Tracing sunspot groups to determine angular momentum transfer on the Sun

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ABSTRACT
In this paper, our goal is to investigate Reynolds stress and to check whether it is plausible that this is responsible for angular momentum transfer towards the solar equator. We have also analysed meridional velocity, rotation velocity residuals and correlation between the velocities. We have used the position measurements of sunspot groups from the Greenwich Photographic Result and the Solar Observing Optical Network/United States Air Force/National Oceanic and Atmospheric Administration data bases, covering the period 1878–2011. In order to calculate the velocities, we used the daily motion of sunspot groups. The sample was also limited to ±58° in the central meridian distance in order to avoid solar limb effects. We have mainly investigated velocity patterns depending on the solar cycle phase and latitude. We have found that the meridional motion of sunspot groups is towards the centre of activity from all available latitudes and in all phases of the solar cycle. The range of meridional velocities is ±10 m s⁻¹. Horizontal Reynolds stress is negative at all available latitudes and indicates that there is a minimum value (q ≈ −3000 m² s⁻²) located at b ≈ ±30°. In our convention, this means that angular momentum is transported towards the solar equator, in agreement with the observed rotational profile of the Sun.

Key words: Sun: activity – Sun: photosphere – Sun: rotation – sunspots.

1 INTRODUCTION
Tracing the motion of sunspot groups has a long history and it is still used frequently today for studies of solar rotation and related phenomena. In this work we have used the daily motion of sunspot groups from the Greenwich Photographic Result (GPR) and Solar Observing Optical Network/United States Air Force/National Oceanic and Atmospheric Administration (SOON/USAF/NOAA) data bases combined into a single data set. The key parameter we investigate is the horizontal Reynolds stress, which might explain the transfer of angular momentum towards the equator. In a review, Schröter (1985) advocated the use of sunspot data because both components of horizontal Reynolds stress can be measured separately. The GPR data set, complemented by SOON/USAF/NOAA, is the longest homogeneous sunspot catalogue and presents a unique opportunity to study long term average properties of the solar velocity field as well as its variations on time-scale of a century. The GPR data set we used was digitized in the frame of several projects and it is not identical to the on-line version.1 Various analyses were carried out by using whole or parts of our GPR data set (Balthasar & Wöhl 1980, 1981; Arévalo et al. 1982, 1983; Balthasar, Vázquez & Wöhl 1986; Balthasar, Wöhl & Stark 1987; Brajša & Wöhl 2000; Wöhl & Brajša 2001; Brajša et al. 2002, 2004, 2007; Ruždjak et al. 2004, 2005; Brajša, Ruždjak & Wöhl 2006).

Balthasar et al. (1986) and Brajša et al. (2006) have given a very comprehensive analysis of cycle-to-cycle variations of rotation velocity for the GPR data set. They have also analysed rotation velocity variations with respect to the phase of the solar cycle. Recently, Willis et al. (2013a,b) and Erwin et al. (2013) have undertaken the comprehensive project of revising the GPR data set in order to correct a number of erroneous measurements and typographical errors; this illustrates the importance of the GPR data set.

Various aspects of solar rotation and related phenomena are used to quantify and constrain solar models. Usually, the main focus of such investigations is on meridional motions and rotation residual velocity. The correlation between two velocities and their covariation has even greater significance. All of these quantities play an important role in our understanding of the solar cycle and its variations from one cycle to another. There has been a series of numerical simulations and theoretical works regarding the transfer of angular momentum towards the equator (Canuto, Minotti & Schilling...
The meridional flow was first observed by Howard & Labonte (1991, 1995) and further confirmed by Rošnak et al. (2000, 2002), Basu & Antia (2003) and others. At latitudes below $b \approx 40^\circ$, bands propagate towards the equator, while each band is about $15^\circ$ wide in latitude. The amplitude of the effect is about $\Delta v_{\text{rot}} \approx 5 - 10$ m s$^{-1}$. Brajša et al. (2006) have shown an interesting analysis of rotation velocity residuals versus the phase of the solar cycle in their fig. 6, showing variations of $\Delta v_{\text{rot}} \approx 0.05$ d$^{-1}$ corresponding to $\Delta v_{\text{rot}} \approx 7$ m s$^{-1}$ at the equator. Transfer of the angular momentum from higher to lower latitudes can be revealed by studying the correlation and covariance between azimuthal and meridional flows. Covariance, denoted as $Q = \langle \Delta v_{\text{rot}} v_{\text{mer}} \rangle$, is a horizontal Reynolds stress. Reynolds stress is thought to be the main generator of maintaining the current differential rotation profile (e.g. Pulkkinen & Tuominen 1998; Rüdiger & Hollerbach 2004). Indeed, observations seem to show that the correct value of $Q$ was observed (Ward 1965; Belvedere et al. 1976; Schröter & Wöhl 1976; Gilman & Howard 1984; Pulkkinen & Tuominen 1998; Vršnak et al. 2003). In addition, other authors (Ward 1965; Gilman & Howard 1984; Pulkkinen & Tuominen 1998; Vršnak et al. 2003) have investigated the latitudinal dependence of Reynolds stress, and have found that it mainly decreases with higher latitudes, with a possible minimum around $b = \pm 30^\circ$.

### 2 DATA AND REDUCTION METHODS

We limited the data to $\pm 58^\circ$ in the central meridian distance (CMD), which corresponds to about 0.85 of projected solar radius (see Balthasar et al. 1986). With such a cut-off, we obtained a sample of 92 091 data pairs from the GPR to obtain rotation rates and meridional velocities. We used two subsequent measurements of individual sunspot groups to obtain one velocity value.

Using the same CMD cut-off, we ended up with a sample of 43 583 data pairs from observations found in the SOON/USAF/NOAA data base in the period 1977–2011. Combining these samples into a single data set, the total number of data points for sunspot groups was 135 674 spanning from 1878 to 2011. In the rest of the paper, we refer to this combined data set as the extended Greenwich result (EGR). In the GPR era (until 1977), the positions of sunspot groups are given with an accuracy of 0.1 in both coordinates, while subsequent measurements were usually taken 1 d apart. After 1977, the positions are usually given with an accuracy of 1.0.

When observed by tracers, solar rotation and related phenomena should be treated statistically, which requires a large number of measurements for proper analysis. While solar rotation velocity has a large signal-to-noise ratio (S/N), solar rotation residuals, meridional velocities and Reynolds stress are significantly weaker effects with lower S/N. So, in this paper, we mostly concentrate on identifying the basic net effect in the various relationships between the phenomena mentioned.

Meridional motion and angular rotation velocity have been calculated from two subsequent position measurements. Because most of the measurements in our data set are 1 d apart, velocities are calculated from the daily shifts of sunspot groups. To obtain rotation velocity residuals, it is necessary to subtract the actual velocity measured from the average rotation velocity at a given latitude. Synodic angular velocities have been calculated by using the daily motion of sunspot groups, and converted to sidereal angular velocities using the procedure described by Rošnak et al. (1995) and Brajša et al. (2002). Because of the latitudinal distribution of sunspots, it is sufficient to use only the first two terms in the standard solar differential rotation equation,

$$\omega(b) = A + B \sin^2 b,$$

where $b$ is the heliographic latitude and $\omega(b)$ is sidereal angular velocity.

For the EGR data set, we obtained $A = 14.499 \pm 0.005$ d$^{-1}$ and $B = -2.64 \pm 0.05$ d$^{-1}$, which we calculated by fitting the above equation to all points in the data set ($n = 135 674$).
After the subtraction was carried out, angular velocity residuals were transformed to linear velocity residuals \((\Delta v_{\text{mer}})\) in units of \(\text{m s}^{-1}\), taking into account the latitudes of the tracers. The solar radius used for conversion from angular velocities to linear velocities was \(R_\odot = 696.26 \times 10^3 \text{ km}\) (Stix 1989).

We limited the calculated sidereal rotation velocity to 8–19 \(\text{d}^{-1}\) in order to eliminate any gross errors, usually resulting from the misidentification of sunspot groups or from typographical errors. Angular meridional velocities were also transformed to linear velocities in \(\text{m s}^{-1}\).

In Section 3, we present several map plots of various quantities depending on latitude, \(b\), and the phase of the solar cycle, \(\phi\). Therefore, it is useful to show the latitudinal distribution of sunspots from the EGR data set with respect to the phase of the solar cycle, \(\phi\), in order to indicate where the results are more reliable (Fig. 1). We folded all the data into one solar hemisphere.

In order to determine the phase, we used the times of minima and maxima of solar activity found in table 1 of Brajša et al. (2009). All points that belong after the minimum of the solar cycle and before the maximum were mapped to the \([0, 0.5]\) phase range. Points after the maximum, but before the minimum of the next cycle, were assigned a phase in the \([0.5, 1]\) range. The phase was calculated in a linear scale:

\[
\phi_i = \frac{t_i - t_{\text{max}/\text{min}}}{t_{\text{max}/\text{min}} - t_{\text{min}/\text{max}}}, \tag{2}
\]

Because the distribution of sunspots in latitude is not uniform, some care must be taken in order not to detect false motion (Olemskoy & Kitchatinov 2005). Calculated velocities need to be assigned to some latitude. Considering that we have two measurements of position for one velocity, we have to decide to which latitude we should assign the velocity. Olemskoy & Kitchatinov (2005) have shown that false flow can arise if the average latitude of the two values is used, because the gradient of the sunspot latitudinal distribution will pollute the result. They have also shown that this false meridional flow is of the right order of magnitude and in the right direction as the results obtained by many authors who have used tracers to detect surface flows on the Sun. However, there is a simple solution to this problem, as follows. If we assign the velocity to the latitude of the first measurement of position, there is no net flow into the latitude bin from other latitudes and we do not have to worry about the non-uniform distribution of sunspots in latitude. Olemskoy & Kitchatinov (2005) also came to the same conclusion.

3 RESULTS

3.1 Meridional flow

We used the convention that negative meridional velocity reflects motion towards the equator: \(v_{\text{mer}} = -\partial b/\partial t\) for the Southern hemisphere, where we have defined southern latitudes as negative values.

In this section, we investigate the properties of \(v_{\text{mer}}\) depending on latitude, \(b\), and the phase of the solar cycle, \(\phi\). In Fig. 2, we show a map plot of \(v_{\text{mer}}\) versus the phase of the cycle and latitude, \(b\), for the EGR data set. All points were folded into one phase diagram and both solar hemispheres were folded together according to our convention above. The map plot is constructed first by binning the data into square bins of width 0.1 in the phase of the cycle, \(\phi\), and height 1° in latitude. Then, we calculated the average values of velocity in each bin, discarding all the bins where the number of data points was less than 10 (see Fig. 1). Finally, we calculated the smoothed averages of each bin with weight given by \(w(d) = 1/(1 + d^2)\), where \(d\) is the distance of each data point from the map grid point. Brighter shades of grey depict poleward motion \(v_{\text{mer}} > 0\), while darker shades show motion towards the solar equator \(v_{\text{mer}} < 0\). On the same plot, we show a contour line of \(v_{\text{mer}} = 0\) shown with a solid line, which clearly separates the two regions of opposite meridional flow.

Such a distinct appearance can easily be confirmed by plotting average values of \(v_{\text{mer}}\) in bins 2° wide in latitude (Fig. 3). This is similar to Fig. 2 but integrated in the phase of the solar cycle, \(\phi\), or in other words integrated in time. In the same figure, we show a linear fit of \(v_{\text{mer}}(b)\) through individual data points, given by

\[
v_{\text{mer}} = (-0.571 \pm 0.038) \text{ m s}^{-1} \phi + (8.61 \pm 0.64) \text{ m s}^{-1}.
\]

From equation (3), we obtain the intersect with the \(x\)-axis, \(v_{\text{mer}}(b) = 0\), for \(b \approx 15°\). In the inset of Fig. 3, we show average values of \(v_{\text{mer}}\) separately for the two solar hemispheres. We can see that for very low latitudes, \(v_{\text{mer}}\) goes back to zero and reverses sign when crossing the solar equator.
Figure 3. Average values of $v_{\text{mer}}$ in bins $\pm 10^\circ$ wide in latitude, $b$. A linear fit (equation 3) is shown by a dashed line. In the inset, we show average values of $v_{\text{mer}}$ separately for the Northern and Southern solar hemispheres.

Table 1. Values of linear fit coefficients (equation 4), intersecting with the $x$-axis, $b_{\text{mer}}=0$, and the average latitude of sunspot groups, $b(\phi)$.

| $\phi$ (°) | $c_1$ (m s$^{-1}$) | $c_2$ (m s$^{-1}$) | $b_{\text{mer}}=0$ (°) | $b(\phi)$ (°) |
|-----------|-----------------|-----------------|-----------------|----------------|
| 0.05      | -0.11 ± 0.22    | 0.3 ± 4.8       | 2.7 ± 43.8      | 19.3           |
| 0.15      | -0.51 ± 0.23    | 10.3 ± 5.5      | 19.9 ± 13.9     | 22.0           |
| 0.25      | -0.94 ± 0.17    | 20.5 ± 3.7      | 21.8 ± 5.5      | 21.1           |
| 0.35      | -0.83 ± 0.12    | 15.4 ± 2.5      | 18.6 ± 4.0      | 19.1           |
| 0.45      | -0.65 ± 0.11    | 10.6 ± 1.9      | 16.3 ± 4.0      | 16.9           |
| 0.55      | -0.63 ± 0.09    | 10.2 ± 1.3      | 16.1 ± 2.9      | 15.0           |
| 0.65      | -0.73 ± 0.10    | 9.6 ± 1.4       | 13.2 ± 2.6      | 13.1           |
| 0.75      | -0.88 ± 0.15    | 9.2 ± 1.8       | 10.4 ± 2.7      | 11.4           |
| 0.85      | -1.20 ± 0.24    | 11.1 ± 2.7      | 9.3 ± 2.9       | 9.9            |
| 0.95      | -0.73 ± 0.30    | 5.8 ± 3.4       | 8.0 ± 5.6       | 9.5            |

We also divided the data into 10 bins in phase, $\phi$, each being 0.1 wide. Then, we calculated $v_{\text{mer}}(b)$ linear fits for those 10 bins. The coefficients, given by

$$v_{\text{mer}} = c_1 b + c_2,$$

are shown in Table 1.

Perhaps the most interesting result is that the contour line from Fig. 2, which represents values of $v_{\text{mer}} = 0$, is very close to the centre of activity in each phase [i.e. the average latitude as a function of phase, $b(\phi)$]. The average latitude, $b(\phi)$, was calculated with $b(\phi) = \sum b_i/n(\phi)$ (the sunspot group area was not taken into account). Therefore, we calculated values of $b_i$, for which $v_{\text{mer}} = 0$, in all 10 phase bins ($b_i = -c_2/c_1$). The values are given in column 4 of Table 1. Both quantities are shown in Fig. 4. The average latitudes, $b(\phi)$, are drawn with a dashed line and solid circles, while $b_i$ is drawn with a thin solid line, solid triangles and error bars. The error bar for phase $\phi = 0.05$ is off the scale. We have also shown the contour line for $v_{\text{mer}} = 0$ obtained from the map plot presented in Fig. 2. Apart from the highly uncertain value of $b_0$ for phase $\phi < 0.1$, the agreement between $b_0$ and $b(\phi)$ is very good.

Regions with latitudes closer to the solar equator move towards the pole, while the centre of activity in each phase roughly marks the line where meridional motion changes to motion towards the equator.

3.2 Torsional oscillations

Torsional oscillations can be described as a pattern in which the solar rotation is speeded up or slowed down in certain regions of latitude. In this section, we investigate whether we can reveal this pattern by examining the relationship of $\Delta v_{\text{rot}}$ with latitude, $b$, and cycle phase, $\phi$.

We have constructed a map plot of $\Delta v_{\text{rot}}$ (Fig. 5) in the same fashion as in the previous section. Thick dashed lines show the contour where $\Delta v_{\text{rot}} = 0$. Darker regions depict slower than average rotation ($\Delta v_{\text{rot}} < 0$), while brighter regions show faster than average rotation ($\Delta v_{\text{rot}} > 0$). Fig. 5 shows a complex pattern of rotation velocity residuals. The pattern does not look like a typical torsional oscillation pattern. In order to investigate whether the typical torsional oscillation pattern is only present in today’s data, we divided our data set into three epochs: early epoch (cycles 12–15), mid-epoch (cycles 16–19) and late epoch (cycles 20–24). Then, we plotted the same type of plot (Fig. 6). However, Fig. 6 does not reveal anything that even resembles the typical torsional oscillation pattern. Moreover, the results from the three different epochs are not consistent and we cannot see any regularity in the changes of the pattern over time. With our data set, it is not possible to make the division into shorter epochs, because as we can see from the top part of Fig. 6 we are already running out of data points at latitudes above 30°.
It is very difficult to actually determine the meridional velocity, $v_{\text{mer}}$, and the rotation velocity residual, $\Delta v_{\text{rot}}$, with precision only. Coupled with the usual 1-d period between the different solar cycles, we do not expect that any dependence of the $v_{\text{mer}}$ and $\Delta v_{\text{rot}}$ relationship on $\varphi$ will show up. In order to calculate the angular momentum $L_{\text{tot}}$ produced by a sunspot group, we need to know the latitude $\varphi$ of the group at the time of observation. The latitudinal position $\varphi$ is usually derived by positioning the group along the bottom edge of the solar disk. This measurement can be biased by the tilt angle of the group. The presence of high latitude groups can affect the observed correlation between $v_{\text{mer}}$ and $\Delta v_{\text{rot}}$ in two ways. First, the latitudinal position $\varphi$ of the group is measured by the position of the group along the bottom edge of the solar disk. This measurement can be biased by the tilt angle of the group. Second, the latitudinal position $\varphi$ of the group is derived from the position of the group along the bottom edge of the solar disk. This measurement can be biased by the tilt angle of the group. The presence of high latitude groups can affect the observed correlation between $v_{\text{mer}}$ and $\Delta v_{\text{rot}}$ in two ways. First, the latitudinal position $\varphi$ of the group is measured by the position of the group along the bottom edge of the solar disk. This measurement can be biased by the tilt angle of the group. Second, the latitudinal position $\varphi$ of the group is derived from the position of the group along the bottom edge of the solar disk. This measurement can be biased by the tilt angle of the group. The tilt angle of the group is irrelevant for the results, because the whole SOON/USAF/NOAA part of the data set, which is usually recorded with $L_{\text{tot}}$ precision only. Coupled with the usual 1-d period between successive measurements, we obtain the horizontal artefacts in the form of horizontal lines, which correspond to steps of $1^\circ$ $L_{\text{tot}}^{-1}$. These are a consequence of poor precision of position in the SOON/USAF/NOAA part of the data set, which is usually recorded with $L_{\text{tot}}$ precision only. Coupled with the usual 1-d period between successive measurements, we obtain the horizontal artefacts in the form of horizontal lines, which correspond to steps of $1^\circ$ $L_{\text{tot}}^{-1}$. These are a consequence of poor precision of position in the SOON/USAF/NOAA part of the data set, which is usually recorded with $L_{\text{tot}}$ precision only.

Howard (1984) criticized the use of sunspot groups to derive the correlation between $v_{\text{mer}}$ and $\Delta v_{\text{rot}}$. Because of the average tilt of about $4^\circ$ of a sunspot group towards the equator (Howard 1991b), it is possible to imagine that as the group evolves, the measured position of the group could be biased towards the tilt line. Consequently, the velocities derived would also be biased and would produce just the type of correlation between $v_{\text{mer}}$ and $\Delta v_{\text{rot}}$ as we see in Fig. 7 and equation (5). However, it is very difficult to actually quantify the magnitude of this effect and to conclude how much it would influence the true correlation between the two velocities. By using coronal bright points (CBPs) as tracers, Vrsnak et al. (2003) derived almost the same correlation. The tilt angle is irrelevant for CBPs and this is our first hint from independent measurements that the bias mentioned by Howard (1984) is negligible or not present at all. The GPR catalogue also contains information about the morphological type of the sunspot group. Quite a significant number of these are actually classified as a single spot. The tilt angle for such groups is meaningless and no such bias can exist for this subset. Therefore, we checked the correlation between $v_{\text{mer}}$ and $\Delta v_{\text{rot}}$ for this subset of single spots and obtained $v_{\text{mer}} = (-0.0803 \pm 0.0036) \Delta v_{\text{rot}} + (-1.40 \pm 0.46) \text{ m s}^{-1}$. (6)

The correlation is virtually identical to the one obtained for all groups (equation 5). This is conclusive proof that the tilt bias of sunspot groups is not significantly affecting the observed correlation when using sunspot groups as tracers. In Fig. 7, there are visible artefacts in the form of horizontal lines, which correspond to steps of $1^\circ$ $L_{\text{tot}}^{-1}$. These are a consequence of poor precision of position in the SOON/USAF/NOAA part of the data set, which is usually recorded with $L_{\text{tot}}$ precision only. Coupled with the usual 1-d period between successive measurements, we obtain the horizontal artefacts separated by $1^\circ$ $L_{\text{tot}}^{-1}$. However, we note that these do not significantly affect the results, because the whole SOON/USAF/NOAA part of the data set was excluded from the calculations given in equation (6) because this part of the data set does not contain information on the morphological type of the sunspot groups.

We have also investigated whether the averages of $\langle \Delta v_{\text{rot}} v_{\text{mer}} \rangle$ grouped in bins of $10^\circ$ will show any dependence of $\varphi$ with latitude. We grouped the data set into four subsets, spanning from $0^\circ$ to $10^\circ$, $10^\circ$ to $20^\circ$, $20^\circ$ to $30^\circ$, and above $30^\circ$ in latitude, and then we simply calculated the averages of $\Delta v_{\text{rot}} v_{\text{mer}}$ product for each bin to obtain $q(\varphi)$ values. The results are given in Table 2 and are shown in Fig. 8. In the same figure, we also show the results obtained by
Table 2. Average value of covariance, $q$, for several bins in latitude, $b$.

| Bin     | $q$ (m$^2$ s$^{-2}$) | $b$ (°) | $n$  |
|---------|---------------------|---------|------|
| $0° < b < 10°$ | $-1404 \pm 128$     | 6.85    | 35 836 |
| $10° < b < 20°$ | $-2113 \pm 98$      | 14.93   | 67 156 |
| $20° < b < 30°$ | $-2896 \pm 160$     | 23.93   | 28 518 |
| $b > 30°$    | $-2446 \pm 397$     | 33.953  | 4164 |

Figure 8. Relationship between covariance, $q$, and latitude, $b$, represented by average values in bins of different width obtained with data from this paper and others. The results from Ward (1965) are labelled Ward and the results from Vršnak et al. (2003) are labelled Vršnak 10 and Vršnak 30, for their bins of 10° and 30°, respectively. The linear fit (equation 7) is shown by a thin solid line.

Figure 9. Map plot of covariance, $q$, versus the phase of the solar cycle, $\phi$, and latitude, $b$. Levels of $q = -2000$, $-2500$ and $-2900$ m$^2$ s$^{-2}$ are shown by solid lines, from thinnest to thickest, respectively.

Vršnak et al. (2003) (their 10° and 30° bins) and Ward (1965) and also a linear fit

$$q = (-76.4 \pm 9.5) \text{ m}^2 \text{ s}^{-2} (\phi)^{-1} b + (-933 \pm 161) \text{ m}^2 \text{ s}^{-2}$$

through the individual data points of our EGR data set.

Finally, in Fig. 9, we show a map plot of covariance, $q$, as a function of the phase of the solar cycle, $\phi$, and latitude, $b$. In the same figure, we outline levels of $q = -2000$, $-2500$ and $-2900$ m$^2$ s$^{-2}$ by the thinnest to thickest solid lines, respectively. This representation of covariance, $q$, is fairly consistent across all phases and latitudes, with larger values of $q$ concentrated around lower latitudes for all phases.

4 DISCUSSION

Before we comment on the results presented, we must clarify the statistical significance of the various types of plots shown in this paper. Map plots have the lowest statistical significance because they show dependence of the variable with respect to two parameters, which means that in each bin there are significantly fewer data points than in other plots. In this work, these plots have been used only as informative and not a single value was calculated from them. However, they do provide a useful first look at the data and various relationships.

The second type of plot shows some empirical curve fitted to the data with one variable and one parameter. These plots are more important because the density of data points is significantly larger in one-dimensional than in two-dimensional parameter space. Nevertheless, caution must be taken before interpretation. Because fitted curves are only empirical, the form of the equation largely depends on how the raw data look, rather than what the data should be like from theoretical modelling. A typical example is shown in Fig. 3, where the linear function is fitted to the data. The look of the function might lead us to believe that there is a positive meridional flow at 0° latitude. However, if we look at the average values in the bins of latitude (even more conclusively shown in the inset of Fig. 3), we see that the flow actually drops to zero for very low latitudes in both hemispheres. The values of parameters of the fitted linear function are dominated by mid-latitude behaviour, which is where most of the data points actually are.

In the third type of plot, we show average values grouped in bins. Together with their errors, these should be the most reliable representation of what a true relationship actually looks like.

4.1 Meridional motions

We have seen that the meridional motions of sunspot groups show a distinct pattern where, at latitudes below the centre of activity, the motion is poleward, while on the other side the motion is predominantly towards the equator. In addition, we have shown that this is valid for all phases of the solar cycle. Flow converging to activity belts from both sides suggests that the plasma is circulating towards the centre of the Sun at these latitudes (sinking). Diverging flow on the equator suggests that the plasma is circulating from below to the surface. There might be another latitude point of diverging flow at about 40°, but our results are inconclusive as to whether this diverging flow is real.

This result contradicts most of the other results generated from tracer measurements (Howard 1991a, 1996; Snodgrass & Dailey 1996; Vršnak et al. 2003). In most of these studies, the flow is opposite to ours (i.e. meridional flow was found to be out from the centre of activity). Although none of the above papers has mentioned to which latitude calculated velocity is assigned, we suggest that their results are a consequence of not properly accounting for the non-uniform latitudinal distribution of the tracers used. Moreover, if we use any of the other two possibilities (i.e. the average latitude or the latitude of the second position measurement), we obtain almost exactly the opposite flow. The approach that we have used in this paper eliminates the need to keep track of tracer distribution and shows the true meridional motion.

Helioseismology measurements usually detect predominant poleward flow at all latitudes (Zhao & Kosovichev 2004; González Hernández et al. 2008, 2010). However, there is a striking similarity between our results for meridional flow (Figs 2 and 3) and residual meridional flow found by helioseismology (Zhao &
Kosovichev 2004; González Hernández et al. 2008, 2010). For example, all details from Fig. 3 are reproduced in González Hernández et al. (2008). This includes the change from motion towards the equator to poleward flow at about 40° latitude, even though our results are highly uncertain at these high latitudes. Zhao & Kosovichev (2004) have explicitly stated that residual meridional flow is converging towards activity belts. This makes us ask why we cannot see the dominant poleward flow in tracer measurements. One possibility is that this flow is not constant in time and that it was different in the past, which averages to net zero flow, and we only observe the residual that is permanent.

Hathaway (2012) has used supergranules as probes of the Sun’s meridional circulation, and has found that surface poleward flow gradually changes to motion towards the equator as we go deeper below the solar surface. An intermediate result between those two extremes might be the result we obtained by tracing sunspot groups. Ruždjak et al. (2004) have suggested that, at their birth, sunspots groups could be coupled to the layer at about \( r = 0.93 \) R⊙, effectively showing the plasma flow from beneath the solar surface.

Another possible explanation for the difference between helioseismology and tracer measurements is that the flow is different around sunspots than in the rest of the solar surface. With tracers, we are confined to a small region of the solar disc around active regions, while helioseismology does not have this limitation.

4.2 Torsional oscillations

A torsional oscillation pattern is usually described as distinct bands of faster and slower than average rotation rates. These bands move towards the equator with time at low latitudes (e.g. Basu & Antia 2003).

Our analysis of rotation velocity residuals reveals a pattern much less distinctive than for the meridional flow. Apart from generally remarking that slower than average flow is obtained around the maximum and faster than average around the minimum, there is hardly anything else we could say about it. Similar behaviour was found by Bračić et al. (2006, 2007). It has no distinct latitudinal dependence and it shows no significant correlation with zones of activity. Zhao & Kosovichev (2004) have shown zonal flows for the period 1996–2002 (solar cycle 23) and it looks as if their results are similar to ours (faster than average in the phases around the minimum of solar activity and slower than average around the maximum). However, we cannot confirm that the typical torsional oscillation pattern is visible in sunspot data.

We have investigated the rotation residual flow in three different epochs in order to see whether the typical torsional oscillation pattern can only be seen in modern data. The pattern does change with time, but it has no resemblance to torsional oscillations. Moreover, we cannot see any regularity in its shape and change from epoch to epoch. Thus, we have been unable to quantify the dependence of rotation residuals with respect to phase and latitude. It is quite possible that we see only random noise.

4.3 Reynolds stress

From Fig. 8, equation (7) and Table 2, it is easily seen that the angular momentum transfer towards the equator is predominant at all latitudes covered by sunspot group data \( q < 0 \). This is consistent with other studies using different methods and/or different data samples (Ward 1965; Gilman & Howard 1984; Vršnak et al. 2003).

The average values of \( q \) in 10° bins of \( b \) (shown in Table 2) show very similar behaviour to the linear fit for the sample of sunspot groups. Vršnak et al. (2003) have shown similar results for 10° bins using the CBP sample (their fig. 6c), but the qualitative behaviour is different from ours. We suggest that this is because their 10° bins have a statistical significance that is too low. This is also implied by their 30° bins, which show different qualitative behaviour much more similar to ours. Moreover, our results for 10° bins show very similar behaviour to that of Ward (1965), both qualitatively and quantitatively.

Another interesting feature visible in Fig. 8 is that there appears to be a minimum in the \( q(b) \) relationship around 25°–30°, both for our sample and for that given by Ward (1965). This is also reminiscent of similar results obtained by, for example, numerical simulations (Pulkkinen et al. 1993) and the results obtained by Canuto et al. (1994) based on theoretical considerations. In order to investigate this a little more, we plotted the average values of covariance, \( q \), with respect to latitude for both solar hemispheres separately (Fig. 10). In this representation, negative values represent angular momentum transfer towards the equator for the Northern solar hemisphere. In the Southern hemisphere, positive values of \( q \) also show momentum transfer towards the equator. So, this figure is consistent with Figs 8 and 9 where we see momentum transfer towards the equator at all available latitudes.

Because the minimum at around \( b = 25°–30° \) of latitude appears in all our plots, we have constructed an empirical relationship between \( q \) and \( b \), which takes into account dominant linear dependence for low latitudes and allows for a possible minimum at some unspecified latitude:

\[
q = (e_1b + e_2) e^{-e_3b^2}.
\]

This shape can produce a minimum, but could also ‘explode’ to \( q = \pm \infty \), if \( e_1 \) turns out to be negative. By fitting through individual data points, we obtained the coefficients of the fit shown in Table 3 and we also show the fit in Fig. 10 as a solid line.

| Coefficient | Value       |
|-------------|-------------|
| \( e_1 \) [m² s⁻² (°)⁻¹] | -169.0 ± 9.7 |
| \( e_2 \) [m² s⁻²] | 69 ± 80     |
| \( e_3 \) [°⁻²] | 0.00060 ± 0.00012 |

Figure 10. The \( q(b) \) relationship for the EGR sample fitted with the exponential cut-off model (solid line) and average values of \( q \) in bins spanning both solar hemispheres.
The shape of the curve is very similar to that given by Canuto et al. (1994) in their fig. 15, including the prediction that \( q(b) \) falls to zero at the poles. The form of our fit function (equation 8) does not guarantee this, because the answers to the questions whether and where (in terms of \( b \)) covariance, \( q \), falls to zero are highly sensitive to fitted coefficients and not constrained by the physical fact that the maximum latitude is \( b = 90^\circ \).

However, we must stress that although the agreement with Canuto et al. (1994) is good and that calculated coefficients seem to be strongly constrained (Table 3), the latitudinal extent of sunspot groups is very limited and anything that happens beyond \( b = 35^\circ - 40^\circ \) is ambiguous considering our data set. Even the location and depth of the minimum are not very reliable, for the same reason. Therefore, it is very important to confirm or disprove our hypothesis about the shape of \( q(b) \) at mid to high latitudes.

The depth of the minimum is also close to the one found by Canuto et al. (1994), considering that \( q \approx -3000 \text{ m}^2 \text{ s}^{-2} \approx -0.15 \left( \text{d}^{-1} \right)^2 \).

5 SUMMARY AND CONCLUSION

The most important results can be summarized as follows.

(i) The meridional motion of sunspot groups clearly shows poleward motion for all latitudes below the centre of activity and motion towards the equator for higher latitudes. This is valid for all phases of the solar cycle with a very strong correlation.

(ii) The variations of \( \nu_{\text{erot}} \) with latitude, \( b \), are approximately in the range of \( \nu_{\text{erot}} = \pm 10 \text{ m s}^{-1} \).

(iii) The rotation velocity residuals show an unusual torsional oscillation pattern. The actual values of rotation residual velocity rarely exceed \( \Delta \nu_{\text{rot}} = 5 \text{ m s}^{-1} \). The map plots of rotation velocity residuals in different epochs show a changing pattern, which we are unable to explain. It is possible that we see only a pattern resulting from random errors.

(iv) The meridional velocities are similar to residual meridional velocities found with time–distance helioseismology (Zhao & Kosovichev 2004; González Hernández et al. 2010).

(v) Reynolds stress is negative at all available latitudes, which, in our convention, corresponds to transfer of angular momentum towards the equator. This supports the idea that the observed rotational profile is actually driven by the Reynolds stress.

(vi) The latitudinal dependence of Reynolds stress suggests a minimum at about \( b \approx 30^\circ \). The value of covariance is \( q \approx -0.15 \left( \text{d}^{-1} \right)^2 \). This is consistent with Canuto et al. (1994).

(vii) The phase and latitudinal dependence of covariance, \( q \), seem to be fairly uniform in all phases of the solar cycle, with the possible exception at phases very late in the solar cycle.

Most authors who have used tracers to track the meridional flow have found meridional velocities that are opposite to ours. We believe that this is a consequence of the improper assignment of latitude to measured velocities. In the approach we have used (assigning velocities to the starting latitude), the latitudinal distribution of tracers is irrelevant because the calculation of averages (and even fit functions) does not need to take into account the number of tracers at a specific latitude; that is, \( n(b) \) is the same for all of them for each particular \( b \). If we were to use average latitude between two successive position measurements, we would have to calculate weighted averages by taking into account from which latitude the tracer actually started, and to assign weight accordingly.

As a test, we have also carried out an analysis by assigning the second latitude of two successive measurements and calculating average meridional motions. We have obtained results very similar to those of, for example, Snodgrass & Dailey (1996) and Vršnak et al. (2003). The assignment of the second latitude or average latitude suffers from exactly the same problem; we would need to take into account the first latitude in order to properly calculate weighted averages. By using the starting latitude as the relevant latitude, we have simply defined the flow as flow from a certain latitude instead of flow into some latitude. There is no loss of generality in doing this, and no difference in physical interpretation. A similar result to ours for meridional velocity was obtained by Olemskoy & Kitchatinov (2005), who pointed out the solution to this problem and used the starting latitudes of the tracers. As a point of interest, even when we used the second latitude in our test, the calculated correlation and covariance of meridional and residual rotation velocities were very similar to the results we have obtained in this paper by using the first latitude. This also explains why our results, regarding correlation and covariance, are similar to the results of other authors who used tracers, even if they found a different average meridional flow than we have.

We can see a clear increase of uncertainties at latitudes larger than 30°, which is a consequence of sunspot latitudinal distribution. Therefore, it is very important to use other methods or tracers (e.g. CBPs) to extend the analysis to higher latitudes.

The absence in our data of predominant poleward meridional flow, which is found in helioseismology, might be explained by several possibilities. The first possibility is that sunspots are anchored at a depth below the surface showing subphotospheric flow. Another possibility is that on longer time-scales, the flow changes from poleward to towards the equator, and consequently averages out in our analysis. Finally, it is possible that the flow in active regions is different than in the rest of the solar disc, so in our analysis we see only this localized flow. Future work might shed some light on these open questions.

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