Mechanism for enhancement of superconductivity in multi-band systems with odd parity hybridization

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Abstract. The study of multi-band superconductivity is relevant for a variety of systems, from ultra-cold atoms with population imbalance to particle physics and condensed matter. As a consequence, this problem has been widely investigated and has brought to light many new and interesting phenomena. In this work we point out and explore a correspondence between a two-band metal with a $k$-dependent hybridization and a uniformly polarized fermionic system in the presence of spin–orbit coupling (SOC). We study the ground state phase diagram of this metal in the presence of an attractive interaction. We find remarkable superconducting properties whenever hybridization mixes orbitals of different parities in neighboring sites. We show that in this case hybridization enhances superconductivity and drives the crossover from weak to strong coupling which is analogous with the SOC in cold atoms. We obtain the quantum phase transitions between the normal and superfluid states, as the intensities of different parameters characterizing the metal are varied, including Lifshitz transitions, with no symmetry breaking associated with the appearance of soft modes in the Fermi surface.

Keywords: Bose–Einstein condensation (theory), quantum phase transitions (theory), superfluidity
1. Introduction

The development and progress of the techniques for studying ultra-cold atomic systems has made them an ideal and clean platform for investigating condensed matter systems. They allow us to tune the relevant interactions over a large range and consequently to explore the phase diagrams of these many-body systems. More recently, the spin–orbit interaction has been implemented in cold atoms [1, 2] revealing still richer phase diagrams. This interaction allows for quantum phase transitions which do not present the usual symmetry-breaking phenomenon of the Landau paradigm and are best characterized in terms of topological transitions.

This work focus on the study of asymmetric superfluids [3], where different types of quasi-particles coexist at a common Fermi surface [4]. These may be atomic systems, with atoms in different nuclear states [5], colored superconductors, as found in the core of neutron stars [6–8], where the particles are different quarks or condensed matter systems where the quasi-particles are electrons arising from different orbitals. A common parameter that characterizes asymmetric superfluids is the mismatch $\delta k_F$ between the Fermi wave-vectors associated with different quasi-particles. Previously, we investigated [9, 10] the quantum phase diagram (i.e. at $T = 0$) of these superfluids in the limit of large mismatches where they are initially in the normal phase. As the mismatch is reduced they present an instability to an inhomogeneous superfluid state, characterized by a space modulated order parameter, known as the Fulde–Ferrell–Larkin–Ovchinnikov (FFLO) phase [11]. In this work we study the opposite limit of small mismatches where the ground state is a homogeneous superfluid.
Although our study has a wider range of applicability, as pointed out above, for concreteness we present and focus our discussions in the context of condensed matter physics. We consider a two-band metal with inter-band attractive interactions and a $k$-dependent hybridization between them. We find that the superfluid properties of these systems have a crucial dependence on the nature of the hybridization between the states.

In our previous work we have shown that a $k$-independent hybridization is detrimental to superconductivity and may even destroy it at a superconducting quantum critical point [12]. Here we show that, even when we take into account a non-local mixing, such that the hybridization acquires a $k$-dependence, if it is symmetric, i.e. $V(-k) = V(k)$, its effect on superconductivity remains deleterious. More important, we show that if hybridization is anti-symmetric, i.e. $V(-k) = -V(k)$, it enhances superconductivity and promotes a crossover from pure Bardeen–Cooper–Schrieffer (BCS) to Bose–Einstein condensation (BEC) of pairs, even for weak couplings. Anti-symmetric or odd-parity hybridization occurs when it mixes orbitals in neighboring sites with different parities. This is the case when the angular momentum $l$ of these orbitals differ by an odd number as for the case with $l$ and $l+1$, like s-p, p-d or d-f orbitals. The case of p-d hybridization is relevant for the high-$T_c$ superconductors [13] and that of d-f for heavy fermion materials [14–16]. Since doping or pressure can control hybridization our results have exciting consequences for these systems.

We find a remarkable correspondence between the two-band problem with anti-symmetric hybridization (ASH) and that of a single band system with broken inversion symmetry in the presence of Rashba spin–orbit coupling (SOC). In fact depending on the functional form of the $k$-dependence of the hybridization there is an exact mapping of one problem onto another. This is quite useful since the problem of superfluidity in the presence of spin orbit interaction has recently received a lot of attention and yielded many new and interesting results [17–25]. The correspondence allows the bridging of the two fields, bringing the results from one to the other.

Technically we adopt here a different strategy to that used to study the SOC problem. There, the general approach is to introduce the helicity basis in which the kinetic part of the Hamiltonian together with the Zeeman and spin–orbital terms are diagonal. Then a BCS decoupling is performed to deal with the many-body attractive interaction which is written in the helicity basis [23]. The superfluid order parameter now contains triplet and singlet contributions arising from pairing states with the same or opposite helicities, respectively [23–27]. In our case, we treat all terms of the Hamiltonian, that consist of the kinetic part, the hybridization (or SOC), the BCS decoupled attractive interaction and the Zeeman term, on the same footing. This allows us to consider a single order parameter instead of several pairing amplitudes that arise in the helicity basis [23]. Of course both methods should yield equivalent results, as we discuss below.

In the course of our investigation we were interested in observing the effects of changing hybridization and/or a magnetic field in our problem. Eventually this gave rise to Fermi surface instabilities that we studied from the point of view of Lifshitz transitions.

Finally, notice that as a mathematical tool, instead of using generalized Bogoliubov transformations, we use Green’s functions and the equations of motion method.
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2. Model and formalism

We consider a model with two types of quasi-particles, $a$ and $b$, arising from different atomic orbitals with an attractive inter-band interaction $g$, and a hybridization term $V(k) = V_k$ that mixes different quasi-particle states [3, 16, 28]. This one-body mixing term $V_k$ is related to the overlap of the wave functions on the same or neighboring sites and can be tuned by external parameters, like pressure or doping. The Hamiltonian is given by

$$H = \sum_{k\sigma} \epsilon_k^a a_{k\sigma}^\dagger a_{k\sigma} + \sum_{k\sigma} \epsilon_k^b b_{k\sigma}^\dagger b_{k\sigma} - g \sum_{k\sigma} a_{k\sigma}^\dagger b_{-k-\sigma}^\dagger b_{-k-\sigma} a_{k\sigma} + \sum_{k\sigma} \left( V_k a_{k\sigma}^\dagger b_{k\sigma} + V_k^* b_{k\sigma}^\dagger a_{k\sigma} \right)$$

(1)

where $b_{k\sigma}^\dagger$ and $b_{k\sigma}^U$ are creation operators for the $a$ and $b$ quasi-particles, respectively and $g > 0$. The dispersion relations $\epsilon_k^l = \frac{\hbar^2 k^2}{2m_l} - \mu_l$ ($l = a, b$), where we allow for different masses and chemical potentials. We set $\hbar = 1$. The motivation for considering inter-band attractive interactions is that, as argued by many authors, in heavy fermions the main contribution to superconductivity is due to hybrid or inter-orbital pairs involving $f$-electrons and conduction electrons, which arise from the dominant Kondo interaction [29]. For the copper oxides in some versions of the $d$-$p$ model [30], it is argued that the $d$-$p$ interaction has a predominant role [31].

We now point out an important feature of the Hamiltonian equation (1). If the bands $a$ and $b$ are taken as the up-spin and down-spin bands of a polarized single band system, formally the hybridization term now mixes different spin states as the spin–orbit interaction. Then, depending on the $k$-dependence and the symmetry properties of $V_k$, the two-band problem above becomes formally identical to that of a non-centrosymmetric system in the presence of Rashba spin–orbit interaction [32]. Specifically, this is the case if hybridization is anti-symmetric i.e. $V(-k) = -V(k)$ and $\left| V(k) \right|^2 = \beta^2 k_\perp^2$, where $k_\perp^2 = k_x^2 + k_y^2$, as we will discuss below.

Thus, the Hamiltonian equation (1) describes either a hybridized two-band system or a polarized single band material with spin-dependent mixing. In both cases, there is an attractive interaction between the different quasi-particles. In spite of the formal similarity, spin degrees of freedom are important and distinguish the two-problems: the hybridization problem mixes fermions in different bands with the same spin; and the spin–orbit Rashba interaction mixes fermions in the same orbital band, but with different spins. This distinction becomes most important in the presence of a magnetic field.

We will consider here the ground state phase diagram of a 3d $s$-wave superfluid described by equation (1). The order parameter that characterizes the superfluid phase is $\Delta_{ab} = g \sum_{k\sigma} \langle b_{-k-\sigma} a_{k\sigma} \rangle$.

Within the BCS approximation, equation (1) can be exactly diagonalized, either using a generalized Bogoliubov transformation or using the equations of motion for the Green’s function [3, 33]. Here we use the latter method and obtain the anomalous correlation functions $\langle b_{-k-\sigma} a_{k\sigma} \rangle$ from the corresponding anomalous Greens function, $\langle \langle a_{k\sigma}, b_{-k-\sigma} \rangle \rangle_{\omega}$ [27, 33].

$^4$ The mapping is $a_{\uparrow}^\dagger = c_{\uparrow}^\dagger$, $a_{\downarrow} = c_{\downarrow}$, $b_{\uparrow}^\dagger = c_{\uparrow}^\dagger$, $b_{\downarrow} = c_{\downarrow}$ for $\sigma = \uparrow$ or $\downarrow$, with $\epsilon_{\uparrow} = \epsilon_{\uparrow} - h$, $\epsilon_{\downarrow} = \epsilon_{\downarrow} + h$ and $V(k) = \lambda(k_{\perp} - ik_{\parallel})$ [32].
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The poles of the Green’s function also yield the spectrum of excitations in the superconducting phase. Excitonic types of correlations that simply renormalize the hybridization \[34\] are neglected. Finally, the anomalous frequency dependent propagator, from which the order parameter can be self-consistently obtained, is given by \[3\],

\[
\langle \{ a_{k\sigma}; b_{-k-\sigma} \} \rangle = \frac{\Delta_{ab} D_k(\omega)}{\omega^4 + C_k \omega^2 + F_k}.
\]

As we will see below, the values of the quantities \(C_k\), \(D_k\) and \(F_k\) depend in a crucial manner on the symmetry properties of the hybridization \(V_k\) under space inversion symmetry. We distinguish between two cases: symmetric hybridization, such that \(V(−k) = V(k)\), and anti-symmetric, where \(V(−k) = −V(k)\). ASH occurs when one mixes orbitals with angular momenta \(l\) and \(l + 1\) in neighboring sites. This is the case of the \(V_{df}\) hybridization between orbitals \(d\) and \(f\) in rare-earth and actinide based systems \[15\] or \(V_{pd}\) as in transition metals oxides \[13\]. Due to the different parities of the orbitals with orbital momenta \(l\) and \(l + 1\), the hybridization breaks the inversion symmetry and is odd in \(k\). This occurs even for centrosymmetric systems, like a cubic lattice, where, assuming for example that \(k\) is in the x-direction, one gets \[15, 35\] \(V_k \propto \sin k_x a\). ASH does not mix states at the band edges \(k = 0\) and \(k = \pi\) (\(a = 1, 2\)). In the former case it is similar to the Rashba spin–orbit coupling.

3. Symmetric hybridization

This is the case where \(V(−k) = V(k)\). Using this property in the equations of motion, where terms of the type \(V(−k)\) arise due to the BCS interaction that mixes states with opposite momenta, the anomalous frequency dependent propagator is given by equation (2) with \[3\],

\[
D_k(\omega) = \Delta_{ab}^2 - |V_k|^2 - (\omega - \epsilon_k^f) (\omega + \epsilon_k^f),
\]

and

\[
C_k = \left[ -\epsilon_k^a + \epsilon_k^b - 2 (\Delta_{ab}^2 + |V_k|^2) \right], \quad F_k = \left[ \epsilon_k^a \epsilon_k^b - \left( |V_k|^2 - \Delta_{ab}^2 \right) \right]^2.
\]

The poles of the propagators yield the energies \((\omega_{1,2}(k)\) and \(\omega_{3,4}(k) = −\omega_{1,2}(k)\)) for the excitations in the superconducting phase. Also from the discontinuity of the Greens function, equation (2), on the real axis we can obtain the anomalous correlation function characterizing the superconducting state.

The condition for having excitations with zero energy is

\[
F_k = \left[ \epsilon_k^a \epsilon_k^b - \left( |V_k|^2 - \Delta_{ab}^2 \right) \right]^2 = 0.
\]

For a constant hybridization \(V_k = V_0\), this occurs for \(V_0 = \Delta_{ab}\), in which case gapless excitations appear at \(k = k^*_F\) and \(k = k^F\), where \(\epsilon_k^a = 0\) and \(\epsilon_k^b = 0\).

The energy of the excitations obtained from the poles of equation (2) are given by

\[
\omega_{1,2}(k) = \sqrt{A_k \pm \sqrt{B_k}}
\]
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with

$$A_k = \frac{\varepsilon_k^2 + \varepsilon_k^2}{2} + \Delta_{ab}^2 + |V_k|^2$$

(6)

and

$$B_k = \left(\frac{\varepsilon_k^2 - \varepsilon_k^2}{2}\right)^2 + |V_k|^2 \left(\varepsilon_k^2 + \varepsilon_k^2\right)^2 + \Delta_{ab}^2 \left(\varepsilon_k^2 - \varepsilon_k^2\right)^2 + 4 |V_k|^2 \Delta_{ab}^2.$$ 

(7)

3.1. A two-band system with hybridization

Let us apply these results for a two-band superconductor in a zero external magnetic field with the ratio of the quasi-particle masses given by \(m_a/m_b = \alpha\). For simplicity we assume that the dispersion relations of these bands are given by \(\varepsilon_k^2 = \alpha \varepsilon_k^2 = \alpha \varepsilon_k\). The condition for the existence of zero energy modes is given by

$$\alpha \varepsilon_k^2 - (|V_k|^2 - \Delta_{ab}^2) = 0.$$ 

(8)

This equation can be conveniently normalized and rewritten as:

$$\alpha(\kbar^2 + \kperp^2 - 1) - (|\tilde{V}_k|^2 - \tilde{\Delta}_{ab}^2) = 0.$$ 

(9)

where \(\kbar = k/k_F\), \(\tilde{V}_k = V_k/E_F\), \(\tilde{\Delta}_{ab} = \Delta_{ab}/E_F\), where \(k_F\) and \(E_F = k_F^2/2m_a\) are the Fermi wave vector and Fermi energy of the unhybridized system, respectively. Also \(k_{\perp} = \sqrt{k_{\perp}^2 + k_{\parallel}^2}\).

For a constant hybridization, equation (9) is satisfied for \(V = \Delta_{ab}\) and \(k_{\parallel}^2 + k_{\perp}^2 = 1\), i.e. at the Fermi surface of the unhybridized system. In this case, when hybridization increases from zero, there is a discontinuous quantum first-order phase transition from the superconductor to the normal state as it reaches the critical value \(V_c = \Delta_{ab}\). This is associated with an instability of the whole Fermi surface of the system with respect to zero energy excitations.

In real systems in many cases mixing occurs among orbitals of different sites and the k-dependence of the hybridization must be taken into account. Let us consider the case of YbAlB\(_2\), where mixing occurs mainly in a plane [16] and can be modeled by \(V_k = \beta k_F^2\). Substituting this expression for \(V_k\) in equation (9), we see that the condition for zero modes is now quite different from the constant \(V\) case. For \(\beta k_F^2 < \Delta_{ab}\) the system is a standard superconductor with gaped excitations. However, at \(\beta k_F^2 = \Delta_{ab}\), where we define \(\tilde{\beta} = \beta k_F^2 / E_F\), there is a zero temperature phase transition to a superconducting state with a line of zero energy excitations at the Fermi surface of the unhybridized system. This line occurs for \(\tilde{k}_\parallel = 1\), \(\tilde{k}_\perp = 0\). As \(\tilde{\beta}\) increases this line splits in two, one in each hemisphere of the Fermi surface, as shown in figure 1.

This quantum phase transition (QPT) occurs without any symmetry breaking since, as shown below, the system remains a superfluid for \(\tilde{\beta} > \tilde{\beta}_c = \tilde{\Delta}_{ab}\). This transition is a Lifshitz transition and the associated quantum critical exponents are well known [36, 37].

Figure 1 shows the surfaces of zero energy modes for \(\tilde{\beta} > \tilde{\beta}_c = \tilde{\Delta}_{ab}\). These surfaces cross the original Fermi surface at two circles, one in each hemisphere, where the
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Figure 1. Contour plots of equation (9) for $\alpha = 0.5$, $\Delta_{ab} = 0.15$ and $\beta = 0.3$ (dotted) and $\beta = 0.16 > \beta_c = \Delta_{ab}$ (thick line). The dashed line is a projection of the Fermi surface of the unhybridized system.

Figure 2. Dispersion relations as functions of $k_z$ for a fixed value of $k_\perp$ for $\alpha = 0.5$, $\Delta_{ab} = 0.15$ and $\beta = 0.3$ for $k_\perp > \beta_c = \Delta_{ab}$. For this value of $k_\perp$ the excitations are gapless at two values of $k_z$.

energy of the excitations vanishes, as shown in this figure. Figure 2 shows the dispersion relations of the excitations for a fixed value of $k_\perp$, using the parameters of figure 1 with $\beta = 0.4 > \beta_c$. For the chosen value of $k_\perp$ there are two circles with zero energy modes.

It is important to emphasize that superconductivity survives the Lifshitz transition, at least at zero temperature. This can be verified using the self-consistent gap equation to calculate the superconducting order parameter. This equation can be written as

$$\frac{1}{\rho g} = \frac{V}{(2\pi)^3} A_\pi \left\{ \int_0^{k_F + \delta} dk_z \int_0^{\sqrt{(k_F - \delta)^2 - k_z^2}} dk_{\perp} \rho_{f_a}(k_z, k_{\perp}) \right. $$

$$- \left. \int_0^{k_F - \delta} dk_z \int_0^{\sqrt{(k_F - \delta)^2 - k_z^2}} dk_{\perp} \rho_{f_a}(k_z, k_{\perp}) \right\}, \tag{10}$$

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\[ \pi \omega = \omega_{\omega} = \begin{vmatrix} \begin{array}{c} f_{kk} k_{T} k_{T} \end{array} \end{vmatrix} + \begin{vmatrix} \begin{array}{c} a z \end{array} \end{vmatrix} + \begin{vmatrix} \begin{array}{c} B_{2} B_{2} \end{array} \end{vmatrix} (1). \]

The energies \( \omega \) above are given by equation (5) and are functions of \( \epsilon_{\kappa} \) and \( \alpha \), since we are using homothetic bands, such that \( \epsilon^{k}_{a} = \epsilon_{a} \) and \( \epsilon^{k}_{b} = \alpha \epsilon_{a} \). Furthermore,

\[ \epsilon_{a} = E_{F}(k_{F}^{2} + k_{\perp}^{2} - 1). \]

We solve the gap equation, equation (10), as a function of \( \beta \) at zero temperature. As shown in figure 3, the order parameter \( \Delta_{ab} \) remains finite even for \( \beta_{c} > \beta_{c} \). However, \( \Delta_{ab} \) is sensitive to the Lifshitz transition and for sufficiently large \( \beta \) superfluidity is eventually destroyed continuously at a quantum critical point [37].

3.2. Weak to strong coupling crossover

In the case when the attractive interaction becomes sufficiently strong, we have to solve self-consistently the number and gap equations to obtain the chemical potential and the order parameter. We consider the two-band case and, as usual, when dealing with the strong coupling limit, we introduce the scattering length \( a_{s} \) as a convenient renormalization that allows us to eliminate the ultraviolet divergence in the gap equation. This can then be written as:

\[ - \frac{m}{4\pi a_{s}} = \sum_{k} \left( \frac{1}{\omega_{1} + \omega_{2}} - \frac{1}{(1 + \alpha) \epsilon_{k}} \right). \]

The energies \( \omega_{1,2}(k) \) are given by equations (5) and as before we use the homothetic relations, \( \epsilon^{k}_{b} = \alpha \epsilon^{k}_{a} = \alpha \epsilon_{a} \). The number equation is given by,

\[ N = \sum_{k} \left( 1 - \frac{(1 + \alpha) \epsilon_{k}}{\omega_{1} + \omega_{2}} \right), \]
where $N = N_a + N_b$ is the total number of electrons in the two bands. Equations (12) and (13) determine the gap and the chemical potential of the two-band system. The calculations are implemented by substituting the sums by integrals, $\sum_k \rightarrow \frac{1}{2\pi^2} \int dk_{\perp} \int dk_{z} k_{\perp}$, where we took a unitary volume, with the limits of the integrals extending to $\infty$, since they now converge because the integrands vanish in this limit.

In figures 4–6, we show the gap and the chemical potential as functions of the ratio $1/k_F a_s$ for different values of the hybridization strength $\tilde{\beta}$. For easier convergence of the integrals we used the form of the hybridization $\tilde{V} = \tilde{\beta} k_{\perp}$, varying linear with $k_{\perp}$ as in the SOC problem. As $\tilde{\beta}$ increases, it requires a minimum value of the attractive interaction for superconductivity to be stabilized in the system. This is in agreement with the weak coupling results that have shown the deleterious effect of the symmetric hybridization in superconductivity. It is then natural to expect that, above a critical value of $\tilde{\beta}$, a

**Figure 4.** Zero temperature order parameter $\Delta = \bar{\Delta}_{ab}$ and chemical potential as functions of the ratio $1/k_F a_s$ for the case when the hybridization $\tilde{V} = \tilde{\beta} k_{\perp}$ with $\tilde{\beta} = 0.1$. The ratio of the masses of the quasi-particles is taken as $\alpha = 0.1$.

**Figure 5.** Zero temperature order parameter $\Delta = \bar{\Delta}_{ab}$ and chemical potential as functions of the ratio $1/k_F a_s$ for $\tilde{V} = \tilde{\beta} k_{\perp}$ with $\tilde{\beta} = 0.5$. The ratio of the masses of the quasi-particles is taken as $\alpha = 0.1$. The crossover to the BEC regime occurs for smaller ratios $1/k_F a_s$ as the hybridization strength $\tilde{\beta}$ increases. Notice the existence of a superconducting quantum critical point (SQCP) at a minimum critical value of the coupling $1/k_F a_s$ for superconductivity to appear.

\begin{align*}
\text{where } N &= N_a + N_b \text{ is the total number of electrons in the two bands. Equations (12) and (13) determine the gap and the chemical potential of the two-band system. The calculations are implemented by substituting the sums by integrals, } \\
\sum_k &\rightarrow \left( \frac{1}{2\pi^2} \right) \int dk_{\perp} \int dk_{z} k_{\perp}, \text{ where we took a unitary volume, with the limits of the integrals extending to } \infty, \text{ since they now converge because the integrands vanish in this limit.}
\end{align*}
minimum value for the attractive interaction is required to stabilize superconductivity. In our case this is clearly associated with the presence of a quantum critical point at a critical value of the coupling \(1/k_Fa_s\). Notice that if \(\tilde{\beta}\) is sufficiently large \(\tilde{\beta} = 0.8\) superconductivity appears already in the BEC region where the chemical potential is negative, as shown in figure 6.

3.3. Lifshitz transitions in a polarized single band system

The problem treated in this section is essentially that of Pauli limiting in superconductors [38] but with additional features brought about by the presence of a \(k\)-dependent hybridization.

Let us now consider the cases \(a\) and \(b\) which are up and down spin-bands with the degeneracy raised by an external longitudinal magnetic field \(h\), such that,

\[
\begin{align*}
\epsilon^a_k &= \epsilon_k + h \\
\epsilon^b_k &= \epsilon_k - h,
\end{align*}
\]

where \(\epsilon_k = k^2/2m - \mu\). Hybridization now mixes different spin bands, but in the symmetric case, i.e. with \(V(-k) = V(k)\), it does not correspond to any real physical interaction in a polarized single band system. As it turns out to be interesting to study this case, we can imagine it arises from an external \(k\)-dependent transverse magnetic field \(h_x(k) = V(k) = \gamma k_\perp\) applied in the \(x\)-direction, alongside the longitudinal uniform Zeeman magnetic field \(h\). Furthermore, since further down we consider the anti-symmetric case that corresponds to a Rashba spin–orbit coupling, comparing the two cases will show the profound influence the symmetry properties of \(V(k)\) have on the phase diagram of the system.

Substituting equations (14) in the expressions for the energy of the quasi-particle excitations, equations (5), these simplify considerably and we get

\[
\omega_{1,2}(k) = E_k \pm \sqrt{|V_k|^2 + h^2},
\]
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Figure 7. The zero temperature order parameter $\Delta = \widehat{\Delta}_{ab}$ as a function of the external longitudinal Zeeman magnetic field for different values of the transverse field. The dotted line marks the Lifshitz transition at which Fermi points appear on the Fermi surface. The arrows point to the fields for which the Lifshitz transitions occur for the case $\tilde{\gamma} = 0.11$. We have used as numerical parameters $g_\rho = 0.25$ and $\delta = 0.05$.

where $E_k = \sqrt{\epsilon_k^2 + \Delta_{ab}^2}$ and $|V_k|^2 = \gamma k_\perp^2$ with $\gamma$ the intensity of the transverse field (we use $\gamma$ instead of $\beta$ to distinguish from conventional hybridization). The condition for having zero energy modes, $F_k = 0$ (see equation (3)), is now given by

$$\epsilon_k^2 - h^2 - \gamma^2 k_\perp^2 + \Delta_{ab}^2 = 0. \quad (16)$$

This equation can be written in the form

$$\left(\tilde{k}_x^2 + k_\perp^2 - 1\right)^2 - \tilde{h}^2 - \gamma^2 k_\perp^2 + \tilde{\Delta}_{ab}^2 = 0, \quad (17)$$

where $\tilde{k} = k / k_F$, $\tilde{h} = h / E_F$, $\tilde{\Delta}_{ab} = \Delta_{ab} / E_F$ and $\tilde{\gamma} = (\gamma k_F) / E_F$, where $E_F$ is the Fermi energy.

Defining the functions,

$$\mathcal{F}^\pm = 4\pi \int_0^{1/\delta} d\tilde{k}_z \int_0^{\sqrt{(1/\delta)^2 - \tilde{k}_z^2}} d\tilde{k}_1 \tilde{k}_1 \left(\tilde{\omega}_1 + |\tilde{\omega}_2|\right), \quad (18)$$

where $\tilde{\omega}_i = \omega_i / E_F$, with $\omega_i$ given by equations (15), and $\delta$ is a momentum cutoff (normalized by $k_F$), the gap equation can be cast in the form,

$$\frac{1}{g\rho} = \mathcal{F}^+ - \mathcal{F}^-. \quad (19)$$

In the absence of the transverse field, $\gamma = 0$, the order parameter is constant up to a critical longitudinal field $h_c = \Delta_{ab}$, at which there is a first-order quantum phase transition to the normal state where the order parameter $\Delta_{ab}$ vanishes abruptly, as shown in figure 7. This instability is that associated with the Pauli limiting in superconductors [38] and arises due to the appearance of zero energy modes at the whole Fermi surface of the non-polarized system. This is also a Lifshitz transition, in this case
associated with a broken symmetry since it is accompanied by the disappearance of superconductivity.

For \( \gamma \neq 0 \), the superconducting phase is also destroyed by the longitudinal magnetic field, though the transition instead of being abrupt becomes rounded due to the transverse field. We have to distinguish between two cases,

\[
\gamma \Delta \sim \frac{a}{b} \quad \text{and} \quad \gamma \Delta \gg \frac{a}{b}
\]

where \( \Delta \sim (h_0, 0) \). In the case \( \gamma \Delta \sim \frac{a}{b} \), the first Lifshitz transition occurs for \( \tilde{\Delta}_{ab} = \tilde{\Delta}_{ab} \left( \tilde{h} = 0, \gamma = 0 \right) \).

In the case \( \gamma \lesssim \Delta_{ab} \), the first Lifshitz transition occurs for \( \tilde{h}^0_t = \sqrt{(\tilde{\Delta}_{ab})^2 - \gamma^2} \), where using the numerical values of the parameters in figure 7, \( \tilde{\Delta}_{ab} = 0.148 \). This Lifshitz transition is associated with the appearance of a line of zero energy modes in the equator of the original Fermi surface, at \( k_\perp = 1, k_z = 0 \). In figure 4 for \( \gamma = 0.11 \) and using the value of \( \tilde{\Delta}_{ab} \) above, this occurs for \( \tilde{h}^0_t \approx 0.1 \). As the field further increases, the line of zero modes splits in two, one in each hemisphere of the Fermi surface (see figure 1) and finally at \( \Delta = \Delta_{ab} \left( h_1 \right) \) these shrink to points in the poles of the spherical Fermi surface.

The case \( \gamma = \Delta_{ab} \) is in its own class since, as can be seen from equation (17), for \( h = 0 \) there is a collapse of the whole Fermi surface \( (k_\perp^2 + k_z^2 = 1 = 0) \) for \( \gamma = \Delta_{ab} \). In this case superconductivity disappears abruptly in the zero external longitudinal field. This first-order transition is similar to that which occurs for \( \gamma = 0 \) and \( \tilde{h} = \Delta_{ab} \), as shown in figure 7.

As \( \gamma \) increases beyond \( \Delta_{ab} \), i.e. for \( \gamma > \Delta_{ab} = 0.148 \), using the parameters of figure 7, the Lifshitz transition now occurs exclusively for \( h_1 = \Delta_{ab} \left( h_1^1 \right) \) and is related to the appearance of Fermi points at \( k_\perp = 0, k_z = \pm 1 \), i.e. on the poles of the original Fermi sphere. The line of these transitions is also shown in figure 7 (straight dotted line). Notice that these transitions occur without necessarily destroying the superconducting phase at least at \( T = 0 \). It is worth pointing out, as can be seen from equation (17), that a zero energy mode can also appear at \( k = 0 \), for a field \( \tilde{h} = \sqrt{1 + \Delta_{ab}^2} \). This field
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is much larger than \( \bar{h}_t^0 \) and \( \bar{h}_t^1 \) considered previously and for reasonable values of the other parameters (smaller than 1), superconductivity has already been destroyed before \( h \) reaches this value.

In order to obtain a complete picture of the influence of the transverse field in the phase diagram, we show in figure 8 the effect of this field on superfluidity. For a zero external longitudinal magnetic field there is a critical value of the transverse field \( \gamma_c \) for which superconductivity disappears. Using the same numerical parameters as in figure 7, we obtain \( \gamma_c = 0.23 \) as shown in figure 8.

Let us consider the dispersion relation of the modes which soften as \( \bar{h} \rightarrow \bar{h}_t \) at \( k_\perp = 1, k_z = 0 \), as for the case \( \gamma \leq \widetilde{\Delta}_{ab} \). This is given by

\[
\omega(k_\perp) = (\bar{h}_t - \bar{h}) + 2\frac{(k_\perp - 1)^2}{\widetilde{\Delta}_{ab}}.
\]

where \( \bar{h}_t^1 = \sqrt{\left(\widetilde{\Delta}_{ab}\right)^2 - \gamma^2} \). This expansion is possible since the order parameter \( \widetilde{\Delta}_{ab} \) remains finite at the Lifshitz transition. The gap vanishes linearly close to this transition with a characteristic exponent \([37]\) \( \nu z \approx 1 \), while the spectrum in this case is quadratic in momentum.

4. Anti-symmetric hybridization

This is the case \( V(\mathbf{-k}) = -V(\mathbf{k}) \). This situation may arise in non-centrosymmetric lattices but more interestingly occurs also in symmetric lattices, if we consider hybridization among orbitals with opposite parities in neighboring sites, such as p-d or d-f hybridization that mixes orbitals with angular momentum \([15]\) \( l \) and \( l + 1 \). The former is relevant for the high-\( T_c \) oxides and the latter for heavy fermion materials and actinide metals in general \([15]\). Furthermore, many of the most interesting heavy fermion systems have tetragonal structures with rare-earths and transition metals in the planes perpendicular to the \( c \)-axis, such that d-f hybridization occurs predominantly.
in this plane. Additional effects due to crystal fields may also constrain mixing to take place mostly in the \( ab \) plane \[16\].

Using \( V(-k) = -V(k) \) in the equations of motion for the Green’s functions, we find that the quantities \( C_k, D_k \) and \( F_k \) in equation (3) are modified and the energy of the excitations in the superconducting phase are now given by \( \pm \omega_{1,2}(k) \), where

\[
\bar{\omega}_{1,2}(k) = \sqrt{\tilde{A}_k} \pm \sqrt{\tilde{B}_k}
\]  

(21)

with

\[
\tilde{A}_k = \frac{\epsilon_k^2 + \epsilon_{1k}^2}{2} + \Delta_{ab}^2 + |V_k|^2
\]

(22)

and

\[
\tilde{B}_k = \left( \frac{\epsilon_k^2 - \epsilon_{1k}^2}{2} \right)^2 + |V_k|^2 \left( \epsilon_k + \epsilon_{1k} \right)^2 + \Delta_{ab}^2 \left( \epsilon_k - \epsilon_{1k} \right)^2.
\]

(23)

The condition for zero energy modes now is given by

\[
\left( \epsilon_k^2 + \Delta_{ab}^2 - |V_k|^2 \right)^2 + 4\Delta_{ab}^2 |V_k|^2 = 0.
\]

(24)

Since this condition for any given \( V_k \) cannot be satisfied, there is no Lifshitz transition in this case. The situation is quite different in the presence of a longitudinal external magnetic field where a pair of Fermi points appears at a Lifshitz transition \[39\].

The gap equation is also modified by the anti-symmetry property of \( V(k) \), which is now given by

\[
\frac{1}{g} = \sum_k \frac{1}{4\sqrt{\tilde{B}_k}} \left[ \frac{\left( \epsilon_k^2 - \epsilon_{1k}^2 \right)^2}{2} \left[ \frac{\tanh(\beta\bar{\omega}_1/2)}{\bar{\omega}_1} - \frac{\tanh(\beta\bar{\omega}_2/2)}{\bar{\omega}_2} \right] 
+ \sqrt{\tilde{B}_k} \left[ \frac{\tanh(\beta\bar{\omega}_1/2)}{\bar{\omega}_1} + \frac{\tanh(\beta\bar{\omega}_2/2)}{\bar{\omega}_2} \right] \right]
\]

(25)

and the number equation (at \( T = 0 \)) given by

\[
N = \sum_k \left\{ 1 - \frac{\epsilon_k^2 + \epsilon_{1k}^2}{2(\bar{\omega}_1 + \bar{\omega}_2)} \bar{\omega}_1 \bar{\omega}_2 + \varphi_k \right\}
\]

(26)

where \( \varphi_k = \epsilon_k^2 \bar{\omega}_1 + \Delta^2 - |V_k|^2 \) and \( N = N_s + N_v \).

Introducing the scattering length, as before we solve self-consistently the equations above at zero temperature to obtain results for the superconducting gap and the chemical potential in the case of nearly two-dimensional (2d) systems where hybridization occurs mostly in a plane. Furthermore we take the functional form \( |V(k)| = \beta_k \) to be similar to the Rashba coupling. This is actually the form of \( |V_{ab}(k)| \) for the square lattice of the CuO\(_2\) planes in the tight-binding approximation \[35\] and in the limit of small \( k \). For simplicity we use the homothetic relations, \( \epsilon_{1k} = \alpha \epsilon_k \) with \( \epsilon_k = \epsilon_{1k} = k^2 / 2m - \mu \).

A remarkable result is shown in figures 9 and 10 where we plot the gap and the chemical potential for a mass ratio \( \alpha = 0.5 \) and \( 1/(k_F a_S) = -0.5 \) as functions of the

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Figure 10. The zero temperature chemical potential $\tilde{\mu}$ as a function of the strength of the hybridization for two mass ratios, $\alpha = 0.5$ (full line) and $\alpha = 0.25$ (dashed line). As hybridization increases and $\tilde{\mu}$ becomes negative, the main mechanism of superconductivity is the condensation of hybridons (see text).

intensity of the hybridization $\tilde{\beta}$ ($| V(k) | = \tilde{\beta} k_\perp$). Differently from the previous case of symmetric hybridization, as $\tilde{\beta}$ increases superconductivity is enhanced as the strength of the ASH $\tilde{\beta}$ increases. This is indicated by the increase of the renormalized gap $\tilde{\Delta}_{ab}$ with $\tilde{\beta}$, as shown in figure 9. Furthermore, as $\tilde{\beta}$ increases the chemical potential drops and becomes negative, signaling a change of regime from BCS superconductivity to BEC of pairs. Notice that this occurs for a value of the interaction $1/(k_F a_{S}) = -0.5$ which is typical of the weak-coupling BCS regime [40]. This behavior was noted previously in the context of cold atomic systems with spin–orbit interactions [26]. In the presence of Rashba SOC there is a bound state of two spin 1/2 fermions for any scattering length, negative and positive. Then the Rashba SOC increases the tendency of the fermions for pairing. The same effect occurs in the two-band problem, with ASH playing a role similar to the Rashba interaction. In the present context, we call these bound states, formed by fermions in different bands due to ASH, hybridons. They are closely connected with the enhancement of superconductivity; the anticipation of the BEC regime shown in figures 9 and 10, respectively. Considering that the two-band problem with ASH and $| V(k) | = \tilde{\beta} k_\perp$ maps exactly onto that of a polarized single band system ($\alpha = 1$) with Rashba SOC, both in the presence of attractive interactions $^5$, it is not unexpected that we find similar behavior in both problems.

On the other hand, the dispersion relations of the superconducting quasi-particles for symmetric and ASH are different (see equations (7) and (23)) and not just by the functional form of $| V(k) |$. Then, in this case there is no reason to expect the same physical behavior.

In the context of condensed matter physics, the phenomenon above acquires significance due to the sensitivity of hybridization on doping and external pressure in multi-band systems. Increasing $V_{pd}$ or $V_{df}$ by either of these mechanisms provides a means not only for enhancing the critical temperatures but also to drive the BCS–BEC crossover in this type of superconductor. The class of materials with tetragonal structures for which non-symmetric hybridization occurs, namely the high-$T_c$ oxides [13] with $V_{dp}(k)$ and the many heavy fermions [15, 16] with $V_{df}(k)$, is of great interest.

$^5$ See footnote 4.

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In spite of the formal similarities pointed out above between the odd parity hybridization case and the spin–orbit problem where the two species of fermions are labeled by spin, there is an important difference between these two problems. In the latter, when one diagonalizes the kinetic energy and the spin–orbit coupling, the new quasi-particle operators involve a linear combination of creation and annihilation operators of electrons with different spins \[41\]. In the former, the new quasi-particle operators that diagonalize the kinetic energy terms plus the hybridization involve also a linear combination of the original band operators but with the same spin. A direct consequence of this difference is that, in the spin–orbit problem, when the interaction terms of the BCS mean field Hamiltonian, \[ H_{\text{BCS}} = -\Delta_{ab} \sum_k \left( a_{k\sigma}^\dagger b_{-k-\sigma} + b_{-k-\sigma} a_{k\sigma} \right) \], are written in terms of the new quasi-particle operators, triplet correlations immediately arise. This is not the case in the mixing problem. If the inter-band interaction acts only in the s-wave channel it continues to do so in the new basis of hybridized states, even though anomalous induced correlations with a p-wave character can arise, as we discuss below.

A final comment concerns the role of self-energy corrections to the problem above. At finite temperatures the correct BEC temperature is obtained in the strong coupling limit of the BCS–BEC crossover only if one goes beyond the mean-field and includes the self-energy, which enters in the calculation by considering fluctuation corrections \[42\]. These corrections affect even the zero temperature behavior but in a quantitative way. However, the main point here is that we have shown that even for a fixed weak-coupling interaction, where fluctuations are negligible, the BCS–BEC crossover can be reached by varying the strength of hybridization, as shown in figure 9.

5. Intra-band interaction

Here we mention briefly the case with attractive intra-band interactions, in the narrow b-band only, and for odd parity hybridization. The condition for zero modes is given by \[3, 39\],

\[(\epsilon_k^a \epsilon_k^b - |V(k)|^2)^2 + \epsilon_k^a \Delta_{bb}^2 = 0,\]

where \(\Delta_{bb}\) is the superconducting order parameter in our notation. Then, in this case there are no zero modes unless the hybridization vanishes at the Fermi wave-vector of the a-band. It turns out from the calculations that the energy of the excitations in the superconducting phase for symmetric and anti-symmetric (odd parity) hybridizations are formally the same and differ only by the specific functional form of \(|V(k)|^2\). Furthermore, considering just the intra-band attractive interaction and a hybridization term, we find induced inter-band pairing correlations due to the hybridization in the form \[39\], \(\Delta_{ab}(k) \propto V(k) \Delta_{bb}\). Then, for odd parity \(V(k)\) the induced inter-orbital pairing is of the p-wave type. A reverse effect occurs in the inter-band case treated here, but with \(\Delta_{ab}(k) \propto V(k) \Delta_{ab}\) \[39\]. Notice that induced gaps do not appear in the zero mode equations.
6. Comparison with other approaches

Instead of diagonalizing the full Hamiltonian, equation (1), with the attractive interaction treated in the BCS approximation, it is a common approach in the literature [18, 22, 23, 32] to use the helicity basis and write the attractive interaction in this basis. The helicity basis is that which diagonalizes the part of the Hamiltonian containing the kinetic energy, the Rashba coupling and the Zeeman term. This has as eigenvalues [23],

$$\epsilon_k^\pm = \epsilon_k \pm h^2 + |V_k|^2,$$

(27)

where $\pm$ refers to helicity states. If one uses a BCS approximation and writes the attractive interaction in this helicity basis, the energies of the quasi-particles in the superconducting state are obtained as [23, 32],

$$\Omega^\pm(k) = \sqrt{(E_k^\pm \sqrt{|V_k|^2 + h^2})^2 + |\Delta^\pm|^2},$$

(28)

where $E_k = \sqrt{\epsilon_k^2 + |\Delta^\pm|^2}$ and $\Delta_{\eta\lambda}$ pair states with the same or different helicities ($\eta, \lambda = \pm$).

On the other hand if we substitute in equations (22) and (23), $\epsilon_k^\pm \rightarrow \epsilon_k - h$ and $\epsilon_k^\pm \rightarrow \epsilon_k + h$, we obtain the same result for the energies with the identification [18, 22, 23, 32],

$$\Delta^+ = \Delta^- = \frac{h}{\sqrt{h^2 + |V_k|^2}} \Delta_{ab},$$

$$\Delta^+ = \Delta^- = \frac{-|V_k|}{2\sqrt{h^2 + |V_k|^2}} \frac{k_x + ik_y}{k_z} \Delta_{ab}. $$

(29)

It is interesting to note that the limits $h \rightarrow 0$ and $V$ or $\gamma \rightarrow 0$ ($V = \gamma k_{\perp}$) of the expressions above do not commute. Indeed, for $h \rightarrow 0$ and $V$ finite, we find,

$$\Delta^{+-} = 0$$

$$\Delta^{++} = \Delta^{*-} = -\frac{k_x + ik_y}{k_z} \Delta_{ab}$$

(30)

while, for $V \rightarrow 0$ and $h$ finite, we obtain,

$$\Delta^{+-} = \Delta_{ab}$$

$$\Delta^{++} = \Delta^{*-} = 0.$$  

(31)

This is related to the fact that space inversion and time reversal operations do not necessarily commute [44]. In our approach, which diagonalizes the full Hamiltonian with BCS, SOC and Zeeman terms, such ambiguity does not arise. The relation $4|\Delta^{++}|^2 + |\Delta^{+-}|^2 = |\Delta_{ab}|^2$ which follows from equations (29) implies that the order parameter $\Delta_{ab}$ used here has contributions from pairing both the same and different helicity states.

7. Conclusions

We have studied the effects of a non-local $k$-dependent hybridization on superconductivity in a two-band system with inter-band interactions. We have focused on the limit
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of small mismatches between the Fermi wave-vectors of these bands, where the system is always a superfluid at \( T = 0 \).

Hybridization is a key concept in chemistry and solid state physics. In the latter case it arises from the mixing of different orbitals by the crystalline potential. It can occur locally at an atomic site for non-orthogonal wave-functions, as in the case of \( s - d \) and \( s - f \) mixing. It also takes place between orbitals in neighboring sites and in this case mixing can involve generic orbitals. Most interesting, as we have shown here, is when hybridization takes place in neighboring sites among orbitals with different parities. This is the case of orbitals with angular momentum \( l \) and \( l + 1 \) which is like \( s-p \), \( p-d \) and \( d-f \) orbitals. In this case \( k \)-dependent hybridizations like \( V_{sp}(k) \), \( V_{pd}(k) \) and \( V_{df}(k) \) are not invariant under space inversion symmetry and have the anti-symmetric property \( V(-k) = -V(k) \), even for inversion symmetric lattices. As we have shown, this property of the hybridization has dramatic effects on superconductivity where the BCS interaction mixes states with opposite momenta\(^6\). We have shown that ASH enhances superconductivity and drives the BCS–BEC crossover, even at weak coupling. Since mixing among the orbitals can be tuned by doping or external pressure, it turns out to be a controllable mechanism for enhancement of superconductivity. Besides, it provides an important parameter for exploring the quantum phase diagrams of systems. ASH includes classes of systems which are of great interest as transition metal oxides in the case of \( V_{dp} \) and heavy fermions for \( V_{df} \) hybridization. We have also shown that the two-band problem with ASH is formally equivalent to that of a polarized single band system with a spin–orbit Rashba coupling between the spin up and down bands. This is a useful analogy since many concepts and results from one field can be easily brought to the other. In particular the mapping between the two problems allows for a deeper understanding of the role of ASH in superconductivity. As an example, the influence of SOC in 2d superconductors has been extensively studied [18–27]. These results allow us to anticipate that the effect of ASH, as that of SOC on 2d superconducting systems, should be particularly strong [10]. In 1d the effects of ASH remain to be investigated.

As in the case of SOC, ASH favors the formation of bound states between the quasiparticles, and this is the main mechanism for enhancing superconductivity in two-band systems.

For completeness, we have also studied the effect of a \( k \)-dependent symmetric hybridization in two-band superconductivity in both weak and strong coupling regimes. Differently from ASH, we have shown this acts in detriment of superconductivity and gives rise to quantum phase transitions, in some cases destroying the superfluid phase, as in the constant \( V \) case [3, 12]. These transitions are associated with changes in the Fermi surface and can be generally identified as Lifshitz transitions.

Finally we point out the possible relevance of the present results for the case of superconducting transition metal oxides where d-p hybridization is important. Since this odd parity d-p hybridization can be increased by conveniently doping the system, we expect from our approach that superconductivity will be enhanced in such materials. However, we have considered here only s-wave pairing, even though the p-wave type of correlations are induced by ASH [45].

\(^6\) The \( V_{df}(k) \) hybridization is essential to understand the magnetism of the series of the actinides (see [15, 44]). However when studying magnetic properties, the anti-symmetry property of \( V_{df}(k) \) does not play a fundamental role (see [15, 44]).
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References

[1] Lin Y J, Compton R L, Perry A R, Phillips W D, Porto J V and Spielman I B 2009 Phys. Rev. Lett. 102 130401
[2] Lin Y J, Jiménez-García K and Spielman I B 2011 Nature 471 83–6
[3] Continentino M A and Padilha I T 2008 J. Phys.: Condens. Matter 20 095216
  Continentino M A and Padilha I T 2012 Phys. Lett. A 376 599
  Padilha I T and Continentino M A 2009 Physica B 404 2920
  Padilha I T and Continentino M A 2009 J. Phys.: Condens. Matter 21 095603
[4] Bedaque P F, Caldas H and Rupak G 2003 Phys. Rev. Lett. 91 247002
  Caldas H 2004 Phys. Rev. A 69 063602
[5] Southwell K 2002 Nature 416 205
[6] Sedrakian A, Clark J W and Alford M (ed) 2006 Pairing in Fermionic Systems (Singapore: World Scientific)
[7] Casalbuoni R and Nardulli G 2004 Rev. Mod. Phys. 76 263
[8] Alford M 2006 arXiv:hep-lat/0610046 and references therein
[9] Caldas H and Continentino M A 2012 Phys. Rev. B 86 144503
  Caldas H and Continentino M A 2013 J. Phys. B: At. Mol. Opt. Phys. 46 155301
  Resende M A, Mota A L, Farias R L S and Caldas H 2012 Phys. Rev. A 86 033603
[10] Caldas H, Farias R L S and Continentino M A 2013 Phys. Rev. A 88 023615
[11] Fulde P and Ferrell R A 1964 Phys. Rev. 135 A550
  Larkin A I and Ovchinnikov Y N 1965 Sov. Phys. — JETP 20 762
[12] Padilha I T and Continentino M A 2009 J. Magn. Magn. Mater. 321 3466
[13] Ramires A and Continentino M A 2011 J. Phys.: Condens. Matter 23 125701
[14] Takagi H and Ishibashi S, Ido T, Uchida S 1990 Phys. Rev. B 41 11657
  Fujimori A 1989 Phys. Rev. B 39 793
  Hifner S, Hossain M A, Damascelli A and Sawatzky G A 2008 Rep. Prog. Phys. 71 062501
[15] Si Q and Steglich F 2010 Science 329 1161
[16] Jullien R, Galleoni d’Agliano E, Coqblin B 1972 Phys. Rev. B 6 2139
  Jullien R and Coqblin B 1973 Phys. Rev. B 8 5263
[17] Ramires A, Coleman P, Nevidomskyy A H and Tsvelik A M 2012 Phys. Rev. Lett 109 176404
[18] Tewari S, Stanescu T D, Sau J D and Sarma S D 2011 New J. Phys. 13 065004
[19] Gong M, Tewari S and Zhang C 2011 Phys. Rev. Lett. 107 195303
[20] Liao R, Yi-Xiang Y and Liu W-M 2012 Phys. Rev. Lett. 108 080406
[21] Seo K, Han L and Sa de Melo C A R 2012 Phys. Rev. Lett. 109 105303
[22] Seo K, Han L and Sa de Melo C A R 2012 Phys. Rev. A 85 033601
[23] Jiang L, Liu X-J, Hu H and Pu H 2011 Phys. Rev. A 84 063618
[24] Aicea J 2010 Phys. Rev. B 81 125318
[25] He L and Huang X-G 2012 Phys. Rev. A 86 043618
  Zhou J, Zhang W and Yi W 2011 Phys. Rev. A 84 063603
  Yu Z-Q and Zhai H 2011 Phys. Rev. Lett. 107 195305
  Hu H, Jiang L, Liu X-J and Pu H 2011 Phys. Rev. Lett. 107 195304
  Anna L D, Mazzarella G and Salasnich L 2011 Phys. Rev. A 84 033633
  Liu X-J, Jiang L, Pu H and Hu H 2012 Phys. Rev. A 85 021603
[26] Sato M and Fujimoto S 2009 Phys. Rev. B 79 094504
[27] Vyasanakere J P and Shenoy V B 2011 Phys. Rev. B 83 094515
  Vyasanakere J P, Zhang S and Shenoy V B 2011 Phys. Rev. B 84 014512
[28] Coleman P and Andrei N 1989 J. Phys.: Condens. Matter 1 4057

doi:10.1088/1742-5468/2014/07/P07015
Mechanism for enhancement of superconductivity in multi-band systems with odd parity hybridization

[29] Dolgov O V, Fetsisov E P and Khomskii D J 1987 Z. Phys. B 67 63
Valadares E C, Troper A and de Menezes O L T 1985 J. Magn. Magn. Mater. 47–48 400
Koo J H and Kim J-J 2000 Phys. Rev. B 61 4289
Koo J H and Kim Y 2001 J. Korean Phys. Soc. 38 618
Caldas H and Continentino M A 2012 Phys. Rev. B 86 144503
[30] Emery V J 1987 Phys. Rev. Lett. 58 2794
Kuiper P, Kruizinga G, Ghijsen J, Grioni M, Sawatzky G A, Weijss P J W, de Groot F H M, Verweij H, Feiner L F and Petersen H 1988 Phys. Rev. B 38 6483
Feiner L F, Jefferson J H and Raimondi R 1996 Phys. Rev. B 53 8751
[31] Koo J H and Kim J-J 2000 Phys. Rev. B 61 4289
Cao T D, Guo S L, Li C Q and Cheng G S 2004 Physica C 402 388
[32] Gong M, Chen G, Jia S and Zhang C 2012 Phys. Rev. Lett. 109 105302
[33] Tyablikov S V 1967 Methods in the Quantum Theory of Magnetism (New York: Plenum) p 221
Zubarev D N 1960 Sov. Phys.—Usp. 3 320
[34] Sarusua L G and Continentino M A 2002 Phys. Rev. B 65 184503
[35] Mishonov T and Penev E 2000 J. Phys.: Condens. Matter 12 143
[36] Volovik G E 2003 The Universe in a Helium Droplet (Oxford: Oxford University Press)
[37] Continentino M A 1993 Phys. Rev. B 47 11587
Continentino M A 2001 Quantum Scaling in Many-Body Systems (Singapore: World Scientific)
[38] Clogston A M 1962 Phys. Rev. Lett. 9 266
Chandrasekhar B S 1962 App. Phys. Lett. 1 7
[39] Continentino M A, Padilha I T, Deus F and Caldas H 2014 Ann. Phys. 348 1
[40] Leggett A J 2006 Quantum Liquids: Bose Condensation and Cooper Pairing in Condensed-Matter Systems (Oxford: Oxford University)
[41] Loder F, Kampf A P and Kopp T 2013 J. Phys.: Condens. Matter 25 362201
Stewart G R 2013 J. Phys.: Condens. Matter 25 025601
[42] Nozières P and Schmitt-Rink S 1985 J Low Temp. 59 195
Sa de Melo C A R, Randeria M and Engelbrecht J 1993 Phys. Rev. Lett. 71 3202
Diener R B, Sensarma R and Randeria M 2008 Phys. Rev. A 77 023626
[43] Silva Neto M B, Castro Neto A H, Kim J S and Stewart G R 2013 J. Phys.: Condens. Matter 25 025601
[44] Lee T D and Wick G C 1966 Phys. Rev. 148 1385
[45] Continentino M A, Deus F and Caldas H 2014 Phys. Lett. A 378 1561

doi:10.1088/1742-5468/2014/07/P07015