Large Gauge Transformations and the Light-Front Vacuum Structure

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Abstract

A residual gauge symmetry, exhibited by light-front gauge theories quantized in a finite volume, is analyzed at the quantum level. Unitary operators, which implement the symmetry, transform the trivial Fock vacuum into an infinite set of degenerate coherent-state vacua. A fermionic component of the vacuum emerges naturally without the need to introduce a Dirac sea. The vacuum degeneracy along with the derivation of the theta-vacuum is discussed within the massive Schwinger model. A possible generalization of the approach to more realistic gauge field theories is suggested.

1 Introduction

Hamiltonian quantum field theory formulated in the light front (space-time and field) variables [1–5] has often been considered as a conceptually very attractive theoretical scheme. Vacuum aspects of the dynamics seem to simplify remarkably (Fock vacuum is to a very good approximation an eigenstate of the full Hamiltonian) at the same time causing problems with understanding chiral properties, vacuum degeneracy and symmetry breaking phenomena. For example, it is not clear how one could reproduce the axial anomaly and the chiral condensate [6,7] in the light-front Schwinger model. These and related difficulties [8] have been usually explained by the “peculiarities” of the quantization on the characteristic surface $x^+=0$ [9].

In the present work, one of the so far missing components of the light-front (LF) gauge field theory, namely the non-trivial vacuum structure, is found to be directly related to a residual “large” gauge symmetry present in the finite-box formulation [1] of the theory. The general idea is of course not new. Gauge transformations with non-trivial topological properties have been shown to be responsible for the vacuum degeneracy in [12–15], e.g.. Their role has been studied also in the light-front literature [10,9,23].
The novel feature in our approach is the quantum-mechanical implementation of large gauge transformations by unitary operators in the context of the “trivial” non-perturbative Fock vacuum of the LF field theory. The unitary operators act on the fields as well as on states in Hilbert space. As a consequence, the “trivial” LF Fock vacuum transforms into an infinite set of non-trivial vacua. They are basically coherent states of both the dynamical gauge-field zero mode and an effective boson field bilinear in dynamical fermi field operators. The multiple vacua can be superimposed to form a unique gauge invariant vacuum. This will be shown with the example of the (massive) Schwinger model, which is known to exhibit in a tractable form many of non-perturbative features expected in QCD. We will argue however that the mechanism could in principle work also for more complicated gauge theories.

2 LF Quantization of the Massive Schwinger Model

Due to specific light-front constraints, it is inevitable to adopt the Dirac-Bergmann (DB) [23] or other similar method to properly quantize the LF massive Schwinger model [24, 25]. Here we quote only those results of the DB analysis which are relevant for our approach to the vacuum problem.

In terms of the LF variables, the Lagrangian density of the two-dimensional spinor field Ψ of mass \( m \) interacting with the gauge field \( A^\mu \) takes the form

\[
L_{LF} = i\bar{\psi}_+ \vec{\partial}_+ \psi_+ + i\bar{\psi}_- \vec{\partial}_- \psi_- + \frac{1}{2}(\partial_+ A^+ - \partial_- A^-)^2 - m(\bar{\psi}_+ \psi_- + \bar{\psi}_- \psi_+) - \frac{e}{2} j^+ A^- - \frac{e}{2} j^- A^+.
\]  

(1)

We choose \( x^+ = x^0 + x^1 \) and \( x^- = x^0 - x^1 \) as the LF time and space variable, correspondingly. The dynamical \( (\psi_+) \) and dependent \( (\psi_-) \) projections of the fermi field are defined as \( \psi_\pm = \Lambda_\pm \Psi \), where \( \Lambda_\pm = \frac{1}{2} \gamma^0 \gamma^\pm, \gamma^\pm = \gamma^0 \pm \gamma^1, \gamma^0 = \sigma^1, \sigma^1 = i\sigma^2 \) and \( \sigma^1, \sigma^2 \) are the Pauli matrices. At the quantum level, the vector current will be represented by normal-ordered product of the fermi operators, \( j^\pm = 2 : \psi_\pm \bar{\psi}_\pm : \).

A suitable finite-interval formulation of the model is achieved by imposing the restriction \(-L \leq x^- \leq L\) and by prescribing antiperiodic boundary conditions for the fermion field and periodic ones for the gauge field. The latter imply a decomposition of the gauge field into the zero-mode (ZM) part \( A_0^\mu \) and the part \( A_n^\mu \) containing only normal Fourier modes. We will work in the usual gauge \( A_n^+ = 0, A_0^- = 0 \), which completely eliminates gauge freedom with respect to small gauge transformations. In a finite volume with periodic gauge field, the ZM \( A_0^+ \) becomes a physical variable [11, 13, 21, 26, 27] since it cannot be gauged away. In quantum theory, it satisfies the commutation relation

\[
\left[ A_0^+(x^+), \Pi A_0^+(x^+) \right] = i\frac{\Lambda}{\mathcal{L}},
\]

(2)

where \( \Pi A_0^+ = \partial_+ A_0^+ \) is the momentum conjugate to \( A_0^+ \). The DB procedure yields the anticommutator for the independent fermi field component

\[
\{ \psi_+(x^-, x^+), \psi_+(y^-, x^+) \} = \frac{1}{2} \Lambda^+ \delta_0(x^- - y^-)
\]

(3)
with the antiperiodic delta function \( \delta_a(x^--y^-) \) being regularized by a LF momentum cutoff \( N \). The fermi-field Fock operators are defined by

\[
\psi_+(x^-) = \frac{1}{\sqrt{2L}} \left[ \sum_{n=\frac{1}{2}}^N (b_n e^{\frac{i}{2} k_n^+ x^-} + d_n^{\dagger} e^{\frac{i}{2} k_n^+ x^-}) \right],
\]

(4)

\[
\{b_n, b_n^{\dagger}\} = \{d_n, d_n^{\dagger}\} = \delta_{n,n'}, \quad n = \frac{1}{2}, \frac{3}{2}, \ldots, \quad k_n^+ = \frac{2\pi}{L} n.
\]

(5)

While the LF momentum operator \( P^+ \) depends only on \( \psi_+ \), the gauge invariant (see below) LF Hamiltonian of the model is expressed in terms of the both unconstrained variables \( \psi_+ \) and \( A_0^+ \) as

\[
P^- = L\Pi^2_{A_0^+} - \frac{e^2}{4} \left[ \frac{+L}{2} \int_{-L}^{+L} dx^- \frac{+L}{2} \int_{-L}^{+L} dy^- j^+(x^-) G_2(x^- - y^-) j^+(y^-) + m^2 \int_{-L}^{+L} dx^- \frac{+L}{2} \int_{-L}^{+L} dy^- \left[ \psi^{\dagger}(x^-) G_a(x^- - y^-; A_0^+) \psi(y^-) + h.c. \right] \right].
\]

(6)

The Green’s functions

\[
G_2(x^- - y^-) = \frac{4}{L} \sum_{m=1}^M \frac{1}{p_{m}^+} \left( e^{-\frac{i}{2} p_{m}^+ (x^- - y^-)} + e^{\frac{i}{2} p_{m}^+ (x^- - y^-)} \right), \quad p_{m}^+ = \frac{2\pi}{L} m,
\]

(7)

\[
G_a(x^- - y^-; A_0^+) = \frac{1}{4i} \left[ \epsilon(x^- - y^-) + i \tan(\frac{eL}{2} A_0^+) \right] \exp \left( -\frac{i e}{2} (x^- - y^-) A_0^+ \right)
\]

(8)

have been used to eliminate the constrained variables \( A_n^- \) and \( \psi_- \), respectively, with \( \epsilon(x^-) \) being twice the sign function, \( \partial_\epsilon(x^-) = 2\delta_a(x^-) \).

The final consequence of the DB analysis is the condition (a first-class constraint) of electric neutrality of the physical states, \( Q|_{\text{phys}} = 0 \).

## 3 Large Gauge Transformations and Theta-Vacuum

It is well known that gauge theories quantized in a finite volume exhibit an extra symmetry not explicitly present in the continuum approach \[11,17,20,27,29,30\]. In the LF formulation, the corresponding gauge function is linear in \( x^- \) with a coefficient, given by a specific combination of constants. These simple properties follow from the requirement to maintain boundary conditions for the gauge and matter field, respectively. The above symmetry is the finite-box analogue \[19,20\] of topological transformations familiar from the continuum formulation. Note that in the LF theory they are restricted to the + gauge field component even in \( 3+1 \) dimensions. This simplifies their implementation at the quantum level.

For the considered \( U(1) \) theory, the corresponding gauge function has the form \( \Lambda_\nu = \frac{\pi}{L} \nu x^- \), is non-vanishing at \( \pm L \) and defines a winding number \( \nu \):

\[
\Lambda_\nu(L) - \Lambda_\nu(-L) = 2\pi \nu, \quad \nu \in \mathbb{Z}.
\]

(9)
Thus, the residual gauge symmetry of the Hamiltonian (3) is [24]

$$A^+_0 \rightarrow A^+_0 - \frac{2\pi}{eL}\nu, \ \psi_+(x^-) \rightarrow e^{i\nu x^-}\psi_+(x^-).$$

(10)

Let us discuss the ZM part of the symmetry first. At the quantum level, it is convenient to work with the rescaled ZM operators [31] $\hat{\zeta}$ and $\hat{\pi}_0$:

$$A^+_0 = \frac{2\pi}{eL}\hat{\zeta}, \ \Pi A^+_0 = \frac{e}{2\pi}\hat{\pi}_0, \ \left[\hat{\zeta}, \hat{\pi}_0\right] = i.$$  \hspace{1cm} (11)

Note that the box length dropped out from the ZM commutator. The shift transformation of $A^+_0$ is for $\nu = 1$ implemented by the unitary operator $\hat{Z}_1$:

$$\hat{\zeta} \rightarrow \hat{Z}_1\hat{\zeta}\hat{Z}_1^\dagger = \hat{\zeta} - 1, \ \hat{Z}_1 = \exp(-i\hat{\pi}_0).$$  \hspace{1cm} (12)

The transformed (displaced) vacuum expressed in terms of the harmonic oscillator Fock states $|n\rangle$ and the corresponding amplitudes $C_n$ [33, 34] can be understood as describing the condensate of zero-mode gauge bosons.

Alternatively, one may consider the problem in quantum mechanical coordinate representation, where $\hat{\pi}_0 = -i\frac{d}{d\zeta}$ and the vacuum wavefunction $\psi_0(\zeta)$ transforms as

$$\psi_0(\zeta) = \pi^{-\frac{1}{4}}\exp\left(-\frac{1}{2}\zeta^2\right) \rightarrow \psi_\nu(\zeta) = \exp\left(-\nu\frac{d}{d\zeta}\right)\psi_0(\zeta) = \pi^{-\frac{1}{4}}\exp\left(-\frac{1}{2}(\zeta - \nu)^2\right).$$

(17)

The ZM kinetic energy term of the LF Hamiltonian (8) is given by

$$P_0^- = -\frac{e^2L}{2\pi^2}\frac{d^2}{d\zeta^2}.$$  \hspace{1cm} (18)
Usually, a Schrödinger equation with the above $P^0_-$ (or its equal-time counterpart) is invoked to find the vacuum energy and the corresponding wave functions subject to a periodicity condition at the boundaries of the fundamental domain $0 \leq \zeta \leq 1$ [11, 21, 35]. Here we are led by simple symmetry arguments to consider instead of the lowest-energy eigenfunction of $P^0_- \sim \hat{\pi}_0^2$ the eigenstates of $a_0$ with a non-vanishing eigenvalue $\nu$ – the ZM coherent states. The corresponding LF energy

$$E_0 = \int_{-\infty}^{+\infty} d\zeta \psi_\nu(\zeta) P_0^- \psi_\nu(\zeta) = \frac{e^2 L}{8\pi^2},$$

(19)

is independent of $\nu$, thus the infinite set of vacuum states $\psi_\nu(\zeta)$, $\nu \in Z$, is degenerate in the LF energy. In addition, they are not invariant under $\hat{Z}_1$, 

$$\hat{Z}_1 \psi_\nu(\zeta) = \psi_{\nu+1}(\zeta)$$

(20)

and those $\psi_\nu(\zeta)$ which differ by unity in the value of $\nu$ have a non-zero overlap. The latter property resembles tunnelling due to instantons in the usual formulation. Note however that in our picture one did not consider minima of the classical action. The lowest energy states have been obtained within the quantum mechanical treatment of the residual symmetry consisting of the c-number shifts of an operator.

Implementation of large gauge transformations for the dynamical fermion field $\bar{\psi}_+ (x^-)$ is based on the commutator

$$[\bar{\psi}_+ (x^-), j^+(y^-)] = \psi_+ (y^-) \delta_a (x^- - y^-)$$

(21)

which follows from the basic anticommutation relation (3). The unitary operator $\hat{F}(\nu) = (\hat{F}_1)^\nu$ that implements the phase transformation (10) is

$$\psi_+ (x^-) \rightarrow \hat{F}(\nu) \psi_+ (x^-) \hat{F}^\dagger(\nu), \quad \hat{F}(\nu) = \exp \left[ -i \frac{\pi}{L} \nu \int_{-L}^{+L} dx^- j^+(x^-) \right].$$

(22)

The Hilbert space transforms correspondingly. But since physical states are states with zero total charge and the pairs of operators $b_k^\dagger d_k^\dagger$, which create these states, are gauge invariant, it is only the vacuum state that transforms:

$$|0\rangle \rightarrow \hat{F}(\nu)|0\rangle = \exp \left[ -\nu \sum_{m=1}^{M} \frac{(-1)^m}{m} (A_m^\dagger - A_m) \right]|0\rangle \equiv |\nu; f\rangle.$$  

(23)

The boson Fock operators $A_m, A_m^\dagger$ [36]

$$A_m = \sum_{k=\frac{1}{2}}^{m+\frac{1}{2}} d_{m-k} b_k + \sum_{k=\frac{1}{2}}^{m} \left[ b_k^\dagger b_{m+k} - d_k^\dagger d_{m+k} \right],$$

$$A_m^\dagger = \sum_{k=\frac{1}{2}}^{m-\frac{1}{2}} b_k^\dagger d_{m-k} + \sum_{k=\frac{1}{2}}^{m} \left[ b_k^\dagger b_{m+k} - d_k^\dagger d_{m+k} \right].$$

(24)
satisfying $[A_m, A_{m'}^\dagger] = \sqrt{mm'}\delta_{m,m'}$ emerge naturally after taking a Fourier transform of $j^+(x^-)$ expressed in terms of fermion modes. This yields

$$j^+(x^-) = \frac{1}{L} \sum_{m=1}^{M} [A_m e^{-i\not{p}_m x^-} + A_m^\dagger e^{i\not{p}_m x^-}]$$

(25)
as well as the exponential operator in Eq. (23). The states $|\nu; f\rangle$ are not invariant under $\hat{F}_1$: $|\nu; f\rangle \rightarrow |\nu + 1; f\rangle$, in analogy with the Eq. (20).

To construct the physical vacuum state of the massive Schwinger model, one first defines the operator of the full large gauge transformations $\hat{T}_1$ as a product of commuting operators $\hat{Z}_1$ and $\hat{F}_1$. The requirement of gauge invariance of the physical ground state then leads to the $\theta$-vacuum, which is obtained by diagonalization, i.e. by summing the degenerate vacuum states $|\nu\rangle = |\nu; z\rangle|\nu; f\rangle$ with the appropriate phase factor:

$$|\theta\rangle = \sum_{\nu=-\infty}^{+\infty} e^{i\theta} |\nu\rangle = \sum_{\nu=-\infty}^{+\infty} e^{i\theta} (\hat{T}_1)\nu |0\rangle, \quad \hat{T}_1 |\theta\rangle = e^{-i\theta} |\theta\rangle, \quad \hat{T}_1 \equiv \hat{Z}_1 \hat{F}_1,$$

(26)

(|0\rangle here denotes both the fermion and gauge boson Fock vacuum). Thus we see that the $\theta$-vacuum $|\theta\rangle$ is an eigenstate of $\hat{T}_1$ with the eigenvalue $\exp(-i\theta)$. In other words, it is invariant up to a phase, which is the usual result [7, 16].

The physical meaning of the vacuum angle $\theta$ as the constant background electric field [37] can be found by a straightforward calculation: $\langle \theta | \Pi_{\mu}^+ | \theta \rangle = \frac{\vartheta}{2\pi}$, where the infinite normalization factor $\langle \theta | \theta \rangle$ has been divided out.

The $|\nu\rangle$-vacuum expectation values of $P^-$ are degenerate due to gauge invariance of the latter. Subtracting the value (19) as well as another constant coming from the normal-ordering of the mass term [25], this vacuum expectation value can be set to zero. Then one has $\langle \theta | P^- | \theta \rangle = 0$, while $\langle \theta | P^+ | \theta \rangle = 0$ and $Q|\theta\rangle = 0$ automatically [23].

Finally, we would like to point out that the fermion component of the theta-vacuum (26), described in terms of the exponential of the effective boson operators $A_m, A_m^\dagger$, introduces a possibility of obtaining a non-vanishing fermion condensate in the LF massive Schwinger model.

## 4 LF Vacuum in Other Gauge Theories

Let us consider briefly the application of the above ideas to more complicated gauge theories. The first example is the two-dimensional $SU(2)$ Yang-Mills theory with colour massive fermion field $\Psi_i(x), i = 1, 2$, in the fundamental representation. The gauge field is defined by means of the Pauli matrices $\sigma^a, a = 1, 2, 3$, as $A^{\mu a}(x) = A^{\mu a}(x) \sigma^a$.

The gauge fixing in the model can be performed analogously to the massive Schwinger model by setting $A_n^+ = 0, A_0^+ = 0$. In the finite volume, the residual gauge symmetry, represented by constant $SU(2)$ matrices, permits to diagonalize $A^a_0$. Consequently, there is only one dynamical gauge field $ZM$ for the $SU(2)$ theory, namely $A_0^+ = \frac{2\pi}{gL} \hat{z}$, where $g$ is the gauge coupling constant.
The LF Hamiltonian, which is a $SU(2)$ generalization of the expression (6), is invariant under residual large gauge transformations

$$A_0^+ \rightarrow A_0^+ - \frac{2\pi \sigma^3}{gL}, \quad \psi_+^i(x^-) \rightarrow e^{i\frac{\pi}{L} x^-} \psi_+^i(x^-), \quad i = 1, 2.$$  

Their implementation in coordinate representation is analogous to the abelian case with one important difference [22, 30]: in order to correctly define the ZM momentum and kinetic energy operators, one has to take into account the Jacobian $J(\zeta)$, which is induced by the curvature of the $SU(2)$ group manifold:

$$P_0^- = -\frac{1}{2} e^{2L} \frac{1}{2} \frac{d}{d\zeta} J \frac{d}{d\zeta}, \quad \Pi_0 = -i \frac{1}{\sqrt{J}} \frac{d}{d\zeta} \sqrt{J} = -i \frac{d}{d\zeta} - i\pi \cot \pi \zeta, \quad J = \sin^2 \pi \zeta.$$  

The presence of the Jacobian has a profound impact on the structure of the ZM vacuum wave functions. Defining again the vacuum state as $(\hat{\zeta} + i\Pi_0)\Psi_0 = 0$, one finds

$$\Psi_0(\zeta) = \pi^{-\frac{1}{4}} e^{-\frac{1}{2} \xi^2} \frac{1}{|\sin \pi \zeta|} \rightarrow \Psi_\nu(\zeta) = e^{-i\nu \Pi_0} \Psi_0(\zeta) = \pi^{-\frac{1}{4}} e^{-\frac{1}{2} (\xi - \nu)^2} \frac{1}{|\sin \pi \zeta|}.$$  

Thus, each wave function is divided into pieces separated by singular points at integer multiples of $\pi$ and individual states are just shifted copies of $\Psi_0(\zeta)$ with no overlap. Consequently, the $\theta$-vacuum cannot be constructed [38, 39]. Further details will be given separately [40].

It is rather striking that the generalization of the present approach to the vacuum problem for the case of the LF QED(3+1), quantized in the (generalized) LC gauge and in a finite volume $-L \leq x^- \leq L$, $-L_\perp \leq x^j \leq L_\perp, j = 1, 2$, appears to be straightforward. The crucial point is that in spite of two extra space dimensions, there is still only one dynamical ZM, namely $A_0^+$ (the subscript 0 indicates the $(x^-, x^j)$-independent component). Indeed, $A_0^-$ can be gauged away (see below) and $A_0^j$ are constrained. Proper zero modes, i.e. the gauge field components $a^+, a^-, a^j$ that have $p^+ = 0, p^j \neq 0$, are not dynamically independent variables either [35] in contrast with the situation in the equal-time Hamiltonian approach [29].

Residual gauge transformations, which are the symmetry of the theory even after all redundant gauge degrees of freedom have been completely eliminated by the gauge-fixing conditions $A_n^+ = 0, A_0^+ = 0, \partial_+ a^+ + \partial_j a^j = 0$ [35], are characterized by the same gauge function $\Lambda_\nu$ as in the Schwinger model, since constant shifts of constrained $A_0^j$ in $j$ directions are not allowed. In this way, we are led to consider essentially the same unitary operators implementing the residual symmetry as in the Schwinger model. For example, defining the dimensionless quantities $\hat{\xi}$ and $\hat{\pi}_0$ by

$$A_0^+ = \frac{2\pi}{g L} \hat{\xi}, \quad \Pi A_0^+ = \frac{1}{(2L_\perp)^2} e^{\frac{\pi}{2} \hat{\pi}_0},$$  

one again recovers the commutator [11], the shift operator $\hat{Z}(\nu)$, etc.

Before being able to make conclusions about the $\theta$-vacuum of the light-front QED(3+1) [41], one needs to better understand the role of constrained zero
modes. Let us emphasize only one point here: the fermion part of the transformed vacuum state acquires again the simple form of Eq. (23) with generalized boson operators $\tilde{A}_m, \tilde{A}^\dagger_m$ ($\sigma$ is the spin projection and $k_\perp \equiv k^j = \pm 1, \pm 2, \ldots$)

$$
\tilde{A}^\dagger_m = \sum_{k=-\frac{1}{2}}^{m-\frac{1}{2}} \sum_{k_\perp=-M_\perp}^{M_\perp} \sum_{\sigma=\pm\frac{1}{2}} \left[ b^\dagger_{m+k,k_\perp,\sigma} b_{k,k_\perp,\sigma} - d^\dagger_{m+k,k_\perp,\sigma} d_{k,k_\perp,\sigma} \right]
$$

(31)

$$
+ \sum_{k=-\frac{1}{2}}^{m-\frac{1}{2}} \sum_{k_\perp=-M_\perp}^{M_\perp} \sum_{\sigma=\pm\frac{1}{2}} \left[ b^\dagger_{k,k_\perp,\sigma} d^\dagger_{m-k,-k_\perp,-\sigma} \right].
$$

(32)

The vacua $|\nu; f\rangle$ (23) with $\tilde{A}^\dagger_m, \tilde{A}_m$ as given above satisfy $\langle \nu; f | P^+ | \nu; f \rangle = 0$, $Q | \nu; f \rangle = 0$, as should.

5 Discussion

The main result of the present work is the demonstration that, despite the apparent “triviality” of the LF vacuum in the sector of normal modes, it is possible to recover the necessary vacuum structure of light-front gauge theories. The principal elements of the approach were the infrared regularization achieved by quantizing in a finite volume and a systematic implementation of the residual large gauge symmetry (specific to the compactified formulation) in terms of unitary operators. An infinite set of non-trivial non-perturbative vacuum states then emerges as the transformed “trivial” Fock vacuum. The requirement of gauge invariance (as well as of the cluster property [42]) of the ground state yields the $\theta$-vacuum in the case of the massive Schwinger model.

Zero-mode aspects of the LF Schwinger model quantized at $x^+ = 0$ have been discussed in the literature before [10, 20, 31, 43]. The massive case has been studied in [24, 44]. Fermionic aspects of the residual symmetry are usually analyzed within the model (rather ad hoc ‘N-vacua’) for the LF fermionic vacuum [10, 24]. Our construction avoids the introduction of the Dirac sea in a natural way. The new insight is that fermion degrees of freedom are inevitably present in the LF ground state – though outside the usual Fock-state description – as a consequence of the residual symmetry under large gauge transformations. It remains to be seen if other non-perturbative features like fermion condensate and axial anomaly can be (at least in the continuum limit) reproduced correctly in this approach, which uses only fields initialized on one characteristic surface. Also, we believe that the physics of the massless model will be recovered in the $m \to 0$ limit of the massive theory.

Furthermore, a possible generalization of the latter to the LF $SU(2)$ gauge theory in two dimensions has been suggested. Structure of the vacuum wave functions, changed by a presence of the non-trivial Jacobian, indicates that no $\theta$-vacuum can be formed in this case, in agreement with previous conclusions [38, 39]. Although the extension of our approach to the vacuum problem of a realistic abelian gauge theory, namely QED(3+1), appeared to be rather straightforward, difficulties related to the renormalization and the presence of non-dynamical zero modes obeying the complicated operator constraints [35]
are to be expected. On the other hand, a more general method of elimination of redundant gauge degrees of freedom by unitary transformations may become a useful alternative to the conventional gauge-fixed formulation of the light-front quantization.

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