Abstract:
We add here compared to our former arXiv version an explicit expression for the descendant ratio along the generations that appeared in the Hebrew version published in BDD (Bar-Ilan University Press) 23, 71 (2010), titled “The distribution route from ancestors to descendants”. The equation that is added here is of an equation that appears in the Hebrew publication in Eq. 11, an explicit descendants ratio \[ 1 - r_n = (1 - r_0)^{2^n} \].

With the approximation \[ \ln(1 - r_0) \approx -r_0 = -1/N_0 \] one gets \[ n_r \approx \log_2 N_0 + \log_2 [\ln(1 / (1 - r_0))] \]. A half of the population becomes descendant after \[ n_{1/2} \approx \log_2 N_0 - 0.53 \] generations and for 99.99% it takes \[ n_{0.9999} \approx \log_2 N_0 + 3.2 \] generations. Otherwise, we don’t change nor show here the paper but leave and refer the reader to the former arXiv version with the above addition or to the published paper in Hebrew in BDD (Bar-Ilan University Press), 23, 71 (2010).
In the former arXiv version we had the abstract:

We study the distribution of descendants of a known personality, or of anybody else, as it propagates along generations from father or mother through any of their children. We ask for the ratio of the descendants to the total population and construct a model for the route of Distribution from Ancestors to Descendants (DAD). The population ratio \( r_n \) is found to be given by the recursive equation \( r_{n+1} = (2 - r_n) r_n \), that provides the transition from the \( n \)-th to the \((n+1)\)th generation. The number of generations it takes to make half the population descendants is \( \log N_0 / \log 2 \) and additional \( \sim 4 \) generations make everyone a descendent (=the full descendant spreading time). These results are independent of the population growth factor even if it changes along generations. As a running example we consider the offspring of King David. Assuming a population between \( N_0 = 10^5 \) and \( 5 \times 10^6 \) of Israelites at King David's time (~ 1000 BC), it took 24 to 26 generations (about 600-650 years, when taking 25 years for a generation) to make every Israelite a King David descendent. We note that this work doesn't deal with any genetical aspect. We also didn't take into account here any geo-social-demographic factor. Nevertheless, along tens of generations, about 120 from King David's time till today, the DAD route is likely to govern the distribution in communities that are not very isolated.

In the new version we direct the reader to the former arXiv version, but we add to the recursive equation 3.7 what is added in the Hebrew version, an explicit expression for the descendants:

\[
\frac{r}{n} = 1 - (1 - \frac{1}{N_0})^{2n}.
\]

From the former arXiv paper we bring below only the introduction, a short summary on the expression for the descendant ratio, a summary section and relevant references.

1. Introduction

The well known Galton-Watson (GW) process [1, 2] investigates the extinction of surnames which propagate from father to son. We consider here a model which unlike the GW model depends on both parents, namely the offspring of a known personality which propagates through father or mother to their children and we ask for the ratio of the descendants to the total population. As an example, we will consider the offspring of King David. King David lived about 3,000 years ago. We assume that a generation (from birth until marriage and children) is 25 years, and each married couple has \( 2g \) children, where \( g \) is the growth factor per generation. Let \( N_n \) denote the number of Israelites at the \( n \)-th generation and \( N_n = N_0 - g^n \) where \( n = 0, 1, 2, \cdots, 120 \) (= 3000/25). Let \( D_n \) denote the number of descendants (male and female) of King David at the \( n \)-th generation and \( C_n = N_n - D_n \) denote the non-descendants. We start at the first generation with \( D_0 = 1 \), the dynasty founder. Our problem is to estimate the ratio \( \frac{D_n}{N_n} \) after \( n \) generations. This will show, in particular, that practically all Israelites today are descendants. It is possible that a family disappears after a few generations (discussed in Section 6), but we assume throughout the paper that it doesn't happen. In our example with what we know about King David and his son Solomon, we do not have to worry about that. However if we are not sure about that we can say, as we discuss later, that either all Israelites are his descendant or none. Therefore if there is one descendant then all are descendants.
It will be shown that for a population of \( N_0 \) it took \( \log N_0 / \log 2 \) generations to make half the population descendants of King David. Additional four generations made all of them his descendants. That is the DAD full spreading time. The transitions region between low to high spreading ratio is very quick, a few generations. It is not only King David; the same relation exists regarding anyone else of his era (or any other early era) whose family survived in the first few generations (discussed in Section 6), including for example less admirable characters in the Bible like Nabal... Assuming a population between \( N_0 = 10^6 \) and \( N_0 = 5 \times 10^6 \) Israelites at King David’s time (~ 1000 BC) [3], it took 24 to 26 generations (600-650 years, when taking 25 years for a generation) to ensure that every Israelite was his 2 descendant. That means that every Israelite living at 400 BC, the beginning of the era of the Second Temple in Jerusalem, was already a descendant of King David.

An interesting feature of the DAD route is that the descendant population ratio and the spreading time depend on \( N_0 \) but not on the population growth factor \( g \) even if \( g \) is generation dependent.

2. The rule of passing from \( D_n \) to \( D_{n+1} \)

We add here a new expression that appeared only in the Hebrew version [4]. It is an explicit expression for the descendant ratio:

\[
R_n = 1 - (1 - r_0)^{2n}.
\]

It adds to the recursive equation for the descendants given in Eq. 3.7 of the former arXiv version.

Conclusion:

We have presented a model for the distribution of descendants along generations, the DAD route. The descendant population and the ratio are given by the recursive equations of the former version (3.6) and (3.7). The descendant ratio, given for a few examples in figure 1, is shown to reach 1 in a relatively few generations. For an initial population of \( N_0 \) the DAD full spreading time is ~ \( \log N_0 / \log 2 + 4 \), that gives about 20 generations (500 years) for \( N_0 = 10^5 \). Every additional factor of 10 in \( N_0 \) adds 1/ \( \log_{10} 2 \) = 3.32 generations to the DAD full spreading time. The basic DAD route behavior, in particular the descendant population ratio does not depend on the population growth factor \( g \), but only on the initial population \( N_0 \).

We have not included here any genetic or geo-social-demographic aspects. It is clear that DAD will not spread into and out of very isolated groups. Nevertheless, we saw how quick the spreading process is. For a small group of \( 10^3 \) or \( 10^4 \) it takes about 14 and 17 generations (350 and 425 years) to reach a full descendant spreading ratio. For all Israelites at King’s David time it took 24-26 generations (600-650 years) to make every Israelite his descendent. That therefore happened already at 400 BC, the beginning of the era of the Second Temple in Jerusalem. Even for the whole world population at King’s David time (1000 BC), estimated as \( 5 \times 10^7 \), it is but 29.5 generations (740 years). Segregation of local communities can slow down the process, but only in a limited way for relatively short time. It would be sufficient that one descendant migrates to another community, say with a similar population number, to make the whole population descendants in a few generations. Therefore, more globally one
might say that even if mankind started with many Adams and Eves, it took relatively very short time to consider the whole founder group at any early era, as a Common Forefather.

A super DAD of all of us.

For the future, the DAD route means that, assuming a reasonable population mobility, each of us on earth today \( N_0 = 5 \times 10^9 \) (beyond the low extinction percentage discussed in Section 6, and assuming that no catastrophic event happens, will be an ancestor of everyone in the world - on Earth in \( \sim \log(5 \times 10^9) / \log 2 + 4 = 36 \) generations (\( \sim 900 \) years) from now. Can this picture lend a philosophical meaning to what is said about a common forefather of mankind? We saw that all of us have common ancestors and eventually we will be ourselves the ancestors of everyone in the future in a relatively short period of time. They all were our Fathers and Mothers and they all will be our Sons and Daughters...

We finally note that after we first deposited the paper in ArXiv, our attention was drawn to the work of Derrida et al. on the same subject [5, 6]. We believe that our paper adds new insight and results treating the process along the generation line rather than the backward way. We have added a short discussion on the difference between the approaches in Sec. 7.

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**References**

[1] F. Galton and H.W. Watson, “On the probability of extinction of familites”, Journal of the Anthropological Institute, (1874), 308–311.

[2] S. Karlin and H. H. Taylor, “A First Course in Stochastic Processes”, Academic Press, New York, 1975.

[3] The Bible, I Chronicles; Chapter 21:5, (New American Standard Bible, 1995): “Joab gave the number of the census of all the people to David. And all Israel were 1,100,000 men who drew the sword; and Judah was 470,000 men who drew the sword.” We therefore estimate the total population to be a few millions.

[4] BDD (Bar-Ilan University Press) 23, 71 (2010), titled “The distribution route from ancestors to descendants”.

[5] B. Derrida, S. C. Manrubia, and D. H. Zanette; “Statistical Properties of Genealogical Trees”, Phys. Rev. Lett. 82, 1987, (1999).

[6] B. Derrida, S.C. Manrubia, D.H. Zanette, Jour. theor. Biol. 203 (2000) 303. (arXiv:physics/0003016v1); [http://arxiv.org/abs/physics](http://arxiv.org/abs/physics)