Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields

Alessandro Mirizzi ‡
Max-Planck-Institut für Physik (Werner Heisenberg Institut)
Föhringer Ring 6,
80805 München, Germany

Daniele Montanino
Dipartimento di Fisica, Università del Salento and Sezione INFN di Lecce
Via Arnesano,
I–73100 Lecce, Italy

Abstract. Very-high energy photons emitted by distant cosmic sources are absorbed on the extragalactic background light (EBL) during their propagation. This effect can be characterized in terms of a photon transfer function at Earth. The presence of extragalactic magnetic fields could also induce conversions between very high-energy photons and hypothetical axion-like particles (ALPs). The turbulent structure of the extragalactic magnetic fields would produce a stochastic behaviour in these conversions, leading to a statistical distribution of the photon transfer functions for the different realizations of the random magnetic fields. To characterize this effect, we derive new equations to calculate the mean and the variance of this distribution. We find that, in presence of ALP conversions, the photon transfer functions on different lines of sight could have relevant deviations with respect to the mean value, producing both an enhancement or a suppression in the observable photon flux with respect to the expectations with only absorption. As a consequence, the most striking signature of the mixing with ALPs would be a reconstructed EBL density from TeV photon observations which appears to vary over different directions of the sky: consistent with standard expectations in some regions, but inconsistent in others.

Keywords: axions, very high-energy gamma-rays.

‡ Current address: II. Institut für theoretische Physik, Universität Hamburg, Luruper Chausse 149, 22761 Hamburg, Germany.
1. Introduction

Axion-like particles (ALPs) with a two-photon vertex are predicted in many extensions of the Standard Model [1–3]. The \( a\gamma\gamma \) coupling allows for ALP-photon conversions in electric or magnetic field. This effect is exploited by the ADMX experiment to search for axion dark matter [4], by CAST to search for solar axions [5–7], and by regeneration laser experiments [8–12].

ALPs also play an intriguing role in astrophysics. Indeed, photons emitted by distant sources and propagating through cosmic magnetic fields can oscillate into ALPs. The consequences of this effect have been studied in different situations [13–23]. In particular, in the last recent years photon-ALP conversions have been proposed as a mechanism to avoid the opacity of the extragalactic sky to high-energy radiation due to pair production on the Extragalactic Background Light (EBL). At this regard, recent observations of cosmologically distant gamma-ray sources by ground-based gamma-ray telescopes have revealed a surprising degree of transparency of the universe to very high-energy (VHE) photons \( (E \gtrsim 100 \text{ GeV}) \) [24, 25]. Surprisingly, data seem to require a lower density of the EBL than expected and/or considerably harder injection spectra than initially thought [26, 27]. Oscillations between very high-energy photons and ALPs could represent an intriguing possibility to explain this puzzle through a sort of “cosmic light-shining through wall” effect. In fact, if VHE photons are converted into ALPS and then regenerated, they should not suffer absorption effects while they propagate as ALPs. In this sense, two complementary mechanisms have been proposed: a) VHE photon-ALP conversions in the magnetic fields around gamma-ray sources [20, 28] and then further back-conversions in the magnetic field of the Milky Way [29] (see also [30]); b) oscillations of VHE photons into ALPs in the random extragalactic magnetic fields [31, 32]. In principle, both the mechanisms can be combined together, as shown in [33]. Currently, the inference of EBL from VHE photons emitted by sources at high redshift [34] is still object of debate, and it is not clear how robust are the conclusions on the absorption effects obtained from the recent gamma data [35]. In this sense, it is also possible that the observed transparency of the universe to VHE photons could be explained without the need of introducing nonstandard mechanisms (see, e.g., [36, 37]). Nevertheless, the seminal works mentioned before have pointed out the nice connection between VHE gamma-astronomy and ALP searches. Therefore, it seems worthwhile to further explore the consequences of this exciting possibility.

In this context, the treatment of the oscillations of VHE photons into ALPs in the extragalactic medium in presence of the absorption on the EBL presents a certain degree of complexity. Extragalactic magnetic fields are supposed to have a turbulent structure which can dramatically affect the development of photon-ALP conversions. Brute force numerical simulations get the solution of the mixing equations along a given photon line of sight by iterating the equations in each domain in which the magnetic field is assumed constant [14, 31]. Since one cannot know the given configuration of magnetic domains crossed by VHE photons during their propagation, in the previous
literature VHE photon-ALP conversions were usually characterized in terms of the mean conversion probability, obtained averaging the resulting conversion probabilities over an ensemble of magnetic field configurations along the photon line of sight. Such a procedure slows down the solution of our problem, since to obtain stable results typically the average has to be performed over more than $10^3$ realizations of the magnetic fields [31]. Moreover, the use of the mean probability as representative value does not appear a priori completely justified, since the variance of the probability distribution could produce relevant deviations from the mean value for conversions occurring in different realizations of the random magnetic fields.

In order to overcome these previous limitations, in this paper we perform a new study of the mixing equations of VHE photons in the turbulent magnetic fields and we provide a user-friendly calculation of the mean and of the variance for the distribution of the photon transfer functions. The plan of our work is as follows. In Section 2 we discuss about the absorption of VHE photons on the extragalactic background light. In Section 3 we characterize VHE photon-ALP mixing in presence of absorption and we present our calculation of the mean photon transfer function averaged over the ensemble of all the possible realizations of random magnetic field configurations. We also show how to calculate the variance of the statistical distribution of the transfer functions. In Section 4 we present our results for the photon transfer function of VHE gamma-rays emitted from distant sources with and without the mixing with ALPs. Contrarily to previous predictions, the presence of a broad variance in the statistical distribution of the photon transfer functions could produce both an enhancement or a suppression of the observed VHE photon flux with respect to the case with only absorption. The resulting photon flux would depend on the particular random magnetic field configuration along the photon line of sight. As a consequence, photon-ALP mixing can not provide an universal mechanism to obtain the transparency of the universe to VHE radiation, but instead they would produce a strong direction-dependent behaviour in the flux of VHE photons from distant sources. In Section 5 we discuss about possible developments of our study and we conclude. There follow two Appendices, in which we present some details for the derivation of the mean photon transfer function (Appendix A) and for the variance of the distribution (Appendix B).

2. Absorption of very high-energy photons on extragalactic background light

The flux of very high-energy (VHE) gamma rays ($E \gtrsim 100$ GeV) from distant sources is attenuated in an energy dependent way by the interaction with background photons in the universe. The main source of absorption for VHE photons is due to the pair production process $\gamma_{\text{VHE}} \gamma_{\text{bkg}} \rightarrow e^+e^-$. In the energy range $100$ GeV $\lesssim E \lesssim 10$ TeV, the absorption is dominated by the interaction with optical/infrared photons of the so called Extragalactic Background Light (EBL), sometimes also referred as Metagalactic Radiation Field (MRF). The absorption rate for such a process in function of the incident
photon energy $E$ is given by [38]

$$\Gamma_{\gamma}(E) = \int_{m_{\gamma}^2/E}^{\infty} d\epsilon \frac{d\eta_{bkg}^{\gamma\epsilon}}{d\epsilon} \int_{-1}^{1} d\xi \frac{1 - \xi}{2} \sigma_{\gamma\gamma}(\beta),$$  

(1)

where the limits of integration in both integrals are determined by the kinematical threshold of the process and

$$\sigma_{\gamma\gamma}(\beta) = \sigma_0 (1 - \beta^2) \left[ 2\beta(\beta^2 - 2) + (3 - \beta^4) \log \frac{1 + \beta}{1 - \beta} \right],$$  

(2)

with $\sigma_0 = 1.25 \times 10^{-25}$ cm$^2$, is the cross section for the pair production process [39], in function of the electron velocity in the center of mass of the interaction $\beta = [1 - 2m_e^2/E\epsilon(1 - \xi)]^{1/2}$, being $\epsilon$ the background photon energy, and $\xi$ the cosine of the angle between the incident and the background photon. For practical purposes, it can be useful to notice that the inner integral in $d\xi$ in Eq. (1) can be evaluated by performing the change of variable $\xi \equiv \xi(\beta)$ and has actually an analytic closed form:

$$4\sigma_0 (1 - \beta_m^2)^2 \left[ \text{Li}_2 \left( \frac{1 - \beta_m^2}{2} \right) - \text{Li}_2 \left( \frac{1 + \beta_m^2}{2} \right) - \frac{\beta_m(1 + \beta_m^2)}{1 - \beta_m^2} \right. 
\left. + \frac{1}{2} \left( \frac{1 + \beta_m^4}{1 - \beta_m^2} - \log \frac{1 - \beta_m^2}{4} \right) \log \frac{1 + \beta_m^2}{1 - \beta_m^2} \right],$$  

(3)

where $\beta_m = (1 - m_e^2/E\epsilon)^{1/2}$ is the maximum electron velocity in the center of mass of the interaction and the function $\text{Li}_2$ is the polylogarithm of order two.

For simple estimations, the background photon spectrum in Eq. (1) can be approximated, at redshift $z = 0$, with a power–law [40]

$$\frac{d\eta_{bkg}^{\gamma\epsilon}}{d\epsilon} = 10^{-3} k \left( \frac{\epsilon}{\text{eV}} \right)^{-2.55} \text{eV}^{-1} \text{cm}^{-3},$$  

(4)

with $0.61 \leq k \leq 1.52$, depending on the model used (see Fig. 1). With this approximation one has

$$\frac{\Gamma_{\gamma}(E)}{\text{Mpc}^{-1}} \approx 1.1 \times 10^{-3} k \left( \frac{E}{\text{TeV}} \right)^{1.55}. $$  

(5)

In the literature are present different realistic models for the photon background. In particular, in the following we will refer to the 2008 Minimal Kneiske Model [41], which provides a strict lower-limit flux for the extragalactic background light from ultraviolet to the far-infrared photon energies. The model parameters are chosen to fit the lower limit data from galaxy number count observations, assuming that the shape and the normalization of the background radiation does not change except for the red-shifting between the source and the observer. This model has the advantage that it is not inferred by an inversion of the VHE photon observed spectra, as e.g. in [25]. We note that this latter procedure would be not reliable in the presence of VHE photon-ALP conversions, since in this case the effects of mixing and absorption on EBL would be entangled, preventing the possibility to extract information on the EBL from VHE photon measured spectra. Our reference model gives us the maximal possible transparency compatible
Figure 1. Spectrum of the EBL relevant for the absorption of VHE photons. The dashed power-law lines correspond to the simplified model of Eq. (4) for the limiting cases of $k = 0.61$ (lower line) and $k = 1.52$ (upper line) respectively. The solid line corresponds to the 2008 Minimal Kneiske Model at redshift $z = 0$. (see the text for details)

with the standard expectations, so that an evidence of a greater transparency would have to be attributed to nonstandard effects in the photon propagation.

The spectral energy distribution at a given redshift $z$ can be inferred by the tabulated power spectrum [41] $P(\lambda, z) = \lambda I_\lambda(\lambda, z)$ (where $I(\lambda, z)$ is the flux at redshift $z$ of energy for unit of solid angle between $\lambda$ and $\lambda + d\lambda$ where $\lambda$ is the comoving wavelength) by

$$\frac{dn^{\text{bkg}}}{d\lambda} = 2P(\lambda(1+z))(1+z)^3.$$  \hspace{1cm} (6)

The corresponding EBL energy spectrum is represented in Fig. 1. From this Figure we see that the realistic Kneiske model has an approximate power-law trend at photon background energies $\epsilon \gtrsim \text{few} \times 10^{-1}$ eV, which are relevant for the absorption of TeV photons. This will be useful to explain the high-energy behaviour of the transfer function.
The absorption function $\Gamma_{\gamma}$ as function of the photon energy on Earth $E$ and of the redshift $z$ is given by Eq. (1) simply changing $E \rightarrow E(1+z)$ in the integrand and in the limit of integration.

Finally, for a given source at distance $L$, the photon spectrum observed on Earth (apart from the geometrical dilution) is given by

$$I_{\text{obs}}(E) = T_{\gamma}(E, L) \cdot I_{\text{source}}(E_0) = \exp \left( -\tau_{\gamma} \right) I_{\text{source}}(E_0),$$

where the initial source spectrum $I_{\text{source}}(E_0)$, with initial photon energy $E_0 = E(1+z)$, is modified due to the effect of the VHE photon absorption, through the transfer function $T_{\gamma}$, expressed in terms of the optical depth

$$\tau_{\gamma} = \int_0^L dx \, \Gamma_{\gamma}(E, x) = \frac{c}{H_0} \int_0^{z_0} \frac{dz}{(1+z)\sqrt{\Omega_\Lambda + \Omega_m(1+z)^3}} \Gamma_{\gamma}(E, z),$$

dependent on the evolution of the Universe through the Hubble constant $H_0 = 73 \text{ km Mpc}^{-1} \text{ s}^{-1}$, the matter density $\Omega_m = 0.24$ [42], and the dark energy density $\Omega_\Lambda = 1 - \Omega_m$ (assuming a flat cosmology). We observe that in absence of photon-ALP conversions the transfer function $T_{\gamma}$ drops exponentially at high energies. We will see that the effect of VHE photon-ALP conversion is to soften this behaviour.

3. Very high-energy photons mixing with axion-like particles

3.1. Equations of motion

The effect of the absorption of VHE photons on the EBL, described in the previous Section, can be strongly modified if photons do mix with axion-like particles (ALPs). Pseudoscalar ALPs couple with photons through the following effective Lagrangian [43]

$$\mathcal{L}_{a\gamma} = -\frac{1}{4} g_{a\gamma} F_{\mu\nu} \tilde{F}^{\mu\nu} a,$$  

where $a$ is the ALP field with mass $m_a$, $F_{\mu\nu}$ the electromagnetic field-strength tensor, $\tilde{F}_{\mu\nu} \equiv \frac{i}{2} \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ its dual, and $g_{a\gamma}$ the ALP-photon coupling. As a consequence of this coupling, ALPs and photons do oscillate into each other in an external magnetic field. For a scalar particle, the coupling is proportional to $F_{\mu\nu} F^{\mu\nu} a$. For definiteness, we limit our discussion to the pseudoscalar case, but similar consequences apply also to scalars.

Let us suppose that a photon with energy $E$ moves in the $x_3$ direction. The transverse component of the external magnetic field is $B_T = B - B_3 e_3$. The evolution equations of the photon-ALP system in presence of mixing and absorption are [14, 43]

$$i \frac{\partial}{\partial x_3} \begin{pmatrix} A_1 \\ A_2 \\ a \end{pmatrix} = \mathcal{H} \begin{pmatrix} A_1 \\ A_2 \\ a \end{pmatrix} = \begin{pmatrix} \Delta_{11} - i \frac{\Gamma_{\gamma}}{2} & \Delta_{12} & \Delta_{a\gamma} c_{\phi} \\ \Delta_{21} & \Delta_{22} - i \frac{\Gamma_{\gamma}}{2} & \Delta_{a\gamma} s_{\phi} \\ \Delta_{a\gamma} c_{\phi} & \Delta_{a\gamma} s_{\phi} & \Delta_a \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ a \end{pmatrix},$$

(10)
where \( c_\phi \equiv \cos \phi = B_T \cdot e_1 / B_T = \sqrt{1 - s_\phi^2} \). The entries \( \Delta_{ij} \) \((i, j = 1, 2)\) that mix the photon polarization states are energy-dependent terms determined by the properties of the medium and the QED vacuum polarization effect. In particular, neglecting the Faraday rotation effects which are not relevant for the high energies of our interest, they read

\[
\begin{align*}
\Delta_{11} &= \Delta_{||} c_\phi^2 + \Delta_\perp s_\phi^2, \\
\Delta_{22} &= \Delta_{||} s_\phi^2 + \Delta_\perp c_\phi^2, \\
\Delta_{12} &= \Delta_{21} = (\Delta_{||} - \Delta_\perp) s_\phi c_\phi,
\end{align*}
\]

with \( \Delta_{||} = \Delta_{\text{pl}} + \frac{2}{7} \Delta_{\text{QED}}, \Delta_\perp = \Delta_{\text{pl}} + 2 \Delta_{\text{QED}} \) and

\[
\begin{align*}
\Delta_{a\gamma} &= \frac{1}{2} g_{a\gamma} B_T \simeq 1.52 \times 10^{-2} \left( \frac{g_{a\gamma}}{10^{-11} \text{GeV}^{-1}} \right) \left( \frac{B_T}{10^{-9} \text{G}} \right) \text{Mpc}^{-1}, \\
\Delta_a &= -\frac{m_a^2}{2E} \simeq -7.8 \times 10^{-4} \left( \frac{m_a}{10^{-10} \text{eV}} \right)^2 \left( \frac{E}{\text{TeV}} \right)^{-1} \text{Mpc}^{-1}, \\
\Delta_{\text{pl}} &= -\frac{\omega_{\text{pl}}^2}{2E} \simeq -1.1 \times 10^{-11} \left( \frac{E}{\text{TeV}} \right)^{-1} \left( \frac{n_e}{10^{-7} \text{cm}^{-3}} \right) \text{Mpc}^{-1}, \\
\Delta_{\text{QED}} &= \frac{\alpha E}{45\pi} \left( \frac{B_T}{m_e^2/e} \right)^2 \simeq 4.1 \times 10^{-9} \left( \frac{E}{\text{TeV}} \right) \left( \frac{B_T}{10^{-9} \text{G}} \right)^2 \text{Mpc}^{-1},
\end{align*}
\]

where \( B_T \) is expressed in Lorentz-Heaviside units and \( \omega_{\text{pl}}^2 = 4\pi \alpha n_e / m_e \) is the plasma frequency of the medium, being \( n_e \) the electron density. For the numerical estimations above, that we will use in the following as benchmark values, we have referred to the following physical input: The strength of widespread, all-pervading \( B \)-fields in the extragalactic medium must be \( B \lesssim 2.8 \times 10^{-7} (l/\text{Mpc})^{-1/2} \text{G} \), coherent on a scale \( l \simeq 1 \text{Mpc} \) \([44]\), as obtained scaling the original bound from the Faraday effect of distant radio sources \([45, 46]\) to the now much better known baryon density measured by the Wilkinson Microwave Anisotropy Probe (WMAP) \([47]\). The mean diffuse intergalactic plasma density is bounded by \( n_e \lesssim 2.7 \times 10^{-7} \text{cm}^{-3} \), corresponding to the recent WMAP measurement of the baryon density \([47]\). Recent results from the CAST experiment give a direct experimental bound on the ALP-photon coupling of \( g_{a\gamma} \lesssim 8.8 \times 10^{-11} \text{GeV}^{-1} \) for \( m_a \lesssim 0.02 \text{eV} \) \([7]\), slightly better than the long-standing globular-cluster limit \([48]\). For ultra-light axions a stringent limit from the absence of \( \gamma \)-rays from SN 1987A gives \( g_{a\gamma} \lesssim 1 \times 10^{-11} \text{GeV}^{-1} \) \([49]\) or even \( g_{a\gamma} \lesssim 3 \times 10^{-12} \text{GeV}^{-1} \) \([50]\). Previous bounds on ALPs can be relaxed if they have a chameleontic nature \([51]\). In this case, the best constraint comes from the structure of starlight polarization: \( g_{a\gamma} \lesssim 10^{-9} \text{GeV}^{-1} \) \([52]\).

The absorption term \( \Gamma_\gamma \) in Eq. \((10)\), due to the VHE photons scattering with the low-energy photons in the background, produces a damping of the oscillations in analogy with the case of the mixing of high-energy neutrinos in an absorbing matter \([53]\). In the presence of this term, the Hamiltonian \( \mathcal{H} \) is no longer hermitian.
3.2. Mean photon transfer function

Assuming an homogeneous magnetic field in a domain of size \( l \), in absence of absorption the probability that a photon will convert into an ALP reads [43]

\[
P_{a\gamma} = \sin^2 2\theta \sin^2 \left( \frac{\Delta_{\text{osc}} l}{2} \right),
\]

where the photon-ALP mixing angle \( \theta \) is

\[
\theta = \frac{1}{2} \arcsin \left( \frac{2 \Delta_{a\gamma}}{\Delta_{\text{osc}}} \right),
\]

and the oscillation wavenumber reads

\[
\Delta_{\text{osc}} = \left[ (\Delta_a - \Delta_{\text{pl}})^2 + 4 \Delta^2_{a\gamma} \right]^{1/2} = 2 \Delta_{a\gamma} \sqrt{1 + \left( \frac{E_c}{E} \right)^2},
\]

in terms of the critical energy

\[
E_c \equiv E \frac{|\Delta_a - \Delta_{\text{pl}}|}{2 \Delta_{a\gamma}} \simeq 2.5 \times 10^{-2} \frac{|m_a^2 - \omega_{\text{pl}}^2|}{(10^{-10}\text{eV})^2} \left( \frac{10^{-9}\text{GeV}}{B_T} \right) \left( \frac{10^{-11}\text{GeV}^{-1}}{g_{a\gamma}} \right) \text{TeV}.
\]

In the high-energy limit \( E \gg E_c \), \( \Delta_{\text{osc}} \simeq 2 \Delta_{a\gamma} \), the photon-ALP mixing is maximal \( (\theta \simeq \pi/4) \) and the conversion probability becomes energy-independent. This is the so-called strong-mixing regime. In this case, if \( \Delta_{a\gamma} l \ll 1 \), the conversion probability on a single domain becomes extremely simple, namely \( P_{a\gamma} = (\Delta_{a\gamma} l)^2 \). In the following, we will work in this regime.

In this situation, we explicitly drop \( \Delta_{||,\perp} \) and \( \Delta_a \) from the equations of motion [Eq. (10)]. Thus, the propagation hamiltonian \( \mathcal{H} \) can be written as \( \mathcal{H} = \Delta - i \mathcal{D} \) where

\[
\Delta = \Delta_{a\gamma} \begin{bmatrix} 0 & 0 & c_\phi \\ 0 & 0 & s_\phi \\ c_\phi & s_\phi & 0 \end{bmatrix},
\]

and \( \mathcal{D} = \frac{\Gamma_{\gamma}}{2} \text{diag}(1,1,0) \) is the damping term associated to the absorption.

Very high-energy gamma-rays propagate in the extragalactic magnetic fields during their route to the Earth. These \( B \)-fields presumably have a turbulent structure. Therefore, for the case under study we need to describe photon-ALP conversions in random magnetic field configurations. Let us now consider the propagation of photons in many domains of equal size \( l \) (\( \simeq 1 \text{ Mpc in our case} \)) in which the magnetic field has (constant) random values and directions. Along a given line of sight, the angles \( \phi \) are randomly distributed in \([0,2\pi)\). In the following, we will work in the formalism of the density matrix

\[
\rho = \left( \begin{array}{c} A_1 \\ A_2 \\ a \end{array} \right) \otimes \left( A_1 A_2 a \right)^*.
\]
For the $k$-th domain the density matrix is given by
\[ \rho_k = e^{-i\mathcal{H}_kl} \cdot \rho_{k-1} \cdot e^{i\mathcal{H}_ll}, \] (19)
where $e^{-i\mathcal{H}_kl}$ is the propagation operator for the $k$-th domain. During their path with a total length $L$, photons cross $k = 1, \ldots, n$ domains ($n = L/l$) representing a given random realization of $B_k$ and $\phi_k$. Since we cannot know this particular configuration, we perform an ensemble average over all the possible realizations on the 1, \ldots, $n$ domains. Defining this ensemble average as $\bar{\rho}_n = \langle \rho_n \rangle_{1 \ldots n}$, we have
\[ \bar{\rho}_n = \langle e^{-i\mathcal{H}_nl} \cdot \rho_{n-1} \cdot e^{i\mathcal{H}_ll} \rangle_{1 \ldots n} = \langle e^{-i\mathcal{H}_nl} \cdot \bar{\rho}_{n-1} \cdot e^{i\mathcal{H}_ll} \rangle_n. \] (20)
For the chosen values of the input parameters as in Eq. (12), we can perform a perturbative expansion up to the second order of the evolution operator in each domain, i.e.
\[ e^{-i\mathcal{H}_nl} \simeq 1 - i\mathcal{H}_nl - \frac{1}{2} \mathcal{H}_nl^2. \] (21)
Performing then the ensemble average, as shown in Appendix A, using that $\bar{\rho}_n - \bar{\rho}_{n-1} \simeq l \partial_{x_3} \bar{\rho}(x_3)$, and summing over the two indistinguishable photon polarization states, we finally arrive at a system of two coupled differential equations
\[ \frac{\partial}{\partial x_3} \begin{pmatrix} T_\gamma \\ T_a \end{pmatrix} = P_{a\gamma} \frac{1}{l} \begin{pmatrix} -\left(\alpha + \frac{1}{2}\right) & 1 \\ \frac{1}{2} & -1 \end{pmatrix} \begin{pmatrix} T_\gamma \\ T_a \end{pmatrix}, \] (22)
where $T_\gamma = \bar{\rho}_{11} + \bar{\rho}_{22}$ and $T_a = \bar{\rho}_{aa}$ are the mean transfer functions for the photon and for the ALP respectively; $P_{a\gamma} = \Delta^2_{\gamma} l^2$ is the average photon-ALP conversion probability in each domain (in absence of absorption and in the limit of strong mixing) and finally $\alpha = \Gamma_\gamma l / P_{a\gamma}$ is the ratio between the absorption probability and the conversion probability.

In realistic astrophysical situations both $P_{a\gamma}$ and $\alpha$ are functions of the distance, due to the redshift dependence of the extragalactic magnetic field and of the EBL. However, taking these parameters as constant, in the hypothesis of only photons in the initial state ($T_\gamma(0) = 1, T_a(0) = 0$) Eq. (22) has a simple analytical solution
\[ T_\gamma(y) = e^{-\nu y} \left[ \cosh \kappa y + \frac{1 - 2\alpha}{4\kappa} \sinh \kappa y \right], \] (23)
where
\[ \nu = \frac{\alpha}{2} + \frac{3}{4}, \]
\[ \kappa = \sqrt{\nu^2 - \alpha}, \]
\[ y = \frac{P_{a\gamma} x_3}{l}. \] (24)
In particular, Eq. (23) gives the two limiting expressions
\[ T_\gamma(y) \simeq \begin{cases} \frac{2}{3} + \frac{1}{3} e^{-3y/2} & \alpha = 0, \\ \frac{1}{2\alpha^2} e^{-y} & \alpha \gg 1. \end{cases} \] (25)
In the absence of absorption, we recover the mean transfer function already found in [54]. Conversely, in the case of strong absorption ($\alpha \gg 1$) we have $T_\gamma \propto (\Gamma_\gamma)^{-2}$. Using
the approximate expression for $\Gamma_\gamma$ given in Eq. (4) we observe that the transfer function would drop as a power of the energy (rather than exponentially as expected without ALP mixing). Moreover, also the attenuation of the transfer function with the distance is less than in the case of absence of conversions. In fact, the argument of the exponential is suppressed by a factor $1/\alpha$ with respect to the no-conversion case [see Eq. (7)].

In Appendix B we report also the calculation of the root mean square $\delta T_\gamma$ for the distribution of the transfer function in different random realizations of the magnetic field. This result is useful to estimate the uncertainty associated with the averaging procedure.

For illustrative purposes, in Fig. 2 we compare the transfer matrix $T_\gamma(y)$ of Eq. (23), with the numerical solution of Eq. (10), for a value of the parameter $\Delta_{a\gamma}$ as in Eq. (12). For simplicity, we choose constant values of the absorption factor $\alpha$, namely $\alpha = 0$ (no absorption) in the upper panel and $\alpha = 1$ in the lower panel. The dashed lines represent $T_\gamma \pm \delta T_\gamma$, where $\delta T_\gamma$ is the dispersion calculated as in Appendix B. In the

![Figure 2](image-url)

**Figure 2.** Photon transfer function $T_\gamma(y)$ for $\alpha = 0$ (no absorption) in the upper panels and for $\alpha = 1$ in the lower panels. The continuous curve corresponds to the mean value obtained by Eq. (23), while the dashed lines correspond to the dispersion around the mean $T_\gamma \pm \delta T_\gamma$, calculated in Appendix B. In the left panels, the dot-dashed lines correspond to a given realization of the random magnetic fields along the photon line of sight, obtained by the numerical integration of Eq. (10). In the right panels we show a scatter plot for $T_\gamma$ corresponding to $N_r = 20$ realizations of the random magnetic fields. (see the text for details)
left panels, the dot-dashed lines correspond to $T_\gamma$ for a given random realization of the magnetic field along the photon line of sight, obtained by the numerical solution of Eq. (10). In absence of absorption, we realize that along a given line of sight, the photon transfer function can present strong deviations with respect to the average value obtained with our analytical calculation. In this case, the dispersion with respect to the average tends to $\delta T_\gamma = 1/3\sqrt{5}$ for $y \to \infty$, as shown in Appendix B. We stress that the presence of a dispersion around the average $T_\gamma$ has not been properly appreciated in the previous literature. In particular, previous studies on the mixing of photons with ALPs emitted from point-like sources into random magnetic fields have presented $2/3$ as limiting value for the photon transfer function (see e.g. [13] for the case of photons emitted by supernovae Ia). In the case of strong mixing, where the oscillations are achromatic and photons of different energies are not dephased, this result is correct only on average, while along a given line of sight one can expect $O(1)$ deviations from this value. This peculiar effect has been also recently recognized in the context of photons emitted by active galactic nuclei [23]. Indeed, the scatter in their luminosities has been interpreted in [23] as a possible hint of the existence of a very light ALP. As a further consequence of this effect, it would be worthwhile to investigate if the presence of these large variations in the photon transfer function could put additional constraints to the mechanism of photon-ALP conversions introduced in [13] to explain the observed dimming of supernovae Ia. In fact, photon conversions into ALP could produce large dispersions in the supernova lightcurves. In the presence of absorption (lower panels) the transfer function (and its dispersion) is suppressed as a power-law for $y \gg 1$, due to the damping of the oscillations. In the right panels, we superimpose on our prescription for $T_\gamma$ a scatter plot corresponding to $N_r = 20$ different realizations of the random magnetic fields and for 500 steps in the variable $y$. Again, we realize that behaviour of the $T_\gamma$ distribution is damped by the effect of the absorption ($\alpha = 1$). However, as we will see in the next Section, the presence of a dispersion around the mean value will have interesting consequences for VHE photons. Finally, we mention that performing the averaging procedure over the transfer functions corresponding the different random realizations, we recover our analytical results (not shown).

4. Transfer function for very high-energy photons

We will apply the results of the previous Section to the case of VHE photons. For the characterization of the EBL, we refer to the 2008 Minimal Kneiske Model [41], discussed in Section 2. Concerning the extragalactic magnetic field, we consider it frozen into the medium. With this assumption, the scaling law of the magnetic field with the redshift $z$ is given by $B(z) = B_0(1+z)^2$ [46] while the size of the magnetic domains scales as $l = l_0/(1+z)$. With this choice, $P_{a\gamma}(z)$ scales as $(1+z)^2$. We assume $B_0 = 1$ nG and $l_0 = 1$ Mpc. Equation (22) can be easily written in terms of the redshift, by means of
Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields

12

the Jacobian

\[ \frac{dx}{dz} = \frac{c}{H_0 (1 + z) \sqrt{\Omega_\Lambda + \Omega_m (1 + z)^3}}. \] (26)

In Fig. 3 we show the photon transfer function \( T_\gamma \) in presence of absorption on the EBL for our reference model, in function of the redshift \( z \) for different values of the observed photon energy \( E \) with (continuous curve) and without ALP mixing (dotted curve). The dashed curves represent the spread \( T_\gamma \pm \delta T_\gamma \) around the mean value. One realizes that at redshift \( z > 0.2 \) the presence of ALPs could produce dramatic modifications in the shape of the photon transfer function, the stronger the effect the higher the photon energy. In this sense, a hint of the mixing of very high-energy photons with ALPs would be the detection of very distant sources otherwise obscure due to the absorption. However, from the spread of the photon transfer function \( \delta T_\gamma \), we also realize that at high redshift \( T_\gamma \) can present relevant deviations with respect to the mean value. In this sense, the effect of mixing with ALPs for VHE photons emitted by distant gamma sources would be strongly dependent on the particular realization of the extragalactic magnetic fields crossed by them during their propagation. Therefore, it is not guaranteed that the mixing with ALPs could provide an universal mechanism to achieve the transparency of distant gamma-sources. Conversely, for particular configurations of the extragalactic magnetic fields crossed by VHE photons, one could also find a suppression of the transfer function stronger than in the standard case. In general, observing VHE photons from far sources one would find strong differences in the measured spectra along different lines of sight. As a further consequence, the presence of ALPs would prevent the possibility to use distant gamma sources at very high energy as cosmological candles [55].

In Fig. 4 we show the photon transfer function as function of the energy for four different values of the redshift of the emitting source (as in Fig. 1 of [41]). We see that in absence of ALP conversions the photon transfer function would be strongly suppressed at energies above \( E \gtrsim 100 \text{ GeV} \), the stronger the suppression the larger the redshift. On the other hand, in presence of conversions, \( T_\gamma \) has an approximate power-law behaviour at high energies, as pointed out in the previous Section. We also realize that the spread in the possible values of \( T_\gamma \) would make difficult to infer strong conclusions about ALP mixing observing only few sources. We also note that in the case of \( z = 0.20 \) the inclusion of the ALPs does not produce any significant change in the photon transfer function. This suggests that it would be difficult to interpret in terms of ALP conversions the presumed transparency to gamma radiations for the sources at \( z = 0.165 \) and \( z = 0.186 \) discussed in [24, 25]. Conversely, ALP conversions could play a significant role for the source 3C279 at redshift \( z = 0.54 \) [34].

Finally, in Figure 5 we show two iso-contours of the photon transfer function \( T_\gamma \) in function of the redshift \( z \) and of the observed photon energy \( E \) with and without ALP conversions. We see that the attenuation of the photon transfer function in the presence of ALP conversions could present a large variation at high redshift with respect to the standard expectations with only absorption.
5. Discussion and conclusions

Very high-energy gamma-ray observations would open the possibility to probe the existence of axion-like particles (ALPs) predicted in many theories beyond the Standard Model. Recent gamma observations of cosmologically distant gamma-ray sources have revealed a surprising degree of transparency of the universe to very high-energy photons. The oscillations between high-energy photons and ALPs in the random extragalactic magnetic fields have been proposed as an intriguing possibility to explain these observations [31, 32]. Apart from the original proposal, the consequences of this mechanism are testable with the measurements of the new generation of Imaging Atmospheric Cherenkov Telescopes, like MAGIC [56], HESS [57], VERITAS [58] or CANGAROO-III [59], covering energies in the range 0.1-20 TeV, and hopefully with the future Cherenkov Telescope Array, reaching energies of 100 TeV [60].

In order to perform a systematic study of these effects, without recurring to brute-force time-consuming numerical simulations, in this paper we have presented a simple calculations of the photon mean transfer function and of its variance in presence of
absorption on the EBL and mixing with ALPs. We have found that our prescription is enough accurate for the case under study and we have shown some numerical examples of VHE photon transfer functions. It results that VHE photon-ALP mixing would produce peculiar deformations in the energy spectra of very high-energy gamma-rays emitted at high redshift. We have also found that the photon transfer function in presence of VHE photon-ALP conversions presents a relevant dispersion around the mean value due to the randomness of the extragalactic magnetic fields crossed by the photons. Due to this fact, the measured flux for VHE photons traveling along different lines of sight can be strongly suppressed or enhanced with respect to the case with only absorption, depending on the particular configuration of random magnetic fields encountered. This would suggest that photon-ALP conversions could not be an universal mechanism to produce the transparency of universe to VHE photons. Conversely, the most striking signature of the mixing with ALPs would be a reconstructed EBL density from TeV photon observations which appears to vary over different directions of the sky: consistent with standard expectations in some regions, but inconsistent in others. To test this effect we would need to collect data from sources along different directions in the sky in order to perform a study of the photon energy distributions, from which we could hope to infer possible hints of ALPs. A further signature of these stochastic conversions
Figure 5. Iso-contours of the photon transfer function $T_\gamma$ in function of the redshift $z$ and of the observed photon energy with only absorption (light curves) and in presence of ALP conversions. In this latter case, we represent the mean value of $T_\gamma$ (black solid curves) and its dispersion $T_\gamma \pm \delta T_\gamma$ (black dashed curves).

would be the detection of peculiar direction-dependent dimming effects in the diffuse photon radiation observable in GeV range, testable with the FERMI (previously called GLAST) experiment [61].

As further developments, we plan to use our calculation to perform a systematic study of ALP signatures in very high-energy gamma-rays, analyzing in details the spectral deformations expected for observed sources at different redshifts and for different models of the extragalactic background light. Thanks to our calculation, this task now appears more doable than before. After that, it will remain to see if elusive
ALPs will show up from the sky.

Acknowledgements

We thank R. Wagner and D. Shaw for fruitful discussions and C. Burrage, G. Raffelt, M. Roncadelli and P. D. Serpico for reading the manuscripts and for useful comments on it. D.M. acknowledges kind hospitality at the Max-Planck-Institut where part of this work was done. D.M. thanks also the organizers of the “5th Patras Workshop on Axions, WIMPs and WISPs” for the kind hospitality and the stimulating discussions. In Lecce, the work of D.M. is partly supported by the Italian MIUR and INFN through the “Astroparticle Physics” research project, and by the EU ILIAS through the ENTApP project.

Appendix A. Calculation of the mean photon transfer function

Here, we present the derivation of the photon transfer function $T_\gamma$ introduced in Sec. 3.2. We have defined $\bar{\rho}_n \equiv \langle \rho_n \rangle_{1...n}$, as an ensemble average of the density matrix $\rho_n$ in the $n$-th domain over all the possible realizations of the random magnetic fields in all the domains from 1 to $n$. Since $H_n$ depends only from the configuration of the $n$-th domain, we have that

$$\bar{\rho}_n = \langle e^{-iH_n l} \cdot \rho_{n-1} \cdot e^{iH_n l} \rangle_{1...n} = \langle e^{-iH_n l} \cdot \bar{\rho}_{n-1} \cdot e^{iH_n l} \rangle_n . \quad (A.1)$$

For the chosen values of the input parameters in Eq. (10), we can perform a perturbative expansion of the evolution operator up to the second order $H_n = \Delta_n - iD_n$ in each domain, i.e.

$$e^{-iH_n l} \simeq 1 - iH_n l - \frac{1}{2} H_n^2 l^2 . \quad (A.2)$$

Since, from Eq. (17) it results that $\langle \Delta_n \rangle_{\phi_n} = 0$, Eq. (A.1) can be written as

$$\bar{\rho}_n = \bar{\rho}_{n-1} - l \left( D_n \bar{\rho}_{n-1} + \bar{\rho}_{n-1} D_n \right)$$

$$+ l^2 \langle \Delta_n \bar{\rho}_{n-1} \Delta_n \rangle_n - \frac{l^2}{2} \left( \langle \Delta_n^2 \rangle_n \bar{\rho}_{n-1} + \bar{\rho}_{n-1} \langle \Delta_n^2 \rangle_n \right) . \quad (A.3)$$

In the previous equation we have neglected the second order terms in $D_n l$ since they give a contribution of the order of $(\Gamma_n l)^2 \lesssim 10^{-6}$ [see Eq. (5)] which is at least two orders of magnitude smaller than $(\Delta_{a\gamma} l)^2 \sim 10^{-4}$, when we use the benchmark values in Eq. (12).

For $\langle \Delta_n^2 \rangle_n$ we have

$$\langle \Delta_n^2 \rangle_n = \frac{1}{4} g_{a\gamma}^2 \left( B_{T,n} \begin{bmatrix} \cos^2 \phi_n & \sin \phi_n \cos \phi_n & 0 \\ \sin \phi_n \cos \phi_n & \sin^2 \phi_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \right)_{\phi_n,B_n}$$

$$\equiv \frac{\Lambda_{a\gamma}^2}{a\gamma} \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} , \quad (A.4)$$
Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields

where we have defined

$$\Delta^2_{a\gamma} = \frac{1}{4}g_{a\gamma}^2\langle B_T^2 \rangle_{B_n} = \left(\frac{1}{2}g_{a\gamma}B_{\text{eff}}\right)^2,$$

(A.5)

where \(B_{\text{eff}} = \langle B_T^2 \rangle_{B_n} = 2\langle |B|^2 \rangle / 3\) due to the projection effect. In the same way we have

$$\langle \Delta_{n-1}^2 \rangle_n = \frac{1}{2}\Delta^2_{a\gamma}\left[\begin{array}{ccc}
\hat{\rho}_{aa} & 0 & \hat{\rho}_{a1} \\
0 & \hat{\rho}_{aa} & \hat{\rho}_{a2} \\
\hat{\rho}_{1a} & \hat{\rho}_{2a} & \hat{\rho}_{11} + \hat{\rho}_{22}
\end{array}\right]_{n-1}.$$

(A.6)

After \(n\) domains, the total distance travelled by the photon is \(x_3 = nl\). Defining \(\bar{\rho}_n \equiv \bar{\rho}(x_3)\), one has \(\bar{\rho}_n - \bar{\rho}_{n-1} \simeq l\partial_{x_3}\bar{\rho}(x_3)\). After a straightforward calculation, we obtain the evolution equation for the averaged density matrix \(\bar{\rho}\)

$$\frac{\partial \bar{\rho}}{\partial x_3} = \frac{P_{a\gamma}}{l}\left[\begin{array}{ccc}
-\mu \hat{\rho}_{11} + \frac{1}{2}\hat{\rho}_{aa} & -\mu \hat{\rho}_{12} & -\nu \hat{\rho}_{1a} + \frac{1}{2}\hat{\rho}_{a1} \\
-\mu \hat{\rho}_{21} & -\mu \hat{\rho}_{22} + \frac{1}{2}\hat{\rho}_{aa} & -\nu \hat{\rho}_{2a} + \frac{1}{2}\hat{\rho}_{a2} \\
-\nu \hat{\rho}_{a1} + \frac{1}{2}\hat{\rho}_{1a} & -\nu \hat{\rho}_{a2} + \frac{1}{2}\hat{\rho}_{2a} & -\hat{\rho}_{aa} + \frac{1}{2}(\hat{\rho}_{11} + \hat{\rho}_{22})
\end{array}\right].$$

(A.7)

where

$$\mu = \alpha + \frac{1}{2},$$
$$\nu = \frac{\alpha}{2} + \frac{3}{4},$$

(A.8)

and we have defined as \(P_{a\gamma} = \Delta^2_{a\gamma}l^2\) the average photon-ALP oscillation probability in each domain (in absence of absorption and in the limit of strong mixing) and \(\alpha \equiv \Gamma l/P_{a\gamma}\) the ratio between the absorption probability and the oscillation probability.

Since we are interested in determining the total final photon and ALP flux, we define the mean transfer functions \(T_\gamma = \bar{\rho}_{11} + \bar{\rho}_{22}\) and \(T_a = \bar{\rho}_{aa}\), for whose evolution we finally obtain

$$\frac{\partial}{\partial x_3}\left(\begin{array}{c}
T_\gamma \\
T_a
\end{array}\right) = \frac{P_{a\gamma}}{l}\left[\begin{array}{cc}
-\mu & 1 \\
\frac{1}{2} & -1
\end{array}\right]\left(\begin{array}{c}
T_\gamma \\
T_a
\end{array}\right).$$

(A.9)

Defining the variable \(dy = P_{a\gamma}dx_3/l\), we can also obtain a second order equation for the function \(T_\gamma\)

$$T_\gamma'' + \left(\alpha + \frac{3}{2}\right)T_\gamma' + (\alpha + \alpha')T_\gamma = 0,$$

(A.10)

where the prime denotes the derivative with respect to \(y\). For constant \(\alpha\) we easily obtain the solution in Eq. (23).

Appendix B. Calculation of higher momenta

In the previous Appendix we have have calculated the photon transfer function \(T_\gamma\) averaged over all the possible magnetic domain configurations. However, photons coming from a single source cross just one (unknown) particular realization of the magnetic field domains. It is thus interesting to evaluate the uncertainty introduced by the procedure of averaging. To do this, we calculate the second order momenta of
Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields

the probability distributions, by means of a procedure similar to the one introduced in Appendix A. We define the “square” of the density matrix as
\[ \rho^{(2)} = \rho \otimes \rho \rightarrow \rho^{(2)}_{ijkl} = \rho_{ij} \rho_{kl}, \]  
and \[ \bar{\rho}^{(2)}_n = \langle \rho^{(2)}_n \rangle_{1...n}. \] We rewrite Eq. (A.7) in the following way
\[ \frac{\partial}{\partial y} \bar{\rho}_{ij} = G_{ijrs} \bar{\rho}_{rs}, \]  
where the tensor \( G_{ijkl} \) can be written as
\[ G_{ijkl} = \begin{cases} 
-\mu & \text{if } ijk = 1111, 1212, 2121, 2222 \\
-1 & \text{if } ijk = aaaa \\
-\nu & \text{if } ijk = 1a1a, a1a1, 2a2a, a2a2 \\
\frac{1}{2} & \text{if } ijk = 11aa, 22aa, aa11, aa22 \\
1 & \text{if } ijk = 1a1a, 2a2a, a11a, a2a2 \\
0 & \text{otherwise}.
\end{cases} \]

Performing the average of \( \rho^{(2)} \) up to the first order in \( \Gamma_\gamma \) and to the second order in \( \Delta_a \gamma \) as in Eq. (A.3), we arrive at the following equation for the evolution of \( \bar{\rho}^{(2)} \)
\[ \frac{\partial}{\partial x_3} \bar{\rho}^{(2)}_{ijkl} = \frac{P_{\alpha\gamma}}{l} \left( G_{ijrs} \bar{\rho}^{(2)}_{rskl} + G_{rskl} \bar{\rho}^{(2)}_{ijrs} \right) \\
- l \langle (\Delta_{ij} \rho_{rj} - \rho_{ir} \Delta_{rj}) (\Delta_{ks} \rho_{sl} - \rho_{ks} \Delta_{sl}) \rangle_{1...n} \\
= \frac{P_{\alpha\gamma}}{l} \left( G_{ijrs} \bar{\rho}^{(2)}_{rskl} + G_{rskl} \bar{\rho}^{(2)}_{ijrs} \right) \\
- l \left( \langle \Delta_{ij} \Delta_{ks} \rangle \bar{\rho}^{(2)}_{rjst} + \langle \Delta_{rj} \Delta_{sl} \rangle \bar{\rho}^{(2)}_{irs} \right) \\
- \langle \Delta_{ij} \Delta_{kl} \rangle \bar{\rho}^{(2)}_{rjks} + \langle \Delta_{rj} \Delta_{kl} \rangle \bar{\rho}^{(2)}_{irs}, \]  
(B.3)
where for simplicity we have dropped the subscript \( n \) from the averages in the last two lines. The last term arises from the linear term in \( l \) in Eq. (20) which is absent in Eq. (A.3) since it averages out \( \langle \Delta \rangle = 0 \). From Eq. (17) we have that \( \langle \Delta_{ij} \Delta_{kl} \rangle = \frac{1}{2} \sum_{\alpha \gamma} \xi_{ijkl} \) with
\[ \xi_{ijkl} = \begin{cases} 
1 & \text{if } ijk = 11aa, 1a1a, 11a1, aa11 \\
2aa2, 2a2a, a112, a22a & \text{otherwise}.
\end{cases} \]

After a long but straightforward derivation, one can extract a subset of 6 independent equations out from the set of the 81 of (B.3):
\[ \partial_y R_\gamma = - (2\alpha + 1) R_\gamma + 2\eta_{a\gamma} - \zeta_{a\gamma} \]
\[ \partial_y R_a = - 2 R_a + \eta_{a\gamma} - \zeta_{a\gamma} \]
\[ \partial_y R_p = - (2\alpha + 1) R_p - \zeta_{a\gamma} \]
\[ \partial_y \zeta_\gamma = - (2\alpha + 1) \zeta_\gamma - \zeta_{a\gamma} \]
\[ \partial_y \eta_{a\gamma} = - \left( \alpha + \frac{3}{2} \right) \eta_{a\gamma} + \frac{1}{2} R_\gamma + R_a + \zeta_{a\gamma} \]
\[ \partial_y \zeta_{a\gamma} = - \left( \alpha + \frac{5}{2} \right) \zeta_{a\gamma} - \frac{1}{2} R_\gamma - 2 R_a + 2 \eta_{a\gamma} - \frac{1}{2} (R_p + \zeta_\gamma), \]  
(B.4)
Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields

where, as usual, \( dy = P_a \gamma dx_3 / l \) and,

\[
R_\gamma = \langle \tilde{\rho}_{1111}^{(2)} + \tilde{\rho}_{2222}^{(2)} + \tilde{\rho}_{1122}^{(2)} + \tilde{\rho}_{2211}^{(2)} \equiv \langle \tilde{\rho}_{11} + \tilde{\rho}_{22} \rangle^2 \rangle \\
R_a = \tilde{\rho}_{aaaa}^{(2)} \equiv \langle \tilde{\rho}_a^2 \rangle \\
R_p = \tilde{\rho}_{1111}^{(2)} + \tilde{\rho}_{2222}^{(2)} - \tilde{\rho}_{1122}^{(2)} - \tilde{\rho}_{2211}^{(2)} \equiv \langle \tilde{\rho}_{11} - \tilde{\rho}_{22} \rangle^2 \rangle \\
\zeta_\gamma = \frac{1}{2} \left( \tilde{\rho}_{11aa}^{(2)} + \tilde{\rho}_{aa11}^{(2)} + \tilde{\rho}_{22aa}^{(2)} + \tilde{\rho}_{aa22}^{(2)} \right) \equiv \langle (\tilde{\rho}_{11} + \tilde{\rho}_{22}) \cdot \tilde{\rho}_{aa} \rangle \\
\eta_a\gamma = \frac{1}{2} \left( \tilde{\rho}_{11aa}^{(2)} + \tilde{\rho}_{aa11}^{(2)} - \tilde{\rho}_{a11a}^{(2)} - \tilde{\rho}_{1aa1}^{(2)} + \tilde{\rho}_{2a2a}^{(2)} + \tilde{\rho}_{a2a2}^{(2)} - \tilde{\rho}_{a22a}^{(2)} - \tilde{\rho}_{2a2a}^{(2)} \right). \tag{B.5}
\]

We can thus give a physical interpretation for some of these quantities: \( R_\gamma \) and \( R_a \) are the square average of the photon and ALP transfer function respectively, \( R_p \) is the average square degree of polarization of the photon, \( \eta_a\gamma \) is the photon-ALP correlation. Since the third and the fourth of Eqs. (B.4) are similar, starting from a completely unpolarized photon state we find also that \( R_p(y) = \zeta_\gamma(y) \), so that we can reduce further the system Eqs. (B.4) to five independent equations.

We finally define the “1σ” uncertainty on the photon transfer function as \( \delta T_\gamma = [R_\gamma - T_\gamma^2]^{1/2} \). However, the distribution of \( T_\gamma \) is not gaussian (and in general is also asymmetrical) so that \( T_\gamma \pm \delta T_\gamma \) should be interpreted just as a qualitative “band” with a not defined confidence level.

For a constant value of \( \alpha \), the system of Eqs. (B.4) can be integrated analytically. For simplicity, we give here only the solution for the case \( \alpha = 0 \)

\[
R_\gamma(y) = \frac{49 + 50e^{-3y/2} + 6e^{-5y}}{105}. \tag{B.6}
\]

For \( y \to \infty \) we have thus the prediction \( T_\gamma = 2/3 \pm 1/3\sqrt{5} \).

References

[1] P. Svrcek and E. Witten, “Axions in string theory,” JHEP 0606, 051 (2006) [hep-th/0605206].

[2] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, “String Axiverse,”

arXiv:0905.4720 [hep-th].

[3] E. Masso, “Axions and their relatives,” Lect. Notes Phys. 741, 83 (2008) [hep-th/0607215].

[4] L. D. Duffy et al., “A High Resolution Search for Dark-Matter Axions,” Phys. Rev. D 74, 012006 (2006) [astro-ph/0603108].

[5] K. Zioutas et al. [CAST Collaboration], “First results from the CERN Axion Solar Telescope (CAST),” Phys. Rev. Lett. 94, 121301 (2005) [hep-ex/0411033].

[6] S. Andriamonje et al. [CAST Collaboration], “An improved limit on the axion-photon coupling from the CAST experiment,” JCAP 0704, 010 (2007) [hep-ex/0702006].

[7] E. Arik et al. [CAST Collaboration], “Probing eV-scale axions with CAST,” JCAP 0902, 008 (2009) [arXiv:0810.4482 [hep-ex]].

[8] C. Robilliard, R. Battesti, M. Fouche, J. Mauchain, A. M. Sautivet, F. Amiranoff and C. Rizzo,

“No light shining through a wall,” Phys. Rev. Lett. 99, 190403 (2007) [arXiv:0707.1296 [hep-ex]].
Stochastic conversions of TeV photons into axion-like particles in extragalactic magnetic fields

[9] A. S. Chou et al. [GammeV (T-969) Collaboration], “Search for axion-like particles using a variable baseline photon regeneration technique,” Phys. Rev. Lett. 100, 080402 (2008) [arXiv:0710.3783 [hep-ex]].

[10] A. Afanasev et al., “New Experimental limit on Optical Photon Coupling to Neutral, Scalar Bosons,” Phys. Rev. Lett. 101, 120401 (2008) [arXiv:0806.2631 [hep-ex]].

[11] M. Fouche et al., “Search for photon oscillations into massive particles,” Phys. Rev. D 78, 032013 (2008) [arXiv:0808.2800 [hep-ex]].

[12] F. Caspers, J. Jaeckel, A. Ringwald, “Feasibility, engineering aspects and physics reach of microwave cavity experiments searching for hidden photons and axions,” arXiv:0908.0759 [hep-ex].

[13] C. Csaki, N. Kaloper and J. Terning, “Dimming supernovae without cosmic acceleration,” Phys. Rev. Lett. 88, 161302 (2002) [hep-ph/0111311].

[14] C. Csaki, N. Kaloper, M. Peloso and J. Terning, “Super-GZK photons from photon axion mixing,” JCAP 0305, 005 (2003) [hep-ph/0302030].

[15] A. Mirizzi, G. G. Raffelt and P. D. Serpico, “Photon axion conversion as a mechanism for supernova dimming: Limits from CMB spectral distortion,” Phys. Rev. D 72, 023501 (2005) [astro-ph/0506078].

[16] A. Mirizzi, G. G. Raffelt and P. D. Serpico, “Photon axion conversion in intergalactic magnetic fields and cosmological consequences,” Lect. Notes Phys. 741, 115 (2008) [astro-ph/0607415].

[17] A. Mirizzi, J. Redondo and G. Sigl, “Constraining resonant photon-axion conversions in the Early Universe,” JCAP 08, 001 (2009) [arXiv:0905.4865 [hep-ph]].

[18] A. Mirizzi, G. G. Raffelt and P. D. Serpico, “Signatures of axion-like particles in the spectra of TeV gamma-ray sources,” Phys. Rev. D 76, 023001 (2007) [arXiv:0704.3044 [astro-ph]].

[19] A. Dupays, C. Rizzo, M. Roncadelli and G. F. Bignami, “Looking for light pseudoscalar bosons in the binary pulsar system J0737-3039,” Phys. Rev. Lett. 95, 211302 (2005) [astro-ph/0510324].

[20] D. Hooper and P. D. Serpico, “Detecting Axion-Like Particles With Gamma Ray Telescopes,” Phys. Rev. Lett. 99, 231102 (2007) [arXiv:0706.3203 [hep-ph]].

[21] A. De Angelis, O. Mansutti and M. Roncadelli, “Axion-Like Particles, Cosmic Magnetic Fields and Gamma-Ray Astrophysics,” Phys. Lett. B 659, 847 (2008) [arXiv:0707.2695 [astro-ph]].

[22] M. Fairbairn, T. Rashba and S. Troitsky, “Photon-axion mixing in the Milky Way and ultrahigh-energy cosmic rays from BL Lac type objects - Shining light through the Universe,” arXiv:0901.4085 [astro-ph.HE].

[23] C. Burrage, A. C. Davis and D. J. Shaw, “Active Galactic Nuclei Shed Light on Axion-like-Particles,” Phys. Rev. Lett. 102, 201101 (2009) [arXiv:0902.2320 [astro-ph.CO]].

[24] F. Aharonian et al. [H.E.S.S. Collaboration], “A Low level of extragalactic background light as revealed by gamma-rays from blazars,” Nature 440, 1018 (2006) [astro-ph/0508073].

[25] D. Mazin and M. Raue, “New limits on the density of the extragalactic background light in the optical to the far-infrared from the spectra of all known TeV blazars,” Astron. Astrophys. 471, 439 (2007) [astro-ph/0701694].

[26] F. W. Stecker and S. T. Scully, “The Spectrum of 1ES0229 + 200 and the Cosmic Infrared Background,” arXiv:0710.2252 [astro-ph].

[27] F. W. Stecker, M. G. Baring and E. J. Summerlin, “Blazar Gamma-Rays, Shock Acceleration, and the Extragalactic Background Light,” Astrophys. J. 667, L29 (2007) [arXiv:0707.4676 [astro-ph]].

[28] K. A. Hochmuth and G. Sigl, “Effects of Axion-Photon Mixing on Gamma-Ray Spectra from Magnetized Astrophysical Sources,” Phys. Rev. D 78, 123011 (2007) [arXiv:0708.1144 [astro-ph]].

[29] M. Simet, D. Hooper and P. D. Serpico, “The Milky Way as a Kiloparsec-Scale Axionscope,” Phys. Rev. D 77, 063001 (2008) [arXiv:0712.2825 [astro-ph]].

[30] N. Bassan and M. Roncadelli, “Photon-axion conversion in Active Galactic Nuclei?,” arXiv:0905.3752 [astro-ph.HE].

[31] A. De Angelis, O. Mansutti and M. Roncadelli, “Evidence for a new light spin-zero boson from
cosmological gamma-ray propagation?,” Phys. Rev. D 76, 121301 (2007) [arXiv:0707.4312 [astro-ph]].

[32] A. De Angelis, O. Mansutti, M. Persic and M. Roncadelli, “Photon propagation and the VHE gamma-ray spectra of blazars: how transparent is the Universe?,” Mon. Not. R. Astron. Soc. 394, L21 (2009) [arXiv:0807.4246 [astro-ph]].

[33] M. A. Sanchez-Conde, D. Paneque, E. Bloom, F. Prada and A. Dominguez, “Hints of the existence of Axion-Like-Particles from the gamma-ray spectra of cosmological sources,” Phys. Rev. D 79, 123511 (2009) [arXiv:0905.3270 [astro-ph.CO]].

[34] J. Albert et al. [MAGIC Collaboration], “Very High-energy gamma rays from a distant Quasar: how transparent is the Universe?,” Science 320 1752 (2008) [arXiv:0807.2822[astro-ph]].

[35] M. A. Sanchez-Conde, D. Paneque, E. Bloom, F. Prada and A. Dominguez, “Hints of the existence of Axion-Like-Particles from the gamma-ray spectra of cosmological sources,” Phys. Rev. D 79, 123511 (2009) [arXiv:0905.3270 [astro-ph.CO]].

[36] F. Aharonian, D. Khangulyan and L. Costamante, “Formation of hard VHE gamma-ray spectra of blazars due to internal photon-photon absorption,” Mon. Not. R. Astron. Soc. 387, 1206 (2008) [arXiv:0801.3198 [astro-ph]].

[37] W. Essey and A. Kusenko, “A new interpretation of the gamma-ray observations of active galactic nuclei,” arXiv:0905.1162 [astro-ph.HE].

[38] R. J. Gould and G. P. Schreder, “Opacity of the Universe to High-Energy Photons,” Phys. Rev. 155, 1408 (1967).

[39] W. Heitler, The Quantum Theory of Radiation, (Dover, 1984) p. 430.

[40] M. H. Salamon, F. W. Stecker and O. C. De Jager, “A New method for determining the Hubble constant from subTeV gamma-ray observations,” Astrophys. J. 423 (1994) L1.

[41] T. M. Kneiske and H. Dole, “A strict lower-limit EBL: Applications on gamma-ray absorption,” AIP Conf. Proc. 1085, 620 (2009) [arXiv:0810.1612 [astro-ph]]. Tables of the EBL spectra at various energies and redshifts can be found at the following URL: http://astroparticle.de

[42] C. Amsler et al. [Particle Data Group], “Review of particle physics,” Phys. Lett. B 667, 1 (2008).

[43] G. Raffelt and L. Stodolsky, “Mixing of the Photon with Low Mass Particles,” Phys. Rev. D 37, 1237 (1988).

[44] P. Blasi, S. Burles and A. V. Olinto, “Cosmological Magnetic Fields Limits in an Inhomogeneous Universe,” Astrophys. J. 514, L79 (1999) [astro-ph/9812487].

[45] P. P. Kronberg, “Extragalactic magnetic fields,” Rept. Prog. Phys. 57, 325 (1994).

[46] D. Grasso and H. R. Rubinstein, “Magnetic fields in the early universe,” Phys. Rept. 348, 163 (2001) [astro-ph/0009061].

[47] WMAP Collaboration, G. Hinshaw et al., “Five-Year Wilkinson Microwave Anisotropy Probe Observations: Data Processing, Sky Maps, & Basic Results,” Astrophys. J. Suppl. 180, 225 (2009) [arXiv:0803.0732 [astro-ph]].

[48] G. G. Raffelt, “Astrophysical axion bounds,” Lect. Notes Phys. 741, 51 (2008) [hep-ph/0611350].

[49] J. W. Brockway, E. D. Carlson and G. G. Raffelt, “SN 1987A gamma-ray limits on the conversion of pseudoscalars,” Phys. Lett. B 383, 439 (1996) [astro-ph/ 9605197].

[50] J. A. Grifols, E. Massó and R. Toldrà, “Gamma rays from SN 1987A due to pseudoscalar conversion,” Phys. Rev. Lett. 77, 2372 (1996) [astro-ph/9606028].

[51] P. Brax, C. van de Bruck and A. C. Davis, “Compatibility of the chameleon-field model with fifth-force experiments, cosmology, and PVLAS and CAST results,” Phys. Rev. Lett. 99, 121103 (2007) [hep-ph/0703243].

[52] C. Burragge, A. C. Davis and D. J. Shaw, “Detecting Chameleons: The Astronomical Polarization Produced by Chameleon-like Scalar Fields,” Phys. Rev. D 79, 044028 (2009) [arXiv:0809.1763 [astro-ph]].

[53] V. A. Naumov, “High-energy neutrino oscillations in absorbing matter,” Phys. Lett. B 529, 199 (2002) [hep-ph/0112249].

[54] Y. Grossman, S. Roy and J. Zupan, “Effects of initial axion production and photon axion oscillation
on type Ia supernova dimming,” Phys. Lett. B 543, 23 (2002) [hep-ph/0204216].
[55] X. J. Bi and Q. Yuan, “Cosmology from very high energy \(\gamma\)-rays,” arXiv:0809.5124 [astro-ph].
[56] E. Lorenz et al. J. Albert et al. [MAGIC Collaboration], “Status of the 17m diameter MAGIC telescope,” New Astron. Rev. 48, 339 (2004).
[57] J. A. Hinton et al. [H.E.S.S. collaboration], “The status of the H.E.S.S. project,” New Astron. Rev. 48, 331 (2004).
[58] T. C. Weekes et al. [VERITAS Collaboration], “VERITAS: the Very Energetic Radiation Imaging Telescope Array System,” Astropart. Phys. 17, 221 (2002) [astro-ph/0108478].
[59] R. Enomoto et al. [CANGAROO Collaboration], “Design study of CANGAROO-III, stereoscopic imaging atmospheric Cherenkov telescopes for sub-TeV gamma-ray detection,” Astropart. Phys. 16, 235 (2002) [astro-ph/0107578].
[60] “Cherenkov Telescope Array: An advanced facility for ground-based gamma-ray astronomy,”
Website: www.cta-observatory.org
[61] N. Gehrels and P. Michelson, “GLAST: The next-generation high energy gamma-ray astronomy mission,” Astropart. Phys. 11, 277 (1999).