Three-dimensional Dynamic Behavior of the Flexible Umbilical Cable System

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Abstract. We extend an idea of the modeling flexible umbilical cable using a nonlinear finite element method to three-dimensional space. The umbilical cable is assembled by Reissner’s geometrically exact beam elements, which take account of the geometric nonlinearities of large rotation and displacement, and effects of axial load, transverse shear, torsion and bending stiffness for modeling of slack cables. The resulting semi-discrete motion equations are solved by the Newmark method. Finally, an example about the three-dimensional dynamic behavior of the umbilical cable system in depth-dependent ocean current with ship motion is presented.

Introduction

Due to the need for submarine operations, such as installations, maintenances and repairments, the deep sea remotely operated vehicles [1] are applied widely, which are typically composed of the supporting ship, umbilical cable, cage and vehicle. The umbilical cable plays an important role in this deep sea operation system, such as physical link, real-time transmission and lifeline. Through the umbilical cable the motion of the supporting ship and the disturbances of ambient ocean current can be passed to the cage, leading to large tension fluctuation in the umbilical, moreover, the safety of submarine operation of the whole underwater system is affected, and the launch and recovery of vehicle will be more difficult.

There are lots of literatures on dynamics modeling of flexible marine cables. Driscoll [2] presented one-dimensional finite element model composed of linear viscoelastic element, it could predicate the heave motion of cage under several different sea states accurately. J.M. Wu and A.T. Chwang [3] proposed a hydrodynamic model of a two-part underwater manoeuvrable towed system using the finite difference method. However, these authors’ literatures have not considered the effect of bending stiffness, when the cable tension is zero, the cable equation will appear a singular point, which will cause the breakdown of calculation. In order to allow the cable equation to be suitable for low tension problem, Buckham[4] considered the effects of the bending and torsional stiffness of the cable, and established a low tension cable model using the lumped mass method, which was proved that it can capture the dynamics at the low tension state by the tank experiment. In addition to the effects of axial load and bending, the deformation of actual cable should include the effects of shear and torsion for completeness, and the actual cable moves in three-dimensional space, but the above works contain these deformation factors incompletely, and mostly are restrained in the two-dimensional plane. These drawbacks can be avoided by using the geometrical exact beam which uses the position coordinate of centerline and the finite rotation of cross-section to describe the cable configuration.

This paper extends the total Lagrangian formulation of the geometrically exact beam theory[5-8] to the dynamic modeling of umbilical cable in three-dimensional space, and studies some important characteristics of the cable under the influences of mother ship movement and surrounding ocean current, such as the dynamic three-dimensional configuration, the tension of both ends of cable and the motion of cage.
Motion Equation

There are some assumptions for developing the motion equation of the umbilical cable system in this paper: (i) Cross-section plane initially perpendicular to the centerline is always a plane, and does not wrap; (ii) Cable material is continuous, linearly elastic and isotropic; (iii) Symmetrical circular cross-section shape; (iv) Umbilical cable is straight and free of stress in the initial state.

**Virtual Work Principle.** There are three types of forces acting on the flexible umbilical cable system, which are external forces, internal forces and acceleration forces. One can apply the principle of virtual work to obtain the weak form of the governing equations of the flexible umbilical cable. The total virtual work can be written as

\[ \delta W = \delta W_{\text{int}} - \delta W_{\text{ext}} - \delta W_{\text{acc}} = 0 \]  

(1)

where \( \delta W_{\text{int}} \) is the internal virtual work, \( \delta W_{\text{ext}} \) is the external virtual work, \( \delta W_{\text{acc}} \) is the virtual work of acceleration forces. The minus on the right side of the Eq.1 describes that the virtual work due to internal and acceleration forces is contrary to the virtual work due to the external forces. The Eq.1 has no direct analytical solution due to its highly nonlinearity, we usually adopt the Newton-Raphson iterative method to access to the real solution with given tolerance. The linearization of total virtual work produces residual terms and linear increment terms, which yield the nodal force vectors and tangential stiffness matrices respectively through applying the finite element interpolation.

**3D Beam Element.** In this section, we discuss the discrete approximation of Eq.1 based on the theory of the geometrically exact beam using finite element method by employing the standard isoparametric interpolations. According to this theory the placement field is measured with respect to a fixed spatial frame. The beam element based on this theory has independent translational and rotational fields, and thus has simple shape functions. Meanwhile, this beam element has only six nodal degrees of freedom (DOF) including three translational and three rotational DOFs. In this paper, we only use the beam element with two nodes and linear shape functions. The nodal internal force vector for the this beam element is given as

\[ f_{\text{int}} = \int_{s_i} B_i^T F_{\text{int}} ds, B_i = BQ_i, B = \begin{pmatrix} \mathbf{R}^T & \mathbf{O} \\ \mathbf{O} & \mathbf{T} \end{pmatrix}, D_{\Psi}^2 \end{pmatrix} \]

(2)

where \( (\cdot)^T = \partial(\cdot)/\partial s \), \( s \) is the initial arc length of the flexible cable, \( x_c \) is the centerline of umbilical cable, \( \Psi \) is the total rotation vector of cross section, \( N_i \) is the nodal shape function, \( R \) is the rotation transformation matrix, \( T \) is the linear transformation matrix, \( D_{\Psi} (\cdot) \) is the Frechet derivative, \( I \) is the identity matrix. The resultant force vector \( F_{\text{ext}} \) is defined as

\[ F_{\text{ext}} = (N \ M)^T = C_{NM} (\Gamma \ K)^T = C_{NM} \left( \mathbf{R}^T x_c^\prime - E_i \ T \Psi \right)^T. \]  

(3)

where \( \mathbf{N} \) and \( \mathbf{M} \) are internal stresses and stress couples, \( C_{NM} = \text{diag} \{ E_i \} \) is the material elastic modulus for the circular cross section, \( \{ E_i \} \) is the basis vector of the material frame, \( \Gamma \) and \( K \) are the linear and curvature strain measures, and can be written in terms of the displacement \( x_c \) and rotation \( \Psi \). The nodal external force of this beam element is

\[ f_{\text{ext}} = \int_{s_i} N_i^T F_{\text{ext}} ds, F_{\text{ext}} = (\bar{n} \ M)^T. \]  

(4)

where \( N_i = N_{..} I_{6 \times 6} \), \( F_{\text{ext}} \) is the resultant external force vector, \( \bar{n} \) and \( \bar{M} \) are the resultant external forces and force couples per unit initial arc length. For the ocean cable, the external forces \( \bar{n} \) mainly include the gravity, buoyancy, and viscous drag force. \( \bar{M} \) usually vanish due to the centerline condition and thus can be neglected in this paper. The nodal acceleration force vector \( f_{\text{acc}} \) can be expressed as
where the effective density is \( \rho = \rho_0 + C_m \rho_w \) (including the added mass), \( \rho_0 \) is the cable density, \( C_m \) is the added mass coefficient, \( \rho_w \) is the density of sea water, \( J \) is the cross section inertia tensor, \( Y = X_\alpha E_\alpha + X_\beta E_\beta \), \( X_\alpha \) is the material coordinate. The nodal tangential stiffness matrix is obtained by the linearization of the internal force vector \( f_{\text{int}} \) and can be given as

\[
K_{\text{el}} = \int_{l_e} B_j^T C_{\text{el}} B_j \, ds + \int_{l_e} Q_j^T K_{\text{geo}} Q_j \, ds.
\]

where \( K_{\text{geo}} \) is the geometrical stiffness. The nodal mass matrix of this beam element can be given by

\[
M_{\text{el}} = \int_{l_e} N_j^T M(\Psi) N_j \, ds, \quad M(\Psi) = \begin{pmatrix} A & 0 \\ 0 & T^T J T \end{pmatrix}.
\]

By combining the foregoing equations, we can obtain the finite element equation of the flexible ocean cable, for it is only discretized in the spatial domain, the whole finite element equation can also be called semi-discrete equation, it’s given by

\[
\bar{R} + \bar{K} \Delta q - \bar{C} \Delta q = \bar{M} \ddot{q}.
\]

where \( q = (\mathbf{x}, \dot{\mathbf{x}})^T \), “( \cdot )” means the value at a given state \( q_n \), \( \bar{R} \) is the total residual force vector, \( \bar{K} \) is the total stiffness tensor, \( \bar{C} \) is the total damping tensor, which usually is not significant and can be ignored unless in high drag loading scenario, while it is incorporated in this work for the purpose of completeness, \( \bar{M} \) is the total mass tensor, \( \Delta q \) is the total displacement increment vector, \( \Delta \dot{q} \) is the total velocity increment vector, \( \ddot{q} \) is the total acceleration vector.

**Newmark Method**

By the variables substitution, there are \( a = \dot{q}, v = \ddot{q}, u = q \). At the time \( t_{n+1} \), the formulation of Eq.8 can be expressed as

\[
M_a a_{n+1} + C_a \Delta v_{n+1} - K_a \Delta u_{n+1} - R_a = 0.
\]

The dynamic problems Eq.9 usually are solved by the direct time integration method, including the explicit method and the implicit method. The main drawback of explicit method is that its step must not be greater than a critical step, or else this method will be unstable, i.e., the explicit method is conditionally stable. With the wider step size and unconditional stability, the implicit methods are usually used in engineering applications with low frequency changes, one of which is the Newmark method, which is often used in structural mechanics and solid mechanics. The approximation of velocity and acceleration of the Newmark method can be written as

\[
a_{n+1} = \alpha_1 (u_{n+1} - u_n) - \alpha_2 v_n - \alpha_3 a_n, \quad v_{n+1} = \alpha_4 (u_{n+1} - u_n) + \alpha_5 v_n + \alpha_6 a_n,
\]

\[
\alpha_1 = \frac{1}{\beta (\Delta t)^2}, \quad \alpha_2 = \frac{1}{\beta \Delta t}, \quad \alpha_3 = \frac{1 - 2 \beta}{2 \beta}, \quad \alpha_4 = \frac{\gamma}{\beta \Delta t}, \quad \alpha_5 = \left[1 - \frac{\gamma}{\beta}\right], \quad \alpha_6 = \left[1 - \frac{\gamma}{2 \beta}\right] \Delta t.
\]

where \( \beta, \gamma \) are parameters that define the Newmark method, \( \Delta t \) is the time step size. The iterative formulation of Eq.9 at time \( t_{n+1} \) can be denoted by

\[
M (u'_{n+1}) a_{n+1} + C (u'_{n+1}) \Delta v_{n+1} - K (u'_{n+1}) \Delta u_{n+1} - R (u'_{n+1}) = 0,
\]

\[
a_{n+1} = \alpha_1 (u'_{n+1} - u_n) - \alpha_2 v_n - \alpha_3 a_n, \quad u_{n+1} = u_{n+1} + \Delta u'_{n+1}, \quad \Delta v'_{n+1} = \alpha_4 \Delta u'_{n+1}.
\]
From the foregoing equations, we can obtain

\[
\begin{align*}
K_T(u_{n+1}^t) \Delta u_{n+1}^t &= H(u_n, v_n, a_n, u_{n+1}^t), \\
K_T(u_{n+1}^t) &= \alpha C(u_{n+1}^t) + \alpha M(u_{n+1}^t) - K(u_{n+1}^t), \\
H(u_n, v_n, a_n, u_{n+1}^t) &= R(u_{n+1}^t) - M(u_{n+1}^t)[\alpha_i(u_{n+1}^t - u_n) - \alpha_2 v_n - \alpha_3 a_n].
\end{align*}
\tag{12}
\]

From the Eq. 12, we can solve that

\[
\Delta u_{n+1}^t = \left[K_T(u_{n+1}^t)\right]^{-1} H(u_n, v_n, a_n, u_{n+1}^t).
\tag{13}
\]

The update schemes of velocity and acceleration are

\[
v_{n+1}^t = v_n + \frac{\gamma}{\beta h} \Delta u_{n+1}^t, \quad a_{n+1}^t = a_n + \frac{1}{\beta h^2} \Delta u_{n+1}^t.
\tag{14}
\]

where \( h = \Delta t = t_{n+1} - t_n, i = 0,1,\ldots, m - 1 \). The iteration stops when the following convergence criteria are satisfied, i.e. \( \|\Delta u_{n+1}^t\| \leq \varepsilon \|u_{n+1}^t\| \), where \( \varepsilon \) is a very small positive number.

**Numerical Example**

We consider the problem of this umbilical cable system in three-dimensional horizontal current with sinusoidal ship motion at the top node of the umbilical. The main physical parameters of the system are given as following: \( EA = 4.6 \times 10^7 \) (N), \( GA = 1.9 \times 10^7 \) (N), \( EI = 2.5 \times 10^3 \) (Nm\(^2\)), \( GJ = 2.1 \times 10^3 \) (Nm\(^2\)), cable diameter is 0.030 (m), cable mass is 3.01 (kg/m), cable weight in sea is 25.4 (N/m), tangent drag coefficient is 2.0, normal drag coefficient is 0.02, bi-normal drag coefficient is 0.02, added mass coefficient is 1.5, cage weight in sea is 43200 (N), and vehicle weight in sea is 300 (N). The ship motion is of amplitude 1.0m and period 8.0s. It is still assumed that the current profile is \( v_x = v_y = v_0 \left(1 - z/h\right), z \in [0, h], v_z = 0 \), where \( h \) is the water depth, and the horizontal components \( v_x, v_y \) are nonzero, \( v_0 \) is taken as 0.5m/s in this example.

**Fig. 1.** Dynamic configuration of the umbilical cable system within 100s.

**Fig. 2.** Motion and tension of both ends of the umbilical cable.

Simulations were performed with the Newmark method of the standard parameters: \( \beta = 0.25, \gamma = 0.5 \). The simulations were run for a maximum of 100s with the initial time step 0.1s. Based on these conditions, we analyzed the complex dynamic characteristics of the umbilical cable system at the depth of 5000m, including the dynamic cable configuration in three-dimensional space and its projections in two-dimensional plane, tension and motion of both ends of the cable, as shown in Fig.1-2.
Fig.1a illustrates the three-dimensional time-dependent dynamic configuration every other 50 time steps through the standard Newmark method. It could be found that with the increasing time, the umbilical cable slowly shifts away from the initial position along the direction of resultant velocity of three-dimensional ocean current. The configuration of umbilical cable becomes $S$ shape due to the hydrodynamic action of the depth-dependent ocean current. Fig.1b shows the projection of dynamic configuration of the umbilical cable in XOZ plane, from which we can find that the projection of horizontal offset of the end node in XOZ plane has reached to 9.09m at the moment of 100s, while the projection of maximum offset has arrived at 21m at the same time. From Fig.1c, it can be found that the angle between horizontal projection of those dynamic configurations and the X axis is exactly 45 degree, which is just the direction of the resultant velocity of the ocean current.

Compared with motion of the supporting ship, the motion of the end node of the umbilical cable (cage) lags behind with obvious mean phase lag of 2.1s at the depth of 5000m, and the amplitudes of the vertical displacement of the same end node (cage) are all amplified with mean amplification factor of 1.5, as shown in Fig.2a. In particular, the peak displacement of the cage is continually increased within 100s due to the action of three-dimensional ocean current. Fig.2b shows the dynamic tension at each end fluctuates around its own static tension, which are 172.7KN and 43.2KN for both ends of the cable, respectively. Meanwhile, it can be seen that the maximum tension at the top node is always less than the maximum working load of umbilical (200KN), which implied even under this condition the cable is still in safe.

Summary

A novel three-dimensional geometrically exact model for umbilical cable system analysis has been presented. The model presented in this paper is comprehensive as it includes the effects of axial load, bend, torsion and shear, therefore, it can be applicable to the high or low tension cables. The resulting motion equations expressed by the second-order differential equations with variable coefficients of matrices are solved by the Newmark algorithm. The numerical results demonstrate the capability of the present model for dynamic analysis of flexible umbilical cable system, and indicate that the importance of ocean current for predication of the umbilical behavior.

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