The Mechanism of High-$T_c$ Superconductivity: ”Nonlinear” Superconductivity

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"True laws of Nature cannot be linear.” - Albert Einstein

The main purpose of the paper is to present an overview of the current situation in the development of understanding of the mechanism of high-$T_c$ superconductivity which arises due to moderately strong, nonlinear electron-phonon interactions and due to magnetic (spin) fluctuations. The former are responsible for electron pairing, and the latter mediate the phase coherence.

I. INTRODUCTION

This Section is a brief reminder of key events which led to understanding of the mechanism of high-$T_c$ superconductivity.

The first observation of what is now called the soliton was made by John Scott Russell near Edinburgh (Scotland) in 1834. He was observing a boat moving on a shallow channel and noticed that, when the boat suddenly stopped, the wave that it was pushing at its prow "rolled forward with great velocity, assuming the form of a large solitary elevation, a rounded, smooth and well defined heap of water which continued its course along the channel apparently without change of form or diminution of speed" [1]. He followed the wave along the channel for more than a mile.

The phenomenon of superconductivity was discovered by Dutch physicist H. Kamerlingh Onnes in 1911. He found that dc resistivity of mercury suddenly drops to zero below 4.2 K [2].

The microscopic theory of superconductivity in metals was proposed by J. Bardeen, L. Cooper and R. Schrieffer in 1957. The central concept of the BCS theory is weak electron-phonon interactions which lead to the appearance of an attractive potential between two electrons [3].

To the best of my knowledge, the soliton (or bisoliton) model of superconductivity was for the first time considered by L. S. Brizhik and A. S. Davydov in 1984 [4] in order to explain the superconductivity in organic quasi-one-dimensional (quasi-1D) conductors [5].

The interest in the research of superconductivity was renewed in 1986 with the discovery of high-$T_c$ superconductivity in copper oxides (cuprates), made by J. G. Bednorz and K. A. Müller [6].

In 1987, L. P. Gor’kov and A. V. Sokol proposed the presence of two components of itinerant and more localized features in cuprates [7]. This kind of microscopic and dynamical phase separation was later rediscovered in other theoretical models.

By using the fact of the absence of the isotope effect in cuprates, which is a hallmark of the BCS mechanism of superconductivity (in fact, there is the isotope effect in cuprates but it is very weak) A. S. Davydov proposed in 1988 the bisoliton mechanism of high-$T_c$ superconductivity [8].

The pseudogap above $T_c$ was observed in 1989 in nuclear magnetic resonance (NMR) measurements [9].

In 1990, A. S. Davydov presented a theory of high-$T_c$ superconductivity, based on the concept of a moderately strong electron-phonon coupling which results in perturbation theory being invalid [10,11]. The theory utilizes the concept of bisolitons, or electron (or hole) pairs coupled in a singlet state due to local deformation of the -O-Cu-O-Cu- chain in CuO$_2$ planes. We shall discuss the bisoliton model below.

In 1994, A. S. Alexandrov and N. F. Mott pointed out that, in cuprates, it is necessary to distinguish the "internal" wave function of a Cooper pair and the order parameter of the Bose-Einstein condensate, which may have different symmetries [12].

In 1995, V. J. Emery and S. A. Kivelson emphasized that superconductivity requires pairing and long-range phase coherence [13]. In conventional superconductors described by the BCS theory, the pairing and the long-range phase coherence occur simultaneously at $T_c$, since the phase stiffness, which measures the ability of the superconducting state to carry supercurrent, is much larger than the energy gap, $\Delta$, which reflects the strength of the binding of electrons into Cooper pairs. In contrast to conventional superconductors, in cuprates, the energy gap and the phase stiffness have similar values [14]. Therefore, the phase stiffness in cuprates is the weak link, and the pairing may occur above $T_c$ without the phase coherence which is established at $T_c$ [14].

In the same year 1995, J. M. Tranquada and co-workers [15] found the presence of coupled, dynamical modulations of charges (holes) and spins in Nd-doped La$_{2-x}$Sr$_x$CuO$_4$ (LSCO) from neutron diffraction. In LSCO, antiferromagnetic stripes of copper spins are separated by periodically spaced quasi-1D domain walls to which the holes segregate. The spin direction in antiferromagnetic domains rotates by 180 degrees on crossing a domain wall.

In 1997, V. J. Emery, S. A. Kivelson and O. Zachar presented the model of high-$T_c$ superconductivity based on the presence of charge stripes in CuO$_2$ planes [16]. They assumed that charge stripes are metallic, and there is the
local separation of spin and charge along an individual stripe. Neutral spinons on a stripe acquire a spin gap via pair hopping between the stripe and its environment creating a singlet bound state. The phase coherence is established due to the Josephson coupling between stripes. It turned out that the model is incorrect, however, it is the first model of high-$T_c$ superconductivity based on the presence of charge stripes in CuO$_2$ planes.

In 1999, on the basis of tunneling neutron scattering measurements, it was found that, in Bi$_2$Sr$_2$CaCu$_2$O$_{8+x}$ (Bi2212) and YBa$_2$Cu$_3$O$_{6+x}$ (YBCO), the phase coherence is established due to spin excitations [15,18] which cause the appearance of the so-called magnetic resonance peak in inelastic neutron scattering spectra [13].

In 2001, tunneling measurements provided evidence that quasiparticle peaks in tunneling spectra obtained in Bi2212 are caused by condensed solitonlike excitations, and the Cooper pairs in Bi2212 seem to be Davydov’s bisolitons [20,21]. The data are discussed below.

### II. NONLINEAR EXCITATIONS: SOLITONS

For a long time linear equations have been used for describing different phenomena. For example, Newton, Maxwell and Schrödinger’s equations are linear, and they take into account only a linear response of a system to an external disturbance. However, the majority of real systems are nonlinear. Most of the theoretical models are still relying on a linear description, corrected as much as possible for nonlinearities which are treated as small perturbations. It is well known that such approach can be absolutely wrong. The linear approach can sometimes miss completely some essential behaviors of the system.

The solitary wave observed by J. S. Russell in 1834 on the water surface is the nonlinear excitation called the soliton. Such waves can not be described by using linear equations. Unlike ordinary waves which represent a spatial periodical repetition of elevations and hollows on a water surface, or condensations and rarefactions of a density, or deviations from a mean value of various physical quantities, solitons are single elevations, such as thickening etc., which propagate as a unique entity with a given velocity. The transformation and motion of solitons are described by nonlinear equations of mathematical physics.

Let us explore the world of solitons. A soliton is the extremely robust, nonlinear excitation localized in space, which has particlelike properties. There are three basic categories of solitons: the Korteweg-de Vries solitons (Russell’s solitons), the Frenkel-Kontorova solitons and the envelope (group) solitons [22]. (The denomination “the Frenkel-Kontorova solitons” is used here as a more general term “the topological solitons”.) The equation established by D. J. Korteweg and G. de Vries in 1895 describes Russell’s solitons which propagate at constant speed. The Frenkel-Kontorova solitons which can be moving or entirely static are described by the sine-Gordon equation.

Yu. I. Frenkel and T. A. Kontorova theoretically predicted in 1939 solitons in chains of atoms and studied their properties [23], however, their real significance as well as their relation to the Russell soliton remained unknown for almost 30 years [24]. The Frenkel-Kontorova soliton is simpler than the Russell soliton. It is encountered in diverse physical systems. The Frenkel-Kontorova soliton has a fixed shape that does not depend on its velocity. It is interesting that the dependence of the energy $E$ of a Frenkel-Kontorova soliton on its velocity $v$ has the same form as for a relativistic particle

$$E = \frac{mv^2}{\sqrt{1 - \frac{v^2}{v_o^2}}},$$

where $m$ is the mass of the Frenkel-Kontorova soliton, and $v_o$ is the longitudinal sound velocity in the chain. Thus, the Frenkel-Kontorova solitons behave like relativistic particles, and they cannot propagate faster than $v_o$. In addition, there also exist antisolitons which are analogous to antiparticles.

Solitons can also be found in the superconducting state. Vortices in type-II superconductors, which appear in the mixed state (Shubnikov’s phase), are solitons. Frenkel-Kontorova solitons exist in long Josephson junctions, called Josephson solitons. A long Josephson junction is analogous to the chain of atoms studied by Frenkel and Kontorova [22].

Solitons are encountered in biological systems in which the nonlinear effects are often the predominant ones [8,12,24]. For example, many biological reactions would not occur without large conformational changes which cannot be described, even approximately, as a superposition of the normal modes of the linear theory.

The shape of a nerve pulse was determined more than 100 years ago. The nerve pulse has the bell-like shape and propagates with the velocity of about 100 km/h (the diameter of nerves in mammals is less than 20 microns) [23]. For almost a century, nobody realized that the nerve pulse is the soliton. So, all living creatures including humans are literally stuffed by solitons. Living organisms are mainly organic and, in principle, should be insulators.

It is important to emphasize that almost all solitons described above are one-dimensional objects. For example, the Russell soliton can be approximately regarded as one-dimensional. There are only a few three-dimensional solitons known today, e.g. vortices and tornados (in fact, tornados are a bound state of a few solitons spiralling around each other). Two-dimensional solitons are not observed experimentally, however, were discussed in a few theoretical papers [23].
III. QUASI-1D ORGANIC CONDUCTORS, DNA, NANOTUBES AND CUPRATES

One may wonder what can be common among quasi-1D organic conductors [5], DNA (deoxyribonucleic acid) [8,12], nanotubes [26] and cuprates. Slightly doped, they all become superconducting at low temperature. (Superconductivity in DNA is, in fact, induced [25], however, if one can find a technique to dope a DNA, it is most likely that the DNA will superconduct at low temperature by itself [27].)

The second feature which is common among quasi-1D organic conductors [8,12], DNA [24] and nanotubes [26,30] is that they transfer electrons (carry current) by solitons. This signifies that electron-phonon interactions in these compounds are relatively strong and nonlinear [8,12].

In the new MgB₂ superconductor, strong and nonlinear electron-phonon interactions are responsible for the high value of $T_c$ [31].

From these analogies, one can suppose that there are strong, nonlinear electron-phonon interactions in cuprates, which may lead to the formation of solitons.

IV. SOLITONS IN CUPRATES

Unexpectedly, tunneling measurements performed on Bi2212 single crystals provided evidence for solitonlike excitations in Bi2212, which form the superconducting condensate [29]. The main point of the evidence is presented here. For more details, the reader is referred to Refs [24,25].

It was found that, in Bi2212, tunneling spectra measured below $T_c$ are composite: they consist of incoherent part from the pseudogap and coherent quasiparticle peaks. The pseudogap which is observed above and below $T_c$ is a normal-state gap (a charge gap). Tunneling $I(V)$ characteristics corresponding exclusively to the quasiparticle peaks differ from the characteristics of BCS-type superconductors, but are in excellent agreement with theoretical curves derived for a bound state of two solitons.

Figure 1 shows two theoretical $I(V)$ characteristics. The first one which corresponds to tunneling between a normal metal and a conventional superconductor is predicted by the so-called BTK theory [32]. In metallic superconductors, the BTK predictions are verified by tunneling experiments. The second curve shown in Fig.1 corresponds to a bound state of two quasi-1D solitons [33]. The difference between the two curves shown in Fig.1 is striking. If, in a conventional superconductor, the asymptotics of the $I(V)$ curve are quasi-linear, and current increases as the bias increases, the asymptotics of the characteristic corresponding to a bound state of solitons are horizontal, constant! If it is difficult to confuse these two $I(V)$ curves, however, their conductance peaks look very similar! (not the backgrounds)

The tunneling measurements performed on underdoped and overdoped single crystals of Bi2212 show that $I(V)$ characteristics of the quasiparticle peaks are similar to the $I(V)$ curve shown in Fig.1(b). This fact indicates that, in Bi2212, there is (quasi-) one dimensionality, and the Copper pairs consist of solitons.

The most natural interpretation of the presence of quasi-one dimensionality in Bi2212 is quasi-1D charge stripes [34] which have been experimentally observed in cuprates, nickelates and manganites [35]. Antiferromagnetic domains which separate the quasi-1D charge stripes are two dimensional [15]. The solitons are quasi-1D objects, so they reside on dynamical charge stripes.

Unfortunately, in cuprates, there is not much evidence for the presence of charge stripes below $T_c$. The reason is simple: it is not easy to detect them directly because they are truly dynamical. Almost all experimental techniques are too slow to observe the charge stripes at rest. The span time (or time of interaction) of a technique which is able to detect them has to be smaller than $1 \text{ ps} = 10^{-12}$ second. An illustration will make it clear. Watching a movie you are not able to realize that every second on the screen "consists of" 24 different exposures (snapshots) because the speed of the film is too fast for your eyes (in fact, for the brain) to see them separately.

Recently, solitons have been observed by tunneling spectroscopy on CuO chains in YBCO [36]. The chains in YBCO are insulating, i.e. there are $2k_F$ charge-density-waves (CDWs) along the chains [37]. The tunneling spectrum averaged along a CuO chain shows that, below $T_c$, there is a weak bound state of solitons inside the CDW gap [38]. The magnitude of induced superconducting gap is about 6 meV.

Solitons in cuprates are of the second type, i.e. they are the Frenkel-Kontorova (topological) solitons. Consequently, the superconductivity in cuprates is the relativistic phenomenon!
V. THE BISOLITON MODEL OF SUPERCONDUCTIVITY

Here we shall briefly sketch Davydov’s bisoliton model of superconductivity. For more details, the reader is referred to Refs [11,12].

A. Electron transfer by solitons

First, we consider the electron transfer by solitons in molecular chains (the main area of Davydov’s research was the physics of biological processes). Many biological phenomena, such as photosynthesis, relate to the electron transfer from donor molecules to acceptor molecules through molecular structures. Davydov and co-workers theoretically showed that the most energetically profitable transfer of electrons in protein molecules occurs when there is a strong interaction of an electron with a chain. The system of an electron surrounded by local chain deformation is called an electrosoliton. When the static electrosoliton is generated, the total energy of the electron and the displacement field is decreased due to their interaction by the quantity

\[ \Delta E = m^*a^2\sigma^4/2k^2\hbar^2, \]

where \( m^* \) is the mass of the electrosoliton (which exceeds the electron mass, \( m \)); \( a \) is the distance between molecules along the chain; \( \sigma \) is the deformation parameter of the chain, and \( k \) is the coefficient of longitudinal elasticity of the chain. As \( \Delta E \) increases, the electrosoliton becomes more stable. Large values of \( \Delta E \), which provide the stability of the soliton under the conduction band of a quasiparticle (hole or electron), damps it strongly in the conduction band.

It is known that, in redox reactions occurring in living organisms, electrons are transferred from one molecule to another in pairs with opposite spins. It is also known that the transport of electrons in the synthesis process of ATP (adenosine triphosphate) molecules in conjugate membranes of mitochondria and chloroplasts is realized by pairs, but not individually.

Two electrons coupled in a singlet state due to local chain deformation is called a bisoliton. In the bisoliton state, both electrons move in the combined effective potential well

\[ U_{\text{eff}}(\zeta) = -2g^2J \text{sech}^2(g\zeta) \]

relative to the coordinate frame \( \zeta = (x - vt)/a \), where \( x \) is the axis along the chain; \( t \) is time, and \( v \) is the velocity of the bisoliton. \( J \) is the energy of the exchange interaction between neighboring molecules in the chain, and \( g \) (and \( G \)) is the dimensionless quantity

\[ g = m^*a^2G/\hbar^2, \quad G = \sigma^2/k(1 - s^2), \]

where \( s = v/v_0 \) (\( v_0 \) is the longitudinal sound velocity in the chain). The dimensionless parameter \( g \) characterizes the coupling of an electron (hole) with the deformation field. The average distance between paired electrons in a bisoliton is determined by

\[ L \approx 2\pi a/g. \]

If \( g \approx 1.5 \), then \( L \approx 4a \).

The energy of pairing is given by

\[ \Delta E(v) = \frac{\hbar^2g^2(1 - 5s^2)}{4ma^2(1 - s^2)^2}, \]

where the effective Coulomb repulsion is neglected. At small electron velocities, a pairing becomes energetically profitable. For this regime of velocities, the energy can be represented as follows

\[ \Delta E(v) \approx ma^2\sigma^4(1 - 2s^2)/4k^2\hbar^2, \quad s^2 \ll 1. \]

The effective mass of paired electrons moving together with a local chain deformation is given by

\[ M_{\text{bis}} = 2m(1 + 2M\sigma^4/3k^3\hbar^2), \]

where \( M \) is the mass of a molecule. The mass of a bisoliton exceeds the mass of two electrons.

If we take into account the Coulomb repulsion as a perturbation, then, at small velocities, a pairing is still energetically profitable if the dimensionless coupling constant \( g \) is greater than some critical value,

\[ g_{cr} \equiv (4e_{\text{eff}}/bJ)^{1/2}, \]

where \( e_{\text{eff}} \) is the effective screened charge, and \( b \) is the average radius of molecules across the chain.

It is interesting to note that the pairing energy of a bisoliton, given by (6), is independent of the mass \( M \) of heavy molecules unlike that of the pairing energy predicted by the BCS theory.

B. Quasi-1D organic conductors

The bisoliton model of superconductivity in quasi-1D organic conductors was proposed by Brizhik and Davydov [4].

In quasi-1D organic conductors, the bonds between plane molecules in stacks refer to weak van der Waals forces. Hence, due to the deformation interaction, a quasiparticle (electron or hole) causes a local deformation of a stack of molecules, which also induces intramolecular atomic displacements in molecules. The deformation interaction results in the nonlinear equations, which were considered above.

If the density of excess pairs of quasiparticles in a stack is small, and the condition \( g < 3 \) takes place, the crystal
becomes superconducting as the temperature decreases. The estimated critical temperature \( T_c \), given by
\[
k_B T_c \approx \sigma^4 / 2J^2 k^2 a^4, \tag{9}
\]
is independent of the mass \( M \) of heavy molecules. \( k_B \) is the Boltzmann constant.

The paired quasiparticles are not localized between the molecules of stacks, but, surrounded by deformation, enveloping some molecules of stacks, move coherently as a unique entity along stacks of organic molecules without resistance.

### C. Superconductivity in cuprates

Davydov has applied the bisoliton model of superconductivity to high-\( T_c \) superconductivity in cuprates. At that time, Davydov did not know about charge stripes which may exist in \( \text{CuO}_2 \) planes. He needed to locate one dimensionality in \( \text{CuO}_2 \) planes. The \( \text{CuO}_2 \) planes in cuprates consist of quasi-infinite parallel chains of alternating ions of copper and oxygen. He assumed that each \(-\text{Cu-O-Cu-O}\) chain in the \( \text{CuO}_2 \) planes can be considered as a quasi-1D system. Therefore, the current flows along parallel chains. He studied the charge migration in one of these chains.

Due to electron-phonon interactions of quasiparticles with the resulting displacements of the positions of elementary cells, there appears local chain deformations which, in turn, lead to a coupling of quasiparticles into singlet spin pairs, \( \text{i.e.} \) into bisolitons propagating along chains with a constant velocity \( v < v_o \). Bisolitons do not interact with acoustic phonons since that interaction is taken completely into account in the coupling of quasi-particles with a local deformation. Therefore, they do not radiate phonons. At low temperature, bisolitons are stable if the gain in the binding energy under their coupling exceeds the screened Coulomb repulsion of their charges. We further assume that this condition holds.

The calculations of the energy gap, \( \Delta \), in the quasiparticle spectrum resulting from a pairing give
\[
2\Delta = g^2 J F(q), \tag{10}
\]
where the function \( F(q) \) is determined by the modulus of the Jacobian elliptic function via the relation
\[
F(q) = \frac{2}{E^3(q)}[2 - q^2 - \Xi(q)E^{-1}(q)], \tag{11}
\]
where
\[
\Xi(q) = \frac{1}{3}[2(2 - q^2)E(q) - (1 - q^2)K(q)]. \tag{12}
\]
The pairing gap is a half of the energy of the formation of a bisoliton. The functions \( K(q) \) and \( E(q) \) are the complete elliptic integrals of the first and the second kind, respectively. The value of the modulus \( q \) is determined by the product of the period \( L/a \) with the dimensionless coupling parameter \( g \) and the elliptic integrals as follows
\[
gL/a = 2E(q)K(q). \tag{13}
\]

The energy of pairing in (10) does not depend on the mass \( M \) of an elementary cell. This mass only appears in the kinetic energy of the bisolitons. Therefore the isotope effect is absent, notwithstanding the fact that the basis of pairing is an electron-phonon interaction.

At rather small densities of quasiparticles, when the inequality \( gL/a \gg 1 \) is valid, the energy gap can be presented as
\[
2\Delta = \frac{2}{3}q^2 J[1 + 4gL/a \exp(-gL/a)]. \tag{14}
\]

At rather high density of quasiparticles, the energy gap which takes the value
\[
2\Delta = 2g^2 J \left( \frac{2a}{\pi} \right)^3 \tag{15}
\]
decreases in comparison with (14). In addition, Davydov did not take into account that \( J \) (more precisely the effective exchange energy \( J_{\text{eff}} \)) may depend on the doping level.

The permissible maximum density of quasiparticles (hole or electrons) is determined by the minimum allowable distance \( L_{\text{min}} \) between two bisolitons, given by
\[
L_{\text{min}} = \pi^2 a / 2g. \tag{16}
\]

In the bisoliton model of high-\( T_c \) superconductivity, \( T_c \) is defined as the temperature at which the energy gap vanishes.

In superconductors described by the BCS theory, the dimensionless coupling parameter \( g \) is the order of \( 10^{-3} - 10^{-4} \), while it is around 1 in cuprates.

### VI. THE BISOLITON MODEL AND EXPERIMENT

Here we discuss the bisoliton model of superconductivity and compare data obtained in cuprates with predictions of the model.

Davydov’s genius is that, from only three experimental facts described below, he immediately understood that the high-\( T_c \) superconductivity is related to soliton excitations. These three facts were: (i) the absence of the isotope effect (now we know that there is the isotope effect in cuprates but it is very weak \([34]\)); (ii) the coherence length in hole-doped cuprates is very short, \( \xi \approx 15–20 \ \text{Å} \), and (iii) cuprates become superconducting only when they are slightly doped. He concluded that the sharp decrease of the coherence length in cuprates
in comparison with metallic superconductors indicates a rather large interaction of quasiparticles (holes or electrons) with the acoustic branch of the lattice vibrations inherent in cuprates.

In the bisoliton model, the mechanism of the establishment of phase coherence among bisolitons is not specified. In the framework of the bisoliton model, phonons cannot mediate the phase coherence because, as emphasized by Davydov, bisolitons do not interact with acoustic phonons. In the BCS theory for conventional superconductors, the phase coherence among the Cooper pairs is established due to the overlap of their wave functions (the wave-function coupling) because the average distance between the Cooper pairs is much smaller than the coherence length (the size of a Cooper pair). In cuprates, the distance between the Cooper pairs (bisolitons) is similar to the size of the bisoliton. Under such conditions, can the wave-function coupling mediate the phase coherence among the bisolitons? Davydov defined $T_c$ as the temperature at which the energy gap vanishes. With such a definition for $T_c$, the wave-function coupling can formally be the mediator of the phase coherence. However, in reality, the wave-function coupling cannot be responsible for mediating the phase coherence among the bisolitons. This was the reason why Davydov avoided to discuss the bell-like shape of $T_c(p)$ dependence (see Fig.3), where $p$ is the hole (electron) concentration in CuO$_2$ planes, because he could not explain it in the framework of the model. The bisoliton theory predicts that by increasing the hole (electron) concentration the magnitude of pairing gap decreases. Consequently, if the wave-function coupling mediate the phase coherence among the bisolitons, then, from $T_c \sim \Delta$, one can obtain that by increasing the hole (electron) concentration $T_c$ will monotonically decrease, contrary to the experiment. Thus, in cuprates, the phase coherence among bisolitons is established due to a non-phonon mechanism which is different from the wave-function coupling.

Experimentally, in hole-doped cuprates, spin fluctuations (magnetic electron-electron interactions) mediate the phase coherence among the Copper pairs [7][8]. So, the bisoliton model is the theory of soliton pairing, but it lacks the mechanism of the establishment of phase coherence. Therefore, we now discuss solely the pairing characteristics.

First, let us estimate the coupling constant $g$ in hole- and electron-doped cuprates. By using the values of Cu-O-Cu bonding length ($a \simeq 3.9 \text{ Å}$) and the coherence length measured in hole-doped ($\xi \approx 15–20 \text{ Å}$) and electron-doped ($\xi \simeq 80 \text{ Å}$) cuprates, from (4), we have $g_h \simeq 1.2–1.6$ and $g_e \simeq 0.3$, respectively. The ratio $g_h/g_e \simeq 4.3–5.3$ is in good agreement with similar ratio estimated for $C_{60}$, $g_h/g_e \approx 5–6$ [10]. This means that, independently from the material, the maximum $T_c$ value will always be higher in hole-doped superconductors.

Let us now estimate the maximum concentration of doped holes (electrons) in cuprates, $p_{\text{max}}$, at which the bisolitons still exist. From (4) and (16), one can easily obtain that, on a single chain (stripe etc.),

$$p_{\text{max}} = \frac{4g}{\pi(\pi + 4)}.$$

To estimate $p_{\text{max}}$ in hole-doped cuprates, let us use $g \approx 1.5$. Then, the calculations give $p_{\text{max}} \approx 0.27$ in hole-doped and $p_{\text{max}} \approx 0.054$ in electron-doped cuprates. The maximum hole/electron concentration at which hole- and electron-doped cuprates are still superconducting is 0.27 and 0.17, respectively. So, in hole-doped cuprates, the prediction of the bisoliton model coincides with the real value. However, in electron-doped cuprates, it is three times smaller than the measured one.

In (14) and (15), the energy gap $\Delta$ depends on the energy of the exchange interaction $J$, the hole (electron) concentration $p$ ($\sim a/L$) and the coupling parameter of the electron-phonon interactions $g$. Both $J$ (i.e. $J_{\text{eff}}$) and $g$ depend themselves on the doping level. Since the coupling parameter depends weakly on $p$, the variations of the energy gap as a function of $p$ is mainly determined by the $J_{\text{eff}}(p)$ dependence. Raman scattering measurements in Bi2212 show that, at different hole concentrations, $\Delta \approx \frac{\Delta}{\sqrt{p}} J_{\text{eff}}$ (see Fig.2 in Ref. [41]). In order to estimate the energy gap from (14) and (15), let us use again $g \approx 1.5$. Then, at low hole concentrations, we have $\Delta \approx \Delta_{\text{eff}} \approx \frac{\Delta}{\sqrt{p}} J_{\text{eff}}$, and, at high hole density, $\Delta \approx \frac{\Delta}{\sqrt{p}} J_{\text{eff}} \approx \frac{\Delta}{\sqrt{p}} J_{\text{eff}}$. So, there is very good agreement between the bisoliton model and the data.

To conclude, in hole-doped cuprates, the bisoliton model of superconductivity is correct in the description of pairing characteristics, however, it lacks the mechanism of the establishment of phase coherence. It is a pity that the bisoliton model did not attract any attention on earlier stages of the development of high-$T_c$ superconductivity. It is also a pity that A. S. Davydov is no longer with us [12].

VII. CHARGE STRIPES

The origin of the driving force for the charge-stripe formation in cuprates is still an open question. Since the charge stripes in cuprates appear immediately after the lattice transformation [13], it is reasonable to assume that electron-lattice interactions are responsible for the formation of charge stripes (see also Ref. [13]). In other words, solitons which later form the bisolitons reside on charge stripes not because the charge stripes existed beforehand, but because solitons and charge stripes appear simultaneously as a consequence of electron-lattice interactions.

In fact, the formation of dynamical charge stripes in cuprates can be viewed as the appearance of dynamical
CDWs \cite{44,45,46} in consequence of electron-phonon interactions.

VIII. THE MECHANISM OF SUPERCONDUCTIVITY IN CUPRATES

In this Section, I present a brief description of the mechanism of high-\(T_c\) superconductivity. For more details, the reader is referred to Refs \cite{24,27}.

The Cooper pairs in cuprates are bisolitons or hole (electron) pairs coupled in a singlet state due to local deformation of the lattice. Thus, moderately strong, nonlinear electron-phonon interactions are responsible for the pairing. The bisolitons are formed above \(T_c\) on charge stripes (on dynamical CDWs).

The long-range phase coherence among the bisolitons is established due to spin fluctuations in local antiferromagnetic domains of CuO\(_2\) planes, which are situated between charge stripes. However, in different cuprates, the phase coherence is established differently. Let us classify all hole-doped superconducting cuprates in the two categories: 40 K and 90 K cuprates. All optimally doped cuprates having the \(T_c\) value less (more) than 40 K (90 K) belong to the first (second) group. \textit{De fait}, the 40 K cuprates have one CuO\(_2\) layer per unit cell. The main difference between the two groups, which defines the \(T_c\) value, is the absence/presence of the so-called magnetic resonance peak in inelastic neutron scattering (INS) spectra. The phase coherence in 40 K cuprates is locked at \(T_c\) via the long-range antiferromagnetic order (or spin-density-wave order). In 90 K cuprates, the phase coherence is established among bisolitons due to spin excitations which cause the appearance of the magnetic resonance peak in INS spectra. The spin excitations seem to be in resonance with the bisolitons.

Such a mechanism of the establishment of phase coherence implies that the magnetic resonance peak can be observed in all 90 K cuprates. The resonance peak has already been detected by INS in the double-layer cuprates YBCO and Bi2212 \cite{17,18} and in the single-layer Tl\(_2\)Ba\(_2\)CuO\(_6\) \cite{5}. The process of bisoliton condensation at \(T_c\), as a matter of fact, is the Bose-Einstein condensation. In cuprates, below \(T_c\), charge, spin and lattice degrees of freedom in the CuO\(_2\) planes are coupled.

The symmetry of the pairing wave function of bisolitons is an anisotropic s-wave while the order parameter which is responsible for the establishment of phase coherence has the \(\Delta_{d_{x^2-y^2}}\) (d-wave) symmetry. This is the reason why all phase-sensitive techniques detect in cuprates the d-wave symmetry. At the same time, tunneling measurements show a s-wave symmetry of the condensate \cite{18,34}, \textit{i.e.} the symmetry of the pairing wave function. The maximum magnitudes of the two gaps in Bi2212 as functions of doping are schematically shown in Fig.2 as well as their temperature dependences.

In cuprates, above and below \(T_c\), there is a normal-state gap, a pseudogap. It is impossible to formulate the origin of the pseudogap because, in fact, there are two pseudogaps, and different techniques observe different pseudogaps. Besides the presence of the bisoliton pairing gap above \(T_c\), there is a charge gap (a CDW gap) on charge stripes, and there is a spin gap in local antiferromagnetic domains situated between charge stripes \cite{34,19}. The superconducting cuprates inherited antiferromagnetic correlations from their parent compounds, antiferromagnetic Mott insulators. Figure 3 schematically shows the characteristic temperatures of charge and spin degrees of freedom in CuO\(_2\) planes of Bi2212.

In transport measurements, the pseudogap which relates to the spin gap \cite{5} vanishes at \(p \approx 0.19-0.2\) \cite{13} because the electronic state in CuO\(_2\) planes above \(p \approx 0.19-0.2\) becomes inhomogeneous in a macroscopic scale \cite{52}. In other words, in the heavily overdoped region, there exists already metallic islands in CuO\(_2\) planes.

The scenario of superconductivity in electron-doped cuprates seems to be similar to the scenario realized in 40 K hole-doped cuprates.

Many data obtained in cuprates can be naturally understood in the framework of such a scenario for high-\(T_c\) superconductivity, as analyzed elsewhere \cite{21}. As an example and additional evidence, let us consider elastic properties of high-\(T_c\) superconductors, which are remarkably different from those of conventional superconductors. In cuprates, the elastic coefficients for various modes reveal not only anisotropic lattice properties in the normal state, but also anisotropic coupling between superconductivity and lattice deformation \cite{33,54}. What is even more striking is the effect of magnetic field ap-
The bisoliton superconductivity requires moderately strong electron-phonon interactions, thus, it cannot occur in pure metals at normal conditions. The first indication of bisoliton superconductivity is the coherence length. As emphasized by Davydov, the short coherence length, of say less than 100 Å, can already be a good sign of bisoliton superconductivity.

By analogy with the isotope effect in conventional superconductors, the study of elastic properties in cuprates and other compounds can serve as a key experiment for revealing the bisoliton superconductivity.

In addition, tunneling measurements always remain a key experiment for any type of superconductivity.

An attribute of the magnetic (spin-fluctuation) mechanism is the temperature dependence of the gap, lying below the BCS curve, as shown in the inset of Fig.2.

X. SUPERCONDUCTIVITY AT 300 K

Due to a clear picture of the mechanism of high-\(T_c\) superconductivity, it is possible to discuss necessary "ingredients" of superconductivity at high temperature. This discussion does not signify that all types of superconductivity which will be discovered in the future have to be based on electron-phonon interactions. However, by taking into account the importance of electron-phonon interactions in the BCS mechanism of superconductivity and in the bisoliton model of high-\(T_c\) superconductivity, it is clear that the electron-phonon interactions will surprise us repeatedly in the future.

Let us discuss necessary ingredients of superconductivity at high temperature (> 200 K) by using the scenario of superconductivity in cuprates as a basis. First, the system has to be hole-doped and quasi-1D. It is important that the system is quasi-1D because one-dimensional systems tend not to be superconducting by themselves like, for example, the chains in YBCO. Second, a rather large interaction of holes with the acoustic branch of lattice vibrations is necessary for pairing at high temperature. Third, the presence of strong magnetic (spin) fluctuations is vital to mediate the phase coherence and, in each system, it is necessary to understand the best conditions for the appearance of the resonance mode. The latter ingredient, probably, will be always the weakest link, the bottleneck in achieving high \(T_c\).

Superconductivity at 300 K, in principle, is possible. As a matter of fact, bisolitons (the Cooper pairs) exist above 300 K in living tissues. The question is how to make them to communicate with each other?
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