Optimization of territory control of the mail carrier by using Hungarian methods

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Abstract. In this paper, the territory control of the mail carrier from the central post office Bandung in delivering the package to the destination location was optimized by using Hungarian method. Sensitivity analysis against data changes that may occur was also conducted. The sampled data in this study are the territory control of 10 mail carriers who will be assigned to deliver mail package to 10 post office delivery centers in Bandung. The result of this research is the combination of territory control optimal from 10 mail carriers as follows: mail carrier 1 to Cikutra, mail carrier 2 to Ujung Berung, mail carrier 3 to Dayeuh Kolot, mail carrier 4 to Padalarang, mail carrier 5 to Situ Saeur, mail carrier 6 to Cipedes, mail carrier 7 to Cimahi, mail carrier 8 to Soreang, mail carrier 9 to Asia-Afrika, mail carrier 10 to Cikeruh. Based on this result, manager of the central post office Bandung can make optimal decisions to assign tasks to their mail carriers.

Keyword: territory control, Hungarian method, optimal decisions, sensitivity analysis

1. Introduction

Territory control of a mail carrier is the understanding of a mail carrier in knowing the existence of the destination area. In a transportation problem, territory control variable can be considered for obtaining the optimal solution in assignment problem. The assignment problem is a special case of the transportation problem. It arises in a variety of decision-making situations [1]. A distinguishing feature of the assignment problem in this case is that one mail carrier is assigned to one and only one post office delivery centers [8].

Many papers have been discussed the assignment problem to solve decision-making problem. Sudradjat et al. [8] studied the Hungarian method to solve the assignment problem based on traveling time for a case study on the central Post Office Bandung. A new approach to one-sided assignment problems was proposed by Sasaki [6]. Maxon and Bhadury [3] discussed the assignment problem with repetitive tasks and tried to introduce a human element into the analysis. A simple random assignment
problem with a unique solution was presented by Bogomolnaia and Moulin [2]. Nuass [4] presented a special purpose branch-and-bound algorithm for solving assignment problems. Sourd [7] discussed the continuous assignment problem with the aim of solving scheduling problems with irregular cost functions. Odior et al. [5] addressed a problem of the effectiveness of feasible solutions of assignment problems.

The concept of assignment problem based on territory control was applied to solve a problem for a case study on the central Post Office Bandung. This research discusses how to optimize territory control problem then solved by using Hungarian method. This optimization process is applied to a case study of the central post office Bandung in assigning employees to deliver the packet to the destination location based on several criteria owned by each employee. It is also conducted a sensitivity analysis of data changes that may occur so as not to change the optimal assignment from the initial problem.

2. Materials and Methods
The concept of assignment problem based on territory control by using Hungarian methods was applied to solve a problem for a case study on the central Post Office Bandung. The Hungarian method is a combinatorial optimization algorithm that solves the assignment problem in polynomial time for a given cost matrix [8].

**Theorem 2.1:** If a number is added to or subtracted from all of the entries of any one row or column of a cost matrix, then an optimal assignment for the resulting cost matrix is also an optimal assignment for the original cost matrix.

**The Hungarian Method:** The following algorithm applies the above theorem to a given $n \times n$ cost matrix to find an optimal assignment.

**Step 1.** Subtract the smallest number in each row from every number in the row. (This is called row reduction.) Enter the results in a new table.

**Step 2.** Subtract the smallest number in each column of the new table from every number in the column. (This is called column reduction.) Enter the results in another table.

**Step 3.** Test whether an optimal set of assignments can be made. You do this by determining the minimum number of lines needed to cover (i.e., cross out) all zeros. Since this minimum number of lines equals the maximum number of assignments that can be made to zero element positions, if the minimum number of lines equals the number of rows, an optimal set of assignments is possible. (If you find that a complete set of assignments to zero element positions is not possible, this means that you did not reduce the number of lines covering all zeros down to the minimum number.) In that case, go to step 6. Otherwise go on to step 4.

**Step 4.** If the number of lines is less than the number of rows, modify the table in the following way:

   a. Subtract the smallest uncovered number from every uncovered number in the table.
   b. Add the smallest uncovered number to the numbers at intersections of covering lines.
   c. Numbers crossed out but not at the intersections of cross-out lines carry over unchanged to the next table.

**Step 5.** Repeat steps 3 and 4 until an optimal set of assignments is possible.

**Step 6.** Make the assignments one at a time in positions that have zero elements. Begin with rows or columns that have only one zero. Since each row and each column needs to receive exactly one assignment, cross out both the row and the column involved after each assignment is made. Then move on to the rows and columns that are not yet crossed out to select the next assignment, with preference again given to any such row or column that has only one zero that is not crossed out. Continue until every row and every column has exactly one assignment and so has been crossed out. The complete set of assignments made in this way is an optimal solution for the problem.

Data collected (in Likert scale) from the central Post Office Bandung is shown in Table 1 below.
Table 1. Territory control each mail carrier to each post office delivery centers (in Likert scale)

| Mail Carrier | I   | II  | III | IV  | V   | VI  | VII | VIII | IX  | X  |
|--------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|
| 1            | 2   | 3   | 2   | 2   | 2   | 2   | 3   | 3   | 3   | 2  |
| 2            | 3   | 3   | 2   | 2   | 2   | 4   | 2   | 4   | 3   | 2  |
| 3            | 2   | 2   | 2   | 2   | 2   | 4   | 2   | 4   | 3   | 2  |
| 4            | 4   | 1   | 2   | 3   | 4   | 3   | 3   | 2   | 2   | 4  |
| 5            | 2   | 1   | 2   | 3   | 4   | 3   | 2   | 2   | 3   | 2  |
| 6            | 4   | 4   | 3   | 3   | 2   | 5   | 2   | 2   | 2   | 4  |
| 7            | 2   | 3   | 2   | 2   | 3   | 4   | 2   | 1   | 2   | 3  |
| 8            | 3   | 2   | 3   | 3   | 2   | 3   | 4   | 2   | 3   | 4  |
| 9            | 2   | 3   | 3   | 3   | 2   | 4   | 3   | 4   | 3   | 3  |
| 10           | 3   | 3   | 3   | 3   | 2   | 3   | 2   | 3   | 4   | 1  |

Table 1 shows that the territory control of 10 mail carriers to deliver mailing packets to 10 post office delivery centers in Bandung that is Cikutra (I), Padalarang (II), Ujung Berung (III), Dayeuh Kolot (IV), Asia-Africa (V), Soreang (VI), Situ Saeur (VII), Cimahi (VIII), Cipedes (IX) and Cikeruh (X). The approach to scaling responses in survey research is used the Likert scale as follows: strongly know = 1, know = 2, neither know nor don’t know = 3, don’t know = 4 and strongly don’t know = 5.

3. Result and Discussion

The goal of the assignment problem is to minimize the territory control of 10 employees to deliver mailing packets to 10 post office delivery centers in Bandung. An important characteristic of this assignment problem is the number of workers is equal to the number of destinations. It is explained in the following way.

- Only one job is assigned to mail carrier.
- Each mail carrier is assigned to exactly one destination.

Based on the data collected, manager of the Central Post Office Bandung has ten mail carriers for ten separate destinations and the territory control of assigning each destination to each mail carrier is given. His goal is to assign one and only destination to each mail carrier in such a way that the territory control of assignment is minimized.

Balanced assignment problem: The number of mail carriers is equal to the number of destinations. The mathematical model of the assignment problem in this case can be written as follows:

\[
\begin{align*}
\text{Min } T = & \sum_{i=1}^{10} \sum_{j=1}^{10} c_{ij}x_{ij} \\
\text{s.t. } & \sum_{j=1}^{10} x_{ij} = 1 \hspace{1cm} ; i = 1, 2, \ldots, 10 \\
& \sum_{i=1}^{10} x_{ij} = 1 \hspace{1cm} ; j = 1, 2, \ldots, 10 \\
x_{ij} = & \begin{cases} 
1, & \text{if mail carrier } i \text{ go to destination } j \\
0, & \text{if mail carrier } i \text{ does not go to destination } j
\end{cases} \hspace{1cm} i = 1, 2, \ldots, 10 \text{ and } j = 1, 2, \ldots, 10.
\end{align*}
\]

Based on data in Table 1, the optimal assignments matrix was produced by applied the Hungarian method in this case as shown in Table 2 below.
Table 2. Optimal Assignment Problem Matrix

| Mail Carrier | I    | II   | III  | IV   | V    | VI   | VII  | VIII | IX   | X   |
|--------------|------|------|------|------|------|------|------|------|------|-----|
| 1            | 0    | 2    | 0    | 0    | 0    | 0    | 1    | 1    | 1    | 0   |
| 2            | 1    | 2    | 0    | 0    | 0    | 1    | 0    | 2    | 1    | 0   |
| 3            | 0    | 1    | 0    | 0    | 0    | 1    | 0    | 2    | 2    | 1   |
| 4            | 2    | 0    | 0    | 1    | 2    | 0    | 1    | 0    | 0    | 2   |
| 5            | 0    | 0    | 0    | 1    | 2    | 0    | 0    | 0    | 1    | 0   |
| 6            | 2    | 3    | 1    | 1    | 0    | 2    | 0    | 0    | 0    | 1   |
| 7            | 1    | 3    | 1    | 1    | 2    | 2    | 1    | 0    | 1    | 2   |
| 8            | 1    | 1    | 1    | 1    | 0    | 2    | 0    | 1    | 2    | 0   |
| 9            | 0    | 2    | 1    | 1    | 0    | 1    | 1    | 2    | 1    | 0   |
| 10           | 2    | 3    | 2    | 2    | 1    | 1    | 1    | 2    | 3    | 0   |

Based on the optimal solution in Table 2, the assignment of mail carrier from Central Post Office Bandung to each post office delivery centers in Bandung can be seen in Table 3 below.

Table 3. Mail carriers assignment determination

| Mail Carrier | Post office delivery centers in Bandung |
|--------------|----------------------------------------|
| 1            | Cikutra                                 |
| 2            | Ujung Berung                            |
| 3            | Dayeuh Kolot                            |
| 4            | Padalarang                              |
| 5            | Situ Saeur                              |
| 6            | Cipedes                                 |
| 7            | Cimahi                                  |
| 8            | Soreang                                 |
| 9            | Asia-Afrika                             |
| 10           | Cikeruh                                 |

In order to determine the data changes range of territory control that does not change the assigned optimal assignment of workers, the sensitivity analysis was done. Sensitivity analysis to change of objective function coefficient from territory control data was obtained as shown in Table 4.

Table 4. Range of territory control coefficient

| Worker | I    | II   | III  | IV   | V    | VI   | VII  | VIII | IX   | X   |
|--------|------|------|------|------|------|------|------|------|------|-----|
| 1      | $[-\infty, 3]$ | $[2, \infty]$ | $[1,3]$ | $[1,3]$ | $[1,3]$ | $[2,4]$ | $[1,\infty]$ | $[1,\infty]$ | $[1, \infty]$ | $[1,3]$ |
| 2      | $[1, \infty]$ | $[2, \infty]$ | $[-\infty,3]$ | $[1,3]$ | $[1,3]$ | $[1,\infty]$ | $[1,3]$ | $[2, \infty]$ | $[1, \infty]$ | $[1,3]$ |
| 3      | $[1,3]$ | $[1,\infty]$ | $[1,3]$ | $[-\infty,3]$ | $[1,3]$ | $[1,\infty]$ | $[1,3]$ | $[2, \infty]$ | $[2, \infty]$ | $[1, \infty]$ |
| 4      | $[2, \infty]$ | $[-\infty,2]$ | $[1,3]$ | $[1,\infty]$ | $[2, \infty]$ | $[2,4]$ | $[1,\infty]$ | $[1,3]$ | $[1,3]$ | $[2, \infty]$ |
| 5      | $[1,3]$ | $[0,2]$ | $[1,3]$ | $[1,\infty]$ | $[2, \infty]$ | $[2,4]$ | $[-\infty,3]$ | $[1,3]$ | $[1, \infty]$ | $[1,3]$ |
| 6      | $[2, \infty]$ | $[3, \infty]$ | $[1,\infty]$ | $[1,\infty]$ | $[1,3]$ | $[2, \infty]$ | $[1,3]$ | $[1,3]$ | $[-\infty,3]$ | $[1, \infty]$ |
| 7      | $[1, \infty]$ | $[3, \infty]$ | $[1,\infty]$ | $[1,\infty]$ | $[2, \infty]$ | $[2, \infty]$ | $[1,\infty]$ | $[1, \infty]$ | $[-\infty,2]$ | $[1, \infty]$ |
| 8      | $[1, \infty]$ | $[1, \infty]$ | $[1,\infty]$ | $[1,\infty]$ | $[1,3]$ | $[-\infty,4]$ | $[2, \infty]$ | $[-\infty,3]$ | $[1, \infty]$ | $[2, \infty]$ |
| 9      | 0 | $[2, \infty]$ | $[1,\infty]$ | $[1,\infty]$ | $[-\infty,3]$ | $[1,\infty]$ | $[1, \infty]$ | $[2, \infty]$ | $[1, \infty]$ | $[1, \infty]$ |
| 10     | $[2, \infty]$ | $[3, \infty]$ | $[2, \infty]$ | $[1,\infty]$ | $[1,\infty]$ | $[1, \infty]$ | $[2, \infty]$ | $[3, \infty]$ | $[-\infty,2]$ | $[1, \infty]$ |
Based on Table 4, it is obtained the range of coefficients changes from the objective function allowed for each mail carrier to reach the destination location does not change the initial assignment.

4. Conclusion
In this paper, the concept of assignment problem has been applied to solve a problem for of the central post office Bandung which had a difficulty in assigning 10 mail carriers to 10 destination locations of post office delivery centers. Based on the data collected, Hungarian Method was used to solve the problem. Optimal assignments of the cases were obtained to the central post office Bandung. It was obtained that, the optimal assignment of mail carriers from Central Post Office Bandung to each post office delivery centers as follows: mail carrier 1 to Cikutra, mail carrier 2 to Ujung Berung, mail carrier 3 to Dayeuh Kolot, mail carrier 4 to Padalarang, mail carrier 5 to Situ Saeur, mail carrier 6 to Cipedes, mail carrier 7 to Cimahi, mail carrier 8 to Soreang, mail carrier 9 to Asia-Afrika, mail carrier 10 to Cikeruh. Based on this result, manager of the central post office Bandung can make optimal decisions to assign tasks to their mail carriers. The Sensitivity Analysis for assignment issues can illustrate the range of coefficient value changes for the allowed objective function so that this does not change the optimal assignment of the initial problem.

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