SOME RESULTS ON UNBOUNDED ABSOLUTE WEAK DUNFORD-PETTIS OPERATORS

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ABSTRACT. In this paper, we characterize Banach lattices on which each Dunford-Pettis operator (or weak Dunford-Pettis) is unbounded absolute weak Dunford-Pettis operator and the converse.

1. Introduction

The notion of unbounded order convergence (uo-convergence, for short) was firstly introduced by Nakano in [14], then it was used and systematically investigated in [8, 9, 10, 12, 17]. After that, A. Bahramnezhad et al. proposed the definition of unbounded order continuous operators in [3]. A closely related notion of unbounded norm convergence (un-convergence, for short) was introduced and systematically studied in [5, 11, 15]. In [11 Section 9], M. Kandić et al. gave the definition of (sequentially) un-compact operators and obtained the relationships between weakly compact operators and sequentially un-compact operators. Recently, O. Zabeti in [19] proposed a new so-called unbounded version convergence (uaw-convergence). And, uaw-Dunford-Pettis operators were introduced and investigated in [6].

In this paper, we will establish some results on uaw-Dunford-Pettis operators. We first present some necessary and sufficient conditions for positive Dunford-Pettis operators being uaw-Dunford-Pettis. More precisely, we will prove that each positive Dunford-Pettis operator from a Banach lattice $E$ into arbitrary Banach lattice $F$ is uaw-Dunford-Pettis if and only if the norm of $E'$ is order continuous or $F = \{0\}$ (Theorem 3.1). We will also give a characterization of Banach lattice $E$ on which each positive operator $T : E \to \ell_1$ is uaw-Dunford-Pettis (Theorem 3.3). After that, we will investigate Banach lattices under which each uaw-Dunford-Pettis operator is Dunford-Pettis. And we
will show that if Banach lattice $E$ is an $AM$-space, then every operator $T$ from $E$ into arbitrary Banach space is uaw-Dunford-Pettis if and only if $T$ is Dunford-Pettis (Corollary 3.7). Finally, we will present the relationships between weak Dunford-Pettis operators and uaw-Dunford-Pettis operators. Whenever Banach lattice $E$ is Dedekind $\sigma$-complete, we will establish that $E$ is reflexive if and only if each positive weak Dunford-Pettis operator from $E$ into $E$ is an uaw-Dunford-Pettis operator (Theorem 4.1). We will also give some sufficient conditions under which each positive uaw-Dunford-Pettis operator is weak Dunford-Pettis (Theorem 4.4).

2. Preliminaries

To state our results, we need to recall some definitions. Recall that a Riesz space $E$ is an ordered vector space in which sup($x$, $y$) exists for every $x$, $y \in E$. A sequence $(u_n)$ of a Riesz space is called disjoint whenever $n \neq m$ implies $u_n \perp u_m$. A Banach lattice is a Banach space $(E, \|\cdot\|)$ such that $E$ is a Riesz lattice and its norm satisfies the following property: for each $x, y \in E$ with $|x| \leq |y|$, we have $\|x\| \leq \|y\|$. By Theorem 4.1 of [1], if $E$ is a Banach lattice, then its norm dual $E'$ is also a Banach lattice.

A norm $\|\cdot\|$ of a Banach lattice $E$ is order continuous if for each net $(x_\alpha)$ in $E$ with $x_\alpha \downarrow 0$, one has $\|x_\alpha\| \downarrow 0$. A Banach lattice $E$ is said to be a KB-space whenever every increasing norm bounded sequence of $E_+$ is norm convergent. Every KB-space has an order continuous norm. A Banach space is said to have the Schur property whenever every weak convergent sequence is norm convergent, i.e., whenever $x_n \overset{w}{\to} 0$ implies $\|x_n\| \to 0$.

Recall that an operator $T$ from a Banach space $X$ to a Banach space $Y$ is Dunford-Pettis if it maps weakly null sequences of $X$ to norm null sequences of $Y$, and is weak Dunford-Pettis if $f_n(T(x_n)) \to 0$ for any weakly null sequence $(x_n)$ in $X$ and any weakly null sequence $(f_n)$ in $Y'$.

Recall that a net $(x_\alpha)$ in a Banach lattice $E$ is said to be unbounded absolutely weakly convergent to $x \in E$, written as $x_\alpha \overset{uaw}{\longrightarrow} x$, if for any $u \in E_+, |x_\alpha - x| \wedge u \overset{w}{\to} 0$ holds.

Definition 2.1. [6] An operator $T$ from a Banach lattice $E$ into a Banach space $X$ is said to be an unbounded absolute weak Dunford-Pettis (uaw-Dunford-Pettis, for short) if for every norm bounded sequence $(x_n)$ in $E$, $x_n \overset{uaw}{\longrightarrow} 0$ implies $\|Tx_n\| \to 0$. 

Every uaw-Dunford-Pettis operator is continuous. In fact, if $T : E \to X$ is a uaw-Dunford-Pettis operator and $\|x_n\| \to 0$, then for each $u \in E^+$, $\|\|x_n\| \wedge u\| \leq \|x_n\|$, i.e., $\|\|x_n\| \wedge u\| \to 0$. That is, $x_n \xrightarrow{uaw} 0$, and so $\|Tx_n\| \to 0$.

All operators in this paper are assumed to be continuous. We refer to [1, 13] for all unexplained terminology and standard facts on vector and Banach lattices. All vector lattices in this paper are assumed to be Archimedean.

3. The relationships with Dunford-Pettis operators

There exist operators which are Dunford-Pettis but not uaw-Dunford-Pettis. For example, the identity operator $Id_{\ell_1} : \ell_1 \to \ell_1$ is Dunford-Pettis since $\ell_1$ has the Schur property, but it is not a uaw-Dunford-Pettis operator. In fact, for the standard basis $(e_n)$ of $\ell_1$, $(e_n)$ is disjoint, so by Lemma 5 of [19], $e_n \xrightarrow{uaw} 0$. However, $\|Id_{\ell_1}(e_n)\| = \|e_n\| = 1$.

The following theorem gives a characterization of Banach lattices $E$ and $F$ under which each positive Dunford-Pettis operator $T : E \to F$ is uaw-Dunford-Pettis.

**Theorem 3.1.** Let $E$ and $F$ be Banach lattices. Then the following assertions are equivalent:

1. Each positive Dunford-Pettis operator $T : E \to F$ is uaw-Dunford-Pettis.
2. Each positive compact operator $T : E \to F$ is uaw-Dunford-Pettis.
3. One of the following conditions is valid:
   (i) The norm of $E'$ is order continuous.
   (ii) $F = \{0\}$.

**Proof.** (1) $\Rightarrow$ (2) It is obvious, since each compact operator is Dunford-Pettis.

(2) $\Rightarrow$ (3) Assume by way of contradiction that the norm of $E'$ is not order continuous and $F \neq \{0\}$. We have to construct a compact operator which is not uaw-Dunford-Pettis.

Since the norm of $E'$ is not order continuous, it follows from Theorem 2.4.14 and Proposition 2.3.11 of [13] that $\ell_1$ is a closed sublattice of $E$ and there exists a positive projection $P : E \to \ell_1$. On the other hand, since $F \neq \{0\}$, there exists a vector $0 < y \in F_+$. Define the operator $S : \ell_1 \to F$ as follows:

$$S(\lambda_n) = \left( \sum_{n=1}^{\infty} \lambda_n \right)y$$
for each \((\lambda_n) \in \ell_1\). Obviously, the operator \(S\) is well defined. Let
\[
T = S \circ P : E \to \ell_1 \to F,
\]
then \(T\) is a compact operator since \(S\) is a finite rank operator (rank is 1). But \(T\) is not an uaw-Dunford-Pettis operator. Let \((e_n)\) be the canonical basis of \(\ell_1\). Obviously, \((e_n)\) is disjoint, by Lemma 5 of [19], we know that \(e_n \xrightarrow{uaw} 0\). However, \(\|T(e_n)\| = \|y\| > 0\). Hence, \(T\) is not an uaw-Dunford-Pettis operator.

(3)(i) ⇒ (1) Follows from Proposition 1 of [6].
(3)(ii) ⇒ (1) Obvious. □

Whenever \(E = F\) in the Theorem 3.1, we get the following characterization:

**Corollary 3.2.** Let \(E\) be a Banach lattice. Then the following assertions are equivalent:

1. Each positive Dunford-Pettis operator \(T : E \to E\) is uaw-Dunford-Pettis.
2. Each positive compact operator \(T : E \to E\) is uaw-Dunford-Pettis.
3. The norm of \(E'\) is order continuous.

The following theorem gives a characterization of Banach lattice \(E\) for which each positive operator \(T : E \to \ell_1\) is uaw-Dunford-Pettis.

**Theorem 3.3.** Let \(E\) be a Banach lattice, then the following assertions are equivalent:

1. Each positive operator from \(E\) into \(\ell_1\) is uaw-Dunford-Pettis.
2. The norm of \(E'\) is order continuous.

*Proof. (1) ⇒ (2) Assume by way of contradiction that the norm of \(E'\) is not order continuous. Then it follows from Theorem 116.1 of [18] that there exists a norm bounded disjoint sequence \((u_n)\) of positive elements in \(E\) which does not weakly convergence to zero. Without loss of generality, we may assume that \(\|u_n\| \leq 1\) for any \(n\). And there exist \(\epsilon > 0\) and \(0 \leq \phi \in E'\) such that \(\phi(u_n) > \epsilon\) for all \(n\). Then by Theorem 116.3 of [18], we know that the components \(\phi_n\) of \(\phi\) in the carriers \(C_{u_n}\) form an order bounded disjoint sequence in \((E')_+\) such that
\[
\phi_n(u_n) = \phi(u_n) \quad \text{for all } n \quad \text{and} \quad \phi_n(u_m) = 0 \quad \text{if} \quad n \neq m.
\]
Define the positive operator \(T : E \to \ell_1\) as follows:
\[
T(x) = \left( \frac{\phi_n(x)}{\phi(u_n)} \right)_{n=1}^{\infty}
\]
for all \(x \in E\). Since
\[
\sum_{n=1}^{\infty} \frac{\phi_n(x)}{\phi(u_n)} \leq \frac{1}{\varepsilon} \sum_{n=1}^{\infty} \phi_n(|x|) \leq \frac{1}{\varepsilon} \phi(|x|)
\]

holds for all \( x \in E \), the operator \( T \) is well defined and it is also easy to see that \( T \) is a positive operator. Hence \( T \) is an uaw-Dunford-Pettis operator. For the norm bounded disjoint sequence \((u_n)\), by Lemma 5 of \[19\], we know that \( u_n \xrightarrow{uaw} 0 \). However, let \((e_n)\) be the standard basis of \( \ell_1 \), then \( \|T(u_n)\| = \|e_n\| = 1 \), which is a contradiction. Therefore, the norm of \( E' \) is order continuous.

(2) \(\Rightarrow\) (1) Since \( \ell_1 \) has the Schur property, each positive operator \( T \) from \( E \) into \( \ell_1 \) is Dunford-Pettis. And since the norm of \( E' \) is order continuous, by Theorem 3.1 we obtain that \( T \) is uaw-Dunford-Pettis. \( \square \)

Based on Theorem 5.29 of \[1\] and Theorem 2.9 of \[7\], we get the following conclusion.

**Corollary 3.4.** Let \( E \) be a Banach lattice, then the following assertions are equivalent:

1. The norm of \( E' \) is order continuous.
2. Each positive operator from \( E \) into \( \ell_1 \) is uaw-Dunford-Pettis.
3. Each positive operator from \( E \) into \( \ell_1 \) is weakly compact, and hence compact.
4. Each positive operator from \( E \) into \( \ell_1 \) is semi-compact.

A Banach lattice is said to have **weakly sequentially continuous lattice operations** whenever \( x_n \xrightarrow{w} 0 \) implies \( |x_n| \xrightarrow{w} 0 \). Every AM-space has this property.

The following theorem gives a characterization of Banach lattices \( E \) and \( F \) for which each uaw-Dunford-Pettis operator \( T : E \to F \) is Dunford-Pettis.

**Theorem 3.5.** Let \( E \) and \( F \) be Banach lattices. Each uaw-Dunford-Pettis operator \( T : E \to F \) is Dunford-Pettis if one of the following assertions is valid:

1. The lattice operations in \( E \) are weakly sequentially continuous.
2. \( E \) is discrete with an order continuous norm.
3. \( T \) is positive and \( F \) is discrete with an order continuous norm.

**Proof.** (1) Let \((x_n)\) be a weakly null sequence in \( E \). Since the lattice operations in \( E \) are weakly sequentially continuous, we have \( |x_n| \xrightarrow{w} 0 \). Then for each \( u \in E_+ \), \( |x_n| \wedge u \xrightarrow{w} 0 \), i.e., \( x_n \xrightarrow{uaw} 0 \). Since \( T \) is an uaw-Dunford-Pettis operator, we get \( \|T(x_n)\| \to 0 \). Hence, the operator \( T \) is Dunford-Pettis.
(2) Suppose that $E$ is discrete with an order continuous norm, then by Corollary 2.3 of [1], the lattice operations in $E$ are weakly sequentially continuous. Hence, following from (1), we get the result.

(3) Let $T : E \to F$ be a positive uaw-Dunford-Pettis operator and $W$ be a relatively weakly compact set in $E$. Let $A$ be the solid hull of $W$ in $E$. For every disjoint sequence $(x_n)$ in $A$, by Lemma 5 of [19], we know that $x_n \stackrel{\text{uaw}}{\longrightarrow} 0$. Since $T$ is uaw-Dunford-Pettis, we get that $\|T(x_n)\| \to 0$. Then by Theorem 4.36 of [1], for each $\varepsilon > 0$, there exists some $u \in E_+$ lying in the ideal generated by $A$ such that $\|T(|x| - u)^+\| < \varepsilon$ holds for all $x \in A$. Following from the equality $|x| = |x| \wedge u + (|x| - u)^+$, we have

$$T(|x|) = T(|x| \wedge u) + T[(|x| - u)^+] + u.$$ 

Let $V$ be the closed unit ball of $F$. Then

$$T(|x|) \in [-T(u), T(u)] + \varepsilon \cdot V$$

for all $x \in A$. Since $T$ is a positive operator, $|T(x)| \leq T(|x|)$. It is easy to see that the set $[-T(u), T(u)] + \varepsilon \cdot V$ is a solid set in $F$. Hence,

$$T(x) \in [-T(u), T(u)] + \varepsilon \cdot V$$

for all $x \in A$, and then

$$T(W) \subset [-T(u), T(u)] + \varepsilon \cdot V.$$ 

Since $F$ is discrete with an order continuous norm, $[-T(u), T(u)]$ is norm compact. Hence, $T(W)$ is a relatively compact set in $F$. Thus $T$ is a Dunford-Pettis operator. \hfill \Box

**Corollary 3.6.** Let $E$ and $F$ be Banach lattices such that the norm of $E'$ is order continuous and $F$ is discrete or its lattice operations are weakly sequentially continuous. Then the following assertions are equivalent:

1. Each positive uaw-Dunford-Pettis operator $T : E \to F$ is Dunford-Pettis.

2. One of the following assertions is valid:
   (i) The lattice operations in $E$ are weakly sequentially continuous.
   (ii) The norm of $F$ is order continuous.

**Proof.** (2)(i) $\Rightarrow$ (1) Follows from Theorem 3.5(1).

(2)(ii) $\Rightarrow$ (1) Based on Corollary 2.3 of [4], if $F$ has an order continuous norm and the lattice operations of it are weakly sequentially continuous, then $F$ is also discrete. Therefore, following from Theorem 3.5(3), we get the result.
(1) ⇒ (2) Let $S : E \to F$ be a operator which satisfies $0 \leq S \leq T$ and $T : E \to F$ is a Dunford-Pettis operator. Since the norm of $E'$ is order continuous, by Theorem 3.1 we get that the operator $T$ is uaw-Dunford-Pettis. Now we claim that $S$ is also uaw-Dunford-Pettis, i.e., uaw-Dunford-Pettis operators satisfy domination. In fact, if $x_n \overset{\text{uaw}}{\to} 0$ holds in $E$, then it is easy to see that $|x_n| \overset{\text{uaw}}{\to} 0$. And so $\|T(|x_n|)\| \to 0$ holds in $F$. By using the inequalities $|S(x_n)| \leq S(|x_n|) \leq T(|x_n|)$, we get that $\|S(x_n)\| \leq \|T(|x_n|)\|$ for all $n$. That is, $S$ is an uaw-Dunford-Pettis operator. Then $S$ is a Dunford-Pettis operator. Following from Theorem 2 of [16], the lattice operations in $E$ are weakly sequentially continuous or the norm of $F$ is order continuous. □

Corollary 3.7. Let $E$ be an AM-space. Then every operator $T$ from $E$ into arbitrary Banach space is uaw-Dunford-Pettis if and only if $T$ is Dunford-Pettis.

Proof. Let $X$ be an arbitrary Banach space and $T : E \to X$ be a continuous operator.

Assume $T$ is an uaw-Dunford-Pettis operator. Since $E$ is an AM-space, by Theorem 4.23 of [1], the dual of $E$ is an AL-space. So the norm of $E'$ is order continuous. Then by Theorem 3.1 we obtain that $T$ is Dunford-Pettis.

Conversely, assume $T$ is a Dunford-Pettis operator. Since $E$ is an AM-space, by Theorem 4.31 of [1], the lattice operations in $E$ are weakly sequentially continuous. Then by Theorem 3.5(1), we obtain that $T$ is uaw-Dunford-Pettis. □

4. The relationships with weak Dunford-Pettis operators

Recall that a Banach space $X$ is said to have the \textbf{Dunford-Pettis property} whenever $x_n \overset{\text{w}}{\to} 0$ in $X$ and $x'_n \overset{\text{w}}{\to} 0$ in $X'$ imply $x'_n(x_n) \to 0$. AL-space and AM-space have the Dunford-Pettis property ([1, Theorem 5.85]). Obviously, if $X$ has the Dunford-Pettis property, then every continuous operator from $X$ to a Banach space $Y$ is weak Dunford-Pettis.

Since each Dunford-Pettis operator is weak Dunford-Pettis, the identity operator $Id_{\ell_1} : \ell_1 \to \ell_1$ is also the example which is weak Dunford-Pettis but not uaw-Dunford-Pettis. Next, we give a characterization of reflexive Banach lattice for which each positive weak Dunford-Pettis operator from $E$ into $E$ is uaw-Dunford-Pettis operator.

\textbf{Theorem 4.1.} Let $E$ be a Dedekind $\sigma$-complete Banach lattice. Then the following assertions are equivalent:

1. $E$ is reflexive.
(2) Each positive weak Dunford-Pettis operator from $E$ into $E$ is uaw-Dunford-Pettis.

Proof. (1) $\Rightarrow$ (2) Since $E$ is reflexive, each weak Dunford-Pettis operator $T$ from $E$ into $E$ is Dunford-Pettis. Based on Theorem 4.70 of [1], the norm of $E'$ is order continuous. Then by Theorem 3.1, we know $T$ is uaw-Dunford-Pettis.

(2) $\Rightarrow$ (1) We first claim that the norm of $E$ is order continuous. Otherwise, it follows from Corollary 2.4.3 of [13] that $E$ contains a sublattice which is isomorphic to $\ell_\infty$ and there exists a positive projection $P : E \to \ell_\infty$. Let $S : \ell_\infty \to E$ be the canonical injection of $\ell_\infty$ into $E$. Define the operator $T$ as follows:

$$T = S \circ P : E \to \ell_\infty \to E.$$  

Since $\ell_\infty$ has the Dunford-Pettis property, $T$ is weak Dunford-Pettis operator. Hence, $T$ is uaw-Dunford-Pettis. Let $(e_n)$ be the standard basis of $\ell_\infty$. Similarly to the proof of Theorem 3.1, $e_n \overset{uaw}{\longrightarrow} 0$. However, $\|T(e_n)\| = \|e_n\| = 1 > 0$, which is a contradiction. Therefore, $E$ has an order continuous norm.

Next, we prove $E$ is a KB-space. If not, it follows from Theorem 2.4.12 of [13] that $E$ contains a sublattice which is isomorphic to $c_0$ and there exists a positive projection $P : E \to c_0$. Let $S : c_0 \to E$ be the canonical injection of $c_0$ into $E$. Define the operator $T$ as follows:

$$T = S \circ P : E \to c_0 \to E.$$  

Since $c_0$ has the Dunford-Pettis property, $T$ is a weak Dunford-Pettis operator. Let $(e_n)$ be the standard basis of $c_0$. Similarly, $e_n \overset{uaw}{\longrightarrow} 0$. However, $\|T(e_n)\| = \|e_n\| = 1 > 0$, we get that $T$ is not an uaw-Dunford-Pettis operator, which is a contradiction. Hence, $E$ is KB-space.

At last, we show that the norm of $E'$ is order continuous. If not, it follows from Theorem 2.4.14 and Proposition 2.3.11 of [13] that $E$ contains a sublattice which is isomorphic to $\ell_1$ and there exists a positive projection $P : E \to \ell_1$. Define the operator $T$ as follows:

$$T = S \circ P : E \to \ell_1 \to E.$$  

Since $\ell_1$ has the Dunford-Pettis property, $T$ is a weak Dunford-Pettis operator. Let $(e_n)$ be the standard basis of $\ell_1$. Similarly, $e_n \overset{uaw}{\longrightarrow} 0$. However, $\|T(e_n)\| = \|e_n\| = 1 > 0$, we obtain $T$ is not an uaw-Dunford-Pettis operator, which is a contradiction. Hence, $E'$ has an order continuous norm.
Following from Theorem 4.70 of [1], we obtain that $E$ is reflexive. □

Whenever $E \neq F$ in Theorem 4.1, we get the following conclusions.

**Corollary 4.2.** Let $E$ and $F$ be Banach lattices. If the norm of $E'$ is order continuous and $F$ is reflexive, then each weak Dunford-Pettis operator from $E$ into $F$ is uaw-Dunford-Pettis operator.

**Proof.** Similarly to the proof of (1) $\Rightarrow$ (2) of the Theorem 4.1. Since $F$ is reflexive, each weak Dunford-Pettis operator $T$ from $E$ into $F$ is Dunford-Pettis. By Theorem 3.1, we get that $T$ is uaw-Dunford-Pettis. □

**Theorem 4.3.** Let $E$ and $F$ be Banach lattices. If each weak Dunford-Pettis operator is uaw-Dunford-Pettis operator, then one of the following assertion is valid:

1. The norm of $E'$ is order continuous.
2. The norm of $F$ is order continuous.

**Proof.** It suffices to establish that if the norm of $E'$ is not order continuous, then $F$ has an order continuous norm.

Since the norm of $E'$ is not order continuous, it follows from Theorem 2.4.14 and Proposition 2.3.11 of [13] that $\ell_1$ is a closed sublattice of $E$ and there exists a positive projection $P : E \rightarrow \ell_1$. We need to show that $F$ has an order continuous norm. By Theorem 4.14 of [1], it suffices to show that each order bounded disjoint sequence $(y_n)$ is norm convergent to 0 in $F$.

Define the operator $S : \ell_1 \rightarrow F$ as follows:

$$S(\lambda_n) = \sum_{n=1}^{\infty} \lambda_n y_n$$

for each $(\lambda_n) \in \ell_1$. Obviously, it is well defined. Let

$$T = S \circ P : E \rightarrow \ell_1 \rightarrow F.$$ 

Since $\ell_1$ has the Dunford-Pettis property, $T$ is a weak Dunford-Pettis operator. Then $T$ is uaw-Dunford-Pettis. Let $(e_n)$ be the standard basis of $\ell_1$, $e_n \xrightarrow{uaw} 0$, so, $\|T(e_n)\| = \|y_n\| \rightarrow 0$. Hence, $F$ has an order continuous norm. □

At last, we give a characterization of Banach lattices for which each positive uaw-Dunford-Pettis operator from $E$ into $F$ is weak Dunford-Pettis operator.

Recall that a Banach lattice is said to have **AM-compactness property** if every weakly compact operator from $E$ to an arbitrary
Banach space is AM-compact. The Banach lattices $c_0$, $\ell_1$, $c$, and $c'$ have AM-compactness property. We have the following conclusion.

**Theorem 4.4.** Let $E$ and $F$ be Banach lattices. Each positive uaw-Dunford-Pettis operator $T : E \to F$ is weak Dunford-Pettis if one of the following assertions is valid:

1. The lattice operations in $E$ are weakly sequentially continuous.
2. $F$ is discrete with an order continuous norm.
3. $F$ has AM-compact property.

**Proof.** Since each Dunford-Pettis operator is weak Dunford-Pettis, it follows from Theorem 3.5, if the lattice operations in $E$ are weakly sequentially continuous or $F$ is discrete with an order continuous norm, every positive uaw-Dunford-Pettis operator $T : E \to F$ is weak Dunford-Pettis.

Next, we only need to show if $F$ has AM-compact property, the assertion is valid. Let $T : E \to F$ be a positive uaw-Dunford-Pettis operator and $W$ be a relatively weakly compact set in $E$, we have to show $T(W)$ is a Dunford-Pettis set in $F$. Let $A$ be the solid hull of $W$ in $E$ and $V$ be the closed unit ball of $F$. It follows from the proof of Theorem 3.5(3), for each $\varepsilon > 0$, there exists some $u \in E_+$ lying in the ideal generated by $A$ such that

$$T(W) \subset [-T(u), T(u)] + \varepsilon \cdot V.$$

Since $F$ has AM-compact property, based on Proposition 3.1 and Lemma 4.1 of [2], we get that $T(W)$ is a Dunford-Pettis set in $F$. Therefore, following from Theorem 5.99 of [11], $T$ is a weak Dunford-Pettis operator.

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