Radar Imaging based on Orthogonal Matching Pursuit via Sparse Constraint

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Abstract: This paper presents constraint generalized orthogonal matching pursuit (C-gOMP) approach to tackle the problem of clutter mitigation for radar imaging in the compressive sensing context. The generalized orthogonal matching pursuit algorithm (gOMP) in radar imaging process is susceptible to environmental noise interference. Therefore, a particular constraint condition is added to the recovery signal, and the noise component is restrained after the greedy iteration. The C-gOMP algorithm uses the cost function to impose more constraints on each coefficient to ensure the convergence of the whole function. To further analyze the performance of C-gOMP algorithm, the effectiveness of this algorithm is demonstrated by simulations. The results show that the proposed approach is very obvious at suppressing unwanted clutter and enhancing the desired targets, and C-gOMP algorithm brings an evident performance improvement and application prospect.

1. INTRODUCTION
The resolution of traditional synthetic aperture radar depends on the signal bandwidth, and the data sampling is limited by the Nyquist sampling rate, which is difficult to meet the high sampling rate requirement. Compressed sensing (CS) theory is an emerging signal compression sampling technique. This theory breaks through the limitations of the Nyquist sampling theorem. Orthogonal matching tracking (OMP) proposed by Joel A et al. has been widely used in radar imaging [1-3, 6]. Compared with the matching pursuit algorithm (MP), the improvement of the OMP algorithm is that orthogonalization is performed on all selected atoms at each step of the decomposition [4,5]. However, the OMP algorithm is susceptible to noise in practice. Fortunately, a large number of OMP improvement algorithms have been proposed. The generalized orthogonal matching pursuit algorithm(gOMP) proposed by H.SUN et al. is a generalization algorithm of the OMP algorithm [7, 8]. The gOMP algorithm directly and chooses S atoms which are most correlated with the residual to iterate, thus speeding up the convergence rate of the algorithm [9]. However, in free-space radar imaging the strong clutter dominate the backscattered radar targets and mask the targets in the formed image. The noise will cause a significant error in the reconstructed signal, and may also cause false alarm detection, thus, prior to image formation, mitigating the noise is vital for target detection and identification. Aiming at the above problems, an improved generalized orthogonal matching pursuit reconstruction algorithm is proposed. This algorithm can effectively suppress the environmental noise and realize the radar to image the target accurately. The reconstruction of the algorithm is verified and analyzed by simulation experiments.
2. SIGNAL MODEL
Compressed sensing radar imaging is a new method for radar imaging based on the finite number of measurements. In actual radar detection, most application scenarios satisfy sparsity. The composition of the scene target matrix are as follows:

\[
X = \begin{pmatrix}
x(1, 1) & \cdots & x(1, M) \\
\vdots & \ddots & \vdots \\
x(N, 1) & \cdots & x(N, M)
\end{pmatrix}
\]  
(1)

The two-dimensional reflection coefficient matrix is concatenated into a one-dimensional matrix:

\[
X = [x(1, 1), \ldots, x(N, 1), \ldots, x(1, M), \ldots, x(N, M)]^T.
\]

The transmitted signal is a chirp signal, the process of constructing an over-complete dictionary as follows.

\[
Y_1(t) = \exp(2\pi(f_0 t + \frac{\beta}{2} t^2))
\]
(2)

\(\beta\) is the impulse width, \(B\) is the signal bandwidth, \(f_0\) is the chirp signal center frequency, the distance between the target and the antenna is \(R\). Then the echo signal expression for the entire scene is.

\[
Y_{(m,n)} = \sum_{i=1}^{\eta} X_i \times \exp\left(\frac{4\pi B R_{(m,n)}}{c B}(t - \tau) + \frac{2 R_{(m,n)}}{c}(2\pi f_0 - \frac{2\pi B R_{(m,n)}}{c B})\right)
\]
(3)

\(X_i\) is the reflection coefficient of the \(i\)-th grid, \(\tau\) is the time delay of the electromagnetic wave, and \(R_{(m,n)}\) is the distance between the \((m, n)\) grid point and the signal source. Definition.

\[
a_{(m,n,i)} = \exp\left(\frac{4\pi B R_{(m,n)}}{c B}(t_i - \tau) + \frac{2 R_{(m,n)}}{c}(2\pi f_0 - \frac{2\pi B R_{(m,n)}}{c B})\right)
\]
(4)

\(t_i\) is the number of fast time sampling points, the over-complete dictionary: \(a(m, n) = [a_{(m,n,1)}, a_{(m,n,2)}, \ldots, a_{(m,n,MN)}]\), We can get the over-complete dictionary.

\[
\Phi = [a(1, 1), \ldots, a(1, N), \ldots, a(M, 1), \ldots, a(M, N)]^T
\]
(5)

\(n\) is the noise interference, the compression-sensing radar imaging expression is:

\[
Y = \Phi X + n
\]
(6)

3. GENERALIZED ORTHOGONAL MATCHING PURSUIT BASED ON CONSTRAINT
A generalized orthogonal matching pursuit algorithm based on constraints (C-gOMP) is proposed for in the paper. After the greedy iteration, the algorithm uses the cost function to impose more constraints on each coefficient to ensure the convergence of the whole function, that is, the noise component is deeply constrained in the reconstruction process. Compared with the traditional gOMP algorithm, the algorithm has apparent advantages in anti-noise performance. The C-gOMP core algorithm flow is as follows:

**Algorithm** constraint generalized orthogonal matching pursuit (C-gOMP)

Explanation of symbols,

\(\emptyset\): indicates an empty set; \(\text{res}\): residual value generated by each iteration;

\(\text{iter}\) indicates the number of iterations; \(\varepsilon\) is microscale, indicating the accuracy of reconstruction.

**Input:** dictionary matrix \(\Phi\), measurement matrix \(Y\)

**Initialization:** The initialization residual \(r_0 = y, A_t = \emptyset, \Lambda_t = \emptyset, t = 0\), setting parameters \(\rho_1, \rho_2, \lambda\) and initial weight \(\omega_0\).

**While:** \(t < \text{iter} + 1\ or \ \text{res}_t \leq \varepsilon\ do\)

\(\text{calculate } u_t = (\text{res}_{t-1}, \Phi_i)\), then select the largest \(S\) values in \(u_t\) and add them to index set \(i\), update the support set \(A_t = A_{t-1} \cup \{i\}; \Lambda_t = \Lambda_{t-1} \cup \{\Phi_i\};\)

Orthogonal projection operator: \(\widetilde{\Theta}_t = \arg \min_{\Theta} \|Y - A_t \cdot \Theta\| = A_t \cdot \left[(A_t)^T \cdot A_t\right]^{-1} \cdot (A_t)^T \cdot Y.

Calculate new residual \(\text{res}_t = Y - A_t \cdot \widetilde{\Theta}_t;\)

\(t = t + 1;\)

**End while**
We can get $\theta = [\theta_1, \theta_2, \cdots, \theta_N]$, $\eta = \begin{bmatrix} \theta_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \theta_t \end{bmatrix}$. $\eta_j$ is the column $j$ of $\eta$.

\begin{itemize}
  \item While: $j < N + 1$
  \item further constrain the solution model: $x_j = \left( \frac{\lambda \| \eta_j \cdot \omega_j \|_1 + (1 - \lambda) \| y - \Phi \eta_j \|_2}{\| \eta_j \|_1 + \rho_2} \right)$
  \item Update weight $\omega_j = \frac{\rho_1 \rho_2}{\| \eta_j \|_1 + \rho_2}$
  \item End while
\end{itemize}

Output: $X = [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_N]$.

### 4. EXPERIMENTAL EVALUATION

In practical applications, large coefficients may correspond to real signals, so small component signals need to be attenuated during the optimization process. Let, $\lambda = \frac{1}{2}$ and the disciplinary value $\rho_1 = \rho_2 = 10^{-5}$, $\lambda$ is mainly used to adjust sparsity constraints and fidelity constraints.

To verify that the two-dimensional distance image model of C-gOMP algorithm has stronger anti-noise performance and higher range resolution, a simulation experiment is deployed to test the performance of C-gOMP. The simulation conditions are as follows:

1. The chirp signal center frequency is $f_0 = 5.6$ GHz, the bandwidth is 1 GHz, and the uncompressed impulse width of the signal is $t_u = 1$ ms.
2. The imaging area is set to a closed geometric space of 6 meters in length and 6 meters in width. There are 9 point targets in the geometric space located at 5 cm to 15 cm in the horizontal and vertical directions.
3. Use 31 transceiver integrated antennas to form an array, evenly distributed them at (-15 cm, 15 cm). And the antenna array is 5 m away from the imaging area.
4. The number of signal observation points is $M = 128$, and the signal sparsity degree is $K = 9$.

The gOMP algorithm and the C-gOMP algorithm are imaged under noiseless conditions and SNR=0dB. Figure 1(a) shows that both the gOMP algorithm and the C-gOMP algorithm can achieve radar imaging in the no-noise environment. However, there is a false target generated at (2m, 2m) imaged by the gOMP algorithm, and the reconstructed real target is slightly blurred. Since the noise will destroy the sparsity in the radar environment, it will affect the reconstruction accuracy of the target. In Figure 1(b), in the noise environment with SNR=0dB, the radar image generated by the gOMP algorithm is corrupted by noise, causing several false points to appear. On the contrary, although the amplitude value of the reconstruction target of the C-gOMP algorithm is partially lost, it has obtained high definition imaging results.

Fig1. Radar imaging results of the gOMP and C-gOMP, $M = 128$, $K = 9$, $\rho_1 = \rho_2 = 10^{-5}$, $\lambda = \frac{1}{2}$ (a) Radar imaging in noiseless condition. (b) Radar imaging with SNR=0dB.
5. CONCLUSION
The C-gOMP algorithm proposed in this paper is an improvement on the traditional gOMP algorithm. In practical application, the large coefficient may be a real signal, so after the greedy iteration, a particular constraint condition is added to the signal, and the noise component is restrained. Simulation and experimental results show that the imaging algorithm proposed in this paper can accurately recover the target position in the case of strong environmental noise.

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