Magnetic tensor gradiometry using Ramsey interferometry of spinor condensates

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We have realized a magnetic tensor gradiometer by interferometrically measuring the relative phase between two spatially separated Bose–Einstein condensates (BECs). We perform simultaneous Ramsey interferometry of the proximate $^{87}$Rb spin-1 condensates in freefall and infer their relative Larmor phase – and thus the differential magnetic field strength – with a common-mode phase noise suppression exceeding 50 dB. By appropriately biasing the magnetic field and separating the BECs along orthogonal directions, we measure the magnetic field gradient tensor of ambient and applied magnetic fields with a nominal precision of 30 µG cm$^{-1}$ and a sensor volume of $2 \times 10^{-3}$ mm$^3$. We predict a spin-projection noise limited magnetic energy resolution of order $\hbar$ for typical Zeeman coherence times of trapped condensates with this scheme, even with the low measurement duty cycle inherent to BEC experiments.

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Precision measurement of magnetic fields underpins applications as diverse as fundamental symmetry tests [1], magnetoencephalography [2] and geophysical exploration [3]. Many of these applications require precise and accurate measurements of the change in magnetic field across a region of space, for example remote sensing of the field due to mineral deposits against the larger but locally homogeneous field of the Earth dipole [4]. In this Letter, we present the first tensor magnetic gradiometer based on atomic magnetometry, measuring $\partial B_i/\partial x_j$ (the gradient of vector components along orthogonal axes) with a sensor volume suitable for resolving microscopic biomagnetic or surface-science sources. Tensor measurements incisively probe magnetic source distributions [3, 5]: the gradient tensor at a single point in space determines the bearing, normalized source-strength, and orientation of a dipole [4]. Tensor gradient magnetometers have to date been macroscopic devices – employing SQUID [6–8] or fluxgate [9, 10] sensors – primarily applicable to geophysics and ordinance detection [11].

Atomic magnetometry is a well-established alternative to SQUIDs and other solid-state sensors, delivering absolute, calibration-free measurement of magnetic fields by measurement of the Larmor precession frequency of atomic spins [12]. Warm atomic vapor magnetometers have sensing volumes ranging from hundreds of cubic millimeters for the most sensitive magnetometers down to a few cubic millimeters achieved with microfabricated glass cells [13]. Colder, denser clouds of atoms in traps, such as Bose-Einstein condensates (BECs) offer the prospect of magnetic measurement on the microscale. Wildermuth et al. [14] used a highly elongated condensate to measure the magnetically-induced trapping potential variations from a current-carrying wire. Vengalattore et al. [15] used a non-destructive phase contrast imaging technique to spatially resolve Larmor precession in a spinor BEC, attaining sensitivities comparable to SQUID-based devices. Elongated clouds of ultracold atoms have been used as time-resolved magnetic gradiometers with sub-nT sensitivity and a spatial resolution of 50 µm [16, 17]. This work establishes tensor gradiometry using highly-sensitive, small-volume atomic magnetometers with a dynamically configurable orientation.

Our gradiometer measures the phase difference between two spin-1 Ramsey interferometers formed from spatially separated $F = 1$ $^{87}$Rb BECs. Each condensate is formed in a separate potential well of a three-beam crossed optical dipole trap and translated to a spatial separation of just less than 1 mm by an acousto-optic modulator (AOM). We can change our dipole trap configuration so that the gradiometer spans two separation axes in a plane (Fig. 1) and apply bias magnetic fields to make the gradiometer sensitive to field components in all spatial directions. A Ramsey pulse sequence probes the phase acquired by each condensate over an interrogation time $T$ due to the magnetic field $B$ at the position of each condensate. The phase difference between fringes from the two interferometers is proportional to the magnetic gradient tensor of ambient magnetic fields and classical detection noise. Simultaneous interrogation of a dual atomic fountain interferometer demonstrated magnetic gradient measurement over a large spatial region [18], with substantial common-mode rejection of noise from drifts and pulse errors. We characterize the gradiometer by measuring applied field gradients, in addition to the gradient tensor of ambient magnetic fields in our laboratory.

The differential interferometer output is a measure of the relative phase acquired during the Ramsey sequence, which begins with a $\pi/2$ spin rotation pulse at $t = t_0$, free evolution over $T$ and a final $\pi/2$-pulse at $T + t_0$:

$$\Delta \phi = \hbar^{-1} \int_{t_0}^{t_0+T} \Delta E \, dt, \quad (1)$$
where $\Delta E$ is the difference in the Zeeman splitting of the atoms in each constituent interferometer (labeled 1 and 2) due to a spatially varying magnetic field $B(r)$:

$$\frac{\Delta E}{\hbar \gamma} = |B(r_1)| - |B(r_2)| \approx \nabla |B(r)| \cdot (r_1 - r_2), \quad (2)$$

with $\gamma$ the gyromagnetic ratio and $r_{12} = (r_1 + r_2)/2$. The differential interferometer is thus sensitive to derivatives of the magnetic field strength $B(r) = |B(r)| = \sqrt{B_x^2 + B_y^2 + B_z^2}$,

$$\frac{\partial B}{\partial x_i} = \frac{B_x}{B} \frac{\partial B_x}{\partial x_i} + \frac{B_y}{B} \frac{\partial B_y}{\partial x_i} + \frac{B_z}{B} \frac{\partial B_z}{\partial x_i}. \quad (3)$$

To achieve vector magnetic field sensitivity along $x$ for example, we experimentally null magnetic field components along $y$ and $z$, leaving a total magnetic field $B \approx B_x \hat{x}$.

$$\frac{\partial B}{\partial x_i} \approx \frac{B_x}{B} \frac{\partial B_x}{\partial x_i} = \text{sign}(B_x) \frac{\partial B_x}{\partial x_i}. \quad (4)$$

In general we measure components of the magnetic field gradient tensor via the differential Ramsey signal:

$$\frac{\partial B_z}{\partial x_i} \approx \text{sign}(B_z) \frac{1}{\gamma} \frac{d(\Delta \phi)}{dT}, \quad (5)$$

where $\Delta x_i = |r_1 - r_2|$ is the separation along the $x_i$ axis.

The differential interferometer is sensitive to any difference in Zeeman energy between the two condensates. We perform the interferometry sequence in freefall to prevent spurious contributions to the measured magnetic field gradients from vector light shifts induced by the trapping beams [19]. Imperfect linear polarization of the trapping beams induces an atomic vector polarizability, the spatial variation of which appears as a synthetic magnetic field gradient. Freefall interferometry eliminates the vector shift at the expense of introducing a trade-off between the maximum Ramsey interrogation time $T$ and the spatial resolution due to gravity-induced blurring of the sensor volume. Alternatively, differential Ramsey interferometry of trapped clouds presents a means of precisely measuring and canceling vector light shifts; this will be the focus of future work.

Our experiment begins by forming two $^{87}$Rb Bose-Einstein condensates in the $|F = 1, m = -1\rangle$ hyperfine ground state in two 1064 nm crossed-beam optical dipole traps. The propagation directions of two intersecting dipole trapping beams $(1/e^2 \text{ radii of } 75 \mu m \text{ and } 89 \mu m \text{ respectively})$ define near-perpendicular horizontal axes $\hat{x}'$ and $\hat{z}'$ [20] as shown in Fig. 1. The amplitude and horizontal position of each beam is controlled using a separate AOM. Driving either one of the AOMs with two radiofrequency (rf) tones from an agile direct-digital synthesizer produces two diffracted orders, resulting in two crossed-beam dipole traps separated along either $\hat{x}'$ or $\hat{z}'$. The rf frequencies determine the separation of the dipole traps [21] along the intersecting beam.

We Bose condense $5 \times 10^4$ atoms in each trap, initially separated by 100 $\mu m$ to maximize loading efficiency from a precursor hybrid optical dipole-magnetic quadrupole trap [22]. The condensates are further separated over 2$s$ with a smooth frequency ramp, achieving a maximum separation of $\Delta x' = 680 \mu m$ ($\Delta z' = 840 \mu m$) when splitting the beam propagating along the $\hat{z}'$ direction ($-\hat{x}'$ direction).
FIG. 3. (Color online) Measurement of a magnetic field gradient using differential atom interferometry. Varying the phase of the second \( \pi/2 \)-pulse of the Ramsey sequence for each fixed interrogation time \( T \) traces an ellipse (top). From each ellipse we extract the magnitude of the phase difference \( |\Delta \phi| \), and determine the field gradient using Eq. (5); \( \partial B_y/\partial z = -5.33(3) \text{ mG cm}^{-1} \) for these data. Statistical uncertainties of \( |\Delta \phi| \) and \( F_{z,i} \) are smaller than the data points.

Using three orthogonal coil pairs, we ensure the magnetic field is oriented along one of the \((x, y, z)\) axes with magnitude in the range 250–600 mG. We extinguish the dipole trapping light and the two condensates begin to fall. After 100 \( \mu \)s freefall we initiate Ramsey interferometry between Zeeman states of the \( F = 1 \) hyperfine ground state using a resonant rf \( \pi/2 \)-pulse. The two falling condensates comprise two independent interferometers. The interferometers are closed with a second \( \pi/2 \)-pulse after an interrogation time \( T \). Spin components \((m = 0, \pm 1)\) of each condensate are separated after further freefall by pulsing a 50 G cm\(^{-1}\) magnetic field gradient for 3 ms. The number of atoms \( N_{m,\alpha} \) in state \( m \) of condensate \( \alpha = 1, 2 \) is determined by absorption imaging with a resonant laser after a total drop time of 23 ms. This constitutes a single realization, or shot, of the experiment from which we compute the normalized spin projection \( F_{z,\alpha} = \sum_m m N_{m,\alpha} / \sum_m N_{m,\alpha} \) for each interferometer.

Ramsey fringes in the phase domain \((F_{z,1}(\phi) \text{ and } F_{z,2}(\phi))\) are clearly resolved for interrogation times \( T < 500 \mu\)s, and are subsequently dominated by phase noise induced by magnetic field fluctuations common to each interferometer. Plotting the interferometer outputs parametrically yields an ellipse, which is immune to common-mode phase noise and whose eccentricity and orientation are related to the phase difference \( \Delta \phi \). This has been utilized in gravity gradiometry where atomic momentum states are interfered, and the common-mode phase noise derives from vibration of the reference platform [23]. We fit an ellipse [24] to a parametric dataset \((F_{z,1}(\phi), F_{z,2}(\phi))\) to extract \( |\Delta \phi| \) [25]; by repeating this process for different interrogation times we compute a magnetic field gradient via \( \partial(\Delta \phi)/\partial T \) in Eq. (5). A measurement of a gradient in the \( y \)-component of the magnetic field is shown in Fig. 3. We infer a nominal precision of 30 \( \mu \text{G cm}^{-1} \) from the statistical uncertainty in the slope of such linear fits.

We quantified the common-mode rejection of the gradiometer by comparing the phase noise from a single interferometer to the uncertainty in the relative phase extracted from the elliptical fits. Assuming our magnetic field noise is baseband, the measured phase noise yields a standard deviation of the Larmor frequency of \( \sigma_{\omega_L} = 2\pi \times 192(11) \text{ Hz} \), representing a common-mode rejection ratio exceeding 50 dB. Fig. 2 shows the deterioration of phase domain fringes at long interrogation times, while the corresponding parametric plots do not exhibit discernible degradation.

To demonstrate the tensor sensitivity of the gradiometer, we measured multiple field derivatives as a function of the current imbalance \( \Delta I_z \) in the \( z \)-bias coils (Fig. 4). This allows us to explicitly evaluate the response of the gradiometer to an applied gradient when biased differently; with a field along \( z \), we measure \( \partial B_y/\partial z \) and \( \partial B_y/\partial x \) proportional to and independent of \( \Delta I_z \), respectively, in agreement with a numerical Biot-Savart calculation. Orienting the magnetic field along the \( y \)-axis renders the gradiometer insensitive to the applied gradient, as this measures \( \partial B_y/\partial x \) and \( \partial B_y/\partial z \) (Eq. (4)), which we find to be \(< \sigma_{\omega_L} / 51(6) \text{ G cm}^{-1} \text{ A}^{-1} \) due to imperfect alignment of the magnetic field along the \( y \)-axis.

The background magnetic field environment of our apparatus has predominantly linear magnetic field gradients originating from equipment within 1 m of the atoms. The magnetic field gradient tensor can be represented by a matrix \( G_{ij} = \partial B_i/\partial x_j \). Using three bias field orientations and baselines that span the horizontal \( x', y' \) plane is sufficient to calculate the full gradient tensor \( G \) in the \( x, y, z \) frame from Maxwell’s laws for magnetic fields in a vacuum. Our experimental conditions permit the use of \( \nabla \times \mathbf{B} = 0 \) and \( \nabla \cdot \mathbf{B} = 0 \) when applying Maxwell’s equations. We can thus determine gradients \( \partial B_i/\partial y \) from the gradients measured in the horizontal plane, resulting in the gradient tensor

\[
G = \begin{pmatrix}
-5.71(7) & -6.92(4) & 14.70(7) \\
-6.92(4) & 15.18(8) & 2.66(4) \\
14.95(3) & 2.66(4) & -9.47(3)
\end{pmatrix} \text{ mG cm}^{-1},
\]

where the inferred values are inside the dashed box. We observe that measurements of gradients over the course of a month do not vary beyond their uncertainties. The gradient tensor has applications in localizing magnetic sources, which has seen widespread use in geophysics and...
FIG. 4. (Color online) Response of the gradiometer to a gradient applied by driving a differential current $\Delta I_{\text{A}}$ through the $z$-bias coils. Four field derivatives $\partial B_y/\partial x$, $\partial B_z/\partial x$, $\partial B_y/\partial z$, and $\partial B_z/\partial z$ are measured by biasing the gradiometer along $y$ or $z$ with baselines along $x'$ or $x''$. The dominant gradient is $\partial B_z/\partial z = -32.8(1)\Delta I_{\text{A}}$ mG cm$^{-1}$; the relative insensitivity of the other measured gradients to $\Delta I_{\text{A}}$ quantifies the alignment of the bias coils along the Cartesian axes.

surveying [3, 7, 26]. The power of the gradient tensor is exemplified by the ability to localize a source from a tensor measurement at a single point in space; the three orthogonal eigenvectors of the matrix $G$ span a coordinate system in which off-diagonal gradient terms vanish, with the eigenvector corresponding to the largest magnitude eigenvalue pointing at the dominant dipole source [4]. For the gradient tensor in Eq. (6), this vector points in the direction of our dominant gradient source, the unshielded permanent magnets of an ion pump. Homogeneous magnetic fields are required to preserve the coherence or robust entanglement of spins in many systems, e.g. the macroscopically entangled singlet state of a spinor quantum gas. Biasing the magnetic field along one of the eigenvectors of $G$ reduces the problem of achieving a uniform field strength to canceling a single diagonal gradient.

Tensor gradiometers make use of Maxwell’s laws to infer the complete tensor from as few as five independent gradient terms [6, 9, 10, 27, 28]. Gradients measured over baselines in the $x$-$z$ plane quantify a systematic error in our inference; the equality of $\partial B_y/\partial x$ and $\partial B_z/\partial z$ is violated by $<0.25(7)$ mG cm$^{-1}$. We attribute this discrepancy to asymmetric sampling of residual field curvature from the bias coils and imperfect cancellation of transverse field components when aligning $\mathbf{B}$ along a given axis (Eq. (4)). These two systematics may be reduced by larger Helmholtz coils, and by applying larger bias fields. While an in-plane measurement is sufficient to determine the gradient tensor, two-axis acousto-optic deflection of the trapping beams could be used to measure all terms independently.

Atomic magnetometers detect the Larmor precession of spins in a magnetic field. For large atom numbers, as in warm vapor magnetometers, spin relaxation and photon shot noise ultimately limit sensitivity. Cold atom systems interrogate smaller, trapped samples, and spin projection noise ($\delta F_z = 1/\sqrt{N}$ at the standard quantum limit for $N$ atomic spins) is relatively more important. As multiple experimental shots are required to impute a differential phase from the elliptical data reduction, a single-shot phase sensitivity is ill-defined here. Nonetheless, the differential phase uncertainty from fitting an ellipse with $M$ points scales with $1/\sqrt{\delta \Delta}$, and thus the quantum limited field sensitivity is $\delta B \sim 1/(\gamma \sqrt{NTDT\text{int}})$ for a total integration time $T_{\text{int}} = MT_{\text{shot}}$, a duty cycle $D = T/T_{\text{shot}}$, and a single-shot duration of $T_{\text{shot}}$.

We measure the spin projection of $N = 10^5$ atom condensates at twice the standard quantum limit, and the field sensitivity per unit bandwidth of this gradiometer is $\delta B\sqrt{T_{\text{int}}} = 240\,\text{pT Hz}^{-1/2}$ for $T_{\text{shot}} = 25\,\text{s}$.

We now estimate the sensitivity of a realistic prospective gradiometer for trapped condensates, provided vector light shifts from the trapping beams are suppressed. The sensitivity of each interferometer scales with differential magnetic moment of the atomic transition, the evolution time, and the atomic density. The $|F = 1, m = 1\rangle \leftrightarrow |F = 2, m = 2\rangle$ microwave transition offers a three-fold improvement in sensitivity compared to the radiofrequency transitions used in this work. Zeeman coherence times of order seconds have been observed in spin-1 [29] and pseudospin-$\frac{1}{2}$ [30] condensates, limited by losses due to density dependent collisions. For a $^{87}$Rb BEC with a peak number density of $10^{14}$ atoms/cm$^{-3}$, an interrogation time of $T = 200\,\text{ms}$ (an order of magnitude lower than the three-body limited lifetime) is foreseeable, corresponding to $N = 10^6$ atoms for the current trap. The prospective in-trap gradiometer could thus achieve a differential field sensitivity per unit bandwidth of $0.2\,\text{pT Hz}^{-1/2}$ at the standard quantum limit, even with the same trap and single-shot duration used here (corresponding to a non-unity duty cycle of $D = 0.008$).

Spatial resolution is conventionally quantified by the sensing volume $V$; a vapor magnetometer with $V = 300\,\text{mm}^3$ attained sub-femtotesla sensitivities [31] whereas NV-center magnetic probes deliver nanoscopic resolution but at much lower field sensitivities [32]. Here, the sensor volume $V = 2 \times 10^{-5}\,\text{mm}^3$ is that swept out by a falling, expanding condensate during the Ramsey interrogation. A commonly used comparative metric for precision magnetometers is the magnetic field energy resolution per unit bandwidth $\epsilon = (\delta B)^2TV/2\mu_0$ [33]. Compared to vapor cell magnetometers, the low duty cycle and number of spins in BEC based measurements limits their sensitivity per unit bandwidth, but the far smaller volume results in a comparable $\epsilon$, with $\epsilon \sim 50–100\,\text{fT}$ for
vapor magnetometers [13, 34], and ε ∼ h for the prospective gradiometer described above.

We have demonstrated magnetic tensor gradiometry using differential Ramsey interferometry of spatially separated BECs in freefall. The gradiometer senses vector components of the magnetic field, rejecting gradient components orthogonal to the biasing direction. The gradiometer is immune to common-mode magnetic noise orders of magnitude larger than the field difference, and operates without field cancellation or screening. The fine spatial control and versatility in baseline length our technique offers allows for precision magnetic surveys of magnetic structures. We anticipate our gradiometer receiving interest as a potential in-vacuum magnetic microscope, a high spatial resolution magnetometer that operates close to the quantum measurement limit.

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