Two-loop effective potentials in general \( \mathcal{N} = 2, d = 3 \) chiral superfield model

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Abstract

We study local superspace contributions to low-energy effective action in general chiral three-dimensional superfield model. The effective Kähler and chiral potentials are computed in explicit form up to the two-loop order. In accordance with the non-renormalization theorem, the ultraviolet divergences appear only in the full superspace while the effective chiral potential receives only finite quantum contributions in the massless case. As an application, the two-loop effective scalar potential is found for the three-dimensional \( \mathcal{N} = 2 \) supersymmetric Wess-Zumino model.

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1 Introduction

The interest to three-dimensional supersymmetric field models is partly motivated by recent achievements in constructing field theories modelling multiple M2 branes (see, e.g., [1, 2, 3, 4]). These are three-dimensional superconformal field theories of Chern-Simons gauge fields interacting with matter in a special way such that superconformal invariance and $\mathcal{N} = 6$ (or even $\mathcal{N} = 8$) extended supersymmetry are respected. There is very significant progress in studying correlation functions and scattering amplitudes in the ABJM-like models (see, e.g., recent papers [5, 6, 7, 8]), but the problem of low-energy effective action in such theories is still purely understood. One of the subtle questions in defining the low-energy effective action in the ABJM theory is how to separate the light and heavy degrees of freedom when the scalar fields acquire some vacuum. Indeed, in [9] it was shown that the Higgs mechanism for the ABJM-like models works differently from the standard case of SYM-matter models. Once the scalar fields acquire some vevs, this novel Higgs mechanism turns the ABJM action into the $\mathcal{N} = 8$ SYM model with higher derivative corrections. So, one is forced to study the effective action in the three-dimensional extended SYM models [10, 11] rather in the ABJM model itself.

The Chern-Simons fields in the ABJM action make the M2 brane description very elegant because the supersymmetries, R-symmetry and conformal invariance become quite explicit [12, 13]. However, these fields are non-dynamical and can be, in principle, eliminated by fixing the gauge symmetry. Once the gauge symmetry is fixed, one is left with a three-dimensional non-linear supersymmetric sigma-model in which the symmetries of the M2 brane become non-manifest. Quantum aspects of such a sigma-model (and, in particular, low-energy effective action) can be investigated by standard methods of quantum field theory.

Keeping these motivations in mind, we initiate the studies of low-energy effective action in $\mathcal{N} = 2, d = 3$ supersymmetric sigma-models. In the present paper we restrict ourself to the model with one chiral superfield $\Phi$ which is described by the following general action

$$S = -\int d^3x d^4\theta K(\Phi, \bar{\Phi}) - \left(\int d^3x d^2\theta W(\Phi) + c.c.\right).$$

(1.1)

Here $K(\Phi, \bar{\Phi})$ is the Kähler potential and $W(\Phi)$ is chiral potential. The sigma-model case corresponds to $W \equiv 0$, but we keep it non-vanishing for the sake of generality. Another particular case with

$$K_{\text{WZ}} = \Phi \bar{\Phi}, \quad W_{\text{WZ}} = \frac{m}{2} \Phi^2 + \frac{\lambda}{6} \Phi^3 + \frac{g}{24} \Phi^4,$$

(1.2)

corresponds to the classical action of $\mathcal{N} = 2, d = 3$ Wess-Zumino model.

The aim of this paper is to study some aspects of superfield quantum effective action in the model under consideration. In general, the effective action is extremely complicated non-local functional of background superfields. To be more precise, we are interested in the local part of the quantum effective action in the model (1.1) which is described by the effective Kähler and chiral potentials,

$$\Gamma = -\int d^3x d^4\theta K_{\text{eff}}(\Phi, \bar{\Phi}) - \left(\int d^3x d^2\theta W_{\text{eff}}(\Phi) + c.c.\right).$$

(1.3)
All non-local terms or the terms with derivatives of background superfields are out of our approximation.\footnote{It means that all possible derivative dependent or space-time non-local contributions to effective action are systematically neglected.} In the present paper we compute both one- and two-loop quantum corrections to $K_{\text{eff}}$ and $W_{\text{eff}}$ and briefly discuss the application of these results to the three-dimensional Wess-Zumino model. In particular, effective scalar potential in the Wess-Zumino models is found.

The effective Kähler and chiral potentials for the four-dimensional analog of the model (1.1) were studied in [14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. In these papers it was shown that the quantum divergences appear only in the sector of the effective Kähler potential while the effective chiral potential is finite. These results are in complete agreement with the non-renormalization theorem [27, 28] which applies for the three-dimensional models as well. In the present work, for the three-dimensional general chiral superfield model we show that UV divergences appear only in the Feynman graphs contributing to the effective Kähler potential while the effective chiral potential UV finite. Such finite quantum corrections in the chiral sector arise only in the massless case.

We point out that the effective Kähler potential in the $\mathcal{N} = 1$, $d = 3$ scalar superfield model was studied in [29, 30, 31]. In the present work we extend some of the results of these papers to the $\mathcal{N} = 2$ supersymmetric case.

The rest of this paper is organized as follows. In Sect. 2 we find one- and two-loop quantum contributions to the effective Kähler potential while similar computations for the effective chiral potential are given in Sect. 3. In Sect. 4 we apply these general results for obtaining effective scalar potential in the $\mathcal{N} = 2$, $d = 3$ Wess-Zumino models. The details of computations of Feynman graphs contributing to the effective Kähler and chiral potentials are given in the Appendices.

Throughout this work we employ the $\mathcal{N} = 2$, $d = 3$ superspace notations from [10, 11].

## 2 Effective Kähler potential

Within the loop expansion of the effective action the effective Kähler potential can be represented by a series

$$ K_{\text{eff}} = K + K^{(1)} + K^{(2)} + \ldots , $$

where $K$ is the classical Kähler potential and $K^{(1)}$, $K^{(2)}$ correspond to one- and two-loop quantum contributions. The dots stand for higher loops. In the present paper we restrict ourself to the two-loop approximation.

### 2.1 One-loop contributions

Let us split the superfield $\Phi$ into the “background” $\Phi$ and “quantum” $\phi$ parts, $\Phi \rightarrow \Phi + \phi$. For computing the one-loop effective action it is sufficient to expand the classical action (1.1) up to the second order in quantum superfields,

$$ S = -\int d^3 xd^4 \theta \left[ \frac{1}{2} \bar{\phi}^2 K_{\Phi \Phi}'' + \frac{1}{2} \phi^2 K_{\Phi \Phi}'' + \phi \phi K_{\Phi \Phi}'' \right] - \left( \int d^3 xd^2 \theta \frac{1}{2} \phi^2 W''(\Phi) + \text{c.c.} \right) + \ldots , $$

(2.2)
where dots stand for higher order terms with respect to the quantum fields. For computing $K_{\text{eff}}(\Phi, \bar{\Phi})$ it is sufficient to consider constant background fields,

$$D_\alpha \Phi = 0, \quad \bar{D}_\alpha \bar{\Phi} = 0.$$  \hspace{1cm} (2.3)

In this case the terms involving $K''_{\Phi \Phi}$ and $K''_{\bar{\Phi} \bar{\Phi}}$ in (2.2) vanish.

The one-loop effective action is given by

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln H,$$  \hspace{1cm} (2.4)

where $H$ is an operator which defines the part of the action which is quadratic with respect to the background superfields,

$$H = \begin{pmatrix} -W'' & \frac{1}{4} K''_{\Phi \Phi} \bar{D}^2 \\ \frac{1}{4} K''_{\Phi \Phi} D^2 & 0 \end{pmatrix}.$$  \hspace{1cm} (2.5)

We represent this operator as a sum of two terms,

$$H = H_0 + H_1, \quad H_0 = \begin{pmatrix} 0 & \frac{1}{4} K''_{\Phi \Phi} \bar{D}^2 \\ \frac{1}{4} K''_{\Phi \Phi} D^2 & 0 \end{pmatrix}, \quad H_1 = \begin{pmatrix} -W'' & 0 \\ 0 & W'' \end{pmatrix},$$  \hspace{1cm} (2.6)

and expand the logarithm in (2.4),

$$\Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln H_0 + \frac{i}{2} \text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (H_0^{-1} H_1)^n.$$  \hspace{1cm} (2.7)

For the constant background fields, the first term in the rhs of (2.7) reads

$$\frac{i}{2} \text{Tr} \ln H_0 = \frac{i}{4} \text{Tr} \ln H_0^2 = \frac{i}{4} \text{Tr} + \ln((K''_{\Phi \Phi})^2) + c.c.$$  \hspace{1cm} (2.8)

Here $\text{Tr}+$ denotes the functional trace over the chiral superfields. The box operator in the rhs in (2.8) originates from the identity

$$\frac{1}{16} \bar{D}^2 D^2 \Phi = \Box \Phi,$$  \hspace{1cm} (2.9)

which holds for any chiral superfield $\Phi$ and follows from the anticommutation relations of the Grassmann derivatives, $\{D_\alpha, \bar{D}_\beta\} = -2i \partial_{\alpha\beta}$.

Using the inverse operator

$$H_0^{-1} = \begin{pmatrix} 0 & \frac{1}{4} K''_{\Phi \Phi} \bar{D}^2 \\ \frac{1}{4} K''_{\Phi \Phi} D^2 & 0 \end{pmatrix},$$  \hspace{1cm} (2.10)

for the second term in the rhs of (2.7) we get

$$\frac{i}{2} \text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \begin{pmatrix} 0 & \frac{1}{4} W'' D^2 \\ \frac{1}{4} W'' \bar{D}^2 & 0 \end{pmatrix}^n = \frac{i}{4} \text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \begin{pmatrix} \frac{W''}{K''_{\Phi \Phi}} & \frac{1}{4} \Box \\ 0 & \frac{W''}{K''_{\Phi \Phi}} \end{pmatrix}^n \begin{pmatrix} W'' \bar{D}^2 & 0 \\ 0 & W'' D^2 \end{pmatrix} = \frac{i}{4} \text{Tr} + \ln \left(1 + \left|\frac{W''}{K''_{\Phi \Phi}}\right|^2 \frac{1}{\Box} \right) + c.c.$$  \hspace{1cm} (2.11)
Combining (2.8) and (2.11) together, we get the following formal expression for the one-loop effective action
\[ \Gamma^{(1)} = \frac{i}{4} \text{Tr} \ln(\Box + M^2) + c.c., \] (2.12)
where the effective mass squared is
\[ M^2 = \left| \frac{W''}{K''_{\Phi\bar{\Phi}}} \right|^2. \] (2.13)

The functional trace over chiral superfields in (2.12) can be written explicitly as
\[ \text{Tr} \ln(\Box + M^2) = \int d^5z_1d^5z_2 \delta_+(z_2, z_1) \ln(\Box + M^2) \delta_+(z_1, z_2). \] (2.14)

To compute this expression we apply the following identity for the chiral delta-function,
\[ \delta_+(z_2, z_1) = \frac{1}{16} \delta_7(z_2 - z_1) \ln(\Box + M^2) \delta_+(z_1, z_2), \] (2.15)
and integrate by parts the \( D^2 \)-operator,
\[ d^7z = -\frac{1}{4} \bar{D}^2 d^5z, \] (2.16)

Now, using standard identity
\[ \delta^4(\theta_1 - \theta_2) \frac{1}{16} D^2 \bar{D}^2 \delta^7(z_1 - z_2) = \delta^7(z_1 - z_2), \] (2.18)
we integrate over one set of Grassmann variables,
\[ \text{Tr} \ln(\Box + M^2) = \int d^3x_1d^3x_2d^4\theta \delta^3(x_1 - x_2) \ln(\Box + M^2) \frac{1}{4} \delta^3(x_2 - x_1). \] (2.19)

Passing to the momentum space, we compute the momentum integral,
\[ -\int \frac{d^3p}{(2\pi)^3} \frac{1}{p^2} \ln \left( 1 - \frac{M^2}{p^2} \right) = \frac{i}{2\pi} |M|. \] (2.20)

As a result, we get the following answer for the one-loop Kähler potential,
\[ K^{(1)}(\Phi, \bar{\Phi}) = \frac{1}{4\pi} |M| = \frac{1}{4\pi} \left| \frac{W''}{K''_{\Phi\bar{\Phi}}} \right|. \] (2.21)

We point out that the one-loop effective action is finite because only two-loop UV quantum divergences can appear in three-dimensional field theories. Hence, (2.21) is a finite renormalization of the classical Kähler potential in (1.1).
2.2 Two-loop contributions

The two-loop Feynman diagrams involve three- and four-point vertices, see Fig. 1. Therefore it is sufficient to expand the classical action up to the fourth order in quantum superfields,

\[
S = S_2 + S_{\text{int}} + \ldots \tag{2.22}
\]

\[
S_2 = -\int d^3 x d^4 \theta \left( K_{\Phi \Phi}^{\prime\prime} \Phi \bar{\Phi} \right) - \left( \int d^3 x d^2 \theta \frac{1}{2} W^\prime \Phi^2 + c.c. \right), \tag{2.23}
\]

\[
S_{\text{int}} = -\frac{1}{2} \int d^3 x d^4 \theta \left( K_{\Phi \Phi}^{(3)} \Phi^2 \bar{\Phi} + K_{\Phi \Phi}^{(3)} \bar{\Phi}^2 \Phi + \frac{1}{2} K_{\Phi \Phi \Phi}^{(4)} \Phi^2 \bar{\Phi}^2 + \frac{1}{3} K_{\Phi \Phi \Phi}^{(4)} \Phi^2 \bar{\Phi} + \frac{1}{3} K_{\Phi \Phi}^{(4)} \bar{\Phi}^3 \Phi \right) - \left( \frac{1}{6} \int d^3 x d^2 \theta [W^\prime \Phi^3 + \frac{1}{4} W^{(4)} \Phi^4] + c.c. \right). \tag{2.24}
\]

In this decomposition we assume that the background fields are constant, \( (2.23) \). Therefore we omitted the terms involving \( D^2 K_{\Phi \Phi}^{(n)} \) and \( D^2 K_{\Phi \Phi}^{(n)} \) since they do not contribute to the effective Kähler potential.

The quadratic action \( (2.23) \) defines the propagator which can be explicitly written for the constant field background,

\[
\begin{pmatrix}
G_{++}(z_1, z_2) & G_{+-}(z_1, z_2) \\
G_{-+}(z_1, z_2) & G_{--}(z_1, z_2)
\end{pmatrix} = \begin{pmatrix}
\frac{\bar{W}^{-}}{(K_{\Phi \Phi}^{(3)})^{1/2} + M^2} \delta_+(z_1, z_2) & -\frac{1}{4K_{\Phi \Phi}^{(3)}} \frac{1}{(K_{\Phi \Phi}^{(3)})^{1/2} + M^2} D^2 \delta_-(z_1, z_2) \\
-\frac{1}{4K_{\Phi \Phi}^{(3)}} \frac{1}{(K_{\Phi \Phi}^{(3)})^{1/2} + M^2} D^2 \delta_+(z_1, z_2) & \frac{\bar{W}^{-}}{(K_{\Phi \Phi}^{(3)})^{1/2} + M^2} \delta_-(z_1, z_2)
\end{pmatrix}. \tag{2.25}
\]

The interaction vertices can be read from \( (2.24) \).

We follow standard procedure of computing the two-loop Feynman diagrams in supersymmetric field theories \( (2.28) \). Each line at Fig. 1 corresponds to one of the elements of the matrix propagators \( (2.24) \). Each vertex can be either (anti)chiral, e.g.,

\[
S_{\Phi^3}(z_1, z_2, z_3) \equiv \frac{\delta^3 S}{\delta \Phi(z_1) \delta \Phi(z_2) \delta \Phi(z_3)} = -W^{\prime \prime}(z_3) \delta_+(z_1, z_2) \delta_+(z_2, z_3), \tag{2.26}
\]

or, non-chiral, e.g.,

\[
S_{\Phi \Phi}(z_1, z_2, z_3) \equiv \frac{\delta^3 S}{\delta \Phi(z_1) \delta \Phi(z_2) \delta \Phi(z_3)} = \frac{1}{4} D_{(3)}^2 \left[ K_{\Phi \Phi}^{\prime \prime}(z_3) \delta_+(z_2, z_3) \right] \delta_+(z_1, z_2), \tag{2.27}
\]

\[
S_{\Phi \Phi}(z_1, z_2, z_3) \equiv \frac{\delta^3 S}{\delta \Phi(z_1) \delta \Phi(z_2) \delta \Phi(z_3)} = \frac{1}{4} D_{(2)}^2 \left[ K_{\Phi \Phi}^{\prime \prime}(z_2) \delta_+(z_1, z_2) \right] \delta_-(z_2, z_3). \tag{2.28}
\]
Each non-chiral vertex has extra $D^2$ or $\bar{D}^2$ operator in comparison with the chiral one. As a consequence, there are many combinatoric possibilities with these propagators and vertices to construct the two-loop diagrams shown at Fig. 1. However, not all these diagrams contribute to the effective Kähler potential. Only those diagrams are eligible which have specific number of the diagrams contribute to the effective Kähler potential. Only those diagrams are eligible which have specific number of the $D^2$ and $\bar{D}^2$ operators which are necessary for restoring full superspace measures at each vertex using (2.16) and then applying the identity (2.18) for each loop. Analyzing the diagrams of topology $A$ at Fig. 1, one can see that only one such diagram contributes,

$$\Gamma_A = -\frac{1}{2} \int d^5z_1 d^5\bar{z}_2 d^5z_3 d^5\bar{z}_4 S^{(4)}_{\phi\phi\phi\phi}(z_1, z_2, z_3, z_4) G_{++}(z_1, z_2) G_{+-}(z_3, z_4). \quad (2.29)$$

Here $S^{(4)}$ is the fourth variational derivative of the classical action written explicitly in (A.1). Among the diagrams of the topology $B$ contributing to the effective Kähler potential, there are five various terms,

$$\Gamma_{B_1} = -\frac{1}{6} \int d^5z_1 d^5\bar{z}_2 d^5z_3 d^5\bar{z}_4 d^5z_5 d^5\bar{z}_6 S^{(3)}_{\phi\phi\phi}(z_1, z_2, z_3) G^{(3)}_{++}(z_1, z_2, z_3), \quad (2.30)$$

$$\Gamma_{B_2} = -\frac{1}{6} \int d^5z_1 d^5\bar{z}_2 d^5z_3 d^5\bar{z}_4 d^5z_5 d^5\bar{z}_6 S^{(3)}_{\phi\phi\phi}(z_1, z_2, z_3) G^{(3)}_{+-}(z_3, z_4), \quad (2.31)$$

$$\Gamma_{B_3} = -\int d^5z_1 d^5\bar{z}_2 d^5z_3 d^5\bar{z}_4 d^5z_5 d^5\bar{z}_6 S^{(3)}_{\phi\phi\phi}(z_1, z_2, z_3) G^{(3)}_{++}(z_1, z_4), \quad (2.32)$$

$$\Gamma_{B_4} = -\frac{1}{2} \int d^5z_1 d^5\bar{z}_2 d^5z_3 d^5\bar{z}_4 d^5z_5 d^5\bar{z}_6 S^{(3)}_{\phi\phi\phi}(z_1, z_2, z_3) G^{(3)}_{+-}(z_2, z_5), \quad (2.33)$$

$$\Gamma_{B_5} = -\frac{1}{2} \int d^5z_1 d^5\bar{z}_2 d^5z_3 d^5\bar{z}_4 d^5z_5 d^5\bar{z}_6 S^{(3)}_{\phi\phi\phi}(z_1, z_2, z_3) G^{(3)}_{++}(z_1, z_4). \quad (2.34)$$

The details of the computations of (2.29) - (2.34) are given in the Appendix [A.1]. The resulting two-loop contributions to the Kähler potential $K_{\text{eff}}^{(2)}$ can be written as the sum of the divergent and finite parts,

$$K^{(2)} = K^{(2)}_{\text{div}} + K^{(2)}_{\text{fin}}, \quad (2.35)$$

where

$$K^{(2)}_{\text{div}} = \frac{1}{64\pi^2\epsilon} \left[ \frac{3 K''_W W''^2}{(K''_\phi)^3} + \frac{1}{3} \frac{|W''|^2}{(K''_\phi)^3} - \frac{K''_W W'' W'' W''}{(K''_\phi)^3}, \quad (2.36) \right]$$

$$K^{(2)}_{\text{fin}} = \frac{1}{32\pi^2} \left[ \frac{K^{(4)}_W |W''|^2}{(K''_\phi)^4} + \frac{1}{2} (\gamma + \ln \frac{M^2}{\mu^2}) \frac{K''_W W'' W''^2 + K''_\phi W'' W''^2}{(K''_\phi)^4} \right. \right.$$

$$- \frac{1}{6} (\gamma + \ln \frac{M^2}{\mu^2}) \frac{|W''|^2}{(K''_\phi)^3} - \left( 1 + \frac{3}{2} \gamma + \frac{3}{2} \ln \frac{M^2}{\mu^2} \right) \frac{|K''_W W''|^2}{(K''_\phi)^5}, \quad (2.37)$$

6
where $M^2 = |W''|^2/(K''_{\Phi \Phi})^2$, $\mu^2$ is a normalization point and $\epsilon$ is the parameter of dimensional regularization, $d = 3 - 2\epsilon$.

3 Effective chiral potential

3.1 General properties

For the four-dimensional $\mathcal{N} = 1$ supersymmetric field theories the non-renormalization theorem says that any contribution to the effective action can be represented by an expression in the full superspace (see e.g. [27, 28]),

$$\int d^4\theta f(\Phi, \bar{\Phi}),$$

with $f(\Phi, \bar{\Phi})$ being some function of superfields (with or without derivatives). One can easily check that all steps of proof of this theorem and general conclusion remain to be true for the $\mathcal{N} = 2, d = 3$ models as well.

Naively, one can think that (3.1) forbids any quantum contributions to the chiral effective potential $W_{\text{eff}}$, but it is well-known for the four-dimensional chiral superfield model [17, 18, 19, 20, 21, 22] that the effective action acquires finite quantum corrections in the full superspace of the form

$$\int d^4\theta f(\Phi) \left(-\frac{D^2}{4\Box}\right) g(\Phi),$$

which do not contradict the non-renormalization theorem. Here $f$ and $g$ are some functions. In the present section we demonstrate that for the three-dimensional chiral superfield model the two-loop Feynman diagrams also contain the terms of the form (3.2) which result in the contributions to the chiral effective potential,

$$\int d^3x d^4\theta f(\Phi) \left(-\frac{D^2}{4\Box}\right) g(\Phi) = \int d^3x d^2\theta f(\Phi) g(\Phi).$$

(3.3)

It is important to note that the terms in the effective action like (3.2) can appear only in massless models. Indeed, the propagator in a massive model involves the operator $(\Box + m^2)^{-1}$ instead of $\Box^{-1}$. As a consequence, in the massive model the relation (3.3) gets modified as

$$\int d^3x d^4\theta f(\Phi) \left(-\frac{D^2}{4(\Box + m^2)}\right) g(\Phi) = \int d^3x d^2\theta f(\Phi) \left(\frac{\Box}{\Box + m^2}\right) g(\Phi),$$

(3.4)

but this expression vanishes in the limit of slowly varying fields unless $m = 0$. Hence, a non-trivial chiral effective potential may be present only in the massless theory. This conclusion is completely analogous to the one for the four-dimensional $\mathcal{N} = 1$ chiral superfield model [19, 20, 21, 22]. Therefore, in this section we assume that the chiral potential $W(\Phi)$ in the classical action (1.1) obeys the constraint

$$W''|_{\Phi=0} = 0.$$  

(3.5)
It is important to note that for deriving the terms like (3.2) we cannot employ the constant field approximation any more. The background chiral superfield $\Phi$ should be arbitrary throughout the computations while the anti-chiral one can be freely sent to zero,

$$\Phi = 0, \quad \Phi \text{ is arbitrary.} \tag{3.6}$$

Only after computing all momentum integrals and passing to the chiral subspace using (3.3), one can apply the constant field approximation to single out the contributions to the chiral effective potential.

### 3.2 Analysis of possible Feynman diagrams contributing to chiral effective potential

The relation (3.3) shows that only those Feynman diagrams contribute to the chiral effective potential which contain one $D^2$ operator on the external lines after performing all the $D$-algebra and one $\Box^{-1}$ operator after computing the momentum integrals. First of all, we point out that the one-loop diagrams cannot contribute to the effective chiral potential as soon as the corresponding momentum integral yields only the odd power of external momenta $|p| = \sqrt{p^mp_m}$,

$$\int d^3k \frac{1}{k^2} \frac{1}{(k+p)^{2n}} \propto \frac{1}{|p|^{2n-1}}. \tag{3.7}$$

Because of (2.9), the $D$-algebra produces only even powers of momenta, $p^{2n}$. Therefore, for the rest of this section we will consider two-loop diagrams only.

As in the previous section, we split the chiral field $\Phi$ into the “background” $\Phi$ and “quantum” $\phi$ parts, $\Phi \to \Phi + \phi$, and consider the decomposition of the classical action (1.1) up to the fourth order in the quantum superfields (c.f. (2.22)–(2.24)),

$$S = S_2 + S_{\text{int,1}} + S_{\text{int,2}} + S_{\text{int,3}} + \ldots, \tag{3.8}$$

$$S_2 = -\frac{1}{2} \int d^3xd^4\theta K^{(3)}_{\Phi\Phi}\phi^2\bar{\phi}, \tag{3.9}$$

$$S_{\text{int,1}} = -\frac{1}{2} \int d^3xd^4\theta \left( K_{\Phi\phi\bar{\phi}}^{(3)} \phi^2\bar{\phi} + K_{\Phi\phi\phi}^{(3)} \phi^2\bar{\phi} + \frac{1}{2} K_{\Phi\phi\bar{\phi}}^{(4)} \phi^2\bar{\phi}^2 \right. \right.$$  

$$\left. + \frac{1}{3} K_{\Phi\phi\bar{\phi}}^{(4)} \phi^3\bar{\phi} + \frac{1}{3} K_{\Phi\phi\phi}^{(4)} \phi^3\phi \right), \tag{3.10}$$

$$S_{\text{int,2}} = -\frac{1}{2} \int d^3xd^2\theta \left( \phi^2(-\frac{1}{4}D^2)K_{\Phi\phi}^{\prime\prime} + \frac{1}{3} \phi^3(-\frac{1}{4}D^2)K_{\Phi\phi}^{\prime\prime\prime} + \frac{1}{12} \phi^4(-\frac{1}{4}D^2)K_{\Phi\phi}^{(4)} \right)$$

$$-\frac{1}{2} \int d^3xd^2\theta \left( \bar{\phi}^2(-\frac{1}{4}D^2)K_{\bar{\Phi}\bar{\phi}}^{\prime\prime} + \frac{1}{3} \bar{\phi}^3(-\frac{1}{4}D^2)K_{\bar{\Phi}\bar{\phi}}^{\prime\prime\prime} + \frac{1}{12} \bar{\phi}^4(-\frac{1}{4}D^2)K_{\bar{\Phi}\bar{\phi}}^{(4)} \right), \tag{3.11}$$

$$S_{\text{int,3}} = -\frac{1}{2} \int d^3xd^2\theta \left( \phi^2W^{\prime\prime} + \frac{1}{3} \phi^3W^{\prime\prime\prime} + \frac{1}{12} \phi^4W^{(4)} \right) + c.c. \tag{3.12}$$

$^2$Recall that the propagator (2.25) was derived in the constant field approximation.
Table 1: Graphical representations of the vertices which are relevant for two-loop computations. Thick lines stand for the expressions depending on the background superfields while the thin ones mean the quantum superfields.

We point out that, owing to (3.6), all derivatives of $K$ and $W$, $\bar{W}$ in this expansion are considered at vanishing antichiral background field $\bar{\Phi}$. Therefore all these factors are chiral.

The action $S_2$ is responsible for the propagator,

$$\langle \phi(z)\bar{\phi}(z') \rangle \equiv G_0(z, z') = -\frac{1}{K_{\phi\Phi}^2} \frac{D^2}{4\Box} \delta_+(z, z') = \frac{1}{K_{\phi\Phi}^2} \frac{D^2\bar{D}^2}{16\Box} \delta^7(z - z'). \quad (3.13)$$

Note that this propagator contains four Grassmann derivatives on the delta-function while the propagators $\langle \phi\phi \rangle$ and $\langle \bar{\phi}\bar{\phi} \rangle$ are traded for the corresponding vertices. This restricts the number of possible Feynman diagrams with these propagators.

The actions (3.10), (3.11) and (3.12) are responsible for the vertices which are relevant for two-loop Feynman graphs. We use the graphical representations of these vertices according to Table 1. It is convenient to distinguish these vertices with respect to the type of superspace over which they are integrated (chiral, antichiral or full $N = 2, d = 3$ superspace). Let us comment on each of these types in more details.

The action $S_{\text{int.1}}$ yields the vertices in the full superspace which involve $K_{\Phi^{(3)}\Phi^{2}\Phi}$, $K_{\Phi^{(2)}\Phi^{2}\Phi}$, $K_{\Phi^{(4)}\Phi^{2}\Phi}$, $\frac{1}{3}K_{\Phi^{(3)}\Phi^{3}\Phi}$ and $\frac{1}{3}K_{\Phi^{(4)}\Phi^{3}\Phi}$. Such vertices bring extra $D^2$ or $\bar{D}^2$ operators as compared with the (anti)chiral vertices, see, e.g., (3.10) and (3.11). These $D^2$-operators can hit both external and internal lines in a diagram. Once they hit the external lines, they do not affect the loop momenta any more. In fact, we need to consider only

\footnote{The terms in the actions (3.11), (3.12) containing $\phi^2$ and $\bar{\phi}^2$ are treated as vertices rather than the propagators for the quantum fields.}
those diagrams which have only one $D^2$ operator on the external lines that is necessary for chiral contributions due to (3.3). If these operators hit the internal lines, they either can be used to restore full superspace measure (2.16) or increase the superficial degree of divergence of the diagram owing to the $D$-algebra (2.9). In the following, by $n_1$ we denote the number of vertices in a Feynman diagram corresponding to the action $S_{\text{int},1}$.

The action $S_{\text{int},2}$ contains chiral and antichiral vertices. Note that the chiral vertices bear the $\bar{D}^2$ operators on the external lines, but we need $D^2$ operator to apply the identity (3.3). Hence, we can neglect all the chiral vertices given in the first line of (3.11) and consider only the antichiral ones in the second line. By $n_2$ we denote the number of vertices corresponding to the second line of (3.11) in a Feynman diagram. It is clear that a diagram contributing to the chiral effective potential should contain no more than one such vertex, $n_2 = 0$ or $n_2 = 1$.

The action $S_{\text{int},3}$ is responsible for the (anti)chiral vertices without $D^2$ or $\bar{D}^2$ operators. Denote the number of such vertices by $n_3$.

Our aim now is to analyze the two-loop diagrams at Fig. 1 and to fix the numbers $n_1$, $n_2$ and $n_3$ which correspond to non-trivial contributions to the chiral effective potential. The strategy of our considerations is as follows:

- Draw all admissible two-loop diagrams with the propagator (3.13) and with the vertices given in Table I.
- Restore full superspace measures at all (anti)chiral vertices using the Grassmann derivatives from the propagators;
- All other Grassmann derivatives can be integrated by parts producing a number of different terms, but we need to consider only those of them which contain exactly two derivatives $D_\alpha$ on the external lines;
- Two operators $\bar{D}^2D^2$ are eaten by the identity (2.18) (by one for each loop);
- The remaining Grassmann derivatives generate the internal momenta which increases the superficial degree of divergence of diagrams;
- Only those diagrams are eligible in which the momentum loop integrals produce the power of external momenta as $p^{-2}$. Then, upon application of the identity (3.3), this diagram contributes to the effective chiral potential.

### 3.2.1 Diagrams of topology $A$

The diagrams of topology $A$ at Fig. 1 have one of the following quartic vertices, $K^{(4)}_{\Phi^2\bar{\Phi}^2\Phi^2\bar{\Phi}^2}$, $\frac{1}{3}K^{(4)}_{\Phi^2\bar{\Phi}^2\Phi^2\bar{\Phi}^2}$, $\frac{1}{3}K^{(4)}_{\Phi^2\bar{\Phi}^2\Phi^2\bar{\Phi}^2}$, $\phi^4W^{(4)}$ or $\phi^4\bar{W}^{(4)}$. Consider a Feynman graph of this topology which involves $n_2$ vertices with $\phi^2W^\prime$ and $n_3$ vertices with $\phi^2W^\prime$. Recall that there is no antichiral $\bar{W}^\prime$ vertex since it vanishes for the considered background (3.6) in the massless theory. Clearly, this diagram should have $n_2 + n_3 + 2$ propagators (3.13) each of which brings the operator $D^2\bar{D}^2\Box^{-1}$.

The non-chiral vertices $K^{(4)}_{\Phi^2\bar{\Phi}^2\Phi^2\bar{\Phi}^2}$, $\frac{1}{3}K^{(4)}_{\Phi^2\bar{\Phi}^2\Phi^2\bar{\Phi}^2}$, $\phi^4W^{(4)}$, $\phi^4\bar{W}^{(4)}$, $\frac{1}{3}K^{(4)}_{\Phi^2\bar{\Phi}^2\Phi^2\bar{\Phi}^2}$ are integrated over the full superspace. Therefore, we need to restore the full superspace measure only for the $n_2$
antichiral vertices with $\bar{\phi}^2(-\frac{1}{4}D^2)K''_{\Phi^2}$ and for $n_3$ chiral vertices with $\phi^2W''$ with the help of the $D$-operators which are present in the propagators. Moreover, doing integration by parts we have to collect $1 - n_2$ operators $D^2$ on the external lines which are required for the identity \((3.3)\). For each loop we have to apply the identity \((2.18)\) which eats the $D^2\tilde{D}^2$ operator. As a result, we are left with $(n_3 + n_2 - 1)$ operators $D^2$ and with $n_2$ operators $\tilde{D}^2$. We cannot put any more Grassmann derivatives on the external lines as we wish to get the chiral effective potential. Hence, all these Grassmann derivatives should recombine into the internal momenta which means that there should be equal numbers of $D^2$ and $\tilde{D}^2$ operators, $n_3 + n_2 - 1 = n_2$. As a result, the eligible diagrams have only $n_3 = 1$ vertices $\phi^2W''$. But for the number $n_2$ the only possibility is $n_2 = 1$ since for $n_2 = 0$ the two-loop diagram with the only external line vanishes automatically. After using all the $D$-operators from the propagators as is described here, we are left with one $D^2$ and one $\tilde{D}^2$ which produce one $\Box$ operator in the nominator, but four box operators stand in the denominators of four propagators. The resulting momentum integral is something like

$$\int \frac{d^4k_1d^4k_2k^2_1}{k^2_1k^2_2(k_1 + p)^2(k_2 + p)^2} \propto p^0,$$

but we need $p^{-2} \rightarrow \Box^{-1}$ to apply \((3.3)\). Hence, the diagrams with $K^{(4)}_{\Phi^2\Phi^2\Phi^2\Phi^2}$, $\frac{1}{3}K^{(4)}_{\Phi^2\Phi^2\Phi^2\Phi^2}$, $\frac{1}{3}K^{(4)}_{\Phi^2\Phi^2\Phi^2\Phi^2}$ vertices do not contribute to the effective chiral potential.

Consider a diagram with $\bar{\phi}^3(-\frac{1}{4}D^2)K''_{\Phi^2}$ vertex which is integrated over the antichiral superspace. Here we already have one $\tilde{D}^2$ on the external line, hence no vertices with $\bar{\phi}^2(-\frac{1}{4}D^2)K''_{\Phi^2}$ are allowed, $n_2 = 0$. Since we have only $\langle \phi\bar{\phi} \rangle$ propagators but neither $\langle \phi\phi \rangle$ nor $\langle \phi\phi \rangle$, the only possibility to build the diagram of the topology $A$ is by attaching one $\phi^2W''$ vertex for each loop, i.e., $n_3 = 2$. From four propagators we take one $D^2$ operator and two $\tilde{D}^2$ operators to restore the full superspace measure at each vertex and we are left with unbalanced number of such operators leading to the null contribution for the effective chiral potential.

Consider now a diagram with the quartic vertex $\phi^4W^{(4)}$. It is easy to see that such a diagram should have at least two $\bar{\phi}^2(-\frac{1}{4}D^2)K''_{\Phi^2}$ vertices because we have only $\langle \phi\bar{\phi} \rangle$ propagator. But this leads to two $D^2$ operators on the external lines while we need only one to apply \((3.3)\). Hence, such diagrams do not contribute to the effective chiral potential.

Finally, we have to consider a diagram with the $\bar{\phi}^4W^{(4)}$ quartic vertex. Assume that it involves also $n_2$ vertices with $\bar{\phi}^2(-\frac{1}{4}D^2)K''_{\Phi^2}$ and $n_3$ vertices with $\phi^2W''$. Clearly, there are $n_2 + n_3 + 2$ propagators each of which brings the $\Box^{-1}D^2\tilde{D}^2$ operator. We need to put on the external lines $1 - n_2$ operators $D^2$ to satisfy \((3.3)\). To restore full superspace measure, we need also $n_2 + 1$ operators $D^2$ and $n_3$ operators $\tilde{D}^2$. One $D^2\tilde{D}^2$ operator is eaten by each loop because of the identity \((2.18)\). As a result we are left with $n_2 + n_3 = 2$ operators $D^2$ and $n_2$ operators $\tilde{D}^2$. These numbers should be equal since we cannot put these derivatives on the external lines, $n_2 + n_3 = 2 = n_2$. Hence, the only possibility is the diagram with $n_2 = 0$ and $n_3 = 2$. The corresponding momentum integral has exactly right power of external momenta to apply \((3.3)\),

$$\int \frac{d^3k_1d^3k_2}{k^3_1k^3_2(p + k_1)^2(p + k_2)^2} = \frac{\pi^6}{p^2}.$$

11
As a result, the diagram given at Fig. 2a can contribute to the effective chiral potential.

### 3.2.2 Diagrams of topology $B$

There are two non-chiral vertices $K^{(3)}_{\phi^2\bar{\phi}}\phi^2\bar{\phi}$ and $K^{(3)}_{\bar{\phi}^2\phi}\bar{\phi}^2\phi$ which are integrated in the full superspace, two chiral vertices $\phi^3W^m$, $\phi^3(-\frac{1}{4}D^2)K^{'''}_{\Phi^3}$ and two antichiral ones, $\bar{\phi}^3W^m$, $\bar{\phi}^3(-\frac{1}{4}D^2)K^{'''}_{\Phi^3}$. Hence, there are many different possibilities to construct the two-loop diagrams of topology $B$ with these vertices. Let us analyze all of them.

Consider a diagram with two vertices in the full superspace, either $K^{(3)}_{\phi^2\bar{\phi}}\phi^2\bar{\phi}$ or $K^{(3)}_{\bar{\phi}^2\phi}\bar{\phi}^2\phi$ and with $n_2$ vertices $\bar{\phi}^3(-\frac{1}{4}D^2)K^{'''}_{\Phi^3}$ and with $n_3$ vertices $\phi^2W^m$. From the propagators we have $3 + n_2 + n_3$ operators $\Box^{-1}D^2\bar{D}^2$ out of which we use $n_2$ operators $D^2$ and $n_3$ operators $\bar{D}^2$ to restore full superspace measures. Two operators $D^2\bar{D}^2$ are eaten owing to the identity \(2.18\), one for each loop. And we need to put $1 - n_2$ operators $D^2$ on the external lines to apply \(3.3\). As a result, we are left with $n_2 + n_3$ operators $D^2$ and with $n_2 + 1$ operators $\bar{D}^2$. These numbers should coincide, $n_2 + n_3 = n_2 + 1$, to produce the internal momenta in the corresponding power. Hence, $n_3 = 1$ and $n_2 = 0$ or $n_2 = 1$. For both these values of $n_2$, the momentum integral results in the same power of external momenta as in \(3.14\). But we need one $\Box^{-1}$ after the integration over the loop momenta to apply \(3.3\). Hence, the diagrams with only the full superspace cubic vertices do not contribute to the effective chiral potential.

Consider a diagram with one full superspace cubic vertex $K^{(3)}_{\phi^2\bar{\phi}}\phi^2\bar{\phi}$ or $K^{(3)}_{\bar{\phi}^2\phi}\bar{\phi}^2\phi$ and one chiral cubic vertex $\phi^3W^m$. As contrasted with the previous case, we use one $\bar{D}^2$ operator to restore full superspace measure in the chiral vertex and we are left $n_2 + n_3$ operators $D^2$ and $n_2$ operators $\bar{D}^2$. Hence, $n_3 = 0$ and $n_2 = 0$ or $n_2 = 1$. For both values of $n_2$ the momentum integral has insufficient power of momenta in the denominator to produce $\Box^{-1}$ operator which is necessary for \(3.3\). The cubic chiral vertex $\phi^3(-\frac{1}{4}D^2)K^{'''}_{\Phi^3}$ can be considered in similar lines with the same negative conclusion.

Consider a diagram with one full superspace cubic vertex $K^{(3)}_{\phi^2\bar{\phi}}\phi^2\bar{\phi}$ or $K^{(3)}_{\bar{\phi}^2\phi}\bar{\phi}^2\phi$ and one antichiral cubic vertex $\bar{\phi}^3W^m$. In contrast with the previously considered case, to restore full superspace measure we need the $\bar{D}^2$ operator instead of $D^2$. Hence, after contracting Grassmann loops to points, we are left with $n_2 + n_3 - 1$ operators $D^2$ and $n_2 + 1$ operators $\bar{D}^2$. From the equation $n_2 + n_3 - 1 = n_2 + 1$ we get $n_3 = 2$ and $n_2 = 0$ or $n_2 = 1$. Take, for instance, $n_2 = 0$, then we have five operators $\Box$ in the denominator and one $\Box$ in the nominator owing to the $D$-algebra. The resulting momentum integral gives exactly $\Box^{-1}$ operator on the external lines which is necessary for the identity \(3.3\). The momentum integral gives the same power for $n_2 = 1$. Hence, both these diagrams $b_1$ and $b_3$ at Fig. 2 can contribute to the effective chiral potential. Their calculation will be performed in the next subsection. Note that similar diagram with the antichiral cubic vertex $\bar{\phi}^3(-\frac{1}{4}D^2)K^{'''}_{\Phi^3}$ does not contribute.

Let us analyze the diagrams with purely chiral or antichiral cubic vertices. Take a diagram with two cubic vertices $\phi^3W^m$. Now we need to use two extra $D^2$ operators to restore full superspace measure in these vertices. As a result we are left with $n_2 + n_3$ operators $D^2$ and $n_2 - 1$ operators $\bar{D}^2$. Comparing these numbers, we get $n_3 = -1$ that is impossible. Therefore such diagrams do not contribute to the effective chiral potential.

Take a diagram with one chiral cubic vertex $\phi^3W^m$ and one antichiral one $\bar{\phi}^3W^m$. The corresponding diagram containing two external chiral vertices $\phi^3W^m$ and one antichiral $\bar{\phi}^3W^m$ does not contribute.
After restoring full superspace measures at these vertices we are left with \(n_3 + n_2 - 1\) operators \(D^2\) and \(n_2\) operators \(\bar{D}^2\). Comparing these numbers we get \(n_3 = 1\) and \(n_2 = 1\). The momentum integral yields the external momenta as \(p^{-2}\) and, hence, this diagram may contribute to the effective chiral potential. Such a diagram is given at Fig. 2 \(b_2\).

However, similar diagram with the \(\bar{\phi}^3D^2\) cubic vertex does not contribute.

Consider a diagram with two antichiral vertices \(\bar{\phi}\bar{\phi}\bar{W}''\) which require two extra \(D^2\) operators for restoring full superspace measure. We are left with \(n_2 + n_3 - 2\) operators \(D^2\) and with \(n_2 + 1\) operators \(\bar{D}^2\). Comparing these numbers we see that \(n_3 = 3\) and \(n_2 = 0\) is required since we have only \(\langle \phi\bar{\phi} \rangle\) propagator. However, from six propagators we get six \(\Box^{-1}\) operators, but only one \(\Box\) operator comes from the remaining \(D\)-algebra.

The resulting loop momentum integral can produce \(\Box^{-2}\) instead of \(\Box^{-1}\) which is required for \(E_3\). Hence, these diagrams do not contribute to the effective chiral potential. Note that similar diagram with one \(\bar{\phi}^3(\frac{1}{4}D^2)\bar{K}_{\bar{\phi}\bar{\phi}}^m\) vertex instead of \(\bar{\phi}^3\bar{W}''\) also gives vanishing contribution to the effective chiral potential.

To summarize, we need to compute only the diagrams depicted at Fig. 2 to find the contributions to the effective chiral potential.

### 3.3 Results for two-loop chiral effective potential

Let us denote the contributions to the effective chiral potential from the supergraphs in Fig. 2 as

\[
W_{\text{eff}}^{(2)} = W_a + W_{b_1} + W_{b_2} + W_{b_3}.
\]

Here the subscripts in the rhs label the contributions from the corresponding diagrams.

The computation of these Feynman graphs is a standard routine in supersymmetric quantum field theory [28] with the only feature: In general, such diagrams give non-local contributions to the effective action, but we need to extract from them only the local pieces in the chiral sector which are relevant for the effective chiral potential. This procedure of extracting local parts from Feynman diagrams is usually referred to as the local limit [19, 20, 21, 22]. Essentially, this is the limit when all the external momenta of a Feynman graph are sent to zero. We point out that only those Feynman graphs contribute to the effective chiral potential which are finite in the local limit and for which this limit is unique. All the diagrams which are singular in the local limit should be systematically neglected as they are essentially non-local and do not contain local parts. As is demonstrated in
the Appendix A.2, only the diagram $a$ at Fig. 2 possesses unique and well defined local limit and contributes to the effective chiral potential as

$$W_a = -\frac{1}{512} \frac{W^{(4)}(W'')^2}{(K''_{\Phi \Phi})^4}. \quad (3.17)$$

On the contrary, the diagrams $b_1$, $b_2$ and $b_3$ do not have uniquely defined local parts and, hence, in accordance with the definition of chiral effective potential we have

$$W_{b_1} = W_{b_2} = W_{b_3} = 0. \quad (3.18)$$

All contributions form these diagrams to the effective action are essentially non-local and are out of our considerations.

Note also that for the Wess-Zumino model (1.2) all contributions to the effective chiral potential from the diagrams $b_1$, $b_2$ and $b_3$ vanish identically because $K''_{\Phi \Phi} = K''_{\Phi \Phi} \Phi = 0$ in this case.

4 Effective potentials in the Wess-Zumino model

The computations of the effective Kähler and chiral potentials are done for arbitrary classical Kähler and chiral potentials in the model (1.1). Now we consider an application on these results for the three-dimensional Wess-Zumino model with $K$ and $W$ given in (1.2). For simplicity, we restrict ourself to the case with

$$m = 0, \quad \lambda = 0, \quad (4.1)$$

which corresponds to the scale invariant classical action.

Consider the one- (2.21) and two-loop (2.37) contributions to the effective Kähler potential in the Wess-Zumino model,

$$K_{\text{eff}} = \Phi \bar{\Phi} \left(1 + \frac{g}{8\pi} - \frac{g^2}{192\pi^2} - \frac{g^2}{96\pi^2} \ln \frac{g \Phi \bar{\Phi}}{2\mu} \right). \quad (4.2)$$

The normalization point $\mu$ can be fixed from the condition

$$\left. \frac{\partial^2 K_{\text{eff}}}{\partial \Phi \partial \bar{\Phi}} \right|_{\Phi = \Phi_0, \bar{\Phi} = \bar{\Phi}_0} = 1, \quad (4.3)$$

which implies

$$-\frac{g}{8\pi} + \frac{g^2}{192\pi^2} - \frac{g^2}{192\pi^2} \ln \frac{g \Phi_0 \bar{\Phi}_0}{2\mu} = 0. \quad (4.4)$$

Expressing $\mu$ from this equation and substituting back into (4.2) we get

$$K_{\text{eff}} = \Phi \bar{\Phi} + \frac{g^2}{96\pi^2} \Phi \bar{\Phi} \left(2 - \ln \frac{\Phi \bar{\Phi}}{\Phi_0 \Phi_0} \right). \quad (4.5)$$
Since for the Wess-Zumino model the following derivatives of the classical Kähler potential vanish, $K_{\Phi^2}'' = 0$, $K_{\Phi^2\bar{\Phi}}'' = 0$, only (3.17) contributes to the effective chiral potential,

$$W_{\text{eff}} = \frac{g}{24} \Phi^4 - \frac{g^3}{2048} \Phi^4.$$  

(4.6)

Note that it is finite similarly as in the four-dimensional case [17, 18, 19, 20]. This quantum contribution means simply a finite renormalization of the coupling constant,

$$g \to g' = g(1 - \frac{3}{256} g^2).$$  

(4.7)

Once this shift is performed, all two-loop quantum contributions to the chiral potential are accounted by this coupling $g'$. In the rest of this section we will use $g$ instead of $g'$, for brevity, assuming that it already takes the quantum corrections into account.

Let us study the effective scalar potential induced by the quantum corrections (4.5). Consider only the constant scalar $\varphi$ and auxiliary $F$ fields,

$$\Phi = \varphi + \theta^2 F, \quad \bar{\Phi} = \bar{\varphi} - \bar{\theta}^2 \bar{F}.$$  

(4.8)

Eliminating the auxiliary fields in the classical Wess-Zumino model gives scale-invariant scalar potential,

$$V_{\text{class}} = \frac{g^2}{36} (\varphi \bar{\varphi})^3.$$  

(4.9)

Analogously, elimination of the auxiliary fields in the effective action with the effective Kähler potential (4.5) yields the effective scalar potential,

$$V_{\text{eff}} = \frac{g^2}{36} (\varphi \bar{\varphi})^3 \left(1 + \frac{g^2}{96\pi^2} \ln \frac{\varphi \bar{\varphi}}{\varphi_0 \bar{\varphi}_0} + \text{higher loop corrections}\right).$$  

(4.10)

Qualitatively, this quantum correction to the effective scalar potential (4.10) is similar to the one in the four-dimensional Wess-Zumino model [28].

5 Summary

We studied the two-loop effective action in the three-dimensional general chiral superfield model which is described by the classical Kähler potential $K(\Phi, \bar{\Phi})$ and chiral potential $W(\Phi)$. In general, this model in non-renormalizable, but as a particular case it includes the three-dimensional Wess-Zumino models with $K$ and $W$ given in (1.2) which is renormalizable. We are interested in the contributions to the effective action up to two-derivative order which are described in superspace by the effective Kähler potential $K_{\text{eff}}$ and chiral potential $W_{\text{eff}}$.

At one loop order the effective Kähler potential receives only finite corrections (2.21) while the divergences start from two loops (2.37). Qualitatively, these two-loop results are analogous to the one-loop Kähler potential in the four-dimensional analog of the model (1.1), [21] [22] [23] [24] [25] [26]. In particular, for the three-dimensional Wess-Zumino model this effective Kähler potential reduces to (4.5) which corresponds to the effective
scalar potential (4.10). This quantum deformation of the classical scalar potential in the three-dimensional Wess-Zumino model is very similar to the four-dimensional case [28].

For the four-dimensional chiral superfield model it is well known that the effective chiral potential receives finite quantum corrections only in the massless case [17, 18, 19, 20, 21, 22, 23, 24]. We show that this conclusion is also true for the three-dimensional chiral superfield model under considerations. The two-loop effective chiral potential in general chiral superfield model is given by (3.16) and (3.17). For the particular case of the Wess-Zumino model (1.2), this effective chiral potential reduces to a finite shift of the coupling constant (4.7). It is interesting to note that only the Feynman graph of topology $A$ at Fig. 1 is responsible for this shift while in the four-dimensional case the effective chiral potential originates form the diagram of topology $B$ [28].

A natural extension of the present considerations might by a study of two-loop effective action in three-dimensional $\mathcal{N} = 2$ supersymmetric electrodynamics and SYM theories with matter.

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A Details of two-loop calculations

A.1 Two-loop effective Kähler potential

Let us consider the computation of the contribution (2.29) which originates from the diagram of topology $A$ at Fig. 1. It involves the fourth variational derivative of the classical action,

$$\frac{\delta^4 S}{\delta \Phi(z_1)\delta \Phi(z_2)\delta \Phi(z_3)\delta \Phi(z_4)} = \frac{1}{4} D^2_{(2)}[K^{(4)}_{\Phi^4\Phi^2}(z_3)\delta_+(z_2, z_3)]\delta_-(z_3, z_4)\delta_+(z_1, z_2).$$  \hspace{1cm} (A.1)

Using these delta-functions we integrate over $z_1$ and $z_2$,

$$\Gamma_A = -\frac{1}{8} \int d^3z_2 d^3z_3 D^2_{(2)}[K^{(4)}_{\Phi^4\Phi^2}(z_2)\delta_+(z_2, z_3)]G_-(z_3, z_2)G_+(z_3, z_2).$$  \hspace{1cm} (A.2)

Now restore full superspace measure using (2.16) and integrate over $z_3$,

$$\Gamma_A = \frac{1}{2} \int d^7z K^{(4)}_{\Phi^4\Phi^2}G_-(z, z)G_+(z, z).$$  \hspace{1cm} (A.3)
The propagator $G_{+-}$ is given in (2.25). At coincident points we use the identity $D^2 \bar{D}^2 \delta^4(\theta_1 - \theta_2) |_{\theta_1=\theta_2} = 16$, and compute the momentum integral,

$$G_{+-}(z, z) = \frac{1}{K''_\phi} \frac{1}{\Box + M^2} \delta^3(x_1 - x_2) |_{x_1=x_2}$$

$$= -\frac{1}{K''_\phi} \int \frac{d^3 p}{(2\pi)^2} \frac{1}{p^2 - M^2} = -i \frac{\Gamma}{4\pi K''_\phi}, \quad (A.4)$$

Recall that $M^2 = |W''|^2/(K''_\phi)^2$. As a result,

$$\Gamma_A = -\frac{1}{32\pi^2} \int d^7 z K''_\phi (W'')^2. \quad (A.5)$$

Consider the computation of the contribution (2.30) which comes from the diagram $B$ at Fig. 1. The vertices in this diagram are given in (2.26) and (2.28). We integrate four delta-functions in these vertices and restore full superspace measure using (2.10),

$$\Gamma_B = -\frac{1}{2} \int d^7 z_2 d^5 z_4 K''_\phi (z_2) W''(z_4) G_{++}(z_2, z_4) G_{++}(z_2, z_4). \quad (A.6)$$

Note that, because of $\delta_+(z_1, z_1) = -\frac{1}{2} D^2 \delta^7(z_1 - z_2)$, the propagators $G_{+-}$ contain effectively four Grassmann derivatives while $G_{++}$ only two. We use two Grassmann derivatives to restore full superspace measure in (A.6) and the remaining eight derivatives are necessary for applying the identity (2.18) twice,

$$\Gamma_B = -\frac{1}{2} \int d^4 \theta d^3 x_1 d^3 x_2 \frac{K''_\phi (z_2) W''(z_4)}{(K''_\phi)^4} \left[ \frac{1}{\Box + M^2} \delta^3(x_1 - x_2) \right]^3$$

$$= -\frac{1}{2} \int d^4 \theta d^3 p_1 d^3 p_2 \frac{K''_\phi (z_2) W''(z_4)}{(K''_\phi)^4} \frac{1}{p_1^2 - M^2} \frac{1}{p_2^2 - M^2} \frac{1}{(p_1 + p_2)^2 - M^2} \quad (A.7)$$

Calculating the momentum integral within the dimensional regularization,

$$\int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} \frac{1}{p_1^2 - M^2} \frac{1}{p_2^2 - M^2} \frac{1}{(p_1 + p_2)^2 - M^2} = \frac{\Gamma(\epsilon)}{32\pi^2 M^{2\epsilon}}, \quad (A.8)$$

we get

$$\Gamma_B = \frac{1}{64\pi^2} \int d^7 z \frac{K''_\phi (z_2) W''(z_4) \Gamma(\epsilon)}{(K''_\phi)^4} \frac{\Gamma(\epsilon)}{M^{2\epsilon}}. \quad (A.9)$$

For computing (2.31) we need the variational derivative of the classical action (2.26) and its conjugate. Integrating all the delta-functions in these vertices, we get

$$\Gamma_B = \frac{1}{6} \int d^5 z_1 d^5 z_4 W''(z_4) G'_{++}(z_1, z_4). \quad (A.10)$$

Restoring full superspace measure and applying the identity (2.18) we get

$$\Gamma_B = \frac{1}{6} \int d^7 z_1 d^7 z_2 \frac{W''(z_2)}{(K''_\phi)^3} \left[ \frac{1}{\Box + M^2} \right] \frac{1}{16} \left[ D^2 \bar{D}^2 \delta^7(z_1 - z_2) \right]^2$$

$$= \frac{1}{6} \int d^4 \theta d^3 x_1 d^3 x_2 \frac{W''(z_4)}{(K''_\phi)^3} \left[ \frac{1}{\Box + M^2} \right] \left[ \frac{1}{\Box + M^2} \delta^3(x_1 - x_2) \right]^3. \quad (A.11)$$
Applying the momentum integral (A.8), we obtain

$$\Gamma_{B_2} = -\frac{1}{192\pi^2} \int d^7z \frac{\hat{W}''W''\Gamma(\epsilon)}{(K''_{\Phi\Phi})^3} \frac{1}{M^{2\epsilon}}. \quad \text{(A.12)}$$

The part of the effective action (2.32) involves the variational derivative of the classical action (2.28) and its conjugate. After integrating out all the delta-functions coming from the vertices, we get

$$\Gamma_{B_3} = -\int d^7 z_1 d^7 z_2 K'''_{\Phi\Phi}(z_1)K'''_{\Phi\Phi}(z_2)G_+^+(z_1, z_2)G_−(z_1, z_2)G_−(z_1, z_2) \quad \text{(A.13)}$$

Taking into account the explicit form of the propagators (2.25), we apply the identity (2.18) and the momentum integral (A.8),

$$\Gamma_{B_3} = \int d^4\theta d^3x_1 d^3x_2 K'''_{\Phi\Phi}(z_1)K'''_{\Phi\Phi}(z_2) \frac{W''\hat{W}''}{(K''_{\Phi\Phi})^5} \left[\frac{1}{\Box + M^2} \delta^3(x_1 - x_2)\right]^3 = -\frac{1}{32\pi^2} \int d^7 z K'''_{\Phi\Phi}(z_2)K'''_{\Phi\Phi}(z_2) \frac{W''\hat{W}''}{(K''_{\Phi\Phi})^5} \frac{1}{M^{2\epsilon}}. \quad \text{(A.14)}$$

The expression (2.33) also involves the vertex (2.28) and its conjugate. After integrating out the delta-functions in these vertices we get

$$\Gamma_{B_4} = -\frac{1}{2} \int d^7 z_1 d^7 z_2 K'''_{\Phi\Phi}(z_1)K'''_{\Phi\Phi}(z_2)G_+^+(z_1, z_2)G_−(z_1, z_2)G_−(z_1, z_2) \quad \text{(A.15)}$$

The propagators here have three $D^2$ and three $\bar{D}^2$ operators altogether. We can shrink down two $D^2\bar{D}^2$ pairs owing to the identity (2.18) and the remaining $D^2\bar{D}^2$ operator generates the Dlembertian operator owing to (2.9). As a result, we have

$$\Gamma_{B_4} = -\frac{1}{2} \int d^4\theta d^3x_1 d^3x_2 K'''_{\Phi\Phi}(x_1)K'''_{\Phi\Phi}(x_2) \frac{\Box}{(K''_{\Phi\Phi})^3} \left[\frac{1}{\Box + M^2} \delta^3(x_1 - x_2)\right]^2 \quad \text{(A.16)}$$

The corresponding momentum integral reads

$$\int \frac{d^3p_1 d^3p_2}{(2\pi)^6} \frac{1}{p_1^2 - M^2} \frac{1}{p_2^2 - M^2} \frac{(p_1 + p_2)^2}{(p_1 + p_2)^2 - M^2} = -\frac{M^2}{16\pi^2} + \frac{M^2}{32\pi^2 M^{2\epsilon}}. \quad \text{(A.17)}$$

As a result, we find

$$\Gamma_{B_4} = \frac{1}{32\pi^2} \int d^7 z K'''_{\Phi\Phi}(z_2)K'''_{\Phi\Phi}(z_2) \frac{W''\hat{W}''}{(K''_{\Phi\Phi})^5} - \frac{1}{64\pi^2} \int d^7 z K'''_{\Phi\Phi}(z_2)K'''_{\Phi\Phi}(z_2) \frac{W''\hat{W}''}{(K''_{\Phi\Phi})^5} \frac{1}{M^{2\epsilon}}. \quad \text{(A.18)}$$

Finally, we point out that the expression (2.34) is complex conjugate to (2.30),

$$\Gamma_{B_5} = (\Gamma_{B,1})^* = \frac{1}{64\pi^2} \int d^7 z W''\hat{W}''K'''_{\Phi\Phi} \frac{\Gamma(\epsilon)}{(K''_{\Phi\Phi})^4} \frac{1}{M^{2\epsilon}}. \quad \text{(A.19)}$$
The divergent and finite parts of these actions can be separated with the help of the identity
\[
\frac{\Gamma(\epsilon)}{M^{2\epsilon}} = \frac{1}{\epsilon} - \gamma - \ln M^2 + O(\epsilon),
\]
where \(\gamma\) is the Euler gamma constant. Summing up all obtained expressions \((A.3), (A.9), (A.12), (A.14), (A.18)\) and \((A.19)\), we find the two-loop contribution to the effective Kähler potential,
\[
\Gamma^{(2)} = \Gamma_{\text{div}} + \Gamma_{\text{fin}},
\]
where the propagator \(G\) is given in \((3.13)\). From these propagators, we use two \(\bar{\Phi}^4 W^{(4)}\) and with one chiral vertex \(\Phi^2 W''\) at each loop,
\[
\Gamma_a = -\frac{1}{8} \int d^5 z_1 d^5 z_2 d^5 z_3 W''(z_1) W''(z_2) W^{(4)}(z_3) G_0(z_1, z_3) G_0(z_1, z_3) G_0(z_2, z_3) G_0(z_2, z_3),
\]
and with one chiral vertex \(\Phi^2 W''\) at each loop,
\[
\Gamma_a = \frac{1}{32} \frac{1}{12} \int d^7 z_1 d^7 z_2 d^7 z_3 (D^2 (K''_{\Phi}(z_1))^2) (K''_{\Phi}(z_2))^2 W^{(4)}
\]
\[
\times \frac{1}{\delta^7(z_1 - z_3)} \frac{1}{\delta^7(z_1 - z_3)} \frac{1}{\delta^7(z_2 - z_3)} \frac{1}{\delta^7(z_2 - z_3)}.
\]
We apply the identity \((2.18)\) twice and pass to the momentum space,
\[
\Gamma_a = \frac{1}{32} \int d^4 \theta d^3 p d^3 p d^3 k_1 d^3 k_2 (D^2 (K''_{\Phi}(p))^2) (K''_{\Phi}(-p))^2 W^{(4)}
\]
\[
\times \frac{1}{k_1^2(k_1 + p)^2} \frac{1}{k_2^2(k_2 - p)^2}.
\]
Computing the momentum integrals,
\[
\int \frac{d^3 k}{k^2(p + k)^2} = \frac{\pi^3}{|p|},
\]
and passing back to the coordinate space we find in the local limit,
\[
\Gamma_a = -\frac{1}{32} \frac{1}{64} \int d^7z \frac{W'' \bar{W}'^4 D^2}{(K_{\Phi^2})^2} \frac{W''}{(K_{\Phi^2})^2}.
\] (A.28)

Hence, the corresponding contribution to the effective chiral potential is
\[
\Gamma_a = \frac{1}{512} \int d^5z \bar{W}'^4 (W')^2 \frac{1}{(K_{\Phi^2})^4}.
\] (A.29)

Note that \( \bar{W}'^4 \) is constant here and \( K_{\Phi^2} \) is considered at \( \Phi = 0 \) and therefore it is chiral.

### A.2.2 Diagram \( b_1 \)

In the diagram \( b_1 \) the vertices "1" and "3" are chiral, "4" is the antichiral vertex and "2" is the vertex in the full superspace. Therefore, this diagram corresponds to the following expression
\[
\Gamma_{b_1} = \frac{1}{2} \int d^5z_1 d^5z_2 d^5z_3 d^5z_4 K''_{\Phi^2} (z_2) W''(z_1) W''(z_3) W''
\times G_0(z_1, z_2) G_0(z_1, z_4) G_0(z_3, z_2) G_0(z_3, z_4) G_0(z_2, z_4).
\] (A.30)

We use one \( D^2 \) and two \( \bar{D}^2 \) operators from the propagators to restore the full superspace measure and integrate by parts another two \( D^2 \) operators,
\[
\Gamma_{b_1} = -\frac{1}{8} \frac{1}{16^2} \int d^7x_1 d^7x_2 d^7x_3 d^7x_4 K''_{\Phi^2} (z_2) \bar{W}'^m 1 \square \delta^7(z_1 - z_2) \frac{1}{\square} \delta^7(z_3 - z_2) \frac{1}{\square} \delta^7(z_2 - z_4)
\times D^2 \left( \frac{W''(z_1)}{(K''_{\Phi^2})^2} \right) \bar{D}^2 D^2 \left( \frac{W''(z_3)}{(K''_{\Phi^2})^2} \right) \delta^7(z_3 - z_4).
\] (A.31)

Using the delta-functions we integrate over the Grassmann variables \( \theta_1 \) and \( \theta_3 \) and distribute the \( D \)-operators in the second line in (A.31) keeping only the terms containing no more than two derivatives \( D_\alpha \) on the external lines,
\[
\Gamma_{b_1} = -\frac{1}{8} \frac{1}{16^2} \int d^3x_1 d^3x_2 d^3x_3 d^3x_4 K''_{\Phi^2} (z_2) \bar{W}'^m 1 \square \delta^3(x_1 - x_2) \frac{1}{\square} \delta^3(x_3 - x_2)
\times \frac{1}{\square} \delta^7(z_2 - z_4) [T_1 + T_2 + T_3],
\] (A.32)
where

\[
T_1 = D^2 \left( \frac{W''(x_1, \theta_2)}{(K''_{\Phi\Phi})^2} \right) \frac{D^2 D^2}{\square} (\delta^3(x_1 - x_4)\delta^4(\theta_2 - \theta_4)) \\
\times W''(x_3, \theta_2) D^2 \left( \frac{\delta^3(x_3 - x_4)\delta^4(\theta_2 - \theta_4)}{(K''_{\Phi\Phi})^2} \right),
\]

\[
T_2 = \frac{W''(x_1, \theta_2)}{(K''_{\Phi\Phi})^2} D^2 \left( \delta^3(x_1 - x_4)\delta^4(\theta_2 - \theta_4) \right) \\
\times D^2 \left( \frac{W''(x_3, \theta_2)}{(K''_{\Phi\Phi})^2} \right) \frac{D^2 D^2}{\square} \left( \delta^3(x_3 - x_4)\delta^4(\theta_2 - \theta_4) \right),
\]

\[
T_3 = -4D^\alpha \left( \frac{W''(x_1, \theta_2)}{(K''_{\Phi\Phi})^2} \right) \partial_{\alpha\beta} D^\beta D^2 \delta^3(x_1 - x_2)\delta^4(\theta_2 - \theta_4) \\
\times D^\gamma \left( \frac{W''(x_3, \theta_2)}{(K''_{\Phi\Phi})^2} \right) \partial_{\gamma\delta} D^\delta D^2 \delta^3(x_3 - x_4)\delta^4(\theta_2 - \theta_4). \tag{A.33}
\]

We integrate by parts the operator $D^2$ in the first line of (A.32) and apply the identity (2.13) twice. Then we take another $D^2$ from the full measure which can hit only the external lines and pass to the momentum space,

\[
\Gamma_{b_1} = \frac{1}{2} \int d^3p_1 d^3p_2 d^3\theta \frac{W''(p_1, \theta)W''(p_2, \theta)K''_{\Phi\Phi}(-p_1 - p_2, \theta)\tilde{W}''}{(K''_{\Phi\Phi})^5} S_1(p_1, p_2), \tag{A.34}
\]

where

\[
S_1(p_1, p_2) = \int \frac{d^3k_1 d^3k_2 p_1^2 k_2^2 + k_1^2 p_2^2 - 2(p_1 k_2)(k_1 k_2) + 2(p_1 k_2)(p_2 k_1) - 2(p_1 k_1)(p_2 k_2)}{k_1^4 k_2^4 (p_1 + k_1)^2 (p_2 + k_2)^2 (k_1 + k_2)^2}.
\tag{A.35}
\]

We are interested in the local contributions to the effective action. To take the local limit we make the inverse Fourier transform for the superfields,

\[
\Gamma_{b_1} = -\frac{1}{2} \int d^3\theta d^3x_1 d^3x_2 d^3x_3 \frac{W''(x_1, \theta)W''(x_2, \theta)K''_{\Phi\Phi}(x_3, \theta)\tilde{W}''}{(K''_{\Phi\Phi})^5} \\
\times \int d^3p_1 d^3p_2 S_1(p_1, p_2) e^{ix_1 p_1} e^{ix_2 p_2} e^{-ix_3(p_1 + p_2)}, \tag{A.36}
\]

and assume that the superfields vary slowly in the Minkowski space,

\[
W''(x_1, \theta)W''(x_2, \theta)K''_{\Phi\Phi}(x_3, \theta) \approx W''(x_1, \theta)W''(x_1, \theta)K''_{\Phi\Phi}(x_1, \theta). \tag{A.37}
\]

The integration over $d^3x_2$ and $d^3x_3$ in (A.36) yields two delta-functions, $(2\pi)^3\delta^3(p_2)$ and $(2\pi)^3\delta^3(p_1 + p_2)$. These delta-functions show that we need to compute the limit

\[
S_1 = \lim_{p_1 \to 0, p_2 \to 0} S_1(p_1, p_2). \tag{A.38}
\]

However, in the Appendix B.2 we show that this limit does not exist, i.e., the value of $S_1$ depends essentially on the way of computing this limit. Hence, we conclude that the diagram $b_1$ in Fig. 2 does not give any local contribution to the effective action in the chiral sector.
A.2.3 Diagram $b_2$

This diagram has the chiral vertices “1”, “3” and the antichiral ones “4”, “2”. The corresponding analytic expression reads

$$\Gamma_{b_2} = -\frac{1}{2} \int d^6z_1 d^6z_2 d^6z_3 d^6z_4 W''(z_1) \left( \frac{1}{4} D^2 K''_{\Phi^2}(z_2) \right) W''(z_3) W''(z_4) \times G_0(z_1, z_2) G_0(z_3, z_2) G_0(z_3, z_4) G_0(z_1, z_4).$$

(A.39)

Restore the full superspace measures using the $D^2$-operators from the propagators,

$$\Gamma_{b_2} = -\frac{1}{2 \cdot 16^3} \int d^7z_1 d^7z_2 d^7z_3 d^7z_4 W''(z_1) \left( \frac{1}{4} D^2 K''_{\Phi^2}(z_2) \right) W''(z_3) W''(z_4) \frac{1}{K''_{\Phi^2}} \delta^7(z_1 - z_2) \times \frac{\bar{D}^2 D^2}{K''_{\Phi^2}} \delta^7(z_3 - z_2) \frac{1}{K''_{\Phi^2}} \delta^7(z_3 - z_4) \frac{\bar{D}^2 D^2}{K''_{\Phi^2}} \delta^7(z_1 - z_4).$$

(A.40)

Note that in the first line of this expression we have the operator $D^2$ on the external line $K''_{\Phi^2}$ and, hence, all other terms with derivatives on the external lines can be neglected. This means that upon integration by parts the Grassmann derivatives do not hit the external lines. The operator $\bar{D}^2 D^2$ on the delta-function $\delta^7(z_3 - z_2)$ in the second line of (A.40) can be transported to the delta functions $\delta^7(z_1 - z_4)$ and then it produces the box operator owing to the identity (2.9),

$$\Gamma_{b_2} = -\frac{1}{2 \cdot 16^2} \int d^7z_1 d^7z_2 d^7z_3 d^7z_4 W''(z_1) \left( \frac{1}{4} D^2 K''_{\Phi^2}(z_2) \right) W''(z_3) W''(z_4) \frac{1}{K''_{\Phi^2}^5} \times \frac{\bar{D}^2 D^2}{K''_{\Phi^2}} \delta^7(z_1 - z_2) \frac{1}{K''_{\Phi^2}} \delta^7(z_3 - z_2) \frac{1}{K''_{\Phi^2}} \delta^7(z_3 - z_4) \left( \frac{\bar{D}^2 D^2}{K''_{\Phi^2}} \delta^7(z_1 - z_4) \right).$$

(A.41)

We integrate over all but one Grassmann variables and apply the identity (2.18) twice,

$$\Gamma_{b_2} = -\frac{1}{2} \int d^4\theta d^3 p_1 d^3 p_2 \frac{W''(-p_1 - p_2, \theta) K''_{\Phi^2}(p_1, \theta) W''(p_2, \theta) W''(x_1, \theta) W''(x_2, \theta) W''(x_3, \theta) W''(x_4, \theta) \times \frac{1}{\theta^3(x_1 - x_2)} \frac{1}{\theta^3(x_2 - x_3)} \frac{1}{\theta^3(x_3 - x_4)} \left( \frac{1}{\theta^3(x_1 - x_4)} \right).$$

(A.42)

In this expression we pass to the chiral subspace for the Grassmann variables and to the momentum representation for the Minkowski space coordinates,

$$\Gamma_{b_2} = -\frac{1}{2} \int d^2\theta \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} \frac{W''(-p_1 - p_2, \theta) K''_{\Phi^2}(p_1, \theta) W''(p_2, \theta) W''(x_1, \theta) W''(x_2, \theta) W''(x_3, \theta) W''(x_4, \theta) \times \frac{1}{\theta^3(x_1 - x_2)} \frac{1}{\theta^3(x_2 - x_3)} \frac{1}{\theta^3(x_3 - x_4)} \left( \frac{1}{\theta^3(x_1 - x_4)} \right)}{K''_{\Phi^2}^5} S_2(p_1, p_2),$$

(A.43)

where

$$S_2(p_1, p_2) = \int d^3 k_1 d^3 k_2 \frac{p_1^2}{(2\pi)^6} (p_1 + p_2 + k_1)^2 (p_2 + k_1)^2 (k_1 + k_2)^2 k_2^2.$$

(A.44)

In this function, the integration over $k_2$ can be done using (A.27),

$$S_2(p_1, p_2) = \int d^3 k \frac{p_1^2}{(4\pi)^3} (p_1 + p_2 + k)^2 (p_2 + k)^2 |k|.$$
In principle, this momentum integral can be computed for general values of the external momenta, but we need to find the local limit to single out the contributions to the effective chiral potential. We make the inverse Fourier transform for the superfields in (A.43),

\[ \Gamma_{b_2} = -\frac{1}{2} \int d^2\theta d^3x_1 d^3x_2 d^3x_3 \frac{W'''(x_1, \theta)K'''_{\Phi_2}(x_2, \theta)W''(x_3, \theta)\bar{W}'''}{(K''_{\Phi})^5} \times \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} e^{-ix_1(p_1+p_2)} e^{ix_2p_1} e^{ix_3p_2} S_2(p_1, p_2), \] (A.46)

and consider the slowly varying fields,

\[ W'''(x_1, \theta)K'''_{\Phi_2}(x_2, \theta)W''(x_3, \theta) \approx W'''(x_3, \theta)K'''_{\Phi_2}(x_3, \theta)W''(x_3, \theta). \] (A.47)

The integration over \( d^3x_1 \) and \( d^3x_2 \) yields two delta functions, \((2\pi)^3\delta^3(p_1 + p_2)\) and \((2\pi)^3\delta^3(p_2)\). These delta-functions mean that we need to compute the limit

\[ \lim_{p_1 \to 0, p_2 \to 0} S_2(p_1, p_2). \] (A.48)

In the Appendix [B.1] we show that this limit does not exist, i.e., its value depend on the way of computing this limit. Hence, we conclude that the Feynman graph \( b_2 \) in Fig. 2 does not contribute to the effective chiral potential.

### A.2.4 Diagram \( b_3 \)

This diagram has the vertex \( \text{“}1\text{“} \) in the full superspace, the vertices \( \text{“}2\text{“} \) and \( \text{“}4\text{“} \) in the chiral subspace while \( \text{“}3\text{“} \) and \( \text{“}5\text{“} \) are antichiral vertices. The corresponding expression reads

\[ \Gamma_{b_3} = \frac{1}{2} \int d^7z_1 d^7z_2 d^7\bar{z}_3 d^7z_4 d^7z_5 K'''_{\Phi_2}(z_1)W''(z_2)(\frac{1}{4} D^2K''_{\Phi_2}(z_3))W''(z_4)\bar{W}'' \times G_0(z_2, z_1)G_0(z_2, z_3)G_0(z_4, z_3)G_0(z_4, z_5)G_0(z_1, z_5)G_0(z_1, z_5). \] (A.49)

As the first step, we restore the full superspace measure in (A.49) by taking the Grassmann derivatives from the propagators,

\[ \Gamma_{b_3} = \frac{1}{2} \frac{1}{16^4} \int d^7z_1 d^7z_2 d^7\bar{z}_3 d^7z_4 d^7z_5 K'''_{\Phi_2}(z_1)W''(z_2)(\frac{1}{4} D^2K''_{\Phi_2}(z_3))W''(z_4)\bar{W}'' \times \frac{\bar{D}^2 D^2}{K''_{\Phi} \delta^7(z_2 - z_1)} \frac{1}{K''_{\Phi} \delta^7(z_2 - z_3)} \frac{\bar{D}^2 D^2}{K''_{\Phi} \delta^7(z_4 - z_3)} \frac{1}{K''_{\Phi} \delta^7(z_4 - z_5)} \times \frac{\bar{D}^2 D^2}{K''_{\Phi} \delta^7(z_1 - z_5)} \frac{\bar{D}^2 D^2}{K''_{\Phi} \delta^7(z_1 - z_5)}. \] (A.50)

In the first line here we have the expression \( \frac{1}{4} D^2K''_{\Phi} \) which shows that we have sufficient number of Grassmann derivatives on the external lines to apply the identity (3.3). Hence, upon integration by parts of the other Grassmann derivatives we can omit all the terms in which these derivatives hit the external lines. For instance, in the second line of (A.50) we
integrate by parts the derivatives $\bar{D}^2 D^2$ acting on $\delta^7(z_4 - z_3)$ such that they hit $\delta^7(z_2 - z_1)$ and cancel one box operator,

$$\Gamma_{b_3} = \frac{1}{2} \frac{1}{16^3} \int d^7 z_1 d^7 z_2 d^7 z_3 d^7 z_4 d^7 z_5 K_{\Phi^* \Phi}(z_1) W''(z_2) \left( \frac{1}{4} D^2 K_{\Phi^* \Phi}(z_3) \right) W''(z_4) \frac{1}{(K_{\Phi^* \Phi})^6}$$

$$\times \bar{D}^2 D^2 \delta^7(z_2 - z_1) \frac{1}{\Box} \delta^7(z_2 - z_3) \frac{1}{\Box} \delta^7(z_4 - z_3) \frac{1}{\Box} \delta^7(z_4 - z_5)$$

$$\times \bar{D}^2 D^2 \delta^7(z_1 - z_5) \frac{1}{\Box} \delta^7(z_1 - z_5). \tag{A.51}$$

In a similar way we integrate by parts the derivatives $\bar{D}^2 D^2$ in the second line of (A.51) such that they hit the delta functions $\delta^7(z_1 - z_5)$ and yield one more box operator,

$$\Gamma_{b_3} = \frac{1}{2} \frac{1}{16^3} \int d^7 z_1 d^7 z_2 d^7 z_3 d^7 z_4 K_{\Phi^* \Phi}(z_1) W''(z_2) \left( \frac{1}{4} D^2 K_{\Phi^* \Phi}(z_3) \right) W''(z_4) \frac{1}{(K_{\Phi^* \Phi})^6}$$

$$\times \frac{1}{\Box} \delta^7(z_1 - z_2) \frac{1}{\Box} \delta^7(z_3 - z_2) \frac{1}{\Box} \delta^7(z_3 - z_4) \frac{1}{\Box} \delta^7(z_1 - z_4) \left( \frac{\bar{D}^2 D^2}{\Box} \delta^7(z_1 - z_4) \frac{\bar{D}^2 D^2}{\Box} \delta^7(z_1 - z_4) \right). \tag{A.52}$$

Now we apply the identity (2.18) two times, integrate all but one Grassmann variables and pass to the momentum space for the Minkowski space coordinates,

$$\Gamma_{b_3} = \frac{1}{2} \int d^3 \theta \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} \frac{K_{\Phi^* \Phi}(p_1 - p_2, \theta) W''(p_1 - p_2, \theta) K_{\Phi^* \Phi}(p_1, \theta) W''(p_2, \theta) \frac{1}{(K_{\Phi^* \Phi})^6}}$$

$$\times S_2(p_1, p_2), \tag{A.53}$$

where the function $S_2(p_1, p_2)$ is given by the momentum integral (A.45). As is proved in the Appendix B.1, this function does not have the local limit (B.12). Hence, we conclude that the diagram $b_3$ does not contribute to the effective chiral potential.

## B Momentum integrals

### B.1 Local limit for the momentum integral (A.44)

It is hard to compute the momentum integral (A.44) for arbitrary values of external momenta $p_1$ and $p_2$. In fact, we need only the local limit (A.48) for this integral. Our aim is to prove that this function $S_2(p_1, p_2)$ does not possess well defined local limit and, hence, does not contribute to the effective chiral potential.

We are going to demonstrate that the limit (A.48) depends essentially on the path tending to the origin of the $(p_1, p_2)$ space. Hence, choosing different ways of computing this limit, the expression (A.48) can be given an arbitrary value, including zero and infinity. In fact, it is sufficient to compute (A.44) for $p_2 = tp_1 \equiv tp$, with some real parameter $t$, and to show that the momentum integral (A.44) acquires different values for different $t$, with arbitrary small $p$.

Let us represent (A.44) as

$$S_2(p_1, p_2) = p_1^2 S(p_1, p_2), \tag{B.1}$$
where
\[ S(p_1, p_2) = \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{1}{(p_1 + p_2 + k_1)^2(p_2 + k_1)^2(k_1 + k_2)^2k_2^2}. \] (B.2)

Using (A.27) we integrate over \( k_2 \),

\[ S(p_1, p_2) = \int \frac{d^3k}{(4\pi)^3} \frac{1}{(p_1 + p_2 + k)^2|k|}. \] (B.3)

Introducing the Feynman parameters we merge together the propagators,

\[ S(p_1, p_2) = \frac{3}{4} \int \frac{d^3k}{(4\pi)^3} \int_0^1 \frac{d\alpha}{\sqrt{\alpha}} \int_0^{1-\alpha} d\beta \frac{1}{[\alpha k^2 + \beta(p_1 + p_2 + k)^2 + (1 - \alpha - \beta)(p_2 + k)^2]^{3/2}} \] (B.4)

where
\[ P = k^2 + 2k(\beta(p_1 + p_2) + (1 - \alpha - \beta)p_2) + \beta(p_1 + p_2)^2 + (1 - \alpha - \beta)p_2^2. \] (B.5)

Using the general formula \[32\]
\[ \int \frac{d^3k}{(k^2 + 2k \cdot Q - M^2)^\alpha} = \frac{i\pi^{3/2}(-1)^{-\alpha}\Gamma(\alpha - 3/2)}{\Gamma(\alpha)(M^2 + Q^2)^{\alpha - 3/2}}, \] (B.6)

we compute the integral over momenta,

\[ S(p_1, p_2) = \frac{1}{64\pi^2} \int_0^1 \frac{d\alpha}{\sqrt{\alpha}} \int_0^{1-\alpha} d\beta \frac{1}{\beta(\beta - 1)p_1^2 + \alpha(\alpha - 1)p_2^2 - 2\alpha(1 - \alpha)p_1 \cdot p_2}. \] (B.7)

The integration over \( \beta \) can be done readily,

\[ S(p_1, p_2) = \frac{1}{64\pi^2 p_1^2} \int_0^1 \frac{d\alpha}{\sqrt{\alpha}} 2M \ln \left| \frac{M - 1/2 + \alpha Y + \alpha M - 1/2 - \alpha Y}{M + 1/2 - \alpha Y - \alpha M + 1/2 + \alpha Y} \right|, \] (B.8)

where
\[ X = \frac{p_2^2}{p_1^2}, \quad Y = \frac{(p_1 \cdot p_2)}{p_1^2}, \quad M = \sqrt{(1/2 + \alpha Y)^2 + \alpha(1 - \alpha)X}. \] (B.9)

The integral over the remaining parameter \( \alpha \) is very complicated. To simplify this, we consider the collinear momenta, \( p_2 = tp_1 \equiv tp \). In this case \( X = t^2, Y = t, M^2 = \frac{1}{4} + \alpha t(t + 1) \), and the integral (B.8) simplifies,

\[ S_2(p, tp) = \frac{1}{32\pi^2 \sqrt{t(t + 1)}} \int_{1/2}^{t+1/2} \frac{dM}{\sqrt{M - 1/2} \sqrt{M + 1/2}} \ln \left| \frac{M - 1/2 + \eta_t}{M + 1/2 - \eta_t} \right|, \] (B.10)

The value of this integral can be expressed in terms of the Euler di-logarithm function,

\[ S_2(t) \equiv S_2(p, tp) = \frac{1}{16\pi^2 \sqrt{t(t + 1)}} \left[ \mathrm{Li}_2 \left( \frac{\sqrt{1 + t - \sqrt{t}}}{\sqrt{1 + t + \sqrt{t}}} \right) - \frac{\pi^2}{4} \right]. \] (B.11)
The function $S_2(t)$ is well defined for $0 < t < \infty$. Here we see that all momenta cancelled out and the remaining function depends only on the parameter $t$ which relates the momenta. Hence, the value of $S_2(p, tp)$ remains the same for arbitrary small momenta, $p \to 0$, but depends essentially on $t$. As a result, along the path $p_2 = tp_1$ the limit (A.48) depends essentially on the parameter $t$ and it can take arbitrary value allowed for the function (B.11). This proves that

$$\lim_{p_1 \to 0, p_2 \to 0} S_2(p_1, p_2) \text{ does not exist.} \quad \text{(B.12)}$$

### B.2 Local limit for the momentum integral (A.35)

Similarly as in the previous subsection we are going to prove that

$$\lim_{p_1 \to 0, p_2 \to 0} S_1(p_1, p_2) \text{ does not exist.} \quad \text{(B.13)}$$

To show this it is sufficient to compute (A.35) for collinear momenta, $p_1 = tp_2 \equiv tp$ for which it simplifies,

$$S_1(tp, p) = p^2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{t^2k_2^2 + k_1^2 - 2t(k_1k_2)}{k_1^2k_2^2(tp + k_1)^2(p + k_2)^2(k_1 + k_2)^2} = I_1 + I_2 + I_3, \quad \text{(B.14)}$$

where

$$I_1 = p^2 t(t + 1) \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{1}{k_1^2(tp + k_1)^2(p + k_2)^2(k_1 + k_2)^2}, \quad \text{(B.15)}$$

$$I_2 = p^2 (t + 1) \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{1}{k_2^2(tp + k_1)^2(p + k_2)^2(k_1 + k_2)^2}, \quad \text{(B.16)}$$

$$I_3 = -tp^2 \int \frac{d^3k_1 d^3k_2}{(2\pi)^6} \frac{1}{k_1^2k_2^2(tp + k_1)^2(p + k_2)^2}. \quad \text{(B.17)}$$

It is easy to see that the last integral is just a constant,

$$I_3 = -\frac{1}{64}, \quad \text{(B.18)}$$

where (A.27) has been used. For the remaining two integrals, the integration over one of the momenta can be done readily,

$$I_1(t) = p^2 t(t + 1) \int \frac{d^3k}{(4\pi)^3} \frac{1}{|k|(p + k)^2(k + p(t + 1))^2}, \quad \text{(B.19)}$$

$$I_2(t) = p^2 (t + 1) \int \frac{d^3k}{(4\pi)^3} \frac{1}{|k|(tp + k)^2(k + p(t + 1))^2} = I_1(1/t). \quad \text{(B.20)}$$

This integrals can be calculated using (B.8) and (B.11),

$$I_1 = \frac{\sqrt{t(t + 1)}}{16\pi^2} \left[ \text{Li}_2 \frac{\sqrt{1 + t} - \sqrt{t}}{\sqrt{1 + t} + \sqrt{t}} - \text{Li}_2 \left( -\frac{\sqrt{1 + t} - \sqrt{t}}{\sqrt{1 + t} + \sqrt{t}} \right) - \frac{\pi^2}{4} \right], \quad \text{(B.21)}$$

$$I_2 = \frac{\sqrt{t + 1}}{16\pi^2 t} \left[ \text{Li}_2 \frac{\sqrt{1 + t} - 1}{\sqrt{1 + t} + 1} - \text{Li}_2 \left( -\frac{\sqrt{1 + t} - 1}{\sqrt{1 + t} + 1} \right) - \frac{\pi^2}{4} \right]. \quad \text{(B.22)}$$
As a result, the value of the integral (B.14) is given by the sum of (B.18), (B.21) and (B.22). It depends only on the parameter \( t \) rather than on the momentum \( p \). Hence, its value remains the same for arbitrary small momentum \( p \), but it changes upon varying the parameter \( t \). This proves (B.13).

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