Obtaining $|V_{ub}|$ exclusively: a theoretical perspective

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1 Introduction

Recent inclusive determinations of $|V_{ub}|$ have uncertainties of approximately 10% [1], as opposed to $\lesssim 2\%$ on $|V_{cb}|$ via $B \to X_s l \nu$ [2]. Exclusive channels provide a competitive alternative route to $|V_{ub}|$, but although experimentally more promising this requires information about hadronic matrix elements via form factors. Form factors are calculable via non-perturbative techniques such as Lattice QCD (see e.g. refs. [3]) or QCD sum rules on the light-cone (LCSR). Predictions are usually confined to a particular region of $q^2$, the momentum transfer squared, i.e. LCSR and Lattice are restricted to large and small recoil energies of the daughter hadron respectively. In LCSR one considers a correlator $\Pi_\mu$ of the time-ordered product of two quark currents, sandwiched between the final state hadron, which is on shell, and the vacuum [4], i.e. for a $B$ decaying to a $\pi$ of momenta $p_B$ and $p$,

$$\Pi_\mu = i m_b \int d^D x e^{-i p_B x} \langle \pi(p) | T \{ \bar{u}(0) \gamma_\mu b(0) \bar{b}(x) i \gamma_5 d(x) \} | 0 \rangle.$$ (1)

This can be expressed on one hand by a light-cone expansion via perturbative hard scattering kernels convoluted with non-perturbative light-cone distribution amplitudes (LCDAs), ordered in increasing twist, or by inserting a sum over excited states, i.e. the $b$ hadron and a continuum of heavier states. Assuming quark hadron duality above a certain continuum threshold $s_0$, one can subtract this continuum contribution from both sides. Borel transforming this relation then ensures that this assumption, and the truncation of the series, have a minimal effect on the resulting sum rule. At present, $|V_{ub,excl}|$ is obtained most precisely from $B \to \pi l \nu$, where in the limit of massless leptons the decay rate for $B \to \pi$ depends on a single form factor $f_+(q^2)$. However by considering other channels, e.g. baryonic decays such as $\Lambda_b \to p l \nu$, one can obtain interesting complementary information[5]. Here I will discuss recent progress in the calculation of the form factors for $B \to \pi l \nu$ [6] and $\Lambda_b \to p l \nu$ [8] using LCSR.

\footnote{In the limit of massless leptons the decay rate for $\Lambda_b \to p l \nu$ depends on four form factors, $f_{1,2}(q^2)$ and $g_{1,2}(q^2)$.}
2 Recent LCSR updates on \( f_+(q^2) \) for \( B \to \pi l \nu \)

There has been much progress in the LCSR calculations of \( f_+(q^2) \) in the last 15 years. The next-to-leading order (NLO) corrections to \( f_+(q^2) \) at leading twist (twist-2) were first calculated in LCSR in ref. [5] and LO corrections up to twist-4 were calculated in ref. [6]. Since the LO twist-3 contribution was found to be large, it was confirmed that the NLO corrections are under control, using both the pole and \( \overline{\text{MS}} \) mass for \( m_b \) [7]. In ref. [8], different values for the moments of the twist-2 LCDA were employed, extracted from latest experimental data for \( F_\pi \) using LCSR. The normalised decay rate integrated over a given range in \( q^2 \),

\[
\Delta \zeta (0, q^2_{\text{max}}) = \frac{1}{|V_{ub}|^2} \int_{0}^{q^2_{\text{max}}} dq^2 \frac{d\Gamma}{dq^2} (\Lambda_b \to pl \nu),
\]

was then predicted to be \( \Delta \zeta (0, 12 \text{GeV}^2) = 4.59^{+1.00}_{-0.88} \text{ps}^{-1} \), which can be combined with experimental predictions, allowing the extraction of \( |V_{ub}| \).

Two-loop corrections to the form factor \( f_+(q^2) \) at twist-2 were recently calculated in ref. [9]. In light of the large two-loop sum rules corrections to \( f_B \) calculated in ref. [10], one aim of this work was to test the argument that, in obtaining \( f_+(q^2) \) via LCSR, radiative corrections to \( f_+ f_B \) and \( f_B \) should cancel when both calculated in sum rules. Due to the technical challenges posed by a full calculation, a subset of two-loop radiative corrections for twist-2 contribution to \( f_+(0) \) proportional to \( \beta_0 \) was considered, as this gauge invariant subset is thought to be a good approximation to the complete next-to-next-to-leading order (NNLO) result. In combination with the experimental result for \( f_+(0) |V_{ub}| \) one can then obtain \( |V_{ub}| \). The necessary diagrams are obtained by inserting a fermion bubble in the gluon propagator of the NLO twist-2 diagrams, further details can be found in ref. [9]. The results for \( f_+(0) \), seen in fig. 1, show that despite the \( \sim 9\% \) positive NNLO corrections to the QCD sum rules result for \( f_B \), the LCSR prediction for \( f_+(0) \) is stable, increasing by \( \sim 2\% \) to \( f_+(0) = 0.261^{+0.020}_{-0.023} \), as shown in fig. 1. This enforces the stability of LCSR with respect to higher order corrections, and could be taken to provide confirmation that \( f_B \) from sum rules, not Lattice should be used here. A recent analysis by BaBar [11] finds \( |V_{ub}| = (3.34 \pm 0.10 \pm 0.05^{+0.29}_{-0.26}) \times 10^{-3} \) using this result, and \( |V_{ub}| = (3.46 \pm 0.06 \pm 0.08^{+0.37}_{-0.32}) \times 10^{-3} \) using \( \Delta \zeta (0, 12 \text{GeV}^2) \) from ref. [8], which are clearly in good agreement.

3 Improvements on form factors for \( \Lambda_b \to p \) decays

Recently there has been increasing work on extracting \( |V_{ub}| \) via \( \Lambda_b \to pl \nu \). A number of complications arise in LCSR when baryons are considered instead of mesons, the first being the choice of the heavy-light baryon interpolating current \( \eta \) described by
\[ \eta = \epsilon^{ijk}(u_i\alpha\overline{C}\Gamma_b d_j)\tilde{\Gamma}_b c_k, \]  

(3)
debated since the 1980s. Additionally, the contribution of the negative parity \( \Lambda_b^* \) baryon, with \( J^P = 1/2^- \), which has a similar mass to \( \Lambda_b \), is difficult to isolate, and in the literature was often included in the continuum [12]. Recently however it was found to be possible to separate the \( \Lambda_b^* \) from the \( \Lambda_b \) contribution in the sum rule, and on comparing results for both \( \Gamma_b = \gamma \gamma_5\gamma \lambda \) and \( \tilde{\Gamma}_b = 1\gamma \lambda \), it was found that the resulting form factors show a reduced dependence on the choice of \( \Gamma_b \) and \( \tilde{\Gamma}_b \) [13].

## 4 Summary and Outlook

Recent progress on the LCSR calculation of form factors for the exclusive determination of \( |V_{ub}| \) was presented. This included recent updates on \( f_+(q^2) \): the 2011 NLO analysis in the \( \overline{MS} \) scheme resulted in \( |V_{ub}| = (3.46 \pm 0.06 \pm 0.08^{+0.37}_{-0.32}) \times 10^{-3} \) and the 2012 \( \mathcal{O}(\alpha_s^2\beta_0) \) result found a \( \sim 2\% \) increase in \( f_+(0) = 0.262^{+0.020}_{-0.023} \), such that \( |V_{ub}| = (3.34 \pm 0.10 \pm 0.05^{+0.29}_{-0.26}) \times 10^{-3} \). New results for the form factors for \( \Lambda_b \rightarrow p\ell\nu \) were also discussed, where it was showed that by isolating and removing the negative parity baryons’ contribution, the form factors show a reduced dependence on the choice of \( \Gamma_b \) and \( \tilde{\Gamma}_b \). Future work should focus on combining the \( \mathcal{O}(\alpha_s^2\beta_0) \) \( f_+(0) \) and Lattice results to determine \( |V_{ub}| \) and calculating remaining twist-2 NNLO corrections to \( f_+(q^2) \) and gluon radiative corrections to the \( \Lambda_b \rightarrow p \) form factors.
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