\[ B \to \tau \nu \text{ in Two-Higgs doublet models with MFV} \]

Gianluca Blankenburg\(^1\), Gino Isidori\(^2\)

\(^1\) Dipartimento di Fisica, Università di Roma Tre, Via della Vasca Navale 84, I-00146 Roma, Italy
\(^2\) INFN, Laboratori Nazionali di Frascati, Via E. Fermi 40, I-00044 Frascati, Italy

Received: date / Revised version: date

Abstract. We analyse the \( B \to \tau \nu \) decay in a generic two-Higgs doublet model satisfying the MFV hypothesis. In particular, we analyse under which conditions \( \mathcal{B}(B \to \tau \nu) \) can be substantially enhanced over its SM value, taking into account the constraints of \( K \to \mu\nu \), \( B \to X\gamma \), and \( B_s \to \mu^+\mu^- \). We find that for large \( \tan \beta \) values and Peccei-Quinn symmetry breaking terms of \( \mathcal{O}(1/\tan \beta) \) a sizable (~50%) enhancement of \( \mathcal{B}(B \to \tau \nu) \) is possible, even for \( m_H \sim 1 \text{ TeV} \).

PACS. 13.20.He 12.60.-i

1 Introduction

The leptonic decays \( B \to \ell \nu \) are particularly interesting probes of the Higgs sector and, particularly, of the Yukawa interaction. The \( \tau \) channel is the only decay mode of this type observed so far. In the experimental world average \( \bar{\mathcal{B}}(B \to \tau \nu) = (1.64 \pm 0.34) \times 10^{-4} \),

\[ \bar{\mathcal{B}}(B \to \tau \nu) = (1.64 \pm 0.34) \times 10^{-4}, \quad (1) \]

has to be compared with the SM prediction

\[ \bar{\mathcal{B}}(B \to \tau \nu)_{\text{SM}} = \frac{G_F^2 m_B m_{\tau}^2}{8 \pi^2} \left( 1 - \frac{m_{\tau}^2}{m_B^2} \right)^2 f_B^2 |V_{ub}|^2 \tau_B, \quad (2) \]

whose uncertainty is mainly due to the determination of \( |V_{ub}| \) and \( f_B \). Using the best fit values of \( |V_{ub}| \) from global CKM fits, the UTfit \(^2\) and CKMfitter \(^3\) collaborations quote

\[ \mathcal{B}(B \to \tau \nu)_{\text{SM}} = (0.79 \pm 0.07) \times 10^{-4} \quad \text{[UTfit]}, \]
\[ \mathcal{B}(B \to \tau \nu)_{\text{SM}} = (0.76 \pm 0.08) \times 10^{-4} \quad \text{[CKMfitter]}. \]

These low values correspond to a 2.5(2.8) \( \sigma \) deviation from the experimental result in Eq. (1). Motivated by this discrepancy, we analyse in this paper the predictions of \( B \to \tau \nu \) in models with an extended Higgs sector.

The helicity suppression in Eq. (2) is a manifestation of the key role played by the Yukawa interaction in the \( B \to \tau \nu \) decay amplitude. For instance, in models with two Higgs doublets coupled separately to up- and down-type quarks (2HDM-II models), the \( B \to \tau \nu \) amplitude receives an additional tree-level contribution from the heavy charged-Higgs exchange, leading to

\[ \frac{\mathcal{B}^{\text{2HDM-II}}(B \to \tau \nu)}{\mathcal{B}^{\text{SM}}(B \to \tau \nu)} = \left[ 1 - \frac{m_{\tau}^2 \tan^2 \beta}{m_H^2} \right]^2, \]

where \( \tan \beta = v_2/v_1 \) is the ratio of the two Higgs vacuum expectation values and \( m_H \) is the charged-Higgs boson mass. For large \( \tan \beta \) values the ratio in Eq. (3) can be substantially different from one. However, within this simple framework the interference sign of SM and non-standard amplitudes is fixed. Taking into account the constraints on \( m_H \) from other processes, this sign implies a suppression of \( \mathcal{B}(B \to \tau \nu) \) in the 2HDM-II compared to the SM, worsening the comparison with the experimental result in Eq. (1). The sign of SM and non-SM contributions can be varied in generic models with new sources of flavour-symmetry breaking. However, in this case the problem is how to justify the good agreement between SM predictions and observations in most other flavour-violating observables.

A consistent assumption usually invoked to avoid large deviations from the SM in flavour-violating processes is the so-called Minimal Flavor Violation (MFV) hypothesis \(^5\) (see also \(^6\)). According to this hypothesis, the quark Yukawa couplings are the only sources of quark flavour-symmetry breaking both within and beyond the SM. As recently discussed in \(^9\)\(^10\), the MFV hypothesis applied to two-Higgs doublet models not only provides a sufficient protection of FCNCs in case of extra scalar degrees of freedom, it can also provide an explanation of the existing tensions in \( \Delta F = 2 \) observables. More explicitly, it has been show that two-Higgs doublet models respecting the MFV hypothesis with the inclusion of flavour-blind CP-violating (CPV) phases (dubbed 2HDM\(_{\text{MFV}}\) framework), can accommodate a large CPV phase in \( B_s \) mixing softening in a correlated manner the observed anomaly in the relation between \( \varepsilon_K \) and \( S_{\psi K_S} \). In this work we analyse under which conditions, within the 2HDM\(_{\text{MFV}}\) framework, the interference sign of SM and non-standard amplitudes can be varied to accommodate the experimental result in Eq. (1).
framework, \( B(B \to \tau\nu) \) can be enhanced over its SM prediction.\(^1\)

2 The 2HDM\(_{\text{MFV}}\) framework

We consider a model with two Higgs fields, \( H_{1,2} \), with opposite hypercharge \( (Y = \pm 1/2) \). The generic form of the Yukawa interaction for such a Higgs sector is

\[
- \mathcal{L}^\text{gen} = \bar{Q}_L X_{d1} D_R H_1 + \bar{Q}_L X_{u2} U_R H_2^c + \bar{Q}_L X_{d2} D_R H_2^c + \bar{Q}_L X_{u1} U_R H_2 + \text{h.c.},
\]

where \( H_2^c \equiv -i\tau_2 H_1^* \). The two real vacuum expectation values (vevs) are defined as \( \langle H_1^\dagger \rangle H_1 \approx v_1^2/2 \), with \( v^2 = v_1^2 + v_2^2 \approx (246 \text{ GeV})^2 \), and, as anticipated, \( \tan \beta = v_2/v_1 \).

The \( X_i \) are 3 \times 3 matrices in flavour space. The general structure implied by the MFV hypothesis for these matrices is a polynomial expansions in terms of the two (left-handed) spurions \( Y_u Y^\dagger_1 \) and \( Y_d Y^\dagger_2 \) [5,9]:

\[
X_{d1} \equiv Y_d ,
X_{d2} = \epsilon_0 Y_d + \epsilon_1 Y_d Y^\dagger_1 Y_d + \epsilon_2 Y_u Y^\dagger_2 Y_d + \ldots ,
X_{u2} \equiv Y_u ,
X_{u1} = \epsilon_0 Y_u + \epsilon_1 Y_d Y^\dagger_1 Y_u + \epsilon_2 Y_u Y^\dagger_2 Y_u + \ldots ,
\]

where the \( \epsilon^{(i)} \) are complex parameters. We work under the assumption \( \epsilon^{(i)} \ll 1 \), as expected by an approximate \( U(1)_{\text{PQ}} \) symmetry that forbids non-vanishing \( X_{u1,2} \) at the tree level, and we assume \( \tan \beta = t_\beta = \delta s_\beta/c_\beta \gg 1 \). For simplicity, we also restrict the attention to terms with at most three Yukawa couplings in this expansion (namely we consider only the terms explicitly shown above) and we assume real \( \epsilon^{(i)} \) since we are interested only in CP-conserving observables. Finally, we assume negligible violations of the \( U(1)_{\text{PQ}} \) symmetry in the lepton Yukawa couplings.

After diagonalizing quark mass terms and rotating the Higgs fields such that only one doublet has a non-vanishing vev, the interaction of down-type quarks with the neutral Higgs fields assumes the form

\[
\mathcal{L}^d_{\text{a.c.}} = -\sqrt{2} \bar{d}_L Z^d I a d_R \phi_0^0 - \frac{1}{s_\beta} \bar{d}_L Z^d \lambda^d d_R \phi_0^0 + \text{h.c.},
\]

where \( \phi_0 \) is the linear combination of \( H_{1,2} \) with non-vanishing (vanishing) vev \( \langle \phi_0^0 \rangle = v/\sqrt{2} \) (\( \langle \phi_0^0 \rangle = 0 \)). The flavour structure of the \( Z^d \) couplings is

\[
Z^d_{ij} = \bar{a} \delta_{ij} + \left[ \bar{a} a V^\dagger \lambda^d_0 V + a_1 a V^\dagger \Delta^2 V + a_2 a V^\dagger V^\dagger V \right]_{ij} ,
\]

where \( V \) is the physical CKM matrix, \( \Delta \equiv \text{diag}(0,0,1) \), and \( \lambda^d_0,a \) are the diagonal Yukawa couplings in the limit of unbroken \( U(1)_{\text{PQ}} \) symmetry, and the \( a_i \) are flavour-blind coefficients (see [5,9] for notations). Similarly, the interaction of the quarks with the physical charged Higgs is described by the following flavour changing effective Lagrangian [5]:

\[
\mathcal{L}_{H^+} = \left[ \bar{U}_L C^H_R \lambda_d D_R + \frac{1}{\tan \beta} \bar{U}_R \lambda_u C^H_L D_L \right] H^+ + \text{h.c.},
\]

where the flavour structure of \( C^H_{R,L} \) is

\[
C^H_{R} = (b_0 V + b_1 V \Delta + b_2 V + b_3 V) ,
C^H_{L} = (b'_0 V + b'_1 V \Delta + b'_2 V + b'_3 V) ,
\]

and the \( b^{(i)}_i \) are flavour-blind coefficients. As explicitly given in [5,9], the \( a_i \), and \( b^{(i)}_i \) depend on the \( \epsilon^{(i)} \), on \( \tan \beta \), and on the overall normalization of the Yukawa couplings. Even if \( \epsilon^{(i)} \ll 1 \), the \( a_i \) and \( b_i \) can reach values of \( \mathcal{O}(1) \) at large \( \tan \beta \) and can be complex, since we allow flavour-blind phases in the model.

2.1 \( B(B \to \tau\nu) \) and other observables

We present here the theoretical expressions of \( B(B \to \tau\nu) \) and a series of other flavour-violating observables, necessary to set bounds on the parameter space, in the 2HDM\(_{\text{MFV}}\) framework.

In order to simplify the notations, we absorb terms proportional to the top and bottom Yukawa coupling into the definition of \( \epsilon^{(i)} \). More explicitly, we redefine \( \epsilon^{(i)}_{1,2} \) as follows:

\[
\epsilon^{(i)}_{1} y_{\tau}^0 \rightarrow \epsilon^{(i)}_{1} ,
\epsilon^{(i)}_{2} y_{\mu}^0 \rightarrow \epsilon^{(i)}_{2} .
\]

With such a notation, the \( b_{R \to u_L} \) and \( s_{R \to u_L} \) interactions with the physical charged Higgs are

\[
\begin{align*}
\mathcal{L}_{b,s \to u}^{b,s \to u} &= m_b \tan \beta \frac{1}{v} \bar{V}_{ub} V_{ub} \frac{1}{1 + (\epsilon_0 + \epsilon_1) \tan \beta} \bar{u}_L b_R H^+ \\
&\quad + m_s \tan \beta \frac{1}{v} \bar{V}_{us} V_{us} \frac{1}{1 + \epsilon_0 \tan \beta} \bar{u}_L s_R H^+ + \text{h.c.}. \quad (10)
\end{align*}
\]

This allows us to derive the following expression for the modification of \( B(B \to \tau\nu) \), relative to the SM, within this framework:

\[
R_{B_{\tau\nu}} = \frac{B(B \to \tau\nu)}{B_{\text{SM}}(B \to \tau\nu)} = 1 - \left( \frac{m_H^2}{m_H^2 + (\epsilon_0 + \epsilon_1) \tan \beta} \right)^2 , \quad (11)
\]

A closely related observable which provide a significant constraint on the parameter space is \( B(K \to \mu\nu) \). In this case from [10] we find

\[
R_{K_{\mu\nu}} = \frac{B(K \to \mu\nu)}{B_{\text{SM}}(K \to \mu\nu)} = 1 - \left( \frac{m_K^2}{m_K^2 + \epsilon_0 \tan \beta} \right)^2 . \quad (12)
\]

---

\(^1\) Recent analyses of \( B \to \tau\nu \) in different models with an extended Higgs sector can be found in Ref. [14,16].
Beside semileptonic charged currents, stringent constraints on the 2HDM parameter space are provided also by the flavour-changing-neutral-current (FCNC) transitions $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$. In principle, also the $B_s - B_s$ mixing amplitude could be used to constrain the parameter space of the model; however, as we will discuss below, it turns out that $B_s - B_s$ constraints are automatically satisfied after imposing the bounds from $B_s \to \mu^+ \mu^-$. In order to implement these bounds, we introduce the FCNC Hamiltonian

$$\mathcal{L}_{b \to s} = -\frac{G_F \alpha_{em}}{2 \sqrt{2} \pi \sin^2 \theta_W} V_{ts}^* V_{tb} \sum_n Q_n \mathcal{Q}_n + \text{h.c.} ,$$

where

$$Q_7 = \frac{e}{\sqrt{2}} m_b s \sigma_{\mu \nu} (1 + \gamma_5) b F_{\mu \nu} ,$$

$$Q_S^{\mu} = s (1 + \gamma_5) b \bar{\mu} (1 - \gamma_5) \mu ,$$

and the complete list of effective operators can be found in Ref. [5]. Following Ref. [17], the experimental constraints on $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$ can effectively be encoded into constraints on $C_7$ and $C_S^{\mu}$. More precisely, we can translate the experimental data into bounds on $\delta C_7 = C_7 (M_W^2) - C_7^{\text{SM}} (M_W^2)$ and $\delta C_S^{\mu} = C_S^{\mu} (M_W^2) - C_S^{\mu, \text{SM}} (M_W^2)$. Working under the hypothesis that the only relevant non-standard contributions are those associated to the heavy Higgs fields, the dominant contributions to $\delta C_7$ are the one-loop contributions from both charged and neutral Higgs exchange. Adopting to our notations the results of Ref. [5] we have

$$\delta C_7 = \frac{1}{D_{012}} \left[ 1 + (\epsilon'_0 + \epsilon'_2) \tan \beta - \frac{\epsilon_2 \epsilon'_1 \tan^2 \beta}{D_{01}} F_7 (x_{qH}^2) \right]$$

$$-\frac{\epsilon_2 \tan^3 \beta \bar{x}_{qH}^2}{D_{012} D_{01}} 36$$

where $x_{qH} = m_{12}^2 / m_{qH}^2$, $D_{012} = 1 + (\epsilon_0 + \epsilon_1 + \epsilon_2) \tan \beta$, $D_{01} = 1 + (\epsilon_0 + \epsilon_1) \tan \beta$, and $F_7(x)$ is defined as in [5].

The effective coupling of $Q_S^{\mu}$ receives contributions from the FCNC component of $\delta C_7$ already at the tree-level:

$$\delta C_S^{\mu} = \frac{m_{q_H}^2 2 \pi \sin^2 \theta_W \epsilon_2 \tan^3 \beta}{m_{q_H}^2} \frac{\alpha_{em}}{D_{01} D_{012}} .$$

### 3 Phenomenological analysis

We are now ready to analyse the parameter space of the model, searching for regions where $\mathcal{B}(B \to \tau \nu)$ is enhanced over its SM prediction and the other low-energy constraints are satisfied. Since the main observables used in CKM fits receive tiny corrections from the extended Higgs sector, we assume that the standard CKM determination remains valid.

The low-energy phenomenological constraints used in our analysis are

$$R_{K_{\ell \nu}} \in (0.98, 1.02) ,$$

$$\delta C_7 \in (-0.14, 0.06) ,$$

$$\delta C_S^{\mu} \in (-0.09, 0.09) .$$

The first input follows from the analysis of semileptonic $K$ decays in [18] (see also [19]), while the range of $\delta C_7$ and $\delta C_S^{\mu}$ follows from the analysis of $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$ performed in [17]. Moreover, since we are interested in substantial enhancements of $\mathcal{B}(B \to \tau \nu)$, we impose

$$R_{B_{\tau \nu}} > 1.2 .$$

On the other hand, given the condition $\epsilon_i \ll 1$ expected from an approximate $U(1)_{PQ}$ symmetry, we will restrict the free parameters of the model to vary in the following interval:

$$\tan \beta \in [40, 60) ,$$

$$m_{H^\pm} [\text{GeV}] \in (150, 1000) ,$$

$$\epsilon_i \tan \beta \in (-2, 2) .$$

#### 3.1 Analytical considerations

In principle the model has enough parameters that allow us to satisfy the three conditions in Eqs. (19) and, at the same time, get the desired enhancement in $B \to \tau \nu$, provided we properly tunes the values of $\epsilon_i \propto \tan \beta$. However, we are not interest in fine-tuned solutions. In particular, while it is natural setting to zero some of the $\epsilon_i$, which are symmetry breaking terms, we consider not natural fine-tuned solutions corresponding to large values of $\epsilon_i \tan \beta$. In this perspective, taking into account the theoretical expressions for the observables presented in the previous section, we find that:

i. Since $\delta C_S^{\mu} \propto \epsilon_2$, the bound from $B_s \to \mu^+ \mu^-$ can easily be satisfied assuming $\epsilon_2 \approx 0$. This “natural” tuning (according to the discussion above) allow us to decouple charged-Higgs and neutral-Higgs flavour-changing amplitudes. Incidentally, this is why we do not get additional significant constraints from $B_s - B_s$ mixing.

ii. Contrary to $B_s \to \mu^+ \mu^-$, we cannot get rid of the $B \to X_s \gamma$ bound without some amount of fine tuning. In particular, setting $\epsilon_2 \approx 0$, the charged-Higgs contribution to $B \to X_s \gamma$ vanishes completely only under the fine-tuned condition

$$\epsilon_2 \tan \beta = -1 .$$

Before analysing how far from the fine-tuned condition in Eq. (22) we can move, it is worth discussing the correlation between $B \to \tau \nu$ and $K \to \mu \nu$ ignoring all other constraints.
In the case of $K \to \mu \nu$, the Higgs-mediated amplitude is always much smaller that the SM one. Imposing $R_{K\mu\nu} \in [0.98, 1.02]$ implies
\[ |\epsilon_0 \tan \beta + 1| > 0.9 \times r \] (23)
where $r = m_1^2 \tan^2 \beta / m_H^2$. For the chosen range of $\tan \beta$ and $m_H$ we have $0.04 < r < 4.3$. For small values of $r$ the above condition is very natural; we only exclude a narrow region around the (unnatural) point $\epsilon_0 \tan \beta = -1$. On the other hand, for growing values of $r$ we are pushed toward a fine tuned configuration. We thus conclude that the $K \to \mu \nu$ bound points toward small values of $r$.

Two solution are possible to generate an enhancement of $B(B \to \tau \nu)$: a destructive interference of SM and charged-Higgs amplitudes, if the latter has been pointed out first in Ref. [13]. However, in the latter work the correlation with the other observables has not been considered in Ref. [16]. The importance of the role of the $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$ bounds, we have plotted with a different color (grey point) the region of parameter space that can be reached only in the fine-tuned configuration where $\epsilon_2 = 0$ and $\epsilon_0, 2$ are fixed to satisfy the condition (22). The black contours mark equally-populated areas resulting from the global sampling (without fine-tuned conditions): the three most inner contours include 50% of the points.

The results of this numerical analysis are shown in Fig. 1. In Fig. 1 we show the points satisfying all constraints in Eqs. (19)–(20). To better quantify the role of the $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$ bounds, we have plotted with a different color (grey point) the region of parameter space that can be reached only in the fine-tuned configuration where $\epsilon_2 = 0$ and $\epsilon_0, 2$ are fixed to satisfy the condition (22). As can be seen from Fig. 1 in general there is no significant constraint on $m_H$ and $\tan \beta$; however, low values of $m_H$ can be obtained only in the in the fine-tuned configuration.

At this point it is worth to comment on the bounds in the $m_H$–$\tan \beta$ plane by direct searches for heavy Higgs bosons at the LHC [12]. A direct implementation of these constraints in our frameworks is not possible, given the former are obtained in the limit limit $\epsilon_i^{(t)} = 0$. Still, it is worth to note that in this limit direct searches set the approximate bound $m_H \gtrsim 420 \text{ GeV} + 6 \times (\tan \beta - 40)$ [12], which does not represent a problem for most of the points in Fig. 1. Only the fine-tuned (gray) points are potentially affected by this constraint, which thus provide a further argument against the tuned configuration with low $m_H$.

In Fig. 2 we show the points satisfying all constraints in the $\epsilon_0$–$\epsilon_1$ plane. We also show the line $1 + (\epsilon_0 + \epsilon_1) \tan \beta = 0$, separating the region of destructive interference (above the line) and constructive interference (below the line) in $B \to \tau \nu$. As can be seen, the region of constructive interference is reached essentially only in the fine-tuned configuration where $\epsilon_2$ and $\epsilon_0, 2$ are fixed to eliminate any non-standard contribution to $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$. Indeed in this region we need large values of $m_H$, that would get in conflict with $B \to X_s \gamma$ and $B_s \to \mu^+ \mu^-$ for generic values of $\epsilon_2$ and $\epsilon_0, 2$. On the other hand, the region of constructive interference is densely populated even in absence of a fine-tuning on $\epsilon_2$ and $\epsilon_0, 2$. As anticipated in the analytical discussion, the absence of points for $\epsilon_0 \tan \beta$.

**Diagram:**

- **Fig. 1.** Allowed regions in the $\tan \beta$–$m_H$ plane. The grey points correspond to regions of the parameter space that can be reached only in the fine-tuned configuration where $\epsilon_2 = 0$ and $\epsilon_0, 2$ are fixed to satisfy the condition (22). The black contours mark equally-populated areas resulting from the global sampling (without fine-tuned conditions): the three most inner contours include 50% of the points.
close to -1 is a consequence of the $K \rightarrow \mu \nu$ bound. Last but not least, we stress the absence of points near the $U(1)_{PQ}$ symmetric point $\epsilon_0 = \epsilon_1 = 0$. This is a simple consequence of combining the $K \rightarrow \mu \nu$ and $B \rightarrow \mu \nu$ constraints in Eqs. (23)–(25).

In Fig. 2 and 3 we show the correlation of $R_{B\tau \nu}$ with the two most significant constraints, namely $B(K \rightarrow \mu \nu)$ and $\delta C_7$ (or $B \rightarrow X_s \gamma$) (in these two figures we do not plot with different colors the fine-tuned points). As can be seen from Fig. 2 the interference between SM and charged-Higgs amplitudes is necessarily destructive in $B \rightarrow X_s \gamma$ (we recall that $C_7^{SM} < 0$). On the other hand, both positive and negative interferences in $K \rightarrow \mu \nu$ are possible, depending on the sign of $1 + \epsilon_0 \tan \beta$. As illustrated in Fig. 2, if the maximal deviation from the SM in $B(K \rightarrow \mu \nu)$ could be reduced to 1%, the parameter space leading to an enhancement of $B(B \rightarrow \tau \nu)$ would be strongly reduced. This also implies that if the precision on $B(K \rightarrow \mu \nu)$ will improve, there are realistic chances to see a deviation from the SM in this mode within this framework. On the contrary, we have checked that for $R_{B\tau \nu} < 2$ the deviations from the SM predictions in $B(B \rightarrow D\tau \nu)$ do not exceed the 20% level, well within the present theoretical and experimental uncertainties.

As a final check of the stability of our findings, we have performed scan of the parameter space allowing arbitrary complex phases for the $\epsilon_i^{(1)}$. As expected, no significant deviations in Fig. 1, 2, and 3 have been observed. Fig. 2 is unaffected provided we interpret it as the $\text{Re}(\epsilon_0) - \text{Re}(\epsilon_1)$ plane.

4 Discussion and Conclusions

Our analysis shows that is possible to accommodate sizable enhancements of $B(B \rightarrow \tau \nu)$ in the 2HDM$_{\text{MFV}}$ framework, despite the tight constraints of other low-energy observables. This is clearly illustrated by the plots discussed in the previous section. However, it must be stressed that this enhancement occurs under a few specific circumstances:

i. At least some of the $\epsilon_i \tan \beta$ must of order one, i.e. sizable deviations from the exact 2HDM-II limit, or from the limit of unbroken $U(1)_{PQ}$ symmetry in the Yukawa sector, are necessary. As shown in Fig. 2, almost no solution survive for $|\epsilon_i \tan \beta| < 0.5$. This conclusion holds independently of the simplifying assumptions on the $\epsilon_i$ adopted in the present analysis.

ii. If we assume $\epsilon_i < 1$, as realised in several explicit models where the $U(1)_{PQ}$ symmetry in the Yukawa sector is broken only by radiative corrections, the need for large $\epsilon_i \tan \beta$ necessarily imply large $\tan \beta$ values.

iii. In addition of being sizable, the $U(1)_{PQ}$ breaking terms $\epsilon_0$ and $\epsilon_1$ should conspire to suppress the combination $1 + (\epsilon_0 + \epsilon_1) \tan \beta$ appearing in the denominator of the $B \rightarrow \tau \nu$ amplitude. The more $m_H$ is large, the more fine tuning on $1 + (\epsilon_0 + \epsilon_1) \tan \beta$ is needed in order to keep the charged-Higgs amplitude at the level of the SM one.

iv. The most likely possibility to enhance $B \rightarrow \tau \nu$, especially if $m_H$ is above 200 GeV, occurs in the case of constructive interference between SM and charged Higgs amplitudes in $B \rightarrow \tau \nu$. This requires value of the $\epsilon_i$ which cannot be obtained in simplified MSSM scenarios, such as the one considered in Ref. [16], but can be obtained in less standard supersymmetric frameworks, such as the "up-lifted" scenario considered in Ref. [20].
If the above conditions are satisfied, a large enhancement of $\mathcal{B}(B \to \tau \nu)$ is compatible with the existing constraints. In absence of fine tuning, this implies non-negligible and potentially visible deviations from the SM in $B \to X_s \gamma$ and $K \to \mu \nu$. The most interesting effects are expected in $\mathcal{B}(K \to \mu \nu)$, as illustrated in Fig. 3. To this purpose, we stress that $\mathcal{B}(K \to \mu \nu)$ is presently measured with a 0.27% relative error \cite{KLOE}. If future lattice determinations of the kaon form factors could allow us to reduce the theoretical error on $\mathcal{B}(K \to \mu \nu)$ at the same level, the $\mathcal{B}(B \to \tau \nu) - \mathcal{B}(K \to \mu \nu)$ correlation would provide a useful tool to test this framework.

Acknowledgments

We thank Stefania Gori for useful comments. This work was supported by the EU ERC Advanced Grant FLAVOUR (267104), and by MIUR under contract 2008XM9HLM. G.I. acknowledges the support of the Technische Universität München – Institute for Advanced Study, funded by the German Excellence Initiative.

References

1. R. Barlow, [arXiv:1102.1267 [hep-ex]]; D. Asner et al. [Heavy Flavor Averaging Group Collaboration], [arXiv:1010.1589 [hep-ex]].
2. A. J. Bevan et al. [UTfit Collaboration], PoS ICHEP2010 (2010) 270. [arXiv:1010.5089 [hep-ph]]; Phys. Lett. B687 (2010) 61-69. [arXiv:0908.3470 [hep-ph]].
3. J. Charles et al. [CKMfitter Collaboration], [arXiv:1106.4041 [hep-ph]].
4. W. -S. Hou, Phys. Rev. D48 (1993) 2342-2344.
5. G. D’Ambrosio, G. F. Giudice, G. Isidori, A. Strumia, Nucl. Phys. B645 (2002) 155-187. [hep-ph/0207036].
6. R. S. Chivukula and H. Georgi, Phys. Lett. B 188 (1987) 99.
7. A. J. Buras, P. Gambino, M. Gorbahn, S. Jager and L. Silvestrini, Phys. Lett. B 500 (2001) 161 [arXiv:hep-ph/0007085].
8. A. L. Kagan, G. Perez, T. Volansky, J. Zupan, Phys. Rev. D80 (2009) 076002. [arXiv:0903.1794 [hep-ph]].
9. A. J. Buras, M. V. Carlucci, S. Gori, G. Isidori, JHEP 1010 (2010) 009. [arXiv:1005.5310 [hep-ph]].
10. A. J. Buras, G. Isidori, P. Paradisi, Phys. Lett. B694 (2011) 402-409. [arXiv:1007.5291 [hep-ph]].
11. J. F. Kamenik, F. Mescia, Phys. Rev. D78 (2008) 014003. [arXiv:0802.3790 [hep-ph]].
12. S. Chatrchyan et al. [CMS Collaboration], [arXiv:1202.4083 [hep-ex]].
13. A. G. Akeroyd, S. Recksiegel, J. Phys. G G29 (2003) 2311-2317. [hep-ph/0306037].
14. B. A. Dobrescu, P. J. Fox, A. Martin, Phys. Rev. Lett. 105 (2010) 041801. [arXiv:1005.4238 [hep-ph]].
15. M. Jung, A. Pich, P. Tuzon, JHEP 1011 (2010) 003. [arXiv:1006.0470 [hep-ph]].
16. B. Bhattacharjee, A. Dighe, D. Ghosh and S. Raychaudhuri, Phys. Rev. D 83 (2011) 094026 [arXiv:1012.1052 [hep-ph]].