Rate-Optimal Streaming Codes Over the Three-Node Decode-And-Forward Relay Network

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Abstract—We study the three-node Decode-and-Forward (D&F) relay network subject to random and burst packet erasures. The source wishes to transmit an infinite stream of packets to the destination via the relay. The three-node D&F relay network is constrained by a decoding delay of $T$ packets, i.e., the packet transmitted by the source at time $i$ must be decoded by the destination by time $i+T$. For the individual channels from source to relay and relay to destination, we assume a delay-constrained sliding-window (DCSW) based packet-erasure model that can be viewed as a tractable approximation to the commonly-accepted Gilbert-Elliott channel model. Under the model, any time-window of width $w$ contains either up to $a$ random erasures or else erasure burst of length at most $b$ ($\geq a$). Thus the source-relay and relay-destination channels are modeled as $(a_1, b_1, w_1, T_1)$ and $(a_2, b_2, w_2, T_2)$ DCSW channels. We first derive an upper bound on the capacity of the three-node D&F relay network. We then show that the upper bound is tight for the parameter regime: $\max\{a_1, b_1\} | (T-b_1-b_2-\max\{a_1, a_2\}+1), a_1 = a_2$ OR $b_1 = b_2$ by constructing streaming codes achieving the bound. The code construction requires field size linear in $T$, and has decoding complexity equivalent to that of decoding an MDS code.

I. INTRODUCTION

Low-latency communication is a critical ingredient of upcoming promising applications such as telesurgery, virtual and augmented reality, industrial automation and self-driving cars [1]. Ultra-Reliable, Low-Latency Communication (URLLC) is one of the three core focus areas of 5G, where latency is measured as the time elapsed between the transmission of a packet from the source and it’s recovery at the receiver. Using ARQ-based schemes to ensure reliability results in an undesirable round-trip delay, which makes it challenging to meet the low latency requirement of URLLC. Physical layer FEC cannot help recover from packet drops arising due to congestion, a wireless link in deep fade or else late packet arrival. Streaming codes represent a packet-level FEC scheme for countering such packet losses.

A. A Brief History of Streaming Codes

Research on streaming codes began with the study of burst-erasure correction under a decoding-delay constraint [2], [3]: the authors argued in favor of packet-extension encoding framework, where the redundancy is added within the packets rather than being transmitted as separate packets to avoid adding to network congestion. A measurement study of mobile video calls over wireless networks [4] indicated that packet erasures occur both in an isolated and bursty fashion. In [5], a delay-constrained sliding-window (DCSW) channel model was introduced as a tractable deterministic approximation to the commonly-accepted Gilbert-Elliott erasure channel model [6]–[10] that is capable of causing burst and random erasures. An $(a, b, w, T)$ DCSW channel imposes a decoding-delay constraint of $T$, and can cause at most $a$ random erasures or else, a burst of $b$ erasures within any sliding window of size $w$ time slots, where $0 < a \leq b \leq T$.

![Fig. 1: Illustrating a permissible erasure pattern in an $(a = 2, b = 3, w = 5, T)$ DCSW channel.](image)

$w = 5$; 2 random erasures

In [5], [11], it was shown that one can without loss of generality, set $w = T + 1$ and hence, we will abbreviate and write $(a, b, T)$ in place of $(a, b, w, T)$. A packet-level code is referred to as an $(a, b, T)$ streaming code if it enables recovery from all the permissible erasure patterns of an $(a, b, T)$ DCSW channel. The coding rate, denoted by $R$, of an $(a, b, T)$ streaming code was shown in [5] to be upper bounded as

$$R \leq \frac{T - a + 1}{T - a + 1 + b} \triangleq C_{a,b,T}. \quad (1)$$

Hence, we will refer to $C_{a,b,T}$ as the point-to-point channel capacity. Code constructions achieving the point-to-point channel capacity for all parameters $(a, b, T)$ can be found in [12]–[14]. The construction in [12] is not explicit, and requires a field size of $q^2$, where $q \geq T + b - a$ is a prime power. [13] provides an explicit construction with a field size that scales quadratically with the delay. Explicit code construction with reduced quadratic field size, $q^2$, where $q \geq T$ is presented in [14]. Streaming codes based on staggered diagonal embedding [15]–[17] having linear field size are rate-optimal for special cases. Streaming codes have also been constructed for channels with unequal source-channel inter-arrival rates [5], multiplicative-matrix channels [18] and multiplexed communication scenarios with different decoding delays for different streams [19], [20]. In [21], the authors consider a setting for
variable-size arrivals. Locally recoverable streaming codes for packet-erasure recovery were constructed in [22]. Other FEC schemes suitable for streaming setting can be found in [23–31]. In contrast to the existing literature on burst and random packet erasure correcting streaming codes which focuses on point-to-point networks, our focus in this paper is on three-node relay network, which consists of a source, a relay and a destination. This kind of topology is often present in content delivery networks [32–34].

B. Paper Outline

The three-node D&F relay network and symbol-wise decode-and-forward (SW D&F) strategy are introduced in Section II. An upper bound on the capacity of the three-node D&F relay network is derived in Section III. The Staggered diagonal embedding (SDE) approach [15] is introduced in Section IV. A rate-optimal streaming code construction using the SDE approach and the SW D&F strategy is provided in Section V. Section VI concludes the paper.

II. THREE-NODE RELAY NETWORK

We follow [33] in assuming a D&F network and further, one in which the encoding function at the relay does not take into account the erasure pattern observed over (s, r) channel. Such a relaying strategy would be preferred in settings where the relay also has an interest in the contents of the packet stream. The network consists of a source, a destination and a relay between them, which are denoted by s, d and r, respectively. The channel between nodes s and r is denoted by (s, r), and the channel between nodes r and d is denoted by (r, d). We consider the case where the (s, r) and (r, d) channels are subject to both random and burst erasures. Thus the channels (s, r) and (r, d) are modeled as (a1, b1, T1) and (a2, b2, T2) DCSW channels, respectively, where T1, T2 < T.

![Diagram of three-node relay network](image)

Fig. 2: Illustrating a three-node D&F relay network under packet-extension framework. Here, (s) denotes the symbol index in a packet. The time instances \(t_0, t_1, \ldots, t_k-1\) \(\in [0, 1]\), and the symbol indices \(s, \ell, \ldots, s, k-1\) \(\in [0, k-1]\).

Node s wishes to transmit an infinite stream of packets \(\{m_i\}_{i=0}^\infty\) to node d via the node r. At any time i, as shown in

\[m_i \in \mathbb{F}_q^n, \quad x_i \in \mathbb{F}_q^n, \quad x^{(r)}_i \in \mathbb{F}_q^n, \quad \hat{m}_{i-T} \in \mathbb{F}_q^n\]

Fig. 3: Illustrating a delay profile for the three-node D&F relay network. Note that the decoding delays are independent of i.

Definition 1. A delay profile for a k-length message packet is defined as

\[d = \left( (t_0, \tau_0), (t_1, \tau_1), \ldots, (t_{k-1}, \tau_{k-1}) \right)\]

where \((t_\ell, \tau_\ell) \in \mathbb{Z}_2^2\) denotes the decoding delay of \(m_\ell \) at (s, r) and (r, d) channels, respectively, for any time \(i\).

The first-hop and second-hop delay profiles are defined to be \((t_0, t_1, \ldots, t_{k-1})\) and \((\tau_0, \tau_1, \ldots, \tau_{k-1})\), respectively.

\[m_i \in \mathbb{F}_q^n, \quad x_i \in \mathbb{F}_q^n, \quad x^{(r)}_i \in \mathbb{F}_q^n, \quad \hat{m}_{i-T} \in \mathbb{F}_q^n\]

Thus, the overall decoding delay can be interpreted as requiring that the delay profile satisfies the following constraint

\[t_\ell + \tau_\ell \leq T, \quad (3)\]

where \(\ell \in [0, k-1]\).

We follow [33] in adopting this convention.
III. AN UPPER BOUND ON THREE-NODE 
DECODE-AND-FORWARD RELAY NETWORK CAPACITY

In this section, we first define the capacity of the three-node D&F relay network and then derive an upper bound on it.

**Definition 2.** The capacity of three-node D&F relay network, denoted by $C_{a_1,b_1,a_2,b_2,T}$, is the maximum rate achievable by $(a_1, b_1, a_2, b_2, T)$ streaming codes, i.e.,

$$
C_{a_1,b_1,a_2,b_2,T} \triangleq \sup \left\{ \frac{k}{\max\{n_1,n_2\}} \left| \begin{array}{l}
\text{There exists an} \\
(a_1, b_1, a_2, b_2, T) \\
\text{streaming code} \\
\text{with parameters} \\
(k,n_1,n_2)q_a
\end{array} \right. \right\}.
$$

**Theorem 1.** For any $(a_1, b_1, a_2, b_2, T)$,

$$
C_{a_1,b_1,a_2,b_2,T} \leq \min\{C_{a_1,b_1,T-b_2}, C_{a_2,b_2,T-b_1}\},
$$

where $C_{a_1,b_1,T_i}$ are the point-to-point channel capacities.

**Proof.** The bound can be derived as follows. If the source transmits at a rate in excess of $C_{a_1,b_1,T-b_2}$, then there is at least one permissible erasure pattern which will make it impossible to decode a packet transmitted at time $i$ by time $i + T - b_2$ at the relay. If the $(r,d)$ channel then experiences a burst of duration $b_2$, then this will make it impossible for the destination to recover packet $i$ by time $(i + T)$. Therefore, $C_{a_1,b_1,a_2,b_2,T} \leq C_{a_1,b_1,T-b_2}$. The upper bound $C_{a_1,b_1,a_2,b_2,T} \leq C_{a_2,b_2,T-b_1}$ follows from noting that an initial burst of duration $b_1$ on the $(s,r)$ channel, is equivalent to reducing the time available for the relay to convey information to the destination by an amount $b_1$. \qed

IV. STAGGERED DIAGONAL EMBEDDING

We use the staggered diagonal embedding (SDE) approach introduced in [15] to construct a rate-optimal code over the three-node D&F relay network. As shown in Section V, the SDE approach enables matching of channel-level delay profiles to satisfy the overall decoding delay constraint.

In [15], for an $(a, b, T)$ DCSW channel, SDE approach was used to construct a rate-optimal streaming code for the parameter regime: $b \mid T - a + 1$, with field size linear in $T$. We briefly describe the SDE approach below.

Let $C$ be an $[n,k]$ linear code in systematic form, with the first $k$ code symbols corresponding to the message symbols. Let $N \geq n$ be an integer and let $S$ be a subset of $[0 : N - 1]$ of size $n$:

$$
S = \{i_0, i_1, \ldots, i_{n-1}\} \subseteq [0 : N - 1],
$$

where $0 \triangleq i_0 < i_1 < \cdots < i_{n-1} \triangleq N - 1$. Let $x_i(\ell)$ denote the $(\ell+1)\text{th}$ symbol of the coded packet $x_i$ for each $\ell \in 0, 1, \ldots, n - 1$. Then for every time $i$, the collection of symbols

$$
(x_{i+i_0}(0), x_{i+i_1}(1), \ldots, x_{i+i_{n-1}}(n-1))
$$

forms a codeword in code $C$. The set $S$ is called as the placement set as it determines the placement in time of the code symbols of $C$ within the packet stream, and $N$ is referred to as the dispersion-span parameter. The packet-level code thus constructed through SDE, has rate $\frac{k}{N}$. The code $C$ is referred to as the base code.

A. Construction of an $(a, b, T)$ streaming code using SDE of an MDS-Base-Code

In [15], it was shown that the SDE approach yields $(a, b, T)$ streaming codes meeting the dispersion-span constraint $N \leq T+1$ for all parameters $(a, b, T)$. Let $T+1 = mb + \delta_1$, with $0 \leq \delta_1 < b$ and set $\delta_2 \triangleq \min\{\delta_1,a\}$. Let the base code $C$ be an $[n = k + a, k]$ MDS code, where $k \in \mathbb{Z}_+$. The authors of [15] selected the value of the parameter $k$ to be the maximum possible, which is $(m-1)a + \delta_2$ but for ensuring optimality over the three-node D&F relay network we require a wider range for $k$, thus we assume that $k \leq (m-1)a + \delta_2$. Additionally let the dispersion-span parameter and dispersion set be given by:

$$
N = n + (b-a) \cdot \left\lfloor \frac{n-1}{a} \right\rfloor, \tag{5}
$$

$$
S = \bigcup_{i=0}^{n-1} \left\{ i + (b-a) \cdot \left\lfloor \frac{i}{a} \right\rfloor \right\}, \tag{6}
$$

respectively. Since $k \leq (m-1)a + \delta_2$, it follows that

$$
N \leq ma + \delta_2 + (b-a) \cdot \left\lfloor \frac{ma + \delta_2 - 1}{a} \right\rfloor = mb + \delta_2 \leq mb + \delta_1 = T+1.
$$

The delay profile simply follows from the dispersion-span parameter and the displacement set; the decoding delay of the $j\text{th}$ message symbol is

$$
t_j = N-1 - j - (b-a) \cdot \left\lfloor \frac{j}{a} \right\rfloor \leq T, \tag{7}
$$

where $j \in [0, k-1]$. Also, as shown in Fig. 4, it can be seen that the erasure of any successive $b$ packets corresponds to the

![Fig. 4: Staggered diagonal embedding of a codeword in [6,3]MDS-base code within the packet stream. The contents of the codeword can be successfully recovered as a burst of length 6 erases at most 3 symbols of the diagonally-embedded codeword.](image-url)
erasure of exactly $a$ code symbols belonging to the base code $C$. Thus, due to the MDS nature of $C$, the packet-level code can tolerate a burst of length $b$ and any $a$ random erasures. Therefore, the packet-level code constructed using the SDE approach is an $(a, b, T)$ streaming code. As the base code is an MDS code, the code construction requires linear field size.

V. RATE-OPTIMAL CODE CONSTRUCTION FOR THE THREE-NODE D&F RELAY NETWORK

We first present an example, shown in Fig. (5), to demonstrate how SDE approach and SW D&F strategy are employed to construct a rate-optimal $(a_1 = 1, b_1 = 2, a_2 = 1, b_2 = 3, T = 8)$ streaming code. From (4), the capacity is upper bounded by

$$C_{a_1, b_1, a_2, b_2} \leq \min \left\{ C_{a_1, b_1, T-b_2}, C_{a_2, b_2, T-b_1} \right\}$$

$$= \min \left\{ \frac{8 - 3 - 1 + 1}{8 - 3 - 1 + 1}, \frac{8 - 2 - 1 + 1}{8 - 2 - 1 + 1} \right\}$$

$$= \min \left\{ \frac{5}{6}, \frac{2}{3} \right\} = \frac{2}{3}.$$

For the $(s, r)$ channel, we use an $[3, 2]$ MDS code as the base code to construct a rate-optimal $(a_1 = 1, b_1 = 2, T_1 = 5)$ streaming code. Similarly, for the $(r, d)$ channel, we use an $[3, 2]$ MDS code as the base code to construct $(a_2 = 1, b_2 = 3, T_2 = 6)$ streaming code. While re-encoding the decoded message symbols at the relay node, the order of the packet symbols within a packet is flipped. Therefore, from (7), the delay profile of the packet is

$$(t_0 = 4, \tau_0 = 3), (t_1 = 2, \tau_1 = 6).$$

From the delay profile, it can be seen that the overall decoding delay for each packet symbol is $\leq 8$ packets. Therefore, the code construction is a rate-optimal $(a_1 = 1, b_1 = 2, a_2 = 1, b_2 = 3, T = 8)$ streaming code.

To construct a rate-optimal $(a_1, a_2, b_1, b_2, T)$ streaming code over the three-node D&F relay network, we require the parameters $k, n_1, n_2$ to satisfy:

$$R_{s, r, d} = \frac{k}{\max\{n_1, n_2\}} = \min\{C_{a_1, b_1, T-b_2}, C_{a_2, b_2, T-b_1}\}.$$ 

It can be shown that this leads to:

$$k = \max\{a_1', a_2'\}, \min\left\{ \frac{T-b_2-a_1+1}{b_1'}, \frac{T-b_2-a_2+1}{b_2'} \right\},$$

(8)

where $a_1' = n_1 - k$ and $a_2' = n_2 - k$. We now describe the code construction.

Construction 1.
Let $a \triangleq \max\{a_1, a_2\}, b_u' \triangleq \max\{b_u, a\}, \alpha \triangleq T + 1 - b_1' - b_2' - a$, for $u \in \{1, 2\}$. We select $T_1 = T - b_2'$ and $T_2 = T - b_1'$ as the channel-level decoding delay constraints for the $(s, r)$ and $(r, d)$ channels, respectively. Thus, the $(s, r)$ and $(r, d)$ channels are modeled as $(a_1, b_1, T - b_2')$ DCSW and $(a_2, b_2, T - b_1')$ DCSW channels. We then construct $(a, b_1', T - b_2')$ and $(a, b_2', T - b_1')$ streaming codes for the $(s, r)$ and $(r, d)$ channels, respectively, using SDE of $[[k] + a, [k]]$ MDS base codes for both the channels, where $k$ is

$$k = \max\{a, a\}, \min\left\{ \frac{T - b_2' - a_1 + 1}{b_1'}, \frac{T - b_2' - a_2 + 1}{b_2'} \right\} = a, \min\left\{ \frac{\alpha}{b_1'} + 1, \frac{\alpha}{b_2'} + 1 \right\} = a, \frac{T - \min\{b_1', b_2'\} - a + 1}{\max\{b_1', b_2'\}}.$$

Let $u \in \{1, 2\}$, from (5), the dispersion-span parameters are

$$N_u = n_u + (b_u' - a). \left\lfloor \frac{n_u - 1}{a} \right\rfloor.$$

While re-encoding the message symbols at the relay node, the order of the message symbols within a packet is flipped following the approach adopted in [33]. From (7), it follows that the decoding delays of the $j^{th}$ message symbol across the $(s, r)$ and $(r, d)$ channels are given by

$$t_j = N_1 - 1 - (b_1' - a). \left\lfloor \frac{a}{a} \right\rfloor,$$

$$\tau_j = N_2 - 1 - (k - 1 - j) - (b_2' - a). \left\lfloor \frac{k - j - 1}{a} \right\rfloor,$$

respectively, where $j \in \{0 : k - 1\}$.

We next present a lemma involving numerics.

Lemma 1. Let $x, y, z \in \mathbb{Z}, z > 0$. Let $x = q_1 z + r_1$ and $y = q_2 z + r_2$, where $q_1 = \left\lfloor \frac{x}{z} \right\rfloor$, $r_1 \equiv x \mod z$, $q_2 = \left\lfloor \frac{y}{z} \right\rfloor$, $r_2 \equiv y \mod z$.

$$\frac{x}{z} - \frac{y}{z} = \begin{cases} q_1 - q_2 & r_1 \geq r_2 \\ q_1 - q_2 - 1 & r_1 < r_2. \end{cases}$$

Proof.

$$\frac{x}{z} - \frac{y}{z} = \begin{cases} q_1 z + r_1 - q_2 z - r_2 \\ q_1 - q_2 + \frac{r_1}{z} - \frac{r_2}{z} \end{cases}$$

$$= q_1 - q_2 + \frac{r_1}{z} - \frac{r_2}{z}$$

from which the result follows

$$= \begin{cases} q_1 - q_2 & r_1 \geq r_2 \\ q_1 - q_2 - 1 & r_1 < r_2. \end{cases}$$

□

Theorem 2. If $T, a_1, a_2, b_1, b_2$ satisfy:

$$\max\{b_1', b_2'\} \mid \alpha, a_1 = a_2 \text{ OR } b_1 = b_2$$

then construction [7] is a rate-optimal $(a_1, a_2, b_1, b_2, T)$ streaming code.

Proof. Since $\max\{b_1', b_2'(T - \min\{b_1', b_2'\}) + 1 - a\}$.

$$\lfloor k \rfloor = k.$$ 

Therefore, when $a_1 = a_2$ or $b_1 = b_2$, Construction [7] in this parameter regime is rate-optimal. Since, in this parameter regime $a \mid k$, it follows that

$$N_u = n_u + (b_u' - a) \left\lfloor \frac{n_u - 1}{a} \right\rfloor.$$
It then follows from the dispersion-span constraint that
\[ n + 1 = 0. \]

It can be verified that the dispersion-span constraint is met
\[ m_i \cdot 1 \]

Therefore, \( k = a + (b_i - a) \frac{k}{\alpha} = a + b'_i \frac{k}{\alpha}. \)

It can be verified that the dispersion-span constraint is met for both the channels. From (3), we have that for \( j \in [0, k - 1], t_j + \tau_j \leq T \) or equivalently,
\[ N_1 + N_2 - k - 1 - (b_i - a) \left( \frac{k - 1 - j}{\alpha} \right) \leq T. \]

Consider the L.H.S. \( \Delta f(j) \) of the above inequality,
\[ f(j) = N_1 + N_2 - k - 1 - (b_i - a) \left( \frac{k - 1 - j}{\alpha} \right) - (b'_i - a) \left( \frac{k - 1 - j}{\alpha} \right). \]

Since \( a \mid k \), we have that \( f(j) \) is equal to
\[ N_1 + N_2 - k - 1 - (b_i - a) \left( \frac{k - 1 - j}{\alpha} \right) - (b'_i - a) \left( \frac{k - 1 - j}{\alpha} \right). \]

From Lemma [1] it follows that \( \left[ \frac{k - 1 - j}{\alpha} \right] - \frac{k - 1 - j}{\alpha} - \frac{1}{\alpha} \right] = -1 - \left[ \frac{k - 1 - j}{\alpha} \right]. \)

Therefore,
\[ f(j) = N_1 + N_2 - 1 - a - b'_i \left( \frac{k}{\alpha} \right) - (b'_i - b'_i) \left( \frac{j}{\alpha} \right) \]
\[ = N_1 - 1 + b'_i \left( 1 - b'_i \right) \left( \frac{j}{\alpha} \right) . \]

For the inequality to hold for all \( j \in [0 : k - 1] \), the following constraint must be satisfied:
\[ \max_{j \in [0 : k - 1]} f(j) \leq T. \]

Case I: \( b'_i = b'_i \geq b'_i \): If \( b'_i - b'_i > 0 \) then \( f(j) \) is maximum at \( j = 0 \), else \( b'_i = b'_i \) then \( f(j) = N_1 - 1 + b'_i \). Therefore,
\[ \max_{j \in [0 : k - 1]} f(j) = N_1 - 1 + b'_i. \]

It then follows from the dispersion-span constraint that
\[ \max_{j \in [0 : k - 1]} f(j) \leq T - b'_i + 1 - 1 + b'_i = T. \]

Case II: \( b'_i < b'_i \): As \( b'_i - b'_i < 0 \), \( f(j) \) is maximum when \( j = k - 1 \). Therefore,
\[ \max_{j \in [0 : k - 1]} f(j) = N_1 - 1 + b'_i - (b'_i - b'_i) \left( \frac{k - 1}{\alpha} \right) . \]

Since \( a \mid k \),
\[ \max_{j \in [0 : k - 1]} f(j) = N_1 - 1 + b'_i - (b'_i - b'_i) \left( \frac{k}{\alpha} \right) \]
\[ = N_1 - 1 + b'_i + (b'_i - b'_i) \left( \frac{k}{\alpha} \right) \]
\[ = a - 1 + b'_i + b'_i \left( \frac{k}{\alpha} \right) \]
\[ = N_2 - 1 + b'_i. \]

It then follows from the dispersion-span constraint that
\[ \max_{j \in [0 : k - 1]} f(j) \leq T - b'_i + 1 - 1 + b'_i = T. \]

Therefore, Construction [1] satisfies the decoding delay constraint. As we have used an \( (a, b'_i, T - b'_i) \) streaming code for the \( (s, r) \) channel, the code can recover from any permissible erasures of \( (a, b'_i, T - b'_i) \) DCSW channel. Similarly, for the \( (r, d) \) channel, as we have used an \( (a, b'_i, T - b'_i) \) streaming code, the code can recover from any permissible erasures of \( (a_2, b'_i, T - b'_i) \) DCSW channel. Therefore, Construction [1] is a rate-optimal \( (a_1, b_1, a_2) \) (\( s, r, d \)) streaming code.

VI. CONCLUSIONS

An upper bound on the capacity of three-node decode-and-forward relay network was derived, and was shown to be tight for the parameter regime: \( \max\{b_1, b_2\} \mid (T - b_1 - b_2 - \max\{a_1, a_2\} + 1), a_1 = a_2 \) OR \( b_1 = b_2 \). A rate-optimal code construction having a field size linear in decoding delay, \( T \), was designed by using staggered diagonal embedding approach and symbol-wise D&F strategy.
