Bare-Excitation Ground State of a Spinless-Fermion – Boson Model and \( W \)-State Engineering in an Array of Superconducting Qubits and Resonators

Vladimir M. Stojanović

Institut für Angewandte Physik, Technical University of Darmstadt, D-64289 Darmstadt, Germany

(Dated: December 30, 2021)

This Letter unravels an interesting property of a one-dimensional lattice model that describes a single itinerant spinless fermion (excitation) coupled to zero-dimensional (dispersionless) bosons through two different nonlocal-coupling mechanisms. Namely, below a critical value of the effective excitation-boson coupling strength the exact ground state of this model is the zero-quasimomentum Bloch state of a bare (i.e., completely undressed) excitation. It is demonstrated here how this last property of the lattice model under consideration can be exploited for a fast, deterministic preparation of multipartite \( W \) states in a readily realizable system of inductively-coupled superconducting qubits and microwave resonators.

Sophisticated quantum-state engineering \(^1\) \( ^2 \) is a prerequisite for the development of next-generation quantum technologies \(^3\) \( ^4 \). In this context, tantalizing progress was made in recent years by utilizing diverse physical platforms \(^5\) \( ^6 \). In particular, owing to their continuously improving scalability and coherence properties superconducting (SC) circuits \(^7\) \( ^8 \) and, among them, circuit-QED systems \(^9\) \( ^10 \), allow accurate preparation of various quantum states of SC qubits and photons alike \(^11\).

The most prominent classes of entangled many-qubit states are maximally-entangled Greenberger-Horne-Zeilinger (GHZ) \(^12\) and \( W \) states \(^13\). An \( N \)-qubit \( W \) state is the equal superposition of states with exactly one qubit in its “up” state, the remaining ones being in their “down” states. In particular, it is known that a \( W \) state and its GHZ counterpart cannot be transformed into each other via local operations and classical communication (LOCC-inequivalence) \(^14\). \( W \) states are also extremely robust with respect to particle loss, remaining entangled even if any \( N - 2 \) parties lose the information about their particle \(^15\). They lend themselves to applications in quantum-information protocols \(^16\) \( ^17 \), which motivates their preparation in various systems \(^18\) \( ^19 \). In particular, owing to their continuous-time evolution, allow the realization of various quantum states of SC qubits and photons at all. What makes this protocol particularly robust is the fact that its target state is the ground state of the system in a parametrically large window of values of its main experimental knob – an external dc flux.

Model and its ground state.– The 1D lattice model under consideration describes a single spinless-fermion excitation interacting with dispersionless bosons through two different nonlocal coupling mechanisms. The noninteracting part of its total Hamiltonian includes the excitation kinetic-energy- and free-boson terms:

\[
H_0 = -t_c \sum_n (c_{n+1}^\dagger c_n + \text{H.c.}) + \hbar \omega_b \sum_n b_n^\dagger b_n .
\]  

(1)

Here \( c_n^\dagger \) (\( c_n \)) creates (destroys) an excitation at site \( n \) \( (n = 1, \ldots, N) \), \( b_n^\dagger \) (\( b_n \)) a boson with frequency \( \omega_b \) at the same site, while \( t_c \) is the excitation hopping amplitude. The interacting \((e-b)\) part is given by

\[
H_{e-b} = g \hbar \omega_b \sum_n \left[ c_n^\dagger c_n (b_n^\dagger b_{n-1} - b_n b_{n+1}) + c_n^\dagger c_n + \text{H.c.} (b_n^\dagger b_{n+1} - b_n b_{n-1}) \right] ,
\]  

(2)

where \( g \) is the dimensionless e-b coupling strength. The first term on the right-hand-side (rhs) of the last equation captures the antisymmetric coupling of the excitation density at site \( n \) with the local boson displacements on the neighboring sites \( n \pm 1 \) (breathing-mode-type coupling) \(^20\). The second term accounts for the linear dependence of the effective excitation-hopping amplitude between sites \( n \) and \( n + 1 \) on the respective boson displacements (Peierls-type coupling) \(^21\) \( ^22 \).

The coupling Hamiltonian \( H_{e-b} \) can be recast in the generic momentum-space form

\[
H_{e-b} = \frac{1}{\sqrt{N}} \sum_{k,q} \gamma_{e-b}(k,q) c_{k+q}^\dagger c_k (b_{-q}^\dagger b_q) .
\]  

(3)

where the coefficients \( \gamma_{e-b}(k,q) \) capture the e-b coupling strength its ground state is the zero-quasimomentum Bloch state of a bare excitation. It is shown here how this property of the model under consideration can be exploited for a fast, deterministic preparation of \( W \) states in an array of inductively-coupled SC qubits and resonators, an analog simulator of this model \(^23\).
Its corresponding e-b vertex function depends on both the excitation- and boson quasimomenta \((k \text{ and } q, \text{ respectively, here expressed in units of the inverse lattice period})\) and is given by

\[
\gamma_{e-b}(k,q) = 2ig\hbar\omega_b \left[ \sin k + \sin q - \sin(k + q) \right].
\]

The ground state of \(H = H_0 + H_{e-b}\) undergoes a sharp level-crossing transition \([34]\) at a critical value \(\lambda_{c-b} \sim 1\) [cf. Fig. 2] of the effective coupling strength \(\lambda_{c-b} = \frac{2g^2\hbar\omega_b}{\pi t_c}\). For \(\lambda_{c-b} < \lambda_{c-b}^*\), the ground state is the 0 eigenvalue of the total quasimomentum operator \(K_{tot} = \sum_k k^c \hat{c}_k + \sum_q q \hat{b}_q\). For \(\lambda_{c-b} > \lambda_{c-b}^*\), on the other hand, the ground state is twofold-degenerate and corresponds to \(K = \pm K_{gs}\) \((K_{gs} \neq 0)\).

A ground state with \(K = 0\) is by no means unusual – in fact, an overwhelming majority of coupled e-b models have such ground states. Yet, the model at hand has the peculiar property that its \(K = 0\) ground state for \(\lambda_{c-b} < \lambda_{c-b}^*\) is the 0 bare-excitation Bloch state \(|\Psi_{k=0}\rangle \equiv c_k^0|0\rangle_c \otimes |0\rangle_b\), where \(|0\rangle_c\) and \(|0\rangle_b\) are the excitation and boson vacuum states. In what follows, it will first be demonstrated explicitly that \(|\Psi_{k=0}\rangle\) is an exact eigenstate of \(H\) for an arbitrary value of \(\lambda_{c-b}\). It will subsequently be shown numerically (see Fig. 2 below) that for \(\lambda_{c-b} < \lambda_{c-b}^*\) this state is the ground state of \(H\).

Given that \(|\Psi_{k=0}\rangle\) is an eigenstate of \(H_0\), to prove that it is an eigenstate of the total Hamiltonian \(H\) it suffices to show that it is also an eigenstate of \(H_{e-b}\). Indeed, by acting with \(H_{e-b}\) [cf. Eq. (3)] on this state and making use of the fact that \(c_k c_k^\dagger|0\rangle_c \equiv \delta_{k,0}|0\rangle_c\), one obtains

\[
H_{e-b}|\Psi_{k=0}\rangle = \frac{1}{\sqrt{N}} \sum_q \gamma_{e-b}(k = 0, q) c_q^\dagger|0\rangle_c \otimes \hat{b}_q^\dagger|0\rangle_b.
\]

Because here \(\gamma_{e-b}(k = 0, q) = 0\) for an arbitrary \(q\) [cf. Eq. (3)], each term in the sum on the rhs of Eq. (4) vanishes, implying that \(H_{e-b}|\Psi_{k=0}\rangle = 0\). Therefore, \(|\Psi_{k=0}\rangle\) is an eigenstate of \(H_{e-b}\) (for an arbitrary \(\lambda_{c-b}\)), the corresponding eigenvalue being equal to zero. This concludes the proof that \(|\Psi_{k=0}\rangle\) is an exact eigenstate of \(H\).

Qubit-resonator system.— The analog simulator of the model under consideration [see Fig. 1(a)] consists of SC qubits \((Q_n)\) with the energy splitting \(\varepsilon_z\), microwave resonators \((R_n)\) with the photon frequency \(\omega_c\), and coupler circuits \((B_n)\) \([35]\) which mediate both qubit-qubit and qubit-resonator interactions in this system. The simulator can be realized with transmons \([12]\) \((E_z^* \sim 100, E_r^* \sim 2\varepsilon_z)\) and gatemons \([35]\) \((E_j^*/E_z^* \sim 25)\). Its \(n\)-th repeating unit is described by the free Hamiltonian \(H_n^f = \varepsilon_z/2 \sigma_n^z + \hbar\omega_c b_n^\dagger b_n\), where the pseudospin-1/2 operators \(\sigma_n\) represent qubit \(n\) and the bosonic operators \((b_n, b_n^\dagger)\) photons in the \(n\)-th resonator.

The upper and lower loops of \(B_n\) are threaded by magnetic fluxes \(\phi_n^u\) and \(\phi_n^l\), respectively [both are expressed in units of \(\Phi_0/2\pi\), where \(\Phi_0 \equiv hc/(2e)\) is the flux quantum]. In particular, the upper loop is subject to ac-driving with the flux \(\pi(\omega_0 t)\). The other contribution to \(\phi_n^u\) originates from the modes of resonators \(n\) and \(n + 1\) and is given by \(\phi_{n, res} = \delta\theta[(b_{n+1} + b_n^\dagger) - (b_n + b_n^\dagger)]\), where \(\delta\theta = [2eA_{eff}/(\hbar d_{0c})] \times (\hbar\omega_c/C_0)^{1/2}\), with \(A_{eff}\) being the effective coupling area, \(C_0\) the resonator capacitance, and \(d_0\) the effective spacing in the resonator \([37]\). Therefore, unlike the much more common capacitive coupling \([38]\), the qubit-resonator coupling in the system at hand is inductive \([11]\). Similarly, \(\phi_n^l\) includes an ac contribution, given by \(-\pi(\omega_0 t)\), and a dc part \(\phi_{dc}\), the main experimental knob in this system.

The Josephson energy of \(B_n\) is given by \(H_j^f = -\sum_{n=1}^3 E_j^s \cos \phi_n^s\), with \(\phi_n^s\) being the respective phase drops on the three Josephson junctions within \(B_n\) and \(E_j\) their energies, here chosen such that \(E_j^3 = E_j^2 \equiv E_j\) and \(E_j^1 = E_j^b \neq E_j\). Using the flux-quantization rules \([3]\), the total Josephson energy \(H_j^f = \sum_n H_j^f\) can be expressed in terms of the gauge-invariant phase variables \(\varphi_n\) of SC islands of different qubits. The latter enter this energy through terms of the type \(\cos(\varphi_n - \varphi_{n+1})\), which in the regime of interest for transmons/gatemons \((E_j^3 \gg E_j^b)\) can be recast (up to an additive constant) as \(\delta\phi_{0}^2 [\sigma_n^z \sigma_{n+1}^z + \sigma_n^+ \sigma_{n+1}^+] - (\sigma_n^z + \sigma_{n+1}^z)/2\), where \(\delta\phi_{0}^2 = (E_j^b/E_j^s)^{1/2}\) \([38]\).

![FIG. 1: (Color online) (a) Schematic of the qubit-resonator system, whose \(n\)-th repeating unit (indicated by the dashed rectangle) comprises SC qubit \(Q_n\), resonator \(R_n\), and coupler circuit \(B_n\). The fluxes from the resonator modes \(n\) and \(n + 1\) thread the upper loops of the coupler circuit \(B_n\), effectively giving rise to an indirect inductive qubit-resonator coupling. (b) Pictorial illustration of the effective lattice model of the system, with the excitation hopping amplitude \(t_0(\phi_{dc})\) and each lattice site hosting dispersionless bosons with the frequency \(\delta\omega = \omega_c - \omega_0\).](image)
Further analysis is carried out in the rotating frame of the drive, where \( \delta \omega \equiv \omega_c - \omega_0 \) is the effective boson frequency and the Josephson-coupling term becomes time-dependent. This time dependence can, however, be disregarded due to its rapidly-oscillating character, in line with the rotating-wave approximation (RWA). The remaining part of \( \hat{H}_n^J \) can succinctly be written as \( \hat{H}_n^J = -E_n^J(\phi_{dc}, \phi_{n, \text{res}}) \cos(\varphi_n - \varphi_{n+1}) \), where

\[
E_n^J = E_J b_1(1 + \cos \phi_{dc}) - E_J J_1(\pi/2) \phi_{n, \text{res}},
\]

and \( J_n(x) \) are Bessel functions of the first kind whose presence in this expression stems from the use of the Jacobi-Anger expansion in conjunction with the RWA. In what follows, without significant loss of generality, \( E_{Jb} \) is chosen to be given by \( 2E_J J_0(\pi/2) \).

The above expression for \( \cos(\varphi_n - \varphi_{n+1}) \) in terms of the operators \( \sigma_n \) implies that the effective interaction between adjacent qubits in this system is of XY type. Through the flux \( \phi_{n, \text{res}} \), the interaction strength acquires a dependence on the boson displacements \( u_n \propto b_n + b_n^\dagger \), whose form is equivalent to that of the XY spin-Peierls model \([28, 41]\). The spinless-fermion – boson coupling that appears (without the constant-energy contribution) in Eq. (6) and Fig. 2, is nonlocal in nature, in contrast to other examples of such couplings in various solid-state systems \([28, 41]\).

**Effective Hamiltonian and its ground states.** To show that the effective system Hamiltonian consists of contributions akin to \( \hat{H}_0 \) and \( \hat{H}_c^\text{res} \) [cf. Eqs. (1) and (2)], one switches to the spinless-fermion representation using the Jordan-Wigner (JW) transformation. The latter reads \( \sigma_n^z = 2c_n^\dagger c_n - 1 \), \( \sigma_n^+ = 2c_n e^{i\pi \sum_{\ell<n} c_{\ell}^\dagger c_{\ell}} \), where the sum in the last exponent defines the JW string \([12]\). In this representation, the noninteracting part of the effective system Hamiltonian comprises the excitation-hopping- and free-photon terms. [Note that \( c_n^\dagger c_n \) terms resulting from the \( \sigma_n^z \) terms in \( \hat{H}_0^\text{res} \) and \( \hat{H}_n^J \) are largely immaterial for further discussion as they only lead to a constant energy offset (band-center energy).] It assumes the form of \( \hat{H}_0 \), with \( \omega_c \to \delta \omega \) and \( t_c \to t_0(\phi_{dc}) \equiv E_J b_0^2 (1 + \cos \phi_{dc}) \), the latter being the effective \( \phi_{dc} \)-dependent hopping amplitude [cf. Fig. 1(b)]. At the same time, the interacting part adopts the form of \( \hat{H}_{bc} \). In particular, via the JW transformation, the Peierls-coupling term is obtained in a manner familiar from the XY spin-Peierls model \([40]\), while the breathing-mode term originates from the \( \sigma_n^+ \) terms in the above expression for \( \cos(\varphi_n - \varphi_{n+1}) \).

The dimensionless coupling strength \( g \) is determined by the system parameters through the relation \( g \delta \omega = \delta \varphi_0^2 E_J J_1(\pi/2) \delta \theta \), while the \( \phi_{dc} \)-dependent – thus in-situ tunable – effective coupling strength is given by

\[
\lambda_{c-b}(\phi_{dc}) = g \frac{J_1(\pi/2) \delta \theta}{J_0(\pi/2)(1 + \cos \phi_{dc})}.
\]

For a typical resonator \( \delta \theta \sim 3.5 \times 10^{-3} \) \([20]\). Besides, for \( \delta \omega \) it is pertinent to take \( \delta \omega/2\pi = 200 - 300 \text{ MHz} \) and also choose \( E_J \) such that \( \delta \varphi_0^2 E_J / 2\pi \hbar = 100 \text{ GHz} \).

![Graph](image-url)

**FIG. 2: Ground-state energy of the system with \( \delta \omega/2\pi = 300 \text{ MHz} \) as a function of the effective coupling strength \( \lambda_{c-b} \).**

For \( \lambda_{c-b} < \lambda_{c-b}^\text{res} \approx 0.72 \) (i.e., \( \phi_{dc} < 0.972\pi \)) the ground state of the system corresponds to a bare excitation, while for \( \lambda_{c-b} \geq \lambda_{c-b}^\text{res} \) it corresponds to a heavily-dressed (polaronic) one.

The ground-state energy of the system, expressed in units of \( E_n \equiv 10^{-3} \delta \varphi_0^2 E_J \), was evaluated through Lanczos-type exact diagonalization \([24, 43]\) and illustrated (without the constant-energy contribution) in Fig. 2. For \( \lambda_{c-b} \geq \lambda_{c-b}^\text{res} \), the system has a polaron-like ground state (strongly boson-dressed excitation), with its energy showing a rather weak dependence on \( \lambda_{c-b} \). On the other hand, for \( \lambda_{c-b} < \lambda_{c-b}^\text{res} \) the ground state corresponds to \( |\Psi_{k=0}\rangle \), i.e., a bare excitation with \( k = 0 \). Its energy \( E_{gs} = -2t_0(\phi_{dc}) \) is the minimum of a 1D cosine-shaped dispersion. The energy separation of this ground state from the first excited state exactly equals \( \hbar \delta \omega \) for any \( \phi_{dc} \) below the critical value. This is consistent with the fact that coupled e-b systems with dispersionless bosons invariably have one-boson continua separated from their ground states by the single-boson energy and in the weak-coupling regime typically feature only one bound state below those continua \([44]\).

**W states and their preparation.** Bearing in mind that JW strings act trivially on \( |0\rangle_e \), so that \( c_n^\dagger |0\rangle_e \equiv S_n^+ |0\rangle_e \) (where \( S_n = \hbar \sigma_n/2 \)), it holds that

\[
c_n^\dagger |0\rangle_e = N^{-1/2} \sum_{n=1}^N e^{-i k n} S_n^+ |0\rangle_e.
\]

The last equation is equivalent to \( |\Psi_k\rangle = |W_N(k)\rangle \otimes |0\rangle_b \), where \( |W_N(k)\rangle \) is a “twisted” \( N \)-qubit W state. Thus, bare-excitation Bloch states coincide with generalized W states, while in particular \( |\Psi_{k=0}\rangle \) – the ground state of the system at hand for \( \lambda_{c-b} < \lambda_{c-b}^\text{res} \) – corresponds to the ordinary \( N \)-qubit W state

\[
|W_N\rangle = \frac{1}{\sqrt{N}} (|10\ldots 0\rangle + |01\ldots 0\rangle + \ldots + |00\ldots 1\rangle).
\]
In the following, a microwave-pumping based protocol for the preparation of an \( N \)-qubit \( W \) state is proposed assuming that the system is initially in the vacuum state \(|0\rangle \equiv |0\rangle_e \otimes |0\rangle_{ph} \). The external driving required for this purpose is assumed to be represented by the operator

\[
\Omega_{\text{prep}}(t) = \frac{\hbar}{\sqrt{N}} \sum_{n=1}^{N} \left( \sigma_n^+ e^{-i\delta \omega n} + \sigma_n^- e^{i\delta \omega n} \right),
\]

where \( \delta \) describes its time dependence and the factors \( e^{\pm i\delta \omega n} \) account for the possibility that flip operations on different qubits are applied with a phase difference. It is important to stress that the general form of driving in Eq. (10) allows one to prepare – through different choices of \( q_\delta \) and \( \delta \) – various states of the proposed system, including its strongly boson-dressed ground states realized when \( \phi_{dc} \) is above the critical value.

The transition matrix element of the operator \( \Omega_{\text{prep}}(t) \) between the initial state \(|0\rangle \) and the target state \(|\Psi_{\text{prep}}\rangle \equiv |W_N\rangle \otimes |0\rangle_{ph} \) evaluates to \( \hbar \delta \rangle \delta_{q_\delta,k=0} \), which indicates that the preparation of this particular target state requires only a global driving field [i.e., \( q_\delta = 0 \) in Eq. (10)]. Thus, in contrast to some other schemes for \( W \)-state preparation \cite{22}, the present one does not require a local qubit control \cite{15, 40}. By assuming that \( \delta \) = \( 2\beta_0 \cos(\omega t) \), where \( \omega \) is the energy difference between the two relevant states, in the RWA these states are Rabi-coupled with the effective Rabi frequency \( \beta_0 \). Thus, starting from the state \(|0\rangle \), the desired state \( W \)-qubit \( W \) state will be prepared within a time interval of duration \( \tau_{\text{prep}} = \pi \hbar / (2 \beta_0) \), which does not depend on \( N \).

Taking the pumping amplitude to be \( \beta_0 / (2 \pi \hbar) = 10 \text{ MHz} \), one finds \( \tau_{\text{prep}} \approx 25 \text{ ns} \), which is three orders of magnitude shorter than typical coherence times of SC qubits (e.g., for transmons \( T_2 \sim 20 - 100 \text{ ms} \) \cite{10}). Thus the proposed protocol should not be affected by a loss of coherence in the system. At the same time, the obtained \( \tau_{\text{prep}} \) is sufficiently long that a leakage outside of the computational subspace of a single qubit can be neglected. Namely, due to the multilevel character of SC qubits, a finite anharmonicity \( \alpha \equiv E_{12} - E_{01} \) (where \( E_{ij} \) is the energy difference between qubit states \( j \) and \( i \)) is required. In order to avoid such a leakage, the minimal pulse duration of \( t_p \sim 1 / |\alpha| \) is necessary. For transmons \( (\alpha \sim -200 \text{ MHz}) \), even a few-nanoseconds-long pulse is frequency selective enough that such a leakage is negligible \cite{13}. The obtained \( \tau_{\text{prep}} \sim 25 \text{ ns} \) suffices even in the case of gatemons, whose typical anharmonicity is by a factor of two smaller than that of transmons \cite{49}.

Importantly, the large energy separation \( h \delta \omega \) between the target state and the lowest-lying excited state of the system ensures that the proposed \( W \)-state preparation will not be hampered by an inadvertent population of undesired states. For instance, for \( \delta \omega / 2 \pi = 200 \text{ (300) MHz} \) this energy separation is equal to \( 2E_u \) (\( 3E_u \)), which represents a significant fraction of the energy difference between the initial and target states (cf. Fig. 2).

The proposed protocol is deterministic in nature and generates \( W \)-type entanglement of all the qubits in parallel. Moreover, in contrast to the typical situation in quantum-state control, where state-preparation times often scale unfavorably with the system size, here \( \tau_{\text{prep}} \) does not depend on the system size at all. Finally, because they represent ground states of the system, multipartite \( W \) states prepared by this protocol can be expected to be extremely robust.

Besides allowing \( W \)-state preparation, the proposed system features an \( XY \)-type qubit-qubit interaction, which opens the possibility for a universal quantum computation \cite{50, 51}. Because the strength of this interaction depends dynamically on the boson degrees of freedom (photons), this system bears a formal similarity to certain trapped-ion systems in which the role of bosons is played by collective motional modes (phonons) \cite{52}. Compared to its trapped-ion counterparts, this system has an added advantage that it merely involves dispersionless bosons of one single frequency, which circumvents the spectral crowding problem resulting from the quasi-continuous character of phonon spectra in large trapped-ion chains \cite{53}.

**Robustness to losses and feasibility.** – It is pertinent to briefly address the robustness of the system at hand to possible deleterious effects of losses. To this end, it is worthwhile to first note that qubit-state flips and displacements of the resonator modes are the two leading sources of decoherence in this system. In addition to the very long \( T_2 \) times of transmon (gatemon) qubits, the damping time of microwave photons in coplanar waveguide resonators can reach the same order of magnitude as \( T_2 \), with the corresponding quality factor being larger than \( 10^7 \) \cite{54}. Besides, the relevant excitation- and photon energy scales in this system \( (\delta \omega, g \delta \omega, t_0 / \hbar) \), expressed in frequency units, are all of the order of several \( 2 \pi \times 100 \text{ MHz} \). Thus, they far exceed the decoherence rates whose state-of-the-art values in this type of systems are \( \gamma \sim 0.01 \text{ MHz} \) \cite{10}. Finally, in this system thermal excitations – which at temperatures typical for such SC-qubit setups \( (T \sim 100 \text{ nK}) \) have characteristic energies of a few GHz – can be safely neglected. Therefore, the loss mechanisms do not pose obstacles to realizing the proposed system.

**Conclusions.** – The present paper proposes a scheme for a fast, deterministic creation of a large-scale \( W \)-type entanglement \cite{55} in a system of inductively-coupled superconducting qubits and microwave resonators. The mechanism behind this scalable entanglement resource – which allows one to engineer \( W \) states with the preparation times independent of the system size – stems from the unconventional ground-state properties of a one-dimensional model describing a nonlocal coupling of a spinless fermion to zero-dimensional bosons. The feasibility and robustness of the underlying state-preparation
protocol—which only requires a global driving field—is demonstrated with realistic system parameters.

This study can be viewed as being complementary to that of Ref. [22], where the preparation of W states of photons—rather than qubits—was proposed. The common denominator of these two proposals is that they both rely on superconducting systems and an *in-situ* tunability of a hopping amplitude, albeit being based on completely different physical mechanisms. These schemes are far more scalable than the conventional ones in which resonator-mediated qubit–qubit interactions are utilized to controllably entangle multiple qubits; such an approach was recently used to prepare a GHZ state of 10 superconducting qubits[7]—the largest entanglement demonstrated so far in solid-state architectures. Thus, the need to demonstrate the envisioned W-state preparation is compelling.

**Acknowledgments.**—This research was supported by the Deutsche Forschungsgemeinschaft (DFG)–SFB 1119—236615297.

* Electronic address: vladimir.stojanovic@physik.tu-darmstadt.de

[1] A. M. Zagoskin, *Quantum Engineering: Theory and Design of Quantum Coherent Structures* (Cambridge University Press, Cambridge, UK, 2011).
[2] Y. Makhlin, G. Schön, and A. Shnirman, Quantum-state engineering with Josephson-junction devices, Rev. Mod. Phys. **73**, 357 (2001).
[3] J. P. Dowling and G. J. Milburn, “Quantum Technology: The Second Quantum Revolution,” Phil. Trans. R. Soc. Lond. A **361**, 1655–1674 (2003).
[4] W. P. Schleich *et al.*, Quantum Technology: from research to application, Appl. Phys. B **122**, 130 (2016).
[5] F. Haas, J. Volz, R. Gehr, J. Reichel, and J. Estève, Entangled states of more than 40 atoms in an optical fiber cavity, Science **344**, 180 (2014).
[6] N. Fiiss *et al.*, Observation of Entangled States of a Fully Controlled 20-Qubit System, Phys. Rev. X **8**, 021012 (2018).
[7] C. Song *et al.*, 10-Qubit Entanglement and Parallel Logic Operations with a Superconducting Circuit, Phys. Rev. Lett. **119**, 180511 (2017).
[8] Z.-L. Xiang, S. Ashhab, J. Q. You, and F. Nori, Hybrid quantum circuits: Superconducting circuits interacting with other quantum systems, Rev. Mod. Phys. **85**, 623 (2013).
[9] For an introduction, see U. Vool and M. Devoret, Introduction to quantum electromagnetic circuits, Int. J. Circ. Theor. Appl. **45**, 897 (2017).
[10] See, e.g., G. Wendin, Quantum information processing with superconducting circuits: a review, Rep. Prog. Phys. **80**, 106001 (2017).
[11] For a recent review, see P. Krantz, M. Kjaergaard, F. Yan, T. P. Orlando, S. Gustavsson, and W. D. Oliver, A Quantum Engineer’s Guide to Superconducting Qubits, Appl. Phys. Rev. **6**, 021318 (2019).
[12] J. Koch, T. M. Yu, J. Gambetta, A. A. Houck, D. I. Schuster, J. Majer, A. Blais, M. H. Devoret, S. M. Girvin, and R. J. Schoelkopf, Charge-insensitive qubit design derived from the Cooper pair box, Phys. Rev. A **76**, 042319 (2007).
[13] For an introduction, see S. M. Girvin, in *Lecture Notes on Strong Light-Matter Coupling: from Atoms to Solid-State Systems* (World Scientific, Singapore, 2013), pp. 155-206.
[14] See, e.g., B. Vlastakis, G. Kirchmair, Z. Leghtas, S. E. Nigg, L. Frunzio, S. M. Girvin, M. Mirrahimi, M. H. Devoret, and R. J. Schoelkopf, Deterministically Encoding Quantum Information Using 100-Photon Schrödinger Cat States, Science **342**, 607 (2013).
[15] D. M. Greenberger, M. A. Horne, and A. Zeilinger, Bell’s Theorem, Quantum Theory, and Conceptions of the Universe (Kluwer Academic, Dordrecht, 1989), pp. 73-76.
[16] W. Dür, G. Vidal, and J. I. Cirac, Three qubits can be entangled in two inequivalent ways, Phys. Rev. A **62**, 062314 (2000).
[17] M. A. Nielsen, Conditions for a Class of Entanglement Transformations, Phys. Rev. Lett. **83**, 436 (1999).
[18] M. Koashi, V. Bužek, and N. Imoto, Entangled webs: Tight bound for symmetric sharing of entanglement, Phys. Rev. A **62**, 050302(R) (1999).
[19] J. Joo, Y.-J. Park, S. Oh, and J. Kim, Quantum telepation via a W state, New J. Phys. **5**, 136 (2003).
[20] H. Häffner *et al.*, Scalable multiparticle entanglement of trapped ions, Nature (London) **438**, 643 (2005).
[21] R. McConnell, H. Zhang, J. Z. Hu, S. Ćuk, and V. Vuletić, Entanglement with negative Wigner function of almost 3,000 atoms heralded by one photon, Nature (London) **519**, 439 (2015).
[22] A. A. Gangat, I. P. McCulloch, and G. J. Milburn, Deterministic Many-Resonator W Entanglement of Nearly Arbitrary Microwave States via Attractive Bose-Hubbard Simulation, Phys. Rev. X **3**, 031009 (2013).
[23] C. Li and Z. Song, Generation of Bell, W, and Greenberger–Horne–Zeilinger states via exceptional points in non-Hermitian quantum spin systems, Phys. Rev. A **91**, 062104 (2015).
[24] F. Reiter, D. Reich, and A. S. Sørensen, Scalable Dissipative Preparation of Many-Body Entanglement, Phys. Rev. Lett. **117**, 040501 (2016).
[25] Y.-H. Kang, Y.-H. Chen, Z.-C. Shi, J. Song, and Y. Xia, Fast preparation of W states with superconducting quantum interference devices by using dressed states, Phys. Rev. A **94**, 052311 (2016).
[26] J. Chen, H. Zhou, C. Duan, and X. Peng, Preparing Greenberger-Horne-Zeilinger and W states on a long-range Ising spin model by global controls, Phys. Rev. A **95**, 032340 (2017).
[27] B. Fang, M. Menotti, M. Lisicidini, J. E. Sipe, and V. O. Lorenz, Three-Photon Discrete-Energy-Entangled W State in an Optical Fiber, Phys. Rev. Lett. **123**, 070508 (2019).
[28] See, e.g., F. M. Souza, P. A. Oliveira, and L. Sanz, Quantum entanglement driven by electron-vibrational mode coupling, Phys. Rev. A **100**, 042309 (2019), and references therein.
[29] V. M. Stojanović and I. Salom, Quantum dynamics of the small-polaron formation in a superconducting analog simulator, Phys. Rev. B **99**, 134308 (2019).
[30] C. Slezak, A. Macridin, G. A. Sawatzky, M. Jarrell, and T. A. Maier, Spectral properties of Holstein and breathing polarons, Phys. Rev. B **73**, 205122 (2006).
[31] V. M. Stojanović, P. A. Bobbert, and M. A. J. Michels,
Nonlocal electron-phonon coupling: Consequences for the nature of polaron states, Phys. Rev. B 69, 144302 (2004).

[32] V. M. Stojanović and M. Vanević, Quantum-entanglement aspects of polaron systems, Phys. Rev. B 78, 214301 (2008).

[33] K. Hannewald, V. M. Stojanović, and P. A. Bobbert, A note on temperature-dependent band narrowing in oligo-acene crystals, J. Phys: Condens. Matter 16, 2023 (2004).

[34] For a general discussion of such transitions, see V. M. Stojanović, Entanglement-spectrum characterization of ground-state nonanalyticities in coupled excitation-phonon models, Phys. Rev. B 101, 134301 (2020).

[35] For other types of tunable couplers, see M. R. Geller, E. Donate, Y. Chen, C. Neill, P. Roushan, and J. M. Martinis, Tunable coupler for superconducting Xmon qubits: Perturbative nonlinear model, Phys. Rev. A 92, 012320 (2015).

[36] T. W. Larsen, F. Kuemmeth, T. S. Jespersen, P. Krogstrup, J. Nygård, and C. M. Marcus, Semiconductor-Nanowire-Based Superconducting Qubit, Phys. Rev. Lett. 115, 127001 (2015).

[37] T. P. Orlando and K. A. Delin, Introduction to Applied Superconductivity (Addison-Wesley, Reading, MA, 1991).

[38] F. Mei, V. M. Stojanović, I. Siddiqi, and L. Tian, Analog superconducting quantum simulator for Holstein polarons, Phys. Rev. B 88, 224502 (2013).

[39] M. Abramowitz and I. A. Stegun, Handbook of Mathematical Functions (Dover Publications, New York, 1972).

[40] See, e.g., L. G. Caron and S. Moukouri, Density Matrix Renormalization Group Applied to the Ground State of the XY Spin-Peierls System, Phys. Rev. Lett. 76, 4050 (1996), and references therein.

[41] See, e.g., A. Pályi, P. R. Struck, M. Rudner, K. Flensberg, and G. Burkard, Spin-Orbit-Induced Strong Coupling of a Single Spin to a Nanomechanical Resonator, Phys. Rev. Lett. 108, 206811 (2012), and references therein.

[42] P. Coleman, Introduction to Many-Body Physics (Cambridge University Press, Cambridge, UK, 2015).

[43] J. K. Cullum and R. A. Willoughby, Lanczos Algorithms for Large Symmetric Eigenvalue Computations (Birkhäuser, Boston, 1985).

[44] See, e.g., V. M. Stojanović, M. Vanević, E. Demler, and L. Tian, Transmon-based simulator of nonlocal electron-phonon coupling: A platform for observing sharp small-polaron transitions, Phys. Rev. B 89, 144508 (2014).

[45] R. Heule, C. Bruder, D. Burgarth, and V. M. Stojanović, Controlling qubit arrays with anisotropic XXZ Heisenberg interaction by acting on a single qubit, Eur. Phys. J. D 63, 41 (2011).

[46] V. M. Stojanović, Feasibility of single-shot realizations of conditional three-qubit gates in exchange-coupled qubit arrays with local control, Phys. Rev. A 99, 012345 (2019).

[47] See, e.g., M. Russ, D. M. Zajac, A. J. Sigillito, F. Borjans, J. M. Taylor, J. R. Petta, and G. Burkard, High-fidelity quantum gates in Si/SiGe double quantum dots, Phys. Rev. B 97, 085421 (2018).

[48] See, e.g., D. M. Zajac, A. J. Sigillito, M. Russ, F. Borjans, J. M. Taylor, G. Burkard, and J. R. Petta, Resonantly driven CNOT gate for electron spins, Science 359, 439 (2018).

[49] A. Kringhøj, L. Casparis, M. Hell, T. W. Larsen, F. Kuemmeth, M. Leijnse, K. Flensberg, P. Krogstrup, J. Nygård, and C. M. Marcus, Anharmonicity of a superconducting qubit with a few-mode Josephson junction, Phys. Rev. B 97, 060508(R) (2018).

[50] N. Schuch and J. Siewert, Natural two-qubit gate for quantum computation using the XY interaction, Phys. Rev. A 67, 032301 (2003).

[51] D. M. Abrams, N. Didier, B. R. Johnson, M. P. da Silva, and C. A. Ryan, Implementation of the XY interaction family with calibration of a single pulse, arXiv:1912.04424.

[52] M. L. Wall, A. Safavi-Naini, and A. M. Rey, Boson-mediated quantum spin simulators in transverse fields: XY model and spin-boson entanglement, Phys. Rev. A 95, 013602 (2017).

[53] K. A. Landsman, Y. Wu, P. H. Leung, D. Zhu, N. M. Linke, K. R. Brown, L. Duan, and C. Monroe, Two-qubit entangling gates within arbitrarily long chains of trapped ions, Phys. Rev. A 100, 022332 (2019).

[54] Z. Wang, S. Shankar, Z. K. Minev, P. Campagne-Ibarcq, A. Narla, and M. H. Devoret, Cavity Attenuators for Superconducting Qubits, Phys. Rev. Appl. 11, 014031 (2019).

[55] F. Fröwis, P. Sekatski, N. Gisin, W. Dürr, and N. Sangouard, Macroscopic quantum states: Measures, fragility, and implementations, Rev. Mod. Phys. 90, 025004 (2018).