Demonstration of long-range correlations via susceptibility measurements in a one-dimensional superconducting Josephson spin chain

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Spin chains have long been considered an effective medium for long-range interactions, entanglement generation, and quantum state transfer. In this work, we explore the properties of a spin chain implemented with superconducting flux circuits, designed to act as a connectivity medium between two superconducting qubits. The susceptibility of the chain is probed and shown to support long-range, cross-chain correlations. In addition, interactions between the two end qubits, mediated by the coupler chain, are demonstrated. This work has direct applicability in near term quantum annealing processors as a means of generating long-range, coherent coupling between qubits.

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INTRODUCTION

Superconducting quantum information platforms have reached a level of maturity where tens of individual qubits, comprising a computational device, can provide proof of principle demonstrations of quantum simulations, quantum algorithms, and basic error correction functionality1. As these devices, and the tasks they seek to address, scale in size and complexity, so does the need for realizing qubit networks with increased dimensionality and expanded connectivity. These two desired features of future quantum processors prompt the development of long-range, qubit coherence preserving interactions4–5. Quantum spin chains have been proposed as an effective medium for qubit interactions with these desired properties4–5. In this article, we explore the possibility of long-range interactions supported by quantum spin chains for superconducting qubits10–12. This architecture has direct application in recently proposed quantum annealing platforms based on superconducting capacitively shunted flux qubits13,14, rf-SQUIDs15, fluxonium qubits16, and flux quantum chains23.

Quantum annealing is emerging as a promising paradigm for near term quantum computing16–21. An initial Hamiltonian, whose ground state is straightforward to prepare, is transformed continuously to the problem Hamiltonian. The prepared state of the problem Hamiltonian is located in the vicinity of the true ground state and is a useful solution to the optimization problem. In the limit of weak coupling to the environment, adiabatic quantum computing has been shown to be immune to dephasing in the energy basis, making it a particularly attractive candidate for near term, noisy quantum computing platforms22. Commercial quantum annealers, based on superconducting Josephson flux qubits23–25, have recently become available to the larger community and are beginning to make their mark as a valuable research tool, see e.g., refs.26–28.

There are strong motivations for improving upon the performance of quantum annealing processors29, in particular with respect to how their constituent qubits interact with one another. Increasing both the graph dimensionality of qubit networks23,30, and improving connectivity31, the degree to which the qubits are coupled to one another, would greatly reduce physical hardware overhead by increasing the types and sizes of optimization problems that can be natively embedded. Existing quantum annealing processors based on superconducting qubits possess either nearest neighbor32 or a combination of inter- and intra- unit cell interactions33 between qubits. Commercial annealers, possessing this combination of inter- and intra- unit cell interactions, currently rely on minor embedding34,35, a procedure of extending logical qubits over multiple physical qubits to implement problems that require higher dimensionality or connectivity than the processor’s hardware natively allows.

As each connection made to a qubit introduces additional noise and decoherence channels, expanding qubit connectivity in quantum annealing processors must be balanced against the need to maintain the qubits’ coherence properties. Developing quantum annealing processors that support improved qubit coherence times would allow greater functionality in computation. Higher precision flux control, afforded by improved coherence, is required by many computational problems of interest36–38. In general, just how much of a computational advantage greater qubit coherence provides in quantum annealing processes is itself an open scientific question39,40. Furthermore, more coherent quantum annealers will enable diabatic annealing protocols that require a greater degree of qubit coherence throughout the annealing process41–43.

These two, often competing, improvements—creating qubit networks with higher dimensionality and expanded connectivity and maintaining qubit coherence—call for further development of long-range qubit interactions. One proposed scheme that accomplishes...
this dual need is utilizing spin chains as the qubit interaction medium\(^4,6,7\). Gapped spin chains, in the context of semiconducting quantum dots\(^5,8,9\), have been proposed to support long-range, Ruderman–Kittel–Kasuya–Yoshida (RKKY) type qubit interactions\(^44\). Recent progress in this direction includes a demonstration of adiabatic quantum state transfer along a linear array of four electron spin qubits\(^45\). In addition to the possibility of supporting coherent coupling between two distant qubits, the spin chain architecture lends itself to higher connectivity schemes. Multiple qubits can be simultaneously interacting with a single 1-D chain\(^5\). Additionally, paramagnetic trees, formed by spin chains forking into multiple paths, offer another possible scheme for higher qubit connectivity\(^10–12\).

This work demonstrates the viability of this coupling scheme in the context of superconducting Josephson qubit hardware. In the following we discuss long-range coupling mediated by quantum spin chains in a hardware independent fashion. This is a more natural language to describe long-range coupling as a consequence of the system's underlying quantum phase transition\(^46–48\) (More precisely, these parameters are where the quantum phase transition happens for an infinite system). Following this discussion, we demonstrate a realization of the quantum spin model with superconducting circuits. To accomplish this, we design a system of two qubits, coupled together through a chain of seven spin units. The spin chain, shown in Fig. 1, is realized by a one-dimensional array of seven tunable rf-SQUIDs\(^13,49–53\) inductively coupled to their nearest neighbor through the SQUIDs' main loops. Each end coupler is inductively coupled to a tunable, capacitively shunted, superconducting flux qubit\(^54–57\). Finally, to illustrate the viability of mediating long-range, coherent qubit interactions with our device, we characterize the non-local susceptibility of the coupler chain, demonstrate long-range qubit–qubit interactions, and identify the parameter region where both long-range correlations exist and the detrimental effects of low frequency flux noise are negligible.

The Hamiltonian for the quantum spin chain is the one-dimensional Ising model. Incorporating the two end qubits, it can be written as

\[
H = H_q + H_c + H_{\text{int}}.
\]  

(1)

with

\[
H_q = \sum_{i=1}^{2} \left( \frac{\epsilon_q}{2} \sigma_z^q_i + \frac{\Delta_q}{2} \sigma_x^q_i \right).
\]  

(2)

\[
H_c = \sum_{j=1}^{7} \left( \frac{\epsilon_c}{2} \sigma_x^c_j + \frac{\Delta_c}{2} \sigma_x^c_j \right) + \sum_{i=1}^{6} J_{c_{c_{i-1}}} \sigma_z^c_{c_{i-1}} \sigma_z^c_{c_i},
\]  

(3)

and

\[
H_{\text{int}} = J_{q_c} \sigma_z^q_1 \sigma_z^c_1 + J_{q_2} \sigma_z^q_2 \sigma_z^c_2.
\]  

(4)

In the previous equations, \(\Delta_q/2 (\Delta_c/2)\) and \(\epsilon_q/2 (\epsilon_c/2)\) are the transverse and longitudinal components of the qubits’ (couplers’)
spin while $J_{c,c'}$ and $J_{q,q'}$ represent the coupling strength between adjacent coupler units and between qubits and their nearest coupler unit. For the remainder of the article, we will assume the coupler units are operated homogeneously, that is $\epsilon_c = \epsilon_q$, $\Delta_c = \Delta_q$, and $J_{c,c'} = J_{q,q'}$.

Virtual excitations of the coupler chain can be integrated over to derive an expression for the coupler-chain-mediated effective qubit–qubit interaction strength, $J_{q,q'}^{\text{eff}}$. By considering the qubit-adjacent coupler unit interaction, $J_{c,q',c'}$, to be a weak perturbation to the coupler Hamiltonian, the interaction energy can be calculated to second order as the shift of the ground state energy of the coupler Hamiltonian. As the operating temperature of the device will be much less than the coupler chain excitation energy, it is reasonable to assume that the coupler chain remains in its ground state and the cross-chain interactions are supported by virtual excitations. This is reminiscent of the RKKY interaction whose long-range interaction between magnetic impurities is mediated by virtual excitations of conduction electrons above the Fermi surface.

By taking these above stated approximations into account it is possible to derive an expression for the chain mediated effective coupling strength between the end qubits (see Supplementary Note 1 for calculation details). The effective Hamiltonian is

$$H_{q,q'}^{\text{eff}} = H_q + J_{q,q'}^{\text{eff}} \sigma_q \sigma_{q'}^\dagger \sigma_{q'} \sigma_q^\dagger,$$

where $\Omega_c$ is the energy gap between the coupler chain ground state and first excited state and $|0_c\rangle$ represents the collective ground state of the unperturbed seven unit coupler chain. Note that an exact expression for the effective coupling between qubits contains the integrals of frequency dependent connected coupler correlation functions. The ground state connected correlation function in Eq. (5) is an approximation assuming a large excitation gap, $\Omega_c$, and that the coupler chain excitation frequencies are sufficiently degenerate. This approximation is strictly valid for the coupler chain in its paramagnetic phase, where the transverse field on each coupler unit is much larger than the exchange interaction between coupler units. Writing this expression in terms of the zero temperature, bulk susceptibility of the response function in the Lehmann representation, $\chi_{c,c'}$, the effective interaction can be expressed as

$$J_{q,q'}^{\text{eff}} = \chi_{c,c'} J_{c,c'} \epsilon_c.$$

The main objective of this work is to measure the quantity $\chi_{c,c'}$ as a function of $J_{c,c'}/(\Delta_c/2)$, the ratio of the inter-coupler longitudinal coupling strength, proportional to $\sigma_c^z \sigma_{c'}^z$, to the individual coupler unit transverse field strength, oriented along $\sigma_c^x$, for the homogeneously tuned chain. Long-range coherent coupling becomes possible when the spin chain is tuned to the vicinity of its quantum critical point. In the case presented here, this occurs when the strength of the transverse fields of the coupler spins and inter-unit longitudinal coupling energies between the nearest-neighbor coupler spins become comparable. The coupler chain susceptibility is determined by measuring the response of the longitudinal fields of the coupler units along the chain when a small longitudinal field, $\delta_c$, is applied to the end coupler unit. It is shown that the response then becomes long-range, that is entirely cross chain, for $J_{c,c'}/(\Delta_c/2) \gtrsim 1$, where the system approaches and enters its ordered phase.

The one-dimensional transverse field Ising spin model can be realized by multilevel superconducting Josephson circuits. With the assumption of negligible state occupation of higher levels, the two lowest energy levels of the circuit define the qubit subspace where the transverse and longitudinal components of the unit’s spin can be determined as a function of the applied magnetic flux. The individual qubit and coupler circuits utilize inductive couplings for implementing the inter-unit interactions $J_{c,c'}$, $J_{c,c'}$, and $J_{q,q'}$. Coupling of this type for single unit coupler circuits has been demonstrated in flux qubits, phase qubits, and fluxmons. The design choice of independent coupler circuits, as opposed to direct coupling between qubits, is particularly appealing for use in annealing processors where it is necessary to independently control the qubit properties and coupling strengths. This single unit method of identifying both the qubit and coupler’s spin components is not as universally applicable as the Schrieffer-Wolff transformation, particularly in the strong coupling regime. However, as we will restrict our analysis to the weak coupling limit, the results of the two methods coincide.$^{16-21}$ With these assumptions, the behavior of the physical device can be mapped to the one-dimensional, transverse field Ising spin model.

Each coupler circuit can be approximately characterized by its susceptibility, $\chi$, which is the change in current induced by a biasing flux. Assuming the coupler remains in its ground state, this is equivalent to the curvature of the ground state energy with respect to the flux in the coupler’s main loop.

$$\chi = \frac{1}{L_{\text{eff}}^2} \frac{d^2 E_c}{d^2 f} \approx \frac{d^2 E_c}{d f} \approx \frac{d^2 E_c}{d f^2} (7)$$

In Eq. (7), $\chi$ represents the ground state expectation value of the current in the coupler’s main loop and $E_c$ is the ground state energy of the coupler unit. The character of these two quantities, $\chi$ and $E_c$, is determined by $f_c$, the magnetic flux in the coupler’s small loop. The coupler circuit’s susceptibility, $\chi$, is optimized as the unit’s $\beta_c \equiv L_c/L_{\text{eff}} = 2m_{\text{c}}/\epsilon_c \approx \epsilon_c$, where $\beta_c$ is the local potential minimum is highly sensitive to biasing flux. As shown in Fig. 2, this occurs in the same $f_c$ region where $J_{c,c'}/(\Delta_c/2) = 1$, a design choice made to optimize the generation of long-range correlations across the device. In addition, the device can be operated in a regime such that the coupler’s minimum excitation energy, larger than $5\text{GHz}$, is much greater than the temperature of the system, approximately $400\text{MHz}$, the strength of the qubit–coupler interaction, which is below $1\text{GHz}$, and the typical qubit excitation frequency, approximately $2\text{GHz}$. This ensures the ground state properties of the coupler dictate its behavior, entanglement between qubits mediated by the coupler is supported, and fast (coupler) and slow (qubit) modes can be separated to preserve the qubit subspace.

The same notions of susceptibility and design constraints can also be applied to a long but finite chain of couplers. The current induced in coupler $j$ when a flux is applied to coupler $i$ is expressed as $\chi_{i,j}$, the inter-chain susceptibility. The design constraints are slightly more involved for the chain when compared to a single coupler. For example, the length of the chain has a closing effect on the size of the gap as the fundamental mode frequency of the chain decreases with length. This introduces a trade-off between the physical range of interaction and the need to preserve the excitation energy gap of the chain.$^{52}$

Taking the same concept of single coupler susceptibility to hold for the coupler chain, as well as adhering to the extra constraints introduced by the many-body chain system, it is possible to construct the form of the effective qubit-qubit interaction, Eq. (6), in terms of circuit parameters. Assuming the coupler gap is much greater than the qubit working frequencies allows one to separate the device spectrum into ‘slow’ qubit-like states and ‘fast’ coupler-like states. Invoking the Born-Oppenheimer approximation restricts the coupler spectrum to its unperturbed ground state. Further restricting our analysis to the weak coupling limit, $M_{qq}/L_c \ll 1$, allows us to write the qubit-chain interaction, in general a complicated nonlinear quantity, as an inductive interaction.
that our measurements of the potential energy of the tunable rf-SQUID coupler circuit. When the geometrical inductance dominates, $\beta_c \ll 1$, the potential energy landscape is approximately harmonic. When the Josephson inductance dominates, $\beta_c \gg 1$, the energy landscape becomes double-welled with each minimum representing oppositely circulating current states. Due to the large energy barrier between the states, moderate changes in $f_z$ do not change the current state of the circuit. The coupling is optimized when the geometrical and Josephson inductances are approximately equal. This results in a wide, shallow energy minimum where even slight changes in $f_z$ can induce strong fluctuations between the oppositely circulating current states of the coupler.

Both the single coupler transverse field, $\Delta_f/2$, and the intercoupler interaction energy, $J_{c1c7} = M_{c1c7} r_x^f f_z^c$, are displayed as a function of the coupler $f_z$ when $f_x = f_0/2$. These parameters are calculated in single coupler simulations and then transcribed into spin model parameters. The equality of these two terms appearing in the transverse field Ising model for $f_x \simeq 0.14 f_0$ signals the location of the quantum critical point, in the vicinity of which we expect long-range correlations to emerge. In addition, the dependence of $\beta_c$ is displayed as a function of the coupler’s $f_z$ for $f_x = f_0/2$. By design, the optimum coupling point, $\beta_c \approx 1$, coincides with the coupler $f_z$ value where we expect critical behavior in the coupler chain.

between qubit currents\textsuperscript{16,64}, given by

$$H_{\text{int}} = \chi_{c1c7}(M_{q1c1} r_x^f)(M_{q7c7} r_x^f) = \chi_{c1c7}(\sigma^c_{q1} \sigma^c_{q7}) \sigma^c_{q1} \sigma^c_{q7}. \tag{8}$$

The final line of Eq. (8) connects the circuit model of the coupler chain to the spin chain model by recognizing that the two versions of the susceptibility are related by $\chi_{c1c7} = \chi_{c1c7}^{\text{c1c7}}$. The symbol $r_x^f$ refers to the persistent current of the $f_x$ coupler, which, when operated at $f_x = f_0/2$, is simply the current dipole moment $\langle 0 | \sigma^c_{q1} | \Phi_0 \rangle^2$\textsuperscript{39}.

Now that the long-range, effective interaction between the two qubits is expressed in terms of circuit parameters, it is possible to measure the response function, $\chi_{c1c7}$, of the coupler chain in a quantitative manner. This task is accomplished by performing two similar measurements. Firstly, only the behavior of the coupler chain units is considered. This measurement is performed with both flux qubits placed at a magnetic flux bias operating point where the circuit has its maximum transition frequency which is much greater than the operating frequencies of the remaining chain units. This decouples the qubits from the coupler chain dynamics and allows the coupler susceptibility to be characterized. Secondly, with knowledge of the chain susceptibility, the two flux qubits are brought into an interacting flux operating point and cross chain qubit-qubit interactions are demonstrated. Finally, to quantitatively measure the effective qubit-qubit interaction strength, $J_{\text{eff}}$, we revisit the coupler chain only measurements in more detail to extract the cross chain susceptibility, $\chi_{c1c7}$. We find that our measurements of $J_{\text{eff}}$ through susceptibility measurements agree well with full device simulations of the qubit-qubit spectral line splitting.

**RESULTS**

**Device details and control**

The coupler chain device consists of two capacitively shunted, tunable flux qubits and seven tunable rf-SQUIDs, all equipped with individual readout resonators. Device characterization, circuit parameter extraction, and wiring details are described in Supplementary Note 4. The device is fabricated using the architecture described in ref.\textsuperscript{65}, and consists of two separate chips - called the qubit layer and the interposer layer. The interposer layer, seated on the device package’s printed circuit board cavity and wirebonded to the exterior control lines, holds the flux bias lines. The qubit layer, hosting the qubits, couplers, resonators, and co-planar waveguide, is indium bump bonded atop. The indium bumps provide structural stability, common ground paths between layers, and a conduit for microwave signals originating on the interposer layer, running through the bumps, and continuing on the qubit layer. This device environment allows greater flexibility than planar devices for distributing flux bias lines, represents a step towards full 3-D integration, and supports an electromagnetic environment suitable for quantum annealing controls.

Each unit, qubit or coupler, possesses a meandering resonator terminated in an rf-SQUID for purposes of readout and calibration. These resonators, when their terminating rf-SQUID is biased to a flux sensitive region, act as magnetic flux detectors, capable of discerning the qubit or coupler unit’s persistent current state. When the terminating rf-SQUID is biased to its flux insensitive operating point, the resonator is exclusively sensitive to the unit’s energy level occupation through the resonator-unit dispersive interaction\textsuperscript{66}. Being able to operate in these two modes alleviates the need for multiple readout structures, further freeing up space on chip.

Gaining full flux control of a device of this size is a difficult task for a number of reasons. Complete individual control of each unit requires 27 flux bias control lines corresponding to the 27 Josephson flux loops. Current in one control line provides magnetic flux for its target Josephson loop but also couples to nearby loops. Hence, it is necessary to determine the full $27 \times 27$ element mutual inductance matrix before one can expect adequate control of this device. In addition, these inductive elements need to be determined while in the presence of spurious interactions between units. Strong inter-unit interactions can easily mask the linear line-loop inductive interaction. In order to
address these points, scalable, device independent, automated methods have been developed and implemented to characterize the bias line to circuit flux inductive matrix to within acceptable errors for device control67.

Determination of the cross chain susceptibility

To explore the behavior of long-range qubit interactions mediated by the coupler chain, it is necessary to characterize the inter-coupler susceptibility, \( \chi_{cc} \). To isolate the coupler chain dynamics, we first flux bias the two end qubits to their high frequency, uncoupled state. Every coupler unit is operated such that its main loop is flux biased at one-half magnetic flux quantum and its small loops are uniformly biased with \( f_z \). In this configuration, Coupler 7’s \( f_z \) is swept across its half quantum point for a range of uniformly biased coupler \( f_z \) values. Instead of directly measuring the current response in the target unit, we observe the shift of the target unit’s effective main loop half-quanta point (see Methods). As the unit’s main loop half-flux quantum operating point corresponds to its minimum transition frequency, the dispersive interaction with the unit’s resonator provides an accurate determination of the unit’s effective half-quanta point in the presence of strong inter-unit and unit-resonator interactions.

The magnitude of the induced fluxes for each coupler unit are displayed in Fig. 3. Recall that \( f_z \) simultaneously controls the unit’s transverse field, \( \Delta \phi \), in the Ising spin model picture as well as the magnitude of the unit’s persistent current, thus the longitudinal coupling strength, \( J_{cc} = M_{cc} f_x f_z \). For larger values of \( f_x \), the flux propagating signal attenuates resonator length scale, due to a feeble interaction point. These results agree well with full device simulations of the equivalent protocol allowing us to track the target unit’s effective main loop half-quanta point yielding results that match well with the experimental outcome (see Supplementary Fig. 8).

By coupling qubits via the coupler chain, \( \chi_{cc} \) is swept across its one-half magnetic flux quantum point and its smaller \( f_x \) loop is flux biased such that its transverse field has strength \( \Delta \phi = 2.3 \text{ GHz} \), approximately where the qubit’s potential becomes double-welled. Qubit 2, the source qubit, is placed at its minimum \( \Delta \phi \approx 10 \text{ MHz} \), deep in its double well regime, and its flux bias \( f_z \) is swept across its one-half magnetic flux quantum point. This measurement protocol is repeated for the different coupler chain operating points described in the coupler chain susceptibility experiment.

The results of the qubit–qubit interaction experiment are shown in Fig. 3. This figure displays the magnitude of the flux signal propagating along the coupler chain and ultimately into the opposite qubit. These results agree well with full device simulations of the equivalent protocol. As shown in Fig. 3c, d, long-range, cross chain interactions become supported at approximately \( f_x \approx 0.15 \) – 0.18 \( \Phi_0 \) in both the coupler chain susceptibility and long-range qubit interaction experiments.

Furthermore, the full results of the coupler-only susceptibility measurements can be used to predict the strength of the effective qubit coupling, \( J'_{cc} \) mediated by the chain. Equations (9) and (10) show how the measured coupler susceptibility, \( \chi_{cc} \), determines the effective qubit interaction strength.

\[
J'_{cc} = \chi_{cc} (M_{cc} f_x f_z) (M_{cc} f_x f_z)
\]

\[
\chi_{cc} = \frac{d(J'_{cc})}{df_x} = \frac{d(J'_{cc})}{df_z}
\]

Shown in Fig. 4 are the effective one-half magnetic flux quantum points of Coupler 1’s main loop as a function of Coupler...
by lowering the longitudinal coupling strength increases, the effect of Coupler interaction limit assumed in Eq. (8) breaks down and the effective represents a simple qubit.

Analysis of the impact of noise on cross chain correlations

As demonstrated, the coherence preserving properties of this long-range interaction. Therefore, it is crucial to identify a flux operating regime for the coupler units where strong, long-range coupling is present and the detrimental effects of flux noise are not amplified across the device.

With this goal in mind, full device simulations were performed with realistic values of low frequency flux noise. Flux noise has been measured, across many different platforms and frequencies\textsuperscript{57,60,70} to be approximately $1/\Phi_0$ in nature, $\alpha=0.9$, with magnitude $1-5 \mu\Phi_0/\text{Hz}^{-1/2}$. For moderate frequency measurements, the effect of this low frequency noise is to effectively add a small random flux offset to the flux operating point of the measurement. This small random flux offset is sampled from a Gaussian distribution whose standard deviation is determined by integrating the noise spectrum over the appropriate frequency range, from measurement repetition rate to pertinent experimental frequencies, as well as accounting for the circuit geometry. This amounts to a typical random flux offset in the tens of $\mu\Phi_0$.

Simulations of this type were performed repeatedly to determine the behavior of the device energy level structure in the presence of low frequency flux noise. As shown in Fig. 5, the energy spectrum of the device is highly susceptible to flux noise in its deeply coupled state. Significant line broadening occurs for the uniformly tuned coupler $f_c^\text{eff}$ between 0 and 0.15 $\Phi_0$. This allows us to identify a region of flux operation, coupler $f_c^\text{eff}$ from 0.15 to 0.18 $\Phi_0$ where significant long-range interactions are present yet the detrimental effects of flux noise are still minimal.

Another source of experimental imperfection is caused by device fabrication variations. In particular, expected variations in Josephson junction critical currents from device to device can cause offsets both in the targeted $\Delta$ and inter-unit coupling strengths, causing inhomogeneities across the coupler chain. Note that these errors are not set at fabrication, unlike the flux offsets due to low frequency flux noise which are fluctuating. Spin model simulations of the effective long-range coupling strength and flux propagation experiments were performed with random inhomogeneities added to the targeted values of $\Delta$, $J_{q1c1}$, $J_{q1c7}$, and $J_{q7c1}$. For errors typical of measured devices, both the energy level splitting and flux propagation signal are robust against these imperfections (see Supplementary Note 5).
broaden the lower qubit-like level’s linewidth. For present but where the effects of there have been proposals to generalize one dimensional spin of a superconducting Josephson system. To build on this idea, Presented here is a preliminary step in this direction in the context range qubit interactions that do not degrade qubit behavior. The improved qubit coherence. Accomplishing this will require long-

dimensional qubit networks, expanded connectivity, and

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DISCUSSION
Quantum annealing processors stand to benefit from higher dimensional qubit networks, expanded connectivity, and improved qubit coherence. Accomplishing this will require long-

range qubit interactions that do not degrade qubit behavior. The use of spin chains as a quantum bus is a promising venue for this. Presented here is a preliminary step in this direction in the context of a superconducting Josephson system. To build on this idea, there have been proposals to generalize one dimensional spin

chains, capable of entangling end qubits, to both paramagnetic trees\(^{10,11}\) and two dimensional spin networks capable of providing entanglement amongst a perimeter of qubits\(^{71,12,71}\). However, as we look to scale this coupling architecture to larger processors, there are important scientific questions to answer. In particular, it is an open question as to how the effective coupling scales with respect to the chain length when operating in the weak coupling, paramagnetic regime. Nonetheless, the coupler chain architecture holds promise for use in scalable, coherent quantum annealing devices with high graph dimensionality.

Susceptibility measurements in quantum systems, such as those performed in this study, have been considered as a possible measure of the system’s entanglement\(^7\). The susceptibility experiment’s close agreement with full device simulations, which also demonstrate qubit energy level splitting in the presence of expected noise levels, suggest the qubits can be prepared in an entangled doublet state. In this view, the susceptibility measurements presented here are a consequence and valid measure of a coherent long-range qubit interaction. This view, however, needs further experimental validation. Future experimental work will address measurement of the coherent coupling enabled by this method using spectroscopic characterization as well as adiabatic transfer protocols. Additionally, the detection of entanglement can be augmented by measuring other observables and witnesses for interacting quantum spin systems\(^9\).

In closing, we have demonstrated long-range interactions in a superconducting Josephson spin bus by probing the device’s response function. Simulations of the device, which agree well with measured quantities, predict significant long-range interaction simultaneous with satisfactory qubit coherence. This device has immediate application in near-term quantum annealing devices where both long-range and coherent qubit couplings are necessary for quantum computation speedup.

METHODS

Device details
This device consists of two highly coherent superconducting tunable capacitively shunted flux qubits and seven similarly designed tunable rf-

SQUIDs all equipped with individual readout. These are aluminum devices with Al/Au/Al junctions. The device is hosted in a two-tier environment, illustrated in the Supplementary Fig. 3, and is similar to the architecture described in ref. \(^{10}\). The interposer tier, sitting in the device package and wirebonded to a PCB which routes signals to the exterior control lines, holds the flux bias lines. The qubit tier, hosting the qubits, couplers, resonators, and co-planar waveguide, is Indium bump bonded atop. Both tiers are built out of a thick silicon substrate layer topped by the high quality Aluminum film. This device environment allows greater flexibility in distributing flux bias lines, represents a step towards full 3-D integration, and supports an electromagnetic environment suitable for quantum annealing controls. The device package is gold-plated copper with aluminum wirebonds and installed within a Leiden dilution refrigerator with base temperature of ~ 20 mK.

Experimental protocol
Both the coupler-only susceptibility measurement and full device qubit interaction demonstration experiments were conducted in similar fashion. Either Coupler 7, for the coupler-only, or Qubit 2, in the full device, acted as the source unit. For uniformly tuned coupler units, all couplers at their z-symmetry point and homogeneously tuned x-flux settings, the source unit’s z-flux is swept across its z-symmetry point, causing the source unit to transition from its ‘left’ to ‘right’ circulating current state. This acts to shift the effective z-symmetry point of the remaining units in a manner dependent on the coupler units x-flux.

The response of the target units is measured by observing the dispersive shift in the target units’ resonator. The target units’ resonators are maintained at zero flux, or their high frequency point. The dispersive interaction between the unit and resonator can be used to ascertain the unit’s z-symmetry point. So, for each source unit’s z-flux setting, the target unit’s resonator is probed over a range of frequencies for a range of target

![Fig. 5 Effects of Noise. a Full device simulation illustrating the effects of low frequency flux noise on the qubit-like energy levels. Displayed are the means and standard deviations of the full device energy levels compiling ten separate calculations with the described random flux operating point offsets. The two lowest energy levels are identified as the two initially degenerate qubit levels, \(\left| + - \right>\) and \(\left| - + \right>\). In the limit of no coupling, or large coupler \(f_x\), they reproduce single qubit behavior. As the coupler \(f_x\) is lowered and cross chain coupling becomes significant, the previously degenerate qubit levels, shown in bold, split in the presence of cross chain coupling. Also shown, in lightly faded color, are the few next higher energy levels of the full device consisting of a mixture of coupler and higher qubit levels. Note that the coupler chain and the qubit levels have become comparable in frequency by coupler \(f_x \leq 0.15 \Phi_0\). In this region, what were initially qubit-like energy levels, are now dressed by the coupler levels and the splitting of these two energy states no longer represents the effective qubit-qubit interaction strength \(J\). b Shown is the linewidth of the lower qubit-like level, calculated as the standard deviation of the transition frequency over multiple simulation runs in the presence of realistic flux noise. There is a region, \(f_x = 0.15 - 0.18 \Phi_0\) where significant cross chain coupling is present but where the effects of flux noise do not significantly broaden the lower qubit-like level’s linewidth. For \(f_x < 0.15 \Phi_0\), the calculated linewidth suggests the coherence times of the qubit have deteriorated to the nanosecond scale.](https://example.com/fig5)
unit's z-flux settings. In this way, the target unit's effective z-symmetry point can be tracked as a function of the source unit's z-flux setting.

This procedure was applied iteratively starting with the unit adjacent to the source unit, and then continued down the chain. Applying this process iteratively also allowed us to fine-tune the target units' z-flux. As the coupler units' x-flux is lowered and the inter-coupler interactions become stronger, slight mis-tunings of the unit's z-flux away from their symmetry point can shift the effective symmetry point of nearby units. Applying the measurement/fine-tuning procedure iteratively allows the units to be tuned correctly even in the presence of slight imperfections further down the chain.

Simulations using the platform described above were used to validate the experimental flux signal propagation results. The $f_z$ symmetry point of the target unit is defined as the unit's $f_z$ where the ground state expectation value of the z-loop current equals zero. To track the magnitude of the flux signal, we recorded the difference in the target unit's z-symmetry point when the source unit was 20 m$\Omega$ on either side of the z-symmetry point. This protocol was followed in both simulation and experiment.

DATA AVAILABILITY
The data that support the findings of this study are available from the corresponding author upon reasonable request.

CODE AVAILABILITY
The device simulation, experimental control, and data analysis code that support the findings of this study are available from the corresponding author upon reasonable request.

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AUTHOR CONTRIBUTIONS

D.M.T., A.J.M., and R.T. performed the measurements. D.M.T., X.D., A.J.M., J.A.G., S.M.D., and J.J.B. built the code base supporting the experiment. A.J.M., D.M., and S.N. designed the device. D.M.T., D.M., M.A.Y., Y.T., S.B., and R.Y. provided analysis of results. R.D., D.K.K., A.J.M., B.M.N., and J.L.Y. fabricated the device. S.J.W., A.J.K., E.M., D.A.L., and A.L. supervised the project.

COMPETING INTERESTS

The authors declare no competing interests.

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