Brain-Swarm Interface (BSI): Controlling a Swarm of Robots with Brain and Eye Signals from an EEG Headset

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Abstract—This work presents a novel marriage of Swarm Robotics and Brain Computer Interface technology to produce an interface which connects a user to a swarm of robots. The proposed interface enables the user to control the swarm’s size and motion employing just thoughts and eye movements. The thoughts and eye movements are recorded as electrical signals from the scalp by an off-the-shelf Electroencephalogram (EEG) headset. Signal processing techniques are used to filter out noise and decode the user’s eye movements from raw signals, while a Hidden Markov Model technique is employed to decipher the user’s thoughts from filtered signals. The dynamics of the robots are modulated by the human user through the brain-swarm interface to move the robots. The method is demonstrated experimentally with a human controlling a swarm of three M3pi robots in a laboratory environment, as well as controlling a swarm of 128 robots in a computer simulation.

I. INTRODUCTION

In this paper we present a new brain machine interface for a human user to control a swarm of robots, which we call a Brain-Swarm Interface (BSI). The BSI uses an off-the-shelf Electroencephalogram (EEG) headset to record brain and muscle activity from the user’s scalp. We use both signals from brain neuron activity, as well as signals from contracting muscles due to eye movement, to generate control signals for the robot swarm. We allow the user to control the dispersion/aggregation of the swarm, as well as the direction of motion of the swarm. The dispersion/aggregation is determined from the user’s brain neuronal signals, and is decoded using an HMM-based method. The direction of motion is decoded from the user’s eye movements with a multi-step signal processing algorithm. The robots maintain a cohesive swarm using a potential-field based swarm controller, and the dispersion/aggregation and direction of motion commands influence the motion of the robots through parameters in their swarm controller.

Brain Computer Interfaces hold great promise for enabling people with various forms of disabilities, from restricted motion due to injury or old age, to severe disabilities like the ALS (Locked-in syndrome), Tetraplegia, and paralysis. Several works have investigated using BCIs for controlling prosthetics\cite{5,6}, and for medical rehabilitation. If implemented effectively, BCI technology may allow people with disabilities the power to manipulate their environment, and to move themselves within their environment\cite{1}. Researchers have also developed brain interfaces for controlling single mobile robot platforms. For example Bin He et al.\cite{2} have demonstrated 3D control of a quadcopter, and Tim Bredl et al.\cite{3} have remotely teleoperated a UAV, both using motor imagery BCI. C J Bell et al.\cite{4} have controlled a humanoid with a non invasive BCI using P300 signals. However, brain computer interfaces for controlling swarms have received little attention in the literature.

Whereas the motivation for a BCI operated prosthetic or wheelchair is evident, the applications for a brain swarm interface may be less obvious. We envision several applications for this technology. Firstly, people who are mobility-impaired may use a swarm of robots to manipulate their environment using a brain-swarm interface. Indeed, a swarm of robots may offer a greater range of possibilities for manipulation than what is afforded by a single mobile robot or manipulator. For example a swarm can reconfigure to suit different sizes or shapes of objects, or to split up and deal with multiple manipulation tasks at once. Another motivation for our work is that using a brain interface may unlock a new, more flexible, way for people to interact with swarms. Currently human swarm interfaces are largely restricted to gaming joysticks with limited degrees of freedom. However, swarms typically have many degrees of freedom, and the brain has an enormous potential to influence those degrees of freedom beyond the confines of a traditional joystick.

We envision that the brain can eventually craft shapes and sizes for swarm, split a swarm into sub swarms, aggregate or disperse the swarm, and perhaps much more. In this work, we take a small step toward this vision.

The brain is arguably the most complex organ in the human body. Brain activity occurs mainly in the chemical and electromagnetic domain. Different areas of the brain are responsible for different thoughts and actions. The EEG is one of the few ways in which brain activity can be monitored in a non-invasive manner. An EEG measures the electrical signals generated by the brain, but it also measures electrical noise and artifacts generated by eye movements and other muscle activity. We use the concept of Hybrid BCI or hBCI popularized by Pfurtscheller et al.\cite{7} and utilize the “artifacts” generated by eye movements, which are usually discarded in BCI systems, as control input to modulate the position and size of a robotic swarm. A survey of other hBCI systems, which do not include application to robotic swarms, can be found in\cite{8}.

One of the key components of a typical BCI paradigm

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In the previous section we stated the problem, and introduced the main elements of our solution as shown in Fig [2]. The following paragraphs will describe the HMM, eye movement detection, and formation control strategies in detail.

A. Decoding Thoughts Using an HMM

Based on a survey by F. Lotte et al. [9] we adopted Hidden Markov Models to train and classify EEG data. Previously, researchers have used HMMs for BCIs. Pfurtscheller et al. [12] used an HMM to classify EEG data, and HMMs have been used in conjunction with other techniques [13], as well. Our novel HMM implementation uses performance metrics generated by the Emotiv software suite as observations and maps them to discrete thoughts. We observe that each thought will corresponds to different signatures of these metrics.
However, these signatures vary greatly across different trials and experimental conditions, so a heuristic approach to classify them is not effective.

Instead we implement a training phase to train the HMM to detect the users though signatures and transition probabilities. Using standard terminology and notation for HMMs, [10], [11], here we will describe our system in detail.

1) HMM: An Overview: A Hidden Markov Model is a joint probabilistic model of a collection of discrete random variables \( O_1, \ldots, O_T, Q_1, \ldots, Q_T \) described by:

\[
P(O, Q) = P(Q_0) \prod_{t=1}^{T} P(Q_t | Q_{t-1}) \prod_{t=0}^{T} P(O_t | Q_t)
\]  
(2)

Many algorithms exist for both learning the transition probabilities and observation probabilities of such a system from data, and determining a likely sequence of states hidden \( \{Q_t\} \) from data. The observations of these states \( O_t \in \mathbb{R}^T \) can be discrete or continuous.

The main components of the model from Equation 2 are:

- \( P(Q_0) \) which is the initial state distribution and is represented by \( \pi_i = P(Q_1 = i) \) where \( i \in \{1, \ldots, m\} \)
- \( P(Q_t | Q_{t-1}) \) is the state transition probability represented by the matrix \( X = \{x_{ij}\} = P(Q_t = j | Q_{t-1} = i) \) where \( i, j \in \{1, \ldots, m\} \)
- \( P(O_t | Q_t) \) is the observation probability represented by the gaussian distribution \( B_i(o_t) = P(O_t = o_t | Q_t = i) = \frac{1}{\sqrt{2\pi} \sigma_{o_t}} \exp[-1/(2\sigma_{o_t})^2] \) with means \( \mu_{i,j} \in \mathbb{R}^T \) and covariance matrix \( R = \{r_{ij}\} \in \mathbb{R}^{T \times T} \).

The model can be completely described by parameters \( \theta = \{\pi, X, \mu, R\} \). The initial phase involves learning these parameters \( \theta \) to generate the model using training data consisting of observations of the expected state space. We employ the Baum-Welch algorithm (a version of Expectation Maximization (EM)) to train the model parameters. After the training phase we use the Forward Algorithm to estimate the state of the model online using the current observations.

We use a two state HMM which represents two distinct thoughts of the user, so \( m = 2 \) in our case. The observation space consists of the EEG output \( U(t) \). In this case, the signal \( U(t) \) is derived from performance metrics provided by the manufacturer of the EEG headset, Emotiv. Emotiv provides six metrics: "Engagement", "Meditation", "Excitement", "Frustration", "Valence" and "Long-Term Excitement" out of which we use the first three metrics, which makes \( l = 3 \) and also \( O_t \in [0, 1] \) for all metrics.

2) The Training Phase with Baum-Welch Algorithm: Training data consisting of the three metrics mentioned before is recorded in a single trial. During the training period the user repeats two thoughts through a pre-defined switching sequence. We enforced this thought sequence using a timed slide presentation which the user observed during training. This training data is fed into the Baum-Welch algorithm which estimates the parameters \( \theta \) for the system.

The EM Algorithm is a two step iterative process (Expectation followed by Maximization), which can be expressed in the single expression

\[
\theta_{k+1} = \arg\max_{\theta} E_{P(Q,O|\theta_k)}[\mathcal{L}(Q, \theta)],
\]  
(3)

where \( \mathcal{L}(Q, \theta) = \ln P(Q,O|\theta) \) is the log likelihood function, \( Q \) is the unobserved state, \( \theta \) is the unknown parameters of the model, and \( O \) is the observed variable. The expectation step can be summarized by calculating the following quantities:

\[
\alpha_i(t) = P(O_t = o_t, \ldots, O_T = o_T | Q_T = i, \theta),
\]  
(4)

where \( \alpha_i(t) \) is known as the forward variable, and is the probability of ending in state \( i \) and seeing the partial observations \( \{o_1, \ldots, o_i\} \) given the model parameters \( \theta \), and

\[
\beta_i(t) = P(O_{t+1} = o_{t+1}, \ldots, O_T = o_T | Q_T = i, \theta),
\]  
(5)

where \( \beta_i(t) \) is known as the backward variable, and is the probability of observing partial sequences \( \{o_t, \ldots, o_T\} \) given the model parameters and state at time \( t \). With \( \alpha \) and \( \beta \) we can compute

\[
\gamma_i(t) = P(Q_t = i | O, \theta) = \frac{\alpha_i(t) \beta_i(t)}{\sum_{j=1}^{m} \alpha_j(t) \beta_j(t)},
\]  
(6)

where \( \gamma_i(t) \) is the probability of being in state \( i \) given the model parameters and observations, as well as

\[
\zeta_{ij}(t) = \frac{P(O_t = i, Q_{t+1} = j, O | \theta)}{P(O | \theta)} = \frac{\alpha_i(t) x_{ij} B_j(o_j+1) \beta_j(t+1)}{\sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i(t) x_{ij} B_j(o_j+1) \beta_j(t+1)},
\]  
(7)

where \( \zeta_{ij}(t) \) is the probability of being in state \( i \) at time \( t \) and state \( j \) at time \( t+1 \).

Using the above defined quantities in the expectation step we can estimate the parameters \( \theta \) in the maximization step of our system as follows:

\[
\mu_{i+1} = \frac{\sum_{t=0}^{T} y_{i}^{(t)}(o_t) o_t}{\sum_{t=0}^{T} y_{i}^{(t)}}
\]  
(8)

\[
R_{i+1} = \frac{\sum_{t=0}^{T} y_{i}^{(t)}(o_t - \mu_{i+1})(o_t - \mu_{i+1})^T}{\sum_{t=0}^{T} y_{i}^{(t)}}
\]  
(9)

\[
X_{i+1} = \frac{\sum_{t=1}^{T} \zeta_{ij}(t-1)}{\sum_{t=1}^{T} y_{i}^{t-1}}
\]  
(10)

\[
\pi_{i+1} = \gamma_i^{(0)}.
\]  
(11)

Naturally, for this procedure to start we need \( \theta_0 \) which is the set of initial model parameters from which recursion begins according to Eqn. 3. We adopt a K-means clustering approach to initialize the model parameters, specifically the mean matrix \( \mu \). We used a two class K-means approach with the observations \( O \) as inputs, which give us the centroids of the two classes that we used to initialize \( \mu \). The other parameters of \( \theta \) are initialized randomly. We stop the iterative process when we observe only minute changes (order of \( 10^{-4} \)) in the estimated parameters from their previous estimations. A typical training signal from our experiments is shown in Fig. 3(a) and the resulting state sequence after
Specifically, we do not know whether we have to relate the HMM states back to the original thoughts. However, we use during training to the thought sequences visited by the user. We assign the abstract states to meaningful "aggregate" thought and "disperse" thought by the user.

The output $h$ is used to determine the control parameters $(a(t), b(t))$ for aggregation and dispersion of the swarm, as described in Sec. III-C below.

### B. Tracking Eye Movements

In traditional EEG research, Eye Movement signals are considered as artifacts and are removed. In contrast, we use these signals as inputs for our system to command the direction of travel for the robots. There are various available methods in the literature for detecting and tracking eye movements, which vary considerably [14]. These methods can be broadly categorized into (i) Contact based tracking which offer high accuracy and sophistication, (ii) Non-contact based optical tracking methods which measure relative positioning remotely with sensors such as cameras, and (iii) measuring surface electrical potentials from skin, also known as Electrooculogram (EOG), near the eyes. Our EEG headset detects these EOG signals related to eye movement, hence we can detect eye movement with no additional hardware.

1) **EOG : An Overview:** The human eye can be modeled as an electrical dipole whose axis is roughly collinear to the axis of the human eye. The electrical dipole rotates with the rotation of the eye causing small differences (in microvolts) between the electrical potential at the skin surface depending on eye position. The order of magnitude of these signals are much larger than signals due to brain activity evident from Fig. 3 and Fig. 5, hence they can be measured and contrasted easily.

EOG typically uses exclusive electrodes around the eyes to measure movements. But our electrode positions are fixed so we employ the four closest electrodes to the eyes: ‘AF3’, ‘AF4’, ‘F7’ and ‘F8’, according to the 10-20 EEG sensor placement system, as shown in the diagram in Fig. 4. Previous methods to detect eye motion have relied on complex classification based algorithms. In contrast, our method uses a simple statistical calculation.

The spatio-temporal signals from these electrodes near the eyes can be described by $u_i(t) \in \mathbb{R}$ where $i \in \{AF3, AF4, F7, F8\}$ denoting the electrodes used. We first normalize the signal by subtracting its mean for each electrode to center the signals about zero. This can be described by

$$u_i(t) = \frac{\sum_{\tau=0}^{\tau} u_i}{\tau},$$

where $\tau$ denotes the number of samples used for the baseline removal.
2) **Horizontal Eye Movement Detection:** Electrodes 'F7' and 'F8' are chosen for horizontal eye movement detection as they are the farthest apart in the horizontal plane while being closest to the eyes. Our algorithm for decoding horizontal directional movement depicted in Fig. 5 is described in Algorithm 1. In Fig. 5, the green ellipses indicate the signal for leftward eye movement and the blue ellipses indicate rightward eye movement. The red ellipse represents blinks which are filtered out.

![Algorithm 1 Horizontal Eye Movement Detection](image)

![Algorithm 2 Vertical Eye Movement Detection](image)

3) **Vertical Eye Movement Detection:** Electrodes 'AF3' and 'AF4' are chosen for vertical eye movement detection. The method used is different from horizontal eye movement detection since we do not have any electrode below the eyes to detect the dipoles in the vertical plane of the head. Eye movement upwards results in a positive deflection in both electrodes whereas eye movement downwards has negative deflection for both electrodes. Our algorithm for decoding vertical eye movement is depicted in Fig. 6 and described in Algorithm 2. In Fig. 6, the green ellipses indicate the signal for vertical eye movement, and the blue ellipses indicate horizontal movement. The red ellipse represents blinks which are filtered out. It should be noted that both the algorithms above also filter out blinks, which look quite similar to the eye movements (see Figs. 6 and 5). In the case of horizontal eye movements, the F7 and F8 electrodes both record blinks with almost equal magnitude since they are located approximately at the same distance from the eyes. Hence the signal \( u_{F7,F8}^\eta(t) \) is automatically devoid of blinks (Eqn 15), as evident from Fig 5. For the vertical eye movement detection, we introduce an upper threshold to filter out the blinks, which typically are much larger than eye movement signals, as can be seen in Fig. 5. Hence, our algorithm effectively discards blinks and measures only the user’s intentional eye movements.

![EEG sensor placement on the human scalp using the 10-20 system. The locations highlighted in orange depict the locations used by the Emotiv EpoC headset](image)

Finally, after the left-right and up-down signals have been extracted form the user’s eye movements, these signals are used to control the left-right and forward-backward motion of the robot swarm through the control parameter \( v(t) \), as described below in Sec. III-C.

### C. Formation Control

We described the general form of the potential field based formation controller above in Sec. II. We refer the reader to [16], [15], [17] for details and proofs about stability and convergence properties. Here we describe how we control the size and motion of the swarm through the parameters \( a, b, v \).

1) **Controlling Size:** Recall that the system is described by the two dimensional state space equation for the \( i^{th} \) agent:

\[
x_i = \sum_{j=1}^{M} f_{ab}(x_i, x_j) + v.
\]

We let the interaction between robots \( i \) and \( j \) be given by

\[
f_{ab}(x_i, x_j) = \frac{a(x_i - x_j)}{\|x_j - x_i\| - 2r} - \frac{b(x_j - x_i)}{\|x_j - x_i\| - 2r}^3
\]

Where \( r \) is the radius of the robot. We can see that the left term provides the attracting field, and the right the repelling field. The \( r \) term introduces a safety region around the robots so collision can be avoided.

There is an equilibrium inter-robot distance for this system, in which attraction and repulsion forces balance. Let
that equilibrium distance be denoted $\delta$, so that $g_a(\|\delta\|) = g_r(\|\delta\|)$. This $\delta$ is governed by the attraction and repulsion gains $a$ and $b$ respectively. In our method we vary the gains to achieve different equilibrium formation. We map the two state output from the HMM to two distinct sets of gains in order to achieve the aggregation and dispersion of the swarm.

2) Controlling size and motion: Now to control the motion of the swarm we rely on the output from the eye movement detection which gives us four possible motion commands: Forward, Backward, Left, and Right. We use these command to assign values to the vector $v$, which drives every robot in the swarm in the same direction. Depending on the eye movement the vector $v$ is assigned preset values which makes all the agents in the swarm move locally in the direction of $v$ independent of the swarms aggregation or dispersion.

IV. SIMULATIONS AND HARDWARE EXPERIMENTS

To demonstrate our brain-swarm interface, we developed a simulation environment in Matlab. We chose a section of the Boston University campus, with a rectangular path around a campus building, as shown in Fig. 8. The path is divided into 4 edges and the swarm has to be driven starting from the left of edge 1 and end on the top of edge 4 following a clockwise motion. At edge 3 (purple path) due to the narrow passageway, the user has to make the swarm aggregate into a tighter swarm by switching thoughts, while in edges 1, 2 and 4 (Blue path) the user makes the swarm disperse.

For the training phase, the user switched between two thoughts at least twice over a period of 60 seconds, during which the EEG signals were recorded and fed into the Baum-Welch Algorithm (Fig. 3) to get the model parameters as described in Sec. III-A. The thoughts used for simulations and experiment were distinct and repeatable: the disperse state was invoked with a relaxed neutral thought, while the aggregate state was invoked by a mentally challenging task (in this case calculating the Fibonacci series). Then EEG signals were streamed live and processed as discussed previously to generate the control inputs $\Theta$.

The simulated swarm consisted of 128 point sized holonomic robots. The attraction gain $a$ was fixed at 1 and the repulsion gain $b$ was calculated according to $b(t) = h(t) * M/2.625$, where $h(t)$ is the estimated state sequence from (12). In our 2 state HMM case according to the previous formula the user’s thought corresponding to state 1 causes the robots to disperse and increase swarm size, and for state 2 causes the robots to converge and aggregate to a smaller size. The swarm reaches its equilibrium size for a particular thought state and stays at that size until the user switches thoughts.

The results of the simulation exercise is summarized in Figs. 7 and 8. Fig. 7 shows the time history of eye movement detection and the mental thought estimation during the motion of the swarm along the 4 legs of the path. It can be seen that the thought estimation remains mostly in the disperse state during legs 1, 2, and 4, and is mostly in the
aggregate state in Sector 3, as intended. Minor inaccuracies can be attributed not only to the stochastic nature of the HMM, but also to the quality of user’s thoughts and noise in the EEG headset. From Fig. 8 which shows the path of the centroid of the swarm, we can see the Eye Movement detection is successful in steering the swarm, with a few misclassifications due to the nature of the noisy EEG signals and non-intentional eye movements. We refer the reader to the accompanying video.

For the hardware experiments we used the m3pi platform with an Mbed controller for mobile swarming robots, and Zigbee radios for communication. The experiments were carried out in an environment with an Optitrack motion capture system to track the motion of the robots (see Fig. 9). The control parameters used were $a = 4$ and $b = 80$ for aggregation, and $a = 2$ and $b = 80$ for dispersion. These two wheeled differential drive robots receive individual motor speeds as control inputs from the computer. A proportional point-offset controller is used to generate the motor speeds from the potential function controller (the details of which can be found in [18]). The control commands for the robots were computed off board the robots, and sent to the robots over Zigbee at an update rate of 30 Hz. Due to computational and hardware complexities the computations were divided among 3 computers (one for Optitrack data acquisition, one for controller implementation, and another other for EEG signal processing and video recording) as shown in the experimental setup in Fig. 10. The tcp/ip protocol was used for communication among them. The experimental area (Fig. 12) was chosen to be a rectangular area divided into 4 legs, similarly to the simulation. In sector 3 the user again must make the swarm aggregate, and in the other legs the swarm should disperse, while navigating in a clockwise manner starting from sector 1. The user obtained visual feedback of the position of the swarm by viewing a live feed of the experimental area from a GoPro camera on a mobile device.

The experimental results are summarized in Figs. 11 and 12. Fig. 11 shows the time history of eye movement detection and the thought state estimation throughout the motion of the swarm along the 4 legs. From Fig. 8 which shows the path of the robots and their centroid, we can see the eye movement detection is successful in steering the swarm. The color map for the thought state estimation again shows that the HMM is able to reliably determine the user’s intention. The system remains in the disperse state with high confidence during legs 1, 2, and 4, and in the aggregate state with high confidence during sector 3. We again refer the reader to the accompanying experiment video.

The biggest challenge in this work is the integration of this complex system with interacting hardware, communication, software, and human components. We used holonomic dynamics during simulation whereas The M3pis are nonholonomic robots with inefficient actuation and communication. In addition, it was quite a mental challenge for the user to concentrate on thoughts, eye movement, and system monitor-
plan to extend the algorithm to control a swarm of quadrotors robots in 3 dimensional space.

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