Role of momentum correlations in the properties of fragments produced in heavy-ion collisions

Sakshi Gautam\textsuperscript{1} and Rajni Kant

Department of Physics, Panjab University, Chandigarh -160 014, India.

The role of momentum correlations is studied in the properties of light and medium mass fragments by imposing momentum cut in clustering the phase space. Our detailed investigation shows that momentum cut has major role to play in the properties of fragments.

\textsuperscript{1}Email: sakshigautm@gmail.com
1 Introduction

Intermediate energy heavy-ion collisions produce rich amount of information on the correlations and fluctuations and eventually on the dynamics and interactions among the nucleons. The breaking of nuclei, i.e., multifragmentation, is one of the rare phenomena that has attracted major attention in recent years [1]. The physics behind multifragmentation is so complicated that many different theoretical approaches have been developed [1–4]. Since no theoretical model simulates fragments, one needs afterburners to identify clusters. Since correlations and fluctuations are the main features of the molecular dynamics model, the quantum molecular dynamics (QMD) model is very successful in explaining the phenomena of multifragmentation. Once the phase space is accessible, one generally clusterize the phase space with simple spatial correlation method where one binds the nucleons in a fragment they lie within a distance of 4 fm. This method is known as minimum spanning tree (MST) method [5]. At the same time fragments formed in MST method will be highly unstable (especially in central collisions) as there the two nucleons may not be well formed and therefore can be unstable that will decay after a while. In order to filter out such unstable fragments, we impose another cut in terms of relative momentum of nucleons. This method, dubbed as minimum spanning tree with momentum cut (MSTP) method was discussed by Puri et al. [6]. In our recent work, we study the role of momentum cut on fragment structure [7]. We also studied the role of colliding geometry on the fragmentation when momentum cut is being imposed. No study exits in literature to see the role of momentum cut on the various fragment properties like the rapidity distribution, $p_t$ spectra and $E_{rat}$. So the the present paper, we plan to see the role of momentum cut on various fragment properties and to investigate how these properties vary with impact parameter. The present study is carried out within the framework of QMD model [2,3] which is described in the following section.
2 The Formalism

2.1 Quantum Molecular dynamics (QMD) model

We describe the time evolution of a heavy-ion reaction within the framework of Quantum Molecular Dynamics (QMD) model [2, 3] which is based on a molecular dynamics picture. This model has been successful in explaining collective flow [8], elliptic flow [9], multifragmentation [10] as well as dense and hot matter [11]. Here each nucleon is represented by a coherent state of the form

\[ \phi_{\alpha}(x_1, t) = \left( \frac{2}{L \pi} \right)^{\frac{3}{2}} e^{-\frac{(x_1-x_{\alpha}(t))^2}{2L^2}} e^{ip_{\alpha}(x_1-x_{\alpha})} e^{-\frac{ip_{\alpha}^2 t}{2m}}. \]  

(1)

Thus, the wave function has two time dependent parameters \( x_\alpha \) and \( p_\alpha \). The total n-body wave function is assumed to be a direct product of coherent states:

\[ \phi = \phi_\alpha(x_1, x_\alpha, p_\alpha, t) \phi_\beta(x_2, x_\beta, p_\beta, t) \ldots, \]

(2)

where antisymmetrization is neglected. One should, however, keep in the mind that the Pauli principle, which is very important at low incident energies, has been taken into account. The initial values of the parameters are chosen in a way that the ensemble \((A_T+A_P)\) nucleons give a proper density distribution as well as a proper momentum distribution of the projectile and target nuclei. The time evolution of the system is calculated using the generalized variational principle. We start out from the action

\[ S = \int_{t_1}^{t_2} L[\phi, \phi^*]d\tau, \]

(3)

with the Lagrange functional

\[ L = \left( \phi \left| i\hbar \frac{d}{dt} - H \right| \phi^* \right), \]

(4)

where the total time derivative includes the derivatives with respect to the parameters. The time evolution is obtained by the requirement that the action is stationary under the allowed variation of the wave function

\[ \delta S = \delta \int_{t_1}^{t_2} L[\phi, \phi^*]dt = 0. \]

(5)

If the true solution of the Schrödinger equation is contained in the restricted set of wave function \( \phi_{\alpha}(x_1, x_\alpha, p_\alpha) \), this variation of the action gives the exact solution of the
Schrödinger equation. If the parameter space is too restricted, we obtain that wave function in the restricted parameter space which comes close to the solution of the Schrödinger equation. Performing the variation with the test wave function \((2)\), we obtain for each parameter \(\lambda\) an Euler-Lagrange equation:

\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{\lambda}} - \frac{\partial L}{\partial \lambda} = 0. 
\]

(6)

For each coherent state and a Hamiltonian of the form,

\[
H = \sum_{\alpha} \left[ T_{\alpha} + \frac{1}{2} \sum_{\alpha\beta} V_{\alpha\beta} \right],
\]

the Lagrangian and the Euler-Lagrange function can be easily calculated

\[
L = \sum_{\alpha} \dot{x}_\alpha p_\alpha - \sum_{\beta} \langle V_{\alpha\beta} \rangle - \frac{3}{2Lm},
\]

(7)

\[
\dot{x}_\alpha = \frac{p_\alpha}{m} + \nabla p_\alpha \sum_{\beta} \langle V_{\alpha\beta} \rangle,
\]

(8)

\[
\dot{p}_\alpha = -\nabla x_\alpha \sum_{\beta} \langle V_{\alpha\beta} \rangle.
\]

(9)

Thus, the variational approach has reduced the n-body Schrödinger equation to a set of \(6n\)-different equations for the parameters which can be solved numerically. If one inspects the formalism carefully, one finds that the interaction potential which is actually the Brückner G-matrix can be divided into two parts: (i) a real part and (ii) an imaginary part. The real part of the potential acts like a potential whereas imaginary part is proportional to the cross section.

In the present model, interaction potential comprises of the following terms:

\[
V_{\alpha\beta} = V_{\text{loc}}^2 + V_{\text{loc}}^3 + V_{\text{Coul}} + V_{\text{Yuk}}
\]

(10)

\(V_{\text{loc}}\) is the Skyrme force whereas \(V_{\text{Coul}}, V_{\text{Yuk}}\) and \(V_{\text{MDI}}\) define, respectively, the Coulomb, and Yukawa potentials. The Yukawa term separates the surface which also plays the role in low energy processes like fusion and cluster radioactivity [12]. The expectation value of these potentials is calculated as

\[
V_{\text{loc}}^2 = \int f_\alpha(p_\alpha, r_\alpha, t)f_\beta(p_\beta, r_\beta, t)V_I^{(2)}(r_\alpha, r_\beta)
\]

\[
\times d^3r_\alpha d^3r_\beta d^3p_\alpha d^3p_\beta.
\]

(11)
\[ V^3_{\text{loc}} = \int f_\alpha(p_\alpha, r_\alpha, t) f_\beta(p_\beta, r_\beta, t) f_\gamma(p_\gamma, r_\gamma, t) \]
\[ \times V^{(3)}_I(r_\alpha, r_\beta, r_\gamma) d^3r_\alpha d^3r_\beta d^3r_\gamma \times d^3p_\alpha d^3p_\beta d^3p_\gamma. \]  

where \( f_\alpha(p_\alpha, r_\alpha, t) \) is the Wigner density which corresponds to the wave functions (eq. 2). If we deal with the local Skyrme force only, we get
\[ V^{\text{Skyrme}} = \sum_{\alpha=1}^{A_{p+4}} \left[ \frac{A}{2} \sum_{\beta=1}^{\rho_0} \left( \frac{\hat{\rho}_{\alpha\beta}}{\rho_0} \right) + \frac{B}{C+1} \sum_{\beta \neq \alpha} \left( \frac{\hat{\rho}_{\alpha\beta}}{\rho_0} \right)^C \right]. \]  

Here A, B and C are the Skyrme parameters which are defined according to the ground state properties of a nucleus. Different values of C lead to different equations of state. A larger value of C (= 380 MeV) is often dubbed as stiff equation of state. The finite range Yukawa \( (V_{\text{Yuk}}) \) and effective Coulomb potential \( (V_{\text{Coul}}) \) read as:
\[ V_{\text{Yuk}} = \sum_{j,i \neq j} t_3 \exp\left\{ -\frac{|r_i - r_j|}{\mu} \right\}, \]  
\[ V_{\text{Coul}} = \sum_{j,i \neq j} \frac{Z_2^2 \epsilon_f^2}{|r_i - r_j|}. \]

The Yukawa interaction (with \( t_3 = -6.66 \text{ MeV} \) and \( \mu = 1.5 \text{ fm} \)) is essential for the surface effects. The relativistic effect does not play role in low incident energy of present interest [13].

The phase space of nucleons is stored at several time steps. The QMD model does not give any information about the fragments observed at the final stage of the reaction. In order to construct the fragments, one needs clusterization algorithms. We shall concentrate here on the MST and MSTP methods.

According to MST method [5], two nucleons are allowed to share the same fragment if their centroids are closer than a distance \( r_{\text{min}} \),
\[ |r_i - r_j| \leq r_{\text{min}}. \]  
where \( r_i \) and \( r_j \) are the spatial positions of both nucleons and \( r_{\text{min}} \) taken to be 4fm.

For MSTP method, we impose a additional cut in the momentum space, i.e., we allow only those nucleons to form a fragment which in addition to equation(16) also satisfy
\[ |p_i - p_j| \leq p_{\text{min}}, \]  
where \( p_{\text{min}} = 150 \text{ MeV/c} \).
Figure 1: The rapidity distribution for free nucleons and LCPs for the reaction of $^{12}\text{C}+^{12}\text{C}$ at incident energy of 100 MeV/nucleon at central (left panel) and peripheral (right) geometries with MST and MSTP methods.

3 Results and Discussion

We simulated the reactions of $^{12}\text{C}+^{12}\text{C}$ and $^{40}\text{Ca}+^{40}\text{Ca}$ at 100 MeV/nucleon at central and peripheral colliding geometries, i.e., at $\hat{b} = 0.0$ and 0.8, respectively. We use a soft equation of state with standard energy-dependent Cugon cross section.
In figure 1, we display the rapidity distribution of free nucleon and LCP’s for the reaction of $^{12}\text{C} + ^{12}\text{C}$ at energy of 100 MeV at central (left panel) and peripheral (right) colliding geometry. The solid and dashed lines indicate the calculations of MST and MSTP methods, respectively. From the figure, we see that there is quantitative difference in the results of MST and MSTP methods, though qualitatively, both methods give similar behaviour of the rapidity distribution of nucleons and fragments.

For central collisions (left panel), we see that the peak of $dN/dY$ plot is pronounced for the MSTP method, thus, indicating enhanced production of free nucleons in MSTP
method as compared to MST method. This is due to the fact that in the MST method, we have a single big fragment because of no restriction is being imposed on the relative momentum of nucleons forming fragments. The production of LCP’s is more with MSTP method compared to MST method which is supported by Ref. [6, 7]. At peripheral collisions, the behaviour of the rapidity plots of free nucleons is similar as for the central
one whereas the trend reverses for the LCP’s plot. For peripheral collisions, we have greater production with MSTP method.

In figure 2, we display the rapidity distribution of free nucleons, LCP’s and IMF’s for the reaction of $^{40}$Ca+$^{40}$Ca. Left(right) panels display the results for $b/b_{\text{max}}=0.0$ (0.8). We find similar behaviour of free nucleons and LCP’s as reported for the reaction of $^{12}$C+$^{12}$C. The IMF’s also follow the similar trend as for LCP’s, i.e., we have more (less) production of IMF’s with MST method at central (peripheral) collisions.

In figures 3 and 4, we display $dN/dp_{t}dp_{t}$ versus $p_{t}$ for the reaction of $^{12}$C+$^{12}$C and

Figure 4: Same as Fig. 3 but for the reaction of $^{40}$Ca+$^{40}$Ca.
Figure 5: The time evolution of $E_{rat}$ for free nucleons and LCP’s for the reaction of $^{12}\text{C}+^{12}\text{C}$. $^{40}\text{Ca}+^{40}\text{Ca}$, respectively. We see that $dN/p_tdp_t$ spectra follow the similar behaviour for both MST and MSTP methods. We have a higher peak in the spectra of free nucleons with MST method at both the colliding geometries. The difference between MST and MSTP methods in spectra of LCP’s is less significant. Similar behaviour is also observed for the reaction of $^{40}\text{Ca}+^{40}\text{Ca}$.

In figure 5, we display the time evolution of $E_{rat}$ of free nucleons and LCP’s for
the reaction of $^{12}\text{C} + ^{12}\text{C}$ at central (left panel) and peripheral (right) colliding geometry. For MST and MSTP methods, we find a significant difference between MST and MSTP methods for both free nucleons and LCP’s. The difference is more for the central collisions as compared to the peripheral one.

In figure 6, we display time evolution of $E_{\text{rat}}$ of free nucleons, LCP’s and IMF’s for the reaction of $^{40}\text{Ca} + ^{40}\text{Ca}$ at central (left panel) and peripheral (right) collisions. The solid (dashed) lines represent the results of MST (MSTP) method. From figure, we find a significant difference of $E_{\text{rat}}$ with MST and MSTP method as in case of the reaction of $^{12}\text{C} + ^{12}\text{C}$.

Figure 6: Same as Fig. 5 but for the reaction of $^{40}\text{Ca} + ^{40}\text{Ca}$. 

![Figure 6](image-url)
\(^{12}\text{C} + ^{12}\text{C}\). We also find that the difference between MST and MSTP reduces at peripheral colliding geometries.

4 Summary

Using the quantum molecular dynamic model, we studied the role of momentum correlations in the properties of fragments. This was achieved by imposing cut in momentum space during the process of clusterization. We find that this cut yields significant difference in the fragment properties of system at all colliding geometries.

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