AUGMENTED ROBUST PCA FOR FOREGROUND-BACKGROUND SEPARATION ON NOISY, MOVING CAMERA VIDEO

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ABSTRACT

This work presents a novel approach for robust PCA with total variation regularization for foreground-background separation and denoising on noisy, moving camera video. Our proposed algorithm registers the raw (possibly corrupted) frames of a video and then jointly processes the registered frames to produce a decomposition of the scene into a low-rank background component that captures the static components of the scene, a smooth foreground component that captures the dynamic components of the scene, and a sparse component that isolates corruptions. Unlike existing methods, our proposed algorithm produces a panoramic low-rank component that spans the entire field of view, automatically stitching together corrupted data from partially overlapping scenes. The low-rank portion of our robust PCA model is based on a recently discovered optimal low-rank matrix estimator (OptShrink) that requires no parameter tuning. We demonstrate the performance of our algorithm on both static and moving camera videos corrupted by noise and outliers.

Index Terms— Robust PCA, foreground-background separation, total variation, denoising, random matrix theory.

1. INTRODUCTION

Video processing methods are an important class of algorithms in computer vision because video data is a rich source of semantic information. In this work, we focus on the problem of robust foreground-background separation, where one seeks to decompose a scene into a static background and dynamic foreground in the presence of noise or other corruptions. Decompositions of this form are useful because the constituent components play important roles in various computer vision problems, such as motion detection [1], object recognition [2], moving object detection [3], [4] and video coding [5].

Decompositions of the form $A_1 p_1 + A_2 p_2 + r$ are useful because the constituent components play important roles in various computer vision problems, such as motion detection [1], object recognition [2], moving object detection [3], [4] and video coding [5].

The vast majority of video data gathered today is captured by moving (e.g., handheld) cameras. As such, it is necessary to register the raw video—i.e., convert it into a common coordinate system—before the frames of the video can be jointly processed. In this work, we adopt the standard perspective projection model [16], in which we relate different views of the same scene via homographic transformations.

2. VIDEO REGISTRATION

The vast majority of video data gathered today is captured by moving (e.g., handheld) cameras. As such, it is necessary to register the raw video—i.e., convert it into a common coordinate system—before the frames of the video can be jointly processed. In this work, we adopt the standard perspective projection model [16], in which we relate different views of the same scene via homographic transformations.

2.1. Registering two frames

Consider a point $(x, y)$ in a frame that is known to correspond to a point $(\tilde{x}, \tilde{y})$ in another frame. Under a planar surface model, one can relate the points via a projective transformation of the form

$$\kappa \tilde{p} = H^T p,$$

where $\tilde{p} = [\tilde{x}, \tilde{y}, 1]^T$, $p = [x, y, 1]^T$, $\kappa \neq 0$ is an arbitrary scaling constant, and $H \in \mathbb{R}^{3 \times 3}$ with $H_{13} = 1$ is the transformation matrix that we would like to estimate. Given $d > 3$ correspondences $\{(x_i, y_i) \mapsto (\tilde{x}_i, \tilde{y}_i)\}_{i=1}^d$, one can estimate $H$ in a least squares sense by minimizing

$$\min_h \|Ah\|^2 \text{ subject to } h_9 = 1,$$

where $h = \text{vec}(H)$, $A^T = [A_1^T, \ldots, A_9^T]$, and

$$A_i = \begin{bmatrix} 0 & p_i^T & -\tilde{y}_i p_i^T \\ p_i^T & 0 & -\tilde{x}_i p_i^T \end{bmatrix} \in \mathbb{R}^{2 \times 9}.$$
The solution to (2) is the (scaled) smallest right singular vector of $A$.

Of course, to estimate $H$ in practice one must also solve the correspondence problem of identifying pairs of candidate correspondences $(x_i, y_i) \mapsto (x'_i, y'_i)$ between the video frames. In this work, we adopt the standard procedure [16] of computing Speeded-Up Robust Features (SURF) [17] for each frame and then using the Random Sample Consensus (RANSAC) [18] algorithm to find a robust subset of correspondences from among the candidate features that produce a solution $H$ to (2) with small cost.

### 2.2. Registering a video

One can readily extend the two-frame registration procedure from Section 2.1 to a video by iteratively constructing homographies $H_k := H_{k \times k + 1}$ between frames $k$ and $k + 1$ of the video and then chaining the homographies together to map all $p$ frames to a common reference perspective (e.g., the middle frame, $k = \lfloor p/2 \rfloor$). Since consecutive frames of a video are highly correlated, the homographies $H_k$ can be computed with high accuracy.

Indeed, let $F_1, \ldots, F_p \in \mathbb{R}^{n \times b}$ denote the frames of a moving camera video, and denote by $H_k := H_{k \times k + 1}$ the linear transformation that applies the projective transformation (1) defined by $H_k$ to each pixel of $F_k$. One can register the frames of the video against an anchor frame $\tilde{k}$ by computing for each $k = 1, \ldots, p$,

$$F_k = \begin{cases} (H_{k-1} \circ H_{k-2} \circ \cdots \circ H_{k+1}) (F_k) & k < \tilde{k}, \\ F_k & k = \tilde{k}, \\ (H_{k-1} \circ H_{k-2} \circ \cdots \circ H_{k+1}) (F_k) & k > \tilde{k}. \end{cases}$$

(4)

The above procedure yields $\tilde{F}_1, \ldots, \tilde{F}_p \in \mathbb{R}^{m \times n}$, a collection of registered frames in a common perspective, where $m$ and $n$ are the height and width of the region defined by the union of the registered frame extents. See Figure 1 for a graphical depiction.

### 3. AUGMENTED ROBUST PCA ALGORITHM

In this section, we describe our augmented robust PCA algorithm for noisy, moving camera video. Given the registered frames $\tilde{F}_1, \ldots, \tilde{F}_p \in \mathbb{R}^{m \times n}$ of a moving camera video, we construct the matrix $Y \in \mathbb{R}^{mn \times p}$

$$Y = [\text{vec}(\tilde{F}_1) \ldots \text{vec}(\tilde{F}_p)],$$

whose columns are the vectorized registered frames. Associated with $Y$, we also define the mask matrix $M \in \{0, 1\}^{mn \times p}$ whose columns encode the support of the registered frames in the aggregate (common) view extent (see Figure 1).

The representation (5) is useful because each row of $Y$ corresponds to a fixed point in space, so we can readily apply standard static-camera models for foreground-background separation. In particular, in this work, we model the observed data $Y$ using the following (approximate) structured low-rank plus sparse model

$$P_M(Y) \approx P_M(L + S_1 + S_2),$$

(6)

where $P_M$ denotes the orthogonal projection onto $M$, defined as

$$[P_M(Y)]_{ij} = \begin{cases} X_{ij} & M_{ij} = 1 \\ 0 & M_{ij} = 0. \end{cases}$$

(7)

In (6), the $L$ component represents the (static) background, which we model as low-rank; $S_1$ represents sparse corruptions, which we model as a sparse matrix; $S_2$ is the foreground, which we model as a smoothly-varying matrix; and we use $\approx$ to allow for additional dense corruptions. To learn a decomposition of the form (6), we propose to solve the augmented robust PCA problem

$$\min_{L, S_{1,2}} \frac{1}{2} \|P_M(Y - L - S_1 - S_2)\|_F^2 + \lambda_L \|L\|_* + \lambda S_1 \|S_1\|_1 + \lambda S_2 \text{TV}(S_2).$$

(8)

Here, $\|\cdot\|_*$ denotes the nuclear norm (sum of singular values), $\|\cdot\|_1$ denotes the element-wise $\ell_1$ norm, and $\text{TV}(\cdot)$ denotes the total variation (TV) regularizer, a popular approach for reconstructing an image from noisy observations [19]. In particular, in this work, given a matrix $X \in \mathbb{R}^{mn \times p}$ whose columns contain the vectorized $m \times n$ spatial frames, we use the weighted anisotropic TV of $X$:

$$\text{TV}(X) := \sum_{i,j,k} \left( w_{i+j}^x |x_{i+j+k} - x_{i+j}| + w_{ijk}^y |x_{i+j+k} - x_{i+j}| + w_{ijk}^z |x_{i+j+k} - x_{i+j+k+1}| \right),$$

(9)

where $x = \text{vec}(X)$ and, with slight abuse of notation, we use $x_{i+j}$ to denote the pixel $(i, j)$ from frame $k$—i.e., the $(i + m(j - 1), k)$ entry of $X$. Here, $w_{i+j} \in \{0, 1\}$ are (fixed) indicator variables that omit first differences involving unobserved pixels, i.e., those that lie outside the extent of the registered frames. The $w_{ijk}$ can be readily computed from mask $M$ (see Figure 1).

### 3.1. Minimization strategy

One can solve (8) iteratively using the proximal gradient method [20], for which the $L$ updates would involve applications of singular value thresholding (SVT) [21]. However, motivated by recent work [22], we consider a modified $L$ update based on an improved low-rank matrix estimator (OptShrink) [23], which has been shown to
produce superior low-rank components in practice. Our proposed (modified) proximal gradient scheme thus becomes
\[
U^{k+1} := P_H(L^{k+1} + S_1^{k+1} + S_2^{k+1} - Y),
\]
\[
L^{k+1} := \text{OptShrink}_k(L^k - \tau^k U^{k+1}),
\]
\[
S_1^{k+1} := \text{soft}_{\psi_{k,\lambda_2}}(S_1^k - \tau^k U^{k+1}),
\]
\[
S_2^{k+1} := \text{TVDN}_{\psi_{k,\lambda_2}}(S_2^k - \tau^k U^{k+1}),
\]
where \(\tau^k\) denotes the step size at the \(k\)-th iteration. In [10], \(\text{OptShrink}_k(\cdot)\) is the low-rank matrix estimator, defined for a given \(r > 0\) as
\[
\text{OptShrink}_k(Z) = \sum_{i=1}^r \left( -2 \frac{D_{\mu_2}(\sigma_i)}{D_{\mu_2}(\sigma_1)} \right) u_i u_i^H, \tag{11}
\]
where \(Z = U \Sigma \Gamma^H\) is the SVD of \(Z\). The OptShrink estimator computes the rank \(r\) truncated SVD of its input and then applies a particular data-driven shrinkage to the leading singular values. See [23] for more details. Note that, since our data \(Y\) is registered, we can ideally model the video background as static, in which case the low-rank component \(L\) would be a rank-1 matrix whose columns are repeated (up to scaling) vectorized copies of the static background image. Thus, the universal parameter \(r = 1\) is a natural choice, and we have essentially eliminated a tuning parameter from our model compared to the SVD approach. Also, \(\text{soft}(\cdot)\) is the element-wise soft thresholding operator
\[
\text{soft}_\lambda(z) = \text{sign}(z)(|z| - \lambda)_+ , \tag{12}
\]
where \((z)_+ = \max(z, 0)\). Finally,
\[
\text{TVDN}_\lambda(Z) := \arg\min_Z \frac{1}{2} \|Z - X\|^2_F + \lambda \text{TV}(X) \tag{13}
\]
is the solution to the (weighted) total variation denoising problem data with \(Z\) (i.e., the proximal operator of \(\text{TV}(\cdot)\)). Problem [13] does not have a closed-form solution, so one must employ an iterative algorithm. To that end, we can equivalently express [12] as
\[
\min_{x} \frac{1}{2} \|z - x\|^2_F + \lambda \|WCx\|_1, \tag{14}
\]
where \(z = \text{vec}(Z)\), \(W\) is a diagonal matrix with \(0/1\) entries encoding the indicator variables \(w_{ij,k}\), and \(C\) is a sparse matrix that computes the first differences along each dimension of the (vectorized) video frame. We solve [14] via the alternating direction method of multipliers [24], which prescribes the updates
\[
x^{k+1} = \arg \min_x \frac{1}{2} \|z - x\|^2_F + \frac{\rho}{2} \|WCx\|_2^2 - v^k + u^k \|_2^2 ,
\]
\[
v^{k+1} = \arg \min_v \lambda \|v\|_1 + \frac{\rho}{2} \|WCx^{k+1} - v + u^k \|_2^2 ,
\]
\[
u^{k+1} = u^k + WCx^{k+1} - v^{k+1},
\]
for some \(\rho > 0\). In the static camera case—when \(w_{ij,k} \equiv 1\) in [10]—and circular boundary conditions are assumed, one can efficiently compute the solution to the \(x\) update in [15] using fast Fourier transform operations. In the general case, the \(x\) update is quadratic and can be computed via many off-the-shelf algorithms (e.g., conjugate gradient). The \(v\)-update is a simple soft thresholding operation, \(v^{k+1} = \text{soft}_{\psi_{k,\lambda_2}}(WCx^{k+1} + u^k)\). Algorithm [1] summarizes the proposed algorithm.

\[\text{See equation (14) of [23] and Algorithm 1 of [23] and the surrounding text for the full description and intuition behind OptShrink.}\]

4. RESULTS

To demonstrate the performance of our proposed algorithm, we first compare to the recent RPCA [8], TVRPCA [13], and DECOLOR [15] algorithms on corrupted static camera videos. We then demonstrate the ability of our algorithm to process corrupted moving camera videos, a scenario that the other methods cannot handle.

4.1. Static camera

We work with the I2R dataset of static camera sequences. Each sequence has between 523 and 3584 frames, each with a subset of 20 frames with labeled foreground masks. We select a subset of several hundred (contiguous) frames from each sequence containing 10 labeled frames.

To evaluate the denoising capabilities of each algorithm, we measure the peak signal-to-noise ratio of the foreground (f-PSNR) and background (b-PSNR) pixels, respectively. We also measure the ability of each algorithm to isolate the foreground by applying a simple thresholding strategy to the foreground component of each algorithm (\(S_2\) for our proposed method, \(S\) for RPCA, and \(F\) for TVRPCA) and then computing the F-measure of these estimated masks with respect to the labeled masks. Here, F-measure is defined in terms of the precision and recall of the estimated mask as
\[
F_{\text{measure}} = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}, \tag{16}
\]
where \(F_{\text{measure}} = 1\) corresponds to perfect accuracy. We tune the parameters of each algorithm for each metric and dataset individually.

Tables [1] and [2] compare the performance of the algorithms on 12R sequences corrupted by 20% outliers (salt and pepper) and Gaussian noise with 30dB SNR, respectively. Tables [3] and [4] show the performance of each algorithm on the Hall sequence as a function of outlier probability and SNR, respectively. Our proposed method performs better than the other methods in most cases.

The performance of RPCA and DECOLOR degrades dramatically when outliers are added because they lack the ability to separate outliers and other non-idealities from the dynamic foreground component. While TVRPCA performs better than these methods in the presence of outliers, our proposed method consistently achieves higher foreground PSNR and F-measure, which suggests that our algorithm can better decompose the scene into foreground and background components.

4.2. Moving camera

Next, we demonstrate the performance of our proposed method on a moving camera sequence from a recent benchmark dataset [25].

\[\text{See http://perception.12r.a-star.edu.sg/bk_model/bk_index.html for DECOLOR, we use the foreground mask returned by the algorithm.}\]
Fig. 2: Proposed algorithm applied to the Tennis dataset corrupted by 30% outliers. \(Y\): registered corrupted frames; \(L\): reconstructed (panoramic) background; \(S_1\): decoupled sparse corruptions; \(S_2\): dynamic foreground; \(L + S_2\): reconstructed scene.

| Sequence   | Proposed | f-PSNR | b-PSNR | F-measure | Proposed | f-PSNR | b-PSNR | F-measure | Proposed | f-PSNR | b-PSNR | F-measure |
|------------|----------|--------|--------|-----------|----------|--------|--------|-----------|----------|--------|--------|-----------|
| Hall       | 38.94    | 37.98  | 0.60   | 27.12     | 32.63   | 0.19   | 36.70   | 34.72   | 0.60   | 27.02  | 31.83  | 0.17       |
| Fountain   | 39.73    | 35.48  | 0.74   | 26.99     | 32.06   | 0.21   | 36.87   | 35.48   | 0.72   | 26.89  | 30.69  | 0.15       |
| Escalator  | 33.15    | 31.56  | 0.72   | 23.45     | 26.27   | 0.35   | 30.91   | 30.96   | 0.69   | 23.27  | 22.17  | 0.25       |
| Water Surface | 42.14 | 36.96  | 0.94   | 22.92     | 31.45   | 0.40   | 40.14   | 36.81   | 0.82   | 22.12  | 20.66  | 0.26       |
| Shopping Mall | 40.26   | 39.83  | 0.74   | 25.06     | 34.62   | 0.31   | 37.43   | 40.88   | 0.73   | 25.01  | 31.42  | 0.26       |

Average: 38.84 36.36 0.75

The Tennis sequence consists of 35 frames, each with resolution 480 × 854, of a camera panning across a tennis court with a player swinging a racket in the foreground.

Figure 2 shows the outputs of Algorithm 1 on the Tennis sequence corrupted by 30% outliers (salt and pepper). The parameters used were \(\tau = 0.33\), \(r = 1\), \(\lambda_{S_1} = 0.001\), and \(\lambda_{S_2} = 0.001\). Our proposed method gracefully aggregates the background information from the corrupted frames to produce a clean panoramic estimate \((L)\) of the full field of view. Also, the TV-regularized component \((S_2)\) is able to estimate the dynamic foreground (person) and decouple it from the sparse corruptions \((S_1)\). None of the methods considered in Section 4.1 can produce comparable results.

5. CONCLUSION

We proposed an augmented robust PCA algorithm for jointly estimating the foreground and background of a scene from noisy, moving camera video. Our proposed approach relies on a recently-developed low-rank matrix estimator (OptShrink) and weighted total variation regularization to recover the respective components of the scene. Our experimental results indicate that our algorithm is robust to both dense and sparse corruptions of the raw video and yields superior foreground-background separations compared to existing methods. In future work, we hope to investigate the usefulness of the foreground components produced by our algorithm for computer vision tasks like object tracking and activity detection.
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