Ternary Hybrid Neural-Tree Networks for Highly Constrained IoT Applications

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Abstract

Machine learning-based applications are increasingly prevalent in IoT devices. The power and storage constraints of these devices make it particularly challenging to run modern neural networks, limiting the number of new applications that can be deployed on an IoT system. A number of compression techniques have been proposed, each with its own trade-offs. We propose a hybrid network which combines the strengths of current neural- and tree-based learning techniques in conjunction with ternary quantization, and show a detailed analysis of the associated model design space. Using this hybrid model we obtained a 11.1% reduction in the number of computations, a 52.2% reduction in the model size, and a 30.6% reduction in the overall memory footprint over a state-of-the-art keyword-spotting neural network, with negligible loss in accuracy.

1 Introduction

Machine learning algorithms, and neural networks (NNs) in particular, are increasingly deployed in Internet-of-Things (IoT) devices. Popular applications include speech interfaces in smart-home devices, predictive maintenance for commercial and industrial machines, health-monitoring in wearables, etc. However, due to the energy, power, storage, and compute limitations of highly-constrained IoT devices, they are frequently limited to simplistic tasks, while more sophisticated requests are off-loaded to a more capable device or to a server. In addition to being computationally constrained, IoT devices frequently have very little available SRAM, tend to be “always-on”, and are often connected to constrained power sources. Because of these constraints, reducing the computation and storage required by ML models for IoT applications is of paramount importance in order to ensure a longer battery life.

To enable this computation and size compression of NN models, one particularly effective technique has been the use of depthwise-separable (DS) convolutional layers. We see these layers being used in a large, image classification application (Howard et al., 2017) and also on a ubiquitous keyword-spotting application (Zhang et al., 2017), showing state-of-the-art or near-state-of-the-art accuracy in both cases.

While DS convolutional layers have been transformative,
low and sparse tree to reduce model size but with powerful
nodes for accurate prediction. It uses more powerful branch-
ing functions than the axis-aligned hyperplanes in standard
decision trees. This is coupled with non-linear predictions
made by internal and leaf nodes on a single, shallow deci-
sion tree learned in a low-dimensional space. Combining
these ideas allows Bonsai trees to learn complex non-linear
decision boundaries using a compact representation. While
the results in (Kumar et al., 2017) show promising results
for smaller applications, our observations were that the tech-
niques do not scale when extended to a more complex use-
case, showing poor prediction accuracy even with a large
model footprint.

We now, therefore, have two ways of compressing models,
each with its own advantages and limitations:

- StrassenNets is effective at reducing the size of a neural
  network model but at a potentially significant cost of
  more addition operations.

- Bonsai tree is effective at reducing the number of oper-
  ations for simple models but cannot easily be extended
to larger models.

Motivated by these observations, we propose a hybrid net-
work architecture capable of giving start-of-the-art accuracy
levels, while requiring a fraction of the model parameters
and considerably fewer operations per inference. The hybrid
architecture makes this possible by leveraging a few neural
DS convolutional layers for feature extraction and then re-
yling on a compute-efficient, shallow Bonsai decision tree
to perform the classification. It then applies StrassenNets
over the overall neural-tree network to reduce its memory
footprint significantly thus enabling a compact compute-
efficient architecture. We apply this hybrid architecture to
a representative IoT application keyword-spotting. The hy-
brid network achieves a 98.89% reduction in multiplications,
a 12.22% reduction in additions (overall 11.1% reduction
in number of operations), a 52.2% reduction in model size,
and a 30.6% reduction in overall memory footprint over a
state-of-the-art keyword-spotting neural model. The hy-
brid network accomplishes this with a very minimal loss
in accuracy of 0.27%. The final network is well within the
constrained compute budget of typical microcontrollers.

The remainder of the paper is organized as follows. Sec-
tion 2 elaborates on the incentives behind this hybrid net-
work architecture for microcontrollers and provides a brief
overview of the neural and tree-based learning algorithms
that it attempts to hybridize along with our observations of
applying them to the keyword-spotting application. Failing
to find a good balance between accuracy and computation
costs shifts our focus towards designing a hybrid neural-tree
network. Section 3 describes our hybrid network. Section 4
presents results. Section 5 compares our hybrid network
against prior works and and Section 6 concludes the paper.

2 Model Compression Limitations
for an IoT Application

2.1 StrassenNets

Given two $2 \times 2$ matrices, Strassen’s matrix multiplication
algorithm computes their product using 7 multiplications
instead of the 8 required with a naïve implementation of
matrix multiplication. It essentially converts the matrix
multiplication operation to a 2-layer sum-product network
(SPN) computation as shown below:

$$vec(C) = W_c[vec(B) \odot vec(A)] \quad (1)$$

$W_a, W_b \in K^{r \times n^2}$ and $W_c \in K^{n^2 \times r}$ are ternary matrices
with $K \in \{-1, 0, 1\}$, $vec(A)$ and $vec(B)$ are the vectoriza-
tion of the two input square matrices $A, B \in R^{n \times n}$; and
$vec(C)$ represents the vectorized form of the product $A \times B$.
$\odot$ denotes the element-wise product. The $(W_b vec(B))$ and
$(W_a vec(A))$ of the SPN compute $r$ intermediate factors
each from additions, and/or subtractions of elements of $A$ and
$B$ realized by the two associated ternary matrices $W_a$
and $W_b$ respectively. The two generated $r$-length interme-
diate factors are then element-wise multiplied to produce the
$r$-length $(W_b vec(B)) \odot (W_a vec(A))$. The outmost ternary
matrix $W_c$ later combines the $r$ elements of the product
$(W_b vec(B)) \odot (W_a vec(A))$ in different ways to generate
the vectorized form of product matrix $C$. Therefore, the
width of the hidden layer of the SPN $r$ depends on the number
of multiplications required for the Strassen’s matrix mul-
tiplication algorithm. For example, given two $2 \times 2$ matrices,
ternary matrices $W_a$ and $W_b$ with sizes of $7 \times 4$ can multiply
them using 7 multiplications and 36 additions. It is impor-
tant to note that Strassen’s algorithm requires a hidden
layer with 7 units here to compute the exact product matrix
that a naïve matrix multiplication algorithm can obtain using
8 multiplications.

Building on top of Strassen’s matrix multiplication algo-

rithm, the StrassenNets work (Tschannen et al., 2018) in-
stead realizes approximate matrix multiplications in DNN
layers using fewer hidden layer units compared to the
standard Strassen’s algorithm to achieve the exact prod-
uct matrix. StrassenNets makes this possible by training a
SPN-based DNN framework end-to-end to learn the ternary
weight matrices from the training data. The learned ternary
weight matrices can then approximate the otherwise exact
matrix multiplications of the DNN layers with significantly
fewer multiplications than Strassen’s algorithm. The approx-
imate transforms realized by the SPNs, adapted to the DNN
architecture and application data under consideration, can
Table 1. Test accuracy along with the number of multiplications, additions, operations and model size for state-of-the-art DS-CNN and strassenified DS-CNN (ST-DS-CNN) on KWS. \(r\) is the hidden layer width of a strassenified convolution layer, \(c_{out}\) is the number of output channels of the corresponding convolution layer.

| Network            | ACC. (%) | MULS. | ADDS. | MACS | Ops | MODEL SIZE |
|--------------------|----------|-------|-------|------|-----|------------|
| DS-CNN             | 94.4     |       |       | 2.7M | 2.7M| 22.07KB    |
| ST-DS-CNN \((r = 0.5c_{out})\) | 93.18    | 0.05M | 2.85M | 2.9M | 16.23KB |
| ST-DS-CNN \((r = 0.75c_{out})\) | 94.09    | 0.06M | 4.09M | 4.15M | 19.26KB |
| ST-DS-CNN \((r = 0.75c_{out})\) | 94.03    | 0.07M | 5.32M | 5.39M | 22.29KB |
| ST-DS-CNN \((r = 2c_{out})\) | 94.74    | 0.11M | 10.25M | 10.36M | 34.42KB |

Table 1 captures our observation with strassenified DS layers in the uncompressed DS-CNN KWS model. Multiply, addition, and multiply-accumulate (MAC) operations typically incur similar execution latencies in modern microprocessors, but different models have different ratios of these operations. They are, therefore, counted individually and aggregated in the “Ops” column. The strassenified network with \(r = 0.75c_{out}\) configuration incurs a negligible loss in accuracy of 0.31% while reducing multiplications by 97.7% but increasing additions by 51.4% (2.7M MACs of DS-CNN vs. 0.06M multiplications and 4.09M additions of ST-DS-CNN with \(r = 0.75c_{out}\)). That means the strassenified network with \(r = 0.75c_{out}\) configuration actually increases the number of total operations to 4.15M when compared to 2.7M operations in the uncompressed DS-CNN network. As shown in Table 1, a number of potential values for the hidden layer width \(r\) were explored and a value of at least \(0.75c_{out}\) was needed to achieve a comparable accuracy to that of the full-precision DS-CNN model. Using fewer hidden units \(r = 0.5c_{out}\) than this incurs an accuracy loss of 1.22%, whereas wider strassenified hidden layers \(r = 2c_{out}\) recover the negligible accuracy loss of the \(r = 0.75c_{out}\) configuration. For sufficiently large \(r\) values, the strassenified network can even out-perform the uncompressed DS-CNN model in accuracy, albeit with a significant increase (about 280% for \(r = 2c_{out}\)) in the number of additions than the DS-CNN model.

2.1.2 Compute inefficiency of StrassenNets for models with DS convolutions

It is important to note here that although the number of additions does increase marginally with strassenifying standard \(3 \times 3\) or \(5 \times 5\) convolutional layers (Tschannen et al., 2018), that trend does not hold true with strassenifying DS layers. This stems from the fact that \(1 \times 1\) pointwise convolutions dominate the compute bandwidth of a neural network with DS layers (Zhang et al., 2017; Howard et al., 2017) and strassenifying a \(1 \times 1\) pointwise convolution requires executing two equal-sized (for \(r = c_{out}\)) \(1 \times 1\) convolution operations (with ternary weight filters) in place of the standard \(1 \times 1\) convolution. This results in a significant increase in additions in comparison to the execution of the standard \(1 \times 1\) convolution. In contrast to that, a \(3 \times 3\) strassenified convolution with \(r = c_{out}\) instead requires executing a \(3 \times 3\) convolution and a \(1 \times 1\) convolution with ternary weight filters, causing a marginal increase in additions compared to the execution of the standard \(3 \times 3\) convolution (Tschannen et al., 2018). This overhead of addition operations with strassenified DS convolutions increases in proportion to the width of the strassenified hidden layers, i.e. to the size of the ternary convolution operations, as observed in Table 1. As a result, a strassenified DS convolution layer may incur enough overhead to offset the benefit of strassenifying a DS convolution layer.

While (Tschannen et al., 2018) demonstrates better trade-offs when strassenifying ResNet-18 architecture, this is not likely to continue once a larger network dominated with DS convolutions (e.g. MobileNetV2) is strassenified. (Tschannen et al., 2018) observes the ResNet-18 architecture with strassenified \(3 \times 3\) convolutions to...
achieve comparable accuracy to that of the uncompressed ResNet-18 on the ImageNet dataset with $r = 2^{c_{out}}$ configuration while requiring a modest (29.63%) increase in additions. A strassenified MobileNets with $r = 2^{c_{out}}$ configuration for the DS layers will give rise to about a 300% increase in additions over the uncompressed MobileNets architecture. This increase in computational costs associated with strassenified DS convolutions in conjunction with the high accuracy and low latency requirements of IoT applications call for a model architecture exploration that can leverage the compute efficiency of DS layers and model size reduction of strassenified convolutions owing to their ternary weights while maintaining acceptable or no increase in additions. As tree-based learning techniques from recent work (Kumar et al., 2017) exhibit accuracy on par with neural models while requiring significantly fewer MAC operations, this motivates us to explore the model accuracy and compute-efficiency of these tree-based techniques for representative IoT applications.

### 2.2 Bonsai Decision Trees

Piece-wise axis-aligned decision boundaries coupled with constant predictions at just the leaf nodes restrict the prediction accuracy of typical tree models when compared to that of their neural counterparts. Tree ensembles are commonly used to improve the accuracy, but they can occupy too large a memory footprint for typical microcontrollers. Recent work on tree models attempt to learn more complex decision boundaries by moving away from learning axis-aligned hyperplanes at internal nodes and constant predictors at the leaves. Bonsai decision trees (Kumar et al., 2017) fall into this paradigm. Using more powerful branching functions than the axis-aligned hyperplanes of standard decision trees in conjunction with non-linear prediction scores in both internal and leaf nodes allows Bonsai to learn a single, shallow tree that can achieve accuracy on par with small neural-based models. For a multi-class classification problem with $L$ targets, Bonsai learns matrices $W_{D \times L}$ and $V_{D \times L}$ at both leaf and internal nodes so that each node now predicts a non-linear prediction score $W^\top Zx \circ \tanh(\sigma V^\top Zx)$. Bonsai reduces model size by projecting each $D$-dimensional input feature vector $x$ into a low $\hat{D}$-dimensional space using a projection matrix $Z_{\hat{D} \times D}$ in which the tree is learned. Once an input feature is projected to a low-dimensional space, Bonsai adds the individual node predictions along the path traversed by the projected input to derive the overall prediction. Owing to a single, shallow tree with powerful nodes and branching functions learned in a low-dimensional space, Bonsai can achieve impressive computation reduction over a typical DNN, while preserving DNN-level accuracy for very small models.

### 2.2.1 Bonsai Tree for KWS

When applied to the KWS application, Bonsai shows poor prediction accuracy even with a significantly large tree with many internal and leaf nodes. As shown in Table 2, Bonsai trees achieve poor accuracies, saturating at about 84%, even when the tree architecture is scaled up with wider projection layers, more tree nodes and trained for longer. Furthermore, a major fraction of the model size (e.g. 69.63% of Bonsai tree with $\hat{D}=64$ and $T=2$) is attributed to the fully-connected (FC) layer used in projecting the incoming input data to low-dimensional space. Clearly, weight quantization and aggressive pruning will reduce the model size further, as described in (Kumar et al., 2017), however it will not be able to recover the significant accuracy loss of Bonsai trees when compared to that of the neural models for KWS (e.g. DS-CNN). It is worth emphasizing that although Bonsai occupies a large memory footprint, its computational costs are very low in comparison to those neural models.

### 2.2.2 The limitations of Bonsai trees for KWS

While (Kumar et al., 2017) shows the effectiveness of Bonsai trees for the applications they considered, our results show that for more complex applications, there might be a fundamental limitation in the expressiveness of Bonsai trees. More specifically, the simple projection matrix that is made of a FC layer in a Bonsai tree is likely not effective in compressing KWS’s initial speech inputs to extract rich useful features. This observation is further corroborated by prior works (Zhang et al., 2017; Arik et al., 2017; Sainath & Parada, 2015) on designing state-of-the-art neural networks for small-footprint KWS that leverages convolutional layers instead to compress complex speech inputs of KWS applications to extract a few rich, meaningful features.

### Table 2. Test accuracies for DS-CNN and Bonsai tree variants on KWS. $D = \text{projected dimension}, T = \text{depth of tree}$

| NETWORK | ACC. (%) | MACs | OPS | MODEL SIZE |
|---------|---------|------|-----|------------|
| DS-CNN  | 94.4    | 2.7M | 2.7M | 22.07KB    |
| Bonsai ($\hat{D}=64, T=2$) | 80.20  | 0.02M | 0.02M | 140.75KB  |
| Bonsai ($\hat{D}=64, T=4$) | 82.92  | 0.04M | 0.04M | 287.75KB  |
| Bonsai ($\hat{D}=128, T=2$) | 81.56  | 0.04M | 0.04M | 281.5KB   |
| Bonsai ($\hat{D}=128, T=4$) | 84.38  | 0.07M | 0.07M | 573.5KB   |

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1Bonsai trees in Table 2 are trained significantly longer than the other networks in this work. The learning rate is initially chosen as 0.001, and later gradually reduced after every 100 epochs.

2Each Bonsai tree weight in Table 2 requires 4 bytes to store.
of a potential hybrid model - one that can use the feature extraction capabilities of a convolutional network while also reducing the amount of compute required for subsequent classification of these features. This hybrid model proposed in this paper exploits Bonsai tree’s strength as a compute-efficient classifier given rich features and couples that with StrassenNets to achieve significant reduction in model size.

3 Hybrid Neural-Tree Architecture

We propose a hybrid neural-tree architecture that can leverage a few convolutional layers to extract the minimal set of necessary local features, and then can rely on powerful branching functions and non-linear Bonsai tree nodes to find global correlation between features and to perform the required classification. As the tree section of the hybrid network is comparatively compute-efficient in terms of MAC operations, use of it to find the global interaction between local features and classifying the voice commands should result in an overall reduction of computational costs compared to a neural-only state-of-the-art network for KWS without compromising its accuracy. DS convolutional layers are used in particular for feature extraction in the hybrid network. Additionally, the matrix multiplications associated with the entire hybrid network are strassenified to reduce multiplications and the overall memory footprint to enable a more compact network.

Architecture. Figure 1 shows the hybrid neural-tree architecture optimized for KWS application with the corresponding parameters. The raw time-domain speech signal is converted to 2-D MFCC (Mel-frequency cepstral coefficients) inputs for succinct representation and efficient training. Speech features are first extracted from the MFCC inputs by one standard convolutional layer followed by two DS convolutional layers which greatly reduce dimensionality of the original speech signal. The low-dimensional compressed speech features are then fed to a single depth 2 Bonsai tree with 3 internal and 4 leaf nodes to provide global interaction and to identify the appropriate keyword in the detected voice command.

Note that the branching functions of the tree’s internal nodes output a probability to influence whether the low-dimensional speech sample should be branched to a node’s left or right child. During inference, non-linear prediction scores are computed for all tree nodes regardless of the most probable tree path traversed by the compressed sample. The tree nodes from the least probable paths contribute insignificantly to the overall prediction score. Computing prediction scores for all nodes certainly increases prediction costs. As the hybrid network for KWS has a shallow depth 2 tree, the incremental costs from computing prediction scores for all nodes is marginal in comparison to the computational costs of the overall hybrid network. However, evaluating the entire tree ensures that the tree computation does not incur any control-flow overhead in the processor from unpredictable branching in internal nodes. This, in turn, results in a more resource-efficient, data-parallel computation pattern and a more efficient utilization of any available SIMD units. For example, in Figure 1, a low-dimensional speech feature sample $D$ finds the leftmost tree path most probable to traverse and as a result the three tree nodes along the leftmost path in the tree contribute the most to the overall prediction score, even though the prediction scores for all tree nodes are computed during inference.

End-to-end training. The three convolutional layers along with all the tree nodes of the hybrid network are trained jointly so as to maximize accuracy. A gradient-descent (GD) based training algorithm is used to train the hybrid network end-to-end. Note that the path traversed by a training point in a standard decision tree is a sharply discontinuous function of parameters of internal branching nodes thereby making gradient based techniques ineffective. In order to make effective use of GD algorithm with a differentiable loss function, the training of a Bonsai tree begins with smooth activation functions of internal branching nodes to allow points to traverse multiple paths in the tree. As training progresses, the activation functions of internal branching nodes are tuned to ensure that points gradually start traversing at most a single path.

Strassenified hybrid network. Finally, in order to reduce the hybrid network’s memory footprint, we apply strassenified matrix multiplications to its convolution layers and tree section to create the strassenified hybrid network. Note that each of the tree nodes of the hybrid network learns two matrices $W$ and $V$ to compute a non-linear prediction score. Computation of this prediction scores at the tree nodes involves matrix multiplications, which are strassenified as
well. The hidden layer width \(r\) of the strassenified hybrid network is set to 0.75\(c_{out}\) for convolution layers (\(c_{out}\) is the number of output channels of a convolution layer), whereas \(r\) for the tree nodes is set to the number of targets \(L\) of the multi-class KWS classification problem.

Training a network with strassenified matrix multiplications essentially involves learning ternary \(W_a, W_b,\) and \(W_c\) matrices for each strassenified network layer. We employ the training procedure described in the StrassenNets (Tschannen et al., 2018) work to train the strassenified hybrid network. Training of the strassenified hybrid network begins with full precision \(W_a, W_b,\) and \(W_c.\) Once the network is sufficiently trained with full-precision \(W_a, W_b,\) and \(W_c,\) the elements of these three strassen matrices are quantized to ensure ternary-valued weights in them and the training continues. Quantization converts the full-precision \(W_b\) to a ternary-valued \(W_b^t\) along with a scaling factor \((W_b = \text{scaling factor} * W_b^t)\). \(W_a\) and \(W_c\) are quantized the same way. Once the training recovers the accuracy loss with quantized strassen matrices, the three strassen matrices are fixed and the scaling factors associated with them are absorbed by full-precision \(\text{vec}(A)\) portion of strassenified matrix multiplication. Note that in the context of strassenified matrix multiplications of a network layer, \(A\) is associated with the weights or filters of the layer and \(B\) is associated with the corresponding activations or feature maps. As a result, after training, \(W_a\) and \(\text{vec}(A)\) can be collapsed into a vector \(\hat{a} = W_a \text{vec}(A),\) as they are both fixed during inference. We follow the weight quantization procedure described in the StrassenNets (Tschannen et al., 2018) work for quantizing all strassen matrices of the hybrid network.

Furthermore, in order to recover any accuracy loss of the hybrid network compressed with strassenified matrix computations, knowledge distillation (KD) is exploited during training, as described in (Tschannen et al., 2018). Using KD, an uncompressed teacher network can transfer its prediction ability to a compressed student network by navigating its training. We use the uncompressed hybrid network as the teacher network and the compressed strassenified network as the student network in this work.

In short, the strassenified hybrid neural-tree architecture essentially combines the strengths of DS convolutions, Bonsai trees, and strassenified matrix computations, with additional strategies applied during training to improve the overall performance, while keeping a small-footprint size.

4 We apply KD while training the strassenified DS-CNN networks in Section 2 as well. The results reported for strassenified networks in Table 1 are obtained using KD.

4 We use hinge loss to train the baseline Bonsai trees in Section 2.2.1, whereas we use standard cross-entropy loss to train the strassenified DS-CNN networks in Section 2.1.1.
from its compute-efficient tree portion. Note that all the baseline networks in Table 3 require 1 byte to store their weights as opposed to the 4 bytes required to store the weights of the uncompressed hybrid network. Consequently, the uncompressed hybrid network requires a larger model size of 94.25KB when compared to other baselines in Table 3. Clearly, straightforward quantization of the weights of the hybrid network to low-precision values will result in reducing its model size. However, as discussed in this work, we instead apply StrassenNets over the entire hybrid network to reduce its full-precision (4 bytes) parameters and resultant model size.

**Strassenified hybrid network training.** We begin by training the strassenified hybrid network (ST-HybridNet) with full-precision strassen matrices \((W_a, W_b, W_c)\) for 135 epochs. The learning rate is initially chosen as 0.001, and later gradually reduced after every 45 epochs. We then activate quantization for these strassen matrices and the training continues. Finally, we fix the strassen matrices to their learned ternary values and continue training for another 135 epochs to ensure that the scaling factors associated with these matrices can be absorbed by full-precision \(\text{vec}(A)\) portion of strassenified matrix multiplication.

**Strassenified hybrid network evaluation.** The testing accuracy of the ST-HybridNet is shown in Table 4, along with the reduction in the number of operations, and the model size in comparison to the uncompressed hybrid network. The ST-HybridNet achieves similar accuracy to that of the uncompressed hybrid network and the baseline DS-CNN while reducing the number of multiplications and additions by 98.89% and 12.22%, respectively, over the baseline DS-CNN network. Of particular note is that it reduces the number of additions to about 2.37M when compared to 4.09M additions of strassenified DS-CNN network described in Section 2. This, in turn, results in fewer overall operations, 2.4M, for the ST-HybridNet when compared to 2.7M operations of the baseline DS-CNN and 4.15M operations of the strassenified DS-CNN. This reduction in operations is primarily attributed to strassenifying a few (three) convolutional layers and a compute-efficient tree as opposed to strassenifying all of the five convolutional layers found in the baseline DS-CNN model. Owing to the ternary weights matrices, the ST-HybridNet reduces the model size to 14.99KB when compared to 22.07KB of the baseline DS-CNN network thus enabling a 32.1% savings in model size for KWS\(^3\). Furthermore, our ST-HybridNet does not incur any accuracy loss over the baseline DS-CNN while achieving reduction in computational costs and model size. The use of KD in training the ST-HybridNet does not result in any tangible change in accuracy.

We perform exhaustive search of feature extraction hyperparameters and model hyperparameters to develop ST-HybridNet. Table 5 summarizes the hyperparameters of different configurations of ST-HybridNet along with their impact on accuracy and computational complexity. A ST-HybridNet with two convolutional layers (one standard convolutional layer followed by one DS convolutional layer) and a single depth 2 Bonsai tree with 7 nodes (3 internal and 4 leaf nodes) reduces computational requirements of a KWS model but at the cost of more than 3% accuracy loss in comparison to the baseline DS-CNN network. Even a ST-HybridNet with three convolutional layers and a depth 1 Bonsai tree with 3 nodes (1 internal and 2 leaf nodes) cannot preserve the baseline accuracy. This hyperparameter search subsequently results in designing the ST-HybridNet with three convolutional layers and a depth 2 Bonsai tree for KWS application in this work.

As ternary \(W_a\) and full-precision \(\text{vec}(A)\) weights of the ST-HybridNet are both fixed during inference, they are learned jointly as collapsed full-precision \(\hat{a}\) \((W_a\text{vec}(A))\) from scratch, and these full-precision \(\hat{a}\) weights along with the bias parameters occupy 7.34KB out of 14.99KB of the ST-HybridNet. The limited memory of microcontroller systems motivates further reducing the numerical precision of the entire network model, including the inputs, outputs, activations, and remaining full-precision weights, to minimize

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\(^3\)During inference, the batch normalization parameters (beta, moving mean, and moving variance) are folded either into the full-precision bias parameters of the preceding convolution layers and/or into the full-precision \(\text{vec}(A)\) parameters of the ST-HybridNet.
Table 5. Different network hyperparameters of ST-HybridNet and their impact on accuracy and number of operations. D = depth of tree, N = number of tree nodes.

| NETWORK     | MODEL HYPERPARAMETERS | ACC. (%) | OPS   |
|-------------|-----------------------|----------|-------|
| ST-HybridNet| 2 CONVOLUTIONAL       | 91.1     | 1.53M |
|             | LAYERS, D=2, N=7      |          |       |
| ST-HybridNet| 3 CONVOLUTIONAL       | 93.15    | 2.39M |
|             | LAYERS, D=1, N=3      |          |       |
| ST-HybridNet| 3 CONVOLUTIONAL       | 94.51    | 2.4M  |
|             | LAYERS, D=2, N=7      |          |       |

Table 6. Model quality and memory footprint after quantizing weights and activations of pre-trained ST-HybridNet. Memory footprint denotes the total memory required for storing weights and activations of a network during inference. 1KB = 1024 bytes.

| NETWORK                               | ACC. (%) | OPS   | MODEL size | TOTAL MEMORY FOOTPRINT |
|---------------------------------------|----------|-------|------------|------------------------|
| DS-CNN                                | 94.4     | 2.7M  | 22.07KB    | 37.7KB                 |
| ST-HybridNet Quantized (Fully 8b activations) | 94.13    | 2.4M  | 10.54KB    | 26.17KB                |
| ST-HybridNet Quantized (mixed 8b/16b activations) | 94.71    | 2.4M  | 10.54KB    | 41.8KB                 |

Quantization of activations and remaining full-precision weights of strassenified hybrid network. Quantization can convert these high-precision floating-point weights and activations of ST-HybridNet to a low-precision fixed-point format more amenable for deployment in resource-constrained microcontrollers. This can also ensure faster inference through the use of fixed-point integer operations rather than floating-point operations.

We follow the quantization procedure described in (Qiu et al., 2016; Zhang et al., 2017) for quantizing the remaining full-precision weights and activations of the pre-trained ST-HybridNet. The full-precision weights and activations of the pre-trained ST-HybridNet are quantized progressively, one layer at a time, by finding the optimal min/max range for each layer that minimizes the loss in accuracy because of quantization. Table 6 captures the accuracy, model size and total memory required for storing the weights and activations of the quantized ST-HybridNet model during inference. We assume that the memory for activations is reused across different layers and, hence, the memory requirement for the activations uses the maximum of two consecutive layers (output activations from a preceding layer and input activations to the following layer). As shown in Table 6, quantizing activations of our ST-HybridNet to 8 bits reduces the model size to 10.54KB and the total memory footprint to 26.17KB, albeit with a very small loss in accuracy of 0.27%. This is primarily attributed to the intermediate activations (activations produced post-convolution with strassen matrix \(W_s\)) of the strassenified depthwise convolutions of two DS layers that require 16 bits to represent their range precisely and preserve baseline accuracy. Quantizing the \(\hat{W}\) weights and the intermediate activations of the depthwise convolution layers to 16 bits and the remaining full-precision weights and activations to 8 bits in our ST-HybridNet not only recovers the small accuracy loss of the quantized ST-HybridNet with fully 8 bits activations, but also achieves marginally better accuracy than the baseline quantized DS-CNN network, possibly owing to better regularization because of quantization. The quantized ST-HybridNet with mixed 8/16 bits activations (16 bits for strassenified depthwise convolution layers) reduces model size to 10.54KB and requires an overall memory footprint of 41.8KB. Out of 41.8KB of total footprint, 31.25KB of memory is primarily attributed to the storage of 16 bits intermediate activations of strassenified depthwise layers. Remaining layers of ST-HybridNet require at most 15.63KB of memory during inference for storing activations of two consecutive layers.

In summary, the quantized ST-HybridNet reduces model size by 52.2% and overall memory footprint by 30.6% while incurring a negligible loss in accuracy when compared to the baseline quantized DS-CNN. It is important to note that Table 6 captures accuracy results with quantizing weights and activations of the pre-trained ST-HybridNet. In other words, the ST-HybridNet here is not retrained post quantization. We believe this 0.27% drop in accuracy with quantizing activations to 8 bits can be recovered via integrating the quantization process into the training procedure of ST-HybridNet. We leave this exploration for future work.

5 COMPARATIVE ANALYSIS

In recent years, numerous research efforts have been devoted to compressing neural networks for deployment in resource-constrained environments through the use of model pruning, quantization, low-rank matrix factorization, compact network architecture design, etc. ST-HybridNet falls into the category of compact architecture design for IoT applications. In order to demonstrate the efficacy of ST-HybridNet over other model compression techniques, we apply state-of-the-art pruning and quantization techniques to the baseline DS-CNN network and present their performance in this section.

Model pruning. Pruning away unimportant connections induces sparsity in a neural network, thereby reducing the number of nonzero-valued parameters in the model. Recent works (Han et al., 2015; Narang et al., 2017; Zhu & Gupta, 2017) on model pruning have shown that common networks have significant redundancy and can be pruned dramatically during training with marginal to no degradation in the model accuracy. By reducing nonzero parameters of a network, model pruning attempts to reap improvements
Table 7. Model size and accuracy tradeoff for DS-CNN, the current state-of-the-art for KWS application.

| SPARSITY | NONZERO PARAMETERS | ACC. (%) |
|----------|--------------------|----------|
| 0%       | 23.18K             | 94.4     |
| 50%      | 11.59K             | 94.03    |
| 75%      | 5.79K              | 92.37    |
| 90%      | 2.31K              | 87.41    |

in inference time and energy-efficiency. In addition to the storage for the nonzero model elements, a pruned model requires to store auxiliary data structures for indexing these elements resulting in additional storage overhead. On top of that the specialized routines involved with sparse matrix computations of a pruned model require considerable sparsity in associated matrices to realize any benefit in runtime owing to their irregular computation pattern and under utilization of any available SIMD units. Typically, a sparsity level of 70% or above is required in order for a sparse matrix computation to observe any benefit in runtime than the corresponding dense matrix computation.

We follow the gradual pruning technique proposed in (Zhu & Gupta, 2017) to prune the parameters of the baseline DS-CNN network. (Zhu & Gupta, 2017) gradually prunes the small magnitude weights to achieve a preset level of network sparsity. Table 7 compares the performance of sparse DS-CNN models pruned to varying extents. As shown in Table 7, although a 50% sparse DS-CNN model causes marginal loss in accuracy, it will be hard for the DS-CNN model to realize any benefit with this sparsity either in runtime, due to the sparse matrix computation, or in model size, due to the overhead from storing indices when compared to ST-HybridNet. Nevertheless, as different model pruning techniques (Guo et al., 2016; Aghasi et al., 2017; Wen et al., 2016; He et al., 2017; Luo et al., 2017; Yang et al., 2018; Gordon et al., 2018) are orthogonal to our compression scheme, they can be used in conjunction with ST-HybridNet to further reduce model size.

Model quantization. As mentioned previously, the baseline DS-CNN network in Table 3 uses an 8-bit fixed-point quantized format to represent weights. In order to observe the impact of binary/ternary quantization (Courbariaux et al., 2015; Rastegari et al., 2016; Lin et al., 2017; Cai et al., 2017; Li & Liu, 2016; Zhu et al., 2016), we apply ternary weight quantization (Li & Liu, 2016) over the baseline DS-CNN network. Ternary quantization of the weights of DS-CNN reduces the model size to 9.92KB but drops prediction accuracy significantly (by 2.27%). Any increase in the size of the DS-CNN network to recover the accuracy loss while using ternary quantization will lead to an increase in the number of MAC operations. Recent work on BinaryCmd (Fernández-Marqués et al., 2018) achieves significant reduction in KWS model size but at the cost of 3.4% accuracy loss compared to the baseline DS-CNN network.

Low-rank matrix factorization. Besides pruning and quantization, low-rank matrix factorization techniques (Jaderberg et al., 2014; Tai et al., 2015; Wen et al., 2017) exploit parameter redundancy to obtain low-rank approximations of weight matrices without compromising model accuracy. Strassen matrices of our ST-HybridNet can adopt these prior proposals to further reduce model size and computational complexity.

Compact network architectures. Much research has been done in recent years on developing compact architectures (Chen et al., 2014; Sainath & Parada, 2015; Sun et al., 2016; Arik et al., 2017; Zhang et al., 2017; Li et al., 2017; Fernández-Marqués et al., 2018; Myer & Tomar, 2018; Coucke et al., 2018) for keyword spotting on resource-constrained environments. Recent work on EdgeSpeechNets (Lin et al., 2018) produces good results albeit with significantly higher computational complexity. It is targeted for mobile processors (Arm Cortex-A53) as it requires at least 10x more MAC operations than our baselines and proposed ST-HybridNet, all of which are primarily targeted for microcontrollers. EdgeSpeechNet is well beyond the constrained compute and storage budget of typical microcontrollers described in (Zhang et al., 2017).

6 Conclusion and Future Work

We have presented a hybrid network architecture for a keyword spotting application capable of giving start-of-the-art accuracy levels while requiring a fraction of the model parameters and considerably fewer operations per inference pass. The hybrid architecture makes this possible by leveraging a few neural DS layers to extract features from the audio input and feeding those features to a shallow Bonsai decision tree to perform the classification. Furthermore, StrassenNets is used to significantly reduce the model size. The reduction in computation from the Bonsai tree, the parameter-efficiency of the DS convolutional layers, and the model footprint reduction provided by StrassenNets all combine to make the KWS model much more amenable to run on a highly constrained IoT device.

In the next iterations of this work, we will explore different algorithmic ways to constrain the number of additions in a strassenified network dominated with DS layers or specifically pointwise convolutions (e.g. MobileNets architecture) and develop architectures or specialized hardware suitable for such changes. This will not only enable a more homogeneous network architecture, but also will pave the way for incorporating StrassenNets into the next generation microcontrollers while maintaining acceptable computational costs and model size. We leave this exploration for future work.
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