MAGNETICALLY LEVITATING ACCRETION DISKS AROUND SUPERMASSIVE BLACK HOLES

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ABSTRACT

In this paper, we report on the formation of magnetically levitating accretion disks around supermassive black holes (SMBHs). The structure of these disks is calculated by numerically modeling tidal disruption of magnetized interstellar gas clouds. We find that the resulting disks are entirely supported by the pressure of the magnetic fields against the component of gravitational force directed perpendicular to the disks. The magnetic field shows an ordered large-scale geometry that remains stable for the duration of our numerical experiments extending over 10% of the disk lifetime. Strong magnetic pressure allows high accretion rates and inhibits disk fragmentation. This in combination with the repeated feeding of magnetized molecular clouds to an SMBH yields a possible solution to the long-standing puzzle of black hole growth in the centers of galaxies.

Key words: accretion, accretion disks – Galaxy: nucleus – ISM: clouds – magnetic fields – magnetohydrodynamics (MHD) – methods: numerical

Online-only material: color figures

1. INTRODUCTION

It is believed that the growth of supermassive black holes (SMBHs) in centers of galaxies is enabled by gas accretion from surrounding disks (Lynden-Bell 1969) which have been observed with increasing precision by modern telescopes (Miyoshi et al. 1995; Jaffe et al. 2004). In the early theoretical work (Lynden-Bell 1969; Shakura & Sunyaev 1973), it has been suggested that magnetic stresses play an important role in driving the accretion by enabling the outward angular-momentum transport through the disk. This suggestion has been put on a firm theoretical footing by the Balbus et al. (1991) discovery of the importance of magnetorotational instability (MRI) in astrophysical disks, and by the subsequent work, which demonstrated the ability of MRI to build and maintain substantial magnetic stresses inside the disk (Brandenburg et al. 1995; Stone et al. 1996; Hirose et al. 2006; Davis et al. 2010). All of the numerical studies to date have demonstrated MRI-generated magnetic stresses which are associated with the sub-thermal magnetic fields in the disk mid-plane. One of the central unresolved issues of feeding SMBHs has been the tendency of all modeled extended gaseous disks to clump due to their self-gravity (Kolykhalov & Syunyaev 1980; Shlosman & Begelman 1987; Shlosman et al. 1990; Goodman 2003; Rafikov 2009). Such choking of the accretion flow is a major obstacle in SMBH growth. It has been conjectured (Shibata et al. 1990; Machida et al. 2000, 2006; Pariev et al. 2003; Begelman & Pringle 2007; Oda et al. 2009) that in some astrophysical disks magnetic stresses may become dominant relative to the mid-plane gas pressure, and that these disks may effectively resist fragmentation. In this paper, we investigate the formation of accretion disks by performing numerical simulations of collisions between magnetized gas clouds and a black hole. It has been suggested that such collisions may be responsible for feeding the SMBHs at the centers of galaxies (King & Pringle 2007; Wardle & Yusef-Zadeh 2008, 2012) and that it may have led to the formation of the stellar disk in our own Galactic center (Sanders 1998; Levin & Beloborodov 2003; Paumard et al. 2006; Bonnell & Rice 2008; Hobbs & Nayakshin 2009). We find that the resulting disks are completely dominated by the magnetic field pressure and display high accretion rates due to the Maxwell stress associated with the large-scale magnetic field the structure of which remains stable over the duration of the simulation. The Toomre–Q factors of these naturally formed magnetically levitating accretion disks (MLADs) indicate their stability to gravitational fragmentation. Therefore, MLADs represent a new class of accretion-disk solutions which may play an important role in feeding the SMBHs.

2. SIMULATIONS SETUP

We model a collision between a magnetized gas cloud and an SMBH using a new moving-mesh ideal MHD scheme (see the Appendix). We choose an equation of state \( P_{\text{gas}} = c_s^2 \rho \), where \( c_s = 0.03 v_K \); here \( v_K \) is Keplerian velocity around an SMBH. The temperature in this setup is \( T = 1.63 \times 10^4 \) K (0.1 pc/R), which, in the absence of magnetic fields, would produce disks with \( H_0/R = 0.03 \). In the presence of magnetic fields the effective scale height is modified by the magnetic pressure, \( H = H_0 \sqrt{1 + \beta^{-1}} \), where \( \beta^{-1} = P_m/P_g \) is the ratio of magnetic to gas pressures. In order to isolate the effects of magnetic fields on the disk formation process, we ignore effects of the gas self-gravity in our calculations.

We conduct three high-resolution simulations with \( 1.6 \times 10^7 \) particles in which a magnetized gas cloud with mass and radius \( 3.5 \) pc and \( 8.8 \times 10^6 M_\odot \), respectively, is collided with a \( 3.5 \times 10^9 M_\odot \) SMBH. The geometry of the initial conditions is taken from Alig et al. (2011) and repeated in Table 1; in particular, we use initial conditions from their simulations C01, V01, and V02, where a molecular cloud has an impact parameter of 2 pc, 3 pc, and 3 pc, respectively. On top of these initial conditions we also impose a uniform magnetic field that threads

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both the cloud and vacuum regions. The magnetic field strength is such that the resulting magnetization inside the cloud is \( \beta = 1 \), which corresponds to \( |B| \approx 100 \mu G \) and dimensionless mass-to-flux ratio \( \zeta \approx 5 \), where \( \zeta = (M/\Phi)/\sqrt{5/(9\pi^2 G)} \) (Mouschovias & Spitzer 1976; Mac Low & Klessen 2004). This field strength corresponds to the large-scale field in the Galactic center (Yusef-Zadeh & Morris 1987; Morris & Yusef-Zadeh 1989; Crocker et al. 2010). The initial magnetic field orientation was such that each of the components of the magnetic field have the same magnitude, namely \( B_x = B_y = B_z = B/\sqrt{3} \). The vacuum is modeled with fluid 10⁶ times less dense than the cloud density (\( \rho \)), which we also use as a floor density to avoid local density contrasts larger than \( 10^7 \) that our code cannot deal with due to use of single-precision floating point arithmetics.

The computational domain is a periodic box with \( 32 \times 32 \times 32 \) pc³ volume, which is large enough that it does not influence physical processes occurring in sub-parsec regions. The mass and distance units were \( [M] = 10^5 M_\odot \) and \( [R] = 1 \) pc, respectively, which set the time unit \([T] \approx 0.047 \) Myr, magnetic field units \( [B] \approx 5.40 \) mG, and the speed unit \([V] \approx 20 \) km s⁻¹. The simulation lasted till \( t_{\text{end}} = 5.0 \), which corresponds to 0.24 Myr or 4.7 orbital periods at \( R = 1 \) pc. The inner boundary conditions are applied only within 0.02 pc from the SMBH, which we regard as the inner disk boundary, as follows. Any particle within 0.01 pc is removed from the computational domain, and in the transition region between 0.01 pc and 0.02 pc we set the density to the floor value, and both the velocity and the magnetic field to zero.

3. RESULTS

3.1. Geometry of the Collision

A collision between a molecular cloud and an SMBH is a violent event occurring on dynamical timescale. Since fluid elements generally have non-zero angular momentum, the natural outcome of such an event is the formation of a disk. Hydrodynamical simulations of such collision event robustly show the formation of an eccentric disk around SMBH with the disk geometry being dependent on the initial conditions (Alig et al. 2011). Our aim in this work is to study a similar event but in a strongly magnetized regime in which the initial magnetic pressure in the cloud is in equipartition with the gas thermal pressure.

In Figure 1 we show snapshots of the gas density in the XY plane at \( t = 0, 47, 94, \) and 240 thousand years. In the first hundred thousand years, the cloud experiences a violent collision with the black hole. In particular, the bow shock, which can be seen in the bottom left panel as a large curved region with density jump just above the disk, is formed by isothermal shock guards the newly formed inner disk from the destructive effect of the incoming fluid. The outcome of this collision event is a formation of a parsec-size gas disk with irregular density structure. Similar disks were formed in other simulations, as can be seen in the top two panels of Figure 2. This can be contrasted with Alig et al. (2011) where simulations C01 and V01 have final differently shaped gas disks. Finally, in the bottom right panel of Figure 2 we show the divergence error, \(|\nabla \cdot B|/|B|\) where \( h \) is the cell size, in the final snapshot of V01 simulation.

In Figure 3 we show magnetic field geometry in different regions of the disk at the end of the V01 simulation, which corresponds to approximately 150 orbital periods at \( R = 0.1 \) pc. The top left panel shows that magnetic field lines originated in the central region of the disk and extend above and below mid-plane. The magnetic field in this regions is dominated by poloidal components. The top right panel shows mid-plane magnetic field structure in the central region (\( R \sim 0.1 \) pc). The field lines appear to be regular and tightly winding in an azimuthal direction, which is the result of strong Keplerian shear inside the disk. In the bottom left panel, we also show the magnetic field in the mid-plane region but further away from the center (\( R \sim 0.5 \) pc). While magnetic field is still stretched in the azimuthal direction, in contrast to the central regions, it shows less regular structure. In the bottom right panel we show the magnetic field in the disk corona, where the magnetic field shows a regular large-scale azimuthal pattern.

3.2. Vertical Structure

This section focuses on the vertical disk structure in our simulations. In particular, we studied the vertical structure of the disk at \( R \approx 0.1 \) pc, which is far enough from the inner boundary and close enough that the disk performed approximately 100 orbital periods by the end of the simulation. One of the crucial properties in magnetized disk simulations is the MRI quality factor, \( Q \), which is related to the number of resolution points, e.g., grid points, particles or mesh cells, per MRI fastest growing mode wavelength. With this number being too low (\( Q \lesssim 8 \)), the simulation may fail to faithfully model MRI (e.g., Hawley et al. 2011). Due to the nature of our simulations, it was impossible a priori to identify which of our simulations can faithfully model long-term disk evolution. As a result, we computed MRI quality factors in vertical and azimuthal directions at the end of our simulations, and determined which of the simulations were able to resolve MRI. Namely, we compute \( Q_z = \lambda_z/h \) and \( Q_\phi = \lambda_\phi/h \), where \( h \) is the size of a resolution element and \( \lambda_z, \phi \approx 2\pi |B_{z,\phi}|/(\sqrt{\rho} \Omega) \).

In Table 2, we show vertically averaged quality factors at \( R \approx 0.1 \) pc. This table demonstrates that all simulations have \( Q_\phi \gtrsim 8 \), which means that they can faithfully model non-axisymmetric MRI. However, only the V01 simulation qualifies when it comes to axisymmetric MRI, and therefore we focus our study of vertical structure on this simulation.

*Notes.* Column 1 (ID) shows the name of a run, Column 2 (\( R_{cl} \)) shows the radius of a cloud, Column 3 (\( v_{cl} \)) shows the cloud infall speed in km s⁻¹, and finally the last column (\( b \)) shows the cloud’s impact parameter.

| ID   | \( R_{cl} \) (pc) | \( v_{cl} \) (km s⁻¹) | \( b \) (pc) |
|------|-------------------|----------------------|-------------|
| C01  | 3.5               | 120                  | 2           |
| V01  | 3.5               | 30                   | 3           |
| V02  | 3.5               | 50                   | 3           |

| ID   | \( Q_z \) | \( Q_\phi \) |
|------|----------|-------------|
| C01  | 3        | 26          |
| V01  | 11       | 60          |
| V02  | 6        | 48          |

\( \approx \) Since the inner region of the disk is formed at approximately \( t \approx 0.05 \) million years, the actual number of disk revolutions at \( R \sim 0.1 \) pc is \( \lesssim 100 \).
We studied the vertical structure of the disk in the V01 simulation at $R \approx 0.1$ pc, which is far enough from the inner boundary and close enough that the disk performed approximately 100 orbital periods by the end of the simulation. We compute scale height, $H$, at this radius by fitting an isothermal density profile in approximately two scale heights. The resulting scale height is $H \approx 0.01$, which gives $H/R \approx 0.1$ (top left panel in Figure 4). The radial temperature dependence is expected to produce disks with the scale height $H/R \approx 0.03 \sqrt{1 + \beta^{-1}}$ which, for $\beta^{-1} = P_m/P_g \approx 10$ found at $R = 0.1$ pc, gives $H/R \approx 0.1$, consistent with the simulation data (bottom right panel in Figure 4). We also studied the deviation of azimuthal velocity from the Keplerian velocity at $R = 0.1$ pc as a function of height, and found that azimuthal velocity variations are less than a percent for $|z| < H$ (top right panel in Figure 4). It is therefore justifiable to assume that the disk angular velocity is constant on cylinders. Finally, in the bottom left panel of the Figure 4, we show Maxwell stress $\alpha_M = -(B_r B_\phi)/P_{tot}$ which is approximately 0.1 within the scale height.

In Figure 5 we show an azimuthally averaged magnetic field. This figure shows that the magnetic field is confined within a few scale heights of the mid-plane. The magnetic field is dominated by the azimuthal component that is an order of magnitude larger than the radial one. The vertical component, $B_z$, is much smaller compared to both $B_r$ and $B_\phi$ for $|z| \lesssim H/2$ (in this figure both $B_r$ and $B_z$ strength are magnified by a factor of 10).

All of our simulations show similar vertical confinement of the field, which can be interpreted as a result of the disk formation: a combination of isothermal shocks that amplify the magnetic field and Keplerian shear which generates a strong azimuthal field component. However, we would like to stress that the field confinement in our best-resolved model (V01) is in good agreement with Johansen & Levin (2008; hereafter JL) who find similar results in their shearing box models. In the JL shearing-box simulations, which were performed with a grid-based Pencil Code, the disk field was initially in equipartition with the gas pressure, but evolved by Parker and MRIs to a magnetic field configuration in the vertical direction similar to what we see in our disk which is formed via a collision of a magnetized gas cloud with the black hole. It is significant that the two simulations that are so different in their approach give vertical structure of $B_\phi$ that is in a good agreement with each other. In particular, the azimuthal
component changes sign at 2–3 scale heights above the mid-plane, and the strength of the mid-plane field is 10 times the value of the reversed field.

It is likely that the process responsible for flux confinement is of a similar nature as described in JL. We sketch it in Figure 6. The strong azimuthal magnetic field is subject to Parker instability. As the fluid elements slide along the field line toward the mid-plane (top left) they are acted upon by the Coriolis force (Hanasz et al. 2002), and due to this the line becomes helical (bottom left). This generates a radial field, which via Keplerian shear renews the azimuthal field that was lost to Parker instability (top right). Further shearing generates oppositely oriented azimuthal fields above the mid-plane which reconnects with the field of the original orientation. This decreases its magnitude or even reverses its direction, whereas the field at the mid-plane is amplified while maintaining its original direction (bottom right).

3.3. Radial Structure

The radial evolution of the disk is driven by the flow of matter from the outer to the inner regions. This flow is enabled by the effective viscosity from magnetohydrodynamical stresses. Within the viscous timescale, the disk reaches steady radial structure which is computed by applying conservation laws and theory of steady thin disks (e.g., Frank et al. 2002). Here we present some numerical evidence for an extra constraint, the conservation of azimuthal magnetic flux, \( \Phi = \int B \phi \, dS \) where \( dS = dz \, v_r \, dt \). The set of equations that describe disk’s radial structure is

\[
\dot{M} = 2 \pi R \Sigma v_t, \tag{1}
\]

\[
\dot{\Phi} = 2 H B_\phi \, v_r, \tag{2}
\]

\[
M = 3 \pi \alpha_{acc} \Sigma H^2 \Omega. \tag{3}
\]

Here, Equation (2) describes the frozen-in condition of magnetic field.\(^8\) In what follows we assume that the right-hand side of

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\(^7\) In Figure 7 of JL the scale height can be increased by \( \sqrt{2} \) due to extra support provided by the magnetic pressure.

\(^8\) The condition can also be derived from the induction equation by consideration of the radial advective flux of the vertically integrated azimuthal magnetic field, \( \int B_\phi \, dz \).
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Figure 3. Magnetic field geometry in different regions of the disk. The density is shown in the XZ plane, with values given in \( \log(\rho/(4.1 \times 10^6 \text{ cm}^3/m_u)) \). The top two panels show magnetic field geometry in the central region of the disk \((R \sim 0.1 \text{ pc})\), whereas two bottom panels focus on the outer regions of the disk \((R \sim 0.5 \text{ pc})\). While magnetic field geometry is mostly uniform, the gas density shows irregular structure.

(A color version of this figure is available in the online journal.)

these equations are constants for steady-state disks. We also assume that accretion viscosity \( \alpha_{\text{acc}} \) is set by Maxwell stresses

\[
\alpha_{\text{acc}} = \alpha_m = -\frac{\langle B_r B_\phi \rangle}{4\pi P},
\]

where \( P = \Omega^2 H \Sigma \) is the total pressure. The disk scale height is

\[
H = H_0 \sqrt{1 + \beta^{-1}},
\]

where \( H_0 = c_s/\Omega \) is a hydrodynamical scale height. According to the numerical results from the previous section, the magnetic pressure is entirely dominated by \( B_\phi \), which gives \( P_m \approx B_\phi^2/8\pi \).

Using these equations and noting that \((1 + \beta) B_\phi^2/8\pi = \Omega^2 H \Sigma \), we derive the radial dependence of disk scale height,

\[
\frac{H}{R} = \left( \frac{3\pi^2 GM (1 + \beta)}{40 \alpha_m \zeta^2 (\Omega R)^3} \right)^{1/5},
\]

where we write \( \Phi = \sqrt{5/(9\pi^2G)}M/\zeta \). In the limit, \( \beta \ll 1 \), the radial dependence of disk magnetization is

\[
\beta^{-1} = \left( \frac{3\pi^2 GM}{40 \alpha_m \zeta^2 (\Omega R)^3} \right)^{2/5} \left( \frac{R}{H_0} \right)^2.
\]

In a general case, \( H_0 \) is self-consistently computed by solving the radiative transfer equation in the vertical direction. Therefore, the magnetization depends on the thermal properties of the disk. However, in our simulations \( H_0 = 0.03 R \) from which we have \( \beta^{-1} \propto R^{2/5} \).

Since the radial dependence of \( \alpha_m \) is not possible to establish from the first principles, we obtain it empirically. In our numerical experiments \( \alpha_m \propto R^{1/2} \) is consistent with the data. This relationship predicts that the disk magnetization should decrease with radius, \( \beta^{-1} \propto R^{-2/5} \), in agreement with our simulations (upper panel in Figure 7). This radial dependence of \( \alpha_m \) is specific to our simulations which we use to check for consistency. However, a similar dependence was found by Flock et al. (2011) in the case of weakly magnetized disks. In a realistic disk, however, we expect that \( \alpha_m \) is a function of the local \( \beta \), which will be the subject of subsequent research.

Using the equations above, we derive the radial dependence of \( B_\phi, B_r, \) and \( \Sigma \). To derive \( B_r \), we use the fact that in our disks the Maxwell stresses in Equation (4) are dominated by the mean field, \( \langle B_r B_\phi \rangle \approx \langle B_r \rangle \langle B_\phi \rangle \). The resulted radial dependences are

\[
\Sigma = \left( \frac{1600 M^3 \zeta^4}{2187\pi^9 G^2(1 + \beta)^2} \right)^{1/5} \left( \frac{\Omega R^{1/5}}{\alpha_m^{3/5} R} \right) \propto R^{-7/5},
\]

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\]
$B_\psi = \left( \frac{512 \sqrt{5}}{27 \pi} \frac{M^2 \zeta}{\sqrt{G(1 + \beta)^3}} \right)^{1/5} \frac{(\Omega R)^{4/5}}{\alpha_m^{2/5} R} \propto R^{-3/5}. \quad (9)$

$B_r = \frac{\alpha_m}{2} B_\psi \propto R^{-11/10}. \quad (10)$

These equations are consistent with our simulations throughout most parts of the disk (bottom panel in Figure 7), except in the regions close to the disk inner boundary (or scale-height resolution limit in case of low-resolution simulations). We also do not expect the model to hold for $R \gtrsim 0.4 \text{ pc}$, where the viscous time estimated using steady thin disk approximation is much larger than the duration of the simulation. Agreement with the analytical model beyond this radius implies that disk evolution occurs at a higher than viscous rate derived from the steady thin disk theory. We also note that the gas density distribution in the disk is not steady, but exhibits clumpy and filamentary structures (right panel in Figure 3). This is also reflected in the irregularity of the surface density profile in Figure 7. Nevertheless, agreement of radial dependence between the model and simulations indicates that azimuthal magnetic flux is conserved in MLADs during accretion.

To verify that the mass accretion is physical, we extract azimuthally and vertically averaged $\Sigma, H, \dot{M}$ at different radial locations and use Equation (3) to calculate $\alpha_{\text{acc}} = \dot{M}/(3\pi \Sigma H^2 \Omega)$. If the accretion is driven by magnetohydrodynamical stresses, this value should be comparable to the azimuthally and vertically averaged sum of Maxwell and Reynolds stresses ($\alpha$). We show the results in Table 3, which shows good agreement between measured ($\alpha$) and derived viscosity ($\alpha_{\text{acc}}$) coefficients. This reinforces our confidence that the mass accretion is indeed driven by magnetohydrodynamical stresses. Finally, using data from this table, we estimate a viscous timescale for a parsec-size accretion disk to be $t_{\text{visc}} = (R/H)^2 \alpha^{-1} \Omega^{-1} \approx 10^8 \text{ years}$, where we use $\alpha = 0.2$ and $H/R = 0.2$ at $R = 1 \text{ pc}$.

4. FRAGMENTATION

The striking result that $\Sigma$ and $H$ do not depend on the thermal properties of MLAD allows a robust estimate of its macroscopic gravitational stability.\footnote{Some of the gas clumps and filaments may form stars even if the disk is globally stable.} Its fragmentation boundary...
is determined by two parameters: $\dot{M}$ and dimensionless mass-to-flux ratio $\zeta$. The latter one allows us to compute the magnetic flux accretion rate from the mass accretion rate, $\Phi \propto \dot{M} / \zeta$, since the magnetic field is frozen in the fluid. Using Equations (6) and (8), the Toomre-$Q$ parameter is (Toomre 1964; Goldreich & Lynden-Bell 1965)

$$Q = \frac{\Omega^2 H}{\pi G \Sigma} = \left[ \frac{6561\tau^6 (1 + \beta)^3 \alpha_m^2 (\Omega R)^6}{64000 \sqrt{G M^2 \zeta^6 \kappa_{es} \epsilon}} \right]^{1/5}. \quad (11)$$

If we write $\dot{M}$ in terms of Eddington luminosity, $I_E = L / L_{\text{Edd}}$, and radiative efficiency, $\epsilon = L / (\dot{M} c^2)$,

$$\dot{M} = \frac{4\pi GM I_E}{\kappa_{es} \epsilon}, \quad (12)$$

where $\kappa_{es} \approx 0.4 \text{ cm}^2 \text{ g}^{-1}$ is electron scattering opacity, and $c$ is speed of light, we obtain

$$Q = \left[ \frac{6561\tau^6 (1 + \beta)^3 \alpha_m^2 (\epsilon I_E)^2}{4^3 10^7 G^2 \zeta^6 \kappa_{es} \epsilon} \right]^{1/5}. \quad (13)$$

Using Equation (13) we find the fragmentation boundary beyond which $Q < 1$,

$$R_{\text{frag}} \approx 2.09 \left( \frac{M_6 \alpha_{0,1} \epsilon_2^{1/2}}{\xi_{10}^{6/2}} \right)^{1/3} \text{ pc}, \quad (14)$$

where we used $\epsilon = 0.1 \epsilon_{0,1}$, $\alpha_m = 0.1 \alpha_{0,1}$, $\xi = 10 \xi_{10}$, and $M = 10^6 M_6 M_6$. The radial dependence of enclosed mass and $H/R$ within the fragmentation radius is given by

$$\frac{M_{\text{disk}}}{M} = \frac{\sqrt{10 \pi r^{9/10}}}{3 \xi} \approx 0.331 \frac{r^{9/10}}{\xi_{10}}, \quad (15)$$

$$\frac{H}{R} = \frac{3 \pi r^{3/10}}{2 \sqrt{10 \xi}} \approx 0.149 \frac{r^{3/10}}{\xi_{10}}, \quad (16)$$

where we define $r = R / R_{\text{frag}}$. It is worth noting that at the fragmentation boundary $M_{\text{disk}}$ and $H/R$ depend only on the mass-to-flux ratio.

In future work, we will use our MLAD solution to model observations of active galactic nucleus accretion disk. Here, we briefly consider a parsec-sized disk in NGC 1068 (Jaffe et al. 2004) as an example. This Seyfert 2 galaxy hosts an $\sim 10^7 M_6$ SMBH with a disk extending to a distance of $\sim 1$ pc. The observed upper bound for $H/R \sim 0.6$ and the hydrogen column density $N_H \sim 10^25 \text{ cm}^{-2}$ (Matt et al. 2004; Köhler & Li 2010), and its luminosity is $\sim 0.4 L_{\text{Edd}}$ (Pier et al. 1994). Here, we assume that this SMBH accretes at an Eddington rate ($I_E \approx 1$) with 10% radiative efficiency ($\epsilon_{0,1} \approx 1$); we set $\alpha_{0,1} = 1$. We find that by setting $\xi = 3$, we are able to obtain values for disk thickness and column density that are consistent with observations. For these parameters, the MLAD thickness at the edge of such disk is $H/R \approx 0.15$. While this is lower than the observed value, it is plausible that strong magnetic fields contribute toward increasing disk thickness. Finally, using the enclosed disk mass at this location and assuming that the disk consists purely of atomic hydrogen, we estimate $N_H \approx 1.3 \times 10^{25} \text{ cm}^{-2}$ which is consistent with the observational data. Furthermore, the fragmentation boundary is located at $\approx 50$ pc, which indicates that a parsec-sized disk is stable to clumping. This MLAD model predicts that such disks should have magnetic field strength of $\sim 100$ mG.

5. CONCLUSIONS

In this paper, we produce from first principles dynamically stable models of accretion disks in a state of magnetic levitation.

Table 3: Accretion Rate and Effective Viscosity

| $R$ (pc) | $M$ ($M_\odot$ yr$^{-1}$) | $\alpha_{\text{acc}}$ | $\alpha$ |
|----------|---------------------------|-----------------------|---------|
| 0.05     | 0.035                     | 0.06                  | 0.03    |
| 0.1      | 0.066                     | 0.09                  | 0.14    |
| 0.2      | 0.070                     | 0.22                  | 0.18    |
| 0.4      | 0.085                     | 0.37                  | 0.51    |
We show that such disks are the natural outcome of in-fall of a massive magnetized molecular cloud onto SMBH. Such MLADs enable large accretion rates due to the large scale height and $\alpha \gtrsim 0.1$. In our simulations, the geometry and strength of the large-scale magnetic field are stable for at least 0.24 Myr, corresponding to several hundred orbits at the disk inner edge. With measured accretion rates of $\approx 0.05 M_\odot \, yr^{-1}$, this is more than 10% of the disk lifetime, supporting the claim that such magnetic field structure is possibly long lived. The viscous timescale of such magnetically levitating disk is estimated to be a few million years. Interestingly, this feature may help solve a theoretical problem that was recently identified by Alexander et al. (2012) with respect to the formation of the stellar disk in our Galactic center. These authors show that if, as according to the currently accepted scenario (Levin & Beloborodov 2003; Nayakshin et al. 2007; Bonnell & Rice 2008), the stellar disk formed as a result of fragmentation of the massive gaseous accretion disk several million years ago, then a substantial gaseous remnant of the accretion disk should survive to the present epoch, due to the expected long viscous time of the standard Shakura–Syunyaev thin disks. Such a remnant is not observed. On the other hand, the expected short lifetime of the MLAD may solve the problem of the missing remnant gas disk.

A unique property of magnetically levitating disks is that their surface density and scale height are independent of the disk’s thermal structure. This is expected because thermal effects are superseded by magnetic properties in determining disk structure. Magnetic levitation allows the disk to withstand its own self-gravity to large distances. A strong dependence of the fragmentation radius on the mass-to-flux ratio of the parent cloud permits a scenario in which a tidal disruption of a magnetized cloud forms a magnetized gas ring. The inner parts spread inward on a timescale determined by global magnetic stresses which fuel fast accretion onto the central SMBH, while the outer part fragments into stars.
A proper understanding of the field confinement requires both local and global analysis. In accordance with JL, the field confinement appears to be a local phenomenon and its stability is likely to depend on the relative strength of the vertical and azimuthal magnetic fields, which itself depends on both kinematics and magnetization of the infalling matter. However, our simulations show that there are non-local processes that generate a global coherent magnetic field structure. Its topology and strength are very important for accretion flows near black hole horizons (Beckwith et al. 2008; Tchekhovskoy et al. 2011; Tchekhovskoy & McKinney 2012). Therefore, it is also important to understand the long-term evolution of the field topology across several decades in the disk radius. We believe that both local and global simulations are essential to our understanding of MLADs.

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APPENDIX

NUMERICAL METHOD

In this Appendix, we demonstrate the ability of our numerical scheme to model MHD flows. Our numerical method combines a moving-mesh approach (Tress 1988; Springel 2010) with a weighted particle MHD scheme (Gaburov & Nitadori 2011). At every time step, a Voronoi mesh is re-built on the set of particles that is used to solve equations of ideal MHD in the same way as in the weighted particle scheme. A similar approach has also been attempted in TESS (Duffell & MacFadyen 2011) and AREPO (Pakmor et al. 2011) moving-mesh codes. In contrast to these two approaches but similarly to Gaburov & Nitadori (2011), we add a source term to the induction equation that restores Galilean invariance of scheme in the case of \( \nabla \cdot \mathbf{B} \neq 0 \). This proved to be crucial to stabilize the numerical scheme in the presence of strongly magnetized super-Alfvénic flows. The numerical code is publicly available for download.\(^{10}\) Here, we present validation of our numerical method by means of two test problems: propagation of circularly polarized Alfvén wave and the linear regime of MRI in a cylindrical disk.

A.1. Circularly Polarized Alfvén Wave

This problem was first presented by Tóth (2000) as an exact nonlinear test problem for an ideal MHD. Following Tóth (2000) we set the following initial conditions. We use a periodic three-dimensional computational domain with the total number of particles equal to \( N_{\text{tot}} = N_x \times N_y \times N_z = 16 \times 16 \times 16 \), where \( N_x = 16, 32, 64 \), and 128. Particles were initially randomly sampled from a uniform distribution and regularized with the Lloyd’s algorithm (e.g., Springel 2010). The initial conditions are \( \rho = 1, P_{\text{gas}} = 0.1, B_x = 1, v_x = 1, B_y = v_y = 0.1 \sin(4\pi x), B_z = v_z = 0.1 \cos(4\pi x) \), which fit two wavelengths into the \( x \)-direction. With these initial conditions, the wavelength is resolved with an average of 6, 13, 26, and 52 mesh points from the lowest to the highest resolution, respectively. The apparent discrepancy from the expected resolutions of 8, 16, 32, and 64 mesh cells per wavelength is due to the initial particle distribution is not being a simple cubic lattice, but rather a random distribution which was relaxed by the Lloyd’s algorithm. This relaxed distribution consists of mesh cells which can be approximated by regular convex polyhedra with large number of faces (\( \gtrsim 15 \)). The effective size of such mesh cell can be approximated by the diameter of a sphere having the same volume as the cell itself, and this in turn increases the effective size of the mesh cell by approximately \( \sqrt{6/\pi} \approx 1.24 \) compared to a simple cubic cell, while keeping the total volume the same.

In Figure 8, we compare the simulated results to the analytical solution. It can be seen that lower resolution simulations have more dissipation but do not introduce phase error in the solution. The dissipation is not the result of the underlying Riemann-solver or a reconstruction method, but rather that of nonlinear monotonicity constraints on the linear reconstruction model which is required for a stable description of discontinuities. The side effect of these constraints is that the scheme becomes first-order accurate at extrema (van Leer 1979; Iwasaki & Inutsuka 2011).

In Figure 9, we show \( L_1 \) error of the simulations solution as a function of the number of mesh points per unit wavelength.

\(^{10}\) http://github.com/egaburov/fvmhd3d.

Figure 8. Circularly polarized Alfvén wave after five crossings of computational domain. The panels show the \( y \)-component of the magnetic field as a function of the \( x \)-coordinate. The solid line demonstrates exact solution, while the open circles show the result of simulations. In the left, middle, and right panels, the wavelength is resolved with an average of 13, 26, and 52 mesh points, respectively. (A color version of this figure is available in the online journal.)
Here, the $L_1$ error is defined as $L_1 = 1/N \sum_j |f_j - f_{ex}|$ where sum is carried out over all $N$ mesh cells, and $|f_j - f_{ex}|$ is absolute deviation of the value in a cells, $f_j$, from the corresponding exact solution, $f_{ex}$. The result demonstrates that the convergence for this problem is consistent with the second-order scheme.

A.2. Magnetorotational Instability in Non-stratified Cylindrical Disk

In this problem, we study the ability of our code to reproduce analytical growth rates of axisymmetric MRI. Our computational domain consists of the three-dimensional non-stratified cylindrical disk. The inner and outer radii of the disk are equal to $R = 1$ and $R = 8$, and the thickness of the disk is $H = 1$. We use periodic boundary conditions in the $z$-direction, and outflow boundaries at $R = 1$ and $R = 8$. The total computational domain is a box with size $[16.6 \times 16.6 \times 1]$.

We simulated three models with an average 14, 20, and 28 mesh points in the $z$-direction. The initial density is set to unity, and we used isothermal equations of state with constant sound speed $c_s = 0.1$. The gravitational potential is equal to $\phi = -1/R$, where $R = \sqrt{x^2 + y^2}$, and the initial velocity is equal to the Keplerian velocity. Initially, we set a uniform magnetic field in $2 < R < 4$ annulus of the disk with such strength that results in fastest growth for $n = 2$ mode at $R \approx 2$. Namely, we have $B_x = B_y = 0$ and $B_z \approx 0.055/n$, where $n = 2$. In other words, at $R \approx 2$ the fastest growing MRI mode has the wavelength $\lambda_{MRI} \approx H/2$. In this setup, the $\lambda_{MRI}$ is resolved with approximated 7, 10, and 14 mesh points in low, medium, and high-resolution simulations, respectively.

In Figure 10, we show time evolution of radial magnetic energy $E = B_r^2/2$ as a function of the number of orbits at $R = 2$ for three different resolutions. All simulations show exponential growth rate after approximately one orbital period, and the low-resolution simulations show growth rate $\approx 0.6 \Omega$, whereas the medium- and high-resolution simulations show growth rates $\approx 0.65 \Omega$ and $\approx 0.75 \Omega$, respectively.

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Figure 9. $L_1$ error as a function of resolution for a circularly polarized Alfvén wave. The open circles connected by the red solid line show results of simulations, and the blue solid line is the expected dependence for the second-order scheme $\propto O(N^{-2})$. The vertical axis shows the $L_1$ error in the solution as a function of number of mesh points, $N$, per wavelength. (A color version of this figure is available in the online journal.)

Figure 10. Radial magnetic energy in an annulus $2 < R < 2 + 1/2$ (vertical axis) as a function of the number of local orbits at $R = 2$ (horizontal axis). The solid red line shows time evolution of radial magnetic energy for low-resolution simulation (on average 7 mesh points per $\lambda_{MRI}$), the green dashed and blue dotted lines show the results for medium (10 mesh points per $\lambda_{MRI}$) and high resolutions (14 mesh points per $\lambda_{MRI}$). The left and right dotted lines show exponential growth with slopes $0.75\gamma$ and $0.65\gamma$, respectively, where $\gamma = 4\pi$. (A color version of this figure is available in the online journal.)
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