Decentralized extended Kalman filter of multi-robot position

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Abstract. This paper will discuss the estimation of the multi-robot position. It may be difficult for all sensors to submit their observations due to communication constraints on the multi-robot system. Thus, in this paper, we will estimate the position of each robot with limited communication between robots using decentralized estimation. Since the multi-robot system is non-linear, the extended Kalman filter is used as an estimation method. The combination of decentralized estimation and extended Kalman filter is called a decentralized extended Kalman filter. In the multi-robot system, each robot is represented as a node that makes its prediction. Assumed that some of the nodes will interact with their closest neighbors, so the measurement information will be transferred. Based on simulation results, the decentralized extended Kalman filter algorithm runs well for estimation in a multi-robot position.

1. Introduction
In recent years, significant developments have taken place in the field of robotics, where researchers have begun to develop their research, from the investigation of a single robot system to a multi-robot system. In practice, the multi-robot system has several advantages.

Localization is one of the most important issues in the study of a multi-robot system, known as position estimation. Multi-robot system communication is an important part of the multi-robot system to estimate the position. In [1], the authors estimate of multi-robot position with distributed communication and combined observation, the approach allows two or more observers to achieve greater effective environmental sensor coverage and enhanced accuracy in estimating multi-robot position. The authors in [2], make an estimate with an extended Kalman filter to track multi-robot movements when the measurement data is minimal.

The centralized estimation integrates model information and real-time measurements for state estimation [3]. However, for certain reasons, such as environment and maps, sometimes communication between robots can not be done as a whole or, in other words, not all robots receive information from other robots. A new approach is needed that can be presented in a multi-robot system called decentralized estimation.

Compared to the centralized scheme, the decentralized Kalman filter [4] allows for the estimation of linear Gaussian state-space models in distributed multi-sensor network configurations without the need for a central communication. In [5] it has been shown that the interlaced extended Kalman filter is a reasonable solution to the problem of estimating the position of a multi-robot with an algorithm of decentralization. In [6], the authors build a decentralized Kalman filter in a sensor network and use a consensus filter to combine information
obtained from other sensors. In [7], the decentralized Kalman filter is capable of tracking the moving target accurately for distributed object tracking.

In this paper, we use a modification of the Kalman filter, extended Kalman filter. Since the multi-robot system is non-linear, we linearized the multi-robot system via the Taylor series to obtain the Jacobi matrices. The matrices are used to compute the covariance and Kalman gain in prediction and update steps.

In the decentralized estimation scheme, each node is making its predictions. We assume that some nodes will transfer the information of measurement with their closest neighbors. Describes the position of each robot and its interactions on the graph so their information can be exchanged. Besides, the decentralized Kalman filter algorithm in [7] is used to complete the decentralized extended Kalman filter.

In the simulation, each of the five nodes of the multi-robot system, initialized with a Gaussian mean equal to zero matrices and covariance is an identity matrix with appropriate size. Perfect communication exists between all nodes for a certain number of time steps, so we can know how close the decentralized results to a centralized one.

2. Multi-robot system
Consider a non-linear of multi-robot system defined by [2].

\[
\begin{align*}
\dot{u}_i &= r_i \cos \omega_i \\
\dot{v}_i &= r_i \sin \omega_i \\
\dot{\omega}_i &= \phi_i
\end{align*}
\]

(1)

with \( u_i, v_i \) is the position of \( i \)-th robot in \( u \) and \( v \) direction respectively, and \( \omega \) is the angle of rotation of \( i \)-th robot, and \( i \) is the number of robot where \( i = 1, 2, ..., N \). For \( i = 1 \), system in (1) is called as the single robot represented in global coordinates as shown in Figure 1.

![Figure 1. Illustration of single robot position.](image)

Given \( u, v, \omega \) which are the system’s global coordinates and robot \( R \) which has state of \( h = [u, v, \omega]^T \). The state variables \( u \) and \( v \) are the direction of the robot, \( \omega \) is the angle between \( u \) and \( u_0 \).
The system in (1) is a continuous multi-robot system. To simplify the process of estimation, control, and computing algorithms, the multi-robot system can be rewritten in the form of a discreet stochastic system because it is assumed that the observed multi-robot system is in an unknown environment, so that ignorance of the environmental conditions can be considered a stochastic disturbance in the system. We get the discretization of the non-linear model (1) in the following:

\[
\begin{align*}
    u_{i,k+1} &= u_{i,k} + \Delta t r_i \cos \omega_{i,k} + \eta_{ui,k} \\
    v_{i,k+1} &= v_{i,k} + \Delta t r_i \sin \omega_{i,k} + \eta_{vi,k} \\
    \omega_{i,k+1} &= \omega_{i,k} + \Delta t \phi_{i,k} + \eta_{wi,k}
\end{align*}
\]  

where \(\eta_{ui,k}, \eta_{vi,k}, \eta_{wi,k}\) are the system process noises assumed to be Gaussian white noises. The discrete model in (2) will be used in the extended Kalman filter algorithm to estimate the position of the multi-robot. In this paper, we use 5 robots \((N = 5)\) and the multi-robot position illustration is given in Figure 2.

**Figure 2.** Illustration of multi-robot position.

From Figure 2, we describe

- \(d_1\) is distance between \(R_1\) and \(R_2\) - \(\theta_1\) is angle of \(R_2\) against \(u_1\)
- \(d_2\) is distance between \(R_2\) and \(R_3\) - \(\theta_2\) is angle of \(R_3\) against \(u_1\)
- \(d_3\) is distance between \(R_3\) and \(R_4\) - \(\theta_3\) is angle of \(R_5\) against \(u_1\)
- \(d_4\) is distance between \(R_4\) and \(R_5\) - \(\Theta_1\) is the angle between \(u_1\) and \(u_2\)
- \(d_5\) is distance between \(R_5\) and \(R_2\) - \(\Theta_2\) is the angle between \(u_1\) and \(u_3\)
- \(d_6\) is distance between \(R_2\) and \(R_4\) - \(\Theta_3\) is the angle between \(u_3\) and \(u_4\)
- \(d_7\) is distance between \(R_2\) and \(R_3\) - \(\Theta_4\) is the angle between \(u_4\) and \(u_5\)
- \(\Theta_5\) is the angle between \(u_2\) and \(u_5\)
From the information above, the measurement model can be created as

\[
d_1 = \sqrt{(u_2 - u_1)^2 + (v_2 - v_1)^2}, \\
\theta_1 = \arctan \frac{-(u_2 - u_1) \sin \omega_1 + (v_2 - v_1) \cos \omega_1}{(u_2 - u_1) \cos \omega_1 + (v_2 - v_1) \sin \omega_1}, \\
\Theta_1 = \omega_2 - \omega_1, \\
d_2 = \sqrt{(u_3 - u_1)^2 + (v_3 - v_1)^2}, \\
\theta_2 = \arctan \frac{-(u_3 - u_1) \sin \omega_1 + (v_3 - v_1) \cos \omega_1}{(u_3 - u_1) \cos \omega_1 + (v_3 - v_1) \sin \omega_1}, \\
\Theta_2 = \omega_3 - \omega_1, \\
d_3 = \sqrt{(u_4 - u_3)^2 + (v_4 - v_3)^2}, \\
\Theta_3 = \omega_4 - \omega_3, \\
d_4 = \sqrt{(u_5 - u_4)^2 + (v_5 - v_4)^2}, \\
\Theta_4 = \omega_5 - \omega_4, \\
d_5 = \sqrt{(u_5 - u_2)^2 + (v_5 - v_2)^2}, \\
\theta_3 = \arctan \frac{-(u_5 - u_2) \sin \omega_2 + (v_5 - v_2) \cos \omega_2}{(u_5 - u_2) \cos \omega_2 + (v_5 - v_2) \sin \omega_2}, \\
\Theta_5 = \omega_4 - \omega_2, \\
d_7 = \sqrt{(u_3 - u_2)^2 + (v_3 - v_2)^2}. \\
\]

We collect the measurements as

\[
g(h_k) = [ d_1 \quad \theta_1 \quad \Theta_1 \quad d_2 \quad \theta_2 \quad \Theta_2 \quad d_3 \quad \Theta_3 \quad d_4 \quad \Theta_4 \quad d_5 \quad \theta_3 \quad d_6 \quad \Theta_5 \quad d_7 ]^T \tag{4} \\
\]

\[
\zeta_k = [ \zeta_{d_1,k} \quad \zeta_{\theta_1,k} \quad \zeta_{\Theta_1,k} \quad \zeta_{d_2,k} \quad \zeta_{\theta_2,k} \quad \zeta_{\Theta_2,k} \quad \zeta_{d_3,k} \quad \zeta_{\Theta_3,k} \quad \zeta_{d_4,k} \quad \zeta_{\Theta_4,k} \quad \zeta_{d_5,k} \quad \zeta_{\Theta_5,k} \quad \zeta_{d_7,k} ]^T \tag{5} \\
\]

\(\zeta_k\) is the measurement noise at time step \(k\), and the measurement model can be written as the sum of (4) and (5).

3. Extended Kalman filter

Considering that the model and measurement processes are non-linear and Gaussian, the extended Kalman filter can be used to estimate the states. The state transition and observation model do not need to be linear in the extended Kalman filter, but can be differentiable functions instead. Given the non-linear stochastic model and measurements as follows.

\[
h_{k+1} = f(h_k, w_k, \eta_k) \\
\begin{align*}
z_k &= g(h_k, \zeta_k) \\
\end{align*} \tag{6}
\]

where \(f, g\) are the state-transition and measurement function respectively, and \(\eta_k, \zeta_k\) are independent Gaussian white noise vectors. Function \(f\) can be used to calculate the predicted
state from the previous estimate, and Function $g$ can also be used to calculate the predicted measurement from the predicted condition. However, $f$ and $g$ can not be extended explicitly to covariance. In a Taylor series, equation in (6) should be linearized, representing the Jacobian matrices.

$$
\tilde{F} = \frac{\partial f}{\partial h},
\tilde{G} = \frac{\partial g}{\partial h}.
$$

For the multi-robot system, we collect the measurements $h_k$, the input $w_k$, and the process noises $\eta_k$ as

$$
h_k = \begin{bmatrix} h_{1,k} \\ h_{2,k} \\ \vdots \\ h_{N,k} \end{bmatrix}, \quad w_k = \begin{bmatrix} r_{1,k} \\ \phi_{1,k} \\ r_{1,k} \\ \phi_{1,k} \\ \vdots \\ r_{N,k} \\ \phi_{N,k} \end{bmatrix}, \quad \eta_k = \begin{bmatrix} \eta_{1,k} \\ \eta_{2,k} \\ \vdots \\ \eta_{N,k} \end{bmatrix}
$$

where $h_i = [u_i, v_i, \omega_i]^T$ ($i = 1, 2, ..., N$). For $N = 5$ we define the Jacobian matrix $\tilde{F}(i, i) = 1$, $i = 1, 2, ..., 15$ and

$$
\begin{align*}
\tilde{F}(1, 3) &= -\Delta t \ r_{1,k} \sin \omega_{1,k}, & \tilde{F}(2, 3) &= \Delta t \ r_{1,k} \cos \omega_{1,k}, \\
\tilde{F}(4, 6) &= -\Delta t \ r_{2,k} \sin \omega_{2,k}, & \tilde{F}(5, 6) &= \Delta t \ r_{2,k} \cos \omega_{2,k}, \\
\tilde{F}(7, 9) &= -\Delta t \ r_{3,k} \sin \omega_{3,k}, & \tilde{F}(8, 9) &= \Delta t \ r_{3,k} \cos \omega_{3,k}, \\
\tilde{F}(10, 12) &= -\Delta t \ r_{4,k} \sin \omega_{4,k}, & \tilde{F}(11, 12) &= \Delta t \ r_{4,k} \cos \omega_{4,k}, \\
\tilde{F}(13, 15) &= -\Delta t \ r_{5,k} \sin \omega_{5,k}, & \tilde{F}(14, 15) &= \Delta t \ r_{5,k} \cos \omega_{5,k}.
\end{align*}
$$

and the other index are equal to zero. For the measurement $\tilde{G}$, we have

$$
\tilde{G} = \begin{bmatrix} C_{1,k} & C_{2,k} & C_{3,k} & C_{4,k} & C_{5,k} \end{bmatrix},
$$

where,

$$
C_{1,k} = \begin{bmatrix}
\frac{u_{1,k} - u_{2,k}}{(u_{1,k} - u_{2,k})^2 + (v_{1,k} - v_{2,k})^2} & \frac{v_{1,k} - v_{2,k}}{(u_{1,k} - u_{2,k})^2 + (v_{1,k} - v_{2,k})^2} & 0 \\
\frac{(u_{1,k} - u_{2,k})^2 + (v_{1,k} - v_{2,k})}{u_{1,k} - u_{2,k}} & 0 & 0 \\
\frac{(u_{1,k} - u_{2,k})^2 + (v_{1,k} - v_{2,k})}{v_{1,k} - v_{2,k}} & 0 & 0 \\
\frac{(u_{1,k} - u_{2,k})^2 + (v_{1,k} - v_{2,k})}{u_{1,k} - u_{3,k}} & 0 & 0 \\
\frac{(u_{1,k} - u_{2,k})^2 + (v_{1,k} - v_{2,k})}{v_{1,k} - v_{3,k}} & 0 & 0
\end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
$$

\[9 \times 1\]
$$C_{2,k} = \begin{bmatrix}
\sqrt{(u_2,k-u_1,k)^2+(v_2,k-u_1,k)^2} & 0 \\
(u_2,k-u_1,k)^2+(v_2,k-u_1,k)^2 & 0 \\
(u_2,k-u_1,k)^2+(v_2,k-u_1,k)^2 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}$$

(11)

$$C_{3,k} = \begin{bmatrix}
\sqrt{(u_3,k-u_1,k)^2+(v_3,k-u_1,k)^2} & 0 \\
(u_3,k-u_1,k)^2+(v_3,k-u_1,k)^2 & 0 \\
(u_3,k-u_1,k)^2+(v_3,k-u_1,k)^2 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}$$

(12)

$$C_{4,k} = \begin{bmatrix}
\sqrt{(u_4,k-u_3,k)^2+(v_4,k-u_3,k)^2} & 0 \\
(u_4,k-u_3,k)^2+(v_4,k-u_3,k)^2 & 0 \\
(u_4,k-u_3,k)^2+(v_4,k-u_3,k)^2 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}$$

(13)

$$C_{5,k} = \begin{bmatrix}
\sqrt{(u_5,k-u_4,k)^2+(v_5,k-u_4,k)^2} & 0 \\
(u_5,k-u_4,k)^2+(v_5,k-u_4,k)^2 & 0 \\
(u_5,k-u_4,k)^2+(v_5,k-u_4,k)^2 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0 \\
\end{bmatrix}$$

(14)
where the measurement for each node of the multi-robot system is described as $L$.

For the decentralized scheme, let $\hat{h}$ be the error initialization of $h$ computed using the measurements up to time step $k$ when the state function $f$ is used in equation [10] with the state function $f$ and measurement function $g$ are linear. In this case, we use $Q_k$, $R_k$, and $P_k$ in diagonal matrix form with appropriate size. The subscript notation $k|k$ is used to indicate association with the (Gaussian) pdf of the state vector $h_k$ at time step $k$ computed using the measurements up to time step $k$.

The algorithm for the extended Kalman filter to estimate multi-robot position with the states of $h_1, h_2, ..., h_N$ at a certain time step. The algorithm is starting with mean and covariance error initialization $\mathcal{N}(h_0; h_{00}, P_{00})$ at time step $k = 0$. First, we go to the prediction step and estimate $\hat{h} = h_{k|k}$ in the update step. The process of predicting and updating runs recursively until certain steps in time.

4. Decentralized extended Kalman filter

In a multi-robot system, there are several robots whose positions have been determined. In this case, 5 robots are used to construct a multi-robot system. When the dynamics of the multi-robot system are quite complex with remote positions, a different approach is needed to be able to estimate the position. The decentralized Kalman filter is a modification of the Kalman filter. Since the multi-robot system is non-linear, we use the extended Kalman filter to get the decentralized scheme, next we call it the decentralized extended Kalman filter.

In the previous section, the centralized extended Kalman filter is obtained by collecting all states and all subsystems (nodes) communicating to obtain complete state information. In this case, not all robots communicate (interaction) with other robots in the multi-robot system so a decentralized scheme in the state estimation issue is required.

In the decentralized extended Kalman filter, each node is making its predictions or collect their measurements $C_{i,k}$ firstly. For simplicity, we assume that each node can use the information from its subsystem. Next, each node processes information received from the connected node to calculate the total estimation. Modifications have been made in the extended Kalman filter update step for the mean $h_k$ and the covariance error $P_k$ in the time step $k$. The results of the modification can be written as follows:

$$P_{k|k}^{-1} = P_{k|k-1}^{-1} + \bar{G}_k R_k^{-1} \bar{G}_k,$$

$$P_{k|k}^{-1} h_{k|k} = P_{k|k-1}^{-1} h_{k|k-1} + \bar{G}_k R_k^{-1} z_k,$$

(16)

For the decentralized scheme, let $L$ be the number of robot (nodes) used to collect measurements, where the measurement for each node of the multi-robot system described as

$$z_{k,l} = \bar{G}_{k,l} h_k + \zeta_{k,l}, \quad l = 1, 2, ..., L,$$

(17)
where $z_{k,l}$ denotes the measurement vectors that collected by node $l$ at time step $k$, $h_k$ is the state vector, and $\zeta_{k,l}$ is a vector measurement noise with Gaussian random with zero mean and covariance matrix $R_k$.

It was assumed that the measurement noise vector is uncorrelated [7]. Since the measurement equation has a block structure, we formulate as

$$G_k^T R_k^{-1} g_k = \sum_{l=1}^{L} G_{k,l}^T R_{k,l}^{-1} G_{k,l},$$

$$G_k^T R_k^{-1} z_k = \sum_{l=1}^{L} G_{k,l}^T R_{k,l}^{-1} z_{k,l}.\tag{18}$$

Substitute (18) into (16), we obtained

$$P^{-1}_{k|k} = P^{-1}_{k|k-1} + \sum_{l=1}^{L} G_{k,l}^T R_{k,l}^{-1} G_{k,l},$$

$$P^{-1}_{k|h_k|k} = P^{-1}_{k|k-1} h_{k|k-1} + \sum_{l=1}^{L} G_{k,l}^T R_{k,l}^{-1} z_{k,l}.\tag{19}$$

Equation (19) represent the update equation of the decentralized extended Kalman Filter [7].

The decentralized extended Kalman filter update step equals the centralized extended Kalman filter update step. The central measuring mechanism was split between the interacted nodes of $L$, so we know that the decentralized extended Kalman filter can be regarded as a counterpart of the centralized extended Kalman filter. The Decentralized extended Kalman filter algorithm is described in Table 1 below. The algorithm in Table 1 is runs on prediction – decentralized -

update recursively.

### Table 1. The decentralized extended Kalman filter algorithm

| System model | $h_k = f(h_{k-1}, w_k) + \eta_k$ |
|--------------|----------------------------------|
| Measurement model | $z_{k,l} = G_{k,l} h_k + \zeta_{k,l}$ |
| Initialization | $N(h_0; h_0^0, P_0^0)$ for $k = 0$, at each node $l$ |

#### Prediction-step

| Estimate | $h_{k|k-1} = f(h_{k-1|k-1})$ |
| Error covariance | $P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k$ |

#### Decentralization-step

| Calculate | $G_{k,l}^T R_{k,l}^{-1} G_{k,l}, G_{k,l}^T R_{k,l}^{-1} z_{k,l}$, and communicate to all other nodes |

#### Update-step

| Error covariance | $P_{k|k} = \left(P_{k|k-1}^{-1} + \sum_{l=1}^{L} G_{k,l}^T R_{k,l}^{-1} G_{k,l}\right)^{-1}$ |
| Estimate | $h_{k|k} = P_{k|k} \left(P_{k|k-1}^{-1} h_{k|k-1} + \sum_{l=1}^{L} G_{k,l}^T R_{k,l}^{-1} z_{k,l}\right)$ |

5. Simulation results

In this section, the simulations are performed to show a centralized extended Kalman filter and decentralized extended Kalman filter of multi-robot position ($u$ and $v$ directions).
Figure 3. Actual multi-robot position.

In Figure 3, we use time step equal to $k = 100$ to simulate the actual position of each robot. Next, we estimate the multi-robot position by centralized extended Kalman filter such as (15). Since all robots have information from others as in Figure 4, we have got a full measurement of the multi-robot system. The initial value and parameters are given by Table 2.

Figure 4. Interconnected of multi-robot system in centralized scheme.

For the decentralized extended Kalman filter, we use the same initial state value and time step as in the centralized extended Kalman filter. We use the decentralized extended Kalman
The decentralized extended Kalman filter is performed at each of the 5 robots. For time step $k < 30$ and $k \geq 80$, since perfect communication occurs between all robots such as in Figure 4, we use centralized extended Kalman filter. For the steps of time $30 \leq k < 80$, the communication of robots is limited. The communication between each robot is described as follows in Figure 5.

![Interconnected of multi-robot system in decentralized scheme.](image)

**Table 2.** The initial state value

| state value | state value | state value | state value | state value |
|-------------|-------------|-------------|-------------|-------------|
| $u_1$: -10 m | $u_2$: 12 m | $u_3$: 10 m | $u_4$: -1 m | $u_5$: -10 m |
| $v_1$: 12 m | $v_2$: -12 m | $v_3$: 10 m | $v_4$: -15 m | $v_5$: -5 m |
| $\omega_1$: $\pi$/4 | $\omega_2$: $-\pi$/2 m | $\omega_3$: $2\pi$/3 | $\omega_4$: $\pi$ | $\omega_5$: $\pi$/5 |
| $\phi_1$: 1.5 rad/s | $\phi_2$: 1.6 rad/s | $\phi_3$: 1.7 rad/s | $\phi_4$: 1.8 rad/s | $\phi_5$: 1.9 rad/s |

has information of measurements from its own and $R_2$.
- $R_2$ has information of measurements from its own, $R_1$, and $R_5$.
- $R_3$ has information of measurements from its own and $R_1$.
- $R_4$ has information of measurement from its own only.
- $R_5$ has information of measurements from its own and $R_2$.

In this simulation, based on Figure 5, the measurement of the multi-robot system consists of $L = 5$ sensor nodes, node 1 has $R_1$ and $R_2$ measurements with dimension 6, node 2 has $R_1$, $R_2$ and $R_5$ measurements with dimension 9, node 3 has $R_1$ and $R_3$ measurements with dimension
6, node 4 only has \( \mathcal{R}_4 \) measurement with dimension 3, and node 5 has \( \mathcal{R}_2 \) and \( \mathcal{R}_5 \) measurements with dimension 6.

First, we collects the information of measurements based on the communication in Figure 5. The measurements for each robots described as

\[
\bar{G}_{k,1} = \begin{bmatrix} C_{1,k}^T & C_{2,k}^T \end{bmatrix}^T \\
\bar{G}_{k,2} = \begin{bmatrix} C_{1,k}^T & C_{2,k}^T & C_{5,k}^T \end{bmatrix}^T \\
\bar{G}_{k,3} = \begin{bmatrix} C_{1,k}^T & C_{3,k}^T \end{bmatrix}^T \\
\bar{G}_{k,4} = \begin{bmatrix} C_{4,k}^T \end{bmatrix}^T \\
\bar{G}_{k,5} = \begin{bmatrix} C_{2,k}^T & C_{5,k}^T \end{bmatrix}^T
\]

(20)

The covariance matrix \( R_k \) is

\[
R_k = \begin{bmatrix} R_{k,1} & 0 & \cdots & 0 \\
0 & R_{k,2} & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & R_{k,5} \end{bmatrix}
\]

(21)

where

\[
R_{k,1} = 3.2 \times 10^{-5} \times [I_{6 \times 6}] \\
R_{k,2} = 3.2 \times 10^{-5} \times [I_{9 \times 9}] \\
R_{k,3} = 3.2 \times 10^{-5} \times [I_{6 \times 6}] \\
R_{k,4} = 3.2 \times 10^{-5} \times [I_{3 \times 3}] \\
R_{k,5} = 3.2 \times 10^{-5} \times [I_{6 \times 6}] 
\]

(22)

It can be seen in the time step \( 30 \leq k < 80 \) Figure 6 shows multi-robot position estimation results at the \( \mathcal{R}_1 \) position with two measurement data i.e. \( \mathcal{R}_1 \) and \( \mathcal{R}_2 \). Figure 7 shows multi-robot position estimation results at the \( \mathcal{R}_2 \) position with three measurement data i.e. \( \mathcal{R}_2 \), \( \mathcal{R}_1 \), and \( \mathcal{R}_5 \). Figure 8 shows multi-robot position estimation results at the \( \mathcal{R}_3 \) position with two measurements data i.e. \( \mathcal{R}_3 \) and \( \mathcal{R}_1 \). Figure 9 shows multi-robot position estimation results at the \( \mathcal{R}_4 \) position with its own measurement. Figure 10 shows multi-robot position estimation results at the \( \mathcal{R}_5 \) position with two measurements data i.e. \( \mathcal{R}_5 \) and \( \mathcal{R}_2 \). In Figure 6-10, for time step \( k < 30 \) and \( k \geq 80 \) all estimation results has similarity to the actual due to perfect multi-robot communication.

To evaluate the accuracy of the decentralized extended Kalman filter, we calculate the relative Mean Absolute Presentation Error (MAPE) between the estimated (centralized and decentralized) and actual data. The formula of MAPE is defined as

\[
E_{rr} = \text{mean} \left\{ \frac{|h_k - \hat{h}_k|}{\max h_k} \right\}.
\]

(23)

The MAPE of centralized extended Kalman filter and decentralized Kalman filter are shown in Table 3.

Because the information in the decentralized extended Kalman filter not as complete in the centralized extended Kalman filter, so the decentralized extended Kalman filter cannot achieve better performance than the best centralized extended Kalman filter. However, the decentralized extended Kalman filter gives a pretty good performance compared with the centralized extended Kalman filter.
Figure 6. Estimation of $R_1$ position.

Figure 7. Estimation of $R_2$ position.
Figure 8. Estimation of $\mathcal{R}_3$ position.

Figure 9. Estimation of $\mathcal{R}_4$ position.
Figure 10. Estimation of $R_5$ position.

Table 3. Comparison of error

| Estimation scheme | Direction | $R_1$  | $R_2$  | $R_3$  | $R_4$  | $R_5$  |
|-------------------|-----------|--------|--------|--------|--------|--------|
| Centralized ($C$) | $u$       | 0.0004 | 0.0003 | 0.0004 | 0.0008 | 0.0004 |
|                   | $v$       | 0.0011 | 0.0007 | 0.0011 | 0.0029 | 0.0037 |
| Decentralized ($D$) | $u$       | 0.0013 | 0.0012 | 0.0025 | 0.0079 | 0.0111 |
|                   | $v$       | 0.0020 | 0.0035 | 0.0022 | 0.0075 | 0.0265 |
| Error difference ($C - D$) | $u$       | 0.0009 | 0.0009 | 0.0022 | 0.0071 | 0.0107 |
|                   | $v$       | 0.0009 | 0.0028 | 0.0011 | 0.0046 | 0.0228 |

6. Conclusion

Due to the environment and communication constraints of robots, measurement data is limited and new approaches are needed to estimate the position of a multi-robot. Decentralized extended Kalman filter is a modification of the Kalman filter that can be used to solve multi-robot position estimation problems with limited measurement data. Each robot in a multi-robot is represented as a node that makes its prediction. Assumed that some of the nodes will interact with their closest neighbors, so the measurement will be transferred. Mathematically, the decentralized extended Kalman filter update step equals the centralized extended Kalman filter update step. Computationally, the central measuring mechanism was split between the interacted nodes.

We use 5 robots with their interactions in the simulation. At time step $30 \leq k < 80$ the interaction between nodes is limited so that we use the decentralized extended Kalman filter at this time and at time step $k < 30$ and $k \geq 80$ the interaction between nodes is perfect so that we use the centralized extended Kalman filter for position estimation. Based on simulation results, the decentralized estimation in particular has a larger error than the centralized ones. The results are reasonable because, in the decentralized scheme, we cannot collect the measurement...
data as complete as the centralized scheme. But these results are pretty good because the error difference between decentralized and centralized extended Kalman filter in position estimation of multi-robot is quite small.

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