We put forward a model, or rather a relatively broad class of models, beyond the Standard Model based on the two main assumptions: MPP) The coupling constants should be fixed such as to ensure that there be many “vacuum states”, i.e. Lorentz invariant states of the fields, with the same energy density. Anti-GUT) At high energy, above an essential desert of only Standard Model interactions, there is the bigger gauge group $SMG \times U(1)_f$ which means that each family of quarks and leptons has its own set of gauge bosons analogous to those in the Standard Model itself, and then there is one extra abelian gauge boson called $U(1)_f$. In addition we make some further more phenomenological assumptions. We succeed in fitting order of magnitudewise most of the (effective) Yukawa couplings observed in the Standard Model as quark and lepton masses and mixing angles. We have more accurate numbers for the three fine-structure constants and the top and Higgs masses, as well as some suggestive understanding of the fine-tuning wonders for the cosmological constant and the $\Theta$-angles. CP-violation is predicted a bit low, but order of magnitudewise in agreement with experiment. In summary, we obtain at least an order of magnitudewise understanding of the Standard Model parameters within our scheme, except for the mysteriously low weak scale compared to the Planck scale, with only 4 continuous parameters being fitted, in addition to our somewhat more discrete choices.

1 Introduction

If one wishes to seek inspiration from experimental data for clues to the theory beyond the Standard Model, one has not much else to work with other than the about 19 Standard Model parameters not fixed by the Standard Model itself. Otherwise, one only has information from neutrino oscillation experiments and cosmological studies, together with the lack of evidence for baryon decay or other new physics deviating from the Standard Model. In the present talk we would like to specify, at least order of magnitudewise, these parameters in a model for the physics beyond the Standard Model. We do not really use a full model, but rather a series of assumptions which partially fix the model \cite{1,2,3,4,5}. The two most characteristic assumptions—we discuss our assumptions in more detail below—for this “fitting” of the Standard Model parameters are:

1. **MPP.** The parameters (coupling constants) must have their values adjusted so that a large number of vacuum states have the same energy density. In fact it is easy to show that the couplings, for which two vacua are degenerate, are just the same ones for which the Euclideanised (imaginary time) version of the theory has a phase transition. So if several vacua are degenerate there is a multiple point. Hence we call our assumption the Multiple Point Principle: MPP.
2. Anti-GUT. Near the Planck scale, the gauge group is the anti-grand unified group $SMG^3 \times U(1)_f$. This means that each of the three quark-lepton generations have got their own gauge group with the same structure as the Standard Model Group $SMG$. In addition there is an abelian flavour group, which we call $U(1)_f$.

In outline our model runs roughly like this: The pure Standard Model, together with its yet to be found Weinberg-Salam Higgs particle, will be valid with high accuracy up to higher energies than most other physicists believe; namely up to about one order of magnitude below the Planck energy scale. That is to say our model would be falsified by the discovery of, for example, supersymmetric partners in any experimentally accessible mass range. Actually the existence of such partners would disturb some of the agreements of our model with experiment. Then up about one order of magnitude below the Planck scale, one should find, in our picture, that the Standard Model Group, called for short $SMG$, is the “diagonal” subgroup of a much bigger gauge group—the Anti-GUT postulate. This non-simple anti-grand unified group $SMG^3 \times U(1)_f$ is supposed to be broken down by a series of Higgs fields, to which we have given the names $W$, $T$, $\xi$ and $S$.

The Grand Unified SU(5) group, often thought to exist beyond the Standard Model, is not a subgroup of our group $SMG^3 \times U(1)_f$. So we do not have SU(5) in our model and the coincidence of the gauge couplings agreeing with the SU(5) prediction, after supersymmetry (SUSY) corrections, must be declared a total accident or explained in a roundabout way, if at all, in our model. Ironically enough, our central predictions for the gauge couplings happen to agree better with simple SU(5) than the Standard Model experimental couplings do. However that is a kind of pure accident, since our calculational accuracy is too low to spot deviations of the order of the SUSY corrections to the gauge coupling differences at the SU(5) scale. This accidental coincidence with the SU(5)-GUT result is seen in figure 1, which shows our anti-GUT predictions at the Planck scale $M_{\text{Planck}} = 1.2 \times 10^{19}$ GeV. Note that these predictions are for the absolute values of the gauge couplings and not just their ratios.

Above the anti-GUT breaking energy scale (an order of magnitude below the Planck scale), there are really three times as many gauge couplings. For example the colour SU(3) group is replaced, at these scales, by three SU(3) groups meaning 3 times 8 gluons, one set of 8 gluons coupling only to the first generation, the next set of 8 gluons only to the second generation, and so on. In addition there is a quite extra $U(1)$ gauge group, which we refer to as $U(1)_f$. In summary we can say: each generation has got its own set of gauge fields/particles quite analogous to those in the Standard Model, i.e. twelve gauge fields for each generation, only coupling to just that generation, and then in addition the $U(1)_f$ gauge field. Altogether there are thus $3 \times 8 = 24$ gluons, $3 \times 3 = 9$ W’s and $3 \times 1 + 1 = 4$ abelian gauge bosons.

Contrary to many popular unifying gauge group proposals, our group is very far from being in the group theoretical sense a simple group; it has lots of nontrivial invariant subgroups. But, in our model, we do not need any unification of the group to make the number of independent gauge coupling constants small. We have a completely different method of predicting the values of coupling constants. We postulate—and this is the MPP postulate—that the various coupling constants,
for some reason or another, have put themselves to just such values that a lot of different Lorentz invariant states of the space and its fields have got the same energy density and pressure. They are degenerate. Naturally the imposition of such a number of equalities among the pressures, or equivalently of the energy densities, of these vacuum states leads to restrictions between the various parameters (coupling constants) of the field theory model. It is these restrictions that replace for us the GUT predictions relating coupling constants due to unification into simple groups. Our predictions come instead from imposing the common energy densities for the supposed many vacua.

It is important for some of these predictions, and it is a part of our model, that there is a fundamental length of the order of magnitude of the Planck length. At that scale there is, what we would like to think of as, a truly existing regularization. We sometimes think and calculate as if it were due to a truly physically existing lattice. However we hope and speculate that, for most purposes, it does not matter what the regularization really is: a lattice, superstring theory, or yet another type of cut-off. But it is important that we have one or another cut-off, in as far as some of the vacua declared degenerate are lattice (or regularization) artifacts. So the idea is in our model, in which some sort of regularization should really exist, that such artifacts should also really exist. Furthermore we shall require, for some as yet not fully understood physical reason, the various regularization artifact vacua to be degenerate; the corresponding requirements on the coupling constants are then expected to be valid in nature (and should be looked for experimentally).

We shall even make an assumption of the type that up near this regularization scale (taken to be the Planck scale), we can essentially find particles with any quantum numbers; so that all allowed amplitudes will exist and correspond to rates of order unity in Planck units.
It should be stressed that our model, especially at the high energy scale, is not so terribly specific, but rather in some sense encompasses a large class of models. We speculate, as an extra assumption, that all the models in this class in practice behave in much the same way. We really do not want to commit ourselves to a specific regularization—lattice or superstring or momentum cut-off say—and we do not really exclude the possible existence of further gauge fields not interfering with the \( SMG^3 \times U(1)_f \) gauge fields. We do not try to specify precisely the large number of possible particles that should be found at the Planck scale. Only the somewhat lighter than Planck mass particles are restricted strongly by our assumptions.

The fundamentally existing regularization could perhaps just be that, at Planck scale distances, the theory is really some superstring theory which is free of divergencies. But to have agreement of our predictions with the experimental couplings, the top quark mass etc., we cannot tolerate supersymmetry to remain unbroken down to even near the electroweak scale. SUSY must be broken already close to the Planck scale. Also our group \( SMG^3 \times U(1)_f \) has to be included in the gauge group of such a string model. Moreover the string gauge group must break down to just our anti-GUT group at the Planck scale, except perhaps for some fully decoupled extra groups.

Let us now spell out some of the details of the above sketched picture of our model, by describing its five most important features or assumptions:

**MPP: Many degenerate vacua.**

In a quantum field theory there can a priori be several Lorentz invariant states. The typical example is a theory with a scalar field for which the effective potential has more than one minimum. For each minimum there is a Lorentz invariant vacuum state in which the scalar field expectation value lies in the chosen minimum. A priori these different vacua have energy momentum tensors proportional to the metric tensor, but with different coefficients for the different vacua. They have different cosmological constants, one could say. These different cosmological constants can formally be calculated for each proposed vacuum and will turn out normally to depend on the various coupling constants (parameters) of the field theory in question. If it were not because of our assumption of a physically existing regularization, a divergent nonsensical number would usually result. However with regularization one should get a meaningful number.

Our assumption of degenerate vacua now says that the coupling constants take just such values that the various cosmological constants for the different vacua become equally large. That obviously should give some equations among the various coupling constants, depending of course on the details of the field theory used.

But why should we expect that these vacua should have the same energy densities? Perhaps the most convincing argument is provided by the successful phenomenological predictions for the gauge coupling constants obtained by using this requirement. In addition we can list four other arguments which carry less weight, but anyway helped motivate us and may eventually lead to a deeper understanding of our approach.

1) An argument essentially due to Susskind: If there is some reason why the cosmological constant in Nature (our vacuum) is so accurately zero, then the same argument is expected to work for the other potential vacua. However it should be
admitted, for example, that if the mechanism of Tsamis and Woodard making the cosmological zero is the reason, Woodard denies that it should also work to make the cosmological constant zero for the scalar field in another minimum. So it seems that the argument does not work for all imaginable mechanisms explaining the vanishing of the cosmological constant.

2) Ice-water analogy: In a microcanonical ensemble for water, i.e. with a definite amount of energy, it does not take so terribly much luck to get such an energy chosen that the ensemble cannot be realized (under the given pressure) as pure liquid water, or as pure ice. It has to be a mixture—slush—but then the temperature has to have a fine-tuned value equal to zero degrees Celsius, or rather the freezing temperature. So actually it can very easily happen that slush is produced and that thus the freezing temperature is very likely to occur. This is due to the large latent heat of fusion of water giving a strongly first order phase transition between ice and water. By analogy with this we want to say that one expects it would be very likely—using somehow similar assumptions for the field theory—that the microcanonical ensemble-like mechanism would work and lead to precisely those couplings for which the different vacua become degenerate. The degeneracy of the vacua is supposedly the analogous fine-tuning to slush being at the freezing point of water.

3) Non-locality: If we generalize the mixed phase (slush) mechanism, as just suggested above, from the case of the statistical mechanics partition function in three dimensions to the four dimensional functional integral formulation of quantum field theory, it turns out that the analogue of the microcanonical ensemble is strictly speaking non-local. According to the principle of locality, the action $S$ used to define the Feynman path integral

$$\int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\phi \exp(iS[A, \psi, \phi])$$  \hspace{1cm} (1)$$

should be an integral over four dimensional space-time of a Lagrangian density. However, the analogue of the microcanonical ensemble is obtained by replacing the exponentiated action $\exp(iS)$, in the integrand of the path integral, by a delta function of an integral over all space-time:

$$\int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\phi \delta(I[A, \psi, \phi] - I_0)$$  \hspace{1cm} (2)$$

where

$$I[A, \psi, \phi] = \int d^4x \mathcal{L}(x)$$  \hspace{1cm} (3)$$

But such a delta function does not obey the principle of locality and, actually, it makes the presence of degenerate vacua a likely occurrence. If you rewrite the delta function path integral eq. (2) as one with an exponentiated action as usual, you would have to make the action a non-linear function of the expression in eq. (3). Alone letting the action be a non-linear function of space-time integrals like eq. (3) is sufficient to make it likely that there will be degenerate vacua, i.e. MPP. Non-linearity in such expressions means non-locality. With this form of mild non-locality the theory can for practical purposes be simulated by a local theory. The only left-over signal that it is fundamentally non-local is the MPP, that is to say...
the effective coupling constants of the simulating local theory will turn out to be adjusted to make several degenerate vacua.

4) Supersymmetry (but strongly broken): Supersymmetry models typically have several states of vacuum type with precisely the same energy density, namely zero. In this way the situation required by our MPP principle naturally arises: the existence of several degenerate vacua. However we must, as must also be done for purely phenomenological reasons, assume that supersymmetry is broken. As we shall see below in section 6, the supersymmetry breaking scale must be quite close to the Planck scale, so as to avoid spoiling the otherwise good agreement between the experimental gauge coupling constants and our model.

**Anti-GUT gauge group.**

Before defining our Anti-GUT group, it should be mentioned that we follow Michel and O’Raifeartaigh in making sense not only of the Lie algebra but even of the Lie group for a Yang-Mills theory. The interpretation of the global properties of the gauge group is that the matter field representations should be only those that could be representations of the gauge group. As a familiar example you may think about the rotation group; while the group SU(2) has both half integer and integer spin representations, the group SO(3), having the same Lie algebra as SU(2), has only the integer spins as its group representations. So we could pack the information that some set of particles has only integer spin into the statement that they are faithful representations of SO(3), but not of SU(2). In an analogous way, O’Raifeartaigh points out that only the representations of the Standard Model Lie algebra which also obey the charge quantisation rule

\[ \frac{y}{2} + \frac{d}{2} + \frac{t}{3} = 0 \pmod{1} \]  

are representations of the group SMG = S(U(3) × U(2)), which has the same Lie algebra as SU(3) × SU(2) × U(1). Here we used the definition of triality \( t \) as the quantum number, counted modulo 3, with the property that it is related to the representation of the particle type in question under the colour group SU(3): if the SU(3) representation is the triplet representation the triality is defined to be 1 (mod 3), and if it occurs in the representation obtained by putting together \( t \) triplet representation particles we say it has triality \( t \). Analogously the “duality” \( d \) is defined as a function, modulo 2, of the representation under the SU(2) algebra: if the weak isospin is half integer \( d \) is odd (say \( d = 1 \)), and if the weak isospin is integer we define \( d \) to be even, \( d = 0 \). The weak hypercharge \( y/2 \) is normalised so that, for example, \( y/2 = 1 \) for the positron. According to the well-known relation \( Q = y/2 + T_3 \) for the electric charge \( Q \), where \( T_3 \) is the third component of the weak isospin, the charge quantisation rule eq. (4) expresses the somewhat complicated way in which the electric charge is quantised in the Standard Model.

One may think of such quantisation rules as the restriction of the Lie algebra representations allowed by the introduction of some type of monopole. There is a direct correspondence between the charge of the magnetic monopole and the invariant discrete subgroup elements to be “divided out” of the covering group, in order to get to the group in question with the prescribed Lie algebra.

It should also be kept in mind that, in line with our philosophy of a real physically existing cut-off, e.g. a lattice (see below), actually the gauge group and not
only the Lie algebra acquires a physical significance.

We therefore think of the extension of the Standard Model into our anti-GUT model as an extension of the group and not only of the Lie algebra. So we also propose an extension of the charge quantisation rule, at the same time as we extend the algebra.

Our proposed group is literally taken as a group $SMG^3 \times U(1)$ (and not only an algebra). That is to say that for each of the three $SMG=S(U(2) \times U(3))$ factors in the cross product—each supposed to couple to just one generation, leaving the other two untouched—there is a separate quantisation rule just for that generation. In addition the extra $U(1)$—the one we call $U(1)_f$—algebra is also supposed to have only integer charge representations, so that it is indeed a representation of the group $U(1)$ and not only of the algebra or of the covering group $R$. Actually it turns out, from our fit to the quark-lepton mass spectrum described in section 5, that the Higgs fields we introduce into our model have somewhat strangely quantised $Q_f$ charges—the abelian $U(1)_f$ charges. The Higgs fields $Q_f$ charges are quantised in units three times smaller than those for the quarks and leptons.

The original motivation for our non-simple anti-grand unified group was inspired by the ideas of “confusion”, but we now think a more convincing argument is the one which we present below in section 4. Also we have noted that the Standard Model group $SMG = S(U(2) \times U(3))$ has a remarkably low number of automorphisms for its size. Then one might speculate that this result is connected with groups tending to get fewer and fewer automorphisms the more they break down by the Higgsing. Such argumentation also points, to some extent, in the direction of our proposed group.

**Desert almost to the Planck scale.**

It should be stressed that it is part of our model that there be essentially no new physics, except for the Weinberg-Salam Higgs particle, until an order of magnitude or so below the Planck energy scale. That is to say that there shall for instance be no supersymmetric partners before this scale, also no leptoquarks or the like. When we say essentially it means that some new particles could perhaps be tolerated in our model provided they do not disturb our predictions. We namely get good predictions without anything new at any accessible scale.

**Physically existing regularization.**

Although quantum field theories are exceedingly successful, it has been shown, for example, that scalar self-interacting theory in 4 dimensions is only consistent over an infinity of scales provided the self-coupling vanishes (the triviality bound). Presumably most quantum field theories are indeed inconsistent, unless they are trivial in the sense of having zero couplings, or asymptotically free. So we can really not easily have field theory without having some new physics beyond the Standard Model that would appear much like a regularization. Especially with gravity becoming strong quantum mechanically at the Planck scale, it would seem strange if indeed there should be no new physics—presumably being finite, so that it could appear as a regularization—at the Planck scale (or below).

In section 6 we shall describe computations made as if the regularization was provided by a real physically existing space-time lattice, but we hope it does not matter too much what form of regularization we use.
Order unity fundamental Yukawa couplings.

The assumption that the various fundamental Yukawa couplings are all of order unity could perhaps have some rationale in our MPP principle, but that would seem a rather stupid way to argue for it. Really it is most natural to simply assume that any couplings, not having a reason to be suppressed or to take on special values, are unity order of magnitudewise.

To these assumptions, we add some details about which Higgs fields might be responsible for the needed breakdown of the gauge group. Also we make some assumptions, which in principle can be checked by lattice calculations, as to the phases which are supposed to meet at the multiple point. In particular the rather complicated choice of quantum numbers for the Higgs fields $W$, $T$, $\xi$ and $S$, responsible for the breaking of our group to the Standard Model group, is really made as a kind of discrete number fitting. In fact it took some time to find a proposal yielding a successful phenomenology.

We would like to stress that it is not essential for all the features of our model to be correct. Even if part of it turns out wrong, it does not necessarily fall apart completely. Really the point is that to produce our results for the Higgs particle and top quark masses (see section 2), we only use the MPP assumption among our two “basic” assumptions. For the ratios of the quark and lepton masses and the quark mixing angles (see section 5), for which we get order of magnitude fits, we only use the other basic assumption, the anti-GUT gauge group. For the fine structure constant predictions (see section 6), however, both basic assumptions are needed. For the fine-tuning questions as to why the cosmological constant and the theta angles for SU(3) and SU(2) are zero, we need, if we can do anything at all, only the MPP assumption, but not the gauge group beyond the Standard Model (see section 3). The most difficult and mysterious parameter—the scale of the weak interaction compared to the Planck scale—is not yet predicted by our model, and it remains a mystery why this scale is so exceedingly low.

2 Higgs Mass 135 ± 9 GeV and Top Quark Mass 173 ± 5 GeV

The application of the MPP to the pure Standard Model (SM) implies that the SM parameters should be adjusted, such that there exists another vacuum state degenerate in energy density with the vacuum in which we live. This means that the effective SM Higgs potential $V_{eff}(|\phi|)$ should, for example, have a second minimum degenerate with the well-known minimum at the electroweak scale $\langle|\phi|\rangle = 246$ GeV. Thus we predict that our vacuum is barely stable and we just lie on the vacuum stability curve; the Higgs particle mass is predicted to take on its lowest allowed value before our vacuum becomes unstable. The form of the SM vacuum stability curve in the top quark mass, Higgs particle mass ($M_t, m_H$) plane depends on the physical cut-off scale $\Lambda$ beyond which the Standard Model is replaced by a more fundamental theory, as illustrated in figure 2. Taking the cut-off scale to be given by the Planck mass $\Lambda \simeq M_{Planck} \simeq 10^{19}$ GeV and the top quark mass from
For the Higgs particle pole mass.

However if the vacuum degeneracy requirement should have a good chance of being physically relevant, the range of $|\phi|$ values, lying between the vacuum expectation values (VEVs) $|\phi_{vac 1}|$ and $|\phi_{vac 2}|$ for the two vacua, should be as large as possible. This condition is analogous to the necessity for an appropriately large latent heat of fusion for water so that slush—partially melted snow or ice—is likely to be found in winter. That is to say we have the strong first order phase transition requirement:

$$|\phi_{vac 2}| - |\phi_{vac 1}| \simeq \Lambda \simeq M_{Planck}$$

(6)

Since $|\phi_{vac 1}| = 246$ GeV, this requires the second vacuum to have a VEV of the order $|\phi_{vac 2}| \simeq M_{Planck}$. If we now impose both the degenerate vacua and first order phase transition requirements, we determine a single point on the vacuum stability curve. In this way we obtain within the pure Standard Model, predictions for both the top quark and Higgs boson pole masses:

$$M_t = 173 \pm 5\,\text{GeV} \quad M_H = 135 \pm 9\,\text{GeV}$$

(7)
There is remarkably good agreement with the experimental value of the top quark mass.

The above results were obtained by studying the renormalisation group improved SM Higgs potential. Including quantum fluctuations, the classical potential picks up loop corrections and we get the effective Higgs potential:

\[
V_{\text{eff}}(|\phi|) = \frac{1}{2} m_{0H}^2 |\phi|^2 + \frac{1}{8} \lambda_0 |\phi|^4 + \frac{1}{2} \text{Tr} \log [\Delta^{-1}_{\text{Bosons}}(\phi)] + \frac{1}{2} \text{Tr} \log [\Delta^{-1}_{\text{Fermions}}(\phi)] + \text{Higher order loop terms} \tag{8}
\]

Here \(\Delta^{-1}_{\text{Bosons}}(\phi)\) and \(\Delta^{-1}_{\text{Fermions}}(\phi)\) denote the (appropriately normalised) inverse propagators for boson and fermion fields in the background Higgs field \(\phi\). The parameters \(m_{0H}^2\) and \(\lambda_0\) are the bare Higgs mass and bare Higgs self-coupling constant respectively. An efficient way of taking the higher order loops into account is to make use of the running coupling constants—by far the most relevant for our work is the Higgs field self-interaction coupling \(\lambda(\mu)\)—as calculated by integrating the renormalisation group equations for the various couplings in the Standard Model. Then, by identifying the scale parameter \(\mu\) with the field value \(|\phi|\), the perturbative expansion is reorganised so that the leading-log contributions from the loop corrections are transferred to the tree level part of the effective potential. The next to leading-log terms can be included by using the one-loop effective potential, with running couplings evaluated using the two-loop renormalisation group beta functions.

For the purposes of our discussion it is sufficient to consider the renormalisation group improved tree level effective potential:

\[
V_{\text{eff}}(\phi) = \frac{1}{2} m_{H}^2 (\mu = |\phi|) |\phi|^2 + \frac{1}{8} \lambda(\mu = |\phi|) |\phi|^4 \tag{9}
\]

In order that \(|\phi_{\text{vac}}| = 246 \text{ GeV}\), the coefficient \(m_{H}^2(\mu)\) has to be of the order of the electroweak scale. We are interested in values of the Higgs field of the order \(|\phi_{\text{vac}}| \approx M_{\text{Planck}}\), which is very large compared to the electroweak scale, and for which the quartic term strongly dominates the \(\phi^2\) term; so to a very good approximation we have:

\[
V_{\text{eff}}(\phi) \approx \frac{1}{8} \lambda(\mu = |\phi|) |\phi|^4 \tag{10}
\]

The running Higgs self-coupling constant \(\lambda(\mu)\) is readily computed by means of the renormalisation group equation:

\[
\frac{d\lambda}{d \ln \mu} = \beta_\lambda(\lambda, g_t, g_1, g_2, g_3) \tag{11}
\]

Here the \(g_i(\mu)\) are the three SM running gauge coupling constants, discussed further in section \ref{6} and \(g_t(\mu)\) is the top quark running Yukawa coupling constant, which satisfies the renormalisation group equation:

\[
\frac{dg_t}{d \ln \mu} = \beta_{g_t}(\lambda, g_t, g_1, g_2, g_3) \tag{12}
\]
Figure 3: Plots of $\lambda$ and $g_t$ as functions of the scale of the Higgs field $\phi$ for degenerate vacua with the second Higgs VEV at the Planck scale $\phi_{\text{vac}2} = 10^{19}$ GeV. We formally apply the second order SM renormalisation group equations up to a scale of $10^{25}$ GeV.

The beta functions are given to first order by:

$$16\pi^2 \beta_\lambda = 12\lambda^2 + 3 \left( 4g_t^2 - 3g_2^2 - g_1^2 \right) \lambda + \frac{9}{4}g_2^4 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4$$

(13)

and

$$16\pi^2 \beta_{g_t} = g_t \left( \frac{9}{2}g_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{12}g_1^2 \right)$$

(14)

The renormalisation group equations are in practice solved numerically, using the second order expressions for the beta functions.

The vacuum degeneracy condition is imposed by requiring:

$$V_{\text{eff}}(\phi_{\text{vac}1}) = V_{\text{eff}}(\phi_{\text{vac}2})$$

(15)

Now the energy density in vacuum 1 is exceedingly small compared to $\phi_{\text{vac}2}^4 \approx M_{\text{Planck}}^4$. So we basically get the degeneracy condition, eq. (13), to mean that the coefficient $\lambda(\phi_{\text{vac}2})$ of $\phi_{\text{vac}2}^4$ must be zero with high accuracy:

$$\lambda(\phi_{\text{vac}2}) = 0$$

(16)

At the same $\phi$-value the derivative of the effective potential $V_{\text{eff}}(\phi)$ should be zero, because it has a minimum there. In the approximation $V_{\text{eff}}(\phi) \approx \frac{1}{8} \lambda(\phi)\phi^4$ the derivative of $V_{\text{eff}}(\phi)$ with respect to $\phi$ becomes

$$\left. \frac{dV_{\text{eff}}}{d\phi} \right|_{\phi_{\text{vac}2}} = \frac{1}{2} \lambda(\phi)\phi^3 + \frac{1}{8} \frac{d\lambda(\phi)}{d\phi} \phi^4 = \frac{1}{8} \beta_\lambda \phi^3$$

(17)

Thus at the second minimum of the effective potential we have:

$$\beta_\lambda(\mu = \phi_{\text{vac}2}) = \lambda(\phi_{\text{vac}2}) = 0$$

(18)
which gives to leading order, setting $\lambda = 0$ in eq. (13), the relationship:

$$\frac{9}{4}g_4^2 + \frac{3}{2}g_2^2g_1^2 + \frac{3}{4}g_1^4 - 12g_t^4 = 0$$

(19)

between the top quark Yukawa coupling and the gauge coupling constants at the scale $\mu = \phi_{vac}^2 \approx M_{Planck}$. We use the renormalisation group equations to relate the couplings at the Planck scale to their values at the electroweak scale. Figure 3 shows the running coupling constants $\lambda(\phi)$ and $g_t(\phi)$ as functions of $\log(\phi)$. Their values at the electroweak scale give our predicted combination of pole masses: $M_t = 173$ GeV and $M_H = 135$ GeV.

The vacuum stability curve has been studied for the Standard Model by several authors [17, 18, 19]. Their results are slightly different but, within errors, are each consistent with the linear fit

$$M_H = 135 + 2(M_t - 173) - 4\frac{\alpha_3 - 0.117}{0.006}$$

(20)

to the vacuum stability curve, in GeV units. This is illustrated by the results of Casas et al. [19] in figure 4. When this degenerate minima condition eq. (20) is combined with the experimental value of the top quark pole mass, $M_t = 175 \pm 6$ GeV, we obtain a rather clean MPP prediction for the Higgs pole mass: $M_H = 139 \pm 16$ GeV.

If we now also impose the strong first order phase transition requirement, which takes the form $|\phi_{vac}^2| \simeq M_{Planck}$, we no longer need the experimental top quark mass as an input, but rather obtain our prediction for both $M_H$ and $M_t$. A change in the scale of the minimum $\phi_{vac}^2$ by an order of magnitude, from $10^{19}$ GeV to $10^{18}$ or $10^{20}$ GeV, gives a shift in the top quark mass of about 2.5 GeV. Since the

Figure 4: SM vacuum stability curve for $\Lambda = 10^{19}$ GeV and $\alpha_s = 0.124$ (solid line), $\alpha_s = 0.118$ (upper dashed line), $\alpha_s = 0.130$ (lower dashed line).
concept of Planck units only makes physical sense w.r.t. order of magnitudes, this
means that we cannot, without new assumptions, get a more accurate prediction
than of this order of magnitude of 2.5 GeV uncertainty in $M_t$ and 5 GeV in $M_H$.
The uncertainty in the strong fine structure constant $\alpha_3(M_Z) = 0.117 \pm 0.006$ leads
to an uncertainty in our predictions of approximately $\pm 2\%$, meaning $\pm 3.5$ GeV in
the top quark mass. So our overall result for the top quark mass is $M_t = 173 \pm 5$
GeV. Combining the uncertainty from the Planck scale only being known in order of
magnitude and the $\alpha_3$ uncertainty with the calculational uncertainty in the vacuum
stability curve, we get an overall uncertainty in the Higgs boson mass of $\pm 9$ GeV.
So our Standard Model multiple point prediction for both the top quark and Higgs
boson pole masses is:

$$M_t = 173 \pm 5 \text{ GeV} \quad M_H = 135 \pm 9 \text{ GeV}$$

3 Some Well-Known Fine-Tuning Problems

By these well-known fine-tuning problems, we allude to the questions as to why the
cosmological constant is zero and why $\Theta_{QCD}$ and presumably also the weak SU(2)
topological term coefficient $\Theta_{weak}$ are zero. The gauge hierarchy problem of why
the W’s and $Z^0$ are so light compared to the unified or Planck scale also belongs
to this class of fine-tuning problems. However we do not yet even have a partial
solution to the gauge hierarchy problem.

We can formally bring the cosmological constant being zero and our MPP prin-
ciple together under the single postulate: All vacua have energy density zero.

As far as we can think of our MPP as saying that the Euclideanised theory has a
phase transition just at the parameters chosen by Nature, we can make use of some
lattice gravity calculations by Ambjørn and Varsted and by Hamber. These show
that there is a phase transition in the space of lattice gravity parameters, where the
cosmological constant is zero. It is not so surprising that there should be a phase
border where the cosmological constant is zero, since for instance the four-volume—
of a universe development—grows to infinity just when the effective cosmological
constant passes through zero.

Another similar fine-tuning problem to the cosmological constant one is the
problem of why the theta-vacuum parameter or topological term coefficient $\Theta$
vanishes. Is it also thinkable that this $\Theta$ is just zero on some phase border? In a
calculation by Schierholz one does find a phase transition at the $\Theta_{QCD} = 0$
hypersurface.

4 The “Anti-GUT” Gauge Group $SMG^3 \times U(1)_f$

As mentioned in the introduction, the second major assumption in our model is that
the gauge group $G$ to be found beyond the Standard Model, close to the Planck scale,
should be $SMG^3 \times U(1)_f$. This group breaks down by the Higgs mechanism to the
SMG as the diagonal subgroup of its $SMG^3$ subgroup. This group $SMG^3 \times U(1)_f$, 
with its $3 \times 12 + 1 = 37$ generators, would at first seem a very arbitrary choice of
“unified group”. However, we shall actually now argue it can be characterized by
postulating 4 to 5 not so unreasonable, nor arbitrary assumptions about the gauge group \( G \) beyond the Standard Model (SM).

As a zeroth postulate, of course, our characterization should have the property that the gauge group \( G \) beyond the Standard Model must contain the Standard Model Gauge group \( SMG \) as a subgroup. In addition it should obey the following 4 postulates—for which we also deliver some hand-waving arguments:

- \( G \subseteq U(45) \), the group of unitary transformations of the known 45 Weyl fermions in the Standard Model.

This assumption is really the postulate that we totally ignore those generators in the gauge group which do not couple to the already known fermions in the Standard Model. At first, it therefore just means that we only look for a factor group \( G \) of the subgroup of the full group not transforming the known fermions into yet to be found ones. In fact \( G \) should be that factor group which is obtained by dividing out the subgroup leaving the known fermions untouched. However, even a part of the gauge group not coupling to the known fermions could influence the interaction of the various Higgs fields in our model below and thus influence our phenomenology—and perhaps more importantly it could influence our gauge coupling calculations in section 6. Thus our restriction to \( G \subseteq U(45) \) involves a physical assumption about the theory and is not simply a convenient definition of \( G \) as the factor group, which couples to the SM fermions, of a presumably bigger group. We may hope though that it is a good approximation for phenomenological applications to ignore the rest of the true group.

In the spirit of using our assumption about many degenerate vacua, one could cook up an argument for why we should already know the fermions to which the gauge fields couple: If there are many phases with the same zero temperature vacuum energy density, then those with the highest number of “light” particle species (i.e. particles that can form a Planck radiation immediately after the Big Bang) will tend to push the other phases aside and come to dominate the early Universe. We would therefore expect the phase, surviving after the Big Bang, to have a maximal number of particle species with mass lighter than the scale of mass relevant for the survival of phases. This would favour the dominance, after the Big Bang, of a phase with exceptionally many of the fermion species remaining mass protected (chiral) after the breakdown of the gauge group \( G \) to the \( SMG \)—the gauge group in the present day phase of the Universe. But, in such a case, we may still see a very large proportion of the fermion species which existed at the outset of the Universe and, thus, our assumption that all the gauge fields belonging to \( G \) couple to some known fermions has an increased reliability.

- No anomalies.

There should neither be gauge nor mixed anomalies (nor discrete anomalies, but it is not relevant here). This is an almost unavoidable assumption, in as far as the existence of anomalies would spoil the gauge symmetries. Rather our real assumption is that we do not allow for a Green-Schwarz type of anomaly cancellation to play any role in our model. In the Standard Model itself there is, of course, no need for any Green-Schwarz anomaly cancellation. This may be taken as some support from phenomenology for our assumption that only rather straightforward anomaly cancellations take place.
• Keep irreducible representations of the Standard Model Group irreducible under the big group $G$.

Grand Unified Theories, like $SU(5)$, combine the SM irreducible representations into larger GUT irreducible representations and thereby obtain symmetry relations between the SM Yukawa couplings. However the exact $SU(5)$ GUT degeneracies of db-quark and charged lepton masses at the unification scale are really not wanted, except for the case of $\tau$ being degenerate with $b$. In fact the unwanted $SU(5)$ predictions, $m_{\mu} = m_s$ and $m_e = m_d$, can only be tolerated by having e.g. Georgi-Jarlskog factors of 3 coming in, by postulating several different Higgs representations to provide the various quark and lepton masses. It would really be an advantage for the GUT model agreement with data if one, as in our model, could replace these exact $SU(5)$ predictions by only order of magnitude degeneracy predictions.

Taking a crude and unbiased look at the spectrum of quark and leptons, its most remarkable feature is that almost every mass has a value deviating by big factors from almost all the other masses. At first sight, there does not seem to be even degeneracy order of magnitudewise. If that were the case—which is not really true even in our model—it would suggest that every irreducible representation should have its own set of approximately conserved quantum number combinations, so that almost no degeneracies even order of magnitudewise should be likely to occur. Thus the best way of ensuring that no degeneracy is realised is to keep each irreducible representation under the SMG in a separate irreducible representation of $G$.

• Maximal G with the above constraints.

In order to obtain the order of magnitude mass splittings mentioned above, we need as many as possible partially conserved quantum numbers to provide different suppressions for each mass term. If we assume, as we do, that these quantum numbers are gauge quantum numbers, we need as big a gauge group as possible. One might also try to justify the maximal $G$ assumption by postulating that, at the Planck level, all the fields that can couple to the Weyl fermions and that are allowed will exist as dynamical degrees of freedom. For each irreducible $n$-dimensional representation of the Standard Model, we could postulate a priori a $U(n)$ gauge group just transforming its $n$ components.

With these four postulates a somewhat cumbersome calculation shows that, modulo permutations of the various irreducible representations in the Standard Model fermion system, we are led to our gauge group $SMG^3 \times U(1)_f$. Furthermore it shows that the $SMG$ is embedded as the diagonal subgroup, as in our AGUT model. We first consider the non-abelian part of the sought-after group $G$. There are $5 \times 3 = 15$ irreducible representations of Weyl fermions in the Standard Model, classifying the 45 Weyl components into 3 sets with six components in each (the left-handed quarks), 6 sets with 3 in each (the right-handed quarks), 3 sets with 2 in each (the left-handed leptons), and finally the 3 lonely components (the right-handed leptons). Our first postulate means that we only consider transformations of these 45 Weyl components. The third postulate means that we do not allow those transformations in $G$, which transform components in one of the mentioned 15 sets of components into another one. So $G$ must be a subgroup of the group of all the transformations inside these 15 sets which do not mix them: $U(6)^3 \times U(3)^6 \times U(2)^3 \times U(1)^3$. Imposing
our second postulate, we must avoid anomalies and thus make some identifications among the $SU(m)$’s and $U(1)$’s contained in the various $U(n)$ groups. However in order to make the resulting total group $G$ as big as possible, as our final postulate claims, we should make as few identifications as possible.

It follows from our zeroth postulate ($SMG \subset G$) that the transformations on the left-handed quarks cannot be restricted more than to lie in the Standard Model. Consequently it turns out that we cannot escape anomalies for the $SU(6)$’s. So we must give up having any $SU(6)$ and must rather be satisfied with their $SU(2) \times SU(3)$ subgroups. We must then identify the $SU(2)$’s and $SU(3)$’s in the left-handed quark transformations with those on the other sets of components. The minimal amount of identification of these non-abelian groups turns out to correspond, up to permutations, to having just three generations of quarks and leptons, each with its own $SU(2)$ and $SU(3)$. At this stage of the argument, apart from the right-handed leptons which have no non-abelian groups acting on them, we have a generation structure in the sense that we have different $SU(3)$’s and $SU(2)$’s for the three proto-generations. However these proto-generations do not have to correspond exactly to the experimentally observed generations—in fact we shall see below that, in our fit, we let the right-handed $2/3$ charge quarks, $c_R$ and $t_R$, be permuted relative to the proto-generation structure.

The most complicated part of the calculation is to decide which identifications of the abelian groups have to be made in order to avoid anomalies, i.e. how big a subgroup of the $U(1)^{15}$ can avoid having anomalies and be allowed in $G$. In searching for the generators of an allowed subgroup, one may expand them in terms of the generators for these 15 $U(1)$’s and they have to obey some first order (linear) relations for the coefficients, in order to avoid anomalies involving also the non-abelian or gravitational fields. Also there are third order relations that have to be satisfied, in order that there be no anomalies involving only the subspace of abelian generators. It turns out that, with the three generations of fermions, there are too many constraints to be solved with an abelian subgroup of dimension higher than 4. It is found that three of the allowed abelian generators in $G$ can be taken to be the 3 weak hypercharges, each defined to act on only one generation. After that choice the scheme becomes so tight that, apart from various rewritings and permutations of the particle names, there is a unique fourth $U(1)$ allowed and that is what we call $U(1)_f$. Several of the anomalies involving this $U(1)_f$ are cancelled by assigning equal and opposite values of the $U(1)_f$ charge to the analogous particles belonging to second and third generations, while the first generation particles have just zero charge. The $U(1)_f$ can in fact be chosen to obey the following rules:

All members of the first generation carry zero $U(1)_f$ charge.

Left-handed particles, i.e. doublets under the $SU(2)$ of the Weinberg-Salam model, carry no $U(1)_f$ charge either.

The right-handed leptons and right-handed dsb-quarks in the same proto-generation carry the same $U(1)_f$ charge, as if they obeyed an $SU(5)$ symmetry.

The $U(1)_f$ charge is opposite on the (right-handed) $2/3$ electric charge quark and the $-1/3$ electric charge quark in the same proto-generation.

Using these rules the $U(1)_f$ charges are totally given, except for an overall normalisation and sign convention. For example, we choose the right-handed b-
quark, and thus also the right-handed \( \tau \) lepton, to have the charge \( Q_f = 1 \). Then the proto-right-handed \( t \)-quark gets the charge \( Q_f = -1 \). However we note that there is a finesse of our fit to the fermion spectrum, according to which the right-handed component of the experimentally observed \( t \)-quark is actually the one having second generation \( SU(3) \) quantum numbers and is thus really the proto-right-handed charm quark. In a similar way the right-handed component of the experimentally observed charm quark has the third generation \( SU(3) \) representation and is really the proto-right-handed top quark. It is only the right-handed top and charm quarks that are permuted in this way, while for example the left-handed components are not.

5 Masses and Mixing Angles for Quarks and Leptons

We now consider the use of the gauge group \( SMG^3 \times U(1)_f \) breaking down to the SMG, embedded as the diagonal subgroup of the \( SMG^3 \), to make a fit to the orders of magnitude of the quark and lepton masses and mixing angles (i.e. the Cabbibo Kobayashi Maskawa matrix). We propose a system of Higgs fields performing the needed breaking to the SMG and a candidate for being identified with the Weinberg-Salam Higgs field of the Standard Model. The idea is that the proposed Higgs fields have expectation values which are small compared to the fundamental scale (the Planck scale), so that the gauge group charges broken by these Higgs fields will be approximately conserved. These partially conserved quantum numbers can now take different values on the right- and the left-handed components of a quark or lepton, thereby making the mass term—or the effective SM Yukawa coupling to the Weinberg-Salam Higgs field—forbidden in first approximation. It is the main idea that this kind of forbiddenness is responsible for the suppression of quark-lepton mass terms or, equivalently, for their effective SM Yukawa couplings being smaller than of order unity.

It is a significant assumption and the philosophy of our model that all the fundamental Yukawa couplings—and other couplings too—in our basic model are of order unity, only the Higgs field expectation values can be small! Since we have decided, for the mass matrices, only to hope to predict—fit—the order of magnitudes of the masses and mixing angles, it means that we have simply set the Anti-GUT Yukawa couplings to unity. However, in the computer calculation of the mass spectrum we provide each mass matrix element with a complex factor of order unity. We use a random number generator to generate these factors, with random phases and magnitudes within a factor of two or three or so from unity. Finally we average the output masses and mixing matrix in a geometrical way (i.e. average the logarithms) to obtain our predictions. But really the order of magnitude results can be estimated with reasonable accuracy by head, because only one or at least very few terms dominate a given quantity. It is precisely our philosophy that “small” Higgs expectation values are very small, in the sense that when you have a series of several products of them, the largest term in the series will be a good enough approximation to the sum of the series.

So we introduce some Higgs fields, named \( W, T \) and \( \xi \), and the Weinberg-Salam Higgs field \( \phi_{WS} \) with small vacuum expectation values compared to the fundamental scale \( M_{Planck} \). As a result, whenever these Higgs fields are required to generate mass
terms, they cause suppressions of those terms. In addition we introduce one Higgs field \( S \) with a vacuum expectation value (VEV) of order unity on this scale. We adjust the quantum numbers of these fields, so as to make a fit of all the orders of magnitude of the matrix elements in the three charged fermion mass matrices, with products of the corresponding suppression factors.

The philosophy really is that there are a huge number of species of vector-like Dirac fermions, with unsuppressed masses \( M_F \) of the order of the fundamental mass \( M_{Planck} \). One can then form chain diagrams of the type shown in figure 5, in which the VEVs of the needed Higgs fields can, for example, cause the effective b-quark mass matrix element to become non-zero. For a given choice of the quantum numbers under \( SMG^3 \times U(1)_f \) for the various Higgs fields, we can estimate the orders of magnitude of the various mass matrix elements. They are given by products of the small numbers denoting the VEVs in the fundamental units of the fields \( W, T, \xi \) and the of order unity VEV of \( S \). With the quantum number choice that seems to fit the data, we find the following orders of magnitude for the effective SM Yukawa coupling matrix elements—but remember that “random” order unity factors are supposed to multiply all the matrix elements—for the uct-quarks:

\[
Y_U = \begin{pmatrix}
S^\dagger W^\dagger T^2 (\xi^\dagger)^2 & W^\dagger T^2 \xi & (W^\dagger)^2 T \\
S^\dagger W^\dagger T^2 (\xi^\dagger)^3 & W^\dagger T^2 & (W^\dagger)^2 T^\dagger \\
S^\dagger (\xi^\dagger)^3 & 1 & W^\dagger T^\dagger
\end{pmatrix}
\] (22)

the dsb-quarks:

\[
Y_D = \begin{pmatrix}
SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 \xi & T^3 \xi \\
SW(T^\dagger)^2 \xi & W(T^\dagger)^2 & T^3 \\
SW^2(T^\dagger)^4 \xi & W^2(T^\dagger)^4 & WT
\end{pmatrix}
\] (23)

and the charged leptons:

\[
Y_E = \begin{pmatrix}
SW(T^\dagger)^2 \xi^2 & W(T^\dagger)^2 (\xi^\dagger)^3 & (S^\dagger)^2 WT^4 \xi \\
SW(T^\dagger)^2 \xi^5 & W(T^\dagger)^2 & (S^\dagger)^2 WT^4 \xi^2 \\
S^3 W(T^\dagger)^5 \xi^3 & (W^\dagger)^2 T^4 & WT
\end{pmatrix}
\] (24)

These Yukawa matrices were calculated using the Higgs field abelian quantum
numbers given below in the form of $U(1)$ charge vectors:

$$\vec{Q} \equiv \left( \frac{y_1}{2}, \frac{y_2}{2}, \frac{y_3}{2}, Q_f \right)$$

(25)

We let the non-abelian representations be consequences of the specified abelian quantum numbers, by imposing the natural generalisation of the Standard Model charge quantisation rule:

$$y_i/2 + d_i/2 + t_i/3 = 0 \pmod{1}$$

(26)

We also require that the non-abelian representations be the smallest possible with the dualities $d_i$ and/or trialities $t_i$ determined from the quantisation rule. The abelian charge vectors are as follows:

$$\vec{Q}_{\phi WS} = (0, 2/3, -1/6, 1)$$

(27)

$$\vec{Q}_W = (0, -1/2, 1/2, -4/3) \quad \vec{Q}_T = (0, -1/6, 1/6, -2/3)$$

(28)

$$\vec{Q}_\xi = (1/6, -1/6, 0, 0) \quad \vec{Q}_S = (1/6, -1/6, 0, -1)$$

(29)

These quantum numbers were chosen with a view to fitting the mass and mixing angle data extrapolated to the Planck scale, using the SM renormalisation group equations for $Y_U$, $Y_D$ and $Y_E$. Actually we have chosen the Weinberg-Salam Higgs field quantum numbers in our gauge group to be such that the mass matrix element suggested to give the top quark mass—namely the transition between the formally generation number 2 right-handed charge $2/3$ Weyl field “$c_R$” and the left-handed third generation field $t_L$—is unsuppressed. In other words, we simply took the quantum numbers for the $\phi WS$ field to be the difference between those of these two Weyl fields.

Next we studied the predictions that follow from our gauge group quantum number pattern, but are not sensitive to the choice of Higgs field quantum numbers. We can for instance find some inequalities, but the most important result is that corresponding proto-diagonal elements in each of the Yukawa matrices $Y_U$, $Y_D$ and $Y_E$ have the same 3 quantum number differences. This feature can, for example with our choice of Higgs field quantum numbers, be seen in the matrices of eqs. (22-24). Along the diagonals of these three matrices, the suppression factors are given by $SW(T^i)^2\xi^2$, $W(T^i)^2$ and $WT$. Well, in the act-matrix it is the hermitean conjugated fields that occur along the diagonal, but it does not matter, because the suppression is the same anyway. It implies that, if the proto-diagonal dominates the eigenvalues, the masses within a generation will be order of magnitudewise degenerate. So, for example, the $u$, the $d$ and the electron are predicted to be order of magnitudewise degenerate, without having to specify the choice of Higgs quantum numbers, provided only that it happens that these proto-diagonal elements dominate.

Proto-diagonal dominance and order of magnitude generation mass degeneracy work well for the leptons being degenerate with the dsb-quarks and for the whole first generation, i.e. both up and down quarks being degenerate with the electron. But clearly the top quark and the charm quark are not order of magnitudewise
degenerate within their generations! It is therefore necessary, in our model, to arrange for the top and the charm quarks to get the dominant contributions to their masses from matrix elements not on the proto-diagonal. But that then means the right- and left-handed components of these two quarks do not, at the proto-level, belong to the same generation. For example the left-handed component of the charm quark couples to the second generation colour group, while its right-handed component couples to the third generation $SU(3)$ group (as well as third generation weak hypercharge). It was for this reason that we had to organize the top quark mass to be given by a proto-off-diagonal matrix element, and then it could be unsuppressed relative to the electro-weak scale.

This rule of the same proto-diagonal in all the three mass matrices gives most of the predictive power for quark and lepton masses in our model. So let us give an idea, as best we can, the reason why there is such a rule: It is well-known that the same Weinberg-Salam Higgs field can provide masses to all the three types of charged fermions in the simple Standard Model. Now, with our $SMG^3$ group the quantum number differences, between the right- and left-handed fermions, on the proto-diagonal become just the quantum numbers of the simple Weinberg-Salam Higgs field, translated to belong to the generation in question. For instance, the quantum number difference needed to be provided to give a mass to the first generation proto-diagonal matrix element in, say, the dsb-quark mass matrix is $\vec{Q} = (1/2, 0, 0, 0)$. Using the charge quantisation rule and small representation rule to translate the abelian quantum numbers into non-abelian ones, this quantum number difference corresponds to no colour but doublet under the $SU(2)$ of the first generation. One namely gets the same quantum numbers as those of the simple Weinberg-Salam Higgs doublet and $y/2 = 1/2$ but just for the first generation, as long as we ask for the first element along the diagonal. We find, of course, the same quantum numbers in uct-quark and charged lepton mass matrices. That is why we have this rule of the same suppressions for the three proto-diagonals.

So for the $SMG^3$ group, we understand this proto-diagonal quantum number difference rule, but what about the $U(1)_f$? We can simply check that, perhaps miraculously, the $U(1)_f$ quantum number differences along the proto-diagonals also turn out to be the same in all the three mass matrices. So the rule becomes general in our model! It is this rule that simulates the GUT $SU(5)$ mass predictions, namely the degeneracy of the dsb-quarks with the charged leptons in the corresponding generations. Note, however, that we only get the prediction of these degeneracies at the Planck scale as far as order of magnitude is concerned, and not exactly! This gives much better agreement with experiment than exact $SU(5)$ predictions, which are rather bad unless more Weinberg-Salam Higgs fields are included a la Georgi-Jarlskog’s factor 3 mechanism. Also note that we in addition predict that the up-quark is degenerate with the down-quark and the electron! This does not follow just from GUT $SU(5)$, although the up-quark is equally, not to say better, degenerate with the electron than the down quark!

As well as the proto-diagonal quantum number difference rule, we have the result that the quantum number difference on the element responsible for the top quark mass is balanced by $\phi_{WS}$. Using these facts we could study various relations between quantum numbers and use experimental mass data to suggest Higgs fields
and their expectation values. In this way we found the system described by the quantum numbers above.

From the Fritzsch rule for the mixing matrix element $V_{12} = \sin \theta_{Cabbibo}$ between the first and second generations, it is suggested that the two off-diagonal matrix elements connecting the d-quark and the s-quark be equally big. We take this to indicate that these two elements in the dsb-quark mass matrix should have essentially the same approximately conserved quantum number differences. We achieve this in our model by introducing a special Higgs field $S$, with quantum numbers equal to the difference between the quantum number differences for these 2 matrix elements in the dsb-quark matrix. Then we postulate that this Higgs field has a VEV of order unity in fundamental units, so that it does not cause any suppression but rather ensures that the two matrix elements get equally suppressed.

The existence of a non-suppressing field $S$ means that we cannot control phenomenologically when this $S$-field is used. Thus all the quantum numbers of the other Higgs fields, found by fitting data, can only have their quantum numbers predicted modulo those of the field $S$. We should therefore somehow, by requiring small representations or the like, seek to guess the best quantum numbers for the fields $W$, $T$, $\xi$ and $\phi_{WS}$ by adding adjustable multiples of $\vec{Q}_S$. The result of such a guess, using small representations as the principle, is provided by the following set of quantum numbers:

$$\vec{Q}_{\phi_{WS}} = (1/6, 1/2, -1/6, 0)$$  \hspace{1cm} (30)
$$\vec{Q}_W = (-1/6, -1/3, 1/2, -1/3) \quad \vec{Q}_T = (-1/6, 0, 1/6, 1/3)$$  \hspace{1cm} (31)
$$\vec{Q}_\xi = (0, 0, 0, 1) \quad \vec{Q}_S = (1/6, -1/6, 0, -1)$$  \hspace{1cm} (32)

With this new pattern of quantum numbers the powers of $S$ in the mass matrices would be changed but, since $S$ is 1 or at least close to 1, it would not change the predictions significantly.

The VEVs of the Higgs fields $W$, $T$ and $\xi$ are numbers to be fitted in our model. Thus we have only three parameters to fit the nine quark and lepton masses, the three mixing angles and the CP-violation. In a way even the overall scale of the masses is predicted, since the top quark Yukawa coupling is not suppressed and therefore of order unity. So we get 9 + 4 numbers out of three parameters, but only as far as order of magnitude is concerned. To the extent that we only care for orders of magnitude it should not matter that, in the computer calculation, we make use of complex random numbers of order unity to average statistically the predictions from our order of magnitude coupling matrices eqs. (22-24). In principle, of course, there is some parameter in the precise specification of the random distribution.

The fit of the suppression factors or Higgs VEVs is presented in table 1, where we used the conventional quark masses. In table 2 we present a fit to the data using recent lattice calculation estimates of the current algebra masses for the lighter quarks. We performed this order of magnitude fit by minimising a "$\chi^2$" function, defined to be the sum of the squares of the natural logarithms of the ratios of the predicted to the "experimental" values of the fermion masses and mixing matrix elements. For order of magnitude fits, both fits agree very well with the data. With the conventional quark masses the fitted values of the suppression
Table 1: Best fit to conventional experimental data. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

| Mass     | Fitted | Experimental |
|----------|--------|--------------|
| $m_u$    | 3.6 MeV | 4 MeV        |
| $m_d$    | 7.0 MeV | 9 MeV        |
| $m_e$    | 0.87 MeV | 0.5 MeV     |
| $m_c$    | 1.02 GeV | 1.4 GeV     |
| $m_s$    | 400 MeV | 200 MeV      |
| $m_{\mu}$ | 88 MeV | 105 MeV      |
| $M_t$    | 192 GeV | 180 GeV      |
| $m_b$    | 8.3 GeV | 6.3 GeV      |
| $m_{\tau}$ | 1.27 GeV | 1.78 GeV   |
| $V_{us}$ | 0.18    | 0.22         |
| $V_{cb}$ | 0.018   | 0.041        |
| $V_{ub}$ | 0.0039  | 0.0035       |

Table 2: Best fit using alternative light quark masses extracted from lattice QCD. All masses are running masses at 1 GeV except the top quark mass which is the pole mass.

| Mass     | Fitted | Experimental |
|----------|--------|--------------|
| $m_u$    | 1.9 MeV | 1.3 MeV      |
| $m_d$    | 3.7 MeV | 4.2 MeV      |
| $m_e$    | 0.45 MeV | 0.5 MeV     |
| $m_c$    | 0.53 GeV | 1.4 GeV     |
| $m_s$    | 327 MeV | 85 MeV       |
| $m_{\mu}$ | 75 MeV | 105 MeV      |
| $M_t$    | 192 GeV | 180 GeV      |
| $m_b$    | 6.4 GeV | 6.3 GeV      |
| $m_{\tau}$ | 0.98 GeV | 1.78 GeV   |
| $V_{us}$ | 0.15    | 0.22         |
| $V_{cb}$ | 0.033   | 0.041        |
| $V_{ub}$ | 0.0054  | 0.0035       |

Factors, proportional to the Higgs field VEVs, become:

$$\langle W \rangle = 0.179 \quad \langle T \rangle = 0.071 \quad \langle \xi \rangle = 0.099 \quad (33)$$

with $\chi^2 = 1.87$. We here fitted the 9 masses and the 3 mixing angles with the three parameters. Thus our $12 - 3 = 9$ predictions each on the average deviate by a factor the squared logarithm of which is $1.87/9 = 0.21$, meaning $\sqrt{0.21} = 46\%$ disagreement typically. The values of the Higgs VEVs—or rather suppression factors—for the fit with the lattice quark masses are:

$$\langle W \rangle = 0.123 \quad \langle T \rangle = 0.079 \quad \langle \xi \rangle = 0.077 \quad (34)$$

and this fit has a larger value of $\chi^2 = 3.81$, corresponding to $\sqrt{3.81/9} = 65\%$ deviations. But even this is good for an order of magnitude fit.

In these fits we did not use the CP-violation information as input, but we predict from the fit the CP-violating area of the “unitarity triangles” to be:

$$J \approx 5.8 \times 10^{-6} \quad \text{for conventional quark masses} \quad (35)$$

and

$$J \approx 1.2 \times 10^{-5} \quad \text{for lattice quark masses} \quad (36)$$

The “experimental” value derived from the observed CP-violation is

$$J \approx 2.0 \times 10^{-5} \quad \text{to} \quad 3.5 \times 10^{-5} \quad (37)$$

Our prediction is about a factor 4 below the data. However we should bear in mind that the quantity $J$ is, in our model, a product of very many factors and is expected to be more uncertainly predicted than most other quantities.
It is worthwhile to see the rather simple relations one obtains from our model by eliminating the suppression factors: First one gets the already mentioned degeneracy of the masses in the same generation, except for the top and the charm quarks (all after transport by the renormalisation group to the Planck scale). In addition we have the following order of magnitude Planck scale relations:

\[
\begin{align*}
& a) \quad m_3^3 = m_t m_c m_s \\
& b) \quad V_{ub} = V_{td} = V_{12} V_{23} \\
& c) \quad J \text{ (for CP-viol.)} = V_{ub} V_{12} V_{23} \\
& d) \quad V_{23} = \frac{m_s^2}{m_c m_b} \\
& e) \quad V_{12} = \sqrt{\frac{m_c}{m_s}}
\end{align*}
\]

6 Degenerate Vacua and Lattice Artifact Confinement; the Fine-Structure Constants

As we have assumed a regularization to truly exist and that we could take it to be lattice regularization without changing our results concerning phase transitions too much, we shall now think of phase diagrams for lattice gauge theories. It is well-known that if a lattice action for a lattice gauge theory depends on a couple of parameters, as in the example of a non-abelian SU(2) Yang-Mills field with both a trace of the doublet representation and a trace of the triplet representation of the plaquette variables occurring in the action, then the coefficients to these trace terms span a 2-dimensional phase diagram in which three phases arise. As we also stated above, the requirement of degenerate vacua becomes, in the Euclideanised formulation, the requirement of various phases coming together in the phase diagram space spanned by the lattice action parameters. With our very complicated non-simple Anti-GUT gauge group $SMG^3 \times U(1)_f$ a huge number of phases are possible.

What we ideally should have done to evaluate the fine-structure constants was the following:

We make a Monte Carlo computer simulation of the lattice gauge theory for our favourite gauge group $SMG^3 \times U(1)_f$ using an action with a rather large number of parameters. There should, say, be terms with real parts of traces of the many different irreducible representations of the gauge group. We then use Monte Carlo methods to map out the phase diagram by measuring various expectation values and observing when they jump. The phase diagram should reveal hypersurfaces separating various phases that sometimes intersect other such hypersurfaces along new hypersurfaces of progressively higher co-dimension. Now we look for a submanifold along which a lot of (maximally many) phases meet. This is a multiple “point”. Next we compute the fine-structure constants very close to this “point”, but just barely in a convening phase that is consistent with the survival of the Standard Model at long distances. Do we get the correct fine-structure constants?

We did not really use Monte Carlo methods nor a group of 37 generators, but rather made analytic estimates for what should have come out of the Monte Carlo calculation outlined above. These analytical calculations suggest that there should be one or more points in action parameter space where many phases meet. The MPP asserts that gauge couplings have values related to this(these) point(s). In our
approximation, the coordinates of this point are found with the help of the results of computer simulations done on some of the subgroups of our Anti-GUT gauge group. We then calculate what we would expect as the gauge couplings extrapolated down to long distances. The results that we obtain are in quite good agreement with the experimental gauge coupling constants.

For use in calculating the gauge couplings, we ideally want to determine a point (or points) in the action parameter space where many phases meet for the whole Anti-GUT gauge group. For the non-abelian subgroups SU(3) and SU(2), we found that a good approximation to such a point is obtained by finding a multiple point for each of the non-abelian invariant subgroups separately. We then simply assume that the different non-abelian groups do not interact at the multiple point(s) for the whole Anti-GUT gauge group. This approach is straightforward since there are already published results with the phase diagram of say an SU(3) gauge group with both a triplet and an octet action term. There are three phases meeting at a point (the multiple point) that we interpret in terms of what is the expected behaviour of the fluctuations very close to the Planck or lattice scale. These phases are not really separate phases, in as far as the one we interpret as the “Coulomb phase” is actually connected to the “confined” phase by going around the phase border that ends at the tricritical point. The third phase is one in which the \( \mathbb{Z}_3 \) subgroup of SU(3) “confines” while the continuous part of the group behaves in a “Coulomb-like” fashion. The phase diagram is indeed very analogous to the water-vapour-ice diagram, where one also has the possibility of going from vapour to water without crossing a phase boundary by going to sufficiently high temperature and pressure. The interpretations we give in terms of “confinement” and “Coulomb” are only valid near the lattice scale, while at long distances they are all really confining. The phase that should be identified phenomenologically with “our phase” is the one we denote as “Coulomb-like”, because “our phase” should not be confined in the high energy regime near the Planck scale.

The SU(2) group has a quite analogous phase diagram to SU(3), with three phases meeting at a point in a phase diagram spanned by two action parameters.

Now, as part of our scheme, the Anti-GUT gauge group breaks down to the diagonal subgroup SMG. The SU(3) group breaks to its diagonal subgroup \( SU(3)_{\text{diag}} = \{(u,u,u)|u \in SU(3)\} \). The inverse fine-structure constant \( 1/\alpha_{3\text{diag}} \) for the diagonal subgroup is given, in first approximation, as the sum of the inverse fine-structure constants for the three SU(3) groups in the Cartesian product subgroup \( SU(3)^3 \) of the Anti-GUT gauge group. The fine-structure constant of each of the three SU(3) factors is just that at the multiple point for the single SU(3) group, as if there were no interaction between the three SU(3)'s (and other SMG subgroups) that convene at the multiple point. It must be admitted that the formula for the diagonal subgroup gauge coupling

\[
1/\alpha_{3\text{diag}} = 1/\alpha_{3,1\text{st\ gen.}} + 1/\alpha_{3,2\text{nd\ gen.}} + 1/\alpha_{3,3\text{rd\ gen.}} \tag{38}
\]

is not invariant in going from one scheme of renormalisation to another one, so it can hardly be very accurate. We hope that if we use it for schemes like MOM or \( \overline{\text{MS}} \) or schemes numerically not far from them, then it should hold approximately, while we do not expect it to work in a true lattice scheme. So we either rewrite
the lattice scale couplings corresponding to the multiple point for SU(3) in one of the schemes in which we trust this formula, or we have to really figure out a more accurate formula. In reality, we assumed that Parisi-correcting the lattice couplings would bring us to one of the schemes for which it should be approximately trustable. The formula is easily derived upon recalling that the Parisi correction

\[ 1/g^2_{\text{Parisi corrected}} = \frac{<\text{tr}U>}{\text{Tr}1} \cdot 1/g^2 \]  

(39)

supposedly gives you an effective action in a continuum formulation.

Then of course the couplings obtained by Parisi correction must be the running couplings, referred roughly to the lattice scale which is only an order of magnitude or so above the scale at which the Anti-GUT group is Higgsed to the diagonal subgroup, by the W, S, T and ξ fields discussed above. Therefore a renormalisation group running down to the experimentally accessible scale is needed. Only in this stage of the calculation do we take into account the influence of fermions; we have assumed it a good approximation that this is the only influence of the fermions on the phase border couplings.

For the U(1)’s it is suspected that we do not find the highest number of meeting phases simply by treating the different U(1) groups independently. Rather we invented a coupling between the three U(1) subgroups of SMG^3 (the extra U(1)_f has been ignored in this calculation) that has a discrete symmetry of the same structure as a certain hexagonal lattice in a three dimensional parameter space. The lattice with this symmetry has nothing to do with the space-time lattice in the lattice regularization. It is rather a lattice in the covering group \( \mathbb{R}^3 \) of the group \( U(1)^3 \), which is the abelian part of our Anti-GUT gauge group once the \( U(1)_f \) is ignored. Having this discrete symmetry, we can use it to permute the various anticipated phases into each other. By imposing the symmetry just at the hoped for multiple point, we can be sure that if a phase is in contact with the multiple point, then the images of this phase under the symmetry are guaranteed to meet there also. In this way we ensure that a rather large number of phases can convene at the multiple point, provided we can construct an action that both has this symmetry and, at the same time, can bring together at the multiple point just some of the hoped for phases. But this requires the inclusion of extra action terms and, thereby, introduces the risk that these are required to have such large coefficients that they cause other unwanted phases to dominate.

The reason that interactions are presumably more relevant for the abelian than for the non-abelian groups is that there are more invariant subgroups for a Cartesian product of abelian groups than for a Cartesian product of simple groups. For abelian groups, all subgroups are automatically invariant. The relevance of invariant subgroups is that they can confine by themselves leaving the rest (i.e. the factor group) in a “Coulomb-like” phase. It is also easier to have interactions between the various abelian subgroup factors of a Cartesian product than in the non-abelian case. This is because the interactions one could have for the non-abelian subgroups would require irrelevant terms that would really be regularization (lattice) artifacts. Such interactions are not easy to make in the continuum. For the abelian groups however, one can even have interactions between different U(1) groups as relevant.
terms:

$$\text{const.} \cdot F_{\mu\nu}^{1\text{st gen.}}(x) F_{\mu\nu}^{2\text{nd gen.}}(x). \quad (40)$$

With such terms in the Lagrangian density, we might arrange that some linear combination of first and second generation $U(1)$ fields could make confining fluctuations while another combination behaves Coulomb-like. By using such possibilities there is the chance of constructing an interaction such that a large number of phases can be made to meet. But how many and which phases actually meet might be sensitive to our approximations.

As we want the Lagrangian for the special case in which all three abelian fields are equal—corresponding to the diagonal subgroup to be identified with the weak hypercharge group in the Standard Model—such interaction terms will contribute to the final $1/\alpha_{1\text{diag}}$. In the picture that we believe corresponds to the ‘best’ multiple point it happens, mainly due to the hexagonal symmetry, that the coefficients of the interaction terms are just like the one on the usual $(F_{\mu\nu}^{1\text{st gen.}})^2$ term. These terms add up so that we get six terms (3 squares and three interaction terms) leading to the $1/\alpha_{1\text{diag}}$ being 6 times as big as the $1/\alpha_1$ found at the multiple point in the diagram for a single $U(1)$ (for the non-abelian case there was instead only a factor 3). Actually, including the corrections from the extra interaction terms leads to a factor between 6 and 7.

Hence we obtained the following values for the fine-structure constants:

|                            | predicted | experimental |
|-----------------------------|-----------|--------------|
| $\alpha_3^{-1}(M_Z)$        | 12 ± 6    | 9.25 ± 0.43  |
| $\alpha_2^{-1}(M_Z)$        | 29 ± 6    | 30.10 ± 0.23 |
| $\alpha_1^{-1}(M_Z)$        | 99 ± 5    | 98.70 ± 0.23 |
| $\alpha^{-1}(0)$            | 137 ± 9   | 137.036...   |

(41)

It should be stressed that the good agreement seen in eq. (41) is achieved using a pure desert renormalisation group extrapolation of the Parisi corrected etc. Planck scale predictions. The theoretical uncertainties, typically of the order ±6, for the inverse fine-structure constants are crude estimates of the reliability of our going to the continuum and also include an estimate of the uncertainty on the Monte Carlo data used. If one introduced supersymmetry broken around the weak scale, the modification of the running of the fine-structure constants would cause our predictions for the inverse fine-structure constants—at the $Z^0$ scale—to go up by about 15 to 20. This corresponds to about 3 standard deviations for each of the three inverse fine-structure constants. That is to say, since we have already (accidentally) very good agreement, inclusion of supersymmetry down to the weak scale would weaken our coupling constant predictions by about 3 standard deviations for each of the three couplings!

So our model, although not by itself necessarily in conflict with supersymmetry at “low” energies, would no longer fit the data with such supersymmetry. Therefore we predict that supersymmetric partners should not be found in any experiments that are even remotely realistic in our time.

As can be seen from figure 1, our predictions are at the Planck scale and thus can only be tested together with an extrapolation model, which we take to be
the desert (minimal Standard Model) almost all the way up to the Planck scale. Since the running is also sensitive to the number of generations, our predicted low energy couplings depend on this number, both because of the renormalisation group running and because of the number of SMG's in the Anti-GUT gauge group being equal to the number of generations $N_{\text{gen}}$. In terms of $N_{\text{gen}}$, our gauge group is written $SMG^{N_{\text{gen}}} \times U(1)^k$ (where the number $k$ of $U(1)$ groups varies in a more complicated way, but is $k=1$ for $N_{\text{gen}} = 2$ and $3$, and $k = 2$ for $N_{\text{gen}} = 4$). It turns out that both dependences make our predicted couplings at the electroweak scale weaker the larger $N_{\text{gen}}$ is. It is, therefore, not surprising that even an older, less sophisticated version of the fine-structure constant calculation part of our model led to a fit indicating that the number of generations $N_{\text{gen}}$ must be three in order that our couplings can fit. In this work, only the non-abelian couplings are treated even approximately the way we do in the present model. Our approach was to see if our fit of the number of generations $N_{\text{gen}}$ led to an integer, namely $3$. It did so and, in this sense, a version of our model showed that it could truly predict the number of generations, at a time when only cosmological fits indicated that $3$ was a little better than $4$ in fitting the abundances of the primordially produced isotopes.

7 Conclusion; Are There Any Chances of Realistic Tests?

We have put forward a model, or scheme, for physics at the Planck scale having a gauge group $SMG^3 \times U(1)^f$, as we call it, and in which it is imposed that the coupling constants be so as to make several vacua degenerate in energy density. In addition we made very data-inspired choices for the Higgs fields that break down this 37-dimensional group to the Standard Model group, in the sense of choosing their quantum numbers under the big 37-dimensional group. We used four such Higgs fields, one of which $S$ has a VEV of order unity in Planck units, and the Weinberg-Salam Higgs field $\phi_{WS}$ which also must be assigned quantum numbers under the bigger group.

The fundamental Yukawa couplings in our model are taken to be of order of magnitude unity. We then obtained a fit to the quark-lepton masses and mixing matrix, in terms of the Higgs field VEVs. One of the VEVs was fixed of order unity, $\langle S \rangle = 1$, in Planck units. So, in addition to the electroweak scale $\langle \phi_{WS} \rangle = 246$ GeV, we used just three new parameters—namely the $W$, $T$ and $\xi$ Higgs field VEV suppression factors. We have no explanation for the order of magnitude of $\langle \phi_{WS} \rangle$: the gauge hierarchy problem remains the most difficult to solve. Otherwise we fit, order of magnitude wise or better, the 19 parameters of the Standard Model. If we take this rather impressive fit as a signal for truth, it is in a way slightly sad in the sense that our model has, in its uncorrupted form, no new physics except the Weinberg-Salam Higgs particle until the Planck scale. So there are not many positive predictions, rather the negative one that you shall find no new physics at accessible scales: No SUSY at experimental scales, no right handed $SU(2)$ etc. rather only the dull Standard Model.

One may wonder: are our predictions really so dull that there is almost no way to settle, by further investigation, whether our model should be right or wrong? Well, there might be a few chances:
Cosmological Strings that can split.

The $U(1)^4$ subgroup in our model is broken down to the single $U(1)$ of the Standard Model. So the Higgs fields in one point of space-time must formally give rise to a non-simply connected space of configurations compatible with being a vacuum locally. This local configuration space has a $\Pi_1$ homotopy group $\mathbb{Z}^3$ which should give rise to several types of stable vortex lines or cosmological strings, and the different types of string would be able to branch into each other. So there would be a network of such cosmological strings, rather than just a single unbranched type of string as is usually considered. This feature may have some cosmological consequences which could be looked for.

Baryogenesis problem.

In our model we have, essentially at least, just the Standard Model interactions up to the Planck scale or, rather, one or two orders of magnitude below it. So we have no way, at the electroweak scale, to generate the phenomenologically determined number of about $10^{-9}$ baryon per photon in the cosmic background radiation. There is insufficient CP violation in the Standard Model. Furthermore, even if created, it would immediately be washed out by sphaleron transitions after the electroweak phase transition since we already know from LEP that the Standard Model Higgs boson mass is greater than 70 GeV. So the only chance in our scheme to get a sufficient number of baryons relative to the photon number is to postulate that there is, at some stage, a violation of the quantum number $B - L$ (= baryon number minus lepton number). This quantum number $B - L$ is anomaly free and exactly conserved by the Standard Model. However, there is an anomaly for this quantum number in our model due to the $U(1)_f$ gauge field. That is to say that, at temperature scales so high that the $U(1)_f$ subgroup is unbroken, there is $B - L$ violation due to the anomaly. So, barring CP-violation and deviation from equilibrium, there will be a wash-out of the $B - L$ to zero even if there were some truly primordial $B - L$. The calculation of any surviving $B - L$ and, consequently, the baryon number produced in our model (after $B + L$ is washed-out by electroweak sphaleron transitions) is a future challenge for us. Really it looks crudely as if we reach a number about four orders of magnitude too low (but we should calculate more carefully):

According to our scenario there is, in the beginning, an anomaly in $B - L$ conservation mainly due to the $U(1)_f$ gauge field. This anomaly keeps washing out any net $B - L$ that might appear, due to CP-violating forces from the Planck scale physics, until the temperature of the Universe has fallen so low that the $\xi$ and other Higgs fields get their VEVs. The non-Standard Model gauge particles of the $SMG^3 \times U(1)_f$ group then obtain masses and the $B - L$ quantum number becomes much better conserved. However $B - L$ is still violated by irrelevant terms, reflecting the Planck scale physics where “everything” happens. We imagine that there are effective irrelevant terms, breaking $B - L$ as well as CP for example, active at the temperature when the physics has already come to first approximation into the desert. A term in the Lagrangian of dimension $d = 5$ which violates $B - L$ conservation is

$$gl^c \bar{l} \phi + h.c.$$  \hspace{1cm} (42)

where $l$ is a left-handed lepton field, e.g. the $(\tau, \nu_\tau)$ field, and $\phi$ is the Weinberg-
Salam Higgs field. By using a coupling $g$ which is complex, CP can be violated in this term. Applying this term in lowest order perturbation theory, there is still no CP-violation in the rates of the processes governed by it. However by interference with a next order term, it is possible to obtain a difference in the cross-sections of say $\phi^+\tau^- \to \phi^-\tau^+$ and the CP conjugate process $\phi^-\tau^+ \to \phi^+\tau^-$. Although with time-reversal breaking there does not have to be detailed balance, the numbers of particles of different sorts will of course be (statistically) stable in thermal equilibrium. In an expanding Universe, however, the density falls and, since the scattering typically delays the out-going particles a bit, the rates in various processes may no longer keep the various species in chemical equilibrium. We should imagine that even in thermal equilibrium, but with time-reversal broken, there is a stationary circulation of particles or pairs of particles around say three channels. For example the particles $\phi^+\tau^-$ may scatter more often into $\phi^-\tau^+$ than the opposite way $\phi^-\tau^+ \to \phi^+\tau^-$. But then there is an opposite reaction going on indirectly via some third channel. Such a third channel may be achieved by using several flavours. For example the particles could circulate through the chain of processes:

$$\phi^+\tau^- \to \phi^-\mu^+ \to \phi^-e^+ \to \phi^+\tau^-$$

Small time delays cause some of the processes in the circulation chain to last a little longer than the others and, in the presence of the Hubble expansion, a non-equilibrium results, which can even lead to an excess of $B-L$. In our estimate of the $B-L$ number produced, we shall assume the cross sections for the processes in this “circulation” were of order unity in Planck units. So the rate for, say, the process $\phi^+\tau^- \to \phi^-\tau^+$ would be given by the probability of finding two particles to scatter within a Planck length and that is $T^6$, where T is the temperature (in Planck units). For the moment we ignore the suppressions due to the selection rules in our model, except for the need to use the $d=5$ terms. Then we estimate the time-reversal violation to be lower than the process rate by yet another factor of T. Ignoring the number of degrees of freedom factor, the time in the early Universe is given by $t = 1/T^2$. Hence the rate $T^7$ for the $B-L$ asymmetric (under CP) violation gives a net production of $T^5_{initial}$, where $T_{initial}$ is the temperature at the moment when the $B-L$ anomaly switches off and production can start.

However, it is not at all correct, in our model, that the irrelevant term processes here used are of order unity in Planck units even if, as we assume, “everything” except gauge symmetry violation goes on at the Planck scale. These processes are in fact suppressed by the approximate conservation of the global charges corresponding to our $SMG^3 \times U(1)_f$ gauge group, even after the Higgs fields $W$, $T$, $\xi$ and $S$ have got their non-zero VEVs. The rates are therefore suppressed similarly to the fermion masses and mixing angles, by such factors as $W$, $T$ etc. It happens that the terms we need are rather closely related to the neutrino mass matrix elements. A very preliminary estimate suggests a suppression factor $W^2T$ for the $\phi^+\tau^- \to \phi^-\mu^+$

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$a$If the energy dependence of a scattering amplitude is Fourier transformed into a function of what is a passage time variable, one easily gets a delay in the sense of some extra averaged passage time needed for the process.

$b$It should be clear from the context whether $T$ refers to a Higgs field suppression factor or temperature.
transition, in analogy to the mass term in the neutrino mass matrix connecting
the $\mu$ neutrino with the $\tau$ antineutrino. The next process $\phi^- \mu^+ \rightarrow \phi^- e^+$ in the
“circulation” described above has a suppression factor of $\xi^3$ in our model. The final
step $\phi^- e^+ \rightarrow \phi^+ \tau^-$ then needed acquires a suppression factor $W^2 T^3 \xi^3$, analogous to
an off-diagonal (Majorana) neutrino mass matrix element. So putting $T_{initial} = \xi$
we get the $B-L$ density to be

$$\xi^5 \times \text{“suppression factors”} = \xi^{11} W^4 T^2 \approx 0.099^{11} \times 0.179^4 \times 0.071^2 = 10^{-16.3}$$

at the time corresponding to this temperature. Now conventionally this is measured
relative to the entropy density, also roughly the density of photons, which at this
time is $T_{initial}^3 \approx 10^{-3}$. Thus the baryon number to photon number ratio is in
our model predicted to be $10^{-13.3}$, which is not so terribly bad compared to the
phenomenological value of $10^{-9}$. Perhaps one may even find a more copious mode
of production within our model.

- Octet companions of the Weinberg-Salam Higgs.

The Weinberg-Salam Higgs must lie in an irreducible representation of the full
gauge group, but this representation is not necessarily also irreducible under the
Standard Model Gauge group. So we can expect partners or companions of the
Weinberg-Salam Higgs field. It is possible the mysterious reason for the Weinberg-
Salam Higgs components getting such a small mass scale may also extend to the
other components of the same $SMG^3 \times U(1)_f$ irreducible representation. In this case
there could be practically observable companions of the Weinberg-Salam Higgs field.
It happens that in our detailed model the Weinberg-Salam Higgs representation
is indeed reducible under the Standard Model and a colour octet companion is
predicted. Actually it is an SU(2) doublet and there will thus be both a neutral
and a charged colour octet companion. They should be looked for experimentally.
Since they are octets under QCD-colour they cause no baryon non-conservation,
unlike the analogous triplet companion in SU(5)-GUT. Thus the octet companions
could be very light, without making themselves felt through proton decay.

- Discrete group flux strings.

One of the major principles in our model—MPP—is that several phases of the
vacuum should coexist. Now, in some phases, discrete subgroups confine while
the continuous part of the group does not follow suit. A priori the flux tubes
representing the discrete gauge fields have string tensions of order unity in Planck
units. However at the phase boundary it just passes through zero and it could be
that the phase transition was so “weak” that this tension was exceptionally small.
In that case there could be experimentally detectable effects on, say, the $Z \rightarrow b\bar{b}$
vertex and the string states could be sufficiently light to be detected.

- Neutrino oscillations may give access to very short distance physics.

Since neutrino oscillations can reflect an exceedingly high mass see-saw particle,
we have via neutrino oscillations a window to the physics at very high energies. So
it may touch on the validity of our model in a way that does not just “see” the
desert and its pure Standard Model interactions. In the unmodified model, we have
a desert up to the Planck scale apart from the VEVs of our Higgs fields $W$, $T$ and
$\xi$, i.e. to just one or at most two orders of magnitude under the Planck scale. We
have also made more detailed neutrino Majorana mass estimates in our model. But
in the most clean version with the see-saw mass scale set by the Planck mass, we predict the neutrinos to be too light to give any practically observable neutrino oscillations! Taken this way our model is already falsified by the present neutrino oscillation experiments. We have, however, proposed how to modify our model with just one extra Higgs field. In this very lightly modified version, we predict the solar neutrinos to come in just half the amount predicted by the no oscillation calculations.

- Several vacua may be found; non-local effects.

If really it is so important that many vacua have the same energy density, you should expect these many vacua to be realized somewhere or sometime. That is to say we might look for some new type of vacuum spreading through the Universe as a bubble. There might be a chance to see some galaxies or the like being mirrored in the surface of such a bubble. Presumably it would move with a speed near that of light and you would see the mirrored galaxies blue shifted. Or should we make a bubble ourselves?

The vacua could come to exist first in the future but, anyway, it seems hard to get a general model making these degenerate vacua unless you use either

1) non-local effects at the fundamental level, like in baby universe theory. In fact we can argue that just having non-locality in the mild form, that the coupling constants are influenced by an average of the fields over both future and past, easily leads to the degenerate vacuum principle. So non-local effects might be what are predicted to get our phenomenological principle explained. Or

2) Supersymmetry, which easily gives different vacua with different physics but the same (zero) energy density.

It should be stressed though that our detailed predictions would be spoiled by supersymmetry at experimental scales at least. So a supersymmetry explanation of the degenerate vacua would have to be based on a very strongly broken supersymmetry—presumably broken at the Planck scale.

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