Probabilistic resumable quantum teleportation in high dimensions

Xiang Chen,† Jin-Hua Zhang,∗† and Fu-Lin Zhang†

†Department of Physics, School of Science, Tianjin University, Tianjin 300072, China
∗School of Mathematical Sciences, Capital Normal University, Beijing 100048, China
‡Department of Physics, Xinzhou Teacher’s University, Xinzhou 034000, China

Teleportation is a quantum information processes without classical counterparts, in which the
sender can disembodied transfer unknown quantum states to the receiver. In probabilistic telepor-
tation through a partial entangled quantum channel, the transmission is exact (with fidelity 1), but
may fail in a probability and the initial state is destroyed simultaneously. We propose a scheme
for nondestructive probabilistic teleportation of high-dimensional quantum states. With the aid
of an ancilla in the hands of the sender, the initial quantum information can be recovered when
teleportation fails. The ancilla acts as a quantum apparatus to measure the sender’s subsystem.
Erasing the information recorded in it can resume the initial state.

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I. INTRODUCTION

Quantum teleportation is one of the processes unique
to the quantum information, which serves as an impor-
tant example for the most intriguing uses of entanglement
[1–4]. It has been widely studied both theoretically and
experimentally since it was put forwards [5–24]. One rea-
son it is gaining so much attraction is that, the original
scheme and its various extensions play key roles in several
contexts in quantum communication, including quantum
repeaters, quantum networks and cryptographic confer-
ces [25–29].

In the original and also the simplest version of telepor-
tation [2], Alice (the sender) can transfer an unknown
state from a qubit to Bob (the receiver), with fidelity 1
and successful probability 1, without physical transmis-
sion of the qubit itself. The key ingredient in the protocol
is a two-qubit Bell state shared by them as a quantum
channel. Alice makes a joint measurement on the two
qubits in her hands, projecting them onto one of the four
Bell states with equal probability. Through a classical
channel, Alice sends Bob two classical bits to inform him
of her outcome. According to the classical information,
Bob can perform appropriate unitary operations on his
qubit to rebuild Alice’s initial state, and thereby accomplis-
hes the teleportation.

A variant of the process called probabilistic telepor-
tation was proposed based upon the consideration of that
a quantum channel may be prepared in a partially en-
tangled pure state in practice [5–9]. Such teleportation
is exact (with fidelity 1), but may sometimes fail as the
the price to pay for the fidelity. In principle, Alice’s joint
measurement is destructive, and it is generally assumed
that the information encoded in the state to be tele-
ported is lost when the teleportation fails. In their recent
work [9], Roa and Groiseau presented a nondestructive
scheme for probabilistic teleportation by introducing an
ancillary qubit. This avoids losing the initial information
and offers the chance to repeat the teleportation process.
The nondestructive scheme has been extended in many
branches, including bidirectional teleportation [30], tele-
portation of an entangled state [31] and multihop sce-
nario [32, 33].

In this work, we present a general protocol for nonde-
structive probabilistic teleportation of high-dimensional
quantum states, in which Alice can resume her initial
state when the teleportation fails. The motivation for
this work is not only to extend the study of Roa and
Groiseau to high-dimensional systems, but also based on
the following considerations. In theory, high-dimensional
channels, especially the partially entangled ones, have
more rich entanglement properties. Our protocol pro-
vides a sample to study the cooperative relationship
among the entanglement, joint measurement and clas-
sical information in a quantum information process. In
practice, teleporting high-dimensional states is required
in the task to completely rebuild the quantum states of
a a real particle remotely. And, recent experiment pro-
gresses [16, 17] show the possibility of implementing our
protocol in optical systems. We make a remark here that, although Fu et al. [32, 33] give a version of non-
destructive probabilistic teleportation in high dimension
using an auxiliary particle in Bob’s hand, we maintain
the approach in [9] with an ancilla belonging to Alice.
In this case, only the sender, Alice, is required to have
the ability of bipartite operations, such as generation and
measurement of entangled states.

In addition, one can notice the similarity of our proto-
col and the quantum two-path interference experiments
[34]. In the latter, interference fringes vanish when the
which-path information is acquired by a detector, while
reappear when the detector returns the information to
the particle. The ancilla in our protocol can be regarded
as a quantum apparatus to measure Alice’s system. The
initial state is recovered by erasing the information it
records after an unambiguous quantum state discrimina-

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†Corresponding author: flzhang@tju.edu.cn
II. TELEPORTATION OF A QUTRIT

Let us start with the teleportation of a qutrit (a three-level quantum system). Suppose Alice wishes to teleport to Bob an arbitrary qutrit state as

$$|\phi\rangle = \alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle,$$

with $|\alpha_0|^2 + |\alpha_1|^2 + |\alpha_2|^2 = 1$. The participants share a two-qutrit entangled state as the quantum channel,

$$|\Phi\rangle_{23} = b_0 |00\rangle_{23} + b_1 |11\rangle_{23} + b_2 |22\rangle_{23},$$

with $|b_0|^2 + |b_1|^2 + |b_2|^2 = 1$. Without loss of generality, we assume that the Schmidt coefficients are non-negative real numbers and $b_0 \leq b_1 \leq b_2$. Here, the three qutrits are identified by the subscripts 1, 2 and 3 respectively, and qutrits 1 and 2 belong to Alice while qutrit 3 is in the hands of Bob. The total state of the three-qutrit system can be written as,

$$|\Psi\rangle_{123} = |\phi\rangle_1 \otimes |\Phi\rangle_{23}$$

$$= \frac{1}{3} \left[ |\psi_{00}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{10}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{20}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{01}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{11}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{21}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{02}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{12}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 + |\psi_{22}\rangle_{12} (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle)_3 \right],$$

where $\omega = e^{\frac{2\pi i}{3}}$ is the triple root, and $|\psi_{nm}\rangle_{12}$ are nine linearly independent two-qutrit states, equivalent to the channel (2) under local unitary transformations, as follows,

$$|\psi_{00}\rangle_{12} = (b_0 |00\rangle + b_1 |11\rangle + b_2 |22\rangle)_{12},$$
$$|\psi_{10}\rangle_{12} = (b_0 |00\rangle + b_1 |11\rangle + b_2 |22\rangle)_{12},$$
$$|\psi_{20}\rangle_{12} = (b_0 |00\rangle + b_1 |11\rangle + b_2 |22\rangle)_{12},$$
$$|\psi_{01}\rangle_{12} = (b_0 |10\rangle + b_1 |21\rangle + b_2 |02\rangle)_{12},$$
$$|\psi_{11}\rangle_{12} = (b_0 |10\rangle + b_1 |21\rangle + b_2 |02\rangle)_{12},$$
$$|\psi_{21}\rangle_{12} = (b_0 |10\rangle + b_1 |21\rangle + b_2 |02\rangle)_{12},$$
$$|\psi_{02}\rangle_{12} = (b_0 |20\rangle + b_1 |01\rangle + b_2 |12\rangle)_{12},$$
$$|\psi_{12}\rangle_{12} = (b_0 |20\rangle + b_1 |01\rangle + b_2 |12\rangle)_{12},$$
$$|\psi_{22}\rangle_{12} = (b_0 |20\rangle + b_1 |01\rangle + b_2 |12\rangle)_{12}.$$

The vector of qutrit 3 in each term (each line) of the total state (3) is equivalent to the initial state (1) under a unitary transformation. However, the corresponding states of qutrits 1 and 2 $|\psi_{nm}\rangle_{12}$ are not orthogonal to each other. To teleportate the initial state exactly, an option is that, Alice performs an unambiguous quantum state discrimination process [6, 7, 12, 13], to distinguish the nine states with no error but a probability of failures. As in the standard teleportation through a maximal entangled quantum channel [16, 17], Bob can rebuild the initial state on his qutrit by performing appropriate unitary operations according to Alice’s outcome. In another scheme, Alice still performs a joint measurement in nine maximally entangled basis, while Bob needs an extracting quantum state process [5]. Here, we follow the former scheme, which only requires Alice’s ability to manipulate two or more particles.

For the sake of brevity, before putting forward our final total state in this section, we only show the states in the hands of Alice. We show the sequential operations of Alice in Fig. 1. To discriminate the nine two-qutrit states (4), Alice factorizes the states by applying a generalized controlled-NOT (GCNOT) gate onto her bipartite system and obtain

$$C_{21} |\psi_{nm}\rangle_{12} = |m\rangle_1 (b_0 |0\rangle + b_1 \omega^n |1\rangle + b_2 \omega^{2n} |2\rangle)_2,$$

with $m, n = 0, 1, 2$. Here, we define the GCNOT gate $C_{ij}$ acting on qutrits $i$ and $j$ as

$$C_{ij} = |0\rangle_i \otimes 1_j + |1\rangle_i \otimes \mathbb{V}_j + |2\rangle_i \otimes \mathbb{V}_j,$$

where $1_j$ is the identity of $j$ and $\mathbb{V}_j = |0\rangle_j \langle 1| + |1\rangle_j \langle 2| + |2\rangle_j \langle 0|$. It shifts the target $j$ clockwise or anticlockwise when the control qutrit $i$ is a $|1\rangle$ or $|2\rangle$.

To distinct the nine states in (5), Alice can perform a von Neumann measurement on qutrit 1 followed by an unambiguous quantum state discrimination on qutrit 2. However, these operations destroy the initial state of qutrit 1 even though the discrimination fails. Here, following the protocol of Roa and Groiseau [9], we introduce an extra auxiliary qutrit 0 which acts as a quantum apparatus to measure qutrit 1. The key point is that, when we erase the information of qutrit 1 recorded in the ancilla, the initial state of qutrit 1 recovers when discrimination fails. Alice applies the inverse of the GC-
NOT gate on the ancilla initial in $|0\rangle_0$ and qutrit 1, and obtain the nine states in her hands as
\[ C_{10}^t |0\rangle_0 |m_z\rangle_1 |\tau_n\rangle_2 = |m_z\rangle_0 |m_z\rangle_1 |\tau_n\rangle_2, \] (7)
where $|\tau_n\rangle_2 = b_0 |0\rangle_1 + b_1 \omega^n |1\rangle_1 + b_2 \omega^{2n} |2\rangle_1$ and $m, n = 0, 1, 2$. Then, qutrit 1 serves as an ancilla in the unambiguous quantum state discrimination of qutrit 2. Let us define two unitary transformations on qutrit 2 to be
\[ U_1 = \frac{b_0}{b_1} |1\rangle_1 + \sqrt{1 - \left(\frac{b_0}{b_1}\right)^2} \mathbb{I}_1, \]
\[ W_1 = \frac{b_0}{b_2} |1\rangle_1 + \sqrt{1 - \left(\frac{b_0}{b_2}\right)^2} \mathbb{I}_1, \] (8)
and a controlled-unitary operation
\[ D_{21} = |0\rangle_2 (0 \otimes \mathbb{I}_1 + |1\rangle_2 (1 \otimes U_1 + |2\rangle_2 (2 \otimes W_1). \] (9)
Alice performs it on qutrits 1 and 2 and obtains
\[ D_{21} |m\rangle_0 |m_z\rangle_1 |\tau_n\rangle_2 = |m\rangle_0 \left[ \sqrt{3} b_0 |m_z\rangle_1 |\kappa_n\rangle_2 \ight. \]
\[ + |m \oplus_3 2\rangle_1 \left( \sqrt{b_1^2 - b_0^2 \omega^n} |1\rangle_2 + \sqrt{b_2^2 - b_0^2 \omega^{2n}} |2\rangle_2 \right), \] (10)
where $\oplus_3$ denotes modulo 3 addition, and $|\kappa_n\rangle = \frac{1}{\sqrt{3}} (|0\rangle_2 + \omega^n |1\rangle_2 + \omega^{2n} |2\rangle_2)$. In the above form, the nine states $|m\rangle_0 |m_z\rangle_1 |\kappa_n\rangle_2$ corresponding to successful discrimination are orthogonal to each other, and orthogonal to the states with qutrits 0 and 1 in $|m\rangle_0 |m \oplus_3 2\rangle_1$. The latter nine are for the failure of discrimination as they are not linearly independent.

The final step of Alice’s unitary operations is to erase the information measured by qutrit 0. Applying the G-C-NOT gate $C_{10}$, Alice can obtain
\[ C_{10} |m\rangle_0 |m_z\rangle_1 = |0\rangle_0 |m_z\rangle_1, \]
\[ C_{10} |m\rangle_0 |m \oplus_3 2\rangle_1 = |1\rangle_0 |m \oplus_3 2\rangle_1. \] (11)
The information recorded on the apparatus qutrit 0 can be erased partially, as it returns $|0\rangle_0$ in the terms corresponding to successful discrimination but becomes $|1\rangle_0$ for the case of failure. This divides the total four-qutrit state into two parts as
\[ |\Delta\rangle = \frac{b_0}{\sqrt{3}} |0\rangle_0 \left[ |0\rangle_1 (|\phi_0\rangle |\phi_00\rangle + |\phi_1\rangle |\phi_10\rangle + |\phi_2\rangle |\phi_20\rangle)_{23} \right. \]
\[ + |1\rangle_1 (|\phi_0\rangle |\phi_01\rangle + |\phi_1\rangle |\phi_11\rangle + |\phi_2\rangle |\phi_21\rangle)_{23} \]
\[ + |2\rangle_1 (|\phi_0\rangle |\phi_02\rangle + |\phi_1\rangle |\phi_12\rangle + |\phi_2\rangle |\phi_22\rangle)_{23} \]
\[ + |1\rangle_0 \left[ \sqrt{b_1^2 - b_0^2} |\phi_0\rangle |1\rangle_1 + \sqrt{b_2^2 - b_0^2} |\phi_0\rangle |2\rangle_1 \right] \]
\[ \left. \right| \square_2. \] (12)
Here $|\phi_{nm}\rangle$ are the states of qutrit 3 multiplied to $|\psi_{nm}\rangle_{12}$ in (3) as
\[ |\phi_0\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle), \]
\[ |\phi_1\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle), \]
\[ |\phi_2\rangle = (\alpha_0 |0\rangle + \alpha_1 |1\rangle + \alpha_2 |2\rangle), \]
\[ |\phi_01\rangle = (\alpha_1 |0\rangle + \alpha_2 |1\rangle + \alpha_0 |2\rangle), \]
\[ |\phi_11\rangle = (\alpha_1 |0\rangle + \alpha_2 |1\rangle + \alpha_0 |2\rangle), \]
\[ |\phi_21\rangle = (\alpha_1 |0\rangle + \alpha_2 |1\rangle + \alpha_0 |2\rangle), \]
\[ |\phi_02\rangle = (\alpha_1 |0\rangle + \alpha_2 |1\rangle + \alpha_0 |2\rangle), \]
\[ |\phi_12\rangle = (\alpha_1 |0\rangle + \alpha_2 |1\rangle + \alpha_0 |2\rangle), \]
\[ |\phi_22\rangle = (\alpha_1 |0\rangle + \alpha_2 |1\rangle + \alpha_0 |2\rangle). \]

Then, the state to be teleported is encoded in qutrit 3 in the first part of $|\Delta\rangle$, but in qutrit 1 in the second part.

Alice performs a von Neumann measurements on qutrit 0 in the standard basis. It projects qutrit 0 to $|0\rangle_0$ in a probability $2 |b_0|^2$. Then, she measures the nine orthogonal direct product states $|0\rangle_0 |m_z\rangle_1$, and informs Bob to perform appropriate unitary operations on his qutrit 3 to rebuild the initial state, and thereby the teleportation succeeds. On the contrary, the qutrit 0 could be projected to $|1\rangle_0$ in a probability $1 - 3 |b_0|^2$, which means the failure of the teleportation. Alice can recover the state $|\phi\rangle_1$ in qutrit 1, by performing a joint unitary transformation $E_{21} = \mathbb{I}_1 \otimes (|0\rangle_2 (0 + |2\rangle_2 (2) \otimes |1\rangle_2) (1)$ on qutrits 1 and 2. Alternatively, Alice could perform a von Neumann measurements on qutrit 2 in the standard basis. She obtains the initial state $|\phi\rangle_1$, by using the operation $V_1$ on qutrit 1 when qutrit 2 was projected to $|2\rangle_2$, while $\mathbb{I}_1$ corresponding to $|0\rangle_2$ and $|1\rangle_2$.

### III. General Protocol

Now we turn to the general protocol for teleporting a quNit (a $N$-level quantum system) through a partially entangled two-quNit quantum channel. Since it is direct to extend the above scheme to $N$-level systems, we present the following results in compact general formulae in this part. Let an initial quNit state to be
\[ |\phi\rangle_1 = \sum_{i=0}^{N-1} \alpha_i |i\rangle_1, \] (14)
where $\alpha_i = 0, \ldots, N-1$ are complex numbers satisfying the normalization condition $\sum_{i=0}^{N-1} |\alpha_i|^2 = 1$. Alice and Bob share a two-quNit entangled state as the quantum channel, which can be generally written in the form of the Schmidt disposition as
\[ |\Phi\rangle_{23} = \sum_{j=0}^{N-1} b_j |j\rangle_2 |j\rangle_3, \] (15)
with the real coefficient $0 \leq b_0 \leq b_1 \leq \ldots \leq b_{N-1}$ and \( \sum_{j=0}^{N-1} b_j^2 = 1 \). The total state of the tripartite system is given by
\[
|\Psi\rangle_{123} = |\phi\rangle_1 |\Phi\rangle_{23}
\]
\[
= \frac{1}{N} \sum_{m=0, n=0}^{N-1} |\psi_{nm}\rangle_{12} \sum_{f=0}^{N-1} \alpha_f e^{-i\phi f} |f\rangle_3,
\]
where \( \oplus \) denotes modulo $N$ addition, and $|\psi_{nm}\rangle_{12}$ are $N^2$ linearly independent bipartite states
\[
|\psi_{nm}\rangle_{12} = \sum_{k=0}^{N-1} b_k e^{i\phi_k/N} |k \oplus m\rangle_1 |k\rangle_2.
\]
In the above form in Eq. (16), the states multiplied to $|\psi_{nm}\rangle_{12}$ are equivalent to the state (14) on quNit 3 under unitary transformations as
\[
\sum_{f=0}^{N-1} \alpha_f e^{-i\phi_f/N} |f\rangle_3 = U_3^{(n,m)} |\phi\rangle_3
\]
(18)
with
\[
U_3^{(n,m)} = \sum_{f=0}^{N-1} e^{-i\phi_f/N} |f\rangle_3 \langle f \oplus m|,
\]
and the subscript $j$ denoting the $j$th subsystem.

Alice’s first two operations are to disentangle subsystems 1 and 2 in the states $|\psi_{nm}\rangle_{12}$, and to measure 2 by using an auxiliary apparatus, quNit 0 initial in $|0\rangle_0$. Here, we define the $N$-level GCNOT gate as
\[
C_{ij} = \sum_{y=0}^{N-1} |y\rangle_i \langle y| \otimes U_j^{(0,y)}.
\]
(20)
The four-partite state becomes
\[
|\Omega\rangle = C_{10}^\dagger \left( |0\rangle_0 C_{21} |\Psi\rangle_{123} \right)
\]
\[
= \frac{1}{N} \sum_{m=0, n=0}^{N-1} |mm\rangle_0 \sum_{j=0}^{N-1} b_j e^{i\phi_j/N} |j\rangle_2 \sum_{f=0}^{N-1} \alpha_f e^{-i\phi_f/N} |f\rangle_3.
\]
To unambiguously discriminate the states of particle 2, Alice applies a joint unitary transformation on 1 and 2
\[
D_{21} = \sum_{y=0}^{N-1} |y\rangle_2 \langle y| \otimes \left( b_0 \not{y} + \sqrt{1 - \left( \frac{b_0}{b_y} \right)^2} U_1^{(0,1)} \right),
\]
obtains the whole system in the state
\[
|\Gamma\rangle = D_{21} |\Omega\rangle
\]
\[
= \frac{b_0}{N} \sum_{m=0, n=0}^{N-1} |mm\rangle_0 \left( \sum_{f=0}^{N-1} \alpha_f e^{-i\phi_f/N} |f\rangle_3 \right) + \sum_{m=0, j=1}^{N-1} \sqrt{b_j^2 - b_0^2} |m\rangle_0 \alpha_j \alpha_{j+m} |m \oplus N-1\rangle_1 |jj\rangle_23.
\]
When Alice erases the information in the auxiliary apparatus, 0, the changes of the quNit 1 are recorded as $|1\rangle_0$, and the total state becomes
\[
|\Delta\rangle = C_{10} |\Gamma\rangle
\]
\[
= |0\rangle_0 \frac{b_0}{N} \sum_{m=0, n=0}^{N-1} |m\rangle_1 \left( \sum_{j=0}^{N-1} e^{-i\phi_{jn}} |j\rangle_2 \right) U_3^{(n,m)} |\phi\rangle_3 + |1\rangle_0 \sum_{j=1}^{N-1} \sqrt{b_j^2 - b_0^2} U_1^{(0,j \oplus 1)} |\phi\rangle_1 |jj\rangle_23.
\]
Obviously, it is divided into two parts of success and failure, which can be collapsed by Alice’s measurements in the standard basis. When Alice’s outcome $|0\rangle_0$ occurs in the probability $N b_0^2$, the teleportation can be accomplished by two local von Neumann measurement on qNits 1 and 2. One the other hand, Alice can recover the initial state by a joint unitary operation, $B_{21} = \sum_{j=0}^{N-1} U_1^{(0,j \oplus 1)} \otimes |j\rangle_2 |j\rangle$, on her two systems, when the task of teleportation fails in the probability $1 - N b_0^2$.

### IV. SUMMARY

We present a scheme for nondestructive probabilistic teleportation of high-dimensional quantum states. A partial entangled pure state serves as the quantum channel, whose smallest coefficient determines the successful probability of exactly teleporting a state. With the aid of an auxiliary particle, Alice can recover her initial state to be teleported when teleportation fails. Compared to the existing results [32, 33], our protocol only requires the sender, Alice, to have the ability to perform bipartite operations, while the dimension of the ancilla needs to be the same as the state to be teleported. In addition, the ancilla acts as a quantum apparatus to measure Alice’s system. The process of resuming the initial state can be regarded as erasing information recorded in the ancilla.

As the following research, it is a fundamental problem to explore the roles of quantum correlations in our four-party procedure, which is a fundamental problem in quantum information. In addition, the relation between our protocol and the theory of extracting information from a quantum system by multiple observers [35] would be interesting, since the quantum correlations in the lat-
ter are studied in many works [36–38]. While we focus here on the teleportation using quantum channels with the same dimension as the state to be teleported, it is a natural extension to apply the present ideas in the case with different dimension [14]. And finally, we hope that the process can be implemented in laboratories with the help of the techniques recently developed in optical systems [16, 17].

[1] Nielsen M A and Chuang I L 2000 Quantum Computation and Quantum Information (Cambridge:Cambridge University Press) pp.571-582
[2] Bennet C H, Brassard G, Crépeau C, Jozsa R, Peres A and Wootters W K 1993 Phys. Rev. Lett. 70 1895
[3] Brunner N, Gisin N and Scarani V 2005 New J. Phys. 7 88
[4] Horodecki R, Horodecki P, Horodecki M and Horodecki K 2009 Rev. Mod. Phys. 81 865
[5] Li W L, Li C F and Guo G C 2000 Phys. Rev. A 61 034301
[6] Banaszek K 2000 Phys. Rev. A 62 024301
[7] Rosa L, Delgado A and Fuentes-Guridi I 2003 Phys. Rev. A 68 022340
[8] Verstraete F and Verschelde H 2003 Phys. Rev. Lett. 90 097901
[9] Rosa L and Groisseau C 2015 Phys. Rev. A 91 012344
[10] Karlsson A and Bourennane M 1998 Phys. Rev. A 58 4394
[11] Li X H and Ghoe S 2014 Phys. Rev. A 90 052305
[12] Zhang F L, Chen J L, Kwek L C and Vedral V 2013 Sci.Rep. 3 2134
[13] Zhang F L and Wang T 2017 Europhys. Lett. 117 10013
[14] Chen X, Shen Y and Zhang F L 2020 preprint arXiv:2101.06693
[15] Huang Y and Yang W 2020 Chinese Journal of Electronics 29 228
[16] Luo Y H, Zhong H S, Erhard M, Wang X L, Peng L C, Krenn M, Jiang X, Li L, Liu N L, Lu C Y, Zeilinger A and Pan J W 2019 Phys. Rev. Lett. 123 070505
[17] Hu X M, Zhang C, Liu B H, Cai Y, Ye X J, Guo Y, Xing W B, Huang C X, Huang Y F, Li C F and Guo G C 2020 Phys. Rev. Lett. 125 230501
[18] Bouwmeester D, Pan J W, Mattle K, Eibl M, Weinfurter H and Zeilinger A 1997 Nature 390 575
[19] Boschi D, Branca S, Martini F D, Hardy L and Popescu S 1998 Phys. Rev. Lett. 80 1121
[20] Furusawa A, Sorensen J L, Brannstein S L, Fuchs C A, Kimble H L and Polzik E S 1998 Science 282 706
[21] Gottesman D and Chuang I L 1999 Nature 402 390
[22] Nolleke C, Neuzner A, Reiserer A, Hahn C, Rempe G and Ritter S 2013 Phys. Rev. Lett. 110 140403
[23] Pfaff W, Hensen B, Bernien H, van Dam S B, Blok M S, Taminiau T H, Tiggelman M J, Schouten R N, Markham M, Twitchen D J and Hanson R 2014 Science 345 532
[24] Torres J M, Bernard J Z and Alber G 2014 Phys. Rev. A 90 012304
[25] Sangouard N, Simon C, de Riedmatten H and Gisin N 2011 Rev. Mod. Phys. 83 33
[26] Biham E, Huttner B and Mor T 1996 Phys. Rev. A 54 2651
[27] Bose S, Vedral V and Knight P L 1998 Phys. Rev. A 57 822
[28] Townsend P 1997 Nature 385 47
[29] Aoun B and Tariﬁ M 2004 arXiv: quantCph/0401076
[30] Gou Y T, Shi H L, Wang X H and Liu S Y 2017 Quantum Inf. Process. 16 278
[31] Wang Z Y, Gou Y T, Hou J X, Cao L K and Wang X H 2019 Entropy. 21 352
[32] Fu F and Jiang M 2020 J. Opt. Soc. Am. B 37 233
[33] Fu F, Li H, Xie S and Jiang M 2020 J. Opt. Soc. Am. B 37 1896
[34] Jacques V, Wu E, Grosshans F, Treussart F, Grangier P, Aspect A and Roh Ch J-F 2008 Phys. Rev. Lett. 100 220402
[35] Bergou J, Feldman E and Hillery M 2013 Phys. Rev. Lett. 111 105001
[36] Pang C Q, Zhang F L, Xu L F, Liang M L and Chen J L 2013 Phys. Rev. A 88 052331
[37] Zhang J H, Zhang F L and Liang M L 2018 Quantum Inf. Process. 17 260
[38] Zhang J H, Zhang F L, Wang Z X, Lai J M and Fei S M 2020 Phys. Rev. A 101 032316