Two-photon correlations as a sign of sharp transition in quark-gluon plasma.

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Abstract

The photon production arising due to time variation of the medium has been considered. The Hamilton formalism for photons in time-variable medium (plasma) has been developed with application to inclusive photon production. The results have been used for calculation of the photon production in the course of transition from quark-gluon phase to hadronic phase in relativistic heavy ion collisions. The relative strength of the effect as well as specific two-photon correlations have been evaluated. It has been demonstrated that the opposite side two-photon correlations are indicative of the sharp transition from the quark-gluon phase to hadrons.
1 Introduction

The formation of the quark-gluon plasma (QGP) in relativistic heavy ion collisions with subsequent transition to hadrons is under discussion for many years. Numerous measurable signals from QGP phase (such as $J/\psi$ suppression) have been suggested. At the same time, the observation of enhanced antibaryon production \cite{1} together with lattice calculations \cite{2} which predict low temperature of the transition are indicative of the fast QGP-hadron transition \cite{3} without formation of the mixed phase. In this paper we consider a new specific mechanism of photon production which is effective in the case of sharp transition from QGP to hadrons, see \cite{4}.

The phenomenon under consideration is the photon production in the course of evolution of strongly interacting matter. Let us consider photons in the medium at initial moment $t_0$ having momentum $k$ and energy $\omega_{in}$. Let the properties of the medium (its dielectric penetrability) change within time interval $\delta \tau$ so that the final photon energy is $\omega_f$. As a result of the energy change the production of extra photons with momenta $\pm k$ takes place these photons having specific two-photon correlations. Moreover the photons with opposite momenta and opposite helicities are produced even in the absence of the initial photons. Analogous processes were considered for mesons \cite{5, 6, 7, 8} and applied to pion production in high-energy heavy ion collisions \cite{9}. The conditions for strong effect are the following: first, the ratio of the energies $\omega_{in}/\omega_f$ must not be too close to unity and second, the transition should be fast enough.

The calculation of the effect requires consideration of the Hamilton equations of motion. So in Section 2 we present in short the corresponding formalism which is extended in Section 3 to the case of time-dependent medium. A simple method of calculation of the photon correlations is presented in Section 4 with subsequent estimation in Section 5 of the polarization operator and the evolution parameter which determines the the strength of the transition effect. In the last Section 6 we calculate the transition effects in heavy ion collisions and present the results of the work.

2 Basic formulation

We are interested in time evolution of photon creation and annihilation operators $a^\dagger_i(k, t)$ and $a_i(k, t)$ in the medium. In this section the medium is taken in its rest frame. The properties of the medium (plasma) will be described by the transverse polarization operator $\Pi(\omega, k, T, \mu, m)$ which depends on energy $\omega$, momentum $k$, temperature $T = 1/\beta$, chemical potential $\mu$ and the mass $m$ of the charged particles of the plasma. To get the evolution law of $a^\dagger_i(k, t)$ and $a_i(k, t)$ we will use the Hamilton formalism.

The Lagrangian of the electromagnetic field in the medium is taken in the
form
\[ L = \frac{1}{2} \int d^3 x \left( \mathbf{E}(\mathbf{x}, t) \hat{\epsilon} \mathbf{E}(\mathbf{x}, t) - \mathbf{H}^2(\mathbf{x}, t) \right) \] (1)

where dielectric penetrability \( \hat{\epsilon} \) acts as the factor \( \epsilon(\omega, \mathbf{k}) \) in the momentum space (otherwise it acts as an operator). The object of quantization is the real valued vector potential \( \mathbf{A}(\mathbf{x}, t) \). In our case we can use the gauge conditions

\[ A_0(\mathbf{x}, t) = 0, \quad \text{div}\mathbf{A}(\mathbf{x}, t) = 0 \] (2)

so that

\[ \mathbf{E}(\mathbf{x}, t) = -\dot{\mathbf{A}}(\mathbf{x}, t), \quad \mathbf{H}(\mathbf{x}, t) = \text{rot}\mathbf{A}(\mathbf{x}, t) \] (3)

The second of Eqs.(2) means that the vector potential is transverse one (\( \mathbf{kA} = 0 \)) having two components \( A_i \) (usual linear polarizations). The transverse dielectric penetrability \( \hat{\epsilon} \) is connected with photon transverse polarization operator \( \hat{\Pi} \) through equation

\[ \epsilon(\omega, k) = 1 - \Pi(\omega, k)/\omega^2 \] (4)

where \( \Pi(\omega, k) \) will be cosidered as the real valued and even function of \( \omega \) and \( k \) (see Section 5). Turning to momentum representation

\[ \mathbf{A}(\mathbf{x}, t) = \int \frac{d^4 k}{(2\pi)^2} e^{-ik_0 t + i\mathbf{k} \cdot \mathbf{x}} \mathbf{A}(\mathbf{k}, k_0) \] (5)

and using (1-5) the action of the system can be written in the form

\[ S = \frac{1}{2} \int d^4 k A_i(-\mathbf{k}, -k_0) \left[ (k_0^2 - \mathbf{k}^2 - k_0^2 - k_0^2) A_i(\mathbf{k}, k_0) \right] \] (6)

Variation of the action provides the equation

\[ \left[ k_0^2 - \mathbf{k}^2 - k_0^2 \right] A_i(\mathbf{k}, k_0) = 0, \quad i = 1, 2 \] (7)

and the energy \( \omega \) of the photon in the medium is given by the usual dispersion equation

\[ \omega^2 - \mathbf{k}^2 - \Pi(\omega, k) = 0 \] (8)

Let us note in advance that \( \Pi(\omega, k) \) will be positive and slowly varying function of \( k \) playing essentially the role of the effective photon mass squared.

Introducing the time and momentum dependent field coordinates \( q(\mathbf{k}, t) \)

\[ \mathbf{A}(\mathbf{x}, t) = \int \frac{d^3 k}{(2\pi)^3} e^{i\mathbf{k} \cdot \mathbf{x}} q(\mathbf{k}, t), \quad q(-\mathbf{k}) = q(\mathbf{k}) \] (9)

and coming one step back in (6) we get the Lagrangian in the form

\[ L = \frac{1}{2} \int d^3 k \left[ \dot{q}_i(-\mathbf{k}, t) \dot{q}_i(\mathbf{k}, t) - \mathbf{k}^2 q_i(-\mathbf{k}, t) q_i(\mathbf{k}, t) - q_i(-\mathbf{k}, t) \Pi(\mathbf{k}) q_i(\mathbf{k}, t) \right] \] (10)
where the polarization operator acts either to the left or to the right giving equivalent results. In this form the polarization term is related to potential energy (unlike $\dot{q}$ terms) ensuring the physically sensible form of the Hamiltonian in the case under consideration. The Lagrange equations are given by variation of (10). Evidently they have the oscillator form

$$\ddot{q}_i(k) + \omega^2 q_i(k) = 0$$

with $\omega$ determined from (8). That is

$$\hat{\Pi}(k)q_i(k) = \Pi(\omega, k)q_i(k)$$

for plane wave solutions of the equations of motion.

Turning to Hamilton formalism for quantum fields, we introduce the canonically conjugated momentum

$$p_i(k,t) = \frac{\delta L}{\delta \dot{q}_i(k,t)} = \dot{q}_i(-k,t), \quad i = 1, 2$$

and postulate canonical equal time commutation relations

$$[q_i(k_1, t), p_j(k_2, t)] = i\delta_{ij}\delta(k_1 - k_2)$$

with all other commutators being zero. Let us note that the presence of the opposite sign of $k$ in the rhs of (13) is necessary also for compatibility of (14) with commutation relations in coordinate space

$$[A_i(x_1, t), \dot{A}_j(x_2, t)] = i\delta_{ij}\delta(x_1 - x_2)$$

Below this opposite sign of the momentum $k$ will result in important physical consequences describing production of photon pairs with opposite directions of momenta of the photons.

The Hamiltonian is introduced in the usual way

$$H = \int d^3k p_i(k)\dot{q}_i(k) - L$$

$$= \frac{1}{2} \int d^3k \left( p_i(-k)p_i(k) + k^2 q_i(-k)q_i(k) + q_i(-k)\hat{\Pi}(k)q_i(k) \right)$$

and the Hamilton equations for Heisenberg operators

$$\dot{q}_i(k) = i [H, q_i(k)] = p_i(-k),$$

$$\dot{p}_i(-k) = i [H, p_i(-k)] = -\omega^2 q_i(k)$$

connect $q_i(k)$ with $p_i(-k)$ being consistent with (11),(12).
Let us at last introduce the photon creation and annihilation operators $a^\dagger(k)$ and $a(k)$ through decomposition

$$A_i(x, t) = \int \frac{d^3k}{(2\pi)^{3/2}} \frac{1}{(2\omega)^{1/2}} \left(a_i(k, t)e^{ikx} + a_i^\dagger(k, t)e^{-ikx}\right)$$

They are defined here for stationary medium in initial and final states with constant $\omega$ and they are connected in these regions with canonical coordinates and momenta by equations

$$q_i(k) = \frac{1}{\sqrt{2\omega}} \left(a_i(k) + a_i^\dagger(-k)\right)$$
$$p_i(-k) = i\sqrt{\frac{\omega}{2}} \left(a_i^\dagger(-k) - a_i(k)\right)$$

as it can be seen from comparison of representations (9) and (18) and their time derivatives. The creation and annihilation operators have simple time dependence

$$a(t) \sim e^{-i\omega t}, \quad a^\dagger(t) \sim e^{i\omega t}$$

corresponding to running waves (photons) and satisfy commutation relations

$$[a_i(k_1), a_j^\dagger(k_2)] = \delta_{ij}\delta(k_1 - k_2)$$

which follow from canonical commutation relations (14).

### 3 Photon evolution in time dependent medium

Let the polarization operator be the time dependent function of its parameters. In this case the Hamilton equations (17) remain valid having time dependent energy $\omega(t)$ (as well as Lagrange equation (11) which is their consequence). This is confirmed by the fact that their solution represents the time dependent canonical transformation conserving commutator (14). Indeed, the solution of (17) can be written in the form (cf [10]):

$$q_i(k, t) = s_1(t)q_i(k, 0) + s_2(t)p_i(-k, 0)$$
$$\dot{q}_i(k, t) = p_i(-k, t) = \dot{s}_1(t)q_i(k, 0) + \dot{s}_2(t)p_i(-k, 0)$$

where $s_1(t)$ and $s_2(t)$ are two linearly independent real valued solutions of the classical equation (11) with time dependent energy $\omega$ and with initial conditions

$$s_1(0) = 1, \quad \dot{s}_1(0) = 0, \quad s_2(0) = 0, \quad \dot{s}_2(0) = 1$$
One can see from (22) that the canonical commutator transforms in the following way:

\[
[q_i(k, t), p_i(k, t)] = W(s_1, s_2) [q_i(k, 0), p_i(k, 0)]
\]  
(24)

where

\[
W(s_1, s_2) = s_1(t) \dot{s}_2(t) - s_2(t) \dot{s}_1(t)
\]  
(25)

is the Wronskian determinant of (11) which does not depend on time for this equation (due to absence of the term with the first order derivative in the equation), this constant determinant being equal 1 due to initial conditions (23).

Let us consider the process of evolution of the photons from the initial state with energy \( \omega_1 \) to asymptotic final state with energy \( \omega_2 \). The final state annihilation and creation operators are given by

\[
a_i(k, t) = \frac{1}{\sqrt{2\omega_2}} (\omega_2 q_i(k, t) + ip_i(-k, t))
a_i^\dagger(k, t) = \frac{1}{\sqrt{2\omega_2}} (\omega_2 q_i(-k, t) - ip_i(k, t))
\]  
(26)

as it follows from (19). We substitute solutions (22) for \( q_i(k, t), p_i(k, t) \) and introduce two linearly independent complex valued classical solutions \( \xi(t), \xi^*(t) \) instead of \( s_1(t), s_2(t) \)

\[
\xi(t) = s_1(t) + i\omega_1 s_2(t), \quad \xi^*(t) = s_1(t) - i\omega_1 s_2(t)
\]  
(27)

with initial conditions

\[
\xi(0) = \xi^*(0) = 1, \quad \dot{\xi}(0) = i\omega_1, \quad \dot{\xi}^*(t) = -i\omega_1
\]  
(28)

Then, using (19) for the initial state, we get the Bogoliubov transformation [11] connecting the creation and annihilation operators in the initial and final states (let us remind that these operators were defined only for asymptotic states having constant energy \( \omega \)):

\[
a_i(k, t) = u(k, t)a_i(k, 0) + v(k, t)a_i^\dagger(-k, 0)
a_i^\dagger(k, t) = v^*(k, t)a_i(-k, 0) + u^*(k, t)a_i^\dagger(k, 0)
\]  
(29)

with

\[
u(k, t) = \frac{1}{2} \sqrt{\frac{\omega_2}{\omega_1}} \left[ \frac{\xi^*(t)}{\xi(t)} + i \frac{\dot{\xi}^*(t)}{\dot{\xi}(t)} \right]
\]

\[
\dot{v}(k, t) = \frac{1}{2} \sqrt{\frac{\omega_2}{\omega_1}} \left[ \frac{\xi(t)}{\xi^*(t)} + i \frac{\dot{\xi}(t)}{\dot{\xi}^*(t)} \right]
\]  
(30)

It follows from (30) that

\[
u^*(t)u(t) - v^*(t)v(t) = \frac{i}{2\omega_1} W(\xi, \xi^*)
\]  
(31)
where
\[ W(\xi, \xi^*) = \xi(t)\hat{\xi}^*(t) - \hat{\xi}(t)\xi^*(t) \] (32)
is again the time independent Wronskian determinant. So
\[ u^*u - v^*v = 1 \] (33)
due to initial conditions (28). In turn, as it follows from (29), the condition (33) ensures conservation of the commutator
\[ \left[ a_i(k, t), a_i^*(k, t) \right] = \left[ a_i(k, 0), a_i^*(k, 0) \right] \] (34)
(the last commutator is equal to $V/(2\pi)^3$ for the system having volume $V$). In view of condition (33) the Bogoliubov coefficients $u, v$ can be represented in the form
\[ u = \cosh r(k)e^{i\alpha_1}, \quad v = \sinh r(k)e^{i\alpha_2} \] (35)
where $r(k)$ is the main parameter which determines the photon production and the phases $\alpha_1, \alpha_2$ do not play important role and they will not be considered below.

To find the coefficients $u, v$ (for fixed momentum $k$) one must turn to classical equations for oscillator having variable frequency (energy). We look for solution of the equation
\[ \ddot{\xi} + \omega^2(t)\xi = 0 \] (36)
In the initial state, where the energy $\omega_1$ is constant, we take a single wave
\[ \xi(t) = e^{i\omega_1 t}, \quad t < t_{in} = 0 \] (37)
At large enough time, $t > t_f$, when the energy $\omega_2$ becomes constant again, the general solution has the form
\[ \xi(t) = C_1 e^{i\omega_2 t} + C_2 e^{-i\omega_2 t}, \quad t > t_f \] (38)
Substituting (37),(38) into (30) we get corresponding Bogoliubov coefficients
\[ u = \sqrt{\frac{\omega_2}{\omega_1}} C_1^* e^{-i\omega_2 t}, \quad v = \sqrt{\frac{\omega_2}{\omega_1}} C_2 e^{-i\omega_2 t}, \quad t > t_f \] (39)
where constants $C_1, C_2$ satisfy relationship
\[ |C_1|^2 - |C_2|^2 = \frac{\omega_1}{\omega_2} \] (40)
due to condition (33).

To find the final expressions for coefficients $u, v$ one must know the full solution of (36) connecting the asymptotics (37) and (38). Here one can use an analogy between the above problem and the problem of the wave propagation through
the one-dimensional potential barrier. In the last case (38) (after substitution of \( x \) for \( t \)) represents incoming and reflected waves and (37) represents outgoing wave. The reflection coefficient

\[ | \frac{C_2}{C_1} |^2 = | \frac{v}{u} |^2 = \tanh^2 r \]  

(41)
gives the desired ratio of the Bogoliubov coefficients (up to phases). Therefore one can use known quantum-mechanical results. So, shifting the initial time \( t_{in} \) to large negative value, we take the reference model of the energy variation:

\[ \omega^2(t) = \frac{\omega_2^2 + \omega_1^2}{2} + \frac{\omega_2^2 - \omega_1^2}{2} \tanh \left( \frac{2t}{\delta \tau} \right) \]  

(42)
The problem with such form of the potential barrier can be found in textbooks [12]. It contains the important parameter \( \delta \tau \) giving characteristic time of the energy variation. The evolution parameter \( r \) is given now by

\[ r = \tanh^{-1} \left| \frac{v}{u} \right| = \frac{1}{2} \ln \left( \frac{\tanh(\pi \omega_2 \delta \tau/4)}{\tanh(\pi \omega_1 \delta \tau/4)} \right) \]  

(43)
For sharp transition \( (\omega_i \delta \tau \ll 1) \) we get from (43):

\[ r = \frac{1}{2} \ln \left( \frac{\omega_2}{\omega_1} \right), \quad \left| \frac{u}{v} \right|^2 = \left( \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1} \right)^2, \quad \omega_i \delta \tau \ll 1 \]  

(44)
This is the case of the most intensive pair production. For large transition time \( \delta \tau \) the process becomes adiabatic one and the evolution parameter falls down exponentially:

\[ r = \left| e^{-\pi \omega_2 \delta \tau/2} - e^{-\pi \omega_1 \delta \tau/2} \right|, \quad \omega \delta \tau \gg 1 \]  

(45)
Let us note that the above results (44) for sharp transition can be found immediately from Hamilton equations (17). Suggesting that the canonical coordinate \( q \) and momentum \( p \) are finite, we see from these equations that \( \dot{q} \) and \( \dot{p} \) are finite as well and so \( q \) and \( p \) are continuous functions of time at the transition point:

\[ q(-\delta t) = q(\delta t), \quad p(-\delta t) = p(\delta t), \quad \delta t \to 0 \]  

(46)
coinciding for both sides of the sharp border between media with different photon energies \( \omega_1 \) and \( \omega_2 \). Being expressed according (19) through creation and annihilation operators \( a^\dagger, a \) at both sides of the border, the equations (46) lead to Bogoliubov transformation (29) between pairs of operators \( a^\dagger, a \) at different sides of the border with coefficients

\[ |u| = \frac{\omega_2 + \omega_1}{2\sqrt{\omega_1 \omega_2}}, \quad |v| = \frac{\omega_2 - \omega_1}{2\sqrt{\omega_1 \omega_2}} \]  

(47)
The last equations correspond to (44).
4 Inclusive photon production in a simple model

Below we will be interested in inclusive photon production in heavy ion collisions. We confine ourselves to symmetric case when the photons having opposite momenta \( \pm k \) are produced in an equivalent way (central collisions of identical nuclei). For simplicity the Bogoliubov coefficients \( u(k), v(k) \) will be taken to be real valued and \( k = |k| \) dependent. To get feeling of the main features of the photon production and their correlations (and for further references and comparison) we formulate in this section a simple model – fast simultaneous transition of large homogeneous system at rest (the movement of the system will be considered in Section 6).

For physical interpretation of the evolution effect it is helpful to introduce the complex valued vectors of the photon circular polarization:

\[
e_\pm = (e_1 \pm ie_2)/\sqrt{2}, \quad ke_\pm = 0
\] (48)

and corresponding components of operators \( a(k), a^\dagger(k) \)

\[
a_\pm = (a_1 \pm ia_2)/\sqrt{2}, \quad a^\dagger_\pm = (a^\dagger_1 \mp ia^\dagger_2)/\sqrt{2}
\] (49)

so that

\[
a(k) = e_+ a_-(k) + e_- a_+(k), \quad a_\pm = e_\pm a
\] (50)

The components \( a_\pm, a^\dagger_\pm \) satisfy the standard commutation relations of the form (21) with \( i = \pm \) and represent the creation and annihilation operators of the photons with definite spin projection \( \pm 1 \) on the direction of the photon momentum (the helicity).

In what follows we will denote the creation and annihilation operators in the final state as \( b^\dagger, b \) leaving notations \( a^\dagger, a \) for the operators in the initial state. In terms of the components \( b_\pm \) the helicity operator is

\[
S_{b3} = i(b_1^\dagger b_2 - b_2^\dagger b_1) = b_+^\dagger b_+ - b_-^\dagger b_- = N_{b+} - N_{b-}
\] (51)

for every momentum \( k \), the photon number operator is

\[
N_b = b_+^\dagger b_+ + b_-^\dagger b_- = N_{b+} + N_{b-}
\] (52)

and the Bogoliubov transformation takes the form:

\[
b_\pm(k) = ua_\pm(k) + va^\dagger_\pm(-k), \quad b_\pm(k) = u^*a^\dagger_\pm(k) + v^*a_\pm(-k)
\] (53)

where indexes \( \pm \) are helicities related to corresponding momenta \( k \) or \(-k\).

The resulting average number of the photons and their correlations depends on the initial state. We suggest that the initial state is the statistical (gaussian)
mixture of the coherent states $|13, 5\rangle$ of the photons of each polarization. Then using (53) we get the photon momentum distributions

$$
\left( \frac{dN}{d^3k} \right)_\pm = \langle N_{b\pm}(k) \rangle = \left( 1 + |v|^2 \right) \langle N_{a\pm}(k) \rangle + |v|^2 \langle N_{a\pm}(-k) \rangle + |v|^2 \frac{V}{(2\pi)^3} \tag{54}
$$

As one can see from (54) the photons are created in pairs having opposite directions of their momenta and opposite spin directions (the same helicities). If both kinds of polarizations are equally represented in the initial state for every momentum $k$, then the same property is valid for the final state,

$$
\langle S_{b\lambda}(k) \rangle = 0 \tag{55}
$$

and the photons produced with opposite momenta have equal helicities. The intensity of the transition production is given by the Bogoliubov coefficient $|v|^2$ and the last term in rhs of (54) represents the result of the ground state rearrangement (the initial ground state is not the ground state for $b$-operators which operate in another medium in the final state). We suggest that $k \to -k$ symmetry takes place. Then the resulting photon momentum distribution can be written in the form:

$$
\frac{dN}{d^3k} = \langle b^\dagger_\lambda(k) b_\lambda(k) \rangle = \frac{2V}{(2\pi)^3} \left[ n(k) + (2n(k) + 1) \sinh^2 r \right] \tag{56}
$$

(sum over helicities $\lambda$) where $n(k)$ is the average level occupation number of the single mode in the initial state and we used the parametrization (35). The photon production in the course of the transition is given by the second term in rhs of (56). It is weak for small evolution parameter $r(k)$ being of the order $r^2$.

The evolution effect is better seen in photon correlations. Two-photon inclusive cross-section is given by

$$
\frac{1}{\sigma} \frac{d^2\sigma}{d^3k_1 d^3k_2} = \langle b^\dagger_\lambda(k_1) b^\dagger_\mu(k_2) b_\lambda(k_1) b_\mu(k_2) \rangle = \langle b^\dagger_\lambda(k_1) b_\lambda(k_1) \rangle \langle b^\dagger_\mu(k_2) b_\mu(k_2) \rangle + \langle b^\dagger_\lambda(k_1) b^\dagger_\mu(k_2) \rangle \langle b_\lambda(k_1) b_\mu(k_2) \rangle \tag{57}
$$

The first term in rhs of (57) is the product of single-photon distributions, the second term gives the Hanbury Brown-Twiss effect (HBT, called also Bose-Einstein correlations) and the third term is essential if the time evolution effect takes place giving opposite side photon correlations.

Production of the photons with opposite directions of their momenta was already noted after (54). However it was implicitly suggested there that the volume of the system is arbitrary large, $V \to \infty$. In real situation we deal with large but finite size of the colliding nuclei. The finite size of the photon source will smooth out the effect. The same is valid for HBT effect responsible for the same side correlations. The special technique was elaborated in a number of papers to describe the source size effect for HBT correlations including the
use of Wigner phase space density \([14]\) and the method of equivalent classical currents \([15]\) which can be used here. Instead we directly modify the creation and annihilation operators \([5]\) in such a way that they are nonzero only inside some region in coordinate space which is described by the function \(f(\mathbf{x})\):

\[
\tilde{b}_\lambda(\mathbf{x}) = b_\lambda(\mathbf{x})f(\mathbf{x}), \quad \tilde{b}_\lambda^\dagger(\mathbf{x}) = b_\lambda^\dagger(\mathbf{x})f(\mathbf{x})
\] (58)

Then the modified (smoothed out) operators in momentum space are:

\[
\tilde{b}_\lambda^\dagger(\mathbf{k}_1) = \int d^3 k f(\mathbf{k}_1 - \mathbf{k}) b_\lambda^\dagger(\mathbf{k}), \quad \tilde{b}_\lambda(\mathbf{k}_2) = \int d^3 k f(\mathbf{k} - \mathbf{k}_2) b_\lambda(\mathbf{k})
\] (59)

where the Fourier transform \(f(\mathbf{k}_1 - \mathbf{k})\) is sharply peaked around the point \(\mathbf{k} = \mathbf{k}_i\) (smoothed \(\delta\)-function).

The introduced operators satisfy modified commutation relations

\[
[\tilde{b}_\lambda(\mathbf{k}_1), \tilde{b}_\lambda^\dagger(\mathbf{k}_2)] = \delta_{\lambda\nu} \int d^3 k f(\mathbf{k}_1 - \mathbf{k}) f(\mathbf{k} - \mathbf{k}_2)
\]

\[
= \delta_{\lambda\nu} \int \frac{d^3 \mathbf{x}}{(2\pi)^3} f^2(\mathbf{x}) e^{i(\mathbf{k}_1 - \mathbf{k}_2)\cdot \mathbf{x}} = \delta_{\lambda\nu} F(\mathbf{k}_1 - \mathbf{k}_2)
\] (60)

Here \(F(\mathbf{k}_1 - \mathbf{k}_2)\) is the form-factor of the photon source (Fourier transform of the source density) and it also represents the smoothed \(\delta\)-function which has the width of the order of the inverse size of the source. In particular

\[
F(0) = V/(2\pi)^3,
\] (61)

where \(V\) is the effective volume of the source at the stage of the transition in the medium. Let us note that above we in fact already used (61) substituting it for \(\delta^3(0)\) in the contribution of the ground state rearrangement to the evolution effect in (54). This factor appears also when one wants to use the level population function \(n(\mathbf{k})\) (see (56)), having its origin in the correspondence of discrete and continuous Fourier decompositions. Using operators modified in three-dimensional coordinate space we suggest that the volume \(V\) changes inessentially in the course of the time transition.

Now one can apply the Bogoliubov transformation (53) to estimate the correlators of the modified operators:

\[
\langle \tilde{b}_\pm^\dagger(\mathbf{k}_1) \tilde{b}_\pm(\mathbf{k}_2) \rangle = \int d^3 k \left[ u^2(k)n(k) + v^2(k)(n(k) + 1) \right] f(\mathbf{k}_1 - \mathbf{k}) f(\mathbf{k} - \mathbf{k}_2)
\]

\[
= \frac{V}{(2\pi)^3} \left[ n(k) + (2n(k) + 1) \sinh^2 r \right] G(\mathbf{k}_1 - \mathbf{k}_2)
\] (62)

and in a similar way

\[
\langle \tilde{b}_\pm(\mathbf{k}_1) \tilde{b}_\pm(\mathbf{k}_2) \rangle = \langle \tilde{b}_\pm^\dagger(\mathbf{k}_1) \tilde{b}_\pm^\dagger(\mathbf{k}_2) \rangle
\]

\[
= \frac{V}{(2\pi)^3} (2n(k) + 1) \sinh r(k) \cosh r(k) G(\mathbf{k}_1 + \mathbf{k}_2)
\] (63)
with other correlators vanishing in the course of statistical averaging in the case of sharply peaked form-factor. In the above equations $G(k_1 \pm k_2)$ is the normalized form-factor ($G(0) = 1$) and we took into account that the functions $f(k_1 - k)$ and $f(k - k_2)$ as well as form-factor $G(k_1 \pm k_2)$ are sharply peaked functions of their arguments (at zero momentum) having characteristic scale of the order of inverse size of the source, this scale being much less than the characteristic scales of the momentum distribution $n(k)$ and the evolution parameter $r(k)$. So the last two functions can be evaluated at any of momenta $k_1, k_2 \approx \pm k$ (we suggest that the process is $k \to k$ symmetric). The equations (62-63) together with (57) show that the correlations arising due to photon identity (HBT effect) are the same side momentum correlations of the photons having the same helicities whereas the photons arising due to transition effect have the opposite side momentum correlations and approximately opposite spin directions (the same helicities again).

Returning to two-photon correlations given by (57) (sum over helicities) we get the relative correlation function which is measured in experiment:

$$C(k_1, k_2) = 1 + \frac{1}{2} G^2(k_1 - k_2) + \frac{1}{2} R_0^2(k) G^2(k_1 + k_2)$$

(64)

with

$$R_0(k) = \frac{(2n(k) + 1) \sinh r(k) \cosh r(k)}{n(k) + (2n(k) + 1) \sinh^2 r(k)}$$

(65)

according to (62-63). As can be seen from (64-65) the transition effect depends strongly on the evolution parameter $r(k)$, see (43-45). Below, after necessary modifications, we apply the above considerations to photon transition radiation in heavy ion collisions.

5 Photons in plasma and the evolution parameter

To find the evolution parameter $r(k)$ one must know the photon energy in plasma. The spectrum of photons in plasma is given by dispersion equation

$$\omega^2_k = k^2 + \Pi(\omega_k, k; T, \mu, m)$$

(66)

Here $\Pi$ is the polarization operator for transverse photons dependent on temperature $T = \beta^{-1}$, chemical potential $\mu$ and the mass $m$ of charged particles. Below we use an approximate form of $\Pi$ extracted from original expression [16]:

$$\Pi(\omega, k) = \omega^2_a \left[ 1 - \frac{\omega^2 - k^2}{k^2} \left( \ln \left( \frac{\omega + vk}{\omega - vk} \right) - 1 \right) \right]$$

(67)
with
\[ \omega_a^2 = \frac{4g\alpha}{\pi\beta} \int_{m\beta}^{\infty} dx \left( x^2 - m^2 \beta^2 \right)^{1/2} n_F(x, \mu\beta) \] (68)
where \( \alpha = 1/137 \), \( v^2 \) is the averaged velocity squared of the charged particles in the plasma, factor \( g \) takes into account the number of the particle kinds and their electric charges (\( g = 5/3 \) for \( u, d \) quarks) and \( n_F \) is the occupation number of the charged particles:
\[ n_F(\beta\omega) = \left( e^{\beta\omega - \beta\mu} + 1 \right)^{-1} + \left( e^{\beta\omega + \beta\mu} + 1 \right)^{-1} \] (69)

(Fermi distribution). The polarization operator for scalar charged particles is approximately a half of that for fermions with substitution of Bose distribution for Fermi distribution. The imaginary part of the polarization is small in the case under consideration (no Landau damping) and it was neglected in the calculations.

For small masses of charged particles, \( \beta m \ll 1 \), the asymptotics of the polarization operator takes the known simple form
\[ \omega_a^2 = \frac{2\pi g\alpha}{3} \left( T^2 + \frac{3}{\pi^2} \mu^2 \right) \] (70)
This form can be used in quark-gluon phase. For large masses of fermions (constituent quarks and nucleons) when \( \beta m > 1 (m > \mu) \) it is convenient to use the expansion
\[ \omega_a^2 = \frac{8g\alpha m}{\pi\beta} \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \cosh(n\beta\mu)K_1(n\beta m) \] (71)
where \( K_1(x) \) is the exponentially decreasing modified Bessel function and the main contribution comes from a few first terms of the series. Analogous expansion was also used for bosons (pions) having no chemical potential:
\[ \omega_a^2 = \frac{4\alpha m}{\pi\beta} \sum_{n=1}^{\infty} \frac{1}{n} K_1(n\beta m) \] (72)

We considered the polarization operator and photon spectrum for three possible kinds of plasma: quark-gluon plasma (QGP) with \( u, d \) light quarks, constituent quark(\( m = 350MeV \))-pion plasma and hadronic (pions and nucleons) plasma. Chemical potential (baryonic one) was taken to be equal to 100MeV per quark corresponding to typical value for CERN-SPS energies (say 160GeV per nucleon in \( Pb – Pb \) collisions). The temperature \( T_c \) of the transition was taken to be equal to rather small value 140MeV characteristic for final hadrons \[17\] (\( T_c = 200MeV \) was also tested giving close results). Under above conditions the asymptotic values \( \omega_a \) in (70-72) are equal to 24, 11.7, 11.2 and 2.5 in MeV units for light quarks, constituent quarks, pions and nucleons correspondingly.
Let us note that the approximation (67) where we use the averaged velocity squared \( \langle v^2 \rangle \), suggests that higher moments of the velocity distribution do not differ significantly from corresponding powers of \( v^2 \). So we calculated the ratio \( \langle v^4 \rangle / \langle v^2 \rangle^2 \) and verified that it differs from unity inessentially being 1 for QGP, 1.11 for constituent quarks (valons), 1.06 for pions and reaching 1.25 for nucleons which last contribution is small by itself. Corresponding values of \( v^2 \) used in our estimations are 1, 0.545, 0.731 and 0.300.

Evidently the polarization operator in (66) plays the role of (momentum dependent) photon mass squared \( m_\gamma^2 \). The termal mass squared at zero photon momentum \( k \) is equal to

\[
\omega_0^2 = \omega_a^2 \left( 1 - \frac{v^2}{3} \right)
\]

and it approaches \( \omega_a^2 \) at large momenta. The slope of the dispersion curve at the origin is:

\[
\left. \frac{d\omega_k^2}{dk^2} \right|_{k=0} = \left( 1 - \frac{1}{5} v^2 \right) / \left( 1 - \frac{1}{3} v^2 \right) = 1 + c
\]

(74)

varying from 1 at \( v = 0 \) to 1.2 at \( v = 1 \). In rough approximation the polarization operator taken at the dispersion curve can be represented by the simple expression

\[
\Pi(\omega_k, k) = m_\gamma^2(k) = \omega_0^2 + \frac{\omega_0^2 k^2}{\omega_0^2 + dk^2}, \quad d = 1 - \frac{2}{5} v^2
\]

(75)

reproducing position of the point \( \omega_0 \), asymptotic value \( \omega_a \) and the slope (74). In the presence of two different sorts of charged particles (say pions and nucleons) the expression (75) should be modified in an appropriate way to take into account the presence of two contributions and to ensure the new correct slope \( d\omega^2/dk^2 \). The simple approximation (75) appears to be rather close to polarization operator (67) taken at the dispersion curve (66) (these two curves have a common point also at some finite \( k \)) and it will be used below for estimation of the evolution parameter \( r(k) \). Zero momentum photon masses \( m_\gamma(0) \) was found to be 19.5, 14.4 and 10.0 in MeV units for QGP, valon-pion and nucleon-pion plasma correspondingly.

The evolution parameter \( r(k) \), which gives the strength of the transition radiation, was determined in (43-45) for the reference model (42). It depends strongly on transition time \( \delta \tau \). We suggest that this time interval is not large being of the order of \( 1 \text{ fm}/c \). Then for small momenta \( k \) \( (\omega(k)\delta \tau \ll 1) \) the parameter \( r(k) \) is universal and it can be well approximated by the expression which follows from (44):

\[
r(k) = \frac{m_{\gamma 1}^2(k) - m_{\gamma 2}^2(k)}{4(\langle m_{\gamma}^2(k) \rangle + k^2)} = \frac{\delta m_{\gamma}^2(k)}{4\langle \omega^2(k) \rangle}, \quad \omega(k)\delta \tau \ll 1
\]

(76)

where \( m_{\gamma i}^2 \) are photon termal masses squared at both sides of the transition and \( \langle m_{\gamma}^2 \rangle \) is their average. Zero momentum evolution parameter \( r(0) \) is equal to 0.324, 0.154 and 0.178 for QGP-hadron, QGP-valon and valon-hadron transitions.
correspondingly. Higher momentum behaviour of \( r(k) \) is model dependent. Below it will be taken in the form

\[
    r(k) = \frac{\delta m^2}{4k^2} \exp \left( -\frac{\pi}{2} k \delta \tau \right)
\]  

(77)

The Eq.(77) is a simple version of the asymptotical form (45) and it can be well sewed together with (76) giving a monotonically decreasing function of the momentum. Let us note that on general grounds one expects that \( r(k) \) falls down exponentially at large \( k \delta \tau \) if the time dependence of the energy \( \omega(k, t) \) in the course of the transition has no singularities (non-analiticity) at real time axis. Below Eqs.(76-77) will be used for estimation of the transition effect in heavy ion collisions. Only QGP-hadron transition will be considered. In view of fact that the evolution parameter \( r(k) \) appears to be small number at all momenta \( k \), all expressions will be estimated in the lowest order in \( r(k) \).

6 Transition effect in heavy ion collisions

Let us now apply the above consideratinos to photon production in heavy ion collisions. Let us suggest that the quark-gluon plasma is formed at the initial stage of the ion collision. Let the plasma undergoes expansion and cooling. The expansion is taken to be longitudinal and boost invariant [18]. Recent lattice calculations [2] show rather low critical temperature of the deconfinement and chiral phase transition, \( T_c \approx 150 \text{ MeV} \) as well as sharp drop of the pressure up to very small value when the temperature approaches \( T_c \) thus provoking instability in the presence of overcooling. So we do not expect long-living mixed phase and consider fast transition from quark to hadron matter with characteristic transition proper time duration \( \delta \tau \) of the order of \( 1 \text{ fm/c} \).

To calculate the transition effect one must shift to rest frame of each moving element of the system and integrate over proper times \( \tau \) and space-time rapidities \( \eta \) of the elements of the system. Then the invariant single-photon distribution in central rapidity region \( y = 0 \) reads:

\[
    \left. \frac{dN}{d^2k_Tdy} \right|_{y=0} = I_{QGP} + I_{tr}^{(1)}
\]

\[
    = \int \tau d\tau \int d\eta \int d^2x_T \frac{dR_\gamma}{dp^0} + \int d\eta \int d^2x_T \frac{2p_{\tau c}}{(2\pi)^3} \gamma^2(p)
\]

(78)

with \( p = k_T \cosh \eta \).

The first term in rhs of (78) describes photon production from hot quark-gluon plasma. Here \( R_\gamma \) is the QGP production rate per unit four-volume in the rest frame of the matter [19].
\[ p_0 \frac{dR_d}{dp} = \frac{5\alpha\alpha_s}{18\pi^2} T^2 \exp(-p/T) \ln(1 + \frac{\kappa p}{T}) \]  
\[ (79) \]

with \( \alpha = 1/137 \), \( \alpha_s = 0.4 \), \( \kappa = 0.58 \). It can be used also for hadron gas as its uncertainty is larger than the difference between the first-order QGP and hadron gas production rates [20]. Contribution from hadronic resonances are not considered here. The second term in rhs of (78) describes photon production due to transition from QGP to hadrons in the vicinity of proper time \( \tau_c \), cf (56). The time duration of the transition is taken to be small in this term in comparison with total time duration of photon production process. The evolution parameter \( r(p) \) is given here by (76-77).

As the last step one must specify the temperature evolution in (78). We suggest that the temperature depends on proper time of the volume element with power-like dependence:

\[ \left( \frac{T}{T_0} \right) = \left( \frac{\tau}{\tau_0} \right)^{-1/b} \]  
\[ (80) \]

where \( \tau_0 \) and \( T_0 \) are initial proper time and initial temperature. For final estimation we use \( b = 3 \) typical for hydrodynamical picture and choose low transition temperature \( T_c = 140 MeV \). After transition the photons live some time in hadronic medium and we suggest termal momentum distribution of the hadrons (modified by the expansion of the system). We neglect termal photon production below \( T_c \) and do not introduce a special freee-out temperature. An alternative way is to introduce the final temperature \( T_f < T_c \). Not pretending to high accuracy we shall not distinguish between \( T_c \) and \( T_f \) below.

Using now the (79-80) the photon production rate in QGP phase in (78) can be integrated over space-time rapidities \( \eta \):

\[ I_{QGP} = \frac{5\alpha\alpha_s}{9\pi^2} \frac{1}{(2\pi)^{1/2} b} \frac{(k_T \tau_c)^2}{(k_T \beta_c)^{2b-2}} \int d^2 x_T J_{QGP} \]  
\[ (81) \]

with

\[ J_{QGP} \approx \int_{k_T \beta_0}^{k_T \beta_c} dx \frac{x^{2b-3} e^{-x}}{(4x+1)^{1/2}} \ln \left( 1 + \kappa x + \frac{\kappa x}{4x+1} \right) \]  
\[ (82) \]

The remaining integral (82) over temperatures can be easily evaluated in different subregions of the photon momentum \( k_T \). Below we will be interested in rather low photon momenta \( k_T \) (up to 500 MeV) where transition effect is expected to be more pronounced. In this momentum region the integral (82) depends mainly on final temperature \( T_c \) (at asymptotically large momenta \( k_T \) it depends on initial temperature \( T_0 \)). The final proper time \( \tau_c \) in (81) depends on initial conditions in general. However for two main variants of the initial conditions used in literature [21]: \( \tau_0 T_0 = 1, T_0/T_c = 3/2 (\tau_0 = 1 fm/c) \) and \( \tau_0 T_0 = 1/3, T_0/T_c = 5/2 (\tau_0 = 0.2 fm/c) \) the proper time \( \tau_c \) changes inessentially, being 3.125 fm/c and
3.375 fm/c correspondingly (for $b = 3$). Below we will use for $\tau_c$ an average value $\tau_c = 3.25 fm/c$.

Let us note that the photon production rate in (78) can be expressed through photon occupation number $n(k)$:

$$ p_0 \frac{dR_\gamma}{d^3p} = \frac{2k_T}{(2\pi)^3} \frac{dn(k_T \cosh \eta)}{d\tau} $$

(with two polarizations included). In particular, taking (83) into account one can see that if the velocities of the volume elements, as well as proper time interval in the first term in rhs of Eq. (78) are small then (78) is reduced to (56) as it should be. The estimation of (79) shows that the photon occupation number $n(k)$ in Eq. (83) is small numerically,

$$ n(k) \ll 1 $$

That means in particular that the transition radiation is dominated not by the photon amplification but by the ground state rearrangement.

The transition contribution $I^{(1)}_{tr}$ in (78) appears essential only at small momenta $k_T$. So dealing with single-photon distributions one can use (76) for evolution parameter $r(k)$. The resulting relative strength of the transition radiation

$$ R_1(k_T) = \frac{I^{(1)}_{tr}}{I_{QGP}} $$

appears sizable only in the momentum region $k_T \leq (20 - 30) Mev$ independently of the time duration of the transition. It is shown at Fig.1.

Mach better the transition effect is seen in photon correlations (cf (64-65)) where it is first order effect with respect to $r(k)$. Let us note that HBT effect for photons has now more complicated form than that in (64) because of finite time duration [15] of the process of photon emission from QGP and it will not be exposed here. We consider only the transition effect (the third term in (57,64) which gives opposite side correlations) estimating its contribution to two-photon correlation function in central rapidity region. Suggesting fast transition we can evaluate the contribution in the vicinity of fixed proper time $\tau_c$. So one only has to shift to the rest frame of each element of the expanding volume and perform $\eta$-integration. Then the extention of the invariant correlator in (63) to the case of expanding volume takes the form:

$$ 2k\langle \tilde{b}_\pm(k_1)\tilde{b}_\pm(k_2) \rangle = G(k_{1T} + k_{2T})I^{(2)}_{tr} $$

with

$$ I^{(2)}_{tr} = \int d^2x_T \int d\eta \frac{2\tau_c k_T \cosh \eta}{(2\pi)^3} r(k_T \cosh \eta) $$

where we neglected $n(k)$ in comparison with unity (see above). Therefore the normalized two-photon correlation function is given by (cf (64-65))

$$ C(k_{1T}, k_{2T})\big|_{y_1 = y_2 = 0} = 1 + C_{HBT} + \frac{1}{2} R_2^2(k_T)G^2(k_{1T} + k_{2T}) $$
with

$$R_2(k_T) = \frac{I_{I_{tr}}^{(2)}}{I_{QGP} + I_{I_{tr}}^{(1)}}$$

(88)

We calculated the ratio $R_2(k_T)$ for different transition times $\delta \tau = 0, \delta \tau = 0.5 \text{fm/c}, \delta \tau = 1 \text{fm/c}$ up to $k_T = 500 \text{MeV}$. The results are shown at Fig.2. In the region $k_T < 100 \text{MeV}$ the ratio $R_2(k_T)$ is sizable for all these $\delta \tau$ being equal 4.94 at $k_T = 0$, reaching maximal value $R \sim 6$ at $k_T \sim 20 \text{MeV}$ and falling down at $k_T = 100 \text{MeV}$ to $R_2 = 1.78$ for $\delta \tau = 0$, $R_2 = 0.95$ for $\delta \tau = 0.5 \text{fm/c}$, $R_2 = 0.55$ for $\delta \tau = 1 \text{fm/c}$. At larger transverse momenta the behaviour of the ratio $R$ depends strongly on transition time $\delta \tau$: in $k_T$ interval $(200 - 500) \text{MeV}$ the ratio $R_2(k_T)$ rises for $\delta \tau = 0$, it is approximately constant for $\delta \tau = 0.5 \text{fm/c}$ and it decreases for $\delta \tau = 1 \text{fm/c}$. The asymptotic behaviour of the ratio $R_2(k_T)$ at large photon momenta $k_T$ depends on relationship of the temperature dependent single-particle production rate (decreasing like $\exp(-\beta k)$) and the transition time dependent evolution parameter (decreasing like $\exp(-\pi k \delta \tau/2)$ in our reference model). In any case, one can hope to see the effect of the transition in the region $k_T \leq 100 \text{MeV}$ where the peak of the ratio $R_2(k_T)$ is always present.

7 Conclusion

Estimation of photon emission accompanying transition between quark-gluon and hadron states of matter in heavy ion collisions shows that opposite side photon correlations can serve as a sign of the transition if transition time is not large.

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Figure captions

**Fig.1** The relative strength of the transition radiation for transition from QGP to hadrons.

**Fig.2** The relative strength of the opposite side two-photon correlations for transition time $\delta \tau = 0, \delta \tau = 0.5fm/c, \delta \tau = 1fm/c$ (from top to bottom), see Eqs.87-88.
Fig1
Fig2