Majorana fusion in interacting one-dimensional Kitaev chains

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We employ a time-dependent real-space local density-of-states method to study the movement and fusion of Majorana zero modes in the 1D interacting Kitaev model, based on the time evolution of many-body states. We study the dynamics and both fusion channels of Majoranas using time-dependent potentials, either Wall or Well, focusing on the local density-of-states and charge-density of fermions varying with time. For a Wall, i.e. repulsive strong potential, after fusion of Majoranas the electron (or hole) forms at $\omega = 0$, whereas for a Well, i.e. attractive deep potential, electron (or hole) forms at $\omega \sim -V$, where $V$ is the Coulomb repulsion. We also describe specific upper and lower limits on the Majorana movement needed to reduce non-adiabatic effects as well as to avoid poisoning due to decoherence, focusing on forming a full electron (or hole) after the fusion.

\textit{Introduction.} Majorana zero modes (MZMs) generate much interest because of potential applications in quantum information and computation \cite{1–3}. MZMs obey non-abelian exchange statistics and being topologically protected from local perturbations and disorder, they are of value as possible qubits \cite{4–6}. Signatures of MZMs are expected to develop in tunneling conductance experiments as zero bias peaks \cite{7–9}. The simplest setup to realize Majoranas are one-dimensional topological quantum wires, where MZMs develop at the two edges \cite{1,15}. For ferromagnetic atomic chains with strong spin-orbit coupling placed over a superconductor, MZMs were indeed reported at the edges in spatially and spectral resolved scanning tunneling experiments \cite{10,11}. In these nanowires, most theoretical work neglects repulsion among particles. However, Coulomb repulsion plays an important role in 1D MZMs because it suppresses the pairing-induced bulk gap and can destroy topological protection \cite{12,13}. Reciprocally, the Coulomb interaction may lower the Zeeman energy in nanowires and Majorana fermions could be accessed over larger regions of chemical potential (i.e. gates) \cite{14}.

In one dimension MZMs behave as Ising non-abelian anyons \cite{4,16} and obey the fusion rule \cite{17}: $\gamma \times \gamma = I + \psi$, which means two MZMs can fuse into a hole $I$, or into an electron $\psi$. The fusion process requires a slow adiabatic movement of Majoranas, which can be achieved by applying properly adjusted time-dependent local gates to the topological superconducting wire \cite{15}. The rapid progress in 1D quantum-wires with tunable local gates set-ups \cite{15,17,18}, provides a promising platform for creation, movement, and fusion of Majoranas \cite{19}.

Motivated by the experimental progress in nanowires \cite{20}, we employ a time-dependent real-space local density-of-states LDOS($\omega,j,t$) method to observe the movement and fusion rules of Majoranas in the Kitaev model with repulsive interaction of strength $V$. Compared to previous studies based on single particle states, here we use the exact-diagonalization method for the time evolution of the many-body states of interacting electrons in the 1D Kitaev model of 12 sites. With sequential application of time-dependent chemical potential gates (to generate either Walls or Wells depending on signs), we perform fusion of Majoranas in both channels (Electron : $\Psi$ and Hole : $I$). For Walls, after fusion we find that the electron (or hole) appears at energy $\omega = 0$, whereas for Wells the electron (or hole) appear at $\omega \sim -V$. We also discuss the minimum required switching time of the local gates to observe a nearly full electron (or hole) in the fusion process for several $V$’s, i.e. the conditions needed for adiabatic movement.

\textit{Model and Method.} We consider the time-dependent interacting 1D Kitaev model for spinless fermions with open boundary condition:

\begin{equation}
H(t) = -t_h \sum_{i=1}^{N-1} \left( \varepsilon_i c_i^\dagger c_{i+1} + H.c. \right) + V \sum_{i=1}^{N-1} (n_i n_{i+1}) + \Delta \sum_{i=1}^{N-1} \left( \varepsilon_i c_i^\dagger c_{i+1} + H.c. \right) + \sum_{i}^{N} \mu_i(t)n_i,
\end{equation}

where $n_i = c_i^\dagger c_i$ and $c_i^\dagger$ ($c_i$) is the fermionic creation (annihilation) operator, $t_h$ is the hopping amplitude, and $\Delta$ is the $p$-wave pairing strength. The time dependence is incorporated in the chemical potential $\mu_i(t)$ as:

\begin{equation}
\mu_i(t) = 0 \ (i < i_0), \quad \mu_i(t) = \mu \ (i > i_0),
\end{equation}

\begin{equation}
\mu_i(t) = \mu \frac{n \Delta t}{\tau} \ (i = i_0),
\end{equation}

where $1/\tau$ is the quenched rate, $\Delta t = 0.001$ is the small time step we used, and $n$ is the integer number of those steps, such that the on-site chemical potential $\mu_i(t)$ at $i = i_0$ increases approximately linearly from 0 to $\mu$ in a time $\tau$ (defined as the switching time of the local gate at site $i = i_0$). The sequential application of onsite gates $\mu_i(t)$ on the right half of the 1D chain, creates a moving Wall for $\mu > 0$ (or moving Well for $\mu < 0$), separating topological from non-topological regions at site $i = i_0$. Equating our number of sites with number of gates in a coarse-grained approach, this process leads to the slow movement of the
right edge Majorana zero mode $\langle \mu_j \rangle$ from the edge $i = N$ to site $i_0 = 1$ in a finite time $t = N_R \tau$, with $N_R$ the number of sites where the chemical potential reached its maximum value (here being $|\mu| = 12$) at time $t$ (Fig. 1).

To calculate the time-dependent local density-of-states (at zero temperature), we first time evolve the ground-state wave function $|\psi(t)\rangle$ up to time $t = N_R \tau$, using the time-dependent Hamiltonian $H(t)$ as:

$$|\psi(t)\rangle = \exp\left(-i \int_0^t H(s) ds\right) |\psi(0)\rangle,$$

where $T$ is the time ordering operator [21]. Then, we calculate the double-time Green function $G(t, t')$ [22], using the time-independent Hamiltonian $H_f = H(t = t_f)$ at time $t_f = N_R \tau$:

$$G_{\text{elec}}(t, t') = \langle \psi(t) | e^{i H_f t'} c_j \rho_j e^{-i H_f t} c_j \rho_j | \psi(t) \rangle.$$

The time-dependent $\text{LDOS}_{\text{elec}}(\omega, j, t)$ for electrons is the Fourier transform with respect to $t'$ of the local-Green function at site $j$:

$$\text{LDOS}_{\text{elec}}(\omega, j, t) = \frac{1}{\pi} \text{Im} \int_0^T dt' e^{i (\omega + in) t'} i G_{\text{elec}}(t, t'),$$

where we use $T = 70$, for the integration and broadening $\eta = 0.1$. Similarly we obtained the $\text{LDOS}_{\text{hole}}(\omega, j, t)$ for holes, using the Fourier transform of the Green function $G_{\text{hole}}(t, t') = \langle \psi(t) | c_j(t') c_j \rho_j e^{-i H_f t'} c_j \rho_j | \psi(t) \rangle$. The total local density-of-states at site $j$ is, thus, $\text{LDOS}(\omega, j, t) = \text{LDOS}_{\text{hole}}(\omega, j, t) + \text{LDOS}_{\text{elec}}(\omega, j, t)$.

**Slow movement of Majoranas.** For fusion or braiding of MZMs, it is required to transfer the Majoranas slowly (i.e. close to the adiabatic limit) [23, 24]. At $t = 0$ with $\mu_i = 0$ (for all sites), and $t_h = \Delta = 1$ for simplicity, the MZMs are localized at the two edges. Figure 2(a) shows the real-space local density-of-states $\text{LDOS}(\omega = 0, j, t = 0)$ vs. site $j$. For small or zero Coulomb repulsion $V$, these peaks are sharply localized at the end sites $(i = 1$ and 12), whereas for nonzero values of $V$ the $\omega = 0$ peaks are slightly delocalized over a few sites. In the inset, we plot $\text{LDOS}(\omega)$ at time $t = 0$ at the end site $j = 12$ and several values of $V$. We find a sharp peak at $\omega = 0$, signaling a MZM mode at the end site. Integrating in $\omega$ the $\text{LDOS}(\omega, j = 12)$ at $V = 0$ gives spectral weight 0.48, very close to the analytically expected value 0.5 [25]. Next, with sequential application of the time-dependent chemical potential $\mu_j(t)$, the right edge MZM $\langle \mu \rangle$ (site $j = 12$) is moved to the middle site $(j = 6)$ in a time $t = N_R \tau$ (i.e. $t/\tau = N_R = 6$ because we travel 6 sites). We study cases $\tau = 36, 60$ and 72 and interaction strengths $V = 0.0, 0.5$ and 1.0. In this case, $|\psi(t)\rangle$ remains close to the degenerate ground-state space (larger values of $V$ require a slower-rate of increase in the onsite $\mu_{i_0}(t)$). As shown in Fig. 2(b), for $\mu = 12$ (i.e. when creating a potential Wall), the $\text{LDOS}(\omega = 0, j, t)$ has peaks at site $j = 1$ and $j = 6$ at time $t/\tau = 6$, indicating that the slow transfer of MZM $\langle \mu \rangle$ from $j = 12$ to $j = 6$ occurred. At $t/\tau = 6$, we also found that the peak height of the electron- and hole-components of the local
density-of-states at \( j = 6 \) are almost equal (satisfying one of the characteristic features of MZM, \( \gamma_i = \gamma_i^t \)) \cite{26}. The average density \( \langle n(j, t) \rangle \) takes values close to zero for \( j \geq 7 \), while it is close to 0.5 for \( j \leq 6 \) (inset of Fig. 2(b)).

Interestingly, at \( \mu_R = -12 \) (i.e. when creating a potential Well), the effect of interaction increases. In the non-topological region (\( j \geq 7 \)), each site is occupied by one fermion, whereas in the topological region (\( j \leq 6 \)) the mean occupancy of fermions is close to 0.5 (for \( V = 0 \)). Without nonzero \( V \), to minimize the Coulomb interaction between fermions at the topological to non-topological boundary, the fermions near the boundary become inhomogeneously distributed (see inset of Fig. 2(c)). This also leads to the delocalization of MZM\(^{t(R)}\) over more sites as \( V \) increases (see Fig. 2(c)).

**Fusion of Majoranas.** For the fusion of Majoranas, we move the right edge MZM\(^{t(R)}\) all the way to the left end (site \( j = 1 \)) using sequential operations of \( \mu(t) \) in a time interval \( t = \tau \) that allows for adiabatic movement. At \( t = 0 \), for \( V = 0 \), \( \nu = \Delta = 1 \), with \( \mu_i = 0 \) (for all sites), the system has degenerate many-body ground states (\( |\psi_1 \rangle \) and \( |\psi_2 \rangle \)). We find at \( t = 0 \), both states have equal occupancy (0.5) of electron at each site. These degenerate ground states can be distinguished using non-local operators \cite{27, 28} \( \hat{O} = i a_N b_N = c_N^1 c_1^1 + c_N^1 c_1^1 + H.c. \), in the standard notation. For \( V = 0 \), \( \nu = \Delta = 1 \), at time \( t = 0 \), we find \( \langle \psi_1 | \hat{O} | \psi_1 \rangle = -1 \) and \( \langle \psi_2 | \hat{O} | \psi_2 \rangle = +1 \) (increasing \( V \) this reduces in absolute value because the Majorana no longer is perfectly localized at one site). At \( t = 0 \), we start the time evolution with those initial states \( |\psi_s \rangle \) (\( s = 1 \) or 2) up to \( t = \tau = 11 \), to confirm both fusion channels (Electron : \( \Psi \) and Hole : \( \bar{\Psi} \)).

For positive chemical potential \( \mu(t) > 0 \) (Wall) and the initial states \( |\psi_s \rangle \) satisfying \( \langle \psi_s | \hat{O} | \psi_s \rangle = 1 \), the electron \( LDOS_{elec} (\omega) \) at \( j = 1 \) shows a sharp peak close to \( \omega = 0 \), for \( V = 0 \), 0.5, and 1 (Fig. 3(a)). Meanwhile, the hole \( LDOS_{hole} (\omega) \) at \( j = 1 \), displays no peak. The time-dependent density \( \langle n(j, 1, t) \rangle \) at site \( j = 1 \) takes values one at \( \tau = 11 \), giving a clear indication of the formation of a single electron (spinless) at site \( j = 1 \) after Majoranas fusion. On the other hand, for the initial state \( |\psi_s \rangle \) satisfying \( \langle \psi_s | \hat{O} | \psi_s \rangle = -1 \), the hole \( LDOS_{hole} (\omega) \) displays a sharp peak close to \( \omega = 0 \), for \( V = 0 \), 0.5, and 1 (Fig. 3(d)). The electron \( LDOS_{elec} (\omega) \) has no peak at \( t = \tau = 11 \), see Fig. 3(e). As shown in Fig. 3(e) the density \( \langle n(j, 1, t) \rangle \) at \( j = 1 \) approaches zero, confirming hole formation at \( t = \tau = 11 \). Because the MZM spread over more than one site as \( V \) grows, density fluctuations occur at site \( j = 1 \) as compared to \( V = 0 \) (Figs. 3(c),f)).

For negative chemical potential \( \mu(t) < 0 \) (Well), and for the initial state \( |\psi_s \rangle \) satisfying \( \langle \psi_s | \hat{O} | \psi_s \rangle = -1 \), the electron \( LDOS_{elec} (\omega) \) shows a sharp peak close to \( \omega = -V \), for \( V = 0.5 \) and 1.0 at \( t = \tau = 11 \) (Fig. 4(a)).
The hole $LDOS_{\text{hole}}(\omega)$ at $j = 1$ shows no peaks at $t/\tau = 11$, Fig. 3(b). These results suggest the formation of an electron at $\omega = -V$ for $\mu < 0$ after Majorana fusion. $\langle n(j = 1, t) \rangle$ takes value one at $t/\tau = 11$, also indicating electron formation after Majorana fusion (Fig. 4(e)). On the other hand, for the initial state satisfying $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim 1$, the hole $LDOS_{\text{hole}}(\omega)$ at $j = 1$ displays a peak at $\omega = -V$ (Figs. 4(d,e)), while the electron $LDOS_{\text{elec}}(\omega)$ has no peak at $j = 1$ and $t/\tau = 11$. $\langle n(j = 1, t) \rangle$ approaches zero at $t/\tau = 11$ (Fig. 4(f)), confirming the formation of a hole after Majorana fusion.

The time-dependent density $\langle n(j = 1, t) \rangle$ at finite $V$ shows larger fluctuations compared to the case $\mu(t) > 0$, as the effect of interactions are quite strong for $\mu(t) < 0$ at the topological-non-topological boundary. For $\mu(t) < 0$ (Wall), at $t/\tau = 11$ and initial state with $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim -1$, each site is occupied with one fermion $|1, 1, 1, 1, \ldots, 1\rangle$ while for the initial states with $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim 1$, at $t/\tau = 11$ the sites are occupied as $|0, 0, 0, \ldots, 1\rangle$. The strong repulsive nearest-neighbor $V$ leads to a larger split in the ground state energies of the instantaneous Hamiltonian. For this reason after the fusion process, the electron or hole form close to $\omega = -V$. In the case of $\mu(t) > 0$ (Wall) at $t/\tau = 11$ the initial state with $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim -1$ at $t/\tau = 11$ each site is empty i.e. $|0, 0, 0, 0, \ldots, 0\rangle$, whereas the initial states with $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim 1$ at $t/\tau = 11$ the occupancy is $|1, 0, 0, 0, \ldots, 1\rangle$. Interactions are less important for fusion and, thus, there is not much splitting in ground state energies. For this reason even at finite $V$, in fusion the electron or hole form close to $\omega = 0$.

**Non-adiabatic Fusion of Majoranas.** In real Majorana nanowire setups, due to the quasi-particle "poisoning" [29, 30] it is necessary to move the Majoranas with sufficient speed (i.e. within the quasi-particle poisoning time) [17, 29–31]. Reciprocally, changing $\mu(t)$ employing a too fast rate (smaller $\tau$), would lead to a too fast movement of MZMs[8] generating non-adiabatic effects [32, 33], because the time-evolved state $|\psi(t)\rangle$ can have finite overlaps to higher excited states (above the gap) of the instantaneous Hamiltonian $H(t)$. Starting the time evolution with initial states $|\psi_s\rangle$, following $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim 1$ for $\mu(t) > 0$ (Wall), the electron $LDOS$ at finite time $t = N_R\tau$ can be written as:

$$LDOS_{\text{elec}}(\omega, t) = \frac{1}{\pi} \text{Im} \left( \sum_{m,n} \frac{\langle \Psi(t)|n\rangle\langle n|c_p^+|m\rangle\langle m|c_0|\Psi(t)\rangle}{e_n - e_m + \omega + i\eta} \right),$$

where $|m\rangle$ and $|n\rangle$ are eigenvectors of the instantaneous Hamiltonian $H(t)$ at time $t = 11\tau$, $\eta = 0.1$, and the rest of the notation is standard. For $V = 0$ ($\Delta = 1.0$), all many-body eigenstates of $H(t = 11\tau)$ develop in pairs and each states of a pair are degenerate [34, 35]. For smaller $\tau$, $|\psi(t)\rangle$ have finite overlaps with these higher $|m\rangle$ states. Due to the degenerate pairs of excited states, we observe spectral weight from many higher energy degenerate-pair states at $\omega = e_n - e_m = 0$ (with $|m-n| = 1$) in $LDOS_{\text{elec}}(\omega, t)$ (Eq. 6) and $LDOS_{\text{hole}}(\omega, t)$. For $LDOS_{\text{hole}}(\omega, t)$ the spectral weight only arise from higher energy degenerate-pair states $|m, n\rangle$.[34, 35]. For this reason, even for smaller $\tau$ (faster-motion of MZM[8]), we have well-defined sharp peaks at $\omega = 0$ in $LDOS_{\text{elec}}(\omega)$ and $LDOS_{\text{hole}}(\omega)$ at $V = 0$ (see Figs. 5(a,b)). Increasing $\tau$ (slower-motion of MZM[8]), the peak at $\omega = 0$ for $LDOS_{\text{elec}}(\omega)$ starts increasing, while the $LDOS_{\text{hole}}(\omega)$ peak value at $\omega = 0$ decreases. At $\tau = 36$, we obtain a sharp electron peak for $V = 0$ close to $\omega = 0$. For $\tau \geq 36$, the contribution in the $LDOS_{\text{elec}}(\omega)$ peak at $\omega = 0$ arises from $m, n = 1$ or 2 in Eq. 6. There is no peak for the hole $LDOS_{\text{hole}}(\omega)$ close to $\omega = 0$, confirming the formation of a full electron (for $\tau \geq 36$).

![FIG. 5. Non-adiabatic fusion of Majoranas for $\mu(t) > 0$ (Wall) at different switching rates $\tau$, starting with an initial state with $\langle \psi_s|\tilde{O}|\psi_s \rangle \sim +1$.](/content/5.png)

For $\tau < 60$, the contribution in the $LDOS_{\text{elec}}(\omega)$ peak at $\omega = 0$ arises from $m, n = 1$ or 2 in Eq. 6. Then, clearly the Coulomb repulsion reduces the topological protection by the reduction of the effective gap. Thus, a substantially slower movement of MZM[8] is needed (compared to $V = 0$) to reach the adiabatic limit.

In terms of SI units the switching time for $V = 0$ corresponds to $\tau h/\Delta \sim 0.13ns$ to 3.9ns (using $\tau = 36$, and $\Delta = 180\mu$eV or $\Delta = 6\mu$eV as in previous litera-
ture [19, 29]). This is the time required per gate, which, as example, was five in [19]. Independently, the quasiparticle “poisoning” time in nano-wire systems has been estimated in a broad range 10ns to 10ms [29, 30]. Because in the worse case of 3.9ns, five gates require a total time 19.5ns to move adiabatically the Majorana, and since this number is very close to the lower bound of poisoning time, we conclude that there should be a time range where moving Majoranas in chains can occur adiabatically before poisoning occurs for \( V = 0 \). As \( V \) increases, the situation deteriorates because at \( V = 2 \) we must use \( \tau = 60 \), but the new adiabatic time needed is only 33ns, still close to the experimental lower poisoning time.

**Conclusion** We performed real-time dynamics and fusion of Majoranas in the interacting 1D Kitaev chain model using sequential application of time-dependent chemical potentials (gates). We show that the movement model using sequential application of time-dependent Majoranas in the interacting 1D Kitaev chain with long hole) after the fusion. Due to advancements in the fabrication of Majorana nanowire devices [20, 36] with long quasiparticle poisoning time [29, 30], and considering our estimation for the times needed for adiabatic movement, we believe proper Majorana movement could be realized in realistic gate-control nanowire devices [20, 37].

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