Rub impact response analysis of Jeffcott rotor for simply supported end conditions and fluid film bearings: a comparative study

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Abstract. In this paper, a comparative study on rub-impact responses of rotor for simply supported end conditions and fluid film bearings is presented. A 2 degree of freedom (dof) Jeffcott rotor model is used. Fluid film bearing is modelled using short bearing approach given by Ocvirk theory. Stiffness damping coefficients of the bearings are incorporated in the rotor equation of motion. The equation of motion is solved by using Runge-Kutta numerical integration method. Rub-impact responses are analysed by using Fast Fourier Transform (FFT) and orbit plots. It is found that with compare to simply supported end conditions 6X, 7X, 8X and 9X frequency components are absent in the Y direction response at operating speed \( \omega = 1.3\omega_n \) for the rotor supported on fluid film bearings. In addition, 2X and 3X frequency components are present in the Z direction response at operating speed \( \omega = 1.3\omega_n \).

1. Introduction
Fault identification and diagnosis in turbo machineries is a powerful technique. The technique is widely accepted in industry which protects machineries, saves investment and maintains safety standards. Rub impact failure of turbo machineries is common. It occurs due to heavy unbalance, bow in the shaft, misalignments etc. Therefore, identification of rub-impact in the rotor bearing system is essential.

A large number of research articles are available. Patel and Darpe [1] have studied the coast-up vibration response for rub detection. Two different conditions: heavy and light rub are considered. Mokhtar et al [2] have investigated bending torsional vibrations of rotor during rotor-stator rub using Lagrange multiplier method. Chen et al [3] have presented dynamic characteristics of rub-impact on rotor system with cylindrical shell using finite element simulation. Yang et al [4] have studied the dynamic characteristics of rotor casing system subjected to axial load and radial rub. Behzad and Alvandi [5] have investigated unbalance induced rub between rotor and compliant segmented stator. Hong et al [6] have presented nonlinear dynamic analysis using the complex nonlinear modes for rotor system with an additional constraint due to rub-impact. Chen et al [7] studied detection of rub-impact fault for rotor-stator systems using adoptive chirp mode decomposition method. Hu et al [8] presented the experimental test-based results on rotor rub and crack. The intrawave frequency modulation has been used. Chen et al [9] have studied the rub impact characteristics of drill string on casing. Yang et al [10] have presented imbalance rub pedestal looseness coupling fault on a geometrically nonlinear rotor system. Numerical and experimental validation has been given. Xu et al [11] presented a general electromagnetic excitation model for electrical machines considering the magnetic saturation and rub impact. Xianga et al [12] presented nonlinear couple dynamics of the asymmetric double disc rotor bearing system under rub impact and oil film forces.
From the literature it is found that most of the research on rub-impact rotor is based on the simply supported shaft end conditions and fluid film bearings. However, a comparative response analysis of rub-impact rotor for both the shaft end conditions is not been attempted.

In this paper, a comparative study of rub-impact responses for simply supported end conditions and fluid film bearings is presented. A 2-dof Jeffcott rotor model is used. Rub-impact rotor phenomenon is modelled using Beatty model. A fluid film bearing in the rotor is modelled using short bearing approach (Ocvirk theory). Rotor responses are obtained for both the conditions and comparison is presented.

2. Mathematical Model

A 2-dof Jeffcott rotor model is considered with two types of boundary conditions as shown in Figure 1 (a) and (b).

![Diagram](image)

Figure 1. (a) Simply supported end conditions (b) Fluid film bearings end support

The rotor equation of motion for simply supported shaft end conditions with rub-impact can be written as follows,

\[
m\ddot{Y} + CY + KY = F_Y + mu_{im}\omega^2 \cos(\omega t) - mg
\]

\[
m\ddot{Z} + CZ + KZ = F_Z + mu_{im}\omega^2 \sin(\omega t)
\]

\[(1)\]

In the above equation, \(m\) is the rotor mass, \(K\) is shaft stiffness, \(C\) is shaft damping coefficient, \(u_{im}\) is unbalance eccentricity, \(\omega\) is angular speed of rotor, \(Y\) and \(Z\) are vertical and horizontal degree of freedom of the rotor.

In equation (1) \(F_Y\) and \(F_Z\) are rubbing forces generated due to interactions between rotor and the stator. When rub-impact occurs, the radial impact force \(F_N\) and the tangential friction force \(F_T\) can be expressed as follows (Beatty [13]),

\[
F_N = \begin{cases} 
0 & \text{for } e_r < \delta \\
(e_r - \delta)K_S & \text{for } e_r \geq \delta
\end{cases}
\]

and \(F_T = \mu F_N\) \(\quad (2)\)

where, \(e_r = \sqrt{Y^2 + Z^2}\) is a radial displacement of the rotor, \(\delta\) is clearance between rotor and stator, \(\mu\) is coefficient of friction and \(K_S\) is stator stiffness. The forces \(F_Y\) and \(F_Z\) depend on the rotor response and \(F_N\) and \(F_T\) (see Figure 2(a)) can be written as,

\[
F_Y = -F_N \left[ \frac{Y}{e_r} - \psi_f F_r \frac{Z}{e_r} \right] \quad \text{and} \quad F_Z = -F_N \left[ \frac{Z}{e_r} + \psi_f F_r \frac{Y}{e_r} \right]
\]

\[(3)\]

In matrix form \(F_Y\) and \(F_Z\) can be written as

\[
\begin{bmatrix} F_Y \\ F_Z \end{bmatrix} = \begin{bmatrix} F_r, \mu \end{bmatrix} \begin{bmatrix} 1 & \psi_f \\ -\psi_f, \mu & 1 \end{bmatrix} \begin{bmatrix} Y \\ Z \end{bmatrix}
\]

\[
\text{for } e_r \geq \delta \quad \text{and} \quad \begin{bmatrix} F_Y \\ F_Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ for } e_r < \delta
\]

\[(4)\]

Where, \(\psi_f\) is the function to decide the direction of frictional forces.

\[
\psi_f = \begin{cases} 
-1 & \text{for } \omega R + v_r > 0 \\
0 & \text{for } \omega R + v_r = 0 \quad \text{and} \quad v_r = \dot{Z} \left( \frac{Y}{e_r} \right) - \dot{Y} \left( \frac{Z}{e_r} \right) \\
1 & \text{for } \omega R + v_r < 0
\end{cases}
\]

\[(5)\]
The rotor equation of motion for fluid film bearing with rub-impact can be written as follows

\[ m\ddot{Y} + C\dot{Y} + KY = F_y + F_{by} + \mu u_m \omega^2 \cos(\omega t) - mg \]
\[ m\ddot{Z} + C\dot{Z} + KZ = F_z + F_{bz} + \mu u_m \omega^2 \sin(\omega t) \]  

where, \( F_{by} \) and \( F_{bz} \) are bearing reaction forces and given as follows,

\[
\begin{bmatrix}
F_{by} \\
F_{bz}
\end{bmatrix} = 
\begin{bmatrix}
K_{YY} & K_{YZ} \\
K_{ZY} & K_{ZZ}
\end{bmatrix}
\begin{bmatrix}
Y \\
Z
\end{bmatrix} - 
\begin{bmatrix}
C_{YY} & C_{YZ} \\
C_{ZY} & C_{ZZ}
\end{bmatrix}
\begin{bmatrix}
\dot{Y} \\
\dot{Z}
\end{bmatrix}
\]  

In the above equation bearing reaction forces are function of stiffness and damping coefficients. The stiffness and damping coefficients are obtained by using short bearing Ocvirk theory [14]. The Sommerfeld boundary condition has been implemented. The Ocvirk number can be written as,

\[ S = \frac{D_o \omega v L}{8 f c_o ^2} \]  

where, \( D_o \) is bearing diameter, \( \omega \) is angular speed, \( v \) is oil viscosity, \( L \) is bearing length, \( f \) is the load on journal, \( c_0 \) is the radial clearance. In order to find the eccentricity ratio for given load and bearing characteristics Ocvirk have given following polynomial,

\[ e^8 - 4e^6 + [6 - S_o^2(16 - \pi^2)]e^4 - [4 + \pi^2 S_o^2]e^2 + 1 = 0 \]  

For the above equation, there will be only two real roots and only one is between 0 & 1. The stiffness and damping coefficients can be obtained using following equations

\[ K_{ZZ} = 4h_o \left[ \pi^2 (2 - e^2) + 16e^2 \right] \]
\[ K_{ZY} = h_o \pi \frac{\pi^2 (1 - e^2)^2 - 16e^4}{\epsilon \sqrt{1 - e^2}} \]
\[ K_{YX} = -h_o \pi \frac{\pi^2 (1 - e^2)(1 + 2e^2) + 32e^2(1 + e^2)}{\epsilon \sqrt{1 - e^2}} \]
\[ K_{YX} = 4h_o \left[ \pi^2 (1 + 2e^2) + \frac{32e^2(1 + e^2)}{(1 - e^2)} \right] \]
\[ C_{ZZ} = h_o \frac{2\pi \sqrt{1 - e^2}(\pi^2 (1 + 2e^2) - 16e^2)}{\epsilon} \]
\[ C_{YX} = C_{ZY} = -8h_o \left[ \pi^2 (1 + 2e^2) - 16e^2 \right] \]
\[ C_{yy} = h_o \frac{2\pi \left( \pi^2 (1-\epsilon^2)^2 + 48\epsilon^2 \right)}{\epsilon \sqrt{1-\epsilon^2}} \]

Where,
\[ h_o = \frac{1}{\left[ \pi^2 (1-\epsilon^2) + 16\epsilon^2 \right]^{3/2}} \] (10)

The dimensional stiffness and damping coefficients can be obtained as follows,
\[ K_e = \frac{f}{c_o} \begin{bmatrix} K_{zz} & K_{zv} \\ K_{yz} & K_{yy} \end{bmatrix} \quad \text{and} \quad C_e = \frac{f}{c_o \omega} \begin{bmatrix} C_{zz} & C_{zv} \\ C_{yz} & C_{yy} \end{bmatrix} \] (11)

| Parameters                     | Values          |
|--------------------------------|-----------------|
| Mass of Disc, m               | 4 kg            |
| Shaft diameter, d             | 0.02 m          |
| Disc diameter, D              | 0.2 m           |
| Shaft stiffness, K            | 2.275×10^5 N/m  |
| Unbalance Eccentricity, u_im  | 1.0×10^{-5} m   |
| Stator stiffness, K_s         | 60×10^6 N/m     |
| Damping coefficient, C        | 95.39 Ns/m      |
| Coefficient of friction, µ    | 0.2             |
| Bearing length, L             | 0.01 m          |
| Oil viscosity, ν              | 0.1 Ns/m²       |
| Bearing load, f               | 39.24 N         |
| Bearing radial clearance, c_o | 173.5×10^{-6} m |
| Rotor-Stator Clearance, δ     | 15×10^{-6} and 173.5×10^{-6} m |

3. Results and Discussion
Rub-impact response analysis of Jeffcott rotor for simply supported end conditions and fluid film bearing has been carried out. It is discussed in the following sections.

3.1. Rub-impact response for simply supported end conditions
As explained in section 2, equation (1) is solved by using Runge-Kutta fourth order numerical integration method and various responses are obtained. The rotor response is obtained at sub critical speed. Figure (3) shows responses obtained at 0.5ω_n and rotor-stator clearance δ=173.5μm. The bending natural frequency of the rotor ω_n is 37.95Hz. From the variation of rubbing forces F_Y and F_Z, it is confirmed that the rubbing occurs between rotor and stator. The response in Y direction shows strong nonlinear behavior. The FFT plot shows frequency components such as 0.5X, 1X, 2X, 2.5X, 3X, 4X, 4.5X, 5X, 5.5X and 6X. The 1X and 2X components are dominants. However, the response in Z direction is single periodic and shows 1X frequency component. The orbit plot shows complex shape.

The Rotor responses are also obtained at super critical speed. Figure (4) shows responses operating at 1.3ω_n and rotor-stator clearance δ=173.5μm. It is found that the rotor response is highly nonlinear in Y direction. As compared to the values obtained at 0.5ω_n, the amplitude of rubbing forces F_Y and F_Z are higher. This may be due to the amplification of unbalance force corresponding to operating speed ω = 1.3ω_n. The Y direction frequency components viz. 1X, 2X, 3X. 4X, 5X, 6X, 7X, 8X, 9X are observed. However, Z direction frequency component is found to be 1X. A triangular orbit shape is observed.
Figure 3. Responses for simply supported end conditions operating at $\omega=0.5\omega_n$ and rotor-stator clearance $\delta=173.5\mu$m (a) Vertical response, (b) Vertical response FFT, (c) Vertical rubbing force, (d) Horizontal response, (e) Horizontal response FFT, (f) Horizontal rubbing force, (g) Orbit plot.
Figure. 4. Responses for simply supported end conditions operating at $\omega=1.3\omega_n$ and rotor-stator clearance $\delta=173.5\mu$m (a) Vertical response, (b) Vertical response FFT, (c) Vertical rubbing force, (d) Horizontal response, (e) Horizontal response FFT, (f) Horizontal rubbing force, (g) Orbit plot.

3.2. Rub-impact response for fluid film bearings.
In order to find the rub-impact response of rotor with fluid film bearings, the stiffness and damping coefficients are required. Equation (8) and (9) are solved for given rotor bearing system. As explained in section 2, equation (6) is solved along with stiffness and damping coefficient using Runge-Kutta numerical integration method. First the rotor response is obtained at sub critical speed. Figure 5 shows responses obtained at $0.5\omega_n$ and rotor-stator clearance $\delta=173.5\mu$m. From the response of rubbing forces $F_Y$ and $F_Z$, it is confirmed that the rubbing is absent between rotor and stator. The horizontal and vertical responses show single periodic motion. The fluid film bearing reaction forces suppress the vibration amplitude and leads to no rub between rotor and stator. The orbit plot shows elongated ellipse.

In order to investigate the rub-impact phenomenon in the rotor supported on fluid film bearings, rotor-stator clearance $\delta$ is varied and simulation is performed. It is found that at $\delta=15\mu$m rub between rotor and stator occurs. Figure (6) shows responses obtained at operating speed $\omega = 0.5\omega_n$ with $\delta=15\mu$m.

From the response of rubbing forces $F_Y$ and $F_Z$, it is confirmed that the rubbing is present between rotor and stator. However, the horizontal and vertical responses show single periodic motion. The fluid film bearing reaction forces suppress the higher order frequency components. The orbit plot shows elongated elliptical shape.

The Rotor responses are also obtained at super critical speed. Figure (7) shows responses obtained at $\omega = 1.3\omega_n$ and rotor-stator clearance $\delta=173.5\mu$m. From the response of rubbing forces $F_Y$ and $F_Z$, it is confirmed that the rubbing is absent between rotor and stator. The horizontal and vertical responses show single periodic motion. The fluid film bearing reaction forces suppress the vibration amplitude.
and leads to no rub between rotor and stator. The orbit plot shows elongated ellipse. Similarly rub-impact responses of rotor operating at $\omega = 1.3\omega_n$ and rotor-stator clearance $\delta = 15\mu\text{m}$ are obtained (see Figure 8).

From the response of rubbing forces $F_Y$ and $F_Z$, it is confirmed that the rubbing is present between rotor and stator. The horizontal and vertical responses show nonlinear behavior. The Y direction response shows 1X, 2X, 3X, 4X and 5X frequency components. However, Z direction response shows 1X, 2X and 3X frequency components. The orbit plot shows banana shape. With compare to simply supported end conditions 6X, 7X, 8X and 9X frequency components are absent in the Y direction response at operating speed $\omega = 1.3\omega_n$. With compare to simply supported end conditions 2X and 3X frequency components are present in the Z direction response at operating speed $\omega = 1.3\omega_n$. At operating speed $\omega = 1.3\omega_n$, triangular and banana shape orbits are observed for simply supported and fluid film bearings end conditions respectively.

In this paper, two types of shaft end condition have been investigated for the rub impact fault. The two types of boundary conditions are simply supported end condition and fluid film bearing end condition. In the literature, simply supported shaft end condition is approximated as anti-friction bearing end condition. However fluid film bearing shaft end condition is more realistic. It is based on the solution of Reynolds equation. The fluid film bearing shaft end condition gives four stiffness and four damping coefficients which are used in the rotor equation of motion. The rub impact response amplitude of rotor is significantly influenced by the damping present in the fluid film bearing.
Figure. 5. Responses for fluid film bearings end conditions operating at $\omega = 0.5\omega_n$ and rotor-stator clearance $\delta = 173.5\mu m$ (a) Vertical response, (b) Vertical response FFT, (c) Vertical rubbing force, (d) Horizontal response, (e) Horizontal response FFT, (f) Horizontal rubbing force, (g) Orbit plot.

Figure. 6. Responses for fluid film bearings end conditions operating at $\omega = 0.5\omega_n$ and rotor-stator clearance $\delta = 15\mu m$ (a) Vertical response, (b) Vertical response FFT, (c) Vertical rubbing force, (d) Horizontal response, (e) Horizontal response FFT, (f) Horizontal rubbing force, (g) Orbit plot.
Figure 7. Responses for fluid film bearings end conditions operating at $\omega = 1.3\omega_n$ and rotor-stator clearance $\delta = 173.5\mu m$ (a) Vertical response, (b) Vertical response FFT, (c) Vertical rubbing force, (d) Horizontal response, (e) Horizontal response FFT, (f) Horizontal rubbing force, (g) Orbit plot.

Figure 8. Responses for fluid film bearings end conditions operating at $\omega = 1.3\omega_n$ and rotor-stator clearance $\delta = 15\mu m$ (a) Vertical response, (b) Vertical response FFT, (c) Vertical rubbing force, (d) Horizontal response, (e) Horizontal response FFT, (f) Horizontal rubbing force, (g) Orbit plot.
4. Conclusion
A comparative study of Jeffcott rotor with rub-impact for simply supported shaft end conditions and fluid film bearings is presented.

- For simply supported end conditions rub-impact shows 0.5X, 1X, 2X, 2.5X, 3X, 4X, 4.5X, 5X, 5.5X and 6X frequency components in Y direction when operating at $\omega = 0.5\omega_n$. At operating speed $\omega = 1.3\omega_n$ it shows 1X, 2X, 3X, 4X, 5X, 6X, 7X, 8X and 9X frequency components. For both the speed, it shows 1X frequency component in Z direction.
- For fluid film bearing, rub-impact shows 1X frequency component in both Y and Z direction when operating at $\omega = 0.5\omega_n$. At operating speed $\omega = 1.3\omega_n$ it shows 1X, 2X, 3X, 4X and 5X frequency components in Y direction and 1X, 2X and 3X components in Z direction.
- With compare to simply supported end conditions 6X, 7X, 8X and 9X frequency components are absent in the Y direction response at operating speed $\omega = 1.3\omega_n$.
- With compare to simply supported end conditions 2X and 3X frequency components are present in the Z direction response at operating speed $\omega = 1.3\omega_n$.
- At operating speed $\omega = 1.3\omega_n$, triangular and banana shape orbits are observed for simply supported and fluid film bearings end conditions respectively.
- The results can be useful for detecting the rub-impact in the rotor bearing system.

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