Multi-Quarks and Two-Baryon Interaction in Lattice QCD

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Abstract. We study multi-quark (3Q, 4Q, 5Q) systems in lattice QCD. We perform the detailed studies of multi-quark potentials in lattice QCD to clarify the inter-quark interaction in multi-quark systems. We find that all the multi-quark potentials are well described by the OGE Coulomb plus multi-Y-type linear potential, i.e., the multi-Y Ansatz. For multi-quark systems, we observe lattice QCD evidences of “flip-flop”, i.e., flux-tube recombination. These lattice QCD studies give an important bridge between elementary particle physics and nuclear physics.

INTRODUCTION

The inter-quark force [1, 2, 3] is one of the most important quantities in hadron physics to understand hadron properties at the quark-gluon level, and also plays an essential role to reveal properties of multi-quarks [4, 5, 6]. Nevertheless, before our studies, nobody knew the exact form of the confinement force in the multi-quark systems directly from QCD. In fact, some hypothetical forms of the inter-quark potential have been used in all the quark model calculations. Then, the lattice QCD study of the inter-quark interaction is quite desired, and it leads to the proper Hamiltonian and a guideline to construct the QCD-based quark model for multi-quark systems.

We show in Fig.1 our global strategy to understand hadron properties from QCD. One way is the direct lattice QCD calculations for low-lying hadron masses [7] and simple hadron matrix elements, although the wave function cannot be obtained in the path-integral formalism and the practically calculable quantities are severely limited. The other way is to construct the quark model from QCD. From the analysis of the inter-quark forces in lattice QCD, we extract the quark-model Hamiltonian. Through the quark model calculation, one can obtain the quark wave-function of hadrons and more complicated properties of hadrons.

MULTI-QUARK INTERACTION FROM LATTICE QCD

In order to clarify the inter-quark force in multi-quark systems, we study the static multi-quark potentials systematically in lattice QCD using the multi-quark Wilson loops [1, 2].
As the results, we clarify that the multi-quark potential is well described as

\[ V = \frac{g^2}{4\pi} \sum_{i<j} T^a_i T^b_j \left| r_i - r_j \right| + \sigma L_{\text{min}} + C, \]

where \( L_{\text{min}} \) is the minimal value of the total length of flux-tubes linking the static quarks.

From lattice QCD, the \( Q\bar{Q} \) potential \( V_{Q\bar{Q}} \) is known to be well described by this form \([1, 3]\). For the 3Q potential \( V_{3Q} \), our accurate calculations clarify that \( V_{3Q} \) is well described by the OGE Coulomb plus Y-type linear potential, i.e., the Y-Ansatz \([1, 2]\), which supports the Y-shaped flux-tube formation \([1, 2, 8]\). Note that the Y-type linear potential implies existence of the three-body interaction. (In usual many-body systems, the main interaction is described by two-body interactions and the three-body interaction is a higher-order contribution. In contrast, as is clarified by lattice QCD study, the quark confinement force in baryons is a genuine three-body interaction.)

![FIGURE 1. Our global strategy to understand the hadron properties from QCD.](image1)

![FIGURE 2. The lattice QCD results for the 3Q, 4Q and 5Q potentials.](image2)
Thus, from a series of our lattice QCD studies, the inter-quark potential is clarified to consist of the OGE Coulomb part and the flux-tube-type linear confinement part in both mesons, baryons and multi-quark hadrons [1, 2].

![Diagram](image)

**FIGURE 3.** The flux-tube formation in multi-quarks indicated by our lattice QCD results.

From the comparison among the Q-Q, 3Q, 4Q and 5Q potentials in lattice QCD, we find the universality of the string tension $\sigma$ [1, 2] as $\sigma_{Q\bar{Q}} \approx \sigma_{3Q} \approx \sigma_{4Q} \approx \sigma_{5Q}$, and the OGE Coulomb coefficient, i.e., $g \approx 1.6$. Here, the OGE Coulomb term is considered to originate from the OGE process, which plays the dominant role at short distances, where perturbative QCD is applicable. The flux-tube-type linear confinement physically indicates the flux-tube picture, where quarks and antiquarks are linked by the one-dimensional squeezed color-electric flux-tube with the string tension $\sigma \approx 0.89\text{GeV/fm}$.

To conclude, the inter-quark interaction would be generally described by the sum of the OGE part and the flux-tube-type linear confinement part with the universal string tension $\sigma$. Thus, based on the lattice QCD results, we propose the proper quark-model Hamiltonian $\hat{H}$ for multi-quark hadrons as

$$\hat{H} = \sum_i \sqrt{\hat{p}_i^2 + M_i^2} + \sum_{i<j} V_{\text{OGE}}(\mathbf{r}_i - \mathbf{r}_j) + \sigma L_{\text{min}},$$

where $V_{\text{OGE}}$ denotes the OGE potential and $M_i$ the constituent quark mass.

To summarize, we have performed the multi-quark potential in lattice QCD to clarify the inter-quark interaction in multi-quark systems. We have found that the 3Q potential is well described by the Y-Ansatz. For 4Q and 5Q systems, $V_{4Q}$ and $V_{5Q}$ are well described by the OGE Coulomb plus multi-Y Ansatz, which indicates the flux-tube picture even for multi-quarks. We have observed a clear lattice QCD evidence of “flip-flop”.

As a successive work, we are now investigating the nuclear force in lattice QCD [9]. The lattice calculations have been done on NEC-SX5(Osaka U.) and SR8000(KEK).

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