Position feedback control of a nonmagnetic body levitated in magnetic fluid

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Abstract. This paper is concerned with the position feedback control of a magnetic fluid actuator which is characterized by the passive levitation of a nonmagnetic body immersed in a magnetic fluid under magnetic fields. First of all, the magnetic fluid actuator is designed based on the ferrohydrostatic relation. After manufacturing the actuator, its static and dynamic characteristics are investigated experimentally. With the aid of the dynamic governing relation obtained experimentally and the proportional-derivative controller, the position tracking control of the actuator is carried out both theoretically and experimentally. As a result, the applicability of the proposed magnetic fluid actuator to various engineering devices is verified.

1. Introduction

Recently, various applications using magnetic fluids has been developed, such as accelerometers, inertial dampers, passive magnetic bearings, pressure rotary seals, loud speaker coolants, and ferrohydrostatic separators [1-2]. Compared with conventional devices, magnetic fluid actuators have many advantages of simplicity in design and construction, high reliability due to the inexistence of any moving mechanical parts, and a large range of electric current for generating a desired magnetic field. Therefore, many investigations have recently been conducted in relation to the development of magnetic fluid actuators [3-7]. However, most of the previous studies have been focused on the basic design of the magnetic fluid actuators or their static characteristics. In order to improve the dynamic characteristics of these actuators and to verify its availability in various engineering applications, the development of an effective controller and the investigation on its control characteristics are inevitable. Nevertheless, the research data on the practicability of the magnetic fluid actuator are insufficient, and there still remains a wide unexplored domain. Consequently, the aim of this work is to derive the dynamic governing relation of the magnetic fluid actuator based on the passive levitation and to verify its control performances both theoretically and experimentally.

2. Design of a magnetic fluid actuator and its performances

2.1. Passive levitation condition
A nonmagnetic body of arbitrary shape, volume $V$, surface area $A$, and density $\rho_b$ immersed in a stationary magnetic fluid of density $\rho_f$ in the presence of an external magnetic field gradient $\mathbf{H}$ experiences the following forces [8]:

(i) Gravitational force: 
\[ \vec{F}_g = \int_V \rho_b \mathbf{g} dV = \rho_b \mathbf{g} V \]  
(1)

(ii) Buoyancy force: 
\[ \vec{F}_b = -\int_V \rho_f \mathbf{g} dV = -\rho_f \mathbf{g} V \]  
(2)

(iii) Magnetic buoyancy force: 
\[ \vec{F}_m = -\int_A \left( \frac{1}{2} \mu_0 M^2 + \mu_0 \int_0^H \mathbf{M} \mathbf{d} \mathbf{H} \right) \mathbf{n} dA \]  
(3)

where $\mathbf{g}$ is the gravitational acceleration, $M$ the magnetization of the fluid, $\mathbf{H}$ the magnetic field intensity, and $\mu_0$ the magnetic permeability of vacuum. Also, $\mathbf{n}$ is the unit vector normal to any point on $A$. Above equations are written in SI units. The last equalities in (1) and (2) are true only if the densities and the gravitational acceleration are uniform over $V$. The first term on right of (3) represents distortions of the magnetic field due to the immersed body. Because the error by these distortions is of the order of $M/\mathbf{H}$, they can be reasonably neglected in the common case of $M << H$. By assuming that the magnetic fluid is isothermal material as well as $M$ and $\nabla \mathbf{H}$ are constant cross $V$, (3) can be simplified as 
\[ \vec{F}_m = -\mu_0 M \nabla MV. \]  
Then, the force acting on the volume unity of the body will be:
\[ \vec{F}/V = (\rho_b - \rho_f) \mathbf{g} - \mu_0 M \nabla H \]  
(4)

where $\mathbf{g}$ is the gravitational acceleration, $M$ the magnetization of the fluid, and $\mu_0$ the magnetic permeability of vacuum. Above equations are written in SI units. As can be seen in above equation, the immersed body sinks when $\rho_b > \rho_f$ in the absence of magnetic field. If the magnetic field gradient has the same orientation to gravitational acceleration, the nonmagnetic body can be levitated at an desired point in the fluid space by a good choice of $\nabla \mathbf{H}$; that is, the conditions for floating-up and stable levitation of the body can be expressed by 
\[ M \nabla H \leq (\rho_b - \rho_f) \mathbf{g} / \mu_0 \equiv \alpha \]  
(5)

2.2. Experimental setup

Figures 1 and 2 show the schematic configuration of a magnetic fluid actuator proposed in this work and the experimental apparatus constructed to investigate its performances. Sample magnetic fluid is a kerosen-based ferricollloid HC-50 (Taiho Industries Co.). Its magnetization curve is shown in Figure 3. For the convenient calculation of $M \nabla H$ in (5), the magnetization equation was obtained
sensibly by a curve fitting method. Considering the viscous effect of the magnetic fluid, three cylindrical nonmagnetic bodies with an identical height of 10mm but different diameters of 10, 13, and 14mm are prepared. They are made of aluminum (the density $\rho = 2700 \text{ kg/m}^3$). In order to measure the displacement $x$ of the body by using an optical displacement transducer, the circular plastic stick with a small plate is attached to the body. A magnetic field gradient suitable for the stable levitation can be obtained by using a pair of hollow solenoids with north poles placed face-to-face with each other. The gap between the magnetic poles is 30mm. Each solenoid is made of 1000 turns of 0.5mm enamel wire (the electric resistance 10.77$\Omega$), and its size is 24mm in inner diameter, 40mm in outer diameter, and 30mm in length. The maximum current available is 2.4A. All experiments were performed at room temperature 20°C. Figure 4 shows the magnetic field intensity obtained both theoretically and experimentally. The coil current was induced to the solenoids in the range of 0.5 to 2.0A by a step of 0.5A, and the magnetic field was measured by a Gauss meter. As predicted, the magnetic field was zero at the center of the gap and gradually stronger as the position was further away from the center. Through the comparison between the experimental results and the simulation analysis, the efficiency of the solenoids was found by approximately 85%.

2.3. Static and Dynamic Performances

Figure 5 shows $M\times H$ curves obtained theoretically according to the coil currents. Considering the properties of the materials used in this work, $\alpha$ in (2) was found by $-1.0453 \times 10^{-10} A^2/m^3$. From this figure, it can be seen that there are two equilibrium points (EPs) corresponding to the equality of (2) at a certain coil current; one point is unstable and the other is stable. At the unstable EP, the nonmagnetic body tends to sink down or float up in the presence of external mechanical or electromagnetic disturbances. Contrarily, at the stable EP, the body in any displacement is subject to a restoring force returning it to the EP.
Figure 6 shows the displacement of the nonmagnetic body in the magnetic fluid with respect to the coil current. The ‘ideal’ curve was obtained analytically, and the ‘simulated’ curve was obtained considering the efficiency of the solenoids. The experiments were performed using the DC current with its range of 1.2 to 2.4A by a step of 0.1A. As shown in Figure 6, the experimental results were not in good accordance with the analytical ones. It is considered that some errors result from the assumptions introduced for the derivation of (1): (i) the magnetic field distortions due to the nonmagnetic body are negligible, (ii) $M$ and $\nabla H$ are constant cross the body volume, and (iii) the magnetic fluid is an isothermal material. Other reason for the disagreement may be inhomogeneity of the magnetic field in the radial direction. Nevertheless, it can be clearly seen that the displacement of the immersed body is increased with increasing the coil current. Therefore, it is concluded that the stable levitation of the nonmagnetic body in the magnetic fluid can be achieved only at the stable EP and its levitating position can be controlled actively by coil currents.

Figure 7 shows the step response of the magnetic fluid actuator. Initially, the nonmagnetic body was levitated stably at the position corresponding to the coil current of 1.4A. When the current of 2.2A was suddenly applied after 2sec, the body behaved as a typical step response with the overshoot and viscous damping. In order to investigate the effect of the body diameter, the obtained data were normalized by each minimum and maximum value. Then, the damping coefficients $\zeta$ and natural frequencies $\omega_n$ for three different bodies were found sensibly through the comparison with simulation results using a second order linear system. From the results, it can be seen that the body with larger diameter has lower natural frequency and larger damping coefficient. This is due to the viscous effect of the fluid at the gap between the body and the container wall. The gap decreased with increasing the body diameter makes the drag force acting on the body strong. Therefore, it is concluded that these design parameters can be optimally determined according to application fields of the proposed actuator.

3. Position Control of the magnetic fluid actuator

3.1. Proportional-Derivative (PD) Controller

The displacement of the body $x$ is measured, and the desired displacement $x_d$ can be determined by substituting the desired current $I_d$ into the relation $x=f(i)$ obtained from curve-fitting results in Figure 5. Also, the proportional-derivative controller is given in the Laplace domain by

$$K(s) = k_p + k_ds$$

where $k_p$ and $k_d$ are the proportional and derivative gains, respectively. Using second order linear system and (6), the characteristic equation of the closed-loop system is obtained as follows:

$$s^2 + (2\zeta\omega_n + k_d\omega_n^2)s + \omega_n^2(1+k_p) = 0$$

Introducing the desired damping coefficient $\zeta_d$ and the desired frequency $\omega_d$, so that the closed-loop system lies on a desired dynamic behaviour, the control gains are analytically found by

$$k_p = (\omega_d^2 - \omega_n^2)/\omega_n^2, \quad k_d = 2(\zeta_d\omega_n - \zeta\omega_n)/\omega_n^2$$

In this research, $\zeta_d = 0.7$ and $\omega_d = 50 rad/s$ are set. Then, the control gains for three different bodies can be determined with the help of $\zeta$ and $\omega_n$ obtained from the dynamic evaluations. In addition, the applied coil current to the magnetic fluid actuator is obtained by

$$I(t) = I_d + f^{-1}\{k_pe(t) + k_de'(t)\} \quad \text{where} \quad e(t) = x_d(t) - x(t)$$

3.2. Experimental Results and Discussion
Figure 8 shows the results of the position tracking control of the magnetic fluid actuator. In all cases, the results using the controller show improved dynamic characteristics: smaller overshoots, faster rising times, and weaker oscillation behaviours. However, there are some differences between the simulation and experimental results. Although the magnetic body force varies with respect to both the levitating position and applied current, the dynamic model of (2) is obtained neglecting the effect of the levitating position for the convenience. The external disturbances, especially the variation of fluid temperature due to the solenoids, are put in other reason. Note that there are various controllers available in real applications. Therefore, it is expected that the using the detailed model of the actuator as well as the advanced controller leading to better control performances allows further improvement in the dynamic characteristics of the actuator.

4. Conclusions
The results obtained are summarized as follows; (i) There is one equilibrium point for the stable levitation of any nonmagnetic body immersed in a magnetic fluid in the presence of sufficient magnetic field gradients. In addition, the levitating position can be effectively controlled with the coil currents. (ii) Even with the conventional controller, the dynamic performances of the proposed magnetic fluid actuator can be improved effectively. (iii) Through the comparison study between the simulation analyses and the experimental results, the applicability of the proposed magnetic fluid actuator to various engineering devices was verified.

5. References
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