Probing dark matter streams with CoGeNT

Aravind Natarajan∗
McWilliams Center for Cosmology, Carnegie Mellon University, Department of Physics, 5000 Forbes Ave., Pittsburgh PA 15213, USA

Christopher Savage†
The Oskar Klein Centre for Cosmoparticle Physics, Department of Physics, Stockholm University, AlbaNova, SE-10691 Stockholm, Sweden

Katherine Freese‡
Michigan Center for Theoretical Physics, University of Michigan, Ann Arbor, MI 48109, USA

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We examine the future sensitivity of CoGeNT to the presence of dark matter streams and find that consideration of streams in the data may lead to differences in the interpretation of the results. We show the allowed particle mass and cross section for different halo parameters, assuming spin-independent elastic scattering. As an example, we choose a stream with the same velocity profile as that of the Sagittarius stream (and in the Solar neighborhood) and find that, with an exposure of ∼10 kg year, the CoGeNT results can be expected to exclude the SHM-only halo in favor of an SHM+stream halo at the 95% (99.7%) confidence level provided the stream contributes 3% (5%) of the local dark matter density. The presence of a significant stream component may result in incorrect estimates of the particle mass and cross section unless the presence of the stream is taken into account. We conclude that the CoGeNT experiment is sensitive to streams and care should be taken to include the possibility of streams when analyzing experimental results.

I. INTRODUCTION

The nature of the dark matter (DM) that comprises the bulk of the mass in the Universe is one of the longest outstanding problems in all of physics. Weakly Interacting Massive Particles (WIMPs) with weak scale cross sections and masses in the GeV to TeV range are among the best motivated dark matter candidates. Recent experiments indicate hints that the DM particle may have been detected. However, interpretation of the data depends on our understanding of the velocities of these particles as they pass through the detectors. The canonical distribution of WIMPs in the Halo of our Galaxy is the isothermal Maxwell-Boltzmann distribution. However, the real Galaxy is not likely to be quite so simple. Numerical simulations suggest that dark matter halos may be triaxial and anisotropic, which can lead to significant differences in the scattering rates and the amplitude and phase of the annual modulation [1, 2].

Direct detection experiments are sensitive to the form of the local dark matter velocity distribution [3–6], and the results may be significantly modified if a cold stream of dark matter particles exists in the solar neighborhood. Tidal disruption of dwarf satellites is expected to produce such cold streams. The late infall of dark matter onto the Galaxy is also expected to result in cold streams contributing ∼a few percent to the local dark matter density [7,9]. Cosmological N-body simulations from the Aquarius project [10, 11] show the presence of tidal streams in the solar neighborhood at the level of ∼1% of the local dark matter density, at ∼20% probability. Larger stream contributions are likely if dense tidal streams pass near the sun’s location. Streams will tend to dominate in the outer halo, rather than the inner halo - as has been found in stellar surveys thus far (for a discussion, see [12]).

Recently, [13] have studied the effect of major mergers and conclude that massive (>10^{10} M_☉) mergers can lead to observable structure in the velocity distribution of dark matter particles. Authors [14, 15] find that such a merger event is very likely. Previously [16, 17] showed that in principle such streams should be visible in data from DM detectors by giving rise to a step in the energy spectrum of the count rate: the count rate would be higher up to a critical energy above which the stream could no longer contribute to the data. Future experiments will be sensitive to a wide range of stream velocities. In particular, as we will show, the CoGeNT experiment may have, over the next few years, the sensitivity to detect such streams.

In 2010, the CoGeNT collaboration [19] reported an excess of events at low energies, which was interpreted as possibly due to scattering of dark matter particles with the target [19–22]. More recently, CoGeNT reported [23] the detection of an annual modulation [24, 25] in the
event rate at the 2.8σ level, with a modulation amplitude of 16.6 ± 3.8%. The CRESST-II collaboration has also seen unexplained events that are compatible with a possible explanation in terms of WIMPs. It is tantalizing to compare these new results with the decade-old annual modulation seen in the DAMA data which has reached 9σ confidence level. All these results indicate a possible ~10 GeV WIMP mass, yet the issue of compatibility between them is unclear. Some authors claim consistency between the different experimental results while others do not. Authors showed that the presence of streams could help to reconcile the DAMA results with other experiments.

Yet more perplexing is the apparent discrepancy of the null results of XENON and CDMS with both the DAMA and CoGeNT experiments; we do not address these null results in this paper, and restrict our discussion only to DAMA and CoGeNT. We further address the issue of compatibility between DAMA and CoGeNT in this paper by extending the discussion, within the context of the standard Maxwellian halo, to allow variation in the two relevant velocities characterizing the distribution: the escape velocity and the dispersion.

The focus of the paper, however, is the search for streams in the CoGeNT data. The low energy threshold (0.47 keVee) and excellent energy resolution (0.05 keVee) obtained by CoGeNT are key to detecting dark matter streams for small particle masses. The planned upgrade to the CoGeNT experiment (C-4) will consist of 4 detectors, of approximately 1.3 kg each, and is expected to start taking data later this year. We test the sensitivity of the CoGeNT upgrade to dark matter streams by performing a number of Monte Carlo simulations and fitting the results. We select a dark matter mass and scattering cross-section consistent with the current CoGeNT results, assuming that the excess events currently seen by CoGeNT at low energies is entirely due to scattering of dark matter particles with the target. We caution the reader that if a significant (and currently unknown) exponential background exists at low energies, our conclusions may be altered. We also do not consider possible ways in which the CoGeNT and DAMA results could be made compatible with the null results of XENON and CDMS.

Consider dark matter particles of mass $m_\chi$ scattering elastically off a nucleus of mass $m_N$. The number of recoil events per unit time, per unit detector mass, and per unit energy, is given by the formula:

$$\frac{dR}{dQ}(t, Q) = \frac{\rho_\chi}{2m_\chi m_{R,p}} A^2 F^2(Q) \int_{v_{\min}(Q)}^\infty \frac{f(v)}{v} dv.$$  

(1)

$\rho_\chi$ is the dark matter density at the earth's location, $A$ is the atomic mass number and $m_{R,p} = m_\chi m_p/(m_\chi + m_p)$ is the WIMP-proton reduced mass. We have assumed here that the WIMP coupling to the proton is the same as the coupling to the neutron. $F(Q)$ is the form factor, containing the momentum dependence of the cross section, and takes the form described in [13][15]. $f(v)$ is the one-dimensional speed distribution of dark matter particles relative to the detector. It is this term that is sensitive to different halo models. $v_{\min}(Q) = \sqrt{Qm_N/2m_R^2}$ is the minimum velocity a particle must have in order to effect a recoil at energy $Q$, where $m_R = m_\chi m_N/(m_\chi + m_N)$ is the WIMP-nucleus reduced mass.

The standard halo model (SHM henceforth) is characterized by a Maxwell-Boltzmann distribution of velocities in the rest frame of the Galaxy given by

$$f_{\text{SHM}}(\vec{v}_{wh}) = \frac{1}{N_{\text{esc}}} \exp \left[ -\frac{\left|\vec{v}_{wh}/v_0\right|^2}{2} \right]$$

for $|\vec{v}_{wh}| < v_{\text{esc}}$

$$= 0 \quad \text{otherwise}$$

(2)

where $v_0$ characterizes the velocity dispersion, $\vec{v}_{wh}$ is the WIMP velocity relative to the halo, $v_{\text{esc}}$ is the escape velocity, and $N_{\text{esc}}$ is a normalization constant chosen such that $\int dv f(v) = 1$.

FIG. 1: (a) The recoil spectrum observed by CoGeNT and several predicted spectra compatible with the observations. The solid (red) curve represents the best fit mass and cross section for $v_0=220$ km/s, $v_{\text{esc}}=600$ km/s. The other panels show 3σ contours in the ($m_\chi$, $\sigma_p$) plane for CoGeNT (solid red) and DAMA (solid black) for (b) varying $v_0$ and (c) varying $v_{\text{esc}}$ (see text). The cross marks indicate the best fit points. Also shown are the exclusion limits from XENON10 (dashed blue), XENON100 (dashed pink), and from the CDMS II low energy analysis (dashed cyan). The XENON10 limit assumes $v_0 = 230$ km/s, $v_{\text{esc}} = 600$ km/s, while the XENON100 limit assumes $v_0 = 220$ km/s, $v_{\text{esc}} = 544$ km/s [39][40]. The CDMS II limit assumes $v_0 = 220$ km/s, $v_{\text{esc}} = 544$ km/s [40][41].
Fig. [1](a) shows the time averaged recoil rate for the low energy bins, from [23]. We have also plotted predicted recoil spectra for WIMP masses $m_{\chi} = 7, 8.5, \text{ and } 10$ GeV, for the SHM assuming $v_0 = 220$ km/s and $v_{\text{esc}} = 600$ km/s. The energy dependence of the Germanium quenching factor is obtained from the measurements reported in [46]. The solid (red) curve represents the best fit ($m_{\chi}, \sigma_p$), with a $\chi^2 = 6.3/14$ d.o.f. Panel (b) shows the $3\sigma$ allowed contours (solid, red) for $v_0 = 180, 220$, and $260$ km/s, from left to right respectively, for an assumed $v_{\text{esc}} = 600$ km/s. The cross marks indicate the parameter values that minimize the value of $\chi^2$ over the 16 lowest energy bins. Also shown are the $3\sigma$ contours for the DAMA results [27, 28, 47], ignoring the possibility of channeling, and assuming a constant quenching factor for Sodium = 0.3 (the recoils off of Iodine are not significant at these masses). We do not consider varying the Sodium quenching factor here, and caution the reader that the contours will be altered if there is a significant uncertainty in the Sodium quenching factor. The 90% exclusion limits [48, 49] for XENON 100 and XENON 10 are plotted. Panel (c) shows the variation with $v_{\text{esc}}$, for fixed $v_0 = 220$ km/s. Shown from left to right are $3\sigma$ contours for $v_{\text{esc}} = 600, 500, \text{ and } 400$ km/s. Note that while comparing the CoGeNT contours with the XENON exclusion limits, care should be taken to match the halo parameters. We provide CoGeNT contours for different values of $v_0$ and $v_{\text{esc}}$, but the XENON and CDMS bounds are for specific values of $v_0$ and $v_{\text{esc}}$. Thus the CoGeNT contour for $v_0 = 260$ km/s is likely excluded when compared to the XENON and CDMS exclusion curves for $v_0 = 260$ km/s (not shown). The dark matter density at the sun’s location was set to 0.3 GeV/cm$^3$.

II. SENSITIVITY TO STREAMS

We now consider the possibility that in addition to a thermal component of dark matter, there exist dark matter streams, i.e. particles with small or negligible velocity dispersion. As mentioned in the Introduction, such streams are expected to occur due to the tidal break-up of small halos, the Sagittarius stream being a well known example. The CoGeNT experiment has also measured an annual modulation in the recoil rate, at the $\sim 2.8\sigma$ level. Fig. [2](a) shows the modulation (mean subtracted) with the expectation for theoretical models with and without streams. Fitting the amplitude to the SHM, we obtain a $\chi_{\text{min}}^2 = 7.8/10$ d.o.f. for $m_{\chi} = 10$ GeV, $\sigma_p = 0.11$ fb. The dashed (blue) curve includes a 5% contribution from the Sagittarius stream for $m_{\chi} = 9.3$ GeV, $\sigma_p = 0.16$ fb. With the Sagittarius stream included, the $\chi_{\text{min}}^2$ improves to 6.8/10 d.o.f. The pink dot-dashed curve is plotted as an example of an unknown stream that fits the phase of the CoGeNT modulation. A 5% contribution due to this stream improves the fit significantly, resulting in a $\chi_{\text{min}}^2 = 2.7/7$ d.o.f., for a small WIMP mass $m_{\chi} = 6$ GeV, and a very large cross section $\sigma_p = 0.69$ fb (for the unknown stream, we allow the 3 velocity components to vary, in addition to the mass and cross section). This stream has a velocity relative to the sun $\vec{v}_{\odot} = (475 \text{ km/s}, \theta = 120^\circ, \phi = 160^\circ)$, and the co-ordinate system is as defined in [17]. For comparison, the Sagittarius stream’s velocity relative to the sun [16] for $v_0 = 220$ km/s, is $(340 \text{ km/s}, \theta = 151^\circ, \phi = 266^\circ)$. We provide this example merely to illustrate that the presence of streams can have a significant effect on the phase of the annual modulation. More accurate results require better data for the annual modulation.

Fig. [2](b) shows the $3\sigma$ contours taking into account both the amplitude over one year, and the average recoil rate. The contour for the SHM is very similar to the contour obtained using the time averaged data alone (Fig. [1], owing to the large error bars in the modulation data. The inclusion of a 5% contribution due to the Sagittarius stream moves the contour only slightly. The best fit values are $m_{\chi} = 8.6$ GeV, $\sigma_p = 0.06$ fb, with a $\chi_{\text{min}}^2 = 14.7$ with 22 degrees of freedom (12 time bins averaged over energy, 12 energy bins averaged over time, and 2 fitting parameters). With the Sagittarius stream included (5% contribution), the best fit values are $m_{\chi} = 9.3$ GeV, $\sigma_p = 0.05$ fb, with a $\chi_{\text{min}}^2 = 14.9$.

With the inclusion of a stream, the velocity distribution $f(v)$ is modified as:

$$f(v) = \xi f_{\text{str}}(v) + (1 - \xi)f_{\text{SHM}}(v)$$

where $\xi$ is the fraction of the dark matter density contributed by the stream to the local dark matter density. For cold streams, $f_{\text{str}}(v) \approx \delta(v - v_{\text{str}})$ where $v_{\text{str}}$ is the stream speed. A finite velocity dispersion may be accounted for by replacing the delta function by a Gaussian. For definiteness, let us consider the Sagittarius stream. The Sagittarius dwarf galaxy is being tidally disrupted by the Milky Way, resulting in tidal tails. Previously, the possibility existed that the leading tidal tail passed through the local neighborhood, allowing for detection by direct detection experiments (for details, we refer the reader to [16] and references therein). It is no longer considered likely that the Sagittarius stream passes near the Earth [45]. However, we will use the Sagittarius stream as a case study, to be representative of detecting a dark matter stream of known direction but unknown density. Recently, it was pointed out [49] that the infall of the Sagittarius dwarf galaxy may contribute to the formation of the spiral arms of the Milky Way.

We study the effect of adding the Sagittarius stream by performing Monte Carlo (MC) simulations of future CoGeNT results, taking the stream to compose a fraction $\xi$ of the local density (which is fixed at 0.3 GeV/cm$^3$). We only consider the time averaged information, as the present data does not constrain the annual modulation effectively. We consider an exposure of 10 kg-year, which may be obtained in $\sim 3$ years with the CoGeNT C-4 detector upgrade. We assume a known background that is modeled by a constant plus a double Gaussian, as done in previous works [22]. The constant and the heights of
Fig. 2: CoGeNT modulation. Shown in (a) are the first 12 time bins from [23], with the mean subtracted. The solid (red) curve is for the SHM. The dashed (blue) curve includes a 5% contribution from the Sagittarius stream, while the dot-dashed (pink) curve includes a 5% contribution from an unknown stream. (b) shows the 3σ contours combining modulation information with the time averaged recoil rate, with and without a stream contribution, assuming \( v_0 = 220 \text{ km/s} \) and \( v_{\text{esc}} = 600 \text{ km/s} \). The cross marks indicate the best fit parameters.

The Gaussian peaks are obtained from [23]. We include the known background in all our MC simulations. For the halo, we take \( v_0 = 220 \text{ km/s} \) and \( v_{\text{esc}} = 600 \text{ km/s} \) and we take for our fiducial dark matter mass and cross-section \( m_\chi = 10 \text{ GeV} \) and \( \sigma_p = 0.05 \text{ fb} \). These values are consistent with a dark matter signal interpretation of the excess CoGeNT events.

To first examine the impact of a stream on an experimental analysis, we perform 1000 MC simulations of the CoGeNT results for a given local stream density \( \xi = 0.05 \) (see Eq. [3]). We fit each of these simulated results to two types of halo models: (i) an SHM+stream model with variable \( \xi \) and (ii) an SHM-only model (\( \xi = 0 \)). Fits are obtained by minimizing the chi-square (using the 10 lowest energy bins in [23]) over the mass \( m_\chi \), cross-section \( \sigma_p \), and, for the SHM+stream model, \( \xi \). Fig. 3(a) shows the minimum chi-square \( \chi^2_{\text{min}} \) obtained for fits to the SHM+stream model (red, solid) and the SHM-only model (blue, dashed). The SHM+stream model fares significantly better with a median \( \chi^2_{\text{min}} \) of 8.2/9 d.o.f. compared to a median \( \chi^2_{\text{min}} \) of 16.7/10 d.o.f. for the SHM-only model. Fig. 3(b) shows the best fit values of \( \xi \) obtained for the different simulations. Figs. 3(c) and (d) show the best fit values of \( m_\chi \) and \( \sigma_p \), respectively, for the SHM+stream model and the SHM only model. The SHM+stream model gives best fit values of \( m_\chi \) and \( \sigma_p \) very close to the true values. The SHM-only model on the other hand underestimates the mass by \( \approx 6\% \) and overestimates the cross section by \( \approx 11\% \). The Sagittarius stream is clearly visible in (b) and results in erroneous values of \( m_\chi \) and \( \sigma_p \) if the presence of the stream is ignored, as in the SHM-only model.

To quantify the ability of CoGeNT to exclude the SHM-only halo model in favor of a halo also containing the Sagittarius stream, we apply a likelihood ratio test, which in this case is equivalent to examining the statistic \( \Delta \chi^2 = \chi^2_{\text{min}}(\text{SHM-only}) - \chi^2_{\text{min}}(\text{SHM+stream}) \), where the two \( \chi^2_{\text{min}} \) are the minimum chi-square obtained using an SHM-only halo (minimized over \( m_\chi \) and \( \sigma_p \) with fixed \( \xi = 0 \)) and an SHM+stream halo (minimized over \( m_\chi \), \( \sigma_p \), and \( \xi \)), respectively. The distribution of this statistic for an SHM-only true halo model, as determined from 10,000 MC simulations, is shown in Fig. 4 (dashed blue). The distribution falls off rapidly for \( \Delta \chi^2 \) above \( \sim 1 \), similar to a \( \chi^2 \)-distribution with 1 d.o.f. [54]. The distribution is peaked about small \( \Delta \chi^2 \) as adding a stream com-
FIG. 4: The distribution of $\Delta \chi^2$ for an SHM-only halo (dashed blue) and an SHM+stream halo with $\xi = 0.03$ (solid red) as determined from MC simulations (see the text). The vertical line indicates the median value of $\Delta \chi^2 = 3.1$ for the SHM+stream halo. The median value is exceeded in only 5.3% of the SHM-only simulations.

FIG. 5: Confidence level at which the SHM-only model may be excluded when the true halo contains a stream of density fraction $\xi$. Exclusion levels are shown for streams with velocity dispersions $v_0 = 0$ (solid red) and 15 km/s (dashed blue). Exclusion level is defined as the median $\Delta \chi^2$ as determined from MC simulations. In other words, there is a 50% chance that the CoGeNT results will exclude the SHM-only model at the given CL or better. The horizontal dashed line represents the 2$\sigma$ level. Thus, the Sagittarius stream is detectable at > 2$\sigma$ with a ∼ 10 kg-year exposure with CoGeNT, provided the velocity dispersion associated with the stream is low, and the stream contributes 3-5% of the local dark matter density.

III. DISCUSSION

In this paper, we studied the ability of a future CoGeNT data set to detect the presence of dark matter streams. We performed Monte Carlo simulations of a halo that consists of both a thermal component, and a cold stream, and fitted 2 models to the simulations: (i) a halo model containing the stream and (ii) the SHM-only model. We then performed simulations of a fully thermal halo (i.e. SHM-only), and fitted the 2 models to the null simulations. We studied the Sagittarius stream as an example, and showed that for stream densities ∼ 3−5% of the local dark matter density, the stream is detectable at the 2$\sigma$ level with an exposure of 10 kg year. Such an exposure is attainable by CoGeNT C-4 within ∼ 3 years. We set the particle mass = 10 GeV, and assumed knowledge of the stream velocity. Let us now briefly consider variations in these parameters.

Varying the particle mass: The particle mass $m_\chi = 10$ GeV provides an acceptable fit to the CoGeNT observation, but lies at the high end of the mass range for $v_0 = 220$ km/s. As the mass is lowered, we lose sensitivity to the stream (for $v_0 = 220$ km/s) and for $m_\chi < 8$ GeV, the Sagittarius stream becomes almost completely invisible as recoil events fall entirely below the energy threshold of 0.47 keVee. This is however, dependent on the assumed values of $v_0$ and $v_{esc}$. We have verified with our simulations that for $m_\chi \geq 8.5$ GeV, we are able to detect the presence of the stream. Other high velocity streams should be visible for smaller WIMP masses.

Varying the stream parameters: In order to test the importance of our knowledge of the stream parameters, we perform fits with a random component added to the stream velocity. The stream speed relative to the sun is chosen at random to lie between ±50 km/s from the true value, while the two angles that describe the stream arrival direction are chosen to lie between ±20 degrees of the true direction. This represents a small uncertainty in our knowledge of the stream parameters. We performed fits to 1000 MC simulations of the SHM+5% Sagittarius stream, with 25 such random velocities, and obtained $\chi^2_{min}$ values ranging from 8.4 to 12.1, with a median value.
of 9.3 with 9 d.o.f. By comparison, knowledge of the true stream parameters yielded a $\chi^2_{\text{min}}$ of 8.2/9 d.o.f. The SHM-only model resulted in a substantially worse fit (for $\xi=0.05$), with a $\chi^2_{\text{min}}$ of 16.7/10 d.o.f. We thus conclude that an approximate knowledge of the stream parameters is still useful when analyzing data from experiments.

We have shown that dark matter streams are potentially detectable by the future CoGeNT C-4 experiment. Ignoring the presence of streams may result in erroneous estimates of $m_\chi$ and $\sigma_p$. For sufficiently large exposures, the annual modulation provides additional information. The annual modulation is an excellent indicator of streams \[3, 4\], and should reveal the presence of the stream at a measured energy near the cutoff energy of the stream. Reconstructing the stream parameters for arbitrary dark matter streams using a combination of the percentage modulation and the total number of recoils will enable us to understand the phase space distribution of dark matter in the solar neighborhood. In previous work \[3, 4\], we showed that streams can alter the phase and structure of the annual modulation. In certain energy bins, the presence of the stream may only be apparent during part of the year, when the stream speed relative to the earth is largest. In these energy bins, the annual modulation due to a $\sim$ few percent stream may result in an annual modulation of a few percent, comparable to the contribution of the entire Maxwellian halo.

Observing the variation in the amplitude of the annual modulation in different energy bins provides valuable information regarding the particle mass [2, 44, 51–53]. The phase of CoGeNT’s annual modulation does not fit the SHM very well and in our next analysis we will investigate whether streams lead to a better fit. We plan to undertake this work at a later stage.

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While the likelihood ratio test statistic is expected to follow a $\chi^2$-distribution in many cases, we note that the distribution here is not actually very well fit by any $\chi^2$-distribution due to the physicality condition $0 \leq \xi \leq 1$, which leads to a pile-up at $\xi = 0$ in the fits (42% of the simulations are best fit by $\xi = 0$).