Students’ misconceptions on the algebraic prerequisites concept: operation of integer numbers and fractions

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Abstract. Numbers operation is one of the algebraic prerequisites concepts that students must understand before understanding the algebraic material. Understanding the wrong prerequisite concept can lead to misconceptions. This research aimed to describe students' misconceptions on the algebraic prerequisites concept, causative factors, and alternative solutions. This research was qualitative descriptive research using diagnostic test methods and interview. Three students who did the most and varied misconceptions in completing the diagnostic test were chosen to be the subject of this research. The results showed the subject misconceptions experienced on the algebraic prerequisites concept, namely the operation of integers and fractions. The misconceptions experienced by the subject occurred in the integer addition, integer subtraction, integer division, multiplication with zero numbers, fractions addition, and fractions division. The factors that cause misconception are preconception, associative thinking, incomplete or incorrect reasoning, wrong intuition, student ability. Alternative solutions to overcome the misconceptions are re-explain and cognitive conflict. The teacher must know the misconceptions, causative factors and alternative solutions to overcome students’ misconceptions on the algebraic prerequisites concept so that students will not experience misconceptions when understanding the algebraic material in the next stages.

1. Introduction
Algebra is one of the branches of mathematics that must be mastered by students from elementary to high school. A good algebraic understanding can help students understand other mathematical materials [1-4] and other subjects [1-3]. Algebra is considered a "gatekeeper" that can be a determinant of someone to succeed in understanding mathematical material [5-8].

In understanding algebra, the concept of algebraic prerequisites must first be understood. There are nine concepts of algebraic prerequisites, namely (1) numbers and numerical operations, (2) ratios and proportions, (3) the order of operations, (4) equality, (5) patterning, (6) algebraic symbolism and letter usage, (7) algebraic equations, (8) functions, and (9) graphing [9]. The concept of number operations has begun to be taught in elementary school. Mastery of the concept of number operations is the main prerequisite for understanding the algebraic concepts in the later stages.

Students' understanding of algebra is still low. Indonesia ranks 38th out of 42 countries participating in TIMSS 2011 [10]. 61% of students in Pennsylvania in 2011 scored below average in algebra, 64% of students in California in 2014 were less proficient in final algebra exams and 30% of students in Michigan in 2008 failed in algebra [7]. Students in South Africa are also lacking in algebra competence.
[1], so that globally students are still lacking in algebraic competencies which can be one of the main barriers for many students in learning mathematics [2, 11].

The lack of understanding of student concepts in algebra can lead to misconceptions. Misconception is an incorrect idea or view of a concept held by a person who is different from concepts that are considered true by experts. Misconception can be interpreted as an incorrect idea or view of a concept that someone has that is not in accordance with the concept that is considered right by experts [12, 13]. Misconception is the result of a lack of understanding or error in applying mathematical rules or generalizations [1, 14-17]. Misconception occurs when new knowledge is not in accordance with the knowledge previously taught [6, 18].

Misconceptions in algebra are still experienced by students. For example, misconceptions made by 7th graders in Indonesia in completing \((2 + x) = 180\) by means of \(x = \frac{180}{90} = 9\) and \(x - 9 = 13\) by means of \(13 = x + 9\) [2]. 8th grades in Malang complete \(\frac{1}{a} + \frac{2}{a} = \frac{4b}{ab}\) and \(8 + 4x = 12x\) [19]. Misconception was also carried out by 8th grades students in Semarang in completing \(2x + 3y = 5xy\) and \(\frac{2}{xy} + \frac{2x}{y} = \frac{4x}{xy}\) [20]. High school and college students in Ghana complete \(\frac{1}{2} - \frac{3}{2x} = \frac{1-3}{2}\) [21]. High school students in Kenya understand \(\frac{1}{3x} + \frac{2}{x} = 7x\) [22]. These findings indicate that students have not mastered the algebraic prerequisites concept, one of them is the operation of integers and fractions.

The misconception of integer operations is still widely found in students, such as misconceptions carried out by 7th grade students in Surakarta who understand \(-90 - 9 = 81\) [23]. 8th graders in Bangkalan Regency misconcept integer addition and subtraction operations namely \(-3 + (-10) = 13\), \(-15 + (-8) = 7\), \(-12 - 7 = -5\), \(-12 - 7 = 5\), \(10 + (-7) = 17\), \(8 - (-14) = 22\) and \(8 - (-14) = 6\) [24]. 7th graders in Palembang complete \(-24 - 8 = 16\), \(19 + (-6) = 25\) and \(-31 + (-8) = 39\) [25]. Misconception also occurs when students do multiplication with zero numbers. Students understand \(9 \times 0 \times 8\) as \(9 \times 8\) so that they get 72 results [26].

In addition, misconceptions were also found in fraction operations. 8th graders of junior high schools in Yogyakarta complete \(\frac{47}{25} + \frac{25}{25} = \frac{72}{25}\) and \(\frac{47}{25} + \frac{25}{25} = \frac{85}{25}\) [27]. 9th graders in Ohio, USA complete \(\frac{2}{3} + \frac{1}{6} = \frac{2}{3} + \frac{1}{6} = \frac{3}{3} \times \frac{1}{6} + \frac{1}{2} = \frac{14}{4} + \frac{3}{4} = \frac{17}{4}\) and \(\frac{20}{3} + \frac{3}{2} = \frac{20}{3} \times \frac{3}{2}\) [28]. 9th grade Brunei Darussalam students understand the fraction \(2\frac{1}{4}\) as \(2 + 4 + 1 = 7\), thus solving the problem \(2\frac{1}{4} \div 6 = 7 \div 6 = \frac{1}{6}\) and there are also students who think \(6 \div 1\) as the opposite of 6 so that it resolves the problem \(2\frac{1}{4} \div 6 = \frac{9}{6} \times \frac{6}{6} = \frac{27}{18} = \frac{13}{2}\) [29].

The misconceptions experienced by students can hinder the acceptance of new students' material and affect students' success in solving mathematical problems [6, 8, 30, 31]. Therefore, it is important for teachers to overcome the misconceptions that occur in students. The thing that can be done to overcome students' misconceptions is to find misconceptions experienced by students, causative factors and alternative solutions to overcome these misconceptions [12, 14-15, 21-22, 32].

One of the factors causing misconception is the factor that comes from the student itself, namely preconception, associative thinking, humanistic thinking, incomplete or incorrect reasoning, wrong intuition, cognitive developmental stages, ability and learning interest [12]. Strategy that most often carried out by teachers in overcoming misconceptions are re-explain and cognitive conflict [33]. Re-explain can be done again by explaining part of each concept or procedure that students have not understood. Cognitive conflict can encourage students to reevaluate the mistakes made because students can identify the contradictions of the mathematical principle between the original answers and the students' answers.

The algebraic prerequisites concept is a concept to understand before studying algebra matter. Students' misconceptions on the algebraic prerequisites concept is an idea or an incorrect view of the algebraic prerequisites concept have the student who is different from the algebraic prerequisites concept.
is considered to be true by experts. Table 1 below shows the possible misconceptions on the algebraic prerequisites concept that occur in students.

| Concept          | Possible Misconceptions that Occur                                                                 |
|------------------|--------------------------------------------------------------------------------------------------|
| Integer Addition | 1. Stating the addition of positive integers with negative integers is done by subtracting positive integers with the inverse of the negative integer but not understanding the sign used for the answer.  
2. Stating that the result of adding two negative numbers is positive.  
3. Stating that the result of adding two negative numbers is by subtracting a smaller number than a larger number. |
| Integer Subtraction | 1. Wrong in completing the subtraction followed by a negative sign.  
2. Solve the problem by subtracting the smaller number than the larger number. |
| Integer Division | 1. Ignores the negative sign when completing the division of negative integers with positive integers.  
2. Read the division sign as a subtraction when completing the division of negative integers with positive integers.  
3. Read division operations such as multiplication operations. |
| Multiplication with zero number | 1. Stating that 0 does not represent anything.  
2. Read multiplication operations such as exponent operations.  
3. Put 0 behind the multiplied number. |
| Fractions Addition | 1. Adds the numerator with the numerator and the denominator with the denominator (without first equating the two denominators).  
2. Could not find the least common multiple.  
3. Finding least common multiple from different denominators but after it does not change the fractions into an equivalent form.  
4. Wrong in changing the fractions into an equivalent form.  
5. Incorrectly change the positive integers to the $\frac{a}{b}$ form when adding the positive integers with fractions. |
| Fractions Division | 1. Incorrectly change the positive integers to the $\frac{a}{b}$ form when dividing the positive integers with fractions.  
2. Divide fractions incorrectly by dividing the numerator with the numerator and then dividing the denominator with the denominator.  
3. Change the division operation to multiplication, but forget to reverse the second fraction.  
4. Stating that the result of the division of the positive integers with the fraction of one numerator is the division of the positive integers with the denominator of the fraction.  
5. Incorrectly change the improper fraction to the $\frac{a}{b}$ form when dividing the improper fraction with the positive integers.  
6. Equate the denominator the same as in addition and subtraction fractions. |

Based on the description above, it is necessary to know what misconceptions students experience on the algebraic prerequisites concept, namely the operation of integers and fractions, causative factors and alternative solutions, so that this study aims to describe students’ misconceptions on the algebraic prerequisite concepts, causative factors and alternative solutions.

2. Method
This research was qualitative descriptive research using diagnostic test methods and interview. This research was conducted at SMPN 1 Koba, class VIII A. This class consists of 32 students, chosen
purposively from six available classes. All students had completed a diagnostic test for 80 minutes. The diagnostic test provided consists of 13 questions about the operation of integers and fractions. Three students who did the most and varied misconceptions in completing the diagnostic test to be the subject of this research, in this research students who had misconceptions of 9, 11 and 12 questions from 13 questions to be the subject of this research. The semi-structured interview was conducted on the subject of the research to find out more about students’ misconceptions on the algebraic prerequisites concepts and causative factors. One week after that, triangulation was carried out to find out the validity of the data.

3. Result and Discussion

3.1. Misconceptions made by S1

Figure 1 (a, b, c) shows that S1 is wrong in solving integer operation questions. S1 is wrong in solving integer addition and subtraction questions, can be seen in Figure 1 (a). S1 understands the addition and subtraction of integers by subtracting large numbers by small numbers and giving a sign for answers by looking at the size of a smaller number of question [34-37]. For question number 1, S1 subtracts 5 with 3 produces 2 and gives a negative sign for 2 because S1 assumes that the sign for the answer is obtained by looking at the size of a smaller number of question, (-3), so the answer is -2, as well as questions number 2, 3 and 4. S1 answers questions number 2, 3 and 4 in the same way as the number 1. The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

Besides, S1 is also wrong in solving integer division questions, can be seen in Figure 1 (b). S1 understands integer division by subtracting the first number with the second number [37]. S1 subtracts 30 with 5 produces 25 and subtracts 24 with 6 produces 18. The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

Figure 1 (c) shows that S1 understands multiplication with zero numbers (question number 7) by putting point 0 then jumps 10 times so that 0 does not represent anything. Any number multiplied by 0 remains the number itself [26]. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.

![Figure 1](a)
In addition, S1 is wrong in solving fraction operation questions, can be seen in Figure 2 (a, b). Figure 2 (a) shows that S1 answers the question number 9, the adding of fractions with fractions whose denominator is different by find least common multiple from different denominators but after finding the least common multiple does not change the fractions into an equivalent form \([36, 38, 39]\). S1 searches least common multiple of 12 and 8, namely 24. After that, S1 adds the numerator with the numerator, 7 with 3 produces 10. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained. S1 is incorrectly change the positive integers to the total form in question number 10 \([36, 38]\). S1 considers the denominator of the first fraction equal to the denominator of the second fraction. The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

Figure 2 (b) shows that S1 is wrong in solving fraction division questions. In question number 11, S1 is incorrectly change the positive integers to the total form \([36, 38]\) and understanding the division of fractions by equalizing the denominator as in addition and subtraction \([38]\). S1 change the positive integers 3 to \(\frac{3}{4}\) then because the denominator is the same, namely 4, S1 only divides the numerator with the numerator, divides 3 with 1. The cause of this misconception is that students experience wrong intuition and associative thinking. S1 tends to just follow his feelings without it first and assume a concept is always the same as other concepts. S1 understands the division of fractions, question number 12, by equalizing the denominator as in the addition and subtraction of fractions \([38]\). S1 equates the denominator by finding least common multiple of 3 and 6, namely 6 then dividing the numerator with the numerator, divides 2 with 1 produces 2. The cause of this misconception is associative thinking that students have. S1 tends to assume a concept is always the same as other concepts. In question number 13, S1 is incorrectly change the improper fractions to the total form \([29]\), incorrectly change the positive integers to the total form \([36, 38]\) and understanding the division of fractions by equalizing the denominator as in the addition and subtraction of fractions \([38]\). S1 change the improper fractions \(\frac{2}{4}\) to \(\frac{2+4+1}{4}\). The numerator is obtained by adding all the digits of the improper fractions the denominator is still 4. Because the denominator is the same, namely 4, S1 only divides the numerator with the numerator, divides 7 with 6. S1 is wrong in dividing 7 by 6 produces 1. The cause of this misconception is that students experience incomplete or incorrect reasoning for information obtained, wrong intuition, having associative thinking and wrong preconceptions on integer division. S1 tends to just follow his feelings without learning it first and assume a concept is always the same as other concepts.
Time triangulation is done to test the validity of the data. Diagnostic test II and the second interview were given to the subject one week after diagnostic test I and the first interview. On diagnostic test 2, S1 is wrong in solving the operation of integers questions, can be seen in Figure 3 (a, b, c). Figure 3 (a) shows that S1 is wrong in solving integer addition and subtraction questions. S1 answers questions number 1, 2, 3 and 4 by subtracts large numbers with small numbers and gives a sign for answers by looking at the sign of a smaller number than the problem \[34-37\]. S1 subtracts 9 with 2 produces 7. S1 assumes that the sign for the answer is obtained by looking at the sign of a smaller number from the question, namely \((-2)\) in the question number 1, so the answer is -7, as well as questions number 2, 3 and 4. S1 The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

Furthermore, S1 also wrong in solving integer division questions. Figure 3 (b) shows that S1 answers questions number 5 and 6 by subtracts the first number with the second number [37]. S1 subtracts 36 with 9 produces 27 and subtracts 27 with 3 produces 24. The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

S1 is wrong in solving multiplication with zero numbers, can be seen in Figure 3 (c). S1 places the point at 0 then because \(20 \times 0\) means to move as much as 20. S1 understands that 0 does not represent anything. Any number multiplied by 0 the result remains that number [26]. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.
S1 is also wrong in solving fraction addition questions, can be seen in Figure 4 (a). S1 understands the adding of fractions with fractions whose denominator is different by finding the least common multiple from the denominator first (least common multiple of 6 and 12, namely 12) then immediately adds the numerator with the numerator, adds 1 with 5 produces 6 without changing the fraction into an equivalent form [36, 38, 39]. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained. S1 is incorrectly change the positive integer to the \( \frac{a}{b} \) form, the denominator of the first fraction equal to the denominator of the second fraction (question number 10) [36, 38]. The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

Figure 4 (b) shows that S1 is wrong in solving fraction division questions. S1 is incorrectly change the positive integers to the \( \frac{a}{b} \) form [36, 38] and the division of fractions is understood by equalizing the denominator as in the addition and subtraction of fractions (question number 10) [38]. S1 change the positive integers 2 to \( \frac{2}{3} \) then divides the numerator with the numerator, divides 2 with 1. The cause of this misconception is that students experience wrong intuition and associative thinking. S1 tends to just follow his feelings without learning them first and assume a concept is always the same as other concepts. In question number 12, S1 divides fractions by equalizing the denominator as in the addition and subtraction of fractions [38]. S1 equates the denominator to 8 then dividing the numerator with the numerator, divides 6 with 3 produces 2. The cause of this misconception is that students have associative thinking. S1 tends to assume a concept is always the same as other concepts. In addition, S1 is incorrectly change the improper fractions to the \( \frac{a}{b} \) form [29], incorrectly change the positive integers to
the $\frac{a}{b}$ form [36, 38] and dividing fractions by equalizing the denominator as in the addition and subtraction of fractions [38] in solving question number 13. S1 change the improper fractions $4\frac{1}{3}$. The numerator is obtained by adding all the digits of the improper fractions, namely $4+3+1$. Then, S1 equates the denominator to 3 and divides 8 with 2 produces 4. The cause of this misconception is that students experience incomplete or incorrect reasoning for information obtained, experience wrong intuition and having associative thinking. S1 tends to just follow his feelings without learning them first and assume a concept is always the same as other concepts.

**Figure 4 (a), (b).** The results of diagnostic tests II about fraction operation carried out by S1.

It appears that there is consistency in research data regarding misconceptions experienced by S1 subjects for answers to questions 1, 2, 3, 4, 5, 6, 7, 9, 10, 11, 12 and 13 on the diagnostic test I and the first interview and the diagnostic test II and the second interview, it can be concluded that the S1 misconception data is valid.
3.2. Misconceptions made by S2

Figure 5 (a, b) shows that S2 is wrong in solving integer operation questions. S2 is wrong in solving the integer addition and subtraction questions, can be seen in Figure 5 (a). S2 understands the addition and subtraction of integers by subtracting large numbers with small numbers and signs for answers are searched by multiplying the sign of first number by the second number [34-37]. For example, question number 1, S2 answers by subtracting 5 by 3. Then because the sign for the first number is positive and the sign for the second number is negative, positive multiplied by negative results are negative, so S2 answers with the answer -2 and so on for questions number 2, 3 and 4. The cause of this misconception is that students experience wrong intuition and associative thinking. S2 tends to just follow her feelings without learning it first and assume a concept is always the same as other concepts.

S2 answers the question number 6, (-24) ÷ 6 = 4, can be seen in Figure 5 (b). S2 ignores the negative sign for the answer [37]. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.

Figure 5 (a), (b). The results of diagnostic tests I about integer operation carried out by S2.

Figure 6 (a, b) shows that S2 is wrong in solving fraction operation questions. S2 understands the addition of fractions by adding the numerator with the numerator and the denominator with the denominator [16, 26, 36-39], can be seen in Figure 6 (a). In question number 8, S2 adds 3 with 2 produces 5 and adds 8 with 8 produces 16 and in question number 9, S2 adds 7 with 3 produces 10 and adds 12 with 8 produces 20. The cause of this misconception is the student's own ability which is difficult in understanding mathematical concepts. In question number 10, S2 is incorrectly change the positive integers to the \( \frac{a}{b} \) form [36, 38]. S2 considers the denominator of the first fraction the same as the denominator of the second fraction. S2 then adds up the fractions by adding the numerator with the numerator and the denominator with the denominator [16, 26, 36-39]. S2 adds 8 with 3 produces 11 and adds 5 with 5 produces 10. The cause of this misconception is that students experience wrong intuition and students' own ability which is difficult in understanding mathematical concepts. S2 tends to just follow her feelings without learning it first.

S2 is wrong in solving fraction division questions, can be seen in Figure 6 (b). S2 is incorrectly change the positive integer to the \( \frac{a}{b} \) form [36, 38] then dividing the fractions by dividing the numerator with the numerator and the denominator with the denominator [36]. S1 changes 3 to \( \frac{3}{4} \) then divides 3 with 1 produces 3 and 4 with 4 produces 1. The cause of this misconception is that students experience wrong intuition and students' own ability which is difficult in understanding mathematical concepts. S2 tends to just follow her feelings without learning it first. S2 understands the division of fraction by dividing
the numerator with the numerator and the denominator with the denominator (question number 12) [36]. S2 divides 2 with 1 produces 2 and 3 with 6 produces 2. S2 is wrong in dividing 3 by 6, namely 2. The cause of this misconception is the ability of students who are difficult in understanding mathematical concepts. S2. In question number 13, S2 is incorrectly change the improper fractions to the $\frac{a}{b}$ form and then incorrectly change the positive integers to the $\frac{a}{b}$ form [36, 38] and dividing fractions by dividing the numerator with the numerator and the denominator with the denominator [36]. S1 changes $\frac{2}{4}$ to $\frac{2\times1}{2\times4}$ produces $\frac{2}{8}$. S2 divides 2 with 6 produces 3 and 8 with 8 produces 1. S2 is wrong in dividing 2 by 6, namely 3. The cause of this misconception is that students experience incorrect or incomplete reasoning of information obtained and the ability of students who are difficult in understanding mathematical concepts.

Time triangulation is done to test the validity of the data. Diagnostic test II and the second interview were given to the subject one week after diagnostic test I and the first interview. On diagnostic test 2, S2 is wrong in solving integers questions, can be seen in Figure 7 (a, b). S2 understands the addition and subtraction of integers questions, can be seen in Figure 7 (a). S2 subtracts large numbers with small numbers and signs for answers are searched by multiplying the sign of first number by the second number [34-37]. The cause of this misconception is that students experience wrong intuition and associative thinking. S2 tends to just follow her feelings without learning it first and assume a concept is always the same as other concepts.

Figure 7 (b) shows that S2 answers questions number 6, $-27 \div 3 = 9$. S2 ignores the negative sign for the answer of integer division questions [37]. S2 divides -27 by 3 produces 9. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.

Figure 6 (a), (b). The results of diagnostic tests I about fraction operation carried out by S2.
S2 is wrong in solving integer addition questions, can be seen in Figure 8 (a). S2 understands the addition of fractions by adding the numerator with the numerator and the denominator with the denominator [16, 26, 36-39]. S2 adds 1 with 2 produces 3 and adds 5 with 5 produces 10 (question number 8). S2 also adds 1 with 5 produces 6 and adds 6 with 12 produces 18 (question number 9). The cause of this misconception is the student's own ability which is difficult in understanding mathematical concepts. S2 answers the question number 10, S2 is incorrectly change the positive integer to the \( \frac{a}{b} \) form [36, 38] and then adding up fractions by adding the numerator with the numerator and the denominator with the denominator [16, 26, 36-39]. S2 changes 6 to \( \frac{6}{1} \) because S2 considers the denominator of the first fraction the same as the denominator of the second fraction. S2 adds 6 with 6 produces 12 and adds 4 with 4 produces 8. The cause of this misconception is that students experience wrong intuition and students’ own ability which is difficult in understanding mathematical concepts. S2 tends to just follow her feelings without learning it first.

Figure 8 (b) shows that S2 is wrong in solving fraction division questions. In question number 11, S2 is incorrectly change the positive integer to to the \( \frac{a}{b} \) form [36, 38], changes 2 to \( \frac{2}{1} \). Then S2 dividing the fractions by dividing the numerator with the numerator [36], divides 2 with 1 produces 2 and the denominator with the denominator, divides 3 with 3 produces 1. The cause of this misconception is that students experience wrong intuition and students’ own ability which is difficult in understanding mathematical concepts. S2 tends to just follow her feelings without learning it first. S2 divides the numerator with the numerator and the denominator with the denominator [36] in question number 12. S2 divides 6 with 3 produces 2 and 8 with 2 produces 4. The cause of misconception is the student’s own ability which is difficult in understanding mathematical concepts. In question number 13, S2 is incorrectly change the improper fraction to the \( \frac{a}{b} \) form and then incorrectly change the positive integer to the \( \frac{a}{b} \) form [36, 38] and dividing fractions by dividing the numerator with the numerator and then the denominator with the denominator [36]. S1 changes \( \frac{4}{3} \) to \( \frac{4 \times 1}{4 \times 3} \) produces \( \frac{4}{12} \). S2 divides 4 with 2 produces 2 and 12 with 12 produces 1. The cause of this misconception is that students experience incorrect or incomplete reasoning of information obtained and the students' own ability which is difficult in understanding mathematical concepts.
The results of diagnostic tests II about fraction operation carried out by S2.

It appears that there is consistency in the research data regarding the misconceptions experienced by S2 subject for answers to questions number 1, 2, 3, 4, 6, 8, 9, 10, 11, 12 and 13 in the diagnostic test I and the first interview and the diagnostic test II and the second interview, it can be concluded that the S2 misconception data is valid.

3.3. Misconceptions made by S3

S3 is wrong in solving integer operation questions, can be seen in Figure 9 (a, b). Figure 9 (a) shows that S3 understands integer subtraction by subtracting large numbers with small numbers and giving a sign for an answer by looking at the sign of a larger number from the problem \[34 - 37\]. The cause of this misconception is that students experience wrong intuition. S3 tends to just follow his feelings without learning it first.

Figure 9 (b) shows that S3 understands integer division by subtracting the first number by the second number [37]. The cause of this misconception is that students experience wrong intuition. Figure 9 (c) shows that S3 answers \(10 \times 0 = 10\) (multiplication with zero numbers) by stating that 0 does not represent anything. Any number multiplied by 0 remains the number itself [26]. The cause of misconception is students experience incorrect or incomplete reasoning of the information obtained.
3. Tentukan hasil dari $5 - (-7)$! Jelaskan bagaimana Anda memperoleh jawabannya.
Jawab:
$$5 - (-7) = 12$$

4. Tentukan hasil dari $-10 - 1$! Jelaskan bagaimana Anda memperoleh jawabannya.
Jawab:
$$-10 - 1 = -11$$

6. Tentukan hasil dari $(-29) : 10$! Jelaskan bagaimana Anda memperoleh jawabannya.
Jawab:
$$(-29) : 10 = -2.9$$

7. Tentukan hasil dari $10 \times 0.1$! Jelaskan bagaimana Anda memperoleh jawabannya.
Jawab:
$$10 \times 0.1 = 1$$

S3 also wrong in solving the addition of fractions, can be seen in Figure 10 (a). S3 understands the addition of fractions by finding least common multiple first, searches least common multiple of 12 and 8, namely 24 and least common multiple of 1 and 5, namely 5 but wrong in changing the fractions into an equivalent form, [27], changes \( \frac{7}{12} \) to \( \frac{24+12+7}{24} \), changes \( \frac{3}{8} \) to \( \frac{24+8+3}{24} \), changes \( \frac{8}{5} \) to \( \frac{5+1+8}{5} \) and changes \( \frac{3}{5} \) to \( \frac{5+5+3}{5} \). The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.

Figure 10 (b) shows that S3 changing the division operation to multiplication but forgetting to reverse the second fraction. S3 solving question number 11 $3 \div \frac{1}{4} = \frac{3}{1} \times \frac{1}{4}$, question number 12 $\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{1}{6}$ and question number 13 $\frac{1}{2} + 6 = \frac{2 \times 4 + 1}{4} \times 6$. S3 understands fraction division by changing the division operation to multiplication, but forgetting to reverse the second fraction [26, 36, 39]. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.

Figure 9 (a), (b), (c). The results of diagnostic tests I about integer operation carried out by S3.

Figure 10 (a), (b). The results of diagnostic tests I about fraction operation carried out by S3.
Time triangulation is done to test the validity of the data. Diagnostic test II and the second interview were given to the subject one week after diagnostic test I and the first interview. On diagnostic test 2, Figure 11 (a, b, c) shows that S3 is wrong in solving integer operation questions. S3 subtracts large numbers with small numbers and gives a sign for answers by looking at the sign of a number that is larger than the question [34-37], can be seen in Figure 11 (a). For question number 1, S3 subtracts 8 by 3 produces 5 and gives a negative sign for 5 because S3 assumes that the sign for the answer is obtained by looking at the sign of a larger number from the question so the answer is -5, as well as questions number 2. The cause of this misconception is that students experience wrong intuition. S1 tends to just follow his feelings without learning it first.

S3 answers the question number 24 by subtracts the first number by the second number [37], can be seen in Figure 11 (b). The cause of this misconception is students experience wrong intuition. S3 tends to just follow his feelings without learning it first.

In addition, S3 wrong in solving multiplication with zero numbers, can be seen in Figure 11 (c). S3 answers 20 × 0 = 20. S3 states any number multiplied by 0 remains the number itself. 0 does not represent anything [26]. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.

Then S3 also wrong in solving the adding of fractions questions, can be seen in Figure 12 (a). In answers questions number 9 and 10, S3 seeks least common multiple first but wrong in changing the fractions into an equivalent form [27]. The cause of misconception is students experience incorrect or incomplete reasoning of the information obtained.

Furthermore, S3 answers questions number 11, 12 and 13 by changes the division operation to multiplication, but forgets to reverse the second fraction [26, 36, 39], can be seen in Figure 12 (b). S3 solving 2 ÷ 1 1/3 to 2 × 3/4, solving 6 ÷ 3/2 to 6 × 2/3 and solving 4 1/3 ÷ 2 to 12 ÷ 4 3/3 × 2. The cause of this misconception is that students experience incorrect or incomplete reasoning of the information obtained.
It appears that there is consistency in research data regarding misconceptions experienced by S3 subjects for answers to questions number 3, 4, 6, 7, 9, 10, 11, 12 and 13 on the diagnostic test I and the first interview and the diagnostic tests II and the second interviews, it can be concluded that the S3 misconception data is valid.

4. Conclusion
Misconceptions in the concept of addition and subtraction of integers made by students are subtracting a large number from a small number and using the wrong sign for an answer by looking at the sign of a smaller number of questions, multiplying the sign of first number by the second number and looking at the sign of a larger number of questions. The misconception of the concept of integer division are to subtract the first number by the second number and ignores the negative sign when completing a negative integer division. The misconception in the concept of multiplication with zero numbers is stating that 0 does not represent anything. The misconception of the adding fraction concept are add the numerator by the numerator and the denominator by the denominator, find the least common multiple from different denominators of fraction but after finding the least common multiple does not change fractions into equivalent forms, incorrectly change the positive integers to the form when adding positive integers with fractions and find least common multiple first but wrong in changing the fractions into an equivalent form. The misconception in the concept of dividing fractions are incorrectly change the positive integers to the form when dividing positive integers with fractions, dividing fractions by fractions by equalizing the denominator as in addition and subtraction of fractions, dividing the numerator by the numerator and the denominator by the denominator, changes the division operation to multiplication but forgets to reverse the second fraction and incorrectly change the improper fraction to the form when dividing improper fractions with positive integers. The causative factor of student misconception are preconception, associative thinking, incomplete or incorrect reasoning, wrong intuition, and ability. Alternative solutions that can be used to overcome misconceptions are re-explain and cognitive conflict.

5. Acknowledgment
The authors thanks to Universitas Negeri Surabaya for supporting this research. The authors also thanks to the head department to our master program, Dr. Yusuf Fuad, M.App. Sc, who teaches and motivates the researchers to learn more about research methodology on research in mathematics education. Furthermore, the authors also thanks to the school and the teachers in SMPN 1 Koba who helped in this research and the students who were willing to be research subjects.
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