Galactic kinematics and dynamics from RAVE stars

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ABSTRACT

We analyse the kinematics of ~ 400 000 stars that lie within ~ 2 kpc of the Sun and have spectra measured in the RAdial Velocity Experiment (RAVE). We decompose the sample into hot and cold dwarfs, red-clump and non-clump giants. The kinematics of the clump giants are consistent with being identical with those of the giants as a whole. Without binning the data we fit Gaussian velocity ellipsoids to the meridional-plane components of velocity of each star class and give formulae from which the shape and orientation of the velocity ellipsoid can be determined at any location. The data are consistent with the giants and the cool dwarfs sharing the same velocity ellipsoids, which have vertical velocity dispersion rising from 21 km s\textsuperscript{-1} in the plane to ~ 55 km s\textsuperscript{-1} at |z| = 2 kpc and radial velocity dispersion rising from 37 km s\textsuperscript{-1} to 82 km s\textsuperscript{-1} in the same interval. At (R, z) the longest axis of one of these velocity ellipsoids is inclined to the Galactic plane by an angle ~ 0.8 arctan(z/R). We use a novel formula to obtain precise fits to the highly non-Gaussian distributions of v\textsubscript{φ} components in eight bins in the (R, z) plane.

We compare the observed velocity distributions with the predictions of a published dynamical model fitted to the velocities of stars that lie within ~ 150 pc of the Sun and star counts towards the Galactic pole. The predictions for the v\textsubscript{z} distributions are exceptionally successful. The model’s predictions for v\textsubscript{φ} are successful except for the hot dwarfs, and its predictions for v\textsubscript{r} fail significantly only for giants that lie far from the plane. If distances to the model’s stars are over-estimated by 20 per cent, the predicted distributions of v\textsubscript{r} and v\textsubscript{z} components become skew, and far from the plane broader. The broadening significantly improves the fits to the data.

The ability of the dynamical model to give such a good account of a large body of data to which it was not fitted inspires confidence in the fundamental correctness of the assumed, disc-dominated, gravitational potential.

Key words: Galaxy: disc - kinematics and dynamics solar neighbourhood - galaxies: kinematics and dynamics -
1 INTRODUCTION

A major strand of contemporary astronomy is the quest for an understanding of how galaxies formed and evolved within the context of the concordance cosmological model, in which the cosmic energy density is dominated by vacuum energy and the matter density is dominated by some initially cold matter that does not interact electromagnetically. This quest is being pursued on three fronts: observations of objects seen at high redshifts and early times, simulations of clustering matter and star formation, and by detailed observation of the interplay between the chemistry and dynamics of stars in our own Galaxy.

As a contribution to this last “Galactic archaeology” strand of the quest for cosmic understanding, the RAdial Velocity Experiment (Steinmetz et al. 2006) has since 2003 gathered spectra at resolution $\sim 7500$ around the CaII near-IR triplet of $\sim 400 000$ stars. The catalogue stars are roughly half giants and half dwarfs, and mostly lie within 2.5 kpc of the Sun (Burnett et al. 2011; Binney et al. 2013). The RAVE survey is complementary to the Sloan Digital Sky Survey (SDSS; York et al. 2000) and the latter’s contamination of the Galaxy, contributes a small but some first moments are non-zero: values of the distributions of the velocity components parallel to the principal axes of the local velocity ellipsoid. The second moments are consistent with our previously derived values, and thick discs, by contrast, completely dominate the RAVE proportion of the stars in the SDSS data releases. The thin and thick discs, by contrast, completely dominate the RAVE catalogue.

Recently Binney et al. (2013) derived distances to $\sim 400 000$ stars from 2MASS photometry and the stellar parameters produced by the VDR4 spectral-analysis pipeline described by Kordopatis et al. (2013). We use these distances to discuss the kinematics of the Galaxy in the extended solar neighbourhood, that is, in the region within $\sim 2$ kpc of the Sun. Since the selection criteria of the RAVE survey are entirely photometric, we can determine the distribution of the velocities of survey stars within the surveyed region without determining the survey’s complete selection function, which is difficult (see Prieto & Steinmetz in preparation, Sharma et al. in preparation).

We characterise the kinematics in several distinct ways. In Section 3 we obtain analytic fits to the variation within the $(R, z)$ plane of the velocity ellipsoid by a technique that avoids binning stars (Burnett 2011). In Section 4 we bin stars to obtain histograms of the distribution of three orthogonal components of velocity. We use a novel formalism to obtain analytic fits to the distributions of the azimuthal component of velocity. We examine the first and second moments of the distributions of the velocity components parallel to the principal axes of the local velocity ellipsoid. The second moments are consistent with our previously derived values, but some first moments are non-zero; values $\sim 1.5 \text{ km s}^{-1}$ are common and values as large as 5 km s$^{-1}$ occur.

In Section 5 we compare our results with the predictions of a dynamical model Galaxy that is based on Jeans’ theorem. Although this model, which was described by Binney (2012; hereafter B12), was not fitted to any RAVE data, we find that its predictions for the distributions of vertical components are extremely successful, while those for the radial components are successful at $|z| < 0.5$ kpc but become less successful further from the plane, where they produce velocity distributions that are too narrow and sharply peaked. In Section 5.3 we investigate the impact of systematically over-estimating distances to stars. When distances to the model’s stars are over-estimated by 20%, the predicted distributions of $v_R$ and $v_z$ acquire asymmetries that are similar to those sometimes seen in the data. Systematic over-estimation of distances brings the model into better agreement with data far from the plane by broadening its $v_R$ distributions.

2 INPUT PARAMETERS AND DATA

Throughout the paper we adopt $R_0 = 8$ kpc as the distance of the Sun from the Galactic centre, $\Theta_0 = 220 \text{ km s}^{-1}$ for the local circular speed and from Schönrich et al. (2010) $(U_0, V_0, W_0) = (11.1, 12.24, 7.25) \text{ km s}^{-1}$ as the velocity of the Sun with respect to the Local Standard of Rest. While our values of $R_0$ and $\Theta_0$ may be smaller than they should be (e.g. McMillan 2011), we adopt these values in order to be consistent with the assumptions inherent in the B12 model.

Proper motions for RAVE stars can be drawn from several catalogues. Williams et al. (2013) compares results obtained with different proper-motion catalogues, and on the basis of this discussion we originally decided to work with the PPMX proper motions (Röser et al. 2008) because these are available for all our stars and they tend to minimise anomalous streaming motions. However, when stars are binned spatially and one computes the dispersions in each bin of the raw velocities $4.73 \mu (s/kpc) + v_{\text{los}}$ from the PPMX proper motions, the resulting dispersions for bins at distances $\sim 0.5$ kpc are often smaller than the contributions to these from proper-motion errors alone. It follows that either our distances are much too large, or the quoted proper-motion errors are seriously over-estimating the true random errors. The problem can be ameliorated by cutting the sample to exclude stars with large proper-motion errors, but there are still signs that the velocity dispersions in distant bins are coming out too small on account of an excessive allowance for the errors in the proper motions of stars that have small proper motions. The errors in the UCAC4 catalogue (Zacharias et al. 2013) are $\sim 60$ percent of those in the PPMX catalogue and the problem just described does not arise with these proper motions, so we have used them. We do, however, exclude stars with an error in $\mu_b$ greater than 8 mas yr$^{-1}$.

In addition to this cut on proper-motion error, the sample is restricted to stars for which Binney et al. (2013) determined a probability density function (pdf) in distance modulus. To belong to this group a star has to have a spectrum that passed the Kordopatis et al. (2013) pipeline with S/N ratio of 10 or more.

3 FITTING MERIDIONAL COMPONENTS WITHOUT BINNING THE DATA

At each point in the Galaxy a stellar population that is in statistical equilibrium in an axisymmetric gravitational potential $\Phi(R, z)$ should define a velocity ellipsoid. Two of
We use four further parameters $\sigma_1$ through $\sigma_3$ and experimentation shows that power series in $t$ work well. Second, it has been conventional to assume exponential dependence of velocity dispersion on $R$ since the scale heights of discs were found to be roughly constant ($\frac{R}{z} \approx \text{constant}$). Moreover, the data cover a significant range in $R$ only at large $|z|$, so we are not in a position to consider elaborate dependence on $R$. The parameters $a_1$ and $a_2$ set the overall velocity scale of $\sigma_1$ and $\sigma_3$, respectively, while $a_3$ and $a_4$ determine how fast these dispersions decrease with increasing radius. The parameter pairs ($a_3$, $a_4$) and ($a_7$, $a_8$) determine how the dispersions vary with distance from the plane.

The lengths of the principal semi-axes of the velocity ellipsoid are of course the principal velocity dispersions

$$\sigma_1(R, z) = \left( \langle \mathbf{v} \cdot \mathbf{e}_1 \rangle^2 \right)^{1/2}$$
$$\sigma_\phi(R, z) = \left( \langle \mathbf{v} \cdot \mathbf{e}_\phi \rangle^2 - \langle \mathbf{v} \cdot \mathbf{e}_z \rangle^2 \right)^{1/2}$$
$$\sigma_3(R, z) = \left( \langle \mathbf{v} \cdot \mathbf{e}_3 \rangle^2 \right)^{1/2}.

In the following we shall use the notation

$$V_1 \equiv \langle \mathbf{v} \cdot \mathbf{e}_1 \rangle \quad \text{and} \quad V_3 \equiv \langle \mathbf{v} \cdot \mathbf{e}_3 \rangle.

We estimate the functional forms of $\sigma_1$ and $\sigma_3$ as follows.

From equations (4) it is straightforward to calculate the derivatives. The code for extracting the values of the $a_i$ from a catalogue of stellar phase-space coordinates was tested as follows. The velocity of each RAVE star was replaced by a velocity chosen at random from a triaxial Gaussian velocity distribution with variances $\sigma_i^2(R, z) + \epsilon^2(V_i)$, where the $\sigma_i$ were derived from plausible values of the $a_i$ and the errors $\epsilon(V_i)$ are the actual errors on that star’s velocity compo-

\[\sum_{\text{stars}} \ln \left[ \sigma_i^2 + \epsilon^2(V_i) \right] = \frac{V_i^2}{\sigma_i^2 + \epsilon^2(V_i)},\]
controls the tilt of the velocity ellipsoid. The parameters $a_i$,

-0.1 from the dwarfs. However, even the of the vertical dispersion, are recovered quite well from the

values of the $a_i$. The conventional $\chi^2$ is

$$\chi^2 = \sum_{\text{stars}} \sum_{i=1,3} \frac{V_i^2}{\sigma_i^2 + e^2(V_i)}.$$  

In all tests the chosen model yielded a value of $\chi^2$ per degree of freedom that differed from unity by less than $3 \times 10^{-4}$.

We have analysed separately four classes of stars: clump giants (0.55 $< J - K < 0.8$ and $1.7 \leq \log g < 2.4$), non-clump giants ($\log g < 3.5$), hot ($T_{\text{eff}} > 6000$ K) dwarfs and cool dwarfs.

The first row of Table 1 shows the parameters from which fitting started, while the bottom row gives the values of the parameters that were used to assign velocities to the stars. The second row shows the parameters values upon which FRPRMN converged with data at the locations of 40175 red-clump stars in the RAVE sample. The third row gives the results obtained using the sample’s 181726 non-clump giants. The fourth and fifth rows give, respectively, results obtained using the 55 398 hot dwarfs and 95 469 cool dwarfs.

Naturally the precision with which the parameters can be recovered from the data increases with the size and spatial coverage of the sample. Hence the cold dwarfs deliver the least, and the giants the most, accurate results. The parameters that are most accurately recovered are $a_1$ and $a_3$, which control the magnitudes of dispersions, and $a_0$, which controls the tilt of the velocity ellipsoid. The parameters $a_2$ and $a_4$, which control the vertical variation of the radial dispersion, and $a_7$ and $a_8$, which control the vertical variation of the vertical dispersion, are recovered quite well from the giants but rather poorly from the dwarfs. Even though the dwarfs yield quite accurate values for the products $a_2^2 a_4$ and $a_7^2 a_8$ that occur in the first non-trivial term in the MacLaurin series of the final brackets of equations (4). The parameters $a_2$ and $a_6$, which control radial gradients are recovered only moderately well by all star classes.

When fitting the measured velocities of RAVE stars, the difference between unity and $\chi^2$ per degree of freedom for the chosen model ranged from $3.5 \times 10^{-3}$ for cold dwarfs to $1.7 \times 10^{-2}$ for non-clump giants. Table 2 shows the parameters of the chosen models. Both classes of giants and the cool dwarfs yield similar values $a_0 \simeq 0.8$ of the parameter that controls the orientation of the velocity ellipsoid. Since this value lies close to unity, the long axis of the velocity ellipsoid points almost to the Galactic centre (Fig. 1) consistent with the findings of Siebert et al. (2008). The hot dwarfs yield a much smaller value, $a_0 \simeq 0.2$, so the long axis of their velocity ellipsoid does not tip strongly as one moves up.

The velocity dispersions in the plane are $\sigma_r = 30a_1 \, \text{km s}^{-1}$ and $\sigma_z = 30a_3 \, \text{km s}^{-1}$. The smallest dispersions, ($\sigma_R, \sigma_z$) = (29.3, 14.0) are for the hot dwarfs and the largest, (37.3, 21.4) are for the giants. For the giants and cool dwarfs we have $\sigma_z/\sigma_R = a_3/a_1 \simeq 0.6$, while for the hot dwarfs we have $\sigma_z/\sigma_R \simeq 0.48$, significantly smaller.

The scale lengths on which the dispersions vary are $R_o = R_0/a_2$ for $\sigma_r$ and $R_o = R_0/a_6$ for $\sigma_z$. For the giants these are $\sim 2.5R_0$, which is surprisingly large: one anticipates $R_o \lesssim 3R_0 \simeq R_0$. The cool dwarfs, by contrast yield $R_o < R_0$. For $\sigma_r$, the hot dwarfs yield $R_o \simeq 1.4R_0$, but for $\sigma_z$ they yield a negative value of $R_z$, implying that $\sigma_z$ increases with radius. Given that the survey volume is a cone that excludes the plane, not only is it hard to disentangle radial and vertical gradients, but stars such as hot dwarfs that are strongly concentrated to the plane do not probe a large volume and consequently are not suited to measuring gradients. Moreover, the longest axis of the velocity ellipsoids of populations of young stars are known not to lie within the $(R, z)$ plane – the “vertex deviation” (e.g. Dehnen & Binney 1998). This phenomenon is evidence that these populations are not in dynamical equilibrium as our methodology assumes, either because they are too young, or because they are strongly disturbed by spiral structure.

The upper panel of Fig. 2 shows the dependencies of $z$ at $R = 8$ kpc of $\sigma_1$ (dashed line) and $\sigma_2$ (full line) that are implied by Table 2 for non-clump giants. The squares and triangles show velocity dispersions estimated by binning the data as described in Section 4 below. The lower panel

| start | Clump giants | Non-clump giants | Hot dwarfs | Cool dwarf |
|-------|-------------|-----------------|------------|------------|
| $a_0$ | 0.506       | 0.491           | 0.459      | 0.587      |
| $a_1$ | 1.011       | 0.998           | 0.994      | 1.003      |
| $a_2$ | 0.414       | 0.482           | 0.611      | 0.541      |
| $a_3$ | 5.355       | 6.519           | 3.329      | 2.905      |
| $a_4$ | 0.549       | 0.462           | 2.194      | 1.841      |
| $a_5$ | 0.493       | 0.499           | 0.499      | 0.499      |
| $a_6$ | 0.307       | 0.347           | 0.448      | 0.210      |
| $a_7$ | 0.307       | 0.347           | 0.448      | 0.210      |
| $a_8$ | 11.425      | 9.768           | 5.241      | 5.505      |

Table 1. Test of the fitting procedure. The bottom row gives the parameters used to choose the velocities, while top row gives the values of the parameters in equation (4) from which FRPRMN started. The second row shows the values of the parameters on which it converged given data at the locations of the 40175 clump giants. The third, fourth and fifth rows give the parameters values similarly obtained using data at the locations of 181726 non-clump giants, 55 398 hot dwarfs and 95 469 cool dwarfs, respectively.
shows the corresponding radial dependencies at $z = 0.22$ and $z = 0.86$ kpc.

In Fig. 3 the full black curves show the runs with $z$ at $R = R_0$ of $\sigma_1$ and $\sigma_3$ for non-clump giants, while the dashed red curves show the same quantities for the cool dwarfs. From these plots we infer that the dispersions of the cool dwarfs are probably consistent with those for non-clump giants except very near the plane where $\sigma_1$ may be lower for the dwarfs. The blue dotted curves show the distinctly lower velocity dispersions of the hot dwarfs: lower dispersions are to be expected of such relatively young stars since they have experienced less stochastic acceleration than older stars.

4 USING BINNED DATA

4.1 Azimuthal velocities

In a disc galaxy, the distribution of $v_\phi$ components is inherently skew and the skewness of the distribution contains essential information about the system’s history and dynamics. Consequently, it is not appropriate to use the machinery described in the last section to fit observed $v_\phi$ distributions.

The $v_\phi$ distributions of the dynamical models described by B12, which will be discussed in Section 5 below, can be fitted extremely well by the following analytic distribution

\[ P(v_\phi) = \text{constant} \times e^{-(v_\phi - b_0)^2/2\sigma_\phi^2}, \]  

(7)

where $\sigma_\phi$ is a cubic in $v_\phi$:

\[ \sigma_\phi(v_\phi) = b_1 + b_2 v_{\phi,100} + b_3 v_{\phi,100}^2 + b_4 v_{\phi,100}^3, \]  

(8)

with $v_{\phi,100} \equiv v_\phi/100$ km s$^{-1}$. The general idea here is that $b_0$ defines a characteristic streaming velocity, while $b_1$ is a basic azimuthal velocity dispersion. The parameters $b_2$ to $b_4$ cause the velocity dispersion $\sigma_\phi$ to increase/decrease as $v_\phi$ moves below/above the circular speed, thus making the $v_\phi$ distribution skew.

In principle functional forms could be adopted for the dependence on $(R, z)$ of the parameters $b_i$ appearing in equations (7) and (8), and then, in strict analogy to the work of the previous section, the values of the parameters appearing in these functional forms could be determined by maximising the likelihood of the data given the distribution (7). Unfortunately, for this scheme to be viable we require an expression for the value of the normalising constant as a function of the parameters, and no such formula is available. Therefore we have determined the $b_i$ by binning the data and doing a least-squares fit of equation (7) convolved with the observational errors to the histogram of the binned data.

Table 2. Velocity ellipsoids from measured velocities. When the values given here are inserted into equations (7) and (8) one obtains expressions for the semi-axis lengths and orientation of the velocity ellipsoids at a general point $(R, z)$. From top to bottom the rows give results for clump giants, non-clump giants, and hot and cool dwarfs.

| $a_0$  | $a_1$  | $a_2$  | $a_3$  | $a_4$  | $a_5$  | $a_6$  | $a_7$  | $a_8$  |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Clump giants | 0.872 | 1.183 | 0.394 | 24.835 | 0.212 | 0.682 | 0.554 | 29.572 | 0.211 |
| Non-clump giants | 0.815 | 1.243 | 0.398 | 25.283 | 0.214 | 0.713 | 0.362 | 34.815 | 0.218 |
| Hot dwarfs | 0.213 | 0.976 | 0.719 | 7.891  | 1.282 | 0.468 | -0.209 | 26.992 | 0.380 |
| Cool dwarfs | 0.815 | 1.153 | 1.142 | 47.112 | 0.169 | 0.711 | 1.572 | 9.852  | 1.200 |

The stars were divided into 8 spatial bins according to whether $R < R_0$ or $R > R_0$ and $|z|$ lay in intervals bounded by (0, 0.3, 0.6, 1, 1.5) kpc for giants or (0, 0.15, 0.3, 0.45, 0.6) kpc for dwarfs. Table 3 gives the parameters that fit the $v_\phi$ distributions of the clump stars (upper block) and non-clump giants (lower block). Table 4 gives values of the parameters for the hot (upper block) and cool dwarfs. The black points in Figs. 4 to 7 show the observational histograms. At the top left of each panel we give the mean values of ($R, |z|$) and $\langle v_\phi \rangle$ for stars in the bin, where the latter is the r.m.s. error for the stars in the given bin. Also given at the top of each panel is the mean velocity $\langle v_\phi \rangle$, which of course is sensitive to our adopted values $\Theta_0 = 220$ km s$^{-1}$ and $v_{\phi,0} = \Theta_0 + 12.24$ km s$^{-1}$. The values of $\langle v_\phi \rangle$ are also given in Tables 3 and 4 where we see that on account of the skewness of the $v_\phi$ distributions, $\langle v_\phi \rangle$ is sys-
tematically smaller than the fit parameter $b_0$, which would be the mean velocity if $\sigma_\phi$ were not a function of $v_\phi$.

In Figs. 4 to 7 bins with $R < R_0$ are shown in the left column, bins with $R > R_0$ are shown in the right column, and $|z|$ increases downwards. The dotted curves show the functions defined by the $b_i$ in Tables 3 and 4 while the full curves show the results of convolving these curves with the Gaussian of dispersion $\sigma(v_\phi)$. The dotted curves are mostly obscured by the full curves because observational errors do not have a big impact on these data. All histograms are fitted to great precision by the full curves.

In Fig. 8 the points for giants show a clear trend for $\langle v_\phi \rangle$ to decline with distance from the plane, as we would expect for a rotating disk. The data points were obtained by fitting the analytic model convolved with the measurement errors to histograms of $v_\phi$ components with the stars placed in seven bins at each of $R < R_0$ and $R > R_0$, and then calculating for each bin the mean velocity of the model distribution before convolution by error. We do not show error bars, but the statistical errors on these points are very small. All these points would move upwards by 20 km s$^{-1}$ if we increased our estimate of the local circular speed from $\Theta_0 = 220$ km s$^{-1}$ to $\Theta_0 = 240$ km s$^{-1}$, and they would move down by 5 km s$^{-1}$ if we decreased our estimate of $v_\phi - \Theta_0$ from 12.24 km s$^{-1}$ to 7.24 km s$^{-1}$. In Fig. 8 the points for giants show a clear trend for $\langle v_\phi \rangle$ to decline with distance from the plane, as we would expect for a rotating disk.

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**Table 3.** Values of the mean streaming velocity and the parameters defined by equations (7) and (8) required to fit the $v_\phi$ distributions of RAVE stars. The upper block refers to red clump stars and the lower one to non-clump giants.

| $(R,|z|)$ | $\bar{\sigma}_\phi$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|----------|-----------------|-------|-------|-------|-------|-------|
| (7.61, 0.19) | 217.9 | 242.2 | 51.0 | -5.79 | -9.78 | 2.81 |
| (8.36, 0.19) | 211.4 | 215.5 | 45.1 | -2.48 | -12.34 | 3.60 |
| (7.51, 0.44) | 210.8 | 222.0 | 58.6 | -14.91 | 0.07 | 0.20 |
| (8.36, 0.43) | 207.9 | 214.7 | 49.7 | -2.48 | -12.34 | 3.60 |
| (7.48, 0.75) | 199.0 | 207.3 | 71.2 | -50.09 | 27.76 | -5.55 |
| (8.41, 0.75) | 200.1 | 211.4 | 62.4 | -18.73 | -0.20 | 0.63 |
| (7.52, 1.18) | 189.3 | 195.8 | 71.2 | -39.27 | 18.50 | -3.27 |
| (8.37, 1.19) | 191.2 | 201.9 | 70.1 | -30.49 | 9.61 | -1.44 |

| (7.66, 0.19) | 215.6 | 223.3 | 53.6 | -14.15 | -12.21 | 3.36 |
| (8.28, 0.19) | 209.8 | 215.1 | 52.8 | -11.90 | -7.53 | 2.74 |
| (7.54, 0.43) | 208.7 | 219.2 | 63.8 | -21.69 | 1.61 | -0.31 |
| (8.34, 0.42) | 206.4 | 213.5 | 57.0 | -12.40 | -7.85 | 2.83 |
| (7.48, 0.75) | 198.2 | 206.7 | 72.0 | -41.92 | 17.72 | -2.96 |
| (8.42, 0.75) | 198.7 | 209.3 | 66.1 | -23.36 | 1.59 | 0.53 |
| (7.50, 1.20) | 186.6 | 193.3 | 76.4 | -42.35 | 16.29 | -2.28 |
| (8.42, 1.20) | 190.2 | 200.3 | 78.0 | -44.79 | 18.81 | -3.20 |

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**Table 4.** The same as Table 3 but for hot (upper block) and cool (lower block) dwarfs.

| $(R,|z|)$ | $\bar{\sigma}_\phi$ | $b_0$ | $b_1$ | $b_2$ | $b_3$ | $b_4$ |
|----------|-----------------|-------|-------|-------|-------|-------|
| (7.85, 0.10) | 220.1 | 224.9 | 69.5 | -44.33 | 10.68 | -0.73 |
| (8.11, 0.11) | 216.5 | 220.1 | 29.3 | 20.80 | -24.86 | 5.67 |
| (7.80, 0.22) | 220.7 | 224.4 | 29.8 | 20.98 | -24.10 | 5.36 |
| (8.13, 0.22) | 217.5 | 221.3 | 29.6 | 21.56 | -25.23 | 5.69 |
| (7.78, 0.36) | 219.5 | 225.0 | 46.9 | -0.85 | -13.53 | 3.59 |
| (8.15, 0.36) | 215.8 | 219.2 | 79.2 | -56.54 | 14.43 | -0.71 |
| (7.79, 0.50) | 217.6 | 223.2 | 46.8 | -3.30 | -10.04 | 2.75 |
| (8.15, 0.50) | 214.3 | 218.7 | 69.6 | -37.94 | 9.23 | 0.74 |

| (7.90, 0.09) | 215.8 | 222.2 | -9.6 | 98.37 | -66.58 | 12.49 |
| (8.06, 0.08) | 213.7 | 219.9 | -18.9 | 111.09 | -72.53 | 13.42 |
| (7.84, 0.21) | 211.1 | 219.7 | 18.8 | 52.04 | -41.09 | 8.08 |
| (8.10, 0.21) | 211.1 | 217.6 | -4.6 | 87.28 | -59.59 | 11.26 |
| (7.81, 0.36) | 211.5 | 219.9 | 19.7 | 58.80 | -47.62 | 9.56 |
| (8.12, 0.35) | 207.7 | 215.2 | 57.3 | -12.98 | 8.21 | 2.90 |
| (7.73, 0.50) | 203.6 | 216.1 | 22.4 | 52.28 | -39.99 | 7.54 |
| (8.16, 0.51) | 210.9 | 218.4 | 8.8 | 87.40 | -67.29 | 13.41 |
Figure 7. As Fig. 4 but for cool dwarfs.

Figure 8. The mean rotation velocity of the giants as a function of distance from the plane. The full curve is for bins at $R < R_0$. The data points are the means of model distributions like those plotted as dotted curves in Fig. 5. The statistical errors on these points are very small.

Figure 9. As Fig. 8 but for the dwarfs: hot (top) and cool (below).

Figure 10. Dotted curve: the contribution to the circular speed from the disc and bulge components; dashed curve: the contribution of the dark halo.

expect given that along this sequence $\sigma_1$ rises and increases the asymmetric drift $v_a \sim \sigma_2^2/v_c$.

In Fig. 8 the point for hot dwarfs at $z \lesssim 50$ pc and $R < R_0$ is $\sim 25$ km s$^{-1}$ larger than the corresponding point at $R > R_0$, so both points are highly anomalous. However, the histograms for the associated bins (which we do not show) indicate that the anomaly is not caused by small-number statistics. The points for larger distances from the plane lie close to the circular speed at $R < R_0$ and fall about 4 km s$^{-1}$ lower at $R > R_0$. These differences could well reflect spiral structure. The points for cool dwarfs show a slight fall with increasing distance from the plane and a tendency to be up to 2 km s$^{-1}$ lower at $R > R_0$ than at $R < R_0$. The fall in $\langle v_\phi \rangle$ between the plane and 0.5 kpc is consistent with that of the giants.

4.2 Moments of the $V_1$ and $V_3$ distributions

The black points in Figs. 11 to 14 show, for hot dwarfs, cool dwarfs, clump and non-clump giants respectively, the distributions of the meridional-plane components $V_1$ and $V_3$ defined by equations (2). At the bottom-centre of each panel the numbers in brackets give the mean values of $R$ and $|z|$ for the stars in each bin, the standard deviation of the data (sD), the value at this location of the relevant velocity dispersion from the Gaussian model of Section 3 (sM), the mean velocity of the stars in the bin (mV) and the rms measurement error for those stars (eV). The agreement between the standard deviations of the data and the model dispersion at the bin’s barycentre is typically excellent.

If the Galaxy were in an axisymmetric equilibrium and we were using the correct value for the Sun’s peculiar velocity, the mean velocities would all vanish to within the discreteness noise, but they do not. All the three older populations show similar trends in mean velocities: the means of $V_3$ tend to be negative at $R > R_0$ and increase in absolute value away from the plane, while the mean values of $V_1$ fall from positive to negative as one moves away from the plane with the largest absolute values occurring for giants near the plane. [Siebert et al. (2011) and Williams et al. (2013) have analysed similar statistically significant mean velocities in velocities of RAVE stars drawn from an earlier spectral-analysis pipeline than that used here. We defer discussion of this phenomenon until Section 5.3.
5 COMPARISONS WITH DYNAMICAL MODELS

It is interesting to compare the observed distributions with ones predicted by the favoured equilibrium dynamical model of B12. This model is defined by a gravitational potential and a distribution function. The potential is generated by thin and thick exponential stellar discs, a gas layer, a flattened bulge and a dark halo. Fig. 10 shows the contributions to the circular speed from the baryonic (dotted curve) and from the dark halo (dashed curve). One sees that this is a maximum-disc model. In fact, 65% of the gravitational force on the Sun is produced by baryons rather than dark matter.

The distribution function (\(df\)) is an analytic function \(f(J)\) of the three action integrals \(J\). The function, which specifies the density of stars in three-dimensional action space, has nine parameters. Four parameters specify each of the thin and thick discs and one parameter specifies the relative weight of the thick disc. Their values are given in column (b) of Table 2 in B12. They were chosen by fitting the model’s predictions for the velocity distribution of solar-neighbourhood stars to that measured by the Geneva-Copenhagen survey (GCS) of F and G stars (Holmberg et al. 2007), and to the vertical density profile of the disc determined by Gilmore & Reid (1983). Hence the data to which this \(df\) was fitted do not include velocities in the region distance \(s > 150\) pc within which most RAVE stars lie, and whatever success the \(df\) has in predicting the velocities of RAVE stars must be considered a non-trivial support for the assumptions that went into the model, which include the use of a particular, disc-dominated, gravitational potential and the functional form of the \(df\).

We have used the B12 \(df\) to generate pseudo-data for each star in the RAVE sample from the model’s velocity distribution as follows. We start by choosing a possible true location \(x'\) by picking a distance \(s'\) from the multi-Gaussian model of the star’s pdf in distance \(s\) that Binney et al. (2013) produced. We then sample the velocity distribution of the dynamical model for that class of star at \(x'\) and compute the corresponding proper motions and line-of-sight velocity \(v_{\text{los}}\). To these observables we add random errors drawn from the star’s catalogued error distributions, and from the modified observables compute the space velocity using the catalogued distance \(s\) rather than the hypothesised true distance \(s'\). This procedure comes very close to reproducing the data that would arise if the Galaxy were correctly described by the model, each star’s distance pdf were sound and the errors on the velocities had been correctly assessed; it does not quite achieve this goal on account of a subtle effect, which is costly to allow for. This effect causes the procedure to overweight slightly the possibility that stars lie at the far ends of their distance pdfs (Sanders & Binney in preparation). We believe the impact of this effect to be small, so our model histograms correctly represent the model’s predictions for a survey with the selection function and errors of RAVE.

We assume that the hot dwarfs are all younger than 5 Gyr (e.g., Fig. 2 of Zwitter et al. 2010) and correspondingly restrict the B12 \(df\) of these objects to the portion of the thin disc that is younger than 5 Gyr. The distributions of clump and non-clump giants and cool dwarfs are (rather arbitrarily) assumed to sample the whole \(df\).

5.1 Azimuthal velocities distributions

The red points in Figs 4-7 show the model’s predictions for the \(v_\phi\) components. Figs 4 and 5 show that the velocities of the clump giants are very similar to those of the non-clump giants. This result is in line with expectations, but serves to increase our confidence in our distance estimates for, as we shall see in Section 5.3, systematic errors in the distances of whole groups of stars distort the derived velocity distributions. Hence consistency between the histograms for clump and non-clump giants suggests that our distances to non-clump giants, which are the hardest to determine, are no more in error than are the distances to clump giants.

In Figs 4 and 5 the models definitely under-populate the wing at \(v_\phi > \Theta_0\), especially away from the plane. This is likely to reflect the model’s thick disc being radially too cool, as discussed below.

A notable difference between the observed and predicted distributions for both the giants and the hot dwarfs (Figs 4 to 7) is that at \(R < R_0\) and \(|z| \sim 0.5\) kpc the black, measured, distribution is shifted to larger values of \(v_\phi\) than the red predicted one. In the case of the hot dwarfs, a similar but distinctly smaller shift is seen at \(R > R_0\). The smaller shift at \(R > R_0\) is clearly connected to the fact that in Fig. 7 the \(\langle v_\phi \rangle\) points for \(R > R_0\) lie below those for \(R < R_0\). At \(|z| < 0.5\) kpc the same phenomenon is evident for giants in Fig. 6. One possible explanation is that the Galaxy’s circular-speed curve is falling with \(R\) relative to that of the model.

While the theoretical distribution depends only on the model’s value 220 km s\(^{-1}\) for the local circular speed \(\Theta_0\), the observed velocities have been derived using both \(\Theta_0\) and a value \(V_0 = 12.24\) km s\(^{-1}\) from Schönrich et al. (2010) for the amount by which the Sun’s \(v_\phi\) exceeds \(\Theta_0\). Hence an offset between the red and black curves in Figs 4 to 7 can be changed by changing the assumed value of \(V_0\); reducing \(V_0\) shifts the black distribution to the left. However, the case for such a change is less than unconvincing because the shift is clear only at \(R < R_0\) and \(|z| \lesssim 0.5\) kpc. Moreover in Fig. 7 for the cool dwarfs the model histograms provide excellent fits to the data. In Fig. 8 for the hot dwarfs the offset between the red and black histograms vanishes at \(R > R_0\) near the plane but grows with \(|z|\).

A more convincing case can be made for an increase in the width of the theoretical distributions of giants away from the plane.

In addition to a possibly incorrect value of \(V_0\), there are four other obvious sources of offsets between the observational and theoretical distributions of \(v_\phi\):

- Spiral arms must generate fluctuations in the mean azimuthal velocity of stars. Judging by oscillations with Galactic longitude in the observed terminal velocity of interstellar gas (e.g. Malhotra 1993), the magnitude of this effect is probably at least as great as 7 km s\(^{-1}\) in a population such as hot dwarfs that has a low velocity dispersion. Moreover, it is now widely accepted that the irregular distribution of Hipparcos stars in the \((U,V)\) plane of velocities (Dehner 1993) is in large part caused by spiral arms perturbing the orbits of stars (De Simone et al. 2004; Antoja et al. 2011; Siebert et al. 2012; McMillan 2013). The large (up to 20°) value of the vertex deviation for hot dwarfs is surely also due to spiral structure. Spiral-induced modulations in \(v_\phi\) will
Figure 11. Distributions of $V_1 \simeq -v_r$ and $V_3 \simeq v_z$ for hot dwarfs. Black points show the RAVE data, red points the predictions of the B12 model when it is assumed that all hot dwarfs are younger than 5 Gyr and as such belong to the thin disc. At the lower middle of each panel are given: the mean $(R, z)$ coordinates of the bin; the standard deviation of the data after correction for error and the velocity dispersion at the mean coordinates of the Gaussian-model described in Section 3; the mean of the data and the rms error of the velocities.

Figure 12. As Fig. 11 but for cool dwarfs. The red points now show the predictions of the B12 model when cool dwarfs are assumed to sample the entire DF. In the last two panels of the top row we show the Gaussian distributions that were fitted in Section 3 to illustrate how well the dynamical model captures the deviations of the observed distribution from Gaussianity.
Figure 13. As Fig. 12 but for clump giants.

vary quite rapidly with radius and thus could make significantly different contributions to \( \langle v_\phi \rangle \) in our bins at \( R < R_0 \) and \( R > R_0 \).

- The mean age of the stellar population is expected to decrease with increasing Galactocentric distance. Such a decrease would introduce a bias into a sample selected to be young such that there were more stars seen near pericentre than near apocentre than in a sample of older stars, so stars in the younger sample would tend to have larger values of \( v_\phi \) than stars in the older sample. This effect could explain why the histograms for hot dwarfs show larger offsets than do those for cool dwarfs.

- We are probably using a value of \( R_0 \) that is too small by \( \sim 3\% \). Changing the adopted value of \( R_0 \) changes the supposed direction of the tangential vector \( e_\phi(\star) \) at the location of a star and thus changes the component of a star’s Galactocentric velocity \( v \) that we deem to be \( v_\phi \). The velocity \( v \) is made up of the star’s heliocentric velocity \( v_\odot \) and the Sun’s largely tangential velocity \( v_\odot = \Theta_0 e_\phi(\odot) + (U_\odot, V_\odot, W_\odot) \). For a star at a given distance, increasing \( R_0 \) diminishes the angle between \( e(\star) \) and \( e(\odot) \), and thus, by diminishing the angle between \( e_\phi(\star) \) and \( v_\odot \), tends to increase \( v_\phi \). Consequently, in Figs 4 to 7 increasing \( R_0 \) moves the black points to the right, away from the model’s predictions.

- We are probably using a value of \( \Theta_0 \) that is too small by \( \sim 9\% \). Increasing \( \Theta_0 \) by \( \delta \Theta \) simply moves the observational histogram to the right by \( \delta \Theta \). However, since the asymmetric drift \( v_a \) of a population that has radial velocity dispersion \( \sigma_r \) scales as \( \sigma_r^2/\Theta_0 \), increasing \( \Theta_0 \) moves the theoretical histogram to the right by

\[
\delta \Theta - \delta v_a = \left( 1 + \frac{\sigma_r^2}{\Theta_0^2} \right) \delta \Theta.
\]

so this upward revision will reduce by \((\sigma_r/\Theta_0)^2\delta \Theta \sim 0.04 \delta \Theta_0\) the offsets we obtained with our traditional choices of \( R_0 \) and \( \Theta_0 \).

5.2 Velocities in the meridional plane

Figs. 11 to 14 are the analogues of Figs. 4 to 7 for components of velocity \( V_1 \) and \( V_3 \) (equation 2) in the meridional plane: black points show observational histograms and red ones the predictions of the B12 model. \( V_1 \) is the component of velocity along the longest principal axis of the velocity ellipsoid at the star’s location according to the Gaussian model fitted in Section 3. The sign convention is such that at the Sun \( V_1 \approx U = -v_R \), \( V_3 \approx W = v_z \) is the perpendicular velocity component. The left two columns are for bins with \( R < R_0 \) while the right two columns are for bins with \( R > R_0 \). At the lower middle of each panel are given: the mean \((R, z)\) coordinates of stars in the bin; the standard deviation of the data
after correction for error ($s_D$) and the velocity dispersion at the mean coordinates of the Gaussian-model described in Section 3 ($s_M$); the mean of the data ($m_D$) and the rms error of the velocities ($e_V$).

All distributions are significantly non-Gaussian (i.e. the distributions are far from parabolic) and the B12 model captures this aspect of the data beautifully. The last two panels in the top row of Fig. 12 illustrate this phenomenon by showing the parabolas of the Gaussian distributions fitted in Section 3. Notwithstanding the non-Gaussian nature of the velocity distributions, in every bin there is good agreement between the standard deviation of the data $s_D$ and the dispersion at of the Gaussian model $s_M$ at the barycentre of the bin. This result implies that equations 1 can be safely used to recover the principal velocity dispersions throughout the studied region.

The model is particularly successful in predicting the $V_3$ distributions of both dwarfs and giants. In the case of the dwarfs, the only blemish on its $V_3$ distributions is a marginal tendency for the distribution of hot dwarfs to be too narrow at high $|z|$.

The principal differences between the model and observed $V_1$ distributions of dwarfs arise from left-right asymmetries in the data. For example, in the third panels from the left in the first and second rows of Fig. 12 for hot dwarfs, the black points lie systematically above the red points for...
Figure 15. The black points and curves are identical to those plotted in Fig. 14. The red model histograms have been modified by supposing that the catalogued distance to each (giant) star is 20% larger than it should be. The values \( s_D \) and \( m_V \) given at the bottom are now the standard deviation and mean of the red histogram.

\( V_1 > 0 \) (inward motion), a phenomenon also evident in the top left panel of that figure. In the first and third panels in the second row of Fig. 12 for cool dwarfs, a similar phenomenon is evident in that the red points lie above the black points at \( V_1 < 0 \). A contribution to these divergences must come from star streams, which Dehnen (1998) showed to be prominent in the local UV plane.

Figs 13 and 14 for clump and non-clump giants show \( V_1 \) and \( V_3 \) distributions in bins that extend to much further from the plane. In both cases the model and observed \( V_3 \) distributions agree to within the errors. Given the smallness of the error bars in the case of the giants and the fact that the data extend to a distance from the plane that is more than ten times the extent of the GCS data to which the B12 model was fitted, the agreement between the observed and theoretical \( V_3 \) histograms in Fig. 14 amounts to a very strong endorsement of the B12 model.

The observed \( V_1 \) distributions for clump and non-clump giants are consistent with one another, and the superior statistics of non-clump giants highlight the deviations from the model predictions. Near the plane the model fits the data well, but the further one moves from the plane, the more clear it becomes that the model distribution of \( V_1 \) is too narrow. This phenomenon arises because in B12, contrary to expectation, the thick disc needed to be radially cooler than the thin disc. The RAVE data are indicating that this was a mistake. In B12 two factors shared responsibility for the radial coolness of the thick disc. One was the ability of the thin-disc df to fit the wings of the \( U \) and \( V \) distributions in the GCS, leaving little room for the thick disc’s contribution there. The other factor was an indication from SDSS that \( \langle v_\phi \rangle \) does not fall rapidly with distance from the plane. Fig. 5 relates to this second point, and indeed the RAVE data show more stars with large \( v_\phi \) than the model, especially at large \(|z|\). In B12 it was demonstrated that there is a clean dynamical trade-off between \( \langle v_\phi \rangle \) and \( \sigma_\phi \) in the sense that an increase in the former has to be compensated by a decrease in the latter. Moreover, \( \sigma_\phi \) is dynamically coupled to \( \langle V_2^2 \rangle^{1/2} \), so if one is reduced the other must be reduced as well. Hence large \( \langle v_\phi \rangle \) implies small \( \langle V_2^2 \rangle^{1/2} \). There is a puzzle here that requires further work.

5.3 Effect of distance errors

Our model predictions already include the effects of random distance (and velocity) errors. Now we investigate how systematic errors in our spectrophotometric distances affect the derived kinematics. This investigation is motivated in part...
by the indication in Binney et al. (2013) from the kinematic test of Schörnich et al. (2012) that distances to giants might be over-estimated by as much as 20%, and distances to the hottest dwarfs under-estimated by a similar amount.

The black points in Fig. [13] are identical to those in the corresponding panels of Fig. [13] but the red model points have been modified by adding $-5 \log_{10}(e) \times 0.2$ to the randomly chosen distance modulus of each star before evaluating the DF. This modification enables us to model the impact on the survey of catalogued distances being on average 20 per cent too large.

The figure shows that such distance errors introduce left-right asymmetry into the model distributions of both $V_1$ and $V_3$ similar to that evident in the $V_1$ distribution of hot dwarfs. The red values of $mV$ at the bottom middle of each panel, show the mean values of $V_1$ and $V_3$ for the model histograms. We see that these values are non-zero and of comparable magnitude to the mean values of the observed histograms given in Fig. [14]. Thus non-zero mean values of $\langle V_1 \rangle$ and $\langle V_3 \rangle$ may arise from distance errors rather than from real streaming motion. However, near the plane our distance errors induce negative mean values of $V_1$ (net outward motion) whereas the data histogram shows a smaller positive mean value of $V_1$.

Physically, over-estimating distances makes the $V_1$ distribution skew to positive $V_1$ because the survey volume is not symmetric in Galactic longitude, and at certain Galactic longitudes proper motion generated by the disc’s differential rotational is wrongly interpreted to be proper motion associated with motion towards the Galactic centre.

The assumption that distances are over-estimated also broadens the model distribution of $V_1$ far from the plane, with the result that, for example, in the third row of Fig. [13] the red and black points for $V_1$ lie significantly closer than in the corresponding panels of Fig. [13].

Fig. [16] is the analogue of Fig. [5] for the case in which the distances to giants have been over-estimated by 20%. In the top left panel for small $|z|$ and $R < R_0$ the agreement between model and data is now less good than it is in Fig. [5] but in every other panel the agreement is at least as good in Fig. [6] and for $R > R_0$ it is distinctly improved. Thus the $v_\phi$ distributions by no means speak against the suggestion that many distances have been over-estimated by $\sim 20\%$.

While in Fig. [13] distance errors have improved the fit to the data only at $|z| > 0.5 \, \text{kpc}$ and weakened the fit closer to the plane, it is perfectly possible that systematic errors are largely confined to more distant stars and/or ones further from the plane. In fact, such an effect is inevitable even if the errors in distances of individual stars were inherently unbiased because stars that happen to pick up a positive distance error will tend to accumulate in the distant bins, and conversely for stars that happen to pick up a negative distance error. When we modified the model’s predictions to allow for random distance errors, we did not capture this effect because the spatial bin to which a star is then assigned is not affected by whether it is supposed to have had its distance over- or under-estimated.

6 DISCUSSION

Siebert et al. (2011) reported a significant radial gradient in the mean $\langle v_R \rangle$ of velocities of stars reduced by the RAVE VDR2 pipeline. Williams et al. (2013; hereafter W13) used data from the VDR3 pipeline to analyse the mean velocity field $(v)$ of clump stars. In a steady-state, axisymmetric Galaxy the only non-vanishing component of this field would be $\langle v_\phi \rangle$ and it would have a maximum in the plane, falling away with $|z|$ symmetrically on each side. Instead Fig. 11 of W13 indicates that the velocity field of the clump stars has both $\langle v_R \rangle$ and $\langle v_z \rangle$ components non-zero and with gradients in both the $R$ and $z$ directions, and there is a lack of symmetry about the plane. W13 strike a cautionary note by showing that the $\langle v_R \rangle$ and $\langle v_z \rangle$ components are sensitive to which proper motions one adopts, but they demonstrate that $(v)$ is insensitive to the adopted absolute magnitude of clump stars.

As W13 show, probing the observed velocity field is made difficult by the complexity of the three-dimensional volume surveyed by RAVE. samples assembled to have a progression of values of one coordinate inevitably differ systematically in another coordinate as well. For this reason it is crucial to compare observational results with the predictions of a model that suffers the same selection effects. W13 compare the observations to mock catalogues selected by the code GALAXIA (Sharma et al. 2010) from the Besançon model (Robin et al. 2003). Our comparisons differ in that (i) we have used a fully dynamical model, based on Jeans’ theorem, rather than the essentially kinematic Besançon model, and (ii) we assign new velocities to existing stars rather than drawing an entirely new sample from the model – this procedure has the great advantage that we do not have to engage with the survey’s complex photometric selection function.

Our emphasis has been different in that we have focused on entire velocity distributions rather than just the distributions’ means. This has been possible because we have a more prescriptive dynamical model, but it has resulted in our using much bigger bins than W13. In particular, we have grouped together stars above and below the plane, which will inevitably wash out some of the structure in the $(R,z)$ plane seen by W13.

Our demonstration that introducing plausible systematic errors in the assumed distances to stars causes the model histograms to acquire mean velocities that are similar in magnitude to those found by Williams et al. (2013) must be a concern even though the particular systematic in distance error that we have considered does not generate the observed pattern of mean velocities. The extent to which distance errors broaden the distributions of $V_1$ is surprising and interesting given the difficulties one encounters finding a dynamical model that is consistent with all the data for $\langle v_\phi \rangle$ and $\langle v_\phi^2 \rangle^{1/2}$ in the absence of systematic distance errors.

7 CONCLUSIONS

We have analysed the kinematics of $\sim 400 \, 000$ RAVE stars for which Binney et al. (2013) have deduced pdfs in distance modulus. The sample divides naturally into clump and non-clump giants, hot and cool dwarfs. For each of these classes, and without binning the data, we have obtained analytic formulae for the structure of the velocity ellipsoid at each point in the $(R,z)$ plane. We are able to map the velocity ellipsoid of the giants to distances $\sim 2 \, \text{kpc}$. 

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from the Sun and find that at \((R, z)\) the direction of the longest axis is inclined to the Galactic plane by an angle \(\sim 0.8 \arctan(z/R)\). The lengths of the \((R, z)\) semi-axes are in the ratio \(\sigma_3/\sigma_1 \approx 0.6\). The velocity dispersions rise with distance from the plane, from \(\sigma_r \approx 37\, \text{km s}^{-1}\), \(\sigma_z \approx 21\, \text{km s}^{-1}\) at \((R_0, 0)\) to \(\sigma_r \approx 82\, \text{km s}^{-1}\), \(\sigma_z \approx 54\, \text{km s}^{-1}\) at \((R_0, 2\, \text{kpc})\). The velocity ellipsoid of the cool dwarfs cannot be traced to great distances, but it is consistent with being the same as that of the giants. In the plane the velocity dispersions of the hot dwarfs are \(\sigma_r \approx 29\, \text{km s}^{-1}\) and \(\sigma_z \approx 14\, \text{km s}^{-1}\) and they increase rather slowly with distance from the plane. From equations (3) and (4) and Table 2 one can compare for any of our four classes of star the structure of the velocity ellipsoid at a general point in the \((R, z)\) plane.

We have used a novel formula to obtain remarkably precise analytic fits to the distinctly non-Gaussian \(v_\phi\) distributions for eight bins in the \((R, z)\) plane. The complete \(v_\phi\) distributions at these points can be recovered for any of the four classes of stars by inserting values from either Table 5 or Table 4 into equations (7) and (8).

We have compared our observational velocity histograms with the predictions of a dynamical model that was fitted to the local velocity distribution and the vertical density profile of the current type should be fitted to the richer body of observational data that is now available using an updated Galactic potential \(\Phi\). Next this \(\Phi\) and these data should be used as a starting point for a re-determination of \(\Phi\) along the lines outlined by McMillan & Binney (2012). Currently the \(\Phi\) is being extended to include chemistry alongside age (Binney & Sanders 2013): this extension should markedly increase our ability to diagnose \(\Phi\) because the requirement that several stellar populations that differ in both their chemistry and their kinematics exist harmoniously in a common potential will strongly constrain \(\Phi\).

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