Self-averaging in the three-dimensional site diluted Heisenberg model at the critical point

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Abstract. We study the self-averaging properties of the three-dimensional site diluted Heisenberg model. The Harris criterion states that disorder is irrelevant since the specific heat critical exponent of the pure model is negative. According to some analytical approaches, this implies that the susceptibility should be self-averaging at the critical temperature ($R_\chi = 0$). We have checked this theoretical prediction for a large range of dilution (including strong dilution) at criticality and we have found that the introduction of scaling corrections is crucial for obtaining ‘self-averageness’ in this model. Finally we have computed critical exponents and cumulants which compare very well with those of the pure model, supporting the universality predicted by the Harris criterion.

Keywords: classical Monte Carlo simulations, critical exponents and amplitudes (theory), finite-size scaling, disordered systems (theory)

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1. Introduction

It is a well known fact that a possible way to obtain new universality classes is to add disorder to pure systems. The Harris criterion [1] states that if the specific heat diverges in the pure system, the disorder will change the critical behaviour of the model, i.e. a new universality class will appear. Conversely, if the specific heat does not diverge in the pure system the critical exponents of the disordered system will not change.

The aim of this paper is to study the dependence of some observables on the disorder (self-averaging properties). The study of the self-averaging properties in disordered systems has generated in the past few years a large amount of both analytical, see [2]–[4], and numerical, see [5]–[7], works. We will focus on the computation at criticality of the quantity $R_\chi$, which will be defined later and is a measure of the ‘self-averageness’ of the susceptibility.

We will study the three-dimensional site diluted Heisenberg model with quenched disorder (in which, according to [1], the disorder is irrelevant) in order to test the analytical predictions.

To obtain accurate measures of critical properties, in particular of $R_\chi$, we will use finite size scaling (FSS) techniques such as the quotient method [8,9] that allows us to work in large lattices at the critical point and to perform infinite volume extrapolations.

We will show results strongly supporting that this $R_\chi$ cumulant is zero at the critical point, but only taking into account the scaling corrections, against some theoretical predictions [3] and supporting others [2,4].

The structure of the paper is as follows. In the next section we summarize some analytical predictions concerning the self-averaging properties of diluted models at criticality. In section 3 we define the model and the observables. In the first part of section 4 we describe our simulation methods, in the second part we analyse deeply the correction to the scaling exponent, $\omega$, while in the last part we give our numerical results.

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self-averaging properties of the susceptibility both in vector and in tensor channels. Results related to the universality of the critical exponents and cumulants are given in the appendix. Finally we give our conclusions in section 5.

2. Analytical predictions

The self-averaging (SA) of the susceptibility is defined in terms of

\[ R_\chi \equiv \frac{\langle M^2 \rangle^2 - \langle M^2 \rangle^2}{\langle M^2 \rangle^2}, \]

(1)

\( \mathcal{M} \) being the total magnetization. The susceptibility is self-averaging if \( R_\chi \to 0 \) as \( L \to \infty \).

In [2], the following picture was found:

1. Away from the critical temperature: \( R_\chi = 0 \). It can be found, on the basis of the renormalization group (RG) or using general statistical arguments, that \( R_\chi \propto (\xi/L)^d \) in a finite geometry, \( L \) being the system size and \( \xi \) the correlation length, which is finite for \( T \neq T_c \): then \( R_\chi \to 0 \) as \( L \to \infty \). This is called strong SA.

2. At the critical temperature, a renormalization group analysis opens two possible scenarios:
   - Models in which according to the Harris criterion the disorder is relevant (\( \alpha_{\text{pure}} > 0 \)): \( R_\chi \neq 0 \). The susceptibility at the critical point is not self-averaging. In particular, reference [2] shows that in these conditions \( R_\chi \) is proportional to the fixed point value of the coupling which induces the disorder in the Hamiltonian, which controls the new universality class. This is called no SA.
   - Models in which according to the Harris criterion the disorder is not relevant (\( \alpha_{\text{pure}} < 0 \)): \( R_\chi = 0 \). The susceptibility at the critical point is self-averaging. In a finite geometry \( R_\chi \) scales as \( L^{\alpha/\nu} \to 0 \), where \( \alpha \) and \( \nu \) are the critical exponents of the pure system, which are the same in the disordered one. This is called weak SA.

The observable \( R_\chi \) has been measured for other diluted models: for example in the four-dimensional diluted Ising model; see [10]. In this model a mean field computation and a numerical one found a non-zero value for \( R_\chi \) although the diluted model was shown to belong to the same universality class as the pure model, contradicting the conclusions of [2]. One can claim that the logarithms which live in the upper critical dimension make the numerical analysis difficult. In particular it was found analytically in the mean field that \( R_\chi = 0.31024 \) and numerically that \( R_\chi \in [0.15, 0.32] \). Because of the logarithms, it was impossible to do an infinite volume extrapolation for the numerical values of \( R_\chi \).

Notice that in this model the only fixed point is the Gaussian one (all the values of the couplings are zero) and, following [2], \( R_\chi \) should be zero.

In addition a two-loop field theory calculation done in [3] predicts a non-zero value for \( R_\chi \) for the diluted Heisenberg model (in which the disorder is irrelevant, \( \alpha_{\text{pure}} = -0.134 \); see [11]). The two-loop field theoretical prediction for \( \alpha \) in the pure case was \( \alpha_{\text{pure}} > 0 \), so, apparently, this work is consistent with the findings of [2]. The starting point in [3] was the mean field computation done in [10], modifying it to take into account the vector degrees of freedom, introducing the fluctuations using the Brezin–Zinn-Justin (BZJ) method, [12]. They found analytically \( R_\chi = 0.022688 \) for the vector channel and

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universally (independent of the dilution for all \( p < 1 \)). It is important to remark that in the BZJ method one fixes from the beginning the temperature of the system to the infinite volume critical one, working in a finite geometry, so in order to compute \( R_x \) in this scheme the following sequence of limits is used:

\[
R^*_x = \lim_{L \to \infty} \lim_{T \to T_c} R_x(L, T),
\]

where \( R^*_x \) is the infinite volume extrapolation at criticality of \( R_x(L, T) \), and \( T_c \) is the infinite volume critical temperature of the system. The other possible limits sequence that can be computed is

\[
\lim_{T \to T_c} \lim_{L \to \infty} R_x(L, T),
\]

which is zero even when the disorder is relevant since \( R_x \propto L^{-d} \) as \( T \neq T_c \).

Hence, in order to test these discrepancies we have simulated numerically the site diluted three-dimensional Heisenberg model computing \( R^*_x \) in the vector and tensor channels. To perform this programme, in particular in doing the infinite volume extrapolations of cumulants and exponents, really important is a proper use of the corrections to scaling.

3. The model

The Heisenberg site diluted model in three dimensions is defined in terms of O(3) spin variables placed at the nodes of a cubic three-dimensional lattice, with Hamiltonian

\[
H = -\beta \sum_{\langle i,j \rangle} \epsilon_i \epsilon_j S_i \cdot S_j,
\]

where the \( S_i \) are three-dimensional vectors of unity modulus, and the sum is extended only over nearest neighbours. The disorder is introduced by the random variables \( \epsilon_i \) which take value 1 with probability \( p \) and 0 with probability \( 1 - p \). An actual \( \{ \epsilon_i \} \) configuration will be called a sample.

In addition, as was done in [8], we define a tensorial channel associated with the vector \( S \) through the traceless tensor

\[
\tau^\alpha_\beta_i = S_\alpha_i S_\beta_i - \frac{1}{3} \delta^\alpha_\beta, \quad \alpha, \beta = 1, 2, 3.
\]

In the following, we shall denote a thermal average with brackets, while the sample average will be overlined. The observables will be denoted with calligraphic letters, i.e. \( \mathcal{O} \), and the double average with italics, \( \mathcal{O} = \langle \mathcal{O} \rangle \). We define the total nearest neighbour energy as

\[
\mathcal{E} = \sum_{\langle i,j \rangle} \epsilon_i \epsilon_j S_i \cdot S_j,
\]

and the normalized magnetization for both channels as

\[
\mathcal{M} = \frac{1}{V} \sum_i \epsilon_i S_i,
\]

\[
\mathcal{M}_T^{\alpha \beta} = \frac{1}{V} \sum_i \epsilon_i \left( S_\alpha_i S_\beta_i - \frac{1}{N} \delta^{\alpha \beta} \right),
\]

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V being the volume (defined as $L^3$, where $L$ is the lattice size). Because of the finite probability of reaching every minimal value for the free energy, the thermal average of equations (7) and (8) is zero in a finite lattice. Therefore, we have to define the order parameters as the $O(3)$ invariant scalars

$$M = \langle \sqrt{\mathcal{M}^2} \rangle, \quad M_T = \langle \sqrt{\text{tr} \mathcal{M}^2_T} \rangle. \quad (9)$$

We also define both susceptibilities as

$$\chi = V \langle \mathcal{M}^2 \rangle, \quad \chi_T = V \langle \text{tr} \mathcal{M}^2_T \rangle. \quad (10)$$

A very useful quantity is the Binder parameter, defined as

$$g_4^V = 1 - \frac{1}{3} \frac{\langle \mathcal{M}^4 \rangle}{\langle \mathcal{M}^2 \rangle^2}, \quad g_4^T = 1 - \frac{\langle (\text{tr} \mathcal{M}^2_T)^2 \rangle}{3 \langle \text{tr} \mathcal{M}^2_T \rangle^2}. \quad (11)$$

Another kind of Binder parameter, meaningless for the pure system, can be defined as

$$g_2^V = \frac{\langle \mathcal{M}^2 \rangle^2 - \langle \mathcal{M}^2 \rangle^2}{\langle \mathcal{M}^2 \rangle^2}, \quad g_2^T = \frac{\langle (\text{tr} \mathcal{M}^2_T)^2 \rangle - \langle \text{tr} \mathcal{M}^2_T \rangle^2}{\langle \text{tr} \mathcal{M}^2_T \rangle^2}, \quad (12)$$

and these are the quantities we are using to estimate the self-averaging properties of the susceptibility ($R_\chi$) in both channels.

A very convenient definition of the correlation length in a finite lattice reads, see [13],

$$\xi = \left( \frac{\chi/F - 1}{4 \sin^2(\pi/L)} \right)^{1/2}, \quad (13)$$

where $F$ is defined in terms of the Fourier transform of the magnetization:

$$\mathcal{F}(k) = \frac{1}{V} \sum_r e^{ik \cdot r} \epsilon_r S_r \quad (14)$$

as

$$F = \frac{V}{3} \langle |\mathcal{F}(2\pi/L, 0, 0)|^2 + \text{permutations} \rangle. \quad (15)$$

The same definition is also valid in the tensorial case. This definition is very well behaved for the finite size scaling (FSS) method that we have employed; see [8]. Finally, we measure the specific heat as

$$C = V^{-1} \langle \mathcal{E}^2 \rangle - \langle \mathcal{E} \rangle^2. \quad (16)$$

4. Simulations

4.1. Description

The lattice sizes $L$ that we have studied are 8, 12, 16, 24, 32, 48, 64, and, only in the pure model, $L = 96$. 

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Between each measure of the observables described in section 3, firstly, we update
the spin variables using a Metropolis method over 10% of the individuals spins, chosen
at random, then we perform a number (growing with \( L \)) of cluster updates using a Wolff
method—see [14]; this is our elementary Monte Carlo step (EMCS). The number of
clusters traced (or Wolff updates) between measures have been chosen to yield a good
value of the self-correlation time, see [14], in our case always \( 1 < \tau < 2 \) (\( \tau \) being the
integrated autocorrelation time of the energy).

In order to work in thermally equilibrated systems we perform a great number of
EMCS. We start the simulation always from random (hot) distributions of the spin
variables, although we have checked that averages do not change if we begin from cold
configurations (i.e. all spins being in the same direction). For concreteness, we have
taken \( 4 \times 10^6 \) measures for the pure model, discarding about \( 10^5 \) of the first measures
for \( L = 8 \) and growing this number with the lattice size. We have performed \( 2 \times 10^4 \)
quenched disorder realizations in the diluted models independently of the dilution and
the lattice size and taking 100 measures per sample, according to Ballesteros \textit{et al} [10]
who demonstrated that the best approach for minimizing the statistical error is to simulate
a great number of samples with a few measures in each one.

To measure the critical exponents, we use the so-called quotients method [8,9], which
allows for great statistical accuracy. The starting point is the equation

\[
Q_O|_{\xi=s} = s^{\nu / \nu} + O(L^{-\omega}).
\]

Here \( Q_O \) is the quotient of the observable \( O \) measured in a pair of lattices of sizes \( L \)
and \( sL \), at the temperature where \( Q_\xi = s \), \( \omega \) being the eigenvalue corresponding to the
first irrelevant operator in the language of the RG theory. We have fixed \( s = 2 \) in our
case. Therefore, firstly we need to estimate by successive simulations the \( \beta \) point where

\[
\frac{\xi(2L, \beta, p)}{2L} = \frac{\xi(L, \beta, p)}{L},
\]

for each pair of lattices \( (L, 2L) \); then we have used reweighting techniques to fine-tune
this condition. These reweighting techniques are used to \( \beta \) extrapolate the observables
and calculate their \( \beta \) derivatives, always before the sample averaging is performed. The
equations used are, see [10,14],

\[
\langle O \rangle(\beta + \Delta \beta) = \frac{\langle O e^{\Delta \beta E} \rangle}{\langle e^{\Delta \beta E} \rangle},
\]

\[
\partial_\beta \langle O \rangle = \langle O E - \langle O \rangle \langle E \rangle \rangle.
\]

These extrapolations are biased; for instance, the expectation value of equation (20),
when the averages are calculated with \( N_m \) measures, is

\[
\frac{1 - \frac{2\tau}{N_m}}{\partial_\beta \langle O \rangle},
\]

and hence we have to correct this bias. We have followed the recipe given in [10]. An
example of the effect of this correction is found in figure 1 (for the greatest lattice size
simulated and \( p = 0.9 \), the same applies to other dilutions and lattice sizes): a great
bias affects the non-corrected numerical data and the importance of taking this effect
into account is clear. In addition, it is clear that the recipe of [10] is working perfectly
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Figure 1. The $g_2$ cumulant in both channels for $L = 64$, $p = 0.9$ with 1000 samples, $\beta_{\text{simulation}} = 0.791 \pm 0.12$, reweighted at $\beta = 0.79082$ as a function of $1/N_m$, $N_m$ being the number of measures in each sample. We report data with $N_m = 50, 100, 500, 1000, 5000, 10\,000$. The data without the bias correction proposed in [10] are marked with triangles while the corrected ones are marked with circles. We also mark with the dotted lines the selection used in this work (which corresponds to $N_m = 100$). Notice the importance of the correction of the bias if one performs reweighting with the data.

For $N_m = 100$, which is the number of measures per sample we have taken in this work. Therefore, we are very confident that all the data presented in this paper, processing with the recipe of [10], are not biased due to the reweighting technique.

On the other hand, we have tried to use the solution obtained in [15], where each sample is split into four parts, but the results were bad; this is due to the small number of measures we take for each sample ($10^2$), which yields big differences between the averages in each quarter.

To compute errors in the averages we have used jackknife methods; see [14]. We have defined 20 jackknife blocks for the pure model in a unique sample and one block for each sample in the diluted ($p < 1$) models.

Calculated observables and critical exponents present sometimes, instead of a stable value, a monotonically decreasing one. For $\eta$, such an evolution with growing $L$ is found, but it is clearly weaker than for $\nu$. In these cases an infinite volume extrapolation is called for. If hyperscaling holds, we expect finite volume scaling corrections proportional to $L^{-\omega}$. This issue will be addressed in the next subsection.

4.2. The scaling exponent $\omega$

As can be seen in tables from A.1 to A.8 in the appendix, especially for the thermal exponents and the cumulants ($g_1$ and $g_2$), there are evident finite volume effects, so we have to use the equation

$$\frac{x_{\nu}}{\nu}\bigg|_\infty - \frac{x_{\nu}}{\nu}\bigg|_{(L,2L)} \propto L^{-\omega}, \quad (22)$$

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Table 1. \( \omega \) values from the \( L \to \infty \) extrapolations of some quantities. In the last row can be seen the weighted average of each column. We disregarded data with error bars bigger than 100\% of the values themselves. Those disregarded data are shown in the table as —.

\[
\begin{array}{lcccc}
\mathcal{O} & \omega_{p=1.0} & \omega_{p=0.9} & \omega_{p=0.7} & \omega_{p=0.5} \\
\eta_{\chi}^{V} & 1.45(52) & - & - & - \\
\eta_{M}^{V} & 1.62(80) & - & - & - \\
\eta_{\chi}^{T} & - & - & 1.2(1.1) & 0.68(46) \\
\eta_{M}^{T} & - & - & - & 0.73(46) \\
\nu_{\partial \phi \xi_{1}^{V}} & - & - & - & - \\
\nu_{\partial \phi \xi_{1}^{T}} & 2.30(61) & - & - & 0.62(47) \\
\nu_{\partial \phi \xi_{1}^{2}} & - & - & - & - \\
\nu_{\partial \phi \xi_{1}^{4}} & 2.12(52) & 1.76(60) & 1.09(40) & 1.34(27) \\
\xi_{1}^{V}/L & 1.08(21) & 1.21(31) & 0.61(12) & 0.45(10) \\
\xi_{1}^{T}/L & - & - & 1.55(76) & 1.64(17) \\
g_{1}^{V} & 0.85(14) & 2.00(61) & 1.21(15) & 1.19(13) \\
g_{1}^{T} & 1.06(14) & - & 1.35(33) & 1.41(42) \\
g_{2}^{V} & - & 0.81(16) & 0.89(9) & 0.94(7) \\
g_{2}^{T} & - & - & 0.63(12) & 0.72(10) \\
\bar{\omega}_{\text{weighted}} & 1.07(9) & 0.92(9) & 0.81(7) & 0.88(4) \\
\end{array}
\]

which is a consequence of the scale hypothesis, first derived in [16]. Consequently, choosing a good value for \( \omega \) is a crucial question.

Exact results and RG calculations tell us that the disorder, being irrelevant in this model, induces scaling corrections with an exponent \( \alpha/\nu \approx -0.188 \) (in \( L \)) [17]. In addition to this new scaling correction one must have that of the pure model, which is related to the coupling of the \( (\phi^2)^2 \) term in the Ginzburg–Landau theory. This exponent is assumed to be 0.8 [18, 19] (for the pure model). Hence, the leading one is the exponent induced by the disorder. We will try to check this scenario by computing the ‘leading’ correction to the scaling exponent from the numerical data.

First of all, we have tried to estimate \( \omega \) just by considering it as another tunable parameter in equation (22) applied to some physical quantities. In these fits, as a first approximation, we disregarded the possible correlations between the data for different \( L \) values; the results can be seen in table 1. If we perform a weighted averaging with these results we obtain \( \omega = 1.07(9) \) for the pure model and \( \omega = 0.92(9), \omega = 0.81(7) \) and \( \omega = 0.88(4) \) for the diluted model with \( p = 0.9, 0.7 \) and 0.5 respectively, in very good agreement with the value from the references given before. However, we think this method is not very reliable because of the variability of the results from one quantity to another as shown in table 1.

Another approach, following [20], is to study the crossing points of scaling functions (such as \( \xi/L \) and \( g_4 \)) measured in pairs of lattices with sizes \( L \) and \( 2L \). The deviation of these crossing points from the infinite volume critical coupling will behave as

\[
\Delta \beta(L, sL) \equiv \beta(L, sL) - \beta_c(\infty) \propto \frac{1 - s^{-\omega}}{s^{1/\nu} - 1} L^{-\omega - (1/\nu)}. \tag{23}
\]

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Figure 2. Upper part: $\chi^2$ as a function of $\omega$ deduced from the fits to $L \to \infty$, equation (23), for the crossing point of $\xi/L$ and $g_4$ for the $(L, 2L)$ pair for the pure model. Also shown is the combined $\chi^2$. Lower part: extrapolated $\beta_c(\infty)$, as an $\omega$ function; the point where the two observables give the same extrapolated value is marked with the dotted line.

With this method we need an additional estimate for the thermal exponent $\nu$; we have used, following [11], the value $\nu = 0.7113(11)$ for the pure model (notice the really small error in $\nu$—so we will discard it in the following; see the comment at the end of this section), which is also a valid value for the diluted models, because of the validity of the Harris criterion and as can be checked with the data in the appendix. Again we fixed $s = 2$. In this approach, we only use the crossing points in the vectorial channel because they are clearer.

Extrapolating this crossing points using equation (23), we can plot the minimum of the $\chi^2$ of the fit as a function of $\omega$, obtaining the upper part of figure 2 and the whole of figure 3. To carry out this extrapolation we have to realize that the measures of the crossing points are correlated in pairs, so we have to use the $\chi^2$ definition that includes the whole self-covariance matrix:

$$\chi^{2}_x = \sum_{i=1}^{N} \sum_{m=1}^{N} (x_i - \text{fit})(\text{cov}^{-1})_{i,m}(x_m - \text{fit}),$$

where $N$ being the number of crossing points, that is to say, the number of simulated $L$ values minus 2; $x_i$ is the value obtained for the observable $x$ ($\xi^V/L$ or $g^V_4$), at the crossing point for $L$ and $2L$, and ‘fit’ is the value fitted to the form of equation (23) for $L$. In addition, $\text{(cov)}_{i,m} = \langle x_m x_i \rangle - \langle x_m \rangle \langle x_i \rangle$ can also be defined in terms of jackknife blocks, see [14], as

$$\text{(cov)}_{i,m} = \frac{N_b - 1}{N_b} \sum_{i=1}^{N_b} (x_{i,j} - \langle x_i \rangle)(x_{m,j} - \langle x_m \rangle)$$

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Figure 3. Upper part: $\chi^2$ as a function of $\omega$ for the diluted $(p = 0.7)$ model. Also shown is the combined $\chi^2$. Lower part: $\chi^2$ as a function of $\omega$ for the diluted $(p = 0.5)$ model.

where $N_b$ is the number of jackknife blocks, $x^{j-K}_{l,i}$ are block variables, where the first subindex runs over $L$ values while the second one runs over jackknife blocks, and $\langle x_l \rangle$ is the average over all block variables, given $L = L_l$.

Also, following [20], we can do a combined fit in $\omega$ of the crossing points of $\xi^V/L$ and $g_4^V$ by defining

$$\chi^2_{\text{combined}} = \chi^2_{\xi^V/L} + \chi^2_{g_4^V} \quad (27)$$

using equation (24) to calculate each of the latter terms and searching for the minimum of $\chi^2_{\text{combined}}$. We can obtain the error in $\omega$ searching for the point $\omega_1$ at which $\chi^2_{\text{combined}}(\omega_1) = \chi^2_{\text{combined}}(\omega_{\text{min}}) + 1$, so the error is $\Delta\omega = |\omega_{\text{min}} - \omega_1|$. The results for the combined fits are shown in the upper part of figure 2 and in the whole of figure 3. We find with this method the values

$$\omega = 0.96(15),$$

for the pure model and

$$\omega = 2.29(70), 0.84(17), 0.64(13),$$

for the diluted models with $p = 0.9$, 0.7 and 0.5 respectively, in agreement with the value obtained with the pure model [8, 11, 18, 19], except in the $p = 0.9$ case, in which the value is far away: two standard deviations from $\omega = 0.8$ [18, 19]. One possibility is that we are computing the leading correction to the scaling exponent but with a large error. Another possibility is that for the $p = 0.9$ model the coefficients of the leading correction (from the numerical point of view) to the scaling vanish or are very small. This result and the change in the slope of the $g_4$ data for $p < 1$ with respect to the $p = 1$ ones, as can be seen in table 2, are evidence of the possible improved action found for $p = 0.9$, see [15], and therefore the $\omega$ exponent that we are measuring in this case could correspond to the third irrelevant operator, instead of the second one (remember that following RG the first one is $\alpha/\nu \simeq -0.188$).
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Table 2. Cumulants for the O(3) model. In the first column is represented the spin density \( p \). All the cumulants are calculated at the crossing points of \( \xi/L \) for \( L \) and \( 2L \). The averages have been computing using \( 10^4 \) samples (except in the \( p = 1 \) case).

| \( p \) | \( L \) | \( g_2^V \) | \( g_2^T \) | \( g_4^V \) | \( g_4^T \) |
|---|---|---|---|---|---|
| 1.0 | 8 | 0 | 0 | 0.62243(4) | 0.5216(1) |
| | 12 | 0 | 0 | 0.62172(5) | 0.5189(2) |
| | 16 | 0 | 0 | 0.62152(6) | 0.5181(2) |
| | 24 | 0 | 0 | 0.62100(5) | 0.5166(2) |
| | 32 | 0 | 0 | 0.62092(3) | 0.5162(1) |
| | 48 | 0 | 0 | 0.62066(5) | 0.5156(2) |
| 0.9 | 8 | 0.0327(4) | 0.0576(7) | 0.6151(2) | 0.5102(3) |
| | 12 | 0.0273(3) | 0.0518(6) | 0.6163(1) | 0.5104(3) |
| | 16 | 0.0253(3) | 0.0499(6) | 0.6166(1) | 0.5100(3) |
| | 24 | 0.0226(3) | 0.0453(6) | 0.6168(1) | 0.5098(3) |
| | 32 | 0.0208(2) | 0.0421(5) | 0.6171(1) | 0.5100(3) |
| 0.7 | 8 | 0.0780(8) | 0.1406(16) | 0.6061(3) | 0.4994(6) |
| | 12 | 0.0610(6) | 0.1177(13) | 0.6108(2) | 0.5039(5) |
| | 16 | 0.0512(5) | 0.1009(11) | 0.6131(2) | 0.5064(4) |
| | 24 | 0.0423(4) | 0.0868(10) | 0.6150(2) | 0.5077(4) |
| | 32 | 0.0371(4) | 0.0770(9) | 0.6160(2) | 0.5089(4) |
| 0.5 | 8 | 0.1130(11) | 0.2061(24) | 0.6006(4) | 0.4999(8) |
| | 12 | 0.0834(8) | 0.1600(18) | 0.6072(3) | 0.5047(6) |
| | 16 | 0.0702(7) | 0.1395(16) | 0.6107(3) | 0.5070(6) |
| | 24 | 0.0553(6) | 0.1138(13) | 0.6138(2) | 0.5085(5) |
| | 32 | 0.0474(5) | 0.0980(11) | 0.6151(2) | 0.5095(4) |

In addition, as [20] also did, we were able to estimate the correct value for \( \omega \) as the one producing the same \( \beta_c(\infty) \) value for \( \xi/L \) and \( g_4 \), as can be seen in the lower part of figure 2, marked with the dotted line for \( \omega = 0.88 \). This approach only works for the pure model in which such a point is found; with another \( p \) value the \( \beta_c(\infty) \) estimates from \( \xi/L \) and \( g_4 \) do not cross each other.

In conclusion, we have shown that our data (both for the pure and for the diluted model) are fully compatible with the value \( \omega = 0.80(1) \), obtained previously both numerically and analytically for the pure model\(^1\). In addition, since the error bars in \( \omega \) are really small (1% of error) we have discarded the uncertainty in \( \omega \) in the analysis presented in this paper; the error bars in the extrapolated quantities are much bigger than the uncertainty caused by the error bars in \( \omega \): so we have fixed \( \omega = 0.80 \). The extrapolations (tables 3, A.1–A.10) and figures 3–7, A.1 shown in the rest of the paper are obtained using this value of \( \omega \).

Finally, it is interesting to note that we have seen in the analysis presented in this subsection no traces of the leading correction to the scaling exponent even for the strongest dilution that we have simulated, which should be \( \alpha/\nu \simeq -0.188 \). One can explain this

\(^1\) In more detail, the field theoretical approaches (both fixed dimension and \( \epsilon \) expansion) provide very accurate values for \( \omega \): 0.782(13) and 0.784(18) (respectively) [18]. Recent numerical simulations bring us the values 0.775(13) and 0.790(13) [19] and 0.64(13) and 0.71(15) [8].
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Figure 4. Combined extrapolation to $L \to \infty$ for the $g_2$ cumulant of the vectorial susceptibility. Extrapolations are carried out by choosing a common value for the first term of equation (22) for all dilutions and by minimizing the combined $\chi^2$. We have disregarded the data with $L = 8$ to obtain a good value for $\chi^2$.

Table 3. Cumulants for the O(3) model with high $p$ values. All the cumulants are computed averaging only $10^5$ samples.

| $p$ | $L$ | $g_2^V$ | $g_2^T$ | $g_4^V$ | $g_4^T$ |
|-----|-----|---------|---------|---------|---------|
| 0.97 | 8   | 0.0108(6) | 0.0181(13) | 0.6201(4) | 0.5187(10) |
|     | 12  | 0.0102(6) | 0.0189(14) | 0.6195(4) | 0.5164(10) |
|     | 16  | 0.0084(6) | 0.0158(12) | 0.6201(4) | 0.5159(10) |
|     | 24  | 0.0072(5) | 0.0146(11) | 0.6205(4) | 0.5162(9)  |
|     | 32  | 0.0074(5) | 0.0152(12) | 0.6206(4) | 0.5152(10) |
| 0.95 | 8   | 0.0179(10) | 0.0290(18) | 0.6180(5) | 0.5158(11) |
|     | 12  | 0.0167(9) | 0.0329(20) | 0.6183(5) | 0.5116(12) |
|     | 16  | 0.0150(9) | 0.0286(18) | 0.6181(5) | 0.5129(11) |
|     | 24  | 0.0117(7) | 0.0228(14) | 0.6186(4) | 0.5135(11) |
|     | 32  | 0.0118(7) | 0.0251(17) | 0.6193(4) | 0.5140(10) |

fact by assuming that the amplitudes of this scaling correction exponent are really small, so we are seeing the next to the leading scaling correction.

4.3. Numerical results on self-averaging

Once we have checked that the value $\omega = 0.80$ can be used to describe the corrections to the scaling for the pure and diluted models we can try to extrapolate the values of the two $g_2$ values to infinite volume.

Numerical results for $g_2$ and $g_4$, in both channels, are shown in table 2 both for pure (only $g_4$) and for diluted models.
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Figure 5. Combined extrapolation to $L \to \infty$ for the $g_2$ cumulant of the tensorial susceptibility, for the form of equation (22).

Figure 6. Extrapolation to $L \to \infty$ for the $g_2$ cumulant of the vectorial susceptibility. The fitting function is in this case of the form $g_2 = aL^{\alpha/\nu} + bL^{-\omega}$.

First of all, we will try to check the non-zero $g_2$ scenario with the correction to the scaling exponent fixed to that obtained in the previous section. We had found that it is possible to extrapolate using the form of equation (22) (performing a joint fit\(^2\)) of the values of $g_2$ to a value (depending only on the channel) which is independent to the dilution, and near the analytical prediction of [3]. However,

\(^2\) We have used for all fits the data set which gives the smallest $\chi^2$ with the biggest number of degrees of freedom.

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Figure 7. Extrapolation to $L \to \infty$ for the $g_2$ cumulant of the tensorial susceptibility for the form $g_2 = aL^{\alpha/\nu} + bL^{-\omega}$.

simulations (with a small number of samples) at dilutions $p = 0.95$ and 0.97 do not follow the scaling found for $p \leq 0.90$ (see table 3). Hence, our numerical data do not support the scenario $g_2 \neq 0$; see figures 4 and 5. Notice, see also table 3, that all the values for these two lowest dilutions are smaller than the extrapolated point and they are decreasing (for both channels and taking into account the error bars).

Secondly, we will check the $g_2 = 0$ scenario. To do this, we extrapolate $g_2$ using the form proposed in [2] ($g_2 \sim L^{\alpha/\nu}$) but also including the term $L^{-\omega}$:

$$g_2 = aL^{\alpha/\nu} + bL^{-\omega}. \quad (28)$$

We obtain the fits shown in figures 6 and 7 for both channels. The $\chi^2$ for these fits are all really good; hence, we have obtained strong evidence supporting this $g_2 = 0$ scenario. Notice that the introduction of the scaling correction has had paramount importance for obtaining a very good $\chi^2$ for all the fits. The numerical data, for the lattice size simulated, do not follow the one-term dependence $g_2 \propto L^{\alpha/\nu}$.

5. Conclusions

We have studied the critical properties of the Heisenberg diluted model for different values of the dilution using the quotient method. Our main aim was to check the self-averaging properties of the susceptibility.

We have studied in a great detail the corrections to the scaling in the diluted Heisenberg model. We have obtained that the numerical data follow the next to the leading correction to the scaling exponent instead the leading one. We will show in the appendix all the critical exponents and cumulants obtained using this next to the leading exponent; also we report that the result of this analysis is fully compatible with the RG predictions and the Harris criterion: our exponents and cumulants are compatible with those of the pure model and independent of the dilution with a high degree of precision.
In addition, we have shown that we obtain non-universal quantities if we assume $\alpha/\nu$ as the main scaling correction even if we add the $\omega$ correction to the scaling exponent (see the appendix), using two corrections to scaling exponents in the analysis.

Finally, we have shown strong evidence for a zero $g_2$ cumulant, both in vector and in tensor channels, in the thermodynamic limit at criticality, contrasting with some analytical predictions [3], and in agreement with the ones obtained in [2]. The introduction of scaling corrections in the analysis has become crucial to obtain the $g_2 = 0$ scenario. In addition, simulations of samples with lower dilution have helped us to discard the $g_2 \neq 0$ scenario.

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**Appendix. Critical exponents and cumulants**

In this appendix we will check the consistency of the $\omega$ exponent computed in the text by means of the computation of critical exponents and cumulants. In addition, we will check

![Figure A.1. Combined extrapolation to $L \rightarrow \infty$ for the $\eta$ exponent deduced from the vectorial susceptibility ($\chi^V$). Extrapolations are carried out by choosing a common value for the first term of equation (22) for all dilutions, and by minimizing the combined $\chi^2$.](image-url)
Figure A.2. Combined extrapolation with all $p$ values to $L \to \infty$ for the $\eta$ exponent deduced from the vectorial magnetization ($M^V$).

Table A.1. Magnetic exponents for the pure O(3) model. The last three rows correspond to the $L \to \infty$ extrapolation (disregarding data with $L = 8$).

| $L$ | $\chi M$ | $\eta^V$ | $\chi_T M_T$ | $\eta_T$ |
|-----|---------|----------|-------------|---------|
| 8   | 0.0301(7) 0.0319(8) | 1.4301(12) 1.4343(13) | \n| 12  | 0.0339(7) 0.0353(8) | 1.4324(11) 1.4352(12) | \n| 16  | 0.0348(7) 0.0358(8) | 1.4310(11) 1.4335(12) | \n| 24  | 0.0361(6) 0.0367(7) | 1.4293(9) 1.4307(10) | \n| 32  | 0.0369(7) 0.0374(7) | 1.4289(11) 1.4300(12) | \n| 48  | 0.0373(6) 0.0378(7) | 1.4271(9) 1.4280(10) | \n| $L \to \infty$ | 0.0391(9) 0.0390(10) | 1.4250(13) 1.4249(15) | \n| $\chi^2/d.o.f$ | 0.138/3 0.354/3 | 1.047/3 1.952/3 | \n| Prob | 0.987 0.950 | 0.790 0.582 | \n
Table A.2. Thermal critical exponents for the pure O(3) model. In the last column we have disregarded data with $L < 16$.

| $L$ | $\partial g^V_\xi$ | $\partial g^T_\xi$ | $\partial g^V_\zeta$ | $\partial g^T_\zeta$ |
|-----|----------------|----------------|----------------|----------------|
| 8   | 0.7016(30) 0.7217(13) | 0.6846(41) 0.7306(14) | \n| 12  | 0.7033(32) 0.7162(14) | 0.6931(49) 0.7188(13) | \n| 16  | 0.7028(35) 0.7123(16) | 0.6830(56) 0.7118(17) | \n| 24  | 0.7061(37) 0.7123(17) | 0.6908(47) 0.7112(18) | \n| 32  | 0.7081(35) 0.7121(19) | 0.7022(61) 0.7116(23) | \n| 48  | 0.7101(41) 0.7118(19) | 0.7125(61) 0.7085(21) | \n| $L \to \infty$ | 0.7109(38) 0.7071(19) | 0.7082(51) 0.7071(35) | \n| $\chi^2/d.o.f$ | 0.667/4 4.104/4 | 7.039/4 5.656/2 | \n| Prob | 0.954 0.392 | 0.134 0.754 |
Table A.3. Magnetic exponents for the diluted O(3) model with \( p = 0.9 \). Extrapolations are carried out without disregarding data.

| \( L \) | \( \chi \) | \( M \) | \( \chi_T \) | \( M_T \) |
|--------|-------|-------|--------|--------|
| 8      | 0.0346(26) | 0.0345(28) | 1.4154(36) | 1.4176(37) |
| 12     | 0.0360(24) | 0.0360(26) | 1.4195(34) | 1.4207(36) |
| 16     | 0.0371(23) | 0.0374(25) | 1.4207(34) | 1.4218(35) |
| 24     | 0.0373(22) | 0.0375(24) | 1.4204(32) | 1.4221(34) |
| 32     | 0.0383(21) | 0.0383(23) | 1.4219(31) | 1.4227(33) |
| \( L \to \infty \) | 0.0397(29) | 0.0399(31) | 1.4245(41) | 1.4252(43) |
| \( \chi^2/d.o.f \) | 0.292/3 | 0.124/3 | 0.544/3 | 0.137/3 |
| Prob   | 0.962 | 0.989 | 0.909 | 0.987 |

Table A.4. Thermal exponents for the diluted O(3) model with \( p = 0.9 \). In the first and third columns we obtain bad results because of the series not monotonically decreasing.

| \( L \) | \( \partial_{\beta} g_4^V \) | \( \partial_{\beta} \xi^V \) | \( \partial_{\beta} g_4^T \) | \( \partial_{\beta} \xi^T \) |
|--------|------|------|------|------|
| 8      | 0.7319(49) | 0.7443(24) | 0.7128(83) | 0.7709(29) |
| 12     | 0.7381(53) | 0.7411(25) | 0.7267(86) | 0.7514(29) |
| 16     | 0.7430(55) | 0.7381(26) | 0.7536(99) | 0.7426(31) |
| 24     | 0.7384(57) | 0.7368(28) | 0.7337(95) | 0.7395(32) |
| 32     | 0.7398(54) | 0.7365(29) | 0.7241(97) | 0.7345(33) |
| \( L \to \infty \) | 0.734(15) | 0.7318(33) | 0.728(17) | 0.7152(39) |
| \( \chi^2/d.o.f \) | 0.134/1 | 0.168/3 | 5.468/2 | 3.156/3 |
| Prob   | 0.714 | 0.983 | 0.065 | 0.368 |

Table A.5. Magnetic exponents for the diluted O(3) model with \( p = 0.7 \). Extrapolations are carried out without disregarding data.

| \( L \) | \( \chi \) | \( M \) | \( \chi_T \) | \( M_T \) |
|--------|-------|-------|--------|--------|
| 8      | 0.0436(38) | 0.0412(41) | 1.3882(52) | 1.3879(53) |
| 12     | 0.0411(34) | 0.0401(36) | 1.4005(48) | 1.4007(49) |
| 16     | 0.0392(31) | 0.0392(34) | 1.4061(45) | 1.4073(46) |
| 24     | 0.0383(29) | 0.0386(31) | 1.4131(41) | 1.4136(43) |
| 32     | 0.0382(27) | 0.0389(29) | 1.4142(40) | 1.4149(41) |
| \( L \to \infty \) | 0.0343(57) | 0.0370(58) | 1.4299(72) | 1.4318(76) |
| \( \chi^2/d.o.f \) | 0.232/3 | 0.059/3 | 0.472/3 | 0.567/3 |
| Prob   | 0.972 | 0.996 | 0.925 | 0.904 |

whether these sets of exponents are universal or not by comparing different dilutions with the pure model. We will use in this analysis the data for \( p = 0.9, 0.7 \) and 0.5 (we have simulated these values of the dilution for \( 10^4 \) samples).

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Figure A.3. Combined extrapolation with all $p$ values to $L \to \infty$ for the $\nu$ exponent deduced from the $\partial_\beta g_4^V$.

Table A.6. Thermal exponents for the diluted O(3) model with $p = 0$. In the second column the fit is obtained disregarding data with $L < 16$.

| $L$ | $\partial_\beta g_4^V$ | $\partial_\beta \xi^V$ | $\partial_\beta g_4^T$ | $\partial_\beta \xi^T$ |
|-----|-----------------|-----------------|-----------------|-----------------|
| 8   | 0.7888(69)      | 0.7881(31)      | 0.8256(143)     | 0.8422(42)      |
| 12  | 0.7810(74)      | 0.7806(33)      | 0.8078(140)     | 0.8067(41)      |
| 16  | 0.7633(70)      | 0.7760(35)      | 0.7739(131)     | 0.7897(43)      |
| 24  | 0.7491(66)      | 0.7628(37)      | 0.7719(146)     | 0.7792(47)      |
| 32  | 0.7400(67)      | 0.7521(42)      | 0.7656(178)     | 0.7627(56)      |
| $L \to \infty$ | 0.7206(88) | 0.723(10)      | 0.729(19)       | 0.7255(61)      |
| $\chi^2$/d.o.f | 2.313/3 | 0.281/1       | 1.314/3        | 1.965/3         |
| Prob | 0.510         | 0.596          | 0.726          | 0.580           |

Table A.7. Magnetic exponents for the diluted O(3) model with $p = 0.5$. In the third and fourth columns we have only used data with $L > 12$.

| $L$ | $\chi$ | $M$ | $\chi_T$ | $M_T$ |
|-----|-------|-----|----------|-------|
| 8   | 0.0505(45) | 0.0461(48) | 1.3435(61) | 1.3431(62) |
| 12  | 0.0448(39) | 0.0439(42) | 1.3684(54) | 1.3702(56) |
| 16  | 0.0421(36) | 0.0417(39) | 1.3877(51) | 1.3896(52) |
| 24  | 0.0396(32) | 0.0406(35) | 1.4033(46) | 1.4053(48) |
| 32  | 0.0399(30) | 0.0414(32) | 1.4126(43) | 1.4152(45) |
| $L \to \infty$ | 0.0346(60) | 0.0378(46) | 1.446(12)   | 1.449(12)    |
| $\chi^2$/d.o.f | 2.225/2 | 2.191/3   | 0.119/1    | 0.327/1     |
| Prob | 0.329   | 0.534   | 0.730     | 0.568      |

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Figure A.4. Combined extrapolation to $L \to \infty$ for the $\nu$ exponent deduced from the $\partial_3 g_4^T$.

Table A.8. Thermal exponents for the diluted $O(3)$ model with $p = 0.5$. In the first column we have only used data with $L > 8$ while in the second and fourth ones we have only used data with $L > 12$.

| $\nu$ | $\partial_3 g_4^V$ | $\partial_3 \xi^V$ | $\partial_3 g_4^T$ | $\partial_3 \xi^T$ |
|-------|-------------------|-------------------|-------------------|-------------------|
| 8     | 0.8102(91)        | 0.8357(46)        | 0.9180(241)       | 0.9540(72)        |
| 12    | 0.8042(90)        | 0.8322(50)        | 0.8880(248)       | 0.8866(71)        |
| 16    | 0.7764(89)        | 0.7862(48)        | 0.8449(242)       | 0.8136(64)        |
| 24    | 0.7702(93)        | 0.7778(52)        | 0.8311(234)       | 0.7952(66)        |
| 32    | 0.7562(91)        | 0.7779(56)        | 0.7812(220)       | 0.7833(70)        |
| $L \to \infty$ | 0.720(16) | 0.764(14) | 0.735(28) | 0.744(17) |
| $\chi^2$/d.o.f | 1.149/2 | 0.208/1 | 1.565/3 | 0.025/1 |
| Prob  | 0.563             | 0.649             | 0.667             | 0.874             |

Equation (17) applied to the operators $\partial_3 \xi$, $\partial_3 g_4$, $M$ and $\chi$ yields respectively the critical exponents $1 + 1/\nu$, $1/\nu$, $(d - 2 + \eta)/2$ and $2 - \eta$. The numerical results are shown in tables A.1 and A.2 for the pure model, tables A.3 and A.4 for $p = 0.9$, tables A.5 and A.6 for $p = 0.7$ and tables A.7 and A.8 for $p = 0.5$. We have also carried out a combined extrapolation for all $p$ values by fixing the same value of the extrapolated exponent (first term in equation (22)) for every $p$ value and then minimizing the combined $\chi^2$. Some of these fits are shown in figures A.1–A.4 and the compared results can be seen in tables A.9 and A.10.

The combined extrapolation of the Binder cumulant $g_4$ is shown in table A.11 and the agreement of our results with the ones obtained in [11] (numerical for the pure model) and [3] (analytical) is really very good. We obtain also complete agreement with previous numerical estimates of the pure model critical exponents; see [11].
Table A.9. Combined extrapolation with all $p$ values for the magnetic exponent $\eta$ compared with the results from [11]. The first three rows correspond to our $L \to \infty$ extrapolation, prob being the probability of finding a larger value for the $\chi^2$ of the fit (it is a measure of the goodness of the fit) and d.o.f. the number of degrees of freedom (the total number of data minus the total number of adjustable parameters in the fitting function).

| $\chi$ | $M$ | $\eta$ | $MT$ |
|--------|-----|--------|------|
| Our results | 0.0390(9) | 0.0389(10) | 1.4251(13) | 1.4251(14) |
| $\chi^2$/d.o.f. | 6.675/12 | 5.104/15 | 9.151/10 | 13.931/11 |
| Prob | 0.878 | 0.991 | 0.518 | 0.237 |
| Ref. [11] | 0.0378(6) | — | — | — |

Table A.10. Combined extrapolation with all $p$ values for the thermal exponent $\nu$ compared with the results from [11].

| $\partial_3 g_4^V$ | $\partial_3 \xi^V$ | $\partial_3 g_4^T$ | $\partial_3 \xi^T$ |
|-------------------|-------------------|-------------------|-------------------|
| Our results | 0.7126(46) | 0.7129(31) | 0.7294(81) | 0.7089(32) |
| $\chi^2$/d.o.f. | 4.831/11 | 6.606/6 | 9.009/13 | 9.609/7 |
| Prob | 0.939 | 0.359 | 0.772 | 0.212 |
| Ref. [11] | 0.7113(11) | — | — | — |

Table A.11. Combined extrapolation to $L \to \infty$ with all $p$ values for the Binder cumulant, $g_4$, defined in equation (11), compared with results from [11] and [3].

| $g_4^V$ | $g_4^T$ |
|---------|---------|
| Our results | 0.620 18(6) | 0.51366(19) |
| $\chi^2$/d.o.f. | 10.324/9 | 5.980/10 |
| Prob | 0.325 | 0.817 |
| Ref. [11] | 0.6202(1) | — |
| Ref. [3] | 0.625783 | — |

We obtain non-universal critical exponents and cumulants if instead $\omega = 0.8$ we use $\omega = -\alpha/\nu$ as the correction to the scaling exponent. In addition, the dilution dependent exponents and cumulants are clearly different from the pure ones. Furthermore, this scenario does not change if we fit the data using both $\omega = -\alpha/\nu$ and $\omega = 0.8$.

References

[1] Harris A B, 1974 J. Phys. C: Solid State Phys. 7 1671
[2] Aharony A and Harris A B, 1996 Phys. Rev. Lett. 77 3700
[3] Chamati H, Korutcheva E and Tonchev N S, 2002 Phys. Rev. E 65 026129
[4] Aharony A, Harris A B and Wiseman S, 1998 Phys. Rev. Lett. 81 252
[5] Wiseman S and Domany E, 1998 Phys. Rev. E 58 2938
[6] Deroulers C and Young A P, 2002 Phys. Rev. B 66 014438

doi:10.1088/1742-5468/2007/06/P06014
Self-averaging in the three-dimensional site diluted Heisenberg model at the critical point

[7] Parisi G, Picco M and Sourlas N, 2004 Europhys. Lett. 66 465
[8] Ballesteros H G, Fernández L A, Martin-Mayor V and Muñoz Sudupe A, 1996 Phys. Lett. B 387 125
[9] Ballesteros H G, Fernández L A, Martin-Mayor V, Muñoz Sudupe A, Parisi G and Ruiz-Lorenzo J J, 1997 Phys. Lett. B 400 346
[10] Ballesteros H G, Fernández L A, Martin-Mayor V, Muñoz Sudupe A, Parisi G and Ruiz-Lorenzo J J, 1998 Nucl. Phys. B 512 681
[11] Camostrini M, Hasenbusch M, Pelissetto A, Rossi P and Vicari E, 2002 Phys. Rev. B 65 144520
[12] Brezin E and Zinn-Justin J, 1985 Nucl. Phys. B 257 FS14, 867
[13] Cooper F, Freedman B and Preston D, 1982 Nucl. Phys. B 210 210
[14] Amit D J and Martín Mayor V, 2005 Field Theory, The Renormalization Group and Critical Phenomena (Singapore: World Scientific)
[15] Hasenbusch M, Toldin F P, Pelissetto A and Vicari E, 2007 J. Stat. Mech. P07016
[16] Fisher y M E and Barber M N, 1972 Phys. Rev. Lett. 28 1516
[17] Pelissetto A and Vicari E, 2002 Phys. Rep. 368 549
[18] Guida R and Zinn-Justin J, 1998 J. Phys. A: Math. Gen. 31 8103 and references therein
[19] Hasenbusch M, 2001 J. Phys. A: Math. Gen. 34 8221
[20] Ballesteros H G, Fernández L A, Martin-Mayor V, Muñoz Sudupe A, Parisi G and Ruiz-Lorenzo J J, 1998 Phys. Rev. B 58 2740