Inclusive Production of Four Charm Hadrons in $e^+e^-$ Annihilation at $B$ Factories

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Measurements by the Belle Collaboration of the exclusive production of two charmonia in $e^+e^-$ annihilation differ substantially from theoretical predictions. Till now, no conclusive explanation for this remarkable discrepancy has been provided. Even the origin of the discrepancy is not identified, yet. We suggest that the measurement of four-charm events in Belle data must provide a strong constraint in identifying the origin of this large discrepancy. Our prediction of the cross section for $e^+e^- \rightarrow c\bar{c}c\bar{c}$, in lowest order in strong coupling constant, at $\sqrt{s} = 10.6$ GeV is about 0.1 fb. If measured four-charm cross section is compatible with the prediction based on perturbative QCD, it is very likely that factorization of hadronization process from perturbative part may be significantly violated or there exists a new production mechanism. If the cross section for the four-charm event is also larger than the prediction like that for the exclusive $J/\psi + \eta_c$ production, perturbative QCD expansion itself will be proved to be unreliable and loses predictive power.

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The Belle Collaboration has measured the cross section for $J/\psi + \eta_c$ by observing a peak in the momentum spectrum of the $J/\psi$ that corresponds to the 2-body final state $J/\psi + \eta_c$. The measured cross section is

$$\sigma[J/\psi + \eta_c] \times B^{\eta_c}[\geq 4] = (33^{+7}_{-6} \pm 9) \text{ fb}, \tag{1}$$

where $B^{\eta_c}[\geq 4]$ is the branching fraction for the $\eta_c$ to decay into at least 4 charged particles. Since $B^{\eta_c}[\geq 4] < 1$, the right side of Eq. (1) is a lower bound on the cross section to produce $J/\psi + \eta_c$. This lower bound is about an order of magnitude larger than the predictions by Braaten and Lee [2], and by Liu, He, and Chao [3] of nonrelativistic QCD(NRQCD) [4] in the nonrelativistic limit. The cross section was calculated previously by Brodsky and Ji [5], and by Liu, He, and Chao [3] of nonrelativistic QCD(NRQCD) [4] in the nonrelativistic limit. The calculation given in Ref. [5] is the calculation given in Ref. [3]. They found exact agreement [7] with the result based on NRQCD [2, 3]. Currently, the cross section for the process $e^+e^- \rightarrow J/\psi + \eta_c$ shows the largest discrepancy between theory and data available within standard model. This presents a challenge to our current understanding of charmonium production based on perturbative QCD framework.

A few theoretical studies have been carried out in order to explain the large discrepancy. Bodwin, Braaten, and Lee proposed [5, 6] that the Belle data for $J/\psi + \eta_c$ may include the $J/\psi + J/\psi$ events because the width of the $\eta_c$ peak in the recoil mass distribution for inclusive $J/\psi$ production measured by the Belle Collaboration is wide enough to accommodate $J/\psi$ events. The large enhancement from photon fragmentation in the two-photon mediated process $e^+e^- \rightarrow J/\psi + J/\psi$ overcomes the suppression factor $\alpha_s^2/\alpha_s^2$ in couplings compared to the $e^+e^- \rightarrow J/\psi + \eta_c$. Brodsky, Goldhaber, and Lee introduced an exotic scenario that the Belle $J/\psi + \eta_c$ signal may include the associated production of $J/\psi$ and spin-J glueball $G_J$, $J = 0, 2$ [10]. The Belle Collaboration carried out an updated analysis [11] motivated by these proposals. According to the Belle analysis, no events for $J/\psi + J/\psi$ have been detected and the upper limit for $\sigma(e^+e^- \rightarrow J/\psi + J/\psi) \times B^{J/\psi}[\geq 2] = 9.1 \text{ fb}$ at the 90% C.L., which is consistent with the prediction given in Refs. [8, 9]. Here $B^{J/\psi}[\geq 2]$ is the branching fraction for the $J/\psi$ to decay into more than 2 charged particles. The measured cross section for $e^+e^- \rightarrow J/\psi + J/\psi$, however, does not explain the large fraction of the $J/\psi + \eta_c$ signals. The Belle Collaboration also analyzed the angular distribution of $J/\psi$ in order to identify if the data include the associated $J/\psi + G_0$ signals. The predicted angular distributions are proportional to $\cos^2 \theta$ for $J/\psi + \eta_c$ [2] and $J/\psi + G_2$ [10], and $\sin^2 \theta$ for $J/\psi + G_0$ [10], where $\theta$ is the scattering angle of the $J/\psi$ in the $e^+e^-$ c.m. frame. The updated analysis show the measured distribution is proportional to $\cos^2 \theta$, which ruled out the spin-0 glueball scenario [11]. Since the angular distribution for $J/\psi + G_2$ has the same form as that for $J/\psi + \eta_c$, spin-2 glueball has not been ruled out, yet.

Another scenario is that higher-order corrections in strong coupling $\alpha_s$ may be huge [12]. If it is true, perturbative expansion is not a proper method to predict the cross section. If it is not, it is probable that the factorization of long-distance factor involving hadronization is
seriously violated or there exists a unknown production mechanism which we do not understand, yet. Once we can estimate the size of the perturbative QCD corrections to this process, it might be easier for us to identify the origin of this large discrepancy. Unfortunately, the next-to-leading-order corrections to the cross section of exclusive $J/\psi + c\bar{c}$ process is not available. A comprehensive review on recent developments in quarkonium physics can be found in Ref. [13].

In this paper, we introduce an economical method to check if the perturbative QCD corrections to the cross section is indeed large enough to explain the discrepancy. If we consider inclusive four-charm-hadron production, the prediction for the cross section can be expressed as inclusive $c\bar{c}c\bar{c}$ production rate $\sigma(e^+e^- \rightarrow c\bar{c}c\bar{c} + X)$. This is analogous to estimating $\sigma(e^+e^- \rightarrow \text{hadrons})$ by $\sum_q \sigma(e^+e^- \rightarrow q\bar{q} + X)$. We expect $\sigma(e^+e^- \rightarrow c\bar{c} + X)$ to be a good approximation at $\sqrt{s} = 10.6$ GeV. Unlike the prediction for $J/\psi + c\bar{c}$ cross section, the prediction for the inclusive four-charm-hadron production rate purely consists of short-distance factor. Corresponding long-distance factor for hadronization is of order 1. Since this process involves the same Feynman diagrams for exclusive $J/\psi + c\bar{c}$ production, the measurement of the cross section for four-charm-hadron production will provide an important information in estimating the size of the short-distance coefficient for $J/\psi + c\bar{c}$ cross section. We present our prediction for inclusive four-charm-hadron production by calculating $\sigma(e^+e^- \rightarrow c\bar{c}c\bar{c} + X)$ in order $\alpha^2\alpha_s^2$, which is leading order in strong coupling constant. If our leading-order prediction is comparable to the measured value, it is very probable that the QCD higher-order corrections to the $J/\psi + c\bar{c}$ cross section is small. Then the large discrepancy in $J/\psi + c\bar{c}$ cross section may be due to the violation of factorization or existence of new production mechanism. If the measured cross section for the four-charm-hadron inclusive production is much larger than our prediction like the case of $J/\psi + c\bar{c}$, it is very likely that perturbative QCD corrections to $J/\psi + c\bar{c}$ cross section is large enough to explain the discrepancy, which leads to the failure of reliability in perturbative expansion.

In leading order in strong coupling $\alpha_s$, $c\bar{c}c\bar{c}$ can be produced at order $\alpha^2\alpha_s^2$. There are two topologically distinct Feynman diagrams generating two pairs of $c\bar{c}$, which are shown as $\mathcal{M}_1$ and $\mathcal{M}_2$ in Fig. 1(a) and 1(b), respectively. Momenta for the involving particles are assigned as $e^-(k_1)e^+(k_2) \rightarrow c(p_1)c(p_2)c(p_3)c(p_4)$. The amplitude for the two diagrams shown in Fig. 1 are

$$-iM_i = i\frac{(4\pi)^2e_c\alpha_s}{s(p_2 + p_3)^2} v_c(k_2)\gamma_\alpha u_c(k_1) \times \bar{u}(p_3)T^a\gamma_3v(p_2)\bar{u}(p_1)T^aH^{a\beta}_i v(p_4),$$

(2)

where $s = (k_1 + k_2)^2$, $e_c = \frac{4}{3}$ is the fractional electric charge of the charm quark, and $a$ is the SU(3) color index for the virtual gluon. The vector indices $\alpha$ and $\beta$ are for the virtual photon and gluon, respectively. We suppress the spin and color indices of the charm quarks in Eq. (2).

For $i = 1$ or 2 the tensors $H^{a\beta}_i$ in Eq. (2), which are matrices in spinor space, are defined by

$$H^{a\beta}_1 = \gamma^\beta \Lambda(p_1 + p_2 + p_3)\gamma^\alpha,$$

(3a)

$$H^{a\beta}_2 = \gamma^a \Lambda(-p_2 - p_3 - p_4)\gamma^\beta,$$

(3b)

where $\Lambda(p) = (p + m_c)/(p^2 - m_c^2)$.

There are 6 more Feynman diagrams that can be obtained from the two amplitudes $\mathcal{M}_1$ and $\mathcal{M}_2$ by exchanging two charm quarks and two antiquarks, respectively, as

$$\mathcal{M}_3 = -P_{1+34}\mathcal{M}_1,$$

$$\mathcal{M}_4 = -P_{1+34}\mathcal{M}_2,$$

$$\mathcal{M}_5 = -P_{2+41}\mathcal{M}_1,$$

$$\mathcal{M}_6 = -P_{2+41}\mathcal{M}_2,$$

$$\mathcal{M}_7 = +P_{3+42}\mathcal{M}_1,$$

$$\mathcal{M}_8 = +P_{3+42}\mathcal{M}_2,$$

(4)

where $P_{i+34}$ is the operator exchanging two particles with momentum indices $p_i$ and $p_3$ shown in Fig. 1. The signs of $\mathcal{M}_3$ through $\mathcal{M}_8$ in Eq. (4) are determined by the antisymmetry of Fermi statistics in exchanging identical fermions among the final-state particles.

The total cross section for the process is expressed as

$$d\sigma = \frac{1}{2s} \sum_i |\mathcal{M}_i|^2 \frac{d\Phi_4}{(2\pi)^2},$$

(5)

where $\mathcal{M} = \sum_{i=1}^{8} \mathcal{M}_i$, and the factor $(2!)^2$ in Eq. (5) is divided in order to avoid double-counting of identical final-state particles. The summation notation $\sum$ in Eq. (5) stands for averaging over initial spin states.
and summation over final color and spin states. The four-body phase-space element \( d\Phi_4 \) in Eq. (5) can be parametrized by

\[
d\Phi_4 = \frac{d\Omega_{ij} d\Omega_{ij}^*}{(2\pi)^8} \times \frac{|P|d\Omega}{4\sqrt{s}} \times \frac{|p_4^*|d\Omega_{ij}^*}{4m_{13}} \times \frac{|p_4|d\Omega_{ij}}{4m_{24}},
\]

where \( m_{ij} \) is the invariant mass of \( p_i + p_j \) and \( d\Omega_{ij} \) is the solid angle element of \( p_i + p_j \) in the rest frame of \( p_i + p_j \). Their physical regions are \( 2m_c < m_{13} < \sqrt{s} - 2m_c \) and \( 2m_c < m_{24} < \sqrt{s} - m_{13} \). The three-momenta \( p_1 \) and \( p_2 \) are defined in \( p_1 + p_2 \) and \( p_2 + p_4 \) rest frames, respectively. The three-momentum \( P \) and solid angle element \( d\Omega \) are for \( p_1 + p_2 \) in the c.m. frame. Integrating the differential cross section \( \Phi_4 \) over the phase space \( \Omega \), we get the total cross section for \( e^+e^- \to c\bar{c}c\bar{c} \).

We compute the \( \sum|\mathcal{M}|^2 \) in Eq. (5) using REDUCE [14] and carry out the phase-space integral in Eq. (5) making use of the adaptive Monte Carlo routine VEGAS [12]. As a check, we carry out the same calculation using ComPhEP [10]. Our analytic result for \( \sum|\mathcal{M}|^2 \) and numerical values for the total cross section agree with those obtained by using ComPhEP.

Our predictions for the inclusive four-charm-hadron cross sections in \( e^+e^- \) annihilation at \( \sqrt{s} = 10.6 \) GeV depending on the charm-quark mass \( m_c \) is shown in Fig. 2. The cross section for \( e^+e^- \to c\bar{c}c\bar{c} \) is very sensitive to the value of \( m_c \). For \( \alpha = 1/137 \), \( \alpha_s = 0.2 \), \( m_c = 1.5 \) GeV \( \sigma(e^+e^- \to c\bar{c}c\bar{c}) \approx 97 \) fb. The cross section varies from 0.31 pb at \( m_c = 1.2 \) GeV to 24 fb at \( m_c = 1.8 \) GeV. The cross section decreases as \( m_c \) increases mainly because available phase space shrinks. If one can increase the c.m. energy of the \( e^+e^- \), the \( m_c \) dependence will decrease. In previous analyses for exclusive two-charmonium production cross sections, the next-to-leading order pole mass \( m_c = 1.4 \pm 0.2 \) GeV has been used for the \( m_c \) [2, 8, 9].

However, the cross section for \( e^+e^- \to c\bar{c} \) is not sensitive to the charm-quark mass \( m_c \). The lowest-order cross section of order \( \alpha^2 \) is \( \sigma(e^+e^- \to c\bar{c}) \approx 1.0 \) nb with relative errors of \( 3 \times 10^{-3} \) for \( m_c = 1.5 \) ± 0.3 GeV at \( \sqrt{s} = 10.6 \) GeV. In Ref. [17] the total cross section for \( e^+e^- \to c\bar{c}c\bar{c} \) is predicted. If we use the input parameters \( \alpha_s = 0.24 \) and \( m_c = 1.4 \) GeV given in Ref. [17], we get 0.210 pb, which is different from the prediction 0.237 pb given in Ref. [17] by about 13%.

In Fig. 3 we show the differential cross section with respect to the invariant mass of \( cc \). This is the prediction for \( d\sigma(e^+e^- \to ccX)/dm_{cc} \) in leading order in \( \alpha_s \). Experimentally, this differential cross section can be compared with the \( \sum H,H' \to d\sigma(e^+e^- \to HH' + ccX)/dm_{HH'} \), where \( H \) and \( H' \) are charm hadrons, which do not include anticharm.

Finally, we estimate the number of four-charm-hadron events that could be detected by the Belle Collaboration. Production rate for baryonic states such as \( \Lambda_c \) will be small and we do not include the contribution in the following rough estimate. Based on heavy-quark spin symmetry, the relative rates for a \( c \) quark fragmenting into the charm mesons are \( D^+ : D^0 : D^{**} : D^{0*} \approx 1 \) : 1 : 3 : 3. Because the two spin-triplet states decay into spin-singlet states by 100% with branching fractions \( \text{Br}[D^{**} \to D^0\pi^+] = 70\%\), \( \text{Br}[D^{**} \to D^0\pi^0] = 30\%\), \( \text{Br}[D^{0*} \to D^0\pi^0] = 62\%\), and \( \text{Br}[D^{0*} \to D^0\gamma] = 38\%\), we may only consider charm meson pairs made of either \( D^+ \) or \( D^0 \). Resulting fragmentation probabilities are approximately \( P[cc \to D^+ + X] \approx \frac{1}{4} \) and \( P[cc \to D^0 + X] \approx \frac{1}{4} \), respectively. Therefore, \( \sigma(e^+e^- \to D^+D^+ + X) \approx \frac{1}{6} \sigma(e^+e^- \to cc + X) \), \( \sigma(e^+e^- \to D^+D^0 + X) \approx \frac{6}{16} \sigma(e^+e^- \to cc + X) \), and \( \sigma(e^+e^- \to D^0D^0 + X) \approx \frac{1}{8} \sigma(e^+e^- \to cc + X) \).
\[ \frac{d}{dP} \sigma[e^+e^- \to cc + X] \]. The detection rate will suffer losses from branching fractions \( \text{Br}[D^+ \to K^-\pi^+\pi^+] = 9.2\% \) and \( \text{Br}[D^0 \to K^-\pi^+] = 3.8\% \), and detection acceptance/efficiency \( \approx 80\% \) for each charged particle in the decay products of \( D^+ \) or \( D^0 \). With \( \sigma[e^+e^- \to cc + X] \approx 0.1 \text{ pb} \) and current integrated luminosity \( L \approx 300 \text{ fb}^{-1} \) we expect roughly 30 events will be detected by the Belle detector. Even if we consider the uncertainties from \( \alpha_s \) and \( m_c \) in our prediction, we expect at least about 10 events will be detected by the Belle Collaboration. If there is a large QCD corrections, the number of events will be increased into several hundreds.

In summary, we have calculated the cross section for \( e^+e^- \) annihilation into \( cc\bar{c}\bar{c} \). Assuming quark-hadron duality, the cross section for the inclusive four charm hadrons is predicted to be about 0.1 pb. The comparison of this prediction with the measured cross section for the four charm hadrons at \( B \)-factories will provide a strong constraint in determining the origin of the large discrepancy between prediction and Belle data for exclusive \( J/\psi + \eta_c \) production in \( e^+e^- \) annihilation at \( \sqrt{s} = 10.6 \text{ GeV} \). The measurement will also provide a useful information in explaining large cross section for \( J/\psi + \bar{c}c + X \) measured by the Belle Collaboration [1] compared to the NRQCD predictions [18].

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