Plasmon-graviton conversion in a magnetic field in TeV-scale gravity.

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Kaluza-Klein (KK) gravitons emission rates due to plasmon-graviton conversion in magnetic field are computed within the ADD model of TeV-scale gravity. Plasma is described in the kinetic approach as the system of charged particles and Maxwell field both confined on the brane. Interaction with multidimensional gravity living in the bulk with n compact extra dimensions is introduced within the linearized theory. Plasma collective effects enter through the two-point correlation function of the fluctuations of the energy-momentum tensor. The estimate for magnetic stars is presented leading to the lower limit of the D-dimensional Plank mass.

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I. INTRODUCTION

Photon-graviton conversion in magnetic field in four-dimensional space-time \( \ ] \) was discussed earlier in a number of papers including the monograph \( \ ] \). It was argued that plasma effects will make the photon (plasmon) – graviton conversion process astrophysically negligible, because the characteristic length of photon-graviton oscillations

\[
_l = \frac{4\pi \omega}{\omega_L} \ll _L, \quad \text{where} \quad \omega_L = \sqrt{\frac{4\pi^2 e^2 n}{m}}
\]

is the plasma electron Langmuir frequency will be much smaller than typical homogeneity distance of magnetic field \( _L \). In fact, the resonance is destroyed since the dispersion relation for the graviton is \( \omega = k \), while for the plasmon \( \omega^2 = k^2 + \omega_L^2 \).

Recently, a lot of interest has been attracted to models with large extra-dimensions \( \ [ \) \[ \ ] \). According to one such scenario, called ADD, the standard model particles live in the four-dimensional subspace (the brane) of the D-dimensional bulk with \( n = D - 4 \) extra dimensions compactified on a torus which are inhabited only by gravity. The D-dimensional Planck mass \( _p = \) is supposed to be TeV-scale, and the large extra dimensions (LED) to have sub-millimeter size. In this scenario there are KK gravitons which are seen in four dimensions as a tower of massive particles (the mass \( _m \) ) with the dispersion relation \( \omega^2 = k^2 + _m^2 \). This can restore the resonance condition for plasmon-graviton conversion in magnetic field if \( \omega_L = _m \).

II. SELF-CONSISTENT KINETIC THEORY FOR MAGNETIZED PLASMA

Consider collisionless plasma consisting of charged particles of types \( \alpha \) with charges and masses \( e_\alpha, _m_\alpha \) described by the microscopic distribution function \( \frac{N_\alpha}{F_\alpha(x,p)} = \sum_{i=1}^{N_\alpha} \delta(r - r_i(t))\delta(p - p_i(t)) \),

\[
f_\alpha(x,p) = \sum_{i=1}^{N_\alpha} \delta(r - r_i(t))\delta(p - p_i(t)), \quad (1)
\]

normalized as \( \int f_\alpha(x,p) d^3x = _N_\alpha \), and satisfying the kinetic equation

\[
\frac{\partial F_\alpha}{\partial t} + v B - e_\alpha E + [v B] \frac{\partial F_\alpha}{\partial p} = 0. \quad (2)
\]

They move in the magnetostatic field \( _B \) and interact via electromagnetic fluctuating field \( E, B \) satisfying the Maxwell equations

\[
\text{Div} E = 4\pi \rho, \quad \text{Rot} E = - \frac{\partial B}{\partial t},
\]

\[
\text{Div} B = 0, \quad \text{Rot} B = 4\pi j + \frac{\partial E}{\partial t}, \quad (3)
\]

with the source terms

\[
\rho(x) = \sum_{\alpha} e_\alpha \int F_\alpha(x,p) dp,
\]

\[
j(x) = \sum_{\alpha} e_\alpha \int \frac{p}{p_\alpha} F_\alpha(x,p) dp. \quad (4)
\]

To solve the system of equations \( \ ) \) we use perturbation theory in terms of the electric charges. First we separate fluctuations from the average distribution using the approach of \( \ ) \:

\[
F(t,r,p) = f_0 + \delta f_0 + \delta f_\alpha, \quad (5)
\]

where \( f_0 \equiv \langle F_\alpha(x,p) \rangle \) - the equilibrium Maxwell distribution function:

\[
f_\alpha = N_\alpha \left( \frac{m}{2\pi T_\alpha} \right)^{\frac{3}{2}} \sqrt{\frac{T_\alpha}{T_\alpha}} \left( e^{\frac{m^2}{2T_\alpha}} - 1 \right), \quad (6)
\]

where the symbols \( \parallel, \perp \) are introduced for transversal and longitudinal thermal velocities with respect to the

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magnetostatic field. Here $\delta f^0_\alpha$ represents fluctuations due to chaotic particle motion ("zero" fluctuations), while $\delta f_\alpha(t, r, p)$ stands for fluctuations arising due to their electromagnetic interaction. Using the fact that $f^0_\alpha$ satisfies the free kinetic equation, we obtain:

$$\frac{\partial \delta f_\alpha}{\partial t} + v \frac{\partial \delta f_\alpha}{\partial r} + e_\alpha(E + \|vB\|) \frac{\partial (f^0_\alpha + \delta f_\alpha)}{\partial p} = 0. \quad (7)$$

Using the Fourier-transformation

$$f(x) = \frac{1}{(2\pi)^3} \int f(k)e^{-ikx}d^3k,$$

the solution of the self-consistent system equations [2] in linear approximation is well known [6-8].

Maxwell equation may be written as

$$\Lambda_{ij}J_i = -\frac{4\pi i}{\omega} J_j,$$

$$\delta f_\alpha(x) = \sum_\alpha e_\alpha \int v \delta f^0_\alpha(x, p)d^3p. \quad (8)$$

Here

$$\Lambda_{ij} = \varepsilon_{ij} - \frac{k^2}{\omega^2} (\delta_{ij} - k_i k_j), \quad (9)$$

where $\varepsilon_{ij}$ is the permittivity tensor, $\hat{k} = \frac{k}{|k|}$, $j^0$ - "zero" fluctuation current density.

For cold plasma in magnetic field satisfying the conditions

$$\frac{k_\perp v_T}{\omega B_0} \ll 1, \quad \frac{k_\perp v_T}{\omega} \ll 1, \quad \frac{\omega \pm \omega B_0}{k_\perp v_T} \gg 1. \quad (10)$$

the permittivity tensor may be presented as:

$$\varepsilon_{ij} = \begin{pmatrix} \varepsilon_\perp & ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix},$$

where

$$\varepsilon_\perp = \varepsilon_{xx} = \varepsilon_{yy} = 1 - \sum_\alpha \frac{\omega^2_\alpha}{\omega^2 - \omega B_0^2},$$

$$\varepsilon_\parallel = \varepsilon_{zz} = 1 - \sum_\alpha \frac{\omega^2_\alpha}{\omega^2},$$

$$\varepsilon_{xy} = -\varepsilon_{yx} = ig = -i \sum_\alpha \frac{\omega^2_\alpha \omega B_0}{\omega(\omega^2 - \omega B_0^2)},$$

$$\varepsilon_{xx} = \varepsilon_{zz} = \varepsilon_{yy} = \varepsilon_{yy} = 0. \quad (11)$$

The average of fluctuational field will be zero, while the quadratic correlation functions are non-zero [9]. Fluctuations can be characterized by correlation functions whose Fourier-transforms exhibit homogeneity, e.g.

$$< j^0_i(r_1, t_1) j^0_j(r_2, t_2) > \equiv < j_i j_j >_{r, t}, \quad r = r_1 - r_2, \quad t = t_1 - t_2. \quad (12)$$

The space-time Fourier transformation will give the spectra of current fluctuations:

$$< j_i j_j >_{k, t} = \int d^3r e^{-ikr + i\omega t} < j_i j_j >_{r, t},$$

$$< j^0_i(k, \omega) j^0_j(k', \omega') > = (2\pi)^4 < j_i j_j >_{k, \omega} \delta(\omega - \omega'). \quad (13)$$

Correlation function for "zero" current $< j_i j_j >_{k, \omega}$ plays the most important role in the magnetized plasma. For the Maxwell distribution function [6] it reads:

$$< j_i j_j >_{k, \omega} = 2\pi\epsilon^2 \int d^3v \sum_n \Pi_{ij} \delta(\omega - k||v|| - l\omega) f_0(v) \quad (14)$$

where $b_\alpha = \frac{k_w}{omega}$. Correlation functions for fluctuational fields may be expressed in terms of zero current correlation function. Due to the equations [8], the following relation holds between the spectral functions $< j_i j_j >_{k, \omega}$ and $< E_i E_j >_{k, \omega}$:

$$< E_i E_j > = \frac{16\pi^2 \lambda_{ip} \lambda_{ip}^*}{\omega^2} \lambda_{ip}^* < j_p j_p^* >, \quad \lambda_{ip} = \Lambda_{ip}^{-1}, \quad \lambda_{ip}^* = \delta_{ps}, \quad (15)$$

where $\lambda_{ij}$ - algebraic adjunct of matrix $\Lambda_{ij}$, $\Lambda = \det|\Lambda_{ij}|$.

**III. PLASMON-GRAVITON TRANSFORMATION**

Formula for the total (integrated over space and time) energy loss on gravitational radiation in ADD model for linearized $D = 4 + d$ dimensional Einstein equations was recently derived in [10]. The corresponding expression for the gravitational emission rate to all graviton KK modes per unit plasma volume per unit time in the stochastic plasma system was given in [11]:

$$P = \frac{\kappa_D^2}{16\pi^2 V_d} \sum_{N \in Z^d} \int dk < \delta T_{ik} \delta T_{i'k'} >_{k} \tilde{\Lambda}^{ikl'}k' \quad |_{\omega = k^0},$$

$$k^0 = \sqrt{|k|^2 + M^2}, \quad V_d = (2\pi L)^{D-4}, \quad \tilde{\Lambda}^{ikl}k' = \frac{1}{2} [\Delta^{i'} \Delta^{kk'} + \Delta^{ik'} \Delta^{kk'}] - \frac{M^2}{D-2} \Delta^{ik} \Delta^{i'k'}, \quad (15)$$

$$\Delta_{ik} = \delta_{ik} - n_i n_k, \quad n = \frac{k}{k^0}, \quad M^2 = (2\pi N/L)^2, \quad (16)$$

where $L$ is the size of the extra dimensions, $\kappa_D^2 = 16\pi G_D$, $G_D = M_D^{-(d+2)}$. In our approach we did not use explicit decomposition of the full multidimensional
metric perturbation $h_{MN}$ in massless and massive modes, these correspond to $N = 0$ and $N \neq 0$ terms in the sum respectively. For massless modes the three-dimensional vector $n$ is the unit vector, but not for massive modes. Note that $h_{MN}$ has bulk components due to the trace term in the wave equation.

The gravitational radiation due to plasmon-graviton transformation possess will be generated by the field part of the energy-momentum tensor:

$$fT_{ij} = -\frac{1}{4\pi} (B_i B_{0j} + B_{0i} B_j - \delta_{ij} (B B_0)),\quad B_i = \frac{1}{\omega} \epsilon_{ijk} k_j E_k,$$

(17)

where $B, E$ are the first order fluctuation fields, $\epsilon_{ijk}$ is unit antisymmetric tensor.

After substitution of (17) into the expression for the radiation power and contraction with the projector $\Lambda_{ik'i'}$, the radiation per unit time per unit volume may be presented as

$$P = \frac{2\pi}{8\pi^2 V_d} \sum_{N \in \mathbb{Z}_d} \int dk \mu^2 \left( (B_0^2 - (n B_0)^2) (\delta_{ss'} - \tilde{k}_s \tilde{k}_{s'}) + \frac{1}{2} (D - 4) [(n^2 - 1)^2 + 8] + [(n^2 + 3)^2 - 16] \right),$$

$$|B_0 k_s| |B_0 \tilde{k}_{s'}| = <E_0 E_{s'}>, \quad \mathbf{k} = \frac{k}{k}, \quad (18)$$

1. Transversal plasmon $\omega^2 = k^2 + \omega_L^2$.

The detailed dispersion relation in a strongly magnetized plasma is more complicated than the simple relation $\omega^2 = k^2 + \omega_B^2$, that was used in the early papers on this subject [12, 13]. The condition of existence of plasma waves (coherent fluctuations) is $Re \Lambda = 0$. In magnetized "cold" plasma the dispersion relation for the coherent fluctuations are resolved analytically only for certain sites of a spectrum, at certain direction with respect to the magnetic field and in general there are not strictly longitudinal or transversal waves. The ordinary transverse high frequency electron wave $\omega^2 = k^2 + \omega_L^2$ propagates strongly across the magnetic field:

$$\omega^2 = k^2 + \omega_L^2, \quad k_\perp = k,$$

$$k_z \ll \frac{\omega_0 (\omega^2 - \omega_B^2)}{\omega_L^2 \omega_B^2}. \quad (19)$$

The field correlation function for coherent fluctuation may be rewritten as

$$<E_i E_j> = \frac{16\pi^3}{\omega^2} \lambda_{ip} \lambda_{jp'} \delta (Re \Lambda) \frac{\lambda_{ip} \lambda_{jp'}}{\lambda_{ip} \lambda_{jp'}}, \quad >j_{p'} > 0, \quad (20)$$

where $|Im \Lambda|$ is determined by the small thermal terms of the anti-hermitean part of the permittivity tensor

$$\delta \varepsilon_{xx}^a = \delta \varepsilon_{zz}^a,$$

$$\delta \varepsilon_{yy}^a = \delta \varepsilon_{xy}^a,$$

$$\delta g^a = \delta g^a + i \sqrt{2\pi} \sum_\alpha \frac{\omega_0^2}{\omega} |\tilde{k}_s \tilde{k}_{s'}| \omega \omega_B^2.$$

According to the condition (19), the imaginary part of the determinant of the Maxwell tensor is equal to

$$Im \Lambda = \left( \varepsilon_1^2 - g^2 - \frac{k^2}{\omega^2} \varepsilon_1 \right) \delta \varepsilon_{||}, \quad (22)$$

and the only nonzero term of the correlation function is

$$< j_z j_z > = \frac{4e^2 N_0}{\pi m} \sqrt{T \omega \omega B \delta \varepsilon_{||}}, \quad (23)$$

and single term for adjunct tensor $\lambda_{zz}$ is

$$\lambda_{zz} = \varepsilon_1^2 - g^2 - \frac{k^2}{\omega^2} \varepsilon_1 = \frac{\omega_0^2 \omega_B^2}{\omega^2 (\omega^2 - \omega_B^2)}. \quad (24)$$

The delta-function entering the expression (20) for the transverse plasmon with $\omega^2 = k^2 + \omega_L^2$ may be presented as

$$\delta(Re \Lambda) = \frac{\omega^4 (\omega^2 - \omega_B^2)}{\omega_0^2 \omega_B^2 |\omega^2 - k^2 + \omega_L^2|^2} \delta (\omega^2 - k^2 - \omega_L^2). \quad (25)$$

Radiation rate (18) for the transformation possess may be then rewritten as

$$P = \frac{2\pi^2}{8\pi^2 V_d} \sum_{N \in \mathbb{Z}_d} \int dk \mu^2 \left( k^2 + \omega_L^2 \right) \delta (M^2 - \omega_L^2). \quad (26)$$

The next step is to sum over the KK modes. Assuming that a large number of modes is excited, one can replace the summation over $N$ by integration and integrating over $k_z$ taking into account the condition (19) we obtain:

$$P = \frac{2\pi^2}{\Gamma \left( \frac{d}{2} \right)} \frac{B_0^2 \sqrt{T \omega B \omega_L^2}}{\omega_B^2} \int \frac{dk}{k^2 + \omega_L^2 \omega_B^2}. \quad (27)$$

Here according to the conditions $\varepsilon_1^2 \omega_1 = < T$ we introduced the cutoff parameter $k_{max} = \frac{1}{\omega_{min}}$.  

2. Astrophysical estimates.

After integration over $k$ let us rewrite expression (27) in the CGSE system of units and for an estimate assume
that $T_{\perp} \simeq T_{\parallel} \equiv T$:

$$P \simeq \frac{6 \cdot 2^{13} \pi^{2}}{7} \frac{B_{0}^{2} \omega_{L}^{2} T^{8}}{M^{d+2} \omega_{B} \alpha^{2}} \frac{\text{erg}}{\text{cm}^{3}s}. \quad (28)$$

Here $\alpha = \frac{1}{137}$, and the Langmuir electron frequency $\omega_{L}$, the cyclotron frequency $\omega_{B}$, the electron temperature $T$ and the D-dimensional Plank mass $M_{D}$ have to be expressed in eV.

In the ADD scenario the massive Kaluza- Klein (KK) gravitons contribute to energy losses of astrophysical objects [14]–[21] leading to bounds on $M_{D}$. Let us estimate contribution of plasmon-graviton transformation in neutron stars using data [22] for millisecond pulsar J0437-4715: $B = 10^{9}$ gauss, $N \sim 10^{24} \text{cm}^{-3}$, $T = 10^{5.6}$ K.

The conservative upper limit of the energy-loss rate of magnetic neutron stars

$$\dot{\varepsilon} \sim 0.5 \cdot 10^{14} \frac{\text{erg}}{\text{cm}^{3}s},$$

gives the following lower bounds for the D-dimensional Plank mass: $M_{D} \sim 1.3 \text{TeV}$ for $d = 2$, $M_{D} \sim 1.09 \cdot 10^{-2} \text{TeV}$ for $d = 3$ and $M_{D} \sim 3.8 \cdot 10^{-4} \text{TeV}$ for $d = 4$. Thus, the resonant conversion plasmon-graviton in magnetic stars has to be included into the list of basic mechanisms leading to astrophysical restrictions on the parameters of the ADD model [14]–[21].

### IV. CONCLUSION

We presented the calculation of the energy-loss rate due to the emission of KK gravitons via plasmon-graviton transformation processes in the magnetized non-relativistic plasma. The calculation was made by using the kinetic approach taking into account the collective effects. The estimates show that this process can compete with other mechanisms of the KK dissipation in magnetic stars such as bremsstrahlung.

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