Optimal control for extraction lipid model of microalgae Chlorella Vulgaris using PMP method

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Abstract. Microalgae is one of the energy to be used as biodiesel. Chlorella Vulgaris is one of the most economical algae to produce biodiesel, because these green algae are rich in carbohydrates, require no special care, and easy to grow. Algae oil obtained for biodiesel production is obtained through a fairly long process, one of which is the process of lipid extraction. In the study of lipid extraction process using solid-liquid or leaching extraction is the transfer of dissolved dissolved components from solids into the solvent. In the study used CO2-expended methanol (CXM). Soxhlet Extraction Method is a method for lipid extraction using hexane, in the process of extraction with Soxhlet tool there is a process of lipid mass transfer contained in Chlorella Vulgaris to flow CXM. This process of displacement has created a diferential equation which can be viewed as a system by Sriati Monalisa Siahaan(2016) to be used as a mathematical model to predict extractable lipid results for subsequent use as the main ingredient of biodiesel production. In her research, Sriati refers to a research conducted by Yi-Hung Yang(2015). In this research aims to perform optimal control with Pontryagin Maximum Principle (PMP) method to produce maximum lipid extraction result. This method is used to obtain the best control of the dynamic system from the initial state to the last by maximizing the objective function and more modern than other methods such as Linear Quadratic Regulator (LQR). Using the PMP method to optimal control in this systems, can increase the value of lipid concentration.

1. Introduction

Biodiesel is an oil fuel made of renewable materials like plants and animals used as an alternative to fossil fuels. Biodiesel has a 10-15 cetane number, higher than diesel, so the combustion process is faster, the engine becomes softer, and not noisy [1]. Plant species that can be used for biodiesel feedstock are palm oil, soybean, and jatropha curcas, and some potentially good plants as the biodiesel feedstock, ie microalgae. The superiority of microalgae compared to other vegetable materials is the process of extracting the oil that is done without milling and directly extracted with the help of solvent [2]. Microalgae are water plants that can be grouped into 2 groups of macroalgae and microalgae. According to Weber Van Boss In the Indonesian waters found 782 species of microalgae scattered throughout the territorial waters of Indonesia. Microalgae are autotrophs that can photosynthesize and thus require sufficient sunlight and carbon dioxide [3]. Choosing microalgae as the basic ingredient of biodiesel is a good alternative based on the amount of production. It is estimated that microalgae are capable of producing 200 times more biodiesel than oil-producing plants (jatropha oil and palm oil).
Chlorella Vulgaris microalgae are recommended for industrial ingredients. In addition to a large number of microalgae in Indonesian waters, this type of microalgae has been widely cultivated in bulk. Research on microalgae as an alternative energy source has been much done like the research done by Nanda Dewi Oktavianti, Optimal Control of Microalgae Growth Through Dilution Levels of Nutrition [4], then research of Hajar, Carbon Dioxide Control in Algae Growth Model [5], Nailul Izzati, Optimal Feeding Strategy on Microalgae Growth in Fed-Batch Bioreactor Model [6] and Mardlijah et al, Optimal control of algae growth by controlling CO2 and nutrition flow using Pontryagin Maximum Principle [7]. Processing microalgae oil into biodiesel requires a fairly long process, one of the important steps is the extraction of lipids. Extraction is the process of separating components from solids with the help of solvents. Liquid-solid extraction or leaching is the transfer of dissolved components diffusion from solids into the solvent, which uses CXM (CO2-Expanded Methanol) solvent in this study. In the extraction process occurs the process of lipid mass transfer contained in Chlorella Vulgaris using Soxhlet tool that uses repetitive filtering and heating. The process of lipid mass transfer from this microalgae will then be formed in a differential equation so it can be used as a mathematical model.

The mathematical model itself is a simple way to translate a problem into a mathematical language using equations, inequalities, or functions. In this research the mathematical model is used to predict the extracted lipid results for subsequent use as the main ingredient of biodiesel production and optimal control is performed by adjusting the volume of solvent to produce maximum lipid extraction. To obtain the maximum lipid extraction, optimal control in accordance with the mathematical model obtained is required, therefore to perform optimal control with Pontryagin Maximum Principle (PMP) method to produce maximum lipid extraction result. This method is used to obtain the best control of the dynamic system from the initial state to the last by maximizing the objective function and more modern than other methods such as Linear Quadratic Regulator (LQR).

2. Mathematics Model of Extraction Lipid Microalgae Chlorella Vulgaris

Based on the mechanism of extraction process, then the formation of subsystem can be described as follows. Microalgae extraction process is the process of releasing the solute (lipid) from the cell matrix into the solvent. This process is a mass transfer process. Solutes that are in the cell matrix will be released and bonded with solvents that diffuse into the cell by dissolution. Furthermore, the dissolved solute diffuses through the pore towards the surface of the particle. On the outside of the microalgae particles there is a film layer separating the concentration on the flow and the concentration at the surface of the particle. Finally, the solute moves through the stagnant film layer around the particles toward the flow of the solvent. The illustration above then becomes the basis for the formation of Mathematical model of lipid mass transfer divided into 3 subsystems expressed as follows [8]:

a. Subsystem 1
   In subsystem 1, the balance process is described in the flow of solvent in the extraction column.

b. Subsystem 2
   In this subsystem, there is a process of mass balance on microalgae particles. The equation for subsystem 2 is described as mass transfer through the outer layer of the film.

c. Subsystem 3
   In this process occurs the absorption of CXM solvent. Adsorption occurs when the solvent is attached to the lipid in the microalgae and subsequently there is a reversible uptake process to exit from the solid phase (solid) of the microalgae particles.
Based on the extraction mechanism on each subsystem that formed the lipid mass transfer process, then obtained the dynamic model of microalgae lipid extraction process *Chlorella vulgaris* as follows:

\[
\begin{align*}
\frac{dC_s}{dt} &= -(1 - \varepsilon_p) \frac{dC_p}{dt} - \frac{S}{\rho V} C_s \\
\frac{dC_p}{dt} &= -k_f a (C_p - C_{sat}^*) \frac{1}{V(1 - \varepsilon_p)} - \frac{dC_l}{dt} \\
\frac{dC_l}{dt} &= k_a C_p - k_d C_l.
\end{align*}
\]  

(1)

**Table 1.** Description of the variables and parameters Model

| Symbol | Description                              | Unit  |
|--------|------------------------------------------|-------|
| *C_s*  | Lipid concentration on the surface of the particle | kg/m³ |
| *C_p*  | Lipid concentration in microalgae particles | kg/m³ |
| *C_s*  | Lipid concentration in the flow of solvent | kg/m³ |
| *K_a*  | Kinetic constant adsorption              | 1/s   |
| *K_d*  | Kinetic constant desorption              | 1/s   |
| *K_f*  | Mass transfer coefficient                | m/s   |
| *C_{sat}^* | Lipid concentration in balance            | kg/m³ |
| *\varepsilon_p* | Porosity of microalgae particles        | -     |
| *S*    | Solvent flow rate                        | kg/s  |
| *a*    | Surface area of microalgae particles (4πr²) | m²    |
| *ρ*    | Mass solvent type                        | kg/m³ |
| *V*    | Volume of solvent                        | m³    |

3. Design Control System using PMP Method

In this study the goal of optimal control completion is to optimize lipid concentration in microalgae particles by controlling the volume. From Equation (1), we will look for optimal control using the maximum principle of pontryagin. The steps are:

1. Form purpose function In accordance with the aim of this study is to maximize lipid concentration of the solvent flow (*C_s*) and lipid concentration in microalgae particles (*C_p*) by minimizing the volume of solvent (*V*) inserted, from this problem then the purpose function as follows:

\[
J(V) = \int_{t_0}^{t_f} (C_s(t) + C_p(t) - \frac{Q}{2} V^2(t))dt
\]  

(2)

2. Settlement Steps Form The Hamilton function

\[
H(C_s, C_p, C_l, V, \lambda) = C_s(t) + C_p(t) - \frac{Q}{2} V^2(t) + \sum_{i=1}^{3} \lambda_i f_i
\]  

(3)

\[
H = C_s(t) + C_p(t) - \frac{Q}{2} V^2(t) + \lambda_1 \left( \frac{1}{\varepsilon} \left( -k_f a (C_p - C_{sat}^*) \frac{V}{V(1 - \varepsilon_p)} - k_a C_p + k_d C_l \right) \right) + \lambda_1 C_s \frac{S}{\rho V} + \lambda_2 k_d C_l - \lambda_2 k_a C_p - \lambda_2 \frac{k_f a (C_p - C_{sat}^*)}{V(1 - \varepsilon_p)} + \lambda_3 k_a C_p - \lambda_3 k_d C_l
\]
Maximize $H$ against vector control $v(t)$

$$\frac{\partial H}{\partial V} = 0$$

$$V(t) = \left( \frac{\lambda_1 - k_f a (C_p - C_{sat})}{\varepsilon} - \frac{k_f a (C_p - C_{sat})}{(1 - \varepsilon_p)} \right)^{-\frac{1}{\lambda}}$$

Determining the optimal $H^*$

$$H^* = C_s(t) + C_p(t) - \frac{Q}{2} V^2(t) + \lambda_1 \left( \frac{- (1 - \varepsilon_p)}{\varepsilon} \left( \frac{- k_f a (C_p - C_{sat})}{V(1 - \varepsilon_p)} - k_a C_p + k_d C_l \right) \right)$$

$$- \lambda_1 C_s S \frac{\rho \varepsilon V}{\rho \varepsilon V} - \lambda_2 k_d C_l - \lambda_2 k_a C_p - \lambda_2 \frac{- k_f a (C_p - C_{sat})}{V(1 - \varepsilon_p)} + \lambda_3 k_a C_p - \lambda_3 k_d C_l$$

Solve the state and costate equations to obtain the optimal system

State Equations:

$$\frac{\partial H^*}{\partial \lambda_1} = \left( \frac{-(1 - \varepsilon_p)}{\varepsilon} \left( \frac{- k_f a (C_p - C_{sat})}{V(1 - \varepsilon_p)} - k_a C_p + k_d C_l \right) \right) - C_s \frac{S}{\rho \varepsilon V}$$

$$\frac{\partial H^*}{\partial \lambda_2} = k_d C_l - k_a C_p - \frac{- k_f a (C_p - C_{sat})}{V(1 - \varepsilon_p)}$$

$$\frac{\partial H^*}{\partial \lambda_3} = k_a C_p - k_d C_l$$

Costate Equations:

$$\frac{\partial \lambda_1}{\partial t} = - \frac{\partial H^*}{\partial C_s} = -(1 - \lambda_1 \frac{S}{\rho \varepsilon V})$$

$$\frac{\partial \lambda_2}{\partial t} = - \frac{\partial H^*}{\partial C_p} = -(1 - \lambda_2 k_a - \lambda_2 \frac{k_f a}{V(1 - \varepsilon_p)} + \lambda_1 \left( \frac{- (1 - \varepsilon_p)}{\varepsilon} \left( \frac{- k_f a}{V(1 - \varepsilon_p)} - k_a \right) \right) + \lambda_3 k_a)$$

$$\frac{\partial \lambda_3}{\partial t} = - \frac{\partial H^*}{\partial C_l} = - \left( \lambda_1 \left( \frac{- (1 - \varepsilon_p)}{\varepsilon} k_d \right) + \lambda_2 k_d - \lambda_3 k_d \right)$$
4. Simulation
Given the value of the parameter to be used in the extraction process as follows:

| Parameter | Value |
|-----------|-------|
| $k_a$     | 0.000002 1/s |
| $k_d$     | 0.000001 1/s |
| $k_f$     | 1.72737 m/s |
| $a$       | 1.256x10^{-11} m^2 |
| $C_{sat}$ | 9.25   |
| $V$       | 9x10^{-5} m^3 |
| $\varepsilon_p$ | 0.3 |
| $S$       | 5x10^{-5} kg/s |
| $\rho$    | 74.79195 kg/m^3 |

Before analyzing the results obtained after optimum control using the mass transfer model of lipid extraction on algae, it will show the initial conditions before being given control. Given the initial conditions ($C_s$, $C_p$, $C_l$)=(0.5, 0.3, 0.1), the lipid concentration of the solvent flow ($C_s$) with concentrations 0.5 kg/m$^3$ decreased until 1800 seconds and headed to 0 kg/m$^3$. Lipid concentration in microalgae particles ($C_p$) and concentration on the surface lipid solvents ($C_l$) with concentrations 0.3 kg/m$^3$ and 0.1 kg/m$^3$ only experienced a relatively small increase of 0,3046 kg/m$^3$ and 0,1009 kg/m$^3$. It can be seen in Figure 1. Figure 2, shows that after the optimal control has increased. Before controlling at $t=1800$ the value of lipid concentration in solvent flow ($C_s$) was headed to 0 kg/m$^3$, then after control at $t=1800$ the value of lipid concentration in solvent flow ($C_s$) amount 7,4138 kg/m$^3$. This indicates that after the existence of the optimal control can affect lipid concentration in the solvent flow.
Figure 3. Lipid concentration in microalgae particles ($C_p$)

Figure 4. Concentration on the surface lipid solvents ($C_l$)

Figure 3, before controlling at $t = 1800$, the value of lipid concentration in microalgae particles ($C_p$) was 0.3046 kg/m$^3$. Then after given of optimal control at $t = 1800$ the value of lipid concentration in microalgae particles ($C_p$) get the final value is 9.0401 kg/m$^3$. This is shows that after given of optimal control to the lipid concentration in microalgae particles has the difference is quite significant. Figure 4, shows that after optimal control has increased. Before controlling at $t = 1800$, the value of concentration on the surface lipid solvents ($C_l$) was 0.1009 kg/m$^3$, then after control at $t = 1800$ the value of concentration on the surface lipid solvents ($C_l$) was 0.1709 kg/m$^3$. This suggests that after optimal control can affect the concentration on the surface lipid solvents.

5. Conclusion
In this paper, PMP method is used as the optimal control method to maximize lipid concentration of the solvent flow ($C_s$) and lipid concentration in microalgae particles ($C_p$) by minimizing the volume of solvent ($v$) inserted, with the initial conditions ($C_s$, $C_p$, $C_l$)=(0.5, 0.3, 0.1). The final time at $t = 1800$ shows that before controlling is ($C_s$, $C_p$, $C_l$)=(0, 0.3046, 0.1009) and after controlling is ($C_s$, $C_p$, $C_l$)=(7.4138, 9.0401, 0.1709). It shows that the optimal control with PMP method can increase the value of lipid concentration.

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