An Analytical Approach to Study the Blood Flow over a Nonlinear Tapering Stenosed Artery in Flow of Carreau Fluid Model

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The current study provides an analytical approach to analyze the blood flow through a stenosed artery by using the Carreau fluid model. The flow governing equations are derived under the consideration of mild stenosis. Mathematical analysis has been carried out by considering the blood as non-Newtonian nature. Then, the analytical solution has been investigated by using the regular perturbation technique. The solutions obtained by this perturbation are up to the second-order in dimensionless Weissenberg number ($We$). The performed computations of various parameter values such as velocity, wall shear stress, shear stress, and resistance impedance at the stenotic throat are discussed in detail for different values of Weissenberg number ($We$). The obtained results demonstrate that for shear-thinning fluid, the fluid velocity increases with the increasing parameter $m$ while opposite behavior is observed with the increase in $We$. Hence, the presented numerical analysis reveals many aspects of the flow by considering the blood as a non-Newtonian Carreau fluid model, and the presented model can be equally applicable to other bio-mathematical studies.

1. Introduction

The study of artery constriction due to the development of stenosis has attained prime importance in fluid dynamics [1–4]. The blood flow in the vessels is a result of the delicate relationship between pressure and area of the fluid. The size of the stenosis determines the flow type. Three types of flow have been studied: mild stenosis as the flow is laminar, moderate stenosis as the flow is a combination of turbulent and laminar, and thirdly the flow depicts turbulent nature when the size of the stenosis is increased. The characteristics of the blood flow depend on the shape and size of the stenosis. Many researchers have analyzed this biomechanical aspect of the flow theoretically and experimentally in recent years. Tang et al. [5] propounded that when the blood pressure is low, stenotic vessels get collapsed and the perfusion of the area beyond stenosis is highly compromised leading to ischemia/infarct. Applications of micropolar fluids in biomedical sciences have received attention for blood flow in arteries [6–8].
In the literature, numerous analytical studies related to blood flow through stenosed arteries have been extensively performed [9–13]. In most of these studies, the flow is considered as laminar with mild stenosis [14]. Considerable work has been done for non-Newtonian fluid models. The Carreau fluid model is one of the generalized Newtonian fluid models which is also considered as a viscosity model. This model helps us to explain the behavior of fluid flow in high shear regions and modeling blood at narrow arteries with low shear rates. Firstly, this model was presented by Carreau [15] in his molecular network theories. Then, Siska et al. [16] have proposed a procedure for the terminal velocity of nonspherical particles by using Carreau fluid in transient flow regions. Such regions are responsible for the development of stenosis in the arteries that could lead to stroke, nausea, back pain, etc., specifically with regard to malfunction of the cardiovascular structures. Therefore, the study of blood vessels especially in stenotic arteries has also attained importance in the fluid dynamics field [17].

The shear-thinning and shear-thickening processes of the fluid are well explained by the Carreau fluid model [18, 19]. This method is a mixture of power-law and Newtonian fluid models. Chhabra and Uhlherr [20] have analyzed the Carreau viscosity equations for shear-thinning elastic liquids. Theoretical analysis of the Carreau fluid model is studied by Bush and Phan-Thien [21]. Later on, Lee [22] discussed the Carreau generalized Newtonian model for error estimations. Tabakova et al. [23] analyzed the flow dynamics in blood vessels by using the Carreau model. They have studied the oscillatory and steady flows and approximated their numerical solutions. Liu et al. [27] constructed a mathematical model to study the evolution of stenosis in the arteries that could lead to transient flow regions. Such regions are responsible for the development of stenosis in the arteries that could lead to stroke, nausea, back pain, etc., specifically with regard to malfunction of the cardiovascular structures. Therefore, the study of blood vessels especially in stenotic arteries has also attained importance in the fluid dynamics field [17].

Graphical results are presented with different values of parameters of interest.

2. Problem Formulation

The fundamental equations used in the derivation of the governing equations for the problem considered are

\[
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \\
\rho \left( \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = \frac{\partial p}{\partial r} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rr} \right) + \frac{\partial}{\partial z} \left( S_{rz} \right) - \frac{S_{00}}{r}, \\
\rho \left( \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = \frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left( r S_{rz} \right) + \frac{\partial}{\partial z} \left( S_{zz} \right).
\]

It is assumed that the properties of blood flowing the cylindrical tube are described by the constitutive relation Carreau model given as [41]

\[
S = \mu \left[ 1 + \left( \Gamma \dot{\gamma} \right)^2 \right]^{(n-1)/2}. \\
\text{Assuming } \Gamma \dot{\gamma} \ll 1, \text{ we can write} \\
S = \mu \left[ 1 + \left( \frac{m-1}{2} \right) \left( \Gamma \dot{\gamma} \right)^2 \right] \dot{\gamma},
\]

where \( \Gamma \) is a time constant of the fluid. The geometrical representation of the constricted portion is shown in Figure 1 and defined as

\[
h(z) = d(z) \left[ 1 - \eta \left( b^{n-1} (z - 1) - (z - a)^n \right) \right], \quad a \leq z \leq (a + b)
\]

\[
d(z) = d_0 + \xi^2 \cdot Z.
\]

In the above equation, \( \xi = \tan \phi \) is called the tapering parameter; \( a \) is the length of the nonstenotic part, and \( b \) is the length of the stenotic section; the radius of the nonstenotic artery in the nonstenotic section is \( d_0 \); and shape parameter is \( m \) which defined the constriction shape, \( n = 2 \) gives the stenosis symmetric behavior. The parameter \( \eta \) is written as

\[
\eta = \frac{\delta h^n (n-1)}{d_0 b^n (n-1)},
\]

where \( \delta \) is the maximum height of the stenosis defined as

\[
z = a + \left( \frac{b}{1/n^{(n-1)}} \right).
\]

The detailed derivation of the presenting problem is given here

\[
V = [u(r, z), 0, w(r, z)].
\]

Thus
Here we defined the rate of strain tensor as given
\[
\dot{\gamma} = L + L^T = \begin{bmatrix} u_r & 0 & u_z \\ 0 & u & 0 \\ v_r & 0 & v_z \end{bmatrix} + \begin{bmatrix} u_r & 0 & v_r \\ 0 & u & 0 \\ u_r & 0 & v_z \end{bmatrix},
\] (12)
or
\[
\dot{\gamma} = \begin{bmatrix} 2u_r & 0 & u_z + v_r \\ 0 & 2u & 0 \\ u_r + v_z & 0 & 2v_z \end{bmatrix},
\] (13)

now
\[
\dot{\gamma} = |\dot{\gamma}| = \sqrt{\frac{1}{2} \sum \sum \dot{\gamma}_{ij} \dot{\gamma}_{ji}}.
\] (14)

Expanding the double sum, we get
\[
\dot{\gamma} = \frac{1}{2} \left( \dot{\gamma}_{11} \dot{\gamma}_{11} + \dot{\gamma}_{12} \dot{\gamma}_{12} + \dot{\gamma}_{13} \dot{\gamma}_{13} + \dot{\gamma}_{21} \dot{\gamma}_{21} + \dot{\gamma}_{22} \dot{\gamma}_{22} + \dot{\gamma}_{23} \dot{\gamma}_{23} + \dot{\gamma}_{31} \dot{\gamma}_{31} + \dot{\gamma}_{32} \dot{\gamma}_{32} + \dot{\gamma}_{33} \dot{\gamma}_{33} \right)^{(1/2)}.
\] (15)

By choosing the values of the component of \( \dot{\gamma} \) from equation (13), we get
\[
\dot{\gamma} = \frac{1}{2} \left[ 4u_r u_r + (u_z + v_r) (u_z + v_r) + 4 \frac{u^2}{r^2} + 4w_z^2 \right]^{(1/2)},
\] (16)

Simplification of the above expression yields
\[
\dot{\gamma} = \frac{1}{2} \left[ 4u_r^2 + 2(u_z + v_r)^2 + 4 \frac{u^2}{r^2} + 4w_z^2 \right]^{(1/2)},
\] (17)
or
\[
\dot{\gamma} = \left[ 2u_r^2 + (u_z + v_r)^2 + 2 \frac{u^2}{r^2} + 2w_z^2 \right]^{(1/2)},
\] (18)

by assuming
\[
M = (\Gamma \dot{\gamma})^2,
\] (19)
we get the following components of the extra stress tensor for the Carreau model:
\[
S_{rr} = 2 \mu \left[ 1 + \left( \frac{m-1}{2} \right) M \right] u_r,
\] (20)
\[
S_{r\theta} = 2 \mu \left[ 1 + \left( \frac{m-1}{2} \right) M \right] \frac{u}{r},
\] (21)
Applying the mild stenotic conditions, we get
\[ M = \Gamma^2 w_r^2, \]
\[ S_{rr} = 0, \]
\[ S_{\theta\theta} = 0, \]
\[ S_{zz} = 0, \]
\[ S_{rz} = \mu \left[ 1 + \left( \frac{m-1}{2} \right) \Gamma^2 (w_r)^2 \right] w_r = S_{zr}. \]

Given the above results, the governing equations (1)–(3) can be rewritten as
\[ \frac{\partial p}{\partial r} = 0, \]
\[ \frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \]
\[ \frac{\partial p}{\partial z} = \frac{\mu}{r} \frac{\partial}{\partial r} \left[ r \left[ 1 + \left( \frac{m-1}{2} \right) \Gamma^2 (w_r)^2 \right] (w_r) \right]. \]

Dimensionless variables are defined as
\[ \tau = \frac{r}{d_0}, \quad \zeta = \frac{z}{b}, \quad w = \frac{w}{u_0}, \quad h = \frac{h}{d_0}, \quad \eta = \frac{\eta}{u_0}, \]
\[ \tilde{P} = \frac{\alpha^{m+1}}{u_0^2 \mu} \rho \tilde{R}, \quad \tilde{S}_{rr} = \frac{b S_{rr}}{u_0^2 \mu}, \quad \tilde{S}_{zr} = \frac{b S_{zr}}{u_0^2 \mu}, \]
\[ \tilde{S}_{zz} = \frac{b S_{zz}}{u_0^2 \mu}, \quad \tilde{S}_{\theta\theta} = \frac{b S_{\theta\theta}}{u_0^2 \mu}. \]

and get the following dimensionless governing equation after dropping the bars for simplicity
\[ \frac{\partial p}{\partial \tau} = 0, \]
\[ \frac{\partial u}{\partial \tau} + \frac{u}{\tau} + \frac{\partial w}{\partial \zeta} = 0, \]
\[ \frac{\partial \tilde{P}}{\partial \zeta} = \frac{1}{r \frac{\partial}{\partial r}} \left[ r \left[ 1 + \frac{(m-1)}{2} \Gamma^2 (w_r)^2 \right] \right] \frac{\partial \tilde{w}}{\partial \tau}. \]

where \( We = \Gamma u_0 / d_0 \) is the dimensionless Weissenberg number.

2.1. Boundary Conditions. The following boundary conditions are applied along with geometrical interpretation:
\[ w = 0, \quad \text{at} \; r = h(z), \]
\[ \frac{\partial w}{\partial r} = 0, \quad \text{at} \; r = 0, \]
where \( h(z) \) is already defined in equation (6).

3. Employing Perturbation Approach

To get the perturbation solution for the above defined mathematical model, equation (26) can be rearranged as
\[ \frac{\partial p}{\partial \tau} \frac{1}{\tau} \frac{\partial}{\partial r} \left[ r \left[ 1 + \frac{(m-1)}{2} \Gamma^2 (w_r)^2 \right] \right] \frac{\partial w}{\partial \tau}, \]
and by integrating equation (29), we get
\[ r^2 \frac{\partial p}{\partial \zeta} + C_1 = \left[ r \left[ 1 + \frac{(m-1)}{2} \Gamma^2 (w_r)^2 \right] \right] \frac{\partial w}{\partial \tau}. \]

The symmetry condition at the center line \( r = 0 \) yields \( C_1 = 0 \), and thus we can write equation (30) as
\[ r \frac{\partial p}{\partial \zeta} = \left[ 1 + \frac{(m-1)}{2} \Gamma^2 (w_r)^2 \right] \frac{\partial w}{\partial \tau}. \]

Due to nonlinearity arising in equation (31), we opt perturbation technique to solve it. We assume
\[ w = w_0 + We^2 w_1 + \cdots, \]
\[ p = p_0 + We^3 p_1 + \cdots, \]
\[ F = F_0 + We^3 F_1 + \cdots. \]

Substituting equations (32) and (33) into equations (27) and (34), we find that
\[
\frac{r}{2} \left( \frac{\partial p_0}{\partial z} + \text{We} \frac{\partial p_1}{\partial z} \right) = \frac{\partial w_0}{\partial r} + \text{We} \left[ \frac{\partial w_1}{\partial r} + \frac{m-1}{2} \left( \frac{\partial w_0}{\partial r} \right)^3 \right] + \text{We}^2 \left[ \frac{m-1}{2} \left( \frac{\partial w_0}{\partial r} \right)^2 + 2 \left( \frac{m-1}{2} \right) \frac{\partial w_1}{\partial r} \left( \frac{\partial w_0}{\partial r} \right)^2 \right] + \text{We}^4 \left[ \frac{m-1}{2} \left( \frac{\partial w_0}{\partial r} \right)^4 + 2 \left( \frac{m-1}{2} \right) \frac{\partial w_1}{\partial r} \left( \frac{\partial w_0}{\partial r} \right)^2 \right] + \text{We}^6 \frac{m-1}{2} \left( \frac{\partial w_1}{\partial r} \right)^5, \]
\]
(35)

Now comparing the coefficients of various powers of \(\text{We}\) in equations (35) and (36), the following systems can be obtained.

System of order \(\text{We}^0\):
\[
\frac{r}{2} \frac{\partial p_0}{\partial z} = \frac{\partial w_0}{\partial r} \quad \text{at} \quad r = h(z).
\]
(37)

System of order \(\text{We}^2\):
\[
\frac{r}{2} \frac{\partial p_0}{\partial z} = \frac{\partial w_1}{\partial r} + \frac{m-1}{2} \left( \frac{\partial w_0}{\partial r} \right)^3 \quad \text{at} \quad r = h(z).
\]
(38)

Solving these system results in the following values of \(w_0\) and \(w_1\):
\[
w_0 = \frac{1}{4} \frac{\partial p_0}{\partial z} \left[ r^2 - h^2 \right],
\]
(39)

\[
w_1 = \frac{1}{4} \frac{\partial p_1}{\partial z} \left[ r^2 - h^2 \right] + \frac{m-1}{64} \left( \frac{\partial p_0}{\partial z} \right)^3 \left[ h^4 - r^4 \right].
\]
(40)

The definition of flow rate allows us to write
\[
F_0 = 2\pi \int_0^h rw_0 dr,
\]
(41)

inserting the value \(w_0\) from equation (39) leads to the following expression:
\[
\frac{\partial p_0}{\partial z} = \frac{8}{\pi h^4 F_0},
\]
(42)

and thus,
\[
\frac{\partial p_1}{\partial z} = \frac{8F_1}{h^4} - (m-1) \frac{16 \times 8}{3\pi^3 h^{10}} (F_0)^3.
\]
(44)

We summarize the results of the perturbation series through an order \(\text{We}^2\). By substituting equations (39)–(44), we get
\[
\frac{\partial p}{\partial z} = -\frac{8 \text{We}^2}{\pi h^{12}} \left( m-1 \right) \left[ h^4 - r^4 \right] (F_0)^3,
\]
(46)
If we define
\[ F^{(2)} = F_0 + We^2 F_1, \]
\[ F_0 = F^{(2)} - We^2 F_1, \] (47)

On substituting these expressions into equations (45) and (46) and retaining only terms up to \( O(We^2) \), we obtain

\[ \frac{\partial p}{\partial z} = \frac{8F}{\pi h^4} - We^2 (m - 1) \frac{16 \times 8}{3\pi^2 h^{10} r^3}, \] (48)

\[ w = -\frac{2F}{\pi h^4} \left[ r^2 - h^2 \right] - \frac{32We^2}{3\pi^2 h^{10} r^3} \left[ r^2 - h^2 \right] (m - 1) - \frac{8We^2}{\pi h^4} F^3 (m - 1) \left[ h^4 - r^4 \right]. \] (49)

Now the pressure drop is
\[ \Delta p = \int_0^{(1/b)} \left( \frac{\partial p}{\partial z} \right) dz, \] (50)

and on substituting the value of \( (\partial p/\partial z) \) from equation (48), we derive

\[ \lambda = \frac{\Delta p}{F} = \int_0^{(a/b)} R(z)|_{h=1+\xi z} dz + \int_0^{(a/b)+1} R(z) dz + \int_0^{(1/b)} R(z)|_{h=1+\xi z} dz, \] (52)

and the nondimensional shear stress for the Carreau law model is given as
\[ S_{rz} = \frac{\partial w}{\partial r} + We^2 \left( -\frac{m - 1}{2} \right) \left( \frac{\partial w}{\partial r} \right)^3, \] (53)

since
\[ w = \frac{2F}{\pi h^4} \left[ r^2 - h^2 \right] - \frac{32We^2}{3\pi^2 h^{10} r^3} \left[ r^2 - h^2 \right] (m - 1) - \frac{8We^2}{\pi h^4} F^3 (m - 1) \left[ h^4 - r^4 \right], \] (54)
we obtain
\[
\frac{\partial \omega}{\partial r} = -\frac{4Fr}{\pi h^4} \left( \frac{64We^2F^3(m-1)r}{3\pi^4h^{10}} + \frac{32r^3We^2}{\pi^3h^{12}}F^3(m-1) \right),
\] (55)

and thus equation (53) gives
\[
S_{rz} = -\frac{4Fr}{\pi h^4} \left( \frac{64We^2F^3(m-1)r}{3\pi^4h^{10}} + \frac{32r^3We^2}{\pi^3h^{12}}F^3(m-1) \right) + \frac{3}{2} We^2 \left( \frac{m-1}{2} \right)^3 \left( \frac{4Fr}{\pi h^4} \frac{64We^2F^3(m-1)r}{3\pi^4h^{10}} + \frac{32r^3We^2}{\pi^3h^{12}}F^3(m-1) \right)^3.
\] (56)
Finally, at maximum height, i.e., \( h = 1 - \delta \), the wall shear stress of the stenosis can be written as

\[
\tau_s = S_{rz}|_{h=1-\delta} = \frac{4}{\pi (1 - \delta)^3} \left[ -F + \frac{16We^2}{3\pi^2 (1 - \delta)^9} (m - 1)F^3 + \frac{8We^2 (m - 1)}{\pi^2 (1 - \delta)^8} F^3 \right].
\]  

(57)

**Figure 5:** Wall shear stress variation for the Carreau model for \( n = 2, L = 1, F = 1 \), and \( We = 0.2 \).

**Figure 6:** Velocity variation for the Carreau model for \( n = 2, L = 1, F = 3 \), and \( m = 0.5 \).
4. Graphical Results and Discussion

In this section, the effect of rheological parameters of the Carreau fluid model, i.e., \( \text{We} \) and \( m \) on velocity profile, resistance impedance, wall shear stress, and shear stress at the stenotic throat, is graphically performed and discussed. The Weissenberg number is the ratio of the relaxation time of the fluid and a specific process time. Relaxation time increases when we increase the Weissenberg number and velocity field easily increases and skin friction decreases. Another aspect of increasing the Weissenberg number is that it reduces the magnitude of the fluid velocity for shear-thinning fluid, while it arises for the shear-thickening fluid. Figure 2 shows that resistance impedance is an increasing function of the Weissenberg number \( \text{We} \).

In Figure 3, it is observed that the resistance impedance increases as there is an increase in power-law index \( m \). It explains the fluid shear-thinning behavior \((m < 1)\) slowly compared to Newtonian \((m = 1)\) and shear-thickening \((m > 1)\) fluids.

The behavior of shear stress for \( \text{We} \) and \( m \) is shown in Figures 4 and 5. It is observed that the magnitude of wall shear stress increases by increasing \( \text{We} \) as shown in Figure 4. The opposite behavior is observed when \( m \) is increased as seen in Figure 5.
The effect of the severity of stenosis on the velocity profile $w$ is shown in Figures 6 and 7. In Figure 6, the velocity profile increases at the center of the channel by increasing $m$.

And in Figure 7, an increase in $w$ decreases the velocity at the channel center. It can be seen that for a fixed value of prescribed flux $F$, the velocity profile $w$ increases near the center, while it decreases near the wall with an increase in the severity of stenosis.

In Figure 8, we noticed that the shear stress at the maximum height of stenosis behaves differently by increasing $We$ and $m$. It is analyzed that shear stress by fixing Weissenberg number $We$ and flow rate $F$ at the stenosis throat increases with an increase in $m$. While shear stress decreases with an increase in $We$ with defined values of power-law index $m$ and flow rate $F$.

5. Conclusion

In this study, we have analyzed the numerical solution of the blood flow with its non-Newtonian nature. Analytical solutions are derived from the given governing equations. The presented computation results of various parameter values, namely, velocity, wall shear stress, shear stress, and resistance impedance at the stenotic throat, are studied in detail for different values of Weissenberg number ($We$) and power-law index $m$. From the analysis of the related facts and figures, the following results are concluded:

1. The velocity profile increases at the center of the channel by increasing $m$. However, an increase in $We$ decreases the velocity at the channel center.
2. Resistance impedance is an increasing function of $We$ and $m$.
3. The magnitude of shear stress increases by increasing $We$, while it shows the opposite behavior when $m$ is increased.
4. Shear stress at the maximum height of stenosis behaves differently by increasing $We$ and $m$.

Conclusively, the present work may be an improvement in the analysis of pulsatile blood flow through a mild stenotic tapering artery.

Nomenclature

$We$: Weissenberg number  
$\Delta$: Time constant of the fluid  
$\xi^*$: Tapering parameter  
$a$: Length of the nonstenotic part  
$b$: Length of the stenotic section  
$d_n$: Radius of the nontapered artery in the nonstenotic section  
n: Shape parameter  
$\delta$: Maximum height of the stenosis  
$\tau_w$: Wall shear stress  
$\Delta p$: Pressure drop  
$\lambda$: Resistance impedance  
$m$: Power-law index  
w: Velocity profile

$F$: Flow rate  
$S_{\text{extr}}$: Shear stress  
$S$: Extra stress tensor  
$\theta$: Tapering angle.

Data Availability

No data were required to perform this research.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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