The Quark Model and $b$ Baryons

Marek Karliner$^a$, Boaz Keren-Zur$^a$, Harry J. Lipkin$^{a,b,c}$, and Jonathan L. Rosner$^d$

$^a$ School of Physics and Astronomy
Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, Tel Aviv 69978, Israel

$^b$ Department of Particle Physics
Weizmann Institute of Science, Rehovoth 76100, Israel

$^c$ High Energy Physics Division, Argonne National Laboratory
Argonne, IL 60439-4815, USA

$^d$ Enrico Fermi Institute and Department of Physics
University of Chicago, 5640 S. Ellis Avenue, Chicago, IL 60637, USA

ABSTRACT

The recent observation at the Tevatron of $\Sigma^\pm_b$ ($uub$ and $ddb$) baryons within 2 MeV of the predicted $\Sigma_b - \Lambda_b$ splitting and of $\Xi^-_b$ ($dsb$) baryons at the Tevatron within a few MeV of predictions has provided strong confirmation for a theoretical approach based on modeling the color hyperfine interaction. The prediction of $M(\Xi^-_b) = 5790$ to 5800 MeV is reviewed and similar methods used to predict the masses of the excited states $\Xi'_b$ and $\Xi''_b$. The main source of uncertainty is the method used to estimate the mass difference $m_b - m_c$ from known hadrons. We verify that corrections due to the details of the interquark potential and to $\Xi_b - \Xi'_b$ mixing are small. For S-wave $qqb$ states we predict $M(\Omega_b) = 6052.1 \pm 5.6$ MeV, $M(\Omega^*_b) = 6082.8 \pm 5.6$ MeV, and $M(\Xi^0_b) = 5786.7 \pm 3.0$ MeV. For states with one unit of orbital angular momentum between the $b$ quark and the two light quarks we predict $M(\Lambda_b[1/2]) = 5929 \pm 2$ MeV, $M(\Lambda_b[3/2]) = 5940 \pm 2$ MeV, $M(\Xi_b[1/2]) = 6106 \pm 4$ MeV, and $M(\Xi_b[3/2]) = 6115 \pm 4$ MeV. Results are compared with those of other recent approaches.

PACS codes: 14.20.Mr, 12.40.Yx, 12.39.Jh, 11.30.Hw
1 Introduction

The first observed baryon with a $b$ quark was the isospin-zero $\Lambda_b$, whose mass has recently been well-measured: $M(\Lambda_b) = 5619.7 \pm 1.2 \pm 1.2$ MeV [1]. Its quark content is $\Lambda_b = bud$, where the $ud$ pair has spin and isospin $S(ud) = I(ud) = 0$. Now the CDF Collaboration has observed candidates for $\Sigma_b^+$ and $\Sigma_b^\pm$ [2] with masses consistent with predictions [3, 4, 5, 6, 7, 8, 9, 10, 11]. D0 and CDF have seen candidates for $\Xi_b^- = bsd$ [12, 13]. The more precise CDF mass lies close to a prediction based on careful accounting for wave function effects in the hyperfine interaction [14].

The CDF sensitivity appears adequate to detect further heavy baryons. The $S$-wave levels of states containing $bsu$ or $bsd$ consist of the $J = 1/2$ states $\Xi_b^{0,-}$ and $\Xi_b^{(0,-)}$ and the $J = 3/2$ states $\Xi_b^{(0,-)}$. Additional baryonic states containing the $b$ quark include $\Omega_b = bss$ ($J = 1/2$), $\Omega_b^* = bss$ ($J = 3/2$), and orbital excitations of $\Lambda_b$ and other $b$-flavored baryons. In this paper we predict the masses of these states and estimate the dependence of the predictions on the form of the interquark potential, extending a previous application to hyperfine splittings of known heavy hadrons [15]. Two observations based on a study of the hadronic spectrum lead to improved predictions for the $b$ baryons. The first is that the effective mass of the constituent quark depends on the spectator quarks [5], and the second is an effective supersymmetry [8] — a resemblance between mesons and baryons where the anti-quark is replaced by a diquark [16]. Parts of this article have appeared previously in preliminary form [14, 17].

We review the predictions for $\Sigma_b$ and $\Sigma_b^*$ in Section 2, and discuss predictions for $M(\Xi_b)$ in Section 3, starting with an extrapolation from $M(\Xi_c)$ without correction for hyperfine (HF) interaction and then estimating this correction. In the $\Xi_b$ the light quarks are approximately in a state with $S = 0$, while another heavier state $\Xi_b^*$ is expected in which the light quarks mainly have $S = 1$. There is also a state $\Xi_b^*$ expected with light-quark spin 1 and total $J = 3/2$. Predictions for $\Xi_b^*$ and $\Xi_b^0$ masses are discussed in Section 4. We estimate the effect of mixing between light-quark spins $S = 0$ and 1 in Section 5, and isospin splittings of the $\Xi_b$ family of states in Section 6. Section 7 is devoted to predictions of $M(\Omega_b)$ and $M(\Omega_b^*)$, while Section 8 treats orbital excitations. Comparisons with other approaches are made in Section 9, while Section 10 summarizes.

2 The $\Sigma_b$ and $\Sigma_b^{\pm}$ states

The $\Sigma_b^{\pm}$ states consist of a light quark pair $uu$ or $dd$ (a “nonstrange diquark”) with $S = I = 1$ coupled with the $b$ quark to $J = 1/2$, while in the $\Sigma_b^{*\pm}$ states the light quark pair and the $b$ quark are coupled to $J = 3/2$. The corresponding $ud$ pair in the $\Lambda_b$ has $S = I = 0$. The experimental $\Sigma_b^- - \Lambda_b$ mass differences [2],

$$M(\Sigma_b^-) - M(\Lambda_b) = 195.5^{+1.0}_{-1.0} \text{ (stat.)} \pm 0.1 \text{ (syst.)} \text{ MeV}$$

(1)
\[ M(\Sigma^+_b) - M(\Lambda_b) = 188.0^{+2.0}_{-2.3} \text{ (stat.)} \pm 0.1 \text{ (syst.)} \text{ MeV} \]

with isospin-averaged mass difference \( M(\Sigma_b) - M(\Lambda_b) = 192 \text{ MeV} \), are to be compared with the prediction \([5, 8]\) \( M_{\Sigma_b} - M_{\Lambda_b} = 194 \text{ MeV} \). Note also:

1. The mass difference between spin-1 and spin-zero nonstrange diquarks governs the splitting between the spin-weighted average \( [2M(\Sigma^*_b) + M(\Sigma_b)]/3 \) and the \( \Lambda_b \),

\[
\frac{M(\Sigma_b) + 2M(\Sigma^*_b)}{3} - M(\Lambda_b) = (205.9 \pm 1.8) \text{ MeV},
\]

where we have used the averages of the differences for \( \Sigma^{(s)}_b \). This should be the same as the corresponding quantity for charmed baryons,

\[
\frac{M(\Sigma_c) + 2M(\Sigma^*_c)}{3} - M(\Lambda_c) = (210.0 \pm 0.5) \text{ MeV},
\]

and that for strange baryons,

\[
\frac{M(\Sigma) + 2M(\Sigma^*)}{3} - M(\Lambda) = (205.1 \pm 0.3) \text{ MeV},
\]

where the masses are from Ref. \([18]\), and an average over the \( \Sigma \) isospin multiplet is taken. In each case the dominant source of error is the mass of the \( I_3 = 0, J = 3/2 \) state, \( \Sigma^{(s)}_c \) or \( \Sigma^{(s)}_b \). The agreement is quite satisfactory.

2. The charge-averaged hyperfine splitting between the \( J = 1/2 \) and \( J = 3/2 \) states involving the spin-1 nonstrange diquark may be predicted from that for charmed particles:

\[
\frac{M(\Sigma^*_c) - M(\Sigma_c)}{M(\Sigma^*_c) - M(\Sigma^*_b)} = \frac{m_c}{m_b} = \frac{1.5 \text{ GeV}}{4.9 \text{ GeV}} = 0.31,
\]

where “constituent” quark masses are from Ref. \([19]\). Using isospin-averaged differences \( M(\Sigma_c) - M(\Lambda_c) = (167.09 \pm 0.13) \text{ MeV} \) and \( M(\Sigma^*_c) - M(\Lambda_c) = (231.5 \pm 0.8) \text{ MeV} \) \([18]\), we predict \( M(\Sigma^*_b) - M(\Sigma_b) = 20.0 \pm 0.3 \text{ MeV} \). This agrees with the observed splitting (see Table \([\text{I}]\)).

In analyzing their data on \( \Sigma^+_b \) and \( \Sigma^{*\pm}_b \), the CDF Collaboration assumed equal mass splittings \( M(\Sigma^+_b) - M(\Sigma^*_b) \) and \( M(\Sigma^*_b) - M(\Sigma^+_b) \). This assumption was found to be valid within the experimental errors. In Ref. \([7]\) a relation \( \Sigma^{*1}_b - \Sigma^{*1}_b = 0.40 \pm 0.07 \text{ MeV} \) was proved between the \( \Delta I = 1 \) mass differences \( \Sigma^{*1}_b \equiv M(\Sigma^+_b) - M(\Sigma^-_b) \) and \( \Sigma^{*1}_b \equiv M(\Sigma^{*+}_b) - (\Sigma^{*-}_b) \).

### 3 \( \Xi_b \) mass prediction

In our model the mass of a hadron is given by the sum of the constituent quark masses plus the color-hyperfine (HF) interactions:

\[
V_{ij}^{HF} = v \frac{\vec{\sigma}_i \cdot \vec{\sigma}_j}{m_i m_j} \langle \delta(r_{ij}) \rangle
\]
where the $m_i$ is the mass of the $i$’th constituent quark, $\sigma_i$ its spin, $r_{ij}$ the distance between the quarks and $v$ is the interaction strength. We shall neglect the mass differences between $u$ and $d$ constituent quarks, writing $u$ to stand for either $u$ or $d$. All the hadron masses (the ones used and the predictions) are for isospin-averaged baryons and are given in MeV.

The $s$ and $u$ quarks in $\Xi_q$ ($q$ standing for $c$ or $b$) are assumed to be in relative spin 0 and the total mass is given by the expression:

$$\Xi_q = m_q + m_s + m_u - \frac{3v\langle\delta(r_{us})\rangle}{m_um_s} \quad (7)$$

The $\Xi_b$ mass can thus be predicted using the known $\Xi_c$ baryon mass as a starting point and adding the corrections due to mass differences and HF interactions:

$$\Xi_b = \Xi_c + (m_b - m_c) - \frac{3v}{m_um_s} \left( \langle\delta(r_{us})\rangle_{\Xi_b} - \langle\delta(r_{us})\rangle_{\Xi_c} \right) \quad (8)$$

The observed masses for the charmed-strange baryons $\Xi_c$, $\Xi'_c$, and $\Xi^*_c$ are [18]:

$$\Xi_c = 2469.5 \pm 0.5 \text{ MeV} \quad \Xi'_c = 2577 \pm 4 \text{ MeV} \quad \Xi^*_c = 2646.3 \pm 1.8 \text{ MeV} \quad (9)$$

### 3.1 Constituent quark mass difference

The mass difference $(m_b - m_c)$ can be obtained from experimental data using one of the following expressions:

- We can simply take the difference of the masses of the $\Lambda_q$ baryons, ignoring the differences in the HF interaction:

  $$m_b - m_c = \Lambda_b - \Lambda_c = (3333.2 \pm 1.2) \text{ MeV} \quad . \quad (10)$$

- We can use the spin averaged masses of the $\Lambda_q$ and $\Sigma_q$ baryons:

  $$m_b - m_c = \left( \frac{2\Sigma^*_b + \Sigma_b + \Lambda_b}{4} - \frac{2\Sigma^*_c + \Sigma_c + \Lambda_c}{4} \right) = (3330.4 \pm 1.8) \text{ MeV} \quad . \quad (11)$$
Table 2: Comparison between experimental data and predictions of the ratio of $u$ and $s$ contact probabilities in $\Xi$ and $\Xi_c$ (Eq. (14)).

|                         | $\langle \delta(r_{us}) \rangle_{\Xi}/\langle \delta(r_{us}) \rangle_{\Xi_c}$ |
|-------------------------|-------------------------------------------------------------------------------------------------|
| **Experimental data [18]** | 1.071 ± 0.069                                                                                   |
| Linear                  | 1.022 ± 0.072                                                                                   |
| Coulomb                 | 1.487 ± 0.002                                                                                   |
| Cornell                 | 1.063 ± 0.047                                                                                   |

- Since the $\Xi_q$ baryon has strangeness 1, it might be better to use masses of mesons with $S = 1$:

$$m_b - m_c = \left( \frac{3B_s^* + B_s}{4} - \frac{3D_s^* + D_s}{4} \right) = (3324.6 \pm 1.4) \text{ MeV} \,.$$  

3.2 HF interaction correction

The HF interaction correction can also be based on $\Xi_c$ baryon experimental data:

$$\frac{v}{m_u m_s} \left( \langle \delta(r_{us}) \rangle_{\Xi_b} - \langle \delta(r_{us}) \rangle_{\Xi_c} \right) = \frac{v}{m_u m_s} \left( \frac{\langle \delta(r_{us}) \rangle_{\Xi_b}}{\langle \delta(r_{us}) \rangle_{\Xi_c}} - 1 \right)$$

$$= \frac{2\Xi_c^* + \Xi'_c - 3\Xi_c}{12} \left( \frac{\langle \delta(r_{us}) \rangle_{\Xi_b}}{\langle \delta(r_{us}) \rangle_{\Xi_c}} - 1 \right) = \left( \frac{\langle \delta(r_{us}) \rangle_{\Xi_b}}{\langle \delta(r_{us}) \rangle_{\Xi_c}} - 1 \right) (38.4 \pm 0.5) \text{ MeV} \,.$$

This expression requires the calculation of the $\delta$ function expectation value using 3-body wavefunctions from a variational method [15]. One only needs the shape of the confining potential, as coupling constants cancel out in the ratio of the $\delta$ function expectation values. The potentials considered here are the linear, Coulomb and Cornell (Coulomb + linear) potentials. We also note results obtained without the HF corrections. For the Cornell potential we have an additional parameter determining the ratio between the strengths of the linear and Coulombic parts of the potential. In these calculations we used the parameters extracted in [20] from analysis of quarkonium spectra (or $K = 0.45$ in the parametrization of [15]).

As a test case we compared the values obtained from experimental data and variational calculations for the ratio of contact probabilities in $\Xi$ and $\Xi_c$:

$$\frac{2\Xi_c^* + \Xi'_c - 3\Xi_c}{2(\Xi^* - \Xi)} = \frac{6v \langle \delta(r_{us}) \rangle_{\Xi_c}}{m_u m_s} = \frac{\langle \delta(r_{us}) \rangle_{\Xi_c}}{\langle \delta(r_{us}) \rangle_{\Xi}} \,.$$  

The results in Table 2 show good agreement between data and theoretical predictions using the Cornell potential.
Table 3: Predictions for the $\Xi_b$ mass with various confining potentials and methods of obtaining the quark mass difference $m_b - m_c$.

|        | $\Lambda_b - \Lambda_c$ | $\Sigma_b - \Sigma_c$ | $B_s - D_s$ |
|--------|--------------------------|------------------------|-------------|
| Eq. (10) | Eq. (11) | Eq. (12) |
| No HF correction | 5803 ± 2 | 5800 ± 2 | 5794 ± 2 |
| Linear | 5801 ± 11 | 5798 ± 11 | 5792 ± 11 |
| Coulomb | 5778 ± 2 | 5776 ± 2 | 5770 ± 2 |
| Cornell | 5799 ± 7 | 5796 ± 7 | 5790 ± 7 |

Table 4: Observations of $\Xi_b^- \rightarrow J/\psi \Xi^-$ at the Fermilab Tevatron. Errors on mass are (statistical, systematic).

|        | D0 [12] | CDF [13] |
|--------|---------|----------|
| Mass (MeV) | 5774 ± 11 ± 15 | 5792.9 ± 2.5 ± 1.7 |
| Width (MeV) | 37 ± 8 | ~ 14 |
| Significance | 5.5σ | 7.8σ |

3.3 Results

The predictions for $M(\Xi_b)$ under various assumptions about constituent quark mass differences and confinement potentials are given in Table 3. In Ref. [15] we find that the Coulomb potential leads to a very strong dependence on quark masses not seen in the data, so one should give these predictions less weight. Ignoring the Coulomb potential, one gets a prediction for $M(\Xi_b)$ in the range 5790–5800 MeV.

The predictions of Table 3 were first presented in Ref. [14]. At that time we learned of the $\Xi_b^-$ observation in the $J/\psi \Xi^-$ decay mode by the D0 Collaboration [12]. Subsequently the CDF Collaboration released their very precise measurement of $M(\Xi_b^-)$ in the same decay channel [13]. The reported masses, Gaussian widths (due to instrumental resolution), and significances of the signal are summarized in Table 4 and in Fig. 1. CDF also sees a significant $\Xi_b^- \rightarrow \Xi_c^0 \pi^-$ signal with mass consistent with that found in the $J/\psi \Xi^-$ mode.

The D0 mass is consistent with all our predictions for the isospin-averaged mass, while that of CDF allows us to rule out the (previously disfavored [15]) prediction based on the Coulomb potential. Both experiments also agree with a prediction in Ref. [4], $M(\Xi_b^-) = M(\Lambda_b) + (182.7 \pm 5.0)$ MeV = (5802.4 ± 5.3) MeV, where differences in wave function effects were not discussed and $m_b - m_c$ was taken from baryons only. (Here we have updated the prediction of Ref. [4] using the recent CDF [1] value of $M(\Lambda_b)$.) In our work the optimal value of $m_b - m_c$ was obtained from $B_s$ and $D_s$ mesons which contain both heavy and strange quarks, as do $\Xi_b$ and $\Xi_c$. See also Refs. [6] and [9] for compilations of earlier predictions for the $\Xi_b$ mass; we shall
Figure 1: Comparison of theoretical predictions and experimental results for the $\Xi_b^-$ mass from D0 [12] and CDF [13] (adapted from [21]). The theoretical predictions are denoted by the two horizontal bands, corresponding to Refs. [4] and [14], respectively.

return to this question in Sec. 9. The dependence of $m_b - m_c$ obtained from $B$ and $D$ mesons upon the flavor of the spectator quark was noted in Ref. [5] where Table I shows that the value is the same for mesons and baryons not containing strange quarks but different when obtained from $B_s$ and $D_s$ mesons.

4 $\Xi_b^*$, $\Xi_b'$ mass prediction

4.1 Spin averaged mass $(2\Xi_b^* + \Xi_b')/3$

The $s$ and $u$ quarks of the $\Xi_q^*$ and $\Xi_q'$ baryons are assumed to be in a state of relative spin 1. We then find

$$\Xi_q^* = m_q + m_s + m_u + v\left( \langle \delta(r_{qs}) \rangle + \langle \delta(r_{qu}) \rangle + \langle \delta(r_{us}) \rangle \right)$$

$$\Xi_q' = m_q + m_s + m_u + v\left( -2\langle \delta(r_{qs}) \rangle + \frac{-2\langle \delta(r_{qu}) \rangle}{m_q m_s} + \frac{\langle \delta(r_{us}) \rangle}{m_u m_s} \right)$$

The spin-averaged mass of these two states can be expressed as

$$\frac{2\Xi_q^* + \Xi_q'}{3} = m_q + m_s + m_u + v\langle \delta(r_{us}) \rangle$$

7
Table 5: Predictions for the spin averaged $\Xi_b'$ and $\Xi_c^*$ masses with various confining potentials and methods of obtaining the quark mass difference $m_b - m_c$.

|                  | $\Lambda_b - \Lambda_c$ | $\Sigma_b - \Sigma_c$ | $B_s - D_s$ |
|------------------|--------------------------|------------------------|-------------|
| Eq. (10)         | 5956 ± 3                 | 5954 ± 3               | 5948 ± 3    |
| Eq. (11)         | 5954 ± 3                 | 5954 ± 4               | 5948 ± 4    |
| Eq. (12)         | 5948 ± 3                 | 5956 ± 3               | 5956 ± 3    |

And as for the $\Xi_b$ case, the following prediction can be given:

$$\frac{2\Xi_b^* + \Xi_b'}{3} = \frac{2\Xi_c^* + \Xi_c'}{3} + (m_b - m_c) + \frac{2\Xi_c^* + \Xi_c'}{12} \left( \frac{\langle \delta(r_{us}) \rangle_{\Xi_b}}{\langle \delta(r_{us}) \rangle_{\Xi_c}} - 1 \right).$$ (17)

The predictions obtained using the same methods described above are given in Table 5. Here the effect of the HF correction is negligible, so the difference between the spin averaged mass $(2\Xi_b^* + \Xi_b')/3$ and $\Xi_b$ is roughly $150 - 160$ MeV.

4.2 $\Xi_b^* - \Xi_b'$

This mass difference will be small due to the large mass of the $b$ quark:

$$\Xi_q^* - \Xi_q' = 3v \left( \frac{\langle \delta(r_{qs}) \rangle}{m_q m_s} + \frac{\langle \delta(r_{qu}) \rangle}{m_q m_u} \right)$$ (18)

We can once again use the $\Xi_c$ hadron masses:

$$\frac{\Xi_b^* - \Xi_b'}{\Xi_c^* - \Xi_c'} = \frac{3v \langle \delta(r_{bs}) \rangle}{m_s m_c} + \frac{\langle \delta(r_{bu}) \rangle}{m_u m_c} = \frac{m_c}{m_b} \frac{\langle \delta(r_{bs}) \rangle_{\Xi_b} + \langle \delta(r_{bu}) \rangle_{\Xi_b}}{\langle \delta(r_{cs}) \rangle_{\Xi_c} + \langle \delta(r_{cu}) \rangle_{\Xi_c}}$$ (19)

This expression is strongly dependent on the confinement model. In the results given in Table 5 we have used $m_s/m_u = 1.5 \pm 0.1$, $m_b/m_c = 2.95 \pm 0.2$.

In the context of $\Xi_b$ and $\Xi_b'$ masses it is worth mentioning two elegant relations among bottom baryons [22] which incorporate the effects of $SU(3)_f$ breaking:

$$\Sigma_b + \Omega_b - 2\Xi_b' = 0 ,$$ (20)

$$(\Sigma_b^* - \Sigma_b) + (\Omega_b^* - \Omega_b) - 2(\Xi_b^* - \Xi_b') = 0 ,$$ (21)

where isospin averaging is implicit.
Table 6: Predictions for \( M(\Xi_b^\ast) - M(\Xi'_b) \) with various confining potentials.

| Potential       | \( \Xi_b^\ast - \Xi'_b \) |
|-----------------|-------------------------------|
| No HF correction| 24 ± 2                        |
| Linear          | 28 ± 6                        |
| Coulomb         | 36 ± 7                        |
| Cornell         | 29 ± 6                        |

5 Effect of light-quark spin mixing on \( \Xi_b \) and \( \Xi'_b \)

In estimates up to this point we have assumed that the light-quark spins in \( \Xi_b \) and \( \Xi'_b \) are purely \( S = 0 \) and \( S = 1 \), respectively. The differing hyperfine interactions between the \( b \) quark and nonstrange or strange quarks leads to a small admixture of the opposite-\( S \) state in each mass eigenstate \([23, 24, 25, 26]\). The effective hyperfine Hamiltonian may be written \([25, 26]\)

\[
H_{\text{eff}} = M_0 + \lambda (\sigma_u \cdot \sigma_s + \alpha \sigma_u \cdot \sigma_b + \beta \sigma_s \cdot \sigma_b),
\]

where \( M_0 \) is the sum of spin independent terms, \( \lambda \sim 1/(m_u m_s) \), \( \alpha = m_s/m_b \), and \( \beta = m_u/m_b \). The calculation of \( M_{3/2} \) is straightforward, as the expectation value of each \( \sigma_i \cdot \sigma_j \) in the \( J = 3/2 \) state is 1. For the \( J = 1/2 \) states one has to diagonalize the \( 2 \times 2 \) matrix

\[
\mathcal{M}_{1/2} = \begin{bmatrix}
M_0 - 3\lambda & \lambda \sqrt{3}(\beta - \alpha) \\
\lambda \sqrt{3}(\beta - \alpha) & M_0 + \lambda (1 - 2\alpha - 2\beta)
\end{bmatrix}.
\]

The eigenvalues of \( H_{\text{eff}} \) are thus

\[
M_{3/2} = M_0 + \lambda (1 + \alpha + \beta),
\]

\[
M_{1/2,\pm} = M_0 + \lambda [- (1 + \alpha + \beta) \\
\pm 2\lambda (1 + \alpha^2 + \beta^2 - \alpha - \beta - \alpha\beta)^{1/2}]
\]

In the absence of mixing (\( \alpha = \beta \)) one would have \( M_{3/2} = M_0 + \lambda (1 + 2\alpha) \), \( M_{1/2,+} = M_0 + \lambda (1 - 4\alpha) \), and \( M_{1/2,-} = M_0 - 3\lambda \).

To see the effect of mixing, we rewrite the expression for \( M_{1/2,\pm} \),

\[
M_{1/2,\pm} = M_0 - \lambda (1 + \alpha + \beta) \pm 2\lambda \left[ (1 - \frac{\alpha + \beta}{2})^2 + \frac{3}{4} (\alpha - \beta)^2 \right]^{1/2}
\]

The effect of the mixing is seen in the term \( \frac{3}{4} (\alpha - \beta)^2 \). Expanding \( M_{1/2,\pm} \) to second order in small \( \alpha - \beta \), we obtain

\[
M_{1/2,\pm} \approx \text{(terms without mixing)} \pm \lambda \cdot \frac{3(\alpha - \beta)^2/4}{1 - (\alpha + \beta)/2}.
\]
For $m_u = 363$ MeV, $m_s = 538$ MeV, and $m_b = 4900$ MeV [27], one has $\alpha \simeq 0.11$, $\beta \simeq 0.07$, while the discussion in the previous section implies $\lambda \simeq 40$ MeV [Eq. (13)]. Hence the effect of mixing on our predictions is negligible, amounting to $\pm 0.04$ MeV.

Since we use the $\Xi_c$ and $\Xi'_c$ masses as input for $\Xi_b$, it is also important to check the mixing effects on the former. Since $m_b/m_c \sim 3$, this amounts to changing in the expressions above $\alpha \rightarrow 3\alpha$, $\beta \rightarrow 3\beta$. The corresponding effect of mixing on $\Xi_c$ and $\Xi'_c$ is $\sim 0.5$ MeV, still negligible.

6 Isospin splittings of $\Xi_b$ states

The $\Xi_b^0$ mass is expected to be measured by the CDF collaboration through the channel $\Xi_b^0 \rightarrow \Xi_c^+\pi^-$, where $\Xi_c^+ \rightarrow \Xi^-\pi^+\pi^+$, $\Xi^- \rightarrow \Lambda\pi^-$, and $\Lambda \rightarrow p\pi^-$ [21].

The source for the isospin splitting ($\Delta I$) is the difference in the mass and charge of the $u$ and $d$ quarks. These differences affect the hadron mass in four ways [28]: they change the constituent quark masses ($\Delta M = m_d - m_u$), the Coulomb interaction ($V^{EM}$), and the spin-dependent interactions – both magnetic and chromo-magnetic ($V^{\text{spin}}$). One can obtain a prediction for the $\Xi_b$ isospin splitting by extrapolation from the $\Xi$ data, which has similar structure as far as EM interactions are concerned (note that for $\Xi_b$ there are no spin-dependent interactions between the heavy quark and the $su$ diquark which is coupled to spin zero):

$$\Delta I(\Xi^*) = \Delta M + \left[V^{EM}_{ssd} - V^{EM}_{ssu}\right] + 2\left[V^{\text{spin}}_{ds} - V^{\text{spin}}_{us}\right] = 3.20 \pm 0.68 \text{ MeV (28)}$$

$$\Delta I(\Xi) = \Delta M + \left[V^{EM}_{ssd} - V^{EM}_{ssu}\right] - 4\left[V^{\text{spin}}_{ds} - V^{\text{spin}}_{us}\right] = 6.85 \pm 0.21 \text{ MeV (29)}$$

$$\Rightarrow \Delta I(\Xi_b) = \Delta M + \left[V^{EM}_{ssd} - V^{EM}_{ssu}\right] - 3\left[V^{\text{spin}}_{ds} - V^{\text{spin}}_{us}\right]$$

$$= \frac{2\Delta I(\Xi^*) + \Delta I(\Xi)}{3} + \frac{\Delta I(\Xi) - \Delta I(\Xi^*)}{2} = \frac{\Delta I(\Xi^*) + 5\Delta I(\Xi)}{6}$$

$$= 6.24 \pm 0.21 \text{ MeV (30)}$$

With the observed value [13] $M(\Xi_b^-) = (5792.9 \pm 2.5 \pm 1.7)$ MeV (the error from the D0 experiment is considerably larger [12]) and this estimate, we predict $M(\Xi_b^0) = 5786.7 \pm 3.0$ MeV.

Another option is to use $\Xi_c$, which has the same spin-dependent interactions, as a starting point:

$$\Delta I(\Xi_c) = \Delta M + \left[V^{EM}_{csd} - V^{EM}_{csu}\right] - 3\left[V^{\text{spin}}_{ds} - V^{\text{spin}}_{us}\right] = 3.1 \pm 0.5 \text{ MeV (31)}$$

$$\Rightarrow \Delta I(\Xi_b) = \Delta M + \left[V^{EM}_{ssd} - V^{EM}_{ssu}\right] - 3\left[V^{\text{spin}}_{ds} - V^{\text{spin}}_{us}\right]$$

$$= \Delta I(\Xi_c) + \left[V^{EM}_{ssd} - V^{EM}_{ssu}\right] - \left[V^{EM}_{csd} - V^{EM}_{csu}\right]$$

10
Table 7: Isospin splittings $\Delta I$ used in calculating $\Delta I(\Xi_b) \equiv M(\Xi_b^-) - M(\Xi_b^0)$.

| Splitting | Value (MeV) |
|-----------|-------------|
| $\Delta I(\Xi)$ | $6.85 \pm 0.21$ |
| $\Delta I(\Xi^*)$ | $3.20 \pm 0.68$ |
| $\Delta I(\Xi_c)$ | $3.1 \pm 0.5$ |
| $\Delta I(\Xi'_c)$ | $2.3 \pm 4.24$ |
| $\Delta I(\Xi''_c)$ | $-0.5 \pm 1.84$ |

$$= \Delta I(\Xi_c) + \frac{2\Delta I(\Xi^*) + \Delta I(\Xi)}{3} - \frac{2\Delta I(\Xi'_c) + \Delta I(\Xi''_c) + \Delta I(\Xi_c)}{4}$$

$$= 6.4 \pm 1.6 \text{ MeV}$$

We summarize the isospin splittings which have been used in these calculations in Table 7. All masses have been taken from the 2007 updated tables of the Particle Data Group [29], and all values of $\Delta I$ are defined as $M(\text{baryon with } d \text{ quark}) - M(\text{baryon with } u \text{ quark})$.

7 $\Lambda_b$ and $\Xi_b$ orbital excitations

Table 8: Masses of $\Lambda$ and $\Xi$ baryon ground states and orbital excitations [29].

|        | $\Lambda$          | $\Lambda_c$         | $\Xi_{c}^+$ | $\Xi_{c}^0$ |
|--------|--------------------|---------------------|-------------|-------------|
| $M(1/2^+)$ | $1115.683 \pm 0.006$ | $2286.46 \pm 0.14$ | $2467.9 \pm 0.4$ | $2471.0 \pm 0.4$ |
| $M(1/2^-)$ | $1406.5 \pm 4.0$ | $2595.4 \pm 0.6$ | $2789.2 \pm 3.2$ | $2791.9 \pm 3.3$ |
| $M(3/2^-)$ | $1519.5 \pm 1.0$ | $2628.1 \pm 0.6$ | $2816.5 \pm 1.2$ | $2818.2 \pm 2.1$ |

In the heavy quark limit, the $(1/2^-)$ and $(3/2^-)$ $\Lambda^*$ and $\Xi^*$ excitations listed in Table 8 can be interpreted as a P-wave isospin-0 spinless diquark coupled to the heavy quark. Under this assumption, the difference between the spin averaged mass of the $\Lambda^*$ baryons and the ground state $\Lambda$ is only the orbital excitation energy of the diquark.

$$\Delta E_L(\Lambda) \equiv \frac{2\Lambda_{[3/2]}^{*} + \Lambda_{[1/2]}^{*}}{3} - \Lambda = 366.15 \pm 1.49 \text{ MeV}$$

$$\Delta E_L(\Lambda_c) \equiv \frac{2\Lambda_{c[3/2]}^{*} + \Lambda_{c[1/2]}^{*}}{3} - \Lambda_c = 330.74 \pm 0.47 \text{ MeV} \quad (33)$$

$$\Delta E_L(\Xi_c) \equiv \frac{2\Xi_{c[3/2]}^{*} + \Xi_{c[1/2]}^{*}}{3} - \Xi_c = 339.11 \pm 1.11 \text{ MeV}$$

The spin-orbit splitting seems to behave like $1/m_Q$:

$$\Lambda_{[3/2]}^{*} - \Lambda_{[1/2]}^{*} = 113.0 \pm 4.1 \text{ MeV}$$
\[
\begin{align*}
\Lambda_{c[3/2]}^* - \Lambda_{c[1/2]}^* &= 32.7 \pm 0.8 \text{ MeV} \\
\Xi_{c[3/2]}^* - \Xi_{c[1/2]}^* &= 26.9 \pm 2.6 \text{ MeV}
\end{align*}
\]

where the \( \Xi_c \) entries are isospin averages.

The orbital excitation energies in Eq. (33) may be extrapolated to the case of excited \( \Lambda_b \) baryons in the following manner. Energy spacings in a power-law potential \( V(r) \sim r^p \) behave with reduced mass \( \mu \) as \( \Delta E \sim \mu^p \), where \( p = -\nu/(2 + \nu) \) \[30\]. For light quarks in the confinement regime, one expects \( \nu = 1 \) and \( p = -1/3 \), while for the \( c\bar{c} \) and \( b\bar{b} \) quarkonium states, with nearly equal level spacings, an effective power is \( \nu \approx 0 \) and \( p \approx 0 \). One should thus expect orbital excitations to scale with some power \(-1/3 \leq p \leq 0 \). One can narrow this range by comparing the \( \Lambda \) and \( \Lambda_c \) excitation energies and estimating \( p \) with the help of reduced masses \( \mu \) for the \( \Lambda \) and \( \Lambda_c \).

\[
\frac{\mu(\Lambda_c)}{\mu(\Lambda)} = \frac{M[ud] m_c}{M[ud] + m_s} = \frac{M(\Lambda) m_c}{M(\Lambda_c) m_s} = 1.55
\]

Now we use the ratio \( \Delta E_L(\Lambda_c)/\Delta E_L(\Lambda) = 0.903 \pm 0.004 \) to extract an effective power \( p = -0.23 \pm 0.01 \) which will be used to extrapolate to the \( \Lambda_b \) system:

\[
\begin{align*}
\Delta E_L(\Lambda_b) &= \Delta E_L(\Lambda_c) \left[ \frac{\mu(\Lambda_b)}{\mu(\Lambda_c)} \right]^p = \Delta E_L(\Lambda_c) \left[ \frac{M(\Lambda_c) m_b}{M(\Lambda_b) m_c} \right]^p \\
&= \Delta E_L(\Lambda_c) \left[ \frac{M(\Lambda_c)[M(\Lambda_b) - M(\Lambda) + m_s]}{M(\Lambda_b)[M(\Lambda_c) - M(\Lambda) + m_s]} \right]^p \\
&= \Delta E_L(\Lambda_c) \left[ \frac{1}{1 - \frac{M(\Lambda) - m_s}{M(\Lambda_b)}} \right]^p = 317 \pm 1 \text{ MeV}
\end{align*}
\]

where the last form of the expression shows the explicit dependence of the result on \( m_s \). Using the value \( M(\Lambda_b) = (5619.7 \pm 1.2 \pm 1.2) \text{ MeV} \) observed by the CDF Collaboration [1], and rescaling the fine-structure splittings of Eq. (34) by \( 1/m_Q \) with \( m_b/m_c = 2.95 \pm 0.06 \), we find

\[
M(\Lambda_{b[3/2]}^*) - M(\Lambda_{b[1/2]}^*) = \frac{m_c}{m_b} (M(\Lambda_{c[3/2]}^*) - M(\Lambda_{c[1/2]}^*)) = (11.1 \pm 0.4) \text{ MeV} ,
\]

\[
M(\Lambda_{b[1/2]}^*) = (5929 \pm 2) \text{ MeV} , \quad M(\Lambda_{b[3/2]}^*) = (5940 \pm 2) \text{ MeV} .
\]

The observed values of the \( \Sigma_b \) masses [2],

\[
\begin{align*}
M(\Sigma_{b}^-) &= 5815.2 \pm 1.0(\text{stat.}) \pm 1.7(\text{syst.}) \text{ MeV} \\
M(\Sigma_{b}^+) &= 5807.8^{+2.0}_{-2.2}(\text{stat.}) \pm 1.7(\text{syst.}) \text{ MeV}
\end{align*}
\]
are sufficiently close to the predicted values of $M(\Lambda_b^{*}[1/2,3/2])$ that the decays $\Lambda_b^{*}[1/2,3/2] \to \Sigma_b^+ \pi^+$ are forbidden. The $\Lambda_b^{*}[1/2,3/2]$ should decay directly to $\Lambda_b \pi^+ \pi^-$. 

A similar calculation may be performed for the orbitally-excited $\Xi_b$ states. Here, to a good approximation, one may regard the $[sd]$ diquark in $\Xi_b^-$ or the $[su]$ diquark in $\Xi_b^0$ as having spin zero, so that methods similar to those applied for excited $\Lambda_b$ states should be satisfactory. We find

$$\Delta E_L(\Xi_b) = \Delta E_L(\Xi_c) \left[ \frac{\mu(\Xi_b)}{\mu(\Xi_c)} \right]^p = \Delta E_L(\Xi_c) \left[ \frac{M(\Xi_c) m_b}{M(\Xi_b) m_c} \right]^p = (322 \pm 2) \text{ MeV}.$$  

(40)

Now we use the observed $\Xi_b^-$ mass $[13] \quad M(\Xi_b^-) = (5792.9 \pm 2.5 \pm 1.7) \text{ MeV}$ and our estimate of isospin splitting $M(\Xi_b^-) - M(\Xi_b^0) = 6.4 \pm 1.6 \text{ MeV}$ to predict the isospin-averaged value $M(\Xi_b) = 5790 \pm 3 \text{ MeV}$. We then rescale the fine-structure splitting $[14]$ and find

$$\Xi_b^{*}[3/2] - \Xi_b^{*}[1/2] = \frac{m_c}{m_b} (\Xi_c^{*}[3/2] - \Xi_c^{*}[1/2]) = (9.1 \pm 0.9) \text{ MeV},$$  

(41)

$$M(\Xi_b^{*}[1/2]) = (6106 \pm 4) \text{ MeV}, \quad M(\Xi_b^{*}[3/2]) = (6115 \pm 4) \text{ MeV}.$$  

(42)

The lower state decays to $\Xi_b \pi$ via an S-wave, while the higher state decays to $\Xi_b \pi$ via a D-wave, and hence should be narrower. Decays to $\Xi_b^0 \pi$ and $\Xi_b^+ \pi$ also appear to be just barely allowed, given the values of $M(\Xi_b^0, \Xi_b^+)$ predicted here.

8 $\Omega_b$ mass prediction

Taking the approach implemented in Sec. 3 for the prediction of the $\Xi_b$ mass, the spin averaged mass of $\Omega_b$ can be obtained by extrapolation from available data for $\Omega_c$ and a correction based on strange meson masses, as listed in Table 9.

$$M(\Omega_b) \equiv \frac{2M(\Omega_b^0) + M(\Omega_b)}{3} = \frac{2M(\Omega_c^0) + M(\Omega_c)}{3} + (m_b - m_c)_{B_s - D_s}$$  

(43)

$$= \frac{2M(\Omega_c^0) + M(\Omega_c)}{3} + \frac{3M(B_s^0) + M(B_s)}{4} - \frac{3M(D_s^0) + M(D_s)}{4}$$

$$= 6068.9 \pm 2.4 \text{ MeV}$$

where $M(\bar{X})$ denotes the spin-averaged mass that cancels out the hyperfine interaction between the heavy quark and the diquark containing lighter quarks.

The HF splitting can be estimated as follows:

$$M(\Omega_b^0) - M(\Omega_b) = (M(\Omega_c^0) - M(\Omega_c)) \frac{m_c}{m_b} = (24.0 \pm 0.7) \text{ MeV},$$  

(44)

where we have used the experimental mass difference $[31] \quad M(\Omega_c^0) - M(\Omega_c) = (70.8 \pm 1.0 \pm 1.1) \text{ MeV} = (70.8 \pm 1.5) \text{ MeV}$ with $m_b/m_c$ taken to be $2.95 \pm 0.06$, as discussed in the Appendix. This gives the following mass predictions:

$$\Omega_b^* = (6076.9 \pm 2.4) \text{ MeV}; \quad \Omega_b = (6052.9 \pm 2.4) \text{ MeV}.$$  

(45)
Table 9: Hadron masses used in the calculation of the $\Omega_b$ mass prediction

| Splitting                                  | Value (MeV) |
|--------------------------------------------|-------------|
| $M(\Omega_c)$                             | 2697.5 ± 2.6 |
| $M(\Omega_c^*)$                           | 2768.3 ± 3.0 |
| $M(\Omega_c^*) - M(\Omega_c)$            | 70.8 ± 1.5  |
| $M(D_s)$                                   | 1968.49 ± 0.34 |
| $M(D_s^*)$                                 | 2112.3 ± 0.5 |
| $M(B_s)$                                   | 5366.1 ± 0.6 |
| $M(B_s^*)$                                 | 5412.0 ± 1.2 |
| $M(B_s^*) - M(B_s)$                       | 45.9 ± 1.2  |
| $M(\Xi_c^0)$                              | 2471.0 ± 0.4 |
| $M(\Xi_b^0)$                              | 5792.9 ± 3.0 |

Taking into account the wavefunction correction as described in [13], one must add the following correction to the spin averaged mass:

$$v\left[\frac{\langle \delta(r_{ss})\rangle_{\Omega_b}}{m_s^2} - \frac{\langle \delta(r_{ss})\rangle_{\Omega_c}}{m_s^2}\right] = v\left[\frac{\langle \delta(r_{ss})\rangle_{\Omega_b}}{m_s^2}\right] - 1 \approx (50 ± 10)\left[\frac{\langle \delta(r_{ss})\rangle_{\Omega_b}}{\langle \delta(r_{ss})\rangle_{\Omega_c}} - 1\right] = (2.0 ± 1.1) \text{MeV}$$ (46)

where the contact probability ratio was computed using variational methods

$$\frac{\langle \delta(r_{ss})\rangle_{\Omega_b}}{\langle \delta(r_{ss})\rangle_{\Omega_c}} = 1.04 ± 0.02$$, (47)

and we used the following calculation to evaluate the strength of the $ss$ HF interaction:

$$50 \text{ MeV} \approx M(\Omega) + \frac{1}{4}(2M(\Xi_c^0) + M(\Xi_b^0) + M(\Xi_c)) - \frac{1}{3}(2M(\Xi^*) + M(\Xi)) - \frac{1}{3}(2M(\Omega_c^*) + M(\Omega_c)) =$$

$$= \left(3m_s + 3v\frac{\langle \delta(r_{ss})\rangle_{\Omega}}{m_s^2}\right) + \left(m_u + m_s + m_c\right)$$

$$- \left(2m_s + m_u + v\frac{\langle \delta(r_{ss})\rangle_{\Xi}}{m_s^2}\right) - \left(2m_s + m_c + v\frac{\langle \delta(r_{ss})\rangle_{\Omega}}{m_s^2}\right)$$

$$\approx v\frac{\langle \delta(r_{ss})\rangle_{\Omega}}{m_s^2}$$ (48)

An alternate derivation of the $\Omega_b$ mass from the $\Xi_b - \Xi_c$ mass difference

Thanks to new measurements of the $\Xi_b^0$ mass [12,13], we now have another way to estimate the spin-averaged $\Omega_b$ mass. Following the approach in previous sections, the
The mass difference can be schematically written as
\[
M(\Xi_b^-) - M(\Xi_c^0) = (m_b - m_c) + \text{(wavefunction correction)} + \text{(EM correction)}
\]
\[
= (m_b - m_c) + (-4 \pm 4) \text{ MeV} + (V_{bsd}^{EM} - V_{csd}^{EM})
\]
where the value of the wave function correction is calculated as described in Sec. 3, and the last term denotes the EM interactions of the relevant quarks.

Similarly, the spin-averaged \(\Omega_b - \Omega_c\) mass difference can be written as
\[
M(\tilde{\Omega}_b) - M(\tilde{\Omega}_c) = (m_b - m_c) + \text{(wavefunction correction)} + \text{(EM correction)}
\]
\[
= (m_b - m_c) + (2.0 \pm 1.1) \text{ MeV} + (V_{bs}^{EM} - V_{cs}^{EM})
\]
where the wave-function correction is given in Eq. (46).

Since the \(b\) and \(s\) quarks have the same charge, the EM contribution \(V_{bs}^{EM} - V_{cs}^{EM}\) to the \(\Omega_b - \Omega_c\) mass difference is almost the same as the EM contribution \(V_{bsd}^{EM} - V_{csd}^{EM}\) to the \(\Xi_b^- - \Xi_c^0\) mass difference, modulo a negligible correction from the change in the mean radius of the relevant baryons. We then immediately obtain
\[
M(\tilde{\Omega}_b) - M(\tilde{\Omega}_c) = M(\Xi_b^-) - M(\Xi_c^0) + (6.0 \pm 4.1) \text{ MeV}
\]
which leads to
\[
M(\tilde{\Omega}_b) = 6072.6 \pm 5.6 \text{ MeV}
\]
(52)
which leads to
\[
M(\tilde{\Omega}_b) = 6072.6 \pm 5.6 \text{ MeV}
\]
(52)
and
\[
M(\tilde{\Omega}_b) = 6070.9 \pm 2.7 \text{ MeV}
\]
from Eqs. (43) and (46).

The consistency of these two estimates, based on different experimental inputs, is a strong indication that both the central values and the error estimates are reliable. Moreover, the estimate in Eq. (52) includes EM corrections, while the estimate Eqs. (43) does not, thus indicating that the EM corrections are likely to be smaller than our error estimate. Consequently, in the following we use the estimate (52).
An alternative derivation of HF splitting from effective supersymmetry

An alternative approach to estimate the HF splitting is to use the effective meson-baryon supersymmetry discussed in [8] and apply it to the case of hadrons related by changing a strange antiquark $\bar{s}$ to a doubly strange $ss$ diquark coupled to spin $S = 1$:

$$
\frac{M(\Omega^*_b) - M(\Omega_b)}{M(B^*_s) - M(B_s)} = \frac{M(\Omega^*_c) - M(\Omega_c)}{M(D^*_s) - M(D_s)} = \frac{M(\Xi^*_b) - M(\Xi)}{M(K^*) - M(K)}
$$

(56)

$$
\approx 0.49 \pm 0.01 \approx 0.54
$$

$$
\Omega^*_b - \Omega_b = (B^*_s - B_s)(0.52 \pm 0.02) = 23.9 \pm 1.1 \text{ MeV}
$$

(57)

This gives

$$
\Omega^*_b = 6080.6 \pm 5.6 \text{ MeV}; \quad \Omega_b = 6056.7 \pm 5.6 \text{ MeV}.
$$

(58)

9 Comparisons with other approaches

We begin by comparing $M(\Sigma^*_b) - M(\Sigma_b) = 20.0 \pm 0.3$ MeV as predicted in Sec. 2 with other predictions and data. The first of Refs. [4] finds $M(\Sigma^*_b) - M(\Sigma_b) = 23.8$ MeV, the second finds 15.8 MeV, Ref. [6] finds 29 MeV, and Ref. [9] finds 26 $\pm$ 1 MeV. The experimental value is $21.2^{+0.9}_{-1.9}$ MeV [2]. A recent analysis [11] uses $M(\Sigma^*_b) - M(\Sigma_b)$ as an input to a sum rule

$$
M(\Sigma^*_b) - M(\Sigma_b) - 2[M(\Xi^*_b) - M(\Xi_b)] + M(\Omega^*_b) - M(\Omega_b) = \pm 0.28 \text{ MeV}.
$$

(59)

Our predictions entail a value of $-7 \pm 12$ MeV for the right hand side. The deviation between these two predictions is significant because they arise from a difference in the sign between the SU(3) breaking contributions.

The sign in our prediction

$$
M(\Sigma^*_b) - M(\Sigma_b) < M(\Omega^*_b) - M(\Omega_b)
$$

(60)

appears to be counterintuitive, since the color hyperfine interaction is inversely proportional to the quark mass. The expectation value of the interaction with the same wave function for $\Sigma_b$ and $\Omega_b$ violates our inequality. When wave function effects are included, the inequality is still violated if the potential is linear, but is satisfied in predictions which use the Cornell potential [15].

This reversed inequality is not predicted by other recent approaches [6, 10, 11] which all predict an $\Omega_b$ splitting smaller than a $\Sigma_b$ splitting.

However the reversed inequality is also seen in the corresponding charm experimental data,

$$
M(\Sigma^*_c) - M(\Sigma_c) < M(\Omega^*_c) - M(\Omega_c)
$$

$$
64.3 \pm 0.5 \text{ MeV} \quad 70.8 \pm 1.5 \text{ MeV}
$$

(61)
This suggests that the sign of the \( SU(3) \) symmetry breaking gives information about the form of the potential. It is of interest to follow this clue theoretically and experimentally.

We compare our results with some other recent approaches [6, 10, 11] and with data in Table I. The results of Ref. [9], based on Heavy Quark Effective Theory and QCD sum rules, typically carry \( \pm 80 \) MeV errors so we omit them here. We also take note of a very recent set of predictions which differ substantially from those in Table I [32]. The main difference between our predictions for \( \Xi_b \) and \( \Omega_b \) states and other recent ones [6, 9, 10, 11] is the use of masses of hadrons containing strange quarks to obtain the quark mass difference \( m_b - m_c \). We also take into account wave function corrections, particularly important for the hyperfine splitting between \( \Omega_b^{*} \) and \( \Omega_b \).

10 Summary

We have predicted the masses of several baryons containing \( b \) quarks, using descriptions of the color hyperfine interaction which have proved successful for earlier predictions. Correcting for wave function effects, we have shown that predictions for \( M(\Xi_b) \) based on the masses of \( \Xi_c, \Xi_c', \) and \( \Xi_c^* \) lie in the range of 5790 to 5800 MeV, depending on how \( m_b - m_c \) is estimated. Wave function differences tend to affect these predictions by only a few MeV. The spin-averaged mass of the states \( \Xi_b' \) and \( \Xi_b^* \) is predicted to lie around 150 to 160 MeV above \( M(\Xi_b) \), while the hyperfine splitting between \( \Xi_b \) and \( \Xi_b^* \) is predicted to lie in the rough range of 20 to 30 MeV.

We have evaluated the isospin splitting of the \( \Xi_b \) states and find \( \Delta I(\Xi_b) \equiv M(\Xi_b^1) - M(\Xi_b^0) = 6.24 \pm 0.21 \) MeV on the basis of an extrapolation from the \( \Xi \) and \( \Xi^* \) states. This value is consistent with one which includes information from the \( \Xi_c \) states, \( \Delta I(\Xi_b) = 6.4 \pm 1.6 \) MeV.

We predict \( M(\Omega_b) = 6052.1 \pm 5.6 \) MeV and \( M(\Omega_b^*) = 6082.8 \pm 5.6 \) MeV. These values differ from some others which have appeared in recent literature because we use hadrons containing strange quarks to evaluate the effective \( b - c \) mass difference, include electromagnetic contributions, and employ different hyperfine splittings.

We have also evaluated the orbital excitation energy for \( \Lambda_b \) and \( \Xi_b \) states in which the light diquark \( (ud \text{ or } us) \) remains in a state of \( L = S = 0 \). Precise predictions have been given for the masses of the states \( \Lambda_b^{*[1/2,3/2]} \) and \( \Xi_b^{*[1/2,3/2]} \).

We look forward to tests of some of the predictions summarized in Table I in experiments at the Fermilab Tevatron and the CERN Large Hadron Collider.

Acknowledgements

J.L.R. wishes to acknowledge the hospitality of Tel Aviv University during the early stages of this investigation. Part of this work was performed while J.L.R. was at the Aspen Center for Physics. We thank Dmitry Litvintsev for providing his figure comparing theoretical predictions with measurements of the \( \Xi_b^0 \) mass. This research was supported in part by a grant from Israel Science Foundation administered by
Table 10: Comparison of predictions for $b$ baryons with those of some other recent approaches [6, 10, 11] and with experiment. Masses quoted are isospin averages unless otherwise noted. Our predictions are those based on the Cornell potential.

| Quantity                  | Refs. [6] | Ref. [10] | Ref. [11] | This work     | Experiment     |
|---------------------------|-----------|-----------|-----------|----------------|----------------|
| $M(\Lambda_b)$           | 5622      | 5612      | Input     | Input          | $5619.7\pm1.7$ |
| $M(\Sigma_b)$            | 5805      | 5833      | Input     | –              | $5811.5\pm2$   |
| $M(\Sigma_b^*)$          | 5834      | 5858      | Input     | –              | $5832.7\pm2$   |
| $M(\Sigma_b^*) - M(\Sigma_b)$ | 29     | 25        | Input     | $20.0\pm0.3$  | $21.2^{+2.2}_{-2.1}$ |
| $M(\Xi_b)$               | 5812      | 5806$^a$  | Input     | $5790-5800$   | $5792.9\pm3.0^b$ |
| $M(\Xi_b^*)$             | 5937      | 5970$^a$  | 5929.7$\pm4.4$ | 5930$\pm5$   | –              |
| $\Delta M(\Xi^b)^c$      | –         | –         | –         | 6.4$\pm1.6$   | –              |
| $M(\Xi_b^*) - M(\Xi_b)$  | 5963      | 5980$^a$  | 5950.3$\pm4.2$ | 5959$\pm4$   | –              |
| $M(\Omega_b)$            | 6065      | 6081      | 6039.1$\pm8.3$ | 6052.1$\pm5.6$ | –              |
| $M(\Omega_b^*)$          | 6088      | 6102      | 6058.9$\pm8.1$ | 6082.8$\pm5.6$ | –              |
| $M(\Omega_b^*) - M(\Omega_b)$ | 23    | 21        | 19.8$\pm3.1$ | 30.7$\pm1.3$ | –              |
| $M(\Lambda_b^{*[1/2]})$  | 5930      | 5939      | –         | 5929$\pm2$    | –              |
| $M(\Lambda_b^{*[3/2]})$  | 5947      | 5941      | –         | 5940$\pm2$    | –              |
| $M(\Xi_b^{*[1/2]})$      | 6119      | 6090      | –         | 6106$\pm4$    | –              |
| $M(\Xi_b^{*[3/2]})$      | 6130      | 6093      | –         | 6115$\pm4$    | –              |

$^a$Value with configuration mixing taken into account; slightly higher without mixing.
$^b$CDF [13] value of $M(\Xi_b^*)$.
$^c$M(state with $d$ quark) – M(state with $u$ quark).
Israel Academy of Science and Humanities. The research of H.J.L. was supported in part by the U.S. Department of Energy, Division of High Energy Physics, Contract DE-AC02-06CH11357. The work of J.L.R. was supported by the U.S. Department of Energy, Division of High Energy Physics, Grant No. DE-FG02-90ER40560.

References

[1] D. Acosta et al. [CDF Collaboration], Phys. Rev. Lett. 96 (2006) 202001.
[2] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 99 (2007) 202001.
[3] E. Bagan, M. Chabab, H. G. Dosch and S. Narison, Phys. Lett. B 278 (1992) 367; B 287 (1992) 176; S. Capstick and N. Isgur, Phys. Rev. D 34 (1986) 2809; R. Roncaglia, D. B. Lichtenberg and E. Predazzi, Phys. Rev. D 52 (1995) 1722; N. Mathur, R. Lewis and R. M. Woloshyn, Phys. Rev. D 66 (2002) 014502; B. Silvestre-Brac, Few Body Syst. 20 (1996) 1; C. Albertus, J. E. Amaro, E. Hernandez and J. Nieves, Nucl. Phys. A 740 (2004) 333.
[4] E. Jenkins, Phys. Rev. D 54 (1996) 4515; ibid. 55 (1997) 10.
[5] M. Karliner and H. J. Lipkin, arXiv:hep-ph/0307243 (unpublished).
[6] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D 72 (2005) 034026; Phys. Lett. B 659 (2008) 612.
[7] J. L. Rosner, Phys. Rev. D 75 (2007) 013009.
[8] M. Karliner and H. J. Lipkin, Phys. Lett. B 660 (2008) 539.
[9] X. Liu, H. X. Chen, Y. R. Liu, A. Hosaka and S. L. Zhu, Phys. Rev. D 77 (2008) 014031.
[10] W. Roberts and M. Pervin, arXiv:0711.2492 [nucl-th].
[11] E. E. Jenkins, Phys. Rev. D 77 (2008) 034012.
[12] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 99 (2007) 052001.
[13] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 99 (2007) 052002.
[14] M. Karliner, B. Keren-Zur, H.J. Lipkin and J.L. Rosner, hep-ph/0706.2163v1.
[15] B. Keren-Zur, Annals Phys. 323 (2008) 631.
[16] D. B. Lichtenberg, J. Phys. G 16, 1599 (1990); J. Phys. G 19 (1993) 1257; D. B. Lichtenberg, R. Roncaglia and E. Predazzi, J. Phys. G 23 (1997) 865.
[17] M. Karliner, B. Keren-Zur, H. J. Lipkin and J. L. Rosner, arXiv:0708.4027 [hep-ph].
[18] W.-M. Yao et al., Journal of Physics G 33 (2006) 1.

[19] W. Kwong, P. B. Mackenzie, R. Rosenfeld and J. L. Rosner, Phys. Rev. D 37 (1988) 3210.

[20] E. Eichten et al., Phys. Rev. D 21, 203-233 (1980).

[21] D. Litvintsev, on behalf of the CDF Collaboration, seminar at Fermilab, June 15, 2007, http://theory.fnal.gov/jetp/talks/litvintsev.pdf.

[22] M. J. Savage, Phys. Lett. B 359 (1995) 189.

[23] K. Maltman and N. Isgur, Phys. Rev. D 22, 1701 (1980).

[24] H. J. Lipkin, unpublished.

[25] J. L. Rosner, Prog. Theor. Phys. 66, 1422 (1981).

[26] J. L. Rosner and M. P. Worah, Phys. Rev. D 46, 1131 (1992).

[27] S. Gasiorowicz and J. L. Rosner, Am. J. Phys. 49, 954 (1981).

[28] J. L. Rosner, Phys. Rev. D 57, 4310 (1998).

[29] Particle Data Group,
   http://pdg.lbl.gov/2007/listings/contents_listings.html

[30] C. Quigg and J. L. Rosner, Phys. Lett. B 71, 153 (1977).

[31] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 97, 232001 (2006).

[32] S. M. Gerasyuta and E. E. Matskevich, arXiv:0803.3497 [hep-ph].