ANALYSIS THE $0^{++}$ NONET MESONS AS FOUR-QUARK STATES WITH THE QCD SUM RULES

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Abstract

In this article, we take the point of view that the $0^{++}$ nonet mesons below $1\text{GeV}$ are diquark-antidiquark states $(qq)(\bar{q}\bar{q})$, and devote to determine their masses in the framework of the QCD sum rules approach with the interpolating currents constructed from scalar-scalar type and pseudoscalar-pseudoscalar type diquark pairs respectively. The numerical results indicate that the $0^{++}$ nonet mesons may have two possible diquark-antidiquark substructures.

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1 Introduction

The light flavor scalar mesons present a remarkable exception for the constituent quark model and the structures of those mesons have not been unambiguously determined yet [1]. Experimentally, the strong overlaps with each other and the broad widths (for the $f_0(980)$, $a_0(980)$ et al, the widths are comparatively narrow) make their spectra cannot be approximated by the Breit-Wigner formula. The numerous candidates with the same quantum numbers $J^{PC} = 0^{++}$ below $2\text{GeV}$ can not be accommodated in one $q\bar{q}$ nonet, some are supposed to be glueballs, molecules and multiquark states [2, 3, 4, 5, 6]. There maybe different dynamics dominating the $0^{++}$ mesons below and above $1\text{GeV}$ which results in two scalar nonets below $1.7\text{GeV}$.

The attractive interactions of one-gluon exchange favor the formation of diquarks in color antitriplet $\bar{3}_c$, flavor antitriplet $\bar{3}_f$ and spin singlet $1_s$. The strong attractions between the states $(qq)_{\pi}$ and $(\bar{q}\bar{q})_3$ in $S$-wave may result in a nonet manifested below $1\text{GeV}$ while the conventional $3P_0$ $q\bar{q}$ nonet would have masses about $1.2 - 1.6\text{GeV}$. In the same energy region, there are two well established siblings $3P_1$ and $3P_2$ $q\bar{q}$ nonets with $J^{PC} = 1^{++}$ and $2^{++}$ respectively. Furthermore, there are enough candidates for the $3P_0$ $q\bar{q}$ nonet mesons, $a_0(1450)$, $f_0(1370)$, $K^*(1430)$, $f_0(1500)$ and $f_0(1710)$ [5, 6].

Taking the diquarks and antidiquarks as the basic constituents, keeping the effects of the $s$ quark mass at the first order, the two isoscalars $udud$ and $\bar{s}_s\frac{ua+dd}{\sqrt{2}}$
mix ideally, the $s\bar{s}\bar{u}d\bar{u}$ degenerate with the isovectors $s\bar{s}\bar{u}d\bar{d}$ and $s\bar{s}\bar{u}d$ naturally. Comparing with the traditional $\bar{q}q$ nonet mesons, the mass spectrum is inverted. The lightest state is the non-strange isosinglet ($\bar{u}\bar{d}ud$), the heaviest are the degenerate isosinglet and isovectors with hidden $s\bar{s}$ pairs while the four strange states lie in between $[\bar{s}s\bar{u}u, \bar{s}s\bar{u}d\sqrt{2}]$. Naturally. Comparing with the traditional $\bar{q}q$ nonet mesons, the mass spectrum is inverted. The lightest state is the non-strange isosinglet ($\bar{u}\bar{d}ud$), the heaviest are the degenerate isosinglet and isovectors with hidden $s\bar{s}$ pairs while the four strange states lie in between $[\bar{s}s\bar{u}u, \bar{s}s\bar{u}d\sqrt{2}]$ and $\bar{s}s\bar{u}u\bar{d}$. The broad isosinglet $S$-wave $\pi\pi$ resonance near 600 MeV can be signed to be the $f_0(600)$ or $\sigma$ meson with quark constituent $\bar{u}\bar{d}ud$. The well established isoscalar $f_0(980)$ and isovector $a_0(980)$ mesons which lie just below the $KK$ threshold have quark constituents $s\bar{s}\bar{u}d\bar{u}$ and $s\bar{s}\bar{u}d\bar{d}$ respectively in the four-quark model. The four light isospin-$\frac{1}{2}$ $K\pi$ resonances near 800 MeV, known as the $\kappa(800)$ mesons, have not been confirmed yet, there are still controversy about their existence [7].

In this article, we take the point of view that the scalar $0^{++}$ nonet mesons below 1 GeV are four-quark states $(\bar{q}q)_3(\bar{q}q)_3$ in the ideal mixing limit, and devote to determine the values of their masses in the framework of the QCD sum rules approach [8, 9, 11].

The article is arranged as follows: in section II, we obtain the QCD sum rules for the masses of the $0^{++}$ nonet mesons; in section III, numerical results; section IV is reserved for conclusion.

2 Masses of the $0^{++}$ nonet mesons with the QCD Sum Rules

In the four-quark models, the structures of the scalar nonet mesons in the ideal mixing limit can be symbolically taken as $[2, 5, 6]$

\[
\sigma(600) = u\bar{d}u\bar{d}, \quad f_0(980) = \frac{us\bar{s} + d\bar{s}d}{\sqrt{2}}, \\
a_0^+(980) = dsu\bar{s}, \quad a_0^+(980) = \frac{us\bar{s} - d\bar{s}d}{\sqrt{2}}, \quad a_0^+(980) = u\bar{s}d, \\
\kappa^+(800) = u\bar{d}u\bar{d}, \quad \kappa^0(800) = u\bar{d}u\bar{s}, \quad \kappa^0(800) = u\bar{d}u, \quad \kappa^-(800) = d\bar{s}u\bar{d}.
\]

The four-quark configurations of the $J^{PC} = 0^{++}$ mesons can give a lot of satisfactory descriptions of the hadron phenomenon, for example, the mass degeneracy of the $f_0(980)$ and $a_0(980)$ mesons, the mass hierarchy pattern of the scalar nonet, the large radiative widths of the $f_0(980)$ and $a_0^+(980)$ mesons, the $D^+$ to $\pi^+\pi^+\pi^-$ and $D_s^+(c\bar{s})$ to $\pi^+\pi^+\pi^-$ decays.

In the following, we write down the interpolating currents for the scalar nonet
mesons below 1GeV based on the four-quark model \([9, 10, 11]\),

\[
J^A_{f_0} = \frac{\epsilon^{abc} \epsilon^{ade}}{\sqrt{2}} \left[ (u^T_b C_\gamma s_c)(\bar{u}_d \gamma_5 C_\gamma s_e^T) + (d^T_b C_\gamma s_c)(\bar{d}_d \gamma_5 C_\gamma s_e^T) \right],
\]

(1)

\[
J^B_{f_0} = \frac{\epsilon^{abc} \epsilon^{ade}}{\sqrt{2}} \left[ (u^T_b C_\gamma s_c)(\bar{u}_d C_\gamma s_e^T) + (d^T_b C_\gamma s_c)(\bar{d}_d C_\gamma s_e^T) \right],
\]

(2)

\[
J^A_{a_0} = \frac{\epsilon^{abc} \epsilon^{ade}}{\sqrt{2}} \left[ (u^T_b C_\gamma s_c)(\bar{u}_d C_\gamma s_e^T) - (d^T_b C_\gamma s_c)(\bar{d}_d C_\gamma s_e^T) \right],
\]

(3)

\[
J^B_{a_0} = \frac{\epsilon^{abc} \epsilon^{ade}}{\sqrt{2}} \left[ (u^T_b C_\gamma s_c)(\bar{u}_d C_\gamma s_e^T) - (d^T_b C_\gamma s_c)(\bar{d}_d C_\gamma s_e^T) \right],
\]

(4)

\[
J^A_{\kappa^0} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma s_c)(\bar{d}_d \gamma_5 C_\gamma s_e^T),
\]

(5)

\[
J^B_{\kappa^0} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma s_c)(\bar{d}_d C_\gamma s_e^T),
\]

(6)

\[
J^A_{\kappa^+} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma d_c)(\bar{d}_d \gamma_5 C_\gamma s_e^T),
\]

(7)

\[
J^B_{\kappa^+} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma d_c)(\bar{d}_d C_\gamma s_e^T),
\]

(8)

\[
J^A_{\kappa^0} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma d_c)(\bar{u}_d \gamma_5 C_\gamma s_e^T),
\]

(9)

\[
J^B_{\kappa^0} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma d_c)(\bar{u}_d C_\gamma s_e^T),
\]

(10)

\[
J^A_{\sigma} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma d_c)(\bar{u}_d C_\gamma d_e^T),
\]

(11)

\[
J^B_{\sigma} = \epsilon^{abc} \epsilon^{ade}(u^T_b C_\gamma d_c)(\bar{u}_d C_\gamma d_e^T),
\]

(12)

where \(a, b, c, \ldots\) are color indices and \(C\) is the charge conjugation matrix. The constituents \(S^a(x) = \epsilon^{abc} u^T_b(x) C_\gamma d_c(x), \epsilon^{abc} u^T_b(x) C_\gamma s_c(x), \epsilon^{abc} d^T_b(x) C_\gamma s_c(x)\) and \(P^a(x) = \epsilon^{abc} u^T_b(x) C d_c(x), \epsilon^{abc} u^T_b(x) C s_c(x), \epsilon^{abc} d^T_b(x) C s_c(x)\) represent the scalar \(J^P = 0^+\) and the pseudoscalar \(J^P = 0^-\) diquarks respectively. They both belong to the antitriplet \(3\) representation of the color \(SU(3)\) group and can cluster together to form \(S^a - \bar{S}^a\) type and \(P^a - \bar{P}^a\) type diquark pairs to give the correct spin and parity for the scalar mesons \(J^P = 0^+\). The scalar diquarks correspond to the \(^1S_0\) states of \(ud, us\) and \(ds\) diquark systems. The one-gluon exchange force and the instanton induced force can lead to significant attractions between the quarks in the \(0^+\) channels \([12]\). The pseudoscalar diquarks do not have nonrelativistic limit, can be taken as the \(3P_0\) states. As the instanton induced force results in strong attractions in the scalar diquark channel and strong repulsions in the pseudoscalar diquark channel, if the effects of the instanton are manifest in the \(0^{++}\) nonet mesons, we shall prefer the \(S^a - \bar{S}^a\) type interpolating currents to the \(P^a - \bar{P}^a\) type interpolating currents \([12, 13]\).

The calculation of the \(a_0(980)\) meson as a four-quark state in the QCD sum rules approach was done originally for the decay constant and the hadronic coupling
constants with the interpolating currents \( J^1_{a_0} \) and \( J^2_{a_0} \),

\[
\begin{align*}
J^1_{f_0(a_0)} &= \sum_{\Gamma=1,\pm \gamma_5} \bar{s} \Gamma s \frac{\bar{u} \Gamma u \pm \bar{d} \Gamma d}{\sqrt{2}}, \\
J^2_{f_0(a_0)} &= \sum_{\Gamma=1,\pm \gamma_5} \bar{s} \Gamma s \frac{\lambda^{a}}{2} \frac{\bar{u} \lambda^{a} u \pm \bar{d} \lambda^{a} d}{\sqrt{2}},
\end{align*}
\]

(13)

where the \( \lambda^a \) is the \( SU(3) \) Gell-Mann matrix. Perform Fierz transformation both in the Dirac spinor and color space, for example, we can obtain

\[
\begin{align*}
J^2_{f_0} &\propto C_A J^A_{f_0} + C_B J^B_{f_0} \cdots, \\
J^2_{a_0} &\propto C_A J^A_{a_0} + C_B J^B_{a_0} \cdots.
\end{align*}
\]

(14)

Here \( C_A \) and \( C_B \) are coefficients which are not shown explicitly for simplicity. In the color superconductivity theory, the one-gluon exchange induced Nambu–Jona-Lasinio like Models will also lead to the \( S^a - \bar{S}^a \) type and \( P^a - \bar{P}^a \) type diquark pairs [16],

\[
G q\gamma^\mu \frac{\lambda^{a}}{2} q \bar{q} \gamma^\mu q \propto C_A S^a \bar{S}^a + C_B P^a \bar{P}^a + \cdots.
\]

(15)

So we can take the point of view that the lowest lying scalar mesons are \( S \)-wave bound states of diquark-antidiquark pairs of \( S^a - \bar{S}^a \) type and \( P^a - \bar{P}^a \) type.

In this article, we investigate the masses of the scalar nonet mesons with two interpolating currents respectively and choose the following two-point correlation functions,

\[
\Pi^i_S(p) = i \int d^4x \ e^{ip \cdot x} \langle 0 | T[J^i_S(x) J^{i\dagger}_S(0)] | 0 \rangle.
\]

(16)

Here the current \( J^i_S \) denotes \( J^A_{f_0}, J^B_{f_0}, J^A_{a_0}, J^B_{a_0}, J^A_{a_0^\dagger}, J^B_{a_0^\dagger}, J^A_{\kappa^+}, J^B_{\kappa^+}, J^A_{\kappa^0}, J^B_{\kappa^0}, J^A_{\sigma} \) and \( J^B_{\sigma} \). According to the basic assumption of current-hadron duality in the QCD sum rules approach [8], we insert a complete series of intermediate states satisfying the unitarity principle with the same quantum numbers as the current operator \( J^i_S(x) \) into the correlation functions in Eq.(16) to obtain the hadronic representation. Isolating the ground state contributions from the pole terms of the nonet mesons, we get the result,

\[
\Pi^i_S(p) = \frac{2 f_{S}^{i 2} m_{S}^{i 8}}{m_{S}^{i 2} - p^{2}} + \cdots,
\]

(17)

where the following definitions have been used,

\[
\langle 0 | J^i_S | S \rangle = \sqrt{2} f_{S}^{i} m_{S}^{i 4}.
\]

(18)

\[2^2\]There is also other work based on the four-quark model with QCD sum rules [15], however, it is not available in NCEPU.
We have not shown the contributions from the higher resonances and continuum states explicitly for simplicity.

The calculation of operator product expansion in the deep Euclidean space-time region is straightforward and tedious, technical details are neglected for simplicity. In this article, we consider the vacuum condensates up to dimension six. Once the analytical results are obtained, then we can take the current-hadron dualities below the thresholds $s_0$ and perform the Borel transformation with respect to the variable $P^2 = -p^2$, finally we obtain the following sum rules,

$$2 f_{f_0(a_0)} A^2 m_{f_0(a_0)} e^{-\frac{m_{f_0(a_0)}^2}{M^2}} = AA,$$

$$2 f_{f_0(a_0)} B^2 m_{f_0(a_0)} e^{-\frac{m_{f_0(a_0)}^2}{M^2}} = BB,$$

$$2 f_{\kappa(\kappa^0)} A^2 m_{\kappa(\kappa^0)} e^{-\frac{m_{\kappa(\kappa^0)}^2}{M^2}} = CC,$$

$$2 f_{\kappa(\kappa^0)} B^2 m_{\kappa(\kappa^0)} e^{-\frac{m_{\kappa(\kappa^0)}^2}{M^2}} = DD,$$

$$2 f_{\sigma} A^2 m_{\sigma} e^{-\frac{m_{\sigma}^2}{M^2}} = EE,$$

$$2 f_{\sigma} B^2 m_{\sigma} e^{-\frac{m_{\sigma}^2}{M^2}} = FF,$$

where

$$AA = \int_{4m_2^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ \frac{s^4}{2^{9/5} \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s}{12\pi^2} + \frac{3\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle}{2^6 \pi^4} m_s s \right\},$$

$$BB = \int_{4m_2^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ -\frac{s^4}{2^{9/5} \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s}{12\pi^2} + \frac{3\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle}{2^6 \pi^4} m_s s \right\},$$

$$CC = \int_{m_2^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ \frac{s^4}{2^{9/5} \pi^6} + \frac{\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle \langle \bar{q}q \rangle}{24\pi^2} s + \frac{3\langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle}{2^7 \pi^4} m_s s \right\},$$

$$DD = \int_{m_2^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ -\frac{s^4}{2^{9/5} \pi^6} + \frac{\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle \langle \bar{q}q \rangle}{24\pi^2} s + \frac{3\langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle}{2^7 \pi^4} m_s s \right\},$$

$$EE = \int_{m_2^2}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ \frac{s^4}{2^{9/5} \pi^6} + \frac{\langle \bar{q}q \rangle^2}{12\pi^2} s + \frac{s^2}{2^9 \pi^4} \frac{\alpha_s G}{\pi} \right\},$$

$$FF = \int_{0}^{s_0} ds e^{-\frac{s}{M^2}} \left\{ -\frac{s^4}{2^{9/5} \pi^6} + \frac{\langle \bar{q}q \rangle^2}{12\pi^2} s - \frac{s^2}{2^9 \pi^4} \frac{\alpha_s G}{\pi} \right\}.$$
here we have taken the same notation $s_0$ for the threshold parameters $s_{f_0(a_0)}^0$, $s_{f_{K^0}^0}$ and $s_{f_0^0}^0$. Differentiate the above sum rules with respect to the variable $\frac{1}{m^2}$, then eliminate the quantities $f_{f_0(a_0)}^A$, $f_{f_{K^0}^0}$, $f_{f_{K^0}^0}$, $f_{f_{K^0}^0}$, $f_{f_{K^0}^0}$ and $f_{f_{K^0}^0}$, we obtain

$$m_{f_0(a_0)}^{A_2} = \int_{m_2^2}^{s_0} d s e^{-\frac{s}{m^2}} \left\{ \frac{s^5}{295! \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s^2}{12 \pi^2} + \frac{3 \langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle}{26 \pi^4} m_s s^2 \\
- \frac{2 \langle \bar{q}q \rangle - \langle \bar{s}s \rangle}{26 \pi^4} m_s s^3 + \frac{s^3}{293! \pi^4} \left( \frac{\alpha_s G}{\pi} \right) \right\} / AA,$$

$$m_{f_0(a_0)}^{B_2} = \int_{m_2^2}^{s_0} d s e^{-\frac{s}{m^2}} \left\{ \frac{s^5}{295! \pi^6} + \frac{\langle \bar{s}s \rangle \langle \bar{q}q \rangle s^2}{12 \pi^2} + \frac{3 \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle}{26 \pi^4} m_s s^2 \\
- \frac{2 \langle \bar{q}q \rangle + \langle \bar{s}s \rangle}{26 \pi^4} m_s s^3 - \frac{s^3}{293! \pi^4} \left( \frac{\alpha_s G}{\pi} \right) \right\} / BB,$$

$$m_{K^0}^{A_2} = \int_{m_2^2}^{s_0} d s e^{-\frac{s}{m^2}} \left\{ \frac{s^5}{295! \pi^6} + \frac{\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle \langle \bar{q}q \rangle s^2}{24 \pi^2} + \frac{3 \langle \bar{q}g_s \sigma Gq \rangle - \langle \bar{s}g_s \sigma Gs \rangle}{27 \pi^4} m_s s^2 \\
- \frac{2 \langle \bar{q}q \rangle - \langle \bar{s}s \rangle}{27 \pi^4} m_s s^3 + \frac{s^3}{293! \pi^4} \left( \frac{\alpha_s G}{\pi} \right) \right\} / CC,$$

$$m_{K^0}^{B_2} = \int_{m_2^2}^{s_0} d s e^{-\frac{s}{m^2}} \left\{ \frac{s^5}{295! \pi^6} + \frac{\langle \bar{q}q \rangle^2 + \langle \bar{s}s \rangle \langle \bar{q}q \rangle s^2}{24 \pi^2} + \frac{3 \langle \bar{q}g_s \sigma Gq \rangle + \langle \bar{s}g_s \sigma Gs \rangle}{27 \pi^4} m_s s^2 \\
- \frac{2 \langle \bar{q}q \rangle + \langle \bar{s}s \rangle}{27 \pi^4} m_s s^3 - \frac{s^3}{293! \pi^4} \left( \frac{\alpha_s G}{\pi} \right) \right\} / DD,$$

$$m_{\sigma}^{A_2} = \int_0^{s_0} d s e^{-\frac{s}{m^2}} \left\{ \frac{s^5}{295! \pi^6} + \frac{\langle \bar{q}q \rangle^2}{12 \pi^2} s^2 + \frac{s^3}{293! \pi^4} \left( \frac{\alpha_s G}{\pi} \right) \right\} / EE,$$

$$m_{\sigma}^{B_2} = \int_0^{s_0} d s e^{-\frac{s}{m^2}} \left\{ -\frac{s^5}{295! \pi^6} + \frac{\langle \bar{q}q \rangle^2}{12 \pi^2} s^2 - \frac{s^3}{293! \pi^4} \left( \frac{\alpha_s G}{\pi} \right) \right\} / FF.$$

It is easy to perform the $s$ integral in Eqs.(25-36), we prefer this form for simplicity.

### 3 Numerical Results

In calculation, the parameters are taken as $\langle \bar{s}s \rangle = 0.8 \langle \bar{u}u \rangle$, $\langle \bar{s}g_s \sigma Gs \rangle = m_0^2 \langle \bar{s}s \rangle$, $\langle \bar{q}g_s \sigma Gq \rangle = m_2^2 \langle \bar{q}q \rangle$, $m_0 = 0.8{\text{GeV}}^2$, $\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = \langle \bar{q}q \rangle = (-219{\text{MeV}})^3$, $\langle \frac{\alpha_s G}{\pi} \rangle = (0.33{\text{GeV}})^4$, $m_u = m_d = 0$ and $m_s = 150{\text{MeV}}$. The main contributions to the sum rules come from the quark condensates terms (i.e. $\langle \bar{q}q \rangle$ and $\langle \bar{s}s \rangle$), here we have taken the standard values and neglected the uncertainties, small variations of those condensates will not lead to large changes about the numerical values. The threshold parameters are taken as $s_{f_0(a_0)}^0 = (1.4 - 1.6){\text{GeV}}^2$, $s_{f_{K^0}}^0 = (1.0 - 1.2){\text{GeV}}^2$ and $s_{f_0}^0 = (0.8 - 1.0){\text{GeV}}^2$ to avoid possible contaminations from the higher resonances and continuum states. The widths of the $f_0(980)$ and $a_0(980)$ mesons are narrow, the threshold parameters $s_{f_0(a_0)}^0 = (1.4 - 1.6){\text{GeV}}^2$ are sufficient to include the contributions from those mesons. Although the existence of the $\sigma$
meson is confirmed, there are still controversy about its mass and width, here we take the point of view that the $\sigma$ meson is the isosinglet $S$-wave $\pi\pi$ resonance near $600\text{MeV}$ and take the largest $s^0_\sigma$ to be the $K\bar{K}$ threshold. As far as the $\kappa(800)$ mesons are concerned, there are still controversy about their existence, here we take them as the $S$-wave isospin $\frac{1}{2}$ $K\pi$ resonance with the Breit-Wigner mass about $800\text{MeV}$ and width about $400\text{MeV}$; our numerical results support this assumption [9]. In the region $M^2 = (1.2 - 3.0)\text{GeV}^2$, the sum rules for $m^A_{f_0} = m^A_{a_0}$, $m^B_{f_0} = m^B_{a_0}$, $m^A_{\kappa^+} = m^A_{\kappa^0}$, $m^B_{\kappa^+} = m^B_{\kappa^0}$, $m^A_\sigma$ and $m^B_\sigma$ are almost independent of the Borel parameter $M^2$, the values of masses for those mesons are shown in Table 1. Due to the special quark constituents and Dirac structures of the interpolating currents, the $f_0(980)$ and $a_0(980)$, the $\kappa^+(800)$ and $\kappa^0(800)$ have degenerate masses respectively. For the $S^a - S^a$ type interpolating currents $J^A_{f_0}$, $J^A_{a_0}$, $J^A_{\kappa^+}$, $J^A_{\kappa^0}$ and $J^A_\sigma$, the values for masses are about $m^A_{f_0} = m^A_{a_0} = (0.96 - 1.02)\text{GeV}$, $m^A_{\kappa^+} = m^A_{\kappa^0} = (0.80 - 0.88)\text{GeV}$ and $m^A_\sigma = (0.72 - 0.80)\text{GeV}$, while for the $P^a - \bar{P}^a$ type interpolating currents $J^B_{f_0}$, $J^B_{a_0}$, $J^B_{\kappa^+}$, $J^B_{\kappa^0}$ and $J^B_\sigma$, the values for masses are about $m^B_{f_0} = m^B_{a_0} = (0.95 - 1.01)\text{GeV}$, $m^B_{\kappa^+} = m^B_{\kappa^0} = (0.79 - 0.87)\text{GeV}$ and $m^B_\sigma = (0.71 - 0.79)\text{GeV}$. In this article, we take the ideal mixing limit for the two isoscalar mesons, the $f_0(980)$ and $\sigma(600)$. We can investigate the mixing with the following substitutions for the interpolating currents,

$$
J^A_\sigma \to \cos\theta J^A_\sigma - \sin\theta J^A_{f_0}, \quad J^A_{f_0} \to \sin\theta J^A_\sigma + \cos\theta J^A_{f_0},
$$

$$
J^B_\sigma \to \cos\varphi J^B_\sigma - \sin\varphi J^B_{f_0}, \quad J^B_{f_0} \to \sin\varphi J^B_\sigma + \cos\varphi J^B_{f_0},
$$

(37)

here $\theta$ and $\varphi$ are mixing angles. From above equations, we can obtain lower masses for the $f_0(980)$ meson and higher masses for the $\sigma(600)$ meson with small mixing angles, which will not potentially change our numerical results. There may be some $q\bar{q}$ components in those nonet scalar mesons, as the $q\bar{q}$ type interpolating currents can also give the correct spin and parity, $J^P = 0^+$. We can explore the mixing between the two quark and four quark components by introducing a free parameter $t$ with mass dimension 3, which can vary between 0 and $\infty$, for example,

$$
J^A_\sigma \to J^A_\sigma + t\frac{\bar{u}u + \bar{d}d}{2}, \quad J^A_{\kappa^+} \to J^A_{\kappa^+} + t\bar{s}u.
$$

(38)

The analysis based on QCD sum rules approach indicates that the masses of the ground states of $q\bar{q}$ type interpolating currents are always larger than $1\text{GeV}$ or about $1\text{GeV}$ [17], small $q\bar{q}$ components will lead to slightly higher masses for those scalar mesons. In the limit $t \to \infty$, we obtain the sum rules for the ground states of $q\bar{q}$ type interpolating currents. Although the values for masses $m^A_{f_0}$, $m^A_{a_0}$, $m^A_{\kappa^+}$, $m^A_{\kappa^0}$ and $m^A_\sigma$ lie a little above the corresponding masses $m^B_{f_0}$, $m^B_{a_0}$, $m^B_{\kappa^+}$, $m^B_{\kappa^0}$ and $m^B_\sigma$, we can not get to the conclusion that the scalar nonet mesons prefer the $S^a - \bar{S}^a$ type interpolating currents $J^A_{f_0}$, $J^A_{a_0}$, $J^A_{\kappa^+}$, $J^A_{\kappa^0}$ and $J^A_\sigma$ to the $P^a - \bar{P}^a$ type.
interpolating currents \( J^B_f, J^B_d, J^B_s, J^B_c, J^B_{\kappa}, J^B_{\kappa^0}, J^B_{\kappa_0} \) and \( J^B_{\sigma} \). Precise determination of what type interpolating currents we should choose calls for original theoretical approaches, the contributions from the direct instantons may do the work. In our recent work, we observe that the contributions from the direct instantons are negligible for the pentaquark state \( \Theta^+(1540) \) [18], however, the contributions from the direct instantons can improve the QCD sum rule greatly in some channels, for example, the nonperturbative contributions from the direct instantons to the conventional operator product expansion can significantly improve the stability of chirally odd nucleon sum rules [19, 20]. Despite whatever the interpolating currents may be, we observe that they can both give the correct mass hierarchy pattern of the scalar nonet mesons below 1GeV, there must be some four-quark constituents in those mesons.

\[
\begin{array}{|c|c|}
\hline
m^2_{f_0(a_0)} & (0.96 - 1.02)\text{GeV}^2 \\
\hline
m^2_{B_0(a_0)} & (0.95 - 1.01)\text{GeV}^2 \\
\hline
m^2_{\kappa^0} & (0.80 - 0.88)\text{GeV}^2 \\
\hline
m^2_{\kappa^+} & (0.79 - 0.87)\text{GeV}^2 \\
\hline
m^2_{\sigma} & (0.72 - 0.80)\text{GeV}^2 \\
\hline
m^2_{\kappa^0} & (0.71 - 0.79)\text{GeV}^2 \\
\hline
\end{array}
\]

Table 1: The values of the scalar nonet mesons

4 Conclusions

In this article, we take the point of view that the 0++ nonet mesons below 1GeV are four-quark states \((qq)(\bar{q}q)\) in the ideal mixing limit, and devote to determine the values of their masses in the framework of the QCD sum rules approach. Due to the special quark constituents and Dirac structures of the interpolating currents, the \( f_0(980) \) and \( a_0(980) \), the \( \kappa^+(800) \) and \( \kappa^0(800) \) have degenerate masses respectively. For the \( S^a - \bar{S}^a \) type interpolating currents \( J^A_f, J^A_{a_0}, J^A_{a_+}, J^A_{\kappa^+}, J^A_{\kappa^0}, J^A_{\kappa^-} \) and \( J^A_{\sigma} \), the values for masses are about \( m^A_{f_0} = m^A_{a_0} = (0.96 - 1.02)\text{GeV} \), \( m^A_{\kappa^0} = m^A_{\kappa^+} = (0.80 - 0.88)\text{GeV} \) and \( m^A_{\sigma} = (0.72 - 0.80)\text{GeV} \), while for the \( P^a - \bar{P}^a \) type interpolating currents \( J^B_f, J^B_{a_0}, J^B_{a_+}, J^B_{\kappa^+}, J^B_{\kappa^0} \) and \( J^B_{\sigma} \), the values for masses are about \( m^B_{f_0} = m^B_{a_0} = (0.95 - 1.01)\text{GeV} \), \( m^B_{\kappa^0} = m^B_{\kappa^+} = (0.79 - 0.87)\text{GeV} \) and \( m^B_{\sigma} = (0.71 - 0.79)\text{GeV} \). Although the values for masses \( m^A_{f_0}, m^A_{a_0}, m^A_{a_+}, m^A_{\kappa^+}, m^A_{\kappa^0} \) and \( m^A_{\sigma} \) lie a little above the corresponding masses \( m^B_{f_0}, m^B_{a_0}, m^B_{a_+}, m^B_{\kappa^+}, m^B_{\kappa^0} \) and \( m^B_{\sigma} \), we can not get to the conclusion that the scalar nonet mesons prefer the \( S^a - \bar{S}^a \) type interpolating currents \( J^A_f, J^A_{a_0}, J^A_{a_+}, J^A_{\kappa^+}, J^A_{\kappa^0} \) and \( J^A_{\sigma} \) to the \( P^a - \bar{P}^a \) type interpolating currents \( J^B_f, J^B_{a_0}, J^B_{a_+}, J^B_{\kappa^+}, J^B_{\kappa^0} \) and \( J^B_{\sigma} \). Despite whatever the
interpolating currents may be, we observe that they can both give the correct mass hierarchy pattern of the scalar nonet, there must be some four-quark constituents in those mesons, our results support the four-quark model and the hybrid model. In the hybrid model, those mesons are four-quark states \((qq)_3(\bar{q}\bar{q})_3\) in \(S\)-wave near the center, with some constituent \(qq\) in \(P\)-wave, but further out they rearrange into \((qq)_1(\bar{q}\bar{q})_1\) states and finally as meson-meson states \[5\]. Precise determination of what type interpolating currents we should choose calls for original theoretical approaches, the contributions from the direct instantons may do the work.

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**References**

[1] S. Godfray and J. Napolitano, Rev. Mod. Phys. **71** (1999) 1411.

[2] R. L. Jaffe and K. Johnson, Phys. Lett. **B60** (1976) 201; R. L. Jaffe, Phys. Rev. **D15** (1977) 267, 281; **D17** (1978) 1444; N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315** (1989) 465; N. N. Achasov and V. V. Gubin, Phys. Rev. **D56** (1997) 4084; Phys. Rev. **D63** (2001) 094007; J. R. Pelaez, Phys. Rev. Lett. **92** (2004) 102001; D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. **D59** (1999) 074026; M. Harada, F. Sannino and J. Schechter, Phys. Rev. **D69** (2004) 034005.

[3] J. Weinstein and N. Isgur, Phys. Rev. Lett. **48** (1982) 659; Phys. Rev. **D27** (1983) 588; Phys. Rev. **D41** (1990) 2236 ; F. E. Close, N. Isgur and S. Kumana, Nucl. Phys. **B389** (1993) 513; R. Kaminski, L. Lesniak and J. P. Maillet, Phys. Rev. **D50** (1994) 3145; N. N. Achasov, V. V. Gubin and V. I. Shevchenko, Phys. Rev. **D56** (1997) 203; Yu. S. Surovtsev, D. Krupa and M. Nagy, [hep-ph/0311195](http://arxiv.org/abs/hep-ph/0311195).

[4] N. A. Tornqvist, Z. Phys. **C68** (1995) 647; M. Boglione and M. R. Pennington, Phys. Rev. Lett **79** (1997) 1998; N. A. Tornqvist, [hep-ph/0008136](http://arxiv.org/abs/hep-ph/0008136) N. A. Tornqvist and A. D. Polosa, Nucl. Phys. **A692** (2001) 259; A. Deandrea, R. Gatto, G. Nardulli, A. D. Polosa and N. A. Tornqvist, Phys. Lett. **B502** (2001) 79; F. De Fazio and M. R. Pennington, Phys. Lett. **B521** (2001) 15; M. Boglione and M. R. Pennington, Phys. Rev. **D65** (2002) 114010; Z. G. Wang, W. M. Yang and S. L. Wan, Eur. Phys. J. **C37** (2004) 223; E. V. Beveren, T. A. Rijken, K.
Metzger, C. Dullemond, G. Rupp and J. E. Ribeiro, Z. Phys. C30 (1986) 615; E. V. Beveren and G. Rupp, Eur. Phys. J. C10 (1999) 469; E. V. Beveren and G. Rupp, Eur. Phys. J. C22 (2001) 493.

[5] F. E. Close and N. A. Tornqvist, J. Phys. G28 (2002) R249; and references therein.

[6] R. L. Jaffe, Phys. Rept. 409 (2005) 1; C. Amsler and N. A. Tornqvist, Phys. Rept. 389 (2004) 61; and references therein.

[7] For example, D. Black, A. H. Fariborz, F. Sannino and J. Schechter, Phys. Rev. D58 (1998) 054012; S. Ishida, M. Ishida, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. 98 (1997) 621; S. Ishida, M. Ishida, H. Takahashi, T. Ishida, K. Takamatsu and T. Tsuru, Prog. Theor. Phys. 95 (1996) 745; S. Ishida, T. Ishida, M. Ishida, K. Takamatsu, T. Tsuru, Prog. Theor. Phys. 98 (1997) 1005.

[8] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979) 385, 448.

[9] L. Maiani, A. Polosa, F. Piccinini and V. Riquer, Phys. Rev. Lett. 93 (2004) 212002.

[10] T. V. Brito, F. S. Navarra, M. Nielsen and M. E. Bracco, Phys. Lett. B608 (2005) 69.

[11] Z. G. Wang and W. M. Yang, hep-ph/0501105.

[12] A. De Rujula, H. Georgi and S. L. Glashow, Phys. Rev. D12 (1975) 147; T. DeGrand, R. L. Jaffe, K. Johnson and J. E. Kiskis, Phys. Rev. D12 (1975) 2060; G. ’t Hooft, Phys. Rev. D14 (1976) 3432 [Erratum-ibid. Phys. Rev. D18 (1978) 2199]; E. V. Shuryak, Nucl. Phys. B203 (1982) 93; T. Schafer and E. V. Shuryak, Rev. Mod. Phys. 70 (1998) 323; E. Shuryak and I. Zahed, Phys. Lett. B 589 (2004) 21.

[13] J. I. Latorre and P. Pascual, J. Phys. G11, (1985) L231.

[14] S. Narison, Phys. Lett. B175 (1986) 88; S. Narison, QCD Spectral Sum Rules, World Scientific Lecture Notes in Physics 26.

[15] V. M. Braun and Y.M. Shabelski, Sov. J. Nucl. Phys. 50 (1989) 306.

[16] R. T. Cahill, J. Praschik and C. Burden Austral.J.Phys. 42 (1989) 161; M. Huang, hep-ph/0409167.

[17] D. S. Du, J. W. Li and M. Z. Yang, hep-ph/0409302; S. Narison, hep-ph/0208081 and references therein.
[18] Z. G. Wang, W. M. Yang and S. L. Wan, hep-ph/0501015.

[19] A. E. Dorokhov and N. I. Kochelev, Z. Phys. C46 (1990) 281.

[20] M. Aw, M. K. Banerjee and H. Forkel, Phys. Lett. B454 (1999) 147; H. Forkel and M. Nielsen, Phys. Rev. D55 (1997) 1471; H. Forkel and M. K. Banerjee, Phys. Rev. Lett. 71 (1993) 484.