Review on Stick-slip Vibration’s Mechanism Analysis and Its Control Strategy in Deep Drilling with Drag Bits

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Abstract. The stick-slip vibration is one of the major causes for drilling quality reduction, shorter drilling tool lifespan, drilling efficiency decrease, it also affect the drilling cost and completion period seriously. The complex situations in the process of drilling result in the slow progress in this aspect, they rarely break through. The recent advances are outlined in stick-slip vibration of the drill string in three areas: theoretical analysis, experiment research and control strategy. The main research direction, methods and existing problem in the progress are introduced, which will provide reference and guidance for further research.

1. Introduction
With the development of petroleum industry technology and the increase of demand for oil and gas resources, oil drilling has gradually developed to deep strata and deep sea areas. In the process of deep well drilling, with the increase of well depth, the confining pressure at the bottom of the well increases the hardness and plasticity of the formation. At the same time, deep well drilling needs to pass through many sets of strata, spans different geographies, and the corresponding geological conditions are complicated. [1] With the increase of drill string length, the equivalent torsional stiffness of drill string decreases and the transmission torque is insufficient. Under the friction action of drill string, bit, sidewall and bottom hole, the stick-slip vibration of drill string system is easy to occur. When slippage occurs on the drill string, the ground torque oscillation frequency decreases, the peak torque increases, and the bottom hole bit stops rotating. At this time, the drill string continues to twist under the rotation of the rotary table. When the accumulated energy in the drill string is enough to overcome the friction torque between the drill bit and the rock, the accumulated energy of the drill string is released instantaneously, and the drill bit will rotate at twice or several times the rotational speed of the rotary table, and the bottom drill tool will vibrate violently. As the swing intensifies, the bit rotation torque fluctuates sharply. Intermittent motion drill string bears alternating tension and compression stress and friction between the positive and negative process and the borehole wall, which can accelerates the passivation process of the drill bit. When it's sliding away, the collision friction between drill string and borehole wall and the interaction with drilling fluid will waste the energy provided by wellhead. In the actual drilling process, stick-slip vibration often occurs simultaneously with other forms of vibration, forming a variety of vibration forms of coupling vibration, which has a great impact on the damage of drilling tools, and seriously affects the penetration rate and completion cycle. At the same time, the irregular rotation of the drill bit and the vibration of the drill string will can reduce the wellbore quality.[2] Overseas research on stick-slip vibration of drill string began in the 1980s. This phenomenon was first discovered when measuring the torque of the rotary table. After decades of
research, its overall research progress is slow, compared with other forms of drill string vibration research, innovative results and practical guidance recommendations are less. The early research on stick-slip vibration of drill string is almost blank in China. Until the beginning of the 21st century, some researchers have carried out the research on stick-slip vibration of drill string one after another, and have achieved stage research results.

2. Mechanism of Drill-string Stick-slip Vibration

2.1. 1-DOF Model
In 1982, Belokobyl’skii and Prokopov[3] found that self-excited torsional vibration caused by friction between bit and rock during rock breaking played an important role in rotary drilling. In this paper, the drill string is regarded as a slender rod whose upper end rotates at constant angular velocity and the lower end of the rod is connected with a rigid cylinder and subjected to a friction torque. A 1-DOF centralized mass torsional pendulum model, which is widely used later, is established. In this paper, the author regards the friction between drill bit and rock as Coulomb friction, gives the vibration equation of stick-slip vibration of drill string and calculates the period of stick-slip vibration. This is the initial study on stick-slip vibration of drill string, but this model does not consider the influence of drill fluid viscous damping. Subsequently, Dawson[4] used the torsional pendulum model similar to Belokobyl’skii to study stick-slip vibration of drill string, but Dawson regarded the friction torque between drill string and borehole wall as a function of the change of drill string speed. Therefore, an important conclusion is drawn: when the rotating speed of the rotary table reaches a certain critical value, the stick-slip vibration phenomenon of the drill string will disappear, which is also consistent with the actual situation.

Kyllingstad and Halsey [5] based on Dawson's research, considering the difference between drill pipe and collar structure, give the equivalent moment of inertia of drill string system, consider bottom drill tool as a flywheel with concentrated mass, and give the motion equation of drill bit. Through the research and analysis of the model, the following conclusions are drawn: 1) the stick-slip vibration frequency of drill string is smaller than the natural frequency of the system, and the stick-slip vibration frequency further decreases with the decrease of rotary speed; 2) when the bit slips off, the rotary speed of the bit is more than twice that of the rotary speed; by controlling the rotary speed, the stick-slip vibration caused by downhole friction can be slowed down or even eliminated, which is also in line with Dawson's conclusion.

Lin and Wang [6] studied the effects of rotational speed of rotary table, natural frequency of drill string system and viscous damping of drilling fluid on stick-slip vibration of bottom-hole assembly by establishing a single-degree-of-freedom torsional pendulum model. The biggest difference between Kyllingstad model and Halsey model is that Lin et al considered the influence of viscous damping of drilling fluid on stick-slip vibration of drill string. Lin and Wang think that stick-slip vibration of drill string is a kind of self-excited vibration caused by non-linear friction between drill string and borehole wall. Due to the restriction of drill string structure and material properties, the critical speed of stick-slip vibration will be very large, so it is difficult to eliminate stick-slip vibration by increasing the speed. The viscous-slip vibration can be effectively reduced by increasing the viscous damping between the drill string and the borehole wall. However, while increasing the viscous damping, in order to ensure that the drill string rotates at the same speed, the turntable needs to provide greater torque.

Rudat [7] established a single-degree-of-freedom torsional pendulum model to predict the stick-slip vibration behavior of drill string by giving a set of drilling parameters, and to mitigate the stick-slip vibration by optimizing drilling parameters (rotating speed and drilling pressure). Mojuwa[8] used Rudat's model to study the torsional vibration response of drill string in viscous damped environment.

In 2001, Huang Genlu and Han Zhiyong [9-10] established a single-degree-of-freedom torsional pendulum model to analyze the stick-slip vibration mechanism of drill string in extended reach wells. The model considers that bit torque and friction torque between drill string and borehole wall are continuous functions of actual angular velocity of drill string. Through the analysis of the steady state of the drill string rotation and the torsional vibration energy of the drill string, it is found that the stick-
slip vibration of the drill string originates from the negative equivalent viscous damping of the drill string system or the vibration energy absorbed by the drill string system from the outside world, and the discriminant formula of whether the stick-slip phenomenon occurs in the drill string is given. Through the analysis of the influence parameters of stick-slip vibration of drill string, the ways and measures to reduce the vibration of stick-slip vibration are given.

2.2. 2-DOF Model

Brett[11] improved Kyllingstad's single-degree-of-freedom torsional pendulum model, established a two-degree-of-freedom concentrated mass torsional pendulum model, and studied the influence of bit structure and characteristics on stick-slip vibration of drill string. Through theoretical and experimental analysis, the following phenomenon of torsional vibration of drill string are explained: 1) The reason of the torsional vibration of drill string occurs in shallow vertical wells with small friction torque; 2) PDC bit is more prone to stick-slip vibration than tricone bit; 3) the bigger the bit pressure is, the blunter the bit is, and the smaller the rotation speed is, the more violent the stick-slip vibration of the bit is.

On the basis of Brett's model, Jansen [12-14] considers the influence of the driving system (the moment of inertia of the motor and the damping of the components of the driving system) on the vibration of the drill string, establishes the mathematical model of the drill string system, and gives the motion equations of the sticky stage and the slippage stage of the drill string system, respectively. In this paper, the dynamic characteristics of stick-slip vibration of drill string system are analysed, and the idea of using active damping control system to eliminate stick-slip vibration of drill string is put forward, that is, by applying damping at the top of drill string to prevent the transmission of torque wave at the turntable, the stick-slip vibration of drill string is eliminated.

By establishing a two-degree-of-freedom torsional pendulum model, Navarro-López[15-16] studied stick-slip vibration of drill string. The greatest difference from previous studies was that the model considered the influence of WOB on stick-slip vibration of drill string. Different friction models were used to describe the interaction between drill bit and rock, and the motion equation of drill string system was given. The length and stick resistance of drill string were analysed by experiments. The influence of viscous damping coefficient, rock properties and bit type on stick-slip vibration of drill string.

Without considering the transverse vibration of drill bit, Patil and Teodoriu[17] studied the stick-slip vibration of drill string in vertical wellbore. By assuming that the mud damping coefficient and motor driving torque are constant, a two-degree-of-freedom torsional pendulum model similar to Jansen's was established.

The system motion equation is similar to that established by Navarro-López, but Patil considers the influence of bit diameter on friction torque when analysing the friction torque of bit. In this paper, the effects of rotational speed of rotary table, drill pressure, stiffness and inertia of drill string and rock strength on the drill string penetration rate are analysed in detail by establishing MATLAB/SIMULINK simulation model. The simulation results are in good agreement with the field data.

Richard [18] established a two-degree-of-freedom lumped mass model to study the self-excited stick-slip vibration of drill string system from a new angle. The biggest difference between the model and the previous model is that the model analysed the axial and torsional vibration of drill string based on the coupling of cutting and friction between drill bit and rock stratum.

Germay[19-20] revised Richard’s model, took into account the quasi-helical effect of the bit during rock breaking, established the dynamic equation which took into account the state-dependent delay effect of rock breaking and introduced discontinuous friction term. The simulation results show that there are some phenomenon in the drill string system, such as axial and torsional stick-slip vibration, quasi-static limit cycle and bit runout. At the same time, it pointed out that the self-excited stick-slip vibration of the drill string is related to the number of blades and rotating speed of the scraper bit.

On the basis of Richard and Germany research, Kovalyshen[21] used the linear stability analysis method to study the source of self-excited stick-slip vibration of drill string. The results show that the cutting process of the bit and the interaction between the cutting plane of the bit blade and the rock are
the main reasons for the torsional vibration of the drill string. At the same time, the author considers that the self-excited stick-slip vibration of the drill string is caused by the weakening effect of the rotational speed of the bit in the process of rock breaking, and designs a series of experiments to verify the theoretical results.

Liu Xianbo of Shanghai Jiaotong University[22] studied the coupled axial and torsional vibration of 1000m drill string system by establishing a two-degree-of-freedom torsional pendulum model. In this paper, the author assumes that the borehole axis is vertical without considering the lateral motion of the drill string, and that the axial motion velocity and rotation velocity of the top of the drill string are constant. The stability of linearized model of drill string system is analysed by semi-discretization method. The parameters include bit angular velocity, cutting coefficient and nominal cutting depth. In addition, the author uses the optimal feedback gain control method to improve the stability of the drill string system, and uses this method to control the stick-slip vibration of the non-linear drill string system, and achieves good control effect.

2.3. Multi-DOF Model
Navarro-López[23] improved the two-degree-of-freedom centralized mass torsional pendulum model, considered each drill pipe and bottom assembly separately, regarded the drill string system as a continuous system with smooth segments, and established a multi-degree-of-freedom torsional pendulum model. The influence of drilling parameters such as drilling pressure, motor output torque and rotary speed on stick-slip vibration of drill string is analyzed and studied. Based on the analysis results, the control method of stick-slip vibration of drill string is put forward. Subsequently, Navarro-López simplified the multi-degree-of-freedom model, considering the movement behavior of drill pipe and collar, and successively established Four-Degree-of-Freedom [24] model of rotary table-drill string-bottom drill bit assembly and three-degree-of-freedom [25] model of drill string system. The results show that the stick-slip vibration of drill string is closely related to drill pressure, rotating speed of rotary table and output torque of motor. Then, the dynamic sliding mode control method is used to eliminate stick-slip vibration of drill string system in vertical wells.

3. Experimental Study on Drill-string’s Stick-slip Vibration
The stick-slip vibration test of drill string includes field test and laboratory test. Field test is to monitor the movement of drill string on the drilling site by measuring instruments while drilling and other measuring tools. When stick-slip vibration occurs, the stick-slip vibration of drill string can be reduced or eliminated by adjusting drilling parameters. Through equivalent and simplified drilling system, a reasonable test device is built to simulate the movement behavior of drill string, and then the mechanism and influencing factors of stick-slip vibration of drill string are explored.

In 1986, Halsey[26] used MWD to measure the variation of bit parameters in a 1000m vertical well, and then studied the torsional and longitudinal coupling vibration of the drill string system, and validated the theoretical research results with field test data.

Dufeyte[27] found that stick-slip was accompanied by stick-slip for more than 50% of the drill string during the process of drilling. The effects of rotary speed, drilling pressure, drilling fluid performance, bit type, drill pipe type and borehole itself (rock characteristics, borehole trajectory and casing) on stick-slip vibration of drill string are analyzed in detail. The measured data show that the maximum speed of the bit in the slippage stage can reach 10 times of the rotating speed of the rotary table, which is also consistent with the theoretical analysis results.

Pavone[28] applied high frequency sampling technique to study stick-slip vibration of drill string. Measurements while drilling data show that stick-slip phenomena occur in drill string at 1060m depth under different bit pressures and rotating speed of rotary table. Through the mathematical modeling of drill string system, two methods, PID control and downhole anti-stick-slip tool are proposed to suppress stick-slip vibration of drill string. The simulation results show that the control effect is good.

Mihajovic[29-30] believes that the negative damping induced by friction in the well is the main reason for the vibration of drill string system. Laboratory tests were carried out to study the causes of torsional vibration of drill string system. The coupling of torsional and transverse vibration of drill string and the effects of dry friction and unbalanced mass on torsional vibration of drill string were
considered. The test device includes power amplifier, DC motor, upper and lower turntable, steel shaft and brake disc. The torsional stiffness of the steel shaft in the device is low, so that the stick-slip vibration of the lower turntable can be produced under the discontinuous friction torque exerted by the brake disc.

Through laboratory tests, Khalife[31] studied the effects of bit stick-slip, drill string contact with borehole wall and drilling fluid on drill string movement. The test device consists of steel frame, DC motor, stainless steel shaft with different aspect ratio and plexiglass tube containing drilling fluid. A magnetic particle tension brake is used to simulate stick-slip vibration of drill string under axial excitation. The effects of rotating speed of rotary table, length-diameter ratio of drill string and viscous damping of drilling fluid on drill string motion are studied. The coupling relationship between different types of vibration of drill string is also analysed.

Forster[32-33] carried out laboratory tests to reduce stick-slip vibration of drill string by applying axial excitation to drill bit. The main feature of the test device is that an axial excitation device is installed at the bit. When the drill string is sticky, the axial excitation device keeps the drill bit rotating by applying external load. The test results show that the stick-slip vibration of drill string can be effectively reduced by applying axial excitation. The higher the frequency of axial excitation applied, the easier it is for the bit to get rid of the sticky state.

Liao[34] of Maryland University, USA, studied the influence of friction coefficient and rotational speed between drill bit and rock on stick-slip vibration of drill string by establishing mechanical model and combining with laboratory test.

In 2014, Kovalys[35] validated his early theoretical research results on stick-slip vibration of drill string through laboratory tests. The experimental results are consistent with the theoretical research results, that is, stick-slip vibration of drill string is caused by the weakening effect of bit speed and self-excited vibration of drill string axis. Jia Xiaoli [36] of China University of Petroleum studied the influence of rotary speed, drilling pressure and rock properties on stick-slip vibration of drill string in deep wells by setting up a stick-slip vibration test rig.

4. Control Strategy of Stick-slip Vibration

Hardware shock absorber can be used to suppress stick-slip vibration. For example, Navarro López, Suarez Cortez and Vigue reported a passive vibration absorber [37-38], which was installed at the bottom of the drill pipe. Another kind of vibration absorber is the water pressure moment drive system proposed by Richardson[39], which is placed in the assembly of drilling tools. However, the use of hardware vibration absorber not only increases the complexity and cost of the drill string system itself, but also requires different vibration absorber tools for different drill string systems. However, there are no such problems when active control is used to suppress stick-slip vibration. Engineering practice shows that the control method of drilling stick-slip vibration should be satisfied: the torsional vibration of drill pipe can be effectively suppressed; because the drill string system itself is an uncertain system, the control strategy should be adaptive and robust; and the implementation of the control method is simple.

In literature [40], reduced-order PI compensator and friction compensator are used to compensate the interference of drilling process. In view of the non-linear characteristics of drill string system, researchers have applied various non-linear control methods to drill string system, such as H∞ control, sliding mode control and back stepping control [41-42], and integrated traditional PID control into it. They have explored the control methods of restraining stick-slip vibration and maintaining the stable operation of drill string system from different aspects. Literature [43] establishes a model of stick-slip vibration of drill string under different conditions, and obtains a discrete control strategy by using bit's drill pressure, torque and angular velocity at the top of drill string; Literature [44] adopts a dynamic sliding mode control to solve the stick-slip problem in vertical drill string system; Literature [45] studies the stick-slip vibration frequency, rotational speed and viscosity coefficient in drill string system of influence on drilling vibration by phase plane method.

Tucker[46] proposed a PI speed controller based on spatial distributed drill pipe model in 1999, but this low-order model cannot completely suppress torsional vibration and has no adaptive ability. Al-Hiddabi[47] designed a full-order controller based on two elastic mass block models. The dependence
of the sensitivity of the controller on the control parameters is also studied. Puebla and Alvarez Ramirez [48] analysed this dependence by adding a non-linear drill pipe parameter estimator to the controller. The use of robust controllers to control stick-slip vibration in drilling has also been studied, such as sliding mode controllers [49], H∞ controllers and so on. A full-order feedback or full-order estimator is needed to adopt the controller and estimator method proposed by Abdulgalil [50].

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