Unsteady Aerodynamic Response of a Rapidly Started Flexible Wing

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ABSTRACT

Effects of camber and camber-change due to elastic deflection for aspect ratio 4.25 wings were examined for the classical unsteady problem of rectilinear translational acceleration. Direct force measurements and flow visualization by laser illumination of fluorescent dye allowed for the tracking of force history vs. evolution of the flowfield of rigid flat, rigid cambered, and flexible membranous wings. At low incidence (10 degrees and below), Wagner’s approximation provides an accurate prediction of the time-evolution of lift for the rigid wings, beyond which flow separation leads to peaks in the force history and the camber-effect is no longer additive to the incidence effect. Both the rigid uncambered and cambered wings reach peak lift at 35 degrees, whereas the flexible wing experiences a form of stall-delay and reaches peak lift at 50 degrees. Due to the aeroelasticity of the flexible membrane, flow over the suction surface remains attached for much higher incidence angles than for the rigid wings. For incidence angles less than 30 degrees, the flexible wing’s peak lift is lower than that of its rigid counterparts. However, beyond 30 degrees, the flexible wing’s peak lift is higher than that of its rigid counterparts.

1. INTRODUCTION

The impact of wing flexibility on aerodynamic performance has become a topic of interest among the micro air vehicle (MAV) community. Research on a wide range of natural fliers, from insects [1-3] to bats [4-5] aims to discover the aerodynamic benefit of membranous wings in the low Reynolds number flight regime. These natural fliers use thin, compliant wings as lifting surfaces, causing their wings to undergo large change in shape during flight [6]. Membrane wings have the potential to reduce the gross weight of micro air vehicles, while also introducing variable geometry (e.g. camber and twist) and improving stability to ensure robust flight characteristics over a wide range of harsh flight conditions.

In the low Reynolds number regime typical of MAV flight (Re < 10⁵), flow separation and vortex formation are common and provide sources of flow unsteadiness. It is important to characterize this unsteadiness in consideration of designing a controller to stabilize the vehicle from potentially damaging gusts or stall conditions. Comparisons of a two-dimensional flexible membrane along with its rigid cambered analog revealed a coupling between unsteady vortex shedding and dynamic structural response for the flexible wing, resulting in delayed stall and enhanced lift [7]. It is important to note that flapping wing MAVs depend heavily on transient, high lift mechanisms such as leading edge vortices (LEV)s in largely separated flow to generate lift.

Studies focused on the effect of wing flexure in impulsively translating wings [8] concluded that wingtip flexure effectively diminishes the influence of the tip vortex (TiV) on the structure and trajectory of the LEV, allowing the LEV to remain connected and close to the wing. This is contrary to the rigid flat plate, and to some extent the rigid cambered wing, in which the LEVs are quickly shed from the leading edge and surface of the wing to subsequently convect downstream, resulting in oscillations in the force history [9]. There is a special interplay between the LEV and TiV in low aspect ratio wings, and it has been shown that the tip vortex can either promote or have little effect on aerodynamic performance in this regime [10-12]. Strength and formation time of the LEV dictates the importance of the TiV to lift production.

The current study examines rigid flat, rigid cambered, and flexible AR = 4.25 wings accelerated
from rest in pure rectilinear translation at fixed angles of attack. The aim of the present work is to quantitatively compare the aerodynamic performance of rapidly accelerated wings over a wide range of incidence angles and identify regions where geometric parameters, i.e. camber and flexibility, have noticeable effect. Direct force measurements will allow for the comparison of force histories over 14 chord-lengths of translation for a wide range of angles of attack. Results are broken down into “early time”, corresponding to the typical travel distance of a flapping wing stroke, and “long convective time”, displaying the long-term effect of a transient flow disturbance.

2. EXPERIMENTAL SETUP

The U.S. Air Force Research Laboratory’s Horizontal Free-surface Water Tunnel (Figure 1) is fitted with a three degree-of-freedom electric rig [13], here used only in translation of the test article. Motion is controlled via a Galil DMC-4040 motion controller achieving <0.1 mm error in translation and <0.2° error in incidence angle. Direct force measurements were obtained at Re = 20,000 to ensure good signal-to-noise ratio via an ATI Nano25 6-component load cell connected to the center of the wing. Force measurements were recorded at 1 kHz and low-pass filtered in hardware at $f = 18$ Hz. Frequency analysis, by means of Fast Fourier transforms (FFT) of both a tap test and the force histories, provided confirmation that filtering at $f = 18$ Hz produced a clean result without masking any underdamped structural vibrations.

![Figure 1. Horizontal Free-Surface Water Tunnel at AFRL. Freestream flows from left to right.](image)

Each of the wings was driven in pure rectilinear translation (i.e., towed) at a fixed angle of attack, $\alpha$. All of the velocity profiles were linear with respect to time, accelerating over 0.84 chord-lengths to a target velocity corresponding to a Reynolds number of 20,000 (Fig. 2). Once the wing reached the target speed, the wing motion continued at this constant velocity until deceleration at the end of the stroke. The present study maintains the same kinematics for all wings. Several angles of attack were investigated to explore the unsteady response of flows ranging from completely attached to largely separated. Static force measurements were also obtained by holding the wing at a fixed angle of attack with the tunnel freestream on for many chords of travel ($s/c > 25$) before the force measurements were recorded. It is of interest to identify how far a wing must travel after a disturbance before it returns to this steady state lift.

The flexible wing, shown at the right of Figure 3 was constructed using a thin plastic membrane with chord-wise stiffeners at seven spanwise locations. The membrane is fixed via a stiff carbon fiber leading edge and is pinned at the mid-span of the trailing edge. The leading and trailing edges are connected by a carbon fiber bow, visible in the bottom center of Figure 3. Figure 3 (bottom left) shows images of the flexible wing at two incidence angles, illustrating how the wing “inflates” and becomes cambered when subject to an incoming flow. From high-speed images of the wing in motion, it was found that, once inflated, the flexible wing maintained a constant camber line. The camber line was reconstructed using these high-speed images. Two rigid wings, one flat (Fig. 1, top left) and one cambered (Fig. 1, top center), of the same planform as the flexible wing were constructed to provide intermediate steps between a flat rigid wing and a flexible membrane wing, thus allowing for the
identification of aerodynamic effects due to camber, flexibility, and aeroelasticity. The cambered wing was modeled to match the camber line of the flexible wing when towed at $\alpha = 45^\circ$.

Flow visualization of the leading edge vortex and flow over the suction side of the wing was performed using fluorescent dye and planar laser fluorescence. A high concentration of Rhodamine 6G in water was injected at the leading edge of the wing at the 3/4 span. The dye was injected by a positive-displacement pump with a prescribed volumetric infusion rate via 0.5 mm internal-diameter rigid lines glued to the leading edge of the wing, and was illuminated by an Nd:YLF 527 nm 50 Hz pulsed laser sheet of 2 mm thickness. Images were recorded with a PCO DiMax high-speed camera through a Nikon PC-E 45 mm Micro lens. A Tiffen orange #21 filter removed the reflected 527 nm laser light. Although force measurements were obtained at a chord-based Reynolds number of Re = 20,000, all flow visualization experiments were performed at a Reynolds number of Re = 2,500. Performing dye flow visualization at Re = 20,000 often resulted in rapid diffusion of dye and did not provide a good visual representation of the flow structures. Slowing the motion down, however, to Re = 2,500 allowed for the dye to remain cohesive and accurately represent the flow structures (i.e. leading edge vortices). Figure 4 provides a qualitative confirmation that Re = 2500, 5000, 10000, and 20000 all produce similar flow structures and could thus all be used with confidence to represent this particular flow regime.
3. RESULTS

Natural flapping wing fliers typically have stroke amplitudes of 3-5 chord lengths [14], rendering force histories over only relatively short convective times of interest for flapping wing flight. However, for the purposes of MAV development and control, it is also of interest to evaluate the long-term aerodynamic response of a non-reciprocating wing after a transient flow disturbance. We therefore consider two regions of wing travel. The “early time” corresponds to that which is applicable to the wing stroke of a typical flapping wing flier, i.e., from the start of wing motion to approximately 5 chord-lengths of travel. The “long time” is more relevant to recovery from a gust encounter or extreme maneuver, and is taken to be past 5 chord-lengths of travel.

3.1 Early Time (s/c = 0-5)

Lift generation in rapidly accelerating plates is due to a combination of circulatory force from the formation of leading edge vortices, bound circulation (if there is attached flow), and non-circulatory force or added mass during wing acceleration [9, 15, 16, 25]. As such, the force history is directly related to the wing geometry, angle of incidence, and kinematics. Each of these affects both the non-circulatory force and the leading edge vortex dynamics. In this study, all wings undergo the same motion profile. Figure 5 shows the measured force histories in terms of lift coefficient, $C_L$, for all three wings over a range of incidence angles. Both rigid wings (a-b) have similar force histories at early time for any given angle of incidence. However, note that the lift curve at $\alpha \approx 15^\circ$ gradually increases through s/c = 3 for the cambered wing but exhibits a sharp local maximum in that region for the flat-plate wing. The presence of the local maximum at s/c = 3 for the flat plate is indicative of separated flow and the formation of lift-enhancing flow features, such as a leading edge vortex [9]. The flow over the cambered wing remains attached at $\alpha = 15^\circ$, explaining the absence of such peak. Lift due to this vortex increases at higher angles of incidence for both wings as flow separates near the leading edge and rolls up into a vortex that convects downstream over the surface of the wing. The flexible wing achieves the same camber as the rigid cambered wing well before three chords traveled but does not contain a clear local force peak until around $\alpha = 25^\circ$, s/c = 3.5. Flow separation at the leading edge is delayed on the flexible wing, as illustrated in Figure 6, leading to greater lift production at higher angles of attack as compared to the rigid models.

Visbal [17] suggested that the strength and trajectory of the LEV on the flexible wing are augmented by the dynamic motion of the membrane, which causes an aeroelastically induced excitation of the shear layer. This would force the shear layer to roll up more rapidly and remain closer to the wing surface, effectively increasing the lift-enhancing effect of the leading edge vortex. This phenomena is clearly shown by the laser fluorescent dye flow in Figure 7. For $\alpha = 20^\circ$, Figure 5(a-b) show a clear force peak at s/c = 3 due to leading edge vortex formation followed by its subsequent shedding downstream. This peak is not present in the flexible wing force history of Figure 5(c), which can be attributed to the fact that the shear layer forms a recirculating region about the leading edge that remains attached to the leading edge throughout the motion, see Figure 6.
Another clear feature of the flexible wing’s force histories is a shifting of the inertial peak (i.e., the first peak after start of motion). The wing is driven such that it accelerates over 0.84 chord-lengths of travel, after which it translates at constant velocity. However, due to the three-dimensional flexibility of the membranous wing, there is a delay in the time it takes for the wing to reach full inflation. Confirmed via high-speed video, the shifting of the inertial peaks in Figure 5 corresponds to the varying travel distances it takes for the wing to reach full inflation at each angle of attack. High-speed video showed several clear steps in the inflation process: flow-induced camber at mid-span, induced camber propagating spanwise outboard, and the manifestation of upward dihedral due to the unconstrained wing tips. There is a non-linear relationship between incidence angle and the resulting magnitude and location of maximum camber, as also reported by Newman [18] and Rojratsirikul [19].

Figure 5 also reveals an interesting phenomenon for the rigid wing cases: no significant lift is gained by increasing incidence angles above 35°. This is supported by Figure 7(a), which shows a clear peak in $C_L$ versus $C_D$ at $\alpha \approx 35^\circ$. The “dynamic” values in Figure 7 are reported at the location of maximum lift immediately following the end of the acceleration phase. Not only is there no lift benefit to increasing the angle of attack of a rigid wing beyond 35°, but Figure 7(b) shows that normal force continues to increase with angle of attack, resulting in potentially undesirable increases in drag with increasing $\alpha$.

The flexible wing shows a considerable improvement in maximum lift coefficient and flight...
efficiency over the rigid wings. Figures 5(c) and 7(a) show that the flexible wing reaches peak $C_L$ at $\alpha = 50^\circ$. Wing flexibility delays the onset of stall and allows for a larger range of attainable lift coefficients over a wider range of incidence angles when compared to a rigid wing of the same planform and camber. Similar instances of flexibility-induced delayed stall and superior lift production have been reported in previous work [7, 17, 19-20]. Bishop [21-22], studying real-time flight of compliant-wing flying mammals, reported lift coefficients of 2.12 and 1.48 for flying squirrels and sugar gliders, respectively, at glide angles of approximately 40°-57°, which are consistent with the findings in this study. They also achieved the same maximum camber, 10% of the chord length, as the wing presented here. Figure 7(c-d) provides lift-to-drag ratios for all three wings in the dynamic and static regions. Beyond approximately 20°, the flexible wing attains consistently higher lift-to-drag than the rigid counterparts, which quickly converge on one another as leading edge flow separation increases. Note that natural flapping wing fliers and their analogous micro air vehicles perform wing strokes at very high incidence angles (>25°). Thus, the result of Figure 7(d) implies that membranous wings could result in more efficient flight of the MAV. Improved lift-to-drag ratio and increased overall

Figure 7. Comparison of static and dynamic force measurements
lift coefficient have been similarly observed in the experimental endeavors of membranous wings of Wrist et al. [25] and Song et al [26].

In previous work, these favorable effects are generally attributed to the aeroelastic motion of the wing membrane which forces the shear layer to remain close to the wing, even at large incidence. Figure 8 shows the instantaneous flowfields upon startup of the flexible wing at high incidence angle, \( \alpha = 45^\circ \). Contrary to rigid wings, the first leading edge vortex that forms over the flexible membrane does not convect off of the wing surface. Instead, it travels streamwise along the surface of the wing until it reaches the trailing edge (see Figure 8). Figure 6(b) shows that even at very high, deep-stall angles of incidence, there exists a vortex-like recirculation region on the flexible wing that is fed from the leading edge, which explains the continuous increase in lift from \( 5^\circ \leq \alpha \leq 50^\circ \).

\[ \text{Figure 8. Dye flow visualization on the flexible wing at } \alpha = 45^\circ \text{ illustrating the trajectory of the initial leading edge vortex, indicated by the red arrow.} \]

### 3.2 Long Convective Time (s/c > 5)

Although flapping wing fliers and MAVs generally have stroke amplitudes of only a few chord-lengths, the stability and control of a micro air vehicle is largely dependent on how it responds to and recovers from external stimuli, such as large-amplitude wind gusts. The rapid wing acceleration applied in this study could also be interpreted as a large change in wind speed or angle of attack of the wing. The previous section focused on the aerodynamic response in early time following a rapid change in wing speed; this section explores the long-term effect of that transient disturbance.

Figure 9 compares the flow over the rigid (a) and flexible (b) wings in a fully developed flow at \( \alpha = 20^\circ \). The images in this figure agree well with the results of Visbal [17] and Rojratsirikul [19], who have demonstrated that flow remains attached on a membranous wing when it would be detached on a rigid wing of similar camber. After a long convective time (s/c > 25) at \( \alpha = 20^\circ \), the rigid wing reaches a fully developed separated flow at the leading edge, whereas the flexible wing retains an attached shear layer over roughly a third of its chord before separating from the wing surface. The effect of the flexible membrane on static forces is also evident in Figure 7(a). Stall is delayed to incidence angles past those of both rigid wings, and static lift is significantly larger.

\[ \text{Figure 9. Dye flow visualization over the (left) rigid cambered wing and (right) flexible wing at } \alpha = 20^\circ \text{ in fully developed flow. The relative flow direction is left to right. Images are rotated by their respective incidence angles to align the chord with the horizontal.} \]
The canonical comparison for impulsively started wings is that of the Wagner approximation, which models the delay in circulation development on a wing impulsively pitching in a steady freestream or accelerating from rest at a fixed angle of incidence [23]. Wagner’s prediction for lift produced by finite aspect ratio, impulsively started wings is calculated as

\[ C_L = 2 \pi \alpha \phi \frac{AR}{AR + 2} \]

where the Wagner function \( \phi \) is given by the Jones approximation as [24]

\[ \phi = 1.0 - 0.165e^{-0.0455s} - 0.335e^{-0.3s} \]

and \( s \) is the distance traveled in chord-lengths. This analysis can be extended to cambered wings through the use of thin airfoil theory. The mean camber line, \( z(x)/c \), can be transformed to polar coordinates using

\[ x = \frac{c}{2} (1 - \cos \theta) \]

and after computing the Fourier coefficients of classical thin airfoil theory,

\[ A_0 = a - \frac{1}{\pi} \int_0^\pi \frac{dz}{dx} d\theta \]
\[ A_1 = \frac{2}{\pi} \int_0^\pi \frac{dz}{dx} \cos \theta d\theta \]

one can calculate the theoretical lift coefficient as a function of chord-lengths traveled, given by

\[ C_L = 2\pi(A_0 + \frac{A_1}{2}) \phi \frac{AR}{AR + 2} \]

Figure 10 shows the measured lift coefficient for each wing out to 14 chords traveled. The wing has stopped accelerating at 0.84c and continues traveling at constant velocity for the remaining 13.16 chord-lengths. For low incidence angles, \( \alpha \leq 10^\circ \), Wagner’s approximation (plotted for the rigid wing cases in Figure 10) provides an accurate prediction of the time-evolution of lift, especially at long times. Beyond 10 degrees, however, the shear layer on the rigid wings begins to separate at the leading edge. Wagner’s model is based on the assumption of completely attached flow, and thus leading edge separation essentially violates the assumptions inherent in Wagner’s prediction. Note that while Wagner’s approximation was derived for an impulsively started wing, previous work has shown that wings accelerating over a small but finite distance exhibit a similar build-up of lift [15].

At high angles of attack, the measured forces are much greater than predicted by Wagner at short times but much lower at long times. For the rigid wings, aggressive vortex interactions lead to a lift peak around \( s/c = 7-8 \), corresponding to the reformation of dominant leading edge circulation, which has been detailed in previous work by Mancini et al. [9, 25]. This second lift peak occurs at \( \alpha = 25^\circ \) for both rigid wings, but not until \( \alpha = 40^\circ \) for the flexible wing. Recalling the flow visualizations shown in Figure 6, the delay in appearance of the second peak in Figure 10 can be attributed to the delay in leading edge separation over the flexible wing. Only at \( \alpha \geq 40^\circ \) is the flow sufficiently separated on the flexible wing to evoke the same vortex dynamics experienced by the rigid wings at moderate angles of attack.

A secondary focus of this work was to determine the long time response of a flow disturbance on the lift history. The physical limit of surge distance in the test section was 14 chord-lengths. Figure 10 shows that for every case, there is an influence from the initial disturbance on lift production that remains present in the force history throughout the entire recorded motion. Thus, the wing must travel farther than 14 chord-lengths in order to reach its final steady state.
4. CONCLUSIONS

The effects of wing camber and elastic deflection for aspect ratio 4.25 wings were examined for the canonical unsteady problem of rectilinear translation with constant acceleration and fixed angle of attack. A rigid flat plate wing was compared to a batten-stiffened membrane of the same planform and rigid wing with fixed camber equivalent to the camber attained by the flexible wing. Direct force measurements and flow visualization were obtained in a water tunnel using a 6 degree-of-freedom force/torque sensor and planar laser illumination of fluorescent dye. These results allowed the lift-

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Figure 10. Lift coefficient versus chords traveled for the (top) flat rigid wing, (middle) cambered rigid wing, and (bottom) flexible wing. The dashed lines correspond to Wagner’s approximation for a finite wing.
coefficient history to be visually explained via the evolution of the flowfield on the wings. Flexibility was found to attenuate the aerodynamic load transients experienced by the wing. The membranous wing "inflated" to a cambered shape very soon after wing motion began, when dynamic pressure reached sufficient levels. The time-delay over which the wing inflated resulted in a delay in lift relative to the rigid wings at very early times. For incidence angles 30 degrees and below, the peak lift on the flexible wing was lower than that on the rigid wings. However, the situation is reversed beyond 30 degrees of incidence. At high angles of incidence, the flexible wing resulted in larger maximum lift coefficients and lift-to-drag ratios, suggesting that flexibility evinces a form of stall delay that may contribute to a superior flight efficiency.

At low incidence angles (10 degrees and below), the effect of wing camber on lift is essentially additive to that of wing incidence, and the time-evolution of lift predicted from Wagner’s function is a reasonable approximation to the measured history. At higher incidence, formation of leading edge vortices was observed. The growth, saturation, ejection, and downstream convection of the initial leading edge vortex (followed by succeeding vortices) correlates with the primary lift peak near conclusion of the accelerated portion of the motion (and subsequent peaks). At sufficiently high incidence where the flow is largely separated, camber is no longer additive to incidence, and both rigid uncambered and cambered wings evinced peak lift at $\alpha = 35^\circ$. However, it should be noted that the flexible wing achieved higher lift at $\alpha > 35^\circ$. For all of the cases tested, it was found that more than 14 chord-lengths of travel are required for the measured lift force to relax from the transient that occurs at the start of wing motion to a truly fully-developed steady-state.

ACKNOWLEDGEMENTS

This work was supported by the U.S. Army Research Laboratory under the Micro Autonomous Systems and Technology (CTA-MAST) program, and the Air Force Office of Scientific Research (AFOSR) under the AFOSR Summer Faculty Fellowship Program.

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