Derivation of Z-R relationship parameter for Alor Setar radar using Traditional Matching Method (TMM)

Mahyun A W 1,2, Kar-Kei Chua 1, Salwa M.Z.M 1,2, Noramirah Abu Seman 1,2, Zulkarnain Hassan 1,2 and Ain Nihla Kamarudzaman 1,2

1Water Research Group (WAREG),Universiti Malaysia Perlis, Perlis, Mslsydis
2Faculty of Civil Engineering Technology, Universiti Malaysia Perlis, Perlis, Malaysia

Email: mahyun@unimap.edu.my

Abstract. The rainfall measurement can be done by using rain gauge and weather radar instruments. However, weather radar does not measure the rainfall depth directly as contrary to rain gauge. Therefore, an empirical relationship between reflectivity (Z) and rainfall rate (R) which is commonly known as reflectivity-rainfall (Z-R) relationship consisting of parameter $A$ and exponential $b$ ($Z = A \cdot R^b$), usually used to convert reflectivity data into rainfall rate for a radar. Presently, the Z-R relationship parameters proposed by Marshall and Palmer (1947) used in Malaysia is seem no longer suitable for Malaysia condition. Hence, a new relationship should be developed. The reflectivity data from year 2006 to year 2007 at Alor Setar radar and gauge rainfall data from 14 rain gauge in the Northern of Peninsular Malaysia were calibrated. By using the Traditional Matching Method (TMM), a new parameter was developed for Alor Setar radar which located in Northern Peninsular Malaysia. By minimizing the errors, a Mean Field Bias Correction (MFB) technique was apply to all selected parameter in this study with adjusting the value $A$ and fixed the value $b$. As a results, new climatological Z-R relationship ($Z=14.30R^{1.9}$) was obtained. To justify the new relationship, validation analysis has been performed by using the five statistical measure. It was found that the validation analysis has given the best results of Mean Error (ME), Mean Absolute Error (MAE), Root Mean Square Error (RMSE), Bias and Pearson Correlation Coefficient ($r$) with value of 0.00, 0.00, 7.54, 1.00 and 0.85 respectively. Concluded that, it is suitable for radar rainfall estimation in the Northern Region of Peninsular Malaysia.

1. Introduction
Radar has been used to measure rainfall for over 40 years [1]. As an engineers, some parts of engineering structures are also built based on weather forecasts as we look at the economic future. For example, during the construction of water dams, engineers should be aware that the estimated amount of water flowing into the reservoir can cause overflow especially during the heavy rainy season.

Estimates of rain and flood relief are still a challenge for engineers to meet the needs of the community. Different methods and applications of techniques are used to predict rainfall and flooding such as using weather radar. The application of radar rain forecasts is not new for engineers, but in Malaysia, the application of radar rainfall is still in its early days, especially in hydrological modelling jobs[2].

To convert the reflectivity into the rainfall intensity, the power empirical equation as described in equation (1) is normally utilize
\[ Z = A \cdot R^b \tag{1} \]

Where:
- \( Z \) = Radar reflectivity factor (mm\(^6\)/m\(^3\))
- \( R \) = Rainfall rate (mm/hr)
- \( A \) and \( b \) = Relationship Parameters

Due to the radar reflectivity varies across many orders of magnitude, the equation of \( Z \) normally expressed in decibels (dB) of reflectivity or dBZ as equation (2).

\[ Z = dBZ = 10 \log_{10} \left( \frac{Z}{1 \text{ mm}^6 \text{ m}^{-3}} \right) \tag{2} \]

According to [3], the relationship \( Z = 200R^{1.6} \) was developed and used since then to this day. This relationship is always a reference as an initial equation before new parameters are developed based on location because the fitting coefficients (parameters \( A \) and \( b \)) usually differ depending on the location and variation of the distribution of raindrops in space and time. According to [4], the \( Z-R \) Marshall and Palmer relationship \( (Z = 200R^{1.6}) \) is no longer suitable for rainfall estimation in Malaysia especially for Alor Setar radar. Therefore, the main objective in this study is to calibrate and validate new \( Z-R \) parameters that suitable for Alor Setar radar.

2. Data collection

2.1 Radar reflectivity data
Radar data is provided by Meteorological Malaysia Department (MetMalaysia). Basically, MetMalaysia is the authority that operated radar networks in Malaysia. Malaysia radar network consisting seven radars in Peninsular while four radars located in East Malaysia. In this study, reflectivity data was collected from Alor Setar radar since Alor Setar radar is closest radar station to Perlis with the scan range can cover for the whole Perlis is less than 100 km.

Radar data which are measured at 10 minutes interval using radar beam with three different angles such as 0.5\(^\circ\), 0.8\(^\circ\) and 1.1\(^\circ\). Station number for Alor Setar radar is 267 and located at 6.18 N\(^\circ\), 100.41 E\(^\circ\), also with 4 meter altitude. Alor Setar radar is a type of S-band radar. S-band radar did not has the attenuation problem. It also has a maximum horizontal coverage of 480 km, thus it is very useful to apply in this study due to its scan range can cover over the catchment area [5].

Reflectivity data obtained from MetMalaysia contains the reflectivity values measures in decibels unit (dBZ). The value of rain rates was computed by using the Marshall and Palmer relationship \( (Z=200R^{1.6}) \).

2.2 Rain gauge data
Department of Irrigation and Drainage (DID) Malaysia provided the rain gauge data. Rainfall data interval is in 10 minutes to make sure it is similar with the time interval of radar. The available rainfall data from 2006 to 2007 collected at 14 hydrological available stations were used for the calibration and validation process of the \( Z-R \) relationship parameters development.

3. Methodology
Calibration and validation process were done to determine the new value of parameters \( A \) and \( b \). According to [6], raindrop size distribution (DSD) or Traditional Matching Method (TMM) is normally used to derive \( Z-R \) relationship. Since the Disdrometer is very limited in Malaysia, TMM was applied in this study to derive the new \( Z-R \) relationship parameters for Alor Setar radar. To improve all existing parameters which selected in this study, the Mean Field Bias Correlation (MFB) technique was applied...
to determine the minimum error between radar reflectivity and rain gauge observations next the best correlation result.

3.1 Z-R Relationship parameters calibration

To determine the suitable Z-R relationship parameters, technique proposed by [6] and [7] that used the daily data were applied. This technique used the typical relationship of $Z = 200R^{1.6}$ proposed by Marshall and Palmer to covert the reflectivity data into the rainfall rate and the accumulated into daily rainfall. To obtain the accumulation daily data, the rainfall accumulation is computed by multiplying the rainfall rates (mm/hr) with the sampling interval. Following that, multiplying radar rainfall data for each interval to become the daily radar rainfall in mm.

In this study, 20 storm events capture by rain gauge stations in Perlis were used to calibrate the most accurate fitting coefficients for Alor Setar radar. The summarization of the steps in the calibration processes are listed:

1. The parameter $A$ and exponential $b$ were fixed at 200 and 1.6 respectively. Then, the $Z = 200R^{1.6}$ was applied to change the reflectivity into the rainfall intensity.
2. The estimated radar rainfall rate and gauge rainfall were gathered into daily rainfall in the unit of millimetre (mm).
3. The mean gauge and radar rainfall for each day were calculated by using Equation (3) and Equation (4) respectively.

Mean Gauge Rainfall, $G_j$ is expressed as

$$G_j = \frac{1}{N} \sum_{i=1}^{N} g_{ij}$$

Where;

$G_j$ = The mean gauge rainfall on day $j$.
$g_{ij}$ = Gauge rainfall at station $i$ and on day $j$.
$N$ = Total rain gauge number.

Mean Radar Rainfall, $R_j$ is expressed as:

$$R_j = \frac{1}{N} \sum_{i=1}^{N} r_{ij}$$

Where;

$R_j$ = the mean radar rainfall on day $j$.
$r_{ij}$ = Radar rainfall accumulation computed by using the $Z = 200R^{1.6}$, for day $j$ at the pixels that contain the $N$ rain gauges.
$N$ = Total rain gauge number.

After finished step 3, five statistical measurements were used to compare the estimated mean gauge rainfall with the mean gauge rainfall as illustrates in table 1 which are suggested by [7].
**Table 1.** Five statistical measures utilized in the calibration processes [7].

| Type of indicator                  | Formula                                                                 |
|-----------------------------------|-------------------------------------------------------------------------|
| Pearson Correlation Coefficient, $r$ | $r = \frac{\sum_{i=1}^{N} (O_i - \bar{O})(F_i - \bar{F})}{\sqrt{\left(\sum_{i=1}^{N} (O_i - \bar{O})^2\right)\left(\sum_{i=1}^{N} (F_i - \bar{F})^2\right)}}$ |
| Mean Error (ME)                   | $ME = \frac{1}{N} \sum_{i=1}^{N} (F_i - O_i)$                         |
| Mean Absolute Error (MAE)         | $MAE = \frac{1}{N} \sum_{i=1}^{N} |F_i - O_i|$                     |
| Root Mean Square Error (RMSE)     | $RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (F_i - O_i)^2}$              |
| Bias                              | $Bias = \frac{\sum_{i=1}^{n} R_i}{\sum_{i=1}^{n} G_i}$                |

*Where; $n$ = number of mean areal rainfalls totals in the records*

Lastly, calculation from step 1 to step 4 are were repeated through applying the other selected Z-R relationship parameters to observe which parameters will give the best results. Whichever relationship gives the minimum of the errors and best correlation will be chosen as the most suitable relationship.

### 3.2 Traditional Matching Method (TMM)

The determination of $A$ and $b$ were done by using linear regression analysis which is known as Traditional Matching Method (TMM). By using TMM, a new Z-R relationship is formed as shown as equation (5).

$$Z = \left(\frac{1}{a}\right)^{\frac{1}{m}} (R)^{\frac{1}{m}}$$  (5)

Where

$$A = \left(\frac{1}{a}\right)^{\frac{1}{m}}$$

$$b = \frac{1}{m}$$

### 3.3 Mean Field Bias correction (MFB)

In order to improve the accuracy of all existing parameters which selected in this study, the Mean Field Bias Correction (MFB) technique was applied to obtain the adjustment factor ($F$) for each relationship. The adjustment factor ($F$) can be calculated as shown in equation (6) (Steiner et al., 1999).

$$F = \frac{\sum_{i=1}^{n} G_i}{\sum_{i=1}^{n} R_i}$$  (6)

Where:

- $G_i$ = Accumulated rain gauge rainfall (mm/h) over the analysis period at gauge $i$.
- $R_i$ = Accumulated initial radar rainfall (mm/h) at gauge $i$, calculated using the climatological Z-R relationship.
\( N = \) The number radar-gauge pairs data available.

The original Z-R relationship \((Z = A \cdot R^b)\) can become a new equation as shown as equation (7).

\[
Z = AF^{-b} \cdot R^b
\]  

(7)

Where the new \( A \) obtained is from the formula \( AF^{-b} \).

4. Result and discussion
The scatter plot of mean radar rainfall using Marshall and Palmer equation \((Z=200R^{1.6})\) and Mahyun et. al (2017) developed parameters are compared and illustrated in figure. 1. Moreover, figure. 1 shows that the radar rainfall data are underestimated than gauge rainfall. It can be observe from figure. 2.

![Figure 1. Comparison between mean radar rainfall based on the relationship \( Z = 200R^{1.6} \) and \( Z = 40R^{1.6} \).](image)
Figure 2. Comparison of time series plot between $Z=200R^{1.6}$ and $Z=40R^{1.6}$ with mean rain gauge rainfall.

Both results shows that current Z-R relationship parameters used by MetMalaysia to convert the reflectivity to rainfall rate should be replace with the equation of $Z=40R^{1.6}$. Besides that, the relationship $Z=40R^{1.6}$ also gives the minimum value of error and better correlation as shown in table 2. The results of statistical measures show that relationship $Z=40R^{1.6}$ gives significant reduction.

Table 2. Comparison of the statistical measures from the different Z-R relationships parameters.

| Statistical Measures | $Z=200R^{1.6}$ | $Z=40R^{1.6}$ | $Z=343R^{1.9}$ |
|----------------------|----------------|----------------|----------------|
| ME                   | -23.22         | -3.84          | 27.99          |
| MAE                  | 23.22          | 3.84           | 27.99          |
| RMSE                 | 21.97          | 8.44           | 26.12          |
| Bias                 | 3.07           | 1.13           | 5.33           |
| Pearson Correlation  | 0.83           | 0.82           | 0.85           |

For the new derived parameters by using the TMM, the equation obtained is displayed as $Z=1.0R^{5.5}$. The comparison between equations $Z=1.0R^{5.5}$ and $Z=200R^{1.6}$ is shown in the scatter plot as in figure. 3. The time series plot shows new relationship has much better relationship between mean gauge rainfall and radar rainfall for calibrated event as illustrated in figure. 4. The time series plot and the scatter plot once again shows that the current equation used by MetMalaysia can be replace with the new equation. Besides that, the relationship $Z=1.0R^{5.5}$ also gives the minimum value of error and better correlation as shown in table 3.
Figure 3. Scatter plot of mean radar rainfall based on the relationship $Z=200R^{1.6}$ and $Z=1.0R^{5.5}$.

Table 3. Comparison of the statistical measures gained from the different Z-R relationships.

| Statistical Measures | $Z=200R^{1.6}$ | $Z=1.0R^{5.5}$ |
|----------------------|----------------|-----------------|
| ME                   | -23.22         | -21.39          |
| MAE                  | 23.22          | 21.39           |
| RMSE                 | 21.97          | 20.43           |
| Bias                 | 3.07           | 2.63            |
| Pearson Correlation  | 0.83           | 0.81            |

Figure 4. Time series plot of mean gauge rainfall and radar rainfall using the relationship $Z=200R^{1.6}$ and $Z=1.0R^{5.5}$. 
The result of statistical measures shows that a new Z-R relationship gives some reduction. However, these are quite a number of error remain in the new developed parameters. Thus, the MFB can be applied to all parameters used in this study in order to obtain new \( A \) for each relationship but remain the value \( b \) at constant value. Following that, the best relationship \( Z=14.30R^{1.9} \) was obtained which is originally adjusted from \( Z=343R^{1.9} \). Figure 5 shows the agreement mean gauge rainfall and radar rainfall for calibrated event. An agreement between estimated radar and gauge rainfall was examined using the RMSE and Pearson correlation coefficient (r). Table 4 represent the relationship \( Z=14.30R^{1.9} \) has given the best results of statistical measures when compare to the other parameter. From the comparison, the result shows that new relationship \( Z=14.30R^{1.9} \) developed in this study gives an accurate results comparing to standard Marshall and Palmer equation.

![Figure 5. Scatter plot of mean radar rainfall based on the relationship \( Z=14.30R^{1.9} \).](image)

**Table 4.** Summary of statistical result for the all new developed Z-R relationship parameters.

| Original Relationships (developed by other researcher) | New Relationships parameter (developed in this research based on the original relationship) | Statistical Measurements |
|---------------------------------------------------------|------------------------------------------------------------------------------------------|--------------------------|
| Marshall and Palmer (Z=200R^{1.6})                    | (Z=33.32R^{1.6})                                                                         | ME 0.13 MAE 0.13 RMSE 8.14 BIAS 1.00 r 0.82 |
| Mahyun et al. (Z=40R^{1.6})                            | (Z=33.12R^{1.6})                                                                         | ME 0.01 MAE 0.01 RMSE 8.14 BIAS 1.00 r 0.82 |
| Mat Kamuzaman et al. (Z=343R^{1.9})                    | (Z=14.30R^{1.9})                                                                         | ME 0.00 MAE 0.00 RMSE 7.54 BIAS 1.00 r 0.85 |
| TMM (Z=1.0R^{5.5})                                     | (Z=0.0048R^{5.5})                                                                        | ME 0.08 MAE 0.08 RMSE 9.93 BIAS 0.10 r 0.81 |

**5. Conclusion**

In conclusion, the calibrated relationship \( Z=14.30R^{1.9} \) is suitable used for an estimation of daily radar rainfall for Perlis river basin. The overall result obtained had answered the objective in this study. New Z-R relationship was developed for Alor Setar radar for Perlis rainfall estimation. The parameters proposed by Marshall and Palmer that was presently used by MetMalaysia shows the underestimated results while the new relationship \( Z=14.30R^{1.9} \) gives more accurate results. The parameters \( A \) was adjusted to minimize the ME, MAE and RMSE. Also, to improve the correlation in terms of Bias and Pearson correlation (r). However, the exponential \( b \) is fixed at 1.9. Time series and scatter plot of mean
radar rainfall and mean gauge rainfall have shown that new equation is suitable applied for Alor Setar radar. Lastly, concluded that the TMM is workable since the relationship $Z=343R^{1.9}$ is actually originally derived from TMM in the research done by [5] and then only become $Z=14.30R^{1.9}$ after applied with adjustment factor (F) based on MFB technique. Perhaps the data provided in this study was not that sufficient, then it might lead to the inaccuracy in rainfall estimation when using the parameters developed by TMM.

References
[1] Islam M R 2005 Improved Quantitative Estimation of Rainfall By Radar Master's thesis
[2] Ramli S and Tahir W 2011 Radar Hydrology: New Z / R Relationships for Quantitative Precipitation Estimation in Klang River IEEE Colloquium on Humanities, Science and Engineering (CHUSER) 2(3)
[3] Marshall J S, Langille R C and Palmer W M K 1947 Measurement of rainfall by radar J. Meteorol. 4(6) 186-192
[4] S and Tahir W 2011 Radar Hydrology: New Z/R Relationships for Quantitative Precipitation Estimation in Klang River Basin, Malaysia Int. J. Env. Sci. Dev. 2(3) 223-227
[5] Kamaruzaman M, Adam M, Yunus F and Abdullah M H 2014 Estimation of Rainfall Rate by Weather Radar Improved Through Increased Data Sampling and the Window Correlation Matching Method of Analysis Hydrol. Earth Syst. Sci. 11(4) 361–1372
[6] Mapiam P P and Sriwongsitanon N 2008 Climatological Z-R relationship for radar rainfall estimation in the upper Ping river basin Sci. Asia 34 215–222
[7] Seed A, Siriwardena L, Sun X, Jordan P and Elliott J 2002 Catchment Hydrology on The Calibration of Australian Weather Radars Research Centre for Catchment Hydrology