Innovative meshless approach for shaped charges applications

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Abstract. We focus here on modelling shaped charges. Combining large deformations, numerous interface productions, and strong damage mechanisms, those events are particularly challenging from a numerical point of view. Eulerian finite element methods are classically used for such modeling. However, they induce very long computation times, accuracy losses (projection algorithms), and difficulties with opening criteria related to jet fragmentation. Among the Lagrangian approaches, the meshless method called Smoothed Particle Hydrodynamics (SPH) appears as a relevant alternative to prevent such shortcomings. Based on a set of moving interpolation points, it disregards any connectivity between its elements which makes it naturally well suited to handle material failure. Nevertheless, SPH schemes suffer from well-known instabilities questioning their accuracy and activating nonphysical processes, such as numerical fragmentation. Many stabilizing tools are available in the literature however, they either raise conservation and consistency issues or drastically increase the computation times. We propose then to use an alternative scheme called γ-SPH-ALE. Based on the ALE framework, it achieves robust and consistent stabilization through an arbitrary description of motion. Thanks to CFL-like conditions obtained through a nonlinear stability analysis, the scheme stability is ensured. By preventing spurious oscillations in elastic waves and correcting the so-called tensile instability, both stability and accuracy are increased regarding classical approaches. Also, taking advantage of GPU computing, such results are achieved in reduced computation times contrary to classical CPU implementations. Its implementation on a “Viper” shaped charge shows that the scheme handles the jet generation process as well as its resulting interaction with a target.

1. Introduction

Historically, the shaped charge appeared in the 18th century, but it has found use in the past century. Indeed, during the Second World War, its penetrating power was exploited, thanks to the addition of the liner. The shaped charge consists of an explosive block in which there is a conical cavity and on the walls of the cavity is a liner. Upon detonation, the gases produced propagate towards the liner and exert pressure on the metal block, which affects the liner jet along the axis. It is this fine, elongated stream that penetrates the target. Historically, the presence of a metallic liner was favored, but recently other materials are beginning to be used to achieve higher jet speeds. Furthermore, the shape of the cavity is not always conical but can have complex shapes.

Regarding numerical modeling, two aspects are crucial to reproduce the jet generation process and its fragmentation. As the phenomenon is mainly in the field of large deformations, historical simulations have favored Eulerian approaches [1-4]. However, facing a huge compromise between accuracy and computation times particularly regarding the jet fragmentation process, Lagrangian and ALE approaches were then evaluated [3,5,6]. Among them is the Smoothed Particle Hydrodynamics method and we focus
here on a recent method called γ-SPH-ALE [7]. The combination of the SPH-ALE formalism [8] and a
finite volume low-Mach scheme [9] provides consistent and robust stabilization to well-known SPH
instabilities such as spurious oscillations and tensile instability. Benefiting from a massively
parallel GPU implementation, it provides convincing and reliable results on numerous academic test
cases going from hydrodynamics [7] to solid dynamics [10] in reduced computation times contrary to
legacy SPH solvers. Thus, we propose to evaluate its capabilities regarding shaped charge modeling. Its
governing equations are detailed in section 2 and its evaluation is then presented in section 3.

2. γ-SPH-ALE
In this section the γ-SPH-ALE scheme is detailed in the context of Euler equations [7]. The general SPH
framework is explained and then applied to the ALE context. The governing equations are presented for
the hydrodynamic version and then extended to solid dynamics.

2.1. SPH basics
SPH relies on approximation features to discretize conservation laws on a particle set \((x_i, \omega_i)_{i \in P}\)
(respectively the particle position and volume). These approximations are achieved thanks to kernel
functions \(W(\|x_i - x_j\|, h) = W_{ij}\) depending on the smoothing length \(h\) (half radius of its compact
support) and the distance between two particles. The kernel gradient can also be evaluated as \(\nabla W_{ij} = \text{grad}_x W_{ij}\) allowing for the smoothed particle approximation of a function \(f\) and its derivative.

\[
I^k(f)_i = \sum_{j \in P} \omega_j f_j W_{ij} \quad \text{and} \quad \nabla I^k(f)_i = \sum_{j \in P} \omega_j f_j \nabla W_{ij} \quad (1)
\]

As introduced above, classical SPH suffers from a lack of interpolation completeness which can be
compensated by using renormalized kernels. Vila [8] proposed to replace \(\nabla W_{ij}\) by \(A_{ij} = B_{ij} \nabla W_{ij}\)
where \(B_{ij} = (B_i + B_j)/2\) and \(B\) is the renormalization matrix (introduced by Johnson et al. [11] and
Vila [8]) defined as

\[
B_i = E_i^{-1} \quad \text{and} \quad \forall (\alpha, \beta) \in \{1,2,3\}, \ E_i^{\alpha \beta} = \sum_{j \in P} \omega_j (x_j^\beta - x_i^\beta) \nabla W_{ij}^\alpha \quad (2)
\]

One set in the following \(n_{ij} = A_{ij}/\|A_{ij}\|\).

To ensure the conservative property of the scheme, alternative derivative operators are used in practice
and combined to symmetric kernels such that \(W_{ij} = W_{ji}\) and \(\nabla W_{ij} = -\nabla W_{ji}\).

\[
D_h f_i = \sum_{j \in P} \omega_j (t_i f_j) A_{ij} \quad \text{and} \quad D_h f_i = \sum_{j \in P} \omega_j (f_j + f_i) A_{ij} \quad (3)
\]

Note that, as proved in [8], the operator \(D_h\) (3) combined to renormalization strongly approximates
the gradient operator under the condition \(\Delta x/h = O(1)\) (where \(\Delta x\) is the particle spacing). It is a crucial
point to ensure the scheme’s consistency. In standard SPH this convergence property depends on the ratio
\(\Delta x/h\) and is only enforced for particular values [12]. Classically \(h = 1.3\Delta x\) in 3D

2.2. ALE Framework
The idea is now to apply these approximation features to a conservation law in ALE formalism. Consider
the following law

\[
L_{\psi v} (\Phi) + \text{div} [F_E (\Phi) - \nu_0 \otimes \Phi] = S(\Phi) \quad (4)
\]

Where \(\nu_0 \in \mathbb{R}^3\) is a regular transport field, \(\Phi \in \mathbb{R}^4\) the vector of the conserved variables, \(F_E = (F_{E_i}^j)^3\)
the Eulerian flux vectors, \(S = (S^j)^3_i\) the source term and \(L_{\psi v}\) the transport operator associated to \(\nu_0\) (5)

\[
L_{\psi v}: \Phi \rightarrow \frac{\partial \Phi}{\partial t} + \sum_{l=1}^3 \frac{\partial (v^l \Phi)}{\partial x^l} \quad (5)
\]

In the case of Euler equations, we get the following equation set

\[
\frac{dx_i}{dt} = \nu_0 (x_i, t) \quad (6)
\]

\[
\frac{d\omega_i}{dt} = \omega_i \text{div} (\nu_0 (x_i, t)) \quad (7)
\]
\[
\frac{d(\omega_i \Phi)}{dt} + \omega_i \sum_{a=1,3} \mathbf{v}_h^a \cdot [F_E^a(\Phi) - \mathbf{v}_0^a]. \Phi = 0
\]  
(8)

Where \(-\mathbf{V}_h^a\) is the adjoint of an operator \(\mathbf{V}_h\) approximating the derivative operator \(\nabla\).

2.3. Hydrodynamic Formulation

The idea is to choose \(\mathbf{V}_h\) to stay conservative and consistent. Several choices are admissible, and we propose to work with the following scheme discretized in space

\[
\frac{dx_i}{dt} = \mathbf{v}_0(x_i, t)
\]  
(9)

\[
\frac{d\omega_i}{dt} = \omega_i \sum_{j \in P} \omega_j \left( (\mathbf{v}_{0j} - \mathbf{v}_{0i}) \cdot A_{ij} \right)
\]  
(10)

\[
\frac{d(\omega_i \rho_i)}{dt} = -\omega_i \sum_{j \in P} \omega_j \left( \rho_j + \rho_i \right)(\mathbf{w}_{ij}, A_{ij})
\]  
(11)

\[
\frac{d(\omega_i \rho_i v_{ij})}{dt} = -\omega_i \sum_{j \in P} \omega_j \left[ (\rho_i v_j + \rho_j v_i)(\mathbf{w}_{ij}, A_{ij}) + Q_{ij} \right]
\]  
(12)

With

\[
\mathbf{w}_{ij} = \frac{1}{2}(\mathbf{w}_i + \mathbf{w}_j) - \Gamma_{ij} \quad \text{and} \quad Q_{ij} = p_i A_{ij} - \pi_{ij}
\]  
(13)

Where the variables \(x, \omega, \rho, \mathbf{v}\) and \(p\) refer respectively to the position, volume, density, velocity, and pressure of the particles. \(\mathbf{w}\) corresponds to the relative ALE velocity defined by \(\mathbf{w} = \mathbf{v} - \mathbf{v}_0\). We also define \(p_{ij} = p_i + p_j\), \(\Gamma_{ij}\) corresponds to the stabilizing term coming from the FV low-Mach scheme and \(\pi_{ij}\) corresponds to the artificial viscosity. \((.,.)\) defines the usual scalar product on \(\mathbb{R}^3\).

A vectorial version of Monaghan’s stabilizing artificial viscosity [13] is proposed (14). It depends on a parameter \(\alpha\) to adjust to minimize the amount of artificial diffusion induced in the scheme. \(c_0\) is the material sound speed and \(\rho_{ij} = \frac{1}{2} (\rho_i + \rho_j)\).

\[
\pi_{ij} = \alpha_{ij} (\mathbf{v}_j - \mathbf{v}_i) \quad \text{and} \quad \alpha_{ij} = \alpha c_0 \rho_{ij} \left\| A_{ij} \right\|
\]  
(14)

The coupling between the SPH version of the FV Low-Mach scheme and the SPH-ALE formalism is achieved by the term \(\Gamma_{ij}\) (20,21) through the mean relative ALE velocity \(\mathbf{w}_{ij}\). We set \(\mathbf{n}_{ij} = A_{ij}/\|A_{ij}\|\)

\[
\Gamma_{ij} = \gamma_{ij} (p_j - p_i) \mathbf{n}_{ij} \quad \text{and} \quad \gamma_{ij} = \frac{\gamma}{2c_0 \rho_{ij}}
\]  
(15)

Disregarding \(\Gamma_{ij}\) and \(\pi_{ij}\), the classical Lagrangian \((w=0)\) SPH formulation with constant masses \(m_i = \omega_i \rho_i\) is recovered. With this ALE formulation particles are moved following the velocity field \(\mathbf{v}_0\) (9). Such field is said to be arbitrary in the sense that, as suggested by Vila [8], a smart choice of \(\mathbf{v}_0\) could increase both stability and robustness of the calculations without impacting the scheme stability conditions. Various methods are available in the literature (for example XSPH method [14]) and the idea is to set \(\mathbf{v}_{0i} = \mathbf{v}_i + \delta \mathbf{v}_i\) where \(\delta \mathbf{v}_i\) stores the regularizing technic.

2.4. Extension to solid dynamics

The proposed \(\gamma\)-SPH-ALE scheme is directly extended to account for material strength. To do so the momentum equation (12) is enhanced by the deviatoric stress tensor contribution. To account for material strength models, we use discrete formulations of strains, to describe the isotropic linear elasticity (through Hooke’s law), and rotation, to ensure the stresses objectivity (through Jaumann’s derivative). Working in an ALE framework, a discretization of the advection component of path-dependent variables, such as stresses, is also considered. Thanks to the Von Mises yield stress criterion and a flow stress model, we are then able to describe the material plastic flow. Aiming to handle the fragmentation process, we also consider a description of the material damage growth, a direct consequence of its deformation, and a precursor to its failure.
To ensure that the proposed scheme is conservative, robust, stable, and consistent, stability conditions on the scheme parameters are exhibited thanks to nonlinear stability analysis. We refer to [7] for the detailed proof in the context of monophasic barotropic Euler equations. These properties are validated on numerous academic test cases going from hydrodynamics to solid dynamics [7,10]. γ-SPH-ALE is also fully implemented in the Impetus Afea Solver® benefiting from massive GPU parallelization allowing it to provide such compelling results 100 times faster than legacy SPH solvers. Thus, γ-SPH-ALE capabilities in terms of stability, robustness, accuracy, and computation times open the door to investigations of complex Multiphysics configurations such as shaped charges.

3. Shaped Charge Modeling Validation

To evaluate γ-SPH-ALE in the context of shaped charge modeling, we propose to study a Viper model [1] in two configurations: jet generation and impact on a target.

3.1. Jet generation of a Viper shaped charge

As a first step, we propose to investigate γ-SPH-ALE in terms of jet generation capabilities on a Viper shaped charge, widely studied in the literature. The reference charge is caliber 65mm length 120mm and consists of a bare LX-14 high explosive (HE) part coating an OFHC copper liner 2mm thick and a 42° cone angle (Figure 1). The charge is initiated thanks to a detonation point at the end of the HE part. We evaluate here the scheme’s ability to reproduce the jet generation process in terms of shape, length, and velocity.

![Detonation point](image)

**Figure 1** - Viper shaped charge - Reference model: LX-14 high explosive, OFHC copper liner.

The HE behavior is reproduced by using a JWL equation of state (EOS). An elastic perfectly plastic material model (initial yield strength 350 MPa) is used in combination with a Gruneisen EOS (S=1.489, gamma=1.99) to simulate the copper liner. The corresponding parameters are displayed in tables 1 and 2. Our γ-SPH-ALE model is fully 3D based on an initial particle spacing dx=150 µm giving 11e6 particles.

| Table 1. Material parameters for HE and Copper liner. | Table 2. LX-14 JWL parameters. |
|-----------------------------------------------|---------------------------------|
| Density kg.m⁻¹ | Young Modulus Pa | Shear Modulus Pa | Poisson Coefficient | A | B | R1 | R2 | Ω | E0 J.m⁻³ |
| HE | 1821 | - | 2.2e9 | - | | | | | | 826.1e9 | 1724e9 | 94.55 | 1.32 | 0.38 | 10.2e9 |

| Copper | 8940 | 120e9 | - | 0.345 |

Figure 2 shows a comparison between γ-SPH-ALE and the experiment [16] at the early stages of the jet generation process (24 and 30 µs after detonation). In both cases, we can see that γ-SPH-ALE is in very good agreement with the experiment by reproducing particularly the dome shape of the jet head as well as a correct jet length. We compare in Figure 3 γ-SPH-ALE results to experimental Xrays at late stages of the jet generation process (90 and 100 µs after detonation). Here again, γ-SPH-ALE succeeds
in reproducing a correct jet regarding the experiment in terms of length, shape and the fragmentation mechanism as well.

**Figure 2** - Comparison between experiment [16] (left) and γ-SPH-ALE (right) at 24 µs (top) and 30 µs (bottom) after detonation.

Regarding this last aspect, a closer look at the jet at 100µs (Figure 4) shows that γ-SPH-ALE can recover the characteristic oval shape of the jet fragments and particularly the jet head. Also, this oval shape of fragments reveals that the fragmentation process is the result of a necking mechanism and not due to numerical errors such as the so-called SPH tensile instability. Finally, the comparison in table 3 of the jet tip velocities provided by experiment [17], reference numerical codes [17], and γ-SPH-ALE confirms the ability of the proposed scheme to correctly handle the jet generation process by reaching the expected velocity range. On this aspect, note that the ignition method has a significant impact on the jet characteristics. By using a booster to detonate the charge rather than a single detonation point (closer to the real ignition conditions), the experimental tip velocity is accurately computed. GPU
implementation of the solver resulted in computation time of only 12 hours on a standard workstation (one GV100). Faced with such compelling results the next step was to investigate $\gamma$-SPH-ALE in terms of jet penetration capabilities in an impact configuration.

![Closeup view of the $\gamma$-SPH-ALE jet at 100 µs. Velocity magnitude map.](image)

**Figure 4** – Closeup view of the $\gamma$-SPH-ALE jet at 100 µs. Velocity magnitude map.

|                | Experiment | CTH  | ALEGRA | $\gamma$-SPH-ALE |
|----------------|------------|------|--------|------------------|
| Velocity (m/s) | 9100       | 9000 | 8600   | 8755 (detonation point) |
|                |            |      |        | 9050 (18mm booster)  |

**Table 3.** Comparison of Jet tip velocity between experiment [17], legacy codes [17], and $\gamma$-SPH-ALE.

3.2. *Viper shaped charge against a steel target*

Using the following experiment performed at Los Alamos [18] consisting of impact of a viper on a cold-rolled steel 1018 target, 150 mm below the face of a shaped charge, to observe the target perforation (Figure 5). The same Viper model described in the previous section (full 3D model consisting of 11e6 particles) was used in the simulation.

![Viper shaped charge impact on a steel target. Experimental setup [18.](image)

**Figure 5** - Viper shaped charge impact on a steel target. Experimental setup [18].
Figure 6 shows a comparison of γ-SPH-ALE results with experimental Xrays at different times before and after impact. At 4 µs before impact, one can see the jet tip arriving against the target. At 21 µs after impact, the continuous jet has penetrated the target. A debris cloud is then generated and expelled on the target front face. Also, the target rear face is deformed resulting from the jet penetration. Finally, at 29 µs after impact, the continuous jet has entirely penetrated the target producing a second expelled debris cloud on the target rear face. One can see that γ-SPH-ALE correctly handles the interaction between the jet and the target by properly reproducing the penetration mechanisms at the expected times. Also notice that as experimentally observed, the proposed scheme preserves the jet continuity even after complete perforation (crucial in terms of penetration capabilities) and reproduces the debris cloud's topology (composition of small fragments and diameter). Thanks to GPU implementation, these results are achieved in only 5 hours on a standard workstation (one GV100).

4. Conclusion
A new meshless scheme called γ-SPH-ALE based on the combination of the SPH-ALE formalism and an SPH version of a finite volume low-Mach scheme was proposed. Free from spurious oscillations and correcting entirely the tensile instability, it provides reliable results by preventing numerical fractures. Fully implemented in the Impetus Afea Solver®, γ-SPH-ALE also benefits greatly from massively parallel processing with GPU Technology which provides such compelling results at 100 times faster than legacy SPH solvers. Armed with a solver technology that demonstrated stability, robustness, accuracy, and fast computation times on academic test cases, the next step was to investigate more complex Multiphysics configurations such as shaped charges.

Through the first study, the ability of γ-SPH-ALE to handle the shaped charge jet generation process was demonstrated. The proposed scheme successfully reproduced the behavior of the HE, the copper liner, and their interaction to generate a correct jet, regarding experiment and reference numerical results, in terms of shape, length, and velocity. γ-SPH-ALE accurately handles the jet fragmentation process induced by its elongation resulting from a necking mechanism which is clearly shown in the experimental results. In the second study, γ-SPH-ALE ability to reproduce the jet penetration capabilities was used to model an impact scenario of a shaped charged jet with a steel target. The results showed that the methodology was able to accurately reproduce the experimental penetration mechanisms at the...
expected times. Also, the jet continuity is preserved even after complete perforation and the debris clouds topology was captured.

Finally, γ-SPH-ALE was able to provide a stable and robust shaped charge simulation with a quasi Lagrangian meshless method in computation times that rival other methods and provide industrial users a extremely fast, cost effective and efficient design tool. Future work will include more complex shaped charges and impact configurations.

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