Electroweak symmetry breaking and precision data

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We study the impact of LEP2 constraints on the dimensionless coefficients of the electroweak chiral Lagrangian (EWCL) on the precision observables using the improved renormalization group equations. We find that the current uncertainty in the triple and quartic gauge boson couplings can accommodate electroweak symmetry breaking models with $S(\Lambda = 1 \text{ TeV}) > 0$.

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There have been recent attempts to constrain the electroweak symmetry breaking (EWSB) models by analyzing the electroweak $S$, $T$, $W$ and $Y$ parameters [1]. However, these studies have not considered the uncertainty in the triple gauge couplings (TGC) from LEP2 and Tevatron. In this letter, we study implications of the TGC constraints from LEP2 and Tevatron on the parameter space of the nonlinearly realized electroweak chiral Lagrangian (EWCL) by taking into account the logarithmic scale dependence of the chiral coefficients.

Restricting our study on the bosonic sector of the electroweak effective field theory we include all the operators upto mass dimension four in EWCL [2] which contribute to the two, three and four point functions. We confine to consider the set of operators consistent with discrete symmetries, $P$, $T$, and $C$. In a similar study, Bagger et. al. [3] considered operators contributing to two point functions only.

In the framework of effective field theory, the dimensionless chiral coefficients of the EWCL, such as the precision parameter $S$ and $T$, depend on the renormalization scale as

$$O(m_Z)^{\text{exp}} = O(A)^{\text{New Phys.}} + \beta_O \ln \left( \frac{\Lambda}{m_Z} \right).$$

We evaluate $\beta_O$ by including all dimensionless chiral coefficients corresponding to $O(m^4)$ operators in our renormalization group equation (RGE) analysis and take into account the bounds of the TGC from the LEP2 measurements as our input. We have extended our earlier study on computation of one loop RGE using background field technique for SU(2) case [4] to improve upon the existing RGE’s [2, 5] for EWCL and are presented in reference [6].

Before presenting the $\beta$ functions of two point chiral coefficients, we describe the experimental or theoretical bounds of all dimensionless chiral coefficients.

Two point function chiral coefficients are extracted from data collected in Z factories. We perform the analysis with the three best measured quantities $m_W = 80.425 \pm 0.038$ GeV, $\sin^2 \theta_W^{\text{eff}} = 0.23147 \pm 0.00017$ and the leptonic decay width of $Z$, $\Gamma_l = 83.984 \pm 0.086$ MeV for the $S$, $T$ and $U$ fitting. The other inputs used are $1/\alpha_{\text{em}}(m_Z) = 128.74$, $m_Z = 91.18$ GeV, and $m_t = 175$ GeV.

The central values with 1$\sigma$ errors of the $S$, $T$, $U$ parameters are found as

$$S = (-0.06 \pm 0.11) \quad T = (-0.08 \pm 0.14) \quad U = (+0.17 \pm 0.15)$$

which roughly agrees with [8].

The fit is based on one loop calculation and performed using the procedure in reference [1]. In order to make a correspondence with the definition of $S - T$ in EWCL, we subtract the contribution of Higgs boson from the Standard Model (SM) at a reference value $m_H^{\text{ass}}$ as given in [8]. The validity of the subtraction method is checked by observing the independence of Higgs mass in the fit.

The relations among the $S - T - U$ parameters with the chiral coefficients $\alpha_1$, $\alpha_0$, and $\alpha_8$ of EWCL are found to be

$$\alpha_1(\mu) = \frac{S(\mu)}{16\pi}, \quad \alpha_0(\mu) = \frac{\alpha_{\text{EM}} T(\mu)}{2}, \quad \alpha_8(\mu) = -\frac{U(\mu)}{16\pi},$$

which are in complete agreement with those in [2]. Provided we take into account the sign difference for $\beta$ parameter accounted for the calculation performed in Euclidean space [4].

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**References:**

[1] [arXiv:hep-ph/0603038v1](http://arxiv.org/abs/hep-ph/0603038v1), 5 Mar 2006.

[2] [arXiv:hep-ph/0603038v1](http://arxiv.org/abs/hep-ph/0603038v1), 5 Mar 2006.

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[7] [arXiv:hep-ph/0603038v1](http://arxiv.org/abs/hep-ph/0603038v1), 5 Mar 2006.

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Since the $S - T - U$ parameters are defined by Z pole data, Eq. 3 is to be read at $\mu = m_Z$, from which $\alpha_1(m_Z) - \alpha_0(m_Z) - \alpha_3(m_Z)$ are determined.

The three point chiral coefficients $\alpha_2$, $\alpha_3$ and $\alpha_9$ are extracted from the LEP2 W pair production measurements. These three chiral coefficients are related to the experimental observable $\delta k_\gamma$, $\delta k_Z$, $\delta g_Z$ as

$$\delta k_\gamma = -(\alpha_1 + \alpha_8 + \alpha_2 + \alpha_3 + \alpha_9)g^2, \quad (4)$$

$$\delta k_Z = -(\alpha_8 + \alpha_3 + \alpha_9)g^2 + (\alpha_1 + \alpha_2)g^2, \quad (5)$$

$$\delta g_Z = -\alpha_3G^2 \text{ where } G^2 = g^2 + g'^2. \quad (6)$$

Due to the difference in the definition of the covariant differential operator, our triple chiral coefficients have extra signs compared with those in Eq. 2. Current precision on TGC allows us to drop the negligible terms induced through the diagonalization and normalization between Z boson and photon.

There are no experimental data relaxing the custodial symmetry except L3 collaboration 4 from where we take $\delta k_Z = -0.076 \pm 0.064$ as one of the inputs. Other inputs $\delta k_\gamma = -0.027 \pm 0.045$ and $\delta g_Z = -0.016 \pm 0.022$ are taken from LEP Electroweak working group 10, 11. All these data are extracted from one-parameter TGC fits as the two-parameter fits on $\delta g_Z$ and $\delta k_\gamma$ show larger errors while three parameter fits do not exist. We found TGC errors are quite large as reported in D0 collaboration 12 at Tevatron.

Further the most stringent constraints data from LEP2 are preferably analyzed relaxing the custodial $SU(2)$ gauge symmetry as it is natural in the framework of the EWCL to have a non-vanishing $\alpha_9$ if the underlying dynamics break this symmetry explicitly 13. Each of these data corresponds to a set of solution for $\alpha_2(m_Z)$, $\alpha_3(m_Z)$, $\alpha_9(m_Z)$ and are assumed to be extracted from independent measurements. Computing the anomalous TGC in EWCL from these data we get

$$\alpha_2 = (-0.09 \pm 0.14)$$

$$\alpha_3 = (+0.03 \pm 0.04) \rho_{co} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & -7 & -3 \\ -12 & -3 & 1 \end{pmatrix} \quad (7)$$

Correlations among the experimental observables affects the $\rho_{co}$ insignificantly, without changing their central values. We observe that $\alpha_3(m_Z)$ is more tightly constrained than $\alpha_2(m_Z)$ and $\alpha_9(m_Z)$. Anomalous TGC are observed to be one order more constrained w.r.t. the tree level unitary bounds from $f_1 f_2 \rightarrow V_1 V_2$ at $\Lambda \geq 1$ (TeV) 14. $|\delta k_\gamma| < \frac{1.86}{\Lambda^2}$, $|\delta k_Z| < \frac{0.85}{\Lambda^2}$, $|\delta g_Z| < \frac{0.87}{\Lambda^2} . \quad (8)$

The four point chiral coefficients or the quartic gauge couplings (QGC) have no experimental data and usually are assumed to be of order one. Partial wave unitary bounds of longitudinal vector boson scattering processes can be used to put bounds on the magnitude of those chiral coefficients. Absence of Higgs boson or other resonances below the UV cutoff $\Lambda$ renders the form factor of these scattering amplitudes to be energy dependent. We use the following five conditions to constrain five chiral coefficients, $\alpha_4$, $\alpha_5$, $\alpha_6$, $\alpha_7$, and $\alpha_{10}$:

$$|4\alpha_4 + 2\alpha_5| < 3\pi \frac{v^2}{\Lambda^4},$$

$$|3\alpha_4 + 4\alpha_5| < 3\pi \frac{v^2}{\Lambda^4},$$

$$|\alpha_4 + \alpha_6 + 3(\alpha_5 + \alpha_7)| < 3\pi \frac{v^2}{\Lambda^4}, \quad (9)$$

$$|2(\alpha_4 + \alpha_6) + \alpha_5 + \alpha_7| < 3\pi \frac{v^2}{\Lambda^4},$$

$$|\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7 + \alpha_{10})| < \frac{4\pi}{5} \frac{v^2}{\Lambda^4},$$

where the bounds are obtained from $W^+_L W^+_L \rightarrow W^+_L W^+_L$, $W^+_L W^-_L \rightarrow W^+_L W^-_L$, $W^+_L W^-_L \rightarrow Z_L Z_L$, $W^+_L Z_L \rightarrow W^+_L Z_L$, and $Z_L Z_L \rightarrow Z_L Z_L$, respectively.

The $\alpha_1$ and $\alpha_8$ are small in magnitude and are dropped. The contributions of TGC and terms proportional to $v^2/\Lambda^2$ are also dropped here, but are included in the numerical analysis. We have avoided a more strict procedure to derive unitary bounds as shown in 12.

Above is our current knowledge on those dimensionless chiral coefficients. Below we analyze how uncertainty in those dimensionless chiral coefficients can affect the value of $S(\Lambda)$-$T(\Lambda)$-$U(\Lambda)$.

In order to determine the values of $S(\Lambda)$-$T(\Lambda)$-$U(\Lambda)$, we need the RGEs of $\alpha_1$-$\alpha_0$-$\alpha_8$, which are given as $8\pi^2 \left[ d \alpha_i / d t \right] = \beta_{\alpha_i}$ while the $\beta_{\alpha_{1,5,6}}$ are

$$\beta_{\alpha_1} = \frac{1}{12} + 4\alpha_1 g^2 - \alpha_8 g^2$$

$$+ \frac{5}{2} \alpha_2 g^2 - \frac{5}{6} \alpha_3 g^2 + \frac{1}{2} \alpha g^2 \quad (10)$$

$$\beta_{\alpha_1} = \frac{\alpha_6}{2} + \alpha_5 g^2 + 12 \alpha_8 g^2$$

$$+ \frac{5}{6} \alpha_2 g^2 - \frac{1}{2} \alpha_3 g^2 + \frac{17}{6} \alpha g^2 \quad (11)$$
where we observe that all TGC contributes to the $\beta_{\alpha_0}$, while QGC contributes only to the $\beta_{\alpha_0}$. This implies QGC do not contribute to the $S$ parameter. The $S(\Lambda)$, $T(\Lambda)$, and $U(\Lambda)$, are computed from the evolved $\alpha_1(\Lambda)$-$\alpha_0(\Lambda)$-$\alpha_8(\Lambda)$ through RGE. The $S(\Lambda)$, $T(\Lambda)$ and $U(\Lambda)$ are the values of the parameters $S$, $T$, and $U$ at the matching scale $\Lambda$, where the EWCL matches with fundamental theories, Technicolor models, extra dimension models, Higgsless models, etc.

How does the uncertainty of TGC affect the value of $S(\Lambda)$? To answer this question we set all QGC to zero at $\Lambda = m_Z$ to the study the effect of TGC on $S(\Lambda) - T(\Lambda)$ plane which is depicted in Fig. 1. We highlight some features of this figure.

(1) In absence of TGC contribution (red contours), $S(\Lambda)$ becomes more negative as $\Lambda$ increases w.r.t. the reference LEP1 fit contour at $\Lambda = m_Z$. This is in agreement with the observation of Ref. [8] and Ref. [11]. Inclusion of TGC contribution as obtained from LEP2 fit (Eq. 7), makes $S(\Lambda)$ almost unchanged (the solid line).

(2) We observe that when TGC with 1σ uncertainty at $\Lambda = m_Z$ are taken into account, $S(1\text{ TeV})$ can vary between $-0.3 \leq S(1\text{ TeV}) \leq 0.12$ which is almost $3\sigma$ away from the prediction of $S(1\text{ TeV})$ without these uncertainties. Analysis with Tevatron data and LEP2 two dimensional TGC fit data would exceed this limit dramatically.

(3) The TGC contributions can at most lower the value of $T(1\text{ TeV})$ by $|\Delta T(1\text{ TeV})| \approx 0.1$. Thus the contribution of TGC is not large enough to cancel the large leading contribution from $3g^2/8$ in the $\beta$ function of $T$ parameter, which makes $T(\Lambda)$ positive for high energy.

Experimental data on the TGC allows the radiative mechanism to render large +ve $T(\Lambda)$. To realize vanishing $T(\Lambda)$ with QGC switched off would require $T(m_Z)$ to be negative $\approx -0.4$ or so, which is in confrontation to the global fit value given in Eq. 2.

Whether is it possible to find a solution in the parameter space of the EWCL? To answer this question, it is worthwhile to understand the evolution of the beta functions of QGC affecting $T(\Lambda)$ parameter. We observe that $\alpha_4, \alpha_5$ terms come along with $g^2$, making them one order weaker w.r.t. those of $\alpha_6, \alpha_7$, and $\alpha_{10}$. Assuming unitarity bounds on all anomalous QGC would be of the same order and $\alpha_{10}$ to dominate among the total QGC contribution. We find that $|\alpha_{10}|$ has to be $\geq 0.03$ to switch the sign of $T(1\text{ TeV})$, which is contradictory to the unitary bound given in Eq.

$$
\beta_{\alpha_0} = -\frac{3g^2}{8} + \frac{9\alpha_0g^2}{4} - \frac{9\alpha_0g^2}{4} + \frac{1}{4} \frac{3g^2}{8} - \frac{\alpha_8}{8} g^4 \\
+ \frac{\alpha_2}{2} \left( \frac{3g^2}{4} - \frac{3g^4}{4} + \frac{1}{2} \frac{3g^2}{2} + \frac{3g^4}{4} \right) + \frac{\alpha_3}{2} \frac{3g^2}{4} + \frac{\alpha_9}{2} \left( -\frac{g^4}{2} + \frac{3g^2g^2}{4} \right) \\
- \frac{\alpha_4}{7} \frac{15g^2g^2}{4} + \frac{15g^4}{8} - \frac{\alpha_5}{2} \left( \frac{3g^2g^2}{2} + \frac{3g^4}{4} \right) \\
- \frac{\alpha_6}{4} \frac{3g^4}{4} + \frac{3G^4}{8} - \alpha_7 \left( 3g^4 + 3G^4 \right) - \alpha_{10} \left( \frac{9G^4}{2} \right),
$$

(12)
at $\Lambda = 1$ TeV with $v = 246$ GeV.

The reason for the subdominant behavior of QGC couplings with increasing energies can be explained from the Table 1. We realize that with the increasing $\Lambda$ the TGC uncertainty $\delta T^{\text{TGC}}$ increases logarithmically while the QGC uncertainty $\delta T^{\text{QGC}}$ decreases rapidly due to the power dependence in the unitary bounds given in Eq. (9). Consequently it is observed that $\delta T^{\text{QGC}}$ and $\delta T^{\text{TGC}}$ dominates the error of $T(\Lambda)$ below and above $\Lambda < 950$ GeV, respectively.

| $\Lambda$ (TeV) | $T(\Lambda) \pm 1\sigma$ | $\delta T_Z$ | $\delta T^{\text{TGC}}$ | $\delta T^{\text{QGC}}$ |
|-----------------|----------------------------|-------------|-------------------------|-------------------------|
| 0.3             | $0.25 \pm 8.91$           | $\pm 0.14$ | $\pm 0.06$              | $\pm 8.91$             |
| 0.5             | $0.29 \pm 1.16$           | $\pm 0.14$ | $\pm 0.08$              | $\pm 1.15$             |
| 1               | $0.40 \pm 0.22$           | $\pm 0.14$ | $\pm 0.12$              | $\pm 0.10$             |
| 3               | $0.60 \pm 0.25$           | $\pm 0.14$ | $\pm 0.17$              | $\pm 0.04$             |

TABLE I: Values of $T(\Lambda)$ and $1\sigma$ errors from $\delta T_Z$, $\delta T^{\text{TGC}}$ and $\delta T^{\text{QGC}}$.

From Table 1, we can conclude that in the constrained EWCL parameter space with $1\sigma$ error in TGC and with unitary bounds on QGC, it is unlikely to have a scenario with vanishing $T(1 \text{ TeV})$ while keeping $T(m_Z) = -0.08$. It is worth mentioning that performing the analysis with two-parameter TGC fits $\delta T^{\text{TGC}}$ becomes larger while $\delta T^{\text{QGC}}$ changes insignificantly. However, there are possible ways to evade this situation: (1) Lowering the UV scale $\Lambda$ down to 700 GeV or so, (2) Relaxing the error of $T(1 \text{ TeV})$ to $2\sigma$ or so, and (3) Generating a large enough positive $T(\Lambda)$ from more fundamental dynamics, as proposed in most Technicolor models when matched with the effective theory.

We summarize our study and conclude that LEP2 data has constrained the anomalous TGC (three point chiral coefficients), but allows regions where the $S(m_Z)$ parameter can be explained by the radiative corrections of the TGC accompanying with a positive $S(\Lambda)$. This letter shows that the negative $S(\Lambda)$ parameter problem can be related to the loosely constrained large anomalous TGC ($\alpha_2$ and $\alpha_0$). With the current experimental and theoretical knowledge, TGC’s and QGC’s uncertainty can undermine our prejudice for discarding or accepting a specific EWSB model. However the upcoming colliders, with higher sensitivity to the TGC, can reduce the parameter space and help to pinpoint the correct model of EWSB.

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