Localization of spin mixing dynamics in a spin-1 Bose-Einstein condensate

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We propose to localize spin mixing dynamics in a spin-1 Bose-Einstein condensate by a temporal modulation of spin exchange interaction, which is tunable with optical Feshbach resonance. Adopting techniques from coherent control, we demonstrate the localization/freezing of spin mixing dynamics, and the suppression of the intrinsic dynamic instability and spontaneous spin domain formation in a ferromagnetically interacting condensate of $^{87}$Rb atoms. This work points to a promising scheme for investigating the weak magnetic spin dipole interaction, which is usually masked by the more dominant spin exchange interaction.

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Dynamic localization is ubiquitous in nonlinear systems, both for classical dynamics as in an inverted pendulum with a rapidly modulating pivot \(1\) or an ion in a Paul trap \(2\), and for quantum dynamics like a one- or two-dimensional soliton in a Bose-Einstein condensate (BEC) when the attractive interaction strength is rapidly modulated \(3\) \([\text{or } 4]\). It is often used to stabilize a dynamically unstable system.

Spin mixing dynamics of a spin-1 atomic condensate are dynamically unstable \(7\) when the spin exchange interaction is ferromagnetic, \(i.e.,\) favoring a ground state with all atomic spins aligned. When confined spatially, the unstable dynamics is known to cause formation of spin domain structures \(8\) \([\text{or } 9]\). For many applications of spinor condensates, from quantum simulation to precision measurement \(10\), it is desirable that spin domain formation is suppressed. In addition, atomic spin dipolar interactions, although weak when compared to typical spin exchange interactions, induce intricate spin textures that are difficult to probe when masked by spin domain structures. Thus the suppressing/freezing of the undesirable dynamics from spin exchange interaction is also important for investigating the effect of dipolar interaction \(11\) \([\text{or } 12]\).

Compared to conventional magnets in solid states, a spin-1 BEC has one unsurpassed advantage: its spin exchange interaction is given by \(c_0 = 4\pi \hbar^2 (a_2 + a_0)/3M\), two to three orders of magnitude larger than \(|c_2| \approx 3\) \([20–22]\). This ensures the validity of single spatial mode approximation (SMA) \(22\) when the number of atoms is small and the magnetic field is low. The spin degrees of freedom and the spatial degrees of freedom become separated within the SMA. This allows one focus on the most interesting spin dynamics free from density dependent interactions.

Within the mean field framework, the spin dynamics of a spin-1 condensate under the SMA is described by \(23\)

\[
\begin{align*}
\dot{\rho}_0 &= \frac{2c}{\hbar} \rho_0 \sqrt{(1 - \rho_0)^2 - m^2 \sin \theta}, \\
\dot{\theta} &= \frac{2c}{\hbar} \left(1 - 2\rho_0 + \frac{(1 - \rho_0)(1 - 2\rho_0)}{\sqrt{(1 - \rho_0)^2 - m^2}} \cos \theta\right),
\end{align*}
\]

where \(\rho_i (i = +, 0, -)\) is the fractional population of component \(|i\rangle\), \(\sum \rho_i = 1\), \(m = \rho_+ - \rho_-\) is the magnetization in a spin-1 Bose condensate, a conserved quantity. \(\theta\) is the relative phase \(23\). \(\phi(\vec{r})\) is a unit normalized spatial mode function under the SMA determined from a scalar Gross-Pitaevskii equation with an \(s\)-wave scattering length of \(a_2\). As before, the effective spin exchange interaction is given by \(c(t) = c_2(t)N \int d\vec{r} |\phi(\vec{r})|^4\), albeit the time dependence, with \(N\) the total number of trapped atoms. Although the system dynamics \(11\) does not con-
serve the total spin energy
\[ \mathcal{E}(t) = c(t)\rho_0 \left[ (1 - \rho_0) + \sqrt{(1 - \rho_0)^2 - m^2 \cos \theta} \right]. \tag{2} \]
due to the temporal modulation, the transversal spin squared \( f_\perp^2 = f_x^2 + f_y^2 = 2\mathcal{E}(t)/c(t) \) remains conserved and is determined solely by the initial condition. Because \( \mathcal{E}(t) \) and \( c(t) \) are modulated exactly in the same manner, replacing \( \theta \) with \( f_\perp^2 \), equation (1) is further simplified to
\[ (\dot{\rho}_0)^2 = \frac{4\gamma^2}{\hbar^2} f_\perp^2 (\rho_u - \rho_0)(\rho_u - \rho_d), \tag{3} \]
where \( \rho_{u,d} = f_\perp^2 \left( \frac{1 \pm \sqrt{1 - f_\perp^2}}{2f} \right) \) with \( f_\perp^2 = f_x^2 + m^2 \). \( \rho_{u,d} \) takes the \((+)(-)\) sign, denoting the largest (smallest) value of \( \rho_0 \) along the orbit and satisfies \( \dot{\rho}_0|_{\rho_{u,d}} = 0 \). It is straightforward to find the solution
\[ \rho_0(t) = \frac{\rho_u + \rho_d}{2} + \frac{\rho_u - \rho_d}{2} \sin[\gamma + \int_0^t \beta(s)ds], \tag{4} \]
where \( \beta(t) = \pm 2c(t)f/\hbar \) is the frequency of the periodic spin evolution and the \( \pm \) sign denotes the forward and backward evolutions, respectively, and
\[ \gamma = \tan^{-1} \left( \frac{\rho_0(0) - [(\rho_u + \rho_d)/2]}{\sqrt{[\rho_u - \rho_0(0)][\rho_0(0) - \rho_d]}} \right) \]
is given by the initial values of \( \rho_0 \) and \( \theta \). The above solution (4) is valid for an arbitrary temporal modulation function \( c(t) \).

To control the spin dynamics, we consider several simple but practical modulations in the following. Based on these examples, we demonstrate that spin dynamics with a modulated \( c \) is very different from the free dynamics and understand how a temporal modulated \( c(t) \) affects the spin dynamics.

First we consider a sinusoidal modulation with \( c(t) = d \cos(\Omega t) \). \( d \) and \( \Omega \) are respectively the modulation amplitude and frequency. The solution Eq. (4) then becomes
\[ \rho_0(t) = \frac{\rho_u + \rho_d}{2} + \frac{\rho_u - \rho_d}{2} \sin[\gamma + \eta \sin(\Omega t)], \tag{5} \]
with \( \eta = \pm 2df/\hbar \Omega \). The corresponding results are illustrated in Fig. 1 for \( \Omega = 0, 1/2, 1, \) and 2. The case of \( \Omega = 0 \) is simply the free evolution without modulation with a period \( T_0 = 2\pi/\beta \) determined by the initial condition [29]. For other cases, irrespective of the values for \( \Omega \), the frequency of oscillation is always \( \Omega \) and the corresponding period is \( 2\pi/\Omega \). As shown in Fig. 1 the oscillation amplitudes show two distinctive regions: one for \( \Omega \leq \Omega_c = 2df/\hbar \) (i.e. \( \eta \geq \pi \)) where the amplitude is the same with/without modulation; another for \( \Omega > \Omega_c \) (i.e. \( \eta < \pi \)) where the amplitude decreases with modulation frequency. In this latter region, it is easy to check that \( A = (\rho_u - \rho_d)(1 - \cos \eta)/2 \) for the case shown in Fig. 1.

Figure 2 illustrates the orbits for the corresponding spin dynamics. A full orbit is occupied if \( \Omega < \Omega_c \) but only partial orbits are occupied when \( \Omega > \Omega_c \). The occupied portion deceases when \( \Omega \) increases. Spin dynamics for the first half period is reversed during the second half period evolution, irrespective of the values for \( \Omega \neq 0 \). This reversal is responsible for a more robust modulated dynamics against various noises as noises are not reversed and their effect can be averaged zero according to coherent control theory [30].

Next we consider a periodic square function modulation with
\[ c(t) = \begin{cases} 
    d, & n(w + \tau) \leq t < n(w + \tau) + w, \\
    0, & n(w + \tau) + w \leq t < (n + 1)(w + \tau),
\end{cases} \tag{6} \]
for \( n = 0, 1, 2, \ldots \). The spin dynamics is halted completely if \( n(w + \tau) + w \leq t < (n + 1)(w + \tau) \) and is unmodulated if \( n(w + \tau) \leq t < n(w + \tau) + w \). The corresponding plots for \( \rho_0 \) and \( \theta \) display interesting step like
features as shown in Figs. 3(a) and 3(b).

The modulation dynamics considered above offers many interesting possibilities. A direct application is to remove a dynamical instability observed in a ferromagnetically interacting spin-1 condensate. This instability is removed whenever the imaginary part of the eigenfrequency for the corresponding Bogoliubov excitation becomes zero in a modulation cycle. We show this instability is indeed suppressed in the following for the cosine modulation \( c_2(t) = d \cos(\Omega t) \). This suppression inhibits the spontaneous formation of spin domains.

We now consider our system of a homogeneous \(^{87}\text{Rb}\) spin-1 condensate starting from an off-equilibrium initial state \( \tilde{\Psi}(0) \). The averaged spin \( \bar{\mathbf{f}} = \langle F_z \rangle \tilde{x} + \langle F_y \rangle \tilde{y} + \langle F_z \rangle \tilde{z} \) is conserved where \( F_{x,y,z} \) are spin-1 matrices. Starting from any stationary point, the evolution of the collective excitations takes a compact form

\[
\frac{i \hbar}{\partial t} \bar{\mathbf{x}} = \mathbf{M}(t) \cdot \bar{\mathbf{x}},
\]

where \( \bar{\mathbf{x}} = (\delta \Psi_+, \delta \Psi_0, \delta \Psi_-, \delta \Psi_0^+, \delta \Psi_0^-) \) with deviations \( \delta \Psi_j \) and \( \delta \Psi_j^* \) from the stationary solution \( \tilde{\Psi} \).

The special solution to Eq. (7) is \( \bar{\mathbf{x}}(t) = U(t,0)\bar{\mathbf{x}}(0) \) where \( U(t_2, t_1) = T \exp[-(i/\hbar) \int_{t_1}^{t_2} ds \mathbf{M}(s)] \) with \( T \) the time ordering operator. For periodic modulation, the solution during \( t \in [p T, (p + 1)T] \) is further simplified to \( \bar{\mathbf{x}}(t) = U(t, p T)U_T \bar{\mathbf{x}}(0) \), where \( p = 0, 1, 2, \cdots \) indexes the number of periods. \( T = 2\pi/\Omega \) is the period of modulation, and \( U_T = U(T, 0) = T \exp[-(i/\hbar) \int_0^T ds \mathbf{M}(s)] \) is the evolution operator for a complete period, \( |\bar{\mathbf{x}}(t)| \) will grow (decay) exponentially with \( p \) if the modulus of \( U_T \) are larger (smaller) than unity. Diagonalizing \( U_T \) and rewriting it as \( U_T = V \exp(-i TF)V^\dagger \) where the Floquet operator \( F \) is a 6-by-6 diagonal matrix, the criteria for stable dynamics reduces to a vanishing imaginary part of \( F_q \) (\( q = 1, 2, \cdots, 6 \)). Unstable dynamics arises if the imaginary part is not zero, while stable dynamics emerges if the imaginary part of all \( F_q \) is exactly zero.

The inset of Fig. 3 shows the imaginary part of a typical spectrum for the system under a cosine modulation of \( c_2 \). We focus on the most unstable mode which in principle dominates the unstable dynamics. Compared to the modulation free results (in dashed lines), we find the most unstable mode (in double contours) is not only suppressed in amplitude but also shifted to larger wave vector. The dependence of \( k_- \) on the modulation frequency \( \Omega \) is illustrated in Fig. 4. The almost independence of \( k_- \) on \( \Omega \) at small values contrasts with a strong monotonic increase at large values of \( \Omega \).

The emergence of spin domains is prohibited due to the modulation. On one hand, the suppression of \( F \) implies a smaller effective spin exchange interaction thus a longer effective spin healing length \( \xi \). On the other hand, the upshift of \( k_- \) means shorter wave length \( \lambda = 2\pi/k_- \). If the unstable mode wave length (potentially domain width) is smaller than the spin healing length, the condensate is able to heal by itself. The domain structure would never appear if \( \xi \) exceeds \( \lambda \).

Because of the modulation, however, the resulting dynamics becomes completely different from the case of \( c_2 = 0 \) or no spin dynamics at all. The modulation does not stop spin mixing dynamics, i.e., as we continue to observe spin waves which is nominally disguised in experiments by the spontaneously formed spin domains. Furthermore, we expect the modulated spin dynamics to be robust against various experimental noises because the periodic modulation effectively cancels uncorrelated noises from alternating modulation periods.

Finally we confirm the above conclusions for a trapped spin-1 condensate with full numerical simulations. We adopt experimental parameters as in Ref. [2]. The initial conditions are as in the experiment [8], with \(^{87}\text{Rb}\) condensates \( n_0(0) = 0.744, \theta(0) = 0, N = 2.0 \times 10^5 \), and \( m = 0 \), in a trap \( V_{\text{ext}}(r) = (M/2)(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) \).
with \( \omega_x = \omega_y = (2\pi)240 \text{ Hz} \) and \( \omega_z = (2\pi)24 \text{ Hz} \). The modulation function is \( c_2(t) = d \cos(\Omega t) \) with \( d = c_2 \), and \( \Omega = \omega_z \) which is about 3 times larger than the free spin evolution frequency \( 2\pi/T_0 \). Two cases will be considered: (1) The modulation is applied immediately (\( \rho_0 \) oscillates around \( \rho_u \)); (2) The modulation is turned on at \( t = 6 (1/\omega_z) \) (\( \rho_0 \) oscillates around \( \rho_d \)).

The results from numerical simulations are shown in Fig. 5. With modulation, the spin dynamics is clearly localized as \( \rho_+ \) (same for \( \rho_- \)) oscillates with a smaller amplitude around its initial value, in contrast to the large amplitude unmodulated result [panel (a)]. In addition, the unmodulated dynamics shows domain structures after \( t \approx 300 \), due to the intrinsic dynamical instability. While for both modulated cases, no spin domains are observed. Thus temporal modulation of spin exchange interaction does suppress the intrinsic dynamical instability and prohibit spontaneous domain formation. In our extensive numerical simulations, we find that when additional white noises are added, spin domains are found to arise quicker for the unmodulated case; yet for the two cases with modulations, almost the same behaviors are observed as if no noise were added.

Although the life time of the condensate at optical Feshbach resonance is reduced dramatically in \(^{87}\text{Rb} \) gases \cite{13}, we notice that there exists at least one magic window of relative frequency, e.g., between resonance \( \beta \) and \( \gamma \) in Fig. 7 of Ref. \cite{15}, where the condensate life time lasts several hundred milliseconds and \( c_2 \) changes sign. On the other hand, the spin domain emerges in a time scale typically shorter than 100 ms \cite{33}. Therefore the suppression of the domain formation in \(^{87}\text{Rb} \) condensate is experimentally feasible.

In summary, we propose to localize the spin mixing dynamics in a spin-1 condensate by temporally modulating the spin exchange interaction. For condensed atoms, the modulation can be facilitated with the technique of optical Feshbach resonance \cite{13,15}. We demonstrate the suppression of the intrinsic instability thus the inhibition of spontaneous spin domain formation in a ferromagnetically interacting spin-1 Bose condensate, such as \(^{87}\text{Rb} \) condensate in the \( F = 1 \) manifold. In addition, the effective freezing of spin mixing dynamics due to spin exchange interaction provides a cleaner approach to investigate magnetic spin mixing dynamics in a spin-1 Bose condensate \cite{11,12}.

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