MHD Natural Convection Ferrofluid Flow in Semi-Annulus Enclosures

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Abstract. This study examines the natural convection flow and the heat transfer of a water-based ferrofluid consisting Fe₃O₄ nanoparticles in a semi-annulus cavity with a circular outer and sinusoidal inner walls under the effect of a magnetic field produced by multiple nodal magnetic sources. The equations governing the ferrofluid flow are numerically solved using the dual reciprocity boundary element method. That is the equations are transformed into integral equations only on the boundary by using the fundamental solution of the Laplace equation and treating all other terms as nonhomogeneity through radial basis function approximation, which results in a discretized system of small size. The numerical computations are performed for several physical parameters namely, Hartmann, Rayleigh and magnetic numbers, solid volume fraction of nanoparticles, the number of undulation and the amplitude of sinusoidal wall. The obtained results show that increasing Rayleigh number, solid volume fraction and amplitude of sine waves enhance the average Nusselt number whereas it decreases with an increase in Hartmann number.

1. Introduction
In literature, many investigations are carried out numerically in the study of convective ferrofluid flow in smooth geometries under the effect of spatially variable magnetic field by using finite element method (FEM) in the works (e.g [1, 2]), finite difference method (FDM) in the works (e.g. [3, 4]), and control volume FEM (CVFEM) in the works (e.g. [5, 6]). However, it is also important to study the ferrofluid flow driven in more complex geometries since ferrofluid flows in cavities with irregular surfaces are widely encountered in many engineering applications. Sheikholeslami and Ganji [7] investigated the effect of a nodal magnetic source on ferrofluid flow and heat transfer in a semi-annulus enclosure with a sinusoidal wall using CVFEM. The fundamental problem of the biomagnetic fluid flow in a channel with stenosis in the presence of magnetic field was also investigated by using FDM in [8].

This study deals with the dual reciprocity boundary element method (DRBEM) solution of the two dimensional steady, laminar flow of viscous, incompressible, ferrofluid in a complex geometry with a sinusoidal inner wall under the effect of multiple nodal magnetic sources. For the mathematical formulation of the problem both magnetization and electrical conductivity effects on ferrofluid are taken into account and thence the combined principles of MHD and FHD are considered. The numerical simulations are carried out to investigate the effect of Rayleigh, Hartmann and magnetic numbers, solid volume fraction of nanoparticles, number of undulation and amplitude of sine waves on the flow and heat transfer characteristics. It is observed that adding nanoparticles to the base fluid and increasing the amplitude of sine waves enhance the average Nusselt number, which indicates that the heat transfer inside the enclosure can be controlled by the shape of the computational domain under various combinations of physical parameters.
2. Statement of The Problem and The Governing Equations

The equations governing the natural convection flow and the heat transfer in the presence of multiple nodal magnetic sources is considered. The problem is defined in a semi-annulus enclosure with a circular outer and sinusoidal inner walls. The shape of inner wall is determined by \( r = r_{in} + A \cos(n \xi) \) in which \( r_{in} \) is the radius of base circle, \( A \), \( n \) and \( \xi \) are the amplitude, number of undulations and rotation angle, respectively. The nodal magnetic sources are placed below and above the mid of the sinusoidal inner wall at the positions \((a_1, b_1)\) and \((a_2, b_2)\), respectively. The equations are consistent with the principles of FHD and MHD and the magnetic Reynolds number is assumed to be so small that the induced magnetic field is neglected. The ferrofluid has constant thermophysical properties and obeys the Boussinesq approximation. Thus, the equations governing the flow of Newtonian and incompressible fluid in the form of stream function \( \psi \), vorticity \( \omega \) and temperature \( \theta \) are given as:

\[
\nabla^2 \psi = -\omega
\]

\[
\nabla^2 \omega = \frac{\rho_{nf}/\rho_f}{Pr_f (\mu_{nf}/\mu_f)} \left( \frac{\partial \psi}{\partial y} \frac{\partial \omega}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} \right) - Mn_f \frac{\rho_{nf}/\rho_f}{\mu_{nf}/\mu_f} \left( \frac{\partial \psi}{\partial y} \right)^2 + \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial \xi} \right) - H_a^2 \frac{\rho_{nf}/\rho_f}{\mu_{nf}/\mu_f} \left( \frac{\partial H}{\partial y} \right)^2 + 2H_a^2 \frac{\partial H}{\partial y} \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial y} \frac{\partial H}{\partial x} + H_a^2 \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial \xi} \right) - Ra \frac{\rho_{nf}/\rho_f}{\mu_{nf}/\mu_f} \left( \frac{\partial \psi}{\partial y} \right)^2 + \frac{\partial \psi}{\partial x} \frac{\partial \omega}{\partial \xi} \right) \right)
\]

\[
\nabla^2 \theta = \frac{(\rho C_p)_{nf}}{k_{nf}} \frac{(\rho C_p)_f}{k_f} \left( \frac{\partial \psi}{\partial y} \frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial y} \right) - H_a^2 Ec \frac{(\rho C_p)_f}{k_f} \left( \frac{\partial \psi}{\partial y} \right)^2 + \frac{\partial \psi}{\partial x} \frac{\partial \theta}{\partial \xi} \right) \right)
\]

where \( H = \sqrt{(H_x)^2 + (H_y)^2} \), \( Ra_f = g \beta_f \ell^3 (T_c - T) / (\alpha_f \nu_f) \), \( Pr_f = \nu_f / \alpha_f \), \( H_a = \ell \mu_0 H_0 \sqrt{\sigma_f / \mu_f} \), \( \epsilon_1 = T_1 / \Delta T \), \( \epsilon_2 = T_c / \Delta T \), \( Ec = (\mu_f \nu_f) / [(\rho C_p)_f \Delta T \ell^2] \) and \( Mn_f = \mu_0 H_0^2 K\ell (H_0 - H_c)^2 / (\mu_f \alpha_f) \) are the strength of magnetic field \( \mathbf{H} = (H_x, H_y) \), Rayleigh number, Prandtl number, Hartmann number arising from MHD, temperature number, Eckert number and magnetic number arising from FHD for the fluid, respectively. The thermo-physical properties of the nanoparticles (Fe_3O_4) (indicated by \( nf \)) and base fluid (water, indicated by \( f \)) are taken as in the work of Sheikholeslami [7].

The corresponding boundary conditions for stream function and temperature are given as follows:

At the inner sinusoidal wall:
\[
\psi = 0, \quad \frac{\partial \theta}{\partial n} = -0.5
\]

At the outer circular wall:
\[
\psi = 0, \quad \theta = 0
\]

At the flat parts of bottom wall:
\[
\psi = 0, \quad \frac{\partial \theta}{\partial n} = 0
\]
The unknown boundary conditions for the vorticity will be obtained from the stream function equation (1) by using radial basis functions through the application of the dual reciprocity BEM.

3. Application of Dual Reciprocity Boundary Element Method

DRBEM aims to transform the governing differential equations into boundary integrals by using the fundamental solution of Laplace equation \( u^* = \frac{1}{2\pi} \ln(\frac{1}{r}) \). In this sense, when Equations (1)-(3) are weighted with \( u^* \) and the Divergence theorem is applied, one can obtain the following equation:

\[
c_i R_i + \int_{\Gamma} \left( q^* R - u^* \frac{\partial R}{\partial n} \right) d\Gamma = - \int_{\Omega} b_R u^* d\Omega
\]

(5)

where \( R \) is used for each unknown \( \psi, \omega \) and \( \theta \). Here \( q^* = \frac{\partial u^*}{\partial n} \), \( \Gamma \) is the boundary of the domain \( \Omega \) and the constant \( c_i \) depends only on the boundary geometry at the point \( i \) under consideration. All terms on the right hand side of Equations (1)-(3) denoted by \( b_R \), are treated as inhomogeneity and they are approximated by using the thin-plate spline radial basis functions \( f_j = r_j^m \ln r_j \) as,

\[
b_R \approx \sum_{j=1}^{N+L} \alpha_{Rj} f_j = \sum_{j=1}^{N+L} \alpha_{Rj} \nabla^2 \tilde{u}_j
\]

where the coefficients \( \alpha_{Rj} \) are undetermined constants and \( f_j \) are linked through the particular solutions \( \tilde{u}_j \) of Poisson equation \( \nabla^2 \tilde{u}_j = f_j \) [9]. Thus, Equation (5) takes the form

\[
c_i R_i + \int_{\Gamma} \left( q^* R - u^* \frac{\partial R}{\partial n} \right) d\Gamma = \sum_{j=1}^{N+L} \alpha_{Rj} \left[ c_i \tilde{u}_{ji} + \int_{\Gamma} (q^* \tilde{u}_j - u^* \tilde{q}_j) d\Gamma \right]
\]

(6)

which contains only the boundary integrals and \( \tilde{q} = \frac{\partial \tilde{u}_j}{\partial n} \). By discretizing the boundary with constant elements, the matrix-vector form of Equation (6) can be expressed as

\[
HR - G \frac{\partial R}{\partial n} = (H\hat{U} - G\hat{Q})F^{-1}b_R.
\]

(7)

The matrices \( \hat{U} \) and \( \hat{Q} \) are constructed by taking each of the vectors \( \tilde{u}_j \) and \( \tilde{q}_j \) as columns, respectively. The coordinate matrix \( F \) of size \((N+L) \times (N+L)\) consists of the vectors \( f_j \) as columns. The components of the DRBEM matrices \( G \) and \( H \) are obtained by taking the integral of the fundamental solution \( u^* \) and its normal derivative along each boundary elements \( \Gamma_j \), respectively. The DRBEM equations (7) are coupled so that they are solved iteratively. In each iteration, the required space derivatives of the unknowns \( \psi, \omega \) and \( \theta \), and also the unknown vorticity boundary conditions are obtained by using the matrix \( F \) [9].

4. Numerical Results

A computational analysis has been done to inspect the effect of magnetic field produced by two nodal magnetic sources (located below the inner wall at \( (a_1 = 0, b_1 = 1.15) \) and above the upper wall at \( (a_2 = 0, b_2 = 2.05) \)) on the flow and heat transfer. The considered parameters are \( Ra(=10^3, 10^4, 10^5) \), \( Ha(=0, 5, 10) \), \( Mn_f(=0, 100) \), solid volume fraction \( \phi(=0, 0.04, 0.08, 0.12) \), \( n(=0, 2, 4, 6) \) and \( A(=0, 0.1, 0.2, 0.3) \). In all calculations, the Prandtl number \( (Pr = 6.8) \), temperature number \( (\epsilon_1 = 0) \) and Eckert number \( (Ec = 10^{-6}) \) are kept fixed and results are presented by means of average Nusselt number along the sinusoidal inner wall.

Figure 1 presents the impact of Hartmann and Rayleigh numbers on \( \overline{Nu} \) when \( n = 4 \) and \( \phi = 0.04 \) for \( Mn_f = 0 \) and \( Mn_f = 100 \). It is observed that \( \overline{Nu} \) increases with an increase in \( Ra \) whereas it decreases as \( Ha \) increases. Further, there is no significant change in \( \overline{Nu} \) when the magnetic number increases.
\( \text{Mn}_f = 0 \)
\( \text{Mn}_f = 100 \)
\( \text{Mn}_f = 0 \)
\( \text{Mn}_f = 100 \)

\[ \text{Nu} \]

\( \text{Nu} \)
\( \text{Nu} \)
\( \text{Nu} \)

Figure 1: Effects of (a) \( Ha \) and (b) \( Ra \) on \( \text{Nu} \): \( n = 4, \phi = 0.04 \).

In Figure 2 (a), the variation of \( \text{Nu} \) with solid volume fraction \( \phi \) is displayed when \( Ra = 10^3, 10^4, 10^5 \) at \( Ha = 5, \text{Mn}_f = 100 \). It is observed that \( \text{Nu} \) increases with an increase in \( \phi \) at each \( Ra \) since adding nanoparticles to the fluid increases the energy transport through the flow and subsequently enhances the heat transfer rate. Similarly, an increase in \( Ra \) results in an increase in \( \text{Nu} \) at each \( \phi \). On the other hand, the variations of \( \text{Nu} \) with the number of undulation \( n \) when \( A = 0.2 \) and with the amplitude of inner wall \( A \) when \( n = 4 \) are shown in Figure 2 (b) and 2 (c), respectively, in the absence of magnetic field (\( Ha = \text{Mn}_f = 0 \)). In general \( \text{Nu} \) is an increasing function of \( n \) when \( n \geq 2 \) due to the increase in the length of the heated portion. However, as \( n \) increases from 0 to 2, \( \text{Nu} \) decreases for each \( Ra \) following the decrease in the length of the inner hot wall as \( n \) increases from 0 to 2. On the other hand, an increase in amplitude results in an increase in the length of the hot surface, which leads to an increase in the heat transfer rate with increasing \( \text{Nu} \) at each \( Ra \).

Figure 2: Variation of \( \text{Nu} \) with (a) \( \phi \), (b) \( n \) and (c) \( A \) at different \( Ra(= 10^3, 10^4, 10^5) \).

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