Load Frequency Control and Real-Time Pricing with Stochastic Model Predictive Control

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Abstract: We developed a load frequency control system with stochastic model predictive control (SMPC) for power systems where the market penetration of wind power generation is high. The controller adjusts the electricity price for heat pump water heaters while at the same time controlling thermal power plants and batteries in order to maintain the frequency in the designated range. We propose an approach for solving SMPC problems on Hammerstein models including affine disturbance feedback parametrization. Simulation results show that SMPC with affine disturbance feedback parametrization outperforms both SMPC without parametrization and deterministic model predictive control in terms of the stage-cost and constraint violation.

Key Words: power system, real-time pricing, stochastic model predictive control.

1. Introduction

The market for renewable energy and sustainable electricity generation is rising in many countries [1]. Solar power and wind power account for a large portion of renewable energy sources [1]. However, they have a drawback in that the output power fluctuates significantly when the weather changes [2]. In an AC power system, it is necessary to always balance the supply and demand of power and control the frequency within a specified range. As the amount of renewable energy increases, it becomes more difficult to cope with power fluctuations only by adjusting the output of existing generators, and this may be a bottleneck to the spread of renewable energy.

A battery is a potential option for compensating for this power fluctuation. However, the amount of battery energy should be small since its cost is high. Research [3] shows that the use of demand response can reduce costs compared with the case in which batteries deal with all power fluctuations. Demand response means regulating the power consumption of the consumer to match the demand with the supply instead of adjusting the supply. Demand response can be achieved, for example, by sending signals indicating the power consumption directly to the consumers’ electrical equipment and forcing the equipment to follow the signal. Although this direct approach is desirable from the viewpoint of the performance of the control, it makes it impossible for consumers to determine the power consumption in accordance with their own intentions. Real-time pricing, which changes electricity prices at every moment as a way to adjust electricity consumption, is preferable for this reason. Moreover, real-time pricing can easily give consumers incentives to adjust their electricity consumption.

In general, frequency control has a hierarchical structure which comprises primary, secondary, and tertiary control reserves [4]. In this paper, we focus on the time scale of primary and secondary control reserves, which is called load frequency control (LFC), and assume that the power system has a high market penetration of wind power. At these time scales, it is important to deal with unpredicted disturbances in demand or renewable energy. We propose a control system that incorporates conventional control of generators and real-time pricing on consumers.

Several approaches to using demand response to stabilize the power balance on such time scales have been proposed [5]–[7]. Jokić et al. [5] developed a method of giving appropriate electricity prices to many interconnected areas conforming to certain constraints on power flow. Kamemoto et al. [6] used nonlinear model predictive control (MPC) for optimizing prices so that the nonlinearity of the models can be directly treated. Satouchi et al. [7] proposed to combine direct control of generators with demand response by real-time pricing. The PI controller proposed by Satouchi et al. sends signals directly to generators and batteries, while the MPC controller presents the price to the consumer’s heat pump water heater (HPWH). However, the proposed controller is completely separated into a PI controller and MPC, and it does not work in an integrated manner. Additionally, the studies mentioned above do not explicitly consider disturbances. If a large amount of wind power were introduced to the power grid, the disturbance would be too large to ignore.

In this paper, we propose an integrated frequency control system with stochastic MPC (SMPC) on the basis of the work of Satouchi et al. [7]. SMPC considers the disturbance of a model explicitly. We assume that the HPWH can adjust its own power consumption in response to the price. A demand function can be used to estimate the HPWH’s power consumption, but it can vary with the weather, time, and season. We use a particle filter proposed by Satouchi et al. [8] in order to estimate the demand function in real time.

Since SMPC can deal with disturbances by using their prob-
ability distributions, it can evaluate the objective function and constraints in a stochastic manner. In SMPC, chance constraints are often considered instead of deterministic constraints. A chance constraint requires that the probability of satisfying an inequality is above a designated bound. In this paper, we impose chance constraints on the frequency and suppress the frequency deviation to within a bound with a designated probability.

Various approaches to SMPC have been developed in the literature [9]. Most of them are for linear systems, while much less work has been done on nonlinear systems [9]. The SMPC approaches for linear systems can be roughly classified into two categories [10]. The first is a class of analytic approximation methods [11]. These methods reformulate the chance constraints and the objective function into deterministic forms in order for them to be included in the MPC formulation. The second class is based on randomized methods [12]. A sufficient number of samples of disturbances are generated, and every sample imposes deterministic constraints instead of chance constraints. For nonlinear systems, a Gaussian-mixture approach is used to describe the evolution of the distributions of states [13]. However, the calculation costs of these methods are generally high [9].

The model derived in this paper is a so-called Hammerstein model [14], in which nonlinearity is included only in the input term. In the case of linear systems, the SMPC problem can be transformed into an equivalent deterministic optimization problem [15] if the cumulative distribution function (CDF) of disturbances is analytically defined. This transformation relies on the linearity of the problem and cannot be applied to nonlinear systems in general. However, in this study, we show that the SMPC problem for Hammerstein models can be transformed into an equivalent deterministic optimization problem by utilizing the inverse function of the nonlinearity in the model. Then, such computationally demanding approaches as randomized methods and Gaussian mixture can be avoided in spite of the nonlinearity in the model. Moreover, open-loop SMPC, which optimizes only the input sequence, is conservative in that it does not consider any feedback on the prediction horizon. Affine disturbance feedback parametrization is often used in linear SMPC in order to relax this sort of conservativeness [16]. We also show that SMPC for Hammerstein models with affine disturbance feedback parametrization can be approximately transformed into a deterministic problem. Therefore, less conservative SMPC can also be realized for Hammerstein models without computationally demanding randomized methods or Gaussian mixtures.

This paper is organized as follows. The electrical power system is described and modeled in Section 2. Two kinds of SMPC (open-loop and closed-loop) are formulated and transformed into deterministic problems in Section 3. The simulation results described in Section 4 verify the effectiveness of the proposed method. Section 5 concludes this paper and outlines future work.

2. Power System Model

The electric power system considered in this paper is shown in Fig. 1. This system consists of thermal power plants, nuclear power plants, a battery energy storage system (BESS), wind power plants, and heat pump water heaters (HPWHs). The controller proposed in this paper controls the thermal power plants and BESS directly and the HPWHs indirectly with real-time pricing. We assume that the HPWHs are installed in households and can regulate the power consumption in response to the electricity price. The controller observes the states of all devices in the system as well as fluctuations in the wind power generation and demand. It is assumed that sufficiently high-speed communication is possible in the future power system and that observation delay is negligible. Nuclear power plants are normally controlled over a long time scale; thus, they are not controlled in this system. All generators are assumed to be synchronized, and each kind of generator and piece of equipment is represented by a single model. Power generation or consumption is represented by the deviation from the initial value.

2.1 Generator Model

Under the assumption that all generators are synchronized, the dynamics of the power system frequency are represented by the following swing equation [17],

$$M_{gm}\dot{\omega} + D_{gm}\omega = P_M - P_L,$$

where $M_{gm}$ and $D_{gm}$ are the sums of the inertia constants of all generators in the grid and the damping constant of the grid, respectively, and $P_M$ and $P_L$ are the mechanical output from the generators and the electrical power consumption of the grid, respectively.

2.2 Thermal Power Generator Model

The thermal power plant is modeled as shown in Fig. 2 with the parameters listed in Table 1. This model consists only of a governor control system and has two inputs; one is governor-free (GF) and is proportionally controlled with the frequency; the other is a command input from the SMPC. The command input and its derivative are restricted to the designated upper and lower bounds.
2.3 BESS Model

The BESS model is as shown in Fig. 3. The BESS tracks the charge (discharge) signal within the designated upper and lower bounds. The battery capacity is limited, but we ignored this limitation because we considered a short time scale. The time constant $T_b$ (network communication delay and control delay) in Fig. 3 was set to 1 s.

2.4 HPWH Model

The HPWH model is as shown in Fig. 4. The HPWH is a water heater using a refrigeration cycle; thus, it cannot be switched on and off frequently. Irie et al. [17] assumed that the HPWH operates at 90% of its rated electricity consumption if uncontrolled and that the consumption can be adjusted by ±10% from 90%. The time constants in Fig. 4 were set to $T_{hpc} = 30$ s and $T_{ig} = 1$ s [17].

2.5 Wind Power Generation and Demand Fluctuation

Wind power and demand fluctuation are considered disturbances. We created patterns of wind power generation data, considering the wind power capacity as described in Section 3. The power spectrum of wind power can be represented by a function proportional to $1/f^5$, where $f$ is frequency [18]. We used MATLAB’s colored noise generator to generate data having this property. Similarly, we generated patterns of demand fluctuation data by referring to the work of Irie et al. [17] and using the same tool. The power spectrum of demand fluctuation is assumed to be proportional to $1/f^2$, based on analysis of data in [19]. Figures 5 and 6 show one set of wind power generation data and demand fluctuation data that we created.

2.6 Model of Consumers

The demand function represents the correspondence between the price and the amount of power consumed. We use the sigmoid function proposed by Satouchi et al. [7] as the demand function:

$$D_h(u_p) = -C_h \left( \frac{1}{1 + e^{-a_h u_p} - \frac{1}{2}} \right),$$  \hspace{1cm} (2)

where $u_p$ is the price, $C_h$ is the adjustable capacity of HPWH, and $a_h$ is price elasticity. Figure 7 shows a graph of the demand function. The HPWHs change their power consumption automatically in response to a change in the price; in particular, they should reduce their power consumption when the price is raised. Also, since the adjustable power range is limited, the power consumption deviation of the whole HPWH saturates as the price is raised or lowered. For these reasons, the use of a sigmoid function is appropriate for representing the important characteristics of the demand function.

3. Control Design

In this section, we formulate the SMPC for the power system modeled in the previous section. In Section 3.1, the models in Section 2 are simplified and discretized so that they can be used in the SMPC. In Section 3.2, we formulate two types of SMPC, i.e., open-loop SMPC and closed-loop SMPC. Then, we transform these SMPC problems into deterministic optimization problems in Section 3.3. The transformed problems can be solved with a common optimization solver. In Section 3.4, we use the particle filter proposed by Satouchi et al. [8] to estimate the demand characteristic in real time.
3.1 Models for SMPC

First, we simplify the models in Section 2 by ignoring terms whose time constant is sufficiently small. That is, all time constants except for $T_3$ and $T_{hp}$ are regarded as zero. We define $i$ to be discrete time and $\Delta t$ to be the control cycle in what follows.

A simplified model of a thermal power generator is obtained as follows:

$$p_t(i) = (1 - K_{tp})x_t(i) + K_{tp}\left(-\frac{100}{\delta} \omega(i) + u_t(i)\right),$$  \hspace{1cm} (3)

where $u_t$ represents the command input to the thermal power generator, $x_t$ represents the output of slow dynamics, and $p_t$ is the output of the generator.

Since the BESS model has a sufficiently fast response, its output ideally follows the input:

$$p_b(i) = u_b(i),$$  \hspace{1cm} (5)

where $p_b$ is positive if the BESS is discharging, and $p_b$ is negative otherwise.

The HPWH is simplified as follows:

$$p_h(i + 1) = p_h(i) + \frac{1}{T_{hp}}(-p_h(i) + u_b(i)\Delta t),$$  \hspace{1cm} (6)

where $u_b$ is the HPWH input (electric power signal), and $p_h$ represents the HPWH output (electric power).

We treat the wind power generation and demand fluctuation as one variable $p_w$ because their power spectra are nearly proportional to $1/f^2$, as discussed in Section 2.5. Since a $1/f^2$ power spectrum is obtained by integrating white noise, we assume that the fluctuation follows the model,

$$p_w(i + 1) = p_w(i) + w(i), \quad w(i) \sim N(0, \sigma).$$  \hspace{1cm} (7)

Finally, we define the state vector $x$, input vector $u$, and output vector $y$ as follows:

$$x = \begin{bmatrix} \omega & p_h & u_t & u_b & p_w & x_t \end{bmatrix}^T, \quad u = \begin{bmatrix} \Delta u_t & \Delta u_b \end{bmatrix}^T, \quad y = \begin{bmatrix} \omega & p_h & p_b \end{bmatrix}^T,$$

where $\Delta u_t$ and $\Delta u_b$ denote the variation of $u_t$ and $u_b$ per control cycle, respectively. We chose $\Delta u_t$ and $\Delta u_b$ as the input variables in order to constrain the variation of the inputs per control cycle. The actual inputs $u_t$ and $u_b$ to the system are obtained by integrating $\Delta u_t$ and $\Delta u_b$ with respect to time.

As a whole, the state equation and output equation are represented as follows:

$$x(i + 1) = Ax(i) + Bg(u(i)) + Cw(i),$$  \hspace{1cm} (10)

$$y(i) = Dx(i),$$  \hspace{1cm} (11)

where

$$A = \begin{bmatrix} a_{11} & -\Delta \frac{T_{pp}}{M_{p}} & \frac{K_p}{M_{p}} & \Delta t & \Delta t & \frac{1}{M_{p}} \Delta t \\ 0 & 1 - \frac{\Delta t}{T_{pp}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ a_{51} & 0 & 0 & 0 & 1 & 0 \\ a_{51} & 0 & 0 & 0 & 1 & 0 \end{bmatrix},$$

$$A_{11} = 1 - \left(D_{gm} - \frac{100}{\delta} K_p\right) \frac{\Delta t}{M_{gm}}, \quad a_{51} = -\frac{100}{\delta} \frac{\Delta t}{T_5}.$$

The state equation (10) is a Hammerstein model [14] because nonlinearity exists only in the input term. Note that $D_0$ is defined in (2); thus, the nonlinear function $g(\cdot)$ is one-to-one and its inverse exists. This property is necessary for transforming it into the deterministic problem described in Section 3.3.

3.2 Formulation of SMPC

3.2.1 Open-loop SMPC

First, we formulate open-loop SMPC, in which only one sequence of inputs over the horizon is optimized for all possible sequences of disturbances. At each time step $i$, open-loop SMPC solves the following problem, setting the present state $\bar{x}(i)$ as the initial state $\bar{x}(0)$. Note that open-loop SMPC is a kind of feedback control, in spite of its name.

**Problem 1** (Open-loop SMPC problem).

$$\min_{u(0),\ldots,u(N-1)} \mathbb{E}[J]$$  \hspace{1cm} (12)

subject to

$$\mathbb{E}[J] = \mathbb{E}\left[\sum_{j=0}^{N} \frac{1}{2} y(j)^T Q y(j) + \sum_{j=0}^{N-1} \frac{1}{2} u(j)^T Ru(j)\right],$$  \hspace{1cm} (13)

$$x(j + 1) = Ax(j) + Bg(u(j)) + Cw(j) \quad (j = 0, \ldots, N - 1),$$  \hspace{1cm} (14)

$$y(j) = Dx(j) \quad (j = 1, \ldots, N),$$  \hspace{1cm} (15)

$$x(0) = \bar{x}(i),$$  \hspace{1cm} (16)

$$\mathbb{P}[E_a x(j) \leq h_a] \geq p_a \quad (j = 1, \ldots, N),$$  \hspace{1cm} (17)

$$E_a u(j) \leq h_u \quad (j = 0, \ldots, N - 1).$$  \hspace{1cm} (18)
Equation (13) represents the objective function, which is a quadratic form of the output and the input. Equations (14) and (15) are the state equation and output equation defined in Section 3.1. Inequality (17) denotes component-wise chance constraints, which require that the probability of satisfying the state inequalities be above a vector-valued designated bound $p_c$. Let $\bar{u}(i)$ denote the actual input applied to the plant at time $i$ in what follows. The controller optimizes the control inputs $\{u(0), \ldots, u(N-1)\}$ at each time step, and the first one $u(0)$ is used as the actual input $\bar{u}(i)$. The procedure above accomplishes feedback control in that it derives an input $\bar{u}(i)$ depending on the present state $\bar{x}(i)$. We call the Problem 1 open-loop SMPC, because it simply optimizes the input sequence, i.e., open-loop control, over the horizon.

### 3.2.2 Closed-loop SMPC

The state sequence $x(1), \ldots, x(N)$ on the prediction horizon is affected by past disturbances and thus has uncertainty. Therefore, the optimal input on the prediction horizon should change depending on the state trajectory earlier than the input. This implies that it is preferable to find an optimal control policy rather than only one sequence of inputs. A control policy can be generally written as a function of states that are observed when the input is actually applied. In particular, for linear systems, it can be parametrized as an affine function of past states:

$$
 u(j) = \sum_{k=1}^{j} M_{jk} x(k) + \gamma_j. 
$$

(19)

On the other hand, future states $x(k)$ are uniquely determined by the state equation and future disturbances. Thus, an input on the prediction horizon can also be written as an affine function of the past disturbances. This parametrization is called affine disturbance feedback parametrization [11],[16] and is represented as follows:

$$
 u(j) = \sum_{k=0}^{j-1} M_{jk} w(k) + \gamma_j. 
$$

(20)

Both parametrizations (19) and (20) are equivalent, while the latter leads to a convex problem that is easy to deal with [16]. In this paper, we call SMPC with affine disturbance feedback parametrization closed-loop SMPC, because it optimizes the feedback policy, i.e., closed-loop control, over the horizon. However, the model we consider is a Hammerstein model with nonlinearity in the input; thus, the parametrization cannot be used directly.

In this case, we propose to parametrize the term $g(u)$ instead of input $u$, as follows:

$$
 g(u(j)) = \sum_{k=0}^{j-1} M_{jk} w(k) + g(\gamma_j). 
$$

(21)

In this paper, the nonlinear function $g(\cdot)$ is one-to-one; thus, the input is represented by the inverse function

$$
 u(j) = g^{-1} \left( \sum_{k=0}^{j-1} M_{jk} w(k) + g(\gamma_j) \right). 
$$

(22)

Consequently, we can formulate a closed-loop SMPC problem as follows. At each time step $i$, the closed-loop SMPC solves the problem below, setting the present state $\bar{x}(i)$ as the initial state $x(0)$.

**Problem 2** (Closed-loop SMPC problem).

$$
 \min_{\{M_{jk}\}} \mathbb{E}[J] 
$$

subject to

$$
 \mathbb{E}[J] = \mathbb{E} \left[ \sum_{j=1}^{N} \frac{1}{2} y(j)^T Q y(j) + \frac{1}{2} u(j)^T R u(j) \right], 
$$

(23)

$$
 x(j+1) = Ax(j) + B g(u(j)) + C w(j) \quad (j = 0, \ldots, N-1), 
$$

(24)

$$
 y(j) = D x(j) \quad (j = 1, \ldots, N), 
$$

(25)

$$
 g(u(j)) = \sum_{k=0}^{j-1} M_{jk} w(k) + g(\gamma_j) \quad (j = 1, \ldots, N), 
$$

(26)

$$
 x(0) = \bar{x}(i), 
$$

(27)

$$
 \mathbb{P}[E_s x(j) \leq h_s] \geq p_s \quad (j = 1, \ldots, N), 
$$

(28)

$$
 \mathbb{P}[E_u u(j) \leq h_u] \geq p_u \quad (j = 0, \ldots, N-1). 
$$

(29)

The controller optimizes the sequences $\{M_{jk}\}$ and $\{\gamma_j\}$ at each time step $i$ and sets the first element of the optimal input sequence $u(0) = \gamma_0$ as $\bar{u}(i) = u(0)$. The above procedure accomplishes feedback control in that it finds an input $\bar{u}(i)$ depending on the present state $\bar{x}(i)$.

### 3.3 Transformation into Deterministic Optimization Problems

The SMPC problems defined in Section 3.2 are hard to deal with because of their stochastic formulations. On Hammerstein models, the chance constraints can be converted into deterministic ones, as in the case of linear models [15]. The converted problems are deterministic nonlinear programming problems that are solvable with a common optimization solver. Since open-loop SMPC can be viewed as a special case of closed-loop SMPC with $M_{jk} = 0$, we consider only closed-loop SMPC in this subsection.

Here, we introduce the following augmented matrices:

$$
 X = \begin{bmatrix} x_1 \\ \vdots \\ x_N \end{bmatrix}, 
 U = \begin{bmatrix} u_0 \\ u_{N-1} \end{bmatrix}, 
 Y = \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}, 
 W = \begin{bmatrix} w_0 \\ w_{N-1} \end{bmatrix}, 
 A = \begin{bmatrix} A \\ \vdots \\ A^N \end{bmatrix}, 
 B = \begin{bmatrix} B & O & \cdots & O \\ AB & B & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}B & A^{N-2}B & \cdots & B \end{bmatrix}, 
 C = \begin{bmatrix} C & O & \cdots & O \\ AC & C & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ A^{N-1}C & A^{N-2}C & \cdots & C \end{bmatrix}, 
 D = \begin{bmatrix} D & O & \cdots & O \\ O & D & \cdots & O \\ \vdots & \vdots & \ddots & \vdots \\ O & O & \cdots & D \end{bmatrix}. 
$$
\[
\Gamma = \begin{bmatrix}
\gamma_0 \\
\vdots \\
\gamma_{N-1}
\end{bmatrix},
\]

\[
M = \begin{bmatrix}
O & O & \cdots & O \\
M_{1,0} & O & \cdots & O \\
M_{2,0} & M_{2,1} & \cdots & O \\
\vdots & \vdots & \ddots & \vdots \\
M_{N-1,0} & M_{N-1,1} & \cdots & M_{N-1,N-1} & O
\end{bmatrix}.
\]

Accordingly, the state equation (25), output equation (26), and affine disturbance feedback parametrization (27) yield the following state equation and output equation:

\[
X = AX_0 + Bg(\Gamma) + (BM + C)W, \quad (31)
\]

\[
Y = DX, \quad (32)
\]

where

\[
g(U) := [g(\mu_0)^T \quad \cdots \quad g(\mu_{N-1})]^T. \quad (33)
\]

The objective function (24), state constraint (29), and input constraint (30) are represented with the augmented matrices:

\[
\mathbb{E}[J] = \mathbb{E}\left[\frac{1}{2}Y^TQY + \frac{1}{2}U^T RU\right], \quad (34)
\]

\[
\mathbb{P}[E_x X \leq h_x] \geq p_x, \quad (35)
\]

\[
\mathbb{P}[E_x U \leq h_x] \geq p_x, \quad (36)
\]

where

\[
Q = \begin{bmatrix}
O & \cdots & O \\
\vdots & \ddots & \vdots \\
O & \cdots & O
\end{bmatrix}, \quad R = \begin{bmatrix}
R & \cdots & R \\
\vdots & \ddots & \vdots \\
R & \cdots & R
\end{bmatrix},
\]

\[
E_x = \begin{bmatrix}
E_x & \cdots & E_x \\
\vdots & \ddots & \vdots \\
E_x & \cdots & E_x
\end{bmatrix}, \quad E_u = \begin{bmatrix}
E_u & \cdots & E_u
\end{bmatrix},
\]

\[
h_x = \begin{bmatrix}
h_1^T \\
\vdots \\
h_1^T
\end{bmatrix}, \quad h_u = \begin{bmatrix}
h_2^T \\
\vdots \\
h_2^T
\end{bmatrix},
\]

\[
p_x = \begin{bmatrix}
p_1^T \\
\vdots \\
p_1^T
\end{bmatrix}, \quad p_u = \begin{bmatrix}
p_2^T \\
\vdots \\
p_2^T
\end{bmatrix}.
\]

The first term of the objective function (34) is transformed into

\[
\mathbb{E}\left[\frac{1}{2}Y^TQY\right] = \frac{1}{2}(AX_0 + Bg(\Gamma))^T D^T QD (AX_0 + Bg(\Gamma))
\]

\[
+ \frac{1}{2}\mathbb{E}[(BM + C)W]^T DQD (BM + C)W
\]

\[
+ \frac{1}{2}\mathbb{E}[(BM + C)W]^T DQD (BM + C)W
\]

\[
= \frac{1}{2}(AX_0 + Bg(\Gamma))^T D^T QD (AX_0 + Bg(\Gamma))
\]

\[
+ \frac{1}{2}\text{tr}\left\{(BM + C)^T D^T QD (BM + C) \cdot \mathbb{E}[WW^T]\right\}.
\]

Assuming that the effect of the \( MW \) term is small enough, the inverse nonlinear function \( g^{-1}(V) \) can be linearly approximated around the nominal point \( V = g(\Gamma) \). Define \( \hat{g}(\Gamma) = \frac{g^{-1}(V)}{\sqrt{\mathbb{V}[g(\Gamma)]}} \); the above term can be approximated as

\[
\frac{1}{2}\mathbb{E}\left[U^TRU\right] \approx \frac{1}{2}\mathbb{E}\left[(\hat{g}(\Gamma)MW)^T R(\hat{g}(\Gamma)MW)\right]
\]

\[
= \frac{1}{2}\mathbb{E}[\hat{g}(\Gamma)MW]^T R\hat{g}(\Gamma)MW
\]

\[
+ \frac{1}{2}\mathbb{E}\left[\hat{g}(\Gamma)MW\right]^T R\hat{g}(\Gamma)MW
\]

\[
= \frac{1}{2}\mathbb{E}[\hat{g}(\Gamma)M]^T R\hat{g}(\Gamma)M \cdot \mathbb{E}[WW^T].
\]

Each row of the chance constraint (35) can be rewritten as

\[
\mathbb{P}[E_x X \leq h_x] \geq p_x.
\]

By substituting the state equation (31), the above inequality becomes

\[
\mathbb{P}[E_x (BM + C)W \leq (h_x) - (E_x A x_0) - (E_x Bg(\Gamma)) \geq (p_x)]. \quad (37)
\]

Letting \( F_{M,\Gamma}(\cdot) \) represent the CDF of \((E_x(BM + C)W)_i\), the inequality (37) can be written as

\[
F_{M,\Gamma}(h_x) - (E_x A x_0) - (E_x Bg(\Gamma)) \geq (p_x).
\]

Then, taking the inverse of \( F_{M,\Gamma} \) yields

\[
(E_x A x_0) + (E_x Bg(\Gamma)) - (h_x) + F_{M,\Gamma}^{-1}(p_x) \leq 0.
\]

This inequality is defined on each component, which finally gives

\[
E_x A x_0 + E_x Bg(\Gamma) - h_x + F_{M,\Gamma}^{-1}(p_x) \leq 0. \quad (38)
\]

The input chance constraint (36) can be transformed as follows. By introducing the linear approximation of \( g^{-1}(V) \) again, the input chance constraint can be approximated as

\[
\mathbb{P}[E_u U \leq h_u] \approx \mathbb{P}[E_u g^{-1}(\hat{g}(\Gamma) + MW) \leq h_u]
\]

\[
\approx \mathbb{P}[E_u (\hat{g}(\Gamma) + MW) \leq h_u] \geq p_u.
\]

Defining the CDF of \((E_u g(\Gamma)MW)_i\), as \( F'_{\Gamma,M,\Gamma} \), the input constraint can be rewritten as

\[
E_u - h_u + F'_{\Gamma,M,\Gamma}(p_u) \leq 0.
\]

In summary, the open-loop SMPC and closed-loop SMPC are transformed into the following problems.

**Problem 3 (Open-loop SMPC).**

\[
\min_{U} \mathbb{E}[J]
\]

subject to

\[
\mathbb{E}[J] = \frac{1}{2}(A x_0 + Bg(U))^T D^T QD (A x_0 + Bg(U))
\]

\[
+ \text{tr}\left\{C^T D^T QDC \cdot \mathbb{E}[WW^T]\right\} + \frac{1}{2}U^TRU,
\]

\[
E_x A x_0 + E_x Bg(U) - h_x + F_{\Gamma}^{-1}(p_x) \leq 0,
\]

\[
E_u U - h_u \leq 0.
\]
Problem 4 (Closed-loop SMPC).

$$\min_{M,T} \mathbb{E}[J]$$
subject to

$$\mathbb{E}[J] = \frac{1}{2} \left[ (A_{x_0} + B_g(\Gamma))^T D_\Psi Q (A_{x_0} + B_g(\Gamma)) \right. \\
+ \frac{1}{2} \text{tr} \left( (BM + C)^T D_\Psi Q (BM + C) \cdot \mathbb{E}[W W^T] \right) \\
+ \frac{1}{2} \text{tr} \left( (\Gamma M)^T R_g(\Gamma) M \cdot \mathbb{E}[W W^T] \right),$$

$$E_x A_{x_0} + E_x B_g(\Gamma) - h_x + F_M^{-1}(p_x) \leq 0,$$

$$E_o \Gamma - h_o + F_M^{-1}(p_o) \leq 0.$$ 

Note that Problem 3 is obtained by setting $M = O$ and $\Gamma = U$ in Problem 4. These are deterministic nonlinear programming problems that can be solved with a common nonlinear optimization solver. Note that $F_M^{-1}(\cdot)$ depends on the decision variables $M$, and $F_M^{-1}(\cdot)$ depends on the decision variables $\Gamma, M$. In the simulation reported below, MATLAB’s `fmincon` function is used to solve the problems; $F_M^{-1}(\cdot)$ and $F_M^{-1}(\cdot)$ must be calculated every time the value of the objective function is requested. These calculations can be performed with MATLAB’s `norminv` function or `icdf` function. Note that, although the objective function and the input constraints are linearly approximated, the state constraints are not approximated. It should also be noted that linear approximations in the objective function and the input constraints vanish in the case of open-loop SMPC (Problem 3).

3.4 Estimation of Demand Characteristic

Since the demand characteristic varies with the weather, time, and season, a particle filter is used to estimate the demand characteristic of the HPWHs in real time. The particle filter estimates the price elasticity $a_0$, from the power consumption of HPWH and the presented price. The dynamics of the HPWHs are assumed to follow the same model as in SMPC (6), except with noise added:

$$p_h(i + 1) = p_h(i) + \frac{1}{\Delta t_{hp}} (p_h(i) + D_h(u_p(i))) \Delta t + w_h(i),$$

$$w_h(i) \sim N(0, \sigma_w).$$

Although the parameter $a_0$ is a constant, we assume that it is affected by noise in the filter at every time step:

$$\hat{a}_h(i + 1) = \hat{a}_h(i) + w_h(i), \quad w_h \sim N(0, \sigma_w).$$

4. Numerical Simulation

4.1 Simulation Settings

We chose the smaller grid of the two interconnected areas in the EAST 30 system model [20] as the model of the electric power system. The capacity and initial output of the generators are shown in Table 2. The model consists of six thermal power plants and two nuclear power plants. However, the nuclear power generators are not controlled in the simulation since the controller is designed for a fast time scale. Although this power system model was originally interconnected to the other grid, the simulation treated it as a single grid in order to simply examine the effect of a supply-demand imbalance on the frequency.

The parameters of the power system are shown in Table 3. The GF and LFC capacities were chosen so that they would represent the typical composition of generators at night, when the effects of wind power fluctuations become apparent and LFC capacity is small. We assumed 2000 MW of wind power on the basis of Japan’s target in 2030 [21].

The BESS capacity was set to 100 MW. The previous paper [17] shows that if BESS has a 200 MW capacity, HPWH is not needed for frequency control. In terms of cost, it is preferable to reduce the BESS capacity. Thus, our goal in this study was to show that HPWH output can still be adjusted to deal with the fluctuation even if the BESS capacity is 100 MW. In particular, we assume 1.26 million HPWHs, so the capacity is 1,512 MW [17]. The previous paper [17] also investigated the number of households in the same area as in this paper and calculated the capacity assuming that 30% of them have HPWHs. As mentioned in Section 2.4, 20% of the rated capacity of HPWH is assumed to be controlled; thus, the controllable capacity is 302.4 MW.

The parameters of the controller, the upper and lower bounds of constraints, and the variance of the disturbance are shown in Table 4. The control cycle of 1 s was chosen to control the responses of the BESS and the thermal power generator. The prediction horizon of $N = 7$ was chosen to avoid an excessive calculation time for closed-loop SMPC. The probability bounds for chance constraints were all set to 90%.

The simulation was carried out in MATLAB/Simulink for 1800 s. The control methods were open-loop SMPC (Problem 3), closed-loop SMPC (Problem 4), and deterministic MPC, which does not consider disturbances. We prepared ten patterns of disturbance data, as mentioned in Section 2.5. We carried out simulations with each disturbance pattern.

### Table 2 Parameters of the generators.

| Rated Capacity (MW) | Initial Output (MW) |
|---------------------|---------------------|
| Nuclear             | 3000                |
| Thermal             | 5560                |
| Wind                | 2000                |

### Table 3 Parameters of the power system.

| Reference frequency (Hz) | 50 |
|--------------------------|----|
| Power system constant (MW/Hz) | 0.09 |
| GF capacity (%) | ±5 |
| (MW) | ±278 |
| LFC capacity (%) | ±1.5 |
| (MW) | ±125 |

### Table 4 Parameters of the controller.

| Time interval | Δτ | 1 s |
|---------------|----|-----|
| Prediction horizon | $N$ | 7   |
| Weight matrices | $Q$ | $\text{diag}(10000, 10, 100, 0.1)$ |
| Constraints | $\Delta M_{\text{max}}$ | 125 MW |
| Constraints | $u_{0\text{max}}$ | 100 MW |
| Constraints | $\Delta w_{\text{max}}$ | 4.63 MW/s |
| Constraints | $\Delta w_{\text{max}}$ | 100 MW/s |
| Standard deviation of $W$ | $\sigma$ | 9.9406 MW/s |
As for the estimation of the demand characteristic, we changed the parameter $a_h$ in the middle of the simulation in order to confirm that the uncertainty in it is properly coped with and does not lead to failure of the frequency control. The initial value was set to $a_h = 5$ and was changed to $a_h = 4$ at 900 s in the simulation. The initial estimate of the estimator was $a_h = 4.5$.

### 4.2 Simulation Results

Here, we show the simulation results for closed-loop SMPC, open-loop SMPC, and deterministic MPC. Note that the price elasticity $a_h$ was estimated on-line with the particle filter in all cases. Figures 8–12 show simulation results for one of the disturbance patterns with closed-loop SMPC. Figure 8 shows the frequency deviation; it is clear that the deviation does not exceed the bounds for most of the simulation time. Figures 9–12 show the outputs from the generators and equipment and the presented price; it can be seen that the controller presents proper inputs within the constraints, considering the characteristics of each generator and piece of equipment. Although the price in Fig. 12 shows spike-like behavior, it is attenuated sufficiently in the HPWH output (Fig. 11) reacting to the price because of the slow dynamics of the HPWH.

Figure 13 shows the estimated parameter $\hat{a}_h$ for the demand characteristic estimation; the dotted line represents the true parameter. The estimated value follows the true value $a_h = 5$ after about 200 s from the initial estimate, $\hat{a}_h = 4.5$, and it follows the change in the parameter to $a_h = 4$ at 900 s after about 200 s from the change. It has been shown by Satouchi et al. [8] that small fluctuations in the estimate do not affect the closed-loop responses significantly.

Next, we compared the responses of the different control methods, i.e., closed-loop SMPC, open-loop SMPC, and deterministic MPC, in terms of two performance measures: (a) the percentage of time that the frequency exceeds the limit $\omega_{\text{max}}$, and (b) the average and variance of the stage cost $\frac{1}{2}(y(i)^TQy(i) + u(i)^TRu(i))$ calculated from the simulation output. Figure 14 plots measure (a) for the three control methods for ten different disturbance patterns. The performance values of the three control methods for the same disturbance pattern are connected with a line. It can be seen that the closed-loop SMPC has the smallest percentage, followed in order by open-loop SMPC and deterministic MPC, for all disturbance patterns. SMPC can predict future state distributions affected by disturbances; thus, it handles the constraints more properly. Additionally, closed-
loop SMPC can consider future feedback; thus, it can deal with constraints even better than open-loop SMPC can.

Figure 15 shows the average stage cost, while Fig. 16 shows the variance of the stage cost. From these figures, it can be seen that the order of control performance is the same between the average and the variance, and the smallest average and variance were achieved by closed-loop SMPC, followed in order by open-loop SMPC and deterministic MPC. One of the reasons for this result is that closed-loop SMPC reduces conservativeness against constraints.

In summary, closed-loop SMPC can avoid violations of constraints more effectively, and at the same time, it can reduce the stage cost. Thus, we conclude that closed-loop SMPC is the best control method of the three. However, a disadvantage is its high calculation cost due to the large number of decision variables. As for the other two methods, their calculation time was below the control cycle of 1 s for our setup (CPU: Intel(R) Xeon(R) E5-2650 v4 2.20 GHz). In contrast, the calculation time for closed-loop SMPC was about 40 s. Note that we used MATLAB’s \texttt{fmincon}, which is an off-the-shelf nonlinear optimization solver, for the calculation; thus, the calculation time may be further reduced if a dedicated algorithm for closed-loop SMPC is developed.

5. Conclusions

We have developed a load frequency control method with SMPC for power systems with high market penetration of wind power. The control system directly sends signals to thermal power plants and a BESS, while it indirectly controls households’ HPWHs by presenting the electricity price in real time. We used a particle filter in order to estimate the demand’s uncertain characteristics.

Regarding the solution methods for SMPC, we showed how to transform the SMPC problems for the Hammerstein model into equivalent deterministic optimization problems. Numerical simulations showed that the proposed method can suppress the frequency deviation to within the designated bounds for almost the entire time, satisfying the input constraints. The simulations also revealed that closed-loop SMPC outperforms open-loop SMPC and deterministic MPC. Our future work will include development of a more efficient algorithm for closed-loop SMPC and simulations on more complicated and interconnected grids.

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References

[1] International Energy Agency: Medium-term Renewable Energy Market Report 2016, 2016.
[2] B. Ernst, B. Oakleaf, M.L. Ahlstrom, M. Lange, C. Moehrlen, B. Lange, U. Focken, and K. Rohrig: Predicting the wind, \textit{IEEE Power and Energy Magazine}, Vol. 5, No. 6, pp. 78–89, 2007.
[3] D.S. Watson, N. Matson, J. Page, S. Kiliccote, M.A. Piette, K. Corfee, B. Seto, R. Masiello, J. Masiello, L. Molander, S. Golding, K. Sullivan, W. Johnson, and D. Hawkins: Fast automated demand response to enable the integration of renewable resources, \textit{California Energy Commission}, 2012.
[4] Y.G. Rebours, D.S. Kirschen, M. Trottignon, and S. Rossignol: A survey of frequency and voltage control ancillary services: Part II: economic features, \textit{IEEE Transactions on Power Systems}, Vol. 22, No. 1, pp. 358–366, 2007.
[5] A. Jokić, M. Lazar, and P.P.J. van den Bosch: Real-time control of power systems using nodal prices, \textit{International Journal of Electrical Power & Energy Systems}, Vol. 31, No. 9, pp. 522–530, 2009.
[6] H. Kamemoto, T. Hashimoto, K. Kashima, and T. Ohtsuka: Model predictive control based real-time pricing for load fre-
frequency control in electric power systems, *Transactions of the Institute of Systems, Control and Information Engineers*, Vol. 27, No. 10, pp. 405–411, 2014 (in Japanese).

[7] R. Satouchi, Y. Kawano, and T. Ohtsuka: Load frequency control by integrating real-time price presentations for consumers and direct commands issued to generators and batteries, *Proceedings of the 2016 IEEE International Conference on Sustainable Energy Technologies (ICSET)*, pp. 396–400, 2016.

[8] R. Satouchi, Y. Kawano, and T. Ohtsuka: Real-time pricing with consumers estimation by a particle filter, *Transactions of the Society of Instrument and Control Engineers*, Vol. 53, No. 8, pp. 463–472, 2017 (in Japanese).

[9] A. Mesbah: Stochastic model predictive control: An overview and perspectives for future research, *IEEE Control Systems Magazine*, Vol. 36, No. 6, pp. 30–44, 2016.

[10] M. Farina, L. Giulioni, and R. Scattolini: Stochastic linear model predictive control with chance constraints: A review, *Journal of Process Control*, Vol. 44, pp. 53–67, 2016.

[11] F. Oldewurtel, C.N. Jones, and M. Morari: A tractable approximation of chance constrained stochastic MPC based on affine disturbance feedback, *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 4731–4736, 2008.

[12] G. Schildbach, L. Fagiano, C. Frei, and M. Morari: The scenario approach for stochastic model predictive control with bounds on closed-loop constraint violations, *Automatica*, Vol. 50, No. 12, pp. 3009–3018, 2014.

[13] A. Mesbah, S. Streif, R. Findeisen, and R.D. Braatz: Stochastic nonlinear model predictive control with probabilistic constraints, *Proceedings of the 2014 American Control Conference*, pp. 2413–2419, 2014.

[14] K.P. Fruzzetti, A. Palazo˘glu, and K.A. McDonald: Nonlinear model predictive control using Hammerstein models, *Journal of Process Control*, Vol. 7, No. 1, pp. 31–41, 1997.

[15] T. Hashimoto: Transformation of a chance constraint in stochastic model predictive control problems, *Systems, Control, and Information*, Vol. 61, No. 2, pp. 63–68, 2017 (in Japanese).

[16] P.J. Goulart, E.C. Kerrigan, and J.M. Maciejowski: Optimization over state feedback policies for robust control with constraints, *Automatica*, Vol. 42, pp. 523–533, 2006.

[17] H. Irie, A. Yokoyama, and Y. Tada: System frequency control by coordination of batteries and heat pump based water heaters on customer side in power system with a large penetration of wind power generation, *The Transactions of the Institute of Electrical Engineers of Japan B*, Vol. 130, No. 3, pp. 338–346, 2010 (in Japanese).

[18] J. Apt: The spectrum of power from wind turbines, *Journal of Power Sources*, Vol. 169, No. 2, pp. 369–374, 2007.

[19] IEE Japan: Recommended practice for simulation models for automatic generation control, CD-ROM, Ver. 1.0, 2016 (in Japanese).

[20] IEE Japan: The standard model in power system, IEEJ Technical Report, No. 754, pp. 1–82, 1999 (in Japanese).

[21] Agency for Natural Resources and Energy: Long-term Energy Supply and Demand Outlook, 2009 (in Japanese).

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