Fusion Cycles in Stars and Stellar Neutrinos

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Abstract

Starting from the early works by Weizsäcker and Bethe about fusion cycles and energy conversion in stars, a brief survey of thermonuclear processes in stars leading to contemporary research problems in this field is given. Special emphasis is put on the physics of stellar and, in particular, solar neutrinos which is at the frontline of current investigations.

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1 Introduction

In the 1920s Eddington formulated the hypothesis that fusion reactions between light elements are the energy source of the stars - a proposition that may be considered as the birth of the field of nuclear astrophysics [1]. It was accompanied by his pioneering work on stellar structure and radiative transfer, the relation between stellar mass and luminosity, and many other astrophysical topics. Atkinson and Houtermans [3] showed in more detail in 1929 - after Gamow [2] had proposed the tunnel effect - that thermonuclear reactions can indeed provide the energy source of the stars: they calculated the probability for a nuclear reaction in a gas with a Maxwellian velocity distribution. In particular, they considered the penetration probability of protons through the Coulomb barrier into light nuclei at stellar temperatures of $4 \times 10^7 \, K$. From the high penetration probabilities for the lightest elements they concluded that the build-up of alpha-particles by sequential fusion of protons could provide the energy source of stars. An improved formula was provided by Gamow and Teller [4].

Hence, hydrogen and helium (which were later - in the 1950s - identified as the main remnants of the big bang) form the basis for the synthesis of heavier elements in stars - but details of the delicate chain reactions that mediate these processes remained unknown until 1938. This is in spite of the fact that rather precise models of the late stages of stellar evolution existed or were soon developed. At that time, white dwarfs were generally considered to be the endpoints of stellar evolution, although Zwicky and Baade had speculated in 1934 that a neutron star could be the outcome of a supernova. In 1939 Oppenheimer and Volkoff presented the first models of neutron stars as final stages of stellar evolution. Together with the so-called "frozen" or "collapsed" stars - which were re-named by Wheeler in 1967 as "black holes" - Chandrasekhar included these results for sufficiently massive progenitors in his book about stellar structure [5]. Eddington strongly rejected the proposal, but it proved to be true when the first rotating neutron star (pulsar) was detected in 1967 by Bell and Hewish.

Probably the most important breakthrough regarding the recognition of fusion cycles occurred in 1937/8 when Weizsäcker [6,7] and Bethe [9] found the CNO-cycle - which was later named after their discoverers Bethe-Weizsäcker-cycle (figure 3) - in completely independent works, and Bethe and Critchfield [8] first outlined the proton-proton chain
(figure 2). After a brief review of thermonuclear reactions in section 2, these nucleosynthesis mechanisms are reconsidered in section 3.

After World War II, stellar nucleosynthesis was studied further by Fermi, Teller, Gamow, Peierls and others, but it turned out to be difficult to understand the formation of elements heavier than lithium-7 because there are no stable nuclei with mass numbers 5 or 8. In 1946 Hoyle interpreted the iron-56 peak in the relative abundances of heavier elements vs. mass (figure 1) as being due to an equilibrium process inside stars at a temperature of $3 \cdot 10^9 K$. Later Salpeter showed that three helium nuclei could form carbon-12 in stars, but the process appeared to be extremely unlikely. To produce the observed abundances, Hoyle predicted an energy level at about 7 MeV excitation energy in carbon-12, which was indeed discovered experimentally, generating considerable excitement and progress in the world of astrophysics. Burbidge, Burbidge, Fowler and Hoyle then systematically worked out the nuclear reactions inside stars that are the basis of the observed abundances and summarized the field in 1957 [10].

The role of stellar neutrinos was considered by Bethe in [9]. Neutrinos had already been postulated by Pauli in 1930 to interpret the continuous beta-decay spectra, but could not be confirmed experimentally until 1952 by Cowan and Reines. Bethe argued in 1938 that fast neutrinos emitted from beta-decay of lithium-4 (which would result from proton capture by helium-3) above an energy threshold of 1.9 MeV might produce neutrons in the outer layers of a star. However, this required the assumption of long-lived lithium-4, which turned out to be wrong. Bethe did not pursue stellar neutrinos further in his early works, and he or Weizsäcker also did not explicitly consider the role of neutrinos in the initial p-p reaction, or in the CNO-cycle at that time.

In a normal star, electron neutrinos that are generated in the central region usually leave the star without interactions that modify their energy. Hence, the neutrino energy is treated separately from the thermonuclear energy released by reactions, which undergoes a diffusive transport through the stellar material that is governed by the temperature gradient in the star. Stellar neutrinos are generated not only in nuclear burnings and electron capture, but also by purely leptonic processes such as pair annihilation or Bremsstrahlung.

Neutrinos from nuclear processes in the interior of the sun should produce a flux of $10^{11}$ neutrinos per $cm^2 second$ on the earth. In 1967 Davis et al. - following suggestions by Pontecorvo, and by Bahcall and Davis to use neutrinos ”.. to see into the interior
of a star and thus verify directly the hypothesis of nuclear-energy generation in stars” - indeed succeeded to measure solar neutrinos with a detector based on 390000 liters of tetrachloroethylene. When electron-neutrinos travelling from the sun hit the chlorine-37 nuclei, they occasionally produced argon-37 nuclei, which were extracted and counted by their radioactive decay. First results were published in 1968 [11]. However, these were neutrinos with higher energies produced in a side branch of the proton-proton chain. More than 90 per cent of the neutrinos are generated in the initial p-p reaction, and these were observed first by the Gallex collaboration in 1992 [12] using gallium-71 nuclei as target in a radiochemical detector. Together with the corresponding results of the Sage collaboration, this confirmed experimentally the early suggestions by Weizsäcker and Bethe that p-p fusion is the source of solar energy.

The measurements [11,12] showed that less than 50 per cent of the solar neutrinos that are expected to arrive on earth are actually detected. The subsequent controversy whether this is due to deficiencies in the solar models, or caused by flavor oscillations was resolved at the beginning of the 21st century by combined efforts of the SuperKamiokande [14] and SNO [15]-collaborations in favor of the particle-physics explanation: Neutrinos have a small, but finite mass, and hence, they can oscillate and therefore escape detection, causing the ”solar neutrino deficit” - with the size of the discrepancy depending on energy. The identification of oscillating solar neutrinos [15] was actually preceded by evidence for oscillations of atmospheric muon-neutrinos - most likely to tau-neutrinos [13]. Origin of atmospheric neutrinos are interactions of cosmic rays with particles in the earth’s upper atmosphere that produce pions and muons, which subsequently decay and emit electron- and muon-neutrinos (or antineutrinos).

Oscillation experiments are, however, only sensitive to differences of squared masses. Hence, the actual value of the neutrino mass is still an open issue, and presently only the upper limit of the mass of the antielectron-neutrino can be deduced from tritium beta decay to be $2.2eV/c^2$ [16]. From neutrinoless double beta decay [17], a lower limit of $0.05 - 0.2eV/c^2$ has been deduced in 2002, but this result is only valid if the neutrino is its own antiparticle, which is not certain.

Solar neutrinos are considered in section 4, and a brief outline of some of the perspectives of the field is given in section 5.
2 Energy Evolution in Stars

Stellar and in particular, solar energy is due to fusion of lighter nuclei to heavier ones, which is induced by thermal motion in the star. According to the mass formula that was derived by Weizsäcker in 1935 [18], and by Bethe and Bacher independently in 1936 [19], the difference in binding energies before and after the reaction - the mass defect - is converted to energy via Einstein’s \( E = mc^2 \) [20], and is then added to the star’s energy balance. The binding energy per nucleon rises with mass number starting steeply from hydrogen because the fraction of the surface nucleons decreases, then it flattens and reaches a maximum at iron-56, the most tightly bound nucleus; afterwards it drops slowly towards large masses. Although this smooth behavior of the fractional binding energy per nucleon is modified by pairing and shell effects, the overall shape of the curve ensures that energy can be released either by fission of heavy nuclei or by fusion of light nuclei, as it occurs in stars, thus providing our solar energy.

In case of main-sequence stars such as the sun there are no rapid changes in the star that could compete with the time-scale of the nuclear reactions and hence, the energy evolution occurs through equilibrium nuclear burning. Most important in the solar case is hydrogen burning, where the transformation of four hydrogen nuclei into one helium-4 nucleus is accompanied by a mass loss of 0.71 per cent of the initial masses, or 0.029\( u \). It is converted into an energy of about 26.2 MeV, including the annihilation energy of the two positrons that are produced, and the energy that is carried away by two electron neutrinos. From the known luminosity of the sun, one can calculate a total mass loss rate of \( 4.25 \cdot 10^9 \text{kg/s} \). At this rate, the hydrogen equivalent of one solar mass could sustain radiation for almost \( 10^{11} \) years.

The reactions between nuclei inside stars are due to the thermal motion, and are therefore called thermonuclear. Before stars reach an explosive final (supernova) stage, the energy release due to these reactions is rather slow. From the hydrostatic equilibrium condition in the sun one derives the central temperature as

\[
T_\odot \leq \frac{8 G \mu M_\odot}{3 R \, R_\odot}.
\]

With the gas constant \( R \), the average number of atomic mass units per molecule \( \mu \) (=0.5 for ionized hydrogen), the gravitational constant \( G \), the solar mass \( M_\odot = 1.99 \cdot 10^{30} \text{kg} \)}
and the solar radius $R_\odot = 6.96 \cdot 10^8 \text{m}$ one finds the central solar temperature

$$T_c \leq 3 \cdot 10^7 \text{K}. \quad (2)$$

Numerical solutions by Bahcall et al. [22] yield a central temperature $T_c = 1.57 \cdot 10^7 \text{K}$ and a central pressure $P_c = 2.34 \cdot 10^{16} \text{Pa}$, with a central solar density of $\rho_c = 1.53 \cdot 10^5 \text{kg/m}^3$. For these large values of temperature, the assumption of an ideal gas is indeed justified. The reaction rates are strongly dependent on temperature (typically $\sim T^{22}$ for the CNO-cycle and $\sim T^4$ for the pp-chain at $T_c$) and therefore, massive stars have much greater luminosities with only slightly higher central temperatures. As was noted by Bethe already in 1938, Y Cygni has $T = 3.2 \cdot 10^7 \text{K}$ and a luminosity per mass unit of $0.12 \text{W/kg}$, whereas the sun’s luminosity per mass unit is only about $2 \cdot 10^{-4} \text{W/kg}$ (the most recent best-estimate value [21] of the total solar luminosity being $3.842 \cdot 10^{26} \text{W}$).

Expressed in units of energy, however, the central solar temperature is only about $1.35 \cdot 10^3 \text{keV}$. This has to be compared with the height of the Coulomb barrier

$$E_{\text{coul}} = \frac{Z_1 Z_2 e^2}{R} \quad (3)$$

with the interaction radius $R$ and the proton numbers $Z_1, Z_2$ of the nuclei that tend to fuse in order to release energy. Since $E_{\text{coul}}(R) \sim Z_1 Z_2 \text{MeV}$, more than a factor of $10^3$ in thermal energy is missing in order to overcome the Coulomb barrier.

Thermonuclear reactions in stars can therefore only occur due to the quantum-mechanical tunneling that was established by Gamow [2]. The tunneling probability is

$$P = p_0 E^{-1/2} \exp(-2G) \quad (4)$$

with the Gamow-factor

$$G = \sqrt{\frac{m}{2} \frac{2\pi Z_1 Z_2 e^2}{\hbar E^{1/2}}}. \quad (5)$$

Here $m$ is the reduced mass and $Z_1, Z_2$ are the respective charges of the fusing nuclei, and $E$ is the energy. The factor $p_0$ depends only on properties of the colliding system. For the pp-reaction at an average energy and at solar temperature, $P$ is of the order of $10^{-20}$. It steeply increases with energy and decreases with the product of the charges. Hence, at solar temperatures only systems with small product of the charges may fuse, and for systems with larger $Z_1 Z_2$ the temperature has to be larger to provide a sizeable
penetration probability. As a consequence, clearly separated stages of different nuclear burnings occur during the evolution of a star in time.

Once the Coulomb barrier has been penetrated, an excited compound nucleus is formed, which can afterwards decay with different probabilities into the channels that are allowed from the conservation laws. The energy of outgoing particles and gamma-rays is shared with the surroundings except for neutrinos, which leave the star without interactions.

Energy levels of the decaying compound nucleus above or below the nucleon removal energy can be of different types, stationary levels of small width which decay via gamma-emission, and short-lived quasi-stationary levels above the removal energy which can also (and more rapidly) decay via particle emission. Their width becomes larger with increasing energy and eventually also larger than the distance between neighbouring levels.

Due to the existence of quasi-stationary levels above the nucleon removal energy, a compound nucleus may also be formed in a resonance when the initial energy matches the one of an energy level in the compound nucleus. At a resonance, the cross-section can become very large, sometimes close to the geometrical value. Astrophysical resonant or non-resonant cross-sections are usually written as

$$\sigma(E) = S \cdot E^{-1} \exp(-2G)$$

with the astrophysical cross-section factor $S$ that contains the properties of the corresponding reaction. Although it can be computed in principle, laboratory measurements are a better option. However, because of the small cross-sections, these measurements are difficult at low energies. Extrapolations to these energies are fairly reliable for non-resonant reactions where $S(E)$ is a slowly varying function of $E$, but this is not true in the case of resonances, which may (or may not) be hidden in the region of extrapolation. The present state of the art for measurements of $S(E)$ in an underground laboratory to shield cosmic rays is shown in figure 4 for the reaction

$$^3\text{He}({^3\text{He},2p})^4\text{He}$$

that is very important in the stellar pp-chain, cf. next section. The solid line is a fit with a screening potential that accounts for a partial shielding of the Coulomb potential of the nuclei due to neighbouring electrons. Data from the LUNA collaboration [23] extend
down to 21 keV, where the Gamow peak at the solar central temperature is shown in
arbitrary units. The peak arises from the product of the Maxwell distribution at a given
temperature $T$ and the penetration probability. Its maximum is at an energy

$$E_G = \left[ \sqrt{\frac{m}{2\pi}} \frac{2\pi Z_1 Z_2 e^2 kT}{\hbar} \right]^{2/3}.$$  \hfill (8)

At $E_G$, the S-factor for the He-3 + He-3 reaction becomes $5.3 MeVb$. The average
reaction probability per pair and second is given by

$$< \sigma v > = \int_0^\infty \sigma(E) v f(E) dE$$ \hfill (9)

where $f(E)$ can be expressed in a series expansion near the maximum. Keeping only the
quadratic terms, the reaction probability becomes [24]

$$< \sigma v > = \frac{4}{3} \left( \frac{2}{m} \right)^{1/2} \frac{1}{(kT)^{1/2}} S_G \cdot \tau^{1/2} \exp(-\tau)$$ \hfill (10)

with the S-factor $S_G$ at the Gamow peak and

$$\tau = 3E_G / (kT).$$ \hfill (11)

The temperature dependence of $< \sigma v >$ may be expressed as

$$\frac{\partial \ln < \sigma v >}{\partial \ln T} = \tau / 3 - \frac{2}{3},$$ \hfill (12)

which can attain values near or above 20. As a consequence of such large values for the
exponent of $T$, the thermonuclear reaction rates become extremely strongly dependent on
temperature, and small fluctuations in $T$ may cause dramatic changes in the energy (and
neutrino) production of a star. The corresponding uncertainty in stellar models created
the long-standing controversy about the origin of the solar neutrino deficit, which has
only recently been decided in favor of the particle-physics explanation, cf. section 4.
3 Hydrogen burning

Due to the properties of the thermonuclear reaction rates, different fusion reactions in a star are separated by sizeable temperature differences and during a certain phase of stellar evolution, only few reactions occur with appreciable rates. Stellar models account in network-calculations for all simultaneously occurring reactions. Often the rate of the fusion process is determined by the slowest in a chain of subsequent reactions, such as in case of the nitrogen-14 reaction of the CNO-cycle.

In hydrogen burning, four hydrogen nuclei are fused into one helium-4 nucleus, and the mass defect of 0.71 per cent is converted into energy (including the annihilation energy of the two positrons, and the energy carried away by the neutrinos):

\[4 \cdot ^1 H \rightarrow ^4 He + 2e^+ + 2\nu_e + 26.2 MeV.\]  

As net result, two protons are converted into neutrons through positron emission (beta\(^+\)-decay), and because of lepton number conservation, two electron neutrinos are emitted. Depending on the reaction which produces the neutrinos, they can carry between 2 and 30 per cent of the energy. Helium synthesis in stars proceeds through different reaction chains which occur simultaneously. The main series of reactions are the proton-proton chain, figure 2, and the CNO-cycle, figure 3.

In the present epoch, the pp-chain turns out to be most important for the sun - the CNO-cycle produces only 1.5 per cent of the luminosity [22]. The pp-chain starts with two protons that form a deuterium nucleus, releasing a positron and an electron neutrino. (With much smaller probability it may also start with the p-e-p process, figure 2). This reaction has a very small cross section, because the beta-decay is governed by the weak interaction. At central solar temperature and density, the mean reaction time is \(10^{10}\) years, and to a certain extent it is due to this huge time constant that the sun is still shining. With another proton, deuterium then reacts to form helium-3. This process is comparably fast and hence, the abundance of deuterons in stars is low.

To complete the chain to helium-4 three branches are possible. The first - in the sun with 85 per cent most frequent - chain (ppI) requires two helium-3 nuclei and hence, the first reaction has to occur twice, with two positrons and two electron neutrinos being emitted. The other two branches (ppII, ppIII) need helium-4 to be produced already.
in previous burnings, or primordially). In the subsequent reactions between helium-3 and helium-4, the additional branching occurs because the product beryllium-7 can react either with an electron to form lithium-7 plus neutrino (ppII), or with hydrogen to form boron-8 (ppIII). The energy released by the three chains differs because the neutrinos carry different amounts of energy with them, and the relative frequency of the different branches depend on temperature, density, and chemical composition. The percentages in figure 2 refer to the standard solar model at the present epoch [22]. Details of the various parts of the chain including the corresponding energy release, the energies carried away by the neutrinos and the reaction rate constants have been discussed by Parker et al. [26] and Fowler et al. [27].

The other main reaction chain in hydrogen burning is the CNO-cycle, figure 3. Here, the carbon, nitrogen and oxygen isotopes serve as catalysts, their presence is required for the cycle to proceed. The main cycle is completed once the initial carbon-12 is reproduced by nitrogen-15 + hydrogen. There is also a secondary cycle (not shown in figure 3 since it is $10^4$ times less probable). It causes oxygen-16 nuclei which are present in the stellar matter to take part in the CNO-cycle through a transformation into nitrogen-14. The CNO-cycle produces probably most of the nitrogen-14 found in nature. For sufficiently high temperatures, the nuclei attain their equilibrium abundances and hence, the slowest reaction - which is nitrogen-14 + hydrogen - determines the time to complete the whole circle (bottom of figure 3).

The CNO-cycle contributes only a few per cent to the luminosity of a star with one solar mass, but it dominates in stars with masses above 1.5 times the solar value because its reaction rates rise much faster with temperature as compared to pp. Details of the Bethe-Weizsäcker cycle have been discussed by Caughlan and Fowler [28]. The cycle had first been proposed by Weizsäcker in [7]. In this work, he abandoned the main reaction path that he had considered in [6], namely, from hydrogen via deuterium and lithium to helium, because the intermediate nuclei of mass number 5 that were supposed to be part of the scheme had turned out to be unstable.

In the first paper of the series [6], he had considered various reaction chains that allow for a continuous generation of energy from the mass defect, and also of neutrons for the buildup of heavy elements. He had confirmed that the temperatures in the interior of stars are sufficient to induce nuclear reactions starting from hydrogen. In the second paper he
modified the results; in particular, he discussed the possibility that some of the elements might have been produced before star formation by another process.

The link between energy evolution in stars and the formation of heavy elements as considered in [6] turned out to end up in difficulties when calculated quantitatively. Hence, he modified his version of the so-called "Aufbauhypothese", according to which the neutrons necessary for the production of heavy elements should be generated together with the energy, and decoupled the generation of energy from the production of heavy elements. He then concluded that stellar energy production should essentially be due to reactions between light nuclei, with the corresponding abundances being in agreement with observations. The CNO-cycle was considered to be the most probable path.

In his independent and parallel development of the CNO-cycle that was published somewhat later [9] and contained detailed calculations, Bethe showed that "... there will be no appreciable change in the abundance of elements heavier than helium during the evolution of the star but only a transmutation of hydrogen into helium. This result...is in contrast to the commonly accepted 'Aufbauhypothese'". Here, he referred to Weizsäcker’s first hypothesis [6] which had, however, already been modified [7].

Together with Critchfield [8], Bethe also investigated essential parts of the pp-chain (which Weizsäcker also mentioned) - in particular, deuteron formation by proton combination as the first step - and came to the conclusion that it "...gives an energy evolution of the right order of magnitude for the sun". Details of the pp-chain were developed much later in the 1950s by Salpeter [29] and others. In 1938/9, however, Bethe was convinced that "... the reaction between two protons, while possible, is rather slow and will therefore be much less important in ordinary stars than the cycle (1)" namely, the CNO-cycle.

In a calculation of the energy production by pp-chain versus CNO-cycle (figure 5), Bethe obtained qualitatively the preponderance of H+H at low and N+H at high temperatures. However, the result had to be modified in the course of time as it became evident that the pp-chain is more important than the CNO-cycle at solar conditions, although the Bethe-Weizsäcker-fraction will increase considerably in the coming 4 billion years, and eventually supersede the contribution from the ppII-chain (figure 6).

Today, detailed solar models allow to calculate the fractions of the solar luminosity that are produced by different nuclear fusion reactions very precisely [22]. The model results not only agree with one another - in the neutrino flux predictions to within about
1 per cent - they are also consistent with precise p-mode helioseismological observations of the sun’s outer radiative zone and convective zone [30]. Moreover, the production of heavier elements up to iron in subsequent burnings at higher temperatures [27], as well as beyond iron in the r- and s-process is rather well-understood [31].

4 Stellar Neutrinos

In stellar interiors, only electron neutrinos play a role. The interaction of neutrinos with matter is extremely small, with a cross-section of

$$\sigma_\nu \simeq (E_\nu / m_e c^2)^2 \cdot 10^{-17} mb.$$  \hspace{1cm} (14)

Hence, the cross-section for neutrinos with $E_\nu \simeq 1MeV$ is $\sigma_\nu \simeq 3.8 \cdot 10^{-17}mb$, which is smaller than the cross-section for the electromagnetic interaction between photons and matter by a factor of about $10^{-18}$. Associated with the cross-section is a mean free path

$$\lambda_\nu = \frac{u}{\rho \cdot \sigma_\nu} \simeq \frac{4 \cdot 10^{20}}{\rho} m$$  \hspace{1cm} (15)

with the atomic mass unit $u = 1.66 \cdot 10^{-27}kg$ and $\rho$ in $kg/m^3$. In stellar matter with $\rho \simeq 1.5 \cdot 10^3kg/m^3$, the mean free path of neutrinos is therefore approximately

$$\lambda_\nu \simeq 3 \cdot 10^{17} m \simeq 10pc \simeq 4 \cdot 10^9 R_\odot$$  \hspace{1cm} (16)

and hence, neutrinos leave normal stars without interactions that modify their energy. This is different during the collapse and supernova explosion in the final stages of the evolution of a star where nuclear density can be reached, $\rho \simeq 2.7 \cdot 10^{17}kg/m^3$ such that the mean free path for neutrinos is only several kilometers, and a transport equation for neutrino energy has to be applied.

Here only the neutrinos from nuclear reactions in a normal main-sequence star like the sun are considered; their energies are (to some extent, since the continuous distributions overlap) characteristic for specific nuclear burnings. The pp-chain which provides most of the sun’s thermonuclear energy produces continuum neutrinos in the reactions ([32]; cf. figure 2)

$$^1H + ^1H \rightarrow ^2H + e^+ + \nu_e \quad (0.420MeV)$$
\[ 8B \rightarrow ^8Be^* + e^+ + \nu_e \quad (14.06\text{MeV}) \]
\[ ^{13}N \rightarrow ^{13}C + e^+ + \nu_e \quad (1.20\text{MeV}) \]
\[ ^{15}O \rightarrow ^{15}N + e^+ + \nu_e \quad (1.74\text{MeV}) \]

where the numbers are the maximum neutrino energies for the corresponding reaction.

In addition to these continuum neutrinos, there are neutrinos at discrete energies from the pp-chain
\[ ^1H + ^1H + e^- \rightarrow ^2H + \nu_e \quad (1.44\text{MeV}) \]
\[ ^7Be + e^- \rightarrow ^7Li^* + \nu_e \quad (0.861\text{MeV} - 90\% \text{percent}) \]
\[ \quad (0.383\text{MeV} - 10\% \text{percent}) \]

(depending on whether lithium-7 is in the ground state, or in an exited state)
\[ ^8B + e^- \rightarrow ^8Be + \nu_e \quad (15.08\text{MeV}) \]

The CNO-cycle (figure 3) which becomes important in stars with masses above 1.5 solar masses, or in later stages of the stellar evolution (figure 8) also produces neutrinos at discrete energies
\[ ^{13}N + e^- \rightarrow ^{13}C + \nu_e \quad (2.22\text{MeV}) \]
\[ ^{15}O + e^- \rightarrow ^{15}N + \nu_e \quad (2.76\text{MeV}) \]

For experiments to detect these neutrinos when they arrive on earth 8.3 minutes after their creation the flux at the earth’s surface is of interest. Neutrinos from the central region of the sun yield a flux of about \(10^7/(m^2\cdot s)\). The precise value as function of the neutrino energy can be calculated from solar models ([25], figure 7). Here, solid lines denote the pp-chain and broken lines the CNO-cycle. The low-energy neutrinos from the initial pp-reaction yield the largest flux. However, the first experiment by Davis et al. [11] that detected solar neutrinos on earth with a large-scale underground tetrachloroethylene tank in 1967/8 - and thus confirmed the theory how the sun shines and stars evolve - made use of the reaction
\[ \nu_e + ^{37}_{17}\text{Cl} \rightarrow e^- + ^{37}_{18}\text{Ar} - 0.814\text{MeV} \]

and hence, only neutrinos with energies above 0.814 MeV could be observed through the decay of radioactive argon nuclei - which are mostly the solar boron-8 neutrinos, cf. figure 7. The rate of neutrino captures is measured in solar neutrino units; 1 SNU corresponds to \(10^{-36}\) captures per second and target nucleus. Experimental runs by Davis et al. during the 1970s, 80s and 90s yielded a signal (after subtraction of the cosmic-ray background 1.6 kilometers underground) of \((2.3 \pm 0.3)\text{SNU}\), whereas the predicted capture
rates from a solar model were 0 SNU for pp (because it is below threshold), 5.9 SNU for boron-8 beta decay, 0.2 SNU for the pep-reaction, 1.2 SNU for beryllium-7 electron capture, 0.1 SNU for nitrogen-13 decay and 0.4 SNU for oxygen-15 decay, totally about 8 SNU.

The observation of less than 50 per cent of the expected neutrino flux created a controversy about the origin of the deficit, which was finally - in 2001 - resolved [15] in favor of the particle-physics explanation that had originally been proposed by Pontecorvo in 1968 [33]: on their way from the solar interior to the earth, electron-neutrinos oscillate to different flavors which escape detection, thus creating the deficit. Although deficiencies in the solar models could have been responsible for the discrepancy (in view of the sensitive dependence of the neutrino flux on the central temperature), it could be confirmed [15] that the models are essentially correct, giving the right value of $T_c$ within 1 per cent.

Before this big step in the understanding of stellar evolution and neutrino properties could be taken, there was substantial progress both in experimental and theoretical neutrino physics. In 1992 the Gallex-collaboration succeeded to measure the pp-neutrinos from the initial fusion reaction, which contributes more than 90 per cent of the integral solar neutrino flux [12]. They used a radiochemical detector with gallium as target, exploiting the reaction

$$\nu_e + ^{71}_{31} Ga \rightarrow e^- + ^{71}_{32} Ge - 0.23 MeV.$$  

The threshold is below the maximum neutrino energy for pp-neutrinos of 0.42 MeV and a large fraction of the pp-neutrinos can therefore be detected in addition to the pep-, beryllium-7 and boron-8 neutrinos. Gallex - which is sensitive to electron neutrinos only - thus provided the proof that pp-fusion is indeed the main source of solar energy. The result $[(69.7 + 7.8/ - 8.1)SNU]$ was confirmed by the Sage experiment $[(69 \pm 12)SNU]$ in the Caucasus [35]. Again this was substantially below the range that various standard solar models predicted (120-140 SNU), and the solar neutrino deficit persisted. At that time, there were clear indications - but no definite evidence yet - that the flux decreases between sun and earth due to neutrino flavor oscillations - most probably enhanced through the MSW-effect [36] in the sun -, "...pointing towards a muon-neutrino mass of about 0.003 eV" [34]. The result was later updated to $(73.9 \pm 6.2)SNU$ and could be assigned to the fundamental low-energy neutrinos from the pp and pep reactions - but then there remained no room to accommodate the beryllium-7 and boron-8 neutrinos.
Solar neutrinos were also detected in real time with the Kamiokande detector [37] in Japan, a water-Cerenkov detector, and precursor to the famous SuperKamiokande detector. Due to the high threshold of about 7.5 MeV, it could see only the most energetic neutrinos from the decay of boron-8 in the solar center. With the Cerenkov light pattern one could measure for the first time the incident direction of the scattering neutrinos, and prove that they do indeed come from the sun. The result of the boron-8 neutrino flux was

\[
\text{flux}^{\text{Observed}}_{\nu_e} / \text{flux}^{\text{Predicted}}_{\nu_e} = 0.54 \pm 0.07,
\]

again confirming the deficit. To solve the solar neutrino problems, a larger target volume and a lower energy threshold was needed: the SuperKamiokande detector in the same zinc mine with a threshold of 5 MeV. Here, 32000 tons of pure water are surrounded by 11200 photomultiplier tubes for observing electrons scattered by neutrinos (many of the tubes were destroyed end of 2001 when one collapsed, emitting an underwater shock wave). The detector was designed to record about 10000 solar neutrino collisions per year - 80 times the rate of its predecessors -, but also atmospheric neutrinos, and possible signs for proton decay. The result [14] for the boron-8 flux can be expressed as

\[
\text{flux}^{\text{Observed}}_{\nu_e} / \text{flux}^{\text{Predicted}}_{\nu_e} = 0.47 \pm 0.02,
\]

which was in agreement with the previous findings, but more precise. There was a massive hint that the deficit could be due to neutrino oscillations, since the SuperKamiokande collaboration found evidence in 1998 [13] that muon neutrinos which are produced in the upper atmosphere by pion and muon decays change their type when they travel distances of the order of the earth's radius due to oscillations into another species, most likely into tau neutrinos. The appearance of the tau neutrinos could not yet be detected directly, but oscillations to electron neutrinos in the given parameter range were excluded since the $\nu_e$-flux was unchanged, and also by reactor data. Accelerator experiments with a long baseline of 700 km between neutrino source and detector are being planned to verify this interpretation [40].

The atmospheric data showed a significant suppression of the observed number of muon neutrinos as compared to the theoretical expectation at large values of $x/E_\nu$, with the travel distance $x$ (large when the neutrinos travel through the earth) and the neutrino energy $E_\nu$ which is in the GeV-range for atmospheric neutrinos and hence, much higher than in the solar case. The observed dependence on distance is expected from the theoretical expression for oscillations into another flavor, which yields in the model case of
two flavors

\[ P = \frac{1}{2} \sin^2(2\theta) \cdot (1 - \cos(2\pi x/L)). \]  

(17)

Here, \( \theta \) is the mixing angle between the two flavors considered, and the characteristic oscillation length (the distance at which the initial flavor content appears again) is

\[ L = 4\pi E_{\nu}/\Delta m^2 \simeq 2.48(E_{\nu}/MeV)/(c^4 \Delta m^2/eV^2)[m] \]  

(18)

with the difference \( \Delta m^2 = |m_2^2 - m_1^2| \) of the neutrino mass eigenstates. Atmospheric neutrino experiments are thus sensitive to differences in the squared masses of \( 10^{-4} \) to \( 10^{-2}eV^2/c^4 \) whereas solar neutrinos are sensitive to differences below \( 2 \cdot 10^{-4}eV^2/c^4 \) due to the lower neutrino energy and the larger distance between source and detector. Whereas such mixings between neutral particles that carry mass had been firmly established many years ago in the case of quarks that build up the \( K^0 \) and \( B^0 \)-mesons and their antiparticles - including the proof that CP is violated [38] for three quark families -, it remained an open question until 1998 whether the corresponding phenomenon [39] exists for leptons.

The atmospheric SuperKamiokande data proved beyond reasonable doubt the existence of oscillations and finite neutrino masses [13, 40], with \( \Delta m^2_{\text{atm}} \simeq 2.5 \cdot 10^{-3}eV^2 \) and maximal mixing \( \sin^2(2\theta_{\text{atm}}) \simeq 1 \). The corresponding step for solar neutrinos followed in 2001: charged-current results \( (\nu_e + d \rightarrow e + p + p; \text{sensitive to electron neutrinos only}) \) from the Sudbury Neutrino Observatory (SNO) in Canada with a \( D_2O \)-Cerenkov detector [15], combined with elastic scattering data from SuperK \( (\nu_\odot + e \rightarrow \nu + e; \text{sensitive to all flavors}) \), established oscillations of solar boron-8 neutrinos.

These results were confirmed and improved \((5.3\sigma)\) in 2002 by neutral current results from SNO \( (\nu_\odot + d \rightarrow \nu + n + p) \); a further improved measurement with salt (NaCl; chlorine-35 has a high n-capture efficiency) is currently underway. The total flux measured with the NC reaction is \((5.09 + 0.64/−0.61) \cdot 10^6 \text{ neutrinos per} \ (cm^2 \cdot s) \). This is in excellent agreement with the value from solar models \((5.05 + 1.01/−0.81) \), proving that stellar structure and evolution is now well-understood. The currently most-favored mechanism for solar neutrino conversion to myon- and tauon-flavors is the ”Large mixing angle” solution, which also implies matter-enhanced (resonant) mixing in the interior of the sun through the MSW-effect [36].
5 Perspectives

Since the early works by Weizsäcker and Bethe, the investigation of thermonuclear processes in stars has developed into considerable detail, and with the advent of stellar neutrino physics an independent confirmation of the origins of solar energy has emerged. Improvements in the precise measurements of all the reaction rates at low energies that are involved in the fusion chains may still be expected, as has been outlined in the model case of the $^3\text{He} + ^3\text{He}$ system within the energy region of the solar Gamow peak [23].

However, not only a good knowledge of the processes involved in equilibrium burnings at energies far below the Coulomb barrier, but also of explosive burning (with short-lived nuclides at energies near the Coulomb barrier) is of interest, because both contribute to the observed abundances of the elements. This requires new experimental facilities.

An improved understanding of the cross-sections will then put the predictions of the solar neutrino flux and spectrum on a better basis. This entire spectrum will be investigated with high precision in the coming decade. In particular, the new detector Borexino will measure the monoenergetic beryllium-7 neutrinos at 862 keV, which depend very sensitively on the oscillation parameters. The LENS experiment will utilize inverse beta-decay to an isomeric state of the daughter nuclide in order to investigate the low-energy solar neutrinos in real time with considerably reduced background. Together with forthcoming SNO and KamLAND results it will then be possible to definitely determine all the mixing parameters in a three-family scheme - and verify, or falsify, the LMA solution.

The more detailed knowledge about the physics of stars will thus be supplemented by considerable progress regarding neutrino properties [40]. Questions to be settled are the individual neutrino masses (rather than the difference of their squares); whether neutrinos are their own antiparticles (to be decided from the existence or non-existence of neutrinoless double beta-decay); whether neutrinos violate CP just as quarks do, or maybe in a different manner that opens up a better understanding of the matter-antimatter asymmetry of the universe than has been possible from the investigation of quark systems (to be decided in experiments with strong neutrino beams).

In any case, the Standard model of particle physics has to accommodate finite neutrino masses, and in future theoretical formulations the relation between quark mixing and neutrino mixing will probably become more transparent.
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References

[1] A.S. Eddington, The Internal Constitution of the Stars
   (Cambridge University Press, 1926).
[2] G. Gamow, Z. Physik 52, 510 (1928).
[3] R. d’E. Atkinson and F.G. Houtermans, Z. Physik 54, 656 (1929).
[4] G. Gamow and E. Teller, Phys. Rev. 53, 608 (1938).
[5] S. Chandrasekhar, An Introduction to the Study of Stellar Structure
   (University of Chicago Press, 1939).
[6] C.F. v. Weizsäcker, Physik. Zeitschr. 38, 176 (1937).
[7] C.F. v. Weizsäcker, Physik. Zeitschr. 39, 633 (1938).
[8] C.L. Critchfield and H.A. Bethe, Phys. Rev. 54, 248, 862 (L) (1938).
[9] H.A. Bethe, Phys. Rev. 55, 434 (1938).
[10] E.M. Burbidge, G.R. Burbidge, W.A. Fowler, F. Hoyle,
    Rev. Mod. Phys. 29, 547 (1957).
[11] R. Davis, D.S. Harmer, K.C. Hoffman, Phys. Rev. Lett. 20, 1205 (1968).
[12] P. Anselmann et al., Phys. Lett. B 285, 376 (1992);
    327, 377 (1994); 342, 440 (1995).
[13] S. Fukuda et al., Phys. Rev. Lett. 81, 1562 (1998); 85, 3999 (2000).
[14] S. Fukuda et al., Phys. Rev. Lett. 86, 5651 (2001).
[15] Q.R. Ahmad et al., Phys. Rev. Lett. 87, 071301 (2001)
    and 89, 011301 (2002).
[16] C. Weinheimer et al., Phys. Lett. B 460, 219 (1999).
[17] L. Baudis al., Phys. Rev. Lett. 83, 41 (1999) and
    Eur. Phys. J. A12, 147 (2001).
[18] C.F. v. Weizsäcker, Z. Physik 96, 431 (1935).
[19] H.A. Bethe and R.F. Bacher, Rev. Mod. Phys. 8, 82 (1936).
[20] A. Einstein, Ann. Physik 18, 639 (1905).
[21] C. Fröhlich and J. Lean, Geophys. Res. Lett. 25, No. 23, 4377 (1998).
[22] J.N. Bahcall, M.H. Pinsoneault and S. Basu,
Astrophys. J. **555**, 990 (2001).

[23] E.C. Adelberger et al., Rev. Mod. Phys. **70**, 1265 (1998); M. Junker et al, LUNA collab., Nucl. Phys. Proc. Suppl. **70**, 382 (1999).

[24] R. Kippenhahn and A. Weigert, Stellar Structure and Evolution, Springer 1990.

[25] J. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. **67**, 1 (1995)
and astro-ph/0010346.

[26] P.D. Parker, J.N. Bahcall and W.A. Fowler, Ap. J. **139**, 602 (1964).

[27] W.A. Fowler, G.R. Caughlan and B.A. Zimmerman, Ann. Rev. Astron. Astrophys. **5**, 525 (1967).

[28] G.R. Caughlan and W.A. Fowler, Ap. J. **136**, 453 (1962).

[29] E.E. Salpeter, Phys. Rev. **88**, 547 (1952).

[30] S.A. Bludman and D.C. Kennedy, Astrophys. J. **472**, 412 (1996).

[31] K.R. Lang, Astrophysical Formulae, chapter 4 (Springer 1980),
and references therein.

[32] J. N. Bahcall, Phys. Rev. **135**, B 137 (1964).

[33] B. Pontecorvo, Sov. Phys. JETP **26**, 984 (1968);
V. Gribov and B. Pontecorvo, Phys. Lett. **28**, 493 (1969).

[34] T. Kirsten in: Proc. 4th Int. Solar Neutrino Conf., Heidelberg;
Ed. W. Hampel. MPI-HD (1997).

[35] J. Abdurashitov et al., Phys. Lett. B**328**, 234 (1994).

[36] S. Mikheyev, A. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985). L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[37] Y. Suzuki et al., Nucl. Phys. B **38**, 54 (1995).

[38] J.H. Christensen, J.W. Cronin, V.L. Fitch and R. Turlay,
Phys. Rev. Lett. **13**, 138 (1964).
M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
H Burkhardt et al., Phys. Lett. B **206**, 169 (1988).

[39] Z. Maki, N. Nakagawa and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[40] Proc. XXth Int. Conf. on Neutrino Physics and Astrophysics, München,
Ed. F. v. Feilitzsch et al., in press (2002).
Figure captions

Fig. 1. Solar system abundances of the nuclides relative to silicon (= 10⁶) plotted as function of mass number. The stellar nuclear processes which produce the characteristic features are outlined. The p-p-chain and the CNO-cycle of hydrogen burning are discussed in the text. Elements up to \( A \leq 60 \) are produced in subsequent burnings at higher temperatures, beyond \( A = 60 \) in supernovae through the r-, s- and p-processes.

Source of the data:
A.G.W Cameron, Space Sci. Rev. 15, 121 (1973).

Source of the Figure:
K.R. Lang, Astrophysical Formulae, p.419. Springer (1980).

Fig. 2. Proton-proton reactions are the main source of stellar energy in stars with masses close to or below the solar value. They were already briefly considered by Weizsäcker [6] and discussed in more detail by Critchfield and Bethe [8]. Today it is known that the ppI branch is supplemented by the ppII branch, and the small, very temperature-dependent ppIII branch. In the latter two branches, additional electron neutrinos of fairly high energy are produced. The approximate partitions refer to the sun.

Source of the Figure:
H. Karttunen et al., Fundamental Astronomy. Springer (1987).

Fig. 3. At temperatures above 20 Million Kelvin corresponding to stars of more than 1.5 solar masses the Bethe-Weizsäcker-cycle is more important than the proton-proton chain because its reaction rate rises faster with temperature. This CNO-cycle was first proposed by Weizsäcker [7] and Bethe [9]. Here, carbon, oxygen and nitrogen act as catalysts.

Source of the Figure:
H. Karttunen et al., Fundamental Astronomy. Springer (1987).
**Fig. 4.** The astrophysical cross-section factor $S(E)$ for the reaction $^3\text{He}(^3\text{He},2p)^4\text{He}$. The solid line is a fit with a screening potential. Data from the LUNA collaboration [23] extend down to 21 keV, where the Gamow peak at the solar central temperature is shown in arbitrary units.

Source of the Figure:
E.C. Adelberger et al., Rev. Mod. Phys. **70**, 1265 (1998).

**Fig. 5.** Stellar energy production in $10^{-4} J/(kg \cdot s)$ due to the proton-proton chain (curve H+H) and the CNO-cycle (N+H), and total energy production (solid curve) caused by both chains. According to this calculation by Bethe in 1938 [9], the CNO-cycle dominates at higher than solar temperatures. Its role at and below solar temperatures as compared to pp is, however, overestimated, cf. fig. 6.

Source of the Figure:
H.A. Bethe, Phys. Rev. **55**, 434 (1938).

**Fig. 6.** Fractions of the solar luminosity produced by different nuclear fusion reactions versus solar age, with the present age marked by an arrow (Bahcall et al. 2001 [22]). The proton-proton chain is seen to generate the largest luminosity fractions - in particular, through the branch that is terminated by the $^3\text{He} - ^3\text{He}$ reaction. The solid curve shows the luminosity generated by the CNO-cycle, which increases with time, but is only a small contribution today.

Source of the Figure:
J.N. Bahcall, M.H. Pinsonneault and S. Basu, Astrophys. J. **555**, 990 (2001).
**Fig. 7.** Spectrum of solar electron neutrinos according to the "Standard Solar Model". The largest contribution is generated by low-energy neutrinos from the p-p chain that have been detected by the Gallex experiment [12,34]. Solid lines indicate neutrinos from the pp-chain, dashed lines from the CNO-cycle. Neutrinos of higher energies had first been observed by Davis et al. [11]. The calculation is by Bahcall and Pinsonneault [25]. (The hep-neutrinos arise from the ppIV-reaction \( {}^3\text{He} + p \rightarrow {}^4\text{He} + \nu_e + e^+ \) which is not shown in figure 2).

Source of the Figure:
J. Bahcall and M.H. Pinsonneault, Rev. Mod. Phys. 67, 1 (1995) and astro-ph 0010346.

**Fig. 8.** The proton-proton, beryllium-7, boron-8 and nitrogen-13 neutrino fluxes as functions of solar age, with the present age marked by an arrow (Bahcall et al. [22]). The Standard Solar Model ratios of the fluxes are divided by their values at 4.57 \( \cdot 10^9 \) y, the present solar age.

Source of the Figure:
J.N. Bahcall, M.H. Pinsonneault and S. Basu, Astrophys. J. 555, 990 (2001).
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