SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory

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We calculate the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ up to $\mathcal{O}(p^4)$ in covariant baryon chiral perturbation theory with dynamical octet and decuplet contributions. We find that the decuplet contributions are of similar or even larger size than the octet ones. Combining both, we predict positive SU(3)-breaking corrections to all the four independent $f_1(0)$'s (assuming isospin symmetry), which are consistent, within uncertainties, with the latest results from large $N_c$ fits, chiral quark models, and quenched lattice QCD calculations.

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I. INTRODUCTION

Hyperon semileptonic decays, parameterized by three vector transition form factors ($f_1$, $f_2$, and $f_3$) and three axial form factors ($g_1$, $g_2$, and $g_3$), have received renewed interest in recent years due to various reasons. In particular, they provide an alternative source [1, 2, 3, 4] to allow one to extract the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $V_{us}$ [5, 6], in addition to kaon semileptonic decays (see, e.g., Ref. [3] for a recent review), hadronic decays of the $\tau$ lepton [7] and the ratio $\Gamma(K^+ \to \mu^+\nu_\mu)/\Gamma(\pi^+ \to \mu^+\nu_\mu)$ [8]. The hyperon vector coupling $f_1(0)$ plays an essential role in order to extract $V_{us}$ accurately.

Due to the Conservation of Vector Current (CVC) $f_1(0)$ is known up to SU(3)-breaking effects, which are of subleading-order according to the Ademollo-Gatto theorem [9]. Theoretical estimates of SU(3)-breaking corrections to $f_1(0)$ have been performed in various frameworks, including quark models [11, 12, 13], large-$N_c$ fits [3], and chiral perturbation theory (ChPT) [14, 15, 16, 17, 18]. These SU(3)-breaking corrections have also been studied recently in quenched lattice QCD (LQCD) calculations for two of the four independent channels (assuming isospin symmetry): $\Sigma^- \to n$ [19] and $\Xi^0 \to \Sigma^+$ [20].

In principle, ChPT provides a model independent way to estimate the SU(3)-breaking corrections to $f_1(0)$. However, it is known that ChPT calculations converge slowly in SU(3) flavor space. This problem becomes even more pronounced in the one-baryon sector, where the physics at a certain order can be blurred by the power-counting restoration procedures, as can be clearly seen in the case of the baryon octet magnetic moments [21]. Fortunately, in the case of $f_1(0)$, the Ademollo-Gatto theorem dictates that up to $\mathcal{O}(p^4)$ no unknown LEC’s contribute and, therefore, no power-counting-breaking terms appear. Consequently, up to this order there is no need to apply any power-counting restoration procedures and a ChPT calculation is fully predictive.

In a recent $\mathcal{O}(p^4)$ calculation performed in Heavy Baryon (HB) ChPT, it was shown that the chiral series with only the octet contributions converge slowly while the convergence is completely spoiled by the inclusion of the decuplet ones [17]. In a later work [18], the infrared version of baryon chiral perturbation theory (IRChPT) [22] was employed and calculations were performed up to $\mathcal{O}(p^4)$ with only the octet contributions. The slow convergence of the chiral series was confirmed but the importance of relativistic corrections was stressed.

In the present work, we perform the first covariant baryon ChPT calculation of $f_1(0)$ up to $\mathcal{O}(p^4)$, including both octet and decuplet contributions. This article is organized as follows. In Sec. 2, we fix our notation and write down the relevant chiral Lagrangians up to $\mathcal{O}(p^4)$. To study the contributions of the decuplet baryons, we adopt the “consistent” coupling scheme for the Rarita-Schwinger description of the spin-3/2 decuplet fields [23]. In Sec. 3, we present our numerical results order by order, contrast them with the corresponding HBChPT and IRChPT results, and study the convergence of the chiral series. We also compare our full results with those obtained in other approaches, including large $N_c$ fits, quark models, and lattice QCD calculations. Finally, we use our results of $f_1(0)$ to extract $V_{us}$ from the experimental values of the decay rates and $g_1(0)/f_1(0)$. Summary and conclusions follow in Sec. 4.

II. FORMALISM

The baryon vector form factors as probed by the charged $\Delta S=1$ weak current $V^\mu = V_{us} \bar{u} \gamma^\mu s$ are defined by

$$\langle B'|V^\mu|B\rangle = V_{us} \bar{u}(p') \left[ \gamma^\mu f_1(q^2) + \frac{2iq^\mu q^\rho}{M_{B'^*} + M_B} f_2(q^2) + \frac{2q^\mu}{M_{B'^*} + M_B} f_3(q^2) \right] u(p),$$

(1)
where \( q = p' - p \). In the SU(3)-symmetric limit, \( f_1(0) \) is fixed by the conservation of the SU(3)\(_V\)-charge \( g_V \). Furthermore, the Ademollo-Gatto theorem states that SU(3)-breaking corrections start at second order in the expansion parameter \( m_s - m \)

\[
f_1(0) = g_V + \mathcal{O}((m_s - m)^2),
\]

where \( m_s \) is the strange quark mass and \( m \) is the mass of the light quarks. The values of \( g_V \) are \(-\sqrt{\frac{2}{3}}, -\frac{1}{\sqrt{2}}, -1, \sqrt{\frac{2}{3}}, \frac{1}{\sqrt{2}}, 1 \) for \( \Lambda \to p, \Sigma^0 \to p, \Sigma^- \to n, \Xi^- \to \Lambda, \Xi^0 \to \Sigma^0 \), and \( \Xi^0 \to \Sigma^+ \), respectively. In the isospin-symmetric limit only four of these channels, which we take as \( \Lambda \to N, \Sigma \to N, \Xi \to \Lambda \), and \( \Xi \to \Sigma \), provide independent information.

We will parameterize the SU(3)-breaking corrections order-by-order in the relativistic chiral expansion as follows:

\[
f_1(0) = g_V \left( 1 + \delta^{(2)} + \delta^{(3)} + \cdots \right),
\]

where \( \delta^{(2)} \) and \( \delta^{(3)} \) are the leading and next-to-leading order SU(3)-breaking corrections induced by loops, corresponding to \( \mathcal{O}(p^3) \) and \( \mathcal{O}(p^4) \) chiral calculations.

### A. Chiral Lagrangians involving only octet baryons and pseudoscalars

The lowest-order SU(3) chiral Lagrangian describing the pseudo-Goldstone bosons in the presence of an external vector current is:

\[
\mathcal{L}^{(2)}_{\phi} = \frac{F_0^2}{4} (\nabla_\mu U^\dagger \nabla^\mu U) + U_i \chi_i U^\dagger + \chi U^\dagger,
\]

where the parameter \( F_0 \) is the chiral-limit decay constant, \( U \) is the SU(3) representation of the meson fields and \( \nabla_\mu \) is its covariant derivative: \( \nabla_\mu U = \partial_\mu - i [v_\mu, U] \), with \( v_\mu \) being the vector source. The explicit breaking of chiral symmetry comes from \( \chi = 2 \tilde{B}_0 \mathcal{M} \) where \( \tilde{B}_0 \) measures the strength of the breaking and \( \mathcal{M} = \text{diag}(m, m, m) \) is the quark mass matrix in the isospin symmetric limit \[24\]. In the above and forthcoming Lagrangians, the symbol \( \langle \ldots \rangle \) denotes the trace in SU(3) flavor space.

The lowest-order chiral Lagrangian describing octet baryons interacting with pseudoscalars and an external vector source reads:

\[
\mathcal{L}^{(1)}_{\phi B} = (\tilde{B} (i \mathcal{D} - M_0) |B \rangle + \frac{D/F}{2} (\tilde{B} \gamma^\mu \gamma_5 [u_\mu, B])_\pm,
\]

where \( B \) denotes the traceless flavor matrix accounting for the octet-baryon fields, \( M_0 \) is the chiral-limit octet-baryon mass, \( D \) and \( F \) are the axial and vector meson-baryon couplings and \( D_\mu B = \partial_\mu B + [\Gamma_\mu, B] \) is the covariant derivative. Furthermore, with \( u^2 \equiv U, v_\mu \) and \( \Gamma_\mu \) are the so-called vielbein and connection:

\[
u_\mu = i (u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) + (u^\dagger v_\mu u - u v_\mu u^\dagger),
\]

\[
\Gamma_\mu = \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) - \frac{i}{2} (u^\dagger v_\mu u + u v_\mu u^\dagger).
\]

The only higher-order chiral Lagrangian that also contributes is through the SU(3)-breaking of the masses of octet baryons\footnote{We have omitted a singlet term indistinguishable from \( M_0 \) for our purposes.}

\[
\mathcal{L}^{(2)}_{\phi B} = b_{D/F} \langle B | \chi_+, B \rangle_\pm
\]

with

\[
\chi_+ = 2 \chi = 4 B_0 \mathcal{M} = 2 \text{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2),
\]

which leads to the following octet baryon masses up to this order:

\[
M_N = M_0 + 4m_K^2 b_{D/F} - 4(m_K^2 - m_\pi^2) b_F, \quad M_\Sigma = M_0 + 4m_K^2 b_{D/F}, \quad M_\Xi = M_0 + 4m_K^2 b_{D/F} + 4(m_K^2 - m_\pi^2) b_F, \quad M_\Lambda = M_0 + \frac{4}{3} (4m_K^2 - m_\pi^2) b_{D/F}.
\]
Fitting the above masses to their corresponding physical values, with \( m_\pi = 0.138 \) GeV, \( m_K = 0.496 \) GeV, and \( m_\eta = 0.548 \) GeV, yields \( M_0 = 1.197 \) GeV, \( b_D = -0.0661 \) GeV\(^{-1}\), and \( b_F = 0.2087 \) GeV\(^{-1}\), which correspond to \( M_N = 0.942(0.939) \) GeV, \( M_S = 1.192(1.193) \) GeV, \( M_\Xi = 1.321(1.318) \) GeV, and \( M_\Omega = 1.112(1.116) \) GeV, with the physical values given in parentheses. It is clear that the differences between the second order fits and the physical values are quite small. Using either of them will be numerically equivalent. In the \( \mathcal{O}(p^4) \) calculation, we will use the second order fits, Eq. (8), to keep track of the SU(3)-breaking pattern. While at \( \mathcal{O}(p^3) \), we use the average mass of the octet baryons, \( M_B = 1.151 \) MeV, without introducing mass splittings.

## B. Chiral Lagrangians involving decuplet baryons

In this work, we adopt the so-called “consistent” couplings to describe the interactions between the decuplet and the octet baryons. Compared to conventional couplings (see, e.g., Refs. 23, 25), the consistent couplings are more stringent due to the requirement that all interactions have the same type of gauge invariance as the kinetic term of the spin-3/2 fields. To calculate \( f_1(0) \) up to \( \mathcal{O}(p^3) \), one only needs the following lowest-order chiral Lagrangians 26:

\[
\mathcal{L}^{(1)}_{\text{DD}} = \bar{T}_\mu (\gamma^{\mu\nu} i D_\nu - M_{D0} \gamma^{\mu\nu} T^\nu, \quad \mathcal{L}^{(1)}_{\text{DB}} = \frac{iC}{m_D} (D^\mu T^\nu \gamma^{\mu\nu} u_\Lambda B + \bar{B} u_\Lambda \gamma^{\mu\nu} D_\mu T^\nu),
\]

where \( M_{D0} \) is the chiral-limit decuplet-baryon mass, \( D^\mu T^\nu = \partial^\mu T^\nu + (\Gamma^\mu)_{\nu\rho} T^{\rho\nu} + (\Gamma^\mu)_{\rho\nu} T^{\rho\nu} + (\Gamma^\mu)_{\rho\sigma} T^{\rho\sigma\nu} \). The spin-3/2 propagator in \( d \) dimensions is

\[
S^{\mu\nu}(p) = -\frac{\not{p} + M_D}{p^2 - M_D^2 + i\epsilon} \left[ g^{\mu\nu} - \frac{1}{d - 1} \gamma^{\mu} \gamma^{\nu} - \frac{1}{(d - 1) M_D} (\gamma^{\mu} p^{\nu} - \gamma^{\nu} p^{\mu}) - \frac{d - 2}{(d - 1) M_D^2} p^{\mu} p^{\nu} \right].
\]

with \( M_D \) the decuplet baryon mass.

To calculate \( f_1(0) \) at \( \mathcal{O}(p^4) \), the following second-order chiral Lagrangian is needed to break the mass degeneracy of the decuplet baryons 3

\[
\mathcal{L}^{(2)}_{\text{DD}} = \frac{\gamma M}{2} T^\mu X + T^\mu,
\]

which leads to

\[
M_\Delta = M_{D0} + 3m_\pi^2 \gamma_M, \quad M_\Sigma^0 = M_{D0} + 2m_K^2 + m_\eta^2 \gamma_M, \quad M_\Xi^0 = M_{D0} + 3m_K^2 + m_\pi^2 \gamma_M.
\]

A fit to the decuplet baryon masses, with the meson masses given above, yields \( \gamma_M = 0.3236 \) GeV\(^{-1}\) and \( M_{D0} = 1.216 \) GeV, which correspond to \( M_\Delta = 1.235(1.232) \) GeV, \( M_\Sigma^0 = 1.382(1.384) \) GeV, \( M_\Xi^0 = 1.529(1.533) \) GeV, and \( M_\Omega = 1.676(1.672) \) GeV. As in the octet case, we use the second order fits in our calculation of the \( \mathcal{O}(p^4) \) results, while in the \( \mathcal{O}(p^3) \) calculation, we use the average of the decuplet baryon masses, \( M_D = 1.382 \) GeV, for all the decuplet baryons.

\(^2\) Note the definition of \( u_\mu \) in Eq. (6) is a factor of 2 different from that of HBChPT in Refs. 17, 27.

\(^3\) As in the octet case, we have omitted a singlet term indistinguishable from \( M_{D0} \) for our purposes.
III. RESULTS AND DISCUSSIONS

A. SU(3)-breaking corrections to $f_1(0)$ due to octet contributions up to $O(p^4)$

All the diagrams contributing to $f_1(0)$ up to $O(p^4)$ are shown in Fig. 1 where the leading and next-to-leading order SU(3)-breaking corrections are given by the diagrams in the first and second row, respectively.

The $O(p^3)$ results are quite compact and have the following structure for the transition $i \rightarrow j$:

$$
\delta_B^{(2)}(i \rightarrow j) = \sum_{M=\pi,\eta,K} \beta_{M}^{BP} H_{BP}(m_M) + \sum_{M=\pi,\eta,K} \beta_{M}^{MP} H_{MP}(m_M,m_K) + \sum_{M=\pi,\eta,K} \beta_{M}^{KR} H_{KR}(m_M)
-\frac{3}{8} \sum_{M=\pi,\eta} H_{TD1}(m_M,m_K) + \frac{3}{8} \sum_{M=\pi,\eta} H_{TD2}(m_M) + \frac{3}{4} H_{TD2}(m_K)
+ \frac{1}{2} \sum_{M=\pi,\eta,K} (\beta_{M}^{WF}(i) + \beta_{M}^{WF}(j)) H_{WF}(m_M),
$$

where $\beta_{M}^{BP}$, $\beta_{M}^{MP}$, $\beta_{M}^{KR}$, and $\beta_{M}^{WF}$ are given in Tables IV, VII, VIII and IX in the Appendix, and the loop functions $H_{BP}$, $H_{MP}$, $H_{KR}$, $H_{TD1}$, $H_{TD2}$, and $H_{WF}$ are also given there. It is interesting to note that although separately these loop functions are divergent (scale-dependent) and some of them contain power-counting breaking pieces ($H_{KR}$ and $H_{MP}$), the overall contributions are finite and do not break power-counting. This is an explicit manifestation of the Ademollo-Gatto theorem.

In the $O(p^4)$ calculation, we have implemented mass-splitting corrections in a similar way as Ref. 18 except that we have used the masses obtained from the second-order ChPT fit, as described above, instead of the physical masses. Similar to the IRChPT study of Ref. 18, the $O(p^4)$ results contain higher-order divergences. We have removed the infinities using the modified minimal-subtraction ($\overline{MS}$) scheme. The analytical results are quite lengthy and will not be shown here. In Fig. 2 we show the scale dependence of the octet contributions, which is rather mild for most cases except for the $\Sigma$ to $N$ transition. The scale dependence can be used to estimate higher-order contributions by varying $\mu$ in a reasonable range. In the following, we present the results by varying $\mu$ from 0.7 to 1.3 GeV. It should be mentioned that if we had adopted the same method as Ref. 17 to calculate the $O(p^4)$ contributions, i.e., by expanding the results and keeping only those linear in baryon mass splittings, our $O(p^4)$ results would have been convergent.

We have checked that our results up to $O(p^3)$ are the same as those obtained in Ref. 14, while in the $M_B \sim \Lambda_{\chi^S_B}$ limit our results recover the HBChPT ones 17 at both $O(p^3)$ and $O(p^4)$ including the $1/M$ recoil corrections. All these are known to explicitly verify the Ademollo-Gatto theorem in the sense of Eq. (2).

Table I shows the SU(3)-breaking corrections in the notation of Eq. (3). For comparison, we also list the numbers obtained in HBChPT 17 and IRChPT 18. The numerical values are obtained with the parameters given in Table II. As in Ref. 21 we have used an average $f_0 = 1.17 f_\pi$ with $f_\pi = 92.4$ MeV. It should be pointed out that the HBChPT and the IRChPT results are obtained using $f_\pi$.

First, we note that in three of the four cases, the $\delta^{(3)}$ numbers are smaller than the $\delta^{(2)}$ ones. The situation is similar in IRChPT but quite different in HBChPT. In the HBChPT calculation 17, the $\delta^{(3)}$ contribution is larger than the $\delta^{(2)}$ one for the four cases. This tells that recoil corrections (in the HBChPT language) or relativistic effects are

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4 What we denote by $\delta^{(3)}$ is the sum of those labeled by $\alpha^{(3)}$ and $\alpha^{(1/M)}$ in Ref. 17.
TABLE I: Values for the masses and couplings appearing in the calculation of the SU(3)-breaking corrections to $f_1(0)$.

|     |       |             |     |             |
|-----|-------|-------------|-----|-------------|
| $D$ | 0.8   | $M_B$       | 1.151 GeV |
| $F$ | 0.46  | $M_D$       | 1.382 GeV |
| $f_\pi$ | 0.0924 GeV | $M_0$ | 1.197 GeV |
| $F_0$ | 1.17$f_\pi$ | $b_D$ | $-0.0661$ GeV$^{-1}$ |
| $m_\pi$ | 0.138 GeV  | $b_F$ | 0.2087 GeV$^{-1}$ |
| $m_K$ | 0.496 GeV | $M_{D0}$ | 1.216 GeV |
| $m_\eta$ | 0.548 GeV | $\gamma_M$ | 0.3236 GeV$^{-1}$ |
| $C$ | 1.0   |             |     |             |

FIG. 2: Scale dependence of the octet contributions to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$. It is clear from Table I that the convergence is slow even in the case of the relativistic calculations, a well known feature of SU(3) baryon ChPT. It is then necessary to have a way to calculate “higher-order” contributions. Going to $O(p^5)$ one needs to introduce unknown LEC’s such that the predictive power of ChPT is lost. An alternative approach is to consider the contributions of dynamical heavier resonances. A basic assumption of ChPT is that these heavier degrees of freedom can be integrated out with their effects incorporated in the LEC’s. However, that may not be totally true in the one-baryon sector since the gap between the lowest baryon octet and the lowest baryon decuplet is only $\sim 0.3$ GeV, not very different from the pion mass and even smaller than the kaon(eta) mass. Therefore, it is necessary to investigate their contributions. In the HBChPT scheme, this task has recently been undertaken in Ref. [17], where it is concluded that the decuplet contributions completely spoil the convergence of the chiral series. We study the contributions of the decuplet baryons in the covariant framework in the following section.

B. SU(3)-breaking corrections to $f_1(0)$ induced by dynamical decuplet baryons up to $O(p^4)$

Fig. 3 shows the diagrams that contribute to SU(3)-breaking corrections to $f_1(0)$ with dynamical decuplet baryons up to $O(p^4)$. It should be noted that unlike in the HBChPT case [17], Kroll-Rudermann (KR) kind of diagrams also contribute. In fact, using the consistent coupling scheme of Ref. [23], there are four KR diagrams: Two are from minimal substitution in the derivative of the pseudoscalar fields and the other two are from minimal substitution in the derivative of the decuplet fields (see Eq. (10) and also Ref. [26]).
TABLE II: Octet contributions to the SU(3)-breaking corrections to $f_1(0)$ (in percentage). The central values of the $\mathcal{O}(p^3)$ results are calculated with $\mu = 1$ GeV and the uncertainties are obtained by varying $\mu$ from 0.7 to 1.3 GeV.

| Transition | present work | HBChPT [17] | IRChPT [18] |
|------------|--------------|-------------|-------------|
| $\Lambda \to N$ | $-3.8$ | $0.2^{+1.2}_{-0.9}$ | $-3.6^{+1.2}_{-0.9}$ | $2.7$ | $-5.7 \pm 2.1$ |
| $\Sigma \to N$ | $-0.8$ | $4.7^{+2.8}_{-2.8}$ | $3.9^{+3.8}_{-2.8}$ | $4.1$ | $2.8 \pm 0.2$ |
| $\Xi \to \Lambda$ | $-2.9$ | $1.7^{+2.4}_{-1.8}$ | $-1.2^{+2.4}_{-1.8}$ | $4.3$ | $-1.1 \pm 1.7$ |
| $\Xi \to \Sigma$ | $-3.7$ | $-1.3^{+0.2}_{-0.2}$ | $-5.0^{+0.3}_{-0.2}$ | $0.9$ | $-5.6 \pm 1.6$ |

The $\mathcal{O}(p^3)$ results are relatively simple and have the following general structure for the transition $i \to j$:

$$
\delta_D^{(2)} (i \to j) = \sum_{M=\pi,\eta,K} \gamma_{BP}^M D_{BP}(m_M) + \sum_{M=\pi,\eta} \gamma_{MP}^M D_{MP}(m_M, m_K) + \sum_{M=\pi,\eta,K} \gamma_{KR}^M D_{KR}(m_M)
$$

$$
+ \frac{1}{2} \sum_{M=\pi,\eta,K} \left( \gamma_{WF}^M (i) + \gamma_{WF}^M (j) \right) D_{WF}(m_M),
$$

where $\gamma_{BP}^M$, $\gamma_{MP}^M$, $\gamma_{KR}^M$, and $\gamma_{WF}^M$ are listed in Tables X, XI, XII, and XIII of the Appendix. The loop functions $D_{BP}^M$, $D_{MP}^M$, $D_{KR}^M$, and $D_{WF}^M$ can be calculated analytically, but they are quite lengthy. In the Appendix, they are given in terms of Feynman-parameter integrals, which can be easily integrated.

To calculate the $\mathcal{O}(p^4)$ chiral contributions, we implement the decuplet-baryon mass splittings in the same way as in the octet case. The $\mathcal{O}(p^4)$ results contain again higher-order divergences, with the scale dependence shown in

FIG. 4: Scale dependence of the decuplet contributions to the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$. 
TABLE III: Decuplet contributions to the SU(3)-breaking corrections to $f_1(0)$ (in percentage). The central values of the $O(p^4)$ result are calculated with $\mu = 1$ GeV and the uncertainties are obtained by varying $\mu$ from 0.7 GeV to 1.3 GeV.

|       | Present work | HBChPT         |
|-------|--------------|----------------|
|       | $\delta^{(2)}$ | $\delta^{(3)}$ | $\delta^{(2)} + \delta^{(3)}$ |
| $\Lambda \to N$ | 0.7 | $3.0^{+0.1}_{-0.1}$ | $3.7^{+0.1}_{-0.1}$ | 1.8 | 1.3 | 3.1 |
| $\Sigma \to N$ | $-1.4$ | $6.2^{+0.4}_{-0.3}$ | $4.8^{+0.4}_{-0.3}$ | $-3.6$ | 8.8 | 5.2 |
| $\Xi \to \Lambda$ | $-0.02$ | $5.2^{+0.4}_{-0.3}$ | $5.2^{+0.4}_{-0.3}$ | $-0.05$ | 4.2 | 4.1 |
| $\Xi \to \Sigma$ | 0.7 | $6.0^{+1.9}_{-1.4}$ | $6.7^{+1.9}_{-1.4}$ | 1.9 | $-0.2$ | 1.7 |

TABLE IV: SU(3)-breaking corrections to $f_1(0)$ up to $O(p^4)$ (in percentage), including both the octet and the decuplet contributions.

|       | $\delta^{(2)}$ | $\delta^{(3)}$ | $\delta^{(2)} + \delta^{(3)}$ |
|-------|----------------|----------------|----------------|
| $\Lambda \to N$ | $-3.1$ | $3.2^{+1.3}_{-1.0}$ | $0.1^{+1.3}_{-1.0}$ |
| $\Sigma \to N$ | $-2.2$ | $10.9^{+4.2}_{-3.1}$ | $8.7^{+4.2}_{-3.1}$ |
| $\Xi \to \Lambda$ | $-2.9$ | $6.9^{+2.8}_{-2.1}$ | $4.0^{+2.8}_{-2.1}$ |
| $\Xi \to \Sigma$ | $-3.0$ | $4.7^{+2.2}_{-1.6}$ | $1.7^{+2.2}_{-1.6}$ |

Fig. 4. The infinities have been removed by the $\overline{MS}$ procedure. The dependence is found to be rather mild except for the $\Xi \to \Sigma$ transition. This in contrast, unlike in the octet case, the divergences cannot be removed by expanding and keeping only terms linear in baryon and decuplet mass splittings. The full $O(p^4)$ analytical results are quite involved and, therefore, they will not be shown here.

The numerical results obtained with the parameter values given in Table I are summarized in Table III. It can be seen that at $O(p^3)$, the decuplet contributions are relatively small compared to the octet ones at the same order. On the other hand, the $O(p^4)$ contributions are sizable and all of them have positive signs.

Using the conventional Lagrangians of Ref. 25, one obtains different numbers and different $\mu$ dependence. In the heavy-baryon limit, however, the results obtained with both coupling schemes are found to be the same and convergent, confirming the fact that the differences induced by the “consistency” procedure are of higher chiral order [23] (see also Ref. 26).

In Table III, the numbers denoted by HBChPT differ from those of Ref. 17. The $\delta^{(2)}$ column would have coincided if we had used the same values for the couplings $C = 0.8$ and $F_0 = 0.0933$ GeV. On the other hand, our $\delta^{(3)}$ contributions due to the octet baryon mass splittings are much smaller than those of Ref. 17. It is interesting to note that unlike in the octet case, the HBChPT results are similar to the relativistic ones.\(^5\)

As the decuplet-octet mass splitting increases, one expects that the decuplet contributions decrease and eventually vanish as the splitting goes to infinity. This is indeed the case, as can be clearly seen from Fig. 5 where the $O(p^3)$ decuplet contributions are plotted as a function of the decuplet-octet mass splitting.

### C. Full results and comparison with other approaches

Summing the octet and the decuplet contributions, we obtain the numbers shown in Table IV. Two things are noteworthy. First, the convergence is slow, even taking into account the scale dependence of the $\delta^{(3)}$ corrections. Second, for three of the four transitions, the $\delta^{(3)}$ corrections have a different sign than the $\delta^{(2)}$ ones.

\(^5\) The HB results are obtained in a slightly different way than the relativistic ones. To obtain the $O(p^3)$ numbers, we have used $M_B = 1.151$ GeV and $M_D = 1.382$ GeV and have performed an expansion in terms of the decuplet-octet mass splitting, $M_D - M_B$. To obtain the $O(p^4)$ ones, we have used physical masses for both the octet and the decuplet baryons and have performed an additional expansion keeping only the terms linear in the octet and the decuplet baryon mass splittings. Although this procedure is the same as that of Ref. 17, we get different results. We find that the discrepancy comes from the octet mass-splitting corrections to the meson-pole diagram of Fig. 3. If we had mistakenly exchanged the masses of the mesons in the loop, we would have obtained the same results as those of Ref. 17.
In Table V, we compare our results with those obtained from other approaches, including large $N_c$ fits, quark models, and two quenched LQCD calculations. The large $N_c$ results in general favor positive corrections, which are consistent with our central values. Two of the quark models predict negative corrections, while that of Ref. 13 favors positive corrections. It is interesting to note that in Ref. 13 the valence quark effects give negative contributions, as in the other two quark models, while the chiral effects provide positive contributions, resulting in net positive corrections. Our numbers also agree, within uncertainties, with the quenched LQCD ones. In principle, LQCD calculations provide another model-independent way to obtain the SU(3)-breaking corrections to $f_1(0)$. At present, however, the quenched LQCD calculations are not yet accurate enough to determine these numbers, due to the large quark masses used in the simulation and other systematic uncertainties.

Finally, we will briefly discuss the implications of our results for the estimation of $V_{us}$. There have been several previous attempts to extract this parameter using hyperon semileptonic decays. A rather clean determination of $f_1 V_{us}$ can be done by using $g_1/f_1$ and the decay rates from experiment and taking for $g_2$ and $f_2$ their SU(3) values. This latter approximation is supported by the fact that their contributions to the decay rate are reduced by kinematic factors (See, for instance, Eq. (10) of Ref. 3). Using the values of $f_1 V_{us}$ compiled in Table 3 of Ref. 4 and our results for $f_1$ we get

$$V_{us} = 0.2177 \pm 0.0030.$$  \hspace{1cm} (16)

This value is consistent with the large $N_c$ fits of Ref. 3 and with the result obtained from $\tau$ decays, and lower than the results of kaon decays and the fits to hyperon decays from Refs. 1, 2. Although the quoted error seems competitive with calculations using other processes, we must remark that the error estimation for Eq. (16) includes only the experimental errors and the uncertainties related to the scale dependence. Other systematic uncertainty sources, like the effect of higher order SU(3)-breaking corrections are hard to estimate and have not been included. Even with these limitations, our results clearly point to positive values for the SU(3)-breaking corrections to $f_1$ and therefore towards relatively small values of $V_{us}$.

### TABLE V: SU(3)-breaking corrections (in percentage) to $f_1(0)$ obtained in different approaches.

| Process      | Present work | Large $N_c$ | Quark model | LQCD         |
|--------------|--------------|-------------|-------------|--------------|
| $\Lambda \rightarrow N$ | $0.1^{+1.3}_{-1.0}$ | $2 \pm 2$ | $-1.3$ | $-2.4$ | $0.1$ |
| $\Sigma \rightarrow N$ | $8.7^{+4.2}_{-3.1}$ | $4 \pm 3$ | $-1.3$ | $-2.4$ | $0.9$ | $-1.2 \pm 2.9 \pm 4.0$ [19] |
| $\Xi \rightarrow \Lambda$ | $4.0^{+2.8}_{-2.1}$ | $4 \pm 4$ | $-1.3$ | $-2.4$ | $2.2$ |
| $\Xi \rightarrow \Sigma$ | $1.7^{+2.2}_{-1.6}$ | $8 \pm 5$ | $-1.3$ | $-2.4$ | $4.2$ | $-1.3 \pm 1.9$ [20] |
IV. SUMMARY AND CONCLUSIONS

We have performed a study of the SU(3)-breaking corrections to the hyperon vector coupling $f_1(0)$ in covariant baryon chiral perturbation theory including both the octet and the decuplet contributions. We confirm earlier findings in HBChPT and IRChPT that the convergence of the chiral series is slow in the case with only dynamical octet baryons. Our study of the decuplet contributions shows that at $\mathcal{O}(p^3)$ they are in general smaller than those of their octet counterparts, while at $\mathcal{O}(p^4)$ they are sizable. Combining both octet and decuplet contributions, we found positive SU(3)-breaking corrections to all the four independent $f_1(0)$'s, which compare favorably with the large $N_c$ fits and those of the quark model taking into account chiral effects.

The fact that the $\mathcal{O}(p^4)$ chiral contributions are comparable to the $\mathcal{O}(p^3)$ ones suggests that the $\mathcal{O}(p^5)$ chiral effects may not be negligible. We have estimated their size by varying $\mu$ from 0.7 to 1.3 GeV. Taking into account these higher-order uncertainties, our results still favor positive SU(3)-breaking corrections to the four $f_1(0)$'s.

An accurate determination of $V_{us}$ from hyperon semileptonic decays depends largely on our knowledge on the value of $f_1(0)$. While the SU(3)-symmetric values have been used in some fits to extract $V_{us}$, most studies have taken into account SU(3)-breaking corrections to $f_1(0)$. We have provided the first covariant baryon ChPT predictions for $f_1(0)$ up to $\mathcal{O}(p^4)$ including both the octet and the decuplet contributions. We encourage their uses in new attempts to extract $V_{us}$ from hyperon decay data.

V. ACKNOWLEDGMENTS

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VI. APPENDIX

A. Octet $O(p^3)$ contributions

In this subsection, we present the coefficients and loop functions appearing in the calculation of the $O(p^3)$ octet contributions, i.e., Eq. \[13\].

\[H_{BP} = \frac{1}{(4\pi F_0)^2} \left[ -3m^2 - \frac{4\cos^{-1}(\frac{m}{2M_B})m^3}{\sqrt{4M_B^2 - m^2}} \left( \frac{m^2}{M_B} - 3 \right) + \frac{2\log\left(\frac{m^2}{M_B^2}\right)m^4}{M_B^2} + 2\log\left(\frac{M_B^2\mu}{m^3}\right)m^2 \right], \tag{17}\]

\[H_{MP} = \frac{1}{(4\pi F_0)^2} \frac{1}{4(m_1^2 - m_2^2)} \times \left[ \frac{8\left(\frac{m_1^2}{M_B^2} - 4\right)\cos^{-1}\left(\frac{m_1}{2M_B}\right)m_1^6}{\sqrt{4m_1^2M_B^2 - m_1^2}} - \frac{4\log\left(\frac{m_1^2}{M_B^2}\right)m_1^6}{M_B^2} + \left( 6\log\left(\frac{m_1^2}{M_B^2}\right) + 2\log\left(\frac{\mu^2}{M_B^2}\right) + 11 \right)m_1^4 \right. \\
\left. - \frac{8\left(\frac{m_2^2}{M_B^2} - 4\right)\cos^{-1}\left(\frac{m_2}{2M_B}\right)m_2^6}{\sqrt{4m_2^2M_B^2 - m_2^2}} + \frac{4\log\left(\frac{m_2^2}{M_B^2}\right)m_2^6}{M_B^2} - \left( 6\log\left(\frac{m_2^2}{M_B^2}\right) + 2\log\left(\frac{\mu^2}{M_B^2}\right) + 11 \right)m_2^4 \right. \\
\left. + 8M_B^2(1 + \log\left(\frac{\mu^2}{M_B^2}\right))(m_1^2 - m_2^2) \right], \tag{18}\]

\[H_{KR} = \frac{1}{(4\pi F_0)^2} \left[ \log\left(\frac{\lambda_B}{m}\right)m^4 - \frac{\sqrt{4M_B^2 - m^2}\cos^{-1}\left(\frac{m}{2M_B}\right)m^3}{M_B^2} + m^2(\log\left(\frac{\mu^2}{M_B^2}\right) + 2) + M_B^2(1 + \log\left(\frac{\mu^2}{M_B^2}\right)) \right], \tag{19}\]

\[H_{TD1} = \frac{1}{(4\pi F_0)^2} \left[ \log\left(\frac{\mu^2}{m_1^2}\right)m_1^4 - \log\left(\frac{\mu^2}{m_2^2}\right)m_2^4 + \frac{3}{2}(m_1^2 + m_2^2) \right], \tag{20}\]

\[H_{TD2} = \frac{1}{(4\pi F_0)^2} \left[ (\log\left(\frac{\mu^2}{m_1^2}\right) + 1)m_1^2 \right], \tag{21}\]

\[H_{WF} = \frac{1}{(4\pi F_0)^2} \left[ 2\log\left(\frac{m^2}{M_B^2}\right)m^4 + \frac{4\left(3 - \frac{m^2}{M_B^2}\right)\cos^{-1}\left(\frac{m}{2M_B}\right)m^3}{\sqrt{4M_B^2 - m^2}} + \left( \log\left(\frac{\mu^2}{m^2}\right) - 2\log\left(\frac{m^2}{M_B^2}\right) - 3 \right)m^2 \right]. \tag{22}\]

**TABLE VI: Coefficients $\beta_{BP}$ appearing in Eq. \[13\].**

| Channel | $\pi$ loop | $\eta$ loop | $K$ loop |
|---------|------------|-------------|----------|
| $\Lambda \to N$ | $-\frac{1}{2}D(D + F)$ | $-\frac{1}{6}D(D - 3F)$ | $-\frac{1}{6}(D - 3F)^2$ |
| $\Sigma \to N$ | $-\frac{1}{2}(D^2 + 3FD + 2F^2)$ | $\frac{1}{6}D(D - 3F)$ | $-\frac{1}{6}(D + F)^2$ |
| $\Xi \to \Lambda$ | $\frac{1}{2}D(F - D)$ | $-\frac{1}{6}D(D + 3F)$ | $-\frac{1}{6}(D + 3F)^2$ |
| $\Xi \to \Sigma$ | $-\frac{1}{6}(D^2 - 3FD + 2F^2)$ | $\frac{1}{6}D(D + 3F)$ | $-\frac{1}{6}(D - F)^2$ |
### TABLE VII: Coefficients $\beta^{MP}$ appearing in Eq. (14).

| Channel | $\pi K$ loop | $\eta K$ loop |
|---------|--------------|---------------|
| $\Lambda \rightarrow N$ | $\frac{1}{4}(3D^2 + 2FD + 3F^2)$ | $\frac{1}{12}(D + 3F)^2$ |
| $\Sigma \rightarrow N$ | $\frac{1}{12}(D^2 - 18FD + 9F^2)$ | $\frac{3}{4}(D - F)^2$ |
| $\Xi \rightarrow \Lambda$ | $\frac{1}{4}(3D^2 - 2FD + 3F^2)$ | $\frac{1}{4}(D - 3F)^2$ |
| $\Xi \rightarrow \Sigma$ | $\frac{1}{12}(D^2 + 18FD + 9F^2)$ | $\frac{3}{4}(D + F)^2$ |

### TABLE VIII: Coefficients $\beta^{KR}$ appearing in Eq. (14).

| Channel | $\pi$ loop | $\eta$ loop | $K$ loop |
|---------|------------|-------------|----------|
| $\Lambda \rightarrow N$ | $-\frac{1}{4}(3D^2 + 2FD + 3F^2)$ | $-\frac{1}{12}(D + 3F)^2$ | $-\frac{1}{6}(5D^2 + 6FD + 9F^2)$ |
| $\Sigma \rightarrow N$ | $-\frac{1}{12}(D^2 - 18FD + 9F^2)$ | $-\frac{3}{4}(D - F)^2$ | $-\frac{1}{6}(D - 3F)(5D - 3F)$ |
| $\Xi \rightarrow \Lambda$ | $-\frac{1}{4}(3D^2 - 2FD + 3F^2)$ | $-\frac{1}{12}(D - 3F)^2$ | $-\frac{1}{6}(5D^2 - 6FD + 9F^2)$ |
| $\Xi \rightarrow \Sigma$ | $-\frac{1}{12}(D^2 + 18FD + 9F^2)$ | $-\frac{3}{4}(D + F)^2$ | $-\frac{1}{6}(D + 3F)(5D + 3F)$ |

### TABLE IX: Coefficients $\beta^{WF}$ appearing in Eq. (14).

| Channel | $\pi$ loop | $\eta$ loop | $K$ loop |
|---------|------------|-------------|----------|
| $\Lambda$ | $D^2$ | $\frac{D^2}{3}$ | $\frac{1}{9}(D^2 + 9F^2)$ |
| $\Sigma$ | $\frac{1}{2}(D^2 + 6F^2)$ | $\frac{D^2}{3}$ | $D^2 + F^2$ |
| $N$ | $\frac{1}{4}(D + F)^2$ | $\frac{1}{12}(D - 3F)^2$ | $\frac{1}{6}(5D^2 - 6FD + 9F^2)$ |
| $\Xi$ | $\frac{1}{8}(D - F)^2$ | $\frac{1}{12}(D + 3F)^2$ | $\frac{1}{6}(5D^2 + 6FD + 9F^2)$ |
B. Decuplet $O(p^3)$ contributions

In this subsection, we provide the coefficients and loop functions appearing in the calculation of the $O(p^3)$ decuplet contributions, i.e., Eq. (15).

\[ D_{BP} = -\frac{C^2}{(4\pi F_0)^2 M_D^2} \int_0^1 dx \frac{M_B^2 (1-x) \left((2x-1)M_B^2 + 2M_B x M_D - ((x-2)M_B^2 + 2m^2) x \right) \times \log \left( \frac{\mu^2}{((x-1)M_B^2 + m^2) x - M_D^2(x-1)} \right)}{\mu^2} - (M_B^2 + 2M_D M_B + M_D^2 - m^2) x) \right), \]  

\[ D_{MP} = -\frac{C^2}{(4\pi F_0)^2 M_D^2} \int_0^1 dx \int_0^{1-x} dy \frac{M_B^2 (2M_B x (x M_B - M_B - M_D) + (-3(x-1) x M_B^2 + 2M_D x M_B - M_D^2 x - m_B^2 y + m_B^2 (x + y - 1)) \log \left( \frac{\mu^2}{((x-1) x M_B^2 + M_D^2 x + m_B^2 y - m_B^2(x + y - 1))} \right)}{\mu^2} \right), \]  

\[ D_{KR} = -\frac{C^2}{(4\pi F_0)^2 M_D^2} \int_0^1 dx \frac{M_B^2 (2M_D + M_B x) \left((x-1)M_B^2 + m^2) x - M_D^2(x-1) \right) \times \log \left( \frac{\mu^2}{M_D^2(x-1) - ((x-1)M_B^2 + m^2) x} \right)}{\mu^2} \right), \]  

\[ D_{WF} = -\frac{C^2}{(4\pi F_0)^2 M_D^2} \int_0^1 dx \frac{M_B^2 (2M_B^2(x-1) x (M_B x - M_B - M_D) + (-5M_B^2(x-1)^2 x + 4M_B^2 M_D (x-1) x + 3M_B(x-1) (m^2(x-1) - M_D^2 x) + 2M_D (M_D^2 x - m^2(x-1))) \log \left( \frac{\mu^2}{m^2(x-1) - x(M_B^2(x-1) + M_D^2)} \right)}{m^2(x-1) - x(M_B^2(x-1) + M_D^2))} \right). \]  

| Channel | $\pi$ loop | $\eta$ loop | $K$ loop |
|---------|------------|-------------|----------|
| $\Lambda \rightarrow N$ | -4 | 0 | -2 |
| $\Sigma \rightarrow N$ | -4 | 0 | -2 |
| $\Xi \rightarrow \Lambda$ | -2 | 0 | -2 |
| $\Xi \rightarrow \Sigma$ | -4 | -2 | -14 |
TABLE XI: Coefficients $\gamma^{MP}$ appearing in Eq. (15).

| Channel   | $\pi K$ loop | $\eta K$ loop |
|-----------|--------------|---------------|
| $\Lambda \to N$ | 1            | 0             |
| $\Sigma \to N$  | -2           | -1            |
| $\Xi \to \Lambda$ | 0            | -1            |
| $\Xi \to \Sigma$ | 1            | 2             |

TABLE XII: Coefficients $\gamma^{KR}$ appearing in Eq. (15).

| Channel   | $\pi$ loop | $\eta$ loop | $K$ loop |
|-----------|------------|-------------|----------|
| $\Lambda \to N$ | 7          | 0           | 3        |
| $\Sigma \to N$  | $\frac{14}{3}$ | 1           | $\frac{13}{3}$ |
| $\Xi \to \Sigma$ | 4          | 1           | 5        |
| $\Xi \to \Sigma$ | $\frac{5}{7}$ | 2           | $\frac{19}{7}$ |

TABLE XIII: Coefficients $\gamma^{WP}$ appearing in Eq. (15).

|        | $\pi$ loop | $\eta$ loop | $K$ loop |
|--------|------------|-------------|----------|
| $\Lambda$ | -3        | 0           | -2       |
| $\Sigma$ | $-\frac{2}{3}$ | -1          | $-\frac{10}{3}$ |
| $\Xi$    | -4         | 0           | -1       |
| $\Xi$    | -1         | -1          | -3       |