Prediction of $\sin^2\theta_W$ in a Conformal Approach to Coupling Unification

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Abstract

The possibility that non-supersymmetric conformal field theories softly broken below 100 TeV may provide an alternative to conventional grand unification is explored. We consider a low energy theory presumed to be of this type arising from the Type IIB superstring compactified on a $AdS_5 \times S_5/\Gamma$ space whose gauge group and the particle content are severely restricted by the compactification process. We present an example of a resulting $SU(4)_C \times SU(2)_L \times SU(2)_R$ with three generations, which leads to coupling unification and a prediction for $\sin^2\theta_W \simeq .227$ and other phenomenology generally consistent with observations.

14.60.Pq; 14.60.St.
I. INTRODUCTION

The idea that all gauge couplings describing the forces in the standard model may eventually merge into one is not only an aesthetically pleasing one, but also has the virtue of making testable predictions at low energies [1]. Two predictions typical of simple grandunification models are \( \sin^2 \theta_W(M_Z) \) and proton life time. The earliest grand unified theories however had the problem of an enormous gauge hierarchy that separated the weak scale from the scale of grand unification. The introduction of supersymmetry not only provided a solution to this problem but also provided a simpler framework for unification of couplings. While this is perhaps the simplest approach to coupling unification, at present there is no experimental evidence for this view. It is therefore interesting to pursue other approaches to coupling unification and explore the possibility of constructing phenomenologically viable models and isolate their tests.

Motivated by conformal invariance of string theories, a new approach to coupling unification has been suggested in a series of recent papers [2]. The basic idea of this approach relies on the fact that a type IIB string theory compactified on an \( AdS_5 \times S_5 \) gives rise to an \( \mathcal{N} = 4 \) SU(N) gauge theory, which has been known for some time [1] to be conformal due to extended global symmetry and nonrenormalization theorems. All of the beta functions in this theory vanish, making the couplings “static”. The \( \mathcal{N}=4 \) supersymmetry is crucial for the conformal invariance. However, it has been shown that by compactifying the theory on \( S_5/\Gamma \), where \( \Gamma \) is a discrete subgroup of the R-symmetry group SU(4), one can obtain conformal invariant SU(N) gauge theories with finite N and lower number of supersymmetries [5,6]. One could then add soft conformal breaking terms in order to introduce the mass scales, say M. The couplings below the scale M will then “run” following the usual rules of field theories. If the constraints of \( \sin^2 \theta_W \) and compactification then allowed for unification of the gauge couplings, then one could consider these theories as alternatives to conventional grand unified theories. In particular, if the scale M is in the TeV range, then one may completely bypass low energy supersymmetry. In [3], it was shown how to use the abelian \( \Gamma = Z_7 \) to arrive at \( \sin^2 \theta_W(\mu) = 3/13 = 0.231 \); here we examine the situation for non-abelian \( \Gamma \).

In a recent paper, an exhaustive classification of possible gauge groups and the particle contents that arise from different compactifications of AdS/S_5 have been presented [8]. In this note, we study one particularly interesting class of models based on the gauge group \( SU(4)_C \times SU(2)_L \times SU(2)_R \) and show that this model can lead to prediction of \( \sin^2 \theta_W \) at the \( M_Z \) scale in agreement with experiment. The nontriviality of this result follows from the fact that group theory and unification allows only a small number of possibilities for the value of \( \sin^2 \theta_W \) at the conformal scale \( M \), which we choose in this case to be of order 100 TeV to be consistent with low energy rare weak processes such as \( K_L \rightarrow e^+ \mu^- \).
II. THE MODEL

We start with an AdS/$S^5$ compactification of a type II B string theory which has $\mathcal{N} = 4$ supersymmetry and is conformal invariant. This theory has an SU(4) global symmetry as its R-symmetry. We start with two D-branes in this theory at a point in $S^5$ so that the starting gauge theory is $SU(2)$. We then consider the nonabelian discrete group $\Gamma = Z_3 \times D_4$ which is a group of order 24. Recall that the group $D_4$ consists of the eight rotations that leave a square invariant; two of the rotations are flips about two lines that bisect the square and the other four are 90, 180, 270 and 360 degree rotations about the perpendicular to the plane of the square. Following the conjecture by Lawrence et al [6], we assume that the gauge group of this theory is $\Pi_i SU(2_d)$, where $d_i$ are representations of the $\Gamma \equiv Z_3 \times D_4$.

The representation of $\Gamma$ are three 2-dim ones and 12 1-dim. which leads to the low energy gauge group $[SU(4)]^3 \times [SU(2)]^{12}$. A diagonal sum of this group can be chosen as the actual gauge group of the model. In particular, we will consider various diagonal sums of the type $SU(4)_C \times SU(2)_L \times SU(2)_R$. Here SU(4) is the diagonal subgroup of $r$ of the SU(4)'s where $r = 1$ or 2 since, as shown in [8], $r = 3$ disallows chiral fermions. The two SU(2)'s are direct sums of $p$ and $q$ of the original SU(2)'s such that $p + q = 12$. The model is left-right symmetric [7] with quarks and leptons unified via $SU(4)_C$ a la Pati and Salam. It then follows trivially that the resulting gauge couplings of the effective SU(4) and the SU(2)'s respectively are: $g_4 = g_U/\sqrt{2r}$ (where we insert a factor $(1/\sqrt{2})$ for the dimensionality $d_i$ of the representation) ; $g_{2L} = g_U/\sqrt{p}$ and $g_{2R} = g_U/\sqrt{q}$, with $p + q = 12$. We have assumed that the gauge couplings become unified at the conformal scale and due to conformal nature of the theory they remain frozen afterwards. For the case of $\mathcal{N} = 0$ supersymmetry, this is a conjecture, which has been checked only to one loop. We assume this to be true to all orders. Note that since $p$ and $q$ are integers, it apriori not at all obvious that this will lead to an acceptable prediction for $\sin^2\theta_W$ at the weak scale. As we show below however, for a unification scale of the order of 100 TeV, one does indeed get an acceptable value.

The fermion and the Higgs content of the group is given by the quiver diagram of the group [8]. They include Higgs as well as fermions in all bi-fundamentals. The fact that the embedding of representations of $\Gamma$ into the 4 of SU(4) are not real guarantees guarantees the existence of chiral fermions [2]. It has been shown that in the $Z_3 \times D_4$ embedding into SU(4) that we are considering, there exists precisely three fermion generations in the bifundamentals: (4,2,1), (4,1,2); any fermion in (1,2,2) representation can be given a bare mass term and be made to decouple from the low energy part of the spectrum.

The Higgs bosons are in the representations (4,2,1), (4,1,2), (1,2,2); let us denote these by $\chi_L$, $\chi_R$ and $\phi$ respectively.

The question we address in this paper is whether with the above particle content, one can have a viable theory. The first point we check is the prediction for $\sin^2\theta_W$ at the scale $M_Z$. Before proceeding to this discussion, we briefly review how the three generation model with the gauge group $SU(4)_C \times SU(2)_L \times SU(2)_R$ emerges in this picture.
III. COMPLEX SCALARS AND CHIRAL FERMIONS

Some explanation of the matter content of complex scalars and chiral fermions will be useful to the reader.

The gauge group arising from the orbifold $S^5/\Gamma$ and $N$ branes is, as already mentioned, $G = \otimes_i SU(Nd_i)$ where the $d_i$ are the dimensionalities of all the irreps of $\Gamma$.

The content of matter fields can be found from the chosen embedding of $\Gamma$ in $SU(4)$, which may be specified from the decomposition of the $4$ of $SU(4)$. The $6$ of $SU(4)$ is then the antisymmetric product $(4 \times 4)_{\text{antisym}}$. For any consistent embedding $\Gamma \subset SU(4)$, the $6$ is necessarily real.

Given the content of the $6$ in terms of irreps of $\Gamma$ the representations of the complex scalars under the gauge group $G$ are derived by producing the $6$ with all irreps of $\Gamma$, using the multiplication table, as provided in e.g. the Appendix of [3]. Each term appearing in the decomposition of such products leads to the survival of a bi-fundamental representation of the corresponding $SU(Nd_i) \times SU(Nd_j)$. In the special case $i = j$, this is to be interpreted as an adjoint plus a singlet of $SU(Nd_i)$. The procedure can be conveniently summarized and facilitated by use of a quiver diagram [8].

For chiral fermions, the same procedure is followed using the $4$ instead of the $6$. To achieve survival of chirality it is necessary that the $4$ not be real or pseudoreal, as proved in [8]. The chiral fermions can be derived by a quiver diagram in which at least some of the “arrows” are oriented (chiral). This is in contrast to the scalar quiver diagram in which all the arrows are non-oriented (self-conjugate).

These considerations have been applied in ref. [10] to show that the gauge group at the conformality scale is $[SU(4)_1 \times SU(4)_2 \times SU(4)_3 \times \otimes_i SU(2)_i]$ with $i = 1, 2, ... 12$. The fermion content of this model consists of

a.) Below we list the 24 bifundamentals $(2_i, 2_j)$ under the $i$th and $j$th $SU(2)$’s and singlets under the rest of the groups. (We will employ this notation throughout this paper; below the bold face numbers to the left of a bracket denote how many such multiplets are there.);

\[2(2_1, 2_2), (2_1, 2_3), (2_1, 2_5); 2(2_3, 2_4), (2_3, 2_5); 2(2_5, 2_6); (2_2, 2_4), (2_2, 2_6); (2_4, 2_6);\]
\[2(2_7, 2_8), (2_7, 2_9), (2_7, 2_{11}); 2(2_9, 2_{10}), (2_9, 2_{11}), 2(2_{11}, 2_{12}), (2_8, 2_{10}), (2_8, 2_{12}), (2_{10}, 2_{12}).\]

b.) three $2(4_i, \bar{4}_i)$’s under the $SU(4)$’s and $(4_1, \bar{4}_2), (4_2, \bar{4}_3), (4_3, \bar{4}_4) + \text{c.c.}$

c.) 12 bifundamentals of the form $(2_i, 4_a)$ under $SU(2)_i \times SU(4)_a$ given by: $(4_1, 2_5) \oplus (4_2, 2_4) \oplus (4_3, 2_3) \oplus (4_1, 2_6) \oplus (4_2, 2_2) \oplus (4_3, 2_4) \oplus (4_1, 2_{11}) \oplus (4_2, 2_7) \oplus (4_3, 2_9) \oplus (4_1, 2_{12}) \oplus (4_2, 2_8) \oplus (4_3, 2_{10})$

d.) Anti-bifundamentals $(\bar{4}_1, 2_4) \oplus (\bar{4}_2, 2_6) \oplus (\bar{4}_3, 2_2) \oplus (\bar{4}_1, 2_9) \oplus (\bar{4}_2, 2_{11}) \oplus (\bar{4}_3, 2_7) \oplus (\bar{4}_1, 2_{10}) \oplus (\bar{4}_2, 2_{12}) \oplus (\bar{4}_3, 2_8) \oplus (\bar{4}_1, 2_3) \oplus (\bar{4}_2, 2_5) \oplus (\bar{4}_3, 2_1)$.

Turning now to the scalar multiplets, we have the following:
1.) Bifundamentals of $(2, 2_j)$ type:
$(2_1, 2_4), (2_1, 2_6), (2_3, 2_2), (2_3, 2_5), (2_5, 2_2), (2_5, 2_4)$;

2.) Bifundamentals of $(2_i, 4_j)$ type and their complex conjugates
$(2_1, 4_2), (2_1, 4_3), (2_3, 4_1), (2_3, 4_3), (2_5, 4_1), (2_5, 4_2); (2_2, 4_2), (2_2, 4_3), (2_4, 4_1), (2_4, 4_3), (2_6, 4_1), (2_6, 4_2);
(2_7, 4_2), (2_7, 4_3), (2_9, 4_1), (2_9, 4_3), (2_{11}, 4_1), (2_{11}, 4_2); (2_8, 4_2), (2_8, 4_3), (2_{10}, 4_1), (2_{10}, 4_3), (2_{12}, 4_1), (2_{12}, 4_2) +
complex conjugates.

3.) Bifundamentals of type $(4_i, 4_j)$ with $i \neq j$ and $i, j = 1, 2, 3$.

It is now easy to see that the one loop $\beta$ functions vanish for all $SU(2)_a$ and $SU(4)_i$ groups as desired. This is of course not a sufficient condition; however, we will assume that
conformality condition is satisfied to all loops.

Now let us note that all the multiplets in a.), b.) are vectorlike and become massive at
the conformality scale. The required generations must therefore come from the rest of the
multiplets. For this purpose, let us first break the $SU(4)_3$ group completely and consider
the low energy $SU(4)$, group as the direct sum of $SU(4)_1 \times SU(4)_2$; also anticipating the
discussion of $\sin^2 \theta_W$ in the next section, let us consider the $SU(2)_L$ group as the direct sum
of $SU(2)_m$’s for $m = 1, 2, 7, 5$ and the $SU(2)_R$ as the sum of the rest of the $SU(2)_m$’s. Then,
three generations surviving at low energies correspond to the “primordial” representations
$(4, 2_1) \oplus (4, 2_2) \oplus (4, 2_7)$ are left $SU(2)_L$ doublets and $(\bar{4}, 2_3) \oplus (\bar{4}, 2_4) \oplus (\bar{4}, 2_6)$ are right
$SU(2)_R$ doublets, where we have denoted $4 = 4_1 \oplus 4_2$. The multiplets of $(4_3, 2_k)$ and $(\bar{4}_3, 2_j)$
type become vector like aftercomplete breakdown of $SU(4)_3$ and pick up mass of order 100
TeV. For instance, $(2_8, 4)$ and $(2_{10}, \bar{4})$ pair up to become massive via the Higgs coupling
$(2_8, 4)(2_{10}, \bar{4})(2_8, 2_{10})_H$. One can write similar couplings involving the fermions of $SU(4)_3$
that make all of them massive.

IV. PREDICTION FOR $\sin^2 \theta_W(M_Z)$

Using the standard evolution equations for nonsupersymmetric standard model [11] and
assuming as just noted the relations between the $SU(4)_c \times SU(2)_L \times SU(2)_R$ gauge couplings
and the unified “static” coupling $\alpha_U$, for the SU(2)’s to be
\begin{align*}
\alpha_{2L}^{-1}(M_U) &= p\alpha_U^{-1}, \\
\alpha_{2R}^{-1}(M_U) &= q\alpha_U^{-1}, \\
\alpha_{4c}^{-1}(M_U) &= 2r\alpha_U^{-1},
\end{align*}
we can relate the observed standard model couplings at the scale $M_Z$ to $\alpha_U$ and the scale
of unification $M_U$. For this purpose, let us remind the reader [11] that in the single scale
breaking model that we are considering, the hypercharge gauge coupling $\alpha_1^{-1}$ (suitably normal-
ized) is related at the unification scale to the $SU(2)_R$ and the $SU(4)_c$ couplings by the relation
\begin{equation}
\alpha_1^{-1} = \frac{2}{5}\alpha_{4c}^{-1} + \frac{3}{5}\alpha_{2R}^{-1}.
\end{equation}
Using Eq. (1) and (2) and the beta functions of the standard model, we can express $\sin^2 \theta_W$
and the QCD fine structure constant $\alpha_s(M_Z)$ in terms of $\alpha_U$ and the unification scale $M_U$
(expressed below as $y = \ln \left(\frac{M_U}{M_Z}\right)$) as follows:
\[
\sin^2 \theta_W(M_Z) = \frac{p - (19/12\pi)y\alpha_U}{p + q + \frac{4}{3}r + (11/6\pi)y\alpha_U};
\]

Using these formulae and using \( \alpha_s(M_Z) = 0.12 \), we find that for \( M_U \simeq 100 \text{ TeV} \) we get \( \sin^2 \theta_W(M_Z) \simeq 0.227 \) for \( p = 4 \) and the simplest case \( r = 2 \) in the one loop approximation. This is close to the central value from present experiments [12]. Clearly Eq.(3) with \( p = 4 \) would give \( \sin^2 \theta_W = 3/11 \) for the too-low conformality scale \( M_U = M_Z \) but this value is beautifully corrected to agree more closely with experiment by the renormalization-group running between \( M_Z \) and the conformality scale \( M_U \simeq 100 \text{ TeV} \). Since \( p \) can take only discrete integer values, we find this remarkable that the prediction for \( \sin^2 \theta_W \) is close to the observed value.

As noted below, 100 TeV is not far from the lowest phenomenologically allowed scale for the \( SU(4)_C \) breaking scale derived from present upper limits on the branching ratio for the process \( K_L \rightarrow \mu^+e^- \) [13].

V. OTHER PHENOMENOLOGICAL ISSUES:

Once one has quark lepton unification at a scale near a 100 TeV, there are several phenomenological issues, one has to deal with: first is the splitting between quarks and leptons; the second question is how to understand the smallness of neutrino masses and finally the question of rare processes. We address them one by one:

**Quark-lepton mass splitting:**

There are two issues that need to be addresses here: first is how the masses of different families of fermions arise and second how the quark masses are split from the lepton masses despite the \( SU(4) \) unification group. We do not address the details of the first question. Our view is that the chiral families arise as a linear combination of large number of high scale fermion representations and in the process, the different families could acquire different Yukawa couplings that may eventually explain the family hierarchy.

We now address the second question which is also a nontrivial one since since the Higgs boson content of the model is severely limited by the requirements of conformal invariance. The allowed Higgs multiplets in addition to those stated earlier (i.e. \( \chi_{L,R} \) and \( \phi \)) are \( \Sigma(15,1,1) \) (this multiplet can arise from \( (4_1,4_2) \) multiplets when the diagonal sum is taken). It is then clear that one can have higher dimensional operators of the form \( \bar{\Psi}_L\phi\Sigma\Psi_R/M \) where \( \Psi = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix} \) represents the fermion multiplet. This operator splits the quarks from leptons and provides a way to understand the quark-lepton mass splitting.
Neutrino masses

The second question that one has to address has to do with the origin of small neutrino masses. The usual seesaw mechanism \[14\] requires a high scale of about \(10^{12} \text{ GeV}\) or more for reasonable values of Dirac masses for neutrinos. In the model, since the highest scale is only a 100 TeV, one must seek an alternative way to understand, why neutrinos are so light. One possibility is to use a generalized seesaw that requires the existence of gauge singlet fermions \[15\]. Suppose there are three singlet neutrinos denoted by \(\nu_{s,i}\) (\(i=1,2,3\)). Then one can write a mass matrix of the form for the neutrinos in the basis: \((\nu_e, \nu_\mu, \nu_\tau; N_e, N_\mu, N_\tau; \nu_{s1}, \nu_{s2}, \nu_{s3})\)

\[
\mathcal{M}_\nu = \begin{pmatrix}
0 & M_D & 0 \\
M_D^T M_N & M_s & \\
0 & M_s^T & \mu_s
\end{pmatrix}
\]

(4)

where \(M_{D,N,s}\) are \(3 \times 3\) matrices. For typical values of the masses in \(M_D\) we assume 100 MeV since they originate at the weak scale and ought to be of order of the charged lepton masses. \(M_N\) is the Majorana mass of the right handed neutrinos and is expected to be order 100 TeV as are the elements of \(M_s\). As far as \(\mu_s\) is concerned, since the theory breaks lepton number (B-L to be precise), radiative corrections can in conjunction with higher dimensional terms can generate small masses for \(\nu_s\) if we assume that they also have lepton number like their electroweak counterparts. The effective “active” neutrino masses are then given by:

\[
\mathcal{M}_\nu \simeq M_D^T M_N^{-1} \mu_s M_N^{-1} M_D
\]

(5)

One can estimate typical magnitudes of the entries in \(\mathcal{M}_\nu\), the active neutrino masses, to be of order \(\sim \text{eV}\) to milli \(\text{eV}\) depending on the magnitudes of entries in \(\mu_s\). Clearly these values are in the right range to be of interest for explaining the current evidences for neutrino oscillations.

Rare decay constraints

A major phenomenological constraint on these models comes from the presence of the gauge leptoquark bosons connecting quarks to leptons which lead to rare decays of the K-meson to \(\mu^+ e^-\) channels. The strength of these couplings is given by \(\sim g_4^2/M_U^2 \sim 10^{-10}\) GeV\(^{-2}\). Present upper limits on \(B(K_L \to \mu^+ e^-) \leq 6 \times 10^{-12}\), imply \(M_U \geq 400\) TeV, which is not far from the unification scale that gives the correct order of magnitude for the \(\sin^2 \theta_W\) as discussed above. Besides our estimate of the unification scale is based on an one loop calculation and could easily be uncertain by a numerical factor of 3 to 4.
VI. CONCLUSION AND DISCUSSIONS

In this note, we have studied the phenomenological implications of a new approach to grand unification that uses presumed conformal invariance of a nonsupersymmetric theory derived from type IIB strings compactified on an ADS/$S_5$. We find it interesting that there is only one model based on that uses compactification on $S_5/\Gamma$ where $\Gamma = Z_3 \otimes D_4$ group which has three families. The gauge group of this model is $SU(4)_C \times SU(2)_L \times SU(2)_R$. We calculate the $\sin^2 \theta_W$ for this model in the one loop approximation and find it to be $0.227$ which is in very good agreement with the observations. We choose the scale to be in the 100 TeV range which is not far from that required by the rare $K_L \to \mu^- e^+$ decay limits. We also discuss some other related phenomenological issues.

ACKNOWLEDGEMENTS

The work of PHF and SS is supported by a grant from US Department of Energy DE-FG02-97ER-41036 and the work of RNM is supported by a grant from the National Science Foundation under grant number PHY-9802551.
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