A model of a virtual community with a decentralized reputation-based peer evaluation

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1. Introduction

This study was motivated by the problem of identifying fake news on the Internet. Generation and distribution of fake news on the Internet has become a powerful tool in politics and commerce \cite{1-3}.

Known approaches to verification of information have limited efficiency. The activity of fact checking sites such as FactCheck.org and Snopes.com is not enough to counter-balance the influx of fake information. The throughput of these sites is constrained by available resources, particularly by the number of people working for them. In addition, the credibility of such sites is questioned since closed groups of people, involved in verification, may develop biases \cite{4}. The aim of the present study is to explore the potential of a decentralized mechanism that would engage millions of people into evaluating information.

There are many computational models of reputation and trust that vary in how individuals interact and use the results of interactions; see for example review \cite{5}. A particular attention has been attracted to peer-to-peer interactions in the context of online commerce as in eBay and Amazon \cite{6-9}. The model considered in this study is different from most of these works in several respects. In our case, individual’s product (piece of information) is directly evaluated not by one but by three peers. Next, in peer-to-peer interactions, when the reputation of an individual decreases, others interact with this individual less and less. In our model, the rate of submitting products is the same for all individuals and does not change over time. Finally, we are not concerned about the reputation of particular individuals. Our model describes time evolution of proportions of individuals with certain levels of reputation.

The proposed evaluation mechanism is inspired by academia peer-reviewing. Two fundamental and time-proven means of obtaining high-quality publications have been borrowed: the mechanism of peer-reviewing and the institute of reputation, which assumes that reputable reviewers are good reviewers and good publications increase reputation. However the process modeled here is simpler than peer-reviewing and because of that is more amenable for analysis: 1) evaluation results are binary – documents are evaluated as either authentic or fake; 2) evaluation process is not hierarchical – each of three reviewers votes pro or contra and the decision is determined by simple majority; 3) evaluators do not deal with rebuttals and resubmissions.

The main assumptions regarding the reputations of the community members are as follows. The reputation of a member increases if the submitted document is evaluated as an authentic and decreases otherwise. The probability of being selected to serve as an evaluator is proportional to the reputation. These features have some similarity to known reputation systems resulting from peer-to-peer interactions \cite{5} but in many respects are different. In our case, an individual’s product – a submitted piece of information – is directly evaluated not by one but by three peers simultaneously. These peers do not select products, they evaluate what they get by chance. In peer-to-peer interactions, when the reputation of an individual decreases the others interact with such individual less. In our model, every member has the right to obtain an evaluation of the documents s/he submits. The rate of submitting documents is the same for all community members and does not change over time. Finally, we are concerned about proportions of members with certain values of reputation but not about particular individuals.

Our analytical and computational results suggest the proposed evaluation mechanism is effective in a wide range of the model’s parameters and even in cases when some members form cliques whose only priority is pursuing their own agenda. The corresponding mathematical model is introduced and explored gradually, step

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by step. We start with a model of a community without a clique. The members of the community have only three possible values of reputation (Section 2). The following Section 3 describes a generalization of the previous model to the case with an arbitrary number of reputation values; results of computer simulations of the model are presented in Section 4. Subsequent Sections 5 and 6 add one and then two cliques to the modeled community; in the latter case, the cliques are antagonistic to each other. Section 7 presents related work, and Section 8 concludes.

2. Communities with three levels of reputation

2.1. Notations and equations

In this section, reputation of the community members has only three values, \( r_0, r_1, r_2 \), where \( r_0 < r_1 < r_2 \); for example, \( r_0 = 0, r_1 = 0.5, r_2 = 1 \). The proportion of the community members with the reputation \( r_k \) is denoted by \( R_k \), \( 0 \leq R_k \leq 1, k = 0, 1, 2 \); \( R_0 + R_1 + R_2 = 1 \). Equivalently, \( R_k \) is the probability of randomly selecting a member with the reputation \( r_k \). Together, \( R_k \) define a probability distribution. It is assumed that new members start with the same value of reputation. Appearance of new members and loss of existing members of the community can be modeled by changes in the corresponding values of \( R_k \).

After the document, submitted by a member, is evaluated, the member’s reputation changes. It increases or remains maximal (equal to \( r_2 \)) if the document is evaluated as authentic. Otherwise the reputation decreases or remains minimal (equal to \( r_0 \)). All the members submit documents with the same rate. These assumptions lead to the equations

\[
\begin{align*}
\frac{dR_0}{dt} & = -R_0 e_0 + R_1 (1 - e_1) \\
\frac{dR_1}{dt} & = R_0 e_0 - R_1 + R_2 (1 - e_2) \\
\frac{dR_2}{dt} & = R_1 e_1 - R_2 (1 - e_2),
\end{align*}
\]

(1)

where \( t \) is time, and \( e_k \), \( 0 \leq e_k \leq 1 \), is the probability that a document, submitted by a member with the reputation \( r_k \), is evaluated as authentic.

The following proposition shows that (1) indeed describes an evolution of a probability distribution \( \{R_0, R_1, R_2\} \).

**Proposition 1.** Let the initial values \( R_k(0) \) of \( R_k \) be such that \( R_0(0) + R_1(0) + R_2(0) = 1 \) and \( 0 \leq R_k(0) \leq 1 \). Then a) \( R_0(t) + R_1(t) + R_2(t) \equiv 1 \), and b) \( 0 \leq R_k(t) \leq 1 \) for all \( t \).

**Proof 1.** Summing up the equations yields \( d(R_0 + R_1 + R_2)/dt \equiv 0 \) which proves a). Part b) follows from the fact that according to (1) once \( R_k \) becomes equal to 0 it can no longer decrease and once it becomes equal to 1 it can no longer increase. Indeed, let for example, \( R_0(t) = 0 \) for some \( t \), while \( 0 < R_1(t) < 1 \) and \( 0 < R_2(t) < 1 \). Then the right hand side of the first equation in (1) is non-negative, or \( dR_0(t)/dt \geq 0 \). On the other hand, if \( R_0(t) = 1 \) for some \( t \) then it implies \( R_1(t) = 0 \) and the right hand side of the equation for \( dR_0(t)/dt \) is non-positive, or \( dR_0(t)/dt \leq 0 \). The cases of \( R_1 \) and \( R_2 \) are similar.

System (1) is essentially two-dimensional since one of the variables \( R_k \) can be excluded using the equality \( R_0 + R_1 + R_2 \equiv 1 \). With the excluded \( R_1 \) system (1) turns into

\[
\begin{align*}
\frac{dR_0}{dt} & = -R_0 e_0 + (1 - R_0 - R_2)(1 - e_1) \\
\frac{dR_2}{dt} & = (1 - R_0 - R_2)e_1 - R_2 (1 - e_2).
\end{align*}
\]

(2)

To define system (2) completely we need to express the probabilities \( e_k \) as functions of \( R_k \). It takes several steps. First of all, each \( e_k \) is a sum of two probabilities. One is the probability that the document is indeed authentic, \( a_k \), and is correctly evaluated as authentic, \( p_c(A) \). The other probability is the probability that the document is fake, \( 1 - a_k \), but incorrectly evaluated as authentic, \( p_m(F) \):

\[
e_k = a_k p_c(A) + (1 - a_k) p_m(F), \quad k = 0, 1, 2.
\]

(3)
Whether the document is authentic or fake is decided by three evaluators. They decide independently. The final decision is determined by the majority. Let \( p_{\text{c,ind}}(A) \) be the probability that a randomly selected evaluator correctly evaluates an authentic document. Then the probability that authentic document is correctly evaluated by the majority

\[
p_c(A) = p_{\text{c,ind}}^3(A) + 3p_{\text{c,ind}}^2(A)(1 - p_{\text{c,ind}}(A)).
\]  

(4)

The first term is the probability that all of the three evaluators correctly evaluate the document as authentic. The second term is the probability that one of the evaluators mistakenly decides the document is fake. Similarly, the probability that the majority mistakenly evaluate a fake document as authentic:

\[
p_m(F) = p_{\text{m,ind}}^3(F) + 3p_{\text{m,ind}}^2(F)(1 - p_{\text{m,ind}}(F)).
\]  

(5)

Here \( p_{\text{m,ind}}(F) \) is the probability that a randomly selected evaluator mistakenly evaluates a fake document as authentic.

Let a member with a reputation \( r_k \) correctly evaluate authentic documents with a probability \( c_k(A) \) and correctly evaluate fake documents as fake with a probability \( c_k(F) \). Since a selected evaluator can have one of the three reputations,

\[
p_{\text{c,ind}}(A) = s_0c_0(A) + s_1c_1(A) + s_2c_2(A),
\]

\[
p_{\text{m,ind}}(F) = s_0(1 - c_0(F)) + s_1(1 - c_1(F)) + s_2(1 - c_2(F)).
\]  

(6)

The probability \( s_k \) of selecting a member with a reputation \( r_k \) is proportional to \( r_k \):

\[
s_k = \frac{r_kR_k}{r_0R_0 + r_1R_1 + r_2R_2}, \quad k = 0, 1, 2.
\]  

(7)

For \( r_0 = 0 \) the formula gives \( s_0 = 0 \) meaning that the members with the zero reputation are never selected to serve as evaluators. Function \( s_0 \) has a discontinuity at \( R_0 = 1, R_1 = 0, R_2 = 0 \). We remove it by setting \( s_0 = 0 \) at this point. Then all \( s_k = 0 \) and \( e_k = 0 \) at \((1,0,0)\), and the system remains at \( R_0 = 1, R_1 = 0, R_2 = 0 \). This point is an equilibrium of (2).

To simplify analysis we assume that evaluators correctly evaluate authentic and fake documents with the same probability:

\[
c_k = c_k(A) = c_k(F), \quad k = 0, 1, 2.
\]  

(8)

The assumption implies \( p_m(F) = 1 - p_c(F) = 1 - p_c(A) \) in (3). Then (3) takes the form

\[
e_k = a_kp_c + (1 - a_k)(1 - p_c), \quad k = 0, 1, 2,
\]  

(9)

where \( p_c = p_c(F) = p_c(A) \) is the probability of correct evaluation of any document.

To complete the model we need to specify \( a_k \) in (3) and \( c_k \) in (6). Let

\[
a_k = a(r_k) = r_k(1 + \alpha(1 - r_k)), \quad c_k = c(r_k) = r_k(1 + \sigma(1 - r_k)), \quad k = 0, 1, 2,
\]  

(10)

where the parameters \( \alpha \) and \( \sigma \) take values from \([-1,1]\). For negative values of these parameters \( a_k \) and \( c_k \) are smaller than \( r_k \); for positive values \( a_k \) and \( c_k \) are greater than \( r_k \) (Fig.1). In all the cases, \( a_k \) and \( c_k \) take on values in \([0,1]\).

System (2) is completely defined now. It is a non-linear system with five parameters: \( r_0, r_1, r_2, \alpha \) and \( \sigma \). Unless mentioned otherwise \( r_0 = 0, r_1 = 0.5, r_2 = 1 \). The values of \( r_k \) are sometimes expressed as percentages. For example, \( r_2 = 1 \) corresponds to 100% reputation.

2.2. Equilibria

The Poincaré-Bendixson theorem [10] implies that the long-term behavior of system (2) as a two-dimensional system, could be either an equilibrium, a limit cycle or a homo-/heteroclinic orbit.

The vector field of system (2) for different values of \( \alpha \) and \( \sigma \) indicates that the system has equilibria at the line \( R_2 = 1 - R_0 \) (Fig.2).
Figure 1: Function $f(x) = x(1 + \alpha(1 - x))$ used in (10). The line $y = x$ corresponds to $\alpha = 0$. The unlabeled values of $\alpha$ are 0.5 and $-0.5$.

**Proposition 2.** The points of the line $R_2 = 1 - R_0$, $R_0 \in [0, 1]$ are equilibria of system (2) for all $\alpha, \sigma \in [0, 1]$. At these points all documents are evaluated correctly, $p_c = 1$.

**Proof 2.** For the proof we need to verify that $-R_0 e_0 + (1 - R_0 - R_2)(1 - e_1) = 0$ and $(1 - R_0 - R_2)e_1 - R_2(1 - e_2) = 0$. The condition $R_2 = 1 - R_0$ simplifies the equations to $R_0 e_0 = 0$ and $R_2(1 - e_2) = 0$. The condition also implies $R_1 = 0$. If $R_0 = 1$ then $e_0 = 0$ and $R_2 = 0$ and the equations hold. If $0 < R_0 < 1$ then according to (7)-(10) $p_c = 1$, $e_0 = 0$ and $e_2 = 1$, which proves the proposition. The reasoning holds for all $\alpha, \sigma \in [0, 1]$.

In fact the proof holds for all the functions $a(r)$ and $c(r)$ such that $a(0) = 0$ and $a(1) = c(1) = 1$. The condition $a(0) = 0$ implies that the members with zero reputation submit only fake documents. The condition $a(0) = 0$ and $a(1) = c(1) = 1$ implies that the members with 100% reputation submit only authentic documents and make no mistakes in evaluation.

The reasons why the states $(R_0, 1 - R_0)$ are equilibria are quite intuitive. Evaluators are chosen only from the members with 100% reputation who make no mistakes. Hence the members with zero reputation, who submit only fake documents, have no chance to increase their reputation, while the members with 100% reputation, who submit only authentic documents, never lose it. It is not the case when there exist members whose reputation is more than zero and less than one. Such members can be selected to evaluate and can make mistakes in evaluation. Because of such mistakes, members with zero reputation can increase it and members with 100% reputation can loose it.

Linear stability analysis does not allow to determine the stability of the equilibria $(R_0, 1 - R_0)$. When $R_0 < 1$, the matrix of the system (2), linearized at $(R_0, 1 - R_0)$, has the eigenvalues $-1$ and $0$, the latter meaning a critical case [11]. Computer simulations suggest that the equilibria $(R_0, 1 - R_0)$, $R_0 \in [0, 1)$, are stable in Lyapunov’s sense, meaning that if the system starts evolving from near an equilibrium it will remain near the equilibrium forever [11] (cf. Fig.2).

### 3. Communities with multiple levels of reputation

System (2) can be directly generalized to the case with more than three levels of reputation. Let $r_k = \Delta k$, $\Delta = 1/L$, $k = 0, \cdots, L$. The equations for $R_k$, $0 \leq R_k \leq 1$, $\sum_{k=0}^{L} R_k = 1$, take the form
\[ \frac{dR_0}{dt} = -R_0 e_0 + R_1 (1 - e_1) \]
\[ \vdots \]
\[ \frac{dR_k}{dt} = R_{k-1} e_{k-1} - R_k + R_{k+1} (1 - e_{k+1}), \quad k = 1, \cdots, L - 1, \]
\[ \vdots \]
\[ \frac{dR_L}{dt} = R_{L-1} e_{L-1} - R_L (1 - e_L). \]

(11)

Definitions (9)-(10) are generalized in a straightforward way. As in the case of system (2), system (11) has two parameters: \( \alpha \) and \( \sigma \).

System (11) has a similar set of equilibria as system (2) for the community with three levels of reputation.

**Proposition 3.** The points of the line \((R_0, 0, \cdots, 0, 1 - R_0)\), \(R_0 \in [0, 1]\), are equilibria of system (11) for all \( \alpha, \sigma \in [0, 1] \). At these points all documents are evaluated correctly, \( p_c = 1 \).

**Proof 3.** For the proof we need to verify that on the line \((R_0, 0, \cdots, 0, 1 - R_0)\) all the right-hand sides of (11) are equal to zero. This is trivially true for all the equations but the first one, \( R_0 e_0 = 0 \), and the last one, \( R_L (1 - e_L) = 0 \). For these two equations the reasoning is the same as in the proof of Proposition 2.

The numerical results presented in the next section suggest that as in the case of system (2) these equilibria are stable in Lyapunov’s case for all \( \alpha, \sigma \in [0, 1] \). However they are not globally stable at least for some values of \( \alpha \) and \( \sigma \).

4. Computer simulations

In what follows \( L = 10 \) and \( r_k = 0.1k, \ k = 0, \cdots, 10 \). Since the dynamical system is ten–dimensional (only ten independent \( R_k \)s) the Poincaré-Bendixson theorem is not applicable. Figure 3 shows the dynamics of the system, typical for the majority of the initial values and values of \( \alpha \) and \( \sigma \). Two 'forces' pull \( R_k \)s in the opposite directions – the members either gain reputation until it reaches the maximal value of 100\%, or loose reputation until it reaches its minimal value of 0\%. The most common initial condition considered is supposed to model the 'birth' of the community, when it consists of new members only, and every new member has the same initial reputation that we set to be 60\%.

The dynamics of system (11) is more complex compared to system (2). For \( \alpha \) and \( \sigma \) near \( \alpha = 1, \sigma = -1 \) the system shows bi-stability – the equilibrium state depends on initial values of \( R_k \). The existence of an equilibrium different from those described in Proposition 3 is shown in Figure 4. The initial conditions are the same as in the case of Fig.3 but \( \alpha = 1, \sigma = -1 \). As the figure shows, the 'force' that makes members of the community
loose their reputation dominates and the equilibrium is attained with the majority of the members having low reputations; the most numerous category of the members have the reputation equal to 30%. At that equilibrium state \( p_c = 0.07 \) meaning the majority of the evaluators is almost always wrong. However, with the initial values \( R_k = 0, k \neq 7, \) and \( R_7 = 1, \) the system converges to a bimodal distribution \((0.0425, 0, \ldots, 0, 0.9575)\) with \( p_c = 1 \) (not shown). The probability of correct document evaluation \( p_c \) at the equilibrium states of system \((\ref{eq:system})\) and various values of \( \alpha \) and \( \sigma \) is shown in Fig\[3\] For all the values of \( \alpha \) and \( \sigma \) initial values are the same as in Figures \[3\] and \[4\].

Computations show that whether and how the system exhibits bi-stability depends on the dimension of the system, and parameters \( \alpha \) and \( \sigma \). In these computations, all the members have initial reputation equal to 0.6. In the case of the 5–dimensional system with the reputation values \( r = (0, 0.2, \ldots, 1) \) the system behaves as in Fig\[3\] and converges to an equilibrium state with \( p_c = 1 \). In the case of the 20–dimensional system with the reputation values \( r = (0, 0.05, \ldots, 1) \) the system behaves as in Fig\[4\] and converges to an equilibrium state with \( p_c = 0.04 \). Making all the initial reputations in the 20–dimensional system equal to 0.65 does not change the behavior quantitatively. However, the equilibrium state changes drastically when all initial reputations are set to 0.7. Then the system behaves as in Fig\[3\] with an equilibrium state at which \( p_c = 1 \).

Figure 3: Evolution of the model with ten levels of reputation to a bimodal distribution. \( R_k \) are the proportions of the members with reputations \( r_k = 0.1k, k = 0, \ldots, 10. \) Initially \( R_6 = 1 \) while all the other \( R_k = 0. \) Over time, all the members attain either zero \((k = 0)\) or 100% \((k = 10)\) reputation, and the probability of correct document evaluation \( p_c = 1 \). \( \alpha = \sigma = 0. \)

Figure 4: Evolution of the model with ten levels of reputation to a unimodal distribution. \( R_k \) are the proportions of the members with reputations \( r_k = 0.1k, k = 0, \ldots, 10. \) Initially \( t = 0, R_6 = 1 \) while all the other \( R_k = 0. \) At the equilibrium, most of the members have low reputations. The members in the most numerous category have reputation 30% \((k = 3)\). At this state the probability of correct document evaluation \( p_c = 0.07 \). \( \alpha = 1, \sigma = -1. \)
5. Community with a clique

In real life there are groups of agents with agendas. For those groups their agendas is the only thing that matters. Groups of this kind are called cliques here. Consider a system with one clique. Let $p_{cl}$ be the proportion of clique members. The remaining $1 - p_{cl}$ proportion of the community are ‘regular’ members. We extend the system with regular members, considered in the previous section, by adding $L + 1$ phase variables $C_k$, $k = 0, \ldots, L$, $0 \leq C_k \leq 1$, for the proportions of the clique members with reputations $r_k$.

Table 1 defines the probabilities of submitting documents of various types by regular and clique members. The documents can have nothing to do with the clique’s agenda (‘Generic’ or ‘g’), supporting the clique’s views (‘Clique’ or ‘c’), and contradicting the clique’s views (‘Anti-clique’ or ‘c’). Each of these three types of documents can be authentic (‘A’) or fake (‘F’). Parameter $p_\lambda$ stands for the proportion of the documents supporting the clique’s views. For simplicity, the proportion of the documents contradicting the clique’s views is set to $p_\lambda$ as well. The proportion of the documents having nothing to do with the clique’s agenda is $1 - 2p_\lambda$. Clique members submit only the documents that support the clique’s view. Probabilities $a_k$ of submitting authentic documents by the members with reputation $r_k$ are defined by (10). Parameter $\gamma$, $0 \leq \gamma \leq 1$, determines to what extent clique members are able to submit authentic documents; for small values of $\gamma$, even highly reputable clique members submit mostly fake documents.

Table 2 defines the probabilities of correct evaluation of particular documents by regular and clique members. The probabilities $c_k$ of correct document evaluation by the members with reputations $r_k$ are defined by (10). In contrast to regular members, who evaluate all the documents in the same way, clique members evaluate documents differently depending whether the documents support the clique’s views or not. They evaluate
Table 2: Probabilities of correct document evaluation by regular and clique members with reputations \( r_k, k = 0, \ldots, L \).

| Document type         | Regular members | Clique members |
|-----------------------|-----------------|----------------|
| Generic, authentic    | \( c_k \)       | 0.5            |
| Generic, fake         | \( c_k \)       | 0.5            |
| Clique, authentic     | \( c_k \)       | 1              |
| Clique, fake          | \( c_k \)       | 1              |
| Anti-clique, authentic| \( c_k \)       | 0              |
| Anti-clique, fake     | \( c_k \)       | 0              |

generic documents by flipping a coin. All the documents that support the clique’s views are evaluated as authentic. All the documents that contradict the clique’s views are evaluated as fake.

The differential equations for \( C_k \) are similar for those for \( R_k \):

\[
\begin{align*}
\frac{dR_0}{dt} &= -R_0 e_0^{(r)} + R_1 (1 - e_1^{(r)}) \\
\vdots \\
\frac{dR_k}{dt} &= R_{k-1} e_{k-1}^{(r)} - R_k + R_{k+1} (1 - e_{k+1}^{(r)}), \quad k = 1, \ldots, L - 1, \\
\vdots \\
\frac{dR_L}{dt} &= R_{L-1} e_{L-1}^{(r)} - R_L (1 - e_L^{(r)}) \\
\frac{dC_0}{dt} &= -C_0 e_0^{(c)} + C_1 (1 - e_1^{(c)}) \\
\vdots \\
\frac{dC_k}{dt} &= C_{k-1} e_{k-1}^{(c)} - C_k + C_{k+1} (1 - e_{k+1}^{(c)}), \quad k = 1, \ldots, L - 1, \\
\vdots \\
\frac{dC_L}{dt} &= C_{L-1} e_{L-1}^{(c)} - C_L (1 - e_L^{(c)}).
\end{align*}
\]

(12)

In (12), \( e_k^{(r)}, 0 \leq k \leq L \), stand for the probability that the documents submitted by the regular members with the reputation \( r_k \) are evaluated as authentic. Similarly, \( e_k^{(c)}, 0 \leq k \leq L \), stand for the probabilities that the documents, submitted by clique members with the reputation \( r_k \), are evaluated as authentic. As in (1) and (11) the summation of all the equations in (12) shows that the sum of all \( R_k \) does not change over time and is always equal to 1 – \( p_{cl} \), and the sum of all \( C_k \) does not change over time, and is always equal to \( p_{cl} \).

In the model of a community without cliques, the probability \( e_k \) of evaluating documents as authentic is equal to the sum of the probabilities that authentic and fake documents submitted by the members with reputation \( r_k \) are evaluated as authentic. Similarly, in the model with a clique we need to specify the probabilities of evaluating possible types of documents as authentic. Following Table 1 we obtain for regular agents...
\[ e_k^{(r)} = (1 - 2p_\lambda)a_k p_c(g, A) + (1 - 2p_\lambda)(1 - a_k)p_m(g, F) \\
+ p_\lambda a_k p_c(c, A) + p_\lambda(1 - a_k)p_m(c, F) \\
+ p_\lambda a_k p_c(\bar{c}, A) + p_\lambda(1 - a_k)p_m(\bar{c}, F), \quad k = 0, \ldots, L. \tag{13} \]

Every term is a product of three probabilities. In the first term, these are the probabilities that the document is generic/authentic/correctly evaluated as authentic: \(1 - 2p_\lambda/a_k/p_c(g, A)\). In the second term, these are the probabilities that the document is generic/fake/mistakenly evaluated as authentic: \(1 - 2p_\lambda/1 - a_k/p_m(g, F)\). In the third term, these are the probabilities that the document supports the clique’s views/authentic/correctly evaluated as authentic: \(p_\lambda/a_k/p_c(c, A)\). In the fourth term, these are the probabilities that the document supports the clique’s views/fake/mistakenly evaluated as authentic: \(p_\lambda/1 - a_k/p_m(c, F)\). In the fifth term, these are the probabilities that the document contradicts the clique’s views/authentic/correctly evaluated as authentic: \(p_\lambda/a_k/p_c(\bar{c}, A)\). Finally, in the sixth term, these are the probabilities that the document contradicts the clique’s views/fake/mistakenly evaluated as authentic: \(p_\lambda/1 - a_k/p_m(\bar{c}, F)\).

Similarly,
\[ e_k^{(c)} = \gamma a_k p_c(c, A) + (1 - \gamma a_k)p_m(c, F), \quad k = 0, \ldots, L. \tag{14} \]

The probability of correct evaluation, \(p_c\), and the probability of false evaluation, \(p_m\) in (13) and (14) depend on the corresponding probabilities for individual evaluators (cf. (11)). Namely,
\[ p_c(g, A) = p_{c,ind}(g, A)(3 - 2p_{c,ind}(g, A)), \quad p_c(g, F) = p_{c,ind}(g, F)(3 - 2p_{c,ind}(g, F)) \]
\[ p_c(c, A) = p_{c,ind}(c, A)(3 - 2p_{c,ind}(c, A)), \quad p_c(c, F) = p_{c,ind}(c, F)(3 - 2p_{c,ind}(c, F)), \tag{15} \]
\[ p_c(\bar{c}, A) = p_{c,ind}(\bar{c}, A)(3 - 2p_{c,ind}(\bar{c}, A)), \quad p_c(\bar{c}, F) = p_{c,ind}(\bar{c}, F)(3 - 2p_{c,ind}(\bar{c}, F)), \]

where, following (6) and Table 2,
\[ p_{c,ind}(g, A) = \sum_{k=0}^L s_k^{(r)} c_k + 0.5 \sum_{k=0}^L s_k^{(c)} c_k + 0.5 \sum_{k=0}^L s_k^{(r)} (1 - c_k), \quad p_{m,ind}(g, F) = \sum_{k=0}^L s_k^{(r)} (1 - c_k) + 0.5 \sum_{k=0}^L s_k^{(c)} c_k, \]
\[ p_{c,ind}(c, A) = \sum_{k=0}^L s_k^{(r)} c_k + \sum_{k=0}^L s_k^{(c)} c_k, \quad p_{m,ind}(c, F) = \sum_{k=0}^L s_k^{(r)} (1 - c_k) + \sum_{k=0}^L s_k^{(c)} c_k, \tag{16} \]
\[ p_{c,ind}(\bar{c}, A) = \sum_{k=0}^L s_k^{(r)} c_k, \quad p_{m,ind}(\bar{c}, F) = \sum_{k=0}^L s_k^{(r)} (1 - c_k). \]

In (10), the probabilities of correct evaluation \(c_k\) are defined as in (10).

The probability of selecting a regular member with a reputation \(r_k\)
\[ s_k^{(r)} = \frac{r_k R_k}{\sum_{i=0}^L r_i (R_i + C_i)}. \tag{17} \]

and, similarly, the probability of selecting a clique member with a reputation \(r_k\)
\[ s_k^{(c)} = \frac{r_k C_k}{\sum_{i=0}^L r_i (R_i + C_i)}. \tag{18} \]

The main characteristic of the system’s functionality is the probability \(p_c\) of correct evaluation of a document. All three types of documents have to be taken into account: generic, ‘g’, supporting the clique’s views, ‘c’, and contradicting the clique’s views, ‘\(\bar{c}\)’:
\[ p_c = Prob(g, A)p_c(g, A) + Prob(g, F)(1 - p_m(g, F)) \\
+ Prob(c, A)p_c(c, A) + Prob(c, F)(1 - p_m(c, F)) \\
+ Prob(\bar{c}, A)p_c(\bar{c}, A) + Prob(\bar{c}, F)(1 - p_m(\bar{c}, F)). \tag{19} \]

Here, the probabilities of authentic, ‘A’, or fake, ‘F’, documents of types \(g, c, \) and \(\bar{c}\) are
\[
\begin{align*}
\text{Prob}(g, A) &= (1 - 2p_\lambda) \sum_{k=0}^{L} R_k a_k, \\
\text{Prob}(g, F) &= (1 - 2p_\lambda) \sum_{k=0}^{L} R_k (1 - a_k), \\
\text{Prob}(c, A) &= p_\lambda \sum_{k=0}^{L} R_k a_k + \sum_{k=0}^{L} C_k \gamma a_k, \\
\text{Prob}(c, F) &= p_\lambda \sum_{k=0}^{L} R_k (1 - a_k) + \sum_{k=0}^{L} C_k (1 - \gamma a_k), \\
\text{Prob}(\bar{c}, A) &= p_\lambda \sum_{k=0}^{L} R_k a_k \\
\text{Prob}(\bar{c}, F) &= p_\lambda \sum_{k=0}^{L} R_k (1 - a_k).
\end{align*}
\] (20)

In summary, the model of a community with a clique has three parameters, additional to the parameters of the model (11) of a community without cliques: \( p_{cl} \), the proportion of the clique in the community, \( p_\lambda \), the proportion of documents matching the clique’s agenda, and \( \gamma \), the factor that stands for decreasing the probability of submitting authentic documents by clique members.

System (12), similar to (11), has easy-to-guess equilibria at which \( p_c = 1 \).

**Proposition 4.** For all admissible values of the parameters system (12) has equilibria at which \( R_L = 1 - p_{cl} - R_0 \), \( R_0 \in [0, 1 - p_{cl}] \), \( C_0 = p_{cl} \) and all the other phase variables are equal to zero. At the equilibria \( p_c = 1 \).

**Proof 4.** Substitution of the equilibrium values of \( R_k \) and \( C_k \) into (17) and (18) gives \( s^{(r)}_L = 1 \) while all the other \( s^{(r)}_k = 0 \) and \( s^{(c)}_k = 0 \). Then (16) yields \( p_{c,\text{ind}}(\cdot, A) = 1 \) and \( p_{m,\text{ind}}(\cdot, F) = 0 \). Next, (13) and (14) imply \( e^{(r)}_k = a_k \) and \( e^{(c)}_k = \gamma a_k \), meaning in particular, \( e^{(r)}_L = 1 \) and \( e^{(c)}_0 = 0 \). These values and the putative equilibrium values of the phase variables make all the right-hand sides of (11) equal to zero. Thus it is indeed an equilibrium. Similarly, it is directly verified that at the points of equilibria \( p_c = 1 \).

![Figure 6: Probability of correct document evaluation \( p_c \) in the model of a community with a clique. At \( t = 0 \), \( R_0 = 1 - p_{cl} \), \( C_0 = p_{cl} \) while all the other \( R_k \) and \( C_k \) are equal to zero. Documents are evaluated correctly, \( p_c = 1 \), only when the relative size of the clique \( p_{cl} \) is small. \( p_\lambda = 0.01, \alpha = \sigma = 0 \).](image)

Figure 6 shows the dependency of the probability of correct document evaluation \( p_c \) on various \( \gamma \) and \( p_{cl} \) given the proportion of documents matching the clique’s agenda \( p_\lambda = 0.01, \alpha = \sigma = 0 \), and \( R_0(0) = 1 - p_{cl} \), \( C_0(0) = p_{cl} \), while all the other \( R_k(0) \) and \( C_k(0) \) are equal to zero.
Consider two extreme cases shown in the figure, when there is no clique and when there are no regular agents. In the absence of a clique, \( p_{cl} = 0 \), the equations for a community with a clique (12) turn into the equations for a community without a clique (11). The value of \( \gamma \) becomes irrelevant, and \( p_{c} = 1 \), as in Fig. 5. In the absence of regular elements, \( p_{cl} = 1 \), the dynamics of the system is very simple. All documents, submitted by clique members, are evaluated by clique members only and therefore are judged as authentic. The reputations of all the members only increase. Eventually, all the members have the maximal reputation, \( r_{L} = 1 \). At this point, for all values of other parameters, \( \text{Prob}(c, A) = \gamma, p_{c}(c, A) = 1, p_{m}(c, F) = 1, \) and therefore \( p_{c} = \gamma \) (Fig. 6).

In general, the system sets to the states with perfect evaluation, \( p_{c} = 1 \), whenever the clique size \( p_{cl} \) does not exceed 20% of the whole community and \( \gamma \leq 0.7 \). When \( \alpha \) and \( \sigma \) both increase, \( p_{c} = 1 \) for greater clique sizes (not shown).

In the simulations above, the proportion \( p_{\lambda} \) of documents, related to the clique, was small, \( p_{\lambda} = 0.01 \). The equilibria with \( p_{c} = 1 \) from proposition 4 should be independent of \( p_{\lambda} \) according to the proposition’s proof. Figure 7 shows this is indeed the case for \( \alpha = \sigma = 0 \), and \( \gamma = 0.5 \). It shows also that the states with the probability of correct document evaluation \( p_{c} \) less than one depend on \( p_{\lambda} \) only a little.

![Figure 7: Probability of correct document evaluation \( p_{c} \) in the equilibrium states weakly depends on the proportion \( p_{\lambda} \) of documents, related to the clique’s agenda. At \( t = 0 \), \( R_{k} = 1 - p_{cl} \), \( C_{k} = p_{cl} \), while all the other \( R_{k} \) and \( C_{k} \) are equal to 0. \( \alpha = \sigma = 0, \gamma = 0.5 \).](image)

6. Two antagonistic cliques

Cliques often form antagonistic pairs. In such a pair, the agendas of a clique and an ‘anti–clique’ are diametrically opposite. The antagonism determines the probabilities of submission and correct evaluation of documents (Tables 3 and 4). All the parameters in the tables play the same role as in the model with one clique.

Table 3 defines the probabilities of submitting authentic and fake documents by regular and clique members. For example, according to the first line, regular member\( S \) with a reputation \( r_{k} \) submit authentic generic documents with the probability \( (1 - 2p_{\lambda})a_{k} \) while members of the cliques never submit such documents. The second line shows that the probability that regular members with a reputation \( r_{k} \) submit fake generic documents is equal to \( (1 - 2p_{\lambda})(1 - a_{k}) \). Cliques members do not submit fake generic documents, and so on.

Table 4 defines the probabilities of correct evaluation of particular documents by regular and clique members. The probabilities \( c_{k} \) of correct evaluation of documents by members with reputations \( r_{k} \) are defined by (10). As the table shows, it is assumed that the members of both cliques, when selected to be evaluators, don’t care about generic documents that not related to their agenda; they evaluate those documents by flipping a coin. All the documents that support the position of one clique the members of the antagonistic clique evaluate as fake. All the documents that contradict the position of one clique the members of the antagonistic clique evaluated as authentic.

Let \( Q_{k}, k = 0, \ldots, L, \) be the proportions of the members of the anti-clique with reputations \( r_{k} \). The equations for \( Q_{k} \) have the same form as for regular members and members of the clique (see 11) since the rules of the community are the same for all the members.
Table 3: Probabilities of submitting various types of documents by regular, clique 1, and clique 2 members with the reputations $r_k$, $k = 0, \cdots, L$.

| Document type          | Regular members | Clique 1 members | Clique 2 members |
|------------------------|-----------------|------------------|------------------|
| Generic, authentic     | $(1 - 2p_\lambda)a_k$ | 0                | 0                |
| Generic, fake          | $(1 - 2p_\lambda)(1 - a_k)$ | 0                | 0                |
| Clique, authentic      | $p_\lambda a_k$ | $\gamma a_k$     | 0                |
| Clique, fake           | $p_\lambda(1 - a_k)$ | $1 - \gamma a_k$ | 0                |
| Anti-clique, authentic | $p_\lambda a_k$ | 0                | $\gamma a_k$     |
| Anti-clique, fake      | $p_\lambda(1 - a_k)$ | 0                | $1 - \gamma a_k$ |

Table 4: Probabilities of correct evaluation of various types of documents by regular and clique members with reputation $r_k$, $k = 0, \cdots, L$.

| Document type          | Regular members | Clique 1 members | Clique 2 members |
|------------------------|-----------------|------------------|------------------|
| Generic, authentic     | $c_k$           | 0.5              | 0.5              |
| Generic, fake          | $c_k$           | 0.5              | 0.5              |
| Clique, authentic      | $c_k$           | 1                | 0                |
| Clique, fake           | $c_k$           | 1                | 0                |
| Anti-clique, authentic | $c_k$           | 0                | 1                |
| Anti-clique, fake      | $c_k$           | 0                | 1                |
\[
\frac{dR_0}{dt} = -R_0 e^{(r)}_0 + R_1 (1 - e^{(r)}_1)
\]
\[
\vdots
\]
\[
\frac{dR_k}{dt} = R_{k-1} e^{(r)}_{k-1} - R_k + R_{k+1} (1 - e^{(r)}_{k+1})
\]
\[
\vdots
\]
\[
\frac{dR_L}{dt} = R_{L-1} e^{(r)}_{L-1} - R_L (1 - e^{(r)}_L)
\]
\[
\frac{dC_0}{dt} = -C_0 e^{(c)}_0 + C_1 (1 - e^{(c)}_1)
\]
\[
\vdots
\]
\[
\frac{dC_k}{dt} = C_{k-1} e^{(c)}_{k-1} - C_k + C_{k+1} (1 - e^{(c)}_{k+1})
\]
\[
\vdots
\]
\[
\frac{dC_L}{dt} = C_{L-1} e^{(c)}_{L-1} - C_L (1 - e^{(c)}_L)
\]
\[
\frac{dQ_0}{dt} = -Q_0 e^{(q)}_0 + Q_1 (1 - e^{(q)}_1)
\]
\[
\vdots
\]
\[
\frac{dQ_k}{dt} = Q_{k-1} e^{(q)}_{k-1} - Q_k + F_{k+1} (1 - e^{(q)}_{k+1})
\]
\[
\vdots
\]
\[
\frac{dQ_L}{dt} = Q_{L-1} e^{(q)}_{L-1} - Q_L (1 - e^{(q)}_L), \quad k = 1, \ldots , L - 1.
\]

In (21), \(e^{(r)}_k\) and \(e^{(c)}_k\) have the same meaning as in (21). Similarly, \(e^{(q)}_k\), \(0 \leq k \leq L\), stand for the probability that the documents, submitted by anti-clique members with a reputation \(r_k\), are evaluated as authentic. It is readily verified that the sum of all \(Q_k\) does not change over time. We set the sum to be equal to \(p_{cl2}\). Similarly, the sum of all \(C_k\) does not change with time. It is set to \(p_{cl1}\). And finally, the sum of all \(R_k\) is always equal to \(1 - p_{cl1} - p_{cl2}\).

System (21) has equilibria similar to those of system (12).

**Proposition 5.** For all admissible values of the parameters system (11) has equilibria at which \(R_L = 1 - p_{cl1} - p_{cl2} - R_0, R_0 \in [0, 1 - p_{cl1} - p_{cl2}], C_0 = p_{cl1}, Q_0 = p_{cl2}\), and all the other phase variables are equal to zero. At the equilibria \(p_c = 1\).

In the model of a community without cliques, the probability \(e_k\) of evaluating documents as authentic was equal to the sum of the probabilities that documents of certain types submitted by the members with reputation \(r_k\) are evaluated as authentic. Similarly, we need to specify here the probabilities of evaluating possible types of documents as authentic. Following Table 1 for regular agents

\[
e^{(q)}_k = \gamma a_k p_c(q, A) + (1 - \gamma a_k) p_m(q, F), \quad k = 0, \ldots , L.
\]

(22)

The probabilities of correct evaluation, \(p_c\), and false evaluation, \(p_m\), in (13) and (14) depend on the corresponding probabilities for individual evaluator (cf. (4)). For example,

\[
p_c(q, A) = p^2_{c,ind}(g, A)(3 - p_{c,ind}(g, A)).
\]

Following (6) and Table 2
\[ p_{c,\text{ind}}(g, A) = \sum_{k=0}^{L} \left( s_k(r) c_k + 0.5 \sum_{k=0}^{L} (s_k(c) + s_k(q)) \right), \]
\[ p_{m,\text{ind}}(g, F) = \sum_{k=0}^{L} \left( s_k(r) (1 - c_k) + 0.5 \sum_{k=0}^{L} (s_k(c) + s_k(q)) \right), \]
\[ p_{c,\text{ind}}(c, A) = \sum_{k=0}^{L} s_k(r) c_k + \sum_{k=0}^{L} s_k(c), \]
\[ p_{m,\text{ind}}(c, F) = \sum_{k=0}^{L} s_k(r) (1 - c_k) + \sum_{k=0}^{L} s_k(c), \]
\[ p_{c,\text{ind}}(\bar{c}, A) = \sum_{k=0}^{L} s_k(r) c_k + \sum_{k=0}^{L} s_k(q), \]
\[ p_{m,\text{ind}}(\bar{c}, F) = \sum_{k=0}^{L} s_k(r) (1 - c_k) + \sum_{k=0}^{L} s_k(q). \]

Here, \( c_k \) are defined as in (10).

The probability of selecting a regular member with a reputation \( r_k \)
\[ s_k(r) = \frac{r_k R_k}{\sum_{i=0}^{L} r_i (R_i + C_i + Q_i)}, \]
and, similarly, the probability of selecting a clique member with a reputation \( r_k \)
\[ s_k(c) = \frac{r_k C_k}{\sum_{i=0}^{L} r_i (R_i + C_i + Q_i)}, \]
and an anti-clique member with a reputation \( r_k \)
\[ s_k(q) = \frac{r_k Q_k}{\sum_{i=0}^{L} r_i (R_i + C_i + Q_i)}. \]

The formula for \( p_c \) is the same as in the case of one clique \[ (23) \]. However, some of the terms change:

\[ \text{Prob}(g, A) = (1 - 2p_\lambda) \sum_{k=0}^{L} R_k a_k, \]
\[ \text{Prob}(g, F) = (1 - 2p_\lambda) \sum_{k=0}^{L} R_k (1 - a_k), \]
\[ \text{Prob}(c, A) = p_\lambda \sum_{k=0}^{L} R_k a_k + \sum_{k=0}^{L} C_k \gamma a_k, \]
\[ \text{Prob}(c, F) = p_\lambda \sum_{k=0}^{L} R_k (1 - a_k) + \sum_{k=0}^{L} C_k (1 - \gamma a_k), \]
\[ \text{Prob}(\bar{c}, A) = p_\lambda \sum_{k=0}^{L} R_k a_k + \sum_{k=0}^{L} Q_k a_k, \]
\[ \text{Prob}(\bar{c}, F) = p_\lambda \sum_{k=0}^{L} R_k (1 - a_k) + \sum_{k=0}^{L} Q_k (1 - \gamma a_k). \]

In summary, the system with two antagonistic cliques has the following parameters: \( p_{cl1} \), the relative size of the first clique, \( p_{cl2} \), the relative size of the antagonistic clique, \( p_\lambda \), the proportion of documents matching the
clique’s agenda, and \( \gamma, 0 \leq \gamma \leq 1 \), the factor that decreases the probabilities of submitting authentic documents. For simplicity in what follows it is assumed that both cliques have the same size: \( p_{cl} = p_{cl1} = p_{cl2} \), \( p_{cl} \leq 0.5 \). When \( p_{cl} = 0.5 \) there are no regular members in the community, \( 1 - 2p_{cl} = 0 \).

The first experiment was to explore whether the presence of an anti-clique improves the quality of document evaluation. The experiment shows that the improvement does take place but is not large. Figure 8 shows a typical behavior of the probability of correct document evaluation \( p_c \) for various values of \( p_{cl} \) and \( \gamma \). In the case, shown in the figure, \( L = 10, \alpha = \sigma = 0, at \ t = 0, R_6 = 0.4, C_6 = 0.3, \) and \( Q_6 = 0.3 \) and all the other \( R_k, C_k, Q_k \) were equal to zero. Comparison with the behavior of the community with one clique (Fig.6) shows that the region of the parameters \( p_{cl} \) and \( \gamma \), for which \( p_c = 1 \), in the case of two cliques is greater.

In accord with Proposition 5 in the states with \( p_c = 1, R_{10} = 1 - 2p_{cl}, C_0 = p_{cl}, Q_0 = p_{cl} \). The system pushes regular agents into just two categories, with zero and 100% reputation. All clique members end up with zero reputation. This type of system evolution is shown for \( p_{cl} = 0.3 \) and \( \gamma = 0.5 \) in Fig.9. These values of \( p_{cl} \) and \( \gamma \) correspond to the left filled circle in Fig.8.

When \( p_{cl} = 0.3 \) and \( \gamma = 1 \) (the evolution of the system is different (these values of \( p_{cl} \) and \( \gamma \) correspond to the right filled circle in Fig.8). In the corresponding steady state there are clique members with non-zero and even 100% reputation (Fig.10). These members can be selected to serve as evaluators. Because of that there are many mistakes in document evaluation, \( p_c \approx 0.7 \).
Variables $P_k$ represent all types of members: a) regular members, $R_k = P_k$, b) members of the first clique $C_k = P_{k+11}$, and c) members of the second (anti-) clique, $Q_k = P_{k+22}$, $k = 0, \cdots, 10$. At $t = 0$, $R_0 = 0.4$, $C_0 = 0.3$, and $Q_0 = 0.3$, while all the other phase variables are equal to zero. Regular members have either 0% or 100% reputation. $\alpha = 0$, $\sigma = 0$, $p_{cl} = 0.3$, $\gamma = 1$.

7. Related work

Most of the related work is done in the analysis of academic peer-reviewing and ecommerce. Academic peer-review models are often considerably more complex compared to the presented model which makes direct comparison with our model challenging. They include academic-specific factors such as the resources available to researchers [12], reputation of a journal [13], impact of rational referees, who might not have incentives to see high quality publications other than their own [14], and so on. Nevertheless, these studies have ideas that could be applied to the model considered. In particular in [15], documents are evaluated by highly qualified ‘experts’, who make few mistakes, and less qualified ‘readers’, who make more mistakes. It has been shown that the evaluations by large numbers of readers (up to 100) have better accuracy compared to the evaluations obtained by several experts. The presented model can be generalized to include not three but any number of evaluators to exploit this mechanism.

Peer-to-peer interactions and associated reputation and trust models is a field that has a lot of interest because of online commerce (e.g. eBay and Amazon) [6, 8, 9, 16–18] and sharing resources and information [7, 19]. The computational models of reputation and trust vary in how individuals interact and use the results of interactions. Most of this work is based on agent-based simulations. Theoretical results are rare. Our approach is most closely related to EigenTrust reputation management system in peer-to-peer networks [8]. In EigenTrust, peers accumulate global trust values from peer-to-peer interactions. Greater global trust values increase the chances of peers to evaluate others. In our model, not individuals but subsets of members who have the same value of reputation are considered. It makes the model more tractable analytically and computationally.

Mathematically, our model belongs to the class of models that describe interacting subpopulations. These models originate from the classic studies of Lotka and Volterra [20]. Such models have been used in numerous contexts, from epidemiology [21] to social group competition [22]. The system in [23] is particularly related to the present study. It describes interactions between altruists and defectors. As in the model considered here, the model in [23] has a continuum of stable equilibrium states that are not asymptotically stable.

8. Conclusions and future work

We describe here a model of a virtual community, in which fake and authentic documents, submitted by the community members, can be effectively evaluated by peers. The mechanism of evaluation is based on the intuition that judgment should be delegated to worthy. Similar mechanisms have been considered earlier [8], but as far as we know not in population models. Population models allowed us to explore possible dynamics of the community assuming large numbers of its members. In traditional agent-based simulations the sizes of communities are limited.

Can a community modeled in the present study be implemented in the real world? It depends on the extent to which the model assumptions are correct. Two assumptions can be met relatively easy. One is that all the
documents are verifiable. For example, if the description says that the photo depicts an event happening at some place in Syria then the image should have corresponding EXIF-encoded GPS coordinates. Similarly, if the description claims that the episode happened at a certain time then this claim should be supported by the timing information. In a real-world community an authentic document is also expected to be novel and not a copy or a slightly changed version of an already posted document. Current technologies allow for meeting these requirements. The second assumption is that the community members, selected to serve as evaluators, should really do it and do it quickly. This is a matter of the community discipline. Some journals, for example from the Frontiers family, show such a discipline is possible to maintain.

We haven’t found data on how reputation effects the probability of submitting authentic documents and the probability to correctly evaluate documents of others. Currently, it is assumed that the members with zero reputation submit only fake documents and the members with 100% reputation submit only authentic documents and make no mistakes in evaluating documents of others. It is also assumed the probability of submitting authentic documents and the probability to correctly evaluate documents of others increase with the increase of reputation. These assumptions look intuitive, however it is important to put them in accord with data.

Future work should deeper explore mathematical properties of the model. Those include multi-stability and its dependency on the model parameters.

We have shown that the proposed model is flexible and can be refined in multiple ways. For example, earning reputation can be harder than losing it. It can be modeled by a transition from $R_k$ not to $R_{k-1}$ but to $R_{k-2}$. Instead of fixed functions of reputation that describe probabilities of submitting authentic documents and correctly evaluating documents of others one can consider various distributions, etc. Practical implementations of the proposed virtual community will determine the most important directions of future research.

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References

[1] C. Chen, K. Wu, V. Srinivasan, X. Zhang, Battling the internet water army: Detection of hidden paid posters, in: Advances in Social Networks Analysis and Mining (ASONAM), 2013 IEEE/ACM International Conference on, IEEE, 2013, pp. 116–120.

[2] M. Balmas, When fake news becomes real, Communication Research 41 (3) (2014) 430–454.

[3] H. Allcott, M. Gentzkow, Social media and fake news in the 2016 election, Tech. rep., National Bureau of Economic Research (2017).

[4] B. Nyhan, J. Reifler, P. A. Ubel, The hazards of correcting myths about health care reform, Medical care 51 (2) (2013) 127–132.

[5] I. Pinyol, J. Sabater-Mir, Computational trust and reputation models for open multi-agent systems: A review, Artificial Intelligence Review 40 (1) (2013) 1–25.

[6] L. Xiong, L. Liu, A reputation-based trust model for peer-to-peer e-commerce communities, in: EEE International Conference on E-Commerce, 2003. CEC 2003., 2003, pp. 275–284.

[7] A. B. Can, B. Bhargava, Sort: A self-organizing trust model for peer-to-peer systems, IEEE transactions on dependable and secure computing 10 (1) (2013) 14–27.

[8] S. D. Kamvar, M. T. Schlosser, H. Garcia-Molina, The eigentrust algorithm for reputation management in p2p networks, in: WWW03, 2003, pp. 640–651.

[9] L. Mui, M. Mohtashemi, A. Halberstadt, A computational model of trust and reputation, in: System Sciences, 2002. HICSS. Proceedings of the 35th Annual Hawaii International Conference on, IEEE, 2002, pp. 2431–2439.
[10] P. Hartman, Ordinary Differential Equations, 2nd Edition, Classics in Applied Mathematics (book 38), Society for Industrial and Applied Mathematics, 2002.

[11] V. I. Arnold, Geometrical methods in the theory of ordinary differential equations, Vol. 250, Springer Science & Business Media, 2012.

[12] J. B. Cabotà, F. Grimaldo, F. Squazzoni, Do editors have a silver bullet? an agent-based model of peer review, in: European Conference on Modelling and Simulation (ECMS 2014), 2014, pp. 725–731.

[13] M. Kovanis, R. Porcher, P. Ravaud, L. Trinquart, Complex systems approach to scientific publication and peer-review system: development of an agent-based model calibrated with empirical journal data, Scientometrics 106 (2) (2016) 695–715.

[14] S. Thurner, R. Hanel, Peer-review in a world with rational scientists: Toward selection of the average., arXiv:1008.4324v1 (2011) 1–5.

[15] D. M. Herron, Is expert peer review obsolete? a model suggests that post-publication reader review may exceed the accuracy of traditional peer review, Surgical Endoscopy 26 (8) (2012) 2275–2280.

[16] F. Hendrikx, K. Bubendorfer, R. Chard, Reputation systems: A survey and taxonomy, Journal of Parallel and Distributed Computing 75 (2015) 184–197.

[17] P. Resnick, K. Kuwabara, R. Zeckhauser, E. Friedman, Reputation systems, Commun. ACM 43 (12) (2000) 45–48.

[18] G. Zacharia, P. Maes, Trust management through reputation mechanisms, Applied Artificial Intelligence 14 (9) (2000) 881–907.

[19] D. Chatzopoulos, M. Ahmadi, S. Kosta, P. Hui, Openrp: a reputation middleware for opportunistic crowd computing, IEEE Communications Magazine 54 (7) (2016) 115–121.

[20] F. Hoppensteadt, Predator-prey model, Scholarpedia 1 (10) (2006) 1563.

[21] H. W. Hethcote, The mathematics of infectious diseases, SIAM Review 42 (4) (2000) 599–653.

[22] D. M. Abrams, H. A. Yagle, R. J. Wiener, Dynamics of social group competition: Modeling the decline of religious affiliation, Phys. Rev. Lett. 107 (2011) 088701.

[23] H. Brandt, K. Sigmund, Indirect reciprocity, image scoring, and moral hazard, Proceedings of the National Academy of Sciences of the United States of America 102 (7) (2005) 2666–2670.