In the paper, a state space model for vibration control with arbitrary controller structures for soft mounted induction motors with sleeve bearings fixed on active motor foot mounts is shown. Besides the mathematical description of the forced vibrations caused by dynamic rotor eccentricity, a procedure is presented to derive the threshold of vibration stability. The challenge of the paper is the use of arbitrary controller structures with different feedback strategies – feedback of the motor feet displacements, velocities or accelerations – in combination with a special vibration system. This specialty is, that the stiffness and damping matrices depend on the rotor angular frequency \( \Omega \), which corresponds to the excitation angular frequency, when analyzing the forced vibration, and depend additionally on the natural angular frequency \( \omega_{\text{stab}} \) of the critical mode, when analyzing the threshold of stability. After the mathematical description is shown, a numerical example of a 2-pole induction motor (power rating 2.4 MW) is presented, in which the threshold of stability is analyzed as well as the forced vibrations due to dynamic rotor eccentricity by investigating the bearing housing vibrations, the foundation vibrations and the actuator forces.

KEYWORDS
active vibration control, actuators, foundation, induction motor, sleeve bearings

1 | INTRODUCTION

Large induction motors with high power ratings (> 1MW) are often mounted on flexible steel frame foundations. This leads sometimes to vibration problems, so that the vibration limits according to ISO 10813-3\(^{[1]} \) cannot be fulfilled. The reason is that such motors are usually designed for operation on massive foundations, according to IEC 60034-14.\(^{[2]} \) Therefore, critical speeds may occur in the operation speed range, if the motor is mounted on a flexible steel frame foundation, leading to resonance problems.\(^{[3–7]} \) An approach to improve the vibration behavior is to implement active vibration control,\(^{[8–26]} \) which is used in many different technical applications, to improve the vibration behavior. The active vibration system consists hereby of active motor foot mounts (actuators), which are positioned between the motor feet and the foundation, vibration sensors, which are mounted at each motor foot and detect the vertical vibrations and lead – depending on the feedback strategy – the vertical displacement or velocity or acceleration of each motor foot to a separate controller. This concept was basically described and
investigated in [27], but only for standard controllers (P-, I-, PI-, PD (ideal)-, PID (ideal)-controller) and only in combination with a feedback strategy that allowed the controller parameters to be implemented in the system matrices – the mass matrix and/or the damping matrix and/or the stiffness matrix. This paper advances this concept by contributing a mathematical description that permits the use of arbitrary controller structures in combination with each feedback strategy. However, in this case the implementation of the controller parameters in the mass matrix and/or damping matrix and/or stiffness matrix may no longer be possible. Therefore, a special state space formulation for the controlled system is used, where arbitrary controller structures in combination with different feedback strategies for a special vibration system can be analyzed. In this system the stiffness and damping matrices depend on the rotor angular frequency \( \Omega \), which corresponds to the excitation angular frequency, when analyzing the forced vibration due to dynamic rotor eccentricity, and depend additionally on the natural angular frequency \( \omega_{stab} \) of the critical mode, when analyzing the threshold of stability.

## 2 | VIBRATION MODEL

The vibration model is a simplified plane multibody model (Figure 1), basically described in [27], which contains two main bodies, with the rotor mass \( m_w \) and the stator mass \( m_s \), with the moment of inertia \( \theta_{ax} \).

The mass of each shaft journal \( m_v \) and of each bearing housing \( m_b \) for each rotor side, as well as the masses of the actuators \( m_{as} \) (stator) and \( m_{aa} \) (armature) and the foundation masses \( m_L \) and \( m_R \) for each motor side are used as additional masses. The rotor rotates with the rotary angular frequency \( \Omega \) and has the stiffness \( c \) and internal damping (rotating damping) \( d_i \). The shaft journals of the rotor are connected to the sleeve bearing housings by the stiffness and the damping matrices \( C_v \) and \( D_v \) of the oil film:

\[
C_v = \begin{bmatrix}
  c_{zz} & c_{zx} \\
  c_{zx} & c_{yy}
\end{bmatrix}, \quad D_v = \begin{bmatrix}
  d_{zx} & d_{zy} \\
  d_{yz} & d_{yy}
\end{bmatrix}
\] 

The bearing housings are connected to the stator by the stiffness and damping matrices \( C_b \) and \( D_b \) of the sleeve bearing housings including the end-shields:

\[
C_b = \begin{bmatrix}
  c_{bz} & 0 \\
  0 & c_{by}
\end{bmatrix}, \quad D_b = \begin{bmatrix}
  d_{bz} & 0 \\
  0 & d_{by}
\end{bmatrix}
\]
The electromagnetism of the induction motor, is considered by the electromagnetic spring and damper matrices $C_m$ and $D_m$:

$$C_m = \begin{bmatrix} c_{md} & 0 \\ 0 & c_{md} \end{bmatrix}; \quad D_m = \begin{bmatrix} d_m & 0 \\ 0 & d_m \end{bmatrix} \quad (3)$$

The stator structure can be assumed to be rigid, due to the soft steel frame foundation. The actuator stiffness matrix $C_a$ and the actuator damping matrix $D_a$ connect the motor feet to the flexible foundation:

$$C_a = \begin{bmatrix} c_{az} & 0 \\ 0 & c_{ay} \end{bmatrix}; \quad D_a = \begin{bmatrix} d_{az} & 0 \\ 0 & d_{ay} \end{bmatrix} \quad (4)$$

The actuator forces, which are induced in the system, are described by $f_{azL}$ and $f_{azR}$. All the values of the actuators are related on one motor side. The foundation stiffness and damping is described by the stiffness matrices $C_fL$ and $C_fR$ and the damping matrices $D_fL$ and $D_fR$:

$$C_fL = \begin{bmatrix} c_{fzL} & 0 \\ 0 & c_{fyL} \end{bmatrix}; \quad C_fR = \begin{bmatrix} c_{fzR} & 0 \\ 0 & c_{fyR} \end{bmatrix}; \quad D_fL = \begin{bmatrix} d_{fzL} & 0 \\ 0 & d_{fyL} \end{bmatrix}; \quad D_fR = \begin{bmatrix} d_{fzR} & 0 \\ 0 & d_{fyR} \end{bmatrix} \quad (5)$$

Because of the planarity of the model ($yz$-plane), only one vibration sensor is positioned on each motor side, measuring the vertical motor feet vibrations and transmitting the signals – motor feet displacements $z_{aL}$ and $z_{aR}$ or velocities $v_{zLaL}$ and $v_{zRaR}$ or accelerations $a_{zLaL}$ and $a_{zRaR}$ (depending on the feedback strategy) – to the controllers. The controllers have arbitrary controller structures, described in the Laplace domain – with the Laplace variable $s$ – by the transfer functions $G_{cL,r}(s)$ and $G_{cR,r}(s)$:

$$G_{cL,r}(s) = \frac{\sum_{\mu L=0}^{mL} b_{\mu L} s^\mu}{\sum_{\nu L=0}^{nL} a_{\nu L} s^\nu}; \quad G_{cR,r}(s) = \frac{\sum_{\mu R=0}^{mR} b_{\mu R} s^\mu}{\sum_{\nu R=0}^{nR} a_{\nu R} s^\nu} \quad (6)$$

where $b_{\mu L}, b_{\mu R}, a_{\nu L}, a_{\nu R}$ are the constants of the polynomial functions.

Three different kinds of dynamic rotor eccentricity are considered here as excitations, eccentricity of rotor mass (mass eccentricity), bent rotor deflection (rotor bow), and magnetic eccentricity (Figure 2), which are occurring with rotor angular frequency $\Omega$.

Referring to [35–38], the oil film stiffness and damping coefficients $c_{ij} = c_{ij}(\Omega)$ and $d_{ij} = d_{ij}(\Omega)$ can be calculated. The electromagnetic stiffness coefficient $c_{md}(\omega_F)$ and damping coefficient $d_m(\omega_F)$, depending on the whirling angular frequency $\omega_F$, can be calculated referring to [27–34]. The mechanical damping coefficients $d_n(d_1, d_{b2}, d_{by}, d_{fzL}, d_{fzR}, d_{fyL}, d_{fyR}, d_{az}, d_{ay})$ can
be derived by the corresponding mechanical loss factor \( \tan \delta_n \), the corresponding stiffness \( c_n \) and by the whirling frequency \( \omega_F \), referring to [7] and [27]:

\[
d_n = \frac{c_n \cdot \tan \delta_n}{\omega_F} \quad \text{with} \quad n = i, bz, by, f z L, f y L, f z R, f y R, az, ay
\]

with the additional definition \( c_i = c \). For analysis of the forced vibration due to dynamic rotor eccentricity, the whirling angular frequency is equal to the rotor angular frequency \( \omega_F = \Omega \). For analysis of natural vibrations with marginal decay, the whirling angular frequency can be defined as the correspondent natural angular frequency of the considered mode, as a simplification, referring to [7].

3 | MATHEMATICAL MODEL

3.1 | Formulation in the time domain

Based on [27], the following differential equation can be described:

\[
M \cdot \ddot{q} + D \cdot \dot{q} + C \cdot q = f_e + f_a
\]  

(8)

with linearization for the motor feet displacements, because of small displacements:

\[
z_{aL} = z_s - \varphi_s \cdot b; \quad z_{aR} = z_s + \varphi_s \cdot b; \quad y_{aL} = y_s = y_s - \varphi_s \cdot h
\]  

(9)

The vector \( q(t) \) contains the coordinates for the displacements of important points and the rotation \( \varphi_s \) of the stator mass:

\[
q(t) = \left[ z_s; z_a; y_s; y_w; \varphi_s; z_v; z_b; z_{fL}; y_v; y_b; y_{fL}; y_{fR} \right]^T
\]  

(10)

The excitation force vector \( f_e \) can be described by:

\[
f_e(t) = \hat{f}_e \cdot e^{i \Omega t} = \left( \hat{f}_{e,u} + \hat{f}_{e,m} + \hat{f}_{e,a} \right) \cdot e^{i \Omega t}
\]  

(11)

with:

\[
\begin{bmatrix}
0 \\
\frac{m_u}{2} \Omega^2 \\
0 \\
-j \cdot \frac{m_u}{2} \Omega^2 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
\ddot{e}_u \cdot e^{i \varphi_u}; \\
\ddot{e}_m \cdot e^{i \varphi_m}; \\
\ddot{e}_a \cdot e^{i \varphi_a}
\end{bmatrix}
\]

(12)

\[
\begin{bmatrix}
-c_{md} + j \Omega d_m \\
c_{md} - j \Omega d_m \\
f_{c_{md}} + j \Omega d_m \\
f_{c_{md}} - j \Omega d_m \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \begin{bmatrix}
0 \\
c \\
0 \\
-j \cdot c \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

The vector \( \hat{f}_{e,u} \) describes the excitation by rotor mass eccentricity, caused e.g. by residual unbalance with the amplitude \( \dot{e}_u \) and the phase \( \varphi_u \) of the mass eccentricity. The vector \( \hat{f}_{e,m} \) describes the excitation by magnetic eccentricity, caused by e.g. deviation of concentricity between the inner diameter of the rotor core and the outer diameter of the rotor core, with the amplitude \( \dot{e}_m \) and the phase \( \varphi_m \) of the magnetic eccentricity. Vector \( \hat{f}_{e,a} \) describes the excitation by bent rotor deflection, which is e.g. caused by
thermal bending of the rotor with the amplitude \( \hat{a} \) and the phase \( \varphi_a \). The actuator force vector can be split into the actuator force vector on the left side \( f_{azL}(t) \) and on the right side \( f_{azR}(t) \) of the motor:

\[
f_a(t) = \hat{f}_a \cdot e^{i\Omega t} = \hat{f}_{azL} \cdot e^{i\Omega t} + \hat{f}_{azR} \cdot e^{i\Omega t} = (\hat{f}_{azL} + \hat{f}_{azR}) \cdot e^{i\Omega t}
\]

(13)

with:

\[
\hat{f}_{azL} = \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
-\hat{b} \\
0 \\
0 \\
-1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} \quad \hat{f}_{azR} = \begin{pmatrix}
1 \\
0 \\
0 \\
0 \\
b \\
0 \\
0 \\
-1 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix}
\]

(14)

Where \( \hat{f}_{azL} \) and \( \hat{f}_{azR} \) are the complex amplitudes of the actuator forces on the left side and on the right side of the motor.

The mass matrix \( \mathbf{M} \) is described by:

\[
\mathbf{M} = \begin{bmatrix}
m_s + 2m_{aa} & 0 & 0 & 0 & 0 & 0 \\
0 & m_o & 0 & 0 & 0 & 0 \\
0 & 0 & m_s + 2m_{aa} & 0 & -2m_{aa} \cdot h & 0 \\
0 & 0 & 0 & m_o & 0 & 0 \\
0 & 0 & -2m_{aa} \cdot h & 0 & \Theta_{sx} + 2m_{aa} (b^2 + h^2) & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2m_v & 0 & 0 & 0 & 0 \\
0 & 0 & m_{as} + m_{fL} & 0 & 0 & 0 \\
0 & 0 & 0 & m_{as} + m_{fR} & 0 & 0 \\
0 & 0 & 0 & 0 & 2m_v & 0 \\
0 & 0 & 0 & 0 & 2m_b & 0 \\
2m_v & 0 & 0 & 0 & 0 & 0 \\
0 & 2m_b & 0 & 0 & 0 & 0 \\
0 & m_{as} + m_{fL} & 0 & 0 & 0 & 0 \\
0 & 0 & m_{as} + m_{fR} & 0 & 0 & 0 \\
0 & 0 & 0 & 2m_v & 0 & 0 \\
0 & 0 & 0 & 2m_b & 0 & 0 \\
0 & 0 & 0 & m_{as} + m_{fL} & 0 & 0 \\
0 & 0 & 0 & 0 & m_{as} + m_{fR} & 0
\end{bmatrix}
\]

(15)
The damping matrix $D$ is described by:

$$
D = \begin{bmatrix}
2(d_{az} + d_{bz}) + d_m & -d_m & 0 & 0 & 0 \\
-d_m & d_m + d_i & 0 & 0 & 0 \\
0 & 0 & 2(d_{ay} + d_{by}) + d_m & -d_m & -2d_{ay}h \\
0 & 0 & -d_m & d_m + d_i & 0 \\
0 & 0 & -2d_{ay}h & 0 & 2(d_{ay}h^2 + d_{az}b^2) \\
0 & -d_i & 0 & 0 & 0 \\
-2d_{bz} & 0 & 0 & 0 & 0 \\
-d_{az} & 0 & 0 & 0 & d_{az}b \\
-d_{az} & 0 & 0 & 0 & -d_{az}b \\
0 & 0 & 0 & 0 & -d_i \\
0 & 0 & -2d_{by} & 0 & 0 \\
0 & 0 & -d_{ay} & 0 & d_{ay}h \\
0 & 0 & -d_{ay} & 0 & d_{ay}h \\
\end{bmatrix}
$$

The stiffness matrix $C$ is described by:

$$
C = \begin{bmatrix}
2(c_{az} + c_{bz}) - c_{md} & c_{md} & 0 & 0 & 0 \\
0 & c - c_{md} & 0 & 0 & d_i\Omega \\
c_{md} & 0 & 0 & 2(c_{ay} + c_{by}) - c_{md} & c_{md} \\
0 & -d_i\Omega & c_{md} & c - c_{md} & 0 \\
0 & 0 & -2c_{ay}h & 0 & 2(c_{ay}h^2 + c_{az}b^2) \\
0 & 0 & -c & 0 & -d_i\Omega \\
0 & -2c_{bz} & 0 & 0 & 0 \\
-c_{az} & 0 & 0 & 0 & c_{az}b \\
-c_{az} & 0 & 0 & 0 & -c_{az}b \\
0 & d_i\Omega & 0 & -c & 0 \\
0 & 0 & -2c_{by} & 0 & 0 \\
0 & 0 & -c_{ay} & 0 & c_{ay}h \\
0 & 0 & -c_{ay} & 0 & c_{ay}h \\
\end{bmatrix}
$$
FIGURE 3 State space model for vibration control of the vibration system with negative feedback of the output vector

\[
\begin{bmatrix}
0 & -2c_{bz} & -c_{az} & -c_{az} & 0 & 0 & 0 & 0 \\
-c & 0 & 0 & 0 & -d_1\Omega & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -2c_{by} & -c_{ay} & -c_{ay} \\
d_1\Omega & 0 & 0 & 0 & -c & 0 & 0 & 0 \\
0 & 0 & c_{az}b & -c_{az}b & 0 & 0 & c_{ay}h & c_{ay}h \\
2c_{zz} + c & -2c_{zz} & 0 & 0 & 2c_{zy} + d_1\Omega & -2c_{zy} & 0 & 0 \\
-2c_{zz} & 2(c_{zz} + c_{by}) & 0 & 0 & -2c_{zy} & 2c_{zy} & 0 & 0 \\
0 & 0 & c_{az} + c_{izL} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c_{az} + c_{izR} & 0 & 0 & 0 & 0 \\
2c_{yz} - d_1\Omega & -2c_{yz} & 0 & 0 & 2c_{yy} + c & -2c_{yy} & 0 & 0 \\
-2c_{yz} & 2c_{yz} & 0 & 0 & -2c_{yy} & 2(c_{yy} + c_{by}) & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & c_{ay} + c_{iyL} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & c_{ay} + c_{iyR}
\end{bmatrix}
\]

(19)

Differing from [27], now a state space formulation, based on [17–20], is used.

The reason why the index “st” is used here for the matrices of the state space is to avoid confusion with the stiffness matrix \( C \) and damping matrix \( D \).

After transferring the differential equation system (8) into the state space model (Figure 3), the following mathematical descriptions can be used, where \( \mathbf{x}(t) \) is the state space vector, described by the coordinate vector \( \mathbf{q} \) and the velocity vector \( \dot{\mathbf{q}} \):

\[
\mathbf{x}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \end{bmatrix}
\]

(20)

The vector \( \mathbf{y}(t) \) is the output vector, described by the coordinate vector \( \mathbf{q} \), the velocity vector \( \dot{\mathbf{q}} \), and the acceleration vector \( \ddot{\mathbf{q}} \):

\[
\mathbf{y}(t) = \begin{bmatrix} \mathbf{q}(t) \\ \dot{\mathbf{q}}(t) \\ \ddot{\mathbf{q}}(t) \end{bmatrix}
\]

(21)

The system matrix \( \mathbf{A}_{st} \), the input matrix \( \mathbf{B}_{st} \), the output matrix \( \mathbf{C}_{st} \), and the straight-way matrix \( \mathbf{D}_{st} \) are described by:

\[
\mathbf{A}_{st} = \begin{bmatrix} 0_{13} & \mathbf{I}_{13} \\ -\mathbf{M}^{-1} \cdot \mathbf{C} & -\mathbf{M}^{-1} \cdot \mathbf{D} \end{bmatrix}
\]

(22)

\[
\mathbf{B}_{st} = \begin{bmatrix} 0_{13} \\ \mathbf{M}^{-1} \end{bmatrix}
\]

(23)

\[
\mathbf{C}_{st} = \begin{bmatrix} \mathbf{I}_{13} & 0_{13} \\ 0_{13} & \mathbf{I}_{13} \\ -\mathbf{M}^{-1} \cdot \mathbf{C} & -\mathbf{M}^{-1} \cdot \mathbf{D} \end{bmatrix}
\]

(24)

\[
\mathbf{D}_{st} = \begin{bmatrix} 0_{13} \\ 0_{13} \end{bmatrix}
\]

(25)
with the zero-matrix $0_{13} \in \mathbb{R}^{13 \times 13}$ and the unit-matrix $I_{13} \in \mathbb{R}^{13 \times 13}$. According to the state space model (Figure 3), the state space formulation can be written as:

$$\dot{x}(t) = A_{st} \cdot x(t) + B_{st} \cdot [f_e(t) + f_a(t)]$$
$$y(t) = C_{st} \cdot x(t) + D_{st} \cdot [f_e(t) + f_a(t)]$$

with the actuator force vector

$$f_a(t) = -T_{st,y} \cdot y(t)$$

described by the output vector $y(t)$ and the controller matrix $T_{st,y}$, which will be defined later.

### 3.2 Forced vibration due to excitation by dynamic rotor eccentricity

#### 3.2.1 General formulation

Using the complex ansatz functions $x(t) = \hat{x} \cdot e^{j\Omega t}$ and $y(t) = \hat{\dot{y}} \cdot e^{j\Omega t}$, the state space formulation can be transferred into the frequency domain:

$$\hat{x} \cdot j\Omega = A_{st} \cdot \hat{x} + B_{st} \cdot (\hat{f}_e - T_{st,y} \cdot \hat{\dot{y}})$$
$$\hat{\dot{y}} = C_{st} \cdot \hat{x} + D_{st} \cdot (\hat{f}_e - T_{st,y} \cdot \hat{\dot{y}})$$

After changing the equation (29) to $\dot{x}$, it follows that:

$$\dot{x} = (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} \cdot (\hat{f}_e - T_{st,y} \cdot \hat{\dot{y}})$$

with the unit-matrix $I_{26} \in \mathbb{R}^{26 \times 26}$.

When inserting (31) into (30) and putting $\hat{\dot{y}}$ on the left side, following equation results:

$$\hat{\dot{y}} + C_{st} \cdot (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} \cdot T_{st,y} \cdot \hat{\dot{y}} + D_{st} \cdot T_{st,y} \cdot \hat{\dot{y}} = \left[ C_{st} \cdot (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} + D_{st} \right] \cdot \hat{f}_e$$

Finally, the complex amplitude output vector $\hat{\dot{y}}$ can be calculated by:

$$\hat{\dot{y}} = \left[ I_{39} + \left( C_{st} \cdot (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} + D_{st} \right) \cdot T_{st,y} \right]^{-1} \left[ C_{st} \cdot (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} + D_{st} \right] \cdot \hat{f}_e$$

with the unit-matrix $I_{39} \in \mathbb{R}^{39 \times 39}$.

Now the index $\gamma$ has to be defined, considering four different cases:

$$\gamma = \begin{cases} 
0 & \text{: No feedback (open control loop)} 

z & \text{: Feedback of the vertical motor feet displacements } z_{al}, z_{aR} 

v & \text{: Feedback of the vertical motor feet velocities } v_{z,al}, v_{z,aR} 

a & \text{: Feedback of the vertical motor feet accelerations } a_{z,al}, a_{z,aR} 
\end{cases}$$

Therefore, the output vector $\hat{\dot{y}}$ becomes dependent on the different cases ($\hat{\dot{y}} \rightarrow \hat{\dot{y}}_{\gamma}$):

$$\hat{\dot{y}}_{\gamma} = \left[ I_{39} + \left( C_{st} \cdot (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} + D_{st} \right) \cdot T_{st,y} \right]^{-1} \cdot \left[ C_{st} \cdot (I_{26} \cdot j\Omega - A_{st})^{-1} \cdot B_{st} + D_{st} \right] \cdot \hat{f}_e$$

Now, the controller matrix $T_{st,y}$ has to be derived. With the complex ansatzes $f_a(t) = \hat{f}_a \cdot e^{j\Omega t}$ and $y(t) = \hat{\dot{y}} \cdot e^{j\Omega t}$ and with the equations (13), (14) and (28) follows:

$$\hat{f}_a = -T_{st,y} \cdot \hat{\dot{y}} = \hat{f}_{azL} + \hat{f}_{azR} = P_{azL} \cdot \hat{f}_{azL} + P_{azR} \cdot \hat{f}_{azR}$$

(36)
Due to the negative feedback loops, the complex amplitudes of the actuator forces can be described, using the cinematic constraints (9):

\[
\begin{align*}
\hat{f}_{aL} &= \begin{cases} 
0 & \text{for } \gamma = 0 \\
-\hat{z}_{aL} \cdot G_{cL,z}(j\Omega) = -(\hat{z}_s - \hat{\phi}_s \cdot b) \cdot G_{cL,z}(j\Omega) & \text{for } \gamma = z \\
-\hat{v}_{aL} \cdot G_{cL,v}(j\Omega) = -(\hat{v}_{z,s} - \hat{\omega}_s \cdot b) \cdot G_{cL,v}(j\Omega) & \text{for } \gamma = v \\
-\hat{a}_{aL} \cdot G_{cL,a}(j\Omega) = -(\hat{a}_{z,s} + \hat{\phi}_s \cdot b) \cdot G_{cL,a}(j\Omega) & \text{for } \gamma = a 
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\hat{f}_{aR} &= \begin{cases} 
0 & \text{for } \gamma = 0 \\
-\hat{z}_{aR} \cdot G_{cR,z}(j\Omega) = -(\hat{z}_s + \hat{\phi}_s \cdot b) \cdot G_{cR,z}(j\Omega) & \text{for } \gamma = z \\
-\hat{v}_{aR} \cdot G_{cR,v}(j\Omega) = -(\hat{v}_{z,s} + \hat{\omega}_s \cdot b) \cdot G_{cR,v}(j\Omega) & \text{for } \gamma = v \\
-\hat{a}_{aR} \cdot G_{cR,a}(j\Omega) = -(\hat{a}_{z,s} + \hat{\phi}_s \cdot b) \cdot G_{cR,a}(j\Omega) & \text{for } \gamma = a 
\end{cases}
\end{align*}
\]

The value \(\hat{z}_s\) is the complex amplitude of the vertical displacement of the stator, \(\hat{\dot{v}}_{z,s}\) is the complex amplitude of the vertical velocity of the stator, and \(\hat{\ddot{a}}_{z,s}\) is the complex amplitude of the vertical acceleration of the stator. The value \(\hat{\phi}_s\) is the complex amplitude of the angular displacement of the stator, \(\hat{\omega}_s\) is the complex amplitude of the angular velocity of the stator, and \(\hat{\phi}_s\) is the complex amplitude of the angular acceleration of the stator.

The arbitrary controller frequency functions – in polynomial formulation – for the left side and the right side of the motor are described by:

\[
G_{cL,\gamma}(j\Omega) = \frac{\sum_{nL=0}^{mL} b_{\mu L,\gamma}(j\Omega)^{\mu L}}{\sum_{nL=0}^{mL} a_{sL,\gamma}(j\Omega)^{s L}}; \quad G_{cR,\gamma}(j\Omega) = \frac{\sum_{nR=0}^{mR} b_{\mu R,\gamma}(j\Omega)^{\mu R}}{\sum_{nR=0}^{mR} a_{sR,\gamma}(j\Omega)^{s R}}
\]

Based on the equations (36), (37), and (38), the controller transfer matrix \(T_{st,\gamma}\) can be now be derived in the frequency domain:

\[
T_{st,\gamma}(j\Omega) = \begin{bmatrix} 0_{13} & 0_{13} & 0_{13} \end{bmatrix} \begin{bmatrix} T_x & 0_{13} & 0_{13} \\ 0_{13} & T_v & 0_{13} \\ 0_{13} & 0_{13} & T_a \end{bmatrix}
\]

with the matrix \(T_{\gamma}\), also described in the frequency domain:

\[
T_{\gamma}(j\Omega) = \begin{bmatrix} 0_{13} & 0_{13} & 0_{13} & b \left[ G_{cR,\gamma}(j\Omega) - G_{cL,\gamma}(j\Omega) \right] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\]

Here it is important to point out, that the stiffness matrix \(C\) and the damping matrix \(D\) depend on the rotary angular frequency \(\Omega\), because the oil film stiffness coefficients \(c_{ij}\) and the oil film damping coefficients \(d_{ij}\) are a function of \(\Omega\). For forced vibration due to dynamic eccentricity, the whirling angular frequency \(\omega_{i}\) for the mechanical damping coefficients \(d_{ii}\), for the magnetic
stiffness coefficient $c_{md}$, and for the magnetic damping coefficient $d_{m}$ is equal to the rotary angular frequency $\Omega$ ($\omega_F = \Omega$). Therefore, the matrices $A_{st}$ and $C_{st}$ are also a function of the rotor angular frequency $\Omega$:

$$c_{ij}(\Omega), d_{ij}(\Omega), c_{md}(\Omega), d_{m}(\Omega) \rightarrow C(\Omega), D(\Omega) \rightarrow A_{st}(\Omega), C_{st}(\Omega)$$  (42)

Therefore, the solution for the complex amplitude output vector $\mathbf{y}$ can be written as:

$$\mathbf{y} = G_y(j\Omega) \cdot \mathbf{f}_e$$  (43)

with:

$$G_y(j\Omega) = \mathbf{I}_{39} + \left( C_{st}(\Omega) \cdot (I_{26} \cdot j\Omega - A_{st}(\Omega))^{-1} \cdot B_{st} + D_{st} \right) \cdot T_{st,y}(j\Omega)^{-1} \cdot \left[ C_{st}(\Omega) \cdot (I_{26} \cdot j\Omega - A_{st}(\Omega))^{-1} \cdot B_{st} + D_{st} \right]^{-1}$$  (44)

The frequency response vector for each single excitation can be written as:

- for mass eccentricity: $G_y,j\Omega u = \mathbf{G}_{y,u}(j\Omega) \cdot \mathbf{P}_{e,u}$  (45)
- for magnetic eccentricity: $G_y,j\Omega m = \mathbf{G}_{y,m}(j\Omega) \cdot \mathbf{P}_{e,m}$  (46)
- for bent rotor deflection: $G_y,j\Omega a = \mathbf{G}_{y,a}(j\Omega) \cdot \mathbf{P}_{e,a}$  (47)

Where $\mathbf{y}_{y,u}$ is the amplitude output vector for excitation due to mass eccentricity, $\mathbf{y}_{y,m}$ is the amplitude output vector for excitation due to magnetic eccentricity, and $\mathbf{y}_{y,a}$ is the amplitude output vector for excitation due to bent rotor deflection.

### 3.2.2 Vibration velocities of bearing housings

To evaluate the vibration quality of an electrical motor, the vibration velocities of the bearing housings are often analyzed. Using the index $\kappa$ for the different kind of excitation

$$\kappa = u, m, a$$  (48)

the response functions for the vibration velocities of the bearing housings in vertical and in horizontal direction for each kind of excitation can be described by:

- vertical direction: $G_{y,\kappa,vb,z}(j\Omega) = G_{y,\kappa}(j\Omega)_{20}$  (49)
- horizontal direction: $G_{y,\kappa,vf,y}(j\Omega) = G_{y,\kappa}(j\Omega)_{24}$  (50)

Where $G_{y,\kappa}(j\Omega)_{20}$ is the 20th element and $G_{y,\kappa}(j\Omega)_{24}$ is the 24th element of the frequency response vector $G_{y,\kappa}(j\Omega)$.

### 3.2.3 Vibration velocities of foundation

Of course, it is also necessary to analyze the vibrations that are induced into the foundation, especially if active vibration control is used.

The response functions for the foundation vibration velocities in vertical and in horizontal direction for each kind of excitation can be described by:

- vertical direction, left side: $G_{y,\kappa,vf,\kappa, L}(j\Omega) = G_{y,\kappa}(j\Omega)_{21}$  (51)
- vertical direction, right side: $G_{y,\kappa,vf,\kappa, R}(j\Omega) = G_{y,\kappa}(j\Omega)_{22}$  (52)
- horizontal direction, left side: $G_{y,\kappa,vf,\kappa, L}(j\Omega) = G_{y,\kappa}(j\Omega)_{25}$  (53)
- horizontal direction, right side: $G_{y,\kappa,vf,\kappa, R}(j\Omega) = G_{y,\kappa}(j\Omega)_{26}$  (54)
3.2.4 | Actuator forces

For the design of the actuators, it is essential to know the actuator forces that are required. The response functions for the actuator forces can be calculated by:

\[
G_{f, \text{azL,R}} (j\Omega) = \begin{cases} 
0 & \text{for } \gamma = 0 \\
-\left[ G_{z,k} (j\Omega) - G_{z,k} (j\Omega) b \cdot G_{cL} (j\Omega) \right] & \text{for } \gamma = z \\
-\left[ G_{v,k} (j\Omega) + G_{v,k} (j\Omega) b \cdot G_{cR} (j\Omega) \right] & \text{for } \gamma = v \\
-\left[ G_{a,k} (j\Omega) + G_{a,k} (j\Omega) b \cdot G_{cL} (j\Omega) \right] & \text{for } \gamma = a 
\end{cases}
\]

(55)

\[
G_{f, \text{azR,R}} (j\Omega) = \begin{cases} 
0 & \text{for } \gamma = 0 \\
-\left[ G_{z,k} (j\Omega) + G_{z,k} (j\Omega) b \cdot G_{cR} (j\Omega) \right] & \text{for } \gamma = z \\
-\left[ G_{v,k} (j\Omega) + G_{v,k} (j\Omega) b \cdot G_{cR} (j\Omega) \right] & \text{for } \gamma = v \\
-\left[ G_{a,k} (j\Omega) + G_{a,k} (j\Omega) b \cdot G_{cR} (j\Omega) \right] & \text{for } \gamma = a 
\end{cases}
\]

(56)

3.3 | Stability analysis

Due to different kinds of sources of instability – the oil film of the sleeve bearings, the rotating damping of the rotor shaft, the electromagnetic field damping, and the control system – a stability analysis is necessary to find the threshold of stability. Therefore, the poles of the system have to be analyzed. The differential equations (26) and (27) have to be transferred from the time domain into the Laplace domain with the initial conditions zero. Setting the external excitation to zero and using relation (28), the following transformation can be done:

\[
X (s) \cdot s = A_{st} \cdot X (s) - B_{st} \cdot T_{st,y} (s) \cdot Y (s)
\]

(57)

\[
Y (s) = C_{st} \cdot X (s) - D_{st} \cdot T_{st,y} (s) \cdot Y (s)
\]

(58)

with the controller transfer matrix \( T_{st,y} \), described now in the Laplace domain by:

\[
T_{st,y} (s) = \begin{bmatrix} 0_{13} & 0_{13} & 0_{13} \\ T_z & 0_{13} & 0_{13} \\ 0_{13} & T_v & 0_{13} \\ 0_{13} & 0_{13} & T_a \end{bmatrix} \quad \text{for } \gamma = 0, z, v, a
\]

(59)

with the matrix \( T_y \), also described in the Laplace domain:

\[
T_y (s) = \begin{bmatrix} 
G_{cL,y} (s) + G_{cR,y} (s) & 0 & 0 & 0 & b \left[ G_{cR,y} (s) - G_{cL,y} (s) \right] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
b \left[ G_{cR,y} (s) - G_{cL,y} (s) \right] & 0 & 0 & 0 & b^2 \left[ G_{cL,y} (s) + G_{cR,y} (s) \right] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\left[ G_{cL,y} (s) \right] & 0 & 0 & 0 & b \cdot G_{cL,y} (s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-\left[ G_{cR,y} (s) \right] & 0 & 0 & 0 & -b \cdot G_{cR,y} (s) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]

(60)
Equation (57) and (58) can be changed to:

\[
X(s) \cdot s - A_{st} \cdot X(s) + B_{st} \cdot T_{st,y}(s) \cdot Y(s) = 0
\]

\[
Y(s) = (I_{39} + D_{st} \cdot T_{st,y}(s))^{-1} \cdot C_{st} \cdot X(s)
\]

Inserting (62) into (61), it follows:

\[
[I_{36} \cdot s - A_{st} + B_{st} \cdot T_{st,y}(s) \cdot (I_{39} + D_{st} \cdot T_{st,y}(s))^{-1} \cdot C_{st}] \cdot X(s) = 0
\]

and the poles can be directly calculated by solving the following equation:

\[
det [I_{36} \cdot s - A_{st} + B_{st} \cdot T_{st,y}(s) \cdot (I_{39} + D_{st} \cdot T_{st,y}(s))^{-1} \cdot C_{st}] = 0
\]

However, this direct procedure is only possible if \( A_{st} \) and \( C_{st} \) are independent of the whirling frequency \( \omega_F \), which corresponds here to the natural angular frequency. This is not the case here, as will be shown:

When calculating the threshold of stability, the rotary angular frequency \( \Omega \) will be increased until the real part of the pole, which will lead to instability, reaches zero. Then the threshold of stability is reached. In this case, the rotary angular frequency \( \Omega \) becomes \( \Omega_{stab} \) and the critical pole becomes \( s_{stab} \):

\[
\Omega = \Omega_{stab}; \quad s_{stab} = \pm j \cdot \omega_{stab}
\]

That means that at the threshold of stability the critical mode vibrates with the natural angular frequency \( \omega_{stab} \), without decaying, because the real part of \( s_{stab} \) is here zero. Therefore, the whirling angular frequency \( \omega_F \) is no longer the rotary angular frequency \( \Omega \), as it was for the forced vibration analysis in section 0. In this special case, the whirling frequency \( \omega_F \) is equal to the natural angular frequency \( \omega_{stab} \) for the considered mode.

\[
\omega_F = \omega_{stab}
\]

The oil film stiffness coefficients \( c_{ij} \) and the oil film damping coefficients \( d_{ij} \) are still functions of \( \Omega \), but the mechanical damping coefficients \( d_n \) are functions of whirl angular frequency \( \omega_F \), and therefore in this case a function of the natural angular frequency \( \omega_{stab} \). The magnetic stiffness coefficient \( c_{md} \) and the magnetic damping coefficient \( d_m \) depend on the rotary angular frequency \( \Omega \) and now additionally on the natural angular frequency \( \omega_{stab} \). Therefore, the stiffness matrix \( C \) and the damping matrix \( D \) are now dependent on the rotary angular frequency \( \Omega \) and on the natural angular frequency \( \omega_{stab} \), and also the matrices \( A_{st} \) and \( C_{st} \).

\[
c_{ij}(\Omega), \quad d_{ij}(\Omega), \quad d_n(\omega_F), \quad c_{md}(\Omega, \omega_F), \quad d_m(\Omega, \omega_F) \rightarrow C(\Omega, \omega_F), \quad D(\Omega, \omega_F) \rightarrow A_{st}(\Omega, \omega_F), \quad C_{st}(\Omega, \omega_F)
\]

Therefore, an iterative solution is necessary to derive the threshold of stability, which is shown in Figure 4.

In the first step, the coefficients \( d_n, d_m, \) and \( c_{md} \), which depend on the whirling angular frequency \( \omega_F \), are set to zero and the poles can be calculated directly. The rotor angular frequency \( \Omega \) will be increased until the real part of the pole, which leads to instability, reaches zero. Here, the rotary angular frequency is \( \Omega_{stab,1} \) - index “1” for the first calculation – and the natural angular frequency of the critical mode \( \omega_{stab,1} \) can be derived from the critical pole.

In the second step, the coefficients \( d_n, d_m, \) and \( c_{md} \) are now calculated with \( \omega_F = \omega_{stab,1} \). Again the rotor angular frequency \( \Omega \) will be increased until the real part of the pole, which leads to instability, reaches zero, leading to \( \Omega_{stab,2} \) and \( \omega_{stab,2} \).

In the third step, the new calculated natural angular frequency \( \omega_{stab,2} \) will be compared to the previous natural angular frequency \( \omega_{stab,1} \). If the ratio (absolute value of the difference, divided by \( \omega_{stab,1} \)) is less than \( \Delta \), which is an arbitrarily chosen value (e.g., 0.05), the calculation is finished and the results are \( \Omega_{stab} = \Omega_{stab,2} \) and \( \omega_{stab} = \omega_{stab,2} \). If the ratio is larger than the chosen value \( \Delta \), a new calculation is necessary and the coefficients \( d_n, d_m, c_{md} \) are now calculated with \( \omega_F = \omega_{stab,2} \) and so a new threshold of stability \( \Omega_{stab,n+1} \) and a new natural angular frequency \( \omega_{stab,n+1} \) are derived. Afterwards the new value \( \omega_{stab,n+1} \) is again compared to the previous value \( \omega_{stab,2} \). If the deviation is still too large, the loop will run through, until the deviation is less than \( \Delta \). With this iterative process the threshold of stability \( \Omega_{stab} \) – respectively the rotor speed \( n_{stab} \) in rpm at the threshold of stability – can be calculated, as well as the corresponding natural angular frequency \( \omega_{stab} \) of the critical mode, for this kind of vibration system.
4 | NUMERICAL EXAMPLE

In this section, a numerical example is presented, where the vibration velocities of the bearing housings and the foundation are analyzed as well as the actuator forces for forced vibrations caused by dynamic rotor eccentricity. Additionally a stability analysis is deduced, where the threshold of stability is calculated.

4.1 | Boundary conditions

The analyzed induction motor is a 2-pole motor with sleeve bearings, converter driven, based on real motor data, but with a stiff rotor design. The load torque of the load machine has a quadratic function in respect to the rotor speed \( n \), which is typical for a pump or ventilator. The operation speed range lies between 300 rpm and 3600 rpm and the motor is driven in this speed range with constant magnetization. If the rotor speed is higher than 3600 rpm – maximum speed is 4500 rpm – the motor is driven in the field weakening range. The most important data are shown in Table 1.

In Figure 5, the magnetic spring value \( c_{md} \) and the magnetic damper value \( d_m \) are shown, depending on the rotor speed \( n \) and on the whirling angular frequency \( \omega_F \). For natural vibrations, the whirling angular frequency \( \omega_F \) becomes the natural angular frequency of the considered mode and may be different to the rotor angular frequency \( \Omega \) (Figure 5a) and c)). Therefore, the magnetic spring value \( c_{md} \) and the magnetic damper value \( d_m \) now depend on the rotor angular frequency \( \Omega \) and on the whirling angular frequency \( \omega_F \).

For forced vibrations due to dynamic rotor eccentricity, the whirling angular frequency \( \omega_F \) becomes the rotor angular frequency \( \Omega \) (Figure 5b) and d)). At a rotor speed of 3600 rpm the field weakening begins, which can be clearly seen in the course of curve. For natural vibrations, the whirling angular frequency \( \omega_F \) becomes the natural angular frequency of the considered mode and may be different to the rotor angular frequency \( \Omega \) (Figure 5a) and c)). Therefore, the magnetic spring value \( c_{md} \) and the magnetic damper value \( d_m \) now depend on the rotor angular frequency \( \Omega \) and on the whirling angular frequency \( \omega_F \).

The oil film stiffness and damping coefficients of the sleeve bearings for each rotor speed in steady state operation are calculated with the program SBCALC from the sleeve bearing supplier RENG AG. The data are shown in Figure 6.

For the control system, a feedback of the vertical motor feet velocities is chosen, so that the index \( \gamma \) becomes \( \varepsilon \). The transfer function for both controllers – which is supposed to be identical for the left side and for the right side of the motor – is
Table 1: Data of the induction motor, sleeve bearings, foundation and actuators

### Data of the motor:
- **Rated power**: \( P_N = 2400 \, kW \)
- **Rated speed**: \( n_N = 3600 \, rpm \)
- **Mass of the stator**: \( m_s = 7040 \, kg \)
- **Moment of inertia of the stator at the x-axis**: \( \theta_s = 1550 \, kgm^2 \)
- **Mass of the rotor**: \( m_w = 1900 \, kg \)
- **Mass of the rotor shaft journal**: \( m_v = 10 \, kg \)
- **Mass of the bearing housing**: \( m_b = 80 \, kg \)
- **Stiffness of the rotor**: \( c = 6.0 \cdot 10^8 \, kg/s^2 \)
- **Height of the centre of gravity**: \( h = 560 \, mm \)
- **Distance between motor feet**: \( 2b = 1060 \, mm \)
- **Horizontal stiffness of bearing housing and end shield**: \( c_{by} = 4.8 \cdot 10^8 \, kg/s^2 \)
- **Vertical stiffness of bearing housing and end shield**: \( c_{bz} = 5.7 \cdot 10^8 \, kg/s^2 \)
- **Mechanical loss factor of the bearing housing and end shield**: \( \tan \delta_b = 0.04 \)

### Data of the sleeve bearings:
- **Bearing shell**: Cylindrical
- **Lubricant viscosity grade**: ISO VG 32
- **Nominal bore diameter / Bearing width**: \( d_b = 110 \, mm / b_b = 81.4 \, mm \)
- **Ambient temperature / Supply oil temperature**: \( T_{amb} = 20^\circ C / T_{in} = 40^\circ C \)
- **Mean relative bearing clearance (DIN 31698)**: \( \Psi_m = 1.6 \, \% \)

### Data of the foundation (for each motor side):
- **Mass left side**: \( m_{fL} = 30 \, kg \)
- **Mass right side**: \( m_{fR} = 30 \, kg \)
- **Vertical stiffness for each motor side**: \( c_{zL} = c_{zR} = 1.5 \cdot 10^8 \, kg/s^2 \)
- **Horizontal stiffness for each motor side**: \( c_{yL} = c_{yR} = 1.0 \cdot 10^8 \, kg/s^2 \)
- **Mechanical loss factor**: \( \tan \delta_f = 0.04 \)

### Data of the actuators (for each motor side):
- **Mass of the stator**: \( m_{as} = 10 \, kg \)
- **Mass of the armature**: \( m_{aa} = 3 \, kg \)
- **Vertical stiffness**: \( c_{zL} = 1.2 \cdot 10^8 \, kg/s^2 \)
- **Horizontal stiffness**: \( c_{yL} = 3.0 \cdot 10^8 \, kg/s^2 \)
- **Mechanical loss factor**: \( \tan \delta_{a} = 0.04 \)

 arbitrarily chosen here as polynomial functions of 2nd degree for the numerator and for the denominator, defined in the Laplace domain:

\[
G_{cL,v} (s) = G_{cR,v} (s) = \frac{b_{2,v} \cdot s^2 + b_{1,v} \cdot s + b_{0,v}}{a_{2,v} \cdot s^2 + a_{1,v} \cdot s + a_{0,v}} \quad (68)
\]

The coefficients are described in Table 2.

As can be seen, the coefficients are only roughly chosen, because it is not the aim of the paper to find the optimal justification of the controllers for this example, but to show the fundamental influence of the control system.

### 4.2 Stability analysis

Before the forced vibrations are analyzed, a stability analysis has to be deduced, according to the procedure in Figure 4. In this analysis it is ignored that rotor speeds above 4570 rpm are not possible from the technical point of view, because at these speeds the load torque would be higher as the breakdown torque of the motor. Table 3 shows that for open control loop operation, the
FIGURE 5 Magnetic spring ($c_{md}$) and magnetic damper ($d_{md}$), depending on the rotor speed $n$ and on the whirling angular frequency $\omega_F$.

FIGURE 6 a) Oil film stiffness coefficients and b) oil film damping coefficients of the sleeve bearings.
TABLE 2  Coefficients of the transfer function of the controllers for feedback of the motor feet velocities

| Coefficients of the numerator | Coefficients of the denominator |
|------------------------------|---------------------------------|
| $b_{0,v} = 1 \cdot 10^8 \text{ [kg/s]}$ | $a_{0,v} = 100 \text{ [-]}$ |
| $b_{1,v} = 1 \cdot 10^6 \text{ [kg]}$ | $a_{1,v} = 10 \text{ [s]}$ |
| $b_{2,v} = 1 \cdot 10^6 \text{ [kg s]}$ | $a_{2,v} = 1 \text{ [s^2]}$ |

TABLE 3  Threshold of stability and natural angular frequency of the critical mode

| Cases                  | Threshold of stability $n_{\text{stab}}$ [rpm] | Natural angular frequency of the critical mode $\omega_{\text{stab}}$ [rad/s] |
|------------------------|-----------------------------------------------|--------------------------------------------------|
| Open control loop      | 6225                                          | 339.7                                            |
| Closed control loop    | 6522                                          | 337.4                                            |

threshold of stability is reached at a rotor speed of 6225 rpm. The natural angular frequency of the critical mode gets 339.7 rad/s. If the control loop is closed, the threshold of stability is shifted to 6522 rpm, while the natural angular frequency of the critical mode changes to 337.4 rad/s. Therefore, for both cases – open control loop and closed control loop – the threshold of stability is far away from the maximum operational rotor speed.

4.3 | Forced vibration analysis

First, the amplitudes of the frequency response functions for the vibration velocities of the bearing housings are calculated and shown in Figure 7.

For open control loop operation, the curves with the solid lines (—) occur. It can be seen that in this case three critical speeds occur regarding the bearing housing velocities, one for vertical direction (blue curves) and two for horizontal direction (red curves). When operating with a closed control loop, the curves with dashed lines (−−) occur, and no critical speeds exist anymore in the operational speed range. The amplitudes of the frequency response functions for the foundation vibration velocities are shown in Figure 8.

For open control loop operation, three critical speeds regarding the vertical foundation vibrations and two critical speeds regarding the horizontal foundation vibrations occur. When closing the loops, again no critical speeds occur anymore. However, for excitation due to rotor eccentricity and due to bent rotor deflection (Figure 8a and c), the vertical foundation vibrations increase strongly for rotor speeds higher than 3000 rpm. In Figure 9 the amplitude and phase responses of the frequency response functions for actuator forces are shown.

4.4 | Comparison of the calculation methods

When the calculation method according to [27] is applied instead the here presented enhanced calculation method, the chosen transfer functions (68) for the controllers cannot be longer be used. Based on the calculation method,[27] only a limited number of controllers (P-, I-, PI-, PD (ideal)- or PID (ideal)-controllers) are applicable for the analysis, in contrast to the arbitrary controller selection permitted by the here presented calculation method. Comparing the structure of these limited controllers, the transfer function of an ideal PID-controller (69) would be the most similar to the arbitrary transfer function (68).

$$G_{cL,v,\text{PID ideal}}(s) = G_{cR,v,\text{PID ideal}}(s) = \frac{K_d \cdot s^2 + K_p \cdot s + K_i}{s}$$

(69)

When comparing the transfer function (69) with (68), it is obvious, that the transmission behavior will be different and therefore also the vibration behavior. Using the same values for the coefficients – except the coefficients $a_{0,v}$ and $a_{2,v}$, which are zero when comparing (69) with (68) – the controller parameters for the ideal PID controller can be defined (Table 4).

An example of the different vibration behavior is shown exemplarily in Figure 10.

However, if for both calculation methods ideal PID-controllers with the same transfer function are used, the results are completely identical.
FIGURE 7  Amplitudes of the frequency response functions for the vibration velocities of the bearing housings in vertical direction (z-direction) and horizontal direction (y-direction) for open control loop (—) and for closed control loop with velocity feedback (−−), depending on the rotor speed, for excitation a) due to rotor eccentricity, b) due to magnetic eccentricity, and c) due to bent rotor deflection

4.5 | Discussion of the results

The stability analysis, which was presented in section 4.2, shows that in the whole speed range (300 rpm to 4500 rpm) stable operation is possible for open control loop and for closed control loop. In section 4.3 the forced vibration due to dynamic rotor eccentricity was investigated. It could be shown that with closed control loop operation critical speeds in the speed range can be avoided. When analyzing Figure 7 and Figure 8, it is obvious, that the differences between the amplitude functions for the vibrations due to mass eccentricity and due to bent rotor deflection are here only marginal. The reason is that for these kinds of vibrations the excitation by bent rotor deflection can be simulated by a superposition of excitation by mass eccentricity and magnetic eccentricity, with $\hat{a} = \hat{e}_u = \hat{e}_m$ and $\varphi_a = \varphi_u = \varphi_m$. Here the amplitudes regarding excitation due to magnetic eccentricity are much lower compared to the amplitudes regarding excitation due to mass eccentricity. However, it would be a wrong assumption that excitation due to magnetic eccentricity can be generally neglected, because the ratio between magnetic eccentricity and mass eccentricity $\hat{e}_m/\hat{e}_u$ for such a kind of induction motor is usually in a range of about 2…40. Therefore, the magnetic excitation may lead to higher vibrations than the excitation due to mass eccentricity. When analyzing the vertical foundation vibrations (Figure 8), it is shown that they are different on the right side and on the left side of the motor for certain rotor speeds, in contrast to the horizontal foundation vibrations, which are identical on the right and left side. The main reasons for the different vertical foundation vibrations (z-direction) are the non-symmetric stiffness matrix of the oil film – the coupling coefficients in the oil film matrices couple the horizontal direction (y-direction) with the vertical direction (z-direction) – and the passive damping of the actuators and the foundation. This fact leads, in combination with the chosen controller structures,
Figure 8

Amplitudes of the frequency response functions for the foundation vibration velocities on the right side and on the left side of the motor, in vertical direction (z-direction) and horizontal direction (y-direction) for open control loop (---) and for closed control loop with velocity feedback (−−), depending on the rotor speed, for excitation a) due to rotor eccentricity, b) due to magnetic eccentricity, and c) due to bent rotor deflection.
FIGURE 9 Amplitudes and phases of the frequency response functions for the actuator forces on the right side and on the left side of the motor, for closed control loop with velocity feedback, depending on the rotor speed, for excitation a) due to rotor eccentricity, b) due to magnetic eccentricity, and c) due to bent rotor deflection.
TABLE 4 Coefficients of ideal PID controller

| Coefficients       | Value  |
|--------------------|--------|
| $K_d$              | $1 \cdot 10^5$ [kg] |
| $K_p$              | $1 \cdot 10^5$ [kg/s] |
| $K_i$              | $1 \cdot 10^7$ [kg/s^2] |

FIGURE 10 Amplitudes of the frequency response functions for the vibration velocities of the bearing housings in vertical direction (z-direction) and horizontal direction (y-direction) for closed control loop with the arbitrary controllers (—) and with ideal PID-controllers (—), depending on the rotor speed, for excitation due to rotor eccentricity

also to different actuator forces on the left side and on the right side of the motor for certain rotor speeds, which can be seen in Figure 9. The disadvantage of closed control loop operation is that high vertical foundation vibrations occur for rotor speeds in the range from 3000 rpm to 4500 rpm (Figure 8) and that high actuator forces are required in this speed range. However, when analyzing Figure 7, Figure 8 and Figure 9, it is obvious that above a rotor speed of about 2700 rpm closed control loop operation is no longer necessary. Therefore, an appropriate control strategy would be, to operate the motor with closed control loop in the speed range from 300 rpm to 2700 rpm and in open control loop if the rotor speed is higher than 2700 rpm. The calculation results, which would be achieved with the calculation method according to [27] – using ideal PID-controllers instead the arbitrarily controllers – are clearly different, because of the different transfer functions. Using ideal PID-controllers with identical transfer functions for both calculation methods, leads to identical results.

5 CONCLUSION

In the paper, a state space model for vibration control with arbitrary controller structures for soft mounted induction motors with sleeve bearings fixed on active motor foot mounts was shown. Besides the mathematical description of the forced vibrations, caused by dynamic rotor eccentricity – rotor mass eccentricity, magnetic eccentricity, and bent rotor deflection – a procedure was presented to find the threshold of stability. The challenge of the paper is the use of arbitrary controller structures with different feedback strategies – feedback of the motor feet displacements or velocities or accelerations – in combination with a special vibration system. This specialty is, that the stiffness and damping matrices depend on the rotor angular frequency $\Omega$, which corresponds to the excitation angular frequency, when analyzing the forced vibration, and depend additionally on the natural angular frequency $\omega_{stab}$ of the critical mode, when analyzing the threshold of stability. After the mathematical description was shown, a numerical example of a 2-pole induction motor (power rating of 2.4 MW) was presented, where the threshold of stability was analyzed as well as the forced vibrations due to dynamic rotor eccentricity. The bearing housing vibrations, the foundation vibrations and the actuator forces have been analyzed. It was shown that critical speeds could be avoided, when operating the motor with closed control loop. To avoid high vertical foundation vibrations and high actuator forces, the closed control loop operation is only useful in the speed range from 300 rpm to 2700 rpm in the case shown. Above a rotor speed of 2700 rpm, operation with open control loop is more useful.

Finally, the benefit of the presented calculation method is that arbitrary controller structures with different feedback strategies can now be analyzed, which is a huge enhancement compared to the calculation method in [27], where only a limited number of controllers (P-, I-, PI-, PD (ideal)-, PID (ideal)-controller) can be investigated. If only these controllers are being used for
simulation both calculation methods lead to the same results. However, if e.g. real PD- or real PID-controllers or arbitrary controller structures should be analyzed, the calculation method presented here is applicable in contrast to [27]. The presented calculation method can also be easily adapted, if the influence of filters or special transmission behavior of the actuators and sensors should be considered in the simulation, which is not possible using the calculation method in [27].

REFERENCES

[1] ISO 10816-3 (International Organization of Standardization), Mechanical vibration - Evaluation of machine vibration by measurements on non-rotating parts - Part 3: Industrial machines with nominal power above 15 kW and nominal speeds between 120 r/min and 15000 r/min when measured in situ 2009.
[2] IEC 60034-14 (International Electrotechnical Commission), Rotating electrical machines - Part 14: Mechanical vibration of certain machines with shaft heights 56 mm and higher - Measurement, evaluation and limits of vibration severity 2007.
[3] G. Genta, Dynamics of Rotating Systems, Springer Science & Business Media 2005.
[4] J. M. Vance, F. J. Zeidan, B. Murphy, Machinery Vibration and Rotordynamics, John Wiley & Sons, Inc. Hoboken, New Jersey 2010.
[5] J. S. Rao, Rotor Dynamics, John Wiley & Sons, New York 1996.
[6] M. I. Friswell, J. E. T. Penny, S. D. Garvey, A. W. Lees, Dynamics of Rotating Machines, Cambridge University Press, Cambridge 2010.
[7] R. Gasch, R. Nordmann, PflügnerH, Rotordynamik, Springer, Berlin-Heidelberg 2002.
[8] O. Tokhi, S. Veres, Active Sound and Vibration Control Theory and Applications, Institution of Electrical Engineers, London 2002.
[9] A. Preumont, Vibration Control of Active Structures: An Introduction, Springer 2011.
[10] K. Janschek, Mechatronic Systems Design: Methods, Models, Concepts, Springer 2012.
[11] I. D. Landau, T.-B. Airimițoae, A. Castellanos-Silva, A. Constantinescu, Adaptive and Robust Active Vibration Control: Methodology and Tests, Springer 2017.
[12] C. R. Fuller, S. J. Elliot, P. A. Nelson, Active Control of Vibration, Academic Press Limited 1996.
[13] C. Ehmann, R. Nordmann, Comparison of control strategies for active vibration control of flexible Structures, Arch. Control Sci. 2012, 13, 303.
[14] F. Dohnal, H. Ecker, A. Tondl, Vibration control of self-excited oscillations by parametric stiffness excitation, Eleventh International Congress of Sound and Vibration 2004, 339.
[15] K. Makihara, H. Ecker, F. Dohnal, Stability analysis of open-loop stiffness control to suppress self-excited vibrations, J. Vib. Control 2005, 11, 643.
[16] F. W. Fairman, Linear Control Theory: The State Space Approach, John Wily and Sons 1998.
[17] H. Unbehauen, Regelungstechnik II, Zustandsregelungen, digitale und nichtlineare Regel- und Identifikationssysteme, Vieweg 2007.
[18] R. L. Williams II, D. A. Lawrence, Linear State Space Control System, John Wily and Sons 2007.
[19] E. Hendricks, O. Jannerup, P. Haase, Linear Systems Control: Deterministic and Stochastic Methods, Springer 2008.
[20] R. K. Yedavalli, Robust Control of Uncertain Dynamic Systems: A Linear State Space Approach, Springer 2014.
[21] N. Skricka, R. Markert, Improvements of the integration of active magnetic bearings, Mechatronics 2002, 12, 1059.
[22] G. Schweitzer, E. H. Maslen, Magnetic Bearings, Springer 2009, 27.
[23] S. Ran, Y. Hu, H. Wu, Design, modeling, and robust control of the flexible rotor to pass the first bending critical speed with active magnetic bearing, Adv. Mech. Eng. 2018, 10, 1.
[24] H. Ulbrich, A comparison of different actuator concepts for applications in rotating machinery, Int. J. Rotating Mach. 1994, 1, 61.
[25] H. M. Chen, P. Lewis, S. Donald, S. Wilson, Active mounts, J. Acoust. Soc. Am. 1998, 91.
[26] W. Sun, H. Gao, B. Yao, Adaptive robust vibration control of full-car active suspensions with electrohydraulic actuators, IEEE Trans. Control Syst. Technol. 2013, 21, 2417.
[27] U. Werner, Analysis of different vibration control strategies for soft mounted induction motors with sleeve bearings using active motor foot mounts, J. Appl. Math. Phys. 2019, 7, 611.
[28] R. Belmans, A. Vandenput, W. Geysen, Calculation of the flux density and the unbalanced pull in two pole induction machines, Electr. Eng. 1987, 70, 151.
[29] R. Belmans, A. Vandenput, W. Geysen, Influence of unbalanced magnetic pull on the radial stability of flexible-shaft induction machines, IEE Proc. B, Electr. Power Appl. 1987, 134, 101.
[30] J. Früchtenicht, H. Jordan, H. O. Seinsch, Exzentrizitätsfelder als Ursache von Lauflaststabilitäten bei Asynchronmaschinen Archiv für Elektrotechnik Bd. 65 Teil 1, pp. 271–281, Teil 2, pp. 283–292 1982.
[31] H. O. Seinsch, Oberfelderscheinungen in Drehfeldmaschinen, Teubner, Stuttgart 1992.
[32] D. G. Dorrell, Sources and characteristics of unbalanced magnetic pull in three-phase cage induction motors with axial-varying rotor eccentricity, IEEE Trans. Ind. Appl. 2011, 47, 12.
[33] A. Arkkio, M. Anttila, K. Pokki, A. Simon, E. Lantto, Electromagnetic force on a whirling cage rotor, IEE Proc. B, Electr. Power Appl. 2000, 147, 353.
[34] T. P. Holopainen, Electromechanical interaction in rotor dynamics of cage induction motors. VTT Technical Research Centre of Finland, Ph.D. Thesis, Helsinki University of Technology, Finland 2004.
[35] A. Tondl, Some Problems of Rotor Dynamics. Chapman & Hall, London 1965.
[36] J. Glienicke, Feder- und Dämpfungskonstanten von Gleitlagern für Turbomaschinen und deren Einfluss auf das Schwingungsverhalten eines einfachen Rotors. Dissertation, Technische Hochschule Karlsruhe 1966.
[37] J. Lund, K. Thomsen A Calculation Method and Data for the Dynamics of Oil Lubricated Journal Bearings in Fluid Film Bearings and Rotor Bearings System Design and Optimization. ASME, New York 1978, pp. 1–28.

[38] T. Someja, Journal Bearing Databook, Springer 2013.

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