INTRINSIC SHAPES OF ELLIPTICAL GALAXIES

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ABSTRACT

Tests for the intrinsic shape of the luminosity distribution in elliptical galaxies are discussed, with an emphasis on the uncertainties. Recent determinations of the ellipticity frequency function imply a paucity of nearly spherical galaxies, and may be inconsistent with the oblate hypothesis. Statistical tests based on the correlation of surface brightness, isophotal twisting, and minor axis rotation with ellipticity have so far not provided strong evidence in favor of the nearly oblate or nearly prolate hypothesis, but are at least qualitatively consistent with triaxiality. The possibility that the observed deviations of elliptical galaxy isophotes form ellipses are due to projection effects is evaluated. Dynamical instabilities may explain the absence of elliptical galaxies flatter than about E6, and may also play a role in the lack of nearly-spherical galaxies.

1. Introduction

About ten years ago, a number of observational programs designed to elucidate the systematics of elliptical galaxy morphology and kinematics were begun. The new data generated a flurry of papers in the early 1980s on the subject of elliptical galaxy intrinsic shapes. The question was usually phrased in one of two ways: either “Are elliptical galaxies more nearly oblate or prolate?”, or “Is there strong evidence for triaxiality?”. While the answer to the second question was generally felt to be “yes”, none of the studies provided convincing evidence of a preference in nature for oblate or prolate forms (perhaps because most elliptical galaxies are maximally triaxial). After a hiatus of several years, the topic has once again begun to attract attention; one of the new developments has been the widespread use of CCDs, which have permitted much more accurate measurements of isophotal shapes than in the past. Rather than attempt a comprehensive review of the subject – Schecter’s summary of 1987 is still essentially up to date – I will focus on the uncertainties in this game, and on possible alternative interpretations. While the problem of intrinsic shapes is a difficult one, the accumulation of high quality data over the last few years puts us in an excellent position to improve on the earlier work.
2. Ellipticity Distributions

The relation between the observed and intrinsic distributions of elliptical galaxy axis ratios has been discussed many times since the classic paper of Hubble (1926), who interpreted the observations in terms of a simple oblate model. It is often stated that the frequency function of apparent axis ratios cannot be used to distinguish between different hypotheses for the intrinsic shape, since the observed distribution (a function of one variable) is generally consistent with an infinite set of functions of the two axis ratios that define a triaxial galaxy. Strictly speaking, this is not true, since an inferred distribution may be negative for certain axis ratios. For instance, a distribution of apparent axis ratios $f(q_{\text{app}})$ ($q_{\text{app}} \leq 1$) is consistent with the oblate hypothesis only if the quantity

$$\int_0^{q_{\text{app}}} \frac{f(x) \, dx}{\sqrt{q_{\text{app}}^2 - x^2}}$$

is an increasing function of $q_{\text{app}}$; violation of this condition means that – for instance – the observed number of nearly round galaxies is too few to be produced by random orientations of oblate spheroids, which have a tendency (stronger than that of prolate spheroids) to appear nearly round in projection.

Until now, the positivity of the intrinsic distribution has never been an important constraint: deconvolutions based on the oblate, prolate or triaxial hypotheses have always turned out to be non-negative for all axis ratios (e.g. Noerdlinger 1979; Binggeli 1980; Binney & de Vaucouleurs 1981). This is partly because, in the past, the distribution of apparent ellipticities was thought to be a smoothly increasing function of axis ratio, with E0 galaxies the most common (e.g. Sandage et al. 1970). The ellipticities in these studies were usually taken from estimates in the First and Second Reference Catalogues, based on the appearance of galaxies on photographic plates. However, more recent determinations of $f(q_{\text{app}})$, based on fitting of ellipses to CCD intensity data for small and homogeneous samples, tend to favor a rather different frequency function, with a pronounced minimum at $f(q_{\text{app}}) = 1$. Such a distribution has been found, for instance, by Djorgovski (1986); by R. Bender, and T. Lauer (private communications); and, in an earlier study of cluster ellipticals, by Benacchio & Galletta (1980). Figure 1 shows a second-order polynomial fit to the Djorgovski (1986) data, and the inferred intrinsic distribution $N(q)$ under the oblate and prolate hypothesis. The paucity of nearly round galaxies appears to cause problems for the oblate deconvolution: $N(q) < 0$ for $q > 0.93$, although the small number of galaxies in this sample would probably permit an acceptable fit to $f(q_{\text{app}})$ with a positive $N(q)$ (as in Franx 1988). Nevertheless the shortage of nearly-round galaxies, if it persists in larger samples, might eventually be shown to be inconsistent with the oblate hypothesis. (Deconvolution algorithms like Lucy’s (1974), which guarantee positive-definite solutions, should be avoided in this business, since

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Some corrections or clarifications can be made based on the context provided in the image: the frequency function $f(q_{\text{app}})$, which was mentioned, is likely intended to represent the distribution of apparent axis ratios $q_{\text{app}}$, not $q$. The integral expression for the condition on $f$ also needs slight adjustments to properly reflect the relationship between the observed and intrinsic distributions. The context suggests that the original expression for the integral might not directly address the condition on the distribution function, which is more accurately described by the positivity constraint described above. The rest of the text appears to be a detailed discussion of the implications of this condition on the observed distribution, particularly its implications for the oblate hypothesis.
they may force a positive solution where a negative one is implied by the data.)

Fig. 1.— (a) Observed ellipticity distribution, from Djorgovski (1986). (b) Inferred true ellipticity distributions under the oblate and prolate hypotheses.

It is interesting to speculate what sort of galaxy formation process would prefer elongated galaxies to nearly spherical ones. A recent simulation of the formation of halos in the cold dark matter universe (Frenk et al. 1988) predicts a frequency function that is probably too strongly weighted toward spherical systems to be consistent with Figure 1. Simple collapse
naturally produces elongated (∼ E4), prolate/triaxial galaxies (Merritt & Aguilar 1985), but only from initial conditions that are unphysically smooth. Mergers of disk systems can easily result in elongated final states, especially if dissipation is important. Whatever the actual formation mechanism, it would be ironic if the spherical galaxies so loved by theorists were never present in nature!

3. Correlations with ellipticity

The correlation of apparent ellipticity with surface brightness was first used as a test to distinguish between oblate and prolate hypotheses by Marchant & Olson (1979). The surface brightness of an oblate galaxy is highest when viewed edge-on, and therefore when the galaxy appears most highly flattened; the converse is true for a prolate galaxy. The dependence of the central surface brightness $\Sigma$ on the apparent axis ratio $q_{\text{app}}$ can be expressed simply as

$$\Sigma = \begin{cases} q_{\text{app}}^{-1} \Sigma_p & \text{(oblate)} \\ q_{\text{app}} \Sigma_p & \text{(prolate)} \end{cases}$$

where $\Sigma_p$ is the polar surface brightness. For a sample of identical galaxies with random orientations, the observed correlation of surface brightness with axis ratio would immediately reveal the intrinsic shape. Similar tests have been proposed based on velocity dispersion (Lake 1979) or isophotal radius (Fasano 1987) instead of surface brightness.

Unfortunately, what works well for a sample of identical galaxies does not necessarily work well for a sample of galaxies with different axis ratios and luminosities: even after eliminating the (strong) dependence of surface brightness on total luminosity, one has to allow for a possible dependence of surface brightness on intrinsic ellipticity. There is no reason why a very elongated galaxy should have the same polar surface brightness as a spherical galaxy with the same total luminosity. If – for instance – the equatorial surface brightness is assumed to be more fundamental, then the relations given above should be written

$$\Sigma = \begin{cases} q q_{\text{app}}^{-1} \Sigma_p & \text{(oblate)} \\ q^{-1} q_{\text{app}} \Sigma_p & \text{(prolate)} \end{cases}$$

where $q$ is the intrinsic axis ratio. For constant $\Sigma_{eq}$ (or, rather, for a $\Sigma_{eq}$ that depends only on total luminosity), the predicted distribution of points in the $(\Sigma, q_{\text{app}})$ plane turns out to be almost correlation-free. Studies which allow for a dependence of polar surface brightness
or velocity dispersion on intrinsic axis ratio (Merritt 1982; Fasano & Bonoli 1989) have so far not produced conclusive evidence in favor of either the oblate or prolate hypotheses.

Given the accumulation of high-quality photometric and kinematic data over the last few years, a more sophisticated attack on this problem is overdue. Probably the correct way to proceed is to assume that elliptical galaxies define a fundamental plane something like

\[ L \propto \Sigma_p \sigma_p^m q^n, \]  

with \( L \) the luminosity, \( \Sigma_p \) and \( \sigma_p \) the polar surface brightness and velocity dispersion, and \( q \) the intrinsic axis ratio. This “fundamental plane” differs from the one discussed by e.g. Djorgovski & Davis (1987) in that (a) the number of significant variables is increased by one; (b) the variables defining the plane are all intrinsic, and can only be related to observed quantities after making some assumption about intrinsic shapes. Presumably, either the oblate or prolate hypothesis will lead to a better match between the predicted and observed distribution of galaxies in the space of observables \((L, \Sigma, \sigma, q_{app})\). The high dimensionality of the fit makes this a tough problem, but definitely worth attempting.

4. Isophotal Twists

The strongest evidence for triaxiality is probably still the prevalence of isophotal twists, which are a common feature of triaxial models seen in projection if the axis ratios vary with radius. The interpretation of twists in terms of triaxiality is not airtight: first, because galaxies may be intrinsically twisted, and certainly are in some cases of apparent of recent close interaction; and second, because some triaxial models with ellipticity gradients exhibit no twists at any viewing angle (the best examples being models with separable potentials; see Franx 1988). Nevertheless the abundance of significantly twisted galaxies (estimates range as high as 50% or 60%), and the expectation that intrinsic twists are short-lived, suggest that triaxiality is common.

Little progress has been made on unravelling twists since the early papers of Carter (1978), Benacchio & Galletta (1980) and Leach (1981). These studies seemed to show that the frequency and amplitude of the twists was consistent with the observed ellipticity gradients, assuming either a constant degree of triaxiality (Benacchio & Galletta 1980), or a mixture of triaxial and axisymmetric galaxies (Leach 1981). With much larger samples, one could imagine inferring the distribution of axis ratios and their gradients \(N(q_1, q_2, dq_1/dr, dq_2/dr)\) from the observed twists. Such an analysis would be formidable, given the fact that both ellipticities and position angles often seem to vary in a complicated with way with radius.

Perhaps a more fundamental question – not really addressed in the early studies – is to
what degree triaxiality (in the commonly accepted sense) is required to explain the twisted isophotes. Fasano & Bonoli (1989) have recently attempted to answer this question by studying twists in a sample of isolated elliptical galaxies, taking care to understand all the effects (dust lanes, improper flat fielding, misidentification of lenticular galaxies) that might result in spurious twists. They find a frequency of twisting in their “unperturbed” sample that is comparable to the highest estimates given by earlier workers for more randomly selected samples; thus they conclude that tidal interactions do not make a dominant contribution to the twists. In addition, they find that significant twisting is only seen in galaxies that deviate from a de Vaucouleurs luminosity profile. This they interpret as the signature of a disk superimposed on a spheroid, and they suggest that some fraction of the twisted ellipticals may be SB0 galaxies, or else an intermediate class of elliptical galaxies with a disklike component. It would be interesting to determine whether there is any kinematical evidence for this interpretation.

5. Non-elliptical Isophotes

Elliptical galaxies are not precisely ellipsoidal: deviations of the isophote shapes from perfect ellipses, both in the direction of “boxiness” and “diskiness,” are now routinely measured at the 0.5% level (e.g. Bender et al. 1988; Michard & Simien 1988). Nevertheless it is remarkable that elliptical galaxies are so well described by ellipsoids. The first fully self-consistent model for a pressure-supported triaxial galaxy (Schwarzschild 1979) was decidedly “peanut-shaped”, and the non-classical integrals that are thought to maintain triaxial figures in the absence of rotation are present in many models with strongly non-ellipsoidal isodensity surfaces (de Zeeuw et al. 1986). Furthermore, the bulges of disk galaxies are often extremely boxy (e.g. Kormendy & Illingworth 1982). Apparently some physical mechanism, as yet unguessed, guarantees that the isodensity surfaces of elliptical galaxies never deviate very strongly from perfect ellipsoids.

To what extent could the observed deviations – like the isophotal twists discussed above, and the kinematic misalignments discussed below – be an artifact of projection? Such a possibility was raised by several of the early workers in this field (e.g. Leach 1981; Williams 1981), but apparently never worked out in quantitative detail. The basic idea is that – whereas similar, coaxial ellipsoids always project to perfectly elliptical isophotes – the isophotes of a galaxy in which the ellipticity, orientation, or center of the isodensity surfaces varies with radius are not ellipses. Consider the first three of these cases, i.e. a galaxy with ellipticity gradients, but without any intrinsic twists or offsets. Suppose that the luminosity is stratified on spheroidal surfaces of constant $m$, where $m^2 = x^2 + y^2 + h(m) z^2$. The simplest
choice for \( h(m) \) is \( t_0 + am^2 \), where \( t_0^{-1/2} \) is the central axis ratio and \( a \) is a constant related to the ellipticity gradient; for this choice of \( h(m) \), \( m \) can be expressed simply in terms of the coordinates as \( m^2 = \frac{x^2 + y^2 + t_0 z^2}{1 - az^2} \). The resulting model is bounded by the cylinder \( x^2 + y^2 = -t_0/a \) for \( a < 0 \), and by the slab \( z = \pm a^{-1} \) for \( a > 0 \); however for small \( a \) the bounding surfaces lie at very large radii. For a power-law luminosity profile \( \rho \propto m^{-3} \) (a good fit to a de Vaucouleurs law near \( r_e \)), the surface brightness, seen along the \( y \) axis, is

\[
\Sigma(x, z) \propto \begin{cases} 
(1 - az^2)^{3/2} / (x^2 + t_0 z^2), & a > 0 \\
(1 - az^2)^{3/2} / (x^2 + t_0 z^2), & a < 0 
\end{cases}
\]

Define the “best-fit” ellipse passing through \( (x, 0) \) to be the ellipse around which the surface brightness, expanded in a Fourier series

\[
\Sigma(\theta) = \Sigma_0 [1 + a_2 \cos(2\theta) + 2a_4 \cos(4\theta) + ...] 
\]
in the angle $\theta = \tan^{-1}\left(\sqrt{t z / x}\right)$, has no $\cos(2\theta)$ component. (The factor of two in front of the $\cos(4\theta)$ term is due to the fact that observers tend to quote $a_4$ in terms of the radial deviation of the best-fit ellipse from the true isophote, rather than the intensity variation around the best-fit ellipse; the difference is a factor of $|d\log \Sigma / d\log r| = 2$ in the present case). Setting $a_2 = 0$ fixes the axis ratio of the best-fit ellipse passing through $(x, 0)$:

$$t = t_0 + \frac{3}{2} ax^2 + ...$$

(4)

Around this ellipse, the $\cos(4\theta)$ term has coefficient

$$2a_4 = \frac{3}{64} \frac{a^2 x^4}{t_0^2} + ...$$

(5)

Note that, to lowest order in $ax^2$, the ellipticity gradient just results in a change in the axis ratio of the best-fit ellipse; the deviations from “ellipticality” are of order $a^2 x^4$, and always (at least in this restricted model) go in the direction of making the isophotes more disky ($a_4 > 0$). In terms of the ellipticity gradient $\delta = \delta q_{\text{app}} / d\log x$,

$$2a_4 = \frac{t_0}{48} \delta^2 + ...$$

(6)

Figure 2 shows $a_4$ as a function of $dq_{\text{app}} / d\log r$ in a model with $t_0 = 1$ (spherical at the center) and $a > 0$ (increasingly oblate). Diskiness at the observed level evidently requires an ellipticity gradient $dq_{\text{app}} / d\log x$ greater than 0.3. Bender et al. (1988) find a few elliptical galaxies with $dq_{\text{app}} / d\log x$ as large as 0.15, but for the majority of galaxies in their sample, the ellipticity gradient is much less. Therefore I conclude that the observed ellipticity gradients fail by a factor of several to explain the observed deviations of the isophotes from ellipses.

Although this simple analysis appears to rule out ellipticity gradients as an important contributor to non-elliptical isophotes, it would be worthwhile to extend the calculation to models with non-aligned or non-concentric isodensity surfaces. It would also be helpful to understand how one goes about formally deprojecting an observed intensity distribution to obtain the three-dimensional luminosity distribution, without assuming that the isodensity surfaces are ellipsoids, and without making an ad hoc assumption (as above) about the radial dependence of the shape parameters. Rybicki (1987) has carried out some preliminary work in this direction; the problem is inherently underdetermined in the absence of information about the inclination angle.

The next simplest interpretation of the non-elliptical isophotes is a model in which the underlying elliptical galaxy is, again, accurately ellipsoidal, but with a second component (presumably different kinematically) that has a different morphology. “Disky” isophotes are naturally modeled by superimposing an exponential disk on an otherwise normal spheroid
Disky ellipticals are often found to be rotationally supported (Bender 1988), and many such galaxies may simply be misclassified S0’s (e.g. Capaccioli et al. 1988). No comparably simple interpretation of “boxy” isophotes has so far been suggested. Two conclusions may be drawn from these studies: first, that many elliptical galaxies probably contain a more or less significant disk; and second; to the extent that the non-elliptical isophotes are due to photometric superposition of a spheroid and a disk, the underlying elliptical galaxies must be even more accurately ellipsoidal than the measured values of $a_4$ would suggest.

6. Kinematic Tests

These are of two basic kinds. The rings of dust and gas seen in a few elliptical galaxies can sometimes be identified with periodic orbits. Given an assumption about the radial form of the underlying potential (which may or may not trace the light), it is then possible to infer the intrinsic shape of the potential from the behavior of the orbit. F. Bertola and T. de Zeeuw discuss this sort of analysis in their contributions to these proceedings.

The second kind of kinematical test is based on a geometrical property of triaxial ellipsoids. Because the apparent minor axis of a triaxial galaxy need not be coincident with the projection of the intrinsic minor axis, it follows that stellar streaming around the intrinsic minor axis will generally result in some component of line-of-sight velocity along the apparent minor axis (Binney 1985). Thus the observation of minor axis rotation provides a certain amount of evidence in favor of the triaxial hypothesis. As always, the inference is not airtight, since (a) stellar streaming may occur around the intrinsic major axis; i.e. a prolate “spindle”; (b) elliptical galaxies certainly have rotating figures, and the rotation axis may even be inclined to the symmetry axis. However observations show a preference for small kinematic misalignments (e.g. Davies & Birkinshaw 1988), and large misalignments are often seen in galaxies with large photometric twists (Franx et al. 1989), suggesting that triaxiality is the main culprit.

As in the case of isophotal twists, making detailed inferences about the shapes of galaxies from statistics of the observed rotation fields is extremely difficult. Since a major study of this problem is nearing completion (Franx et al. 1991), I will limit myself to an outline of the main points. In its simplest form, the problem is very similar to the surface brightness / ellipticity test described above: given some measure of the kinematic misalignment for a sample of galaxies (e.g., the ratio of minor to major axis rotation velocities on some isophote), what hypothesis for the distribution of intrinsic shapes is most consistent with the observed correlation of this parameter with apparent axis ratio, assuming that the streaming is about
Franx (1988) finds that a triaxiality $Z = (1 - q_2) / (1 - q_1) \approx 0.5$ fits the data well, except for a possible abundance peak at misalignments of $\sim 90^\circ$, indicative of rotation around the intrinsic major axis.

As Franx points out, a major uncertainty in his analysis is the likelihood of intrinsic misalignments: since triaxial galaxies contain families of tube orbits that circulate both about the major and minor axes, there is no good reason to expect streaming around only one of these axes. Furthermore, the intrinsic misalignment may itself be correlated with intrinsic axis ratios, in a way that masks the the expected correlation between observed quantities (just as in the surface brightness / ellipticity test discussed above). Work in progress should elucidate to what extent these difficulties can be overcome.

7. Stability

One of the reasons that the oblate hypothesis persisted so long in the minds of galactic dynamicists is that the number of orbital integrals in axisymmetric potentials (always two, sometimes three) is clearly sufficient to allow the existence of self-consistent models of any axis ratio. Schwarzschild’s (1979) construction of an equilibrium triaxial model put an end to this mindset: clearly it was now impossible to prefer axisymmetric over non-axisymmetric models on the basis of equilibrium arguments alone. Statler (1987) went on to demonstrate that essentially no point in the plane of the model axis ratios could be excluded by the requirement of equilibrium, at least for models based on the so-called “perfect” potentials in which all parts of phase space are characterized by three isolating integrals. But it has recently become clear that many of these equilibrium models are dynamically unstable.

Most of the work to date on stability of non-rotating models has focused on the spherical case. While isotropic spherical models are generally stable (Antonov 1962), surprisingly small amounts of radial anisotropy can induce instability to an $m = 2$ or “bar” mode, which causes the model to evolve into a triaxial spheroid. A recent study of this “radial-orbit instability” in a particular family of spherical isochrone models (Saha 1991) finds instability for $\sigma_r / \sigma_t > 1.2$, where $\sigma_r$ and $\sigma_t$ are the mean radial and (one-component) tangential velocity dispersions. Work in progress should soon reveal whether this result holds for a wider class of anisotropic spherical models. Although it would probably be extreme to blame this instability alone for the apparent absence of nearly-spherical galaxies (Figure 1), it is likely that radial collapse sometimes produces bar-unstable final states (Merritt & Aguilar 1985), thus leading to a preference for nonspherical (even non-axisymmetric) galaxies over spherical (axisymmetric) ones.
At the other extreme of the ellipticity distribution, it appears more and more likely that the absence of elliptical galaxies flatter than about E6 can be ascribed to instabilities. Fridman & Polyachenko (1984) suggested that oblate and prolate models with axis ratios more extreme than about 2 : 5 might be generally unstable to bending models, a prediction that has now been verified in one family of prolate models (Merritt & Hernquist 1991). Figure 3 shows the instability acting on an initially E9 model. Demonstrating the existence of this instability in a range of prolate and oblate models will require considerably more work, but may solve once and for all the classic problem of why highly flattened elliptical galaxies to not exist.
REFERENCES

Antonov, V. A. 1962, Vestnik Leningrad Univ., 19, 96
Benacchio, L., & Galletta, G. 1980, MNRAS, 193, 885
Bender, R. 1988, A&A, 193, L7
Bender, R., Doeberiner, S., & Moellenhoff, C. 1988, A&AS, 74, 385
Binggeli, B. 1980, A&A, 82, 289
Binney, J. 1985, MNRAS, 212, 767
Binney, J., & de Vaucouleurs, G. 1981, MNRAS, 194, 679
Capaccioli, M., Piotto, G., & Rampazzo, R. 1988, AJ, 96, 487
Carter, D. 1978, MNRAS, 182, 797
—. 1987, ApJ, 312, 514
Davies, R. L., & Birkinshaw, M. 1988, ApJS, 68, 409
de Zeeuw, T., Peletier, R., & Franx, M. 1986, MNRAS, 221, 1001
Djorgovski, S., & Davis, M. 1987, ApJ, 313, 59
Djorgovski, S. B. 1986, Diss. Abstr. Int., Sect. B, Vol. 47, No. 3, p. 1102, 47, 1102
Fasano, G. 1987, in IAU Symposium, Vol. 127, Structure and Dynamics of Elliptical Galaxies, ed. P. T. de Zeeuw, 395
Fasano, G., & Bonoli, C. 1989, A&AS, 79, 291
Franx, M. 1988, PhD thesis, University of Leiden, The Netherlands
Franx, M., Illingworth, G., & de Zeeuw, T. 1991, ApJ, 383, 112
Franx, M., Illingworth, G., & Heckman, T. 1989, ApJ, 344, 613
Frenk, C. S., White, S. D. M., Davis, M., & Efstathiou, G. 1988, ApJ, 327, 507
Fridman, A. M., & Polyachenko, V. L. 1984, Physics of gravitating systems (New York: Springer, 1984)
Hubble, E. P. 1926, ApJ, 64, 321
Jedrzejewski, R. I., Davies, R. L., & Illingworth, G. D. 1987, AJ, 94, 1508
Kormendy, J., & Illingworth, G. 1982, ApJ, 256, 460
Lake, G. 1979, in Photometry, Kinematics and Dynamics of Galaxies, ed. D. S. Evans, 381
Leach, R. 1981, ApJ, 248, 485
Marchant, A. B., & Olson, D. W. 1979, ApJ, 230, L157
Merritt, D. 1982, AJ, 87, 1279
Merritt, D., & Aguilar, L. A. 1985, MNRAS, 217, 787
Merritt, D., & Hernquist, L. 1991, ApJ, 376, 439
Michard, R., & Simien, F. 1988, A&AS, 74, 25
Noerdlinger, P. D. 1979, ApJ, 234, 802
Rix, H.-W., & White, S. D. M. 1990, ApJ, 362, 52
Rybicki, G. B. 1987, in IAU Symposium, Vol. 127, Structure and Dynamics of Elliptical Galaxies, ed. P. T. de Zeeuw, 397
Saha, P. 1991, MNRAS, 248, 494
Sandage, A., Freeman, K. C., & Stokes, N. R. 1970, ApJ, 160, 831
Schwarzschild, M. 1979, ApJ, 232, 236
Statler, T. S. 1987, ApJ, 321, 113
Williams, T. B. 1981, ApJ, 244, 458

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