CUTOFF FOR THE MEAN-FIELD ZERO-RANGE PROCESS

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We study the mixing time of the unit-rate zero-range process on the complete graph, in the regime where the number $n$ of sites tends to infinity while the density of particles per site stabilizes to some limit $\rho > 0$. We prove that the worst-case total-variation distance to equilibrium drops abruptly from 1 to 0 at time $n \left( \rho + \frac{1}{2}\rho^2 \right)$. More generally, we determine the mixing time from an arbitrary initial configuration. The answer turns out to depend on the largest initial heights in a remarkably explicit way. The intuitive picture is that the system separates into a slowly evolving solid phase and a quickly relaxing liquid phase. As time passes, the solid phase dissolves into the liquid phase, and the mixing time is essentially the time at which the system becomes completely liquid. Our proof combines meta-stability, separation of timescales, fluid limits, propagation of chaos, entropy, and a spectral gap estimate by Morris [1].

References

1. Ben Morris. Spectral gap for the zero range process with constant rate. *Ann. Probab.*, 34(5):1645–1664, 2006. MR2271475