Kinematics, dynamics and energy balance of electric arc in a magnetic field

V A Bityurin, A N Bocharov, I A Moralev, T N Kuznetsova
Joint Institute of High Temperatures, Russian Academy of Sciences
Izhorskaya 13. Bd 2, Moscow, Russia, 125412

valentin.bityurin@gmail.com

Abstract. Electric arcs (EA) are widely used in various applications as a high-density energy source, providing heating of the environment to temperatures of several thousand degrees. The presence of an external magnetic field perpendicular to the arc current vector causes a ponderomotive force normal to the plane of the electric current and magnetic field vectors. Thus, in addition to the thermal effect, there is also a dynamic one. It is these properties that motivate the use of EA as an MHD actuator to control the flow characteristics of gaseous working bodies in various technological processes. MHD actuator is most often considered as a control element (1) in combustion processes to intensify the mixing of fuel-air mixtures and maintain combustion due to the volume and constant high-temperature energy supply and (2) to control the characteristics of the boundary layer in the flow of the working surfaces of the devices. In both cases, the key issue is the efficiency of mixing (relative displacement of neighboring elements of the medium). Within the framework of the standard "Eulerian" description of the gas-dynamic fields evolution in the laboratory reference frame, the movements of individual elements of the medium are not explicitly available. In this regard, this work is focused on the trajectories analysis of the individual mass elements exposed to the MHD of the actuator.

1. Introduction
Electric arcs, in addition to the thermal effect, could provide a dynamic effect, which predetermines the possibility of developing various technologies for converting energy and momentum: MHD generators and accelerators, current breakers, railguns, mixers, heaters, etc. These features make it possible to consider the use of EA as an MHD actuator to control the characteristics of the flow of gaseous working bodies in various technological processes.

At the present time the MHD actuator is considered most often as a means of control either in combustion processes to intensify the mixing of fuel-air mixtures and maintain combustion due to the volume and constant high-temperature energy supply [1-3], or to control the characteristics of the boundary layer when flowing around the working surfaces of devices [3-5]. In both cases, the key issue is the efficiency of mixing (relative displacement of neighboring elements of the medium). Within the framework of the standard "Eulerian" description of the gas-dynamic field evolution in the laboratory reference frame, the movements of individual elements of the medium are not explicitly available. In this regard, this work, initiated by large-scale experimental and computational-theoretical studies of MHD actuators for various applications in modern aerodynamics [3-8], is focused on the trajectories analysis of...
individual mass elements affected by the MHD actuator. To this end, a standard numerical experiment using a complete two-dimensional unsteady system of Navier-Stokes equations for laminar flow of real air with simplified equations of magnetic gas dynamics (an extended characteristic of the model used with the implementation in the PlasmAero package is presented, for example, in [8]), supplemented by a "Lagrangian" stage that allows tracking the trajectories of individual elements of the medium. Note that Lagrangian processing does not affect the solution of the main problem in any way. Formally, the volumetric ponderomotive force $F = \mathbf{J} \times \mathbf{B}$ appears together with the pressure gradient and the viscous stress tensor. In the considered configuration with a limited area of current existence in a transverse magnetic field, the ponderomotive force is fundamentally vortex, while the pressure gradient is a potential vector. Consequently, these two factors cannot be balanced, and the vortex viscous terms are small, at least at the initial point in time after the interaction is switched on. The result is the appearance of an inertial force $\rho \mathbf{v}$. It begins the formation of the velocity field up to the date of termination of the ponderomotive force (the external current or the magnetic field is switched off). It is obvious that the stabilization of the process can be provided only by viscous forces, which seems unlikely because of the fundamentally different structure of these two dynamic factors. In addition, for environments with temperature dependence of conductivity, the energy balance, determined primarily, Joule heat dissipation is an essential factor. Below it will be shown that the heat exchange of the arc channel with the external environment is one of the determining factors of the evolution of the arc channel in the transverse magnetic field. It is precisely these properties that motivate the use of EA as an MHD actuator to control the characteristics of the flow of gaseous working bodies in various technological processes. An important factor is also that the initiation of motion of a medium outside the scope of the ponderomotive force is caused by inhomogeneous pressure field, as a reaction to applied ponderomotive force just inside the arc channel. It is clear that the target task in the problems of mixing intensification is initially the relative displacement of neighboring elements of the medium, which is achieved at a certain time interval. The measure of this criterion can be extracted from trajectories analysis of the certain family of mass elements motion, for this it is necessary to integrate the velocity field in time with the registration of the individual mass elements position of medium. It is the construction of trajectories on the velocity field obtained in a standard way that forms the basis of the "Lagrangian" stage declared above. As our experience has shown, the direct use of mass coordinates to construct a solution seems inefficient because of strong deformations of the mass grid.

2. Problem statement
To reduce multi-factor effects (such as effects of wall close, the effect of initial velocity field of the medium, etc.) with the desire to match a particular experiment, we consider here the movement of the electric arc in a homogeneous initially stationary air at normal conditions. It is assumed that a cylindrical arc channel sufficiently extended along the current is placed in a uniform transverse magnetic field. Below, it is believed that the arc current is controlled by an external circuit. As a rule, a constant current pulse at a finite time interval is considered (the values of these parameters are chosen from the conditions of compliance with the experiment [1]). It can be shown that under these conditions there is constant everywhere the external electric field generated by the power supply and the arc current is determined by the integral conductivity of the arc cross-section:

$$I = E \int \int \sigma (P(x,y),T(x,y)) \, dx \, dy$$

at a given current, the electric field value is defined from the previous as

$$E = I / \int \int \sigma (P(x,y),T(x,y)) \, dx \, dy$$

The initial parameters of the arc channel should ensure the maintenance of the required current (usually several tens of amperes) at reasonable values of the voltage on the power source—several hundred volts. The used procedure requires setting the initial perturbation in some small region to initiate an
electric discharge. Usually one of two methods is used: isobaric-temperature at the level $T = 6000-9000$ K and density $\rho = P/RT$, $P = 10^5$ Pa, or $T$ isochoric, $p = \rho RT$ with density under normal conditions.

This paper does not aim to study systematically the characteristics of the arc channel evolution in a transverse magnetic field in a wide range of changes in the determining parameters of the process as a whole. Therefore, referring to empowerment in the interpretation of the results of experiment presented in [1], will stay in this stage for the example of [1]: the arc current is given as a rectangular pulse $I_{arc} = 30 - 60$ with duration $130 - 260 \mu s$, the magnetic field is constantly and uniformly $B = (0, B_y, 0)$ $T = 0.33 – 0.66 T$, environment – air at normal conditions $p = 10^5$ Pa, $T = 300$ K.

3. Numerical simulation

The following assumptions are made in the numerical analysis. The modeling is performed in a 2D approach. It means that all flow variables are assumed to be homogeneous in $z$-direction and all $\partial/\partial z$ derivatives are zero. Electric field strength is assumed to be uniform in $xy$-plane. The arc plasma is assumed to be in local thermodynamic equilibrium (LTE), with the conductivity being a function of temperature only. Finally, the Hall current in $z$ direction, that can lead to the 3D effects onset, was omitted in current approach. With these assumptions, the mathematical formulation for the MHD actuator reads as follows.

3.1. Governing equations

The flow field can be found from the solution of two-dimensional Navier-Stokes equations enclosed with the appropriate equations of state (1) – (4).

$$\frac{\partial \rho}{\partial t} + \nabla (\rho U) = 0 \quad (1)$$

$$\frac{\partial \rho U}{\partial t} + \nabla (\rho U \otimes U) + \nabla \tau = -\frac{\partial p}{\partial r} + J \times B \quad (2)$$

$$\frac{\partial \rho e_0}{\partial t} + \nabla (\rho Uh^0) + \nabla (U \tau) + \nabla q = \rho E + Q_{rad} \quad (3)$$

$$h^0 = e^0 + \frac{P}{\rho}, e^0 = e + \frac{v^2}{2}, (\gamma - 1)e = \frac{P}{\rho}, P = \frac{\rho}{M} R^0 T \quad (4)$$

$$\frac{\partial}{\partial r} = \eta_x \frac{\partial}{\partial x} + \eta_y \frac{\partial}{\partial y}, \nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} \quad (5)$$

In equations (1) – (4) $t$ is time, $\rho$ is the mass density, $U(U_x, U_y)$ is the velocity, $P$ is the thermodynamic pressure, $T$ is the temperature, $e_0$ is the specific total energy, $e$ is the specific internal energy, $h_0$ is the specific total enthalpy, $h$ is the specific internal enthalpy, $M$ is the molecular weight. Operators in (5) designate gradient and divergence, respectively. $\eta_x, \eta_y$ are unit base vectors.

To specify LTE air plasma characteristics the approximations [9] are used:

$$h = h_{eq}(P, T), M = M_{eq}(P, T), \gamma = \gamma_{eq}(P, T) \quad (6)$$

The heat flux $q$ and viscous stress tensor $\tau$ read as:

$$q = -\lambda \frac{\partial T}{\partial r} \quad (7)$$

$$\tau_{ij} = \frac{2}{3} \eta \delta_{ij} \nabla U - \eta \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (8)$$

In (7), (8) $\eta$ is the molecular viscosity coefficient, $\lambda$ is molecular heat conductivity, $\delta_{ij}$ is Kronecker-Capelli symbol. Transport coefficients $\lambda$ and $\eta$ are calculated from approximations [9].

3.2 Electrodynamic part

Electrodynamic part of the problem reduces to the relations for total arc current and electric field strength assuming Ohm’s law. Namely,

$$I_x = \sigma E_x \quad (9)$$

$$I_x(t) = \int I_x dS = \int \sigma E_x dS = E_x \int \sigma dS, \quad (10)$$
3.3. Real gas properties

Total arc current $I_a(t)$ is set as an external parameter, so (10) is used for determining average electric field strength $E_a$. After this, source terms in equations (2) and (3) can be found. Electric conductivity $\sigma(x,y)$ is calculated in accordance with [9]. Radiative losses in (3) are estimated as follows: $Q_{rad} = \chi \sigma_{SB} T^4$, where $\chi$ is the absorption coefficient, $\sigma_{SB}$ is the Stefan-Boltzmann constant. Absorption coefficient is calculated from tables given in [10].

3.4 Computational details

Equations (1) – (3) are discretized on a structured grid in conservative variables ($\rho$, $\rho U$, $\rho e^0$). AUSM ([11],[12]). The technique [13] is used to obtain spatial reconstruction of higher order flow variables. Viscous flows (7), (8) are calculated, as, for example, in [13,14]. The explicit Euler method is used for time integration of equations (1) - (3).

The solution was constructed on a rectangular grid. To reduce the influence of far field conditions, an exponentially increasing grid belt was introduced along the perimeter of the base grid. Initialization of the counteragent current channel was carried out either by Isobaric, isochoric or perturbation, respectively, of temperature or pressure in a small (usually $\sim 100 \times 100$ mm$^2$) region. The kinematics of the process is studied in the framework of the proposed here the option "Euler-Lagrangian" approach. For this purpose, along with the main stationary grid, which is the integration of two-dimensional nonstationary Navier-Stokes equations, is introduced moving "Lagrangian" mesh, which is associated with the mass elements of the medium at some initial time, then the position of nodes of the moving grid is determined by integrating in time the velocity field, taught from the solution of the original problem on the Euler grid. Note that the Lagrangian stage has no influence on the solution of the main problem and is only an additional information element.

Dynamic and energy characteristics: the fields of pressure, velocity, density and different types of energy are the main result of the solution on a stationary grid.

4. Results

The qualitative interaction process characteristic of the current channel with the external magnetic field and with the medium is given in Fig.1, which shows the temperature, pressure and velocity fields evolution in the vicinity of the initially initiated isochoric perturbation of temperature and density in the region of 120-120 mm$^2$, $T_0 = 10000$ K, $P_0 = 3.33$ MPa over the entire range of the current pulse. The main features of the process are determined, firstly, by the initial conditions-highly inhomogeneous distributions of pressure and temperature and, secondly, determined by the current dynamic and thermal effects on the current channel. Changes in gas-dynamic fields outside the current channel are determined by the transfer of momentum and energy by "fast" shock-wave and relatively slow diffusion processes.

Under the position of the arc here and further we will mean the position of the maximum temperature point, which qualitatively corresponds to intuitive experimental practice. Differentiating the time of this ham allows you to determine the "speed" of the arc. In addition to a certain value of the arc velocity, the numerical solution gives the true velocity of the medium at the point of maximum temperature. The evolution of all these parameters is shown in Figure 2. The most important fact is almost two-fold excess of the medium true velocity above the "phase" speed the movement of "hot" points, and specifies the displacement of the hot spot on the medium, i.e. the speed of the hot spot are the sum of the velocity environment and movement of high temperature in the environment. It is this circumstance that motivated the use of "Lagrangian" procedures to increase the information content of the computational and the theoretical analysis.
Figure 1. Time evolution of temperature (contours) and velocity (arrows) on left, and pressure on right.
Figure 2. Time evolution of gas in “hot” point and velocity of “hot point”.

Figure 3. Temperature field on Euler grid (left) and on Lagrangian one (right) 150 μs after current switch off (t\text{break}=130\mu s).

Figure 3 shows an example of the temperature distribution and velocity vector on the fixed (left) and mobile (Lagrangian – right) grids at some point in time after switching off the arc current. Unfortunately, the strong deformation of the moving mesh in the evolution process makes it almost impossible to visually analyze the process. In this regard, we will continue to use only a small fragment of the movable grid, in which the arc was initialized. Figure 4 is an example of arc evolution in Lagrangian coordinates is presented. At the initial moment of time, the place of the isochoric perturbation initiating the arc is fixed. In this example, this region is a rectangular element of the Euler (fixed) grid size 5\times5 cells, which is given a 30-fold increase in temperature and pressure. The gas mass with disturbing parameters is marked for further tracking simply by the temperature field, at this initial point in time. This temperature field is represented here and in the future by a blue-pink color palette. The actual field of true temperature (changing during the evolution of the arc) is represented in a green-red color palette. Note that the upper figure represents the situation not at the initial, but at a somewhat later (+ 10 microseconds) point in time. The next two frames will represent the moment of magnetic field shutdown-approximately 65 microseconds after initialization, and at the moment of arc dock shutdown – approximately 130 microseconds. Figure 4 clearly shows that the active current-carrying mass is outside the zone of initial shock-wave expansion, and the initially superheated high-pressure mass has cooled to temperatures below 6000 K, at which the air ceases to be any significant conductor.
Figure 4. Position of arc-related mass (pink-blue) and active mass (red-green) at three time instances (from top to bottom): 10 μs, 65 μs, 130 μs.

Figure 5. Trajectories of mass-elements of current channel at 410 μs after initialization of the arc.

Another kinematic characteristic is the mass element trajectory of the medium. Figure 5 presents three beams of trajectories, the starting points of which are selected as follows. Three groups of initial coordinates are located on vertical segments of straight lines, which are symmetric with respect to the horizontal axis of the current spot, the central group lies on the vertical diameter of the current channel, and the left and right - symmetrically with respect to the Central one. The initialization area is conventionally shown in Figure 5 painted rectangle. The central (red) trajectories are divided into two groups. The two central trajectories are almost rectilinear and limit the "through channel", in which there are several, also almost rectilinear trajectories of mass elements of the external undisturbed medium. The external trajectories of the initially hot volume undergo a certain circulating phase, similar to the trajectories of the mass elements of an ideal liquid when flowing around a circular cylinder. The group of
right elements moves similarly to the last, just to passage free for the axial jet of hot and cold elements. Apparently, the main contribution to the mixing function is made by this axial tube, the contribution of external remote elements to the mixing seems negligible.

![Figure 6. Integral power characteristics of the discharge.](image)

The energy efficiency of the MHD actuator can be estimated from the data shown in Figure 6. In this picture, the solid curves represent the values scaled on the left axis and the dashed ones on the right. The modification of the velocity field (in the case of the initial state of rest) can be estimated simply as the total kinetic energy of the entire domain of consideration. It is useful to compare this value with the full operation of the ponderomotive force, which is several times greater by the time the current is turned off, as can be seen. Moreover, the contribution of the total energy (the upper envelope curve) is about 50 times greater than the work of the ponderomotive force. The feasibility of this technology should be justified by additional criteria. In addition, a detailed analysis of the gas-dynamic field formation and peculiarities of the trajectories of mass elements of the environment and possible ways of process of optimization to control the MHD actuator functioning.

5. Concluding remarks
This preliminary study of the arc evolution in transverse uniform magnetic field reveals that the experimentally visualized the arc motion velocity is rather a kind of “phase” propagation velocity of the maximal temperature (or maximal intensity of radiation). Thus, such a value couldn’t be used directly in arc dynamics characterization. The momentum and energy balances require the accurate accounting for the convection fluxes through some arc border.

The analysis of the Lagrangian grid deformation and the corresponding trajectory of mass elements shows that the strongest relative displacements of the media are located inside the arc and its close environment. In far field the Langrangian grid deformation is rather low that means that mixing effectiveness there is also low.

Energy balance of the arc is only 2-3% related to the creation of motion generation, but mostly connected to the heat deposition into the media.

Euler-Lagrangian approach used in this study has shown rather new promising features in the fast running process with intensive momentum and energy conversion studies.

Acknowledgement
This work is supported by the PROJECT ГР АААА-116051810079-7
References

[1] F. P. N. Kazansky, I. A. Moralev, A. A. Firsov, V. A. Bityurin, A. N. Bocharov, S. B. Leonov, experimental study of vortex generation with a pair of counter-moving pulse arcs, AIAA Scitech forum 2019, January 7-11, 2019, San Diego, California; DOI 10.2514/6.2019-0051. Yang et al., “Low-voltage circuit breaker arcs - Simulation and measurements,” J. Phys. D. Appl. Phys., vol. 46, no. 27, 2013.

[2] V. A. Bityurin, A. Likhachev, S. A. Medin, and G. A. Lyubimov, “On the Dynamics of a nonUniform Conducting Flow in an MHD Generator Channel.”

[3] V. Bityurin and A. Bocharov, “Advanced MHD assisted mixing of reacting streams,” in 39th Aerospace Sciences Meeting and Exhibit, American Institute of Aeronautics and Astronautics, 2001.

[4] A. N. Bocharov, V. A. Bityurin, I. B. Klement, and S. B. Leonov, “A STUDY OF MHD ASSISTED MIXING AND COMBUSTION,” in 41th AIAA Aerospace Sciences Meeting and Exhibit, 2003, p. AIAA-2003-0378.

[5] A. N. Bocharov, V. A. Bityurin, I. B. Klement, and S. B. Leonov, “EXPERIMENTAL AND THEORETICAL STUDY OF MHD ASSISTED MIXING AND IGNITION IN CO- FLOW STREAMS,” 2002, pp. 1–12.

[6] J. Sirohi, B. Pafford, and L. L. Raja, “Experimental Characterization of the RailPAc Plasma Flow Actuator,” 44th AIAA Plasmadynamics Lasers Conf., pp. 1–16, 2013.

[7] T. Fujino, S. Hirayama, and M. Ishikawa, “Numerical Simulation of Unsteady Behavior of Rotary Arc Plasma under Magnetic Field,” in 41st Plasmadynamics and Lasers Conference, 2010, no. July, p. AIAA 2010-4641.

[8] A. N. Bocharov, V. A. Bityurin, Studies of Hypersonic MHD Flows // Lambert Academic Publishing, 2017, pp. 219(In Russian)

[9] A. D’Angola, G. Colonna, C. Gorse, and M. Capitelli. Thermodynamic and trans temperature range. Eur. Phys. J. D 46, 129–150 (2008), DOI: 10.1140/epjd/e2007-00305-4.

[10] Yasuhro Wada and Meng-Sing Liou. An Accurate and Robust Flux Splitting Scheme for Shock and Contact Discontinuities // SIAM J. Sci. Comput., May 1997. V.18, No.3, pp.633-657

[11] Meng-Sing Liou. A Sequel to AUSM: AUSM+ // J. Comp. Phys, 1996. 129, pp.364-382.[12]. Barth T.J., and Jesperson D.C. The Design and Application of Upwind Schemes on Unstructured Meshes // AIAA Paper 89-0366, Jan 1989.

[12] Mathur S.R., Murthy J.Y. All Speed Flows on Unstructured Meshes Using a Pressure Correction Approach // AIAA Paper 99-3365, 1999.

[13] Ferziger J.H., Peric M. Computational Methods for Fluid Dynamics // Springer-Verlag Berlin, 1996.