MAJORANA NEUTRINO MAGNETIC MOMENTS

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The presence of trilinear $R$-parity violating interactions in the MSSM lagrangian leads to existence of quark–squark and lepton–slepton loops which generate mass of the neutrino. By introducing interaction with an external photon the magnetic moment is obtained. We derive bounds on that quantity being around one order of magnitude stronger than those present in the literature.

Thanks to more than 40 years of intensive work oscillations of neutrinos are treated as a well established experimental fact. Up to our best knowledge this situation implies that neutrinos are massive particles and therefore the Standard Model (SM) of elementary particles and interactions should be extended. All such attempts to go beyond SM are called non-standard physics and include supersymmetry (SUSY), theories of grand unification (GUT), extra dimensions and others.

The problem of generation of very small neutrino masses has recently received a great deal of attention. There is, among others, a mechanism that relies on the $R$-parity violation in supersymmetric Standard Model. The $R$-parity is defined as $R = +1$ for ordinary particles and $R = -1$ for their supersymmetric partners. $R$-parity conservation makes life easier because SUSY particles cannot decay into ordinary particles and vice-versa. Theoretically, however, nothing motivates such behavior, therefore many models consider violation of $R$-parity. This implies, in turn, non-conservation of lepton and baryon numbers, which opens the possibility for exotic nuclear processes to appear.

We will use the minimal supersymmetric standard model (MSSM), with supersymmetry broken by supergravity (SUGRA). The model is fully described by the following superpotential and lagrangian. The $R$-parity conserving part of the superpotential has the form

$$W^{\text{MSSM}} = \epsilon_{ab}[ (Y_E)_{ij} L^a_i H^b_1 \bar{E}_j + (Y_D)_{ij} Q^a_i Q^b_x H^b_1 \bar{D}_j + \epsilon_x (Y_U)_{ij} Q^a_i Q^b_x H^b_2 \bar{U}_j + \mu H^a_1 H^b_2 ],$$

(1)
while its $\mathcal{R}$-parity violating part reads

$$W^{\mathcal{R}pV} = \epsilon_{ab} \left[ \frac{1}{2} \lambda_{ijk} L_i^a L_j^b \bar{E}_k + \lambda'_{ijk} L_i^a Q_j^b \bar{D}_k \right] + \frac{1}{2} \epsilon_{xyz} \lambda''_{ijk} \bar{U}_x^i \bar{D}_y^j \bar{D}_z^k + \epsilon_{ab} \kappa^i L_i^a H_2^b. \quad (2)$$

Here $Y$'s are $3 \times 3$ Yukawa matrices, $L$ and $Q$ stand for lepton and quark left-handed $SU(2)$ doublet superfields while $\bar{E}, \bar{U}$ and $\bar{D}$ denote the right-handed lepton, up-quark and down-quark $SU(2)$ singlet superfields, respectively. $H_1$ and $H_2$ mean two Higgs doublet superfields. We have introduced color indices $x, y, z = 1, 2, 3$, generation indices $i, j, k = 1, 2, 3$ and the SU(2) spinor indices $a, b, c = 1, 2$.

The introduction of $\mathcal{R}$-parity violation implies the existence of lepton or baryon number violating processes, like the unobserved proton decay and neutrinoless double beta decay ($0\nu2\beta$). Fortunately one may keep only one type of terms and it is not necessary to have both non-zero at one time. In order to get rid of too rapid proton decay and to allow for lepton number violating processes it is customary to set $\lambda'' = 0$.

We present here for completeness the mass term for scalar particles:

$$\mathcal{L}^{mass} = m_{H_1}^2 h_1^1 h_1^1 + m_{H_2}^2 h_2^1 h_2^1 + q^q m_{q}^2 q + l^l m_{l}^2 l + u^d m_{u}^2 d + e^e m_{e}^2 e, \quad (3)$$

soft gauginos mass term ($\alpha = 1, ..., 8$ for gluinos):

$$\mathcal{L}^{gaug.} = \frac{1}{2} \left( M_1 B^l \bar{B} + M_2 W_i^1 \bar{W}_i^1 + M_3 G_\alpha^1 \bar{G}_\alpha^1 + h.c. \right), \quad (4)$$

as well as the SUGRA mechanism, by introducing the soft supersymmetry breaking Lagrangian

$$\mathcal{L}^{soft} = \epsilon_{ab} \left[ (\mathbf{A} E)_{ij} \bar{d}_i^a \bar{d}_j^b + (\mathbf{A} D)_{ij} \bar{q}_i^{ax} \bar{h}_1^b \bar{d}_j^x ight. + \left. (\mathbf{A} U)_{ij} \bar{q}_i^{ax} \bar{h}_2^b \bar{d}_j^x + B_\mu h_1^\mu h_2^\mu + B_2 \epsilon \bar{L}_1^a \bar{H}_2^b \right], \quad (5)$$

where lowercase letters stand for scalar components of respective chiral superfields, and $3 \times 3$ matrices $\mathbf{A}$ as well as $B_\mu$ and $B_2$ are the soft breaking coupling constants.

![Feynman diagrams leading to Majorana neutrino masses. By attaching photon to the internal line of the loop one gets the neutrino magnetic moment.](image-url)
The loops showed in Fig. 1 lead to Majorana neutrino mass term. Detailed calculation gives for the quark–squark loop the following contribution:

\[ M^q_{ii'} = \frac{3}{16\pi^2} \sum_{jk} \lambda'_{ijk} \lambda'_{i'kj} \left[ m_q \frac{\sin(2\theta^k)}{2} f(x^j_2, x^j_1) + (j \leftrightarrow k) \right], \quad (6) \]

where \( f(a, b) = \log(a)/(1 - a) - \log(b)/(1 - b) \), \( m_q \) is the \( i \)-th generation down quark mass, \( \theta^k \) is the squark mixing angle between the \( k \)-th squark mass eigenstates \( M_{\tilde{q}_1, 2} \), and \( x^j_1, 2 = m^2_{d_i} / M^2_{\tilde{q}_1, 2} \).

A similar contribution comes from loops containing lepton–slepton pairs. It reads:

\[ M^\ell_{ii'} = \frac{1}{16\pi^2} \sum_{jk} \lambda_{ijk} \lambda_{i'kj} \left[ m_e \frac{\sin(2\phi^k)}{2} f(y^j_2, y^j_1) + (j \leftrightarrow k) \right], \quad (7) \]

where all the quantities are defined in complete analogy with the previous case, by replacing squarks with sfermions and quarks with fermions. The factor 3 in Eq. (6) comes from summation over three colors of quarks and therefore it is absent in the case of fermions.

The left-hand sides of Eqs. (6) and (7) may be obtained by considering neutrino oscillations phenomenology or by using data from neutrinoless double beta decay experiments. The half-life of this exotic process depends on the effective neutrino mass \( \langle m_\nu \rangle = \sum_{i=1}^{3} |U_{ei}|^2 m_i \), where \( U \) is the neutrino mixing matrix and \( m_i \) are neutrino mass eigenstates. There is no firm experimental evidence for observation of the 0ν2β process. The conservative constraint coming from the Heidelberg–Moscow experiment is for now \( T^{0\nu}_{1/2} \geq 1.9 \times 10^{25} \) years. By assuming nuclear matrix elements of Ref. 8 this value translates into \( \langle m_\nu \rangle \leq 0.55 \text{ eV} \), which in turn implies the neutrino mass matrix in the form

\[ |M^{HM} | \leq \begin{pmatrix} 0.55 & 0.78 & 0.66 \\ 0.78 & 0.56 & 0.69 \\ 0.66 & 0.69 & 0.81 \end{pmatrix} \text{ eV}. \quad (8) \]

The entries of \( M^{HM} \) may be used as the left-hand side of Eqs. (6) and (7), constraining non-standard physics parameters.

By attaching a photon to the internal line of the loop one has an effective neutrino–neutrino–photon vertex, which allows to calculate the neutrino magnetic moment. In the case of Majorana neutrino, the CPT theorem allows only for the transitional magnetic moment between two neutrinos of different flavors. It is described by the effective hamiltonian \( H_{\text{eff}} = \mu_{ii'} \bar{\nu}_{iL} (\sigma^{ab}/2) \nu_{i'R} F_{ba} \), \( F \) being the electromagnetic tensor.

After evaluating the loop amplitude one gets the following expression for the magnetic moment:

\[ \mu^q_{ii'} = (1 - \delta_{ii'}) \frac{3}{16\pi^2} m_e Q_{d} \sum_{jk} \Lambda'_{ijk} \Lambda'_{i'kj} \left[ \frac{2\sin(2\theta^k)}{m_q} g(x^j_2, x^j_1) - (j \leftrightarrow k) \right] \mu_B, \quad (9) \]
$m_e$ being the electron mass, $g(a, b) = (a \log(a) - a + 1)/(1 - a)^2 - (b \log(b) - b + 1)/(1 - b)^2$, $Q_d = 1/3$ is the down-quark electric charge, and $\mu_B$ the Bohr magneton. A similar expression may be obtained for the lepton–slepton contribution.

We have calculated the low energy MSSM spectrum by starting from certain unification conditions at the GUT scale $\sim 10^{16}$ GeV and evolving the running masses and coupling constants down to the $m_Z$ scale using renormalization group equations (RGE). The so-obtained spectrum is tested against various constraints on SUSY masses, proper electroweak symmetry breaking, FCNC phenomenology and others. The procedure has been described in detail elsewhere.\(^3\) We use the so-called conservative approach, assuming that only one mechanism dominates at a time. It means that we perform calculations for some given $j, k$ without doing the summation. We pick up the highest obtained value, which constitutes the “most optimistic” result:

$$
\begin{align*}
\mu_{e\mu}^q &\leq 4.0 \times 10^{-17} \mu_B, \\
\mu_{\tau\tau}^q &\leq 3.4 \times 10^{-17} \mu_B, \\
\mu_{e\tau}^q &\leq 3.6 \times 10^{-17} \mu_B, \\
\mu_{e\mu}^\ell &\leq 1.6 \times 10^{-16} \mu_B, \\
\mu_{\tau\tau}^\ell &\leq 1.4 \times 10^{-16} \mu_B, \\
\mu_{e\tau}^\ell &\leq 1.4 \times 10^{-16} \mu_B.
\end{align*}
$$

These values were obtained for the following GUT conditions: $A_0 = 100$ GeV, $m_0 = m_{1/2} = 150$ GeV, $\tan(\beta) = 19$, $\mu > 0$, where $A_0$ denotes the common value of soft breaking couplings, $m_0$ and $m_{1/2}$ the common scalar and fermion masses, and $\tan(\beta)$ is the ratio of Higgs vacuum expectation values. By taking, for example, $A_0 \sim 1000$ GeV, the results become smaller by at least one order of magnitude. We would like to mention at this point that our bounds are around one order of magnitude stronger than those published earlier.\(^2\) It is due to the more exact procedure (GUT and RGE) and partially due to inclusion of sparticle mixing. It is also worth mentioning that the present experimental limits are $\mu_{ii'} \leq 10^{-10} \mu_B$.

In further discussion one should analyze the influence of quark mixing on $\mu_{i\nu}$, which may give important corrections. This discussion is, however, beyond the scope of the present work and will be postponed to a separate regular paper.

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