Multidimensional gravity interacting with intersecting electric and magnetic $p$-branes is considered for fields depending on a single variable. Some general features of the system behaviour are revealed without solving the field equations. Thus, essential asymptotic properties of isotropic cosmologies are indicated for different signs of spatial curvature; a no-hair-type theorem and a single-time theorem for black holes are proved (the latter makes sense in models with multiple time coordinates). The validity of the general observations is verified for a class of exact solutions known for the cases when certain vectors, built from the input parameters of the model, are either orthogonal in minisuperspace, or form mutually orthogonal subsystems. From the non-existence of Lorentzian wormholes, a universal restriction is obtained, applicable to orthogonal or block-orthogonal subsystems of any $p$-brane system.

1. Introduction

In the weak field limits of the bosonic sectors of supergravities [1], superstring and M-theory, their generalizations and modifications [2–6] there naturally appear multiple self-gravitating scalar dilatonic fields and antisymmetric forms, associated with $p$-branes.

This paper continues the studies of such models on the basis of a general action, see (1), without fixing the total space-time dimension $D$ or other input parameters [7]–[16], thus to a large extent abstracting from the details of specific underlying models, but with a hope to predict some features of new models, unformulated by now. We will here deal with the one-variable sector of the model, where all fields depend on a single coordinate: time in cosmological models, a radial coordinate in spherically symmetric models, etc. In this case the model reduces to a Toda-like dynamical system in minisuperspace, see (15), (16).

Much work has been devoted to searches for exact solutions and their subsequent analysis. Thus, in Ref. [15], the most general one-variable solution was presented for the case when certain vectors $Y_s$ in the target space, built from the input parameters of the model, form an orthogonal system (OS). This solution describes a set of intersecting electrically and magnetically charged $p$-branes and generalized many previously found ones, beginning with Schwarzschild and Reissner-Nordström and ending with dilatonic and some more special $p$-brane solutions [17, 18, 19, 20, etc.]. The OS solution was further generalized to models where $Y_s$ form a block-orthogonal system (BOS). The OS solution is recovered when each block consists of a single vector. Other families of exact solutions have been found for cases when $Y_s$ form bases of integrable Toda models, see [14, 21] and references therein. Many solutions are known beyond the one-variable sector and references therein).

The exact solutions have disclosed many features of interest of physically relevant configurations, such as cosmological models and black holes. The generality of these features remains, however, questionable, since the equations of motion can be solved exactly only for special (though numerous) choices of the input parameters. To have an idea of what can and what cannot be expected from yet unknown solutions, it makes sense to try to extract some information directly from the equations. Such an attempt is undertaken here.

It appears possible to reveal some important properties of $p$-brane cosmologies, namely, the nature of asymptotics for different signs of spatial curvature. For spherically symmetric configurations, among other results, two theorems about black holes (BHs) are proved: a “no-hair theorem”, that a BH is incompatible with the so-called
The starting point is, as in Refs. [10–15], the model action for $D$-dimensional gravity with several scalar dilatonic fields $\varphi^a$ and antisymmetric $n_s$-forms $F_s$:

$$ S = \frac{1}{2\kappa^2} \int_{\mathcal{M}} d^Dz \sqrt{|g|} \left\{ R[g] - \delta_{ab} g^{MN} \partial_M \varphi^a \partial_N \varphi^b - \sum_{s \in S} \frac{n_s}{n_s!} e^{2\lambda_sa} F_s^2 \right\}, $$

in a (pseudo-)Riemannian manifold $\mathcal{M} = \mathbb{R}_u \times \mathcal{M}_0 \times \ldots \times \mathcal{M}_n$ with the metric

$$ ds^2 = g_{MN} dz^M dz^N = w e^{2\alpha(u)} du^2 + \sum_{i=0}^n e^{2\beta_i(u)} ds_i^2, \quad w = \pm 1, $$

where $u$ is a selected coordinate ranging in $\mathbb{R}_u \subseteq \mathbb{R}$; $g^i = ds_i^2$ are metrics on $d_i$-dimensional factor spaces $\mathcal{M}_i$ of arbitrary signatures $\varepsilon_i = \text{sign} g^i$; $|g| = |\det g_{MN}|$ and similarly for subspaces; $F_s = F_s, M_1, \ldots, M_{n_s}; \lambda_{sa}$ are coupling constants; $n_s = \pm 1$ (to be specified later); $s \in S$, $a \in A$, where $S$ and $A$ are some finite sets. All $\mathcal{M}_i, i > 0$ are assumed to be Ricci-flat, while $\mathcal{M}_0$ is allowed to be a space of constant curvature $K_0 = 0, \pm 1$.

In the one-variable sector, $\varphi^a = \varphi^a(u)$. The set of indices $S = \{ s \}$ in (1) will be used to jointly describe essentially $u$-dependent electric ($F_{el}$) and magnetic ($F_{mt}$) $F$-forms, to be associated with different subsets $I = \{ i_1, \ldots, i_k \} \ (i_1 < \ldots < i_k)$ of the set of numbers labelling the factor spaces: $\{ i \} = I_0 = \{ 0, \ldots, n \}$. Thus one can write

$$ S = \{ s \} = \{ eI_s \} \cup \{ mI_s \}. $$

A given $F$-form may have several essentially (non-permutatively) different components, both electric and magnetic; such a situation is sometimes called “composite p-branes” [16]. Conformally, we will nevertheless treat essentially different components of the same $F$-form as individual (“elementary”) $F$-forms. A subsequent reformulation to the composite ansatz is straightforward.

So, by construction, nonzero components of $F_{el}$ carry coordinate indices of $u$ and the subspaces $\mathcal{M}_i, i \in I$, those of $F_{mt}$ — the indices of $\mathcal{M}_i$, $i \in T \stackrel{\text{def}}{=} I_0 \setminus I$ since a magnetic form is built as a form dual to a possible electric one. Therefore

$$ n_{el} = \text{rank } F_{el} = d(I) + 1, \quad n_{mt} = \text{rank } F_{mt} = D - \text{rank } F_{el} = d(T), $$

where $d(I) = \sum_{i \in I} d_i$ are the dimensions of the subspaces $\mathcal{M}_I = \mathcal{M}_{i_1} \times \ldots \times \mathcal{M}_{i_k}$.

Several electric and/or magnetic forms (with maybe different coupling constants $\lambda_{sa}$) can be associated with the same $I$ and are then labelled by different values of $s$. (The index $s$ by $I$ is, however, sometimes omitted when this cannot cause confusion.)

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1There is an exception: two components having only one noncoinciding index, cannot coexist since in this case there emerge nonzero off-block-diagonal components of the energy-momentum tensor (EMT), while the Einstein tensor in the l.h.s. of the Einstein equations is block-diagonal. See more details in Ref. [16].
This problem setting covers various classes of models: isotropic and anisotropic cosmologies, \( u \) is a timelike coordinate and \( w = -1 \); static models with various spatial symmetries (spherical, planar, pseudospherical, cylindrical, toroidal), where \( u \) is a spatial coordinate, \( w = +1 \), and time is selected among \( \mathcal{M}_t \); and Euclidean models with similar symmetries or models with a Euclidean “external” space-time, where also \( w = +1 \).

A simple analysis shows that a positive energy density \(-T^t_t\) of the fields \( F^t \) is achieved in all Lorentzian models with the signature \((-+++\cdots+)\) if one chooses in (1), as usual, \( \eta_s = 1 \) for all \( s \). In more general models, with arbitrary \( \varepsilon_i \), the requirement \(-T^t_t > 0\) is fulfilled if

\[
\begin{align*}
\eta_{eI} &= -\varepsilon(I)\eta(I), & \eta_{mI} &= -\varepsilon(I)\eta(I), \\
\varepsilon(I) &= \prod_{i\in I}\varepsilon_i, & \varepsilon(I) &= \begin{cases} 1, & \mathbb{R}_t \subseteq \mathcal{M}_I, \\
-1, & \text{otherwise}
\end{cases}
\end{align*}
\]

where \( \mathbb{R}_t \) is the time axis. If \( \varepsilon(I) = 1 \), we are dealing with a genuine electric or magnetic field, while otherwise the \( F \)-form behaves as an effective scalar or pseudoscalar in the physical subspace. The latter happens, in particular, in isotropic cosmologies and their Euclidean counterparts where the time coordinate is \( u \) and \( \mathbb{R}_t = \mathbb{R}_u \), unrelated to any subset \( I \). \( F \)-forms with \( \varepsilon(I) = -1 \) will be called quasiscalar.

**Example:** consider a spherically symmetric configuration, with \( D = 6 \), \( \mathcal{M} = \mathbb{R}_0 \times \mathbb{R}_1 \times S^2 \times \mathbb{R}_4 \times \mathbb{R}_5 \), where the coordinate indices 0, 1, 4, 5, refer to time, radius and two extra dimensions, 2 and 3 to the spherical angles, respectively; thus \( \mathbb{R}_0 = \mathbb{R}_t \) and \( R_1 = R_4 \). Then, for rank \( F = 3 \), the component \( F_{012} \) is electric, \( I \mapsto (0,5) \); \( F_{234} \) is magnetic, \( I \mapsto (0,5) \); \( F_{145} \) is electric quasiscalar, \( I \mapsto (4,5) \); \( F_{023} \) is magnetic quasiscalar, \( I \mapsto (4,5) \), where the figures in parentheses are coordinate indices of the respective subspaces \( \mathcal{M}_I \).

Let us now, as in (9) and many later papers, choose the harmonic \( u \) coordinate \((\nabla^M \nabla_M u = 0)\), such that

\[
\alpha(u) = \sum_{i=0}^n d_i\beta_i^0 \equiv d_0\beta^0 + \sigma_1(u), & \sigma_1(u) \equiv \sum_{i=1}^n d_i\beta^i.
\]

The Maxwell-like equations due to (1) for the \( F \)-forms are easily integrated, giving

\[
\begin{align*}
F^u_{el}M_{1 \cdots d(I)} &= Q_{eI} e^{-2\alpha - 2\varepsilon(I)} e^{M_{1 \cdots d(I)} / \sqrt{|g_I|}}, & Q_{eI} = \text{const}, \\
F^m_{II, M_{1 \cdots d(I)}} &= Q_{ml} \varepsilon_{M_{1 \cdots d(I)}} / \sqrt{|g_T|}, & Q_{ml} = \text{const},
\end{align*}
\]

where \(|g_I| = \prod_{i\in I}|g_i|\), \( Q_s \) are charges and overbars replace summing in \( a \). In what follows we will restrict the set \( S = \{s\} \) to such \( s \) that the charges \( Q_s \neq 0 \).

Consequently, at the r.h.s. of the Einstein equations due to (1) \( R^N_M - \frac{1}{2}\delta^N_M R = T^N_M \), the energy-momentum tensor (EMT) \( T^N_M \) takes the form

\[
e^{2\alpha}T^N_M = -\frac{w}{2} \sum_{s} \varepsilon_s Q_s^2 e^{2\varepsilon(I)} \chi_s \nabla \nabla \delta^{(j)}_I, \quad \text{where } j = 1, \ [1], \ [-1]_I
\]

where the first place on the diagonal belongs to \( u \) and the symbol \([f]_I\) means that the quantity \( f \) is repeated along the diagonal for all indices referring to \( \mathcal{M}_j, \ j \in I; \sigma(I) \equiv \sum_{i\in I} d_i\beta^i; \) the sign factors \( \varepsilon_s \) and \( \chi_s \) are

\[
\varepsilon_{el} = -\eta_{eI}\varepsilon(I), & \varepsilon_{ml} = w\eta_{ml}\varepsilon(I); & \chi_{el} = +1, & \chi_{ml} = -1,
\]

so that \( \chi_s \) distinguishes electric and magnetic forms.

Let us suppose, as is usually (and reasonably) done in \( p \)-brane studies, that neither of \( I_s \) such that \( Q_s \neq 0 \) contains the index 0, that is, neither of the branes “lives” in the subspace \( \mathcal{M}_0 \), interpreted as the external space or its subspace. (This means that, e.g., in the spherically symmetric case there is no electric or magnetic field along a coordinate sphere \( \mathcal{M}_0 = S^{d_0} \).) Then each constituent EMT and hence the total EMT possess the property \( T^n_u + T^n_z = 0 \) if \( z \) belongs to \( \mathcal{M}_0 \). As a result, the corresponding combination of the Einstein equations has a Liouville form and is integrated:

\[
\begin{align*}
\ddot{\alpha} - \dot{\beta}^0 &= wK_0(d_0 - 1)^2 e^{2\alpha - 2\beta^0} = 0, \\
e^{\beta^0} &= (d_0 - 1)S(wK_0, \ k, \ u),
\end{align*}
\]
where \( k \) is an integration constant (IC) and we have introduced the notation

\[
S(1, h, t) = \begin{cases} 
  h^{-1} \sinh ht, & h > 0, \\
  t, & h = 0, \\
  h^{-1} \sin ht, & h < 0; 
\end{cases}
\]

\[
S(-1, h, t) = h^{-1} \cosh ht; \quad h > 0;
\]

\[
S(0, h, t) = e^{ht}, \quad h \in \mathbb{R}.
\] (13)

Another IC is suppressed by properly choosing the origin of the \( u \) coordinate.

With (12) the \( D \)-dimensional line element may be written in the form (\( \overline{d} \equiv d_0 - 1 \))

\[
ds^2 = \frac{e^{-2\sigma_1/\overline{d}}}{\overline{d}S(wK_0, k, u)^{2/\overline{d}}} \left[ \frac{w \, du^2}{\overline{d}S(wK_0, k, u)^2} + ds_0^2 \right] + \sum_{i=1}^{n} e^{2\beta_i} ds_i^2
\] (14)

Let us treat the remaining set of unknowns \( \beta^i(u), \varphi^a(u) \) as a real-valued vector function \( x^A(u) \) (so that \( \{A\} = \{1, \ldots, n\} \cup \mathcal{A} \) in an \( (n + |\mathcal{A}|) \)-dimensional vector space \( V \) (target space). The field equations for \( \beta^i \) and \( \varphi^a \) can be derived from the Toda-like Lagrangian

\[
L = G_{AB} \dot{x}^A \dot{x}^B - V_Q(y) = \sum_{i=1}^{n} (\dot{\beta}^i)^2 + \frac{\delta_{i1}^2}{d_0 - 1} + \delta_{ab} \varphi^a \dot{\varphi}^b - V_Q(y),
\]

\[
V_Q(y) = - \sum_s \epsilon_s Q_s^2 e^{2y_s}
\] (15)

with the “energy” constraint

\[
E = G_{AB} \dot{x}^A \dot{x}^B + V_Q(y) = \frac{d_0}{d_0 - 1} K, \quad K = \begin{cases} k^2 \text{sign } k, & wK_0 = 1; \\
k^2, & wK_0 = 0, -1.
\end{cases}
\] (16)

where the IC \( k \) has appeared in (12). The nondegenerate symmetric matrix

\[
(G_{AB}) = \begin{pmatrix} 
  d_i d_j / \overline{d} + d_i \delta_{ij} & 0 \\
  0 & \delta_{ab}
\end{pmatrix}
\] (17)

defines a positive-definite metric in \( V \); the functions \( y_s(u) \) are defined as scalar products:

\[
y_s = \sigma(I_s) - \chi_s \overline{\lambda} \overline{\varphi} = Y_s, A x^A, \quad (Y_s, A) = \left( d_i \delta_{I_s}, -\chi_s \lambda_{sa} \right),
\] (18)

where \( \delta_{II} = 1 \) if \( i \in I \) and \( \delta_{II} = 0 \) otherwise. The contravariant components and scalar products of the vectors \( \bar{Y}_s \) are found using the matrix \( G^{AB} \) inverse to \( G_{AB} \):

\[
(G^{AB}) = \begin{pmatrix} 
  \delta^{ij} d_i - 1 / \overline{d} & 0 \\
  0 & \delta^{ab}
\end{pmatrix}, \quad (Y^{s, A}) = \left( \delta_{II} - \frac{d(I)}{\overline{d}}, -\chi_s \lambda_{sa} \right);
\] (19)

\[
Y_{s, A} Y_{s'}^{-A} = \bar{Y}_s \bar{Y}_{s'} = d(I_s \cap I_{s'}) - \frac{d(I_s) d(I_{s'})}{\overline{d}} + \chi_s \chi_{s'} \overline{\lambda}_s \overline{\lambda}_{s'}, \quad \overline{d} = D - 2.
\] (20)

The equations of motion in terms of \( \bar{Y}_s \) read

\[
\ddot{x}^A = \sum_s q_s Y^{s, A} e^{2y_s}, \quad q_s = \epsilon_s Q_s^2
\] (21)

### 3. General properties of brane systems

The positive energy requirement (11) that fixes the input signs \( \eta_s \), can be written as follows for Lorentzian models using the notations (11):

\[
\epsilon_s = \epsilon_I (I_s).
\] (22)

The corresponding requirement for Euclidean models is obtained by applying the conventional Wick rotation to Lorentzian cosmologies. This rotation of the time \( t \) changes \( w \) but preserves all \( \eta_s \) as well as \( \epsilon(I) \) since \( \mathbb{R} \not\subset \mathcal{M}_t \),
Then by (1), \( \epsilon_{el} \) remain invariable while \( \epsilon_{mI} \) change. This distinction between electric and magnetic forms is also connected with the property of the duality transformation to change the sign of the EMT in Euclidean models [24, 25].

Table 1 shows the sign factors \( wK_0 \) and \( \epsilon_s = \text{sign} q_s \) for \( F \)-forms in different classes of models under the above positive energy requirement.

| \( wK_0 \) | \( w = -1 \) | \( w = +1 \) | \( w = +1 \) |
|---|---|---|---|
| electric | none | +1 | none |
| magnetic | none | +1 | none |
| electric quasiscalar | -1 | -1 | -1 |
| magnetic quasiscalar | -1 | -1 | +1 |

In what follows, we restrict ourselves to the model described in Sec. 2 with the sign factors specified in Table 1, unless specially indicated.

One general statement, to be taken into account in the subsequent proofs, can be formulated as a lemma:

**Lemma 1.** At any regular point of the space-time, for all \( a \in A \) and \( s \in S \),

\[
e^{-2\alpha (\dot{\varphi}^a)^2} < \infty, \quad e^{-2\alpha + 2\nu_s} < \infty.
\]

Indeed, regularity implies finite values of all curvature invariants, including \( R \) and \( R_N^N R_N^M \); by virtue of the Einstein equations, one must have \( T_M^N T_N^M < \infty \). Since \( T_M^N \) has a block-diagonal structure, the latter invariant can be written as a sum of squares, where each summand must thus be finite, including \((T_1^1)^2\). The component \( T_1^1 \) is in turn, due to (21), a sum of negative-definite terms, corresponding to scalar fields \( \varphi^a \) and \( F \)-forms \( F_s \). Therefore every such term must be finite, leading to (23).

### 3.1. Isotropic cosmology

Table 1 shows that in isotropic cosmologies, when \( u \) is a time coordinate and \( M_0 \) is identified with the physical space (conventionally \( d_0 = 3 \), \( \epsilon_s = -1 \): there are only quasiscalar forms since a true electric or magnetic field would violate the spatial isotropy.

The logarithm of the extra-dimension volume factor, \( \sigma_1 \), by virtue of (21) obeys the equation

\[
\dot{\sigma}_1 = \frac{d_0 - 1}{D - 2} \sum_s d(I_s) Q_s^2 e^{2\nu_s},
\]

whence \( \dot{\sigma}_1 < 0 \). So this volume factor cannot have a minimum and, moreover, if it tends to a finite value \( e^{\sigma_{10}} \) as \( u \to \pm \infty \), at other values of \( u \) it is smaller than \( e^{\sigma_{10}} \). This feature is unfavourable for obtaining models with the so-called dynamical compactification, where the size of extra dimensions decreases to microscopic scales in the course of the evolution.

Next, due to \( \epsilon_s = -1 \), both terms in the expression (11) for \( E \) are positive-definite, so that nontrivial solutions correspond to \( k > 0 \). The range of \( u \) is \( \mathbb{R} \) for \( K_0 = 0 \), +1 and (without loss of generality) \( u > 0 \) for \( K_0 = 1 \). By (13) and (14), the model asymptotics are characterized as follows.

For any \( K_0 \), at the asymptotic \( u \to +\infty \) the total volume factor \( e^{d_0 \rho^0 + \sigma_{10}} \) (which, by (11), coincides with \( e^\alpha \)) tends to zero. Although separately the physical scale factor \( a(u) = e^{\sigma_1} \) and the “internal” one, \( e^{\sigma_{10}} \), may have various limits, the behaviour of \( e^\alpha \) indicates that this asymptotic is singular. Moreover, since asymptotically \( \alpha \sim cu \), \( c = \text{const} \), \( \sigma_{10} \), the proper time \( t = \int e^\alpha du < \infty \): the singularity occurs at finite proper time.

For \( K_0 = +1 \) the other asymptotic \( u \to -\infty \) is like the one just described, due to the symmetry of the function \( \cosh k u \) in (13). Thus closed models evolve in a finite proper time interval between two singularities where the total volume of the Universe tends to zero.

For \( K_0 = 0 \), the asymptotic \( u \to -\infty \) corresponds to an infinitely growing total volume factor \( e^\alpha \) while the proper time \( t \) is also infinite. In the special case when \( \sigma_1 \to \sigma_{10} \) = const, the physical scale factor \( a \) obeys the law \( a \sim |t|^{1/d_0} \).
For $K_0 = -1$ the second asymptotic is $u \to 0$, and this is a regular point of the equations of motion (21) determining $x^A$. So the metric behaviour is (now in the general case) governed by the function $S(1,k,u) \approx u$ in (14), while all $e^{\beta^i}$, $i > 0$ and consequently $\sigma_1$ tend to finite limits. As $u \to 0$,
\[
e^\alpha \sim u^{-1-\sqrt{7}}, \quad |t| \sim u^{-1/\sqrt{7}} \to \infty, \quad a(t) = e^{\theta_0} \sim |t|,
\]
corresponding to linear expansion or contraction of the physical space.

Finally, Eq. (16) with $V_Q > 0$ implies that all $\dot{x}^A$ are bounded above, hence $x^A(u)$ are finite for all finite $u$ and cannot create a singularity. Therefore the above description of the asymptotics is quite general and applies to all isotropic cosmologies in the field model under consideration.

It should be noted, however, that this discussion concerns the model behaviour in the $D$-dimensional Einstein conformal frame, in which the action (1) was postulated. See further comments in Sec. 6.

3.2. Static spherical symmetry: general observations

In static, spherically symmetric models, where $u$ is a radial coordinate, $w = +1$, $M_0 = S^{d_0}$, $K_0 = +1$, among other $\mathcal{M}_i$ there should be a one-dimensional subspace, say, $\mathcal{M}_1$, which may be identified with time: $\varepsilon_1 = +1$. The sign factor $wK_0$ in (12) is +1, while $\varepsilon_1$ is, due to (22), +1 for normal electric and magnetic forms $F_I$ and −1 for quasiscalar ones.

By construction, see Eqs. (13), (14), spatial infinity corresponds to $u = 0$ (where the usual “area function” $e^{\theta_0} \sim u^{1/\sqrt{7}}$) and, without loss of generality, the range of $u$ is
\[
0 < u < u_{\text{max}}
\]
where $u_{\text{max}}$ is either $+\infty$, or the smallest value of $u$ where the fields lose regularity.

The experience of dealing with particular models belonging to the class (1) indicates that a generic spherically symmetric solution exhibits a naked singularity. Possible exceptions can be (i) black holes (BHs), (ii) wormholes (WHs) or wormhole-like objects with a neck and a second nonsingular asymptotic, (iii) configurations with a regular centre (a soliton-like object, which might be expected for an interacting field system) and, finally, (iv) a situation where the coordinate patch we use is incomplete, terminates at a regular sphere $u = u_{\text{max}}$ (which may be even infinitely remote in our static frame of reference), and a possible continuation may reveal either a singularity, or one of the opportunities (i)–(iii).

One can show, however, that for our model only the BH opportunity is viable. Lorentzian WHs do not exist according to (16) (see also Sec. 5), while variants (iii) and (iv) are ruled out by the following theorem:

**Theorem 1.** The present model does not admit solutions describing a static, spherically symmetric configuration (a) with a regular centre or (b) where $u = u_{\text{max}}$ corresponds to a regular surface such that $\mathcal{M}_0$ is a sphere of finite radius.

**Proof.** (a) A regular centre implies local flatness of the metric at some $u = u^*$ where $e^{\beta_0} = 0$, while other $\beta^i$ remain finite. One easily shows that with (14) it may happen only when $k = 0, u^* = u_{\text{max}} = \infty$ (otherwise the correct radius-to-circumference ratio for small circles around the centre cannot be achieved). Then due to (3), since $|\sigma_1| < \infty$,
\[
e^{\beta_0} \sim u^{1/(d_0-1)}, \quad e^\alpha \sim u^{-d_0/(d_0-1)} \quad \text{as} \quad u \to \infty.
\]
On the other hand, the EMT regularity requirement (2) (see Lemma 1) leads to $|\varphi| < \infty$ as $u \to \infty$. Therefore at such a centre the $F$-forms behave like free fields exhibiting (see (8)–(10)) a singularity, with infinite values of the EMT invariants. Item (a) is proved.

The assumption (b) means that both $\beta_0$ and $\sigma_1$ are finite at $u = u_{\text{max}}$. This cannot happen at $u_{\text{max}} < \infty$ since there would be no reason to stop at this value of $u$; and at $u_{\text{max}} = \infty$ this means that $S(1,k,\infty) < \infty$, contrary to the definition (13).

3.3. Black holes: no-hair and single-time theorems

We see that the only positive-energy Lorentzian spherically symmetric configurations without naked singularities are BHs. BH solutions of various models belonging to the class (1) have been studied in numerous recent papers (see [2, 15, 19] and references therein). However, exact solutions have been (and probably can be) only obtained

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2One might just require $|\varphi| < \infty$ as part of the centre regularity conditions. Our proof, however, also rules out a hypothetical situation when an infinite $\varphi$ value, due to the factor $e^{2\lambda\varphi}$, modifies the behaviour of $F$-forms, leading to a regular geometry.
for a small subset of the whole set of models \(^1\), and it makes sense to look for general properties of BH solutions which may be discovered without solving the equations. Two such properties, having the form of restrictions generalizing the previously observed properties of specific solutions \(\text{[15, 16]}\), are proved here.

In what follows, the word “horizon” will mean a nonsingular surface \(u = u_*\) in \(\mathcal{M}\) where some scale factors \(e^{\beta^i} = 0\) (corresponding to possibly multiple time coordinates), while other \(\beta^i\) remain finite. A BH solution is a static, spherically symmetric solution containing a horizon. These working definitions, though incomplete, are sufficient for our purposes.

An immediate observation is

**Lemma 2.** BH solutions can only exist for \(k \geq 0\) and the horizon is then at \(u = \infty\).

Indeed, at a horizon, the function \(\sigma_1\) defined in \(\text{(7)}\) tends to \(-\infty\) along with a part of its constituents, another part remaining finite. According to \(\text{(14)}\), to obtain a finite value of \(\beta^0\), one has then to require that \(S(1, k, u_*) = +\infty\), which by \(\text{(13)}\) is only possible when \(k > 0\) and \(u_* = \infty\).

Another result applies to BHs in manifolds \(\mathcal{M}\) with several time coordinates, as suggested in some recent unification models (see \(\text{[5, 26]}\) and references therein). If there is another time coordinate, some branes can evolve with it. The following theorem shows, however, that in our framework, even in a space-time with multiple times, a BH can only exist with its unique preferred, physical time, while other times are not distinguished by the metric behaviour from extra spatial coordinates.

**Theorem 2 (Single-Time Theorem).** Any BH solution with \(k > 0\) contains precisely one coordinate \(t\) such that \(g_{tt} = 0\) at the horizon.

**Proof.** Suppose that \(u = \infty\) is a horizon where some \(e^{\beta^i} \to 0\), \(i \in I_t \subseteq (I_0 \setminus 0)\). As follows from \(\text{(7)}\), at the asymptotic \(u \to \infty\) one has \(\alpha \to -\infty\) and, moreover, the finiteness of \(\beta^0\) means (see \(\text{(14)}\)) that \(\alpha \sim -ku\). On the other hand, the condition \(\text{(23)}\) holds only if for all \(F\)-forms, at most, \(e^{2y_i} = O(e^{-2ku})\). (28)

The equations of motion \(\text{(21)}\) then show that, as \(u \to \infty\),

\[
\dot{x}^A = -c^A + o(1), \quad c^A = \text{const} \tag{29}
\]

where \(c^i > 0\) for \(i \in I_t\) and \(c^i = 0\) for other \(A\) (see Remark 2).

In the constraint \(\text{(16)}\), the potential \(V_Q(u) \to 0\) due to \(\text{(28)}\), therefore

\[
G_{AB}c^A c^B = \frac{d_0}{d_0 - 1}. \tag{30}
\]

The asymptotic of \(\alpha\) and the condition \(\text{(7)}\) show that, simultaneously,

\[
\sum_{i \in I_t} d_i c^i = k, \tag{31}
\]

so that \(c_i \leq k\). From \(\text{(24)}\) with \(\text{(31)}\) and \(\text{(17)}\) it follows

\[
\sum_{i \in I_t} d_i c^i = k^2. \tag{32}
\]

Combined, Eqs. \(\text{(31)}\) and \(\text{(32)}\) lead to

\[
\sum_{i \in I_t} d_i c^i(k - c^i) = 0, \tag{33}
\]

which is compatible with \(\text{(23)}\) for \(0 \leq c^i \leq k\) only when the sum consists of one term, to be labelled \(i = 1\), such that \(d_1 = 1\) and \(c^1 = k\). This proves the theorem.

One more theorem shows that BH solutions can contain only true electromagnetic \(F\)-forms rather than quasiscalar ones.

**Theorem 3 (No-Hair Theorem).** All \(F\)-forms in a BH solution with \(k > 0\) possess the property \(\delta_i I_t = 1\), where the number \(i = 1\) refers to the time axis.

The proof rests on Lemma 1, which, applied to \(F\)-forms, leads again to \(\text{(28)}\). Now, according to Theorem 2, at a horizon \((u \to \infty)\) only \(\beta^1 \to -\infty\), while other \(\beta^i\) are finite. As is directly verified, \(\text{(28)}\) holds in the
case $\delta_1 t = 1$ (for true electromagnetic forms), while for quasiscalar ones one has finite limits for $e^{y_s}$, leading to infiniteness of the corresponding EMT constituent.

**Remark 1.** The regularity of the scalar fields, $x^A = \varphi^a$, at $u \to \infty$ was not required in the conditions of Theorems 2 and 3; for $k > 0$ it follows from (32). Under the additional requirement $\varphi^a < \infty$ as $u \to \infty$, Theorem 3 is easily proved for $k = 0$ as well.

**Remark 2.** For $k = 0$ we have no Theorem 2; moreover, Theorem 3 is not proved for $k = 0$ without assuming $\varphi^a < \infty$. Nevertheless, for BH solutions with $k = 0$ obtainable as a limit of ones with $k > 0$, the statements of both theorems remain valid. (For known exact BH solutions, $k = 0$ corresponds to the extreme limit of minimal mass for given charges.) Meanwhile, the existence of exceptional BH solutions with $k = 0$, nonzero quasiscalar forms and/or multi-time horizons is not ruled out by our study; such solutions may perhaps exist with infinite limits of scalar fields that balance the infinity of $e^{-\alpha}$ in the EMT of $F$-forms.

**Remark 3.** If there are BH solutions, there are also others, where the scale factor showing a zero value is associated, instead of physical time, with one of the extra coordinates (such solutions are obtained from BH ones by simple re-denoting). One thus finds the so-called T-holes, where crossing a horizon leads to changing the signature of the external, physical space from $(- + + \cdots)$ to $(- - + \cdots)$. Possible properties of such objects are discussed in more detail elsewhere [3, 27] within the frames of dilaton gravity, but the considerations thereof are valid as well for the more general model (1). Theorems 2 and 3 are valid for T-holes after proper re-formulation.

4. Some exact solutions

4.1. Orthogonal systems (OS)

The field equations are entirely integrated if all $\vec{Y}_s$ are mutually orthogonal in $\nu$, that is,

$$\vec{Y}_s \vec{Y}_{s'} = \delta_{ss'}/N_s^2, \quad 1/N_s^2 = d(I)[1-d(I)/D] + \lambda_s^2 > 0. \quad (34)$$

Then the functions $y_s(u)$ obey the decoupled Liouville equations $\ddot{y}_s = b_s e^{2y_s}$, with $b_s \equiv \epsilon_s Q_s^2/N_s^2$, whence

$$e^{-y_s(u)} = \sqrt{|b_s|} S(\epsilon_s, h_s, u + u_s), \quad (35)$$

where $h_s$ and $u_s$ are ICs and the function $S(\epsilon, \ldots)$ has been defined in (33). For the sought functions $x^A(u) = (\beta^a, \varphi^a)$ we then obtain:

$$x^A(u) = \sum_s N_s^2 Y_s A_y^A(u) + c A u + \zeta^A, \quad (36)$$

where the vectors of ICs $\vec{c}$ and $\vec{\zeta}$ are orthogonal to all $Y_s$: $c A Y_s A = \zeta^A Y_s A = 0$, or

$$c^i d_i \delta_{I_s} - c^\alpha \lambda_\alpha = 0, \quad \zeta^i d_i \delta_{I_s} - c^\alpha \lambda_\alpha = 0. \quad (37)$$

The solution is general for the properly chosen input parameters; the number of independent charges equals the number of $F$-forms.

4.2. Block-orthogonal systems (BOS)

Suppose now [11] that the set $S$ splits into several non-intersecting non-empty subsets,

$$S = \bigcup_\omega S_\omega, \quad |S_\omega| = m(\omega), \quad (38)$$

such that the vectors $\vec{Y}_{\mu(\omega)}$ ($\mu(\omega) \in S_\omega$) form mutually orthogonal subspaces $\nu_\omega$ in $\nu$:

$$\vec{Y}_{\mu(\omega)} \vec{Y}_{\nu(\omega')} = 0, \quad \nu \neq \omega'. \quad (39)$$

Suppose, further, that, for each fixed $\omega$, all $\vec{Y}_\nu$ (where $\nu = \nu(\omega)$) are linearly independent and the charge factors $q_\nu = \epsilon_\nu Q_\nu^2 \neq 0$ satisfy the set of linear algebraic equations

$$(\vec{Y}_\nu - \vec{Y}_{\nu'}) \vec{Z}_\omega = 0, \quad \vec{Z}_\omega \equiv \sum_{\mu \in S_\omega} q_{\mu} \vec{Y}_{\mu}, \quad (40)$$
for each pair \((\nu, \nu')\). Then the function \(y_\omega(u) := Y_{\mu(\omega)}A^A\) is the same for all \(\mu \in S_\omega\) and satisfies the Liouville equation \(\ddot{y}_\omega = b_\omega e^{2y_\omega}\). As a result, we obtain a solution to the equations of motion, generalizing (33), (36):

\[
e^{-y_\omega} = \sqrt{b_\omega} S(\text{sign} b_\omega, h_\omega, u + u_\omega),
\]

\[
x^A = \sum_\omega N_\omega^2 Y_\omega(u) + c^A u + \zeta^A,
\]

where \(h_\omega\) and \(u_\omega\) are ICs; the constants \(c^A\) and \(\zeta^A\) satisfy the same orthogonality relations (17) as for OS, that is, the vectors \(\vec{c}\) and \(\zeta\) are orthogonal to each individual \(\vec{Y}_\omega\), even if it is a member of a BOS subsystem. We have used the notations

\[
\begin{align*}
 b_\omega &= \vec{Y}_\nu(\omega) \vec{Z}_\omega; & \vec{Y}_\omega &= \vec{Z}_\omega \hat{q}_\omega; & N_\omega^{-2} &= \vec{Y}_\omega^2 = \frac{b_\omega}{\hat{q}_\omega}; & \hat{q}_\omega &= \sum_{\mu \in S_\omega} q_{\mu
}
\end{align*}
\]

Here \(b_\omega\) is nonzero and independent of \(\nu(\omega) \in S_\omega\) due to (11); moreover, \(\hat{q}_\omega \neq 0\) since \(\hat{q}_\omega = \vec{Z}_\omega^2 / b_\omega\) while the nonzero vector \(\vec{Z}\) is determined up to extension by (10).

The linear independence of \(\vec{Y}_\mu(\omega)\) thus guarantees that Eqs. (11) yield \(q_{\mu(\omega)}\) for a given subsystem up to a common factor. Therefore, unlike the OS solution, the BOS one is special: the number of independent charges coincides with \(|\{\omega\}|\), the number of subsystems; however, we thus gain exact solutions for more general sets of input parameters, e.g. a one-charge solution can be obtained for actually an arbitrary configuration of branes with linearly independent \(\vec{Y}_\mu\) (except possible cases when the solution of (11) leads to at least one zero charge).

When \(m = 1\), we have a single vector \(\vec{Y}_\omega = \vec{Y}_s\) orthogonal to all others, with the norm \(N_\omega^{-2} = N_s^{-2}\), and the charge factor is \(b_\omega = b_s\). Thus single branes and BOS subsystems are represented in a unified way, and the OS solution is a special case \((m(\omega) = 1, \forall \omega)\) of the BOS one.

The metric has the form (14), where the function \(\sigma_1\) is

\[
\sigma_1 = -\frac{d_0 - 1}{D - 2} \sum_\omega N_\omega^2 y_\omega(u) \sum_{\mu \in S_\omega} q_{\mu(\omega)} d(I_\mu) + u \sum_{i=1}^n c^i + \sum_{i=1}^n c^i.
\]

For OS \((\omega \mapsto s)\) the sum in \(\mu\) reduces to \(d(I_s)\). The “conserved energy” (14) is

\[
E = \sum_\omega N_\omega^2 h_\omega^2 \sign h_\omega + c A C^A = \frac{d_0}{d_0 - 1}.
\]

In the special case \(m = 2\), \(\vec{Y}_1^2 = \vec{Y}_2^2\), one easily obtains \(b_1 = b_2\), as was shown in [13] for a single \(F\)-form. By definition of \(b_\mu\) that means not only \(Q_1^2 = Q_2^2\), but also a coincidence of the sign factors \(\sign b_\mu = \sign \epsilon_s\). For instance, in spherical symmetry, the \(F\)-fields must be either both true electric/magnetic ones \((\sign b_s = 1)\), or both quasiscalar ones \((\sign b_s = -1)\).

### 4.3. On cosmological and black-hole solutions

There is a large number of exact cosmological solutions to special cases of the model (1), see [12, 21, 33] and references therein. It can be seen that the description of Sec. 3.1 (which is certainly confirmed by exact solutions) actually exhausts all general features of the model, since other details, such as, e.g., the particular behaviour of the physical scale factor \(a(t)\), depend on the choice of integration constants.

BHs are obtained as special spherically symmetric solutions when \(h_\omega > 0\), \(u_{\text{max}} = \infty\). The functions \(\beta^i\) \((i = 0, 2, \ldots, n)\) and \(\varphi^a\) remain finite as \(u \rightarrow \infty\) under the following constraints on the ICs:

\[
\begin{align*}
 h_\omega &= k, & \forall \omega; & c^A &= k \sum_\omega N_\omega^2 Y_\omega A - k d_1^A,
\end{align*}
\]

where \(A = 1\) corresponds to \(i = 1\) (time), \(d_1 = 1\) (according to Theorem 2). The constraint (45) then holds automatically.

The subfamily (10) exhausts all BH solutions under OS or BOS assumptions, except the extreme case \(k = 0\); extreme BHs are obtained by subsequently passing to the limit \(k \rightarrow 0\). One can notice that exceptional extreme BH solutions, whose possibility was mentioned in Sec. 3.3, are not found in this way.

General explicit forms of OS and BOS BH solutions have been presented in Refs. [15] and [16], respectively. The BH properties stated in Theorems 2 and 3 are confirmed for the OS and BOS solutions and, moreover, have been first observed [15, 16] for these solutions.
5. Wormholes

5.1. Wormhole existence conditions

Wormhole-like configurations which can appear as special OS or BOS solutions, have an infinite “external radius” \( e^{\beta(u)} \) at both ends \( u_\pm \) of the \( u \) range are regular between them; all \( \beta'(u_\pm) \) \((i > 0)\) and \( \varphi''(u_\pm) \) are finite. This happens when \( k < 0 \) and the solution behaviour is governed by the function \( \sin ku \) (so that \( u - u_\pm = 0 \) and \( u_\pm = \pi/|k| \)) and is possible if the first positive zero of the function \( \sin|hx|(u - u_s) \) is greater than \( \pi/|k| \) for any \( s \) such that \( h_s < 0 \). See Fig. 1.

In the cosmological setting, this behaviour would correspond to nonsingular, bouncing models, which are, however, absent according to Sec. 3.1 (due to \( k > 0 \)). The static and Euclidean cases are not \textit{a priori} excluded.

As is evident from Fig. 1, any WH solution is characterized by \( |k| > |h_\omega| \) for all \( h_\omega \) which are negative. Due to (45), for \( k < 0 \) at least some \( h_\omega \) should be negative as well. Furthermore, for \( k < 0 \) and \( h_\omega < 0 \) it is necessary to have \( wK_0 = 1 \) and \( b_\omega > 0 \), respectively.

Table 1 shows that WHs can exist in static or Euclidean models only with spherical rather than pseudospherical or planar symmetry. In cosmology we have no fields capable to give negative \( h_s \) or \( h_\omega \), which again confirms the absence of nonsingular “bounced” models. In static spherical symmetry the necessary \( F \)-forms are true electric and magnetic ones. In Euclidean models, magnetic quasiscalar forms are needed.

Suppose \( k < 0 \). Since in \( c^2 \geq 0 \), the requirement \( |k| > |h_\omega| \) means that

\[
\sum_{\{\omega : h_\omega < 0\}} N_\omega^2 > \frac{d_0}{d_0 - 1}
\]

This inequality is not only necessary, but also \textit{sufficient} for the existence of WHs with given input parameters: \( d_i \) and the vectors \( \vec{\varphi}_x \). Indeed, put \( c^4 = 0 \) and turn to zero the charges \( Q_{\mu(\omega)} \) in all subsystems with \( \tilde{q}_\omega < 0 \) (note that, by (43), \( \text{sign} \tilde{q}_\omega = \text{sign} b_\omega \)). Choose all \( h_\omega \) to be negative and equal, then due to (47) \( |h_\omega| < |k| \). It is now an easy matter to choose the ICs \( u_\omega \) in such a way that \( \sin[H_\omega(u + u_\omega)] > 0 \) on the whole segment \( [0, \pi/|k|] \) — and this results in a WH solution.
5.2. Lorentzian wormholes and a universal restriction for brane systems

In general relativity static [28] and even dynamic [29] traversable WHs are known to violate the null energy condition. It can be verified [16] that, under the present positive energy requirement, the model (1) after reduction to \( d_0 + 2 \) dimensions by integrating out all \( \mathcal{M}_i \), \( i > 1 \) and a transition to the Einstein conformal frame, reduces to general relativity with a set of material fields whose EMT satisfies the null energy condition, which rules out static WHs. On the other hand, given a static WH in \( D \) dimensions as described in the previous subsection, it would also appear as a static WH in \( d_0 + 2 \)-dimensional Einstein frame since the relevant conformal factor (the volume factor of extra dimensions) is everywhere finite and nonzero. We have to conclude that static WHs are absent in our model.

This means in turn that the sufficient condition [13] must be violated, and a properly formulated opposite inequality must hold. We arrive at the following theorem for brane systems having an orthogonal subsystem:

**Theorem 4.** Consider a vector space \( \mathcal{V} \), with a scalar product defined by the metric ([17]), where \( d_i \in \mathbb{N}, \ i = 0, \ldots, n, \ d_0 > 1, \ d_1 = 1, \ \mathcal{D} = \sum_{i=0}^{n} d_i - 1 \), and a set of nonzero vectors \( \bar{Y}_s, \ s \in \mathcal{S} \), defined in ([18]) \( (I_s \subset \{1, \ldots, n\}, \ \chi_s \lambda_{sa} \in \mathbb{R}) \). Let there be a subset \( \mathcal{S}_\perp \subset \mathcal{S} \) such that \( \bar{Y}_s \bar{Y}_s' = 0 \) for \( s \neq s', \ s, s' \in \mathcal{S}_\perp \). Then the following inequality holds:

\[
\sum_{s \in \mathcal{S}_\perp} \delta_{sI_s} N_s^2 \leq \frac{d_0}{d_0 - 1}, \quad \text{or for } \lambda_{sa} = 0:
\]

\[
\sum_{s \in \mathcal{S}_\perp} \delta_{sI_s} \left( d(I_s) \left( 1 - \frac{d(I_s)}{D - 2} \right) \right)^{-1} \leq \frac{d_0}{d_0 - 1}.
\]

(48)

The factor \( \delta_{sI_s} \) in ([18]) excludes quasiscalars. For \( \mathcal{S}_\perp = \mathcal{S} \) the theorem has been already proved by the above reasoning. If there are \( \bar{Y}_s \notin \mathcal{S}_\perp \), their influence can be ruled out by turning to zero the corresponding charges \( Q_s \), and then, as before, assuming the contrary of ([13]), we immediately obtain a Lorentzian WH solution.

**Comment.** The formulation of Theorem 4 does not mention \( F \)-forms, time, or any other physical entities and is actually of purely geometric (or even combinatorial) nature. From the combinatorial viewpoint it is essential that in the set \( I_0 = \{0, \ldots, n\} \) there is a distinguished number, in our case 1, with \( d_1 = 1 \), included in all subsets \( I_s \) entering into the sum. Our proof, however, rests on physically motivated analytical considerations.

A similar theorem for a brane system with a BOS subsystem is readily obtained:

**Theorem 4a.** Consider the model described in Sec. 2, under the conditions specified in the first sentence of Theorem 4. Let there be a subset \( \mathcal{S}' \subset \mathcal{S} \) such that the vectors \( \bar{Y}_s, \ s \in \mathcal{S}' \) form a block-orthogonal system with respect to the metric ([17]). Then the following inequality holds for \( s \in \mathcal{S}' \):

\[
\sum_{\{\omega: \ q_\omega > 0\}} N_\omega^2 \leq \frac{d_0}{d_0 - 1}
\]

(49)

where \( q_\omega \) and \( N_\omega^2 \) are defined in ([4]) and, for all \( q_s \) included in the sum, \( \epsilon_s = \text{sign} q_s = -1 + 2\delta_{sI_s} \).

According to the latter condition, \( \epsilon_s \) depend on the inclusion or non-inclusion of the distinguished one-dimensional factor space \( \mathcal{M}_1 (= \mathbb{R}_t \text{ in Lorentzian models}) \) into the world volume of specific BOS members. Thus, unlike the OS case, the sum may include \( F \)-forms with different \( \epsilon_s \), but in such a way that the combined factor \( q_0 = \sum_{\mu \in \omega} q_\mu \) be positive for each \( \omega \).

6. Concluding remarks

1. Some general restrictions on the behaviour of brane systems described by the action ([1]) have been obtained, independent of specific space-time symmetry and signature: cosmological asymptotics, some BH properties and a universal restriction on the parameters of possible orthogonal or block-orthogonal subsystems (Theorems 4 and 4a).

Throughout the paper, the \( D \)-dimensional Einstein (D-E) conformal frame was used, although in such a general setting of the problem there is no evident reason to prefer one frame or another. For any specific underlying theory that leads to ([1]) in a weak field limit, two conformal frames are physically distinguished: one where the theory is originally formulated and another, providing the weak equivalence principle (or geodesic motion) for ordinary
matter in 4 dimensions; the latter depends on how fermions are introduced in the underlying theory \[18, \, 31, \, 33\]. The first one should be used when discussing such issues as singularities or topology of a model, etc. (what happens as a matter of fact), while the second one, the so-called atomic system of measurements, is necessary for formulating observational predictions (what seems to us). They are, generally speaking, different.

Among the present results, however, only cosmological ones are conformal frame-dependent if different frames are connected by exponentials of the internal scale factors $\beta^i$ and dilatonic fields $\varphi^a$. Indeed, such factors, being regular everywhere including horizons and asymptotics, cannot change the BH or WH nature of a given metric. (The only exceptions are hypothetic exceptional extreme BH solutions mentioned in Remark 2.)

The conclusions of Sec. 3.1 on cosmological asymptotics are directly applicable to theories formulated in the outset in the D-E frame, like the weak-field bosonic sector of $D = 11$ supergravity following from M-theory [3], where the action (truncated by neglecting the Chern-Simons term) has the form \[\Box\] with a single antisymmetric 4-form and no scalar fields.

2. Unlike Lorentzian ones, Euclidean WHs (EWHs) are not ruled out, and the reason is (taking, say, OS solutions as an example) that, when selecting the $F$-forms (branes) for WH construction, in the Euclidean case we are no more restricted to $I_s$ containing a distinguished number, connected with $R_\ell$, while now $R_\ell = R_u$. So there is a wider choice of $I_s$ able to give $h_s < 0$ and to fulfill the WH necessary and sufficient condition \[27\].

As seen from Table 1, EWHs corresponding to \[1\], if any, may be built only with the aid of magnetic forms $F_\ell$, though the existence of electric forms in a WH solution is not excluded.

The situation is well exemplified for $D = 11$ supergravity. Indeed, the orthogonality conditions \[37\] are satisfied by 2-branes, $d(I_s) = 3$, and 5-branes, $d(I_s) = 6$, if the intersection rules hold:

$$d(3 \cap 3) = 1, \quad d(3 \cap 6) = 2, \quad d(6 \cap 6) = 4.$$  \[50\]

(the notations are evident); for all $F$-forms $N_2^2 = 1/2$. In particular, with $d_0 = 2$ or $d_0 = 3$ and other $d_i = 1$, there is a maximal OS of seven 2-branes \[12, \, 13\]:

$$a: \quad 123, \quad d: \quad 345, \quad e: \quad 246,$$

$$b: \quad 147, \quad f: \quad 257, \quad c: \quad 156, \quad g: \quad 367.$$  \[51\]

where the figures 1, . . . , 7 label 1-dimensional factor spaces, and for static models “1” refers to the time axis $R_\ell$. Only three of these $I_s$ ($a, b, c$) have $\delta_{I_s} = 1$, i.e., describe true electric or magnetic fields in a static space-time. Lorentzian WHs are absent since \[27\] requires $\sum_s N_s^2 > 2$ for $d_0 = 2$ and $> 3/2$ for $d_0 = 3$.

In the Euclidean case we can have as many as 7 magnetic 2-branes, each with $N_2^2 = 1/2$, and WHs are easily found. Though, the latter is true if one considers $F_{m\ell}$ of rank 7. If one remains restricted, as usual, to $F_\ell$ of rank 4 \[3\], then for magnetic forms $d(I) = 6$, and EWHs cannot be obtained. Examples of EWHs have been found \[14\] for $D = 12$ theory \[8\].

By construction, classical EWHs possess finite actions and are related to possible quantum tunneling processes. Explicit expressions for their action and throat radii in the case of symmetric WHs described by OS and BOS solutions, have been calculated \[16\] explicitly in a general form for WHs which are symmetric with respect to their throats.

3. The present conclusions rest on the positive energy requirement that seems quite natural as long as we deal with classical fields. Thus, in particular, the well-known singularity theorems of general relativity actually work as well in multidimensional $p$-brane cosmology. Meanwhile, the low energy limit of the unification theories is believed to work at scales from Planckian to subatomic and in the early universe where quantum effects of both gravity and material fields must be of importance (e.g. the Casimir effect due to compactification of extra dimensions), and a classical treatment is only a tentative, though necessary, stage in studying such systems. One can mention some papers discussing the relevant quantum effects: the Wheeler-DeWitt equation for $p$-branes \[13, \, 22\] and the Casimir effect in cosmology \[32\]. Some non-quantum effects able to prevent a cosmological singularity are discussed by Kaloper et al. \[33\] and Gasperini \[34\]: see also references therein. All such effects necessarily violate the usual energy requirements and can therefore create traversable Lorentzian wormholes.

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