Symbol Sense of Mathematics Students in Solving Algebra Problems

Ariati Dwi Prasetya Rini*, Saddam Hussen², Herlina Hidayati¹, and Anwar Muttaqien¹

¹STKIP Muhammadiyah Sampit, Sampit, Indonesia
²Faculty of Teacher Training and Education, University of Jember, Jember, Indonesia

*ariati_dwi@stkipmsampit.ac.id

Abstract. Symbol sense is a sensibility or mastery of someone against the use of symbols and an understanding of the situation when symbols can be used. This study attempts to describe the extent to which the use of symbol sense students STKIP Muhammadiyah Sampit to solve the algebra problem based on the steps of Polya’s problem-solving. The results showed that understanding the problem step of students in the first semester was still less able to relate the symbol to the problem that given, and the students still used words instead of symbols in understanding the problem. Then the third-semester students have fulfilled all the indicators of symbol sense. Furthermore, fifth-semester students are less precise in writing symbols according to their meaning in the problem. Steps of planning the problem solving all students can use the symbol sense properly. Then in the step of implementing the problem-solving plan, students in the third and fifth semesters can fulfill all the indicators symbol sense to the maximum. However, first semester students bring up or make some new symbols to solve problems. Steps of looking back at the problem solving has been obtained, all students meet all the symbol indicators well. So, the first semester students are still lacking in using their symbol sense in solving algebra problems.

Keywords—symbol sense, algebra problem, polya's problem-solving

1. Introduction

Mathematics is inseparable from symbols as representatives of abstract things. Mathematical problems are represented in the form of symbols, diagrams or others to find a solution. Symbol Sense or sensibility symbols have an important role in understanding mathematical problems that contain many symbols, one of which is the problem of algebra. Symbols used in algebra such as variables, coefficients, and constants represent a number or number in the universal set [1]. A student with good Symbol Sense can appreciate the power of symbols, knowing when to use appropriate symbols and being able to manipulate and understand symbols in various contexts [2][3][4]. According to previous research, the ability of mathematics Symbol Sense in middle-level students is generally in the low category [1]. Starting from this, further research needs to be done on the ability of Symbol Sense to students in algebra problems. Considering that students are prospective teachers who are required to be critical, creative and collaborative as well as professionals to face the industrial revolution 4.0.

Arcavi [4] introduces the term symbol sense and categorizes that there are six components in symbol sense, namely friendliness with symbols, manipulations and reading through symbols, engineer symbolic relationships, select one possible symbolic representation for a problem, check for the symbol meanings during the implementation of a procedure, and symbols can play different roles in different
contexts. Furthermore, according to Polya, there are four steps to solving mathematical problems, namely understanding the problem, devising a plan, carrying out the plan, and looking back at the problem solving has been obtained [5].

2. Method
This study aims to describe the extent to which the use of the symbol sense of students of STKIP Muhammadiyah Sampit in solving algebra problems. Students of the Mathematics Education Study Program at STKIP Muhammadiyah Sampit were the subjects in this study. The number of students in the mathematics education study program consists of 8 students in the first semester, 14 students in the third semester, and 10 students in the fifth semester. From all of these students, students were asked to work on algebra problems. Based on the results of these answers will be selected one student from each level of the semester who can solve algebra problems properly and correctly and have good communication skills.

The main instrument in this study is the researcher himself. While supporting instrument is a test algebra and interview. An instrument sheets test algebra used to get data in the shape of the subject of study during the deal algebra. This algebra problem test sheet contains algebraic factorization, algebra simplification, and story problems to determine the area and width of a rectangle. Next, interviews were conducted to gain knowledge about student problem-solving behavior and gain a deeper understanding of students' cognitive development in solving algebra problems. Audio recording is also done during the interview process. Verbatim from the audio recording transcribed. The indicators used in the study can be seen in Table I.

| Table 1. SymbolSense Indicators | Poly's Problem-solving Steps | Symbol Sense Indicators |
|--------------------------------|----------------------------|-------------------------|
| 1. Understanding the problem   | a. Mention symbols that can be used in problem-solving | a. Associate symbols with problems |
|                                | b. Identify the meaning of a symbol in the problem | b. Write symbols according to their meaning in the problem |
|                                | c. Associate symbols with problems | c. Write symbols according to their meaning in the problem |
|                                | d. Write symbols according to their meaning in the problem | d. Write symbols according to their meaning in the problem |
| 2. Devising a plan of problem solving | a. Choose the right symbol to solve the problem | a. Choose the right symbol to solve the problem |
|                                | b. Expressing symbols in mathematical models that have been created in the problem | b. Expressing symbols in mathematical models that have been created in the problem |
|                                | c. Explain the meaning of mathematical models that have been created in the problem | c. Explain the meaning of mathematical models that have been created in the problem |
|                                | d. Choosing the right method of representation of the symbol chosen in the problem | d. Choosing the right method of representation of the symbol chosen in the problem |
| 3. Carrying out the plan of problem solving | a. Use the method chosen to solve the problem | a. Use the method chosen to solve the problem |
|                                | b. Use mathematical models to solve problems | b. Use mathematical models to solve problems |
|                                | c. Use symbols correctly in every step of problem-solving | c. Use symbols correctly in every step of problem-solving |
|                                | d. Explain the relationship between symbols used in solving problems | d. Explain the relationship between symbols used in solving problems |
| 4. Looking back of problem solving | a. Proving the suitability of symbols used during the implementation of problem-solving procedures | a. Proving the suitability of symbols used during the implementation of problem-solving procedures |
|                                | b. Explaining the symbols used will have different meanings in different problems | b. Explaining the symbols used will have different meanings in different problems |

3. Results and discussion
Symbol sense in this study is a person's sensibility to determine work strategies and the ability to use symbols in solving problems. Symbols of students' sense in solving algebra problems by using steps according to Polya which include understanding the problem, devising a plan, carrying out the plan, and looking back at the problem solving has been obtained.

The problems given to students are as follows.
A rectangle has a length of 20 cm and a width of b cm. If the width is reduced by 4 cm, then the area becomes 160 cm². Determine the width and width of the initial rectangle!

Based on the results of tests and interviews, researchers describe the results of the study from representatives of each semester. These student representatives were chosen because they were able to solve the algebraic problems given correctly and had different completion steps and were able to communicate well. S1 represents subjects from semester one, S2 represents subjects from semester three and S3 represents subjects from semester five. The following is a description of the results of research conducted by the three students.

3.1 Step One: Understanding the problem

Symbol sense that will be described at this step is the sensibility or ability of students to mention symbols that can be used in problem-solving such as variables, coefficients, constants or signs, associating symbols with problems, identifying the meaning of symbols in problems and writing symbols according to their meaning in problems. Figure 1 is the result of S1 work in the step of understanding the problem.

Figure 1. S1 Work Results in Understanding Problems

Based on Figure 1, it can be seen that subject S1 only writes symbols that are known to the problem. S1 subjects write the symbol \( p \) to express the length of the rectangle and write the symbol \( l \) as the width of the rectangle. To find out the meaning of the symbols written by the subject, interviews were conducted as follows.

P : Do you understand this problem?
S1 : Yes, understand.
P : From this problem, can you find symbols like variables, coefficients, constants or signs (algebraic operations) that you can use to solve the problem? Try to mention it!
S1 : Yes I can. The variables that I use are \( p \) and \( l \).
P : What information can you use to find the symbol?
S1 : A rectangle has a length of 20 cm and a width of \( b \) cm. So, for the length, I suppose with the variable \( p \) and for the width, I say with the variable \( l \).
P : Is there any other information that you can use to find the symbols?
S1 : Yes, mom. But I am confused to write it.

Based on the interview passage on the subject of S1 at the understanding of the problem step, S1 can mention the symbol used to solve, identify the meaning of the symbol and write the symbol according to its meaning in the problem given in the initial known information. However, subject S1 has difficulty in mentioning symbols for other information contained in the problem given. S1 subjects understand other information using words not using symbols. Thus, S1 subjects are less able to associate symbols with problems.

Furthermore, the subject S2 in the step of understanding the problem has fulfilled all four indicator symbol sense. Figure 2. is the result of S2 work in understanding the problem.
Based on Figure 2, subject S2 has written symbols $p_1$, $p_2$, $l_1$, $l_2$, and $L_2$ to solve the problem. Subject S2 writes the symbol $p_1$ to express length and $l_1$ to express width. Then, subject S2 writes the symbol $l_2$ to express the width of the rectangle if the width that has been known reduce by 4 and writes the symbol $p_2$ expressed as length. Next, write the symbol $L_2$ to express the area of the rectangle with the known width minus 4. S2 subjects use words when writing what is asked not to use predetermined symbols. Based on the written answers, an interview was conducted to find out the symbol sense in understanding the problem.

P : Do you understand this problem?
S2 : Yes, understand. So, this problem is like there are two rectangles.
P : From this problem, can you find symbols like variables, coefficients, constants or signs (algebraic operations) that you can use to solve the problem? Try to mention it!
S2 : Yes, I can. The variables I use are $p$ and $l$.
P : What information can you use to find the symbol?
S2 : A rectangle has a length of 20 cm and a width of $b$ cm. So, for the first rectangle, I suppose the length with variable $p_1$ and for the width, I suppose with the variable $l_1$. Then for the second rectangle, I suppose length with variable $p_2$ and for the width, I suppose with variable $l_2$. Now, for the area of the rectangle, 160 cm$^2$ if the width is reduced by 4 cm. Means $L_2=160$ cm$^2$.
P : Is there any other information that you can use to find the symbols?
S2 : Yes, mom, to find the initial width and initial area.
P : Where is the symbol?
S2 : Oh yes mom, it should mean using the symbols $l_1$ and $L_1$ for this question.

Based on the interview passage, it appears that the subject S2 has fulfilled all the indicators of sense symbol at the step of understanding the problem. Even though the subject S2 in understanding other information did not use symbols, the subject S2 clarified during the interview that the information should be stated using symbols. Thus, subject S2 can associate symbols with problems.

Furthermore, the S3 sense symbol in the step of understanding the problem only fulfills indicators 1.a, 1.b and 1.c. Figure 3 shows the work results of the S3 subject in the step of understanding the problem.
Subject S3 in Figure 3 appears that in understanding the problem by making a drawing in advance of the given problem. Subject S3 writes the symbol \( p \) to express the length and width expressed by the symbol \( l \). Then subject S3 rewrites the symbol \( l \) with a different definition that is stated with \( b \) minus 4. Furthermore, subject S3 writes the symbol \( L \) to express the area of the rectangle. Besides, subject S3 writes the symbols \( b \) and initial \( L \). Based on the answers written in Figure 3, an interview was conducted to uncover the S3 sense symbol subject in the step of understanding the problem.

P : Do you understand this problem?
S3 : Yes, understand.
P : Can you explain again what you understand?
S3 : So there is a rectangle that has a length of 20 cm and a width of \( b \) cm. Then the width is reduced by 4 cm. So the width that was known reduced by 4 cm. Asked \( b \) and the initial area of rectangle.
P : From this problem, can you find symbols like variables, coefficients, constants or signs (algebraic operations) that you can use to solve the problem? Try to mention it!
S3 : Yes, I can. The variables I use are \( p, l, b, L \) and \( L_{\text{initial}} \).
P : What information can you use to find the symbol?
S3 : A rectangle has a length of 20 cm and a width of \( b \) cm. So, the variable \( p \) shows the length of the rectangle that is 20 cm. Then \( l \) as width. In that case, the value of \( l \) is unknown, so I still write it as \( b \). If the width is reduced by 4 cm the width becomes 160 cm\(^2\). So here I suppose the variable \( l \) as its width with \( b \) minus 4.
P : So the \( l \) here (while pointing to the equation \( l = b \) cm) with the \( l \) here (while pointing to the equation \( l = b - 4 \) cm) means the same?
S3 : Different.
P : So it's different, right? Can you explain the meaning?
S3 : Yes, different, mom. Oh yeah, it should be like this, Mom \( L_{\text{initial}} = b \) cm is the initial width and 
\[
l_{\text{after}} = b - 4 \text{ cm}
\]
which will be 160 cm\(^2\).
P : Is there any other information that you can use to find the symbols?
S3 : Yes, mom. From the sentence of the question. in this question, we ask to find the initial width and initial area, so I symbolize the initial width to \( b \) and initial area to \( L_{\text{initial}} \).

Based on the interview passage, the subject S3 can mention the symbol used to solve the problem, associate the symbol with the problem and identify the meaning of the symbol in the problem. However, the subject S3 in writing symbols does not correspond to their meaning in the problem given in the initial known information. Subject S3 in writing symbols to define the width of the rectangle is only fixated on one symbol that is using the symbol \( l \), whereas the use of symbols should be different because it has a different meaning.

Based on the analysis of the results of work and interviews at the step of understanding the problems of the students in the first semester are less able to associate symbols with problems. Students do not know the meaning of symbols used is the cause. Then fifth-semester students use the
same symbol for different meanings. This is following Thompson, et al. [6] which states that when students do not understand symbols well, students have difficulty in interpreting symbols.

Kanakasaby and Naidoo [7] also said that students trying to interpret information on a problem as a variable require broader algebraic knowledge. Lack of understanding of the concept of algebraic letters and symbol manipulation can affect students' symbol sense. This is supported by Skemp [8], Kastberg [9] that if students understand a concept, students can identify and solve problems. Students understand the problem given by writing down what is known and what is asked. Following the opinion of Polya [10] that at the step of understanding the problem students must understand, what is not known, what is already known, give a picture of the problem if there are pictures that are related to the problem and show the unknown part, and use the notation corresponding.

3.2 Second Step: Devising a plan of problem solving

Symbol sense that will be described at the step of device a plan of problem-solving is the sensibility of students in choosing the right symbols to solve problems, stating symbols that have been made in mathematical models in problems, explaining the meaning of mathematical models that have been made in problems and choosing the right method of representation of symbols chosen in problems. Devising a plan of problem-solving steps can be revealed through the interview process. The following are excerpts of interviews with S1 subjects in planning algebra problem-solving.

P : Why do you use the symbols $p$ and $l$?
S1 : Because it's nice to use $p$ and $l$ and it'll be easy to find the area.
P : Can you replace this variable with another? Explain the reason!
S1 : Yes, Mom, but if I use other variables, I find it difficult to find the area, because the rectangular area formula is length times width.
P : Please show me for example?
S1 : Can use variables $k$ and $l$, and must be different from the existing variables in the problem so as not to get confused when working on the problem.
P : What is your strategy/way to solve this problem?
S1 : Find the width of the rectangle that is known to be 160 cm$^2$. Then after knowing the width, I add 4, because the area of 160 cm$^2$ is the area of the rectangle if the width is reduced by 4. So, the initial width can be known. Then look for an initial area with an added width of 4.

Based on the interview excerpt, S1 subjects fulfill indicators 2.a, 2.c and 2.d at the step of devising a plan of problem-solving. S1 subject has difficulty in expressing other information that is known to use symbols that have been determined into the mathematical model. So, subject S1 does not fulfill indicator 2.b.

Then, subject S2 fulfills all the indicators at the step of devising a plan of problem-solving. The following are the results of interviews with S2 subjects.

P : Why do you use the symbols $p_1$, $p_2$, $l_1$, $l_2$, and $L_2$?
S2 : Because previously it was known that there were two rectangles, so for the first rectangle, I used $p_1$ and $l_1$ and for the second rectangle, I used $p_2$, $l_2$, and $L_2$. I adjust these symbols to the rectangular formula $L = p \times l$.
P : Can you replace this variable with another? Explain the reason!
S2 : Yes, I can, Mom
P : Please show me for example?
S2 : Can use variables $a_1$ and $b_1$.
P : What is your strategy/way to solve this problem?
S2 : First I wrote the formula to find the area of a rectangle that is length times width. Then I look for the area of the second rectangle because the length and area are already known, so I can find the width. I can't use the first rectangle because it's only known for its length, while the width and area are unknown. So, look for the width of the second rectangle to find the width of the first rectangle. Then you can find the initial area with the added width 4.
Based on the interview passage S2 subject at the step of devising a plan of problem solving, choose the symbols symbols $p_1$, $p_2$, $l_1$, $l_2$, and $L_2$ to solve the problem. Then, the subject of S2 has no difficulty in writing other information contained in the problem using the symbols that have been determined into mathematical mathematics. Besides, Subject S2 explains the problem-solving plan by using predetermined symbols. So that the subject of S2 has fulfilled all the indicators of sense symbol at the step of planning problem-solving.

Furthermore, the subject S3 fulfills all the indicator symbol sense in devising plan of problem-solving. The following are the results of interviews with S3 subjects.

P: Why do you use the symbols $p$ and $l$?
S3: Because the length is usually symbolized by small $p$ and width with small $l$.

P: Can you replace this variable with another? Explain the reason!
S3: It could be, Mom. I can use another variable.

P: Please show me for example?
S3: Can use variables $a$ and $b$, oops (pause and think) ... but $b$ symbolized is the same as the question. So you can use variables $a$ and $c$, variable $a$ as length and $c$ as width.

P: What is your strategy/way to solve this problem?
S3: I use the rectangular area formula which is length times width to find the initial width. Then substitution from what is known, I replace the length with 20 and the width I replace with $b - 4$ to find the width of the rectangle that is known to be 160 cm$^2$. After obtaining $b$ or initial width, I look for the initial area by length times the initial width obtained earlier.

Based on the interview passage at the step of devising a plan of the problem solving subject S3 chooses a symbol to solve the problem, using $p$ and $l$ variables. This is because S3 subjects are accustomed to using these symbols in solving problems. Besides, subject S3 had no difficulty in writing other information contained in the problem using predefined symbols in the mathematical model. S3 subjects can choose the right method from the chosen symbol representation and can explain the problem-solving plan by using predetermined symbols.

Therefore, subject S3 has fulfilled all the symbol sense indicators at the step of planning problem-solving.

Students recall the concept of the rectangular area before devising a plan of problem-solving. This is supported by Musser [11] which states that students must use pre-existing cognitive skills and abilities to find problem-solving. Then, students devise a plan of problem-solving by looking for the relationship between the information provided on the problem and what will be sought from the problem. Polya [10] said that students must look for relationships between existing information and unknown/sought after. Both are needed to find ideas for problem-solving. Fifth-semester students draw pictures of problems to make it easier to solve problems. In line with Polya [10] which says that making diagrams or drawings and writing equations/mathematical modeling is a strategy for devising a plan of problem-solving. Furthermore, the use of mathematical symbols is also needed to translate words to fit mathematical problems [11].

3.3 Third Step: Carrying out the plan of the problem solving

In carrying out the plan of the problem-solving, the symbol sense that will be considered is the sensibility/ability of students to use the chosen method, using mathematical models, using symbols appropriately in every step, and the ability to explain the relationships between symbols used to solve problems. The results of the work of S1 subjects in carrying out the plan of the problem-solving can be seen in Table II.

| Num. | Polya's Problem-solving Steps | Symbol Sense Indicators |
|------|-----------------------------|-------------------------|

Table 2. Work Results and S1 Subject Interviews
| Num. | Polya's Problem-solving Steps | Symbol Sense Indicators |
|------|-----------------------------|-------------------------|
| 1.   | Write the rectangular area formula | Subject S1 writes the rectangular area formula by writing the symbol $L = p \times l$. The following are excerpts of interviews with S1 subjects. P: You wrote the symbol $L$, is this symbol different from the predetermined $l$? S1: Yes it's different, Mom. This $L$ symbol is for the area of the rectangle. |
| 2.   | Substitute/replace variables with values that are known to the given problem | Subject S1 substitutes/replaces the symbol $L$ with a value of 160, the symbol $p$ with a value of 20 and there is no change in the symbol $l$. |
| 3.   | Determine the value of $l$ | Subject S1 determines the value of $l$ by changing the form of the equation of the left-hand segment to $l$ and the right-hand segment into a constant 160/20 and obtains the value of $l = 8$ |
| 4.   | Determine the initial width of the rectangle | Subject S1 determines the initial width of the rectangle by adding the value of $l$ obtained by 4. Then subject S1 writes the symbol $l_a$ as the initial width. The following are excerpts of interviews showing the use of the $l_a$ symbol. P: What does the $l_a$ symbol mean here? S1: I use this $l_a$ to represent the initial broad word. P: Will this $l_a$ symbol help you solve problems? S1: Yes, mom. Later I will use to search for the initial area. |
| 5.   | Look for initial area | - Subject S1 determines the initial area of the rectangle by writing the symbol $p \times l_a$. - Subject S1 substitutes/replaces the symbol $p$ with the known value of the problem and the symbol $l_a$ with the value of the results obtained is 12 cm - Subject S1 wrote the symbol $L_a$ as the initial area in question. The following are excerpts from researchers' interviews with S1 subjects in determining the $L_a$ symbol. P: You write the $L_a$ symbol, what's the difference with the $l_a$ symbol? Explain! S1: Different ma'am, if I use this $L_a$ to determine the initial area, while for the symbol $l_a$ is the initial width. |
| 6.   | Make conclusions about problem solving from carrying out a problem solving plan | Subject S1 writes the answer to the problem given. |
Based on the results of the work of S1 subjects and interviews it can be seen that S1 subjects can carry out the plan of problem-solving by using selected methods, using mathematical models, using symbols appropriately in each step, and being able to explain the relationships between symbols used to solve problems. However, the subject of S1 in carrying out the plan of the problem-solving raises/creates several new symbols such as \( L_a \) and \( L_a \). So that the subject S1 fulfills all the symbol sense indicators at the step of carrying out the plan of the problem-solving.

Then the subject S2 fulfills all the indicator symbol sense in carrying out the plan of problem-solving. The results of the work and interviews of the subject S2 in carrying out the plan of problem-solving can be seen in Table 3.

**Table 3. Work Results and S2 Subject Interviews**

| Num. | Polya's Problem-solving Steps | Symbol Sense Indicators |
|------|-------------------------------|-------------------------|
| 1.   | Write the rectangular area formula | Subject S2 writes the rectangular area formula by writing the symbol \( L = p \times l \). |
| 2.   | Substitute/replace variables with values that are known to the given problem | Subject S2 substitutes/replaces the symbol \( L_2 \) with a value of 160, the symbol \( p_2 \) with a value of 20 and \( l_2 \) with a value of \( b - 4 \). |
| 3.   | Determine the value of \( b \) which is the initial width of the rectangle | Subject S2 determines the value of \( b \) by using the distributive property of 2, the reduction is continued by changing the form of the equation of the left-hand segment to \( b \) and the right-hand segment into the \( 240/20 \) constant by using the division property with the same number and obtaining the value of \( b = 12 \) cm. |
| 4.   | Determine the initial area of the rectangle | - Subject S2 determines the initial area of the rectangle by writing the symbol \( L_2 = p_2 \times l_2 \).  
- Subject S2 substitutes/replaces the symbol \( p_1 \) with the known value of the problem that is 20 and the symbol \( l_2 \) with the value of the results obtained is 12 cm  
- Subject S2 wrote the results of the answer to the problem given that the initial area was 240 cm\(^2\) |

Based on the results of the work of the subject S2 and the interview in Table III. It appears that the subject S2 can carry out problem-solving plans by using the methods that have been selected, using mathematical models, using symbols appropriately in every step, and being able to explain the relationships between symbols used to solve problems.
Furthermore, subject S3 fulfills all the symbol indicators in carrying out the plan of problem-solving. The results of the work and interviews of S3 subjects in carrying out the plan of problem-solving can be seen in Table 4.

### Table 4. Work Results and S2 Subject Interviews

| Num. | Polya's Problem-solving Steps | Symbol Sense Indicators |
|------|------------------------------|-------------------------|
| 1.   | Write the rectangular area formula | Subject S3 writes the rectangular area formula by writing the symbol \( L = p \times l \). |
| 2.   | Substitute/replace variables with values that are known to the given problem | Subject S3 substitutes/replaces the symbol \( L \) with a value of 160, the symbol \( p \) with a value of 20 and \( l \) with a value of \( b - 4 \). |
| 3.   | Determine the value of \( b \) which is the initial width of the rectangle | Subject S3 determines the value of \( b \) by using the distributive property of the reduction followed by changing the form of the equation of the right segment to \( b \) and the segment left to being 240/20 using the division property with the same number and obtaining the value of \( b = 12 \). |
| 4.   | Conclusion in determining initial width | Subject S3 writes a conclusion from the calculation results looking for the initial width. |
| 5.   | Determine the initial area of the rectangle | - Subject S3 determines the initial area of the rectangle by writing the symbol \( L = p \times l \).  
- Subject S3 substitutes/replaces the symbol \( p \) with the known value of the problem that is 20 and the symbol \( l \) with the value of the results obtained is 12 cm  
- Subject S3 wrote the results of the answer to the problem given that the initial area is 240 cm² |
| 6.   | Conclusions in determining initial area | Subject S3 wrote a conclusion from the calculation results looking for initial area. |

Based on Table IV, it appears that subject S3 can carry out the plan of problem-solving by using the method chosen, using a mathematical model, using symbols appropriately in each step, and being able to explain the relationships between symbols used to solve problems.

Based on the description of the results of work and interviews, students have carried out a plan to solve the problem that has been planned. Students do not experience difficulties in solving problems given. However, first semester students bring up/make new symbols. The use of this new symbol is because the symbol that has been determined at the beginning of understanding the problem is still lacking. This is following Polya [10] that if in carrying out a strategy or problem-solving plan is not successful then students can look for other clues or place the problem on the problem that has been learned. In this case, students use a new symbol. Symbols have different meanings in different contexts [4]. Students can solve problems well using predetermined symbols. The use of symbolic representations of problems enables students to solve problems well [12].
3.4 Fourth step: Looking back of problem solving

The step of looking back at the problem solving, the symbol sense that will be revealed is to prove the suitability of the symbols used during the implementation of the problem-solving procedure, explaining the symbols used will have different meanings in the problem. The following are excerpts of interviews with S1 subjects in looking back of problem-solving has been obtained.

P : Are the symbols you use from the beginning to the end according to their meaning?
S1 : Yes, mom.
P : Why you sure?
S1 : Because I checked, mom.
P : How do you prove it?
S1 : Yes, that was earlier, mom. I checked from the beginning to the end there was something wrong or not written. Then I read again what is known from the problem and try to substitute the value obtained earlier and it is appropriate.
P : Do the symbols such \( L \), \( l_a \), and \( L_a \) that you write have different meanings?
S1 : It's all different, mom. This symbol \( L \) is for the area of the rectangle, this symbol \( l_a \) is for the initial width and this \( L_a \) is for the area of the initial rectangle that is sought in this problem.

Based on interview passages at the step of looking back of problem solving subject S1 checks the suitability of the symbols used from beginning to end and can explain the symbols used have different meanings. Thus, the subject S1 fulfills all the symbol indicators in checking the problem resolution again.

Then the subject S2 in the fourth step also fulfills all the symbol indicators. The following are the results of interviews with S2 subjects.

P : Are the symbols you use from the beginning to the end according to their meaning?
S2 : Ehmm…Yes, mom.
P : Why you sure?
S2 : Because I checked, mom.
P : How do you prove it?
S2 : I adjust to the problem given earlier. So, I input the value obtained in the formula area \( L = p_2 \times l_2 \) and it is appropriate that if the width is 8 cm and the length is 20 cm then the area is 160 cm². Then try to substitute the initial width value obtained earlier and is appropriate.
P : Do the symbols you write have different meanings?
S2 : It's all different, mom.

Based on the interview passage at the step of looking back at the problem-solving subject S2 checks the suitability of the symbols used from beginning to end and can explain the symbols used will have different meanings. The same thing happened to subject S3. Subject S3 meets all the fourth step indicators. The following are excerpts of the S3 subject interview in looking back at the problem solving has been obtained.

P : Are the symbols you use from the beginning to the end according to their meaning?
S3 : Yes, mom.
P : Why you sure?
S3 : Because I checked, mom.
P : How do you prove it?
S3 : Yes, that was earlier, mom. I check from the beginning to the end for operations, especially in the division. Then I read again what is known from the problem and try to substitute the value obtained earlier and it is appropriate.
P : Do the symbols you write have different meanings?
S3 : It's all different, mom. It's just that I used the wrong symbol for initial width (pointing symbol = \( b \)) and width after (pointing symbol \( l = b - 4 \)), I should distinguish the symbol.

Based on the interview passage at the stage of looking back at the problem solving, subject S3 checks the suitability of the symbols used from beginning to end. S3 subject can explain the symbols
used will have different meanings. However, S3 subjects only realized that there were differences in the meaning of the symbols used during the interview. Thus, the subject of S3 in looking back at the problem solving has been obtained using symbol sense to the full.

The step of looking back at the problem solving has been obtained by the first, third and fifth semester students have fulfilled all the indicators of sense symbol well. Even though at this step, the fifth-semester students only realize that they have used the same symbol for different meanings in solving problems. Then the student is convinced that the results obtained are correct. In line with Polya [10] which states that in looking back at the problem solving has been obtained by the students must further convince themselves that the results obtained are true and correct. Besides, at this step students verify the results that have been obtained if an error occurs while implementing the problem-solving plan.

4. Conclusion

Based on the analysis of the results of work and interviews, the student's sense symbol in solving algebraic problems based on Polya's steps can be concluded that at the step of understanding the problems of the first semester students only fulfill two indicator sense symbols. Students are still lacking in associating symbols with given problems, so students in understanding problems still use words instead of predetermined symbols. Then the third-semester students have fulfilled all the indicator symbol sense. Furthermore, fifth-semester students fulfill all three indicator symbols sense well. However, fifth-semester students are less precise in writing symbols according to their meaning in the problem.

The second step of Polya is devising a plan of problem-solving. All students in this second step can properly fulfill all indicator sense symbols. Then in the step of carrying out the plan of problem-solving, all students can fulfill all the symbol indicators to the maximum. However, first semester students bring up/make some new symbols to solve problems. The student in the first-semester, third-semester, and fifth-semester fulfill all the indicator symbols sense well in looking back at the problem solving have been obtained step.

5. Suggestion

Symbol Sense has an important role in understanding mathematical problems that many use symbols, one of which is the problem of algebra. Therefore, it is expected for educators to know the students' sense symbols and develop them. To practice the students' symbol sense, questions are given that can stimulate students to think more deeply about the use of symbols when students are confronted with problems involving many symbols.

The study of this study is still limited to the sensed symbol in solving algebraic problems in students. For other researchers who are interested in conducting a similar study, it should be examined more deeply about the student's sense symbols from different reviews. The results of this study can also be developed into further research on how to bring up the symbol sense in students, especially for students in the field of science.

References

[1] H. Sugilar, R. Kariadinata, and N. Sobarnigsih, “Spektrum Symbol dan Structure Sense Matematika Siswa Madrasah Tsanawiyah,” KALAMATIKA J. Pendidik. Mat., vol. 4, no. 1, pp. 37–48, 2018.
[2] M. . Samo, “Students’ Perceptions about the Symbols, Letters and Signs in Algebra and How Do These Affect Their Learning of Algebra: A Case Study in a Government Girls Secondary School Karachi,” Int. J. Math. Teach. Learn., 2009.
[3] S. Turşucu, J. Spandaw, and M. J. de Vries, Search for Symbol Sense Behavior: Students in Upper Secondary Education Solving Algebraic Physics Problems. 2018.
[4] A. Arcavi, “Developing and Using Symbol Sense in Mathematics,” Learn. Math., vol. 25, no. 2, pp. 42–47, 2005.
[5] A. Rini, “Proses Berpikir Siswa SMA dalam Menyelesaikan Masalah Program Linear Berdasarkan Tipe Kepribadian Hippocrates-Galenus,” Universitas Negeri Malang, 2017.
[6] Thompson, W. Patrick, R. Cheepurupalli, B. Hardin, C. Lienert, and A. Selden, “Cultivating Symbol Sense in Your Calculus Class,” IM&E Workshop, 2010.

[7] Kanakasaby, K. Sagaren, and Naidoo, “An Investigation of Learners’ Symbol Sense and Interpretation of Letters in Early Algebraic Learning,” University of the Witwatersrand, 2009.

[8] R. Skemp, Relational Understanding and Instrumental Understanding Mathematics Teaching, no. 77. 1979.

[9] S. E. Kastberg, “Understandings Mathematical Concepts: The Case of Logarithmic Function,” University of Georgia, 2002.

[10] G. Polya, How To Solve It. New Jersey: Princeton University Press., 1973.

[11] G. L. Musser, B. E. Peterson, and W. F. Burger, Mathematics for Elementary Teacher A Contemporary Approach. United Stated of America, 2011.

[12] I. Papadopoulos, “Using mobile puzzles to exhibit certain algebraic habits of mind and demonstrate symbol-sense in primary school students .” J. Math. Behav., vol. 53, pp. 210–227, 2018.