Ground-State Masses and Magnetic Moments of Heavy Baryons

Zahra Ghalenovi\textsuperscript{a,b}, Ali Akbar Rajabi\textsuperscript{b}, Si-xue Qin \textsuperscript{a}, Dirk. H. Rischke\textsuperscript{a,c}

\textsuperscript{a}Institute for Theoretical Physics, Johann Wolfgang Goethe University, Frankfurt am Main, Germany
\textsuperscript{b}Physics Department, Shahrood University of Technology, Shahrood, Iran
\textsuperscript{c}Frankfurt Institute for Advanced Studies, Frankfurt am Main, Germany

Abstract
In this work we study single, double, and triple heavy-flavor baryons using the hypercentral approach in the framework of the non-relativistic quark model. Considering two different confining potentials and an improved form of the hyperfine interaction, we calculate the ground-state masses of heavy baryons and also the ground-state magnetic moments of single charm and beauty baryons with $J^P = 3/2^+$. The obtained results are in good agreement with experimental data and those of other works.

Key words: Heavy baryons, hypercentral approach, confining potential, non-relativistic quark model.

PACS Nos.: 14.20.Mr, 14.2.Lq, 12.39.Pn

1 Introduction

The properties of heavy-flavor baryons have recently received much attention, both experimentally and theoretically \cite{1, 4, 2, 3}. The investigation of the properties of such hadrons is not only important to understand the dynamics of quantum chromodynamics (QCD) at hadronic energy scales, but also interesting in view of the recent progress in studying heavy-flavor hadrons by different experimental groups like BaBar, BELLE, BESIII, CLEO, and SELEX. Different methods based on the constituent quark model (CQM) have been used to investigate heavy-flavor baryons. Ebert et al. studied heavy baryons in the quark-diquark model in the relativistic limit \cite{5}. Reference \cite{4} investigated heavy-flavor baryons by using the Bethe-Salpeter equation in the heavy-quark limit and calculated the Isgur-Wise function. Albertus et al. evaluated different properties of single heavy-flavor baryons using heavy-quark symmetry in the non-relativistic quark model \cite{6}. Flynn et al. studied charmed baryons and spin-splittings in quenched lattice QCD \cite{7}. Faessler et al. considered ground-state magnetic moments of heavy baryons in the relativistic quark model using heavy-hadron chiral perturbation theory \cite{8}. Patel et al. used the non-relativistic quark model with a hypercentral Coulomb plus linear potential and obtained masses and magnetic moments of heavy-flavor baryons \cite{9, 10}.

In the present work we calculate the ground-state masses and magnetic moments of heavy baryons in the hypercentral approach \cite{11, 12, 13, 14, 15, 16, 17, 18, 19}. We study the three-body problem, particularly the baryons containing one, two, and three charm (beauty) quarks. The potential is assumed to be a combination of a long-range confinement part and a short-range potential which is a Coulombic one, depending on the color charge.

The solution of a three-body problem in three spatial dimensions is rather difficult. Here, we employ the hypercentral approach where the Schrödinger equation of the three-body system depends only on a single variable. We solve this one-dimensional Schrödinger equation numerically. We also introduce a non-confining interquark potential, namely a spin-isospin dependent part, as hyperfine interaction. We study the baryonic systems using two types of potentials. First, we introduce the Cornell potential, $bx - c/x$, as confining potential between quarks and obtain the masses of heavy baryons. Second, we add a harmonic oscillator term to the confining potential and then compare the obtained baryon masses to the results without this term, and also to those of other works. The obtained masses and magnetic moments are close to experimental data and other theoretical predictions.
This paper is organized as follows. In Sec. 2 we introduce the interquark potential. In Sec. 3 we simplify the three-body problem using the hypercentral approach. We present our method to obtain masses and magnetic moments of baryons in Sec. 4. Numerical results are shown and compared to those of other works in Sec. 5. Finally, a summary is given in Sec. 6.

2 Interaction Potential

In principle, the potential between quarks could be of any confining form (e.g. linear, logarithmic, power law, etc.). The interquark potential usually contains a linear part which describes confinement in QCD and is supplemented by a Coulomb term which may be attributed to one-gluon exchange. The Coulomb term alone is not sufficient because it would allow ionization of quarks from the system. As a first case (in the following termed “case I”), we consider the Cornell potential \[20, 21\]:

\[
V(x) = bx - \frac{c}{x},
\]

where \(x\) is the relative coordinate of the quark pair, and \(b, c\) are constants. In many practical applications a harmonic oscillator (h.o.) potential yields spectra not much different from those for Eq. (1) \[20\]. Therefore, as a second case (termed “case II”) we also consider a potential which is a combination of Eq. (1) and the h.o. potential which has the form \(ax^2\):

\[
V(x) = ax^2 + bx - \frac{c}{x},
\]

where \(a\) is another constant. In addition, we introduce a spin- and isospin-dependent potential as hyperfine interaction for the baryons. This combination of potentials yields spectra which are very close to the experimental results and other theoretical predictions.

The non-confining spin-spin interaction potential is proportional to a \(\delta\)-function which is an illegal operator term \[22\]. We modify it to a Gaussian function of the relative distance of the quark pair,

\[
H_S = A_S \frac{\hat{s}_1 \cdot \hat{s}_2}{(\sqrt{\pi} \sigma_S)^3} \exp \left( -\frac{x^2}{\sigma^2_S} \right),
\]

where \(s_i\) is the spin operator of the \(i^{th}\) quark (\(\hat{s}_i = \hat{\sigma}_i/2\), with \(\hat{\sigma}_i\) being the vector of Pauli matrices) and \(A_S\) and \(\sigma_S\) are constants.

Other spin-, as well as isospin-dependent interaction potentials can arise from quark-exchange interactions. We conclude that two additional terms should be added to the Hamiltonian for quark pairs which result in hyperfine interactions similar to Eq. (3). The first one depends on isospin only and has the form \[22, 23\]:

\[
H_I = A_I \frac{\hat{t}_1 \cdot \hat{t}_2}{(\sqrt{\pi} \sigma_I)^3} \exp \left( -\frac{x^2}{\sigma^2_I} \right),
\]

where \(t_i\) is the isospin operator of the \(i^{th}\) quark, and \(A_I\) and \(\sigma_I\) are constants. The second one is a spin-isospin interaction given by \[22, 23\]:

\[
H_{SI} = A_{SI} \frac{(\hat{s}_1 \cdot \hat{s}_2)(\hat{t}_1 \cdot \hat{t}_2)}{(\sqrt{\pi} \sigma_{SI})^3} \exp \left( -\frac{x^2}{\sigma^2_{SI}} \right),
\]

where \(s_i\) and \(t_i\) are the spin and isospin operators of the \(i^{th}\) quark, respectively, and \(A_{SI}\) and \(\sigma_{SI}\) are constants. Then, from Eqs. (3-5) the hyperfine interaction (a non-confining potential) is given by

\[
H_{int}(x) = H_S(x) + H_I(x) + H_{SI}(x).
\]

The parameters of the hyperfine interaction (6) are given in Table 1.
Table 1. Constituent quark masses and hyperfine - potential parameters used in cases I and II [11, 25].

| parameter | value    |
|-----------|----------|
| $m_u$     | 330 MeV  |
| $m_d$     | 335 MeV  |
| $m_s$     | 469 MeV  |
| $m_c$     | 1600 MeV |
| $m_b$     | 4980 MeV |
| $\sigma_S$ | 2.87 fm |
| $A_S$     | 67.4 fm² |
| $\sigma_{SI}$ | 2.31 fm |
| $A_{SI}$  | 106.2 fm²|
| $\sigma_I$ | 3.45 fm |
| $A_I$     | 51.7 fm² |

3 The Hypercentral Approach

In the quark model, a baryon is a three-body bound state made of quarks. The mathematical description of a three-body system is more complicated than that of a two-body system. Several methods have been used by different authors to solve three-body problems [17, 18, 19, 22, 23, 24, 25, 26].

In order to describe the baryon as a bound state of three constituent quarks, we define the configuration of three particles by two Jacobi coordinates $\vec{\rho}$ and $\vec{\lambda}$ as

$$\vec{\rho} = \frac{1}{\sqrt{2}}(\vec{r}_1 - \vec{r}_2), \quad \vec{\lambda} = \frac{1}{\sqrt{6}}(\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3),$$

such that

$$m_\rho = \frac{2m_1m_2}{m_1m_2}, \quad m_\lambda = \frac{3m_3(m_1 + m_2)}{2(m_1 + m_2 + m_3)}.$$  

Here $m_1$, $m_2$, and $m_3$ are the constituent quark masses. Instead of $\rho$ and $\lambda$, one can introduce hyperspherical coordinates which are given by the angles $\Omega_\rho = (\theta_\rho, \phi_\rho)$ and $\Omega_\lambda = (\theta_\lambda, \varphi_\lambda)$, respectively, together with the hyperradius $x$ and the hyperangle $\zeta$, defined by

$$x = \sqrt{\rho^2 + \lambda^2}, \quad \zeta = \arctan\left(\frac{\rho}{\lambda}\right).$$

Therefore, the Hamiltonian will be

$$H = \frac{P_\rho^2}{2m_\rho} + \frac{P_\lambda^2}{2m_\lambda} + V(\rho, \lambda) = \frac{P^2}{2m} + V(x).$$

In the hypercentral constituent quark model (hCQM), the quark potential, $V$, is assumed to depend only on the hyperradius $x$. Therefore, in the three-quark wave function one can factor out the hyperangular part which is given by hyperspherical harmonics. The remaining hyperradial part of the wave function is determined by the hypercentral Schrödinger equation

$$\left[\frac{d^2}{dx^2} + \frac{5}{x} \frac{d}{dx} - \frac{\gamma(\gamma + 4)}{x^2}\right] \psi_\gamma(x) = -2m[E_\gamma - V(x)]\psi_\gamma(x),$$

where $\psi_\gamma(x)$, $E_\gamma$, and $\gamma$ are the hyperradial part of the wave function, the energy eigenvalues, and the grand angular quantum number, respectively. The latter is given by $\gamma = 2\nu + l_\rho + l_\lambda$ where $l_\rho$
and $l_\lambda$ are the angular momenta associated with the $\rho$ and $\lambda$ variables and $\nu$ is a non-negative integer number. The quantity $m$ in Eqs. (10,11) is the reduced mass,

$$m = \frac{2m_\rho m_\lambda}{m_\rho + m_\lambda}.$$  \hfill (12)

We use the transformation

$$\psi(x) = x^{-5/2} \phi(x)$$  \hfill (13)

to bring Eq. (11) into the form

$$\left[\phi''(x) - \frac{(2\gamma + 3)(2\gamma + 5)}{4x^2}\right] \phi(x) = -2m[E - V(x)] \phi(x).$$  \hfill (14)

Substituting the potentials (1) and (2) into Eq. (11) we obtain the following equations:

(I) In case I we only consider the Cornell potential (1) as confining interaction. Using the hyperradial approximation used in Ref. [11], the Schrödinger equation for the baryons is given as

$$\phi''(x) + 2\mu \left[-bx + \frac{c}{x} - \frac{(2\gamma + 3)(2\gamma + 5)}{8\mu x^2}\right] \phi(x) = -2\mu E \phi(x),$$  \hfill (15)

where $\mu = m$. As in Ref. [11], in the following we shall consider $\mu$ as a free parameter which is fitted to the baryon spectrum.

(II) In case II, we add the h.o. term to the confining interaction. Then, using the potential (2) and substituting it into Eq. (14) we obtain the following equation:

$$\phi''(x) + \left[-a_1 x^2 - b_1 x + \frac{c_1}{x} - \frac{(2\gamma + 3)(2\gamma + 5)}{4x^2}\right] \phi(x) = -\varepsilon \phi(x),$$  \hfill (16)

where

$$\varepsilon = 2mE, \quad a_1 = 2ma, \quad b_1 = 2mb, \quad c_1 = 2mc.$$  \hfill (17)

4 Heavy Baryon Masses and Magnetic Moments

We take the non-confining potential $H_{int}$, Eq. (6), as a perturbation of the energy eigenvalues obtained by solving Eqs. (15,16). To first order in perturbation theory, the correction can be computed using the unperturbed wave function for the ground state,

$$\langle H_{int} \rangle = \frac{\int \psi_0 H_{int} \psi_0 x^5 dx d\Omega_\rho d\Omega_\lambda}{\int \psi_0^2 x^5 dx d\Omega_\rho d\Omega_\lambda}.$$  \hfill (18)

Note that in Eqs. (3-5) the spin-isospin dependent interaction potentials of a two-quark system are actually functions of the relative distance between the quarks. In the hypercentral approach, however, we take the same form for these potentials, but replace the relative distance by the hyperradius $x$ which is the average relative distance between the three quarks. We believe that this is a reasonable approximation, at least for quarks with the same mass. The spin-spin term $\vec{s}_1 \cdot \vec{s}_2$ in Eq. (3) is replaced by the average of $\sum_{i<j} \hat{s}_i \cdot \hat{s}_j$, and similarly for Eqs. (4,5).

The mass of the baryon is then obtained as the sum of the masses of the constituent quarks, the ground-state energy eigenvalue $E_0$, and $\langle H_{int} \rangle$,

$$M_B = m_1 + m_2 + m_3 + E_0 + \langle H_{int} \rangle.$$  \hfill (19)
where $E_0 + \langle H_{\text{int}} \rangle$ depends on the type of confining interaction used. The effective quark mass is defined as

$$m_i^{\text{eff}} = m_i \left(1 + \frac{E_0 + \langle H_{\text{int}} \rangle}{\sum_i m_i} \right),$$

(20)
such that the mass of the baryon is

$$M_B = \sum_i m_i^{\text{eff}}.$$  

(21)

The physical interpretation of the effective quark mass (20) is that, within a baryon, the mass of a quark may get modified due to its interactions with the other quarks.

In the quark model, the magnetic moment of the baryon is obtained as [9, 24]

$$\mu_B = \langle \phi_{s_f} | M_z | \phi_{s_f} \rangle,$$  

(22)

where $| \phi_{s_f} \rangle$ represents the spin-flavor wave function of the respective baryonic state and

$$\vec{M} = \sum_i \frac{g e_i \vec{s}_i}{2m_i^{\text{eff}}}.$$  

(23)

Here, $g = 2$ is the spin $g$–factor, $e_i$ is the electric charge, and $\vec{s}_i$ the spin of the $i$th quark.

5 Discussion

In Refs. [23, 25] heavy-flavor baryons were studied in the hypercentral approach with the confining interaction (2) and the hyperfine interactions (3-5). The Schrödinger equation was solved analytically to obtain masses of heavy-flavor baryons. In Ref. [11] heavy-flavor baryons were also studied in the hypercentral approach, but with the confining interaction (1) and the hyperfine interactions (3-5). The Schrödinger equation was solved using a variational method to obtain masses of single, double, and triple heavy-flavor baryons. Patel et al. used the non-relativistic quark model with hypercentral Coulomb plus linear potential [9, 24] and Coulomb plus harmonic oscillator potential [26] and obtained heavy-flavor baryon masses.

Table 2. The confining potential parameters used in case I and II [11, 25].

| parameter | value |
|-----------|-------|
| caseI b   | 1.61 $fm^{-2}$ |
| c         | 4.59  |
| a         | 0.73 $fm^{-3}$ |
| caseII b  | 0.81 $fm^{-2}$ |
| c         | 2.12  |

In the present work we use the same potentials and the hypercentral approach, but we solve the Schrödinger equation numerically. We study baryonic systems using the confining interactions (1) and (2), respectively. The quark masses and potential parameters used in both case I and II are obtained from our corresponding works [11] and [25], respectively, and are listed in Table 1. The confining potential parameters for the two cases are listed in Table 2. In case I, the parameter $\mu$ of Eq. (15) is obtained by fitting the experimental mass of the $\sum_{c}^{++}$ baryon (resulting in $\mu = 0.844 \, fm^{-1}$). Using $\mu$ as a fit parameter instead of identifying it with the reduced mass $m$ allows to make an accurate comparison between the results of our present work and previous results [11].

In Tables 3-8 the results for the masses and magnetic moments are presented and compared with other works [24, 25, 26, 27, 28, 29, 30, 29] and experimental data [31, 32, 33, 34, 35]. From Tables 3-8...
Table 3. Single charm baryon masses (masses are in MeV). The last two columns show the relative errors of cases I and II in comparison to experimental data.

| Baryon | Case I | Case II | Exp. | Ref. [11] | Ref. [23] | Ref. [25] | Ref. [26] | Error I | Error II |
|--------|--------|---------|------|-----------|-----------|-----------|-----------|---------|----------|
| ∑⁺c   | 2454   | 2459    | 2454 | 2318      | 2452      | 2454      | 2425      | 0.0%    | 0.2%     |
| ∑⁺⁺c  | 2492   | 2508    | 2518 | 2446      | 2581      | 2526      | 2488      | 1.0%    | 0.4%     |
| ∑⁺*c  | 2459   | 2461    | 2453 | 2323      | 2457      | 2458      | 2442      | 0.2%    | 0.3%     |
| ∑⁺*⁺c | 2497   | 2510    | 2518 | 2451      | 2586      | 2530      | 2507      | 0.8%    | 0.3%     |
| ∑⁰c   | 2464   | 2464    | 2454 | 2328      | 2461      | 2460      | 2460      | 0.4%    | 0.3%     |
| ∑⁰⁺c  | 2503   | 2512    | 2518 | 2456      | 2591      | 2533      | 2526      | 0.6%    | 0.2%     |
| ∑⁰⁺⁺c | 2576   | 2504    | 2468 | 2467      | 2466      | 2545      | 2512      | 4.4%    | 1.5%     |
| ∑⁰*c  | 2634   | 2583    | 2647 | 2577      | 2596      | 2614      | 2584      | 0.5%    | 2.4%     |
| ∑⁰*⁺c | 2581   | 2506    | 2471 | 2453      | Input     | 2547      | 2529      | 4.5%    | 1.4%     |
| Ξ⁺c   | 2576   | 2504    | 2468 | 2467      | 2466      | 2545      | 2512      | 4.4%    | 1.5%     |
| Ξ⁺⁺c  | 2634   | 2583    | 2647 | 2577      | 2596      | 2614      | 2584      | 0.5%    | 2.4%     |
| Ξ⁰c   | 2581   | 2506    | 2471 | 2453      | Input     | 2547      | 2529      | 4.5%    | 1.4%     |
| Ξ⁰⁺c  | 2634   | 2583    | 2647 | 2577      | 2596      | 2614      | 2584      | 0.5%    | 2.4%     |
| Ω₀𝑐   | 2715   | 2566    | 2697 | 2587      | 2476      | 2631      | 2601      | 0.7%    | 4.9%     |
| Ω₀⁺c  | 2773   | 2648    | 2768 | 2716      | 2476      | 2631      | 2601      | 0.7%    | 4.9%     |

Table 4. Single beauty baryon masses (masses are in MeV). The last two columns show the relative errors of cases I and II in comparison to experimental data.

| Baryon | case I | case II | Exp. | Ref. [11] | Ref. [23] | Ref. [25] | Ref. [26] | Error I | Error II |
|--------|--------|---------|------|-----------|-----------|-----------|-----------|---------|----------|
| ∑⁺b   | 5834   | 5808    | 5807 | 5700      | Input     | 5816      | 5772      | 0.5%    | 0.0%     |
| ∑⁺⁺b  | 5872   | 5858    | 5829 | 5826      | 5936      | 5888      | 5793      | 0.7%    | 0.5%     |
| ∑⁺*b  | 5839   | 5810    | 5811 | -         | -         | 5819      | 5793      | 0.5%    | 0.0%     |
| ∑⁺*⁺b | 5877   | 5860    | 5832 | -         | -         | 5890      | 5816      | 0.8%    | 0.5%     |
| ∑⁰b   | 5844   | 5811    | 5815 | 5708      | 5818      | 5821      | 5816      | 0.4%    | 0.1%     |
| ∑⁰⁺b  | 5882   | 5861    | 5836 | 5836      | 5946      | 5892      | 5840      | 0.8%    | 0.4%     |
| ∑⁰*b  | 5956   | 5848    | 5787 | 5828      | 5821      | 5886      | 5880      | 2.9%    | 1.1%     |
| ∑⁰*⁺b | 6014   | 5928    | -    | 5957      | 5956      | 5972      | 5907      | -       | -        |
| Ξ⁰b   | 5961   | 5849    | 5792 | 5833      | 5826      | 5887      | 5903      | 2.9%    | 1.0%     |
| Ξ⁰⁺b  | 6019   | 5930    | -    | 5962      | 5956      | 5974      | 5931      | -       | -        |
| Ω₀b   | 6095   | 5903    | 6054 | 5967      | -         | 5986      | 5994      | 0.7%    | 2.5%     |
| Ω₀⁺b  | 6135   | 5986    | -    | 6096      | 5961      | 6049      | 6028      | -       | -        |

we see that the results of the present work are in good agreement with experimental data and other predictions. A comparison between the results of case I and the previous work [11] shows that the results obtained in case I are closer to experimental data. Note that the results of case II are very close to the ones obtained by Refs. [23, 25].

By comparing the results of cases I and II, we find that, apart from the Ω𝑐 and Ω𝑏 baryons, the results obtained in case II are overall closer to experimental data and other predictions than the ones obtained in case I and also in previous works. Also Tables 6-8 show that the predicted masses of double and triple heavy-flavor baryons in case II are closer to the results of other works.

6 Summary

In this paper we have studied masses and magnetic moments of heavy-flavor baryons containing one, two, and three heavy-flavor quarks in the ground-state (γ = 0) for the different confining potentials (1) and (2). Using the hypercentral approach we have simplified the three-body problem and solved the Schrödinger equation numerically to obtain the ground-state energy eigenvalues and eigenfunctions
Table 5. Magnetic moments of single charm and single beauty baryons with $J^P = 3/2^+$ in terms of the nuclear magneton $\mu_N$.

| Baryon | caseI | caseII | Ref. [9] | Ref. [23] | Ref. [28] | Ref. [29] |
|--------|-------|--------|----------|----------|----------|----------|
| $\Sigma^+_{cc}$ | 4.10 | 3.766 | 3.842 | 3.739 | 3.407 | 3.560 |
| $\Sigma^0_{cc}$ | 1.32 | 1.220 | 1.252 | 1.210 | 1.130 | 1.170 |
| $\Xi^+_c$ | -1.44 | -1.333 | -0.848 | -1.322 | -1.146 | -1.230 |
| $\Xi^0_c$ | 1.04 | 1.503 | 1.513 | 1.485 | 1.264 | 1.430 |
| $\Omega^+_c$ | -1.18 | -1.124 | -0.688 | -1.111 | -0.986 | -1.000 |
| $\Xi^0_{cc}$ | -0.92 | -0.903 | -0.865 | -0.887 | -0.833 | -0.770 |
| $\Sigma^+_{bb}$ | 3.69 | 3.588 | 3.234 | 3.570 | 3.082 | - |
| $\Sigma^0_{bb}$ | 0.90 | 0.865 | 0.791 | 0.861 | 0.724 | - |
| $\Xi^+_b$ | -1.91 | -1.859 | -1.655 | -1.849 | -1.634 | - |
| $\Xi^0_b$ | 1.157 | 1.136 | 1.041 | 1.127 | 0.875 | - |
| $\Omega^-_{bb}$ | -1.65 | -1.621 | -1.095 | -1.609 | -1.477 | - |
| $\Omega^0_{bb}$ | -1.38 | -1.380 | -1.199 | -1.365 | -1.292 | - |

Table 6. Double and triple charm baryon masses (in MeV).

| Baryon | caseI | caseII | Ref. [11] | Ref. [23] | Ref. [30] | Ref. [29] |
|--------|-------|--------|----------|----------|----------|----------|
| $\Xi^{++}_{cc}$ | 3703 | 3532 | 3579 | 3583 | 3510 | 3676 |
| $\Xi^{++}_{uc}$ | 3765 | 3623 | 3708 | 3722 | 3548 | 3753 |
| $\Omega^+_{cc}$ | 3846 | 3667 | 3718 | 3592 | 3719 | 3815 |
| $\Omega^+_{uc}$ | 3904 | 3758 | 3847 | 3731 | 3746 | 3876 |
| $\Omega^{++}_{ccc}$ | 5035 | 4880 | 4978 | 4842 | 4803 | 4965 |

Table 7. Double and triple charm baryon masses (in MeV).

| Baryon | caseI | caseII | Ref. [11] | Ref. [23] | Ref. [30] | Ref. [29] |
|--------|-------|--------|----------|----------|----------|----------|
| $\Xi^{00}_{bb}$ | 10467 | 10334 | 10339 | 10284 | 10130 | 10340 |
| $\Xi^{00}_{ub}$ | 10525 | 10431 | 10468 | 10427 | 10144 | 10367 |
| $\Omega^0_{bb}$ | 10606 | 10397 | 10478 | 10293 | 10422 | 10454 |
| $\Omega^0_{ub}$ | 10664 | 10495 | 10607 | 10436 | 10432 | 10486 |
| $\Omega^{++}_{bbb}$ | 15175 | 15023 | 15118 | 14810 | 14569 | 14834 |

Table 8. Beauty and charm baryon masses (in MeV).

| Baryon | caseI | caseII | Ref. [11] | Ref. [23] | Ref. [27] | Ref. [30] |
|--------|-------|--------|----------|----------|----------|----------|
| $\Omega^+_c$ | 7087 | 6988 | 6959 | 6935 | 6928 | 6792 |
| $\Omega^{++}_c$ | 7145 | 7083 | - | 7076 | - | 6827 |
| $\Omega^{00}_b$ | 7226 | 7103 | 7098 | 6945 | 7013 | 6999 |
| $\Omega^{00}_b$ | 7284 | 7200 | - | 7085 | - | 7024 |
| $\Omega^+_b$ | 8357 | 8190 | 8229 | 8038 | - | 8018 |
| $\Omega^{++}_b$ | 8415 | 8290 | 8358 | 8186 | - | 8025 |
| $\Omega^0_{bb}$ | 11737 | 11542 | 11609 | 11363 | - | 11280 |
| $\Omega^0_{bb}$ | 11795 | 11643 | 11738 | 11512 | - | 11287 |
of baryonic systems. Hyperfine spin- and isospin-dependent interactions result in small shifts of the baryon energy. Our results are similar to those of other works. The confining interaction including a harmonic-oscillator term seems to give results closer to experimental data, especially for double and triple heavy baryons. Our approach can also be used to study other three-body systems in the fields of nuclear, atomic, and molecular physics.

References

[1] C. Albertus, E. Hernandez, J. Nieves and J. M. Verde-Velasco, Eur. Phys. J. A 32 (2007) 183 [Erratum-ibid. A 36 (2008) 119]
[2] D. Ebert, R. N. Faustov, V. O. Galkin and A. P. Martynenko, Phys. Rev. D 66 (2002) 014008
[3] J. -R. Zhang and M. -Q. Huang, Phys. Rev. D 78 (2008) 094015
[4] X. H. Guo and T. Muta, Mod. Phys. Lett. A 11 (1996) 1523
[5] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 72 (2005) 034026
[6] C. Albertus, J. E. Amaro, E. Hernandez and J. Nieves, Nucl. Phys. A 755 (2005) 439
[7] J. M. Flynn et al. [UKQCD Collaboration], JHEP 0307 (2003) 066
[8] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij, D. Nicmorus and K. Pumsa-ard, Phys. Rev. D 73 (2006) 094013
[9] B. Patel, A. K. Rai and P. C. Vinodkumar, J. Phys. G 35 (2008) 065001 [J. Phys. Conf. Ser. 110 (2008) 122010]
[10] B. Patel, A. Majethiya and P. C. Vinodkumar, Pramana 72 (2009) 679
[11] Z. Ghalenovi, A. A. Rajabi and M. Hamzavi, Acta Phys. Polon. B 42 (2011) 1849
[12] N. Isgur and G. Karl, Phys. Rev. D 18 (1978) 4187
[13] S. Capstick and N. Isgur, Phys. Rev. D 34 (1986) 2809
[14] M. M. Giannini, Rept. Prog. Phys. 54 (1990) 453
[15] L. Y. Glozman and D. O. Riska, Phys. Rept. 268 (1996) 263
[16] N. Salehi, A. A. Rajabi and Z. Ghalenovi, Acta Phys. Polon. B 42 (2011) 1247
[17] H. Hassanabadi, A. A. Rajabi and S. Zarrinkumar, Mod. Phys. Lett. A 23 (2008) 527
[18] H. Hassanabadi and A. A. Rajabi, Few-Body Syst. 41 (2005) 201
[19] A. A. Rajabi, Few-Body Syst. 37 (2005) 4
[20] G. S. Bali et al. [TXL and T(X)L Collaborations], Phys. Rev. D 62 (2000) 054503
[21] G. Plante and A. F. Antippa, J. Math. Phys. 46 (2005) 062108
[22] M. M. Giannini, E. Santopinto and A. Vassallo, Prog. Part. Nucl. Phys. 50 (2003) 263
[23] Z. Ghalenovi, A. A. Rajabi and A. Tavakolinezhad, Int. J. Mod. Phys. E 21 (2012) 1250057
[24] B. Patel, A. Majethiya and P. C. Vinodkumar, Phys. At. Nucl. 65 (2002) 917
[25] Z. Ghalenovi and A. Akbar Rajabi, Eur. Phys. J. Plus 127 (2012) 141
[26] B. Patel, A. K. Rai and P. C. Vinodkumar, Pramana 70 (2008) 797
[27] C. Albertus, E. Hernandez and J. Nieves, Phys. Lett. B 683 (2010) 21
[28] A. Majethiya, B. Patel and P. C. Vinodkumar, Eur. Phys. J. A 38 (2008) 307
[29] W. Roberts and M. Pervin, Int. J. Mod. Phys. A 23 (2008) 2817
[30] A. P. Martynenko, Phys. Lett. B 663 (2008) 317
[31] W. M. Yao et al. [Particle Data Group Collaboration], J. Phys. G 33 (2006) 1
[32] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 99 (2007) 202001
[33] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 99 (2007) 052002
[34] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 80 (2009) 072003
[35] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 107 (2011) 102001