A simplified approach for Chaplygin-type cosmologies

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A new class of accelerating cosmological models driven by a one-parameter version of the general Chaplygin-type equation of state is proposed. The simplified version is naturally obtained from causality considerations with basis on the adiabatic sound speed $v_s$ plus the observed accelerating stage of the universe. We show that very stringent constraints on the unique free parameter $\alpha$ describing the simplified Chaplygin model can be obtained from a joint analysis involving the latest SNe type Ia data and the recent Sloan Digital Sky Survey measurement of baryon acoustic oscillations (BAO). In our analysis we have considered separately the SNe type Ia gold sample measured by Riess et al. (2004) and the Supernova Legacy Survey (SNLS) from Astier et al. (2006). At 95.4% (c.l.), we find for BAO + gold sample, $0.91 \leq \alpha \leq 1.0$ and $\Omega_M = 0.28^{+0.043}_{-0.048}$ while BAO + SNLS analysis provides $0.94 \leq \alpha \leq 1.0$ and $\Omega_M = 0.27^{+0.048}_{-0.045}$.

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I. INTRODUCTION

The impressive convergence of recent observational facts along with some apparently successful theoretical predictions seem to indicate that the simple approach provided by the standard cold dark matter (CDM) model is insufficient to describe the present stage of our universe. From these results, the most plausible picture for our world seems to be a nearly flat scenario dominated basically by CDM and an exotic component endowed with large negative pressure, usually named dark energy. Despite the good observational indications for the existence of these two components, their physical properties constitute a completely open question at present, which gives rise to the so-called dark matter and dark energy problems (see [1] for a recent review on this topic).

Among the many candidates for the dark energy component, a very interesting one was suggested by Kamenshchik et al. [2] and developed by Bilić et al. [3] and Bento et al. [4]. Such an exotic fluid, named generalized Chaplygin gas (C-gas), can be macroscopically characterized by the equation of state (EoS)

$$p_C = -\frac{A}{\rho_C^\alpha},$$

where $\alpha = 1$ and $A$ is a positive constant related to the present-day Chaplygin adiabatic sound speed, $v_s^2 = \frac{\alpha A}{\rho_C^{1+\alpha}}$ ($\rho_C$ stands for the current C-gas density). In actual fact, the above equation for $\alpha \neq 1$ constitutes a generalization of the original C-gas EoS proposed by Bento et al. in Ref. [4].

In the last few years, the possibility of describing the unknown dark energy component using the C-gas-type EoS above has provoked a considerable debate in the literature. Theoretical connections between the C-gas and string theory, supersymmetric generalizations, self-interacting, and even a tachyonic fluid representation has also been investigated. Another interesting feature of the above EoS comes from the fact that the C-gas becomes pressureless at high redshifts, which suggests a possible unification scheme for the cosmological “dark sector”, an interesting idea which has been considered in different contexts [5].

Observational aspects of the above C-gas scenarios have also been largely investigated in the literature. Cosmological tests involving type Ia supernovae (SNe Ia) data [6–11], the shape of the matter power spectrum [12], statistical properties of gravitational lenses [13], the age of the Universe [14], cosmic microwave background (CMB) measurements [15–18], galaxy clusters X-ray [19], and gamma-ray bursts data [20] have been discussed. In general, to perform such analyses, besides the present value of the C-gas density parameter ($\Omega_C$), the above barotropic EoS implies that one needs to constrain two additional free parameters, namely, $A$ and $\alpha$. Therefore, in the context of the Friedman-Robertson-Walker (FRW) cosmologies with CDM plus a C-gas, there are at least 4 parameters to be constrained by the data. Actually, this number can be reduced to 3 if one assumes a flat geometry, i.e., $\Omega_M = 1 - \Omega_C$ or if a unified dark matter/energy picture involving only the C-gas and baryons is assumed from the very beginning (in this case, the baryonic density ($\Omega_b$) may be fixed a priori by using, for instance, nucleosynthesis [22] or the recent Cosmic Microwave Background (CMB) observations [23]). However, even in this latter cases, there are so many parameters to be constrained by the data, that a high degree of degeneracy on the parametric space becomes inevitable.
Many generalizations of the original C-gas \[24, 25, 26, 27, 28\], or even of its extended version \[29\] have appeared in literature. In these cases, the number of free parameters is usually increased, and, as consequence, the models become mathematically richer although much less predictive from a physical viewpoint. In this work by following the opposite direction, we propose a simplified version for the generalized C-gas-type EoS which diminishes one of its free parameters. By an additional physical condition, the allowed range of the remaining parameter is also restricted a priori, which makes not only the relevant parametric space bi-dimensional but also (and more important) the model more easily discarded or confirmed by the present set of observations since the range of its free parameter is physically limited from causality considerations. We test the viability of this simplified C-gas approach by discussing the constraints imposed from current SNe Ia observations and Large Scale Structure (LSS) data.

II. A SIMPLIFIED C-GAS SCENARIO

Let us consider a homogeneous and isotropic Universe whose energy components are cold dark matter plus the generalized C-gas fluid. Since both components are separately conserved, by inserting Eq. (1) into the energy conservation law \(\dot{\rho}_C = -3H(\rho_C + p_C)\), one obtains the following expression for the density of the C-gas \[4, 9, 19\]

\[
\rho_C = \rho_{C_0} \left[ A_s + (1 - A_s)a^{3(1+\alpha)} \right]^{\frac{1}{1+\alpha}},
\]

where \(a(t)\) is the cosmological scale factor and \(A_s = \frac{\alpha}{\rho_{C_0}^{1+\alpha}}\) is a convenient dimensionless constant (as usual, the subscript “0” denotes present-day quantities). As one may check, the above C-gas evolving in the FRW metric can be modeled as a quintessence, that is, a scalar field model described by an ordinary Lagrangian density, \(\mathcal{L}_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\), with the following potential

\[
V(\phi) = \frac{1}{2}\rho_{C_0} A_s^{\frac{1}{1+\alpha}} \left[ \cosh \sqrt{6\pi} m_{pl}(\alpha + 1)\phi \right]^{\frac{1}{1+\alpha}}
\]

\[
+ \left[ \cosh \sqrt{6\pi} m_{pl}(\alpha + 1)\phi \right]^{-\frac{2}{1+\alpha}},
\]

where \(m_{pl}\) is the Planck mass.

In a flat geometry, the Friedmann equation for a conserved C-gas plus cold dark matter is given by \[19\]

\[
\mathcal{H} = \Omega_M a^{-3} + \Omega_C [A_s + (1 - A_s)a^{-3(\alpha+1)}]^{\frac{1}{1+\alpha}},
\]

where \(\mathcal{H} = \frac{H(a)^2}{H_0^2}\). Note that besides the Hubble parameter \(H_0\) we still have 3 additional parameters in this case (\(\alpha, A_s, \Omega_M\)), even using the flat condition \(\Omega_C = 1 - \Omega_M\). Therefore, an interesting question to be answered at this point is how to reduce the C-gas parameters based on reasonable physical constraints?

In order to answer the above question, we first notice that the dimensionless constant \(A_s\) appearing in the above expressions encodes the basic information coming from the original parameter \(A\) [see Eq. (1)]. On the other hand, the Chaplygin adiabatic sound speed reads

\[
v_s^2 = \frac{dp}{d\rho} = \alpha A/\rho_{C_0}^{1+\alpha},
\]

which must positive definite for a well-behaved gas (zero in the limit case of dust). Note also that the present day Chaplygin adiabatic sound speed is \(v_{so}^2 = \alpha A/\rho_{C_s}^{1+\alpha}\) or, equivalently,

\[
v_{so}^2 = \alpha A/\rho_{C_s}^{1+\alpha} = \alpha A_s.
\]

Therefore, if the \(A_s\) parameter is a function of \(\alpha\), the number of free parameters is naturally reduced, and, as an extra bonus, the positiviness of \(v_s^2\) at any time, as well as its thermodynamic stability, is naturally guaranteed. Among many possible relations (e.g., \(A_s = \alpha^n\), clearly the simplest choice is \(A_s = \alpha (n = 1)\). In this case, \(v_{so}^2 = \alpha^2\), or more generally, \(v_s^2 = \alpha^2(\rho_{C_0}/\rho)\). Note also that, since the light speed is a natural cutoff for the sound propagation, it follows that \(v_{so} = |\alpha| \leq 1\), thereby restricting \(\alpha\) to the interval \([-1,1]\). An additional constraint can still be imposed to this parameter. In fact, with \(A_s = \alpha\), the simplified C-gas EoS (1) becomes

\[
\rho_C = -\alpha\rho_{C_0} \left( \frac{\rho_{C_0}}{\rho_C} \right)^\alpha,
\]

so that a negative pressure is obtained only for positive values of \(\alpha\). In other words, this accounts to saying that the combined requirements from causality along with the observed accelerating stage of the Universe limit naturally the parameter \(\alpha\) to the interval \([0,1]\).

Note that the the simplified Chaplygin gas above (from now on SC-gas) preserves the unifying character of the original C-gas, i.e., it behaves as a pressureless fluid (non-relativistic matter) at high-z while, at late times, it approaches the quintessence behavior, which now is fully characterized by the \(\alpha\) parameter (for a unified dark matter (dark energy description of the above scenario, see \[21\]). However, note also that, even in this limiting case, the sound speed is positive. In other words, the Universe evolution resembles the one driven by a quintessence component but the thermodynamic behavior does not present the pathologies of such scenarios.

In this simplified approach, Eq. (4) is rewritten as

\[
\mathcal{H} = \Omega_M a^{-3} + \Omega_C [\alpha + (1 - \alpha)a^{-3(\alpha+1)}]^{\frac{1}{1+\alpha}},
\]

so that the parameter \(\alpha\) is actually the unique unknown constant related to this SC-gas model. In what follows, we confront this simplified approach with the most recent SNe Ia and Large Scale Structure (LSS) data.
FIG. 1: Supernova results. Panel (a) displays the residual magnitude with respect to an empty ($\Omega_T = 0$) universe for the HZST (filled circles) and SNLS (open circles) samples, respectively. The solid curves are the predictions for the SG-gas-type models characterized by ($\Omega_M, \alpha$). For comparison, we also display the predictions of the open cold dark matter (OCDM) and Einstein-de Sitter Universes. Panels (b) and (c) show 68.3%, 95.5% and 99.73% confidence contours on the space parameter ($\Omega_M, \alpha$) from the HZST and SNLS supernova data, respectively. The horizontal dotted and dashed lines correspond to the constraints arising from the SDSS baryon acoustic oscillations detection. For SNe Ia data alone, if $\alpha$ is greater than 0.91, values of $\Omega_M$ smaller than 0.15 are ruled out by the two samples. Constraints from BAO contribute to increase the allowed values of $\Omega_M$.

III. OBSERVATIONAL CONSTRAINTS

A. SNe Ia

Let us first investigate the bounds arising from SNe Ia observations on the SC-gas scenario described above. To this end we use the most recent SNe Ia observations, namely, the High-Z SN Search (HZS) Team \cite{30} and the Supernova Legacy Survey (SNLS) Collaboration data \cite{31}.

The so-called gold sample from the HZS team is a selection of 157 SNe Ia events distributed over the redshift interval $0.01 \lesssim z \lesssim 1.7$, and constitutes the compilation of the best observations made so far by them and by the Supernova Cosmology Project plus 16 new events observed by Hubble Space Telescope (HST). The current data from SNLS collaboration correspond to the first year results of its planned five year survey. The total sample includes 71 high-$z$ SNe Ia in the redshift range $0.2 \lesssim z \lesssim 1$ plus 44 low-$z$ SNe Ia. This data set is arguably (due to multi-band, rolling search technique and careful calibration) the best high-$z$ SNe Ia compilation to date, as indicated by the very tight scatter around the best fit in the Hubble diagram and a careful estimate of systematic uncertainties. Another important aspect to be emphasized on the SNLS data is that they seem to be in a better agreement with WMAP results than the gold sample (see, e.g., \cite{32} for a discussion). The two SNe Ia samples are illustrated on a residual Hubble Diagram with respect to the empty universe model ($\Omega_T = 0$) in Fig. 1a.

The predicted distance modulus for a supernova at redshift $z$, given a set of parameters $\mathbf{p}$, is

$$
\mu_p(z|\mathbf{p}) = m - M = 5\log d_L + 25, \quad (9)
$$

where $m$ and $M$ are, respectively, the apparent and absolute magnitudes, the complete set of parameters is $\mathbf{p} = (H_o, \Omega_M, \alpha)$ and $d_L$ stands for the luminosity distance (in units of megaparsecs),

$$
d_L = c(1 + z) \int_{x'}^1 \frac{dx}{x^2 H(x; \mathbf{p})}, \quad (10)
$$

with $x' = (1 + z)^{-1}$ being a convenient integration variable and $H(x; \mathbf{p})$ the expression given by Eq. (8).

We estimated the best fit to the set of parameters $\mathbf{p}$ by using a $\chi^2$ statistics

$$
\chi^2 = \sum_{i=1}^N \frac{[\mu_p(z|\mathbf{p}) - \mu_o(z|\mathbf{p})]^2}{\sigma_i^2}, \quad (11)
$$

with the parameters $\Omega_M$ and $\alpha$ spanning the interval $[0,1]$ in steps of 0.01. In the above expression, $N = 157$ and 115 for gold and SNLS samples, respectively, $\mu_o(z|\mathbf{p})$ is given by Eq. (9). $\mu_o(z|\mathbf{p})$ is the extinction corrected distance modulus for a given SNe Ia at $z_i$, and $\sigma_i$ is the
uncertainty in the individual distance moduli. In our analysis, \(H_0\) is considered a *nuisance* parameter so that we marginalize over it.

In Figures (1b) and (1c) we show the results of our statistical analysis. Contours of constant likelihood (99.73%, 95.4% and 68.3%) are shown in the parametric space \(\alpha - \Omega_M\). Panel (1b) displays the results for the HZS *gold* sample. Compared to Fig. 4 of Ref. [1], the parameter \(\alpha\) is now considerably more restricted than in the standard C-gas approach. In particular, note that for any value of the matter density parameter, models with \(\alpha \lesssim 0.63\) are ruled out at 99.73% level. The best-fit model for this analysis occurs for \(\Omega_M = 0.0\) and \(\alpha = 0.79\) with \(\chi^2_{\text{min}}/\nu = 1.13\) (\(\nu = \) degrees of freedom). At 95.4% c.l. we also find \(\Omega_M \lesssim 0.36\) and \(0.71 \leq \alpha \leq 1.0\). Panel (1c) shows a similar analysis for the SNLS data. The best-fit parameters in this case are \(\Omega_M \simeq 0.2\) and \(\alpha = 0.96\) with \(\chi^2_{\text{min}}/\nu = 1.0\). Note that, when compared with recent dynamical estimates of \(\Omega_M\), this latter value for the matter density parameter seems to be more realistic than the one provided by the *gold* sample analysis. The SNLS sample also imply \(\Omega_M \lesssim 0.34\) and \(0.75 \leq \alpha \leq 1.0\) at 95.4% (c.l.).

### B. SNe Ia + LSS analysis

The recent detection of a peak in the large scale correlation function at 100\(h^{-1}\) Mpc separation [38] provide not only a remarkable confirmation of the big bang cosmology but also a kind of “ruler” with which cosmological scenarios can be tested. The peak detected (from a sample of 46748 luminous red galaxies selected from the SDSS Main Sample) is predicted to arise precisely at the measured scale and is basically due to baryon acoustical oscillations (BAO) in the primordial baryon-photon plasma prior to recombination. Here, this measurement is characterized by

\[
A \equiv \frac{\Omega_M^{1/2}}{H(z_*)^{1/3}} \left[ \frac{1}{z_*} \Gamma(z_*) \right]^{2/3} = 0.469 \pm 0.017, \tag{12}
\]

where \(z_* = 0.35\) is the redshift at which the acoustic scale has been measured, and \(\Gamma(z_*)\) is the dimensionless comoving distance to \(z_*\).

The dotted lines in Figs. (1b) and (1c) represent the constraints from SDSS BAO measurements on the parameter space \(\Omega_M - \alpha\). Note that they are approximately orthogonal to those arising from SNe Ia data, which indicates that possible degeneracies in the \(\Omega_M - \alpha\) plane may be broken from a joint analysis involving these observational data sets. This is exactly what we show in Panels (2a) and (2b) for the BAO+*gold* and BAO+SNLS samples, respectively. Note that the available parametric plane in both cases is considerably reduced relative to the former analyses (Figs. 1b and 1c). Note also that, although compatible with the data, the region \(\alpha > 1\) (forbidden from thermodynamic stability and causality considerations) should be disregarded from the analysis since these arguments lead to the physical bound \(0 \leq \alpha \leq 1\). For the BAO+*gold* sample we find \(\Omega_M = 0.28^{+0.048}_{-0.034}\) and \(\alpha \geq 0.916\) (with the best fit \(\alpha = 0.98\)) at 95.4% (c.l.) while for the BAO+SNLS sample the best-fit model happens at \(\Omega_M = 0.27\) and \(\alpha = 1.0\). This latter best-fit scenario corresponds to an accelerating universe with \(q_0 \simeq -0.5\), a total age of the Universe of \(t_\text{o} \simeq 10.2h^{-1}\) Gyr, and a D/A redshift transition (from deceleration to acceleration) \(z_{\text{D/A}} \simeq 0.75\). At 95.4% c.l., the BAO+SNLS analysis also provides \(0.94 \leq \alpha \leq 1.0\) and \(\Omega_M = 0.27^{+0.045}_{-0.045}\).
IV. DISCUSSION AND CONCLUSIONS

As widely known, there are many theoretical approaches for describing the exotic dark energy component accelerating the Universe. However, until the present, the available battery of cosmological tests was not capable to decide which is the best theoretical representation. We have argued here that one of such candidates, the so-called Chaplygin type gas (whose equation of state depends on two parameters $A_{\alpha}$ and $\alpha$), may have a very simplified description. We postulate that $A_{\alpha}$ is a function of $\alpha$ and for simplicity we have taken $A_{\alpha} = \alpha$. Thus, similarly to the concordance model ($\Lambda$CDM), the resulting flat cosmology is completely described only by a pair of parameters ($\alpha$, $\Omega_M$). This SC-gas cosmology mimics the dynamics of the $X$-matter models with an extra bonus, namely: the fluid stability and other thermodynamic features are guaranteed from the very beginning.

By considering this particular parameterization we have investigated constraints on the $\alpha$ parameter from the most recent SNe Ia (gold and SNLS samples) and LSS data. We have found that the limits arising from this particular combination of the data are much more restrictive on this simplified approach than on the generalized C-gas version. In particular, for the the BAO+SNLS combination we found $0.94 \leq \alpha \leq 1.0$ and $\Omega_M = 0.27^{+0.04}_{-0.05}$ (at 95% c.l.), which is in agreement with recent estimates of the clustered matter. Naturally, it should be interesting to investigate whether current CMB data and other independent observations can or cannot discard the simplified scenario proposed here.

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