Optimal Design of Airbag Landing System without Rebound

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Abstract. Airbag system has so many advantages including small volume, superior cushioning performance and easy to control that it has been widely used in many fields such as heavy cargo airdrop, soft landing of spacecraft and so on. In this paper, an optimal design method of the airbag is proposed. First, based on the law of thermodynamics and the deformation assumption of airbag, a mathematical model of airbag landing process is established. The results of the model calculation of the cylindrical airbag is preferably consistent with the results of finite element analysis, which shows that the airbag mathematical model is reasonable and accurate. Second, on the basis of this model, the optimal design method of the airbag without rebound is proposed to solve the problem of rebound which will result in uncontrollable attitude and secondary shock in the landing processes. In this method, the evaluation index of airbag cushioning performance is determined, then the key design parameters which have significant impact on airbag cushioning performance are studied, and the optimization model of airbag under constraint with no rebound is subsequently established and solved. Third, by this method, a cylindrical airbag without rebound is obtained. Compared with the non-optimized one, the maximum impact acceleration of the optimized cylindrical airbag is smaller. Consequently, the effectiveness of the proposed optimal design method is verified.

1. Introduction

Although there are various types of cushion airbags, they can be classified into three types according to the cushioning mechanism: non-vented airbag, vented airbag and combined airbag. Non-vented airbag does not exhaust externally, and consumes energy mainly through internal gas compression and multiple bounces of the system [1]. Therefore, the payload does not directly touch the ground during landing process. Due to the inevitable rebound and rollover, the airtight airbag system will result in an uncontrollable final attitude of the payload, increasing the risk of landing. Thus the facilities for attitude adjustment, airbag removal and extra protection have to be added. And the vented airbag is first compressed during the landing process, then the airbag venting mechanism is opened when the internal pressure is increased to the threshold pressure [2]. The exhaust gas will consume most of the energy of the system so that the payload can land without rebound. Compared with the non-vented airbag, the vented airbag can land only through one compression with higher cushioning efficiency, but the system is likely to be hard landing at a certain terminal speed, and the payload will be impacted. The combined airbag is usually composed of two airbags, the outer one is a main-venting airbag, and the inner one is a non-venting anti-bottoming airbag (AB airbag) [3]. It combines the advantages of the first two airbags to eliminate system bounce and avoid the danger of direct contact with the ground surface. However,
regardless of the venting airbag or combination airbag, if the airbag parameters are unreasonable, the system may rebound. Therefore, this must be avoided when designing the airbag.

The research methods of airbag mainly include theoretical analysis, simulation calculation and experimental design. Because of the high cost and time-consuming, experiments are usually used to verify the validity of the designed airbag model. The mathematical model of the Mars PATHFINDER airbag system was established by Cola [4] to analyze its cushioning process, and the calculation results were in good agreement with the experimental results. Using the same thermodynamic theory as the mathematical model of the Mars PATHFINDER airbag, Alizadeh [5] constructed five airbag models of different shapes. And a comparative study of the cushioning performance of these airbags was presented. The software LS-DYNA was used by Welch [6] to simulate the landing process of the airbag system of the crew detection aircraft (CEV), and the simulation results were basically consistent with the experimental results. The geometric parameters of the airbag have a great influence on its cushioning performance. Anthony [7] established the finite element model of Beagle II airbag landing system. By analyzing and optimizing the configuration and parameters of the airbag, the configuration of the spherical airbag was changed from two sections to three sections, resulting in better cushioning performance. Next-2 airbag landing system was optimized by Tutt [8] using the sequential response surface method, and the maximum impact acceleration was finally reduced from 6.2g to 5g. Here, it can be seen that optimization is an effective scheme to improve the cushioning performance of airbag.

It is feasible to compare with various specifications airbags and choose the one with better performance from them, but the efficiency is lower, the result is not optimal, and it is difficult to meet all cushioning performance requirements at the same time. In this paper, based on the law of thermodynamics and the deformation assumption of airbag, a mathematical model of landing process is established suitable for different shape of vented airbag, and on the basis of this model, the optimal design method of the airbag without rebound is proposed. In this method, the optimization model of airbag under constraint with no rebound is established and solved. The cushioning airbag with no rebound and less impact acceleration is obtained.

2. Mathematical model of the airbag system
For the vented airbag system, there are four main dynamic stages in the whole landing process: inflating, freefall, compressing and venting. As shown in Figure 1.

(1) Airbag inflation stage (a-b): when the whole system falls to a certain height, the airbag begins to inflate.
(2) Free fall stage (b-c): the whole system falls until the airbag contact with the ground surface; after the completion of inflation, the thermodynamic state of the airbag remains unchanged.

(3) Airbag compression stage (c-d): after contact with the ground, the parachute disengages and the airbag begins to compress, resulting in the increase of internal pressure.

(4) Airbag venting stage (d-f): once a set pressure threshold is exceeded, the vents built into the airbag are opened immediately and the airbag begins to vent. Throughout the duration of this process, the speed of the payload decreases until it falls safely, and the vented gas carries most of the energy away from the system.

Therefore, the cushioning performance of the airbag is mainly reflected in the compression and venting stages. Then, the two stages are studied in detail to establish the mathematical model of the airbag system.

2.1. Construction of Airbag mathematical model

The compression and venting process of airbag are analyzed from four subprocesses: system dynamic process, airbag deformation process, gas thermodynamic process and orifice flow process. Then the mathematical model of the whole cushioning process is established by taking the relationship between each subprocess into account.

![Airbag deformation process](image)

**Figure 2.** Airbag deformation process

2.1.1. System Dynamics Analysis. Assuming that the payload falls vertically and there is no horizontal velocity, according to Figure 2, performing a force equilibrium calculation with the forces present in this single degree of freedom system in the vertical direction (upward is positive) yields the following system dynamics equation:

\[ Ma + (P_{\text{bag}} - P_{\text{atm}})A_t = Mg \]  

Where \( M \) is the mass of the payload, \( P_{\text{bag}} \) is the internal pressure of the airbag, \( P_{\text{atm}} \) is the atmospheric pressure, and \( A_t \) is airbag footprint area. This equation is the basis for determining the system dynamic state at each time step within the airbag cushioning model. Rearranging Equation (1) yields the acceleration of the payload:

\[ a = g - \frac{(P_{\text{bag}} - P_{\text{atm}})A_t}{M} \]  

Then, the change in payload velocity over each time step \( \Delta t \) can be expressed as:

\[ \Delta u = a \cdot \Delta t \]  

Therefore, the velocity of the payload is obtained from:

\[ u_t = u_{t-1} + \Delta u \]  

Where \( u_{t-1} \) is the velocity of the payload in the previous time step.
And the displacement of the payload is obtained from:

\[ h_i = h_{i-1} + u_i \Delta t + \frac{1}{2} a \Delta t^2 \]  \hspace{1cm} (5)

Where \( h_{i-1} \) is the displacement of the payload in the previous time step.

2.1.2. Airbag deformation assumption. The efficiency of the energy transfer between the payload and the airbags is directly related to the manner in which this geometry changes during the compression process. Using a finite element method to investigate the fluid-structure interaction effects occurring between the airbag material and the operating gas, the deformation can be more accurately predicted [9]. However, it takes too long time to solve, so the shape function equations of the airbag is often assumed in the design stage.

In the study of cylindrical airbags, Esgar and Morgan [10] assumed that the axial length of the cylindrical airbag and the circumference of the airbag cross section both remains constant throughout the compression process. As a result, it only focus on the changing cross section of the cylindrical airbag from its initial circular shape, as shown in Figure 2. Hence, this can be expressed as:

\[ \pi D_0 = \pi D_i + 2L_i \]  \hspace{1cm} (6)

Where \( D_0 \) is the cross-section initial diameter of cylindrical airbag, \( D_i \) is the height of the deformed airbag, and \( L_i \) is the airbag footprint length.

According to the dynamic equation of the system, the displacement of the payload is obtained. Then, the height of the deformed airbag is obtained from:

\[ D_i = D_0 - h_i \]  \hspace{1cm} (7)

Combining Equations (6) and (7) yields a relationship for the airbag footprint length as a function of the payload displacement. That is:

\[ L_i = \frac{\pi}{2} (D_0 - D_i) = \frac{\pi}{2} h_i \]  \hspace{1cm} (8)

On the other hand, the cross sectional area of the airbag in the stroked state is the sum of the areas of a rectangle and two semi-circles, as shown in Figure 2. Thus:

\[ S_i = \frac{\pi}{4} D_i^2 + D_i L_i = \frac{\pi}{4} (D_0 - h_i)^2 + (D_0 - h_i)L_i \]  \hspace{1cm} (9)

Since the cylindrical airbag axial length is assumed to remain constant, the cylindrical airbag volume and contact surface area (or footprint area) can be obtained by multiplying Equations (8) and (9) by this fixed length. That is:

\[ A_i = \frac{\pi}{2} L_i h_i \]  \hspace{1cm} (10)

And

\[ V_i = L_o (D_0 - h_i) \left[ \frac{\pi}{4} (D_0 - h_i) + L_i \right] \]  \hspace{1cm} (11)

Where \( L_o \) is the fixed cylindrical airbag axial length.

Similar to the above method, the deformation assumptions can be given for airbags of different shapes, based on which the airbag volume and footprint area in the cushioning process can be also obtained. As shown in Table 1, the deformation assumptions of six different shapes of airbags during cushioning processing are presented.

2.1.3. Gas thermodynamic analysis. With the expression of the airbag volume now obtained, standard gas dynamics equations can be used to obtain the internal pressure of the airbag and determine whether the conditions required for the airbag venting mechanisms to open are met.
Table 1. Deformation assumptions for different shapes of airbags

| Airbag type                  | Deformation assumptions                                                                 |
|------------------------------|-----------------------------------------------------------------------------------------|
| Cylindrical (horizontal)    | Esgar and Morgan’s assumption [10].                                                      |
| Cylindrical (vertical)      | Assuming that the footprint area remains constant during the compression process, the change of the airbag volume only depends on the change of the airbag height. |
| cubic [1]                   | Similar to the cylindrical airbag (horizontal) deformation assumption.                   |
| Two truncated pyramid       | Assuming that the footprint area remains constant during the compression process, the change of the airbag volume only depends on the change of the airbag height. However, for an inverse type, it is assumed that the footprint area starts from the area of the lower surface and increases to the area of the upper surface [5]. |
| Toroidal [11]               | Since the cross section is a circular surface, the same deformation assumption as that of a cylinder airbag (horizontal) can be used. |
| Truncated cone              | Similar to the deformation assumption of the two truncated pyramid.                      |

To simplify the analysis of the model, the following assumptions are made:

1. The operating medium within the airbags acts as an ideal gas.
2. The whole landing process is isentropic.

Because of its fast speed and short time, there is no heat exchange of airbag with the environment. As a result, the compression and venting process of airbag can be regarded as an adiabatic process. On the other hand, the venting port is short and the friction resistance can be neglected, so the impact of irreversible effect is relatively small [9]. Consequently, the isentropic process assumption is appropriate.

The ideal gas equation is given for the airbag system as:

$$P_{bag}V_t = m_t R_{GAS} T_t$$  \hspace{1cm} (12)

Where $m_t$ is the mass of the gas in the airbag at the current time step, $T_t$ is the operating temperature at the current time step, and $R_{GAS}$ is the specific gas constant.

With this, the isentropic process equation is given as:

$$\frac{T_t}{T_0} = \left( \frac{P_{bag}}{P_0} \right)^{\gamma-1} = \left( \frac{\rho_t}{\rho_0} \right)^{\gamma-1}$$  \hspace{1cm} (13)

Where the subscripts 0 indicate the initial state of the system, $\gamma$ is the ratio of specific heats of the operating medium, and $\rho_t$ is the gas density which can be obtained from:

$$\rho_t = \frac{m_t}{V_t}$$  \hspace{1cm} (14)

During the compression stage, the airbag does not exhaust and the gas mass remains unchanged. Thus:

$$m_t = m_0$$  \hspace{1cm} (15)

Where $m_0$ is the initial mass of the gas in the airbag.

And during the venting phase, the mass of the gas in the airbag at the current time step is obtained from:

$$m_t = m_{t-1} - \Delta m$$  \hspace{1cm} (16)

Where $m_{t-1}$ is the mass of the gas in the previous time step, and $\Delta m$ is the mass of gas through the orifice at the current time step, which can be obtained according to the orifice flow equation established in the next section.
Therefore, the internal pressure of the airbag at each time step can be obtained by combining the above equations, subsequently which is compared with the pressure threshold $P_{\text{open}}$ to determine the opening state of the airbag venting mechanism.

### 2.1.4. Orifice flow analysis

During the airbag venting phase, the flow of a gas at the location of the orifice can be modeled by the mass flow equation, as given by:

$$\frac{\Delta m}{\Delta t} = C_d A_{\text{or}} \rho_{\text{or}} u_{\text{or}}$$  \hspace{1cm} (17)

Where $C_d$ is the discharge coefficient, $A_{\text{or}}$ is the orifice area during the current time step, $\rho_{\text{or}}$ is the gas density at the location of the orifice, and $u_{\text{or}}$ is the flow velocity through the orifice.

Discharge coefficient is a factor representing the inefficiencies inherent to orifice flow because of the losses due to frictional and fluidic viscous effects as the gas flows through the orifice, which varies as a function of the pressure ratio across the orifice $[9]$. That is:

$$C_d = -3.8399 \lambda^6 + 9.4363 \lambda^5 - 7.2326 \lambda^4 + 1.6972 \lambda^3 - 0.2908 \lambda^2 - 0.013 \lambda + 0.8426$$  \hspace{1cm} (18)

Where $\lambda$ is the pressure ratio at the orifice and upstream from it. Since the average flow of gas in the airbag is close to zero, a standard nozzle flow equation can be used to relate the flow velocity through the orifice, to the pressure ratio. Assuming that the upstream pressure is equivalent to the airbag pressure, and the orifice pressure is equal to the downstream atmospheric pressure $[9]$, $\lambda$ can be expressed as:

$$\lambda = \frac{P_{\text{atm}}}{P_{\text{bag}}} = \left[ 1 + \frac{(\gamma - 1)M^2_{\text{or}}}{2} \right]^{\frac{1}{\gamma - 1}}$$  \hspace{1cm} (19)

Where $M_{\text{or}}$ is the Mach number of the gas flowing through the orifice.

Rearranging Equation (19) yields a relationship for the Mach number as a function of the airbag internal pressure. That is:

$$M_{\text{or}} = \left\{ \frac{2}{\gamma - 1} \left[ \frac{P_{\text{atm}}^{\frac{1}{\gamma}}}{P_{\text{bag}}^{\frac{1}{\gamma}}} - 1 \right] \right\}^{\frac{1}{2}}$$  \hspace{1cm} (20)

And the velocity of gas is obtained from:

$$u_{\text{or}} = M_{\text{or}} a_{\text{or}}$$  \hspace{1cm} (21)

Where $a_{\text{or}}$ is the speed of sound through a medium. For an isentropic process of the ideal gas, the speed of sound can be expressed as:

$$a_{\text{or}} = \sqrt{\gamma \frac{R_{\text{gas}}}{T_{\text{or}}}}$$  \hspace{1cm} (22)

Where $T_{\text{or}}$ is the temperature of the gas at the orifice.

It is assumed that the gas at the orifice experiences an isentropic process from the initial system state to the current state, and since the orifice is initially closed, the initial pressure and temperature of the gas at the orifice is the same as that of the gas within the airbag $[9]$. That is:

$$T_{\text{or}} = T_{\text{atm}} \left( \frac{P_{\text{atm}}}{P_{\text{or}}} \right)^{\frac{1}{\gamma}}$$  \hspace{1cm} (23)

And density form of the ideal gas law applied at the orifice is expressed as:

$$\rho_{\text{or}} = \frac{P_{\text{or}}}{\gamma R_{\text{gas}} T_{\text{or}}}$$  \hspace{1cm} (24)

Substituting equations (20) to (24) into equations (17), and considering that the flow velocity of air through the orifice may be equal to the sound speed, the final form of this orifice flow equation can be obtained $[4]$.

For the subsonic flow ($\lambda \geq 0.528$):
\[
\Delta m / \Delta t = C_D A_w \rho_0 \left(\frac{1}{R_{\text{Gas}} T_1}\right)^2 \left[\frac{2\gamma}{\gamma - 1} \left(\frac{P_1}{P_0}\right)^{\gamma - 1}\right]^{\frac{1}{2}} \left[\frac{P_{\text{bag}}}{P_{\text{gaseous}}} - 1\right]^{\frac{1}{2}}
\]  

(25)

For the sonic flow (\lambda < 0.528):

\[
\Delta m / \Delta t = C_D A_w \rho_1 \left(\frac{1}{R_{\text{Gas}} T_1}\right)^2 \left[\gamma \left(\frac{2}{\gamma + 1}\right)^{\frac{\gamma - 1}{\gamma}}\right]^{\frac{1}{2}} \left[\frac{P_{\text{bag}}}{P_1} - 1\right]^{\frac{1}{2}}
\]  

(26)

2.1.5. **Single airbag model calculation process.** According to the above analysis, the calculation flow of the single airbag cushioning model can be established, as shown in figure 3.
By the system dynamics analysis, the acceleration of the payload is obtained. And the displacement of the payload can be calculated during the current time step. Subsequently, based on the airbag deformation assumption, the footprint area and volume of the airbag are obtained. Then, the information such as the density and internal pressure of the gas in the airbag is obtained by thermodynamic analysis. If the internal pressure exceeds the threshold pressure, the airbag venting mechanism is opened and the mass of gas through the orifice during the current time step is obtained by the orifice flow equation. In this case, an iterative solution is used to obtain information on the airbag after venting, because the orifice flow analysis requires airbag pressure information from the gas dynamics analysis, which in turn requires knowledge of the mass of gas within the airbag, which is dependent on the orifice flow conditions. After that, the system information for the next time step will be calculated. If the internal pressure is less than the threshold pressure, the next state is calculated directly and there is no iterative solution. In particular, once the venting mechanism is opened, it will not be closed. Consequently, before the venting mechanism is opened, the internal pressure of the airbag should be compared with the threshold pressure to determine whether to exhaust; and after the venting mechanism is opened, it is necessary to compare with the atmospheric pressure to determine whether to exhaust. In addition, the mass of the gas within the airbag is judged at the end of each time step. If the gas within the airbag is completely discharged ($m \leq 0$), the calculation is stopped. In this paper, a time step of 0.001 seconds was used.

2.2. Model calculation and verification
The model parameters of the cylindrical airbag (horizontal type) are shown in Table 2. It is assumed that the orifice area remains constant during the venting process. The cushioning performance of the airbag system is analyzed using the established mathematical model, which is compared with the calculation results of LS-DYNA for the same model, as shown in Figure 4. And the finite element model of the cylindrical airbag (horizontal type) is shown in Figure 5.

| Table 2. Cylindrical airbag model parameters |
|---------------------------------------------|
| **Model** | **Parameter** | **Symbol** | **Quantity** | **Unit** | **Remarks** |
| Cylindrical airbag | Initial diameter | $D_0$ | 1.00 | m | Horizontal cylindrical airbag |
| | Airbag axial length | $L_0$ | 1.50 | m | |
| | Orifice diameter | $D_{or}$ | 0.1 | m | |
| | Fabric thickness | $d$ | 0.002 | m | |
| | Density | $\rho$ | 1.205 | kg/m$^3$ | |
| | Initial pressure | $P_0$ | 101325 | Pa | |
| | Initial temperature | $T_0$ | 293.15 | K | |
| | Threshold pressure | $P_{open}$ | 130000 | Pa | |
| | Specific gas constant | $R_{GAS}$ | 286.9 | J/kg/K | |
| | Ratio of specific heats | $\gamma$ | 1.4 | | |
| Payload | Mass | $M$ | 500 | kg | Corresponds to the nominal velocity of the Orion CEV[12] |
| Speed parameter | Initial velocity | $u_0$ | 7.62 | m/s | |
| Environment | Pressure | $P_0$ | 101325 | Pa | Earth |
| Gravity parameter | Gravity acceleration | $g$ | 9.8 | m/s$^2$ | |
According to Figure 4, in the process of airbag landing, the six parameters of airbag internal pressure, airbag volume, gas mass, payload acceleration, payload speed and payload displacement calculated by the model in this paper are in good agreement with the calculation results of the finite element model. Here, the deviation of the internal pressure of the airbag is the largest, about 10.03%, which is within an acceptable range. Since the deformation of the airbag is obtained by the assumption, the airbag volume
calculated by the mathematical model is smaller, resulting in a larger airbag internal pressure. The deviation of the maximum impact acceleration is 3.06%, which is smaller than the deviation of the internal pressure of the airbag. This is because the acceleration of the payload is determined not only by the internal airbag pressure but also by the footprint area. And the speed and displacement of the payload obtained by this two methods are in good agreement. These results show that the airbag mathematical model is reasonable and accurate. On the other hand, according to the time-displacement curve of the payload, as shown in figure 4(f), it can be seen that the airbag system rebounded during the cushioning process.

3. Optimization design method

3.1. Optimized mathematical model

3.1.1. Determination of optimization objective. In general, the cushioning performance of the airbag can be evaluated from four aspects: airbag fabric strength, impact acceleration, payload touchdown speed, and weather to rebound. However, these performance parameters should be quantified in theoretical analysis or simulation calculation.

Whether the airbag will burst during the cushioning process is mainly affected by the internal pressure. Therefore, the maximum internal pressure of the airbag is selected as an evaluation index to meet the strength requirement of the airbag fabric. The impact load that the body or equipment can withstand is limited, so the maximum impact acceleration during the cushioning process can be chosen as the evaluation index. In the venting process, if the gas flow rate is too high and the energy of the payload is not fully absorbed, the payload will contact with the ground surface at high speed, thereby resulting in a large impact, so the touchdown speed cannot be too high, which is able to be an evaluation index. Conversely, if the gas flow rate is too slow, it may accelerate the payload in the reverse direction, which causes rebound. Since the payload does not directly contact with the ground surface, the touchdown speed in this case is defined as zero. Obviously, when the payload displacement is reversed, it indicates that the system has rebounded, hence the rebound information about the system can be obtained by judging whether the payload displacement is positive or negative at each time step.

As shown in Table 3, the evaluation index of the airbag cushioning performance and its solving method are finally determined. In addition, these evaluation index can be selected as the optimization target or constraint condition of the optimized design. Under different usage requirements and working conditions, the optimization objective may be different, and can be selected according to actual working conditions and engineering requirements.

| Cushioning performance | evaluation index | Symbol | Solving method |
|------------------------|------------------|--------|----------------|
| Fabric strength        | maximum internal pressure | $P_{\text{max}}$ | Obtain maximum internal pressure in the airbag |
| Impact acceleration    | maximum impact acceleration | $a_{\text{max}}$ | Obtain maximum impact acceleration of the payload |
| Touchdown speed        | payload touchdown speed | $u_L$ | Obtain the payload touchdown speed; if the system rebounds, $u_L = 0$ |
| Weather to rebound     | whether the payload displacement is positive at each time step | $F_R$ | If the displacement during current time step is positive, indicating that the system has rebounded , that is $F_R = 1$ ; otherwise, the system does not rebound, then $F_R = 0$ |

Table 3. Evaluation index of airbag cushioning performance and its solving method
3.1.2. Selection of design variables. Once the optimization goal has been selected, the design variables should be determined. According to Table 2, the design parameters of the airbag mainly include geometric parameters and gas parameters. As there are many design parameters of airbag, it is not only difficult but also time consuming to optimize directly. Therefore, it is necessary to study the key influence parameters of the optimization target before optimization, and the design parameters with greater influence should be selected as the design variables.

Through the analysis of influencing factors, the degree of influence of each design parameter on the optimization objective can be obtained. The main steps are as follows: firstly, the experimental design of the design parameters of the airbag is carried out, and then the calculation results are analyzed to obtain the influence degree by range analysis method, finally the main influence parameters are selected by the order of the degree of influence.

The range analysis method (R method) includes two steps of calculation and judgment. Suppose that the orthogonal table \( L_{n^m} \) is designed. \( K_{jk} \) is the sum of the responses of experimental groups whose factor serial number is \( j \) (\( j \leq m \)) and level serial number is \( k \) (\( k \leq n \)), and \( \bar{K}_{jk} \) is the average of \( K_{jk} \), which is called the index value. The difference between the maximum value and the minimum value of the index values is defined as the range of experimental groups whose factor serial number is \( j \). That is:

\[
R_j = \max(K_{j1}, K_{j2}, \ldots, K_{jm}) - \min(K_{j1}, K_{j2}, \ldots, K_{jm})
\]

(27)

R_j reflects the fluctuation of the response when the level serial number is changed of the experimental groups whose factor serial number is \( j \). Therefore, the larger the \( R_j \), the greater the impact of this factor on the response, and the more important it is. In addition, the influence degree \( P_{R_j} \) of the factor whose serial number is \( j \) on the response can be obtained by normalization. That is:

\[
P_{R_j} = \frac{R_j}{\sum_{i=1}^{m} |R_i|} \times 100\%
\]

(28)

Therefore, by comparing the magnitude of the range \( R_j \) (or the influence degree \( P_{R_j} \)) of each design parameter, the main parameters influencing on the response can be obtained. Moreover, the positive and negative of the influence can be obtained according to the positive and negative of the difference between the two levels corresponding to the range. The positive influence indicates that the larger the value of the factor, the larger the response value, and the negative effect is just the opposite.

3.1.3. Construction of optimization model. For the optimized design of the landing cushioning airbag, its mathematical model can be expressed as:

\[
\begin{align*}
\min f(X), & \quad x_i \in R \quad (i = 1, 2, \ldots, n) \\
\text{s.t.} \quad g(X) & \leq 0 \\
x & \leq x_i \leq \bar{x} \quad (i = 1, 2, \ldots, n)
\end{align*}
\]

(29)

Where \( f(X) \) is the objective function and \( g(X) \) is the constraint function, which can be selected from Table 3 according to the specific situation, \( x_i \) is the design variable, that is, the design parameters of the airbag, which can be determined according to engineering experience or influencing factors analysis, \( \bar{x} \) and \( \underline{x} \) are the upper and lower limits of the variable values respectively.

3.2. Optimal solution method
For optimization problems, the choice of optimization method is also crucial, and it is necessary to take both convergence accuracy and computational efficiency into account. Obviously, airbag optimization is a nonlinear optimization problem. NLQPL is one of the best algorithms for dealing with small and medium scale nonlinear programming problems. It has universal versatility for engineering design in
aerospace. The algorithm is stable with a good mathematical foundation, which has rapid convergence and high convergence efficiency for nonlinear optimization problems. Therefore, this paper chooses the NLPQL algorithm to solve the optimization problem of airbag cushioning.

In the optimization design, the airbag mathematical model established in this paper is used to solve the airbag cushioning performance. The calculation process is shown in Figure 6.

4. Optimal design of cylindrical airbag without rebound
Taking the cylindrical airbag shown in Table 2 as an example, an optimized design of airbag landing system without rebound is presented. Firstly, the optimized design variables are obtained through the analysis of the influencing factors. And then the optimization model is constructed. Finally, the calculation results of the model are analyzed.

4.1. Optimization model of cylindrical airbag
The main design parameters of the cylindrical airbag are the airbag cross-sectional diameter ($D_0$), the airbag axial length ($L_0$), the orifice diameter ($D_{or}$), and the threshold pressure ($P_{open}$). An orthogonal table $L_{16}(4^4)$ is designed for these parameters. The cushioning performance of the 16 groups of airbags was solved by using the airbag mathematical model established in this paper. The calculation results are shown in Table 4. Here, the absolute value of the touchdown speed is taken, regardless of the direction (upward is the positive).

Through the range analysis of the results of the 16 sets of experimental design, the influence degree of the airbag design parameters on its cushioning performance is obtained, as shown in Figure 7. It can be seen that the orifice diameter is the most influential parameters of the four indexes of the airbag cushioning performance, which has negative influence on the three indexes of the maximum pressure, the maximum impact acceleration and the rebound performance, but it has positive influence on the payload touchdown speed. Consequently, it indicates that the larger the orifice diameter, the smaller the maximum pressure and impact acceleration, the less likelihood of the system to rebound, and the higher the touchdown speed. This is because the larger the orifice diameter, the more gas is discharged, and the airbag is compressed faster, so the internal pressure in the airbag will be lower, which lead to the smaller impact acceleration and less likelihood to rebound during the cushioning process. It can also be directly observed from Table 4 that if the system does not rebound, the maximum impact acceleration of the payload is smaller.
On the other hand, it is precisely because the gas flow rate is too high and the energy of the payload is not fully absorbed, the payload will contact with the ground surface at high speed. This reflects the relationship between the touchdown speed and whether or not the system rebounds. If the payload directly touches the ground, the system will not rebound; if the system rebounds, the payload will not directly touch the ground. And this relationship can also be seen from Table 4. That is to say, it can be judged whether the system rebounds by judging whether the speed of the payload is reversed, which is equivalent to evaluating whether the displacement is reversed. In addition, the airbag axial length has less influence on the four indexes of the cushioning performance of the airbag, and it has little influence on whether or not the system rebounds.

| Group |  |  |  |  |  |  |  |  |
|-------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 1     | 0.75                        | 1.50                        | 110000.0                    | 0.10                        | 212442.0                    | 28.79                       | 0                           | 1                           |
| 2     | 0.75                        | 1.75                        | 120000.0                    | 0.15                        | 165858.9                    | 19.20                       | 0                           | 1                           |
| 3     | 0.75                        | 2.00                        | 130000.0                    | 0.20                        | 139396.7                    | 12.22                       | 1.055                       | 0                           |
| 4     | 0.75                        | 2.25                        | 140000.0                    | 0.25                        | 138980.4                    | 8.16                        | 1.927                       | 0                           |
| 5     | 1.00                        | 2.00                        | 110000.0                    | 0.15                        | 153206.8                    | 20.05                       | 0                           | 1                           |
| 6     | 1.00                        | 2.25                        | 120000.0                    | 0.10                        | 163129.0                    | 24.91                       | 0                           | 1                           |
| 7     | 1.00                        | 1.50                        | 130000.0                    | 0.25                        | 129819.1                    | 5.38                        | 2.401                       | 0                           |
| 8     | 1.00                        | 1.75                        | 140000.0                    | 0.20                        | 141340.1                    | 12.16                       | 0.777                       | 0                           |
| 9     | 1.25                        | 2.25                        | 110000.0                    | 0.20                        | 134114.0                    | 15.80                       | 0                           | 1                           |
| 10    | 1.25                        | 2.00                        | 120000.0                    | 0.25                        | 124653.1                    | 9.72                        | 0.807                       | 0                           |
| 11    | 1.25                        | 1.75                        | 130000.0                    | 0.10                        | 157277.9                    | 20.78                       | 0                           | 1                           |
| 12    | 1.25                        | 1.50                        | 140000.0                    | 0.15                        | 151035.8                    | 16.04                       | 0                           | 1                           |
| 13    | 1.50                        | 1.75                        | 110000.0                    | 0.25                        | 123888.8                    | 10.23                       | 0.876                       | 0                           |
| 14    | 1.50                        | 1.50                        | 120000.0                    | 0.20                        | 131649.6                    | 11.23                       | 0                           | 1                           |
| 15    | 1.50                        | 2.25                        | 130000.0                    | 0.15                        | 138543.5                    | 18.38                       | 0                           | 1                           |
| 16    | 1.50                        | 2.00                        | 140000.0                    | 0.10                        | 146291.0                    | 28.79                       | 0                           | 1                           |

Table 4. Orthogonal experimental design results

According to Figure 7(a), the four design variables have a negative influence on the maximum pressure of the airbag. In addition to the orifice diameter, the cross-sectional diameter of the airbag also has a great influence on the maximum internal pressure of the airbag during the cushioning process. This is because an increase in the cross-sectional diameter will increase the airbag volume and the cushioning stroke, resulting in a decrease in internal pressure.

According to Figure 7(b), the maximum impact acceleration of the payload is mainly affected by the orifice diameter. And the airbag axial length has a positive influence on it. When the airbag axial length $L_0$ increases, the airbag volume increases, leading to a decrease in the maximum internal pressure of the airbag during the cushioning process. But the footprint area also increases, so the maximum impact acceleration increases. The results show that the maximum impact acceleration of the payload is more sensitive to the change of footprint area than the change of airbag internal pressure.

According to Figure 7(c), in addition to the orifice diameter, the cross-sectional diameter of the airbag and the opening pressure of the exhaust valve also have a great influence on the maximum internal pressure of the airbag during the cushioning process.

And according to Figure 7(d), whether the system rebounds is mainly affected by the orifice diameter. By comparing Figure 7(c) and 7(d), it is easy to see that the four design parameters have opposite influences on the touchdown speed and whether the system rebounds, that is, when they have a positive influence on the touchdown speed, they have a negative influence on whether the system rebounds.

In conclusion, the orifice diameter, cross-sectional diameter and threshold pressure of the airbag have a great influence on the airbag cushioning performance. Therefore, these three parameters are selected as optimized design variables. The maximum impact acceleration of the payload during the cushioning
process is selected as the optimization target, while the other three indicators are determined as constraints. And the cylindrical airbag model shown in Table 2 is used as the initial model for optimization. Consequently, a mathematical model for the optimization of a non-rebounding cylindrical airbag is established. That is:

$$\begin{align*}
\min & \quad a_{\text{max}}(D_0, D_{\text{or}}, P_{\text{open}}) \\
\text{s.t.} & \quad P_{\text{max}} \leq 176.738 \text{kPa} \\
& \quad u_t \leq 1.0 \text{ m/s} \\
& \quad F_R = 0 \\
& \quad 0.75m \leq D_0 \leq 1.5m \\
& \quad 110.0 \text{kPa} \leq P_{\text{open}} \leq 140.0 \text{kPa} \\
& \quad 0.10m \leq D_{\text{or}} \leq 0.25m
\end{align*}$$

(30)

(a) Influence on the maximum internal pressure in the airbag
(b) Influence on the maximum impact acceleration of the payload
(c) Influence on the touchdown speed of the payload
(d) Influence on whether or not the system rebounds

Figure 7. Influence of design variables on the airbag cushioning performance
4.2. Optimization results and analysis
The optimization model established is solved by using the NLPQL algorithm, and the iteration converges fast, as shown in Figure 8. The optimized maximum impact acceleration of the airbag is about 7.26g, and as can be seen in Table 4, the average maximum impact acceleration of the cylindrical airbag without rebound is about 10.0g. The optimized airbag parameters are shown in Table 5. Compared with the non-optimized one, the cross-sectional diameter of the optimized airbag is increased by 35%, and the orifice diameter is 2.5 times as large as the original, resulting in an increase in the mass flow rate of the exhaust gas. As a result, the maximum internal pressure of the airbag and the maximum impact acceleration of the payload decrease by 26.44% and 67.12% respectively. As the orifice area becomes larger, the payload touchdown speed is increased, but less than 1.0 m/s.

Table 5. Key model parameters and cushioning performance evaluation indexes of the cylindrical airbag before and after optimization

|                | \(D_0\) (m) | \(P_{\text{span}}\) (Pa) | \(D_{\text{or}}\) (m) | \(P_{\text{max}}\) (Pa) | \(a_{\text{max}}\) (g) | \(u_x\) (m/s) | \(F_R\) |
|----------------|--------------|--------------------------|----------------------|--------------------------|------------------------|-------------|--------|
| Non-optimized  | 1.000        | 130000.0                 | 0.10                 | 176738.1                 | 22.08                  | 0.0         | 1      |
| Optimized      | 1.350        | 130000.0                 | 0.25                 | 130000.0                 | 7.26                   | 0.987       | 0      |

The comparison of payload displacement between optimized and non-optimized airbag is shown in Figure 9. It is obvious that the optimized model does not rebound. At the same time, the optimized airbag system model is analyzed and calculated by finite element method. According to Figure 9, the results of the model calculation are basically consistent with the results of finite element analysis. Therefore, the cushioning performance of the optimized airbag is better, indicating the effectiveness and feasibility of the optimized design method.

Figure 8. Iterative curve of payload acceleration

Figure 9. Payload displacement of optimized and non-optimized airbag

5. Conclusion
In this paper, the deformation assumptions of six different shapes of airbags are proposed, and a mathematical model suitable for various venting airbag system is established. The model is used to analysis the cushioning performance of the cylindrical airbag. The calculation results are in good agreement with the finite element analysis results, which indicates that the mathematical model is reasonable and accurate.

Aiming at the rebound problem of the airbag landing system, based on the mathematical model of airbag cushioning, an optimal design method of the airbag without rebound is proposed. Using this method, a cylindrical airbag with no rebound during the cushioning process is obtained. Compared with the non-optimized airbag, the maximum internal pressure of the airbag is reduced by 26.44%, and the
maximum impact acceleration of the payload is reduced by 67.12%. So, the airbag cushioning performance has been greatly improved, which shows the feasibility and effectiveness of the proposed optimization method.

In particular, four indexes for evaluating the cushioning performance are determined: the maximum airbag internal pressure, the maximum impact acceleration, the payload touchdown speed, and whether the payload has a reverse displacement. In the optimization of the cylindrical airbag, the influencing factors analysis method is applied to study the key design parameters affecting the cushioning performance of the airbag. The results show that the orifice diameter has the greatest influence on the four indexes of the cushioning performance of the airbag. On the contrary, airbag axial length has little effect on all four indexes. Consequently, the three parameters of the orifice diameter, cross-sectional diameter and threshold pressure are selected as optimized design variables.

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