Bubble domains in disc-shaped ferromagnetic particles

S. Komineas, C.A.F. Vaz, J.A.C. Bland
Cavendish Laboratory, Madingley Road, Cambridge CB3 0HE, United Kingdom.

N. Papanicolaou
University of Crete, and Research Centre of Crete, Heraklion, Crete, Greece.
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We study the fundamental magnetic states of disc-shaped ferromagnetic particles with a uniaxial anisotropy along the symmetry axis. Besides the monodomain, a bidomain state is also identified and studied both numerically and theoretically. This bidomain state consists of two coaxial oppositely magnetized cylindrically symmetric domains and remains stable even at zero bias field, unlike magnetic bubbles in ferromagnetic films. For a given disc thickness we find the critical radius above which the magnetization configuration falls into the bidomain bubble state. The critical radius depends strongly on the film thickness especially for ultrathin films. The effect of an external field is also studied and the bidomain state is found to remain stable over a range of field strengths.

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Current developments in experimental techniques allow the engineering of magnetic particles to very small length scales, typically tens or a few hundred of nanometres \[^1\]. In such systems the characteristic length scales of the magnetic domains may be comparable to the system size. The particles should thus support only a small number of magnetic domains and these should be usually biased by the symmetry of the particle to give a small number of distinct magnetic configurations. Examples are vortex states observed in disc particles \[^1,2,3\] as well as vortex and onion states observed in magnetic rings \[^4,5\]. This situation is in marked contrast to a bulk ferromagnet where we have a large number of magnetic domains with significantly different sizes.

Recently, much research effort has focused on the investigation of narrow ferromagnetic rings where it is found that single, highly symmetric domain structures mediate the switching between the high moment (onion) state and low moment (vortex) states \[^6\]. Motivated by the work on rings, in the present case we search for the existence of high symmetry stable states in circular disc structures with perpendicular anisotropy, in view of the recent development of materials with a large uniaxial anisotropy \[^7,8\] and experiments in permalloy discs \[^9\]. In particular, we identify bidomain states which exist even in the absence of a bias field but are otherwise the analogues of the magnetic bubbles observed in ferromagnetic films. In addition, the bidomain states remain stable for a range of applied fields and can be manipulated in a controlled manner. Furthermore, we estimate the size of the magnetic domains supported in the particle. The importance of the present findings is that, again, very simple, high symmetry domain structures are found to be stable and to mediate the magnetic switching process, in contrast to the complex behaviour which usually occurs in small elements.

Static as well as dynamical properties of the magnetization \(m\) are governed by the Landau-Lifshitz equation. The constant length of the magnetization is normalized to unity: \(m^2 = 1\). An important length scale of the system is the exchange length

\[
\ell_{ex} = \sqrt{\frac{A}{2\pi M_0^2}},
\]

where \(A\) is the exchange constant and \(M_0\) is the saturation magnetization. In the following, we shall use \(\ell_{ex}\) as the unit of length. Another important quantity is the dimensionless quality factor

\[
\kappa = \frac{K}{2\pi M_0^2},
\]

where \(K\) is the anisotropy constant. The significance of the quality factor can be seen in two important quantities. First, the domain wall width is \(\ell_{ex}/\sqrt{\kappa}\). Second, \(\kappa\) controls the relative strength of the magnetostatic field which has a demagnetizing effect, with respect to the anisotropy field which favours alignment along a direction perpendicular to the film. We shall suppose here that \(\kappa > 1\) which means that the anisotropy is, in general, stronger than the demagnetising field.

The Landau-Lifshitz equation is the basis for all our calculations and we are interested only in its static solutions. We find such magnetic configurations by a relaxation algorithm the details of which were explained in \[^10,11\]. We introduce a Gilbert damping term in the equation and feed the algorithm with an initial guess state. This eventually converges to a static solution at a local minimum of the energy functional.

The most demanding part of the method is by far the calculation of the magnetostatic field which requires the solution of a boundary value Poisson problem. This makes the numerical simulation of most realistic problems practically impossible to achieve in three dimensions. However, a huge reduction of the numerical calculations is obtained if we confine our interest to axially symmetric configurations, and in the rest of this paper we shall be concerned only with such magnetic...
states. We expect that this is not a serious constraint at least for the most basic magnetic states of small particles which will be our main focus. Indeed, it is reasonable to assume that the lowest lying states of the system will have the symmetry of the geometry of the particle. We call $z$ the axis of symmetry of the disc which is also the direction of the easy axis. We go to cylindrical coordinates and suppose that the radial ($m_\rho$), azimuthal ($m_\phi$) and longitudinal ($m_z$) components of the magnetization vector are functions of $\rho$ and $z$ only: $m_\rho = m_\rho(\rho, z)$, $m_\phi = m_\phi(\rho, z)$, $m_z = m_z(\rho, z)$.

As a first step we calculate the simplest possible state. This is expected to be a single-domain state in which all spins are driven by anisotropy and lie roughly along the symmetry $z$ axis. We use the uniform $m = (0, 0, 1)$ state as an initial guess in the relaxation algorithm. This then quickly converges to a quasi-uniform static state, at least for strong anisotropy $\kappa > 1$. The magnetization vector deviates from the $z$ axis only around the edges of the disc. The quasi-uniform state is a monodomain state and is thus expected to be the ground state for sufficiently small particles. The transformation $m \to -m$ gives a second monodomain state.

As the size of the particle becomes larger it is anticipated that the magnetic configuration will break up into domains. We conjecture a bidomain state that is axially symmetric. This consists of an inner cylindrical domain of “down” magnetization surrounded by the outer domain of “up” magnetization. A domain wall has to separate the two domains. We shall call these axially symmetric bidomain states “bubbles” because they bear some essential similarities to the so-called magnetic bubbles observed in abundance in ferromagnetic films.

Ferromagnetic films with a strong perpendicular anisotropy were studied experimentally and theoretically around the 70s. These early studies were largely driven by technological interest in magnetic bubbles whose statics and dynamics were analysed in detail [12]. A magnetic bubble is a circular domain of opposite magnetization in an otherwise uniformly magnetized film with magnetization perpendicular to the film. The presence of an external bias field is essential for the stabilisation of these structures [12, 14]. If the bias field is lifted then the magnetostatic field destroys the bubble which expands and eventually transforms into stripe domains. In contrast, the bubble states calculated here for sufficiently small ferromagnetic particles remain stable even in the absence of a bias field.

We thus return to our conjecture and proceed to test it numerically. As a standard example we choose the following values for the quality factor, the film thickness, and the radius:

$$\kappa = 2, \quad d = 8 \ell_{\text{ex}}, \quad R = 24 \ell_{\text{ex}},$$

and this choice will be explained later in the text. In the case of the FePt films of Ref. [7] where $M_0 = 1150$ cmu/cm$^3$, $A = 10^{-6}$ erg/cm, the exchange length is $\ell_{\text{ex}} = 3.5$ nm and thus the values of Eq. (3) are translated to $d = 28$ nm, $R = 84$ nm. The value $\kappa = 2$ for the quality factor corresponds to $K = 1.6 \times 10^7$ erg/cm$^3$ for the anisotropy constant, as is typical in FePt films. We feed our numerical algorithm with an initial condition which has the gross features of the bubble state described above with a domain wall of the Bloch type smoothly connecting the domains. The algorithm converges to a static bubble state which has a complicated domain wall structure shown in Fig. 1. Also, the magnetization deviates to some extent from the $z$ direction at the side surface of the particle. The profile of this structure is sufficiently interesting and deserves attention. The magnetostatic energy is the driving force here and it clearly favours a bidomain state with opposite magnetization where the total magnetization would roughly vanish. On the other hand, the anisotropy and exchange energies are significant at the domain wall and they put a tension on it to shrink. In the final result the bubble has an inner domain with volume smaller than the outer domain. Thus the total magnetization points along the symmetry axis and it is nonzero. As mentioned already, contrary to the situation in films, we suppose here that no external bias field is present.

The domain wall resembles those discussed in the literature in related calculations [10, 12]. It is Bloch in the central plane (set at $z = 0$ here) and it progressively becomes Néel towards the surfaces. The Néel wall is significantly wider than the Bloch wall. The radius of the bubble is larger at the centre than near the surfaces, but this is a small effect. It is easy to understand that for this type of wall the magnetostatic energy and the total energy density are larger near the surfaces than at the disc centre. In short, the surfaces disfavour the domain wall and subsequently also the bubble state. The final and important result is that, for the parameters [7], the

![FIG. 1: The bubble state illustrated at the middle ($z = 0$) plane of the disc and at the top ($z = d/2$). The magnetization is such that $m_z = -1$ at the center and $m_z \approx 1$ in the outer domain. The arrows show the projection of the magnetization on the ($x, y$) plane which has a significant value at the domain wall. This is a Bloch-like wall in the middle ($z = 0$) plane and it turns almost Néel-like at the top and bottom surfaces ($z = \pm d/2$). The magnetization satisfies the parity relations $m_\rho(\rho, z) = -m_\rho(\rho, -z)$, $m_\phi(\rho, z) = m_\phi(\rho, -z)$, $m_z(\rho, z) = m_z(\rho, -z)$.

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FIG. 2: Energy per unit volume (in units of $2\pi M_0^2$) of the monodomain (dashed lines) and of the bubble state (solid lines) as a function of the disc radius $R$ for three values of the disc thickness $d$ ($R$ and $d$ in exchange length units). The bubble exists only for $R > R_1$, where $R_1$ is a critical radius which corresponds to the intersection of the two lines for each value of $d$. Both $R_1$ and $R_c$ depend on $d$.

The energy per unit volume of the bubble state as a function of the disc radius is given in Fig. 2 for three values of the thickness $d$. For $R > R_c$ the bubble has a lower energy than the monodomain state.

We now proceed to a systematic numerical study of the bubble state. We first fix the disc thickness at $d = 8.0$ and vary the radius $R$. We find a bubble state when the radius is larger than some critical radius $R_1 \approx 14$. For smaller radii $R < R_1$ our algorithm always converges to the quasi-uniform state irrespective of the initial condition. Our results thus indicate that the inner bubble domain cannot be sustained if it is too small. For a disc radius slightly larger than $R_1$ the bubble radius is small and the total magnetization of the structure is large. As the radius of the disc increases the inner bubble domain expands and the absolute value of the total magnetization decreases.

The energy per unit volume of the bubble state as a function of the disc radius is given in Fig. 2 for three values of the thickness $d$, along with the corresponding energy for the monodomain state. The latter exists as a local minimum of the energy for any radius up to the largest that we checked. The energy per unit volume of the bubble is greater than that of the monodomain state at the lowest radius $R_1$ where the bubble first appears. It decreases for larger radii and becomes lower than that of the monodomain state above a critical radius $R_c$. It eventually becomes an increasing function of the radius but apparently remains lower than the energy of the monodomain state for all $R > R_c$. We consider $R_c$ as marking the size for the break up of the magnetization configuration into domains. We could find the bubble as a local minimum of the energy for all radii $R > R_1$ that we have checked. However, it is expected that a multidomain state will eventually set in for a sufficiently large radius, with energy lower than the energy of both the monodomain and the bubble.

In order to study the dependence of $R_c$ on the thickness $d$ we have repeated our calculation for a few values of $d$. From Fig. 2 one can extract the critical radii for three values of $d = 6, 8, 15$. In Fig. 3 we give the $R_c$ as a function of $d$ inferred from six values of $d$. For small $d$ the critical radius significantly exceeds the particle thickness because surface effects become important and disfavor the formation of a domain wall. On the other hand, the critical radius $R_c$ levels off for higher values of $d$. The thickness for which $R_c$ attains a minimum appears to be close to $d = 8$ for which $R_c \approx 18$. This is actually the reason for choosing $d = 15$ as our standard parameters along with the fact that for $d = 8$ the energy of the bubble has a minimum at $R \approx 24$ as is seen in Fig. 2.

We have also repeated our calculation for the particle sizes employed in the experiment of Ref. 4 and have confirmed the existence of a bubble state. However, a detailed quantitative comparison cannot be made before one determines the strength of the deposition induced anisotropy in the permalloy used in the experiment.

Once we have established the existence of a bubble we would like to know how it behaves under an externally applied field. Apart from the apparent practical implications, this is an interesting question also because the field will affect the intricate balance of energies that is responsible for the stabilization of the bubble. The field is applied along the symmetry axis of the disc, i.e., it is of the form $h_{ext} = (0, 0, h_{ext})$.

We apply the field on a particle which is already in a bubble state. As a specific example, we choose our standard parameter values (3). Our results are given in Fig. 4. For $h_{ext} = 0$ the total magnetization per unit
The total magnetization $\mu$ per unit volume as a function of the applied field, for a bubble state in a disc of thickness $d = 8$ and radius $R = 24$.

 volume, $\mu = 1/V \int m_z \, dV$, is non-zero. If we choose the inner domain to point down then $\mu$ is positive (Point C in Fig. 4). Applying a positive external field favours the outer domain, which expands at the expense of the inner domain. The system does reach a new equilibrium state which is again a bubble state but with a smaller radius. This corresponds to an increased value for $\mu$. For a high enough magnetic field the bubble becomes too small and it cannot be sustained by the system. In our example $\mu$ jumps to unity for $h_{ext} = 0.13$ which signals that the bubble shrinks to zero radius resulting in a monodomain state with magnetization pointing up (Point B). On the other hand, if we start from point C but now reduce $h_{ext}$ to negative values the inner domain is favoured and pushes the domain wall to a larger radius, which is reflected in a smaller value for $\mu$. The total magnetization $\mu$ crosses zero for $h_{ext} = -0.08$, it then becomes negative and eventually $\mu$ jumps to minus unity for $h_{ext} = -0.26$ which means that the system is in the monodomain state pointing down (Point A). Below $h_{A}$ the domain wall is attracted by the side surface and is expelled from the disc. The dashed line in Fig. 4 corresponds to the equivalent physical situation obtained by the symmetry transformation $\mathbf{m} \rightarrow -\mathbf{m}$, $h_{ext} \rightarrow -h_{ext}$.

The bubble is stable in the range $h_{A} < h_{ext} < h_{B}$ in which reversible behaviour occurs. For $h < h_{A}$ and $h > h_{B}$ irreversible jumps in the magnetization occur which correspond to the domain wall being attracted to the edge (point A) or shrink to the center of the disc (point B). The size of the jumps in the magnetization reflect the size of the domain wall being annihilated (larger when the inner domain expands) and constitutes yet another example of how the geometry of the element constrains the shape of the domain wall and the details of the switching process [16, 17]. On the other hand, if a particle with $R > R_{c}$ is saturated by a strong in-plane field, it will eventually relax into a bubble state after the field is removed [9].

In conclusion, we have studied the fundamental states of disc-shaped magnetic particles with uniaxial anisotropy along the axis of the disc. A magnetic bubble state has been identified within our numerical calculation and has been studied in detail. The bubble is a particularly simple axially symmetric state and has apparently been observed experimentally [9].

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