Axial and gauge anomalies in a theory with one and two-form gauge fields

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Abstract

We study the problem of axial and gauge anomalies in a reducible theory involving vector and tensor gauge fields coupled in a topological way. We consider that vector and axial fermionic currents couple with the tensor field in the same topological manner as the vector gauge one. This kind of coupling leads to an anomalous axial current, contrarily to the results found in literature involving other tensor couplings, where no anomaly is obtained.

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1 Introduction

The interest for tensor gauge fields dates back to more than twenty years. They were considered by Kalb and Ramond \[\text{[1]}\] with the motivation that they could carry the force among string interactions. At the same time, Cremmer and Scherk \[\text{[2]}\] considered these fields coupled in a topological way with the usual vector gauge one, with the purpose in obtaining a kind of dynamical breaking of the gauge symmetry and a consequent mass generation for the vector field. This same problem has been considered nowadays in a version where the mass generation is carried out as an effective theory when the tensor field is conveniently eliminated \[\text{[3, 4, 5]}\]. We mention that antisymmetric tensor fields also appear as one of the massless solutions of string theories, in company with photons, gravitons etc. \[\text{[6]}\]

It is also opportune to mention that the particular structure of constraints involving tensor fields is an interesting subject for its own rights. They constitute a natural example of reducible theory, in a sense that the first-class constraints \[\text{[7]}\] are not all independent. Many developments have been done in this direction too \[\text{[8, 9]}\].

Our purpose in this paper is to study the problem of anomalies, where the fermionic vector and axial currents also couple to the tensor field. We consider that this coupling has the same topological nature of the vector-tensor gauge ones \[\text{[2]}\]. It is important to emphasize that this differs from the usual tensor coupling that appear in the literature, where the tensor field (considered as an external field) couples with a tensor current \[\text{[10]}\]. Using the Fujikawa path integral formalism \[\text{[11]}\], we show that this topological coupling leads to a contribution for the axial current anomaly. This result is new comparing with the ones found in literature where no contribution for the axial current anomaly is found. These developments are done at Sec. 2.

In Sec. 3, we show that the \(U(1)\) and the tensor gauge symmetries are not obstructed in the considered model. It is interesting to note, however, that if one considers chiral couplings between the vector and tensor fields with the fermionic currents, the \(U(1)\) gauge symmetry is obstructed due to anomalies. Here, to perform these calculations, we use the field-antifield formalism \[\text{[12]}\], the best known method to treat reducible theories in a covariant way. As in the usual axial current anomaly case, the contribution of the tensor sector to the \(U(1)\) gauge anomaly is not trivial. However it keeps the form of a total derivative times the \(U(1)\) ghost, when a convenient regularization is adopted.

2 Axial current anomaly

Let us start from the action involving vector and tensor gauge fields \[\text{[4]}\]
\[ S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{2} m \epsilon_{\mu\nu\rho\lambda} A^\mu \partial^\nu B^{\rho\lambda} \right] \] (2.1)

where \( F_{\mu\nu} \) and \( H_{\mu\nu\rho} \) are totally antisymmetric tensors written in terms of the potentials \( A_\mu \) and \( B_{\mu\nu} \) (also antisymmetric) through the curvature tensors

\[
F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \\
H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\nu B_{\rho\mu} + \partial_\rho B_{\mu\nu}
\] (2.2)

We notice that the vector and tensor gauge fields are coupled in a topological way. It is a well known fact that the system represented by \( S \) is invariant under the gauge transformations

\[
\delta A_\mu = \partial_\mu \Lambda \\
\delta B_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu
\] (2.3)

\[
\delta B_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\] (2.4)

Although (2.3) is the usual irreducible \( U(1) \) gauge symmetry, (2.4) is reducible, since \( \delta B_{\mu\nu} \) vanishes identically if the vector parameter is the gradient of a scalar. At quantum scenario the symmetries (2.3) and (2.4) are not obstructed. Integrating out the tensor fields leads to a non-local \( U(1) \) gauge invariant but massive effective vector theory [4].

Let us now introduce matter field in this theory. We consider that the fermionic vector current also has a topological coupling with the tensor field, with action given by

\[
S_0 = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{2} m \epsilon_{\mu\nu\rho\lambda} A^\mu \partial^\nu B^{\rho\lambda} + i \bar{\psi} D \psi \right] \] (2.5)

where \( D_\mu \) is a covariant derivative that also contains tensor gauge fields,

\[
D_\mu = \partial_\mu + i e A_\mu + \frac{1}{2M} \epsilon_{\mu\nu\rho\lambda} \partial^\nu B^{\rho\lambda}
\] (2.6)

The parameter \( 1/M \) that appears in the Eq. (2.6) is the coupling between \( B_{\mu\nu} \) and the vector current. This kind of coupling means that the theory described by Eq. (2.5) is nonrenormalizable.

In this section, we consider the axial current anomaly by using the Fujikawa path integral technique [11]. As it was already previously mentioned, we emphasize that the study of anomaly involving tensor couplings that is found in literature differs from the one we are going to develop here. Usually, one considers the tensor field as an external field and coupled to a tensor current [10]. We notice that in our case,
the tensor field has dynamics and is coupled in a topological way to the same vector current coupled to the vector potential.

The axial current anomaly arises in the Fujikawa approach from the fact that the measure $[d\bar{\psi}] [d\psi]$ is not invariant under the chiral gauge transformations

$$
\psi(x) \rightarrow \psi'(x) = e^{i\epsilon(x)\gamma_5} \psi(x) \\
\bar{\psi}(x) \rightarrow \bar{\psi}'(x) = \bar{\psi}(x) e^{i\epsilon(x)\gamma_5}
$$

(2.7)

It can be shown that [11]

$$
[d\bar{\psi}][d\psi] = [d\bar{\psi}'][d\psi'] \exp 2ie \int d^4 x \epsilon(x) I(x)
$$

(2.8)

for infinitesimal transformations $\epsilon(x)$. $I(x)$ is a divergent quantity given by

$$
I(x) = \sum_n \phi^\dagger_n(x) \gamma_5 \phi_n(x)
$$

(2.9)

where $\phi_n(x)$ is an orthonormal set of eigenfunctions of some hermitian operator. This quantity needs to be regularized. We use the operator (2.1) (conveniently Wick rotated to an hermitian form) in order to do so. It is not necessary to go into details to do this. We can just consider the final result given in literature [11] and make the replacement

$$
A_\mu \rightarrow \tilde{A}_\mu = A_\mu - \frac{i}{2eM} \epsilon_{\mu\nu\rho\lambda} \partial^\nu B^{\rho\lambda}
$$

(2.10)

Since the generating functional must be independent of the parameter $\epsilon(x)$, we thus obtain

$$
\partial_\mu j^\mu_5 = \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\lambda} \tilde{F}_{\mu\nu} \tilde{F}_{\rho\lambda}
$$

(2.11)

where $\tilde{F}_{\mu\nu}$ is the field strength defined in terms of the $\tilde{A}_\mu$. The combination of (2.10) and (2.11) gives

$$
\partial_\mu j^\mu_5 = \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\lambda} F_{\mu\nu} F_{\rho\lambda} + \frac{1}{24\pi^2 M^2} \epsilon^{\mu\nu\rho\lambda} \partial^\nu H_{\mu\nu\lambda} \partial^\beta H_{\alpha\mu\beta} \\
- \frac{ie}{8\pi^2 M} F_{\mu\nu} \partial_\rho H^{\rho\mu\nu}
$$

(2.12)

We notice in the relation above the contribution for the axial current anomaly originated from the tensor coupling we have considered. These terms can also be written as a total derivative as it occurs in the usual anomalous case.
3 Gauge anomalies

In the preceding section we have analyzed the anomalous divergence of the fermionic chiral current when both vector and tensor fields are coupled to non-chiral fermions. It is interesting to argue if there are quantum obstructions to the gauge symmetries associated to the tensor sector are reducible, and also because we are interested in keeping covariance at each stage, it is useful to search for gauge anomalies with the aid of the field-antifield formalism [12]. The case involving only tensor fields can be found in References [9, 13]. The case where vector and tensor fields are topologically coupled was considered in [4]. The inclusion of fermions induces only simple modifications regarding the results found in [4]. We get for the field-antifield action

\[ \bar{S} = S_0 + \int d^4x \left( i A_\mu^a \partial^\mu c + \bar{c}^a b - ie \psi^* c \psi + i e \bar{\psi} \psi^* + i B_{\mu \nu}^a \partial^\mu d^\nu \right. \\
\left. + d_\mu^a \partial^\mu d + \bar{d}_\mu^a e^\mu + i \bar{d}^* f + i \eta^* f \right) \] (3.1)

where \( S_0 \) is given by (2.4). In the expression above we have introduced the gauge fixing term for the vector gauge and Dirac fields, consisting of ghosts, trivial pairs and corresponding antifields, which essentially represent the sources for the BRST transformations of the field sector. We have also considered the gauge fixing for the tensor field that is a bit more involved due to its reducibility. It was demanded the introduction of ghosts for ghosts and the corresponding antifields, besides trivial pairs for the implementation of the gauge fixing. For completeness, let us introduce the parities and ghost numbers of these fields

\[ \epsilon \left[ A^\mu, B^{\mu \nu}, b, d, \bar{d}, \psi^*, \bar{\psi}^*, f^*, \bar{f}^*, e^\mu, c^*, e^\mu, c^*, d_\mu^a, \bar{d}_\mu^a, \eta \right] = 0 \]

\[ \epsilon \left[ \psi, \bar{\psi}, A_\mu^a, B_{\mu \nu}^a, b^*, d^*, \bar{d}^*, f, \bar{f}, e_\mu^a, c, \bar{c}, d_\mu^a, \bar{d}_\mu^a, \eta^* \right] = 1 \] (3.2)

\[ \text{gh}(d^a) = -3 \]

\[ \text{gh}(c^*, d_\mu^a, \bar{d}, f^*) = -2 \]

\[ \text{gh}(A_\mu^a, B_{\mu \nu}^a, \psi^*, \bar{\psi}^*, c^*, b^*, \bar{d}_\mu^a, e_\mu^a, \eta^*, \bar{f}) = -1 \]

\[ \text{gh}(A^\mu, B^{\mu \nu}, \psi, \bar{\psi}, c, \bar{c}, b, d^\mu, e^\mu, \eta, \bar{f}^*) = 0 \]

\[ \text{gh}(c, d^\mu, \bar{d}^*, f) = 1 \]

\[ \text{gh}(d) = 2 \] (3.3)

The quantum theory is defined through the generating functional

\[ Z_{\Psi} [J] = \int \prod [d\phi^A][d\phi_\Lambda] \delta \left[ \phi_A^* - \frac{\delta \Psi}{\delta \phi^A} \right] \exp \left( i \hbar \left( W[\phi^A, \phi_\Lambda^*] + J_A \phi^A \right) \right) \] (3.4)
where $\phi^A$ and $\phi_A^*$ respectively represent all the fields and antifields appearing in (3.2) and (3.3) and $W$ is a quantum action constructed starting from (3.1). The gauge fixing fermionic function can be chosen to be

$$\Psi = -\int d^4x \left[ \bar{c} (\partial_\mu A^\mu - \frac{\alpha}{2} b^\mu) + \bar{d}_\mu \left( \partial_\rho B^{\rho\mu} - \frac{\beta}{2} e^\mu \right) + \bar{d} \partial_\mu d^\mu + \eta \partial^\mu \bar{d}_\mu \right]$$

and the expectation value for an operator $X$ is given by

$$< X >_{\Psi,J} = \int \prod [d\phi^A] X \exp \left( \frac{i}{\hbar} W[\phi^A,\phi_A^*] + J_A \phi^A \right)$$

The condition that (3.4) is independent of specific gauge choices for null external sources, or equivalently, in the same situation, that it must be invariant under admissible changes in $\Psi$, implies that the quantum master equation

$$< \frac{1}{2} (W,W) - i\hbar \Delta W >_{\Psi,J} = 0$$

must be satisfied. In Eq. (3.7) the antibracket is defined as $(X,Y) = \frac{\delta X}{\delta \phi^A} \frac{\delta Y}{\delta \phi_A^*} - \frac{\delta X}{\delta \phi_A^*} \frac{\delta Y}{\delta \phi^A}$ and the operator $\Delta$ as $\Delta \equiv \frac{\delta}{\delta \phi^A} \frac{\delta}{\delta \phi_A^*}$. As can be observed, the operator $\Delta$ is potentially singular and its action must be regularized. In this sense, the master equation at loop order equal or greater than one is just formal unless a regularization scheme is introduced. Expanding $W[\phi,\phi^*]$ in powers of $\hbar$ gives $W[\phi^A,\phi_A^*] = S[\phi^A,\phi_A^*] + \sum_{p=1}^{\infty} \hbar^p M_p[\phi^A,\phi_A^*]$ and consequently the master equation (3.4) can be written in loop order. The first terms are

$$< S,S > = 0$$

$$< M_1,S > = i \Delta S$$

If we adopt a Pauli-Villars regularization with fermionic mass terms and with the usual form for those of Dirac fields [14], it is not difficult to show that the action of the $\Delta$ operator on $S$ is trivial, and so the theory is anomalous free. This result is not surprising because we know that $QED_4$ is anomalous free and also because the actual theory has a fermionic covariant derivative that reduces to that one of $QED_4$ under the correspondence (2.10). Since there is no obstruction of gauge symmetries, $W$ and $S$ can be identified and we can integrate over the antifields to obtain the gauge fixed version of (3.1) as an effective action:
\[ \bar{S} = S_0 + \int d^4 x \left[ i \partial_\mu \bar{c} \partial^\mu c + (\partial_\mu A^\mu - \alpha \frac{b}{2}) b ight. \\
+ i \partial_\mu \bar{d} \partial^\mu d + \partial_\mu \bar{d} \partial^\mu d \\
+ (\partial_\mu B^{\nu \mu} - \frac{\beta}{2} \epsilon^\mu - \partial^\mu \eta) e\mu \\
\left. - i \partial_\mu d^\mu \tilde{f} + i f \partial^\mu \bar{d}_\mu \right] \quad (3.10) \]

If now we integrate out the tensor degrees of freedom, it is not difficult to see that
the effective action obtained in [4] is generalized to

\[ \bar{S} = \int d^4 x \left[ \frac{1}{2} \bar{A}_\mu (\Box - m^2) A^\mu - \frac{1}{2} \partial_\mu \bar{A}^\mu \left( 1 - \frac{1}{\alpha} - \frac{m^2}{\Box} \right) \partial_\nu \bar{A}^\nu \right] + S_{\text{ghost}} \quad (3.11) \]

where \( \bar{A}_\mu = A_\mu + i \frac{\bar{\psi}}{m_M} \gamma^5 \bar{\psi} A_\mu \) is essentially the vector gauge field shifted by the
vectorial current. So it appears an effective mass term for the vector fields, as can
be read from the inverse of the operator appearing in (3.11), but also current-current
self interactions in the fermionic (effective) sector.

The situation becomes completely different if we consider chiral couplings with
\( \bar{A}_\mu \). First it is necessary to replace the covariant derivative \( D_\mu \) appearing in (2.5)
and (2.6) by

\[ \hat{D}_\mu = \partial_\mu + i e P_+ \bar{A}_\mu , \quad (3.12) \]

where \( P_\pm = \frac{1}{2} (1 \pm \gamma^5) \). The gauge invariances appearing in the vector and tensor
sectors do not change, but we need to consider the changes in the Dirac sector. To
do this, it is enough to replace in Eq. (3.1) \( -ie\bar{\psi}^c c\psi + ie\bar{\psi} c\bar{\psi} \) by
\( -ie\bar{\psi}^c c\psi + ie\bar{\psi} c\bar{\psi} + ie\bar{\psi} c P_+ \bar{\psi} \). The remaining gauge fixing terms are not affected. The quantum master
equation, however, is not satisfied anymore. By using the same kind of Pauli-Villars
regularization previously adopted, we can see that [14]

\[ \Delta S_{\text{Reg}} = -\frac{e^3}{16\pi^2} \int d^4 x c e^{\mu \nu \rho \lambda} \tilde{F}_{\mu \nu} \tilde{F}_{\rho \lambda} , \quad (3.13) \]

which is a similar expression to the one already developed in the previous section.
So we conclude that the \( U(1) \) gauge symmetry is obstructed if chiral couplings are
present, but the contribution of the tensor field to the anomaly is yet the one induced
by correspondence (2.10). It is opportune to mention that the nonrenormalizable
vertex given by Eq. (2.5) does not spoil the result above because it does not enter
in the triangle diagram.
4 Conclusion

We have shown that introducing fermions in a vector-tensor gauge theory, coupled in a topological way with the tensor field, modifies the quantum expression for the divergence of the axial Noether current in a nontrivial manner, when compared to the usual vector case. The anomalous divergence expression, however, is yet a total derivative, which is a consequence of the form of the chosen coupling between fermions and the tensor field.

It was also shown that although the proposed theory presents this kind of anomalous current divergence, it does not present $U(1)$ or tensor gauge anomalies. This fact permit us to integrate out the tensor degrees of freedom, what generates mass for the vector sector. It also introduces effective current-current couplings, but in such a way that the $U(1)$ gauge symmetry is explicitly kept.

We have also observed that when chiral couplings are permitted, true gauge anomalies are generated, within a form that reduces to the usual $FF^*$ one, once we redefine the gauge vector field by a particular shift depending on the tensor fields.

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