Cosmological constant, semiclassical gravity, and foundations of quantum mechanics

Hrvoje Nikolić  
*Theoretical Physics Division, Rudjer Bošković Institute, P.O.B. 180, HR-10002 Zagreb, Croatia*

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The old cosmological-constant (CC) problem indicates an inconsistency of the usual formulation of semiclassical gravity. The usual formulation of semiclassical gravity also seems to be inconsistent with the conventional interpretation of quantum mechanics based on the discontinuous wave-function collapse. By reformulating semiclassical gravity in terms of Bohmian deterministic particle trajectories, the resulting semiclassical theory avoids both the old CC problem and the discontinuous collapse problem of the usual semiclassical theory. The relevance to the new CC problem and to particle creation by classical gravitational fields is also discussed.

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I. INTRODUCTION

A. Problems with semiclassical gravity

As the correct theory of quantum gravity is not yet known, there is some hope that at least a semiclassical approximation could work. In this approximation, gravity is treated classically, while all other forms of matter are quantized. The semiclassical theory is usually formulated as a semiclassical Einstein equation

\[
G_{\mu\nu}(x) = 8\pi G_N \langle \hat{T}_{\mu\nu}(x)|\Psi\rangle,
\]

where \( G_{\mu\nu} \) is the Einstein tensor, \( G_N \) is the Newton constant, \( \hat{T}_{\mu\nu} \) is a quantum operator representing the symmetric energy-momentum tensor of matter, and \( |\Psi\rangle \) is the quantum state. However, as \( \hat{T}_{\mu\nu} \) is calculated from quantum field theory (QFT), it contains a huge contribution from the vacuum energy of the field, leading to a huge contribution to the cosmological constant, many orders of magnitude larger than the measured one. This represents the core of the cosmological-constant (CC) problem. In the old formulation of the problem [1, 2] one would like to find a theoretical mechanism that makes this vacuum contribution to the cosmological constant vanishing, while in the new, more ambitious, formulation of the problem [3, 11] one would like to explain why the sum of all possible contributions to the cosmological constant, including that of the vacuum energy, is of the same order of magnitude as the matter density of the universe.

Another, seemingly unrelated, problem with the semiclassical equation [11] concerns the fundamental interpretational problems of quantum mechanics (QM) itself. When \( |\Psi\rangle \) in [11] is a superposition of two macroscopically distinct states, then experiments show that [11] is wrong [6]; the measured gravitational field is not given by the average value of the energy-momentum in the superposition \( |\Psi\rangle \), but rather by the actual measured value of the energy-momentum. One could take this effect into account by reformulating [11] in terms of a quantum state \( |\Psi(t)\rangle \), in which the extra time dependence corresponds to quantum “collapses” of \( |\Psi\rangle \) induced by quantum measurements. However, according to the standard interpretation of QM, the “collapses” are discontinuous processes that change \( |\Psi\rangle \) instantaneously and nonlocally. Consequently, owing to the extra time dependence, the energy-momentum in [11] ceases to be a smooth function, which implies that it cannot satisfy the local conservation equation \( \nabla^\mu \langle \Psi(t)|\hat{T}_{\mu\nu}(x)|\Psi(t)\rangle = 0 \). On the other hand, the left-hand side is a classical quantity that satisfies \( \nabla^\mu G_{\mu\nu}(x) = 0 \), suggesting an inconsistency of [11]. We refer to this problem as the discontinuous collapse problem.

Both problems with the semiclassical equation [11] indicate that the semiclassical approximation is not an appropriate framework to deal with interactions between gravity and matter. For that reason, it is very likely that, in order to have a consistent theory, gravity must also be quantized. Nevertheless, we believe that it is too early to completely give up the attempts to construct a satisfying semiclassical theory that avoids the problems outlined above. The aim of this work is just to propose such a reformulated semiclassical theory that avoids these problems.

B. Main ideas for a solution

To avoid the discontinuous collapse problem, we first need to replace the usual notion of instantaneous discontinuous wave-function collapse in QM with something smooth and continuous. Fortunately, there already exists such a formulation of QM - the Bohmian formulation [7, 8, 9, 10, 11]. (For a comparison with other formulations, see also [12].) In the case of a completely quantum description of a physical system, the Bohmian formulation of QM, just as any other formulation, leads to the same statistical predictions as the usual formulation. Nevertheless, in general, a theoretical concept of a “semiclassical approximation” is somewhat ambiguous, so different approaches to a semiclassical approximation...
may not be equivalent. In fact, among various formulations of QM [12], the Bohmian formulation is the most similar to classical mechanics, so it seems reasonable that the Bohmian approach could be the most suitable for a satisfying formulation of a semiclassical approximation. Besides, the Bohmian interpretation of quantum gravity [13, 14, 15, 16] has already been found useful for certain cosmological applications [17, 18, 19, 20, 21]. Therefore, we base our semiclassical formulation of gravity interacting with matter on a Bohmian description of quantum matter.

In the case of first quantization of particles, the Bohmian interpretation assumes that particles are point-like objects with continuous and deterministic trajectories. However, the force on the particle depends on the wave function, which makes these trajectories different from the classical ones. The particle positions at any time are completely determined by the initial conditions. However, if an observer is ignorant of the actual initial particle positions, one completely restores the effective standard probabilistic rules of QM. Although this hidden-variable formulation of QM is conceptually appealing and consistent with observations, most physicists do not use the Bohmian formulation in practice, mainly because it is technically more complicated than the standard formulation, with the same measurable statistical predictions for purely quantum systems. However, the application of the Bohmian formulation to a semiclassical approximation may lead to measurable predictions that cannot be obtained with other formulations.

In the case of QFT, the Bohmian formulation is constructed in an analogous way, but with the crucial difference that now the fundamental objects having a continuous and deterministic dependence on time are not point-like particles, but continuous fields. Indeed, in high-energy physics, the dominating point of view is that the fundamental quantized objects are not particles but fields. Still, many phenomenologically oriented particle physicists view QFT merely as a mathematical tool useful only for calculation of properties of particles. Moreover, it seems that it is possible to construct a consistent particle-scattering formalism that completely avoids any referring to fields [22]. In fact, there is no real proof that fields (or particles) are more fundamental objects than particles (or fields) [23]. In the Bohmian formulation, where particles or fields are supposed to objectively exist even when they are not measured, the field-or-particle dilemma is even sharper than that in the standard formulation. To reproduce all good results of both nonrelativistic first quantization and relativistic QFT, in the Bohmian formulation it can be assumed that both particles and fields exist separately, such that, in particle-physics experiments, particles are objects that are really observed, whereas fields play a role in governing continuous deterministic processes of particle creation and destruction [24, 25].

If both particles and fields exist separately, then, in the Bohmian formulation, both particles and fields generate separate continuously and deterministically evolving energy-momentum tensors. However, the total energy-momentum tensor cannot be a sum of these two tensors, because it would correspond to a double counting. Instead, either only particles or only fields determine the energy-momentum tensor on the right-hand side of a semiclassical Einstein equation. Is that the energy-momentum of fields, or that of particles? While it is difficult to answer this question by using purely theoretical arguments, it is important to notice the following essential difference between these two choices: Whereas the field energy-momentum contains an infinite (or huge) vacuum contribution, the particle energy-momentum does not contain this vacuum contribution at all. Of course, particles in an external potential may also have a nonzero ground-state energy, but such a particle ground-state energy is finite and usually small. The huge vacuum energy-momentum can be removed for fields as well, e.g., by normal ordering, but such a removal is theoretically artificial. On the other hand, by assuming that fundamental objects that determine the energy-momentum tensor are not fields but particles, the vacuum contribution removes automatically. This is how the quantum theory formulated in terms of Bohmian particle trajectories avoids two fundamental problems of [11] at the same time: the discontinuous collapse problem and the old CC problem.

Before presenting details of such a Bohmian formulation, the following remarks are in order. First, it is often claimed that the existence of the Casimir effect is a proof that the vacuum energy is real, so that it is unphysical to ignore it. However, the fact is that the Casimir effect can be derived even without referring to vacuum energy [20], so the existence of the Casimir effect cannot really be taken as a proof that vacuum energy is physical. Instead, the Casimir force can be treated as a van der Waals-like force, the energy-momentum of which can be described by a small potential between real particles that constitute the conductive plates.

Second, in curved spacetime, which the semiclassical theory of gravity is supposed to describe, QFT particle states cannot be defined in a unique way [27], which is a problem for a theory with an ambition to deal with particles as fundamental objectively existing entities. However, this problem can be avoided by an introduction of a preferred frame that allows to define particles in an objective and local-covariant manner [28]. Moreover, it is possible that a preferred frame is generated dynamically in a covariant way (for a concrete proposal see [29]), which, at least, makes the idea of a preferred frame less unpleasant.

In the next section we formulate the theory with first quantization of particles, while the effects of QFT, including the effects of particle creation and destruction, are studied in Sec. III. Some further physical implications, including the relevance to the new CC problem and to the problem of backreaction associated with Hawking radiation, are qualitatively discussed in Sec. IV.

In the paper, we use units in which $\hbar = c = 1$, while
the signature of spacetime metric is \((+−−−)\).

II. BOHMIAN SEMICLASSICAL GRAVITY IN FIRST QUANTIZATION

A. Bohmian particle trajectories

Consider the Klein-Gordon equation for a massive spin-0 particle in curved spacetime

\[
(\nabla^{\mu} \partial_{\mu} + m^2)\psi(x) = 0,
\]

where \(\nabla^{\mu}\) is the covariant derivative, and the fact that \(\nabla^{\mu} \psi = \partial^{\mu} \psi\) is used. Eq. (2) implies the local conservation law

\[
\nabla^{\mu} \left( \frac{i}{2} \psi^{*} \partial_{\mu} \psi \right) = 0,
\]

which implies that the norm

\[
(\psi, \psi) = \int_{\Sigma} d\Sigma^{\mu} \frac{i}{2} \psi^{*} \partial_{\mu} \psi
\]

(4)

(where \(d\Sigma^{\mu} = d^3x \sqrt{|g^{(3)}|} n^{\mu}\) and \(n^{\mu}\) is a unit vector normal to \(\Sigma\) does not depend on the choice of the spacelike hypersurface \(\Sigma\). We consider a solution \(\psi\) for which the norm (4) is positive and equal to 1.

By writing \(\psi = Re^{iS}\), where \(R\) and \(S\) are real functions, the complex Klein-Gordon equation (2) is equivalent to two real equations

\[
\nabla^{\mu} (R^{2} \partial_{\mu} S) = 0,
\]

\[
-(\partial^{\mu} S)(\partial_{\mu} S) + \frac{m^2}{2} + Q = 0,
\]

where

\[
Q = \frac{1}{2m} \frac{\nabla^{\mu} \partial_{\mu} R}{R}
\]

is the quantum potential. Eq. (5) is the conservation equation (3). Thus, the fact that (4) is unit can be written as

\[
\int d^3x \sqrt{|g^{(3)}|} R^2 \omega = 1,
\]

where \(\omega(x) = -n^{\mu}(x) \partial_{\mu} S(x)\) is the “local frequency”. This shows that \(R^2 \omega\) can be interpreted as a probability density of particle positions, provided that \(R^2 \omega\) is nonnegative. (For the case in which it is locally negative, see \[31\].) Eq. (5) can be viewed as a quantum Hamilton-Jacobi equation, differing from the classical relativistic Hamilton-Jacobi equation in containing an additional \(Q\)-term. Indeed, in physical units with \(\hbar = 1\), the right-hand side of (7) attains an additional factor \(\hbar^2\), which shows that \(Q \to 0\) in the classical limit.

In the Bohmian interpretation of relativistic QM, the particle is a pointlike object having a continuous trajectory \(X^\mu(s)\) satisfying the deterministic equation \[24, 31, 32\]

\[
\frac{dX^\mu(s)}{ds} = -\frac{1}{m} \partial^\mu S,
\]

where it is understood that the right-hand side is evaluated at \(x = X\) and \(s\) is an affine parameter along the trajectory. Using the identity

\[
\frac{d}{ds} \frac{dX^\mu}{ds} = \frac{d^2X^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dX^\alpha}{ds} \frac{dX^\beta}{ds},
\]

as well as Eqs. (9) and (9), one finds the equation of motion

\[
m \frac{D^2X^\mu}{Ds^2} = \partial^\mu Q,
\]

where

\[
\frac{D^2X^\mu}{Ds^2} \equiv \frac{d^2X^\mu}{ds^2} + \Gamma^\mu_{\alpha\beta} \frac{dX^\alpha}{ds} \frac{dX^\beta}{ds}.
\]

The right-hand side of (11) describes the quantum force, i.e., the deviation of the particle trajectory from a motion along a geodesic.

B. Energy-momentum tensor

To construct the conserved energy-momentum tensor associated to the particle equation of motion (11), we use the methods developed in \[33\]. The energy-momentum tensor written in a manifestly covariant form turns out to be

\[
T^{\mu\nu}(x) = \int ds \frac{\delta^4(x - X(s))}{\sqrt{|g^{(3)}|(x)|}} \times \left[ m \frac{dX^\mu}{ds} \frac{dX^\nu}{ds} - g^{\mu\nu}(x)Q(x) \right].
\]

For a timelike trajectory \(X^\mu(s)\), the physical meaning of (13) is more manifest when coordinates are chosen such that \(g_{00} = 0\) and \(X^0(s) = s/\sqrt{g_{00}}\). In this case, (13) can be written as

\[
T^{\mu\nu}(x) = \frac{\delta^4(x - X(s))}{\sqrt{|g^{(3)}|(x)|}} \times \left[ m \frac{dX^\mu}{ds} \frac{dX^\nu}{ds} - g^{\mu\nu}(x)Q(x) \right],
\]

which is nonvanishing only along the particle trajectory \(X(s)\). Using (11) (see also \[34\]) one finds

\[
\nabla_{\nu} \int ds \frac{\delta^4(x - X(s))}{\sqrt{|g^{(3)}|(x)|}} m \frac{dX^\mu}{ds} \frac{dX^\nu}{ds} = \int ds \frac{\delta^4(x - X(s))}{\sqrt{|g^{(3)}|(x)|}} m \frac{D^2X^\mu}{Ds^2},
\]

(15)
\[ \nabla^\flat \int ds \frac{\delta^4(x - X(s))}{\sqrt{|g(x)|}} g^{\mu\nu}(x)Q(x) = \int ds \frac{\delta^4(x - X(s))}{\sqrt{|g(x)|}} \partial^\mu Q(x). \] (16)

Thus, when the equation of motion (11) is satisfied, then the energy-momentum tensor (13) is conserved:

\[ \nabla^\flat T^{\mu\nu}(x) = 0. \] (17)

Therefore, it is consistent to introduce a semiclassical Einstein equation as

\[ G_{\mu\nu}(x) = 8\pi G_N T_{\mu\nu}(x). \] (18)

Note that the definition of \( T^{\mu\nu} \) as above in terms of pointlike particles is not in spirit of the usual formulation of QM. Nevertheless, assuming that one does not know the actual position of the particle, one may obtain an expression more in spirit of the usual formulation of QM by averaging over all possible particle positions. Assuming that \( \psi \) is a wave packet localized within a small 3-volume \( \sigma \subset \Sigma \), one makes the replacement

\[ T^{\mu\nu} \rightarrow \langle T^{\mu\nu} \rangle, \] (19)

where \( \langle T^{\mu\nu} \rangle \) is the energy-momentum averaged over the unknown particle positions and attributed to the small region \( \sigma \). The average energy-momentum \( \langle T^{\mu\nu} \rangle \) is obtained from \( T^{\mu\nu} \) in (14) by making a replacement

\[ \frac{\delta^3(x - X)}{\sqrt{|g(3)(x)|}} \rightarrow \frac{1}{v} \int_\sigma d^3x \sqrt{|g(3)(x)|} R^2(x) \omega(x), \] (20)

where \( v = \int_\sigma d^3x \sqrt{|g(3)|} \) and \( dX^\mu/\partial s \) is replaced by \( -m^{-1}\partial^\mu S\). Note, however, that the semiclassical Einstein equation with such an averaged energy-momentum is not physically viable when \( \psi \) is not a localized wave packet. For example, if \( \psi \) is a superposition that corresponds to two macroscopically separated lumps, then such a semiclassical Einstein equation with an energy-momentum averaged over both lumps contradicts experiments [8]. This indicates that the gravitational field responds to the actual (not to the average) particle position, so, in general, Eq. (15) seems more viable as a satisfying semiclassical theory of gravity.

C. Generalization to the many-particle case

Let us also briefly generalize the results above to the case of \( n \) particles with mass \( m \) described by a wave function \( \psi_n(x_1, \ldots, x_n) \). The wave function satisfies the many-particle generalization of (2)

\[ \sum_{a=1}^n (\nabla_a^\flat \partial_{a\mu} + m^2) \psi_n(x_1, \ldots, x_n) = 0. \] (21)

Thus, all equations above generalize in a trivial way by adding an additional label \( a \). In particular, (7) generalizes to

\[ Q_n = \frac{1}{2m} \sum_{a=1}^n \nabla_a^\flat \partial_{a\mu} R_n, \] (22)

(11) generalizes to

\[ m \frac{d^2X_n^\mu}{ds^2} = \partial_{\mu} Q_n, \] (23)

and (13) generalizes to

\[ T_n^{\mu\nu}(x) = \sum_{a=1}^n \int ds \frac{\delta^4(x - X_a(s))}{\sqrt{|g(x)|}} \times \left[ m \frac{dX_n^\mu}{ds} \frac{dX_n^\nu}{ds} - g^{\mu\nu}(x)Q_n(x) \right]. \] (24)

This provides a semiclassical theory of gravity for the case in which the number of particles \( n \) is fixed. However, to consider the possibility of particle creation and destruction, first quantization is not sufficient. The processes of particle creation and destruction can be described by QFT, which we do in the next section.

III. BOHMIAN SEMICLASSICAL GRAVITY IN QFT

A. Particles from QFT

As an example, consider a real field \( \phi \) in curved spacetime with a self-interaction described by the interaction Lagrangian density \(-\frac{\lambda}{4!} \phi^4\). In the Heisenberg picture, the field operator \( \hat{\phi}(x) \) satisfies

\[ \nabla^\flat \partial_{\mu} \hat{\phi}(x) + m^2 \hat{\phi}(x) + \frac{\lambda}{3!} \hat{\phi}^3(x) = 0. \] (25)

As outlined in the Introduction and references cited therein, we assume that a preferred foliation of spacetime defines a preferred notion of particles. Therefore, an arbitrary QFT state \( \ket{\Psi} \) can be written as a superposition of n-particle states as

\[ \ket{\Psi} = \sum_{n=0}^{\infty} c_n \ket{\Psi_n}, \] (26)

where \( \ket{\Psi_n} \) is a normalized n-particle state. The normalized n-particle wave function is then defined as [24, 34]

\[ \psi_n(x_1, \ldots, x_n) = \frac{S(x_n)}{\sqrt{n!}} \ket{\hat{\phi}(x_1) \cdots \hat{\phi}(x_n) \ket{\Psi_n}} \]

\[ = \frac{S(x_n)}{c_n \sqrt{n!}} \ket{\hat{\phi}(x_1) \cdots \hat{\phi}(x_n) \ket{\Psi}}, \] (27)
where $|0\rangle \equiv |\Psi_0\rangle$ and $S_{(x_a)}$ denotes the symmetrization over all $x_a$, $a = 1, \ldots, n$, which is needed because the field operators do not commute for nonequal times. For $\lambda = 0$, Eq. (25) implies that the wave function (27) satisfies the $n$-particle Klein-Gordon equation (21). To see an effect of the self-interaction term in (25) on the wave functions, we consider an immediate consequence of (25):

$$\left(\nabla^\mu \partial_\mu - m^2 + \frac{\lambda}{3!} \phi^3(x)\right) |\Psi\rangle = 0. \quad (28)$$

Eqs. (28) and (27) then imply

$$c_1 [\nabla^\mu \partial_\mu + m^2] \psi_1(x) + \frac{\lambda}{\sqrt{3!}} c_3 \psi_3(x, x, x) = 0. \quad (29)$$

Thus the nonlinear equation (25) for the field operator implies a linear equation for the wave functions, such that the nonlinearity transforms into a linear interaction between wave functions for different numbers of particles. Eq. (29) also shows under which conditions the particle described by $\psi_1$ behaves as a free particle satisfying the free Klein-Gordon equation (2); the interaction is non-negligible only when all 4 particles (1 particle described by $\psi_1(x)$ and 3 particles described by $\psi_3(x_1, x_2, x_3)$) are “close to each other”, in the sense that the wave packets described by $\psi_1$ and $\psi_3$ have a significant overlap. This is, indeed, consistent with the phenomenological picture according to which particles need to come close to each other in order to interact by an interaction such as the $-(\lambda/4!) \phi^4$ theory.

By writing

$$\psi_1(x) = R_1(x)e^{iS_1(x)},$$
$$\psi_3(x, x, x) = R_3(x)e^{iS_3(x)}, \quad (30)$$

and, for simplicity, by assuming that $c_3/c_1$ is real, the complex equation (29) is equivalent to a set of two real equations

$$-\frac{(\partial^\mu S_1)(\partial_\mu S_1)}{2m} + \frac{m}{2} + Q = 0, \quad (31)$$

$$\nabla^\mu (R_1^2 \partial_\mu S_1) = J, \quad (32)$$

where

$$Q \equiv \frac{1}{2m} \left[ \nabla^\mu \partial_\mu R_1 + \frac{\lambda}{\sqrt{3!} c_1 R_1} c_3 R_3 \cos(S_1 - S_3) \right], \quad (33)$$

$$J \equiv \frac{\lambda}{\sqrt{3!} c_1} R_1 R_3 \sin(S_1 - S_3). \quad (34)$$

The Bohmian particle trajectory associated with the wave function $\psi_3(x)$ can be introduced in the same way as in [13] with $S \to S_1$, but now with a modified quantum potential [33]. Consequently, the associated energy-momentum tensor $T^\mu_\nu$ is given by the expression (13), in which $Q$ is given by (33). In a similar way, it is straightforward to derive a modified expression for $T^\mu_\nu$ in (24) for an $n$-particle wave function (27). (The expression for $Q_n$ in (22) attains additional terms proportional to $\lambda$ similar to that in (33), but we do not write them explicitly as the explicit expression for general $n$ is rather cumbersome.)

In this way one can define $T^\mu_\nu$ for any $n \geq 1$, but not for $n = 0$. The absence of the $n = 0$ term is a simple consequence of the fact that, by definition, the energy-momentum is that of particles (not of fields), so the no-particle-state (the vacuum) does not contribute to the energy-momentum. Perhaps a vacuum contribution to the energy-momentum could be introduced by hand, but here it would be a rather artificial procedure. This should be contrasted with the usual field-theoretic approach where the fields (not the particles) are regarded as fundamental objects, so that the vacuum contribution appears naturally in the field energy-momentum tensor, leading to the old CC problem. Here, in our approach with particles regarded as more fundamental than fields, the old CC problem simply does not appear. Turning this argument round, the fact that the measured cosmological constant is many orders of magnitude smaller than the one predicted by the field energy-momentum indicates that the particles (not the fields) might be the fundamental objects existing in nature.

In this picture, quantum fields are merely auxiliary mathematical objects useful for calculation of certain particle processes, such as particle creation and destruction. (For a somewhat similar view of QFT, see also [33].)

Note also that Eq. (22) indicates that $R_2^2 \omega_1$ is not the probability density for the particle described by $\psi_1$ when the overlap with $\psi_3$ is significant. Nevertheless, the probability density can be calculated in principle by explicitly calculating the trajectories for a large sample of initial particle positions, provided that the initial overlap is negligible, so that the initial probability density is given by $R_2^2 \omega_1$.

B. The effects of particle creation and destruction

To explicitly take into account the effects of particle creation and destruction, it is more convenient to work in the Schrödinger picture [24, 30]. In this picture, the QFT state is denoted as $\Psi(\phi; t)$, which is a functional with respect to $\phi(x)$ and a function with respect to $t$. Eq. (20) is now written as

$$\Psi(\phi; t) = \sum_{n=0}^{\infty} \tilde{\Psi}_n(\phi; t), \quad (35)$$

where the tilde above $\tilde{\Psi}_n$ denotes that the norm of it may be smaller than unit. In the processes of particle creation and destruction this norm changes with time. The field $\phi$ may also be interpreted in a Bohmian deterministic manner [10, 11]. By writing $\Psi = \Re e^{iS}$, one finds an
expression analogous to (9)

\[ \frac{\partial \Phi(x, t)}{\partial t} = \frac{\delta S}{\delta \phi(x)}. \]  

(36)

where it is understood that the right-hand side is evaluated at \( \phi = \Phi \). The Bohmian effectivity \( e_n \) of the \( n \)-particle sector of (33) is \[ |\Psi_n(\Phi)\rangle \] 

(37)

The effectivity \( e_n \) is a number between 0 and 1 and satisfies \( \sum_{n=0}^{\infty} e_n = 1 \). As shown in [24], when the number of particles is measured, then \( e_n \) becomes \( e_n = 1 \) for one \( n \) and \( e_{n'} = 0 \) for all other \( n' \). This corresponds to an effective collapse of (33) to one of \( \Psi_n \)'s, which is induced by the quantum measurement. The probability for such an effective collapse is exactly equal to the corresponding probability predicted by the standard probabilistic rules of QFT [24]. However, when the number of particles is not measured, i.e., when more than one \( e_n \) is different from 0, then all \( T^\mu_\nu \) for which \( e_n \neq 0 \) contribute to the total energy-momentum. Thus, the total energy-momentum is

\[ T^\mu_\nu = \sum_{n=1}^{\infty} e_n T^\mu_\nu_n + U^\mu_\nu. \] 

(38)

The additional term \( U^\mu_\nu \) is a compensating term that provides the conservation of \( T^\mu_\nu \) even when the effectivities \( e_n \) change with time. Since \( \nabla_\nu T^\mu_\nu_n = 0 \) by construction, the requirement

\[ \nabla_\nu T^\mu_\nu = 0 \] 

(39)

leads to the equation

\[ \nabla_\nu U^\mu_\nu = j^\mu, \] 

(40)

where

\[ j^\mu \equiv -\sum_{n=1}^{\infty} (\partial_\nu e_n) T^\mu_\nu. \] 

(41)

We see that \( j^\mu \) can be viewed as a collection of pointlike sources nonvanishing only along the trajectory. However, in [38] we do not want \( U^\mu_\nu \) to be nonvanishing only along the particle trajectories, because then \( U^\mu_\nu \) would simply cancel the pointlike energy-momentum of new created particles described by the first term in [38], so that the new created particles would not influence the gravitational field. Instead, we want equation (41) to describe a continuous field \( U^\mu_\nu(x) \) produced by the pointlike sources \( j^\mu \). This makes \( U^\mu_\nu \) in (41) similar to the electromagnetic field described by the Maxwell equations, but with an important difference consisting in the fact that \( U^\mu_\nu \) is a symmetric tensor, whereas the electromagnetic field is an antisymmetric tensor. Therefore, we assume

\[ U^\mu_\nu = \nabla^\mu V^\nu + \nabla^\nu V^\mu, \] 

(42)

where \( V^\mu(x) \) is a vector field analogous to the electromagnetic potential. Now [40] becomes

\[ \nabla_\nu \nabla^\mu V^\nu + \nabla_\nu \nabla^\nu V^\mu = j^\mu, \] 

(43)

which describes the propagation of the field \( V^\mu \), the source of which is a collection of pointlike sources described by \( j^\mu \). Eq. (43) represents a set of 4 equations for 4 unknowns \( V^\mu \), which further justifies the ansatz (42).

In some cases, the solution of (43) can be found explicitly. For example, assume (i) that spacetime can be approximated by a flat spacetime and (ii) that \( \partial_\nu e_n \) changes slowly, so that one can use the approximation \( \partial_\mu \partial_\nu e_n \approx 0 \). In this case, (43) can be written as

\[ \partial_\nu \partial_\mu V^\nu + \partial_\nu \partial_\nu V^\mu = j^\mu, \] 

(44)

while \( j^\mu \) is approximately conserved:

\[ \partial_\nu j^\mu = -\sum_{n=1}^{\infty} (\partial_\nu \partial_\nu e_n) T^\mu_\nu \approx 0. \] 

(45)

Introducing the well-known retarded Green function \( G(x - x') \) satisfying

\[ \partial_\nu \partial_\mu G(x - x') = \delta^4(x - x'), \] 

(46)

the explicit solution of (44) is

\[ V^\nu(x) = \int d^4 x' G(x - x') j^\nu(x'). \] 

(47)

Indeed, (45) implies that (47) satisfies the Lorentz condition

\[ \partial_\mu V^\nu(x) = \int d^4 x' G(x - x') \partial_\nu j^\mu(x') \approx 0, \] 

(48)

so (44) reduces to \( \partial_\mu \partial_\nu V^\mu = j^\mu \), which, indeed, is satisfied by (47).

Now the final semiclassical Einstein equation reads

\[ G_{\mu\nu}(x) = 8\pi G_N T_{\mu\nu}(x), \] 

(49)

where the quantum matter energy-momentum tensor \( T_{\mu\nu}(x) \) is given by [38]. Of course, we have explicitly analyzed only the contributions from massive spinless uncharged particles corresponding to the hermitian field \( \phi \), but the contributions from other types of particles can be introduced in a similar way. Some additional physical features of the resulting semiclassical theory are qualitatively discussed in the next section.
IV. DISCUSSION AND CONCLUSION

As we have seen, by regarding particles as more fundamental objects than fields, the usual field energy-momentum tensor no longer represents the physical energy-momentum, which automatically solves (or at least avoids) the old CC problem, simply because only particles contribute to the physical energy-momentum. However, it is important to emphasize that, by discarding the field ground-state energy, we do not discard the particle ground-state energy. The QFT ground state containing no particles is physically very different from the particle ground state. The best known example of the latter is a single particle in a one-dimensional harmonic-oscillator potential $V(x) = m \omega^2 x^2 / 2$, where the ground state having the nonrelativistic energy $\omega/2$ is still a one-particle (not zero-particle) state. Indeed, such a particle ground-state energy is included in the particle energy-momentum. In fact, the second term in $\mathcal{H}$ proportional to $g^{\mu\nu} Q$ is exactly of the form of a cosmological term. Moreover, in a nonrelativistic limit one may expect that $\partial^\mu \partial_\mu R \sim \pm m^2 R$, so (7) implies

$$|Q| \sim m.$$

This means that particles with a mass $m$ may contribute to the cosmological constant by a contribution of the order of $mn_e$, where $n_e$ is the number of particles per unit volume. It is tempting to speculate that this could have something to do with the coincidence problem, i.e., with the new CC problem. Note, however, that a plane wave $e^{-i k \cdot x}$ has a constant $R$, so (7) vanishes for a plane wave. Nevertheless, it is conceivable that the so-called dark energy might consist of particles described by a nontrivial wave function that leads to a nontrivial quantum potential $Q$, so that (i) the energy-momentum of these particles is dominated by a cosmological term $\propto g^{\mu\nu} Q$ and (ii) the quantum force described by (11) prevents these particles from forming structures. Such a wave function should be a wave packet with a width larger than typical scales associated to cosmological structures. (The needed large width might be a natural consequence of inflation.) However, a more serious investigation of such a possibility would require a further theoretical input, which would go beyond the scope of the present paper.

Concerning the issue of the new CC problem, we recall that a term proportional to $\lambda$ also survives in (33). This demonstrates that a nontrivial field potential may also influence the cosmological constant. In particular, it means that the quintessence models of dark energy may also play a role for the new CC problem, provided that they are reinterpreted in terms of particle wave functions, analogously to that in [29]. A similar remark applies also to scalar-field potentials supposed to drive the early cosmological inflation.

Another new physical ingredient that we want to discuss is the physical meaning of $U^{\mu\nu}$ in (35). Unlike the first term in (33), $U^{\mu\nu}$ represents a continuously distributed contribution to the total energy-momentum. Thus, it is a nonparticle contribution to the energy-momentum, but the particles are the source for it. More precisely, from (10) and (11) we see that $U^{\mu\nu}$ is created only when the effective cosines $e_n$ change with time. Physically, this means that a particle that gets destroyed compensates it by emitting positive $U$-energy, while a particle that gets created compensates it by emitting negative $U$-energy. In fact, in most physical processes with particle creation and destruction (usually described by the S-matrix formalism in elementary-particle physics) the energy-momentum of the initial particles is exactly equal to the energy-momentum of the final particles. This means that $U^{\mu\nu}$ averaged over a large volume vanishes in the initial as well as in the final state of such a process. The creation of $U^{\mu\nu}$ as described by (10) is only a transient phenomenon, not directly observable in typical particle collision and decay processes. On the other hand, when particles are created from an unstable vacuum, then the conservation of $T^{\mu\nu}$ implies that average $U^{\mu\nu}$ must be nonzero even in the final state. In particular, this provides a backreaction mechanism for the process of Hawking radiation, in which particles are created from the vacuum in a background of a classical black-hole [27]. Thus, Eq. (49) may be applied to a new analysis of the process of Hawking radiation with backreaction, but a detailed analysis of such a process is beyond the scope of the present paper. It is also fair to note that the ansatz (42) is not necessarily the only possibility.

To conclude, the formulation of semiclassical gravity in terms of Bohmian particle trajectories has several advantages over the usual formulation. First, regarding particles (rather than fields) as the fundamental physical objects automatically avoids the old CC problem. Second, the use of the Bohmian formulation of quantum theory avoids the discontinuous collapse problem. Besides, this formulation suggests new approaches to the solution of the new CC problem and of the backreaction problem associated to particle creation by classical gravitational fields. Thus, we believe that our new approach to semiclassical gravity is worthwhile of further investigation.

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