Chiral expansion in the dual (string) model of the Goldstone meson scattering

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Abstract

We consider in details the dual models for the Goldstone mesons (pions) scattering in the presence of the explicit chiral symmetry breaking caused by non-zero current quark mass. New method of incorporation of the quark masses into the dual model is suggested. In contrast to the previously considered in the literature methods, the dual amplitude obtained by this method is consistent with all low–energy theorems following from the Effective Chiral Lagrangian (EChL) to the $O(p^4)$ order and simultaneously it does not contain states with negative width. The resonance spectrum of the model and its implications for the fourth and sixth order EChL in large $N_c$ limit are discussed. We argue that the possible relations between large $N_c$ QCD and some underlying string theory can be revealed by studying interactions of hadrons at low–energies.

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1. The success of the Regge phenomenology and applications of the finite energy sum rules led in the late sixties to the formulation of the duality principle for the hadronic amplitudes as a certain relation between two ways of describing scattering amplitudes: the Regge pole exchange at high energies and resonance dominance at low energies. Almost immediately it was found that the duality arises naturally in the quantum theory of extended objects – strings. In other words the string theory appeared originally to describe some striking features of strong interactions. With the advent of the Quantum Chromodynamics it was found that QCD in the large colour number limit \( N_c \to \infty \) shares many qualitative features with string theories. Let us just (sporadically) list the evidences of deep relations between QCD and some string theory:

- The perturbation expansion in the large \( N_c \) limit of QCD can be written as a sum over surfaces which may correspond to a sum over string world sheets.
- The strong coupling expansion for lattice gauge theory strongly resembles a string theory.
- The Wilson loop expectation values in the large \( N_c \) limit satisfy equations which are equivalent to those for one or another specific string theory strings.
- 2D QCD can be rewritten as a string theory.

All these facts instill belief that QCD in the large \( N_c \) limit corresponds to some string theory, though the particular form of such theory is not found. This task is especially difficult because it involves comparing field theory (QCD) in which we can not compute hadron amplitudes, with a string theory in which basically all one can do is to compute \( S \)-matrix in the narrow resonance approximation. Probably the most natural place to make such a comparison is a low energy region, where the dynamics is governed by soft pions. For the latter both QCD and dual (string) models made quantitative predictions.

Another portion of facts in favour of possible underlying string dynamics can be found in regularities of hadronic spectrum and interactions at low energies. We again give only a short (incomplete) list of them:

- The relative meson/baryon abundance might be explained by string modular invariance.
- The Ademolo, Veneziano and Weinberg (AVW) mass relations found in early days of string theory for hadrons linked by the \( S \)-wave pion emission \( \alpha'(M^2_\star - M^2) = \frac{1}{2} \mod(1) \) was rederived from general structure and properties of conformal field theory amplitudes.
- The Weinberg’s mended symmetries (algebraic realization of the chiral symmetry) can be obtained as a consequence of Kac–Moody algebra on the string world sheet.
- The effective chiral lagrangian to the \( O(p^6) \) order obtained by the low–energy reduction of the dual resonance model for pions is in agreement with phenomenology and non–topological chiral anomaly of QCD.

\(^3\)Recent general discussion of the relation between chiral symmetry and duality see in ref.
In this paper we study in details the dual 4–point amplitude for the (pseudo)Goldstone mesons. The main our concern is a proper inclusion of the current quark masses into the model, i.e. effects related to the deviations from the chiral limit. We require that the sought–for amplitude satisfies the following conditions:

1. In the chiral limit it is reduced to the Lovelace–Shapiro amplitude \[14, 15\] and inclusion of the mass does not spoil duality properties of the Lovelace–Shapiro amplitude.
2. The low–energy limit of the amplitude is consistent with the effective chiral lagrangian to the $O(p^4)$ order.
3. There are no ghosts (states with negative residue).

In the late sixties Lovelace \[14\] and Shapiro \[15\] proposed the following $\pi\pi$ scattering amplitude (in the chiral limit)

$$M^{abcd} = \text{tr}(\tau^a \tau^b \tau^c \tau^d)V(s,t) + \text{non-cyclic permutations},$$

$$V(s,t) = \lambda \frac{\Gamma(1 - \alpha_{\rho}(s))\Gamma(1 - \alpha_{\rho}(t))}{\Gamma(1 - \alpha_{\rho}(s) - \alpha_{\rho}(t))}, \quad (1)$$

where the $\rho$-meson Regge trajectory and the constant $\lambda$ are chosen to be

$$\alpha_{\rho}(s) = \frac{1}{2} + \frac{s}{2M_{\rho}},$$

$$\lambda = \frac{M_{\rho}^2}{\pi F_0^2}, \quad (2)$$

in order to ensure the low-energy theorem for the amplitude:

$$\lim_{s,t \to 0} V(s,t) = \frac{s + t}{F_0^2} + O(p^4), \quad (3)$$

where $F_0 = \lim_{m_q \to 0} F_\pi$ is a pion decay constant in the chiral limit. The amplitude (1), besides correct low–energy properties, satisfies a Regge asymptotic restrictions at high energies and the positions and residues of the resonance poles are in a good agreement with the phenomenological ones. Hence the basic phenomenological features of the hadron interactions are implemented by a simple dual amplitude (1). Let us stress that the amplitude (1) could not serve for the $\pi\pi$ scattering amplitude in the real QCD, if for no other reason than severe violation of unitarity. It could be rather considered as an amplitude for some limiting case (large $N_c$) of the real QCD. Indeed, the amplitude (1) possess all properties peculiar to the hadronic amplitudes in the large $N_c$ limit: it has only pole singularities on the real axis of kinematical variables, the OZI rule is absolute. In the present paper we shall confront the dual models for pions not with the experimental data but with the low–energy theorems at large $N_c$ limit.

Recently the amplitude (1) and its many point generalization was derived in the composite superconformal string model suggested by V. Kudryavtsev \[16\]. Although the chiral properties of this model are still not studied.
In pioneering works of Lovelace [14] and Shapiro [15] the departure from the strict chiral limit was achieved by shifting the intercept of the $\rho$-meson Regge trajectory from $\frac{1}{2}$ to $\frac{1}{2} - \frac{M^2}{2M_m^2}$ to reproduce the Adler zero of the off-shell amplitude. Let us stress that the Adler condition is an off mass shell one, whereas the dual (string) amplitudes can be defined and constructed consistently only on mass shell, and a continuation of those to unphysical region is ambiguous. Another problem is the appearance in this scheme of states with negative width. Following ref. [12] we shall use a new way of introducing quark masses (explicit chiral symmetry breaking) into the dual model. Instead of using the Adler condition we shall impose on mass shell low energy theorems on the dual amplitudes for the $\pi \pi$ scatterings.

2. In the lowest order of momentum expansion $O(p^2)$ the interactions of (pseudo)Goldstone mesons (pions, kaons and eta mesons) are described by the famous Weinberg lagrangian [17, 18]:

$$L^{(2)} = \frac{F_\pi^2}{4} \text{tr} (L_\mu L^\mu) + \frac{F_0^2 B_0}{4} \text{tr} (\chi),$$

where $\chi = 2B_0 (\hat{m} U + U^\dagger \hat{m})$, $L_\mu = iU \partial_\mu U^\dagger$, $\hat{m} = \text{diag}(m, m, m)$ is a quark mass matrix and $F_0$ and $B_0$ are low-energy coupling constants carrying an information about long-distance behaviour of the QCD. The latter are related to the pion decay constant and the quark condensate in the chiral limit as follows

$$F_0 = \lim_{m_q \to 0} F_\pi \approx 88 \text{ MeV},$$
$$B_0 = -\lim_{m_q \to 0} \frac{\langle \bar{\psi} \psi \rangle}{F_\pi}.$$

The chiral field $U(x)$ is a unitary $3 \times 3$ matrix and is parametrized in terms of eight pseudoscalar meson fields $\pi, K$ and $\eta$:

$$U(x) = e^{i\Pi},$$
$$\Pi = \begin{pmatrix}
\frac{\pi^0}{F_\pi} + \frac{\eta}{\sqrt{3}F_\eta} & -\sqrt{2}\pi^+ / F_\pi & -\sqrt{2}K^+ / F_K \\
-\sqrt{2}\pi^- / F_\pi & \frac{\pi^0}{F_\pi} + \frac{\eta}{\sqrt{3}F_\eta} & -\sqrt{2}K^0 / F_K \\
-\sqrt{2}K^- / F_K & -\sqrt{2}K^0 / F_K & -\frac{2\eta}{\sqrt{3}F_\eta}
\end{pmatrix},$$

with decay constants normalized as $F_\pi = 93.3$ MeV, $F_K \approx 1.2 F_\pi$.

In the next $O(p^4)$ order the interactions of the (pseudo)Goldstone mesons are described by the following EChL (we write only terms surviving in the large $N_c$ limit and contributing to $\pi \pi$ scattering amplitude) [20]:

$$L^{(4)} = (2L_2 + L_3) \text{tr} (L_\mu L^\mu L_\nu L^\nu) + L_2 \text{tr} (L_\mu L_\nu L^\mu L^\nu) + L_5 \text{tr} (L_\mu L^\mu \chi) + L_8 \text{tr} (\chi^2).$$

For the parameters of the fourth order EChL we use here notations of Gasser and Leutwyler [21]. We see that the fourth order EChL has in the large $N_c$ limit four independent parameters.\footnote{Without taking the large $N_c$ limit it depends on eight parameters [20].}
The last decade was a significant progress in understanding the structure of the fourth order EChL. In particular, the values of the corresponding constants was determined from the experiment [19, 20, 25] with a good accuracy, these values were also calculated in various approaches to the low–energy QCD [21, 22, 23, 24, 26, 27]. All these calculations relied on large $N_c$ limit.

Let us consider the elastic $\pi\pi$–scattering process

$$\pi_a(k_1) + \pi_b(k_2) \rightarrow \pi_c(k_3) + \pi_d(k_4)$$

( $a, b, c, d = 1, 2, 3$ are the isotopic indices and $k_1, .., k_4$ — pion momenta). Its amplitude $M^{abcd}$ can be written in the form:

$$M^{abcd} = \delta^{ab}\delta^{cd}A + \delta^{ac}\delta^{bd}B + \delta^{ad}\delta^{bc}C,$$  \hspace{1cm} (7)

where $A, B, C$ are the scalar functions of Mandelstam variables $s, t, u$:

$$s = (k_1 + k_2)^2, \quad t = (k_1 - k_3)^2, \quad u = (k_1 - k_4)^2,$$  \hspace{1cm} (8)

obeying the Bose–symmetry requirements:

$$A(s, t, u) = A(s, u, t),$$

$$B(s, t, u) = A(t, s, u),$$

$$C(s, t, u) = A(u, t, s).$$  \hspace{1cm} (9)

At low momenta one can expand the (iso)scalar amplitude $A$ in power series of invariant kinemantical variables around point $s = 0, t = 0$:

$$A(s, t) = \sum_{i,j} A_{ij} F_0^{2(i+j)} s^iT^j.$$  \hspace{1cm} (10)

The expansion coefficients $A_{ij}$ can be computed as a series in the quark mass $m$ with help of EChL (4), (8). The tree level (large $N_c$) computations give:

$$A_{00} = -\frac{2mB_0}{F_0^2} + \frac{64m^2B_0^2}{F_0^4}(3L_2 + L_3) + O(m^3),$$

$$A_{10} = 1 - \frac{32mB_0}{F_0^2}(2L_2 + L_3) + O(m^2),$$

$$A_{20} = 4(2L_2 + L_3) + O(m),$$

$$A_{02} = 8L_2 + O(m).$$  \hspace{1cm} (11)

Non–analytical contributions to these low–energy theorems appear due to the loop contributions and hence suppressed by $1/N_c$. We shall use the low–energy theorems at large $N_c$ eqs. (11) to restrict the parameters of the dual models and simultaneously to determine the parameters $L_i$ of the fourth order EChL. Let us stress that the low–energy theorems (11), in contrast to the Adler zero conditions, deal with the on–mass shell amplitude.
3. In the presence of the small explicit chiral symmetry breaking (non-zero quark mass \( m \))
we can generically write down the dual amplitude for pions as a series in the powers of the quark mass \( m \):

\[
V(s,t) = -\frac{M^2_{\rho}}{2\pi F_0^2} (1 + a_1 m + a_2 m^2 + \ldots) \left\{ \frac{2\Gamma(1 - \alpha_{\rho}(s))\Gamma(1 - \alpha_{\rho}(t))}{\Gamma(1 - \alpha_{\rho}(s) - \alpha_{\rho}(t))} \right\} \sum_{(k,n,p) \neq (1,1,1)} \left[ \frac{\Gamma(n - \alpha_{\rho}(s))\Gamma(k - \alpha_{\rho}(t))}{\Gamma(p - \alpha_{\rho}(s) - \alpha_{\rho}(t))} + (k \leftrightarrow n) \right],
\]

(12)

where \( k, n, p \) are natural numbers, such that

\[
1 \leq n \leq k \leq p \leq n + k.
\]

(13)

The conditions (13) ensure that satellites we add do not spoil the dual properties of the resulting amplitude (15). The indices \( k, n, p \) label the unknown expansion coefficients \( b_i^{(k,n,p)} \).

The latter will be fixed by use of the low-energy theorems (11) and by a requirement of positivity of the resonance widths. The intercept of the \( \rho \)-meson Regge trajectory has also mass corrections:

\[
\alpha_{\rho}(x) = \frac{1}{2} (1 + i_1 m + i_2 m^2 + \ldots) + \frac{x}{2M^2_{\rho}},
\]

(14)

\( M_{\rho} \) is the mass of the \( \rho \) meson in the chiral limit and the coefficients \( i_k \) enter with the mass corrections to the intercept of the \( \rho \) meson trajectory and simultaneously the quark mass corrections \( 5 \) to the \( M_{\rho} \). Using the \( SU(3)_{fl} \) symmetry it is easy to relate the coefficient \( i_1 \) to the linear in strange quark mass \( m_s \) correction to the \( K^*(892) \) mass (assuming \( m_u = m_d = 0 \)):

\[
M^2_{K^*} = M^2_{\rho}(1 - \frac{m_s i_1}{2} + O(m_s^2)).
\]

(15)

In (12) all quantities are understood as a chiral expansion in the quark mass \( m \). In the chiral limit the amplitude (12) coincides with (1).

4. The partial width of the spin-\( l \) resonance of mass \( M_r(N) \) with isospin \( I \) is given by

\[
\Gamma_I(N,l) = -k_I \frac{q_{c.m.}}{16\pi^2 M^2_r(N)} \int_{-1}^{1} dz \ P_l(z) \gamma(N, z),
\]

(16)

where

\[
k_I = \begin{cases} 
3/2, & I = 0 \\
1, & I = 1 \\
0, & I = 2,
\end{cases}
\]

\( P_l(z) \) are the Legendre polynomials, the \( q_{c.m.} \) is the c.m. momentum given by

\[
q_{c.m.} = \frac{1}{2} \sqrt{M^2_r(N) - 4M^2_{\pi}},
\]

\( 5 \) We do not include corrections to the slope of the trajectory because they can be absorbed by a redefinition of the \( \rho \)-mass.

\( \)
and the residue of the pole at $\alpha(s) = N$ is given by

$$
\gamma(N, z) = \text{Res}_{\alpha(s)=N} V(s, t) =
$$

$$
= -\frac{2M^4_\rho}{\pi F_0^2} (-1)^{N+1} (1 + a_1 m + a_2 m^2 + \ldots) \left\{ \frac{-2\Gamma(1 - \alpha_\rho(t))}{\Gamma(N) \Gamma(1 - N - \alpha_\rho(t))} \right\}
$$

$$
+ \sum_{(k,n,p)\neq(1,1,1)} \left( b_1^{(k,n,p)} m + b_2^{(k,n,p)} m^2 + \ldots \right)
$$

$$
\times \left[ \frac{(-1)^n \Gamma(k - \alpha_\rho(t))}{\Gamma(N - n + 1) \Gamma(p - N - \alpha_\rho(t))} + (k \leftrightarrow n) \right],
$$

(18)

$$
\times \left[ \frac{(-1)^n \Gamma(k - \alpha_\rho(t))}{\Gamma(N - n + 1) \Gamma(p - N - \alpha_\rho(t))} + (k \leftrightarrow n) \right].
$$

(19)

where the Mandelstam variables are expressed in terms of cms momentum $q$ and cms scattering angle $z = \cos \theta$:

$$
s = 4(q^2 + M^2_\rho), \quad t = -2q^2(1 - z), \quad u = -2q^2(1 + z).
$$

(20)

The mass of the resonances associated with this pole is given by

$$
M^2_\rho(N) = 2M^2_\rho(N - \frac{1}{2}(1 + i_1 m + i_2 m^2 + \ldots)).
$$

(21)

The positivity of the resonance widths can be formulated as a positivity of the following integral:

$$
- \int_{-1}^{1} dz \ P_l(z) \ \gamma^I(N, z) = I^{(0)}(N, l) + mI^{(1)}(N, l) + \ldots,
$$

(22)

where $I^{(0)}$, $I^{(1)}$ denote the expansion coefficients in the quark mass. It is known that for any $N$ and $l$ all integrals $I^{(0)}(N, l)$ are non-negative [13]. In order to ensure the positivity of the widths (absence of ghost) one should satisfy at least two conditions:

1. $I^{(1)}(N, l) \geq 0$ for $N$ and $l$ such that $I^{(0)}(N, l) = 0$ (if the $I^{(0)}(N, l)$ is positive then the chiral corrections are not able to change a sign of the leading contribution $I^{(0)}(N, l)$).

   Actually $I^{(0)}(N, l) = 0$ only for $N = 2$ and $l = 0$ [13].

2. The ratio $I^{(1)}(N, l)/I^{(0)}(N, l)$ should not rise with $N \to \infty$ at fixed $l$, otherwise one could not rely on chiral counting for states with mass $\sim M^2_\rho/m$.

We checked that the second condition is satisfied only for $b_1^{112}, b_1^{121}$, and $b_2^{213}$ satellites. For the others the ratio $I^{(1)}(N, l)/I^{(0)}(N, l)$ grows with $N$ and moreover oscillates in sign.

Furthermore the satellites $b_1^{112}, b_1^{212}$, and $b_1^{213}$ are not independent due to the identities

$$
C^{112} = 2C^{213}
$$

$$
C^{111} + C^{112} - 2C^{212} = 0
$$

where

$$
C^{knp} \equiv \left[ \frac{\Gamma(n - \alpha_\rho(s)) \Gamma(k - \alpha_\rho(t))}{\Gamma(p - \alpha_\rho(s) - \alpha_\rho(t))}\right] + (k \leftrightarrow n).
$$

(23)

This observation leaves us with the following form of the generalized dual amplitude
\[ V(s, t) = -\frac{m^2}{\pi F^2_0} \left( 1 + a_1 m + a_2 m^2 + \ldots \right) \left\{ \frac{\Gamma(1 - \alpha_{\rho}(s))\Gamma(1 - \alpha_{\rho}(t))}{\Gamma(1 - \alpha_{\rho}(s) - \alpha_{\rho}(t))} \right\} + \left( b^{112}_1 m + b^{112}_2 m^2 + \ldots \right) \frac{\Gamma(1 - \alpha_{\rho}(s))\Gamma(1 - \alpha_{\rho}(t))}{\Gamma(2 - \alpha_{\rho}(s) - \alpha_{\rho}(t))} \}. \] (24)

This amplitude was used in [12] in order to determine the parameters of the fourth and sixth order EChL. Here we address additionally the problem of the resonance widths, in particular, the absence of ghosts.

It is known that the Lovelace-Shapiro amplitude does not contain ghosts [15]. Including the mass corrections (14) to the intercept or addition of satellites to (1) may cause appearance of ghosts. Since all chiral corrections are proportional to the quark mass, they are responsible only for the small mass corrections and therefore they can not compensate the contributions from the leading term \( \Gamma^{(0)}_I (N, l) \) when it is positive and non–zero. Thus the problem of the positivity of the resonance widths is equivalent to the study of the mass corrections \( I^{(1)}(N = 2, l = 0) \), the case when the leading order result is zero.

Taking into account the chiral expansion at large \( N_c \) of the pion mass

\[ M^2_\pi = 2mB_0 + O(m^2), \] (25)

we obtain from (16) and (24) for the resonance width \( \Gamma_I(N = 2, l = 0) \) to the linear order of the chiral expansion

\[ \Gamma^{(0)}_I(N = 2, l = 0) = 0, \]

\[ \Gamma^{(1)}_I(N = 2, l = 0) = \frac{M^3_\rho}{4\pi F^2_0} (i_1 + b^{(112)}_1), \] (26)

and we require this quantity to be positive, i.e. \( i_1 + b^{(112)}_1 \geq 0 \).

The unknown constants in (26) are fixed by use of the low-energy theorems (11). Namely, at low momenta one can expand the amplitudes in power series of invariant kinemantical variables. Then we compare the low-energy expansion coefficients \( A_{ij} \) from the dual resonance amplitude (24)

\[ A_{00} = \frac{M^2_\rho}{F^2_0} (i_1 - b^{112}_1) + O(m^2), \] (27)

\[ A_{10} = 1 + \frac{m}{2} [2a_1 + 8 \log(2) i_1 + \frac{16 \log(2) B_0}{M^2_\rho} - 4 \log(2) b^{112}_1] + O(m^2), \] (28)

\[ A_{20} = 0 + O(m), \] (29)

\[ A_{02} = \frac{F^2_0}{M^2_\rho} \log(2) + O(m), \] (30)

with the low–energy theorems (11). This comparison gives the following results for the parameters of the fourth order EChL [28, 12].
\[ L_2 = 2L_1, \]  
\[ L_3 = -2L_2 \]  
\[ L_2 = \frac{F_0^2}{8M_{\rho}^2} \log(2) \approx 1.25 \times 10^{-3}. \]

The relation (31) is identical to the one following from the large \( N_c \) conditions for the meson scattering amplitude [20]. These conditions are “built in” in the dual resonance models through the Chan–Paton isotopic factor. The relation (32) is exactly the relation predicted by integration of the non-topological chiral anomaly [21, 23, 22, 24]. Actually the chiral anomaly relation \( 2L_2 + L_3 = 0 \) holds in the dual model due to the existence of the zero trajectories in these models. These trajectories arise in the dual models to prevent a double pole in the amplitude. The single-term amplitude (1) has simultaneous s- and t-channel poles. Hence an intersection zero must occur to prevent a double pole. This is provided by the lines of zeros:

\[ \alpha_{\rho}(s) + \alpha_{\rho}(t) = l \geq 1. \]  

The zero trajectory with \( l = 1 \) is associated with the Adler zero (in the chiral limit, we are considering now, the Adler zero is located in the physical region), so that, in a sense, one has in the dual model not only the Adler zero but rather a line of zeros, what influences higher order EChL. These zero patterns have been studied by Odorico [29] who suggested that they may not rely on a specific dual model, but have a more general nature.

Moreover the numerical value of \( L_2 \) (33) is close to that found by Gasser and Leutwyler in ref. [20] \( L_2 = (1.7 \pm 0.7) \cdot 10^{-3} \), to recent determination of this constant from analysis of the \( K_{14} \) decay [23] \( L_2 = (1.35 \pm 0.3) \cdot 10^{-3} \) and simultaneously to that obtained by integration of the non–topological chiral anomaly \( L_2 = 1.58 \cdot 10^{-3} \). The values of the combination \( 2L_2 + L_3 \) obtained in refs. [20, 23] are consistent with zero.

Also the comparison with the low–energy theorems gives the value of the parameters entering (24), i.e.

\[ b_1^{112} = \frac{2B_0}{M_{\rho}^2} + i_1, \]  
\[ a_1 = -2 \log(2) \left( \frac{2B_0}{M_{\rho}^2} + i_1 \right). \]

Thus, inserting (35) into (26) we obtain the following condition for the positivity of the resonance width

\[ i_1 + \frac{B_0}{M_{\rho}^2} \geq 0. \]  

Let us stress that this condition differs from the one following from the requirement of the Adler zero [14]. In the latter case it is

\[ 6 \text{For the direct check of the relation } 2L_2 + L_3 = 0 \text{ dictated by non-topological chiral anomaly of QCD and dual (string) models one need to repeat the fitting procedure of ref. [23] using, among others, variable } 2L_2 + L_3. \]
\[
\frac{i_1 + 2B_0}{M^2_\rho} = 0, \quad (38)
\]
what evidently contradicts the condition (37).

Using the relation of the \(i_1\) to the \(K^*(892)\) mass eq. (15) and the leading order expression for the \(K^-\) meson mass:

\[
M^2_K = m_s B_0 + O(m^2_s), \quad (39)
\]
one gets the following positivity condition:

\[
M^{(1)}_{K^*} - M^2_\rho - M^2_K/2 \leq 0. \quad (40)
\]
Here we introduce the notation:

\[
M^{(1)}_{K^*} \equiv M^2_\rho (1 - \frac{m_s i_1}{2}). \quad (41)
\]

Using \(M_\rho = 760\) MeV and \(M^2_K \approx m_s B_0 \approx (495)^2\) MeV\(^2\) we obtain from (40) the restriction \(M^{(1)}_{K^*} \leq 837\) MeV. We see that the maximal possible mass \(M^{(1)}_{K^*}\) differs from the \(K^*(892)\) mass by corrections of order \(O(m^2_s)\) which can easily explain the difference in 55 MeV between \(M^{(1)}_{K^*} = 837\) MeV and \(M_{K^*} = 892\) MeV.

The widths of resonances can be now easily calculated. For example, the width of the \(\rho\)-meson in this model is given by

\[
\Gamma(N = 1, l = 1) = \frac{M^2_\rho}{8\pi^2 F^2_0} \frac{1}{3} q_{c.m.} \left[1 - m \left\{ \frac{4B_0}{M^2_\rho} (\ln 2 + (2 \ln 2 + 1)i_1) \right\} \right], \quad (42)
\]
where \(q_{c.m.} = \frac{1}{2} \sqrt{M^2_\rho - 4M^2_\pi}\). The width of the \(f_2\)-meson is obtained as

\[
\Gamma(N = 2, l = 2) = \frac{M^4_\rho}{8\pi^2 F^2_0} \frac{3q_{c.m.}}{10M^2_{f_2}} \left[1 - m \left\{ \frac{4B_0}{M^2_\rho} (\ln 2 + \frac{4}{3}) + 2(\ln 2 + \frac{1}{3})i_1 \right\} \right], \quad (43)
\]
where \(q_{c.m.} = \frac{1}{2} \sqrt{M^2_{f_2} - 4M^2_\pi}\).

The numerical values of masses and widths for some other resonance states are shown in Table 1. For numerical calculations we choose \(M_\rho = 760\) MeV, \(F_0 = 88\) MeV and \(i_1 = -\frac{B_0}{M^2_\rho}\); the latter value corresponds to the maximal possible value of \(M^{(1)}_{K^*} = 837\) MeV.

5. Summarizing, we made a detailed study of the implications of spontaneous chiral symmetry breaking for dual (string) models of Goldstone boson scattering. We suggested a new method of inclusion of the explicit chiral symmetry breaking by the non–zero current quark mass into the dual (string) model. This method, in contrast to the one suggested in the original papers [14, 15], does not lead to appearance of resonances with negative width (ghosts). Constructed by this method \(4\pi\) dual amplitude on the one hand respects all low–energy theorems for soft pions, on the other hand it gives particular predictions for the higher orders of the effective chiral lagrangian (EChL). The most, in our view, remarkable prediction is the relations \(2L_2 + L_3 = 0\) for the parameters of the fourth order EChL. This relation is
exactly the same as that followed from the integration of the chiral non–topological anomaly \cite{21, 22, 23} and instanton model of the QCD vacuum \cite{24}. The deeper understanding of this amazing coincidence could shed a light on possible relations between quark–gluon dynamics and strings.

Let us once more attract reader’s attention to the idea that the natural region to look for evidences of “QCD strings” (possible string theory underlying QCD) is a soft pion physics. Both strings and QCD make particular quantitative predictions in this region and hence can be confronted there. In other words one could search for “string structure” of chiral expansion.

To realize this program the efforts from three sides are needed. First, more precise measurements of Goldstone boson scattering could give an additional information on the “string” chiral counting \footnote{The string effects are shadowed by loop $1/N_c$ corrections, but the latter are under quantitative theoretical control of chiral perturbation theory \cite{30}.}. Also the more precise and detailed meson spectroscopy (e.g. search for scalar state with mass $\sim 1300$ MeV and with coupling constant to $\pi\pi$ vanishing faster then $1/\sqrt{N_c}$ in the large colour number limit\footnote{Note the remarkable resemblance to elusive glueball !}) would help in quest of “QCD strings”. Second, from the QCD side some additional efforts in understanding of the sixth order EChL are required in order to confront sensibly “QCD chiral expansion” with the “string” one. The third, construction of the many pion string amplitudes and study of their low–energy behaviour are also needed. Actually the failure of attempts to generalize the Lovelace–Shapiro amplitude to the many pions led to loosing the interest to the strings as a theory of strong interactions. Very promising achievements in construction of such generalization were recently made by V.A. Kudryavtsev \cite{15}, although the chiral properties of suggested n–point pion amplitudes were not yet studied.

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Table 1: Masses and $\pi\pi$ widths (in MeV) of resonances predicted by dual model with mass corrections.

| $J^P$ | 770  | 1327 | 1710 | 2022 | 2293 | 2534 | 2755 | 2959 | 3150 | 3330 |
|-------|------|------|------|------|------|------|------|------|------|------|
| 10$^+$|      |      |      |      |      |      |      |      | 2    | 1.5  |
| 9$^-$ |      |      |      | 5    |      |      | 4    |      | 5    | 4.5  |
| 8$^+$ |      |      |      |      | 5    | 4    | 5    |      |      |      |
| 7$^-$ |      |      |      |      |      |      |      | 7    | 4    | 7.5  |
| 6$^+$ |      |      |      |      |      | 11   | 9    | 10   | 10   | 9    |
| 5$^-$ |      |      |      |      |      |      |      |      | 9    | 7    |
| 4$^+$ |      |      |      |      |      | 12   | 10   | 9    | 9    | 7    |
| 3$^-$ | 32   | 27   | 15   | 17   | 10   | 12   | 7    | 9    |      |      |
| 2$^+$ | 81   | 73   | 25   | 35   | 16   | 22   | 12   | 16   | 10   |      |
| 1$^-$ | 93   | 100  | 15   | 32   | 11   | 18   | 8    | 13   | 7    | 10   |
| 0$^+$ | 493  | 0    | 68   | 15   | 34   | 13   | 23   | 11   | 17   | 9    |

Table 1: Masses and $\pi\pi$ widths (in MeV) of resonances predicted by dual model with mass corrections.