Application of ternary logic for digital signal processing

I Moldakhan¹, D B Shaltikova², Z M Egemberdyeva¹ and I E Suleimenov²

¹ Almaty University of Power Engineering and Telecommunications, 126/1 Baitursynova Street, Almaty, Kazakhstan
² Institute of information and computational technologies, 125 Pushkina Street, Almaty, Kazakhstan

imoldakhan@gmail.com

Abstract. The article shows that digital processing of slowly changing signals based on ternary logic has significant advantages compared to processing based on binary logic. It is shown that for slowly changing signals, digital processing can be performed on the basis of dividing analysed signal into frequency bands. In particular, information on derivatives of a slowly varying signal which spectrum lies in the frequency band from 0 to \( \omega_0 \), can be restored basing on the analysis of its component lying in the frequency band from \( \omega_0/3 \) to \( \omega_0 \).

1. Introduction

There are a number of reports [1-3] which prove that in relation to digital signal processing ternary logic has a number of advantages compared to binary. The main advantage is a reduction of operations’ numbers required to convert an analogue signal into digital form. According to available estimates, the number of operations required to convert a signal into digital form using ternary logic is approximately in 1.5 times less than operations numbers that must be performed to carry out a similar procedure using binary logic.

However, the most clearly examples showing benefits of ternary logic over binary are related to digital processing of signals that relatively slowly change in time (more precisely with processing of signals which time derivative does not exceed a certain value).

For such signals can be formed \( \varepsilon \)-cover. In other words it is possible to choose \( \Delta u_\varepsilon \) subinterval by levels when a signal value \( \Delta u_{(i+1)} \) on the \((i+1)\)-st beat will differ from value of \( \Delta u_{i} \) on the \(i\) beat no more than \( \Delta u_\varepsilon \). That is if the signal on \((i+1)\)-st beat corresponded to the discrete level number \(j\) then the signal on the \((i+1)\)-st beat will correspond the level with one of numbers \(j-1, j, j+1\).

Obviously, an amount of information contained in such signal is most conveniently measured in trit (unit of information [4], which name is built by analogy with the name of generally accepted unit “bit”, but relating not to binary, but to ternary logic). Obviously, that information quantity which exhaustively describes signals of the type under consideration is equal to \( N \) trit, where \( N \) is a number of beats (with the accuracy to information which contained in an initial value of signal).

However, by historical reasons, ternary logic has not become widespread yet, although there are a number of tasks that require digital processing of slowly changing signals. An example on this regard is any system which applied to track values that are changing relatively slowly in time (in particular, it refers to any systems which built on measurements of inertial quantities, i.e. the temperature of sufficiently massive bodies [5]). Also, a typical example are adaptive optics systems which intend for use in the solar energy industry. Specifically, to track a position of the Sun in the sky [6-8]. In this article provides an additional evidence of the benefits of ternary logic. More precisely, it is shown that the
digital processing of signals which have ε-coverage, in terms of ternary logic, can be performed using frequency filters (that is, using the separation of the original signal in frequency ranges).

2. Problem statement

Currently, digital methods are widely used for signal processing. In the vast majority of cases, digital signal processing uses binary logic, which is why analog-to-digital converters are used. These converters must be applied in the same way both to the fast-changing signal and to the slow-changing signal, which was discussed above. Using such converters does not allow you to use the fact that the derivative of these signals is limited. The processing of each clock cycle proceeds independently. This leads to the fact that the existing types of analog-to-digital converters require a significant number of operations, as follows from the materials of this work. If we take into account the specifics of slowly changing signals, the number of operations that are necessary to translate them into the digital form will be significantly reduced [9-13]. There is every reason to believe that reducing the number of operations in digital signal processing is becoming an increasingly urgent task due to the development of the Internet of things [14], big data [15] and digitalization of the economy as a whole [16].

The question of further development of ternary logic is also relevant from a philosophical point of view, in particular, for dialectical positivism [17,18]; within the framework of modern mathematical logic, very non-trivial logics are also considered [19,20], the construction of which differs significantly from the logic of Aristotle and Buhl.

3. Prerequisites for development of a spectral method for digital processing of slowly varying signals

Obviously, that if the signal is approximately replaced by its model, consistent to ε-cover, then it automatically supposed using of discrete beats (time intervals) during which the value of signal is considered constant. Duration of all beats without losing of generality can be considered as the same. A spectrum of such model signal is intentionally limited. Specifically, it belongs to a range from 0 to $\frac{1}{2T_0}$ by a frequency scale, where $T_0$ - is a duration of separate beats.

A signal $u(t)$ with a limited spectrum can be presented as follows

$$u(t) = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} \exp(i\omega t)A(\omega)\,d\omega$$  \hspace{1cm} (1)

where $\omega$ - angular frequency, $A(\omega)$ - signal spectrum, this function takes complex values, $\omega_0$ - liminal angular frequency.

At the same time, if the signal changes slowly then to describe it in a vicinity of some selected point (of point in time) correctly representation through a Taylor series is fair to use

$$u(t_i \pm \Delta t) = u(t_i) \pm \frac{du}{dt}|_{t=t_i} \Delta t + \frac{d^2 u}{dt^2}|_{t=t_i} (\Delta t)^2$$  \hspace{1cm} (2)

which within transition to discrete quantity take the form

$$u_{i\pm1} = u_i \pm \frac{du}{dt}|_{t=t_i} \Delta t_0 + \frac{d^2 u}{dt^2}|_{t=t_i} (\Delta t_0)^2$$  \hspace{1cm} (3)

Considering that the values of $u_i(\pm1)$ for the signals of concerned type can differ from $u_i$ no more than to one discrete level and the value of $\Delta t_0$ - is a constant, so using of formula (3) de facto means that it should be found some procedure of assessment of the first and second derivative. In essence, representation of the following form is constructed

$$\left( \pm \frac{du}{dt} |_{t_i}, \frac{d^2 u}{dt^2} |_{t_i} \right) \to q$$  \hspace{1cm} (4)

where q can take values -1, 0, +1.

From the formula (4), even without detailed calculations, follows that while considering a discrete signals in expansion into Taylor’s series - it is necessary to consider elements up to the second derivative inclusive. Because using only the first derivative will not reflect, for example, a situation where both the value $u_i(i+1)$ and the value $u_i(i-1)$ are excess of one level the value of the signal $u_i$.

The question arises - exist there a connection between conceptions one of the same discretely varying signal through formula (1) and through formula (2). In other words, is it possible to determine characteristics of displaying (4) by analyzing frequency spectrum of the signal?
4. Evidence of applicability of a spectral method of digital processing to slowly varying signals

Let us divide a spectral interval, in which the spectrum of the function under consideration is defined, into the following three subintervals $[-\omega_0, -\frac{1}{3}\omega_0]$, $[-\frac{1}{3}\omega_0 \frac{1}{3}\omega_0]$ and $[\frac{1}{3}\omega_0, \omega_0]$. Let us calculate contribution to the integral (1) for each of these intervals individually. For interval $[\frac{1}{3}\omega_0, \omega_0]$, changing a variable

$$\omega_1 = \omega - \frac{2}{3}\omega_0$$

we have

$$u_1(t) = \exp\left(i \frac{2}{3}\omega_0 t\right) \int_{\frac{1}{3}\omega_0}^{\omega_0} \exp(i\omega_1 t)A(\omega_1) \, d\omega_1 = \exp\left(-i \frac{2}{3}\omega_0 t\right) \bar{u}_1$$

Obviously, the integral in formula (6) in its form looks identical to the integral (1) i.e. it also describes a signal, which is equivalent to a system of equidistant counting, but following with period in three times upper $T_1 = 3T_0$.

Considering an interval $[-\omega_0, -\frac{1}{3}\omega_0]$ in the same way let us to make a change of variable

$$\omega_3 = \omega + \frac{2}{3}\omega_0$$

one can obtain

$$u_2(t) = \exp\left(-i \frac{2}{3}\omega_0 t\right) \int_{-\omega_0}^{\omega_0} \exp(i\omega_3 t)A(\omega_3) \, d\omega_3 = \exp\left(-i \frac{2}{3}\omega_0 t\right) \bar{u}_2$$

where the period of sequence of counts of signal defined by the integral appearing on the right side of (8) is also tripled comparing to the original $T_1 = 3T_0$.

For interval $[-\frac{1}{3}\omega_0, \frac{1}{3}\omega_0]$ the formula (1) is applied directly and here also $T_1 = 3T_0$ takes place.

$$u_3(t) = \frac{1}{2\pi} \int_{-\frac{1}{3}\omega_0}^{\frac{1}{3}\omega_0} \exp(i\omega t)A(\omega) \, d\omega$$

Summing up formulas (6), (8) and (9), we obtain the following representation for the initial signal

$$u(t) = \bar{u}_1(nT_1)\exp\left(-i \frac{2}{3}\omega_0 t\right) + \bar{u}_3(nT_1)\exp\left(-i \frac{2}{3}\omega_0 t\right) + u_2(nT_1)$$

where all functions $\bar{u}_1, \bar{u}_3, u_2$ have a limited spectrum in a frequency band up to $\frac{1}{3}\omega_0$. This, in particular, means that for three beats of the initial signal, there is only one count of these functions. In other words each of them can be associated with a value that will remain unchanged for three beats.

Comparing the result obtained in (10) with the formula (3) which directly derives from expansion into Taylor series, it is possible to see that information on the value of the second and first derivatives actually turns out to be in the frequency band from $\frac{1}{3}\omega_0$ to $\omega_0$, which can be separated from a frequency band that allows to restore the function $u_2$ (and this can be done using typical electronic tools).

We emphasize that considerations forcing to take into account precisely the first two derivatives are fundamental. That is why the range of frequencies in which the spectrum of the signal considered above fits was divided into two unequal parts.

Let us consider a harmonic signal modulated by a signal in a certain frequency band. As is well known from radio engineering, in this case a signal spectrum contains combination frequencies, in other words, a band occupied by this signal becomes twice as wide as the band of the original signal. Nevertheless, information transmitted by such signal corresponds to a width of the original band, since in this case opportunities of the double bandwidth capability is not fully utilized. Along with modulation of the harmonic signal, let us say sinusoidal, in order to transmit information it is possible to use modulation of harmonica signal opposite in phase, i.e. in this case cosine one. This signal is capable of carrying the same information, in other words, information can be transmitted in a doubled frequency band by modulating two antiphase signals.

For that very reason, we used a quite specific division of frequency band into two unequal parts, related to each other in width as two to one. It can be seen that in this case information on signal is actually contained in three frequency bands, each of which is $1/3$ of the original. However, since bands from
from $\frac{2}{3}\omega_0$ to $\omega_0$ are actually “mixed”, it is needed to combine and consider them together, concurrently taking into account the first and second derivatives.

The diagram shown in Fig. 1 illustrates how information obtained by radio-frequency analysis of the first and second derivatives can be used to restore behavior of a function with $\varepsilon$-cover over a period of three beats.

Figure 1. Illustration of decoding of relationship between a nature of coding sequences for a three-stroke range and signs of the first and second derivative; examples of coding sequences $(-1,1)$ (a) and $(-1,0)$ (b)

This change can obviously be encoded in a sequence of two symbols, each of them can take the value 1, 0 and -1.

These symbols correspond to changes in the signal level during transition from the first beat to the second and from the second one to the third beat (three possible particular cases are shown in Fig. 1). The same figure also shows examples of circuits illustrating the nature of deviation of the signal from the center beat (the right parts of figures). For example, the first signal with a change of the signal in accordance with the $-1,1$ sequence, is schematically represented by drawing (the right part of Fig. 1a), on which there are two arrows pointing up. This means that deviation of the signal from the value that it takes on the central beat is positive for the beat located on the left and for the beat located on the right.

Nine possible combinations of a sequence of change of levels correspond to nine possible combinations of values of the second and first derivatives, which are assigned the value either +1, or 0, or -1.

Exactly this consideration can be taken as a basis for digital radio engineering processing of slowly changing signals (more precisely, signals with $\varepsilon$-cover). Namely, as follows from considerations above, in order to establish the nature of the signal change over three selected beats it is sufficient to identify only the signs of the first and second derivatives.

This way, dividing the signals into frequency bands allows to highlight the behavior of the signal over three beats by analyzing the sign of the signal in a well-defined frequency range. Moreover, to highlight the sign of cosine and sinusoidal signals, it is quite possible to use standard radio engineering techniques.

5. Conclusion

Hereby, digital processing of signals based on ternary logic has really very serious advantages compared to binary, especially if we consider signals with $\varepsilon$-cover. In this case, comprehensive information about derivatives of a given signal in a group of three beats can be established basing on an analysis of its component lying in a frequency band from $\omega_0/3$ to $\omega_0$, where $\omega_0$ - the frequency corresponding by the time to discretization step of the original signal. Wherein, in order to restore the signal, it is sufficient to establish values of the derivatives in digital form i.e. correlate them with values corresponding to a ternary logic: “-1”, “0”, “+1”.

The proposed approach can be implemented in hardware since its implementation is essentially based on conventional spectral methods. It is enough to select the signals in the corresponding frequency band and determine whether they exceed the specified threshold or not. Thus, further work in this direction will significantly simplify the digital processing of slowly changing signals. We emphasize that we do not intend to contrast ternary logic with binary logic in this work. On the contrary, we can predict that in the future there should be flexible platforms that, depending on the need, will use one or another logic.
to reduce the number of operations, which is an urgent task in the modern world due to the development of the digital economy.

6. References

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