A study of thermal vorticity in PICR hydrodynamic model

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Abstract. With a Yang-Mills field, stratified shear flow initial state and a high resolution (3+1)D Particle-in-Cell Relativistic (PICR) hydrodynamic model, we calculate thermal vorticity for peripheral Au+Au collisions at different energies $\sqrt{s} = 7.7 - 200$ GeV. Based on the thermal vorticity calculations, we investigate the two puzzles in $\Lambda$ polarization studies: the global polarization splitting between $\Lambda$ and $\bar{\Lambda}$, and local polarization/vorticity structure along the beam direction. Based on the vector meson field mechanism, we calculate the polarization splitting between $\Lambda$ and $\bar{\Lambda}$, the results fit to the experimental results fairly well. We also confirm that thermal vorticity along the beam direction has a quadrupolar structure on transverse space plane $[x, y]$. Interestingly the quadrupolar structure takes time to form and will significantly weaken at later times. Besides, we find that the magnitude of $z$-directed thermal vorticity actually increases with the collision energy.

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1 Introduction

Non-central heavy ion collisions create a participant system of extremely hot and dense matter, carrying substantial angular momentum that is perpendicular to the reaction plane $[1,2,3]$. Through the spin-orbital coupling, just as the Einstein-de-Hass effect $[4]$ and Barnet effect $[5]$ had revealed, the initial fireball angular momentum will eventually give rise to the spin alignment of final particles, such as $\Lambda$ hyperons $[6,7]$. The $\Lambda$ hyperon reveals its polarization by emitting preferentially the weak decay products along its spin direction, and thus is a fairly good choice of polarization measurement in experiments $[8,9,10]$. Many theories and simulations were also addressing this topic $[6,11,12,13]$. Recently, the STAR collaboration measured the non-vanishing $\Lambda$ polarization for Au-Au collisions at different energies $\sqrt{s_{NN}} = 7.7$ GeV - 200 GeV $[14,15,16]$, and as far as we know, the results conform with the theoretical predictions and simulations in two significant aspects: the global polarization of both $\Lambda$ and $\bar{\Lambda}$ aligns with the initial angular momentum, and decreases with the energy; the local polarization along the beam direction shows quadrupolar structure on transverse momentum plane.

However, there still exist some puzzles in this field $[17]$. Globally, the magnitude of $\bar{\Lambda}$ polarization is larger than that of $\Lambda$ polarization. Some might argue that due to the large errors in measurements, it is not sure that whether this polarization splitting really exists, but at least for collision energy of $\sqrt{s} = 7.7$ GeV, this splitting clearly exists and the difference could be 3% at least (see Fig. 1). This splitting effect has raised large interests. It was proposed that the magnetic field induced by the charged spectators can give rise to the polarization splitting between $\Lambda$ and $\bar{\Lambda}$, but this will require a magnetic field that is long lasting and has a large magnitude. These are not realized. A recent suggestion is that the magnetic field can also be induced by charged particles in vortical Quark-Gluon-Plasma (QGP), and in this scenario the magnetic field could last long enough until freeze-out, but problem still exists: the charge density might not be large enough to produce a magnetic field that is strong enough. E.g., the upper limits of the estimated polarization difference at 7.7GeV is below 1%, which is far away from the lower boundary of experimentally observed 3% $[18]$ difference.

Another novel mechanism was proposed by Ref. $[19]$, that the vector meson’s strong interaction ”magnetic” field, induced by the baryon vorticity at freeze-out, can split the polarization. However, the polarization splitting formula therein is driven mainly by the directed flow coefficient $\langle c_1 \rangle$ and the shear flow coefficient $\langle c_3 \rangle$ $[19]$. The coefficient $C$, which is proportional to $\Delta c = c_1 - c_3$, is actually a free parameter. Therefore, in this paper, we are going to revisit the theory in Ref. $[19]$ and modify the splitting formula therein, (mainly by removing the free parameter $C$). Then based on this vector meson field mechanism, we use the high resolution (3+1)D Particle-in-Cell Relativistic (PICR) hydrodynamic model, used in several earlier estimates, to simulate and calculate the polarization splitting effect.
Locally, the longitudinal polarization on transverse momentum plane, from model simulations of both a multiple phase transport (AMPT) model [20] and the hydrodynamic simulations [21], exhibits opposite signature to the experimentally observed quadrupolar structure. However, our recent work [22] using the PICR hydrodynamic model to calculate the polarization at 200GeV Au-Au, shows a fairly good agreement to the experimentally observed longitudinal polarization, in two aspects: 1) the sign distribution is (+, +, -, -) counting from the first quadrant to fourth quadrant of transverse momentum coordinate, 2) the peak value at transverse momentum $p_t = 1.4$ GeV has similar magnitude with the global polarization.

Besides, some interesting points emerge: the longitudinal polarization in our model was found to increase with collision energy, which differs with the energy dependence of the global polarization and contradicts a previous prediction [21] that the z-directed polarization tends to decrease with collision energy. In our model the first term in the polarization vector formula arising from the classical thermal vorticity, still has a sign structure of (-, +, -, +), but will be suppressed by the second term’s opposite signature and resulting in a smaller but correct magnitude. Thus, in this paper, we are going to explore the classical thermal vorticity in the beam direction, showing its spatial structure, time evolution and magnitude changing tendency, etc.

Therefore, in this paper we want to explore the above two puzzles in the $\Lambda$ polarization studies. The paper is organized as follows. Firstly, we revisit the theory in Ref. [19], and then use the PICR hydrodynamic model to simulate and calculate the polarization splitting at different energies $\sqrt{s_{NN}} = 7.7 - 200$ GeV. The averaged thermal vorticity along the $-y$ direction at freeze out as a function of energy are also shown. Secondly, subsequent to our previous work [22], we study the classical thermal vorticity along the beam direction. Finally, a summary is drawn. Throughout this paper, we use the natural units: $\hbar = c = k_B = 1$.

## 2 Polarization splitting induced by baryonic vorticity

### 2.1 Revisiting the theory

Considering the strong interaction of any fermions mediated by any bosonic fields, one could always write down a general equation of Lagrangian density

$$\mathcal{L} = \mathcal{L}_f + \mathcal{L}_b + \mathcal{L}_{int} ,$$

where $\mathcal{L}_f$ denotes the Lagrangian density for the fermions, $\mathcal{L}_b$ represents the Lagrangian density for the bosons, and $\mathcal{L}_{int}$ is the interaction Lagrangian density between them. In a simplest case, this equation can be written as:

$$\mathcal{L} = \sum_1^g \bar{\psi}_i (i\gamma^\mu \partial_\mu - m_i + f_\sigma g_\sigma \sigma - f_V g_V V) \psi_i + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} V^\mu\nu V_\mu V_\nu + \frac{1}{2} m_V^2 V^\mu V_\mu ,$$

where the first line corresponds to $(\mathcal{L}_f + \mathcal{L}_{int})$, denoting the Lagrangian density of Dirac field for fermions with a Yuwaka interaction coupling. The second line corresponds to $\mathcal{L}_b$, being the Lagrangian density for the scalar boson $\sigma$ and vector boson $V_\mu$. Here, $g_\sigma$ is the coupling constant between fermion $\psi_i$ (of species $i$) and the scalar boson $\sigma$, and $g_V$ is the coupling constant between the fermion $\psi_i$ and the vector boson $V_\mu$. $m_i$, $m_\sigma$ and $m_V$ are respectively the mass of baryon, scalar meson and vector meson. The vector meson tensor is: $V^{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. The two constants, $f_\sigma$ and $f_V$ in the Yuwaka interaction term are parameters that should be determined case by case.

In relativistic heavy ion collisions, the hyperons are created at the chemical freeze out and then interact with other particles during the hadronic scattering phase. Given that the strong interaction of hyperons (including hyperons) with other particles is mediated by a scalar meson field $\sigma$ and a vector meson field $V^\mu$, then with the constants $f_\sigma = f_V = 1$, and following from the Euler-Lagrange equations, one finds the equations of motion for these fields:

$$[\gamma^\mu (i\partial_\mu - g_V V_\mu) - (m_i - g_\sigma \sigma)] \psi = 0 , \quad \partial_\mu V^{\mu\nu} + m_V^2 V^\nu = \sum_i g_V J_i^\nu ,$$

where $n_{yi} = \langle \bar{\psi}_i \psi_i \rangle$ is the scalar density of species $i$, and $J_i^\mu = \langle \bar{\psi}_i \gamma^\mu \psi_i \rangle$ is the baryon current of species $i$. These equations are actually the Dirac field equations with scalar and vector field coupling, the Klein-Gordon equation and the Proca equations. The detailed treatments of the above three equations has been demonstrated in Ref. [19]. In this subsection we will simply revisit the theory and modify it in a way that is suitable to our simulations.

For the Proca equation [5], analogous to Maxwell equations of massless photon field, it could be decomposed into Maxwell-Proca equations for vector mesons

$$\mathbf{\nabla} \cdot \mathbf{E}_V = \bar{g}_V \rho - \rho_\sigma V_0 , \quad \mathbf{\nabla} \cdot \mathbf{B}_V = 0 \quad (6)$$
$$\mathbf{\nabla} \times \mathbf{E}_V + \frac{\partial \mathbf{B}_V}{\partial t} = 0 , \quad \mathbf{\nabla} \times \mathbf{B}_V - \frac{\partial \mathbf{E}_V}{\partial t} = \bar{g}_V \mathbf{J}_B + m_V \mathbf{V}, \quad (7)$$

where $\bar{g}_V$ is the mean coupling constant of vector meson, the baryon density is $\rho_B = \sum_i \psi_i^\dagger \psi_i$ and the baryon (three-)current is $\mathbf{J}_B$. These are components of the baryon (four-)current $\mathbf{J}_i^\mu = \langle \bar{\psi}_i \gamma^\mu \psi_i \rangle$. Here the $\mathbf{E}_V$ and $\mathbf{B}_V$ are the ‘electric’ and ‘magnetic’ components of the vector meson field, defined as:

$$E_i \equiv V_i - \partial_i V_0 - \partial_0 V_i = (\mathbf{\nabla} V_0 - \frac{\partial V}{\partial t} ,) , \quad (8)$$
$$B_i \equiv -\frac{1}{2} \varepsilon_{ijk} V^{jk} = -\frac{1}{2} \varepsilon_{ijk} (\partial^i V^k - \partial^k V^i) = (\mathbf{\nabla} \times \mathbf{V}) , \quad (9)$$
where $i, j, k = 1, 2, 3$. Let us take the curl of Maxwell-Proca equations (7), and we obtain

$$\frac{\partial^2 E_V}{\partial t^2} - \nabla^2 E_V + m^2 E_V = - \hat{g}_V (\nabla \rho_B + \frac{\partial J_B}{\partial t}),$$

(10)

$$\frac{\partial^2 B_V}{\partial t^2} - \nabla^2 B_V + m^2 B_V = \hat{g}_V (\nabla \times J_B).$$

(11)

An analytic solution is possible, e.g. for magnetic fields:

$$B_V(x) = \int dy \hat{g}_V (\nabla \times J(y)) \int \frac{d^4p}{(2\pi)^4} e^{-ip(x-y)}$$

(12)

where $x, y$ are the space-time points, and $p, m$ are the four-momentum and mass of the vector meson $V$'s. However, a simple solution was obtained:

$$B_V = \frac{\hat{g}_V}{m_V^2} (\nabla \times J_B).$$

(13)

by neglecting the derivatives in eqs. (10, 11) due to large meson mass, $m_V = 783$ MeV and $m_\sigma = 550$ MeV. Assuming local equilibrium of the system during the hadron scattering, i.e. the small gradients, $\nabla \rho \approx 0$, then for the current $J_B = \rho_B (x,t) \nu(x,t)$,

$$\nabla \times J_B = \rho_B (\nabla \times \nu) = \rho_B \omega,$$

(14)

where $\omega$ is the vorticity of baryon current. Therefore, we can see that the vortical baryon current will induce a vector meson’s ‘magnetic’ field, which, together with the vector meson’s ‘electric’ field, follow from the Maxwell-Proca equations (6,7) and definition equations (8,9).

Then the Zeeman energy term in the Foldy-Wouthuysen (FW) Hamiltonian for the hyperon particle’s spin (with effective mass $M_H$) and the vector meson’s magnetic fields was written as [19]:

$$H_{\text{spin}} = - \frac{g_{\nu H}}{M_H} \beta \cdot B_V.$$

(15)

where it was argued that the constant matrix $\beta$ will result in the opposite signs for $A$ and $\hat{A}$, thus it may be responsible for the polarization splitting.

Supposing that spin-$1/2$ hyperons are at local equilibrium, one could add into the density matrix of the system, $\rho$, an extra term $\rho_s \sim \exp (\hat{S} \cdot \hat{\Omega}/T)$, where $\Omega = \mu_B V/\nu$ is the vector meson’s ‘magnetic moment’ with $\mu = -(g_{\nu H}/M_H)\beta$ being the ‘magneton’. The ensemble average of the spin vector of spin-$1/2$ particles are given as $\bar{S} = \text{tr}(\rho S)$ where $S$ is the spin operator. Then the ensemble averaged polarization vector can be obtained as [23]:

$$P = 2 \bar{S} = \tanh \left( \frac{\Omega}{2T} \right) \hat{\Omega} \simeq \frac{\Omega}{2T} = - \frac{g_{\nu H} \beta V}{M_H T},$$

(16)

where $\hat{\Omega}$ is the unit vector along $\Omega$ direction. Taking eqs. (13) and (14) into the above equation, the polarization splitting would be

$$\Delta P = P_H - P_{\hat{H}} = \frac{g_{\nu H} \hat{g}_V}{M_H m^2_V} \rho_B \omega.$$

(17)

Hence, if the baryons in high energy nuclear collisions have a vortical flow motion, the scalar and vector meson interactions given above can provide a mechanism for hyperon polarization splitting.

### 2.2 Polarization splitting at $\sqrt{s_{NN}} = 7.7 - 200$ GeV

The nucleus-nucleus impact in our initial state is divided into many slab-slab collisions, and Yang-Mills flux-tubes. These are assumed to form streaks [24,25]. In this scenario, the initial state naturally generates longitudinal velocity shear flow, which when placed into the subsequent high resolution (3+1)D Particle-in-Cell Relativistic (PICR) hydrodynamic model, will develop into substantial vorticity. Since our initial state+hydrodynamic model describes the shear and vorticity in heavy ion collisions fairly well, its simulations to the $\Lambda$ polarization was also successful.

Therefore, we use the PICR hydrodynamic model to simulate the Au+Au collisions at RHIC BES energy region $\sqrt{s_{NN}} = 7.7 - 200$ GeV, and calculate the polarization difference between the $\Lambda$s and $\bar{\Lambda}$s, based on eq. [17]. The coefficients in eq. [17] are kept the same as in Ref. [19]: $M_A = 1115.6$ MeV, $M_V = 780$ MeV, $\hat{g}_V = 5$, and $g_{\nu A} \approx 0.55g_{\nu V} \approx 4.76$.

![Fig. 1](image-url) (Color online) The polarization difference between $\Lambda$s and $\bar{\Lambda}$s for Au+Au collisions at $\sqrt{s_{NN}} = 7.7 - 200$ GeV with impact parameter ratio $b_0 = 0.7$. The black squares represent the polarization difference as a function of collision energy, with the freeze-out time being fixed to $t_{\text{FZ}} = 7.24$ fm/c. The red circles correspond to the case of varied freeze-out time, i.e. $t = 5.9 - 7.9$ fm/c for the energy range of $\sqrt{s} = 7.7 - 200$ GeV. The experimental data denoted by cross symbols with error bars are extracted from Ref. [17].

For the purpose of continuity, we do not perform a new simulation, but just use the same data in our previous Rapid Communication [20], which was then the first
work to show the energy dependence of global polarization $\Pi_{\text{by}}$, and exhibited fairly good agreement with the experimental data. In that work, the simulation parameters were set as follows: the impact parameter ratio is $b_0 = b / b_{\text{max}} = 0.7$, (where $b$ is the impact parameter and $b_{\text{max}}$ is the maximum impact parameter); the cell size is $0.343^3$ fm$^3$, the time increment is 0.0423 fm/c; the freeze-out time is chosen as $7.24$ fm/c = $2.5 + 4.74$ fm/c (2.5 fm/c for the initial state's stopping time and 4.74 fm/c corresponds to the hydro-evolution time).

The simulation and calculation results are shown in Fig. 1 which exhibits the polarization difference between $\Lambda$ and $\bar{\Lambda}$ for Au+Au collisions at $\sqrt{s_{NN}} = 7.7 – 200$ GeV. The black and red symbols respectively represent the polarization difference with freeze-out time $t_{FZ}$ being fixed to $7.24$ fm/c, and with varied freeze-out time increasing from 5.9 to 7.9 fm/c (when collision energy increases from 7.7 to 200 GeV). The experimental data denoted by cross symbols with error bars are extracted from Ref. [17]. Obviously, our simulation results, by using the PICR hydrodynamic model, show good agreement with the STAR’s experimental data.

For the case of 7.7 GeV, the difference from our model could be as significant as 2.3%, which is more than 2 times larger than the upper boundary estimate in Ref. [18]. However, our value of 2.3% is still smaller than the lower boundary of experimental measurement of 3%. Presently several mechanisms were proposed, and quantitative calculations were performed, to explain and the $\Lambda$ and $\bar{\Lambda}$ polarization splitting [19,18,27,28], but none of them can achieve 3% difference at 7.7 GeV. If the experimental results are true, this might indicate the polarization splitting phenomenon is not induced by a single effect and needs a combined theory to explain.

Specifically, the thermal vorticity $\varpi_y$ in our model for $\sqrt{s} = 39\text{-}200$ GeV is about 0.5 - 0.6, corresponding to the vorticity value of 0.15 - 0.25 in AMPT model. Due to the large magnitude of vorticity created in our model, the

$$\langle \varpi_y \rangle = \langle \nabla \times \frac{h\vec{\nu}}{T} \rangle_y, \quad (18)$$

as a function of collision energy ($h$ was absorbed into the calculation to have a dimensionless vorticity). It is not surprising that the $y$-directed thermal vorticity decreases with the collision energy, but the magnitude of the thermal vorticity in our model is larger than that from the AMPT simulation [29,30].

Fig. 2 shows the thermal vorticity along the $y$ direction, (averaged over the whole volume.)

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calculated splitting effect could be as significant as the experimental data, as demonstrated in Fig. 1.

3 Thermal vorticity along the beam direction

The longitudinal polarization as a function of azimuthal angle was observed to have a sine structure [17,32], as predicted by theory and simulations [33,21], that the longitudinal polarization at central rapidity shows a quadrupolar structure on transverse momentum plane.

One could see that, it is until \( t=7.24 \text{ fm/c} \), i.e. the freeze-out time, that the \( \varpi_z \) shows a quadrupolar structure on the transverse space plane. Before freeze-out at \( t = 3.5 \text{ fm/c} \), the quadrupolar structure has not formed yet, and after freeze-out at \( t = 9.27 \text{ fm/c} \), the quadrupolar structure is significantly weakens, and tends to vanish. Another interesting point is that, it seems that the \( z \)-directed thermal vorticity’s magnitude peaks around the freeze-out time, i.e. either pre- or post-freeze-out, its magnitude is smaller. (We want to clarify that the freeze-out time in our model can be chosen freely, but in order to fit the global \( \Lambda \) polarization value measured at STAR 200GeV Au-Au collisions, the freeze-out time should be around \( t=7.24 \text{ fm/c} \)).

Then we also extract the maximum value of \( \varpi_z \) on the transverse plane \([x, y]\) at freeze-out for different energies \( \sqrt{s} = 11.5, 39, 200 \text{ GeV} \), as shown in Fig. 5. This figure indicates that the thermal vorticity along the beam direction increases with collision energy, which differs from the energy dependence of \( y \)-directed vorticity as shown in Fig. 2. However, this is in line with our previous prediction that the longitudinal polarization’s magnitude increases with collision energy [22]. If our results are true, the longitudinal polarization could be a non-trivial signal in higher energy collisions beyond RHIC energy region.

4 Summary and Conclusion

With the PICR hydrodynamic model, we study thermal vorticity along the \( y \) direction, \( \varpi_y \), and along the beam direction, \( \varpi_z \), as a function of collision energy. We found that the \( y \)-directed thermal vorticity, \( \varpi_y \), is larger than the...
one from the AMPT model, and when applied to calculate the polarization splitting between \( A \) and \( \bar{A} \), the results fit to the experimental data fairly well. We also confirm that the \( z \)-directed thermal vorticity \( \varpi_z \) has a quadrupolar structure on transverse space plane, but interestingly the quadrupolar structure takes time to form and will be significantly weaken at late freeze-out. Besides, we find that the magnitude of \( z \)-directed thermal vorticity, \( \varpi_z \), actually increases with the collision energy.

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