A Simple Model for the Turbulence Intensity Distribution in Atmospheric Boundary Layers

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Abstract. In the current work, we examine the distribution of turbulence intensity (TI) in simulations of atmospheric boundary layers (ABL) for wind turbine applications. We relate the turbulent fluctuations to the mean wind shear profile for neutrally stable ABL based on turbulent boundary layer theory. The results are then compared to corresponding LES data for various shear conditions. From this, a model for the TI distribution across a rotor disc is devised and calibrated. In addition, a second simpler version of the model is derived by assuming a power law for the velocity profile. The models are related to the Mann model and shown to have a similar scaling, however they are simpler and based on assumptions for turbulent boundary layers rather than homogeneous turbulence. The models are then tested for a range of stability conditions and shear values of the ABL including waked conditions from upstream wind turbines that stretch the assumptions made.

1. Introduction

In many current models for wind turbine design and operation, the hub-height turbulent intensity (TI) is used as an indicator for the expected velocity fluctuations for the turbine blades. However, a significant variation of the TI across the rotor disc impacts the loads and performance of each blade as it rotates. As rotor diameters increase, the distribution of TI becomes a larger concern.

Many different models for the atmospheric boundary layer turbulence currently exist and are used in wind turbine design. For instance, the standard Kaimal model [1] provides a common method for determining the turbulence spectrum at given wind speeds, but does not provide information on its spatial variation. Other observations by Barthelmie et al [2] have noted that the TI decreases with height based on measurements in offshore wind farms, but did not connect the variation with other wind properties. Finally, the Mann model [3], [4] provides an analytic model for the turbulence in the presence of uniform linear shear using rapid distortion theory, but the model is strictly speaking not valid in the logarithmic layer of the turbulent wall-bounded boundary layer that wind turbines are frequently exposed to.

The current work analyzes the variation of turbulence fluctuations in neutral and unstable atmospheric boundary layers. A relatively simple model is formulated which allows the TI to be estimated from the mean velocity shear profile. The model is then calibrated with Large Eddy Simulations (LES) data. Comparisons are shown indicating the potential for such an approach. Moreover, it is common in turbine system analysis to assume a power law velocity profile that is fitted through hub height mean velocity [5] (measured or computed) or an equivalent single value such as a rotor-averaged wind speed. With this in mind, a second even simpler model is also devised and assessed. Finally, the models are tested for waked conditions in a wind farm.
2. Model for turbulence intensity distribution

2.1. Formulation

A relatively simple model for the turbulence intensity distribution under neutrally stable conditions can be developed using turbulent boundary layer theory. We consider the transport equation for the turbulent kinetic energy $k$

$$\frac{Dk}{Dt} = \nabla \cdot (\nu_t \nabla k) + P - \varepsilon$$

with turbulent viscosity $\nu_t$, a constant turbulent Prandtl number $\sigma_k$, turbulent kinetic energy production $P$ and dissipation rate $\varepsilon$. For a simple shear flow [6], the production term can be written as

$$P = -(u_i u_j) \frac{\partial U_i}{\partial x_j} = \nu_t \left( \frac{\partial U_i}{\partial z} \right)^2 = c k^{1/2} l_m \left( \frac{\partial U_i}{\partial z} \right)^2$$

where, in accordance with mixing length theory [7], the turbulent viscosity is assumed to be

$$\nu_t = c k^{1/2} l_m$$

for some mixing length $l_m$ and constant $c$. Similarly, the dissipation rate term can be modeled as

$$\varepsilon = c_d k^{3/2} / l_m.$$

Near the surface of the boundary layer, the kinetic energy convection and diffusion terms can be neglected in equation (1), leaving only a balance between the production and dissipation terms. This balance remains valid as long as the velocity profiles fall within the log layer region, and is critical to the accuracy of the model (see section 3.1). Following this assumption leads to the conclusion that

$$k = \frac{c_l^2}{c_d} \left( \frac{\partial U}{\partial z} \right)^2,$$

or that the turbulent fluctuations should vary as $u_{rms} \sim l_m \left( \frac{\partial U}{\partial z} \right)$. In this work, we relate the fluctuating velocities and the vertical mean shear via

$$u_{rms} = u_0 + C_0 l_m \left( \frac{\partial U}{\partial z} \right)$$

for some constants $u_0'$ and $C_0$. For wall-bounded shear flows, a simple but often sufficient model for the mixing length $l_m$ can be written as linearly varying with height in the inner part of the boundary layer and a constant in the outer part [8]

$$l_m(z) = \delta_{BL} C_\mu \left( \frac{2}{z_l} f_{blend}(z - z_l) + f_{blend}^+(z - z_l) \right)$$

where $\delta_{BL}$ is the boundary layer height and the blending function is defined as

$$f_{blend}^+(z) = \left\{ \begin{array}{ll} 1 & \text{for} \ 0 \leq z \leq z_l \\ \frac{1}{2} (1 \pm \tanh(z)) & \text{else} \end{array} \right.$$

From equation (2), one can model the turbulence intensity profile in a similar manner

$$TI(z) = TI_0 + \frac{u_{rms}}{U(z)}$$

where $TI_0$ is a constant for calibration purposes. The values of $TI_0$, $u_0'$, and $C_0$ are discussed in section 3.1.

While mixing length analyses depend heavily on the assumptions used in modeling the length scale, the simplicity of equations (2) and (5) allows for a quick estimate of the TI variation. Furthermore, in many situations the velocity profile used in the analysis of wind turbine dynamics is fitted to a power-law profile. As discussed in section 3.1, using such a velocity profile in a properly calibrated model further simplifies the estimates and removes the need for simulations or measurements that provide the shear information.

2.2. Comparison with the Mann model for neutral atmospheric surface layer turbulence
From the analysis above, we find that the turbulent kinetic energy with in the logarithmic layer is approximately proportional to the square of the mean shear. Here we compare this result with the analysis of Mann [3], who considered the effect of uniform linear shear on homogeneous turbulence using rapid distortion theory. Specifically, we examine the limit of low shear, when the non-dimensional time $\beta = \frac{du}{dz} \times \tau$ is relatively small for a particular timescale $\tau$. In this regime, we find that the diagonal components of the velocity spectral tensor $\Phi_{ij}$ at wavenumber $\mathbf{y}$ reduce to

$$
\Phi_{11}(\mathbf{y}) \approx \frac{E(y_0)}{4\pi y_0^4}(y_0^2 - y_1^2 - 2y_1y_30\beta F_1(y) + (y_1^2 + y_2^2)\beta^2 F_2(y))
$$

$$
\Phi_{22}(\mathbf{y}) \approx \frac{E(y_0)}{4\pi y_0^4}(y_0^2 - y_2^2 - 2y_2y_30\beta F_2(y) + (y_1^2 + y_2^2)\beta^2 F_2(y))
$$

$$
\Phi_{33}(\mathbf{y}) = \frac{E(y_0)}{4\pi y_0^4}(y_1^2 + y_2^2)
$$

for a given energy spectrum $E(\mathbf{y})$ and $\mathbf{y}(t = 0) = y_0$. The functions $F_1(\mathbf{y})$ and $F_2(\mathbf{y})$ are given by

$$
F_1 = \frac{y_1^2(y_0^2 - 2y_30)}{y^2(y_1^2 + y_2^2)} - \frac{y_1y_2}{(y_1^2 + y_2^2)^{3/2}}
$$

$$
F_2 = \frac{y_2^2(y_0^2 - 2y_30)}{y_1[y^2(y_1^2 + y_2^2)]} + \frac{y_1y_2}{(y_1^2 + y_2^2)^{3/2}}
$$

To calculate the turbulent kinetic energy $k$, we integrate the trace of $\Phi_{ij}$ over the entire wavenumber space

$$
k = \frac{1}{2} \int (\Phi_{11} + \Phi_{22} + \Phi_{33}) d^3y.
$$

From this we find that the turbulent kinetic energy also varies

$$
k \sim \beta^2 \sim \left(\frac{dU}{dz}\right)^2 \tau^2,
$$

which is consistent with the result from the previous section. In the case of uniform shear with blocking, also considered by Mann, the velocity spectral tensor is modified close to the wall, but the quadratic dependence on shear remains. However, it is worth noting that while Mann’s model considers the effect of uniform shear on the turbulence, the model described in section 2.1. assumes instead a logarithmic mean velocity profile exists, i.e. a high Reynolds number turbulent boundary layer.

3. Results

3.1. Calibration of model using LES data

To examine the suitability of the current model and determine the necessary calibration constants, we first applied it to a series of LES calculations of neutrally stable atmospheric boundary layers (ABL) in the absence of wind turbines or other complex terrain. These ABL calculations covered hub-height wind speeds from 7 m/s to 14 m/s, hub-height shear exponents from 0.11 – 0.19, and included turbulent intensities ranging from 8% to 28% at a hub-height of $z=80$ m (see Table 1). These conditions were chosen to represent the range of wind speeds and wind shears typically encountered in neutrally stable atmospheric boundary layers. All calculations were done using the SOWFA LES code for wind farm simulations [9] on a 4 km x 4 km x 1 km flat domain using a 400 x 400 x 100 size mesh. All SOWFA calculations used a one-equation eddy viscosity subgrid scale model and a wall shear stress model from Schumann [10] to model the surface stress at the ground. This wall shear stress model incorporates on a rough wall log law at the bottom boundary, and depends on an aerodynamic roughness height parameter $z_0$. The roughness parameter $z_0$ can then be adjusted to target a specific hub-height shear in the atmospheric boundary layer.
The atmospheric boundary layer solver in SOWFA uses a finite-volume formulation with Rhie-Chow flux interpolation and a pressure implicit splitting operation (PISO-SIMPLE) scheme to solve the momentum and pressure equations. Time advancement is performed using a second order backward Euler scheme. Periodic boundary conditions are applied to the north, south, east and west boundaries. A capping inversion layer controls the boundary layer height, and for the neutrally stable atmospheric cases, a zero temperature gradient at the wall condition was applied. The wind direction was arbitrary set to come from 235 degrees southwest, the initial velocity profile was set using a logarithmic mean velocity profile. During the simulation, the applied pressure gradient was adjusted until the desired hub-height wind velocity was achieved, and the simulation was then continued until the mean flow statistics converged.

| Wind Speed (m/s) | Hub-height shear | Hub-height TI | $z_0$ [m] | Stability |
|------------------|------------------|----------------|-----------|-----------|
| 7                | 0.11             | 8.3%           | 0.15      | Neutral   |
| 10               | 0.12             | 9.0%           | 0.01      | Neutral   |
| 10               | 0.16             | 18.8%          | 0.10      | Neutral   |
| 10               | 0.19             | 14.5%          | 0.15      | Neutral   |
| 12               | 0.19             | 17.6%          | 0.15      | Neutral   |
| 14               | 0.19             | 27.8%          | 1.00      | Neutral   |

![Figure 1. Horizontal wind profiles for the atmospheric boundary layer profiles in Table 1. The hub height location (dashed line) is located at $z=80m$.](image)

The horizontal wind profiles, shown in Figure 1, were collected from each LES after averaging the domain in both horizontal directions. In addition, statistics regarding the vertical mean shear and turbulent intensity profiles were also gathered. By plotting the mean horizontal profiles on a log-linear scale, we can also check the assumption that the region of interest in the wind profile falls within the log-layer region. As illustrated in Figure 2, the velocity profiles near the rotor disc altitudes for the 10 m/s and 12 m/s case fall within the log-layer region, where we expect the production and dissipation
terms to roughly balance. For these cases we would then expect the model in equation (5) to be reasonable accurate. The location of the turbine hub-height fell within a similar region for the 7 m/s and 14 m/s profiles, although the log-layer region was smaller at the highest velocities.

Using the mean flow information from the LES calculations, the model described in section 2.1 was then calibrated. In this particular case, the parameter $u'_0$ was set to be equal to the variance of the measured horizontal velocity fluctuations $u'_0 = \sqrt{(u'_{rms})^2 + (v'_{rms})^2}$ for each profile at the hub-height location. The values of the calibration constants $T_{I_0} = -0.057$ and $C_o = 0.48$ were then determined using a linear regression fit between the computed shear $\frac{dU}{dz}$ and $TI$.

![Figure 2. Comparison of the wind speed profile (blue lines) versus the log layer profile (green line) for the (a) neutral 10 m/s, (b) neutral 12 m/s, (c) unstable 10 m/s, and (d) unstable 12 m/s ABL simulations. The horizontal mean velocities are non-dimensionalized by the friction velocity $u^*$ and the vertical heights by the surface roughness $z_0$, and the hub height position at $z=80m$ is given by the vertical dashed line.](image)

The resulting comparison between the calibrated model for the turbulence intensity and the LES turbulence intensity is shown in Figure 3. Here we see that using the computed vertical shear $\frac{dU}{dz}$ in equation (5) provides a fairly accurate representation of the TI profile for the neutral boundary layers. Of the six configurations considered in Table 1, the TI variation was best captured for the hub-height wind velocities at 10 m/s and 12 m/s, and slight deviations were observed for the ABL at 7 m/s and 14 m/s.

We can also consider applying the model in equation (5) to reconstruct the entire TI profile using information only gathered at the hub-height location. To accomplish this, we use a power-law fit to the horizontal wind velocity profile

$$U_{power}(z) = U_{HH} \left( \frac{z}{z_{HH}} \right)^{\alpha},$$

given a specific hub-height $z_{HH}$, wind speed $U_{HH}$ and shear exponent $\alpha$. These values can be readily obtained from meteorological mast measurements and nacelle anemometer data. From this wind profile, the wind shear can be determined and used to compute the resulting TI profile. The results computed using $U_{power}(z)$, also shown in Figure 3, are similar to those using the LES computed wind speed profiles. The majority of the differences occur directly near the ground, where $\alpha$ varies from the hub-height measured value and the power law profile in ) is not likely to fall within the log-layer region.

The information on the TI profiles presented so far can also be used to quantify the variation in turbulence that a wind turbine should expect to encounter across the rotor disc. In Table 2 we compute the maximum and minimum values of TI experienced by the rotor at the top and bottom of the disc, and the difference between the two. Here we have assumed a hub height of 80 m and a rotor diameter of 77 m, corresponding to a GE-1.5SLE turbine. For wind speeds from 7 m/s to 12 m/s, the difference
in TI was generally between 3-6%. However, at the highest wind speed considered here (14 m/s), the difference in TI was 15%, significantly higher than at the other conditions.

![Figure 3. Comparison of the computed and modelled TI distribution for neutral ABL simulations. Legend: Solid blue lines (LES computed TI), red dashed lines (model TI using LES velocity profile), black dots (model TI using power law velocity).](image)

\[ U_{\text{power}}(z) = U_{HH} \left( \frac{z}{z_{HH}} \right)^{\alpha} \]

Table 2. TI differences across the rotor disc for a 77 meter rotor diameter. The TI is based on the local mean horizontal velocity

| Wind Speed | Hub-height shear | Rotor Top TI [%] | Rotor Bottom TI [%] | Difference [%] |
|------------|-----------------|------------------|---------------------|----------------|
| 7 m/s      | 0.11            | 6.6              | 12.4                | 5.8            |
| 10 m/s     | 0.12            | 7.8              | 11.1                | 3.3            |
| 10 m/s     | 0.16            | 16.9             | 21.6                | 4.7            |
| 10 m/s     | 0.19            | 12.7             | 17.7                | 5.0            |
| 12 m/s     | 0.19            | 15.5             | 21.4                | 5.9            |
| 14 m/s     | 0.19            | 23.6             | 38.8                | 15.2           |

3.2. Application to the unstable atmospheric boundary layer

Table 3. Overview of the weakly unstable boundary layer cases.

| Wind Speed | Hub-height shear | Hub-height TI | \( z_0 \) [m] | \( q_{wall} \) [K/m] | Stability |
|------------|-----------------|---------------|---------------|---------------------|-----------|
| 10 m/s     | 0.08            | 13.8%         | 0.15          | -0.025              | Unstable  |
| 12 m/s     | 0.03            | 22.8%         | 0.15          | -0.25               | Unstable  |

In addition to the neutrally stable cases analysed in the previous section, we also computed a series of
weakly unstable boundary layers with wind speeds of 10 m/s and 12 m/s (see Table 3). For these cases, the temperature gradient near the wall was also applied to change the temperature profile near the ground. For these cases, the same procedure was used in SOWFA to reach the desired condition: the applied pressure gradient was adjusted until the target hub-height wind speed was achieved. Compared to the previous neutrally stable cases, the unstable boundary layer profiles can be characterized by lower values of the hub-height shear exponent. This is to be expected, as the majority of the shear was observed to exist near the ground. The differences between the horizontal wind speed profiles are shown in Figure 4.

![Figure 4](image-url)  
**Figure 4.** Horizontal wind profiles for the neutral (dashed lines) and weakly unstable (solid lines) atmospheric boundary layer profiles given in Table 3. The hub height location (black dashed line) is located at $z=80m$.

![Figure 5](image-url)  
**Figure 5.** Comparison of the computed and modelled TI distribution for the unstable ABL. Legend: Solid blue lines (LES computed TI), red dashed lines (model TI using LES velocity profile), black dots (model TI using power law velocity and hub-height shear exponent), black dash-dot (model TI using power law velocity and an averaged, near-ground shear exponent). The hub height location (black dashed line) is located at $z=80m$.

Similar to the comparisons shown in section 3.1, we can also apply the modelled TI profiles from equation (5) to the unstable cases, using the same calibrated constants as before. As shown in Figure 2, the logarithmic law does not hold for these flow conditions. As a consequence, the model fails to capture the TI distribution as accurately as the neutrally stable boundary layers. In Figure 5, we see that the unstable cases display greater turbulent kinetic energy generation, especially near the ground, which is missed by equation (5). Replacing the LES computed wind shear with the power-law wind
profile and $\alpha$ matched at hub-height yields a similar discrepancy. However, by using the power law with a value of $\alpha$ averaged from the ground to the bottom of the rotor disc, we find that the model can better captures the TI profiles in this case. Note however that no effort was made to recalibrate the model coefficients for unstable conditions.

3.3. Wind farm application

In the previous sections, the model for TI variation was calibrated and evaluated for precursor simulations over flat domains. These cases did not include the presence of the wind turbine, and did not consider the effect of axial induction or turbine wakes on the boundary layer profiles. To examine the behaviour of this model under more realistic conditions, we now investigate the atmospheric boundary in a two turbine, waked configuration commonly seen in wind farms.

The conditions of the LES calculations of this two turbine configuration were chosen to match previous internal studies of wake interactions between turbines. The SOWFA simulations were run with a pair of GE 1.5SLE turbines, each with a $D=77$ meter rotor diameter and a hub height of 80 meters. The turbines are spaced 5.5 rotor diameters apart and aligned with the incoming wind direction, as depicted in Figure 6. The average hub-height wind speed was set at 7m/s originating from 235 degrees southwest (here north corresponds to 0 degrees and is aligned with the positive y-axis). Although the majority of the dataset used to calibrate the model used wind speeds of 10-12 m/s, here we adopt the same calibration constants to see if the model remains valid at the lower wind velocities. After an initial precursor simulation was run to establish the atmospheric boundary layer, the flow field and boundary conditions were used in a refined grid simulation which included both turbines. Data from the LES were collected at 16 sampling planes upstream and downstream of the two turbines (see Figure 6, right). Mean velocity data was averaged over a ten minute sampling period and further spanwise averaged to obtain velocity profiles as a function of height. The ten minute sampling period corresponds to the standard window length used to gather statistics in the field.

Figure 7 shows the mean horizontal wind speed profile at planes 1, 2, 3, located 3D, 2D, and 1D upstream of turbine 1 respectively, as well as the downstream planes 6, 8, and 9, located 1.5D, 3.5D, 4.5D downstream, respectively. At the upstream locations, the wind speed profile follows an approximate power-law behaviour with relatively minor differences between each station. However, at plane 6, located 1.5D downstream of turbine 1, the effects of the turbine rotor are clearly visible and a velocity deficit emerges. This wake deficit diffuses and is less prevalent at planes 8 and 9, but still causes some deviations from the boundary layer profile.

The turbulent intensity levels can also be calculated from the TKE statistics gathered in the SOWFA simulations. Comparisons of the LES TI levels with the levels predicted using equation (2)
are shown in Figure 8. At the upstream plane locations 1, 2, and 3, the agreement between the two is relatively good. Immediately downstream of the turbine at plane 6, the model is invalid and mispredicts the turbulence at the edges of the rotor disc. Further downstream at plane 8, the flow still deviates from a simple boundary layer, and the model does not completely capture the effects of the rotor wake. At plane 9 downstream, the wake depth has decreased, and the model captures some qualitative features of the TI distribution. For this particular case, we see that the top of the rotor disc experiences a TI level of 9%, while at the bottom the TI was about 14.7%. This difference is consistent with the TI variations discussed in section 3.1.

4. Summary
As a part of this work, a simple relation was developed allowing the turbulent intensity to be estimated across the rotor using the mean wind shear profiles and a mixing length model. This analysis was then calibrated and evaluated using SOWFA wind simulations for the neutral atmospheric boundary layer. The model was shown to provide good agreement to the LES computed TI. Further simplification was also shown to be possible if we assume a power law velocity profile, which yielded similar results using only measurements of the shear exponent and hub-height TI. The models were then also applied outside the modelling assumptions to unstable ABL and a wind farm simulation. For the first, larger deviations of the model from LES data were observed. For the latter, favourable results were obtained for the wind profiles upstream of the turbine, and some qualitative similarity was seen in the downstream profiles.

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Figure 7 – The mean horizontal wind speed profile $U$ (in m/s) at sampling planes 1, 2, 3 (located 3D, 2D, and 1D upstream), and planes 6, 8, 9 (1.5D, 3.5D, 4.5D downstream) relative to turbine 1. The turbine hub height and rotor disc limits are shown with the dashed and dashed-dot lines, respectively.

Figure 8 – Comparison of the turbulent intensity $T_I$ profiles between the LES calculation (solid blue lines) and modelled $T_I$ (dashed red lines) at planes 1, 2, 3 upstream and planes 6, 8, 9 downstream. The turbine hub height and rotor disc limits are shown with the dashed and dashed-dot lines, respectively.