SPHERICAL ACCRETION ONTO NEUTRON STARS REVISITED: ARE HOT SOLUTIONS POSSIBLE?

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ABSTRACT

Stationary, spherical accretion onto an unmagnetized neutron star is here reconsidered on the wake of the seminal paper by Zel’dontovich & Shakura (1969). It is found that new “hot” solutions may exist for a wide range of luminosities. These solutions are characterized by a high temperature, $10^9$ – $10^{11}$ K, and arise from a stationary equilibrium model where the dominant radiative mechanisms are multiple Compton scattering and bremsstrahlung emission. For low luminosities, $\lesssim 10^{-2} L_E$, only the “cold” (à la Zel’dovich and Shakura) solution is present.

Subject headings: accretion, accretion disks – stars: neutron – X-rays: stars

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I. INTRODUCTION

Even before the observational evidence that Galactic X-ray sources are mostly binary systems containing an accreting neutron star, Zel’dovich & Shakura (1969, ZS in the following) studied in some detail the spectrum of radiation produced by stationary, spherical accretion onto an unmagnetized neutron star and compared their results with the (poor) data available at the time for Sco X–1. The pioneering paper of ZS shows that the resulting spectrum depends on two parameters, the accretion rate (luminosity) and the penetration length of the accreting ions in the outermost neutron star layers. The outcome can be described, in essence, as a black body with a high energy tail due to the Compton heating of thermal photons in the hot, external part of the atmosphere surrounding the neutron star.

ZS’s analytical work was pushed further by Alme & Wilson (1973), making use of numerical methods. Shapiro & Salpeter (1973) adopted essentially ZS solution at the inner boundary and explored the possibility that in the surrounding region a shock is formed, which may modify the resulting spectrum. While models of spherical accretion evolved substantially since then, considering, for instance, the role of nuclear reactions induced in the crust by the bombardment of accreting ions (see e.g. Bildsten, Salpeter & Wasserman 1992), the basic picture proposed by ZS has been maintained.

In this letter we reconsider the issue, adopting a set of equations for the neutron star atmosphere which essentially coincides with those of ZS. We show that ZS’s solution for the temperature profile is not unique: for large enough accretion rates, another solution, at considerably higher temperatures, may exist. The presence of these solutions per se, and the transition between “hot” and “cold” solutions can have interesting astrophysical consequences.
II. THE MODEL

In order to explore the possible existence of high temperature solutions, we use a very simplified model which follows closely the original work of Zel’dovich & Shakura. In particular we assume that the accreting flow impinges onto a spherically–symmetric, static atmosphere which surrounds the neutron star, and is decelerated as it penetrates into the atmosphere. The kinetic energy released by the incident protons goes mainly into electron thermal energy and is then re–emitted as free–free radiation. The details of the flow braking fall within the domain of plasma physics and are still far from a thorough understanding. For this reason, following ZS, we treat the total column density of the atmosphere required to stop the incoming beam, $y_0$, as a free parameter and the column density

$$y = \int_R^\infty \rho dR$$

(1)

is used as the independent coordinate in place of the radial distance $R$; here $\rho$ is the matter density.

The heat injected by the infalling protons per unit time and mass in the atmosphere is assumed to be constant and is related to the total luminosity observed at infinity by

$$W = \frac{L_\infty}{4\pi \int_0^{y_0} R^2 dy}.$$  

(2)

In the inner region where $y > y_0$, $W = 0$; in all our models $5 < y_0 < 20$. If the flow velocity $v$ exceeds the electron thermal velocity, $v_{th}$, these values of the proton penetration length are appropriate to describe the stopping of the incoming proton beam in a hydrogen plasma where only Coulomb interactions take place (e.g. Alme and Wilson 1973). On the contrary, if $v \lesssim v_{th}$, the proton stopping through repeated Coulomb scatterings is less effective and other collective processes (e.g. plasma oscillations) need to be considered to keep $y_0$ within this interval.

The transfer of radiation in the atmosphere is governed by the equations for the radiative luminosity $L$ and the radiation energy density $U$ which, in spherical symmetry and using the Eddington approximation, can be written as

$$\frac{dL}{dy} = -4\pi R^2 W$$

(3)
\[
\frac{1}{3}\frac{dU}{dy} = \kappa_1 \frac{L}{4\pi R^2 c}.
\] (4)

The only radiative processes taken into account are scattering and bremsstrahlung, so that the flux mean opacity can be conveniently expressed as

\[
\kappa_1 = \kappa_{es} + 6.4 \times 10^{22} \varrho T^{-7/2} \text{ cm}^2 \text{ g}^{-1}
\]

where \( \kappa_{es} = 0.4 \text{ cm}^2 \text{ g}^{-1} \) and \( T \) is the gas temperature. Since, as numerical models show, the atmosphere does not expand considerably even for high temperature solutions, the Eddington approximation is reasonable. The appropriate boundary condition for the radiation field at the outer edge of the non–illuminated medium is \( U = L_\infty /2\pi R^2 c \). The inner boundary condition, at \( y_{in} \gg y_0 \), is fixed by the requirement that all the observed radiative flux must be generated within the atmosphere, that is, \( L = 0 \). This is the same condition used by ZS and is appropriate if the atmosphere is effectively thick close to \( y_{in} \).

The runs of pressure, \( P \), and temperature are obtained from the hydrostatic balance and radiative energy equilibrium

\[
\frac{dP}{dy} = \frac{GM_*}{R^2}
\]

\[
\frac{W}{c} = \kappa_P (aT^4 - U) + 4\kappa_{es}U \frac{K T}{m_e c^2} \left(1 - \frac{T_\gamma}{T}\right),
\] (6)

where \( M_* \) is the mass of the neutron star. The matter density \( \varrho \) is calculated from the perfect gas equation of state assuming that the material is completely ionized hydrogen. Since we study the general properties of spherical accretion onto neutron stars for luminosities below the Eddington limit, in equation (5) we neglected the radiative force and the ram pressure exerted by the incoming protons, which is typically two orders of magnitude below the gravitational term. In equation (6) \( \kappa_P \) is the Planck mean opacity and \( T_\gamma \) is the radiation temperature which is defined as the mean photon energy. In general \( T_\gamma \) can be computed only solving the full frequency–dependent transfer problem and will depend on \( y \). In ZS, \( T_\gamma \) was taken equal to \([U(y)/a]^{1/4}\), which is appropriate in LTE. Being interested also in solutions in which multiple Compton scattering becomes important, we derive the radiation temperature from the equation (see Wandel, Yahil & Milgrom 1984; Park & Ostriker 1989).
\[
\frac{y}{T} \frac{dT_\gamma}{dy} = 2Y_c \left( \frac{T_\gamma}{T} - 1 \right)
\]  \hspace{1cm} (7)

where \(Y_c = (4KT/m_e c^2) \max(\tau_{es}, \tau_{es}^2)\) is the Comptonization parameter and \(\tau_{es} = \kappa_{es} y\). Use of equation (7) requires some care since it is meant to describe the variation of the radiation temperature when multiple Compton scattering is the dominant mechanism to exchange energy between photons and electrons. Equation (7) therefore does not apply if either \(\tau_{es} < 1\) or true emission–absorption are important. On the other hand, equation (7) gives the correct limit for low optical depth (\(Y_c \ll 1\) implying \(T_\gamma \approx \text{const}\)), so that one can extend the validity of equation (7) to all regimes, provided that no physical significance is attached to \(T_{es}\) where the effective optical depth \(\tau_{eff} > 1\).

From equation (1), it follows immediately that

\[
\frac{dR}{dy} = -\frac{1}{\vartheta}.
\]  \hspace{1cm} (8)

The boundary conditions for equations (5) and (8) are \(P = 0\) at \(y = 0\) and \(R = R_\ast\) at \(y = y_{in}\). We notice that a boundary condition for \(T_\gamma\) must be also imposed because the radiation temperature obeys a differential equation; models were obtained specifying a value of \(T_\gamma\) at \(y = 0\). The solution to equations (3)–(8) provides the variation of \(L, U, P, T, T_\gamma\) and \(R\) as functions of the column density \(y\) and is found numerically; all models refer to a neutron star of \(R_\ast = 10\) km and \(M_\ast = 1\ M_\odot\).

We find that two distinct kinds of solutions, “hot” and “cold”, always exist for any \(y_0\) provided that the luminosity exceeds a certain limit, which depends on \(y_0\). The thermal properties of the atmosphere are illustrated in figures 1 and 2, where the run of \(T\) versus column density is shown for different luminosities in the case \(y_0 = 20\). The “cold” solutions of figure 1 are just those already found by ZS and are obtained setting \(T_\gamma(y = 0) = [L_\infty/(4\pi R_\ast^2 \sigma)]^{1/4}\). The “hot” solutions of figure 2 exist for values of \(T_\gamma\) satisfying the condition \(T_\gamma > T_{crit}(L_\infty, y_0)\); here \(T_\gamma(y = 0) = 2 \times 10^9\) K. Temperature is close to \(T_\gamma\) in the outer region (\(y \lesssim 23\ \text{g cm}^{-2}\)), while in the dense layers close to \(y_{in}\) LTE is attained at \(T \sim 10^7\) K. The temperature profile in the hot region resembles that one of static atmospheres around X–ray bursting neutron stars (see e.g. London,
Taam & Howard 1986) where the same radiative processes dominate. Cold, thermal photons do not propagate outwards because \( L = 0 \) for \( y_0 < y < y_{in} \), so the hot and the cold zone are thermally decoupled, at least radiatively. On the other hand, we have checked that imposing either an ingoing or an outgoing flux at \( y_{in} \) does not alter our picture significantly if \( |L(y_{in})|/L_\infty \lesssim 0.1 \).

The presence of two possible regimes has a simple interpretation in terms of the relative efficiency of the two radiative processes we have considered, Compton scattering and free–free emission–absorption. The static atmosphere must, in fact, radiate a given luminosity and, for doing that in the scenario we are proposing, there are two ways. A first possibility is that a lot of soft bremsstrahlung photons are produced in a low–temperature, dense medium in which the effective depth is large. This gives rise to a spectrum which is essentially blackbody and corresponds to the “cold” solution. Comptonization is never dominant because temperature is low and the scattering depth is not large enough to make \( Y_c > 1 \). The “hot” solutions represent the opposite case, in which much less energy is generated through bremsstrahlung emission in a low–density, hot plasma far from LTE. Comptonization, however, is now so efficient that matter and radiation temperatures are everywhere very close and the same energy output can be obtained.

It is possible to get an insight on the existence of high–temperature solutions and to give an estimate of the limiting value \( T_{crit} \) by means of simple analytical considerations using, for the sake of simplicity, a plane–parallel geometry for the atmosphere. For \( y < y_0 \) we can safely neglect free–free absorption in equation (4) and we get the expressions for \( L, U \) and \( \rho \) as functions of \( y \) and \( T \):

\[
L = L_\infty \frac{y_0 - y}{y_0} \tag{9}
\]

\[
U = \frac{W}{c} \left[ 2y_0 + 3k_{es}y \left( y_0 - \frac{y}{2} \right) \right] \tag{10}
\]

\[
\rho = D \frac{y}{T} \tag{11}
\]

where \( D = GM_\ast m_p/2kR_\ast^2 = 8.1 \times 10^5 \), in c.g.s. units for \( R_\ast = 10^6 \) cm and \( M_\ast = 1 M_\odot \). Neglecting the term \( \kappa_P U \), the energy equation becomes a cubic equation in \( x = T^{1/2} \), that can be studied analytically for given values of \( y_0 \).
and \( L_\infty \) and treating \( T_\gamma \) as a free parameter. Equation (6) can be cast into the form

\[ x^3 + p x + q = 0 \] (12)

with

\[
p = -\left( T_\gamma + A \frac{l_\infty}{U} \right) \sim -T_\gamma
\]

\[
q = B \frac{y}{U(y)} ,
\]

where \( l_\infty \) is the total luminosity observed at infinity in units of the Eddington luminosity; \( A = 6.3 \times 10^{22} \) and \( B = 5.1 \times 10^{25} \), again in c.g.s. units. The approximated expression for \( p \) holds only for \( T_\gamma \gg 10^8 \)K, while, in order to make the analytical treatment affordable, in the inner part of the atmosphere \( q \) is set approximately equal to its maximum value, \( q_{\text{max}} = q(y_0) \). Once \( y_0 \) and \( l_\infty \) are fixed, equation (12) has one or three real roots, according to the sign of the discriminant, and it is easy to prove that all the roots are real only if

\[
T_\gamma \geq \frac{2.5 \times 10^8}{(1 + \frac{2}{3} \kappa_{es} y_0)^{2/3} l_\infty^{2/3}} . \] (13)

It can also be shown that if just one root is present it is \( T \gtrsim T_\gamma \), while, when condition (13) is satisfied, the three roots have magnitudes \( T \gtrsim T_\gamma \), \( T \ll T_\gamma \) and \( T \ll T_\gamma \), respectively. The solution \( T \gtrsim T_\gamma \) is unacceptable since \( T > T_\gamma \) will produce a negative radiation temperature gradient (see eq. [7]) and also \( T \ll T_\gamma \) must be discarded because it is inconsistent with our starting assumption that absorption could be neglected because the plasma is very hot. Finally, the root \( T \ll T_\gamma \) is the “hot” solution, which exists only when the radiation temperature exceeds the limit given by (13), which represents the analytical estimate of \( T_{\text{crit}} \).

In figure 3 we plot the mean energy of the outgoing photons as a function of the total luminosity for \( y_0 = 20 \). For “hot” solutions only the lower bound \( T_{\text{crit}} \) (dashed line) is shown. As can be seen, while the mean photon energy of the “cold” solutions (crosses) monotonically increases with the luminosity, the lower bound \( T_{\text{crit}} \) for “hot” solutions shows the opposite behaviour. Moreover the numerical analysis indicates that high temperature solutions may exist only for high enough values of \( l_\infty \) and that the critical luminosity, \( l_{cr} \), under which
no “hot” solutions exist depends on $y_0$: for $y_0 = 5$, $l_{cr} = 2 \times 10^{-2}$, while for $y_0 = 20$, $l_{cr} = 6 \times 10^{-3}$.

The anticorrelation between $T_{crit}$ and $l_\infty$, and the lack of “hot” solutions at low luminosities can be explained as follows. As $l_\infty$ decreases, $W$ and $U$ start to decrease (see eqs. [2] and [10]) and, since Compton heating (which is dominant over cooling) becomes progressively smaller in magnitude, the free–free emissivity ($\kappa \rho a T^4 \propto g T^{1/2} \propto P/T^{1/2}$) must also decrease for the energy equation to be satisfied. This can be achieved only by an increase of temperature (and a decrease of density, see eq. [11]) because the pressure profile is nearly independent on the thermal properties of the gas. The lack of “hot” solutions at small luminosities is due to the fact that, when density is very low, the envelope becomes photon starved and even a strong Comptonization is unable to produce the required luminosity.

The existence of both “cold” and “hot” solutions for the same values of the flow parameters has been already found in black hole accretion (Park 1990; Nobili, Turolla & Zampieri 1991) when both free–free and Compton scattering are present. An alternative picture for the production of hard X–rays ($\sim 100$ keV) from accreting neutron stars has been proposed by Kluzniak & Wilson (1991). In their model matter coming from the inner edge of an accretion disk hits the stellar surface at a shallow angle, creating a hot equatorial belt in which Compton cooling is very efficient.
III. DISCUSSION

The basic limitations of our approach are obviously that the analysis is stationary and the spectrum is described just in terms of a mean photon energy. The fact that “hot” models may exist only for high enough radiation temperatures, $T_{\gamma} \gtrsim T_{\text{crit}} \approx 10^9$K at the outer boundary, suggests that, in order to get started, an extra energy input, different from that produced by the incoming beam, must be supplied, the actual value of $T_{\gamma}$ depending on the preparation of the system. A physically consistent scenario should be investigated in a time dependent picture. Furthermore temperatures are mildly relativistic and the expression we used for the Compton heating–cooling term is just an approximation. Moreover, pair production–annihilation can not be neglected for $l_{\infty} > 10^{-2}$ when the compactness parameter becomes $\gtrsim 10$. Above $l_{\infty} \sim 0.1$, the dynamical effects of radiation pressure and bulk motion Comptonization become also important (on this regard see e.g. Zampieri, Turolla & Treves 1993 and references therein) and are not included in our model.

Our present results can be summarized as follows. At very low accretion rates (the threshold depends on $y_0$) there is a unique solution, the “cold” one discovered by ZS (see figure 3). This essentially indicates that the black body approximation at very low luminosities, as expected for instance in isolated neutron stars accreting the interstellar medium is reasonable (see e.g. Ostriker, Rees & Silk 1970; Treves & Colpi 1991; Blaes & Madau 1993). The “hot” and “cold” solutions coexist for a certain range of luminosities and the mean photon energies of the two modes approach each other for increasing $l_{\infty}$. The “cold” solution becomes hotter for increasing accretion rate while the “hot” one softens and this behaviour is opposite to the one exhibited by the hot, shocked solutions of Shapiro & Salpeter.

Even in the absence of frequency–dependent calculations it is tempting to associate the “cold” solution with the spectral states observed in the Soft X–Ray Transient source Aql X–1, either in outburst or in quiescence (Verbunt et al. 1993). The existence of “hot” solutions for intermediate luminosities with temperatures $\sim 100$ keV suggests that a class of hard X–ray sources, which possibly have not yet been discovered, may exist. The transition between the “hot” and the “cold” regime, even at luminosities where the two solutions are rather different, may be expected in a time–dependent scenario. Non–
stationary calculations are also needed to explore the stability properties of the two solutions.

In conclusion the discovery of “hot” solutions in spherical accretion onto neutron stars deserves further theoretical consideration, a program on which we are presently actively working.

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FIGURE CAPTIONS

Figure 1. Temperature vs. column density of “cold” solutions for $l_\infty = 7 \times 10^{-3}$ (continuous line), $l_\infty = 2 \times 10^{-2}$ (dashed line) and $l_\infty = 7 \times 10^{-2}$ (dashed–dotted line); here $y_0 = 20$.

Figure 2. Same as in figure 1 for “hot” solutions.

Figure 3. Mean energy of the outgoing photons vs. total emitted luminosity for $y_0 = 20$; crosses refer to “cold” models. The dashed line represents the lower limit for the existence of “hot” solutions given by equation (13).