Identification of the parallel 3-PRRR manipulator parameters considering the workspace boundaries and the passive orthosis movements

D I Malyshev¹, A V Nozdracheva² and L R Kholoshevskaya¹

¹Belgorod State Technological University named after V. G. Shukhov, 46, Kostyukova str., Belgorod, 308012, Russia
²Gamaleya National Research Center for Epidemiology & Microbiology, 18, Gamalei str., Moscow, 123098, Russia

E-mail: ribgtu@gmail.com

Abstract. The article discusses a system of rehabilitation of the lower extremities based on a passive orthosis in the form of a sequential RRRR mechanism and an active parallel 3-PRRR robot. Effective numerical methods and algorithms were developed and tested that made it possible to determine the minimum geometric parameters of the active parallel mechanism that ensure the movement of a passive orthosis within the working area in accordance with clinical data when simulating walking. The simulation results are presented by visualizing the exported workspaces in the STL format.

1. Introduction

Robotic mechanotherapy is a relatively new direction in the rehabilitation of patients with injuries of the musculoskeletal system. Its basis is the use of special robotic systems to restore the functions of the upper and lower extremities through passive and active movements with feedback. Although robotic devices do not cancel and do not replace traditional physiotherapy exercises, most authors note that robotic mechanotherapy has significant advantages in restoring the skills of individual limb movements and generally walking patients with severe lesions of the neurohumoral regulation and musculoskeletal system [1-3].

Robotic devices currently used for neuro locomotor rehabilitation can be divided into two groups - manipulators and exoskeletons. Manipulators, as a rule, come into contact only with the distal part of the limb, controlling it through a working point (contact point) without affecting certain joints. In severe motor impairment, limb movements follow the attachment point of the manipulator passively, which reduces the effectiveness of rehabilitation. In the case of moderate motor impairment, when the patient’s ability to motor activity is preserved, the manipulators can work in an assistive mode using residual muscle activity.

Recently, in the early rehabilitation after injuries and surgical interventions on the musculoskeletal system, CPM therapy (Continuous Passive Motion) has become widespread. It is a rehabilitation technique associated with the prolonged passive development of various joints of a person. The basis of this technique is the implementation of long-repeating movements in the joints with the use of a specialized robot simulator without the participation of the patient’s own muscle strength. The development of anthropomorphic robotics has led to the creation of an exoskeleton - a device that repeats
human biomechanics and increases its physical strength due to the external frame [4]. Exoskeletons that directly control movements in the joints of the limbs are more appropriate means of rehabilitation, in any case, with severe motor impairment. The advantage of robot therapy is a higher quality of training compared to classical rehabilitation due to their longer duration, the accuracy of repetitive cyclic movements, a constant training program, the availability of tools to evaluate the success of the classes with the possibility of demonstration to the patient [5].

2. Formulation of the problem
The conceptual design of the system for the rehabilitation of the lower extremities is shown in Figure 1. The device consists of two mechanisms: a passive orthosis and an active parallel robot. The active 3-PRRR parallel robot proposed by Kong and Gosslen [6] and also known as Isoglyde has three degrees of freedom - translational movement along each axis. The mechanism consists of three kinematic chains $A_i B_i C_i D_i$. The position of the rehabilitation platform, which is an equilateral triangle $D_1 D_2 D_3$ centered at point $P$ and the radius of the circumscribed circle $R$, is determined by linear displacements $q = (q_1, q_2, q_3)$. We introduce the following notation: $a_i$ is the distance between points $A_i$ and $B_i$, $b_i$ is between $B_i$ and $C_i$, $c_i$ is between $C_i$ and $D_i$, $d_i$ is between $B_i$ and $D_i$. The RRRR mechanism is used to secure a person’s legs. Its hinges correspond to the patient’s joints: Two rotational hinges $E$ - to the patient’s hip joint, its angle of rotation relative to the horizontal axis $X$ is denoted $\alpha$, relative to the vertical axis $Z - \psi$, the rotational hinge $F$ with the angle of rotation $\gamma$ relative to the link $EF$ and $\beta$ relative to the horizontal axis $Y$ corresponds to the knee joint.

![Figure 1. 3D model and design diagram of the active mechanism.](image)

The process of rehabilitation [7] requires providing the following angles in the hip, knee and ankle joints while simulating walking: 1 step phase - flexion in the hip joint from $0^\circ$ to $20^\circ$, flexion in the knee joint from $0^\circ$ to $60^\circ$, extension in the ankle joint from $0^\circ$ to $10^\circ$; 2 phase - $0^\circ$, $0^\circ$ and $90^\circ$ respectively; 3 phase - extension in the hip joint from $0^\circ$ to $10^\circ$, in the knee joint $0^\circ$, flexion in the ankle joint from $0^\circ$ to $20^\circ$. Also, during rehabilitation, it is necessary to provide leg abduction in the hip joint by $25^\circ$. Flexion and extension in the ankle joint are provided by an additional rotary joint with a drive located at point P. This movement does not affect the dimensions of the active robot. We define the minimum geometric dimensions of the links of the parallel mechanism to ensure the movement of the passive orthosis within the working area in accordance with clinical data when simulating walking.

3. A mathematical model of a rehabilitation system.
Consider the basic kinematic relationships of manipulators. As the output link, we consider the point P, which is located in the center of the circumscribed circle of the moving platform of the active parallel
robot. It coincides with the center of the rotational hinge of the ankle joint P in the passive mechanism. The position of the output link of the active manipulator is determined from the direct kinematics problem in the form

\[
\begin{align*}
    x_p &= q_1 - \frac{\sqrt{7}}{2} R \\
y_p &= q_2 - \frac{R}{2} \\
z_p &= q_3
\end{align*}
\]  

(1)

In this case, the coordinates of the points \( D_i \) are defined as:

\[
D_1 = \begin{bmatrix}
x_p + \frac{\sqrt{7}}{2} R \\
y_p + \frac{R}{2} \\
z_p
\end{bmatrix}, \quad D_2 = \begin{bmatrix}
x_p - \frac{\sqrt{7}}{2} R \\
y_p + \frac{R}{2} \\
z_p
\end{bmatrix}, \quad D_3 = \begin{bmatrix}
x_p \\
y_p - R \\
z_p
\end{bmatrix}
\]  

(2)

When analyzing the working area, structural limitations must be considered. For the robot under consideration, we distinguish the following:

- restrictions on drive coordinates \( q_i \): \( q_i \in [q_{\min}, q_{\max}] \):
  \[
  q_1 = x_{D1} = x_p + \frac{\sqrt{7}}{2} R, \\
  q_2 = y_{D2} = y_p + \frac{R}{2}, \\
  q_3 = z_{D3} = z_p
\]  

(3)

(4)

(5)

- restrictions on the angles of rotation in the hinges of \( C_i \) and \( B_i \): \( \varphi_i \in [\varphi_{\min}, \varphi_{\max}] \), \( \theta_i \in [\theta_{\min}, \theta_{\max}] \):
  \[
  \varphi_i = \cos^{-1}\left(\frac{b^2 + c^2 - d_i^2}{2bc}\right), \quad \varphi_i \in [0; \pi],
\]  

(6)

where \( d_1 = \sqrt{(y_p + \frac{R}{2} - a)^2 + (z_p)^2} \), \( d_2 = \sqrt{(x_p - \frac{\sqrt{7}}{2} R)^2 + (z_p - a)^2} \), \( d_3 = \sqrt{(x_p - a)^2 + (y_p - R)^2} \),

\[
\begin{align*}
  \theta_1 &= \cos^{-1}\left(\frac{z_p(b^2 - s_3^2 - s_1(y_{D1} - a))}{d_1 b}\right), \quad \theta_1 \in [0; 2\pi], \\
  \theta_2 &= \cos^{-1}\left(\frac{-x_{D2} b^2 - s_2^2 - s_2(x_p - a)}{d_2 b}\right), \quad \theta_2 \in [0; 2\pi], \\
  \theta_3 &= \cos^{-1}\left(\frac{-y_{D3} b^2 - s_3^2 - s_3(x_{D3} - a)}{d_3 b}\right), \quad \theta_3 \in [0; 2\pi],
\end{align*}
\]  

(7)

(8)

(9)

The position of the output link will be obtained from the direct kinematics of the sequential manipulator (passive orthosis) and set in the form

\[
\begin{align*}
    x_p &= x_E + \sin \psi \left( L_{thi} \cos \alpha + L_{crus} \cos \beta \right) \\
y_p &= y_E - \cos \psi \left( L_{thi} \cos \alpha + L_{crus} \cos \beta \right) \\
z_p &= z_E - L_{thi} \sin \alpha - L_{crus} \sin \beta
\end{align*}
\]  

(10)

Replace the variables in equation (10).

\[
    x_{PE} = x_p - x_E, \quad y_{PE} = y_p - y_E, \quad z_{PE} = z_p - z_E.
\]  

(11)

When designing a rehabilitation system in order to ensure its compactness, it is important to determine the minimum possible overall dimensions at which it will perform its functions with the given parameters. In this case, these parameters are the sizes of the range of drive coordinates \( q_i \), lengths of links \( b_i \) and \( c_i \). To calculate the optimal dimensions of the parallel robot, it is necessary to first determine the working area of the RRRR mechanism.

4. Determination of geometric parameters of a parallel mechanism.

Consider the task of determining the optimal parameters of a parallel mechanism that provides the movement of the output link of the RRRR mechanism within a given work area. The proposed approach
consists in checking the set of parallelepipeds that form the shell of the working area of the RRRR mechanism for the entry of the active parallel mechanism into the working area. The first condition for verification is the value of the range of drive coordinates equal to or greater than the overall dimensions of the working area of the passive mechanism

\[
\begin{align*}
Q_{PE,\text{min}} & \leq x_{PE,\text{max}} - x_{PE,\text{min}} \\
Q_{PE,\text{min}} & \leq y_{PE,\text{max}} - y_{PE,\text{min}} \\
Q_{PE,\text{min}} & \leq z_{PE,\text{max}} - z_{PE,\text{min}}
\end{align*}
\]

(12)

In this case, the minimum and maximum values of \(x_{PE}, y_{PE}\) and \(z_{PE}\) are determined using the set of parallelepipeds \(P\) that describe the shell of the workspace.

\[
\begin{align*}
x_{PE,\text{max}} &= \max_{B \in F_j} x_{PE}, \\
x_{PE,\text{min}} &= \min_{B \in F_j} x_{PE}, \\
y_{PE,\text{max}} &= \max_{B \in F_j} y_{PE}, \\
y_{PE,\text{min}} &= \min_{B \in F_j} y_{PE}, \\
z_{PE,\text{max}} &= \max_{B \in F_j} z_{PE}, \\
z_{PE,\text{min}} &= \min_{B \in F_j} z_{PE}.
\end{align*}
\]

(13-15)

In the event that conditions (12) are not satisfied, the active parallel working area is guaranteed to be insufficient to cover the entire set of positions of the output link during rehabilitation.

Using (13-15), we calculate the coordinates of point \(E\).

\[
\begin{align*}
x_E &= \frac{Q_1 + Q_2}{2} - \frac{x_{PE,\text{max}} + x_{PE,\text{min}}}{2} + x', \\
y_E &= \frac{Q_2 + Q_3}{2} - \frac{y_{PE,\text{max}} + y_{PE,\text{min}}}{2} + y', \\
z_E &= \frac{Q_3 + Q_4}{2} - \frac{z_{PE,\text{max}} + z_{PE,\text{min}}}{2} + z',
\end{align*}
\]

(16-18)

where \(x', y', z'\) are additional offsets along each axis. For values \(x' = y' = z' = 0\), the center of the described parallelepiped of the working area of the RRRR mechanism coincides with the center of the parallelepiped of the \(Q\) ranges of the active mechanism.

When changing the values \(x', y', z'\), the coordinates of the point \(E\) change, therefore, the working areas of the mechanisms move relative to each other. This allows you to adjust the relative position of the mechanisms to determine the location at which the working area of the passive mechanism enters the active working area. Moreover, the values \(x', y', z'\) do not exceed the maximum distance from the boundary of the working area of the passive mechanism to the boundaries of the intervals of the driving coordinates of the active mechanism

\[-T_{max} \leq x' \leq T_{max}, -T_{max} \leq y' \leq T_{max}, -T_{max} \leq z' \leq T_{max}, \text{ i.e. } T_{max} = \max_{i \in 1...6} f_i,\]

(19)

where

\[
\begin{align*}
f_1 &= x_{PE,\text{min}} + \frac{Q_1 + Q_2}{2} - \frac{x_{PE,\text{max}} + x_{PE,\text{min}}}{2} - Q_1, \\
f_2 &= y_{PE,\text{min}} + \frac{Q_2 + Q_3}{2} - \frac{y_{PE,\text{max}} + y_{PE,\text{min}}}{2} - Q_2, \\
f_3 &= z_{PE,\text{min}} + \frac{Q_3 + Q_4}{2} - \frac{z_{PE,\text{max}} + z_{PE,\text{min}}}{2} - Q_3, \\
f_4 &= Q_1 - \frac{x_{PE,\text{max}} + (Q_1 + Q_2)}{2} - \frac{x_{PE,\text{max}} + x_{PE,\text{min}}}{2}, \\
f_5 &= Q_2 - \frac{y_{PE,\text{max}} + (Q_2 + Q_3)}{2} - \frac{y_{PE,\text{max}} + y_{PE,\text{min}}}{2}, \\
f_6 &= Q_3 - \frac{z_{PE,\text{max}} + (Q_3 + Q_4)}{2} - \frac{z_{PE,\text{max}} + z_{PE,\text{min}}}{2}.
\end{align*}
\]

For each of the parallelepipeds of the working area of the RRRR mechanism for \(x' = y' = z' = 0\), it is required to calculate the values of geometric parameters that impose design restrictions on the working area of the active 3-PRRR mechanism and check for the occurrence of the calculated intervals in the given intervals of geometric parameters: for the intervals \(X_{PE}, Y_{PE} \text{ and } Z_{PE}\), taking into account
formulas (11), (16) - (19), we define the intervals $X_p, Y_p$ and $Z_p$, for which the intervals $Q'_i$ of the change of drive coordinates are calculated by formulas (3) - (5), the intervals $\Phi'_i$ of the rotation angles in the joints $C_i$ according to the formula (6), the intervals $\Theta'_i$ of the rotation angles in the hinges $B_i$ according to formulas (7) - (9). The system, including the conditions for the occurrence of the calculated intervals in the given, has the form

$$\begin{align*}
Q'_i & \subseteq Q_i \\
\Phi'_i & \subseteq \Phi_i \\
\Theta'_i & \subseteq \Theta_i
\end{align*}$$  \hspace{1cm} (20)

The number $N$ of parallelepipeds for which system (20) is not satisfied determines

If $N > 0$, then the intervals $Q'_i$, $\Phi'_i$, $\Theta'_i$ are calculated for other values $x'_E, y'_E, z'_E$. The task is to determine the minimum ranges $Q_i$, for which $N = 0$.

5. Algorithm Synthesis

To solve the problem of approximating the set of solutions to a system of nonlinear inequalities, an algorithm was synthesized based on the concept of non-uniform coatings [8-12], which works with two lists of four-dimensional parallelepipeds $\mathbb{P}, \mathbb{P}_j$ and a list of three-dimensional parallelepipeds $\mathbb{P}_i$. Each of the dimensions of the parallelepipeds in the list $\mathbb{P}_j$ corresponds to the intervals $X_{PE}, Y_{PE}$ and $Z_{PE}$, the fourth dimension of the lists $\mathbb{P}$ and $\mathbb{P}_j$ corresponds to the interval $B$. The algorithm works as follows:

1. Set the geometric parameters of the mechanisms, the accuracy of the approximation $\delta$, and the initial value of the intervals $Q_i$.
2. Calculate $x_{PE, min}, x_{PE, max}, y_{PE, min}, y_{PE, max}, z_{PE, min}, z_{PE, max}$.
3. If $x_{PE, max} - x_{PE, min} > Q_1$ or $y_{PE, max} - y_{PE, min} > Q_2$ or $z_{PE, max} + z_{PE, min} > Q_2$, then go to step 25.
4. Assign $x'_E = y'_E = z'_E = x'_E, y'_E = y'_E, z'_E = z'_E, 0 = 0$.
5. Calculate $x_E', y_E', z_E'$.
6. Calculate $Q'_i, \Phi'_i$ and $\Theta'_i$ for all $B_i \in \mathbb{P}_j$.
7. Calculate the number $N$ of parallelepipeds for which $Q'_i \not\subseteq Q_i$ or $\Phi'_i \not\subseteq \Phi_i$ or $\Theta'_i \not\subseteq \Theta_i$.
8. Assign $N_0 = N$.
9. If $N_0 = 0$, then the algorithm completes its work.
10. Calculate $T_{max}$.
11. Assign $\Delta x'_E = \Delta y'_E = \Delta z'_E = T_{max}$.
12. Repeat steps 5-7 for each of $j, k$ combinations $x'_E, y'_E$ and $z'_E$, where $x'_E$ take the values $x'_E, 0 = x'_E, 0 + \Delta x'_E, x'_E, 0 + \Delta x'_E, x'_E, 0 + \Delta x'_E$.
13. If $N_j < N_0$ for at least one $j \in 1, k$, then assign $N_0 = N_j, x'_E, 0 = x'_E, j, x'_E, 0 = x'_E, j, x'_E, 0 = x'_E, j$.
14. In other cases, assign $\Delta x''_E = \frac{\Delta x'_E}{2}, \Delta y''_E = \frac{\Delta y'_E}{2}, \Delta z''_E = \frac{\Delta z'_E}{2}$.
15. If $\Delta x''_E > \delta$, then repeat steps 12-15.
16. In other cases, increase $Q_i : Q'_i = Q_i + \delta$ and go to step 3.

The algorithm is implemented in the C++ programming language using the Snowgoose interval analysis library [13]. Visualization of the results of modeling the workspace is carried out by converting the list of three-dimensional parallelepipeds that describe the workspace into a universal format of 3D models - an STL file.

6. Simulation results

A computational experiment was conducted for the following geometric parameters of the mechanism: $A = [-20°, 10°]$, $\Gamma = [-60°, 0°]$, $\Psi = [0°, 25°]$, $B = [-80°, 10°]$, $L_{\text{high}} = L_{\text{crus}} = 450 \text{ mm}$, $a = 100 \text{ mm}$, $q_{\min} = 0 \text{ mm}$, $b_1 = c_1 = \frac{q_{\max} + q_{\min}}{4}$, $b_2 = c_2 = \frac{q_{\max} + q_{\min}}{4}$, $b_3 = c_3 = \frac{q_{\max} + q_{\min}}{4}$,
\[ \phi_{\text{min}} = \theta_{\text{min}} = 10^\circ \text{ mm, } \phi_{\text{max}} = \theta_{\text{max}} = 170^\circ. \]

The following simulation results were obtained:

\[ q_{1\text{max}} = 598 \text{ mm, } q_{2\text{max}} = 696 \text{ mm, } q_{3\text{max}} = 1196 \text{ mm, } x_E = 161 \text{ mm, } y_E = 1000 \text{ mm, } z_E = 162 \text{ mm. } \]

Visualization of the results is presented in Figure 2. The calculation time for the approximation accuracy \( \delta = 1 \text{ mm} \) on a personal computer was 7 minutes 54 seconds.

7. Conclusion

In conclusion, it can be noted that for the proposed lower limb rehabilitation system based on a passive orthosis and an active 3-PPRRR parallel robot, effective numerical methods and algorithms were developed and tested that made it possible to determine the minimum geometric parameters of the active parallel mechanism. The calculation time for the approximation accuracy \( \delta = 8 \text{ mm} \) on a personal computer was 7 minutes 54 seconds, on a personal computer was 7 minutes 54 seconds.

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