ON THE P-ADIC LOCAL INVARIANT CYCLE THEOREM

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ABSTRACT. For a proper, flat, generically smooth scheme $X$ over a complete DVR with finite residue field of characteristic $p$, we define a specialization morphism from the rigid cohomology of the geometric special fibre to $D_{crys}$ of the $p$-adic étale cohomology of the geometric generic fibre, and we make a conjecture ("$p$-adic local invariant cycle theorem") that describes the behavior of this map for regular $X$, analogous to the situation in $l$-adic étale cohomology for $l \neq p$. Our main result is that, if $X$ has semistable reduction, this specialization map induces an isomorphism on the slope $[0,1)$-part.

1. Introduction

1.1. Let $R$ be a complete discrete valuation ring with quotient field $K$ and finite residue field $k$ of characteristic $p$. We choose a separable closure $\bar{K}$ of $K$ (resp. $\bar{k}$ of $k$) and set $S = \text{Spec}(R)$, $\eta = \text{Spec}(K)$, $s = \text{Spec}(k)$, $\bar{\eta} = \text{Spec}(\bar{K})$ and $\bar{s} = \text{Spec}(\bar{k})$. We denote by $I \subseteq G := \text{Gal}(\bar{K}/K)$ the inertia subgroup, by $W(k)$ the ring of Witt vectors of $k$, and by $K_0$ (resp. $\bar{K}_0^{ur}$) the fraction field of $W(k)$ (resp. $W(\bar{k})$).

For any scheme $X$ over $S$, we have a geometric special fiber $X_{\bar{s}}$ over $\bar{k}$ and a geometric generic fiber $X_{\bar{\eta}}$ over $\bar{K}$, i.e. a commutative diagram

$$
\begin{array}{ccc}
X_{\bar{s}} & \longrightarrow & X \\
\downarrow & & \downarrow \\
\bar{s} & \longrightarrow & S
\end{array}
\quad \quad \quad
\begin{array}{ccc}
& & X_{\bar{\eta}} \\
& & \downarrow \\
& & \bar{\eta}
\end{array}
$$

where the squares are Cartesian. If $X \to S$ is moreover proper and flat one has the specialization morphism on $l$-adic étale cohomology groups:

$$
\text{sp} : H^i(X_{\bar{s}}, \mathbb{Q}_l) \to H^i(X_{\bar{\eta}}, \mathbb{Q}_l)^I
$$

which is the composition

$$
H^i(X_{\bar{s}}, \mathbb{Q}_l) \cong H^i(X', \mathbb{Q}_l) \to H^i(X'_{\bar{\eta}}, \mathbb{Q}_l) \to H^i(X_{\bar{\eta}}, \mathbb{Q}_l)^I
$$

where $X'$ is the base change of $X$ to a strict Henselization of $S$ at $\bar{s}$ and the first isomorphism results from proper base change. This map $\text{sp}$ is $G$-equivariant. We note that all Frobenius eigenvalues on both source and target are known to be Weil numbers, so we have a natural increasing weight filtration $W_j$ on both sides.

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**Conjecture 1.1.** ("local invariant cycle theorem") Assume $X$ is regular and $l \neq p$. Then

1. $\text{sp}$ is an epimorphism
2. $\text{sp}$ induces an isomorphism on $W_{i-1}$

Since by [8] one has $W_i H^i(X_s, \mathbb{Q}_l) = H^i(X_\bar{s}, \mathbb{Q}_l)$, Conjecture 1.1 implies that the kernel of $\text{sp}$ is pure of weight $i$. In [8] (3.6), Deligne has proved the local invariant cycle theorem in the equal characteristic case $\text{char}(K) = p$. In the mixed characteristic case $\text{char}(K) = 0$, it is well known that the local invariant cycle theorem is implied by Deligne’s conjecture on the purity of the monodromy filtration (the "monodromy weight conjecture"). We refer to [18] for this implication in the semistable case and to [11] in the general regular case. Thus the local invariant cycle theorem also holds in the mixed characteristic case if $\text{dim}(X_\eta) \leq 2$ by the results of Rapoport and Zink [23] and in many more cases by the recent work of Scholze [24]. Further unconditional, and probably well known, results were recorded by Flach and Morin in [11]: If $X$ is regular and $l \neq p$ then $\text{sp}$ induces an isomorphism on $W_1$ and is an isomorphism for $i = 0, 1$.

1.2. From now on and for the rest of this paper we assume that $K$ has characteristic 0 and that $X \to S$ is proper and flat. For $l = p$, it is then natural to ask about a local invariant cycle theorem for $p$--adic cohomology. In the case where $X \to S$ is smooth it was shown in [12] (4.1) that the map $\text{sp} : H^i(X_s, \mathbb{Q}_p) \to H^i(X_\bar{\eta}, \mathbb{Q}_p)$ is an isomorphism as a consequence of Fontaine’s $C_{\text{crys}}$-comparison isomorphism in $p$-adic Hodge theory proven by Fontaine-Messing [12] and Faltings [10].

However, it is well known that the geometric $p$-adic étale cohomology of varieties over $k$ only describes the slope 0 part of the full $p$-adic (Weil) cohomology, which is Berthelot’s rigid cohomology $H^i_{\text{rig}}(X_s/k)$. To have a context which is fully analogous to the $l$-adic situation we need to construct an enlarged specialization map as following. Here and in the following we refer to [13], [14] for Fontaine’s functors $D_{\text{crys}}, D_{\text{st}}, D_{\text{pst}}, D_{dR}$ and the corresponding period rings.

**Proposition 1.1.** If $X \to S$ is proper, flat and generically smooth, then there is a functorial $\phi$-equivariant map

\[ \text{sp}' : H^i_{\text{rig}}(X_s/k) \to D_{\text{crys}}(H^i(X_\bar{\eta}, \mathbb{Q}_p)) \]

and a commutative diagram of $\text{Gal}(\bar{k}/k)$--modules, we also have isomorphisms

\[ \lambda_s : H^i(X_s, \mathbb{Q}_p) \cong (H^i_{\text{rig}}(X_s/k) \otimes_{K_0} \hat{K}_0^{\text{ur}})^{\phi \otimes \phi = 1} =: H^i_{\text{rig}}(X_s/k)^{\text{slope0}} \]

and

\[ \lambda_\eta : H^i(X_\bar{\eta}, \mathbb{Q}_p)^I \cong (D_{\text{crys}}(H^i(X_\bar{\eta}, \mathbb{Q}_p)) \otimes_{K_0} \hat{K}_0^{\text{ur}})^{\phi \otimes \phi = 1} =: D_{\text{crys}}(H^i(X_\bar{\eta}, \mathbb{Q}_p))^\text{slope0}. \]

In the case where $X$ has semistable reduction the map $\text{sp}'$ is the composite of a map

\[ H^i_{\text{rig}}(X_s/k) \to H^i_{\text{HK}}(X/S) \]
constructed by Chiarellotto in [6], and the $N = 0$-part of Fontaine’s $C_{st}$-comparison isomorphism

$$H^i_{HK}(X/S) \cong D_{st}(H^i(X_\bar{\eta}, \mathbb{Q}_p))$$

proved by Tsuji [26], among others. Here $H^i_{HK}(X/S)$ is the log-crystalline cohomology defined by Hyodo and Kato [16]. In general, we construct $sp'$ by descent from the semistable case using de Jong’s alterations [9]. We believe that the $p$-adic Hodge theory of rigid analytic spaces that is currently being developed by various authors [25], [2] should ultimately give a more sheaf theoretic construction of $sp'$ which then also works if $f$ is only proper and flat. We note that a map

$$H^i_{rig}(X_s/k) \to H^i_{dR}(X_\eta/K) \cong D_{dR}(H^i(X_\bar{\eta}, \mathbb{Q}_p))$$

is more or less immediate from the definition of rigid cohomology as de Rham cohomology of a tube (see [1][Thm. 6.6] for the general proper, flat case where this map is called a cospecialization map).

Now note that the eigenvalues of the $k$-linear Frobenius $\phi^{[k:F_p]}$ on $H^i_{rig}(X_s/k)$ are Weil numbers, and following the argument of [11](section 10) in the $l$-adic case one proves that the same is true for $D_{pst}(H^i(X_\bar{\eta}, \mathbb{Q}_p))$. Hence one deduces $\phi$-stable weight filtrations $W_j$ on both $H^i_{rig}(X_s/k)$ and

$$D_{crys}(H^i(X_\bar{\eta}, \mathbb{Q}_p)) = D_{st}(H^i(X_\bar{\eta}, \mathbb{Q}_p))^{N=0} = D_{pst}(H^i(X_\bar{\eta}, \mathbb{Q}_p))^{G,N=0}.$$

The full $p$-adic analogue of Conjecture 1.1 (and the results mentioned after it) is then the following conjecture.

**Conjecture 1.2.** ("$p$-adic local invariant cycle theorem") If $X$ is regular then

1. $sp'$ is an epimorphism
2. $sp'$ induces an isomorphism on $W_{i-1}$
3. $sp'$ is an isomorphism for $i = 0, 1$

For semistable $X$ part (1) and (3) were also conjectured by Chiarellotto [6][§4] and proven by him by assuming the $p$-adic monodromy weight conjecture stated in [22][Conj. 3.27]. This conjecture is known if $\text{dim}(X_\eta) \leq 2$ and for $i = 0, 1$ in general. So part (3) holds for semistable $X$. It can also be seen from his paper that $sp'$ is an isomorphism for $i = 0, 1$ in the semistable reduction case.

For general regular $X$ it seems difficult to prove Conjecture 1.2 without a different construction of the map $sp'$ that embeds it into a suitable long exact Clemens-Schmid sequence. It would also be interesting to say something more about the slopes of eigenvalues that occur in the kernel of $sp'$ for regular $X$. Conjecture 1.2 says that such eigenvalues are of weight $i$.

The main result of this article is then the following Theorem.

**Theorem 1.1.** If $X$ has semistable reduction, then the map $sp'$ in (1.1) induces an isomorphism on the slope $[0, 1)$-part, i.e. $H^i(X_s, W\mathcal{O}_{X_s})\cong D_{crys}(H^i(X_\bar{\eta}, \mathbb{Q}_p))^{[0, 1]}$. 

The following conjecture is the generalization of Theorem 1.1.
Conjecture 1.3. If $X$ is regular, the map $sp'$ in (1.1) induces an isomorphism on the slope $[0,1)$-part.

Note that this conjecture implies that: if $X$ is regular then the map $sp' : H^i(X, \mathbb{Q}_p) \to H^i(X, \mathbb{Q}_p')$ is an isomorphism.

Conjecture 1.3 is equivalent to a "compatibility of trace maps", i.e. the commutativity of the following diagram

(1.2)

where $\tau^s_{(0,1)}$ is the trace map defined by Berthelot, Esnault and Rülling in [5], and $\tau_\eta$ is the trace map induced from the lemma 3.4.

This paper is organized as follows. We first give some preliminaries on $\phi$-modules and construct $\lambda_s$, $\lambda_\eta$ in section 2, then we prove Proposition 1.1 and Theorem 1.3 in section 3.

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2. Preliminaries on $\phi$-modules and construction of $\lambda_s$, $\lambda_\eta$

As in the introduction we let $\phi$ be the absolute Frobenius on either $K_0$ or $\hat{K}_0^{ur}$, and a $\phi$-module over either field is a finite dimensional vector space with a bijective $\phi$-semilinear endomorphism $\phi$. The Dieudonné-Manin classification [20] tells us that the category of $\phi$-modules over $\hat{K}_0^{ur}$ is semisimple and that the simple objects

$$E_q = E_{r,s} := \hat{K}_0^{ur} / ((\phi^r - p^s))$$

are parametrized by the set of rational numbers $q = \frac{r}{s} \in \mathbb{Q}$, called slopes in this context. For $q \in \mathbb{Q}$ and a $\phi$-module $D$ over $K_0$, the $q$-isotypical part of $D \otimes_{K_0} \hat{K}_0^{ur}$ descends to $K_0$ and is called the slope $q$-part of $D$. Denote this $\phi$-submodule of $D$ by $D[q]$. It is canonically a direct summand complemented by the sum of $D[q']$ for $q' \neq q$. If $I \subseteq \mathbb{Q}$ is any interval (open, closed or half-open) one defines $D^I$ to be the sum of $D[q]$ for $q \in I$.

For any $\phi$-module $D$

$$V(D) := (D \otimes_{K_0} \hat{K}_0^{ur})^{\phi^0=1}$$

is a finite-dimensional $\mathbb{Q}_p$-vector space with a continuous Gal($\bar{k}/k$)-action, and the functor $V$ gives a Fontaine-style equivalence of categories between such representations and $\phi$-modules $D$ over $K_0$ of slope 0, i.e. such that $D = D[0]$. For any $D$ one has $V(D) = V(D[0])$ and for us it is
more convenient to define the slope 0-part by this formula, i.e. we set
\[ D^{slope_0} := V(D). \]

Of course the Gal($\bar{k}/k$)-action on $V(D)$ again just amounts to a single automorphism Frob$_{\bar{k}} = 1 \otimes \phi^{[k:F_p]} = \phi^{-[k:F_p]} \otimes 1$.

\textbf{Lemma 2.1.} Let $V$ be a finite dimensional $\mathbb{Q}_p$-vector space with a continuous $G := \text{Gal}(\bar{K}/K)$-action, and such that $D_{dR}(V)/\text{Fil}^0 D_{dR}(V) = 0$. Then we have an isomorphism
\[ V^I \simeq D_{crys}(V)^{slope_0}. \]

\textit{Proof.} Let $L$ be the finite extension of $\hat{K}_0^{ur}$ fixed by $I$. Then
\begin{equation}
(2.1) \quad D_{dR,L}(V) = (B_{dR} \otimes_{\mathbb{Q}_p} V)^I \cong D_{dR}(V) \otimes_K L = \text{Fil}^0 D_{dR}(V) \otimes_K L = (B_{dR}^0 \otimes_{\mathbb{Q}_p} V)^I
\end{equation}
by assumption. Moreover $(B_{crys} \otimes_{\mathbb{Q}_p} V)^I \cong D_{crys}(V) \otimes_{K_0} \hat{K}_0^{ur}$ and
\[ (B_{crys}^{\phi=1} \otimes_{\mathbb{Q}_p} V)^I \cong (D_{crys}(V) \otimes_{K_0} \hat{K}_0^{ur})^{\phi \otimes \phi^{-1}} = D_{crys}(V)^{slope_0}. \]

By (2.1) the image of $(B_{crys}^{\phi=1} \otimes_{\mathbb{Q}_p} V)^I$ in $(B_{dR} \otimes_{\mathbb{Q}_p} V)^I$ lies in $(B_{dR}^0 \otimes_{\mathbb{Q}_p} V)^I$, i.e. in $(\text{Fil}^0 B_{crys}^{\phi=1} \otimes_{\mathbb{Q}_p} V)^I$. But by [13] [Thm. 5.3.7] we have $\text{Fil}^0 B_{crys}^{\phi=1} = \mathbb{Q}_p$, hence we obtain $V^I \simeq D_{crys}(V)^{slope_0}$. \hfill \Box

\textbf{Corollary 2.1.} For any variety $X_\eta$ over $\text{Spec}(K)$ one has a functorial isomorphism
\[ \lambda_\eta : H^i(X_\eta, \mathbb{Q}_p)^I \cong D_{crys}(H^i(X_\eta, \mathbb{Q}_p))^{slope_0}. \]

\textit{Proof.} By the $C_{dR}$-isomorphism (see for example [3] for the case of arbitrary $X_\eta$) we have $D_{dR}(H^i(X_\eta, \mathbb{Q}_p)) \cong H^i_{dR}(X_\eta/K)$ and one always has $\text{Fil}^0 H^i_{dR}(X_\eta/K) = H^i_{dR}(X_\eta/K)$. \hfill \Box

For any proper variety $X_s$ over $\text{Spec}(k)$ Berthelot, Bloch and Esnault [4] have defined a functorial morphism of $\phi$-modules
\begin{equation}
(2.2) \quad H^i_{rig}(X_s/k) \rightarrow H^i(X_s, WO_{X_s})_\mathbb{Q}
\end{equation}
where $H^i(X_s, WO_{X_s})_\mathbb{Q} = \lim_n H^i(X_s, W_n O_{X_s}) \otimes \mathbb{Q}$ is Witt vector cohomology, and they show [17] [Thm. 1.1] that this morphism induces an isomorphism
\begin{equation}
(2.3) \quad H^i_{rig}(X_s/k)^{(0,1)} \cong H^i(X_s, WO_{X_s})_\mathbb{Q}.
\end{equation}

\textbf{Lemma 2.2.} For a proper variety $X_s$ over $\text{Spec}(k)$ one has a functorial isomorphism
\[ \lambda_s : H^i(X_s, \mathbb{Q}_p) \cong H^i_{rig}(X_s/k)^{slope_0}. \]

\textit{Proof.} On $X_s_{et}$ one has an Artin-Schreier type short exact sequence [17] [Prop.3.28]
\[ 0 \rightarrow \mathbb{Z}_p \rightarrow WO_{X_s} \xrightarrow{1-\phi} WO_{X_s} \rightarrow 0 \]
whose induced long exact cohomology sequence splits into short exact sequences after tensoring with $\mathbb{Q}$ by \cite[Lemma 5.3]{17}. Hence using the scalar extension of (2.3) to $\hat{K}^ur_0$ we get

$$H^i(X_s, \mathbb{Q}_p) = H^i(X_s, W\mathcal{O}_{X_s})^\phi = (\hat{H}^i_{\text{rig}}(X_s/k))^0 \simeq (H^i_{\text{rig}}(X_s/k) \otimes_{K_0} \hat{K}^ur_0)^{\phi \otimes \phi} = H^i_{\text{rig}}(X_s/k)^{\text{slope}0}.$$

\[\square\]

Combining the above lemmas, we get the isomorphisms $\lambda_s$ and $\lambda_\eta$ in Proposition \[1.1\].

3. Construction of the $p$-adic specialization map

It remains to construct $sp'$. We do this first for semistable $X$ and then use de Jong’s alteration to descend to the general case.

3.1. The semistable case. In this section we assume that $X$ is semistable, i.e. its special fibre $X_s =: Y = \cup_{i \in I} Y_i$ is a reduced, normal crossing divisor with smooth proper irreducible components $Y_i$. As usual, we let

$$Y^{(j)} = \prod_{\{i_1, \ldots, i_j\} \subseteq I} Y_{i_1} \cap \cdots \cap Y_{i_j}$$

be the disjoint union of the intersections of $j$ components, which is a smooth and proper $k$-scheme.

3.1.1. Hyodo-Kato cohomology. The Hyodo-Kato cohomology of $X/S$ (or log-crystalline cohomology of the log smooth morphism $X_s \to \text{Spec}(k)$ where these schemes are endowed with the log-structure induced by the log structures on $X$ and $S$ given by the special fibre) can be computed by the Hyodo-deRham-Witt complex $W\omega_Y^\bullet$.

$$H^i_{HK}(X/S) \cong H^i(Y_{\text{et}}, W\omega_Y^\bullet)_\mathbb{Q}$$

on $Y_{\text{et}}$ which is described in detail in \cite[(1.1), Thm 4.19]{16}. The complex

$$W\omega_Y^\bullet = \lim_{\longleftarrow n} W_n\omega_Y^\bullet$$

is a pro-complex where each $W_n\omega_Y^q$ is a coherent $W_n\mathcal{O}_{Y_{\text{et}}}$-module and by definition one has

$$W_n\omega_Y^q = W_n\mathcal{O}_{Y_{\text{et}}}.$$

The morphism of pro-complexes $W\omega_Y^\bullet \to W\mathcal{O}_{Y_{\text{et}}}[0]$ then induces a map analogous to (2.2)

$$H^i_{HK}(X/S) \to H^i(Y_{\text{et}}, W\mathcal{O}_{Y_{\text{et}}})_\mathbb{Q} \cong H^i(X_s, W\mathcal{O}_{X_s})_\mathbb{Q}$$

and one has the following analogue of (2.3) and of Lemma \[2.2\].
Lemma 3.1. For a semistable scheme \( X/S \) the map (3.3) induces a functorial isomorphism

\[
H^i_{HK}(X/S)[0,1) \simeq H^i(X_s, W_{\mathcal{O}_{X_s}})_{\mathbb{Q}}
\]

and a functorial isomorphism

\[
\tilde{\lambda}_s : H^i(X_s, \mathbb{Q}_p) \cong H^i_{HK}(X/S)^{\text{slope}0}.
\]

Moreover, we have \( H^i_{HK}(X/S)^N = 0,0,1) \simeq H^i_{HK}(X/S)[0,1) \).

Proof. The first statement follows from the degeneration at \( E_1 \) of the slopes spectral sequence

\[
E_1^{qr} = H^r(Y_{et}, W_{\omega_{Y}^q})_{\mathbb{Q}} \Rightarrow H^{r+q}(Y_{et}, W_{\omega_{Y}^q})_{\mathbb{Q}}
\]

proven in [19][Thm. 3.1] (see the remark in loc. cit. after (3.1.1)). The second statement follows from the first by exactly the same proof as that of Lemma 2.2. The third statement is an easy consequence of the relation \( N \phi = p \phi N \) and the fact that \( H^i_{HK}(X/S) \) has no negative slopes. 

3.1.2. The Hyodo-Steenbrink complex. In this section we relate rigid and Hyodo-Kato cohomology of \( X_s \) following [6]. By [22][Cor. 3.17] one has a quasi-isomorphism

\[
W_n \omega_Y^• \cong W_n \mathcal{A}^•
\]

where \( W_n \mathcal{A}^• \) is the simple complex associated to the (Hyodo-Steenbrink) bicomplex \( W_n \mathcal{A}^{i,j} \), \( i, j \geq 0 \), of sheaves in \( Y_{et} \) given by

\[
W_n \mathcal{A}^{i,j} = \frac{W_n \tilde{\omega}_{Y}^{i+j+1}}{P_j W_n \tilde{\omega}_{Y}^{i+j+1}}.
\]

Here \( W_n \tilde{\omega}_{Y}^• \) is a complex defined in [16][(1.4)] which carries a weight filtration \( P_j W_n \tilde{\omega}_{Y}^• \) defined in [22][Sec. 3.5]. Moreover there is a certain global section \( \theta \) of \( W_n \tilde{\omega}_{Y}^1 \) and the isomorphism (3.5) is given by multiplication with \( \theta \) [22][Prop. 3.15]. For example, using (3.2), the projection to the degree 0 part of both complexes in (3.5) gives rise to a commutative diagram

\[
\begin{array}{ccc}
W_n \omega_Y^• & \longrightarrow & W_n \mathcal{A}^• \\
\downarrow & & \downarrow \\
W_n \mathcal{O}_{Y_{et}} & \wedge \theta & W_n \tilde{\omega}_{Y}^1/P_0 W_n \tilde{\omega}_{Y}^1
\end{array}
\]

Let \( \nu_n \) be the endomorphism induced on the simple complex by the endomorphism on \( W_n \mathcal{A}^• \) given by the natural projection \( W_n \mathcal{A}^{i,j} \to W_n \mathcal{A}^{i-1,j+1} \) multiplied with \((-1)^{i+j+1} \). By [22][Prop. 3.18] the inverse limit

\[
\nu = \lim_{\leftarrow} \nu_n
\]

induces the monodromy operator \( N \) in Hyodo-Kato cohomology via (3.5) and (3.1). It is clear that the kernel of \( \nu_n \) on \( W_n \mathcal{A}^• \) is the simple complex associated to the double subcomplex

\[
\frac{P_{j+1} W_n \tilde{\omega}_{Y}^{i+j+1}}{P_j W_n \tilde{\omega}_{Y}^{i+j+1}} \subseteq W_n \mathcal{A}^{i,j}.
\]
On the other hand by \cite{22}(3.7) one has an isomorphism of complexes

\[
\text{Res} : \text{Gr}_j^W W_n \Omega^*_{Y(j)} \to W_n \Omega^*_{Y(j)}(-j)
\]

where \( W_n \Omega^*_{Y(j)} \) is the usual de Rham-Witt complex of \( Y(j) \), thought of as a complex on \( Y_{et} \) via the natural finite morphism \( Y(j) \to Y, (-j) \) is the Tate-shift related to the Frobenius structure \cite{17} and \( \text{Gr}_j = P_j/P_{j-1} \). This leads to the following identification of the kernel of \( \nu \) due to Chiarellotto.

**Proposition 3.1.** \cite{6}, Prop.1.8 The kernel of the operator

\[
\nu_n : W_n A^* \to W_n A^*
\]

is isomorphic to the simple complex associated to the double complex \( W_n \Omega^*_{Y(j)}(\bullet) \)

\[
0 \to W_n \Omega^*_{Y(j)} \to W_n \Omega^*_{Y(j+1)} \to \cdots
\]

on \( Y_{et} \), where \( \rho_j : W_n \Omega^*_{Y(j)} \to W_n \Omega^*_{Y(j+1)} \) is defined by

\[
\rho_j = (-1)^j \sum_{1 \leq i \leq j+1} (-1)^i \delta_i^j
\]

and \( \delta_i : Y(j+1) \to Y(j) \) is the inclusion

\[
Y_{t_1} \cap \cdots \cap Y_{t_j} \hookrightarrow Y_{t_1} \cap \cdots \cap Y_{t_{i-1}} \cap Y_{t_{i+1}} \cap \cdots \cap Y_{t_j}.
\]

For the proper, smooth \( k \)-schemes \( Y(j) \) we have

\[
H^i_{\text{rig}}(Y(j)/k) \cong H^i_{\text{cris}}(Y(j)/k) \cong H^i(Y_{et}, W \Omega^*_{Y(j)})_Q
\]

and the simplicial scheme \( Y(j) \) with boundary maps \( \delta_i \) is a proper, smooth hypercovering of \( Y \).

Since rigid cohomology satisfies cohomological descent \cite{27} we obtain an isomorphism

\[
H^i_{\text{rig}}(Y/k) \cong H^i(Y_{et}, W \Omega^*_{Y(\bullet)})_Q \cong H^i(Y_{et}, \ker(\nu^*))_Q
\]

and the exact sequence of complexes

\[
0 \to \ker(\nu^*) \to W A^* \overset{\nu^*}{\to} W A^*
\]

then induces Chiarellotto’s map

\[
H^i_{\text{rig}}(Y/k) \to H^i(Y_{et}, W A^*)_Q \cong H^i_{H_K}(X/S)^{N=0}.
\]

Using Fontaine-Jannsen’s \( C_{st} \)-comparison isomorphism proven by Tsuji \cite{26}

\[
H^i_{H_K}(X/S) \otimes_{K_0} B_{st} \cong H^i(X_{\bar{q}}, Q_p) \otimes_{Q_p} B_{st}
\]

we obtain an isomorphism

\[
c_{st} : H^i_{H_K}(X/S) \cong D_{st}(H^i(X_{\bar{q}}, Q_p))
\]

and hence our \( p \)-adic specialization map

\[
\text{sp} : H^i_{\text{rig}}(X_S/k) \to D_{st}(H^i(X_{\bar{q}}, Q_p))^{N=0} = D_{\text{cris}}(H^i(X_{\bar{q}}, Q_p)).
\]
Composing (3.8) with (3.3) we obtain a map

\[(3.9) \quad H_{rig}^{i}(Y/k) \rightarrow H_{HK}^{i}(X/S)^{N=0} \rightarrow H^{i}(X_{s}, W\mathcal{O}_{X_{s}})_{Q} \]

**Lemma 3.2.** The map (3.9) coincides with the map (2.2) defined by Berthelot, Bloch and Esnault in [4]. In particular, we have a commutative diagram

\[
\begin{array}{ccc}
H_{rig}^{i}(X_{s}/k)_{\text{slope}0} & \overset{\lambda_{s}}{\rightarrow} & H_{HK}^{i}(X/S)^{N=0, \text{slope}0} \\
\downarrow & & \downarrow \\
H^{i}(X_{s}, Q_{p}) & \overset{\lambda_{s}}{\rightarrow} & H^{i}(X_{s}, Q_{p})
\end{array}
\]

**Proof.** One has maps of spectral sequences

\[
\begin{array}{ccc}
E_{1}^{q,p} = H_{rig}^{p}(Y^{(q)}/k) & \Rightarrow & H_{rig}^{p+q}(Y/k) \\
\downarrow & & \downarrow \\
E_{1}^{q,p} = H^{p}(Y_{et}, W A^{\bullet,q})_{Q} & \Rightarrow & H_{HK}^{p+q}(X/S) \\
\downarrow & & \downarrow \\
E_{1}^{q,p} = H^{p}(Y_{et}, W\mathcal{O}_{Y^{(q)}})_{Q} & \Rightarrow & H^{p+q}(Y_{et}, W\mathcal{O}_{Y})_{Q}
\end{array}
\]

The first vertical map is induced by the inclusion of double complexes

\[W\Omega_{Y^{(q+1)}}^{\bullet} \cong \ker(\nu_{n})^{\bullet \bullet} \subseteq W_{n}A^{\bullet \bullet}\]

and filtering both complexes “vertically”, and using (3.7). The second vertical map is obtained by projecting complexes onto the degree 0 term. Now the map (2.2) of Berthelot, Bloch and Esnault is functorial and is induced by projecting the de Rham-Witt complex onto its degree 0 term if the scheme is proper and smooth. Hence the composite map of spectral sequences equals (2.2) on the initial term and therefore also on the end term. \(\square\)

By these we get Theorem 1.1.

### 3.2. General case

We now define the map \(sp' : H_{rig}^{i}(X_{s}/k) \rightarrow D_{cris}(H^{i}(X_{\bar{q}}, Q_{p}))\) in the case where \(X\) is proper, flat and generically smooth over \(S\). Recall from [9] that an alteration \(g : Y' \rightarrow Y\) is a proper, surjective morphism of schemes for which there exists a dense Zariski open \(U \subseteq Y\) so that \(g^{-1}(U) \xrightarrow{\bar{g}} U\) is finite étale. One has the following fundamental theorem [9][6.5]

**Theorem 3.1.** If \(Y\) is a proper, flat scheme over \(S\) then there exists an alteration \(Y' \rightarrow Y\) so that \(Y'\) is a semistable family (over the possibly non-connected regular base scheme \(S' = \text{Spec}(\Gamma(Y', \mathcal{O}_{Y'}))\)).
We call such a morphism $Y' \to Y$ a \textit{semistable alteration}. Using this theorem we can construct a commutative diagram

$$
\begin{array}{cccccc}
X_{s(2)}^{(2)} & \longrightarrow & X^{(2)} & \longleftarrow & X_{\bar{\eta}}^{(2)} \\
\downarrow & & \downarrow & & \downarrow \\
X_{s(1)}^{(1)} \times X_s X_{s(1)}^{(1)} & \longrightarrow & X^{(1)} \times X^{(1)} & \longleftarrow & X_{\bar{\eta}}^{(1)} \times X_{\bar{\eta}} X_{\bar{\eta}}^{(1)} \\
\downarrow & & \downarrow & & \downarrow \\
X_{s(1)}^{(1)} & \longrightarrow & X^{(1)} & \longleftarrow & X_{\bar{\eta}}^{(1)} \\
\downarrow & & \downarrow & & \downarrow \\
X_s & \longrightarrow & X & \longleftarrow & X_{\bar{\eta}} \\
\end{array}
$$

(3.10)

where $X^{(1)} \to X$ is a semistable alteration and $X^{(2)} \to X^{(1)} \times_X X^{(1)}$ is a proper morphism so that $X^{(2)} \to (X^{(1)} \times_X X^{(1)})_{\bar{\eta}}$ is a semistable alteration onto the closure of the generic fibre. Here we denote by $S^{(j)}$ the base of the semistable family $X^{(j)}$ for $j = 1, 2$. From this diagram we then deduce a commutative diagram for cohomology

$$
\begin{array}{ccccccc}
H^i_{\text{rig}}(X_{s(2)}/k) & \longrightarrow & H^i_{\text{HK}}(X^{(2)}/S^{(2)})_{N=0} \xrightarrow{c_{st}} & D_{\text{crys}}(H^i(X_{\bar{\eta}}^{(2)}/Q_p)) \\
\uparrow & & & \uparrow & & & \\
H^i_{\text{rig}}(X_{s(1)}^{(1)} \times X_s X_{s(1)}/k) & \longrightarrow & D_{\text{crys}}(H^i(X_{\bar{\eta}}^{(1)} \times X_{\bar{\eta}} X_{\bar{\eta}}^{(1)}/Q_p)) \\
\uparrow & & & \uparrow & & & \\
H^i_{\text{rig}}(X_{s(1)}/k) & \longrightarrow & H^i_{\text{HK}}(X^{(1)}/S^{(1)})_{N=0} \xrightarrow{c_{st}} & D_{\text{crys}}(H^i(X_{\bar{\eta}}^{(1)}/Q_p)) \\
\uparrow & & & \uparrow & & & \\
H^i_{\text{rig}}(X_s/k) & \longrightarrow & D_{\text{crys}}(H^i(X_{\bar{\eta}}/Q_p)) \\
\end{array}
$$

(3.11)

where the first arrow $c$ in each row is Chiarellotto’s morphism $\text{[3.8]}$ and $D_{\text{crys}} = D_{\text{crys, K}}$ is Fontaine’s functor over $K$. Note here that we have

$$
H^i_{\text{rig}}(X_{s(j)}/k^{(j)}) \cong H^i_{\text{rig}}(X_{s(j)}/k^{(j)})
$$

for $j = 1, 2$, and the restriction of the base field from $k^{(j)}$ to $k$ just amounts to considering the $\phi$-module $H^i_{\text{rig}}(X_{s(j)}/k^{(j)})$ over $K_0$ instead of $K^{(j)}_0$ (assuming $S^{(j)}$ is connected for the moment). Moreover we have an isomorphism of $G$-representations

$$
H^i(X_{\bar{\eta}}^{(j)}/Q_p) \cong \text{Ind}_{G^{(j)}}^G H^i(X_{s(j)}^{(j)}/Q_p)
$$

where $G^{(j)} = \text{Gal}(\bar{K}/K^{(j)}) \subseteq G$ and an isomorphism

$$
D_{\text{crys}, K^{(j)}}(V) \cong D_{\text{crys}, K}($$

for a $G^{(j)}$-representation $V$. If $S^{(j)}$ is not connected these considerations apply to each connected component. So source and target of the horizontal maps in (3.11) coincide with the groups
discussed in the previous paragraph and the horizontal maps coincide with $sp'$ in the semistable case.

**Proposition 3.2.** With the above notation, the equalizer of

$$H^i(X^{(1)}_\eta, \mathbb{Q}_p) \Rightarrow H^i(X^{(2)}_\eta, \mathbb{Q}_p)$$

is $H^i(X_\eta, \mathbb{Q}_p)$ and the equalizer of

$$D_{\text{cris}}(H^i(X^{(1)}_\eta, \mathbb{Q}_p)) \Rightarrow D_{\text{cris}}(H^i(X^{(2)}_\eta, \mathbb{Q}_p))$$

is $D_{\text{cris}}(H^i(X_\eta, \mathbb{Q}_p))$.

**Proof.** Clearly, the second statement follows from the first since $V \mapsto D_{\text{cris}}(V)$ is a left exact functor. The quickest way to see the first statement is to refer to the theory of cohomological descent [7] for varieties over fields of characteristic 0. The diagram $X^{(2)}_\eta \Rightarrow X^{(1)}_\eta \rightarrow X_\eta$ can be extended to a proper smooth hyper-covering $X^{(\bullet+1)}_\eta$ of $X_\eta$ and the corresponding spectral sequence

$$E^{r,s}_1 = H^s(X^{(r+1)}_\eta, \mathbb{Q}_p) \Rightarrow H^{r+s}(X_\eta, \mathbb{Q}_p)$$

induces the weight filtration on $H^{r+s}(X_\eta, \mathbb{Q}_p)$ in the sense of [7]. But since $X_\eta$ is assumed proper and smooth, $H^i(X_\eta, \mathbb{Q}_p)$ is pure of weight $i$ and the spectral sequence degenerates at $E_2$, inducing an isomorphism

$$E^{0,i}_2 = H^i(H^i(X^{(\bullet+1)}_\eta, \mathbb{Q}_p)) \cong H^i(X_\eta, \mathbb{Q}_p)$$

which is just a reformulation of the statement that $H^i(X_\eta, \mathbb{Q}_p)$ is the equalizer of

$$H^i(X^{(1)}_\eta, \mathbb{Q}_p) \Rightarrow H^i(X^{(2)}_\eta, \mathbb{Q}_p).$$

□

Now the left hand column in (3.11) induces maps

$$H^i_{\text{rig}}(X_*/k) \rightarrow \text{eq} \left( H^i_{\text{rig}}(X^{(1)}_{s(1)}/k) \Rightarrow H^i_{\text{rig}}(X^{(1)}_{s(1)} \times_{X_*} X^{(1)}_{s(1)}/k) \right)$$

$$\rightarrow \text{eq} \left( H^i_{\text{rig}}(X^{(1)}_{s(1)}/k) \Rightarrow H^i_{\text{rig}}(X^{(2)}_{s(2)}/k) \right)$$

and by the commutativity of (3.11) this last equalizer maps to the equalizer of

$$D_{\text{cris}}(H^i(X^{(1)}_\eta, \mathbb{Q}_p)) \Rightarrow D_{\text{cris}}(H^i(X^{(2)}_\eta, \mathbb{Q}_p))$$

which coincides with $D_{\text{cris}}(H^i(X_\eta, \mathbb{Q}_p))$ by the Lemma. Hence we obtain our map $sp'$ by descent from the map $sp'$ constructed in the semistable case. Note that $H^i_{\text{rig}}(X^{(1)}_{s(1)} \times_{X_*} X^{(1)}_{s(1)}/k) \rightarrow H^i_{\text{rig}}(X^{(2)}_{s(2)}/k)$ may not be injective and in fact all three groups in (3.12) may be distinct.

**Lemma 3.3.** The map $sp' : H^i_{\text{rig}}(X_*/k) \rightarrow D_{\text{cris}}(H^i(X_\eta, \mathbb{Q}_p))$ is independent of the choice of alteration $X^{(1)} \rightarrow X$ and is functorial in $X$. 
Proof. In fact, given two semistable alterations \( X^{(1)} \to X \) and \( \tilde{X}^{(1)} \to X \) we can consider the disjoint union
\[
\tilde{X}^{(1)} := X^{(1)} \amalg \tilde{X}^{(1)} \to X \amalg X \xrightarrow{\nabla} X
\]
which is also a semistable alteration. Denoting the specialization maps induced from these three alterations by \( \text{sp}' \), \( \tilde{\text{sp}}' \) and \( \tilde{\tilde{\text{sp}}} \) respectively, we have a diagram
\[
\begin{array}{ccc}
H_{\text{rig}}^{i}(\tilde{X}_{\tilde{g}(1)}/k) & \xrightarrow{\tilde{c}} & D_{\text{crys}}(H^{i}(\tilde{X}_{\tilde{g}(1)}, \mathbb{Q}_{p})) \\
\downarrow & & \\
H_{\text{rig}}^{i}(X_{s(1)}/k) \oplus H_{\text{rig}}^{i}(\tilde{X}_{\tilde{g}(1)}/k) & \xrightarrow{c \oplus \tilde{c}} & D_{\text{crys}}(H^{i}(X^{(1)}, \mathbb{Q}_{p})) \oplus D_{\text{crys}}(H^{i}(\tilde{X}_{\tilde{g}(1)}, \mathbb{Q}_{p})) \\
\downarrow & & \\
H_{\text{rig}}^{i}(X_{s}/k) & \xrightarrow{\text{sp}' \oplus \tilde{\text{sp}}'} & D_{\text{crys}}(H^{i}(X^{(1)}, \mathbb{Q}_{p})) \oplus D_{\text{crys}}(H^{i}(\tilde{X}_{\tilde{g}(1)}, \mathbb{Q}_{p})) \\
\downarrow & & \\
H_{\text{rig}}^{i}(Y_{s}/k) & \xrightarrow{\text{sp}'} & D_{\text{crys}}(H^{i}(Y^{(1)}, \mathbb{Q}_{p}))
\end{array}
\]
where \( \Delta \) is the diagonal map \( x \to (x, x) \) and \( c, \tilde{c} \) and \( \tilde{\tilde{c}} = c \oplus \tilde{c} \) are the lower horizontal maps in \#3.1 for the respective alterations. By construction of the specialization map, all squares except the bottom one commute. An easy diagram chase using the injectivity of \( \alpha \) then implies that the bottom square commutes as well, hence \( \text{sp}' = \tilde{\text{sp}}' = \tilde{\tilde{\text{sp}}} \).

If \( Y \to X \) is an \( S \)-morphism between generically smooth, proper flat \( S \)-schemes, \( X^{(1)} \to X \) a semistable alteration we can choose a semistable alteration \( Y^{(1)} \to (Y \times_{X} X^{(1)})_{\eta} \) onto the closure of the generic fibre. Then \( Y^{(1)} \to Y \) will also be a semistable alteration and we obtain a commutative diagram
\[
Y^{(1)} \longrightarrow X^{(1)} \\
\downarrow \downarrow \\
Y \longrightarrow X
\]
inducing a cubical diagram of maps similar to the one above, the bottom of which is the diagram
\[
H_{\text{rig}}^{i}(X_{s}/k) \xrightarrow{\text{sp}'} D_{\text{crys}}(H^{i}(X^{(1)}, \mathbb{Q}_{p})) \\
\downarrow \\
H_{\text{rig}}^{i}(Y_{s}/k) \xrightarrow{\text{sp}'} D_{\text{crys}}(H^{i}(Y^{(1)}, \mathbb{Q}_{p})).
\]
All other sides commute by known functorialities and the injectivity of
\[
D_{\text{crys}}(H^{i}(Y^{(1)}, \mathbb{Q}_{p})) \to D_{\text{crys}}(H^{i}(Y^{(1)}, \mathbb{Q}_{p}))
\]
then implies that the bottom diagram commutes as well. \( \square \)

By these we construct \( \text{sp}' : H_{\text{rig}}^{i}(X_{s}/k) \to D_{\text{crys}}(H^{i}(X^{(1)}, \mathbb{Q}_{p})) \) in general and get Proposition \#3.1.

Remark 3.1. Note that we have trace maps on the generic fiber induced by the following lemma.
Lemma 3.4. Let $\xi : V' \to V$ be a surjective morphism between smooth proper varieties of the same dimension $n$ over a field $K$ of characteristic 0. Then there is a trace map $\text{Tr} = \text{Tr}_\xi : R\xi_*\mathbb{Q}_p \to \mathbb{Q}_p$ so that the composite $\mathbb{Q}_p \to R\xi_*\mathbb{Q}_p \xrightarrow{\text{Tr}} \mathbb{Q}_p$ is multiplication by the generic degree $d_\xi$ (a locally constant function on $V$).

Proof. This follows from the six functor formalism on the derived category of $p$-adic étale sheaves. If $\eta : V \to \text{Spec}(K)$, $\eta' = \eta \circ \xi : V' \to \text{Spec}(K)$ denote the structure maps, we have

$$\mathbb{Q}_p = (\eta')^!\mathbb{Q}_p(-n)[-2n] = \xi^!\eta^!\mathbb{Q}_p(-n)[-2n] = \xi^!\mathbb{Q}_p$$

since $\eta$ and $\eta'$ are smooth, and we have $R\xi_! = R\pi_*$ as $\xi$ is proper. So the trace map is simply $R\xi_*\mathbb{Q}_p \cong R\xi\xi^!\mathbb{Q}_p \to \mathbb{Q}_p$ where the last arrow is the adjunction. If $\xi$ is finite étale, the composite $\mathbb{Q}_p \to R\xi_*\mathbb{Q}_p \xrightarrow{\text{Tr}} \mathbb{Q}_p$ is multiplication by $d_\xi$ [21][V.1.12] and in general this is true on the dense open $U \subseteq V$ where $\xi$ is finite étale. But maps between (locally) constant $p$-adic sheaves agree if they agree on a dense open subset. \qed

As we mentioned, Conjecture 1.3 follows from the compatibility diagram of trace maps (1.2). A possible approach is to show that these trace morphisms satisfy some required homotopy identities. In fact, we mention here that Proposition 3.2 can also be proved via homotopy argument of trace maps on the generic fiber, and we expect analogue results of Proposition 3.2 for Witt vectors cohomology in the special fiber, i.e. the equalizer of $H^1(X^{(1)}, W_n\mathcal{O}_{X^{(1)}}) \Rightarrow H^1(X^{(2)}, W_n\mathcal{O}_{X^{(2)}})$ is $H^1(X, W_n\mathcal{O}_X)$. However, the proof does not follow directly from the one on generic fiber, the difficulty is the following: Given a semistable alteration $Y \to X$, the scheme $Y \times_X Y$ might have irreducible components lying entirely in the special fibre. Hence one cannot choose a second semistable alteration $Y' \to Y \times_X Y$ that is surjective. Another difficulty is that $Y \times_X Y \to Y$ is not necessarily a local complete intersection morphism, and hence there is a priori no trace map.

Moreover, if we consider the cospecialization map

$$H^i_{\text{rig}}(X_s/k) \to H^i_{dR}(X_{\bar{\eta}}/K) \cong D_{dR}(H^i(X_{\bar{\eta}}, \mathbb{Q}_p))$$

defined in [1], the compatibility (1.2) is then the corollary of a more general compatibility of trace maps:

$$
\begin{array}{ccc}
H^i_{\text{rig}}(X_s^{(1)}/k) & \longrightarrow & D_{dR}(H^i(X_{\bar{\eta}}^{(1)}, \mathbb{Q}_p)) \\
\tau_{\xi} & & \tau_{\eta} \\
\downarrow & & \downarrow \\
H^i_{\text{rig}}(X_s/k) & \longrightarrow & D_{dR}(H^i(X_{\bar{\eta}}, \mathbb{Q}_p))
\end{array}
$$

provided we could define the trace map $\tau_{\eta} : H^i_{\text{rig}}(X_s^{(1)}/k) \to H^i_{\text{rig}}(X_s/k)$ for rigid cohomology. The compatibility of the trace map would then implies the compatibility (1.2) and deduce Conjecture 1.3, also some results in [5].

In some special cases, e.g. $X^{(1)} \to X$ is finite and flat, Grosse-Klönen in [15] defined a trace map for rigid cohomology which satisfies the compatibility. The construction of a trace map for rigid cohomology in a general case is still unknown.
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