Super-Poissonian shot noise in the resonant tunneling due to coupling with a localized level

Ivana Djuric
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030

Bing Dong
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030
Department of Physics, Shanghai Jiaotong University, 1954 Huashan Road, Shanghai 200030, China

H. L. Cui
Department of Physics and Engineering Physics, Stevens Institute of Technology, Hoboken, New Jersey 07030
School of Optoelectronics Information Science and Technology, Yantai University, Yantai, Shandong, China

Abstract

We report our studies of the shot noise spectrum in tunneling through an interacting quantum dot when an additional single-level quantum dot without tunnel coupling to leads is coherently side-connected to it. We show that the zero-frequency shot noise could reach a super-Poissonian value for appropriate ratios between dot-dot hoppings and dot-lead couplings, but the current is independent on the hopping. Moreover, the frequency spectrum of shot noise shows an obvious peak at the Rabi frequency, which is controllable by tuning the dot-lead couplings.
A measurement of the stationary I-V characteristic does not always provide enough information to describe a charge transport mechanism through a mesoscopic system. Shot noise, i.e. fluctuation of the current in time due to the discrete nature of electrons, characterizes the degree of correlation between charge transport events and it can be used as an additional diagnostic tool to distinguish various transport mechanisms possibly resulting in the same mean current.

The resonant tunneling (RT) through a single localized state (quantum dot) has been a subject of intensive investigations. It has been found that the current and the Fano factor, which quantifies correlations with respect to the uncorrelated Poissonian noise, are given by the relations\(^\text{2,3}\)

\[ I = \frac{e \Gamma_L \Gamma_R}{\Gamma_L + \Gamma_R}, \]

and

\[ F = \frac{S_I}{2eI} = 1 - \frac{2\Gamma_L \Gamma_R}{(\Gamma_L + \Gamma_R)^2}. \]

Here, \(S_I = 2 \int_{-\infty}^{\infty} dt e^{i\omega t} \langle (I(t)I(0)) - \langle I \rangle^2 \rangle\) is a current noise power spectrum and \(\Gamma_{L(R)}\) are the tunneling rates between resonant level and left (right) reservoir. The Fano factor has a values from 0.5 for symmetric coupling (\(\Gamma_L = \Gamma_R\)) to 1 for significantly different rates. Suppression of shot noise (\(F < 1\) for asymmetric coupling) is caused by the Pauli exclusion principle which forbids the tunneling of an electron from reservoir into the resonant level as long as the resonant level is occupied. This results in negative correlations between two consecutive electron transfers and, therefore, suppression of shot noise. On the other hand, Coulomb interactions between particles may decrease or increase noise correlations, depending on the physical regimes.

In this work we present a theoretical study of time-dependent fluctuations of the RT current through a two tunnel-coupled quantum dots (QD) in the Coulomb blockade regime when only one of the QD is connected with the leads. This theoretical model, as depicted in Fig. 1, can describe a RT through the quantum well in presence of an impurity inside the well or a RT through the QD when a conducting level is tunnel-coupled with the other localized level of the QD. The shot noise is studied within a bias-voltage and temperature dependent quantum rate equation\(^\text{4}\) approach from our earlier paper\(^\text{5}\).

Both systems can be described by the usual model Hamiltonian for two coupled QDs.
(CQD):

\[
H = \sum_{\eta, k} \epsilon_{\eta k} c_{\eta k}^\dagger c_{\eta k} + \epsilon_A c_A^\dagger c_A + \epsilon_B c_B^\dagger c_B + U c_A^\dagger c_A c_B^\dagger c_B \\
+ t (c_A^\dagger c_B + c_B^\dagger c_A) + \sum_{\eta, k} (V_{\eta k} c_{\eta k}^\dagger c_A + \text{H.c.}),
\]

where \( c_{\eta k}^\dagger (c_{\eta k}) \) are the creation (annihilation) operators for electron with momentum \( k \) and energy \( \epsilon_{\eta k} \) in the lead \( \eta (\eta = L, R) \), \( c_{A(B)}^\dagger (c_{A(B)}) \) are creation (annihilation) operators for an electron in QD A(B), respectively. \( \epsilon_i (i = A, B) \) is the bare energy level of electrons in the \( i \)th QD. Here we assume that only one bare energy level in each dot is involved in transport. The intradot electron-electron Coulomb interactions are assumed to be infinite but the interdot interaction \( U \) is finite. Namely, the state of two electrons occupied in the same QD is forbidden but two electrons dwelling in different QDs is permitted. The \( V_\eta \) describes the coupling between the QD A and lead \( \eta \).

Under the assumption of weak coupling between the QD and the leads, and applying the wide band limit in the two leads, electronic tunneling through this system in sequential regime can be described by the bias-voltage and temperature dependent quantum rate equations for the dynamical evolution of the density matrix elements, \( \rho_{ij}(t) \). The statistical expectations of the diagonal elements of the density matrix, \( \rho_{ii} (i = 0, 1, 2, d) \), give the occupation probabilities of the states of the dots, namely: \( \rho_{00} \) is the probability of finding both dots unoccupied, \( \rho_{11}, \rho_{22}, \rho_{dd} \) are the probabilities of finding dot A, dot B, and both of two dots occupied, respectively. The off-diagonal density matrix elements \( \rho_{12} = \rho_{21}^* \) describe the coherent superposition of the two levels in different QDs. The resulting quantum rate equations are:

\[
\dot{\rho}_{00} = (\Gamma_L^- + \Gamma_R^-)\rho_{11} - (\Gamma_L^+ + \Gamma_R^+)\rho_{00}, \quad (4a)
\]
\[
\dot{\rho}_{11} = (\Gamma_L^+ + \Gamma_R^+)\rho_{00} - (\Gamma_L^- + \Gamma_R^-)\rho_{11} + it(\rho_{12} - \rho_{21}), \quad (4b)
\]
\[
\dot{\rho}_{22} = (\tilde{\Gamma}_L^- + \tilde{\Gamma}_R^-)\rho_{dd} - (\tilde{\Gamma}_L^+ + \tilde{\Gamma}_R^+)\rho_{22} - it(\rho_{12} - \rho_{21}), \quad (4c)
\]
\[
\dot{\rho}_{12} = i(\epsilon_1 - \epsilon_2) + it(\rho_{11} - \rho_{22}) - \frac{1}{2} [\Gamma_L^+ + \Gamma_R^+ + \tilde{\Gamma}_L^+ + \tilde{\Gamma}_R^+] \rho_{12}, \quad (4d)
\]
\[
\dot{\rho}_{dd} = (\tilde{\Gamma}_L^+ + \tilde{\Gamma}_R^+)\rho_{22} - (\tilde{\Gamma}_L^- + \tilde{\Gamma}_R^-)\rho_{dd}, \quad (4e)
\]
together with the normalization relation \( \rho_{00} + \rho_{11} + \rho_{22} + \rho_{dd} = 1 \). The temperature-dependent tunneling rates are defined as \( \Gamma^+ = \Gamma_\eta f^+_\eta(\epsilon_1) \) and \( \Gamma^- = \Gamma_\eta f^-_\eta(\epsilon_1 + U) \), where \( \Gamma_\eta \) are the tunneling constants, \( f^\pm(\omega) = \left[ 1 + e^{(\omega-\mu_\eta)/k_BT} \right]^{-1} \) is the Fermi distribution function of the \( \eta \) lead and \( f^-_\eta(\omega) = 1 - f^+_\eta(\omega) \). Here, \( \Gamma^+_\eta \) (\( \Gamma^-_\eta \)) describes the tunneling rate of electrons into (out of) the QD A from (into) the \( \eta \) lead without the occupation of QD B, while \( \tilde{\Gamma}^+_\eta \) (\( \tilde{\Gamma}^-_\eta \)) stands for the corresponding rates due to the Coulomb repulsion, when the QD B is already occupied by an electron. The current flowing through the system is given by:

\[
I_L = \Gamma^+_L \rho_{00} + \tilde{\Gamma}^+_L \rho_{22} - \Gamma^-_L \rho_{11} + \tilde{\Gamma}^-_L \rho_{dd}.
\] (5)

In order to calculate the noise power spectrum we employed the procedure from Ref. [5]. In the basis \( (\rho_{11}, \rho_{22}, \rho_{00}, \rho_{12}, \rho_{21}, \rho_{dd}) \), the current operator, \( \hat{\Gamma}_L \), has a matrix form with non-zero elements: \( (\hat{\Gamma}_L)_{13} = \Gamma^+_L \), \( (\hat{\Gamma}_L)_{31} = -\Gamma^-_L \), \( (\hat{\Gamma}_L)_{25} = -\tilde{\Gamma}^-_L \), and \( (\hat{\Gamma}_L)_{52} = \tilde{\Gamma}^+_L \).

In the following calculations we set the hopping \( t = 1 \) as the unit of energy, and \( \epsilon_A = \epsilon_B = 1 \), the Coulomb interaction \( U = 5 \), and the thermal energy \( kT = 0.1 \). If \( t = 1 \mu eV \), then \( \Gamma \sim \mu eV \), and \( kT = 0.1 \) meV, which correspond to a typical experimental situation. The Coulomb interaction between the two dot, for this set of parameters, would be \( U = 5 \mu eV \).

The zero of energy is chosen to be the Fermi level of the leads in the equilibrium condition \( (\mu_L = \mu_R = 0) \). The bias voltage, \( V \), between the source and the drain is considered to be applied symmetrically, \( \mu_L = -\mu_R = eV/2 \).

In Fig. 2, we plot the calculated current and Fano factor as functions of bias-voltage for different dot-lead couplings \( \Gamma = 4, 2, 1, \) and 0.5. At small bias, \( eV \ll kT \), the noise is dominated by thermal noise which leads to the divergence \( 2kT/eV \) of the Fano factor occurring at \( V = 0 \). When Fermi level of the source, \( \mu_L \), is well below \( \epsilon_A \) the transport is exponentially suppressed because there are very few tunneling events. In this region, electron transport is limited only by thermally activated tunneling, thus, tunneling events are uncorrelated and Fano factor is Poissonian. The transport through the system becomes energetically allowed when the Fermi level of the source, \( \mu_L \), crosses the discrete level \( \epsilon_A \) (for \( eV/2 \approx \epsilon_A \)). In this region current starts to increase from zero to a constant value \( I_1 \) (the first current plateau):

\[
I_1 = e \frac{\Gamma_L \Gamma_R}{\Gamma_R + 2\Gamma_L},
\] (6)

which is independent on the hopping between two QDs. In contrast to the current, Fano factor \( F_1 \) (Fano factor corresponding to the first current plateau) is dependent on the hopping
Thus, the valuable information can be obtained through the shot noise measurement. The second term in Eq. (7) describes the suppression of the Fano factor below unity, whereas the third term gives a positive contribution. The electron flow through the system is possible by the two different paths: the direct path \((L \rightarrow A \rightarrow R)\) and the indirect one \((L \rightarrow A 
rightarrow B \rightarrow A \rightarrow R)\). For weak coupling between QD A and leads, i.e. \(\Gamma_{L,R} \ll t\), electron performs fast oscillation between two levels with the Rabi frequency \(\omega \sim 2t\). In such case, electron flowing through the indirect path has the same contribution to tunneling as electron through the direct path and the system becomes two interacting tunneling levels model. Clearly, the Fano factor is reduced to \(F_1 = 1 - \frac{4\Gamma_L \Gamma_R}{(\Gamma_R + 2\Gamma_L)^2} + \frac{\Gamma_L^2 \Gamma_R^2}{2t^2 (\Gamma_R + 2\Gamma_L)^2}\). On the contrary, in the case of strong coupling \((\Gamma_{L,R} \gg t)\), the difference between two paths is distinct: electron flowing through the direct path has a fast characteristic time \(\sim \Gamma_{L,R}^{-1}\), while electron tunneling through the indirect path has a slow characteristic time \(\sim (2t)^{-1}\). In the limit where the Coulomb interactions prevent a double occupancy of the central region, there is competition between these two paths. Consequently, the slow flowing of electrons through the indirect path modulate the transport through the direct path, which leads to a bunching of tunneling events and results in super-Poissonian noise, as shown in Fig. 1(b). The super-Poissonian noise as a consequence of the partial blockade of an electronic channel by another one has also been found and described in Ref. 7.

The second step in current-voltage characteristic occur when the Fermi level \(\mu_L\) crosses \(\epsilon_d + U\), i.e. when the system becomes doubly occupied. In this region, the current \(I_2\) and the Fano factor \(F_2\) reduce to the results for one-level system coupled to the leads, Eqs. (1) and (2), respectively. This result can be easily understood as follows: An electron becomes "stuck" in dot B and makes this dot be effectively disconnected from the system.

This theoretical model can be used to explain the experimental results obtained in Refs. 8 and 9, where the investigation of the noise properties of resonant transport through an impurity situated within the quantum well of a tunneling structure has been performed. The measured current-voltage characteristic show a pronounced current step with a Fano factor \(F \approx 0.6\) and an additional second weak structure with \(F > 1\). Our studies claim that the two-step \(I-V\) curve and super-Poissonian noise can be caused by a localized level
coupled with the conducting one.

More information about this system could be obtained through the frequency-dependent shot noise studies. The frequency-dependent Fano factor in the Coulomb blockade regime \((eV/2 < \epsilon_d + U)\), for different couplings \(\Gamma_R\) and \(\Gamma_L\), is plotted in Fig. 3. It is observed that the Fano factor displays an obvious peak at the characteristic frequency \(\omega = 2t\), because the Rabi oscillation has the most pronounced effect on enhancing the deviation of instant current from its average value at this value of frequency. For small coupling between QD A and the right reservoir, \(\Gamma_R < 2t\), there is large probability for an electron inside the CQD to tunnel back and forward many times between two levels before it exits into the right lead. This causes a remarkably sharp peak in the Fano factor at the Rabi frequency \(\omega = 2t\). With increasing \(\Gamma_R\), the probability for an electron to tunnel out increases, which plays a role to destroy oscillating between two levels and induces broadening and decreasing the amplitude of the peak. On the other hand, increasing \(\Gamma_L\) increases the probability to have an electron inside the system, leading to significant enhancement of shot noise.

In conclusion, we have analyzed the shot noise properties of resonant tunneling through coupled QDs when only one of the dots is coupled to the reservoirs. The additional current step and the super-Poissonian shot noise in this region have been found. We have further suggested that experimental results in Ref. \[8\] can be explained through this model. We have also suggested that frequency dependent noise measurement performed on this system could provide more information about hopping strength between levels.

This work was supported by the DURINT Program administered by the US Army Research Office.

1 For an overview of quantum shot noise, please refer to Ya.M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).
2 L.Y. Chen and C.S. Ting, Phys. Rev. B 46, 4714 (1992).
3 Y.V. Nazarov and J.J.R. Struben, Phys. Rev. B 53, 15466 (1996)
4 Bing Dong, H.L. Cui, and X.L. Lei, Phys. Rev. B 69, 35324 (2004).
5 Ivana Djuric, Bing Dong, and H.L. Cui, IEEE Transactions on Nanotechnology, 4, 71 (2004); Ivana Djuric, Bing Dong, and H.L. Cui, cond-mat/0411091 (2004).
6 J. M. Elzerman, R. Hanson, L. H. Willems van Beveren, B. Witkamp, L. M. K. Vandersypen, and L. P. Kouwenhoven, Nature, 430, 431 (2004).

7 A. Cottet and W. Belzig, Europhys. Lett. 66, 405 (2004); A. Cottet, W. Belzig, and C. Bruder, Phys. Rev. Lett. 92, 206801 (2004); A. Cottet, W. Belzig, and C. Bruder, Phys. Rev. B 70, 115315 (2004).

8 A. Nauen, I. Hapke-Wurst, F. Hohls, U. Zeitler, R.J. Haug, and K. Pierz, Phys. Rev. B 66, R161303 (2002); Nauen, F. Hols, J. Königmann, and R.J. Haug, Phys. Rev. B 69, 113316 (2004).

9 S.S. Safonov, A.K. Savchenko, D.A. Bagrets, O.N. Jouravlev, Y.V. Nazarov, E.H. Linfield, and D.A. Ritchie, Phys. Rev. Lett. 91, 136801 (2003).
Figure Caption

FIG.1: Schematic picture of the system. Quantum dot A is connected to the leads L and R via tunneling junctions with rates $\Gamma_L$ and $\Gamma_R$, respectively. It is also coherently coupled to a localized level in quantum dot B with a hopping rate $t$.

FIG.2: Current (a) and Fano factor (b) vs. the bias-voltage in the CQDs calculated for different dot-lead couplings $\Gamma_L = \Gamma_R = \Gamma = 4, 2, 1, \text{and } 0.5$. Other parameters are: $\epsilon_d = 1$, $U = 5$, $kT = 0.1$.

FIG.3: Fano factor vs. frequency in the Coulomb blockade regime calculated for different couplings $\Gamma_R$ (a) and $\Gamma_L$ (b). Other parameters are: $\Gamma_L = 1$ (a), $\Gamma_R = 1$ (b), $\epsilon_d = 1$, $U = 5$, $V = 6$, $kT = 0.1$. 
FIG. 1: Schematic picture of the system. Quantum dot A is connected to the leads L and R via tunneling junctions with rates $\Gamma_L$ and $\Gamma_R$, respectively. It is also coherently coupled to a localized level in quantum dot B with a hopping rate $t$. 
FIG. 2: Current (a) and Fano factor (b) vs. the bias-voltage in the CQDs calculated for different dot-lead couplings $\Gamma_L = \Gamma_R = \Gamma = 4, 2, 1, \text{ and } 0.5$. Other parameters are: $\epsilon_d = 1$, $U = 5$, $kT = 0.1$. 
FIG. 3: Fano factor vs. frequency in the Coulomb blockade regime calculated for different couplings $\Gamma_R$ (a) and $\Gamma_L$ (b). Other parameters are: $\Gamma_L = 1$ (a), $\Gamma_R = 1$ (b), $\epsilon_d = 1$, $U = 5$, $V = 6$, $kT = 0.1$. 