Charmless hadronic $B \to (f_1(1285), f_1(1420))P$ decays in the perturbative QCD approach

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We study twenty charmless hadronic $B \to f_1P$ decays in the perturbative QCD(pQCD) formalism with $B$ denoting $B_s$, $B_d$, and $B_s$ mesons, $P$ standing for the light pseudoscalar mesons, and $f_1$ representing axial-vector mesons $f_1(1285)$ and $f_1(1420)$ that resulting from a mixing of quark-flavor $f_{1q}^{[su,C,du]}$ and $f_{1s}[s\bar{s}]$ states with the angle $\phi_{f_1}$. The estimations of CP-averaged branching ratios and CP asymmetries of the considered $B \to f_1P$ decays, in which the $B_s$ and $f_1P$ modes are investigated for the first time, are presented in the pQCD approach with $\phi_{f_1} \sim 24^\circ$ from recently measured $B_{d/s} \to J/\psi f_1(1285)$ decays. It is found that (a) the tree(penguin) dominant $B^+ \to f_1\pi^+(K^+)$ decays with large branching ratios$(O(10^{-6}))$ and large direct CP violations around $14\% \sim 28\%$ in magnitude) simultaneously are believed to be clearly measurable at the LHCb and Super-B factory experiments; (b) the $B_d \to f_1K^0_S$ and $B_s \to f_1(\eta, \eta')$ decays with nearly pure penguin contributions meanwhile safely negligible tree pollution also have large decay rates in the order of $10^{-6} \sim 10^{-5}$, which can be confronted with the experimental measurements in the near future; (c) as the alternative channels, the $B^+ \to f_1(\pi^+, K^+)$ and $B_d \to f_1K^+_S$ decays have the supplementary power in providing more effective constraints on the Cabibbo-Kobayashi-Maskawa weak phases $\alpha$, $\gamma$, and $\beta$ correspondingly, which are explicitly analyzed through the large decay rates and the direct and mixing-induced CP asymmetries in the pQCD approach and are expected to be stringently examined by the measurements with high precision; (d) the weak annihilation amplitudes play important roles in the $B^+ \to f_1(1420)K^+$, $B_d \to f_1(1420)K^0_S$, $B_s \to f_1(1420)\eta'$ decays and so on, which would offer more evidences, once they are confirmed by the experiments, to identify the soft-collinear effective theory and the pQCD approach on the evaluations of annihilation diagrams and to help further understand the annihilation mechanism in the heavy $B$ meson decays; (e) combined with the future precise tests, the considered decays can provide more information to further understand the mixing angle $\phi_{f_1}$ and the nature of the $f_1$ mesons in depth after the confirmations on the reliability of the pQCD calculations in the present work.

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I. INTRODUCTION

It is well known that non-leptonic weak decays of heavy $B$ (specifically, $B_s$, $B_d$, and $B_s$) mesons can not only provide the important information to search for CP violation and further constrain the Cabibbo-Kobayashi-Maskawa(CKM) parameters in the standard model(SM), but also reveal the deviations from the SM, i.e., the signals of exotic new physics beyond the SM. Furthermore, comparison of theoretical predictions and experimental data for the physical observables may also help us understand the hadronic structure of the involved bound states deeply [1]. In contrast to the traditional $B \to PP, PV$ and $VV$ decays, the alternative channels such as $B \to AP(A$: axial-vector mesons) decays to be largely detected at the experiments in the near future may give the additional and complementary information on exclusive non-leptonic weak decays of heavy $B$ mesons [2], e.g., due to $V_{tb}V_{ts} = -V_{cb}^\dagger V_{cs}(1 + O(\lambda^2))$, the $b \to sq\bar{q}$ penguin-dominated decays have the same CKM phase as the $b \to cc\bar{s}$ tree level decays [3]. Therefore, the $b \to sq\bar{q}$ mediated $B \to AP$ decays such as $B^0 \to \phi_1(1260)[b_s(1235)]K^0_S\pi K_1(1270)[K_1(1400)]$, $f_1K^0_S$ etc. can provide sin $2\beta(\beta$: CKM weak phase) measurements in the SM complementarily.

Very recently, the Large Hadron Collider beauty(LHCb) Collaboration reported the first measurements of $B_{d/s} \to J/\psi f_1(1285)$ decays [4], where the final state $f_1(1285)$ was observed for the first time in heavy $B$ meson decays. In the conventional two quark structure, $f_1(1285)$ and its partner $f_1(1420)$ [5, 6] (Hereafter, for the sake of simplicity, we will use $f_1$ to denote both $f_1(1285)$ and $f_1(1420)$ unless otherwise stated.) are considered as the orbital excitation of $qq\bar{q}$ system, specifically, the light $p$-wave axial-vector flavorless mesons. In terms of the spectroscopic notation $(2S+1)L_J$ with $J = L + S$, both $f_1$ mesons belong to $3P_1$ nonet carrying the quantum number $J^{PC} = 1^{-+}$ [3]. Just as the $\eta - \eta'$ mixing in the pseudoscalar sector [3], these two $f_1$ mesons are believed to be a mixture resulting from the mixing between nonstrange $f_{1q}^{[su,C,du]} \equiv [u\bar{u} + d\bar{d}] / \sqrt{2}$ and strange $f_{1s} \equiv s\bar{s}$ states in the popular quark-flavor basis with a single mixing angle $\phi_{f_1}$. And for the mixing angle $\phi_{f_1}$, there are several explorations that have been performed from theory and experiment sides. However, the value of $\phi_{f_1}$ is still in

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decays, in which hadronic and considered their mixing with two different sets of angles perturbatively. As is well known, the pQCD approach is free of endpoint singularity and the Sudakov formalism makes it the upper limits on the decay rates have been placed at the 90% confidence level as \[ |\langle f_1(1285) \rangle - f_1(1420)\rangle| < 4.1 \times 10^{-6}, \]
for \( B^+ \to f_1(1285)K^+ \) decay and
\[ Br(B^+ \to f_1(1420)K^+) \cdot Br(f_1(1420) \to K^+ K) < 2.9 \times 10^{-6}. \]
for \( B^+ \to f_1(1420)K^+ \) decay, respectively. However, due to the lack of the information on the decay rates of \( f_1(1420) \to K^+ K \) and \( f_1(1420) \to \eta\pi \) decays, the upper limits of \( Br(B^+ \to f_1(1420)K^+) \) cannot be extracted effectively. But, this status will be greatly improved at the present and future experiments, notably at running LHCb and forthcoming Super-B. And also other \( B \to f_1 P \) decays are expected to be detected with good precision at the relevant experiments in the near future.

In this work, we will therefore study twenty charmless hadronic \( B \to f_1 P^d \) decays, in which \( B \) stands for \( B_u, B_d, \) and \( B_s, \) respectively, and \( P \) denotes the light pseudoscalar pion, kaon, \( \eta \) and \( \eta' \) mesons. From the experimental point of view, up to now, only two penguin-dominated \( B^+ \to f_1 K^+ \) decays have been measured by BABAR Collaboration in 2007 [13]. The preliminary upper limits on the decay rates have been placed at the 90% confidence level as [3],
\[ Br(B^+ \to f_1(1285)K^+) < 2.0 \times 10^{-6}, \]

for \( B^+ \to f_1(1285)K^+ \) decay and
\[ Br(B^+ \to f_1(1420)K^+) \cdot Br(f_1(1420) \to K^+ K) < 4.1 \times 10^{-6}, \]
\[ Br(B^+ \to f_1(1420)K^+) \cdot Br(f_1(1420) \to \eta\pi) < 2.9 \times 10^{-6}. \]

from the theoretical point of view, to our best knowledge, G. Calderón et al. have carried out the calculations of \( B_{u,d} \to f_1 P \) decays in the framework of naive factorization with the form factors of \( B \to f_1 \) obtained in the improved Isgur-Scora-Gribov-Wise quark model [2], while Cheng and Yang have studied the decay rates and direct CP asymmetries of \( B_{u,d} \to f_1(\pi, K) \) modes within the framework of QCD factorization(QCDF) with the form factors evaluated in the QCD sum rule [14]. Note that the \( B_{u,d} \to f_1 P \) decays have never been studied yet in any methods or approaches up to this date. And, it should be stressed that the predictions of the branching ratios for \( B_{u,d} \to f_1 P \) decays in naive factorization are so crude that we cannot make effective comparison for relevant \( B \to f_1 P \) modes. For \( B^+ \to f_1 K^+ \) decays, for example, on one hand, the authors did not specify \( f_1(1285) \) and \( f_1(1420) \) [2], which then could not provide effectively the useful information on the mixing angle \( \theta_{f_1} \) from these considered decays; on the other hand, as discussed in Ref. [14], the \( f_1(1285) \) meson behaves analogously to the vector meson, it is then naively expected that \( Br(B^+ \to f_1(1285)K^+) \sim Br(B^+ \to \omega K^+) \) and \( Br(B^+ \to f_1(1420)K^+) \sim Br(B^+ \to \phi K^+) \) if \( f_1(1285) \) and \( f_1(1420) \) are significantly dominated by the \( f_{1q} \) and \( f_{1a} \) components, respectively. Furthermore, in principle, in view of the \( f_1(1285) \to f_1(1420) \) mixing, the branching ratios of \( B^+ \to f_1(1285)K^+ \) and \( f_1(1420)K^+ \) are generally a bit smaller than those of \( B^+ \to \omega K^+ \) ones correspondingly. However, the branching ratio of \( B^+ \to f_1 K^+ \) predicted in the naive factorization is around \( 3 \times 10^{-5} \), which is much larger than that of the corresponding \( V P \) modes, i.e., \( B^+ \to \omega K^+ \) and \( B^+ \to \phi K^+ \) [3]. As for the investigations of \( B^+ \to f_1 K^+ \) decays in QCDF approach [14], the authors specified \( f_1(1285) \) and \( f_1(1420) \) and considered their mixing with two different sets of angles \( \theta_{3P_1} \sim 27.9^\circ \) and \( 53.2^\circ \) in the flavor singlet-octet basis. And the decay rates are barely consistent with the preliminary upper limits within very large errors. However, the pattern exhibited from the numerical results with \( \theta_{3P_1} \sim 53.2^\circ \) is more favored by the available upper limits. As aforementioned, because of the similar behavior between vector meson and \( f_1(1285) \) axial-vector meson, the relation \( Br(B^+ \to f_1(1285)K^+) < Br(B^+ \to f_1(1420)K^+) \) is expected to be highly preferred as it should be.

In order to collect more information on the nature of both \( f_1 \) mesons and further understanding the heavy flavor \( B \) physics, we will study the physical observables such as CP-averaged branching ratios and CP-violating asymmetries of twenty charmless hadronic \( B \to f_1 P \) decays by employing the low energy effective Hamiltonian [15] and the pQCD approach [10-12] based on the \( k_T \) factorization theorem. Though some efforts have been made on the next-to-leading order pQCD formalism [16, 17], we here will still consider the perturbative evaluations at leading order, which are believed to be the dominant contributions perturbatively. As is well known, the pQCD approach is free of endpoint singularity and the Sudakov formalism makes it more self-consistent by keeping the transverse momentum \( k_T \) of the quarks. More important, as the well-known advantage of the pQCD approach, we can explicitly calculate the weak annihilation types of diagrams without any parameterizations, apart from the traditional factorizable and nonfactorizable emission ones, though the different viewpoint on the evaluations and

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1 In the literature [9], two of us(X. Liu and Z.J. Xiao) have studied the \( B_c \to f_1 P \) decays occurring only via the pure annihilation diagrams in the SM within the framework of perturbative QCD(pQCD) factorization approach [10-12].
the magnitudes\(^2\) of weak annihilation contributions has been proposed in the soft-collinear effective theory \([21]\). It is worth
stressing that the pQCD predictions on the annihilation contributions in the heavy \(B\) meson decays have been tested at various
aspects, e.g., see Refs. \([10, 11, 22–26]\). Typically, for example, the evaluations of the pure annihilation \(B_d \to K^+K^-\) and
\(B_s \to \pi^+\pi^-\) decays in the pQCD approach \([23–25]\) are in good consistency with the recent measurements by both CDF and
LHCb Collaborations \([27–29]\). Therefore, the weak annihilation contributions to the considered \(B \to f_1 P\) decays will be
explicitly analyzed in this work, which are expected to be helpful to understand the annihilation mechanism in the heavy \(B\)
meson decays.

The paper is organized as follows. In Sec. II, we present the formalism, hadron wave functions and perturbative calculations of
the considered twenty \(B \to f_1 P\) decays in the pQCD approach. The numerical results and the corresponding phenomenological
analyses are addressed in Sec. III. Finally, Sec. IV contains the main conclusions and a short summary.

II. FORMALISM AND PERTURBATIVE CALCULATIONS

For the considered \(B \to f_1 P\) decays, the related weak effective Hamiltonian \(H_{\text{eff}}\) \([15]\) can be written as

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \left\{ V_{ub}^* V_{uD} [C_1(\mu) O_1^u(\mu) + C_2(\mu) O_2^u(\mu)] - V_{tb}^* V_{tD} \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right\} + \text{H.c.},
\]

with \(D\) the light down-type quark \(d\) or \(s\), the Fermi constant \(G_F = 1.16639 \times 10^{-5}\text{GeV}^{-2}\), CKM matrix elements \(V\),
and Wilson coefficients \(C_i(\mu)\) at the renormalization scale \(\mu\). The local four-quark operators \(O_i(i = 1, \cdots, 10)\) are written as

1. current-current(tree) operators

\[
O_1^u = (\bar{D}_\alpha u_\alpha)_{V\to A}(\bar{u}_\beta b_\beta)_{V\to A}, \quad O_2^u = (\bar{D}_\alpha u_\alpha)_{V\to A}(\bar{u}_\beta b_\beta)_{V\to A};
\]

2. QCD penguin operators

\[
O_3 = (\bar{D}_\alpha b_\alpha)_{V\to A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V\to A}, \quad O_4 = (\bar{D}_\alpha b_\beta)_{V\to A} \sum_{q'} (\bar{q}'_\beta q'_\alpha)_{V\to A},
O_5 = (\bar{D}_\alpha b_\beta)_{V\to A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V\to A}, \quad O_6 = (\bar{D}_\alpha b_\beta)_{V\to A} \sum_{q'} (\bar{q}'_\beta q'_\beta)_{V\to A}.
\]

3. electroweak penguin operators

\[
O_7 = \frac{3}{2} (\bar{D}_\alpha b_\alpha)_{V\to A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V\to A}, \quad O_8 = \frac{3}{2} (\bar{D}_\alpha b_\beta)_{V\to A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\alpha)_{V\to A},
O_9 = \frac{3}{2} (\bar{D}_\alpha b_\alpha)_{V\to A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V\to A}, \quad O_{10} = \frac{3}{2} (\bar{D}_\alpha b_\beta)_{V\to A} \sum_{q'} e_{q'}(\bar{q}'_\beta q'_\beta)_{V\to A}.
\]

with the color indices \(\alpha, \beta\)(not to be confused with the CKM weak phases \(\alpha \) and \(\beta\)) and the notations \((\bar{q}' q')_{V\pm A} = \bar{q}' \gamma_{\mu}(1 \pm \gamma_5)q'\). The index \(q'\) in the summation of the above operators runs through \(u, d, s, c,\) and \(b\). The standard combinations \(a_i\) of the
Wilson coefficients \(C_i\) are defined as follows:

\[
a_1 = C_2 + \frac{C_1}{3}, \quad a_2 = C_1 + \frac{C_2}{3}, \quad a_i = C_i + C_{i \pm 1}/3, \quad i = 3 - 10.
\]

where \(C_2 \sim 1\) is the largest one among all Wilson coefficients and the upper (lower) sign applies, when \(i\) is odd (even). It is
noted that, though the next-to-leading order Wilson coefficients have already been available \([15]\), we will still adopt the leading
order ones to keep the consistency, since the short distance contributions of the considered decays are calculated at leading
order[\(O(\alpha_s)\)] in the pQCD approach. This is also the consistent way to cancel the explicit \(\mu\) dependence in the theoretical

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\(^2\) As a matter of fact, the recent works \([18, 19]\) in the framework of QCDF confirmed that there should exist complex annihilation contributions with large
imaginary part in \(B_{u,d,s} \to \pi \pi, \pi K, K K\) decays by fits to the experimental data, which support the concept on the calculations of the annihilation diagrams
in the pQCD approach \([20]\) to some extent.
formulas. For the renormalization group evolution of the Wilson coefficients from higher scale to lower scale, the expressions are directly taken from Ref. [11].

Nowadays, the pQCD approach has been one of the popular factorization methods that based on the QCD theory to evaluate the hadronic matrix elements in the heavy B meson decays. The unique point of the pQCD approach is that it keeps the transverse momentum $k_T$, which will act as an infrared regulator and smear the end-point singularity when the quark momentum fraction $x$ approaches to 0, of the valence quarks in the calculation of the hadronic matrix elements. Then, all the $B$ meson transition form factors, non-factorizable spectator and annihilation contributions are calculable in the framework of the $k_T$ factorization. The decay amplitude of $B \to f_1 P$ decays in the pQCD approach can be conceptually written as,

$$A(B \to f_1 P) \sim \int d^4k_1 d^4k_2 d^4k_3 \text{Tr} \left[ C(t) \Phi_B(k_1) \Phi_{f_1}(k_2) \Phi_P(k_3) H(k_1, k_2, k_3, t) \right],$$

where $k_i$’s are the momenta of (light) quarks in the initial and final states, $\text{Tr}$ represents the trace over Dirac and color indices, $C(t)$ is the Wilson coefficient which results from the radiative corrections at short distance. In the above convolution, $C(t)$ includes the harder dynamics at larger scale than $m_B$ scale and describes the evolution of local 4-Fermi operators from $m_W$ (the W boson mass) down to $t \sim \mathcal{O}(\sqrt{\Lambda_{\text{QCD}} m_B})$ scale, where $\Lambda_{\text{QCD}}$ is the hadronic scale. The $\Phi$ stands for the wave function describing hadronization of the quark and anti-quark to the meson, which is independent of the specific processes and usually determined by employing nonperturbative QCD techniques such as Lattice QCD(LQCD) or other well measured processes. The function $H(k_1, k_2, k_3, t)$ describes the four quark operator and the spectator quark connected by a hard gluon with the hard intermediate scale $\mathcal{O}(\sqrt{\Lambda_{\text{QCD}} m_B})$. Therefore, this hard part $H$ can be calculated perturbatively.

Since the $b$ quark is rather heavy, we thus work in the frame with the $B$ meson at rest for simplicity, i.e., with the $B$ meson momentum $P_B = \frac{m_B}{\sqrt{2}} (1, 1, 0, 0)$ in the light-cone coordinate. For the considered $B \to f_1 P$ decays, it is assumed that the $f_1$ and $P$ mesons move in the plus and minus $z$ direction carrying the momentum $P_2$ and $P_3$, respectively. Then the momenta of the two final states can be written as

$$P_2 = \frac{m_B}{\sqrt{2}} (1, r_{f_1}, 0, 0), \quad P_3 = \frac{m_B}{\sqrt{2}} (0, 1, -r_{f_1}, 0),$$

respectively, where $r_{f_1} = m_{f_1}/m_B$ and the mass of light pseudoscalar pion, kaon, $\eta$ and $\eta'$ has been neglected. For the axial-vector meson $f_1$, its longitudinal polarization vector $e_{\perp} = \frac{m_B}{\sqrt{2}m_{f_1}} (1, -r_{f_1}, 0, 0)$. By choosing the quark momenta in $B$, $f_1$ and $P$ mesons as $k_1$, $k_2$, and $k_3$, respectively and defining

$$k_1 = (x_1 P_2^+, 0, 0), \quad k_2 = x_2 P_2 + (0, 0, 0), \quad k_3 = x_3 P_3 + (0, 0, 0),$$

then, the integration over $k_{1\perp}, k_{2\perp}$, and $k_{3\perp}$ in Eq. (9) will give the more explicit form of decay amplitude in the pQCD approach,

$$A(B \to f_1 P) \sim \int dx_1 dx_2 dx_3 dx_4 db_1 db_2 db_3 db_4 \text{Tr} \left[ C(t) \Phi_B(x_1, b_1) \Phi_{f_1}(x_2, b_2) \Phi_P(x_3, b_3) H(x_1, b_1, x_2, b_2, x_3, b_3, t) S_t(x_1) e^{-S(t)} \right]$$

where $b_i$ is the conjugate space coordinate of $k_{iT}$, and $t$ is the largest running scale in the hard kernel $H(x_i, b_i, t)$. The large logarithms $\ln(m_W/t)$ are included in the Wilson coefficients $C(t)$. Note that $S_t(x_1)$ and $e^{-S(t)}$ are the two important elements in the perturbative calculations with the pQCD approach. The former is a jet function from threshold resummation, which can strongly suppress the behavior in the small $x$ region [30, 31]; while the latter is a Sudakov factor from $k_T$ resummation, which can effectively suppress the soft dynamics in the small $k_T$ region [32, 33]. These resummation effects therefore guarantee the removal of the endpoint singularities. Thus it makes the perturbative calculation of the hard part $H$ applicable at intermediate scale. We will calculate analytically the function $H(x_i, b_i, t)$ for the $B \to f_1 P$ decays at LO in $\alpha_s$ expansion and give the convoluted amplitudes in next section.

The heavy $B$ meson is usually treated as a heavy-light system and its light-cone wave function can generally be defined as [10, 11, 34]

$$\Phi_B = \left\{ (P + m_B) \gamma_5 \phi_B(x, k_T) \right\}_{\alpha\beta};$$

in which $\alpha, \beta$ are the color indices, $P(m)$ is momentum(mass) of the $B$ meson, $N_c$ is the color factor, and $k_T$ is the intrinsic transverse momentum of the light quark in $B$ meson.

In Eq. (13), $\phi_B(x, k_T)$ is the $B$ meson distribution amplitude, which satisfies the following normalization condition,

$$\int_0^1 dx \phi_B(x, b = 0) = \frac{f_B}{2\sqrt{2}N_c}.$$
For the pseudoscalar $P$ meson, the light-cone wave function can generally be defined as [35, 36],

$$
\Phi_P(x) = \frac{i}{\sqrt{2N_c}} \gamma_5 \left\{ P \phi_P^A(x) + m_0^P \phi_P^T(x) + m_0^P (\bar{\psi} - 1) \phi_P^T(x) \right\}_{\alpha\beta}
$$

(15)

where $\phi_P^A$ and $\phi_P^{P,T}$ are the twist-2 and twist-3 distribution amplitudes, and $m_0^P$ is the chiral enhancement factor of the meson, while $x$ denotes the momentum fraction carried by quark in the meson and $n = (1, 0, 0_T)$ and $v = (0, 1, 0_T)$ are the dimensionless light-like unit vectors.

The light-cone wave function of the axial-vector $f_1$ mesons has been given in the QCD sum rule as [37, 38]

$$
\Phi_{f_1}^L = \frac{1}{\sqrt{2N_c}} \left\{ m_{f_1} \phi_{f_1}^L(x) + f_L^P \phi_{f_1}^T(x) + m_{f_1} \phi_{f_1}^T(x) \right\}_{\alpha\beta},
$$

(16)

for longitudinal polarization with the polarization vector $\epsilon_L$, satisfying $P \cdot \epsilon = 0$, where $\phi_{f_1}$ (not to be confused with the angle $\phi_{f_1}$ in the mixing of $f_1$ mesons) and $\phi_{f_1}^{s,t}$ are the twist-2 and twist-3 distribution amplitudes, respectively. All the explicit forms of the aforementioned hadronic distribution amplitudes in the considered $B \to f_1 P$ decays can be seen in Appendix A.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{feynman_diagrams.png}
\caption{Typical Feynman diagrams contributing to $B \to P f_1$ decays in the pQCD approach at leading order.}
\end{figure}

Now we come to the analytically perturbative calculations of the factorization formulas for the $B \to f_1 P$ decays in the pQCD approach. From the effective Hamiltonian (4), there are eight types of diagrams contributing to the $B \to P f_1$ decays as illustrated in Fig. 1. For the factorizable emission $(f_e)$ diagrams, with Eq. (12), the analytic expressions of the decay amplitudes from different operators are given as follows,

- $(V - A)(V - A)$ operators:

$$
F_{f_e} = -8\pi C_{FP} f_P m_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \left\{ \left( 1 + x_3 \right) \phi_{f_1}^s(x_3) + r_{f_1} \left( 1 - 2x_3 \right) \phi_{f_1}^t(x_3) \right\} h_{f_e}(x_1, x_3, b_1, b_3) E_{f_e}(t_a) + 2 r_{f_1} \phi_{f_1}^s(x_3) h_{f_e}(x_3, x_1, b_3, b_1) E_{f_e}(t_b) \right\},
$$

(17)

- $(V - A)(V + A)$ operators:

$$
F_{f_e}^{P1} = -F_{f_e},
$$

(18)

- $(S - P)(S + P)$ operators:

$$
F_{f_e}^{P2} = -16\pi C_{FP} f_P m_B^2 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_B(x_1, b_1) \left\{ \left[ \phi_{f_1}(x_3) + r_{f_1} \left( 2 + x_3 \right) \phi_{f_1}^s(x_3) - x_3 \phi_{f_1}^t(x_3) \right] h_{f_e}(x_1, x_3, b_1, b_3) E_{f_e}(t_a) + 2 r_{f_1} \phi_{f_1}^s(x_3) h_{f_e}(x_3, x_1, b_3, b_1) E_{f_e}(t_b) \right\};
$$

(19)
where $r_T^P = m_T^P/m_B$ and $C_F = 4/3$ is a color factor. The convolution functions $E_t$, the running hard scales $t_i$, and the hard functions $h_i$ can be referred to Ref. [39].

For the non-factorizable emission($nfa$) diagrams in Fig. 1(c) and 1(d), the corresponding decay amplitudes can be written as

- $(V - A)(V - A)$ operators:

$$M_{nfe} = -\frac{32}{\sqrt{6}} \pi C_F m_B^2 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 b_2 b_3 b_4 \phi_B(x_1, b_1) \phi_B^\dagger(x_2) \times \left\{ \left[ (1 - x_2) \phi_{f_1}(x_3) - r_{f_1} x_3 (\phi_{f_1}^\dagger(x_3) - \phi_{f_1}(x_3)) \right] E_{nfe}(t_e) h_{nfe}^c(x_1, x_2, x_3, b_1, b_2) \right. \right. \nonumber
- \left. \left. \left[ (x_2 + x_3) \phi_{f_1}(x_3) - r_{f_1} x_3 (\phi_{f_1}^\dagger(x_3) + \phi_{f_1}(x_3)) \right] E_{nfe}(t_d) h_{nfe}^d(x_1, x_2, x_3, b_1, b_2) \right\} ,
 \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \r...
For the last two diagrams in Fig. 1, i.e., the factorizable annihilation ($f_a$) diagrams 1(g) and 1(h), we have

- $(V-A)(V-A)$ operators:

\[
F_{fa} = -8\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_2 b_3 \phi_{f_1}(x_3) - [1 - x_3] \phi_{f_1}(x_3) - 2r_0 r_f (x_3 - 2) \phi_{f_1}(x_3) \times E_{fa}(t_f) h_{fa}(t_f) \left\{ \begin{array}{l}
[x_2 \phi_{f_1}^4(x_2) - 2r_0 r_f (x_2 + x_2 - 1) \phi_{f_1}^2(x_2)]
\times E_{fa}(t_f) h_{fa}(1 - x_3, x_2, b_3, b_3)
\end{array} \right\}
\]

(26)

- $(V-A)(V + A)$ operators:

\[
F_{fa}^{P1} = F_{fa}
\]

(27)

- $(S-P)(S + P)$ operators:

\[
F_{fa}^{P2} = -16\pi C_F m_B^2 \int_0^1 dx_2 dx_3 \int_0^\infty b_2 b_2 b_3 \phi_{f_1}(x_3) - [1 - x_3] \phi_{f_1}(x_3) - 2r_0 r_f (x_3 - 2) \phi_{f_1}(x_3) \times E_{fa}(t_f) h_{fa}(t_f) \left\{ \begin{array}{l}
[x_2 \phi_{f_1}^4(x_2) - 2r_0 r_f (x_2 + x_2 - 1) \phi_{f_1}^2(x_2)]
\times E_{fa}(t_f) h_{fa}(1 - x_3, x_2, b_3, b_3)
\end{array} \right\}
\]

(28)

When we exchange $P$ and $f_1$ in Fig. 1, we can obtain the new eight diagrams contributing to the considered $B \to f_1 P$ decays. The corresponding factorization formulas can be easily obtained with the simple replacements in Eqs. (17)-(28) as follows,

\[
f_P \leftrightarrow f_1, \quad \phi_P^A \leftrightarrow \phi_{f_1}, \quad \phi_P^P \leftrightarrow \phi_{f_1}, \quad \phi_P^T \leftrightarrow \phi_{f_1}, \quad r_P^\epsilon \leftrightarrow r_f,
\]

(29)

where $F'$ and $M'$ will be used to denote the Feynman amplitudes from these new diagrams. Note that, due to \( \langle f_1 | S \pm P | 0 \rangle = 0 \), the factorizable emission amplitude $F_{fa}^{P2}$ is therefore absent naturally.

Before we write down the total decay amplitudes for the $B \to f_1 P$ modes, it is essential to give a brief discussion about the $\eta - \eta'$ mixing and $f_1(1285) - f_1(1420)$ mixing, respectively. The $\eta - \eta'$ mixing have been discussed in different bases: quark-flavor basis [40] and octet-singlet basis [41], and the related parameters have been effectively constrained from various experiments and theories (for a recently detailed overview. See [42] and references therein). In the present work, we adopt the quark-flavor basis with the definitions [40]: $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_q = s\bar{s}$. Then the physical states $\eta$ and $\eta'$ can be described as the mixtures of two quark-flavor $\eta_q$ and $\eta_q$ states with a single mixing angle $\phi$.

\[
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix} = U(\phi) \begin{pmatrix}
\eta_q \\
\eta_q
\end{pmatrix} = \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
\eta_q \\
\eta_q
\end{pmatrix}.
\]

(30)

It is assumed that the $\eta_q$ and $\eta_q$ states have the same light-cone distribution amplitudes just as that of the pion but with different decay constants $f_{\eta_q}$ and $f_{\eta_q}$ and different chiral enhancement factors $m_{\eta_q}$ and $m_{\eta_q}$. The $f_{\eta_q}$, $f_{\eta_q}$, and $\phi$ in the quark-flavor basis have been extracted from various related experiments [40, 41]

\[
f_{\eta_q} = (1.07 \pm 0.02) f_\pi, \quad f_{\eta_q} = (1.34 \pm 0.06) f_\pi, \quad \phi = 39.3^\circ \pm 1.0^\circ.
\]

(31)

And the chiral enhancement factors are chosen as

\[
m_{\eta_q}^2 = \frac{1}{2m_q} \left[ m_q^2 \cos^2 \phi + m_{\eta'}^2 \sin^2 \phi - \sqrt{2} f_{\eta_q}^2 (m_{\eta'}^2 - m_q^2) \cos \phi \sin \phi \right],
\]

(32)

\[
m_{\eta_q}^2 = \frac{1}{2m_s} \left[ m_s^2 \cos^2 \phi + m_{\eta'}^2 \sin^2 \phi - \frac{f_{\eta_q}}{\sqrt{2} f_{\eta_q}} (m_{\eta'}^2 - m_s^2) \cos \phi \sin \phi \right].
\]

(33)

with no isospin violation, i.e., the mass $m_q = m_u = m_d$. It is worthy of mentioning that the effects of possible gluonic component of $\eta'$ meson will not be considered here since it is small in size [39, 43–45].

Likewise, by considering both $f_1$ mesons as the mixed quark flavor states, then this $f_1(1285) - f_1(1420)$ mixing can also be described as a $2 \times 2$ rotation matrix with a single angle $\phi_{f_1}$ in the quark-flavor basis [4], although there also have two mixing schemes for the $f_1(1285) - f_1(1420)$ mixing system [5, 8, 14, 37, 46, 47].

\[
\begin{pmatrix}
f_1(1285) \\
f_1(1420)
\end{pmatrix} = \begin{pmatrix}
\cos \phi_{f_1} & -\sin \phi_{f_1} \\
\sin \phi_{f_1} & \cos \phi_{f_1}
\end{pmatrix} \begin{pmatrix}
f_{1q} \\
f_{1s}
\end{pmatrix}.
\]

(34)
As discussed in Ref. [7], since the axial-vector mesons have the similar behavior as that of the vector ones, and the vector meson ρ and ω have the same distribution amplitudes, except for the different decay constants fρ and fω, we therefore assume that the f1q distribution amplitude is the same as a1(1260) with decay constant f1q = 0.193±0.043+0.023−0.026 GeV [48]. For the f1s state, for the sake of simplicity, we adopt the same distribution amplitude as flavor singlet f1(not to be confused with the abbreviation f1 of f1(1285) and f1(1420) mesons) state [7] but with decay constant f1s = 0.230 ± 0.009 GeV [48]. For the masses of two f1q and f1s states, we adopt mTf1q ∼ mTf1(1285) and mTf1s ∼ mTf1(1420) for convenience. Another more important factor is the value of the mixing angle φf1, which is less constrained from the experiments currently. Up to now, we just have some limited information on φf1 still in controversy at the theoretical and experimental aspects: (1) (15.8±5.0)° provided by Mark-II detector at SLAC from the ratio of \( \frac{\Gamma(f_1(1285)\to \gamma\pi^0)}{\Gamma(f_1(1420)\to \gamma\pi^0)} \) [49]; (2) (15.8±4.5)° extracted from the radiative f1(1285) → φγ and ργ decays [46] phenomenologically; (3) (27 ± 2)° from the updated LQCD calculations [50]; and (4) (24.0±3.2)° measured first from the B_d→J/ψf1(1285) decays by LHCB collaboration [4] very recently. In view of the good consistency for the values of φf1 between the latest measurements in B meson decays and the latest calculations in LQCD, we will adopt φf1 = 24° as input in the numerical evaluations.

Thus, by combining various contributions from different diagrams, the total decay amplitudes for twenty charmless hadronic B → f1P decays in the pQCD approach can be written as

1. \( B^+ → f_1(1285)(\pi^+, K^+) \) decays

\[
A(B^+ → f_1(1285)\pi^+) = \left\{ a_1(f_F f_e + f_B F_f) + [a_2]f_{f_{1q}} F_{f_e} + [C_1](M_{nfa} + M_{nfa} M_{nfa}) \right. \\
+ [C_2] \{ \lambda_4^{\phi} \{ a_4 + a_{10} \} \} \{ f_{f_{1q}} F_{f_e} + f_B F_f \} + [a_6 + a_8] \\
\left. \right\} \right. \\
\left\{ \right. \\
\left. \right\} \right.
\]

\[
A(B^+ → f_1(1285)K^+) = \lambda_4^{\phi} \left\{ a_1 \left( f_K F_{f_e} + f_B F_{f_a} \right) \right. \\
+ [a_2]f_{f_{1q}} F_{f_e} \left. \right\} \right. \\
\left\{ \right. \\
\left. \right\} \right.
\]

\[
A(B^+ → f_1(1285)(\pi^+, K^+) \) decays

\[
A(B^+ → f_1(1285)\pi^+) = \left\{ a_1 \left( f_K F_{f_e} + f_B F_{f_a} \right) K_{f_{1q}} + f_B F_{f_2} K_{f_{1q}} \right. \\
+ \left[ a_2 \right] f_{f_{1q}} F_{f_e} K_{f_{1q}} + \left. \right\} \right. \\
\left\{ \right. \\
\left. \right\} \right.
\]

\[
A(B^+ → f_1(1285)K^+) = \lambda_4^{\phi} \left\{ a_1 \left( f_K F_{f_e} + f_B F_{f_a} \right) K_{f_{1q}} + f_B F_{f_2} K_{f_{1q}} \right. \\
+ \left[ a_2 \right] f_{f_{1q}} F_{f_e} K_{f_{1q}} + \left. \right\} \right. \\
\left\{ \right. \\
\left. \right\} \right.
\]
2. $B^0_d \to f_1(1285)(\pi^0, \eta, \eta')$ decays

$$\sqrt{2} \mathcal{A}(B^0_d \to f_1(1285)\pi^0) = \left\{ [a_2](f_{\pi F_{fe}} + f_B F_{fa} + f_B F'_{fa} - f_{f_{1}} F'_{fa}) + [C_2](M_{nfa} + M_{nfa} + M'_{nfa} - M'_{nfa}) \right\}$$

\begin{align*}
&\cdot \lambda^d_{f_{1}} \zeta_{f_{1}} - \lambda^f_{f_{1}} \zeta_{f_{1}} \left\{ [-a_4 - \frac{1}{2}(3a_7 - 3a_9 - a_{10})]f_{\pi} F_{fe} + [-a_4 + \frac{1}{2}(3a_7 + 3a_9 + a_{10})]f_{f_{1}} F'_{fa} \right. \\
&\cdot (f_B F_{fa} + f_B F'_{fa}) - [2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})]f_{f_{1}} F'_{fa} - [a_6 - \frac{1}{2}a_8](f_{\pi} F_{fe} + f_B F_{fa} + f_B F'_{fa} + f_B F'_{fa}) + [C_3 - \frac{1}{2}C_7]
\end{align*}

\begin{align*}
&\cdot \left( M_{nfa} + M_{nfa} \right) \zeta_{f_{1}} + M'_{nfa} \zeta_{f_{1}} + [C_5 - \frac{1}{2}C_7](M_{nfa} + M'_{nfa} \zeta_{f_{1}} + M'_{nfa} \zeta_{f_{1}}) \\
&\cdot \left( [2a_3 - 2a_5 - \frac{1}{2}(a_7 - a_9)]f_{f_{1}} F'_{fa} + [2C_4 + \frac{1}{2}C_7]M_{nfa} + [2C_6 + \frac{1}{2}C_8]M_{nfa} \right) \zeta_{f_{1}} \\
&\cdot \left( [a_3 + a_4 - a_5 + \frac{1}{2}(a_7 - a_9 + a_{10})]f_{f_{1}} F'_{fa} + [C_3 + a_4 - \frac{1}{2}(a_7 + a_9)]f_{f_{1}} F'_{fa} \right)
\right\} \zeta_{f_{1}}, (37)

\begin{align*}
\mathcal{A}(B^0_d \to f_1(1285)K^0) = \lambda^d_{f_{1}} \left\{ [a_2](f_{\eta K_{fe}} + f_B F_{fa} + f_B F'_{fa} + f_{f_{1}} F'_{fa}) + [C_2](M_{nfa} + M_{nfa} + M'_{nfa} + M'_{nfa}) \right\} \\
&\cdot \zeta_{f_{1}} - \zeta_{f_{1}} \left\{ [2a_3 + a_4 - a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})]f_{\eta K_{fe}} + f_{f_{1}} F'_{fa} + [2a_3 + a_4 + 2a_5 + \frac{1}{2}(a_7 + a_9)]f_{\eta K_{fa}} + f_{f_{1}} F'_{fa} + f_B F'_{fa} + f_B F'_{fa} \right. \\
&\cdot [C_3 + 2C_4 - \frac{1}{2}(C_9 - C_{10})](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) + [C_5 - \frac{1}{2}C_7](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) \\
&\cdot M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa} + [2C_6 + \frac{1}{2}C_8](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) \right\} \cdot \zeta_{f_{1}} - \zeta_{f_{1}} \\
&\cdot \left( [a_3 + a_4 - a_5 + \frac{1}{2}(a_7 - a_9)]f_{\eta K_{fa}} + f_{f_{1}} F'_{fa} + f_B F'_{fa} + f_B F'_{fa} + f_B F'_{fa} + f_B F'_{fa} \right) \\
&\cdot [C_6 + \frac{1}{2}C_8](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) \zeta_{f_{1}}, (38)
\end{align*}

\begin{align*}
\mathcal{A}(B^0_d \to f_1(1285)\eta) = \lambda^d_{f_{1}} \left\{ [a_2](f_{\eta_{1} F_{fe}} + f_B F_{fa} + f_B F'_{fa} + f_{f_{1}} F'_{fa}) + [C_2](M_{nfa} + M_{nfa} + M'_{nfa} + M'_{nfa}) \right\} \\
&\cdot \zeta_{f_{1}} - \zeta_{f_{1}} \left\{ [2a_3 + a_4 - a_5 - \frac{1}{2}(a_7 - a_9 + a_{10})]f_{\eta_{1} F_{fe}} + f_{f_{1}} F'_{fa} + [2a_3 + a_4 + 2a_5 + \frac{1}{2}(a_7 + a_9)]f_{\eta_{1} F_{fa}} + f_{f_{1}} F'_{fa} + f_B F'_{fa} + f_B F'_{fa} \right. \\
&\cdot [C_3 + 2C_4 - \frac{1}{2}(C_9 - C_{10})](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) + [C_5 - \frac{1}{2}C_7](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) \\
&\cdot M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa} + [2C_6 + \frac{1}{2}C_8](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) \right\} \cdot \zeta_{f_{1}} - \zeta_{f_{1}} \\
&\cdot \left( [a_3 + a_4 - a_5 + \frac{1}{2}(a_7 - a_9)]f_{\eta_{1} F_{fa}} + f_{f_{1}} F'_{fa} + f_B F'_{fa} + f_B F'_{fa} + f_B F'_{fa} + f_B F'_{fa} \right) \\
&\cdot [C_6 + \frac{1}{2}C_8](M_{nfa} + M'_{nfa} + M_{nfa} + M'_{nfa}) \zeta_{f_{1}}, (39)
\end{align*}
\[ A(B_d^0 \rightarrow f_1(1285)\eta') = \left\{ [a_2](f_{\eta}F_{\eta} + f_BF_{fa} + f_BF'_{fa}) + f_{f_{1A}}(M_{nfa} + M_{nfa}') + [C_2](M_{nfa} + M_{nfa} + M_{nfa}') \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ [2a_3 + a_4 - 2a_5 - \frac{1}{2}(a_7 - a_9 + a_10)](f_{\eta}F_{\eta} + f_{f_{1A}}F'_{fe}) + [2a_3 + a_4 + 2a_5 - \frac{1}{2}(a_7 - a_9 + a_10)](f_{\eta}F_{fa} + f_BF_{fa}) + [a_6 - \frac{1}{2}a_8](f_{\eta}F_{P2} + f_BF_{P2}) + f_BF_{P2} + [C_3 + 2C_4 - \frac{1}{2}(C_9 - C_10)](M_{nf} + M_{nf}' + M_{nf}') + [C_5 - \frac{1}{2}C_7] \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ [a_7 - a_9]f_{\eta}F_{fa} + f_BF_{fa} + \frac{1}{2}(a_7 + a_9)](f_{\eta}F_{fa} + f_BF_{fa}) \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ f_{\eta}F_{fa} + f_BF_{fa} \right\} + \frac{3}{2}(a_7 - a_9) + \frac{3}{2}(a_7 + a_9) + \frac{3}{2}C_10 \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ f_{\eta}F_{fa} + f_BF_{fa} \right\} + \frac{3}{2}(C_8) \left\{ M_{nf}F_{f_{1A}} + (M_{nf}'F_{f_{1A}}) \right\} \]

\[ (40) \]

3. \( B_s^0 \rightarrow f_1(1285)(\pi^0, K^0, \eta, \eta') \) decays

\[ \sqrt{2}A(B_s^0 \rightarrow f_1(1285)\pi^0) = \left\{ [a_2](f_{\pi}F_{\pi} + f_BF_{fa} + f_BF'_{fa}) + [C_2](M_{nf}F_{f_{1A}} + (M_{nf} + M_{nf}')) \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ [a_4 - \frac{1}{2}a_{10}]f_{f_{1A}}F'_{P2} + f_BF'_{fa} \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ [a_6 - \frac{1}{2}a_8](f_{f_{1A}}F'_{P2} + f_BF_{P2}F_{f_{1A}}) \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ [a_7 - a_9]f_{f_{1A}}F'_{P2} + f_BF_{P2}F_{f_{1A}} \right\} \]

\[
\cdot \lambda^d \cdot \zeta f_{1A} \cdot \zeta f_{1A} \lambda^d \left\{ [2a_3 + a_4 + 2a_5 - \frac{1}{2}(a_7 - a_9 + a_10)]f_{f_{1A}}F'_{P2} + f_BF_{P2}F_{f_{1A}} \right\} \]

\[ (41) \]
\[ A(B_s^0 \rightarrow f_1(1285)\eta) = \chi_s^2 \left\{ \zeta_{f_{1s}} \cdot \zeta_{\eta_s} \left[ (a_2) f_{\eta \eta} f_{f_{1s}} + [C_2] M_{n_{f_{1s}}} + \zeta_{\eta_s} \cdot \zeta_{f_{1s}} \left[ (a_2) f_{f_{1s}} f_{f_{1s}'} + [C_2] M_{n_{f_{1s}}}' \right] + \zeta_{\eta_s} \cdot \zeta_{f_{1s}} \right] \right\} - \chi_s^1 \left\{ \zeta_{\eta_s} \cdot \zeta_{f_{1s}} \left( (a_3 + a_4 - a_5 + \frac{1}{2} \right) \right\} \]

\[ \cdot \left( a_7 - a_9 - a_{10} \right) (f_{\eta \eta} f_{f_{1s}} + f_{f_{1s}} f_{f_{1s}'} + (a_6 - \frac{1}{2} a_8) (f_{\eta \eta} f_{f_{1s}} + f_{f_{1s}} f_{f_{1s}'} + f_{f_{1s}} f_{f_{1s}''}) + (C_3 + C_4 - \frac{1}{2} (C_9 + C_{10})) (M_{n_{f_{1s}}} + M_{n_{f_{1s}}}') + (C_5 - \frac{1}{2} C_7) (M_{n_{f_{1s}}} + M_{n_{f_{1s}}}') + M_{n_{f_{1s}}} + M_{n_{f_{1s}}}') + M_{n_{f_{1s}}} + M_{n_{f_{1s}}}' \right) + (a_3 + a_4 + a_5 - \frac{1}{2} \right) \right\} + \zeta_{\eta_s} \cdot \zeta_{f_{1s}} \left( (2C_4 + \frac{1}{2} C_{10}) (M_{n_{f_{1s}}} + M_{n_{f_{1s}}}') \right) + \zeta_{\eta_s} \cdot \zeta_{f_{1s}} \left( (2a_3 - 2a_5 - \frac{1}{2} (a_7 - a_9)) f_{f_{1s}} f_{f_{1s}'} + (2C_4 + \frac{1}{2} C_{10}) M_{n_{f_{1s}}} + (2C_6 + \frac{1}{2} C_8) M_{n_{f_{1s}}}' \right) \right\}; \quad (44) \]

where \( \chi_s^{d(s)} = V_{ub}^* V_{ud}^{(s)} \) and \( \chi_s^{d(s)} = V_{ub}^* V_{cd}^{(s)} \), \( \zeta_{f_{1s}} = \cos \phi_{f_{1s}} / \sqrt{2} \) and \( \zeta_{f_{1s}} = -\sin \phi_{f_{1s}} \) and \( \zeta_{\eta_s} = \cos \phi / \sqrt{2} \) and \( \zeta_{\eta_s} = -\sin \phi \), and \( \zeta_{f_{1s}} = \sin \phi / \sqrt{2} \) and \( \zeta_{f_{1s}} = \cos \phi \). When we make the replacements with \( \zeta_{f_{1s}} \rightarrow \zeta_{f_{1s}}' \sim \sin \phi_{f_{1s}} / \sqrt{2} \) and \( \zeta_{f_{1s}} \rightarrow \zeta_{f_{1s}}' \sim \cos \phi_{f_{1s}} \) in the above equations, i.e., Eqs. (35)-(44), the decay amplitudes of \( B \rightarrow f_1(1420) P \) modes will be easily obtained.

### III. NUMERICAL RESULTS AND DISCUSSIONS

In this section, we will present the theoretical predictions on the CP-averaged branching ratios and CP-violating asymmetries for the considered twenty \( B \rightarrow f_1 P \) decay modes in the pQCD approach. In numerical calculations, central values of the input parameters will be used implicitly unless otherwise stated. The relevant QCD scale (GeV), masses (GeV), and \( B \) meson
lifetime( ps) are the following \[10, 11, 37, 51\]

\[
\Lambda^{(f=4)}_{\text{MS}} = 0.250, \quad m_W = 80.41, \quad m_B = 5.28, \quad m_{B_s} = 5.37, \quad m_b = 4.8; \quad f_\pi = 0.13, \quad f_K = 0.16, \quad m_{f_1(1285)} = 1.2812, \quad m_{f_1(1420)} = 1.4264; \quad m_0^i = 1.4, \quad m_0^K = 1.6, \quad m_0^b = 1.08, \quad m_0^0 = 1.92, \quad f_{f_1 = 24.0^\circ}; \quad \tau_{B_s} = 1.641, \quad \tau_{B_d} = 1.519, \quad \tau_{B_s} = 1.497. \tag{45}
\]

For the CKM matrix elements, we adopt the Wolfenstein parametrization and the updated parameters \(A = 0.811, \lambda = 0.22535, \bar{\rho} = 0.131^{+0.026}_{-0.013}, \) and \(\bar{\eta} = 0.345^{+0.013}_{-0.014} \[51\].

### A. CP-averaged branching ratios of \(B \to f_1P\) decays in the pQCD approach

For the considered \(B \to f_1P\) decays, the decay rate can be written as

\[
\Gamma = \frac{G_F m_B^3}{32\pi} (1 - r_f^2)|A(B \to f_1 P)|^2, \tag{46}
\]

where the corresponding decay amplitudes \(A\) have been given explicitly in Eqs. \((35)-(44)\). Using the decay amplitudes obtained in last section, it is straightforward to calculate the CP-averaged branching ratios with uncertainties for the considered decay modes in the pQCD approach. The pQCD predictions for the CP-averaged branching ratios of the considered \(B \to f_1P\) decays have been collected in Tables I and II. Based on these numerical results, some phenomenological discussions are given in order:

#### TABLE I. The CP-averaged branching ratios for \(B^+ \to f_1(\pi^+, K^+)\) decays in the pQCD approach.

| Channels                        | CP-averaged branching ratios   |
|---------------------------------|--------------------------------|
| \(B^+ \to f_1(1285)\pi^+\)     | \(4.0^{+1.1}_{-0.8}(\omega_b)^{+1.8}_{-1.4}(f_{f_1})^{-0.2}_{+0.1}(a_1^M)^{+0.2}_{-0.3}(\phi_{f_1})^{-0.1}_{+0.2}(a_t) \times 10^{-6}\) |
| \(B^+ \to f_1(1420)\pi^+\)     | \(7.4^{+2.0}_{-1.5}(\omega_b)^{-0.2}_{+0.6}(f_{f_1})^{-0.2}_{+0.4}(a_1^M)^{-0.1}_{+0.2}(\phi_{f_1})^{-0.2}_{+0.1}(a_t) \times 10^{-7}\) |
| \(B^+ \to f_1(1285)K^+\)      | \(1.6^{+0.4}_{-0.3}(\omega_b)^{-0.2}_{+0.8}(f_{f_1})^{-0.1}_{+0.1}(a_1^M)^{-0.2}_{+0.3}(\phi_{f_1})^{-0.1}_{+0.2}(a_t) \times 10^{-6}\) |
| \(B^+ \to f_1(1420)K^+\)      | \(5.1^{+1.0}_{-0.8}(\omega_b)^{-0.3}_{+0.7}(f_{f_1})^{-1.4}_{+0.4}(a_1^M)^{-0.3}_{+0.7}(\phi_{f_1})^{-0.7}_{+0.1}(a_t) \times 10^{-6}\) |

(1) The theoretical errors of these predictions in the pQCD approach are induced mainly by the uncertainties of the shape parameters \(\omega_b = 0.40 \pm 0.04(\omega_b = 0.50 \pm 0.05)\) GeV for the \(B_{u,d}(B_s)\) meson wave function, of the combined \(f_{f_1}\) from the axial-vector \(f_{1q(s)}\) state decay constant \(f_{1q(s)} = 0.193^{+0.042}_{-0.038}(f_{1q(s)} = 0.230 \pm 0.009)\) GeV of the combined Gegenbauer moments \(a_i^M\) from \(a_i^{\perp}(i = 1, 2)\) for the axial-vector \(f_{1q(s)}\) states in the longitudinal polarization and \(a_i^{P}(i = 1, 2)\) for the pseudoscalar \(P\) meson, and of the mixing angle \(\phi_{f_1} = (24.0^{+3.2}_{-2.7})^\circ\), respectively. We also investigate the higher order contributions simply through exploring the variation of the hard scale \(t_{\text{max}}\), i.e., from 0.87 to 1.27 (not changing \(1/b_i, i = 1, 2, 3\), in the hard kernel, which have been counted into one of the source of theoretical uncertainties. Note that very small effects induced by the variation of the CKM parameters appear in the CP-averaged branching ratios of these considered \(B \to f_1P\) decays and thus have been safely neglected.

(2) The considered \(B \to f_1P\) decays can be classified into two kinds of transitions, i.e., \(b \to d(\Delta S = 0)\) and \(b \to s(\Delta S = 1)\), respectively. The former transition includes ten \(B_{u,d} \to f_1(\pi, \eta, \eta')\) and \(B_s \to f_1K^0\) modes, while the latter transition contains ten \(B_{u,d} \to f_1K\) and \(B_s \to f_1(\pi^0, \eta, \eta')\) channels.

(a) For \(\Delta S = 0\) decays, it is found that most of the branching ratios are in the order of \(10^{-8} \sim 10^{-7}\) in the pQCD approach, except for the 

\(Br(B^+ \to f_1(1285)\pi^+) \approx 4.0^{+3.1}_{-2.4} \times 10^{-6}, \quad Br(B^+ \to f_1(1420)\pi^+) \approx 0.7^{+0.6}_{-0.5} \times 10^{-6}\); \tag{47}

which are around \(\mathcal{O}(10^{-6})\) within large errors, where various errors as specified previously have been added in quadrature. It is noted that these two \(B^+ \to f_1\pi^+\) decays are dominated by the color-allowed tree amplitudes, while the other eight \(B^0_{d/s} \to f_1(\pi^0, \eta, \eta')/K^0\) processes are basically penguin dominant with color-suppressed tree contributions. In particular, for the \(B^0_{d/s} \to f_1K^0\) channels, the tree pollution is such tiny that can be neglected safely for the predictions of the CP-averaged branching ratios.
(b) For $\Delta S = 1$ decays, contrary to the $\Delta S = 0$ ones, it is observed that most of the branching ratios are in the order of $10^{-6} \sim 10^{-5}$ in the pQCD approach, apart from the $B_s^0 \to f_1 \pi^0$ channels with the decay rates as

$$Br(B_s^0 \to f_1(1285)\pi^0) \approx 2.7^{+2.0}_{-1.6} \times 10^{-8} , \quad Br(B_s^0 \to f_1(1420)\pi^0) \approx 1.4^{+1.0}_{-0.7} \times 10^{-7} ;$$

in which the theoretical errors from the input parameters have also been added in quadrature. In contrast to the above case, it is worthwhile to stress that all of the $b \to s$ transition processes are determined by the penguin contributions dramatically just with generally very small tree contaminations.

The relation of the CP-averaged branching ratios between these two $\Delta S = 0$ and $\Delta S = 1$ transitions can be understood naively through the involved CKM hierarchy [3], apart from the the interferences between $f_1 P$ and $f_1 sP$ states: $|\lambda_f^B| : |\lambda_f^D| : |\lambda_f^u| \sim 0.09 : 0.02 : 0.22 : 1$, which means that when the decays are dominated by the penguin contributions, then we must observe at least one order difference as roughly anticipated because of the value around 21 of $|\lambda_f^B/\lambda_f^D|^2$. It is known that the $B_d^0 \to K^+K^-$ with decay rate $1.3 \pm 0.5 \times 10^{-7}$ and the $B_d^0 \to \pi^+\pi^-$ with branching ratio $7.6 \pm 1.9 \times 10^{-7}$ have been detected by the experiments [3]. Therefore, the decay modes with the branching ratios in the order of $10^{-6}$ and larger are generally expected to be accessed more easily at the running LHCb and forthcoming Super-B factory experiments in the near future.

(3) By careful analysis on the decay amplitudes, it is found that the $B^+ \to f_1 \pi^+ (\Delta S = 0)$ decays are almost dominated by the contributions from factorizable emission diagrams. Moreover, based on Eqs. (34), (35) and (37), and the numerical results of the branching ratios in Table I, one can straightforwardly see the constructive/destructive effects to the $B_{u,d} \to f_1(1285)[f_1(1420)]\pi$ decays.

Theoretically, these four decays have also been studied in the QCDF\(^3\), and the numerical results can be read as (in units of $10^{-6}$) [14],

$$Br(B^+ \to f_1(1285)\pi^+) = \begin{cases} 5.2^{+1.5}_{-1.0} \pm 1.3 \times 10^{-6}, & Br(B^0 \to f_1(1285)\pi^0) = \begin{cases} 0.26^{+0.32}_{-0.11}, & 0.20^{+0.27}_{-0.09} \end{cases} \\ 4.6^{+1.3}_{-0.9} \pm 0.5 \times 10^{-6}, & Br(B^+ \to f_1(1420)\pi^+) = \begin{cases} 0.003^{+0.005}_{-0.003}, & 0.05^{+0.05}_{-0.03} \end{cases} \\ 0.06^{+0.01}_{-0.00} \pm 0.09 \times 10^{-6}, & Br(B^0 \to f_1(1420)\pi^0) = \begin{cases} 0.003^{+0.005}_{-0.003}, & 0.05^{+0.05}_{-0.03} \end{cases} \end{cases} \; (49)$$

Note that the predictions of the branching ratios for $B_{u,d} \to f_1\pi$ decays in the QCDF correspond to two different sets of $\theta_{3P_1}$ in the flavor singlet-octet basis, i.e., $27.0^\circ$(first entry) and $53.2^\circ$(second entry). One can easily find the good agreement between the pQCD predictions with $\phi_{f_1} \sim 24^\circ$ and the QCDF predictions with $\theta_{3P_1} \sim 53.2^\circ$ for the $B_{u,d} \to f_1\pi$ decays within errors.

According to Ref. [47], the mixing of $f_1(1285) - f_1(1420)$ system in the singlet-octet and quark-flavor bases can be written as the following form,

$$\left( \begin{array}{c} |f_1(1285)\rangle \\ |f_1(1420)\rangle \end{array} \right) = \left( \begin{array}{c} \cos \theta_{3P_1} \sin \theta_{3P_1} \\ -\sin \theta_{3P_1} \cos \theta_{3P_1} \end{array} \right) \left( \begin{array}{c} |f_1^+\rangle \\ |f_1^0\rangle \end{array} \right) = \left( \begin{array}{c} \cos \alpha_{3P_1} \sin \alpha_{3P_1} \\ -\sin \alpha_{3P_1} \cos \alpha_{3P_1} \end{array} \right) \left( \begin{array}{c} |f_1(1285)\rangle \\ |f_1(1420)\rangle \end{array} \right) \; (51)$$

\[\text{TABLE II. Same as Table I but for } B^0_{d/s} \to f_1(\pi^0, K^0, \eta, \eta') \text{ decays.}\]

| Channels | CP-averaged branching ratios | Channels | CP-averaged branching ratios |
|----------|-----------------------------|----------|-----------------------------|
| $B_d^0 \to f_1(1285)\pi^0$ | $1.4^{+0.4}_{-0.2} \times 10^{-7}$ | $B^0 \to f_1(1285)\pi^0$ | $2.5^{+0.9}_{-0.7} \times 10^{-8}$ |
| $B_d^0 \to f_1(1420)\pi^0$ | $1.1^{+0.6}_{-0.4} \times 10^{-7}$ | $B_d^0 \to f_1(1285)\pi^0$ | $1.0^{+0.5}_{-0.4} \times 10^{-7}$ |
| $B_d^0 \to f_1(1285)K^0$ | $1.8^{+1.3}_{-1.1} \times 10^{-6}$ | $B^0 \to f_1(1420)\pi^0$ | $2.5^{+0.9}_{-0.7} \times 10^{-8}$ |
| $B_d^0 \to f_1(1420)K^0$ | $4.8^{+1.0}_{-0.9} \times 10^{-6}$ | $B_d^0 \to f_1(1285)\pi^0$ | $1.0^{+0.5}_{-0.4} \times 10^{-7}$ |
| $B_d^0 \to f_1(1285)\eta$ | $1.7^{+0.9}_{-0.6} \times 10^{-7}$ | $B^0 \to f_1(1420)\pi^0$ | $7.4^{+2.7}_{-1.8} \times 10^{-8}$ |
| $B_d^0 \to f_1(1420)\eta$ | $3.3^{+2.1}_{-1.2} \times 10^{-8}$ | $B_d^0 \to f_1(1285)\pi^0$ | $4.8^{+2.0}_{-1.6} \times 10^{-8}$ |
| $B_d^0 \to f_1(1285)\eta'$ | $5.0^{+0.6}_{-0.6} \times 10^{-8}$ | $B^0 \to f_1(1420)\pi^0$ | $5.9^{+2.0}_{-1.6} \times 10^{-8}$ |

\(^3\text{As stressed in the Introduction, the branching ratios of the } B \to f_1 P \text{ decays given in the naive factorization are very crude. Thus we will only compare our predictions with that obtained in the QCDF theoretically.}\)
where $f_1$ and $f_8$ are the flavor singlet and flavor octet, respectively, and the mixing angle $\alpha_{3p_1}$ in the quark-flavor basis satisfies the relation $\alpha_{3p_1} = 35.3^\circ - \theta_{p_1}$ and measures the deviation from ideal mixing. Then the $\alpha_{3p_1} \sim -17.9^\circ$ can be derived from second entry in the QCDF, which thus leads to the same mixing form as that adopted in this work, i.e., Eq. (34) while with a positive value of mixing angle.

Furthermore, a reasonable deduction obtained more naturally is that the $f_1(1285)[f_1(1420)]$ is basically determined by the component $f_{1u}[f_{1s}]$ based on the following ratios (central values) between the branching ratios of $B^+ \to f_1 \pi^+$ decays in the pQCD and QCDF approaches,

\[
R_{f_1(1285)\pi} = \frac{Br(B^+ \to f_1(1285)\pi^+)}{Br(B^+ \to f_1(1285)\pi^+)_pQCD} \approx 1.15 \sim \frac{|\cos \alpha_{3p_1}|^2}{\cos \phi_{f_1}} \approx 1.09 , \tag{52}
\]

\[
R_{f_1(1420)\pi} = \frac{Br(B^+ \to f_1(1420)\pi^+)}{Br(B^+ \to f_1(1420)\pi^+)_pQCD} \approx 0.80 \sim \frac{|\sin \alpha_{3p_1}|^2}{\sin \phi_{f_1}} \approx 0.58 ; \tag{53}
\]

Notice that the above relations cannot be easily deduced from the $B^0_d \to f_1 \pi$ modes. The underlying reason is that the former $B^+ \to f_1 \pi^+$ decays are with the dominant tree (color-allowed) contributions and negligible penguin pollution, while the latter $B^0_d \to f_1 \pi^0$ channels embrace the small tree (color-suppressed) and more important penguin contributions. The predictions of the CP-averaged branching ratios for $B_{u,d} \to f_1 \pi$ decays in the pQCD approach with the corresponding phenomenological discussions are expected to be tested by the near future experiments at LHC.

(4) For the penguin-dominated $B_{u,d} \to f_1 K$ decays, the destructive [constructive] interferences between $f_{1u} K$ and $f_{1s} K$ result in the approximately equal branching ratios for $B_{u,d} \to f_1(1285)[f_1(1420)]$ $K$ decays,

\[
Br(B^+ \to f_1(1285)K^+) = 1.6^{+2.2}_{-1.4} \times 10^{-6} \sim Br(B^0_d \to f_1(1285)K^0) = 1.8^{+2.5}_{-1.7} \times 10^{-6} , \tag{54}
\]

\[
Br(B^+ \to f_1(1420)K^+) = 5.1^{+3.1}_{-1.7} \times 10^{-6} \sim Br(B^0_d \to f_1(1420)K^0) = 4.8^{+4.1}_{-1.7} \times 10^{-6} , \tag{55}
\]

which indicate that the tree contributions are highly suppressed because of $|\lambda^u_1| : |\lambda^s_1| \sim 0.02$. Of course, it is worth stressing that, in terms of the central values of the decay rates, the color-allowed tree contributions (around 10%) of $B^+ \to f_1 K^+$ decays are larger than those color-suppressed ones (almost 0%) of $B^0_d \to f_1 K^0$ ones, though which are negligible relative to dominant penguin contributions in both sets of decay modes. The predictions on the branching ratios have also been presented in the framework of QCDF (in units of $10^{-6}$) [14],

\[
Br(B^+ \to f_1(1285)K^+) = 5.2^{+9.7}_{-10.1} , \quad Br(B^+ \to f_1(1420)K^+) = 13.8^{+18.4}_{-7.8} , \tag{56}
\]

\[
Br(B^0_d \to f_1(1285)K^0) = 5.2^{+4.5}_{-2.2} , \quad Br(B^0_d \to f_1(1420)K^0) = 13.1^{+17.5}_{-7.3} . \tag{57}
\]

In view of the better consistency observed from the $B_{u,d} \to f_1 \pi$ decays theoretically, we here only quote the second entry of the branching ratios for $B_{u,d} \to f_1 K$ decays in the QCDF for clarification. It can be seen that the theoretical predictions in both pQCD and QCDF approaches are basically consistent with each other within still large uncertainties. However, as far as the central values are considered, $Br(B_{u,d} \to f_1 K)_{QCDF}$ are a bit larger than $Br(B_{u,d} \to f_1 K)_{pQCD}$ with a factor near 3.

As mentioned in the Introduction, there are just the preliminary upper limits of branching ratios for the $B^+ \to f_1 K^+$ decays made by BABAR collaboration [13],

\[
Br(B^+ \to f_1(1285)K^+) < 2.0 \times 10^{-6} , \tag{58}
\]

and

\[
Br(B^+ \to f_1(1420)K^+) \cdot Br(f_1(1420) \to \bar{K}^*K) < 4.1 \times 10^{-6} , \tag{59}
\]

\[
Br(B^+ \to f_1(1420)K^+) \cdot Br(f_1(1420) \to \eta\pi\pi) < 2.9 \times 10^{-6} . \tag{60}
\]

We can find that the prediction for $Br(B^+ \to f_1(1285)K^+)$ in the pQCD approach is in good agreement with the preliminary upper limit, while that in the QCDF is barely consistent with the experimental limit within large theoretical errors. Because there are no any accurate values of the decay rates of $f_1(1420) \to \bar{K}^*K$ and $\eta\pi\pi$ modes currently, which consequently results in no available upper bound for $B^+ \to f_1(1420)K^+$ channel. But, it can be imagined that we can extract phenomenologically the information on the decay rates of $f_1(1420) \to \bar{K}^*K$ and $\eta\pi\pi$ decays once our predictions of $Br(B^+ \to f_1(1420)K^+) \sim O(10^{-6})$ could be confirmed by the measurements at LHCb and Super-B experiments in the near future. Of course, we need await enough data sample to test our theoretical predictions firstly.
In order to observe the dependence on the mixing angle $\phi_{f_1}$ of the branching ratios of $B^+ \rightarrow f_1K^+$, we simply examine the central values of the branching ratios in the pQCD approach as a function of $\phi_{f_1}$ in the range of $[0, 90]^\circ$, which can be seen in Fig. 2. One can observe that the $\phi_{f_1}$ dependence of the $B^+ \rightarrow f_1(1420)K^+$ mode is very opposite to that of the $B^+ \rightarrow f_1(1285)K^+$ directly from Fig. 2. Moreover, we also present the branching ratios of $B^+ \rightarrow f_1K^+$ decays in the pQCD approach with $\phi_{f_1} \sim 15^\circ$ and $20^\circ$ as a reference.

$$Br(B^+ \rightarrow f_1(1285)K^+) = 2.4^{+2.9}_{-1.9} \times 10^{-6}, \quad Br(B^+ \rightarrow f_1(1420)K^+) = 4.3^{+1.6}_{-1.4} \times 10^{-6}$$

with $\phi_{f_1} \sim 15^\circ$ and

$$Br(B^+ \rightarrow f_1(1285)K^+) = 1.9^{+2.6}_{-1.6} \times 10^{-6}, \quad Br(B^+ \rightarrow f_1(1420)K^+) = 4.8^{+1.7}_{-1.5} \times 10^{-6}$$

with $\phi_{f_1} \sim 20^\circ$. According to the brief review of the $f_1(1285) - f_1(1420)$ mixing in the last section, in terms of the central values of the currently existing mixing angle $\phi_{f_1}$ from both theoretical and experimental sides, one can find that the angle $\phi_{f_1}$ lies in the range of $[15^\circ, 27^\circ]$ [47]. Similarly, if the preliminary upper limit for the branching ratio of $B^+ \rightarrow f_1(1285)K^+$ mode could be considered as the central value of the experimental measurement, then we can find a rough constraint of the mixing angle $\phi_{f_1}$ through the numerical evaluations in the pQCD approach, i.e., $\phi_{f_1} \in [20^\circ, 27^\circ]$.

(5) The CP-averaged branching ratios of $B^0_d \rightarrow f_1(\eta, \eta')$ decays in the pQCD approach are presented in Table II. As a matter of fact, it is noted that these decays include two sets of destructive and/or constructive effects simultaneously due to $\eta - \eta'$ mixing and $f_1(1285) - f_1(1420)$ mixing. We find that $Br(B^0_d \rightarrow f_1(1285)\eta) \sim 5 \times Br(B^0_d \rightarrow f_1(1420)\eta)$ and $Br(B^0_d \rightarrow f_1(1285)\eta') \sim Br(B^0_d \rightarrow f_1(1420)\eta')$ within errors. While in terms of their central values of the branching ratios, we can easily find the constructive [destructive] interferences in $B^0_d \rightarrow f_1(1420)\eta$ decays and the slightly destructive [constructive] effects in $B^0_d \rightarrow f_1(1285)\eta'$ ones. And the similarly interesting phenomena can be found correspondingly in $B^0_d \rightarrow f_1(1285)\eta|\eta'|$ and $B^0_d \rightarrow f_1(1420)\eta|\eta'|$ decays. Because of the similar behavior in both vector and axial-vector mesons and this interesting pattern also occurring in the $B^0_d \rightarrow (\omega, \phi)(\eta, \eta')$ decays [43], it is reasonable to conjecture that the $f_1(1285)[f_1(1420)]$ is dominated by the $f_{1\eta}[f_{1\eta'}]$. However, all magnitudes of these four branching ratios are such small that the current experiments cannot observe them in a short period, which then have to be detected in the future.

(6) To our best knowledge, the $B^0 \rightarrow f_1P$ decays are studied for the first time in the pQCD approach and their estimations on the physical observables such as CP-averaged branching ratios and CP-violating asymmetries have been collected in the Tables II and IV.

(a) As shown in Table II, the CP-averaged branching ratios of $B^0 \rightarrow f_1(\pi^0, K^0)$ decays are very small around the order of $10^{-8} \sim 10^{-7}$ in the pQCD approach, which cannot be easily reached in the near future experiments. Relative to $B^0_d \rightarrow f_1K^0$ decays, the $B^0_s \rightarrow f_1K^0$ ones are also the penguin-dominated processes with dramatically small tree amplitudes through the $\Delta S = 0$ transitions. Due to the CKM hierarchy, the moduli of $\lambda^d_i$ is just about 22% of that of $\lambda^s_i$, which consequently leads to $Br(B^0_d \rightarrow f_1K^0) < Br(B^0_s \rightarrow f_1K^0)$ as naive expectations. Different from $B^0_d \rightarrow f_1\pi^0$ decays, the $B^0_s \rightarrow f_1\pi^0$ decays have no $B^0_d \rightarrow \pi^0$ transitions and are nearly determined by the
factorizable emission contributions via \(B_s^0 \to f_1 s\) transitions. Based on the Eq. (34), the coefficients \( - \sin \phi_{f_1} \) and \( \cos \phi_{f_1} \) can be found in the \( B_s^0 \to f_1(1285)\pi^0 \) and \( B_s^0 \to f_1(1420)\pi^0 \) decays, respectively, which thus result in the smaller (larger) branching ratio of the former (latter) mode with \( \sin^2(2\phi) \sim 0.17(\cos^2(2\phi) \sim 0.83) \). The similar[contrary] decay pattern between \( B_s^0 \to f_1 K^0 \) and \( B_s^0 \to f_1 K^0\pi^0 \) and \( B_s^0 \to f_1(1285)\) modes can also been seen from Table II clearly.

(b) The CP-averaged branching ratios of \( B_s^0 \to f_1(\eta,\eta') \) modes completely dominated by the penguin contributions in the pQCD approach are large in the order of \( 10^{-6} \sim 10^{-5} \), which are expected to be easily accessed by the ongoing LHCb and forthcoming Super-B experiments. Without the so-called tree contaminations, the central values of the decay rates of these four channels remain unchanged in the pQCD approach as presented in Table II. Similar to \( B_s^0 \to f_1(\eta,\eta') \) decays, the \( B_s^0 \to f_1(\eta,\eta') \) ones also embrace two sets of constructive and/or destructive interferences because of the \( \eta_0 - \eta_0 \) mixing and \( f_1_q - f_1_s \) mixing. But, in contrast to the decay pattern of \( B_s^0 \to f_1(\eta,\eta') \), as far as the central values are considered, we find the weakly constructive [destructive] effects to the \( B_s^0 \to f_1(1285)\eta' \) and \( B_s^0 \to f_1(1420)\eta' \) decays, while the strongly destructive [constructive] interferences in the \( B_s^0 \to f_1(1285)[f_1(1420)]\eta \) and \( B_s^0 \to f_1(1285)[f_1(1420)]\eta' \) ones. By considering the theoretical errors, we can obtain the relations \( Br(B_s^0 \to f_1(1285)\eta) \sim Br(B_s^0 \to f_1(1285)\eta') \sim O(10^{-6}) \) and \( Br(B_s^0 \to f_1(1420)\eta) \sim Br(B_s^0 \to f_1(1420)\eta') \sim O(10^{-5}) \) approximately. It is therefore of great interests to examine these \( B_s^0 \to f_1(\eta,\eta') \) decays with \( 10^{-6} \) even larger branching ratios and interesting phenomenologies at the experimental aspects.

(7) We also explore some ratios of the CP-averaged branching ratios of the considered \( B \to f_1 P \) decays in the pQCD approach. For simplicity, we just present the ratios of decay modes with large branching ratios. Therefore, the relevant ratios can be read as follows,

\[
R_1 \equiv \frac{Br(B^+ \to f_1(1420)\pi^+)}{Br(B^+ \to f_1(1285)\pi^+)} = 0.18^{+0.20}_{-0.16}, \quad (63)
\]

\[
R_2 \equiv \frac{Br(B^+ \to f_1(1285)K^+)}{Br(B^+ \to f_1(1420)K^+)} = 0.31^{+0.45}_{-0.29}, \quad (64)
\]

\[
R_3 \equiv \frac{Br(B_s^0 \to f_1(1285)K^0)}{Br(B_s^0 \to f_1(1420)K^0)} = 0.38^{+0.55}_{-0.38}, \quad (65)
\]

\[
R_4 \equiv \frac{Br(B_s^0 \to f_1(1285)\eta)}{Br(B_s^0 \to f_1(1420)\eta)} = 0.30^{+0.26}_{-0.21}, \quad (66)
\]

\[
R_5 \equiv \frac{Br(B_s^0 \to f_1(1285)\eta')}{Br(B_s^0 \to f_1(1420)\eta')} = 0.31^{+0.20}_{-0.15}. \quad (67)
\]

One can directly observe that the ratio \( R_1 \) from \( B^+ \to f_1 \pi^+ (\Delta S = 0) \) is very different from the other four similar ratios \( R_{2-5} \) from \( B_{u,d} \to f_1 K (\Delta S = 1) \) and \( B_s^0 \to f_1(\eta,\eta')(\Delta S = 1) \). The measurements of these ratios will be helpful to understand the mixing angle \( \phi_{f_1} \) of the \( f_1(1285) - f_1(1420) \) system effectively and further determine the definite components of both \( f_1 \) mesons.

(8) As mentioned in the Introduction, the contributions from weak annihilation diagrams play important roles in the heavy \( B \) meson decays, which are complex with sizable strong phase proposed by the pQCD approach and supported by the QCDF approach through fitting to the data, although the contrary viewpoint has been stated by soft-collinear effective theory. We will therefore analyze the annihilation contributions in these twenty \( B \to f_1 P \) decays. For the sake of simplicity, we here will only take the central values of the branching ratios in the pQCD approach for clarifications.

(a) For the \( \Delta S = 0 \) processes, when the weak annihilation contributions are neglected, then the branching ratios of the ten \( b \to d \) transitions can be presented as follows,

\[
Br(B^+ \to f_1(1285)\pi^+) = 3.9 \times 10^{-6}, \quad Br(B^+ \to f_1(1420)\pi^+) = 7.1 \times 10^{-7}; \quad (68)
\]

\[
Br(B_s^0 \to f_1(1285)\pi^0) = 1.2 \times 10^{-7}, \quad Br(B_s^0 \to f_1(1420)\pi^0) = 2.1 \times 10^{-9}; \quad (69)
\]

\[
Br(B_s^0 \to f_1(1285)\eta) = 8.7 \times 10^{-8}, \quad Br(B_s^0 \to f_1(1420)\eta) = 5.1 \times 10^{-9}; \quad (70)
\]

\[
Br(B_s^0 \to f_1(1285)\eta') = 4.3 \times 10^{-9}, \quad Br(B_s^0 \to f_1(1420)\eta') = 4.3 \times 10^{-9}; \quad (71)
\]

\[
Br(B_s^0 \to f_1(1285)K^0) = 5.5 \times 10^{-8}, \quad Br(B_s^0 \to f_1(1420)K^0) = 5.3 \times 10^{-7}; \quad (72)
\]

(b) For the \( \Delta S = 1 \) channels, when the weak annihilation contributions are turned off, then the decay rates of the ten
\[ b \to s \text{ transitions can be given as follows,} \]
\[
\begin{align*}
Br(B^+ \to f_1(1285)K^+) &= 1.8 \times 10^{-6}, & Br(B^+ \to f_1(1420)K^+) &= 3.5 \times 10^{-6}; \\
Br(B^0_d \to f_1(1285)K^0) &= 2.0 \times 10^{-6}, & Br(B^0_d \to f_1(1420)K^0) &= 3.5 \times 10^{-6}; \\
Br(B^0_s \to f_1(1285)\eta) &= 4.2 \times 10^{-6}, & Br(B^0_s \to f_1(1420)\eta) &= 1.2 \times 10^{-5}; \\
Br(B^0_s \to f_1(1285)\eta') &= 3.4 \times 10^{-6}, & Br(B^0_s \to f_1(1420)\eta') &= 7.8 \times 10^{-6}; \\
Br(B^0_d \to f_1(1285)\pi^0) &= 2.7 \times 10^{-8}, & Br(B^0_s \to f_1(1420)\pi^0) &= 1.3 \times 10^{-7}.
\end{align*}
\]

Compared with the values listed in Tables I and II, one can find that the decays such as \( B^+ \to f_1\pi^+, f_1(1285)K^+, B^0_d \to f_1(1285)(\pi^0, K^0, \eta), \) and \( B^0_s \to f_1(1285)(\pi^0, \eta, \eta') \) are not significantly sensitive to the weak annihilation contributions. However, it is important to note that the modes such as \( B_{u,d} \to f_1 (1420) K, B^0_d \to f_1(1285)\eta', f_1(1420)(\pi^0, \eta, \eta'), \) and \( B^0_s \to f_1(1285)K^0, f_1(1420)\eta' \) suffer from sizable annihilation effects, specifically, without the contributions from annihilation diagrams, the branching ratios decrease correspondingly by around 30\% for \( B_{u,d} \to f_1 (1420) K \) and \( B^0_s \to f_1(1420)\eta' \), 70\% \( \sim \) 90\% for \( B^0_d \to f_1\eta', f_1(1420)(\pi^0, \eta) \), and 26\% for \( B^0_s \to f_1(1285)K^0 \), respectively. Of course, the reliability of the contributions from the annihilation diagrams to these considered decays calculated in the pQCD approach will be carefully examined by the relevant experiments in the future.

9. Frankly speaking, as the most important inputs in the calculations of pQCD approach, the currently less constrained light-cone distribution amplitudes of the axial-vector \( f_1 \) mesons result in the theoretical predictions of the branching ratios for the considered twenty \( B \to f_1 P \) decays with relatively large uncertainties, which are expected to be greatly improved by the LQCD calculations and/or large numbers of related experiments in the future.

\[ \text{TABLE III. The direct CP violations} \]

\[ A_{\text{CP}}^{\text{dir}} \text{ for } B^+ \to f_1(\pi^+, K^+) \text{ decays in the pQCD approach. Apart from the last error induced by the variations of CKM parameters } \beta \text{ and } \bar{\eta}, \text{ the sources of the main uncertainties have been specified in the discussions of CP-averaged branching ratios.} \]

| Channels | direct CP violations(%) |
|----------|-------------------------|
| \( B^+ \to f_1(1285)\pi^+ \) | \( 18.3^{+2.0}_{-1.9}(\omega_1)^{+0.3}_{-0.4}(f_1)_{+3.5}_{-2.5}(\phi_{f_1})_{+3.2}_{-2.7}(a_t)_{+0.7}_{-0.7}(V_1) \) |
| \( B^+ \to f_1(1420)\pi^+ \) | \( 28.2^{+2.8}_{-2.6}(\omega_1)^{+1.8}_{-1.8}(f_1)_{+3.5}_{-2.5}(\phi_{f_1})_{+1.1}_{-1.0}(a_t)_{+0.7}_{-0.7}(V_1) \) |
| \( B^+ \to f_1(1285)K^+ \) | \( -21.2^{+1.9}_{-2.0}(\omega_1)^{+3.3}_{-1.9}(f_1)_{+12.8}_{-2.4}(a_t)_{+1.3}_{-1.3}(V_1) \) |
| \( B^+ \to f_1(1420)K^+ \) | \( -13.6^{+0.6}_{-0.5}(\omega_1)^{+3.3}_{-1.9}(f_1)_{+2.3}_{-2.0}(a_t)_{+0.5}_{-0.5}(V_1) \) |

Now we come to the evaluations of the CP-violating asymmetries of \( B \to f_1 P \) decays in the pQCD approach. For the charged \( B^+ \to f_1(\pi^+, K^+) \) decays, the direct CP violation \( A_{\text{CP}}^{\text{dir}} \) can be defined as,

\[ A_{\text{CP}}^{\text{dir}} = \frac{|\langle A_f e^{i\phi_f} \rangle|^2 - |\langle A_f \rangle|^2}{|\langle A_f \rangle|^2 + |\langle A_f e^{i\phi_f} \rangle|^2}, \]

where \( A_f \) stands for the decay amplitudes of \( B^+ \to f_1\pi^+ \) and \( B^+ \to f_1K^+ \), respectively, while \( \langle A_f \rangle \) denotes the charge conjugation \( B^- \to f_1\pi^- \) and \( B^- \to f_1K^- \) ones correspondingly. Using Eq. (78), the pQCD predictions for the direct CP-violating asymmetries of \( B^+ \to f_1(\pi^+, K^+) \) modes have been collected in Table III, in which we can easily find the large direct CP violations for the four charged \( B^+ \to f_1\pi^+ \) and \( f_1K^+ \) decays within errors as follows,

\[
\begin{align*}
A_{\text{CP}}^{\text{dir}}(B^+ \to f_1(1285)\pi^+) &= (18.3^{+4.8}_{-3.7})\% , & A_{\text{CP}}^{\text{dir}}(B^+ \to f_1(1420)\pi^+) &= (28.2^{+7.4}_{-6.0})\% ; \\
A_{\text{CP}}^{\text{dir}}(B^+ \to f_1(1285)K^+) &= (-21.2^{+13.8}_{-24.2})\% , & A_{\text{CP}}^{\text{dir}}(B^+ \to f_1(1420)K^+) &= (-13.6^{+3.1}_{-2.7})\% ;
\end{align*}
\]

where various errors from the variations of the input parameters have been added in quadrature. These large direct CP-violating asymmetries combined with the large CP-averaged branching ratios[O(10^{-6})] are believed to be clearly measurable at the LHCb and Super-B factory experiments.

As for the CP-violating asymmetries for the neutral \( B^0_{d(s)} \to f_1 P \) decays, the effects of \( B^0_{d(s)} - \bar{B}^0_{d(s)} \) mixing should be considered. The CP-violating asymmetries of \( B^0_{d(s)}(\bar{B}^0_{d(s)}) \to f_1(\pi^0, K^0, \eta, \eta') \) decays are time dependent and can be defined
\begin{align*}
A_{\text{CP}} &= \frac{\Gamma \left( B_d^{0 \to f_{\text{CP}}} \right) - \Gamma \left( B_d^{0 \to f_{\text{CP}}} \right)}{\Gamma \left( B_d^{0 \to f_{\text{CP}}} \right) + \Gamma \left( B_d^{0 \to f_{\text{CP}}} \right)} \\
&= A_{\text{CP}}^{\text{dir}} \cos(\Delta m_{d(s)} \Delta t) + A_{\text{CP}}^{\text{mix}} \sin(\Delta m_{d(s)} \Delta t),
\end{align*}

where $\Delta m_{d(s)}$ is the mass difference between the two $B_d^{0}$ mass eigenstates, $\Delta t = t_{\text{CP}} - t_{\text{tag}}$ is the time difference between the tagged $B_d^{0}$ mass eigenstates, and the accompanying $B_d^{0}$ mass eigenstates with opposite $b$ flavor decaying to the final CP-eigenstate $f_{\text{CP}}$ at the time $t_{\text{CP}}$. The direct and mixing induced CP-violating asymmetries $A_{\text{CP}}^{\text{dir}}(C_f)$ and $A_{\text{CP}}^{\text{mix}}(S_f)$ can be written as

\begin{align*}
A_{\text{CP}}^{\text{dir}} &= C_f = \frac{|\lambda_{\text{CP}}|^2 - 1}{1 + |\lambda_{\text{CP}}|^2}, \\
A_{\text{CP}}^{\text{mix}} &= S_f = \frac{2\text{Im}(\lambda_{\text{CP}})}{1 + |\lambda_{\text{CP}}|^2},
\end{align*}

with the CP-violating parameter $\lambda_{\text{CP}}$

\begin{equation}
\lambda_{\text{CP}} = \eta_f \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}} \frac{\langle f_{\text{CP}} | H_{\text{eff}} | B_d^{0} \rangle}{\langle f_{\text{CP}} | H_{\text{eff}} | B_d^{0} \rangle},
\end{equation}

where $\eta_f$ is the CP-eigenvalue of the final states. Moreover, for $B_d^{0}$ meson decays, a non-zero ratio $(\Delta \Gamma / \Gamma)_{B_d^{0}}$ is expected in the SM [52, 53]. For $B_d^{0} \to f_1(\pi^0, K^0, \eta, \eta')$ decays, the third term $A_{\Delta \Gamma}$ related to the presence of a non-negligible $\Delta \Gamma$ to

| Channels | CP asymmetries(%) | Channels | CP asymmetries(%) |
|----------|------------------|----------|------------------|
| $B_d^0 \to f_1(1285)\pi^0$ | $70.8^{+12.5}_{-17.0}$ | $B_d^0 \to f_1(1285)\eta$ | $-80.9^{+29.0}_{-15.4}$ |
| $-2.6^{+38.9}_{-32.1}$ | $-42.2^{+2.4}_{-43.9}$ | $-1.0^{+0.9}_{-1.6}$ | $\sim 100$ |
| $B_d^0 \to f_1(1285)\eta'$ | $-29.0^{+24.1}_{-24.6}$ | $B_d^0 \to f_1(1285)\eta'$ | $-2.5^{+0.9}_{-0.9}$ |
| $-44.2^{+17.6}_{-16.5}$ | $-1.5^{+0.5}_{-0.8}$ | $\sim 100$ | $\sim 100$ |
| $B_d^0 \to f_1(1420)\pi^0$ | $42.0^{+51.8}_{-78.6}$ | $B_d^0 \to f_1(1285)K^0$ | $26.0^{+34.9}_{-30.0}$ |
| $86.1^{+13.6}_{-48.8}$ | $70.0^{+3.3}_{-2.9}$ | $-70.9^{+36.3}_{-19.8}$ | $65.5^{+28.7}_{-41.0}$ |
| $B_d^0 \to f_1(1420)K^0$ | $2.3^{+2.5}_{-2.9}$ | $B_d^0 \to f_1(1420)K^0$ | $-2.9^{+3.7}_{-4.3}$ |
| $0.6^{+1.0}_{-0.5}$ | $69.9^{+2.8}_{-2.2}$ | $-67.9^{+6.1}_{-5.8}$ | $73.4^{+5.3}_{-6.0}$ |
| $B_d^0 \to f_1(1285)K^0$ | $26.0^{+34.9}_{-30.0}$ | $B_d^0 \to f_1(1420)\eta$ | $-1.4^{+0.4}_{-0.5}$ |
| $-93.6^{+20.8}_{-9.2}$ | $-13.3^{+51.8}_{-48.0}$ | $0.3^{+1.2}_{-0.9}$ | $\sim 100$ |
| $-13.3^{+51.8}_{-48.0}$ | $-1.0^{+0.9}_{-1.6}$ | $\sim 100$ | $\sim 100$ |
describe the CP violation can be defined as follows [53]:

\[
A_{\Delta \Gamma_v} = \frac{2 \text{Re}(\lambda_{\text{CP}})}{1 + |\lambda_{\text{CP}}|^2}.
\]  

(84)

The three quantities describing the CP violation in \(B_s^0\) meson decays shown in Eqs. (82) and (84) satisfy the following relation,

\[
|A_{\text{dir}}^4|^2 + |A_{\text{mix}}^\pi|^2 + |A_{\Delta \Gamma_v}|^2 = 1.
\]  

(85)

Then, with the decay amplitudes for \(B_{d(s)}^0 \to f_1(\pi^0, K_S^0(K_{S}^0), \eta, \eta')\) decays as shown in the last section and the definitions in the above Eqs. (82)-(84), the direct and mixing-induced CP-violating asymmetries have been calculated in the pQCD approach within large theoretical errors and displayed in Table IV. Some remarks are in order:

1. As observed clearly from Table IV, almost all of the \(b \to d\) transition processes have the large direct CP violations with still large uncertainties, while most of the \(b \to s\) transition ones get the very small direct CP asymmetries except for \(B_s^0 \to f_1 \pi^0\) modes.

2. The relation of \(A_{\text{CP}}^4(B_d^0 \to f_1(1285)K_S^0) \sim 4 \times A_{\text{CP}}^4(B_d^0 \to f_1(1420)K_S^0)\) can be found straightforwardly from Table IV. The underlying reason is with different contributions from tree diagrams because of the dominance of \(f_{1s}(f_{1q})\) component in \(f_1(1420)[f_1(1285)]\) in the current mixing form. The same explanation can also be counted for the relation \(A_{\text{CP}}^4(B^+ \to f_1 K^+) \gg A_{\text{CP}}^4(B_d^0 \to f_1 K_S^0)\) in magnitudes. Of course, as emphasized in the item on the discussions of the branching ratios of \(B^+ \to f_1 K^+\) and \(B_{d(s)}^0 \to f_1 K_{S}^0\) decays, the latter modes are more like purely penguin-dominated channels.

3. It is interesting to note that those decays associated with very small direct CP violations but with much large CP-averaged branching ratios are almost purely penguin-dominated modes, whose tree pollution are such tiny that the numerical values of the decay rates remain unchanged when just the penguin contributions are taken into account. Actually, these mentioned decays, i.e., \(B_s^0 \to f_1 K_S^0\) and \(B_d^0 \to f_1(\eta, \eta')\), are induced by the \(b \to s q \bar{q}\) mediated transitions with \(q = u, d, s\) at the quark level. For the latter modes, in principle, we can utilize the mixing-induced CP asymmetries to study the \(B_s^0 \to B_d^0\) mixing phase \(\phi_s\). Unfortunately, however, these predictions in the pQCD approach suffer from significantly large theoretical errors arising from the much less constrained hadronic parameters. Therefore, this issue have to be lefted for the future studies when the effective constraints are available from the experiments and/or nonperturbative techniques such as LQCD calculations. In the next subsection, we will analyze the \(B_d^0 - B_s^0\) mixing phase \(\phi_d\) explicitly through the \(B_d^0 \to f_1 K_S^0\) modes.

4. The third CP-asymmetric observables \(A_{\Delta \Gamma_v}\) for the \(B_s^0\) meson decays are also listed in the Table IV, in which we can find near 100% for most of the \(B_s^0\) decay modes within large errors, apart from \(B_s^0 \to f_1(1420)K_S^0\) channel around 70%. These interesting predictions in the pQCD approach and the resultant phenomenologies are expected to be examined by the highly precise measurements at the running LHCb and forthcoming Super-B experiments in the future.

C. Information on CKM weak phases \(\alpha, \beta\) and \(\gamma\) from \(B \to f_1 P\) decays

![FIG. 3. (Color online) Dependence on the CKM weak phase \(\alpha(\gamma)\) of central values of the CP-averaged branching ratios for \(B^+ \to f_1 \pi^+(K^+)\) decays in the pQCD approach. The red-solid[blue-dashed] line corresponds to the \(B^+ \to f_1(1285)[f_1(1420)]\pi^+\) decay and the magenta-dotted[gray-dotdashed] one corresponds to the \(B^+ \to f_1(1285)[f_1(1420)]K^+\) mode, respectively.](image)
It is of great interests to note that the $B^+ \to f_1 \pi^+$ decays are $b \to d(\Delta S = 0)$ transitions dominated by the tree diagrams while the $B^+ \to f_1 K^+$ decays are $b \to s(\Delta S = 1)$ ones determined by the penguin contributions. These unique properties exhibited in the $B^+ \to f_1(\pi^+, K^+)$ decays motivate us to further explore more useful information on the CKM weak phases $\alpha$ and $\gamma$ by employing the careful investigations on the large CP-averaged branching ratios and the large direct CP asymmetries of $B^+ \to f_1(\pi^+, K^+)$ decays in the pQCD approach.

We know that the decay amplitudes $A_f$ of $B^+ \to f_1(\pi^+, K^+)$ can be further written as the following forms,

$$A_f(B^+ \to f_1 \pi^+) = \lambda_u^d T - \lambda_t^d P = \lambda_u^d T(1 + r \exp[i(\alpha + \delta)])$$, \hspace{2cm} (86)

$$A_f(B^+ \to f_1 K^+) = \lambda_u^s T' - \lambda_t^s P' = \lambda_u^s T'(1 + r' \exp[i(\gamma' + \delta')])$$; \hspace{2cm} (87)

where $T(T')$ and $P(P')$ denote the tree and penguin decay amplitudes of $B^+ \to f_1 \pi^+(K^+)$ decays, and $r(r')$ and $\delta(\delta')$ represent the ratios of penguin to tree contributions $|\lambda^d_u|/|\lambda^d_t|$ and $|\lambda^s_u|/|\lambda^s_t|$ and the relative strong phases between the corresponding tree and penguin diagrams. The weak phase $\alpha$ come from the identity $\alpha = 180^\circ - \beta - \gamma$ with the definitions $V_{td} = |V_{td}| \exp(-i\beta)$ and $V_{ub} = |V_{ub}| \exp(-i\gamma)$, and the $\gamma'$ is defined as $\arg[-V_{us}^2 V_{ub}^* / V_{td}]$. Then the decay amplitudes $\bar{A}_f$ of the charge conjugated modes $B^- \to f_1(\pi^-, K^-)$ can be easily written as,

$$\bar{A}_f(B^- \to f_1 \pi^-) = (\lambda_u^d)^* T - (\lambda_t^d)^* P = (\lambda_u^d)^* T(1 + r \exp[-i(\alpha + \delta)])$$, \hspace{2cm} (88)

$$\bar{A}_f(B^- \to f_1 K^-) = (\lambda_u^s)^* T' - (\lambda_t^s)^* P' = (\lambda_u^s)^* T'(1 + r' \exp[-i(\gamma' + \delta')])$$; \hspace{2cm} (89)

Therefore, the CP-averaged branching ratios can be read as

$$Br(B^+ \to f_1 \pi^+) \equiv \frac{|\bar{A}_f|^2}{|A_f|^2} = \frac{\lambda_u^d T^2}{1 + 2r \cos \alpha \cos \delta + r^2}$$, \hspace{2cm} (90)

$$Br(B^+ \to f_1 K^+) \equiv \frac{|\bar{A}_f|^2}{|A_f|^2} = \frac{|\lambda_u^s T'|^2}{1 + 2r' \cos \gamma \cos \delta' + r'^2}$$; \hspace{2cm} (91)

in which $T(t)$, $r(r')$ and $\delta(\delta')$ are all perturbatively calculated in the pQCD approach, besides $\lambda_u^d,s$ are determined from the experiments. Thus, Eqs. (90) and (91) can provide a possible way to determine the CKM angles $\alpha$ and $\gamma$ potentially by measuring the branching ratios, respectively. In Fig. 3, we show the central values of the CP-averaged branching ratios for $B^+ \to f_1(1285)\pi^+$(red-solid line) and $B^+ \to f_1(1420)\pi^+$(blue-dashed line) $[B^+ \to f_1(1285)\bar{K}^+$(magenta-dotted line) and $B^+ \to f_1(1420)\bar{K}^+$ (gray-dotted line)] decays as a function of the CKM weak phase $\alpha(\gamma)$ in the pQCD approach. One can easily see the strong (weak) dependence on $\alpha$ for $B^+ \to f_1(1285)|f_1(1420)|\pi^+$ decay and the moderate dependence on $\gamma$ for $B^+ \to f_1 K^+$ decays in the pQCD approach from Fig. 3. One can also directly observe from Fig. 3 that the central values of the branching ratios for the considered decays in the pQCD approach correspond to the central values of $\alpha$ and $\gamma$ as around $89^\circ$ and $70^\circ$, respectively, which are consistent with the constraints from the various experiments [3].

More information on the CKM angles $\alpha$ and $\gamma$ can also be hinted from the large direct CP asymmetries of $B^+ \to f_1 \pi^+(K^+)$ decays in the pQCD approach. With Eqs. (86)-(89), the direct CP-violating asymmetry Eq. (78) for $B^+ \to f_1 \pi^+(K^+)$ can be described as the function of $\alpha(\gamma)$,

$$A_{CP}^{dir}(B^+ \to f_1 \pi^+) = \frac{2r \sin \alpha \sin \delta}{1 + 2r \cos \alpha \cos \delta + r^2}$$, \hspace{2cm} (92)

$$A_{CP}^{dir}(B^+ \to f_1 K^+) = \frac{2r' \sin \gamma \sin \delta'}{1 + 2r' \cos \gamma \cos \delta' + r'^2}$$, \hspace{2cm} (93)

Again, as aforementioned, the ratios $r(r')$ and the relative strong phases $\delta(\delta')$ can be explicitly calculated in the pQCD approach. Undoubtedly, the former Eq. (92) is a function of $\sin \alpha$ and $\cos \alpha$, and the latter Eq. (93) is a function of $\sin \gamma$ and $\cos \gamma$. In particular, if one mode like $B^+ \to f_1(1420)\pi^+$ is almost completely tree-dominated, i.e., $r \ll 1$, then Eq. (92) can be further written approximately as

$$A_{CP}^{dir}(B^+ \to f_1(1420)\pi^+) \sim 2r \sin \alpha \sin \delta$$, \hspace{2cm} (94)

Analogously, if one mode like $B^+ \to f_1(1420)\bar{K}^+$ is nearly pure penguin contributions, i.e., $r' \gg 1$, then Eq. (93) can be further described approximately as

$$A_{CP}^{dir}(B^+ \to f_1 K^+) \sim \frac{2}{r'} \sin \gamma \sin \delta'$$; \hspace{2cm} (95)

Thus, the large direct CP-violating asymmetries driven by these two equations, i.e., Eqs. (94) and (95), will give rise to the effective constraints more easier on the CKM phases $\alpha$ and $\gamma$ from the experimental data with high precision. Certainly, based on Eqs. (94) and (95), the large strong phases $\delta$ and $\delta'$ required by the large direct CP asymmetries can also be deduced naturally.
The central values of the large direct CP violations for the $B^+ \to f_1(1285)\pi^+$ (red-solid line) and $B^+ \to f_1(1420)\pi^+$ (blue-dashed line) $[B^+ \to f_1(1285)K^+$(magenta-dashed line) and $B^+ \to f_1(1420)K^+$(gray-dotdashed line)] decays as a function of the CKM weak phase $\alpha(\gamma)$ in the pQCD approach have also been shown in Fig. 4. One can find straightforwardly from Fig. 4 that $A_{\text{CP}}^{\text{mix}}(B^+ \to f_1\pi^+)$ are large and positive, while $A_{\text{CP}}^{\text{dir}}(B^+ \to f_1K^+)$ are large and negative, which are expected to be tested by the experiments in the near future.

It is important to note that the mixing-induced CP-violating asymmetries of the $B_d^0 \to f_1K_S^0$ decays are with the very small uncertainties as seen in Table IV clearly, which, as the alternative channels, are expected to have the supplementary power in reducing the errors of the CKM weak phase $\beta$. We can write the expression of the CP-violating parameter $\lambda_{\text{CP}}(f_1K_S^0)$ in an explicit form,

$$\lambda_{\text{CP}}(f_1K_S^0) = -\exp (-2i\beta) \frac{\lambda_6^s |T_{f_1K_S^0}| \exp (-i\gamma) - |\lambda_5^s| P_{f_1K_S^0}}{\lambda_6^s |T_{f_1K_S^0}| \exp (i\gamma) - |\lambda_5^s| P_{f_1K_S^0}} , \quad (96)$$

Here, $|\lambda_6^s| \sim 0.02 \cdot |\lambda_5^s|$ and $T_{f_1K_S^0}$ is the decay amplitude arising from the color-suppressed tree diagrams, which will consequently result in the negligible tree pollution relative to the much larger penguin contributions in the $B_d^0 \to f_1K_S^0$ decays, then $\lambda_{\text{CP}}(f_1K_S^0) \approx -\exp (-2i\beta)$, i.e., $A_{\text{CP}}^{\text{mix}} = S_f \sim \sin (2\beta_{\text{eff}})$. In principle, the results should be identical to those measuring the $S_f = -\frac{\eta_f}{16} \sin 2\beta$ from the tree-dominated $b \to c\bar{c}s$ transitions, such as the theoretically cleanest $B_d^0 \to J/\psi K_{S,L}^0$. However, the $b \to sq\bar{q}$ decays are potentially contaminated by the indeed existed tree pollution. Then the deviation between $S_{\text{penguin}}$ and $S_{\text{tree}}$ can be defined as $\Delta S \equiv S_{\text{penguin}} - S_{\text{tree}}$, which will be helpful to justify the discrepancies as promising new physics signals. Up to now, the world average value of the $S_{\text{tree}}$ at the experimental aspect is $[3]$}

$$\sin 2\beta = 0.682 \pm 0.019 , \quad (97)$$

Then our pQCD predictions of $\sin 2\beta_{\text{eff}}$ for the $B_d^0 \to f_1K_S^0$ decays deviate to the sin $2\beta$ as,

$$\Delta S_{f_1(1285)K_S^0} \approx 0.018^{+0.036}_{-0.035} , \quad \Delta S_{f_1(1420)K_S^0} \approx 0.017^{+0.033}_{-0.029} . \quad (98)$$

which are well below the bound at most $O(0.1)$ [54] and can be confronted with the stringent tests by the future experiments.

IV. CONCLUSIONS AND SUMMARY

In this work, we have studied the CP-averaged branching ratios and the CP-violating asymmetries of twenty charmless hadronic $B \to f_1P$ decays within the framework of pQCD approach. We explicitly evaluated the nonfactorizable spectator and annihilation types of diagrams, except for the traditional factorizable emission ones. Based on the quark-flavor mixing of $f_1(1285) - f_1(1420)$ system with the angle $\phi_{f_1} \sim 24^o$ extracted first from the $B$ meson decays, we calculated the numerical results for the considered physical observables and made the phenomenological discussions correspondingly. The main conclusions of the present paper are as follows:

(1) For the four charged $B^+ \to f_1\pi^+(\Delta S = 0)$ and $f_1K^+(\Delta S = 1)$ decays, the large CP-averaged branching ratios $[O(10^{-6})]$ together with the large direct CP asymmetries predicted in the pQCD approach are believed to be clearly
measurable at the running LHC and forthcoming Super-B factory experiments in the near future. Furthermore, it is expected that they could provide supplementary constraints on the CKM weak phase $\alpha(\gamma)$ because of the correspondingly tree(penguin)-dominant contributions to the former(latter) decays. Of course, inferred from the numerical results for the large decay rates theoretically and the preliminary upper limits for the branching ratios of $B^+ \to f_1 K^+$ modes experimentally, the region of angle $\phi_{f_1}$ can be deduced as $\phi_{f_1} \in [20^\circ, 27^\circ]$ by combining with the earlier phenomenological analysis, experimental measurements and updated LQCD calculations, which provides more evidences for the dominance of $f_{1\gamma}[f_{1s}]$ in $f_1(1285)[f_1(1420)]$.

(2) Based on the CP-averaged branching ratios of $B \to f_1(\pi, K)$ decays calculated in the pQCD approach, the destructive or constructive interferences between $f_{1\gamma}(\pi, K)$ and $f_{1s}(\pi, K)$ states can be clearly observed and are expected to be confronted with the future experiments. While besides the effects from the $f_{1\gamma} - f_{1s}$ mixing, the $B^0_{d/s} \to f_1(\eta, \eta')$ modes embrace another set of interferences from $\eta_q - \eta_s$ mixing for $\eta - \eta'$ system simultaneously, which makes the more complicated interactions among the four $B^0_{d(s)} \to f_{1\gamma}\eta_q, f_{1\gamma}\eta_s, f_{1s}\eta_q$ and $f_{1s}\eta_s$ states.

(3) For the eight neutral $B^0_{d,s} \to f_1(\pi^0, \eta, \eta', K^0)$ decays, they are mediated by the $b \to d$ transitions and dominated by the penguin amplitudes just with small color-suppressed tree contributions, which then lead to the small CP-averaged branching ratios in the order of $10^{-8} \sim 10^{-7}$ that cannot be measured by the experiments in short period.

(4) While the rest eight neutral $B^0_{s,d} \to (\pi^0, \eta, \eta', K^0_S)$ modes decay through $b \to s$ transitions and have large CP-averaged branching ratios in the order of $10^{-6} \sim 10^{-5}$ except for $B^0_q \to f_1\pi^0$ decays. The channels with large decay rates are all contributed by the nearly pure penguin amplitudes with tiny and safely negligible tree pollution, which can be easily accessed at the ongoing LHCb experiments in the near future.

(5) Though, in principle, $B^0_d \to f_1 K^0_S$ and $B^0_q \to f_1(\eta, \eta')$ modes can serve as the alternative channels to provide more information on the $B^0_d - \bar{B}^0_d$ and $B^0_q - \bar{B}^0_q$ mixing phases from the mixing-induced CP asymmetries $S_f$, respectively. However, the latter $B^0_q$ decays suffer from large theoretical uncertainties that consequently results in the less effective constraints on the mixing phase $\phi_s$. Fortunately, the former $B^0_d$ ones induced by the $b \to sq\bar{q}$ decays have large mixing-induced CP-violating asymmetries but with very small errors. The resultant deviations of $\Delta S$ for $B^0_d \to f_1(1285)K^0_S$ and $f_1(1420)K^0_S$ are around 0.02, which will be stringently examined by the experiments with high precision.

(6) The weak annihilation contributions to these twenty $B \to f_1 P$ decays have been examined in the pQCD approach. The numerical results show that the sizable effects from annihilation diagrams play important roles in the $B_{u,d} \to f_1(1420)K, B^0_d \to f_1\eta', f_1(1420)(\pi, \eta)$, and $B^0_s \to f_1(1285)K^0, f_1(1420)\eta'$ decays. And the rest channels do not depend sensitively on the weak annihilation contributions. The reliability of the evaluations of the weak annihilation diagrams made in the pQCD approach should be strictly examined by the future experiments, which can help the people to distinguish the different viewpoints on calculating the annihilation diagrams proposed by the pQCD approach and soft-collinear effective theory, and then to further understand the annihilation decay mechanism in the heavy $b$-flavored meson decays.

(7) Admittedly, our pQCD results suffer from large theoretical errors induced by the less constrained hadronic parameters, in particular, from the axial-vector $f_1$ mesons’ wave function presently. Meanwhile, only the short-distance contributions at leading order without considering the final state interactions have been taken into account. However, the channels such as $B^+ \to f_1(\pi^+, K^+), B^0_d \to f_1 K^0_S$ and $B^0_s \to f_1(\eta, \eta')$ with large branching ratios are easily accessible in the near future measurements with precision at LHCb and/or Super-B factory experiments, which are expected in turn to provide useful information on improving the input quantities, on the other hand, can help the people to understand the mixing angle $\phi_{f_1}$ and the nature of both $f_1$ mesons better and to identify the reliability of the perturbative evaluations of QCD factorization and pQCD approach in these decays involving axial-vector mesons.

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Appendix A: Hadrons’ distribution amplitudes

For $B$ meson, the distribution amplitude in the impact $b$ space has been proposed as [10, 11]

$$\phi_B(x, b) = N_B x^2 (1 - x)^2 \exp \left[ -\frac{1}{2} \left( \frac{x m_B}{\omega_b} \right)^2 - \frac{\omega_b^2 b^2}{2} \right], \quad (A1)$$

where the normalization factor $N_B$ is related to the decay constant $f_B$ through Eq. (14). The shape parameter $\omega_b$ has been fixed at $\omega_b = 0.40$ GeV by using the rich experimental data on the $B_{u,d}$ mesons with $f_{B_{u,d}} = 0.19$ GeV based on lots of calculations of form factors [34] and other well-known decay modes of $B_{u,d}$ mesons [10, 11] in the pQCD approach in recent years. Because the $s$ quark is heavier than the $u$ or $d$ quark, then the momentum fraction of $s$ quark should be a little larger than that of $u$ or $d$ quark in the $B_{u,d}$ mesons. Therefore, by considering a small SU(3) symmetry breaking, we adopt the shape parameter $\omega_b = 0.50$ GeV [24] with $f_B = 0.23$ GeV for $B_s$ meson and the corresponding normalization constant is $N_B = 63.67$. In order to analyze the uncertainties of theoretical predictions induced by the inputs, we can vary the shape parameters $\omega_b$ and $\omega_{bs}$ by 10%, i.e., $\omega_b = 0.40 \pm 0.04$ GeV and $\omega_{bs} = 0.50 \pm 0.05$ GeV, respectively.

The twist-2 pseudoscalar meson distribution amplitude $\phi_{\pi,K}^A$, and the twist-3 ones $\phi_{\pi,K}^P$ and $\phi_{\pi,K}^T$ have been parametrized as [35, 36, 55],

$$\phi_{\pi,K}^A(x) = \frac{f_{\pi,K}}{2\sqrt{2}N_c} 6x(1-x) \left[ 1 + a_1^{\pi,K} C_1^{3/2}(2x-1) + a_2^{\pi,K} C_2^{3/2}(2x-1) + a_4^{\pi,K} C_4^{3/2}(2x-1) \right], \quad (A2)$$

$$\phi_{\pi,K}^P(x) = \frac{f_{\pi,K}}{2\sqrt{2}N_c} \left[ 1 + \left( 30\eta_3 - \frac{5}{2} \rho_{\pi,K}^2 \right) C_1^{1/2}(2x-1) - 3 \left( \eta_3 \omega_3 + \frac{9}{20} \rho_{\pi,K}^2(1+6a_2^{\pi,K}) \right) C_4^{1/2}(2x-1) \right], \quad (A3)$$

$$\phi_{\pi,K}^T(x) = \frac{f_{\pi,K}}{2\sqrt{2}N_c} (1-2x) \left[ 1 + 6 \left( 5\eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho_{\pi,K}^2 - \frac{3}{5} \rho_{\pi,K}^2 a_2^{\pi,K} \right) (1-10x+10x^2) \right], \quad (A4)$$

with the Gegenbauer moments $a_1^{\pi} = 0, a_1^{K} = 0.17 \pm 0.17, a_2^{\pi,K} = 0.115 \pm 0.115, a_4^{\pi,K} = -0.015$, the mass ratio $\rho_{\pi,K} = m_{\pi,K}/m_0^{\pi,K}$ and $\rho_{\pi,(s)} = 2m_{q(s)}/m_{q(s)s}$, and the Gegenbauer polynomials $C_n^m(t)$,

$$C_2^{1/2}(t) = \frac{1}{2} (3t^2 - 1) , \quad C_4^{1/2}(t) = \frac{1}{8} (3 - 30t^2 + 35t^4) , \quad (A5)$$

$$C_1^{3/2}(t) = 3t , \quad C_2^{3/2}(t) = \frac{3}{2} (5t^2 - 1) , \quad C_4^{3/2}(t) = \frac{15}{8} (1 - 14t^2 + 21t^4) .$$

In the above distribution amplitudes for kaon, the momentum fraction $x$ is carried by the $s$ quark. For both the pion and kaon, we choose $\eta_3 = 0.015$ and $\omega_3 = -3$ [35, 36].

For the axial-vector states $f_{1q}(s)$, its leading twist light-cone distribution amplitude in the longitudinal polarization can generally be expanded as the Gegenbauer polynomials [37]:

$$\phi_{f_{1q}(s)}(x) = \frac{f_{f_{1q}}(x)}{2\sqrt{2}N_c} 6x(1-x) \left[ 1 + a_{2f_{1q}}^{\parallel} \frac{3}{2} (5(2x-1)^2 - 1) \right], \quad (A6)$$

For twist-3 light-cone distribution amplitudes, we use the following form [38]:

$$\phi_{f_{1q}(s)}^A(x) = \frac{f_{f_{1q}}(x)}{4\sqrt{2}N_c} \frac{d}{dx} \left[ 6x(1-x)(a_{1f_{1q}(s)} \frac{3}{2} (2x-1)(3(2x-1)^2 - 1) \right] , \quad (A7)$$

$$\phi_{f_{1q}(s)}^T(x) = \frac{f_{f_{1q}}(x)}{2\sqrt{2}N_c} \left[ \frac{3}{2} a_{1f_{1q}(s)}(2x-1)(3(2x-1)^2 - 1) \right] , \quad (A8)$$

where the Gegenbauer moments are quoted from Ref. [37]

$$f_{1q} \text{ state : } \quad a_{2}^{\parallel} = -0.92 \pm 0.62 , \quad a_{1}^{\parallel} = -1.04 \pm 0.34 ; \quad (A9)$$

and

$$f_{1s} \text{ state : } \quad a_{2}^{\perp} = -0.94 \pm 0.03 , \quad a_{1}^{\perp} = -1.06 \pm 0.36 . \quad (A10)$$
where the values are taken at $\mu = 1$ GeV.
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