Panchromatic Hubble Andromeda Treasury XIII: The Cepheid period-luminosity relation in M31

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ABSTRACT
Using Hubble Space Telescope Advanced Camera for Surveys (HST/ACS) and Wide Field Camera 3 (WFC3) observations from the Panchromatic Hubble Andromeda Treasury (PHAT), we present new period–luminosity (P-L) relations for Cepheid variables in M31. Cepheids from several ground-based studies are identified in the PHAT photometry to derive new P-L and Wesenheit P-L relations in the near infrared and visual filters. We derive a distance modulus to M31 of 24.51 ± 0.08 in the IR bands and 24.32 ± 0.09 in the visual bands, including the first P-L relations in the F475W and F814W filters for M31. Our derived visual and IR distance moduli disagree at slightly more than a 1σ level. Differences in the P-L relations between ground-based and HST observations are investigated for a subset of Cepheids. We find a significant discrepancy between ground-based and HST P-L relations with the same Cepheids, suggesting adverse effects from photometric contamination in ground-based Cepheid observations. Additionally, a statistically significant radial trend in the P-L relation is found which does not appear to be explained by metallicity.

Key words: stars: variables: Cepheids – galaxies: individual – Local Group – distance scale.

1 INTRODUCTION
The pulsating class of variable stars known as Cepheids has been studied for a number of reasons, the most prominent of which is the existence of a relation between their periods and luminosities. The period–luminosity (P-L) relation for Cepheid variables casts these stars as standard candles, making them ideal for determining distances. As a result, Cepheids play a vital role in the cosmological distance ladder, being observable from the Local Group to distances of tens of Mpc, and overlapping with secondary distance indicators.

More accurate distances from the Cepheid P-L relation lead to improved calibration of stellar luminosities, constraints on stellar population synthesis models, and measurements of the Hubble Constant ($H_0$). Improved precision of the P-L relation has become increasingly important in light of the tension between the Cepheid-based determination of $H_0$ and that from the Planck mission (Riess et al. 2011; Planck Collaboration XVI 2014).

The possibility of using M31 as a local anchor for the P-L relation in the future could benefit the effort to improve the distance scale. Although the further distance degrades accuracy of measurements compared to more local distance estimates (i.e. Milky Way and Large Magellanic Cloud), M31 has the opportunity to diversify the methods used to determine distances. Compared to the Large Magellanic Cloud (LMC), M31 is more akin to external galaxies used for even further distance measurements. The use of M31 as an anchor could potentially benefit distance determinations for the extragalactic distance scale, and perhaps assist in addressing the discrepancy between $H_0$ values.

The Panchromatic Hubble Andromeda Treasury (PHAT) survey of M31 provides a unique opportunity to image a significant number of Cepheids previously only observed from ground-based telescopes. Obtaining accurate Hubble Space Telescope (HST) magnitudes for these variable stars promises to reduce the error in the P-L relation for M31, leading the way to a more precise value of the distance. Previous ground-based surveys have detected thousands of Cepheids in and near the M31 disc (e.g. Stanek et al. 1998; An et al. 2004; Kodric et al. 2013), but prior to PHAT, only small samples of Cepheids in M31 have been observed with HST (Macri et al. 2001; Riess, Fliri & Valls-Gabaud 2012). The extensive coverage of the PHAT programme opens new opportunities to improve the distance to M31 via observations of larger numbers of Cepheid variables with HST.

There are three main contributors to the Cepheid distance uncertainty: metallicity effects, blending and crowding, and the uncertainty in the distance to the LMC (Riess et al. 2009a;
2 DATA

The data used in this paper are obtained from the PHAT photometry of M31. The PHAT survey coverage, design, and photometry are described at length in Dalcanton et al. (2012). The observations utilize the Wide Field Camera 3 (WFC3) in both the ultraviolet-visible (UVIS) mode and the infrared (IR) mode and the Advanced Camera for Surveys (ACS) Wide Field Channel (WFC). They cover about one third of the disc of M31 across six filters ($F775W$ and $F336W$ in the UV with WFC3; $F110W$ and $F160W$ in the IR with WFC3; $F475W$ and $F814W$ in the visual bands with ACS). Each HST pointing is two orbits: one orbit for WFC3/UVIS and one orbit for WFC3/IR (with ACS/WFC in parallel mode). The coverage of M31 by PHAT is divided into 23 `bricks’, each composed of 18 HST fields of view (in a $6 \times 3$ layout, each brick covering a $3 \times 1.5$ kpc area). Photometry is performed on each field and brick using DOLPHOT (Dolphin 2000) and the subsequent photometry files are filtered to reject low-quality and non-stellar objects, as described in Dalcanton et al. (2012) and Williams et al. (2014).

To obtain the largest sample of Cepheid observations possible, we use multiple Cepheid catalogues: PAndromeda (Kodric et al. 2013), DIRECT (Kuznzy et al. 1998, 1999; Stanek et al. 1998, 1999; Bonanos et al. 2003), and Cepheids already identified in the PHAT images (Riess et al. 2012).

The DIRECT survey imaged M31 from 1996 to 1997 with the McGraw Hill Telescope at the Michigan-Dartmouth-MIT Observatory and from 1996 to 2000 with the 1.2-m telescope at the F. L. Whipple Observatory. The Cepheids in the DIRECT survey were observed more than 130 times in the Johnson $V$ band but less frequently in the Johnson $B$ and Cousins–Kron $I$ filters. All stars have $V$ photometry, but not necessarily $B$ or $I$ magnitudes. 6 fields were imaged by the survey, 5 of which overlap with the PHAT coverage of M31, including 87 Cepheids. Further details about the acquisition and reduction procedures can be found in the DIRECT papers (Kuznzy et al. 1998, 1999; Stanek et al. 1998, 1999; Bonanos et al. 2003).

The Pan-STARRS (Panoramic Survey Telescope and Rapid Response System) survey of M31, also referred to as PAndromeda, uses the 1.8-m Panoramic Survey Telescope and Rapid Response System with the Giga Pixel Camera in Haleakala, Maui, Hawaii (Kaiser et al. 2002; Hodapp et al. 2004; Tonry & Onaka 2009). From mid-2010 to late-2011, 183 epochs of data were gathered with a half hour of observing each night in the $r_P$ and $i_P$ bands where $P$ refers to the filter set used on the Pan-STARRS 1 system. Details of the observations as well as reduction procedures can be found in Kodric et al. (2013) and Lee et al. (2012). A similar study examining these stars in the NIR filters from the PHAT images from MAST can be found in Kodric et al. (2015), though with a smaller sample size of about 111 long-period Cepheids. Photometry of 67 Cepheids from the first year of PHAT data was examined by Riess et al. (2012). The initial identification of these Cepheids was through the POMME survey (Fiiri & Valls-Gabaud 2012), with an additional Cepheid from the DIRECT survey (Kuznzy et al. 1998, 1999; Stanek et al. 1998, 1999; Bonanos et al. 2003), and two more Cepheids from the PAndromeda survey (Kodric et al. 2013). Using all three sources, we produce a data set containing a total of 175 distinct variables with published periods longer than 10 d, detailed in Table 1 and whose locations are shown in Fig. 1. We restrict our sample of Cepheids to those with periods longer than 10 d because the presence of a break in linearity in the P-L relation at a period of approximately 10 d has been well documented (Simon 2010a), the effort to reduce the error and dispersion in the P-L relation has made large gains in the past two decades (Madore & Freedman 1992; Macri 2005; Freedman & Madore 2010), though further improvements are on the horizon (Gerke et al. 2011).

There is as yet no clear consensus on the universality of the P-L relation and its dependence (Bono et al. 2008), or lack thereof (Majaess, Turner & Gieren 2011), on different passbands or on the metallicity of the stellar population. If metallicity does affect the P-L relation, any uncertainties are likely to be minimized in the near-infrared (NIR) bands as compared to the visual (Madore & Freedman 1991). Additionally, obtaining Cepheid P-L relations in the NIR reduces the impact of dust on magnitude measurements. Both metallicity effects and extinction effects have driven the push towards observations of Cepheids at longer wavelengths.

Blending and crowding can contaminate the photometry of Cepheids, making true magnitudes more difficult to obtain. This problem is significant for ground-based observations, which may be biased by up to 0.2 mag by blending (Mochejska et al. 2000; Vilardeul, Jordi & Ribas 2007). However, point spread function (PSF) magnitudes obtained from HST greatly reduce these effects, thereby reducing the error in the measurements of Cepheid magnitudes and thus the uncertainty in the distance and Hubble constant as well. Furthermore, making use of a Wesenheit magnitude (an index combining magnitude and colour) can help to account for reddening variations from star to star by taking individual Cepheid colours into account (Madore 1982; Opolski 1983; Moffett & Barnes 1986; Madore & Freedman 1991; Caputo, Marconi & Musella 2000; Leonard et al. 2003; Ngeow & Kanbur 2005; Fiorentino et al. 2007; Bono et al. 2008, 2010; Ngeow 2012).

Lastly, uncertainty in the LMC distance is thought to account for about 5 per cent of the uncertainty in the cosmological distance scale (Freedman & Madore 2010a; Riess et al. 2011; Freedman, Madore & Scowcroft 2012; Inno et al. 2013). By using NGC 4258 as an anchor for the distance scale instead of the LMC, this uncertainty can be reduced to slightly more than 3 per cent (Macri et al. 2006; Riess et al. 2009b, 2011; Fiorentino, Musella & Marconi 2013). Humphreys et al. (2013) have used masers to update the distance to NGC 4258 from 7.60 ± 0.17 ± 0.15 Mpc, which reduces the error of using the LMC as an anchor as well as reducing the error to NGC 4258 from previous studies (Herrnstein et al. 1999).

In this work, we present magnitudes of Cepheids in M31 from the PHAT survey and use P-L relations to re-determine the distance modulus of M31. Section 2 presents the details of the data we have analysed. The methods and the analysis of the data to construct P-L relations are discussed in Section 3, where we decrease the dispersion in the visual P-L relations and increase the sample of Cepheids with NIR photometry with respect to Riess et al. (2012) and Kodric et al. (2015). In Section 4, we examine the benefits from the photometric precision from the PHAT survey. As we show by examining a subset of our Cepheids which were also observed by the DIRECT survey (see Section 2 for details), magnitudes obtained via HST in the visual bands are superior to recent ground-based surveys (e.g. Stanek et al. 1998; An et al. 2004; Kodric et al. 2013). In Section 5, we discuss the determination of distances from the PHAT photometry. The subset of stars from DIRECT is used to compare PHAT-determined distances to ground-based distance estimates. We explore the relationship between metallicity, distance, and radial location for the Cepheids in Section 6. The implications of our results are described in Section 7 while the conclusions are summarized in Section 8.
Table 1. Complete Cepheid sample.

| RA     | Dec.     | F475W | \(\sigma_{F475W}\) | F814W | \(\sigma_{F814W}\) | F110W | \(\sigma_{F110W}\) | F160W | \(\sigma_{F160W}\) | Period (d) | Source     |
|--------|----------|-------|---------------------|-------|---------------------|-------|---------------------|-------|---------------------|------------|------------|
| 10.91937 | 41.21261 | 22.948 | 0.052              | 20.682 | 0.040              | 19.982 | 0.073              | 19.303 | 0.100              | 10.045     | PAndromeda |
| 11.27651 | 41.69481 | 20.764 | 0.037              | 19.608 | 0.053              | 19.333 | 0.057              | 18.883 | 0.046              | 10.054     | PAndromeda |
| 11.01628 | 41.62777 | 21.076 | 0.009              | 19.796 | 0.033              | 19.213 | 0.081              | 18.786 | 0.088              | 10.101     | PAndromeda |
| 11.46423 | 42.14078 | 21.448 | 0.006              | 19.823 | 0.004              | 19.173 | 0.052              | 18.719 | 0.063              | 10.234     | PAndromeda |
| 11.02203 | 41.23451 | 22.039 | 0.036              | 20.254 | 0.025              | 19.686 | 0.105              | 19.134 | 0.111              | 10.29      | DIRECT     |
| 11.09179 | 41.35495 | 21.736 | 0.014              | 19.940 | 0.021              | 19.490 | 0.101              | 18.989 | 0.114              | 10.296     | PAndromeda |
| 11.18490 | 41.92719 | 21.163 | 0.005              | 19.796 | 0.017              | 19.452 | 0.097              | 18.895 | 0.108              | 10.3       | Riess      |
| 11.17153 | 41.40722 | 22.208 | 0.037              | 20.239 | 0.024              | 19.399 | 0.100              | 18.858 | 0.105              | 10.35      | DIRECT     |
| 11.34817 | 42.03007 | 21.452 | 0.034              | 19.829 | 0.017              | 19.666 | 0.027              | 19.076 | 0.033              | 10.371     | PAndromeda |
| 11.38078 | 41.88077 | 21.902 | 0.035              | 20.033 | 0.020              | 19.530 | 0.044              | 19.056 | 0.070              | 10.43      | Riess      |

Figure 1. The locations of 175 Cepheids from the complete data set are shown as circles and the solid line shows the outline of the PHAT footprint and coverage of M31.

& Lee 1981; Tammann & Reindl 2002; Tamman et al. 2002; Kanbur & Ngeow 2004; Sandage, Tamman & Reindl 2004; Ngeow et al. 2005; Kodric et al. 2015). The distribution of these periods, separately by publication and altogether, is shown in Fig. 2.

To construct the data set used in this paper, Cepheid locations were extracted from the PAndromeda and DIRECT surveys as well as from Riess et al. (2012). The positions were used to locate the photometry in the PHAT data base. Because it is the largest data set, positions were taken from the PAndromeda survey first, then from Riess et al. (2012), ignoring duplicates from PAndromeda, and lastly from the DIRECT survey, ignoring duplicates from the other two surveys. We adopt periods from these surveys in the same sequence.

Because the FWHM of the ground-based photometry is significantly larger than that of HST, there can be multiple sources that are plausible matches based solely on position. To identify the most likely Cepheid counterpart in the HST data, we first identify sources within 1 arcsec of the published right ascension and declination. Of these, the final match was based on choosing sources with magnitudes within \(\pm 1\) mag of the source catalogue. Objects were rejected if they did not show variability beyond the photometric error in two PHAT epochs. Objects were also thrown out if there were multiple objects within 1 mag of the expected, published magnitude(s). For two objects published only in the DIRECT data set, the matching radius was extended to 1.2 arcsec when no match was initially found within 1 arcsec (the expansion to a larger radius is not surprising given the large FWHM of the DIRECT photometry). A comparison of the PHAT Cepheids locations to their published locations is shown in Fig. 3. The mean offsets are approximate 0.11 arcsec in right ascension and \(-0.06\) arcsec in declination.

Through this method, magnitudes in the PHAT data set were obtained for each Cepheid in each available visual and NIR filter. The total sample is composed of 175 Cepheid variables with visual and 174 Cepheids with NIR magnitudes (see Table 1; one Cepheid is on the very edge of the PHAT coverage and falls outside the WFC3/IR footprint). The locations of the Cepheids as determined by PHAT are given in columns 1 and 2 of Table 1. In columns 3–10 of the table, we give the PHAT magnitudes in the visual filters (F475W and F814W) and the infrared filters (F110W and F160W), along with their photometric uncertainties from photon counts (uncertainties can be significantly larger due to crowding; see Dalcanton et al. 2012). In column 11, we give the published
The magnitudes obtained from the PHAT photometry are not time-averaged mean magnitudes, due to insufficient temporal coverage. These magnitudes are therefore snapshots of the Cepheid at a single phase in its variation. These ‘random phase’ magnitudes may lead to deviations from a given P-L relation due to the amplitudes of the Cepheids’ variation and the phase of observation.

The complete data set is used for the primary analysis of this paper. However, as we describe in Section 4, we separately analyse the DIRECT sample to examine differences in the P-L relations resulting from ground-based and space-based photometry. The properties of the DIRECT subsample are seen in Table 2. The DIRECT variable name is given in column 1, with the right ascension and declination as determined by the PHAT data given in columns 2 and 3. The $F475W$ and $F814W$ magnitudes from the PHAT survey are given in columns 4 and 6 with their corresponding photometric errors (columns 5 and 7). Column 8 gives the period published by the DIRECT survey. Out of the 85 Cepheids in the DIRECT sample whose positions overlap with the PHAT footprints, only 80 are used due to matching problems, equivalent to a 6 per cent rejection rate.

### Table 2. DIRECT Cepheid sample.

| DIRECT ID | RA     | Dec.   | $F475W$ | $\sigma_{F475W}$ | $F814W$ | $\sigma_{F814W}$ | Period (d) |
|-----------|--------|--------|---------|------------------|---------|------------------|------------|
| V5343     | 11.022 | 41.234 | 22.039  | 0.036            | 20.254  | 0.025            | 10.29      |
| V8515     | 11.091 | 41.354 | 21.736  | 0.014            | 19.940  | 0.021            | 10.308     |
| V13153    | 11.171 | 41.407 | 22.208  | 0.037            | 20.239  | 0.024            | 10.35      |
| V2293     | 11.138 | 41.626 | 21.120  | 0.004            | 19.548  | 0.006            | 10.567     |
| V6363     | 11.368 | 41.659 | 22.249  | 0.008            | 20.258  | 0.007            | 10.593     |
| V410      | 11.091 | 41.664 | 21.800  | 0.035            | 19.999  | 0.020            | 10.792     |
| V13042    | 11.169 | 41.399 | 21.772  | 0.035            | 20.141  | 0.022            | 10.847     |
| V3773     | 10.987 | 41.240 | 23.130  | 0.070            | 19.874  | 0.020            | 10.938     |
| V7381     | 11.092 | 41.321 | 20.808  | 0.029            | 19.586  | 0.019            | 10.943     |
| V4733     | 11.332 | 41.788 | 22.337  | 0.006            | 20.241  | 0.006            | 10.971     |

2.1 Uncertainty determination

We use artificial star tests to determine the true photometric uncertainties in all four relevant filters. For each field and camera in the PHAT survey, $10^5$ artificial stars have been inserted and reprocessed through the PHAT pipeline (as detailed in Dalcanton et al. 2012 and Williams et al. 2014). Artificial stars are inserted individually and the photometry is re-run in the immediate vicinity of that star and the resulting magnitude is recorded. This procedure allows us to fully characterize non-trivial noise from the photometric measurements, which in particular includes blends and completeness.

From the artificial star tests for the corresponding brick and field for each Cepheid, we choose artificial stars with a recovered magnitude within 0.5 mag of the Cepheid’s observed magnitude and locations within 20 arcsec of the Cepheid’s location. The estimated dispersion and systematics of our measurements are then added in quadrature to produce the total photometric uncertainty for each Cepheid.

2.2 Comparison with Riess et al. (2012)

There are 67 Cepheids in Riess et al. (2012) for which we have independently obtained photometry from the PHAT photometry pipeline. We compare the $F110W$ and $F160W$ magnitudes for each of these Cepheids in Fig. 4. We find a median offset of 0.259 ± 0.028 in $F110W$ and 0.035 ± 0.010 in $F160W$. These offsets are similar to those noted by Kodric et al. (2015). Our $F160W$ photometry and that of Kodric et al. (2015) differ by an average of 0.016 and our $F110W$ photometry differs by 0.001. Kodric et al. (2015) and the PHAT team both utilize PSF photometry, and thus achieve very similar results. The discrepant $F160W$ point seen in Fig. 4 is the same Cepheid shown to be a misidentification in Kodric et al. (2015).

Fig. 5 shows the $F110W$ and $F160W$ colour differences for the Riess et al. (2012) photometry, the PHAT photometry, and theoretical colours from a grid of isochrones. Our set of models is composed of Girardi isochrones over an age range of 4 Myr–1 Gyr and metallicity range of $Z = 0.0001–0.05$, constrained to the canonical instability strip defined in Bono et al. (2005). The Riess et al. photometry is shown in the top panel and the PHAT photometry in the middle panel; the offset between the two observed data sets is clear. The offset is due to an unsquared energy fraction correction rather than an encircled energy fraction correction, the latter of
which is the correct application for PSF photometry (Kodric et al. 2015). Whereas the ensquared energy fraction gives the energy over a certain number of pixels, the encircled energy fraction is the light within a certain radii. The difference between using the two energy fraction corrections is $\sim 0.258$, which accounts for the majority of the discrepancies in photometry we see between our photometry and Riess et al. (2012). The range and mean of $F110W-F160W$ colours observed in the PHAT photometry mimic those seen in the theoretical colours in the bottom panel of Fig. 5.

It is important to note this offset between Riess et al. (2012) and our photometry, as it informs the process by which we determine a distance with the NIR filters (Section 5.2).

3 P-L RELATIONS FROM PHAT PHOTOMETRY

We construct optical P-L relations using the $F475W$ and $F814W$ filters and NIR P-L relations using the $F110W$ and $F160W$ filters. The measurements used in these relations are ‘random phase’ magnitudes rather than time-averaged mean magnitudes. Although mean magnitudes lead to less scatter in the P-L relations, the PHAT data alone does not have enough temporal coverage to derive mean magnitudes, unlike optimized ground-based Cepheid surveys.

To determine whether we can accurately recover the mean P-L relation and avoid biased distance estimates even with random phase magnitudes, we did Monte Carlo simulations of 165 light-curve random samples to simulate our Cepheids, using templates from Pejcha & Kochanek (2012). We then took the mean magnitude (for both $B$ and $I$) of these 165 random samples and compared it to the true mean magnitude of the template. This test was repeated 100 times and the results are shown in Fig. 6. The mean difference between the 165 random samples and the true magnitude is 0.0003 for $B$ and $-0.0001$ for $I$; the difference for the $B-I$ colour is 0.0004. While there may be additional scatter around the mean due to sampling a set of Cepheids at random phase, there is no apparent bias of the mean in either magnitude or colour. This suggests that random phase observations do not bias our mean distances.

3.1 Optical P-L relation

We construct an optical P-L relation using the random phase magnitudes from PHAT for the Cepheids in Table 1. We determine a Wesenheit index for the $F475W$ and $F814W$ filters using...
extinction parameters from the appendix of Schlafly & Finkbeiner (2011) with an $R_V = 3.1$ extinction law. The Wesenheit index takes both the brightness and colour of each Cepheid into account; this index is commonly used in variable studies to correct for star-to-star differences in extinction and typically reduces the overall scatter in the P-L relation.

$$W_{F475W,F814W} = F814W - 0.879(F475W - F814W).$$ (1)

We use iterative 2.5σ clipping with respect to the Wesenheit($F475W,F814W$) relation for the 175 Cepheids with $F475W$ and $F814W$ magnitudes, leaving the final sample with 163 Cepheids, rejecting 7 per cent. We employ an iteratively re-weighted least squares, allowing the slope to float during the fitting and clipping process as opposed to clipping with respect to a fixed slope. As we sigma-clip with respect to the Wesenheit relation rather than the individual bands, we do not see a bias flattening the slope in the higher dispersion relations. The P-L relations are seen in the right-hand panel of Fig. 7. Table 3 gives the slopes and intercepts of the error-weighted linear regression relations for $F475W$, $F814W$, and the Wesleyneht magnitudes ($W_{F475W,F814W}$, as defined in equation 1). Most notably, the dispersion in the Wesenheit($F475W,F814W$) relation is only $\sim$0.17 mag, which is the lowest published dispersion for an optical P-L relation in M31. Moreover, this dispersion is comparable to that found in the NIR bands (see Section 3.2).

However, due to the lack of published P-L relations in HST native filters, distance determinations are difficult. Thus, we convert the $F475W$ and $F814W$ magnitudes to $B$ and $I$ magnitudes (respectively) in order to find distances from the optical P-L relations. We use the relations from table 2 of Saha et al. (2011) to determine $B$ and $I$ magnitudes. We then calculate the P-L relationship using the Wesenheit magnitude system. We adopt

$$W_{BI} = B - 1.866(B - I)$$ (2)

from Fouqué et al. (2007) with an $R_V = 3.1$ reddening law.

Note that the colour multiplier in the Wesenheit calculation is a function of the ratio of extinction in the $I$ and $B$ filters. However, our $B$ magnitudes are magnitudes from the $F475W$ passband converted to $B$, using semi-empirical relations that are functions of stellar temperature and typically very low extinction. To measure the impact of varying extinction on the accuracy of the Wesenheit relation using $B$ and $I$ magnitudes transformed from $F475W$ and $F814W$ magnitudes, we used Girardi et al. (2008) isochrones, based on synthetic spectra, to compare the Wesenheit magnitudes at different extinctions ($A_V = 0$ and 1) with $R_V = 3.1$.

To begin, we identified points in solar-metallicity isochrones (Girardi et al. 2000; Marigo et al. 2008) falling in the instability strip in $F475W$ and $F814W$ magnitudes. The $F475W$ and $F814W$ magnitudes were converted to $B$ and $I$ magnitudes using the Saha et al. (2011) relations. We used the $B$ and $I$ magnitudes to compute Wesenheit magnitudes in the same way as our observed Cepheid magnitudes for $A_V = 0$ and 1. This process was repeated for Wesenheit magnitudes computed directly from $B$ and $I$ magnitudes from the same isochrone points inside the instability strip. We found that our Wesenheit $BI$ magnitudes are not extinction independent, but instead have a dispersion of 0.06 mag between $A_V = 0$ and 1. In contrast, the same experiment done with synthetic $B$ and $I$
magnitudes shows a dispersion of 0.01 mag. We fold an error of 0.06 into our Wesenheit magnitude uncertainty to reflect the error associated with making the filter transformation.

For the 175 Cepheids with visual magnitudes, we use a 2.5σ clipping with respect to the Wesenheit relation, leaving the final sample with 163 Cepheids seen in Fig. 7, rejecting 7 per cent. As with the native filters, we allow the slope to float while fitting the P-L relations and do not observe a flattening of the slope in the noisier relations. Table 3 gives the slopes and intercepts of the error-weighted linear regression relations for $B$, $I$, and the Wesenheit magnitudes ($W_{BI}$, as defined in equation 2). The dispersion in the Wesenheit $BI$ relation is only 0.19 mag, which is comparable to the dispersion in the $W_{F475W,F814W}$ relation and also similar to the dispersion in the NIR filters (see Section 3.2).

Our dispersion is less than other visual PLW (Wesenheit) relations from recent studies (e.g.: $>0.34$ mag from the DIRECT survey, 0.33 mag from Kodric et al. 2013), thereby reducing the uncertainty in the eventual distance determination. Additionally, while the sample size is smaller than many ground-based surveys, it is the largest sample size of Cepheids observed in M31 with HST in the visual bands. The high quality of HST photometry significantly reduces the dispersion seen in the P-L and PLW relations, attributed primarily to the reduction of blending and crowding with HST observations. Together, these two improvements lead to gains in constraining the uncertainty in the distance modulus to M31.

We find a slope of $-3.33 \pm 0.09$ for the $W_{BI}$ relation, which is shallower than the slope found by Fouqué et al. (2007) with Galactic Cepheids ($-3.600 \pm 0.079$) and their slope with LMC Cepheids ($-3.454 \pm 0.011$). However, Fouqué et al. (2007) includes a period range from $\log P_{20}(P) \approx 0.6$ to 1.8 in determining a relation, which may contribute to some discrepancy, as including shorter period Cepheids tends to steepen the P-L slope.

### 3.2 Infrared P-L relation

In Fig. 8, we show the P-L relation from the random phase magnitudes for the Cepheids in $F110W$ and $F160W$ WFC3 filters, and in

![Figure 8. NIR P-L relations for 160 of the 175 Cepheids with magnitudes in $F110W$ and $F160W$ and $W_{IR}$. The $F160W$ magnitudes are shifted brighter by 2 mag and the Wesenheit magnitudes are shifted brighter by 4 mag to make the three relations clearly visible. Weighted least-squares linear fits to each relation after 2.5σ cuts are plotted on top of the data. Uncertainties determined by artificial star tests are included, though for some points they are too small to see clearly.](https://academic.oup.com/mnras/article-abstract/451/1/724/1352831/fig8)

| Magnitude  | Slope $(\text{mag/ Log } P)$ | Intercept $(P = 10 \text{ d})$ | Dispersion to fit $(\text{mag})$ |
|------------|-----------------------------|--------------------------------|-----------------------------|
| $F110W$    | $-2.92 \pm 0.11$            | $19.58 \pm 0.03$               | 0.246                       |
| $F160W$    | $-2.96 \pm 0.09$            | $19.02 \pm 0.02$               | 0.187                       |
| Wesenheit  | $-3.38 \pm 0.08$            | $18.45 \pm 0.02$               | 0.173                       |

Wesenheit magnitudes, calculated as in Riess et al. (2012) with an $R_V = 3.1$ reddening law:

$$W_{IR} = F160W - 1.54(F110W - F160W).$$

As with the optical bands, we fit linear, error-weighted relations to the data. The resulting slopes, intercepts, and dispersions are given in Table 4. Out of the 175 Cepheids with IR magnitudes, 160 are left after 2.5σ clipping with respect to the Wesenheit relation (allowing the slope to float, producing a 9 per cent rejection). Although we present the NIR P-L relations in Table 4, we do not directly use these fits to determine a distance in Section 5.2. As expected, the individual NIR bands give a much smaller scatter in the P-L relation, due to lower extinction and smaller overall amplitudes of the Cepheid light curves. However, although the amplitudes in the NIR are generally less than those in the visual, they are not completely negligible (Welch et al. 1984; Persson et al. 2004; Testa et al. 2007; Monson & Pierce 2011).

The dispersion in the Wesenheit relation is similar to previous studies and comparable to that of the visual Wesenheit dispersion. Our results show slightly higher dispersion in $F110W$ (0.246 mag) than found by Riess et al. (2012, 0.20 mag, but very similar to that of Kodric et al. (2015, 0.243 mag). It is possible the discrepancy in the dispersions between our study and Riess et al. (2012) is related to the differences in photometry. Similar dispersions in the $F160W$-P-L are found in all three studies: 0.187 mag in this paper; 0.17 mag in Riess et al. 2012; 0.178 in Kodric et al. (2015). We find a dispersion of 0.173 mag in the PLW relation, between 0.22 from Riess et al. (2012) and 0.147 Kodric et al. (2015), although we note that Kodric et al. use a different definition of the Wesenheit magnitude. Save for $F110W$, our dispersions are also consistent with the expected random phase dispersions as noted by Riess et al. (2012) from the study of Persson et al. (2004) of LMC Cepheids.

Generally, the IR bands provide an opportunity to improve upon the dispersion of the PL relation typically seen in the visual bands. However, we do not necessarily see a significant difference between the dispersions of PLW relations in the visual and in the IR, although the individual bands show immense improvement compared to the visual as the wavelength gets longer, as expected. By increasing the sample size of long-period Cepheids in the IR bands over Riess et al. (2012, 68 Cepheids) and Kodric et al. (2015, 111 Cepheids), we have further reduced the statistical uncertainty in the distance measurement. Comparing the linear fits of the $F160W$ filter from Table 4 to Kodric et al. (2015), we find a similar slope ($-2.96 \pm 0.09$ from this paper and $-2.779 \pm 0.171$) and intercept $(19.02 \pm 0.02$ from this paper and $18.960 \pm 0.028$ from Kodric et al. 2015). However, the $F110W$ slope from Kodric et al. (2015) of $-2.497 \pm 0.209$ is significantly flatter than the slope we find of $-2.92 \pm 0.11$. The intercepts are reasonably similar within 1.5σ ($19.58 \pm 0.03$ from this paper and $19.476 \pm 0.037$ from Kodric et al. 2015).

We find that our IR Wesenheit slope of $-3.38 \pm 0.08$ is consistent with Riess et al. (2012), who found a slope of $-3.43 \pm 0.17$ and with Persson et al. (2004) for the LMC, who found a slope of $-3.38 \pm 0.09$. The period range of Persson et al. (2004)
(extending to $\log_{10}(P) \approx 2$) is comparable to that of Riess et al. (2012) and our own complete sample in the NIR (extending to $\log_{10}(P) \approx 1.9$, see Figs 2 and 8). Our slope is steeper than that of Kodric et al. (2015) at $-3.172 \pm 0.117$; however, they make a different choice in calculating the Wesenheit magnitude so the comparison is incomplete.

4 GROUND VERSUS HST P-L RELATIONS

We follow a similar procedure as Section 3.1 to analyse the subset of DIRECT Cepheids in the PHAT photometric catalogue. From this, a comparison of ground-based and HST photometry can be made. We use the PHAT photometry and published DIRECT periods and magnitudes to analyse the differences between space-based random phase magnitudes and ground-based mean magnitudes. For each of the 80 Cepheids in Table 2, we proceed as follows. First, we note that, because the DIRECT survey is mainly in the $V$ and $I$ (only about half the Cepheids have $B$ magnitudes) bands, the $V$-band and $I$-band magnitudes must be inferred from the $HST$ data to make a comparison of the PHAT survey to the DIRECT survey.

To transform $F814W$ magnitudes to $I$ magnitudes, we use the Saha et al. (2011) relations. We determine a colour transformation from the $F475W$ and $F814W$ filters to the $V$-band Johnson–Cousin filter through the use of Girardi isochrones at a variety of ages (1–10 Gyr) and extinctions ($A_V$ from 0 to 2 mag). The transformation is restricted to the $F475W - F814W$ colour range of our DIRECT sample ($F475W - F814W$ from 0.9 to 3.2). The conversion we determine is

$$V - F814W \approx 0.104 + 0.541(F475W - F814W)$$

$$+ 0.032(F475W - F814W)^2$$

with an dispersion of 0.03 mag in the transformation. The PHAT transformed $V$ and $I$ magnitudes are used to construct P-L relations and compared to the P-L relations using the published $V$ and $I$ magnitudes from the DIRECT survey.

We calculate Wesenheit magnitudes for each Cepheid via the following relation from Fouqué et al. (2007), where $W_V$ is the Wesenheit magnitude, $V$ is as in equation (4), and $I$ is transformed via Saha et al. (2011) from $F814W$:

$$W_{VI} = V - 2.55(V - I).$$

The PHAT P-L relations for $V$, $I$, and Wesenheit magnitudes are shown in the right-hand panel of Fig. 9. For comparison, P-L relations from the DIRECT ground-based mean magnitudes are shown in the left-hand panel of Fig. 9. We use 2.5σ clipping with respect to the Wesenheit magnitudes and allow the slope to float. Out of the 80 Cepheids in the sample, this leaves 77 Cepheids in the DIRECT photometry (a 4 per cent rejection rate) and 75 Cepheids in the PHAT photometry (a 6 per cent rejection rate). Along with their dispersions, the linear, error-weighted fits to the relations in $V$, $I$, and Wesenheit magnitudes from Fig. 9 are given in Table 5.

As illustrated in Fig. 10, the ground-based DIRECT P-L relations have much shallower slopes and brighter intercepts compared to the $HST$ P-L relations. A shallower slope leads to a longer distance modulus; however, a brighter intercept leads to a shorter distance modulus. These two effects may balance out in distance determinations, but it is evident that ground-based observations are susceptible to heavily biased slopes and intercepts and caution should be taken when drawing conclusions from ground-based P-L relations. This behaviour suggests that blending and crowding may be the most significant limitation for ground-based Cepheid studies; these issues are improved by the spatial resolution of ACS/WFC. We note, however, that the level of crowding in ground-based surveys of M31 is comparable to that expected for $HST$ surveys of more distant galaxies. Therefore, we expect many $HST$ Cepheid studies in more distant galaxies may be affected by the same biases affecting the DIRECT observations of M31 (Stanek & Udalski 1999; Mochejska et al. 2000; Chavez, Macri & Pellerin 2012).

As Fig. 10 shows, the difference between the slopes and the intercepts between the ground-based P-L relations and the PHAT P-L relations grows as we move from the $V$ band to the $I$ band. The difference is most stark when comparing the Wesenheit slopes and intercepts. This implies that the ground-based bias significantly affects both the slope and the intercept of the P-L and PLW relations, thereby jeopardizing accurate distance estimates. The existence of
Table 5. DIRECT ground-based P-L relations.

| Magnitude | Slope (mag/Log P) | Intercept (P = 10 d) (mag) | Dispersion to fit (mag) |
|-----------|-------------------|-----------------------------|------------------------|
| DIRECT ground-based | V | −0.67 ± 0.33 | 20.89 ± 0.09 | 0.508 |
| | I | −1.32 ± 0.21 | 19.76 ± 0.06 | 0.339 |
| | Wesenheit | −2.33 ± 0.30 | 18.01 ± 0.08 | 0.440 |
| PHAT random phase | V | −1.22 ± 0.44 | 21.23 ± 0.11 | 0.541 |
| | I | −2.03 ± 0.24 | 20.13 ± 0.06 | 0.327 |
| | Wesenheit | −3.30 ± 0.12 | 18.31 ± 0.03 | 0.187 |

Figure 10. Left-hand panel: a comparison of intercept values across several filters; circles denote intercepts from the DIRECT ground-based P-L relation, triangles indicate the intercepts from the DIRECT sample as observed by PHAT, and squares indicate the intercepts from the complete PHAT sample (I band only). The left section shows intercepts from the V-band P-L relation, the middle from the I band, and the right section from the W band P-L relation. Error bars are also plotted for each intercept. Right-hand panel: a comparison of slope values across several filters; circles denote slopes from the DIRECT ground-based P-L relation, triangles indicate the slopes from the DIRECT sample as observed by PHAT, and squares indicate the slopes from the complete PHAT sample (I band only). The left section shows slopes from the V-band P-L relation, the middle from the I band, and the right section from the W band P-L relation. Error bars are also plotted for each slope.

5 DISTANCE MODULI

5.1 Distances: visual

To determine a distance modulus for the complete data set in the visual bands, we use the Wesenheit BI relation as defined by Fouqué et al. (2007), calibrated to Galactic Cepheids. The distance modulus in these filters is defined by

$$M_{W(BI)} = -3.600(\pm0.079) \log_{10}(P) - 2.401(\pm0.023)$$

and

$$\mu_{M31_{BI}} = m_{W(BI)} - M_{W(BI)}.$$  \hspace{1cm} (7)

where $M_{W(BI)}$ is the magnitude of the Wesenheit BI relation from Fouqué et al. (2007) evaluated at $\log_{10}(P) = 1.26$, the mean period of our sample, and $m_{W(BI)}$ is the magnitude of the Wesenheit BI from our observations, also evaluated at $\log_{10}(P) = 1.26$. The result is a distance modulus of 24.34 ± 0.10.

We also determine a distance modulus using the LMC-based relation from Fouqué et al. (2007), using the same formalism as equations (6) and (7), except with a slope of $-3.454 \pm 0.011$ and an intercept of $15.928 \pm 0.003$ (adopting a distance to the LMC of 18.486 ± 0.065 as in Riess et al. 2011). The result is a distance modulus of 24.31 ± 0.07. We average our two visual band distance determinations to obtain a modulus of 24.32 ± 0.09.

5.2 Distances: IR

To obtain a distance modulus to M31 for the NIR Cepheids, we tie the distance modulus of M31 to that of NGC 4258, as defined in Riess et al. (2009b, 2011) (revisited by Riess et al. 2012 and Fiorentino et al. 2013). From equation 7 of Riess et al. (2009b), we solve for the distance modulus to M31 as

$$\mu_{M31} = \mu_{4258} - z_{4258} + m_{w} - b_{w} \log_{10}(P - 1) + Z_{w} \Delta \log_{10}[O/H],$$

where

$$\mu_{M31} = \mu_{4258} - z_{4258} + m_{w} - b_{w} \log_{10}(P - 1) + Z_{w} \Delta \log_{10}[O/H],$$

\hspace{1cm} (8)

this bias stresses the need for more HST observations of Cepheids, even if they are only at random phases.

The slope of the visual Wesenheit VI relation from the PHAT data is in agreement with previous studies. We find a slope of $-3.30 \pm 0.12$, comparable to that found by Benedict et al. (2007) and Fouqué et al. (2007) for Galactic Cepheids ($-3.31 \pm 0.17$ and $-3.377 \pm 0.023$) and that of Udalski et al. (1999) for the LMC ($-3.28 \pm 0.14$), although the period ranges differ. The periods in Benedict et al. (2007) range from very low period ($\log_{10}(P) = 0.4$) to longer periods ($\log_{10}(P) = 1.6$), but the data points are extremely sparse between $\log_{10}(P) = 1$ and their longest period Cepheid at $\log_{10}(P) = 1.6$. The sample in Fouqué et al. (2007) is better populated in period, with a range from approximately $\log_{10}(P) = 0.6$ to $\log_{10}(P) = 1.7$. For the Cepheids in the complete data set, the period coverage is out to $\log_{10}(P) = 1.9$, a bit farther than Fouqué et al. (2007). However, we do not include the short period (<10 d) Cepheids in our determination of the P-L relations.
where $\mu_{4258}$ is the distance to the water maser galaxy NGC 4258 (7.6 ± 0.17 ± 0.15 Mpc from Humphreys et al. 2013), $z_{4258}$ is the intercept of the P-L relation for Cepheids in NGC 4258, $m_w$ are the observed NIR Wesenheit magnitudes of the Cepheids, $b_W$ is the slope of the global Wesenheit NIR P-L relation, and $P$ are the periods (> 10 d). The term $Z_w \Delta \log_{10}([O/H])$ is the metallicity dependence term; $\Delta \log_{10}([O/H])$ are the metallicities of the Cepheids relative to the LMC and $Z_w$ is the global metallicity slope. To obtain the values of $z_{4258}$, $b_W$, and $Z_w$, we use a simultaneous linear fit to the Cepheids in both M31 and NGC 4258. The simultaneous linear fit allows a reduction in the final distance uncertainty compared to using the NIR P-L relations alone.

First, we should note that same filters were not used to observe Cepheid colours in NGC 4258 $(V - I)$ as the bands observed in the PHAT survey $(F160W - F110W)$. The Wesenheit magnitudes in Riess et al. (2009a, 2011) were calculated using $W_R = F160W - 1.54(V - I)$. To account for the difference in defining the Wesenheit magnitude, a correction has been applied to the $(F110W - F160W)$ based extinction correction to give them the same mean colour in $(V - I)$ as M31 Cepheids. As in Riess et al. (2012), we use the following to determine a correction $(X)$ to the Wesenheit magnitude calculation of NGC 4258:

$$0.504(V - I) = 1.54(F110W - F160W - X).$$

With a mean $(V - I)$ colour of 1.23 (from the DIRECT Cepheids) and a mean $(F110W - F160W)$ colour of 0.56 from the PHAT photometry, we obtain a value of $X = 0.156$. Because of the difference in $F110W$ photometry between PHAT and Riess et al. (2012), we use a different $X$ than Riess et al. (2012) to place our Wesenheit magnitudes on the same scale as that of the NGC 4258 photometry. Errors in their NGC 4258 photometry are incorporated into the analysis and error budget.

To solve for the distance, we also must determine the values of $\Delta \log_{10}([O/H])$ for the Cepheids to include the possibility that differing metallicities of Cepheids could introduce scatter into the P-L relation. To examine possible metallicity effects, we use the relation from Zaritsky, Kennicutt & Huchra (1994) for M31 to determine $\log_{10}([O/H])$ for each Cepheid from its radial location in the disc and compare it to the solar value of 8.9 (as in Riess et al. 2009a, 2011).

We adopt a centre for M31 of RA $= 0^h42^m44^s.31$ and Dec. $= 41^d16^m9^s.4$ (Cotton, Condon & Ariztizabal 1999), a disc scalelength of $\rho_d = 77.44$ (Zaritsky et al. 1994), and position angle $\phi = 37.715$ (Baade & Arp 1964; Haud 1981). These values are used to calculate the $12 + \log_{10}([O/H])$ metallicity term from the relation determined by Zaritsky et al. (1994), as seen in equation (10):

$$12 + \log_{10}([O/H]) = 9.03(\pm 0.09) - 0.28(\pm 0.10)(\rho/\rho_0 - 0.4).$$

(10)

The deprojected radius, $\rho$, is calculated using equations (11) through (13):

$$x = (\delta - \delta_0) \cos(\phi) + (\alpha - \alpha_0) \sin(\phi) \cos(\delta_0)$$

(11)

$$y = (\delta - \delta_0) \sin(\phi) - (\alpha - \alpha_0) \cos(\phi) \cos(\delta_0) \cos(i)$$

(12)

$$\rho = \frac{(x^2 + y^2)^{1/2}}{\rho_0}.$$  

(13)

The resulting $12 + \log_{10}([O/H])$ values are compared to the IR Wesenheit magnitude in Fig. 11. The slope of a linear fit relating the metallicity of the Cepheids to their magnitudes, is $0.22 \pm 0.60$ mag
dex$^{-1}$. A two-sided student $t$-test with a value of $t = 0.37$ and 159 degrees of freedom informs that this is not a statistically significant relation at the 0.05 significance level. Our slope of $0.22 \pm 0.60$ magn dex$^{-1}$ is different than Riess et al. (2012), who obtained a slope of $-0.65 \pm 0.73$, but with large uncertainty.

We see a range of $-0.3$ dex in $12 + \log_{10}([O/H])$ from approximately 8.82 to 9.12, similar to Riess et al. (2012), who find a range of 8.87–9.05. This range is similar to the $-0.4$ dex that we expect from the $-0.018$ dex kpc$^{-1}$ gradient of Zaritsky et al. (1994) over the range of the disc. The Cepheids in M31 do not have a clear magnitude-metallicity dependence, as has been found in previous studies (Freedman & Madore 1990; Riess et al. 2012). However, the values of $12 + \log_{10}([O/H])$ for M31 Cepheids will be used in conjunction with those in NGC 4258 to find a global fit.

Using multiple error-weighted linear regression, we simultaneously fit a P-L relation to the Cepheids in M31 and those in NGC 4258 to determine values for $z_{4258}$, $b_w$, and $Z_w$. These values are then used to determine the distance modulus to M31 as in equation (8). We determine a $b_w$ slope of $-3.24 \pm 0.10$, $z_{P4258}$ of $23.26 \pm 0.06$, and $Z_w$ of $0.16 \pm 0.26$. These are comparable to the values determined by the global fit in Riess et al. (2011, see Table 6).

Note that while Riess et al. (2011) use all observed supernovae hosts for a global fit, we use NGC 4258 with M31 only. With the values in Table 6, we obtain a distance of $24.51 \pm 0.08$ to M31.

The derived distance moduli for the complete data set in the visual (with MWG and LMC anchors) and in the IR are given in Table 7. There is an offset between the visual and the IR

Table 6. Global fit values.

| Variable          | This paper (equation 8) | Riess et al. (2011) |
|-------------------|------------------------|---------------------|
| $\mu_{4258}$      | 7.6 ± 0.17 ± 0.15      | 7.2 ± 0.2 ± 0.3     |
| $z_{4258}$        | 23.26 ± 0.06           | 26.32 ± 0.03 (23.10 at $P = 10 \text{ d}$) |
| $b_w$             | -3.24 ± 0.10           | -3.21 ± 0.03        |
| $Z_w$             | 0.16 ± 0.26            | -0.10 ± 0.09        |
Table 7. Distance moduli.

| Data set sample | Filter      | Anchor       | $\mu$     |
|-----------------|-------------|--------------|-----------|
| Complete        | PHAT visual ($B, I$) Galactic Cepheids | 24.34 ± 0.10 |
|                 | LMC         | 24.31 ± 0.07 |
| PHAT NIR ($F110W, F160W$) NGC 4258 | 24.51 ± 0.08 |
| DIRECT          | Ground-based ($V, I$) Galactic Cepheids | 24.21 ± 0.11 |
|                 | LMC         | 24.21 ± 0.09 |
| PHAT visual ($V, I$) Galactic Cepheids | 24.24 ± 0.10 |
|                 | LMC         | 24.24 ± 0.08 |

Figure 12. Individual IR distances (described in Section 5.2) with propagated errors are plotted against period. No correlation between the distance modulus and the period of variability is seen. The dashed line indicates the mean distance.

Table 5. Distance moduli.

| Data set sample | Filter      | Anchor       | $\mu$     |
|-----------------|-------------|--------------|-----------|
| Complete        | PHAT visual ($B, I$) Galactic Cepheids | 24.34 ± 0.10 |
|                 | LMC         | 24.31 ± 0.07 |
| PHAT NIR ($F110W, F160W$) NGC 4258 | 24.51 ± 0.08 |
| DIRECT          | Ground-based ($V, I$) Galactic Cepheids | 24.21 ± 0.11 |
|                 | LMC         | 24.21 ± 0.09 |
| PHAT visual ($V, I$) Galactic Cepheids | 24.24 ± 0.10 |
|                 | LMC         | 24.24 ± 0.08 |

6 METALLICITY AND THE DISTANCE–RADIUS RELATION

A slight trend of Cepheid distance modulus with radial location from the centre of the galaxy has been seen in M101 and M81, among others (see Friedman & Madore 1990; Kennicutt et al. 1998; Macri et al. 2006; McComas et al. 2009; Gerke et al. 2011). The discrepancy in distance between Cepheids in the inner and outer regions of these galaxies is often attributed, at least in part, to metallicity effects on the P-L relation, although the cause is still debated (Gould 1994; Vilardell et al. 2007; Gerke et al. 2011; Majaess, Turner & Gieren 2011; Kudritzki et al. 2012).

We use our complete set of Cepheids from Table 1 to explore the relationship between distance and radius in M31, determining the radius of each Cepheid by using the right ascension and declination to determine its de-projected position in the galaxy, $\rho$, as seen in equation (13). We convert $\rho$ to kpc using the mean IR distance modulus determined in Section 5. The individual distances in the IR bands ($\mu_{gB}$) for each Cepheid are compared to the de-projected radii in Fig. 13. A linear fit yields a gradient of 0.007 ± 0.003. The two-sided student-$t$ test value for this relation is 2.71, making it significant beyond the 99.5 per cent level with 159 degrees of freedom. We apply Chauvenet’s criterion to the data to determine the sensitivity of individual points on the relation. We find six outlying points with a significance level of 0.05. While this shallows the slope slightly, the slope remains statistically significant. This suggests that
The distance–radius relation for the 160 Cepheids in M31 that were used in the final P-L relations in Section 3.2 using the complete data set. The black points indicate the individual Cepheids’ IR distances with propagated errors. The horizontal dotted line shows the mean distance determination from Section 5, and the solid line shows the linear fit to the data. The linear fit has a slope of 0.007 ± 0.003, and a two-sided student t-test value of 2.71 (with 159 degrees of freedom), making it significant at the 99.5 per cent level.

As previously discussed, we use the relation derived by Zaritsky et al. (1994) to calculate the $12+\log_{10}[O/H]$ values for each Cepheid as a proxy for the metallicity effects in M31. Zaritsky et al. (1994) found a radial metallicity gradient of $-0.018 \pm 0.006$ from their observations of H ii regions in M31, which is quite shallow compared to many of the galaxies in that study. This is the gradient employed by this study and by Riess et al. (2009a, 2011, 2012) in defining their distance scale. Sanders et al. (2012) find a metallicity gradient of $-0.0195 \pm 0.005$ in H ii regions comparable to that found by Zaritsky; however, they do not find the trend echoed in their observations of planetary novae (PNe). Several studies have examined the globular clusters in M31 to search for a metallicity gradient. Barmby et al. (2000) looked at both spectroscopic and photometric metallicities of globular clusters, and while overall found no trend, observed a slight gradient of $-0.023 \pm 0.01$ when restricting their sample to clusters with spectroscopic metallicities. Huxor et al. (2011) use CMD metallicities but assert the lack of a metallicity gradient, suggesting that the gradient was driven primarily by metal-poor clusters. Freedman & Madore (1990) use Cepheids in three different fields of M31 and, despite there being a slight difference in distance moduli between the inner and outer fields, conclude that their data are consistent with there being no dependence of the P-L zero-point on metallicity. However, Gould (1994) re-analysed the same $BVRi$ data used by Freedman & Madore (1990) to show the M31 distance should be corrected for metallicity to avoid introducing additional errors or systematics. Lee et al. (2013) use 17 beat Cepheids to trace the metallicity in M31 and find a gradient of $-0.008 \pm 0.004$, shallower than that of Zaritsky et al. (1994) and Sanders et al. (2012), but similar to the PNe gradient from Kwitter et al. (2012) of $-0.011 \pm 0.004$. While the metallicity gradient in M31 may not be particularly steep, there is a wealth of data supporting its existence and direction.

Previous empirical studies have suggested metallicity as the cause of the distance–radius relationship. However, there is disagreement and find seven outlying points with a significance level of 0.05; however, the slope is not significantly affected. The radial trend in magnitude is causing the change in distance modulus; however, the cause of the inwards brightening of Cepheids is unclear.

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Previous empirical studies have suggested metallicity as the cause of the distance–radius relationship. However, there is disagreement and find seven outlying points with a significance level of 0.05; however, the slope is not significantly affected. The radial trend in magnitude is causing the change in distance modulus; however, the cause of the inwards brightening of Cepheids is unclear.

As previously discussed, we use the relation derived by Zaritsky et al. (1994) to calculate the $12+\log_{10}[O/H]$ values for each Cepheid as a proxy for the metallicity effects in M31. Zaritsky et al. (1994) found a radial metallicity gradient of $-0.018 \pm 0.006$ from their observations of H ii regions in M31, which is quite shallow compared to many of the galaxies in that study. This is the gradient employed by this study and by Riess et al. (2009a, 2011, 2012) in defining their distance scale. Sanders et al. (2012) find a metallicity gradient of $-0.0195 \pm 0.005$ in H ii regions comparable to that found by Zaritsky; however, they do not find the trend echoed in their observations of planetary novae (PNe). Several studies have examined the globular clusters in M31 to search for a metallicity gradient. Barmby et al. (2000) looked at both spectroscopic and photometric metallicities of globular clusters, and while overall found no trend, observed a slight gradient of $-0.023 \pm 0.01$ when restricting their sample to clusters with spectroscopic metallicities. Huxor et al. (2011) use CMD metallicities but assert the lack of a metallicity gradient, suggesting that the gradient was driven primarily by metal-poor clusters. Freedman & Madore (1990) use Cepheids in three different fields of M31 and, despite there being a slight difference in distance moduli between the inner and outer fields, conclude that their data are consistent with there being no dependence of the P-L zero-point on metallicity. However, Gould (1994) re-analysed the same $BVRi$ data used by Freedman & Madore (1990) to show the M31 distance should be corrected for metallicity to avoid introducing additional errors or systematics. Lee et al. (2013) use 17 beat Cepheids to trace the metallicity in M31 and find a gradient of $-0.008 \pm 0.004$, shallower than that of Zaritsky et al. (1994) and Sanders et al. (2012), but similar to the PNe gradient from Kwitter et al. (2012) of $-0.011 \pm 0.004$. While the metallicity gradient in M31 may not be particularly steep, there is a wealth of data supporting its existence and direction.
between empirical studies and pulsation theory on how strong the metallicity effect is, as well as whether a higher metal content leads to a brighter or fainter Cepheid (Freedman & Madore 1990; Korn et al. 1998; Baraffe & Alibert 2001; Macri et al. 2006; Bono et al. 2008, 2010; Caputo 2008; Romaniello et al. 2008, and references therein). Most theoretical approaches predict that metal-rich Cepheids should be fainter (Baraffe & Alibert 2001; Caputo 2008). Conversely, most observational studies, including indirect and direct metallicity measurements, suggest that the more metal-rich Cepheids are brighter (Kovtyukh, Wallerstein & Andrievsky 2005; Macri et al. 2006; Bono et al. 2008), although Romaniello et al. (2008) presented spectroscopic evidence based on observations of Cepheid iron lines that disagrees with most empirical studies. An alternative viable explanation to metallicity effects could be crowding and blending, causing inner Cepheids to appear brighter (leading to shorter distances) compared to Cepheids in outer fields. Our results show a statistically significant distance–radius relationship in the IR bands, but without a significant metallicity component, suggesting that there may be another factor at least partially responsible for the difference. Majaess et al. (2011) argue that crowding is the cause of brighter Cepheids in the inner regions. In comparing ground based and HST, Mochejska et al. (2000) and Chavez et al. (2012) both show that blending can be a significant effect, especially as seeing increases in the ground-based observations. Bono et al. (2008) that blending will be a stronger effect in the central region of a galaxy and thus lead to a decrease in distances. Based on our results, crowding and blending could both be likely candidates for the cause of the brightening of inner Cepheids in the case of M31. We do not see a trend with magnitude and metallicity, but we do see a trend of magnitude (and distance) with radius. When we examine the photometric uncertainties from artificial star tests, we see the magnitude error and crowding parameter increase towards the centre of M31. The average trend in uncertainty (∼0.002 mag kpc⁻¹) is sufficient to account for the observed gradient in magnitude and distance with radius. This suggests that a brightening bias due to blending or crowding, rather than metallicity, could be the driving factor of the trend between radial distance and magnitude trends in M31.

7 DISCUSSION

We present our distance estimates for the entire data set (from Table 1) in Table 7. This data set has the advantage of having the largest sample size of Cepheids in M31 in the visual bands from HST, as well as having a larger sample in the NIR bands than previous studies (Riess et al. 2012; Kodric et al. 2015). Our results are summarized in Table 7 and in Fig. 15.

Table 7 and Fig. 10 show there are significant biases in the slopes and intercepts of the P-L relations of the ground-based DIRECT survey compared to the PHAT survey. The observed differences suggest that Cepheid observations obtained from ground-based surveys, while comprehensive in scope, are not ideal tools in the era of precision cosmology, and that Cepheid observations of more distant galaxies with HST may suffer from similar effects. Surprisingly, the biases do appear to cancel somewhat, leading to only very modest changes in the mean distance modulus.

In Table 8, we give recent values from other studies for the distance modulus of M31. For a full compilation and discussion of M31 distances, we refer the reader to de Grijs & Bono (2014). The suggested M31 distance modulus from their study, incorporating various distance measurement techniques and statistically weighted errors, is 24.46 ± 0.10 (assuming an LMC distance modulus of 18.50). Our distance modulus of 24.51 ± 0.08 determined from the NIR P-L relations is in agreement with this range.

In contrast to the NIR results, the visual Wesenheit (B/I) distance modulus of 24.32 ± 0.09 (an average of the MWG and LMC distances) falls outside of the 1σ range of de Grijs & Bono (2014). The distance moduli determined from the WBI relations for the complete Cepheid sample and WBI relations for the DIRECT Cepheids with PHAT magnitudes are shorter than published distances, for both LMC and MWG anchors (see Table 7). They also disagree with our NIR distance determination of 24.51 ± 0.08. Although our mean visual distance determination is within 1σ of several recent studies (Freedman et al. 2001; Ribas et al. 2005; Vilardell et al. 2007; Joshi et al. 2010, among others), it does fall on the short side of the range of recently published distances, as well as slightly outside the determination of de Grijs & Bono (2014).

Unfortunately, there are no other similar published HST studies in comparable visual filters for Cepheids in M31 for a comparison. As examined by Mochejska et al. (2000), blending in the visual bands in ground-based observations may cause underestimates in Cepheid distances by 6–9 per cent. Although ground-based studies are more affected by these effects than HST, if further blending or crowding effects remain it could partially account for distances that remain shorter than expected. Williams et al. (2014) show that there is a brightening bias for the PHAT photometry in M31, especially for the inner region and fainter magnitudes. For a typical Cepheid, the infrared bands and F814W magnitudes can be biased up to 0.1 mag (Williams et al. 2014). The F475W magnitudes are less
affected, exhibiting a brightening of $<0.05$ magnitudes; colours also appear to have minimal bias at the level of a few hundredths of a magnitude. Magnitudes may be brightened up to 0.1 in the inner regions compared to $\sim 0.02$ at 25 kpc from the centre of M31. These biases reflect the crowding and blending problems present.

In a more direct comparison, we can compare our distance result in the infrared to similar NIR Cepheid studies in M31 with HST. Riess et al. (2012) obtain a distance modulus of $24.38 \pm 0.064$. When we account for the increase in the distance to NGC 4258 by Humphreys et al. (2013) from 7.4 to 7.6 Mpc, this pushes the Riess et al. (2012) value to a 0.06 farther distance. This leads to a difference between the Riess et al. (2012) value and our NIR distance of $1\sigma$. The difference may be explained by differing global fits, where our slightly steeper slope and fainter intercept pushes towards a further distance. Our distance estimate from the long period (greater than 10 d) Cepheids using the scale from Riess et al. (2009a, 2011) is also consistent with the updated Galactic and LMC distance calibrations done by Bhardwaj et al. (2015) in the infrared. The distance determination by Kodric et al. (2015) gives a 0.068 closer distance to M31 relative to the Riess et al. (2012) value. This puts their distance approximately $1-\sigma$ lower than Riess et al. (2012) and within 2-$\sigma$ of our distance estimate, when accounting for the difference in the updated maser distance. Kodric et al. (2015) obtain their Wesenheit magnitudes slightly differently, making a direct comparison incomplete. Additionally, they compare their PL fit to Riess et al. (2012) at log(P) = 1.2, whereas we re-solve the system of M31 and NGC 4258 Cepheids.

Derived from the complete sample of Cepheids, our NIR distance estimate of $24.51 \pm 0.08$ and visual distance of $24.32 \pm 0.09$ disagree slightly beyond the $1\sigma$ level; it is unclear from where this disagreement stems. We do not find evidence for bias from random sampling and the optical bands face less crowding, as shown in Williams et al. (2014). It is also possible that blending from nearby companions has affected the visual filters more than the infrared filters, causing the visual distance estimate to be shorter. Previous studies suggest that the contamination due to blending may be greater in the visual than in the infrared, though more rigorous studies are needed to determine the effects and extent of blending on observations (Mochejska et al. 2000; Gieren et al. 2008).

We are inclined to trust the NIR distance estimate over the visual band estimate for several reasons. The visual bands are not in the same native photometric system as their calibrating relations from the Milky Way and LMC and the transformations to $B$, $V$, and $I$ can be tricky. Although the NIR Wesenheit relation is also in a different photometric system than the original Riess et al. (2009a, 2011) papers, it is in the same system as Riess et al. (2012). Additionally, it is commonly thought that the longer wavelengths of the infrared reduce the effects of extinction, amplitude, and metallicity on Cepheid observations (Madore & Freedman 1991; Persson et al. 2004). Due to the absence of significant scatter in the Wesenheit $BI$ relation, extinction effects are probably not severe. However, the remaining effects could introduce greater variations and possible biases in the visual bands than the infrared. Therefore, we suggest that our NIR distance estimate is a more reliable distance determination.

8 CONCLUSIONS

We have analysed Cepheid variables located in the PHAT data from the PAndromeda, DIRECT, and POMME (as available in Riess et al. 2012) ground-based surveys of M31. Through analysing the visual and NIR magnitudes obtained from HST random phase observations, we make the following conclusions.

(1) We use a sample of Cepheids to construct a P-L relation using highly accurate HST magnitudes obtained by the PHAT survey. In particular, the visual Wesenheit relation from the complete sample $(W_0)$ shows less scatter over previous studies in the visual bands, leading to smaller random uncertainties in the distance modulus of M31.

(2) The dispersions in the visual Wesenheit and the NIR Wesenheit relations are very similar. The dispersion in the NIR relations is comparable to that derived by Riess et al. (2012), but with a $2.5 \times$ larger Cepheid sample that further reduces the uncertainty in the M31 distance modulus. Although our dispersion is slightly larger than that of a similar study by Kodric et al. (2015), they use a different calculation of the Wesenheit magnitude.

(3) We obtain a value of the distance modulus for M31 of $24.32 \pm 0.09$ from the visual filters and $24.51 \pm 0.08$ in the NIR using ACS and WFC3 photometry. These values are both consistent with recently published distance moduli. However, they disagree with each other by slightly more than $1\sigma$ due to the distance estimate at visual wavelengths being on the shorter end of published distances.

(4) The PHAT survey provides highly accurate HST magnitudes of 175 Cepheids in M31. The superiority of the PHAT photometry from HST is clear in this data set from the significantly smaller dispersions than that found using ground-based data, despite the fact that the Cepheid magnitudes are random phase. This is likely due to the enormous improvement in photometric quality and resolution as compared to ground-based surveys.

(5) We find a statistically significant magnitude radial trend leading to a distance modulus–radius relationship, which does not appear to be explained by a metallicity correction but may be due to crowding or blending. Further work must be done to confirm or rule out metallicity as a cause of distance discrepancies between inner field and outer field Cepheids.

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