Spin-induced nonlinearities in the electron magnetohydrodynamic regime

Martin Stefan¹, Gert Brodin and Mattias Marklund
Department of Physics, Umeå University, SE-901 87 Umeå, Sweden
E-mail: martin.stefan@physics.umu.se

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Abstract. The influence of electron spin on the nonlinear propagation of whistler waves is studied in this paper. For this purpose, a recently developed electron two-fluid model, where the spin-up and spin-down populations are treated as different fluids, is adapted to the electron magnetohydrodynamic (MHD) regime. A nonlinear Schrödinger equation is then derived for the whistler waves and the coefficients of nonlinearity with and without spin effects are compared. The relative importance of spin effects depends on the plasma density and temperature as well as the external magnetic field strength and wave frequency. The significance of our results for various plasmas is discussed.

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¹ Author to whom any correspondence should be addressed.
The dynamics of dense, ionized matter, cold matter in strong magnetic fields and nonlinear degenerate plasmas has applications in both the laboratory and in naturally occurring systems. Many such matter states come under the collective term ‘quantum plasmas’. A multitude of studies devoted to quantum plasma effects can be found in the literature, many inspired by works such as [1, 2]. In the last decade, there has been a surge of interest in quantum plasmas [3]–[12]. Interesting applications of this field can be found in, for example, plasmonics [13, 14], quantum wells [15] and ultracold plasmas [16]. Common to such applications are rather ‘extreme’ parameters compared to most laboratory and space plasmas. More specifically, the plasma densities need to be very high and/or the temperatures correspondingly low. For astrophysical plasmas, the situation is somewhat different since the strong magnetic fields [17, 18] may induce various types of quantum effect. In most of the above studies, the spin effects play little or no dynamic role. The inclusion of collective spin dynamics [19, 20] gives rise to new modes in plasma, both at the fluid [9, 21] and the kinetic scale [12]. Indeed, even dusty plasmas can show interesting magnetization effects [22, 23]. In [11], the picture outlined above, concerning the necessary parameter space for quantum effects to be important, was to some extent modified, as it was shown that the spin properties of electrons can be important in plasmas even outside the high density/low temperature regime, and for moderate magnetic field strengths. Moreover, the recent focus on the nonlinear regime in quantum plasmas [24]–[26] makes the question of magnetization nonlinearities interesting.

Motivated by the above, we will in the present work further extend the analysis put forward in [11]. In that work, the electrons were described using a two-fluid model, where the spin-up and spin-down populations relative to the magnetic field were treated as different fluids in the standard magnetohydrodynamic (MHD) regime. Here, we will extend that treatment to cover the electron-MHD (EMHD) regime. In particular, we will study weakly nonlinear whistler waves and derive a nonlinear Schrödinger (NLS) equation for the slowly varying amplitude, both using classical weakly relativistic theory and the recent two-fluid spin model, adapted for the EMHD regime. By comparing the nonlinear coefficients in the different models and their dependence on the plasma parameters (temperature, density and external magnetic field strength), the relative importance of the electron spin effects in various regimes can be deduced. The previous result suggesting that electron spin effects can be important in other regimes, as compared to certain much studied quantum effects (such as the Bohm–de Broglie potential (see also [27] for a discussion) and the Fermi pressure), is confirmed. Finally, we compare the relative importance of electron spin effects in the EMHD regime to that observed in the standard MHD regime.

2. A two-fluid model with spin

The purpose of our present work is to compare the nonlinearities from classical and quantum effects, respectively, in the EMHD regime. As a starting point, we follow [28] and derive the EMHD model. We assume that the timescale of interest is small enough for the ions’ motion, because of their relatively large mass compared to electrons, to be neglected. We also neglect the displacement current in Ampere’s law. Furthermore, by assuming an isothermal pressure
model and no dissipation, the governing equation can be written as
\[
\frac{\partial}{\partial t} \left( B - d_e^2 \Delta B \right) = -\alpha \nabla \times \left[ (\nabla \times B) \times (B - d_e^2 \Delta B) \right],
\]  
where \( B \) is the magnetic field, with \( d_e^2 = c^2 / \omega_{pe}^2 \), \( \omega_{pe}^2 = q_e^2 n / m_e \epsilon_0 \) being the electron plasma frequency and \( \alpha = 1 / n q_e \mu_0 \). Studying the linear modes of this equation propagating parallel to the magnetic field, one finds whistler waves with the dispersion relation \( \omega(k) = \omega_c c^2 k^2 / \omega_{pe}^2 \).

However, since this model does not allow any density fluctuations, an attempt to derive an NLS equation shows that the model does not give rise to any cubic nonlinearities. To still be able to make the intended comparison, we continue to use the assumptions corresponding to the EMHD regime but perform a more general treatment, allowing for relativistic particle velocities and using a multifluid model that permits density perturbations.

The quantum model used in the comparison is obtained formally by starting from the Pauli Hamiltonian as in [8], using ensemble averaging to obtain the fluid equations
\[
\frac{\partial}{\partial t} n + \nabla \cdot (n v) = 0, \tag{2}
\]
\[
m_e n \left( \frac{\partial}{\partial t} + v \cdot \nabla \right) v = q_e n (E + v \times B) - \nabla p + F_{\text{spin}}, \tag{3}
\]
where
\[
F_{\text{spin}} = \pm \mu_B n \nabla B, \tag{4}
\]
with the sign depending on the spin orientation \( S_\pm = \pm (n \hbar 2) \hat{B} \) relative to the magnetic field. Thus, assuming a two-electron fluid model, for which the distinction between the fluids is through their relative spin orientation, we have
\[
m_e n_\pm \frac{dv_\pm}{dt} = n_\pm q_e (E + v_\pm \times B) \pm \mu_B n_\pm \nabla B - k_B T \nabla n_\pm, \tag{5}
\]
\[
\frac{\partial}{\partial t} n_\pm = -\nabla \cdot (n_\pm v_\pm), \tag{6}
\]
where the subscript \( \pm \) denotes spin orientation parallel or anti-parallel to the external magnetic field, respectively, and \( B = |\hat{B}| \). In case the spin-up and spin-down populations are not equal, there will be a net magnetization and a corresponding magnetization current. Thus, within this model, the total current density to be used in Ampere’s law is written as
\[
j = q_e (n_+ v_+ + n_- v_-) + \mu_B \nabla \times (n_+ \hat{B} - n_- \hat{B}), \tag{7}
\]
where the last term is the magnetization current due to the spin and \( \hat{B} = B / B \) is a unit vector in the direction of the magnetic field.

3. Linear theory

Linearizing the momentum and fluid equations around a constant magnetic field \( B_0 = B_0 \hat{z} \) and assuming transversal waves propagating parallel to this external magnetic field, we can deduce
\[
v = \hat{\sigma} \hat{E} \tag{8}
\]
from equation (5), where

\[ \hat{\sigma} = \frac{q_s}{m_s} \hat{M}^{-1} = \frac{q_s}{m_s} \left( \omega_{cs} - \omega^2 \right) \begin{pmatrix} -i\omega & \omega_{cs} \\ -\omega_{cs} & -i\omega \end{pmatrix} \]

is the conductivity tensor, and \( \omega_{cs} = q_s B_0 / m_s \) is the cyclotron frequency for particle species \( s \), and we let the vectors here just contain the parts perpendicular to \( \hat{z} \). Note that since the variations of \( B \) are nonlinear in amplitude for parallel propagation, the spin effects do not enter here. Furthermore, when the thermal energy \( k_B T \) is much larger than the energy difference between the spin states, \( \mu_B B_0 / m_s \), the difference between the number densities of the spin-up and spin-down populations, \( n_{0+} - n_{0-} \), in the thermodynamic ground state is exponentially small (proportional to \( \exp(-\mu_B B_0 / k_B T) \)), and hence we can omit the linearized part of the magnetization current. Thus, in this approximation, no quantum effects remain in the linearized theory. From the Maxwell equations we then obtain

\[ \hat{D} \hat{E} = 0 \] (9)

where

\[ \hat{D} = \begin{bmatrix} -k^2 c^2 \hat{1} + \hat{I} - \sum_s \frac{\omega^2_{ps}}{\omega^2} \left( \begin{array}{cc} \omega_{cs} & -i\omega_{cs} \\ -i\omega_{cs} & -\omega \end{array} \right) \end{bmatrix} \] (10)

Thus, we obtain the general dispersion relation

\[ \omega^2 = k^2 c^2 \left[ 1 + \sum_s \frac{\omega^2_{ps}}{\omega^2_{cs} - \omega^2} \left( 1 - \frac{\omega_{cs}}{\omega} \right) \right]^{-1} \] (11)

for this geometry.

To obtain the EMHD limit, we disregard the displacement current in Ampere’s law and regard the ions as fixed. This corresponds to neglecting the unit matrix in equation (9) and letting \( \omega_{ce} \ll \omega \). By this procedure, we get the dispersion relation

\[ \omega(k) = \frac{\omega_{ce}}{\omega_{pe}^2} c^2 k^2 = \frac{B_0 k^2}{q_s n_e \mu_0} \equiv \alpha B_0 k^2 \] (12)

for small amplitude whistler waves propagating parallel to the external magnetic field.

The dispersion relation (12) was obtained by starting from the two-fluid model and then taking the EMHD limit. If one started directly from the EMHD plasma equation, the corresponding result would be

\[ \omega(k) = \frac{\alpha B_0 k^2}{1 + c^2 k^2 / \omega_p^2}. \] (13)

This difference is due to the fact that for the EMHD assumptions to apply, the parallel (to the external magnetic field) wavenumber \( k_{||} \) must obey \( k_{||} \ll \omega_p / c \). Thus in our case with parallel propagation (\( k = k_{||} \)) we must approximate the denominator with unity, in which case the different expressions agree. As a side note, for general directions of propagation the factor \( (1 + c^2 k^2 / \omega_p^2)^{-1} \) appears correctly from the EMHD theory, but since only the perpendicular part of the wavenumber \( k_\perp \) is allowed to be comparable to the inverse skin depth (i.e. \( k_\perp \sim \omega_p / c \)), we have \( (1 + c^2 k^2 / \omega_p^2)^{-1} = (1 + c^2 k_\perp^2 / \omega_p^2)^{-1} \) in that case.
As usual, by using an ansatz of a weakly modulated amplitude in the EMHD model equation (1), neglecting nonlinear terms and higher order dispersion we obtain the linear part

\[
\left[ i \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) + \frac{v'_g}{2} \frac{\partial^2}{\partial z^2} \right] \tilde{B}_x = 0
\]  

(14)

of the one-dimensional NLS equation. Here, \( v_g \) is the group velocity and \( v'_g = \frac{d v_g}{d k} \) is the group velocity dispersion. Since the spin effects do not enter linear theory, these coefficients are the same in our classical and quantum mechanical models.

4. Classical nonlinear theory

To explore nonlinearities because of relativistic effects, the momentum equation is modified by letting \( v_s \rightarrow \gamma v_s \) in the left-hand side, and it thus reads

\[
n_s \left( \frac{\partial}{\partial t} + v_s \cdot \nabla \right) \gamma v_s = \frac{q_s}{m_s} n_s (E + v_s \times B) - v_{st}^2 \nabla n_s,
\]

(15)

where \( \gamma = 1/(1 - v^2/c^2) \), and we have introduced the thermal velocity \( v_{st} = (k_B T / m_s)^{1/2} \) for species \( s \). The \( \gamma \)-factor can be Taylor expanded to first order and will thus result in purely cubic nonlinearity in the velocity. Including this nonlinearity only, it is straightforward to deduce the NLS equation

\[
\left[ i \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) + \frac{v'_g}{2} \frac{\partial^2}{\partial z^2} + \frac{2 \alpha^3 B_0 k^6}{\omega_p^2 (1 + c^2 k^2 / \omega_p^2)^2} |\tilde{B}_x|^2 \right] \tilde{B}_x = 0.
\]

(16)

The above nonlinear term will be complemented by nonlinear density modifications induced by the ponderomotive force. Within a model that only includes the electron dynamics, the density modifications will be limited due to the general tendency of charge neutrality. However, for sufficiently long pulses, the low-frequency ion dynamics will start to contribute to the nonlinear behavior of the electrons, and it turns out that a fair comparison between quantum and classical nonlinearities must include this effect. To capture the ponderomotive nonlinearities, we start with equation (15) (for simplicity, omitting the relativistic contribution that we already know), for a two-fluid ion-electron model. Again neglecting the displacement current in Ampere’s law, linearizing as previously and using the Maxwell equations, we obtain the system

\[
\text{det}[\hat{D}_{\text{op}}] \begin{pmatrix} \nu_{elf} \\ E_{lf} \\ n_{el} \\ \nu_{lf} \\ n_{lf} \end{pmatrix} = \text{Adj}[\hat{D}_{\text{op}}] \begin{pmatrix} -2 \frac{\partial}{\mu_0 m_e \partial z} |\tilde{B}_x|^2 \\ 0 \\ 0 \\ -2 \frac{\partial}{\mu_0 m_i \partial z} |\tilde{B}_x|^2 \\ 0 \end{pmatrix},
\]

(17)
for the low-frequency variables, where

\[
\hat{D}_{op} = \begin{pmatrix}
  n_0 \frac{\partial}{\partial t} - \frac{q_e n_0}{m_e} k_b T_e \frac{\partial}{\partial z} & 0 & 0 \\
  0 & \frac{\partial}{\partial z} - \frac{q_e}{\epsilon_0} & 0 & -\frac{q_i}{\epsilon_0} \\
  n_0 \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial t} & 0 & 0 \\
  0 & -\frac{q_i n_0}{m_i} & n_0 \frac{\partial}{\partial t} & k_b T_i \frac{\partial}{\partial z} & 0 \\
  0 & 0 & 0 & n_0 \frac{\partial}{\partial z} & \frac{\partial}{\partial t}
\end{pmatrix},
\]

(18)

and we can read that

\[\nu_{\text{elf}} = \kappa_e |\vec{B}_x|^2,\]

(19)

\[\nu_{\text{lf}} = \kappa_i |\vec{B}_x|^2.\]

(20)

The coefficients \(\kappa_e\) and \(\kappa_i\) are determined by solving the corresponding differential equation, obtained from equation (17), using Green’s function techniques, and the result is

\[\kappa_e \approx -2v_b \frac{\omega_p^2/m_e + \omega_{pe}^2/m_i}{n_0 \mu_0} \frac{\omega_p^2 v_g^2 - \omega_{pe}^2 (v_e^2 - v_i^2)}{\omega_p^2 (v_g^2 - v_e^2)}.\]

(21)

Now that the low-frequency perturbations have been determined, the back reaction on the original timescale can be calculated. We note that on this fast timescale, we return to neglecting ion motion. We then obtain

\[\nabla \times \left[ n_0 \nu_{\text{elf}} \frac{\partial}{\partial z} \left( \frac{1}{\mu_0 q_e n_0} \frac{\partial}{\partial z} \hat{z} \times \vec{B} \right) + \frac{q_e}{m_e} n_\text{elf} \vec{v} \times \vec{B}_0 \right] = i \left( \frac{k^3}{\mu_0 q_e} - \frac{k^2 B_0}{m_e \mu_0 v_g} \right) \kappa_e |\vec{B}_x|^2 \vec{B}_x e^{i\theta},\]

(22)

where the subscript \(x\) indicates the \(\hat{x}\)-component of the vector. Combining this result with the linear theory, we obtain an NLS equation that reads

\[i \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) \vec{B}_x + \frac{v_g}{2} \frac{\partial^2 \vec{B}_x}{\partial z^2} + \left[ \frac{q_e B_0 k_e^2 c^2/m_e v_g - k^3 c^2}{\omega_p^2 (1 + c^4 k^2/\omega_p^2)} \kappa_e \right] |\vec{B}_x|^2 \vec{B}_x = 0.\]

(23)

5. Fully nonlinear theory

As opposed to the case of a normal EMHD plasma, in the quantum case we have one equation for each electron spin direction, and the extra term \(\pm \mu_B n_\pm \nabla B\) due to the spin’s influence on the magnetization. One can note that if the spin populations are exactly equal in density, when adding the two force equations this term will vanish, and this corresponds to the classical case. However, if there is a slight difference in number density, nonlinear fluctuations in the magnetic field will be induced. To explore this effect, we try to derive an EMHD model, but now using the two-fluid spin model. Equation (1) is then replaced by

\[2Nq_e \frac{\partial \vec{B}}{\partial t} = \frac{1}{\mu_0} \nabla \times [(\nabla \times \vec{B}) \times \vec{B}] - 2\mu_B \nabla \times [(\nabla \times (n\hat{\vec{B}})) \times \vec{B}] + 2\mu_B \nabla \times (n \nabla B),\]

(24)
where the average number of electrons \( N = (n_+ + n_-)/2 \) tends to deviate little from the unperturbed density (due to charge neutrality), but the difference between the electron species \( n = (n_+ - n_-)/2 \) can vary more. Furthermore, we introduce the notation \( \mathbf{V} = \mathbf{v}_e + \mathbf{v}_\gamma \) and \( \mathbf{v} = \mathbf{v}_e - \mathbf{v}_\gamma \). Here we point out that, in the approximation considered, where the unperturbed density difference is neglected, as before the linear treatment shows agreement with the classical case.

Next, to calculate the low-frequency perturbations of \( n \), we need the difference between equations (5) for the two species, which is written as

\[
0 = -[(N\mathbf{v} + n\mathbf{V}) \times \mathbf{B}] - 2N \frac{\mu_B}{q_e B_0} \frac{\partial}{\partial z} (B_{1x}^2 + B_{1y}^2) + \frac{2k_B T}{q_e} \frac{\partial n}{\partial z}.
\]

Filtering out the low-frequency timescale, we obtain

\[
2 \left( \frac{\partial^2}{\partial z^2} - \frac{\omega_p^2}{v_e^2} \right) N_{lf} = - \frac{1}{\mu_0 m v_e^2} \frac{\partial^2}{\partial z^2} \left( |\tilde{B}_x|^2 + |\tilde{B}_y|^2 \right)
\]

and

\[
n_{lf} = 2N \frac{\mu_B}{k_B T B_0} \left( |\tilde{B}_x|^2 + |\tilde{B}_y|^2 \right),
\]

where we have introduced the electron thermal velocity, \( v_e = (k_B T/m_e)^{1/2} \). Due to the reduced geometry of the problem, with parallel propagating circularly polarized modes, all second harmonic density perturbations can be neglected, and all second-order nonlinearities in the magnetic field also vanish. Furthermore, \( N_{lf} \ll n_{lf} \) as a consequence of the system tending toward charge neutrality, and thus only \( n_{lf} \) needs to be considered here.

Inserting this now in equation (24) and considering the original timescale, the only non-vanishing contribution to the nonlinear constant is

\[
-\mu_B \nabla \times [(\nabla \times (n\hat{\mathbf{B}})) \times \mathbf{B}]_x = 16iN \frac{\mu_B^2 k^2}{k_B T B_0} |\tilde{B}_x|^2 \tilde{B}_x.
\]

Thus, the full NLS equation including all the effects discussed above (relativistic nonlinearity, classical density perturbations induced by the ponderomotive force, and a spin-dependent density modification, driven by the nonlinear magnetic dipole force) will be (cf equation (23))

\[
i \left( \frac{\partial}{\partial t} + v_g \frac{\partial}{\partial z} \right) \tilde{B}_x + \frac{v'_g}{2} \frac{\partial^2 \tilde{B}_x}{\partial z^2} + \frac{4\mu_B^2 k^2}{k_B T B_0 q_e} |\tilde{B}_x|^2 \tilde{B}_x
\]

\[
+ \left[ \frac{2\omega_p^2 B_0 k^6}{\omega_{pe}^2 (1 + c^2 k^2/\omega_{pe}^2)^2} + \frac{m_e v_e}{\omega_{pe}} B_0 k^2 c^2 - k^3 c^2 \right] |\tilde{B}_x|^2 \tilde{B}_x = 0.
\]

This equation is the main result of this paper. From the magnitude of the nonlinear coefficient, one can determine the regimes in which the spin terms can dominate and be responsible for e.g. soliton formation.

6. Discussion

In the present paper, we have studied weakly nonlinear whistler waves propagating along the magnetic field. An NLS equation has been derived for the case of classical nonlinearities.
The nonlinear coefficient then receives two contributions; from relativistic effects and from low-frequency density modifications induced by the ponderomotive force. Taking spin effects into account, within an electron two-fluid spin model, it is found that the low-frequency part of the magnetic dipole force separates the spin-up and spin-down populations. Owing to the different magnetization currents from the two populations, a spin contribution to the nonlinear coefficient then arises. Firstly, comparing the spin contribution to the nonlinear coefficient with the relativistic contribution, we see that the former is larger provided that

$$1 \lesssim \frac{m_i \hbar^2 \omega^2_{pe}}{k_B T m_e^2 C_A^2}.$$  \hspace{1cm} (30)

Here, we have used the lowest frequency allowed by the model $\omega \sim \omega_{ci}$ to obtain a condition that is relatively easy to fulfill. However, we must also compare the spin-induced nonlinearity against the contribution from the nonlinear density oscillations induced by the ponderomotive force. It is found that the former dominates when

$$1 \lesssim \frac{\hbar \omega_{ce} \hbar \omega_{ce}}{m_i C_A^2 m_e v_{ti}^2},$$  \hspace{1cm} (31)

where we have used a maximum value of $k_c$ that is roughly $\omega_{pe}$, due to the limitations imposed by the geometry in combination with the EMHD approximation. The factor $\hbar \omega_{ce}/m_i C_A^2$ is the condition for nonlinear spin effects to dominate, when a similar comparison is made in the standard MHD regime, according to [11]. To obtain a more favorable comparison (i.e. a condition that is easier to reach under laboratory conditions) than in these previous works, the second factor, $\hbar \omega_{ce}/m_i v_{ti}^2$, must be larger than unity. This is unfortunately not the case for the parameters usually found in laboratory conditions. However, astrophysical plasmas with parameters fulfilling both conditions (30) and (31) can be found, e.g. in the vicinity of pulsars or magnetars [17], and thus we note that effects associated with the electron spin can be more important than the classical relativistic and ponderomotive nonlinearities in such environments.

The present study has focused on the EMHD regime. While it is shown that the spin effect certainly can be important during e.g. astrophysical plasma conditions, our study suggests that the standard MHD regime [11] can be affected more by electron spin properties under laboratory conditions. However, much work remains to be done in order for this conclusion to be settled, as the picture may change when a more general geometry is considered or when kinetic effects [12] are taken into account.

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