Components of an algebraic solution of the multichannel problem of low-energy n-\textsuperscript{12}C scattering plus sub-threshold (\textsuperscript{13}C) states

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Abstract

The effects of components in an assumed model interaction potential, as well as of the order to which its deformation is taken, upon resonances in the low-energy cross sections and upon sub-threshold bound states of the compound nucleus (\textsuperscript{13}C) are discussed.

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TABLE I: The parameter values defining the base potential

|                | $V_{\text{central}}$ | $V_{ll}$  | $V_{ls}$  | $V_{Is}$  |
|----------------|-----------------------|-----------|-----------|-----------|
| Odd parity     | -49.144               | 4.559     | 7.384     | -4.770    |
| Even parity    | -47.563               | 0.610     | 9.176     | -0.052    |
| Geometry       | $R_0 = 3.09$ fm       | $a = 0.65$ fm | $\beta_2 = -0.52$ |

In a recent paper [1], we specified in detail a multichannel algebraic scattering theory (MCAS) for nucleons scattering from a nucleus. The approach is noteworthy because it allows a) a systematic determination of the sub-threshold bound states and compound resonances and b) the inclusion in the collective-model excitation of the target of the non-local effects due to the Pauli principle. The theory was built upon sturmian expansions of a model interaction matrix of potential functions with the dimensionality of the problem rapidly increasing with the number of closed and open channels to be taken into consideration. Thus the first application was of low-energy neutron-$^{12}$C scattering since then only three states sensibly were needed in the evaluations; namely the ground $0^+_1$, $2^+_1$ (4.44), and the $0^+_2$ (7.65) states in $^{12}$C.

We chose a collective-model representation for the interaction potential matrix with deformation taken to second order. The potential function involved of a set of operators, and with the parameter values specified in the Table I we obtained excellent results for the total elastic scattering cross section to 4 MeV excitation. Of particular note was that both broad and narrow resonances of the correct spin-parity were found at the correct excitation energies with appropriate widths and maxima resting upon a smooth and realistic background.

The same theory can be applied using negative energies. Then the resonance finding process discussed in ref. [1] determined the sub-threshold bound states of the compound nucleus; in our case $^{13}$C. With the potential parameters listed, agreement with the sub-threshold bound state spectrum resulted. Notably we found the right number of states, with the right spin-parities and at energies in quite good agreement with observation. But it was crucial to allow for Pauli blocking to find that result for Pauli blocking to find that result.

We have repeated those calculations, both of the cross section from the elastic scattering of neutrons from $^{12}$C and of the sub-threshold spectra for $^{13}$C, but now with a view to identify the importance of each component in the interaction potential. We also study the effects of the order to which the deformation is taken; specifically we compare results found when nuclear surface deformation is taken at 0th, 1st, and 2nd order (the result given in Ref. [1]).

The effects of the components of the base interaction potential upon the sub-threshold ($^{13}$C) bound states are displayed in Fig[1]. In this diagram, and as used throughout, the complete interaction results as published previously [1], are designated as “Base”. Those labeled by “No $\ell \cdot s$” are the results obtained by setting the $V_{ls}$ parameters to zero leaving the others unaltered. Likewise those results labeled “No $\ell \cdot \ell$” and “No $I \cdot s$” are ones found by setting just the $V_{ll}$ and $V_{Is}$ parameters to zero respectively, with all others as in Table I. The spectra of $^{13}$C determined on using the complete interaction is shown at the extreme left. The three spectra to the right of that are the results obtained when each of the three operator terms as identified above are omitted from calculation. While the $I \cdot s$ component
of the interaction is a fine tuning effect on the sub-threshold states, the $\ell \cdot s$ and the $\ell \cdot \ell$ components have large influence. They have offsetting effects so that the final spectrum has its ground state at a much lesser (and proper) binding. The two terms of course influence different attributes in the spectrum; the spin-orbit governing the splitting of results for the two values of $j = l \pm \frac{1}{2}$ while the orbit-orbit controls the splitting of orbits on the basis of their $l$-values. Relating changes in binding energies to the result designated as Base, the excision of the $\ell \cdot s$ component causes an increase in the binding of the negative parity states by $\sim 2.5$ MeV, reflecting the component of each of those states formed by coupling of a $p$-wave neutron to $^{12}$C. However the $\frac{1}{2}^+$ state is but slightly shifted; indicating a weak $d$-wave neutron coupling in the description of that state. On the other hand the $\frac{5}{2}^+$ state is markedly shifted into the continuum; consistent with it being formed strongly by the coupling of a $d$-wave neutron with the $2^+$ state in $^{12}$C. The effect of excising the $\ell \cdot \ell$ term also is marked. The negative parity states are all much more bound; this time by $\sim 5.5$ MeV, while again the $\frac{1}{2}^+$ state is but slightly changed in binding energy. These effects are consistent with the dominance of $p$-wave coupling forming the negative parity states while it is $s$-wave coupling that is predominant with the $\frac{1}{2}^+$ state. Now however, a second $\frac{1}{2}^-$ state is bound, being brought down from the continuum. Finally the $\frac{5}{2}^+$ state again is more bound, once more reflecting a strong $d$-wave coupling character.

The cross sections that result when components of the interaction are excised selectively are displayed in Fig. 2. Therein the restricted potential results have been shifted by 3 and 6 b as indicated so that the comparisons are clearer. Omitting the $I \cdot s$ component gives values little different from the Base result (solid curve) and so they are not shown. Note, however, that the limited sensitivity of the cross sections upon the $I \cdot s$ term was expected since the cross-section values are dominated by positive parity amplitudes and the positive parity $I \cdot s$ potential strength is small. Also the $I \cdot s$ term does not contribute to interactions involving

![FIG. 1: The spectra of $^{13}$C determined from the different calculations described in the text.](image)
FIG. 2: Total elastic cross sections for n-^{12}C scattering as functions of neutron energy showing
the effects of omitting single components of the input interaction.

the ground state (0^+); interactions that greatly influence the background scattering.

But the cross sections are very different when the ℓ · s and the ℓ · ℓ terms are omitted (the
dashed and the dot-dashed curves respectively). Not only are the resonances shifted in their
location but also they have widths quite different to those found with the base interaction.
Again the interference between the two operator components is crucial in finding the result
that agrees well with data.

The spectra shown in Fig. 3 and labeled 1^{st} order and 0^{th} order, result when deformation
is taken only to first order or not at all. The first we achieved by setting to zero the value of
β_2^2 (∝ κ^2) in the potential matrices of Eq. B.3 in Appendix B of ref. 1. Of course β_2 (∝ κ)
was taken as -0.52 therein still. The second resulted from a calculation made using β_2 = 0
everywhere.

In 0^{th} order coupling, we obtain single particle states of the base potential. Of those, the
p-shell terms are reasonable when compared to the known spectrum of ^{13}C. In ref. 1, the
p − sd model shell structures for these were listed as dominantly a p-shell neutron coupled
to the ground state of ^{13}C. Allowing coupling to first order in the deformation compresses
the spectrum as now the calculations allow for an appreciable, and realistic, component of
p − sd-shell particles coupled to the 2^+ state of ^{12}C in the description of the states in ^{13}C. The
spectrum obtained is very similar to that resulting when deformation is taken to second order
(the Base result). Treating deformation to second order increases the number of contributing
terms quite markedly as is evident when one considers the entries in Eq. (B.3) of ref. 1
that have the scale quantity κ^2. But the corrections they make to the sub-threshold spectra
are relatively minor. Notably the positive parity pair of states are slightly more separated
and the $\frac{3}{2}^-$ state slightly less bound.

In Fig. 4 we display the cross sections for neutron-$^{12}$C elastic scattering that result when deformation is neglected (dashed line), is taken to first order (long-dashed line) and is taken to second order (the Base result as displayed by the continuous line). Again to add in clarity of viewing, the restricted calculation results have been shifted (again by 3 and 6 b) so that the curves can more easily be distinguished. As is evident, there is little to distinguish between the results when deformation is taken to first and to second order. Overall there is a slight inward shift of the resonances with energy; the more so with the broader $\frac{3}{2}$ resonances in the 3 to 5 MeV region. But neglect of deformation has a most marked effect. Not only do the narrow resonances become bound states in the continuum, but also the large shape resonances vanish.

Finally we consider the effects of changes in the value of the deformation parameter. Values of -0.52 (Base), -0.2, -0.1, -0.05, -0.01, and of -0.0001 have been used in calculations of both the sub-threshold bound states as well as of the scattering cross sections. The bound states (of $^{13}$C) vary with the values of $\beta_2$ as shown in Fig. 5. There is a rapid decrease of the energies of the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states as $\beta_2$ is reduced to a value of -0.2 and thereafter, those states slowly increase in excitation above the $\frac{1}{2}^-$ ground state. The two negative parity states, on the other hand, vary little in their binding with change in $\beta_2$, being slightly compressed from what values they have from the base calculation.

The cross sections that result when different $\beta_2$ values were used in our calculations are shown in Fig. 6. Therein each value is scaled upward overall by a ‘n’ barn, where n is the number of the sequence above the Base result (solid curve) as follows: the lower long-dashed curve is the result on using $\beta_2 = -0.4$, that displayed by the small dashed curve had $\beta_2 = -0.2$, the dot-dashed curve portrays the result with $\beta_2 = -0.1$ the next (a solid curve) was found on using $\beta_2 = -0.05$, while the topmost (long dashed) curve is the cross section found with $\beta_2 = -0.01$. The general trend is that the background cross section values near
threshold increase in size and that is consistent with the strong sub-threshold s-wave $\frac{1}{2}^+$ state moving closer to threshold as $\beta_2$ decreases. Then what was just a sub-threshold state, the $\frac{5}{2}^-$ in the Base calculation moves into the positive energy regime with decrease in $\beta_2$. That state has an extremely small width ($\leq 10^{-11}\text{MeV}$) so that its existence is known but the strength has not been ascertained. An arbitrary cross section spike has been added to these results at the known energies ($\leq 0.3\text{ MeV}$). Then as $\beta_2$ decreases, while some resonance features of the cross section move to higher excitation, notably the $\frac{5}{2}^+, \frac{3}{2}^+, \frac{3}{2}^+, \frac{7}{2}^+, \frac{7}{2}^+$ resonances, others essentially remain unchanged in energy. That is indicated in the figure by the dashed lines showing the energy movement of the $\frac{5}{2}^+$ and $\frac{1}{2}^+$ resonances. Also the broad $\frac{3}{2}^+$ (shape) resonances shrink until, with $\beta = 0$ they disappear.

Summarizing, the MCAS approach to analyze (low-energy) nucleon-nucleus scattering is built from a model structure of the interaction potentials between a nucleon and each of the target states taken into consideration. The process finds all resonances (narrow and broad) in the selected (positive) energy range, as well as defining the background cross section. The method also can be used with negative energies to specify the sub-threshold bound states of the compound nucleus.

With an interaction matrix of potentials defined by a collective model of the structure of the ground ($0^+$), $2^+$ (4.44 MeV), and $0^+_2$ (7.65 MeV) states in $^{12}\text{C}$, and with deformation therein taken to second order, MCAS calculations of neutron scattering have been made with the complete (Base) results well matching the observed scattering data and producing sub-threshold bound states very like those known in the spectrum of $^{13}\text{C}$. Note however,
as stressed before [1], allowance for the influence of the Pauli principle was crucial to that achievement. Once the non-locality due to Pauli blocking was adequately treated, good agreement with respect to sub-threshold spectra and scattering cross sections were obtained with a phenomenological (coupled-channel) interaction. However, in addition to usual central and \( s \cdot \ell \) (spin-orbit) potentials, we have found that additional operator terms of the type \( \ell \cdot \ell \) (orbit-orbit) and \( s \cdot I \) (spin-spin) were necessary. These latter terms provide contributions to the usual interaction, particularly for negative parity, to get the appropriate separation energies of sub-threshold bound and resonance states.

In addition we have shown how the results depend on the coupling deformation parameter and, further, we have shown that both the spectrum (of \(^{13}\text{C}\)) as well as the specific resonance structure of the cross sections converge well when the collective model interactions are taken to second order in that deformation.

[1] K. Amos, L. Canton, G. Pisent, J. P. Svenne, and D. van der Knijff, Nucl. Phys. A728, 65 (2003).
[2] F. Ajzenberg-Selove, Nucl. Phys. A523, 1 (1991).
FIG. 6: Total elastic cross sections for n-\(^{12}\)C scattering as functions of neutron energy showing the effects of reducing the value of \(\beta_2\) as described in the text.