Calculation of radius of plastic zone in surrounding rock of roadway

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Abstract. Mohr-Coulomb yield criterion and Drucker-Prager yield criterion are two commonly used strength criteria in geotechnical analysis. The former is considered to be more suitable for geotechnical media. In this paper, the plastic zone radius of roadway surrounding rock is compared with two yield criteria under different influence factors. Finally, the different characteristics of two yield criteria and the application in engineering design are obtained.

Key words: Mohr-Coulomb; Drucker-Prager; Yield; Plastic zone.

1. Yield conditions commonly used in rock and soil

1.1. Mohr-Coulomb yield condition

Mohr yield condition refers to the yielding of materials when the shear stress reaches a certain limit. It can be expressed as:

\[ \tau_n = f(C, \phi, \sigma_n) \]  

Where: \( \tau_n \) ——Ultimate shear strength; 
\( \sigma_n \) ——Normal stress on shear plane, pressure is the normal stress; 
\( C, \phi \) ——Cohesion and internal friction angle of materials.

In order to overcome the shortcomings of Mohr strength envelope and make the strength envelope more concise, Coulomb proposed the strength envelope given by straight line formula, that's Coulomb's law. The formula is as follows:

\[ \tau_n = c + \sigma_n \tan \phi \]  

The expression of Mohr condition can also be expressed as follows:

\[ \sigma_1 = \sigma_3 + \frac{1 + \sin \phi}{1 - \sin \phi} \frac{2c \cos \phi}{1 - \sin \phi} \]
1.2. Drucker-Prager yield condition
The Mohr-Coulomb model does not consider the intermediate principal stress effect. The yield surface
has a serious deficiency in the principal stress space. As long as the stress falls near the edges or edges,
the derivative of the yield function along the outer normal direction of the surface is not easy to
determine, and there is also a discontinuity problem at the corner point. Therefore, in 1952, on the basis
of the Mises criterion, Drucker-Prager put forward the revised yield criterion:

\[ f = \alpha I_1 + \sqrt{I_2} - k = 0 \] 

(4)

Among them: \( \alpha = \frac{\sin \varphi}{\sqrt{3} \sqrt{3 + \sin^2 \varphi}} \), \( k = \frac{\sqrt{3} \cos \varphi \cdot c}{\sqrt{3 + \sin^2 \varphi}} \)

Where: \( I_1 \) —— First stress invariants;
\( J_2 \) —— Second stress deviation invariants;

2. Mechanics model

2.1. Model establishment
Assuming that the original rock stress is \( p_0 \), support reaction force is \( p_1 \), the radius of roadway is \( a \),
the radius of plastic zone is \( R_0 \). (As shown in Figure 1)

(1) Plastic zone: the internal diameter is \( a \), the external diameter is \( R_0 \), the internal pressure is \( p_1 \),
the external pressure is \( \sigma_{R_0} \);

(2) Elastic zone: the internal diameter \( R_0 \), the outer diameter is infinite, the internal pressure is \( \sigma_{R_0} \),
The external pressure is \( p_0 \).

(3) When \( r = a, \sigma_r = p_1 \).

![Fig. 1 Elastic-Plastic Zone Surrounding Rock](image)

According to the knowledge of elastic-plastic mechanics, the following conclusions can be obtained:
(1) The plastic zone should conform to the stress balance equation and the plastic condition.
(2) The elastic area should satisfy the stress balance equation and the elastic condition.
(3) The boundary of elastoplastic zone: it satisfies both plastic conditions and elastic conditions.
2.2. Stress in elastic zone of surrounding rock

The elastic zone stress around the roadway can be obtained from the knowledge of thick walled cylinders in elasticity.

\[
\begin{align*}
\sigma_{rr} &= p_0 \left(1 - \frac{R_o^2}{r^2}\right) + \sigma_{R_o} \frac{R_o^2}{r^2} \\
\tau_{r\theta} &= 0 \\
\sigma_{\theta\theta} &= p_0 \left(1 + \frac{R_o^2}{r^2}\right) - \sigma_{R_o} \frac{R_o^2}{r^2}
\end{align*}
\] (5)

2.3. Stress analysis of surrounding rock plastic zone

As shown in Figure 2, any element in the surrounding rock should satisfy the equilibrium condition in the radial direction.

\[
\sigma_r \cdot rd\theta + 2\sigma_\theta dr \sin \frac{d\theta}{2} - (\sigma_r + d\sigma_r)(r + dr)d\theta = 0
\] (6)

The higher order trace is removed, and the equilibrium differential equation in polar coordinates is obtained.

\[
\frac{dr}{r} = \frac{d\sigma_r}{\sigma_\theta - \sigma_r}
\] (7)

![Fig. 2 Balance Analysis of Unit](image)

3. Calculation of plastic zone radius

3.1. Calculation of plastic zone radius by Mohr-Coulomb yield condition

According to the introduction and analysis of the foregoing, joint radius (3), (5) and (7) can get the plastic zone radius under the Mohr-Coulomb yield criterion:

\[
R_0 = a \left[ \left( \frac{p_0 + c \cdot \cot \varphi}{p_i + c \cdot \cot \varphi} \right) \right]^{\frac{1 - \sin \varphi}{2 \sin \varphi}}
\] (8)

3.2. Calculation of plastic zone radius by Drucker-Prager yield condition

According to the introduction and analysis, joint radius (4), (5) and (7) can get the plastic zone radius under the Drucker-Prager yield criterion.

\[
R_0 = a \left[ \left( \frac{p_0 + k/3\alpha}{p_i + k/3\alpha} \right) \right]^{\frac{1 - 3\alpha}{6\alpha}}
\] (9)
3.3. Analysis and comparison of two plastic radius

According to the formula (8) and type (9) obtained above, the main factors affecting the radius of plastic zone include: rock stress, supporting resistance, internal friction angle, cohesive force and roadway radius. The influence of various factors on the radius of plastic zone under different yield criteria is analyzed separately. By using one of the factors as independent variables and the rest as constants, the radius of plastic zone of surrounding rock is compared by using Matlab tool. As shown in Fig.3-7.

It can be seen from figures 3 to 7, the radius of plastic zone and the radius of plastic zone obtained by Drucker-Prager yield condition according to Morh-Coulomb yield condition. However, the radius of plastic zone obtained by Drucker-Prager Yield condition is generally larger than that obtained by morh-Coulomb yield condition, which is mainly because Drucker Prager Yield condition considers the influence of intermediate stress.

![Fig. 3 The Relationship Between Radius of Plastic Zone and Original Rock Stress](image1)

![Fig. 4 The Relationship Between Radius of Plastic Zone and Support Resistance](image2)

![Fig. 5 The Relationship Between Radius of Plastic Zone and Internal Friction Angle](image3)

![Fig. 6 The Relationship Between Radius of Plastic Zone and Cohesion](image4)

![Fig. 7 The Relationship Between Radius of Plastic Zone and Tunnel Radius](image5)
4. Conclusion
Morh-Coulomb yield criterion and Drucker-Prager yield criterion are two commonly used strength criteria in geotechnical analysis. They are widely used in geotechnical engineering design and research, especially in geotechnical engineering numerical analysis. From the above comparison, it can be found that the radius of plastic zone obtained by Drucker-Prager Yield Criterion is larger than that by morh-Coulomb yield criterion. Therefore, Drucker Prager Yield Criterion is more conservative in engineering practice and can serve engineering design better.

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