Type-2 fuzzy controller’s performance index. Case study: tank level control

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Abstract. This paper deals with the design and implementation of type-2 fuzzy controller, through a tank level control plant as a pilot system to analyze the response using performance indexes. The controller was developed using type-2 fuzzy logic from a mathematical approach and utilizing the identification of the process through a parametric method. Additionally, a classical proportional, integral, and derivative controller was used to generate the bases for the linguistic variables required by the fuzzy controller implementation. Finally, a statistical analysis of the transient and steady-state response margins is performed, which aims to provide a numerical index of the system response performance.

1. Introduction
A type-2 fuzzy logic system is a system based on its structure and components in a type-1 fuzzy logic system [1]. Type-1 logic is a type of logic that allows simulating the human rationing process through inference mechanisms, which can be expressed mathematically [2]. With the use of this, the uncertainty of human cognitive processes can be modeled, and therefore programmed into a computer. In Fuzzy logic, various concepts are grouped, including fuzzy set theory, if-then rules, arithmetic and others [3]. The term type-2 fuzzy set was first introduced in 1975 by Zadeh as an extension to the concept of type-1 fuzzy sets [4]. A type-2 fuzzy set is defined by a fuzzy membership function, where the value or degree of membership for each element in this set is a fuzzy set in the interval (0,1) rather than a crisp value [5]. If the circumstances are so fuzzy that there are even problems in defining the degree of membership as a point value, then a type-2 fuzzy set is used [6]. The additional degrees of freedom make the membership functions of type II fuzzy sets three-dimensional functions [7]. Therefore, type-2 fuzzy sets can handle more types of uncertainties with larger magnitudes using a smaller rule base than their type-1 counterparts [8].

Today's designs of type-2 fuzzy systems are many, Ojha, Abraham and Václav [9] carry out a review of 3 decades of research on the heuristic design of fuzzy inference systems. The applications of type-2 fuzzy logic are many, in fact, due to its properties has made it possible to solve many complex problems and is much more adapted to the reality of situations [10]. Allowing to significantly expand the areas of application of the same in comparison to the type-1 fuzzy logic, although it has a large area of applications, even more, are the applications that have the type-2 fuzzy logic [11]. Regarding the development of controllers, according to Marco A. Márquez [12], type-2 fuzzy logic serves to justify the shape of fuzzy sets. Allowing us to achieve better results than with the now called type-1 fuzzy logic. But with the disadvantage that it is required to program a larger controller, with more parameters and with a higher computational cost.
In a conventional fuzzy controller (also called type-1 fuzzy controller or T1FLC) it is not possible to deal with the uncertainties associated with the controller parameters. That is why Zadeh developed the theory of type-2 fuzzy systems (T2FL) and higher to eliminate the paradox that the limits of the membership functions of a type-1 fuzzy system were themselves real numbers, that is, crisp values. This leads to the inability to handle the uncertainty associated with these parameters [13]. The application of type-2 fuzzy systems in system control is known as type-2 fuzzy controllers (T2FLC). The possibilities given by a type 2 fuzzy system in the control of processes strengthen them against conditions of uncertainty, that is why this research focuses on evaluating the robustness of a type 2 fuzzy controller applied to one of the most common processes of the industry, the liquid level control of a tank, for which the responses of the system are studied under normal conditions and noise conditions, and by means of a deviation analysis it is known whether or not the system is affected by noise and in what magnitude does it.

2. Type-2 fuzzy control system
The Figure 1 shows a type-2 fuzzy system. At first the clear input signal is converted to fuzzy input with the help of various membership functions. A set of rules is formed to map the input with the output together, known as the inference mechanism. Type-2 output sets cannot be used directly to convert to a sharp value due to computational limitations. For this reason, type-2 fuzzy sets are converted to type-1 fuzzy sets, which is known as a type reduction operation, then the reduced type sets are converted back to a sharp value using various techniques of defuzzification.

The generic implementation of the fuzzy controller in terms of type-2 fuzzy sets like type-1 has two input variables: the error e(t), the difference between the reference signal and the process output; and the error variation Δe(t). Which are observed in Equation (1) and Equation (2) [14].

\[ e(t) = r(t) - y(t), \]  
\[ \Delta e(t) = e(t) - e(t - 1). \]  

The Figure 2 shows the block diagram of the control scheme. It is denoted that the variables defined above, the error and delta error, are the result of the difference between the reference signal and the process output; and the error variation Δe(t). Which are observed in Equation (1) and Equation (2). Ent to the sense, as a fuzzy control is observed, it is nothing more than a fuzzy system in which its inputs feedback information of the product of its output on the process.

3. Experimental setup
To carry out the implementation of the type-2 fuzzy controller, a tank level control plant is taken as a case study. For this, a pilot plant was developed, which by means of the plant identification method, it was obtained that the transfer function that best represents the system, shown by Equation (3).

\[ G(s) = \frac{K_p e^{-3mS}}{tS + 1} = \frac{0.2274 e^{0.85}}{146.65S + 1} = \frac{0.2274}{146.65S + 1}. \]
Once the transfer function of the plant has been obtained, the fuzzy type-2 controller is characterized. It begins with the selection of the type of fuzzy system where the fuzzy interval system was chosen in this case. Subsequently, the linguistic variables are dimensioned, it is determined that the error of its universe of discourse is ± 17 cm. For the delta error, it is ± 2 cm, since the first data of the delta are not considered since they are mathematical impressions and for the output a control action from 0 PWM to 255 PWM.

The linguistic variables are the error, the delta error, and the control action. For error and delta error they are defined as Gaussian functions and in the control action singleton functions. For fuzzy rules, they are determined by using the critical points for a total of 13 rules, which are shown in Table 1. Each of the 13 observed rules define the output of the system according to the combination of the inputs. Where BN is big negative, MN is medium negative, SN is small negative, Z is zero, SP is small positive, MP is medium positive, BP is big positive, VVH is very-very high, VA is very high, H is high, M is medium, L is low, VL is very low and VVL is very-very low.

| No. | Rule | Output |
|-----|------|--------|
| 1   | If e \( \rightarrow \) BN and \( \Delta e \rightarrow Z \) | VVL |
| 2   | If e \( \rightarrow \) MN and \( \Delta e \rightarrow Z \) | VL |
| 3   | If e \( \rightarrow \) SN and \( \Delta e \rightarrow Z \) | L |
| 4   | If e \( \rightarrow \) Z and \( \Delta e \rightarrow Z \) | M |
| 5   | If e \( \rightarrow \) SP and \( \Delta e \rightarrow Z \) | H |
| 6   | If e \( \rightarrow \) MP and \( \Delta e \rightarrow Z \) | VH |
| 7   | If e \( \rightarrow \) BP and \( \Delta e \rightarrow Z \) | VVH |
| 8   | If e \( \rightarrow \) Z and \( \Delta e \rightarrow BN \) | VVH |
| 9   | If e \( \rightarrow \) Z and \( \Delta e \rightarrow MN \) | VH |
| 10  | If e \( \rightarrow \) Z and \( \Delta e \rightarrow SN \) | H |
| 11  | If e \( \rightarrow \) Z and \( \Delta e \rightarrow SP \) | L |
| 12  | If e \( \rightarrow \) Z and \( \Delta e \rightarrow MP \) | VL |
| 13  | If e \( \rightarrow \) Z and \( \Delta e \rightarrow BP \) | VVL |

To implement the type-2 fuzzy controller the professional software MATLAB® was used, to create the fuzzy controller. The input variables (error and delta error) and the output variable, their types of functions with the respective operating ranges, and other configurations such as the fuzzification mechanism, the inference system, and the type reduction method were assigned. The fuzzy controller was characterized and thus the simulation was carried out using Simulink of MATLAB®, as can be seen in Figure 3.

![Figure 3. Real control loop with IT2 fuzzy controller.](image-url)
It can be seen the correspondence of the designed model with the generic model, of course, already applied controller in a real case, in which there are polynomials to give a wide range of application for different reference values, the blocks that are responsible for sending the output of the control to the physical system and the blocks that will be in charge of capturing the process data for feedback control.

4. Response performance index

Figure 4 shows the general response of the system under normal conditions and shows the general response of the system to noise.

It is clear that with the effect of noise the system lost speed on the ascent, but even so the behavior in steady state is acceptable, the deviation of the response in the system under normal conditions obtained an average of ± 0.01731 cm and under noise conditions an average of ± 0.0399 cm, this indicates that the noise only affected the response of the system on average 0.022 cm over the response under normal conditions. The steady-state error under normal conditions is observed in Figure 5.

It is considered that the expected value is 15 cm. In general, an average error in steady state of 2.24% was obtained. Denoting that although the response has an overshoot in normal conditions, its slight oscillations around the settlement point are stable. The steady-state error under noise conditions is observed in Figure 6.

It is taken into account that the expected value is 15 cm. In general, an average error in steady-state of 2.96% was obtained. Obtaining a difference with respect to the system in normal conditions of 0.72 percentage points, which corroborates the robustness of the controller to noise conditions. The response to the disturbance is observed in Figure 7. As can be seen in Figure 7 at the instant of time 140 sec. a disturbance of approximately 0.71 cm was entered into the system. Bringing the tank level to a value of 15.75 cm and remaining for 10 seconds. During this time, the controller only manages to reduce the
disturbance by 30%, bringing the system to a value of 15.54 cm. Resulting in the fail to compensating the value of the disturbance. The results were summarized in Table 2.

\[ \text{\% affectation} = \frac{|\text{normals conditions} - \text{before noise}|}{\text{normals conditions}} \times 100\% \] \quad (4)

| Experiment | td (s) | tr (s) | tp (s) | Mp (cm) | ts (s) | Sd (cm) | Sp (cm) | A (%)  |
|------------|--------|--------|--------|---------|--------|---------|---------|--------|
| Normal     | 41.5   | 76.6   | 0.0    | 0.0     | 180    | 0.0173  | 15.34   | 2.24   |
| Noise      | 50.6   | 97.8   | 0.0    | 0.0     | 200    | 0.0399  | 15.43   | 2.97   |

Results before disturbance

- Stabilizes: Yes, or Not
- Response time to disturbance

In 10 seconds, it is compensated by 30%

*sd: Standard deviation in steady state; **Sp: Setting point; ***A: Average in steady state

It can be observed in the table how different indicators were identified in transitory and stationary regimes, and that these identifications are given under the different experimental conditions of noise and disturbance, from which it is necessary to compare the changes in the control with noise, evaluating how the noise affects its dynamics. For which the percentage of control affectation after the effect of noise is calculated. It is not considered whether or not the change generates better margins, but rather as a change in its dynamics in the presence of noise. The results of the analysis can be seen in Table 3 and were calculated based on Equation (4).

| Experiment | td (s) | tr (s) | tp (s) | Mp (cm) | ts (s) | Sd (cm) | Sp (cm) | A (%)  |
|------------|--------|--------|--------|---------|--------|---------|---------|--------|
| Normal     | 41.5   | 76.6   | 0.0    | 0.0     | 180    | 0.02    | 15.34   | 2.24   |
| Noise      | 50.6   | 97.8   | 0.0    | 0.0     | 200    | 0.04    | 15.43   | 2.97   |

% of affectation

21.93 26.63 0.00 0.00 11.11 130.64 0.59 32.59

*sd: Standard deviation in steady state; **Sp: Setting point; ***A: Average error in steady state

It is observed how Table 2 and Table 3 provide the necessary information to know the affectation that the noise generates on the system in relation to the behavior under normal conditions, results which are discussed below in the conclusions.
5. Conclusions
As it could be observed in the start-up of the experimental setup, it is feasible from a few relatively simple steps to create a functional type-2 fuzzy controller. This does not guarantee that it is indeed the best of the controllers, but it does allow us to obtain a guide to implement controllers of this magnitude and treat them as more recurring case studies. Figure 4 shows the response of the type-2 fuzzy controller subjected to noise, the nature of the response curve was minimally affected, which is verified when analyzing the response indexes in Table 3. It was observed that in the transitory regime the noise caused the process to become slightly slower and shifting the stability time from 180 seconds under normal conditions to 200 seconds before noise.

On the other hand, in the statistical study of the steady-state, a slight worsening in its characteristics is observed, the most relevant being the error in steady-state, taking the system from an error of 2.24 to 2.96% with a variation in the settlement point of 0.59%. This allows us to conclude that the fuzzy controller favorably supports conditions where high noise levels are present. The standard deviation in a stable regime with noise was ± 0.04 cm, that is, the values varied on average ± 0.04 cm around the settlement point, an indicator that refers to the non-vulnerability of the system to noise with the implementation of systems of control based on type-2 fuzzy logic.

Based on the theoretical foundations, it was known in advance that the type-2 fuzzy controller, when implemented with a singleton, would present a slow response from the system. Since being a control based on points of fixed output magnitude and not on a curve as all controllers are commonly used. The change between its response magnitudes would be considered slow, this could be observed exactly when subjecting the system to disturbances where, as shown in Figure 7. The system only managed to compensate 30% for the disturbance in 10 seconds. Therefore, it is concluded that this control is not favorable when the system to be controlled may present recurrent disturbances or the type-2 fuzzy control that is based on singleton outputs.

References
[1] Almaraashi M 2012 Learning of Type-2 Fuzzy Logic Systems Using Simulated Annealing by Declaration of Authorship (Germany: De Montfort University)
[2] Rios Y, Garcia J, Sanchez E, Durán C 2018 Revista Colombiana de Tecnologías de Avanzada 2(32) 26-33
[3] Cantillo A, Gualdron O, Ortiz J 2018 Revista Colombiana de Tecnologías de Avanzada 2(32) 139-147
[4] Wu D, Wan W 2006 ISA Transactions 45(4) 503-516
[5] Castillo O 2012 Type-2 Fuzzy Logic in Intelligent Control Applications (Berlin: Springer-Verlag Berlin Heidelberg)
[6] Marín L G 2016 Introducción a Sistemas Difusos: Modelo Computacional Sistemas Difusos (Chile: Universidad de Chile)
[7] Chaoui H, Gueaieb W, Biglarbegian M, Yagoub M C E 2013 Robotics 2(2) 66-91
[8] Corzo C, Velazco D 2018 Revista Colombiana de Tecnologías de Avanzada 2(32) 104-108
[9] Ojha V, Abraham A, Snášel V 2019 Engineering Applications of Artificial Intelligence 85 845-864
[10] Perez J, Castellanos M 2020 Pistas Educativas 132 176-192
[11] Ascanio J, Sandoval C 2018 Revista Colombiana de Tecnologías de Avanzada 2(32) 129-138
[12] Antonio M, Samantha N 2018 Revista Simulación Computacional 1(1) 9-17
[13] Rivera A V, Enrique L, Miranda A, Alejandro L, Oropeza F, López E 2013 Técnicas de Implementación de Controladores Utilizando Lógica Difusa Tipo 2 (México: Universidad Autonoma de Aguas Calientes)
[14] Castillo O, Melin P 2014 Information Sciences 279 615–631