Calculation model of the plasma load matching with the current sources based on explosive magnetic generator

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Abstract. In this paper, we consider the model of engineering calculation for the matching of the explosive-magnetic generators (EMG) and pulsed plasma accelerator (PPA) at amplitudes of currents reaching 2.5 MA. The main features of the model are taking into account the dynamics of change in the inductance of PPA and the using of the concentrated mass approximation for the current shell. The taken assumptions were checked by experimental data on real PPA powered by EMG. It is shown that the built model has sufficient accuracy for preliminary calculations for design and installation of such technique.

1. Introduction

The need for efficient powering energy of pulsed plasma loads imposes strict requirements on the front and amplitude of the current generated in the load [1, 2]. Investigations of a pulsed current source based on an explosive generator (EMG) show its advantages over a capacitive storage device primarily due to the growing character of the power of this type of current source. The experiments presented in [3] confirm this fact. However, it should be noted that a correct engineering calculation of the parameters of the EMG and matching of its work with a variable plasma load are required for the implementation of these advantages in practice at the design stage of the system. The need for such models is especially great for explosive experiments with EMG, which are relatively expensive.

The use of electrical models for the analysis of linear circuits is impossible in this case because of the pulsed character of the sources, and nonlinear dynamics of current – voltage characteristics of the plasma load. The main nonlinearity is related to the change in space and time of the current shell shape of the plasma in the load, which is mainly inductive. At the same time, magneto hydrodynamic calculation for spatial geometry is complicated and provides extra information for engineering and design engineering. In this paper, the problem is considered for the matching scheme of EMG and pulsed plasma accelerator at the amplitudes of discharge current up to 2.5 MA. Experiments have shown that the dynamics of the inductance of the load is a critical parameter in this problem. Due to the self-consistency of the parameters of the system, which includes an explosive generator, switching elements and plasma load, the calculation of the pulse current in engineering models requires reasonable simplifications. The main assumption taken as a basis in this model is the use of a time-dependent shell model with a concentrated mass. The specified simplification allows us to reduce the model of the motion of the current shell to a one-dimensional model of the motion of the concentrated
mass used, which greatly simplifies performing a series of calculations and optimization of EMG design under a specific load.

The dynamics of the shell mass change is consistent with the experimental data. The obtained data are compared with experimental data obtained in real experiments with the pulsed plasma accelerator (PPA) powered from EMG. It is shown that the built model has sufficient accuracy for preliminary calculations of matching the operation of the EMG with the dynamics of PPA. On the basis of the constructed model, recommendations for the coordination of a specific plasma load for currents up to 5 MA were developed.

2. Model of the motion of the current shell during powering energy from the capacitive storage

As a rule, the stage of studying the parameters of the load powered by energy from a capacitive storage source precedes experiments with the use of EMG. Therefore, at the first stage, a math model of motion of a current shell with a concentrated mass was investigated when powering from a capacitive storage device. It allows simplifying the electrical component of the model and using a large amount of experimental data with use capacity stored sources.

In this paper, we consider the operation of a plasma accelerator with a pulsed injection of the working gas. The specified accelerator is shown schematically in figure 1. Its distinctive feature is the variable mass of the current shell. On the one hand, this is due to the pulsed nature of the gas inject and, accordingly, to the heterogeneity of its distribution. On the other hand it is with the geometry of the current shell. Since the system is coaxial, the pondermotive force at the inner electrode is always higher than that of the outer electrode. This leads to the curvature of the current shell and significant emissions of gas mass to the external electrode [4, 5]. It should be added that other types of plasma loads such as plasma focus, plasma, breakers, etc. also have the specified feature.

In its motion, the plasma shell limits the area occupied by the magnetic field associated with the discharge current. Therefore, the current inductance of the load depends on the position and shape of the shell. Due to the fact that the magnetic field of internal electrode is much greater than the external, it makes a greater contribution to the inductance of the load. In this case, taking into account the Hall Effect near the internal electrode [6] the current shell in this area partially is aligned of its shape. The above reasoning determines the application of the shell model with variable concentrated mass as an effective model for calculating the dynamics of the plasma shell. However, it is worth noting that the use of this model involves a sufficiently high degree of cylindrical symmetry of the initial breakdown of the electrode gap and the movement of the current shell, which is not always performed in practice.

With the known geometry of the electrode system, the acceleration of the current shell depends on the current flowing through the shell, the derivative of the inductance with respect to coordinate x at this site of the electrode system and the mass of the shell:

\[ a = \frac{I^2 \frac{dL}{dx}}{2m} \]  

(1)

As the experimental data show, for an adequate description of the dynamics of change in the position of the current shell, its mass in the process of motion should reduced by at least 5 times, and quite quickly. In other words, there is a rapid release of a mass of gas that ceases to participate in the acceleration. This leads to a sharp acceleration of the current shell and, accordingly, to an increase in the inductance of the load.

In the first approach, the dependence of mass on time is used. In this case, this calculation model allows us to describe the experimentally observed dynamics of the load inductance change for each experiment. However, when changing the operating mode of the load (for example, when changing the mass of the gas in the gap or increasing the current in the system), the time dependence of its mass changes significantly. The moment of a sharp decrease in the mass of the shell changes in the first place. This indicates that the specified mass setting way cannot be scaled from one mode to another. However, good model scalability is especially important when load switching to power from EMG,
because EMG can potentially provide currents of significantly greater amplitude than capacitive storages.

In the second approach, the dependence of the mass on the position of the shell within the electrode system was used to determine the variable mass of the gas. This approach is justified because there is a high degree of similarity of the current shell shape for different load modes and different currents in the system. The moment the shell reaches a certain position in space primarily depends on the current and the absolute value of the mass. In turn, the shape of the shell and the relative change in its mass are more dependent on the geometry of the electrode system and the initial gas distribution. In addition, the moment of mass ejection can be tied to the geometry of the electrode system. Thus, the transition from one mode of operation of the accelerator to another one is characterized by a change in the velocity of the shell at a similar shape, which causes a change in the moment of mass ejection. At the same time, the position of the shell where the specified event occurs remains almost invariable. Since the electrode system of the used accelerator is coaxial, not cylindrical, it has critical points that can provoke a sharp acceleration of the current shell. First of all, it refers to the place where the cylindrical part of the electrode system transits to the cone part. The electrical circuit, which was used in the calculation model, when operating from a capacitive storage source, is shown in figure 1.

![Electrical circuit of PPA powering from the capacitor.](image)

**Figure 1.** Electrical circuit of PPA powering from the capacitor. The red line is the motion of the plasma front in the PPA of coaxial type. $C$ is the capacity of the inductive source 1 MF, 24 kV; $R$ – ohmic resistance of the circuit, $L_{pass}$ is the passive inductance in the circuit, $S1$ is the closing key to the capacitive storage.

The system of equations describing the self-consistent task of discharge of a capacitive storage source on a load with a moving plasma shell in the considered approximation is as follows:

$$
\begin{align*}
\frac{dQ}{dt} &= -I \\
\frac{dI}{dt} &= \frac{Q}{C} - R I - I \frac{\partial L}{\partial x} \\
\frac{dx}{dt} &= V \\
\frac{dV}{dt} &= \frac{j^2 dL}{2m}
\end{align*}
$$

where $Q$ is the charge accumulated in the capacitive storage with capacity $C$; $R$ is ohmic resistance of the circuit, $L_{pass}$ passive inductance in the circuit, $L$ is the inductance of the load, in the form of plasma accelerator, $V$ and $m$ are the speed and mass of the plasma current shell.
When analyzing the experimental data, the optimal dependence of the shell mass on the coordinate was chosen, this most closely describes the experimentally obtained load inductance dynamics. Comparisons of current and voltage pulses obtained in the calculation model and in the real experiments are shown in figure 2. The selected dependence of the shell mass on the coordinate is shown in the upper left corner. It is worth noting a significant discrepancy between the calculation results and the experiments after the moment of reaching the maximum current. As can be seen from the above comparison, in the experiment, the current decline after reaching the maximum is significantly slower, and the voltage increase associated with the increase in the inductance of the load is "cut off". The fact is that the concentrated shell model stops working after reaching the maximum current. If as the current increases, the magnetic field behind the current shell continuously increased and, accordingly, the magnetic pressure on the inner surface of the shell increased, then at the stage of the current decline, the boundary condition changes significantly. Reducing the field on the inner surface of the shell, which has a significant conductivity, leads to the fact that the magnetic field diffused into the shell at the stage of current growth, begins to diffuse from the shell back. This provokes the formation of closed current lines that on the one hand slow down the inner surface of the current shell, on the other – accelerate the outer. Such current configurations are reliably fixed by magnetic probes both in our experiments and in the experiments of other authors [7].

![Graph showing comparison of calculated and experimental data of current and voltage dynamics](image)

**Figure 2.** Comparison of calculated and experimental data of current and voltage dynamics, when PPA is powered by the capacitor. The dependence \( m(x) \) taken in the calculation is in the upper left corner. The blue lines are the current. Here the solid line shows the calculation; the dotted line is the experimental values. The brown lines are the voltage on the plasma accelerator manifold, where the solid line is calculation, the dotted line is experiment. The line of dots is the position along the \( x \) current shell.

The described processes lead to the fact that the shell significantly expands at a sufficiently high speed. Figure 3 qualitatively illustrates the current shell decay scheme. In this case, we can no longer talk about the concentrated mass of the shell, since the current is distributed over a very wide plasma formation. Since that time, the dynamics of inductance growth is no longer described by the model of a plasma shell with a concentrated mass.
Figure 3. The decay scheme of the thin current shell of the plasma after passing the maximum current (before – left, after – right). $I_1$, $I_2$ are the currents on the inner and outer surface of the shell; $F_1$, $F_2$ are the forces acting on the inner and outer surface of the shell; $B$ is the field behind the shell, $B_i$ is the field inside the shell.

Despite the above limitation of the applied model, it well describes the current growth stage, which is the most important in terms of power supply and load matching. In practice, the most important parameter of the described loads is just the maximum achievable amplitude of the current until its sharp decline (the so-called "special feature"). After this point in the system, as a rule, it is not possible significantly to increase the magnetic energy, which also applies to the efficiency of the load operation. The above reasoning substantiates the computational model of the shell motion and the boundaries of its application to solve the problem of matching the power supply with plasma loads of the selected type.

3. The results of numerical simulations of the current pulse when powered by EMG

To describe the experiment with plasma accelerator powered by an explosion-magnetic generator (EMG), a model of the current shell motion in the PPA, powered by a capacitive storage device, was used. At the same time, the modes of both the load (plasma accelerator) and the pulse gas injection were similar. The electrical circuit of the EMG with a variable load used for theoretical evaluations is shown in figure 4.

Figure 4. Electrical circuit of PPA powering from the EMG. $C_0$ is the capacitance of an initial power supply for EMG, $S_3$ is the key closing circuit of $C_0$ on EMG, $S_1$, $S_2$ are closing keys circuit EMG when the liner starts and the load connects. $L_{emg}$ is inductance of EMG, $L_{pass}$ and $R_{pass}$ are passive inductance and the ohmic resistance of the load circuit, $L_r$ and $R_r$ are the inductance and the ohmic resistance of the circuit breaker.

The calculation of the EMG is performed in two stages. In the beginning the calculation of EMG output inductance with using the open numerical complex FEMM 4.2 was performed. This complex allows two-dimensional stationary calculations of magnetic fields at a constant frequency by the finite
element method. The possibility of using scripts to rebuild the geometry allowed calculating the high-frequency inductance for each position of the liner. At the same time, the dynamics of the liner was set manually according to the corresponding experimental data. The final dynamics of the output inductance, which was used in the calculation of the electrical circuit of the generator on the load, is shown in figure 5.

Figure 5. Dynamics of output inductance for EMG (right). Dynamics of changes in the resistance of the explosive breaker (left).

The circuit 1 was shorted to EMG before the load is connected to the generator. This greatly simplifies the equations for its calculation. When the load is connected, the current first flows through both circuits: 1 and 2. At the same time, in the first circuit, when the explosive switch is triggered, its resistance increases in time. The dynamics of the specified resistance determines the process of switching the current to the load. The dependence of the breaker resistance on time was approximated with use the experimental data by the function of the form:

\[
R_r = R_0 \times \exp \left( \left( \frac{T}{T_0} \right)^k \times \ln(n) \right),
\]

(3)

where \(R_0\) is the initial resistance of the breaker at temperature \(T_0\), \(k\) and \(n\) are the approximation coefficients. This formula (3) well describes the experimentally measured resistance dynamics of the developed breaker (see figure 5). However, it should be noted that for higher breaking currents (~2.5–3 MA for our design) when the aluminum foil in the explosive breaker is heated by the current VMG to the melting point, this function may not accurately describe the growth of the resistance of an explosive circuit breaker. In this case, the geometric dimensions of the explosive breaker should be increased. However, the function of the increase in the resistance of the explosive breaker and the dynamics of the inductance output (in figure 5) allow for a set of preliminary calculations to determine the output parameters of the EMG current. Thus, the described model of the EMG allows us to conduct a complete and complex calculation of the scheme powering a plasma load from EMG in conjunction with the model of the motion of the current shell.

The final system of equations describing the dynamics of current switching to the load with its restructuring agreement is presented below:

\[
\begin{align*}
\frac{dl}{dt} &= \frac{-l\left\{R + f \frac{dL_{emg}}{dt} \right\} \left\{(Ln + Lr) + Ln \cdot Rr\right\} - ln \left\{Lr \left( \frac{dLn}{dx} \frac{dx}{dt} \right) - Ln \cdot Rr\right\}}{L \left\{(Ln + Lr) + Ln \cdot \frac{Ln + Lr}{Lr} \right\} + \frac{R_0 \cdot Lr \cdot \frac{dLn}{dx} \frac{dx}{dt}}{L \left(\frac{Ln + Lr}{Lr} + Ln + Lpass\right)}} \\
\frac{dlm}{dt} &= \frac{Rr \cdot Lr \cdot \frac{dLn}{dx} \frac{dx}{dt}}{L \left(\frac{Ln + Lr}{Lr} + Ln + Lpass\right)} \\
\frac{dx}{dt} &= V \\
\frac{dV}{dt} &= \frac{i^2 \frac{dl}{dx}}{2m}
\end{align*}
\]

(4)
where \( f \) is the coefficient of EMG perfection, and

\[
    f = \ln\left(\frac{I_n}{I_{emg}}\right) / \ln\left(\frac{L_{res} + L_n}{L_n}\right),
\]

which reflects the degree of magnetic flux saving during EMG operation; \( I_n, I_{emg} \) are the currents through the load and EMG respectively; \( L_{res} \) is residual inductance of EMG, \( L_n \) is inductance of load; \( I \) is the current through the circuit breaker, \( I_n \) is the current through the load, and \( L_r, R_r \) are breaker inductance and resistance.

The formulated model allows varying the parameters of the load, EMG and explosive breaker in a wide range. It makes the possible to optimize the design in order to harmonize the mode of current input to the load with the dynamics of the current shell movement directly in the plasma load. The specific characteristics of the various details of design are refined using experimental data obtained in preliminary experiments. Semi-empirical formulas include dependence: mass of the shell from the coordinates of the resistance of an explosive circuit breaker EMG time and the distribution coefficient of perfection of EMG along the length of the spiral. Figure 6 illustrates the results of calculations of the parameters of the current pulse and the load voltage for the developed design of the generator and their comparison with experimental data.

![Figure 6](image)

**Figure 6.** Comparison of calculated and experimental data of current and voltage dynamics at plasma accelerator (PPA) powering from EMG. Blue lines are the current. Solid line is the load current and dotted line is the current through the breaker, obtained in the experiment. Line of points is load current. Brown lines are the voltage at the collector plasma accelerator PPA. The solid line is the experimental value; the dotted line is the calculated value.

The obtained curves confirm that the constructed model well describes the dynamics of the shell up to the maximum current, as in the case of a capacitive storage source. After this moment the simulation and experimental results differ significantly. However, as the analysis of the time diagram of the experiment shows, by the time the maximum current is reached, the generator has already finished its work and completely used up its inductance. Therefore, the differences between the experiment and theory based on the concentrated mass model are insignificant from the point of view of the matching the EMG and the investigated load during its work.
4. Conclusion
Two semi-empirical models are built and self-consistent. These are a model of the motion of the current shell within the electrode system of the plasma accelerator, and a model of the operation of the explosion-magnetic generator. Simulation of the dynamics of the shell motion during the operation of the plasma accelerator powered by EMG was carried out using these models. The comparison of the simulation and experimental results showed a good agreement of the results up to the maximum current. It is worth noting that, due to the empirical character of the built models, the significant changes as in the design EMG and plasma accelerator require a preliminary series of experiments, as the study of work load and separate details of EMG design. Due to the high cost of the explosive experiment with EMG, it is advisable to check the load operation with capacitive storage sources. The proposed approach allows minimizing the cost of resources for the development of EMG to supply a specific plasma load.

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