Effects of isospin and momentum dependent interactions on thermal properties of asymmetric nuclear matter

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Thermal properties of asymmetric nuclear matter are studied within a self-consistent thermal model using an isospin and momentum dependent interaction (MDI) constrained by the isospin diffusion data in heavy-ion collisions, a momentum-independent interaction (MID), and an isoscalar momentum-dependent interaction (eMDYI). In particular, we study the temperature dependence of the isospin-dependent bulk and single-particle properties, the mechanical and chemical instabilities, and liquid-gas phase transition in hot asymmetric nuclear matter. Our results indicate that the temperature dependence of the equation of state and the symmetry energy are not so sensitive to the momentum dependence of the interaction. The symmetry energy at fixed density is found to generally decrease with temperature and for the MDI interaction the decrement is essentially due to the potential part. It is further shown that only the low momentum part of the single-particle potential and the nucleon effective mass increases significantly with temperature for the momentum-dependent interactions. For the MDI interaction, the low momentum part of the symmetry potential is significantly reduced with increasing temperature. For the mechanical and chemical instabilities as well as the liquid-gas phase transition in hot asymmetric nuclear matter, our results indicate that the boundary of these instabilities and the phase-coexistence region generally shrink with increasing temperature and is sensitive to the density dependence of the symmetry energy and the isospin and momentum dependence of the nuclear interaction, especially at higher temperatures.

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I. INTRODUCTION

The recent progress in developing advanced radioactive beam facilities offers a great opportunity to explore in terrestrial laboratories properties of nuclear matter and/or nuclei with large isospin asymmetries. As a result, studies on the role of the isospin degree of freedom have recently attracted much attention in both nuclear physics and astrophysics. The ultimate goal of such studies is to extract information on the isospin dependence of in-medium nuclear effective interactions as well as the equation of state (EOS) of isospin asymmetric nuclear matter. The latter, especially the nuclear symmetry energy term, is important for understanding not only many aspects of nuclear physics, but also a number of important issues in astrophysics. Information about the symmetry energy at zero temperature is important for determining ground-state properties of exotic nuclei and properties of cold neutron stars at β-equilibrium, while the symmetry energy or symmetry free energy of hot neutron-rich matter is important for understanding the liquid-gas (LG) phase transition of asymmetric nuclear matter, the dynamical evolution of massive stars and the supernova explosion mechanisms.

Although the nuclear symmetry energy at normal nuclear matter density for cold asymmetric nuclear matter is known to be around 30 MeV from the empirical liquid-drop mass formula, its values at other densities, especially at supra-normal densities, are poorly known. Predictions based on various many-body theories differ widely at both low and high densities. Empirically, the incompressibility of asymmetric nuclear matter is essentially undetermined, even though the incompressibility of symmetric nuclear matter at its saturation density \( \rho_0 \approx 0.16 \text{ fm}^{-3} \) has been determined to be \( 231 \pm 5 \text{ MeV} \) from nuclear giant monopole resonances (GMR) and the EOS at densities of \( 2\rho_0 < \rho < 5\rho_0 \) has also been constrained by measurements of collective flows in nucleus-nucleus collisions. Fortunately, heavy-ion reactions, especially those induced by radioactive beams, provide a unique means to investigate the isospin-dependent properties of asymmetric nuclear matter, particularly the density dependence of the nuclear symmetry energy. Indeed, significant progress has recently been made both experimentally and theoretically in extracting the information on the behaviors of nuclear symmetry energy at sub-saturation density from the isospin diffusion data in heavy-ion collisions from the NSCL/MSU. Using the isospin and momentum-dependent IBUU04 transport model with in-medium nucleon-nucleon (NN) cross sections, the isospin diffusion data were found to be consistent with a density-dependent symmetry energy of \( E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)\gamma \) with \( \gamma = 0.69 - 1.05 \) at subnormal density, which has led to the extraction of a value of \( L = 88 \pm 25 \text{ MeV} \) for the slope parameter of the nuclear symmetry energy at saturation density and a value of \( K_{\text{asy}} = -500 \pm 50 \text{ MeV} \) for the isospin-dependent part of the isobaric incompressibility of isospin asymmetric nuclear matter. The extracted symmetry energy further agrees with the symmetry energy \( E_{\text{sym}}(\rho) \approx 31.6(\rho/\rho_0)^{0.68} \) recently obtained from the isoscaling analyses of isotope ratios in
Theoretical, a lot of efforts have been devoted to the study on the properties of cold asymmetric nuclear matter. However, the properties of hot asymmetric nuclear matter, especially the temperature-dependence of the nuclear symmetry energy, has received so far much less attention. For finite nuclei at temperatures below about 3 MeV, the shell structure and pairing as well as vibrations of nuclear surfaces are important and the symmetry energy was predicted to increase slightly with the increasing temperature. Interestingly, an increase by only about 8% in the symmetry energy in the range of $T$ from 0 to 1 MeV was found to affect appreciably the physics of stellar collapse, especially the neutralization processes. At higher temperatures, one expects the symmetry energy to decrease as the Pauli blocking becomes less important when the nucleon Fermi surfaces become more diffused at increasingly higher temperatures. On the other hand, due to the van der Waals behavior of the nucleon-nucleon interaction, it is expected that the so-called LG phase transition should occur in nuclear matter. Since the early work, see, e.g., Refs. and many investigations have been carried out to explore properties of the nuclear LG phase transition both experimentally and theoretically over the last three decades. For a recent review, see, e.g., Refs. Most of these studies focused on investigating features of the LG phase transition in symmetric nuclear matter. New features of the LG phase transition in asymmetric nuclear matter are expected. In particular, in a two-component asymmetric nucleonic matter, there are two conserved charges of baryon number and the third component of isospin. The LG phase transition was suggested to be of second order. This suggestion together with the need to understand better the properties of asymmetric nuclear matter have stimulated a lot of work recently, see, e.g., Refs. While significant progress has been made recently, many interesting questions about properties of hot asymmetric nuclear matter remain open. Some of these questions can be traced back to our poor understanding about the isovector nuclear interaction and the density dependence of the nuclear symmetry energy. With the recent progress on the constraints of the density dependence of nuclear symmetry energy, it is therefore interesting to investigate how the constrained symmetry energy may allow us to better understand the thermal properties of asymmetric nuclear matter. Moreover, both the isovector (i.e., the nuclear symmetry potential) and isoscalar parts of the single nucleon potential should be momentum dependent due to the non-locality of nucleon-nucleon interaction and the Pauli exchange effects in many-fermion systems. However, effects of the momentum-dependent interactions on the thermal properties of asymmetric nuclear matter have received so far little theoretical attention.

In the present work, we study systematically the effects of isospin and momentum dependent interactions on the thermal properties of asymmetric nuclear matter, including the isospin-dependent bulk and single-particle properties, mechanical and chemical instability, and LG phase transition, within a self-consistent thermal model using three different interactions. The first one is the isospin and momentum dependent MDI interaction constrained by the isospin diffusion data in heavy-ion collisions. The second one is a momentum-independent interaction (MID) which leads to a fully momentum independent single nucleon potential, and the third one is an isoscalar momentum-dependent interaction (eMDYI) in which the isoscalar part of the single nucleon potential is momentum dependent but the isovector part of the single nucleon potential is momentum independent by construction. We note that the MDI interaction is realistic, while the MID and eMDYI interactions are only used as references in order to explore effects of the isospin and momentum dependence of the nuclear interactions.

The paper is organized as follows. In Section II we briefly introduce the MDI, MID and eMDYI interactions and discuss some relevant thermodynamic quantities. Results on thermal effects on the isospin-dependent bulk and single-particle properties of asymmetric nuclear matter, such as the nuclear symmetry energy, the nuclear symmetry potential and isospin-splitting of nucleon effective mass, are presented in Section III. The mechanical and chemical instabilities of hot neutron-rich nuclear matter are then discussed in Section IV. In Section V we present the results on the LG phase transition of hot asymmetric nuclear matter. All the results are calculated from the three interactions, and the effects of isospin and momentum dependence the nuclear interactions are analyzed. Finally, a summary is given in Section VI.

II. THEORETICAL MODELS

A. Isospin and momentum-dependent MDI interaction

The isospin- and momentum-dependent MDI interaction is based on a modified finite-range Gogny effective interaction. In the MDI interaction, the potential energy density $V(\rho, T, \delta)$ of a thermal equilibrium asymmetric nuclear matter at total density $\rho$, temperature $T$
and isospin asymmetry $\delta$ is expressed as follows,$^{16,50}$

$$V(\rho, T, \delta) = \frac{A_u \rho n p_\delta}{\rho_0} + \frac{A_l}{2} \frac{\rho_n^2 + \rho_p^2}{\rho_0^2} + \frac{B}{\rho_0} \rho^{\sigma+1} + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \times (1 - x^2) + \frac{1}{\rho_0} \sum_{\tau, \tau'} C_{\tau, \tau'} \times \int d^3 p \rho^3 p^2 \frac{f_\tau(\vec{r}, \vec{p}) f_{\bar{\tau}}(\vec{r}, \vec{p})}{1 + (\vec{p} - \vec{p})^2 / \Lambda^2} \tau^2.$$ (1)

In the mean field approximation, Eq. 1 leads to the following single particle potential for a nucleon with momentum $\vec{p}$ and isospin $\tau$ in the thermal equilibrium asymmetric nuclear matter, i.e., $^{16,50}$

$$U(\rho, T, \delta, \vec{p}, \tau) = A_u(x) \frac{\rho - \rho_0}{\rho_0} + A_l(x) \frac{\rho_x}{\rho_0} + B(\frac{\rho}{\rho_0})^\sigma (1 - x^2) - 8 \tau x \frac{B}{\sigma + 1} \rho^{\sigma - 1} \rho_{\rho - \tau} + 2 C_{\tau, \tau'} \int d^3 p' f_\tau(\vec{r}, \vec{p}) + 2 C_{\tau, \tau'} \int d^3 p' f_{\bar{\tau}}(\vec{r}, \vec{p}) \tau = \frac{1 + (\vec{p} - \vec{p})^2 / \Lambda^2 + 1 + (\vec{p} - \vec{p})^2 / \Lambda^2. \tau (2)

In the above $\tau = 1/2$ $(-1/2)$ for neutrons (protons); $\sigma = 4/3$; $f_\tau(\vec{r}, \vec{p})$ is the phase space distribution function at coordinate $\vec{r}$ and momentum $\vec{p}$. The parameters $A_u(x), A_l(x), B, C_{\tau, \tau'}, C_{\tau, \tau'}$ and $\Lambda$ have been assumed to be temperature independent and are obtained by fitting the momentum-dependence of $U(\rho, T, 0, \delta, \vec{p}, \tau)$ to that predicted by the Gogny Hartree-Fock and/or the Brueckner-Hartree-Fock calculations, the zero temperature saturation properties of symmetric nuclear matter and the symmetry energy of 31.6 MeV at normal nuclear matter density $\rho_0 = 0.16$ fm$^{-3}$ $^{50}$. The incompressibility $K_0$ of cold symmetric nuclear matter at saturation density $\rho_0$ is set to be 211 MeV. The parameters $A_u(x)$ and $A_l(x)$ depend on the $x$ parameter according to

$$A_u(x) = -95.98 - x \frac{2B}{\sigma + 1}, \quad A_l(x) = -120.57 + x \frac{2B}{\sigma + 1}.$$ (3)

The different $x$ values in the MDI interaction are introduced to vary the density dependence of the nuclear symmetry energy while keeping other properties of the nuclear equation of state fixed $^{16}$ and they can be adjusted to mimic predictions on the density dependence of nuclear matter symmetry energy by microscopic and/or phenomenological many-body theories. The last two terms of Eq. 2 contain the momentum-dependence of the single-particle potential. The momentum dependence of the symmetry potential stems from the different interaction strength parameters $C_{\tau, \tau'}$ and $C_{\tau, \tau'}$ for a nucleon of isospin $\tau$ interacting, respectively, with unlike and like nucleons in the background fields. More specifically, we use $C_{\tau, \tau'} = -103.4$ MeV and $C_{\tau, \tau'} = -11.7$ MeV. We note that the MDI interaction has been extensively used in the transport model for studying isospin effects in intermediate energy heavy-ion collisions induced by neutron-rich nuclei $^{16,51,52,53,54,53,56,57,58}$. In particular, the isospin diffusion data from NSCL/MSU have constrained the value of $x$ to be between 0 and -1 for nuclear matter densities less than about 1.2$\rho_0$ $^{16,17}$, we will thus in the present work consider the two values of $x = 0$ and $x = -1$. We note that the zero-temperature symmetry energy for the MDI interaction with $x = 0$ and $x = -1$ can be parameterized, respectively, as 31.6($\rho/\rho_0)^{0.69}$ MeV and 31.6($\rho/\rho_0)^{1.06}$ MeV $^{16}$, and thus $x = 0$ gives a softer symmetry energy while $x = -1$ gives a stiffer symmetry energy.

B. Momentum-independent MID interaction

In the momentum-independent MDI interaction, the potential energy density $V_{\text{MDI}}(\rho, \delta)$ of a thermally equilibrated asymmetric nuclear matter at total density $\rho$ and isospin asymmetry $\delta$ is written as

$$V_{\text{MDI}}(\rho, \delta) = \frac{\alpha \rho^2}{2 \rho_0} + \frac{\beta \rho^{1+\gamma}}{1+\gamma} + \rho E_{\text{sym}}^\text{pot}(\rho, x) \delta^2.$$ (4)

The parameters $\alpha$, $\beta$ and $\gamma$ are determined by the incompressibility $K_0$ of cold symmetric nuclear matter at saturation density $\rho_0$ $^{33}$

$$\alpha = -29.81 - 46.90 \frac{K_0 + 44.73}{K_0 - 166.32} \text{ (MeV)} \quad (5)$$

$$\beta = 23.45 \frac{K_0 + 255.78}{K_0 - 166.32} \text{ (MeV)} \quad (6)$$

$$\gamma = \frac{K_0 + 44.73}{211.05} \quad (7)$$

and $K_0$ is again set to be 211 MeV as in the MDI interaction. To fit the MDI interaction at zero temperature, the potential part of the symmetry energy $E_{\text{sym}}^\text{pot}(\rho, x)$ is parameterized by $^{16}$

$$E_{\text{sym}}^\text{pot}(\rho, x) = F(x) \frac{\rho}{\rho_0} + [18.6 - F(x)] \left( \frac{\rho}{\rho_0} \right)^{G(x)}$$ (8)

with $F(x = 0) = 129.981$ MeV, $G(x = 0) = 1.059$, $F(x = -1) = 3.673$ MeV, and $G(x = -1) = 1.569$. We note that the MID interaction reproduce very well the EOS of isospin-asymmetric nuclear matter with the MDI interaction at zero temperature for both $x = 0$ and $x = -1$. The single nucleon potential in the MID interaction can be directly obtained as

$$U_{\text{MDI}}(\rho, \delta, \tau) = \alpha \frac{\rho}{\rho_0} + \beta \frac{\rho}{\rho_0} \gamma + U_{\text{asy}}(\rho, \delta, \tau), \quad (9)$$

with

$$U_{\text{asy}}(\rho, \delta, \tau) = \left[ 4F(x) \frac{\rho}{\rho_0} + 4(18.6 - F(x)) \left( \frac{\rho}{\rho_0} \right)^{G(x)} \right] \tau \delta$$

$$+ (18.6 - F(x)) (G(x) - 1) \left( \frac{\rho}{\rho_0} \right)^{G(x)} \delta^2.$$ (10)

Therefore, the single nucleon potential in the MID interaction is fully momentum-independent. It also leads to
the fact that the potential energy density and the single nucleon potential in the MID interaction are independent of the temperature.

C. Extended MDYI (eMDYI) interaction

The momentum-dependent part in the MDI interaction is also isospin dependent while the MID interaction is fully momentum independent. In order to see the effect of the momentum dependence of the isovector part of the single nucleon potential (nuclear symmetry potential), we can construct an isoscalar momentum-dependent interaction, called extended MDYI (eMDYI) interaction since it has the same functional form as the well-known MDYI interaction for symmetric nuclear matter [60]. In the eMDYI interaction, the potential energy density \( V_{\text{eMDYI}}(\rho, T, \delta) \) of a thermally equilibrated asymmetric nuclear matter total density \( \rho \), temperature \( T \) and isospin asymmetry \( \delta \) is expressed as

\[
V_{\text{eMDYI}}(\rho, T, \delta) = A \frac{\rho^2}{\rho_0} + B \frac{\rho^{1+\sigma}}{\rho_0} + \frac{C}{\rho_0} \int d^3 p d^3 p' \frac{f_0(\vec{r}, \vec{p}) f_0(\vec{r}', \vec{p}')}{{1 + (\vec{p} - \vec{p}')^2}/\Lambda^2} + \rho E_{\text{sym}}^{\text{pot}}(\rho, \delta) \delta^2. \tag{11}
\]

Here \( f_0(\vec{r}, \vec{p}) \) is the phase space distribution function of symmetric nuclear matter at total density \( \rho \) and temperature \( T \). \( E_{\text{sym}}^{\text{pot}}(\rho, x) \) has the same expression as Eq. (5).

We set \( A = A_{\text{MDYI}} + A_{\text{MD}} \) and \( C = C_{\text{MDI}} + C_{\text{MD}} \), and \( B, \sigma \) and \( \Lambda \) have the same values as in the MDI interaction, so that the eMDYI interaction gives the same EOS of asymmetric nuclear matter as the MDI interaction at zero temperature for both \( x = 0 \) and \( x = -1 \). The single nucleon potential in the eMDYI interaction can be obtained as

\[
U_{\text{eMDYI}}(\rho, T, \delta, \vec{p}, \tau) = U^0(\rho, T, \vec{p}) + U^{\text{asy}}(\rho, \delta, \tau), \tag{12}
\]

where

\[
U^0(\rho, T, \vec{p}) = A \frac{\rho^2}{\rho_0} + B \frac{\rho^{1+\sigma}}{\rho_0} + \frac{2C}{\rho_0} \int d^3 p d^3 p' \frac{f_0(\vec{r}, \vec{p})}{1 + (\vec{p} - \vec{p}')^2}/\Lambda^2 \tag{13}
\]

and \( U^{\text{asy}}(\rho, \delta, \tau) \) is the same as Eq. (10) which implies that the symmetry potential is identical for the eMDYI and MID interactions. Therefore, in the eMDYI interaction, the isoscalar part of the single nucleon potential is momentum dependent but the nuclear symmetry potential is not. For symmetric nuclear matter, the single nucleon potential in the eMDYI interaction is exactly the same as that in the MDI interaction. We note that the same strategy has been used to study the momentum dependence effects in heavy-ion collisions in a previous work [52].

D. Thermodynamic quantities of asymmetric nuclear matter

At zero temperature, \( f_\tau(\vec{r}, \vec{p}) = \frac{1}{\mathcal{Z}} \Theta(p_f(\tau) - p) \) and all the integrals in previous expressions can be calculated analytically [59], while at a finite temperature \( T \), the phase space distribution function becomes the Fermi distribution

\[
f_\tau(\vec{r}, \vec{p}) = \frac{2}{\hbar^3} \frac{1}{\exp\left(\frac{\sqrt{\vec{p}^2 + U_{\tau} - \mu_\tau}}{\hbar T}\right) + 1} \tag{14}
\]

where \( \mu_\tau \) is the proton or neutron chemical potential and can be determined from

\[
\rho_\tau = \int f_\tau(\vec{r}, \vec{p}) d^3 p. \tag{15}
\]

In the above, \( m_\tau \) is the proton or neutron mass and \( U_{\tau} \) is the proton or neutron single nucleon potential in different interactions. For fixed density \( \rho \), temperature \( T \), and isospin asymmetry \( \delta \), the chemical potential \( \mu_\tau \) and the distribution function \( f_\tau(\vec{r}, \vec{p}) \) can be determined numerically by a self-consistency iteration scheme [24, 60]. From the chemical potential \( \mu_\tau \) and the distribution function \( f_\tau(\vec{r}, \vec{p}) \), the energy per nucleon \( E(\rho, T, \delta) \) can be obtained as

\[
E(\rho, T, \delta) = \frac{1}{\rho} \left[ V(\rho, T, \delta) + \sum_\tau \int d^3 p \frac{p^2}{2m_\tau} f_\tau(\vec{r}, \vec{p}) \right]. \tag{16}
\]

Furthermore, we can obtain the entropy per nucleon \( S_\tau(\rho, T, \delta) \) as

\[
S_\tau(\rho, T, \delta) = -\frac{8\pi}{\rho \hbar^3} \int_0^\infty \frac{p^2 |n_\tau \ln n_\tau + (1-n_\tau) \ln (1-n_\tau)|}{p} dp \tag{17}
\]

with the occupation probability

\[
n_\tau = \frac{1}{\exp\left(\frac{\sqrt{\vec{p}^2 + U_{\tau} - \mu_\tau}}{\hbar T}\right) + 1}. \tag{18}
\]

Finally, the pressure \( P(\rho, T, \delta) \) can be calculated from the thermodynamic relation

\[
P(\rho, T, \delta) = \left[ T \sum_\tau S_\tau(\rho, T, \delta) - E(\rho, T, \delta) \right] \rho + \sum_\tau \mu_\tau \rho_\tau. \tag{19}
\]

III. THERMAL EFFECTS ON THE ISOSPIN-DEPENDENT BULK AND SINGLE-PARTICLE PROPERTIES OF ASYMMETRIC NUCLEAR MATTER

A. Nuclear symmetry energy

As in the case of zero temperature, phenomenological and microscopic studies [21, 22, 24, 26] indicate that
the EOS of hot asymmetric nuclear matter at density $\rho$, temperature $T$, and an isospin asymmetry $\delta$ can also be written as a parabolic function of $\delta$, i.e.,

$$E(\rho, T, \delta) = E(\rho, T, \delta = 0) + E_{\text{sym}}(\rho, T)\delta^2 + O(\delta^4).$$  \hspace{1cm} (20)

This nice feature of the empirical parabolic law for the EOS of hot asymmetric nuclear matter is very useful and convenient for the calculation of the nuclear symmetry energy. With the empirical parabolic law, the temperature and density dependent symmetry energy $E_{\text{sym}}(\rho, T)$ for hot asymmetric nuclear matter can thus be extracted from

$$E_{\text{sym}}(\rho, T) \approx E(\rho, T, \delta = 1) - E(\rho, T, \delta = 0).$$  \hspace{1cm} (21)

In this sense, the symmetry energy $E_{\text{sym}}(\rho, T)$ gives estimation of the energy cost to convert all protons in symmetry matter to neutrons at fixed temperature $T$ and density $\rho$. The parabolic approximation has been justified for MDI interaction in Ref. [24]. We note that the parabolic approximation also holds very well for the MID and eMDYI interactions.

![Density dependence of energy per nucleon](image)

**FIG. 1:** (Color online) Density dependence of energy per nucleon for symmetric nuclear matter (upper panels) and pure nuclear matter (lower panels) in MDI, MID and eMDYI interactions at $T = 0, 10, 30$ and $50$ MeV.

Shown in Fig. 1 is the density dependence of $E(\rho, T, \delta)$ for symmetric nuclear matter and pure neutron matter at $T = 0, 10, 30$ and $50$ MeV using the MDI, MID and eMDYI interactions with $x = 0$ and $-1$. For symmetric nuclear matter ($\delta = 0$), the parameter $x = 0$ would give the same results as the parameter $x = -1$ as we have discussed above, and thus the curves shown in upper panels of Fig. 1 are the same for $x = 0$ and $-1$. However, for pure neutron matter, the parameters $x = 0$ and $-1$ display different density dependence for the energy per nucleon $E(\rho, T, \delta)$, which just reflects that the parameters $x = 0$ and $-1$ give different density dependence of the nuclear symmetry energy as will be discussed in the following. From Fig. 1 one can see that the energy per nucleon $E(\rho, T, \delta)$ increases with increasing temperature $T$. The increment of the energy per nucleon $E(\rho, T, \delta)$ with the temperature reflects the thermal excitation of the nuclear matter due to the change of the phase-space distribution function $f_\tau(\vec{r}, \vec{p})$ or the occupation probability $n_\tau(\vec{r}, \vec{p})$. With the increment of the temperature, more nucleons move to higher momentum states and thus lead to larger internal energy per nucleon. Furthermore, the temperature effects are seen to be stronger at lower densities while they become much weaker at higher densities. At lower densities, the Fermi momentum $p_f(\tau)$ is smaller and thus temperature effects on the energy per nucleon $E(\rho, T, \delta)$ are expected to be stronger.

![Occupation probability distribution](image)

**FIG. 2:** (Color online) Occupation probability distribution for symmetry nuclear matter and pure neutron matter at $\rho = 0.5\rho_0$ and $T = 10$ MeV in MDI, MID and eMDYI interactions with $x = 0$ and $x = -1$. The corresponding Fermi momentum $p_f(\tau)$ at zero temperature is indicated.

In order to understand better the above results, we show in Fig. 2 the momentum dependence of the occupation probability $n_\tau(\vec{r}, \vec{p})$ for symmetric nuclear matter and pure neutron matter at $\rho = 0.5\rho_0$ and $T = 10$ MeV using the MDI, MID and eMDYI interactions with $x = 0$ and $-1$. For comparison, the corresponding Fermi momentum $p_f(\tau)$ at zero temperature is also indicated. The occupation probability $n_\tau(\vec{r}, \vec{p})$ at finite temperatures is self-consistently determined and from which other quantities can be calculated. For symmetric nuclear matter, as mentioned in Section 3, the MDI interaction has the same occupation probability as the eMDYI interaction, and also the occupation probability is independent of the $x$ parameter. One can see clearly from Fig. 2 that compared with the case of zero temperature, more nucleons move to higher momentum states at finite temperature.
of $T = 10$ MeV. In addition, the results indicate that for symmetric nuclear matter, the occupation probability distribution of MID interaction is more extended than those of MDI and eMDYI interactions. For pure neutron matter the result of MID interaction is very close to that of the MID interaction, while the eMDYI interaction seems to have a steeper occupation probability distribution. Meanwhile, the value of $x$ parameter seems to have little effects on the shape of the occupation probability distribution even though for pure neutron matter. Although here we only show the case of $\rho = 0.5 \rho_0$, we note that these properties hold for all the densities.

In addition, it is seen from Fig. 1 that the MDI, MID and eMDYI interactions give almost the same EOS for cold nuclear matter even at high densities though their parameters are constrained only at saturation density as discussed in Section II. Meanwhile, it is interesting to see that the MDI, MID and eMDYI interactions also produce quite similar EOS at finite temperatures, even at high densities, for both $x = 0$ and $x = -1$. This is due to the fact that only low momentum parts of the single-particle potential is lifted at finite temperatures as we will show in the following while these momentum parts do not contribute much to the total energy of system through integration with respect to the momentum. Although the three interactions have different occupation probability distributions at finite temperatures, the potential energy makes self-consistently the balance and leads to that they have the similar EOS for symmetric nuclear matter and pure neutron matter at finite temperatures. This feature implies that the interaction of the simple momentum-independent MID type is enough to describe the EOS of asymmetric nuclear matter at finite temperatures. However, as will be discussed in following, we note that the chemical potential, which is a quite important quantity in the study of the mechanical and chemical instabilities as well as liquid-gas phase transition in hot asymmetric nuclear matter, can be significantly different for the three kinds of interactions.

Now let’s see the temperature dependence of the nuclear symmetry energy. In Fig. 3 we show the density dependence of the nuclear symmetry energy at $T = 0$, 10, 30 and 50 MeV using the MDI, MID and eMDYI interactions with $x = 0$ and $-1$. For different choice of the parameter $x = 0$ and $-1$, $E_{\text{sym}}(\rho, T)$ display different density dependence with $x = 0$ ($-1$) giving larger (smaller) values for the symmetry energy at lower densities while smaller (larger) ones at higher densities for a fixed temperature. For all the three interactions with both $x = 0$ and $-1$, it is seen that the symmetry energy decreases with increasing temperature. At higher temperatures, one expects the symmetry energy $E_{\text{sym}}(\rho, T)$ to decrease as the Pauli blocking (a pure quantum effect) becomes less important when the nucleon Fermi surfaces become more diffused at increasingly higher temperatures [21, 22, 23, 24].

Within the present self-consistent thermal model, because the single particle potential is isospin and momentum dependent with the MDI interaction, the potential part of the symmetry energy with the MDI interaction is thus temperature dependent as shown in Eq. (11). On the other hand, the potential part of the symmetry energy with the MID and eMDYI interactions does not depend on the temperature by the construction as seen in Eq. (3) and Eq. (11). It is thus interesting to study how the potential and kinetic parts of the symmetry energy $E_{\text{sym}}(\rho, T)$ may vary respectively with temperature, which will reflect the effects of isospin and momentum dependence of the nuclear interaction. Fig. 4 displays the temperature dependence of the symmetry energy $E_{\text{sym}}(\rho, T)$ as well as its potential and kinetic energy parts using the MDI, MID and eMDYI interactions with $x = 0$ at $\rho = 1.0 \rho_0$, 0.5$\rho_0$ and 0.1$\rho_0$. With the
parameter \( x = -1 \), we note the same conclusion is obtained. For the MDI interaction, it is seen that both the total symmetry energy \( E_{\text{sym}}(\rho, T) \) and its potential energy part decrease with increasing temperature at all three densities considered. Meanwhile, one can see that the kinetic contribution increases slightly with increasing temperature at low temperature and then decreases with increasing temperature at high temperature for \( \rho = 1.0\rho_0 \) and \( 0.5\rho_0 \), while it decreases monotonically for \( \rho = 0.1\rho_0 \). These features observed for the MDI interaction are uniquely determined by the isospin and momentum dependence in the MDI interaction within the present self-consistent thermal model. On the other hand, for MID and eMDYI interactions the kinetic part of the total symmetry energy decreases with increasing temperature at all the densities while the potential contribution is independent of temperature and it has the same value for the MID and eMDYI interactions. These features indicate that the temperature dependence of the total symmetry energy is due to both the potential contribution and kinetic contribution for MDI interaction, but it is only due to the kinetic contribution for the MID and eMDYI interactions.

It should be mentioned that for the MDI interaction, the decrement of the kinetic energy part of the symmetry energy with temperature at very low densities is consistent with predictions of the free Fermi gas model at high temperatures and/or very low densities \cite{22,26,11,42}. Interestingly, we can see that the temperature dependence of the total symmetry energy \( E_{\text{sym}}(\rho, T) \) is quite similar for all the three interactions except that the MDI interaction exhibits a little stronger temperature dependence at higher temperatures. This is due to the fact that the phase space distribution function will vary self-consistently according to whether the single particle potential is momentum dependent or not. In addition, we note that as shown in Ref. \cite{26}, both the potential and kinetic parts of the symmetry energy \( E_{\text{sym}}(\rho, T) \) can decrease with temperature for all the densities considered there by using the isospin- and momentum-dependent BGBD interaction developed by Bombaci \cite{11} based on the well known Gale-Bertsch-Das Gupta formalism \cite{61}. The different temperature dependence of the potential and kinetic parts of the symmetry energy between the MDI and BGBD interaction is due to the fact that the MDI and BGBD interactions have different forms of the energy density functional and the MDI interaction leads to a more complicated momentum dependence of the single-particle potential. This feature implies that the temperature dependence of the potential and kinetic parts of the symmetry energy depends on the isospin and momentum dependence of the nuclear interactions.

### B. Nuclear symmetry potential

To understand more clearly the effects of the isospin and momentum dependence of nuclear interactions, we discuss the single particle potential and its temperature dependence with the three models. In particular, we study the thermal effects on the nuclear symmetry potential. The nuclear symmetry potential refers to the isovector part of the nucleon mean-field potential in isospin asymmetric nuclear matter. Besides the nuclear density, the symmetry potential of a nucleon in nuclear matter also depends on the momentum or energy of the nucleon. In hot asymmetric nuclear matter, the symmetry potential of a nucleon can also depend on the temperature. The nuclear symmetry potential is different from the nuclear symmetry energy as the latter involves the integration of the isospin-dependent mean-field potential of a nucleon over its momentum. Both the nuclear symmetry potential and the nuclear symmetry energy are important for understanding many physics questions in nuclear physics and astrophysics. Various microscopic and phenomenological models have been used to study the symmetry potential \cite{7,50,51,52,62,63,64,65,66,67,68,69,70,71,72,73,74} and the predicted results vary widely as in the case of the nuclear symmetry energy. In particular, whereas most models predict a decreasing symmetry potential with increasing nucleon momentum albeit at different rates, a few nuclear effective interactions used in some models give an opposite behavior. However, all the above studies on the nuclear symmetry potential are for zero-temperature and the temperature dependence of the nuclear symmetry potential has received so far little theoretical attention. The density, temperature and momentum dependent nuclear symmetry potential can be evaluated from

\[
U_{\text{sym}}(\rho, \vec{p}, T) = \frac{U_n(\rho, \vec{p}, T) - U_p(\rho, \vec{p}, T)}{2\delta}
\]

where \( U_n(\rho, \vec{p}, T) \) and \( U_p(\rho, \vec{p}, T) \) represent, respectively, the neutron and proton single-particle potentials in hot asymmetric nuclear matter. In the present work, we use \( \delta = 0.5 \) in the calculation for the symmetry potential and we note that the result can vary within only several percents at most by choosing different values of \( \delta \).

In order to see the temperature effects on the nuclear symmetry potential, we first study the temperature dependence of the nucleon single-particle potential in hot nuclear matter. In Fig.\ref{fig:1} we show the momentum dependence of the single particle potential of symmetric nuclear matter at \( T = 0, 10, 30 \) and \( 50 \) MeV and \( \rho = 0.5\rho_0, 1.0\rho_0 \) and \( 1.5\rho_0 \) for the MDI or eMDYI interaction. As mentioned in Section\ref{sec:II} for symmetric nuclear matter the MDI and eMDYI interactions are exactly the same while the MID interaction is completely momentum-independent and thus temperature-independent as well. Therefore, we do not discuss here the single particle potential for the MID interaction. From Fig.\ref{fig:1} it is seen that the single particle potentials increase with increasing momentum and saturate at high momenta. In addition, the shape of single-particle potentials are steeper with increasing density and so the momentum dependence becomes stronger at higher densities. It is interesting to
see that only low momentum parts of the potentials are lifted with increasing temperature, and this feature is quite reasonable and expected since the nucleons with high momenta can hardly be affected by the temperature.

FIG. 5: (Color online) Momentum dependence of the single particle potential in symmetric nuclear matter at $\rho = 0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ and $T = 0$, 10, 30 and 50 MeV in the MDI or eMDYI interaction.

shown in Fig. 6 is the momentum dependence of single particle potentials of protons and neutrons in asymmetric nuclear matter with the isospin asymmetry of $\delta = 0.5$ at $T = 0$, 10, 30 and 50 MeV and $\rho = 0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ for MDI (upper panels) and eMDYI (lower panels) interactions with $x = 0$. The similar results are shown in Fig. 7 for $x = -1$. The temperature and density effects are seen to be very similar to that in the case of symmetric nuclear matter as shown in Fig. 5 i.e., only low momentum parts of the potentials are lifted with increasing temperature. In contrast to the results in the case of symmetric nuclear matter, however, the neutron and proton single particle potentials in asymmetric nuclear matter at a fixed temperature becomes different from each other. For the eMDYI interaction, the potentials at a fixed temperature are just momentum-independently shifted for protons and neutrons with a higher potential for neutrons while lower potential for protons, and the shifted value is sensitive to the density and the $x$ value. For the MDI interaction, the isospin and momentum dependence of single particle potentials is somewhat complicated. At a fixed temperature, for MDI interaction with $x = 0$, the potential of neutrons is larger than that of protons at low momenta but the opposite result is obtained at high momenta, which indicates a steeper potential (stronger momentum dependence) for protons than for neutrons. For different $x$ values, the single particle potentials of MDI interaction are also seen to be shifted and the shifted value depending on the density and this is due to the fact that the term with $x$ in Eq. (2) is momentum-independent and depends only on the density.

FIG. 6: (Color online) Momentum dependence of the single particle potential of protons and neutrons in asymmetric nuclear matter with isospin asymmetry $\delta = 0.5$ at $\rho = 0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ and $T = 0$, 10, 30 and 50 MeV in the MDI and eMDYI interactions with $x = 0$.

FIG. 7: (Color online) Same as Fig. 6 but for $x = -1$.

The above discussions imply that the nuclear symmetry potential for the eMDYI interaction does not depend on the momentum while that for the MDI interaction does. Show in Fig. 8 is the momentum dependence of the nuclear symmetry potential at $T = 0$, 10, 30 and 50 MeV and $\rho = 0.1\rho_0$, $0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ using the MDI interaction with $x = 0$ and $x = -1$. It is seen that the symmetry potential decreases with increasing momentum for both $x = 0$ and $x = -1$. Empirically, for the momentum dependence of the nuclear symmetry potential at zero temperature, a systematic analysis of a large number of nucleon-nucleus scattering experiments and (p,n) charge-exchange reactions at beam energies up to about 100 MeV has shown that the data can be very well described by the parametrization $U_{\text{sym}} = a - bE_{\text{lab}}$ with $a \approx 22 - 34$ MeV and $b = 0.1 - 0.2$ MeV. Although the uncertainties in both parameters $a$ and $b$...
are large, the nuclear symmetry potential at saturation density, i.e., the Lane potential $U_{\text{Lane}}$ \cite{79}, clearly decreases approximately linearly with increasing beam energy. This provides a stringent constraint on the low energy behavior of the nuclear symmetry potential at saturation density. We note that for the MDI interaction with both $x = 0$ and $x = -1$ at saturation density, the symmetry potential agrees very well with the empirical Lane potential. In addition, the high energy behavior of the nuclear symmetry potential from the MDI interaction with the high momenta \[\text{MeV/c}\], the temperature effect on the symmetry potential is quite weak due to the fact that the nucleons with high momenta can hardly be affected by the temperature as mentioned above.

C. Isospin-splitting of nucleon effective mass

One of the important single-particle properties of nuclear matter is the nucleon effective mass which characterizes the momentum dependence of the single-particle potential of a nucleon. The nucleon effective mass $m^*_\tau$ is defined by

$$m^*_\tau = \frac{m_\tau}{U_\tau} = \left(1 + \frac{m_\tau}{p} \frac{dU_\tau}{dp}\right)^{-1}. \quad (23)$$

where $\epsilon_\tau$ represents the single-particle energy while $U_\tau$ is the single-particle potential. In such a way, the nucleon effective mass $m^*_\tau$ is related to the density of states $m^*_\tau/(2\pi\hbar)^3$ in asymmetric nuclear matter. By definition, the nucleon effective mass generally depends on the density, isospin asymmetry of the medium, and the momentum of the nucleon \[66, 50, 51\]. In hot nuclear medium, it depends on the temperature as well. At zero temperature, when the nucleon effective mass is evaluated at the Fermi momentum $p_\tau = p_f(\tau)$, Eq. (23) yields the Landau mass which is related to the $f_1$ Landau parameter of a Fermi liquid \[66, 50, 51\]. A detailed discussion about different kinds of effective masses can be found in Ref. 80.

With the single-particle potential in Eq. (2), since the momentum-dependent part of the nuclear potential is independent of the parameter $x$, the nucleon effective masses are independent of the $x$ parameter too. The neutron and proton effective masses are usually different in asymmetric nuclear matter due to the fact that the momentum dependence of the single-particle potential is different for neutrons and protons in asymmetric nuclear matter. The isospin-splitting of nucleon effective mass in asymmetric nuclear matter, i.e., the difference between the neutron and proton effective masses is currently not known empirically \[88\]. We note that theoretically the neutron-proton effective mass splitting is still highly controversial within different approaches and/or using different nuclear effective interactions \[64, 71, 72, 73\]. Being phenomenological and non-relativistic in nature, the neutron-proton effective mass splitting in the MDI interaction is consistent with predictions of all non-relativistic microscopic models, see, e.g., \[66, 70, 81\], and the non-relativistic limit of microscopic relativistic many-body theories, see, e.g., Refs. \[62, 64, 65\]. Recent transport model studies indicate that the neutron/proton ratio at high transverse momenta and/or rapidities is a potentially useful probe of the neutron-proton effective mass splitting in neutron-rich matter \[51, 74\]. Since the momentum dependence of the single-particle potential is usually temperature dependent, it is thus interesting to see the temperature effect on the nucleon effective mass.

In the upper panels of Fig. [9] we show the momentum dependence of nucleon effective mass in symmetric nuclear matter at $\rho = 0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ and $T = 0$, 10, 30 and 50 MeV in the MDI interaction with $x = 0$ and $x = -1$.

![FIG. 8: (Color online) Momentum dependence of the symmetry potential at $\rho = 0.1\rho_0$, $0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ and $T = 0$, 10, 30 and 50 MeV in the MDI interaction with $x = 0$ and $x = -1$.](image-url)
FIG. 9: (Color online) Momentum dependence of the effective mass of protons and neutrons in symmetric nuclear matter ($\delta = 0$, upper panels) and asymmetric nuclear matter ($\delta = 0.5$, middle panels) at $\rho = 0.5\rho_0$, $1.0\rho_0$ and $1.5\rho_0$ and $T = 0$, 10, 30 and 50 MeV in the MDI and eMDYI interactions. The corresponding results for the reduced isospin-splitting of the nucleon effective mass, i.e., $(m_n^* - m_p^*)/m$ in asymmetric nuclear matter ($\delta = 0.5$) are also shown in the lower panels.

In order to see the temperature effect on the isospin-splitting of the nucleon effective mass, we show the corresponding momentum dependence of the reduced isospin-splitting of the nucleon effective mass, i.e., $(m_n^* - m_p^*)/m$ in the lower panels of Fig. 9. It is indicated that the temperature effect on the isospin-splitting of the nucleon effective mass displays some complicated behaviors. At lower densities, the temperature effect seems to reduce the isospin-splitting of the nucleon effective mass for a fixed momentum. However, at higher densities, it depends on the momentum, i.e., at higher momenta, the temperature effect reduces the isospin-splitting of the nucleon effective mass while it increases the isospin-splitting at lower momenta. These features reflect the complexity of the temperature effects on the momentum dependence of the neutron and proton single-particle potential in hot asymmetric nuclear matter for the MDI interaction. It should be mentioned that, at zero temperature, the nucleon effective mass is usually calculated at the Fermi momentum $p_F = P_f(\tau)$, and thus the isospin-splitting is generally a function of the density and the isospin asymmetry.

IV. MECHANICAL AND CHEMICAL INSTABILITIES

The mechanical and chemical instabilities of hot asymmetric nuclear matter have been extensively studied based on various theoretical models [37, 38, 43, 90, 91, 92, 93, 94, 95]. However, effects of the momentum-dependent interactions on the mechanical and chemical instabilities have received so far little theoretical attention. In the following, we study the mechanical and chemical instabilities using the MDI, MID, and eMDYI interactions and mainly focus on the effects of the isospin and momentum dependence of nuclear interactions.

A. Mechanical instability

The mechanical stability condition for a hot asymmetric nuclear matter is

$$\left( \frac{\partial P}{\partial \rho} \right)_{T,\delta} \geq 0. \quad (24)$$

If the above condition is not satisfied, any growth in density leads to the decrement of pressure. As the pressure is lower than its background, the nuclear matter will be compacted, which leads to the further growth of the density. In such way, any small density fluctuations can grow and the nuclear matter becomes mechanically unstable.

As an example to see the picture of the boundary of mechanical instability and the critical case, we show in Fig. 10 the isothermal line in the $P \sim \rho$ plane for asymmetric nuclear matter at $T = 10$ MeV and different values of the isospin asymmetry $\delta$ using the MDI interaction with $x = 0$. For smaller isospin asymmetries, the mechanical stability condition can be violated (below the dotted line). When $\delta$ is higher than the critical value (about 0.59 in this case), the pressure increases monotonically with density and Eq. (24) is satisfied for all densities. In the critical case, we have

$$\left( \frac{\partial P}{\partial \rho} \right)_{T,\delta_c} = \left( \frac{\partial^2 P}{\partial \rho^2} \right)_{T,\delta_c} = 0, \quad (25)$$
where $\delta_c$ is the critical isospin asymmetry. Below the critical isospin asymmetry, the extrema of the $P \sim \rho$ curve at different isospin asymmetries form the boundary of the mechanical instability region, namely, the isothermal spinodal (ITS).

Figs. 11 and 12 display the boundary of mechanical instability, i.e., ITS for the MDI, MID and eMDYI interactions at $T = 5, 10$ and $15$ MeV for the MDI, MID and eMDYI interactions with $x = 0$ and $x = -1$.

Figs. 11 and 12 display the same curves in the $P \sim \rho$ plane at $T = 5, 10$ and $15$ MeV with $x = 0$ and $x = -1$ in the $\rho \sim \delta$ plane while Figs. 13 display the same curves in the $P \sim \rho$ plane with the inclusion of the curves at constant isospin asymmetries of $\delta = 0, 0.5, 1$ and $\delta_c$. From Fig. 11 and 12 one sees that nuclear matter in the left region of the boundary of mechanical instability indicated by ITS is mechanically unstable, and the critical isospin asymmetry as well as the area of the mechanical instability region decreases with increasing temperature. For each interaction, the boundaries overlap at $\delta = 0$ for different values of $x$, since for symmetric nuclear matter the three interactions are independent of the value of $x$. For the MDI and eMDYI interactions, the ITS has the same value at $\delta = 0$, as for symmetric nuclear matter they are exactly the same model as mentioned in Section II while for MID interaction it is shifted to smaller densities at $\delta = 0$. The critical isospin asymmetry is seen to be sensitive to the density dependence of the symmetry energy, which can be seen more clearly from Figs. 13 and 15 where the value of $\delta_c$ is indicated exactly by the dash-dotted lines. At $T = 5$ and $10$ MeV the critical isospin asymmetry is larger for $x = -1$ than for $x = 0$, while at $T = 15$ MeV it is smaller for $x = -1$ than $x = 0$. These phenomena indicate that the density dependence of nuclear symmetry energy and the temperature are two important factors to determine the critical isospin asymmetry and
the area of mechanical instability. In addition, it is seen from Figs. 11 and 12 that the critical isospin asymmetry and the area of mechanical instability are also sensitive to the isospin and momentum dependence of the nuclear interaction, especially at higher temperatures. Detailed comparison indicates that the critical isospin asymmetry of the MDI interaction is very similar to that of the MID interaction at low and moderate temperatures, while it is similar to that of the eMDYI interaction at high temperatures. Meanwhile, the area of the mechanically unstable region is seen to be the largest for the MID interaction while the smallest for the eMDYI interaction.

In the above calculations, we fixed the temperature while varied the isospin asymmetry and thus got the critical isospin asymmetry. In Fig. 16 we fix the isospin asymmetry at $\delta = 0$ and $\delta = 0.5$, respectively, and change the temperature. The case of MDI interaction with $x = 0$ is shown as an example. It is clearly shown that increasing the temperature at fixed isospin asymmetry is just like increasing the isospin asymmetry at fixed temperature, and the mechanical stability condition Eq. (24) is satisfied in all the densities when the temperature is larger than a critical temperature $T_c$ (about 15.6 MeV at $\delta = 0$ and 11.7 MeV at $\delta = 0.5$). The density at the inflection point, which satisfies

$$\left( \frac{\partial P}{\partial \rho} \right)_{T_c,\delta} = \left( \frac{\partial^2 P}{\partial \rho^2} \right)_{T_c,\delta} = 0,$$  \hspace{1cm} (26)$$
is the critical density $\rho_c$, and the pressure at the inflection point is named after as the inflection pressure $P_i$. In Fig. 17 we show the isospin asymmetry dependence of the critical temperature $T_c$, the critical density $\rho_c$, the inflection pressure $P_i$ for the MDI, MID and eMDYI interactions with $x = 0$ and $x = -1$. It is indicated that the critical temperature, critical density and the inflection pressure decrease with increasing isospin asymmetry. Below the curves the system can be mechanically unstable. The critical temperature for symmetric nuclear matter is 15.6 MeV for the MDI and eMDYI interactions and 16.2 MeV for the MID interaction with both $x = 0$ and $x = -1$. For $x = 0$ the system is stable above a certain high isospin asymmetry (0.9 for MDI and MID model and 0.84 for eMDYI model), but for $x = -1$ it can be mechanically unstable even for pure neutron matter. These features indicate again that the boundary of mechanical instability is quite sensitive to the value of $x$.

Furthermore, it is clearly seen from Fig. 17 that the MDI interaction is similar to the MID interaction at low temperatures, while it is similar to the eMDYI interaction at high temperatures.

To see more clearly the effect of the density dependence of the symmetry energy on the critical temperature, critical density and inflection pressure, we show separately their isospin asymmetry dependence for each interaction with $x = 0$ and $x = -1$ in Fig. 18. In each case the critical temperature for $x = 0$ seems to be a little higher.
isospin asymmetry inequality in Eq. (27) is violated. For example, if the isospin asymmetry $\delta$ has a small growth in the region of $x = 0$ and $x = -1$, the system becomes chemical instable if either of the inequalities in Eq. (27) is not satisfied.

\[
\left( \frac{\partial \mu_0}{\partial \delta} \right)_{P,T} > 0 \quad \text{and} \quad \left( \frac{\partial \mu_p}{\partial \delta} \right)_{P,T} < 0, \quad (27)
\]

and the system becomes chemical instable if either of the inequalities in Eq. (27) is violated. For example, if the isospin asymmetry $\delta$ has a small growth in the region of chemical instability, it will even grow since more neutrons will move into the system from the background, because the low neutron chemical potential will lower the total energy of the whole system. This also holds true for the case of protons. So any isospin fluctuations will make the system unstable if either of the inequalities in Eq. (27) is not satisfied.

To analyze the chemical instability, we calculate the chemical potential isobar for neutrons and protons at fixed temperature and pressure. We do this by searching for the cross point between the fixed pressure line and the $P \sim \rho$ curves with fixed isospin asymmetry. In Fig. 19 we can calculate the densities and the chemical potentials of the cross points, just for one isospin asymmetry at fixed pressure and temperature. By changing the isospin asymmetry from 0 to 1 thus get the whole chemical potential isobar at a fixed temperature and pressure. We note that for different regions of pressure, the number of the cross points can be one, two or three for a fixed isospin asymmetry, which will be reflected in the shape of the resulting chemical potential isobar. The critical isospin asymmetry of mechanical instability is 0.59 for the MDI interaction at $T = 10$ MeV with $x = 0$ and the corresponding curve in $P \sim \rho$ plane is plotted by dotted line in Fig. 19. The pressure of the inflection point is 0.121 MeV, above which the mechanical instability disappears and only exists the chemical instability, and the chemical potential isobar can only be one branch for all values of $\delta$.

Fig. 20 displays the chemical potential isobar calculated at the pressure of $P = -0.05, 0.03, 0.10, 0.121, 0.20$ and 0.265 MeV/fm$^3$. One sees that the shape of the curves is different for different pressures. The extrema of curves are just the boundary of the chemical instability, or diffusive spinodals (DS), which is indicated in Fig. 20 by solid circles. $P = 0.265$ MeV/fm$^3$ is the critical pressure $P_c$ in the case of MDI interaction at $T = 10$ MeV.
with \( x = 0 \), above which the chemical potential of neutrons (protons) increases (decreases) monotonically with \( \delta \) and the chemical instability disappears. The inflection point which satisfies

\[
\left( \frac{\partial \mu^L}{\partial \delta} \right)_{P_c,T} = \left( \frac{\partial^2 \mu^L}{\partial \delta^2} \right)_{P_c,T} = 0 \]  

(28)

is also plotted in the figure. We note here that the extrema of \( \mu_n \) and \( \mu_p \) correspond to the same \( \delta \) value for the MDI and MID interaction and thus the critical pressure is achieved simultaneously for neutrons and protons, but for the eMDY1 interaction the chemical potential isobar shows an asynchronous behavior for neutrons and protons, as will be shown in the following. This asynchronous behavior is also different for different temperatures and values of \( x \). [25]

The diffusive spinodals for the MDI, MID and eMDY1 interactions with \( x = 0 \) and \( x = -1 \) at \( T = 5, 10 \) and 15 MeV are also shown in Fig. [11] and [12] in the \( \rho \sim \delta \) plane, and in Figs. [13], [14] and [15] in the \( P \sim \rho \) plane. It is clearly shown that the diffusive spinodal envelops the region of mechanical instability and extends further out into the plane and the area of chemical instability region decreases with increasing temperature. It should be noted that the region of chemical instability is between the curves of ITS and DS. At \( \delta = 0 \), the DS and ITS overlap for the MDI and eMDY1 interaction, and the curves with \( x = 0 \) and \( x = -1 \) overlap as well. The boundary of chemical instability is seen to be sensitive to the density dependence of the symmetry energy. At \( T = 5 \) MeV the maximum isospin asymmetry is larger for \( x = -1 \) than \( x = 0 \), while at \( T = 10 \) and 15 MeV it is smaller for \( x = -1 \) than \( x = 0 \). Furthermore, it is seen from Figs. [11] and [12] that the critical isospin asymmetry and the area of chemical instability are also sensitive to the isospin and momentum dependence of the nuclear interaction, especially at higher temperatures. The maximum \( \delta \) value in the MDI interaction is similar to that in the MID interaction at low temperature, but it becomes similar to that in the eMDY1 interaction at high temperature. This feature also holds in the case of mechanical instability as discussed before. In addition, the shape of the DS curve in the eMDY1 interaction with \( x = -1 \) at \( T = 5 \) MeV observed in Figs. [11], [12] and [13] exhibits some unusual behaviors and this is due to the asynchronous behavior of the chemical potential isobar for neutrons and protons.

V. LIQUID-GAS PHASE TRANSITION

A. Chemical potential isobar

With the above theoretical models, we can now study the LG phase transition in hot asymmetric nuclear matter. The phase coexistence is governed by the Gibbs conditions and for the asymmetric nuclear matter two-phase coexistence equations are

\[
P^L(T, \rho^L, \delta^L) = P^G(T, \rho^G, \delta^G), \tag{29}
\]

\[
\mu^L_n(T, \rho^L, \delta^L) = \mu^G_n(T, \rho^G, \delta^G), \tag{30}
\]

\[
\mu^L_p(T, \rho^L, \delta^L) = \mu^G_p(T, \rho^G, \delta^G), \tag{31}
\]

where \( L \) and \( G \) stand for liquid phase and gas phase, respectively. The Gibbs conditions [29], [30] and [31] for phase equilibrium require equal pressures and chemical potentials for two phases with different concentrations and isospin asymmetries. For a fixed pressure, the two solutions thus form the edges of a rectangle in the proton and neutron chemical potential isobars as a function of isospin asymmetry \( \delta \) and can be found by means of the geometrical construction method [25, 37, 40].

The way to calculate the chemical potential isobar is already explained in above, and here we take the case of \( T = 10 \) MeV as an example to show how we study the LG phase transition from the chemical potential isobar. The solid curves shown in Fig. [21] are the proton and neutron chemical potential isobars as a function of the isospin asymmetry \( \delta \) at a fixed temperature \( T = 10 \) MeV and pressure \( P = 0.090 \) MeV/fm\(^3\) by using the MDI and MID interactions with \( x = 0 \). The resulting rectangles from the geometrical construction are also shown by dotted lines in Fig. [24] from which one can see that different interactions give different shapes for the chemical potential isobar. When the pressure increases and approaches the critical pressure \( P_c \), an inflection point will appear for proton and neutron chemical potential isobars, i.e.,

\[
\left( \frac{\partial \mu}{\partial \delta} \right)_{P_c,T} = \left( \frac{\partial^2 \mu}{\partial \delta^2} \right)_{P_c,T} = 0. \tag{32}
\]

Above the critical pressure, the chemical potential of neutrons (protons) increases (decreases) monotonically with \( \delta \) and the chemical instability disappears. In Fig. [21] we also show the chemical potential isobar at the critical pressure by the dashed curves. At the critical pressure,
the rectangle is degenerated to a line vertical to the $\delta$ axis as shown by dash-dotted lines in Fig. 21. The values of the critical pressure are 0.265 and 0.230 MeV/fm$^3$, respectively, for the MDI and MID interaction with $x = 0$. It is interesting to see that the different interactions give different values of the critical pressure. We note that critical pressure is also sensitive to the density dependence of nuclear symmetry energy with the stiffer symmetry energy ($x = -1$) gives a smaller critical pressure 24.

With the eMDYI interaction, the resulting single nucleon potential is momentum dependent but its momentum dependence is isospin-independent. Comparing the results with those of the MDI interaction, we can extract information about the effects of the momentum dependence of the symmetry potential while the effects of the momentum dependence of the isoscalar part of the single nucleon potential can be studied by comparing the results with those of the MID interaction. Shown in Fig. 22 is the chemical potential isobar as a function of the isospin asymmetry $\delta$ at $T = 10$ MeV by using the eMDYI interaction with $x = 0$. Compared with the results from the MDI and MID interactions, the main difference is that the left (and right) extrema of $\mu_n$ and $\mu_p$ do not correspond to the same $\delta$ but they do for the MDI and MID interactions as shown in Fig. 21. In particular, for the eMDYI interaction, the chemical potentials of protons and neutrons are seen to exhibit asynchronous variation with pressure. We note that this behavior depends on the temperature and the value of $x$. At $T = 10$ MeV the chemical potential of neutrons increases more rapidly with pressure than that of protons. At $T = 5$ MeV the chemical potential of neutrons increases more rapidly with $x = 0$ but vice versa with $x = -1$, while at $T = 15$ MeV the asynchronous behavior seems not quite obvious. This asynchronous variation is uniquely determined by the special momentum dependence in the eMDYI interaction within the present self-consistent thermal model. Actually, it is this asynchronous variation that leads to the fact that the left (and right) extrema of $\mu_n$ and $\mu_p$ correspond to different values of $\delta$.

At lower pressures, for example, $P = 0.090$ MeV/fm$^3$ as shown in Fig. 22 (a), the rectangle can be accurately constructed and thus the Gibbs conditions (29), (30) and (31) have two solutions. Due to the asynchronous variation of $\mu_n$ and $\mu_p$ with pressure, we will get a limiting pressure $P_{lim}$ above which no rectangle can be constructed and the coexistence equations (29), (30) and (31) have no solution. Fig. 22 (b) shows the case at the limiting pressure with $P_{lim} = 0.205$ MeV/fm$^3$ for $x = 0$. In this limit case, we note that the left edge of the rectangle actually corresponds to the left extremum of $\mu_p$. With increasing pressure, namely, at $P = 0.260$, $\mu_n$ passes through an inflection point while $\mu_p$ still has a chemically unstable region and this case is shown in Fig. 22 (c). When the pressure is further increased to $P = 0.305$ MeV/fm$^3$, as shown in Fig. 22 (d), $\mu_p$ passes through an inflection point while $\mu_n$ increases monotonically with $\delta$. We note that the asynchronous variation of $\mu_n$ and $\mu_p$ with pressure also depends on the value of $x$. 23.

B. Binodal surface

For each interaction, the two different values of $\delta$ correspond to two different phases with different densities and the lower density phase (with larger $\delta$ value) defines
a gas phase while the higher density phase (with smaller \( \delta \) value) defines a liquid phase. Collecting all such pairs of \( \delta(T, P) \) and \( \delta'(T, P) \) thus forms the binodal surface. Fig. 23 displays the binodal surface for the MDI, MID and eMDYI interactions at \( T = 5, 10 \) and \( 15 \) MeV with \( x = 0 \) and \( x = -1 \) in the \( P \sim \delta \) plane. As expected, for MDI and MID interactions the binodal surface has a critical pressure, while for the eMDYI interaction the binodal surface is cut off by a limit pressure. Above the critical pressure or below the pressure of equal concentration (EC) point no phase-coexistence region can exist. The EC point indicates the special case that symmetric nuclear matter with equal density coexists, which is called “indifferent equilibrium” \cite{37}. The maximal asymmetry (MA) also plays an important role in LG phase transition. The left side of the binodal surface is the region of liquid phase and the right side the region of gas phase, and within the surface is the phase-coexistence region.

Now let’s see what affects the critical or limit pressure and the region of phase-coexistence. The critical pressure is sensitive to the stiffness of the symmetry energy, with a softer symmetry energy (with \( x = 0 \)) gives a higher critical pressure and a larger area of phase-coexistence region. For the case of limit pressure with eMDYI interaction this holds true at \( T = 10 \) MeV and \( T = 15 \) MeV, but the opposite result is obtained at \( T = 5 \) MeV. For symmetric nuclear matter the different values of \( x \) give the same EC point. As the MDI and eMDYI interactions are the same for symmetric nuclear matter, they have the same EC point, but for the MID interaction the EC point is lower, and the amount seems to increase with increasing temperature. Below the limit pressure the binodal surface is quite similar for the MDI and eMDYI interactions. Comparing the MDI and MID interactions, the isospin and momentum dependence seems to increase the critical pressure in a larger amount. At \( T = 5 \) MeV and \( T = 10 \) MeV, the area of phase-coexistence region for the MDI interaction is larger than that for the MID interaction, but at \( T = 15 \) MeV the opposite result is observed. The critical or limit pressure seems not to change monotonically with temperature, but it is clear that the area of phase-coexistence region decreases with increasing temperature and the pressure of EC point increases with increasing temperature.

FIG. 23: (Color online) The binodal surface at \( T = 5, 10 \) and \( 15 \) MeV in the MDI, MID and eMDYI interactions with \( x = 0 \) and \( x = -1 \). The critical pressure (CP), the limiting pressure (LP), the points of equal concentration (EC) and maximal asymmetry (MA) are also indicated.

FIG. 24: (Color online) The phase coexistence boundary (CE) in \( P \sim \rho \) plane for the MDI, MID and eMDYI interactions with \( x = 0 \) and \( x = -1 \) at \( T = 5 \) MeV. The isothermal spinodals (ITS) and diffusive spinodals (DS) are also shown for comparison.

FIG. 25: (Color online) Same as Fig. 24 but for \( T = 10 \) MeV.

Fig. 24\,25\,26 display the same curves (the phase coexistence boundary (CE)) in \( P \sim \rho \) plain, and the isothermal spinodals (ITS) and diffusive spinodals (DS) are also included for comparison. For MDI and MID interactions the critical pressure is the same for chemical instability and binodal surface. For the eMDYI interaction as the binodal surface is cut off by the limit pressure, the phase-coexistence region can not extend the region of chemical instability.
C. Maxwell construction

The binodal surface we show in the previous section provide much information about the LG phase transition. As discussed in Ref. [37], we can analyze the process of LG phase transition in hot asymmetric nuclear matter by Maxwell construction. We take the case of MDI interaction with \( x = -1 \) at \( T = 10 \) MeV as an example.

In the left panel of Fig. 27 the system is compressed at a fixed total isospin asymmetry \( \delta = 0.5 \). The system begins from gas phase, and encountered the two-phase region at the point A. Then a liquid phase with higher density emerges from the point B, with infinitesimal proportion. As the system is compressed, the gas phase evolves from A to D, while the liquid phase evolves from B to C. In this process the gas phase and the liquid phase coexist and the proportion of each phase changes, but the total isospin asymmetry is fixed. At the point C the system totally changes from gas phase to liquid phase and leaves the phase-coexistence region.

We can analyze this process in the phase-coexistence region by solving the following equations

\[
\lambda \delta L \rho L + (1 - \lambda) \delta G \rho G = \delta \rho, \quad (33)
\]

\[
\lambda \rho L + (1 - \lambda) \rho G = \rho, \quad (34)
\]

where \( \delta^{L(G)} \) and \( \rho^{L(G)} \) are the isospin asymmetry and density of liquid (gas) phase. The total isospin asymmetry \( \delta \) in this case is 0.5. The fraction of the liquid phase \( \lambda \) and the total density \( \rho \), from which Maxwell construction can be produced, can be obtained by solving the above equations. The corresponding isotherms are drawn in the left panel of Fig. 28. The dotted line connecting A and C obtained by direct calculation is unphysical. The nearly straight line connecting A and C is produced by Maxwell construction and corresponds to the realistic process. The fraction of the liquid phase \( \lambda \) from A to C is also shown in the inset and it changes monotonically from 0 to 1.

The geometry of the binodal surface offers a second possibility for the LG phase transition process. The situation is displayed in the right panel of Fig. 27, where the system is compressed at fixed total isospin asymmetry \( \delta = 0.7 \), which is larger than the isospin asymmetry of the CP point. The system begins from gas phase, and encountered the two-phase region at the point \( A' \). Then a liquid phase with infinitesimal fraction emerges from the point \( B' \). As the system is compressed, the gas phase evolves from \( A' \) to \( C' \), while the liquid phase
evolves from $B'$ to $D'$. The system crosses the phase-coexistence region, but at the point $C'$ it remains at gas phase and leaves the binodal surface on the same branch. The corresponding isotherms are shown in the right panel of Fig. 28. The solid line rather than the dotted one connecting $A'$ and $C'$ corresponds to the real process of LG phase transition. In this case the fraction of the liquid phase $\lambda$ increases from 0 to $\lambda_{\text{max}}$ (about 0.13) and then drops to 0 again.

**D. Order of LG phase transition**

In the following we consider the order of LG phase transition and focus on the realistic MDI interaction by observing the behavior of thermodynamical quantities under fixed pressure. Here we use the pressure of $P = 0.05$ MeV/fm$^3$ and we note that there are no qualitative changes if other pressures below the critical pressure are used. We choose a relatively low pressure just to see more clearly the effects of the phase transition on the thermodynamical quantities. In the upper panels in Fig. 29 we show the evolution of entropy per nucleon with temperature at fixed pressure $0.05$ MeV/fm$^3$ for isospin asymmetries of $\delta = 0$ and 0.5 using the MDI interaction with $x = 0$ and $x = -1$ respectively. The method is quite similar to calculating the chemical potential isobar above, so we do not go into the details about the calculation method. In the upper panels in Fig. 29 the dashed line is obtained by direct calculation and it is unphysical, while the solid line corresponds to the realistic process and is obtained by Maxwell construction. We calculate the chemical potential isobar at every temperature of the phase-coexistence region under fixed pressure, then find the densities and isospin asymmetries of the coexistence phase from Gibbs conditions, and thus obtain the fraction of each phase by using Eqs. (33) and (34). The total entropy per nucleon $S$ in the coexistence phase is then calculated from

$$S(\rho, \delta, T) = \lambda S^L(\rho^L, \delta^L, T) + (1-\lambda) S^G(\rho^G, \delta^G, T),$$

where $S^{L(G)}$ can be obtained from $\rho^{L(G)}$ and $\sigma^{L(G)}$ by using Eq. (17). From the upper panels in Fig. 29 we can see that at $\delta = 0$ the entropy jumps at $T = 10.1$ MeV, which clearly indicates that the LG phase transition for symmetric nuclear matter under the pressure of $0.05$ MeV/fm$^3$ (which is below the critical pressure) is of first order. The transition temperature is $T_c = 10.1$ MeV, and we note that this value depends on the fixed pressure used. On the other hand, the curves of $\delta = 0.5$ is continuous. To be more specific, we calculate the heat capacity per nucleon under fixed pressure from

$$C_p(T) = T \left( \frac{\partial S}{\partial T} \right)_{p, \delta},$$

where the entropy per nucleon is obtained from Eq. (35). The lower panels in Fig. 29 display the heat capacity per nucleon as a function of temperature under fixed pressure $0.05$ MeV/fm$^3$ for isospin asymmetries of $\delta = 0$ and 0.5 using the MDI interaction with $x = 0$ and $x = -1$ respectively. For both cases of $x = 0$ and $x = -1$ at $\delta = 0.5$, the heat capacity is continuous but its first derivative is not continuous, which indicates that the LG phase transition for asymmetric nuclear matter is of second order according to Ehrenfest’s definition of phase transitions [9]. These results are consistent with those in Ref. [37] with a different model. Although here we only consider the MDI interaction, we note that the order of LG phase transition does not depend on the isospin and momentum dependence of the nuclear interaction.

**VI. SUMMARY**

Within a self-consistent thermal model we have studied in detail and systematically effects of the isospin and momentum dependent interactions on thermal properties of asymmetric nuclear matter. We put the special emphasis on the temperature dependence of the isospin-dependent bulk and single-particle properties, the mechanical and chemical instabilities, and the LG phase transition of the hot asymmetric nuclear matter. In our analyses we used the isospin and momentum dependent MDI interaction constrained by the isospin diffusion data in heavy-ion collisions, the momentum-independent MID interaction, and an isoscalar momentum-dependent eMDYI interaction. Our results indicate that the EOS and the symmetry energy are quite similar for the three interactions at finite temperature, which implies that their temperature dependence is not so sensitive to the momentum dependence of the interaction. In particular, the symmetry energy at fixed density is found to generally de-
crease with temperature for the three interactions. For the MDI interaction, both the kinetic and potential parts of the symmetry energy are temperature dependent and the decrement of the symmetry energy with temperature is essentially due to the decrement of the potential energy part of the symmetry energy with temperature. On the other hand, for the MID and eMDYI interactions, the decrement of the symmetry energy with temperature is only due to the kinetic part of the symmetry energy since the potential part does not depend on the temperature. Compared with the MID and eMDYI interactions, the single-particle potential of the MDI interaction is isospin and momentum dependent, which leads to the fact that the symmetry potential is momentum dependent and the nucleon effective mass is splitted in asymmetric nuclear matter for the MDI interaction. It is further shown that only the low momentum part of the single-particle potential and the nucleon effective mass is significantly lifted with increasing temperature for the MDI and eMDYI models. For the MDI interaction, the low momentum part of the symmetry potential is significantly reduced with increasing temperature.

We have also analyzed the boundaries of both mechanical and chemical instabilities. We found that the area of both mechanical and chemical instabilities generally decreases with increasing temperature. Meanwhile, the boundaries are shown to be sensitive to the density dependence of nuclear symmetry energy. In the case of mechanical instability the critical isospin asymmetry is larger for stiffer symmetry energies (e.g., $x = -1$) at low and moderate temperatures and vice versa at high temperatures. While for the chemical instability the maximum asymmetry is larger for stiffer symmetry energies at low temperatures and vice versa at high and moderate temperatures. Furthermore, it is indicated that the mechanical and chemical instabilities are also sensitive to the isospin and momentum dependence of the nuclear interaction, especially at higher temperatures. The boundaries obtained with the MDI interaction are similar to those obtained with the MID interaction at low temperatures and with the eMDYI interaction at high temperatures.

Finally, we have explored in detail the effects of isospin and momentum dependence of the nuclear interaction on the liquid-gas phase transition in hot asymmetric nuclear matter. The boundary of the phase-coexistence region is shown to be sensitive to the density dependence of the nuclear symmetry energy with a softer symmetry energy giving a higher critical pressure and a larger area of phase-coexistence region. The critical pressure and the area of phase-coexistence region is also quite sensitive to the isospin and momentum dependence of the nuclear interaction by comparing the cases of MDI and MID interactions. For the eMDYI interaction, a limiting pressure above which the LG phase transition cannot take place has been found and it is shown to be sensitive to the stiffness of the symmetry energy as well. Furthermore, the area of phase-coexistence region decreases with increasing temperature, and the pressure of the EC point increases with increasing temperature for all of the three interactions considered here. The phase transition process and the order of phase transition have also been analyzed by using the Maxwell construction. Our results indicate that the LG phase transition for symmetric nuclear matter is of first order but it becomes second order for the asymmetric nuclear matter.

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