SpeedyIBL: A Solution to the Curse of Exponential Growth in Instance-Based Learning Models of Decisions from Experience

Thuy Ngoc Nguyen, Duy Nhat Phan, Cleotilde Gonzalez*

Abstract Computational cognitive modeling is a useful methodology to explore and validate theories of human cognitive processes. Often cognitive models are used to simulate the process by which humans perform a task or solve a problem and to make predictions about human behavior. Cognitive models based on Instance-Based Learning (IBL) Theory rely on a formal computational algorithm for dynamic decision making and on a memory mechanism from a well-known cognitive architecture, ACT-R. To advance the computational theory of human decision making and to demonstrate the usefulness of cognitive models in diverse domains, we must address a practical computational problem, the curse of exponential growth, that emerges from memory-based tabular computations. When more observations accumulate, there is an exponential growth of the memory of instances that leads directly to an exponential slow down of the computational time. In this paper, we propose a new Speedy IBL implementation that innovate the mathematics of vectorization and parallel computation over the traditional loop-based approach. Through the implementation of IBL models in many decision games of increasing complexity, we demonstrate the applicability of the regular IBL models and the advantages of their Speedy implementation. Decision games vary in their complexity of decision features and in the number of agents involved in the decision process. The results clearly illustrate that Speedy IBL addresses the curse of exponential growth of memory, reducing the computational time significantly, while maintaining the same level of performance than the traditional implementation of IBL models.

Keywords Instance-Based Learning · Cognitive Models · Efficient Computation

*Corresponding author
Thuy Ngoc Nguyen · Duy Nhat Phan · Cleotilde Gonzalez
5000 Forbes Ave., Pittsburgh, 15213 PA, USA
E-mail: ngocnt@cmu.edu, dnphan@andrew.cmu, coty@cmu.edu
1 Introduction

A cognitive theory is a general postulation of mechanisms and processes that are globally applicable to families of tasks and types of activities rather than being dependent on a particular task. Cognitive models are very specific representations of part or of all aspects of a cognitive theory that apply to a particular task or activity [10]. Specifically, normative and descriptive theories of choice often rely on utility theory [37, 28] or aim at describing the psychological impact of perceptions of probability and value on choice [21, 42]. In contrast, models of decisions from experience (DfE) are often dynamic computational representations of sequential choices that are distributed over time and space and that are made under uncertainty [13].

Cognitive models of DfE can be used to simulate the interaction of theoretical cognitive processes with the environment, representing a particular task. These models can make predictions regarding how human choices are made in such tasks. These predictions are often compared to data collected from human participants in the same tasks using interactive tools. The explicit comparison of cognitive models’ predictions to human actual behavior is a common research approach in the cognitive sciences and in particular in the study of decision making [10]. Cognitive models are dynamic and adaptable computational representations of the cognitive structures and mechanisms involved in decision making tasks such as DfE tasks under conditions of partial knowledge and uncertainty. Moreover, cognitive models are generative, in the sense that they actually make decisions in similar ways like humans do, based on their own experience, rather than being data-driven and requiring large training sets. In this regard, cognitive models differ from purely statistical approaches, such as Machine Learning or Bayesian models, that are often capable of evaluating stable, long-term sequential dependencies from existing data but fail to account for the dynamics of human cognition and human adaptation to novel situations.

There are many models of DfE as evidenced by past modeling competitions [9, 8]. Most of these models often make broadly disparate assumptions regarding the cognitive processes by which humans make decisions [9]. For example, the models submitted to these competitions are often applicable to a particular task or choice paradigm rather than presenting an integrated view of how the dynamic choice process from experience is performed by humans. Associative learning models are a class of models of DfE that conceptualize choice as a learning process that stores behavior-outcome relationships and are contingent on the environment [18]. Generally speaking, these kinds of models rely on learning from reinforcement and the contingencies of the environment as in the Skinnerian tradition [39, 40]. These models have shown to be successful at representing human learning over time based on feedback.

In contrast to many of the associative learning models, Instance-Based Learning (IBL) models rely on a single decision theory: Instance-Based Learning Theory [14]. IBLT emerged from the need to explain
the process of dynamic decision making, where a sequence of interdependent decisions were made over time. IBLT provides a single general algorithm and mathematical formulations of memory retrieval that rely on the ACT-R cognitive architecture [2]. The theory proposes a representation of decisions in the form of instances, which are triplets involving state, action, and utilities; the theory also provides a process of retrieval of past instances based on their similarity to a current decision situation, and the generation of accumulated value (from experience) based on a mechanism called Blending, which is a function of the payoffs experienced and the probability of retrieving those instances from memory [24,12].

Many models have been developed based on IBLT. From its inception, the theory was demonstrated in a highly complex, dynamic decision-making task representing the complex process of dynamic allocation of limited resources over time and under time constraints in a “water purification plant” [14]. Since then, IBLT has been used to demonstrate human DfE in a large diversity of contexts and domains, from simple and abstract binary choice dynamics [12,24] to highly specialized and complex tasks such as cyber defense [14] and anti-phishing detection [7]. Also, IBL models have been created to account for group and network effects, where each individual in a group is represented by an IBL agent [11]; more recently, this IBL algorithm has also been applied to multi-state gridworld tasks [31,32,34] in which the agents execute a sequence of actions with delayed feedback.

The recent applications of IBL cognitive models have led to significantly more complex and realistic tasks, where multi-dimensional state-action-utility representations are required, where extended training is common, and where multi-agents interact to solve such tasks. Such an increase in the task complexities and the number of agents modeled with IBL leads to a practical computational problem, the curse of exponential growth, (c.f. [3]) [22]. The curse of exponential growth is a common problem for every modeling approach that relies on the accumulation of data over time and on tabular computation, such as reinforcement learning models (RL) [11]. Deep reinforcement learning, a combination of RL and deep learning, enables to move from simple representations to more realistic complex environments in games such as Atari, Go, Starcraft [27,38]. However, as summarized in a recent overview of the challenges in multi-agent RL models, these algorithms become less efficient with the increase in the dimensions of the state-action space, and as the number of agents increases [14]. The problem becomes even more complex under nonstationary environments and under uncertainty where information is incomplete. Dynamic conditions significantly increase the diversity and number of states as it is needed for every dynamic decision making task [13].

In this paper, we propose a solution to the curse of exponential growth by innovating the mathematics of vectorization and parallel computation over the traditional loop-based approach [23]. We propose this new method in a new Speedy IBL implementation. Importantly, we demonstrate how SpeedyIBL is increasingly more efficient than traditional IBL model implementations (PyIBL for Python-IBL, [29]), as the dimension-
ality and dynamics of the problems increase. The costs of computation in the PyIBL implementation grow exponentially as the dimensions of the representation increase and the number of agents and their interactions increase. The benefits of SpeedyIBL over regular PyIBL models, therefore, increase also exponentially.

2 Instance based Learning Theory

The general decision process proposed in IBLT is illustrated in Figure 1, and the mechanisms are made mathematically concrete in Algorithm 1 [14]. The process starts with the observation of the environmental state, and the determination of whether there are past experiences (i.e., instances) that are similar to the current environmental state (i.e., Recognition). Whether there are similar past instances will determine the process used to generate the expected utility of a decision alternative (i.e., Judgment). If there are past experiences that are similar to the current environmental state, the expected utility of such an alternative is calculated via a process of Blending past instances, but if there are no similar past instances, then the theory suggests that a heuristic is used instead. After Judgment, the option with the highest expected utility is maintained in memory and a decision is made as to whether to stop the exploration of additional alternatives and execute the current best decision (e.g., Choice). When the exploration process ends, a choice is implemented, which changes the environment (i.e., Execution). Feedback (e.g., reward) is received at any time from the environment, with or without delay from the execution of a choice. Such feedback is used to update past experiences since the last time feedback was received through a credit assignment mechanism.

In IBLT, an “instance” is a memory unit that results from the potential alternatives evaluated. These memory representations consist of three elements which are constructed over time: a situation state $s$ which is composed of a set of features $f$; a decision or action $a$ taken corresponding to an alternative in state $s$; and an expected utility or experienced outcome $x$ of the action taken in a state.

Each instance in memory has an Activation value, which represents how readily available that information is in memory, and it is determined by the similarity to past situations, recency, frequency, and noise according to the Activation equation in ACT-R [2]. Activation of an instance is used to determine the probability of retrieval of an instance from memory which is a function of its activation relative to the activation of all instances corresponding the same state in memory. The expected utility of a choice option is calculated by blending past outcomes. This blending mechanism for choice has its origins in a more general blending formulation [20], but a simplification of this mechanism is often used in models with discrete choice options, defined as the sum of all past experienced outcomes weighted by their probability of retrieval [12,24]. This formulation of blending represents the general idea of an expected value in decision making, where the
probability is a cognitive probability, a function of the activation equation in ACT-R. Algorithm 1 provides a formal representation of the general IBL process.

Algorithm 1: Pseudo Code of Instance-based Learning process

Input: default utility \( x_0 \), a memory dictionary \( \mathcal{M} = \{ \} \), global counter \( t = 1 \), step limit \( L \), a flag delayed to indicate whether feedback is delayed.

1. repeat
2. Initialize a counter (i.e., step) \( l = 0 \) and observe state \( s_l \)
3. while \( s_l \) is not terminal and \( l < L \) do
4. | Execution Loop
5. | Exploration Loop \( a \in A \) do
6. | Compute activation values \( A_{i(s_l^{i},a)}^t \) of instances \( (s_l^{i},a), x_i(s_l^{i},a), T_i(s_l^{i},a) \) by \([1]\)
7. | Compute retrieval probabilities \( P_i(s_l^{i},a)^t \) by \([2]\)
8. | Compute blended values \( V_{(s_l,a)}^t \) corresponding to \((s_l,a)\) by \([3]\)
9. | end
10. | Choose an action \( a_l \in \arg \max_{a \in A} V_{(s_l,a)}^t \)
11. | end
12. Take action \( a_l \), move to state \( s_l+1 \), observe \( s_{l+1} \), and receive outcome \( x_{l+1} \)
13. If delayed is true, update outcomes using a credit assignment mechanism
14. \( l \leftarrow l + 1 \) and \( t \leftarrow t + 1 \)
15. end
16. until task stopping condition
Concretely, for an agent, an option \( k = (s, a) \) is defined by taking action \( a \) after observing state \( s \). At time \( t \), assume that there are \( n_{kt} \) different considered instances \((k_i, x_{ik,t})\) for \( i = 1, ..., n_{kt} \), associated with \( k \). Each instance \( i \) in memory has an Activation value, which represents how readily available that information is in memory and expressed as follows [2]:

\[
A_{ik,t} = \ln \left( \sum_{t' \in T_{ik,t}} (t - t')^{-d} \right) + \alpha \sum_j Sim_j(f_j^k, f_j^{k_i}) + \sigma \ln \frac{1 - \xi_{ik,t}}{\xi_{ik,t}},
\]

(1)

where \( d \), \( \alpha \), and \( \sigma \) are the decay, mismatch penalty, and noise parameters, respectively, and \( T_{ik,t} \subset \{0, ..., t - 1\} \) is the set of the previous timestamps in which the instance \( i \) was observed, \( f_j^k \) is the \( j \)-th attribute of the state \( s \), and \( Sim_j \) is a similarity function associated with the \( j \)-th attribute. The second term is a partial matching process reflecting the similarity between the current state \( s \) and the state of the option \( k_j \). The rightmost term represents a noise for capturing individual variation in activation, and \( \xi_{ik,t} \) is a random number drawn from a uniform distribution \( U(0, 1) \) at each timestep and for each instance and option.

Activation of an instance \( i \) is used to determine the probability of retrieval of an instance from memory. The probability of an instance \( i \) is defined by a soft-max function as follows

\[
P_{ik,t} = \frac{e^{A_{ik,t}/\tau}}{\sum_{j=1}^{n_{kt}} e^{A_{jk,t}/\tau}},
\]

(2)

where \( \tau \) is the Boltzmann constant (i.e., the “temperature”) in the Boltzmann distribution. For simplicity, \( \tau \) is often defined as a function of the same \( \sigma \) used in the activation equation \( \tau = \sigma \sqrt{2} \).

The expected utility of option \( k \) is calculated based on Blending as specified in choice tasks [24,12]:

\[
V_{kt} = \sum_{i=1}^{n_{kt}} P_{ik,t} x_{ik,t}.
\]

(3)

The choice rule is to select the option that corresponds to the maximum blended value. In particular, at the \( l \)-th step of an episode, the agent selects the option \((s_l, a_l)\) with

\[
a_l = \arg \max_{a \in A} V_{(s_l, a), t}
\]

(4)

The flag delayed on line 14 of Algorithm 1 is true when the agent knows the real outcome after making a sequence of decision without feedback. In such case, the agent updates outcomes by using one of the credit assignment mechanisms [35]. It is worth noting that when the flag delayed is true depends on a specific task. For instance, delayed can be set to true when the agent reaches the terminal state, or when the agent receives a positive reward.
3 SpeedyIBL Implementation

From the IBL algorithm \[1\] we observe that its computational cost revolves around the computations on lines \(6\) (Eq. 1), \(7\) (Eq. 2), \(8\) (Eq. 3), and the storage of instances with their associated time stamps on line \(13\). Clearly, when the number of states and action variables (dimensions) grow, or the number of IBL agent objects increases, the execution of steps \(6\) to \(3\) in algorithm \[1\] will directly increase the execution time. The “speedy” version of IBL (i.e., SpeedyIBL) is a library focused on dealing with these computations more efficiently.

SpeedyIBL algorithm is the same as that in Algorithm \[1\]. The innovation is in the Mathematics. Equations \[1\], \[2\] and \[3\] are replaced with Equations \[6\], \[7\] and \[8\], respectively (as explained below). Our idea is to take advantage of vectorization, which typically refers to the process of applying a single instruction to a set of values (vector) in parallel, instead of executing a single instruction on a single value at a time. In general, this idea can be implemented in any programming language. We particularly implemented these in Python, since that is how PyIBL is implemented \[29\].

Technically, the memory in an IBL model is stored by using a dictionary \(M\) that, at time \(t\), represented as follows:

\[
M = \{ k_i : \{ x_{ik,t} : T_{ik,t}, \ldots \}, \ldots \},
\]

where \((k_i, x_{ik,t}, T_{ik,t})\) is an instance \(i\) that corresponds to selecting option \(k_i\) and achieving outcome \(x_{ik,t}\) with \(T_{ik,t}\) being the set of the previous timestamps in which the instance \(i\) is observed.

To vectorize the codes, we convert \(T_{ik,t}\) to a NumPy array on which we can use standard mathematical functions with built-in Numpy functions for fast operations on entire arrays of data without having to write loops.

After the conversion, we consider \(T_{ik,t}\) as a NumPy array. In addition, since we may use a common similarity function for several attributes, we assume that \(f\) is partitioned into \(J\) non-overlapping groups \(f_{[1]}, \ldots, f_{[J]}\) with respect to the distinct similarity functions \(Sim_1, \ldots, Sim_J\), i.e., \(f_{[j]}\) contains attributes that use the same similarity function \(Sim_j\). We denote \(S(f^k, f^{k_i})\) the second term of (1) computed by:

\[
\text{set } S(f^k, f^{k_i}) = 0
\]

\[
\text{for } j = 1 \text{ to } J \text{ do}
\]

\[
S(f^k, f^{k_i}) += \text{sum}((Sim_j(f^k_{[j]}, f^{k_i}_{[j]}))
\]

\[
\text{end for}
\]

\[1\] https://numpy.org/doc/stable/
Hence, the activation value (see Equation 1) can be fast and efficiently computed as follows:

\[
A_{ik,t} = \log(\sum(x_{T_k,t} - d)) + \alpha * S(f^k, f^{k_i}) + \sigma * \log((1 - \xi_{ik,t})/\xi_{ik,t}).
\]  

(6)

With the vectorization, the operation such as pow can be performed on multiple elements of the array at once, rather than looping through and executing them one at a time. Similarly, the retrieval probability (see Equation 2) is now computed by:

\[
P_{kt} := \left[ P_{1k,t}, \ldots, P_{nk,kt} \right] = v/\sum(v),
\]  

(7)

where \( v = \exp([\Lambda_{1k,t}, \ldots, \Lambda_{nk,kt}] / \tau) \). The blended value (see Equation 3) is now computed by:

\[
V_{kt} = \sum(x_{kt} * P_{kt}),
\]  

(8)

where \( x_{kt} := [x_{1kt}, \ldots, x_{nk,kt}] \) is a NumPy array that contains all the outcomes associated with the option \( k \).

4 Experiments: Demonstration of the Curse of Exponential Growth and SpeedyIBL solution

To demonstrate the efficiency of SpeedyIBL, we evaluate its performance against a regular implementation of the IBL algorithm (Algorithm 1) in Python (PyIBL [30]), in six different tasks that were selected to represent different dimensions of complexity in dynamic decision making tasks [15]. The codes are available at https://github.com/nhatpd/SpeedyIBL.

4.1 General Simulation Methods

The parameter values configured in the IBL models with SpeedyIBL and PyIBL implementations were identical. In particular, we used the decay \( d = 0.5 \) and noise \( \sigma = 0.25 \). The default utility values generally set to be higher than the maximum value obtained in the task, to create exploration as suggested in [24] (see the task descriptions below for specific values), and they were set the same for PyIBL and SpeedyIBL.

For each of the six tasks, we compared the performance of PyIBL and SpeedyIBL implementations in terms of (i) running time measured in seconds and (ii) performance. The performance measure is identified within each task.

We conducted 1000 runs of the models and each run performed 100 episodes for the Binary choice and Insider attack. Given the running time required for PyIBL, we only ran 100 runs of 100 episodes for the
remaining tasks. We note that an episode of the *Binary choice* and *Insider attack* tasks has one step (trial) while the remaining tasks have 2500 steps within each episode.

The credit assignment mechanisms in IBL are being studied in [31]. In this paper we used an equal credit assignment mechanism for all tasks. This mechanism updates the current outcome for all the actions that took place from the current state to the last state where the agent started or the flag delayed was true.

4.2 Tasks

Table 1 provides an overview of the dimensions of the tasks with respect to the number of agents, actions, states, partial matching mechanism, feedback delays, and number of choice options. There are 4 single agent tasks, one task with two agents, and one task with 3 agents. The tasks have between 2 to 9 potential actions and the number of states and choice options also vary from just a few to a significant large number. We also include a task to illustrate the partial matching (similarity) process of equations 1 and 3 and a task with no feedback delay.

We start with a repeated *Binary choice* task that has only one state and two options, followed by an *Insider attack* two-stage game in which players choose one of six targets after observing their features to advance. We then scale up to a larger number of states and actions in significantly more complex tasks. A *Minimap* task involving a search and rescue mission and *Ms. Pac-man* task have a larger number of the discrete state-action variables. Next, we scale up to two multi-agent tasks: the *Fireman* task has two agents and four actions, and a *Cooperative Navigation* task in which three agents navigate and cooperate to accomplish a goal. The number of agents increases the memory computation, since each of those agents adds their own variables to the joint state-action space. Importantly, all these demonstrations use the same IBL algorithm 1 and the implementation of such algorithm with the Speedy equations described in Section 3. Details for each task follow below. Based on these dimensions of increasing complexity, we expect that SpeedyIBL’s benefits over PyIBL will be larger with increasing complexity of the task.

| Task                 | # Agents | # Actions | # States | # Options | Partial Matching | Delayed Feedback |
|----------------------|----------|-----------|----------|-----------|------------------|------------------|
| Binary choice        | 1        | 2         | 1        | 2         | No               | No               |
| Insider attack game  | 1        | 6         | 4        | 24        | Yes              | Yes              |
| Minimap              | 1        | 4         | $\approx 10^{41}$ | $4 \times 10^{41}$ | No               | Yes              |
| Ms. Pac-man          | 1        | 9         | $\approx 10^{347}$ | $9 \times 10^{347}$ | No               | Yes              |
| Fireman              | 2        | 4         | $\approx 10^{15}$ | $4 \times 10^{15}$ | No               | Yes              |
| Cooperative navigation | 3      | 4         | $\approx 10^{7}$   | $4 \times 10^{7}$  | No               | Yes              |

Table 1: Task Summary
4.2.1 Binary choice

In each trial, the agent is required to choose one of two options: Option A or Option B. A numerical outcome drawn from a distribution after the selection, is the immediate feedback of the task. This is a well-studied problem in the literature of risky choice task [19], where individuals make decisions under uncertainty. Unknown to the agent is that the options A and B are assigned to draw the outcome from a predefined distribution. One option is safe and it yields a fixed medium outcome (i.e., 3) every time it is chosen. The other option is risky, and it yields a high outcome (4) with some probability 0.8, and a low outcome (0) with the complementary probability 0.2.

An IBL model of this task has been created and reported in various past studies, including [12][24]. Here, we conducted the simulations of 1000 runs of 100 trials. We also run the experiment with 5000 trials to more clearly highlight the difference between PyIBL and SpeedyIBL. The default utility $x_0$ was set to 4.4. For each option $s$, where $s$ is either A or B, we consider all the generated instances taking the form of $(s, x)$, where $x$ is an outcome. The performance is determined by the average proportion of the maximum reward expectation choice (PMax).

![Fig. 2: Binary choice](image)

4.2.2 Insider attack game

The insider attack game is an interactive task designed to study the effect of signaling algorithms in cyber deception experiments (e.g., [5]). Figure 3 illustrates the interface of the task, including a representation of the agent (insider attacker) and the information of 6 computers. Each of the six computers is “protected” with some probability (designed by a defense algorithm). Each computer displays the monitoring probability and potential outcomes and the information of the signal. When the agent selects one of the six computers, a signal is presented to the agent (based on the defense signaling strategy); which informs the agent whether the computer is monitored or not. The agent then makes a second decision after the signal: whether to continue or withdraw the attack on the pre-selected computer. If the agent attacks a computer that is monitored, the player loses points, but if the computer is not monitored, the agent wins points. The signals are, therefore, truthful or deceptive. If the agent withdraws the attack, it earns zero points.
In each trial, the agent must decide which of the 6 computers to attack, and whether to continue or withdraw the attack after receiving a signal. An IBL model of this task has been created and reported in past studies (e.g., [7, 5]). We perform the simulations of 1000 runs of 100 episodes. For each option \((s, a)\), where the state \(s\) is the features of computers including reward, penalty and the probability that the computers is being monitored (see [7] for more details), and \(a \in \{1, ..., 6\}\) is an index of computers, we consider all the generated instances taking the form of \((s', a, x)\) with \(s'\) being a state and \(x\) being an outcome. The performance is determined by the average collected reward.

### 4.2.3 Search and rescue in Minimap

The Minimap task is inspired by a search and rescue scenario, which involves an agent being placed in a building with multiple rooms and tasked with rescuing victims [33]. Victims have been scattered across the building and their injuries have different degrees of severity with some needing more urgent care than others. In particular, there are 34 victims grouped into two categories (24 green victims and 10 yellow victims). There are many obstacles (walls) placed in the path forcing the agent to look for alternative routes. The agent’s goal is to rescue as many victims as possible. The task is simulated as a \(93 \times 50\) grid of cells which represents one floor of this building. Each cell is either empty, an obstacle, or a victim. The agent can choose to move left, right, up, or down, and only move one cell at a time.

The agent receives a reward of 0.75 and 0.25 for rescuing a yellow victim and a green victim, respectively. Moving into an obstacle or an empty cell is penalized by 0.05 or 0.01 accordingly. Since the agent might have to make a sequence of decisions to rescue a victim, we update the previous instances by a positive outcome that once the agent receives.

An IBL model of this task has been created and reported in past studies [10]. Here we created the SpeedyIBL implementation of this model to perform the simulation of 100 runs of 100 episodes. An episode
Fig. 4: Search and rescue map. The empty cells are white and the walls are black. The yellow and green cells represent the locations of the yellow and green victims respectively. The cell with the red color represents the start location of the agent.

terminates when a 2500-trial limit is reached or when the agent successfully rescues all the victims. After each episode, all rescued victims are placed back at the location where they were rescued from and the agent restarts from the pre-defined start position.

In this task, a state $s$ is represented by a gray-scale image (array) with the same map size. We use the following pixel values to represent the entities in the map: $s[x][y] = 240$ if the agent locates at the coordinate $(x, y)$, $150$ if a yellow victim locates at the coordinate $(x, y)$, $200$ if a green victim locates at the coordinate $(x, y)$, $100$ if an obstacle locates at the coordinate $(x, y)$, and $0$ otherwise. For each option $(s, a)$, where $s$ is a state and $a$ is an action, we consider all the generated instances taking the form of $(s, a, x)$ with $x$ being an outcome. The default utility was set to 0.1. The flag delayed is set to true if the agent rescues a victim, otherwise false. The performance is determined by the average collected reward.

4.2.4 Ms. Pac-man

The next task considered in the experiment is Ms. Pac-man game, a benchmark for evaluating agents in machine learning, e.g. [17]. The agent maneuvers Pac-Man in a maze while Pac-Man eats the dots (see Fig. 5).

In this particular maze, there are 174 dots, each one is worth 10 points. A level is finished when all dots are eaten. To make things more difficult, there are also four ghosts in the maze who try to catch Pac-Man, and if they succeed, Pac-Man loses a life. Initially, she has three lives and gets an extra life after reaching 10,000 points. There are four power-up items in the corners of the maze, called power dots (worth 40 points). After Pac-Man eats a power dot, the ghosts turn blue for a short period, they slow down and try to escape from Pac-Man. During this time, Pac-Man is able to eat them, which is worth 200, 400, 800, and 1600 points, consecutively. The point values are reset to 200 each time another power dot is eaten, so the agent would
want to eat all four ghosts per power dot. If a ghost is eaten, his remains hurry back to the center of the maze where the ghost is reborn. At certain intervals, a fruit appears near the center of the maze and remains there for a while. Eating this fruit is worth 100 points.

We use the MsPacman-v0 environment developed by Gym OpenAI\(^2\), where a state is represented by a color image. Here, we developed an IBL model for this task and created the SpeedyIBL implementation of this model to perform the simulation of 100 runs of 100 episodes. An episode terminates when either a 2500-step limit is reached or when Pac-man successfully eats all the dots or loses three lives. Like in the Minimap task, for each option \((s, a)\), where \(s\) is a state and \(a\) is an action, we consider all the generated instances taking the form of \((s, a, x)\) with \(x\) being an outcome. The parameter \(delayed\) is set to true if Pac-man receives a positive reward, otherwise it is set to false. The performance is determined by the average collected reward.

### 4.2.5 Fireman

The Fireman task replicates the coordination in firefighting service wherein agents need to pick up matching items for extinguishing fire. This task was used for examining deep reinforcement learning agents [36]. In the experiment, the task is simulated in a gridworld of size 11 × 14, as illustrated in Fig. 6. Two agents A1 and A2 located within the gridworld are tasked with locating an equipment pickup area and choosing one of the firefight items. Afterwards, they need to navigate and find the location of the fire (F) to extinguish it. The task is fully cooperative as both agents are required to extinguish one fire. More importantly, the location of the fire is dynamic in every episode.

\(^2\) https://gym.openai.com/envs/MsPacman-v0/
The agents receive the collective reward according to the match between their selected firefighting items, which is determined by the payoff matrix in Table 2. The matrix is derived from a partial stochastic climbing game [26] that has a stochastic reward. If they both select the equipment E2, they get 14 points with the probability 0.5, and 0 otherwise. This Fireman task has both stochastic and dynamic properties.

|   | Agent 2 |
|---|---------|
|   | E1 | E2 | E3 |
| Agent 1 | E1 | 11 | -30 | 0 |
|         | E2 | -30 | 14/0 | 6 |
|         | E3 | 0  | 0   | 5 |

Table 2: Payoff matrix.

Here we developed an IBL model for this task. We created the SpeedyIBL implementation of this model to perform the simulations of 100 runs of 100 episodes. An episode terminates when a 2500-trial limit is reached or when the agents successfully extinguish the fire. After each episode, the fire is replaced in a random location and the agents restart from the pre-defined start positions.

Like in the search and rescue Minimap task, a state $s$ of the agent A1 (resp. A2) is represented by a gray-scale image with the same gridworld size using the following pixel values to represent the entities in the gridworld: $s[x][y] = 240$ (resp. 200) if the agent A1 (resp. A2) locates at the coordinate $(x, y)$, 55 if the fire locates at the coordinate $(x, y)$, 40 if equipment E1 locates at the coordinate $(x, y)$, 50 if equipment E2 locates at the coordinate $(x, y)$, 60 if equipment E3 locates at the coordinate $(x, y)$, 100 if an obstacle locates at the coordinate $(x, y)$, 0 otherwise. Moreover, we assume that the agents cannot observe the relative positions of the other, and hence, their states do not include the pixel values of the other agent. For each option $(s, a)$, where $s$ is a state and $a$ is an action, we consider all the generated instances taking the form...
of \((s, a, x)\) with \(x\) being an outcome. The flag \textit{delayed} is set to true if the agents finish the task, otherwise false. The performance is determined by the average collected reward.

### 4.2.6 Cooperative Navigation

In this task, three agents (A1, A2 and A3) must cooperate through physical actions to reach a set of three landmarks (L1, L2 and L3) shown in Fig. 7 see [25]. The agents can observe the relative positions of other agents and landmarks, and are collectively rewarded based on the number of the landmarks that they cover. For instance, if all the agents cover only one landmark L2, they receive one point. By contrast, if they all can cover the three landmarks, they get the maximum of three points. Simply put, the agents want to cover all landmarks, so they need to learn to coordinate the landmark they must cover.

![Fig. 7: Cooperative navigation](image)

Here we developed an IBL model for this task. We created the SpeedyIBL implementation of this model to perform the simulations of 100 runs of 100 episodes. An episode terminates when a 2500-trial limit is reached or when each of the agents covers one landmark. After each episode, the fire is replaced in a random location and the agents restart from the pre-defined start positions.

In this task, a state \(s\) is also represented by a gray-scale image with the same gridworld size using the following pixel values to represent the entities in the environment: \(s[x][y] = 240\) if the agent A1 locates at the coordinate \((x, y)\), \(200\) if the agent A2 locates at the coordinate \((x, y)\), \(150\) if the agent A3 locates at the coordinate \((x, y)\), \(40\) if the landmark L1 locates at the coordinate \((x, y)\), \(50\) if the landmark L2 locates at the coordinate \((x, y)\), \(60\) if the landmark L3 locates at the coordinate \((x, y)\), 0 otherwise. For each option \((s, a)\), where \(s\) is a state and \(a\) is an action, we consider all the generated instances taking the form of \((s, a, x)\) with \(x\) being an outcome. The flag \textit{delayed} is set to true if the agents receive a positive reward, otherwise false. The performance is determined by the average collective reward.
5 Results

In this section, we present the results of the SpeedyIBL and PyIBL models across all the considered tasks. The comparison is provided in terms of the average running time and performance.

5.1 Average Running time and Performance

Table 3 shows the overall average of computational time and Table 4 the average performance across the runs and 100 episodes. The Ratio in Table 3 indicates the speed improvement from running the model in SpeedyIBL over PyIBL.

| Task                   | PyIBL          | SpeedyIBL     | Ratio |
|------------------------|----------------|---------------|-------|
| Binary choice          | 0.0087         | 0.0076        | 1.14  |
| Insider Attack Game    | 0.1411         | 0.0652        | 2.2   |
| Minimap                | 21951.88 (≈ 365 mins ≈ 6 hours) | 78.4 (≈ 1.3 mins) | 279   |
| Ms. Pac-man            | 162372.58 (≈ 2706.2 mins ≈ 45 hours) | 111.98 (≈ 1.86 mins) | 1450  |
| Fireman                | 23 743.36 (≈ 395.72 mins ≈ 6.6 hours) | 37.72 (≈ 0.62 mins) | 629   |
| Cooperative Navigation | 9741.37 (≈ 162 mins ≈ 2.7 hours) | 2.59 (≈ 0.04 mins) | 3754  |

Table 3: Average running time in seconds of a run

The ratio of PyIBL running time to SpeedyIBL running time in Table 3 shows that the benefit of SpeedyIBL over PyIBL increases significantly with the complexity of the task. In a simple task such as binary choice, SpeedyIBL performs 1.14 faster than PyIBL. However, the speed-up ratio increases with the higher dimensional state space tasks; for example, in Minimap SpeedyIBL was 279 times faster than PyIBL; and in Ms. Pac-man SpeedyIBL was 1450 times faster than PyIBL.

Furthermore, the multi-agent tasks exhibit the largest ratio benefit of SpeedyIBL over PyIBL. For example, in the Cooperative Navigation task, PyIBL took about 2.7 hours to finish a run, but SpeedyIBL only takes 2.59 seconds to accomplish a run.

In all tasks, we observe that the computational time of SpeedyIBL is significantly shorter than running the same task in PyIBL; we also observe that there is no significant difference in the performance of SpeedyIBL and PyIBL ($p > 0.05$). These results suggest that SpeedyIBL is able to greatly reduce the execution time of an IBL model without compromising its performance.
| Task                  | Metric              | PyIBL | SpeedyIBL | $t$-test            |
|----------------------|---------------------|-------|-----------|---------------------|
| Binary choice        | PMax                | 0.8333| 0.8275    | $t = -0.83, p = 0.4 > 0.05$ |
| Insider Attack Game  | Average Reward      | 1.3828| 1.3751    | $t = -0.38, p = 0.69 > 0.05$ |
| Minimap              | Average Reward      | 4.1021| 4.2641    | $t = 0.87, p = 0.38 > 0.05$ |
| Ms.Pac-man           | Average Reward      | 228.357| 228.464  | $t = 0.72, p = 0.47 > 0.05$ |
| Fireman              | Average Reward      | 4.7825| 4.9456    | $t = 1.07, p = 0.28 > 0.05$ |
| Cooperative Navigation | Average Reward   | 2.7049| 2.7261    | $t = 0.69, p = 0.48 > 0.05$ |

Table 4: Average performance of a run

5.2 Learning curves

Figure 8 shows the comparison of average running time (middle column) and average performance (right column) between PyIBL (Blue) and SpeedyIBL (Green) across episodes for all the six tasks.

In the Binary choice task, it is observed that there is a small difference in the execution time before 100 episodes; where SpeedyIBL performs slightly faster than PyIBL. To illustrate how the benefit of SpeedyIBL over PyIBL implementation increases significantly as the number of episodes increase, we ran these models over 5000 episodes. Figure 9 illustrates the curse of exponential growth very clearly, where PyIBL exponentially increases the execution time with more episodes. The benefit of SpeedyIBL over PyIBL implementation is clear with increased episodes. The PMax of SpeedyIBL and PyIBL overlap, suggesting the same performance.

In the Insider Attack game as shown Figure 8b, the relation between SpeedyIBL and PyIBL in terms of computational time shows again, an increased benefit with increased number of episodes. We see that their running time is indistinguishable initially, but then the difference becomes distinct in the last 60 episodes. Regarding the performance (i.e., average reward), again, their performance over time is nearly identical. Learning in this task was more difficult, given the design of this task, and we do not observe a clear upward trend in the learning curve due to the presence of stochastic elements in the task.

In all the rest of the tasks, the Minimap, Ms.Pac-man, Fireman, and Cooperative Navigation, given the multi-dimensionality of these tasks representations and the number of agents involved in Fireman, and Cooperative Navigation tasks, the curse of exponential growth is observed from early on, as shown in Figure 8c. The processing time of PyIBL grows nearly exponentially over time in all cases. The curve of SpeedyIBL also increases, but it appears to be constant in relation to the exponential growth of PyIBL given the significant difference between the two, when plotted in the same scale.

The performance over time is again indistinguishable between PyIBL and SpeedyIBL. Depending on the task, the dynamics, and stochastic elements of the task, the models’ learning curves appear to fluctuate over
Fig. 8 The comparison between SpeedyIBL (Green line) and PyIBL (Blue line) over time in the considered tasks.

(a) Binary Choice

(b) Insider Attack

(c) Minimap

(d) Ms. Pac-man

(e) Fireman

(f) Cooperative Navigation
Fig. 9: The comparison between SpeedyIBL and PyIBL in 5000 playing episodes of binary choice task.

but when the scenarios are consistent over time, the models show similar learning curves for both, PyIBL and SpeedyIBL.

6 Discussion and Conclusions

The curse of exponential growth is an important computational problem that emerges in many modeling approaches involving tabular and loop computations: as more observations accumulate and the dimensions of a task and the number of agents modeled in a task increase, the execution time of such a model will also increase. Models slow down with the increased number of computations that need to be done in a model.

In this paper, we demonstrate the curse of exponential growth problem and propose a solution to that problem. We demonstrate the problem and solutions within cognitive models, in particular Instance-Based Learning models [14]. We chose IBL models because it is possible to demonstrate how models constructed in agreement with the same theory can demonstrate different behaviors according to the complexity, number of agents, and hyper-dimensionality of the decision tasks.

We propose a new implementation for IBL models in SpeedyIBL, a Python library that allows to create multiple IBL agents with fast processing and response time without compromising the performance. SpeedyIBL relies on the same IBL algorithm [14] but innovates the PyIBL implementation of this algorithm [30] with the mathematics of vectorization and parallel computation [23]. The underlying idea of the SpeedyIBL implementation is to speed up the performance by using a data structure to store memory more efficiently and by leveraging vectorization in computation.

We have demonstrated the robustness of SpeedyIBL by comparing it with PyIBL, a widely used Python implementation of IBLT, on a wide range of tasks that vary in their increased complexity. We demonstrate how SpeedyIBL model implementation, based on the same theory can be exponentially beneficial compared
to the traditional PyIBL implementation. We demonstrate tasks that range from a single-agent, single-state, to single-agent multi-state, and to multi-agent multi-state settings. The results show that SpeedyIBL is able to perform significantly faster than PyIBL while keeping the performance as good as PyIBL. Moreover, the difference in the running time of the SpeedyIBL and PyIBL becomes large, especially in multi-agent domains and high-dimensional state spaces.

With the fast processing time, SpeedyIBL can not only be used in simulation experiments, but also can be integrated into a browser-based application in which IBL agents can interact with human subjects. Given that research on human–machine behavioral has attracted much attention lately, we are convinced that the implementation of SpeedyIBL will bring real benefits to researchers in the area.

Acknowledgements
This research was partly sponsored by the Defense Advanced Research Projects Agency and was accomplished under Grant Number W911NF-20-1-0006 and by AFRL Award FA8650-20-F-6212 subaward number 1990692 to Cleotilde Gonzalez.

References

1. Aggarwal, P., Thakoor, O., Mate, A., Tambe, M., Cranford, E.A., Lebiere, C., Gonzalez, C.: An exploratory study of a masking strategy of cyberdeception using cybervan. In: Proceedings of the Human Factors and Ergonomics Society Annual Meeting, vol. 64, pp. 446–450. SAGE Publications Sage CA: Los Angeles, CA (2020)
2. Anderson, J.R., Lebiere, C.J.: The atomic components of thought. Psychology Press (2014)
3. Bellman, R.: Dynamic programming, princeton univ. Press Princeton, New Jersey (1957)
4. Cranford, E.A., Gonzalez, C., Aggarwal, P., Cooney, S., Tambe, M., Lebiere, C.: Toward personalized deceptive signaling for cyber defense using cognitive models. Topics in Cognitive Science 12(3), 992–1011 (2020)
5. Cranford, E.A., Gonzalez, C., Aggarwal, P., Tambe, M., Cooney, S., Lebiere, C.: Towards a cognitive theory of cyber deception. Cognitive Science 45(7) (2021)
6. Cranford, E.A., Lebiere, C., Gonzalez, C., Cooney, S., Vayanos, P., Tambe, M.: Learning about cyber deception through simulations: Predictions of human decision making with deceptive signals in stackelberg security games. In: C. Kalish, M.A. Rau, X.J. Zhu, T.T. Rogers (eds.) Proceedings of the 40th Annual Meeting of the Cognitive Science Society, CogSci 2018, Madison, WI, USA, July 25-28, 2018 (2018)
7. Cranford, E.A., Lebiere, C., Rajivan, P., Aggarwal, P., Gonzalez, C.: Modeling cognitive dynamics in (end)-user response to phishing emails. Proceedings of the 17th ICCM (2019)
8. Erev, I., Ert, E., Plonsky, O., Cohen, D., Cohen, O.: From anomalies to forecasts: Toward a descriptive model of decisions under risk, under ambiguity, and from experience. Psychological review 124(4), 369 (2017)
9. Erev, I., Ert, E., Roth, A.E., Haruyu, E., Herzog, S.M., Hau, R., Hertwig, R., Stewart, T., West, R., Lebiere, C.: A choice prediction competition: Choices from experience and from description. Journal of Behavioral Decision Making 23(1), 15–47 (2010)
10. Gonzalez, C.: Decision-making: a cognitive science perspective. The Oxford handbook of cognitive science 1, 1–27 (2017)
11. Gonzalez, C., Ben-Asher, N., Martin, J.M., Dutt, V.: A cognitive model of dynamic cooperation with varied interdependency information. Cognitive science 39(3), 457–495 (2015)
12. Gonzalez, C., Dutt, V.: Instance-based learning: Integrating decisions from experience in sampling and repeated choice paradigms. Psychological Review 118(4), 523–51 (2011)
13. Gonzalez, C., Fakhari, P., Busemeyer, J.: Dynamic decision making: Learning processes and new research directions. Human factors 59(5), 713–721 (2017)
14. Gonzalez, C., Lerch, J.F., Lebiere, C.: Instance-based learning in dynamic decision making. Cognitive Science 27(4), 591–635 (2003)
15. Gonzalez, C., Vanyukov, P., Martin, M.K.: The use of microworlds to study dynamic decision making. Computers in human behavior 21(2), 273–286 (2005)
16. Gulati, A., Nguyen, T.N., Gonzalez, C.: Task complexity and performance in individuals and groups without communication. In: AAAI Fall Symposium on Theory of Mind for Teams (2021)
17. Hasselt, H.v., Guez, A., Silver, D.: Deep reinforcement learning with double q-learning. In: Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, AAAI’16, p. 2094–2100. AAAI Press (2016)
18. Hertwig, R.: Decisions from experience. The Wiley Blackwell handbook of judgment and decision making 1, 240–267 (2015)
19. Hertwig, R., Barron, G., Weber, E.U., Erev, I.: Decisions from experience and the effect of rare events in risky choice. Psychological Science 15(8), 534–539 (2004)
20. Kahneman, D., Tversky, A.: The dynamics of cognition: An act-r model of cognitive arithmetic. Kognitionswissenschaft 8, 5–19 (1979)
21. Kahneman, D., Tversky, A.: Prospect theory: An analysis of decision under risk. Econometrica 47(2), 363–391 (1979)
22. Kuo, F.Y., Sloan, I.H.: Lifting the curse of dimensionality. Notices of the AMS 52(11), 1320–1328 (2005)
23. Larsen, S., Amarasinghe, S.: Exploiting superword level parallelism with multimedia instruction sets. ACM SIGPLAN Notices 35(5), 145–156 (2000)
24. Lejarraga, T., Dutt, V., Gonzalez, C.: Instance-based learning: A general model of repeated binary choice. Journal of Behavioral Decision Making 25(2), 143–153 (2012)
25. Lowe, R., Wu, Y., Tamar, A., Harb, J., Abbeel, P., Mordatch, I.: Multi-agent actor-critic for mixed cooperative-competitive environments. In: Proceedings of the 31st International Conference on Neural Information Processing Systems, NIPS’17, p. 6382–6393. Curran Associates Inc., Red Hook, NY, USA (2017)
26. Matignon, L., Laurent, G.J., Fort-Piat, N.L.: Independent reinforcement learners in cooperative markov games: a survey regarding coordination problems. Knowl. Eng. Rev. 27(1), 1–31 (2012). DOI 10.1017/S026988912000057. URL https://doi.org/10.1017/S026988912000057
27. Mnih, V., Kavukcuoglu, K., Silver, D., Graves, A., Antonoglou, I., Wierstra, D., Riedmiller, M.: Playing atari with deep reinforcement learning. arXiv preprint arXiv:1312.5602 (2013)
28. Morgenstern, O., Von Neumann, J.: Theory of games and economic behavior. Princeton university press (1953)
29. Morrison, D., Gonzalez, C.: Pyibl: A python implementation of iblt. URL https://www.cmu.edu/dietrich/sds/ddmlab/downloads.html. Accessed: 2021-09-27
30. Morrison, D., Gonzalez, C.: Pyibl python implementation of ibl. URL http://pyibl.ddmlab.com/ Version 4.1
31. Nguyen, T.N., Gonzalez, C.: Cognitive machine theory of mind. In: Proceedings of CogSci (2020)
32. Nguyen, T.N., Gonzalez, C.: Effects of decision complexity in goal-seeking gridworlds: A comparison of instance-based learning and reinforcement learning agents. In: Proceedings of the 18th intl. conf. on cognitive modelling (2020)
33. Nguyen, T.N., Gonzalez, C.: Minimap: A dynamic decision making interactive tool for search and rescue missions. Tech. rep., Carnegie Mellon University (2021)
34. Nguyen, T.N., Gonzalez, C.: Theory of mind from observation in cognitive models and humans. Topics in Cognitive Science (2021). DOI https://doi.org/10.1111/tops.12553. URL https://onlinelibrary.wiley.com/doi/abs/10.1111/tops.12553
35. Nguyen, T.N., McDonald, C., Gonzalez, C.: Credit assignment: Challenges and opportunities in developing human-like AI agents. Tech. rep., Carnegie Mellon University (2021)

36. Palmer, G., Savani, R., Tuyls, K.: Negative update intervals in deep multi-agent reinforcement learning. In: E. Elkind, M. Veloso, N. Agmon, M.E. Taylor (eds.) Proceedings of the 18th International Conference on Autonomous Agents and MultiAgent Systems, AAMAS ’19, Montreal, QC, Canada, May 13-17, 2019, pp. 43–51. International Foundation for Autonomous Agents and Multiagent Systems (2019)

37. Savage, L.J.: The foundations of statistics. Naval Research Logistics Quarterly (1954)

38. Silver, D., Huang, A., Maddison, C.J., Guez, A., Sifre, L., Van Den Driessche, G., Schrittwieser, J., Antonoglou, I., Panneershelvam, V., Lanctot, M., et al.: Mastering the game of go with deep neural networks and tree search. nature 529(7587), 484–489 (2016)

39. Skinner, B.F.: Contingencies of reinforcement: A theoretical analysis, vol. 3. BF Skinner Foundation (2014)

40. Sutton, R.I., Staw, B.M.: What theory is not. Administrative science quarterly pp. 371–384 (1995)

41. Sutton, R.S., Barto, A.G.: Reinforcement learning: An introduction. MIT press (2018)

42. Tversky, A., Kahneman, D.: Advances in prospect theory: Cumulative representation of uncertainty. Journal of Risk and Uncertainty 5(4), 297–323 (1992)

43. Vinyals, O., Babuschkin, I., Chung, J., Mathieu, M., Jaderberg, M., Czarnecki, W.M., Dudzik, A., Huang, A., Georgiev, P., Powell, R., et al.: Alphastar: Mastering the real-time strategy game starcraft ii. DeepMind blog 2 (2019)

44. Wong, A., Bäck, T., Kononova, A.V., Plaat, A.: Multiagent deep reinforcement learning: Challenges and directions towards human-like approaches. arXiv preprint arXiv:2106.15691 (2021)