a mode fixed by the dispersion relation [17] that is unstable. This fourth-order algebraic equation admits two real solutions with positive and negative sign, respectively. There are other two solutions that can be complex conjugated or real depending on the values of the parameters; anyhow, the real part of such solutions is always greater than zero, resulting in two further unstable, in case oscillating, modes. In Figure 1, we plot the positive unstable solution which, in the limit of small unstable, in case oscillating, modes. In Figure 1, we plot the values of the parameters; anyhow, the real part of such interactions and small scales of the disk plasma, such instability is relevant for the local behavior of the configuration profile. However, the value of this evolution rate is at least two orders of magnitude below the MRI growth and therefore it is expected to be a higher order correction to the main unstable modes associated to the coupling of the differential disk rotation with the central object magnetic field, having a growth rate $\gamma \sim \Omega_K$. Nonetheless, the relevance of TMII we fixed above consists essentially in its intrinsic different morphology with respect to MRI, especially because it is associated to compressional unstable modes which can survive even in the region $\tilde{\beta} \sim 1$, where MRI is instead strongly suppressed. In such a regime, the obtained instability is expected to be a dominant effect and it provides a reliable alternative scenario to MRI in determining a micro-scale turbulence. In other words, when $\tilde{\beta} \sim 1$, TMII replaces MRI and in the outer regions of the disk its growth rate is comparable to the one MRI would have for higher $\tilde{\beta}$ values.

Concluding, we observe that, at very small spatial scales in the disk, for instance for characteristic wavelengths of the perturbations having the size of a meter, then such an effect strongly dominates the typical MRI behavior, becoming a really important feature in the disk micro-structure. Furthermore, considering such small scales is legitimated by the observation that the disk Debye length is always below the millimeter scale. Thus, no questions can arise on the predictive nature of our approach at the scale of the meter.

5 RESISTIVE EFFECTS

It is a well-known feature that dissipative terms correspond to a general damping effect on the perturbation modes. In the following, we focus our attention on the electric and thermal resistivity contributions to the fluctuation profile. In this scheme, the generalized Ohm law [3] and dissipative energy density flux rewrite as

$$E + v \times B/c = J/\sigma + N^2B \times \nabla T,$$

$$(21)$$

$$q = NTB \times J - \chi(\nabla T),$$

$$(22)$$

respectively. Here, the coefficients $\sigma$ and $\chi$ stand for electric and thermal conductivities, respectively. We note that the temperature gradient retains only the radial component and, therefore, $\chi$ has to be intended as referred to the orthogonal direction. The approximate values of these parameters are given in [Lifshitz, Pitaevski (1981); Braginski (1965)]. Moreover, the perturbed magnetic induction and temperature equations allow us to estimate: $B_1/T_1 = \sqrt{3}/4 \ T_0/\beta_0$. This relation permits to evaluate the parameter range in which each single term is relevant with respect to the Nernst contribution in the perturbation scheme. Interestingly, the conditions for neglecting both the resistive and thermal conductivity effects lead indeed to restrictions on the $\beta$ parameter. In fact, it is easy to show that the thermal resistivity needs sub-thermal magnetic fields for it to contribute significantly to the system, i.e., $\tilde{\beta} \gg 1$. Conversely, when $\tilde{\beta} \ll 4/3$, dissipation is mainly due to the electric resistivity ($\sigma$ takes almost the same value in correspondence to the orthogonal and parallel direction of the magnetic field).
Since we are interested in analyzing the plasma behavior in the small-$\beta$ limit, we will consider the sole presence of electric conductivity. The derivation of the new dispersion relation follows the same steps we took for the ideal case. Using Eq. (11), and deriving with respect to time the axial component of the new induction equation,

$$\partial_t B_{\perp} + \nabla \cdot (\nabla \cdot \mathbf{v}_1 + cN\nabla^2 T_1) + c^2 \partial^2 B_{\perp} / 4\pi \sigma = 0,$$

we obtain a set of coupled differential equations in $\xi$ and $B_{\perp 1}$. It is easy to verify that the ensuing dispersion relation stands as

$$y^4 + Z_R y^3 + \left(1 - \frac{4Z_R^2}{3\beta} \right) y^2 + \left[Z_R \left(1 - Z_R^2 \right) - \frac{4}{3} Z_R^2 Z_N \right] y - \frac{4}{3} Z_R^2 \left[1 - \left(1 + \frac{2}{3}\right) Z_N^2 \right] = 0.$$

where we introduced the quantity $Z_R = c^2 k^2 / 4\pi \sigma \Omega$. The positive solution of the equation above is depicted in Figure 2. The overall effect of electric resistivity is, as expected, a damping one of about the order of 50% with respect to the ideal case.

Finally, about the thermo-electromotive contribution to the Ohm law, we observe that, for perturbations propagating along the radial direction in a purely vertical field, the only surviving term is $\alpha_\perp (\nabla T)_\perp$ and it is easy to check that, in the considered parameter region, it can be neglected with respect to the Nernst term. In fact, the orthogonal thermo-electromotive coefficient takes the expression (see Lifshitz, Pitaevskii [1981], Braginskii [1965])

$$\alpha_\perp \simeq 0.36 \left( \frac{\mu_0}{\omega_{\text{Be}}} \right)^2.$$

The radial temperature gradient appears in both these two terms; thus, we have to compare $\alpha_\perp$ with respect to the modulus of the Nernst coefficient $N$ times the background magnetic field intensity $B_0$ which, by the definition of the Nernst coefficient, reads

$$|N|B_0 \simeq \frac{1}{2\pi e} \left( \frac{\mu_0}{\omega_{\text{Be}}} \right).$$

or equivalently

$$|N|B_0 \simeq \frac{T_0^2}{2\pi e^2 \sqrt{n_0 m_e \mu_0}} \left( \frac{\mu_0}{\omega_{\text{Be}}} \right)^2.$$

Hence, it is easy to verify that $|N|B_0 \gg \alpha_\perp$ holds true in typical accretion disks. As an example, considering a layer in which $T = 10^8 K$ and $n_0 = 10^5 cm^{-3}$, leads us to

$$|N|B_0 \simeq 1.16 \times 10^{10} \left( \frac{\mu_0}{\omega_{\text{Be}}} \right)^2.$$

However, in the case a parallel thermo-electromotive contribution were present, i.e., $\alpha_\parallel (\nabla T)_\parallel$, the situation is reversed, in the sense that this effect would overcome the corresponding Nernst contribution. Such a statement follows directly from the expression of $\alpha_\parallel$, i.e.,

$$\alpha_\parallel \simeq \frac{1}{e} \left( \frac{\mu_0}{T} - 4 \right),$$

$\mu_0$ being the electron chemical potential. Even when $\mu_0 \ll T$, this term clearly dominates the Nernst contribution but is associated with the vertical gradient of the temperature, as far as the inequality $\nu_{ie} \ll \omega_{\text{be}}$ is recognized, as for the present analysis. Nonetheless, this issue does not imply that the parallel thermo-electromotive effect would dominate also the Nernst term associated to the radial temperature gradient, when a vertical dependence of the perturbations is allowed. In fact, if we introduce such $z$-dependence, i.e., $k \cdot r = k_r r + k_z z$, it is easy to realize that our analysis holds as far as we have $k_z/k_r \ll \nu_{ie}/\omega_{\text{Be}}$. By other words, the contribution due to $\alpha_\parallel$ remains negligible as long as the vertical temperature gradient remains sufficiently smaller than the radial one.

6 CONCLUDING REMARKS

Concerning the linear behavior of a thin rotating plasma disk, the present analysis has demonstrated the existence of a TMI, whose main feature is the striking complementarity to MRI, namely it is relevant at very small scales in the disk and also for low values of the $\beta$ parameter. The emergence of this instability is just a direct consequence of the kinetic properties of a plasma embedded in a sufficiently strong magnetic field. In fact, the obtained unstable normal modes are triggered by a non-zero Nernst coefficient, as fixed by a first-order expansion of the Boltzmann equation in inverse powers of the cyclotron frequency $\omega_{\text{Be}}$. This effect is therefore present in any real disk for which such frequency is much larger than the ion-electron collision one, i.e., $\omega_{\text{Be}} \gg \nu_{ie}$. In terms of the plasma parameters, this is equivalent to impose that $T_0^2 / \sqrt{\beta \rho_0} \gg \sqrt{2\pi} ce^2 \ell$. This condition is verified in a sufficiently hot plasma with a relatively low density, but in any case it stands for an enough small $\beta$ value. We stress that the constraint above is consistent with the present analysis.

Despite the small value the Nernst coefficient takes in a typical accretion disk surrounding a compact astrophysical stellar object, TMI reaches growth rates even greater than the typical one of the MRI modes, as far as the scale of the linear fluctuations becomes of the order of the meter. This fact is even of large impact if we stress how, at such low spatial scales, MRI is completely suppressed. This new type of instability suggests that the small-scale evolution,
especially in view of a possible turbulent behavior, can play a crucial role in the large-scale behavior too. In fact, the energy associated with the small-scale fluctuation due to TMI is comparable or greater than that present in the MRI on astrophysical size.

Furthermore, we have demonstrated that the TMI is, differently from MRI, present in that region of the thermodynamic and magnetic parameters of the disk, which corresponds to very small values of the $\beta$ parameter. In fact, although in this regime the resistivity damping can not be neglected, TMI is not affected for a factor greater than one-half and it conserves all its efficiency in destabilizing the small scale profile of the disk. Putting together all these considerations, we are naturally led to regard the instability here derived as a significant new effect in characterizing the main mechanisms for the angular momentum transport across the plasma disk configuration and the correspondingly accretion rate.

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