Vibration isolation of rolling bearings in electric machines

H G Shekyan\textsuperscript{1}\textsuperscript{*} and A V Gevorgyan\textsuperscript{2}\textsuperscript{**}

\textsuperscript{1}Institute of Mechanics of the National Academy of Sciences of Armenia, Yerevan, Armenia
\textsuperscript{2}Elektromash GAM, Yerevan, Armenia

E-mail: *hamlet@mechins.sci.am, **elektromash@mail.ru

Abstract. In the paper forced vibrations of high-speed electric machines with rotors on the rolling bearings are considered. It is shown that in high-speed rotor machines the vibration loadings, which bring to overstrain of contact surfaces of the rolling and violation of the bearings work regime, are one of the main reasons of the bearings failure.

1. Introduction

Let a rigid rotor of an elastic machine rotate on elastic supports in an amortized housing, in magnetic field.

The presence of the magnetic field displaces own frequencies of the system towards the low frequencies, that is why it is advisable to keep the concept of the equivalent rigidity of elastic supports, taking into account the rigidity of the elastic elements and conditional rigidity of the magnetic field \[4\], reduced to geometric air gap.

The reduced conditional rigidity of the magnetic field, according to \[5\] will be:

\[ C_M = \frac{7}{\delta} \frac{d}{\delta} l B \frac{1}{\delta} \frac{\text{kg}}{\text{cm}}, \]  

where \( d \) is the outside diameter of the rotor pack in cm, \( l \) is the length of the pack in cm, \( \delta \) is a one-sided geometric gap in cm, \( B \) is the magnetic induction in the gap.

The equivalent rigidity of the elastic supports will be \[5\] \( C_2 = C'_2 - C_M \), where \( C'_2 \) is the rigidity of the elastic supports.

2. Problems formulations

The magnetic unilateral attraction with certain initial uneven gap of the rotor with stator will be considered continuous \[4, 5\]. If the machine rotor is displaced parallel to itself in the stator by the value of \( e_0 \), then the established value of the unilateral magnetic attraction force will be \[5\]:

\[ P_M = 7dlB^2 \frac{e_0}{\delta} \frac{\text{kg}}{\text{cm}}. \]  

The perturbed force \( P_u \sin(\omega t) \) from rotor imbalance causes change of the rotor initial displacement by the value of \( f \sin(\omega t) \), and the force of unilateral attraction changes by the value of

\[ P_M - P_{0_M} = 7dlB^2 \frac{f}{\delta} \sin(\omega t) = \Delta P_M \sin(\omega t), \]  

where
where \( P_M \) is unilateral magnetic attraction force taking into account the displacement of the rotor in the stator, \( f \) is the deflection of elastic supports from maximum perturbing force of the rotor imbalance, if that force had been applied on the rotor statically.

Then the total perturbing force will be:

\[
P(t) = P_u \sin(\omega t) + \Delta P_M \cos \varphi \sin(\omega t),
\]

(4)

where \( P = P_u + \Delta P_M \cos \varphi \) is the maximum perturbing force, \( \varphi \) is the angle between \( P_M \) and \( Z \).

The small vibrations of two mass (rotor \( M_2 \) and stator \( M_1 \)) system with two degrees of freedom in the direction of axis \( Z \) (figure 1) may be described by the system of differential equations:

\[
\begin{align*}
M_1 \ddot{Z}_1 + (1 + i\delta_{m_1})C_1 Z_1 + (1 + i\delta_{m_2})C_2(Z_2 - Z_1) + f(\mu_1 \dot{Z}) &= 0, \\
M_2 \ddot{Z}_2 + (1 + i\delta_{m_2})C_2(Z_2 - Z_1) + f[\mu_1(\dot{Z}_2 - \dot{Z}_1)] &= P \sin(\omega_1 t),
\end{align*}
\]

(5)

where \( \delta_{m_1} \) is the coefficient of the inelastic resistance in the material of the bumper, \( \delta_{m_2} \) is the coefficient of the inelastic resistance material of the elastic element, \( \omega_1 \) is the rotation frequency of the rotor, \( C_1 \) is the bumper rigidity, \( C_2 \) is the equivalent rigidity of elastic supports, taking into account the damping in the magnetic field.

By virtue of the absence of the determination method of functions \( f(\mu_1 \dot{Z}) \) and \( f[\mu_1(\dot{Z}_2 - \dot{Z}_1)] \) in the paper it is admitted that \( f(\mu_1 \dot{Z}) = f[\mu_1(\dot{Z}_2 - \dot{Z}_1)] = 0 \), then equation (5) has the form:

\[
\begin{align*}
M_1 \ddot{Z}_1 + (1 + i\delta_{m_1})C_1 Z_1 + (1 + i\delta_{m_2})C_2(Z_2 - Z_1) &= 0, \\
M_2 \ddot{Z}_2 + (1 + i\delta_{m_2})C_2(Z_2 - Z_1) &= P \sin(\omega_1 t).
\end{align*}
\]

(6)

From the solution of deformation equations (6) the vibration amplitude of the housing will be

\[
A_1^2 = \frac{a_1 + \delta_{m_2}^2 a_1}{b + \delta_{m_1}^2 a_2}.
\]

(7)

The graph of this function (figure 2) illustrates an equilateral hyperbole with coordinates, asymptotically parallel to the axis and mixed, according to the latters on the distance:

\[
x_0 = -\frac{b}{a_2}, \quad y_0 = \frac{a_1}{a_2},
\]

(8)

where \( a_1, a_2, \) and \( b \) are the coefficients depending on the perturbing force, rotor mass, and housing, as well as on the parameters of the elastic supports.

As it is seen in the graph (figure 2) function (7) does not have a turning point. Amplitude \( A_1^2 \) is of great significance when damping approaches value \( x_0 \). With the increase of damping value amplitude \( A_1^2 \) decreases, striving to the limit, equal to \( y_0 \). So, no matter how much the damping of the system increases, to decrease \( A_1^2 \) less than \( y_0 \) is not possible.

To assess the degree of vibration isolation of the machine housing with elastic supports, a concept of the coefficient of power transfer from rotor to housing, expressed by correlation, is introduced:

\[
\mu = \frac{P'}{P},
\]

(9)

where \( P' \) is the amplitude of power, transferred to the machine housing, \( P \) is the amplitude of the perturbing force.
To decrease the boundary frequency, partial frequencies $\omega_1$ and $\omega_2$ should be decreased, by decreasing rigidity $C_1$ and $C_2$, or increasing mass $M_1$ and $M_2$.

The presence of the second resonance is the lack of such a system. Moreover, the boundary frequency in the dual-mass system is usually higher, than in the equivalent single-mass system, which decreases the frequency zone of the effective decrease of power transmission coefficient.

It follows that when applying in electric machines elastic and elastically damped supports, it is necessary to refuse from bumpers, which are usually installed under the machine housing.

Let us consider two basic types of damping – viscous and due to the internal power absorption in the material of the elastic element. Introducing six main coordinates $q_i$ and corresponding to them generalized perturbing forces $P_i(t)$, $i=1, 2, \ldots, 6$, considering the damping forces linearly depending on velocity and applying Lagrange equation, we obtain six independent differential equations of forced vibrations with viscous damping

$$\beta_i \ddot{q}_i + K_i \dot{q}_i + \alpha_i q_i = P_i(t) \quad (10)$$

and six differential equations, taking into account damping due to internal power absorption in the material of the pore element:

$$\beta_i \ddot{q}_i + (1 + i\delta_m) \alpha_i q_i = P_i(t), \quad (11)$$

where $K_i$ is the coefficient of the viscous friction along $i$ main coordinate, $\beta_i$ and $\alpha_i$ are the coefficients, depending on the parameters of the rotor mass and elastic support, $P_i(t)$ is the generalized perturbing force — $P_i(t) = P_{mi} \sin(\omega t)$.

Having solved equations (10) and (11), we get correlations connecting complex amplitudes of the displacement and perturbing forces with viscous damping:

$$P_{mi} = \bar{A} \sqrt{(\alpha_i - \beta_i \omega_i \omega_i^2 + \omega_i^2 K_i^2}, \quad (12)$$

and with damping account in the material:

$$P_{mi} = \bar{A} \sqrt{(\alpha_i - \beta_i \omega_i)\omega_i^2 + \delta_{mi} \omega_i^2 \alpha_i^2}, \quad (13)$$

where $\bar{A}$ is the complex amplitude of displacement, $\omega_i$ is own frequency of the system by $i$ main coordinate, $\delta_{mi}$ is the coefficient of the inelastic resistance of the support elastic element.

The generalized forces, acting on the support in the direction of coordinate $q_i$ will be:
Figure 3. Dependence of the power transmission coefficient on the perturbation frequency for dual-mass system

- with viscous damping:
  \[ P_{mi}' = \frac{\partial U}{\partial q_i} + \frac{\partial F}{\partial q_i} = \bar{A} \sqrt{\alpha_i^2 + \omega_i^2 K_i^2}; \]  
  \[ (14) \]

- with damping in the material:
  \[ P_{mi}' = \frac{\partial U}{\partial q_i} = \bar{A} \alpha_i \sqrt{1 + \delta_{mi}^2}, \]
  \[ (15) \]

where \( U \) is the potential energy of the system, \( F \) is the scattering function.

In case of viscous damping the energy, turning into heat is characterized by scattering function, which similarly with kinematic and potential energy may be represented in the form of:

\[ F = \frac{1}{2} \sum_{i=1}^{6} K_i q_i^2. \]

The coefficient of the power transfer for the coordinate \( q_i \) will be:

- in case of viscous damping:
  \[ \mu_i = \sqrt{\frac{1 + 4 \delta_i^2 K_i^2}{(1-\delta_i^2)^2 + \delta_{mi}^2}}, \]  
  \[ \]  
  \[ (16) \]

- and in case of damping in the material:
  \[ \mu_i = \sqrt{\frac{1 + \delta_{mi}^2}{(1-\delta_i^2)^2 + \delta_{mi}^2}}, \]
  \[ \delta_i = \frac{\omega_i}{\omega_0}, \]  
  \[ (17) \]

where \( K_i \) is the coefficient of the viscous damping:

\[ K_i = \frac{\gamma_i^2}{\sqrt{4\pi^2 + \gamma_i^2}}, \]

\( \gamma_i \) is the logarithmic decrement of attenuation of the elastic support in the direction of vibrations by the main coordinate:

\[ \gamma_i = \frac{\pi}{\sqrt{\alpha_i \beta_i / K_i^2 - 1/4}}. \]
Figure 4. Dependence of power transfer coefficient and its affectivity under different values of viscous damping

The graphs of expressions (16) are represented in figure 4. Here a dotted curve of effectivity for using elastic supports, expressed by dependence is plotted:

\[ K_E = (1 - \mu)100\% , \]

Conclusions

The analysis of the curves (figure 4) permits us to make the following conclusions:

- If the perturbation frequency \( \omega_i \) is small, compared with the frequency \( \omega_{0i} \), then the power transfer coefficient differs slightly from one and the application of the elastic supports, in this case, has no sense.

- When the frequency ratio approaches one, i.e. \( \omega_{0i} \) is near to \( \omega_i \), the power transfer coefficient increases and under the small damping the vibration amplitude takes large values (resonance).

- For all the values of the coefficient of damping the power transfer coefficient becomes smaller one at \( \delta_i > \sqrt{2} \).

- With decreasing \( \delta_i \), which is equivalent to the rigidity decrease of the elastic element support for the established regime of the rotor friction, the value \( \mu \) vanishes, and the affectivity of the power transfer coefficient decrease increases. Beginning with \( \delta_i = 5 \) the curve slope \( K_E \) decreases so that there is no essential increase of coefficient \( \mu \) with the further rigidity decrease of the support elastic element. In main cases it is sufficient, in order \( \delta_i \approx 2.5 \). At \( \delta_i = 2.5 \) \((K = \delta_m = 0 \text{ — zero damping})\), the elastic support consumes 81% of efforts. So, with correct choice of elastic supports rigidity dynamic loadings, acting on the bearings, essentially decrease. Herewith the elastic support should have enough solidity in order to withstand the weight of the rotor.

- At \( \delta_i > \sqrt{2} \) value \( \mu \) is as less as the damping coefficient. Yet this conclusion may only be applied for the electric machines working in stationary conditions and perturbing forces are harmonic functions of time.
References

[1] Alexandrovsky V V 1965 Elastic supports for the rotors of electric machines In Proc. VNIIEM. Vol. 20. Mechanics of electrical machines (Moscow: VNIIEM) pp. 35–9 [in Russian]

[2] Poznyak E L 1971 Nonlinear oscillations of unbalanced vertical rotors on rolling bearings Mashinoved. No. 1 23–31

[3] Poznyak E L, Kosmachev A N, and Raykhлина B B 1965 Damping of forced bending vibrations of elastic rotors In Vibration and Solidity under Variable Stresses pp. 53–79 [in Russian]

[4] Shekyan G G 1983 To the question on factors, decreasing the bearing life of high-speed electric machines In Interuniversity Thematic Collection of Scientific Papers. Ser. “Machinebuilding” (Yerevan) pp 19–24 [in Russian]

[5] Shekyan G G 2004 Dynamics of Rotor Machines (Yerevan: Gitutyun) [in Russian]