The analysis of the discovery learning implementation and its affect to the students conjecturing skills in solving a resolving domination number

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Abstrak. Conjecturing plays a role in mathematics learning, namely: (1) conjecturing as a way of solving problems (2) conjecturing as a process that helps students in understanding material, and (3) conjecturing as a process that trains students in reasoning. One learning model that can be applied in lectures in order to improve students' conjecturing skills in solving expected mathematical problems is a discovery learning model, this paper will describe the development of learning material using discovery learning models and the implementation to know the effect of student conjecturing skills on combinatorics problems. The problem will be focused on discrete modeling with dominating metric dimension number studies. Further we analyze their conjecturing skills by using a triangulation methods, namely a combination of qualitative and quantitative methods. The subjects of this study are students majoring in Mathematics Education at Jember University. Research subjects using experimental class and control class. The result are 22% of students in the control class have fulfilled the completeness criteria study. While in the experimental class there are 29% has met the criteria for mastery learning. It means after the application of discovery learning, the students conjecturing skills are improved signicantly. Overall the results of the study indicate that the stages in conjecturing skill were done sequentially although not all steps were done.

1. Introduction
Mathematics is one of the sciences which has been develop rapidly now days. The functions of mathematics is to develop the ability in counting, measuring, finding and using mathematical formulas that can support the understanding of students' concepts in their daily lives. One of the basic sciences that is very important in mastering science and technology is mathematics, both its applied aspects and reasoning. This means that mathematics needs to be mastered by all Indonesian citizens, because it provides provisions for structuring reason and forming attitudes and mentality. Mathematics also provides provisions for students to be able to apply mathematics in various fields of life such as business, economics, engineering and others.

In addition, mathematics is closely related to problem solving. Problem solving and conjecturing are important parts of mathematical activities. Many researchers have suggested that problem solving and conjecturing are important parts of mathematical activities. Problem solving is one part of school mathematics standards [1]. He further explained that mathematical thinking and reasoning skills,
including making conjectures are important because they serve as a basis for developing new insights and enhancing further study. Conjecturing plays a role in mathematics learning, namely: (1) conjecturing as a way of solving problems (2) conjecturing as a process that helps students understand the material, and (3) conjecturing as a process that trains students in reasoning [2][3].

The current learning paradigm has characteristics including: active teachers, passive students; teacher-centered learning; the teacher transfers knowledge to students; student understanding tends to be instrumental; learning is mechanistic; and students are silent (physically) and full of concentration (mental) pay attention to what the teacher teaches. This will have an impact on lectures at the tertiary level, including the lack of students' conjecturing skills in solving mathematical problems. One learning model that can be applied in lectures in order to improve students' conjecturing skills in solving expected mathematical problems is the discovery learning model. The reason for choosing the discovery learning model is because this learning model in applying lecturers provides an opportunity for students to actively learn and be able to find formulas or concepts from the material being studied, previously.

The Conjecturing process carried out this study includes the conjecturing empirical induction from finite numbers in discrete cases. Based on [4, 9], there are 7 indicators of conjecturing. The seven indicators can be divided into sub indicators, they are as follows: 1) observing the problem (ID1) with the sub-indicator : students draw the graph (ID1A) and name the graph (ID1B); 2) organizing the problem (ID2) with the sub-indicator : students give label in the graph (ID2A) and predict to solve the problem (ID2B); 3) Finding and Predicting (ID3) with the sub-indicator: students investigate information based on the problem (ID3A) and determine the resolving domination number (ID3B); 4) formulating the conjecture (ID4) with the sub-indicator: students determine the pattern of resolving domination number (ID4A) and clarify the resolving domination number (ID4B); 5) validating the conjecture with the sub-indicator students test the working (ID5); 6) generalizing the conjecture with the sub-indicator: students determine the generalization of the resolving domination number (ID6); 7) validating the conjecture (ID7) with the sub-indicator: students validate the generalization of the pattern (ID7A) and students compare the working (ID7B).

In designing learning, several things need to be considered, among others: activating and attracting students to improve students' innovative skills in appropriate learning and using the media that is around. One of the models that will be applied is discovery learning. The concept of learning in this model students are given the authority to compile, and implement learning programs and evaluate the program independently, so that students not only have the skills but also a good way of solving problems. themselves. “Discovery Learning is an inquiry-based approach in which students are given a question to answer, a problem to solve, or a set of observations to explain, and then work in a largely self-directed manner to complete their assigned task and draw appropriate inferences from the outcomes, discovering the desired factual and conceptual knowledge in the process ” [5].

In discovery learning, one of the characteristic is students will construct knowledge about new information and data collected by them in an explorative learning environment [6][7]. There are six procedures that must be carried out in learning activities to apply discovery learning, namely: 1) Stimulation, 2) Problem statement, 3) Data collection, 4) Data processing, 5) Verification and 6) Generalization [8]. For more details can be seen in table 1.

| The Stage | The Activity |
|-----------|--------------|
| 1. Observing the Case | At this stage students are faced with something that causes confusion without giving generalizations to cause students to want to investigate themselves. This stage serves to provide conditions for learning interactions that can develop and assist students in exploring material. Lecturers must master techniques in providing stimulus to students so that the goal of activating students to explore can be achieved. |
The Stage | The Activity
---|---
2. Problem statement | At this stage the lecturer provides the opportunity for students to identify as many agendas as possible that are relevant to the subject matter and then hypothesize one of them.
3. Data collection | At this stage students are given the opportunity to gather various relevant information by reading literature, observing objects, interviewing informants (actively asking questions), conducting tests, and so on.
4. Data processing | At this stage students process data and information obtained. The data is processed, randomized, classified, tabulated, and calculated in certain ways. From this process students will get new knowledge about alternative solutions that need to be logically proven.
5. Verification | Students do a careful examination to prove whether or not a formula or concept is found with the results of data processing.
6. Generalization | This stage is the process of drawing conclusions that can be used as general principles and applies to all the same events or problems, taking into account the results of verification.

As a problem statement, the purpose of this study is developing of discovery learning implementation and their influence on students' conjecturing skills in solving the resolving dominating number of a graph.

The material used in this study is resolving dominating number. Domination number is the smallest of a dominating set is called a minimum dominating set and its size is called as the domination number. The domination number is the minimum cardinality of a dominating set and denoted by \(\gamma_d(G)\). The metric dimension is one of the subjects in graph theory. The issue of the release of the metric dimension was first introduced by Slater in 1975. Slater introduced the license and used the location set and location number for what we asked for the set resolution and metric dimensions. He discussed the usefulness of these concepts in working with the US Sonar and Coast Guard stations [10]. Harary and Melter [11] proposed the same concept in their paper On the Metric Dimension of a Graph Independently, Harry and Melter discovered the concept of the location number as well and called it the metric dimension.

2. Methodology
This research is a type of research with a mixture of embedded type methods simultaneously which is a method that combines the use of quantitative and qualitative research methods simultaneously together, but the weight of this method is different. This research focuses qualitative research methods as primary methods, and quantitative methods as secondary methods. Quantitative research in research, as supporting data to analyze the alleged skills of Mathematics Education at the Faculty of Mathematics and Natural Sciences, University of Jember in Bondowoso Campus.

2.1. Population
There are two classes being the population, class A and class B. Class A with 41 students were a control class and class B with 41 were experimental class. The same indicator that we will see is the conjecturing skill of the two classes so that the same pre-test and post-test are given. Two group of the class were selected randomly (R) and given different treatments. The control group uses a conventional teaching model (lecture-centered), while the experimental group receives discovery learning model. After solving the problems given, can be seen the effectively result.

| Class         | Pre-Test | Treatment                        | Post-Test |
|---------------|----------|----------------------------------|-----------|
| Eksperimen, n = 41 | R₁       | Discovery Learning               | R₂        |
| Control, n = 41    | R₃       | Conventional teaching model      | R₄        |

(lecture-centered)
2.2. Instruments
For the Instruments used pre-test, post-test, observation and interview. Rating scale in this research used 0-100 on pre-test and post-test. While, a rating scale of 0-4 is used on the observation sheet which can be seen in the table 4 as follows:

|Rating scale| Category         |
|------------|-----------------|
|4           | Very Conjecture |
|3           | Conjecture      |
|2           | Less Conjecture |
|1           | No Conjecture.  |

![Figure 1. The triangulation model of qualitative and quantitative](image)

2.3. Task
The students got the task among others : firstly pre-test, secondly worksheet and the last post-test. All of the task given to students are worksheets that have indicators to evaluate conjecturing of the students. After both classes got a pre-test, the experimental class got a worksheet but not the control class. In all assignments, students were instructed to determine the resolving domination number in a graph.

The minimum cardinality of a dominating set is called the domination number . The value of the domination number is always \( \Upsilon(G) \subseteq V(G) \). In Respect of the upper bound of the number which dominates is the number of vertices of the graph. When the dominating set needs at least one vertex , then \( 1 \leq \Upsilon(G) \leq n \) for each graph that have the order n. Let \( D \subseteq V \), If each vertex of graph G without the set graph that dominates G is adjacent to one of the vertex in graphs that supports G, then D will be a dominating set of graph G [11].
According to Harry and Melter, the subset \( W \) is a resolving set if \( r(v|W) \) for every two vertices of \( G \) have distinct representations. A resolving set of minimum cardinality for graph \( G \) is called a minimum resolving set or a basis for \( G \). The metric dimension of \( G \), denoted by \( \dim(G) \) [10]. A Set of vertices of a graph \( G \) that is both resolving and dominating is a resolving dominating set. The minimum cardinality of a resolving dominating set is called resolving dominating set \( \Upsilon_r(G) \).

**Figure 2.** Observe the following figure friendship graph!

![Friendship Graph](image)

**Figure 3.** Determine the cardinality which includes labeling each vertex and edge, number of vertices and edges,

\[
V = \{x_{ij} ; 1 \leq i \leq n; 1 \leq j \leq 2 \} \cup \{z\}
\]

\[
E = \left\{x_{ij}x_{i+1j} ; 1 \leq i \leq n; j = 1 \right\}
\]

\[|V| = 2n + 1\]

\[|E| = n + 2n = 3n\]

**Figure 4.** Find and represent the domination vertex!

![Domination Vertex](image)

Based on the Figure 2 and Table 4, \( S = \{x_{11}, x_{21}\} \) we can see that the resolving domination number is \( |S| = 2 \).

So for friendship graph (\( Fn \)) where \( n = \) the number of sub graphs, applies: \( |S| = n \)

| \( v \) | \( r(v|S) \) | condition |
|---|---|---|
| \( x_{11} \) | \((0, 2, 0, 2, \ldots, 2)\) | \( n \geq 2 \) |
| \( x_{12} \) | \((1, 2, 0, 2, \ldots, 2)\) | \( n - 1 \) |
| \( x_{21} \) | \((2, 0, 2, 0, 2, \ldots, 2)\) | \( n \geq 2 \) |
| \( x_{22} \) | \((2, 1, 2, 0, 2, \ldots, 2)\) | \( n \geq 2 \) |
| \( x_{i1} \) | \((2, 0, 2, 0, 2, \ldots, 2)\) | \( i - 1 \leq n - i + 2 \) |
| \( x_{i1} \) | \((2, 0, 2, 0, 2, \ldots, 2)\) | \( n - 1 \) |
| \( Z \) | \((0, 2, \ldots, 2)\) | \( n \geq 2 \) |

**2.4 Collection and analysis of data**

Because in this study using a mixed method, the analysis came to use qualitative and quantitative analysis. To analyze students’ quantitative data, inferential statistics were needed, one of which is using homogeneity test, normality test, independent test and the last one to see the difference between the two classes (control class and experimental class) t-test is used. Meanwhile, to analyze qualitative data it is not necessary to use inferential statistics, it is enough to use observational, interview and ordinal data throughout the study. For the independent samples t-test with a significance at 0.05 level.
3. Results and discussion
The main objective in this study is to value the students’ conjecturing skills in determining the resolving domination number on the graph from two different classes, namely in the experimental and control classes using different treatments. The application of research was carried out after testing the validity and reliability of the instrument. After that, the experimental and control classes were given a pre-test to find out their conjecturing skills.

After conducting pre-test in the control and experiment class, learning by applying discovery learning model learning is held in the experimental class while conventional learning is held in the control class where the data obtained will be analyzed using SPSS. By using the SPSS application the following analysis results are obtained.

| Table 5. Results of question validity |
|--------------------------------------|
| PROB_1 | PROB_2 | PROB_3 | PROB_4 | PROB_5 | TOTALS |
| Pearson Correlation | .439** | .421** | .475** | -.030 | .657** |
| Sig. (2-tailed) | .005 | .007 | .002 | .855 | .000 |
| N | 40 | 40 | 40 | 40 | 40 |
| Pearson Correlation | .439** | 1 | .556** | .424** | .093 | .764** |
| Sig. (2-tailed) | .005 | .000 | .006 | .569 | .000 |
| N | 40 | 40 | 40 | 40 | 40 |
| Pearson Correlation | .421** | .556** | 1 | .290 | .033 | .670** |
| Sig. (2-tailed) | .007 | .000 | .069 | .839 | .000 |
| N | 40 | 40 | 40 | 40 | 40 |
| Pearson Correlation | .475** | .424** | .290 | 1 | -.009 | .628** |
| Sig. (2-tailed) | .002 | .006 | .069 | .958 | .000 |
| N | 40 | 40 | 40 | 40 | 40 |
| Pearson Correlation | -.030 | .093 | .033 | -.009 | 1 | .469** |
| Sig. (2-tailed) | .855 | .569 | .839 | .958 | .002 |
| N | 40 | 40 | 40 | 40 | 40 |
| Pearson Correlation | .657** | .764** | .670** | .628** | .469** | 1 |
| Sig. (2-tailed) | .000 | .000 | .000 | .000 | .002 |
| N | 40 | 40 | 40 | 40 | 40 |

Based on the output from Table 5, on question 1 the value of the \( r_{count} \) is 0.657; question 2 is 0.764; question 3 is 0.670, question 4 is 0.628 and question 5 While \( r_{table} \) for \( n=40 \) is 0.393 and \( r_{count} > r_{table} \) so all question are valid. When we see table 8, the reliability statistics is 0.740 \( r_{table} \) form the significance level of 5% with \( dk=N-1=39 \), \( r_{table}= 0.3978 \). Because \( r_{count} > r_{table} \) so that the instruments item are reliable.

| Table 6. Reliability Statistics |
|--------------------------------|
| Cronbach’s Alpha | N of Items |
| 0.740 | 6 |

In the study conducted in two classes with a total of 82 students, gave different results. This can be seen from the results of the tests given. The test given measures the student's conjecturing skills on the material determining the resolving domination number of graph. The questions in the pretest contain a problem solving the resolving domination number consisting of 7 indicators, as well as the post-test consisting. Before carrying out further research, researchers conducted an analysis of the pre-test
results for both classes. It must be ensured that both classes have the same variant and it is true that both the control class and the experimental class have the same variant.

**Chart 1. The result of control class and experimental class in pre-test**

![Chart 1](image1.png)

There were 82 students from the control and experimental class who had conducted the pre-test got the following results. The results shown in the pre-test in the control class are the percentage of each indicator was 57.52% to observe the problem (ID 1), to organize the problem (ID 2) was 53.74%, to find and predict patterns (ID 3) was 56.10%; to formulate the conjecture (ID 4) was 54.36%; 55.05% to validate the conjecture (ID 5); to generalize of pattern (LV 6) was 57.64%; and the last, to validate the alleged indicators (ID 7) was 54.64%. Meanwhile, when compared to other classes, namely the experimental class, both classes gave results that were not too far away. The results are as follows: there were 55.64% to observe the case (ID 1) is; 54.56% to organize the problem (ID 2), 54.79%, to find and predict pattern (ID 3), 55.06%, to formulate the conjecture (ID 4); 57.98% to validate the conjecture (ID 5); 52.03% to generalize pattern (ID 6), 54.02% to validate conjecture (ID 7).

There are four categorizes from the results of the overall pre-test and post-test from analyzing students' conjecturing skills which are high conjecturing, conjecturing, fair conjecturing and low conjecturing. When we see the results of the analysis, the percentage of in the experimental class were 16% in high conjecturing category, 25% in conjecturing category, 32% in fair conjecturing category, and 27% in less conjecturing category. While in control class was as follows: high conjecturing category was 15%, conjecturing category was 23%, fair conjecturing category was 36%, and less conjecturing category was 26%.

**Chart 2. The result of conjecturing skill for pre-test in control class**

![Chart 2](image2.png)

From the experimental class from 41 students, there were different level of their conjecturing skill. And the result of their conjecturing skill after giving pretest were 18 % students in the very high level; 27 % students in high level; 29 % students in fair level and 26 % students low level. We can see from chart 2 as follow.
Chart 3. The result of conjecturing skill for pre-test in experimental class

After getting the result of pretest, the next step was the analyzing used quantitative method to find out the difference of variance of discovery learning from the students. Firstly, the analysis of homogeneity test should be checked by using SPSS from control class and experimental class.

From Table 7, the homogeneity test result of pre-test in control class and experimental class the significance value of 0.459 was obtained. Because the result of significant was bigger than 0.05 this could have consequences that the variance of the data from control class and experimental class was homogens.

Table 7. The homogeneity test result of pre-test

|           | Levene Statistic | df1 | df2 | Sig. |
|-----------|------------------|-----|-----|------|
|          | .555             | 1   | 80  | .459 |

Table 8. The Results of independent pre-test tests in the control class and experimental class

| Class         | N   | Mean     | Std. Deviation | Std. Error Mean |
|---------------|-----|----------|----------------|-----------------|
| Score Control Class | 41  | 60.3415  | 7.56508        | 1.18147         |
| Experimental Class | 41  | 61.7073  | 8.38524        | 1.30955         |

Table 9. Independent Samples Test in the control class and experimental class

| Score      | F   | Sig.  | t    | df  | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
|------------|-----|-------|-----|-----|-----------------|------------------|-----------------------|-----------------------------------------|
| Equal variances assumed | .555 | .459  | -.774 | 80  | .441            | -1.36585         | 1.76375                | -4.87582 - 2.14411 |
| Equal variances not assumed | -.774 | 79.167 | .441 | 1.76375 | -1.36585 | 4.87639 | 2.14468 |

The statistical test result from Table 8 indicated the mean of control class was 60.3, whereas the mean of experimental class was 61.70. In this table the average in the control class is less than the average in experimental class. After checking the independent test in table 10, the number of significant (2-tailed) was 0.441. Because the number of \( t_{count} > t_{table} \) whereas \( P_{value} \) was higher than 0.05, so the control class and experimental class were the same.
The next step of the research was giving the different treatment for each class. The control class used conventional learning models (teacher-centered) and otherwise the experimental class used discovery learning. Then was continued by post-test to find out the changes of their conjecturing skill. The result is shown in control class: high conjecturing category was 22%, conjecturing category was 35%, fair conjecturing category was 30%, and less conjecturing category was 13%.

Chart 4. The students distribution of conjecturing skill in control class for post test

When looking at the results of the analysis it was found that the distribution of students’ skills in the experimental class for the post-test after using discovery learning based on graph 4 are: 29% in the high conjecturing, 35% in the conjecturing, 24% in the fair conjecturing, and 12% in the low conjecturing.

Chart 5. The result in experimental class for post-test

When looking at the results of the analysis it was found that the distribution of students’ skills in the experimental class for the post-test after using discovery learning based on graph 4 are: 29% in the high conjecturing, 35% in the conjecturing, 24% in the fair conjecturing, and 12% in the low conjecturing.

Table 10. The number of normality of post-test in the two classes

| Score | Statistic | df  | Sig. | Statistic | df  | Sig. |
|-------|-----------|-----|------|-----------|-----|------|
|       | Kolmogorov-Smirnov* | Shapiro-Wilk |
|       | Statistic | df  | Sig. | Statistic | df  | Sig. |
| Class | .125  | 41  | .107 | .964  | 41  | .212 |
|       | .121  | 41  | .138 | .951  | 41  | .074 |

The result of the control and experimental classes based on Table 10 showed 0.107 and 0.138, because both of the value bigger than 0.05, therefore the data from control and experimental classes had normal distribution.
Table 1. Independent Samples t-test of post-test

| Score        | F     | Sig. | t    | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower  | Upper  |
|--------------|-------|------|------|----|-----------------|-----------------|-----------------------|--------|--------|
| Equal variances assumed | 6.472 | 0.13 | -4.811 | 80 | 0.001          | -6.34146        | 1.31822               | -8.96481 | -3.71811 |
| Equal variances not assumed | -4.811 | 71.231 | 0.001 | -6.34146 | 1.31822 | -8.96978 | -3.71315 |

Based on Table 11, Independent sample t-test of post test from control and experimental classes had sig(2-tailed) 0.001, otherwise it is smaller than 0.05. So the result from the post-test between control using conventional method and experimental classes was significant different after the implementation of using discovery learning.

For homogeneity and independent t-tests that have been carried out in pre-tests revealed that there were no significant differences between the two classes using the SPSS program. Therefore, it can be concluded that the two classes used in his research have almost the same abilities. After that, the two classes are given different treatments. Learning activities in the control class using conventional methods while in the experimental class using discovery learning based learning.

The analysis of student determine resolving dominating numbers to find out the completion process of the worksheet and find the correct end result can be seen as following. After getting the results of learning in two classes, then the researcher needs to dig further information to provide an analysis of each student's work results on the worksheets provided. Extracting information is done by conducting interviews with students.

Portrait phase was needed to show the processes in learning. The portrait phase was representation the processes from conjecturing skill, we selected six objects from each experimental and control groups, and we only represent three subjects as the illustrations in this research. The interview have been done on selected subjects to determine the process of thinking in completing the resolving dominating number.

The conjecturing skill conducted by students in solving the problem of resolving domination number of a graph shows that there is a difference between one subject and another subject. This difference in the conjecturing skill because of their knowledge and previous experience. Based on the results of interviews and analysis of student answer sheets obtained an overview of the conjecturing skill in solving the problem of resolving domination number in each student.

Figure 5. The Result of subject 1 (High Conjecturing Skill)

The result of subject 1 (M1) was shown in figure 5. Subject 1 labeled each edge with and vertex of path graph (P₆). Every vertex of graph is labeled by $x₁, x₂, x₃, x₄, x₅, x₆$ and continued by determining the cardinality of the graph. Then M1 took the next step which was to determine the resolving
domination number of graph P_6. M1 The results of interviews conducted on the subject M1 showed that M1 understood the purpose of the questions given. It was evident from the results of the interview of researcher (R) with M1 about mind mapping with the problem of resolving dominating number. This interview has the aim to show the way of thinking and the level of conjecturing skills of students when determining the resolving domination number problem. Excerpts from the interview are as follows:

R : How do you get about determining the resolving domination number of P_6?
M1 : Initially I did not understand the material resolving domination number because I did not get the material, but after I practiced and worked on the worksheet that was given I finally learned how to do it.

R : Did you get it?
M1 : Yes I Did

R : Could you explain, how do you find the problem solving in resolving domination number step by step?
M1 : Firstly, I drew the graph P_6 and labeled each vertex of the graph. I tried to guess what is the minimum resolving set of graph P_6 and I searched the cardinality each vertex and edge of graph P_6. Next, I check the representation of each vertex. I had to make sure that I had found the minimum dominating vertex that has different distance. Finally I found the Resolving dominating number.

R : What is the easier way to find this problem solving?
M1 : I made several possible answers to determine the point that dominates with the smallest number. Then after that, just re-correct which points have different distances.

R : Did you find difficulty in this problem solving?
M1 : Yes I did, it was so hard when finding the function. I had to find the pattern and did one by one carefully.

Figure 6. The portrait of phase of subject 1

The portrait of phase M1 based on Figure 6, showed the student’s conjecturing skill in finding the problem solving with the material resolving domination number of graph P_6. After drawing (ID 1A) and giving label (ID 2A), M1 skipped to Declaring the resolving domination number (ID 3B). The next interview is subject 2 (M2):

R : How do you get about the material?
M2 : Determining the resolving domination number of graph?
R : Did you get the material?
M2 : No I did not. It was so hard to get it.

R : What is the difficult thing from the steps that you have to do. Could you explain it each step?
M2 : I drew the graph, labeled each vertex and the edges of the graph. Next I named the graph and continued by finding the cardinality each vertex and edge of graph P_6. The last I tried to determine the resolving domination number of graph, but I could not find it.

R : Did you find some difficult thing in the problem solving?
M2 : Yes, I did Miss, it was so hard and confusing when finding the resolving domination
number of the graph. So I was not finished yet.

Based on the conversation with M2 when working on an assignment, it is shown portrait of phrase in sub indicator as the following figure.

![Figure 7. The portrait of phrase of subject 2](image)

Interviews conducted on subjects 3 (M3) showed that M3 observed cases by observing cases by drawing graphs and immediately looking for and predicting patterns continued to look for functions to determine the resolving set on graph P6, with the knowledge of graphs.

The results of interviews conducted on subject 4 (M4) showed that M4 did not understand the purpose of the problem given, but M4 knows that graph P6 has cardinality for each point and side. M4 cannot proceed to the next stage, because it does not understand the purpose of the problem.

Figure 6 presents a combination of several phase portraits of M1, M2, M3. The combined portrait phase can show students' conjecturing skills in general, and certainly each student has a different and unique way of solving a problem of resolving domination number.

![Figure 8. The portrait of phase of subject 1,2,3 and 4](image)

From the description of the results of interviews and analysis of each student's answer sheet, it can be described the process of implementing the conjecturing stages in solving the resolving domination number as shown in figure 8. The description of the process shows how students think in solving the resolving domination number problem based on the stages of the conjecturing process. only happens online, but the conjecturing process also occurs with zigzag as done by M1, M2, M3 and M4 and some even do not proceed to the next stage because the subject does not understand the given problem.

Research that has been done shows whether discovery learning can have an influence on the level of students' conjecturing skills. In addition, in this study the researcher wanted to train students in the resolving domination number material and the findings in the post will be poured into a monograph. After observations made on student activities during the study, the researcher can show that through discovery learning can have an effect on the conjecturing skills and student learning achievement of students in completing the resolving domination number. The result from the student activity in experimental class had increasing. The result of student activities during the implementation of discovery learning in the experimental class shows 39% were highly active, 52% were medium active, 15% were rarely active, 14% were non-active. The existing facts can be shown by the linearity of this research by the study of the actual implementation of discovery learning [13, 14, 15].
4. Discussion
In the learning process, students must be given stimuli to develop their conjecturing skills in resolving domination. The main objective of this study is to analyze the implementation of discovery learning in the conjecturing skills of students to solve the problem of solving dominance. The finding is that classes with learning using discovery learning (experimental class) have significant changes in students’ conjecturing skills. In addition, the results of research conducted have seen an increase in student learning outcomes in the material if bound from the post test.

The post-test results from the experimental class that used discovery learning showed a significant and better change in improving students’ conjecturing skills. The results of the control class study are as follows: 22% in the high conjecturing, 35% in the conjecturing, 30% in fair conjecturing and 13% in the low conjecturing. Whereas, the experimental class showed better results, 29% in the high conjecturing, 24% in fair conjecturing, and 12% in low conjecturing. This research was conducted to analyze the ability to think creatively and the application of discovery learning. Independent test results were obtained for variants of sig scores. (2-tailed) is 0.001. Because the value of sig. 0.001 is smaller than 0.05, it can be concluded that there is a significant difference between the control class and the experimental class after the application of discovery learning. Discovery learning ask the students to construct the knowledge by managing experimentation and discovering principles from the results [16]. The results of this study are in the same result with the theory converted by [14] that the results of research using discovery learning are very helpful for students to be able to achieve learning targets. It shows the learning objectives have an important role in solving students’ problems. Discovery learning model is able to provide opportunities for students to be more active in the learning process, in addition to the application of discovery learning can improve students' creative thinking skills.

5. Conclusions
The results of the research that has been done, it appears that the application of classroom experiment Discovery learning has a significant impact on student skill conjecturing. From the results of the post-test in the study also showed that the application of discovery learning showed an increase in student learning outcomes and conjecturing skills. On the other hand, the application of the method of learning curriculum can also improve students' conjecturing skills higher than the control class that uses conventional methods. Therefore discovery learning is extremely great for improving students' alleged skills.

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