Neutrino-electron scattering in a dense magnetized plasma

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Abstract. The process of neutrino-electron scattering in a dense plasma and magnetic field
of arbitrary strength, where electrons can occupy the states corresponding to excited Landau
levels, is analyzed. The total probability of this process, summarized over all initial states of
the plasma electrons which is only physically meaningful, is calculated. Possible astrophysical
applications are discussed.

1. Introduction
Neutrino physics plays a decisive role in astrophysical cataclysms such as supernova explosions
and coalescence of neutron stars, and also in the early universe. Consequently, studies of
neutrino interactions and in particular neutrino-electron processes in an external active medium
are of considerable interest. At the same time, an investigation of neutrino processes under
such extreme physical conditions is interesting from the conceptual viewpoint since it affects
fundamental problems of quantum field theory.

A correct analysis of the neutrino propagation process in a hot dense plasma in the presence
of a strong magnetic field requires to consider the complete set of neutrino-electron processes.
Then only the probability of the process summarized over all initial states of the plasma electrons
is physically meaningful. In Ref. [1], numerical calculations of the differential cross-section of the
neutrino-electron scattering in dense magnetized plasma were performed in the limit of rather
weak magnetic field \(B, eB < \mu E\), where \(\mu\) is the plasma chemical potential, \(E\) is the typical
neutrino energy. In Refs. [2, 3], the probability of the \(\nu e \rightarrow \nu e\) process and the volume density
of the neutrino energy and momentum losses, summarized over all initial states of the plasma
electrons, were evaluated under the physical situation where the magnetic field is moderate, while
the density of plasma is large, so the conditions are satisfied: \(\mu^2 > eB \gg (T^2, E^2) \gg m_e^2\), where
\(T\) is the plasma temperature, and \(eB \gg \mu E\). Calculations were performed for the case when
both initial and final electrons occupy the same Landau levels, because, as it was concluded
in Refs. [2, 3], such transitions were dominating. The purpose of the present research is to
calculate analytically the probability of this process for a more general case when the initial and
final electrons could occupy any physically allowed Landau levels. Some details of the calculation
technique can be found e.g. in Refs. [4, 5, 6].

We use natural units in which \(c = \hbar = 1\), \(e > 0\) is the elementary charge.
2. Solutions of the Dirac equation for an electron in a magnetic field

It appears to be convenient to use the electron wave functions as the eigenstates of the operator \( \hat{\mu}_z \) \cite{7, 8}:

\[
\hat{\mu}_z = m_e \Sigma_z - i\gamma_0 \gamma_5 [\Sigma \times P]_z ,
\]

where \( P = -i\nabla + eA \). We take the frame where the field is directed along the \( z \) axis, and the Landau gauge where the four-potential is: \( A^\lambda = (0, 0, xB, 0) \). In this approach, the electron wave functions have the form

\[
\Psi_{p,n}^s(X) = \frac{e^{-i(\epsilon n t - p_y y - p_z z)}}{\sqrt{4\pi M_n (\epsilon_n + M_n)(M_n + m_e)L_y L_z}},
\]

where \( \epsilon_n = \sqrt{M_n^2 + p_z^2} \), \( M_n = \sqrt{m_e^2 + 2\beta n} \), \( \beta = eB \), \( \xi = \sqrt{\beta} (x + py/\beta) \). The functions \( \Psi_{p,n}^s(X) \) satisfy the equation:

\[
\hat{\mu}_z \Psi_{p,n}^s(X) = s M_n \Psi_{p,n}^s(X) (s = \pm 1).\]

The bispinors \( U_n^\pm(\xi) \) in Eq. (2) take the form:

\[
U_n^- (\xi) = \begin{pmatrix}
-\text{i}\sqrt{2M_n} p_z V_{n-1}(\xi) \\
(\epsilon_n + M_n) (M_n + m_e) V_n(\xi) \\
-\text{i}\sqrt{2M_n} (\epsilon_n + M_n) V_{n-1}(\xi) \\
-p_z (M_n + m_e) V_n(\xi)
\end{pmatrix}, \quad U_n^+ (\xi) = \begin{pmatrix}
(\epsilon_n + M_n) (M_n + m_e) V_{n-1}(\xi) \\
-\text{i}\sqrt{2M_n} p_z V_n(\xi) \\
\text{i}\sqrt{2M_n} (\epsilon_n + M_n) V_{n-1}(\xi) \\
\text{i}\sqrt{2M_n} (M_n + m_e) V_n(\xi)
\end{pmatrix}.
\]

Here, \( V_n(\xi) (n = 0, 1, 2, \ldots) \) are the well-known normalized harmonic oscillator functions, which are expressed in terms of the Hermite polynomials \( H_n(\xi) \).

In this case, the process amplitude will have an explicit Lorentz invariant structure.

3. The process of the \( \nu_e \rightarrow \nu_e \) scattering

The effective local Lagrangian is:

\[
\mathcal{L} = \frac{G_F}{\sqrt{2}} [\bar{e} \gamma_\alpha (C_V - C_A \gamma_5) e] [\bar{\nu} \gamma^\alpha (1 - \gamma_5) \nu], \quad C_V = \pm \frac{1}{2} + 2 \sin^2 \theta_W, \quad C_A = \pm \frac{1}{2},
\]

where the upper signs correspond to \( \nu_e \) and the lower signs correspond to \( \nu_{\mu,\tau} \).

The \( S \) matrix element of the subprocess \( \nu e^-_{(i)} \rightarrow \nu ee^-_{(n)} \) takes the form

\[
S = 1 \frac{G_F}{\sqrt{2}} \frac{(2\pi)^3}{\sqrt{2E\nu_{(n)} L_y L_z}} \frac{\delta(p_{y} - p_{y} - q_{y}) \delta(p'_{z} - p_{z} - q_{z})}{\sqrt{2E\nu_{(n)} L_y L_z}} e^{-q_{z}^{2}/4eB - i\epsilon_{(P')} / 2eB} \times [\bar{u}(p') j(C_V - C_A \gamma_5) u(p)],
\]

where \( q = P - P' = p' - p, \epsilon_{(i)} \) and \( \epsilon_{(i)}' \) are the energies of the initial and final electrons, \( q_{\perp} \) is the projection of the vector \( q \) on the plane perpendicular to the vector \( B \), \( q_{\perp}^2 = q_{x}^2 + q_{y}^2 \), and \( j_{\alpha} = \bar{\nu}(P') \gamma_{\alpha} (1 - \gamma_{5}) \nu(P) \) is the Fourier transform of the current of the left-handed neutrinos.

The total process probability per unit time can be presented in the form

\[
W(\nu e^- \rightarrow \nu ee^-) = \frac{1}{T} \sum_{\ell} \sum_{n} \sum_{s,s'} \int |S|^2 \, dn_{(n)} \, dn_{(n)}' \frac{q_{\perp}^{2} V}{(2\pi)^{3}} (1 - f(E')), \]

where \( dn_{(n)} = f(\epsilon_{(i)}) \, dp_{y} \, dp_{z} \, L_y \, L_z / (2\pi)^2, \, dn_{(n)}' = (1 - f(\epsilon_{(i)}')) \, dp_{y} \, dp_{z} \, L_y \, L_z / (2\pi)^2 \), \( f(\epsilon_{(i)}) \) is the fermion distribution function, e.g. for initial electrons it is: \( f(\epsilon_{(i)}) = \frac{e^{\epsilon_{(i)} - \mu} / T}{[e^{\epsilon_{(i)} - \mu} / T + 1]}^{-1}; T \) is the total time of interaction, \( V = L_x L_y L_z \) is the total volume of the interaction region.
For the probability per unit time of the subprocess $\nu e^-_{(\ell)} \rightarrow \nu e^-_{(n)}$ one obtains

$$W_{\ell n} = \frac{\beta}{(2\pi)^416E} \int \frac{d^3\rho'}{E'} \left(1 - f(E')\right) \int \frac{dp_\perp}{\varepsilon'_{\perp}\varepsilon'_{\ell}} \delta(\varepsilon'_n - \varepsilon'_\perp - q_0) f(\varepsilon_\ell) (1 - f(\varepsilon'_\ell)) \sum_{s,s'} |M_{ss'}^{\ell n}|^2. \quad (7)$$

Taking into account possible polarisation states of the initial and final electrons, there exist four invariant polarization amplitudes $M_{ss'}^{\ell n}$, where $s, s' = \pm 1$, which can be presented in the form:

$$M_{ss'}^{\ell n} = \eta \frac{G_F}{2\sqrt{2}} \left\{ A_{ss'}^{\ell n} (j\tilde{\varphi}q) + B_{ss'}^{\ell n} (j\tilde{\Lambda}q) + C_{ss'}^{\ell n} (j\Lambda q) + D_{ss'}^{\ell n} (j\varphi q) \right\}, \quad (8)$$

where $\eta$ is an inessential phase factor, $\varphi_{\alpha\beta} = F_{\alpha\beta}/B$ is the dimensionless tensor of the external magnetic field, $\tilde{\varphi}_{\alpha\beta} = \frac{1}{2} \hat{\varepsilon}_{\alpha\beta\mu\nu} \rho^{\mu\nu}$ is the dual dimensionless tensor; the four-vectors with the indices $\perp$ and $\parallel$ belong to the Euclidean $\{1, 2\}$-subspace and the Minkowski $\{0, 3\}$-subspace, correspondingly, and $\Lambda_{\alpha\beta} = (\varphi\varphi)_{\alpha\beta} = \text{diag}(0, 1, 1, 0)$, $\tilde{\Lambda}_{\alpha\beta} = (\tilde{\varphi}\tilde{\varphi})_{\alpha\beta} = \text{diag}(1, 0, 0, -1)$. In Eq. (8), auxiliary functions are introduced: $A, B, C, D$. To illustrate, we give here their explicit form for one of the possible sets of polarization, $s = s' = -1$:

$$A_{ss'}^{\ell n} = \sqrt{\left[1 + \frac{m}{M_\ell}\right] \left[1 + \frac{m}{M_n}\right]} \mathcal{I}_{\ell,n} (C_V \tau + C_A \kappa)$$

$$+ \sqrt{\left[1 - \frac{m}{M_\ell}\right] \left[1 - \frac{m}{M_n}\right]} \mathcal{I}_{\ell-1,n-1} (C_V \tau - C_A \kappa), \quad (9)$$

$$B_{ss'}^{\ell n} = \sqrt{\left[1 + \frac{m}{M_\ell}\right] \left[1 + \frac{m}{M_n}\right]} \mathcal{I}_{\ell,n} (C_V \kappa + C_A \tau)$$

$$+ \sqrt{\left[1 - \frac{m}{M_\ell}\right] \left[1 - \frac{m}{M_n}\right]} \mathcal{I}_{\ell-1,n-1} (C_V \kappa - C_A \tau), \quad (10)$$

$$C_{ss'}^{\ell n} = \sqrt{q_\perp^2} \left[ - \sqrt{\left[1 - \frac{m}{M_\ell}\right] \left[1 + \frac{m}{M_n}\right]} \mathcal{I}_{\ell,n-1} (C_V u + C_A v) 

- \sqrt{\left[1 + \frac{m}{M_\ell}\right] \left[1 - \frac{m}{M_n}\right]} \mathcal{I}_{\ell-1,n} (C_V u - C_A v) \right], \quad (11)$$

$$D_{ss'}^{\ell n} = \sqrt{q_\perp^2} \left[ \sqrt{\left[1 - \frac{m}{M_\ell}\right] \left[1 + \frac{m}{M_n}\right]} \mathcal{I}_{\ell,n-1} (C_V u + C_A v) 

- \sqrt{\left[1 + \frac{m}{M_\ell}\right] \left[1 - \frac{m}{M_n}\right]} \mathcal{I}_{\ell-1,n} (C_V u - C_A v) \right], \quad (12)$$

where, for $n \geq \ell$

$$\mathcal{I}_{\ell,n}(x) = \sqrt{\frac{\ell!}{n!}} x^{-n/2} x^{(n-\ell)/2} L_n^{n-\ell}(x), \quad \mathcal{I}_{\ell,n}(x) = (-1)^{n-\ell} \mathcal{I}_{n,\ell}(x), \quad (13)$$
incorrect. This means that the results for the probability of the $\nu_e$ process obtained in \cite{4} were underestimated.

The analysis shows, that the assumption made in previous calculations, that the subprocesses were dominating where both initial and final electrons occupied the same Landau levels, was incorrect. This means that the results for the probability of the $\nu e \to \nu e$ process obtained in Refs. \cite{2, 3}, were underestimated.

4. Conclusions

The probability of the $\nu e \to \nu e$ process in a dense magnetized plasma is calculated analytically, for a general case when the initial and final electrons could occupy any physically allowed Landau levels. The analysis shows, that the assumption made in previous calculations, that the subprocesses were dominating where both initial and final electrons occupied the same Landau levels, was incorrect.

In astrophysical applications, the mean values of the neutrino energy and momentum losses could be more interesting: $Q^\alpha = E (P - P')^\alpha dW = -E (I, F)$. Here, $dW$ is the total differential probability of the process. The zeroth component of $Q^\alpha$ is connected with the mean energy lost by a neutrino per unit time due to the process considered, $I = dE/dt$. The space components of the four-vector $Q^\alpha$ are similarly connected with the mean neutrino momentum loss per unit time, $F = dP/dt$. An analysis of the four-vector $Q^\alpha$ in a general case for the magnetic field of arbitrary strength, where electrons can occupy the states corresponding to excited Landau levels, now is in progress. The force density $\mathbf{F}$ could lead to a very interesting consequences if a strong toroidal magnetic field is generated in the supernova envelope, providing an asymmetry of the supernova explosion and, in particular, it can explain the phenomenon of high pulsar kick-velocities, for details see Ref. \cite{4}.

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References

\begin{itemize}
  \item{} Bezchastnov V G and Haensel P 1996 Phys. Rev. D 54 3706
  \item{} Mikheev N V and Narynskaya E N 2000 Mod. Phys. Lett. A 15 1551
  \item{} Mikheev N V and Narynskaya E N 2003 Centr. Eur. J. Phys. 1 145
  \item{} Kuznetsov A and Mikheev N 2013 Electroweak Processes in External Active Media (Berlin, Heidelberg: Springer-Verlag)
  \item{} Kuznetsov A V, Rumyantsev D A and Savin V N 2014 Int. J. Mod. Phys. A 29 1450136
  \item{} Kuznetsov A V, Rumyantsev D A and Savin V N 2016 J. Phys.: Conf. Ser. 675 032019
  \item{} Sokolov A A and Ternov I M 1968 Synchrotron Radiation (Oxford: Pergamon)
  \item{} Melrose D B and Parle A J 1983 Aust. J. Phys. 36 755
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