Decoherence framework for Wigner’s friend experiments

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The decoherence interpretation of quantum measurements is applied to Wigner’s friend experiments. A framework in which all the experimental outcomes arise from unitary evolutions is proposed. All the (apparent) wave-function collapses, and the corresponding randomness, result from tracing out relevant parts of the states in which the observers live. The main effect of this framework is ruling out all the inconsistences ensuing from Wigner’s friend experiments. Contrary to what is stated in [D. Frauchiger and R. Renner, Quantum theory cannot consistently describe the use of itself, Nat. Comm. 9, 3711 (2018)], this framework makes compatible the conclusions obtained by all the observers. And contrary to what is shown in [C. Bruckner, A no-go theorem for observer-independent facts, Entropy 20, 350 (2018)], it makes possible to assign joint truth values to the observations made by all the agents. This framework also narrows down the requisites for such experiments, making virtually impossible to apply them to conscious (human) beings.

I. INTRODUCTION

In 1961, Eugene Wigner proposed a thought experiment to show that a conscious being must have a different role in quantum mechanics than that of an inanimate device [1]. This experiment consists of two observers playing different roles. The first one, Wigner’s friend, performs a measurement on a particular quantum system in a closed laboratory; as a consequence of it, she observes one of the possible outcomes of her experiment. The second one, Wigner himself, measures the whole laboratory from outside. If quantum theory properly accounts for what happens inside the laboratory, Wigner observes that both his friend and the measured system are in an entangled superposition state. Hence, the conclusions of both observers are incompatible. For Wigner’s friend, the reality consists in a definite state equal to one of the possible outcomes of her experiment; for Wigner, it consists in a superposition of all these possible outcomes.

Since then, a large number of discussions, interpretations and extensions have been done. Among them, this work focuses on a recent extended version of this experiment, from which two different no-go theorems have been formulated. The first one shows that different agents, measuring on and reasoning over the same quantum system, are bound to get contradictory conclusions [2]. The second one establishes that it is impossible to assign joint truth values to the observations made by all the agents [3]. This extended version of the Wigner’s friend experiment consists of two closed laboratories, each one with an observer inside, and two outside observers dealing with a different laboratory. All the measurements are performed on a pair of entangled quantum systems, each one being measured in a different laboratory. An experiment to prove the second no-go theorem has recently been done [4].

The key point of the original and the extended versions of the Wigner’s friend experiment is the quantum treatment of the measurements performed inside the closed laboratories. It is assumed that Wigner’s friend observes a definite outcome from her experiment, but the wavefunction of the whole laboratory in which she lives remains in an entangled superposition state. This is somehow in contradiction with the spirit of the Copenhagen interpretation of quantum mechanics, since the measurement does not entail a non-unitary collapse. Its main shortcoming is not providing a specific procedure to determine whether a proper measurement has been performed. It is not clear at all whether an agent has observed a definite outcome, or just a simple quantum correlation, implying no definite outcomes, has been crafted. But, at the same time, it can be useful in the era of quantum technologies, because it can describe the evolution of a quantum machine able to perform experiments, infer conclusions from the outcomes, and act as a consequence of them.

The aim of this work is to provide a framework which keeps the quantum character of all the measurements, while supplying a mechanism for the (apparent) wavefunction collapse that the agents perceive. This is done by means of the decoherence interpretation of quantum measurements [5]. The key element of this interpretation is that a third party, besides the measured system and the measuring apparatus, is required to complete a quantum measurement. It consists in an uncontrolled environment, which cannot be the object of present or future experiments, and which is the ultimate responsible of the (apparent) wavefunction collapse. Hence, the laboratory in which Wigner’s friend
lives must include three different objects: the measured system, the measuring apparatus, and the uncontrolled environment. The last one, not present in standard Wigner’s friend setups \([1,3]\), determines to which states the part of the laboratory observed by Wigner’s friend collapses. And, at the same time, it guarantees the unitary evolution of the whole laboratory, and therefore makes it possible for Wigner to observe the system as an entangled superposition of his friend, the measuring apparatus, and the environment. Notwithstanding, our aim is not to support this framework against other possibilities, like wave-function collapse theories, for which the collapse is real and due to slight modifications in the quantum theory that only become important for large systems \([3]\). We just intend to show that this framework rules out all the inconsistencies arising from the standard interpretations of Wigner’s friend experiments, and narrows down the circumstances under which such experiments can be properly done.

Our first step is to build a simple model for the interaction between the measuring apparatus and the environment. This model allows us to determine the properties of the interaction and the size of the environment required to give rise to a proper measurement, as discussed in \([5]\). Then, we profit from it to discuss the original Wigner’s friend experiment \([1]\), and the no-go theorems devised in \([2,3]\). As we have pointed above, we conclude that the decoherence framework rules out all the inconsistencies arising from the usual interpretations of these experiments. The key point is that any kind of measurement, performed either by Wigner’s friends, or by Wigner himself, changes the global state in which all these agents live. By means of the decoherence formalism, these changes consist in unitary evolutions which can be easily tracked to eliminate all the inconsistencies. Notwithstanding, we also show that the usual interpretation can be maintained under very generic circumstances.

To avoid all the difficulties that conscious (human) beings entail, all the observers are considered quantum machines, that is, devices operating in the quantum domain, and programmed with algorithms allowing them to reach conclusions from their own observations. This choice facilitates the challenge of the experimental verification (or refutation) of the results that the decoherence framework provides, in contrast to, for example, the predictions of wave-function collapse models \([6]\).

The paper is organised as follows. Sec. II is devoted to the decoherence interpretation of quantum measurements. A simple numerical model is proposed to guide all the discussions. In Sec. III the original Wigner’s friend experiment is studied in terms of the decoherence framework. A numerical simulation is used to illustrate its most significative consequences. In Sec. IV the consistency of the quantum theory is discussed, following the argument devised in \([2]\). Sec. V refers to the possibility of assigning joint truth values to all the measurements in an extended Wigner’s friend experiments, following the point of view published in \([3]\). Finally, conclusions are gathered in Sec. VI.

II. DECOHERENCE FRAMEWORK

A. Decoherence interpretation of quantum measurements

In all the versions of Wigner’s friend experiments, the protocol starts with a measurement performed by a certain agent \(I\). Let us consider a single photon in the state

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|h\rangle + |v\rangle),
\]

where \(|h\rangle\) denotes that it is horizontally polarised, and \(|v\rangle\), vertically polarised.

If the measurement is quantum, it consists in a unitary evolution, given by the Hamiltonian that encodes the dynamics of the system and the measuring apparatus. It transforms the initial state, in which system and apparatus are uncorrelated, onto a final state in which the system and the apparatus are perfectly correlated

\[
\frac{1}{\sqrt{2}} (|h\rangle + |v\rangle) \otimes |A_0\rangle \rightarrow \frac{1}{\sqrt{2}} (|h\rangle \otimes |A_h\rangle + |v\rangle \otimes |A_v\rangle),
\]

where \(|A_0\rangle\) represents the state of the apparatus before the measurement, and \(|A_h|A_v\rangle = 0\). In \([1]\), an ancillary photon plays the role of the apparatus. In general, such a measurement can be performed by means of a C-NOT gate. As the choice of \(|A_0\rangle\) is arbitrary, we can consider that \(|A_0\rangle \equiv |A_h\rangle\), and thus the corresponding Hamiltonian is given by

\[
H = \frac{g}{2} |v\rangle \langle v| \otimes (|A_h\rangle \langle A_h| + |A_v\rangle \langle A_v| - |A_h\rangle \langle A_v| - |A_v\rangle \langle A_h|),
\]

where \(g\) is a coupling constant. This Hamiltonian performs Eq. (2), if it is applied during an interaction time given by \(g\tau = \pi/2\) \([5]\). The resulting state, which we denote

\[
|\Psi\rangle = \frac{1}{2} (|h\rangle |A_h\rangle + |v\rangle |A_v\rangle)
\]
for simplicity, entails that if the photon has horizontal polarisation, then the apparatus is in state $|A_h\rangle$, and if the photon has vertical polarisation, then the apparatus is in state $|A_v\rangle$. That is, it is enough to observe the apparatus to know the state of the photon.

In both the original and the extended versions of the Wigner’s friend experiment, the interpretation of this measurement is the following. The observer inside the laboratory, $I$, sees that the outcome of the experiment is either $h$ or $v$, with probability 1/2, following the standard Born rule: it sees the reality as consisting in a definite state corresponding either to $|h\rangle$ or $|v\rangle$. Even more, it can write that its observation has been completed, making possible for an external observer, $E$, to know that $I$ is seeing a definite outcome,

$$|\Psi_I^1\rangle = \left[\frac{1}{\sqrt{2}} (|h\rangle A_h + |v\rangle A_v)\right] \otimes \text{Observation}. \quad (5)$$

This implies that $I$ has observed a definite outcome, whereas $E$ still observes the system in a superposition state, despite knowing that $I$ sees the photon either in horizontal or vertical polarisation, and not in such a superposition state.

This conclusion is the basis of all the versions of the Wigner’s friend experiment. Notwithstanding, it suffers from two important shortcomings. The first one is that the complete laboratory consists just in the measured system and the measuring apparatus. Hence, there is no place for a quantum device able to act as a consequence of its measurement —the reasonings to infer contradictory conclusions, as discussed in [2], require a complex machine, not just a qubit signaling whether the measured photon is vertically or horizontally polarised. Therefore, as is pointed out in [4], the consideration of Eq. (4) as a proper measurement is questionable.

The second one is the basis ambiguity problem [5]. The very same state in Eq. (4), $|\Psi_1\rangle$, can be written in different basis,

$$|\Psi_1\rangle = \frac{1}{\sqrt{2}} (|\alpha\rangle A_\alpha + |\beta\rangle A_\beta), \quad (6)$$

where

$$|\alpha\rangle = \sin \theta |h\rangle + \cos \theta |v\rangle, \quad (7a)$$

$$|\beta\rangle = -\cos \theta |h\rangle + \sin \theta |v\rangle, \quad (7b)$$

$$|A_\alpha\rangle = \sin \theta |A_h\rangle + \cos \theta |A_v\rangle, \quad (7c)$$

$$|A_\beta\rangle = -\cos \theta |A_h\rangle + \sin \theta |A_v\rangle. \quad (7d)$$

That is, the final state of the very same measuring protocol, starting from the very same initial condition, also consists in the superposition given in Eq. (6) for arbitrary values of $\theta$. This problem blurs the usual interpretation of all the versions of the Wigner’s friend experiment. Why does $I$ see that the outcome of its measurement is either $h$ or $v$, instead of $\alpha$ or $\beta$? The unitary evolution giving rise to the measurement, Eq. (2), does not determine a preferred basis for the collapse of the wavefunction. A physical mechanism to determine what agent $I$ has observed is still required.

There are several ways to solve this problem. One of them consists in modifying the Schrödinger equation to model the wavefunction collapse and to choose the corresponding preferred basis [6]. These theories are based on the fact that superpositions have been experimentally observed in systems up to $10^{-6}$ g, whereas the lower bound for a classical apparatus is around $10^{-6}$ g. This means that the Schrödinger equation is just an approximation, which works pretty well for small systems, but fails for systems as large as measurement devices. A real collapse would change all the dynamics of Wigner’s friend experiments, presumably ruling out all their inconsistences.

Another possibility, the one which is the object of this work, is that Eq. (2) is not a complete measurement, but just a pre-measurement —a previous step required for any observation [5,7]. Following this interpretation, the observation is not completed before a third party, an environment which is not the object of the measurement, becomes correlated with the measured system and the measuring apparatus. This correlation is given again by a Hamiltonian, and therefore consists in a unitary evolution. If such an environment is continuously monitoring the system [8], the state of the whole system becomes

$$|\Psi_2\rangle = \frac{1}{\sqrt{2}} (|h\rangle |A_h\rangle |\varepsilon_1(t)\rangle + |v\rangle |A_v\rangle |\varepsilon_2(t)\rangle), \quad (8)$$

where the states of the environment $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$ change over the time, because the apparatus is continuously interacting with it. Note that Eq. (8) entails that the correlations between the system and the apparatus remain untouched despite the continuous monitoring by the environment. Hence, the states $|A_h\rangle$ and $|A_v\rangle$ are called
pointer states, because they represent the stable states of the apparatus $[5]$. Furthermore, if such an apparatus-environment interaction implies $\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle = 0$, $\forall t > \tau$, where $\tau$ can be understood as the time required to complete the measurement, the following affirmations hold:

(i) There is no other triorthogonal basis to write the state given by Eq. $[8]$ $[9]$. That is, the basis ambiguity problem is fixed by the action of the uncontrolled environment.

(ii) As the observer $I$ cannot measure the environment, all the further experiments it can perform on the system and the apparatus are compatible with the following mixed state

$$\rho = \frac{1}{2} \left( |h \rangle \langle h | A_h \langle h | + |v \rangle \langle v | A_v \langle v | \right),$$

(9)

independently of the particular shapes of both $| \varepsilon_1(t) \rangle$ and $| \varepsilon_2(t) \rangle$. That is, the observer $I$ sees the system as if it were randomly collapsed either to $|h \rangle | A_h \rangle$ or to $|v \rangle | A_v \rangle$, even though the real evolution of the complete system, including itself!, is deterministic and given by Eq. $[2]$. Relying on the decoherence framework, such an observer can deduce that the real state of the system, the apparatus and itself must be Eq. $[2]$, but it cannot prove it by means of further experiments. Randomness arises through this lack of knowledge.

These two facts are the basis of the decoherence interpretation of the quantum measurements $[5]$. This interpretation says that the observation is completed when the state given by Eq. $[8]$ is reached. In other words, if the observer sees a collapsed state is because an uncontrolled environment is monitoring the system (including itself!), and thus the complete wavefunction is given by Eq. $[8]$. The decoherence interpretation of quantum measurements also provides a framework to derive the Born rule from fundamental postulates $[7]$. Notwithstanding, all this work is based just on the previous facts (i) and (ii), and therefore the possible issues in this derivation of the Born rule are not relevant.

This scenario also fixes the first shortcoming. The quantum device in charge of acting upon the outcomes can consist in part of the environmental degrees of freedom.

Summarizing, if an observer follows the decoherence interpretation of quantum measurements, its conclusions cannot refer to the state of the whole system, but just to the outcomes of present and future measurements done in the same circumstances, that is, tracing out the same environmental degrees of freedom. The observer cannot determine the exact state of the whole system in which it lives; it can just deduce that a certain environment exists and is responsible for its outcomes.

### B. A simple model for the laboratories

The laboratories in which agents $I$ perform their measurements are quantum machines evolving unitarily. Their Hamiltonians must consist of: (i) a system-apparatus interaction, performing the pre-measurements; and (ii) an apparatus-environment interaction, following the decoherence proposal. For (i) we consider the logical C-NOT gate given in Eq. $[3]$. Following $[5]$, for (ii) we propose a model

$$H = |A_h \rangle \langle A_h | \sum_{n,m} V^h_{nm} | \varepsilon_n \rangle \langle \varepsilon_m | +$$

$$+ |A_v \rangle \langle A_v | \sum_{n,m} V^v_{nm} | \varepsilon_n \rangle \langle \varepsilon_m |,$$

(10)

where $V^h$ and $V^v$ are the coupling constants giving rise to the interaction. Independently of their particular shapes, the Hamiltonian given by Eq. $[10]$ guarantees that the correlations $| h \rangle | A_h \rangle$ and $| v \rangle | A_v \rangle$ remain unperturbed, that is, $| A_h \rangle$ and $| A_v \rangle$ are the pointer states resulting from this interaction, and the state given by Eq. $[8]$ holds for any time.

To build a simple model, we consider that both $V^h$ and $V^v$ are random matrices of the Gaussian Orthogonal Ensemble (GOE), which is the paradigmatic model for quantum chaos $[10]$. They are symmetric square matrices of size $N$, with independent Gaussian random elements with mean $\mu(V_{nm}) = 0$, $\forall n, m = 1, \ldots, N$, and $\sigma(V_{nm}) = 1$, $\forall n = 1, \ldots, N$ (diagonal elements); and $\sigma(V_{nm}) = 1/\sqrt{2}$, $\forall n \neq m = 1, \ldots, N$ (non-diagonal elements).

In panel (a) of Fig. 1 we show how the overlap between the two states of the environment, $| \varepsilon_1(t) \rangle$ and $| \varepsilon_2(t) \rangle$, evolves with time; in panel (b) how it evolves with the environment size. To perform the calculations, we have considered that the environment consists in $N$ qubits, and hence the dimension of its Hilbert space is $d = 2^N$. In all the cases, the initial state is a tensor product

$$| \Psi(0) \rangle = \frac{1}{\sqrt{2}} \left( | h \rangle | A_h \rangle + | v \rangle | A_v \rangle \right) \otimes | \epsilon_0 \rangle,$$

(11)
where $|\psi_0\rangle$ is the first element of the environmental basis (as the interaction is a GOE random matrix, the particular shape of the basis is irrelevant [10]). All the results are averaged over 50 different realizations. We have considered $\hbar = 1$.

Panel (a) of Fig. 1 shows the results of $|\langle \xi_1(t)|\xi_2(t)\rangle|^2$ for $N = 1$ ($d = 2$), $N = 3$ ($d = 8$), $N = 5$ ($d = 32$), $N = 7$ ($d = 128$), and $N = 9$ ($d = 512$). We clearly see that, the larger the number of environmental qbits, the smaller the value of $|\langle \xi_1(t)|\xi_2(t)\rangle|^2$ at large times, and the smaller the characteristic time $\tau$ required to complete the measurement process. Therefore, the condition $|\langle \xi_1(t)|\xi_2(t)\rangle|^2 \sim 0$ is almost reached if the number of the environmental qbits is $N \sim 10$. The results plotted in panel (b) of the same figure confirm this conclusion. We show there the long-time average of $|\langle \xi_1(t)|\xi_2(t)\rangle|^2$, calculated for $2 \leq t \leq 10$, as a function of the number of environmental qbits. It is clearly seen that the overlap between these states decreases fast with this number. As a consequence, we can safely conclude that an agent $I$ operating within a laboratory described by Eq. (10) will observe a state given by Eq. (9).

These results imply that the laboratories in which all the agents perform their measurements must have the structure summarized in Tab. I. It is worth to note that this structure is independent from any further evolution of the measured system, after the pre-measurement is completed. For example, let us imagine that the measured system has its own

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**TABLE I.** Parts of laboratories in which the agents $I$ perform their measurements in a Wigner’s friend experiment, following the decoherence framework.

| L1 | The measured system. |
| L2 | The measuring apparatus. |
| L3 | An internal environment, with a chaotic interaction like the one given by Eq. (10), and large enough to guarantee $|\langle \xi_1(t)|\xi_2(t)\rangle|^2 \sim 0$. |
FIG. 2. Panel (a), value of $C(\tau)$ as a function of $\tau$, for environments composed by different number of qbits. The solid curves show, from the upper one to the lower one, $N = 1$, $N = 3$, $N = 5$, $N = 7$ and $N = 9$. Panel (b), finite-size scaling for the long-time average of $C(\tau)$, as a function of the number of qbits composing the environment, $N$.

Hamiltonian, and therefore the time evolution for the whole system is governed by

$$H = H_S \otimes I_{A\epsilon} + I_S \otimes H_{A\epsilon},$$

where $H_S$ is the Hamiltonian for the measured system, $H_{A\epsilon}$ represents the environment-apparatus interaction, given by Eq. (10), and $I_S (I_{A\epsilon})$ is the identity operator for the system (environment-apparatus). As the two terms in this Hamiltonian commute pairwise, the time evolution of the whole system is

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}} [h(t) |a_{h}\rangle |\varepsilon_{1}(t)\rangle + \sqrt{\frac{1}{2}} [v(t) |a_{v}\rangle |\varepsilon_{2}(t)\rangle],$$

and thus all further measurements of the same agents are well described by

$$\rho(t) = \frac{1}{2} [h(t) |a_{h}\rangle \langle h(t)| + |v(t)\rangle |a_{v}\rangle \langle v(t)|].$$

That is, all the possible experiments that agent $I$ can perform in the future are compatible with the system collapsing onto either $|h\rangle$ or $|v\rangle$ after the measurement, and unitarily evolving from the corresponding initial condition. In other words, this framework is fully compatible with the Copenhagen interpretation, but the wave-function collapse being just a consequence of ignoring the environmental degrees of freedom. It is worth to remark that this is not a subjective interpretation, but the result of a unitary time evolution including a number of degrees of freedom that cannot be measured by the same observer.

As the key point in Wigner’s friend experiments consists in further interference measurements, performed from outside this laboratory, a study of the complexity of the state resulting from the time evolution summarized in Fig. [1] is necessary. Such a study can be made by means of a correlation function $C(\tau) = |\langle \varepsilon_{1}(t) |\varepsilon_{1}(t+\tau)\rangle|^{2}$. If $C(\tau) \sim 1$,
R1 A perfect knowlede of the interaction between the system and the apparatus, $H$, given by Eq. (10).

R2 A perfect knowlede of the environmental initial state, $|\varepsilon_0\rangle$.

R3 A perfect knowlede of the time at which agent $I$ performs its measurement, $t_I$.

R4 A perfect choice of the time at which agent $E$ performs its interference experiment, $t_E$.

TABLE II. Requisites for an extended Wigner’s friend experiment in which the external agent, $E$, performs an interference experiment involving only two states.

then the time evolution of the environmental state $|\varepsilon_1(t)\rangle$ is quite simple; its only possible change is an irrelevant global phase. Such a simple evolution would facilitate further interference experiments. On the contrary, if $C(\tau)$ quickly decays to zero, the same evolution is highly involved, implying that the state of the whole laboratory is complex enough to make very difficult further interference experiments.

Results are summarized in Fig. 2. Panel (a) shows $C(\tau)$ for the same environments displayed in the same panel of Fig. 1. It has been obtained after a double average: over 50 different realizations, and over $10^4$ different values of the time $t$. Panel (b) of Fig. 2 displays a finite size scaling of $C(\tau)$ for large values of time versus the number of environmental qubits, calculated averaging over $\tau \geq 10$. It is clearly seen that the results shown in this Figure are correlated with the ones displayed in Fig. 1. That is, if the environment is large enough to give rise to $|\langle \varepsilon_1(t)|\varepsilon_2(t)\rangle|^2 \sim 0$, then the environmental states fulfill $C(\tau) \sim 0$; the smaller the overlap between $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$, the smaller the value of the correlation function $C(\tau)$. It is also worth to note that $C(\tau)$ decays very fast to zero; for $N = 9$, $C(\tau) \sim 0$ for $\tau \approx 10^{-1}$. This means that the state of the environment is changing fast, and therefore the state of the whole laboratory, including the measured system, the measuring apparatus and the environment, is very complex.

As we have pointed out above, the key point of all the versions of Wigner’s friend experiments consist in further interference measurements performed by an external agent, for which the whole laboratory evolves unitarily following Eq. (8). Both in its original [1] and its extended versions, discussed in [2–4], the external agents perform interference experiments involving only two states, $|A_h\rangle |h\rangle$ and $|A_v\rangle |v\rangle$. The Hilbert spaces of the simplified versions of the laboratories discussed in these papers are spanned by $\{|A_h\rangle |h\rangle , |A_v\rangle |v\rangle , |A_h\rangle |v\rangle , |A_v\rangle |h\rangle \}$. Notwithstanding, the last two states are never occupied, and hence such two-state interference experiments are feasible [4]. The situation arising from the decoherence framework is far more complex. The dimension of the whole laboratory, composed by the measured system, the measuring apparatus, and an environment with $N$ qubits, is $d = 2^{N+2}$. From the results summarized in Fig. 2 we conjecture that all the $2^N$ states of the environment are populated, and therefore $2^{N+1}$ states of the whole laboratory become relevant for further interference experiments. Hence, the first consequence of the results discussed in this section is that experiments like the ones in [1][4] become extremely difficult. However, as $|\langle \varepsilon_1(t)|\varepsilon_2(t)\rangle|^2 \sim 0$, it is true that only two states, $|h\rangle |A_h\rangle |\varepsilon_1(t)\rangle$ and $|v\rangle |A_v\rangle |\varepsilon_2(t)\rangle$, are populated at each time $t$; the rest of the Hilbert space is irrelevant at that particular value of the time $t$. Unfortunately, these states change very fast with time, and in a very complex way. Therefore, an interference experiment involving only two states, $|h\rangle |A_h\rangle |\varepsilon_1(t)\rangle$ and $|v\rangle |A_v\rangle |\varepsilon_2(t)\rangle$, would require a very restrictive protocol, whose main requisites are summarized in Tab. 11. Only if such requisites are fulfilled, the external agent $E$ can rely on a simplified basis, composed by $|h(\tau)\rangle \equiv |h\rangle |A_h\rangle |\varepsilon_1(\tau)\rangle$ and $|v(\tau)\rangle \equiv |v\rangle |A_v\rangle |\varepsilon_2(\tau)\rangle$, where $\tau = t_E - t_I$, $t_I$ the time at which agent $I$ performs its measurement, and $t_E$ the same for agent $E$. A small error in points R1-R4 would imply that the real state of the laboratory, $|\Psi(t)\rangle$, had negligible overlaps with both $|h(\tau)\rangle$ and $|v(\tau)\rangle$, and therefore any interference experiments involving just these two states would give no significative outcomes.

Before applying these conclusions to the original and the extended versions of the Wigner’s friend experiments, it makes sense to test if these conclusions depend on the particular model we have chosen for the apparatus-environment interaction. To tackle this task, we consider more general random matrices $V^n$ and $V^v$ in Eq. (10), in which $\mu(V_{nm}) = 0, \forall n,m = 1,\ldots,N$, $\sigma(V_{nm}) = 1, \forall n = 1,\ldots,N$ (diagonal elements); and $\sigma(V_{nm}) = 1/(\sqrt{2}\sqrt{n-m}^{|n-m|}) \forall n \neq m = 1,\ldots,N$ (non-diagonal elements). If the parameter $\alpha$ is large, then only very few non-diagonal elements are relevant, and hence the interaction becomes approximately integrable. On the contrary, if $\alpha = 0$, GOE (chaotic) results are recovered.

We fix our attention in the degree of chaos of the resulting Hamiltonian. To do so, we study the ratio of consecutive level spacings distribution, $P(r)$, where $r_n = s_{n+1}/s_n$ and $s_n = E_{n+1} - E_n$, $\{E_n\}$ being the energy spectrum of the system. It has been shown [11] that the distribution for standard integrable systems is $P(r) = 1/(1 + r)^2$, whereas it is $P(r) = 27(1 + r^2)/(8(1 + r^2)^{5/2})$ for GOE systems.

In Fig. 3 we show the results for four different values of $\alpha$, $\alpha = 0.5, \alpha = 1, \alpha = 2, \alpha = 4$. They consist in the
FIG. 3. Ratio of consecutive level spacings distribution, $P(r)$, for $\alpha = 0.5$ [panel (a)], $\alpha = 1$ [panel (b)], $\alpha = 2$ [panel (c)], and $\alpha = 4$ [panel (d)]. Solid histograms show the numerical results for 2000 matrices with dimension $d = 512$; green dashed line, the result for a GOE system, $P(r) = 27(r + r^2)/(8(1 + r + r^2)^{5/2})$; and the blue dashed line, the result for an integrable system, $P(r) = 1/(1 + r^2)$.

average over 2000 realizations of matrices of dimension $d = 512$. The case with $\alpha = 0$ (not shown) exactly recovers the GOE result, as expected. The case with $\alpha = 0.5$ [panel (a)] is also fully chaotic; its ratio of consecutive level spacings distribution, $P(r)$, is identical to the GOE result. Things become different for larger values of $\alpha$. The case $\alpha = 1$ [panel (b)] is yet different from the GOE result, although its behavior is still highly chaotic. The cases $\alpha = 2$ [panel (c)] and $\alpha = 4$ [panel (d)] are very close to the integrable result.

In Fig. 4, we show how the long-time average of $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle |^2$, calculated for $2 \leq t \leq 50$, scales with the number of environmental qubits, $N$, for five different values of $\alpha = 0, 0.5, 1, 2,$ and 4. The results are averaged over 50 different realizations. It is clearly seen that the two fully chaotic cases, $\alpha = 0$ (circles) and $\alpha = 0.5$ (squares), behave in the same way; the overlap $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle |^2$ decreases with the number of environmental qubits, and therefore we can expect $|\varepsilon_1(t)\rangle$ and $|\varepsilon_2(t)\rangle$ to become orthogonal if the environment is large enough. The behavior of the case with $\alpha = 1$ (upper triangles) is different. First, the overlap $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle |^2$ decreases with $N$, but it seems to reach an asymptotic value for $N \gtrsim 7$. This fact suggests that a fully chaotic apparatus-environment interaction is required for the scenario described by the decoherence framework. This conclusion is reinforced with the results for $\alpha = 2$ (lower triangles) and $\alpha = 4$ (diamonds). These two cases correspond with (almost) integrable Hamiltonians, and their overlaps $|\langle \varepsilon_1(t) | \varepsilon_2(t) \rangle |^2$ remain large independently of the number of environmental qubits.

C. Summary of results

The results discussed in the previous section narrow down the circumstances under which Wigner’s friend experiments are feasible, if we take into account the decoherence interpretation of quantum measurements. First,
FIG. 4. Finite-size scaling for the long-time average of $|\langle \varepsilon_1(t) \varepsilon_2(t) \rangle|^2$, as a function of the number of qbits composing the environment, $N$. Solid circles represent the case $\alpha = 0$; solid squares, $\alpha = 0.5$; solid upper triangles, $\alpha = 1$; solid lower triangles, $\alpha = 2$, and solid diamonds, $\alpha = 4$.

**F1** After the measurement performed by agent $I$ is completed, the real state of the measured system, the measuring apparatus and the surrounding environment (which includes the agent itself) is given by Eq. (8), with $|\langle \varepsilon_1(t) \varepsilon_2(t) \rangle|^2 \sim 0$.

**F2** All the results obtained by the agent $I$ are compatible with the mixed state given by Eq. (9). That is, it sees the system as if it were collapsed onto one of the possible outcomes of its experiment, despite fact F1.

TABLE III. Summary of the facts consequence of the decoherence interpretation of quantum measurements, for Wigner’s friend experiments.

| Fact | Description |
|------|-------------|
| **F1** | After the measurement performed by agent $I$ is completed, the real state of the measured system, the measuring apparatus and the surrounding environment (which includes the agent itself) is given by Eq. (8), with $|\langle \varepsilon_1(t) \varepsilon_2(t) \rangle|^2 \sim 0$. |
| **F2** | All the results obtained by the agent $I$ are compatible with the mixed state given by Eq. (9). That is, it sees the system as if it were collapsed onto one of the possible outcomes of its experiment, despite fact F1. |

In this section we discuss the consequences of the decoherence framework in the standard Wigner’s friend experiment [1]. Let us consider that an internal agent $I$ has performed a measurement on an initial state given by Eq. (1). As we have explained above, independently of the outcome it observes, the resulting state is given by Eq. (8). To simplify the notation, we consider the whole state of the laboratory as follows,

$$|h(t)\rangle = |h\rangle |A_h\rangle |\varepsilon_1(t)\rangle,$$

$$|v(t)\rangle = |v\rangle |A_v\rangle |\varepsilon_2(t)\rangle,$$
where both $|h(t)\rangle$ and $|v(t)\rangle$ may in general change with time. Thus, the state after the measurement by agent $I$ is

$$|\Psi_1(t)\rangle = \sqrt{\frac{1}{2}} (|h(t)\rangle + |v(t)\rangle).$$ (16)

Following the protocol proposed by Wigner [1], an external agent, $E$, performs a measurement on $|\Psi_1(\tau)\rangle$, at a particular instant of time $\tau$. Let us consider that the four requisites, R1-R4, of Tab. [1] are fulfilled, and therefore an interference experiment can be performed with a two-state basis, $\{|\alpha(\tau)\rangle, |\beta(\tau)\rangle\}$, given by

$$|\alpha(\tau)\rangle = \sin \theta |h(\tau)\rangle + \cos \theta |v(\tau)\rangle,$$

$$|\beta(\tau)\rangle = -\cos \theta |h(\tau)\rangle + \sin \theta |v(\tau)\rangle,$$

for an arbitrary value of the angle $\theta$. In this basis, the state $|\Psi_1(\tau)\rangle$ reads,

$$|\Psi_1(\tau)\rangle = \sqrt{\frac{1}{2}} (\sin \theta + \cos \theta) |\alpha(\tau)\rangle +$$

$$+ \sqrt{\frac{1}{2}} (\sin \theta - \cos \theta) |\beta(\tau)\rangle.$$ (18)

Therefore, following the decoherence formalism, the state resulting from agent $E$ measurement is

$$|\Psi_2(\tau)\rangle = \sqrt{\frac{1}{2}} (\sin \theta + \cos \theta) |\alpha(\tau)\rangle |A'_\alpha\rangle |\varepsilon'_1(\tau)\rangle +$$

$$+ \sqrt{\frac{1}{2}} (\sin \theta - \cos \theta) |\beta(\tau)\rangle |A'_\beta\rangle |\varepsilon'_2(\tau)\rangle,$$ (19)

where $A'$ represents its apparatus, and $\varepsilon'$ the environment required by the decoherence framework.

Up to now, we have considered that both the measurement and the correlation between the apparatus $A'$ and the environment $\varepsilon'$ happen at time $\tau$. But this consideration is not relevant. Taking into account that both the internal, $A$, and the external, $A'$, apparati are continuously monitored by their respective environments, the former state unitarily evolves with a Hamiltonian $H = H_I \otimes I_E + I_I \otimes H_E$, where $I_I$ ($I_E$) represents the identity operator for the internal (external) laboratory. Therefore, in any moment after the measurement the resulting state is

$$|\Psi_2(t)\rangle = \sqrt{\frac{1}{2}} (\sin \theta + \cos \theta) |\alpha(t)\rangle |A'_\alpha\rangle |\varepsilon'_1(t)\rangle +$$

$$+ \sqrt{\frac{1}{2}} (\sin \theta - \cos \theta) |\beta(t)\rangle |A'_\beta\rangle |\varepsilon'_2(t)\rangle,$$ (20)

with $|\langle \varepsilon'_1(t) | \varepsilon'_2(t) \rangle|^2 \sim 0$. And hence, any further experiment performed by agent $E$, in which the external environment is not measured, is compatible with the state:

$$\rho_E = \frac{1}{2} (\sin \theta + \cos \theta)^2 |\alpha(t)\rangle \langle A'_\alpha| |\alpha(t)\rangle +$$

$$+ \frac{1}{2} (\sin \theta - \cos \theta)^2 |\beta(t)\rangle \langle A'_\beta| |\beta(t)\rangle.$$ (21)

Two remarks are useful at this point. First, as we have pointed above, the real state of the system is given by Eq. (20): the mixed state given by Eq. (21) is only a description of what agent $E$ sees, that is, of what agent $E$ can infer from any further measurements performed by itself. Second, the interpretation of Eq. (21) is independent of the precise forms of $|\alpha(t)\rangle$ and $|\beta(t)\rangle$. The fact that the internal laboratory changes with time has no influence on agent $E$ conclusions because its apparatus remains pointing at either $\alpha$ or $\beta$.

An interesting question at this stage is what does agent $I$ observe now? The measurement performed by agent $E$ has changed the state of the system from

$$|\Psi_1(t)\rangle = \sqrt{\frac{1}{2}} (|h(t)\rangle + |v(t)\rangle) \otimes |A'_0\rangle |\varepsilon'_0\rangle,$$ (22)

where $|A'_0\rangle$ and $|\varepsilon'_0\rangle$ are the (irrelevant) initial states of agent $E$ apparatus and the external environment, to Eq. (20). The decoherence framework establishes that agent $I$ sees the system as if it were collapsed either onto $|h\rangle$ or $|v\rangle$ (both
with probability \( p_h = p_v = 1/2 \) as a consequence of tracing out the degrees of freedom of \( \varepsilon, A' \) and \( \varepsilon' \) from Eq. (22). But, as the global state has changed onto Eq. (20) as a consequence of agent \( E \) measurement, a change of how agent \( I \) perceives the reality is possible. To answer this question, we can rewrite Eq. (20) using the basis \( \{ |h\rangle, |v\rangle \} \). The resulting state is

\[
|\Psi_2(t)\rangle = \frac{\sin \theta}{\sqrt{2}} (\sin \theta + \cos \theta) |h\rangle |A_h\rangle |\varepsilon_1(t)\rangle |\varepsilon'_1(t)\rangle + \\
+ \frac{\cos \theta}{\sqrt{2}} (\sin \theta + \cos \theta) |v\rangle |A_v\rangle |\varepsilon_2(t)\rangle |\varepsilon'_2(t)\rangle + \\
+ \frac{\cos \theta}{\sqrt{2}} (\cos \theta - \sin \theta) |h\rangle |A_h\rangle |\varepsilon_1(t)\rangle |\varepsilon'_2(t)\rangle + \\
+ \frac{\sin \theta}{\sqrt{2}} (\sin \theta - \cos \theta) |v\rangle |A_v\rangle |\varepsilon_2(t)\rangle |\varepsilon'_1(t)\rangle.
\]

As any further measurements performed by agent \( I \) will involve neither its environment, \( \varepsilon \), nor agent \( E \) apparatus, \( A' \), nor agent \( E \) environment, \( \varepsilon' \), the resulting outcomes can be calculated tracing out all these three degrees of freedom. The result is

\[
\rho_I = \frac{1}{4} (2 - \sin 4\theta) |h\rangle \langle h| |A_h\rangle \langle A_h| + \\
+ \frac{1}{4} (2 + \sin 4\theta) |v\rangle \langle v| |A_v\rangle \langle A_v|.
\]

This is the first remarkable result due to the decoherence framework. We are used to assuming that external interference experiments do not alter the perceptions of the agents living inside the observed laboratories. We see now that, as a consequence of the measurement performed by the external agent \( E \), agent \( I \) still sees the system as if it were collapsed either onto \(|h\rangle \) or \(|v\rangle \), but the probabilities of any of the corresponding outcomes have changed. The most impressive fact happens for \( \theta = \pi/8 \). Then, the vision of agent \( I \) changes from both outcomes being equally probable, to the photon being horizontally polarised with \( p = 1/4 \), and vertically polarised with \( p = 3/4 \).

This result is very difficult to interpret from the point of view of a conscious (human) being. However, it is just the consequence of a unitary evolution governed by the interaction between the laboratory in which agent \( I \) lives, and the measuring apparatus used by agent \( E \). To delve into this point, we perform now a numerical simulation covering all the protocol. We study the case with \( \theta = \pi/8 \), and we consider that both environments are composed by 6 qubits —the total size of the Hilbert space is \( 2^{11} = 32768 \). We start from the state resulting from agent \( I \) pre-measurement

\[
|\Psi_0\rangle = \sqrt{\frac{1}{2}} (|h\rangle \langle A_h| + |v\rangle \langle A_v|) |\varepsilon_1\rangle |\varepsilon'_1\rangle,
\]

where \( \varepsilon_1 \) and \( \varepsilon'_1 \) represent the first states of the basis used to model the internal and the external environments, respectively. Note that we have considered the state \( |A'_0\rangle \) as the zero state of the apparatus, but the results do not depend on this particular choice. From this state, the system passes through three stages:

**Stage 1.** From \( t = 0 \) to \( t = \tau_1 \), the internal environment interacts with apparatus \( A \) to complete the measurement. Even though the external agent \( E \) has not performed any measurement yet, we also consider a similar interaction for the external environment —in such a case, the external agent \( E \) would see a definite outcome pointing to zero, that in this case corresponds to the outcome \( \alpha \). The corresponding Hamiltonian is

\[
H_1 = \left( |A_h\rangle \langle A_h| \sum_{n,m} V_{nm}^h |\varepsilon_n\rangle \langle \varepsilon_m| + |A_v\rangle \langle A_v| \sum_{n,m} V_{nm}^v |\varepsilon'_n\rangle \langle \varepsilon'_m| \right) \otimes I_E + \\
+ \left( |A'_0\rangle \langle A'_0| \sum_{n,m} V_{nm}^a |\varepsilon'_n\rangle \langle \varepsilon'_m| + |A'_\beta\rangle \langle A'_\beta| \sum_{n,m} V_{nm}^\beta |\varepsilon'_n\rangle \langle \varepsilon'_m| \right) \otimes I_I,
\]

where \( I_I \) represents the identity operator over the laboratory in which agent \( I \) lives, and \( I_E \) the identity operator over the degrees of freedom corresponding to \( A' \) and \( \varepsilon' \).

**Stage 2.** From \( t = \tau_1 \) to \( t = \tau_2 \), agent \( E \) performs its pre-measurement. We consider that the interaction with the external environment is switched off, to model that this part of the measurement is purely quantum. However, the interaction between the internal apparatus and the internal environment continues to exist, because the monitorization
is always present after a measurement is completed. The corresponding Hamiltonian is

\[
H_2 = \left( |A_h\rangle \langle A_h| \sum_{n,m} V_{nm}^h |\varepsilon_n\rangle |\varepsilon_m\rangle \right) + |A_v\rangle \langle A_v| \sum_{n,m} V_{nm}^v |\varepsilon_n\rangle |\varepsilon_m\rangle \right) \otimes \mathcal{I}_E +
\]

\[
g |\beta(\tau_1)\rangle \langle \beta(\tau_1)| \left[ |A'_h\rangle \langle A'_h| + |A'_v\rangle \langle A'_v| - |A''_h\rangle \langle A''_h| - |A''_v\rangle \langle A''_v| \right) \otimes \mathcal{I}_I.
\]

(27)

It is worth remarking that the requirements R1-R4 of Tab. 1 have been explicitly taken into account. The interaction leading to agent E pre-measurement is based on \(|\beta(\tau_1)\rangle\rangle\), which is the exact state of the internal laboratory at time \(t = \tau_1\). The duration of this stage is exactly \(\tau_2 - \tau_1 = \pi/(2g)\).

**Stage 3.** From \(t = \tau_2\) on, the external environment gets correlated with apparatus \(A'\), to complete the measurement performed by agent E. Hence, the Hamiltonian is again given by Eq. (26).

In summary, the system evolves from \(|\Psi(0)\rangle\rangle\), given by Eq. (27), by means of \(H_1\), given by Eq. (26), from \(t = 0\) to \(t = \tau_1\); by means of \(H_2\), given by Eq. (27), from \(t = \tau_1\) to \(t = \tau_2\); and by means of \(H_1\) again, from \(t = \tau_2\) on. Agent I point of view is directly obtained from the real state of the whole system, \(|\Psi(t)\rangle\rangle\), by tracing out the degrees of freedom corresponding to \(\varepsilon, A'\) and \(\varepsilon'\). The resulting state can be written

\[
\rho_I(t) = C_{hh}(t) |h\rangle \langle h| + C_{hv}(t) |h\rangle \langle v| + C_{vh}(t) |v\rangle \langle h| + C_{vv}(t) |v\rangle \langle v|.
\]

(28)

If \(C_{hv} \sim 0\) and \(C_{vh} \sim 0\), agent I sees the system as if it were collapsed onto either \(|h\rangle\rangle\), with probability \(C_{hh}\), or \(|v\rangle\rangle\), with probability \(C_{vv}\).

Following the same line of reasoning, agent E point of view is obtained from \(|\Psi(t)\rangle\rangle\) by tracing out the external environment, \(\varepsilon'\). The resulting state can be written

\[
\rho_E(t) = C_{\alpha\alpha}(t) |\alpha(t)\rangle \langle \alpha(t)| + C_{\alpha\beta}(t) |\alpha(t)\rangle \langle \beta(t)| + C_{\beta\alpha}(t) |\beta(t)\rangle \langle \alpha(t)| + C_{\beta\beta}(t) |\beta(t)\rangle \langle \beta(t)|.
\]

(29)

The interpretation is the same as before. If \(C_{\alpha\beta} \sim 0\) and \(C_{\beta\alpha} \sim 0\), agent E sees the reality as it if were collapsed onto either \(|\alpha(t)\rangle\rangle\), with probability \(C_{\alpha\alpha}\), or \(|\beta(t)\rangle\rangle\), with probability \(C_{\beta\beta}\). It is worth to note that the states of the internal laboratory \(|\alpha(t)\rangle\rangle\) and \(|\beta(t)\rangle\rangle\), change with time, but this is not relevant for agent E point of view.

In panel (a) of Fig. 5 we show the results from agent I point of view. The coupling constant is set \(g = 100\); \(\tau_1 = 10\), and \(\tau_2 - \tau_1 = \pi/200\). The non-diagonal element, \(C_{nd} = \sqrt{|C_{hv}|^2 + |C_{vh}|^2}\) (dotted blue line), is significantly large only at the beginning of the simulation; from results in Fig. 1 we expect that larger environments give rise to smaller values for \(C_{nd}\) (see Fig. 5 for a deeper discussion). Hence, our first conclusion is that agent I point of view is compatible with the photon collapsing either to horizontal or to vertical polarization. The measurement performed by agent E, that starts at \(\tau_1 = 10\), does not alter this fact. However, as we clearly see in the inset of the same panel, this measurement does change elements \(C_{hh}\) (violet line) and \(C_{vv}\) (green line). In the main part of the panel, we display the expected values, given in Eq. (24), \(C_{hh} = 1/4\), \(C_{vv} = 3/4\), as black dashed-dotted lines; we can see that these values are fast reached. Furthermore, we can also see in the inset that this is a smooth change, due to the physical interaction between the laboratory and the apparatus \(A'\). Therefore, agent I point of view continuously changes during this small period of time. As we have pointed above, this is a remarkable consequence of the decoherence framework, which is not present in standard interpretations of Wigner’s friend experiments. We will discuss later the important role that this fact plays in the recently proposed extended versions of the experiment 21 3.

Panel (b) of Fig. 5 represents agent E point of view. Before performing the measurement, its apparatus points \(\alpha\) because this is chosen as zero. Then, at \(t = \tau_1\) this point of view starts to change. \(C_{\alpha\alpha}\) (solid violet line) changes to \(C_{\alpha\alpha} = 0.854\), the expected value from Eq. (21), and equally \(C_{\beta\beta}\) (solid green line) changes to \(C_{\beta\beta} = 0.146\). During the first instants of time after the pre-measurement, the non-diagonal element \(C_{nd} = \sqrt{|C_{\alpha\beta}|^2 + |C_{\beta\alpha}|^2}\) (blue dotted line) is significantly different from zero; but, after the external environment has played its role, agent E point of view becomes compatible with the laboratory collapsed either to \(\alpha\) (with probability \(p = 0.854\)) or to \(\beta\) (with probability \(p = 0.146\)) as expected.

A finite-size scaling analysis of the non-diagonal element of \(\rho_I\) is given in Fig. 6. Due to the huge size of the whole Hilbert space, it is not possible to reach large environmental sizes. However, we clearly see in the inset how the size
FIG. 5. Panel (a), matrix elements $C_{hh}$ (solid, violet line), $C_{vv}$ (solid green line), and $C_{nd} = \sqrt{|C_{hv}|^2 + |C_{vh}|^2}$ (dashed blue line), from Eq. (28). Dotted-dashed lines show the expected values at stage 3. The inset show $C_{hh}$ and $C_{vv}$ around stage 2.

Panel (b), matrix elements $C_{aa}$ (solid, violet line), $C_{bb}$ (solid green line), and $C_{nd} = \sqrt{|C_{a\beta}|^2 + |C_{\beta a}|^2}$ (dashed blue line), from Eq. (29). Dotted-dashed lines show the expected values at stage 3. The inset show $C_{aa}$ and $C_{bb}$ around stage 2. The number of qbits of both environment is $N = 6$, $g = 10^2$, $\tau_1 = 10$, and $\tau_2 - \tau_1 = \pi/200$.

of this non-diagonal element, $C_{nd}$, averaged from $t = 3$ to $t = 100$, decays with the number of environmental qbits. Furthermore, a visual comparison between the cases with $N = 3$ (green line) and $N = 6$ (red line), given in the main panel of the same figure, corroborates this impression. Therefore, we can conjecture that both agents $I$ and $E$ see their measured systems as if they were collapsed, provided that their corresponding environments are large enough.

Finally, we study how the results depend on the coupling constant between the external apparatus, $A'$, and the laboratory whose state is measured by agent $E$. In Fig. 7 we show $C_{hh}$ for $N = 6$ and $g = 1$ (blue line), $g = 10$ (green line), and $g = 100$ (violet line), together with the expected value, $C_{hh} = 1/4$ (dotted-dashed black line). We conclude that this expected value is reached only if $g$ is large enough. The explanation is quite simple. If $g$ is small, the time required for the external apparatus $A'$ to complete the pre-measurement is large compared with the characteristic correlation time of the laboratory, given in Fig. 2. Therefore, the state $\beta(\tau_1)$, used in Eq. (27), ceases to be the real state of the laboratory while the external apparatus, $A'$, is still performing the pre-measurement. As a consequence, the resulting measurement is not correct, and neither agent $E$ nor agent $I$ reach the expected results. This is an important fact that difficulties a bit more the external interference measurements trademark of Wigner’s friend experiments. Besides the requirements R1-R4 of Tab. 1, it is also mandatory that the external pre-measurement is shorter than the characteristic time of the internal dynamics of the measured laboratory. As it is shown in Fig. 2, the larger the internal environment, the shorter this time. Hence, if agent $I$ is a conscious (human) being, composed by a huge number of molecules, the external interference pre-measurement must be completed in a tiny amount of time.
FIG. 6. $C_{nd} = \sqrt{|C_{nh}|^2 + |C_{vh}|^2}$, from Eq. (28), for an environment with $N = 3$ qbits (light green line) (dashed blue line), and for an environment with $N = 6$ qbits (dark red line). In the inset, scaling analysis for the time average of $C_{nd}$ obtained from $t = 3$ to $t = 100$.

FIG. 7. $C_{hh}$ from Eq. (28) for different coupling constants $g$ in Eq. (27): $g = 1$ (blue line), $g = 10$ (green line), $g = 100$ (violet line). Dotted-dashed line shows the expected value for stage 3.

The first conclusion we can gather from all these results is that, according to the decoherence framework, the perceptions of all the agents involved in a Wigner’s friend experiment will generically change after the actions of any other agents. As we will see in next sections, this is the clue to interpret the extended versions of the experiment.

Notwithstanding, agent $I$ still sees the reality as if the measured photon were either horizontally or vertically polarised —not in a superposition of both states. Even more, states $\rho_E$, given by Eq. (21), and $\rho_I$, given by Eq. (24), seem incompatible at a first sight. But this is just a consequence of the differences between the experiments performed by these two agents. Agent $I$ sees the universe as if it were in state $\rho_I$, because it ignores $\varepsilon$, $A'$ and $\varepsilon'$. On the other hand, agent $E$ sees the universe as if it were in state $\rho_E$, because it just ignores $\varepsilon'$, and therefore has relevant information about $A'$ and $\varepsilon$. And, even more important, both agents agree that their perceptions about the reality are linked to the limitations of their experiments, and that the real state of the universe is a complex, entangled and superposition state involving the measured photon, both their apparatus, both the environments that surround them, and themselves —neither $\rho_I$, nor $\rho_E$.

This constitutes a good example of how the decoherence framework removes the inconsistencies usually found in Wigner’s friend experiments. Nevertheless, it is worth to remark that, if all the agents adopt a classical point of view, and infer conclusions about the real state of the universe from their outcomes, they are bound to contradict themselves only if requirements R1-R4 of Tab. 11 are strictly fulfilled, and the external interference pre-measurement is completed fast enough. In other words, only under very specific (and somehow artificial) circumstances, a classical
point of view is headed for failure. Under generic circumstances, a two-state interference experiment, like the one performed by agent $E$, is not possible.

IV. CONSISTENCE OF THE QUANTUM THEORY

The aim of this section is to discuss the thought experiment proposed in [2] within the framework presented above. A number of comments and criticisms have been already published, including [3] itself, and some others [13–16]. This work deals with the original proposal in [2].

A. No-go theorem and original interpretation

Both no-go theorems discussed in [2, 3] share a similar scheme:

(a) A pair of entangled quantum systems is generated. In [2] it consists in a quantum coin, with an orthogonal basis given by $\{\text{head}_R, \text{tail}_R\}$, and a 1/2-spin, spanned by $\{\downarrow_S, \uparrow_S\}$. The initial entangled state is

$$|\Psi\rangle = \sqrt{\frac{1}{3}} |\text{head} \rangle_R |\downarrow \rangle_S + \sqrt{\frac{2}{3}} |\text{tail} \rangle_R |\rightarrow \rangle_S ,$$

where $|\rightarrow \rangle_S = \sqrt{\frac{2}{3}} (|\downarrow \rangle_S + |\uparrow \rangle_S)$.

To simplify the notation and make it compatible with [3, 4], the following changes are made: (i) instead of the quantum coin and the spin in Eq. (29), two polarised photons are used; (ii) the first photon is denoted by the subindex $a$, and the second one, by the subindex $b$; (iii) the superpositions of vertical and horizontal polarisation are denoted $|+\rangle = \sqrt{\frac{1}{2}} (|h \rangle + |v \rangle)$ and $|-\rangle = \sqrt{\frac{1}{2}} (|h \rangle - |v \rangle)$, respectively. With this notation, the initial state in [2] reads

$$|\Psi\rangle = \sqrt{\frac{1}{3}} |h \rangle_a |v \rangle_b + \sqrt{\frac{2}{3}} |v \rangle_a |+\rangle_b .$$

(b) Photon $a$ is sent to a closed laboratory $A$, and photon (b), to a closed laboratory $B$.

(c) An observer $I_A$, inside laboratory $A$, measures the state of photon $a$; and an observer $I_B$, inside laboratory $B$, measures the state of photon $b$.

(d) An external observer $E_A$ measures the state of the whole laboratory $A$, and an external observer $E_B$ measures the state of the whole laboratory $B$.

Both no-go theorems [2, 3] deal with the observations made by $I_A$, $I_B$, $E_A$, and $E_B$. The one formulated in [2] is based upon the following assumptions:

Assumption Q.- Let us consider that a quantum system is in the state $|\Psi\rangle$. Then, let us suppose that an experiment has performed on a complete basis $\{|x_1\rangle, \ldots , |x_n\rangle\}$, giving an unknown outcome $x$. Then, if $\langle \Psi | \pi_m |\Psi\rangle = 1$, where $\pi_m = |x_m\rangle \langle x_m|$, for a particular state of the former basis, $|x_m\rangle$, then I am certain that the outcome is $x = x_m$.

Assumption C.- If I am certain that some agent, upon reasoning within the same theory I am using, knows that a particular outcome $x$ is $x = x_m$, then I am also certain that $x = x_m$.

Assumption S.- If I am certain that a particular outcome is $x = x_m$, I can safely reject that $x \neq x_m$.

The theorem says that there exist circumstances under which any quantum theory satisfying these three assumptions is bound to yield contradictory conclusions. The extended version of the Wigner’s friend experiment discussed in [2] constitutes one paradigmatic example of such circumstances.

Before continuing with the analysis, it is worth to remark that the theorem focuses on particular outcomes that happen for certain —with probability $p = 1$. It does not refer to the real state of the corresponding system. Hence, its most remarkable feature is that contradictions arise as consequences of simple observations.

Let us review now all the steps of the experiment from the four agents point of view. We do not go into details about the assumptions required to reach each conclusion; we refer the reader to the original paper [2] for that purpose.

Step 1.- Agent $I_A$ measures the initial state, given by Eq. (31), in the basis $\{|h\rangle_a, |v\rangle_a\}$.

Fact 1: Given the shape of the initial state, agent $I_A$ concludes that, if it obtains that photon $a$ is vertically polarised (outcome $v_a$), then, a further measurement of the laboratory $B$ in the basis $\{|+\rangle_B, |-\rangle_B\}$ will lead to the outcome $+B$, provided that requirements R1-R4 of Tab. [11] are fulfilled.
In [2], this conclusion is reached without considering that agent \(I_A\) measurement requires an environment. However, this is not important to follow the same line of reasoning. From fact F1 of Tab. III the resulting state from Step 1 is

\[
|\Psi_1\rangle = \sqrt{\frac{1}{3}} |h\rangle_a |v\rangle_b |A_h\rangle_a |\varepsilon_1(t)\rangle_a + \\
+ \sqrt{\frac{2}{3}} |v\rangle_a |\mp\rangle_b |A_v\rangle_a |\varepsilon_2(t)\rangle_a .
\] (32)

This expression can be simplified considering the whole state of the laboratory \(A\) which consists in the photon \(a\), the measuring apparatus \(A_a\), and the environment \(\varepsilon_a\). Hence, let us denote

\[
|h(t)\rangle_A = |h\rangle_a |A_h\rangle_a |\varepsilon_1(t)\rangle_a ,
\]

\[
|v(t)\rangle_A = |v\rangle_a |A_v\rangle_a |\varepsilon_2(t)\rangle_a .
\] (33a, 33b)

And, therefore, the state after this measurement is

\[
|\Psi_1\rangle = \sqrt{\frac{1}{3}} |h(t)\rangle_A |v\rangle_b + \sqrt{\frac{2}{3}} |v(t)\rangle_A |+\rangle_b .
\] (34)

Fact 1 seems compatible with this state. There is a perfect correlation between state \(|v(t)\rangle_A\), which represents the case in which agent \(I_A\) has observed that the photon \(a\) is vertically polarised, and state \(|+\rangle_b\). Thus, in principle, agent \(I_A\) might conclude that a further measurement on laboratory \(B\) will yield \(+\rangle_B\), subjected to its own outcome is \(v_a\).

Step 2.- Agent \(I_B\) measures photon \(b\) in the basis \(|h\rangle_b, |v\rangle_b\).

Fact 2: If agent \(I_B\) observes that the photon is horizontally polarised, then the outcome of agent \(I_A\) cannot correspond to a horizontally polarised photon.

Again, this fact seems compatible with the decoherence framework. Using the same notation as before (applied to laboratory \(B\)), the state after agent \(I_B\) completes its measurement is

\[
|\Psi_2\rangle = \sqrt{\frac{1}{3}} |v(t)\rangle_A |h(t)\rangle_B + \\
+ \sqrt{\frac{2}{3}} |v(t)\rangle_A |v(t)\rangle_B + \sqrt{\frac{1}{3}} |h(t)\rangle_A |v(t)\rangle_B .
\] (35)

Therefore, there is a perfect correlation between \(|h(t)\rangle_B\) and \(|v(t)\rangle_A\); the probability of observing \(h_b\) and \(h_a\) in the same realization of the experiment is zero. Hence, all the previous conclusions are well supported.

Step 3.- Agent \(E_A\) measures laboratory \(A\) in the basis \(|+(\tau)\rangle_A, |-(\tau)\rangle_A\}, where

\[
|+(\tau)\rangle_A = \sqrt{\frac{1}{2}} (|h(\tau)\rangle_A + |v(\tau)\rangle_A) ,
\]

\[
|-(\tau)\rangle_A = \sqrt{\frac{1}{2}} (|h(\tau)\rangle_A - |v(\tau)\rangle_A) ,
\] (36a, 36b)

and \(\tau\) is the instant at which this measurement is performed.

This is the first step at which the decoherence framework, discussed in Sec. II, plays a relevant role. Any further conclusion from this measurement requires that conditions R1-R4 of Tab. II are fulfilled. So, let us suppose again that this happens, that is, agents \(I_A\) and \(E_A\) act in perfect synchronisation, and \(E_A\) knows all the details about laboratory \(A\). Then, let us consider the state after this measurement

\[
|\Psi_3\rangle = \sqrt{\frac{2}{3}} |+(\tau)\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |v(\tau)\rangle_B + \\
+ \sqrt{\frac{1}{6}} |+(\tau)\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |h(\tau)\rangle_B - \\
- \sqrt{\frac{1}{6}} |-(\tau)\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |h(\tau)\rangle_B ,
\] (37)

where \(A'\) is the measuring apparatus used by agent \(E_A\), and \(\varepsilon'_A\) the corresponding environment. From this state, we obtain:
Fact 3a: If the outcome obtained by agent $E_A$ is $-A$, then agent $I_B$ has obtained an horizontally polarised photon, $h_b$, in its measurement.

Fact 3b: Given facts 3b and 2, the outcome $-A$, obtained by agent $E_A$ determines that agent $I_A$ could not obtain an horizontally polarised photon.

Fact 3c: Given the facts 3b and 1, the outcome $-A$ determines that a further measurement on laboratory $B$, in the basis $\{|+(\tau)\}_B, \{-(\tau)\}_B\}$ will necessary yield $+B$.

The main conclusion we can infer from these sequential reasonings is that, if agent $E_A$ observes $-A$, then $E_B$ is bounded to observe $+B$. Therefore, it is not possible that outcomes $-A$ and $-B$ occur in the same realization of the experiment. Furthermore, as it is discussed in detail in [2], relying on assumptions Q, S, and C, it is straightforward to show that the four agents agree with that.

The contradiction that (presumably) establishes that quantum theory cannot consistently describe the use of itself consists in that the probability of obtaining $-A$ and $-B$ in the same realization of the experiments is $1/12$, even though all the agents, relying on assumptions Q, C and S, agree that such probability must be zero. This can be easily inferred from the final state of the system after measurements by all the agents (including $E_B$) are completed,

$$|\Psi\rangle = \sqrt{\frac{3}{4}} |+(\tau)\rangle_A |A^+_1(\tau)\rangle_A |\varepsilon'_1(\tau)\rangle_A |+(\tau)\rangle_B |A^+_1(\tau)\rangle_B |\varepsilon'_1(\tau)\rangle_B -$$

$$- \sqrt{\frac{1}{12}} |+(\tau)\rangle_A |A^+_1(\tau)\rangle_A |\varepsilon'_1(\tau)\rangle_A |-(\tau)\rangle_B |A^-_1(\tau)\rangle_B |\varepsilon'_2(\tau)\rangle_B -$$

$$- \sqrt{\frac{1}{12}} |-(\tau)\rangle_A |A^-_1(\tau)\rangle_A |\varepsilon'_2(\tau)\rangle_A |+(\tau)\rangle_B |A^+_1(\tau)\rangle_B |\varepsilon'_1(\tau)\rangle_B -$$

$$- \sqrt{\frac{1}{12}} |-(\tau)\rangle_A |A^-_1(\tau)\rangle_A |\varepsilon'_2(\tau)\rangle_A |-(\tau)\rangle_B |A^-_1(\tau)\rangle_B |\varepsilon'_2(\tau)\rangle_B .$$

B. Discussion relying on the decoherence framework

The first element that the decoherence framework introduces is that requisites R1-R4 from Tab. [11] together with the fast-enough realization of the external interference experiments, are mandatory to reach the previous conclusion. Hence, assumptions Q, S and C might only lead to contradictory conclusions if the experiment is performed under very specific circumstances. Results in Fig. [2] suggest that, the larger the laboratories $A$ and $B$ are, the more specific the circumstances of the experiment must be. Thus, if agents are not small quantum machines, composed by just a few qubits, but human beings, composed by a huge number of particles, the probability that such a contradiction might arise is virtually zero. This conclusion is fully compatible with the spirit of the decoherence interpretation of quantum mechanics [5]. This framework proposes that the universe is quantum, and therefore it is composed by weird superpositions in which a living organism can be dead and alive at the same time; but the probability that such a weird situation arises from a measurement is totally negligible. Hence, the first conclusion we can reach is that we are almost free of potentially contradictory situations.

However, the most important question discussed in [2], can quantum theory consistently describe the use of itself?, is still unanswered. The main aim of this section is to deal with this point. To tackle this task, we focus on steps 2 and 3, and the corresponding facts; a short comment about fact 1 will be given afterwards. As we did in Sec. [11] we will follow the point of view of all the agents, and their possible changes after each step.

After step 2, both agents $I_A$ and $I_B$ have completed their measurements. The resulting state, given by Eq. (35), reads

$$|\Psi_2\rangle = \sqrt{\frac{1}{3}} |v\rangle_a |A_v\rangle_a |\varepsilon_2(t)\rangle_a |h\rangle_b |A_h\rangle_b |\varepsilon_1(t)\rangle_b +$$

$$+ \sqrt{\frac{1}{3}} |v\rangle_a |A_v\rangle_a |\varepsilon_2(t)\rangle_a |v\rangle_b |A_v\rangle_b |\varepsilon_2(t)\rangle_b +$$

$$+ \sqrt{\frac{1}{3}} |h\rangle_a |A_h\rangle_a |\varepsilon_1(t)\rangle_a |v\rangle_b |A_v\rangle_b |\varepsilon_2(t)\rangle_b ,$$

if all the apparati and environments are explicitly shown.

As we have discussed in Sec. [11] the results of further experiments on this state depend on the ignored environmental degrees of freedom. In a standard experiment involving a pair of entangled photons, both agents communicate among themselves. This communication implies that both agents see both apparati, and therefore trace out both
environments. Under such conditions, any further experiments performed on the system are compatible with the mixed state given by \[17\]

\[
\rho_2 = \frac{1}{3} |v\rangle_a \langle A_v|_a |h\rangle_b \langle A_h|_b \langle v|_a \langle A_v|_a \langle h|_b \langle A_h|_b + \\
+ \frac{1}{3} |v\rangle_a \langle A_v|_a |v\rangle_b \langle A_v|_b \langle v|_a \langle A_v|_a \langle v|_b \langle A_v|_b + \\
+ \frac{1}{3} |h\rangle_a \langle A_h|_a |v\rangle_b \langle A_v|_b \langle h|_a \langle A_h|_a \langle v|_b \langle A_v|_b .
\]

That is, any further experiments performed by these two agents are compatible with fact 2, at this stage of the experiment.

Let us now proceed with step 3. After agent \(E_A\) completes its measurement, the state of the system, given by Eq. \[37\], reads

\[
|\Psi_3(\tau)\rangle = \frac{\sqrt{2}}{3} |+(\tau)\rangle_A |A_{3+}\rangle_A |\varepsilon_1(\tau)\rangle_A |v\rangle_b \langle A_v|_b |e_2(\tau)\rangle_b + \\
+ \frac{\sqrt{2}}{6} |+(\tau)\rangle_A |A_{3+}\rangle_A |\varepsilon_1(\tau)\rangle_A |h\rangle_b \langle A_h|_b |e_1(\tau)\rangle_b - \\
- \frac{1}{\sqrt{6}} |-(\tau)\rangle_A |A_{3-}\rangle_A |\varepsilon_2(\tau)\rangle_A |h\rangle_b \langle A_h|_b |e_1(\tau)\rangle_b .
\]

Eq. \[41\] represents the state of the whole system after the measurements performed by agents \(I_A\), \(I_B\) and \(E_A\) are completed. The way that these agents perceive this state depends on the further experiments they are able to perform, that is, on the degrees of freedom they trace out. A joint vision of agents \(I_B\) and \(E_A\) is obtained tracing out the environments \(\varepsilon_b\) and \(\varepsilon'_A\), leading to

\[
\rho_3(\tau) = \frac{2}{3} |+(\tau)\rangle_A |A_{3+}\rangle_A |v\rangle_b \langle A_v|_b |+(\tau)\rangle_A \langle A_{3+}\rangle_A |v\rangle_b \langle A_v|_b + \\
+ \frac{1}{6} |+(\tau)\rangle_A |A_{3+}\rangle_A |h\rangle_b \langle A_h|_b |+(\tau)\rangle_A \langle A_{3+}\rangle_A |h\rangle_b \langle A_h|_b + \\
+ \frac{1}{6} |-(\tau)\rangle_A |A_{3-}\rangle_A |h\rangle_b \langle A_h|_b |-(\tau)\rangle_A \langle A_{3-}\rangle_A |h\rangle_b \langle A_h|_b .
\]

This state is fully compatible with fact 3b. At this stage of the experiment, any further experiment performed by \(I_B\) and \(E_A\) together is incompatible with the outcomes \(-A\) and \(v_b\) at the same time. This implies that both agents \(I_B\) and \(E_A\) agree that, if the latter has obtained \(-A\), then the former has obtained \(h_b\).

To follow with the reasoning, agent \(E_A\) relies on fact 2, first to determine the outcome obtained by agent \(I_A\), and finally to conclude what \(E_B\) is bound to obtain in a further measurement. The key point is that, this fact has been obtained as a consequence of Eq. \[35\], which is not the real state of the system at this stage —Eq. \[35\] represents the real state of the system before agent \(E_A\) has performed its measurement. As we have seen in Sec. \[11\], the state of a laboratory, and therefore the corresponding perceptions of the observers living inside, change as a consequence of external interference experiments. Hence, none of the agents involved in the experiment can rely on a previous state of the system to reach safe conclusions about either its present or its future, if the whole system is going to be measured in between.

This important point is illustrated as follows. If we re-write the current state of the system, Eq. \[41\], in a basis
including \( \{ |h\rangle_a |h\rangle_b, |v\rangle_a |v\rangle_b \} \), we obtain

\[
|\Psi_3(\tau)\rangle = \frac{1}{\sqrt{3}} |h\rangle_a |A_h\rangle_a |\varepsilon_1(\tau)\rangle_a |A'_h\rangle_a |\varepsilon'_1(\tau)\rangle_a |v\rangle_b |A_v\rangle_b |\varepsilon_2(\tau)\rangle_b +
\]

\[
+ \frac{1}{3} |v\rangle_a |A_v\rangle_a |\varepsilon_2(\tau)\rangle_a |A'_v\rangle_a |\varepsilon'_1(\tau)\rangle_a |h\rangle_b |A_h\rangle_b |\varepsilon_1(\tau)\rangle_b +
\]

\[
+ \frac{\sqrt{3}}{12} |h\rangle_a |A_h\rangle_a |\varepsilon_1(\tau)\rangle_a |A'_h\rangle_a |\varepsilon'_1(\tau)\rangle_a |h\rangle_b |A_h\rangle_b |\varepsilon_1(\tau)\rangle_b +
\]

\[
+ \frac{\sqrt{3}}{12} |v\rangle_a |A_v\rangle_a |\varepsilon_2(\tau)\rangle_a |A'_v\rangle_a |\varepsilon'_1(\tau)\rangle_a |h\rangle_b |A_h\rangle_b |\varepsilon_1(\tau)\rangle_b
\]

(43)

To determine the joint vision of agents \( I_A \) and \( I_B \) at this stage of the experiment we have just to trace out \( \varepsilon_a, \varepsilon_b, A'_A \) and \( \varepsilon'_A \) from the density matrix arising from this wavefunction. This leads to

\[
\rho_3 = \frac{1}{3} |h\rangle_b |A_h\rangle_b |v\rangle_a |A_v\rangle_a \langle h|_b \langle A_h|_b \langle v|_a \langle A_v|_a +
\]

\[
+ \frac{1}{3} |v\rangle_b |A_v\rangle_b |v\rangle_a |A_v\rangle_a \langle v|_b \langle A_v|_b \langle v|_a \langle A_v|_a +
\]

\[
+ \frac{1}{6} |h\rangle_b |A_h\rangle_b |h\rangle_a |A_h\rangle_a \langle h|_b \langle A_h|_b \langle h|_a \langle A_h|_a +
\]

\[
+ \frac{1}{6} |v\rangle_b |A_v\rangle_b |h\rangle_a |A_h\rangle_a \langle v|_b \langle A_v|_b \langle h|_a \langle A_h|_a
\]

(44)

This is probably the most remarkable consequence of the decoherence framework. Before agent \( E_A \) performs its measurement, agents \( I_A \) and \( I_B \) agree that, if \( I_B \) has observed a horizontally polarised photon, then agent \( I_A \) has necessarily obtained a vertical polarisation. However, as a consequence of the interaction between the laboratory \( A \) and the apparatus \( A'_A \), this correlation has been lost and the joint view of agents \( I_A \) and \( I_B \) is now given by Eq. (44). Therefore, now, the outcome \( h_b \) does not determine the outcome obtained by agent \( I_A \), which can be either \( h_a \) or \( v_a \). And consequently, fact 3b is no longer true, and thus cannot be used to infer a conclusion about future outcomes.

As we have pointed out in Sec. [III], such a result is very difficult to interpret from the point of view of conscious (human) beings. Does it imply that agents \( I_A \) and \( I_B \) do not remember fact 2 anymore, but they naturally agree that \( h_b \) has always been compatible with both \( h_a \) and \( v_a \)? Although interesting, this question exceeds the purpose of this work. As illustrated in Sec. [III], such a change is smooth and univocally determined by the initial state and the Hamiltonian governing the quantum measurement: therefore, it is not speculation, but a well-supported result.

It is worth to note that the current unapplicability of fact 2 is not a contradictory consequence of assumptions Q, S and C, but it is due to the changes that agent \( E_A \) measurement entails. Indeed, all the agents can rely on the same assumptions to infer the outcomes of further experiments, conditioned to the correctness of the first sentence of assumption Q — system S is in the state \( |\Psi\rangle \) (at a particular instant of time). In other words, assumptions Q, S and C are valid if they are applied to the correct state of the system, and all the agents are aware of its change with time. Doing so, the reasoning of agent \( E_A \) can be summarized as follows:

(i) From Eq. (12), relying on assumption Q, I am certain that, if I read \( \neg A \) from my measurement, then agent \( I_B \) has obtained \( h_b \) in its experiment.

(ii) From Eq. (44), which represents agents \( I_A \) and \( I_B \) joint view after measurements of agents \( I_A \), \( I_B \) and mine are completed, the outcome \( h_b \) is compatible with either \( v_a \) or \( h_a \), for agent \( I_A \). Then, relying on assumption C, I am certain that, if I have obtained \( \neg A \), agent \( I_A \) sees the universe as if photon a were collapsed either to \( |v\rangle_a \) or to \( |h\rangle_a \), but I cannot safely state which of them.

(iii) As the contradiction shown in [2] crucially depends on the fact that my outcome \( \neg A \) implies that agent \( I_A \) outcome is \( v_a \), I am not bound to obtain such a contradiction. On the contrary, if I follow the same line of reasoning, that is, I rely on assumptions Q, S, and C, I will obtain that it is possible for \( E_B \) to obtain \( \neg B \) after I have obtained \( \neg A \) (with probability \( p = 1/12 \)).

This reasoning invalidates the proof of the no-go theorem presented in [2]. If assumptions Q, S and C are used within the decoherence framework, agents \( I_A, I_B, E_A \) and \( E_B \) do not reach the contradictory conclusion that \( \neg A \)
implies $+_{B}$. As we have pointed out above, the decoherence framework shows that this contradiction is due to an incorrect mixture of different states within the same deduction.

Prior to the end of this section, a brief comment on fact 1 is appealing. By means of the decoherence framework, it is straightforward to show that fact 1 would be correct if agent $E_{B}$ outcome were obtained immediately after the one obtained by agent $I_{B}$, that is, without the action of agent $E_{A}$. The reason is the same that invalidates fact 2 after agent $E_{A}$ measurement—the interaction between apparatus $A'_{A}$ and laboratory $A$ modifies the correlations between both laboratories. On the contrary, the final state of the experiment, Eq. (58), shows that a joint vision of agents $I_{A}$ and $E_{B}$ after all four measurements are completed is compatible with the outcomes $v_{a}$ and $-b$ happening at the same time, with probability $p = 1/12$.

Finally, it is worth to remark that we have not proved that the decoherence framework is free from inconsistencies. We have just shown that the proof of the theorem proposed in [2] is not valid if the decoherence framework is taken into account. But the main statement of the theorem can be still considered as a conjecture.

V. OBSERVER-INDEPENDENT FACTS

This section deals with the no-go theorem discussed in [3]. This theorem has been experimentally confirmed in [4].

A criticism is published in [3].

A. Original version of the experiment and no-go theorem

The structure of this experiment has been already discussed in Sec. IV A. The only difference is the initial state, which consists in a pair of polarised photons, spanned by $\{|h\rangle, |v\rangle\}$, and reads

$$|\Psi\rangle_{\beta} = \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} (|h\rangle_{a} |v\rangle_{b} + |v\rangle_{a} |h\rangle_{b}) +$$

$$+ \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} (|h\rangle_{a} |h\rangle_{b} - |v\rangle_{a} |v\rangle_{b}).$$

(45)

This state is used to illustrate a no-go theorem that establishes that the following four statements are incompatible, that is, are bound to yield a contradiction:

Statement 1.- Quantum theory is valid at any scale.

Statement 2.- The choice of the measurement settings of one observer has no influence on the outcomes of other distant observer(s).

Statement 3.- The choice of measurement settings is statistically independent from the rest of the experiment.

Statement 4.- One can jointly assign truth values to the propositions about outcomes of different observers.

The proof of this theorem is based on the initial state given by Eq. (45), and the observations given by the following operators:

$$A_{0} = |A_{h}\rangle_{a} \langle A_{h}|_{a} - |A_{v}\rangle_{a} \langle A_{v}|_{a},$$

(46)

$$B_{0} = |A_{h}\rangle_{b} \langle A_{h}|_{b} - |A_{v}\rangle_{b} \langle A_{v}|_{b},$$

(47)

$$A_{1}(\tau) = +(|\tau\rangle\rangle_{A} \langle +|\tau\rangle\rangle_{A} - -|\tau\rangle\rangle_{A} \langle -|\tau\rangle\rangle_{A},$$

(48)

$$B_{1}(\tau) = +(|\tau\rangle\rangle_{B} \langle +|\tau\rangle\rangle_{B} - -|\tau\rangle\rangle_{B} \langle -|\tau\rangle\rangle_{B},$$

(49)

where $|+(\tau)\rangle\rangle_{A}$ is given by Eq. (36a): $|-(\tau)\rangle\rangle_{A}$ is given by Eq. (36b), and equivalent relations determine $|+(\tau)\rangle\rangle_{B}$ and $|-(\tau)\rangle\rangle_{B}$. Again, $\tau$ is the time at which the interference measurements are performed, according to points R1-R4 of Tab. III

Observables $A_{0}$ and $B_{0}$ represent the internal agents, $I_{A}$ and $I_{B}$, points of view; from the corresponding outcomes, these agents see their photons as they were collapsed onto either horizontal or vertical polarization. Due to the perfect correlations between the photons $a$ and $b$, and the corresponding apparatus, $A_{a}$ and $A_{b}$, they are totally equivalent to $|h\rangle_{a} |h\rangle_{a} \langle h_{a}|_{a} \langle h_{a}|_{a} - |v\rangle_{a} |v\rangle_{a} \langle v_{a}|_{a} \langle v_{a}|_{a},$

(50)

$$B_{0} = |h\rangle_{b} \langle h_{b}|_{b} |h\rangle_{b} \langle h_{b}|_{b},$$

(51)

Observables $A_{1}$ and $B_{1}$ represent the external agents, $E_{A}$ and $E_{B}$, points of view. Again, the corresponding outcomes imply that these agents see the laboratories $A$ and $B$ as they were collapsed on either $|+(\tau)\rangle$ or $|-(\tau)\rangle$. 
The no-go theorem is formulated in terms of correlations between all these observables, applied to the initial state of the system. Statements 1–4 imply the existence of a joint probability distribution \( p(A_0, B_0, A_1, B_1) \) whose marginals satisfy the Claude-Horne-Shimony-Holt (CHSH) inequality \[19, 20\]

\[ S = \langle A_1 B_1 \rangle + \langle A_1 B_0 \rangle + \langle A_0 B_1 \rangle - \langle A_0 B_0 \rangle \leq 2. \] (52)

In \[13\] is theoretically shown that the initial state given by Eq. (45) leads to \( S = 2\sqrt{2} \); in \[4\] this result is confirmed by an experiment. The conclusion is that these resuls are incompatible with statements 1–4, and therefore, assuming that statements 2 (non-locality) and 3 (freedom of choice) are compatible with quantum mechanics \[8, 14, 20\], quantum theory is incompatible with the existence of observer-independent well established facts.

### B. The role of the decoherence framework

The previous analysis accounts neither for the structure of laboratories summarized in Tab. \[11\] nor for the measuring protocol given in Tab. \[11\]. Hence, from the decoherence point of view, applying the observables \( A_0, A_1, B_0 \) and \( B_1 \) directly to the initial state, Eq. (53), implies that none of the agents has performed any measurement. In other words, the previous result, \( S = 2\sqrt{2} \), just refers to an entangled state involving four photons (two corresponding to the measured systems, and the other two to the measuring apparatus), and not to a laboratory including observers acting upon their outcomes. This is exactly what has been measured in \[14\]—the non-definite character of a standard entangled state, not of a set of quantum measurements from which some agents perceive the reality as if it were collapsed onto definite outcomes.

To apply the decoherence framework, a complete description of the four measurements is mandatory. We consider four different situations, each one corresponding to one of the four correlations involved in Eq. (52). As \( A_1 \) and \( B_1 \) correspond to external interference experiments performed by agents \( E_A \) and \( E_B \), respectively, we consider that \( A_1 \) requires that agent \( I_A \) has completed its measurement, and the same for \( B_1 \).

**Case 1.** Agent \( I_A \) measures the state of photon \( a \) in a basis given by \( \{|\hbar\rangle_a, |v\rangle_a\} \), and \( I_B \) measures the state of photon \( b \) in a basis given by \( \{|\hbar\rangle_b, |v\rangle_b\} \). Without explicitly taking into account the external apparatus and environment, which are not entangled with laboratories \( A \) and \( B \) at this stage, the resulting state is

\[
\Psi_1 = \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} |\hbar\rangle_a |A_h\rangle_a |\varepsilon_1(t)_a |v\rangle_b |A_v\rangle_b |\varepsilon_2(t)_b + \\
+ \sqrt{\frac{1}{2}} \cos \frac{\pi}{8} |v\rangle_a |A_v\rangle_a |\varepsilon_2(t)_a |\hbar\rangle_b |A_h\rangle_b |\varepsilon_1(t)_b + \\
+ \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} |\hbar\rangle_a |A_h\rangle_a |\varepsilon_1(t)_a |\hbar\rangle_b |A_h\rangle_b |\varepsilon_1(t)_b + \\
+ \sqrt{\frac{1}{2}} \sin \frac{\pi}{8} |v\rangle_a |A_v\rangle_a |\varepsilon_2(t)_a |v\rangle_b |A_v\rangle_b |\varepsilon_2(t)_b .
\] (53)

**Case 2.** From Eq. (53), agent \( E_A \) measures the state of laboratory \( A \) in the basis \( \{|+(\tau)_A, -(\tau)_A\} \), considering requisites R1-R4 of Tab. \[11\]. The resulting state is

\[
\Psi_2 = \sqrt{\frac{1}{2}} \left( \cos \frac{\pi}{8} - \sin \frac{\pi}{8} \right) |+(\tau)_A |A^+_A \rangle_a |\varepsilon_1'(\tau)_A |v\rangle_b |A_v\rangle_b |\varepsilon_2(\tau)_b + \\
+ \sqrt{\frac{1}{2}} \left( \cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right) |-(\tau)_A |A^-_A \rangle_a |\varepsilon_2'(\tau)_A |v\rangle_b |A_v\rangle_b |\varepsilon_2(\tau)_b + \\
+ \sqrt{\frac{1}{2}} \left( \cos \frac{\pi}{8} + \sin \frac{\pi}{8} \right) |+(\tau)_A |A^+_A \rangle_a |\varepsilon_1'(\tau)_A |\hbar\rangle_b |A_h\rangle_b |\varepsilon_1(\tau)_b + \\
+ \sqrt{\frac{1}{2}} \left( \cos \frac{\pi}{8} - \sin \frac{\pi}{8} \right) |-(\tau)_A |A^-_A \rangle_a |\varepsilon_2'(\tau)_A |\hbar\rangle_b |A_h\rangle_b |\varepsilon_2(\tau)_b .
\] (54)

**Case 3.** From Eq. (53) again, agent \( E_B \) measures the state of laboratory \( B \) in the basis \( \{|+(\tau)_b, -(\tau)_b\} \), considering requisites R1-R4 of Tab. \[11\]. Note that this case is not subsequent to case 2; it represents a different
experimental protocol. The resulting state is

\[ |\Psi_3\rangle = \frac{1}{\sqrt{2}} \left( \cos \left( \frac{\pi}{8} \right) - \sin \left( \frac{\pi}{8} \right) \right) |v\rangle_a |A_v\rangle_a |\varepsilon_2(\tau)\rangle_a |+\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B + \]

\[ + \frac{1}{\sqrt{2}} \left( \cos \left( \frac{\pi}{8} \right) + \sin \left( \frac{\pi}{8} \right) \right) |v\rangle_a |A_v\rangle_a |\varepsilon_2(\tau)\rangle_a |-\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B + \]

\[ + \frac{1}{\sqrt{2}} \left( \cos \left( \frac{\pi}{8} \right) + \sin \left( \frac{\pi}{8} \right) \right) |h\rangle_a |A_h\rangle_a |\varepsilon_1(\tau)\rangle_a |+\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B + \]

\[ + \frac{1}{\sqrt{2}} \left( \cos \left( \frac{\pi}{8} \right) - \sin \left( \frac{\pi}{8} \right) \right) |h\rangle_a |A_h\rangle_a |\varepsilon_1(\tau)\rangle_a |-\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B. \]

(55)

**Case 4.** Agent \( E_A \) measures the state of laboratory \( A \) in the basis \{\( |+\rangle_A, |-\rangle_A \}\), considering requisites R1-R4 of Tab. I and agent \( E_B \) measures the state of laboratory \( B \) in the basis \{\( |+\rangle_B, |-\rangle_B \}\), following the same procedure. This case is subsequence to either case 2 or case 3. The resulting state is

\[ |\Psi_4\rangle = \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{8} \right) |+\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |+\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B - \]

\[ - \frac{1}{\sqrt{2}} \cos \left( \frac{\pi}{8} \right) |-\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |-\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B + \]

\[ + \frac{1}{\sqrt{2}} \sin \left( \frac{\pi}{8} \right) |+\rangle_A |A'_+\rangle_A |\varepsilon'_1(\tau)\rangle_A |-\rangle_B |A'_-\rangle_B |\varepsilon'_2(\tau)\rangle_B + \]

\[ + \frac{1}{\sqrt{2}} \sin \left( \frac{\pi}{8} \right) |-\rangle_A |A'_-\rangle_A |\varepsilon'_2(\tau)\rangle_A |+\rangle_B |A'_+\rangle_B |\varepsilon'_1(\tau)\rangle_B. \]

(56)

The main consequence of the decoherence framework is that the state of the system changes after each measurement. There exist two possible sequences compatible with the protocol devised in [3]: (i) \( |\Psi_1\rangle \rightarrow |\Psi_2\rangle \rightarrow |\Psi_4\rangle \), and (ii) \( |\Psi_1\rangle \rightarrow |\Psi_3\rangle \rightarrow |\Psi_4\rangle \). Each sequence is determined by a particular time-dependent Hamiltonian, representing the operations performed by the agents. The CHSH inequality can be applied at any particular instant of time, just by considering the corresponding state. If we want to test the independence of the four agents point of view, the logical choice is \( |\Psi_4\rangle \), which is the state that the system has after all the four agents have completed their measurements. We obtain the following results

\[ \langle \Psi_4 | A_0 B_0 | \Psi_4 \rangle = 0, \]

(57)

\[ \langle \Psi_4 | A_1 B_0 | \Psi_4 \rangle = 0, \]

(58)

\[ \langle \Psi_4 | A_0 B_1 | \Psi_4 \rangle = 0, \]

(59)

\[ \langle \Psi_4 | A_1 B_1 | \Psi_4 \rangle = 1/\sqrt{2}. \]

(60)

Therefore, the CHSH inequality applied to \( |\Psi_4\rangle \) leads to \( S = 1/\sqrt{2} < 2 \).

The main conclusion obtained from this analysis is the following. If the decoherence framework is properly taken into account, the experiment devised in [3] is compatible with the four statements discussed above — quantum theory is valid at any scale; the choice of the measurement settings of one observer has no influence on the outcomes of other distant observers; the choice of the measurement settings is independent form the rest of the experiment, and one can jointly assign truth values to the propositions about the outcomes of different observers. In other words, these statements do not imply a contradiction in this experiment, if the role of all the parts of each laboratory, given in Tab. I and the physical mechanisms giving rise to each outcome, are considered.

Exactly as it happens with the no-go theorem formulated in [2], our result does not prove that the use of statements 1 – 4 is free from contradictions in any circumstances. We have just shown that the particular setup used to prove the no-go theorem in [3] does not lead to contradictions if the decoherence framework is properly taken into account. But again, the main statement of the theorem can be still considered as a conjecture.

To finish this section, it is worth to discuss why the decoherence framework leads to \( S = 1/\sqrt{2} \) instead to \( S = 2\sqrt{2} \). The simplest answer is that this last result is recovered if CHSH is applied to a mixture of different states. In particular

\[ \frac{\sqrt{S}}{2} = \langle \Psi_4 | A_1 B_1 | \Psi_4 \rangle + \langle \Psi_2 | A_1 B_0 | \Psi_2 \rangle + \]

\[ + \langle \Psi_3 | A_0 B_1 | \Psi_3 \rangle - \langle \Psi_1 | A_0 B_0 | \Psi_1 \rangle = 2\sqrt{2}. \]

(61)

This is a very remarkable result. It shows that both the contradictions discussed in [2] and [3] are due to misleading mixtures of states. The agents involved in the experiment devised in [2] are bound to obtain contradictory results.
if they rely on conclusions reached by other agents from previous states, which do not represent the current state of the system. The experiment proposed in [3] leads to $S = 2\sqrt{2}$, that is, to the inexistence of observer-independent well established facts, if the CHSH inequality is not applied to the final state of the system, but each of its terms is evaluated in a different state.

VI. CONCLUSIONS

The main conclusion of this work is that neither the original Wigner’s friend experiment, nor the extended version proposed in [2], nor the one in [3] (and its corresponding experimental realization, [4]) entail contradictions if the decoherence framework is properly taken into account.

This framework consists in considering that a quantum measurement and the corresponding (apparent) wave-function collapse are a consequence of the interaction between the measuring apparatus and an uncontrolled environment. In this work, we have relied on a simple model to show that a chaotic interaction is necessary to induce such an apparent collapse, but, at the same time, a quite small number of environmental qbits suffices for that purpose. This implies that any experiment on any quantum system can be modeled by means of a unitary evolution, and therefore all the time evolution, including the outcomes obtained by any observers, is univocally determined by the initial state, the interaction between the system and the measuring apparatus, and the interaction between such apparatus and the corresponding environments. Seeing the reality as if a random wave-function collapse had happend is due to the lack of information suffered by the observers —only the system as a whole evolves unitarilly, not a part of it. This is a somehow paradoxical solution to the quantum measurement problem: ignoring an important piece of information about the state in which the observer lives is mandatory to observe a definite outcome; taking it into account would lead to no observations at all. But, besides the ontological problems arising for such an explanation, the resulting framework is enough for the purpose of this work. Its main consequence is that the global state of the whole experiment changes in a deterministic way after each measurement, and that such changes can entail that the involved agents also change their perceptions of the reality.

When we take all these facts into account, the contradictions discussed in [2] and [3] become the consequence of wrongly mixing outcomes obtained from different states. If the three assumptions devised in [2] to test the consistency of the quantum theory are applied to the real state in which the whole experimental setup is after all the outcomes are obtained, the conclusions reached by all the agents are not contradictory at all. And the same happens regarding the experiment proposed in [3]: if the CHSH inequality is applied to the state at which the whole system is at the end of the protocol, the resulting value is compatible with the existence of observer-independent facts.

However, this is not enough to dismiss the main statements of the no-go theorems formulated in such references. The conclusion of this work is that the examples used to prove these theorems are not valid within the decoherence framework, but we have not proved that this framework is totally free of similar inconsistencies. Hence, these statements can be still considered as conjectures. Further work is required to go beyond this point.

It is also worth to remark that the decoherence formalism also narrows down the conditions under which the external interference measurements, trademark of Wigner’s friend experiments, are expected to work. This means that, if the decoherence framework results to be true, we can safely avoid such strange situations, except in a very few bizarre circumstances.

Finally, the conclusion of this work must not be understood as a strong support of the decoherence framework. It just establishes that such a framework does not suffer from the inconsistencies typically ensuing Wigner’s friend experiments. However, there is plenty of space for theories in which the wavefunction collapse is real [6]. These theories predict a totally different scenario, since after each measurement the wave function of the whole system collapses, and therefore becomes different from the predictions of the decoherence framework. Hence, experiments like the ones discussed in this work might be a way to test which of this proposals is correct —if any. The huge development of quantum technologies, expected for the coming years, can help us to deal with this task.

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