Uncertainty evaluation of efficiency measurement in laboratory conditions

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Abstract. Uncertainty is a parameter associated with the result of a measurement, which characterizes the dispersion of the values that could reasonably be attributed to the measurand. All measurements are subject to uncertainty and a measurement result is complete only when it is accompanied by a statement of the associated uncertainty, such as standard deviation or uncertainty band. The identification of uncertainty of individual sensors and standardization in calibration process is under progress in Turbine Testing Lab (TTL) at Kathmandu University (KU). The lab aims to establish a Francis turbine test rig and procedures that meet the standards set by International Electrotechnical Commission (IEC). This needs a major investment on calibration systems and development of procedures to estimate the uncertainty in measurement more precisely than ever before. This study aims to identify the sources of uncertainty and their quantification for efficiency measurement systems at laboratory condition. The study is done on the 14 k\text{W} Francis turbine project at TTL, KU. Based on different published papers regarding uncertainties, type A and type B uncertainties were evaluated separately and then combined to get combined uncertainty of each components. The uncertainty in efficiency was then calculated using the rules of uncertainty calculation. In this paper, the standard method is illustrated to evaluate the uncertainties in sensor-based measurement system. The same method is used in investigation of uncertainty to the extent it was possible and feasible in the lab i.e. study could be mainly focused only on torque transducer. The value of efficiency at 1300 rpm is calculated to be 66.7\% $\pm$ 0.35\%, where $\pm$ 0.35\% represents the uncertainty in efficiency for the same rpm value. The results indicate uncertainty level unforeseen close to meet the IEC standards which could be result of including only parts of uncertainties. All uncertainties should be evaluated for future research papers.

1. Introduction

Uncertainty is a parameter associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand. Uncertainty of
measurement comprises, in general, several components including calculated from series of repeated observations and evaluated using available knowledge. Some of these components calculated from series of repeated observations may be evaluated from the statistical distribution of the results of series of measurements and can be characterized by standard deviations. The other components such evaluated using available knowledge, which also can be characterized by standard deviations, are evaluated from assumed probability distributions based on experience or other information. Uncertainty of measurement does not imply doubt about the validity of a measurement; on the contrary, knowledge of the uncertainty implies increased confidence in the validity of a measurement result [1].

Measurement uncertainty is a quantitative indication of the quality of measurement results, without which they could not be compared between themselves, with specified reference values or to a standard [2]. Uncertainty evaluation is essential to guarantee the metrological traceability of measurement results and to ensure that they are accurate and reliable. In addition, measurement uncertainty must be considered whenever a decision has to be taken based on measurement results, such as in accept/reject or pass/fail processes [3]. Regardless of how careful or scientific, all measurements are subject to error and uncertainty. We can never be absolutely certain that a measurement result is true or finite. Therefore, we must establish a boundary for which we are confident that the measurement result will lie between.

1.1. Types of uncertainties
According to ISO GUIDE 5168-2005, uncertainty is classified as Type A and Type B [4].

1.1.1. Type A uncertainty. It is the evaluation of a component of measurement uncertainty by a statistical analysis of measured quantity values obtained under defined measurement conditions such as standard deviation [4]. Type ‘A’ evaluation of standard uncertainty applies to the situation when several independent observations have been made for any of the input quantities under the same conditions of measurement. If there is sufficient resolution in the measurement process, there will be an observable scatter or spread in the value obtained. Whilst no correction can be made to remove random components of uncertainty, their associated uncertainty becomes less as the number of observation/measurement increases [5]. Type A evaluations in uncertainty estimates using statistics usually from repeated readings [6].

1.1.2. Type B uncertainty. It is estimated from any other information. This could be information from past experience of the measurements, from calibration certificates, manufacturer’s specifications, from calculations and published information [6]. Type B evaluation of uncertainty is those carried out by means other than the statistical analysis of series of observation/measurement. Therefore, it cannot be reduced by increasing the number of measurements if the equipment and conditions of measurements remain unchanged. The associated estimated the standard uncertainty is evaluated by scientific judgment based on all of the available information on the possible variability. It includes the information from: (a) Previous measurement data; (b) Experience with or general knowledge of the behavior and properties of relevant materials and instruments; (c) Manufacturer's specifications; (d) Data provided in calibration and other certificates [5].

1.2. Error and uncertainty
Error is defined as the difference between an individual result and the true value of the measurand. As such, error is a single value. In principle, the value of a known error can be applied as a correction to the result. Error is an idealized concept and errors cannot be known exactly. An error is regarded as having two components, namely, a random component and a systematic component.

Uncertainty, on the other hand, takes the form of a range, and, if estimated for an analytical procedure and defined sample type, may apply to all determinations so described. In general, the value of the uncertainty cannot be used to correct a measurement result. The uncertainty of the result of a
measurement should never be interpreted as representing the error itself, nor the error remaining after correction [7]. The result of an analysis after correction may by chance be very close to the value of the measurand, and hence have a negligible error. However, the uncertainty may still be very large, simply because the analyst is very unsure of how close that result is to the value [1]. The types of errors are classified as:

1.2.1. Systematic error. Systematic error is defined as a component of error which, in the course of a number of analyses of the same measurand, remains constant or varies in a predictable way. It is independent of the number of measurements made and cannot therefore be reduced by increasing the number of analyses under constant measurement conditions [1]. Systematic error is an error that affects all measurements in the same way. When random errors are small, it is possible to identify systematic errors.

1.2.2. Random error. Random error is an error that is uncorrelated, which affects the results of repeated measurements. Sources of random error add a component to the measurement result that is unknown. Random error typically arises from unpredictable variations of influence quantities. These random effects give rise to variations in repeated observations of the measurand. The random error of an analytical result cannot be compensated for, but it can usually be reduced by increasing the number of observations [1].

1.2.3. Spurious errors. These are errors such as human errors, or instrument malfunction, which invalidate a measurement. For example, the presence of pockets of air in the pipe line connected to pressure transducer. Such errors are not incorporated into any statistical analysis. Also such errors cannot be completely eliminated and hence are generally discarded [5].

All measurements have a degree of uncertainty regardless of precision and accuracy. This is caused by two factors, the limitation of the measuring instrument (systematic error) and the skill of the experimenter making the measurements (random error). Uncertainties reduce accuracy in results. There might be common misunderstanding of error and uncertainty among general reader. The brief description of error and uncertainty along with their types aims to solve those misunderstandings. This research only focuses on the standardization and evaluation of uncertainty.

Few papers have been published which includes study of uncertainties around guide vanes of Francis turbine. However, the study and evaluation of uncertainties of other testing setup components has not been done yet. Also, lack of standardization in calibration procedure leads the need of quantification of the uncertainty for valid result. This opens a vacant opportunity to study about it in detail. The main objectives of this research is to identify the source of uncertainties and its quantification for efficiency measurement at laboratory conditions; to develop the evaluation process of uncertainties and verify for efficiency measurement at laboratory conditions; and to study the uncertainties and recommend the solutions to the efficiency measurement and calibration systems available at the TTL. The flow calibration system facility available for 14 kW Micro Francis turbine in TTL was used as model testing set up for this research.

1.3. Standard of testing
The most widely accepted standards related to large hydropower projects are mainly developed according to standards developed by the IEC. The IEC publishes globally relevant technical guidelines that allow millions of devices and systems that use, produce or store electricity to work safely together, anywhere in the world. The IEC 60193 is used for model acceptance tests of hydraulic turbines, storage pumps and pump turbines. The main objective of this standard is to specify methods of testing and of measuring, computation and comparison of the quantities involved, in order to ascertain the hydraulic performance of the model. According to IEC standards, the measurement is acceptable only if the uncertainty is less than 0.2 % [8].
2. Methodology

2.1. Experimental setup
The experimental set up of simplified 14 kW Francis turbine consisted of an electromagnetic flowmeter, two pressure sensors, a torque transducer and rpm sensor. The high-pressure tank available at the TTL was used to pressurize the water thus maintaining the desired head and flowrate. The flow was passed through an electromagnetic flowmeter and through a decreasing trapezoidal cross-sectional casing that with no guide vanes [9].

The design parameters for Simplified 14 kW Francis Turbine are listed below:

Table 2.1 Design Specification. [10]

| Parameter     | Value | Unit |
|---------------|-------|------|
| Flow (Q)      | 0.1   | m³/s |
| Head (H)      | 16    | m    |
| Power (P)     | 14    | kW   |
| Rotational Number | 0.667 |       |

![Figure 2.1 Experimental setup of 14kW Francis turbine. [9]](image)

The inlet pressure was measured at the inlet of the turbine casing by pressure sensor while the torque and speed/rpm were measured with the help of a torque transducer. The outlet pressure was measured at the outlet of the turbine (i.e. at the inlet of draft tube) [9]. The test was performed for various scenarios such as variable rpm, flow rate. The parameters were recorded using a user defined program in LabView. The curves for efficiency calculation and rpm vs efficiency curve were developed as a part of Master’s thesis by D.R. Dahal [9].

2.2. Data analysis
The data acquired from the LabView program required to be processed so as to achieve the desired output. Uncertainty can be represented either in absolute or in relative form. The absolute uncertainty is presented in number form while relative uncertainty is always represented in percentage.

For the processing of uncertainty, following rules of uncertainty calculation must be considered:
- While adding or subtracting the measurements, absolute uncertainty is added.
- While multiplying or dividing the measurements, percentage uncertainty is added.

The type A and type B uncertainty are combined by RSS method. Further, the main statistical methods required for the uncertainty calculation are listed in table 2.2.

Table 2.2. Description of statistical methods. [4] [11]

| Parameter       | Description |
|-----------------|-------------|
| Average         | $\overline{X}(x_i) = \frac{1}{n} \sum_{m=1}^{n} X(x_{i,m})$ |
| Standard Deviation | $S(x_i) = \sqrt{\frac{1}{n-1} \sum_{m=1}^{n} (X_{i,m} - \overline{X}_i)^2}$ |
| Absolute Random Uncertainty | $S(\overline{X}_i) = \pm \frac{S(x_i)}{\sqrt{n}}$ |
| % Random Uncertainty of Mean | $\varepsilon = \pm \frac{S(x_i)}{\overline{X}(x_i)} \times 100$ |
| Running Average | $\overline{X} = \frac{1}{n} \sum_{m=1}^{n} \overline{X}(x_i)$ |
| Running Standard Deviation | $\overline{S} = \frac{1}{n} \sum_{m=1}^{n} S(x_i)$ |
2.3. Development of steps for uncertainty calculation

For the calculation of uncertainty, a proper guideline is required. This guideline is developed and illustrated in table 2.3.

Table 2.3. Block diagram to calculate combined uncertainty of efficiency measurement. [4] [5] [11]

| Type B uncertainty evaluation | Type A uncertainty evaluation |
|-------------------------------|-------------------------------|
| Data Block B                 | Data Block A                  |
| Primary Data                 | Primary Data                 |
| Noise Filtration: *Using Limits* | Noise Filtration: *Using Limits* |
| Percentage Uncertainty from Calibration Certificate provided by Manufacturer | Parameters Calculation |
| Convergence Test             | • Mean                       |
| Reliability Test             | • Standard Deviation         |
| Hysteresis Test              | • Absolute Random Uncertainty |
| Calibration Equation         | • % Random Uncertainty of Mean |
|                               | • Running Average            |
|                               | • Running Standard Deviation |
| Combined Relative Uncertainty for |                     |
| • Torque                     | Combined Uncertainty of Efficiency |
| • Pressure in and Pressure Out |                               |
| • Speed                      |                               |
| • Flow                       |                               |
3. Uncertainty analysis in TTL

3.1. Uncertainty in torque measurement

3.1.1. Type B Uncertainty from calibration of torque transducer. During the calibration of torque transducer, the 20 known weights are added in the arm, and each corresponding voltage is recorded in LabView. The known weights are standard weights of 500 gm ± 0.1 % each. The absolute uncertainty in measuring length of arm is ± 0.5 mm. The sampling frequency of the torque transducer is 2000. The sample is recorded for 29 seconds providing total 58000 number of samples.

During the step 1 of data preparation, the noise filtration test is done and a graph is developed as shown by figure 3.1. The points shown by the graph are the raw data voltage values obtained from torque transducer through LabView. The upper and lower limit is determined at confidence level 95%. All the values outside the range of upper and lower limit as seen in the figure is considered noise which are undesirable values.

The data are now filtered and reduced to a smaller number of samples. The running average and error % of random uncertainty with respect to average is also calculated for each value. The convergence test is done to determine minimum number of samples required to meet the IEC standard. It is the number of samples after which the increment in number of samples wouldn’t affect the whole data or any other calculated statistical parameter. It also signifies the point after which the random error is minimized.

| Table 3.1 Necessary Data. |
|---------------------------|
| Parameters | Value            |
|--------------|------------------|
| Length of the arm | 0.49886 m       |
| Confidence Level  | 95 %            |
| Coverage Factor  | 1.96             |
| No. of weights  | 20               |
| Each weight     | 500 g            |

**Figure 3.1** Noise Filtration Test for 4th load.  **Figure 3.2** Convergence test for 4th load.

Similar process is carried out for all the 20 loads, first in loading order then in unloading order. The process is carried out a couple of time for better result to minimize random and spurious errors.

Then, the hysteresis test for the data is carried out to check the dependency of the results. The difference between loading and unloading for each torque provides the hysteresis of each data.

The developed graph showed minimum hysteresis with R² = 1 which signifies the loading and unloading curve follows similar path. This resulted for the consideration of the average of two tests to develop a linear calibration equation: \( y = 49.77x - 0.4481 \)

Similarly, the uncertainty associated with each value is also calculated to develop the uncertainty bars in the graph. Then joining the ends of uncertainty bars provides the curve for uncertainty.
Fig. 3.3 Calibration and uncertainty bar for torque transducer calibration.

The uncertainty at first point seems maximum. At this point, the torque is non. So, the uncertainty is displayed as unjustifiable.

As discussed earlier, type B uncertainty is determined from calibration certificate but not by standard deviation. It is not to be misinterpreted. The above steps are followed to calibrate the torque transducer. From calibration certificate of torque transducer, the type B uncertainty, $u(\tau)_b$ is ± 0.09 %.

### 3.1.2. Type A Uncertainty from Test of Micro Francis Turbine
Similarly, for the data from the test of Micro Francis Turbine is filtered by similar approach. The filtered data is then converted using calibration equation into torque in Nm. Then, the parameters in table 3.3 are calculated. The % random uncertainty of mean represents type B uncertainty of torque, $u(\tau)_b$.

#### Table 3.2 Calculated Parameters for 1000 rpm.

| Parameters                  | Value    |
|-----------------------------|----------|
| Total no. of samples        | 346026   |
| Average                     | 40.99 N m|
| Standard deviation, $\sigma$| 13.86    |
| Absolute Random Uncertainty | 0.02 N m |
| % Random Uncertainty of Mean, $\varepsilon$ | 0.06 |

#### 3.1.3. Combined uncertainty of torque
Combined uncertainty for torque is calculated by combining type A and type B uncertainty.

\[
\text{Combined uncertainty of torque, } u(\tau)_c = \sqrt{(u(\tau)_a)^2 + (u(\tau)_b)^2} = \pm 0.11 \% \quad [7]
\]

### 3.2. Uncertainty in Pressure Measurement
The Type B uncertainty of pressure from calibration certificate, $u(P)_b$ is used. The calibration equation and Type A analysis is not conducted for this research which should be conducted in similar manner as of uncertainty in torque measurement. In this case, the combined uncertainty for pressure measurement $u(P)_c$ is calculated in similar manner.

### 3.3. Uncertainty in Flow Measurement
The Type B uncertainty of pressure from calibration certificate, $u(Q)_b$ is used. The calibration equation and Type A analysis is not conducted for this research which should be conducted in similar manner as of uncertainty in torque measurement. In this case, the combined uncertainty for pressure measurement $u(P)_c$ is calculated in similar manner.

### 3.4. Uncertainty in Speed Measurement
The speed is measured through the same torque transducer used for torque measurement. The source of uncertainty is same for both the measurements. Since the uncertainty analysis for torque is already conducted, this process is not repeated. However, if the speed is measured through different device the analysis should be conducted in similar manner as torque transducer.

The uncertainty in pressure, flow and speed should be evaluated in similar manner as that of torque transducer for correct evaluation.

### 3.5. Uncertainty in Efficiency calculation

The thesis by D.R. Dahal is as source for the efficiency [9].

For example, Efficiency at 1000 RPM = 59.29% [9]

Similarly, efficiency at different rpm were taken as well the graph associated with it. The combined uncertainty for the efficiency is calculated.

Combined Uncertainty in Efficiency at 1000 rpm, \( u(\eta) = u(\tau) + u(P) + u(Q) = 0.39 \% \)

Similarly, efficiency and uncertainty of efficiency is calculated for various RPM values. The efficiency vs RPM graph developed to determine BEP by D.R. Dahal and et. all [9] is used to plot uncertainty bars as shown in figure 3.4 to illustrate the value of uncertainty at each point.

![Figure 3.4](image-url)  
Note: Uncertainty % is magnified by 10 times for the visibility of bars.

The slight unusual dip in efficiency is seen in the graph specially around 1400 rpm. This pattern is visible in every experiment conducted for the newly developed turbine model. This pattern is discussed in detail in the thesis [9] and conference proceeding [10] by D.R. Dahal and et. all.

### 4. Conclusion

Evaluation of uncertainty in measurement system is very essential for every test. Since, TTL was initiated with the sole motive to test different turbines, standardization in calibration was necessary. The research worked on identification of the sources of uncertainties and quantification for the measurement systems at laboratory conditions. It also worked in development of the evaluation process of uncertainties and verification for measurement system at laboratory conditions as per IEC standards. The research was only limited to calibration of torque transducer for shaft torque measurement due to lack of equipment and resources.
The efficiency of the 14 kW Francis turbine at 1300 rpm is calculated in the MS by research thesis to be 66.7 ± 0.35 %. The value of uncertainty required to meet IEC standards is less than ± 0.2 %. Although the result obtained is unforeseen close to IEC standard, it should not be compared to the IEC standards as the process followed is not complete. The calculated uncertainty only includes parts of the relevant uncertainties. The required uncertainty by the IEC Standard of 0.2 % includes all uncertainties and this signifies that TTL needs to work on standardization more to meet the IEC standards.

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