Determination of $V_{ub}$ from $B \to \pi l \bar{\nu}$ on the lattice

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We present results of a lattice study of the form factors in the decay $B \to \pi l \bar{\nu}$. We attempt to disentangle the dependence of the form factors on the light quark masses and the momentum transfer. Using models of the $q^2$ dependence we calculate the total decay rate, and compare to the experimental measure to extract $V_{ub}$. This study was performed in the quenched approximation at $\beta = 6.2$ on a $24^3 \times 48$ lattice, with a non-perturbatively improved SW fermion action.

1. INTRODUCTION

Semileptonic decays of mesons containing a $b$ quark play an important role in the determination of Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The transition amplitude of the decay $B \to \pi l \bar{\nu}$ factorizes into leptonic and hadronic parts. This hadronic matrix element can be parameterised by two form factors

$$
\langle \pi(\bar{k})|V^\mu|B(\bar{p})\rangle = f_+(q^2)(p + k - q\Delta_{m^2})^\mu + f_0(q^2)q^\mu\Delta_{m^2}
$$

where $\Delta_{m^2} = (m_B^2 - m_\pi^2)/q^2$ and $q = p - k$. In the limit of zero lepton mass, the total decay rate is given by

$$
\Gamma = \frac{G_F^2|V_{ub}|^2}{192\pi^3m_B^3} \int_0^{\eta^2} [\lambda(q^2)]^{3/2}|f_+(q^2)|^2 dq^2
$$

where $\eta^2 = (m_B - m_\pi)^2$ and

$$
\lambda(q^2) = (m_B^2 + m_\pi^2 - q^2)^2 - 4m_B^2m_\pi^2.
$$

We can determine the decay rate from the $q^2$ dependence of the form factor $f_+(q^2)$ and then compare to the experimental measure of the decay rate to extract $V_{ub}$.

2. DETAILS OF THE CALCULATION

The 216 gauge quenched configurations were generated using the Wilson action on a $24^3 \times 48$ lattice. The quark propagators were calculated using an $O(a)$ improved action, where the coefficient $c_{SW}$ has been determined non-perturbatively \cite{3} (NP). We use four heavy quarks with masses around charm, $(\kappa_H = 0.1200, 0.1233, 0.1266, 0.1299)$. Three light quarks with masses around strange ($\kappa_L = 0.1346, 0.1351, 0.1353$) are used for the active propagator, and the heaviest two for the spectator. The heavy quarks were smeared \cite{2} and the light quarks fuzzed. The chiral limit has been determined to be \cite{1} $\kappa_{\text{crit}} = 0.135815$, and the physical value of $m_\pi/m_\rho$ corresponds to $\kappa_n = 0.13577$. The lattice spacing is set by $m_\rho$ and $a^{-1} = 2.64$ GeV.

We obtain the form factors from the heavy-to-light three-point correlation functions, using the masses and amplitudes from the heavy-light and light-light two-point correlation functions. The general method is given in \cite{1}. We place the operator for the heavy-light pseudoscalar meson at $T = 20$ rather than the mid-point of the lattice to check for contamination from different time orderings. We use eight different combinations of $\vec{p}$ and $\vec{k}$ determine the $q^2$ dependence; 0 $\to$ 0, 0 $\to$ 1, 0 $\to$ $\sqrt{2}$, 1 $\to$ 0, 1 $\to$ 1, 1 $\to$ $1_{\perp}$, 1 $\to$ $1_{\perp}$ and 1 $\to$ $\sqrt{2}_{\perp}$ in lattice units. There is no 0 $\to$ 0 channel for $f_+$.

2.1. Mass dependent renormalisation

We can also remove all $O(a)$ errors from matrix elements of on-shell states by an appropriate definition of the currents. For the vector current
Table 1
The effective matching coefficient. The current does not depend on $\kappa_S$.

| $am_Q$ | $\kappa_{S1}$ | $\kappa_{S2}$ | $Z_V(1+b_V am_Q)$ |
|--------|---------------|---------------|-------------------|
| 0.4852 | 1.316$^{+1}_{-1}$ | 1.317$^{+1}_{-1}$ | 1.335            |
| 0.2680 | 1.093$^{+5}_{-5}$ | 1.087$^{+5}_{-5}$ | 1.093            |

for degenerate quarks of mass $m_Q$, we have

$$V_\mu^R = Z_V(1+b_V am_Q)\{V_\mu+c_V a\frac{1}{2}(\partial_\nu+\partial_\nu^*)T_{\mu\nu}\} \tag{4}$$

where $V_\mu$ and $T_{\mu\nu}$ are the local lattice vector and tensor currents respectively. Both $b_V$ and $Z_V$ have been determined non-perturbatively \[7\]. Preliminary results for a non-perturbative determination of the mixing coefficient $c_V$ exist, but we use the one-loop perturbative estimate, which is small. Defining

$$Z_V^{\text{eff}} \equiv Z_V(1+b_V am_Q). \tag{5}$$

For the forward degenerate matrix element, we can calculate $Z_V^{\text{eff}}$ from our data. We show the comparison to $Z_V^{\text{eff}}$ evaluated for these quark masses using the non-perturbative $Z_V$ and $b_V$ in Table 2.1. The excellent agreement suggests higher order discretisation effects are limited.

3. CHIRAL EXTRAPOLATION

To evaluate the form factor $f_i$ at physical quark masses we must consider both the intrinsic dependence of $f_i$ and the indirect mass dependence arising from the change in $q^2$:

$$f_i = f_i(q^2,\kappa_A,\kappa_S). \tag{6}$$

In previous UKQCD analyses \[7\] the $q^2$ dependence was modelled by an extra term. This is potentially difficult to control. Here we extrapolate whilst holding $q^2$ fixed. This approach yields a more reliable extrapolation. This is discussed in more detail in \[8\].

We first interpolate the form factors to a chosen set of $q^2$ values for each quark mass combination. The values of $q^2$ are chosen such that we interpolate for each light quark combination and that for different heavy quark masses, the sets of $q^2$ values correspond to the heavy quarks having the same velocity. This is discussed in the section on the heavy extrapolation.

The form of the interpolation function is motivated by pole dominance models,

$$f_i(q^2) = \frac{f_i(0)}{(1-q^2/m_i^2)^n}, \tag{7}$$

where $i$ is either $+$ or 0. However, as we interpolate in $q^2$, any model dependence in the chiral extrapolation is mild, this is shown in figure 1. We then extrapolate the form factors at fixed $q^2$ to $\kappa_n$ with the light quarks non-degenerate:

$$f(\kappa_S,\kappa_A) = \alpha + \beta \left(\frac{1}{\kappa_S} - \frac{1}{\kappa_{\text{crit}}}\right)$$

$$+ \gamma \left(\frac{1}{\kappa_S} + \frac{1}{\kappa_A} - \frac{2}{\kappa_{\text{crit}}}\right). \tag{8}$$

4. HEAVY QUARK MASS EXTRAPOLATION

Heavy quark effective theory (HQET) is used to motivate the form of the extrapolation to the $B$ meson scale. The scaling relations, $f_+ \sim \sqrt{M}$ and $f_0 \sim 1/\sqrt{M}$ are determined at fixed four-velocity, $v$. Defining the recoil variable,

$$v \cdot k = \frac{M_P^2 + m_r^2 - q^2}{2M_P} \tag{9}$$
we can then extrapolate the form factors at fixed $v \cdot k$ to the $B$ meson scale:

$$C f_i(v \cdot k) M_P^{s_i/2} = \gamma_i \left( 1 + \frac{\delta_i}{M_P} + \epsilon_i \right)$$  \hspace{1cm} (10)

where $s_i = -1$ when $i = +$, and $s_i = +1$ when $i = 0$. The coefficient $C$ is the logarithmic matching factor,

$$C(M_P, m_B) = \left( \frac{\alpha_s(m_B)}{\alpha_i(M_P)} \right)^{2/\beta_0}$$  \hspace{1cm} (11)

and $\beta_0 = 11$ in quenched QCD.

5. RESULTS

The resulting form factors are plotted in figure 2. Pole dominance models, equation 7, combined with the heavy quark scaling relations suggest that $n_+ = n_0 + 1$. Light-cone scaling further suggests $n_0 = 1$. We also impose the kinematic constraint $f_0(0) = f_+(0)$, to parameterise the form factors by a pole for $f_0$ and a dipole for $f_+$. A slightly more sophisticated pole/dipole parameterisation for $f_0$ and $f_+$, consistent with the same constraints, has been suggested by Becirevic and Kaidalov (BK) [9]:

$$f_+(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/m_B^2)(1 - \alpha q^2/m_B^2)}$$

$$f_0(q^2) = \frac{c_B(1 - \alpha)}{(1 - q^2/\beta m_B^2)}.$$  \hspace{1cm} (12)

We fit both parameterisations to the form factors. This is shown in figure 2. We can now use these models to calculate the total decay rate from equation 8. The results are

$$\Gamma(B \to \pi l \nu)/|V_{ub}|^2 = 9.0 \pm 3.0 \pm 3.2 \text{ps}^{-1}$$  \hspace{1cm} (13)

where the first error is statistical and the second is systematic. The systematic errors are estimated by trying different interpolation functions for the chiral extrapolation, a linear fit to the heaviest three quarks for the heavy extrapolation and estimates of the lattice spacing from different quantities, i.e. $r_0$. We can use this model dependent result to extract $V_{ub}$ from experimental data [1].

$$|V_{ub}| = (3.7 \pm 0.5 \pm 0.7 \pm 0.7) \times 10^{-3}.$$  \hspace{1cm} (14)

The third error is the experimental error in the branching ratio.

This is a preliminary model dependent result. The form factors are well determined in the range $16 - 22 \text{ GeV}^2$. The total decay rate is dominated by low $q^2$ due to phase space. Here we have no data and are reliant on models of $q^2$. The differential decay rate could be used to extract $V_{ub}$ in a model independent manner, but there is no experimental data available yet. This work was supported by EPSRC grant GR/K41663 and PPARC grant GR/L29927.

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