Minimal Immersions and the Spectrum of Supermembranes

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Abstract

We describe the minimal configurations of the compact $D = 11$ Supermembrane and D-branes when the spatial part of the world-volume is a Kähler manifold. The minima of the corresponding hamiltonians arise at immersions into the target space minimizing the Kähler volume. Minimal immersions of particular Kähler manifolds into a given target space are known to exist. They have associated to them a symplectic matrix of central charges. We reexpress the Hamiltonian of the $D = 11$ Supermembrane with a symplectic matrix of central charges, in the light cone gauge, using the minimal immersions as backgrounds and the $Sp(2g, \mathbb{Z})$ symmetry of the resulting theory, $g$ being the genus of the Kähler manifold. The resulting theory is a symplectic noncommutative Yang-Mills theory coupled with the scalar fields transverse to the Supermembrane. We prove that both theories are exactly equivalent. A similar construction may be performed for the Born-Infeld action. Finally, the noncommutative formulation is used to show that the spectrum of the regularized Hamiltonian of the above mentioned $D = 11$ Supermembrane is a discrete set of eigenvalues with finite multiplicity.

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1 Introduction

Recently, the spectrum of the regularized compactified $D = 11$ Supermembrane \cite{2} with non-trivial central charge, or equivalently the Supermembrane with non-trivial wrapping on the compactified directions of the target space, was shown to be a discrete set of eigenvalues with finite multiplicity \cite{1, 3} see also \cite{4, 5}. The spatial world-volume was assumed to be a torus while the compactified part of the target space was taken as $S^1 \times S^1$. The regularization was performed using the matrix model constructed in \cite{6}. It was proven several years ago that the spectrum of the regularized $D = 11$ Supermembrane in a Minkowski target space is continuous from $[0, \infty)$ \cite{7, 8}. Meanwhile the spectrum of the compactified $D = 11$ Supermembrane, including configurations without non-trivial central charges, would be continuous according to \cite{9}, although there is no rigorous proof in this case.

The Hamiltonian of the compactified $D = 11$ Supermembrane with non-trivial central charge has local minima for each winding of the Supermembrane, characterized by an integer $n$, on the compactified directions. When we consider a fixed $n$ the one-forms $dX^r$, where $X^r$ are the maps from the world-volume to the target space, may be decomposed into the minima of the Hamiltonian $\hat{X}^r$ plus an exact one form describing the degrees of freedom of the Supermembrane with non-trivial central charge $n$. The resulting action for those degrees of freedom was shown to be a symplectic noncommutative Yang-Mills theory on the spatial world-volume $\mathcal{M}$ \cite{10, 11}. The equivalence between the Supermembrane with non-trivial central charge and the symplectic noncommutative Yang-Mills theory is exact, there is no approximation in the construction.

In this paper we generalize this approach. We look for non-trivial minima of the hamiltonians of the $D = 11$ Supermembrane and D-branes \cite{12}. Those are finite action solutions of the field equations. We completely characterize those solutions when $\mathcal{M}$ is a Kähler manifold. It was already shown in \cite{13} that the extended self-dual configurations are finite action solutions of the Born-Infeld theory, where the metric on $\mathcal{M}$ is taken fixed and equal to the one constructed from the two-form curvature and the almost complex structure in the usual way. The minimal solutions correspond to particular solutions saturating the Bogomolnyi bound. States saturating this bound were extensively used in the analysis of string dualities \cite{14, 15, 16}. In our analysis of the quantum spectrum of the hamiltonian of the supermembrane we will use very specific properties of the solutions which were derived mainly in the mathematical literature. After the analysis of the minimal solutions
we will consider the Supermembrane. Once we have characterized the minimal solutions we will consider the Supermembrane on this background and show that the resulting theory is a symplectic noncommutative Yang-Mills theory on the compactified directions of the target space which we will consider to be a flat torus. We do so, because in these cases the minimal solutions can be explicitly constructed. The central charges associated to those solutions are expressed as a symplectic matrix times the non-trivial winding of the membrane. We also prove in those cases that the quantization of the Supermembrane with a symplectic matrix of central charge is exactly equivalent to the quantization of the symplectic noncommutative theory and show that their regularized hamiltonians have discrete spectrum. The analysis of the minimal solutions for other compactifications like a $G_2$ holonomy manifold, where $G_2$ is the automorphism group of the octonions, which are very relevant in M-theory is under investigation.

2 Minimal solutions

The extended self-dual configurations constructed in [13] were defined on manifolds of even dimension $2n$ where a curvature two-form satisfy the conditions

$$^*P_m \sim P_{n-m}$$

for all possible $m$, where $P_m = F \wedge F \wedge \ldots \wedge F$ ($m$ factors) and $^*$ denotes the Hodge dual. That is, the set $\{P_m\}$ is left invariant by the Hodge dual operation. In four dimensions ($n = 2$) it reduces to $^*(F \wedge F) =$ constant and $^*F \sim F$ the well known self duality condition which solves the Maxwell equations. In two dimensions $^*F =$ constant represents a monopole configuration.

In [13] it was shown that those configurations solve the field equations of the Born-Infeld action. Moreover, it was proven they are minima of the corresponding Hamiltonian.

In this paper we will consider the spatial world-volume $\mathcal{M}$ to be a Kähler manifold. The Kähler form is an extended self-dual configuration with the corresponding Kähler metric, hence, at least as the field strength is concern, it is a solution of the Born-Infeld field equation. This is a good motivation to consider the Born-Infeld action over Kähler manifolds. Also, most of the work on the $D = 11$ Supermembrane has been performed for the case $\mathcal{M}$ is a Riemann surface, which are Kähler manifold. We then restrict our analysis to that case.
The Born-Infeld action of a D-brane [17] is a functional of two fields over the D-brane world-volume \( M \); they are the set of components of a map \( M \to N \), where \( N \) is the target space, and an \( U(1) \) gauge field \( A \). The action is

\[
S[X, A] = \int_M d^{p+1}\sigma \sqrt{-\det E_{\hat{\mu}\hat{\nu}}} \tag{1}
\]

where

\[
E_{\hat{\mu}\hat{\nu}} = G_{\hat{\mu}\hat{\nu}} + F_{\hat{\mu}\hat{\nu}} \tag{2}
\]

\[
G_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}}X^M \partial_{\hat{\nu}}X^N g_{\hat{M}\hat{N}}(X) \tag{3}
\]

\[
F_{\hat{\mu}\hat{\nu}} = \partial_{\hat{\mu}}A_{\hat{\nu}} - \partial_{\hat{\nu}}A_{\hat{\mu}} \tag{4}
\]

\( \hat{M} = 0, M \), \( \hat{M} = 1, \ldots, \dim N - 1 \) \( \hat{\mu} = 0, \mu \), \( \mu = 1, \ldots, p \)

\( g_{\hat{M}\hat{N}} \) denotes a metric on \( N \) and \( X^\hat{M} \) are the components of the map \( X \).

Equations of motion derived from (1) are

\[
(E^{-1})^{\hat{\mu}}_{\hat{\nu}} \partial_{\hat{\mu}}X^\hat{M} + \frac{1}{\sqrt{-\det E_{\hat{\sigma}\hat{\rho}}}} \partial_{\hat{\mu}} \left[ \sqrt{-\det E_{\hat{\sigma}\hat{\rho}}(E^{-1})^{\hat{\mu}}_{\hat{\nu}}} \right] \partial_{\hat{\nu}}X^\hat{M} 
\]

\[
+ (E^{-1})^{\hat{\mu}}_{\hat{\nu}} \partial_{\hat{\mu}}X^\hat{L} \partial_{\hat{\nu}}X^\hat{K} \Gamma^M_{KL} = 0 \tag{5}
\]

\[
\partial_{\hat{\mu}} \left[ \sqrt{-\det E_{\hat{\sigma}\hat{\rho}}(E^{-1})^{\hat{\nu}}_{\hat{\mu}}} \right] = 0 \tag{6}
\]

where \((E^{-1})_+\) and \((E^{-1})_-\) are the symmetric and antisymmetric parts of \( E^{-1} \) respectively and \( \Gamma^M_{KL} \) are the Christoffel symbols of the metric \( g \) on the target. The first (second) equation is obtained by taking variations of (1) with respect to \( X^\hat{M}(A_{\hat{\mu}}) \).

We assume the foliation of the world-volume \( M = \mathcal{M} \times \mathbb{R} \), where \( \mathcal{M} \) is an Riemannian manifold. The first step in constructing our solutions is to impose the following conditions on the fields

\[
X^0 = \sigma^0 \hspace{1cm} X^M = X^M(\sigma^\alpha) \tag{7}
\]

\[
g_{00} = -1 \hspace{1cm} g_{0M} = 0 \tag{8}
\]

\[
A_0 = 0 \hspace{1cm} A_\mu = A_\mu(\sigma^\alpha) \tag{9}
\]

The resulting equations over \( \mathcal{M} \) are

\[
(E^{-1})^{\mu\nu} \partial_\mu X^M + \frac{1}{\sqrt{\det E_{\sigma\rho}}(E^{-1})^{\mu\nu}} \partial_\mu \left[ \sqrt{\det E_{\sigma\rho}}(E^{-1})^{\nu\mu} \right] \partial_\nu X^M 
\]

\[
+ (E^{-1})^{\mu\nu} \partial_\mu X^L \partial_\nu X^K \Gamma^M_{KL} = 0 \tag{10}
\]

\[
\partial_\mu \left[ \sqrt{\det E_{\sigma\rho}(E^{-1})^{\nu\mu}} \right] = 0 \tag{11}
\]
We also suppose the target space to be the product of a Minkowski manifold times a compact Riemann manifold $\mathcal{N}$. For the solution we fix those components of the map corresponding to the Minkowski manifold equal to zero. Therefore, the remaining degrees of freedom are $X^r$, $r = 1, \ldots, \dim\mathcal{N}$, and $A_\mu$, being both fields over $\mathcal{M}$. We consider $X^r$ to be the components of an immersion $\mathcal{M} \to \mathcal{N}$, hence, we assume that $G_{\mu\nu} = \partial_\mu X^r \partial_\nu X^s g_{rs}$ is a Riemannian metric on $\mathcal{M}$.

Let $\mathcal{M}$ be a Kähler manifold of real dimension $p = 2n$. Let $J$, $K$ and $\Omega (U, V) = K (JU, V)$ be the almost complex structure, the Kähler metric and the Kähler form of $\mathcal{M}$ respectively. We suppose that $A_\mu$, $X^M$ and $g_{MN}$ verify

$$G = K, \quad F = \Omega. \tag{12}$$

If (12) is verified, the Born-Infeld tensor becomes $E_{\sigma\rho} = (\delta^\alpha_\sigma + J^\alpha_\sigma) K_{\alpha\rho}$ and then we get the remarkable results

$$\det E_{\sigma\rho} = 2^n \det K_{\sigma\rho} \tag{13}$$
$$\left(E^{-1}\right)^{\mu\nu}_+ = \frac{1}{2} K^{\mu\nu}, \quad \left(E^{-1}\right)^{\mu\nu}_- = -\frac{1}{2} \Omega^{\mu\nu} \tag{14}$$

where Greek indices are raised and lowered with the Kähler metric.

Note that eqs. (11) are automatically solved by (12). The fact that the Kähler geometry ansatz (12) solves the Born-Infeld equation (11) is a particular case of the most general result about extended self-dual configurations, as we mentioned previously. Moreover, the Kähler form of any Kähler manifold may be locally expressed in terms of the Kähler potential $K$, $\Omega = i \partial \bar{\partial} K$ where $\partial$ and $\bar{\partial}$ are Dolbeault operators. This ensures the local existence of an one-form

$$\alpha \equiv \frac{i}{2} (\bar{\partial} - \partial) K \tag{15}$$

which locally verifies $\Omega = d\alpha$, hence $\alpha$ may be taken as $A$.

Under (12), equations (10) yields

$$K^{\nu\mu} \partial_\nu X^r - K^{\mu\nu} \partial_\sigma X^r \gamma^\sigma_{\mu\nu} + K^{\nu\mu} \partial_\nu X^t \partial_\nu X^u \Gamma^r_{tu} = 0 \tag{16}$$

where $\gamma$ is the Christoffel symbol of the Kähler metric $K$. The first two terms form the Laplacian defined with the Kähler metric for each component $X^r$. It is straightforward to show that (10) and (11) are exactly the same equations to be satisfied by the minima of the Hamiltonian of the Born-Infeld action. Hence, (12) and (16) describe the minima of the Hamiltonian over a Kähler spatial world-volume.
Equation (16) can be derived from the volume action
\[
\int_{\mathcal{M}} d^p \sigma \sqrt{\text{det} K_{\sigma \rho}}
\]
where
\[
K_{\mu \nu} = \partial_\mu X^r \partial_\nu X^s g_{rs}.
\]
Therefore, the solutions for the remaining equation (16) represent actually minimal immersions, in the mathematical nomenclature, of the Kähler manifold in the target space.

The condition of being an immersion is a nontrivial one. The search for minimal immersions, which in turn are directly related to harmonic maps between Riemannian manifolds, has been followed in the mathematical literature [18]. Interesting results has been obtained when the target space is a flat torus. In this case minimal immersions of certain Kähler manifolds are known to exist. An explicit example is the Albanese map of a compact Kähler manifold \(\mathcal{M}\) whose cotangent bundle is ample [19, 20]. In particular the Albanese map of a Riemannian surface of positive genus into a flat torus is a minimal immersion. This minimal immersion is the one we will consider in the construction of a noncommutative gauge theory.

3 Noncommutative Yang-Mills from compactified \(D = 11\) membranes

In this section we will show that using the minimal immersion as a background we can reexpress the Born-Infeld action or the \(D = 11\) Supermembrane action as a symplectic noncommutative Yang-Mills theory. We will perform explicit calculations for the \(D = 11\) Supermembrane, although the same arguments follow directly for the Born-Infeld action.

We consider the Hamiltonian of the \(D = 11\) Supermembrane in the light cone gauge [21, 22]. Its bosonic part is given by
\[
\mathcal{H} = \frac{1}{2 \sqrt{W}} [P_m P^m + \text{det} (\partial_\mu X^m \partial_\nu X_m)]
\]
subject to the area preserving diffeomorphisms constraints
\[
\epsilon^{\mu \nu \rho} \partial_\mu \left( \frac{1}{\sqrt{W}} P_m \partial_\nu X^m \right) = 0
\]
where \(X^m, m = 1, \ldots, 9\) are maps from \(\mathcal{M}\) a Riemannian surface of genus \(g\) to a flat target space. \(P_m\) are its conjugate momenta, the transverse indices
$m$ are raised and lowered with the flat metric. $\sqrt{W}$ is a scalar density introduced in the Light Cone Gauge fixing procedure.

The local isolated minima of the Hamiltonian $H$ are immersions satisfying

$$P_m = 0$$

and

$$G^{\mu\nu} \partial_\mu \partial_\nu X^m - G^{\mu\nu} \partial_\sigma X^m \gamma^\sigma_{\mu\nu} + G^{\mu\nu} \partial_\mu X^l \partial_\nu X^k \Gamma^m_{kl} = 0$$

the same equations (16), the third term being zero in the case we consider a flat target space. In this particular case this equation, when rewritten in terms of a local complex coordinate $z$ and its complex conjugate, reduces to

$$\partial_z \partial_{\bar{z}} X^m = 0.$$  

Hence, the minimal immersions are harmonic maps. The condition of having an immersion is in any case a nontrivial one.

We will assume that the target space is compactified and that the Supermembrane has non-trivial winding over it. Although several of our arguments are general we will specifically consider Supermembranes with non-trivial winding over $M_9 \times S^1 \times S^1$ and $M_7 \times S^1 \times S^1 \times S^1$, where $M_9$ and $M_7$ are Minkowski spaces while $S^1 \times S^1$ and $S^1 \times S^1 \times S^1 \times S^1$ are flat torus $T$. The case $M_9 \times S^1 \times S^1$ was considered extensively in [4, 10, 3, 1]. A rigorous proof was given showing that the spectrum of the $D = 11$ Supermembrane with a fixed non-trivial central charge is discrete. The analysis for more general target spaces requires the introduction of a minimal immersion of $M$ onto $T$. This ingredient of the construction becomes straightforward in the case $T$ is $S^1 \times S^1$, and is implicit in the previous works. However, it becomes a nontrivial step in the construction for more general compactified target spaces.

The condition describing a map from $M$ to $M \times T$ is expressed by

$$\oint_C dX^r = m^r$$  (18)

where $m^r$ are integers, while $C$ represents a basis of homology over $M$. $X^r$ denote the compactified directions on the target space. The topological index describing the condition of nontrivial winding is the integral over $M$ of the pull-back of the symplectic form on the torus $T$:

$$\int_\Sigma dX^r \wedge dX^s = 2\pi n \omega^{rs}$$  (19)
where $\omega^{rs}$ is the canonical symplectic matrix on $T$. The area of $M$ has been normalized to $2\pi$. We will consider all maps satisfying conditions (18) and (19). They ensure the membrane has non-trivial central charges.

We know, from the previous section, the existence of a minimal immersion from any compact Riemann surface of genus $g > 0$ to the corresponding Jacobian variety. This map is also known in the literature as the Albanese map. In order to be specific, we will consider $M$ to be a Riemann surface of genus 2 such that its Jacobian variety is $S^1 \times S^1 \times S^1 \times S^1$. The minimal immersion is obtained from the harmonic one-forms $d\hat{X}^r$, $r = 1, \ldots, 4$, over $M$ satisfying condition (19). They have the property that

$$\partial_\mu \hat{X}^r \partial_\nu \hat{X}^s \omega_{rs} d\sigma^\mu \wedge d\sigma^\nu$$

where $\sigma^\mu$, $\mu = 1, 2$, are local coordinates on $M$, is the Kähler two-form $\Omega$ over $M$ satisfying

$$^*\Omega = n$$

(20)

where the Hodge dual is constructed with the Kähler metric

$$K_{\mu\nu} = \partial_\mu \hat{X}^r \partial_\nu \hat{X}^s \delta_{rs} \, .$$

We may now consider a general map from $M$ to $T$, it can always be decomposed as

$$dX^r = m^r_s d\hat{X}^s + \delta^{rs} dA_s \, ,$$

(21)

where as before $d\hat{X}^s$, $s = 1, \ldots, 4$, denotes a basis of harmonic one-forms realizing the minimal immersion from $M$ to $T$ while $dA_s$ are exact one forms. $m^r_s$ must be integers in order to satisfy conditions (18). Moreover, from (19), they must satisfy

$$m^t_r m^u_s \omega_{tu} = \omega_{rs} \, ,$$

hence the matrices with elements $m^r_s$ belong to $Sp(4,\mathbb{Z})$ the modular group for a Riemann surface of genus 2. We notice from (20) that the diffeomorphisms over $M$ which change the basis of harmonic one-forms by elements of $Sp(4,\mathbb{Z})$ preserve $\sqrt{W}$ since

$$\epsilon^{\mu\nu} \partial_\mu \hat{X}^r \partial_\nu \hat{X}^s \omega_{rs} = n\sqrt{W} \, ,$$

where $W$ has been identified with the determinant of the Kähler metric over $M$, hence those diffeomorphisms are gauge symmetries of the theory. We may then consider in (21) a fixed basis of harmonic one forms satisfying condition (19). That is we can always consider

$$dX^r = d\hat{X}^r + \delta^{rs} dA_s \, .$$

(22)
The degrees of freedom of the $D = 11$ membrane satisfying the topological condition (19) are then described by the exact one-forms $dA_r (\sigma)$ and $dX^m$, $r = 1, \ldots, 4$, $m = 1, \ldots, 5$, modulo the area preserving gauge transformations.

We now replace (22) into the Hamiltonian (17). The action of the Supermembrane is then exactly expressed as the value of the Supermembrane action at the minimal immersion, a finite action solution of the field equations plus a positive Hamiltonian for the $A_r$ and $X^m$, $r = 1, \ldots, 4$, $m = 1, \ldots, 5$, fields. It turns out that this is the pull-back to $\Sigma$ of a symplectic noncommutative Yang-Mills theory in $M \times T$.

We first note that the kinetic terms $P\dot{X}$ may be rewritten as

$$ P_r \dot{X}^r + P_m \dot{X}^m = (\delta^{rs} P_s) \dot{A}_r + P_m \dot{X}^m $$

since $\dot{X}^r$ do not depend on $\tau$. We will denote $\pi^r \equiv \delta^{rs} P_s$ the conjugate momenta to $A_r$. We then introduce the derivatives

$$ D_r \equiv D_r + \{A_r, \} $$

where

$$ D_r = e^\mu_r \partial_\mu , $$
$$ e^\mu_r \equiv \Omega^{\mu\nu} \partial_\nu \dot{X}^s \delta_{sr} $$

and

$$ \{\phi, \varphi\} = \Omega^{\mu
u} \partial_\mu \phi \partial_\nu \varphi = \frac{2}{n} \omega^{rs} D_r \phi D_s \varphi . $$

The symplectic noncommutative curvature is then given by

$$ F_{rs} = D_r A_s - D_s A_r + \{A_r, A_s\} . $$

The final form of the bosonic Hamiltonian density, in terms of the above geometrical objects, is

$$ \mathcal{H} = \frac{1}{2\sqrt{W}} \left( P_m P^m + \pi^r \pi_r + W D_r X^m D^r X_m + \frac{1}{2} W F_{rs} F^{rs} ight. $$
$$ + \frac{1}{2} W \{X^m, X^n\}^2 \left. + \frac{1}{8}\sqrt{W} n^2 + \lambda (D_r \pi^r + \{X^m, P_m\}) \right) $$

where indices are raised and lowered with the corresponding flat metrics. $\lambda$ is the Lagrange multiplier associated to the volume preserving constraint which is now interpreted as the Gauss constraint for the noncommutative formulation of the $D = 11$ Supermembrane with non-trivial fixed central charge.
The above Hamiltonian density has the same expressions as the one analyzed in [3] although the range of indices are different. Following [3] one may show: i) The non-existence of string-like spikes for $\mathcal{H}$. ii) The regularized Hamiltonian has a potential bounded below and becoming infinite at infinity in every possible direction on the configuration space. Consequently, using lemma 1 and the results of section 5 and the general criteria discussed in the conclusions of [1], one can show that the spectrum of the complete quantum regularized Hamiltonian, including the fermionic terms, is a discrete set of eigenvalues with finite multiplicity.

If we now integrate out the conjugate momenta $P_m$ and $\pi^r$ from the corresponding functional integral we obtain:

$$
\mathcal{L} = -\sqrt{W} \left[ \frac{1}{2} (\mathcal{D}_r X^m)^2 + \frac{1}{4} (\mathcal{F}_{\hat{r}})^2 + \frac{1}{4} \{X^m, X^n\}^2 \right] - \frac{1}{8} \sqrt{W} n^2
$$

where

$$
\hat{r} = 0, r , \quad r = 1, \ldots, 4
$$

and $\mathcal{D}_r$, $\mathcal{F}_{rs}$ are defined as before. The final expression of the Lagrangian is the pull-back to $\mathcal{M}$ of a noncommutative Yang-Mills in the space-time constructed with the light cone $\tau$ and the compactified subspace of the target space.

4 Conclusions

We described the minimal configurations of the compact Supermembrane and D-brane theories, when the spatial part of the world-volume $\mathcal{M}$ is a Kähler manifold. We showed that the minima of their hamiltonians take place at immersions from $\mathcal{M}$ into the target space minimizing the Kähler volume. The explicit construction of some minimal immersions of Kähler manifold with ample cotangent bundle has been described in the mathematical literature. We follow that construction for the case the target space is the product of a Minkowski space times a flat torus. We prove that a $D = 11$ Supermembrane with non-trivial central charges on a target space of that kind is exactly equivalent to the pullback of a symplectic noncommutative Yang-Mills theory over the compactified sector of the target space coupled to the transverse scalar fields. The construction makes explicit use of the
minimal immersion which is taken as background to reexpress the Supermembrane action as a symplectic noncommutative one. We emphasize that there is no approximation in the construction. We expect the same result for any target space of the form $M \times T$ provided there exists a minimal immersion from $M$ into $T$.

The analysis was performed for the bosonic sector of the Supermembrane. The addition of the fermionic terms does not change any of the results. Moreover, applying the propositions proven in [1] we conclude that the spectrum of the regularized $D = 11$ Supermembrane with target space $M \times T$, $T = S^1 \times S^1 \times S^1 \times S^1$ and with a symplectic matrix of central charges given by eq. (19) consists of a discrete set of eigenvalues with finite multiplicity.

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