CONFINING STRINGS

A.M. Polyakov

Physics Department, Princeton University,
Jadwin Hall, Princeton, NJ 08544-1000.
E-mail: polyakov@puhep1.princeton.edu

Abstract

We propose a hypothesis that all gauge theories are equivalent to a certain non-standard string theory. Different gauge groups are accounted for by weights ascribed to the world sheets of different topologies. The hypothesis is checked in the case of the compact abelian theories, where we show how condensing monopole-instanton fields are reproduced by the summation over surfaces. In the non-abelian case we prove that the loop equations are satisfied modulo contact terms. The structure of these terms unfortunately remains undetermined.
The theory of quark confinement remains an unsolved mystery from the theoretical point of view. The present status of the subject is the following. More than twenty years ago K. Wilson [1] realized that in the lattice gauge theory, treated in the strong coupling approximation charges are confined, because they are connected by color-electric strings, formed by the Faraday flux lines. Yet this approximation is inadequate for the continuous theory and thus the problem of confinement is not resolved by this important result.

The next step was taken by the present author in [2, 3], where the compact $U(1)$ gauge theories have been analyzed. It was shown that the instanton configurations of the gauge fields (which were identified as monopoles in $2+1$ dimensions and as monopole rings in $3+1$) indeed lead to confinement. In the first case this regime persists for all values of the gauge coupling, while in the second it applies only to the couplings above some critical value, due to the condensation of the monopole world lines.

Similar conclusions have been independently reached by 't Hooft [4] and Mandelstam [5] on a qualitative level. These authors argued that in the "dual superconductor" the vortex lines must confine electric charges. Of course this picture is identical to the instanton condensation described above.

The quantitative theory developed for the $U(1)$ case cannot be generalized to the non-abelian gauge groups. This was the rationale to attack the problem from a different angle [6, 7], where it was conjectured that there exists an exact duality between gauge fields and strings. More precisely, it was postulated that there exists a string theory, governing the behavior of the Faraday flux lines and thus equivalent to the Yang-Mills theory. The analyses was based on the equations in the loop space. The intention was to look for their solution in the form of string functional integrals. Closely related ideas were discussed in [8, 9].

Although this approach has been developed extensively, (see e.g. [10-13]) the subject is still hovering in limbo.

To summarize, we have two approaches to confinement. One is field-theoretic, trying to identify relevant field configurations, while the other is based on the string representation. The main purpose of the present paper is to build a bridge between these two approaches.
We will establish the string ansatz in the case of the abelian gauge groups and make conjectures concerning its non-abelian generalization.

Let us begin with a review of the main points of ref. [2]. The action in this theory is simple

\[ S(A) = \frac{1}{4e^2} \int (\partial_\mu A_\nu - \partial_\nu A_\mu)^2 d\vec{x} \]  

(1)

and the Wilson loop is given by:

\[ W(C) = \int DA_\mu e^{-S(A)} \exp i \oint A_\mu dx_\mu \]  

(2)

The key point of [2] is that for the compact \( U(1) \) one must include the multivalued monopole configurations of the \( A \) fields. As a result the Wilson loop has a representation

\[ W(C) = W_0(C) W_M(C) \]  

(3)

Here the first factor comes from the Gaussian integration over the \( A \) field while the second factor is the contribution of the point-like monopoles in 3d or monopole loops in 4d. Let us analyze the 3d case first. A monopole located at the point \( \vec{x} \) contributes to \( W_M(C) \) as following,

\[ W_M^1(x, C) \propto \exp \left(-\frac{\text{const}}{e^2}\right) \exp i\eta(x, C) \]  

(4)

where

\[ \eta(x, C) = \int_C A_\mu^{(\text{mon})}(x - y) dy_\mu = \int_{\Sigma_C} \frac{(x - y)_\mu}{|x - y|^3} d^2 \sigma_\mu(y) \]  

(5)

In (4) the first factor is the value of the monopole action , while \( \eta(x, C) \) is the solid angle formed by point \( x \) and the contour \( C \). \( \Sigma_C \) is an arbitrary surface bounded by the contour \( C \). As we change this surface the phase \( \eta \) may jump by \( 2\pi \) leaving \( W(C) \) unchanged. As was explained in [3] , summation over all possible monopole configurations leads to the following representation:

\[ W_M(C) \propto \int D\varphi(x) \exp -e^2 \int \left[ \frac{1}{2}(\partial \varphi)^2 + m^2 (1 - \cos (\varphi + \eta)) \right] d\vec{x} \]  

(6)
with \( m^2 \propto e^{-\text{const}} \)

In order to find a string representation for \( W(C) \) we have to recast these formulae into a different form. Let us consider the following functional integral,

\[
W(C) \propto \int DB_{\mu\nu} D\phi e^{-\Gamma}
\]

with the effective action given by

\[
\Gamma = \int d\tilde{x} \left[ \frac{1}{4e^2} B_{\mu\nu}^2 + i\phi \wedge dB + e^2 m^2 (1 - \cos \phi) \right] + i \int_{\Sigma_C} B_{\mu\nu} d\sigma_{\mu\nu}
\]

where we have introduced a rank two antisymmetric tensor field \( B \). This field is easy to eliminate by the Gaussian integration. Let us show that this procedure returns us to the old expression (6) with an extra factor equal to \( W_0(C) \). Thus, this formula unifies the monopole and the free field configurations. Indeed, by minimizing \( \Gamma \) with respect to \( B \) we obtain

\[
B_{\mu\nu} = i\epsilon_{\mu\nu\lambda} (\partial_\lambda \phi + \int_{\Sigma_C} \delta(x-y)d^2\sigma(\lambda))
\]

Using the identity

\[
\int_{\Sigma_C} \delta(x-y)d^2\sigma(\lambda) = \partial_\lambda \eta + \epsilon_{\lambda\rho\sigma} \partial_\rho a_\sigma
\]

where \( \eta(x,C) \) is given by (5) and \( a_\mu(x,C) \) is determined from the equation

\[
\partial^2 a_\mu = \oint_C \delta(x-y)dy_\mu
\]

we reduce the integral (8) to the product \( W(C) = W_0(C)W_M(C) \). We also notice that \( \phi = \varphi + \eta \). Since \( \phi \) has no kinetic energy in (8), it can be eliminated by the Legendre transform, leaving us with the effective action for the massive \( B \) field. As a result we obtain

\[
W(C) = \int DB e^{-S(B)} \exp i \int_{\Sigma_C} B d\sigma
\]

with the axion action given by

\[
S(B) = \int d\tilde{x} \left[ \frac{1}{4e^2} (B_{\mu\nu}^2 + f(H)) \right]
\]

\(^1\)Massless Kalb-Ramond fields, interacting with strings were considered in a different context in [15]
Here $H = dB$ and:

$$f(H) = H \arcsin \frac{H}{m^2} - \sqrt{m^4 - H^2}$$

(14)

The multivaluedness of this curious action is a reflection of $\phi$-periodicity in (8).

Now we confront a very important puzzle. Namely, we have claimed before that the surface $\Sigma_C$ is arbitrary. At the same time if we attempt to compute the Wilson loop by expanding the action (14) in $H$, which is justified for the large enough loops, we find a non-trivial dependence on this surface. Let us demonstrate this explicitly.

In the weak field limit the above action takes the form:

$$S(B) = \frac{1}{4e^2} \int (B^2 + m^{-2}(dB)^2)d\vec{x}$$

(15)

In this approximation we obtain:

$$W(C) = e^{-F(C,\Sigma_C)}$$

$$F = \int_\Sigma d\sigma_{\mu\nu}(x)d\sigma_{\lambda\rho}(y)D_{\mu\nu,\lambda\rho}(x - y)$$

(16)

In this formula the $D$ function is the propagator for the massive axion field $B$. In the large loop limit we have the following local expansion for $F$

$$F = c_1e^2m \int d^2\xi \sqrt{g} + c_2e^2m^{-1} \int d^2\xi(\nabla t_{\mu\nu})^2 \sqrt{g} + \ldots$$

(17)

where

$$g_{ab} = \partial_a \vec{x} \partial_b \vec{x}$$

$$t_{\mu\nu} = \epsilon^{ab} \partial_a x_\mu \partial_b x_\nu$$

(18)

It is easy to check that higher order corrections can change the values of the constants $c_{1,2}$ but not the structure of this expression.

This is the action of the rigid string. What is surprising here is that we have found an explicit dependence on the surface $\Sigma$, while originally it was introduced as an unphysical object.

The origin of the paradox is the multivaluedness of the action (14). When expanding it in $H$ we took into account only one branch of the $(\arcsin \frac{H}{m^2})$ at each space-time point. The surface independence would be restored had we summed over all possible branches.
The summation over branches can be replaced by the summation over surfaces. This is the heart of the connection between fields and strings in this problem.

Let us begin with proving this fact in the saddle point approximation. Consider once again the action (8). We have to prove that there are two equivalent options. First we may allow \( \phi \) to vary from \(-\infty\) to \(+\infty\) and minimize this action with respect to \( \phi \) and \( B \). The result will depend on \( \Sigma \) but it is easy to see that for two different surfaces we have

\[
\Gamma(\tilde{\Sigma}) = \Gamma(\Sigma) + 2\pi i N(\Sigma|\tilde{\Sigma})
\]

with \( N \) being an integer. This follows from the fact that the translation

\[
\phi \rightarrow \phi + 2\pi
\]

insures that

\[
dB = 0 \text{ (mod } Z)\]

This is just another reflection of the fact that \( H = dB \), which measures the departure from the abelian Bianchi identity, is created by the integer charged monopoles.

So in this first option we get an expression for \( W(C) \) which does not depend on the shape of the surface but involves all possible branches of the \( (\arcsin(H/m)) \) summed over separately at each space-time point – a rather awkward prescription.

We claim that an alternative to this procedure is to restrict ourselves to the main branch defined by

\[
|\phi(x)| \leq \pi
\]

After that the action, which now explicitly depends on \( \Sigma \), must be minimized with respect to this surface. Of course, as we go beyond the saddle-point approximation, this procedure must be replaced by the summation over surfaces.

To prove the above statement, let us notice that the surface variation of \( \Gamma \) is given by

\[
\delta \Gamma = i \delta \int B_{\mu\nu}(x) d\sigma_{\mu\nu}(x) = i \int H(x(\xi)) \epsilon_{\mu\nu\lambda} \delta x_\mu(\xi) d\sigma_{\nu\lambda}(\xi)
\]

(20)

(where we parametrised the surface by \( x_\mu = x_\mu(\xi_1, x_2) \). Hence the stationarity condition with respect to the surface is just

\[
H(x(\xi)) = 0
\]

(21)
For a generic function $H(x)$ this equation indeed defines a two-dimensional surface. It is convenient now to switch to the $\phi$ variable. We have

$$-\partial^2 \phi + m^2 \sin \phi = 0$$

$$H(x) \propto \sin \phi(x)$$

(22)

with the conditions:

$$\lim_{x \to \infty} \phi(x) = 0$$

$$\phi_+ - \phi_-|_\Sigma = 2\pi$$

(23)

where $\phi_\pm$ are the values of $\phi$ on the two sides of the surface. This can also be formulated as a isomonodromy problem for the sine-gordon equation - we require $\phi$ to change by $2\pi$ as we go around the contour $C$.

On the extremal surface, $\phi$, which starts from zero, approaches the value $\pi$ on one side of the surface and jumps to ($-\pi$) on the other side. This means that on both sides $H \propto \sin \phi = 0$ (which is our stationarity condition).

If we take any other surface, the value of $|\phi|$ is necessarily less than $\pi$ on one side of the surface and greater than $\pi$ on the other side. Hence in this case we are pushed out of the fundamental region $|\phi| \leq \pi$.

To summarize, when solving the isomonodromy problem, the surface of the discontinuity may be chosen arbitrarily. However among these surfaces the one that minimizes the action is "more equal than the others" because in this case the solution lies inside the fundamental region.

The implication of this result is very interesting. It means that at least quasiclassically the surface $\Sigma_C$ due to its connection with the $\phi$-field becomes dynamical. As a result, some of the degrees of freedom of the $\phi$ field become stringy. Roughly speaking the dynamical surface appears as a solution of eq. (21). As we integrate over the $B$ field, we also integrate over two-dimensional surfaces defined by this equation.

To see this more explicitly, let us return to eq. (22). Suppose that we have a solution of this equation. Consider now the spectrum of small perturbations. It is defined by the
following eigenvalue problem,

\[- \partial^2 \psi_n + m^2 (\cos \phi_{cl}) \psi_n = E_n \psi_n \]  \hspace{1cm} (24)

We claim that there are two classes of the eigenvalues in the case of the large loops. The first class includes the ordinary excitations of the field \( \phi \). For them \( E_n \sim m^2 \) and they will not be of interest to us. The second class is formed by the string modes which have

\[ E_n \sim \frac{1}{R^2} \]

(where \( R \) is the size of the contour.)

The string action (17) must be viewed as a low energy lagrangian for these modes. In order to check these statements let us recall the solution of (22) found in (20) for the large loops. It has the form,

\[ \phi_{cl} = 4 \text{sgn}(z) \arctan e^{-m|z|} \]  \hspace{1cm} (25)

and eq.(24) becomes

\[ \partial_z^2 \psi + 2m^2 (\cosh mz)^{-2} \psi = -(E - m^2) \psi \]  \hspace{1cm} (26)

(here \( z \) -is a coordinate ,normal to the surface.) It is easy to see that there is a zero mode with \( E = 0 \) given by:

\[ \psi_0 \propto (\cosh mz)^{-1} \]

. Its presence is slightly surprising since the double layer discontinuity (23) apparently breaks the translational invariance and the usual Goldstone -like argument is not directly applicable. However, the periodicity of the cosine and the quantization of the double layer restore the translational symmetry.

The above solution is valid for infinite loops. For finite loops we get \( E \sim R^{-2} \ll m^2 \). An approximate expression for the field \( \phi \) accounting for this mode is given by

\[ \phi = \phi_{cl} + a(x, y) \psi_0(z) \]  \hspace{1cm} (27)

(where \((x, y)\) are the longitudinal coordinates on the surface.).
The amplitude $a$ has a natural interpretation as the transverse mode of string oscillations. The effective action for this mode may be obtained by passing to the Monge gauge in the covariant action (17).

We do not have a complete proof of this fact. Because of the difficulties related to the definition of the summation over surfaces (described below in more details) we will limit ourselves to a somewhat heuristic argument based on the lattice theory. Let us consider first the partition function, $Z$, given by:

$$Z = \int DB e^{-S(B)} \sum_{\text{all closed surfaces}} \exp i \oint B_{\mu\nu} d\sigma_{\mu\nu}$$

(28)

The latter sum on a lattice can be rewritten as:

$$\sum_{\text{all closed surfaces}} e^{i \oint B d\sigma} = \sum_{[N\vec{x}]} \exp i \sum_{\vec{x}} N_{\vec{x}} H_{\vec{x}} = \sum_{[q\vec{x}]} \delta(H_{\vec{x}} - 2\pi q_{\vec{x}})$$

(29)

Here the $N$-s and the $q$-s are integers.

In (29) we assumed that the surface (which is not necessarily connected) is represented by a collection of cells in such a way that a cell centered at point $\vec{x}$ is covered $N_{\vec{x}}$ times. With this definition of the surface, it has no folds, since oppositely oriented components of the fold cancel each other. At the same time, for simple surfaces without folds and self-intersections, this definition coincides with the standard one. For example, a cube located at point $\vec{x}$ corresponds to $N_{\vec{x}} = 1$. Eq.(29) is a consequence of the Stokes theorem. The monopole representation of the partition function (6) follows immediately, with the $q$-s being the monopole charges. Open surfaces, describing the Wilson loops are defined and treated analogously.

This argument is not a proof because the continuous formulation of the above "foldless" surfaces is absent. The main difficulty with such a formulation lies in the fact that our surfaces have an unusual extended reparametrisation invariance which must be gauge-fixed and renormalized. To see the problem, consider the Wilson loop, $W = W[\vec{c}(s)]$, for a contour parametrized by $\vec{c} = \vec{c}(s)$. It is invariant under the transformations:

$$\vec{c}(s) \Rightarrow \vec{c}(\alpha(s))$$

. In the standard string theory we require that $\alpha$ must be a diffeomorphism, i.e. $\frac{d\alpha}{ds} \alpha > 0$. However, the Wilson loop is invariant under the extended reparametrizations for which the
preceding condition is not necessarily satisfied. Hence, for the confining strings described by the world surface \( \vec{x} = \vec{x}(\xi_1, \xi_2) \) we need the action and the measure on the phase space which are invariant under the extended transformations of the \( \xi \)-space, with the Jacobian not necessarily positive. The string action (12) clearly has this symmetry, but the measure is still to be defined. Among other things, this extended symmetry eliminates folds, as we have already indicated. These problems are not quite solved yet and will be treated elsewhere.

In spite of this deficiency, we can draw a number of interesting conclusions. First of all let us notice that the ultraviolet cut-off for our string theory is set by the \( B \)-field mass, \( m \), while the string tension, \( \sigma \), according to (17) is given by

\[
\sigma \propto m^2 \sim \alpha M_W m \gg m^2 \quad \text{and} \quad \alpha = \frac{e^2}{M_W} \ll 1 ,
\]

where we defined a dimensionless coupling constant, \( \alpha = \frac{e^2}{M_W} \ll 1 \), and \( M_W \) is the \( W \)-boson mass coming from the broken non-abelian theory). The resonance spectrum is determined by the string tension, and so we expect to find resonance states in this theory with the masses \( M_{\text{res}} \) given by:

\[
M_{\text{res}} \sim (\alpha M_W m)^{1/2} \quad m \sim M_W e^{-\frac{\text{const}}{\alpha}}
\]

with \( m \ll M_{\text{res}} \ll M_W \).

It is not clear how narrow these states are but it is obvious that there is an interesting dynamics in the intermediate range introduced above.

Second, we conclude that the confining strings have mixed nature combining a separate massive \( B \)-field with the stringy degrees of freedom created by the oscillations of the surface \( \Sigma \). In the abelian case treated above they are well separated.

So far we have discussed the 3d abelian theory. It is straightforward to generalize this discussion to the 4d abelian case along the lines of [2]. As was explained there, the confining phase related to the condensation of the monopole rings which begins at some critical coupling, \( e_{\text{cr}} \), and we have to consider couplings close to this value. The action (8) is replaced by

\[
\Gamma = \int d\vec{x} \left[ \frac{1}{4e^2} B^2_{\mu \nu} + i \phi \wedge dB \right] + e^2 m^2 \sum_{\vec{\xi}_\mu} (1 - \cos \phi_{\vec{\xi}_\mu}) + i \int_{\Sigma_C} B_{\mu \nu} d\sigma_{\mu \nu}
\]

(31)
Here the field $\phi$ is a 1-form due to the fact that the instantons in this case are the monopole rings which are one dimensional objects. We also used the lattice regularization in this case, which is needed since the coupling isn’t small. All the rest proceeds just as in the previous case. When we restrict the field $\phi$ to its fundamental domain, $|\phi_{\vec{x}_\mu}| < \pi$, we pay for that with the summation over surfaces. Again, the field $\phi$ doesn’t have kinetic energy and can be excluded, leading to a certain action for the $B$-field. Let us notice that the $\phi$-field is a dual gauge field interacting with the monopoles.

Let us now proceed to the most interesting case of the non-abelian gauge theories. We would like to put forward a strong conjecture (already discussed in some form in the previous work[14]). According to this conjecture all gauge theories are described by the same universal confining string theory with the action (12). Different gauge groups are accounted for by different weights ascribed to surfaces with the different topology. Thus in the $U(1)$-case we have to sum over all topologies indiscriminately, with equal weights, while in the $U(\infty)$-case only the simplest disk topology contributes to the Wilson loop. In general, for the $U(N)$ case we expect the t’Hooft factor $N^{-\chi}$ ($\chi$ being the Euler character) to appear in the summation.

In order to test this conjecture we shall use the loop equations derived in [7, 10]. The origin of these equations is simple. They arise from an attempt to find the loop differentiation which, being applied to the non-abelian Wilson loop

$$W(C) = \langle TrP \exp \oint_C A_\mu dx_\mu \rangle$$

will give the Yang-Mills equations of motion inside the brackets. It is easy to see that this is achieved by the following differential operation on the loop space

$$\frac{\partial^2}{\partial^2 x(s)} = \lim_{\epsilon \to 0} \int_{-\epsilon}^{\epsilon} dt \frac{\delta^2}{\delta x_\mu(s + \frac{t}{2})\delta x_\mu(s - \frac{t}{2})}$$

The key feature of this operation is

$$\frac{\partial^2}{\partial^2 x(s)} W(C) = \left\langle TrP(\nabla_\mu F_{\mu\nu}(x(s))\left(\frac{d}{ds}x_\nu(x(s))\right)\exp \oint_C A_\mu dx_\mu \right\rangle \approx 0$$

Here $F_{\mu\nu}$ is the standard Yang-Mills field strength, $\nabla$ is the covariant derivative, and the sign $\approx$ here and below means ”modulo contact terms”, i.e. terms which are nonzero only when the correlation function under consideration contains coincident space-time points.
One of the reasons for our universality conjecture is that according to the above formula the wave operator acting on the Wilson loop is universal, while the contact terms, closely related to the topology of the surfaces involved, are sensitive to the gauge group. In what follows we will show that our string ansatz indeed satisfies eq.(34). In order to prove this statement we have to derive some variational formulas. Consider a surface parametrised by the equation

\[ \vec{x} = \vec{x}(\xi; [\vec{c}(s)]) \]  

where we have introduced explicit dependence on the boundary contour which is parametrized by \( \vec{x} = \vec{c}(s) \). Our aim is to apply the wave operator to our string ansatz. The first variation gives:

\[ \frac{\delta}{\delta c_\mu(s)} \int B_{\lambda\rho}(x) d\sigma_{\lambda\rho} = B_{\mu\rho}(c(s)) \dot{c}_\rho(s) + \int (dB)_{\alpha\beta\gamma} \frac{\delta x_\alpha}{\delta c_\mu(s)} d\sigma_{\beta\gamma} \]  

The second step is to apply the second derivative \( \frac{\delta}{\delta c_\mu(u)} \) and to extract from this expression the term proportional to \( \delta(s - u) \). As is clear from our definition, our loop operator is just the coefficient in front of this \( \delta \)-function. Using this fact we obtain:

\[ \frac{\partial^2}{\partial^2 c(s)} \int B_{\lambda\rho}(x) d\sigma_{\lambda\rho} = \frac{\partial}{\partial c_\mu(s)} B_{\mu\rho}(c(s)) \dot{c}_\rho(s) + \int (dB)_{\alpha\beta\gamma} \frac{\partial^2 x_\alpha}{\partial^2 c(s)} d\sigma_{\beta\gamma} \]  

The partial derivative \( \frac{\partial}{\partial c_\mu(s)} \) is again defined as a coefficient in front of the corresponding \( \delta \)-function. It reduces to the ordinary partial derivative when acting on functions, but in our case it is important to remember that \( B_{\mu\nu} \) is actually a functional. When deriving these formulae we have dropped several terms containing the products of two variational derivatives in the integrand of (37). This is allowed because for smooth surfaces such terms never contain the factor \( \delta(s - u) \) which we are looking for. Essentially, the formula (37) means that our wave operator, despite its appearance, is of the first order and satisfies the Leibnitz rule. The properties of these loop derivatives were discussed in ref[7] in more details.

Let us show now that if we use the equations of motion for \( \vec{x}(\xi) \) and \( B_{\mu\nu}(x) \) the right hand side of (37) is indeed zero modulo contact terms. First of all, as was already noticed, equations of motion for \( \vec{x}(\xi) \) have the form:

\[ dB(\vec{x}(\xi)) = 0 \]  

(38)
and thus exterminate the second term in (37). In order to deal with the first term we need the equations of motion for the $B$-field. If the action has the form:

$$S(B) = \frac{1}{4e^2} \int d\vec{x} [B_{\mu\nu}^2 + m^{-2}(dB)^2 + \ldots] + i \int B_{\mu\nu} d\sigma_{\mu\nu}$$

we get the following classical value of $B$:

$$B_{\mu\nu}(\vec{x}; \{c(u)\}) \propto \int \delta(x - y)d\sigma_{\mu\nu}(y)$$

$$\frac{\partial}{\partial x_{\mu}} B_{\mu\nu}(x, c) \propto \oint_C \delta(x - y)dy_{\nu}$$

(40)

We took the limit $m \to \infty$ in these formulae, since, as was explained above, the mass of the $B$-field serves as an ultraviolet cut-off for the string modes. When computing the derivative $\frac{\partial}{\partial c_{\mu}(s)}$ we have to add the above value of $\partial_{\mu}B_{\mu\nu}(x, c)$ at the point $\vec{x} = \vec{c}(s)$ to the result of the variation with respect to $\vec{c}(s)$. Somewhat lengthy computation shows that in the large mass limit the leading term vanishes:

$$\lim_{m \to \infty} \frac{\partial}{\partial c_{\mu}(s)} B_{\mu\nu} = 0$$

(41)

All this means is that, on a superficial level, our string ansatz satisfies the loop equations. However, we are far from claiming that our gauge field -string hypothesis is proved by the above computations. The main difficulty lies in our present inability to analyze the contact terms which as usual appear when equations of motion are used in the functional integral. Correct gauge fixing and renormalization are crucial for this task. At present the best we can do is to hope that these terms are essentially fixed by the topology of the surface and by dimensional counting. If this is true they must be the same in the Yang-Mills theory and in our string theory, thus proving our hypothesis.

Despite the incompleteness of these results, I feel that some progress has been made in establishing relations between fields and strings.

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