Modelling of a Permanent Magnet Synchronous Machine Using Isogeometric Analysis

Prithvi Bhat, Herbert De Gersem
Technische Universität Darmstadt
Darmstadt, Germany

Zeger Bontinck, Sebastian Schöps
Theorie Elektromagnetischer Felder
Graduate School of CE
Technische Universität Darmstadt
Darmstadt, Germany

Jacopo Corno
Theorie Elektromagnetischer Felder
Graduate School of CE
Technische Universität Darmstadt
Darmstadt, Germany

Abstract—Isogeometric Analysis (IGA) is used to simulate a permanent magnet synchronous machine. IGA uses Non-Uniform Rational B-splines (NURBS) to parametrise the domain and to approximate the solution space, thus allowing for the exact description of the geometries even on the coarsest level of mesh refinement. Given the properties of the isogeometric basis functions, this choice guarantees a higher accuracy than the classical Finite Element Method (FEM).

For dealing with the different stator and rotor topologies, the domain is split into two non-overlapping parts on which Maxwell’s equations are solved independently in the context of a classical Dirichlet-to-Neumann domain decomposition scheme. The results show good agreement with the ones obtained by the classical finite element approach.

Index Terms—Electromagnetic field simulation, Permanent magnet machines, Numerical analysis, Finite element method, Isogeometric analysis

I. INTRODUCTION

Electric machines are usually modelled through the magnetostatic approximation of Maxwell’s equations discretised on the machine cross section, which requires the solution of a 2D Poisson problem. Its numerical solution commonly amounts to triangulating the domain and applying the FEM. The acting electromotive forces and the torque can be determined by a post-processing procedure, e.g. invoking the electromotive force (EMF). The higher order regularity of FEM solutions is typically limited by the C0 continuity across the elements [3].

An exact parametrisation of the air gap is not possible within a classical FEM framework since elements of any order rely on polynomial mappings which are unable to represent conic sections such as circles and ellipses. Furthermore, the regularity of FEM solutions is typically limited by the C0 continuity across the elements [3].

IGA [4] is able to overcome these issues. It chooses Computer Aided Design (CAD) basis functions such as e.g. B-splines and NURBS for the approximation spaces and is thus able to represent CAD geometries exactly even on the coarsest level of mesh refinement. A higher regularity at the mesh interfaces can also be obtained by applying k-refinement, which results in smoother basis functions compared to FEM and increases the accuracy of the solution accordingly [5]. Another benefit of IGA is that it provides an elegant way to do geometric optimisation [6] and to handle uncertainties [7].

This work aims at using IGA to model and simulate a permanent magnet synchronous machine (PMSM). The stator and rotor models parts are inevitably resolved by multiple patches, which in turn necessitates the use of a domain decomposition approach across the air gap. We employ a classical iterative procedure based on a Dirichlet-to-Neumann map [8].

In the following section, we introduce the 2D model electric machine model and the IGA method used for its spatial discretisation. In section III, the iterative domain decomposition scheme is explained and, finally, in section IV, we show the results for the simulation of a PMSM.

II. SOLVING MAXWELL’S EQUATIONS FOR A PMSM

A. Magnetostatics

In early design steps, it is sufficient to model electric machines by the 2D magnetostatic approximation of the Maxwell’s equations. Let \( \Omega = \Omega_{st} \cup \Omega_{rt} \) (see Fig. 1) depict the computational domain. One has to solve the following Poisson equation

\[
\begin{align*}
-\nabla \cdot (\nu \nabla A_z) &= J_{src,z} + J_{pm,z}, \\
A_z|_{r_1} &= 0, \\
A_z|_{r_2} &= -A_z|_{r_1},
\end{align*}
\]

where \( \nu = \nu(x,y) \) is the reluctivity, \( A_z = A_z(x,y) \) is the \( z \)-component of the magnetic vector potential, \( J_{src,z} = J_{src,z}(x,y) \) is the source current density in the coils and \( (0,0,J_{pm,z}) = (0,0,J_{pm,z}(x,y)) = -\nabla \times \vec{H}_{pm} \) is the current density according to the magnetisation \( \vec{H}_{pm} = \vec{H}_{pm}(x,y) \) of the permanent magnets. On \( \Gamma_d \), a Dirichlet boundary condition is applied and on \( \Gamma_1 \) and \( \Gamma_2 \), an anti-periodic boundary condition holds.

Applying the loading method [9] on the solution of (1) gives us the spectrum of the electromotive force (EMF). The first harmonic \( E_1 \) is the EMF of the machines.
harmonics are used to calculate the total harmonic distortion (THD), which is defined as

\[
\text{THD} = \sqrt{\frac{\sum_{p=2}^{\infty} |E_p|^2}{|E_1|}},
\]

with \( p \) the harmonic order and \( E_p \) the EMF of order \( p \).

B. Discretisation in the Isogeometric Framework

For the discretisation of \( A_z \), a linear combination of basis functions \( w_j = w_j(x, y) \) is used, i.e.,

\[
A_z = \sum_{j=1}^{N_{\text{DoF}}} u_j w_j,
\]

where \( u_j \) are the unknowns and \( N_{\text{DoF}} \) depicts the number of unknowns. Using the Ritz-Galerkin approach, a system of equations is obtained, i.e.,

\[
K_{\nu} \mathbf{u} = J_{\text{src}} + J_{\text{pm}},
\]

where \( K_{\nu} \) has the entries

\[
k_{\nu,ij} = \int_{\partial \Omega} \left( \nu \frac{\partial w_i}{\partial x} \frac{\partial w_j}{\partial x} + \nu \frac{\partial w_i}{\partial y} \frac{\partial w_j}{\partial y} \right) \, d\Omega,
\]

and the entries for the discretised current density are

\[
j_{\text{src},i} = \int_{\partial \Omega} J_z w_i \, d\Omega,
\]

\[
j_{\text{pm},i} = \int_{\partial \Omega} \bar{H}_{\text{pm}} : \left( -\frac{\partial w_i}{\partial y}, \frac{\partial w_i}{\partial x} \right) \, d\Omega.
\]

The choice of the basis functions depends on the method one favours. The simplest case in the well established FEM is the use of linear hat functions \([10]\). This paper proposes the use of the IGA framework to model the electrical machine. In this method, the basis functions are defined by NURBS \([11]\).

Let \( p \) depict the degree of the basis functions and let

\[
\Xi = [\xi_1 \ldots \xi_{n+p+1}]
\]

be a vector that partitions \([0, 1]\) into elements, where \( \xi_i \in \Omega \equiv [0, 1] \). Then, the Cox-de Boor’s formula \([11]\) defines \( n \)}
where we have removed the indices \( p_1, p_2 \) for simplicity. We will always assume the same polynomial degree in both directions.

This choice of basis functions guarantees a higher regularity of the solution across the elements. The continuity at the patch boundaries reverts to the FEM case since only \( C^0 \) continuity is imposed (see [12]).

III. ITERATIVE STATOR-ROTOR COUPLING

Since the rotor and the stator have a different topology, we parametrise them independently as multipatch NURBS entities with non-conforming patches at the air gap interface \( \Gamma_{ag} \). The idea is to follow a classical non-overlapping domain decomposition approach based on a Dirichlet-to-Neumann map (see e.g. [8]).

Let us consider the circular arc \( \Gamma_{ag} \) in the air gap and split the domain \( \Omega \) such that \( \Omega_{st} \cup \Omega_{rt} = \Omega \) and \( \Omega_{ag} \cap \Omega_{st} = \Gamma_{ag} \) (see Fig. 1). Given \( k = 1 \) and an initialisation \( \lambda^0 \), we solve:

\[
\begin{align*}
-\nabla \cdot (\nu \nabla A_{st}^{k+1}) &= J_z, \\
A_{st}^{k+1}|_{\Gamma_a} &= 0, \\
A_{st}^{k+1}|_{\Gamma_1} &= -A_{st}^{k+1}|_{\Gamma_2}, \\
A_{st}^{k+1}|_{\Gamma_{ag}} &= \lambda^k,
\end{align*}
\]  
(11)

\[
\begin{align*}
-\nabla \cdot (\nu \nabla A_{rt}^{k+1}) &= J_z, \\
A_{rt}^{k+1}|_{\Gamma_a} &= 0, \\
A_{rt}^{k+1}|_{\Gamma_2} &= -A_{rt}^{k+1}|_{\Gamma_1}, \\
\nu \nabla A_{rt}^{k+1}|_{\Gamma_{ag}} \cdot \vec{n}_{ag} &= \nu \nabla A_{st}^{k+1}|_{\Gamma_{ag}} \cdot \vec{n}_{ag},
\end{align*}
\]  
(12)

where \( \vec{n}_{ag} \) is a unit vector perpendicular to the air gap interface. The two problems are solved iteratively with the update

\[
\lambda^{k+1} = \alpha A_{st}^{k+1} + (1 - \alpha) \lambda^k,
\]  
(13)

where \( \alpha \in [0, 1] \) is a relaxation parameter (in general, the method is not ensured to converge if \( \alpha = 1 \) [8]).

As a stopping criterion for the method, the \( L^2 \) error between two successive iterations is required to be below a specified tolerance both in the rotor and the stator, i.e.,

\[
\varepsilon_{rt} = \frac{\| A_{rt}^{k+1} - A_{rt}^{k} \|_{L^2(\Omega_{rt})}}{\| A_{rt}^{k+1} \|_{L^2(\Omega_{rt})}} < tol,
\]

\[
\varepsilon_{st} = \frac{\| A_{st}^{k+1} - A_{st}^{k} \|_{L^2(\Omega_{st})}}{\| A_{st}^{k+1} \|_{L^2(\Omega_{st})}} < tol.
\]

The discretisation of problems (11)-(12) is straightforward in the IGA framework presented above.

IV. RESULTS

For testing the suitability of IGA for machine simulation, we first consider the domain \( \Omega_{rt} \) and we solve a test problem with homogeneous Dirichlet boundary conditions on \( \Gamma_1 \cup \Gamma_{ag} \) and anti-periodic boundary conditions connecting \( \Gamma_1 \) and \( \Gamma_2 \). The only source present in the model is the magnetisation of the permanent magnets. The IGA solution for the magnetic vector potential is compared to the one obtained using a fine FEM discretisation with first order elements. We solve IGA basis functions of degree 1 and 2 and for an increasing mesh refinement, and we depict the difference to the FEM solution in \( L^2 \) in Fig. 3.

For both discretisation degrees, the curves approach a fixed value, which is expected since the IGA method is solving the problem for the exact geometry in contrast to FEM which introduces an additional geometrical error related to the triangulation of the geometry.

Secondly, the full problem is considered, incorporating the Dirichlet-to-Neumann coupling between stator and rotor introduced in the previous section. In Fig. 4, the convergence of the algorithm is shown for a simulation with degree 2, with approximately 3200 total degrees of freedom. After 29 iterations, both solutions show an incremental error below the prescribed tolerance \( tol = 10^{-7} \).

The simulation results are shown in Fig. 5 where the flux lines in the machine are shown. In the post-processing the spectrum of the EMF has been calculated. In Fig. 6 the first 32 modes of the spectrum of the EMF of the machine are shown. Both methods result in similar spectra. This is also depicted in Tab. I where it is shown that the relative difference of the THD is below 6%.

V. CONCLUSIONS

Isogeometric Analysis (IGA) recently emerged as a promising alternative to the Finite Element Method (FEM) for highly accurate electromagnetic field simulation. IGA is capable of exactly resolving circles and, hence, avoids any geometric approximations errors in contrast to FEM, which is of particular importance in the air gap region. Our IGA model for
a permanent magnet synchronous machine consists of two separate multipatch IGA discretisations for stator and rotor, glued together with a Dirichlet-to-Neumann map in the air gap. IGA (degree 2) attains the prescribed accuracy with a number of degrees of freedom which is substantially smaller (≈ 20 times) than for lowest order FEM. Therefore, IGA is a valuable extension to an engineer’s simulation toolbox, especially when fast and accurate machine simulation tasks are due.

ACKNOWLEDGMENT

This work is supported by the German BMBF in the context of the SIMUROM project (grant nr. 05M2013), by the DFG (grant nr. SCHO1562/3-1) and by the ’Excellence Initiative’ of the German Federal and State Governments and the Graduate School of CE at TU Darmstadt.

REFERENCES

[1] D. Howe and Z.Q. Zhu, “The influence of finite element discretisation on the prediction of cogging torque in permanent magnet excited motors”, IEEE Trans. Magn., vol. 28, no.2, pp. 1080–1083, 1992.
[2] I. Tsukerman, “Accurate computation of ’ripple solutions’ on moving finite element meshes”, IEEE Trans. Magn., vol. 31, no. 3, pp. 1472–1475, 1995.
[3] T. Tarnhuvud and K. Reichert, “Accuracy problems of force and torque calculation in FE-systems”, IEEE Trans. Magn., vol. 24, no. 1, pp. 443–446, 1988.
[4] T. J. R. Hughes, J.A. Cottrell and Y. Bazilevs, “Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement”, Comput. Meth. Appl. Mech. Eng., vol. 194, pp. 4135–4195, 2005.
[5] J. A. Evans, Y. Bazilevs, I. Babuška and T. J. R. Hughes,”n-Widths, sup-inf, and optimality ratios for the k-version of the isogeometric finite element method”, Comput. Meth. Appl. Mech. Eng., vol. 198, no. 21–26, pp. 1726–1741, 2009.
[6] A. Pels, Z. Bontinck, J. Corno, H. De Gersem and S. Schöps, “Optimization of a Stern-Gerlach Magnet by magnetic field-circuit coupling and isogeometric analysis,” IEEE Trans. Magn., vol. 51, no. 12, pp. 1–7, 2015.
[7] J. Corno, C. de Falco, H. De Gersem and S. Schöps,”Isogeometric Analysis simulation of TESLA cavities under uncertainty,” Electromagnetics in Advanced Applications (ICEAA), 2015 International Conference on. IEEE, 2015.
[8] A. Quarteroni and A. Valli, “Domain decomposition methods for partial differential equations”, Oxford University Press, 1999.
[9] M.A. Rahman and P. Zhou, “Determination of saturated parameters of PM motors using loading magnetic fields,” IEEE Trans. Magn., vol. 27, no. 5, pp. 3947–3950, 1991.
[10] S.J. Salomon, Finite element analysis of electrical machines, Boston USA: Kluwer academic publishers, 1995.
[11] L. Piegl and W. Tiller. “The NURBS book”, Monographs in Visual Communication, 1997.
[12] J.A. Cottrell, T.J.R. Hughes and Y. Bazilevs, “Isogeometric Analysis: Toward integration of CAD and FEA”, John Wiley & Sons, Ltd, 2009.