LY$\alpha$ SIGNATURES FROM DIRECT COLLAPSE BLACK HOLES

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ABSTRACT

“Direct collapse black holes” (DCBHs) provide possible seeds for supermassive black holes that exist at $z \sim 7$. We study LY$\alpha$ radiative transfer through simplified representations of the DCBH scenario. We find that gravitational heating of the collapsing cloud gives rise to a LY$\alpha$ cooling luminosity of up to $\sim 10^{38} (M_{\text{gas}}/10^6 M_{\odot})^2$ erg s$^{-1}$. Photoionization by a central source boosts the LY$\alpha$ luminosity to $L_{\text{LY}\alpha} \sim 10^{43} (M_{\text{BH}}/10^6 M_{\odot})$ erg s$^{-1}$, where $M_{\text{BH}}$ denotes the mass of the black hole powering this source. We predict that the width and velocity offsets of the LY$\alpha$ spectral line range from a few tens to few thousands km s$^{-1}$, depending sensitively on the evolutionary state of the cloud. We apply our predictions to observations of CR7, a luminous LY$\alpha$ emitter at $z \sim 7$, which may be associated with a DCBH. If CR7 is powered by a black hole, then its LY$\alpha$ flux requires that $M_{\text{BH}} > 10^7 M_{\odot}$, which exceeds the mass of DCBHs when they first form. The observed width of the LY$\alpha$ spectrum favors the presence of only a low column density of hydrogen, $\log[N_{\text{HI}}/\text{cm}^{-2}] \sim 19–20$. The shape of the LY$\alpha$ spectrum indicates that this gas is outflowing. These requirements imply that if CR7 harbors a DCBH, then the physical conditions that enabled its formation have been mostly erased, which is in agreement with theoretical expectations. These constraints weaken if the observed LY$\alpha$ emission represents the central peak of a more extended halo.

Key words: cosmology: theory – dark ages, reionization, first stars – galaxies: high-redshift – quasars: supermassive black holes – radiative transfer – scattering

1. INTRODUCTION

The “Direct Collapse Black Hole” (DCBH) scenario presents a channel for forming supermassive black holes that exist in the universe at $z \gtrsim 6$ (see e.g., Johnson & Haardt 2016, for a review). In the DCBH scenario primordial gas inside dark matter halos with $T_{\text{vir}} \geq 10^4$ K collapses directly into a $\sim 10^4\sim 10^6 M_{\odot}$ black hole, without any intermediate star formation. DCBH formation requires primordial gas to collapse isothermally at $T \sim 10^4$ K (e.g., Bromm & Loeb 2003), which has been shown to prevent fragmentation (e.g., Li et al. 2003; Omukai et al. 2005). Isothermal collapse at $T \sim 10^4$ K is possible when primordial gas is kept free of molecular hydrogen, $H_2$, for example as a result of a strong photo-dissociating radiation background (e.g., Bromm & Loeb 2003; Shang et al. 2010; Wolcott-Green et al. 2011; Van Borm & Spaans 2013; Fernandez et al. 2014; Regan et al. 2014; Sugimura et al. 2014; Agarwal et al. 2016b; Inayoshi & Tanaka 2015; Latif et al. 2015) and/or magnetic fields (Sethi et al. 2010; Van Borm & Spaans 2013).

The DCBH formation process is a complex problem, which causes theoretical predictions for the number density of DCBHs span to span orders of magnitude (e.g., Dijkstra et al. 2014a; Habouzit et al. 2016). For this reason it is extremely valuable to have observational signposts of this process. Agarwal et al. (2013) presented predictions for the broad-band colors of DCBH host galaxies, under the assumption that the spectrum emitted by the accretion disk surrounding the DCBH is a multi-colored disk. Under this assumption, DCBH host galaxies are characterized by blue UV slopes ($\beta \sim -2.3$), which are similar to those predicted for metal-poor, young stars. More recently, Dijkstra et al. (2016) have shown that the physical conditions that are required for DCBH formation are optimal for LY$\alpha$ pumping of the $2p$ level of atomic hydrogen (see Pottasch 1960; Field & Partridge 1961), and that the $2p$ levels are overpopulated relative to the $2s$ level. These inverted level populations give rise to stimulated $2p_{3/2} \rightarrow 2s_{1/2}$ emission at a rest-frame wavelength of $\lambda = 3.04$ cm. Dijkstra et al. (2016) show that in simplified models of the DCBH scenario, the resulting maser amplifies the background background Cosmic Microwave Background (CMB) by a factor of up to $10^5$, which renders these clouds detectable with the Square Kilometer Array. Dijkstra et al. (2016) performed detailed LY$\alpha$ radiative transfer calculations to compute their signal. The goal of this paper is to focus on the LY$\alpha$ line itself, and to demonstrate that there is useful information in both the total LY$\alpha$ flux and its spectral line shape. The motivation for this work is as follows.

1. LY$\alpha$ is theoretically expected to be the most prominent spectral feature in the spectrum emerging from clouds collapsing into or onto a DCBH (see e.g., Inayoshi et al. 2016, Pallottini et al. 2015, Agarwal et al. 2016a). Importantly, the LY$\alpha$ radiative transfer problem is simplified enormously in the DCBH formation scenario: ordinarily, LY$\alpha$ transfer through the interstellar medium of galaxies depends sensitively on the distribution, composition and kinematics of cold interstellar gas, all of which are closely connected to stellar feedback processes (see Barnes et al. 2014; Dijkstra 2014; Hayes 2015, for extended reviews). In contrast, in the DCBH scenario gas fragmentation is suppressed, and no star formation has taken place, which makes stellar feedback (chemical, kinematical) irrelevant. That is, the major challenges that one ordinarily faces when modeling interstellar LY$\alpha$ transfer are absent in the DCBH scenario. It is worth stressing that an immediate consequence of this difference is that the LY$\alpha$ line emerging from gas in the DCBH scenario is likely different than that emerging from star-forming galaxies.
The physical conditions of the gas that enabled DCBH formation persist briefly after the black hole has formed. Merging with nearby halos occurs over timescales of tens of Myr (e.g., Hartwig et al. 2015, also see Tanaka 2014), which would likely mostly erase these conditions. We will also perform Lyα transfer calculations through clouds that are illuminated by a luminous ionizing source, under the assumption that the gas properties are the same as those that enabled DCBH formation. This also provides us with observational signposts of the DCBH formation process.

2. The recent observation of CR7, a luminous Lyα emitting source at z ~ 6.6 (Matthee et al. 2015; Sobral et al. 2015), with a large Lyα equivalent width, and an accompanying luminous, large equivalent width HeI640 line (Sobral et al. 2015). The fact that Hubble Space Telescope imaging of CR7 reveals that it is composed of a blue galaxy close to a more massive galaxy has further fueled speculation that CR7 is associated with a DCBH (Hartwig et al. 2015; Pallottini et al. 2015; Sobral et al. 2015; Agarwal et al. 2016a). Previous works have focused on the observed line fluxes and equivalent widths of Lyα and HeI640. The observed ratio of the flux in these lines favor photoionization by sources with very hard spectra (with $T_{\text{eff}} > 10^5$ K, see Sobral et al. 2015, Hartwig et al. 2016, also see Johnson et al. 2011), and possibly requires a significant fraction of the emitted Lyα flux not to have been detected (Sobral et al. 2015; Agarwal et al. 2016a).

Because CR7 has such a bright Lyα emission line, there is excellent data on the Lyα spectral line shape (Sobral et al. 2015). In this paper we demonstrate that it is possible to extract unique physical information from the Lyα line. Moreover, our analysis also highlights the physical ingredients that must be considered when modeling Lyα line emission.

The outline of this paper is as follows. In Section 2 we describe our model and list the relevant Lyα radiative processes, including Lyα emission processes (Section 2.2) and radiative transfer processes (Section 2.3). We present our main results such as the predicted Lyα luminosity (Section 3) and spectra (Section 4) of clouds collapsing into or onto a DCBH. We compare our predictions to observations of CR7 in Section 5, before presenting our conclusions and outlook in Section 6.

2. MODEL

2.1. Geometry and Useful Physical Quantities

Our analysis focuses on spherical and ellipsoidal gas clouds. As we discuss in Section 3, our predicted luminosities are unlikely to be affected by this simplifying assumption. However, our predicted spectra will change (see Section 4). Throughout the paper, we derive numerical values in our equations assuming a uniform gas density profile, which is characterized by a single number density, $n$. For a given gas mass $M_{\text{gas}}$, this gives a cloud radius $R$. Results for uniform clouds are easy to verify and interpret. Adopting a uniform gas density allows us to simplify the Lyα transfer enormously, and occasionally use analytic result from earlier work. Importantly, we show in Appendix A.3 that these results differ only slightly from those obtained assuming a (cored) isothermal density profile (see, e.g., Shang et al. 2010, Pacucci & Ferrara 2015). In addition, the density profile changes the average density at which certain processes become important compared to calculations that assume a uniform density, but otherwise does not affect our results.

For an isothermal density profile the gas density becomes very large toward the center of the collapsing gas cloud, which may lead to the formation of a quasi-star, a supermassive star, or a DCBH in the center of the cloud. We therefore also consider models which contain a central source of ionizing radiation, as in Dijkstra et al. (2016). That is, our models assume that the gas is collapsing either into a black hole, or onto a central massive black hole that formed out of the inner (denser) gas within the cloud (accretion rates onto the central black hole can be up to $\sim 0.1 M_\odot \ yr^{-1}$, e.g., Latif & Volonteri 2015).

As we justified in Section 1, we focus on the emission properties of gas with properties that enabled the DCBH scenario, i.e., the gas is pristine, no fragmentation occurred, and no stars have formed. For these reasons, we expect our predictions to be most relevant only during the early phase of DCBH formation. As the dark matter halo merger rates increase as $(1 + z)^{3/2}$ (e.g., Tanaka 2014), the DCBH host halo is expected to merge with another halo on a short timescale (~tens of Myr; see Hartwig et al. 2015). In this case, the changes in gas conditions will likely make the Lyα more complex, and more reminiscent of what is happening in ordinary metal-poor star-forming galaxies. This will not affect our predicted (intrinsic) Lyα luminosities, but it will affect our predicted spectra (also see Section 4.5).

Finally, it is useful to recall that the mass of a dark matter halo with a virial temperature of $T_{\text{vir}} = 10^4$ K is $M_{\text{halo}} = 10^8 (\mu/0.6)^{-3/2} (1 + z)/11)^{-3/2} M_\odot$ ($\mu$ denotes the mean particle mass in units of the proton mass), which has a virial radius of $R_{\text{vir}} = 1.8(1 + z)/11 (M_{\text{halo}}/10^9 M_\odot)^{1/3}$ kpc (Barkana & Loeb 2001). The average number density of hydrogen atoms/nuclei at virialization is $\bar{n} = 0.048 (1 + z)/11^2 \ cm^{-3}$. Linear contraction by a factor of $x$ increases the number density by an additional factor of $x^3$.

2.2. Lyα Emission

The DCBH scenario involves the collapse of a gas cloud, which cools via atomic line cooling, $f_\alpha \approx 40\%$ of which is in the form of Lyα emission (e.g., Dijkstra 2014). Lyα line cooling compensates for the change in gravitational binding energy, denoted with $U_{\text{bind}}$, and is therefore of the order of

$$I_{\alpha, \text{int}} \approx f_\alpha \frac{dU_{\text{bind}}}{dt} = f_\alpha \frac{3GM_{\text{gas}}^2}{5R^2} \dot{R}.$$

$$\approx 4 \times 10^{36} \ erg \ s^{-1} \left(\frac{M_{\text{gas}}}{10^6 M_\odot}\right)^2 \left(\frac{R}{44 \ pc}\right)^2 \left(\frac{\dot{R}}{10 \ km \ s^{-1}}\right),$$

$$\approx 4 \times 10^{36} \ erg \ s^{-1} \left(\frac{M_{\text{gas}}}{10^6 M_\odot}\right)^2 \left(\frac{n}{100 \ cm^{-3}}\right)^2/3 \left(\frac{\dot{R}}{10 \ km \ s^{-1}}\right).$$

(1)

where we assumed that dark matter does not contribute to the gravitational potential, which is appropriate when the gas has collapsed to the high densities that we consider in this paper. A gas cloud of mass $M_{\text{gas}} = 10^6 M_\odot$ with radius $R = 44$ pc has a mean enclosed number density of hydrogen atoms that is
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\(n \approx 100 \text{ cm}^{-3}\). The total Ly\(\alpha\) gravitational cooling luminosity that is \textit{produced} increases as \(\propto n^{2/3}\). However, as we discuss in Section 2.3 Ly\(\alpha\) photons generally are destroyed and do not escape for sufficiently large \(n\) (even in the complete absence of dust). The predicted Ly\(\alpha\) cooling luminosity that can be observed therefore differs from the produced—also known as the “intrinsic” (hence the subscript “int”)—Ly\(\alpha\) luminosity.

It is possible to boost the Ly\(\alpha\) luminosity if the gas is (kept) ionized by radiation coming from the accretion disk surrounding the central BH (e.g., Haiman & Rees 2001). The total Ly\(\alpha\) luminosity recombination is

\[
L_{\alpha,\text{rec}} \approx 0.68E_{\text{Ly}\alpha} \alpha B(T) \int dV \, n_e n_p
\]

\[
\approx 3 \times 10^{41} \text{ erg s}^{-1} \left( \frac{M_{\text{gas}}}{10^6 M_\odot} \right)^2 \left( \frac{R}{44 \text{ pc}} \right)^{-3} \left( \frac{T}{10^4 \text{ K}} \right)^{-0.7},
\]

\[
\approx 3 \times 10^{41} \text{ erg s}^{-1} \left( \frac{M_{\text{gas}}}{10^6 M_\odot} \right)^2 \left( \frac{n}{100 \text{ cm}^{-3}} \right) \left( \frac{T}{10^4 \text{ K}} \right)^{-0.7},
\]

(2)

where \(\alpha B(T) \approx 2.6 \times 10^{-13}(T/10^4 \text{ K})^{-0.7} \text{ cm}^3 \text{ s}^{-1}\) denotes the case-B recombination coefficient (in our calculations we take the more accurate approximation from Hui & Gnedin 1997). The factor “0.68” denotes the number of Ly\(\alpha\) photons produced per recombination event (see Dijkstra 2014, for details). The Ly\(\alpha\) recombination luminosity is orders of magnitude larger than the signal expected from gravitational heating. The total recombination rate is \(N_{\text{rec}} = L_{\alpha,\text{rec}}/E_{\text{Ly}\alpha}\), where \(E_{\text{Ly}\alpha} = 10.2 \text{ eV}\) denotes the energy of a Ly\(\alpha\) photon. The cloud is fully ionized when the recombination rate is less than the production rate of ionizing photons by the accretion disk, \(N_{\text{ion}}\). In the case of \(N_{\text{rec}} > N_{\text{ion}}\), the \(\text{H} \text{II}\) sphere does not extend to the edge of the cloud, and the ionized gas is “ionization bound.” In the other case \((N_{\text{rec}} < N_{\text{ion}})\), the \(\text{H} \text{II}\) sphere extends right to the edge of the cloud, and the ionized gas is “density bound.”

When the cloud is not fully photoionized, then a significant fraction of the high-energy ionizing photons (X-rays) can be absorbed in the neutral gas, which would then be photoheated. This gas efficiently radiates away this energy, again mostly in Ly\(\alpha\). In this case an even larger fraction of the bolometric luminosity of the accretion disk can be converted into Ly\(\alpha\) radiation inside the cloud (see Section 3 for additional discussion).

When the intense photodissociating radiation field that is required for the DCBH formation scenario is sourced by a nearby star-forming galaxy (e.g., Dijkstra et al. 2008), then this galaxy also radiates the collapsing cloud with X-ray photons (Inayoshi & Tanaka 2015). These X-ray photons provide an additional heat source that powers Ly\(\alpha\) emission. To estimate this Ly\(\alpha\) luminosity we consider a star-forming galaxy that

\[
L_{X} \sim 3 \times 10^{36}(R/100 \text{ pc})^{-2} \text{ erg s}^{-1},
\]

which can become comparable to the gravitational cooling luminosity during early stages of the collapse.

### 2.3. Ly\(\alpha\) Transfer

Gas with HI column densities in excess of \(10^{20} \text{ cm}^{-2}\) is considered “extremely” optically thick to Ly\(\alpha\) radiation (e.g., Adams 1972; Neufeld 1990). Ly\(\alpha\) transfer through extremely opaque media has been studied for decades (see Dijkstra 2014, for an extended review). The line center optical depth, \(\tau_0\), of gas with an HI column density \(N_{\text{HI}}\) is

\[
\tau_0 = 5.9 \times 10^6 \left( \frac{N_{\text{HI}}}{10^{20} \text{ cm}^{-2}} \right) \left( \frac{T}{10^4 \text{ K}} \right)^{-1/2},
\]

(3)

where \(T\) denotes the gas temperature. Ly\(\alpha\) photons typically scatter\(^4\) of the order of \(N_{\text{esc}} \sim \tau_0\) before escaping from a static medium (Adams 1972; Harrington 1973; Neufeld 1990).

The number of scattering events can be reduced tremendously in multiphase media and/or media in which velocity gradients are present. In the DCBH formation scenario, fragmentation is suppressed and we expect the gas distribution to be smooth. Velocity gradients only reduce \(N_{\text{esc}}\) by a factor of \(~\Delta v/\nu_{\text{th}}\), where \(\nu_{\text{th}} = 12.9(T/10^4 \text{ K})^{1/2} \text{ km s}^{-1}\) is the thermal velocity of hydrogen and \(\Delta v\) denotes the maximum velocity difference between the center and the edge of the cloud (Bonilha et al. 1979, also see Dijkstra et al. 2016 for a more quantitative analysis).

Ly\(\alpha\) scattering is limited by a number of other processes, which we briefly describe here (for a more detailed description the reader is referred to Dijkstra et al. 2016). Ly\(\alpha\) photons can be destroyed in the following ways.

1. Molecular hydrogen (H\(_2\)) has two transitions that lie close to the Ly\(\alpha\) resonance: (a) the \(v = 1 \rightarrow 2P(5)\) transition, which lies \(\Delta v = 99 \text{ km s}^{-1}\) redward of the Ly\(\alpha\) resonance, and (b) the \(1 \rightarrow 2P(6)\) transition, which lies \(\Delta v = 15 \text{ km s}^{-1}\) redward of the Ly\(\alpha\) resonance. Vibrationally excited H\(_2\) may therefore convert Ly\(\alpha\) photons into photons in the H\(_2\) Lyman bands (Neufeld 1990, and references therein), and thus effectively destroy Ly\(\alpha\).

2. Photoionization by Ly\(\alpha\) of the (tiny) fraction of atoms in the 2\(p\) and 2\(s\) states.

3. Ly\(\alpha\) photons can detach the electron from the H\(^+\) ion. This process is (slightly) less important than destruction via photoionization.

4. Induced transitions 2\(p \rightarrow 2s\) by the CMB can be important when the 2\(p\) and 2\(s\) level populations are inverted. These induced transitions remove Ly\(\alpha\) photons (see Dijkstra et al. 2016). This effect however, can become important at higher densities where \(f_{\text{esc}}\) is already tiny due to collisions (see the next process).

\(^4\) The \textit{trapping time} of Ly\(\alpha\) photons equals \(t_{\text{trapping}} = B_{\text{esc}}\), where \(t_{\text{esc}} \sim R/c\) denotes the light crossing time, and \(B \approx 12(N_{\text{HI}}/10^{20} \text{ cm}^{-2})^{1/3}(T/10^4 \text{ K})^{-1/3}\) (Adams 1975; Dijkstra et al. 2016). For typical cloud sizes of \(\sim 100 \text{ pc}\), and \(\log \eta \approx 8\)–12, we have \(t_{\text{trapping}} \sim 10^{10} \text{ years}\) (longer trapping times for lower densities), which is significantly less than the collapse time of the halo. While Ly\(\alpha\) photons undergo a tremendous number of scattering events, their escape is not affected by them getting “trapped” in the scattering medium. This is because the vast majority of scattering events occurs in the line core, where the mean time is very short, \(t_{\text{mean}} = (c\eta n_{\text{HI}})^{-1} \sim 0.1 n_{\text{HI}}/(10^3 \text{ cm}^{-3})^{-1} \text{ s}\). These Ly\(\alpha\) photons can exert pressure on the hydrogen gas (Smith et al. 2016, also see Dijkstra & Loeb 2008).
5. Collisions between a proton and the atom in the excited 2p state, which put it in the 2s state, at which point radiative transitions to the ground state are only permitted via emission of two continuum photons (whose combined energy is 10.2 eV). This last process is most important in limiting the escape fraction of Lyα photons (see Dijkstra et al. 2016), and we ignore the other processes hereafter.

The lifetime of the hydrogen atom in the 2p-state is \( t_{2p} = \tau_{2p} = \frac{1}{A_\alpha} \), where \( A_\alpha = 6.25 \times 10^6 \text{ s}^{-1} \) denotes the Einstein-A coefficient of the Lyα transition. The collisional deexcitation rate (in s\(^{-1}\)) is \( \frac{n_p q_{2p2s}}{n_p q_{2p2s}} \), where \( n_p \) denotes the number density of free protons and \( q_{2p2s} = 1.8 \times 10^{-4} \text{ cm}^3\text{ s}^{-1} \) denotes the 2p \( \rightarrow \) 2s “collision strength,” which depends only weakly on temperature (e.g., Dennison et al. 2005). The probability that a Lyα photon is eliminated via a collisional deexcitation event is then

\[
P_{\text{dest}}(n, T) = \frac{n_p(T) q_{2p2s}(T)}{n_p(T) q_{2p2s}(T) + A_\alpha}.
\]

The fraction of Lyα photons that escape, \( f_{\text{esc}} \), equals the probability that a Lyα photon is not destroyed after \( N_{\text{cat}} \) scattering events. The Lyα escape fraction therefore equals

\[
f_{\text{esc}}(n, T) = \left[ 1 - P_{\text{dest}}(n, T) \right]^{N_{\text{cat}}}.
\]

Because \( P_{\text{dest}} \ll 1 \), we can approximate Equation (5) as

\[
f_{\text{esc}} \approx 1 - N_{\text{cat}} P_{\text{dest}}. \]

We can obtain the maximum number of scattering events, \( N_{\text{cat}}^{\text{max}} \), by setting \( f_{\text{esc}} = 0 \), which yields

\[
N_{\text{cat}}^{\text{max}} = \frac{1}{P_{\text{dest}}} = \frac{n_p q_{2p2s} + A_\alpha}{n_p q_{2p2s}} \approx 10^{11} \left( \frac{n}{50 \text{ cm}^{-3}} \right)^{-1}.
\]

Here, we assumed collisional ionization equilibrium. Under this assumption, \( n_p = 3.5 \times 10^3 n \) for \( T = 10^4 \text{ K} \). These numbers depend strongly on temperature; for \( T = 8000 \text{ K} \) \( N_{\text{cat}}^{\text{max}} \) is increased by two [five] orders of magnitude. The temperature dependence of \( N_{\text{cat}}^{\text{max}} \) is shown in Figure 7 in the Appendix. We take this temperature dependence into account when we present our results in Section 3, by matching the cooling rate of the gas (which depends strongly on \( T \)) to the heating rate.

It is worth stressing that \( N_{\text{cat}}^{\text{max}} \) decreases as the cloud contracts as \( N_{\text{cat}}^{\text{max}} \propto n_p^{-1} \propto n^{-0.9} \) (as \( n_p \propto n^y \) with \( y = 0.88 \) to a good approximation, see Appendix A.2). For comparison, the cloud column density—and hence the line center optical depth, and consequently the average number of scatterings before escape—increases as \( N_{\text{II}} \propto n^{2/3} \). This implies that there exists a critical density of the cloud, \( n_{\text{max}} \), above which it abruptly stops emitting Lyα radiation. In Section 3 for example we find that collisional deexcitation becomes important at \( n_{\text{max}} \approx 10^4 - 10^6 \text{ cm}^{-3} \), where the precise number depends on the gas temperature.

Lyα radiative transfer through extremely optically thick gas can be described accurately as a diffusion process in both real and frequency space (e.g., Rybicki & dell’Antonio 1994; Higgins & Meiksin 2012). That is, as the photons scatter diffuse outward through the cloud, they also diffuse away from line center, which facilitates their escape. Scattering thus broadens the spectral distribution of Lyα photons. The characteristic broadening of the line through a static medium is (Adams 1972; Harrington 1973; Neufeld 1990)

\[
\frac{\text{FWHM}}{10^3 \text{ km s}^{-1}} \approx 1.5 \left( \frac{N_{\text{II}}}{10^{22} \text{ cm}^{-2}} \right)^{1/3} \left( \frac{T}{10^4 \text{ K}} \right)^{1/6} \\
\approx 1.5 \left( \frac{n}{50 \text{ cm}^{-3}} \right)^{2/9} \left( \frac{M_{\text{gas}}}{10^6 M_\odot} \right)^{1/9} \left( \frac{T}{10^4 \text{ K}} \right)^{1/6}.
\]

For static gas clouds the line is centered and symmetric around the Lyα resonance. For contracting gas clouds, however, the Lyα line is blueshifted relative to the line center by an amount that depends on the HI column density and the infall velocity profile (e.g., Zheng & Miralda-Escudé 2002; Dijkstra et al. 2006). In Section 4 we will present predictions for the line profile, and compare these to observations of CR7 in Section 5.

3. RESULTS I: LYα LUMINOSITY

Figure 1 shows the Lyα luminosity as a function of density \( n \). This plot contains two lines:

1. Gravitational Cooling Luminosity. For a uniform gas density profile, the gravitational binding energy \( U_{\text{bind}} = \frac{G M_\odot n}{5 m_{\text{H}}} \). At each \( n \) we compute the gas temperature \( T \) by setting the gravitational heating rate equal to the total cooling rate, i.e.,

\[
\frac{dE_{\text{cool}}}{dt} = \int f_{\text{esc}}(T) n_{\text{II}}(T) C(T),
\]
in which \( C(T) \) denotes the total gas cooling rate (which includes the relevant cooling rates taken from Hui & Gnedin 1997). Once we have determined \( T \), we compute \( F_{\text{heat}} (n, T) \), and \( f_{\text{esc}} (n, T) \) (see Equations (4) and (5)). Figure 1 shows the total Ly\( \alpha \) cooling luminosity

\[
L_{\alpha-\text{cool}} = f_{\text{esc}} (T, n) L_{\alpha-\text{cool}}^{\text{int}} .
\]

Figure 1 shows that the gravitational cooling signal reaches a maximum\(^5\) of \( L_{\alpha-\text{cool}} \approx 10^{38} \text{ erg s}^{-1} \) at a critical density (which we introduced in Section 2.3) \( n_{\text{max}} \approx 10^6 \text{ cm}^{-3} \), beyond which it rapidly decreases. This rapid decrease is due to the rapid decrease in \( f_{\text{esc}} (T, n) \). Intermediate results of our calculations are shown in Figure 7 in Appendix A.2.

2. Recombination Radiation. The predicted Ly\( \alpha \) luminosity powered by recombination events equals

\[
L_{\alpha} = f_{\text{esc}} (T, n) L_{\alpha}^{\text{rec}} \times \begin{cases}
L_{\alpha}^{\text{rec, int}}, & L_{\alpha}^{\text{rec, int}} \leq L_{\alpha}^{\text{rec, max}} \\
L_{\alpha}^{\text{rec, max}}, & L_{\alpha}^{\text{rec, int}} > L_{\alpha}^{\text{rec, max}} .
\end{cases}
\]

where the intrinsic Ly\( \alpha \) recombination rate \( L_{\alpha}^{\text{rec, int}} \) is given in Equation (2). The Ly\( \alpha \) recombination luminosity is bound by the total photoionization rate in the cloud, and hence by the total ionizing photon production rate around the black hole. We assumed that the accretion disk around the DCBH is radiating at Eddington luminosity, and that its spectrum is identical to that of unobscured, radio-quiet quasars. Under these assumptions, the total ionizing photon emission rate is \( \dot{N}_{\text{ion}} \) (Bolton et al. 2011)

\[
\dot{N}_{\text{ion}} = 6.5 \times 10^{53} \left( \frac{M_{\text{BH}}}{10^6 M_\odot} \right) \text{s}^{-1} .
\]

The Ly\( \alpha \) recombination luminosity then saturates at

\[
L_{\alpha}^{\text{rec, max}} = F_{\text{Ly}\alpha} \dot{F}_{\text{Ly}\alpha} \dot{N}_{\text{ion}} 
\sim 2 \times 10^{43} \left( F_{\text{Ly}\alpha} \right) \left( \frac{M_{\text{BH}}}{2.0} \right) \left( \frac{M_{\text{BH}}}{10^6 M_\odot} \right) \text{ erg s}^{-1} .
\]

Here, \( F_{\text{Ly}\alpha} \) denotes the mean number of Ly\( \alpha \) photons that are produced per ionizing photon. If each ionizing photon ionized one hydrogen atom, then \( F_{\text{Ly}\alpha} \approx 0.68 \) (as we mentioned above). However, photoionization by high-energy photons creates energetic electrons that can collisionally ionize additional atoms.\(^5\) The boost in the production rate of Ly\( \alpha \) photons scales with the mean photon energy above 13.6 eV (Raifer et al. 2010). Our fiducial model assumes a boost by a factor of \( \sim 3 \), and therefore \( F_{\text{Ly}\alpha} \sim 3 \times 0.68 \approx 2 \).

Figure 1 shows \( L_{\alpha}^{\text{rec}} \) as a function of density \( n \). This maximum luminosity\(^7\) \( L_{\alpha}^{\text{rec, max}} \) is reached at a density of \( n \approx 10^4 \text{ cm}^{-3} \). Figure 1 shows that the recombination luminosity plummets for \( n \gtrsim 10^4 \text{ cm}^{-3} \), which is due to the reduction in \( f_{\text{esc}} (n, T) \) as a result of X-ray heating: the Ly\( \alpha \) escape fraction goes to zero at lower density compared to the case of gravitational heating because higher energy photons can penetrate the neutral gas which increases its temperature. We compute this temperature by setting the total cooling rate of the gas equal to the total radiative heating rate, which we obtain by assuming that a fraction \( f_{\text{heat}} \) of the X-ray luminosity goes into heating. The total X-ray heating rate is then

\[
H^\gamma = f_{\text{heat}} f_{\gamma} L_c
= 1.3 \times 10^{43} \frac{f_{\text{heat}}}{1.0} \frac{f_{\gamma}}{0.1} \left( \frac{M_{\text{BH}}}{10^6 M_\odot} \right) \text{ erg s}^{-1} ,
\]

where the choice \( f_{\gamma} \approx 10\% \) is based on observations of luminous quasars for which 10\% of their total bolometric luminosity is in the 0.5–10 keV band (e.g., Marcon et al. 2004; Lusso et al. 2012). The choice \( f_{\text{heat}} = 100\% \) is high. The enhanced temperature of this photo heated neutral gas gives rise to a larger residual ionized fraction, which enhances collisional deexcitation. We have verified that the choice \( f_{\text{heat}} = 10\% \) would result in a lower equilibrium temperature, a lower residual neutral fraction, which in turn would cause \( f_{\text{esc}} \) to drop to zero at a higher density, but otherwise would leave our results unchanged.

The previous discussion and Figure 1 highlights that we can only observe Ly\( \alpha \) signatures from DCBH in cases in which accretion onto the DCBH is already occurring, as the Ly\( \alpha \) cooling signal in response to gravitational heating is orders of magnitude fainter than what can be detected now, and in the near future (see e.g., Dijkstra 2014). To reach detectable Ly\( \alpha \) luminosities in excess of \( L \sim 10^{42} \text{ erg s}^{-1} \) we need the gas surrounding the central massive DCBH (\( M \gtrsim 10^5 M_\odot \)) to be within a very specific density range. In the next section, we predict the spectrum of Ly\( \alpha \) photons emerging from gas in this particular state.

It is worth stressing that our predicted luminosities are set entirely by the energy considerations: the total cooling rate is set by the rate of gravitational heating. The total binding energy of the gas changes for more realistic/complex gas distributions, but only by a factor of the order of unity. The total recombination rate of Ly\( \alpha \) power is limited by the rate at which ionizing photons are emitted into the gas, irrespective of the gas distribution. The gas distribution affects the mean density at which the escape fractions go to zero for both processes, and at which the intrinsic Ly\( \alpha \) recombination rate saturates. The gas distribution more strongly affects the emerging spectra, as we discuss in Section 4.

4. RESULTS II: Ly\( \alpha \) SPECTRA

Predicting the emerging Ly\( \alpha \) spectrum from the clouds is more difficult than predicting the Ly\( \alpha \) luminosity, as they—unlike our predictions for luminosity—depend more strongly on the adopted geometry. In this section we focus on the spectra emerging from different sets of models. We show calculations of Ly\( \alpha \) spectra emerging from a suite of spherically symmetric, collapsing gas clouds in Section 4.1. We show how these spectra are affected by subsequent radiative transfer in the intergalactic medium (IGM) in Section 4.2. We study the directionally dependent spectra...
emerging from flattened clouds without IGM in Section 4.3 and with IGM in Section 4.4. We briefly discuss how more complex gas distributions affect our results in Section 4.5.

4.1. Spherical Clouds

Figure 2 shows Lyα spectra emerging from spherical clouds characterized by (i) the HI column density ($N_{\text{HI}}$), (ii) a “Hubble-like” contraction of the form $v \propto R$, with a maximum contraction velocity $v_{\text{coll}}$ at the edge of the cloud, and (iii) the emissivity profile. We assume either a uniform Lyα emissivity, or a central source fully surrounded by the gas. In all models we set $T = 10^4$ K, and that all photons are emitted at line center. These assumptions do not affect our calculations at all.

We discuss each model below.

1. The black solid line shows a model with $N_{\text{HI}} = 10^{22}$ cm$^{-2}$, $v_{\text{coll}} = 20$ km s$^{-1}$, and uniform emission. The column density is on the lower end of values associated with the maximum Lyα luminosity in Figure 1. We have chosen $v_{\text{coll}} = 20$ km s$^{-1}$ (instead of $v_{\text{coll}} = 10$ km s$^{-1}$ that was adopted in Equation (1)) to better illustrate the impact of collapse on the emerging line profile. This spectrum is broadened into the characteristic double peaked profile that is associated with scattering through static, extremely opaque media. Infall enhances the blue peak compared to the red as expected (see Section 2.3).

2. The red line shows a model with $N_{\text{HI}} = 10^{23}$ cm$^{-2}$, $v_{\text{coll}} = 20$ km s$^{-1}$, and uniform emission. The boost in HI column density by a factor of 10 increases the line center optical depth by the same factor, which further broadens the line by a factor $10^{1/3} \sim 2.2$ (see Equation (7)), which is consistent with the broadening shown in Figure 2.

3. The blue line shows a model with $N_{\text{HI}} = 10^{19}$ cm$^{-2}$, $v_{\text{coll}} = 20$ km s$^{-1}$, and uniform emission. This model represents the special moment in which the ionized region is ionization bound, but only by a tiny shell of neutral gas. In this case, scattering still gives rise to the double peaked profile, but the peaks are separated only by $\Delta v \sim 130$ km s$^{-1}$. This spectrum can also be thought of as representing the spectrum when the cloud is seen along a low column density sightline. We return to discuss this in Section 4.3.

4. The green line shows a model with $N_{\text{HI}} = 10^{22}$ cm$^{-2}$, $v_{\text{coll}} = 60$ km s$^{-1}$, and uniform emission. The main purpose of this plot was to show that the contraction velocity has no major impact on our results. The enhanced contraction velocity enhances the blue peak relative to the red peak. The overall shape of the peaks is mostly preserved.

5. The gray line shows a model with $N_{\text{HI}} = 10^{22}$ cm$^{-2}$, $v_{\text{coll}} = 20$ km s$^{-1}$, and central emission. This plot shows that the precise radial emissivity profile also affects the relative importance of the blue and red peaks, in a similar way as the contraction velocity.

These plots show that a generic prediction of the models is a double peaked line profile, with an enhanced blue peak. The precise FWHM of the line depends most strongly on $N_{\text{HI}}$ (and hence, the evolutionary stage of the cloud) closely in line with expectations from analytic arguments, but also on the Lyα emissivity profile, and infall velocity profile. At face value, these predicted spectra differ greatly from line profiles that are commonly observed in Lyα emitting galaxies (including CR7; see discussion in Section 4.2), which typically show asymmetric, redshifted line profiles, indicative of scattering through outflowing gas (see Dijkstra 2014; Hayes 2015, for reviews). However, we stress that the intergalactic medium (IGM) strongly affects especially those photons that emerge on the blue side of the line. We model the impact of the IGM below.

4.2. Spherical Clouds, with IGM

The IGM is optically thick to photons emerging blueward of the Lyα resonance at $z \gtrsim 4$. To first order, the IGM transmits all flux redward of the Lyα resonance, and suppresses the flux blueward of it. In reality, intergalactic radiative transfer of Lyα photons is more complicated as galaxies reside in overdense regions of the universe, which is more opaque to Lyα radiation than what has been inferred from the Lyα forest observations (e.g., Dijkstra et al. 2007; Laursen et al. 2011). In addition, the opacity of the IGM is highest at frequencies close to the Lyα resonance (see Laursen et al. 2011), which makes spectrally narrower emission lines centered on the line center more subject to intergalactic radiative transfer (also see Zheng et al. 2010). We approximate intergalactic RT at the redshifts of DCBH formation ($z \gtrsim 6$), using the models of Dijkstra et al. (2011), which suppress all flux blueward, and up to $\sim 100$ km s$^{-1}$ redward of the Lyα resonance. This latter accounts for infall of intergalactic gas onto the dark matter halo that hosts the cloud (Dijkstra et al. 2007; Iliev et al. 2008). The model of Dijkstra et al. (2011) accounts for the inhomogeneous distribution of diffuse neutral intergalactic gas that is characteristic of a patchy reionization process. \(^8\) For more precisely, the “effective” optical depth in the Lyα forest exceeds unity, $\tau_{\alpha} > 1$ at $z \gtrsim 4$ (see, e.g., Figure 3 of Faucher-Giguère et al. 2008). \(^9\) The models of Dijkstra et al. (2011) account for the inhomogeneous nature of the reionization process. Their IGM transmission curves in these models were calculated for massive, $M \sim 10^{10}$–$10^{11}$ M$_\odot$, dark matter halos. While DCBH occurs in much less massive halos, the requirement of having a luminous nearby neighbor typically places these halos in close proximity to a more massive halo (Dijkstra et al. 2009, but see Agarwal et al. 2012, Visbal et al. 2014 where the LW flux is provided by low-mass nearby galaxies).
2. Lyα recombination photons will likely scatter, and efficiently escape along the same paths as the ionizing photons, thus leading to a slight beaming of Lyα flux (see Behrens et al. 2014). Similarly, Lyα photons produced inside the HI gas (following collisional excitation) escape in directions of low HI column densities. This beaming will partially compensate\(^\text{10}\) the reduced Lyα production rate.

3. Anisotropic escape of Lyα along low column density directions will suppress the importance of collisional deexcitation at a fixed density, and will lead to collisional deexcitation becoming important at higher densities than what is shown in Figure 1.

We thus expect our predicted maximum Lyα luminosity to be practically the same for more realistic gas distributions, but that they likely apply for a wider range of densities and a subset of viewing angles (also see Section 4.5). The assumed geometry has a bigger impact on the predicted Lyα spectrum. The biggest impact of geometry occurs when the ionized region “breaks” out of the cloud only in certain directions. We refer to sightlines to the DCBH that do not encounter any neutral gas as density-bound sightlines, while sightlines that do intersect neutral hydrogen as ionization-bound sightlines. Here we quantify how geometry affects the emerging spectra in more detail.

We place a fully ionized sphere with radius \(R_{\text{ion}}\) within a ellipsoid of neutral gas on a Cartesian grid of 256\(^3\) cells in a box with box-size of (100, 100, 20) pc. The ellipsoid is characterized by three half-axes given by \(R_{\text{ell,a}} = R_{\text{ell,b}} = 50\) pc and \(R_{\text{ell,c}} = 10\) pc. We focused on the two following setups: (i) The ionized sphere is touching the surface of the ellipsoid \((R_{\text{ion}} = R_{\text{ell,c}})\). We refer to this model as the “no break-out” model. (ii) The ionized sphere breaks out of the ellipsoid \((i.e., R_{\text{ion}} = 2R_{\text{ell,c}})\), which we call “break-out” model. We chose the hydrogen number density so that the maximum column density between the center and the outside is \(10^{22} \text{ cm}^{-2}\), i.e., \(n_{\text{H}}(R_{\text{ell,a}} - R_{\text{ion}}) = 10^{22} \text{ cm}^{-2}\). We assumed that the gas temperature is \(T = 10^4 \text{ K}\).

In both models, we place three different sources: a central source (exponential distribution with scale length 0.1 pc, this emission gives rise to the “central spectrum”), a uniform distribution inside the ionized region (the “inner spectrum”), and a uniform distribution outside the ionized region (the “outer spectrum”). The intrinsic spectrum is a Gaussian with \(\sigma = 12.9 \text{ km s}^{-1}\) in all cases. The inner/central and outer spectra can be interpreted as being associated with Lyα emission powered by recombination and X-ray/gravitational heating, respectively. Figure 4 shows the resulting spectra as seen by an observer viewing the ellipsoid face-on or edge-on, prior to processing of these spectra by intergalactic radiative transfer. The figure also includes sketches of the geometrical setups in which the emitting regions are marked in the same color as the corresponding spectrum. Figure 5 shows how intergalactic radiative transfer alters these spectra. Our main results are as follows.

1. The spectra are doubly peaked and broad in the no break-out scenario. The width of the spectra depends only weakly on viewing angle and emission site. The width of\(^\text{10}\) Or possibly overcompensate. The amount of beaming depends on \(\Omega_{\text{ion}}\), the gas kinematics, etc. Beaming can boost the flux in certain directions, but by a factor of ~ a few at most (Behrens et al. 2014).

### 4.3. Ellipsoidal Clouds

Our previous calculations assumed spherical symmetry. Here, we quantify how our results change if the gas distribution surrounding the DCBH were flattened (or—say—in a thick disk). This affects the predicted Lyα luminosity in several ways:

1. Ionizing radiation can escape in certain directions, but not in others. If we denote the solid angle along which ionizing photons escape with \(\Omega_{\text{ion}}\), then the total production rate of Lyα photons inside the cloud is reduced by a factor of \((1 - \Omega_{\text{ion}})\).
Figure 4. Lyα spectrum arising from an ionized sphere embedded in a ellipsoid filled with hydrogen. The left column shows spectra from the “no break-out” (i.e., $R_{\text{out}} = R_{\text{ell}}$) scenario whereas the right column shows spectra from the “break-out” setup ($R_{\text{out}} = 2R_{\text{ell}}$) as sketched in the panels placed in the top row. The observer is placed face-on (so that the column density to the center is maximized) in the central row and edge-on (minimizing the column density) in the bottom row. In each panel we show the spectra resulting from three different sources indicated in the sketches with the corresponding color: a central source (green dashed line), a uniform luminosity within the ionized region (red solid line), and, a uniform luminosity in the region where the HI is placed (blue broken line). Note, that this curve is rescaled in the “break-out” scenario by a factor of five due to representation purposes. See Section 4.3 for details.

Figure 5. Same as Figure 4, but after processing the lines through the IGM (as in Figure 3).
the spectrum is set approximately by the lowest column density through the flattened cloud. One way to see this is that the HI column density along the shortest axis (\(R_{\text{eff}}, a\)) is \(\sim R_{\text{eff}}^2 \times 10^{22} \text{ cm}^{-2} \sim 2 \times 10^{21} \text{ cm}^{-2}\). This column density translates to a line center optical depth \(\tau_0 \approx 1.2 \times 10^6 (T/10^4 \text{ K})^{-0.5}\). For a spherical cloud with this line center optical depth, we would expect Ly\(\alpha\) spectra to exhibit peaks at \(\Delta v \pm v_{th}(a,\tau_0)^{1/3} \approx \pm 490 (T/10^4 \text{ K})^{1/6} \text{ km s}^{-1}\) (where \(a_1 = 4.7 \times 10^{-3} (T/10^4 \text{ K})^{-0.5}\) denotes the Voigt parameter), which is close to the true location of the peaks. Ly\(\alpha\) photons thus diffuse outward in real-space until they reach the edge of the cloud. The typical column density of HI gas they encountered corresponds to the column density associated with the shortest distance to the edge of the cloud, i.e., the photons typically escape along the paths of “least resistance.” The emerging spectra depends only weakly on viewing angle simply because the escape direction of photons is set by the last-scattering event, which does not depend on cloud geometry. The spectral shape depends weakly on where the Ly\(\alpha\) photons were emitted.

2. The previous discussion, which highlighted that Ly\(\alpha\) photons follow paths of least resistance, also allows us to understand results from the break-out models. The presence of low column density (\(N_{\text{HI}} < 10^{17} \text{ cm}^{-2}\)) sightlines provides escape routes for Ly\(\alpha\) photons, which do not require (and/or allow) them to diffuse far into the wings of the line profile. Indeed, the spectra associated with break-out models are typically narrow (peak separation \(\sim 100 \text{ km s}^{-1}\)). In addition to the characteristic double peaks, some spectra exhibit a third peak around line center. The importance of this component depends strongly on viewing angle. In particular, the single peaked component is due to photons encountering no (or very little) neutral hydrogen along their path and is thus only present if a direct sightline to the source exists and weakened with \(\mu \to 0\).

4.4. Ellipsoidal Clouds, with IGM

Figure 5 shows how the IGM eliminates the blue peaks from our predicted spectra, which causes the spectra to appear redshifted. As we noted earlier, it is therefore difficult to distinguish clouds collapsing onto a DCBH from ordinary star-forming galaxies based on the Ly\(\alpha\) spectral line alone, especially when the spectrally broad component is subdominant to the narrow component. However, the presence of broad redshifted wings (extending to \(\sim 1000 \text{ km s}^{-1}\) redward of line center), and possibly a second redshifted peak, may be indicative of diffusion of Ly\(\alpha\) photons through extremely opaque, dust-free, atomic hydrogen gas, which is associated with DCBH formation. Interestingly, the fact that intergalactic radiative transfer only weakly affects photons that emerge from the cloud far in the red wing of the line profile (also see Dijkstra & Wyithe 2010) may make it easier to detect these objects deep into the Epoch of Reionization.

4.5. More Complex Gas Distributions

Our previous analyses and discussion applies to scattering through smooth, metal-free gas, which represent key properties that enabled DCBH formation. The situation becomes more complicated once the halo that hosts the DCBH merges with other (likely polluted) halos, and/or when feedback from accretion onto the black hole affects the gas hydrodynamically, and/or when in situ star formation starts to occur. When these more complicated processes occur, we are no longer in the regime where Ly\(\alpha\) transfer can be addressed from first principles, and our predictions naturally no longer apply.

Anticipated changes to our predicted spectra include (i) the presence of metals, and therefore, dust reduces the Ly\(\alpha\) escape fraction from the clouds; (ii) the presence of complex gas flows, gas fragmentation and clumping generally increases Ly\(\alpha\) escape compared to homogenous gas distributions that we adopted in our models. These processes would likely boost \(\mathcal{F}_{\text{esc}}(n, T)\) at a given mean density. This may increase the Ly\(\alpha\) cooling luminosity as Ly\(\alpha\) photons may escape from denser gas. The anticipated boost is difficult to predict as fragmentation will lead to in situ star formation, which in turn leads to stellar feedback occurs. At this point, ab initio predictions for the emerging Ly\(\alpha\) spectra become unreliable.

5. COMPARISON TO CR7

5.1. Luminosity

CR7 is a bright Ly\(\alpha\) emitting source at \(z \sim 6.6\) that has been associated with a DCBH (Pallottini et al. 2015; Sobral et al. 2015; Agarwal et al. 2016a). The total Ly\(\alpha\) flux observed from CR7 implies a Ly\(\alpha\) luminosity of \(L_{\alpha} = 10^{44} \text{ erg s}^{-1}\). The total intrinsic Ly\(\alpha\) luminosity is likely higher, as the observed Ly\(\alpha\) luminosity has been suppressed by intergalactic scattering, and possibly by interstellar dust. Agarwal et al. (2016a) argue that in order to reproduce the observed ratio of flux in the Ly\(\alpha\) and He1640 lines, the observed Ly\(\alpha\) luminosity only represents one-sixth of the total intrinsic luminosity. Our calculations in Section 3 imply that in order to explain its observed Ly\(\alpha\) luminosity, CR7 must be powered by a black hole with mass \(M \gtrsim 10^7 M_\odot\).

Previous works have attributed the Ly\(\alpha\) luminosity of CR7 to black holes with mass \(M_{BH} \sim \text{a few} \times 10^6 M_\odot\) (e.g., Pallottini et al. 2015; Agarwal et al. 2016a). The bolometric Eddington luminosity of an \(M_{BH} \sim 10^6 M_\odot\) black hole equals \(L_{\text{bol}} \sim 1.3 \times 10^{44} \text{ erg s}^{-1}\), which makes it problematic to account for the total observed Ly\(\alpha\) luminosity of CR7 of \(L_{\alpha} \sim 10^{44} \text{ erg s}^{-1}\) with (sub)Eddington accretion. This problem worsens when the intrinsic Ly\(\alpha\) flux is required to be \(\sim 6\) times larger (see above).\(^{11}\)

5.2. Ly\(\alpha\) Spectrum

CR7’s rest-frame ultra-violet spectrum was obtained with both X-SHOOTER on the Very Large Telescope and DEIMOS on Keck (see Sobral et al. 2015) with slits of 0.9 and 0.75 respectively and resolutions of \(R \sim 7500–10,000\), and with a total integration time of 3.75 hr. As detailed in Sobral et al. (2015), no continuum is detected either blueward or redward of Ly\(\alpha\) (but rest-frame Lyman–Werner continuum is detected at rest-frame 916–1017 Å; see Sobral et al. 2015).

\(^{11}\) Boosting \(F_{\text{Ly}\alpha}\) (see Equation (11)) further enhances the Ly\(\alpha\) luminosity at fixed \(M_{BH}\). Physically this corresponds to requiring that high-energy photons enhance the Ly\(\alpha\) production rate. However, the Ly\(\alpha\) luminosity remains bound by \(L_{\alpha} \lesssim \mathcal{F}_{\text{esc}} f_{\text{HII}} L_{\text{bol}}\). To reach \(L_{\alpha} \sim 10^{44} \text{ erg s}^{-1}\) with \(M_{BH} < 10^7 M_\odot\) requires \(f_{\text{HII}} > 0.1\) or even \(f_{\text{HII}} > 0.6\) when constraints for the ratio of He1640 to Ly\(\alpha\) line fluxes are considered. Especially this last constraint is unphysical.
The observed FWHM of the Ly$\alpha$ line from CR7 is FWHM $\sim 260$ km s$^{-1}$ (Sobral et al. 2015), which is a factor $\sim 2$ smaller than in the models with $N_{\text{HI}} = 10^{22}$ cm$^{-2}$ (see Section 4). Analytic considerations (Equation (7)) imply that this FWHM thus translates to an HI column density in the scattering medium that is $\sim 2^3$ times smaller (i.e., $\log N_{\text{HI}}$ cm$^{-2} \sim 21$) to explain the observed FWHM. We show below that the actual column density of the scattering medium is likely well below this. The required HI column density, $\log N_{\text{HI}}$ cm$^{-2} \ll 22$, in these models implies a very specific evolutionary state in which the cloud is mostly ionized, and surrounded by a thin skin of HI gas.

In previous sections we predicted Ly$\alpha$ spectra associated with the DCBH formation scenario. We compared these predictions to observations to CR7 in several places. Here, we perform a more quantitative analysis of the observed spectrum. We perform a more quantitative analysis of the observed spectrum (Sobral et al. 2015) by fitting “shell models” to the data. The shell model represents another class of spherically symmetric models that has been routinely adopted to describe Ly$\alpha$ scattering on interstellar scales. The shell model is a simple, six-parameter model that has been proven to be surprisingly successful in reproducing observed Ly$\alpha$ spectra (see, e.g., Ahn 2004; Verhamme et al. 2008). The model consists of a central Ly$\alpha$ and continuum emitting source surrounded by a shell of outflowing hydrogen (and dust). The source can be characterized by the Ly$\alpha$ equivalent width $E_{W}$ and the width of the intrinsic line $\sigma_{t}$. The shell content is described fully by its hydrogen column density $N_{\text{HI}}$, the dust optical depth $\tau_{d}$, the (effective) temperature $T$ (or, equivalently by the Doppler parameter $b$), and, the outflow velocity $v_{\text{exp}}$.

Here, we constrain the shell model parameters that provide the best description of the spectrum of CR7.

We use the pipeline described in Gronke et al. (2015): a set of 10,800 pre-computed shell model spectra in combination with a post-processing procedure allows us to obtain the best-fit values, and also the uncertainties and potential degeneracies between the parameters. The results of the fitting procedure are shown in Figure 6 as one- and two-dimensional projections of the posterior likelihood distribution sampled by a Monte-Carlo sample. We use the affine-invariant Monte-Carlo sampler emcee (Foreman-Mackey et al. 2013) with 1600 steps and 400 walkers. We do not fit for the Ly$\alpha$ equivalent width $E_{W}$ since no continuum was detected (Sobral et al. 2015). Instead, we simultaneously fit for the redshift $z$ using a Gaussian prior with $\mu = 6.600$, $\sigma = 0.0005$, which corresponds to the redshift of the non-resonant He1640 line.

The red solid line in Figure 6 shows the best-fit model to the data. The best-fit shell model parameters and their uncertainties are listed in the top left corner. Two parameters are of particular interest: we obtain $\log N_{\text{HI}}$ cm$^{-2} = 20.10^{+0.08}_{0.08}$ and $\tau_{d} = 3.74_{-0.26}^{+0.26}$. These parameters differ from the values we would naturally associate with DCBH formation (namely, $\tau_{d} = 0$ and $N_{\text{HI}} \gg 10^{20}$ cm$^{-2}$). This tension is qualitatively.$^{12}$

$^{12}$Analytic considerations prefer $\log N_{HI}/cm^2 \sim 21$ because this assumes that the Ly$\alpha$ photons are all initially emitted at line center. In contrast, in the best-fit shell models the initial Ly$\alpha$ spectrum is a Gaussian with $\sigma \approx 150$ km s$^{-1}$. The spectral distribution of Ly$\alpha$ is therefore broad to begin with, which then requires a smaller HI column density to broaden the line to match the observations. The fact that formally the data prefers a high-$\sigma$, low $N_{\text{HI}}$ solution is entirely due to the spectral shape of the line.

Figure 6. Ly$\alpha$ spectrum of CR7 (Sobral et al. 2015). The solid lines represent our best-fit shell models, while the dotted lines show the Ly$\alpha$ lines prior to scattering. The red solid line shows our best-fit shell model that we obtain by only using a prior on CR7’s redshift $z$. The blue solid line shows our best-fit model if we force the scattering medium to have a low dust content by adding a prior on the dust content (see text). Both models reproduce the observed shape and shift of the Ly$\alpha$ spectral line shape. The best-fit shell model parameters for both models are shown. Shell-model fitting shows that the observed Ly$\alpha$ spectrum of CR7 favors the Ly$\alpha$ source to be surrounded by a scattering medium with a column density of $\log[N_{\text{HI}}/cm^2] \sim 19-20$, which is outflowing at a rate $v_{\text{exp}} \sim 200-300$ km s$^{-1}$. These parameters correspond well to those inferred for lower redshift Ly$\alpha$ emitting galaxies, and may suggest that the interstellar medium of CR7 is already shaped by star formation and stellar feedback.
consistent with our analysis of the spectra emerging from the contracting gas cloud, in which the observed width of the Lyα line favored low HI column densities. The high value of $\tau_{\alpha}$ is preferred due to the sharp drop-off of the spectrum on the blue side.

However, the high $\tau_{\alpha}$ value is inconsistent with the upper limit on the metallicity of the gas of $Z \lesssim 0.02 Z_\odot$ found by Hartwig et al. (2016) due the non-detection of the C iii line: for an SMC/LMC type an HI column density of $\log N_{\text{HI}/\text{cm}^{-2}} \sim 20.1$ translates to a dust optical depth $\tau_d \sim 0.04 - 0.07$ ($\tau_d \sim 0.07$) for a higher metallicity $Z \gtrsim 0.02 Z_\odot$ (see Laursen et al. 2009). Moreover, this model also gives rise to a very low escape fraction of $f_{\text{esc}} \ll 10\%$. Due to these tensions we have repeated our fitting procedure, adding a narrow prior on $\tau_{\alpha}(\mu, \sigma) = (0, 0.05)$; in addition we truncated the z-prior at $z \gtrsim 6.6005$ to exclude solutions for which the intrinsic Lyα line was offset significantly from the He1640 line, which yields $\tau_d \sim 0.08^{+0.04}_{-0.03}$. In this case, the line profile favors an even lower column density$^{13}$ of $\log N_{\text{HI}/\text{cm}^{-2}} = 19.33^{+0.09}_{-0.07}$.

The inferred shell model parameters lie within the range of values inferred for lower redshift Lyα emitters: preferred shell column densities in $z \sim 2.2$ Lyα emitters are $\log N_{\text{HI}/\text{cm}^{-2}} \sim 19$ (Hashimoto et al. 2015), and in green pea galaxies at $z \sim 0.3$ are $\log N_{\text{HI}/\text{cm}^{-2}} \sim 19-20$ (Yang et al. 2016). The required shell velocity or CR7 is consistent with these lower redshift galaxies (but see Vanzella et al. 2010 for an example of a galaxy at $z \sim 5.6$ where the shell velocity and HI column density are both higher). The scattering medium of CR7 is therefore similar to that of lower redshift Lyα emitting galaxies.

This implies that if CR7 hosts a DCBH, then the conditions that enabled DCBH formation may have been mostly erased. This may not be surprising as the required black hole mass of $M \gtrsim 10^9 M_\odot$ exceeds the mass of DCBHs when they first form (which is $\sim 10^5-10^6 M_\odot$, e.g., Latif et al. 2013; Shlosman et al. 2016). This requires that the black hole has grown significantly since it first formed (see Agarwal et al. 2016a; Hartwig et al. 2016), which in turn implies that the host galaxy likely remained since the BH formed. It is then difficult to prove that CR7 is indeed powered by a BH that formed via direct collapse, or instead via a BH that grew from a stellar mass seed. Instead, Hartwig et al. (2016) show that the non-detection of the C iii] line puts an upper limit on the metallicity of the gas at $Z \lesssim 0.02 Z_\odot$, and that this condition on metallicity makes it more likely for the BH to have first formed as a massive seed via direct collapse.

5.3. Constraints from the He1640 Line

As we mentioned earlier, the equivalent width of the He1640 line provides a direct measure of the hardness of the source illuminating the gas cloud$^{14}$ (e.g., Sobral et al. 2015; also see Johnson et al. 2011). In addition, constraints imposed by the ratio of line fluxes in He1640 to Lyα have been explored in previous works (Johnson et al. 2011, Sobral et al. 2015, Agarwal et al. 2016a, Hartwig et al. 2016). The He1640 line provides a third constraint, as it constrains both the systemic velocity of CR7 and the width of non-resonant nebular lines, and thus the width of the Lyα spectral line prior to scattering. This information in turn constrains the Lyα transfer process (see Section 5.2 above).

It is interesting that our best-fit model favors the intrinsic FWHM of the Lyα line to be $\sim 2.35 \times \sigma_\alpha \sim 250$ km s$^{-1}$, which is a factor $\sim 2$ broader than the FWHM of the He1640 line. This problem is also encountered for Lyα emitters at $z \sim 2.2$, which also required $\sigma_\alpha$ to be larger than the observed width of the Hα line (see Hashimoto et al. 2015), and is still not understood.

6. DISCUSSION AND CONCLUSIONS

We have modeled the relevant radiative processes of the Lyα line through simplified representations of the “DCBH” scenario. The suppressed gas fragmentation, the absence of star formation and stellar feedback—all key characteristics of the DCBH formation scenario—simply the Lyα radiative transfer problem, which is known to depend sensitively on all these processes. Our main results are as follows.

1. Gravitational heating of the collapsing cloud gives rise to a Lyα cooling luminosity of up to $\sim 10^{38} (M_{\text{gas}}/10^5 M_\odot)^2$ erg s$^{-1}$. Though gravitational heating can increase the Lyα production much further during the later stages of collapse, collisional deexcitation efficiently suppresses the emerging Lyα flux. During these later stages, the cloud may be visible through its two-photon continuum emission (Dijkstra 2009; Inayoshi et al. 2016), or through stimulated 3 cm emission (Dijkstra et al. 2016).

$^{13}$ We did not include the IGM in our fitting procedure. We have also repeated our fitting procedure only for the data points redward of 9246 Å (i.e., ignoring the wavelengths which are likely affected by IGM absorption). This fitting procedure—without putting a prior on the dust content—favors even lower HI column densities of $\log N_{\text{HI}/\text{cm}^{-2}} \sim 18-20$.

$^{14}$ It is interesting to point out that the detection of the He1640 line formally directly rules out models that purely invoke gravitational heating, though these models were also ruled out based on their predicted Lyα luminosity.

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Figure 7. Density dependence of various quantities associated with a collapsing gas cloud of uniform density that is heated by the gravitational collapse. The top panel shows the line center optical depth $\tau_0$—which equals (approximately) the average number of scattering events a Lyα photon undergoes before escaping—as a function of $n$ (black solid line). The red dashed line shows the maximum allowed number of scattering events, before collisional deexcitation becomes important. The two are equal at $n \sim 10^6$ cm$^{-3}$, which implies that at higher densities, collisional deexcitation suppresses the emerging Lyα flux. The central/bottom panel shows that the temperature/residual ionized fraction decreases toward higher density.
2. Photoionization of the cloud by a central source can boost the clouds’ Lyα luminosity to values as large as \( L_{\alpha} \sim 10^{45} (M_{BH}/10^9 M_\odot) \text{erg s}^{-1} \) (irrespective of \( M_{\text{gas}} \)) over a limited range of densities. Here, \( M_{BH} \) denotes the mass of the black hole powering this source. For lower densities the ionized gas is density bound, which reduces the conversion efficiency of ionizing radiation into Lyα. For higher densities the cloud is ionization bound, and the collisional deexcitation in the neutral gas again reduces the Lyα luminosity. This luminosity, and that for gravitational cooling, are not affected by our adopted simplified geometry, and are set entirely by the energetic considerations.

3. We computed Lyα spectra for a suite of spherical and flattened collapsing gas clouds. The predicted spectra are double peaked, with an enhanced blue peak. The width and velocity offsets of the peaks range from a few tens to few thousands km s\(^{-1}\), and are set by the HI column density through the cloud in close agreement with analytic predictions. We process the emerging spectra through the IGM, and find that the IGM completely eliminates the blue peak. The resulting spectra are asymmetric, and redshifted, and can look remarkably like those typically associated with scattering through a galactic outflow. However, the predicted velocity offset and FWHM can exceed \( \sim 1000 \text{km s}^{-1} \), which is larger than what is typically found for high-redshift Lyα emitting galaxies, and thus possibly an interesting observational signature.

4. In flattened clouds, the predicted FWHM and overall offset of the Lyα spectral line is set closely by the minimum HI column density associated with the cloud. An example of this consists of the extreme case in which a central source ionized the cloud in certain directions, but not others. In this case, the predicted Lyα spectral line is narrow, with flux emerging at line center. Moreover, the emerging spectrum and the fraction of the flux that emerges at line center depends on viewing angle. Specifically, sightlines that allow ionizing photons to escape are associated with most Lyα photons escaping at line center (also see Behrens et al. 2014, Verhamme et al. 2015).

5. We compared to observations of CR7, a luminous Lyα emitter at \( z \sim 7 \), which is potentially associated with a DCBH (Sobral et al. 2015). Our analysis implies that the Lyα line luminosity of CR7 alone implies that if it is indeed associated with a DCBH, then it must be associated with a black hole of mass \( M_{BH} \gtrsim 10^7 M_\odot \). If CR7 is indeed powered by such a massive black hole, then it must have accreted most of this mass after its formation, as the maximum mass of a newly formed DCBH is \( \sim 10^5-10^6 M_\odot \) (e.g., Latif et al. 2013; Shlosman et al. 2016). Supply of this gas to the black hole required mergers with other nearby halos (Hartwig et al. 2015; Agarwal et al. 2016a). These mergers would likely have erased the very specific conditions in the gas that were required for DCBH formation. This may explain that the spectral line shape of CR7 favors a low HI column density, \( \log [N_{\text{HI}}/\text{cm}^{-2}] \approx 19-20 \), outflowing scattering medium. These parameters are consistent with those inferred for lower redshift Lyα emitting galaxies, which may reflect that the interstellar medium of CR7 is already shaped by star formation and stellar feedback.

Alternatively, our radiative transfer calculations through simplified representations of the DCBH formation process can reproduce the observed spectral line shape of CR7 when the gas cloud is in a very specific evolutionary state (again associated with a low column density of HI), and that the IGM eliminated the blue side—which contains most flux—of the emitted Lyα flux out of our line of sight. In this case, CR7 should be surrounded by an Lyα halo that extends much further than the present observations with a luminosity (significantly) larger than \( 10^{44} \text{erg s}^{-1} \).

The implication that the interstellar medium of CR7 may already be shaped by star formation and stellar feedback begs the question whether these stars can be a “pure” \( \sim 3 \) Myr old Population III stellar population (as discussed by Sobral et al. 2015, Visbal et al. 2016). It is then interesting to consider that the Lyα spectral line shape favors scattering through outflowing gas (see Section 5.2), which implies that its ISM is being enriched, and that enough time must have elapsed for the effects of feedback to have commenced. If the Lyα line was actually intrinsically symmetric and the observed asymmetry is entirely due to the IGM (as discussed above), then we again expect CR7 to be surrounded by an extended, still undetected Lyα halo with a luminosity (significantly) larger than \( 10^{44} \text{erg s}^{-1} \).

Finally, the association of broad redshifted Lyα lines with the DCBH formation scenario makes this Lyα flux less susceptible to scattering in the IGM compared to narrower emission lines that have been observed from star-forming galaxies (Dijkstra & Wyithe 2010). This reduced sensitivity to the neutral IGM may increase the likelihood of finding DCBHs among the high-redshift Lyα emitting population: a source with an apparent Lyα luminosity of \( \sim 10^{42} \text{erg s}^{-1} \) at \( z = 10 \) (corresponding to a flux of \( 0.8 \times 10^{-18} \text{erg s}^{-1} \text{cm}^{-2} \)) can be detected with the James Webb Space Telescope (JWST) in \( 10^4 \) s at \( \sim 3 - 2 \sigma \) significance\(^{15} \). This—combined with the increased sensitivity of NIRSpec toward longer wavelengths—suggests that JWST at least has the sensitivity to detect broad Lyα lines from \( M_{BH} \gtrsim 10^5 M_\odot \) DCBHs out to redshift well beyond \( z = 10 \).

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APPENDIX

MORE DETAILS ON THE CALCULATIONS

A.1. Temperature Dependence Ionized Fraction

Collisional ionization equilibrium forces the recombination rate to equal the collisional ionization rate, i.e., \( n_e n_p \alpha_B(T) = n_e n_{HII} q_{\text{ion}}(T) \). Here, \( \alpha_B(T) \) is the case-B recombination coefficient and \( q_{\text{ion}} \) denotes the collisional ionization

\(^{15}\text{https://jwst.stsci.edu/science-planning/performance-simulation-tools-1/sensitivity-overview}\)
Figure 8. This plot shows density dependence of various quantities associated with a collapsing gas cloud of uniform density that is heated by the gravitational collapse. The top panel shows the line center optical depth \( \tau_0 \sim N_{\text{scat}} \) which equals (approximately) the average number of scattering events a Ly\( \alpha \) photon undergoes before escaping — as a function of \( n \) (black solid line). The red dashed line shows the maximum allowed number of scattering events, before collisional de-excitation becomes important. The two are equal at \( n \sim 10^4 \text{ cm}^{-3} \), which implies that at higher densities, collisional de-excitation suppresses the emerging Ly\( \alpha \) flux. The central/bottom panel shows that the temperature/residual ionized fraction decreases towards higher density.

rate coefficient. The equilibrium value is

\[
x_{e} = \frac{n_e}{n} = \frac{n_p}{n} = \frac{q_{\text{ion}}(T)}{\sigma_B(T)},
\]

where we have taken fitting formulae for \( \sigma_B(T) \) and \( q_{\text{ion}}(T) \) from Hui & Gnedin (1997).

A.2. Temperature Evolution of Cloud

In Section 3 we computed the Ly\( \alpha \) luminosity, and the Ly\( \alpha \) escape fraction as a function of density \( n \). Figure 7 shows some intermediate results of these calculations. The red dashed line in the upper panel shows the line center optical depth \( \tau_0 = N_{\text{HI}} \sigma_0 \) through the cloud, where \( \sigma_0 = 5.88 \times 10^{-14} (T/10^4 \text{ K})^{-1/2} \text{ cm}^2 \) denotes the absorption cross-section at line center. The column density increases as \( N_{\text{HI}} \propto n^{2/3} \), and therefore \( \tau_0 \) also increase almost as \( \tau_0 \propto n^{2/3} \). The reason that \( \tau_0 \) does not increase exactly as \( \tau_0 \propto n^{2/3} \) is because of the mild temperature dependence of \( \sigma_0 \) (as discussed below and shown in the central panel of Figure 7, the temperature changes with \( n \)). Finally, the maximum number of scatterings \( N_{\text{scat}} \propto n_{p}^{-1} \), and therefore decreases with overall number density \( n \) as the ionized fraction changes quite slowly with density \( n \) (see the lower panel). The total number of scatterings equals the maximum at \( n \sim 10^5 \text{ cm}^{-3} \). This implies that at higher densities, collisional de-excitation suppresses the emerging Ly\( \alpha \) flux, which was also apparent in Figure 1.

The central panel shows the temperature evolution of the cloud with \( n \), under the assumption that the gravitational heating rate is balanced by cooling via collisional excitation (~40% of which is Ly\( \alpha \) cooling). The gravitational heating rate increases toward higher densities as \( \propto n^{2/3} \) (see Equation (1)), but the collisional cooling rate increases slightly faster (as \( n^{2} \)), which is compensated by a reduction in temperature toward higher \( n \) (because the collisional cooling rate is so sensitive to the gas temperature, this decrease in \( T \) is only small). Finally, the lower panel shows how the residual ionized fraction also decreases with \( n \) as a result of the decreasing temperature \( T \). Specifically, \( x_{p} \) changes by \( \sim 1 \) order of magnitude over \( \sim 8 \) orders of magnitude in \( n \). This implies that \( x_{p} \propto n^{-0.12} \), and that therefore \( n_{p} = n x_{p} \propto n^{y} \), where \( y = 0.88 \). We used this dependence in the paper.

A.3. Isothermal Density Profiles

Throughout the paper we assumed isothermal density profile. This greatly simplified our analysis, which improved the clarity of the presentation and which allowed us to apply analytic results obtained for Ly\( \alpha \) transfer. Here, we repeat some of our calculations for a cored isothermal density profile, which is a more realistic description of the density field (at least in 1D hydrodynamical simulations; see Pacucci & Ferrara 2015). This density profile is

\[
\rho(r) = \begin{cases} 
A & \text{for } r \leq R; \\
0 & \text{for } r > R
\end{cases} 
\]

where \( A \) is a normalization constant, and \( r_{c} \) is the core radius, which we assume to be \( r_{c} = 0.1R \), where \( R \) denotes the edge of the cloud (as was the case for the uniform cloud). The total mass enclosed within radius \( r \) is

\[
M_{\text{gas}}(<r) = \int_{0}^{r} dr 4\pi r^2 \frac{A}{1 + (r/r_c)^2},
\]

\[
A = \frac{4\pi \rho_{\text{gas}}}{3} \left( \frac{R}{r_c} - \arctan \left( \frac{R}{r_c} \right) \right).
\]

We can express the HI column density through the cloud as

\[
N_{\text{HI}} = \frac{A e_{\text{HI}}}{\mu m_p} \arctan \left( \frac{R}{r_c} \right) = \frac{100 R K(R/r_c)}{3} \approx 5.5\tilde{n}R,
\]

\[
K(x) = \frac{\arctan x}{x - \arctan x},
\]

where we plugged in \( R/r_c = x = 10 \), which translates to \( K(x) \approx 0.17 \). That is, for a fixed cloud size and mass, the HI column density through a cored isothermal profile (with \( r_c = 0.1R \)) is about 5.5 larger than in the uniform case.

The total recombination rate in turn can be expressed as

\[
N_{\text{rec}}^{\text{iso}} = \frac{M_{\text{gas}}^2}{V (\mu m_p)^2} \mathcal{N}(R/r_c)
\]

\[
= N_{\text{rec}}^{\text{uni}} \mathcal{N}(R/r_c) \approx 6.7 N_{\text{rec}}^{\text{uni}},
\]

\[
\mathcal{N}(x) = \frac{x^3 \arctan x}{3 (x - \arctan x)^2},
\]

where again we plugged in \( x = 10 \). The total recombination rate for a given gas mass and size is thus \( \sim 7 \) times larger than in the uniform case. We stress that this does not boost the maximum Ly\( \alpha \) luminosity from the cloud, as this is set by the total production rate of ionizing photons. The enhanced
The total gravitational binding energy at fixed $M$ and $R$ is slightly more negative than in the uniform case because the gas has settled deeper in the gravitational potential well compared to the uniform density case.

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