Frequency response calculations of carbon nanotube based nanothermophones

Hanping Hu¹, Kai Zhang and Dongdong Wang
Department of Thermal Science and Energy Engineering, University of Science and Technology of China, Hefei, Anhui 230027, China

¹E-mail: hphu@ustc.edu.cn

Abstract. Due to the extraordinary high thermal conductivity and low heat capacity, carbon nanotube (CNT) has shown a great potential as a thermo-acoustic (TA) material recently. It can be used to fabricate novel TA loudspeaker free of magnets and moving parts, a promising device that directly converts thermal energy into acoustic energy. In this paper, we have systemically analysed the frequency responses of the dot/wire/film TA emission from CNTs by using the equations we derived for acoustic field of TA emission from point source and arbitrary source. Formulas used for TA calculation from near-field to far-field can be the same. The dependence of sound pressure level (SPL)-frequency response of all kinds of CNT based nanothermophones on distance and angle are investigated in detail. It is found that TA sound pressure fluctuates in near-field, while increases linearly with logarithmic frequency in far-field. The amplitude of near-field fluctuation increases with increasing distance, but the frequency cycle of the fluctuation decreases and tends to be a constant depending on angle. These studies lay the foundation for further development of CNT based nanothermophones.

1. Introduction
Thermo-acoustic effect was first reported by W. H. Preece [1], who attached an alternating current to thin metal wires or foils at the end of 19th century. Subsequently, Arnold and Crandall used a 700nm-thin platinum film as a thermo-acoustic source, and got the original TA equation [2]. On account of the weak TA sound and backward sound testing tools, TA emission hasn’t received a lot of attention until H. Shinoda et al. in 1999 first proposed to put a conducting 30nm-thin aluminum film on porous silicon (PS) as TA ultrasonic source [3]. In 2002, T. Migita et al. found that TA signals have a sufficient stability in a long-time continuous wave operation and a fast response capability in a relatively wide frequency range not only for pulsed but also impulsive operation [4]. A. O. Niskanmen and his colleagues use suspended metal wire array as aera source of TA sound in 2009 [5]. We also made some theoretical researches around the TA emission from PS, and a serial of TA equations were derived by using a fully thermally-mechanically coupled TA model [6-8].

With the rapid development of material science and nanotechnology, many new functional materials are coming into our life. One of them is the CNT discovered by S. Iijima in 1991 [9]. Because the feature size is hardly to achieve below 20nm, the silicon technology is faced with a bottleneck [10, 11], more and more researchers are seeking for a new material instead of silicon. So, CNT has received a lot of attention [12], due to its prominent high thermal conductivity and low heat capacity per unit area. In 2008, Xiao et al took CNT thinfilm as the TA material [13], and found its good application prospect by measuring CNT thinfilm TA emission in different kinds of gas [14]. But their theoretical equation is just...
the improvement of Arnold and Crandall’s, which can’t suit for near field. After that, Kozlov et al. tested the TA emission from carbon nanotube assemblies [15]. Alievi and his colleagues investigated TA effect in different situations, such as CNT thinfilms are put under water and in a variety of gases [16, 17]. C. W. Li et al also researched thermally induced ultrasonic emission from the suspended CNT thinfilms and listed the formulas of near field and far field based seperately on the expression of plane wave and spherical wave [18]. Tong et al. proposed a theoretical equation of gas-filled encapsulated CNT film TA transducer [19] by taking the window vibrations into consideration. H. Tian et al. reported the graphene TA sound-emitting devices [20-22], which also have promising application prospects. Many researchers did a large number of experiments of TA emission, and their theoretical explainations are mainly based on the Arnold and Crandall’ theory. But in 2010, V. Vesterinen et al. formulated an analytical equation by a way of Green’s function formalism [23], and concluded that the input power and excitation frequency have a linear relationship with sound pressure. However, those studies are simply about the TA emission from a plane, and discussed the features of the TA plane-wave. Few researches concentrate on TA emission from line, and the TA sound emitted from line array is actually resembled to the plane-wave. In this paper, we have devoted ourselves to investigating the characteristics of thermophones which are made of CNT point, line or thinfilm by putting forward a united formula for TA field so that their features and relations can be disclosed more completely.

2. Theoretical formulas
We have derived an equation of TA emission from a small element consist of backing, sample, and heating film [24]:

\[ p_g = \frac{j \omega Q}{4 \pi r^2} \frac{e_g}{v_g} e^{-\frac{j \omega (y-1) \lambda_f}{2v_g}} \]

where \( M = \frac{\epsilon^{s, i} \left( \kappa_i \sigma_b + \kappa_i \sigma_s \right) + e^{-\sigma_i, i} \left( \kappa_i \sigma_b - \kappa_i \sigma_s \right)}{\epsilon^{s, i} \left( \kappa_i \sigma_b + \kappa_i \sigma_s \right) - e^{-\sigma_i, i} \left( \kappa_i \sigma_b - \kappa_i \sigma_s \right)} \), \( \sigma_b = \frac{j \omega}{\alpha_b}, \sigma_s = \frac{j \omega}{\alpha_s} \), subscript \( b, s \) stand for backing, sample respectively. \( L_s \) is the thickness of the sample, and \( r \) is the distance from TA source to the measure point. \( v_g = \sqrt{\rho_b / \rho_s} \) is the sound velocity in gas, \( \gamma \) is the specific heat ratio of gas, \( \kappa_i \) is the thermal conductivity, and thermal diffusivity \( \alpha_i = \kappa_i / \rho_i c_{p,i} \), thermal effusivity \( e_i = \sqrt{\kappa_i \rho_i c_{p,i}} \), subscript \( i \) can take \( s, g \), and \( f \) for the sample, gas, and heating film, respectively. \( C_f = \rho_f \delta_f c_{f} \), \( \delta_f \) is the thickness of the heating film. The periodical heat flow \( q = q_s \left( 1 + \exp( j \omega t) \right) \) is applied to the surface of TA material, and for a small area \( ds \) of heating film, \( Q = q_s ds \).

If the influence of gas viscosity \( \mu_g \) is taken into consideration, Above TA equation can be written as:

\[ dp_g = \frac{j \omega Q}{4 \pi r^2} \frac{e_g}{v_g} e^{-\frac{j \omega (y-1) \lambda_f}{2v_g}} \frac{Q e^{-j \omega (y-1) \lambda_f}}{2v_g \mu_f} \frac{2 \omega \mu_f}{3 v_g \rho} \]

If there is no backing and sample, by replacing the subscript \( b, s \) with \( g \), the Equation (2) is simplified as:

\[ dp_g = \frac{j \omega Q}{2 \pi r^2} \frac{e_g}{2 v_g^2} e^{-\frac{j \omega (y-1) \lambda_f}{2v_g}} \frac{Q e^{-j \omega (y-1) \lambda_f}}{2v_g \mu_f} \frac{2 \omega \mu_f}{3 v_g \rho} \]

The essential TA formulas of nanothermophone can therefore be expressed by superposition of point source as follows:

1. Nanowire thermophone
\[ p_g = \int \frac{j \omega \gamma - 1}{2 \pi r^2 e_g} \frac{1}{2 e_g + \sqrt{j \omega \rho C_f}} q \, e^{\frac{-j \alpha}{2 \pi}} e^{\frac{\alpha^2 (\gamma - 1) \mu_f}{2 \pi^2}} e^{\frac{2 \alpha^2 \mu_f}{3 \pi^2}} \, dl \]  

(4)

2. Nanofilm thermophone

\[ p_g = \int \frac{j \omega \gamma - 1}{2 \pi r^2 e_g} \frac{1}{2 e_g + \sqrt{j \omega \rho C_f}} q \, e^{\frac{-j \alpha}{2 \pi}} e^{\frac{\alpha^2 (\gamma - 1) \mu_f}{2 \pi^2}} e^{\frac{2 \alpha^2 \mu_f}{3 \pi^2}} \, ds \]  

(5)

In following analysis of the characteristics of TA emission from CNT point, line or thinfilm, we assumed that the device of CNT nanothermophone is placed in the air at normal atmospheric pressure and room temperature, and its schematic is shown in Figure 1. Probing positon can be either in vertical direction or in non-vertical direction according to it is in or not the midperpendicular of CNT line/film.

Figure 1. Schematic diagrams of the TA emission from CNT: (a) The single CNT line used for nanowire thermophone; It has length \( L \), and \( t \) is the distance from the measuring position to the midpoint of CNT line; \( \alpha \) is the angle between the midperpendicular of CNT line and the straight line connects the measuring position and the midpoint of CNT line; (b) The square CNT thinfilm with length of side \( L \); \( \theta \) is the altitude angle, and \( \gamma \) is the azimuth angle.

3. Calculation and discussion

To verify above formulas, the comparison between the theoretical and experimental results in vertical direction for TA emission from a suspended CNT thinfilm was conducted. As shown in Figure 2, a good agreement can be seen whether in near-field or in far-field. So, our equations are credible, moreover, no separation of near- and far-field formula is needed, which are convenient for investigating the full-field features of TA emission from nanothermophone.

Figure 2. Sound pressure-frequency response for a suspended CNT thinfilm in air: (a) In near-field; (b) In far-field.

3.1. Frequency response in vertical direction of TA emission from point, line, and areal source

Many researchers studied the sound pressure-frequency response, but seldom explored the relationship of frequency responses among different shapes of TA sources. Figure 3 shows that the change of frequency response in vertical direction of TA emission from CNT point to thinfilm is an evolution
process. There is no wideband flat frequency response for point source, but there is a slowly upward stage for line source, on the contrary, a slowly downward for thinfilm. In addition, when the measuring point is located in a relatively distant place, the curve of SPL from line or thinfilm is same as that of point source.

![Figure 3](image_url)

**Figure 3.** Frequency response changing with the measuring position in vertical direction: (a) point source; (b) line source; (c) areal source.

![Figure 4](image_url)

**Figure 4.** Frequency response changing with distance in non-vertical direction for TA source of (a)(b) nanowire and (c)(d) nanofilm.

3.2. Frequency response in non-vertical direction of TA emission from line and areal source

3.2.1. **TA Frequency response in non-vertical direction changing with distance.** Frequency response of TA wave in non-vertical direction has its special characteristics (see Figure 4). It is found that when the angle is fixed, increasing the distance t, the curve of SPL moves down not only for CNT line but also thinfilm. They all have a fluctuation segment of frequency response, and the amplitude of the fluctuation increases with the increasing measure distance from source. The frequency cycle of the fluctuation
decreases with the increase of measuring distance and tends to be a constant associated with angle (see Figure 4(b) and 4(d), 4(d) is the enlarger image of Figure 4(c)). However, it needs to point out that, for areal TA emitting, the regular cycle of the fluctuation exists only at the measuring position with projection on the axis of symmetry of thinfilm.

3.2.2. TA Frequency response in non-vertical direction of line source changing with angle. The SPL-frequency response of TA wave also changes with angle. Because of different spatial dimension, TA effect should be considered separately for CNT line and thinfilm. Figure 5 shows the angle dependence of frequency response for CNT line. When the measuring distance \( t \) is too closer to the center of line, the sound pressure increases with increasing angle \( \alpha \) (see Figure 5(a)), because, the closer to the CNT line, the more CNT line is regard as infinite, and increasing measured angle \( \alpha \) actually means reduce the measuring distance \( t \). However, with the increase of measured distance \( t \), the SPL curve decreases with increasing angle, especially at high frequency (see Figure 5(b) - 5(c)). This discrepancy can owe to the interference effect at high frequency. In addition, the amplitude of SPL fluctuation increases with increasing angle.

![Figure 5](image)

**Figure 5.** Frequency response changing with angle in non-vertical direction (CNT nanowire).

3.2.3. TA Frequency response in non-vertical direction of area source changing with angle. The influence of angle change on TA signal from a suspended CNT thinfilm includes that of altitude angle and azimuth angle. Figure 6 shows that, the smaller altitude angle, the lower SPL at high frequency, but there is no obvious difference at low frequency (<5 KHz). By damping factor \( \exp(-\omega^2(\gamma - D - \frac{\alpha_s r}{2v_g^3} - \frac{2\omega^2 v_g^3}{3v_s^3}\rho)) \) in Equation (5), the higher the frequency or the larger the measuring distance, the more the SPL attenuation. Reducing altitude angle means enlarging the distance from measuring position to the suspended thinfilm, thus, causes TA decay, and the frequency induced attenuation is quadratic, hence, significant at high frequency. High frequency oscillation may be irregular, as seen in Figure 6(b), if the projection of measuring position is not located in the axis of symmetry of TA source. The influence of azimuth angle is similar to that of altitude angle (see Figure 7), and the change of azimuth has little effect on the SPL in the case of large altitude angle (see Figure 7(c)).

![Figure 6](image)

**Figure 6.** Frequency response changing with altitude angle (suspended CNT thinfilm).
Support from NSFC (Grant No. 10974194, 51276178) is acknowledged.

**4. Conclusions**

In this paper, the theoretical formulas for calculating TA sound field of all kinds of nanothermophone are given and verified, and the frequency response features with distance and angles for point-, line-, and areal TA source are compared with each other and studied for in-depth understanding. All shapes of CNT as TA material have some common characters that the SPL is in proportion to applied heat flux, and inversely proportion to measuring distance in vertical direction. However, if measuring in non-vertical direction, that is, at an angle with midperpendicular of TA source, when measuring point is just in the symmetry plane of TA source and the distance is enough, there exists a fixed frequency cycle of SPL fluctuation. These findings may have significant for developing CNT based nanothermophones.

**Acknowledgement**

Support from NSFC (Grant No. 10974194, 51276178) is acknowledged.

**References**

[1] Preece W H 1879-1880 Proc. R. Soc. London 30 408-411
[2] Arnold H D and Crandall L B 1917 Phys. Rev. 10 22-38
[3] Shinoda H, Nakajima T, Ueno K and Koshida N 1999 Nature 400 853-855
[4] Migita T, Shinoda H and Koshida N 2002 Jap. J. Appl. Phys. 41 2588
[5] Niskanen A O, Hassel J, Tikander M, Maji J, Maijala P, Grönnberg L and Helistö P 2009 Appl. Phys. Lett. 95(16) 163102
[6] Hu H, Zhu T and Xu J 2010 Appl. Phys. Lett. 96 214101
[7] Hu H, Wang Y and Wang Z 2012 J. Phys. D. 45 345401
[8] Hu H, Wang Z, Wu H and Wang Y 2012 AIP Advances 2 032106
[9] Iijima S 1991 Nature 354 56-58
[10] Schulz M 1999 Nature 399(6738) 729-730
[11] Powell J R 2008 Proceedings of the IEEE 96(8) 1247-1248
[12] Venkataraman R 2010 Nature 463(7281) 619-619
[13] Xiao L, Chen Z, Feng C, Liu L, Bai Z Q, Wang Y and Fan S 2008 Nano Lett. 8 4539-4545
[14] Xiao L, Liu P, Liu L, Li Q, Feng Z, Fan S and Jiang K 2011 J. Appl. Phys. 110 084311
[15] Kozlov M E, Haines C S, Oh J, Lima M D and Fang S 2009 J. Appl. Phys. 106 124311
[16] Aliev A E, Lima M D, Fang S and Baughman R H 2010 Nano Lett. 10(7) 2374-2380
[17] Aliev A E, Gartstein Y N and Baughman R H 2013 Nanotechnol. 24 235501
[18] Lim C W, Tong L H and Li Y C 2013 J. Sound and Vibration 332(21) 5451-5461
[19] Tong L H, et al. 2013 J. Vib. Acoust. 135 051033
[20] Tian H, Ren T L, et al. 2011 ACS nano 5(6) 4878-4885
[21] Tian H, Xie D, Yang Y, Ren T L, et al. 2012 Nanoscale 4(7) 2272-2277
[22] Tian H, Xie D, Yang Y, Ren T L, et al. 2012 Nanoscale 4(11) 3345-3349
[23] Vesterinen V, Niskanen A O, Hassel J and Helisto P 2010 Nano Lett. 10 5020-5024
[24] Hu H, Wang D and Wang Z 2014 AIP Advances 4(10) 107114

![Figure 7. Frequency response changing with azimuth angle (suspended CNT thin film).](image-url)