Analysis of the spark ignition engine performances by using a theoretical model

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Abstract. This paper proposes an analysis of the spark ignition engine performance for the road vehicles. The intake, compression, combustion, expansion and evacuation processes are taken into account, being modelled according to the engine load, for several engine speed values, thus determining each of the pressure and temperature parameters. The model developed by the authors also considers the influence of the filling coefficient, \( \eta_v \), defined in relation to the amount of fresh load entering the cylinder during the intake process.

1. Introduction
The engine's working regime is defined by the two functional parameters, namely engine speed and load. As is known from the literature, the engine load can be expressed by the load factor \( \chi \), given by absolute values, more precisely fractions, or by percentages of the reference load [1].

2. Theoretical model
A first process within the internal combustion engine is the intake, being represented by the two characteristic parameters, pressure and temperature. The inlet pressure at the end of the intake process, currently denoted by \( p_a \), is the pressure of the mixture between the fresh load and the residual gases in the cylinder at the end of the intake stroke, when the gas volume in the cylinder is maximum [4].

In turn, the fill or fill rate, often denoted by many authors as fill efficiency or volumetric efficiency, significantly influences the intake process and engine performance. It is defined according to the literature as the ratio between the amount of fresh load, actually retained into the cylinder of the engine under the conditions of a filling accompanied by gaso-dynamic and thermal losses, that is, the quantity that fills the volume \( V_a \) having the parameters \( p_a < p_0 \) and \( T_a > T_0 \), denoted \( G_a \) and \( G_0 \), representing the amount of fresh load which could be retained in the engine cylinder, under the conditions of an optimum filling, if it would fill the volume \( V_a \) and being represented by the parameters \( p_0 \) and \( T_0 \), pressure and temperature [1, 2].

The way the filling coefficient varies, at the speeds in the stable running of the engine, is illustrated in the form of a straight family of various slopes in figure 1. This variation was adopted in the present paper.
The study of the internal combustion engines with piston highlights the character of the real process of the fresh load compression from the cylinder. Compression is a polytrophic process whose exponent continuously varies due to the existence of a permanent heat exchange between the gas mixture in the cylinder and the external environment, represented by the walls that delimit the process space [4, 6]. In order to simplify the study of this process, it will be considered an average polytrophic exponent with a constant value throughout the compression process period.

Factors such as cylinder bore, combustion chamber geometry, engine speed and load, engine cooling system, initial fresh load temperature, but also the excess air coefficient and compression ratio influences the value of the mean polytrophic exponent, \( e_c \) [3, 4, 6].

Modern engines have an improved cylinder seal, reduced free play between the valves and the guides, as well as a wider engine speed range, which leads to a significant decrease of the gases leaks [3, 4]. Analyzing this aspect from a thermodynamic point of view, the amount of heat lost to the...
outside environment, which assimilates the gasses leakage, tends to decrease, which leads to higher values of the average polytrophic compression exponent, compared to the previous generation engines.

The calculation expression of the spark ignition engine compression parameters is developed by the authors of this article as following:

\[
p_c = \Phi_0 \cdot (1 - \varepsilon^{-1}) \cdot (1 - \varphi_{pu} + \gamma) \cdot \left\{ \begin{array}{l} -278 \cdot \chi^2 + 473 \cdot \chi + 225 \\ -167 \cdot \chi^2 + 284 \cdot \chi + 263 \\ -28 \cdot \chi^2 + 48 \cdot \chi + 311 \\ -56 \cdot \chi^2 + 95 \cdot \chi + 299 \end{array} \right\} \cdot \left\{ \begin{array}{l} 0,69 \cdot \chi + 0,13 \\ 0,656 \cdot \chi + 0,212 \\ 0,594 \cdot \chi + 0,277 \\ 0,475 \cdot \chi + 0,4 \end{array} \right\}
\]

\[
p_c = \Phi_0 \cdot (1 - \varepsilon^{-1}) \cdot (1 - \varphi_{pu} + \gamma) \cdot \left\{ \begin{array}{l} -278 \cdot \chi^2 + 473 \cdot \chi + 225 \\ -167 \cdot \chi^2 + 284 \cdot \chi + 263 \\ -28 \cdot \chi^2 + 48 \cdot \chi + 311 \\ -56 \cdot \chi^2 + 95 \cdot \chi + 299 \end{array} \right\} \cdot \left\{ \begin{array}{l} \varepsilon_1^{0,3775}, \ n = 5 \cdot 10^3 \\ \varepsilon_1^{0,3883}, \ n = 4 \cdot 10^3 \\ \varepsilon_1^{0,397}, \ n = 2 \cdot 10^3 \end{array} \right\}
\]

The natural evolution of the combustion process involves the gradual burning of the homogeneous mixture, starting from an initial furnace, located in the area of the spark plug electrode of the combustion chamber, with the occurrence of an ignition front moving at moderate speeds to the areas in the cylinder filled with fresh air, speed values which, in the case of spark-ignition engines, can reach speeds up to 40 [m/s] [5].

The representative equations of the combustion process are represented by the evolution of the pressure \(p_z\) and the temperature \(T_z\).

\[
p_z = p_c \cdot \frac{T_c}{T_z}
\]

\[
p_z = p_c \cdot \frac{T}{T_z}
\]

In order to introduce in the calculation the temperature at the end of the combustion process, \(T_c\), a matrix \(B\) will be used, constituted by the values of the excess air coefficient, \(\lambda\). These values were determined experimentally by the authors. As a read mode, from left to the right, the evolution on each line represents the values of the excess air coefficient for the engine speed values of 5000, 4000, 3000 and 2000 [rpm], and from top to bottom, the values of \(\lambda\) for engine loads from 0,1 to 1 (10% to 100%).

\[
B = \begin{bmatrix} 0,50 & 0,45 & 0,40 & 0,35 \\ 0,63 & 0,58 & 0,53 & 0,48 \\ 0,72 & 0,72 & 0,60 & 0,62 \\ 0,85 & 0,82 & 0,74 & 0,76 \\ 1,00 & 0,90 & 0,85 & 0,80 \\ 1,06 & 1,02 & 0,90 & 0,84 \\ 1,08 & 1,04 & 0,88 & 0,96 \\ 1,10 & 1,04 & 0,86 & 0,95 \\ 1,00 & 1,00 & 0,99 & 0,94 \\ 0,96 & 0,91 & 0,86 & 0,81 \end{bmatrix}
\]

Depending on the value of the excess air coefficient, \(\lambda\), the temperature \(T_z\) will use different calculation expressions. The temperature \(T_z\) represents the positive root of the second degree equation,
noted (7). This equation recommended by the literature derives from the energy balance of the constant volume burning process.

\[ a_2 \cdot T_z^2 + a_1 \cdot T_z + a_0 = 0 \]  \hspace{1cm} (7)

Depending on the working mode of the engine, if \( \lambda < 1 \), the equation (7) coefficients can be expressed as the following:

\[ a_2 = (0.21 + 1.05 \cdot \lambda) \cdot 10^{-3} \]  \hspace{1cm} (8)
\[ a_1 = 1.49 - 0.21 \cdot 10^{-3} \cdot T_0 + (11.47 - 1.05 \cdot 10^{-3} \cdot T_0) \cdot \lambda \]  \hspace{1cm} (9)
\[ a_0 = -\left( \frac{\partial p_0}{\partial a} \cdot \frac{(6912.31 \cdot \lambda - 1953.31) \cdot 10^3 \cdot \lambda}{(57.83 \cdot \lambda + 1) \cdot (1 + \gamma_f)} + (19.67 + 2.51 \cdot 10^{-3} \cdot T_c) \cdot (T_c - T_0) \right) + (1.49 + 11.47 \cdot \lambda) \cdot T_0 \]  \hspace{1cm} (10)

If however, \( \lambda > 1 \), the equation (7) coefficient become:

\[ a_2 = (0.42 + 0.85 \cdot \lambda) \cdot 10^{-3} \]  \hspace{1cm} (11)
\[ a_1 = 1.99 - 0.41 \cdot 10^{-3} \cdot T_0 + (10.98 - 0.85 \cdot 10^{-3} \cdot T_0) \cdot \lambda \]  \hspace{1cm} (12)
\[ a_0 = -\left( \frac{\partial p_0}{\partial a} \cdot \frac{(4959 \cdot \lambda - 1953.31) \cdot 10^3 \cdot \lambda}{(57.83 \cdot \lambda + 1) \cdot (1 + \gamma_f)} + (19.67 + 2.51 \cdot 10^{-3} \cdot T_c) \cdot (T_c - T_0) \right) + (1.99 + 10.98 \cdot \lambda) \cdot T_0 \]  \hspace{1cm} (13)

Therefore, the expression of temperature calculation \( T_z \) is:

\[ T_z = \frac{-a_1 \pm \sqrt{a_1^2 - 4 \cdot a_2 \cdot a_0}}{2 \cdot a_2} \]  \hspace{1cm} (14)

By denoting the following expression with \( D \), we can develop the following relation:

\[ D = \sqrt{a_1^2 - 4 \cdot a_2 \cdot a_0} \]  \hspace{1cm} (15)

Once again, depending on the value of the excess air coefficient, \( \lambda \), we will note the coefficient \( D \) with \( D_1 \), for \( \lambda < 1 \), respectively \( D_2 \), for positive values of \( \lambda \). Two calculation relations can be obtained:

\[ D_1 = \sqrt{1.49 - 0.21 \cdot 10^{-3} \cdot T_0 + (11.47 - 1.05 \cdot 10^{-3} \cdot T_0) \cdot \lambda^2 - 4 \cdot a_0 \cdot (0.42 + 0.85 \cdot \lambda) \cdot 10^{-3}} \]  \hspace{1cm} (16)
\[ D_2 = \sqrt{1.99 - 0.41 \cdot 10^{-3} \cdot T_0 + (10.98 - 0.85 \cdot 10^{-3} \cdot T_0) \cdot \lambda^2 - 4 \cdot a_0 \cdot (0.42 + 0.85 \cdot \lambda) \cdot 10^{-3}} \]  \hspace{1cm} (17)

The expressions of the combustion temperature, \( T_z \), can be rewritten as follows:

\[ T_{z,\lambda<1} = \frac{-(1.49 - 0.21 \cdot 10^{-3} \cdot T_0 + (11.47 - 1.05 \cdot 10^{-3} \cdot T_0) \cdot \lambda) + D_1}{2 \cdot a_2} \]  \hspace{1cm} (20)
\[ T_{z,\lambda>1} = \frac{-(1.99 - 0.41 \cdot 10^{-3} \cdot T_0 + (10.98 - 0.85 \cdot 10^{-3} \cdot T_0) \cdot \lambda) + D_2}{2 \cdot a_2} \]  \hspace{1cm} (21)

the positive results being the value of the combustion temperature.
Considering the permanent heat exchange between the gases evolving in the engine cylinder during the engine cycle, the power stroke can be assimilated to a polytropic transformation with a constant polytropic exponent, \( e_d \)[4].

To determine the pressure at the end of the power stroke, \( p_d \), the compression ratio it is introduced, \( \varepsilon = \frac{V_a}{V_c} \), but also the value of the pressure at the end of the combustion. The relation is similarly obtained:

\[
p_d = \frac{p_z}{\varepsilon^{e_d}}
\]

(22)

where the polytropic exponent of the power stroke, \( e_d \), is given through a matrix, having different values for different engine speed values.

\[
e_d = \begin{bmatrix} 1.246 & n = 5 \cdot 10^3 [\text{rpm}] \\ 1.252 & n = 4 \cdot 10^3 [\text{rpm}] \\ 1.263 & n = 3 \cdot 10^3 [\text{rpm}] \\ 1.285 & n = 2 \cdot 10^3 [\text{rpm}] \end{bmatrix}
\]

The temperature parametric of the power stroke can be calculated using the following expression:

\[
T_d = \frac{T_z}{\varepsilon^{e_d-1}}
\]

(23)

In order to determine the performance of the spark ignition engine, based on the developed model, it is interesting to determine the mean indicated pressure, \( p_i \). Considering the roundness coefficient of the actual engine cycle, \( \varphi_n \), as well as the expression recommended for the mean indicated pressure of the real cycle, \( p_i \) parameter can be expressed as follows:

\[
p_i = \varphi_t \cdot \frac{p_c}{\varepsilon - 1} \cdot \left[ \frac{\lambda_z}{e_d - 1} \cdot \left( 1 - \frac{1}{\varepsilon^{e_d - 1}} \right) - \frac{1}{e_c - 1} \cdot \left( \frac{1}{\varepsilon^{e_c - 1}} \right) \right]
\]

(24)

By successively introducing the functions obtained in \( p_i \) calculation expression, we obtain:

\[
\Phi_0 \cdot (1 - \varepsilon^{-1}) \cdot (1 - \varphi_{pu} + \gamma_t) \cdot \left\{ \begin{array}{l}
-273 \cdot \chi^2 + 473 \cdot \chi + 225 \\
-167 \cdot \chi^2 + 284 \cdot \chi + 263 \\
-28 \cdot \chi^2 + 48 \cdot \chi + 311 \\
-56 \cdot \chi^2 + 95 \cdot \chi + 299
\end{array} \right\}
\]

\[
\left[ \frac{\lambda_z}{e_d - 1} \cdot \left( 1 - \frac{1}{\varepsilon^{e_d - 1}} \right) - \frac{1}{e_c - 1} \cdot \left( \frac{1}{\varepsilon^{e_c - 1}} \right) \right]
\]

(25)

respectively:

\[
\Phi_0 \cdot (1 - \varepsilon^{-1}) \cdot (1 - \varphi_{pu} + \gamma_t) \cdot \left\{ \begin{array}{l}
-273 \cdot \chi^2 + 473 \cdot \chi + 225 \\
-167 \cdot \chi^2 + 284 \cdot \chi + 263 \\
-28 \cdot \chi^2 + 48 \cdot \chi + 311 \\
-56 \cdot \chi^2 + 95 \cdot \chi + 299
\end{array} \right\}
\]

\[
\left[ \frac{\mu_t \cdot T_z}{e_d - 1} \cdot \left( 1 - \frac{1}{\varepsilon^{e_d - 1}} \right) - \frac{1}{e_c - 1} \cdot \left( \frac{1}{\varepsilon^{e_c - 1}} \right) \right]
\]

(26)

and:
Later, it can be studied the evolution of the indicated engine power, \( P_i \), of the indicated moment, \( M_i \), and the indicated mechanical work, \( L_i \), using the following expression:

\[
\Phi_0 \cdot (1 - \varepsilon^{-1}) \cdot (1 - \varphi_{pu} + \gamma_r) \cdot \frac{1}{\varepsilon - 1} \cdot \left[ \frac{\mu_1 \cdot \frac{\mu_2}{\varepsilon} - 1}{\varepsilon e - 1} \cdot \frac{1}{\varepsilon e - 1} \right]
\]

(27)

\[
p_i = \varphi_i \cdot \left[ \frac{\mu_1 \cdot \frac{\mu_2}{\varepsilon} - 1}{\varepsilon e - 1} \cdot \frac{1}{\varepsilon e - 1} \right]
\]

Later, it can be studied the evolution of the indicated engine power, \( P_i \), of the indicated moment, \( M_i \), and the indicated mechanical work, \( L_i \), using the following expression:

\[
P_i = \frac{p_i \cdot V_s \cdot n \cdot i}{30 \cdot \tau}
\]

(28)

\[
M_i = 9550 \cdot \frac{p_i \cdot V_s \cdot i}{30 \cdot \tau}
\]

(29)

\[
L_i = p_i \cdot V_s \cdot 10^3
\]

(30)

3. Results and conclusions

The model developed by the authors contributes to the study and analysis of the indicated power of an internal combustion engine with spark ignition. Next, it is proposed to use the previously presented model for an engine with the following data:

- \( \varepsilon = 9.5 \) (compression ratio)
- \( i = 4 \) (cylinders)
- \( S = 80.5 \text{ (stroke) [mm]} \)
- \( D = 79.5 \text{ (bore) [mm]} \)
- \( \tau = 4 \) (stroke number)
- \( n = 5500 \text{ (nominal speed) [rpm]} \)
- \( V_s = 0.3995 \text{ (displacement) [dm}^3\text{]} \)

For the above-mentioned data engine, the variations of the indicated torque, indicated power, and mechanical speed are shown in the graphs from figure 2, figure 3, respectively, figure 4.

![Figure 2](image)

Figure 2. The evolution of the engine torque, \( M_i \), dependent on the engine speed value.
The analysis of diagrams and theoretical results confirms that the author's model offers an adequate degree of precision to estimate performance and to optimize the operation of spark ignition engines used in propulsion of road vehicles. Taking into account the influence of the fill rate, defined in...
relation to the fresh load entering the cylinder during the intake process, is an additional element that increases the accuracy of theoretical estimates

4. References

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