Isospin violation in $J/\Psi \to \phi \pi^0 \eta$ decay and the $f_0 - a_0$ mixing

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The isospin violating $J/\Psi \to \phi \pi^0 \eta$ decay is thought to be dominated by the mixing of the $f_0(980)$ and $a_0(980)$ scalar resonances. We make a theoretical evaluation of the $J/\Psi \to \phi \pi^0 \eta$ decay extending our previous model for other $J/\Psi$ decays into one vector meson and two pseudoscalars using the techniques of the chiral unitary approach. The scalar resonances are dynamically generated through the final state interaction of the pseudoscalar mesons implementing unitarity from the lowest order ChPT amplitudes. Besides the direct $J/\Psi \phi PP$ vertex, other mechanisms like the sequential exchange of vector and axial-vector mesons are shown to be important in order to obtain the actual strength of the mixing. We get a very good agreement with the $\pi^0 \eta$ invariant mass distribution and branching ratio with recent BESIII data. Quantification of the $f_0(980) - a_0(980)$ mixing is done and compared with results from several experiments.

I. INTRODUCTION

The nature of the scalar mesons has been the subject of controversial debate for more than thirty years and a final consensus has not been reached so far. The problem stems from the fact that it is very difficult to accommodate them within the $q\bar{q}$ picture. Inspired in the four quarks interpretation \cite{4} a second kind of molecule picture was developed, in which only mesonic degrees of freedom were considered \cite{5}. In the past decade new light has been shed by the chiral unitary coupled channel approach \cite{6,7,8} which successfully describes meson-meson interaction in coupled channels up to energies far beyond the limit of applicability of standard chiral perturbation theory (ChPT). With the only input of the lowest order ChPT Lagrangians, the implementation of unitarity in coupled channels and the exploitation of the analytic properties of the scattering amplitudes, many hadronic resonances appear dynamically as poles of the unitarized scattering amplitudes, e.g. \cite{9,12}. These are usually called molecules or dynamically generated resonances, like the $f_0(980)$ and $a_0(980)$ \cite{12,23}, of interest for the present work.

One interesting issue of the $a_0$ and $f_0$ resonances is that they can mix violating isospin, as it was first suggested in ref. \cite{24}. This mixing is expected to help clarify the nature of these resonances. It is thought \cite{24,25} to be enhanced and only relevant in the narrow 8 MeV region between the $K^+K^-$ and $K^0\bar{K}^0$ thresholds, (987.4 MeV and 995.2 MeV respectively). Several reactions could be sensitive to the $f_0 - a_0$ mixing, for example $pn \to d\eta\pi^0$ \cite{26}, $\gamma p \to p\pi^0\eta$ \cite{27} or $\pi^- p \to \pi^0\eta$ \cite{28}. Very recently the isospin violation in $\eta(1405)$ decays into $f_0(980)\pi^0$ have been experimentally measured \cite{29} and theoretically studied in a simultaneous work \cite{30} to the present one and using also techniques of the chiral unitary approach.

Until very recently \cite{31} there has been no conclusive experimental information on this mixing. In ref. \cite{31} the $\pi^0\eta$ invariant mass distribution of the $J/\Psi \to \phi \pi^0\eta$ decay has been measured experimentally by the BESIII collaboration in Beijing and a clear peak in the window between the $K^+K^-$ and $K^0\bar{K}^0$ thresholds has been observed. The branching ratio of the $J/\Psi \to \phi f_0(980) \to \phi a_0(980) \to \phi \pi^0\eta$ is determined in ref. \cite{31} to be of the order of $3.3 \times 10^{-6}$ with a significance of about 3$\sigma$ which represents the most significant evidence of the isospin violating $f_0 - a_0$ mixing to date. From the theoretical side a first estimation of the mixing in the $J/\Psi \to \phi \pi^0\eta$ decay was done in ref. \cite{32} using a simple model with explicit scalar resonance propagators coupling to the pseudoscalar pairs. In ref. \cite{33} the $J/\Psi \to \phi \pi^0\eta$ was calculated from the implementation of the final rescattering of the two pseudoscalars in the direct $J/\Psi \phi PP$ vertex using the chiral unitary approach.

In the work of ref. \cite{34} we developed a very detailed and thorough model (one of the most comprehensive to date) for the decay of the $J/\Psi$ into a vector meson and two pseudoscalars, in particular for the isospin conserving decays into $\omega \pi^+\pi^-$, $\phi \pi^+\pi^-$, $\omega K^+K^-$ and $\phi K^+K^-$. Besides the direct $J/\Psi \phi PP$ vertex and the implementation of the final state interaction of the two pseudoscalars other new mechanisms were considered in ref. \cite{34} where the $J/\Psi$ decays into a vector or axial-vector and a pseudoscalar meson and the vector or axial-vector subsequently decays into the final vector and another pseudoscalar. We call these mechanisms sequential vector and axial-vector exchange. In this sequential mechanisms also the final pseudoscalar-pseudoscalar interaction is implemented from which the scalar resonances are naturally generated dynamically. We found a very good reproduction of the experimental mass distribution also in the $\sigma(500)$ and $f_0(980)$ regions, not trivial at all.

In view of the power of the model of ref. \cite{34}, in the present work we are going to extend it to the $J/\Psi \to \phi \pi^0\eta$ decay in order to analyze the $f_0 - a_0$ mixing. The isospin violation will be implemented essentially from leading order quark mass differences and electromagnetism, similarly as in ref. \cite{34}. In practice it will come essentially from the difference between the masses of the charged and neutral kaons and pions in the loops and the meson-meson scattering amplitudes. One of the main differences with ref. \cite{34} is the new sequential mechanisms...
of ref. [34], which, advancing some results, turn out to be important due to strong interferences with the dominant loops from direct production.

II. THE $f_0 - a_0$ MIXING THROUGH MESON-MESON UNITARIZATION

In the literature several unitarization procedures have been used to obtain a scattering matrix fulfilling exact unitarity in coupled channels, like the Inverse Amplitude Method (IAM) [6, 35] or the N/D method [7]. In this latter work the equivalence with the Bethe-Salpeter equation used in [36] was established. In the present work we use the Bethe-Salpeter equation which leads to the following unitarized amplitude in coupled channels

$$ t = [1 - VG]^{-1}V. \quad (1) $$

Diagrammatically Eq. (1) is equivalent to resum the series expressed by the thick dot in Fig. 2. In Eq. (1) $G_l$ is a diagonal matrix with the l–th element, $G_l$, being the two meson loop function containing the two pseudoscalar mesons of the l–th channel:

$$ G_l(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P - q)^2 - M_l^2 + i\epsilon} \frac{1}{q^2 - m_l^2 + i\epsilon}, \quad (2) $$

with $P$ the total incident momentum, which in the center of mass frame is $(\sqrt{s}, 0, 0, 0)$. The $G_l$ integral above is divergent, and hence it has to be regularized, which can be done either with a three momentum cutoff, $q_{\text{max}}$, or with dimensional regularization. The connection between both methods was shown in Refs. [6, 31]. We use the cutoff regularization since this was the one used in ref. [34] with a value $q_{\text{max}} = 1 \text{ GeV}$ which satisfactorily describes a wide phenomenology for meson-meson interaction [34].

In Eq. (1) $V$, the kernel of the BS equation, is a matrix containing the s-wave projected scattering amplitude for the two pseudoscalar mesons. Since the main source of isospin violation is the different mass between the different charge states, specifically the kaons, it is much more convenient to work in the charge, or particle, basis instead of the isospin basis used in ref. [34] and in typical chiral unitary approach works. Thus the different channels considered are $K^+ K^+$, $K^0 \bar{K}^0$, $\pi^+ \pi^-$, $\pi^0 \pi^0$, $\eta \eta$ and $\eta' \eta$.

Like in ref. [35] the tree level vertex $V$, in Eq. (1) are obtained from the lowest order chiral Lagrangian with isospin-breaking/electromagnetic effects [36], from where the potential between the different channels can be evaluated. The explicit expressions can be found in Eq. (D2) of ref. [33] but all the amplitudes have a difference sign with respect to our notation and the amplitudes involving a $\pi^0 \pi^0$ or $\eta \eta$ must be multiplied by an $1/s$ factor to agree with the unitary normalization of ref. [19]. As explained in ref. [33], these elementary meson-meson amplitudes express the isospin breaking effects in terms of the $\pi^0 \eta$ mixing angle $\epsilon$, and the charge to neutral pion and kaon mass difference, $\Delta \pi = m_{\pi^+} - m_{\pi^-}$ and $\Delta K = m_{K^+} - m_{K^0}$ respectively.

The use of these tree level amplitudes and the different masses between different charge states in the two meson loop functions is what generates the isospin violation in the unitarized amplitudes.

For an illustration of this effect we show in Fig. 1 the modulus of the amplitudes $I=0 < \bar{K}K|t|\pi\pi>_{I=0}$, $I=1 < \bar{K}K|t|\pi\eta>_{I=1}$ and $I=0 < \bar{K}K|t|\pi\eta>_{I=1}$. In the conserving isospin amplitudes we see clear peaks corresponding to the $f_0$ and $a_0$ resonances for the isospin $I = 0$ and $I = 1$ respectively which are dynamically generated. The $I=0 < \bar{K}K|t|\pi\eta>_{I=1}$ amplitude manifests the $f_0 - a_0$ mixing dynamically obtained within the chiral unitary approach. Note the difference between the strength of the isospin violating amplitude with respect to the conserving ones which gives an idea of the size of the isospin violation through $f_0 - a_0$ mixing.

III. THE $J/\Psi \rightarrow \phi \pi^0 \eta$ MODEL WITHIN THE CHIRAL UNITARY APPROACH

In this section we explain the model we use for the evaluation of the $J/\Psi \rightarrow \phi \pi^0 \eta$, which is based in our previous work of ref. [34], and how to implement into it the isospin violation explained in the previous section. Our model of ref. [34] successfully identified and established the relevant mechanisms that contribute to the decay of the $J/\Psi$ into one vector meson and two pseudoscalars. In particular, ref. [34] focused on the isospin conserving channels $\omega \pi \pi$, $\phi \pi \pi$, $\omega K \bar{K}$ and $\phi K \bar{K}$. The main contribution was found to be the mechanisms containing the “direct” $J/\Psi VPP$ vertex implementing also the final state interaction of the pseudoscalar pair using the techniques of the chiral unitary approach from where the scalar mesons appear dynamically in the meson-meson interactions.
scattering amplitudes without the need to include them as an explicit degree of freedom. The actual shape of the amplitudes in the complex plane, and in particular in the physical real axis, couplings of the scalars to the different channels, etc, come naturally with the only input of the lowest order chiral Lagrangian and the implementation of unitarity in coupled channels. Only one free parameter (for the regularization of the two meson loop function) is fitted to the experimental meson-meson data. (See for instance refs. [8, 12] and the references cited in the introduction). The model also includes other mechanisms where the \( J/\Psi \) decays into a vector and or axial-vector and a pseudoscalar meson and the vector subsequently decays into the final vector an the other final pseudoscalar. Also the final state interaction is implemented as before. These new mechanism where found to be crucial in order to obtain the actual shape and strength of the mass distributions in the different channels because, in spite that they are small by themselves the interference with the dominant mechanism is very important.

In this section we extend and adapt the model of ref. [33] to the \( J/\Psi \rightarrow \phi \pi^0 \eta \) stressing the differences of this channel with those studied in ref. [34]. We refer the reader to [34] for the details and here we only address the particularities, differences and the points of special interest for the present work.

The \( J/\Psi \rightarrow \phi \pi^0 \eta \) decay can proceed only through processes involving isospin violation. We do not consider tree level \( J/\Psi \phi \pi^0 \eta \) vertex since in ref. [33] was shown that isospin violation in the production operator is very small, of the order of 4%. Other possible source of isospin violation like the soft photon exchange in the meson propagation was also found to be small in ref. [33]. Thus the main contribution to isospin violation comes from the \( f_0 - a_0 \) mixing which in our formalism is generated essentially from the loops and potentials implicit in the meson-meson unitarized scattering amplitudes, thick dots of Figs. 2 and 3.

For the evaluation of the diagrams of Fig. 2 which we call "direct" production in the following, we need first the \( J/\Psi \phi PP \) vertex which in ref. [34] it was constructed using \( SU(3) \) arguments to relate the different channels and also parametrizing the amplitudes in a way which manifest the OZI rule violation, in a similar way as in ref. [35]. This introduces two a priori unknown constants: an overall coupling \( \tilde{g} \) and the OZI violation parameter, \( \lambda_0 \). These free parameters were fitted in ref. [34] to the invariant mass distributions of the \( J/\Psi \rightarrow \omega \pi^+ \pi^- \) and \( J/\Psi \rightarrow \phi \pi^+ \pi^- \).

Following the procedure of ref. [34] we obtain

\[
t_{J/\Psi \phi K+K-} = -\frac{\tilde{g}}{3}(2\nu + 1) \epsilon^* \cdot \epsilon
\]

\[
t_{J/\Psi \phi \pi^+ \pi^-} = t_{J/\Psi \phi a^0 a^0} - \frac{2\tilde{g}}{3}(\nu - 1) \epsilon^* \cdot \epsilon
\]

with \( \nu = \sqrt{\lambda^2 + 2\lambda_0} \).

After the implementation of the rescattering of the final pseudoscalar pair, the amplitude for the direct production is given by

\[
t_{J/\Psi \rightarrow \phi \pi^0 \eta} = -\tilde{g} \epsilon \times \left[ \frac{1 + 2\nu}{3}(G_{K+K-}\epsilon_{K+K-\rightarrow \pi^0 \eta} + G_{K^0 K^0}\epsilon_{K^0 K^0\rightarrow \pi^0 \eta}) + \frac{\nu - 1}{3}(2G_{\pi+\pi-}\epsilon_{\pi+\pi-\rightarrow \pi^0 \eta} + 2G_{\pi^0 \pi^0}\epsilon_{\pi^0 \pi^0\rightarrow \pi^0 \eta}) + \frac{2}{9}(2 + \nu)G_{\eta \eta}\epsilon_{\eta \eta\rightarrow \pi^0 \eta} \right]
\]

\[ (3) \]

The mechanisms with sequential exchange of vectors and axial-vector mesons for the \( J/\Psi \rightarrow \phi \pi^0 \eta \) decay are shown in Fig. 3. For this particular decay the vector exchanged is a \( K^*(892) \) and the axial-vectors are the \( K_1(1270) \) and \( K_1(1400) \). We need the vertices \( J/\Psi \rightarrow \text{vector-pseudoscalar} \ (J/\Psi V P) \), \( J/\Psi \rightarrow \text{vector-axial} \ (J/\Psi A P) \) and \( J/\Psi \rightarrow \text{axial-axial} \ (J/\Psi A P) \)

1 In Eq. (3) the amplitudes involving \( \pi^0 \pi^0 \) or \( \eta \eta \) are in good normalization, not in the unitary normalization used in the Bethe-Salpeter equation, Eq. (4).
which we obtain from the Lagrangians of refs. [34, 38, 34] and [39] respectively. After implementing the final state interaction of the pseudoscalar pair the amplitudes for the sequential exchange of the $K^*$ meson is

$$ t^{\mu \nu} = G G t^{\mu \nu} (t_{K^+} + t_{K^0} K^0, \bar{p} \bar{p}) $$

(4)

where $G$ is the coupling in the $VVP$ Lagrangian, $G$ the coupling in the $J/\Psi VP$ one, and $t^{\mu \nu}$ contains the three meson loop function, $K^* K$, including the momentum structure of the different vertices. (See ref. [34] for details and values of the constant $G$.

For the $K_1(1270)$ axial-vector exchange we get

$$ t^{\mu \nu} = \frac{8}{M_{J/\Psi} m_0 m_{K_1(1270)}} cD(cD + sF) $$

$$ \times \left\{ (q_{J/\Psi} \cdot q_0 t^{\mu \nu} - q_0^\mu \cdot q_{J/\Psi} G_{KK} t^{\mu \nu} \right\} $$

$$ \times \left\{ (t_{K^+} K^-, \bar{p} \bar{p} + t_{K^0} K^0, \bar{p} \bar{p}) \right\} $$

(5)

where $D$ and $F$ are couplings in the $AVP$ Lagrangian, $D$ is the coupling of the $J/\Psi AP$, $s$ and $c$ parameterize the mixing between the isospin 1/2 members of the axial-vector octet to give the physical $K_1(1270)$ and $K_1(1400)$ states and $t^{\mu \nu}$ contains the three meson loop function, $K_1(1270) K K$, including the momentum structure of the different vertices [34]. For the diagrams with $K_1(1400)$ intermediate state the amplitude is the same but changing $m_{K_1(1270)} \rightarrow m_{K_1(1400)}$, $F \rightarrow -F$, $c \rightarrow s$ and $s \rightarrow c$ and replacing the masses and widths of the $K_1(1270)$ by those of the $K_1(1400)$ in the evaluation of $t^{\mu \nu}$.

**IV. RESULTS**

The main observable that we are going to evaluate is the differential decay width of the $J/\Psi$ with respect to the $\pi^0 \eta$ invariant mass, $M_{\pi^0 \eta}$, which in the $J/\Psi$ rest frame can be evaluated as

$$ \frac{d\Gamma}{dM_{\pi^0 \eta}} = \frac{M_{\pi^0 \eta}}{64 \pi^3 m_{J/\Psi}^2} \int_{m_{\pi^0}}^{m_{J/\Psi}} dm_{\pi^0} d\omega_{\pi^0} \sum |t|^2 \Theta (1 - \cos \theta) $$

where $\Theta$ is the step function, $\cos \theta = ((m_{J/\Psi} - \omega_{\phi} - \omega_{\pi})^2 - m_\pi^2 - |q_{\phi}|^2 / 2)/((q_{\phi}^2 \omega_{\pi}))$, where $\theta$ is the angle between $p_{\pi^0}$ and $q_{\phi}$ and $\omega_{\pi} = \sqrt{q_{\pi}^2 + m_{\pi}^2}$. The final polarization sum and initial average is

$$ \sum |t|^2 = \frac{1}{3} \sum_{\mu \nu \mu' \nu'} \left( -g_{\mu \nu} + \frac{q_{\phi}^\mu q_{\phi}^{\mu'}}{m_{J/\Psi}^2} \right) \left( -g_{\nu' \nu'} + \frac{q_{\phi}^{\nu'} q_{\phi}^{\nu}}{m_{\phi}^2} \right) $$

(6)

To obtain the total scattering amplitude, $t = \epsilon_{\mu}^* \epsilon_{\nu} t^{\mu \nu}$, all the mechanisms explained in the previous section must be added.

**FIG. 4.** Different contributions to the $\pi^0 \eta$ mass distribution in the $J/\Psi \rightarrow \phi \pi^0 \eta$ decay. In the theoretical calculation we have added the experimental background, dotted line, as given in [31] to ease the comparison with the data.

**FIG. 5.** Same as Fig. 4 but zoomed around the peak region

In figs. 4 and 5 we show the contribution of the different mechanisms to the $\pi^0 \eta$ mass distribution, $\frac{d\Gamma}{dM_{\pi^0 \eta}}$. (Fig. 5 is just a zoom of Fig. 4 around the peak region). The experimental data is the result from BESIII [31]. The dotted line represents the background given in the experiment [31]. Since it is not subtracted in the original experimental data, we have added this background to our theoretical results shown in the figure for the sake of comparison with the BESIII data. The data provided in ref. [31] is given in number of events but, since ref. [31] also provides the total number of $J/\Psi$ produced and detector efficiencies, it is possible to obtain the absolute normalization for the observable shown in the figure. The solid line represents our full theoretical result. The dashed line represent the contribution of the direct $J/\Psi VP$ mechanism, fig. 2. The dashed-dotted...
in the final state. In our formalism it has no sense to strictly isolate the $f_0$ or $a_0$ contributions by themselves since the chiral unitary approach generates the full amplitude from where the definition of the resonance is a matter of convention. Thus, the meaningful observable is the ratio

$$\xi_{fa}(M) = \frac{d\Gamma_{J/\Psi \rightarrow \phi \pi^0 \eta}}{d\Gamma_{J/\Psi \rightarrow \phi \pi \pi}}$$

(9)

which close to the $f_0$ mass region should be very similar to Eq. (8). The calculation for the $J/\Psi \rightarrow \phi \pi \pi$ within our formalism was done in Ref. [34].

The result for the mixing intensity is shown in Fig. 7. The mixing intensity can also be obtained from other production processes, like e.g. radiative $\phi$ decays $\phi \rightarrow \pi^0 \eta \gamma$ and $\phi \rightarrow \eta \gamma$. In order to compare the mixing intensity with experimental data, in ref. [41] the mixing intensity at $M = 991.4$ MeV, which is the two kaon threshold for average kaon masses, was obtained for different experiments [42–47]. From Fig. 7 we obtain $\xi_{fa}(M = 991.4$ MeV) $= 0.020^{+0.003}_{-0.011}$ to be compared with the experimental results, quoted in [41], 0.088, 0.034, 0.019, 0.027 for SND [42, 43], KLOE [44, 45], BNL [46] and CB [47] respectively.

V. SUMMARY

We have studied theoretically the isospin violating $J/\Psi \rightarrow \phi \pi^0 \eta$ extending our previous model [34] on the $J/\Psi \rightarrow \phi PP$ decays. The main contribution of the model are the mechanisms containing the "direct" $J/\Psi VP$ vertex implementing also the final state interaction of the pseudoscalar pair. This final state interaction is
evaluated using the techniques of the chiral unitary approach, which implements unitarity in coupling channels with the only input of the lowest order pseudoscalar-pseudoscalar chiral Lagrangian. At the relevant energies of the present work, the scalar \( f_0(980) \) and \( f_0(980) \) appear naturally (dynamically) in the unitarized scattering amplitudes without the need to include them as explicit degrees of freedom. Due to the strong interference with the previous mechanisms, we find also very relevant for the final results the mechanisms where the \( J/\Psi \) decays into a vector and or axial-vector and a pseudoscalar meson and the vector subsequently decays into the final vector and the other final pseudoscalar, where the final state interaction is also implemented.

The isospin violation comes essentially from the difference between the masses of the charged and neutral kaons and pions in the loops and the meson-meson scattering amplitudes.

We find a very good agreement with the experimental BESIII results for the \( \pi^0 \eta \) mass distribution and branching ratio. It is worth noting that without the inclusion of the sequential exchange mechanisms we would have obtained results a factor about 2 larger.

The \( f_0 - a_0 \) mixing intensity, as defined in ref. \[41\], is also evaluated from the ratio between the \( J/\Psi \rightarrow \phi \pi \pi \) decays and we obtain a fair agreement with most of the experimental results evaluated in ref. \[41\].

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