In the process of extracting the characteristic frequency of the fault signal of the rolling bearing in the mechanical system, the signal transmission paths between the fault points inside the bearing and the sensor are diversified, which will produce compound faults of different forms and combinations. These composite faults will lead to a series of serious consequences such as distortion and aliasing of the fault characteristic spectrum. In order to solve these problems, this paper proposes a composite fault feature separation method for rolling bearing based on the mixed function decoupling model. Firstly, the functional series model of the fault system is established, and some kernel functions of functional series are obtained. Secondly, the coupling frequency relationship is established and the recursive search algorithm of frequency energy is applied to decouple the compound fault coupling frequency into a single fault frequency feature set. Finally, the threshold method of fault characteristic frequency energy entropy is utilized to optimize the single fault frequency feature set and identify the true and false features, so as to identify each characteristic frequency point of the composite fault. Experimental results show that, under the condition of low SNR, the proposed method can decouple no more than three typical composite fault signals without depending on the signal filter.

1. Introduction

Rolling bearings are one of the important parts of mechanical equipment. They are widely used in processing, transmission, and other types of mechanical equipment. Due to the harsh working environment of some mechanical equipment and drastic load changes, rolling bearings are prone to various types of failures. These situations may lead to equipment failure, which not only leads to a decline of economic benefits but may also result in unnecessary casualties. Although the fault diagnosis for rolling bearing is concerned extensively, most of the relevant researches are merely focused on a single type of fault diagnosis, which leads to early failure detection being ignored. This is of particular importance since initial failure characteristics are often accompanied by a series of weakly mixed failures from multiple sources [1]. Therefore, it is of great significance to detect and diagnose rolling bearing faults and improve the safe operation of the equipment [2].

Due to the complex operating conditions in the actual engineering environment, rolling bearings in medium and large mechanical equipment tend to have time-varying weakly mixed faults from multiple fault sources. Furthermore, rolling bearing fault frequency coupling, multipath transmission, various types of mechanical impact, and noise are altered with time. There exists strong nonlinear coupling characteristic in the frequency of fault vibration signals, resulting in that the fault frequency point aliasing is hard to be decoupled. Therefore, the issue on the decoupling of
composite fault features with multiple fault frequency coupling still remains open.

Currently, vibration analysis is used to detect single fault features of rolling bearing. Various signal processing methods are used to extract the fault features, such as signal filtering, time-domain analysis, and frequency domain analysis. To list a few, Xu [3] performed signal processing on the basis of SET time-spectrum and proposed a reconstruction method of SET signal components based on Order Statistical Filter (OSF). Zhao et al. [4] proposed a fault feature extraction method for rolling bearings based on Complementary Ensemble Empirical Mode Decomposition (CEEMD). Zhang [5] applied band-pass filtering and envelope on the time-domain signal to improve the signal-to-noise ratio so as to realize the fault diagnosis of rolling bearing under variable speed conditions. Zhang [6] used pattern recognition by constructing time-domain and frequency domain space state matrices. Zhou [7] proposed an improved convolutional neural network fault diagnosis method to study the multistate feature information of rolling bearing faults. Dai [8] effectively separated the fault characteristics of different vibration signals by calculating the sample quantile arrangement entropy of vibration signals. Then the entropy value is composed of eigenvectors, and a classifier is constructed to realize the fault diagnosis of rolling bearings. Wang et al. [9] established a Volterra series model to extract the vibration signal information. Then, they applied the model for the fault diagnosis of the inner ring of rolling bearings. For single faults, the model shows promising results. However, a single fault of a rolling bearing may lead to other faults, and the coupling between faults may increase the difficulty of single fault diagnosis methods. Therefore, the required theoretical frequency points cannot be obtained when using the above method in the actual working environment; consequently, an increased number of researchers focus on fault coupling and decoupling information problems required to meet the needs of complex mechanical system fault diagnosis.

The coupling between complex faults makes it is not easy to extract the fault information. Most solutions focus on pattern recognition [10], identifying fault by means of the fault information as well as expert knowledge base, and establishing models for solving the mixed fault classifier group. Han et al. [11] proposed rolling bearing fault identification based on variational mode decomposition (VMD) and autoregressive (AR) model parameters. Liu et al. [12] studied the correlation vector machine optimized by quantum genetic algorithm and applied it to the Volterra fault diagnosis of rolling bearings. Lei et al. [13] established a hybrid intelligent diagnosis model with the multisymptom domain feature set and multiple classifiers. The model achieved desirable results in the application of mixed locomotive faults. Jiang et al. [14] proposed a new method based on empirical wavelet transform-Duffin oscillator (EWTDOD) for decoupling diagnosis of composite faults of rolling bearings. He et al. [15] presented an adaptive redundant multiwavelet packet (ARMP) method for compound fault diagnosis to detect the compound faults of bevel and planetary gearboxes. Li et al. [16] used Differential-Based Ensemble Empirical Mode (DEEMD) for multifault decoupling of the turbine expander rotor system. EMD can also adaptively detect IMF from signals, while problems of model mixing or overestimation may occur during the decomposition process [17]. The decoupling process of composite fault signals requires signal separation technology and fault characteristic parameter index with a distinguishing degree.

Considering the advantages of previous research of Volterra series models, the research object is the gears in bearings or planetary gearboxes. By building a Volterra series model, a single fault can be distinguished. In fault diagnosis, Volterra series model can solve the problem by considering the second or third order. In mixed faults, the Volterra series model of a single fault is truncated to solve the low-order kernel numerical calculation problem. This method reduces the amount of calculation and simplifies the series structure, which leads to the gap between the established model and the actual one. When a mix of faults occurs inside the device, the faults interact with each other and with the noise, resulting in the need for more cores for analysis. In order to make the model as realistic as possible, the order needs to be continuously increased, which leads to an exponential growth in the number of Volterra series kernels [18]. Since frequencies can be represented by kernels of Volterra series, the curse of dimensionality encountered in the computation and identification of kernels translates into a frequency problem. Then, the fault is diagnosed by analyzing the frequency relationship between the faults.

Based on the research on Volterra, a decoupling diagnosis method for generalized frequency mixed faults is proposed for rolling bearings. The purpose of this method is to diagnose mixed faults based on the frequency coupling relationship. First, the theoretical analysis based on Volterra series multifault frequency is modeled, and the frequency relationship is established by the multifrequency signal method. Then, the vibration signal is collected for envelope analysis, and the set that satisfies the frequency relationship is found according to the envelope diagram. Next, the frequency base is obtained by the traversal method. Finally, the correct frequency base is obtained according to the information entropy, the fault frequency is determined, and the related process of traditional fault diagnosis is carried out.

The rest of this paper is arranged as follows. In Section 2, the multifault frequency Volterra series modeling theory and power spectrum entropy analysis are presented. Moreover, the multifault frequency extraction method is introduced and the optimal frequency is determined using the power spectrum entropy. In Section 3, the specific steps of the proposed method are provided and the simulation results are presented to prove the effectiveness of the proposed scheme. In Section 4, experimental verification is carried out, which further proves the reliability of the method. The conclusions are included in Section 5.
2. Preliminary Knowledge and Research Methods

2.1. Multifault Frequency Volterra Series Modeling Theory

Volterra series theory was first applied to mathematical functional analysis and then gradually extended to other fields [19]. Mathematically, the Volterra series is essentially the expansion of the functional series of nonlinear time-invariant systems. It can also be regarded as the extension of the one-dimensional convolution of linear systems on the multidimensional convolution space. For weakly nonlinear systems, the Volterra series can approximate the nonlinear systems, the Volterra series can be divided into the generalized impulse response function and generalized frequency response function, both of which can be applied in engineering.

The output of a weakly nonlinear system can be described by a series:

\[ Y(t) = \sum_{n=0}^{\infty} Y_n(t), \]  

where

\[ Y_n(t) = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, ..., \tau_n) \prod_{i=1}^{n} u(t - \tau_i) d\tau_i, \]  

where \( Y_n(t) \) is the \( n \)-th order output response of the system, \( u(t - \tau_i) \) is the input of the system, and \( h_n(\tau_1, \tau_2, ..., \tau_n) \) is the impulse response function of the system or the \( n \)-order time-domain kernel of the nonlinear system.

When analyzing a linear system, in order to understand the essence of a weak nonlinear system more easily, the system characteristics can be expressed by using the frequency response function of the system. The Volterra series can be divided into the generalized impulse response function and generalized frequency response function, both of which can be applied in engineering.

The multidimensional Fourier transform of the time-domain kernel \( h_n(\tau_1, \tau_2, ..., \tau_n) \) of the Volterra series can be expressed as

\[ H_n(\omega_1, \omega_2, ..., \omega_n) = \int_{-\infty}^{\infty} ... \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, ..., \tau_n) e^{-j(\omega_1\tau_1 + \omega_2\tau_2 + ... + \omega_n\tau_n)} d\tau_1 d\tau_2 ... d\tau_n, \]  

where \( H_n \) is the \( n \)-th kernel frequency domain of the nonlinear system or the frequency response function of the system.

The multidimensional convolution of linear systems on the multidimensional convolution space. For weakly nonlinear systems, the Volterra series can approximate the nonlinear system in finite order and completely reflect the characteristics of the system, and its time domain and frequency domain cores have clear physical significance. Volterra series can be divided into the generalized impulse response function and generalized frequency response function, both of which can be applied in engineering.

When analyzing a linear system, in order to understand the essence of a weak nonlinear system more easily, the system characteristics can be expressed by using the frequency response function of the system. The Volterra series can be divided into the generalized impulse response function and generalized frequency response function, both of which can be applied in engineering.

If the input signal is assumed as a multifault signal, the frequency domain of the nonlinear system in the frequency domain can be expressed as

\[ Y(\omega) = \sum_{n=1}^{\infty} Y_n(\omega), \]  

\[ Y(\omega) = \frac{1}{(2\pi)^n} \int_{-\infty}^{\infty} H_n(\omega_1, ..., \omega_n) \prod_{i=1}^{n} U(\omega_i) d\sigma_m, \]  

where \( \sigma_m \) represents the integration domain of frequency domain satisfying \( \omega_1 + ... + \omega_n = \omega \), \( \omega_n \) is the frequency of the signal, and \( H_n(\omega_1, ..., \omega_n) \) is the kernel function of the system. Frequency \( Y(\omega) \) and \( U(\omega) \) are the result of the Fourier transform from \( y(t) \) and \( u(t) \).

The general methods to solve the kernel are least square method, impulse response method, and neural network. Based on the author’s previous research [20, 21], the kernel function can be solved in this paper.

Generally, in a rolling bearing mixed fault of mechanical equipment, the output frequency component contains the fundamental and fault and coupling frequencies, as shown in Figure 1.

If the input signal is assumed as a multifault signal, the frequencies of fault signals are different, but the values are similar. Thus, the input signal can be composed of \( L \) frequencies, which can be expressed as

\[ u(t) = \sum_{i=1}^{L} 2a_i \cos(\omega_i t + f_i). \]  

According to Euler’s formula, (5) can be rewritten as

\[ u(t) = \sum_{i=1}^{L} a_i e^{j(\omega_i t + \theta_i)}. \]  

If \( A_i = |A_i|e^{j\theta_i}, A_i^* \) is the conjugate complex number of \( A_i = A_i^* \), \( \omega_i \) is the frequency of the signal, then

\[ u(t) = \sum_{i=0}^{L} A_i e^{j\omega_i t}. \]  

Under the action of this input signal, the corresponding system output reaches

\[ y_n(t) = \sum_{s_1=0}^{L_1} \sum_{s_2=0}^{L_2} \sum_{s_3=0}^{L_3} ... \sum_{s_n=0}^{L_n} A_{s_1} A_{s_2} ... A_{s_n} H_n(j\omega_{s_1}, j\omega_{s_2}, ..., j\omega_{s_n}) e^{j\omega_{s_1} t}, \]  

where \( \omega_{s_k} = \omega_{s_1} + \omega_{s_2} + ... + \omega_{s_n} \).

By combining (7) and (8), \( \omega_k \) can be given by

\[ \omega_k = \sum_{i=0}^{L} \omega_i = \sum_{i=1}^{L} (m_i - m_{s_1}) \omega. \]  

According to the relationship of nuclear symmetry, one has
where \( m_i \) is a nonnegative integer.

If the included frequency contains \( \omega_1, \omega_2, \ldots, \omega_L \) in the input signal, according to (9) and (10), it can be deduced that the first-order kernel of system output contains frequency components \( \{\omega_i\}_{i=1}^{L} \). The second-order kernel should contain frequency components \( \{2\omega_i, |\omega_i \pm \omega_j|\}_{i,j=1}^{L} \), while the third-order kernel should contain frequency components \( \{3\omega_i, |\omega_i \pm 2\omega_j|, |\omega_i \pm \omega_j \pm \omega_k|\}_{i,j,k=1}^{L} \) when \( i \neq j \neq k \), \( i,j \neq k \), \( i \neq k \neq j \), \( i\neq j \neq k \), and \( i,j \neq k \).

According to the multifrequency relationship of series, when the fundamental frequency and the fault frequency of the system are taken as input signals, the frequency spectrum of the system output signal will contain a fundamental frequency, fault frequency, and various coupling frequencies between the rolling bearing fault frequencies. The set of all coupling frequency relations between fundamental frequency and fault frequency can be found through the frequency relations that can be deduced from (9) and (10), and then the decoupling analysis can be carried out one by one. Due to the relative complexity of the multifault coupling relationship, the actual frequency combination that satisfies the coupling frequency relationship in the system output signal spectrum is not unique. However, the type of fault is unique and certain; i.e., only one of the found frequency base sets satisfies the requirements. Therefore, it is indispensable to evaluate the frequency base set for effective and correct screening. In this paper, information entropy can be used to discriminate the power spectrum entropy of frequency base.

2.2. Related Knowledge of Entropy. For system faults, the signal spectrum contains a lot of fault information. Information entropy has strong characterizing ability in characterizing the signal, so information entropy is used to describe the characteristic quantity of a signal only by the signal amplitude information. It is necessary to combine modern signal processing methods with information entropy. As one of the signal processing methods, power spectrum entropy has often been used in practical engineering. For evaluation purposes, the power spectrum entropy model is selected to describe possible signal combination feature sets between the rolling bearing complex fault coupling frequencies separated and obtained by the proposed method.

The steps of feature extraction of power spectrum entropy are as follows:

Step 1: The time-domain signal is set as \( x(t) \), and the discrete Fourier transform leads to
\[
X(\omega) = \frac{1}{2\pi N} \sum_{t=1}^{N-1} x(t)e^{-j\omega t}.
\]

Step 2: The power spectrum is estimated as
\[
S(\omega) = \frac{1}{2\pi N} |X(\omega)|^2.
\]

Step 3: Calculate the power spectrum entropy \( H_F \).

According to the energy conservation of the signal transformed from the time domain to the frequency domain, it can be obtained that
\[
\sum x^2(t)\Delta t = \sum |X(\omega)|^2 \Delta \omega.
\]

It can be known from (14) that the total energy of the signal is equal to the sum of the energy of each frequency component. Thus, the power spectrum can be regarded as a measure of division \( S = \{s_0, s_1, \ldots, s_{N-1}\} \).

\[
H_F = -\sum_{i=0}^{N-1} q_i \log q_i = \frac{s_1}{\sum_{i=0}^{N-1} s_i},
\]

where \( q_i \) is the proportion of the \( i \)-th spectrum in the entire spectrum.

The information entropy method can be used to describe the characteristics of operating state of system. The power spectrum entropy model is an important signal description method for frequency domain information entropy, which describes the distribution of the signal energy within the entire frequency domain. If the signal energy is more evenly distributed in frequency components, the signal is more complex and irregular. The obtained signal power spectrum entropy is also larger. On the contrary, if the signal has a higher degree of order and power, the spectral entropy value is decreased. Therefore, the information entropy method of the collected signal is evaluated in this paper.

2.3. Complex Fault Feature Separation Method. Combined with the theoretical knowledge described in 2.1 and 2.2, taking the rolling bearing as the research object, the fault signal is collected. Then, the frequency satisfying the frequency relationship is obtained according to the frequency conditions. For nonstationary signals, it is inadequate to describe the characteristic quantity of a signal solely by the signal amplitude information. It is necessary to combine modern signal processing methods with information entropy. As one of the signal processing methods, power spectrum entropy has often been used in practical engineering. For evaluation purposes, the power spectrum entropy model is selected to describe possible signal combination feature sets between the rolling bearing complex fault coupling frequencies separated and obtained by the proposed method.

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relationship of the series, and the frequency base satisfying
the frequency relationship is obtained by the traversal
method. Due to the relative complexity of the signal cou-
lping, the frequency base selected only according to the
frequency relationship is not unique. In this paper, the
power spectrum entropy is used to evaluate the optimal
frequency base. Finally, correct frequency base is found; the
fault is accurately identified based on the frequency. In
Figure 2, the flow of the above process is shown.

The selection steps of the method are as follows.

Step 1. The vibration signal \( x(t) \) is collected, envelope
analysis on \( x(t) \) is conducted, and the envelope signal is
obtained for the spectrogram.

Step 2. The basic frequency \( \omega_j \) is determined. The frequency
base should include \( n \) frequencies, the order of the selected
frequency, and the termination condition.

Step 3. According to the conditions determined in Step 2,
the frequencies included in the spectrogram are sequentially
selected from the maximum amplitude value of \( n-1 \) fre-
quencies. Then, the frequency base \( \omega_k \) is determined, where
\( \omega_k = \{ \omega_i, \omega_j \} \quad i = 1, 2, \ldots, n-1 \) is selected as the frequency
base \( \omega = \{ \omega_k | k = 1, 2, \ldots, l \} \) that meets the frequency
relationship.

Step 4. If the selected frequency base \( \omega_k \) does not find a set in
the envelope diagram that satisfies the frequency rela-
tionship or the frequency is already a coupling relationship in the
selected frequency, the selected \( \omega_k \) is directly eliminated.

Step 5. All frequency bases that satisfy the frequency rela-
tionship are found, and the selection is terminated. In order
to determine the reliability and practicability of the method,
the simulation signal is taken as an example to introduce the
specific steps of the method.

3. Simulation Analysis Verification

The input signal is set as \( x(t) \) in MATLAB, and the output
signal is set as \( y(t) \). The input signal consists of two fault
signals with different frequencies and Gaussian white noise
with low signal-to-noise ratio, simulating the rolling bearing
signal mixed into the noise. Among them, the rotation
frequency of the rolling bearing is 0.2 Hz, the noise signal-to-
noise ratio is -6 dB, and the fault signal is a pulse signal with a
frequency of 4 Hz and 5 Hz. After a nonlinear system, the
output \( y(t) \) is obtained. The simulation graphs are shown in
Figures 3–5.

In Figure 3, the signal input is shown. The signal is a
simulated signal that simulates a fault. The basic frequency
\( f_0 \) is set to 0.2 Hz. Figure 4 represents the output \( y(t) \) of the
signal passing through the nonlinear system, and Figure 5
represents the frequency spectrum of the output signal \( y(t) \)
after the envelope analysis. \( f_{\text{max}} \) represents the frequency
point of the highest amplitude, \( f_{\text{max}} = 1 \) Hz.

The frequency selection step is performed according to
the selection principle of the ergodic method. The basic
frequency is known to be 0.2 Hz, and the selection starts
from the highest frequency amplitude, i.e., 0.8 Hz, 1 Hz.
Based on equations (7)–(9), the first three-order frequency
sets of 0.2 Hz, 0.8 Hz, and 1 Hz are obtained. By comparison
with envelope analysis, it is found that the frequency set of
this group meets the requirement of frequency base. Then go
down and find all sets that satisfy the frequency relation. The
results are shown in Table 1.

The power spectrum entropy value is calculated
according to (15), and the calculation results are shown in
Table 2. As can be seen from the table, group c has the largest
entropy value. The greater the entropy, the greater the signal
instability, i.e., the greater the possibility of failure. Thus, it
can be concluded that group c satisfies optimal frequency
base requirements. Compared with the simulation signal, the
frequency selected by the frequency base is consistent with
the fault frequency. In other words, the fault can be evaluated
by selecting the frequency and then calculating the entropy
value.

4. Experimental Analysis Verification

In order to verify the effectiveness of the method, the rolling
bearing fault simulation experimental platform is set up as
shown in Figure 6. The experimental device is composed of
AC motors, sensors, and faulty bearings. The numbers in
Figure 6 represent: 1-acceleration sensor, 2-thermocouple
sensor, and 3-AC motor, while 4, 5, 6, and 7 represent faulty
bearings. The bearing model used in the experimental device
is 6205-2RS, the motor speed is 1800 rpm, and the sampling
frequency is 12000 Hz. APCB 353B33 acceleration sensor is
installed in the horizontal and vertical directions of each
bearing, and the NI6062E acquisition card is used for data
acquisition. Theoretically, the characteristic frequency of the
rolling bearing can be obtained according to the fault
characteristic equation of the rolling bearing. The results are
shown in Table 3, where \( f_r, f_o, f_i, f_b \), respectively, represent
the rotation frequency of the bearing, the fault frequency of
the outer bearing ring, the fault frequency of the bearing
inner ring, and the rolling frequency.

The composite fault of the inner and rolling
element are taken as an example. The simulation analysis is
performed in the MATLAB2016b environment, and the
vibration signal shown is obtained in Figures 7–9.

Figure 7 represents the collected original mixed fault
vibration signal. Figure 8 represents the frequency spectrum
analysis of the vibration signal, and Figure 9 represents the
evelope analysis of the vibration signal. Within Figure 9,
\( f_{\text{max}} \) represents the frequency point of the highest am-
plitude, \( f_{\text{max}} = 161.9 \) Hz.

This paper compares EEMD with the composite fault
decomposition method proposed in this paper. It can be seen
from Figure 10 that the first six IMF components analyzed
by EEMD are not very effective in identifying composite
faults, and the frequency coupling is difficult to identify.
Obviously, the effect of Figure 9 is significantly better than
that of Figure 10. Mixed fault vibration signals are more
complex, and only a single signal analysis method is im-
perfect. Therefore, this paper adopts the envelope analysis
Fault signal collection
Envelope spectrum analysis
Speed determines the base frequency $\omega_j$

Volterra series model $\omega_1, \omega_2, ..., \omega_n +$ Coupling frequency

Traversal selection

Calculation and screening

The frequency base set that satisfies the condition

Failure frequency group

Entropy comparison $H_F = -\sum q_i \log q_i$

Table 1: Screening of frequency relations.

| Order | First order, Hz | Second order, Hz | Third order, Hz |
|-------|----------------|------------------|-----------------|
| Group a | 0.2 0.8 1 | 0.2 0.4 0.6 0.8 1 | 1.2 1.6 1.8 2 | 0.2 0.4 0.6 0.8 1 | 1.2 1.4 1.6 1.8 2 | 2.2 2.4 2.6 2.8 3 |
| Group b | 0.2 2.8 3 | 0.4 1.8 2.2 2.6 4 | 5.8 7.6 | 0.2 0.6 1.6 2 2.4 3.4 3.8 4.2 | 5.6 6 7.4 7.8 9.6 | 11.4 |
| Group c | 0.2 4 5 | 0.4 1 3.8 4.2 4.8 | 5.2 8 9 10 | 0.2 0.6 0.8 1 2 3 3.6 4 4.4 4.6 5 5.4 6 7.8 8.2 8.8 9.2 9.8 | 10.2 12 13 14 15 |
and Volterra series kernel to separate and diagnose the fault characteristic frequency points from the perspective of decoupling.

According to the method in Section 3, the frequency base set is first extracted from Figure 9, and the frequency set is searched according to the traversal method. All the sets satisfying the frequency are found from the highest frequency amplitude. Secondly, the power spectrum entropy of each frequency component was calculated. Finally, a set of frequency bases with the highest entropy value is calculated. Simulation examples are shown to demonstrate specific operations. Step 1: The frequency base is selected according to the spectrum diagram of the envelope analysis to find all frequency sets that meet the conditions. Step 2: The total power spectrum entropy value of the first three orders is calculated according to the power spectrum entropy. Step 3: The power spectrum entropy values under different frequency bases are compared, and the optimal frequency base is obtained. The process is shown in Figure 11.

There are ten sets of frequency bases that meet the requirements. The previous three sets of frequency bases are taken as an example. As shown in Table 4, the first three-

| Grouping | a     | b     | c     |
|----------|-------|-------|-------|
| Entropy  | 0.501 | 0.378 | 0.652 |

| Parameter | $f_t$ | $f_o$ | $f_i$ | $f_b$ |
|-----------|-------|-------|-------|-------|
| Numerical value | 30 Hz | 161 Hz | 107 Hz | 141 Hz |

Table 2: Power spectrum entropy value.

Table 3: Failure frequency.

![Figure 6: Rolling bearing experiment platform.](image)

![Figure 7: Time-domain signal.](image)

![Figure 8: Frequency domain signal.](image)

![Figure 9: Envelope analysis.](image)
Figure 10: EEMD time-domain signal.

Image 1.

Figure 11: Fault diagnosis process after signal processing.

Image 2.

Figure 6: Rolling bearing experiment platform
Table 4: Screening and calculation of the frequency base
Figure 13: Comparison of power spectrum entropy values of frequency bases

Image 3.
order frequencies are calculated separately. All frequency sets are detailed in Table 5.

According to the selected frequency base set, the power spectrum entropy values are calculated, respectively, and the results are shown in Figure 12.

It can be seen from Figure 12 that the power spectrum entropy value of the frequency base set of group b is the largest; that is, 30.0Hz, 161.1Hz, and 141.4Hz are the most suitable frequency base sets. It is verified that the fundamental frequency is 30Hz, and the failure frequency points are 161Hz and 141Hz, which are basically the same as the actual frequency points.

Combined with the experimental content, it can be concluded that the composite faults of rolling bearings can be effectively judged from the perspective of frequency coupling. The type of fault can also be analyzed by data, which introduces a new fault diagnosis method. For weak fault characteristic signals with similar fault frequency values, filtering methods may not be able to separate the signals well. The processing method of this paper is not realized by filtering but directly decomposes the signal and also does not need to prepare too much prior knowledge, which is relatively easy to implement.

### 5. Conclusions

In this paper, a new method for weak compound fault diagnosis is proposed. Functional Volterra series are used to solve the problem of coupling in the frequency of mixed
faults. The coupling between faults may lead to inaccurate extraction of characteristic information, and comprehensive diagnosis must be combined with the signal processing methods. The advantages of functional Volterra series are utilized for frequency decoupling. Combined with the envelope analysis for nonstationary signals processing, the fault type is evaluated by using information entropy. Simulation and experimental results show that the proposed method is feasible for fault signals with low SNR. The proposed method has better performance in fault feature extraction and diagnosis when compared to other separation methods. The advantage of this method is that filtering and too much prior knowledge are not required. However, there are some problems; for example, the frequency filtering method is relatively simple, and traversing all the data takes time. In the follow-up study, it should be improved according to the actual situation.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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