Conditional evolution of vacuum state in dynamical Casimir effect

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Abstract. We analytically investigate the process of photons creation in the empty cavity with trembling ideal conductive boundary. Quantum state of the intracavity field is monitored by the two-level atom passing through the system and undergoing the subsequent state measurement in the ionizing chamber. The resulted field evolution conditioned by atomic state detection result is of the most interest.

1. Introduction

The dynamical Casimir effect (DCE) is the process of photons creation out of vacuum due to the motion of the cavity mirrors. The most intensive studies in this area were made about two decades ago [1, 2, 3] and accompanied with observation of Casimir force in laboratory [4]. The main difficulty in experimental verification of DCE was the very high frequency variation of moving boundaries [5], which prevented photons creation using mechanically driven cavity mirrors. In the consequence of this fact different ideas based on imitation of boundary moving were proposed [6], [7]. Recently, Wilson and coworkers reported about the successful experiment on photon creation using DCE effect in superconductive circuit [8]. That work initiated a discussion about the most effective measurement protocol of created quanta detection and modification of intracavity field statistics due to the interaction with measurement apparatus [9]. In [10] the approximate analytical solution for the problem of created photon detection was done and the main values, which characterized the field statistics were obtained for the case of interaction with two-level atom.

Here we estimate analytically the conditional state evolution of created intracavity field in the process of indirect photodetection with two-level atom-pointer in use. The measurement protocol was organized as follows: two-level atom-pointer prepared in its ground state passing through the cavity with trembling boundary and interacts with generated Casimir photons. After interaction it undergoes the state detection in ionizing chamber. The unitary atom-field evolution is under investigation. For the simplest case of pure squeezing we obtain the Jaynes-Cummings Hamiltonian in the renormalized vacuum representation. Under this approximation we get the field evolution operators, conditioned by the atomic state detection result. Thereby we can predict the field state and all its statistical properties. Also it is possible to investigate the quantum trajectory for the field state using atomic state detection statistics, repeating the interaction many times.
2. The model

In [3] it was shown that in general there are two factors, which contribute to the process of photon creation: squeezing and relativistic acceleration due to the fast boundary motion. Here we will investigate the conditional dynamics of intracavity quantum field state, assuming that acceleration and mode interaction effects are small compared to squeezing and consider the last one as an only source of Casimir photons.

Following [10] we start our description of the system dynamics using Hamiltonian in its simplest form:

\[ H_0 = \omega(t) a^\dagger a + \frac{\Omega}{2} \sigma_z + i \xi(t) \left( a^\dagger a - a^2 \right) + g \left( a^\dagger \sigma_- + a \sigma_+ \right), \quad (1) \]

where \( \omega(t) \) is instantaneous frequency of intracavity field, \( \Omega \) is the frequency of atomic transition, \( a \) is the field annihilation operator, \( \sigma_+ = |e\rangle \langle g| = \sigma_\dagger - \) is the atomic operator and \( \xi(t) \) is squeezing parameter [1]:

\[ \xi(t) = \frac{1}{4\omega(t)} \frac{d\omega}{dt}. \quad (2) \]

Let us choose the time dependence of cavity field frequency in the harmonic low \( \omega(t) = \omega_0 (1 + \varepsilon \sin \eta t) \), with frequency modulation \( \eta \) and modulation depth \( \varepsilon \). We’ll describe the regime corresponding to the exponentially growth of cavity photons \( \varepsilon \omega_0 t \gg 1 \) [8], which gives

\[ \xi(t) \approx \frac{\varepsilon \eta}{4} \cos(\eta t). \quad (3) \]

Then we write Hamiltonian (1) in the form, corresponding to the following state transformation [10]

\[ |\varphi\rangle = V(t) |\psi\rangle, \quad V(t) = \exp \left[ -i t \eta / 2 \left( a^\dagger a + \sigma_z / 2 \right) \right], \quad (4) \]

which gives us \( H = V^\dagger(t) H_0 V(t) - i V^\dagger(t) \dot{V}(t) \) or

\[ H = x a^\dagger a + \frac{\Delta + x}{2} \sigma_z + i \frac{\varepsilon \nu}{4} \left( a^\dagger a - a^2 \right) + \left( a^\dagger \sigma_- + a \sigma_+ \right) \sigma_-. \quad (5) \]

Here \( \nu = \eta / 2, \; x = \omega_0 - \nu, \; \Delta = \Omega - \omega_0 \) and the rotating wave approximation (RWA) was applied.

Unitary transformation of (5) with squeezing operator \( S = \exp \left[ \frac{1}{2} (\beta^* a^2 - \beta a^\dagger a) \right] \), where

\[ \beta = re^{i\theta}, \quad r = \frac{1}{2} \arctanh \frac{\varepsilon \nu}{2x}, \quad \theta = \pi / 2, \quad (6) \]

gives the following form of Hamiltonian \( H_B = S^\dagger H S \) with renormalized vacuum state:

\[ H_B = \mu \left( a^\dagger a + \frac{1}{2} \right) + \frac{\alpha}{2} \sigma_z + \]
\[ + g \left[ \left( a \cosh(r) - i a^\dagger \sinh(r) \right) \sigma_+ + \left( a^\dagger \cosh(r) + i a \sinh(r) \right) \sigma_- \right], \quad (7) \]

where \( \alpha = \Delta + x \) and

\[ \mu = \frac{\sqrt{4x^2 - \varepsilon^2 \nu^2}}{2}. \]

From (6) it is easy to obtain that only for \( |\varepsilon \nu / 2x| < 1 \) this unitary transformation is allowable.
3. In the bounds of Jaynes-Cummings model

Being in the region of parameters which allows the system Hamiltonian in the form (7) we can obtain its simplest form assuming that $|\mu - \alpha| \ll \mu, \alpha$. Writing (7) in interaction picture and applying RWA we reach the following expression for $H_B$ which has the form of Jaynes-Cummings Hamiltonian:

$$H_{J-C} = G \left(e^{i\delta t}a\sigma_+ + e^{-i\delta t}a^\dagger\sigma_-\right),$$

where $\delta = \mu - \alpha$ and $G = g \cosh(r)$.

From expression (8) it is easy to obtain the operator of conditional field state evolution. Expanding evolution operator $U(t)$, which correspond to the solution of Schrodinger equation with Hamiltonian (8) in the following composition

$$U(t) = \sum_{p,q \in \{g,e\}} U_{pq}(t) |q\rangle \langle p|,$$

it is easy to obtain the system of operator-valued differential equation [11]:

$$i\frac{d}{dt} U_{pq}(t) = \sum_{r \in \{g,e\}} \langle p| H_{J-C} |r\rangle U_{rq}(t).$$

This system has a simple analytical solution. Namely, for the atom prepared in its ground state $|g\rangle$ and detected in ground $|g\rangle$ or excited state $|e\rangle$ we obtain

$$U_{gg}(t) = \frac{e^{i\delta t/2}}{\sqrt{\gamma}} \left(\sqrt{\gamma} \cos \frac{\sqrt{\gamma} t}{2} - i\delta \sin \frac{\sqrt{\gamma} t}{2}\right), \quad \gamma = \delta^2 + 4G^2a^\dagger a,$$

$$U_{ge}(t) = -i \frac{Ge^{-i\delta t/2}}{\sqrt{\delta^2/4 + G^2a^\dagger a}} \sin \left(t\sqrt{\delta^2/4 + G^2a^\dagger a}\right) a.$$  

Operators $U_{gr}, r \in \{g,e\}$ have the following physical interpretation: they map initial state $|\varphi(0)\rangle$ of intracavity field onto the final state $|\varphi(t)\rangle$, conditioned by detection result $r$. This final state however is not normalized and full transformation is

$$|\varphi_r(t)\rangle = \frac{U_{gr}(t)|\varphi(0)\rangle}{P_r(t)}, \quad P_r(t) = \langle \varphi_r(t) | \varphi_r(t) \rangle.$$  

Here $P_r(t)$ is the probability to detect the atom in state $|r\rangle$ after interaction lasted time $t$.

4. Analytical and numerical results

In this section we represent the results followed from the numerical analysis of the evolution generated by Hamiltonian (5). In particular the basic statistical characteristics of the monitored intracavity field will be investigated under the assumption that atom was found in its excited state. The system parameters was chosen in a such way to satisfy the condition which gives approximate analytical formula (8). Namely, the following values of parameters are used (in units $10^9$ Hz):

$$x = 1, \quad \epsilon \nu = 1.7, \quad \Delta = 0.36, \quad g = 0.1.$$  

The probability $P_r(t)$ of event, corresponded to the atomic state detection result $|e\rangle$ is given by Eq. (13) and is shown on Fig. 1(a) as a function of dimensionless time $gt$. It increases monotonically and reaches the value 0.015 on the concerned interval. The mean cavity photon number $\langle n(t) \rangle = \langle \varphi_e(t) | a^\dagger a | \varphi_e(t) \rangle$ as a function of time is shown in Fig.1 (b). This quantity, conditioned by detection result overcome the value 6 at $gt \approx 2$. For this instant of time the
mean photon number probability distribution $P_e(n)$ is presented in Fig. 1(c). This distribution contain the nonvanishing probability only for odd numbers of photons, which may be explained easily. The matter is that in the absence of atom the intracavity field is in squeezed vacuum state (which has the distribution with only even photons). When atom is found in its excited state, the photon number in cavity decreases by one.

In Fig. 2(a–c) we show the Mandel factor (a) $Q = \left[\langle (\Delta n)^2 \rangle - \langle n \rangle \right] / \langle n \rangle$, second order correlation function $g^{(2)} = \langle a^\dagger a^\dagger a a \rangle / \langle n \rangle^2$, and field quadratures $\langle (\Delta X)^2 \rangle$ and $\langle (\Delta P)^2 \rangle$, where $X = (a + a^\dagger)/2$ and $X = (a - a^\dagger)/2i$ as a functions of time. This quantities describe the nonclassiallity of the generated intracavity field interacted with atom during certain time interval. From here it is easy to see that statistical properties of the field changes dramatically due to the interaction with atom. Indeed, in the beginning of this process Mandel factor $Q < 0$ and $\langle (\Delta P)^2 \rangle$ decreases to the values, corresponded to squeezed state. Then $Q$ increases and at the point $gt \approx 0.5$ ($g^{(2)} \approx 1$) become equal to unity, which corresponds to the Poissonian field statistics. For $gt > 0.5$ statistics of the field become super-Poissonian.

**Figure 1.** Excited atomic state detection: probability (a) and mean photon number (b) as a functions of dimensionless time $gt$; (c) photon number probability distribution for $gt \approx 2$. All parameters were taken from (14); $r = 0.63$ and $\mu = 0.53$ in units $10^9$ Hz.

**Figure 2.** Excited atomic state detection: Mandel factor $Q$ (a), second order correlation function $g^{(2)}$ (b), field quadratures $\langle (\Delta X)^2 \rangle$ and $\langle (\Delta P)^2 \rangle$ (c) as a functions of dimensionless time $gt$. All parameters were taken from (14); $r = 0.63$ and $\mu = 0.53$ in units $10^9$ Hz.
5. Conclusion
We investigated the conditional state evolution of the field, generated in dynamical Casimir effect under the indirect photodetection protocol. Using two-level atom as a microscopic part of measurement apparatus allows us to obtain the approximate analytical expressions for conditional evolution operators and to estimate basic statistical characteristics of monitored intracavity field. For the following it would be interesting to investigate the case of nonunitary interaction between atom and field due to the presence of environment.

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