Scaling approach to higher-twist corrections

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Abstract

Bjorken scaling is violetd. At large $x$-values ($0.7 \lesssim x \leq 1$) the violation is mainly attributed to $\propto 1/Q^2$ (higher-twist) corrections. We discuss how to incorporate such corrections by using a new scaling variable $\bar{x}(x, Q^2)$ which accounts for the non-perturbative effects due to the confining parton interaction in the final state. The approach can accommodate also the remaining QCD logarithmic corrections and the behaviour of the $F_2(x, Q^2)$ structure functions is reproduced, in a quite natural way, within a wide kinematical range.

The existing data for hadron structure functions, $F_2(x, Q^2)$, show considerable $Q^2$-dependence, which is mainly attributed to the QCD logarithmic corrections to Bjorken scaling. However, at $x \to 1$ the scaling violations are dominated by power corrections $\propto 1/Q^2$ (higher twist and target mass effects):

$$F_2(x, Q^2) = F_2^{as}(x, Q^2) + \frac{B(x)}{Q^2} + \cdots,$$

where $F_2^{as}(x, Q^2) = F_2(x, Q^2) \gg |B(x)|$ and the remaining $Q^2$-dependence in $F_2^{as}(x, Q^2)$ is to be attributed to QCD logarithmic corrections only.

Power corrections can be incorporated in the first term of Eq. (1) by using a different scaling variable,
\[ \dot{x} = \phi(x, Q^2) = x + \frac{b(x)}{Q^2} + \cdots, \]  

so that

\[ F_2(x, Q^2) = F_2 \left( \phi^{-1}(\dot{x}, Q^2), Q^2 \right) \simeq F_2^{as}(\dot{x}, Q^2). \]  

The coefficient \( B \), which determines the value of the power correction in Eq. (1), is thus related to the structure function by \( B(x) = b(x) \partial F_2^{as}(x, Q^2)/\partial x \).

Actually, an analysis of data in terms of an appropriate scaling variable appears to be more convenient, than the direct evaluation of power corrections. For instance, it is common to use for an analysis of data the Nachtmann variable \( \dot{x} \equiv \xi \),

\[ \xi = \frac{2x}{1 + \sqrt{1 + 4M^2x^2/Q^2}}, \]  

which is expected to account for the effects of the target mass (\( M \)).

Besides the target mass effects, there are important nonperturbative effects from the confining interaction of the partons in the final state. Indeed, the partons are never free, so that the system possesses a discrete spectrum in the final state. Although in the Bjorken limit the struck quark can be considered a free particle, the discreteness of the spectrum manifests itself in power corrections to asymptotic structure functions. One can anticipate that these corrections are significant in particular at large \( x \), where lower-lying excitations should play an important role.

A general analysis performed in the framework of Bethe-Salpeter equation shows that in the case of a local confining final state interaction the higher twist and target mass effects can be effectively accounted for by taken the struck quark with the same off-shell mass before and after the virtual photon absorption. As a result, the Bjorken scaling variable \( x \) is replaced by a new scaling variable \( \bar{x} \equiv \bar{x}(x, Q^2) \), which is the light-cone fraction of the off-shell struck quark. Explicitly,

\[ \bar{x} = \frac{x + \sqrt{1 + 4M^2x^2/Q^2} - \sqrt{(1 - x)^2 + 4m_s^2x^2/Q^2}}{1 + \sqrt{1 + 4M^2x^2/Q^2}}, \]  

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where $M$ is the target mass and $m_s$ is the invariant mass of spectator partons (quarks and gluons). For $Q^2 \to \infty$ or for $x \to 0$ the variable $\bar{x}$ coincides with the Nachtmann variable $\xi$, Eq. (4). However, at finite $Q^2$ these variables are quite different.

It follows from Eq. (5) that $\bar{x}$ depends on the invariant spectator mass, $m_s$. The latter can be considered a function of the external parameters only. In the limit $x \to 1$ (elastic scattering) no gluons are emitted, and thus $m_s \to m_0$, which is the mass of a two-quark system (diquark). When $x < 1$, the spectator mass $m_s$ increases due to gluon emission. For $x$ close to 1, $m_s$ can be approximated

$$m_s^2 \simeq m_0^2 + C(1 - x),$$  

(6)

where the coefficient $C \sim (\text{GeV})^2$. In the following we regard it as a phenomenological parameter, determined from the data.

Let us consider the nucleon structure functions in the region of large $x$, where the power corrections to the scaling are dominant. At present, the only available large-$x$ data for proton and deuteron structure functions, $F^p_2(x, Q^2), F^d_2(x, Q^2)$, are the SLAC data [4-7], taken at moderate values of momentum transfer, $Q^2 \lesssim 30 \ (\text{GeV/c})^2$. (The nucleon structure functions for higher values of momentum transfer ($Q^2 \lesssim 250 \ (\text{GeV/c})^2$) are extracted from BCMDS [8] and NMC [9] data, yet only for $x \leq 0.75$). The SLAC data for the proton and deuteron structure functions for $x \geq 0.7$ and $5 \lesssim Q^2 \lesssim 30 \ (\text{GeV/c})^2$ are shown in Fig. 1 as a function of $x$. Also shown is the value of $F^p_2(x, Q^2)$ and $F^d_2(x, Q^2)$ for $Q^2 = 230 \ (\text{GeV/c})^2$ and $x = 0.75$ taken from the BCDMS data [8]. The data points close to the region of resonances were excluded by a requirement that the invariant mass of the final state $(M + \nu)^2 - q^2$ is greater than $(M + \Delta)^2$, where $\Delta = 300$ MeV. In addition, we excluded the data points with $x > 0.9$ from the deuteron structure only (Fig. 1b). The reason is that the deuteron structure function can not be represented as an average of the proton and neutron structure functions for $x \gtrsim 0.9$. Indeed, the calculations of Melnitchouk et al. [10] show that the ratio $2F^d_2/(F^p_2 + F^n_2)$ is about 1.13 for $x = 0.9$ and $Q^2 = 5 \ (\text{GeV/c})^2$, and it rapidly increases for $x > 0.9$. However, for $x < 0.85$, this ratio is within 5% of unity [11].
One finds from Fig.1 that the structure functions show no scaling in the Bjorken variable $x$. Also, very poor scaling is obtained when the data are plotted as a function of the Nachtmann variable $\xi$, Eq. (4) [7]. However, the situation is different if we display the same data as a function of the variable $\bar{x}$, Eq. (5). It appears that the scaling in the $\bar{x}$-variable is strongly dependent on the value of diquark mass, $m_0$, Eq. (6), but is much less sensitive to variation of the coefficient $C$. For instance, Fig. 2 shows the data as a function of $\bar{x}$ for $m_0 = 600$ MeV and $C = 3$ (GeV)$^2$, i.e. by considering the spectator as build up from constituent quarks. The data display very poor scaling, although it is slightly better than that shown in Fig. 1. The scaling deteriorates even more when $m_0 > 600$ MeV. On the other hand, the scaling is very good both for the proton and deuteron data, by taking $m_0 = 0$, i.e. by considering the spectator build up by current quarks [3]. The results are shown in Fig. 3, where the data are plotted as a function of $\bar{x}$ for $m_0 = 0$ and $C = 3$ (GeV)$^2$. Note, that the high-$Q^2$ data points from BCDMS data [8] are very close to the SLAC data points, taken at much lower values of $Q^2$. Also note that $\bar{x} \to 1$ when $x \to 1$ for $m_0 = 0$, Eq. (5) but $\bar{x}(x, Q^2) < x$ for $x < 1$. As a result the data, plotted as a function of $\bar{x}$-variable are extended in a wider region, than the same data plotted as a function of the $x$-variable (cf. Figs. 1 - 3).

Now by using the scaling variable $\bar{x}$, Eq. (5) for $m_0 = 0$ we are going to analyze the proton structure function for smaller values of $x$, where both power and logarithmic corrections to the Bjorken scaling play an important role. In this region ($0.35 < x < 0.75$) the existing BCDMS [8] and NMC [9] data are extended up to much larger values of momentum transfer than the previously considered high-$x$ SLAC data. It allows us to check our predictions in a wide $Q^2$ range.

QCD (logarithmic) evolution effects on $F_2$ are taken into account at Next-to-Leading Order (NLO) [12] evolving back, in $Q^2$, the structure functions starting from an asymptotic value of momentum transfer where the condition $F_2(x, Q^2) \simeq F_2^{as}(x, Q^2)$ (cf. Eq. (1)) is fulfilled (in the present case we choose $Q^2 = 230$ (GeV/c)$^2$, which is the highest value of the momentum transfer in the BCDMS data [8]). At that value of $Q^2$ the functional
form of $F_2$ is parametrized following the prescriptions of the recent NMC fit \cite{13} and valid for a wide range of $x$ ($0.006 < x < 0.9$). For the $x$-region we are interested in, namely $x \geq 0.35$, one can assume that the *valence* contribution to $F_2$ dominates and one can consider evolution of the nonsinglet (NS) components only. The accuracy of such an assumption can be deduced from the recent CTEQ parametrization \cite{14} of the parton distributions. The ratio $r = F_2^{NS}(x, Q^2)/F_2(x, Q^2)$ is $r \approx 0.96$ for $x = 0.35$ and $r > 0.995$ for $x \geq 0.55$ and $Q^2 \geq 5 \text{ (GeV/c)}^2$. In fact, since the only assumptions we are making is due to the fact that the (small) singlet component evolves in $Q^2$ in a different way with respect the dominant nonsinglet part, the inaccuracy is less than 4% for $x = 0.35$ and less than 0.5% for larger $x$. For the actual calculations we use the NLO procedure developed in ref. \cite{15}. Under the renormalization group equation (RGE) the moments of the nonsingle t components of the nucleon structure function, $\langle F_2^{NS}(Q^2) \rangle_n = \int_0^1 dx x^{n-2} F_2^{NS}(x, Q^2)$, evolve according to

$$\langle F_2^{NS}(Q^2) \rangle_n = \langle F_2^{NS}(Q_0^2) \rangle_n \left(1 + \frac{\alpha_s(Q^2)}{4\pi} R_{2,n}^{NS} \right)^{\gamma_n^{0,NS}/(2\beta_0)}$$

(7)

where

$$R_{2,n}^{NS} = \frac{\gamma_n^{1,NS}}{2\beta_0} - \frac{\beta_1}{2\beta_0} \gamma_n^{0,NS} + C_{2,q}^{1,n}$$

(8)

and we used the expansion

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2)} - \frac{\beta_1}{\beta_0} \ln\ln(Q^2/\Lambda^2) - \beta_3 \frac{1}{[\ln(Q^2/\Lambda^2)]^2},$$

(9)

$\beta_0 = 11 - 2 f/3$, $\beta_1 = 102 - 38 f/3$ and $f$ is the number of active flavours. The values of all the other parameters of the calculation (like the anomalous dimensions $\gamma_n^{0,NS}$, $\gamma_n^{1,NS}$ and the Wilson coefficient $C_{2,q}^{1,n}$) have been compiled in refs. \cite{12,16}. The form (7) guarantees complete symmetry for the evolution from $Q_0^2$ to $Q^2 > Q_0^2$ and back. In our case $Q_0^2 = 230 \text{ (GeV/c)}^2$ and $5 < Q^2 < 230 \text{ (GeV/c)}^2$. The NLO factorization scheme implicitly selected in Eq.(8) is the so called DIS scheme where the structure function assumes the form $F_2^p(x, Q^2) = \sum_q e_q^2 x (q(x, Q^2) + \bar{q}(x, Q^2))$. 
The procedure of perturbative QCD evolution previously sketched implies that the power corrections are already extracted from the structure function $s$, so that the evolution has to be actually applied to $F_2^{as}$, Eqs. (1)-(2)

$$F_2^{as}(x, Q^2) = F_2(x, Q^2) - \frac{B(x)}{Q^2} - \cdots \simeq F_2 \left( \bar{x}^{-1}(x, Q^2), Q^2 \right),$$

(10)

where we used the scaling variable $\phi(x, Q^2) \equiv \bar{x}(x, Q^2)$, Eq. (5) for $m_0 = 0$ and $C = 3 \ (\text{GeV})^2$ that effectively incorporates the power corrections. It implies that the evolution of structure functions back from $Q^2 = 230 \ (\text{GeV}/c)^2$, should be calculated with a shifted value of Bjorken variable, namely

$$\Delta_1(x, Q^2) = F_2 \left( \bar{x}^{-1}(x, Q^2), Q^2 \right) - F_2 \left( \bar{x}^{-1}(x, Q^2), 230 \right).$$

(11)

The influence of this shift would be quite important for low and moderated values of $Q^2$.

The corresponding variation of structure functions with $Q^2$ for fixed $x$ due to power corrections can be evaluated as

$$\Delta_2(x, Q^2) = F_2 \left( \bar{x}(x, Q^2), 230 \right) - F_2 \left( \bar{x}(x, 230), 230 \right).$$

(12)

Finally the $Q^2$-dependence of structure functions due to logarithmic and power corrections to Bjorken scaling is given by

$$F_2(x, Q^2) = F_2(x, 230) + \Delta_1(x, Q^2) + \Delta_2(x, Q^2).$$

(13)

The results are shown in Figs. 4 and 5 for proton and deuteron structure functions respectively. The data points are from SLAC and BCDMS data bins [1,8]. The dotted lines show the $Q^2$-dependence of the structure functions due to power corrections only, Eq. (12). The total $Q^2$-dependence of structure functions due to the power and the logarithmic NLO corrections, Eq. (13), is shown by the dashed and continuous lines for $\Lambda = 100 \ \text{MeV}$ and $\Lambda = 200 \ \text{MeV}$ respectively. One finds from Figs. 4 and 5 that Eq. (13) reproduces the experimental data in a large $Q^2$-range for both values of $\Lambda$, although the agreement is slightly better for $\Lambda = 100 \ \text{MeV}$. In addition, since the results are strongly dependent on the spectator mass
it is remarkable that the same parameters $m_0 = 0$ and $C = 3 \text{ (GeV)}^2$ do reproduce the $Q^2$-behavior of the structure functions both for large and moderate $x$-values.

In conclusion we have presented a detailed investigation of the $F_2$ experimental data, both for proton and deuteron, analysed by means of the scaling variable $\bar{x}$ recently proposed in ref. [3]. Such variable includes the non-perturbative effects due to the confining interactions of the partons in the final state and contains, in an effective way, higher-twist corrections. These contributions show up at large $x$ and at low and moderate momentum transfer $Q^2$. The scaling behavior of the experimental data is improved in a wide range of $x$ and $Q^2$. The additional inclusion of the QCD radiative logarithmic corrections allows us the investigation of the structure function in an even larger kinematical range with the extra bonus of a rather precise identification of the spectator diquark mass.

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FIGURE CAPTIONS

Fig 1. The SLAC data [4–7] \((5 \lesssim Q^2 \lesssim 30 \text{ (GeV/c)^2})\) for proton (a) and deuteron (b), are shown as a function of the Bjorken variable \(x\). Three high-statistics data sets [8] for \(Q^2 \approx 5.7, 7.6,\) and 9.5 \((\text{GeV/c})^2\) are marked by “+”, “x”, and “#” respectively. The point at \(Q^2 = 230 \text{ (GeV/c)^2}\) and \(x = 0.75\) is from ref. [8].

Fig 2. The data of Fig. 1 are shown as function of the \(\bar{x}(x,Q^2)\) — the scaling variable of Eq. (5) — assuming \(m_0 = 600 \text{ MeV}\) and \(C = 3(\text{GeV})^2\) for the spectator mass \(m_s\), Eq. (6).

Fig 3. As in Fig. 2 assuming \(m_0 = 0\) and \(C = 3 \text{ (GeV)^2}\).

Fig 4. The proton structure function \(F^p_2(x,Q^2)\) is shown as a function of \(Q^2\) at different \(x\)-values. The dotted lines include power corrections only. They are evaluated according to Eq. (12) and the scaling variable \(\bar{x}\) of Eqs. (5), (6) with \(m_0 = 0\) and \(C = 3 \text{ (GeV)^2}\). The additional QCD logarithmic corrections evaluated at NLO according to the procedure of Eq. (11), (13) for different \(\Lambda\) scales are shown by the dashed \((\Lambda = 100 \text{ MeV})\) and continuous lines \((\Lambda = 200 \text{ MeV})\).

Fig 5. As in Fig. 4 for the deuteron structure function \(F^d_2(x,Q^2)\).
Fig. 1

(a) $F_2^p(x, Q^2)$

(b) $F_2^d(x, Q^2)$

$Q = 230 \text{(GeV/c)}^2$
Fig. 2

(a) $F_2^p(x, Q^2)$

(b) $F_2^d(x, Q^2)$

$m_0 = 600$ MeV
Fig. 3
Fig. 4

\[ F_2^p(x, Q^2) \]

\[ Q^2 \text{ (GeV/c)}^2 \]

\[ x = 0.35 \]
\[ x = 0.45 \]
\[ x = 0.55 \]
\[ x = 0.65 \]
\[ x = 0.75 \]
Fig. 5