Exact correlation functions in $SU(2) \, \mathcal{N} = 2$ superconformal QCD

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We report an exact solution of 2- and 3-point functions of chiral primary fields in $SU(2) \, \mathcal{N} = 2$ super-Yang-Mills theory coupled to four hypermultiplets. It is shown that these correlation functions are non-trivial functions of the gauge coupling, obeying differential equations which take the form of the semi-infinite Toda chain. We solve these equations recursively in terms of the Zamolodchikov metric that can be determined exactly from supersymmetric localization on the four-sphere. Our results are verified independently in perturbation theory with a Feynman diagram computation up to 2-loops. This is a short version of a companion paper that contains detailed technical remarks, additional material and aspects of an extension to $SU(N)$ gauge group.

INTRODUCTION

Quantum field theories often possess exactly marginal deformations along which the data of the theory (spectrum, correlation functions, etc.) may change continuously. A characteristic well-studied example in four dimensions is $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory. In this case an exactly marginal deformation interpolates between weak coupling (where the theory can be analyzed with standard perturbation theory) and strong coupling (where standard perturbative methods are inadequate). It is of great interest to develop non-perturbative techniques that allow us to describe (analytically) properties of the theory at any value of the marginal couplings.

Supersymmetric theories are an opportune context for the development of such techniques. They often possess special sectors that exhibit dynamics with non-trivial, but exactly computable, coupling constant dependence. An exact solution in these sectors can provide useful intuition, or a solid starting point, towards an analysis of the more general properties of the theory.

In this note we will concentrate on a specific example of a four-dimensional conformal field theory with $\mathcal{N} = 2$ supersymmetry: $\mathcal{N} = 2$ SYM theory with gauge group $SU(2)$ coupled to 4 hypermultiplets in the fundamental representation (in short, $SU(2) \, \mathcal{N} = 2$ superconformal QCD, or simply SCQCD). By definition, this theory is invariant under 8 real supercharges. The special sector of interest comprises of (scalar) superconformal chiral primaries $\phi_I$ (to be specified explicitly in a moment) annihilated by the four supercharges of right chirality. The conjugate fields annihilated by the supercharges of left chirality will be denoted as $\bar{\phi}_I$. $\mathcal{N} = 2$ superconformal field theories (SCFTs) are also invariant under the global $SU(2)_R \times U(1)_R$ R-symmetry. The chiral primaries $\phi_I$ are singlets of the $SU(2)_R$, but have non-zero $U(1)_R$ charge $R$ [1]. Their scaling dimension $\Delta$ obeys the relation $\Delta = \frac{R}{2}$. (For anti-chiral primaries $\Delta = -\frac{R}{2}$).

It is well known that the operator product expansion (OPE) of chiral primary fields is non-singular

$$\phi_I(x) \phi_J(0) = C^K_{IJ} \phi_K(0) + \ldots .$$ (1)

It forms a ring structure known as the chiral ring [2]. Two important sets of data in the chiral ring are the 2-point functions

$$\langle \phi_I(x) \bar{\phi}_J(0) \rangle = \frac{g_{\tau IJ}}{|x|^2 \Delta}$$ (2)

and the 3-point functions

$$\langle \phi_I(x) \phi_J(y) \bar{\phi}_K(z) \rangle = \frac{C_{IJK}}{|x-y|^2 \Delta_{IJ,K} |y-z|^2 \Delta_{JK,I} |z-x|^2 \Delta_{IK,J}}$$ (3)

where $\Delta_{IJ,K} = \Delta_I + \Delta_J - \Delta_K$. There is an obvious relation between the OPE and 2- and 3-point function coefficients $C_{IJK} = C_{IJ}^L g_{LJK}$.

In our example there is a single exactly marginal deformation labelled by a complex parameter $\tau$ (the complexified gauge coupling constant). The 2- and 3-point function coefficients $g_{\tau IJ}$, $C_{IJK}$ are non-trivial functions of $\tau$, receiving corrections at all orders in perturbation theory as well as from instanton effects. (The scaling dimensions $\Delta_I$ are fixed by the non-renormalized $U(1)_R$ charge $R_I$ as described above). We will present exact formulae for these data combining methods of supersymmetric localization (in particular, [3, 4]) with certain exact relations between chiral ring correlation functions [5] that are four-dimensional analogs of the $tt^*$ equations in two
We have verified the resulting expressions with an independent computation in perturbation theory up to 2-loops [8].

We point out that analogous correlation functions in $\mathcal{N} = 4$ SYM theory are non-renormalized [9–19] and are therefore trivial functions of the gauge coupling that can be determined at tree-level. $\mathcal{N} = 2$ dynamics is clearly more interesting and the results in this paper indicate that there is a considerable amount of new data that are tractable analytically compared to previous knowledge. The techniques presented here are useful in $\mathcal{N} = 2$ theories with exactly marginal deformations beyond the specific example analyzed in this note. A detailed explanation of the general properties of these techniques and extensions to more general examples are discussed in a companion paper [8].

$SU(2)\mathcal{N} = 2$ SCQCD

The main example of this note is $\mathcal{N} = 2$ SYM theory with gauge group $SU(2)$ coupled to 4 hypermultiplets (at the origin of the Coulomb branch). This is a gauge theory whose field content includes: (a) the $\mathcal{N} = 2$ vector multiplet fields, namely the gauge bosons $A_\mu$, a complex scalar field $\varphi$ and four Weyl fermions (all in the adjoint representation); (b) the $4\mathcal{N} = 2$ hypermultiplets that comprise of 4 complex bosons and 8 Weyl fermions (all in the fundamental representation). The global symmetry group is $U(4) \times SU(2)_R \times U(1)_R$. $U(4)$ is a flavor symmetry rotating the hypermultiplets. The standard Yang-Mills Lagrangian of this theory is summarized, for instance, in appendix B of [8] whose conventions we are also following here.

The single exactly marginal coupling of this theory is the complexified Yang-Mills coupling $\tau = \theta \pi + \frac{4\pi i}{g^2_{YM}}$, where $\theta$ is the $\theta$-angle and $g_{YM}$ is the Yang-Mills coupling. We will work in conventions in which the infinitesimal exactly marginal deformation of the action takes the form

$$S \rightarrow S + \frac{\delta \tau}{4\pi^2} \int d^4x \, \mathcal{O}_\tau(x) + \frac{\delta \bar{\tau}}{4\pi^2} \int d^4x \, \bar{\mathcal{O}}_\tau(x)$$

where $\Delta = 4$ operators $\mathcal{O}_\tau, \bar{\mathcal{O}}_\tau$ are descendants of $\Delta = 2$ (anti)chiral primary fields

$$\mathcal{O}_\tau = Q^4 \cdot \phi_2, \quad \bar{\mathcal{O}}_\tau = \bar{Q}^4 \cdot \bar{\phi}_2.$$ (5)

The notation $Q^4 \cdot \phi_2$ is shorthand notation for the nested (anti)-commutator of four supercharges of left chirality. The Lorentz and $SU(2)_R$ indices of the supercharges are combined to give a Lorentz and $SU(2)_R$ singlet. $\phi_2$ is the lowest dimension $\mathcal{N} = 2$ chiral primary field

$$\phi_2 = \frac{\pi i}{8} \text{Tr}[\varphi^2].$$ (6)

The overall normalization in (5) is fixed so that $\langle \mathcal{O}_\tau(x) \bar{\mathcal{O}}_\tau(0) \rangle = \nabla^2_x \nabla^2 \langle \phi_2(x) \bar{\phi}_2(0) \rangle$.

The chiral ring of the $SU(2)$ theory can be freely generated by the chiral primary field $\phi_2$ by repeated multiplication. The explicit checks reported below verify the consistency of this picture. We will normalize the generic chiral primary $\phi_{2n} \propto \text{Tr}[\varphi^2]^n$ by requiring the OPE

$$\phi_2(x) \phi_{2n}(0) = \phi_{2n+2}(0) + \ldots.$$ (7)

This choice fixes all the non-vanishing OPE coefficients

$$C^{2(2n+m)}_{2n \ 2m} = 1$$ (8)

and the normalization of all the higher order chiral primaries $\phi_{2n}$ ($n > 1$) which are multi-trace.

To summarize, the (chiral ring) sector of interest in this paper comprises of a sequence of fields $\phi_{2n}$ with scaling dimensions $\Delta_{2n} = 2n$.

We will denote the 2-point functions of these fields as

$$\langle \phi_{2n}(x) \bar{\phi}_{2n}(0) \rangle = \frac{g_{2n}(\tau, \bar{\tau})}{|x|^{4n}}.$$ (9)

The 2-point function coefficients $g_{2n}$ (as well as the corresponding 3-point function coefficients $C^{2m 2n 2m+2n}_{2m 2n 2m+2n}$) are non-trivial functions of the complexified coupling $\tau$ that we will determine exactly.

Notice that $g_2$ is directly related to the coefficient $G_2$ of the 2-point function $\langle \mathcal{O}_\tau(x) \bar{\mathcal{O}}_\tau(0) \rangle$. $G_2$ is the so-called Zamolodchikov metric on the space of exactly marginal couplings. For $\mathcal{N} = 2$ theories this space is known to be a complex Kähler manifold. Hence, (specializing to the case at hand) there is a scalar function $K$, the Kähler potential, such that

$$G_2 = \partial_\tau \partial_{\bar{\tau}} K = 192 \ g_2.$$ (10)

EXACT CORRELATION FUNCTIONS

Ref. [5] formulated a set of exact relations between the OPE and 2-point function coefficients for general four-dimensional $\mathcal{N} = 2$ theories with exactly marginal directions. These relations, which take the form of systems of differential equations on the marginal couplings, are direct analogs of the $tt^*$ equations in two-dimensional $\mathcal{N} = (2, 2)$ superconformal theories derived in [6, 7] with the method of the topological-antitopological fusion. Ref. [5] derived such relations in four dimensions with the judicious use of superconformal Ward identities.

Applying the general $tt^*$ equations of [5] in the case of interest here in the so-called holomorphic gauge and the related above-mentioned normalization conventions (see [8] for an exposition of all the pertinent details) we arrive at the following relations for the 2-point function coefficients $g_{2n}$ (9)

$$\partial_\tau \partial_{\bar{\tau}} g_{2n} = \frac{g_{2n+2} - g_{2n}}{g_{2n-2}} - g_2.$$ (11)
where \( n = 1, 2, \ldots \) and \( g_0 = 1 \) by definition. By unitarity all \( g_{2n} > 0 \) and this infinite sequence of differential equations can be recast as the more familiar semi-infinite Toda chain

\[
\partial_{\tau} \partial_{\bar{\tau}} q_n = e^{q_{n+1}-q_n} - e^{q_n-q_{n-1}}, \quad n = 2, \ldots \quad (12)
\]

by setting \( g_{2n} = \exp(q_n - \log\left(\frac{K}{192}\right)) \). \( K \) is the Kähler potential in (10) and the factor of 192 follows from the normalization conventions of the previous section.

It is interesting to ask what is the general solution of the system (11) subject to positivity over the entire space of marginal couplings and whether positivity and some other ‘boundary conditions’ from perturbation theory at weak coupling can fix the solution uniquely. We will not try to answer this question here. Instead, we will use the system of equations (11) recursively, writing

\[
g_{2n+2} = g_{2n} \partial_{\tau} \partial_{\bar{\tau}} \log g_{2n} + \frac{g_{2n}^2}{g_{2n-2}} + g_2 g_{2n}, \quad n = 1, 2, \ldots, \quad (13)
\]

to determine all the higher 2-point functions \( g_{2n} (n > 1) \) from the lowest one \( g_2 \).

**Exact 2-point functions.** Recent work [4] has determined the exact quantum Kähler potential of \( \mathcal{N} = 2 \) SCFTs in terms of the partition function \( Z_{S^4} \) of the theory on the four-sphere \( S^4 \). The precise relation is

\[
K = 192 \log Z_{S^4}. \quad (14)
\]

Notice that the marginal operators \( \mathcal{O}_\tau \) are normalized differently in [4], i.e. \( \mathcal{O}_{here} = 4 \mathcal{O}_{there} \). This explains the factor \( 192 = 12 \times 4 \times 4 \) as opposed to 12 in [4]. Combining with (10) we obtain

\[
g_2 = \partial_{\tau} \partial_{\bar{\tau}} Z_{S^4}. \quad (15)
\]

For the \( SU(2) \) SCQCD theory there is a well-studied integral expression for the sphere partition function \( Z_{S^4} \) that has been determined using supersymmetric localization [3]

\[
Z_{S^4}(\tau, \bar{\tau}) = \int_{-\infty}^{\infty} da \ e^{-4 \pi \text{Im}(\tau) a^2} \frac{H(2ia)H(-2ia)}{H(ia)H(-ia)^2} Z_{\text{inst}}(a, \tau)^2. \quad (16)
\]

\( H(z) = G(1+z)G(1-z) \) in terms of the Barnes \( G \)-function [20], and \( Z_{\text{inst}} \) is the Nekrasov partition function [21] that incorporates the contribution from all the instanton sectors. For further details we refer the reader to [3].

Combining the expressions (13), (15) and (16) we are able to determine recursively any of the 2-point function coefficients \( g_{2n} \) in terms of higher derivatives of the \( S^4 \) partition function.

**Exact 3-point functions.** The non-vanishing 3-point function coefficients \( C_{2m2n \frac{m+n}{2}} \) follow immediately from the general relation \( C_{\ell_1 \ell_2} = |\mathcal{I}^{\ell_1 \ell_2}_{\ell_1 \ell_2}|, \) equation (8), and the above solution of the 2-point function coefficients

\[
C_{2m2n \frac{m+n}{2}} = C_{2(m+n)}^{2m2n} g_2(m+n) = g_2(m+n). \quad (17)
\]

Notice that, although the normalization conventions of the previous sections are very convenient for the above computations, in conformal field theory it is common to work instead with orthonormal primary operators \( \phi_{2n} \) for which \( \langle \phi_{2n}(x) \phi_{2m}(0) \rangle = \frac{\delta_{m,n}}{2|x|^2} \). In these alternative conventions, the 2-point function coefficients are trivial but the OPE coefficients are non-trivial and

\[
C_{2m2n \frac{m+n}{2}} = \sqrt{\frac{g_2(m+n)}{2g_{2m}g_{2n}}}. \quad (18)
\]

**More general extremal correlators.** With a conformal transformation of the form \( x' = \frac{x - y}{|x-y|^2} \) it is possible to recast the general ‘extremal correlator’

\[
\langle \phi_{2m_1}(x_1) \cdots \phi_{2m_n}(x_n) \rangle \quad (19)
\]

with \( m = \sum_{i=1}^{n} m_n \) as

\[
\langle \phi_{2m_1}(x'_1) \cdots \phi_{2m_n}(x'_n) \rangle = \frac{\delta_{m,n}}{|x_1 - y|m_1} \cdots |x_n - y|m_n \quad (20)
\]

Using superconformal Ward identities one can prove that the correlation function on the numerator of (20) is independent of the positions \( x_i \). Consequently, it can be evaluated in any particular limit; in particular, we can make use of the above known OPEs and 2-point functions \( g_{2n} \) to determine the exact \( \tau \)-dependence of such extremal correlators as well, as explained in more detail in [8].

**PREDICTIONS FOR PERTURBATION THEORY**

We can use the above results to make very specific predictions for the weak coupling, \( g_M v \ll 1 \), expansion of 2- and 3-point functions in the chiral ring. As an illustration, here we present explicit examples in the 0-instanton and 1-instanton sectors.

**0-instanton sector.** Working with the perturbative (0-instanton) part of the \( S^4 \) partition function (16)

\[
Z_{S^4}^{(0)} = \int_{-\infty}^{\infty} da \ e^{-4 \pi \text{Im}(\tau) a^2} \frac{H(2ia)H(-2ia)}{H(ia)H(-ia)^2} \quad (21)
\]

our exact formulae provide, e.g. for the first three chiral
primaries, the perturbative expansions

\begin{align*}
g_2^{(0)} &= \frac{3}{8} - \frac{135 \zeta(3)}{32 \pi^2 (\text{Im} \tau)^2} + \frac{1575 \zeta(5)}{64 \pi^3 (\text{Im} \tau)^3} + \ldots , \\
g_4^{(0)} &= \frac{15}{32} - \frac{945 \zeta(3)}{64 \pi^2 (\text{Im} \tau)^2} + \frac{7875 \zeta(5)}{64 \pi^3 (\text{Im} \tau)^3} + \ldots , \\
g_6^{(0)} &= \frac{315}{256} - \frac{76545 \zeta(3)}{1024 \pi^2 (\text{Im} \tau)^2} + \ldots .
\end{align*}

The superscript 0 denotes that this is the 0-instanton contribution. We wrote down contributions only up to 3-loops, but it is easy to go to any desired order. We have verified the validity of the predicted \( g_{2n}^{(0)} \) for all values of the positive integer \( n \), with an independent computation in perturbation theory up to 2-loops \([8]\). This provides an independent 2-loop perturbative check of the \( t^t \) equations \([13]\), but also a check of the recent proposal of Ref. \([4]\) that identifies the quantum Kähler potential of \( \mathcal{N} = 2 \) theories with the \( S^4 \) partition function.

Equivalently, in the alternative basis with orthonormal 2-point functions formula \((18)\) provides very specific results for the non-trivial 3-point function coefficients \( C_{2m}^{(0)} \). As an illustration the first few coefficients are

\begin{align*}
\hat{C}_{22}^{(0)} &= \sqrt{10} \left( 1 - \frac{9 \zeta(3)}{2 \pi^2 (\text{Im} \tau)^2} + \frac{525 \zeta(5)}{8 \pi^3 (\text{Im} \tau)^3} + \ldots \right), \\
\hat{C}_{24}^{(0)} &= \sqrt{7} \left( 1 - \frac{9 \zeta(3)}{\pi^2 (\text{Im} \tau)^2} + \frac{675 \zeta(5)}{4 \pi^3 (\text{Im} \tau)^3} + \ldots \right), \\
\hat{C}_{26}^{(0)} &= \sqrt{12} \left( 1 - \frac{27 \zeta(3)}{2 \pi^2 (\text{Im} \tau)^2} + \frac{2475 \zeta(5)}{8 \pi^3 (\text{Im} \tau)^3} + \ldots \right) .
\end{align*}

1-instanton sector. As an example, we consider the 1-instanton contribution to the \( S^4 \) partition function \((16)\)

\begin{equation}
Z_{S^4}^{(1)} = \cos \theta \exp \left( -\frac{8 \pi^2}{g_{YM}^2} \right) \left( 1 - \frac{1}{8 \pi \text{Im} \tau} - \frac{45 \zeta(3)}{16 \pi^2 (\text{Im} \tau)^2} + \frac{105 (\zeta(3) + 10 \zeta(5))}{128 \pi^3 (\text{Im} \tau)^3} + \ldots \right)
\end{equation}

and from this we obtain expansions of \( g_{2n} \) in the 1-instanton sector

\begin{align*}
g_2^{(1)} &= \cos \theta \exp \left( -\frac{8 \pi^2}{g_{YM}^2} \right) \left[ \frac{3}{8 (\text{Im} \tau)^2} + \frac{3}{16 \pi (\text{Im} \tau)^3} \right] , \\
g_4^{(1)} &= \cos \theta \exp \left( -\frac{8 \pi^2}{g_{YM}^2} \right) \left[ \frac{15}{16 (\text{Im} \tau)^4} + \frac{15}{32 \pi (\text{Im} \tau)^5} \right] .
\end{align*}

If desired it is straightforward to extend these results to higher \( n \), higher instanton number \( \ell \) and higher order in the perturbative expansion around any given instanton sector. It would be interesting to confirm them with an independent perturbative computation in the general \( \ell \)-instanton sector. Moreover, it would be interesting to verify the expected positivity of the resulting expressions at general \( n \).

OUTLOOK

We reported exact non-perturbative formulae for 2- and 3-point functions of chiral primary fields in the SU(2) \( \mathcal{N} = 2 \) SCQCD theory. A detailed exposition of the employed technology, of the perturbative 2-loop check, as well as an extension to the \( \mathcal{N} = 2 \) SCQCD theory with more general \( SU(N) \) gauge group can be found in the companion paper \([8]\). Currently, we do not have a full solution of the \( t^t \) equations in the \( SU(N) \) case, but we find preliminary signs of an underlying structure that remains to be understood better.

The present results indicate the possibility that the dependence of the chiral ring structure of four-dimensional \( \mathcal{N} = 2 \) theories on their marginal couplings is exactly computable despite being highly non-trivial. It would be interesting to extend the application of the 4d \( t^t \) equations \([5]\) to other known classes of \( \mathcal{N} = 2 \) theories and to determine general conditions (e.g. positivity constraints) that fix their solution uniquely. Such solutions are expected to have wider implications. For example, we have already seen that the explicit knowledge of 2- and 3-point functions implies also the exact form of general extremal correlation functions in the chiral ring. In a different direction one can envision using these results as input in a more general bootstrap program in \( \mathcal{N} = 2 \) SCFTs aiming to determine larger classes of correlation functions, spectral data etc. Clearly, more remains to be done.

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