Baryogenesis with QCD Domain Walls

R. Brandenberger$^{1,2}$, I. Halperin$^2$ and A. Zhitnitsky$^2$

$^{1}$Department of Physics, Brown University, Providence, RI 02912, USA;
$^{2}$Department of Physics and Astronomy, University of British Columbia, Vancouver, BC, V6T 1Z1, CANADA

We propose a new baryosymmetric mechanism for baryogenesis which takes place at the QCD scale and is based on the existence of domain walls separating the metastable vacua from the lowest energy vacuum. The walls acquire fractional negative and positive baryon charges, while the observed baryon asymmetry is due to a non-zero value of the $\theta$ angle at the temperatures near the QCD chiral phase transition. The regions of metastable vacuum bounded by walls carry a negative baryon charge and may contribute a significant fraction of the dark matter of the Universe.

PACS numbers: 98.80Cq, 11.27.+d, 12.38.Lg

1. The origin of the asymmetry between baryons and antibaryons, and, more specifically, the origin of the observed baryon to entropy ratio $n_B/s \sim 10^{-9}$ ($n_B$ being the net baryon number density, and $s$ the entropy density) is still a mystery. In order to explain this number, from symmetric initial conditions in the early Universe, it is generally assumed that three criteria, first laid down by Sakharov $^1$ must be satisfied: (1) there must exist baryon number violating processes; (2) these processes must involve $C$ and $CP$ violation; and (3) they must take place out of thermal equilibrium.

Many mechanisms for baryogenesis have been proposed over the past twenty years (for recent reviews see e.g. $^2$: 4), but all involve new physics, often at extremely high energy scales. A potential problem for all baryogenesis mechanisms which operate at scales higher than the electroweak scale $\Lambda_{EW}$ is that electroweak sphaleron effects wash out any pre-existing baryon number unless other quantum numbers such as $B - L$ (where $B$ and $L$ are baryon and lepton number, respectively) are violated during baryogenesis. Therefore, in recent years electroweak baryogenesis has received a lot of attention, see e.g. $^3$ 4. In the standard model, however, $CP$ violation is too small to generate the observed value of $n_B/s$. In addition, given the present bounds on the Higgs mass, it appears unlikely that in the standard model the electroweak phase transition is sufficiently strongly first order to generate the out-of-equilibrium bubbles required to satisfy the third Sakharov criterion. This problem can be circumvented within topological defect-mediated electroweak baryogenesis $^3$, but only at the cost of introducing new physics above the electroweak scale.

Recently, it was suggested $^7$ that the Sakharov criteria could be satisfied at the QCD scale. In this paper, we propose an explicit mechanism which can explain the observed value of $n_B/s$ and which takes place at the QCD scale without introduction of any new physics. Our mechanism, however, is strictly speaking not a baryogenesis, but rather a charge separation mechanism, since the total baryon current is exactly conserved. In this aspect, our approach makes use of ideas proposed in the context of toy models in $^8$ 9.

The first essential ingredient in our scenario is the realization that there exist several vacua in QCD whose energy degeneracy is broken only by a very small amount $\Delta E \sim m_q$ ($m_q$ is a quark mass) compared to the heights of the barriers between the vacua. This leads to the domain walls which provide the locus of out-of-thermal equilibrium required by the third Sakharov criterion.

This picture of the vacuum structure is based on the analysis of the general properties of the $\theta$ dependence in QCD. It is known that physical values depend on the strong $CP$ parameter $\theta$ through $\theta/N_f$ (or $\theta/N_c$ in pure gluodynamics, $N_f$ and $N_c$ are the number of flavors and colors, respectively) and, at the same time, they must be $2\pi$ periodic functions of $\theta$. This is only possible if an effective potential is multi-valued, and in the partition function one sums over all branches $^11$. This yields the vacuum structure described above. In terms of the original theory this prescription means summation over all topological classes $Z \sim \sum_n \int DA \exp \left[ -S_0 + iQ(\theta + 2\pi n) \right]$ imposing the topological charge $Q$ quantization. This picture arises in supersymmetric $^1$ and non-supersymmetric $^2$ theories. The appearance of domain walls in gauge theories was also advocated by Witten within different frameworks $^1$.

The second crucial input is the fact that the domain walls may carry a nonvanishing fermion number, a well-known $^1$ $^2$ $^3$ $^4$ property of many solitonic configurations. The domain walls are also locations of maximal $CP$-violation $^7$ due to the phase difference of the chiral condensate across the wall, demonstrating that the second Sakharov criterion is satisfied. In what follows we use a specific low energy effective Lagrangian for QCD $^10$, though one can expect that the general properties described above are inherent in QCD, and should appear in any approach.

Making use of these ingredients in the context of the early Universe, it follows by the Kibble-Zurek mechanism $^11$ $^12$ that at the QCD phase transition at the temperature $T_c \sim 200$ MeV a network of domain walls separating
different vacua will form. Because of the energy bias \( \Delta E \), walls surrounding regions of metastable vacua will tend to contract. However, when coupled to fermions, they can be stabilized, yielding some compact objects, not necessarily of spherical form, to be called \( B\)-shells in what follows. (This name is due to the fact that the (anti-) baryon charge is concentrated on the surface of such a compact object. This bears some resemblance to the SLAC bag \(^1\).) The compensating positive baryon number is left behind in the bulk. The \( B\)-shells, in turn, can contribute a substantial fraction of the dark matter of the Universe.

2. We start by describing the effective potential \(^2\) which allows one to analyze the vacuum properties and domain walls in QCD. In this approach, the Goldstone fields are described by the unitary matrix \( U_{ij} \) corresponding to the \( \gamma_5 \) phases of the chiral condensate: \( \langle \Psi_L^i \Psi_R^j \rangle = -|\langle \Psi_L^i \Psi_R^j \rangle| U_{ij} \). The effective chiral potential is periodic in \( \theta \) and takes the form

\[
W_{\text{QCD}}(\theta, U) = -\lim_{V \to \infty} \frac{1}{V} \log \sum_{l=0}^{p-1} \exp[VE \cos \Phi + \frac{1}{2} V Tr(MU + M^+U^+)],
\]

(1)

where \( V \) is the 4-volume and

\[
\Phi = -\frac{q}{p} \theta + \frac{i}{p} \log Det U + \frac{2m}{q} l, \quad M = \text{diag}(m_\lambda(\Psi^i \Psi^i))
\]

\( m_\lambda \) are the quark masses), while \( E \) is the nucleon mass in the chiral limit and \( k \) labels different vacua. This is related to the fact that the chiral condensates vary by an amount \( \sim m_q \) in different vacua. \( N_f \) is the number of flavors, but we here keep them as free parameters.

One can argue \(^3\) that Eq. (1) represents the anomalous effective Lagrangian realizing broken conformal and chiral symmetries of QCD. The arguments are the following: (a) Eq. (1) correctly reproduces the Di Vecchia-Veneziano-Witten effective chiral Lagrangian \(^4\) in the large \( N_c \) limit; (b) it reproduces the anomalous conformal and chiral Ward identities of QCD; (c) it contains the built-in topological charge quantization, which shows up in \( \theta \) via the sum over the integers \( l \) and presence of cusp singularities at certain values of the fields.

Thus, the effective chiral potential (1) satisfies all general requirements on the theory and leads to the above picture of the vacuum structure and the related domain walls. For the domain walls interpolating between the true vacuum of lowest energy and the first excited state at \( \theta = 0 \), the surface energy density \( \sigma \) of the wall for the potential (1) has been calculated in \(^5\): \( \sigma = \frac{4p}{q\sqrt{N_f}} f_\pi \sqrt{E} \left( 1 - \cos \frac{\pi}{2p} \right) + 0(m_q f_\pi^2), \) \( \) 

(2)

where \( f_\pi = 133 \text{ MeV} \). The energy splitting between the ground state and the metastable state at \( \theta = 0 \) is

\[
\Delta E = m_q N_f |\langle \Psi \Phi \rangle| \left( 1 - \cos \frac{2\pi}{qN_f} \right) + 0(m_q^2).
\]

(3)

Due to this energy difference, regions of the metastable state tend to disappear, unless they are stabilized by coupling to fermions. In this case, one expects to end up with blobs of the metastable state (B-shells).

3. Next we would like to argue that the above domain walls may acquire a fractional baryon charge when coupled to the baryons. As our domain wall configuration is a flavor singlet (the \( \eta' \) domain wall \(^6\)), we keep only the relevant part of the matrix \( U = \exp[\alpha(z)] \), where \( \alpha(z) \) has a solitonic shape written explicitly in \(^6\). Neglecting the isospin, we consider the following simplified Lagrangian for the nucleon \( N \) interacting with the external, non-fluctuating chiral field \( U \) and additionally via a four-fermion interaction:

\[
\mathcal{L}_4 = \tilde{N} i \partial_\tau \gamma_\mu N - m_N \tilde{N}_L U N_R - m_N \tilde{N} N_U^+ N_L - \lambda (\tilde{N}_L N_R) (\tilde{N} N_L)
\]

(4)

(the choice of the sign of \( \lambda > 0 \) corresponds to the repulsion in the U(1) channel).

The important point is that the nucleon mass \( m_N \) takes different values in different vacua: \( m_N = m_N^{(0)} + m_q f(\theta, k) \), where \( m_N^{(0)} \) is the nucleon mass in the chiral limit and \( k \) labels different vacua. This is related to the fact that the chiral condensate varies by an amount \( \sim m_q \) in different vacua. Given this, estimates of a function \( f(\theta, k) \) are possible but will not be considered here. Thus, in the adiabatic approximation to the interaction with the domain wall, \( m_N \) should be considered as a slowly varying function of \( z \). Fortunately, its precise form is irrelevant for our purposes, only the asymptotics enter the final answer, see Eq. (5) below. Because of the planar symmetry, the problem of computing the charge \( B(4) = \int \tilde{N} \gamma_0 N dx \) in our four dimensional (4D) theory (4) reduces in the mean field approximation to the calculation of the corresponding charge \( B(2) = \int \bar{\psi} \gamma_0 \psi dz \) of a two dimensional (2D) theory. Omitting the details of this calculation which will be given elsewhere, we here present the final answer for the 2D baryon number on the wall:

\[
B(2) = \frac{1}{\pi} \arccos \frac{m_N}{\lambda} \bigg| \frac{z = +\infty}{z = -\infty},
\]

(5)

where \( \lambda \) has dimension of mass and can be found in terms of \( \lambda, m_N^{(0)} \) and a degeneracy factor \( n = N/S \), see below.

To find the original 4D baryon charge \( B(4) \), we should take account of a degeneracy related to the symmetry under the shifts along the wall plane. In many body physics
the definition of the charge is
\[ Q = B^{(4)} = B^{(2)} g \int \frac{dxdydp_xdp_y}{(2\pi)^2} \equiv B^{(2)} N, \]
where \( g = 2 \cdot 2 \) describes the degeneracy in spin and isospin. For a fixed number of quantum states \( N = gS \int \frac{d^2p}{(2\pi)^2} = \frac{gS}{4\pi} \) (\( S \) is the area of the wall), the fermion energy \( E_F \) of the domain-wall fermions is determined by
\[ E_F = gS \int \frac{pd^2p}{(2\pi)^2} = \frac{2}{3} Np_F = \frac{4}{3} \frac{\sqrt{N}}{g} \sqrt{S}. \]
The size of the surface which can accommodate the fixed number of fermions \( N \) can be found from the minimization equation \( \frac{dE_0}{dS} |_{N=const} = 0 \), where the total energy of the fermions residing on a surface \( S \) is given by
\[ E_0 = \sigma S + \frac{4}{3} \frac{\sqrt{N}}{g} \sqrt{S}. \]
The minimization then relates the density of fermions per unit area \( n = N/S \) to the tension \( \sigma \). We obtain
\[ Q = -ST_c^2 \alpha_1, \quad \alpha_1 = \frac{\sigma^{2/3}}{T_c} \left( \frac{g_\star}{4\pi} \right)^{1/3} B^{(2)}. \]

4. Now we are ready to introduce our proposed baryogenesis (charge separation) mechanism. At the QCD phase transition at the temperature \( T_c \approx 200 \text{ MeV} \), the chiral condensate forms. Because of the presence of nearly degenerate states, a network of domain walls will arise immediately after \( T_c \). At \( \theta = 0 \), there are two degenerate metastable states \( |B \rangle \) and \( |C \rangle \) above the true vacuum of lowest energy \( |A \rangle \). For simplicity, we ignore metastable states of higher energy, although they might be important for evolution of the domain wall network\[20\]. A \( CP \) transformation exchanges the states \( |B \rangle \) and \( |C \rangle \). Respectively, the baryon charge of a \( A - B \) wall will be opposite to that of a \( A - C \) wall, and no baryon asymmetry can be produced in this case. The situation becomes different, however, when \( \theta \neq 0 \) at the temperatures close to \( T_c \). This case will be considered below. (It is implied that the strong \( CP \) problem will be cured by an axion at lower temperatures. At \( T \sim T_c \), the axion field does not yet settle in its ground state, and thus \( \theta(T_c) \) might be of order unity. Note that as long as an initial value \( \theta(T_c) \) is the same in the entire observable Universe, so will be the sign of the baryon asymmetry. This occurs if the Universe undergoes inflation after or during the Peccei-Quinn symmetry breaking.) In this case there is a splitting \( \Delta E \sim \theta m_q \) between the energies of lowest metastable vacua, which translates into the splitting \( \Delta M \sim \theta M \) for the masses of B-shells with negative and positive baryon charges (here \( M \) stands for the B-shell mass at \( \theta = 0 \)). If the B-shells reach thermal equilibrium at a temperature \( T \), the relative densities of negative and positive B-shells will be fixed by the Boltzmann formula, and one type of B-shells will thus be very strongly suppressed. (A similar interplay between the spontaneous and explicit breaking of \( CP \) was considered in \[21\] within a different scheme.) Assuming that this is the case, we will therefore discuss the evolution of the domain walls of only one (negative) type.

The initial wall separation depends on the details of the chiral phase transition. We will assume that an infinite domain wall network will exist until a temperature \( T_d \), after which it decays into a number of finite clusters of the false vacuum. The typical wall separation \( \xi = \xi(T_d) \) at this temperature depends on the initial wall separation at formation, the details of the damping mechanism and the interplay between the energy bias \( B \) and the surface pressure in the walls \[13\]. For the time being, we will use the value of \( \xi = \xi(T_d) \) as an unknown parameter in our scenario, and relegate a brief discussion of possible values of \( \xi \) to the end of the paper.

Given the typical wall separation \( \xi \), the total area in the walls at this temperature is \( \langle S \rangle \approx V/\xi \), where \( V \) stands for the Hubble volume at \( T_d \). Note that this is parametrically larger by \( V^{1/3}/\xi \) than the Hubble area \( V^{2/3} \). This enhancement means that the baryon charge on the wall is in fact not a surface effect, as Eq. \[9\] would naively suggest, but rather a volume effect. Thus, the total (anti-) baryon charge stored in the (negative) walls at \( T_d \) is fixed by Eq. \[9\], while the compensating baryon charge is left in the bulk (the positive walls and their subsequent decay products are effectively ascribed to the bulk). As the entropy density is \( s = g_s T_d^3 \), where \( g_s \sim 10 \) is the number of spin degrees of freedom in the radiation bath at \( T_d \), we may estimate the baryon to entropy ratio in the bulk at \( T_d \):

\[ \frac{n_B}{s} \mid_{T_d} \approx \frac{\alpha_1 T_d^2}{g_s \xi T_d}. \]

After \( T_d \), the domain wall network will break up into finite clusters of the false vacuum (“B-shells”). Although the question of stability of these B-shells requires a detailed study, qualitatively we expect that the bubbles will shrink, but not decay completely since they will eventually be stabilized by the fermions. As the non-relativistic baryons can hardly cross the wall, we expect the shells to be stable against the escape of baryons from the interior, but able to lose heat by baryon pair annihilation and emission of the photons and/or neutrinos. The quantum stability of the B-shells will be addressed in \[22\]. Generally, one expects an exponential suppression of quantum decays by the baryon charge of the surface.

To proceed, we introduce two dimensionless constants \( \alpha_2 \) and \( \alpha_3 \) by parametrizing the total area and volume of the B-shells as \( \alpha_2^2 \langle V/\xi \rangle \) and \( \alpha_3 V \), respectively. (This
parametrization does not imply any specific assumption about the form of the B-shells.) In addition, we neglect the change of the Hubble radius between the time corresponding to $T_d$ and the time at which the B-shells are formed. We thus obtain

$$\frac{n_B}{s} \simeq \alpha_1 \alpha_2^2 g_*^{-1} \frac{T_d^2}{\xi T_d} = \alpha_2 \frac{n_B}{s} |_{T_d}. \quad \text{(11)}$$

The energy density $\rho_B$ in the blobs of the metastable state will redshift as matter, i.e. $\rho_B(T) \sim T^3$. Hence, the B-shells can contribute to the dark matter of the Universe. Their contribution to the energy density

$$\Omega_B = \frac{\rho_B(t_{eq})}{\rho_r(t_{eq})} = \frac{\rho_B(t_{eq})}{g_* T_d^4 t_{eq}}, \quad \text{(12)}$$

(where $t_{eq}$ is the time of equal matter and radiation, and $\rho_r$ is the energy density in radiation which, until $t_{eq}$, dominates the total energy density) is roughly,

$$\Omega_B \simeq 8 \alpha_3 \frac{2 \pi^2 m_{\Psi \bar{\Psi}}}{9 N_f} \frac{1}{g_* T_d^4 t_{eq}}, \quad \text{(13)}$$

where we used the expansion of the cosine in Eq. 3. This estimate implies that the dominant contribution to the energy density $\rho_B$ inside the B-shells is due to the false vacuum energy (3), which is larger than the non-vacuum contribution $\sim \alpha_3 m_N n_B$ with $n_B$ given by (1). (Here we assume that the baryon density $n_B$ is of the same order of magnitude in the bulk and interior, while the redundant baryons and anti-baryons with zero net $n_B$ will eventually annihilate and leave the B-shells together with a flux of photons and neutrinos.)

Comparing (11) and (13), we see that the resulting baryon asymmetry and contribution of the B-shells to the dark matter density are related with each other via the geometrical parameters $\alpha_2, \alpha_3$ of the distribution of shells sizes. To calculate them is a complicated task which is at the moment beyond our ability. However, it is of interest to reverse the argument and estimate the parameters $\alpha_2, \alpha_3$ assuming that the B-shells do provide the observed baryon asymmetry $n_B/s \sim 10^{-9}$ and contribute significantly to the dark matter, $\Omega_B \sim 1$. As can be seen from (11) and (13), this requires $\alpha_3 \sim 10^{-7}$ and $\xi T_d \simeq 10^6 \alpha_2^2$, if $\alpha_1 \sim 10^{-3}$ and $T_d \sim T_c$. As $\alpha_2 < 1$ and $\xi T_d > 1$ for the thin wall approximation to work, this leaves us with the window $T_c^{-1} \ll \xi < 10^6 T_c^{-1}$ for the proposed mechanism to be operative. This corridor is consistent with the Kibble-Zurek scenario for the topological defects formation in the early Universe [13,14].

In conclusion, qualitative as our arguments are, they suggest that baryogenesis can proceed at the QCD scale, and might be tightly connected with the origin of the dark matter in the Universe. An alternative scenario could also be considered, in which the charge separation proceeds due to different interactions of particles and anti-particles with a CP-breaking domain wall with zero baryon charge, similarly to what happens in electroweak baryogenesis. The blobs of metastable vacua might be stabilized in this case by the fermi pressure of anti-baryons trapped inside during the evolution of the walls. We emphasise that the suggested ideas can be, in principle, experimentally tested at RHIC. A more quantitative and detailed analysis will be given in [23].

This work was supported in part by the Canadian NSERC and by the U.S. Department of Energy under Contract DE-FG02-91ER40688, TASK A. We wish to thank D. Schwarz, E. Shuryak, A. Vilenkin and L. Yaffe for valuable comments.