N-body simulations of stars escaping from the Orion nebula

Alessia Gualandris, 1,2⋆ Simon Portegies Zwart1,2 and Peter P. Eggleton3

1 Astronomical Institute 'Anton Pannekoek', University of Amsterdam, the Netherlands
2 Section Computational Science, University of Amsterdam, the Netherlands
3 Lawrence Livermore National Laboratory, Livermore, CA

Accepted by MNRAS.

ABSTRACT
We study the dynamical interaction in which the two single runaway stars AE Aurigæ and µ Columbæ and the binary ι Orionis acquired their unusually high space velocity. The two single runaways move in almost opposite directions with a velocity greater than 100 km s⁻¹ away from the Trapezium cluster. The star ι Orionis is an eccentric (e ≃ 0.8) binary moving with a velocity of about 10 km s⁻¹ at almost right angles with respect to the two single stars. The kinematic properties of the system suggest that a strong dynamical encounter occurred in the Trapezium cluster about 2.5 Myr ago. Curiously enough, the two binary components have similar spectral type but very different masses, indicating that their ages must be quite different. This observation leads to the hypothesis that an exchange interaction occurred in which an older star was swapped into the original ι Orionis binary. We test this hypothesis by a combination of numerical and theoretical techniques, using N-body simulations to constrain the dynamical encounter, binary evolution calculations to constrain the high orbital eccentricity of ι Orionis and stellar evolution calculations to constrain the age discrepancy of the two binary components. We find that an encounter between two low eccentricity (0.4 < e < 0.6) binaries with comparable binding energy, leading to an exchange and the ionization of the wider binary, provides a reasonable solution to this problem.

Key words: methods: N-body simulations - stars: binaries: spectroscopic - stars: individual: HD 34078, HD 37043, HD 38666

1 INTRODUCTION
OB runaways are a subgroup of spectral type O and B stars which are recognized by their unusually high velocity (≥ 30 km s⁻¹), often away from known star forming regions (Blaauw, 1961). Stone (1979) also included stars at large distance (up to several kpc) from the Galactic plane for which no parent association could be found. About 40 per cent of O stars and 5-10 per cent of B stars are runaways (Stone 1991), and only one in ten OB runaways has a known binary companion (Gies & Bolton 1986).

The peculiar velocities of about 50 OB runaways have been determined and the trajectories of ~ 20 can be traced back to a nearby association (Hoogerwerf et al. 2000, 2001), supporting the idea that these stars originated in stellar clusters from which they were later ejected with high velocities.

The two main mechanisms that have been proposed to explain the high velocities of OB runaways are:

1. ejection upon a supernova in a binary system (Zwicky 1957; Blaauw 1961);
2. ejection by a close multiple encounter in a star cluster (Poveda, Ruiz & Allen, 1967).

We refer to scenario 1 and 2 as supernova ejection and dynamical ejection, respectively.

In the supernova ejection scenario the OB runaway gets a high velocity from the explosion of the companion star. If the binary is dissociated by the supernova, the companion star is ejected with a velocity of the order of its orbital velocity while if the binary remains bound, the mass loss in the supernova induces a recoil velocity in the center of mass of the binary (Blaauw 1961; Hills 1980; Tauris & Takens 1998). This scenario predicts that about 30 per cent of the OB runaways should still be in a binary with a compact companion. Large natal kicks (up to 1000 km s⁻¹) are ob-
served in isolated pulsars (Lyne & Lorimer 1994; Cordes & Chernoff 1998), but are not sufficient to explain the low binarity fraction observed for OB runaways. Even if the formation of a neutron star is accompanied by a kick drawn from a Maxwellian distribution with a mean of 600 km s\(^{-1}\), about 30 per cent of the binaries remain bound (Portegies Zwart 2000). Selection effects may be important in preventing the observation of compact companions to the runaways (e.g. Colin et al. 1996), but the supernova ejection scenario seems insufficient to explain all the known OB runaways (Portegies Zwart 2000).

The supernova ejection scenario, however, satisfactorily explains the high velocities (\(\sim 50\) km s\(^{-1}\)) of some known high-mass X-ray binaries (van den Heuvel et al. 2000). In one of these, Vela X-1, the high space velocity is inferred from the morphology of a bow shock (Kaper et al. 1997) while the neutron star is visible as an X-ray pulsar. The only known example of a binary dissociated by a natal kick is given by the O9.5V star \(\zeta\) Oph and the pulsar PSR J1932+1059. The large rotational velocity (Penny 1996) and helium abundance (Herrero et al. 1992) of \(\zeta\) Oph indicate that a phase of mass transfer occurred, as expected for runaways ejected by a supernova explosion. Under this hypothesis, a natal kick of about 400 km s\(^{-1}\) can be inferred for the pulsar (Hoogerwerf, de Bruijne & de Zeeuw 2001).

An alternative scenario to the supernova ejection is the dynamical ejection scenario (Poveda, Ruiz & Allen 1967; Gies & Bolton 1986; Leonard 1990), in which close encounters between binaries and/or single stars in dense stellar regions can lead to the ejection of stars with high velocity. Young stellar clusters and OB associations in early evolutionary stages can have high stellar densities (\(\sim 10^5\) pc\(^{-3}\)) in their cores and therefore close encounters between single stars, binaries or higher order systems can be very frequent. These encounters can result in the ejection of stars with high velocities. O and B stars are more likely to participate in strong encounters than low mass stars because of mass segregation and gravitational focusing. As a consequence, the number of close encounters involving O and B type stars can be high despite their short lifetime, especially when the cluster is young and the binary fraction is high.

To help understanding the runaway mechanism we define the kinematical age \(\tau_{\text{kin}}\) as the time since the ejection from the parent association. This time can be derived integrating the trajectory of the runaway in the Galactic potential. At first order \(\tau_{\text{kin}} = d/v\), where \(d\) is the distance traveled from the cluster and \(v\) the velocity of the runaway. The kinematical age can help to identify the ejection mechanism. A runaway which is ejected by a supernova explosion has a kinematical age smaller than the age of the parent cluster because the primary star in the original binary evolved for a few million years before it exploded ejecting its companion. Dynamical ejection is likely to occur when the cluster is very young and therefore in this case the kinematical age is often similar to the age of the parent system.

An example of stars ejected by a dynamical encounter is provided by the single runaways AE Aurigae and \(\mu\) Columbae and the spectroscopic binary \(\iota\) Orionis, whose trajectories in the Galactic potential have been traced back in time and intersect about 2.5 Myr ago at the expected location of the Trapezium cluster (Hoogerwerf et al. 2001). The runaways AE Aurigae and \(\mu\) Columbae show no evidence of binary evolution, like a high rotational velocity or an increased helium abundance, and move in almost opposite directions away from the Trapezium cluster. The kinematical and evolutionary properties of the system suggest the hypothesis that the stars were involved in a 4-body encounter that ejected them with high velocity.

In this paper we describe a combined use of direct N-body simulations and stellar and binary evolution calculations to identify the type of encounter that ejected AE Aurigae, \(\mu\) Columbae and \(\iota\) Orionis. The paper is organized as follows: In section § 2 we describe the main observational properties of the stars and a possible evolutionary model for \(\iota\) Orionis, AE Aurigae and \(\mu\) Columbae and we propose a binary-binary encounter able to reproduce the current positions, velocities and physical properties of the four stars; in section § 3 we describe the code used to perform simulations of 4-body encounters and the classification of the outcomes; in section § 4 we describe the choice of initial conditions for the simulations, we present the main results obtained from the scattering experiments and we compare them with the observed properties of the stars while in section § 5 we present aspects of the evolution of the \(\iota\) Orionis binary after the encounter that can affect the model for the dynamical encounter.

## 2 THE RUNAWAY STARS AE AURIGAE, \(\mu\) COLUMBAE AND THE RUNAWAY BINARY \(\iota\) ORIONIS

### 2.1 The kinematics

The single runaway stars AE Aur and \(\mu\) Col are O type stars moving away from the Orion star forming region. Blaauw & Morgan (1954) recognized them as a remarkable pair of runaways having similar spectral types and moving in almost opposite directions with space velocities of about 100 km s\(^{-1}\). At a distance of about 250 pc from the Trapezium cluster, the kinematical age of both stars is about 2.5 Myr.

The same kinematical age and parent cluster for the two stars together with their relative straight angle of motion suggest a common origin in a dynamical encounter. Gies & Bolton (1986) advanced the hypothesis of a binary-binary interaction that ejected AE Aurigae and \(\mu\) Columbae and left \(\iota\) Orionis as the surviving binary. They noticed that \(\iota\) Ori is one of the most massive objects in the Orion nebula, has a high eccentric orbit and its component stars have relative orbital velocities similar to the space velocities of AE Aur and \(\mu\) Col.

Parallaxes and proper motions provided by Hipparcos together with radial velocity measurements (Turon et al. 1992) are shown in Table 1 and allow precise determinations of the Galactic trajectories of the binary and the single stars. The integration of the trajectories of AE Aur, \(\mu\) Col and \(\iota\) Ori (Hoogerwerf et al. 2001) in the Galactic potential shows that 2.5 \(\pm\) 0.2 Myr ago all the stars had a minimum separation of about 4 pc. The position and velocity of the parent cluster, derived from the orbit integration and the 4-body center of mass velocity, are consistent with those of the Trapezium cluster in the Orion nebula.

We here summarize the arguments in favor of a common
origin of the runaways AE Aur, µ Col and the υ Ori binary in the Trapezium cluster:

1. The trajectories of the single stars and the binary intersect 2.5 Myr ago in the Trapezium cluster.
2. The velocity of the center of mass of the 4-body system is compatible with the velocity of the Trapezium cluster.
3. The ages of all the four stars (see section 2.2 and 5.2) are consistent with the range in ages of the stars in the cluster (Palla & Stahler 1999).
4. The high stellar density (> 2 × 10^6 pc^-3, McCaughrean & Stauffer 1994) in the core of the Trapezium favors dynamical interactions.

### 2.2 The stellar evolution

The υ Orionis binary consists of a O9 III primary and a B0.8 III-IV secondary with a mass ratio of q = 0.5 (Stickland et al. 1987) or q = 0.57 (Marchenko et al. 2000). Using the single-star evolutionary tracks by Schaller et al. (1992), Baguñalo, Riddle, Gies 2001 estimate a difference in age of roughly a factor of two in the components and conclude that the system did not co-evolve. Therefore they propose that the system originated in a binary-binary encounter in which an exchange interaction occurred. The age difference cannot be explained by a phase of mass transfer because of the high eccentricity of the binary: mass transfer would have circularized the orbit of the binary. Exchange encounters are known to alter the eccentricity of the interacting binaries and could provide a natural explanation for the high eccentricity of the system. Observational data on υ Ori, AE Aur and µ Col are presented in Table 2.

Mason et al. (1998) list υ Ori with a speckle companion at a separation of about 0.1 ″. This could mean that υ Ori is the primary component of a hierarchical triple system. If the speckle companion is indeed bound to the binary, it has considerable consequences for the proposed origin of υ Ori, as we discuss in § 5.1. However, at the moment it is not clear whether the third component is bound to the binary.

A possible evolutionary scheme for the system is shown in Table 3. Starting from the currently observed parameters reported in Table 2, the stellar and binary evolution package SeBa (see Portegies Zwart & Verbunt 1996) is used to infer masses and radii of each single star at the moment of the encounter.

### 2.3 The encounter

We explore the hypothesis of a dynamical ejection for AE Aur, µ Col and υ Ori using numerical simulations of binary-binary encounters and try to infer the interaction that is needed to produce the observed properties of the stars. First of all, one binary must be ionized during the encounter in order to eject the two single stars. In addition, an exchange is needed to solve the age discrepancy in the υ Orionis binary. An example of a dynamical encounter producing a system like υ Ori, AE Aur and µ Col is shown in Figure 1. A binary consisting of υ Ori B and µ Col inter-
Figure 1. Example of a binary-binary encounter leading to the ionization of a binary and the exchange of one star. The two binaries are initially in the top right and bottom left corner and approach in the direction indicated by the arrows. The binary containing $\mu$ Columbae is unbound releasing $\mu$ Columbae. As a result, the $\iota$ Ori acts with a binary containing $\iota$ Ori A and AE Aur. In the encounter the softer binary is unbound releasing $\mu$ Columbae and $\iota$ Ori B, which is exchanged in the other binary and together with $\iota$ Ori A forms the $\iota$ Ori system while AE Aur and $\mu$ Columbae are ejected as singles.

acts with a binary containing $\iota$ Ori A and AE Aur. In the encounter the softer binary is unbound releasing $\mu$ Columbae and $\iota$ Ori B, which is exchanged in the other binary and together with $\iota$ Ori A forms the $\iota$ Ori system while AE Aur and $\mu$ Columbae are ejected in almost opposite directions.

3 BINARY-BINARY SCATTERINGS

3.1 The code and the setup of initial conditions

The numerical simulations described in this paper are carried out with the scatter package included in the STARLAB software environment (McMillan & Hut 1996; Portegies Zwart et al. 2001; http://www.ids.ias.edu/~starlab). An N-body encounter is resolved integrating the equations of motions of all the particles under the influence of their mutual gravitational forces using a fourth-order Hermite predictor-corrector scheme (Makino & Aarseth 1992).

We consider a target binary composed of stars of mass $M_{11}$ and $M_{12}$, semi-major axis $a_{11}$ and eccentricity $e_{11}$, and a projectile binary composed of stars of mass $M_{p1}$ and $M_{p2}$, semi-major axis $a_{p}$ and eccentricity $e_{p}$. Standard N-body units (Heggie & Mathieu, 1986) are used in which $M_{11} + M_{12} = 1$, $a_{11} = 1$ and $G = 1$. The relative velocity $v_{\infty}$ is given in units of the critical velocity $V_{c}$ for which the total energy of the system in the 4-body center of mass reference frame is zero:

$$V_{c} = \frac{G}{\mu} \left( \frac{M_{11}M_{12}}{a_{11}} + \frac{M_{p1}M_{p2}}{a_{p}} \right)^{1/2}.$$  

where $\mu = (M_{11} + M_{12})(M_{p1} + M_{p2})/M_{tot}$ is the reduced mass and $M_{tot}$ the total mass of the system. Additional parameters like the orbital phases and the relative orientation of the two binaries are randomly drawn from uniform distributions (Hut & Bahcall 1983). The initial eccentricities are drawn from a thermal distribution $P(e) = 2e$ (Heggie 1975).

The impact parameter $b$ is chosen between zero and a maximum value according to an equal probability distribution for $b^2$. The maximum value $b_{\text{max}}$ is determined automatically for each experiment. The code defines different cylindrical shells with the same cross-sectional area and performs an arbitrary number of scatterings in successive shells until all the encounters in the outermost shell result in a preservation of the binaries. The limiting impact parameter of the outermost shell defines the value of $b_{\text{max}}$ specific to the experiment under consideration.

Energy conservation in our simulations is typically better than one part in $10^{6}$. The error in energy conservation is checked in each experiment and if it exceeds $10^{-5}$ the encounter is rejected. The accuracy in the code is chosen in such a way that at most a few per cent of the encounters are rejected.

3.2 Classification system for binary-binary encounters

When the integration of the equations of motion is stopped (see appendix B for the stopping criteria) the encounters are classified. During the calculation we keep track of the nearest neighbors of all the stars and we compute the distances between them. If two stars approach each other within a distance smaller than the sum of their radii we classify the encounter as a collision.

Many outcomes are possible in binary-binary interactions: two bound systems can be left, or one binary and two single stars, or a triple and a single star, or four single stars. The encounters are classified as follows: (i) preservation or flyby if the two original binaries remain bound, even though strongly perturbed; (ii) exchange if one star is exchanged by the binaries and two new binaries are formed; (iii) ionization if one binary is unbound leaving a binary and two single stars recoiling to infinity; (iv) exchange-ionization if one binary is unbound and one star is exchanged in the other binary; (v) triple if one binary is unbound and one of the stars is captured by the other binary in a bound stable orbit while the other star is ejected as single; (vi) total ionization if both binaries are unbound and four single stars recede to infinity.

In Figure 2 we present different possible outcomes (and corresponding Feynman diagrams) of binary-binary interactions.

3.3 Testing the code

In order to test our scattering package, we performed a series of binary-binary scattering experiments and compared the results with those of Mikkola (1983). He used a hybrid code containing three different integration methods: the hierarchy method, where one or more of the relative motions in the system are integrated as a perturbed keplerian orbit, direct integration and Heggie’s general N-body regulariza-
Figure 2. Examples of different outcomes in binary-binary scattering events and the corresponding Feynman diagrams. In the diagrams the paths of two stars are placed close together when they are bound, and the wavy lines represent gravitational perturbations.

We also computed the cumulative cross-section for close approach during the encounter between two identical circular equal-mass binaries and compared them with those of Bacon, Sigurdsson & Davies (1996) and Cheung et al. (in preparation). The comparison with the most recent results by Bacon et al. and with the results by Cheung et al. indicates that the cross-sections are consistent within the numerical uncertainties.

To test our 4-body scattering package against the 3-body package we reproduced the results by Rasio, McMillan and Hut (1995) regarding the formation of the triple system PSR B1620-26 in M4. Branching ratios and cross-sections obtained simulating binary-binary encounters are well con-
sistent with the ones obtained in the case binary-single star encounters.

4 NUMERICAL SIMULATIONS

4.1 Initial conditions

We consider interactions between a target binary composed by AE Aur and the primary component of ι Ori (ι Ori A) and a projectile binary composed by μ Col and the secondary component of ι Ori (ι Ori B). The values of masses, radii and ages used as initial conditions for the simulations are reported in table 3. After the two binaries and their relative orbits are selected we integrate the equations of motion until the encounter is resolved (see appendix B). The relative velocity at infinity between the centers of mass of the two binaries is set equal to the mean dispersion velocity in the Trapezium cluster (2 km s\(^{-1}\), Herbig & Tendrup 1986). Conservation of total energy and momentum in the 4-body system center of mass frame is applied to derive the binding energies of the initial binaries. Only the sum of the two binding energies can be derived from the conservation laws, so the ratio between the two remains as a free parameter: \(\alpha = E_b/E_p\).

For each value of \(\alpha\) between 0.1 and 10 we perform 2000 scattering experiments and the resulting branching ratios and cross-sections are presented in Figure 3. Most of the encounters lead to the disruption of one binary, possibly accompanied by an exchange or a capture in a stable triple. Only about 10 per cent of the encounters result in an ionization, one of the component stars can be captured by the target binary which contains the most massive star of the system. The distribution of velocities relative to the center of mass of the 4-body system for ι Ori, AE Aur and μ Col (solid line) compared with the observed values (dashed line) for \(\alpha = 2\).

Table 4. Comparison of the frequencies of different types of outcomes with the results by Mikkola (1983). For the scattering experiments we consider binaries with four equal-mass stars and equal semi-major axes.

| \(T_{\infty}\) | \(v_{\infty}\) | unresolved | flyby | stable triple | single ionization | total ionization |
|----------|---------|------------|-------|---------------|-----------------|-----------------|
|          |         | Starlab    | Mikkola | Starlab       | Mikkola         | Starlab         |
| 0.10     | 0.316   | 7          | 0      | 358           | 350             | 38              |
| 0.25     | 0.5     | 6          | 0      | 387           | 409             | 28              |
| 0.50     | 0.707   | 0          | 0      | 430           | 436             | 5               |
| 0.75     | 0.866   | 2          | 0      | 452           | 451             | 2               |
| 1.00     | 1.0     | 1          | 0      | 450           | 442             | 0               |
| 1.50     | 1.225   | 0          | 0      | 466           | 452             | 0               |

4.2 The point particle limit

We first analyse the properties of the stars resulting from the simulations in the approximate case of point particles. The final velocity distributions for ι Ori, AE Aur and μ Col are shown in Figure 4. The velocities are relative to the center of mass of the 4-body system and the observed values (see Table 1) are presented as dashed lines. Due to momentum conservation the two single stars recoil with a velocity \(\sim 3\) times higher than the binary. Typical velocities for the stars range from 30 to 100 km s\(^{-1}\), with an average of the order of their orbital velocities (\(\sim 60 - 70\) km s\(^{-1}\)) in the initial binaries. The velocity of the binary has a peak at about 25 km s\(^{-1}\), close to the observed velocity 18 ± 1 km s\(^{-1}\).

We observe a correlation between the velocity of the binary and that of the single stars, though with a large scatter, while there is no apparent correlation between the space velocities of the two single stars (see Figure 5). This might be due to the many degrees of freedom present in 4-body scatterings.

In Figure 6 we show the distribution of the semi-major axis of the binary after the encounter. The observed value (Stickland 1987; Marchenko et al. 2000) is consistent with the most probable value in the distribution.

The distribution of velocities and semi-major axis does
Figure 3. Left: branching ratios for different types of outcome as a function of the ratio \(\alpha\) between the initial binding energies of the two binaries. Only flybys, exchanges, ionizations, triples and encounters leading to systems like AE Aur, \(\mu\) Col and \(\iota\) Ori are shown. Right: normalized total cross-section as a function of the parameter \(\alpha\).

Figure 5. Velocity of AE Aur versus velocity of \(\iota\) Ori (left) and versus velocity of \(\mu\) Col (right). The density of points over squared areas is indicated through a gray shaded scale. The error boxes of the measured velocities are shown as filled black rectangles.

not change for \(\alpha < 3\) but become significantly different for larger values of \(\alpha\). The consistency of the distributions with the observed values reflects the right choice of the total energy in the encounter. The total energy of the 4-body system is known at present and its conservation before and after the encounter was used in the choice of the initial conditions. Different choices of the total energy available in the interaction lead to different distributions for the velocities and the binary semi-major axis. This seems to exclude the possibility that a fifth body was involved in the encounter.

In order to verify the possibility of producing two single runaways moving in almost opposite directions, as is observed for AE Aur and \(\mu\) Col, we study the distribution of the angular displacement of the two stars with respect to the center of mass of the 4-body system. We compute the angle \(\theta\) between the velocity vectors of the two single stars and report its distribution \(f_\theta\) in Figure 7. We find that about 35\% of all encounters result in an angular displacement \(\theta\) in the 45 degrees range \(135^\circ \leq \theta < 180^\circ\).

The eccentricity of the binary after the encounter is consistent with a thermal distribution and therefore more than 50 per cent of the encounters result in a binary with \(e > 0.67\).

We computed the absolute fractions and the cross-sections of different types of encounters using the procedure described in appendix A. If \(\sigma_X\) is the cross-section for process X, a normalized cross-section can be defined as

\[
\tilde{\sigma}_X = \frac{\sigma_X}{\pi \left( a_1^2 + a_2^2 \right)} \left( \frac{v_\infty}{V_c} \right)^2 ,
\]  

\(c \copyright 2003\) RAS, MNRAS, 000, 1–14
where \( v_\infty \) represents the relative velocity between the centers of mass of the interacting binaries. As shown in Table 5, about 1 in 6 encounters results in the ionization of one binary and the exchange of one specific star, leading to systems like AE Aur, \( \mu \) Col and \( \iota \) Ori.

From the cross-section it is possible to derive the typical time-scale of the process under consideration if the stellar density and the stellar velocity dispersion of the region are known

\[
T_X \sim \frac{1}{\pi (a_1^2 + a_2^2)} \frac{v_\infty}{V} \frac{1}{n f_b \sigma_X},
\]

where \( n \) represents the mean stellar density in the cluster and \( f_b \) is the fraction of binaries in the core. Substituting typical parameters of the Trapezium cluster (with a binary fraction \( f_b \sim 0.6 \); Duquennoy & Mayor 1991) we obtain

\[
T_X \sim 3 \text{Myr} \left( 5 \times 10^4 \text{pc}^{-3} \right) \left( \frac{\text{AU}^2}{a_1^2 + a_2^2} \right) \left( \frac{v}{0.01} \right)^2 \left( \frac{5 \text{km s}^{-1}}{v_\infty} \right)
\]

where \( v = v_\infty/V_c \). In the case \( \alpha = 2 \) the typical time-scale for a binary-binary encounter resulting in an exchange-ionization is about 3 Myr.

### 4.3 Collisions in binary-binary encounters

In this section we relax the hypothesis of point mass stars and investigate the effects of physical radii on the outcomes of binary-binary encounters. In the simulations described in the previous sections, we keep track of the minimum distances between all stars. In this way we are able to use the same data and test the effect of finite stellar radii, assuming that stars collide if they approach each other within some minimum distance \( d_{\text{coll}} \).

Since the initial eccentricities of both binaries are randomly drawn from a thermal distribution between 0 and 1, stars in very eccentric orbits may collide at the first pericenter passage. This would not be realistic, and therefore we limit the range in initial eccentricities. In a highly eccentric orbit two stars with total radius \( R \) collide if \( R \geq 0.75a(1 - e) \) (Kopal 1959). Inverting this equation leads to our definition of \( e_{\text{max}} \).

\[
e_{\text{max}} = 0.9.
\]

We therefore exclude binaries with initial eccentricity \( e \geq 0.9 \) from further consideration.

Figure 8 shows the cumulative distribution for minimum distances between any two stars for the \( \sim 8000 \) encounters which remained after selecting the systems with \( e < e_{\text{max}} \) (solid curve) and for circular initial binaries (dashed curve). Since circular orbits are very rare in our standard initial conditions, we performed a separate simulation of 8000 encounters. If we assume stellar radii from Tab.3 at the moment of the encounter, a collision occurs if \( r_{\text{min}}/a \leq 0.07 \). In that case about half of the initially circular binaries result in a collision, whereas only 25 per cent of the eccentric systems survive merging. In figure 9 we show the fraction of binaries that experience a collision as a function of \( e_{\text{max}} \). The two figures 8 and 9 show similar trends; high initial eccentricities enhance the collision rate.

In Figure 10 we show the proportion of ionized systems
Figure 8. Cumulative distribution for minimum distances $r_{\text{min}}$, between stars during binary-binary encounters. The solid line represents the result for systems with $e < 0.9$ from our standard run. The dashed line is for binaries with initially circular orbits. With the observed stellar radii a collision would occur at about $r_{\text{min}}/a \lesssim 0.07$.

Figure 9. Fraction of collisions as a function of $e_{\text{max}}$.

Figure 10. Branching ratios as a function of $e_{\text{max}}$. The solid line gives the results for encounters in which one binary was ionized. The dashed line gives results for a subset of these, namely the ones which lead to systems like $\epsilon$ Ori (see § 4.1 for the specifics of this encounter).

Figure 11. The fraction of collisions (dotted line) and the branching ratio for ionizations (solid curve) as a function of $\beta$. Our system of interest is represented with the dashed line.

and systems like $\epsilon$ Ori which resulted from the standard run. There seems to be a slight preference for initial eccentricities in the range $0.4 \leq e \leq 0.6$.

To further study the dependence of the collision probability on the stellar radii we define $\beta \equiv d_{\text{coll}}/r$, the dimensionless effective radius for collisions. In Figure 11 we show the branching ratios for collisions, ionizations and the sub-type of $\epsilon$ Ori as a function of $\beta$. Changing the stellar radii (or $\beta$) by a factor of a few does not effect the collision rate much, and therefore the choice of the stellar radii is not critical to the results.

5 THE EVOLUTION OF $\epsilon$ ORIONIS AFTER THE ENCOUNTER

Until now we have considered the implications of a binary-binary encounter on the dynamical properties of AE Aur, $\mu$ Col and $\epsilon$ Ori. The current observations of the binary allow us to constrain its evolution in the $\sim 2.5 \text{ Myr}$ elapsed from the moment of the encounter till now. There are at least three further aspects that have to be integrated into a wholly plausible model of the interaction under consideration. They are:
(i) ι Ori might be a triple system, and possibly even quadruple or quintuple (see § 5.1);

(ii) stellar evolution models make it difficult to confirm that ι Ori B is of the same age as any of the other three stars (see § 5.2);

(iii) tidal friction must operate within the present ι Ori system, and we have to check whether the observed eccentricity can have been maintained for the 2.5 Myr since the collision (see § 5.3).

We deal with these in turn.

### 5.1 Is ι Orionis a multiple system?

The spectroscopic binary ι Ori is the brightest (component A) of three members of the multiple star ADS 4193. The other two members (B and C) are 11′′ and 50′′ distant. At the distance of ι Ori (∼400 pc, see Table 1) these are quite widely separate, and for the time being we discount them as gravitationally bound companions. But ADS 4193A is also a speckle binary, CHARA 250Aa/Ab, with a separation of 0.10″ (Mason et al. 1998). This suggests an orbital period ≳40 yr. The brighter component (Aa) of this speckle pair is the spectroscopic binary. Strictly speaking we should refer to the O9II and B0.8III-IV components of this 29 d binary as ι Ori Aa1 and ι Ori Aa2, and we do so from now on.

The close speckle companion raises at least two questions. Is it reasonable that the companion was carried along during the four-body (or rather 5-body) encounter that we have supposed? And could it affect the eccentricity of the spectroscopic orbit, by the mechanism of Kozai cycles (Kozai 1962)?

We have not attempted to model 5-body encounters because of the enormous parameter space that they can be drawn from. We can imagine it would be possible for the companion to remain bound after an encounter but we doubt the energy released would be enough to eject AE Aur and µ Col with high speed. It might be worth investigating this possibility in the future and testing these predictions.

In a triple system in which the outer orbit is fairly highly inclined to the inner orbit, the third body can have a major long-term effect on the inner pair, causing its eccentricity to cycle between a low and a high value. For instance, if the mutual inclination is 60°, ε can cycle between 0 and 0.76, or between 0.5 and 0.86 (Kiseleva, Eggleton & Mikkola 1998). The period of the cycle is \( \sim \frac{P_{\text{out}}}{P_{\text{in}}}/ \sim \frac{20000 \text{yr}}{\text{in this case}} \). If the triple were formed early in the life of the cluster by some random encounter, an inclination of 60° would be quite likely; alternatively, the kind of encounter that we think is needed to create the binary ι Ori Aa would probably randomize its inclination relative to the (ι Ori Aa, ι Ori Ab) speckle pair. Thus it is possible that the observed high eccentricity of the spectroscopic binary ι Ori Aa is not a result of the encounter, but of the third body ι Ori Ab.

### 5.2 Stellar evolution models for AE Aurigae, µ Columbae and ι Orionis

We now use stellar evolution model to constrain the ages of the four stars. Figure 12 is a theoretical HR diagram for masses in the range required. The positions of all four stars, according to their effective temperatures and gravities as given by Bagmolo et al. (2001), are indicated. Two isochrones are indicated, for 5 and 10 Myr. It is clear that the apparent ages of ι Ori Aa1 and ι Ori Aa2 are substantially different: we estimate 5 and 10 Myr respectively. The uncertainties appear to be of the order of ±1 Myr, but could be substantially greater on account of unquantifiable systematic errors. The possibility that discrepant ages can be accounted for by Roche-lobe overflow can almost certainly be discounted by the high eccentricity of the system. The ages of AE Aur and µ Col are comparable to that of ι Ori Aa1, though AE Aur might be a little younger and µ Col a little older.

The Trapezium Cluster, from which the four stars appear to have been ejected, is a well studied young cluster, the core of the Orion Nebula Cluster, which contains about a dozen OB stars and many more fainter stars. It is estimated that these stars are ≤1 Myr old (Herbig & Terndrup 1986; Brown, de Geus & de Zeeuw 1994; Prosser et al. 1994). Stars later than B2 are often though not always above the main sequence. We may therefore wonder if the two giants of ι Ori are still evolving towards the main sequence rather than away from it. This seems improbable, because their ejection from the cluster 2.5 Myr ago is amply long enough for contraction to the main sequence at their high luminosities. This suggests, incidentally, that not all the Trapezium stars are as young as 1 Myr, and we appear to require that some massive stars have been forming over the last 10 Myr. An age spread from 1 to 10 Myr has already been observed for stars ≤6M⊙ (Palla & Stahler 1999), and we suggest that there may be a similar spread for OB stars.

An alternative possibility is that the parent cluster is one of the older subgroups in Orion such as subgroup 1a or 1c (see Brown et al. 1998). These subgroups have a kinematics very similar to the one of subgroup 1d (which contains the Trapezium cluster) and overlap in position on the sky with ι Ori.

The uncertainties in the theoretical HR diagram are of course substantial, and hard to quantify. The models were
computed using the code of Pols et al. (1997), which was found to give good agreement with the observational data of Andersen (1991). The theoretical uncertainties have to be convolved with observational uncertainties but very substantial error must be involved if ι Ori Aa2 is to be of the same age as any of the other three stars.

A rather long shot is the possibility that the speckle companion is in fact the missing 10 Myr-old companion of ι Ori B. This would require a binary-triple interaction of the form:

\[(ι \text{ Ori Aa2}, ι \text{ Ori Ab}) + (ι \text{ Ori Aa1}, (AE Aur, μ Col)) \rightarrow \]

\[\rightarrow (ι \text{ Ori Aa1}, ι \text{ Ori Aa2}), ι \text{ Ori Ab}) + AE Aur + μ Col.\]

It is hard to assess the likelihood of this but in any event we should not ignore the possible existence of a fifth body in a complete analysis.

5.3 The effects of tidal friction on the binary orbit

An important aspect to consider in the post-encounter evolution is the effect of tidal friction on the spectroscopic orbit. The sum of the radii that we infer is 60% of the periastron separation, which would seem sufficient for tidal effects to be important. We have used the equilibrium-tide model of Hut (1981) as further developed by Eggleton, Kiseleva & Hut (1998). This model was successfully tested on the SMC radio-pulsar binary 0045-7319 (Eggleton & Kiseleva (2001)).

The orbital evolution expected since the encounter 2.5 Myr ago is shown in Figure 13. We find that we could reach the present eccentricity and orbital period if we started with P=110 days and e=0.91. A high eccentricity is common for binaries which undergo exchange-ionization encounters as the final eccentricity distribution is expected to be thermal (see § 4.2). On the other hand, a long-period binary as a final outcome implies an even wider initial target binary, which means a lower binding energy available in the encounter. It is unlikely to obtain recoil velocities as high as 100 km s\(^{-1}\) with a pre-encounter total energy which is smaller than the current one. A tight binary (with an orbital period similar to the observed one) is needed in order to reproduce the velocities of the single runaways.

Figure 13 shows that parallelization (the alignment of the spin axis of the two binary components) and pseudo-synchronization took ~2 Myr to achieve. Given the uncertainty in the viscous time-scale – at least a factor of 10 – it is possible that they have actually not yet been achieved. The pseudo-synchronous rotation period (Hut 1981) is a fraction of the orbital period dependent only on eccentricity: the factor is 9.51 for e=0.76, i.e. \(P_{\text{rot}} \sim 3\text{ d}\) currently. With the radii computed from the evolutionary tracks of Figure 12, the rotation speeds we expect are \(V \sin i \sim 150\) and 115 km s\(^{-1}\) for the primary and the secondary respectively. These values are somewhat greater than the values (110 and 70 km s\(^{-1}\)) quoted by Marchenko et al. (2000), and may indicate that pseudo-synchronization has not yet been reached.

However we have already noted that the high eccentricity of the spectroscopic binary may not in fact have been generated by the collision but may be an artifact of Kozai cycles driven by the third (speckle) body (see § 5.1). In Kozai cycles, while the eccentricity fluctuates the semimajor axis is unperturbed (to lowest order), and as a result the two components will in the long term seldom be as close together as they are at the periastron of the current orbit. Thus tidal friction might be negligible in this case.

The possibility that the eccentricity is due to the third body rather than to the interaction could mean that we have to revise our earlier conclusion that the apparently non-coeval nature of the spectroscopic binary is not due to Roche lobe overflow, since the main argument against there having been any Roche lobe overflow is that the orbit is eccentric. However Kozai cycles are normally suppressed if the quadrupolar distortion of the stars in the close binary is large; and it usually is large enough as a star approaches Roche lobe overflow. Once a star fills its Roche lobe it normally grows larger still, until a very late stage of evolution when it has lost ~70% of its mass. Thus once Kozai cycles are suppressed they are unlikely to re-establish themselves till this late stage, when the donor would be a small helium star rather than a B giant.

We hope that in the near future more information will be available on the speckle orbit, which should move appreciably in ~10 yr. The spectroscopic orbit may also be on the margin of detectability by speckle or adaptive optics; a direct measure of its inclination could limit the possibilities substantially. It will be desirable to search for the speckle companion in the spectrum of ι Ori. If it can be detected it may modify significantly the spectroscopic elements of the 29 d orbit.

6 SUMMARY

We tried to reconstruct the event that ejected the runaways AE Aur and μ Col and the binary ι Ori from their parent cluster about 2.5 Myr ago. We here summarize different models that could explain the high velocities of the single stars and the discrepant ages of the binary components.

1. The single stars and the binary are all unrelated. As shown by Gies and Bolton (1986) and Hoogerwerf et al. (2000, 2001), this can be excluded on the basis of their kinematical and evolutionary properties. The same dynamical encounter must have ejected the stars from the Trapezium about 2.5 Myr ago. In order to explain the high velocities of the single stars and the age difference in the binary, the encounter must have involved an ionization and an exchange.

2. The stars were ejected as a result of a binary-binary interaction that occurred in the Trapezium cluster about 2.5 Myr ago. This model is based on the observation that the velocity of the 4-body system is comparable to the velocity of the Trapezium cluster. The velocities of the stars and the period of the binary after the encounter are consistent with the observed values, so that we can conclude that this model can well reproduce the kinematics of the 4-body system. If we consider stars with physical radii, a fraction of encounters result in a collision between two stars. We thus expect to find collision products in dynamically active young clusters like the Trapezium. The fraction of collisions strongly depends on the initial eccentricity of the two binaries and therefore we consider it likely that ι Ori, AE Aur and μ Col were ejected in an encounter between two low-eccentricity binaries. In this case, the final properties of the system like the recoil velocities of the stars and the orbital
period and eccentricity of ι Ori do not differ from the ones obtained for highly eccentric initial binaries. The occurrence rate of this type of encounter, once corrected for the collision probability, is about one in 10 Myr.

A possible difficulty in this model is the short circularization time-scale for the orbit of the binary after the encounter. Our calculations show that only extremely eccentric post-encounter orbits (e ~ 0.9) evolve in an eccentricity like that of ι Ori after 2.5 Myr.

In addition, the model relies on the assumption that ι Ori B and µ Col are in the same binary before the encounter, implying that the two stars must be coeval. Our evolutionary calculations show that µ Col might be too hot, or equivalently too early in spectral type, to be on the same isochrone as ι Ori B. However, our knowledge about the temperatures and gravities of the stars is sufficiently imprecise that we can assume coevality for all four stars.

3. The stars were ejected as a result of a triple-binary interaction. This model is based on the possibility that ι Ori is the brightest component of a speckle pair, making it a stable hierarchical triple. This would require a 5-body encounter with the same total energy as our standard 4-body encounter resulting in the ejection of a triple and two single stars. According to this model, the high eccentricity of ι Ori may be a result of Kozai cycles driven by the third body. We haven’t tested this possibility but we can argue that this type of encounter would be rather rare and not energetic enough to explain the velocities of the runaways. It may certainly be worth investigating this scenario to test these predictions as well as the ones about corotation and pseudo-synchronisation.

The last word has evidently not been said on the interesting interaction between ι Ori, AE Aur and µ Col, but we think it remains a fascinating combination of orbital dynamics, cluster dynamics, stellar evolution and tidal effects. In addition to more elaborate numerical simulations, high accuracy parameters for the stars are needed in order to settle the co-evality issue. New interferometric speckle observations, possibly resolving the spectroscopic binary too, will be crucial for a better understanding of the dynamical encounter that ejected the runaway stars.

7 ACKNOWLEDGMENTS

We would like to thank Douglas Gies, William Bagnulo, Lex Kaper, Anthony Brown, Ronnie Hoogerwerf and Michela Mapelli for interesting discussions and comments on the manuscript. This work was supported by the Netherlands Organization for Scientific Research (NWO), the Royal Netherlands Academy of Arts and Sciences (KNAW) and the Netherlands Research School for Astronomy (NOVA). This work was performed under the auspices of the U.S. Department of Energy, National Nuclear Security Administration by the University of California, Lawrence Livermore National Laboratory under contract No. W-7405-Eng-48.

REFERENCES

Andersen, J. 1991, A&A Rev., 3, 91
The procedure described in section §3.1 for the computation of the maximum impact parameter in each scattering experiment is incorporated in the actual computation of cross-sections. The cross-section for an event $X$ is derived from (McMillan & Hut 1996)

$$\sigma_X = \sum_{i=0}^{t_{\text{max}}} \pi \left( b_{i+1}^2 - b_i^2 \right) \left( \frac{N_{X,i}}{N_i} \right),$$

(A1)

where $N_{X,i}$ is the total number of scatterings resulting in an outcome $X$ in the shell $i$ and $N_i$ is the total number of scatterings performed in the shell $i$. The uncertainty in the cross-section is given by

$$\Delta \sigma_X = \sum_{i=0}^{t_{\text{max}}} \pi \left( b_{i+1}^2 - b_i^2 \right) \left( \sqrt{N_{X,i}} / N_i \right).$$

(A2)

In analogy to the case of binary-single star scattering (Hut & Bahcall 1983; Sigurdsson & Phinney 1993) and in order to compute physical time-scales for different types of interactions, we define a cross-section

$$\bar{\sigma}_X = \frac{\sigma_X}{\pi \left( a_1^2 + a_2^2 \right)} \left( \frac{v_{\infty}}{c} \right)^2$$

(A3)

normalized to the geometric cross-section and corrected for gravitational focusing.

A characteristic time-scale for a process $X$, defined as the mean time between subsequent occurrences of $X$, can be evaluated from (Bacon, Sigurdsson & Davies 1996)

$$T_X \sim \frac{1}{\pi \left( a_1^2 + a_2^2 \right)} \frac{1}{c^2} \frac{1}{n_f \bar{\sigma}_X}$$

(A4)

where $n_f$ represents the mean stellar density in the cluster and $f_b$ is the fraction of binaries in the core.

**APPENDIX B: CRITERIA FOR STOPPING THE INTEGRATION**

During the numerical integration of a binary-binary encounter the status of the system is analysed after each 20 orbital periods of the initial target binary and the simulation is stopped if one of the following criteria is met:

1. only two stars remain, in which case their orbits are determined and the system is classified as a binary if it is bound or as two single stars if it is unbound;
2. three stars remain, in which case the system is considered ionized if the total energy is positive, the stars are widely separated and receding from each other while it is considered as a binary and a single star if the total energy is negative but the single star is escaping from the binary center of mass;

3. the 4-body system is split in binaries whose centers of mass are unbound, widely separated and receding. In the other cases the system is considered bound and the integration continues until a stopping criterion is satisfied or a maximum integration time is reached.