Scheme for Bell state measurement in a g-factor engineered double dot

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Abstract. We proposed a possible protocol for Bell state measurements of spin qubits in a double quantum dot (QD). Our scheme consists of Pauli spin blockade measurements and biaxial electron spin resonance. It is assumed that the two QDs here are designed so that the Landé g-factors satisfy $g_1 = -g_2$ with the use of g-factor engineering. The opposite sign of the g-factors cancel the average of the Zeeman fields on the two QDs, and enable us to swap three spin-triplet states for a spin-singlet state sequentially. We showed that it is possible to implement the full Bell state measurement by the sequential spin-to-charge conversions in the QD-based spin qubits.

1. Introduction

The discovery of quantum teleportation opened the door to secure quantum communication [1]. The quantum information is carried by photons because they travel with little decoherence. However, as the communication distance increases, the efficiency for the teleportation attenuates exponentially due to optical fiber losses. In order to overcome this difficulty, the usage of the devices called quantum repeaters were proposed for entanglement swappings [2]. The quantum repeater consists of a qubit storage and a Bell state analyzer. The storage is made of a matter qubit, and saves quantum states of photons for an instant. Recently Kosaka \textit{et al.} demonstrated quantum state transfer from photons to electron spins in a semiconductor [3, 4]. On the other hand, the implementation of the Bell state analyzer for the spin qubits is still elusive.

In this work, we propose a scheme for full Bell state measurement in a coupled quantum dots (QDs) shown in Fig. 1(a). For two qubit spins in a double QD, the Bell basis set is composed of the four states, i.e., one spin-singlet state $|S\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ and three spin-triplet states $|T_0\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ and $|T_\pm\rangle = (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2}$. Engel and Loss proposed a partial Bell state analyzer which determines the parity of the two spins [5]. The partial Bell state measurement is sufficient for universal quantum computing, but not for quantum repeaters. This is because the total efficiency of the repetitive entanglement swappings decreases as $1/4^N$ at the $N$-th repeater. Our scheme consists of Pauli spin blockade measurements [6, 7, 8] and biaxial electron spin resonance (ESR) [8]. When the two QDs are arranged to have Landé g-factors with opposite signs ($g_1 = -g_2$), it is possible to swap the three triplet states for the singlet state independently. We show that the sequential spin-to-charge conversions implement the full Bell state measurement in g-factor engineered QDs.
oscillating magnetic field $B$

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where $\hat{H}_c$ means Hermitian conjugate and $\delta_j$ is useful [11]. Here we apply a static magnetic field along $D$ direction, i.e., $\mu_B g_D f_{\text{factors}}$ in the QDs, i.e., $\mu_B g_D f_{\text{factors}}$. We assume that the double QD is designed to have opposite sign of $g_D f_{\text{factors}}$ in the QDs, i.e., $\mu_B g_D f_{\text{factors}}$ and reverse the sign in $A_l$ As quantum well by changing Al concentration spatially [10]. In such a double QD, the average of the Zeeman fields on the two QDs Vanishes, and the Hamiltonian for the Zeeman energy can be written as

$$\hat{H}_Z = \delta h^x |S\rangle \langle T_0| - i \delta h^y |S\rangle \langle T_+| - \delta h^z |S\rangle \langle T_-| + \text{H.c.,}$$

(1)

where H.c. means Hermitian conjugate and $\delta h = (h_1 - h_2)/2$ is the inhomogeneity of the Zeeman fields. Here $h_j = -\mu_B g_D f_{\text{factors}}$ with $g_j$ and $\mu_B$ being the Landé $g$-factor and the Bohr magneton. One can see that it is possible to sequentially swap the three triplet states for the singlet $(1,1)$ state by controlling the inhomogeneity $\delta h$.

In order to swap the two-electron spin states, the method of electron spin resonance (ESR) is useful [11]. Here we apply a static magnetic field along $z$-axis, i.e., $B_0 = (0,0,B_0)$, and oscillating magnetic field $B_{ac}(t) = B_1 \cos \omega t$ in $x$-$y$ plane. The ac magnetic field makes the electron spins flip when it is resonant with the spin precession due to $B_0$. For instance, we consider the case that the ac magnetic field in the $x$-direction $B_{ac}^x(t)$ is applied. In the rotating frame of reference, the two-electron spin state can be written as

$$|\psi(t)\rangle = \exp \left[ -i \frac{1}{\hbar} \sum_j (g_j \mu_B B_0 \hat{s}_j^z) t \right] |\psi(0)\rangle,$$

(2)

where $\hat{s}_j$ is the spin-1/2 operator. When $h\omega = \mu_B |g_1|B_0 = \mu_B |g_2|B_0$, the electron spin in the $j$-th QD rotates in the rotating frame following

$$|\uparrow (\downarrow)\rangle_j^z \rightarrow R_{\uparrow}^z(t)|\uparrow (\downarrow)\rangle_j^z = \cos \frac{\theta_j^z}{2} |\uparrow (\downarrow)\rangle_j^z + i \sin \frac{\theta_j^z}{2} |\downarrow (\uparrow)\rangle_j^z,$$

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where $\theta_j = \Omega_j t$ is the rotation angle with Rabi frequency $\Omega_j = g_j \mu_B B_1^2 / 2\hbar$. It should be noted that the electron spins in the two QDs rotate in the opposite directions.

When one electron is confined in each QD, arbitrary two-electron state is expressed by superposing the Bell states, i.e., $|\psi\rangle = c_S |S\rangle + c_0 |T_0\rangle + c_+ |T_+\rangle + c_- |T_-\rangle$. In order to accomplish

2. Scheme for full Bell state measurement

We assume that the double QD is designed to have opposite sign of $g$-factors in the QDs, i.e., $g_1 = -g_2$. It is known that so-called $g$-factor engineerings enable us to prepare such a situation. Indeed, zero $g$-factor ($|g| < 0.01$) was experimentally achieved in an AlGaAs/GaAs/AlGaAs quantum well of 5 nm thick [3]. It is also possible to make the inhomogeneity of the $g$-factor [9] and reverse the sign in Al$_x$ Ga$_{1-x}$As quantum well by changing Al concentration spatially [10].

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the full Bell state measurement, we have to perform projection measurement of |ψ⟩ in all the Bell states {⟨S⟩, ⟨T0⟩, ⟨T+⟩, |T−⟩}. For that purpose, we propose a method using sequential spin-to-charge conversions, in which the Pauli spin blockade measurements [6, 7, 8] and the single-triplet swappings [8] are repeated alternately (see Fig. 1(b)).

First we perform the projection measurement in |S⟩. The charging energy in the QD is controlled by gate voltage \( V_g \) (see Fig. 1(a)). The energy levels of the two-electron states are described by the Hamiltonian

\[
\hat{H}_T = -\epsilon(t)|0,2 \rangle \langle 0,2| + \sqrt{2T}|(\langle 0,2 \rangle S \rangle \langle S| + |S⟩ \langle 0,2 \rangle S)|,
\]

where \( \epsilon \) and \( T \) are the detuning energy and the hopping integral between the QDs, respectively. The state |(0,2)S⟩ is the spin-singlet state with (0, 2) charge configuration. The label \( (m,n) \) denotes the excess electron number in the first and second QDs. Here we neglect the spin-triplet (0, 2) state because it contains the first excited orbital in the second QD. Assuming that initially the detuning energy is a large negative \( \epsilon = \epsilon_0 \ll -\sqrt{2T} \), the state |(0,2)S⟩ lies far above the four Bell states. As the gate voltage on the first QD is raised to make the system in spin blockade regime, i.e., \( \epsilon = \epsilon_1 > \sqrt{2T} \), the singlet component of |ψ⟩ segues into |(0,2)S⟩. Note that we should sweep the gate adiabatically for the interdot tunneling \( \sqrt{2T} \), but non-adiabatically for the inhomogeneity \( \Delta h_z = -g_1 \mu_B B_0 \) to inhibit the mixing of |S⟩ and |T0⟩ [7, 12, 13]. Therefore, the interdot tunneling is detected as a current into the drain lead, which indicates that |S⟩ is postselected [6, 7, 8].

On the other hand, if no current is detected, the postmeasurement state becomes |ψ⟩ = \( N(c_0|T0⟩ + c_+|T+⟩ + c_-|T−⟩) \). Here \( N = 1/(1 - |c_s|^2)^{1/2} \) is the normalization factor. Thus we proceed to the next step. For the projection measurement to |T−⟩, we apply the ac magnetic field \( B_{ac}^r(t) \) common to both the QDs after setting the system in Coulomb blockade regime \( \epsilon = \epsilon_0 \). After the ESR duration \( \tau_{ESR} \) is set to be \( \theta_1(\tau_{ESR}) = \pi/2 \), |T−⟩′′ is swapped into |S⟩′′ in the rotating frame. In the laboratory frame of reference, however, the Rabi oscillation between |S⟩ and |T0⟩ is present due to \( \Delta h_z \). Then, after the ESR sequence, we let the oscillation evolve for the time \( t_0 \) to become \( \Delta h_z(t_0 + \tau_{ESR})/\hbar = 2\pi n \) with \( n \) being an integer. As a consequence, the two-electron state becomes |ψ⟩ → \( \mathcal{R}^z(t_0)\mathcal{R}^x(\tau_{ESR})|ψ⟩ = N(c_0|T0⟩ + c_+|T+⟩ - ic_-|S⟩) \). Therefore the Pauli spin blockade measurement determines whether |T−⟩ is postselected or not. When we obtain the null result again, we swap |T+⟩ for |S⟩ by the magnetic field \( B_{ac}^l(t) \), and determine which of |T+⟩ and |T0⟩ is postselected.

3. Effect of nuclear spins

We investigate the dephasing effect during ESR sequence due to nuclear spins in semiconductors. The nuclei produce random and nonstationary hyperfine field \( \mathbf{I}(t) \) [14, 15, 16]. Here, in order to capture the basic picture, we include only the time-dependent z-component of the hyperfine field, which acts on both the QDs in the same way [16]. Thus it is taken into account by introducing the phase \( \phi(t) = \phi_1 = \phi_2 = \int dt I(t')/2\hbar \). It should be noted that the engineered g-factors does not couple to the hyperfine field, and causes to rotate the two-electron states in the spin-triplet subspace. Here we consider the case that the hyperfine field varies in time scale much shorter than the ESR duration time. Then the effect of the dephasing can be estimated by averaging over a Gaussian random distribution. In addition it is assumed that the hyperfine field can be treated as an uncorrelated white noise, i.e., \( \langle I(t)I(t') \rangle = \sigma^2 \delta(t - t') \). Thus, the phase-phase correlation is found to be [16]

\[
\langle \phi(t)\phi(0) \rangle = \frac{1}{(2\hbar)^2} \int_0^t dt' dt'' \langle I(t')I(t'') \rangle = Dt,
\]

(5)
where $D = (\sigma/2\hbar)^2$ is the dephasing rate. One can see that the fidelity of the swapping, for example, between $|S\rangle$ and $|T^-\rangle$ decreases exponentially as $\exp(-\pi\hbar D/|g_1|\mu_B B_1^x)$.

Recently much effort has been devoted to realizing the steady-state nuclear polarization [13, 17]. In 2008, the inhomogeneous dephasing time for the two-electron spin state is extended beyond $\mu$s. On the other hand, in experiment of Ref [8], a $\pi/2$ spin rotation takes 27 ns for a GaAs QD with $|g| = 0.4$ and $B_1 = 1.9$ mT. Then the effect of the dephasing seems to be non-exhaustive. However our approach is too simple to be directly checked from the experiments, e.g., the nuclear spin environment was found to be non-Markovian in contrast to the treatment here [8]. Therefore we have to proceed further investigation on the dephasing due to the hyperfine fields.

4. Conclusion
We proposed a Bell state analyzer for spin qubits in a coupled QDs which are designed to have opposite sign of g-factors. It was shown that biaxial ESR and Pauli spin blockade measurements enable us to conduct the full Bell state measurement. Since our scheme consists of the well-established experimental techniques, it will be possible to verify the proposed scheme experimentally in near future.

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