Constraint on Lorentz symmetry breaking in Einstein-bumblebee theory by quasi-periodic oscillations

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Abstract

We have studied quasi-periodic oscillations frequencies in a rotating black hole in Einstein-bumblebee gravity by relativistic precession model. We find that in the case with non-zero spin parameter both of the periastron and nodal precession frequencies increase with the Lorentz symmetry breaking parameter, but the azimuthal frequency decreases. In the non-rotating black hole case, the nodal precession frequency disappears for arbitrary Lorentz symmetry breaking parameter. With the observation data of GRO J1655-40, we constrain the parameters of the rotating black hole in Einstein-bumblebee gravity, and find that the Lorentz symmetry breaking parameter is almost negative in the range of $1\sigma$. The negative breaking parameter, comparing with the usual Kerr black hole, leads to that the rotating black hole in Einstein-bumblebee gravity owns the higher Hawking temperature and the stronger Hawking radiation, but the lower possibility of exacting energy by Penrose process.

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I. INTRODUCTION

Lorentz invariance has been great of importance in general relativity and the standard model of particle physics. However, according to the development of unified gauge theories and the signals from high energy cosmic rays [1, 2], Lorentz symmetry may spontaneously break in the more fundamental physics at a higher scale of energy. And then studying Lorentz violation is also expected to obtain a deeper understanding of nature. In general, the direct test of Lorentz violation is impossible because their high energy scale is unavailable in the current experimentations. However, recent investigations also show that some signals related to Lorentz violation could emerge at lower energy scales so that their corresponding effects could be observed in experiments [3].

Einstein-bumblebee gravity [4] is a simple effective theory of gravity with Lorentz violation where the spontaneous breaking of Lorentz symmetry is induced by a nonzero vacuum expectation value of bumblebee vector field $B_\mu$ with a suitable potential. The black hole solutions in Einstein-bumblebee gravity and the corresponding effects of Lorentz violation have been extensively studied in the past years [5–15]. R. Casana et al firstly found an exact solution of a static neutral black hole and discussed its some classical tests [3]. And then, the gravitational lensing [16], the Hawking radiation [17] and quasinormal modes [18] have been addressed in this black hole spacetime. Moreover, other spherically symmetric black hole solutions, containing global monopole [19], cosmological constant [20], or Einstein-Gauss-Bonnet term [21], and the traversable wormhole solution in the framework of the bumblebee gravity theory [22] have also been found. The cosmological implications of bumblebee gravity model are further investigated in [23]. Furthermore, the rotating black hole solution [24] is also obtained in Einstein bumblebee gravity, and the corresponding shadow [24, 25], accretion disk [26], superradiant instability of black hole [27] and particle’s motion [28] around the black hole are studied. A Kerr-Sen-like black hole with a bumblebee field has also been investigated [29]. These investigations are useful in testing Einstein bumblebee theory and detecting the effects caused by the Lorentz symmetry breaking originating from bumblebee vector field.

Quasi-periodic oscillations is a promising arena to test the nature of the compact objects, which appear as peaks in the observed X-ray power density spectrum emitted by accreting black hole binary systems [30, 31] and hold important information about gravity in the strong field region. Generally, the frequency range of the quasi-periodic oscillations changes from mHz to hundreds of Hz. There are various theoretical models proposed to account for such peaks in power density spectrum, but the essence of quasi-periodic oscillations
is still unclear at present. The relativistic precession model is a highly regarded model of explaining quasi-periodic oscillations in which the oscillation frequencies are believed to associate with three fundamental frequencies of a test particle around a central object \[32–37\]. In this model, the azimuthal frequency \(\nu_\phi\) and the periastron precession frequency \(\nu_{\text{per}}\) of the test particle are explained, respectively, as the twin higher frequencies quasi-periodic oscillations. And the nodal precession frequency \(\nu_{\text{nod}}\) of the particle is identified with the low-frequency quasi-periodic peak in the power density spectrum of low-mass X-ray binaries. Thus, the low-frequency quasi-periodic signal is assumed to be emitted at the same orbit of the test particle where the twin higher frequencies signals are generated. Together with the observation data of GRO J1655-40 \[32\], the constraint on the black hole parameters in various theories of gravity have been performed by quasi-periodic oscillations within the relativistic precession model \[38–51\]. The main purpose of this paper is to constrain the Lorentz symmetry breaking parameter for a rotating black hole in Einstein-bumblebee theory of gravity by using of quasi-periodic oscillations with the observation data of GRO J1655-40. Finally, we present a summary.

The paper is organized as follows: In Sec.II, we will review briefly the rotating black hole in Einstein-bumblebee theory of gravity \[24\]. In Sec.III, we study quasi-periodic oscillations in the above black hole spacetime and then make a constraint on the Lorentz symmetry breaking parameter with the observation data of GRO J1655-40. Finally, we present a summary.

### II. A ROTATING BLACK HOLE IN EINSTEIN-BUMBLEBEE THEORY OF GRAVITY

In this section we review briefly a rotating black hole in Einstein-bumblebee theory \[24\]. In the framework of the bumblebee gravity theory, the spontaneous Lorentz symmetry breaking is induced by a vector \(B_\mu\) with a non-zero nonzero vacuum expectation value. Through a coupling, the bumblebee vector field \(B_\mu\) would affect the dynamics of the gravitational field. The action describing such kind of Lorentz symmetry breaking is \[3–6\]

\[
S = \int d^4\sqrt{-g}\left[\frac{1}{16\pi}(R + \xi B^{\mu\nu}R_{\mu\nu}) - \frac{1}{4} B^{\mu\nu} B_{\mu\nu} - V(B_\mu B^\mu \pm b^2)\right],
\]

(1)

where \(\xi\) is the coupling constant with the dimension \(M^{-2}\) and the bumblebee field strength \(B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu\). The potential \(V\), inducing Lorentz violation, has a minimum at \(B_\mu B^\mu \pm b^2 = 0\) (where \(b\) is a real positive constant). The condition \(B_\mu B^\mu \mp b^2 = 0\) is satisfied when the vector field has a nonzero vacuum value \(\langle B_\mu \rangle = b_\mu\) with \(b_\mu b^\mu = b^2\). The extended vacuum Einstein equations in this model with Lorentz symmetry
breaking becomes

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}, \]  

(2)

with

\[ T_{\mu\nu} = B_{\mu\alpha} B^\alpha_{\nu} - g_{\mu\nu} \left( \frac{1}{4} B_{\alpha\beta} B^{\alpha\beta} + V \right) - 2 B_{\mu} B_{\nu} V' + \frac{\xi}{8\pi} \left[ \frac{1}{2} g_{\mu\nu} B_\alpha B^\alpha - B_\mu B^\alpha R_{\alpha\nu} - B_\nu B^\alpha R_{\alpha\mu} \right. \]

\[ + \frac{1}{2} \nabla_\alpha \nabla_\mu (B^\alpha B_\nu) + \frac{1}{2} \nabla_\alpha \nabla_\nu (B^\alpha B_\mu) - \frac{1}{2} \nabla^2 (B_\mu B_\nu) - \frac{1}{2} g_{\mu\nu} \nabla_\alpha \nabla_\beta (B^\alpha B_\beta) \right]. \]  

(3)

The Einstein equations (2) admits a rotating black hole solution with a metric [24]

\[ ds^2 = \left( 1 - \frac{2Mr}{\rho^2} \right) dt^2 - \frac{4Mar\sqrt{l+1}\sin^2 \theta}{\rho^2} dtd\phi + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 \]

\[ + \frac{\sin^2 \theta}{\rho^2} \left[ (r^2 + (l+1)a^2)^2 - \Delta(l+1)a^2 \sin^2 \theta \right] d\phi^2, \]  

(4)

where

\[ \rho^2 = r^2 + (l+1)a^2 \cos^2 \theta, \quad \Delta = \frac{r^2}{l+1} - \frac{2Mr}{l+1} + a^2. \]  

(5)

Here \( M \) is the ADM mass and \( a \) is the spin parameter of black hole. The form of the bumblebee field is \( b_\mu = (0, b_\rho, 0, 0) \), and the parameter \( l = \xi b^2 \) depends on the spontaneous Lorentz symmetry breaking of the vacuum of the Einstein-bumblebee vector field. As in the Kerr black hole case, the singularity lies at \( \rho^2 = 0 \) and the horizon locates at \( \Delta = 0 \). However, the horizon radius becomes

\[ r_\pm = M \pm \sqrt{M^2 - (l+1)a^2}, \]  

(6)

which depends on the spontaneous Lorentz symmetry breaking parameter \( l \). With the increase of the absolute value of \( l \), the outer horizon radius increases for the positive \( l \) and decreases for the negative one. Thus, comparing with the usual Kerr black hole, the negative \( l \) leads to that the rotating black hole [4] owns the higher Hawking temperature and the stronger Hawking radiation [24]. Moreover, for a rotating black hole [4], its the mass and spin parameter must satisfy \( \frac{|a|}{M} \leq \frac{1}{\sqrt{l+1}} \). The negative \( l \) broadens the range of black hole spin parameter so that \( |a| > M \), which differs quite from the Kerr case in general relativity.

III. CONSTRAINT ON PARAMETERS OF A ROTATING BLACK HOLE IN EINSTEIN-BUMBLEBEE THEORY BY QUASI-PERIODIC OSCILLATIONS

In this section, we will apply quasi-periodic oscillations to make a constraint on parameters of a rotating black hole [4] in Einstein-bumblebee theory. For a general stationary and axially symmetric spacetime, the
metric of a rotating black hole with bumblebee field \( \text{(4)} \) can be written as a common form
\[
 ds^2 = g_{tt} dt^2 + g_{rr} dr^2 + 2 g_{t\phi} dt d\phi + g_{\theta\theta} d\theta^2 + g_{\phi\phi} d\phi^2. 
\]
Obviously, the metric coefficients in Eq. \( \text{(4)} \) are independent of the coordinates \( t \) and \( \phi \). Thus, the geodesic motion of particle in the black hole spacetime \( \text{(4)} \) exists two conserved quantities, i.e., the specific energy at infinity \( E \) and the conserved \( z \)-component of the specific angular momentum at infinity \( L_z \), and the forms of \( E \) and \( L_z \) can be expressed as
\[
 E = -p_t = -g_{tt}\dot{t} - g_{t\phi}\dot{\phi}, \quad L_z = p_\phi = g_{t\phi}\dot{t} + g_{\phi\phi}\dot{\phi}. \quad \text{(8)}
\]
With above two conserved quantities, the timelike geodesics can be further simplified as
\[
 \dot{t} = \frac{g_{\phi\phi}E + g_{t\phi}L_z}{g_{\phi\phi} - g_{tt}g_{\phi\phi}}, \quad \dot{\phi} = \frac{g_{t\phi}E + g_{tt}L_z}{g_{tt}g_{\phi\phi} - g_{t\phi}}, \quad g_{rr}\dot{r}^2 + g_{\theta\theta}\dot{\theta}^2 = V_{\text{eff}}(r, \theta; E, L_z), \quad \text{(11)}
\]
where \( V_{\text{eff}}(r, \theta; E, L_z) \) is the effective potential with the form
\[
 V_{\text{eff}}(r, \theta; E, L_z) = \frac{E^2 g_{\phi\phi} + 2EL_z g_{t\phi} + L_z^2 g_{tt}}{g_{\phi\phi} - g_{tt}g_{\phi\phi}} - 1. \quad \text{(12)}
\]
Here the overhead dot represents a derivative with respect to the affine parameter \( \lambda \). Actually, the radial component of the timelike geodesic equations
\[
 \frac{d}{d\lambda}(g_{rr}\dot{r}) = \frac{1}{2} \left( \partial_r g_{tt}\dot{t}^2 + 2\partial_r g_{t\phi}\dot{t}\dot{\phi} + \partial_r g_{\phi\phi}\dot{\phi}^2 + (\partial_r g_{rr})\dot{r}^2 + (\partial_r g_{\theta\theta})\dot{\theta}^2 \right). \quad \text{(14)}
\]
We here consider only the case where a particle moving along a circular orbit in the equatorial plane, i.e., \( r = r_0 \) and \( \theta = \pi/2 \), which means that \( \dot{r} = \dot{\theta} = 0 \). Thus, for the circular equatorial orbit case, Eq. \( \text{(14)} \) can be simplified as
\[
 (\partial_r g_{tt})\dot{t}^2 + 2(\partial_r g_{t\phi})\dot{t}\dot{\phi} + (\partial_r g_{\phi\phi})\dot{\phi}^2 = 0, \quad \text{(15)}
\]
which gives the orbital angular velocity \( \Omega_\phi \) of particle moving in the circular orbits
\[
 \Omega_\phi = \frac{d\phi}{dt} = \frac{-g_{t\phi,rr} \pm \sqrt{(g_{t\phi,rr})^2 + g_{tt,rr}g_{\phi\phi,rr}}}{g_{\phi\phi,rr}}, \quad \text{(16)}
\]
here the sign is $+(-)$ for corotating (counterrotating) orbits. The corresponding azimuthal frequency $\nu_\phi = \Omega_\phi/(2\pi)$. For a timelike particle moving along circular orbits in the equatorial plane, the timelike conditions $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -1$ gives another relationship between $\dot{t}$ and $\dot{\phi}$

$$g_{tt}\dot{t}^2 + 2g_{t\phi}\dot{t}\dot{\phi} + g_{\phi\phi}\dot{\phi}^2 = -1.$$  \hfill (17)

From two independent equations (15) and (17), one can obtain

$$\dot{t} = \frac{1}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega_\phi - g_{\phi\phi}\Omega_\phi^2}}.$$  \hfill (18)

Together with Eq.(8), one can find that the specific energy $E$ and the conserved $z$-component of the specific angular momentum $L_z$ are expressed respectively as

$$E = -\frac{g_{tt} + g_{t\phi}\Omega_\phi}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega_\phi - g_{\phi\phi}\Omega_\phi^2}},$$

$$L_z = \frac{g_{t\phi} + g_{\phi\phi}\Omega_\phi}{\sqrt{-g_{tt} - 2g_{t\phi}\Omega_\phi - g_{\phi\phi}\Omega_\phi^2}}.$$  \hfill (19)

Assuming a small perturbation around a circular equatorial orbit [38–51], i.e.,

$$r(t) = r_0 + \delta r(t), \quad \theta(t) = \frac{\pi}{2} + \delta \theta(t).$$  \hfill (20)

one can find that the perturbations $\delta r(t)$ and $\delta \theta(t)$ satisfy the following differential equations

$$\frac{d^2\delta r(t)}{dt^2} + \Omega_r^2 \delta r(t) = 0, \quad \frac{d^2\delta \theta(t)}{dt^2} + \Omega_\theta^2 \delta \theta(t) = 0,$$  \hfill (21)

with

$$\Omega_r^2 = -\frac{1}{2g_{rr}t^2} \frac{\partial^2 V_{eff}}{\partial r^2} \bigg|_{r=r_0,\theta=\frac{\pi}{2}}, \quad \Omega_\theta^2 = -\frac{1}{2g_{\theta\theta}t^2} \frac{\partial^2 V_{eff}}{\partial \theta^2} \bigg|_{r=r_0,\theta=\frac{\pi}{2}}.$$  \hfill (22)

The radial epicyclic frequency $\nu_r$ and the vertical epicyclic frequency $\nu_\theta$ can be written as $\nu_r = \Omega_r/2\pi$ and $\nu_\theta = \Omega_\theta/2\pi$, respectively. Inserting metric functions (31) into Eq.(16), we can find the azimuthal frequency

$$\nu_\phi = \frac{1}{2\pi r^{3/2}} \frac{M^{1/2}}{a^* M^{3/2} \sqrt{l+1}}.$$  \hfill (23)

where $a^* \equiv a/M$. It is easy to find that the azimuthal frequency $\nu_\phi$ decreases with the Lorentz symmetry breaking parameter $l$ for the rotating case. As $a = 0$, one can find that $\nu_\phi$ is independent of the parameter $l$.

Similarly, substituting metric functions (31) into Eqs.(18) and (22), one has

$$\nu_r = \nu_\phi \left[ 1 \frac{1}{l+1} - \frac{6M}{(l+1)r} + \frac{8a^* M^{3/2}}{\sqrt{l+1} r^{3/2}} - 3a^* M^2 r^2 \right]^{1/2},$$  \hfill (24)

$$\nu_\theta = \nu_\phi \left[ 1 \frac{4a^* \sqrt{l+1} M^{3/2}}{r^{3/2}} + 3a^* (l+1) \frac{M^2 r^2}{r^2} \right]^{1/2}.$$  \hfill (25)
FIG. 1: The change of the frequencies $\nu_\phi$, $\nu_{\text{per}}$ and $\nu_{\text{nod}}$ with the parameter $l$ in a rotating black hole spacetime in Einstein-bumblebee theory. Here we set $M = 1$ and $r = 6.5$.

Obviously, in the rotating case $a \neq 0$, the frequencies $\nu_r$ and $\nu_\theta$ depend on the Lorentz symmetry breaking parameter $l$. As in the non-rotating case with $a = 0$, one can find that only the frequency $\nu_r$ is related to the parameter $l$ since $\nu_\theta$ is identical with $\nu_\phi$ in this case and they are not functions of the parameter $l$. The properties of above three frequencies make it possible to constrain effect from the Lorentz symmetry breaking by quasi-periodic oscillations. As $l = 0$, it is easy to find that these three frequencies reduce to those in the usual Kerr black hole spacetime [32–35]. Furthermore, the periastron and nodal precession frequencies can be expressed as

$$
\nu_{\text{per}} = \nu_\phi - \nu_r, \quad \nu_{\text{nod}} = \nu_\phi - \nu_\theta,
$$

respectively. In Fig. 1, we plot the change of the frequencies $\nu_\phi$, $\nu_{\text{per}}$ and $\nu_{\text{nod}}$ for the rotating black hole spacetime in Einstein-bumblebee theory [4]. It is shown that in the case with $a \neq 0$ both of the periastron and nodal precession frequencies ( $\nu_{\text{per}}$ and $\nu_{\text{nod}}$ ) increase with the Lorentz symmetry breaking parameter $l$, but the azimuthal frequency $\nu_\phi$ decreases. We also find that as $a = 0$ the nodal precession frequency $\nu_{\text{nod}}$ is zero for arbitrary $l$ as expected. With the increase of the spin parameter $a$, the frequencies $\nu_\phi$ and $\nu_{\text{per}}$ decrease, but the frequency $\nu_{\text{nod}}$ increases.

According to the relativistic precession model, three simultaneous quasi-periodic oscillations frequencies are generated at the same radius of the orbit in the accretion flow. For a rotating black hole spacetime [4] in Einstein-bumblebee gravity, there are three parameters to describe black hole spacetime. Thus, we have to resort to the $\chi^2$ analysis and best-fit the values of these variables. According to the current observations of GRO J1655-40, there are two set of data about these frequencies ($\nu_\phi, \nu_{\text{per}}, \nu_{\text{nod}}$) [32, 38]:

$$
(441^{+2}_{-2}, 298^{+4}_{-4}, 17.3^{+0.1}_{-0.1}) \quad \text{and} \quad (451^{+5}_{-5}, -1, 18.3^{+0.1}_{-0.1}).
$$
FIG. 2: Constraints on the parameters of a rotating black hole in Einstein-bumblebee theory with GRO J1655-40 from current observations of QPOs within the relativistic precession model. The red, blue and gray regions represent the contour levels $1\sigma$, $2\sigma$ and $3\sigma$, respectively. The black dots in the panels correspond the best-fit values of parameters: $M = 5.4002 M_\odot$, $a^* = 0.2976$ and $l = -0.1048$.

Moreover, the mass of the black hole is also independently measured by a dynamical method: $M_{\text{dyn}} = 5.4 \pm 0.3 M_\odot$. Therefore, there are five free parameters: mass $M$, spin parameter $a$, Lorentz symmetry breaking parameter $l$, the radius $r_1$ and $r_2$ correspond the observations with three frequencies and two frequencies, respectively. With these data, we can constrain the parameters of a rotating black hole spacetime in Einstein-bumblebee gravity through the relativistic precession model as in ref. Applying the $\chi^2$ analysis, we obtain the minimum $\chi^2_{\text{min}} = 0.1946$ and fit the black hole parameters

$$M = 5.4002_{-0.0562}^{+0.0478} M_\odot, \quad a^* = 0.2976_{-0.0119}^{+0.0233}, \quad l = -0.1048_{-0.1316}^{+0.1678} (28)$$

at the confidence level 68.3%. Moreover, we find that the best-fit values of the radius of circular orbital corresponding two sets of quasi-periodic oscillations are $r_1 = 5.6194 M = 1.1140 r_{\text{ISCO}}$ and $r_2 = 5.5155 M = 1.0934 r_{\text{ISCO}}$, respectively. Here, the innermost stable circular orbit $r_{\text{ISCO}}$ is obtained for the rotating black hole spacetime with the best-fit values. Our fitted result show that the circular orbit of quasi-periodic oscillations lies in the strong gravitational-field region of the black hole. In Fig.(2), we show the contour levels of $1\sigma$, $2\sigma$ and $3\sigma$ for the black hole parameters $M$, $a$ and $l$. Our results show that the Lorentz symmetry breaking parameter is almost negative in the range of $1\sigma$, which means that the negative Lorentz symmetry breaking parameter and the negative coupling constant $\xi$ between the bumblebee field and gravitational field are favored by the observation data of GRO J1655-40 in the range of $1\sigma$. The negative best-fit value of $l$ means that the spacetime described gravitational field of the source of GRO J1655-40 allows $|a|/M > 1$ for a black hole, which means that the range of black hole spin parameter $a$ is larger than that in the Kerr case in general relativity. Comparing with the usual Kerr black hole spacetime, the negative $l$ leads to that both the outer ergosurface radius $r_{\text{outer}}$ and the outer horizon radius $r_+$ increase, but the width between the
outer ergosurface and the outer horizon \( r_{\text{outer}} - r_+ = \frac{a^2 \sin^2 \theta}{\sqrt{M^2 - (l+1)a^2} + \sqrt{M^2 - (l+1)a^2 \cos^2 \theta}} \) decreases for fixed \( \theta \), which yields the lower possibility of exacting energy by Penrose process for a rotating black hole in Einstein-bumblebee gravity [4]. Moreover, the negative \( l \) means that the black hole [4] owns the higher Hawking temperature and the stronger Hawking radiation than the Kerr black hole.

**IV. SUMMARY**

With relativistic precession model, we have studied quasi-periodic oscillations frequencies in a rotating black hole in Einstein-bumblebee gravity [4]. The black hole owns three parameters: mass \( M \), spin \( a \) and the Lorentz symmetry breaking parameter \( l \). We find that in the case with \( a \neq 0 \) both of the periastron and nodal precession frequencies (\( \nu_{\text{per}} \) and \( \nu_{\text{nod}} \)) increase with the Lorentz symmetry breaking parameter \( l \), but the azimuthal frequency \( \nu_{\phi} \) decreases. In the non-rotating black hole case, the nodal precession frequency \( \nu_{\text{nod}} \) is zero for arbitrary \( l \) since \( \nu_{\theta} = \nu_{\phi} \) in this case and they are independent of the parameter \( l \). With the increase of the spin parameter, the frequencies \( \nu_{\phi} \) and \( \nu_{\text{per}} \) decrease, but the frequency \( \nu_{\text{nod}} \) increases. With the observation data of GRO J1655-40, we constrain the parameters of the rotating black hole in Einstein-bumblebee gravity [4], and find that in the range of 1\( \sigma \) the Lorentz symmetry breaking parameter \( l \) is almost negative. This implies the negative Lorentz symmetry breaking parameter and the negative coupling constant \( \xi \) between the bumblebee field and gravitational field are favored by the observation data of GRO J1655-40. Comparing with the usual Kerr spacetime, the negative \( l \) leads to that the black hole [4] in Einstein-bumblebee gravity owns the higher Hawking temperature and the stronger Hawking radiation than the Kerr black hole, but the lower possibility of exacting energy by Penrose process.

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