Familon emission by dense magnetized plasma

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Abstract

Emission of a familon caused by the processes $e^- \rightarrow e^- + \phi$, $e^- \rightarrow \mu^- + \phi$ in dense magnetized plasma is investigated in the model in which a familon have both direct and no direct coupling to leptons via plasmon. The process probabilities and the integral familon action on plasma are calculated. It is shown that the $P$ odd interference phenomenon in the process $e^- \rightarrow \mu^- + \phi$ leads to the familon force acting on plasma along the magnetic field.

The familon, the Nambu – Goldstone boson, associated with the spontaneous breakdown of a global family symmetry is of interest not only in theoretical aspect of elementary particle physics, but in some astrophysical and cosmology applications \cite{1}. In particular, through coupling to electron and photons, familon could give a contribution to the energy and momentum losses by stellar object. By this means, the investigations of the familon involving processes under extreme conditions, high matter densities and strong magnetic field, are important for an analysis of some astrophysical cataclysms such as a supernova explosion.

Here we study forbidden in vacuum processes of the familon cyclotron emission $e^- \rightarrow e^- + \phi$, transition $e^- \rightarrow \mu^- + \phi$ and their contributions into the energy losses by magnetized plasma. We consider the physical situation when the typical energy of the plasma electrons, $E$, is the largest physical parameter:

$$E^2 \gg eB \gg m_e^2.$$ (1)

The condition (1) corresponds to the relatively weak magnetic field, when plasma electrons occupy highest Landau levels. At the same time the magnetic field is still strong enough in comparison with the Schwinger value $B \gg B_e$, $B_e = m_e^2/e \simeq 4.41 \times 10^{13}$ G. Such extreme conditions: high density of matter $\sim 10^{14} g/cm^3$, large electron chemical potential $\mu \sim 500m_e$, strong magnetic field up to $B \sim 10^{17} G$ could exist, for example, in the core of the exploding supernova. Notice, that in an external magnetic field, the result of calculations depends not only on typical kinematical invariants like $m^2$ and $p^2$, but also on the field invariant

$$e^2(FF) = -2e^2B^2,$$

and dynamical field invariant

$$e^2(pFFp) = e^2B^2E^2\sin^2\theta,$$

\footnote{1 We use natural units in which $c = \hbar = 1$, $e > 0$ is the elementary charge.}
where \( p_\mu \) is the particle 4-momentum, \( F_{\mu\nu} \) is the tensor of the external magnetic field, \( \theta \) is the angle between the particle momentum \( p \) and the magnetic field direction. Inside the parentheses the tensor indices are contracted systematically, for example \((pF\bar{F}p) = p_\alpha F_{\alpha\beta}F_{\beta\nu}p_\nu\).

Thus the conditions (1) can be rewritten in the invariant form:

\[
e^2(pF\bar{F}p) \gg \left[e^2(FF)\right]^{3/2} \gg m_e^6.
\]

It is known, that the problem of studies of the quantum processes under the conditions (1) reduces to a calculation in the constant crossed field \([2]\). It is because that in the rest frame of a high energy electron, a relatively weak and smooth external electromagnetic field looks very close to the constant crossed field \((\vec{B} \perp \vec{E}, |\vec{B}| = |\vec{E}|)\), where \((FF) = (\bar{F}\bar{F}) = 0\), \(\bar{F}_{\mu\nu}\) is the dual tensor of the external field. So, the result depends actually on the dimensionless dynamical parameter \( \chi \) only

\[
\chi^2 = \frac{e^2(pF\bar{F}p)}{m_e^6}, \tag{2}
\]

where \( m \) is the mass of a particle.

### 1 Familon cyclotron emission.

The familon cyclotron emission by plasma electron has two possible channels shown in Fig.1. The process \( e^- \rightarrow e^- + \phi \) due to the direct familon – electron coupling (Fig.1a) can be described by the effective Lagrangian in the following form:

\[
L_{\phi e} = \frac{g_{\phi e}}{2m_e} \bar{\Psi}_e \gamma_\alpha \gamma_5 \Psi_e \partial_\alpha \Phi, \tag{3}
\]

where \( \Phi \) and \( \Psi_e \) are the familon and electron fields respectively, \( g_{\phi e} = 2m_e/F \), \( F \) is the family symmetry breaking scale. The astrophysical constraint gives \( g_{\phi e} < 1.4 \times 10^{-13} \) \((F > 7 \times 10^9 GeV)\) \([3]\).

It is important that magnetic field induces a new effective interaction between a familon and a photon of the type:

\[
L_{\phi\gamma} = g_{\phi\gamma} (\partial_\alpha A_\beta) \tilde{F}^{\alpha\beta} \Phi, \tag{4}
\]

where \( A_\mu \) is the four potential of the quantized electromagnetic field, \( g_{\phi\gamma} \) is the effective familon – photon coupling constant in the presence of external field, which could be derive from the

\[\begin{align*}
\text{(a)} & \quad e^-(p) \quad \rightarrow \quad e^-(p') \\
\text{(b)} & \quad e^-(p) \quad \rightarrow \quad e^-(p')
\end{align*}\]

Figure 1: The Feinman diagrams for the familon cyclotron emission by plasma electron in the presence of a magnetic field.
Figure 2: The effective familon-photon interaction in an external magnetic field

diagram of Fig.2, where double lines indicate that the influence of the external field is taken into account in the propagators of virtual fermions $f$. This constant $g_{\phi\gamma}$ could be extracted from the paper [4] where the effective field–induced interaction of a pseudoscalar particle with photons was investigated. Notice that if the magnetic field is not so strong ($B \ll B_\mu = m_\mu^2/e \simeq 1.8 \times 10^{18} G$, $m_\mu$ is the mass of the muon), only the virtual electron gives a contribution to the $g_{\phi\gamma}$, so for the $g_{\phi\gamma}$ coupling one can obtain the following result:

$$g_{\phi\gamma} = \frac{\alpha}{\pi} \frac{g_{\phi e}}{m_e}.$$ 

It should be stressed that the familon–photon interaction becomes possible in the presence of external magnetic field only. This is due to the fact that familon does not have both anomalous $\Phi G\tilde{G}$ and $\Phi F\tilde{F}$ coupling in vacuum ($G$ and $F$ are gluonic and electromagnetic fields, respectively). As a result, the emission of a familon via plasmon intermediate state (Fig.1b) becomes possible in the presence of both components of active medium: plasma and magnetic field.

The $S$–matrix element of the process $e^- \rightarrow e^- + \phi$ is:

$$S = \frac{1}{F\sqrt{2\omega V}} \left[ \frac{2\alpha e}{\pi} (q F G L^{\text{(em)}}) - (q I^{(5)}) \right],$$

where $I^{\text{(em)}} = \int d^4x \bar{\psi}_e(p', x) \gamma_\mu \psi_e(p, x) e^{iqx}$, $I^{(5)} = \int d^4x \bar{\psi}_e(p', x) \gamma_\mu \gamma_5 \psi_e(p, x) e^{iqx}$, $\psi_e$ is the solution of the Dirac equation in the constant crossed field $\mathbf{B}$, $q^\alpha = (\omega, \mathbf{q})$ is the familon 4-momentum, while $p^\alpha = (E, \mathbf{p})$ and $p'^\alpha = (E', \mathbf{p}')$ are the four-momenta of the initial and final electrons, $G_{\alpha\beta}^L$ is the longitudinal plasmon propagator.

At first glance, the amplitude of the familon emission via plasmon contains the suppression associated with the fine structure constant $\alpha$. However, as the analysis show, the contribution of both familon emission channels into the process could be of the same order. The main motivation why the familon emission via plasmon intermediate state could be expected to be nonnegligible is that this channel has a resonant character at a particular energy of the emitted familon. It is provided by the fact that intermediate plasmon is longitudinal. Similarly to the axion–plasmon interplay [6] the familon and the longitudinal plasmon dispersion curves always cross for certain energy $\omega = \omega_0$. At the same time the contribution from the transverse intermediate plasmon turn out to be negligible small in the case of ultrarelativistic plasma electrons. The longitudinal plasmon propagator $G_{\alpha\beta}^L$ in the limit of weak magnetic field (1) can be written as

$$G_{\alpha\beta}^L \simeq \frac{l_\alpha l_\beta}{q^2 - \Pi^L}, \quad l_\alpha = \sqrt{\frac{q^2}{(uq)^2 - q^2}} \left( u_\alpha - \frac{(uq)}{q^2} q_\alpha \right).$$

Here $l_\alpha$ and $\Pi^L$ are the eigenvector and eigenvalue of the polarization operator corresponding to the longitudinal plasmon, respectively, $u_\alpha$ is the four-vector of the velocity of the medium.

In order to obtain the probability of the cyclotron familon emission by a plasma electron we need to integrate over the phase space of final particles taking into account the electron statistical factor.
The result of our calculations can be presented in the medium rest frame in the following form:

\[
W_{(e^{-}\rightarrow e^{-}\phi)} \simeq \frac{1}{2\pi^2 F^2 E} \int_{e^{-}\omega/E}^{E} \frac{(E\omega) d\omega}{e^{(E\omega/E + \omega)/T} + 1} \times \left\{ \frac{4\alpha^2(eB)^2}{3\pi} \cos^2 \theta \right.
\]

\[
\left. \frac{\omega^2}{\omega^2(1 - \Pi L/q^2)^2} + 3^{1/6} \Gamma(2/3) m_e^2 \left( \frac{\omega^2 eB \sin \theta}{E^2 (E - \omega)^2} \right)^{2/3} \right\},
\]

where \( \theta \) is the angle between the initial electron momentum \( \vec{p} \) and the magnetic field direction.

### 2 Transition \( e^{-} \rightarrow \mu^{-} + \phi \).

The phenomenon of fermion mixing gives rise to flavor – nondiagonal familon – fermion interaction. As a result, the processes with lepton number violation of the type \( e^{-} \rightarrow \mu^{-} + \phi \), \( \mu^{-} \rightarrow e^{-} + \phi \) becomes possible. In this section we investigate the transition \( e^{-} \rightarrow \mu^{-} + \phi \) as an additional channel of the familon emission by plasma electron (Fig. 3). This process was studied earlier [7] in the strong magnetic field limit under the condition of smallness of the difference \( (\mu - m_\mu) \), when \( \mu^2 - m^2_\mu \ll eB \), so the final muons are produced in the lowest Landau level only. In contrast to [7] we consider as an example the conditions in a supernova core with the relatively weak magnetic field, when \( \mu^2 - m^2_\mu \gg eB \), and therefore muons can occupy a great number of excited Landau levels.

The interaction of familon with electron and muon is described by effective Lagrangian in the form:

\[
L = \frac{1}{F} [\overline{\Psi}_\mu \gamma_\alpha (a + b \gamma_5) \Psi_e + \overline{\Psi}_e \gamma_\alpha (a + b \gamma_5) \Psi_\mu] \partial_\alpha \Phi,
\]

where \( \Psi_\mu \) is the muon field, \( a^2 + b^2 = 1 \).

The \( S \) – matrix element of the process \( e^{-} \rightarrow \mu^{-} + \phi \) can be obtained immediately from the Lagrangian (6) by means of substitution of solutions of the Dirac equation in the crossed field:

\[
S = \frac{-1}{\sqrt{2\omega \sqrt{F}}} \int d^4x \overline{\psi}_\mu (p', x) q(a + b \gamma_5) \psi_e (p, x) e^{i qx},
\]

where \( p'^\alpha = (E', \vec{p}') \), \( p^\alpha = (E, \vec{p}) \) are the muon and electron 4 – momenta respectively.

Under the conditions we consider the muon dynamical parameter \( \chi \), \( \chi^2 = e^2 (pFp) / m^6_\mu \), is rather small and in this case the result for the probability has a simple form:

\[
W_{(e^{-}\rightarrow \mu^{-}\phi)} = \frac{m_\mu eB}{18 \sqrt{3\pi} F^2} \sin \theta \ e^{-\sqrt{3}/\chi},
\]

Figure 3: The Feinman diagram for transition \( e^{-} \rightarrow \mu^{-} + \phi \) in the presence of magnetized plasma.
At first glance the probability (7) is exponentially small in comparison with (5), however, as it will be shown below, the plasma energy loss via the familon emission caused by cyclotron process \( e^- \rightarrow e^- + \phi \) and transition \( e^- \rightarrow \mu^- + \phi \) turn out to be of the same order. This is due to the fact that expression (7) in contrast to (5) does not contain a suppression associated mainly with the smallness of electron mass \( m_e \) or fine structure constant \( \alpha \).

3 Familon emissivity.

In the studies of the familon involving processes, not only probabilities but also the integral familon action on plasma is of practical interest for astrophysics. To illustrate possible astrophysical applications of the result obtained we estimate the energy losses of plasma via the familon emission:

\[
\dot{\varepsilon} = \frac{1}{(2\pi)^3} \int \frac{dW}{d\omega} \omega d\omega d^3p \frac{d^3p}{e(E-\mu)/T + 1},
\]

where \( \dot{\varepsilon} \) is volume density of the plasma energy losses per unit time, \( \omega \) is the energy of the emitted familon, \( dW/d\omega \) is the differential probability of the process considered.

The volume density of the plasma energy losses caused by the familon emission can be presented as the sum of two contribution:

\[
\dot{\varepsilon}_\phi = \dot{\varepsilon}_{e^-\rightarrow e^-+\phi} + \dot{\varepsilon}_{e^-\rightarrow \mu^-+\phi}.
\]

Upon integrating over the phase space of initial electron and familon energy in the case of degenerate plasma, \( \mu \gg T \), for the cyclotron process contribution we find:

\[
\dot{\varepsilon}_{e^-\rightarrow e^-+\phi} \simeq \frac{1}{\pi^4 F^2} \left[ c_f m_e^2 \left( \frac{eB}{\mu} \right)^{2/3} T^{13/3} + \frac{\alpha}{12} \frac{(eB)^2 \omega_0^3}{(e\omega_0/T - 1)} \right],
\]

where

\[
c_f = \frac{14}{81} \left( \frac{3}{4} \right)^{1/6} \left( \frac{1}{3} \right) \zeta \left( \frac{13}{3} \right) \simeq 3.38,
\]

and \( \omega_0 \) is the energy where the familon and longitudinal plasmon dispersion curves cross [6]:

\[
\omega_0^2 \simeq \frac{4\alpha}{\pi} \mu^2 \left( \ln \frac{2 \mu}{m_e} - 1 \right).
\]

The first term in (9) describes the contribution from the process with direct familon – electron interaction and the second term defines the familon emission via the longitudinal plasmon intermediate state.

For a contribution of the process \( e^- \rightarrow \mu^- + \phi \) to the plasma energy losses we obtain the following result:

\[
\dot{\varepsilon}_{e^-\rightarrow \mu^-+\phi} \simeq \frac{\sqrt{2} m_e^4 \mu^3}{216 \pi^{5/2} F^2} I(y) y^{3/2},
\]

\[
I(y) = \int_0^\infty x^{7/2} \frac{e^{-y/x} dx}{e^x (x-1) + 1}, \quad y = \sqrt{3} \frac{m^2_\mu}{eB\mu}.
\]

The integral \( I(y) \) in (10) can be easily calculated in two limiting cases:

i) the case of zero temperature limit

\[
\dot{\varepsilon}_{e^-\rightarrow \mu^-+\phi} \simeq \frac{\sqrt{2} m_e^4 \mu^3}{216 \pi^{5/2} F^2} \frac{e^{-y}}{y^{3/2}},
\]
ii) relatively hot relativistic plasma \((T > \mu/y)\)

\[
\dot{\varepsilon}_{\text{e} \rightarrow \mu \phi} \simeq \frac{m_\mu^4 \mu^3}{216 \pi^2 F^2} \sqrt{2y} \left(\frac{T}{\mu}\right)^{5/2} e^{-y(2-1/1)/t}, \quad t = \sqrt{\frac{yT}{\mu}}.
\]

We estimate the familon emissivity under the conditions which could be realized in a supernova explosion. As an example we take \(\mu = 250\,\text{MeV}, T = 35\,\text{MeV}\) and \(B = 10^{17}\,\text{G}\):

\[
\dot{\varepsilon}_{\phi} \simeq 10^{27} \left[0.56(e \rightarrow e\phi) + 0.81(e \rightarrow e\phi)(\gamma) + 0.82(e \rightarrow \mu\phi)\right] \left(\frac{7 \times 10^9\,\text{GeV}}{F}\right)^2 \left(\frac{\text{erg}}{\text{cm}^3\text{sec}}\right).
\] (11)

It is seen from (11) that the contributions from process \(e^- \rightarrow \mu^- + \phi\) and familon cyclotron emission \(e^- \rightarrow e^- + \phi\) are of the same order for the parameters considered, while the familon luminosity \(L_\phi \sim 10^{46}\,\text{erg/sec}\) is much less than the neutrino luminosity \(L_\nu \sim 10^{52}\,\text{erg/sec}\) from supernova core during first few seconds after collapse.

It should be pointed that at the cooling stage when the temperature becomes of order of \(\text{MeV}\) the process \(e^- \rightarrow \mu^- + \phi\) begins to dominate over the cyclotron emission \(e^- \rightarrow e^- + \phi\):

\[
\dot{\varepsilon}_{\phi} \simeq \dot{\varepsilon}_{e \rightarrow \mu \phi} \simeq 0.92 \times 10^{25} \left(\frac{7 \times 10^9\,\text{GeV}}{F}\right)^2 \left(\frac{\text{erg}}{\text{cm}^3\text{sec}}\right)
\]

and could provide a competition with the neutrino energy losses at this stage, \(\dot{\varepsilon}_\nu \sim 10^{26}\,\text{erg/cm}^3\text{sec}\) \([1]\).

The another interesting feature of the processes considered is the asymmetry of familon emission:

\[
A = \frac{1}{\dot{\varepsilon}} \int \frac{q_3 d^3p}{(2\pi)^3} \frac{1}{e^{(E-\mu)/T} + 1} dW,
\]

where \(q_3\) is the component (parallel to field) of the familon momentum. Our calculations show that only the transition \(e^- \rightarrow \mu^- + \phi\) gives a contribution into this asymmetry which has very simple form:

\[
A \simeq \frac{ab eB}{3 m_\mu^2}.
\] (12)

\[ V, \text{km/sec} \]
\[ B, \text{Gauss} \]

\[ 2 \times 10^{16} \quad 6 \times 10^{16} \quad 10^{17} \]

Figure 4: The kick velocity as a function of the magnetic field strength.
We emphasize that asymmetry (12) is caused by the interference of the vector and axial-vector couplings in the effective Lagrangian (6).

As one can see, the dependence of $A$ on plasma parameters – the electron chemical potential $\mu$ and temperature $T$, totally cancelled. Remind, however, the result (12) was obtained under assumption that $\mu$ is the largest physical parameter of the task.

The asymmetry of familon emission leads to the familon force action on the magnetized plasma along the magnetic field which in turn leads to the kick velocity of a supernova remnant. The result of our estimation of supernova remnant kick velocity is presented in Fig.4.

As one can see from Fig.4, on the scale of the magnetic field up to $10^{17}$ Gauss the kick velocity does not exceed 100 km/sec. So, the familon emission asymmetry cannot solve the problem of proper velocity of the pulsars.

In conclusion, we have studied the familon emission by plasma electrons via the processes $e^- \rightarrow \mu^- + \phi$, $e^- \rightarrow e^- + \phi$. Due to very weak interaction of familon with matter the above processes could be important in astrophysics and cosmology. As a specific application of the results obtained we have calculated the plasma energy-momentum losses via the familon emission in a supernova explosion.

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References

[1] G.G. Raffelt, *Stars as Laboratories for Fundamental Physics* (University of Chicago Press, 1996).

[2] V.I.Ritus, *Tr.Fiz.Inst.im. P.N. Lebedeva, Akad. Nauk SSSR, Moskow* 168 (1986)

[3] Partial Datas Group (C. Groom et.al.) *Eur. Phys. J.*, 402 (2001).

[4] N.V. Mikheev, A.Ya. Parkhomenko and L.A. Vassilevskaya *Phys. Rev.* D60, 035001 (1999).

[5] V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii *Quantum electrodynamics* (Pergamon Press, Oxford, 1982).

[6] N.V. Mikheev, G.G. Rafelt and L.A. Vassilevskaya *Phys. Rev.* D58, 055008 (1998).

[7] A.V. Averin, A.V. Borisov and A.I. Studenikin *Phys. Lett.* B231, 280 (1989).