Quantum interference with photon pairs using two micro-structured fibres

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Abstract. We demonstrate a quantum interference experiment between two photons coming from non-degenerate pairs created by four-wave mixing in two separated micro-structured fibres. When the two heralded photons are made indistinguishable a 95% visibility is demonstrated.

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1. Introduction

We have developed a high brightness sources of correlated photon pairs using four-wave mixing (FWM) in a photonic crystal fibre (PCF) [1, 2]. Bright photon pair sources are essential for future multiphoton experiments [3]–[5] and as sources of heralded single photons and entangled states for quantum communications [6, 7]. Multiphoton interference is central to the operation of linear optical quantum gates [8]–[10] and to the development of optical cluster state computation schemes [11]. All of these experiments are built on interference effects between separate photons. Thus the simplest experimental test of the suitability of a single photon source for multiphoton logic is the demonstration of high visibility interference between separate photons at a beam-splitter (BS). This destructive interference effect was first demonstrated by Hong et al in 1987 [12, 13] between photons from degenerate correlated pairs. Later this was demonstrated between heralded single photons and a weak coherent state [14, 15] and recently between heralded photons from sources pumped by separate lasers [16]. The visibility of the interference suppression in all of these experiments is governed by the indistinguishability of the two photons after the BS. For this the photons must be in a pure state, that is in a single spatial and temporal mode with well defined polarisation.

In our source the photon pairs are automatically created within a single spatial mode. They are also polarised, narrowband and extremely bright. However, the source is pulsed (photons are \( \sim 3 \) ps long) and the natural bandwidth although narrow is not Fourier transform limited. Effectively, we start with several coherence times (temporal ‘modes’) within the pulse length. However, suitable band pass filters are chosen to stretch the coherence length to that of the pulse and we are able to show high visibility interference. Previous multi-photon experiments are limited in brightness to less than one four photon event per second mainly due to the limited pump laser power available. Our source is much more efficient and we are easily able to reach hundreds of four-fold coincidences per second. However at these high rates we enter a regime where higher order photon emission is no longer negligible and interference visibility is decreased.

In the first part of this paper (section 2), we briefly describe our photon pair source and how we use it as a heralded single photon source (HPS). In the second part (section 3), we look at the theoretical expectations of this non-classical interference using FWM compared to other processes such as parametric down-conversion (PDC). We then focus on the details of an experiment using two spatially separated PCF pair sources and compare our theoretical expectations to the measured results (section 4). Finally, we assess our source for quantum logic applications and suggest future improvements required (section 5).

2. PCF: from a photon pair source to an HPS

Quantum interference experiments involving several photons created as two or more pairs require pair photon sources pumped by ultra-short laser pulses where the bandwidth is of order nanometres and also requires a high efficiency of collection [17]. Our source of photon pairs using a PCF appears to be ideal in these terms as it demonstrates all these requirements [1, 2]. The main nonlinear process that has to be taken into account is FWM where two pump photons are converted into a pair of photons following phase
matching and conservation of energy:

$$2k_p - k_s - k_i - 2\gamma P = 0,$$

and

$$2\omega_p - \omega_s - \omega_i = 0.$$  \hspace{1cm} (2)

Figure 1 shows the microstructured fibre used in our experiment. It has a zero dispersion point in the near infra-red (715 nm) and is pumped with a picosecond pulsed laser set a few nanometres into the normal dispersion regime at 708 nm. This generates narrowband polarised photon pairs visible to efficient silicon-based photon counting detectors. The short wavelength sits at 583 nm and the corresponding idler at 900 nm with 6 and 11 nm FWHM bandwidths, respectively. Due to the guided configuration, our source is exposed to Raman scattering effects. However, the wide separation of the generated pair wavelengths means that the corresponding Raman noise comes from a sixth-order process and is thus quite weak. Furthermore, the number of created photon pairs is proportional to the square of the peak intensity while the spontaneous Raman scattering grows roughly linearly (at these low pump powers). Hence, the use of a picosecond laser in our experiment fits the requirements for quantum interference experiments but also allows us to improve the brightness while reducing Raman background noise to negligible levels (<10%). Consequently, in this configuration and thanks to the high lumped efficiency we could measure up to $2 \times 10^5$ net coincidences per second with pump power of 1 mW and 20 cm of fibre length. This represents one of the brightest source of photon pairs reported so far.

It has already been demonstrated that photon pair sources can be used to approximate the generation of single photons \([18]-[23]\). The photons of a pair being emitted simultaneously, the detection of one photon can be used to herald the emission of the second. The set-up of an HPS using a PCF is depicted in figure 2. A pulsed laser is used to pump the fibre. At the output, a dichroic mirror centred at 700 nm divides the pair into two arms, one corresponding to the signal channel and the other to the idler. Each photon of the pair is then launched into a single mode fibre. Once suitably filtered to remove the pump beam, the heralded single photons can then be fed into any quantum communication system (see figure 2). In terms of single photon
generation, the coincidence rate reported previously then corresponds to a number of heralded photons detected. These HPSs play an important role in quantum information applications, however, they can be used provided that the emitted photons fit the following requirements. Quantum mechanics predicts indeed that interference effects can be observed when overlapping two independent single photons on a BS as long as they are indistinguishable in spectrum, spatial and temporal modes [17]. This effect is due to the bosonic behaviour of photons which says that if two ‘indistinguishable’ photons arrive at the same time on a 50 : 50 BS, they will stick together and will always take the same output port. This interference effect can then be observed by looking at the coincidence rate between the two output ports while scanning along the temporal overlap of the photons on the BS. The resulting plot is commonly called the Hong–Ou–Mandel dip, referring to the first experiment of this kind [12]. The visibility of this dip will then give informations on the quality of the indistinguishability, while the width corresponds to the actual coherence length of the interfering photons. To confirm the quality of our source, we thus decided to perform this interference experiment using two separated PCF’s and mixed the heralded photons coming from each fibre on a 50 : 50 BS. This configuration is also a more realistic demonstration of the role that our source could play in multi-photon experiments used in QI systems where the number of single photon sources is getting more and more important. A schematic of the set-up is shown on figure 3. Here, the observation of the two photon interference effect turns into the measurement of the four-fold coincidence rate

Figure 2. HPS using a PCF fed into a quantum information system.

Figure 3. Simplified set-up of the interference experiment using two HPSs.
between pairs created from both fibres. The next section is dedicated to the theoretical study of this non-classical interference experiment.

3. Quantum interference: theoretical expectations

The origin of the Hong–Ou–Mandel interference effect comes from the indistinguishability and destructive interference between two paths to a coincidence detection. We can represent a BS (see figure 4) with amplitude transmission and reflection coefficients $t$ and $e^{i\pi/2}r = ir$ ($r$ real). The phase shift on reflection guarantees energy conservation.

When we populate the input modes $a_1$ and $a_2$ with single photons

$$|1\rangle_1|1\rangle_2 \rightarrow (t^2 - r^2)|1\rangle_3|1\rangle_4 + i rt \sqrt{2}(|2\rangle_3|0\rangle_4 + |0\rangle_3|2\rangle_4).$$

When $r = t = 1/\sqrt{2}$ (the 50 : 50 BS) $t^2 - r^2 = 0$ and the probability of detecting one photon at each output port is zero [24] which is clearly non-classical. This effect was first seen when the coincident photon pairs created in PDC were mixed at a BS [12, 13].

The above simple treatment implies that the photons are indistinguishable when viewed from the detectors but contains no spatial or temporal modes explicitly. In the experiment, we have a pulsed source of photon pairs with a finite bandwidth set by external filters created in a single spatial mode. A more realistic treatment of our interference experiment must include the finite bandwidth and resulting finite temporal overlap of our photons. To do this, we follow the treatment first described in [17] where the overlap of heralded photons created in PDC was studied. The main difference here is that the process of FWM implies two pump-photons are converted into a pair of signal and idler photons. Let us start with an effective two-photon wavefunction at the detectors given by

$$\Psi(\vec{t}_s, \vec{t}_i) = \langle \text{vac} | \hat{A}_s(\vec{t}_s) \hat{A}_i(\vec{t}_i) | \Psi \rangle,$$

where $\hat{A}_{s,i}$ are electric field operators for the signal and idler modes ($s, i$). These operators reflect the finite bandwidth and duration of the pairs through,

$$\hat{A}(\vec{t}) = \int d\omega f(\omega)e^{i\omega\vec{t}}$$

and reduced time $\vec{t} = t - X/c$ includes the propagation time from a fibre to detectors.
We label the two pump beams \( p_1, p_2 \) and detector arms \( i_1, i_2, s_3 \) and \( s_4 \) as shown in figure 3 then before the BS

\[
\Psi_{j,k}(t_{s,j}, t_{i,k}) = \alpha \int d\omega_p d\omega_p d\omega_p d\omega_p f_p(\omega_p) f_p(\omega_p) f_s(\omega_s) f_s(\omega_s) \phi(\omega_s, \omega_s) \\
\times \delta(\omega_s + \omega_t - \omega_p_1 - \omega_p_2)e^{i(\omega_s/t_{s,j} + \omega_s/t_{i,k})} \tag{6}
\]

with \( j = 3, 4 \) and \( k = 1, 2, f_{s,j,k} \) being the function describing the narrowband filters in front of each detector, \( f_p \) the pump envelope function corresponding to the spectrum of the pump beam, and \( \phi(\omega_s, \omega_t) \) the phase matching function. We identify \(|\alpha|^2\) with the probability that a pulse contains a pair (i.e. the mean number of pairs per pulse \( \bar{n} = |\alpha|^2 \)). We also ignore the vacuum component of the wavefunction \( \sqrt{1 - |\alpha|^2}\text{vac} \) which completes the normalisation.

When only one photon appears in each output, the wavefunction at detectors \( D_{i1}, D_{i2}, D_{s3} \) and \( D_{s4} \) is given by

\[
\Psi(i_1, i_2, s_3, s_4) = \eta_{i1} \eta_{i2} \eta_{s3} \eta_{s4} \int dt_{i1} dt_{i2} dt_{s3} dt_{s4} H(t_{i1} - t_0, \Delta t) H(t_{i2} - t_0, \Delta t) H(t_{s3} - t_0, \Delta t) \times H(t_{s4} - t_0, \Delta t) |\Psi(i_1, i_2, s_3, s_4)|^2, \tag{7}
\]

The probability of seeing a four-fold coincidence detection is then:

\[
P(i_1, i_2, s_3, s_4) = \eta_{i1} \eta_{i2} \eta_{s3} \eta_{s4} \int dt_{i1} dt_{i2} dt_{s3} dt_{s4} H(t_{i1} - t_0, \Delta t) H(t_{i2} - t_0, \Delta t) H(t_{s3} - t_0, \Delta t) H(t_{s4} - t_0, \Delta t) \times H(t_{s4} - t_0, \Delta t) |\Psi(i_1, i_2, s_3, s_4)|^2, \tag{8}
\]

where \( \eta_{i1,i2,s3,s4} \) are detector efficiencies and \( H(t_i - t_0, \Delta t) (l = 1 - 4) \) is a normalised detector response function centred on \( t_0 \) that falls to zero when \( t_i - t_0 > \Delta t \).

The temporal overlap of the two heralded photons on the BS must be adjusted to within the pump pulse length with fine-tuning \( \delta X \) performed on one of the pump beam versus the other one. We thus get

\[
\tilde{t}_{s13} = \tilde{t}_{s3}; \quad \tilde{t}_{s23} = \tilde{t}_{s3} + \delta X/c; \quad \tilde{t}_{s14} = \tilde{t}_{s4}; \quad \tilde{t}_{s24} = \tilde{t}_{s4} + \delta X/c. \tag{9}
\]

Following the same calculation steps as in [17] and using identical approximations such as energy-matched Gaussian filter functions where

\[
f_p(\omega_p) = \frac{1}{N} e^{(\omega_p_0 - \omega_p)^2/\sigma_p^2}, \tag{10}
\]

\[
f_{i1}(\omega_i) = \frac{1}{N} e^{(\omega_{i0} - \omega_i)^2/\sigma_i^2} = f_{i2}(\omega_i),
\]

\[
f_{s3}(\omega_s) = \frac{1}{N} e^{(\omega_{s0} - \omega_s)^2/\sigma_s^2} = f_{s4}(\omega_s),
\]

\[
2\omega_{p0} = \omega_{i0} + \omega_{s0}.
\]

with \( \sigma \) the filter bandwidth and \( \sigma_p \) the pump bandwidth, we finally get

\[
P(i_1, i_2, s_3, s_4) = \bar{n}^2 N \left( r^4 + t^4 - 2Vr^2t^2e^{-\Delta T^2\sigma^2/2(1+\sigma^2/2\sigma_p^2)} \right), \tag{11}
\]

where \( N \) is a normalising factor, \( V \) the interference visibility with:

\[
V_{\text{max}} = \frac{\sqrt{1 + \sigma^2/2\sigma_p^2}}{1 + \sigma^2/2\sigma_p^2}. \tag{12}
\]
4. Quantum interference: experiment and results

An accurate set-up of the experiment is depicted in figure 5. A mode-locked picosecond Ti:sapphire pump laser (Spectra Physics–Tsunami) set at 708 nm, emitting 1.5 ps pulses with a repetition rate of 82 MHz is sent on to a 50 : 50 BS. The output modes are then used to pump two 12 cm PCF used as HPSs. This length of fibre was chosen so as to minimise the walk-off (which could lead to temporal distinguishability) between the pump and signal pulses. We chose to perform the interference experiment using the signal photons of each PCF. In order to get the spectral indistinguishability (time-bandwidth limited), a bandpass filter of width $\sim 0.28$ nm (corresponding to the pump pulse duration) is used in the signal arm while a $\sim 2$ nm width filter is used in the idler. As the photon pairs are matched in energy a narrowband filter on one photon of the pair will act as a non-local filtering on the other photon. The idler photons of each source are then launched into single mode fibres and the signal photons into a single-mode 50 : 50 coupler with a polarisation controller (PC) set in one of its arms. All the outputs are finally connected to four silicon avalanche photodiodes (APD) linked to a four-fold coincidence measurement apparatus (Ortec Quad 4-Input Logic Unit CO4020 + PC based counter card). The strong filtering process reduces significantly the lumped efficiencies. Nevertheless, we can use the brightness of the source and increase the pump power to $P_p = 8$ mW which compensate the loss and allows us to achieve a coincidence rate of $10^5 \text{s}^{-1}$ within this narrow bandwidth.

A retro-reflector mounted on a micrometric translation stage in one arm of the interferometer allowed us to scan through the temporal overlap $\delta T$ between the two signal...
Figure 6. Raw four-fold coincidence rate per 60 s as a function of temporal overlap in ps.

photons. Figure 6 shows the raw four-fold coincidence rate as a function of the position of the retro-reflector for a very low pump power \( P_p = 1.4 \) mW fibre\(^{-1}\). At \( \delta T = 0 \), a significant reduction of the raw four-fold coincidence count rate was demonstrated. The term ‘raw’ here means that no correction was applied on the result presented. The visibility of this dip was calculated taking into account the fact that our coupler is not perfectly 50 : 50. Using (11) (with \( t = 0.54 \) and \( r = 0.46 \)) and fitting the experimental data allowed us to get back to the actual visibility of 88%. The drift in the four-fold coincidence rate comes from a drop in the launch efficiencies of the PCFs. These very low power experiments run over more than 8 h, which means that misalignment problems are very likely to happen. Since the four-fold coincidence rate using FWM, is proportional to the fourth power in laser power, the effect is thus rapidly significant.

We then increased the pump power to \( P_p = 8 \) mW per fibre and plotted on figure 7 the net four-fold coincidence rate as a function of the temporal overlap of the photons on the coupler. Unlike the previous results, this plot shows the net count rate which implies that the accidental four-fold coincidences (\( \sim 80 \) per second per fibre) have been subtracted. Due to the high brightness of our source, which demonstrates 240 raw four-fold coincidences per second for \( \sim 8 \) mW of pump power per fibre, multi-photon events lead to accidental four-fold coincidences. These events can be subtracted from the raw rate in order to get the true visibility of the interference. Experimentally this background is measured by blocking one input port of the coupler (and subsequently the other one) and looking at the four-fold coincidence rate coming from each fibre at a time revealing the amount of multi-photon events. Once corrected and considering again the defects of our 50 : 50 coupler, we could retrieve the actual visibility of the dip which came out to be 95%. This is close to the maximum theoretical visibility of 97% we can get predicted from the filter widths (see (12)). Finally, besides this great visibility it is also worth noting the high net four-fold counting rate we can achieve of about 80 s\(^{-1}\). This is comparable to the best coincidence rates achieved in bulk crystal multiphoton experiments [25] but could actually be greater since we are not limited in pump power. Although this result
Figure 7. Net four-fold coincidence rate as a function of temporal overlap in ps.

Figure 8. Raw four-fold coincidence rate as a function of temporal overlap in picoseconds. Taken from the same data set as figure 7 but without background correction.

definitely highlights the quality of this source for quantum information purposes, we realise here that the post-selection method to make a single photon source is spoilt at high pump power. In figure 8, we show data of figure 7 but without background correction. The raw visibility of 40% is above the classical limit of 33% for sources with Gaussian field statistics, which is induced by our filtering \[26\]. By ignoring the heralding entirely we get the result shown in figure 9 where we see 25% visibility, below the maximum expected 33%.
Figure 9. Raw two-fold coincidence rate as a function of temporal overlap in picoseconds. Result taken from the same data set as figure 6 but without heralding.

5. Discussion

We can definitely achieve high raw visibilities at low pump power but at higher pump powers we are dogged by a background that arises mainly from cases where two pairs are created in one fibre and one pair in the other. In our data analysis, we are able to measure this background and subtract it simply by alternately blocking the inputs to the BS. In the appendix, we present a brief analysis of this background term and reserve a full analysis to a future paper. In the limit of small $\eta_s$ and low lumped efficiencies ($\eta_i$) we then have

$$V \approx \frac{1 + 8\eta_s}{1 + 12\eta_s},$$

(13)

We then look at the rate of (non-interfering) four-fold coincidence detections and in the limit of small $\eta$ then we have

$$C_4 = R\eta_s^2\eta_i^2/2,$$

(14)

where $R$ is the laser repetition rate.

Clearly to get high raw visibility ($V > 0.9$) we must limit $\eta_s < 0.025$. At the same time we would like to use as high as possible $\eta$ to obtain a high four-fold coincidence rate. We can increase four photon rate without reducing visibility by increasing the lumped efficiency of detection (in fact this also has a small effect on improving visibility, see appendix). We are also able to increase $C_4$ by increasing the repetition rate $R$ of our laser as we are not limited in output power. In our experiment effective detection efficiencies are $\eta_s \sim 0.034$ and $\eta_i \sim 0.05$ with laser repetition rate $R = 8.2 \times 10^7$ s$^{-1}$ and limiting $\eta_s \sim 0.025$ we get $V \sim 0.89$ and see a four-fold coincidence rate $C_4 \sim 4/60$ s close to what we measure.
In the future, we can expect to increase efficiency to 0.1 in both channels by use of improved filters, better coupling and by increased detector efficiency (using wavelengths closer to detector efficiency maximum) which would make 6-photon experiments more realistic. With an average number of pairs per pulse of 0.1 and a repetition rate of 164 MHz (twice our current repetition rate) a raw six-fold coincidence rate up to \(0.164 \, \text{s}^{-1}\) would be obtained (before any BS). This coincidence rate would already be similar to the best rate reported so far in a 6-photon experiment \[25\]. Finally, in a more optimistic point of view where we could get efficiencies of 0.2 in each channel, the previous result would be multiplied by 2\(^6\) leading to more than 10 six-fold coincidences per second.

### 6. Conclusion

In conclusion, we have demonstrated a bright source of pure state photon pairs which can be used in multiphoton interference experiments showing high visibility interference between separate photons. We can easily generate hundreds of four-fold coincidence events per second and are effectively no longer limited by pump laser power. Our experiments are, however, limited by a strong background coincidence rate arising from pairs of pairs generated in each fibre. These higher order terms become significant when the average number of pairs generated per pulse reaches \(\bar{n} \sim 0.025\) and are worsened by the Gaussian statistics of the narrowband filtered source. The short term solution to this problem is to increase the pulse repetition rate and reduce \(\bar{n}\). Further improvement can be achieved by increasing the lumped collection and detection efficiency possibly by using better quality filters or developing an all fibre experiment. We can improve the heralding by using photon number resolving detectors to suppress the multi-photon events. We can also look at this phenomenon as a good indication that future six-fold and even eight-fold photon coincidence experiments are possible.

### Appendix. Background analysis

We can write the state generated in the FWM process up to two generated pairs

\[
|\Psi\rangle = \mathcal{N}[|0, 0\rangle_{s,i} + \alpha|1, 1\rangle_{s,i} + \alpha^2|2, 2\rangle_{s,i} + O(\alpha^3)],
\]

where again \(|\alpha|^2 = \bar{n}\) is the mean number of pairs generated per pulse and \(\mathcal{N}\) is a normalising factor. The idler photon is detected in a detector to herald the arrival of the signal photon. Our generic detector has lumped efficiency \(\eta\) (including filter and transmission loss) and fires in response to an \(n\) photon input with probability

\[
P_n(1) = (1 - (1 - \eta)^n).
\]

After detection of one idler photon, the density operator for the heralded state is

\[
\rho \approx \frac{1}{1 + 2\bar{n}\gamma} |1\rangle_s\langle 1| + \frac{2\bar{n}\gamma}{1 + 2\bar{n}\gamma} |2\rangle_s\langle 2| + O(\alpha^2),
\]

where \(\gamma = (1 - \eta_i/2)\). At this stage, we assume no losses in the signal arm and apply all losses later as lumped detector efficiencies.
We can then generalise the BS transformation to arbitrary number states at both inputs
\[ |n\rangle_1 |m\rangle_2 \rightarrow (a_3^+ + ir a_4^+)^n (ir a_3^+ + t a_4^+)^m |0\rangle_3 |0\rangle_4 / \sqrt{n!m!} \]
\[ = \sum_j \sum_{k=0}^m t^{m-k+j} (ir)^{n-k+j} \sqrt{C^n_j C^m_k} C^{(j+k)}_{(n+m-j-k)} |(j+k)\rangle_3 ((n+m-j-k))_4, \]
(A.4)

where \(a_k^+\) is the creation operator in the mode \(a_k\) with \(k = 3, 4\).

We can apply now the two inputs of heralded single photons as expressed by (A.3) using (A.4) and then calculate the probability of seeing a coincidence detection in the outputs \(a_3, a_4\). In the ideal case this would be zero but here we see
\[ P(|1\rangle_3, |1\rangle_4)_{\text{int}} = \eta_i^2 \left[ \frac{2 \bar{n} \gamma \gamma'}{1 + 2 \bar{n} \gamma} \right] \]
(A.5)
to first-order in \(\bar{n}\) and with \(\gamma' = (1 - \eta_s/2)\). When the photons do not overlap we must apply separate BSs (A.4) to each heralded state with \(n/m = 0\) in the second port and then sum the probabilities in the outputs to obtain
\[ P(|1\rangle_3, |1\rangle_4)_{\text{noint}} = \eta_i^2 \left[ \frac{1}{2} + \frac{6 \bar{n} \gamma \gamma'}{1 + 2 \bar{n} \gamma} \right]. \]
(A.6)

As we define the visibility as \(V = [P(|1\rangle_3, |1\rangle_4)_{\text{noint}} - P(|1\rangle_3, |1\rangle_4)_{\text{int}}] / P(|1\rangle_3, |1\rangle_4)_{\text{noint}}\) then we find
\[ V \approx \frac{1 + 8 \bar{n} \gamma \gamma'}{1 + 12 \bar{n} \gamma \gamma'}. \]
(A.7)

A calculation of the visibility at \(\bar{n} \geq 0.1\) is possible but requires a more careful definition of the heralded state that we hope to present in a much more detailed paper. However, we can illustrate the order of magnitude of the background in figure 8 from the following. According to equation (A.2), when two pairs come from one fibre, the probability of one idler photon being detected is given by
\[ P_2(1) = (1 - (1 - \eta_i^2)^2) \approx 2 \eta_i. \]
(A.8)

A four-fold coincidence arising from three-pair events (two pairs from ‘fibre1’ and one pair from ‘fibre2’) is then given by
\[ C'_{\text{from } 6} \approx R \bar{n}^3 \eta_i^2 (2 \eta_i) \eta_i / 2. \]
(A.9)

Since a similar four-fold coincidence event can happen with two pairs from ‘fibre2’ and one pair from ‘fibre1’, the total four-fold coincidence arising from three-pair events is
\[ C_{\text{from } 6} \approx 2 \times C'_{\text{from } 6} \approx 2 R \bar{n}^3 \eta_i^2 \eta_i^2 \approx 4 \bar{n} C_4. \]
(A.10)

where \(C_4\) is given by (14).

In our high power case we believe \(\bar{n} \approx 0.5\), then \(C_{\text{from } 6} \approx 2 \times C_4\), which is a good approximation of what we measure. However, we should also consider higher terms such as three pairs–one pair and two pairs–two pairs as these are of similar magnitude. The heralding process is only deleting the vacuum terms so at high \(\bar{n}\) we should see little difference from the interference of two beams with the statistics of a thermal source. This would lead to a dip of 33% visibility [26].
References

[1] Fulconis J, Alibart O, Wadsworth W J, Russell P S J and Rarity J G 2005 High brightness single mode source of correlated photon pairs using a photonic crystal fibre Opt. Express 13 7572–82
[2] Alibart O, Fulconis J, Wong G K L, Murdoch S G, Wadsworth W J and Rarity J G 2006 Photon pair generation using four-wave mixing in a microstructured fibre: theory versus experiment New J. Phys. 8 67
[3] Bouwmeester D, Pan J-W, Daniell M, Weinfurter H and Zeilinger A 1999 Observation of three-photon Greenberger-Horne-Zeilinger entanglement Phys. Rev. Lett. 82 1345
[4] Eibl M, Gaertner S, Bourennane M, Kurtsiefer Ch, Zukowski M and Weinfurter H 2003 Experimental observation of four-photon entanglement from down-conversion Preprint quant-ph/0302042
[5] Zhao Z, Chen Y-A, Zhang A-N, Yang T, Briegel H J and Pan J-W 2004 Experimental demonstration of five-photon entanglement and open-destination teleportation Nature 430 54
[6] Gisin N, Ribordy G, Tittel W and Zbinden H 2002 Quantum cryptography Rev. Mod. Phys. 74 145195
[7] Weinfurter H 2000 Quantum communication with entangled photons Adv. At. Mol. Opt. Phys. 42 489
[8] Knill E, Laflamme R and Milburn G J 2001 A scheme for efficient quantum computation with linear optics Nature 409 46
[9] Rarity J G 2003 Quantum communication and beyond Phil. Trans. R. Soc. Lond. A 361 1507–18
[10] Gasparoni S, Pan J-W, Walther P, Rudolph T and Zeilinger A 2004 Realization of a photonic controlled-not gate sufficient for quantum computation Phys. Rev. Lett. 93 020504
[11] Walther P, Resch K J, Rudolph T, Schenck E, Weinfurter H, Vedral V, Aspelmeyer M and Zeilinger A 2005 Experimental one-way quantum computing Nature 434 169
[12] Hong C K, Ou Y and Mandel L 1987 Phys. Rev. Lett. 59 2044
[13] Rarity J G and Tapster P R 1988 Photons and Quantum Fluuctuations ed E R Pike and H Walther (London: Adam Hilger) p 122
[14] Rarity J G and Tapster P R 1989 J. Opt. Soc. Am. B 6 1221
[15] Rarity J G and Tapster P R 1996 Interference between a single photon and a superposition of coherent states Quantum Interferometry ed F De Martini, G Denardo and Y Shih (Wenheim: VCH) p 211
[16] Rarity J G, Tapster P R and Loudon R 2005 Non-classical interference between independent sources J. Opt. B: Quantum Semiclass. Opt. 7 S171
[17] Kaltenbaek R, Blauensteiner B, Zukowski M, Aspelmeyer M and Zeilinger A 2006 Experimental interference of independent photons Phys. Rev. Lett. 96 240502
[18] Rarity J G 1995 Interference of single photons from separate sources Fundamental Problems in Quantum Theory ed D M Greenberger and A Zeilinger (Ann. NY Acad. Sci. 755 624)
[19] Hong C K and Mandel L 1986 Experimental realization of a localized one-photon state Phys. Rev. Lett. 56 58
[20] Rarity J G, Tapster P R and Jakeman E 1987 Observation of sub-poissonian light in parametric down conversion Opt. Commun. 62 201
[21] Kurtsiefer C, Oberparleiter M and Weinfurter H 2000 High efficiency entangled pair collection in type II parametric fluorescence Phys. Rev. Lett. 85 290–3
[22] U’Ren A B, Silberhorn C, Banaszek K and Walmsley I A 2004 Efficient conditional preparation of high-fidelity single photon states for fiber-optic quantum networks Phys. Rev. Lett. 93 093601
[23] Pittman T B, Jacobs B C and Franson J D 2005 Heralding single photons from pulsed parametric down-conversion Opt. Commun. 246 545
[24] Alibart O, Tanzilli S, Ostrowsky D B and Baldi P 2005 Guided wave technology for a telecom wavelength heralded single photon source Opt. Lett. 30 1539
[25] Feam H and Loudon R 1987 Opt. Commun. 64 485–90
[26] Zhang Q, Goebel A, Wagenknecht C, Chen Y-A, Zhao B, Yang T, Mair A, Schmiedmayer J and Pan J-W 2006 Experimental quantum teleportation of a two-qubit composite system Nat. Phys. 2 678–82
[27] de Riedmatten H, Marcikic I, Titel W, Zhbinden H and Gisin N 2003 Quantum interference with photon pairs created in spatially separated sources Phys. Rev. A 67 022301