Hadronization correspondence of Hawking-Unruh radiation from rotating and electrically charged black holes

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Abstract

The proposed correspondence between the Hawking-Unruh radiation mechanism in rotating, electrically-charged, and electrically-charged-rotating black holes and the hadronization process in high-energy collisions is assumed here. This allows us to determine the well-profound freezeout parameters characterizing the heavy-ion collisions. Furthermore, black holes thermodynamics is found analogous to that of the high-energy collisions. We also introduce a relation expressing the dependence of the angular momentum and the angular velocity deduced from rotating black holes on the chemical potential. The novel phase diagram for rotating, electrically-charged, and electrically-charged-rotating black holes are found in an excellent agreement with the phase diagrams drawn for electrically-charged black holes and also with the ones mapped out from the statistical thermal models and the various high-energy experiments. Moreover, our estimations for the freezeout conditions $\alpha$ and $s/T^3$ are in excellent agreement with the ones determined from the hadronization process, especially at $\mu \approx 0.3$ GeV.

Keywords: thermodynamics of black holes, phase transition in statistical mechanics and thermodynamics, quark-gluon plasma

(Some figures may appear in colour only in the online journal)

1. Introduction

From various high-energy experiments, such as the ones at the Relativistic Heavy-Ion Collider (RHIC) and the Large Hadron Collider (LHC) [1–3], an essential framework for the strongly interacting matter could be constructed [4–8]. Accordingly, it is conjectured that the quark-gluon plasma (QGP), the colored partonic matter, has been created [3, 9], i.e. deconfinement phase transition took place. The created fireball of colorless partons expands and then cools down, rapidly [10]. The critical temperature of such hadronization process depends on the net-baryon density of the colliding system or the baryon chemical potential [11–14]. The latter can phenomenologically be related to both collision energy and critical temperature [10, 15].

From solutions to the general theory of gravity, a classical theory, it is well known that the gravitational fields of the black holes (BHs) are conjectured to capture anything approaching the event horizon. Also, nothing can escape from BHs. But when integrating quantum mechanical effects such as tunneling processes, it turns out that the BHs can radiate particles and/or radiations [16–19]. Such a quantum process is known as Unruh mechanism [17, 20]. The Hawking-Unruh radiation is related to the BHs mass so that heavier BHs radiate more particles/radiations than the lighter ones [16, 17]. It should emphasized that there is no lower bound limiting or ending such an emission process [21, 22].

Such a picture assumes an analogy to the hadronization process in the high-energy collisions [15, 20, 23–29] that the
corresponding pair creation and string breaking could be related to each other [30–32]. With this regard, there is another correspondence to be highlighted, namely the equivalence between the BHs metric with a uniform acceleration with the surface gravity and the metric of a high-energy system, such as Rindler metric, with a uniform acceleration [33].

As introduced in [19, 34–36], such a correspondence could be used to suggest solutions for many still-unsolved puzzles of QCD thermalization processes and also the thermodynamic characteristics of the chemical freezeout stage, temperature and chemical potential [19, 28, 29]. Furthermore, the freezeout temperature of the hadronization process has been studied in BHs with a negative cosmological constant [37]. It was found that the temperature decreases with the increase in the chemical potential due to the effect of the cosmological constant [37]. Similar results were also reported in [28, 29].

At vanishing chemical potential, the freezeout conditions $(E)/\langle N \rangle = \sqrt{2/\pi} \sigma \simeq 1.09$ GeV and $s/T^3 = 3 \pi^2/4 \simeq 7.4$ have been determined, at the freezeout temperature $T_f = \sqrt{\sigma/2 \pi} \simeq 165$ MeV [38]. It was concluded that the values determined agree well with the ones obtained from other statistical thermal models [38]. This analogous was extended to finite chemical potentials and implemented on electrically-charged black holes [28]. An expression for the dependence of the BHs electric charge on the baryon chemical potential was proposed and then used to calculate various freezeout conditions. An excellent agreement with the heavy-ion collisions was concluded, especially at $\mu \leq 0.3$GeV [28].

In the present script, we propose to associate the variation of charges of the black holes with the change in the baryon chemical potential in high-energy collisions, while the variation of the angular momentum of black holes is related to the change in the angular velocity of the spectators of the offcentral high-energy collisions [19]. We also propose that the angular velocity can be expressed by the isospin chemical potential, equations (20), which is related to the rotating particles in the high-energy collisions, section 4. Thus, the present script proposes that the temperature of the Hawking-Unruh radiation from all types of black holes is to be related to the number density and/or the spin related to the principle of collectivity [19].

The present paper is organized as follows. The analogy between the Hawking-Unruh radiation and the hadronization in the high-energy collisions is reviewed in section 2. The rotating, electrically-charged, and electrically-charged-rotating black holes shall be discussed in section 3. In section 4, black holes thermodynamics shall be outlined. Also, the dependence of the angular momentum and the angular velocity of the black holes on the chemical potential is introduced in section 4. In section 5, the freezeout diagram and both freezeout conditions $(E)/\langle N \rangle$ and $s/T^3$ shall be elaborated. The results are discussed in section 6. Section 7 concludes this work.

2. Analogy between Hawking-Unruh radiation and QCD hadronization

Over the last four decades, we have been conducting various high-energy experiments. They have been designed for the study the strongly interacting matter under extreme conditions of high temperatures and large densities [8]. Throughout, the colliding hadrons are deconfined into QGP, which then expands and cools down, rapidly. At critical temperatures, confined hadrons are reproduced, again [39]. This phase transition from QGP to hadrons is known as a hadronization process. The black holes, especially in the sense that they emit particles/radiations, are conjectured to have a similarity with the partonic QGP. This emission or radiation process is characterized by Hawking temperature which corresponds to Unruh temperature. The main assumption of the present script is that the latter could be related to the hadronization temperature. The concept of an analogy between the Hawking-Unruh radiation and the QCD hadronization was already proposed in literature [19, 34–36]. The idea is that both theories are based on the confinement/deconfinement properties. The tunneling quantum process is assumed to form hadrons (hadronization) [18]. A similarity is also assumed for the thermal radiation emitted from the black holes and from the high-energy collisions [18]. The thermal radiation stemming from the BHs is conjectured to be produced as a consequence of the uniform acceleration $\alpha$ of the event horizon [17, 18]. Unruh showed that the temperature of this thermal radiation can be expressed as $T = \alpha/2\pi$ [18]. As discussed, it was concluded that the Hawking-Unruh temperature depends on the baryon number density and on the angular velocity of the deconfined system [19].

The measurements from high-energy collisions and the calculations based on the statistical thermal models in grand-canonical ensemble, for instance, could be used to quantify, as much as possible, the hadronization process and also to estimate the freezeout parameters, the temperature and the chemical potential [10]. Accordingly, the chemical potential $\mu$ can be estimated as $\mu = n_b \mu_B + n_s \mu_S + n_I \mu_I$, where $n_b$, $n_s$, and $n_I$ are baryon, strangeness, and isospin quantum numbers of the system, respectively [40, 41]. $\mu_B$, $\mu_S$ and $\mu_I$ are baryon, strangeness, and isospin chemical potential, respectively. Therefore, $\mu_B$ in the high-energy collisions could be related to $\mu_Q$ from the electrically-charged BHs and $\mu_I$ to $\mu_J$ from rotating BHs.

The consideration of adding new BH properties, such as the electric charge, in order to extend the chemical potential $\mu$ from vanishing to finite values and accordingly to relate Hawking-Unruh radiation with the hadronization temperature was already introduced in [28, 42]. Also, the dependence of spin or angular momentum in the BH quantum tunneling processes which could then be related to the angular velocity was proposed in [19]. In doing this, a new component shall be added to the chemical potential $\mu$. This is the isospin chemical potential $\mu_I$ which enabling us to express the change in spin of the black holes (whether just rotating or electrically-charged-rotating) due to the change in the angular velocity.

3. Proposed black holes

A black hole could be formed as a result of the gravitational collapsing of giant stars [43]. The black hole has three basic properties; mass $M$, electric charge $Q$, and spin or angular momentum $J$. Accordingly, there are different types of black
holes depending on these properties. Also, there are other features such as the event horizon metric and the Hawking-Unruh temperature which could be expressed for the various types of black holes [19].

The event horizon is formed by the strong gravitational attraction, at which a divergence of Schwarzschild metric. The spacetime line element \( ds^2 \) reads

\[
ds^2 = (1 - 2GM/r) \, dt^2 - \frac{1}{1 - 2GM/r} \, dr^2,
\]

where \( t \) is the time. Obviously, at the Schwarzschild radius, \( ds^2 \) diverges \( r_S = 2GM \), where \( G \) is the gravitational constant, \( G = 1/2\sigma = 2.631 \) with \( \sigma \) is known as the string tension, \( \sigma = 0.19 \text{GeV}^2 \).

The field strength of the interaction can be obtained from the coefficient

\[
f(r_S) = 1 - \frac{2GM}{r_S}
\]

The radiation temperature is to be estimated as \( f'(r_S)/4\pi \) [44]

\[
T = \frac{1}{2\pi} \frac{GM}{r_S^2},
\]

where \( r_S = 2GM \). The temperature of the Hawking-Unruh radiation is thus given as

\[
T_{BH}(M, 0) = \frac{1}{8\pi GM},
\]

where \( T_{BH}(M, 0) \) is the temperature of Hawking-Unruh radiation emitted from Schwarzschild black hole, i.e. non-rotating and uncharged black hole.

In the sections that follow, we extend the discussion to cover rotating, electrically-charged, and electrically-charged-rotating black holes. We determined \( ds^2 \), and how \( T_{BH} \) varies.

### 3.1. Electrically-charged black holes

If the black hole has an electric charge \( Q \), then the corresponding Coulomb repulsion is conjectured to weaken the gravitational attraction so that the event horizon experiences modifications [45]. The Reissner-Nordström metric (for electrically-charged black holes) can be written as

\[
ds^2 = \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right) dt^2 - \left(1 - \frac{2GM}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2.
\]

The divergence leads to smaller Reissner-Nordström radius

\[
r_K = \frac{r_S}{2} \left(1 + \sqrt{1 - \frac{Q^2}{GM^2}}\right).
\]

At \( Q = 0 \), \( r_K \) reduces to \( r_S \).

The temperature of the Reissner-Nordström radiation becomes [28, 43, 46]

\[
T_{BH}(M, Q) = T_{BH}(M, 0) \sqrt{\frac{2}{1 - \frac{Q^2}{GM^2}}},
\]

where \( T_{BH}(M,0) \) is the temperature of the Schwarzschild radiation, \( T_{BH}(M, 0) = (8\pi GM)^{-1} \) [19].

### 3.2. Rotating black holes

If the black hole rotates around an axis, so it will have an angular momentum, which makes the centripetal force opposes the gravitational attraction, which in turn weakens its strength. The resulting Kerr metric for rotating black holes, where \( J \neq 0 \) and \( Q = 0 \), [19] is given as

\[
ds^2 = \left(1 - \frac{2GMr_K}{r^2} + \frac{Q^2}{r^2}\right) dt^2 - \left(\frac{r^2}{r^2 - 2GMr_K + a^2}\right) \, d\theta^2 - \left(\frac{r^2 + a^2 \cos^2 \theta}{r^2 - 2GMr_K + a^2}\right) \, d\phi^2 - \frac{4S}{r^2} \, \sin^2 \theta \, d\phi \, d\theta
\]

\[
- \left(\frac{r^2 + a^2 \cos^2 \theta}{r^2 - 2GMr_K + a^2}\right) \, dr^2.
\]

where \( \theta \) is the polar axis and here \( \theta = 0 \) [19]. Also, \( J \) is defined as the angular momentum of the black hole, which is obviously given by \( J = aM \), with \( a \) is the angular momentum parameter. At \( a = 0 \), the metric reduces to the Schwarzschild one.

For rotating black holes, there are two event horizons; actual (internal surface) and ellipsoid (external surface). The region between these two boundaries is known as the ergosphere. In the present calculations, we take into consideration the actual event horizon, only, which refers to the ‘outer’ confinement [19]. The Kerr radius defines the actual event horizon of the rotating black holes [19]

\[
r_K = \frac{r_S}{2} \left(1 + \sqrt{1 - \frac{a^2}{G^2M^2}}\right)
\]

The temperature of the Kerr radiation reads

\[
T_{BH}(M, J) = \frac{4GMr_K^2 - 2GMa^2 - 2GMr_K^2}{4\pi (r_K^2 + a^2)^2} = T_{BH}(M, 0) \sqrt{\frac{2}{1 - \frac{a^2}{G^2M^2}}},
\]

which obviously reduces to the temperature characterizing the Schwarzschild black holes, at \( a = 0 \).

### 3.3. Electrically-charged-rotating black holes

For black holes having both electric charge and spin, known as Kerr-Newman (\( Q \neq 0 \) and \( J \neq 0 \)), the corresponding Kerr-Newman metric reads [47]

\[
ds^2 = \left(1 + \frac{Q^2G - 2GMr_{KN}}{r_{KN}^2 + a^2 \cos^2 \theta}\right) dt^2 - \left(\frac{r_{KN}^2 + a^2 \cos^2 \theta}{r_{KN}^2 - 2GMr_{KN} + a^2 + Q^2G}\right) \, d\phi^2
\]

\[
- \left(\frac{r_{KN}^2 + a^2 \cos^2 \theta}{r_{KN}^2 - 2GMr_{KN} + a^2 + Q^2G}\right) \, dr^2.
\]
The actual event horizon is given as
\[ r_{KN} = \frac{r_S}{2} \left[ 1 + \sqrt{1 - \frac{Q^2}{GM^2} - \frac{a^2}{G^2M^2}} \right], \] (12)
and the temperature of the Kerr-Newman radiation is expressed as \[ T_{BH}(M, Q, J) = \frac{2GM^2 - 2GMa^2 - 2G^2r_{KN}^2}{4\pi(r_{KN}^2 + a^2)^2} \]
\[ \times \left\{ \frac{2r_{KN} - GM}{\sqrt{1 - (GQ^2 + a^2)/G^2M^2}} \right\} \] (13)

4. Thermodynamics of black holes

For thermodynamics of rotating black holes, we start with the first law of thermodynamics
\[ dE = TdS + \Omega dJ, \] (14)
where \( S \) is the entropy. The angular velocity \( \Omega \) reads \[ \Omega = \frac{4\pi\alpha}{S}, \] (15)
where the acceleration \( \alpha \) can also be given as \( \alpha = GM/r_{BH}^2 \). As a result of the analogy with the Hawking-Unruh radiation mechanism, i.e. we relate the temperature to \( T_{BH}(M, J) \) and the angular velocity to \( \mu_1 \), then the first law of thermodynamics can be rewritten as
\[ dM = T_{BH}(M, J) dS_K + \mu_1 dJ, \] (16)
where \( S_K = \pi(r_K^2 + a^2)/G \) is the entropy of Kerr black holes. The isospin chemical potential \( \mu_1 \) is the parameter associated with the variation of the angular momentum of the black holes.

Next, we propose an expression relating the chemical potential with rotating BHs (isospin chemical potential) and both angular velocity and angular momentum. In general, the rotational kinetic energy of an object can be given as
\[ E_R = QV, \] (17)
where \( E_R = \frac{1}{2}I\Omega^2 = \frac{1}{2}\alpha\Omega^2 \), with \( I \) is the moment of inertia and \( V \) is the potential which will be exchanged by the chemical potential \( \mu \). Then
\[ \Omega = \frac{2Q\mu}{a}, \] (18)
Also, we substitute \( Q \) from the relation \( \mu = 1.4Q/r \) [28], so that
\[ \Omega = \frac{2\mu^2 r}{1.4a}. \] (19)

The dependence of \( \mu \) on \( \Omega \) can be given
\[ \mu_1 \sim \frac{\Omega a}{\sqrt{0.05 r_{KN}}}, \] (20)
where \( \mu_1 \) and \( \Omega \) are expressed in GeV-units, while \( a \) is a dimensionless quantity, and \( r_{KN} \) has is given in GeV\(^{-1}\)-units. Expression (20) was proposed in classical theory from the relation between the angular momentum and the potential of a particle. We recall an expression for the rotational kinetic energy of a rotating object in a medium in order to obtain an expression for the relation between \( \mu \) and \( \Omega \). This expression obviously gives the potential of adding or removing a particle from the rotating black hole (isospin chemical potential) and how this depends on the variation of the black hole angular momentum. The chemical potential \( \mu \) sums up the various kinds of chemical potentials related to the quantum numbers considered for the system of interest. The correctness of the proposed expression shall be examined in section 6.

Additional to equation (16), the first law of thermodynamics of Kerr-Newman black holes can also be reexpressed due to the analogy with the Hawking-Unruh radiation
\[ dM = T_{BH}(M, Q, J) dS_K + \mu_1 dQ + \mu_2 dJ, \] (21)
where \( S_K = \pi(r_K^2 + a^2)/G \) and \( Q \) are the entropy and the electric charge of the Kerr-Newman black holes, respectively. \( \mu_1 = 1.4Q/r_{KN} \) is the value of the chemical potential related to the variation of the black hole’s electric charge [28, 29]. Thus, the chemical potential reads \( \mu = \mu_1 + \mu_2 \), which expresses the change in the electric charge and the angular momentum of the black holes
\[ \mu = \frac{1.4 Q}{r_{KN}} + \frac{\Omega a}{\sqrt{0.05 r_{KN}}}. \] (22)

So far, we conclude that the correspondence between black hole thermodynamics and thermodynamics of Hawking-Unruh radiation can be understood as a variation of black hole’s electric charge, which apparently refers to an absorbtion (or emission) of a particle with a specific baryon chemical potential, while the variation of the black hole angular momentum (causes changing angular velocity of BH) indicates an insertion of a new quantum number, isospin angular momentum, to which we assign an isospin chemical potential.

5. Freezeout conditions

For the hadron-parton phase transition, we restrict the discussion to the stage of chemical freezeout [28, 48, 49]. There are different conditions suggested for the description of the freezeout phase diagram [28, 48–54]. All these conditions are investigated from various statistical thermal models to determine the freezeout parameters \( T \) and \( \mu \), which in turn have been obtained from measuring particle yields and/or ratios in various high-energy experiment, which is then confronted to statistical thermal model calculations [10], in which \( T \) and \( \mu \) are taken as free parameters [48–51, 55].
We first start with the condition of averaged energy per averaged particle, at vanishing $\mu$ [38]
\[
\frac{\langle E \rangle}{\langle N \rangle} \bigg|_{\mu=0} = \sigma_{rS}.
\] (23)
At finite $\mu$ (or finite $Q$ and/or finite $J$), the string tension of rotating, electrically-charged, and electrically-charged-rotating black holes are assumed to take the same phenomenological behaviour [33]
\[
\sigma(\mu) \simeq \sigma(\mu = 0) \left[1 - \frac{\mu}{\mu_0}\right],
\] (24)
where $\sigma(\mu = 0) = 0.19\text{ GeV}^2$ and $\mu_0 \approx 1.2\text{ GeV}$ is a free parameter [33].

- At finite $J$, $\mu = \mu_J$, then $r_{S}$ could be replaced by $r_K$, equation (9), which modifies the proposed freezeout condition
\[
\frac{\langle E \rangle}{\langle N \rangle} \bigg|_{\mu=0} = \sigma(\mu)\{r_K + a\},
\] (25)

- At finite $Q$ and $J$, $\mu = \mu_Q + \mu_J$, then $r_{S}$ should be replaced by $r_{KN}$, equation (12), and the freezeout condition becomes
\[
\frac{\langle E \rangle}{\langle N \rangle} \bigg|_{\mu=0} = \sigma(\mu)\{r_{KN} + a\}.
\] (26)

Then, conclude that the resulting $\langle E \rangle/\langle N \rangle$ agrees with the values proposed for the best description for the chemical freezeout in the high-energy collisions.

Then, we move to the second freezeout condition, namely, the entropy density $s = S/V$ normalized to cubic temperature, which was suggested to have a constant value along the entire line of particle production covering vanishing to finite baryon chemical potential [49, 53]. The volume $V$ could be modelled as $V = 4\pi r^3 (r^3 + a^3)/3$. Similarly the volume of the black holes could also assumed as a sphere. In the Hawking-Unruh radiation from Schwarzschild black hole [38], this condition was estimated as $s/T^3_{\mu=0} = 3/8 G^2 M T_{\mu=0}^2$ [28]. This condition was also calculated for natural and electrically-charged black holes and it was concluded that the result agree with the one for the particle production [28, 38].

The expressions used to determined $s/T^3$ can be detailed as follows.

- For rotating black holes, we propose that the proposed correspondence of the Hawking-Unruh radiation and the QCD hadronization is also valid at finite density $\mu = \mu_J$ or (finite $Q$ and $J$)
\[
\frac{s}{T^3} \bigg|_{\mu=0} = \frac{s}{T^3} \bigg|_{\mu=0} \left\{1 + \sqrt{1 - \frac{a^2}{G^2 M^2}} \right\} \left\{1 - \frac{a^2}{G^2 M^2} \right\}^{3/2}.
\] (27)

\textbf{Figure 1.} The freezeout temperature $T$ is depicted in dependence on the chemical potential $\mu$. The calculations from equations (10), (20) and equations (13), (22) are given as solid and double-dotted curves, respectively. The previous results for electrically-charged BHs are represented by the dotted curve, [28]. The dashed curve refers to the HRG results obtained at $s/T^3 = 7$. The symbols depict the freezeout parameters deduced from the particle ratios by using different statistical thermal models: Andronic et al [56], Cleymans et al [57], Tawfik et al [55], and from experimentally measurements for particle ratios: HADES [58] and FOPi [59].

- For electrically-charged and rotating black holes, we introduce that the proposed correspondence of the Hawking-Unruh radiation and the QCD hadronization is valid at finite density $\mu = \mu_Q + \mu_J$ or (finite $Q$ and $J$)
\[
\frac{s}{T^3} \bigg|_{Q, J=0} = \frac{s}{T^3} \bigg|_{\mu=0} \left\{1 + \sqrt{1 - \frac{a^2}{G^2 M^2}} \right\} \left\{1 - \frac{a^2}{G^2 M^2} \right\}^{3/2}.
\] (28)

The results, which shall be discussed in the section that follows, suggest that $s/T^3 \approx 7$ which was originally proposed of chemical freezeout in high-energy collisions is also resulted in from electrically-charged and rotating black holes.

\textbf{6. Results and discussion}

Figure 1 presents the dependence of the freezeout temperature $T$ on the chemical potential $\mu$; the freezeout phase diagram. The solid curve represents the present calculations for the rotating black holes, equation (10), where the variation of $J$ is associated with $\mu_Q$ as given in equation (20). The double-dotted curve depicts the results for the electrically-charged and rotating black holes, equation (13), where the variation of $Q$ and $J$ are associated to $\mu$ as given in equation (22). Here, we use both the metric for rotating BHs, equation (8), and the metric for the electrically-charged and rotating BHs, equation (11), but at...
θ = 0. All present results are compared with the calculations from the electrically-charged black holes (dotted curve) [28], with the hadron resonance gas (HRG) model at $s/T^3 = 7$ (dashed line) and with the freezeout parameters (symbols) deduced from the various particle ratios measured and then fitted to the statistical thermal models: Andronic et al [56], Cleymans et al [57], Tawfik et al [55], and from experimental measurements for particle ratios: HADES [38], and FOPI [59].

As a result of the variation in the spin of both types of BHs, the corresponding chemical potentials experiences a change and accordingly affects the resulting freezeout temperature. We notice that there is an excellent agreement between the present freezeout parameters for both types of black holes [28, 29]. It is obvious that our present results seem improving the calculations reported in [28, 29], where as hadronization correspondence was also utilized on other types of BHs, as well. It is obvious that taking into consideration the rotation of the black hole besides the electric charge likely counts for this improvement, which is apparently determined from the direct comparison with the freezeout parameters deduced from the statistical thermal models, figure 1.

Figure 2 depicts both freezeout conditions $s/T^3$ (solid curve) and $(E)/(N)$ (dashed curve) as functions of the chemical potential $\mu$, equations (25), (27), as deduced from rotating BHs, left-hand panel. For the sake of interpretation, we also draw the entropy density $s$ (dotted curve) as a function of the chemical potential $\mu$. The same dependence is also drawn in the right-hand panel, but here for electrically-charged and rotating black holes using, equations (26) and (28).

In both types of black holes, we notice that the $s/T^3$ remains constant $\sim 7$, especially at $\mu \leq 0.3$ GeV, which agrees well with the value conducted from the particle production in the various high-energy experiments [49, 53]. At $\mu > 0.3$ GeV, we find that the entropy density normalized to $T^3$ increases with a further increase in $\mu$, while the entropy density $s$ is nearly independent on the change in spin and/or electric charge of the black holes or in the chemical potential $\mu$. Thus, the increase in $s/T^3$, at $\mu > 0.3$ GeV, can be understood due to the decrease in the denominator with respect to the numerator or the increase in the fraction within the bracket of the denominator of equations (27), (28). Such a condition could be fulfilled with increasing $Q$ and $J$. We also conclude that the results obtained for $s/T^3$ apparently agree well with the electrically-charged black holes [28]. In the present study, we also have taken into consideration the same possibilities proposed in [28] for rotating and electrically-charged-rotating black holes. The mass of black holes which describes well the correspondence between the Hawking-Unruh radiation in both types of black holes and the hadronization in the high-energy collisions is also assumed as 1.3606 GeV. In our calculations for electrically-charged-rotating black hole, we have fixed the charge of the black hole to $Q = 0.15$ GeV.

7. Conclusions

The correspondence between the hadronization process in high-energy collisions and the black holes are examined, here. Concretely, we have studied the analogous correspondence for two types of black holes, rotating and electrically-charged-rotating black holes with results deduced from high-energy collisions. We have proposed an expression for the dependence of the angular momentum and the angular velocity on the chemical potential. The freezeout diagram for rotating and electrically-charged-rotating black holes are found in an excellent agreement with the ones deduced from the electrically charged black holes and from the different statistical thermal models. We have calculated two freezeout conditions, $E/N$ (solid curve) and $(E)/(N)$ (dashed curve) as functions of the chemical potential $\mu$, equations (25), (27), as deduced from rotating BHs, left-hand panel. For the sake of interpretation, we also draw the entropy density $s$ (dotted curve) as a function of the chemical potential $\mu$. The same dependence is also drawn in the right-hand panel, but here for electrically-charged and rotating black holes using, equations (26) and (28).

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7. Conclusions

The correspondence between the hadronization process in high-energy collisions and the black holes are examined, here. Concretely, we have studied the analogous correspondence for two types of black holes, rotating and electrically-charged-rotating black holes with results deduced from high-energy collisions. We have proposed an expression for the dependence of the angular momentum and the angular velocity on the chemical potential. The freezeout diagram for rotating and electrically-charged-rotating black holes are found in an excellent agreement with the ones deduced from the electrically charged black holes and from the different statistical thermal models. We have calculated two freezeout conditions, $(E)/(N)$ (solid curve) and $(E)/(N)$ (dashed curve) as functions of the chemical potential $\mu$, equations (25), (27), as deduced from rotating BHs, left-hand panel. For the sake of interpretation, we also draw the entropy density $s$ (dotted curve) as a function of the chemical potential $\mu$. The same dependence is also drawn in the right-hand panel, but here for electrically-charged and rotating black holes using, equations (26) and (28).

In both types of black holes, we notice that the $s/T^3$ remains constant $\sim 7$, especially at $\mu \leq 0.3$ GeV, which agrees well with the value conducted from the particle production in the various high-energy experiments [49, 53]. At $\mu > 0.3$ GeV, we find that the entropy density normalized to $T^3$ increases with a further increase in $\mu$, while the entropy density $s$ is nearly independent on the change in spin and/or electric charge of the black holes or in the chemical potential $\mu$. Thus, the increase in $s/T^3$, at $\mu > 0.3$ GeV, can be understood due to the decrease in the denominator with respect to the numerator or the increase in the fraction within the bracket of the denominator of equations (27), (28). Such a condition could be fulfilled with increasing $Q$ and $J$. We also conclude that the results obtained for $s/T^3$ apparently agree well with the electrically-charged black holes [28]. In the present study, we also have taken into consideration the same possibilities proposed in [28] for rotating and electrically-charged-rotating black holes. The mass of black holes which describes well the correspondence between the Hawking-Unruh radiation in both types of black holes and the hadronization in the high-energy collisions is also assumed as 1.3606 GeV. In our calculations for electrically-charged-rotating black hole, we have fixed the charge of the black hole to $Q = 0.15$ GeV.
obtained from the electrically-charged black holes which in turn means that the analogy between the hadronization process in high-energy collisions and the Hawking-Unruh radiation from all types of black holes can be used to estimate the values of the freezeout conditions.

The other freezeout condition, $(E)/(N) \approx 1.35 \text{ GeV}$, at $\mu \lesssim 0.3 \text{ GeV}$. At higher $\mu$, the average energy per particle seems slightly decreasing as a result of the variation of the spin and electric charge of the black holes. This result agrees with the value proposed in [40, 50, 60] and also was confirmed in [28, 29].

We can now confirm the conclusions drawn in [28, 29] that the freezeout parameters $T$ and $\mu$ can be explained even by the immense gravitational deconfinement as that of the black holes and also that the Hawking-Unruh radiation emitted from rotating, electrically-charged and electrically-charged-rotating black holes finds a correspondence in the particle production. We also conclude that the degrees of freedom measured by constant $s/T^3$, for instance, seem to remain conserved during the black hole radiation process, which is accompanied by a mass reduction. We mean that the charge and the angular momentum of BH are conjectured not to affect the reduction of the mass of BH, when the radiation is emitted from BH. With decreasing $T$, the BH mass increases preventing BH from the entire dissolving. But when $J \neq 0$ and/or while $Q \neq 0$, the rotational force and the Coulomb repulsion oppose the gravitational attraction. In the case that these forces become able to overcome the gravitational attraction force, BH dissolves although the BH temperature decreases.

In light of present results, we could draw a conclusion that the back hole thermodynamics, classical theory of general relativity, seems to have almost the same correspondence with the high-energy collisions, which is well described by lattice QCD and statistical thermal models for strong interactions.

In a future work, we plan to use the proposed correspondence between QCD and the Hawking-Unruh radiation process in order to study the charm production in both high-energy physics and Hawking-Unruh radiation.

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