Delay and Power consumption Analysis for Queue State Dependent Service Rate Control in WirelessHart System

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Abstract—To solve the problem of power supply limitation of machines working in wireless industry automation, we evaluated the workload aware service rate control design implanted in the medium access control component of these small devices and proposed a bio-intelligence based algorithm to optimise the design regarding the delay constraint while minimizing power consumption. To achieve this, we provide an accurate analysis of the delay cost of this design and for the first time pinpoint an exact departure process model in order to evaluate the overall delay cost in consideration of the medium access time.

I. Introduction

Energy conservation has become an increasing concern in the system design for mobile network. The topic is considered important, impending and imperative. Broadly speaking, energy issue has already become a public talk of town on media for years. Every individual discipline has its own methodology and agenda to tackle the problem within its own domain. It is interesting to notice that no good and sound interdisciplinary subject has been successfully established on the issue yet. From another perspective, it implies the essence of the issue is fundamental. Practically speaking, widely promoted nowadays is the smart city and smart home concept that from an engineering perspective, coordinates small portable devices to communicate intelligently with each other [1, 2, 3]. These devices can be line powered or battery powered and they request efficient mechanism to balance the power consumption by themselves and the user perception of the services by the device holders. Down to the earth, within the specific arena of mobile network, power consumption conservation schemes have been limited to adjust the transmission power of the mobile devices, particularly during routing selections when the overall network performance in terms of energy conservation has to be taken to evaluate against the throughput and QOS metrics [4,5,6,7,8,9]. Energy aware routing can be a very much complicated research cases as the issue, once formulated as an optimization problem can have numbers of constraints and the approach to solve the problem can be NP-hard and requires heuristic algorithms or bio-intelligence functioning mechanisms [10,11].

Few works have been conducted on the inner-device power consumption conservation potentials. The works are much onto experimental analysis of the power consumption pattern of the devices, whether or not employed with the dynamic frequency scaling techniques. These consumption patterns go aligned with different applications on the top of the stack. The most trendy ones are the video streaming and downloading [12].

According to paper [13,14], for any given network configuration, the rate of power consumption of the devices shows less than 15% variation with the load. It is our interest to break down the energy dissipation per functional block on the chip of the mobile assets so as to make an analytical model of energy consumption. A typical layout of the functional blocks of a mobile asset is as below by paper [15]. It is composed of Buffer Read, IPSec/Management, Encryption, Framing and Modulation. For some functional blocks, the processing time per packet is fixed and for others, the processing time per packet varies according to the packet size, protocol used and etc. It will be expected that the speed of the underlying clock can be adjusted so as to slow down the processing while the workload is low and speed up while the workload is high.

![Functional Blocks](image)

**Fig. 1. Functional Blocks**

Thanks to the maturity of the dynamic frequency scaling techniques, it is assumed that the techniques to scale the clock speed is hardware achievable. In paper [16], the clock speed is being mapped to five different regions of buffer queue length. Given a fixed number of frequency
options, the buffer size is being demarcated equally into several regions, each region covering a contiguous set of buffer states which corresponds to a given frequency linearly ordered from highest to lowest. The system is working at the pace of per packet service time. Once a packet is dispatched from the buffer, the decision maker will read the buffer size and adjust the clock frequency if needed.

Dynamic frequency scaling has been introduced into the wireless mac layer design in paper [17], and investigated further in paper [18]. Dynamic frequency scaling itself has reached technological maturity that can be readily implemented in the chip design for wireless devices [18][19]. In paper [17], workload predictive Ethernet device design has been proposed in that the optimisation scheme based on queueing analysis proves to outperform legacy ones by experiments. In [16], state dependent service rate queueing model has been borrowed to evaluate the threshold aware energy efficient wireless access network design. It also provides a greedy algorithm to find the optimal solution that can balance the trade-off between energy minimization and delay constraints. Many works have been conducted theoretically on state dependent service rate queue [20][21], while with few focus on engineering problems in wireless field. [22]’s paper serves to be a rare exception and nevertheless with the application of N-policy queue. Paper [23] applied an recursive algorithm to compute iteratively the equilibrium probability distributions, the cost function it used is generic and standard in the field of queuing control. Paper [24] utilised the precondition of system stability to converge the solutions for a multi-relay network. Paper [25] also discussed approach of optimize the multi-relay network in terms of energy consumption and delays. It circumvents the problem to find an exact solution by achieving an estimation of the upper bound for the delays. Paper [26] approaches this multi-hop queueing problem by utilisation of fluid limit techniques in order to justify the design’s optimality and stability. Paper [27] presents an analysis to the departure process of a sleep-control queue and based on that, it evaluates the system performance based on different threshold levels.

III. Problem Formulation

The service time of a customer depends on the queue length at service initiation epoch of the customer. The queue is partitioned based on different number of thresholds. Concretely, we place the threshold values \(L_k \in \mathbb{R} \) for the queue system embedded in the mobile device. Furthermore, the work details the analytical model to evaluate the system performance under a given frequency region demarcation and borrows particle swarm optimization technique to solve the optimization problem.

The novelty of the work lies as follows:

- It proposed a novel model to analyze queue length dependent service time system embedded in the mobile device
- It presented a probabilistic analysis of the service time per packet within the mobile system supported by WirelessHart mechanism
- It validates the model and analysis by NS2 simulations.

II. Background and Related Works

Dynamic frequency scaling has been introduced into the wireless mac layer design in paper [17], and investigated further in paper [18]. Dynamic frequency scaling itself has reached technological maturity that can be readily implemented in the chip design for wireless devices [18][19]. In paper [17], workload predictive Ethernet device design has been proposed in that the optimisation scheme based on queueing analysis proves to outperform legacy ones by experiments. In [16], state dependent service rate queueing model has been borrowed to evaluate the threshold aware energy efficient wireless access network design. It also provides a greedy algorithm to find the optimal solution that can balance the trade-off between energy minimization and delay constraints. Many works have been conducted theoretically on state dependent service rate queue [20][21], while with few focus on engineering problems in wireless field. [22]’s paper serves to be a rare exception and nevertheless with the application of N-policy queue. Paper [23] applied an recursive algorithm to compute iteratively the equilibrium probability distributions, the cost function it used is generic and standard in the field of queuing control. Paper [24] utilised the precondition of system stability to converge the solutions for a multi-relay network. Paper [25] also discussed approach of optimize the multi-relay network in terms of energy consumption and delays. It circumvents the problem to find an exact solution by achieving an estimation of the upper bound for the delays. Paper [26] approaches this multi-hop queueing problem by utilisation of fluid limit techniques in order to justify the design’s optimality and stability. Paper [27] presents an analysis to the departure process of a sleep-control queue and based on that, it evaluates the system performance based on different threshold levels.

IV. Stationary probability distribution

We derive the queue length distribution just after departure epochs. Let \( \omega_n (n \geq 1) \) be the epoch of successive departures with \( \omega_0 = 0 \). Let \( Y_n \) be the random variable denoting the system length at time \( \omega_n \). Then, the process \( \{Y_n, n \geq 0\} \) forms a Markov chain with finite state space \( \{0,1,2,\cdots\} \). The following probabilities are defined,

\[ p_i = \lim_{n \to \infty} \Pr\{Y_n = i\}, \quad 0 \leq i \leq K. \]

Let \( p_{ij}^k \) be the conditional probability that the \( n+1 \)-th departure leaves behind \( j \) customers in the system given \( n \)-th departure left \( i \) customers in the system during the service time of \( S_k \).

\[ p_{ij}^k = \lim_{n \to \infty} \Pr\{Y_{n+1} = j | Y_n = i\} = \alpha_j^k \cdot 1, \quad \text{where} \quad 0 \leq i \leq j \leq K \]

where \( \alpha_k^i \) denotes the arrival of \( n \) number of customers during a service period of a customer when the server has chosen \( k \)-th type of service.

1) From 0,1,\cdots,\ell_1 : service type \( G_1 \)
2) From \( \ell_1 + 1,\ell_1 + 2,\cdots,\ell_2 : service type \( G_2 \)
3) From \( \ell_2 + 1,\ell_2 + 2,\cdots,\ell_3 : service type \( G_3 \)

Let’s describe the transition probability matrix for the three threshold: \( \ell_1, \ell_2 \).
where $\pi^T_i$ represents the probability of arriving $i$ and more number of customers during the service time of one customer while $j$ type of service is chosen. In other words, $\pi^T_i = \sum_{k=i}^{\infty} a^T_k$. The balanced equation at embedded epoch are:

$$p_j = p_0 a^T_j + \sum_{i=1}^{j} p_i a^T_{j-i+1}, \quad 0 \leq j \leq L_1 - 1,$$

$$p_j = p_0 a^T_j + \sum_{i=1}^{j} p_i a^T_{j-i+1} + \sum_{i=L_1+1}^{j} p_i a^T_{j-i+1} + \cdots + \sum_{i=L_{k-1}+1}^{j} p_i a^T_{j-i+1}, \quad L_k - 1 \leq j \leq L_k - 1,$$

$$p_L = p_0 \pi^T_L + \sum_{i=1}^{L_1} p_i \pi^T_{L-i+1} + \sum_{i=L_1+1}^{L_2} p_i \pi^T_{L-i+1} + \cdots + \sum_{i=L_{k-1}+1}^{L_k} p_i \pi^T_{L-i+1},$$

A. Mean waiting time analysis

Let us define a quantity $T_{\text{mean}}$ to denote the mean time interval between two successive departure epochs. The departure may take place while the system-length may take any value among the various threshold that is configured in the system. Departure may also take place when an arriving packet finds the system empty. Hence, $T_{\text{mean}}$ can be expressed by:

$$T_{\text{mean}} = p_0 (1 + E[S_1]) + \sum_{i=1}^{L_1} p_i E[S_i] + \sum_{i=L_1+1}^{L_2} p_i E[S_2] + \cdots + \sum_{i=L_{k-1}+1}^{L_k} p_i E[S_T].$$

The carried load $\rho'$ is defined as the proportion of time the system is busy, which can be expressed as

$$\rho' = \frac{p_0 E[S_1] + \sum_{i=1}^{L_1} p_i E[S_i] + \sum_{i=L_1+1}^{L_2} p_i E[S_2] + \cdots + \sum_{i=L_{k-1}+1}^{L_k} p_i E[S_T]}{T_{\text{mean}}} + \frac{\sum_{i=L_{k-1}+1}^{L_k} p_i E[S_T]}{T_{\text{mean}}}.$$

Let us denote the Laplace-Stieltjes transform (LST) of $i$-th type service as $B_i^*(s)$. The mean system-time can be expressed as

$$W^*(s) = \sum_{i=0}^{L_1-1} p_i [B_i^*(s)]^{i+1} + \sum_{i=L_1}^{L_2-1} p_i [B_i^*(s)]^{i+1-L_1} [B_1^*(s)]^{L_1},$$

$$E[W] = \sum_{i=0}^{L_1-1} p_i E[S_i] + \sum_{i=L_1}^{L_2-1} p_i [(i+1-L_1)E[S_2] + L_1 E[S_1] +$$

$$+ \cdots + \sum_{i=L_{k-1}+1}^{L_k} p_i [(i+1-L_k)E[S_k] + L_k E[S_1] +$$

$$+ (L_k - 1)E[S_2] + L_1 E[S_1]]$$

V. Steady-state distribution at arbitrary epoch

In this section we derive the expression for steady-state system length distribution at arbitrary epochs. We have applied the classical argument based on renewal theory (see Chaudhury and Templeton [25]) which relates the steady-state system length distribution at an arbitrary epoch to that at the corresponding post-departure epoch. Let $c_k' (k \geq 0)$ are the probability that $k$ customers has arrived during residual service time when type of service is chosen. Hence for all $k \geq 0$, we have

$$c_k' = \frac{1}{E[S]} \int_0^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} (1 - B_i(t)) dt$$

Let us derive the p.g.f. of $c_k'$ which is denoted by $C(z)$. We use the LST of probability density function (pdf) of residual service time as given in Kleinrock [28],

$$\int_0^{\infty} \frac{1}{E[S]} e^{-st} (1 - B_i(t)) dt = \frac{1}{E[S]} \frac{1 - B_i'(s)}{s}.$$

Therefore,

$$C(z) = \sum_{k=0}^{\infty} c_k' \int_0^{\infty} \frac{(\lambda t)^k}{k!} e^{-\lambda t} (1 - B_i(t)) dt$$

$$= \frac{1}{E[S]} \int_0^{\infty} e^{-(\lambda - \lambda z)t} (1 - B_i(t)) dt = \frac{1}{E[S]} \frac{1 - B_i'(\lambda - \lambda z)}{\lambda - \lambda z}.$$
System length at an arbitrary epoch $\pi_i$ can be obtained from post-departure epoch probability $p_i$ ($\forall i \geq 0$) by employing $c_k$ in Eq. (1) - (4), which is described below:

$$\pi_j = \rho' p_0 c_j^1 + \rho' \sum_{i=1}^{j+1} p_i c_{j-i+1}^1, \quad 0 \leq j \leq L_1 - 1,$$

(8)

$$\pi_j = \rho' p_0 c_j^1 + \rho' \sum_{i=1}^{L_1} p_i c_{j-i+1}^1 + \rho' \sum_{i=L_1+1}^{j+1} p_i c_{j-i+1}^2,$$

$$L_1 \leq j \leq L_2 - 1,$$

(9)

$$\vdots$$

$$\vdots$$

(10)

$$\pi_j = \rho' p_0 c_j^1 + \rho' \sum_{i=1}^{j+1} p_i c_{j-i+1}^1 + \rho' \sum_{i=L_k-1+1}^{L_k} p_i c_{j-i+1}^1 + \rho' \sum_{i=L_k+1}^{L_k} p_i c_{j-i+1}^2 + \cdots,$$

$$+ \rho' \sum_{i=L_k+1}^{L_k} p_i c_{j-i+1}^2,$$

(11)

where $\tau_i^j$ represents the probability of arriving $i$ and more number of customers during the residual service time of one customer while $j$ type of service is chosen. In other words, $\tau_i^j = \sum_{k=i}^j c_k$.

$$\pi_{L_k} = \rho' p_0 c_{L_k}^1 + \rho' \sum_{i=1}^{L_k} p_i c_{L_k-i+1}^1 + \rho' \sum_{i=L_k+1}^{L_k} p_i c_{L_k-i+1}^2 + \cdots,$$

$$+ \rho' \sum_{i=L_k+1}^{L_k} p_i c_{L_k-i+1}^2,$$

(12)

VI. Departure Process Analysis

The departure process of a segmented queue consist of two streams of sub processes that interleaved with each other probabilistically. One is that the arrival occurs when the queue is empty. That means a packet arrives after the previous packet in queue is departed. The departure time is the addition of the arrival time and the service time. The other is that the arrival occurs when the queue is not empty. That means a packet arrives before the previous packet in queue is departed. The departure time is the addition of the previous packet’s departure time and the service time. The above statement is mathematically presented below.

$$t_d^k = \begin{cases} t_d^{k-1} + t_s^k & \text{if } t_d^k < t_d^{k-1} \\ t_d^{k-1} + t_s^k & \text{else if } t_d^k > t_d^{k-1} \end{cases}$$

(13)

Where $t_d^k$ is the departure time of the $k$th packet, $t_s^k$ is the time spent at the server for the $k$th packet and $t_a^k$ is the arrival time of the $k$th packet.

Henceforth, the inter-departure time is as follows:

$$\Delta t_d^k = \begin{cases} t_d^k - t_d^{k-1} & \text{if } t_d^k < t_d^{k-1} \\ t_d^{k-1} - t_d^k + t_s^k & \text{if } t_d^k > t_d^{k-1} \end{cases}$$

(14)

The mean service time of the segmented queue is as below:

$$E[t_s] = \begin{cases} \frac{1}{\mu_1} & \text{if } X < L_1 \\ \frac{1}{\mu_2} & \text{if } L_1 < X < L_2 \\ \frac{1}{\mu_3} & \text{else} \end{cases}$$

(15)

Assume the chance of a packet arriving at an empty queue is $p_0$, it is when $t_d^k > t_d^{k-1}$, otherwise the chance is $1 - p_0$. When a packet arrives at an empty queue that $t_d^k > t_d^{k-1}$, the expectation of the inter-departure time is as follows

$$E[\Delta t_d^k] = E[t_d^k] - E[t_d^{k-1}] + E[t_s^k]$$

(16)
Within the equation, the first term \( E[t_d] = \frac{k}{\lambda} \), the second term is as follows

\[
E[t_d^{k-1}] = p_0 * E[t_d^{k-1} + t_s^{k-1}] + (1 - p_0) * E[t_d^{k-2} + t_s^{k-2}] 
\]

(17)

\[
= p_0 * E[t_d^{k-1}] + (1 - p_0) * E[t_s^{k-2}] 
\]

(18)

\[
+ p_0 * E[t_s^{k-1}] + (1 - p_0) * E[t_s^{k-2}] 
\]

(19)

(20)

Here the work aims to derive the departure moments when the system enters a state of stability as \( k \rightarrow \infty \). Hence, the third term \( E[t_s] \) is the average departure time for packet \( k \) which depends on the queue length upon the packet arrival. As the system goes into equilibrium state, the queue length distribution is time invariant. From section, the stable queue length distribution is attained as \([p_i]^K\).

\[
E[t_s] = (\frac{L_1}{\mu_1}) + (\sum_{L_1+1}^{L_2} p_i) + \frac{1}{\mu_2} + (\sum_{L_2+1}^{K} p_i) \frac{1}{\mu_3} 
\]

(21)

Let \( E[t_s] = t_E \), then \( E[d_s] = p_0 * E[t_s] = (1 - p_0) * E[t_s] \)

The following formula can be derived. Please kindly refer to the appendix for details.

\[
E[\Delta t_d] = \frac{1}{\lambda} 
\]

(22)

The below plot shows that the derived result is consistent with simulation.

![Fig. 5. Model Validation - Inter-departure Time](image)

Equivalently, according to (??), the expectation of inter-departure time is decomposed into the probabilistic weighted summation of expectations of two exclusive and complimentary events.

\[
E[\Delta t_d] = (1 - p_0)A + p_0B 
\]

(23)

A is the inter-departure time expectation of the event when a packet arrives at an empty queue while B is when a packet arrives at an non-empty queue. \( A = E[t_E] \), \( B = E[t_d^{k^2} - t_d^{k^2-1} + t_E] \),

\[
B = \frac{1}{p_0 \lambda} - \frac{t_E}{p_0} + t_E 
\]

(24)

(25)

For derivation details, please refer to the appendix. The below plot shows that the derived formulas for both event A and event B are consistent with simulation.

![Fig. 6. Model Validation - Inter-departure Time (E-NE)](image)

### A. Departure Process Distribution

From the above analysis, we arrive to the below hypothesis that the departure process distribution consists of two parts. That’s

\[
f(n) = f_E(n) + f_{NE}(n) 
\]

(26)

Given the condition that \( \sum f_E(n) = 1 - p_0 \) and \( \sum f_{NE}(n) = p_0 \) Where \( f_E(n) \) is the probability mass function when the packet arrives at an empty queue and \( f_{NE}(n) \) is the PMF when the packet arrives at an non-empty queue.

It is easy to deduce that \( f_{NE}(n) \) is the sum of probabilistic weights of three delta function \( \delta(n - n_k) \) with \( t_k \) corresponding to the three service time options. To be precise

\[
f_{NE}(n) = \sum_n p_n \delta(n - n_k) 
\]

(27)

And

\[
p_n = \begin{cases} 
\frac{\sum_{i=1}^{L_1} p_i}{1 - p_0}, & \text{if } n = n_1 \\
\frac{\sum_{i=L_1+1}^{L_2} p_i}{1 - p_0}, & \text{if } n = n_2 \\
\frac{\sum_{i=L_2+1}^{K} p_i}{1 - p_0}, & \text{else} 
\end{cases} 
\]

(28)

The following plot shows that the above analytical formula is consistent with simulation.
For $f_E(n)$, we have the following graph by simulation.

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We assume $f_E(t) = Be^{-\beta t}[u(t-t_0) - u(t-t_1)] + Ce^{-\alpha t} u(t-t_1)$ with $\alpha = \lambda, \beta = \frac{2\lambda \mu}{\lambda + \mu}, t_0 = \bar{E}(t_E)$ and $t_1 = 0.011$. As the inter-departure time PMF when the queue is empty can be further decomposed into weighted summation of 1. the PMF when the consecutive two packets arrive at an empty and 2. the PMF when a packet arrives at an empty queue and the previous packet arrives at a non-empty queue.

For the first term, the inter-departure time is equivalent to the inter-arrival time as there is no log in the queue. Henceforth, we set the following:

$$\alpha = \lambda$$

(29)

For the second term, the inter-departure time is affected by both the inter-arrival time and the service time. Tentatively we set the following:

$$\beta = \frac{2\lambda \mu}{\lambda + \mu}$$

(30)

$t_1 = 0.011$ is an experimental observatory constant factor, the physical interpretation of which can not be explicitly explained.

The validation of the above analysis is demonstrated through the below plot.

VII. Channel Contention Time Analysis

In the wireless Hart system, the server attends to each queue in a cyclic order with a quota of one packet each time. In current work, we assume the server will still visit the queue in sequence even if the queue is empty as it is not aware of the queue status before arrival. Suppose the server processes a packet with a rate of mean value $\mu$ and there are only three queues (equivalent to three mobile terminals in the system). We further assume all the three queues have the same arrival rate, the process of which follows the analysis in the previous section. Then it is easy to arrive that for one particular queue, the server attends with a rate of $\frac{\mu}{3}$ and the system follows the G/M/1 queue model, given the waiting room capacity is unlimited. The system diagram is as below:

To attain the value of $B$ and $C$, we have the below two equations: 1. The summation of the partial probability mass function is equal to $p_0$ and 2. The expectation by the partial PMF is equal to $\frac{1}{A p_0} - \frac{\mu}{p_0} + t_E$.
Where \( \sigma = L_{fa}(\mu(1-\sigma)) \). \( f_a \) is the arrival process to the channel, which is equivalent to the departure process from the thresholded queue. Hence \( f_a = f(n) \).

\[
W = \frac{\sigma}{(1-\sigma)\mu}
\]

(31)

Fig. 11. Model Validation - Waiting Time

VIII. Selective Thresholds for Optimisation

The system waiting time \( W_{system} \) is the summation of the queuing time \( W_{queue} \) and the channel access time \( W_{channel} \).

\[
W_{system} = W_{queue} + W_{channel}
\]

And the average normalised power consumption \( NP \) is

\[
NP = \frac{(\sum L_1 p_1)P_1 + (\sum L_2 p_1)P_2 + (\sum L_2 p_2) P_3}{P_1}
\]

(32)

(33)

where \( p_1 \) is the probability distribution of the queue length and \( P_1 = \gamma C_{capacity} V^2 f_i = \gamma f_i \) by dynamic frequency scaling formula. In current setting \( f_1 = 100000000, f_2 = 150000000 \) and \( f_3 = 200000000 \). The optimisation objective function is as follows:

\[
\min_{L_1, L_2} \quad NP \quad \text{s.t.} \quad W_{system} \leq W_{bound}, L_1 \leq L_2
\]

(34)

Suppose the queue limit is \( K \) as has been previously defined, the size of the overall possible solution set is \( \frac{K(K-1)}{2} \) for \( N = 2 \) thresholds. The problem is not NP-hard. We can use brute force method to locate the optimal options. Somehow to even trim the volume of computational workload, we apply PSO algorithm, which achieves a workload that is lighter in terms of time efficiency.

Result: \( L_{1}^{opt}, L_{2}^{opt} \)

initialization of the \( n \) positions and set the local optimal;
while Iteration do
| Calculate \( L_1 \) increment |
| Calculate \( L_2 \) increment |
| if \( L_1 \) and \( L_2 \) incorrect then |
| \( NP = \infty \) |
| if Wall Touched then |
| Reverse the increment direction |
| end |
| if \( W_{System} > W_{bound} \) then |
| \( NP = \infty \) |
| end |
| Update local optimal |
| Update global optimal |
end
Algorithm 1: PSO Algorithm to Find the optimal threshold options

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IX. Acknowledgement

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X. Appendix

A. The expectation value of the inter-departure time

\[
E[\Delta t_s^k] = (1 - p_0)E[t_s^{k-1}] + p_0[E[t_s^k] - E[t_s^{k-1}]] + E[t_s^0]
\]

= \frac{1}{\lambda} + \frac{1}{\lambda} [kp_0 - (k-1)p_0^2 - p_0(1 - p_0)]t_E

+ \frac{1}{\lambda} [kp_0 - (k-1)p_0^2 - p_0(1 - p_0)(k-2) - p_0^2(1 - p_0)^2(k-3)]

- p_0(1 - p_0)^3 \sum_{i=1}^{k-3} (1 - p_0)^i

E[\Delta t_d^k] = \frac{1}{\lambda} [kp_0 - (k-1)p_0^2 - p_0(1 - p_0)]t_E

+ \frac{1}{\lambda} kp_0 \sum_{i=0}^{k-3} (1 - p_0)^i

- p_0(1 - p_0)^{k-1} \sum_{i=1}^{k-2} (1 - p_0)^{(i-1)}

= \frac{1}{\lambda} [kp_0 - (k-1)p_0^2 - p_0(1 - p_0)]t_E

+ \frac{1}{\lambda} kp_0(1 - p_0)^{k-2} - p_0(1 - p_0)^{k-2}t_E

+ \frac{1}{\lambda} (kp_0(1 - p_0)^{k-2} - p_0(1 - p_0)^{k-2})t_E

= k \rightarrow \frac{1}{\lambda}
B. The expectation of the inter-departure time when a packet arrives an empty queue

\[
E\left[ t^k_d - t^{k-1}_d \right] \\
= t^k_d - p_0 t^{k-1}_d - (1 - p_0) t^{k-2}_d - t_E \\
= t^k_a - p_0 t^{k-1}_a - (1 - p_0) p_0 t^{k-2}_a \\
- (1 - p_0)^2 t^{k-3}_d - (1 - p_0) t_E - t_E \\
= \frac{k}{\lambda} - p_0 \left( \frac{k-1}{\lambda} + (1 - p_0) \frac{k-2}{\lambda} + \cdots + (1 - p_0)^{k-2} \frac{2}{\lambda} \right) - \frac{t_E}{p_0} \\
= \frac{k}{\lambda} - \frac{k}{\lambda p_0} + \frac{1}{\lambda p_0} - \frac{t_E}{p_0} \\
= \frac{1}{p_0 \lambda} - \frac{t_E}{p_0}
\]