Mathematical model of a radial bearing with a low-melting metal coating of design models of hydrodynamic viscoelastic lubricant formed by melting the surface of a bearing bush coated with a metallic low-melting coating

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Abstract. The work is based on equations of motion of viscoelastic lubricant (Maxwell fluid) for the case of “thin layer”, continuity and the formula of the rate of dissipation of mechanical energy. We found asymptotic solution to the degrees of small parameter K, characterizing the rate of dissipation of mechanical energy. This allows determining the molten surface of the bearing sleeve covered with a fusible metal melt, taking into account the dependence of the viscosity of the lubricant and the shear modulus of pressure in the adiabatic process. For zero approximation under adiabatic process conditions, i.e. without taking into account the melt surface, and a solution for the first approximation, i.e., taking into account the melt surface of the bearing sleeve covered with a fusible metal melt, the exact self-similar solution of the problem with incomplete (pre-emergency) filling of the working gap with a lubricant having viscoelastic properties is found. On the basis of the results obtained for the zero and first approximation, the bearing capacity and friction force are found at incomplete filling of the working gap with a lubricant. In addition, for verification calculations of the obtained theoretical models, a numerical calculation was carried out and as a result, graphs showing a decrease in the friction coefficient by about 20% were constructed.

1. Introduction
Tribosystems of modern machines operate under high load-speed regimes. In these conditions, the use of liquid friction provided by liquid lubricants is the most promising. And in this lubrication mode the absolute advantage belongs to the hydrodynamic process.

It is known [1] that in heavily loaded tribosystems, local centers of melting can occur. This is confirmed by Bowden and Tabor’s "welding bridges" [2].

It should be noted that the oxygen of the ambient air very quickly oxidizes the contact surface of mercury and the value of the friction force for 2 hours of operation increases more than 20 times. In addition, mercury, like any metal melt, is a surface-active liquid that amalgam metals being in contact with it. Therefore, the melt area must be isolated from the environment.

The operation of various valve systems in the primary heat removal circuit of nuclear reactors in the melt of alkali metal is also known.
The wide application of hydrodynamic lubrication in a wide variety of machines and mechanisms required the development of a significant range of new highly effective lubricating liquids [3-9] such as micro-polar, viscoelastic, viscoplastic, electrically conductive and melt fusible metal alloys. The practice of using these liquid lubricants significantly outstrips the development of models and theoretical calculations necessary for their wide application in practice.

Lubrication with liquid metals is used at temperatures at which conventional lubricating media undergo irreversible physical and chemical changes. The advantage of melt lubrication is that the lubricant is formed in the contact area where it is necessary. Melting delivers a sufficient amount of lubricant to the friction zone; there are no mechanical and structural difficulties associated with its supply. Melt lubrication has been studied in many applications, particularly in metal forming and cutting processes. A large number of works are devoted to hydrodynamic calculation of the radial bearing of infinite length in the conditions of incomplete filling of the working gap with lubricant [10-11]. A significant disadvantage of the friction pair under consideration, working on the melt lubrication, is the low bearing capacity. In addition, the lubrication process is not self-sustaining.

Thus, the development of a design model of sliding bearings operating on lubricants in the form of metal melts, taking into account the above aspects of operation, is of particular importance and scientific interest. An attempt to develop a general methodology for calculating the hydrodynamic lubrication regime for different bearing designs, lubricated with liquids with different physical and mechanical properties, will significantly accelerate their bringing to industrial application.

2. Materials and methods
A mathematical model is proposed on the basis of equations describing the motion of an incompressible viscoelastic lubricant for a "thin layer", continuity and expression of the rate of dissipation of mechanical energy to determine the profile of the molten surface of the bearing sleeve covered with a metal fusible melt also having viscoelastic properties.

3. Results
New multiparametric expressions are developed for the basic performance characteristics of the radial sliding bearing with incomplete filling of the working gap with a viscoelastic lubricant, taking into account the rheological properties, as well as the melt surface of the bearing sleeve coated with a metal fusible coating in the adiabatic process.

Expressions for the load capacity and friction force are obtained taking into account the rheological properties of viscoelastic lubricant and the melt of a fusible metal coating.

The paper summarizes the influence of factors that describe the process of incomplete (pre-emergency) filling of the working gap with a lubricant having viscoelastic properties, taking into account the melt surface of the bearing sleeve coated with a low-melting metal coating, taking into account the dependence of the viscosity of the viscoelastic lubricant and shear modulus on the pressure in the adiabatic process, which significantly complicates the formulation and solution of the problem, but makes its solution interesting and in demand.

3.1. Task setting
The steady flow of an incompressible viscoelastic lubricant in the gap of an infinite radial sliding bearing coated with a low-melting metallic coating is considered in the conditions of the adiabatic process.

The shaft rotates at angular velocity Ω (fig. 1), and the bearing bush is stationary. It is assumed that the space between the eccentrically located shaft and the bearing is not completely filled (pre-fault condition) with a viscoelastic lubricant, and the bearing bush is made of a material with a low melting point.
In the polar coordinate system $r, \theta$, the pole of which is located in the center of the bearing bush, the equation of the shaft contour, the surface of the bearing bush covered with a metal melt, and the molten surface of the bearing bush covered with a low-melting metallic melt will be written as follows:

$$r' = r_0(1 + H), \quad r' = r, \quad r' = r + \lambda f(\theta),$$

where $H = e \cos \theta - \frac{1}{2} e^2 \sin^2 \theta + \ldots; \quad e = \frac{e}{r_0}; \quad r_0$ is a shaft radius; $r_1$ is radius of the bearing bush covered with a metal melt; $e$ is eccentricity; $e$ is relative eccentricity; $\lambda f(\theta)$ is the bounded function at $\theta \in [0 + 2\pi]$ subject to definition.

Conditions of the motion of an infinite radial sliding bearing are considered under the following assumptions:

1. All the heat released in the lubricating film goes to the melting surface of the material of the bearing bush.
2. The radial velocity component is slightly less than its circumferential component.
3. The pressure is constant throughout the thickness of the lubricating film.
4. The flow of a Maxwell fluid is given by the equation:

$$\frac{\partial \nu_0'}{\partial r^2} = \frac{\tau'}{\mu'} + \frac{\nu'}{G'} \frac{\partial}{\partial \theta} = 0.$$  

Figure 1. Design scheme.

The dependence of the shear modulus and lubricant viscosity on the pressure is expressed by the dependences:

$$G' = G_0 e^{\alpha' p}, \quad \mu' = \mu_0 e^{\alpha' p'},$$

where $\mu_0$ is characteristic viscosity, $\mu'$ is coefficient of dynamic viscosity of the lubricant, $p'$ is hydrodynamic pressure in the lubricating layer, $\alpha'$ is a constant, $G_0$ is characteristic value of the shear modulus, $G'$ is shear modulus.

3.2. Initial equations and boundary conditions

As the initial equations, based on the assumptions made, a system of equations of the motion of a lubricant with viscoelastic properties (the Maxwell liquid) for the case of a "thin layer" with allowance for (1), the continuity equation, as well as the energy dissipation rate formula for determining the function due to the molten surface bearing bush covered with a low-melting metal melt, is taken:

$$\frac{\partial^2 \nu_0'}{\partial r^2} = \frac{1}{\mu'} \frac{d \nu_0'}{d \theta} + \frac{\Omega}{G'} \frac{d^2 \nu_0'}{d \theta^2} + \frac{\partial \nu_0'}{\partial r'} + \frac{1}{\alpha' p'} \frac{\partial \nu_0'}{\partial \theta} = 0, \quad \frac{d \lambda f(\theta)}{d \theta} = 2 \mu' \int_{\theta_0}^{\theta} \frac{d}{d \theta} \left( \frac{\partial \nu_0'}{\partial r'} \right)^2 d\theta,$$

where $L'$ is the specific heat of fusion per unit volume.

The boundary conditions in the case under consideration within terms $O(e^2)$ will be written as:

$$\nu_0' = 0, \quad \nu_0' = 0 \text{ at } r' = r_1 + \lambda f(\theta), \quad \nu = r_0 \Omega, \quad v_0' = -e \sin \theta \text{ at } r' = r_0 + e \cos \theta; \quad p'(\theta_1) = p'(\theta_2) = P_s'. $$

(5)
where \( \theta_1 \) and \( \theta_2 \) are respectively, the angular coordinates of the starting and finishing point of the free surface of the lubricant, \( \lambda' f(\theta) = \Phi(\theta) \).

When forming an analytical expression for hydrodynamic pressure, we transform the boundary conditions, assuming that the lubricant undergoes a shift at the entrance to the loaded zone. We believe that the lubricant enters the zone of hydrodynamic friction at full relaxation. Then the corresponding boundary conditions for the hydrodynamic pressure can be written in the form:

\[
p'(\theta_1) = C'(\theta_1) = 0.
\] (6)

Relations between dimensionless and dimensional variables are given in the form:

\[
p' = p \; p'; \; \mu' = \mu_0 \mu; \; \mu' = G_0 G; \; C' = C^* \; C; \; C^* = \frac{\mu_0 \Omega_0}{v}; \; \alpha' = \frac{\alpha}{\mu}; \; p' = \frac{\mu_0 \Omega_0^2}{2 \delta}.
\] (7)

Substituting (7) into the system of differential equations (4) and the boundary conditions (5) - (6), we obtain the following system of differential equations:

\[
\frac{\partial^2 v}{\partial r^2} + \beta e^{-ap} \frac{dp}{d\theta} + \beta e^{-ap} \frac{d^2 p}{d\theta^2} \cdot \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} = 0, \quad \frac{d \Phi(\theta)}{d \theta} = K \int_{-\phi(\theta)}^{\phi(\theta)} \left( \frac{\partial v}{\partial \theta} \right)^2 dr,
\] (8)

where \( \beta = \frac{\mu_0 \Omega_0}{\delta} \) is the Deborah number; \( u, v \) are components of the velocity vector of the lubricating medium; \( K = \frac{2 \mu_0 \Omega_0}{\delta L'} \) is the parameter characterizing the dissipation rate of mechanical energy; \( h(\theta) = 1 - \eta \cos \theta \) the lubricating layer thickness.

And boundary conditions

\[
\begin{align*}
u = 1, & \quad v = -\eta \sin \theta \text{ at } r = h(\theta); \quad v = 0, & \quad u = 0 \text{ at } r = -\Phi(\theta); \\
p(\theta_1) = p(\theta_2) = 0; & \quad p'(\theta_1) = 0; \quad C'(\theta_1) = 0,
\end{align*}
\] (9)

where \( \eta = \frac{e}{\delta}; \quad \eta' = \frac{\lambda'}{\delta}; \quad \Phi(\theta) = \eta f(\theta) \).

Let us introduce the designations, let \( Z = e^{-ap} \). Having differentiated both sides of the equation within terms \( O(\eta^p) \), the first equation of system (4) takes the form:

\[
\beta \frac{d^2 Z}{d \theta^2} + \frac{dZ}{d \theta} = -\alpha \left( \frac{\bar{c}_1}{h^2(\theta)} + \frac{\bar{c}_2}{h^3(\theta)} \right)
\] (10)

Then the system of equations (8) takes the following form:

\[
\beta \frac{d^2 Z}{d \theta^2} + \frac{dZ}{d \theta} = -\alpha \left( \frac{\bar{c}_1}{h^2(\theta)} + \frac{\bar{c}_2}{h^3(\theta)} \right); \quad \frac{\partial u}{\partial r} + \frac{\partial v}{\partial \theta} = 0; \quad \frac{Z d \Phi(\theta)}{d \theta} = K \int_0^1 \left( \frac{\partial v}{\partial \theta} \right)^2 dr
\] (11)

with the corresponding boundary conditions:

\[
\begin{align*}
u = 1, & \quad v = -\eta \sin \theta \text{ at } r = h(\theta); \quad v = 0, & \quad u = 0 \text{ at } r = -\Phi(\theta); & \quad Z(\theta_1) = Z(\theta_2) = 1; \quad Z'(0) = 0.
\end{align*}
\] (12)

Taking \( K \) as a small parameter, due to the melt and the rate of energy dissipation, we seek the function \( \Phi(\theta) \) as:

\[
\Phi(\theta) = -K \Phi_1(\theta) - K^2 \Phi_2(\theta) - K^3 \Phi_3(\theta) - \ldots = H.
\] (13)

Boundary conditions for dimensionless velocity components \( u \) and \( v \) on the contour \( r = -\Phi(\theta) \) can be written as:

\[
v(0 - H(\theta)) = v(0) - \left( \frac{\partial v}{\partial r} \right)_{r=0} \cdot H(\theta) - \left. \left( \frac{\partial^2 v}{\partial r^2} \right) \right|_{r=0} \cdot H^2(\theta) - \ldots = 0;
\]
\[ u(0 - H(\theta)) = u(0) - \left( \frac{\partial u}{\partial r} \right)_{r=0} \cdot H(0) - \left( \frac{\partial^2 u}{\partial r^2} \right)_{r=0} \cdot H^2(\theta) = 0. \] (14)

We seek the asymptotic solution of the system of differential equations (11) with allowance for the boundary conditions (12) and (14) in the form of series in powers of the small parameter \( K \):

\[ v = v_0(r, \theta) + K v_1(r, \theta) + K^2 v_2(r, \theta) + \ldots; \quad u = u_0(r, \theta) + K u_1(r, \theta) + K^2 u_2(r, \theta) + \ldots; \]

\[ \Phi(\theta) = -K \Phi_1(\theta) - K^2 \Phi_2(\theta) - K^3 \Phi_3(\theta) - \ldots; \quad Z = Z_0 + K Z_1(\theta) + K^2 Z_2(\theta) + K^3 Z_3(\theta) - \ldots. \] (15)

Performing the substitution (15) into the system of differential equations (11), taking into account the boundary conditions (12) and (14), we obtain the following equations:

- for zeroth approximation:
  \[ \frac{\partial^2 v_0}{\partial r^2} = \beta \frac{d^2 Z_0}{d\theta^2} + \frac{dZ_0}{d\theta} \cdot \frac{\partial u_0}{\partial r} + \frac{\partial v_0}{\partial \theta} = 0. \] (16)

With boundary conditions:

\[ u_0 = 1, \quad v_0 = -\eta \sin \theta \text{ at } r = 1 - \eta \cos \theta; \quad u_0 = 0, \quad v_0 = 0 \text{ at } r = 0; \]

\[ K \Phi_0(\theta) = K \Phi_0(\theta) \text{ at } r = 0; \quad Z_0(\theta_1) = Z_0(\theta_2) = 1; \quad Z_0^* (\theta_1) = 0. \] (17)

- for the first approximation:
  \[ \frac{\partial^2 v_1}{\partial r^2} = \beta \frac{d^2 Z_1}{d\theta^2} + \frac{dZ_1}{d\theta} \cdot \frac{\partial u_1}{\partial r} + \frac{\partial v_1}{\partial \theta} = 0; \quad Z_0 \frac{d\Phi_1(\theta)}{d\theta} = K \int_0^1 \left( \frac{\partial v_0}{\partial r} \right)^2 dr. \] (18)

With boundary conditions:

\[ \nu_1 = \left( \frac{\partial v_1}{\partial r} \right)_{r=0} \cdot \Phi_1(\theta); \quad u_1 = \left( \frac{\partial u_1}{\partial r} \right)_{r=0} \cdot \Phi_1(\theta); \]

\[ u_1 = 0, \quad v_1 = 0 \text{ at } r = 1 - \eta \cos \theta; \quad Z_1(\theta_1) = Z_1(\theta_2) = 0; \quad Z_1^* (\theta_1) = 0, \]

\[ K \Phi_1(\theta_1) = K \Phi_1(\theta_2) = \frac{\eta}{\alpha}. \] (19)

The exact self-similar solution of the problem for the zeroth approximation will be sought in the form:

\[ v_0 = \frac{\partial \psi_0}{\partial r} + V_0(r, \theta); \quad u_0 = -\frac{\partial \psi_0}{\partial \theta} + U_0(r, \theta); \quad \psi_0(r, \theta) = \overline{\psi}_0(\xi); \quad \xi = -\frac{r}{h(\theta)}; \]

\[ V_0(r, \theta) = \overline{V}_0(\xi); \quad U_0(r, \theta) = -\overline{u}_0(\xi) \cdot h'(\theta). \] (20)

Substituting (20) into the system of differential equations (16), taking into consideration the boundary conditions (17), we get the following system of differential equations:

\[ \overline{\psi}_0''(\xi) = \frac{\alpha}{2} \overline{C}_1; \quad \overline{v}_0''(\xi) = \frac{\alpha}{2} \overline{C}_1 + \overline{\psi}_0''(\xi) = 0; \quad \beta \frac{d^2 Z_0}{d\theta^2} + \frac{dZ_0}{d\theta} = -\alpha \left( \frac{\overline{C}_1}{h'(\theta)} + \frac{\overline{C}_2}{h'(\theta)} \right). \] (21)

And boundary conditions:

\[ \psi_0'(0) = 0, \quad \psi_0(1) = 0, \quad \overline{\psi}_0(1) = -\eta \sin \theta, \quad \overline{v}_0(0) = 0; \quad Z_0(\theta_1) = Z_0(\theta_2) = 1; \]

\[ \overline{\psi}_0(0) = 0, \quad \overline{v}_0(1) = 1, \quad \int_0^1 \overline{v}_0(\xi) d\xi. \] (22)

By direct integration we obtain:

\[ \overline{\psi}_0(\xi) = \frac{\overline{C}_1}{2} (\xi^2 - \xi); \quad \overline{v}_0(\xi) = \frac{\xi^3}{2} - \left\{ 1 + \frac{\overline{C}_1}{2} \right\} \xi + 1; \quad \overline{C}_1 = 6. \] (23)
3.3. Determination of hydrodynamic pressure

Solving the equation for hydrodynamic pressure
\[ \beta \frac{d^{2}Z_{0}}{d\theta^{2}} + \frac{dZ_{0}}{d\theta} = -\alpha \left( \frac{\tilde{C}_{1}}{h^{2}(\theta)} + \frac{\tilde{C}_{2}}{h^{4}(\theta)} \right) \]

taking into account the boundary condition \( Z_{0}(\theta_{1}) = Z_{0}(\theta_{2}) = 1; \) \( Z_{0}''(\theta) = 0 \) we obtain:
\[
Z_{0} = \sup_{b \in [\theta_{1}; \theta_{2}]} \left| 1 - \alpha \left[ (6 + \tilde{C}_{2})(0 - \theta_{1}) - \frac{6\eta(4 + \tilde{C}_{2})}{1 + \beta^{2}} \times \left( -\beta \sin \left( \theta_{1} + \theta_{2} \right) \sin \left( 0 - \theta_{2} \right) + \cos \left( \theta_{1} + \theta_{2} \right) \sin \left( 0 - \theta_{2} \right) \right) - \alpha C_{2} \left( e^{-\beta} - e^{-\beta'} \right) \right] \right|,
\]

where
\[
C_{2} = \frac{\alpha e^{-\beta}}{1 + \beta^{2} - e^{-\beta}} \left( \theta_{2} - \theta_{1} \right) \left( 0 - \theta_{1} \right) \left( 6 - \tilde{C}_{2} \right) \alpha \eta \left( 4 + \tilde{C}_{2} \right) \beta \sin \left( 0 - \theta_{2} \right) \sin \left( \theta_{1} + \theta_{2} \right) + \cos \left( 0 - \theta_{2} \right) \sin \left( \theta_{1} + \theta_{2} \right) \right). \]

\[
\tilde{C}_{2} = -\frac{1}{3\eta \alpha} \left( \beta \cos \theta_{1} - \sin \theta_{1} \right)
\]

In order to define \( \Phi_{1}(\theta) \) taking into account equation (23), we arrive at the following equation:
\[
\frac{d\Phi_{1}(\theta)}{d\theta} = \frac{h(\theta)}{Z_{0}} \int_{0}^{\theta} \left( \frac{\hat{\psi}_{0}(\xi)}{h^{2}(\theta)} + \frac{\hat{\psi}_{0}(\xi)}{h(\theta)} \right)^{2} d\xi.
\]

Integrating equation (25), we obtain:
\[
\Phi_{1}(\theta) = \frac{1}{Z_{0}} \left( \int_{0}^{\theta} \Delta_{1} d\theta + \int_{0}^{\theta} \Delta_{2} d\theta \right),
\]

where
\[
\Delta_{1} = \int_{0}^{1} \left( \psi_{0}(\xi) d\xi \right)^{2} + \tilde{C}_{2} \quad \Delta_{2} = \int_{0}^{1} 2\psi_{0}(\xi) \cdot \psi_{0}(\xi) d\xi = \tilde{C}_{2} \quad \Delta_{3} = \int_{0}^{1} \left( \psi_{0}(\xi) \right)^{2} d\xi = 4.
\]

Solving equations (26) with allowance for (27) and condition \( K\Phi_{1}(\theta_{1}) = K\tilde{\alpha} \), we obtain:
\[
\Phi_{1}(\theta) = \frac{1}{Z_{0}} \left( \tilde{C}_{1} + \frac{\tilde{C}_{2} + 4}{12} \right) \left( 0 - \theta_{0} \right) + \cos \left( \frac{\theta - \theta_{0}}{2} \right) \sin \left( \frac{\theta - \theta_{0}}{2} \right) \left( 9\eta \tilde{C}_{2} + 8\eta \right) + \tilde{\alpha} \right). \]

Then, for the first approximation, we obtain:
\[
\psi_{i} = \frac{\partial \psi_{i}}{\partial r} + \psi_{i}(r, \theta); \quad u_{i} = -\frac{\partial \psi_{i}}{\partial \theta} + U_{i}(r, \theta); \quad \psi_{i}(r, \theta) = \hat{\psi}_{i}(\xi); \quad \xi = \frac{r}{h(\theta)};
\]
\[
V_{i}(r, \theta) = \hat{v}(\xi); \quad U_{i}(r, \theta) = -\hat{u}(\xi) \cdot h'(\theta).
\]

Substituting (29) into the system of differential equations (18), taking into consideration the boundary conditions (19), we get the following system of differential equations:
\[
\hat{\psi}_{0}(\xi) = \tilde{C}_{2}; \quad \hat{\psi}_{0}(\xi) = \tilde{C}_{1}; \quad \hat{u}_{0}(\xi) + \xi \hat{v}_{0}(\xi) = 0; \quad \beta \frac{d^{2}Z_{0}}{d\theta^{2}} + \frac{dZ_{0}}{d\theta} = -\alpha \left( \frac{\tilde{C}_{1}}{h^{2}(\theta)} + \frac{\tilde{C}_{2}}{h^{4}(\theta)} \right);
\]

and boundary conditions:
\[
\psi_{0}'(0) = 0; \quad \psi_{0}(1) = 0; \quad \psi_{0}(1) = 0; \quad \psi_{0}(0) = 0; \quad \hat{v}_{0}(0) = M; \quad \int_{0}^{1} \hat{v}_{0}(\xi) d\xi = 0; \quad Z_{0}(\theta_{0}) = Z_{0}(\theta_{2}) = 0; \quad Z_{0}''(\theta_{1}) = 0.
\]

\[ (30) \]
From conditions \( Z_1(\theta) = Z_1(\theta) = 0 \). \( Z'_1(\theta) = 0 \), we obtain:

\[
Z_i = \alpha \left( 6M + \tilde{C}_2 \right) (\theta - \theta_i) - \frac{6\eta \alpha \left( 4M + \tilde{C}_2 \right)}{1 + \beta^2} \times \sin \left( \frac{\theta - \theta_i}{2} \right) \left( \beta \sin \left( \frac{\theta + \theta_i}{2} \right) + \cos \left( \frac{\theta + \theta_i}{2} \right) \right),
\]

where

\[
\tilde{C}_2 = \frac{-1}{\beta^2} C_2 e^{-\frac{\theta}{\beta}} - 4,
\]

\[
M = \sup_{\theta \in [0, \pi]} \left| \frac{\partial v_c}{\partial r} \right| \Phi_1(\theta) = \sup_{\theta \in [0, \pi]} \left| \frac{-\eta \sin \theta}{1 - \cos \theta} \right| - \frac{3\eta \beta^2 (1 + \tilde{C}_2)}{4(1 + \beta^2)} \times \left[ -2\sin \left( \frac{\theta - \theta_i}{2} \right) \cos \left( \frac{\theta + \theta_i}{2} \right) + \right.
\]

\[
+ 2 \cos \frac{(\theta - \theta_i)}{2} \cos \frac{(\theta + \theta_i)}{2} - 3 \left( 6 + \tilde{C}_2 \right)(1 - \cos \theta) \times \left[ \frac{\alpha}{2} \sin \left( \frac{\theta + \theta_i}{2} \right) - \frac{\alpha \beta}{2} \sin \left( \frac{\theta - \theta_i}{2} \right) + \cos \left( \frac{\theta - \theta_i}{2} \right) \right] + \]

\[
\left. + \sin \left( \frac{\theta - \theta_i}{2} \right) \cos \left( \frac{\theta + \theta_i}{2} \right) - \sin \left( \frac{\theta + \theta_i}{2} \right) \right] - \frac{\alpha \beta}{2} \sin \left( \frac{\theta - \theta_i}{2} \right) \cos \left( \frac{\theta + \theta_i}{2} \right) \times \left[ \frac{\alpha}{2} \sin \left( \frac{\theta + \theta_i}{2} \right) + \cos \left( \frac{\theta - \theta_i}{2} \right) \right] + \]

\[
\times \left[ \tilde{C}_2 + 4 \tilde{C}_2 \left( \theta - \theta_i \right) + \frac{9\eta \tilde{C}_2}{2} + 3 \eta \tilde{C}_2 \right] + \frac{1}{2} \tilde{C}_2. \]

Then for \( Z = Z_0 + KZ_1 \) we obtain the following expression:

\[
Z = 1 + \alpha (\theta - \theta_i) \left( 6 + \tilde{C}_2 + K \left( 6M + \tilde{C}_2 \right) \right) + \frac{6\eta \alpha}{1 + \beta^2} \left( \sin \left( \frac{\theta - \theta_i}{2} \right) - \cos \left( \frac{\theta + \theta_i}{2} \right) \right) \times \left( 4 + \tilde{C}_2 + K \left( 4M + \tilde{C}_2 \right) \right) - \alpha C_2 \left( e^{-\frac{\theta}{\beta}} - e^{\frac{\theta}{\beta}} \right)
\]

or

\[
Z = 1 + \alpha F(\theta),
\]

where

\[
F(\theta) = (\theta - \theta_i) \left[ 6 + \tilde{C}_2 + K \left( 6M + \tilde{C}_2 \right) \right] + \frac{6\eta \alpha}{1 + \beta^2} \left( \sin \left( \frac{\theta - \theta_i}{2} \right) - \cos \left( \frac{\theta + \theta_i}{2} \right) \right) \times \left( 4 + \tilde{C}_2 + K \left( 4M + \tilde{C}_2 \right) \right) - \alpha C_2 \left( e^{-\frac{\theta}{\beta}} - e^{\frac{\theta}{\beta}} \right)
\]

Then applying Taylor series expansions \( e^{-\frac{\theta}{\beta}} \), we obtain:

\[
1 - \alpha p + \frac{\alpha^2 r^2}{2} = 1 + \alpha F(\theta).
\]

Solving equation (34) within terms \( O(\alpha^3), F(\theta) \) for hydrodynamic pressure, we obtain:

\[
p = 1 + \alpha F(\theta).
\]

3.4. Results of study and their discussion

Let us now turn to the determination of the basic operating characteristics of the radial bearing. Taking into account (16), (18) and (35) for the component of the supporting force vector and the frictional force, we obtain:
\[ R_x = \frac{\mu \Omega_0^2}{\delta^2} \int_{\omega_1}^{\omega_2} p \cos \theta d\theta = \frac{\mu \Omega_0^3}{\delta^3} \times \left[ \sin \omega_2 - \sin \omega_1 + \alpha \left[ \omega_1 \left( \sin \omega_2 - \sin \omega_1 \right) - \left( \omega_2 \sin \omega_2 - \omega_1 \sin \omega_1 + \cos \omega_2 - \cos \omega_1 \right) \right] \right] \times \]
\[ \times \left[ 6 + C_2 + K \left( 6M + \tilde{C}_2 \right) \right] + \frac{6\eta}{\left(1 + \beta^2\right)^2} \left( 4 + C_2 + K \left( 4M + \tilde{C}_2 \right) \right) \times \left[ \sin \omega_1 \left( \sin \omega_2 - \sin \omega_1 \right) - \frac{1}{2} \left( \sin 2\omega_2 - \sin 2\omega_1 \right) - \cos \omega_1 \left( \sin \omega_2 - \sin \omega_1 \right) + \frac{1}{2} \left( \sin 2\omega_2 - \sin 2\omega_1 \right) \right] - C_2 \left( \sin \omega_2 - \sin \omega_1 \right) \left\{ e^{\frac{\omega_1}{\beta}} - e^{-\frac{\omega_1}{\beta}} \right\} \right] \times \]
\[ \left[ \sin \omega_2 - \sin \omega_1 \right] - \frac{3\eta \beta^2}{4 \left(1 + \beta^2\right)^2} \left[ \sin \omega_2 - \sin \omega_1 \right] - \frac{6\eta \beta \left( 4 + \tilde{C}_2 \right)}{2 \left(1 + \beta^2\right)^2} \times \left[ \sin \omega_2 - \sin \omega_1 \right] - \frac{\omega_2}{2\beta} \left( \omega_2 - \omega_1 - \eta \left( \sin \omega_2 - \sin \omega_1 \right) \right) \right] \]
\[ \eta \left( \cos \omega_2 - \cos \omega_1 \right) - \frac{e^{\frac{\omega_1}{\beta}} - e^{-\frac{\omega_1}{\beta}}}{2\beta} \left\{ \omega_2 - \omega_1 - \eta \left( \sin \omega_2 - \sin \omega_1 \right) \right\} \]  (36)

For the verification calculations, the following values are used based on the obtained theoretical models:

\[ \mu_0 = 0.085 \text{ Hs/m}^2; \eta = 0.3\ldots1 \text{ m; } n_0 = 0.0199995 \ldots 0.04939 \text{ m; } \Omega = 100 \ldots 1800 \text{ s}^{-1}; \quad p = 2 \text{ MPa; } \]
\[ \delta = 0.05 \cdot 10^{-3} \ldots 0.07 \cdot 10^{-3} \text{ m; } K = 0.00052 \ldots 0.0000022; \quad \alpha = 0 \ldots 1; \quad L' = 35.33 \ldots 38.1 \text{ H/m}^2. \]

Based on the results of numerical calculations, the graphs shown in Fig. 2–3 were made.

**Figure 2.** Dependence of components of the supporting force on parameter \( \alpha \), the lubricant characterizing dependence of viscosity on pressure, and on the parameter \( \theta_2 - \theta_1 \) characterizing the
extent of the loaded area of lubricant.

![Figure 3](image)

**Figure 3.** Dependence of a frictional force on parameter $\alpha$, the lubricant characterizing dependence of viscosity on pressure, and on the parameter $\beta$ characterizing the extent of the loaded area of lubricant.

In the experimental study, a sliding bearing with low-melting metal babbite alloy No. 83 and No. 88 is considered (table 1). According to the results of the experiments, the friction coefficient was determined, which allows to judge the presence of hydrodynamic friction mode both when the bearing is working on a lubricant having viscoelastic properties, and on the melt of a fusible babbite coating. The temperature regime and transition of hydrodynamic friction to boundary friction were also determined. Analysis of experimental studies shows that the melt of a fusion babbite coating affects the coefficient of friction 2-5 times more intense than the rheological properties of the lubricants used. The complex of experimental studies, which confirmed the reliability of the developed theoretical computational models and the data of their numerical analysis, in the considered range of design and operational parameters of tribosystems with low-melting metal melt, as a result of satisfaction convergence of theoretical and experimental results.

| Table 1 Results of experimental studies of babbitt covering No. 83 ,No. 88 |
|-------------------------------------------------|------------------|------------------|------------------|
| | Theoretical research | Experimental research |
| | A truly viscous lubricant | Viscoelastic lubricant | Viscoelastic lubricant |
| | low-melting metal coating | low-melting metal coating | low-melting metal coating |
| Coefficient of friction | B83 | B83 | B83 |
| No. | 1 | 0,0021 | 0,0018 | 0,0021 |
| | 2 | 0,0022 | 0,0020 | 0,0025 |
| | 3 | 0,0024 | 0,0022 | 0,0026 |
| | 4 | 0,0028 | 0,0027 | 0,0030 |
| | 5 | 0,0029 | 0,0029 | 0,0030 |

4. **Conclusion**
1. A refined design model of a radial sliding bearing operating under conditions of hydrodynamic lubrication with a melt of a low-melting coating is obtained, taking into consideration the dependence of viscosity on pressure under steady conditions of a sliding friction during inexact filling of a working gap with lubricant.

2. A significant contribution of the constructive parameter K due to the melt is shown. With an increase in the design parameter K, the friction coefficient decreases by 60%, and the bearing capacity increases by 16%.

Dependence of the coefficient of friction on the design parameter K, caused by the melt, is close to linear in the range of 0.0014–0.003.

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