Conversion of Transverse to Longitudinal Plasma Waves Induced by Radiation Reaction

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Abstract. We investigate the evolution of a circularly polarized wave in a cold plasma in a regime when radiation reaction is essential. A new effect is that circularly polarized transverse waves become unstable with respect to partial conversion to the longitudinal ones. This effect appears due to radiation reaction entirely and in principle could be implemented for particle acceleration. We provide analytical expression for arising longitudinal field in the case, when it is much smaller, than transverse one.

1. Introduction

One of the promising possible applications of the new powerful laser facilities such as ELI Beamlines [1] in Czech Republic or VULCAN [2] in the UK is particle acceleration by longitudinal electric fields generated by nonlinear propagation of intense laser pulses through plasma. For example, in the most popular LWFA scheme [3–5] these fields are generated by pushing plasma electrons via ponderomotive force of a short focused laser pulse.

As is well known, both longitudinal and circularly polarized transverse modes are capable for propagation through a cold plasma [6] and are independent in linear regime. Since for modern facilities Lorentz factor of electron quiver oscillations becomes already of the order of hundreds, radiation reaction effects should also come into play [7]. In particular, we will show that in such highly ultrarelativistic regime circularly polarized transverse waves in plasma become unstable with respect to conversion to longitudinal modes, and that this effect is provided entirely by radiation reaction. As a result, we suggest a new radiation reaction based mechanism for generation of longitudinal field in plasma, which should compete with the usually employed ponderomotive mechanism and in principle could be implemented for particle acceleration.

Note that previously impact of radiation reaction was studied in a number of single particle problems, see e.g. [7–11], but here we examine how it affects collective motion in plasma. Here we investigate evolution of a circularly polarized one dimensional wave in a cold plasma in a regime when radiation reaction is essential, but can be still treated as perturbation with respect to the Lorenz force. It corresponds to dimensionless field amplitude a of the order of few hundreds, i.e. to laser intensity of around \(10^{22–10^{23}}\) W/cm\(^2\), i.e. the value that is expected to be attainable with the laser facilities of the near future. Besides, in this case we can neglect quantum corrections and use classical picture of particle motion based on Lorenz equation with...
Landau-Lifshitz radiation friction force [12], leaving in addition just the dominant contribution to the latter.

In what follows, we are using a cold plasma model, which in hydrodynamic approximation is described by the equations

$$\nabla \times E = -\frac{\partial B}{\partial t},$$  \hspace{1cm} (1)
$$\nabla \cdot E = -4\pi e(n - n_0),$$  \hspace{1cm} (2)
$$\nabla \times B = \frac{\partial E}{\partial t} - 4\pi ev,$$  \hspace{1cm} (3)
$$\nabla \cdot B = 0,$$  \hspace{1cm} (4)
$$\frac{\partial p}{\partial t} + (v \cdot \nabla)p = -e (E + v \times B) + F_{RR},$$  \hspace{1cm} (5)
$$F_{RR} = -\frac{2e^4}{3m^2} \gamma^2 \left\{(E + v \times B)^2 - (E \cdot v)^2\right\} v.$$  \hspace{1cm} (6)

Here we assume such units that $c = 1$; $e > 0$ and $m$ are the magnitude of electron charge and mass respectively; $E$ and $B$ are electric and magnetic fields; $n_0$ is the density of ions (which we assume immobile); $n$, $v$, $p$ and $\gamma$ are local density, velocity, momentum and $\gamma$-factor of electron fluid.

2. One dimensional wave solution

We will seek for a solution of equations (1)–(6) in the form of functions, which depend only on the phase $\xi = \omega_p(t - z/\beta)$, where $\omega_p = \sqrt{4\pi e^2 n_0/m}$ is the plasma frequency, $\beta$ is the phase velocity of a plasma wave propagating along $z$ direction. Then it follows from Maxwell equations that $B_y = E_z/\beta$, $B_x = -E_y/\beta$, and $n = \frac{n_0}{1 - v_0^2/\beta^2}$. Besides we introduce dimensionless field $a = eE/m\omega_p$ and four-velocity $u = p/m$ and take into account that particles motion is ultrarelativistic, $u \gg 1$, so that $\gamma \approx u$. Then we obtain equations for transverse and longitudinal motions in the form

$$\dot{u}_\perp = -a_\perp + f_{RR\perp}, \hspace{1cm} \dot{a}_\perp = \frac{u_\perp}{(1 - 1/\beta^2)(u - u_z/\beta)},$$  \hspace{1cm} (7)
$$\dot{u}_z = -\frac{ua_z + u_\perp a_\perp/\beta}{u - u_z/\beta} + f_{RRz}, \hspace{1cm} \dot{a}_z = \frac{u_z}{u - u_z/\beta},$$  \hspace{1cm} (8)

where

$$f_{RR} = -Ku \left[(u - u_z/\beta)|a_\perp|^2 + \frac{(ua_z + u_\perp a_\perp/\beta)^2 - (ua_\perp)^2}{u - u_z/\beta}\right], \quad K = \frac{2}{3}e^2\omega_p/m.$$  \hspace{1cm} (9)

Note that parameter $K$ is actually very small. For $\omega_p$ in optical range $K$ is of the order of $10^{-8}$.

2.1. Radiation reaction turned off

First consider the case when radiation reaction can be completely neglected. If the wave was initially transverse, $u_z(0) = a_z(0) = 0$, and circularly polarized, then following [6] Eq. (7) has the solution

$$u_x = -u_0 \sin \omega_0 \xi, \hspace{1cm} u_y = u_0 \cos \omega_0 \xi, \hspace{1cm} a_x = \omega_0 u_0 \cos \omega_0 \xi, \hspace{1cm} a_y = u_0 \omega_0 \sin \omega_0 \xi.$$  \hspace{1cm} (10)
which is a transverse circularly polarized wave with the frequency\(^1\)

\[ \omega_0 = \frac{1}{\sqrt{u_0(1 - 1/\beta^2)}}. \]  

(11)

Since \(u_\perp a_\perp = 0\), longitudinal component does not appear according to Eq. (8).

2.2. Radiation reaction turned on

The situation changes when we take radiation reaction into account. To solve equations (7), (8) we will use several assumptions:

- Radiation reaction force is much smaller than the Lorentz force

\[ Ku_\perp a_\perp^2 \ll a; \]  

(12)

- Generated longitudinal field is still much smaller than the transverse field \(a_z \ll a_\perp\);

- Phase velocity is close to the speed of light, \(\beta \approx 1\).

Then the radiation reaction term can be rewritten in the form

\[ f_{RR} = -ru, \quad r = Ka_\perp^2 u_\perp, \]  

(13)

where we introduced \(u_\perp = u - u_z\).

First consider transverse motion, which is described by the equation

\[ \ddot{u}_\perp + r\dot{u}_\perp + \omega_\perp^2 u_\perp = 0, \quad \omega_\perp = \frac{1}{\sqrt{1 - 1/\beta^2} u_\perp}, \]  

(14)

where we neglected \(r\) against \(\omega_\perp^2\) (it follows from the condition \(a_z \ll a_\perp\), as we will show below). Eq. (14) has the WKB solution

\[ u_x = -u_0^{3/4} u_\perp^{1/4} e^{-\frac{1}{2} \int r d\xi} \sin \int \omega_\perp d\xi, \quad u_y = u_0^{3/4} u_\perp^{1/4} e^{-\frac{1}{2} \int r d\xi} \cos \int \omega_\perp d\xi, \]  

(15)

and for transverse field we obtain

\[ a_x = \frac{u_0^{3/4}}{u_\perp^{1/4} \sqrt{1 - 1/\beta^2}} e^{-\frac{1}{2} \int r d\xi} \cos \int \omega_\perp d\xi, \quad \quad a_y = \frac{u_0^{3/4}}{u_\perp^{1/4} \sqrt{1 - 1/\beta^2}} e^{-\frac{1}{2} \int r d\xi} \sin \int \omega_\perp d\xi. \]  

(16)

It follows from (7), (8) that

\[ \dot{u}_\perp = a_z - ru_\perp, \]  

(17)

then from the conditions \(a_z \ll a_\perp\) and (12) we obtain

\[ \frac{\sqrt{1 - 1/\beta^2} |\dot{u}_\perp|}{\sqrt{a_\perp}} \ll 1, \]  

(18)

or \(|\omega_\perp/\omega_\perp^2| \ll 1\). Hence our assumptions provide validity of WKB approximation as well as the condition \(r \ll \omega_\perp^2\) assumed above.

\(^1\) Note, that if plasma wave is generated by external electromagnetic wave of frequency \(\omega\), then Eq. (11) actually defines phase velocity \(\beta = (1 - (\omega/\omega_0)^2/\omega_0)^{-1/2}\).
Figure 1. $u_-(\xi)$ for $u_0 = a_0 = 10^2$, $1 - 1/\beta^2 = 10^{-2}$. Solid blue line: numerical solution of Eqs. (19), (20); dotted orange line: solution (21) for single electron a circularly polarized plane wave [8]; dashed red line: self-consistent formula (22).

Consider now longitudinal component of a plasma wave. The equation for $u_-$ can be represented in the form

$$
\ddot{u}_- = \frac{1}{2} \left[ \left( \frac{u_0}{u_-} \right)^{3/2} R - 1 \right] - \frac{3 K u_0^{3/2}}{2(1 - 1/\beta^2)} \dot{u}_- \sqrt{u_- R} + \frac{K^2 u_0^3}{(1 - 1/\beta^2)^2} u_-^2 R^2,
$$
(19)

$$
\dot{R} = -K R^2 \sqrt{u_-} \frac{u_0^{3/2}}{1 - 1/\beta^2},
$$
(20)

where we defined $R = e^{-\int r d\xi}$.

In [8] A. Di Piazza obtained an exact solution for electron motion in an external plane wave field with radiation reaction taken into account. In our notations

$$
u_-^{DP} = \frac{u_0}{1 + Ku_0a_0^2\xi},
$$
(21)

where $a_0$ is the dimensionless external field amplitude. In our case this formula cannot be implemented straightforwardly, because damping of an external plane wave field was not taken into account in [8]. However, if we substitute into Eq. (22) $a(\xi)$ from the numerical solution of Eqs. (7), (8), then the self-consistent version of the expression (22)

$$
u_-^{SC} = \frac{u_0}{1 + Ku_0a^2(\xi)\xi},
$$
(22)

perfectly fits the numerical solution, see Fig. 1.

In order to solve equations (19), (20), note that the derivative

$$
\frac{d}{d\xi} \left( Ru_-^{-3/2} \right) = -K R^2 \frac{u_0^{3/2}}{u_- (1 - 1/\beta^2)} - \frac{3}{2} \frac{\dot{u}_-}{u_-^{5/2}} R \ll 1
$$
(23)

for those values of $u_-$ that radiation reaction force is important, therefore we can assume that in the zeroth order $Ru_-^{-3/2}$ remains constant. Taking into account that $u_-(0) = u_0$ and $R(0) = 1$,
Numerical solution
Analytical solution
50 100 150 200 250 300 ξ
20
40
60
80
100

Figure 2. Left panel: numerical solution for \( u_-(\xi) \) (solid blue) vs. analytical solution (26) (dashed red). Right panel: numerical solution for \( a_z(\xi) \) (solid blue) vs. analytical solution (27) (dashed red). \( u_0 = 10^2 (\alpha \sqrt{u_0} \approx 0.13), 1 - 1/\beta^2 = 10^{-2}. \)

as a zeroth approximation we get

\[
R^{(0)} = \left( \frac{u^{(0)}}{u_0} \right)^{3/2}
\]  

Substituting this expression into Eq. (20), we obtain the solution

\[
u^{(0)} = \frac{u_0}{\sqrt{1 + \alpha \xi}}, \quad R^{(0)} = \frac{1}{(1 + \alpha \xi)^{3/4}},
\]

where \( \alpha = \frac{4}{3} \frac{K \rho_0^2}{1 - 1/\beta^2}. \)

Solving Eqs. (19), (20) to the next order we obtain

\[
u_- = \frac{u_0}{\sqrt{1 + \alpha \xi}} - \frac{\alpha u_0^{3/2}}{2\sqrt{3}(1 + \alpha \xi)^{11/16}} \sin \left[ \frac{2\sqrt{3}}{5\alpha \sqrt{u_0}} (1 + \alpha \xi)^{5/4} - 1 \right],
\]

and from (17) derive the expression for the longitudinal field

\[
a_z = \frac{\alpha u_0}{4(1 + \alpha \xi)^{3/2}} \left( 1 - (1 + \alpha \xi)^{17/16} \cos \left[ \frac{2\sqrt{3}}{5\alpha \sqrt{u_0}} (1 + \alpha \xi)^{5/4} - 1 \right] \right).
\]

These analytical formulas are compared with numerical solutions in Figs. 2 and 3.

Note that the expansion parameter in eq. (26) is \( \alpha \sqrt{u_0} \), and that the condition \( \alpha \sqrt{u_0} \ll 1 \) is stronger than (18). However, analytical expressions (26) and (27) remain qualitatively valid and give correct estimations for longitudinal field amplitude even for \( \alpha \sqrt{u_0} > 1 \), see Fig. 3.

In order to illustrate the obtained results, consider the plasma wave generated by a circularly polarized electromagnetic pulse with a smooth front, so that one can neglect impact of the ponderomotive mechanism on longitudinal mode generation. We can estimate \( \dot{u}_\perp \sim a_0 \frac{\omega}{\omega_p}, \)

where \( a_0 \) is the dimensionless electric field of the external pulse (normalized as usually to the wave frequency). Hence \( \sqrt{u_0}/(1 - 1/\beta^2) \sim a_0 \omega/\omega_p \) and \( u_0 \sim a_0 \). Then from (27) we can estimate the amplitude of longitudinal mode as

\[
a_z \sim \alpha u_0 \sim K a_0^4 \left( \frac{\omega}{\omega_p} \right)^2.
\]
Clearly, amplitude of the longitudinal field has very strong scaling with $a_0$. This, in a view of the novel experimental capabilities [1,2], may be interesting for possible applications to electron acceleration. Besides, the frequency of longitudinal oscillations scales as $\omega_z \sim \omega_p/\sqrt{a_0}$, implying that accelerating particles can spend more time in accelerating phase of the longitudinal field.

3. Conclusion

We examined the impact of radiation reaction on initially transverse circularly polarized plasma wave and showed that radiation reaction could effectively convert transverse waves into longitudinal plasma modes. We obtained analytical approximation for coupled transverse and longitudinal plasma waves. This approximate solution is a self-consistent generalization of the exact solution [8] for a particle moving in a plain wave with account for radiation reaction.

Normally radiation reaction acts like hindrance and inhibits acceleration. However, the effect of conversion of transverse to longitudinal plasma waves induced by radiation reaction could be interesting as a novel alternative mechanism for electron acceleration. Such mechanism may be more effective than conventional LWFA for high laser field amplitudes $a_0 \gg 1$ due to a stronger scaling ($a_0^4$ vs. $a_0^2$).

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