Harmonic expressions for the eigen mode fields of step and graded index fibers

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Abstract. For clarifying the mode field characteristics of step and graded index fibers, the Helmholtz’s equation and local planar wave approximation are engaged, a novel idea that the eigenfunction of the traveling wave field in radial grade index dielectric is similar to positive or negative half order Hankel function is advanced. Then, the eigenfunction of core layer of graded index fiber is similar to the combination of half order Bessel function and half order Neumann function, the eigenfunction of cladding layer of graded index fiber is similar to the first kind half order imaginary parameter Hankel function, are suggested. The phase shift for total reflection effect at the turning point of grade fiber is proposed. The angular phase shift π for the ray propagating along the fiber circumference is given. The eigen equation of new improved WKB method for analysing the mode field characteristics of graded index fiber is presented. Moreover, the eigen equation of new improved WKB method for analyzing the mode field characteristics of step index fiber too, it indicates that, the eigen equation of new improved WKB method is of universal significance.

1. Introduction
The fiber mode field theory is established creatively by Snitzer E in 1961 [1], and its simple description in weakly guiding approximation is described by Gloge D in 1971 [2]. In the classical theoretic system, the integral order Bessel function is the eigenfunction of fiber core layer, the integral order imaginary parameter Hankel function is the eigenfunction of fiber cladding layer, the hypothesis of false total reflection or caustic surface in fiber core layer is used for explaining the phase shifts π in the ray method of fiber characteristic analysis [3, 4]. A verdict of mode field theory for step index fiber appears to be formed [1-12]. But some questions of the fiber characteristics are presented in some literatures [13-16], several new ideas are presented for clarifying the essence of fiber, the theories of the electromagnetic adjoint transformation and the Sommerfeld’s formula for spherical wave are brought forward to describe the characteristics of step index fiber by Yu S M et al [13, 14], the zero and first order Bessel functions, Neumann functions which modified by Dirac function and the imaginary parameter Hankel functions are put forward to describe the eigenfunctions of mode field distribution in step index fiber, the combination of half order Bessel function and half order Neumann function, the first kind half order imaginary parameter Hankel function are advanced to rewrite the eigenfunctions of mode field distribution in step index fiber by Guo F Y et al recently [15, 16].
For analysing the characteristics of graded index fiber, the classical and modified WKB methods are applied to analyse the mode field distribution and propagation constant frequently [12, 17-23]. The hypothesis of false total reflection in the core layer is engaged, the phase shifts of the inner knee point and outer turn point are equal to $\pi/2$ is adopted, and the angular phase shift $\pi$ isn’t explained. However, in the graded index planar waveguide domain, the calculating precision of the propagation constant by the classical WKB method is doubted by Cao Z Q et al, and the transfer matrix method was used for improving the calculating precision of the propagation constant [24-26], nevertheless, the physical significance of WKB method is unaccountability yet [26]. As a matter of fact, the classical WKB method, which stemmed from the solution of the Schrödinger equation in the quantum mechanics domain [27], is fit for solving the scalar issue only. As the light wave in the graded index dielectric is vector wave, if the classical WKB method is engaged to analyse the travelling wave in graded index dielectric, some questions would arise at the turning point, so, a new improved WKB method for analysing the symmetrical planar waveguide is suggested by Guo F Y et al lately [28, 29].

As a result of that the local planar wave approximation [3] is engaged to analyse the travelling wave in graded index dielectric, a novel expression of the travelling wave field in radial grade index dielectric is suggested. In succession, a new improved WKB method for analysing the mode field characteristics of the graded index fiber is presented. Moreover, when the index profile of grade index dielectric degenerate into step form, the eigen equation of new improved WKB method for solving the characteristics of grade index fiber is effective for solving the characteristics of step index fiber also.

2. The travelling wave in radial graded index dielectric

In the radial graded index dielectric, the refractive index $n$ change with the radial coordinate $r$ solely, the schematic of the TE travelling wave in the meridian plane of the radial graded index dielectric is shown in figure 1, and the schematic of the TM travelling wave is as the same form as the TE wave, but interchange the symbols of the electric field $E$ and the magnetic field $H$.

![Figure 1. The travelling wave in the meridian plane of radial graded index dielectric.](image)

In general, the electric field intensity of travelling wave is expressed by the electric components $E_x$, $E_y$ and $E_z$, the magnetic field intensity of travelling wave is expressed by the magnetic components $H_x$, $H_y$ and $H_z$, they are the solutions of the Helmholtz’s equation [7, 8], and they are the functions of radial coordinate $r$, angular coordinate $\varphi$ and axial coordinate $z$ in cylindrical coordinate system, and time parameter $t$. Based on the Helmholtz’s equation and the method of separation of variables, the expressions of electric components and magnetic components are expressed by the identical form $\Psi(r, \varphi, z, t)$, it is suggested as follows:

$$\Psi(r, \varphi, z, t) = C \eta_j(r) R(r) \exp(\mp i m \varphi) \exp(\pm ik_z z - i\omega t)$$  \hspace{0.5cm} (1)

where, $C$ is a constant, $\eta_j(r)$ is a coefficient, subscripts $j=E, H$, here, $j=E$ labels for the coefficient of electric components $E_x$, $E_y$ and $E_z$, $\eta_j(r)=1, j=H$ labels for the coefficient of is used for magnetic components $H_x$, $H_y$ and $H_z$, $\eta_j(r)=k_0 \eta_j(r)/(\omega \mu_0)$, respectively. $k_0$ is the wave number of the light wave with wavelength $\lambda_0$ in vacuum, $k_0=2\pi/\lambda_0$, $m(r)$ is the distribution function of refractive index in radial graded index dielectric, $\omega$ is the angular frequency of light wave, $\mu_0$ is the permeability of light wave in vacuum, $i$ is the square root of $-1$, namely, $i=(−1)^{1/2}$, $m$ is the order number of angular parameter, $k_z$ is the component of wave number $k$ in the $z$-axis direction, $k=k_0 \eta_j(r)$, $R(r)$ is radial function.
As the Helmholtz’s equation and equation (1) are engaged, the radial function $R(r)$ satisfies the following equation approximately:

$$\frac{d^2[R(r)r^{1/2}]}{dr^2} + Q(r)R(r)r^{1/2} = 0$$

(2)

where, $Q(r)$ is a parameter.

If the distribution function of the refractive index in radial graded index dielectric $n(r)$ is slowly varying function, $dn/dr\to 0$, the parameter $Q(r)$ is a slowly varying real function also, it is given by the approximate expression [21], as follows:

$$Q(r) = k_0^2n^2(r) - \beta^2 - \frac{m^2 - 1/4}{r^2}$$

(3)

According to the classical WKB theory [21], the approximate eigenfunction of the traveling wave field in radial grade index dielectric is expressed as follows:

$$\Psi(r,\varphi,z,t) = \frac{C}{\sqrt{Q(r)}} \frac{1}{\sqrt{r}} \exp \left[ \pm i \int_0^r \sqrt{Q(\xi)} d\xi \right] \exp(\pm im\varphi) \exp(\pm ik_z z - i\omega t)$$

(4)

where, $m$ is integer, i.e. $m=0, 1, 2, \ldots$.

In the classical WKB theory [21], the eigenfunction of grade index fiber core layer is expressed by the standing wave form of equation (4), and the eigenfunction of grade index fiber cladding layer is expressed by the evanescent wave form of equation (4).

From equation (3), for a given radial coordinate $r$, the parameter $Q(r)$ may tend to zero, it means that, the amplitude of electromagnetic component $\Psi(r,\varphi,z,t)$ in equation (4) may close to the infinity, it breaks the law of energy conversation, so, equation (4) is a false expression of the electromagnet field distribution of traveling wave in the radial graded index dielectric.

For the expression of field distribution of traveling wave in the radial graded index dielectric is significative, the local planar wave approximation is employed, namely, the light wave in the fraction of the radial graded index dielectric is deemed as local planar wave [3] approximately, and the novel idea that the order number $m$ is the positive or negative half is suggested, i.e. $m=\pm 1/2$, then, the parameter $Q(r)$ is taken on a clear physical significance, it is the square of the component of the wave number $k$ in the radial direction, namely, $Q(r) = k_0^2 n^2(r)$, and, the component $k_0(r)$ is expressed as following equation:

$$k_0(r) = \sqrt{k_0^2n^2(r) - k_z^2}$$

(5)

As radiant energy of traveling wave in the radial graded index dielectric is in need of satisfying the energy conversation law, from equation (2), equation (3) and $m=\pm 1/2$, the radial function $R(r)$ is described by the harmonic expression approximately, it is suggested as follows:

$$R(r) = \frac{1}{\sqrt{k_0n(r)r}} \exp \left[ \pm i \int_0^r \sqrt{k_0^2n^2(\xi) - k_z^2} d\xi \right]$$

(6)

Then, the eigenfunction of the traveling wave field in radial grade index dielectric is suggested as harmonic expression approximately [29]:

$$\Psi(r,\varphi,z,t) = \frac{C\eta_j(r)}{\sqrt{k_0n(r)r}} \exp \left[ \pm i \int_0^r \sqrt{k_0^2n^2(\xi) - k_z^2} d\xi \right] \exp(\pm i\theta_0) \exp(\pm ik_z z - i\omega t)$$

(7)

According to the characteristics of the Hankel functions, the first kind and second kind positive half order Hankel functions are expressed by: $H_{1/2}^{(1,2)}(x) = \pm (-i)[2/(\pi x)]^{1/2}\exp(\pm ix)$, the first kind and second
3. A new improved WKB method for the radial graded index fiber

A given graded index fiber for example, the schematic of refractive index profile of fiber is shown as solid line in figure 2. In figure 2, \( n_1 \) is the maximal refractive index of fiber core layer for \( r=0 \), \( n_2 \) is the minimal refractive index of fiber cladding layer for \( r\to\infty \), \( \beta \) is the propagation constant of the fiber, namely, it is the component of wave number \( k \) in the \( z \)-axis direction, \( \beta=\kappa n_{eff} \), here, \( n_{eff} \) is the equivalent index of fiber, \( a \) is the coordinate of turning point, \( a_m \) is the potential ultimate coordinate of turning point.

![Figure 2. Schematic of the refractive index profile of grade index and step index fibers](image)

In the graded index fiber, the standing wave is affirmed for the condition \( k_0 n(r)\geq\beta \), and evanescent wave is validated for the condition \( k_0 n(r)\leq\beta \) also. According to characteristics of the Bessel functions, Neumann functions and first kind imaginary parameter Hankel functions, the positive and negative half order Bessel functions are expressed by: \( J_{1/2}(x)=[2/(\pi x)]^{1/2}\sin(x) \) and \( J_{-1/2}(x)=[2/(\pi x)]^{1/2}\cos(x) \), the positive and negative half order Neumann functions are expressed by: \( Y_{1/2}(x)=[2/(\pi x)]^{1/2}\cos(x) \) and \( Y_{-1/2}(x)=[2/(\pi x)]^{1/2}\sin(x) \), the positive and negative half order first kind imaginary parameter Hankel functions are expressed by: \( K_{1/2}(x)=[2/(\pi x)]^{1/2}\exp(-x) \), and from equation (7), the eigenfunction of core layer of graded index fiber is similar to the combination of half order Bessel function and half order Neumann function, the eigenfunction of cladding layer of graded index fiber is similar to the first kind half order imaginary parameter Hankel function.

If the distribution function of the refractive index in graded index fiber \( n(r) \) is slowly varying function, the difference between maximal refractive index \( n_1 \) and minimal refractive index \( n_2 \) is far less than minimal refractive index \( n_2 \), namely, \( n_1-n_2<<n_2 \), then, the weakly guiding approximation is engaged, and the scalar linear polarization (LP) mode theory of step index fiber [7, 8] is used for reference, the electromagnetic components of fiber core layer is equal to the electromagnetic components of fiber cladding layer in the turning point \( r=a \) with \( k_0 n(r)=\beta \), so, the transverse eigen field distribution of core layer of graded index fiber \( \Psi_1(r,\varphi,z,t) \) and the transverse eigen field distribution of cladding layer of graded index fiber \( \Psi_2(r,\varphi,z,t) \) are expressed by following expressions approximately [29]:

\[
\Psi_1(r,\varphi,z,t) = \frac{C_1}{\sqrt{k_0 n(r)r}} \cos \left[ \int_0^r \sqrt{\gamma_n^2 - \beta^2} \, d\xi - \varphi_n \right] \exp(\pm i \frac{\varphi}{2}) \exp(\pm i k_0 \xi - i \omega t) \tag{8}
\]

\[
\Psi_2(r,\varphi,z,t) = \frac{C_2}{\sqrt{k_0 n(r)r}} \exp \left[ - \int_0^r \sqrt{\beta^2 - k_0^2 n^2(\xi)} \, d\xi \right] \exp(\pm i \frac{\varphi}{2}) \exp(\pm i k_0 \xi - i \omega t) \tag{9}
\]

where, \( \varphi_n \) is the initial phase of eigen field distribution for the \( N \) order mode of graded index fiber in fiber center (\( z \) axis), \( N \) is the mode order of graded index fiber, \( U \) is the normalized standing wave parameter of graded index fiber.
For $N$ order mode of graded index fiber, the initial phase of transverse eigen field distribution in fiber center $\phi_N$ is suggested as follows [16, 29]:

$$\phi_N = (2N + 1)\pi/4$$  \hspace{1cm} (10)

where, $N$ is integer, i.e. $N=0, 1, 2, \ldots$.

As the concept of the normalized standing wave parameter of the step index fiber [7, 8] is used for reference, the normalized standing wave parameter of the graded index fiber [29] is suggested by following equation:

$$U = \int_0^a \sqrt{k_n^2n^2(r) - \beta^2} \, dr$$  \hspace{1cm} (11)

In the interest of deducing the eigen equation of the WKB method, another equation is in need of establishing. As an example, $\Psi_1(r, \varnothing, z, t)$ and $\Psi_2(r, \varnothing, z, t)$ represent the transverse electric components $E_y^1$ and $E_y^2$ respectively, then, the longitudinal electric components of fiber core layer is equal to the longitudinal electric components of fiber core layer, namely, equation $E_z^1=E_z^2$, is a condition of deducing the eigen equation, According to the Maxwell’s equations, the equation $\nabla \cdot E=0$ is engaged, a special equation is presented as follows:

$$\sqrt{\beta^2 - k_n^2n^2(r)} = \sqrt{k_n^2n^2(r) - \beta^2} \tan \left[ \int_0^a \sqrt{k_n^2n^2(r) - \beta^2} \, dr - \phi_N \right].$$  \hspace{1cm} (12)

But, this equation can not work well.

According to the classical WKB method, the eigen equation for analysing the radial graded index fiber is expressed by:

$$4\int_0^a \sqrt{k_n^2n^2(r) - \beta^2} \, dr + 2\Phi(a) = 2N\pi$$  \hspace{1cm} (13)

where, $\Phi(a)$ is the phase shift for total reflection effect at the turning point of the graded index fiber.

The value of phase shift $\Phi(a)$ is calculated by the formula of the Goos-Hänchen shift [30] with the reference points $r=a-\tau$ and $r=a+\tau$. As the parameter $\tau \to 0$ is considered in the classical WKB method, the classical value of the phase shift in turn point $\Phi(a)$ is $\pi/2$, i.e., $\Phi(a)=\pi/2$. If the angular phase shift $\pi$ is ignored, namely, the $\phi_N=\pi/2$ is a hypothesis for equation (12), then, equation (12) is consistent with equation (13). Whereas, the value of phase shift $\Phi(a)$ for planar graded index waveguide is doubted by Cao Z Q et al [24-26] and Guo F Y et al [28], namely, the phase shift $\Phi(a)=\pi/2$ can not work well.

As equation (8) and equation (9) are the exact solutions of the Helmholtz’s equation for the points $r=0$ and $r \to \infty$ with $dn(r)/dr=0$, and approximate solutions for others points with $dn(r)/dr \neq 0$ merely, so, in the new improved WKB method for analysing the characteristics of radial graded index fiber, the reference points for calculating phase shift $\Phi(a)$ are suggested as $r=0$ and $r \to \infty$, then, the approximate computing formula of phase shift for total reflection effect at the turning point is recommended as follows [29]:

$$\Phi(a) = -2\arctan \left[ \sqrt{\beta^2 - k_n^2n_2^2} / \sqrt{k_n^2n_1^2 - \beta^2} \right]$$  \hspace{1cm} (14)

As the ray method of fiber characteristic analysis [3, 4] is used for reference, the angular phase shift which caused by the light ray propagating along the circumference is considered, the eigen equation of new improved WKB method for analysing the characteristics of graded index fiber is suggested by following equation:

$$4\int_0^a k_n(r) \, dr + 2\Phi(a) + \Phi_p = 2N\pi,$$  \hspace{1cm} (15)
In above equation, the angular phase shift which caused by the light ray propagating along the circumference $\Phi_\phi$ is calculated by:

$$\Phi_\phi = \int_0^{2\pi} k_\phi r d\phi$$  \hspace{1cm} (16)$$

where, $k_\phi$ is the component of the wave number $k$ in the angular direction.

As $k_\phi=m/r$ and $m=\pm 1/2$ are employed, then, $\Phi_\phi=\pm \pi$. This implies that the value of phase shift which caused by the ray propagating along the circumference [29] is $\pm \pi$.

From equation (14) to equation (16), the approximate eigen equation of new improved WKB method [29] for analysing the graded index fiber is suggested as follows,

$$\sqrt{\beta^2 - k_\phi^2 n_z^2} = \sqrt{k_0^2 n_1^2 - \beta^2} \tan \left[ \int_0^\infty \sqrt{k_0^2 n_1^2(r) - \beta^2} dr - \phi_N \right]$$  \hspace{1cm} (17)$$

The normalized frequency $V$, it is called the normalized structure parameter also, is one of the main parameters of fiber. For clarifying the normalized meaning of the normalized parameters of the radial graded index fiber, the concepts of the normalized frequency of the step index fiber [7, 8] is used for reference. As the value of normalized frequency $V$ is the ultimate value of normalized standing wave parameter $U$, from equation (11), the normalized frequency of the graded index fiber [29] is suggested by following equation:

$$V = k_0 \int_0^\infty \sqrt{n^2(r) - n_z^2} dr$$  \hspace{1cm} (18)$$

From equation (17) and equation (18), the condition for $N$ order mode appeared in the grade index fiber [29] is commended as follows:

$$V \geq (2N+1)\pi/4$$  \hspace{1cm} (19)$$

Namely, the cut-off normalized frequency of $N$ order mode of graded index fiber is $(2N+1)\pi/4$. It means that, the value of cut-off normalized frequency of nonzero order of graded index fiber is close to the corresponding value of the LP mode of step index fiber [29], but the cut-off normalized frequency of fundamental mode of graded index fiber is $\pi/4$, only the value of $\pi/4$ is an inconvincible solution, it is in need of making further discussion.

As the weakly guiding approximation is engaged for analysing the characteristics of radial graded index fiber, from equation (19), the mode number of multimode graded index fiber [29] is suggested as follows:

$$M = \text{Round}(2V/\pi + 1/2)$$  \hspace{1cm} (20)$$

where, Round is the function of round off operation.

4. New expressions for analysing the homogeneous dielectric and step index fiber

In the homogeneous dielectric, the distribution function of refractive index $n(r)$ is a constant, $n(r)=n_0$, as the Helmholtz’s equation and the method of separation of variables are engaged, equation (2) is transformed to the Bessel’s equation, the exact solutions of this equation are the first kind or second kind Hankel functions, i.e. the radial function $R(r)=H^{(1,2)}_m(k,r)$, here, $m$ is the order number.

In the classical fiber theory, the order number $m$ is integer, the integral order Bessel function and integral order first kind imaginary parameter Hankel function are selected as the eigenfunctions of core layer and cladding layer of step index fiber. As the source function of integral order Bessel function and integral order first kind imaginary parameter Hankel function is integral order Hankel function, so, the integral order Hankel function ought to be the eigenfunction of radial traveling wave field in homogeneous dielectric. Owing to the Hankel function isn’t the harmonic function, it isn’t used for expressing radial traveling wave field in literatures.
A special expression is presented for expressing cylindrical traveling wave field in the homogeneous dielectric by Iizuka K [31], it is as follows:

$$\Psi(r, \varphi, z, t) = C \frac{1}{\sqrt{r}} \exp(\pm ikr - i\omega t)$$  \hspace{1cm} (21)

Although the expression in the right side of equation (21) is a harmonic expression, equation (21) isn’t a solution of the Helmholtz’s equation.

As the distribution function of refractive index in homogeneous dielectric is a constant, i.e., \(n(r)=n_0\), equation (7) is degenerated into the following equation [16, 29]:

$$\Psi(r, \varphi, z, t) = \frac{C \eta_e}{\sqrt{k_0 n_0 r}} \exp(\pm ik_0 r)\exp(\pm i\varphi)\exp(\pm ik_0 z - i\omega t)$$  \hspace{1cm} (22)

where, \(k_r=(k_0^2 n_0^2 - k_z^2)^{1/2}\), it is a degenerated form of equation (5), the coefficient of electric components \(\eta_e=1\), the coefficient of magnetic component \(\eta_h=k_0 n_0/(\omega \mu_0)\).

The expression in the right side of equation (22) is a harmonic expression, and equation (22) is an exact solution of the Helmholtz’s equation. So, the eigenfunction of the radial traveling wave field in homogeneous dielectric is expressed by positive or negative half order Hankel function accurately.

A given step index fiber for example, the schematic of refractive index profile of fiber is shown as dotted line in figure 2. In figure 2, \(n_1\) and \(n_2\) are the refractive indexes of core layer and cladding layer respectively, \(a_0\) is the radius of fiber core layer.

For the step index fiber, if the condition \(n_1-n_2<<n_2\) is satisfied, the weakly guiding approximation is engaged, the scalar LP mode theory [7, 8] is used for reference, then, the transverse field distribution of fiber core layer \(\Psi_1(r, \varphi, z, t)\) and the transverse field distribution of fiber cladding layer \(\Psi_2(r, \varphi, z, t)\) are degenerated from equation (8) and equation (9), they are expressed as follows [16, 29]:

$$\Psi_1(r, \varphi, z, t) = \frac{C \eta_e}{\sqrt{k_0 n_0 r}} \cos(U r/a_0 - \varphi_N)\exp(\pm i\varphi)\exp(\pm i\beta z - i\omega t)$$  \hspace{1cm} (23)

$$\Psi_2(r, \varphi, z, t) = \frac{C \eta_e}{\sqrt{k_0 n_0 r}} \cos(U r/a_0 - \varphi_N)\exp[-W(r/a_0 - 1)]\exp(\pm i\varphi)\exp(\pm i\beta z - i\omega t)$$  \hspace{1cm} (24)

where, \(U=(k_0^2 n_1^2 - \beta^2)^{1/2}a_0\), it is a degenerated form of equation (11), \(W\) is the normalized evanescent wave parameter of fiber cladding layer, \(W=(\beta^2-k_0^2 n_2^2)^{1/2}a_0\).

From equation (17), the eigen equation for analysing the characteristic of step index fiber [16, 29] is become as following equation:

$$W = U \tan(U - \varphi_N)$$  \hspace{1cm} (25)

It is similar to the form of the eigen equation for analysing the characteristic of TE mode in planar waveguide, but the value \(\varphi_N\) is different, the value of \(\varphi_N\) for analysing the characteristic of step index fiber is more than the corresponding value for analysing the characteristic of planar waveguide, the difference between them is \(\pi/4\).

For the step index fiber, the normalized frequency \(V=k_0(n_1^2 - n_2^2)^{1/2}a_0\), it is a degenerative form of equation (18). The existential condition for \(N\) order mode [16, 29] is calculated by equation (19), the cut-off normalized frequency of fundamental mode is \(\pi/4\), this value is still an inconvincible solution, and the mode number \(M\) of multimode step fiber [16, 29] is calculated by equation (20) also.

5. Conclusions
The approximate expressions of eigenfunction of the traveling wave field in radial grade index dielectric and eigenfunctions of core layer and cladding layer in grade index fiber are suggested, then,
a new improved WKB method for analysing the mode characteristics of grade index fiber is presented. Moreover, the eigen equation of new improved WKB method for solving the characteristics of grade index fiber is effective for solving the characteristics of step index fiber also.

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