Variable-Speed-of-Light Cosmology from Brane World Scenario

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Abstract

We argue that the four-dimensional universe on the TeV brane of the Randall-Sundrum scenario takes the bimetric structure of Clayton and Moffat, with gravitons traveling faster than photons instead, while the radion varies with time. We show that such brane world bimetric model can thereby solve the flatness and the cosmological constant problems, provided the speed of a graviton decreases to the present day value rapidly enough. The resolution of other cosmological problems such as the horizon problem and the monopole problem requires supplementation by inflation, which may be achieved by the radion field provided the radion potential satisfies the slow-roll approximation.

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1 Introduction

Variable-Speed-of-Light (VSL) cosmological models were proposed [1, 2] as an alternative to inflation [3, 4, 5] for solving the cosmological problems of the Standard Big Bang (SBB) model. VSL models assume that the speed of light initially took a larger value and then decreased to the present value at an early time. Although the basic idea may be controversial and the theoretical foundation is not yet well-developed, VSL models are appealing in that not only the cosmological problems solved by the inflationary models but also the cosmological constant problem can be solved [1, 2, 6, 7, 8, 9, 10].

The claim in Ref. [11] of the experimental evidence for a time-varying fine structure constant $\alpha = e^2/(4\pi\hbar c)$ suggests that the speed of light may indeed vary with time. More recent observational evidence for a time-varying fine structure constant can be found, for example, in Refs. [13, 14, 15, 16, 17, 18, 19]. Furthermore, the recent works [20, 21, 22, 23, 24, 25, 26, 27] on the Lorentz violation in the brane world scenario hint at the possibility of naturally realizing VSL models within brane world scenario.

In the original models by Moffat [1] and by Albrecht and Magueijo [2], the speed of light (in the action), a fundamental constant of nature, is just assumed to vary with time during an early period of cosmic evolution and thereby the Lorentz symmetry becomes explicitly broken. It is therefore assumed in such models that there exists a preferred frame in which the laws of physics take standard forms (of the Lorentz invariant theories) with the constant speed of light $c$ replaced by a field $c = c(x^\mu)$, the so-called principle of minimal coupling. Although many cosmological problems can be solved through such approach, such radical modification of standard physics may be questionable. In the previous paper [28], we applied the approach of Refs. [1, 2, 6, 8] to the cosmological models of the Randall-Sundrum (RS) scenario [29, 30, 31] for the purpose of studying its consequences.

Clayton and Moffat [32, 33] proposed an ingenious dynamical mechanism by which the speed of light can vary with time in a diffeomorphism invariant manner and without explicitly breaking the Lorentz symmetry. (See also Ref. [34] for an independent development.) Their models therefore avoid the need to introduce a global preferred frame into spacetime. They introduce two metrics into the spacetime manifold, one being associated with gravitons and the other providing the geometry on which matter fields, including photons, propagate. These two metrics are nonconformally related by a scalar field (called a biscalar) or a vector field (called a bivector). The causal structures determined by the two metrics are therefore different, thereby photons propagate at different speed from that of gravitons.

In this paper, we argue that the bimetric mechanism by Clayton and Moffat can be naturally realized within the brane world scenarios. If we define the radion as the distance between the two branes, rather than in terms of the extra spatial component of the bulk metric, then the radion nonconformally relates the induced metric on the TeV brane, to which matter fields on the TeV brane are minimally coupled, to the

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2 Other possibility of time-varying electric charge $e$ was considered in Ref. [12].
gravity metric of the TeV brane in the same manner in which a biscalar does, while the radion varies with time. So, the radion can be regarded as a biscalar. Since the biscalar term in the induced metric comes with the opposite sign from that of Ref. 33, gravitons rather appear to propagate faster than photons on the TeV brane before the radion is stabilized, rather than photons traveling faster than gravitons, as was the case in Ref. 33. This difference allows the bimetric model resulting from the brane world scenario to solve the flatness and the cosmological constant problems (which the original bimetric models were not able to solve by itself), provided that the radion potential has an appropriate form giving rise to rapid enough decrease of the speed of gravitons to the present value. When the speed of a graviton does not decrease rapidly enough, the quasi flatness and the quasi cosmological constant problems are expected to be solved instead. On the other hand, since the speed of a photon remains constant with the natural choice of the time coordinate for the TeV brane observer (or decreases with the choice of the comoving time coordinate of the gravity metric), our bimetric model cannot by itself solve other cosmological problems, such as the horizon problem and the monopole problem, which the original VSL models can solve. To solve such problems, our brane world bimetric model has to be supplemented by inflation. Provided the radion potential has a region satisfying the slow-roll approximation, the radion may also be used as an inflaton. Although our bimetric model may turn out to be insufficient for solving the cosmological problems, it can at least provide with a brane world scenario explanation for a time-varying fine structure constant observed in our universe.

The paper is organized as follows. In section 2, we derive the effective Friedmann equations for cosmological model of the RS1 model and show that gravitons and photons travel at different speeds while the radion varies with time. In sections 3 and 4, we argue that our bimetric model can solve the flatness and the cosmological constant problems, provided the speed of gravitons decreases to the present value rapidly enough. In section 5, we comment on other cosmological problems.

2 Effective Friedmann Equations with Time-Varying Radion

In this section, we obtain the effective Friedmann equations on the TeV brane while the radion varies with time. The details on derivation can be found in Ref. 33 (see also Ref. 30), which we follow closely. However, we rederive the equations since we choose to define the radion differently from Ref. 33.

We define the extra spatial coordinate $y$ such that one of the branes, which we choose to be the Planck brane, is at rest at $y = 0$. In such coordinate system, the TeV brane moves along the $y$-direction before the radion gets stabilized. We choose to encode the distance between the two branes entirely with the location $y = R(t)$ of the TeV brane. With this convention, the radion is defined as the relative distance $R(t)$
between the two branes instead of the metric component along the extra dimension. Namely, the time evolution of the size of the extra space is described not by that of the extra spatial component of the bulk metric, but by that of the location \( y = R(t) \) of the TeV brane. This coordinate choice was also previously considered in Ref. [37] for the purpose of studying the radion dynamics in brane cosmology.

The general Ansatz for the bulk metric describing the expanding brane universe in our convention is given by

\[
\hat{g}_{MN} dx^M dx^N = -n^2(t, y) c^2 dt^2 + a^2(t, y) \gamma_{ij} dx^i dx^j + dy^2,
\]

where \( \gamma_{ij} \) is the metric for the maximally symmetric three-dimensional space given in the Cartesian and the spherical coordinates by

\[
\gamma_{ij} dx^i dx^j = \left(1 + \frac{k}{4} \delta_{mn} x^m x^n \right)^{-2} \delta_{ij} dx^i dx^j = \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2),
\]

with \( k = -1, 0, 1 \) for the three-dimensional space with the negative, zero and positive spatial curvature, respectively. The tensions of the Planck and the TeV branes are respectively denoted as \( \sigma_1 \) and \( \sigma_2 \). The Planck and the TeV branes contain matter fields with the Lagrangian densities \( L_1 \) and \( L_2 \), respectively. We assume that the radion potential \( V_r(R) \) has been generated through some mechanism such as the Goldberger-Wise mechanism [38, 39].

When there is no matter on the branes, the equations of motion are solved by the static brane solution [29, 30] with the metric components

\[
n(y) = a(y) = e^{-m_0 |y|}, \quad \gamma_{ij} = \delta_{ij}.
\]

The brane tensions take the fine-tuned values given by

\[
\sigma_1 = \frac{3c^4 m_0}{4\pi G_5} = -\sigma_2, \quad \Lambda = -\frac{3c^4 m_0^2}{4\pi G_5}.
\]

When matter fields are included on the branes, the brane universe undergoes cosmological evolution. Since the solution for the expanding brane universe should reduce to the static solution (3) in the limit of vanishing mass densities \( \rho_{1,2} \) and pressures \( \rho_{1,2} \) of the matter fields, it is reasonable to parametrize the solution in terms of the linear expansion around the static solution in the following way:

\[
n(t, y) = \Omega(y) (1 + \delta n(t, y)),
\]

\[
a(t, y) = a(t) \Omega(y) (1 + \delta a(t, y)),
\]

with the perturbations \( \delta n \) and \( \delta a \) assumed to be of the order of \( \rho_{1,2} \). Here, \( \Omega(y) \equiv e^{-m_0 |y|} \). To obtain the effective Friedmann equations describing the expanding brane universe as observed on the brane, we either take the averages of the Einstein’s equations as \( \int_0^{R(t)} dy \Omega^4 \mathcal{G}^M_N = \frac{8\pi G_5}{c^4} \int_0^{R(t)} dy \Omega^4 \mathcal{T}^M_N \) or compute the four-dimensional effective
action, with the linear perturbations (5) substituted \[35\]. In doing so, we keep only terms leading order in \(\tilde{\kappa}_1\), \(\tilde{\kappa}_2\).

Following the section 4.1 of Ref. \[35\], we drop the \(\delta n\) and \(\delta a\) perturbations when calculating the effective action, as they contribute at \(\mathcal{O}(\tilde{\kappa}_1^2, \tilde{\kappa}_2^2)\). Namely, we consider the following form for the bulk metric:

\[
n(y) = e^{-m_0|y|}, \quad a(t, y) = a(t)e^{-m_0|y|}.
\] (6)

The induced metrics on the Planck and the TeV branes are then respectively

\[
g_{1\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a^2(t)\gamma_{ij}dx^idx^j,
\] (7)

\[
g_{2\mu\nu}dx^\mu dx^\nu = -\left[\Omega_R^2c^2 - \dot{R}^2\right]dt^2 + a^2(t)\Omega_R^2\gamma_{ij}dx^idx^j,
\] (8)

where \(\Omega_R \equiv \Omega(R(t)) = e^{-m_RR(t)}\) and the overdot stands for derivative w.r.t. \(t\). The four-dimensional effective action therefore takes the form

\[
S_{\text{eff}} = \frac{3c^3}{8\pi G_5 m_0} \int dx^0 \sqrt{\gamma} a^3(1 - \Omega_R^2) \left[\frac{\ddot{a}}{a} + \frac{kc^2}{a^2}\right] + c \int dx^0 \sqrt{\gamma} a^3 V_r(R) + c \int dx^0 \sqrt{\gamma} a^3 \sqrt{\Omega^2_R - \dot{R}^2 / c^2 \Omega_R^3} L_2,
\] (9)

where \(x^0 = ct\) and \(\gamma \equiv \det \gamma_{ij}\). Noting that the Ricci scalar for the four-dimensional Robertson-Walker metric \(g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a(t)^2\gamma_{ij}dx^idx^j\) is given by

\[
\mathcal{R} = \frac{6}{c^2} \left[\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2 + \frac{kc^2}{a^2}\right],
\] (10)

we can put the above effective action in a conventional looking form:

\[
S_{\text{eff}} = \int dx^0 \sqrt{-g} \left[\frac{c^4}{16\pi G_{eff}} \mathcal{R} + V_r(R) + L_1 + \sqrt{\Omega_R^2 - \dot{R}^2 / c^2 \Omega_R^3} L_2\right],
\] (11)

where \(g = \det g_{\mu\nu}\) and \(G_{eff} \equiv m_0 G_5/(1 - \Omega_R^2)\). Therefore, we see that \(a(t)\) will satisfy the four-dimensional Friedmann equations with the contribution to the mass density and the pressure coming from \(L_{1,2}\).

To obtain the effective Friedmann equations satisfied by \(a(t)\) from the effective action \(\Box\), we have to define the mass densities \(\rho_{1,2}\) and the pressures \(\rho_{1,2}\) of the matter fields (described by the Lagrangian densities \(L_{1,2}\)) on the Planck and the TeV branes. The energy-momentum tensors for the matter fields on the branes are defined as

\[
T_{\mu\nu}^{1,2} = -\frac{2}{\sqrt{-g_{1,2}}} \frac{\delta \left(\sqrt{-g_{1,2}} L_{1,2}\right)}{\delta g_{1,2\mu\nu}},
\] (12)

where \(g_{1,2}\) are determinants of the induced metrics \(\Box\) on the Planck and the TeV branes. Note, we defined the energy-momentum tensors in terms of the induced metrics
$g_{1,2 \mu \nu}$, since the induced metrics are the physical metrics for the matter fields on the branes. Modeling the matter fields as perfect-fluid, we can put these energy-momentum tensors into the following standard forms:

$$T_{1,2}^{\mu \nu} = \left( \rho_{1,2} + \frac{\phi_{1,2}}{c^2} \right) U_{1,2}^{\mu} U_{1,2}^{\nu} + \varphi_{1,2} g_{1,2}^{\mu \nu}. \quad (13)$$

Since the four-velocities $U_{1,2}^{\mu}$ of the fluid on the Planck and the TeV branes are normalized as $g_{1,2 \mu \nu} U_{1,2}^{\mu} U_{1,2}^{\nu} = -c^2$, the nonzero components of $U_{1,2}^{\mu}$ in the comoving coordinates are given by

$$U_{1}^{t} = 1, \quad U_{2}^{t} = \frac{1}{\sqrt{\Omega_{R}^2 - \dot{R}^2/c^2}}. \quad (14)$$

So, the nonzero components of the energy-momentum tensors for the matter fields on the Planck and the TeV branes are respectively

$$T_{1}^{tt} = \rho_{1}, \quad T_{1}^{ij} = \frac{\rho_{1}}{a^2} \gamma^{ij}, \quad (15)$$

$$T_{2}^{tt} = \rho_{2} \frac{\Omega_{R}^2}{\Omega_{R}^2 - \dot{R}^2/c^2}, \quad T_{2}^{ij} = \frac{\rho_{2}}{a^2 \Omega_{R}^2} \gamma^{ij}. \quad (16)$$

For the purpose of putting the Friedmann equations into simple and suggestive forms, we reparametrize the mass density and the pressure of the matter fields on the Planck brane in the following way:

$$\rho_{1} = \frac{\tilde{\rho}_{1} \Omega_{R}}{\sqrt{\Omega_{R}^2 - \dot{R}^2/c^2}}, \quad \varphi_{1} = \frac{\tilde{\varphi}_{1} \sqrt{\Omega_{R}^2 - \dot{R}^2/c^2}}{\Omega_{R}}. \quad (17)$$

After the radion is stabilized, i.e., $\dot{R} = 0$, we have $\tilde{\rho}_{1} = \rho_{1}$ and $\tilde{\varphi}_{1} = \varphi_{1}$. Making use of these facts, we obtain the following effective Friedmann equation:

$$\left( \frac{\dot{a}}{a} \right)^2 + \frac{kc^2}{a^2} = \frac{8 \pi m_{0} G_{5}}{3(1 - \Omega_{R}^2)} \left[ \frac{\tilde{\rho}_{1} \Omega_{R} + \tilde{\varphi}_{2} \Omega_{R}^5}{\sqrt{\Omega_{R}^2 - \dot{R}^2/c^2}} + V_{r}/c^2 \right]. \quad (18)$$

Note, the physical metric for the matter fields on the TeV brane, on which we are assumed to live, is given by the induced metric $g_{5}$. So, the cosmic scale factor on the TeV brane is actually given by $a_{R} = a \Omega_{R}$. In terms of the physical cosmic scale factor $a_{R}$, the effective Friedmann equation (18) takes the form:

$$\left( \frac{\dot{a}_{R}}{a_{R}} \right)^2 + 2 m_{0} \ddot{a}_{R} a_{R}^{2} + \frac{k c^2 a_{R}^2}{a_{R}^2} = \frac{8 \pi m_{0} G_{5}}{3(1 - \Omega_{R}^2)} \left[ \frac{\tilde{\rho}_{1} \Omega_{R} + \tilde{\varphi}_{2} \Omega_{R}^5}{\sqrt{\Omega_{R}^2 - \dot{R}^2/c^2}} + V_{r}/c^2 \right]. \quad (19)$$

By defining the cosmic time $\tau$ of the TeV brane in the following way,

$$d\tau^2 \equiv \left[ \Omega_{R}^2 - \dot{R}^2/c^2 \right] dt^2, \quad (20)$$

$$\left( \frac{\dot{a}_{R}}{a_{R}} \right)^2 + 2 m_{0} \ddot{a}_{R} a_{R}^{2} + \frac{k c^2 a_{R}^2}{a_{R}^2} = \frac{8 \pi m_{0} G_{5}}{3(1 - \Omega_{R}^2)} \left[ \frac{\tilde{\rho}_{1} \Omega_{R} + \tilde{\varphi}_{2} \Omega_{R}^5}{\sqrt{\Omega_{R}^2 - \dot{R}^2/c^2}} + V_{r}/c^2 \right]. \quad (19)$$

By defining the cosmic time $\tau$ of the TeV brane in the following way,
we can bring the induced metric (8) on the TeV brane into the following standard comoving frame form for the Robertson-Walker metric:

\[ g_{\mu\nu}dx^\mu dx^\nu = -c^2d\tau^2 + a_R^2(\tau)\gamma_{ij}dx^i dx^j. \]  
(21)

In terms of the cosmic time \( \tau \), the effective Friedmann equation (19) takes the form:

\[
\left( \frac{\dot{a}_R}{a_R} \right)^2 + 2m_0\dot{R}\frac{\dot{a}_R}{a_R} + m_0^2\ddot{R}^2 + \frac{k(c^2 + \dot{R}^2)}{a_R^2} = \frac{8\pi m_0 G_5(1 + \dot{R}^2/c^2)^{3/2}}{3\Omega_R^2(1 - \Omega_R^2)} \left[ \bar{\rho}_1 + \phi_2 \Omega_R^4 + \frac{V_r}{c^2\sqrt{1 + \dot{R}^2/c^2}} \right],
\]
(22)

where the overdot now stands for derivative w.r.t. \( \tau \).

We have to keep in mind that the parameter \( c \) appearing in the conventional Friedmann equations (through the terms \( kc^2/a^2 \) and \( \phi/c^2 \)) corresponds to the speed of gravitons for the obvious reason that the metric, in which the same \( c \) appears, describes the geometry on which gravitons propagate. So, we see from Eq. (22) that the effective speed of gravitons \( c_4 \), as well as the effective four-dimensional Newton’s constant \( G_4 \), is observed to change with time on the TeV brane in the following way:

\[
c_4 = \sqrt{c^2 + \dot{R}^2}, \quad G_4 = \frac{m_0 G_5(1 + \dot{R}^2/c^2)^{3/2}}{\Omega_R^2(1 - \Omega_R^2)}. \]
(23)

The speed of a graviton is therefore observed on the TeV brane to take a larger value than the present day value while the radion varies with time. So, there will be a time lag between transmission of gravitational waves and that of photons along the null surfaces of their respective physical metrics, while the radion varies with time. After the radion is stabilized, the speed of a graviton decreases to the present value, coinciding with the speed of a photon. This is expected to be a generic feature of any brane world scenarios involving two branes.

In the Friedmann equation (22), the radion \( R \) is mixed with the massless graviton through the interaction term \( \sim R \dot{a}_R \). To separate the fields, we perform the conformal transformation on the metric:

\[
a_R(\tau) = e^{m_0(R_0 - R)}a_R, \quad d\bar{\tau} = e^{m_0(R_0 - R)}d\tau,
\]
(24)

where \( R_0 \) denotes the present day value of \( R \). The difference between the new frame and the original frame is important only for large departure of \( R \) from \( R_0 \). In this new frame, the effective Friedmann equation (22) on the TeV brane takes the form:

\[
\left( \frac{\dot{a}_R}{a_R} \right)^2 + \frac{k(c^2 + \dot{R}^2)}{a_R^2} = \frac{8\pi m_0 e^{2m_0 R_0} G_5(1 + \dot{R}^2/c^2)^{3/2}}{3(1 - \Omega_R^2)} \left[ \bar{\rho}_1 + \phi_2 \Omega_R^4 + \frac{V_r}{c^2\sqrt{1 + \dot{R}^2/c^2}} \right],
\]
(25)
where the overdot from now on stands for derivative w.r.t. \( \tau \) and the subscript \( \tau \) denotes derivative w.r.t. \( \tau \). So, in this new frame the effective speed of gravitons \( \bar{c}_4 \) remains the same but the effective Newton’s constant \( \bar{G}_4 \) is rescaled:

\[
\bar{c}_4 = \sqrt{c^2 + R^2}, \quad \bar{G}_4 = \frac{m_0 c^{2m_0} R_5 G_5 (1 + R^2/c^2)^{3/2}}{1 - \Omega_R^2},
\]

which is expected as properties of conformal transformation. The other Friedmann equation in this new frame is given by

\[
\ddot{a}_R = -\frac{4\pi \bar{G}_4}{3} \left[ \rho + \frac{3}{c^4} \phi - \frac{2V_r}{c^2 \sqrt{1 + R^2/c^2}} \right],
\]

where

\[
\rho \equiv \tilde{\rho}_1 + \rho_2 \Omega_R^4, \quad \phi \equiv \tilde{\phi}_1 + \phi_2 \Omega_R^4.
\]

From now on, for the purpose of simplifying the equations, we shall assume that the radion potential terms in the above effective Friedmann equations are absorbed into \( \rho \) and \( \phi \), i.e., \( \rho \to \rho + V \) and \( \phi \to \phi - V/c^4 \) \( (V \equiv V_r/(c^2 \sqrt{1 + R^2/c^2})) \), and the radion \( R \) varies with time as specified by the radion potential.

We have therefore shown that the speed of gravitons, as well as the Newton’s constant, is observed to vary with time on the TeV brane while the radion field varies with time. This fact can be understood as follows. Gravitons propagate in the bulk whose geometry is described by the bulk metric \( \hat{g}_{MN} \) given by Eq. (1). Therefore, on the TeV brane, gravitons are observed to propagate on the geometry described by

\[
g^{\text{grav}}_{\mu\nu} dx^\mu dx^\nu = \hat{g}_{\mu\nu}|_{y=R(t)} dx^\mu dx^\nu \approx -\Omega_R^2 c^2 dt^2 + a^2 \Omega_R^2 \gamma_{ij} dx^i dx^j.
\]

On the other hand, the physical metric to which matter fields, including photons, on the TeV brane are minimally coupled is given by the induced metric (8) on the TeV brane. These two metrics are nonconformally related when \( \dot{R} \neq 0 \) and coincide when \( \dot{R} = 0 \). Thereby, the four-dimensional universe on the TeV brane takes bimetric structure, proposed by Clayton and Moffat [32, 33], with the radion \( R \) identified as a biscalar. The four-velocity vector \( v^\nu \) of a photon, which is null w.r.t. the induced metric \( g_{2\mu\nu} \), i.e., \( g_{2\mu\nu} v^\mu v^\nu = 0 \), is timelike w.r.t. the gravitational metric, i.e., \( g^{\text{grav}}_{\mu\nu} v^\mu v^\nu = -(v^i \partial_i R)^2 < 0 \) when \( \partial_i R \neq 0 \). Gravitons therefore appear to propagate faster than photons on the TeV brane before the radion is stabilized. As pointed out in the above, the parameter \( c \) appearing in the conventional Friedmann equations corresponds to the speed of gravitons. On the other hand, \( c \) appearing in the Robertson-Walker metric [21] corresponds rather to the speed of a photon, because this metric describes geometry on which matter fields, including photons, on the TeV brane propagates.

\footnote{Note, for the bimetric models of Refs. [32, 33] the four-velocity vector of a photon is spacelike w.r.t. the gravitational metric, because of the difference in the sign in the biscalar term of the matter metric.}
This $c$ is the same $c$ that appears in the effective Friedmann equations \([23, 27]\). Since the four-velocity vectors $v^\mu_g$ and $v^\mu_p$ for a graviton and a photon are null w.r.t. $g_{\mu\nu}^{grav}$ and $g_{2\mu\nu}$, respectively, the ratio of the speed of a graviton $c_g$ to the speed of a photon $c_p$ is

$$\frac{c_g}{c_p} = \frac{\Omega_R}{\sqrt{\Omega_R^2 - (\partial_\tau R)^2/c^2}} = \sqrt{1 + (\partial_\tau R)^2/c^2},$$

(30)

which leads to the same expression for the effective speed of a graviton on the TeV brane given in Eqs. \((23, 26)\) upon identifying $c_p = c$. Indeed, the gravitational metric \((29)\) expressed in terms of the cosmic time $\tau$ for the matter metric \((21)\) takes the following comoving coordinate form with the time-varying speed of a graviton $c_g = c_4$ given by Eq. \((23)\):

$$g_{\mu\nu}^{grav} dx^\mu dx^\nu = -c_4^2(\tau) d\tau^2 + a_4^2(\tau) \gamma_{ij} dx^i dx^j.$$

(31)

Note that having a constant speed of a photon but a time-varying speed of a graviton is a frame-dependent statement. Since we choose to measure speeds with the time coordinate $\tau$ or $\tilde{\tau}$, for which the constant $c$ corresponds to the speed of a photon, we should regard the speed of a graviton as changing with time and taking larger value than the speed of a photon before the radion is stabilized. Had we chosen to use the time coordinate (defined by $d\tau^2 \equiv \Omega_R^2 dt^2$) which brings the gravity metric \((29)\) to the comoving frame form to measure speeds, the constant $c$ would have corresponded to the speed of a graviton, and the speed of a photon should be regarded as changing with time and taking smaller value than the speed of graviton before the radion is stabilized. Indeed, with a choice of such time coordinate the resulting effective Friedmann equations will have constant speed of graviton.

We have mapped the bimetric model resulting from the brane world scenario to a model with varying fundamental constants proposed in Refs. \([1, 2]\), in the sense that the parameter $c$ in the Friedmann equations that is assumed to take a large value at an early period in Refs. \([1, 2]\) is actually the speed of a graviton, not the speed of a photon. So, our bimetric model can resolve the flatness and the cosmological constant problems, which are shown to be resolved in Refs. \([1, 2]\) through rapid enough decreasing $c$ in the Friedmann equations. On the other hand, the resolution of some of cosmological problems, such as horizon problem, through the VSL models requires a larger value of the speed of a photon at an early period of cosmic evolution. So, strictly speaking, the bimetric VSL models cannot by itself solve all the cosmological problems that were originally claimed \([1, 2]\) to be solved by the VSL models with time-varying fundamental constants, since resolution of some of the problems requires faster speed of a graviton (namely, a larger $c$ in the Friedmann equations) and the others faster speed of a photon. Nevertheless, our bimetric model has an advantage over the previous bimetric models, in the sense that the only cosmological problem that the VSL models are claimed to solve but inflaton cannot is the cosmological constant problem, which requires larger value of speed of a graviton at an early time. When supplemented with inflation, our
bimetric model therefore has potential of solving all the cosmological problems that are originally claimed to be solved by VSL models.

In Ref. [40] it was pointed out that the VSL cosmology [1, 2, 3, 4, 8, 9] has limited ability to resolve the Planck problem and can make it worse, because a variable speed of light affects the Planck scale. Strictly speaking, \( c \) appearing in the Planck mass

\[
m_{pl} = \sqrt{\frac{\hbar c}{G}}
\]

the Planck length

\[
l_{pl} = \sqrt{\frac{\hbar G}{c^3}}
\]

and the Planck time

\[
t_{pl} = \sqrt{\frac{\hbar G}{c^5}}
\]

are speed of graviton, because these quantities have to do with the quantum gravity effect. In our bimetric model, also the speed of graviton varies with time, taking larger value than the (constant) speed of photon, while the radion varies with time. However, for our case, the effective four-dimensional Newton’s constant, given in Eq. (26), also varies with time, taking larger value than the present value. With the effective four-dimensional speed of graviton and Newton’s constant given in Eq. (26) substituted, the Planck mass \( m_{pl} \) takes smaller value than the present value, the Planck length \( l_{pl} \) remains constant and the decrease in the Planck time \( t_{pl} \) becomes less severe than the previous VSL cosmological models with varying fundamental constants, while the radion varies with time. So, first of all, our bimetric model does not make the hierarchy problem worse, unlike the previous VSL models with varying fundamental constants. However, the Planck density \( \sim m_{pl}/l_{pl}^3 \) rather decreases in our case, so our bimetric model by itself cannot resolve the Planck density problem.

From the effective Friedmann equations (25, 27) (of course, with the radion potential terms absorbed into \( \varrho \) and \( \varphi \)), we obtain the following generalized conservation equations:

\[
\dot{\varrho} + 3 \left( \varrho + \frac{\varphi}{c^4} \right) \frac{\dot{a}_R}{a_R} = -\varrho \frac{\ddot{G}_4}{G_4} + \frac{3k\varphi c_4}{4\pi G_4 a_R^2}.
\]

So, with the time-varying radion field \( R \), which causes the time-variation of the effective speed of gravitons and the effective Newton’s constant, mass is not conserved on the TeV brane, implying that matter is created in the brane universe. Even if the comoving time coordinate for the gravitational metric \( g_{\mu\nu}^{grav} \) were chosen, the mass appears to be not conserved on the TeV brane due to the time variation of the effective Newton’s constant. This result is in contrast with the bimetric models of Refs. [32, 33], for which the mass density of the ordinary matter is conserved. This difference may be attributed to the following reason. In Refs. [32, 33], it is assumed from the outset that the energy-momentum tensor for the ordinary matter satisfies the conservation law (w.r.t. the matter metric) and then shown that the conservation law is consistent with the field equations and the Bianchi identities. In our case, the conservation law for the matter fields on the branes is not compatible with the four-dimensional effective theory while the radion field varies with time. This is due to the fact that the four-dimensional effective Newton’s constant (and the speed of gravitons with a suitable choice of frame) in the four-dimensional effective action is time-dependent while the radion varies with time. As was elaborated in Refs. [1, 2], the energy-momentum tensor conservation law is incompatible with the Bianchi identities \( G_{\mu\nu} = 0 \) of the Einstein tensor when \( \kappa \sim G/c^4 \) is time-dependent.
3 The Flatness Problem

In this section, we examine whether the flatness problem can be resolved by the bimetric model resulting from the brane world cosmology with the time-varying radion field. The critical density \( \rho_c \), the mass density that gives rise to the flat universe \((k = 0)\) for a given \( \dot{a}_R / a_R \), of the brane universe is given by

\[
\rho_c = \frac{3}{8\pi G_4} \left( \frac{\dot{a}_R}{a_R} \right)^2.
\]  

(33)

We define the deviation of \( \rho \) from \( \rho_c \) as \( \epsilon \equiv \rho / \rho_c - 1 \). Then, the \( \epsilon < 0 \), \( \epsilon = 0 \), and \( \epsilon > 0 \) cases respectively correspond to the open \((k = -1)\), flat \((k = 0)\) and closed \((k = 1)\) universes. In order for the flatness problem to be resolved, \( \epsilon = 0 \) therefore has to be a stable attractor. The time derivative of \( \epsilon \) is

\[
\dot{\epsilon} = 2\epsilon \left( \frac{\dot{c}_4}{c_4} - \frac{\ddot{a}_R}{\dot{a}_R} \right),
\]  

(34)

where \( \dot{c}_4 / c_4 = e^{2m_0(R_0 - R)} R_t R_{\tau\tau} / (c^2 + R_t^2) \). So, the rapid enough decrease of the speed of a graviton \((26)\) to the present value \( c \) makes the terms in the bracket to be negative, causing \( \epsilon = 0 \) to be an attractor. This can be achieved by the radion potential with very steep region around the minimum, which allows rapid settling down of the radion to the minimum of the potential, causing sudden decrease of the speed of a graviton from a very large value to the present value.

We now comment on the important difference of our case from Refs. [32, 33, 41, 42, 43, 44]. It is claimed in Refs. [41, 42] that any bimetric implementation of cosmological models does not by itself solve the flatness problem. The claim in Refs. [41, 42] is based on the fact that the speed of gravitons in their bimetric cosmological models is assumed to be constant (while the speed of photons varies with time) and therefore the first term in the bracket of Eq. (34) vanishes. So, in their bimetric models, the flatness problem can be solved only when \( \ddot{a}_R > 0 \), i.e., when the universe inflates by violating the strong energy condition. Another important difference of the bimetric models considered in Refs. [32, 33, 43, 44] from our bimetric model is that, as we mentioned previously, in their case the speed of a photon varies with time, taking larger value than the speed of a graviton, or the speed of a graviton varies with time, taking smaller value than the speed of a photon, depending on the choice of the time coordinate, while the biscalar field varies with time. So, in their case the speed of a graviton increases to the present day value while the biscalar field settles down to the minimum of the biscalar potential, causing the terms in the bracket of Eq. (34) to be always positive unless the universe inflates rapidly enough. This fact is in accordance with the claim in Refs. [11, 12] that the bimetric models of Refs. [32, 33, 43, 44] can solve the flatness problem only when the strong energy condition is violated. To sum up, the reason

\[ \text{On the other hand, it is claimed in Refs. [13, 14] that the flatness problem can be resolved because} \]
why our bimetric cosmological model resulting from the brane world scenario can solve the flatness problem, contrary to the negative claim in Refs. [41, 42], is that in our case the speed of gravitons varies with time (unlike the models considered in [41, 42]) and becomes larger than the speed of photons (unlike the models considered in Refs. [32, 33, 43, 44]) while the biscalar or the radion varies with time.

4 The Cosmological Constant Problem

In this section, we examine whether our bimetric model can solve the cosmological constant problem. First of all, it is important to keep in mind that unlike the case of the conventional cosmology the mass density satisfying \( \rho = \frac{-\varphi}{\bar{c}^2} \) is not directly related to the cosmological constant of the brane universe but rather to the brane tension. However, if we assume that the brane tensions initially took the fine-tuned values which give rise to zero effective four-dimensional cosmological constant and thereby the effective Friedmann equations of the forms (25,27) without cosmological constant term (of course, except for the radion potential term), then we can regard the nonzero mass density \( \rho_{\delta\sigma} \) satisfying \( \rho_{\delta\sigma} = \frac{-\varphi_{\delta\sigma}}{\bar{c}^2} \) as being due to the correction \( \delta\sigma \) to the fine-tuned brane tensions which gives rise to nonzero effective four-dimensional cosmological constant in the brane universe.

We now express the total mass density of the brane universe as the sum \( \rho = \rho_m + \rho_{\delta\sigma} \) of the mass density \( \rho_m \) of the ordinary matter fields on the brane and \( \rho_{\delta\sigma} = \delta\sigma/\bar{c}^2 \). Then, the generalized conservation equation (32) is modified to

\[
\dot{\rho}_m + 3 \left( \rho_m + \frac{\rho_m}{\bar{c}^2} \right) \frac{\dot{a}_R}{a_R} = -\dot{\rho}_{\delta\sigma} - \frac{\dot{G}_4}{\bar{G}_4} + \frac{3k\bar{c}\dot{c}_4}{4\pi\bar{G}_4 a_R^2}. \tag{35}
\]

The time derivative of the ratio \( \epsilon_{\delta\sigma} = \frac{\rho_{\delta\sigma}}{\rho_m} \) of \( \rho_{\delta\sigma} \) to \( \rho_m \) is given by

\[
\dot{\epsilon}_{\delta\sigma} = \epsilon_{\delta\sigma} \left( \frac{\dot{\rho}_{\delta\sigma}}{\rho_{\delta\sigma}} - \frac{\dot{\rho}_m}{\rho_m} \right). \tag{36}
\]

Assuming that the brane matter satisfies the equation of state of the form \( \varphi_m = w \rho_m \bar{c}^2 \) with a constant \( w \), we obtain

\[
\frac{\dot{\rho}_{\delta\sigma}}{\rho_{\delta\sigma}} = \frac{-2\dot{c}_4}{\bar{c}_4}, \tag{37}
\]

\[
\frac{\dot{\rho}_m}{\rho_m} = -3\frac{\dot{a}_R}{a_R} (1 + w) - \frac{2\dot{c}_4 \rho_c}{\bar{c}_4 \rho_m} + \frac{2\dot{c}_4 \varphi + \rho_{\delta\sigma}}{\bar{c}_4 \rho_m} - \frac{\rho}{\rho_m \bar{G}_4}. \tag{38}
\]

of the inflationary behavior of the bimetric models not due to an inflaton potential but due to the fact that the light cone of the matter metric was initially wider than that of the gravity metric and then contracted. However, for our bimetric model, the light cone for the matter metric was initially narrower, so our bimetric model cannot solve the flatness problem in the manner described in Refs. [33, 14].
Therefore, Eq. (36) takes the form

\[
\dot{\epsilon} = \epsilon \left[ 3 \frac{\dot{a}_R}{a_R} (1 + w) + 2 \frac{\dot{\epsilon}}{\epsilon} + \left( \frac{\dot{G}_4}{G_4} - 4 \frac{\dot{C}_4}{C_4} \right) (1 + \epsilon) \right]. \tag{39}
\]

With the effective four-dimensional speed of a graviton and Newton’s constant given in Eq. (26), we have

\[
\frac{\dot{C}_4}{C_4} = e^{2m_0(R_0 - R)} \frac{R_\tau R_{\tau \tau}}{c^2 + R_\tau^2}, \tag{40}
\]

\[
\frac{\dot{G}_4}{G_4} = e^{2m_0(R_0 - R)} \frac{R_\tau [3(\Omega_R^2 - 1)R_{\tau \tau} + 2m_0\Omega_R^2(c^2 + R_\tau^2)]}{(\Omega_R^2 - 1)(c^2 + R_\tau^2)}. \tag{41}
\]

Making use of approximations \( \Omega_R \approx 0 \) and \( e^{2m_0(R_0 - R)} \approx 1 \) assumed in the RS models, we can put Eq. (33) into the form:

\[
\dot{\epsilon} \approx \epsilon \left[ 3 \frac{\dot{a}_R}{a_R} (1 + w) + (1 + \epsilon) \frac{1}{1 + \epsilon} \right]. \tag{42}
\]

So, as long as \( \epsilon < 1 \), the rapid enough decrease of the speed of a graviton to the present value will cause the correction \( \delta \sigma \) to the fine-tuned brane tensions to be driven to zero rapidly, thereby the brane tensions being pushed back to the fine-tuned values giving rise to zero cosmological constant in the brane universe. For the case when the speed of a graviton does not decease rapidly enough, \( \delta \sigma \) is expected to approach a small constant value (while the radion is being stabilized) that gives rise to a small effective four-dimensional cosmological constant, solving the quasi cosmological constant problem in the manner proposed in Ref. [45]. Our bimetric model thereby can be used to bring the quantum corrections to the fine-tuned brane tensions after the SUSY breaking under control, just like the mechanism for self-tuning brane tension [10, 11]. In this process, the correction \( \delta \sigma \) to the fine-tuned to brane tensions is converted into ordinary matter.

Once again it is essential in this mechanism that the speed of a graviton varies with time, taking the value larger than the present value, while the radion varies with time. The reason why the bimetric models in Refs. [32, 33, 41, 42, 43, 44] cannot solve the cosmological constant problem is that in their case the speed of a graviton either remains constant or takes smaller value than the present value while the biscalar varies with time.

5 Other Cosmological Problems

The cosmic microwave background data indicates that photons emitted from the opposite sides of the sky appear to be in thermal equilibrium, although according to the
SBB model those regions are out of causal contact at the time of last scattering, the so-called the horizon problem of the SBB model. The horizon problem is solved in inflationary models with sufficient amount of inflation, since the Hubble length measured in the comoving coordinates decreases during inflation. The VSL models proposed in Refs. [1, 2] also solve the horizon problem, since the particle horizon scale at the last time of scattering \( t^* \), given by

\[
\frac{d}{H(t^*)} = \int_0^{t^*} \frac{c_p(t)dt}{a(t)},
\]

can become larger than the coordinate distance to the last scattering, given by

\[
\frac{d}{H(t^*, t_0)} = \int_{t^*}^{t_0} \frac{c_p(t)dt}{a(t)},
\]

where \( t_0 \) denotes the present epoch, if the speed of a photon \( c_p \) were large enough during an initial period of cosmic evolution. We have to keep in mind that the horizon scale is defined in terms of the speed of a photon \( c_p \) instead of the speed of a graviton \( c_g \), since we are considering the distance over which photons travel and transport energy. Unfortunately, our bimetric model cannot solve the horizon problem through the mechanism proposed in Refs. [1, 2], because the speed of a photon \( c_p \) always remains constant with the choice of the time coordinate \( \tau \) or takes smaller value than the present day value \( c \) with the choice of the comoving time coordinate for the gravity metric \( g_{\mu \nu}^{grav} \), while the radion varies with time. Therefore, we have to incorporate inflation into our bimetric model in order to solve the horizon problem. Although an extra scalar field can be introduced as an inflaton, the radion may be used as an inflaton, if the radion potential has a region satisfying the slow-roll approximation.

Modern particle theory models predict unwanted relics such as magnetic monopoles, domain walls, moduli fields, etc, during very early stage of cosmological evolution. Since these relics get diluted more slowly than the (relativistic) ordinary matter as the universe expands, they become the dominant component of our present universe, which is in contraction with the observational data. The problem of unwanted relics is solved in inflationary models if the temperature of the universe during the reheating was not high enough to produce the relics, because the relics, as well as matter, get diluted to the negligible level compared to the inflationary potential during the rapid expansion of the inflationary stage. The VSL models can also solve the problem for the following reason. According to the Kibble mechanism, topological defect densities are inversely proportional to powers of the correlation length of the Higgs fields, which are generally bounded above by the Hubble distance \( c_p/H \). So, if topological defects were created before the speed of a photon \( c_p \) decreased to the present value, the upper bound on the densities of topological defects is weakened at the time of their production due to very large Hubble distance. Since the mechanism for resolving the problem of the unwanted relics in the VSL models involves a larger value of speed of a photon during an early period, once again our bimetric model cannot by itself solve the problem. The
resolution of the problem therefore necessitates incorporation of other mechanism such as inflation.

Finally, we comment on the explanation for the huge amount of entropy appears to be present in our universe. The inflationary model explains the entropy problem by assuming that the adiabaticity condition is violated during the inflation: While the universe supercools (due to the inflation) to some temperature \( T_s \) and then reheats to \( T_r \), the entropy density is increased by a factor of \((T_r/T_s)^3\). In the case of the VSL models, the large production of entropy can be achieved through creation of particles while the speed of light, as well as the Newton’s constant, changes to the present value, due to the nonconservation of the energy-momentum tensor (cf. see Refs. [8, 9]). In order for particles to be created while the radion varies with time, the RHS of the generalized conservation equation (32) has to be positive. Certainly, this can be achieve by the flat \((k = 0)\) and the open \((k = -1)\) universes. However, if the universe were closed \((k = 1)\) while the radion varies with time, particles would be take away, thereby entropy decreasing. The production of sufficient amount of entropy may require the supplementation of our bimetric model by inflation.

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