Resistive method for measuring the disintegration speed of Prince Rupert’s drops

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Abstract
We have successfully applied the resistance grid technique to measure the disintegration speed in a special type of glass objects, widely known as Prince Rupert’s drops. We use a fast digital oscilloscope and a simple electrical circuit, glued to the surface of the drops, to detect the voltage changes, corresponding to the breaks in the specific parts of the drops. The results obtained using this method are in good qualitative and quantitative agreement with theoretical predictions and previously published data. Moreover, the proposed experimental setup does not include any expensive equipment (such as a high-speed camera) and can therefore be widely used in high schools and universities.

Keywords: Prince Rupert’s drops, disintegration, resistance grid technique, International Physicists’ Tournament

(Some figures may appear in colour only in the online journal)

1. Introduction
Prince Rupert’s drops are widely known tear-shaped glass objects with a thin tail obtained by dropping hot molten glass into water. During the process of their formation the surface of the molten glass is quickly cooled, while the inner portion of the drop remains significantly hotter. After complete cooling this leads to large compressive stresses on the surface, while the core of the drop is in the state of tensile stress.

The earliest study of Prince Rupert’s drops was performed by Robert Hooke after they were introduced to the Royal Society of London in 1660 by King Charles II. These glass
objects were named after his nephew, Prince Rupert of Bavaria, who had brought these droplets of molten glass from Germany to England and showed them to the King. Later, a detailed illustration of Prince Rupert’s drops appeared in Hooke’s *Micrographia* [1], where he described the process of their formation and cooling. The further history of these peculiar objects can be found in a review by Brodsky *et al* [2], while experimental and analytical characterization of temperatures, residual stresses and densities of Prince Rupert’s drops for various types of glass and at different stages of formation can be found in the paper by Johnson and Chandrasekar [3].

One special feature that Prince Rupert’s drops possess is the ability to withstand large mechanical pressure applied to their head without any deformation, and yet they turn into glass powder after the smallest crack at the tail of the droplet. After the initial crack is formed, the destruction process moves from the tail to the head of the drop with high speed (in the order of kilometres per second).

The first precise measurements of this disintegration speed were performed by Chandrasekar and Chaudri [4] using a high-speed camera, which was able to shoot up to a million frames per second. This is an expensive piece of equipment, which is rarely available for regular students at most universities, especially in developing countries. That is why we decided to apply another method, known as the resistance grid technique [5] to measure the disintegration velocity of Prince Rupert’s drops and investigate how it depends on the environmental conditions during the drop formation and properties of the glass used.

This experiment was initially designed and performed for the International Physicists’ Tournament—a team competition for students [6], which was held in Warsaw in April 2015. Measuring the disintegration speed of Prince Rupert’s drops was among 17 experimental and theoretical problems that a team of six students had to solve within several months and present their solutions afterwards. Due to this fact, all the work described here was done in a small group of 4–5 highly motivated students under the guidance of 2–3 faculty members during several weeks. That’s why we suggest using this experiment mostly for demonstrations, as an extended laboratory experiment or as a part of extracurricular activities.

2. Theoretical overview

In this brief overview we focus on the physical picture behind the peculiar cracking behaviour of Prince Rupert’s drops. As described above, the ability of Prince Rupert’s drops to withstand large mechanical pressure and quickly disintegrate after breaking their tail is due to the process of their formation, when a liquid glass drop flies through air and comes into contact with water. After cooling quickly, the outside of the drop becomes compressed, while the inner part is under tension. This leads to a large amount of elastic energy, stored in the drop, that is released when the tail is broken, which leads to fast disintegration of the whole drop.

There were several attempts to theoretically estimate the disintegration speed of Prince Rupert’s drops, but the exact mechanism behind the cracking process still remains the topic of ongoing research. The foundation of the crack’s dynamics in brittle materials was set by Griffith [7] and Mott [8], followed by the work of Yoffe [9], and the estimation of the crack’s limiting velocity was given by Roberts and Wells [10] in the case of static stress (0.38[c]₀, where [c]₀ = \sqrt{E/\rho} is the longitudinal sound velocity) and by Steverding and Lehnigk [11] in the case of pulsed stress (0.52[c]₀, where [c]₀ is the velocity of the Rayleigh surface waves). All this work gave rise to the field of fracture mechanics, which was recently reviewed by Bouchbinder *et al* [12].
To address the question of the complicated structure of the crack, which looks similar to fractals, another approach was used, called fractal fracture mechanics [13, 14]. The estimation of the propagation speed of the crack in the framework of fractal fracture mechanics was given by Yavari and Khezzadreh [15] in the range $[0.318c_0, 0.321c_0]$.

Prince Rupert’s drops can be also viewed as a specific example of a broader phenomena, known as failure waves or self-sustaining fracture waves [16–18], that occur in glass and other brittle materials.

However, unlike ordinary tempered glass, which breaks into small cubical fragments, Prince Rupert’s drops explode upon rupture and the precise mechanism of this disintegration still has not been explained. The high-speed photographic studies [4, 19] suggested crack bifurcation as the main mechanics and the experimental statistical analysis of the disintegrated drops, done by Silverman et al [20], tested various theoretical fragmentation models that predict the form of the particle, mass densities and the fractal dimension of the set of fragments. An important contribution to finding the disintegration mechanism of Prince Rupert’s drops comes from computer simulations of such structures [21, 22]. The next step to answer this puzzle would be to investigate drops, exploded within some sort of confining matrix to permit statistical analysis of the fragments drawn separately from areas of tension and compression.

3. Experiment

3.1. Fabrication of samples

To produce the drops we heated glass sticks with a propane burner. The temperature of the flame was crucial at this point it had to be greater than the melting temperature and less than boiling temperature of the glass used. If the temperature of the flame is too high then boiling leads to the formation of local irregularities in the structure of the drop, which causes its immediate destruction upon contact with cooling liquid. Typical temperature thresholds for regular glass would be within 500 °C–900 °C.

After melting the drop falls into a tank, filled with liquid, and gets cooled in it. We used four different types of glass (soda-lime, borosilicate, blue- and red-colored) to produce the
drops and both water (warm and cold) and liquid nitrogen to cool the drops. Using optical spectroscopy, we determined that colored glass was doped by heavier elements such as cobalt for blue glass and selenium for red glass. The stress distribution in the created Prince Rupert’s drops was obtained using crossed polarisers and is shown in figure 1(b). With the help of this inspection method one can see if there are no stresses inside the drop, which makes it unsuitable for further experiments.

3.2. Experimental setup

The main idea behind the method used is to convert the disintegration of a glass drop into an electrical signal that could be easily observed with an average oscilloscope. This can be achieved by a simple electric circuit with resistors that could be turned ‘on’ or ‘off’ by the propagating fracture wave. Imagine that the glass drop (covered with some conductive layer) is part of an electric circuit and having finite resistance it affects the overall resistance of the circuit. Then, if a part of the drop gets disintegrated it affects the resistive properties of the conductive layer on its surface and hence the total resistance of the circuit, which could be translated into an electric signal recorded by an oscilloscope. Based on this assumption, we proposed the following setup, which is schematically shown in figure 2(a).

The setup consists of a voltage source driving current through a chain of resistors connected in series. The first resistor limits the overall current in the circuit. All other resistors are connected to a common conductive bus that is formed on the surface of the measured sample. A digital oscilloscope measures the overall voltage drop on control resistors $R_i, \ldots R_4$. The nominal resistance for all resistors used in the experiment was chosen to be 1 kΩ.

In the initial state all electrical current flows through the bus on the drop’s surface ignoring the control resistors (provided their resistance is high in comparison with the resistance of the bus). When the drop is disintegrating the fracture wave moves from its tail and gradually breaks the conductive bus, which sequentially adds control resistors into the circuit. That is why the electric current starts to flow through the resistors and causes an abrupt increase in the control voltage.

Figure 2. Proposed experimental setup for measuring the disintegration velocity of Prince Rupert’s drops (a) and its hands-on implementation (b).
The assembled experimental setup is shown in figure 2(b). It consists of an electrical circuit with wires attached to control resistors hanging freely in the air above the mount point of a studied drop. Those wires are attached to the drop’s surface using conductive glue. Drops are fixed to the base with cyan-acrylic glue in order to maintain the same position during preliminary preparations. The conductive bus is also formed with the same conductive glue on the surface of the drop, connecting all wires. In order to decrease noise from the voltage source a 1.5 V battery was used to drive the electric current through the circuit.

After the setup is ready to take measurements, the disintegration process is initiated by breaking the free-hanging tail of the drop with a wire-cutter. An oscilloscope (RIGOL DS1102E) with a time span of 10 μs and temporal resolution of 4 ns was used to record changes of electrical flow through the control resistors. Knowing the time between the disintegration of the neighbouring control wires and the distance between them it is possible to estimate the propagation speed of the fracture wave.

3.3. Safety precautions

While conducting experiments with Prince Rupert’s drops one should follow strict safety instructions. The most dangerous part of the experimental setup is the glass drop itself. Slight damage to the brittle tail or internal flaw growth can lead to sudden glass explosion with a big burst area. Therefore protective goggles or face shields must be worn during all operations. It is also recommended to place the drop inside an enclosed plexiglass box and wear suitable rubber gloves while breaking the drop’s tail and measuring the produced signal. Although the process is explosive and glass shreds can fly as far as a couple of meters, its energy is usually not enough to penetrate skin.

Special precautions should be taken while fabricating Prince Rupert’s drops. Check the gas burner that you are going to use for possible gas leakage and carry out glass melting in a specially designated area. Verify that there are no flammable materials around and floors are protected from hot glass. Injury can result both from fire and explosion of the drop. Wear clothes without openings or gaps, and flame-proof aprons and fire resistant gloves of sufficient thickness are highly recommended. The burner flame is a powerful source of light and heat, so it is preferable to use goggles with darkened lenses to protect the eyes from glare and
heat. Be careful taking drops out from the cooling liquid, use pliers and try not to touch the thin tails.

3.4. Measured data

With the help of a digital oscilloscope we recorded several sets of data, the typical look of which is presented in figure 3. One can see distinct voltage steps, each of which corresponds to the next section of wires being torn away and disconnected from the circuit. Knowing the lengths of conductive sections on the surface of the glass and time between consequential wire disconnections we can estimate disintegration speed of Prince Rupert’s drop.

To fit the obtained individual data traces (red line in figure 3) we used the following expression, taking into account non-zero bus resistance

\[ U_{\text{sig}} = \frac{U_0 R_{\text{sum}}}{R_0 + R_{\text{sum}}}, \]  

Figure 4. Disintegration speed of Prince Rupert’s drops, estimated from measured oscilloscope traces, for borosilicate (a), blue (b) and red glass (c). All samples for this experiment were cooled at \( T_{\text{cool}} \approx 18 ^\circ \text{C}. \)
where $R_{\text{sum}}$ is the equivalent resistance of control resistors in parallel to the finite resistance of the conductive bus, which varies for different samples. For the data set, shown in figure 3, the values of additional bus resistances are $r_1 = 0.3$ kΩ, $r_2 = r_3 = r_4 = 0.05$ kΩ, where $r_i$ is the resistance of the corresponding bus segment.

One can notice that there are some additional effects influencing the recorded signal. First of all, breaking of the conductive bus takes some time, so there is a finite slope of the transition between two voltage plateaus. That is why we have chosen the inflection point of each transition to measure time intervals between consecutive breaks, while also accounting for this source of imprecision in the error estimation. Moreover, we think that other irregularities in the signal are mostly of the same origin. Nevertheless, it would be interesting to check our assumptions and see how the breaking process of the drop with a conductive layer differs from the one without it, using a high-speed camera (frame rate of at least 500 000 fps is required). Unfortunately, we did not possess this kind of equipment, so it remains a topic for future research.

We conducted the same experiment independently varying the type of glass from which the samples were made and the cooling conditions during the formation of the drops, and estimated the disintegration speed for each case. The main results are presented in figures 4 and 5 and in table 1. All the measured velocities are around 2000 ms$^{-1}$, which is in good agreement with 1450–1900 ms$^{-1}$, measured in [4, 19], and with theoretical estimates.
mentioned earlier, resulting in velocities from 1265 ms\(^{-1}\) to 2145 ms\(^{-1}\) for various types of glass.

As one can see from figure 4, the measured disintegration velocities do not depend much on the material used to fabricate the samples. However, with a better measurement precision we would expect to see lower fracture wave velocity in glasses containing more inclusions of heavy elements, since it increases density and decreases Young’s module where the speed of sound \(c_0 \approx \sqrt{E/\rho}\). This might be the reason for the higher disintegration speed in soda-lime glass (figure 5(a)) in comparison to the other three types of glass with heavier inclusions. For the case of different cooling conditions, shown in figure 5, we observe a higher disintegration speed when cooling the drops in cold water, due to larger stresses in the produced samples.

We tried to increase stresses even more and to make the temperature contrast even larger by cooling the falling drops in liquid nitrogen at \(T \approx 77\) K, but the produced drops were not disintegrating in an explosive manner as regular Prince Rupert’s drops do. This probably happens due to fast evaporation of liquid nitrogen, forming a gaseous layer around the drop (Leidenfrost effect) which prevents further heat exchange. As a consequence, the drop is cooling in liquid nitrogen more gradually than in water, and the resulting stress inside the glass is not enough for explosive destruction. As we mentioned in section 3.1, one can use crossed polarisers to inspect the drop before performing the experiment to see if it is likely to disintegrate or not.

### 4. Discussion

As we have shown, such a simple setup, as demonstrated in figure 2, allows measurement of the disintegration speed of Prince Rupert’s drops using the resistance grid technique. However, to achieve maximum precision using this approach one should consider the following remarks.

The biggest influence on the measurement results is made by relations between control resistors. If their nominal values are chosen equal as shown above it leads to a significant decrease of signal amplitude as the fracture wave propagates. When the \(n\)th bus connection is broken the corresponding voltage step decreases as \(n^{-1}\). This behaviour limits the maximum number of contacts that can be placed on a drop.

In order to achieve an equal per-step voltage drop one should use control resistors with different nominal values. A simple algorithm could be used to estimate the maximum number of steps achievable for a specific setup. For this estimation we assume that time distortion of a single step is negligible compared to its length and resistance of the conductive bus is close to zero.

In this case the only limiting factor is the electric noise that can be seen on the screen of the oscilloscope. Having its amplitude one should define the minimal voltage step that can be distinguished from noise. This level corresponds to a single step on the plot which limits the maximum number of observable steps \(N\). Applying Ohm’s law to the circuit one can achieve a non-recurrent expression for the value of the \(n\)th control resistor

\[
R_n = \frac{N}{(N-n)(N-n+1)} R_0.
\]  

Considering the small size of a typical object of investigation it would not always be reasonable to equalize the steps due to both limited contacts density and step front distortion. However this algorithm could provide a great benefit when applied to big drops that were
made with glass-melting furnaces or oxygen torches since it could allow putting dozens of contacts provided there is enough space.

In our experiment we had five contacts equally spread on the drop with an average size not greater than 20 mm. Provided the same contact density this would result in 20–30 contacts for a 150 mm-long drop. This number of experimental measures per sample could provide extra high precision of measuring the disintegration speed of the drop.

The second biggest point of consideration is decreasing front distortions. This effect corresponds to the fact that the bus connection with the linking wires takes some time to be broken. It depends on the physical size of the wire, width of the conducting bus strip, the thickness of its layer, physical dimensions of the glue drop that links the wire with the bus and on the fact that the disintegration front itself is distorted.

There are several ways to suppress this effect. One should consider making both the thickness and width of the conductive bus on the surface of the drop as small as possible. The best way to achieve this might be to deposit the bus as a metal film by evaporation. Another way is to use mechanical fixtures instead of glue, proposed in our experiment. But this method requires being careful and keeping stress on the drop’s walls several orders of magnitude less than its inner tension. Otherwise it may result in local stresses that may distort the propagation speed of the fracture wave.

Another way to get more data from a single measurement is to track several paths simultaneously. An easy way to do this would be to connect several circuits to a multi-channel oscilloscope or to encode different paths into various heights of the voltage steps using several control sub circuits connected in parallel to one power source. There is one principal limitation of the technique used: it is impossible to measure the propagation time of the wavefront between two last contacts (closest to the ‘head’ of the drop). One should develop the map of contacts bearing this fact in mind.

Also, one could think of using a smartphone or a tablet as a digital oscilloscope [23], which would make the experiment even cheaper and more accessible. However, our analysis shows that modern devices do not provide sampling rate high enough to measure such fast signals. For example, if we take the sampling rate of 40 kHz (standard for modern mobile devices) and propagation speed of the fracture wave around 2 km s$^{-1}$, it would result in a spatial resolution of around 50 mm. Although it is possible to make the drop around 150 mm long, as described above, the fabrication would require a special heating furnace or a powerful oxygen torch. The samples we had were made with a cheap burner, available in conventional shops.

Another limiting factor is the low-pass filters that are usually applied to mobile devices which would increase the minimal requirements even further. On the other hand, external soundcards could be used in this experiment. For example, one could easily find devices working at 196 kHz sample rate, which makes their use possible even with our ‘small’ samples. However, for the same price one could get a low-end USB oscilloscope with a greater sample rate around 100 MHz.

Finally, we would like to emphasize once again, that a crucial thing to take care of while performing this experiment is safety, as Prince Rupert’s drops are disintegrating into numerous sub-millimeter size shards traveling at very high velocities. Please refer to section 3.3 for the list of necessary safety precautions before trying to conduct this experiment.
5. Conclusion

We showed that the resistance grid technique can be used to measure disintegration speed of Prince Rupert’s drops. Moreover, the obtained results are in good agreement with theoretical estimates and are close to the values obtained using other measurement methods, such as filming the process with a high-speed camera. We also discussed how to make further improvements to achieve an even higher measurement precision.

The proposed method of measuring fast processes could be used not only to measure the disintegration speed of Prince Rupert’s drops, but of any other fast disintegration processes such as various explosions. Aside from scientific research this method is ideally suited for demonstration and training experiments in a laboratory setting at universities and high-schools, due to its simplicity, low cost and visual attraction.

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