u^c d^c d^c-Based Affleck-Dine Baryogenesis

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Abstract

We consider the possibility of a successful Affleck-Dine mechanism along the $u^c d^c d^c$ direction in R-parity symmetric extensions of the minimal supersymmetric Standard Model (MSSM) which contain a gauge singlet superfield $\phi$. Such gauge singlets commonly occur in extensions of the MSSM, for example in models which seek to account for neutrino masses. We consider a two scalar Affleck-Dine mechanism, with the flat direction stabilized by a non-renormalizable superpotential term of the form $\frac{1}{M}\phi u^c d^c d^c \sim \frac{1}{M}\phi \psi^3$, where $\psi$ corresponds to the gauge non-singlet flat direction. We give approximate solutions of the scalar field equations of motion which describe the evolution of the condensates and show that the final baryon asymmetry in this case is suppressed relative to that expected from the conventional single scalar Affleck-Dine mechanism, based on a superpotential term of the form $\frac{1}{4M}\psi^4$, by a factor $(\frac{m_s}{m_\phi + m_s})^{1/2}$, where $m_s$ is the soft supersymmetry breaking scalar mass and $m_\phi$ is the supersymmetric $\phi$ mass. It is possible for the model to generate a baryon asymmetry even in the limit of unbroken B-L, so long as the gauge singlet condensate doesn’t decay until after anomalous electroweak B+L violation is out of equilibrium following the electroweak phase transition. This condition is generally satisfied if all Dirac neutrino masses are less than around 10keV. This class of Affleck-Dine models can, in principle, be experimentally ruled out, for example by the observation of a Dirac mass for the $\mu$ or $\tau$ neutrino significantly larger than around 10keV together with a mostly Higgsino LSP.
1. Introduction

In supersymmetric (SUSY) models [1], the occurrence of flat directions in the renormalizable scalar potential of the minimal SUSY Standard Model (MSSM) and many of its extensions naturally leads to the possibility of generating the baryon asymmetry of the Universe via the decay of scalar field oscillations along such flat directions. This possibility is the well-known Affleck-Dine (A-D) mechanism for baryogenesis [2]. Although in the limit of unbroken SUSY the renormalizable potential along these flat directions is completely flat, once soft SUSY breaking terms and non-renormalizable terms consistent with the symmetries of the model are added there will be a non-trivial potential. In the original A-D scenario [2] it was assumed that the soft SUSY-breaking terms are the same as the zero temperature SUSY-breaking terms, which are characterized by a mass scale $m_s$ of the order of 100GeV-1TeV [1]. However, it has recently become clear that the large energy density which exists in the early Universe will also break SUSY, resulting in soft SUSY breaking terms characterized by a mass scale typically of the order of the Hubble parameter $H$ [3]. This large mass scale for the SUSY breaking terms radically alters the evolution of the scalar fields during and after inflation [4]. In the original A-D mechanism, because the scalar field masses are much smaller than $H$ during inflation, the classical scalar fields are overdamped and effectively frozen in at their initial values on horizon crossing, as generated by quantum fluctuations [2, 5]. Therefore on the scale of the observable Universe there is a large constant scalar field over the whole Universe with an essentially random phase. This then evolves into a coherently oscillating scalar field, corresponding to a Bose condensate with a roughly maximal asymmetry in the condensate particle number density. The subsequent decay of the condensate was shown to be easily able to account for the baryon asymmetry of the Universe [2]. However, once mass terms of the order of $H$ for the scalar particles are introduced, this picture completely changes [4]. Now the classical scalar fields can evolve to the minimum of their potentials on a time scale of the order of $H^{-1}$. As a result, at the end of inflation, all the scalar fields will be at the minimum of
their potentials, with quantum fluctuations having an effect only on the scale of the horizon at the end of inflation, which is much smaller than the scale of the observable Universe. Therefore the baryon asymmetry coming from the A-D condensate in this case will be determined \textit{dynamically} by the evolution of the scalar fields during the post-inflation era, with the scalar fields starting out at the minimum of their potentials at the end of inflation. Since the A-D mechanism is now dependent upon the details of the scalar potential, one has to consider each case individually in order to determine the magnitude of the resulting asymmetry. The asymmetry will be particularly sensitive to which flat direction the scalar fields oscillate along and to the form of the non-renormalizable superpotential terms which determine the minimum of the scalar potential and introduce the CP violation necessary in order to generate the baryon asymmetry \cite{4}.

In order to generate a baryon asymmetry from a condensate which decays prior to the electroweak phase transition (when anomalous B + L violation is in thermal equilibrium \cite{6}) it is necessary for the condensate to carry a non-zero B-L asymmetry. The lowest dimension operators which characterize the B-L violating flat directions in the MSSM are the dimension 2 (d=2) operator $LH_u$ and the d=3 operators $u^c d^c d^c$, $d^c Q_L$ and $e^c L_L$ \cite{4}. (These operators may be thought of as the superpotential terms responsible for lifting the flat directions or as the scalar field operators which are responsible for introducing explicit B-L violation into the scalar field equations of motion. These are naturally connected by the relationship between the soft SUSY breaking terms and the superpotential terms \cite{1, 4}). These operators characterize the flat directions in the sense that the scalar field operator characterizing a particular flat direction will have a non-zero expectation value along that direction.

We will refer to the flat direction which gives a non-zero expectation value to $LH_u$ and to $u^c d^c d^c$ as the "$LH_u$ direction" and the "$u^c d^c d^c$ direction" respectively. (The A-D mechanism along the $d^c Q_L$ and $e^c L_L$ directions will be essentially the same as that along the $u^c d^c d^c$ direction; we will concentrate on the $u^c d^c d^c$ direction in the following). The $LH_u$ direction and the $u^c d^c d^c$ direction are orthogonal in the sense
that they cannot both be flat simultaneously \[4\]. The LH\(_u\) direction has recently been considered by a number of authors \[4, 7\]. In the present paper we will focus on the u\(^c\)d\(^c\)d\(^c\) direction.

The simplest implementation of the A-D mechanism along the u\(^c\)d\(^c\)d\(^c\) direction would involve adding to the MSSM superpotential a d=3 term of the form u\(^c\)d\(^c\)d\(^c\). However, this term would be phenomenologically dangerous, as it would introduce large B violation into the MSSM unless its coupling was extremely small. (For a review of the constraints on B and L violating couplings in the MSSM see reference \[8\]). For example, squark mediated proton decay imposes the constraint \(|\lambda'\lambda''| \lesssim 10^{-24}\) for the light quark generations, where \(\lambda'\) is the d\(^c\)QL coupling and \(\lambda''\) is the u\(^c\)d\(^c\) coupling \[8\]. Such dangerous B and L violating terms are usually eliminated from the MSSM by imposing R-parity (R\(_p\)) \[1, 8\]. Imposing R\(_p\) implies that the first B-L violating operator in the MSSM which is nonzero along the u\(^c\)d\(^c\)d\(^c\) direction is a dimension 6 operator, u\(^c\)u\(^c\)d\(^c\)d\(^c\)d\(^c\) \[4\]. However, as discussed in reference \[4\] (and briefly reviewed in the present paper), in A-D models where the natural scale of the non-renormalizable terms is the Planck scale, d \(\equiv 6\) A-D models can have an acceptably low B asymmetry only for very low reheating temperatures \(T_R, \theta T_R \lesssim 10\text{GeV}\), where \(\theta\) is a CP violating phase. d \(\equiv 4\) A-D models, on the other hand, can be compatible with a much wider range of reheating temperatures, up to \(10^9\text{GeV}\) or more \[4\]. Thus with only the particle content of the MSSM, the u\(^c\)d\(^c\)d\(^c\) direction would be disfavoured in the simplest models (those based on Planck scale non-renormalizable terms) relative to the LH\(_u\) direction, which can utilize the R\(_p\)-conserving d=4 operator (LH\(_u\))^2. However, if we were to consider extensions of the MSSM which involve the addition of an R\(_p\)-odd gauge singlet superfield \(\phi\), then we could form the R\(_p\) conserving d=4 operator \(\phi u^cd^c d^c\). The addition of such a gauge singlet superfield to the MSSM is a very common and natural feature of many extensions of the MSSM. In particular, in models which seek to account for neutrino masses, the gauge singlet superfield would correspond to a right-handed neutrino superfield. It is the purpose of the present paper to determine whether it
is possible to generate the observed B asymmetry along the \( u^c d^c d^c \) direction via the operator \( \phi u^c d^c d^c \) and, if so, to compare the resulting asymmetry with that coming from the more conventional LH\_u direction.

The paper is organized as follows. In section 2 we discuss the model and the minimization of its scalar potential. In section 3 we consider the scalar field equations of motion and the formation of the coherently oscillating scalar field condensates. In section 4 we discuss the condensate particle asymmetries and the resulting baryon asymmetry. In section 5 we discuss the constraints on the reheating temperature after inflation. In section 6 we discuss the thermalization and decay of the condensates and the upper limits on Dirac neutrino masses in the limit of unbroken B-L. In section 7 we give our conclusions.
2. d=4 Affleck-Dine mechanism along the u^c \cdot d^c \cdot d^c direction

We will consider throughout the simplest scenario, in which it is assumed that inflation occurs with an energy density consistent with the density perturbations observed by COBE, corresponding to $H \approx 10^{14} \text{GeV}$ [9], with the inflaton $\Phi$ subsequently undergoing coherent oscillations about the minimum of its potential. We will also require that the reheating temperature $T_R$ is low enough not to thermally regenerate gravitinos [11], which implies that $T_R$ is less than about $10^{10} \text{GeV}$, corresponding to $H$ not much larger than 1GeV. After reheating we will assume that the Universe is radiation dominated throughout, with no further significant increase in entropy. In general, when inflation ends $H$ will not be much smaller that its value during inflation (even in $\Phi^2$ chaotic inflation, the value of $H$ when the inflaton $\Phi$ starts oscillating is not much smaller that $10^{12} \text{GeV}$ [14]). Therefore the coherent oscillations of the A-D field, which will begin once $H \approx m_s \approx 100 \text{GeV}$, will begin during a matter dominated era, with the energy density of the Universe dominated by inflaton oscillations [4].

It is now understood that in most supergravity models, the energy density that exists in the early Universe will break SUSY, introducing soft SUSY breaking terms characterized by a mass scale typically of the order of $H$ [3]. We will therefore consider in the following soft SUSY breaking terms of the form

$$V_{\text{soft}} = (m_s^2 - c_i H^2)|\phi_i|^2 + (B_a W_{2 \cdot a} + \text{h.c.}) + (A_a W_{n \cdot a} + \text{h.c.}) \quad (2.1),$$

where $W_{2 \cdot a}$ are superpotential terms bilinear in the fields and $W_{n \cdot a}$ are terms of order $n$ in the fields. $A_a$ and $B_a$ are defined by $A_a = A_a + a_a H$ and $B_a = B_a + b_a H$, where $A_a$ and $B_a$ may be thought of as the zero temperature soft SUSY breaking terms from a hidden sector of N=1 supergravity [1] whilst $a_a H$ and $b_a H$ are due to SUSY breaking by the energy density in the early Universe. We will assume throughout that $A_a \approx B_a \approx m_a$. We will also assume that $a_a^2 \approx b_a^2 \approx |c_a|$. In most supergravity models we expect that $|c_a| \approx 1$ [13], although in some models $|c_a|$ may be smaller; for example $|c_a| \approx 10^{-2}$ occurs in supergravity models with a Heisenberg symmetry [12]. Then for the case with $c_a > 0$ the A-D scalar, corresponding to a renormalizable
flat direction in the scalar potential, will have a non-zero value at the minimum of its scalar potential at the end of inflation, with the potential being stabilized by the contribution from the non-renormalizable terms in the superpotential \[4\].

To implement the d=4 A-D mechanism along the \(u^c d^c d^c\) direction we will consider an \(R_p\) symmetric extension of the MSSM defined by the superpotential \(W = W_{sm} + W' + W_\nu\), where \(W_{sm}\) is the MSSM superpotential, \(W'\) is defined by

\[
W' = \frac{m_\phi}{2} \phi^2 + \frac{\lambda}{M} \phi u^c d^c d^c + \frac{\eta}{4M} \phi^4 \tag{2.2}
\]

and \(W_\nu\) is given by

\[
W_\nu = \lambda_\nu \phi H_u L \tag{2.3}
\]

In addition to these terms we would expect terms of the form \(\phi d^c Q L\) and \(\phi e^c L L\). For simplicity we will not include these terms explicitly. The operator \(u^c d^c d^c \equiv \epsilon_{\alpha\beta\gamma} u^c_\alpha d^c_\beta d^c_\gamma\) (where \(\alpha, \beta\) and \(\gamma\) are colour indices and generation indices are, for now, suppressed) is antisymmetric in the \(d^c\) scalar fields. Therefore the \(d^c\) should be from different generations, which we will denote as \(d^c\) and \(d^c'\). With \(u^c\), \(d^c\) and \(d^c'\) having different colour indices, the F-term contribution to the scalar potential is then given by

\[
V_F = \sum \left| \frac{\partial W}{\partial \phi_i} \right|^2 = m_\phi^2 |\phi|^2 + \frac{|\lambda|^2}{M^2} |u^c d^c d^c'|^2 + \frac{|\eta|^2}{M^2} |\phi|^6
\]

\[
+ \frac{|\lambda|^2}{M^2} |\phi|^2 \left[ |d^c d^c'|^2 + |u^c d^c|^2 + |u^c d^c'|^2 \right]
\]

\[
+ \left[ m_\phi^4 \phi \left( \frac{\lambda}{M} u^c d^c d^c' + \frac{\eta}{M} \phi^3 \right) + \frac{\lambda^4 |\eta|^2}{M^2} (u^c d^c d^c')^T \phi^3 + \text{h.c.} \right] \tag{2.4}
\]

The direction with only \(< u^c >, < d^c >, < d^c' >\) and \(< \phi >\) non-zero is F-flat in the MSSM, with only the terms in \(W'\) lifting this flatness. The D-term contribution to the scalar potential, \(V_D = \sum \frac{g_i^2}{2} |\Phi^a T_i^a \Phi|^2 \) \(2.5\),

where the \(\Phi\) are the multiplets of the gauge group \(i\) with generators \(T_i^a\), also vanishes so long as \(u^c\), \(d^c\) and \(d^c'\) have different colour indices and \(|u^c|^2 = |d^c|^2 = |d^c'|^2 = v^2\).

The phases of \(u^c\), \(d^c\) and \(d^c'\) are not, however, fixed by the MSSM F- and D-flatness.
conditions. We will see in the following that the important phases for the A-D mechanism are $\delta_v$ and $\delta_\phi$, where $< u^c d^c d'^c > = v^3 e^{i \delta_v}$ and $< \phi > = v_\phi e^{i \delta_\phi}$.
2.1. Potential minimization in the $\eta \to 0$ limit

We first note that in the SUSY limit with $H=0$ and with $m_\phi \neq 0$, there is a minimum with $v_\phi \neq 0$ which is degenerate with the $v_\phi = 0$ minimum and which could be phenomenologically dangerous. To be precise, from equation (2.4) we see that the SUSY minima correspond to $\phi = 0$ and $\phi^2 = -\frac{m_\phi M}{\eta}$ (with $u^c = d^c = d^c' = 0$ at both minima). The $\phi \neq 0$ minimum results in a dangerous $u^c d^c d^c$ superpotential term with coupling $\frac{|\lambda|}{|\eta|^{1/2}} \left( \frac{|m_\phi|}{M} \right)^{1/2}$. (In general we would expect similar couplings for the $d^cQL$ and $e^cLL$ terms). Thus, if we consider $M \lesssim M_{Pl}$ (where $M_{Pl}$ is the Planck scale) and $|m_\phi| \gtrsim m_s \gtrsim 10^2$GeV, then we see that $\frac{|\lambda|}{|\eta|^{1/2}} \left( \frac{|m_\phi|}{M} \right)^{1/2} \gtrsim \frac{|\lambda|}{|\eta|^{1/2}} \times 10^{-8}$, which, for $|\lambda|$ and $|\eta|$ not much smaller than 1, would result in an unacceptable squark mediated proton decay rate [8]. In order to avoid this danger we must therefore ensure that there exists a minimum of the scalar potential which has $v \neq 0$ for large $H$ but which evolves to the $v = v_\phi = 0$ minimum as $H$ tends to zero.

With regard to the scale of the non-renormalizable terms $M$, we will set this to equal the Planck scale by convention. Then the coupling $\lambda$ can take values small or large compared with 1, depending on the natural mass scale of the non-renormalizable terms. Values much larger than 1 would correspond to the case where the natural mass scale of the non-renormalizable terms is much smaller than the Planck scale, for example a grand unification scale. On the other hand, if the natural mass scale of the non-renormalizable terms was of the order of the Planck scale, then we would expect that $|\lambda| \lesssim 1$.

Typically, we would not expect $\lambda$ and $\eta$ to be very different in magnitude. However, we would like to be able to minimize the potential analytically. We find that we can do this for the case of $|\eta|$ small compared with $|\lambda|$ ($|\eta| \lesssim 0.1|\lambda|$ is sufficient), in which case it may be shown that the terms in the scalar potential proportional to $\eta$ can, to a good approximation, be neglected. We expect that the possibly more likely case with $|\eta| \approx |\lambda|$ will be qualitatively similar. In the following we will consider the minimization of the potential in the $\eta \to 0$ limit.
In the $\eta \to 0$ limit the scalar potential becomes

$$V = (|\mu|^2 + m_s^2 - c_H^2)v_\phi^2 + (m_s^2 - c_H^2)v^2 + 3|\lambda|^2 M^2 v_\phi^2 v^4 + \frac{|\lambda|^2}{M^2} v^6 + \left[\frac{B \mu}{2} v_\phi^2 e^{2i\phi} + \text{h.c.}\right]$$

$$+ \left[\frac{m^\dagger \lambda}{M} v_\phi v^3 e^{i(\delta_\phi - \delta_\nu)} + \text{h.c.}\right] + \left[\frac{A \lambda}{M} v_\phi v^3 e^{i(\delta_\phi + \delta_\nu)} + \text{h.c.}\right]$$

(2.6).

By an choice of the phases of the scalar fields we can make $m_\phi$ and $\lambda$ real. We may also choose $m_\phi$ to be positive. To a reasonable approximation we can neglect the term proportional to $B_\phi$. This only contributes a term of the order of $m_\phi (m_s + a_i H) v_\phi^2$, which is less than or of order of the $(m_\phi^2 + m_s^2 - c_H^2) v_\phi^2$ term. The phases $\delta_\phi$ and $\delta_\nu$ will then adjust to minimize the cross-terms (where we use ”cross-terms” to denote terms which are the sum of a term and its hermitian conjugate). The potential will then have the form

$$V = (m_\phi^2 + m_s^2 - c_H^2)v_\phi^2 + (m_s^2 - c_H^2)v^2 + 3|\lambda|^2 M^2 v_\phi^2 v^4 + \frac{|\lambda|^2}{M^2} v^6 - 2\tilde{m}_\phi |\lambda| M v_\phi v^3$$

(2.7),

where $\tilde{m}_\phi \equiv m_\phi + |A_\lambda|$. In general, the minimum of this potential is given by

$$v_\phi = \frac{|\lambda|}{M} \tilde{m}_\phi v^3$$

(2.8)

and

$$v(v^4 - \frac{\tilde{m}_\phi M}{|\lambda|} v_\phi v + 2v_\phi^2 v^2 + \frac{M^2}{3\lambda^2} (m_s^2 - c_H^2)) = 0$$

(2.9).

We next consider how the minimum evolves from an initially large value of $c_H^2$.

(i) $c_H^2 > m_\phi^2 + m_s^2$

In general the minimum in this case is given by

$$v^4 \approx \frac{c_H}{(1 - \alpha)} \frac{M^2 H^2}{3\lambda^2}$$

(2.10)

and

$$v_\phi \approx \frac{1}{\sqrt{3} H} \frac{\tilde{m}_\phi (1 - \alpha)^{1/2} v}{c_\phi^{1/2}} \approx \frac{1}{\sqrt{3} H} \frac{a_\phi (1 - \alpha)^{1/2}}{c_\phi^{1/2}} v$$

(2.11),

where $\alpha$ is the solution of

$$\alpha^3 - (1 - \frac{a_\phi^2}{c_\phi} - \frac{c_\phi}{c_\phi} \alpha^2 - \frac{5 a_\phi^2}{3 c_\phi} \alpha + \frac{2 a_\phi^2}{3 c_\phi} = 0$$

(2.12),
as may be seen by taking the H^2 terms large compared with the mass terms and substituting (2.10) into (2.9) and (2.8). Typically (1 − α) is of the order of 1. For example, if a^2 is small compared with c_v then α = (1 − cφ/c_v), whilst if a^2 = cφ = c_v then α = −2. Therefore we can roughly say, with c_v ≈ cφ ≈ a^2, that

\[ v ≈ vφ ≈ c^{1/4} \left( \frac{(MH)^{1/2}}{|λ|^{1/2}} \right) \]  

(2.13).

Thus v and vφ are initially of the same order of magnitude.

(ii) \( cφH^2 < m^2 φ + m^2 s \)

In this case we find that it is consistent to assume that m^2 φ + m^2 s > \( \frac{3|Aλ|^2}{M^2} v^4 \), in which case vφ is given by

\[ vφ ≈ \frac{|λ|}{M} \frac{\tilde{m}_φ v^3}{(m^2 φ + m^2 s)} \]  

(2.14).

Solving (2.9) for v, we find solutions v± given by

\[ v^4 ± = -\frac{1}{4} \frac{M^2 (m^2 φ + m^2 s)^2}{\tilde{m}_φ^2} \left( 1 - \frac{\tilde{m}_φ^2}{(m^2 φ + m^2 s)} \right) \left[ 1 ± \left( 1 - \frac{8 \tilde{m}_φ^2 (m^2 s - c_v H^2)}{3 (m^2 φ + m^2 s - \tilde{m}_φ^2)^2} \right) \right]^{1/2} \]  

(2.15).

For c_v H^2 > m^2 s, v^4 is negative, and so there is a minimum at v+ with no barrier between v = 0 and v = v+. Once c_v H^2 is less than m^2 s, a barrier appears at v−. The minimum at v+ subsequently becomes unstable once

\[ m^2 s - c_v H^2 > \frac{3}{8} \frac{|Aλ| m φ + |Aλ|^2 - m^2 s)^2}{(m φ + |Aλ|)^2} \]  

(2.16).

For \( mφ \) large compared with |Aλs| and m_s we see that we must have m^2 s > \( \frac{3}{4} |Aλ|^2 \) in order that the dangerous v+ ≠ 0 minimum becomes destabilized as H → 0. As mφ → 0, this condition becomes \( m^2 s > \frac{3}{8} \frac{|Aλ|^2 (m^2 φ)}{|Aλ|^2} \). Thus we see that it is non-trivial for the dangerous v ≠ 0 minimum to become destabilized as H → 0. So long as the potential can be destabilized, however, it will generally destabilize once \( m^2 s \) is greater than c_v H^2 up to a factor of order 1. The values of the fields when the v+ minimum becomes unstable are then given by

\[ v^4_+ ≈ \frac{1}{4} \frac{M^2}{λ^2} \left( \frac{m^2 φ + m^2 s}{\tilde{m}_φ^2} \right) (\tilde{m}_φ^2 - m^2 φ - m^2 s) \]  

(2.17).
and
\[ v_\phi \approx \frac{1}{2} \frac{1}{(m^2_\phi + m^2_s)^{1/2}} v \] (2.18).

Noting that \(|A_\lambda| \approx m_s\) for \(c_v H^2 \lesssim m_s^2\), we find that, for \(m_\phi \gtrsim m_s\),
\[ v_\phi \approx \frac{1}{\sqrt{2}} \left( \frac{m_s}{m_\phi} \right)^{1/2} v \] (2.19),

whilst for \(m_\phi < m_s\) \(v_\phi \approx v\). So for \(m_\phi > m_s\) we find that \(v_\phi\) becomes suppressed relative to \(v\).
3. Bose condensate formation

To discuss the formation of the Bose condensates, that is to say, the way in which the scalar fields start oscillating freely about the minimum of their potentials, we consider the equations of motion of the scalar fields. Strictly speaking we have four scalar fields; \( u^c \), \( d^c \), \( d^c' \) and \( \phi \). However, since we are considering the evolution of the classical fields along a D-flat direction, we may impose that the \( u^c \), \( d^c \) and \( d^c' \) fields have the same magnitude. Although the phases of these fields could be different, the equations for \( u^c \), \( d^c \) and \( d^c' \) are identical and the initial values of the fields are the same. Therefore we may assume that their phases remain equal throughout.

The equations of motion are then given by,

\[
\ddot{\psi}_R + 3H \dot{\psi}_R = -[(m_s^2 - c_\phi H^2)\psi_R + \frac{2\lambda^2}{M^2} \phi_R^2 \psi_R^3 + \frac{\lambda^2}{M^2} \dot{\psi}_R^5] \\
- \beta \psi_R^2 \phi_R - \alpha(t)(c_\theta \phi_R \psi_R^2 + s_\theta(\psi_R^2 \phi_R + 2\psi_R \phi_R \psi_R)) (3.1),
\]

\[
\ddot{\psi}_I + 3H \dot{\psi}_I = -[(m_s^2 - c_\phi H^2)\psi_I + \frac{2\lambda^2}{M^2} \phi_R^2 \psi_I^3 + \frac{\lambda^2}{M^2} \dot{\psi}_I^5] \\
- \beta(\psi_R^2 \phi_I - 2\psi_R \tau_R \phi_R) - \alpha(t)(s_\theta \phi_R \psi_R^2 - \alpha(\psi_R^2 \phi_R + \psi_R^2 \phi_R)) (3.2),
\]

\[
\ddot{\phi}_R + 3H \dot{\phi}_R = -[(m_\phi^2 + m_s^2 - c_\phi H^2)\phi_R - \beta \phi_R^3 + \frac{3\lambda^2}{M^2} \psi_R^4 \phi_R] \\
- \gamma(c_\phi \phi_R + s_\phi \phi_I) - \alpha(c_\theta \psi_R^3 + s_\theta \psi_R^2 \psi_I)) (3.3),
\]

and

\[
\ddot{\phi}_I + 3H \dot{\phi}_I = -[(m_\phi^2 + m_s^2 - c_\phi H^2)\phi_I - \beta \phi_R^3 + \frac{3\lambda^2}{M^2} \phi_I^4 \phi_I] \\
- \gamma(s_\phi \phi_I - c_\phi \phi_I) - \alpha(s_\theta \psi_R^3 - c_\theta \psi_R^2 \psi_I)) (3.4),
\]

where \( \psi \) represents the \( u^c \), \( d^c \) and \( d^c' \) fields and we define \( \alpha \), \( \beta \) and \( \gamma \) by \( \alpha = \left| \frac{\lambda \lambda^*}{M} \right| \), \( \beta = \left| \frac{m_\phi \phi}{M} \right| \) and \( \gamma = \left| B_\phi m_\phi \right| \), where \( m_\phi \phi = -\beta \) (we choose this to be real and negative by an choice of phase), \( (\frac{\lambda \lambda^*}{M})^\dagger = -\alpha e^{i\theta} \) and \( (B_\phi m_\phi)^\dagger = -\gamma e^{i\epsilon} \). \( c_\theta \) (\( c_\epsilon \)) and \( s_\theta \) (\( s_\epsilon \)) denote \( \cos \theta \) (\( \cos \epsilon \)) and \( \sin \theta \) (\( \sin \epsilon \)) respectively. In writing these equations we have assumed that the real parts of the fields are large compared with the imaginary parts, which turns out to be a reasonable approximation. Throughout our discussion of the equations of motion we will focus on the most likely form for the soft SUSY
breaking terms in the early Universe, corresponding to the case $a_1^2 \approx b_1^2 \approx c_1 \approx 1$.[3, 4]

Before the $v \neq 0$ minimum is destabilized, the fields will be at the minimum of their potentials, corresponding to setting the right hand side (RHS) of the equations of motion to zero. The $v \neq 0$ minimum at $v = v_+$ will become destabilized once $c_v H^2 \lesssim m_s^2$ and the $\psi_R$ field will begin to roll once $H^2 \lesssim m_s^2$. Once $\psi_R$ starts to roll, we will see that the other fields follow the minimum of their potentials as a function of $\psi_R(t)$ and oscillate about this minimum until they become freely oscillating about the $\psi_R = 0$ minimum of their potentials.

We first consider the evolution of the real parts of the fields, beginning with $\phi_R$. We consider the solution of the $\phi_R$ equation of motion in the limit where the terms proportional to $\psi_R^4 \phi_R$, $\gamma$ and $s_\theta$ are neglected, which turns out to be a reasonable approximation. We will also treat $\theta$ as a time-independent constant. (We will comment on this later). The $\phi_R$ equation of motion is then approximately given by

$$\ddot{\phi}_R + 3H \dot{\phi}_R \approx -[(m_\phi^2 + m_s^2 - c_\phi H^2)\phi_R - (\alpha c_\theta + \beta)\psi_R^3] \quad (3.5).$$

From now on we will neglect the $c_i H^2$ mass terms in the equation of motion as these will quickly become negligible as the Universe expands. We will also neglect the $3H \dot{\phi}$ and $3H \dot{\psi}$ damping terms, since we are considering $m_s \gtrsim H$. The effects of damping will, however, be included in the time dependence of the amplitude of oscillation of the fields.

We wish to show that in the solution of this equation $\phi_R$ oscillates around the minimum of its potential as a function of $\psi_R$. To see this let $\phi_R = \overline{\phi}_R + \delta \phi_R$, where

$$\overline{\phi}_R = \frac{(\alpha c_\theta + \beta)}{m_\phi^2 + m_s^2} \psi_R^3 \equiv \eta \psi_R^3 \quad (3.6)$$

is the minimum of the potential as a function of $\psi_R(t)$. Then the $\delta \phi_R$ equation of motion is given by

$$\delta \ddot{\phi}_R + 6\eta \psi_R \dot{\psi}_R^2 \overline{\phi}_R \approx -(m_\phi^2 + m_s^2)\delta \phi_R + 3\eta m_s^2 \psi_R^3 \quad (3.7).$$

In this we have used $\overline{\psi}_R \approx -m_s^2 \psi_R$, which will be shown later to be true. $\delta \phi_R$ will then grow from an initial value of zero until the mass term on the RHS of
\((3.7)\) proportional to \(\delta \phi_R\) becomes dominant, after which \(\delta \phi_R\) will oscillate about the minimum \(\phi_R = \bar{\phi}_R\) with frequency \(\approx (m_\phi^2 + m_s^2)^{1/2}\), the terms on the RHS proportional to \(\psi_R^3\) being rapidly damped by the expansion of the Universe. The initial value of the \(\delta \phi_R\) oscillation amplitude is therefore given by \(\delta \phi_{R0}\), where

\[
\delta \phi_{R0} \approx \frac{3 \eta m^2}{m_\phi^2 + m_s^2} \psi_{R0}^3 \quad (3.8).
\]

It is straightforward to show that \(\frac{\delta \phi_R}{\phi_{R0}} \approx \frac{m_s^2}{m_\phi^2 + m_s^2}\), which is less than or about equal to 1. Eventually the amplitude of \(\delta \phi_R\) will become larger than that of \(\bar{\phi}_R\), in which case \(\delta \phi_R \approx \phi_R\) will effectively oscillate freely around \(\phi_R = 0\) with frequency \((m_\phi^2 + m_s^2)^{1/2}\).

We next consider the solution of the \(\psi_R\) equation of motion. On introducing \(\phi_R = \delta \phi_R + \bar{\phi}_R\), we find that, for \(m_s^2 \gtrsim c_v H^2\), \(\psi_R\) begins oscillating with a frequency approximately equal to \(m_s\). This is not at first obvious since the \(\psi_R^2\phi_R\) and \(\psi_R^5\) terms on the RHS of the \(\psi_R\) equation of motion are initially large \((\sim (m_s m_\phi)\psi_R)\) compared with the \(m_s^2\psi_R\) term. However, it turns out that there is a cancellation between these higher-order terms, such that the sum of these terms on the RHS of \((3.1)\) contributes initially only \(\sim m_s^2\psi_R\) and then rapidly becomes small compared with the \(m_s^2\psi_R\) term as \(\psi_R\) decreases with the expansion of the Universe. Therefore \(\psi_R\) will essentially oscillate with frequency approximately equal to \(m_s\) once the \(v = v_+\) minimum becomes unstable.

Thus we can summarize the evolution of the real parts of the fields by

\[
\psi_R(t) \approx A_\psi(t) \cos(m_s t) \quad (3.9)
\]

and

\[
\phi_R(t) \equiv \bar{\phi}_R + \delta \phi_R \approx \eta \psi_R^3(t) + A_\phi(t) \sin((m_\phi + m_s)t) \quad (3.10),
\]

where the time dependence of \(A_\psi(t)\) and \(A_\phi(t)\) is due to the expansion of the Universe during inflaton matter domination, \(A(t) \propto a(t)^{-3/2}\), where \(a(t)\) is the scale factor.

We next consider the evolution of the imaginary parts of the fields. The evolution of \(\phi_I\) and \(\psi_I\) is similar to the evolution of \(\phi_R\) i.e. they follow the minimum of their
potentials as a function of $\psi_R(t)$. We first consider the solution of the $\psi_I$ equation of motion.

In the $\psi_I$ equation of motion we may roughly absorb the terms proportional to $\alpha c_0$ into the terms proportional to $\beta$. Thus we may neglect these terms for now. We will also set the $\beta \psi_R^2 \phi_I$ term to zero for now. With these assumptions only the phase $\theta$ contributes to the imaginary parts of the fields. We will comment on these assumptions later.

Suppose the $\psi_R$ field starts oscillating at $t_0$. (For convenience we will set $t_0$ equal to 0 throughout). Initially, by a choice of the phase of the scalar fields, we can set the phase $\theta$ to zero at $t_0$. The subsequent evolution of the phase $\theta(t)$ is then found from

$$\alpha e^{i \theta(t)} = - \left( \frac{(A_{\lambda s} + a_{\lambda o} He^{i \sigma}) \lambda}{M} \right) \dagger (1 + a_{\lambda o} e^{i \sigma})^{-1}$$

where $\sigma$ is the phase difference between the $A_{\lambda s}$ and $a_{\lambda} \equiv a_{\lambda o} e^{i \sigma}$ terms. Since during matter domination

$$H = \frac{H_o}{(1 + \frac{3}{2} H_o t)}$$

where $H_o \equiv H(t_o) \approx m_s$, we see that the phase $\theta(t)$ will reach its maximum roughly during the first $\psi_R$ oscillation cycle, in a time $\delta t \approx H_o^{-1} \approx m_s^{-1}$. Since the condensates will form during the first few oscillations of $\psi_R$, it is a reasonable approximation to set $\theta(t)$ to its constant maximum value throughout. With the above assumptions the $\psi_I$ equation of motion can be reasonably approximated by

$$\ddot{\psi}_I \approx - \left[ m_s^2 \psi_I + \frac{\lambda^2}{M^2} \psi_R^4 \psi_I - \alpha s_\theta \eta \psi_R^5 \right]$$

From this we see that the minimum of the $\psi_I$ potential as a function of $\psi_R$ is given by

$$\overline{\psi}_I = \frac{\alpha s_\theta \eta M^2}{\lambda^2 \left( 1 + \frac{m_s^2 M^2}{\lambda^2 \psi_R^4} \right)} \psi_R$$

Thus, noting that, at $H \approx m_s$, $\frac{m_s^2 M^2}{\lambda^2 \psi_R^4}$ is small compared with 1, we see that $\overline{\psi}_I$ is initially proportional to $\psi_R$. This will continue until $\psi_R$ decreases during its oscillation to the point where $\frac{m_s^2 M^2}{\lambda^2 \psi_R^4} \gtrsim 1$. 

To understand the evolution of $\psi_I$ let $\psi_I = \overline{\psi}_I + \delta\psi_I$. Substituting into the $\psi_I$ equation of motion, we find that, for $\frac{m_2^2 M^2}{\lambda^2 \psi_R} \lesssim 1$, $\delta\psi_I$ satisfies

$$\delta\ddot{\psi}_I \approx \frac{\alpha s_\theta \eta M^2}{\lambda^2} m_2^2 \psi_R - \frac{\lambda^2}{M^2} \psi_R^4 \delta\psi_I \quad (3.15).$$

Thus $\delta\psi_I$ will grow from $\delta\psi_I = 0$ to a value given by

$$\delta\psi_I \approx \frac{\alpha s_\theta m_2^4 M^4}{\lambda^4 \psi_R^3} \quad (3.16).$$

We see that the condition $\frac{m_2^2 M^2}{\lambda^2 \psi_R} \lesssim 1$ is equivalent to $\delta\psi_I \lesssim \overline{\psi}_I$. Initially $\delta\psi_I$ has a value

$$\delta\psi_{I,0} \approx s_\theta |A\lambda|(|A\lambda| + m_\phi) \left( \frac{m_s}{m_\phi + m_s} \right) \psi_R \quad (3.17).$$

Thus initially $\frac{\delta\psi_{I,0}}{\psi_{I,0}} \approx \left( \frac{m_s}{m_\phi + m_s} \right)^2 s_\theta$, which is small compared with 1 for $m_\phi$ large compared with $m_s$ or $\theta$ small compared with 1. During the subsequent $\psi_R$ oscillation, we see that, so long as $\frac{m_2^2 M^2}{\lambda^2 \psi_R} \lesssim 1$, $\phi_I$ will be proportional to $\psi_R$. Therefore $\psi_I$ will initially be in phase with $\psi_R$. However, as $\psi_R$ decreases, for a period $\delta t$ during the $\psi_R$ oscillation $\delta\psi_I$ will become larger than $\overline{\psi}_I$ and the approximate equation of motion (3.15) will no longer be valid. During this time the $m_2^2 \psi_I$ term in the $\psi_I$ equation of motion will dominate and the $\psi_I$ oscillation will continue with frequency $\approx \frac{m_s}{m_\phi + m_s}$. However, since during this period the effective mass term in the $\psi_I$ equation of motion will differ from $m_s$ by a factor of order 1, it will be possible for the $\psi_I$ phase to shift relative to $\psi_R$ by approximately $m_s \delta t$. As a result, for a fraction $m_s \delta t$ of the total $\psi_I$ oscillation, there will be a phase shift approximately given by $m_s \delta t$. Thus the average phase shift between $\psi_I$ and $\psi_R$ over the period of the $\psi_I$ oscillation, which, as discussed in the next section, is relevant for determining the $\psi$ asymmetry, is given by $\delta_p \approx (m_s \delta t)^2$. $\delta t$ corresponds to the time during which $\psi_R^4 \lesssim \frac{m_2^2 M^2}{\lambda^2}$. For a matter dominated Universe, with $\psi_R \propto a(t)^{-3/2}$, we find that $\delta_p \approx (m_s \delta t)^2 \approx \left( \frac{m_s}{m_\phi + m_s} \right)^{1/2} \left( \frac{m_s}{H} \right)^2$. $\delta_p$ reaches its largest value, $\delta_p \approx 1$, once $H \approx \left( \frac{m_s}{m_\phi + m_s} \right)^{1/4} m_s$. We also note that for $\theta$ small compared with 1, $\frac{\psi_I}{\psi_R} \approx \frac{\overline{\psi}_I}{\overline{\psi}_R} \approx \left( \frac{m_s}{m_\phi + m_s} \right) s_\theta$ is small compared to 1, as has been assumed throughout.

These results hold if (i) $\alpha c_\theta$ is small compared with $\beta$ and (ii) if the $\beta \phi_I \psi_R^2$ term in the $\psi_I$ equation of motion can be neglected. The effect of the terms proportional
to $\alpha c\theta$ will be to effectively multiply the terms proportional to $\beta$ by a factor $\sim (1 + \frac{m_s}{m_\phi} c\theta)$. However, we can still reasonably ignore the $\beta \psi_R \psi_I \phi_R$ term in the $\psi_I$ equation of motion, even with this factor. The effect of including the $\beta \phi_R \psi_I^2$ term in the $\psi_I$ equation of motion (together with the factor from (i)) turns out to be to approximately replace $s_\theta$ by $s_\tilde{\theta}$, where $s_\tilde{\theta} = s_\theta + \left(\frac{m_\phi}{m_\phi + m_s}\right) s_\epsilon$. Thus an imaginary part for $\psi$ can also be generated by the phase $\epsilon$ so long as $\phi$ has a mass term in the superpotential.

We finally consider $\phi_I$. With $\phi_I = \overline{\phi_I} + \delta \phi_I$, where

$$\overline{\phi_I} = \frac{(k \alpha s_\theta + \gamma \eta s_\epsilon)}{(m_\phi^2 + m_s^2)} \psi_R^3$$

(3.18)

and

$$k \approx 1 + \frac{3(A \lambda c\theta + m_\phi)}{(m_\phi^2 + m_s^2)} m_\phi$$

(3.19),

the $\phi_I$ equation of motion can be written approximately as

$$\delta \ddot{\phi}_I + 6 \frac{(k \alpha s_\theta + \gamma \eta s_\epsilon)}{(m_\phi^2 + m_s^2)} \psi_R \dot{\psi}_R^2 \approx -(m_\phi^2 + m_s^2) \delta \phi_I + 3m_s^2 \frac{(k \alpha s_\theta + \gamma \eta s_\epsilon)}{(m_\phi^2 + m_s^2)} \psi_R^3$$

(3.20).

Thus $\delta \phi_I$ will increase from zero to an initial oscillation amplitude given by

$$\delta \phi_{I_0} \approx \frac{3m_s^2(k \alpha s_\theta + \gamma \eta s_\epsilon)}{(m_\phi^2 + m_s^2)^2} \psi_R^3$$

(3.21)

and will subsequently oscillate freely with frequency $(m_\phi^2 + m_s^2)^{1/2}$. In general $\delta \phi_{I_0} \lesssim \overline{\phi}_{I_0}$. It is straightforward to show that $\delta \phi_R$ and $\delta \phi_I$ reach their maximum values and begin oscillating in a time $\delta t \sim (m_\phi^2 + m_s^2)^{-1/2} \lesssim m_s^{-1}$ and so will become freely oscillating within the first few oscillations of the $\psi_R$ field.

It is important to emphasize that it is the oscillations of $\delta \phi_R$ and $\delta \phi_I$ about the $\overline{\phi}_R$ and $\overline{\phi}_I$ minima which evolve into the $\phi$ Bose condensate as $\overline{\phi}_R$ and $\overline{\phi}_I$ become smaller than $\delta \phi_R$ and $\delta \phi_I$. This occurs once $H \lesssim \left(\frac{m_s}{m_\phi + m_s}\right) m_s$. We will show in the next section that the average $\phi$ asymmetry over the $\psi$ oscillation period $m_s$ comes purely from $\delta \phi$ and not from $\overline{\phi}$.

In summary, the $\psi$ and $\phi$ fields will begin to oscillate at $H \approx m_s$, with the initial values of the oscillating scalar field amplitudes relevant for the formation of
condensate particle asymmetries given by

\[ \psi_{R_0} \approx v_+ \approx \frac{M^{1/2}}{|\lambda|^{1/2}}(m_s^2 + m_\phi m_s)^{1/4} \quad (3.22), \]

\[ \psi_{I_0} \approx \left( \frac{\alpha s_\theta \eta M^2}{\lambda^2} \right) \psi_{R_0} \approx s_\theta \left( \frac{m_s}{m_\phi + m_s} \right) \psi_{R_0} \quad (3.23), \]

\[ \delta \phi_{R_0} \approx \left( \frac{3\eta m_s^2}{m_\phi^2 + m_s^2} \right) \psi_{R_0}^3 \approx \frac{m_s^2(m_s^2 + m_\phi m_s)^{1/2}}{(m_\phi + m_s)^{3/2}} \psi_{R_0} \quad (3.24), \]

and

\[ \delta \phi_{I_0} \approx \left( \frac{k \alpha s_\theta + \gamma \eta s_\theta}{m_\phi^2 + m_s^2} \right) \psi_{R_0}^3 \approx \frac{s_\theta m_s^3(m_s^2 + m_\phi m_s)^{1/2}}{(m_\phi^2 + m_s^2)^2} \psi_{R_0} \quad (3.25), \]

where in the final expressions we have set \( |A_\lambda| \approx |B_\phi| \approx m_s \) in order to show the dependence on \( m_s \) and \( m_\phi \). However, as noted above and discussed further in the next section, although the \( \psi \) scalar begins oscillating at \( H \approx m_s \), for the case where \( m_\phi > m_s \) the full phase difference \( \delta \phi \) between \( \psi_R \) and \( \psi_I \) and the associated \( \psi \) particle asymmetry will only form once \( H \) is smaller than \( \left( \frac{m_s}{m_\phi + m_s} \right)^{1/4} m_s \).

The approximations used in obtaining these results are good for \( m_\phi \gg m_s \). For \( m_\phi \lesssim m_s \) some of the assumptions made are only marginally satisfied or even slightly violated (although not strongly violated). However, we expect that the above results for the initial amplitudes will still be qualitatively correct, giving the correct order of magnitude for the resulting particle asymmetries.
4. Particle and Baryon Asymmetries.

In the limit where we retain only the mass terms in the $\psi$ and $\phi$ equations of motion, there is a global U(1) symmetry. This is broken by the B and L violating terms coming from the non-renormalizable terms in the scalar potential, which give rise to a non-zero U(1) charge in the condensate, corresponding to an asymmetry in the number of $\psi$ and $\phi$ particles. We first consider the $\psi$ asymmetry. The asymmetry in the number density of $\psi$ particles is given by

$$n_\psi = i(\psi^i d\psi - d\psi^i \psi) \equiv -2(\psi_R^i \dot{\psi}_I - \psi_I^i \dot{\psi}_R) \quad (4.1).$$

Note that in the case where $\psi_R$ and $\psi_I$ oscillate with the same frequency, there must be a phase difference between the oscillating $\psi_R$ and $\psi_I$ fields in order to have a non-zero asymmetry. As discussed in the last section, there is a time-dependent phase difference $\delta_p$ between $\psi_R$ and $\psi_I$ (averaged over the period of oscillation $m_s^{-1}$), which for the case $\delta_p \lesssim 1$ is given by

$$\delta_p \approx \left( \frac{m_s}{m_\phi + m_s} \right)^{1/2} \left( \frac{m_s}{H} \right)^2 \quad (4.2)$$

and by $\delta_p \approx 1$ otherwise. We can characterize the $\psi$ particle asymmetry by the asymmetry at $H \approx m_s$ that would evolve to the correct $\psi$ asymmetry at present,

$$n_\psi \approx 2\delta_p m_\psi \psi_R \psi_I \quad (4.3),$$

where the value of $\delta_p$ is determined by the value of $H$ at which the condensate thermalizes or decays.

For the case of the $\phi$ asymmetry, since, for $m_\phi > m_s$, the $\delta\phi_{R,I}$ oscillate with a greater frequency than the $\bar{\phi}_{R,I}$, substituting $\phi = \bar{\phi} + \delta\phi$ into (4.1) shows that the non-zero average asymmetry over the period $H^{-1} \gtrsim m_s^{-1} \gtrsim (m_\phi^2 + m_s^2)^{-1/2}$ will be that purely due to $\delta\phi_R$ and $\delta\phi_I$. Thus we find that the number density asymmetry in $\phi$ particles is given by

$$n_\phi \approx \left( \frac{m_s}{m_\phi + m_s} \right)^5 \frac{n_\psi}{\delta_p} \quad (4.4).$$
The suppression of $n_\phi$ relative to $n_\psi$ when $m_\phi \gtrsim m_s$ is significant, since in the limit where $m_\phi \to 0$ and $\delta_p \to 1$ we may define an unbroken B-L asymmetry by assigning B=1 to $\phi$. This is broken by $m_\phi$, suppressing $n_\phi$ and so preventing any possibility of a cancellation between the B-L asymmetries coming from decay of the $\psi$ and $\phi$ condensates.

We next compare the $\psi$ asymmetry from the above $\frac{\lambda}{M} \phi \psi^3$-type model with that expected from the more conventional A-D mechanism based on a single field with a B violating term of the form $\frac{\lambda}{M} \psi^4$ together with the above form of H corrections to the SUSY breaking terms. In this case the initial values of the $\psi_R$ and $\psi_I$ fields are given by

$$\psi_{R_0} \approx \left(\frac{1}{48}\right)^{1/4} \frac{(m_s M)^{1/2}}{|\lambda|^{1/2}} \quad (4.5),$$

and

$$\psi_{I_0} \approx \frac{4\alpha s_\theta \psi^3_R}{m_s^2} \quad (4.6),$$

with $\psi_R$ and $\psi_I$ subsequently oscillating with a phase difference of the order of 1. Thus in this case we find that

$$n_\psi \approx \frac{\alpha s_\theta M}{|\lambda|} \psi^2_{R_0} \quad (4.7),$$

with the asymmetry being fully formed at $H \approx m_s$. Therefore, for the case where the CP violating phases $s_\theta$ and $s_\tilde{\theta}$ and the coupling $\lambda$ have the same values in both cases, we find that the asymmetry from the $\frac{\lambda}{M} \phi \psi^3$-type superpotential term is related to that from the $\frac{\lambda}{M} \psi^4$ term by

$$n_\psi \approx \delta_p \left(\frac{m_s}{m_\phi + m_s}\right)^{1/2} n_{\psi_0} \quad (4.8),$$

where $n_{\psi_0}$ is the asymmetry expected from the conventional single field A-D mechanism. Thus we see that, even with $\delta_p \approx 1$, for $m_\phi > m_s$ there is a suppression of the $\psi$ asymmetry by a factor $\left(\frac{m_s}{m_\phi + m_s}\right)^{1/2}$ relative to that expected from the conventional single-field A-D mechanism.

The baryon asymmetry from the $\psi$ condensate is simply the asymmetry $n_\psi$ multiplied by a factor for the number of baryons produced per $\psi$ decay. Since we
have in fact three condensates, corresponding to $u^c$, $d^c$ and $d^c'$, each of which carries baryon number 1/3, we see that $n_B = n_\psi$. The baryon asymmetry to entropy ratio after reheating is then found by noting that the ratio of the $\psi$ number asymmetry to the inflaton energy density during inflaton oscillation domination is constant. Since the reheating temperature is given by $\rho_1 = k_T T_R^4$ and the entropy density by $s = k_s T_R^3$ (with $k_T = \frac{\pi^2 g(T)}{30}$ and $k_s = \frac{2\pi^2 g(T)}{45}$, where $g_b(T) = g_b(T) + \frac{7}{8} g_f(T)$ and $g_b(T)/(g_f(T))$ are the number of bosonic (fermionic) degrees of freedom in thermal equilibrium at temperature $T$), it follows that the baryon-to-entropy ratio is given by

$$\frac{n_B}{s} = \frac{k_s n_B}{k_T T_R \rho_1} \quad (4.9)$$

Thus, with the energy density dominated by inflaton oscillations when the Bose condensate forms at $H \approx m_s$ and with $n_B = n_\psi \approx \frac{s \delta m_p^2}{|\lambda|} \left( \frac{m_s}{m_\psi + m_s} \right)^{1/2}$, we find that

$$\frac{n_B}{\rho_1} \approx \frac{8\pi}{3} \frac{s \delta_p}{|\lambda| M_{Pl}} \left( \frac{m_s}{m_\phi + m_s} \right)^{1/2} \quad (4.10),$$

where we have used $M \equiv M_{Pl}$ and $|\lambda| \approx m_s$. Therefore

$$\frac{n_B}{s} \approx \frac{2\pi s \delta_p}{|\lambda|} \left( \frac{m_s}{m_\phi + m_s} \right)^{1/2} \frac{T_R}{M_{Pl}} \quad (4.11).$$

Comparing this with the observed asymmetry, $\frac{n_B}{s} \approx 10^{-10}$, and noting that the thermal gravitino regeneration constraint implies that $\frac{T_R}{M_{Pl}} \lesssim 10^{-9}$, we see that $m_\phi$ cannot be too large compared with $m_s$ if $\frac{s \delta_p}{|\lambda|}$ is not large compared with 1, as we would expect if the natural scale of the non-renormalizible terms was less than or of the order of $M_{Pl}$. On the other hand, even if $\frac{s \delta_p}{|\lambda|}$ is large compared with 1, the suppression factor $\left( \frac{m_s}{m_\phi + m_s} \right)^{1/2}$ would still allow $T_R$ to be consistent with the observed B asymmetry for values of $T_R$ up to the gravitino constraint, so long as $m_\phi$ was sufficiently large.

Thus we can conclude that, in the $R_p$ symmetric MSSM extended by the addition of a gauge singlet scalar, it is indeed possible to have a successful d=4 two-scalar $\phi \psi^3$-type A-D mechanism along the $u^c d^c d^c'$ direction. The resulting baryon asymmetry, assuming that the asymmetries are able to fully form and do not thermalize or
decay before the phase $\delta_p(t)$ has reached its maximum value (we discuss this possibility in the next section), receives an overall suppression by a factor approximately $\left(\frac{m_\phi}{m_\phi+m_\sigma}\right)^{1/2}$ relative to that expected in the case of a conventional $d=4$ $\psi^4$-type A-D mechanism based on a single A-D scalar field. This suppression factor allows for a wider range of non-renormalizable couplings and reheating temperatures to be compatible with the observed baryon asymmetry than in the case of the conventional A-D mechanism.
5. No-Evaporation Constraint

An important constraint on the reheating temperature comes from the requirement that the interaction of the condensate scalars with the radiation energy density due to inflaton decays prior to reheating does not lead to the condensate thermalizing via scattering with the background plasma before the particle asymmetries can be established \[4\]. We refer to this as the no-evaporation constraint.

We first consider the $\psi$ condensate. In general, there are two ways in which the condensate can be destroyed at a given value of $H$: thermalization and decay. Thermalization of the condensate will occur if (i) the rate of scattering of the thermal plasma particles from the condensate scalars, $\Gamma_s$, is greater than $H$ and (ii) if the mass of the initial and final state particles are such that the scattering process is kinematically allowed and the scattering particles in the plasma are not Boltzmann suppressed. Decay of the condensate via tree level two-body decays will occur if (i) the decay rate of the condensate scalars in their rest frame, $\Gamma_d$, is greater than $H$ and (ii) if the final state particles, of mass $\lambda_\psi < \psi >$, where $\lambda_\psi$ is a gauge or Yukawa coupling and $< \psi >$ is the amplitude of the $\psi$ oscillation, are lighter than the condensate scalars. Since the time over which the real and imaginary parts of the fields start rolling and so the asymmetries start to develop is of the order of $H^{-1}$ at $H \approx m_s$, we must ensure that thermalization and decay does not occur on a time scale small compared with $m_s^{-1}$.

Prior to reheating, the radiation energy density coming from inflaton decays during inflaton matter domination corresponds to a background plasma of particles at a temperature $T_r$, where, assuming that the decay products thermalize, $T_r$ is given by \[4\] \[5\]

$$T_r \approx k_r (M_{Pl} HT_R^2)^{1/4} \quad (5.1),$$

where $k_r = \left( \frac{3}{50\pi k_T} \right)^{1/8} \approx 0.4$ (using $g(T) \approx 100$).

For the case of the $\psi$ condensate, the particle asymmetry begins to form at $H \approx m_s$, at which time $\delta_p(t) \approx \left( \frac{m_s}{m_s + m_\phi} \right)^{1/2} \left( \frac{m_\phi}{H} \right)^2 \approx \left( \frac{m_s}{m_s + m_\phi} \right)^{1/2}$, and subsequently grows until $\delta_p \approx 1$ at $H \approx \left( \frac{m_\phi}{m_\phi + m_\psi} \right)^{1/4} m_s$. 

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Thermalization of the $\psi$ condensate is possible if the rate of scattering of the plasma particles in equilibrium at temperature $T_r$ from the condensate scalars is sufficiently large. For the case of t-channel scattering of condensate scalars from plasma fermions via SU(3) gauge boson or Yukawa fermion exchange interactions, the scattering rate is given by $\Gamma_s = k_\Gamma \sigma \lambda_\psi^4 T_r$, where $\lambda_\psi$ is the gauge or Yukawa coupling, $\sigma = \frac{1}{x(1+2x)}$ for the gauge boson exchange and $\sigma = \frac{1}{4} \log \left( \frac{1}{x} \right)$ for the Yukawa fermion exchange, with $x = \frac{m_A^2}{9T^2}$ and $m_A$ the mass of the exchanged gauge boson or fermion, and where for scattering from a single Dirac fermion $k_\Gamma \approx \frac{1}{12\pi^3} \approx 3 \times 10^{-3}$.

The condition for the plasma to be able to thermalize the condensate is then that $\Gamma_s \gtrsim H$, which implies that

$$\lambda_\psi \gtrsim 5 \times 10^{-2} \left( \frac{H}{m_\psi} \right)^{3/16} \left( \frac{10^{10} \text{GeV}}{T_r} \right)^{1/8} \left( \frac{1}{\sigma} \right)^{1/4}. \tag{5.2}$$

We see that this will be satisfied by the gauge couplings, the top quark Yukawa coupling and, marginally, the bottom quark Yukawa coupling. Thus, in order to prevent the thermalization of the condensate, we must require that $\lambda_\psi < \langle \psi \rangle \gtrsim 3T$ for these couplings, to ensure that the associated scattering processes are kinematically suppressed. To be safe, we will conservatively require that $\lambda_\psi < \langle \psi \rangle \gtrsim 30T$, in order to suppress scattering from plasma particles with energy larger than the mean thermal energy $3T$. For the case of the $u^c d^c d^c$ direction, we note that the smallest Yukawa coupling to the condensate scalar $\psi$, which will typically involve a linear combination of all three generations of down squark, will equal the b quark Yukawa coupling up to a factor of the order of 1. Thus, in general, kinematically suppressing the b quark Yukawa interaction will ensure that the $\psi$ condensate is not thermalized. Although the b quark Yukawa coupling, $\lambda_b \approx 4 \times 10^{-2}$ for the MSSM with equal Higgs doublet expectation values, only marginally satisfies the condition for thermalization, and so might allow the condensate to form without large suppression of the asymmetry, if we were to consider t-channel fermion exchange scattering from light gauginos and squarks in the plasma, then the constraint (5.2) would apply to the combination $\lambda_\psi \approx (g \lambda_b)^{1/2}$, where $g$ is a gauge coupling. This would exceed the lower bound (5.2). Thus we will conservatively
assume that the b quark Yukawa interaction must be kinematically suppressed in order to avoid thermalization of the condensate on a time scale small compared with $H^{-1}$ and so to allow the asymmetries to form.

The condition $\lambda_\psi < \psi > \gtrsim \kappa T_r$ corresponds to
\[
T_R \lesssim \frac{\lambda_\psi^2}{\kappa^2 k_r^2} (m_\phi + m_s) M_{Pl}^{1/2} \left( \frac{H}{m_s} \right)^{3/2}, \tag{5.3}
\]
where during matter domination,
\[
< \psi > = \left( \frac{a(t)}{a_0} \right)^{3/2} \psi_{R o} \approx \frac{M^{1/2} (m_s^2 + m_s m_\phi)^{1/4}}{\lambda^{1/2} \left( \frac{H}{m_s} \right)}, \tag{5.4}
\]
with $a_0$ the scale factor at $H \approx m_s$. Using equation (4.11) together with the observed B asymmetry, $\frac{m_s}{s} \approx 10^{-10}$, we find that
\[
T_R \lesssim \frac{10^{-5} M^{3/4} m_s^{1/4} \lambda_\psi}{\sqrt{2 \pi s_0^{1/2}}} \frac{\kappa k_r}{(m_\phi + m_s)^{1/2}} \left( \frac{H}{m_s} \right)^{3/4} \tag{5.5}
\]
Thus, with $\lambda_\psi \equiv \lambda_b \approx 4 \times 10^{-2}$ for the b quark Yukawa coupling and with $\kappa = 30$, we find that the condensate will survive if
\[
T_R \lesssim \frac{1}{\sqrt{80}} \left( \frac{m_\phi + m_s}{m_s} \right)^{3/4} \left( \frac{H}{m_s} \right)^{7/4} 10^7 \text{GeV} \tag{5.6}
\]
where we have used $m_s \approx 10^2 \text{GeV}$. The form of this constraint depends on whether we impose the no-evaporation constraint before the asymmetry has fully formed or not. If we consider the constraint to apply at $H \approx m_s$, when the $\psi$ asymmetry is minimal, then the $T_R$ upper bound is proportional to $(\frac{m_\phi + m_s}{m_s})^{3/4}$. On the other hand, if we allow the $\psi$ asymmetry to grow to its maximum value before thermalization, then the $T_R$ upper bound is proportional to $(\frac{m_\phi + m_s}{m_s})^{5/16}$. In both cases the upper bound on $T_R$ is weakened by having $m_\phi > m_s$. We see that, with $m_\phi \gtrsim m_s$, $T_R$ can take values up to around $\frac{10^7 \text{GeV}}{\sqrt{80}}$ without preventing the formation of the asymmetry. With $m_\phi > m_s$ this constraint becomes weaker, allowing any reheating temperature up to the thermal gravitino limit to be compatible with the no-evaporation constraint, regardless of the value of $\tilde{\theta}$, so long as $m_\phi$ is sufficiently large.
We also have to check that the $\psi$ condensate does not decay before the condensates fully form at $H \approx (m_s / (m_s + m_\phi))^{1/4} m_s$. If $\lambda \psi < \psi > \lesssim m_s$, then the two-body tree-level decay will be kinematically suppressed. This occurs if

$$\frac{\lambda \psi}{\sqrt{\lambda}} > \left( \frac{m_s}{M_{Pl}} \right)^{1/2}$$

which will almost certainly be satisfied. The higher-order decay modes will then be suppressed by a factor of at least $\left( \frac{m_s}{\lambda \psi < \psi >} \right)^4$, which gives $\Gamma_d \ll H$ at $H \approx m_s$. Thus $\psi$ decay is ineffective at $H \approx m_s$ and the no-evaporation constraint is the correct condition for the initial $\psi$ asymmetry to be able to form.

A second, perhaps less important, constraint on the reheating temperature comes from the requirement that $\frac{m_s}{s} \approx 10^{-10}$ can be consistent with non-renormalizable operators whose natural mass scale is the Planck scale, corresponding to $|\lambda| \lesssim 1$. In fact, from (4.11), we see that, with $\frac{m_s}{s} \approx 10^{-10}$, the reheating temperature is given by

$$T_R \approx \frac{|\lambda|}{2\pi s_\theta \delta_\rho} \left( \frac{m_\phi + m_s}{m_s} \right)^{1/2} 10^9 \text{GeV}$$

(The conventional $\psi^4$ d=4 models give the same result but with $m_\phi \to 0$ and $\delta_\rho \approx 1$ [4].) Thus we see that $|\lambda| \lesssim 1$ is necessary in order for $T_R$ to be consistent with the thermal gravitino bound. Although $|\lambda| \lesssim 1$ is possible even if the mass scale of the non-renormalizable operators is small compared with $M_{Pl}$, it is most natural for the case of Planck scale operators. Thus d=4 models are most naturally consistent with the thermal gravitino bound when the mass scale of the non-renormalizable operators corresponds to the Planck mass. For the case of a conventional single field A-D mechanism with d=6 operators, the reheating temperature is given by [4]

$$T_R \approx \left( \frac{|\lambda|^{1/2}}{s_\theta} \right) 10 \text{GeV}$$

Thus in this case the reheating temperature is expected to be very low compared with the thermal gravitino bound for the case of Planck scale non-renormalizable operators, although for non-renormalizable operators with a smaller mass scale, which would naturally have $|\lambda| \gg 1$, larger reheating temperatures would be possible.
Therefore we see that $d=4$ models are favoured for the case of Planck scale operators, naturally allowing for a much wider range of reheating temperatures than $d=6$ models, whilst $d=6$ models are favoured if the natural mass scale of the non-renormalizible operators is much smaller than the Planck scale.
6. Condensate Thermalization and Decay after Reheating and an Upper Limit on Dirac Neutrino Masses.

After the inflaton decays, the Universe will be radiation dominated with \( H = \frac{k_H T^2}{M_{Pl}} \), where \( k_H = \left( \frac{4\pi^3 g(T)}{45} \right)^{1/2} \approx 17 \). We first show that \( \psi \) will typically thermalize at a temperature large compared with the temperature of the electroweak phase transition \( T_{EW} \approx 10^2 \text{GeV} \). \( \Gamma_s > \sim H \) occurs if \( \lambda \psi > \sim \left( \frac{k_H}{k_T} \right)^{1/4} \left( \frac{T}{M_{Pl}} \right)^{1/4} \left( \frac{1}{\sigma} \right)^{1/4} \). Since \( T_R \approx 10^{10} \text{GeV} \), this will be satisfied if \( \lambda \psi > 5 \times 10^{-2} \left( \frac{1}{\sigma} \right)^{1/4} \). Thus so long as the scattering process in not kinematically or Boltzmann suppressed, corresponding to \( \frac{\lambda \psi < \psi >}{3T} \) smaller than 1, the condensate will thermalize. With, for \( T < T_R \),

\[
< \psi > \approx 10^{-2} \left( \frac{T_R}{10^{10} \text{GeV}} \right)^{1/2} \left( \frac{m_{\phi} + m_s}{m_s} \right)^{1/4} \left( \frac{T}{m_s} \right)^{3/2} \tag{6.1}
\]

where we are using \( m_s \approx 10^2 \text{GeV} \) throughout, \( \frac{\lambda \psi < \psi >}{T} \approx 1 \) requires that

\[
\frac{\lambda \psi}{\sqrt{\lambda}} \lesssim 10^4 \left( \frac{m_s}{m_{\phi} + m_s} \right)^{1/4} \left( \frac{10^{10} \text{GeV}}{T_R} \right)^{1/2} \left( \frac{m_s}{T} \right)^{1/2} \tag{6.2}
\]

This will typically be satisfied for some value of \( T \) larger than \( T_{EW} \).

For the case of the gauge singlet \( \phi \) condensate, we first note that since the \( \phi \) scalar is \( R_p \) odd, it can decay only if it is not the lightest supersymmetric partner (LSP). The most rapid possible \( \phi \) decay will correspond to a tree-level two-body decay to a left-handed neutrino and a neutralino via the neutrino Yukawa coupling \( \lambda_\nu \). This will be kinematically allowed so long as one of the neutralinos has a mass less than the \( \phi \) scalar mass. In particular, this will occur if one of the neutralinos is the LSP, as is strongly favoured by the possibility of neutralino cold dark matter. The \( \phi \) decay rate will then depend on the proportion of light mass eigenstate neutralino(s) contained in the weak eigenstate fermion in \( H_u \). This will in turn depend on whether the light neutralino in question is mostly gaugino or Higgsino. If it is mostly Higgsino, then the decay will occur via the neutrino Yukawa coupling with coupling strength approximately equal to \( \lambda_\nu \). On the other hand, if it is mostly gaugino, then we would expect the coupling of the \( \phi \) scalar to the light neutralino to have an additional suppression factor of the order of \( \frac{m_{\phi}}{\mu} \), where \( \mu \) corresponds to the \( \mu H_u H_d \) term in
the MSSM superpotential [1]. Typically, for \( \mu \lesssim 1 \text{TeV} \), this factor will not be much smaller than about 0.1.

Thus in the case of a mostly Higgsino light neutralino (or, more generally, for the case where the mass term \( \mu \) is small compared with the \( \phi \) scalar mass), tree-level two-body \( \phi \) decay will occur via the neutrino Yukawa coupling \( \lambda \) if \( \Gamma_d \approx \alpha_\nu \sim \frac{\lambda_\nu^2}{4\pi} \). This is satisfied if

\[
\lambda_\nu \gtrsim \left( \frac{16\pi k_H}{(m_\phi + m_s) M_{Pl}} \right)^{1/2} T \quad (6.3).
\]

The decay is kinematically allowed so long as \( \lambda_\nu < \phi > \lesssim (m_\phi^2 + m_s^2)^{1/2} \). With, for \( T \lesssim T_R \),

\[
< \phi > \approx \frac{T_R^2}{m_s M_{Pl}} \frac{m_s^2 (m_s^2 + m_s m_\phi)^{3/4}}{(m_\phi + m_s)^3} \left( \frac{M_{Pl}}{\lambda} \right)^{1/2} \left( \frac{T}{T_R} \right)^{3/2} \quad (6.4),
\]

this condition requires that

\[
\frac{\lambda_\nu}{\sqrt{\lambda}} \lesssim 10^4 \left( \frac{10^{10} \text{GeV}}{T_R} \right)^{1/2} \left( \frac{m_\phi + m_s}{m_s} \right)^{13/4} \left( \frac{m_s}{T} \right)^{3/2} \quad (6.5).
\]

It is straightforward to show that, in the cases of most interest to us here, the decay of the \( \phi \) condensate typically occurs before it can thermalize by scattering. The condition for the condensate to thermalize by scattering via the Yukawa coupling \( \lambda_\nu \) is that \( \Gamma_s \approx k_T \sigma \lambda_\nu^4 T > H = \frac{k_H T^2}{M_{Pl}} \), where \( \sigma \approx \frac{1}{2} \log \left( \frac{3T}{\lambda_\nu <\psi>} \right) \). This implies that

\[
\lambda_\nu \gtrsim 5 \times 10^{-4} \left( \frac{T}{m_s} \right)^{1/4} \left( \frac{1}{\sigma} \right)^{1/4} \quad (6.6).
\]

This lower bound is typically larger than the lower bound on \( \lambda_\nu \) coming from \( \phi \) decay. It is possible that thermalization could occur by scattering from light sleptons and SU(2) gauginos in the plasma, which would replace \( \lambda_\nu \) by \( (\lambda_\nu g)^{1/2} \), where \( g \approx 0.6 \) is the SU(2) gauge coupling. However, the lower bound on \( \lambda_\nu \) will still typically be larger than that coming from \( \phi \) decay. In particular, this is true for the important case of \( \phi \) decay below the temperature of the electroweak phase transition, which we discuss below. Another possibility for thermalizing the condensate is via inverse decays and related \( 2 \to 1 \) processes, which are expected to have a rate \( \Gamma_{inv} \approx \kappa \lambda_\nu^2 T \),
where $\kappa \ll 1$. Although at high temperature this rate can be large compared with the $\phi$ decay rate, for temperatures at or below the electroweak phase transition temperature, which are of most interest to us here, the direct decay rate will be much larger than the rate of thermalization via inverse decays. In all this we have assumed that the $\phi$ decay occurs via the neutrino Yukawa coupling. It is also possible that the $\phi$ condensate could decay via the non-renormalizable superpotential coupling $\frac{\lambda}{M} \phi u^c d^c d^c$ once $<\psi>$ is introduced, which gives an effective $\phi\psi\psi$ coupling. However, it is straightforward to check that this effective coupling is in general much smaller than the typical values of $\lambda_\nu$ considered in neutrino mass models, and so may be neglected when discussing $\phi$ condensate decay.

Thus we see that the $\phi$ condensate can evade decay until $T < T_{\text{EW}}$ if $\lambda_\nu$ is sufficiently small. To see what this implies for the baryon asymmetry and for neutrino masses, we first note that the gauge singlet scalar $\phi$ will typically correspond to a linear combination of the three right-hand sneutrino generations. Thus $\lambda_\nu$ will typically correspond to the largest neutrino Yukawa coupling up to a factor of around $\frac{1}{\sqrt{3}}$. In the $m_\phi \to 0$ limit, corresponding to the case where the neutrinos have Dirac masses, we can define an unbroken B-L asymmetry by defining $\phi$ to have B=1. However, so long as the $\phi$ condensate decays after the electroweak phase transition has occurred, the effect of anomalous electroweak B+L violation, which is in thermal equilibrium at temperatures larger than $T_{\text{EW}}$, will be to alter only the B asymmetry coming from the thermalized $\psi$ condensate and so prevent a cancellation of the B asymmetry coming from the $\psi$ and $\phi$ condensate, even though the net B-L asymmetry will be zero. Thus so long as the $\phi$ condensate decays after the electroweak phase transition has occurred, the B asymmetry will be essentially the same as that previously calculated from the decay of the $\psi$ condensate alone, since the magnitude of the net B asymmetry will, up to a factor of the order of 1, equal that of the B-L asymmetry coming from $\psi$ thermalization at $T \approx T_{\text{EW}}$. The coupling $\lambda_\nu$ is related to the heaviest neutrino mass in the $m_\phi \to 0$ limit by $m_\nu \approx 10^2 \lambda_\nu \text{GeV}$. From this and (6.3) we find that, for the case of a mostly
Higgsino light neutralino, the $\phi$ condensate decays at a temperature $T_{d\phi} \approx 10^7 m_\nu$. Thus the condition $T_{d\phi} < T_{EW}$ implies that all Dirac neutrino masses should satisfy

$$m_\nu \lesssim 10\text{keV} \quad (6.7),$$

assuming that $\phi$ corresponds to a roughly equal combination of the three sneutrino generations, as we would generally expect. This is true for the case of a light neutralino which is mostly Higgsino, or more generally for the case where the $\phi$ decay via the Yukawa coupling $\lambda_\nu$ is completely unsuppressed. From this we see that so long as the Dirac neutrino masses are all below about 10keV, the $\phi$ condensate will generally decay below the temperature of the electroweak phase transition and so a baryon asymmetry will be generated even in the limit of B-L conservation. On the other hand, for the case of, for example, a mostly gaugino LSP this upper bound could be increased to around 100keV or more, depending on the particular gaugino LSP mass eigenstate and the $\mu$ parameter. These upper bounds should be compared with the present experimental upper bounds on the neutrino masses, $m_{\nu_e} < 24\text{MeV}$, $m_{\nu_\mu} < 160\text{keV}$ and $m_{\nu_\tau} < 5.1\text{eV}$ [13]. From these we see that the requirement that a non-zero B asymmetry can be generated in the limit of unbroken B-L in the case where the LSP is a neutralino imposes a non-trivial upper bound on $m_{\nu_\mu}$ and $m_{\nu_\tau}$.

In particular, we see that it would be possible, in principle, to experimentally rule out this class of Affleck-Dine models, for example if neutrinos with Dirac masses significantly larger than around 10keV were found to exist together with an LSP which was mostly Higgsino. We also note that an unbroken B-L asymmetry would rule out the possibility of the $d=4$ LH$_u$ direction, leaving the $d=4$ u$^c$d$^c$d$^c$ direction as the unique $d=4$ possibility in the case of B-L conserving models with non-zero neutrino masses.

For the case with $m_\phi \neq 0$, the neutrinos will gain Majorana masses via the see-saw mechanism [14], with $m_\nu \approx \frac{\lambda_\nu^2 100\text{GeV}}{m_\phi}$. Thus in this case $T_{d\phi}$ is given by

$$T_{d\phi} \approx 10^4 \left( \frac{m_\nu}{1\text{eV}} \right)^{1/2} \left( \frac{m_\phi}{100\text{GeV}} \right)^{1/2} \left( \frac{m_\phi + m_s}{100\text{GeV}} \right)^{1/2} \text{GeV} \quad (6.8).$$

Therefore typically the $\phi$ condensate will decay at $T > T_{EW}$ in this case.
Throughout the above discussion we have assumed that the Universe is radiation dominated. It is straightforward to show that this is indeed the case. The $\psi$ condensate would dominate the energy density only once $T$ satisfies

$$T < \frac{(m_{s}^{2} + m_{\phi}^{2})^{1/2}}{\lambda} \frac{T_{R}}{M_{Pl}} \quad (6.9),$$

which is typically satisfied only for temperatures less than around $10^{-7}$GeV. For the $\phi$ condensate the energy density is even less than the $\psi$ condensate, by a factor $\left( \frac{m_{s}}{m_{\psi} + m_{s}} \right)^{3}$. Thus the Universe will be radiation dominated when the condensates thermalize or decay.
7. Conclusions

We have considered the possibility of generating the observed baryon asymmetry via an Affleck-Dine mechanism based on the renormalizable F- and D-flat u’d’d’c’ direction of the SUSY Standard Model. In order to avoid breaking $R_p$ whilst allowing a d=4 superpotential term to lift the flatness and drive baryogenesis, we considered extensions of the SUSY Standard Model which have additional gauge singlets $\phi$, such as commonly occur in models which seek to account for neutrino masses. In such models the u’d’d’ direction becomes potentially as important as the more commonly considered LH$_u$ direction. We have shown that the A-D mechanism based on the d=4 operator $\phi \psi^3$, where $\psi$ is a gauge non-singlet A-D field, can indeed (for an appropriate choice of parameters) generate the baryon asymmetry whilst allowing the scalar fields to evolve to a phenomenologically acceptable minimum. The resulting asymmetry is suppressed relative to the asymmetry coming from the more conventional $\psi^4$-based A-D mechanism (such as the LH$_u$ direction) by a factor $\left(\frac{m_{\phi}}{m_{\phi} + m_s}\right)^{1/2}$, all couplings and CP phases being taken equal, where $m_{\phi}$ is the SUSY $\phi$ mass term and $m_s$ is the soft SUSY breaking mass scale. This suggests that $m_{\phi}$ cannot be much larger than $m_s$, if the observed baryon asymmetry is to be generated without requiring very small couplings in the non-renormalizable terms. The requirement that the initial condensate particle asymmetry can form before the condensate is thermalized imposes an upper bound on the reheating temperature of $\frac{10^7 \text{GeV}}{\sqrt{\theta}}$ in the limit where $m_{\phi} \lesssim m_s$, where $\theta$ is the CP violating phase responsible for the baryon asymmetry. The upper bound becomes weaker if $m_{\phi} \gtrsim m_s$. Thus with a sufficiently large $m_{\phi}$ or small $\theta$ the whole range of reheating temperatures up to the thermal gravitino constraint can be compatible with the initial formation of an asymmetry. The $\psi$ condensate will typically thermalize before the electroweak phase transition occurs. Then in the limit of unbroken B-L (for which case the u’d’d’c’ direction is the unique d=4 possibility), which corresponds to $m_{\phi} \rightarrow 0$, a B asymmetry can be generated only if the $\phi$ condensate decays below the temperature of the electroweak phase transition, when anomalous electroweak B+L violation is
out of thermal equilibrium. This will generally be true if all neutrino masses are less than around 10keV. In the case where the LSP is a neutralino, or more generally where there is a neutralino mass eigenstate lighter than the $\phi$ scalar, the $\phi$ decay condition imposes a non-trivial upper limit on Dirac neutrino masses. For example, for the case of a mostly Higgsino LSP, the upper bound is around 10keV, whilst for a mostly gaugino LSP this bound could increase to around 100keV or more, depending on the $\mu$ parameter of the MSSM and the particular gaugino LSP mass eigenstate. Thus the observation of a Dirac mass for the $\mu$ or $\tau$ neutrino significantly larger than 10keV together with a mostly Higgsino LSP, for example, would experimentally rule out this class of Affleck-Dine models.

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References

[1] H.P.Nilles, Phys.Rep. 110 (1984) 1

[2] I.Affleck and M.Dine, Nucl.Phys. B249 (1985) 361

[3] E.Copeland, A.Liddle, D.Lyth, E.Stewart and D.Wands, Phys.Rev. D49 (1994) 6410,
M.Dine, L.Randall and S.Thomas, Phys.Rev.Lett. 75 (1995) 398,
G.Dvali, preprint hep-ph/9503259 (1995),
E.D.Stewart, Phys.Rev.D51 (1995) 6847

[4] M.Dine, L.Randall and S.Thomas, Nucl.Phys. B458 (1996) 291

[5] E.W.Kolb and M.S.Turner, The Early Universe, (Addison-Wesley, Reading MA (1990))

[6] V.A.Kuzmin, V.A.Rubakov and M.E.Shaposhnikov, Phys.Lett. 155B (1985) 36

[7] E.D.Stewart, M.Kawasaki and T.Yanagida, preprint hep-ph/9603324 (1996)

[8] The phenomenology of B and L violating coupling in the MSSM was first discussed by F.Zwirner, Phys.Lett. 135B (1984) 516. For a review of the constraints on $R_p$ violating couplings see F.Visssani, preprint hep-ph/9602393

[9] R.K.Schaefer and Q.Shafi, Phys.Rev.D47 (1993) 1333,
A.R.Liddle, Phys.Rev. D49 (1994) 739

[10] J.Ellis, A.Linde and D.Nanopoulos, Phys.Lett. 118B (1982) 59,
M.Yu.Khlopov and A.Linde, Phys.Lett. 138B (1984) 265,
J.Ellis, J.E.Kim and D.V.Nanopoulos, Phys.Lett. 145B (1984) 181

[11] A.D.Linde, Phys.Lett. 129B (1983) 177

[12] M.K.Gaillard, H.Murayama and K.A.Olive, Phys.Lett. 355B (1995) 71

[13] Particle Data Group, Phys.Rev.D50 (1994) 1173
[14] M.Gell-Mann, P.Ramond and R.Slansky, in: Supergravity, eds. D.Z.Freedman and P.van Nieuwenhuizen (North-Holland, Amsterdam (1980)).