Adaptive Fuzzy-Sliding Consensus Control for Euler–Lagrange Systems with Time-Varying Delays

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This paper presents an adaptive fuzzy sliding-mode controller for multiple Euler–Lagrange systems communicated with directed topology. Based on the graph theory and Lyapunov–Krasovskii functions, a delay-dependent sufficient condition for the existence of sliding surfaces is given in terms of linear matrix inequalities. The asymptotic stability is analyzed by using the Lyapunov method in the presence of unknown parametric dynamics, actuator faults, and time-varying delays. The usage of adaptive techniques is to adapt the unknown parameters so that the objective of globally asymptotic stability is achieved. Finally, simulation results are provided to illustrate the effectiveness of the proposed control scheme.

1. Introduction

Cooperative control of multiagent systems has attracted much attention in recent years, such as consensus [1–4], formation [5–8], and flocking problems [9–12]. The solving of the consensus problem is an essential and interesting topic of cooperative multiagent research. Basically, the idea of consensus implies that a group of agents can reach an agreement on certain quantities of interest. In graph-based approaches, a directed or indirected graph is popularly applied to describe the communication topology of networked multiagent systems. In the last few years, several methods have been proposed to deal with the consensus problems of the multiagent systems [13–17]. In [13], adaptive synchronization protocols for a heterogeneous multiagent network were investigated, where the interaction between agents is represented by a direct graph. In [14], the leaderless consensus problem over strict-feedback nonlinear multiagent systems with unknown model parameters and control directions was investigated. Also, a robust continuous-time optimization algorithm was presented for multiagent systems with guaranteed fixed-time convergence. Zhang et al. presented a robust consensus tracking strategy for multiple unmanned underwater vehicles with switching topology [16]. In [17], the leader-following average consensus problem was addressed for linear multiagent systems.

Besides the first-order or second-order linear models, one important class of multiagent control systems is the so-called Euler–Lagrange systems, which generally describes the dynamic properties of robot manipulators, rigid body systems, and so on. In [18], the leader-following consensus problem was studied of multiple Euler–Lagrange (EL) systems subject to an uncertain leader. In [19], an adaptive sliding mode control technique was proposed for the EL systems with actuator faults and system uncertainties. In addition, a distributed optimal consensus strategy based on an event-triggered scheme for EL multiagent systems was investigated [20]. The model-free optimal consensus problem was addressed for networked Euler–Lagrange systems without velocity measurements [21]. Chen et al. proposed a robust adaptive finite-time tracking control scheme for Euler–Lagrange systems subject to nonparametric uncertainties, unknown disturbances, and input saturation [22]. In [23], a robust adaptive finite-time tracking control...
scheme was proposed for Euler–Lagrange systems subject to nonparametric uncertainties, unknown disturbances, and input saturation.

The sliding-mode method has been studied for nonlinear systems because of some attractive features, such as robustness to parameter variations and good transient performance. Recently, a sliding-mode controller has been developed to deal with nonlinearities and uncertainties for multiagent systems [24–28]. In [24], the finite-time consensus tracking of multirobotic systems with disturbances was investigated via utilizing integral sliding mode control. In [25], an optimal sliding mode control approach was presented for the consensus of nonlinear discrete-time high-order multiagent systems. Jina et al. investigated the consensus control problem of Takagi–Sugeno fuzzy multiagent systems by using an observer-based distributed adaptive sliding mode control [26]. In addition, the event-triggered sliding control problem of second-order uncertain multiagent systems was addressed by utilizing the distributed sliding-mode control approach [27]. In [28], the consensus tracking problem of networked control systems with disturbances was discussed, where an integral sliding mode protocol was developed to achieve the consensus in a setting time. Recently, fuzzy sliding-mode control has attracted much attention, where the fuzzy mechanism is useful to decrease the chattering behaviors. In [29], an adaptive backstepping fuzzy neural network controller using a fuzzy sliding mode controller was designed to suppress the harmonics of a shunt active power filter. In addition, a fuzzy sliding mode control method was proposed to improve the ability of magnetic levitation force platform subject to external disturbance [30]. In [31], a fuzzy sliding-mode control was developed to deal with unmodeled dynamics and external disturbances in a human-exoskeleton system.

On the other hand, in multiagent systems, actuator failures generally result in poor system performance or even cause the instability. In [32], an adaptive fixed-time controller was designed for a class of uncertain nonstrict feedback multiagent systems subject to actuator faults and external disturbances. In [33], a robust consensus control strategy was addressed for nonlinear second-order multiagent systems against actuator faults and uncertainties. Dong et al. presented an augmented control system for a quadrotor unmanned aerial vehicle with parameter uncertainties, external disturbance, and the partial loss of actuator effectiveness [34]. In [35], a cooperative fault tolerant control was presented for linear leader-follower networks subject to actuator faults. Moreover, the fault-tolerant leader-following consensus problem was discussed for multiagent systems with input saturation and actuator faults [36]. In [37], the consensus problem was investigated for a class of nonaffine nonlinear multiagent systems with actuator faults of partial loss of effectiveness.

Because of the interactive communication in multiagent systems, the coupling delays between agents become more crucial due to practical considerations. In [38], the consensus problem of discrete-time linear multiagent systems was addressed with unbounded time-varying delays. In [39], the containment control problem of the double-integrator multiagent systems was investigated with time-varying communication delays. Tan et al. discussed the output feedback control problem for a class of nonlinear multiagent systems governed by the high-order strict-feedback model with time delays [40]. Also, the finite-time consensus of leader-following multiagent systems was addressed with multiple time delays over time-varying topology [41]. The leader-following consensus problem was discussed for discrete-time multiagent systems with time-varying delays [42]. In [43], the second-order multiagent networks with time-varying delays were investigated, where a sufficient condition was presented to make all agents asymptotically reach consensus using the linear matrix inequality theory.

In this paper, an adaptive fuzzy sliding-mode fault-tolerant controller (AFSFC) is presented for multiple Euler–Lagrange systems. Also, the parametric uncertainties, actuator faults, and time-varying communication delays are considered. The proposed control scheme is based on adaptive sliding-mode techniques combining with the fuzzy logic strategy. An adaptive algorithm is provided to estimate the unknown parametric vector. Moreover, by employing the Lyapunov–Krasovskii function and linear matrix inequalities (LMIs), a sufficient condition is established such that the resulting sliding-mode dynamics is stable. The main contributions of this paper are stated as follows: (1) an adaptive fuzzy sliding-mode controller is proposed for networked Euler–Lagrange systems with communication time-varying delays. The fuzzy sliding mode and adaptive controller are combined to deal with the parametric uncertainties. (2) For the multiagent systems, provided with controller parameters and communication topology, the maximum tolerated delay of all agents can be determined by using the Lyapunov–Krasovskii analysis and LMIs. (3) The proposed control scheme can be applied to a group of agents with a directed communication topology. The tracking errors are shown to be asymptotically convergent with a directed spanning tree of communication topology. (4) The unknown parametric dynamics and actuator faults can be estimated online using adaptive strategies. (5) The overall closed-loop stability can be preserved using Lyapunov stability analysis in both fault-free and faulty situations. Moreover, the allowable communication delays can be obtained and formulated as some LMIs.

This paper is organized as follows: in Section 2, the dynamic model of Euler–Lagrange systems is presented with the consideration of partial loss of effectiveness faults. The stability of the sliding motion of multiagent systems is investigated in Section 3. In Section 4, an adaptive fuzzy sliding-mode controller for multiagent systems with time-varying delays and actuator faults is discussed. In Section 5, the simulation results are provided for performance comparisons. Finally, the concluding remarks are given in Section 6.

2. Preliminaries

2.1. Fundamentals of Graph Theory. A directed graph $G = (V, E)$ consists of a vertex set $V = \{v_1, v_2, \ldots, v_n\}$ and an edge set $E \subseteq V \times V$, where $(v_j, v_i) \in E$ means that the $i$th
node can receive information from the $j$th node. $N_j = \{v_j \in V: (v_j, v_i) \in E\}$ denotes the neighboring set of $v_j$. The adjacent matrix $A = [a_{ij}] \in \mathbb{R}^{n \times n}$, where $a_{ij} = 1$, if $(v_j, v_i) \in E$, or $a_{ij} = 0$, otherwise. The degree matrix $D = \text{diag}(d_1, d_2, \ldots, d_n) \in \mathbb{R}^{n \times n}$, $d_i = \sum_{j \in N_i} a_{ij}$ of a digraph $G$ is a diagonal matrix.

**Lemma 1** (see [44]). The graph $G$ has a directed spanning tree if and only if there is at least one node with a directed path to all other nodes. If a graph $G$ has a spanning tree, then a right eigenvector $L$ associated with the zero eigenvalue is $1_n = [1, 1, \ldots, 1]^T$, i.e., $L1_n = 0$.

**Assumption 1.** The graph $G$ has a directed spanning tree.

2.2. Dynamic Models of Multiagent Systems. This paper considers a group of $n$-agent Euler–Lagrange systems, which can be represented as

$$
M_i (q_i) \ddot{q}_i + C_i (q_i, \dot{q}_i) \dot{q}_i + g_i (q_i) = \tau_i,
$$

(1)

where $q_i \in \mathbb{R}^p$ is the vector of joint positions, $M_i (q_i) \in \mathbb{R}^{p \times p}$ is the inertia matrix, $C_i (q_i, \dot{q}_i) \in \mathbb{R}^{p \times p}$ is the Coriolis matrix, $g_i (q_i) \in \mathbb{R}^p$ is the gravitational vector, $\tau_i \in \mathbb{R}^p$ is the vector of input torques, $i = 1, 2, \ldots, n$. The actuator fault model considered can be described as

$$
\tau_i^f = \sigma_i \tau_i + \Delta \tau_i,
$$

(2)

where $\sigma_i$ is the effectiveness factor, $\Delta \tau_i$ is an additive fault, $0 < \sigma_i \leq 1$, and $i = 1, 2, \ldots, n$. With fault model (2), the Euler–Lagrange dynamics (1) of $i$th agent can be rewritten as follows:

$$
M_i (q_i) \ddot{q}_i + C_i (q_i, \dot{q}_i) \dot{q}_i + g_i (q_i) = \sigma_i \tau_i + \Delta \tau_i.
$$

(3)

Let $\sigma_i$ be the lower bound of $\sigma_i$, $0 < \sigma_i \leq \sigma_i$, in which $\sigma_i$ is an unknown positive constant. The actuators are fault-free when $\sigma_i = 1$ and $\Delta \tau_i = 0$, and $\sigma_i \in (0, 1)$ corresponds to the cases with partial loss of effectiveness (PLOE) faults.

**Property 1.** The Euler–Lagrange dynamics (1) is linearly parameterizable as

$$
M_i (q_i) \ddot{q}_i + C_i (q_i, \dot{q}_i) \dot{q}_i + g_i (q_i) = Y_i (q_i, \dot{q}_i, \ddot{q}_i) \Theta_i,
$$

(4)

where $Y_i (\cdot) \in \mathbb{R}^{p \times p}$ is the regression matrix and $\Theta_i \in \mathbb{R}^p$ is a vector of unknown constant parameters.

**Property 2.** The matrix $M_i (q_i) \dot{q}_i - 2C_i (q_i, \dot{q}_i)$ is skew symmetric.

**Lemma 2** (see [45]). Given a positive definite matrix $Z$, the following inequality holds:

$$
2A^T B \leq A^T Z A + B^T Z^{-1} B,
$$

(5)

where $A$ and $B$ are two matrices with proper dimensions.

**Assumption 2.** There exists a positive constant $\gamma_f$ such that $\|\Delta \tau_i\| \leq \gamma_f$. 

**Lemma 3** (see [46]). Let $O = \begin{bmatrix} O_{11} & O_{12} \\ O_{12}^T & -O_{22} \end{bmatrix}$ be a matrix with proper dimensions, $O_{22} > 0$. Then, the following equation holds,

$$
O \lesssim 0 \iff O_{11} + O_{12} O_{22}^{-1} O_{12}^T \lesssim 0.
$$

3. Stability of Sliding Motion

For the time-delayed multiagent systems (1), a sliding-mode controller will be designed so that the corresponding sliding motion is asymptotically stable. The sliding surface for the $i$th agent is defined as

$$
s_i = \hat{q}_i + K_i \epsilon_i,
$$

(7)

where $K_i \in \mathbb{R}^{p \times p}$ is a constant positive diagonal matrix, $\epsilon_i = \sum_{j \in N_i} a_{ij} (q_i(t - d(t)) - q_j(t - d(t)))$, $i = 1, 2, \ldots, n$.

**Assumption 3.** The communication time-varying delay $d(t)$ satisfies that

$$
0 \leq d(t) \leq \bar{d}, \dot{d}(t) \leq \mu \bar{d},
$$

(8)

where $\bar{d}$ and $\mu$ are positive constants.

In the following, the notations $q_{i,d}$ and $q_{i,d}$ stand for $q_i(t - d(t))$ and $q_j(t - d(t))$, respectively, for simplicity. We denote $s = [s_1^T, s_2^T, \ldots, s_n^T]^T$. The sliding surface of multiagent systems is summarized as

$$
s = q + K (\mathcal{D} \otimes I_p) q_d.
$$

(9)

where $q = [q_1^T, q_2^T, \ldots, q_n^T]^T$, $q_d = [q_{1,d}^T, q_{2,d}^T, \ldots, q_{n,d}^T]^T \in \mathbb{R}^n$, and $K = \text{diag}[K_1, K_2, \ldots, K_n] \in \mathbb{R}^{n \times n}$. When the sliding mode is achieved, (9) can be equivalently described as

$$
\dot{q} = -K (\mathcal{D} \otimes I_p) q_d.
$$

(10)

Let the error function between the 1st agent and other agents as

$$
e_i = q_i - q_r, r = 2, 3, \ldots, n.
$$

(11)

Moreover, (11) can be rewritten as the following augmented form:

$$
e = q_d \otimes 1_{n-1} + \mathcal{E} q_d.
$$

(12)

where $e = [e_{13}^T, e_{15}^T, \ldots, e_{n,n}^T]^T$, $q_d = [q_{1,d}^T, q_{2,d}^T, \ldots, q_{n,d}^T]^T$, $1_{n-1} = [1, 1, \ldots, 1]^T$, and

$$
\mathcal{E} = \begin{bmatrix}
1_p & -1_p & 0_p & \cdots & 0_p \\
1_p & 0_p & -1_p & \cdots & 0_p \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
1_p & 0_p & 0_p & \cdots & -1_p
\end{bmatrix} \in \mathbb{R}^{(n-1) \times np},
$$

(13)

in which $1_p$ is the $p$-dimension identity matrix, and $0_p$ is the $p$-dimensional matrix with all zeros. Thus, (11) can be rewritten as

$$
q = q_d \otimes 1_n + Fe,
$$

(14)

where
The derivative of (12) along (10) and (13) is derived as

\[ e(t) = -E(K(\mathcal{L} \otimes I_p)(q_{d1} \otimes 1_n) - E(K(\mathcal{L} \otimes I_p) F e_d), \]

where the notation \( e_d \) stands for \( e(t - d(t)) \). According to Lemma 1, it is obtained as

\[ \dot{e} = -\Psi e_d, \]

where \( \Psi = E(K(\mathcal{L} \otimes I_p) F). \)

**Lemma 4** (See [47]). The matrix \( \Psi \) is Hurwitz if and only if the communication topology \( G \) has a directed spanning tree.

**Theorem 1.** Suppose that the communication graph of a multiagent Euler–Lagrange system (1) has a directed spanning tree. From Assumption 3, the error dynamics of (15) is asymptotically stable, if the following inequalities hold:

\[
\begin{bmatrix}
-P\Psi - \Psi^T P + Q & P \\
\Psi^T P & -((\bar{\mu}R)^{-1})
\end{bmatrix} < 0,
\]

where \( P, Q \) and \( R \) are symmetric positive definite matrices of \( \mathcal{G}^{(n-1)\cup(n-1)} \) and \( \bar{\mu} = 1 - \mu \).

**Proof.** The Lyapunov–Krasovskii function [48] is chosen as

\[ V_e = \epsilon^T P \epsilon + \int_0^{\tau_d(t)} \epsilon^T(s) Q e(s) ds + \int_0^{\tau_d(t)} \epsilon^T(s) \tilde{\epsilon}^T(s) R \tilde{\epsilon}(s) ds d\theta. \]

The time derivative of \( V_e \) along (15) is derived as

\[ \dot{V}_e \leq -2\epsilon^T P \Psi e_d + \epsilon^T Q e - \bar{\mu} \epsilon^T Q e_d + \tilde{\epsilon}^T \Psi^T R \Psi \tilde{\epsilon}_d \
- \int_0^{\tau_d(t)} \epsilon^T(s) \tilde{\epsilon}^T(s) R \tilde{\epsilon}(s) ds. \]

It is noted that \( e_d = e - \int_0^{\tau_d(t)} e(s) ds \). Then, from Lemma 2 and Assumption 3, it leads to the following inequality:

\[ -2\epsilon^T P \Psi e_d = -2\epsilon^T P \Psi \left( e + \int_0^{\tau_d(t)} \dot{\epsilon}(s) ds \right) \leq -2\epsilon^T P \Psi e + \tilde{\epsilon}^T \Psi^T R \Psi \tilde{\epsilon}_d + \int_0^{\tau_d(t)} \epsilon^T(s) \tilde{\epsilon}^T(s) R \tilde{\epsilon}(s) ds. \]

Substituting (19) into (18), it results in

\[ V_e \leq -2\epsilon^T \Psi P \epsilon + \epsilon^T Q e - \bar{\mu} \epsilon^T Q e_d + \tilde{\epsilon}^T \Psi^T R \Psi \tilde{\epsilon}_d \]

4. **Consensus Controller Design**

4.1. **Fault-Free Cases.** Note that in the fault-free case, i.e., \( \sigma_i = 1 \) and \( \Delta \tau_i = 0 \), the torques acting on the dynamic system (1) are designed as

\[ \tau_i = \tau_{eq,i} + \tau_{sw,i}, \]

where \( \tau_{eq,i} \) is the equivalent control action and \( \tau_{sw,i} \) is the switching action. To obtain the equivalent control action \( \tau_{eq,i} \), the state trajectory is desired to stay in the sliding surface, i.e., \( \dot{s}_i = 0 \). From (7), it gives that \( \dot{s}_i = \dot{q}_i + \dot{\theta}_i = 0 \).

From (1), with the equivalent control action \( \tau_i = \tau_{eq,i} \), the equivalent control action \( \tau_{eq,i} \) can be derived as

\[ \tau_{eq,i} = -M_i K_i \dot{\theta}_i - C_i K_i \dot{\epsilon}_i + g_i, \]

where \( \dot{\theta}_i = (1 - d(t)) \sum_{j \in N_i} \dot{q}_{i,j} - \dot{\theta}_{i,j} \). We consider the unknown parameter vector \( \Theta_i \) of (4). Let the estimation of \( \Theta_i \) be defined as

\[ \hat{\Theta}_i = \Theta_i - \tilde{\Theta}_i, \]

where \( \tilde{\Theta}_i \) is the estimation of \( \Theta_i \). The adaptive law of \( \hat{\Theta}_i \) is designated as follows:

\[ \dot{\hat{\Theta}}_i = \Gamma \epsilon^T(s) (q_i, \dot{q}_i, \dot{\theta}_i, \dot{\epsilon}_i) s_i, \]

where \( \Gamma \) is a constant positive definite matrix.

Let \( s_i \) and \( \tau_{sw,i} \) be the input and output variables of a switching control system, respectively. Therefore, the switching system is represented by a single input-output fuzzy logic system. The fuzzy system is a collection of the fuzzy IF-THEN rules in the form of

Rule k : IF \( s_{ik} \) is \( M_{ki} \), THEN \( \tau_{sw,ki} \) is \( F_{(k,i)} \), \( k = 1, 2, \ldots, 5 \).

where the \( M_{ki} \) and \( F_{(k,i)} \) are the input and output fuzzy sets, \( s_i = [s_{i1}, s_{i2}, \ldots, s_{ip}]^T \), \( \tau_{sw,i} = [\tau_{sw,1,i}, \tau_{sw,2,i}, \ldots, \tau_{sw,pi}]^T \), \( r = 1, 2, \ldots, p \), respectively. The triangular input and singleton output membership functions are shown in Figure 1.
By using the centroid defuzzification technique, the output $\tau_{su,i}$ of the fuzzy system is

$$
\tau_{su,i} = \frac{\sum_{k=1}^{5} g_{ki} \mu_{ki}(s_{ri})}{\sum_{k=1}^{5} \mu_{ki}(s_{ri})} = -\sum_{k=1}^{5} |g_{ki}| \text{sgn}(s_{ri}) \mu_{ki}(s_{ri}),
$$

(28)

where $g_{ki}$ is the value of the corresponding fuzzy output and $\mu_{ki}(s_{ri})$ is the firing strength of the antecedent membership function, and $\text{sgn}(.)$ is a standard sign function. We rewrite (25) as the following augmented form:

$$
\tau_{su,i} = -\prod_{i} \text{sgn}(s_{i}),
$$

(29)

where $\text{sgn}(s_{i}) = [\text{sgn}(s_{1}),\text{sgn}(s_{2}),\ldots,\text{sgn}(s_{pi})]^T$ and

$$
\prod_{i} = \text{diag}\left\{\sum_{k=1}^{5} |g_{ki}| \mu_{ki}(s_{1}),\sum_{k=1}^{5} |g_{ki}| \mu_{ki}(s_{2}),\ldots,\sum_{k=1}^{5} |g_{ki}| \mu_{ki}(s_{pi})\right\}.
$$

(30)

**Remark 2.** Triangular and Gaussian membership functions are typical membership functions chosen for the process of fuzzy inference. In this paper, the reason of choosing triangular functions as the input membership functions is to reduce the computation complexity in the calculation of firing strengths in (25).

**Theorem 2.** We consider a multiagent Euler–Lagrange system of (1) with a directed spanning-tree communication graph. From (7), the state trajectories of (1) will be driven onto the sliding surface $s_{i} = 0$ with the adaptive fuzzy sliding-mode controller (AFSC) (21), (22), (26), and the adaptive law (24).

**Proof.** Let the Lyapunov function be chosen as

$$
V = \sum_{i=1}^{n} \left( s_{i}^T M_{i} s_{i} + \Omega_{i}^T \Gamma^{-1} \Omega_{i} \right).
$$

(31)

The time derivative of $V$ can be expressed as

$$
\dot{V} = \sum_{i=1}^{n} \left( s_{i}^T \dot{M}_{i} s_{i} + \frac{1}{2} s_{i}^T \dot{s}_{i} + \Omega_{i}^T \Gamma^{-1} \Omega_{i} \right)
$$

(32)

$$
= \sum_{i=1}^{n} \left( s_{i}^T (\tau_{su,i} + Y_{i}(q_{i}, \dot{q}_{i}, e_{i}, \dot{e}_{i}) \Omega_{i} - C_{i}s_{i}) \right.
$$

$$
+ \frac{1}{2} s_{i}^T \dot{M}_{i} s_{i} - \dot{\Omega}_{i}^T \Gamma^{-1} \dot{\Omega}_{i} \right).
$$

(33)

From Property 1, (29) can be rewritten as

$$
\dot{V} = \sum_{i=1}^{n} \left( s_{i}^T (\tau_{su,i} + Y_{i}(q_{i}, \dot{q}_{i}, e_{i}, \dot{e}_{i}) \Omega_{i} - \dot{\Omega}_{i}^T \Gamma^{-1} \dot{\Omega}_{i} \right). \quad (34)
$$

Therefore, it implies that $s_{i} = 0$ and that the system trajectory is enforced on the sliding surfaces. The proof is completed.

**4.2. Fault Cases.** In this section, we consider the controller design for multiagent systems with PLOE and unknown effectiveness fault, i.e., $0 < \sigma_{i} < 1$ and $\Delta \tau_{f} \neq 0$. From (7), the dynamical system (3) can be rewritten as

$$
M_{i} \dot{s}_{i} + C_{i} s_{i} - M_{i} K_{i} \dot{e}_{i} - C_{i} K_{i} e_{i} + g_{i} = \sigma_{i} \tau_{f} + \Delta \tau_{f}.
$$

(35)

For the multiagent systems of $n$ agents, a faulty actuator of the $i$th agent can be described as

$$
\tau_{i} = \tau_{ef,i} + \tau_{su,i} + \tau_{cf,i},
$$

(36)

where the auxiliary controller $\tau_{cf,i}$ is provided to compensate the faulty influences.

The auxiliary controller can be expressed as

$$
\dot{\tau}_{cf,i} = -\left( 1 \quad \text{sgn}(s_{i}) \| \tau_{f} \| + \tilde{\tau}_{i} \text{sgn}(s_{i}) \right).
$$

(37)

The adaptive algorithms of fault are given as

$$
\dot{\tilde{\tau}}_{i} = c_{i} \| s_{i} \|, \quad (38)
$$

$$
\tilde{\tau}_{i} = c_{i} \| s_{i} \|, \quad (39)
$$

where $\phi_{i}$ and $\zeta_{i}$ are positive constants. To make the design concepts more concise and clearer, the overall adaptive fuzzy sliding-mode fault-tolerant controller (AFSFC) structure is shown in Figure 2.

**Theorem 3.** We consider a multiagent Euler–Lagrange system of (3) with a directed spanning-tree communication graph. From (7), the state trajectories of the system (32) will be driven onto the surface $s_{i} = 0$ with the AFSC (33), the auxiliary controller (34), and the adaptive laws (24), (35), (36).

**Proof.** Let the Lyapunov function be chosen as

$$
V_{f} = \sum_{i=1}^{n} \left( \frac{1}{2} \| \phi_{i} \|^{2} + \frac{1}{2} \zeta_{i} \| \tau_{f} \|^{2} \right). \quad (40)
$$

where $\phi_{i} = \tau_{i} - \tilde{\phi}_{i}$, and $\zeta_{i} = \tau_{f} - \tilde{\tau}_{i}$. The time derivative of $V_{f}$ can be expressed as
where \( \sigma - \sigma_j < 0 \).
From (24), (35) and (36), one has $$\ddot{\Theta}^T \Gamma^{-1} \ddot{\Theta} = -s_j^T Y_1(q_i, q_i, \dot{e}_i, \dot{e}_i) \ddot{\Theta}^T, \quad 1/\gamma \ddot{r}_i = \ddot{r}_i / \| r_i \|, \text{ and } 1/\phi \ddot{\Theta} = -\ddot{\Theta} (\ddot{\Theta})^{-1} \| \Theta \| \Sigma_r.$$ Therefore, (42) can be rewritten as

$$\dot{Y}_j = \sum_{i=1}^n s_j^T r_{m_i} = -\sum_{i=1}^n s_j^T \Pi \Sigma r_i < 0. \quad (46)$$

Therefore, it implies that $$s_j = 0$$ so that the system trajectory is enforced on the sliding surfaces. The proof is completed. \[\square\]

Remark 3. In summary, with the sliding-mode control, the switching control action $$r_{m_i} (26)$$ will drive the $$i$$th agent to the sliding surface $$s_i$$. As the agents converge to the sliding surface, the equivalent controller $$r_{eqi} (22)$$ will ensure the agents stay on the sliding surface. Moreover, the existence of unknown parameters and actuator faults can be resolved by using the adaptive law (24) and auxiliary controller (34).

5. Simulation Results

The simulations are conducted for 2-DOF (degree of freedom) with six robot agents. The Euler–Lagrange model (1) is obtained as \[49\]

$$M_i(q_i) = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}, \quad C_i(q_i, \dot{q}_i) = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix},$$

$$G_i = 0, \quad q_i = \begin{bmatrix} q_{1i} \\ q_{2i} \end{bmatrix},$$

where $$M_{11} = \alpha_1 + 2\alpha_2 \sin(q_{2i}) + 2\alpha_3 \sin(q_{2i}), \quad M_{21} = \alpha_1 + \alpha_2 \cos(q_{2i}) + \alpha_3 \sin(q_{2i}), \quad M_{22} = \alpha_2, \quad C_{11} = -[\alpha_1 \sin(q_{2i}) + \alpha_2 \cos(q_{2i})]q_{1i},$$

$$C_{12} = -[\alpha_1 \sin(q_{2i}) + \alpha_2 \cos(q_{2i})]q_{2i}, \quad C_{21} = [\alpha_3 \sin(q_{2i}) + \alpha_4 \cos(q_{2i})]q_{1i}, \quad C_{22} = 0,$$

in which $$\alpha_1 = I_1 + m_1 l_{1c}^2 + I_c + m_2 l_{2c}^2, \quad \alpha_2 = I_1 + m_1 l_{1c}^2, \quad \alpha_3 = m_2 l_{2c} \cos(\sigma), \quad \alpha_4 = m_2 l_{2c} \cos(\sigma).$$ The parameters of the multiagent system are set as $$m_1 = 1, m_2 = 1, l_c = 1, l_{1c} = 0.5, l_{2c} = 0.8, \sigma_c = \pi/6, I_c = 0.25, I_1 = 0.25,$$ and $$I_2 = 0.15.$$ The six-agent connected network is shown in Figure 3.

Form Figures 3(a)–3(f), the directed graph exists the spanning tree. According to Theorem 1, the agents of the proposed topology in Figure 3(a) are asymptotically stable using the AFSCF scheme with the delay bound $$\delta = 0.16$$ (sec). The parameters of the AFSCF are set as $$\bar{\delta} = 0.5, K_i = \text{diag}[1, 1], f_{2i} = f_{3i} = 1, f_{3i} = 0.5, g_{3i} = 0, g_{3i} = 0, g_{4i} = 12, g_{5i} = 1.5, \text{ and } I_{1i} = \text{diag}[2, 2].$$ The adaptive laws are set as $$\Theta_i = [\bar{a}_1, \bar{a}_2, \bar{a}_3, \bar{a}_4] \Sigma_i, \quad \phi_i = 1 \times 10^{-4}, \quad c_i = 1, \text{ and }$$

$$Y_{ij} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14}^T \\ Y_{21} & Y_{22} & Y_{23} & Y_{24}^T \end{bmatrix}. \quad (48)$$

where $$Y_{11} = k_{11} \epsilon_{1i}, Y_{12} = k_{21} \epsilon_{2i}, Y_{13} = 0, Y_{14} = k_{11} \epsilon_{1i} + k_{32} \epsilon_{2i},$$

$$Y_{21} = k_{11} \cos(q_{2i}) \epsilon_{1i} + k_{11} \sin(q_{2i}) \epsilon_{1i}, \quad Y_{22} = k_{11} \sin(q_{2i}) \epsilon_{1i} + k_{32} \epsilon_{2i}, \quad \text{and }$$

$$Y_{13} = 2k_{11} \cos(q_{2i}) \epsilon_{1i} + 2k_{11} \sin(q_{2i}) \epsilon_{2i} \rightarrow k_{11} \epsilon_{1i} \sin(q_{2i}) \epsilon_{2i} e_2, \quad Y_{14} = 2k_{11} \sin(q_{2i}) \epsilon_{1i} + 2k_{11} \sin(q_{2i}) \epsilon_{2i} \rightarrow -k_{32} \epsilon_{2i} \cos(q_{2i}) \epsilon_{1i}.$$

In this paper, the time delay is chosen as $$d(t) = 0.08 + 0.08 \sin(t) \text{ (sec) for simulations. The initial states of the six agents are set as } q_1 = [-1, 1]^T, q_2 = [1, 0.5]^T, q_3 = [-2, 1.5]^T, q_4 = [2, -0.5]^T, q_5 = [2.5, 1.5]^T, q_6 = [-2, 1.5]^T, \text{ and the initial velocities and accelerations are zero}. The following indices are considered for the combined formation errors, the integral absolute error (IAE), the integral time absolute error (ITAE), the integral square error (ISE), and the integral time square error (ITSE) \[50\].

Remark 4. The indices IAE, ITAE, ISE, and ITSE are considered for the performance comparisons. The IAE and ISE indicate the accumulated consensus error of agents, where the weights in the transient and steady-state stages are equal. On the other hand, ITAE and ITSE indicate the time-weighted consensus errors, and these two indices can be better used to highlight the performance superiority in the steady state.

5.1. Fault-Free Cases

The comparisons between the adaptive fuzzy sliding-control (AFSC) and the proposed adaptive fuzzy sliding-mode fault-tolerant controller (AFSCF) are also given in Figs. 4–9. Figures 4 and 7 show the responses of multiagent systems with the ASC and proposed AFSCF, respectively. Lines A1–A6 are the trajectories of agents 1–6, the symbol “o” is initial state positions, and the symbol “×” is final state positions. Figures 5 and 8 show the states $$q_{1i}$$ and $$q_{2i}$$ of multiagent systems with the ASC and proposed AFSCF, respectively. Figures 6 and 9 show the control inputs $$r_{1i}$$ and $$r_{2i}$$ of multiagent systems with the AFSCF and proposed AFSCF, respectively. The position errors are summarized in Table 1. In this case, the AFSCF and AFSCF method can support a certain degree of consistence in position responses.

5.2. Fault Cases

In this case, the effectiveness fault $$\sigma_i$$ is present randomly in $$[0.01, 0.5]$$, and the additive faults are set to be $$\Delta = [2 \text{rand}(.) 2 \text{rand}(.)]$$, where rand(.) is a random number uniformly distributed in $$[1, -1]$$. The comparisons between the AFSCF and the proposed AFSCF are also given in Figs. 10–15. Figures 10 and 13 show the response of multiagent systems with the ASC and proposed AFSCF. Figures 11 and 14 show the states $$q_{1i}$$ and $$q_{2i}$$ of multiagent systems with the AFSCF and proposed AFSCF, respectively. All the agents can achieve consensus. The states $$q_{1i}$$ and $$q_{2i}$$ are more stable by the proposed AFSCF compared with the AFSCF.
Figures 12 and 15 show the control inputs $\tau_{1i}$ and $\tau_{2i}$ of multiagent systems with the AFSC and AFSFC, respectively. From Figures 12 and 15, the amplitude of the control inputs is smaller in the first few seconds with the proposed AFSFC. In summary, from Figures 10–15, the performance of the proposed AFSFC is obviously better than the AFSC. Subsequently, the position errors are summarized in Table 2. It can be observed that the proposed AFSFC method extends the performance improvement from 7.07% to 86.84%.

Remark 5. From Figs. 6, 9, 12, and 15, it can be observed that greater inputs are required to deal with the problem of actuator faults. In practical applications, the control inputs should have prescribed limits. The consensus stability issues of multiagent systems with input constraints open a new theoretical problem, which cannot be solved at the current stage and needs to be investigated separately.
Figure 6: Simulation results of control inputs $\tau_1$ and $\tau_2$ and the fault-free case (AFSC).

Figure 7: Multiagent trajectories and the fault-free case (AFSFC).
Figure 8: Simulation results of states $q_1$ and $q_2$ and the fault-free case (AFSFC).

Figure 9: Simulation results of control inputs $\tau_1$ and $\tau_2$ and the fault-free case (AFSFC).

Table 1: Performance comparisons (fault-free case).

|       | IAE  | ITAE | ISE   | ITSE  |
|-------|------|------|-------|-------|
| AFSC  | 19454| 38888| 55358 | 50697 |
| AFSFC | 20697| 42587| 61819 | 63843 |
Figure 10: Multiagent trajectories and the fault case (AFSC).

Figure 11: The states $q_{1i}$ and $q_{2i}$ and the fault case (AFSC).
Figure 12: The control inputs $\tau_{1i}$ and $\tau_{2i}$ and the fault case (AFSC).

Figure 13: State trajectories and the fault case (AFSFC).
6. Conclusions

This paper has proposed an adaptive fuzzy sliding-mode fault-tolerant controller for Euler–Lagrange systems in the presence of unknown parametrics, actuator faults, and communication time-varying delays. In the design of the fuzzy sliding-mode fault-tolerant controller, a delay-dependent sufficient condition is derived, and the allowable bound of time delays can be obtained in the form of linear matrix inequalities. Based on the Lyapunov stability theory, the overall stability of the multiagent system is guaranteed such that the desired consensus of agents can be asymptotically attained. Simulation results indicate that the proposed control scheme has superior responses, compared to the AFSC method. Especially, the proposed AFSFC method provides significant improvement in the case of actuator faults. In practical applications, the control inputs should have prescribed limits. Thus, the systematic analysis and synthesis of input-constrained multiagent systems are an interesting topic in the future.

Data Availability

The data that support the findings of this study are available on request from the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] Y. Zou, Z. Meng, and K. Xia, “Consensus of cooperative–antagonistic multi-agent networks with asynchronous three-option decision mechanism,” *Automatica*, vol. 140, Article ID 110258, 2022.

[2] W. Cho, J. Qino, and M. Sun, “Consensus Control via iterative learning for singular multi-agent systems with switching topologies,” *IEEE Access*, vol. 9, Article ID 81412, 2021.

[3] H.-J. Yoo, T.-T. Nguyen, and H.-M. Kim, “Consensus-based distributed coordination control of hybrid AC/DC microgrids,” *IEEE Transactions on Sustainable Energy*, vol. 11, no. 2, pp. 629–639, 2020.

[4] M. A. Shahab, B. Mozafari, S. Soleymani, N. M. Dehkordi, H. M. Shourkaei, and J. M. Guerrero, “Distributed consensus-based fault tolerant control of islanded microgrids,” *IEEE Transactions on Smart Grid*, vol. 11, no. 1, pp. 37–47, 2020.

[5] B. Kada, M. Khalid, and M. S. Shaikh, “Distributed cooperative control of autonomous multi-agent UAV systems using smooth control,” *Journal of Systems Engineering and Electronics*, vol. 31, no. 6, pp. 1297–1307, 2020.

[6] O. Mechali, L. Xu, and X. Xie, “Nonlinear homogeneous sliding mode approach for fixed-time robust formation tracking control of networked quadrotors,” *Aerospace Science and Technology*, vol. 126, Article ID 1107639, 2022.

[7] H. Fahham, A. Zaraki, G. Tucker, and M. W. Spong, “Time-optimal velocity tracking control for consensus formation of multiple nonholonomic mobile robots,” *Sensors*, vol. 21, no. 23, Article ID 7997, 2021.

[8] L. He, J. Zhang, Y. Hou, X. Liang, and P. Bai, “Time-varying formation control for second-order discrete-time multi-agent systems with directed topology and communication delay,” *IEEE Access*, vol. 7, Article ID 33517, 2019.

[9] T. Yan, X. Xu, Z. Li, and E. Li, “Flocking of multi-agent systems with unknown nonlinear dynamics and heterogeneous virtual leader,” *International Journal of Control, Automation and Systems*, vol. 19, no. 9, pp. 2931–2939, 2021.

[10] Y. Zou, Q. An, S. Miao, S. Chen, X. Wang, and H. Su, “Flocking of uncertain nonlinear multi-agent systems via...
distributed adaptive event-triggered control,” Neuro-
computing, vol. 465, no. 20, pp. 503–513, 2021.
[11] H. Wei and X.-B. Chen, “Flocking for multiple subgroups of
multi-agents with different social distanc-ing,” IEEE Access,
vol. 8, Article ID 164705, 2020.
[12] J. Zhou, D. Zeng, and X. Lu, “Multi-agent trajectory-tracking
flexible formation via generalized flocking and leader-average
sliding mode control,” IEEE Access, vol. 8, Article ID 36089,
2020.
[13] M. F. Arevalo-Castiblanco, D. Tellez-Castro, J. Sofrony,
and E. Mojica-Nava, “Adaptive synchronization of heterogeneous
multi-agent systems: a free observer approach,” Systems &
Control Letters, vol. 146, Article ID 104804, 2020.
[14] H. Rezaee and F. Abdollahi, “Adaptive leaderless consensus
control of strict-feedback nonlinear multiagent systems with
unknown control directions,” IEEE Transactions on Systems,
Man, and Cybernetics: Systems, vol. 51, no. 10, pp. 6435–6444,
2021.
[15] M. Firouzabahrami and A. Nobakhti, “Cooperative fixed-time/
finite-time distributed robust optimization of multi-agent
systems,” Automatica, vol. 142, Article ID 110358, 2022.
[16] W. Zhang, J. Zeng, J. Zhang, and Z. Li, “H∞ consensus
tracking of recovery system for multiple unmanned under-
water vehicles with switching networks and disturbances,”
Ocean Engineering, vol. 245, Article ID 110589, 2022.
[17] Y. Ren, Q. Wang, and Z. Duan, “Optimal distributed leader-
following consensus of linear multi-agent systems: a dynamic
average consensus-based approach,” IEEE Transactions on
Circuits and Systems II: Express Briefs, vol. 69, no. 3,
pp. 1208–1212, 2022.
[18] S. Wang and J. Huang, “Adaptive leader-following consensus
for multiple Euler–Lagrange systems with an uncertain leader
system,” IEEE Transactions on Neural Networks and Learning
Systems, vol. 30, no. 7, pp. 2188–2196, 2019.
[19] Z. Song and K. Sun, “Adaptive sliding mode tracking control
for uncertain Euler-Lagrange System,” IEEE Access, vol. 7,
2019.
[20] Q. Wang, J. Chen, B. Xin, and X. Zeng, “Distributed optimal
consensus for Euler–Lagrange systems based on event-trig-
ered control,” IEEE Transactions on Systems, Man,
and Cybernetics: Systems, vol. 51, no. 7, pp. 4588–4598,
2021.
[21] H. Zhang, J. H. Park, and W. Zhao, “Model-free optimal
consensus control of networked Euler-Lagrange systems,”
IEEE Access, vol. 7, Article ID 100771, 2019.
[22] C. Chen, G. Zhu, Q. Zhang, and J. Zhang, “Robust adaptive
finite-time tracking control for uncertain Euler-Lagrange
Systems with input saturation,” IEEE Access, vol. 8, Article ID
187608, 2020.
[23] D. Li, W. Zhang, W. He, C. Li, and S. S. Ge, “Two-layer
distributed formation-containment control of multiple Eu-
ler–Lagrange systems by output feedback,” IEEE Transactions
on Cybernetics, vol. 49, no. 2, pp. 675–687, 2019.
[24] S. He, Y. Xu, Y. Wu, Y. Li, and W. Zhong, “Adaptive con-
sensus tracking of multi-robotic systems via using integral
sliding mode control,” Neurocomputing, vol. 455, no. 30,
pp. 154–162, 2021.
[25] L. Yuan and J. Li, “Consensus of discrete-time nonlinear
multiagent systems using sliding mode control based on
optimal control,” IEEE Access, vol. 10, Article ID 47275, 2022.
[26] Z. Jin, Z. Wang, and X. Zhang, “Cooperative control problem
of Takagi-Sugeno fuzzy multiagent systems via observer based
distributed adaptive sliding mode control,” Journal of the
Franklin Institute, vol. 359, no. 8, pp. 3405–3426, 2022.
[27] D. Yao, H. Li, R. Lu, and Y. Shi, “Distributed sliding-mode
tracking control of second-order nonlinear multiagent sys-
tems: an event-triggered approach,” IEEE Transactions on
Cybernetics, vol. 50, no. 9, pp. 3892–3902, 2020.
[28] S. He, Y. Liu, Y. Wu, and Y. Li, “Integral sliding mode
consensus of networked control systems with bounded dis-
turbances,” ISA Transactions, vol. 124, pp. 349–355, 2022.
[29] Y. Fang, J. Fei, and T. Wang, “Adaptive backstepping fuzzy
neural controller based on fuzzy sliding mode of Active Power
Filter,” IEEE Access, vol. 8, Article ID 96027, 2020.
[30] L. Yipeng, L. Jie, Z. Fengge, and Z. Ming, “Fuzzy sliding mode
control of magnetic levitation system of controllable excita-
tion linear synchronous motor,” IEEE Transactions on In-
dustry Applications, vol. 56, no. 5, pp. 5585–5592, 2020.
[31] L. Teng, M. A. Gull, and S. Bai, “PD-based fuzzy sliding mode
control of a wheelchair exoskeleton robot,” IEEE, vol. 25,
no. 5, pp. 2546–2555, 2020.
[32] M. Mohit and M. Shahrokhi, “Adaptive fixed-time consensus
control for a class of non-strict feedback multi-agent systems
subject to input nonlinearities, state constraints, unknown
control directions, and actuator faults,” European Journal
of Control, vol. 66, Article ID 100649, 2022.
[33] X. Jin, S. Wang, J. Qin, W. X. Zheng, and Y. Kang, “Adaptive
fault-tolerant consensus for a class of uncertain nonlinear
second-order multi-agent systems with circuit implementa-
tion,” IEEE Transactions on Circuits and Systems I: Regular
Papers, vol. 65, no. 7, pp. 2243–2255, 2018.
[34] J. Dong, Y. Zhang, and X. Liu, “Attitude compensation control
for quadrotor under partial loss of actuator effec-
tiveness,” IEEE Access, vol. 10, Article ID 22568, 2022.
[35] X. Wang and G.-H. Yang, “Fault-tolerant consensus tracking
control for linear multiagent systems under switching di-
rected network,” IEEE Transactions on Cybernetics, vol. 50,
no. 5, pp. 1921–1930, 2020.
[36] Y. Yin, F. Wang, Z. Liu, and Z. Chen, “Finite-time leader-
following consensus of multiagent systems with actuator
defaults and input saturation,” IEEE Transactions on Systems,
Man, and Cybernetics: Systems, vol. 52, no. 5, pp. 3314–3325,
2022.
[37] J. Qin, G. Zhang, W. X. Zheng, and Y. Kang, “Neural network-
based adaptive consensus control for a class of nonaffine
nonlinear multiagent systems with actuator faults,” IEEE
Transactions on Neural Networks and Learning Systems,
vol. 30, no. 12, pp. 3633–3644, 2019.
[38] S. Zong and Y.-P. Tian, “Consensus of multi-agent systems
with unbounded time-varying delays,” IEEE Transactions on
Automatic Control, vol. 55, 2010.
[39] Y. Yang and W. Hu, “Containment control of double-inte-
grator multi-agent systems with time-varying delays,” IEEE
Transactions on Network Science and Engineering, vol. 9,
no. 2, pp. 457–466, 2022.
[40] L. Tan, C. Li, X. He, and T. Huang, “Distributed output
feedback leader-following consensus for nonlinear multiagent
systems with time delay,” Nonlinear Dynamics, vol. 105,
no. 2, pp. 1673–1687, 2021.
[41] F. Sun, M. Tao, J. Kurths, and W. Zhu, “Finite-time consensus
of leader-following multi-agent systems with multiple time
delays over time-varying topology,” International Journal of
Control, Automation and Systems, vol. 18, no. 8, pp. 1985–
1992, 2020.
[42] H.-W. Liu, T. Sun, and C. Q. Zhong, “New results on con-
sensus of multi-agent systems with time-varying delays: a
cyclic switching technique,” IEEE Access, vol. 9, Article ID
91402, 2021.
[43] Q. Cui, J. Sun, Z. Zhao, and Y. Zheng, “Second-order consensus for multi-agent systems with time-varying delays based on delay-partitioning,” IEEE Access, vol. 8, Article ID 91228, 2020.

[44] W. Ren and R. Beard, Distributed Consensus in Multi-Vehicle Cooperative Control: Theory and Applications, Springer-Verlag, London, UK, 2007.

[45] J. Hu, Z. Wang, H. Gao, and L. K. Stergioulas, “Robust sliding mode control for discrete stochastic systems with mixed time delays, randomly occurring uncertainties, and randomly occurring nonlinearities,” IEEE Transactions on Industrial Electronics, vol. 59, no. 7, pp. 3008–3015, 2012.

[46] Y. G. Sun, L. Wang, and G. Xie, “Average consensus in networks of dynamic agents with switching topologies and multiple time-varying delays,” Systems & Control Letters, vol. 57, no. 2, pp. 175–183, 2008.

[47] H. Du, S. Li, and P. Shi, “Robust consensus algorithm for second-order multi-agent systems with external disturbances,” International Journal of Control, vol. 85, no. 12, pp. 1913–1928, 2012.

[48] M. Wu, Y. He, and J. She, Stability Analysis and Robust Control of Time-Delay Systems, Science Press, Beijing, China, 2010.

[49] M. Wu, Y. He, and J. She, Stability Analysis and Robust Control of Time-Delay Systems, Science Press, Beijing, China, 2010.

[50] Y.-H. Chang and W.-S. Chan, “Adaptive dynamic surface control for uncertain nonlinear systems with interval type-2 fuzzy neural networks,” IEEE Transactions on Cybernetics, vol. 44, no. 2, pp. 293–304, 2014.