DIAMAGNETIC AND EXPANSION EFFECTS ON THE OBSERVABLE PROPERTIES OF THE SLOW SOLAR WIND IN A CORONAL STREAMER

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ABSTRACT

The plasma density enhancements recently observed by the Large-Angle Spectrometric Coronagraph (LASCO) instrument on board the Solar and Heliospheric Observatory (SOHO) spacecraft have sparked considerable interest. In our previous theoretical study of the formation and initial motion of these density enhancements it is found that beyond the helmet cusp of a coronal streamer the magnetized wake configuration is resistively unstable, that a traveling magnetic island develops at the center of the streamer, and that density enhancements occur within the magnetic islands. As the massive magnetic island travels outward, both its speed and width increase. The island passively traces the acceleration of the inner part of the wake. In the present paper a few spherical geometry effects are included, taking into account both the radial divergence of the magnetic field lines and the average expansion undergone by a parcel of plasma propagating outward, using the expanding box model (EBM), and the diamagnetic force due to the overall magnetic field radial gradients, the so-called melon-seed force. It is found that the values of the acceleration and density contrasts can be in good agreement with LASCO observations, provided the spherical divergence of the magnetic lines starts beyond a critical distance from the Sun and the initial stage of the formation and acceleration of the plasmoid is due to the Cartesian evolution of MHD instabilities. This result provides a constraint on the topology of the magnetic field in the coronal streamer.

Subject headings: MHD — solar wind — Sun: corona — Sun: magnetic fields

1. INTRODUCTION

One of the most interesting findings of the LASCO instrument on board the Solar and Heliospheric Observatory (SOHO) spacecraft has been the observation of a continuous outflow of material in the solar streamer belt. An analysis performed using a difference image technique (Sheeley et al. 1997; Wang et al. 1998) has revealed the presence of plasma density enhancements, called “blobs,” accelerating away from the Sun. These plasmoids are seen to originate just beyond the cusps of helmet streamers as radially elongated structures a few percent denser than the surrounding plasma sheet, of approximately 1 \( R_{\odot} \) in length and 0.1 \( R_{\odot} \) in width. They are observed to accelerate radially outward maintaining constant angular spans at a nearly constant acceleration up to the velocity of 200–450 km s\(^{-1}\), in the spatial region between about 5 and 30 \( R_{\odot} \). It has been inferred that the blobs are “tracers” of the slow wind, being carried out by the ambient plasma flow (Sheeley et al. 1997).

The solar streamer belt is a structure consisting of a magnetic configuration centered on the current sheet, which extends above the cusp of a coronal helmet. The region beneath the cusp is made up of closed magnetic structures, with the cusp representing the point where separatrices between closed and open field lines intersect. Further from the Sun, at solar minimum, the streamer belt around the equator appears as a laminar configuration consisting of a thick plasma sheet with a density about 1 order of magnitude higher than the surrounding plasma, in which much narrower and complex structures are embedded. As a first approximation, moving from the center of the streamer in polar directions at radii greater than the radius of the cusp, the radial component of the magnetic field increases from zero, having opposite values on the two sides of the current sheet. As far as the flow distribution is concerned, the fast solar wind originates from the unipolar regions outside the streamer belt, while the slowest flows are located at the center of the sheet.

Explaining how both the slow and the fast component of the solar wind are accelerated is one of the outstanding problem in solar physics. Although the association between the slow solar wind and the streamer belt (e.g., Gosling et al. 1981) and between the fast wind and the polar coronal holes is broadly recognized, the mechanism that leads to such accelerations is still a matter of debate. Einaudi et al. (1999) developed a magnetohydrodynamic model (later extended to the compressible case in Einaudi et al. 2001) that accounts for many of the typical features observed in the slow component of the solar wind: the region above the cusp of a helmet streamer is modeled as a current sheet embedded in a broader wake flow. This and previous studies (Dahlburg et al. 2005).
1998; Dahlburg 1998) show that reconnection of the magnetic field occurs at the current sheet and that in the nonlinear regime, when the equilibrium magnetic field is substantially modified, a Kelvin-Helmholtz instability develops, leading to the acceleration of density-enhanced magnetic islands (Einaudi et al. 2001).

The problem we study has a roughly spherical symmetry, but in our simulations we approximate the region beyond the cusp of a helmet streamer with a rectangular box, and the geometry of the fields with a Cartesian geometry, instead of a spherical one. To perform the numerical calculations we use a two-dimensional numerical code with periodic boundary conditions in the streamwise direction (the spatial direction aligned with the mean flow) and nonreflecting boundary conditions in the cross-stream direction (the spatial direction along which the initial mean flow varies). A key feature, at the same time an advantage and a limitation of our analysis, is that thanks to the periodic boundary conditions that we use in the direction of the flow we are allowed to proceed in all our numerical simulations in a pseudo-Lagrangian fashion: i.e., the size of the computational box is fixed, and then the time evolution of the spatial region moving with the box is followed in time, simply because what goes out from one side comes in from the other. In this case we follow the development of the magnetic island during its acceleration outward along the radial direction. With respect to spatially developing studies of the solar wind, our time developing method is able to reach a higher resolution. In our case, in fact, the computational domain is smaller and we are allowed to increase the grid resolution along the current sheet in order to resolve the small scales developing during the nonlinear stage, thus enabling us to study the development of a resistive-like instability that would be lost in a spatially developing study with a much lower resolution. In addition, so far no one has been able to study this phenomenon in spherical geometry, and we are planning to do it as the next step in the development of our model.

In the present paper we introduce two major effects related to the spherical symmetry of the problem we study. In what follows we suppose that when the magnetic island, the “blob,” moves outward along the radial direction it undergoes a spherical expansion due to the approximate spherical topology of the physical fields (Fig. 1). Although the subsequent algebra is quite complicated, as this approach enables us to attach the problem thanks to the availability of enough grid points, we still want to use the previous pseudo-Lagrangian time developing technique. This necessarily requires the use of a rectangular computational box and fields with a Cartesian geometry, but we can include the expansion undergone by a parcel of plasma propagating outward by making the box expand at the correct rate. This is done by inserting a suitable force term in the momentum equation, a generalization to the variable velocity regime of the expanding box model (EBM) developed by Grappin & Velli (1996) and Grappin et al. (1993), as described in the following sections. The other spherical effect included in the present paper is the so-called melon-seed force. This is a diamagnetic force that a region of closed magnetic field lines undergoes when it is embedded in a radially divergent background magnetic field (see Parker 1954, 1957, and Schmidt & Cargill 2000). While in a spherical geometry
this force would rise naturally, in our Cartesian approximation we take it into account by inserting a suitable force term in the momentum equation, in analogy with the EBM.

Section 2 describes the governing equations as well as the initial and boundary conditions, the last ones described with more details in the Appendix. In § 3 we detail our numerical results. Section 4 contains a discussion of our results, as well as some concluding remarks.

2. PHYSICAL MODEL

2.1. Governing Equations

In our analysis and simulations we use the compressible, isothermal, dissipative, three-dimensional magnetohydrodynamic (MHD) equations, which we write here in dimensionless form

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \]

\[ \rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla P + \mathbf{B} \cdot \nabla \mathbf{B} + \frac{1}{\mathcal{R}} \nabla \cdot \mathbf{\xi}, \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{\mathcal{R}_M} \nabla^2 \mathbf{B}, \]

supplemented by the equation of state \( P = \rho T \). In the above equations \( \rho(x, t) \) is the mass density, \( \mathbf{u}(x, t) \) is the flow velocity, \( P(x, t) \) is the thermal pressure, \( \mathbf{B}(x, t) \) is the magnetic induction field, \( T(x, t) \) is the plasma temperature and \( \mathbf{\xi} = (\partial \mathbf{u}/\partial x_j) + (\partial \mathbf{u}_j/\partial x) - \frac{2}{3} \delta_{ij} \nabla \cdot \mathbf{u} \) is the viscous stress tensor. To render the equations nondimensional, we used the characteristic quantities \( L^*, u^*, \rho^* \) and the related quantities \( t^*, \mathcal{R}^*, \mathcal{R}_M^* \):

\[ t^* = \frac{L^*}{u^*}, \quad \mathcal{R}^* = m_p u^*^2, \quad \mathcal{R}_M^* = \frac{4 \pi u^* L^*}{\eta c^2}. \]

To ensure the solenoidality of the magnetic field, and supposing that there is no variation of the fields along \( z \), we introduce the magnetic potential \( \phi \) defined by

\[ \mathbf{B} = \nabla \times (\mathbf{\phi} e_z) + \mathbf{B}_e e_z \]

and replace equation (3) with

\[ \frac{\partial \phi}{\partial t} = \sum_{j=1}^{3} u_j \frac{\partial \phi}{\partial x_j} + \frac{1}{\mathcal{R}_M} \left( \frac{\partial^2 \phi}{\partial x_j \partial x_j} + \frac{\partial^2 \phi}{\partial \eta \partial \eta} \right), \]

\[ \frac{\partial \mathbf{B}_e}{\partial t} = -\mathbf{B}_e \sum_{j=1}^{3} u_j \frac{\partial \phi}{\partial x_j} + \sum_{j=1}^{3} B_j \frac{\partial \phi}{\partial \eta} \frac{\partial u_j}{\partial \eta} + \sum_{j=1}^{3} \frac{\partial B_j}{\partial \eta} \frac{\partial u_j}{\partial \eta} + \frac{1}{\mathcal{R}_M} \left( \frac{\partial^2 B_j}{\partial x_j \partial x_j} + \frac{\partial^2 B_j}{\partial \eta \partial \eta} \right). \]

The equations solved in our numerical simulations depend on the problem that is studied: equations (1)–(2) and (7)–(8) are solved when we do not take into account the expansion and the diamagnetic force, whereas when we consider these last effects some new force terms are added to equations (1)–(2) and (7)–(8), and a coordinate transformation is implemented for a matter of convenience, as described below in this section. In any case the numerical problem is simplified by assuming that there is no variation in one of the spatial coordinates (\( z \)). In the following we refer to the spatial coordinate aligned with the mean flow as the streamwise direction (\( x \)), the spatial coordinate along which the mean flow varies as the cross-stream direction (\( y \)), and the remaining (spanwise) direction as \( z \). The system has periodic boundary conditions in the streamwise direction, along which a Fourier pseudospectral method is used in the numerical computations, and nonreflecting boundary conditions, achieved via the method of projected characteristics (Thompson 1987, 1990; Vanajakshi et al. 1989), in the cross-stream direction, where a compact finite difference scheme of the sixth order is used (Lele 1992). This method, coupled with a hyperbolic tangent mesh stretching around the current sheet, allows reasonable Reynolds numbers to be achieved without increasing the number of grid points dramatically. We solve the equations in a two-dimensional box whose dimensions are \( l_x \times l_y = \pm 10.63 \times 2 \pi / \alpha \), where \( \alpha \) is the streamwise wavenumber (the value of the box length along the \( x \)-direction is the result of the mesh stretching). Time is discretized with a third-order Runge-Kutta method. In the simulations that we present in this paper we used a numerical grid with \( n_x \times n_y = 401 \times 256 \) points, and for all simulations the Reynolds numbers are \( \mathcal{R} = \mathcal{R}_m = 200 \). For a better description of the numerical techniques used in our code see Einaudi et al. (2001).

The region where the acceleration of the solar wind occurs would be more aptly described in spherical coordinates, but in order to perform our numerical simulations we do a Cartesian approximation; i.e., we approximate the fields topology with a Cartesian one. In this way we can use our pseudo-Lagrangian technique using a two-dimensional code. As previously said, we include the spherical expansion undergone by a parcel of plasma propagating outward by making our computational box expand at the correct rate. The region we study (see Fig. 1) is a spherical sector of angular extent \( \pm 90 \) in the cross-stream direction (\( y \)) and whose distance from the center of the Sun is \( R(t) \), where \( R \) changes in time while the plasma flows outward along the radial direction. We approximate this region with a parallelepiped whose cross lengths are, respectively, \( a \) and \( b \) along the directions \( x \) and \( z \), and whose length in the radial direction is \( L \). We suppose that when the plasma present in this region moves outward it undergoes an average expansion that we approximate with the expansion of the box (see Fig. 1). When following the evolution in the comoving frame of reference a noninertial force term arises. In the Appendix it is shown that the force field for a unit mass that, inserted in the momentum equation, yields the correct expansion of the computational box is

\[ f_{\text{exp}}(x, t) = \frac{\tilde{R}(t)}{R(t)} \delta(x, 0, z). \]

In the previous equation the velocity of the box and its distance from the center of the Sun are defined as

\[ \tilde{R}(t) = \frac{1}{L_y} \int_0^{L_y} u_y(0, y, 0, t) \, dy, \]

\[ R(t) = R_0 + \int_0^t \tilde{R}(t') \, dt'. \]
i.e., we define the velocity of the box as the average velocity in the \( y \)-direction along the center of the computational box. The basic numerical algorithm was developed and tested by Grappin et al. (1993) and Grappin & Velli (1996), who showed that the EBM gives a correct description of the evolution of waves (exact wave-action conservation) and turbulence in an expanding medium as well as capturing basic features of the expanding solar wind such as the formation of corotating interaction regions and forward and reverse shocks between high- and low-speed solar wind. The EBM does not conserve energy because in an expanding flow work is done by the plasma. Angular momentum and magnetic flux are conserved exactly, however (see Grappin & Velli [1996] for a detailed description and tests of the physical model). The force term (eq. [9]) introduces a variation along the \( x \)- and \( z \)-coordinates, but this explicit coordinate dependence cancels when we make a coordinate transformation to a self-similarly expanding transverse coordinate \( x'(x, t) \) defined by

\[
x' = \frac{R_0}{R(t)} x, \quad y' = y, \quad z' = \frac{R_0}{R(t)} z.
\]

(12)

It can be easily verified that in this way the expanding computational domain coordinate is transformed to a fixed computational grid. The equations solved by our two-dimensional numerical code and all the mathematical details are extensively explained in the Appendix.

When we make our Cartesian approximation, we approximate the radially diverging magnetic field lines with straight ones, losing the radial nonhomogeneity of the solar magnetic field. An effect that is lost with this approximation is the so-called melon-seed force, a diamagnetic force acting on a region of closed magnetic field lines. It is known (see Schmidt & Cargill 2000; Pneuman & Cargill 1985; Parker 1954, 1957) that a region of closed magnetic field lines embedded in a background radially diverging magnetic field is subject to such force. If we model this region as a prolate spheroid of volume \( V \) elongated in the radial direction it can be demonstrated that the value of the total force acting on the spheroid is approximately

\[
F_m = -V \left[ \frac{d}{dR} \left( \frac{B_z^2}{8\pi} \right) \right] \hat{y},
\]

(13)

where \( B_z = B_c(R_0/R)^2 \) is the background magnetic field, \( B_c \) being constant. In our model axial invariance is assumed; i.e., there is no variation in the \( z \)-direction in the Cartesian approximation. Hence the prolate spheroid is modeled as a cylinder with an elliptical cross section, and the same force for unit volume is used. The force given in equation (13) is the total force acting on this plasmoid. The force per unit volume derived from this equation and used in our computations is given by

\[
f_m(x, y, t) = \Theta(x, y, t) \frac{B_{c0}^2}{2\pi R_0} \left( \frac{R_0}{R} \right)^5 \hat{y},
\]

(14)

which, written in nondimensional form, is given by

\[
f_m(x, y, t) = \Theta(x, y, t) \frac{2B_{c0}^2}{R_0} \left( \frac{R_0}{R} \right)^5 \hat{y},
\]

(15)

in which \( \Theta \) is a field whose value is \( \Theta = 1 \) inside magnetic island and \( \Theta = 0 \) outside. In this way the field (eq. [14]) integrated over the volume of the plasmoid gives as a result the total force (eq. [13]).

The region above the cusp of a helmet streamer is assumed to be in local hydrodynamic equilibrium; therefore, gravity, magnetic curvature forces, and the flow all balance out. Once density fluctuations arise, the gravitational force reappears as an Archimedes buoyancy force acting on regions with a different density than the average. This force field can be written in nondimensional form as

\[
f_G = -k_G \frac{\Delta \rho}{R^2}, \quad \text{where} \quad k_G = \frac{GM_\odot}{L^*(u^*)^2},
\]

(16)

\( G \) being the gravitational constant and \( M_\odot \), the solar mass. Taking as characteristic length and velocity, the width of the wake \( L^* = \alpha v = 0.1 R_c \) and the velocity of the fast solar wind \( u^* = u_{\infty} \approx 6 \times 10^8 \text{ m s}^{-1} \), we obtain the value \( k_G = 5.3 \). We can now compare the relative strength of this buoyancy force with that of the melon-seed force (eq. [15]). As shown in the next paragraph \( \Lambda \approx 2.5, \quad R_0 = 60 \) (i.e., 6 radii in conventional units, and supposing \( \Delta \rho = 1 \)), we obtain near the cusp of the helmet streamer (\( R = R_0 \)):

\[
\frac{f_m}{f_G} \sim \frac{2A^2}{3R_0} \sim 100;
\]

(17)

i.e., the buoyancy force is negligible with respect to the melon-seed force near the cusp of the helmet streamer and in the region of interest of the simulations presented in the following paragraphs.

In the next section we introduce as initial conditions the fields (eqs. [20]–[22]). These fields are a one-dimensional equilibrium for the region beyond the cusp of a helmet streamer; i.e., the variation of the fields is only along the cross-stream direction (the \( x \)-coordinate) and solar corona Reynolds numbers are so high that they are substantially an equilibrium solution of equations (1)–(7) over the timescales of our simulations. But in our computational domain, Reynolds numbers are limited by resolution to values around \( R = R_m = 200 \), values so small that unphysical diffusion of equilibrium fields would take place. In the foregoing equations the pattern of the diffusion terms is

\[
\frac{\partial f}{\partial t} = \ldots + \frac{1}{R} \frac{\partial^2 f}{\partial x^2},
\]

(18)

where \( f \) is a generic field. As the variation of the equilibrium fields is only along the \( x \)-coordinate, i.e., as a Fourier series in the \( y \)-direction they have only a \( k = 0 \) mode, we avoid such an unphysical diffusion, using, instead of equation (18), the following one:

\[
\frac{\partial f}{\partial t} = \ldots + \frac{1}{R} \frac{\partial^2 f}{\partial x^2} (f - \tilde{f}_0).
\]

(19)

In this way the modes \( k = 0 \) of the Fourier series in the \( y \)-direction, which we indicate with \( \tilde{f}_0 \), do not diffuse. This could have potentially given rise to development of excessively small scales in \( x \) on the \( k = 0 \) harmonic but empirically the numerical filtering used along this direction has been enough.

2.2. Initial Conditions

We model a planar section of the solar streamer belt, in the region beyond the cusp of a helmet streamer, as a magnetic current
sheet of thickness $a_B$ embedded in the center of a broader wake flow of thickness $a_V$. The plasma flows at the speed of the fast solar wind at the edges and at a much lower velocity at the sheet. If we take as characteristic length the thickness of the wake $L^* = a_V$ and as characteristic density and velocity those of the fast wind $\rho^* = \rho_{\infty}$, $u^* = u_{\infty}$, the basic fields, written in non-dimensional form, are then given by

$$u_{2y}(x) = 1 - \text{sech}(x),$$

$$B_{2y}(x) = A \tanh(\delta x), \quad B_{2x}(x) = A \text{sech}(\delta x),$$

$$\rho = 1, \quad T = \frac{1}{M^2},$$

where $\delta = a_V/a_B$ is the ratio of the two widths, and $A$ and $M$ are, respectively, the Alfvén number (the ratio between the Alfvén speed $c_{Alf}$ and the flow speed $u_{\infty}$) and the sonic Mach number (the ratio between the flow speed $u_{\infty}$ and the sound speed $c_{s\infty}$) of the system at the edges, i.e., of the fast wind:

$$A = \frac{B_{\infty}}{\sqrt{4\pi \rho_{\infty} u_{\infty}}} = \frac{c_{Alf}}{u_{\infty}}, \quad M = \frac{u_{\infty}}{\sqrt{T_{\infty}}} = \frac{u_{\infty}}{c_{s\infty}}. \quad (23)$$

Our wake-current sheet model is then characterized by three parameters, $A$, $M$, and $\delta$, which vary considerably in the solar corona as a function of the distance from the Sun. In this paper we are interested in studying the time and spatial evolution of a parcel of plasma starting from the region immediately beyond the cusp of a helmet streamer. In this region the typical Alfvén velocity $c_{Alf}$ is approximately 1000–1500 km s$^{-1}$, the sound speed $c_{s\infty}$ is about 100 km s$^{-1}$, and the velocity of the fast solar wind $u_{\infty}$ is $\sim 600$ km s$^{-1}$. It follows that $A = c_{Alf}/u_{\infty} \sim 1500/600 = 2.5$, and $M = u_{\infty}/c_{s\infty} \sim 600/100 = 6$. The plasma $\beta$ is also a useful quantity, defined as $\beta = 2\rho_0|B|^2 = 2c_{s\infty}^2/c_{Alf}^2 = 2/(AM)^2$. In this case we have $\beta \sim 0.0089$. In our numerical calculations we use values of these parameters that are close to these estimates but computationally more accessible. Thus, in our runs $A = 1.5$ and $M = 2$ (hence $\beta \sim 2/9 = 0.22$). Unfortunately, there is no direct observation of the heliospheric current sheet close to the Sun ($r < 10$ Rs$)\); hence, there is no measurement of the parameter $\delta$. But from the topology of both the magnetic and the velocity fields it is reasonable to suppose $\delta \gg 1$, i.e., a width of the wake flow much bigger than that of the current sheet (see Stibring et al. 1996; Einaudi et al. 1999). Such a high value of $\delta$ is not computationally accessible; moreover, we have seen in Einaudi et al. (1999) that the impact of the value of $\delta$ is mainly to change the critical value of $A$ below which the dynamics is dominated by the flow. Therefore, in this paper we have chosen $\delta = 5$, which corresponds to a magnetically dominated regime.

3. RESULTS

This section presents the results of our numerical simulations. In § 3.1 we show for reference the result of a numerical simulation in which both the melon-seed force and the spherical expansion are neglected. In § 3.2, in order to show the influence of the expansion on the dynamics of the wake-current sheet system, we present the results of a numerical simulation in which the geometrical expansion is implemented while the melon-seed force is neglected. At last, in § 3.3 we show the results of a simulation in which both the melon-seed force and the spherical expansion are implemented.

If there were no magnetic field ($A = 0$), the wake-current sheet system (eqs. [20]–[22]) would be unstable for a Kelvin-Helmholtz–like instability. Two unstable modes exist in this limit: a varicose (sausage-like) mode and a sinuous (kink-like) mode. The stabilizing effect of a magnetic field parallel to the flow over the Kelvin-Helmholtz instability is well known (e.g., Chandrasekhar 1961), and in fact our linear studies (Dahlburg et al. 1998; Einaudi et al. 1999) have found that increasing the value of the Alfvén number $A$ beyond a critical value the two ideal modes are stabilized. On the other hand, increasing the strength of the magnetic field leads the system to be unstable to a tearing-like instability. This resistive varicose mode has the same spatial symmetry of the varicose fluid mode, and while in the linear regime the dynamics is not substantially modified by the presence of the flow, developing like a classic tearing instability, in the nonlinear regime the presence of the flow has a dramatic influence. In fact, as the perturbed magnetic and velocity fields attain finite amplitude, the resulting nonlinear stresses along the magnetic island borders (Dahlburg 1998) lead to a transfer of momentum between the flow and the magnetic island. As a result the magnetic island and the central part of the wake are accelerated. The coupling between magnetic reconnection and Kelvin-Helmholtz instability that develops in the dynamics of this mode has been studied in Einaudi et al. (1999).

In all calculations a small amplitude perturbation of the resistive varicose type (see Dahlburg et al. 1998) at the wavenumber $\alpha = 0.42$ was added to the initial state. Although our numerical code was developed to study the nonlinear dynamics, we can estimate (via a fitting during the linear regime) the growth rates of the perturbations at various wavenumbers. Figure 2 shows the dispersion relation for resistive instabilities obtained with this method. It is evident from this figure that we have not chosen the fastest growing wavenumber for our nonlinear calculations. The reason is related to the coupling of the magnetic reconnection instability and of the Kelvin-Helmholtz instability. The initiation of the Kelvin-Helmholtz instability occurs...
when the resistive instability is well into its nonlinear regime. The maximum desired value for $\alpha$ is limited by the well-known fact (e.g., Ray 1982; Ferrari & Trussoni 1983) that the Kelvin-Helmholtz instability is stabilized for wavelengths approaching the thickness of the shear layer (in our case the amplitude of the wake flow $a_V$), in this way the wavelengths that give rise to a more efficient acceleration of the wake are the longer ones (compared to the amplitude of the wake). The minimum desired value for $\alpha$ is determined by the number of subharmonics we want to include, to allow for vortex merging. For this reason, based on the Kelvin-Helmholtz dynamics, we have picked up the value $\alpha = 0.42$ and not the wavenumber of the fastest growing resistive mode. A most comprehensive study of this mechanism is underway, in particular, a multi-wavelength one.

While the numerical computations have been carried out using nondimensional equations, for a matter of clarity the results will be presented in the ordinary dimensional units. As in this paper we are interested in studying the time and spatial evolution of a parcel of plasma starting from the region immediately beyond the cusp of a helmet streamer, i.e., $4\sim6 R_c$, we have chosen as characteristic quantities those characterizing this region. We have chosen as characteristic length the thickness of the wake $L' = 0.1 R_c$, as characteristic velocity that of the fast wind $u' = 600 \text{ km s}^{-1}$, and as characteristic density the current sheet electron density at $\sim 5 R_c$, i.e., $n = 5 \times 10^6 \text{ cm}^{-3}$ (Guhathakurta et al. 1996). From these quantities we derive the characteristic time $t' = L'/u' \sim 116 \text{ s}$. Particular care should be taken when considering the times presented in the following simulations. Our numerical simulations have been in fact performed using Reynolds numbers values ($R = 200$) much lower than coronal values. While most of the nonlinear dynamics is due to ideal processes (i.e., Kelvin-Helmholtz instability), which are little affected by dissipation, the growth rate of the tearing-like instability developing in the linear regime is strongly affected by the Lundquist number; hence, the timescale of the linear stage must be rescaled.

With a Lundquist number of the order $S = 10^9$ the linear phase should last approximately 5 hours (Einaudi et al. 1999). This time should be added to the times of the simulations presented in the following paragraphs.

3.1. Run A—No Expansion or Melon-Seed Force

In this simulation we consider neither the expansion nor the diamagnetic force. While the parameters are somewhat different, we reproduce the essential evolutionary features reported by Einaudi et al. (1999, 2001). Figure 3 shows the formation and acceleration of density-enhanced magnetic islands. These magnetic islands form with streamwise length equal to the perturbation wavelength. The magnetic island cross-stream width grows from a small amplitude in the linear regime to the order of $a_g = 0.02 R_c$, the width of the current sheet, at the beginning of the nonlinear stage ($t \sim 77 \text{ minutes}$). While magnetic reconnection continues to occur in the nonlinear regime, the Kelvin-Helmholtz instability also is triggered, leading to the acceleration of the central part of the wake (Fig. 4) up to the value $\sim 500 \text{ km s}^{-1}$ at $t = 309 \text{ minutes}$.

The value of the density enhancement ($n_{\text{max}}/n_b$, defined as the ratio of the maximum density inside the magnetic island over the background density value) steadily increases from an initial uniform value ($n_{\text{max}} = n_b = 5 \times 10^5 \text{ cm}^{-3}$) up to a value that is roughly 20 times greater ($n_{\text{max}} \sim 9.7 \times 10^6$) at $t = 309 \text{ minutes}$. Whereas the formation and evolution of this density-enhanced magnetic island is very similar to that of the observed “blobs” (i.e., we have a dense, accelerated plasmoid), both the density enhancement and the acceleration profile result in larger values than the ones observed (Sheeley et al. 1997; Wang et al. 1998). This has been one of the main reasons to study the more realistic configurations described in the next two sections. We show in the next section that when expansion effects are included, such a peaked density does not occur because, as expected, while the magnetic island travels outward there is a rarefaction of the island due to the expansion.
Fig. 4.—Run A; streamwise component of the velocity ($u_y$) averaged in the $y$-direction as a function of $x$ (the cross-stream direction) at selected times. When the nonlinear regime develops (after $t \sim 58$ minutes), while magnetic reconnection continues to occur, a Kelvin-Helmholtz–like instability is triggered, leading to the acceleration of the central part of the wake up to the value $\sim 500$ km s$^{-1}$.

Fig. 5.—Run B; expansion but no melon-seed force included in this simulation. Time evolution of magnetic islands is shown. Magnetic field lines (white lines) and electron density filled contours (colors show the value of the density in accordance with the color scale) at selected times show density-enhanced magnetic islands forming and accelerating outward. The $x$- and $z$-axes are, respectively, along the vertical and horizontal direction and are measured in solar radii units ($R_\odot$). The initial distance between the box and the center of the Sun ($R_0$) is $6 \ R_\odot$, during time evolution this distance increases, and at the selected times showed in the Figure the values of $R(t)$ are given by 6.02, 6.2, 7.11, 9.32, 11.86 $R_\odot$. The parameter $n_b$ is the average electron density at the top and bottom $z$ boundaries, as the box expands its value decreases as $(R_0/R)^2$. A $\sim 5$ hour time should be added to the times showed to keep count of the rescaled linear timescale.

Fig. 6.—Run B; streamwise component of the velocity ($u_y$) averaged in the $y$-direction as a function of $x$ (the cross-stream direction) at selected times. Expansion effects imply a weakening of cross-stream gradients of magnetic and velocity fields, leading to a lower acceleration profile.
3.2. Run B—Expansion but No Melon-Seed Force

This run provides us with information on the effects of the geometrical expansion on the dynamics of the wake-current sheet model for slow solar wind formation and acceleration. In this section we neglect the diamagnetic force. We account for this force in § 3.3.

We used as starting radius $R_0 = 6 R_\odot$, approximately the radius of the region located near the cusp of a helmet streamer. During the time evolution the radius $R$ increases its value up to $R \sim 12 R_\odot$ at $t = 309$ minutes. From Figure 1 we can clearly notice that while the plasmoid moves outward, the box undergoes an expansion. Its volume increases as $(R/R_0)^2$, leading to an average decrease of the mass density as $(R_0/R)^2$, as the values of the background density $n_b$ indicated in Figure 5 show.

The dynamical evolution (Fig. 5) for the expansion case exhibits some crucial differences from that described in the previous section for the nonexpanding case. Again we have the formation of a density-enhanced magnetic island, but as now expansion is occurring the average density of matter decreases as $(R_0/R)^2$, and the density enhancement is lower. The density enhancement $n_{\text{max}}/n_b$ for the selected times shown in Figure 5 range between 2.5 and 5. This value is closer, with respect to the nonexpanding simulation presented in the previous paragraph, to the observed one, which is of the order of 7% ± 2% of the background K corona (Sheeley et al. 1997). Dahlburg (1998) showed that the transfer of energy to the perturbed fields from the background magnetic and velocity fields depends strongly on the cross-stream gradients of these fields. On the other hand the expansion of the box implies flow velocities orthogonal to the radial direction,
which lead to a weakening of these field's gradients (Grappin & Velli 1996) and as a consequence to a lower acceleration profile, shown in Figure 6.

In Figure 7 we plot the average streamwise velocity at selected $x_n$-values [here $x_n$ is the nonexpanding numerical grid, related to the expanding physical grid $x$ by the relation $x = x_nR(t)/R_0$] as a function of the distance between the box and the center of the Sun, which reaches the value $R \sim 12 R_\odot$ in conventional units, at $t \sim 309$ minutes. This velocity profile is in good quantitative agreement with the one obtained by observations (compare with Fig. 4 on page L166 of Wang et al. 1998).

### 3.3. Run C—Expansion and Melon-Seed Force

In this section we present the results of a numerical simulation in which both the geometric expansion and the diamagnetic, or melon-seed, force are implemented.

As detailed at the end of § 2.1 the region above the cusp of a helmet streamer is assumed to be in local hydrodynamic equilibrium, therefore gravity, magnetic curvature forces and the flow all balance out. Once density fluctuations arise, the gravitational force reappears as an Archimedes buoyancy force acting on regions with a different density compared to the average. We have shown that this buoyancy force is negligible with respect to the melon-seed force in the region of interest of our simulation.

The expression for the melon-seed force (eq. [15]) holds for a magnetic island embedded in an external magnetic field $B_e = B_{e0}(R(t)/R_0)^2$. We compute the value of $B_e$ as the value of the magnetic field at the outer border of the magnetic island. Hence the value of the melon-seed force is very small during the linear stage, but as non-linear stage occurs, both the Kelvin-Helmholtz and the diamagnetic forces synergistically act to accelerate the magnetic island, leading to an over-acceleration.

On the other hand, the expression of the diamagnetic force (eq. [15]) holds for a plasmoid in a radial divergent background magnetic field. Indeed we have implemented this diamagnetic force supposing that the magnetic field in the region just beyond the cusp of a helmet streamer diverges radially. Unfortunately, there is no direct observation of the topology of the magnetic field in this region. But as the resulting acceleration profile is not observed (Sheeley et al. 1997; Wang et al. 1998), we conclude from the results of this simulation that in such a region the magnetic field lines do not diverge (in other words, the melon-seed force must be much smaller than the semiempirical estimate used in our simulations). Both more refined observations of this region and theoretical studies, including high-resolution simulations in a fully spherical geometry, are desirable for establishing what kind of magnetic topology we have in this region.

### 4. DISCUSSION

In this paper, we have refined the model presented in Einaudi et al. (1999, 2001), for the formation and acceleration of the slow component of the solar wind within a coronal streamer. In particular, we have been concerned with the study of the initial stage of the process. The main results of the present paper can be summarized as follows:

1. The expanding box generalization of the wake-current sheet model exhibits the formation and outward acceleration of density-enhanced magnetic islands. The plasmoid width, length, and acceleration profiles are in good agreement with LASCO observations.
2. The geometrical expansion by itself has a stabilizing effect on the Kelvin-Helmholtz instability developed in the nonlinear regime. It also is responsible for a rarefaction of the accelerated plasmoid.

3. From our simulations there is a strong indication that the topology of the magnetic field lines in the region close but beyond the cusp of a helmet streamer is not radially divergent. This divergence would lead to as yet unobserved acceleration and density profiles due to the diamagnetic force. Note, however, that other effects (i.e., drag from the denser current sheets) might be important as well.

In conclusion, these computations reinforce the scenario that the structure and acceleration of the slow solar wind can be attributed primarily to the intrinsic instability and subsequent evolution of the underlying wake-current sheet system. To reproduce the observational details properly, further theoretical work and additional solar data are needed. From the theoretical point of view, we need a study of more complex and realistic basic configurations, where an initial density profile peaked on the current sheet is present, and a non-force-free magnetic field is chosen.

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APPENDIX

As previously said, we suppose that when the magnetic island moves outward the plasma present in the region in and around it undergoes a spherical expansion due to the approximate spherical topology of the physical fields (Fig. 1). When a particle or a fluid element moves outward along the radial coordinate \( r \) at fixed azimuthal and polar angles, the ratios \( x/r \) and \( z/r \) are constant in time. For the sake of simplicity consider a system of spherical coordinates where \( \theta \) is the polar angle from the \( y \)-axis (with \( 0 \leq \theta \leq \pi \)), \( \phi \) is the azimuthal angle in the \( xy \)-plane from the \( z \)-axis (with \( 0 \leq \phi < 2\pi \)), and \( r \) is the distance from the origin. As usual, the relations between the spherical and the Cartesian coordinates are given by

\[
y = r \cos \theta, \quad z = r \sin \theta \cos \phi, \quad x = r \sin \theta \sin \phi. \tag{A1}
\]

It can be easily verified that the ratios \( x/r \) and \( z/r \) depend only by the angles \( \theta \) and \( \phi \); hence, for a particle or a fluid element moving along the radial coordinate these ratios are constant in time. In our case \( y = R(t) \) and for such a fluid element we can write

\[
\frac{x(t)}{R(t)} = \frac{x_0}{R_0}, \quad \frac{z(t)}{R(t)} = \frac{z_0}{R_0}, \tag{A2}
\]

where \( x_0, z_0, \) and \( R_0 \) are the initial values of these quantities. Differentiating with respect to time, we have

\[
\dot{x}(t) = \frac{x_0}{R_0} \dot{R}(t) = \frac{x(t)}{R(t)} \ddot{R}(t), \quad \dot{z}(t) = \frac{z_0}{R_0} \dot{R}(t) = \frac{z(t)}{R(t)} \ddot{R}(t). \tag{A3}
\]

This means that the cross-component of the velocity is given by

\[
u_\perp = \left( \frac{\dot{R}(t)}{R(t)} \right) \left[ x(t), \ 0, \ z(t) \right]. \tag{A4}
\]

This holds for a particle or a fluid element moving outward along the radial coordinate. To apply this to our subject, we replace this velocity with the fluid velocity and look for a force able to cause this fluid motion. In other words, in our picture a fluid that is expanding spherically is characterized by a velocity field whose cross-component is given by

\[
u_\perp(x, \ t) = \left( \frac{\dot{R}(t)}{R(t)} \right) (x, \ 0, \ z). \tag{A5}
\]

Hence we introduce a new force in the momentum equation corresponding to the spherical expansion effects, i.e., the force term whose solution is given by the previous term. To find this force term we simply insert in the momentum equation the velocity field \( u_\perp(x, \ t) \), which gives us the force field for unit mass:

\[
f_{\exp}(x, \ t) = \frac{\partial u_\perp}{\partial t} + (u_\perp \cdot \nabla)u_\perp = \left( \frac{\dot{R}(t)}{R(t)} \right) (x, \ 0, \ z). \tag{A6}
\]
We have chosen to define the velocity of the box $\dot{R}$ as the average velocity in the $y$-direction along the center of the computational box (see eq. [10]) and its distance from the center of the Sun $R$ accordingly (see eq. [11]).

In order to perform the numerical simulations the computational box must be fixed. For this reason we consider the coordinate transformation $x'(x, t)$, defined by

$$x' = \frac{R_0}{R(t)} x, \quad y' = y, \quad z' = \frac{R_0}{R(t)} z. \quad (A7)$$

It can be easily verified that this coordinate transformation maps the expanding physical domain (with the law given in eq. [A2]) into a fixed one. This coordinate transformation implies a transformation of the fields. To the field $f(x, t)$ corresponds a new field $g(x', t)$ defined by

$$f(x, t) = g(x'(x, t)). \quad (A8)$$

The following relations can be easily verified:

$$\frac{\partial f}{\partial t}(x, t) = \frac{\partial g}{\partial t}(x'(x, t), t) - [U(x'(x, t), t) \cdot \nabla'] g(x'(x, t), t) \quad (A9)$$

$$\nabla f(x, t) = \nabla' g(x'(x, t), t) \quad (A10)$$

where

$$U(x', t) = \left[ \frac{\dot{R}(t)}{R_0} x', 0, \frac{\dot{R}(t)}{R_0} z' \right] = \frac{\dot{R}(t)}{R_0} (x', 0, z') \quad (A11)$$

$$\nabla' = \left( \frac{R_0}{\dot{R}(t)} \frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{R_0}{\dot{R}(t)} \frac{\partial}{\partial z'} \right) \quad (A12)$$

If we define the transformed fields $\rho$, $T$, $u$, and $B$ with a primed index, for example,

$$u(x, t) = u'(x'(x, t)), \quad (A13)$$

and notice that the transformed expansion force field (eq. [A6]) has the following form:

$$f'_{\text{exp}}(x', t) = \frac{\ddot{R}(t)}{R(t)} (x', 0, z') = \frac{\partial U}{\partial t} (x', t), \quad (A14)$$

then the MHD equations (for clarity we explicitly include the isothermal equation) can be written in the new coordinate system and with the transformed fields as

$$\frac{\partial \rho'}{\partial t} = (U \cdot \nabla') \rho' - \nabla' \cdot (\rho'u'), \quad (A15)$$

$$\frac{\partial u'}{\partial t} = (U \cdot \nabla') u' + f'_{\text{exp}} - (u' \cdot \nabla') u' - \frac{1}{\rho'} \left[ \nabla' \left( p' + \frac{B'^2}{2} \right) - (B' \cdot \nabla') B' \right] + \frac{1}{\rho' R} \nabla' \cdot \xi', \quad (A16)$$

$$\frac{\partial B'}{\partial t} = (U \cdot \nabla') B' + \nabla' \times (u' \times B') + \frac{1}{R_{M}} \nabla'^2 B', \quad (A17)$$

$$\frac{\partial T'}{\partial t} = (U \cdot \nabla') T' - (u' \cdot \nabla') T', \quad (A18)$$

where

$$\xi'_{ij} = \xi_{ij} - \frac{2}{3} \nabla' \cdot u' \delta_{ij}, \quad \epsilon'_{ij} = \partial'_{ij} + \partial'_{ji}, \quad P' = \rho' T'. \quad (A19)$$

In order to simplify the previous equation, note that

$$\nabla' \cdot U = \frac{\dot{R}}{R}, \quad (A \cdot \nabla') U = \frac{\dot{R}}{R} \nabla'}, \quad (A20)$$
where \(A\) is a generic vector field and \(P_{\perp}\) is the projector in the cross-stream direction; i.e., \(P_{\perp}A = (A_1, 0, A_3)\). To render more simple our equations, we introduce the new velocity field \(\hat{u}(\mathbf{x}', t) = \mathbf{u}'(\mathbf{x}', t) - \mathbf{U}(\mathbf{x}', t)\). The previous equations can be rewritten as

\[
\frac{\partial \rho'}{\partial t} = -\nabla' \cdot (\rho' \hat{u}) - 2\rho' \hat{R} \hat{R},
\]

\[
\frac{\partial \hat{u}}{\partial t} = \left(f_{\exp} - \frac{\partial \hat{U}}{\partial t}\right) - (\hat{u} \cdot \nabla')\hat{u} \frac{\hat{R}}{R} P_{\perp} \hat{u} - \frac{1}{\rho'} \left[\nabla' \left( p' + \frac{B'^2}{2} \right) \right] - \frac{1}{\rho' R} \nabla' \cdot \chi',
\]

\[
\frac{\partial B'}{\partial t} = \nabla' \times (\hat{u} \times B') - 2 \hat{R} B' + \frac{\hat{R}}{R_{M}} P_{\perp} B' + \frac{1}{R_{M}} \nabla'^2 B',
\]

\[
\frac{\partial T'}{\partial t} = -(\hat{u} \cdot \nabla')T'.
\]

In particular, the first term between round brackets in the momentum equation (A22) cancels, thanks to equation (A14). In this way we have a problem that numerically is invariant along the z-direction and hence can be solved with a two-dimensional numerical code.

To simplify further the equations we introduce the average scalings of the physical fields obtained from the matter conservation law and the Alfvén theorem. The new fields are those with the hat:

\[
\rho = \left(\frac{R_0}{R}\right)^2 \hat{\rho}, \quad B_x = \left(\frac{R_0}{R}\right) \hat{B}_x, \quad B_y = \left(\frac{R_0}{R}\right) \hat{B}_y, \quad B_z = \left(\frac{R_0}{R}\right) \hat{B}_z,
\]

and for a matter of convenience we introduce the scalings

\[
\hat{u}_x = \left(\frac{R_0}{R}\right)^{-1} \hat{u}_x, \quad \hat{u}_y = \hat{u}_y, \quad \hat{u}_z = \left(\frac{R_0}{R}\right)^{-1} \hat{u}_z,
\]

and

\[
\hat{\nabla} = \left(\frac{\partial}{\partial x'}, \frac{\partial}{\partial y'}, \frac{\partial}{\partial z'}\right).
\]

Note that for two functions \(f\) and \(g\) for which holds

\[
g = \left(\frac{R_0}{R}\right)^{\alpha} f,
\]

for the time derivative it follows that

\[
\frac{\partial g}{\partial t} = \left(\frac{R_0}{R}\right)^{\alpha} \left(\frac{\partial f}{\partial t} - \alpha \frac{\hat{R}}{R} f\right).
\]

Furthermore, it can be easily shown that

\[
\hat{u} \cdot \nabla' = \hat{\nabla} \cdot \hat{u}, \quad \nabla' \cdot \hat{u} = \hat{\nabla} \cdot \hat{u}.
\]

At last, the equations that we solve with our two-dimensional numerical code are given by (in what follows all the fields components should have a hat, which we omit for the sake of simplicity)

\[
\frac{\partial \rho}{\partial t} = \left(\frac{\partial \rho}{\partial t}\right)_x - \hat{u}_y \frac{\partial \rho}{\partial y} - \rho \frac{\partial \hat{u}_y}{\partial y},
\]

\[
\frac{\partial \hat{u}_x}{\partial t} = \left(\frac{\partial \hat{u}_x}{\partial t}\right)_x - \hat{u}_y \frac{\partial \hat{u}_x}{\partial y} + \left(\frac{R_0}{R}\right)^2 B_y \frac{\partial B_x}{\partial y} - \frac{2\hat{R}}{R} u_x + \frac{1}{\rho R} \left(\frac{4}{3} \frac{\partial^2 u_x}{\partial x'^2} + \frac{1}{3} \frac{\partial^2 u_x}{\partial y'^2} + \frac{1}{3} \frac{\partial^2 u_x}{\partial z'^2}\right),
\]

\[
\frac{\partial \hat{u}_y}{\partial t} = \left(\frac{\partial \hat{u}_y}{\partial t}\right)_x - \hat{u}_y \frac{\partial \hat{u}_y}{\partial y} - \left(\frac{T}{\rho} \frac{\partial \hat{u}_y}{\partial y} + \frac{B_x}{\rho} \frac{\partial B_y}{\partial y} - \frac{B_y}{\rho} \frac{\partial B_x}{\partial y} + \frac{1}{\rho R} \left(\frac{6}{3} \frac{\partial^2 u_y}{\partial x'^2} + \frac{1}{3} \frac{\partial^2 u_x}{\partial y'^2} + \frac{1}{3} \frac{\partial^2 u_x}{\partial z'^2}\right)\right),
\]

\[
\frac{\partial \hat{u}_z}{\partial t} = \left(\frac{\partial \hat{u}_z}{\partial t}\right)_x - \hat{u}_y \frac{\partial \hat{u}_z}{\partial y} + \left(\frac{R_0}{R}\right)^2 B_y \frac{\partial B_z}{\partial y} - \frac{2\hat{R}}{R} u_x + \frac{1}{\rho R} \left(\frac{\partial^2 u_z}{\partial x'^2} + \frac{\partial^2 u_x}{\partial y'^2}\right).
\]
\[
\frac{\partial B_z}{\partial t} = \left( \frac{\partial B_z}{\partial t} \right)_x - B_z \frac{\partial u_x}{\partial y} + B_y \frac{\partial u_x}{\partial y} - u_y \frac{\partial B_z}{\partial y} + \frac{1}{\mathcal{R}_M} \left( \frac{\partial^2 B_z}{\partial x^2} + \frac{\partial^2 B_z}{\partial y^2} \right),
\]
(A35)

\[
\frac{\partial B_x}{\partial t} = \left( \frac{\partial B_x}{\partial t} \right)_x - u_y \frac{\partial B_y}{\partial y} + \frac{1}{\mathcal{R}_M} \left( \frac{\partial^2 B_x}{\partial x^2} + \frac{\partial^2 B_x}{\partial y^2} \right),
\]
(A36)

\[
\frac{\partial T}{\partial t} = \left( \frac{\partial T}{\partial t} \right)_x - u_y \frac{\partial T}{\partial y},
\]
(A37)

\[
\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial y} (u_y B_x - u_x B_y) + \frac{1}{\mathcal{R}_M} \left( \frac{\partial^2 B_y}{\partial x^2} + \frac{\partial^2 B_y}{\partial y^2} \right),
\]
(A38)

where with the subscript \(x\) outside parentheses we indicate the nondimensional terms containing only the partial \(x\) derivatives:

\[
\left( \frac{\partial \rho}{\partial t} \right)_x = -u_x \frac{\partial \rho}{\partial x} - \rho \frac{\partial u_x}{\partial x},
\]
(A39)

\[
\left( \frac{\partial u_x}{\partial t} \right)_x = -u_x \frac{\partial u_x}{\partial x} - \left( \frac{R_0}{R} \right)^2 \left( \frac{\partial T}{\partial x} + \frac{T}{\rho} \frac{\partial \rho}{\partial x} \right) - \left( \frac{R_0}{R} \right)^2 \frac{1}{\rho} \left[ B_x \frac{\partial B_z}{\partial x} + \left( \frac{R_0}{R} \right)^2 B_y \frac{\partial B_z}{\partial x} \right],
\]
(A40)

\[
\left( \frac{\partial u_y}{\partial t} \right)_x = -u_x \frac{\partial u_y}{\partial x} + \left( \frac{R_0}{R} \right)^2 \frac{B_z}{\rho} \frac{\partial B_y}{\partial x},
\]
(A41)

\[
\left( \frac{\partial u_z}{\partial t} \right)_x = -u_x \frac{\partial u_z}{\partial x} + \left( \frac{R_0}{R} \right)^2 \frac{B_z}{\rho} \frac{\partial B_x}{\partial x},
\]
(A42)

\[
\left( \frac{\partial B_x}{\partial t} \right)_x = -B_z \frac{\partial u_x}{\partial x} + B_x \frac{\partial u_x}{\partial x} - u_x \frac{\partial B_x}{\partial x},
\]
(A43)

\[
\left( \frac{\partial B_y}{\partial t} \right)_x = -B_z \frac{\partial u_y}{\partial x} + B_y \frac{\partial u_y}{\partial x} - u_y \frac{\partial B_y}{\partial x},
\]
(A44)

\[
\left( \frac{\partial T}{\partial t} \right)_x = -u_x \frac{\partial T}{\partial x}.
\]
(A45)

In equations (A31)–(A38) also the diffusive terms would be affected by expansion factors, but we neglect these factors because in numerical codes diffusive terms play a numerically stabilizing role, which would be affected by such expansion factors.

From equations (A39)–(A45) one constructs the characteristic polynomial for the \(x\)-derivative terms, which we use to implement the nonreflecting boundary conditions (see Einaudi et al. 2001). This yields the projected entropy and the Alfvén, fast, and slow characteristic speeds for the expanding box model (EBM) \(\lambda\) in nondimensional form as

\[
\lambda_0 = u_x, \quad \lambda_{a^+} = u_x \pm \xi a'_s, \quad \lambda_{f^+} = u_x \pm f', \quad \lambda_{s^+} = u_x \pm s',
\]
(A46)

where \(a'\) and \(c'\) are the modified Alfvén and sound speeds

\[
c' = \frac{R_0}{R} \sqrt{T}, \quad a'_s = \frac{R_0 B_x}{R \sqrt{\rho}}, \quad a'_s = \left( \frac{R_0}{R} \right)^2 \frac{B_y}{\sqrt{\rho}}, \quad a'_s = \frac{R_0 B_z}{R \sqrt{\rho}},
\]
(A47)

and \(\xi = a'_s / |a'_s|\). The fast and slow speeds are defined by

\[
f'^2 = \frac{1}{2} \left[ (c'^2 + a'^2) + \sqrt{\left( c'^2 + a'^2 \right)^2 - 4a'^2 c'^2} \right],
\]
(A48)

\[
s'^2 = \frac{1}{2} \left[ (c'^2 + a'^2) - \sqrt{\left( c'^2 + a'^2 \right)^2 - 4a'^2 c'^2} \right].
\]
(A49)

Indicating with \(\alpha'_s, \alpha'_s, \) and \(a'_s\) the quantities

\[
\alpha'_s = \sqrt{\frac{f'^2 - a'_s^2}{f'^2 - s'^2}}, \quad \alpha'_s = \frac{f'^2 - c'^2}{f'^2 - s'^2}, \quad a'_s = \sqrt{a'_s^2 + a'_s^2},
\]
(A50)
the projected characteristics are then given by the following expressions: the entropy characteristic,

\[ \mathcal{L}_0 = \frac{u_x c'}{T} \frac{\partial T}{\partial x}, \]  

(A51)

the Alfvén characteristics,

\[ \mathcal{L}_{a+} = (u_x + \xi a_x') \left[ \frac{-R_0 a_x' \partial u_x}{R a_x'} \frac{\partial T}{\partial x} + \frac{a_x' \partial u_x}{a_x'} \frac{\partial T}{\partial x} - \frac{\xi (R_0 / R)}{\sqrt{\rho}} \left( \frac{a_x' \partial B_z}{a_x'} \frac{\partial T}{\partial x} - \frac{a_x' \partial B_x}{a_x'} \frac{\partial T}{\partial x} \right) \right], \]

(A52)

\[ \mathcal{L}_{a-} = (u_x - \xi a_x') \left[ \frac{R_0 a_x' \partial u_x}{R a_x'} \frac{\partial T}{\partial x} + \frac{a_x' \partial u_x}{a_x'} \frac{\partial T}{\partial x} + \frac{\xi R_0}{\sqrt{\rho}} \left( \frac{a_x' \partial B_z}{a_x'} \frac{\partial T}{\partial x} - \frac{a_x' \partial B_x}{a_x'} \frac{\partial T}{\partial x} \right) \right], \]

(A53)

the slow mode characteristics,

\[ \mathcal{L}_{s+} = (u_x + s') \left( -\alpha'_2 \left[ \frac{R_0}{R} \frac{1}{c'} \left( \frac{T \partial \rho}{\rho} + \frac{\partial T}{\partial x} \right) + \xi \frac{a_x' \partial u_x}{f' \frac{\partial T}{\partial x}} \right] + \alpha'_1 \left\{ \frac{c'}{f' \sqrt{\rho}} \left[ \frac{R_0 a_x' \partial B_z}{R a_x'} \frac{\partial T}{\partial x} + \frac{R_0}{R} \left( \frac{2 a_x' \partial B_y}{a_x'} \frac{\partial T}{\partial x} \right) \right] - \xi \frac{a_x' \partial u_x}{\frac{\partial T}{\partial x}} \right\} \right), \]

(A54)

\[ \mathcal{L}_{s-} = (u_x - s') \left( -\alpha'_2 \left[ \frac{R_0}{R} \frac{1}{c'} \left( \frac{T \partial \rho}{\rho} + \frac{\partial T}{\partial x} \right) - \xi \frac{a_x' \partial u_x}{f' \frac{\partial T}{\partial x}} \right] + \alpha'_1 \left\{ \frac{c'}{f' \sqrt{\rho}} \left[ \frac{R_0 a_x' \partial B_z}{R a_x'} \frac{\partial T}{\partial x} + \frac{R_0}{R} \left( \frac{2 a_x' \partial B_y}{a_x'} \frac{\partial T}{\partial x} \right) \right] + \xi \frac{a_x' \partial u_x}{\frac{\partial T}{\partial x}} \right\} \right), \]

(A55)

and the fast mode characteristics,

\[ \mathcal{L}_{f+} = (u_x + f') \left( \alpha'_1 \left[ \frac{R_0}{R} \frac{1}{f'} \left( \frac{T \partial \rho}{\rho} + \frac{\partial T}{\partial x} \right) + \frac{\partial u_x}{\partial x} \right] \right.

+ \alpha'_2 \left\{ \frac{1}{\sqrt{\rho}} \left[ \frac{R_0 a_x' \partial B_z}{R a_x'} \frac{\partial T}{\partial x} + \frac{R_0}{R} \left( \frac{2 a_x' \partial B_y}{a_x'} \frac{\partial T}{\partial x} \right) \right] - \alpha'_1 \left[ \frac{a_x' \partial u_x}{\frac{\partial T}{\partial x}} \right] \right\} \right), \]

(A56)

\[ \mathcal{L}_{f-} = (u_x - f') \left( \alpha'_1 \left[ \frac{R_0}{R} \frac{1}{f'} \left( \frac{T \partial \rho}{\rho} + \frac{\partial T}{\partial x} \right) - \frac{\partial u_x}{\partial x} \right] \right.

+ \alpha'_2 \left\{ \frac{1}{\sqrt{\rho}} \left[ \frac{R_0 a_x' \partial B_z}{R a_x'} \frac{\partial T}{\partial x} + \frac{R_0}{R} \left( \frac{2 a_x' \partial B_y}{a_x'} \frac{\partial T}{\partial x} \right) \right] + \alpha'_1 \left[ \frac{a_x' \partial u_x}{\frac{\partial T}{\partial x}} \right] \right\} \right). \]

(A57)

In terms of characteristics, the time derivatives (eqs. [A39]–[A45]) are then given explicitly by

\[ \left( \frac{\partial \rho}{\partial t} \right)_x = \frac{\rho}{c'} \mathcal{L}_0 + \rho \left[ \frac{\alpha'_2}{c'} \left( \frac{\mathcal{L}_{s+} + \mathcal{L}_{s-}}{2} \right) - \frac{\alpha'_1}{f'} \left( \frac{\mathcal{L}_{f+} + \mathcal{L}_{f-}}{2} \right) \right], \]

(A58)

\[ \left( \frac{\partial u_x}{\partial t} \right)_x = \frac{s'}{c'} \alpha'_2 \left( \frac{\mathcal{L}_{s+} - \mathcal{L}_{s-}}{2} \right) - \alpha'_1 \left( \frac{\mathcal{L}_{f+} - \mathcal{L}_{f-}}{2} \right), \]

(A59)

\[ \left( \frac{\partial u_x}{\partial t} \right)_x = \frac{R a_x'}{R_0 a_x'} \left( \frac{\mathcal{L}_{A+} + \mathcal{L}_{A-}}{2} \right) + \frac{R a_x'}{R_0 a_x'} \left[ \xi \alpha'_1 \left( \frac{\mathcal{L}_{S+} - \mathcal{L}_{S-}}{2} \right) + \frac{a_x'}{f'} \alpha'_2 \left( \frac{\mathcal{L}_{F+} - \mathcal{L}_{F-}}{2} \right) \right], \]

(A60)

\[ \left( \frac{\partial u_x}{\partial t} \right)_x = - \frac{a_x'}{a_x'} \left( \frac{\mathcal{L}_{A+} + \mathcal{L}_{A-}}{2} \right) + \frac{a_x'}{a_x'} \left[ \xi \alpha'_1 \left( \frac{\mathcal{L}_{S+} - \mathcal{L}_{S-}}{2} \right) + \frac{a_x'}{f'} \alpha'_2 \left( \frac{\mathcal{L}_{F+} - \mathcal{L}_{F-}}{2} \right) \right], \]

(A61)
\[
\left( \frac{\partial \mathcal{B}_r}{\partial t} \right)_x = \xi \frac{R}{R_0} \sqrt{\beta} \frac{a'_r}{a_\perp} \left( \frac{\mathcal{L}_{A^+} - \mathcal{L}_{A^-}}{2} \right) - \frac{R}{R_0} \sqrt{\beta} \frac{a'_r}{a_\perp} \left[ \frac{c'}{f'} \alpha_1 \left( \frac{\mathcal{L}_{S^+} + \mathcal{L}_{S^-}}{2} \right) + \alpha_2' \left( \frac{\mathcal{L}_{F^+} + \mathcal{L}_{F^-}}{2} \right) \right],
\]
\[
\left( \frac{\partial \mathcal{B}_\perp}{\partial t} \right)_x = -\xi \left( \frac{R}{R_0} \right)^2 \sqrt{\beta} \frac{a'_r}{a_\perp} \left( \frac{\mathcal{L}_{A^+} - \mathcal{L}_{A^-}}{2} \right) - \left( \frac{R}{R_0} \right)^2 \sqrt{\beta} \frac{a'_r}{a_\perp} \left[ \frac{c'}{f'} \alpha_1' \left( \frac{\mathcal{L}_{S^+} + \mathcal{L}_{S^-}}{2} \right) + \alpha_2' \left( \frac{\mathcal{L}_{F^+} + \mathcal{L}_{F^-}}{2} \right) \right],
\]
\[
\left( \frac{\partial T}{\partial t} \right)_x = -\frac{T}{c'} \mathcal{L}_0.
\]

Nonreflecting boundary conditions are obtained by setting the values of the characteristics to zero for the inward propagating waves in equations (A58)–(A64) (see Thompson 1987, 1990, Vanajakshi et al. 1989, and Roe & Balsara 1996).

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