The influence of crystal anisotropy on the critical state stability and flux jump dynamics of a single crystal of La$_{1.85}$Sr$_{0.15}$CuO$_4$

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Abstract

We studied the critical state stability of a large cubic sample of single-crystalline La$_{1.85}$Sr$_{0.15}$CuO$_4$ for different sample orientations with respect to the external magnetic field as well as the dynamics of the flux jumps. It is shown that thermomagnetic avalanches develop under dynamic conditions, which are characterized by the magnetic diffusivity being significantly lower than the thermal case. In this case, the critical state stability depends strongly on the cooling conditions. We compared predictions from the isothermal model and from the model for a weakly cooled sample with experimental results. In both models, the field of the first flux jump decreases with increase of the sweep rate of the external magnetic field. We also investigated the influence of the external magnetic field on the dynamics of the following stages of the thermomagnetic avalanche. It is shown that the dynamics of the flux jumps is correlated with the magnetic diffusivity, which is proportional to the flux flow resistivity.

1. Introduction

Under certain conditions, the critical state of a type-II superconductor may become unstable [1]; then a thermomagnetic avalanche may develop. During the thermomagnetic avalanche, the temperature of the superconducting sample increases rapidly and a large amount of magnetic flux enters into the sample volume. Thermomagnetic avalanches, called flux jumps, are problematic from the viewpoint of practical application of superconductors, because they may drive a superconducting sample into a normal or, at least, a resistive state.

The stability of the critical state depends strongly on the relation between the thermal ($D_\ell$) and magnetic ($D_m$) diffusivities, $D_\ell = \frac{\kappa}{c_V}$, where $\kappa$ is the thermal conductivity, and $c_V$ is the specific heat at constant volume. $D_m = \frac{\rho_n}{\mu_0}$. In the flux flow regime, $\rho = \rho_n = \rho_n(\frac{H}{H_{c2}})$, where $\rho_n$ is the flux flow resistivity, $\rho_n$ is the normal state resistivity, $H_{c2}$ is the second critical field and $\mu_0$ is the magnetic permeability of vacuum. In the case of hard conventional superconductors, usually $\tau = \frac{D_\ell}{D_m} \ll 1$. Such conditions are called ‘locally adiabatic’. The most commonly studied parameter, which determines the stability of the critical state, is the field for the first (after cooling the sample in zero magnetic field) flux jump, $H_{fj1}$. This parameter is closely correlated with the critical penetration depth, $L_c$. If the critical current density, $J_c$, does not depend on the magnetic field, $L_c = \frac{H_{fj1}}{J_c}$. This parameter is very important from the viewpoint of the practical application of superconductors, because it is...
possible to avoid thermomagnetic avalanches if the width of the superconducting sample is smaller than 2\(L_c\). We can also express the condition for the stability of the critical state in the form \(H_{\text{fj1}} > H_p\), where \(H_p\) is the field of full penetration, \(H_p = j \kappa \alpha\), and \(\alpha\) is half-width of the sample investigated. In the simplest approximations, the sample is usually assumed to have the shape of an infinite slab, of width 2\(a\), and an external magnetic field is applied parallel to its surface. In locally adiabatic conditions, the field of the first flux jump can be expressed using the formula

\[
H_{\text{fj1}} = \sqrt{\frac{2\nu j_c}{\mu_0 |\partial |}}.
\]

(1)

The thermal parameter, which determines the critical state stability for locally adiabatic conditions, is the specific heat \(c_V\). In locally adiabatic conditions, the stability of the critical state is not influenced by the cooling conditions at the surface of the superconducting sample.

The situation is different in the case where \(\tau \gg 1\). Such conditions usually occur in superconducting composites and are called ‘dynamic’. In dynamic conditions, the thermal parameter which determines the stability of the critical state is the thermal conductivity, \(\kappa\), or the thermal boundary conductivity, \(h\) [2].

Some problems connected with the critical state stability of high temperature superconductors (HTSC) have not been clarified yet. The problems are connected with the complex structure of these materials. The analysis of the critical state stability of HTSC is usually performed within approximations developed for conventional superconductors [1], which sometimes lead to wrong results. Because of the increasing number of applications of HTSC, it is very important to understand all aspects of the thermomagnetic avalanche development in these materials. One of the specific features of HTSC is their strong crystallographic anisotropy.

Experimental data show that high temperature superconductors are usually more stable against flux jumping than conventional superconductors. The larger stability of the high temperature superconductors against flux jumping can be correlated with the flux creep phenomenon, which is usually strong in these materials [3, 4]. In studies of the magnetization of superconductors, flux creep manifests itself as a relaxation of the magnetic moment. The process of magnetic relaxation strongly reflects nonlinear current–voltage characteristics of the superconducting sample. These characteristics can be approximated using the formula

\[
j(E) = j_c + \frac{j_c}{n} \ln \left( \frac{E}{E_0} \right),
\]

(2)

where \(E_0\) is the voltage criterion at which the critical current density, \(j_c\), is defined, and \(n\) is a dimensionless parameter. One usually assumes that \(E_0 = 10^{-5} \text{ V m}^{-1}\). In this case, \(n \gg 1\).

The influence of nonlinear current–voltage characteristics on the critical state stability, in the case of a weakly cooled sample (for \(Bi = \frac{\delta T}{T} \ll 1\); \(Bi\) is the so-called Biot number), was analyzed in [5]. It was shown that in such conditions the critical state stability depends on the external magnetic sweep rate, \(\frac{dH_{\text{ext}}}{dt}\):

\[
H_{\text{fj1}}^n = j_c \sqrt{\frac{2\mu_0 h}{|\partial | \frac{dH_{\text{ext}}}{dt}}}.
\]

(3)

In previous works [6, 7], we studied the influence of the external magnetic sweep rate on the critical state stability for crystals of a high temperature Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\) superconductor. It is extremely difficult to obtain thick crystals of Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\), and we have shown [7] that the thickness of the crystal strongly influences the critical state stability. On the other hand, it is relatively easy to obtain large cubic samples of La\(_{1.85}\)Sr\(_{0.15}\)Cu\(_4\). For this reason, we have chosen this system for studying the influence of crystal anisotropy on the critical state stability.

A strong influence of the cooling condition and a role of crystal anisotropy for the critical state stability in the case of textured YBaCuO were considered in [8, 9]. The effects of thermal insulation on the critical state stability of conventional NbTi superconductor were investigated in [10].

We paid special attention to the dynamics of the flux jumps, which remain poorly understood to date. These dynamics are also important from the viewpoint of superconductor applications, because the large power of the heat released during short avalanches may, under certain conditions, lead to the destruction of superconducting devices. In the present work, we also studied the dynamics of the magnetic flux changes in a single crystal of La\(_{1.85}\)Sr\(_{0.15}\)Cu\(_4\) during thermomagnetic avalanches. In particular, we focused on the influence of the external magnetic field on the dynamics of the following stages of the thermomagnetic avalanche.

2. Experiment

In our investigations, we used a large 3 \times 3 \times 3 \text{ mm}^3 single crystal of La\(_{1.85}\)Sr\(_{0.15}\)Cu\(_4\), obtained by the floating zone technique. In order to study the influence of the external magnetic sweep rate on the critical state stability, we used a Quantum Design PPMS system with the extraction magnetometer option and a maximal external magnetic field attainable of 9 T. After cooling the sample in zero magnetic field, the external magnetic field was swept with constant rate from 0 to 9 T. In the following experiments, we varied the sweep rate from the minimal (10\(^{-3}\) \text{ T s}^{-1}\) to the maximal (2 \times 10\(^{-2}\) \text{ T s}^{-1}\) one attainable in our system. The distance between the following data points taken in each magnetization curve was limited by the sweep rate and the time needed to take each data point. At the maximal sweep rate (2 \times 10\(^{-2}\) \text{ T s}^{-1}\), we were not able to obtain a distance smaller than about 0.1 T. This distance has limited the accuracy of \(H_{\text{fj1}}\) determined in our experiments.

In order to determine the current–voltage characteristics of our sample, we performed studies of the magnetic moment relaxation. After cooling in zero external magnetic field, an external magnetic field was first swept, with minimal sweep rate, up to 0.5 T, and then the relaxation of the magnetic
Figure 1. Virgin magnetization curves taken using the extraction magnetometer (PPMS) with the sweep rate of $2 \times 10^{-2}$ T s$^{-1}$ at different temperatures for both sample orientations studied in our experiments.

moment was registered under constant external magnetic field for approximately 1 h.

The studies of $H_{fj1}$, as well as of the magnetic relaxation, were performed both for the external magnetic field parallel to the $c$-axis and for the external magnetic field parallel to the $ab$-plane.

The dynamics of the flux jumps was studied using the Cryogenics 12 T magnet system with variable temperature insert for external magnetic field parallel to the $c$-axis. The pick-up coil consisted of six turns of copper wire wound around the sample investigated and connected to a data acquisition board (DAQ) in the computer. Because of the small inductance of the applied coil and the large internal resistance of the DAQ, we were able to register the changes of the voltage with time resolution better that $10^{-6}$ s. The time resolution of the DAQ was about $10^{-7}$ s. We registered the time dependence of the coil voltage during the following flux jumps. Additionally, we used two miniature Hall probes to register the magnetic field dependence of the surface self-field, $H_{\text{self-surf}}$, of the sample investigated. One of the probes was put in the center of the surface of the sample investigated. The second probe measured the external magnetic field. The differential signal from the two probes was proportional to $H_{\text{self-surf}}$. The cryogenic Hall probes were made from tin doped InSb films. The probe was supplied with the current of 10 mA. The sensitivity of the Hall probe was about 5 mV T$^{-1}$.

The rate of external magnetic field sweep was approximately 1 T min$^{-1}$. We also performed magnetization measurements in the 12 T magnet system, using a vibrating sample magnetometer (VSM).

It is important to note that the heat exchange conditions in the two systems used in our experiments were very different.

In the PPMS system, the sample was put into a standard PPMS holder, which was surrounded by helium gas with low pressure (about 0.5 Torr). In the 12 T magnet system, during the experiments at 2 and 4.2 K, the sample was immersed in liquid helium. During experiments at higher temperatures, the sample was surrounded by flowing helium gas with approximately atmospheric pressure. The sample was either glued to a quartz plate (for studies of the flux jump dynamics) or put into a Teflon holder (for the VSM).

3. Results

The range of flux jump occurrence, the field of the first flux jump and the distance between the following flux jumps depend on the sample orientation, temperature, sweep rate and experimental system. Figure 1 presents some of the virgin magnetization curves taken using the extraction magnetometer (PPMS) with the sweep rate of $2 \times 10^{-2}$ T s$^{-1}$ at different temperatures for the two sample orientations studied in our experiments. Magnetization curves on the left side in figure 1 are cut off for signal amplitude above 20 emu, because of the limitation of the signal in the extraction magnetometer. Nevertheless, in all our PPMS experiments, we were able to determine the value of $H_{fj1}$. The temperature range of the flux jump occurrence depends on the sample orientation. For the external magnetic field parallel to the $c$-axis, flux jumps occur even at 9.5 K (see figure 1(c)). For the external magnetic field parallel to the $ab$-plane, flux jumps are disappearing already at 5 K (see figure 1(f)). With the increase of temperature, we observed an increase of $H_{fj1}$ and of the distance between the following flux jumps. We also observed a decrease of $H_{fj1}$ with increase of the external magnetic field.
sweep rate (see figure 2). Hence, we observed the flux jumps in the largest temperature range, for the highest sweep rate ($2 \times 10^{-2}$ T s$^{-1}$). At the lowest sweep rate, $10^{-3}$ T s$^{-1}$, we did not observe any flux jumps at temperatures higher than 4 K for the external magnetic field parallel to the c-axis, or higher than 2.2 K for the external magnetic field parallel to the ab-plane. Figure 2 shows the changes of $H_{fj1}$ with the sweep rate at different temperatures and for different sample orientations—the data were taken from PPMS experiments. The results are shown on a double-logarithmic scale. If $H_{fj1}$ changes with the sweep rate according to the power law, we expect a straight line in such a plot. Dashed lines show the predictions of equation (3), where $H_{fj1}$ is inversely proportional to the square root of the sweep rate.

The experiments using the 12 T magnet (with Hall probes or a VSM) were all performed with the external magnetic field sweep rate approximately the same as the highest sweep rate in the PPMS system. However, we observed here the flux jumps only at the lowest temperature of about 2.2 K and for the external magnetic field parallel to the c-axis (see figure 3(a)—the experiment with the Hall probes). No jumps were present at the temperature of 4.2 K or at higher temperatures.

In figures 3(b) and (c), one can see the time dependence of the voltage per turn of the coil wound around the sample investigated, registered during the succeeding flux jumps. The numbers on the curves correspond to the numbers denoting successive flux jumps presented in figure 3(a). The typical duration of the flux jumps was several microseconds. The dependence shown in figure 3(b) corresponds to the lower (ascending) branch of the hysteresis loop. The dependence shown in figure 3(c) corresponds to the upper (descending) branch of the hysteresis loop, shown in figure 3(a). Before and after each flux jump, the voltage of the coil was close to zero. However, in order to present all the data in one figure, the initial voltages of the following flux jumps were shifted. The voltage of the coil is proportional to the time derivative of the magnetic flux in the sample investigated. We can recognize two characteristic stages of the flux jump. During the first stage, the rate of the magnetic flux changes increases. During the second stage, the rate of the magnetic flux changes decreases. We have denoted the durations of these two stages as $t_{bm}$ (‘before maximum’) and $t_{am}$ (‘after maximum’), respectively (see figure 3(b)). We have determined the duration of the following stages of the flux jump at a noise level, which in our system was about 30 mV. One can see that the voltage of the observed impulses for the ascending branch of the hysteresis loop (figure 3(b)) has the opposite sign to those for the descending branch (figure 3(c)). This is because, for the ascending branch, the magnetic flux enters into the sample investigated during the succeeding flux jumps, while for the descending branch, we observe flux exit.

4. Analysis

4.1. The current–voltage characteristics

In order to compare the models of the critical state stability with our experimental results, it is necessary to know the current–voltage characteristics of the sample investigated. These characteristics can be determined using the data for the magnetic moment relaxation. In order to obtain these characteristics, we applied the following procedure. We used the model of an infinite slab sample. Additionally, we assumed the screening current density to be the same for the whole sample volume penetrated by the magnetic flux.

The time dependent screening current density can be in this case correlated with the magnetization. The electric field at the surface of the slab is proportional to the time derivative of the screening current density. Treating the time as a parameter, one can obtain the current–voltage dependence, $j(E)$.

The current–voltage characteristics obtained by the procedure described above were fitted according to equation (2). We applied this procedure for both sample orientations studied in our experiments. Our fitting procedure gave critical current densities of the order of $10^5$ A m$^{-2}$ and $10^6$ A m$^{-2}$ and $n$-parameters (see equation (2)) of about 40 and 30 for the external magnetic field parallel to the ab-plane and parallel
Our fitting procedure was performed only over the range of current density in the direction perpendicular to the c-axis, respectively. In the range of the flux jump occurrence, the \( n \)-parameter was only weakly dependent on temperature. Hence, we did not consider this dependence in our further analysis.

The temperature dependence of the critical current density was estimated using the widths of the magnetization hysteresis loops in the external magnetic field of 1 T, which was close to the field of the first flux jump, and the formulas from [12]. We used here the loops taken using the VSM in the 12 T magnet system. Using the VSM system, we did not observe any flux jumps either at 4.2 K or at higher temperatures, and there was no limitation of the signal above 20 emu like for the PPMS extraction magnetometer. The limitation of the signal as well as the large number of flux jumps observed in the PPMS experiments made determination of the critical current density (using PPMS data) impossible.

The experimentally obtained temperature dependence of the critical current density was fitted according to the exponential formula: \( j_c(T) = j_{c0} \exp(-\frac{T}{T_c}) \), where \( T^*_c = \frac{T}{T_c} \), 
\( g(T) = 1 - \left(\frac{T}{T_c}\right)^2 \) and \( T_c = 35.5 \) K. We have found the following fitting parameters. For the external magnetic field parallel to the c-axis, \( j_{c0} = 5.6 \times 10^5 \) A m\(^{-2}\) and \( T_0 = 7.4 \) K (the critical current density in the direction parallel to the c-axis). For the external magnetic field parallel to the ab-plane, \( j_{c0} = 7.3 \times 10^8 \) A m\(^{-2}\) and \( T_0 = 4.3 \) K (the critical current density in the direction perpendicular to the ab-plane). Our fitting procedure was performed only over the range 4–15 K, which was the most interesting from the viewpoint of flux jump occurrence.

4.2. The \( \tau \)-parameter and the Biot number

In order to determine the conditions of the critical state stability and the thermomagnetic avalanche development, it is necessary to find the relation between the thermal and magnetic diffusivities or the parameter \( \tau = \frac{D_{\text{th}}}{\mu_0 \kappa_c} = \frac{\sigma}{c_V} \). In the case of the flux creep model, \( \sigma = \frac{\kappa_{\text{th}}}{\kappa_{\text{in}}} \). In the case of an infinite slab model, the electric field at the surface of the slab is

\[
E = \mu_0 \alpha H - \frac{\partial H}{\partial t} \quad \text{for} \ H < H_p \quad \text{and} \quad E = \mu_0 \alpha \frac{\partial H}{\partial t} \quad \text{for} \ H > H_p.
\]

The maximal value of the electric field which can be, under given conditions, induced in the superconducting sample is \( E_{\text{max}} = \mu_0 \alpha \frac{H_{\text{eff}}}{\partial t} \). Using the maximal value of the electric field we can calculate the minimal value of \( \tau \), \( \tau_{\text{min}} = \frac{\kappa_{\text{in}}}{\kappa_{\text{in}} \alpha} \).

In our analysis, we used the temperature dependence of the thermal parameters \( \kappa \) and \( c_V \) from [13] and [14], respectively.

In all our calculations, we used the value of the in-plane thermal conductivity, which is approximately an order of magnitude higher than the out-of-plane thermal conductivity [13]. As a result, the ability of the sample investigated to remove thermal fluctuations is limited by the in-plane thermal conductivity.

We have found that in the case of all our experiments, \( \tau > \tau_{\text{min}} > 10^2 \gg 1 \). Hence, we can assume dynamic conditions for thermomagnetic avalanche development.
We expect the critical state stability for dynamic conditions to be dependent on the cooling conditions. This is consistent with the results of our experiments. In order to analyze this problem, we calculated the Biot number, $Bi = \frac{ah}{c}$. The Biot number depends on the thermal boundary conductivity, $h$. This parameter is very difficult to evaluate experimentally [2]. For the sample immersed in liquid helium at normal pressure and in the absence of a boiling crisis (12 T magnet experiments), we expect $h$ to be of the order of $10^4$ W m$^{-2}$ K$^{-1}$ [1].

In the case of PPMS experiments, we expect this parameter to be several orders of magnitude lower. The criteria for the critical state stability can be easily derived for two limiting cases: (1) $Bi \ll 1$—a weakly cooled sample (this case was considered in [5]); (2) $Bi \gg 1$—so-called isothermal conditions.

We have found that the approximation of the weakly cooled sample cannot be applied in the case of our sample immersed in liquid helium (in this case, $Bi$ ≫ 1), which seems to be reasonable to use this approximation for the case of PPMS experiments, where we expect $Bi$ to be several orders of magnitude lower.

**4.3. The weakly cooled sample—the PPMS experiment**

We assumed the dynamic model for the weakly cooled sample ($Bi \ll 1$) to describe the critical state stability in the case of our experiments performed using the PPMS. Hence, we used equation (3), and our experimental data, to calculate the thermal boundary conductivity, $h$. If the boundary conductivity is governed by phonon processes, we expect this dependence to be described by a power function with a power of about 3 [2]. Hence, we assumed $h = cT^3$. For the highest sweep rate $2 \times 10^{-2}$ T s$^{-1}$, we found, from the fit to experimental data, $c = 0.25$ W m$^{-2}$ K$^{-4}$ and $c = 2.8$ W m$^{-2}$ K$^{-4}$ for the external magnetic field parallel to the $c$-axis and to the $ab$-plane, respectively. At 4.2 K the thermal boundary conductivity calculated according to our procedure was 2–3 orders of magnitude lower than that for the sample immersed in liquid helium at normal pressure. Thus our assumption of a low value of the Biot number seems to be correct.

However, it is necessary to note that equation (3) does not describe precisely the experimentally observed dependence of $H_{c1}$ on the sweep rate (see figure 2). If $H_{c1}$ changes with the sweep rate according to equation (3), we expect the dashed lines in figure 2 to be parallel to the experimental curves. We observed the largest disagreement for the external magnetic field parallel to the $ab$-plane.

**4.4. The critical state stability and flux creep in the isothermal approximation**

In order to derive the critical state stability criterion for isothermal conditions ($Bi \gg 1$), we used the following approximations. Let us assume the infinite slab to be penetrated by the magnetic flux to the depth $d$. To simplify further calculations, we assume the space coordinate $x = d$ at the sample border. According to the arguments presented in [5] for dynamic conditions for an infinite slab model, we can correlate the fluctuation of the electric field $\delta E(x)$ with the fluctuation of the temperature $\delta T(x)$ according to the formula

$$\delta E(x) = \frac{nE(x)}{\kappa} \frac{\partial j_c}{\partial T} \delta T(x). \quad (4)$$

The background electric field, $E(x)$, is induced by the sweep of the external magnetic field: $E(x) = \mu_0 \frac{\partial H_{ext}}{\partial x}$. In the case of a weakly cooled sample ($Bi \ll 1$), one can assume the thermal fluctuation to be independent of the space coordinate. The stability criterion (equation (3)) can be derived in this case from the inequality $\int_0^d \delta E dx < h\delta T(d)$, where the left side of the inequality is the heat generated by the fluctuation in the sample volume and the right side is the heat removed from the sample through its border [5].

We can assume that for isothermal conditions ($Bi \gg 1$), because of the strong cooling of the sample, the fluctuation of the temperature at the sample border will be equal to zero, $\delta T(d) = 0$. In our approximation, we assumed the space distribution of the thermal fluctuation to be described by a square function $\delta T(x) = \delta T_0(1 - \frac{1}{d^2}x^2)$, where $\delta T_0 = \delta T(x = 0)$ is the maximal temperature fluctuation at the penetration depth. The maximal heat power generated in the sample by the fluctuation, which can be removed from the sample volume, is in this case limited by the thermal conductivity, and the critical state stability criterion can be described using the inequality $\delta j_c(x) \leq -\kappa \frac{\partial T}{\partial x} \delta T(x)$. Hence, $\delta j_c(x) = n\mu_0 \frac{\partial H_{ext}}{\partial x} \frac{\partial T}{\partial x} \delta T_0 x(1 - \frac{1}{d^2}x^2) = -\kappa \frac{\partial^2 T}{\partial x^2} \delta T_0(1 - \frac{1}{d^2}x^2) = 2\kappa \delta T_0$. The left side of this inequality has a maximum for $x = \frac{d}{\sqrt{3}}$. As a result, the critical state stability criterion is given by the condition $d \leq L_c = \frac{\sqrt{\frac{\mu_0 k}{\kappa}}} {n \frac{\partial H_{ext}}{\partial x}}$, or

$$H_{c1}^2 = j_c \frac{\gamma \mu_0 k}{n \frac{\partial H_{ext}}{\partial x}}, \quad (5)$$

where $\gamma = \frac{1}{T}(0 + \sqrt{3}) \approx 2.48$. It is important to note that, in the case of the isothermal approximation too, we expect the field of the first flux jump to decrease with increase of the sweep rate, $\frac{\partial H_{ext}}{\partial t}$.

**4.5. Comparison of the theoretical models with the experimental results**

In figure 4, we present the comparison of the predictions from the three theoretical approximations discussed in the present work, i.e. the local adiabatic approximation and the two dynamic approximations (of a weakly cooled sample and isothermal conditions) with the experimentally obtained temperature dependence of $H_{c1}$. The experimental data were taken for both sample orientations studied in our experiments: with the external magnetic field parallel to the $c$-axis (figure 4(a)) or parallel to the $ab$-plane (figure 4(b)), taken at the sweep rate of $2 \times 10^{-2}$ T s$^{-1}$. For the external magnetic field parallel to the $c$-axis, we present the results both for the
Figure 4. Temperature dependence of the field of the first flux jump, $H_{fj1}$, at the sweep rate of $2 \times 10^{-2}$ T s$^{-1}$ and with the external magnetic field parallel to the c-axis (a) and parallel to the ab-plane (b) of the $3 \times 3 \times 3$ mm$^3$ single crystal of La$_{1.85}$Sr$_{0.15}$CuO$_4$. The predictions from the approximated models of the critical state stability—see equations (1)–(3)—are compared with the experimental data. The dashed line shows the temperature dependence of the field of full penetration, $H_p(T)$.

4.6. Dynamics of the flux jumps

The durations of the two characteristic stages of the flux jumps denoted in figure 3(b) as $t_{bm}$ and $t_{am}$ decrease with increase of the external magnetic field, and $t_{bm}$ was approximately three times shorter than $t_{am}$ (see figure 5(a)).

The exponential character of the final stage of the flux jump suggests that it can be analyzed in terms of the magnetic diffusion. The process of the magnetic diffusion depends on the initial magnetic field distribution in the sample. It is possible to decompose this distribution into a series of functions characteristic for the sample geometry (e.g. a series of cosines in the case of an infinite slab geometry and a series of Bessel functions in the case of a sample with cylindrical symmetry). Each harmonic of this decomposition has specific time dependence. From the sum of the time dependent harmonics, one can obtain the time dependent distribution of the magnetic flux in the sample investigated. The voltage of the coil, wound around the sample investigated, is proportional to the time derivative of the magnetic flux. We assumed the sample to have the shape of an infinite slab.

For sufficiently long times in comparison to the parameter

$$t_0 = \frac{4l^2}{\pi^2D_m},$$

we can assume the voltage of the coil to be $U(t) \sim \exp(-t/t_0)$.

The parameter $l$ is a characteristic length of the diffusion process. If the slab is fully penetrated by the magnetic flux, $l = a$. Hence, when one fits the time dependence of the coil
the coefficient voltage at the final stage of the flux jump, one can find the end of the flux jumps. During the flux jumps observed in our experiment, the sample was wholly penetrated by the magnetic flux (see figure 3(a)). Hence, we assumed as the characteristic length of the diffusion process (in equation (6)), at the end of the flux jump, the parameter \( a \); this means half of the sample width. On the other hand, at the beginning of the flux jump, only a part of the sample is penetrated by the magnetic flux. Hence at this stage, one should assume the characteristic length parameter to be equal to the penetration depth, which is smaller than \( a \). We have found the magnetic field dependence of the parameter \( \rho_t \) for the beginning of the flux jump to be similar to that for the end of the jump. These two parameters coincide very well if we assume in equation (6) \( l = 0.5a \) (see figure 5(b)) for the beginning of the flux jump.

The characteristic diffusivity found in our experiment was of the order of \( 1 \text{ m}^2 \text{s}^{-1} \) (see figure 5(b)). Such an order of diffusivity is characteristic for the magnetic diffusion in our system, provided that the sample is in the normal state or in the flux flow regime. The resistivity of \( \text{La}_{0.85}\text{Sr}_{0.15}\text{CuO}_4 \) is anisotropic. However, we studied the dynamics of the flux jumps only for the external magnetic field parallel to the \( c \)-axis of the crystal investigated. For such a sample orientation, screening currents flow only in the \( ab \)-plane. If we assume the in-plane normal state resistivity \( \rho_n > 0.25 \text{ m}\Omega \text{ cm} \) [16], then \( D_m = \rho_n/\mu_0 > 2 \text{ m}^2 \text{s}^{-1} \). On the other hand, if one takes data for the thermal conductivity \( (\kappa) \) [13] and the specific heat \( (c_V) \) [14], one can estimate the in-plane thermal diffusivity \( (D_h = \kappa/c_V) \) to be of the order of \( 10^{-3} - 10^{-2} \text{ m}^2 \text{s}^{-1} \). The out-of-plane thermal diffusivity is approximately an order of magnitude lower than the in-plane thermal diffusivity [13].

5. Discussion

The critical state stability of single-crystalline \( \text{La}_{0.85}\text{Sr}_{0.15}\text{CuO}_4 \) is influenced by a large number of parameters. All of them must be considered in order to determine the critical state stability in the framework of existing theoretical models. For \( \text{La}_{0.85}\text{Sr}_{0.15}\text{CuO}_4 \) crystal, most of these parameters are anisotropic. Hence, the stability of the critical state depends strongly on the sample orientation with respect to the external magnetic field.

In our analysis, we consider both the anisotropy of critical current density and the anisotropy of the current–voltage characteristic (the anisotropy of the \( n \)-parameter). In the present analysis, we assumed the thermal conductivity of the sample investigated to be limited only by the in-plane thermal conductivity, which is approximately one order of magnitude higher than the out-of-plane thermal conductivity. According to our results for the case of \( \text{La}_{0.85}\text{Sr}_{0.15}\text{CuO}_4 \) crystal, the anisotropy of the \( n \)-parameter is relatively weak. Hence, the anisotropy of the critical state stability is connected mainly with the anisotropy of screening currents. The difference between the in-plane and out-of-plane screening currents, at a given temperature, is about one order of magnitude.

In both sample orientations studied in our experiments, the thermomagnetic avalanche develops under dynamic

![Figure 5](image_url)

Figure 5. (a) The magnetic field dependence of the characteristic times \( t_{bm} \) and \( t_{am} \), before and after the maximum of the coil voltage. (b) The magnetic field dependence of the magnetic diffusivity, estimated from the exponential fit of the coil voltage at the beginning and at the end of the flux jumps, respectively.
conditions. Our sample is characterized by a strong flux creep phenomenon. Hence, the critical state stability is influenced both by the external magnetic field sweep rate and the cooling conditions. These cooling conditions were very different in the two cryostats used in our experiments.

In PPMS experiments, where the sample is surrounded by helium gas of low pressure, the critical state stability can be analyzed under the approximation of weak cooling or low Biot number. In such conditions, the thermal parameter that influences the critical state stability is the thermal boundary conductivity. This parameter is difficult to determine experimentally. It depends on, e.g., the roughness of the surface of the sample investigated. In the case of PPMS experiments, the sample investigated is put additionally into a holder, which makes an analysis of the heat exchange conditions even more difficult. Nevertheless, it seems to be slightly surprising that, according to our analysis, the thermal boundary conductivity changes by an order of magnitude upon change of the sample orientation.

It is possible that such results are, to some extent, connected with the approximations assumed in the models used in our analysis. In our analysis we used models of an isotropic infinite slab sample. In order to determine (in the framework of these models) the critical state stability criterion, it is necessary to know a large number of parameters. Each of these parameters must be determined experimentally and all parameters have some experimental errors. As a result, the uncertainty of the critical state stability criterion obtained can be relatively large. The models used in our analysis can be applied only for some limiting cases (\(Bi \ll 1\) or \(Bi \gg 1\)). In order to determine the critical state stability in an intermediate case, numerical calculations are necessary. Such calculations were performed in e.g. [17].

In order to determine the current–voltage characteristics, we study the relaxation of the magnetic moment. Although equation (2) can be applied to describe the current–voltage characteristics over a wide range of the electric field, one must bear in mind that the electric field induced in the superconducting sample during magnetic relaxation experiments is significantly lower than the electric field induced by the external magnetic field sweep.

One should also bear in mind that the accuracy of determination of \(H_{\text{cr}}\) for the highest sweep rate (\(2 \times 10^{-2} \, \text{T}\)) in our experiment was only about \(\pm 0.1 \, \text{T}\). This experimental error has a special significance for the sample orientation with the external magnetic field parallel to the \(ab\)-plane, because for this orientation all observed values of \(H_{\text{cr}}\) were in the range between 0.5 and \(1 \, \text{T}\) (see figure 2).

Finally, more accurate analysis of the critical state stability of an anisotropic material should also take into account the value of the out-of-plane thermal conductivity.

Our experimental results, as well as the data for textured YBaCuO [8], show that on improvement of the cooling conditions, i.e. on immersing the sample in liquid helium, the critical state stability increases significantly. Both dynamic models used in our analysis predict the dependence of \(H_{\text{cr}}\) on the external magnetic sweep rate (see equations (3) and (5)). These equations were derived assuming the critical current density to be independent of the magnetic field and predict this dependence to be described by a power function with the power of \(-\frac{1}{2}\) and \(-\frac{1}{4}\) for the approximation of weak cooling and the isothermal approximation, respectively. However, if the critical current density is strongly dependent on the magnetic field, the dependence of \(H_{\text{cr}}\) on the sweep rate can be modified [5]. The comparison of the predictions of equation (3) with the experimental results can be seen in figures 2 and 4. One can observe the largest inconsistency for the lowest temperatures and for the external magnetic field parallel to the \(ab\)-plane.

The influence of the heat exchange conditions on the critical state stability, and a strong dependence of \(H_{\text{cr}}\) on the sweep rate show that thermomagnetic instabilities are initiated under dynamic conditions, in which the magnetic diffusivity is smaller than the thermal one. On the other hand, the effective diffusivity describing the dynamics of the flux jumps (see figure 5(b)) is very high—meaning that it is comparable to the magnetic diffusivity in the normal state. The magnetic field dependence of this effective diffusivity suggests that it can be correlated with the flux flow resistivity. In order to understand this discrepancy, one should explain that the initial stage of the avalanche develops at the voltage level that is induced by the external magnetic field sweep. We estimate that the sweep rates used in our experiments induced in the coil a voltage of the order of \(10^{-8} \ldots 10^{-6} \, \text{V}\), which is six or four orders of magnitude lower than the noise level registered by the acquisition board. For this reason, we were not able to register this very early stage of the avalanche. Our experimental results show that after this initial stage of the avalanche the resistivity of the sample rapidly increases and the dynamics of the flux jumps is governed by the magnetic diffusivity, which depends of the flux flow resistivity.

6. Conclusions

The stability of the critical state in a large crystal of La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) is influenced by its anisotropy, and it depends on the sample orientation with respect to the external magnetic field. The most relevant factor is the anisotropy of the critical current. At relatively slow magnetic field sweep rates, thermomagnetic avalanches are initiated under dynamic conditions. Hence, one observes a strong influence of the heat exchange conditions as well as of the sweep rate on the critical state stability. In the case of the experiments, where the sample investigated is surrounded by helium gas with low pressure (PPMS), the critical state stability criterion can be derived using the approximation of a weakly cooled sample. However, this approximation cannot be applied in the case of large crystals immersed in liquid helium. Flux jumps in single-crystalline La\(_{1.85}\)Sr\(_{0.15}\)CuO\(_4\) are very short; a typical duration is several microseconds. The dynamics of the jumps is governed by the magnetic diffusivity, which is proportional to the flux flow resistivity.
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