Remarks on the use of group theory in quantum optics

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Abstract: The relationships between certain important nonclassical states of the quantized field and the coherent states associated with the SU(2) and SU(1,1) Lie groups and associated Lie algebras is briefly reviewed. As an example of the utility of group theoretical methods in quantum optics, a method for generating maximally entangled photonic states is discussed. These states may be of great importance for achieving Heisenberg-limited interferometry and in beating the diffraction limit in lithography.

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1. Introduction

The use of group theoretical methods in various branches of the theoretical physics is by now so well established that it hardly seems necessary to draw special attention to this fact. However, there was a time, years ago, when those such as Wigner and Weyl, who
promoted the use of group theoretical methods in quantum physics, were anointed the “Gruppen Pest” [1], a good indication of the status of group theory in physics at the time. But such reactionary attitudes can no longer be seriously maintained. After all, the currently accepted standard models of the strong and electro-weak interactions are expressed in terms of gauge theories associated with some internal symmetry group. It is no stretch to say that the methods of group theory have been essential to progress in the area of high-energy physics, the very forms of the interactions being determined by the structure of some group. But group theory can and has been used to great advantage in many other areas of physics such as condensed matter, atomic, molecular, and nuclear physics. This article, written in honor of the 65th birthday of Joseph Eberly, highlights the application of group theoretical methods to quantum optics. In fact, one of the earliest papers on the application of these methods to quantum optics is that of Wodkiewicz and Eberly [2].

2. Symmetry groups and dynamical groups

The first, and still by far the prevalent, use of group theory in physics was in regard to geometric symmetries and their connection to degeneracies of energy levels. After the discovery of quantum mechanics it was realized that the formalism of angular momentum was intimately connected with the representations of the rotation group SO(3), a continuous group, a Lie group, associated with the rotational invariance of central potentials. But it soon became evident that certain problems, such as the Coulomb problem and the isotropic harmonic oscillator, exhibited a much higher degree of degeneracy than could be explained on the basis of geometric symmetries alone. These extra degeneracies were often called “accidental” but, in the cases of the two important systems mentioned, turned out to be harbingers of a non-geometric kind of symmetry, a hidden symmetry that depends on the form of the potential energy function. These symmetries are now properly called dynamical symmetries and the related groups are called dynamical symmetry groups. For the Coulomb problem the dynamical symmetry group is SO(4) while for the isotropic harmonic oscillator in 3 dimensions it is SU(3).

But there is yet another context in which Lie groups appear, unrelated to symmetries. These are the dynamical groups, or non-invariance groups, and the associated Lie algebras are known as spectrum generating algebras. For the symmetry Lie groups the elements of the Lie algebra commute with the Hamiltonian whereas for the dynamical groups the Hamiltonian may be expressed in terms of the elements of the Lie algebra, usually as a simple linear combination, and thus knowledge of the relevant unitary representations yields the spectrum. Important examples are the SO(2,1) for the radial Coulomb and radial isotropic harmonic oscillator problems and SU(1,1), which is locally isomorphic to SO(2,1), for the one dimensional harmonic oscillator, and SU(2), locally isomorphic to SO(3), for the bound states of the Morse oscillator. The groups relevant to quantum optics are essentially dynamical groups, not symmetry groups, although they have often been erroneously referred to as the latter. In any case, in the balance of this paper we illustrate the use of the groups SU(1,1) and SU(2) and their associated Lie algebras for select problems of interest in quantum optics.

3. SU(2) and SU(1,1) in a nutshell

In the abstract, the group SU(2) consists of the set of all the two-dimensional unitary matrices (of determinant 1) that preserve the quadratic form

\[ \left| z_1 \right|^2 + \left| z_2 \right|^2, \] (1)
where $z_1$ and $z_2$ are complex numbers. In contrast, the group $SU(1,1)$ consist of the set of all two-dimensional pseudo-unitary matrices (of determinant 1) preserving the quadratic form

$$\left|z_1\right|^2 - \left|z_2\right|^2. \quad (2)$$

$SU(2)$ is a compact group while $SU(1,1)$ is a non-compact group. The Lie algebra associated with the group $SU(2)$, denoted $su(2)$, consist of the commutation relations

$$[J_x, J_z] = 2J_y, \quad [J_y, J_x] = -2J_z. \quad (3)$$

the familiar angular momentum algebra. Furthermore, the operator

$$C_2 = J_3^2 + \frac{1}{2}(J_+J_- + J_-J_+), \quad (4)$$

which is just the square of the total angular momentum, commutes with all the elements of the Lie algebra and in the language of group theory is known as a Casimir operator. Elements of the Lie group (i.e. rotations) are obtained, or generated, through the exponentiation of the elements of the Lie algebra, hence the elements of the Lie algebra are called the generators of the corresponding Lie group. The Lie algebra associated with $SU(1,1)$, denoted $su(1,1)$, is given by

$$[K_x, K_z] = -2K_0, \quad [K_0, K_\pm] = \pm K_\pm. \quad (5)$$

Note that the $su(2)$ and $su(1,1)$ algebras differ only in the sign on the right hand side of the first commutation relation of each algebra. The Casimir operator for this algebra is

$$C_{11} = K_0^2 - \frac{1}{2}(K_+K_- + K_-K_+). \quad (6)$$

The unitary irreducible representations of the $SU(2)$ are just the familiar angular momentum states $|j, m\rangle$ satisfying the relations

$$C_2 |j, m\rangle = j(j + 1) |j, m\rangle, \quad J_z |j, m\rangle = m |j, m\rangle$$

$$J_\pm |j, m\rangle = \sqrt{(j \mp m)(j \pm m + 1)} |j, m \pm 1\rangle,$$

$$j = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \quad m = -j, -j + 1, \ldots, j. \quad (7)$$

Note that the representations are finite dimensional, the dimension for a given $j$ being $2j + 1$.

For $SU(1,1)$ there are many unitary irreducible representations, and because $SU(1,1)$ is a noncompact group, they are all of infinite dimension. Some of the representations are, in fact, continuous but here we shall only deal with the representations known as the positive
discrete series for which the operator $K_0$ is diagonal and has a discrete spectrum. The basis states of these representations we denote as $|k,m\rangle$, where the number $k$ is known as the Bargmann index. These states satisfy the relations

$$C_{11}|k,m\rangle = k(k-1)|k,m\rangle, \quad K_0|m,k\rangle = (m+k)|k,m\rangle,$$

$$K_-|k,m\rangle = \sqrt{(m+1)(m+2k)}|k,m\rangle,$$

$$K_+|k,m\rangle = \sqrt{m(m+2k-1)}|k,m\rangle.$$

$$k = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots; \quad m = 0, 1, 2, \ldots, \infty. \quad (8)$$

Note that the Bargmann index $k$ carries the fractional part of the spectrum of the operator $K_0$.

3. SU(1,1) in quantum optics

The relevance of Lie algebras to physical problems becomes manifest through the realization of the generators of the group in terms of the operators of the underlying physical system. As a first example, we consider a single mode field described by the usual annihilation (creation) operators $a(a^\dagger)$ satisfying the boson algebra $[a,a^\dagger]=1$. A realization of the su(1,1) algebra in terms of these operators is

$$K_0 = \frac{1}{2}(a^\dagger a + \frac{1}{2}), \quad K_+ = \frac{1}{2}a^\dagger a, \quad K_- = \frac{1}{2}a^2,$$

and for the Casimir operator one finds $C_{11} = -3/16$ which in turn yields Bargmann indices $k = 1/4, 3/4$. Note that these fall outside the list given above and thus the relevant representations are not, but can be considered as “continuations” of, the standard ones. The complete Hilbert space of the single mode field given in terms of the eigenstates of the number operator, the $a^\dagger a$ Fock states $|n\rangle$, becomes mapped onto two representations of SU(1,1) according to parity. The representation associated with the Bargmann index $k = 1/4$ consists of only the even numbered Fock states and that for $k = 3/4$ of only the odd numbered Fock states. This is easy to verify from Eqs. (8) and (9). To summarize, we have the correspondence

$$|n\rangle \Leftrightarrow |k,m\rangle \quad \text{for} \quad n = 2(m+k) - 1/2. \quad (10)$$

We now discuss two types of SU(1,1) coherent states of great relevance to quantum optics for single mode fields. The first is the Perelomov [3] form of generalized coherent states which for SU(1,1) are defined in analogy with the displacement operator definition of the usual coherent states. These are given by

$$|\xi, k\rangle = S(z)|k,0\rangle$$

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where the operator \( S(z) = \exp(zK_+ - z^* K_-) \) is an element of SU(1,1) often called the squeeze operator, \( z = -(\theta/2)e^{i\phi} \), \( \xi = -\tanh(\theta/2)e^{-i\phi} \), and where \( \theta \) is a hyperbolic angle \( (0 \leq \theta < \infty) \) and \( \phi \) is an azimuthal angle \( (0 \leq \phi \leq 2\pi) \). Expanded in terms the SU(1,1) states we have

\[
|\xi, k\rangle = \left(1-|\xi|^2\right)^k \sum_{m=0}^{\infty} \frac{\Gamma(2k+m)}{m!\Gamma(2k)} \xi^m |k,m\rangle
\]  

(12)

For the case where \( k = 1/4 \) the squeeze operator acts on the vacuum state and the resulting coherent state is just the familiar squeezed vacuum state,

\[
|\xi_{sv}\rangle = \frac{1}{\sqrt{\cosh(\theta/2)}} \sum_{m=0}^{\infty} (-1)^m \sqrt{2m+1} m! e^{-i\phi} \left(\tanh(\theta/2)\right)^m |2m\rangle,
\]

(13)

where number states appear on the right hand side. The parameter \( \theta/2 \) is sometimes written as \( r \) and is known as the squeeze parameter. For the case \( k = 3/4 \), the corresponding SU(1,1) Perelomov coherent state is just the squeezed one-photon state.

The generation of the Perelomov states is accomplished by unitary evolution from the “ground” state with a driving interaction Hamiltonian of the form

\[
H_i = i\hbar \left( \lambda a^+ - a \right) = 2i\hbar \left( \lambda K_+ - K_- \right)
\]

(14)

which represents either a degenerate parametric down-converter or a degenerate four-wave mixer, both with classical pumping fields assumed to be strong. Clearly, the evolution operator associated with this Hamiltonian,

\[
U_i(t) = \exp\left[-itH_i / \hbar\right] = \exp\left[2t \left( \lambda K_+ - K_- \right) \right],
\]

(15)

is of the form of the squeeze operator with the identification \( |\xi| = 2|\lambda| t \) and \( \phi = 2\text{arg}(\lambda) \).

Another important form of SU(1,1) coherent states of relevance to quantum optics is that first discussed by Barut and Girardello [4]. These are defined as eigenstates of the su(1,1) lowering operator according to

\[
K_- |\eta, k\rangle = \eta |\eta, k\rangle,
\]

(16)

where \( \eta \) is an arbitrary complex number, in obvious analogy to the annihilation operator eigenstate definition of the ordinary coherent states. (But unlike those states, SU(1,1) coherent states are inequivalent under different definitions.) The solution to Eq. (16) is
where

\[
N_k = \left[ \Gamma(2k)|\eta|^{-2k+1} I_{2k-1}(2|\eta|) \right]^{1/2},
\]

\(I_{2k-1}\) being a modified Bessel function. However, the solutions may economically be written as superpositions of the ordinary coherent states \(|\alpha\rangle\) and \(|-\alpha\rangle\) as

\[
|\eta, \frac{1}{2}\rangle = N_+ (|\alpha\rangle + |-\alpha\rangle),
\]

\[
|\eta, \frac{3}{2}\rangle = N_- (|\alpha\rangle - |-\alpha\rangle),
\]

where \(\eta = \alpha^2/2\) and where

\[
N_\pm = \left[ 2 \pm 2 \exp\left(-2|\alpha|^2\right) \right]^{1/2}.
\]

Thus the Barut-Girardello form of SU(1,1) coherent states are nothing but special cases of the so-called Schrödinger cat states, the even and odd coherent states. Unlike the Perelomov states however, they cannot be generated by unitary evolution. One possible generational method is from the competition between the unitary two-photon parametric interactions described the Hamiltonian of Eq. (14) and the nonunitary process of two-photon absorption by a collection of atoms, an irreversible processes [5]. The master equation for the density operator for these competing interactions is of the form

\[
\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar} [H_1, \rho] - \frac{\kappa}{2} \left( a^\dagger a \rho a^2 \rho - 2a^\dagger a \rho a^2 + \rho a^2 a^\dagger \right)
\]

\[
= 2 \left[ \lambda K_+ - \lambda K_- , \rho \right] - 2\kappa \left( K_+ K_+ \rho - 2K_+ \rho K_+ + \rho K_+ K_- \right),
\]

where \(\kappa\) is proportional to the cross section for two photon absorption. The steady-state long-time solutions, for which \(\partial \rho/\partial t = 0\), must satisfy \((K_+ - \lambda/\kappa)\rho = 0\). The particular steady-state solution depends highly on the initial state. Because parity is conserved by the interactions, if the initial state of the field is a pure state containing only even number states, typically this might be just the vacuum, the steady state solution is \(\rho(\infty) = |\eta,1/4\rangle \langle \eta,1/4|\) and if only odd number are in the initial state we shall have \(\rho(\infty) = |\eta,3/4\rangle \langle \eta,3/4|\) where in both cases \(\eta = \lambda/\kappa\). Many other schemes have been discussed for producing these states and we refer the reader to relevant reviews [6].

A realization of su(1,1) in terms of two modes of the field, denoted \(a\) and \(b\), is given by
\[ K_0 = \frac{1}{2} \left( a^+ a + b^+ b + 1 \right), \quad K_+ = a^+ b^+, \quad K_- = ab \] 

where the Casimir operator is given by
\[ C_{11} = \frac{1}{4} \left( \Delta^2 - 1 \right), \quad \Delta = a^+ a - b^+ b. \] 

The two-mode Fock states \( |n_1, n_2 \rangle \equiv |n_1 \rangle \otimes |n_2 \rangle \) organize themselves into representations where the photon number states of the two modes are tightly correlated, differing by the eigenvalue of the number difference operator \( \Delta \). Denoting these eigenvalues by the letter \( q \), where without loss of generality we take \( q \) to be positive, \( q = 0, 1, 2, \ldots \), the su(1,1) basis states are identified as
\[ |n + q, n \rangle \Leftrightarrow |k, m \rangle, \quad k = \frac{1}{2} (1 + q), \quad m = n. \] 

The Perelomov coherent state for this two-mode realization is then
\[ |\xi \rangle \langle \xi| = \left(1 - |\xi|^2\right)^{(1+q)/2} \sum_{n=0}^{\infty} \frac{(n+q)!}{n!q!} |\xi|^n |n+q, n \rangle. \] 

For the special case \( q = 0 \) we just have the familiar two-mode squeezed vacuum state
\[ |\xi\rangle_{n sv} = \frac{1}{\cosh \theta} \sum_{n=0}^{\infty} (-1)^n e^{in\phi} \left[ \tanh(\theta/2) \right]^n |n, n \rangle. \] 

Obviously, the two-mode Perelomov coherent states can be generated from two-mode parametric processes described by Hamiltonians of the form
\[ H_I = i\hbar (\lambda a^+ b^+ - \lambda ab) = i\hbar (\lambda K_+ - \lambda K_-). \] 

On the other hand, the Barut-Girardello coherent states for the two-mode realization are given by
\[ |\eta \rangle \langle \eta| = \left(1 - |\eta|^2\right)^{(1+q)/2} \sum_{n=0}^{\infty} \frac{\eta^n}{n!(n+q)!} |n+q, n \rangle. \] 

where
\[ N_q = \left[ q \|\eta\|^2 \sum_{n=0}^{\infty} |\eta|^n \left(2|\eta|\right)^n \right]^{-1/2}. \] 

These are the pair coherent states discussed by Agarwal [7]. The pair coherent states can be generated by competition between the nondegenerate parametric interaction and
nondegenerate two-photon absorption described by an equation similar to Eq. (21). An initial pure state of the form \(|q,0\rangle\) results in the pure pair coherent state of Eq. (28).

There are many other types of SU(1,1) coherent states that have been considered and are relevant to interesting states of the field, such as the intelligent states, and states associated with the direct product group SU(1,1) \(\otimes\) SU(1,1) which is isomorphic to the group SO(2,2). A discussion of these matters is beyond the scope of this review.

4. SU(2) in quantum optics for two-mode fields

The group SU(2) appears in quantum optics in at least two ways: in connection with the Dicke model of a collection of two-level atoms, and in connection with coupled two mode fields through the Schwinger realization of the su(2) algebra for which

\[
J_3 = \frac{i}{2}(a^+a - b^+b), \quad J_+ = a^+b, \quad J_- = ab^+, \quad J_0 = \frac{1}{2}(a^+a + b^+b)
\]  

(30)

where the operator \(J_0\) commutes with all the others and where \(C_j = J_a(J_a + 1)\). The two-mode number states map onto angular momentum states according to the rule

\[
|j,m\rangle \leftrightarrow |n_a\rangle |n_b\rangle, \quad j = \frac{1}{2}(n_a + n_b), \quad m = \frac{1}{2}(n_a - n_b).
\]  

(31)

where \(J_0|j,m\rangle = j|j,m\rangle\). It is well known that certain passive optical devices such as beam splitters can well be described as “rotations” typically represented by the unitary operator of the form [8]

\[
U_{\frac{\pi}{2j}}(\theta) = \exp(-i\theta J_{\frac{\pi}{2j}})
\]  

(32)

where \(J_i = (J_+ + J_-)/2\) and \(J_2 = (J_+ - J_-)/2i\). Also, parametric frequency converters can be represented by Hamiltonians of the form

\[
H_i = \hbar(\tilde{\alpha}a^+b - \tilde{\alpha}^*ab^+) = \hbar(\tilde{\lambda} J_+ - \tilde{\lambda}^* J_-).
\]  

(33)

In either case, if the initial field state is the number state \(|0\rangle_N\rangle = |j = N/2, m = -N/2\rangle\) then the output state is just the SU(2) coherent state defined as [9]

\[
|\zeta, j\rangle = \exp(\beta J_+ - \beta^* J_-)|j, -j\rangle
\]

\[
= (1+|\zeta|^2)^{-j} \sum_{m=-j}^{j} \binom{2j}{j+m}^{1/2} \zeta^{j+m} |j, m\rangle
\]
\[
\left(1 + \zeta^2\right)^{-N} \sum_{n=0}^{N} \left(\begin{array}{c} N \\ n \end{array}\right)^{1/2} \zeta^n |n, N-n\rangle
\]  

where \( N \) is the total photon number, \( \zeta = \tan\left(\beta/2\right)\exp(-i\psi) \), and \( \psi = \arg(\beta) \). The \( N \) photons are binomially distributed over the two modes.

5. Nonlinear four-wave mixer and generation of maximally entangled photonic states

In the previous sections we have related various well known quantum states to coherent states associated with the Lie groups SU(2) and SU(1,1). Here we wish to illustrate how the group theoretical approach can lead to new, perhaps unsuspected, useful physical results. In the example presented below, it is in the comparison of the dynamics of two dissimilar physical systems, but with a common Lie algebra, that brings about results of potential applicability to interferometry and lithography.

So far we have considered only cases where the interactions have been linear in the elements of a Lie algebra. Some time ago, Yurke and Stoler [10] considered a nonlinear four-wave mixing device which could be described by the Hamiltonian

\[
H_i = \hbar \frac{\Omega}{4} \left(a^+b + ab^+\right)^2 = \hbar \Omega J_z^2.
\]  

Using the machinery of the rotation group, these authors showed that for an input state of the form \( |n\rangle |0\rangle \) and for an interaction time \( t = \pi/\Omega \) the output state is \( i|0\rangle |n\rangle \) (for \( n \) even) or \( \exp(-i\pi/4)|n\rangle |0\rangle \) (for \( n \) odd). Thus the device acts as an even-odd filter, switching the even photon numbers from one mode to the other. Obviously, under these operating conditions it can be used as a device to measure parity without counting the photon number; it is sufficient to detect any photons in either of the output channels. On the other hand, as the present author has shown [11], if the interaction time is \( t = \pi/2\Omega \) the output state will have the form

\[
\frac{1}{\sqrt{2}} \left(|n\rangle |0\rangle + e^{i\Phi_n} |0\rangle |n\rangle\right), \quad \Phi_n = (n + 1) \pi/2,
\]  

a maximally entangled state for \( n \) photons. How did we know that the maximally entangled states could be generated by the above interaction? Well, it turns out that Mølmer and Sørensen [12] had previously studied an interaction of exactly the form as above, that is, a nonlinear spin model as on the right side of Eq. (35), but involving the internal states of a collection of two-level ions. The interaction is produced by the proper application of various laser beams to the system of trapped ions. These authors showed that a maximally entangled state of the ions, a superposition of states with all the ions excited and all de-excited, important for high-resolution spectroscopy, could be generated for the proper interaction time. Thus by recognizing the mathematical similarity between the two rather different systems, possible only because of the use of the su(2) Lie algebra in the optical problem, it was possible to discern the generation of the maximally entangled photonic state of Eq. (36). This state is important because in interferometry one can attain the Heisenberg limit in the phase fluctuations, \( \Delta \phi = 1/n \), and in lithography it may be used...
in beating the diffraction limit [13]. The point of the discussion here is that by using the Lie algebraic form of the nonlinear four-wave mixing interaction, it was possible to easily deduce a potentially important result just by noting the similarity of the interaction forms between two physically dissimilar systems, a result that might otherwise have been missed.

But there is a caveat, however. In order for the above scheme to generate to work in generating the maximally entangled states for achieving Heisenberg-limited interferometry, even number states are required as initial states. But photon number states in general are not easy to produce and, of course, the higher number states, yielding lower phase uncertainty, are particularly difficult to generate. One way around this is to use as input states superpositions of only even number states. A prime example of such a state is, of course, the squeezed vacuum state of Eq. (13). We have been able to numerically show that with the squeezed vacuum as the input state we have Heisenberg-limited phase uncertainty in the form \( \Delta \phi = 1/n_{sv} \) where \( n_{sv} = \sinh^2(\theta/2) \) is the average photon number in the squeezed vacuum state. A similar numerical relationship holds for the even coherent state but such a state is much harder to generate.

6. Conclusion

In this article, we have given a sketch of the application of group theoretical methods to quantum optics, noting the connections between certain well known nonclassical states and the Lie groups SU(1,1) and SU(2) and their associated Lie algebras. Furthermore we have exhibited an example of how such methods may be of great utility in the possible generation of maximally entangled photonic states. However, we have only scratched the surface of the application of group theoretical methods to quantum optics. A more thorough review is in preparation and will be published elsewhere.

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