Diversity considerations in wideband radar detection of migrating targets in clutter

François LE CHEVALIER* & Nikita PETROV*

Department Microwave Sensing, Signals and Systems (MS3), Delft University of Technology, Delft 2628 CD, The Netherlands

Received 2 November 2018/Revised 18 December 2018/Accepted 5 January 2019/Published online 20 February 2019

Abstract: Wideband radars have been proposed for detection of moving targets, with unique capability of non-ambiguous detection due to range migration. Moreover, frequency diversity has long been used for mitigating the fading effects caused by target and clutter fluctuations. The real benefits of wideband radars are difficult to analyze, since they derive from the combined effects of target resolution in range, migration over successive cells of clutter, Doppler resolution and instantaneous bandwidth, and residual ambiguities in Doppler. In order to contribute to a better understanding of the benefits of agile transmissions for detection of moving targets, clutter cancelling performances of wideband radars are examined, demonstrating clear benefits from diversity on clutter and target, primarily – but not only – obtained through target migration effects. Special attention is given to long-range surveillance and tracking, and new results on detection of moving targets in clutter will be provided to demonstrate the effectiveness of these new architectures for small targets detection at long range, in difficult environments. Finally, recommendations for system designs that improve the discrimination of moving targets against fixed and diffuse clutter are presented.

Keywords: radar, extended target, frequency diversity, wideband detection, target migration, clutter cancelling

1 Introduction

1.1 Objectives

In order to design a surveillance radar1), a critical point is the “illumination time”, also called the “time on target”: this time duration should be long enough to allow Doppler analysis, and to gain a sufficient signal to noise ratio (SNR), but without compromising the fast update rate required by the user. This well-known trade-off between update rate and velocity resolution also involves the antenna beamwidth (the wider the beam, the better the velocity resolution, for a given update rate), and the clutter rejection capability (the wider the beam, the higher the clutter level). Moreover, it directly affects the power budget (the wider the beam, the lower the antenna gain, but also the higher the coherent integration gain, for a given update rate).

These intricate relations between beamwidths, velocity resolution, and power budget (hence, range) become even more complex when taking into account the fluctuation characteristics of the targets and

* Corresponding author (email: F.LeChevalier@TUDelft.nl, N.Petrov@TUDelft.nl)

1) This paper focuses on the basic issue of detection of moving targets with a ground surveillance radar, taking into account ground clutter.
Le Chevalier F, et al. Sci China Inf Sci April 2019 Vol. 62 040302:2

Figure 1 Detection and false alarm probabilities after imposing a threshold $T$ on the detected quantity $X$ [1].

Clutter, since performances can be improved through an increased averaging of clutter and target echoes – averaging which may itself be eased through widening of the beam, or longer illumination time. Such improvements are often more difficult to analyze, because they arise through modifications of the clutter and targets distribution functions, more complex than mere mean or standard deviation modifications: clutter and targets radar cross-sections being generally not gaussian, averaging several samples usually changes the shape of the resulting distribution functions.

Moreover, modern radar systems generally operate over significant relative bandwidths – typically 10% – which can be exploited either coherently, with a wide instantaneous bandwidth (providing a fine range resolution), or non-coherently, with a collection of measurements in different sub-bands. Again, these different operating modes have consequences on the power budget, but also on the distribution functions of targets and clutter.

The purpose of this paper is to try and clarify, with intuitive reasoning rather than precise equations, the order of magnitudes of these competing effects, so as to provide the designer with some basic insight necessary for building new radar architectures, involving multiple transmitting/receiving channels and arbitrary waveform agility.

1.2 Canonical problem

Detection being a 2-hypothesis problem ($H_0$: no target, $H_1$: a target), it basically comes down to comparing a certain quantity $X$, function of the received signals and of the expected situations (e.g., energy of the output of a matched filter), to a threshold depending on the required probability of detection $P_d$ and probability of false alarm $P_{fa}$. This situation is shown in Figure 1, where the position of the threshold $T$ defines the probability of detection $P_d$ (shaded with oblique lines) and the probability of false alarm $P_{fa}$ (area with horizontal lines).

Obviously, the shape of the probability density $p_{x/H_0}$ (probability of the received signal, under hypothesis $H_0$) is critical here. Using diversity is a means to improve the separation: generally speaking, averaging quantities is a way to reduce the spread (and change the shape) of each probability density functions, and to bring it closer to a Gaussian (central limit theorem); Using coherent integration, or more generally matched filtering, is a way to increase the mean value of $X$ under hypothesis $H_1$. Both techniques thus improve the separation, in different ways: narrowing each distribution function, or shifting them along the horizontal axis. Our objective here is to clarify these effects and assess their consequences in typical situations.
2 Standard detection [1, 2]

Statistical detection of radar fluctuating targets in the presence of noise is limited by the presence of noise and by the fact that the target may provide only very small signals for certain presentation angles or frequencies of illumination (a phenomenon also known as target fading in the literature). In order to mitigate target fading most radars use frequency agility:

(1) Successive bursts are transmitted at different carrier frequencies;
(2) When received, each burst is coherently processed as usual (Doppler filtering in each range cell);
(3) The outputs of these coherent summations are non-coherently summed (sum of the moduli, or the squared moduli), before final detection thresholding is applied.

This way improves the signal to noise ratio through each coherent burst processing. The resulting non-coherent summation allows consideration of observations at different frequencies, involving different target radar cross sections (RCSs). In practice, these different bursts also generally use different repetition frequencies, allowing removal of the ambiguities in range and Doppler [1].

For a high required probability of detection, “some” non-coherent integration is preferable, in order to avoid getting trapped in a low RCS zone, especially for highly fluctuating targets (e.g., Swerling 1 targets, in the standard classification of targets fluctuations [1, 3]). “Some” means that coherent integration must first be used to get a sufficient SNR which should typically be larger than 0 dB after coherent integration\(^2\), so that it is not too much degraded by the modulus operation which, as every nonlinear operation, would severely reduce the detection capability if it were done at low SNR. This explains the often used “golden rule”: first improve SNR through coherent integration, and then mitigate the low RCS zones by sending a few bursts with frequency agility from burst to burst. The price to pay for that noncoherent integration (and the associated diversity gain) when the available time-on-target is limited, is a lower Doppler resolution because of shorter coherent bursts.

The next question is: how many bursts are required during the time on target? Figure 2 plots the required SNRs per burst as functions of detection probability for \(N = 6\) and 10 bursts on Swerling targets [1, 3, 4]. The diversity gain can be defined as the ratio of the required SNRs per sample in the coherent (numerator) and non-coherent (denominator) summation cases. In the Swerling cases 1 and 2 (top panel of Figure 2), the diversity gain is clearly higher for \(N = 6\) (ranging from 2.6 to 7.0 dB, depending on the required probability of detection) than for \(N = 10\). Although the diversity can improve at higher resolutions, the gain is much smaller than at lower resolution.

Similar analyses with Swerling case 3 and 4 targets (bottom panel of Figure 2) show that the gain is lower – as expected, since the fluctuations of Swerling 3 targets are smaller than those of Swerling 1 – but still exists, at least for detection probabilities larger than 0.8 (i.e., gain between 2.7 and 1 dB). The diversity gain would then become a loss for very high resolution of Swerling 3 targets (1 dB loss for a target analyzed in 30 cells and a required \(P_d = 0.8\), not shown in the figure).

This basic analysis of diversity leads to the following conclusions:

- The more the target fluctuates, the higher the diversity gain;
- The diversity gain increases when the probability of detection \(P_d\) requirement is higher;
- The number of bursts should be between 5 and 10, and should not exceed 10.

A very similar reasoning could apply in the angular/spatial domain, as just shown here in the range/frequency domain. For a multistatic system with a few radar sites, some non-coherent integration will nicely complement the coherent integration of the signals received by each site. Depending on the exact signature of the target, spatial diversity, or frequency diversity, could be preferable: frequency diversity when the scatterers are distributed in range, spatial diversity when the scatterers are distributed in angle. A good solution, if possible, consists in combining both, for example with two or three frequencies per site, and two or three transmitting and/or receiving sites. However, it should be emphasized that frequency diversity is very generally an existing feature on most medium/long range monostatic radars (because most of them use multi-bursts operation, for ambiguity/eclipses removal), whereas spatial diversity, requiring

\(^2\) As known from standard probability, signal to noise after quadratic detection is equivalent to the square of the signal to noise at the input, for low SNR – whereas it is equivalent to the input SNR minus 3 dB for high SNR.
multisite implementations, is only applicable for specific situations, such as passive radars as discussed by Cherniakov [5] and Chernyak [6].

3 Wideband radar detection of moving targets

An essential limitation for standard narrowband radars using bursts of periodic pulses comes from the well-known pulsed radar range-Doppler ambiguity relation, which relates the ambiguous velocity $V_a$ to the ambiguous range $D_a$ through the relation:

$$D_a \times V_a = \left(\frac{c}{2}\right) F_\gamma \times \left(\frac{\lambda}{2F_\gamma}\right) = \lambda \times c/4.$$  \hfill (1)

That relation means many ambiguities, either in range or velocity (or both), need consideration. This in turn implies the transmission of successive pulse trains with different repetition frequencies, requiring more time to be spent on target for ambiguity and blind speeds removal (without a corresponding gain in Doppler resolution, since the successive coherent pulse trains are then processed incoherently).

An alternative solution [1,7] consists in improving the range resolution (through increasing the instantaneous bandwidth) so that the moving target range variation (range walk or range migration) during the pulse train becomes non-negligible compared with the range resolution. Such radars may use bursts with a low pulse repetition frequency (no range ambiguities) and wideband pulses such that the range walk phenomena during the whole burst is significant enough to remove the velocity ambiguity (the range walk being a non-ambiguous measurement of the radial velocity). It then becomes possible to detect the target and measure range and velocity with only one long coherent (and wideband) pulse burst (Figure 3).

With such wideband radars, using Parseval’s theorem, we first observe that the energy in the squared modulus of the impulse response (range profile) is the same as the energy in the squared modulus of the frequency response. So, summing the energy of the impulse response along the length of the target is equivalent, from a detection point of view, to summing the energy of the corresponding frequency response – which is usually done by summing the squared outputs of the successive integrated pulse trains (coherent integration of each burst, followed by non-coherent summation over the successive bursts).

Integration along the range profile of the target for a pre-assumed length of the targets of interest (e.g., 15 m for air targets) is a way to combine coherent integration, used to obtain the range profile
with its associated Doppler analysis in each range cell, with noncoherent integration. In other words, for wideband radars, coherent integration time – and the clutter separation in Doppler that it provides – needs not be reduced to take benefit of diversity gain. Thus summing the squared outputs of the successive integrated bursts, in each range cell, of a narrowband agile radar, is equivalent to summing the squared samples of the range profile of a high range resolution radar.

An appropriate detector for such wideband radars has been designed by Petrov et al. [8], ensuring CFAR (constant false alarm rate) performance with respect to the clutter texture, the speckle correlation matrix, and the target velocity. Comparison in Figure 4 includes the CFAR detector DIM-LRT (for dependent interference model – likelihood ratio test), and the clairvoyant detector (assuming known correlation matrix of clutter), for a point target. The loss of the proposed detector in comparison to the clairvoyant one is about 1 dB in each scenario. The analysis shows that target detection performance does not depend on clutter spatial correlation $\gamma$, but it depends on target velocity. Thus, the detection gain for the point target moving at $v_0 = 15$ m/s, with a range-walk of about 3 range cells during the coherent processing interval, with respect to the stationary one, is about 7 dB in $K$-distributed clutter (a frequently used model for high resolution clutter [9]) with shape parameter $\nu = 0.5$.

This phenomenon – improved detection of fast moving targets – can be well explained by the diversity of clutter, obtained by coherent integration of the target response during its migration over a few range cells (See Figure 3). The faster the target, the more it migrates during the whole burst, the lower the probability of missing the target due to a possible clutter spike in one range cell, so the higher the probability of detection: target range-walk (migration) along non-Gaussian clutter thus provides a new way to exploit clutter diversity. This behavior is similar to detection of range-extended targets in CG clutter, where the detection performance depends on the target extent (see e.g., [10]). The observed diversity gain is not linear and saturates as the number of range cells increases: we have observed that
the major improvement is obtained by the first 3 range cells migration, and fully saturates when the range-walk exceeds 5 range cells.

4 Diffuse clutter suppression

It is interesting to look in more detail at diffuse clutter (the clutter component which includes slowly moving elements, such as tree branches, grass) suppression. An appropriate model for clutter [11] is described on Figure 5: it includes a stationary DC component, and an exponential spectral component. Using this model, a perfromance criterion for clutter suppression can be derived, as a signal to clutter + noise ratio (SCNR loss).

The signal to clutter + noise ratio SCNR is defined as the output signal divided by the residual clutter (after whitening):

$$SCNR = \frac{||a||^2}{\sigma_n^2} a^H (I + \frac{\sigma_c^2}{\sigma_n^2} Q)^{-1} a,$$

(2)

where $a$ is the target steering vector (including the migration effect over a certain number of range cells, assumed to be lower than $K$, the maximum number of range cells migrated by the target), $\alpha$ is the amplitude of the target echo, $\sigma_n^2$ is the variance of thermal noise, $\sigma_c^2$ is the variance of clutter, $Q$ is the correlation matrix of clutter.

The normalized clutter covariance matrix $S$ defines the temporary correlation properties of clutter. Therefore, the clutter is modeled as a $(KM \times 1)$ zero-mean Gaussian vector with normalized covariance matrix $Q = R \otimes S$, Kronecker product of the spatial ($R$) and temporal ($S$) covariances. Here $M$ defines the number of pulses.

The performance degradation due to clutter presence in the scene is characterized by the SCNR loss factor $L$, which is defined as the ratio of the SCNR in the clutter limited scenario to the SNR in the noise limited scenario

$$L = \frac{SCNR}{SNR} = \frac{a^H (I + \frac{\sigma_c^2}{\sigma_n^2} Q)^{-1} a}{a^H (I)^{-1} a} = \frac{1}{M} a^H \left( I + \frac{\sigma_c^2}{\sigma_n^2} Q \right)^{-1} a.$$

(3)

Concerning the ratio $r$ between the power of the stationary and diffuse components, it could be argued that wideband reduces the diffuse clutter more than the stationary clutter, since diffuse clutter is more evenly distributed. In other words, reducing the range cell always reduces the diffuse clutter by the same factor, whereas the stationary clutter is reduced by a lower factor (especially if it is spiky). Therefore, this comparison can be considered as a worst case scenario for wideband radar.


### Table 1  Characteristics of wideband and narrowband radars

| Parameter | WB radar | NB radar |
|-----------|----------|----------|
| $f_c$     | 10 GHz   | 10 GHz   |
| $B$       | 1 GHz    | 10 MHz   |
| $T_r$     | 1 ms     | 1 ms     |
| $M$ (pulses) | 64      | 16       |
| $V_a$     | 15 m/s   | 15 m/s   |

The loss is then a function of the target velocity $v$, through the steering vectors $a$.

Using this criterion, a comparison can now be made between a narrowband (NB: 10 MHz) modern radar and a wideband (WB: 1 GHz) radar, whose parameters are given on Table 1. The central focus of this paper being to compare standard narrowband radars with standard wideband radar, we will make this comparison using two radars with the same time on target (64 ms). The narrowband radar uses 4 bursts of 16 pulses (non-coherently summed for ambiguity removal), whereas the wideband radar uses one burst of 64 wideband pulses, coherently processed for migration compensation.

The results are shown on Figure 6, as residual clutter after adaptive filtering, and on Figure 7, as loss factor for different wind values according to model [11]. In this example, only diffuse clutter is assumed, and the input clutter to noise ratio is assumed to be 20 dB higher for narrowband case, compared to the wideband case, because the range resolution is 20 dB larger in NB than in WB radar.\(^4\)

Under these conditions of strong diffuse clutter, the wideband mode improves the clutter rejection by 7 dB (on average) relative to the narrowband mode, with stronger rejections for weaker clutter levels.

Turning then to situations where both stationary and diffuse clutter are present, Figure 8 shows that the performances are indeed essentially limited by the diffuse clutter component. In the presence of stationary clutter, the loss at the first residual ambiguity is limited at less than 10 dB. Of course for wideband radar, the diffuse component is generally smaller than the stationary one, because the range cells are small, and the Doppler resolution is high (as stated above, we are comparing radars with the

---

\(^4\) Actually, strictly speaking, for wideband radar, the longer integration time also decreases diffuse clutter with respect to the target RCS, in each Doppler cell. This does not change the CNR, since the noise is decreased similarly, but does decrease the clutter to target ratio. Therefore, this comparison can again be considered as a worst case scenario for wideband radar.
same time on target, so the Doppler resolution of the narrowband multibursts is better than that of the wideband single burst).

Migration then appears to improve detection of targets in clutter, both through diversity gain (target passing over different patches of clutter), and through reduction of clutter after ambiguity removal, especially for stationary clutter – which is the main clutter component remaining for wideband radar (improving the range resolution decreases the diffuse clutter proportionately, whereas stationary clutter is less reduced, due to the existence of point-like scatterers).

5 Extended targets and clutter diversity

Given that a target observed by a wideband radar is actually extended, let us now consider different extended target models, as shown in Table 2. For such a target, the residual target to clutter ratio is defined as: SCR = $Aa^HQ^{-1}a$, where $A = \sum_{r=1}^{4} w_r |\alpha_r|^2$, $w_r$ being the weight of the $r$th coefficient and $Q$ being the correlation matrix of clutter. In this example, $K$ (maximum number of range cells migration) = 8, $M$ (number of pulses) = 32, $B$ = 1 GHz, $f_c$ = 10 GHz.

The obtained results are shown on Figure 9, for the cases of a migrating target and a non-migrating target: they clearly show that diversity, obtained for an extended target by integration over the target extent (assumed known a priori, and equal to 4 range cells) can be alternatively achieved by integrating a migrating target along its range-walk (migration = 1 m, i.e., 6 range cells, in this example). When the target is migrating, all types of targets provide similar detection performances – and these performances are improved by more than 5 dB, compared to the non-migrating case, because of the benefits of clutter diversity (target passing over different patches of clutter).

Generally speaking, the radar range resolution should be selected such, that the target of interest
Table 2  Four different target extent situations; in each case, the target extent is assumed to be 4 range cells for signal processing (non-coherent summation along the target extent profile)

| Model number | Cell number |
|--------------|-------------|
| 1            | 1/4         |
| 2            | 1/2         |
| 3            | 3/4         |
| 4            | 1           |
| 2            | 1/4         |
| 3            | 1/4         |
| 4            | 0           |
| 3            | 0           |
| 4            | 0           |

Figure 9  (Color online) Residual clutter after adaptive filtering of stationary (a) and moving (b) targets in the K-distributed model of clutter.

(given its expected dimensions and radial velocity) is spread over 5–10 range cells (consistent with the values in this example, and with the “saturation effect” mentioned at the end of Section 3), as a result of its range extent and its range migration. So meter resolution of the surveillance radar is sufficient for detection of typical air targets with a spatial diversity.

6 Discussion

The above analysis raises the following question: how to combine diversity effects when using agile waveforms? Let us take a baseline example, with a typical modern radar using digital beamforming in elevation only, a chirp waveform with pulse length 100 µs, and a pulse repetition frequency 1 kHz. Any designer would like to benefit from:

1. High Doppler resolution, for visibility of slow and/or weak targets;
2. High angular resolution, in elevation (for altitude measurement) and azimuth (for tracking);
3. Diversity on the target, for improved detection in thermal noise;
4. Diversity on clutter, for improved detection in clutter.

The first point requires long coherent integration time – but anyway this coherent integration time is limited by the fluctuations of the aspect angle of the target, typically to less than 100 ms\(^5\).

The second point requires accurate angular measurements (monopulse) with narrow beams on transmit and receive.

The third point requires observations at different carrier frequencies, or from different aspect angles (multistatic system), or integration along a high resolution range profile.

The fourth point requires the target to be superposed to different patches of clutter, either through range migration or range extent of the target, or through multi-bursts with different range ambiguities (so that the target folds over different clutter patches).

5) There may also exist rapid fluctuations of the target due to moving parts (jet engines, rotating blades, wheels); however, for moving targets detection, these parts have lower radar cross-sections than the cell of the target, so they can be ignored in this general assessment.
These requirements cannot usually be met by standard solutions, such as pencil beam with low range resolution (because of a limited velocity resolution due to a short time on target), or standard digital beam forming with no ambiguity in range (because of a limited angular resolution due to the wide beam on transmit, and limited diversity on clutter).

Facing those trade-offs, several baseline solutions can be sketched:

(a) Pencil beam, high range resolution, unambiguous in range (low PRF). This solution satisfies all requirements if the coherent integration time is sufficient – and also provides valuable target analysis capabilities, made possible through exploitation of the high resolution range-Doppler signatures.

(b) Space-time coding, low range resolution, ambiguous in range (high/medium PRF). This also satisfies all requirements; pure circulating codes [4] could be a preferred solution in strong clutter environments, providing very low sidelobes everywhere.

(c) Space-time coding with high range resolution, unambiguous in range (low PRF). Low sidelobes, high diversity, combined with valuable target analysis capabilities, exploiting the high resolution range-Doppler signatures.

These baseline descriptions should of course not be considered as definitive solutions to the very complex task of defining a multifunction radar (for instance, multistatic solutions could also make sense, possibly combined with space-time coding for solving the “rendez-vous” issue [6]). The objective was rather, as outlined in introduction, to highlight and clarify some specificities of diversity effects which have to be considered when designing future systems. Many other aspects, from complexity and cost to multifunction requirements, have also to be taken into consideration – and should also bring out different advantages of high resolution and space-time coding for surveillance radars.

Acknowledgements This work was partially supported by STW (now Toegepaste en Technische Wetenschappen (TTW)) (Grant No. 12219).

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

References

1 Le Chevalier F. Principles of Radar and Sonar Signal Processing. Norwood: Artech House, 2002
2 De Maio A, Greco S. Modern Radar Detection Theory. Raleigh: SciTech Pub, 2016
3 Richards M A, Scheer J A, Holm W A, et al. Principles of Modern Radar. Raleigh: SciTech Pub, 2010
4 Le Chevalier F. Wideband wide beam motion sensing. In: Advanced Ultrawideband Radar: Targets, Signals and Applications, Boca Raton: CRC Press, 2016
5 Cherniakov M. Bistatic Radar: Principles and Practice. Hoboken: Wiley, 2007
6 Chernyak V. Fundamentals of Multisite Radar Systems. Boca Raton: CRC Press, 1998
7 Bidon S, Lasserre M, Le Chevalier F. Bayesian sparse estimation of radar targets in the compressed sensing framework. In: Compressive Sensing of Earth Observations. Boca Raton: CRC Press, 2017
8 Petrov N, Le Chevalier F, Bogdanovic N, et al. Range migrating target detection in correlated compound-Gaussian clutter. In: Proceedings of the 25th European Signal Processing Conference (EUSIPCO), 2017
9 Ward K D, Tough R J A, Watts S. Sea clutter: scattering, the K-distribution and radar performance. Electron Lett, 1981, 17: 561–563
10 Gerlach K. Spatially distributed target detection in non-Gaussian clutter. IEEE Trans Aerosp Electron Syst, 1999, 35: 926–934
11 Billingsley J B. Exponential Decay in Windblown Radar Ground Clutter Doppler Spectra: Multifrequency Measurements and Model. Technical Report, 1996