Quark clusters in quark stars and possible astrophysical implications

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Abstract

A quark-cluster state, rather than the color-super-conductivity state, may appear in matter with low-temperature but high density, since the phase transition of chiral symmetry broken and that of color-confinement could not occur simultaneously. Such quark clusters might be stable in strange quark matter. Quark stars would have temperatures to be low enough to freeze by residual color interaction between the clusters, and become then solid ones. The charge numbers and the sizes of quark clusters, as well as the residual interactions in-between are investigated. The solid state properties of quark stars are constrained by observations.

Key words: pulsars, neutron stars, elementary particles

1 Introduction

Since E. Witten suggested that strange quark matter may be the true ground state of color interaction system (Witten, 1984), it has been of great interest to find such matter in reality. There are hitherto many kinds of attempts, which can be classified into two ways. One is to detect directly strange quark matter on earth or in space (e.g., the ground and the AMS-II observations of strangelets in cosmic rays), the other is to study distant objects by means of astrophysics (e.g., to identify quark stars with strangeness). As for the nature of quark matter of strange stars, current researchers are focusing on quark condensation in momentum space (e.g., 2SC, CFL and LOFF states). Recently it has been proposed alternatively that quarks in strange quark matter with high density and low temperature may condense in position space (i.e., forming quark clusters which are not color-confined), and may be in a solid state when the temperature is much low (Xu, 2003, 2004). This kind of solid quark stars could be applied to explain well the discrepancy between the observed glitches and free-precessions of pulsars (Xu, 2004; Zhou et al., 2004).
In the following sections, based on the idea of quark clustering, observational constrains on the nature of quark matter with quark clusters are derived.

2 Charge numbers and cluster sizes constrained by the featureless spectrum

The 500ks Chandra observation of RXJ1856.5-3754 (Drake et al. 2002) shows a featureless Plank-like spectrum, which is not natural to be explained in the atmosphere models of normal neutron star. However, for the simplicity nature, the spectrum might be fitted well in a solid quark star model (Zhang et al. 2004). But if a cluster possesses more charges, it may behave like an atom which should have complex energy levels and would thus show corresponding atomic features in the spectrum. Because the observational low-band of thermal X-ray spectra is \( \sim 0.1 \) keV, this kind of “atomic features” can not be observed if the charge number per cluster, \( Z \), is not greater than \( \sim 2 \). In order to explain the observational fact of featureless spectra, the number of quarks per cluster, \( n \), can’t be too large, because \( Z \) can be simply written as \( n \cdot n_e/(3n_b) \). In order to know the upper limit of \( n \), we calculate electron density, \( n_e \), as well as baryon density \( n_b \) near the surface of the star. As will be shown, these two parameters can be obtained by solving the charge neutrality equation and state equation of zero pressure (Alcock et al. 1986), and are determined by three physical constant from MIT bag model.

The charge neutrality equation is

\[
\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0, \tag{1}
\]

and the zero-temperature equation of state is

\[
\frac{1}{3}(n_u + n_d + n_s) - \sum_i (\Omega_i + \mu_i n_i) = B, \tag{2}
\]

where \( i (= u, d, s, e) \) denotes up, down, and strange quarks, and electron; \( \Omega_i \), the thermal dynamic potentials; \( \mu_i \), the chemical potentials; and \( n_i \), the number densities. Here we take the formulae in the appendix of Alcock et al. (1986) to relate \( \Omega_i \) and \( \mu_i \),

\[
\Omega_u = -\frac{\mu_u^4}{4\pi^2}(1 - \frac{2\alpha_s}{\pi}), \tag{3}
\]

\[
\Omega_d = -\frac{\mu_d^4}{4\pi^2}(1 - \frac{2\alpha_s}{\pi}), \tag{4}
\]
\[ \Omega_s = -\frac{1}{4\pi^2} \left( \mu_s (\mu_s^2 - m_s^2)^{1/2} [\mu_s^2 - \frac{5}{2}m_s^2] + \frac{3}{2}m_s^4 \ln \left[ \frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s} \right] \right) 
- \frac{2\alpha_s}{\pi} \left[ 3\{\mu - s(\mu_s^2 - m_s^2)^{1/2} - m_s^2 \ln \left[ \frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{\mu_s} \right] \}^2 - 2(\mu_s^2 - m_s^2)^2 \right] 
- m_s^4 \ln \frac{m_s}{\mu_s} + 6 \ln \frac{\mu_s}{\mu_e} \{\mu_s m_s^2 (\mu_s^2 - m_s^2)^{1/2} - m_s^4 \ln \left[ \frac{\mu_s + (\mu_s^2 - m_s^2)^{1/2}}{m_s} \right] \} \right), \]

\[ \Omega_e = -\frac{\mu_e^4}{12\pi^2}. \] 

(5)

Beside eqs.(1)-(6), as a result of weak-interaction equilibriums between quarks and electrons, the chemical potential \( \mu \) of them have the following relations,

\[ \mu_d = \mu_s = \mu, \] 

(7)

\[ \mu_u + \mu_e = \mu. \] 

(8)

We have also the relations between the number densities and the thermodynamic potentials,

\[ n_i = -\frac{\partial \Omega_i}{\partial \mu_i}. \] 

(9)

Using above equations, the number densities of quarks and electrons can be obtained, which are found to be dependent only on three parameters of MIT bag model: the vacuum energy density \( B \), strange quark mass \( m_s \), and strong interaction coupling constant \( \alpha_s \). Our results show that for some value of these parameters the electron density can drop to zero very fast. Beyond these parameter values, no solution for \( n_e \) exists any more. Fig 1 shows the electron densities as parameters of \( m_s \) and \( \alpha_s \) for different \( B \). It is found that low \( m_s \) and/or high \( \alpha_s \) favor low \( n_e \). This means that the electric field occurred on the quark-star surface could be very weak if \( m_s \) is low and/or \( \alpha_s \) is high.

With the charge density we now are able to constrain the number of quarks per cluster \( n \) from X-ray observation of RXJ1856.5-3754. In order to reproduce the featureless thermal X-ray spectra, the maximum value of quark number in a clusters, \( n_{max} \), is calculated. Fig 2 shows the results of \( n_{max} \) for different parameters in the MIT bag model (we choose \( Z = 2 \) in the computations). We find that the maximum quark number in clusters can vary in a very large range of values. For \( 50 < m_s/\text{MeV} < 200 \) and \( 0 < \alpha_s < 0.6 \), we have \( 10^3 < n_{max} < 10^8 \).
Fig. 1. Contour plots of electron density in unit of cm$^{-3}$ (solid lines) and of baryon number density in unit of fm$^{-3}$ (dashed lines). The left panel is for $B^{1/4} = 140$ MeV, and the right for $B^{1/4} = 170$ MeV.

Fig. 2. The left panel shows the contour plot of $n_{\text{max}}$ for $B^{1/4} = 140$ MeV, and the right for $B^{1/4} = 170$ MeV.

3 Cluster size and interaction strength limited by glitches

It was pointed out by Zhou et al. (2004) that if strange quark stars are in a solid state, pulsar glitches could be understood well by a sudden release of elastic energy accumulated when the shape of a star changes as its spin slows down. The elastic energy of a solid quark star is microscopically the sum of bond-energies of all links between clusters, and is determined by the strength of interactions and the sizes (i.e., $n$) of clusters. Unfortunately up to now the strength of interaction can’t be calculated from first principles. We suppose thus its value to be of the same order of the chemical potential of quarks; that is within 1MeV − 100MeV. By input different strengths of interactions by hand, we can obtain the magnitude of corresponding elastic energy.

A strange star should be in a solid state before a glitch occurs due to increasing elastic energy. There are generally two kinds of movement that are against the solidification of strange stars; one is thermal motion, and the other is the quantum uncertainty (i.e., the zero-point energy). From the X-ray observation,
we know that the thermal energy may be at the magnitude of $0.1 - 1 \text{KeV}$. The zero-point energy can be written as $\hbar^2/(2ml^2)$ if quark clusters are moving non-relativistically, where $l \simeq [n/(3n_b)]^{1/3}$ is the average distance between clusters. From these expressions we can find that, for small clusters which have less then thousands or hundreds of baryon numbers, the effect of quantum uncertainty could be larger than that of thermal motion. This zero-point energy should be much smaller than the depth ($V_p$) of potential well between clusters, $V_p \gg \frac{135n^{-5/3}n_b^{2/3}}{n^2} \text{MeV}$, where $n_b$ is in unit of $\text{fm}^{-3}$ and $m \sim 300n \text{MeV}$ is suggested. We have $V_p \gg \frac{56n^{-5/3}}{n^2} \text{MeV}$ if $n_b = 0.26\text{fm}^{-3}$ is chosen. This result means that, for $n \leq 11$, the bond-energy between clusters must be much higher than 1 MeV; otherwise the matter should become a quantum fluid. In an other word, if the interactions between clusters do exceed much from 1 MeV, then it is natural for quark matter to be solidified even when the clusters have only $\sim 10^2$ quarks.

To generate glitches, a solid quark star should have shear modulus (Zhou et al., 2004) $\mu \sim 10^{30-34} \text{erg/cm}^3$. The shear modulus $\mu$ can be derived from the Young’s modulus $E$ via $\mu = E/[2(1 + \nu)]$, where $\nu$ is called Poisson ratio, $0 < \nu < 0.5$. We see $\mu$ and $E$ are of a same order of magnitude. For simplicity we can assume the potential of interaction is Liénard-Jones potential, $V(r) = -A/r^6 + B/r^{1/2}$, and we have then $l = (2B/A)^{1/6}$ and $V_p = -V(l) = A^2/(4B)$. For finding a linear coefficient which cause the Hook’s law, we extract the Taylor expansion near the bottom of the potential well,

$$V(r) = \frac{-A^2}{4B} + \frac{9}{2^{3/2}B^{3/2}} \left[ r - \left( \frac{2B}{A} \right)^{1/6} \right]^2 + O\left[ r - \left( \frac{2B}{A} \right)^{1/6} \right]^3,$$  

from which we find that the second order term can be written as $9/2^{1/3} \cdot A^{7/3}B^{-4/3}(r-l)^2$. Using $l$ and $V_p$ to eliminate $A$ and $B$, the second order term becomes $36V_p(\Delta l/l)^2$. At the same time, by considering a cylinder of length $L$ and cross section $S$, the number of links between clusters at the direction of length in this volume is $SL \cdot (3n_b/n)$; thus the value of elastic energy can be written as,

$$E_e \sim \frac{3n_b}{n} S L 36V_p(\Delta l/l)^2 = \frac{1}{2} Y(\frac{\Delta L}{L})^2.$$

We can then easily get the Young’s modulus $Y = 72V_p n_b/n$. The typical value of shear modulus is $(1/3 \sim 1/2)Y$ in which $n_b \sim 0.26 \text{fm}^{-3}$, consequently,

$$\mu \sim 10^{34} n^{-1}(V_p/\text{MeV}) \text{ (erg/cm}^3).$$

As mentioned before, from observations, we have shear modulus $\mu \sim 10^{30-34} \text{erg/cm}^3$. The number of quarks per cluster, $n$, has resultantly an upper limit, which is $10^4$ for $V_p = 1\text{MeV}$ and $10^5$ for $V_p = 10\text{MeV}$.
4 Conclusions and Discussions

We have obtained the upper limits of the quark number per cluster, $n$, from featureless spectra and pulsar glitches. The maximum value of $n$, $n_{\text{max}}$, constrained from featureless spectra is $\sim 10^{1-8}$ for parameters of $50 < m_s/\text{MeV} < 200$, $0 < \alpha_s < 0.6$, and reasonable bag constants $B$. The upper limit of $n$ for generating pulsar glitches is $10^{5}$ if bond-energy between clusters is $10\text{MeV}$. We conclude therefore that the upper limit of quark number per cluster could be $\sim 10^{4-5}$ in order for bare strange stars to reproduce both featureless X-ray spectra and pulsar glitches.

What have been done in this paper are based on the solid quark star model. This model is only a phenomenological model, into which it is very necessary to research from first principles. Nevertheless, the conclusion of our paper may provide helpful information about the detail processes of quark clustering and matter solidifying.

One problem in our paper is that we had used an equation of state derived from the traditional concept of free-quark matter, not clustered-quark matter. This problem may affect the result significantly if clustering changes the equation of state too much. Although this issue needs to be treated seriously, the deviation of equation of state may be slight, since color-confinement does not occur yet in the quark-clustering matter and the property of the vacuum could thus be not much different from that described by Alcock et al. (1986).

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