Recovering Coherence via Conditional Measurements

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(March 31, 2022)

We show that conditional measurements on atoms following their interaction with a resonant cavity field mode can be used to effectively counter the decoherence of Fock-state superpositions due to cavity leakage.

PACS numbers: 03.65.Bz, 89.70.+c, 42.50.Dv, 32.80.-t

I. INTRODUCTION

Decoherence of non-classical states of a quantum system via coupling to a reservoir is of fundamental interest, as it constitutes the mechanism that yields the classical limit of the system dynamics \(^{1}\). Recently it has also become a topic of great applied importance, because it determines the feasibility of quantum information storage, encoding (encrypting) and computing \(^{2}\). In numerous current theoretical proposals, the irreversibility of decoherence processes in quantum computing is combatted by two generic means. One is the filtering out of the ensemble portion which has not decohered, i.e., has remained intact. This approach has been suggested for two-mode fields \(^{3}\), but not for single-mode cavity fields. The other means is encoding the state (qubit) by means of several ancillas, decoding the result after a certain time, checking the ancillas for error syndromes and correcting them \(^{4}\). Although the latter approach is in principle applicable to arbitrary errors, only extremely small error probabilities (per qubit or gate, per time step) can afford fault tolerant quantum computation \(^{5}\). Instead of the “high level” unitary transformation approach to error correction in quantum computing—which involves substantial overhead in qubits and gates—the countering of decoherence of stored quantum information, e.g., in between computation steps, may be achieved by a “low level” approach: applying simple physical manipulations to the quantum storage device, which take advantage of its specific physical realization. Such approach has been advocated recently \(^{6}\), relying on continuous monitoring of the dissipation channel for quantum jumps, with perfect photodetection efficiency, and on instantaneous feedback for the inversion of their effect.

The nature of quantum computing requires that decoherence be corrected without knowing which state is in error during the computation. There is, however, a simpler but still important problem: how to protect from decoherence the input states, prior to the onset of computation. Here we suggest a non-unitary approach to counter decoherence, which can be used to safely store quantum field states in dissipative cavities, in order to subsequently use them as \textit{apriori known} input in information processing or in signal transmission. The basic idea is to \textit{restore} the decohered field state by entangling it with an atom, and then projecting the entangled state onto a superposition of atomic eigenstates, whose phase and amplitudes are specifically tailored for the field state we wish to recreate. Such projection amounts to post-selection of the appropriate atomic state, i.e., to a conditional measurement (CM) \(^{7}\). The specific scheme we put forward is based on modification of our optimized CM strategy \(^{8}\) for cavity-mode state preparation by resonant interaction with atoms, in the Jaynes-Cummings (JC) model, followed by projection onto selected atomic states. In the present problem we set the initial (unspoilt) superposition of zero-photon up to \(N\)-photon states as our target state, and work in Liouville space instead of Hilbert space, so as to account for the state decoherence. The results demonstrate that a few \textit{highly-probable} CMs, in this simple model, can drastically reduce even a large error. One of our objectives is to find the optimal tradeoff between the CM probability and the error size, which grows in the course of dissipation.

The ability to \textit{approximately restore any mixture to any pure state} (in our \(N+1\)-dimensional Hilbert space) is the advantage of our post-selection CM approach, compared to the non-selective measurement (tracing) approach: Mixed states can only evolve into the special “cotangent” and “tangent” pure states by a large number of JC interactions with atoms initially prepared in superposition states (under the atomic excitation-trapping condition) followed by tracing over the atomic states \(^{9}\).

II. DECOHERENCE MINIMIZATION BY CONDITIONAL MEASUREMENTS

We consider a single-mode cavity in which the quantized electromagnetic field is initially prepared in a finite superposition of Fock states,

\[
|\psi(0)\rangle = \sum_{n=0}^{N} c_n |n\rangle .
\]  

(1)

To model the effect of dissipation we assume the cavity field to be coupled to a zero-temperature heat bath. The master equation describing such coupling, in the interaction picture, is
\[ \dot{\rho}_p = \gamma (2\bar{a}\rho_p \bar{a}^\dagger - \bar{a}^\dagger \bar{a} \rho_p - \rho_p \bar{a}^\dagger \bar{a}) , \]

where \( \rho_p = \rho_p(t) \) is the density matrix of the cavity field, \( \bar{a} \) and \( \bar{a}^\dagger \) are the annihilation and creation operators of the field, and \( \gamma \) is the damping constant of the cavity.

The solution of Eq. (3) after dissipation over time \( \bar{t} > 0 \) can be shown to have the form

\[ \rho_{n,m}(\bar{t}) = \sum_{k=0}^{\infty} \rho_{n+k,m+k}(0) \sqrt{\binom{n+k}{n} \binom{m+k}{m} (1 - e^{-2\gamma \bar{t}})^{k}} \]

\times \left( e^{-2\gamma \bar{t}} \right)^{n} (1 - e^{-2\gamma \bar{t}})^{k} , \]

written here in Fock basis, \( \rho_{n,m}(\bar{t}) = \langle n|\rho_p(\bar{t})|m \rangle \).

In order to recover the original state of the field we propose to apply an optimized CM (or a sequence thereof) to the cavity as follows: Using a classical field we prepare a two-level atom in a chosen superposition\(^9,12\)

\[ |\phi^{(i)}\rangle = \alpha^{(i)}|e\rangle + \beta^{(i)}|g\rangle \]

of its ground \( |g\rangle \) and excited \( |e\rangle \) states, and let it interact with the field for a time \( \tau \) by sending it through the cavity with controlled speed. The field-atom interaction is adequately described by the resonant Jaynes-Cummings (JC) model\(^13\). We assume the field-atom interaction time \( \tau \) to be much shorter than the cavity lifetime, \( \gamma \tau \ll 1 \), so that we may neglect dissipation during each CM. Upon exiting the cavity the atom is \textit{conditionally measured}, using a second classical field, to be in a state

\[ |\phi^{(f)}\rangle = \alpha^{(f)}|e\rangle + \beta^{(f)}|g\rangle , \]

differing in general from the initial atomic state \( |\phi^{(i)}\rangle \). This means that we post-select, using the same setup as in ref.\(^9\), the atomic superposition state\(^9\) which is \textit{correlated} to a cavity field state that is as close as possible to the original state\(^13\).

The effect the applied CM has on the cavity field is then calculated as follows: Initially, at the time the atom enters the cavity, the density matrix of the field-atom system is

\[ \rho_{FA}(\bar{t}) = \rho_p(\bar{t}) \otimes |\phi^{(i)}\rangle \langle \phi^{(i)}| . \]

It then evolves unitarily by the JC interaction of duration \( \tau \) into

\[ \rho_{FA}(\bar{t} + \tau) = \hat{U}(\tau) \rho_{FA}(\bar{t}) \hat{U}^\dagger(\tau) , \]

where \( \hat{U}(\tau) \) is the interaction picture evolution operator

\[ \hat{U}(\tau)|n\rangle|e\rangle = C_n|n\rangle|e\rangle - iS_n|n+1\rangle|g\rangle \]
\[ \hat{U}(\tau)|n\rangle|g\rangle = C_{n-1}|n\rangle|g\rangle - iS_{n-1}|n-1\rangle|e\rangle , \]

with \( C_n = \cos \left( \lambda \tau \sqrt{n+1} \right) \) and \( S_n = \sin \left( \lambda \tau \sqrt{n+1} \right) \), \( \lambda \) being the field-atom coupling constant (known as the vacuum Rabi frequency). Finally, the conditional measurement of the atom in the state \( |\phi^{(f)}\rangle \) results in a density matrix of the field given by

\[ \rho_p(\bar{t} + \tau) = \text{Tr}_A \left[ \rho_{FA}(\bar{t} + \tau)|\phi^{(f)}\rangle \langle \phi^{(f)}| \right] / P , \]

where

\[ P = \text{Tr}_p \text{Tr}_A \left[ \rho_{FA}(\bar{t} + \tau)|\phi^{(f)}\rangle \langle \phi^{(f)}| \right] \]

is the success probability of the CM. The explicit expressions for \( \rho_p(\bar{t} + \tau) \) and \( P \) are given in the Appendix for an initial superposition of \( |0\rangle \) and \( |1\rangle \) states.

To nearly recover the original state of the field, we use the dependence of \( \rho_p(\bar{t} + \tau) \) on the initial and final atomic states and the field-atom interaction time, choosing optimal parameters \( \alpha^{(i)}, \beta^{(i)}, \alpha^{(f)}, \beta^{(f)} \) and \( \tau \) such that

\[ \rho_p(\bar{t} + \tau) \approx \rho_p(0) \]

holds (see Appendix for an explicit form of this condition), along with high CM success probability\(^10\). These optimal CM parameters are found by minimizing the cost function\(^8\)

\[ G = \frac{d(\rho_p(\bar{t} + \tau), \rho_p(0))}{P r} , \]

where \( d \) is a distance function between two density matrices, defined as

\[ d(\rho^{(1)}, \rho^{(2)}) = \sqrt{\sum_{nm} (\rho_{nm}^{(1)} - \rho_{nm}^{(2)})^2} , \]

\( P \) is the CM success probability\(^10\), and the adjustable exponent \( r > 0 \) determines the relative importance of the two factors in \( G \). If this CM does not bring us as close to the original state as our experimental accuracy permits, we can repeat the process over and over again, as long as the distance to the original state keeps decreasing, while the CM success probability remains high. The atomic states\(^8\) and\(^9\) are determined by the minimization of\(^8\) at each step. Let us note here that the application of each CM may introduce widening of the photon-number distribution by one photon, and yet the \textit{optimized} CMs are capable of avoiding this widening and, moreover, of restoring the field to its initial pure state. Eqs. (A10e-A10g) in the Appendix exemplify the widening-avoidance requirements which are implicit in condition\(^11\). These requirements amount to an effective control of a large Fock-state subspace.

III. EXAMPLES

We illustrate our approach with two examples below, using the Q-function\(^8\)

\[ Q_{\rho_p}(\alpha, \alpha^*) = \langle \alpha|\rho_p|\alpha \rangle \]

being a coherent state of complex amplitude \( \alpha \), to visualize the error-correction process:

1) Let us take as the original field state an equal-amplitude superposition of our basis states, \( \text{e.g.,} \)

\[ |\psi(0)\rangle = (|0\rangle + e^{i\pi/3}|1\rangle)/\sqrt{2} , \]
whose $Q$-function is shown in Fig. 1(a). Dissipation by $\gamma t = 0.3$ renders the error matrix $\rho_{\psi}(t) - \rho_{\psi}(0)$ of considerable magnitude, as seen in Fig. 1(b). After the application of one CM (\( |\phi(1)\rangle = \cos(3\pi/8)\rangle_e + \sin(3\pi/8)e^{i5\pi/4}\rangle_g \), $\lambda \tau = 37.95$, \( |\phi(1)\rangle = \cos(3\pi/8)\rangle_e + \sin(3\pi/8)e^{i\pi/4}\rangle_g \)), optimized to yield high success probability ($r = 2$), the remaining error matrix $\rho_{\psi}(t+\tau) - \rho_{\psi}(0)$ is roughly 2.5 times smaller than before the correction, as seen in Fig. 1(c). The success probability of the CM is roughly 2.5 times smaller than before the correction, of 33% (respectively 16%).

Stronger error reduction is obtainable at the expense of success probability: the application of 4 CMs optimized for $r = 1$ (respectively $r = 0$) yields an error reduction factor of 11 (respectively 28) with sequence probability of 33% (respectively 16%).

2) If the original field state is a strongly unequal superposition of the basis states, such as

$$|\psi(0)\rangle = 10^{-1}|0\rangle + e^{i\pi/3}\sqrt{1-10^{-2}}|1\rangle \quad (15)$$

(Fig. 2(a)), the error matrix after dissipation by $\gamma t = 0.3$ is again significant (Fig. 2(b)). Successive application of 4 CMs, optimized for $r = 2$, reduces this error by a factor of 30 (Fig. 2(c)), which means that the recovered state is practically indistinguishable from the original state. The success probability of the total CM sequence, 50%, is markedly high (Fig. 3). If we ignore success probability in Eq. (12) ($r = 0$) we obtain a higher error-reduction factor of 75, with success probability of 28%.

In Fig. 3 we plot the distance $d_K = d(\rho^K, \rho_{\psi}(0))$ (Eq. (13)) between the recovered state and the original state and the CM sequence probability $P_{\text{seq},K} = \prod_{i=1}^{K} P_i$, with $P_i$ given by (14), as a function of the number of CMs performed. It shows that the first CMs achieve a strong reduction of such a distance, whereas after a few successive CMs saturation sets on, in terms of both distance and success probability.

It is interesting to compare the success probability in our approach with the theoretical probability to find the original state in the dissipation-spoilt state, namely, $\text{Tr}_F [\rho_{\psi}(0)\rho_{\psi}(t)]$, which we call the filtering probability. In Table I we list the success probability of a sequence of 4 CMs (optimized for $r = 2$), $P_{\text{seq},K=4}$, and the corresponding filtering probability for various values of the dissipation parameter $\gamma t$, taking as the original state the state \( |\psi(0)\rangle \) of example 2. The probability $\text{Tr}_F [\rho_{\psi}(0)\rho^K=4]$ of finding the original state in the recovered state is 0.99 or higher for all entries.

IV. DISCUSSION

In conclusion, we have demonstrated here the effectiveness of simple JC-dynamics CMs as a means of reversing the effect of dissipation on coherent superpositions of Fock-states of a cavity field: the application of a small number of optimized CMs recovers the original state of the field with high success probability, which is comparable or even surpasses the filtering probability. The simplest tactics may employ a single highly-probable trial to achieve nearly-complete error correction. As noted above, although we have only five control parameters at our disposal for each CM, our optimization procedure is able to effectively control the amplitudes in a large Fock-state subspace.

Among the experimental imperfections that can degrade the effectiveness of any CM approach, realistic atomic velocity fluctuations (of 1%) and cavity-temperature effects (below 1°C) are relatively unimportant, and especially so in the present scheme which makes use of a single or few CMs so that the effect of experimental imperfections is linear in the input errors. Only atomic detection efficiency is an experimental challenge. Although the detection efficiency is currently low, it is expected to rise considerably in the coming future.

Extensions of this approach to field-atom interaction Hamiltonians with more controllable degrees of freedom can make a single trial within this correction procedure effective for highly complicated states, encoding many qubits of information. Nevertheless, even in its present form the suggested approach has undoubted merits: (a) it can yield higher success probabilities than the filtering approach; (b) it is not limited to small errors as “high level” unitary-transformation approaches are; (c) it corrects errors after their occurrence, with no reliance on ideal continuous monitoring of the dissipation channel and on instantaneous feedback; and (d) it is realistic in that it can counter combined phase-amplitude errors which arise in cavity dissipation, and is of general applicability—not restricted to specific models of dissipation.

ACKNOWLEDGMENTS

The support of the German-Israeli Foundation (GIF) is acknowledged. M.F. thanks the European Economic Community (Human Capital and Mobility programme) for support.

APPENDIX:

The reduced density matrix of the field resulting from its interaction with the atom followed by the conditional measurement on the latter can be found using the formula (Eqs. (11,10))

$$\rho_{\psi}(t + \tau) = \text{Tr}_A \left[ \hat{U}(\tau) \rho_{\psi}(t) \otimes \langle \phi^{(i)} \rangle \langle \phi^{(j)} | \hat{U}^\dagger(\tau) | \phi^{(j)} \rangle | \phi^{(j)} \rangle \right] / P,$$

(A1)
where the normalization constant $P$ is the success probability of the conditional measurement and is given by

$$P = \text{Tr}_r \text{Tr}_\lambda \left[ \tilde{U}(\tau) \rho_\tau (\tilde{t}) \otimes |\phi^{(i)}\rangle \langle \phi^{(i)}| \tilde{U}^\dagger (\tau) |\phi^{(f)}\rangle \langle \phi^{(f)}| \right].$$  
(A2)

In the simple case where the initial field state is a superposition of the vacuum and one-photon states

$$|\psi(0)\rangle = c_0 |0\rangle + c_1 |1\rangle, \quad \rho_\tau (0) = |\psi(0)\rangle \langle \psi(0)| ,$$  
(A3)

the density matrix resulting from dissipation over time $\tilde{t}$ is

$$\rho_\tau (\tilde{t}) = \rho_{00}(\tilde{t}) |0\rangle \langle 0| + \rho_{01}(\tilde{t}) |0\rangle \langle 1| + \rho_{10}(\tilde{t}) |1\rangle \langle 0| + \rho_{11}(\tilde{t}) |1\rangle \langle 1| ,$$  
(A4)

with

$$\rho_{00}(\tilde{t}) = |c_0|^2 (1 - e^{-2\gamma \tilde{t}}) |c_1|^2,$$

$$\rho_{01}(\tilde{t}) = e^{-\gamma \tilde{t}} c_0 c_1^*,$$

$$\rho_{10}(\tilde{t}) = e^{-\gamma \tilde{t}} c_1 c_0^*,$$

$$\rho_{11}(\tilde{t}) = e^{-2\gamma \tilde{t}} |c_1|^2 .$$  
(A5)

The explicit expressions for $\rho_\tau (\tilde{t} + \tau)$ and the success probability $P$ are then

$$\rho_\tau (\tilde{t} + \tau) = \rho_{00}(\tilde{t}) |0\rangle \langle 0| + \rho_{01}(\tilde{t}) |0\rangle \langle 1| + \rho_{10}(\tilde{t}) |1\rangle \langle 0| + \rho_{11}(\tilde{t}) |1\rangle \langle 1| ,$$  
and

$$P = |\alpha^{(i)}|^2 (A + F) + \alpha^{(i)} \beta^{(i)} (M + S) + \beta^{(i)} \alpha^{(i)} (C + I) + \beta^{(i)} |2\rangle \langle 2| + \beta^{(i)} |1\rangle \langle 1| + \beta^{(i)} |\tilde{U}| + \beta^{(i)} |2\rangle \langle 2| .$$  
(A7)

The coefficients $A, B, \ldots$ here are given by

$$A = \rho_{00}(\tilde{t}) |\alpha^{(i)}|^2 C_0^2 + i \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} C_0 S_0 + \rho_{10}(\tilde{t}) \beta^{(i)} |2\rangle \langle 2| - i \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0 C_0 ,$$  
(A8a)

$$B = \rho_{00}(\tilde{t}) |\alpha^{(i)}|^2 C_0 C_1 + i \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} C_1 S_0 + \rho_{10}(\tilde{t}) |\beta^{(i)}|^2 C_0^2 + i \rho_{11}(\tilde{t}) |\beta^{(i)}|^2 S_0 C_0 ,$$  
(A8b)

$$C = \rho_{00}(\tilde{t}) |\alpha^{(i)}|^2 C_0 C_1 + i \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} C_1 S_0 ,$$  
(A8c)

$$D = i \rho_{00}(\tilde{t}) |\alpha^{(i)}|^2 C_0 S_0 - i \rho_{11}(\tilde{t}) |\beta^{(i)}|^2 S_0 C_0 + \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 C_0^2 + \rho_{10}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0^2 ,$$  
(A8d)

$$F = \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 C_1^2,$$

$$G = \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0 S_1 + i \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 C_0 S_1 ,$$  
(A8e)

$$H = \rho_{10}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0 ,$$  
(A8f)

$$I = \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} C_1 C_0 + i \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 C_1 S_0 ,$$  
(A8g)

$$K = i \rho_{00}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0 + \rho_{10}(\tilde{t}) |\beta^{(i)}|^2 C_0 ,$$  
(A8h)

$$L = i \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 C_1 S_1 ,$$  
(A8j)

$$O = \rho_{00}(\tilde{t}) |\beta^{(i)}|^2 ,$$  
(A8k)

$$Q = i \rho_{10}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0 ,$$  
(A8l)

$$U = \rho_{00}(\tilde{t}) |\alpha^{(i)}|^2 S_0^2 - i \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} S_0 C_0 + \rho_{10}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 S_0 C_0 ,$$  
(A8m)

$$V = i \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 \beta^{(i)} C_0 S_1 + \rho_{01}(\tilde{t}) |\alpha^{(i)}|^2 S_0 S_1 ,$$  
(A8n)

$$Z = \rho_{11}(\tilde{t}) |\alpha^{(i)}|^2 S_1^2 ,$$  
(A8o)

with the following relations holding between them

$$A = A^*, \quad B = E^*, \quad C = M^*, \quad D = R^*$$  
(A9a)

$$F = F^*, \quad G = W^*, \quad H = N^*, \quad I = S^*$$  
(A9b)

$$L = X^*, \quad O = O^*, \quad K = T^*, \quad Q = Y^*$$  
(A9c)

$$U = U^*, \quad V = J^*, \quad Z = Z^*.$$  
(A9d)

(The coefficients $K$ and $N$ in this appendix bear no relation to $K$ and $N$ mentioned in the main text).

The explicit form of condition (11) for recovering the original field state is given by the following list of approximation relations:

$$\rho_{00}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 A + \alpha^{(i)} \beta^{(i)} M + \beta^{(i)} \alpha^{(i)} C + |\beta^{(i)}|^2 O \right] \approx |c_0|^2 \ (A10a)$$

$$\rho_{01}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 B + \alpha^{(i)} \beta^{(i)} N + \beta^{(i)} \alpha^{(i)} D \right] \approx c_1 c_0^* \ (A10b)$$

$$\rho_{10}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 E + \alpha^{(i)} \beta^{(i)} R + \beta^{(i)} \alpha^{(i)} I \right] \approx c_0 c_0^* \ (A10c)$$

$$\rho_{11}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 F + \alpha^{(i)} \beta^{(i)} S + \beta^{(i)} \alpha^{(i)} L + |\beta^{(i)}|^2 \tilde{U} \right] \approx c_1 c_0^* \ (A10d)$$

$$\rho_{02}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 G + |\beta^{(i)}|^2 Q \right] \approx 0 \ (A10e)$$

$$\rho_{20}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 W + |\beta^{(i)}|^2 Y \right] \approx 0 \ (A10f)$$

$$\rho_{12}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 L + |\beta^{(i)}|^2 V \right] \approx 0 \ (A10g)$$

$$\rho_{21}(\tilde{t} + \tau) = P^{-1} \left[ |\alpha^{(i)}|^2 X + |\beta^{(i)}|^2 J \right] \approx 0 \ (A10h)$$

$$\rho_{22}(\tilde{t} + \tau) = P^{-1} \left[ |\beta^{(i)}|^2 Z \right] \approx 0 \ (A10i)$$
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\[ Q(\alpha) = |\psi(0)\rangle\langle\psi(0)| = 10^{-1}|0\rangle + e^{i\pi/3}\sqrt{1-10^{-2}}|1\rangle; \]

(b) error after dissipation, \( \rho_F(t) - \rho_F(0) \); (c) error after 4 consecutive optimized CMs, each minimizing Eq. \((12)\).