Own fluctuations of technological systems

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Abstract. Own fluctuations in the drives of technological equipment based on a three-mass model of fluctuating torsional chain system are analyzed. Differential equations of motion of a mechanical system with three degrees of freedom are obtained. An analysis of the forms of fluctuating processes in the system is conducted. Conditions of orthogonality of own torsional fluctuation forms are considered, dependencies of own frequencies on system rigidity coefficients are determined.

1. Introduction
Nowadays, high demands are placed on the quality and precision of machining of mechanical engineering products. The possibilities of technological equipment, which has the ability to perform operations with high productivity, have increased significantly. The increase of working speeds of technological machines makes us pay special attention to revealing of dynamic factors on which accuracy indicators of equipment work depend [1, 2].

Structurally, technological machines have several basic functional parts. These include, above all, various drives consisting of motors, mechanical transmission, executive and working elements as well as control systems.

Internal factors and external influences arising from the functioning of the machine's mechanical system lead to the appearance of various types of fluctuations, which worsen the conditions of normal exploitation of the equipment, reduce the accuracy of operations, distort the trajectories of the working elements [3, 4, 5, 6].

The majority of machining in mechanical engineering is done on high-performance CNC machines. Here significant dynamic loads appear, caused by the interaction of cutting tools and work-piece material in the formation of complex CNC trajectories.

The analysis of torsional fluctuations is of great importance for the solution of the problem of increasing the accuracy of machining on CNC machines. These torsional fluctuations appear in the main motion and feed rate drives during start-up, during acceleration or deceleration, related to changes in the direction of motion, as well as during the engaging of tools into the work-piece material [3, 5, 6, 7, 8, 10]. The methods of technological quality assurance and wear resistance implemented during the production of structural elements of production equipment are also essential for reducing vibration activity during the operation of the machines [9, 10].

2. Materials and methods
The most interesting for engineering calculations are the dynamic models with concentrated parameters and especially three-mass fluctuating systems, which by relatively simple means are able to adequately reflect the fluctuating processes in the technological system [6, 10, 11].
Let us consider the three-mass fluctuating chain system presented in Figure 1. The model includes three disks with moments of inertia $J_1$, $J_2$ and $J_3$, which are fastened to an elastic shaft. The shaft sections between the discs have torsional rigidity coefficients $C_1$ and $C_2$.

The model shows the mechanical drive system consisting of motor rotor 1, the transmission mechanism 2 and the working element 3. The mechanical drive system has three degrees of freedom. The fluctuating processes in the system will be considered in the case of the absence of viscous friction. The rotation angles of the discs $\phi_1$, $\phi_2$ and $\phi_3$, calculated from any position of the system in case of no deformation of the shaft, will be used as generalized coordinates.

For the model of three-mass mechanical fluctuating torsional chain system, the dependence of inertia moments of links on rotation angles looks like [6]:

$$J_1 \dot{\phi}_1 + J_2 \dot{\phi}_2 + J_3 \dot{\phi}_3 = \text{const.}$$

(1)

Figure 1. Three-mass mechanical fluctuating system.

When $\text{const} \neq 0$ this equation defines the rotation of the system as a whole rigid with a constant angular velocity $\omega_0$,

$$\phi_1 = \phi_2 = \phi_3 = \phi_0 + \omega_0 t,$$

(2)

where $\phi_0$ and $\omega_0$ are from the initial conditions.

In the considered mechanical fluctuating system, it is also possible to have fluctuations of disks around the non-deformed state of the shaft, described by functions:

$$\begin{align*}
\phi_1 & = A_1 \sin (kt + \alpha), \\
\phi_2 & = A_2 \sin (kt + \alpha), \\
\phi_3 & = A_3 \sin (kt + \alpha)
\end{align*}$$

(3)

The functions (3) define such a motion of the system, where the disks fluctuate with the equal frequency $k$ and the initial phase $\alpha$, but with different amplitudes.

Depending on the initial conditions, the movement of the system may occur in one of the following ways:

1. The discs rotate around the shaft axis at an angular speed of $\omega_0$, and the discs do not fluctuate. The movement of the system is described by the function (2). Let us conditionally say that the system is in the main frequency fluctuation $k_0 = 0$. The own form is determined by the coefficients $\mu_{10} = \mu_{20} = \mu_{30} = 1$.

2. The discs fluctuate at the frequency of $k_1$ and do not participate in rotational motion at an angular speed of $\omega_0$. The functions (3) are written in this case as follows:
The first digit of the index of rotation angle or amplitude represents the number of the disk (coordinates), and the second digit represents the number of frequency.

This movement is called the main frequency fluctuation $k_1$. The own form of this main fluctuation is determined by the coefficients $\mu_{11}$, $\mu_{31}$.

3. The discs are fluctuating with the frequency $k_2$, and do not participate in rotational motion at an angular velocity of $\omega_0$. The functions (3) are written as follows:

$$\varphi_{12} = A_{12} \sin (k_2 t + \alpha_2),$$
$$\varphi_{22} = A_{22} \sin (k_2 t + \alpha_2),$$
$$\varphi_{32} = A_{32} \sin (k_2 t + \alpha_2)$$

This movement is called the main frequency fluctuation $k_2$. The own form of this main fluctuation is determined by the coefficients $\mu_{12} = 1$, $\mu_{22}$ and $\mu_{32}$.

4. The discs participate in both rotational motion with an angular velocity of $\omega_0$, and in the superimposed fluctuations with frequencies $k_1$ and $k_2$. In this case, the rotation angles of the discs are determined by the dependencies:

$$\varphi_1 = \varphi_{10} + \varphi_{11} + \varphi_{12} = \varphi_{10} + \omega_0 t + A_{11} \sin (k_1 t + \alpha_1) + A_{12} \sin (k_2 t + \alpha_2)$$
$$\varphi_2 = \varphi_{20} + \varphi_{21} + \varphi_{22} = \varphi_{20} + \omega_0 t + \mu_{21} A_{11} \sin (k_1 t + \alpha_1) + \mu_{22} A_{12} \sin (k_2 t + \alpha_2)$$
$$\varphi_3 = \varphi_{30} + \varphi_{31} + \varphi_{32} = \varphi_{30} + \omega_0 t + \mu_{31} A_{11} \sin (k_1 t + \alpha_1) + \mu_{32} A_{12} \sin (k_2 t + \alpha_2)$$

This general solution contains six recurring integrations: $\varphi_0$, $\omega_0$, $A_{11}$, $A_{12}$, $\alpha_1$ and $\alpha_2$. They are determined by the initial conditions:

$$t = 0,
\begin{align*}
\varphi_1(0) &= \varphi_{10}, \quad \varphi_2(0) = \varphi_{20}, \quad \varphi_3(0) = \varphi_{30}, \\
\dot{\varphi}_1(0) &= \dot{\varphi}_{10}, \quad \dot{\varphi}_2(0) = \dot{\varphi}_{20}, \quad \dot{\varphi}_3(0) = \dot{\varphi}_{30}.
\end{align*}$$

Let us consider the orthogonality of the fluctuations’ own form.

Own forms must meet the condition of orthogonality. Let us write down a system of algebraic equations relative to the unknown amplitudes first for $l^{th}$ and then for $r^{th}$ main fluctuations:

$$\begin{align*}
\left(C_1 - J_1 k_1^2\right) A_{l1} - C_1 A_{2l} &= 0 \\
-C_1 A_{l1} + \left(C_1 + C_2 - J_2 k_1^2\right) A_{2l} - C_2 A_{3l} &= 0; \\
-C_2 A_{l1} + \left(C_2 - J_3 k_1^2\right) A_{3l} &= 0
\end{align*}$$

$$\begin{align*}
\left(C_1 - J_1 k_2^2\right) A_{r1} - C_1 A_{2r} &= 0 \\
-C_1 A_{r1} + \left(C_1 + C_2 - J_2 k_2^2\right) A_{2r} - C_2 A_{3r} &= 0; \\
-C_2 A_{r1} + \left(C_2 - J_3 k_2^2\right) A_{3r} &= 0
\end{align*}$$

We multiply the first, second and third lines of the equations (8) by $A_{1r}$, $A_{2r}$, and $A_{3r}$, respectively, and then summarize the left and right parts of the equations. We get:
\[ C_1 A_1 A_r - C_1 A_2 A_r - C_1 A_3 A_r + (C_1 + C_2) A_2 A_r - \\
- C_2 A_1 A_r - C_2 A_2 A_r + C_2 A_3 A_r = k_i^2 \left( J_1 A_1 A_r + J_2 A_2 A_r + J_3 A_3 A_r \right). \] (10)

Then we multiply the first, second and third lines of the equations (9) by \( A_1, A_2 \) and \( A_3 \) respectively, and then form the second sum similar to the first one:

\[ C_1 A_1 A_{l_1} - C_1 A_2 A_{l_1} - C_1 A_3 A_{l_1} + (C_1 + C_2) A_2 A_{l_1} - \\
- C_2 A_1 A_{l_1} - C_2 A_2 A_{l_1} + C_2 A_3 A_{l_1} = k_i^2 \left( J_1 A_1 A_{l_1} + J_2 A_2 A_{l_1} + J_3 A_3 A_{l_1} \right). \] (11)

Let us subtract from the expression (11) the expression (10), we get:

\[ (k_i^2 - k_i^2) \left( J_1 A_1 A_r + J_2 A_2 A_r + J_3 A_3 A_r \right) = 0. \] (12)

Because \( k_r \neq k_l \), we get the relation:

\[ J_1 A_1 A_{l_1} + J_2 A_2 A_{l_1} + J_3 A_3 A_{l_1} = 0, \] (13)

determining the condition for the orthogonality of the \( l \)th and \( r \)th own forms. We divide each term of the expression (13) by \( A_{l_1} A_{l_1} \), and having in mind that

\[ \mu_{1} = \frac{A_{1}}{A_{l_1}}, \mu_{2} = \frac{A_{2}}{A_{l_1}}, \mu_{3} = \frac{A_{3}}{A_{l_1}}, \mu_{1} = \frac{A_{1}}{A_{l_1}}, \mu_{2} = \frac{A_{2}}{A_{l_1}}, \mu_{3} = \frac{A_{3}}{A_{l_1}}, \]

we get:

\[ J_1 + J_2 \mu_{21} \mu_{2r} + J_3 \mu_{31} \mu_{3r} = 0. \] (14)

For \( l = 0 \) and \( r = 1 \) we obtain the condition for orthogonality of the own forms of the zero and first frequencies:

\[ J_1 + J_2 \mu_{20} \mu_{21} + J_3 \mu_{30} \mu_{31} = 0. \] (15)

For \( l = 0 \) and \( r = 2 \) we obtain the condition for orthogonality of the own forms of the zero and second frequencies:

\[ J_1 + J_2 \mu_{20} \mu_{22} + J_3 \mu_{30} \mu_{32} = 0. \] (16)

For \( l = 1 \) and \( r = 2 \) we obtain the condition for orthogonality of the own forms of the first and second frequencies:

\[ J_1 + J_2 \mu_{21} \mu_{22} + J_3 \mu_{31} \mu_{32} = 0. \] (17)

Using the general solution (6), we introduce new generalized main coordinates:

\[ \psi_0 = \varphi_0 + \omega_0 t, \]
\[ \psi_1 = A_1 \sin (k_1 t + \alpha_1), \]
\[ \psi_2 = A_2 \sin (k_2 t + \alpha_2). \] (18)

The generalized coordinate \( \psi_0 \) corresponds to the rotation of the torsional system with angular velocity \( \omega_0 \), the generalized coordinate \( \psi_1 \) – system fluctuation with frequency \( k_1 \), the coordinate \( \psi_2 \) – system fluctuation with frequency \( k_2 \). Taking into account (6) we have:

\[ \varphi_1 = \psi_0 + \psi_1 + \psi_2, \]
\[ \varphi_2 = \psi_0 + \mu_2 \psi_1 + \mu_2 \psi_2, \]
\[ \varphi_3 = \psi_0 + \mu_3 \psi_1 + \mu_3 \psi_2. \] (19)

We calculate the time derivatives of the dependencies (19) and substitute them into the expression determining the kinetic energy of the system:
\[
T = \frac{1}{2} \left[ J_1 (\psi'_0 + \psi'_1 + \psi'_2)^2 + J_2 (\psi'_0 + \mu_2 \psi'_1 + \mu_2 \psi'_2)^2 + J_3 (\psi'_0 + \mu_1 \psi'_1 + \mu_3 \psi'_2)^2 \right] = \\
= \frac{1}{2} \left[ (J_1 + J_2 \mu_1^2 + J_3 \mu_3^1) \psi'_1^2 + (J_1 + J_2 \mu_2^2 + J_3 \mu_3^2) \psi'_2^2 + (J_1 + J_2 + J_3) \psi'_0^2 + \\
+ 2\psi'_1 \psi'_2 (J_1 + J_2 \mu_2 \mu_2, J_3 \mu_3(\mu_3 + 2) + 2\psi'_1 \psi'_2 (J_1 + J_2 \mu_2, J_3 \mu_3 + 2) + \\
+ 2\psi'_1 \psi'_2 (J_1 + J_2 \mu_2 \mu_2, J_3 \mu_3 \mu_3) \right].
\]

Using the expressions (15), (16) and (17), we introduce the notation:
\[
J_1 = J_1 + J_2 \mu_1^2 + J_3 \mu_3^1, \\
J_2 = J_1 + J_2 \mu_2^2 + J_3 \mu_3^2, \\
J_0 = J_1 + J_2 + J_3.
\]

Now, the kinetic energy of the system can be written as:
\[
T = \frac{1}{2} J_1 \psi'_1^2 + \frac{1}{2} J_2 \psi'_2^2 + \frac{1}{2} J_0 \psi'_0^2.
\]

The first term defines the kinetic energy of the main fluctuation with frequency \(k_1\), the second term is the kinetic energy of the main fluctuation with frequency \(k_2\) and the third term is the kinetic energy of the total rotational motion of the discs with angular velocity \(\omega_0\). The values of \(J_1, J_2, \text{ and } J_0\) determine the inertial coefficients of the system corresponding to the frequencies \(k_1, k_2\) and \(k_0\). Thus, we have obtained a known result from the general theory of small fluctuations: the kinetic energy of the system is equal to the sum of kinetic energy of the main fluctuations. We substitute (19) in the expression of potential energy:
\[
\Pi = \frac{1}{2} \left[ C_1 (\phi_1 - \phi_2)^2 + C_2 (\phi_2 - \phi_3)^2 \right] = \\
= \frac{1}{2} \left[ C_1 \left( (1 - \mu_21) \psi_1 + (1 - \mu_22) \psi_2 \right)^2 + C_2 \left[ (\mu_21 - \mu_31) \psi_1 + (\mu_22 - \mu_32) \psi_2 \right]^2 \right] = \\
= \frac{1}{2} \left[ C_1 \left( (1 - \mu_21)^2 + C_2 \left( (\mu_21 - \mu_31)^2 \psi_1^2 + C_1 \left( (1 - \mu_22)^2 + C_2 \left( (\mu_22 - \mu_32)^2 \psi_2^2 + \\
+ 2C_1 (1 - \mu_21)(1 - \mu_22) + C_2 (\mu_21 - \mu_31)(\mu_22 - \mu_32) \right) \right) \psi_1 \psi_2 \right].
\]

The sums in the left side of the expressions, obtained when the condition of orthogonality of own forms is derived, are shown after simple transformations:
\[
C_1 A_{1i} A_{1r} - C_1 A_{2i} A_{2r} - C_1 A_{3i} A_{3r} + (C_1 + C_2) A_{2i} A_{2r} - C_2 A_{3i} A_{3r} - C_2 A_{2i} A_{3r} + C_2 A_{3i} A_{3r} = \\
= C_1 (1 - \mu_32)(1 - \mu_31) + C_2 \left( (\mu_21 - \mu_31) \mu_22 - \mu_32 \right).
\]

For \(l = 1\) and \(r = 2\) due to the orthogonality of their own forms, we get:
\[
C_1 (1 - \mu_21)(1 - \mu_22) + C_2 \left( (\mu_21 - \mu_31) \mu_22 - \mu_32 \right) = 0.
\]

The coefficient of the product of the generalized coordinates \(\psi_1\) and \(\psi_2\) is equal to zero. By entering the symbols:
\[
C_1 = C_1 \left( (1 - \mu_21)^2 + C_2 \left( (\mu_21 - \mu_31)^2 \right) \right. \quad \text{and} \quad C_2 = \left. C_1 \left( \mu_21 - \mu_31 \right)^2 + C_2 \left( \mu_22 - \mu_32 \right)^2 \right),
\]
we will write down the potential energy of the system in the form of:
\[
\Pi = \frac{1}{2} C_1 \psi_1^2 + \frac{1}{2} C_2 \psi_2^2.
\]

The values of \(C_1\) and \(C_2\) they determine the quasi-elastic system coefficients corresponding to the
frequencies \( k_1 \) and \( k_2 \). The generalized coordinate \( \psi_0 \) is not included in the expression of potential energy. Such a coordinate is called a cyclic one.

Thus, by means of the linear transformation \((19)\) it was possible to record the kinetic and potential energy of the system in such a way that in \((20)\) and \((21)\) there are no components containing products of either generalized coordinates \( \psi_1 \cdot \psi_2 \cdot \psi_3 \) or generalized velocities \( \dot{\psi}_1 \cdot \dot{\psi}_2 \cdot \dot{\psi}_3 \). This transformation of coordinates is called orthogonal, and it is possible due to the orthogonality of own forms. The new generalized coordinates \( \psi_1, \psi_2 \) and \( \psi_0 \) are called the main (normal) coordinates.

Using the Lagrange equations, we will get:

\[
\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{\psi}_l} \right) - \frac{\partial T}{\partial \psi_l} = -\frac{\partial \Pi}{\partial \dot{\psi}_l} \quad (l = 0, 1, 2). \tag{28}
\]

With the help of functions \((22)\) and \((27)\) differential equations of motion of the system in the main coordinates are obtained:

\[
\begin{align*}
J_1 \ddot{\psi}_1 + C_1 \psi_1 & = 0 \\
J_2 \ddot{\psi}_2 + C_2 \psi_2 & = 0 \\
J_0 \ddot{\psi}_0 & = 0
\end{align*} \tag{29}
\]

Each of the system equations \((29)\) can be integrated independently. This is one of the conveniences provided by the main coordinates when studying the dynamics of the system. The first two equations determine the free fluctuations of the system: in the first one with frequency \( k_1 = C_1 \sqrt{J_1} \), and in the second with the frequency \( k_2 = C_2 \sqrt{J_2} \) of the main fluctuations.

Integration of the third equation leads to the solution of the following kind:

\[
\begin{align*}
\psi_0 & = \omega_0 t, \\
\psi_0 & = \omega_0 t + C,
\end{align*}
\]

which determines the total rotation of disks with angular velocity \( \omega_0 \).

3. Results and Discussion

Let us consider an example of determining the own frequencies of fluctuations of three-mass torsional systems.

Input data: moments of inertia \( J_1 = 200 \text{ kg m}^2; J_2 = 100 \text{ kg m}^2; J_3 = 150 \text{ kg m}^2; \) rigidity coefficients \( C_1 = 2 \times 10^5 \text{ N m}; C_2 = 1.10^5 \text{ N m}. \)

Own frequencies are derived from the frequency equation \([6]\):

\[
k^2 \left[ k^4 - \left( C_1 \frac{J_1 + J_2}{J_1 \cdot J_2} + C_2 \frac{J_2 + J_3}{J_2 \cdot J_3} \right) k^2 + C_1 C_2 \frac{J_1 + J_2 + J_3}{J_1 \cdot J_2 \cdot J_3} \right] = 0.
\]

By substituting the values of inertia moments and rigidity coefficients, we obtain:

\[
k^2 \left[ k^4 - 46667k^2 + 3 \times 10^{-6} \right] = 0,
\]

where

\[
k_1 = 27.7 \text{ s}^{-1}; k_2 = 62.4 \text{ s}^{-1}; k_0 = 0 \text{ s}^{-1}.
\]

Let us determine the form coefficients:

\[
\mu_{10} = 1, \mu_{20} = 1, \mu_{30} = 1, \mu_{11} = 1,
\]

\[
\mu_2 = \frac{C_1 J_1 C_2 J_3}{C_1 J_1 J_2 J_3} = \frac{2 \times 10^5 - 200 \times 27.7^2}{2 \times 10^5} = 0.23;
\]

\[
\mu_1 = \frac{C_2 J_2}{C_1}.
\]
\[ \mu_{31} = \frac{-C_1 + C_2 - J_2 k_1^2}{C_2} \mu_{21} = \frac{-2 \cdot 10^5 - 100 \cdot 27.7}{1 \cdot 10^5} \cdot 0.23 = -1.49; \]
\[ \mu_{12} = \frac{C_1 - J_2 k_2^2}{C_1} \mu_{22} = \frac{2 \cdot 10^5 - 200 \cdot 62.4^2}{2 \cdot 10^5} = -2.9; \]
\[ \mu_{52} = \frac{-C_1 + C_2 - J_2 k_2^2}{C_2} \mu_{22} = \frac{-2 \cdot 10^5 - 100 \cdot 64.4^2}{1 \cdot 10^5} \cdot (-2.9) = 0.6. \]

Using dependencies (15), (16) and (17), we write the conditions of orthogonality:
\[ J_1 + J_2 \mu_{20} \mu_{21} + J_3 \mu_{30} \mu_{31} = 200 + 100 \cdot 0.23 + 150 \cdot (-1.49) = 0; \]
\[ J_1 + J_2 \mu_{20} \mu_{22} + J_3 \mu_{30} \mu_{32} = 200 + 100 \cdot (-2.9) + 150 \cdot 0.6 = 0; \]
\[ J_1 + J_2 \mu_{20} \mu_{22} + J_3 \mu_{30} \mu_{32} = 200 + 100 \cdot (-2.9) \cdot 0.23 + 150 \cdot (-1.49) \cdot 0.6 = 0. \]

Let us calculate the inertia \( J_I \) and \( J_{II} \) and quasi-elastic \( C_I \) and \( C_{II} \) coefficients:
\[ J_I = J_1 + J_2 \mu_{21}^2 + J_3 \mu_{31}^2 = 200 + 100 \cdot (0.23)^2 + 150 \cdot (-1.49)^2 = 538.4 \text{ kg} \cdot \text{m}^2; \]
\[ J_{II} = J_1 + J_2 \mu_{22}^2 + J_3 \mu_{32}^2 = 200 + 100 \cdot (-2.9)^2 + 150 \cdot (0.6)^2 = 1095 \text{ kg} \cdot \text{m}^2; \]
\[ C_I = C_1 \left( 1 - \mu_{21} \right)^2 + C_2 \left( \mu_{21} - \mu_{31} \right)^2 = 2 \cdot 10^5 \left( 1 - 0.23 \right)^2 + 1 \cdot 10^5 \left( 0.23 + 1.49 \right)^2 = 4.15 \cdot 10^5 \text{ N} \cdot \text{m}; \]
\[ C_{II} = C_1 \left( 1 - \mu_{22} \right)^2 + C_2 \left( \mu_{22} - \mu_{32} \right)^2 = 2 \cdot 10^5 \left( 1 - 2.9 \right)^2 + 1 \cdot 10^5 \left( -2.9 - 0.6 \right)^2 = 42.67 \cdot 10^5 \text{ N} \cdot \text{m}. \]

Using the obtained values, we determine the own frequencies of the three-mass system:
\[ k_1 = \sqrt{\frac{C_I}{J_I}} = \sqrt{\frac{4.15 \cdot 10^5}{538.4}} = 27.7 \text{ s}^{-1}; \]
\[ k_2 = \sqrt{\frac{C_{II}}{J_{II}}} = \sqrt{\frac{42.67 \cdot 10^5}{1095}} = 62.4 \text{ s}^{-1}. \]

The graphs show the dependencies \( k_1(C_1), k_2(C_1) \) at \( C_2 = \text{const} \) and \( k_1(C_2), k_2(C_2) \) at \( C_1 = \text{const} \) (Figure 2).

![Figure 2](image-url)
4. Conclusion
A method for calculating the own fluctuations in the drives of technological equipment using a mathematical model of a three-mass fluctuating torsional chain system is presented.

Differential equations of motion for a mechanical system with three degrees of freedom are obtained. The forms of fluctuating processes in the system are analyzed. Using the conditions of orthogonality of own forms of torsional fluctuations, the frequency equation is solved and the dependencies of own frequencies on the rigidity coefficients of the system are determined. It is shown that with the increase in the rigidity of the torsional system the frequency of own fluctuations increases.

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