Amplitude scaling behavior of band center states of Frenkel exciton chains with correlated off-diagonal disorder

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Abstract

We report the amplitude scaling behavior of Frenkel exciton chains with nearest-neighbor correlated off-diagonal random interactions. The band center spectrum and its localization properties are investigated through the integrated density of states and the inverse localization length. The correlated random interactions are produced through a binary sequence similar to the interactions in spin glass chains. We produced sets of data with different interaction strength and "wrong" sign concentrations that collapsed after scaling to the predictions of a theory developed earlier for Dirac fermions with random-varying mass. We found good agreement as the energy approaches the band center for a wide range of concentrations. We have also established the concentration dependence of the lowest order expansion

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coefficient of the scaling amplitudes for the correlated case. The correlation causes unusual behavior of the spectra, i.e., deviations from the Dyson-type singularity.

1 Introduction

Low-dimensional disordered excitonic systems have generated renewed interest recently. For example, the possibility of using optically active polymers in various photonic applications such as flat panel displays and light emitting diodes has led to increased interest in their electronic and optical properties \[1,2,4\]. We study here numerically the dynamics of Frenkel excitons near the band center in a system with nearest-neighbor off-diagonal random interactions. This study is, we believe, the first to treat correlated disorder in the off-diagonal dipolar interactions in an exciton system which focuses on the behavior near band center \( E = 0 \). However, an analysis of a similar problem in the correlated electronic system has appeared \[5\] but the authors considered correlations those were different from ours, and the interactions were not of the dipolar type \[3\]. Many efforts have also been spent on the same off-diagonal tight-binding system using supersymmetric methods (SUSY) \[6\] where the interactions are formally similar to our dipolar interactions but the authors eventually worked with continuous variables whereas ours are discrete. Later, correlated disorder, in particular the exponential type, was included for the same tight binding model using the SUSY \[7,8\]. Our results can thus be compared with what has been developed for the SUSY methods. This study is important because most realistic random disordered systems have nonlocal correlations.

The nature of the excitations at the band center of random off-diagonal exciton models has been discussed for a long time, and it has been found that the spectrum has a singularity of the Dyson type \[9\] in which spectral properties in the vicinity of the band center are functions of \( \ln(E) \) where \( E \) is the energy relative to the center of the band.
The question of the band center localization has also been argued at length, and it was claimed that the band center mode may be weakly localized because of the strong fluctuations \[11,12,10\]. Ziman \[13,14\] analyzing various uncorrelated distributions of the random coupling found the low-energy behavior of the integrated density of states (IDOS)

\[
IDOS - \frac{1}{2} = \frac{V^U}{|\ln(E^2)|^2},
\]

and the inverse localization length (ILL)

\[
ILL = \frac{V^U}{|\ln(E^2)|},
\]

in which \(E\) is the energy and the logarithmic variance \[13,14\] \(V^U = \frac{1}{2} \left[ \langle (\ln(J^2))^2 \rangle - \langle \ln(J^2) \rangle^2 \right] \) is the amplitude factor, \(J\) being the off-diagonal coupling specified below. For definitions of the ILL and the IDOS see Ziman’s work \[13\] particularly his discussion after the equation of the motion. Notice that the averages above are independent of the lattice site \(n\) since uncorrelated random variables are involved; otherwise they are site dependent. Concerning the shape of the IDOS and the ILL, according to Ziman’s analysis \[13\], they should depend on disorder only through a multiplicative constant, the variance in the distribution of \(\ln(J^2)\). As a result, the spectra near \(E = 0\) have the same form for all sets of independent, identically distributed random variables \(J\). In this context, such a property is referred to as scaling. When there is scaling, the data can be collapsed onto a single curve by dividing through by a factor that depends on the properties of the disorder but is independent of the energy. Although the above mentioned results were developed for uncorrelated disorder, our analysis shows that qualitatively similar scaling behavior for limited range of concentrations and energy can be obtained for a system with exponential correlations but with a modified amplitude factor \(V^C\). Moreover, for a certain range of correlation lengths a higher order expansion of the Dyson singularity can reproduce the numerical data.
better than just a single term.

A disordered 1D exciton model with nearest-neighbor interactions has the following equation of motion

\[ J_{n+1}U_{n+1} + J_{n-1}U_{n-1} = EU_n, \]  

(1.3)

where \( U_n \) denotes the usual exciton creation operator and \( J_n \) are the couplings. This is equivalent to the XY model in a strong applied field in the \( z \)-direction (\( < S_z > = S \)). In our numerics we calculated iteratively the Lyapunov exponent \([13,14]\) \( \gamma(E) = \frac{1}{N} \ln \frac{U_N}{U_1} \) (see appendix), where \( N \) is the number of sites \([13,14]\), whose real part is related to the ILL while the imaginary is related to the IDOS. We consider couplings that have correlated or uncorrelated (for comparison) disorder and are of a dipolar type.

Since the displacements of the atoms in quasi-one dimensional systems are likely to be small compared to the lattice constants, it is usually enough to consider the first order term in the expansion of \( J \) in terms of the displacements \([15]\). For the first order expanded coupling we have \( J_n = 1 + A \xi_n \) with unit lattice constant where \( A \) is the strength of the disorder and \( \xi_n \) is a random sign variable that is exponentially correlated \( < \xi_n \xi_m > = e^{-|m-n|/l(c)} \) where \( l(c) \) is the correlation length as a function of ”wrong sign” concentration \( c \) (see below). Thus the dipolar coupling \( J_n \) has a non fluctuating part and a fluctuating part as in Dirac fermions with random-varying mass \([7,8]\).

The generation of random binary sequences with correlation was devised by implementing the convolution method of constructing a random sequence. Such sequence can be used in various applications as in designing low-dimensional devices with desired properties \([7,16]\). A detailed discussion on this topic recently appeared \([17]\). However, a more efficient and easily handled correlated sequence, related to the spin glass chain problem \([18,19]\), can be obtained through uncorrelated random numbers as outlined below. The correlated random number at site \( n \) follows the relation
\[ \xi_n = \xi_{n-1} x_n = x_1 x_2 x_3 \ldots x_{n-1} x_n \] with the distribution (of \( x_i \) within \([1, n]\))

\[ P(x_i) = (1 - c) \delta(x_i - 1) + c \delta(x_i + 1), \quad (1.4) \]

The \( x_i \) are uncorrelated between different sites and \( c \) is the concentration of "wrong signs" such that \( c = 0 \) and \( c = 1 \) mean no disorder. Clearly \( \xi_n \) is exponentially correlated \( \langle \xi_n \xi_m \rangle = (1 - 2c)^{|m-n|} \) where one can define a correlation length \( l(c) = -1/\ln|1 - 2c| \). For the numerical computation uncorrelated \( x_i \) are obtained from the uniformly distributed random number generator sequence. For the uncorrelated results we set \( \xi_n = x_n \) at the particular concentration \( c = 0.5 \) where both correlated and uncorrelated cases produce identical results since \( l(0.5) = 0 \).

It should be emphasized that while the model for uncorrelated disorder and the model for correlated disorder involve the parameter \( c \), in the latter model \( c \) determines both the fraction of wrong sign bonds and the correlation length. The two models intersect at \( c = 0.5 \) where there are equal numbers of \( \pm \) interactions and the correlation length is 0. We have carried out extensive numerical studies to verify the equation for the correlation length given above.

We note here that unlike the off-diagonal case above, many works have recently appeared on the diagonal tight binding model with correlated disorder dealing with various aspects of the problem \([20][21][22]\). It was revealed that even short range correlated disorder at the band edge caused the anomalies in the spectrum \([23]\). Frenkel excitons in off-diagonal and diagonal models have markedly different spectra, i.e., the former has a Dysonian singularity \([13][14]\) at the band center while the latter has a power-law singularity at the band edge. Hence the short-range correlations are expected to influence off-diagonal spectra more than in the diagonal case.
Fig. 1. The integrated density of states (IDOS, left panel) and the inverse localization length (ILL, right panel) plots for the correlated distributions of the dipolar type couplings with $A = 0.25, 0.50, 0.75$ bottom to top. Symbols are data for a chain of $10^8$ sites for respective concentrations divided by $f(c) = \frac{c}{1-c}$. The lines, shown as guides to the eye, are plots of the amplitude function $f(0.5)((\sqrt{2} \tanh^{-1} A)^2 \div \ln(E^2))$ for the ILL or its square for the IDOS. The parameter $c$ is related to the correlation length $l(c)$ by the equation $l(c) = -1/\ln|1-2c|$.

2 Results

The distribution and localization of the exciton modes are characterized by the Lyapunov exponent. In the case of one-dimensional arrays with nearest-neighbor interactions, the Lyapunov exponent can be determined by making use of mode-counting techniques [25,26]. Recently we studied [27] similar excitons with orientationally disordered couplings in conjugated polymers and calculated the full spectrum and optical line shapes. The goal here is to look at the effects of exponentially correlated disorder on the behavior of the IDOS and ILL near the center of the band ($E = 0$). We have produced three sets of data for each case varying the correlation length i.e., $c$) and the strength of the disorder, $A$, with $A = 0.25, 0.50, 0.75$.

Figure 1 displays the results of the ILL and IDOS for exponentially correlated distributions of the couplings. For fixed $A$, we divided the data produced for a particular
concentration $c$ by a scaling factor $f(c) = c/(1 - c)$ (see next section and the appendix). The data for selected concentrations are shown in Fig. 1 for the indicated values of $A$. We expect first that for a range of concentrations the data should collapse to a Dysonian singularity for fixed $A$. However, Fig. 1 reveals that as $c$ increases, the $ILL$ and the $IDOS$ take values that differ increasingly from predictions based on Ziman’s analysis [13] and a slow approach to the band center is observed. In this limit, the data clearly show qualitative scaling behavior but deviations with varying magnitude exist as $c \to 1$ and $c \to 0$ where the correlation length is infinite. Notice also that the deviations are stronger for the $IDOS$ than for the $ILL$.

Clearly as energy decreases, the scaling gets better; however, the deviations become worse as one gets closer to $c \to 0, 1$. To see whether longer chains and/or lower energies improve the situation, we checked further and found that the longer chain did not improve the agreement appreciably, but as the energy is lowered, the results follow the scaling behavior more closely up to a certain limiting $c$. Similar disagreement between the data and the Dysonian (with a single term) singularity were encountered in the SUSY [7] off-diagonal tight binding case for continuous exponential correlations. The analytical investigations [8] revealed that the spectra for the exponential correlation can be expanded for the $IDOS$ as

$$IDOS - \frac{1}{2} = a_1/|\ln(E^2)|^2 + a_2/|\ln(E^2)|^3 + a_3/|\ln(E^2)|^4 + \ldots,$$

(2.1)

and for the inverse localization length ($ILL$)

$$ILL = b_1/|\ln(E^2)| + b_2/|\ln(E^2)|^2 + b_3/|\ln(E^2)|^3 \ldots,$$

(2.2)

where $a_1, a_2, a_3, b_1, b_2$ are given as of functions of the correlation length multiplied by the strength [7,8] of the exponential correlations. The equations (1.1)–(1.2) are only the first order term in the expansion and $a_1$ and $b_1$ should correspond to correlated amplitude factor $V^C(c)$. Figure 2 clearly shows the improved results obtained using
Fig. 2. The integrated density of states (IDOS, left panel) and the inverse localization length (ILL, right panel) plots for the correlated distributions of the dipolar type couplings with $A = 0.50$. Symbols are data for a chain of $10^8$ sites for particular concentrations (where significant deviations occur) $c = 0.01, 0.05, 0.1, 0.9, 0.95$ from bottom the top and the lines are the scaling function $f(c)(\sqrt{\tanh^{-1} 0.5})^2$ divided by $\ln(E^2)$ for the ILL or its square for the IDOS plus the higher terms given in Eqs. (2.1)–(2.2) where $f(c) = \frac{c}{1-c}$. The parameter $c$ is related to the correlation length $l(c)$ by the equation $l(c) = -1/\ln|1-2c|$.

the Eqs. (2.1)–(2.2) for the selected concentrations where the worst deviations followed from the first order result: $c = 0.01, 0.05, 0.1, 0.9, 0.95$. Unfortunately, for $c < 0.01$ and $c > 0.95$, this type of expansion appears to fail; however, other forms of fitting with fractional powers can give better results such as tried in the SUSY case [7,8]. We will not here discuss significance of this fractional fitting since there doesn’t seem to be any analytic theory that predicts it.

3 Discussion

To shed some light on the scaling behavior for the correlated case, we first compute the binary uncorrelated case. A simple calculation of the logarithmic variance, the amplitude of the spectrum, for the uncorrelated case given in Eqs. (1.1)–(1.2) yields
the result

\[ V^U(c) = 4c(1 - c) \left( \sqrt{2} \tanh^{-1} A \right)^2. \]  

(3.1)

The amplitude factor for the uncorrelated disorder clearly displays a scaling behavior in concentration \( c \) as well as in \( A \). We can identify the scaling functions for the concentration \( g(c) = 4c(1 - c) \) and the strength of the deviation \( h(A) = (\sqrt{2} \tanh^{-1} A)^2 \).

The top inset in Fig. 3 displays the uncorrelated data vs concentrations. The three lines are the calculated amplitudes \( V^U(c) = h(A)g(c) \) presented in Eq. (3.1) for \( A = 0.25, 0.50, 0.75 \). As expected, the theory for the uncorrelated case works well

The amplitude factors in Fig. 3 (large panel) are found from the best fit to the Dysonian singularity. They are obtained matching the smallest value of the energy rather than the value obtained by fitting over a range of energies. In this way, we will be more likely to get a value characterizing the asymptotic region. The calculated amplitude scaling function is also shown in Fig. 3 for the correlated case \( V^C(c) = c(1 - c)^{-1} \left( \sqrt{2} \tanh^{-1} A \right)^2 \) for the three specified \( A \) values above. In the appendix we will argue why this formula holds. We notice immediately that the concentration dependence has a different form \( f(c) = \frac{c}{1-c} \) as compare to \( g(c) = 4c(1 - c) \) of the uncorrelated case while the scaling on \( A \) is the same for both cases. \( h(A) = \left( \sqrt{2} \tanh^{-1} A \right)^2 \). This asymptotic fit can further be checked by plotting the \( a_1 \) and \( b_1 \) in the expansion above since they correspond the coefficient of the Dysonian singularity.

The bottom inset displays the \( a_1 \) and \( b_1 \) obtained by the best fit as compare to the theoretical line \( V^C(c) \) (for \( A = 0.5 \)). Both plots clearly support the asymptotic amplitude scaling.

Although the results obtained by asymptotic fit are clearly reproduced by the theoretical amplitude factor, there are striking differences between correlated and uncorrelated cases. What is particularly interesting and not expected is that the results for correlated disorder is not symmetric about \( c = 0.5 \) like those of the uncorrelated
Fig. 3. The asymptotic amplitude factors for correlated disorder vs concentration $c$ with $A = 0.25, 0.50, 0.75$ bottom to top. In the main plot, the lines show the correlated amplitude $V^C(c) = f(c)(\sqrt{2}\tanh^{-1} A)^2$ where $f(c) = \frac{c}{1-c}$. The top inset is the same plot for uncorrelated case for the same $A$ values. The lines are a plots of the uncorrelated amplitude $V^U(c) = g(c)(\sqrt{2}\tanh^{-1} A)^2$ where $g(c) = 4c(1-c)$. The bottom inset shows $f(c)(\sqrt{2}\tanh^{-1} A)^2$, the expansion coefficient of the IDOS and the ILL $a_1$ and $b_1$, respectively vs $c$ for $A = 0.5$. The parameter $c$ is related to the correlation length $l(c)$ by the equation $l(c) = -\frac{1}{\ln |1-2c|}$.

Also, from our scaling analysis the ILL coefficient becomes large as $c \to 1$, but vanishes as $c \to 0$. This is surprising since the modes for $c = 1$ are extended just like the modes for $c = 0$. The divergence of the amplitude factor as $c \to 1$, is not inconsistent with the results for $c = 1$ since the energy interval over which scaling holds shrinks to zero in the same limit. This can be seen from Eqs. (1.1) – (1.2) by setting either the IDOS or the ILL to a constant value and solving for the cut-off energy as as a function of $c$. As shown in the appendix, the amplitude factors perform a correlated random walk in parameter space. For our case, the square of the length of the random walk is proportional to $cN/(1-c)$ not $N$ alone as in the uncorrelated case. That is the source of the marked difference between the two cases. As the concentration approaches 1, divergent behavior occurs. However, we note that we cannot take $c$ arbitrarily close to 1; beyond a certain point, the random walk sequence can not be constructed \[14\].
4 Summary

We have investigated the dynamics of 1D Frenkel exciton systems with correlated off-diagonal disorder. We have used a negative eigenvalue counting technique \cite{25} which provides a simple and physically transparent analysis of the IDOS and the ILL. We investigated the question of scaling when there was correlated off-diagonal disorder such as might occur, for example, when the off-diagonal interaction depended on the distance between two ions and thus would be affected by small displacements of the ions from their equilibrium positions \cite{15}. Our numerical data indicate that in the correlated case the scaling behavior found by Ziman \cite{13} is followed only for a limited range of \( c \) and very small \( E \). We compared our results with the scaling predictions of Ziman \cite{13} and with the similar tight binding correlated electronic case \cite{7,8}. We computed the asymptotic amplitude factors as a function of concentration and found that they are equal to the uncorrelated variance at \( c = 0.5 \) multiplied by the factor \( \frac{1}{1-c} \) (see appendix). This factor \cite{18} has played a very interesting scaling role in number of unrelated problems. We found that with the new variance for correlated distributions rather unexpected behavior was obtained as compare to that of the uncorrelated case. In particular, the asymptotic amplitude factor is not symmetric about \( c = 0.5 \) while the ILL diverges for concentrations approaching 1 in contrast to what is observed in the uncorrelated case. This amplitude factor can explain only the first order expansion in a Dyson-type singularity as supported by the SUSY model \cite{7,8}. As shown in the appendix, if the random walk observation of Eggarter et al. is implemented, with correlation, the obtained amplitude factor can reproduce the data within a shrinking energy range as \( E \to 0 \) as \( c \) is increased. Finally, we should emphasize that our work is a numerical study supplemented by the approximate theoretical analysis outlined in the Appendix. As mentioned previously, the surprising result is the discovery that near the center of the exciton band, a model with correlated disorder showed asymptotic behavior that is similar to the behavior
of systems without correlations, the only difference being in the amplitude factor appearing in the limiting expressions for the IDOS and ILL. Like those of essentially all numerical studies, our results are approximate. We hope that the availability of the numerical findings together with our analytical results will stimulate rigorous analyses of the model that may shed light on the origin of the similarity.

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6 Appendix

In this section we present the amplitude scaling related to the first order expansion coefficients $a_1$ and $b_1$ in Eqs. (2.1)–(2.2). Similar scaling but in different cases has appeared before [29,30,31,18,28]. However the off-diagonal problem here is more involved than those cases. First the Lyapunov exponent at the band center $E = 0$ is calculated using Eq. (1.3) that can be rearranged [5,13,14]

$$\gamma(0) = 1 \frac{1}{N} \ln \left( \frac{U_N}{U_1} \right) = \frac{1}{N} \sum_{n=1} \ln \left( \frac{U_{2n+1}}{U_{2n}} \right) = \frac{1}{N} \sum_{n=1} \ln \left( \frac{J_{2n+1}}{J_{2n}} \right) + \frac{i\pi}{2}. \quad (6.1)$$

The bipartite nature (the chiral symmetry) of the Eq. (1.3) is responsible for the logarithmic variance as rigorously shown in [32] and eventually the Dyson- type singularity [14] since $\gamma(0)$ executes a random walk with a step $\Delta_n = \ln(J_{2n+1}) - \ln(J_{2n})$. For the uncorrelated case the averages yield $<\Delta_n> = 0$ and $<\Delta_n^2> = \frac{1}{N} [<(\ln(J^2))^2> - <\ln(J^2)>^2]$. The Lyapunov exponent [14,32] takes this form $\gamma(0) = \sqrt{\frac{1}{N} < (\sum_{n=1} \Delta_n)^2 > + \frac{i\pi}{2}}$. The imaginary part here $IDOS = \frac{\Im \gamma(0)}{\pi} = 0.5$ and the
real part is related to the ILL. To make our point, we can attack the problem with a different angle. The average of the summation $< \sum_{n=1}^{\Delta_n} > = 0$ for both correlated and uncorrelated cases; whereas, the average of the squared summation $\sum_{n=1}^{\Delta_n^2} + 2 \sum_{m \neq n} \Delta_n \Delta_m$. It further reduces to $N < \Delta^2 >$ since the second sum is zero for the independent random numbers but non zero for the correlated random numbers. For the correlated disorder the summation is not easy. The complication arises when correlation present since $< \ln(J_n) >$ is not equal to its ensemble average rather it dependents on $n$. But this complication can be avoided in a following way. Let us calculate the Taylor expansion of $\ln(J_n) = \ln(1 + A \xi_n)$ which reads

$$\ln(1 + A \xi_n) = A \xi_n - (A \xi_n)^2/2 + (A \xi_n)^3/3 - (A \xi_n)^4/4 + (A \xi_n)^5/5 + \ldots \quad (6.2)$$

As $\xi_n^{2k} = 1$ but for odd powers $\xi_n^{2k+1} = \xi_n$. Arranging even and odd powers we get

$$\ln(1 + A \xi_n) = (A + A^3/3 + A^5/5 + A^7/7 + \ldots) \xi_n - (A^2/2 + A^4/4 + A^6/6 + \ldots). \quad (6.3)$$

The first sum (with odd powers of $A$) is just the expansion of $\tanh^{-1}A$ and let the second sum (with even powers) be $D$ which will be eliminated when it is inserted in $\Delta_n = \tanh^{-1}A(\xi_{2n+1} - \xi_{2n})$. The same expansion can be used to calculate the uncorrelated case as well; the step takes the form $\Delta_n = \tanh^{-1}A(x_{2n+1} - x_{2n})$ (recall that number of steps $N/2$). Using Eq. (1.4), the variance is given by $V^U(c) = < \frac{2}{N} \sum_n \Delta_n^2 > = 2 < \Delta^2 >= 4c(1-c)(\sqrt{2}\tanh^{-1}A)^2$ so that the ILL becomes $\Re\gamma(0) = \sqrt{V^U(c)/N} = \sqrt{2}\tanh^{-1}A\sqrt{4c(1-c)/N}$. For the correlated case, we have $V^C(c) = \frac{2}{N} < (\sum_n \Delta_n)^2 > = (\tanh^{-1}A)^2 < (\sum_n(\xi_{2n+1} - \xi_{2n}))^2 >$. Taking the square, we get three terms $< (\sum \xi_{odd/even})^2 >$ and twice $< \sum \xi_{odd} \sum \xi_{even} >$, hence the summation results in $N \frac{c}{1-c}$ where inverse of this factor is encountered in previous works [28][18][29][31]. The correlated random walk results in $V^C(c) = \frac{c}{1-c}(\sqrt{2}\tanh^{-1}A)^2$ and the ILL = $\Re\gamma(0) = \sqrt{V^C(c)/N} = \sqrt{2}\tanh^{-1}A\sqrt{\frac{c}{(1-c)N}}$. In contrast, the IDOS
value at the band center is not affected by the correlations since the distribution in eigenvalues is symmetric about the mid-point. The calculation above holds at the band center; however Eggarter et al. [14] have shown that for a certain range of $E$ close to the band center, the random walk behavior still held and the $ILL$ and $IDOS$ can be represented as in Eqs. (2.1)–(2.2) where the amplitude factor is the same as calculated at the band center.

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According to the Frenkel exciton model, the transfer integral usual dipole-dipole interaction has the inverse cube of the distance between the dipoles. Offdiagonal transfer integral disorder is due to the distribution of bond length. Assuming the transition dipoles, the intersite transfer dipole-dipole interaction is given as

$$J_{n,m} = \frac{1}{|a_0 n + \delta_n - (a_0 m + \delta_m)|^3}$$

where $\delta_n$ is the positional deviation from their equilibrium position $a_0 n$ of transition dipoles. The nearest distance between the atoms is $a_0$ and $n$ is the site index. Since the interaction decreases rapidly over the $n$ to $m$ distance, regarding nearest-neighbor couplings is sufficient as a first approximation. Assuming unit neighbor distance the tranfer integral reduces to

$$J_{n,n+1} = \frac{1}{|1 - B \xi_n|^3}$$

where $B$ is the strength and $\xi_n = (\delta_{n+1} - \delta_n)/B$ is the random sign of the deviation from the regular lattice point. Notice that $J_{n,n+1}$ is used in the text as $J_n$. The deviations from the equilibrium positions introduces a correlation. If $B$ is small, one can expand the coupling as $J_n \simeq 1 + 3B\xi_n$. Since $B$ is a free parameter, one can replace $3B$ with $A$ without loss of generality. Note that using the approximate form in place of the cubic form does not make an essential change in the spectra apart from a numerical factor since the spectra are related to averages of the logarithm, $<\ln(J_n^2)>$ and our numerical study confirmed this.

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