Equations of State for \( n \)-Hexadecane and \( n \)-Docosane

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Abstract
Equations of state for \( n \)-hexadecane (\( \text{C}_{16}\text{H}_{34} \)) and \( n \)-docosane (\( \text{C}_{22}\text{H}_{46} \)) have been developed as functions of the Helmholtz energy with independent variables of temperature and density. The equations were developed based on experimental values of density, speed of sound, isobaric heat capacity, and vapor pressure. With these equations, all thermodynamic properties of \( n \)-hexadecane and \( n \)-docosane can be calculated. For \( n \)-hexadecane, the uncertainty in vapor pressure is 0.5 %. The uncertainty of the saturated liquid density is 0.05 % from the triple point up to 400 K, and 0.2 % at higher temperatures. The uncertainty in densities is within 0.5 %. The speed of sound and isobaric heat capacity can be calculated within 0.25 %. The uncertainties of the properties calculated with the equation for \( n \)-docosane are 5 % for vapor pressure, 0.1 % for saturated liquid density, 1 % for density, 0.5 % and 1 % for speed of sound at atmospheric pressure and higher pressures, respectively, and within 3 % for heat capacity.

Keywords Equations of state · Helmholtz energy · \( n \)-hexadecane · \( n \)-docosane · Thermodynamic properties

1 Introduction
\( n \)-Hexadecane (\( \text{C}_{16}\text{H}_{34} \)) and \( n \)-docosane (\( \text{C}_{22}\text{H}_{46} \)) are normal alkanes of interest in the petroleum industry for multiple applications. They are generally used as constituents in mixtures for fuel, especially for aviation. In this work, fundamental equations of state, in terms of the Helmholtz energy, are presented for both liquids. The equations of state are valid over the whole fluid region, and through them all thermodynamic properties can be calculated. Measurements of vapor pressure, density, speed of sound, and heat capacity are available in the literature for both...
n-hexadecane and n-docosane. Although the amount of data available for n-hexadecane is quite large, n-docosane is less studied, as with other heavy alkanes. The experimental data (reported in references 1–84) were used to develop the equations of state and estimate their uncertainties.

2 Critical and Triple Point Values

The critical values are among the most important parameters in the development of equations of state, such as for use as the reducing parameters in the equations. In the work of Lemmon and Goodwin [85], equations for the critical temperatures and pressures of normal alkanes, as functions of the carbon number, were determined. In the present work, the critical temperature values calculated by Lemmon and Goodwin were adopted for both fluids; the critical densities $\rho_c$ were determined during the fitting process of the equations of state. The critical pressures $p_c$ were calculated from the final equations of state as a fixed point at the critical temperature and density.

The values of the critical point determined in this work for $n$-hexadecane are

\[
T_c = 722.1 \text{ K},
\]

\[
p_c = 1.4799 \text{ MPa},
\]

\[
\rho_c = 1.000 \text{ mol} \cdot \text{dm}^{-3}.
\]

The triple point temperature of $n$-hexadecane is 291.329 K, and its molar mass is 226.441 g·mol$^{-1}$ [86].

The critical values of $n$-docosane are

\[
T_c = 792.2 \text{ K},
\]

\[
p_c = 1.1740 \text{ MPa},
\]

\[
\rho_c = 0.723 \text{ mol} \cdot \text{dm}^{-3}.
\]

The triple point temperature of $n$-docosane is 587.6 K and its molar mass is 310.601 g·mol$^{-1}$ [86].

3 Ancillary Equations

The boundaries between the liquid and vapor phases are defined by saturation states that can be estimated through the use of ancillary equations. These give close estimates for the pressures and densities required in the iterative process to find the saturation
states. The ancillary equations were developed by fitting calculated values of the saturation states (determined with the application of the Maxwell criteria applied to the equations of state) [87].

3.1 Ancillary Equations for n-hexadecane

The ancillary form for the vapor pressure $p_\sigma$ is

$$\ln \left( \frac{p_\sigma}{p_c} \right) = \frac{T_c}{T} [N_1 \theta + N_2 \theta^{1.5} + N_3 \theta^{2.8} + N_4 \theta^{6.7} + N_5 \theta^{8.9} + N_6 \theta^{15.5}],$$

where $\theta = 1 - \frac{T}{T_c}$, $N_1 = -10.4856$, $N_2 = 3.8226$, $N_3 = -8.6727$, $N_4 = -4.1440$, $N_5 = 0.8801$, and $N_6 = -5.7224$.

The saturated liquid density $\rho'$ can be calculated by the following ancillary equation

$$\rho' = 1 + N_1 \theta^{0.39} + N_2 \theta^{0.84} + N_3 \theta^{1.27} + N_4 \theta^{1.72} + N_5 \theta^{2.26},$$

where $N_1 = 3.43$, $N_2 = -4.008$, $N_3 = 8.4779$, $N_4 = -7.894$, and $N_5 = 3.4824$.

The ancillary equation that represents the saturated vapor density $\rho''$ is

$$\ln \left( \frac{\rho''}{\rho_c} \right) = N_1 \theta^{0.44} + N_2 \theta^{2.32} + N_3 \theta^{1.75} + N_4 \theta^{4.4} + N_5 \theta^{9.97} + N_6 \theta^{20.9},$$

where $N_1 = -5.0096$, $N_2 = 0.9061$, $N_3 = -15.2865$, $N_4 = -61.4138$, $N_5 = -143.5222$, and $N_6 = -369.0229$.

3.2 Ancillary Equations for n-docosane

The ancillary form representing the vapor pressure $p_\sigma$ is

$$\ln \left( \frac{p_\sigma}{p_c} \right) = \frac{T_c}{T} [N_1 \theta + N_2 \theta^{1.5} + N_3 \theta^{2.7} + N_4 \theta^{5.5} + N_5 \theta^{14.1} + N_6 \theta^{52.1}],$$

where $N_1 = -12.3834$, $N_2 = 2.8818$, $N_3 = -11.6292$, $N_4 = -2.7357$, $N_5 = -7.3103$, and $N_6 = 1188.9117$.

The saturated liquid density $\rho'$ can be calculated with

$$\rho' = 1 + N_1 \theta^{0.5} + N_2 \theta^{0.8} + N_3 \theta^{1.2} + N_4 \theta^{1.8} + N_5 \theta^{2.5},$$

where $N_1 = 6.6254$, $N_2 = -11.0123$, $N_3 = 13.6452$, $N_4 = -8.8244$, and $N_5 = 3.1241$.

The ancillary equation that represents the saturated vapor density $\rho''$ is

$$\ln \left( \frac{\rho''}{\rho_c} \right) = N_1 \theta^{0.5} + N_2 \theta^{24.0} + N_3 \theta^{1.7} + N_4 \theta^{4.2} + N_5 \theta^{10.3} + N_6 \theta^{10.6},$$
where $N_1 = -5.9790$, $N_2 = -586.6421$, $N_3 = -14.3725$, $N_4 = -71.0676$, $N_5 = -213.3123$, and $N_6 = 15.7901$.

4 Equation of State

The form of the equation of state presented in this work is based on the Helmholtz energy as a function of density and temperature $a(\rho, T)$, which is the most commonly used form for the calculation of thermodynamic properties of pure fluids and mixtures with low uncertainties. All thermodynamic properties can be estimated as derivatives of the Helmholtz energy, for example, pressure is calculated as

$$p = \rho^2 \left( \frac{\partial a}{\partial \rho} \right)_T. \tag{7}$$

The derivatives of the Helmholtz energy required to calculate other thermodynamic properties not reported are given elsewhere, e.g., [88].

The functional form for the reduced Helmholtz energy $\alpha$, as a function of the dimensionless density and temperature, is

$$\frac{a(\rho, T)}{RT} = \alpha(\delta, \tau) = \alpha^0(\delta, \tau) + \alpha^r(\delta, \tau), \tag{8}$$

where $\delta = \rho/\rho_c$, $\tau = T/T_c$, and $R$ is the molar gas constant equal to 8.314462618 J·mol$^{-1}$·K$^{-1}$ [89].

The reduced Helmholtz energy $\alpha$ is the contribution from the ideal gas contribution $\alpha^0$, which represents the ideal gas properties, and the residual or real Helmholtz energy $\alpha^r$ that accounts for the interactions between molecules.

4.1 Properties of the Ideal Gas

The ideal gas Helmholtz energy has the following form [90]:

$$\alpha^0 = \ln \delta + (c_0 - 1) \ln \tau + \sum_{k=1}^{2} a_k \tau^k + \sum_{k=1}^{2} v_k \ln \left[1 - e^{-\frac{k \tau}{T_c}}\right], \tag{9}$$

where the coefficients for $n$-hexadecane and $n$-docosane are given in Tables 1 and 2, respectively.
In order to calculate thermodynamics properties, a model for the ideal gas isobaric heat capacity is necessary. The expression for the ideal gas isobaric heat capacity $c_p^0$ used in this work that is required to derive the ideal gas Helmholtz energy is

$$\frac{c_p^0}{R} = c_0 + \sum_{k=1}^{2} v_k \left( \frac{u_k}{T} \right)^2 \frac{e^{u_k/T}}{[e^{u_k/T} - 1]^2}. \tag{10}$$

The $u_k$ coefficients contained in the Einstein functions used in this equation give the proper shape of the ideal gas heat capacity similar to that derived from statistical mechanical models.

The values of $a_k$, $v_k$, and $u_k$ are given in Table 1 for $n$-hexadecane and in Table 2 for $n$-docosane.

### 4.2 Properties of the Real Gas

The functional form often used till about the year 2000 for the residual Helmholtz energy equation was

$$a^r(\delta, \tau) = \sum N_k \delta^{d_k} \tau^{\alpha_k} + \sum N_k \delta^{d_k} \tau^{\alpha_k} \exp(-\delta^{d_k}). \tag{11}$$

A form containing additional Gaussian bell-shaped terms is now typically used, including those for $n$-hexadecane and $n$-docosane in this work, and is expressed as

$$a^r(\delta, \tau) = \sum_{k=1}^{k_1} N_k \delta^{d_k} \tau^{\alpha_k} + \sum_{k=k_1+1}^{k_2} N_k \delta^{d_k} \tau^{\alpha_k} \exp\left[-\delta^{d_k}\right]$$

$$+ \sum_{k=k_2+1}^{k_3} N_k \delta^{d_k} \tau^{\alpha_k} \exp\left[-\eta_k (\delta - c_k)^2 - \beta_k (\tau - y_k)^2\right], \tag{12}$$

where the coefficients and exponents are reported in Table 3 for $n$-hexadecane and in Table 4 for $n$-docosane and the values of $k_1$, $k_2$, and $k_3$ for the equations in this work are 5, 10, and 15. The Gaussian terms are useful in the determination of the fluid properties in the critical region.

| $c_0$ | $k$ | $a_k$ | $i_k$ | $v_k$ | $u_k$ |
|------|-----|-------|------|-------|-------|
| 33.9 | 1   | 66.73 | 0    | 61.6  | 1000  |
| 2    | -44.17 | 1    | 77.7 | 2400  |
Fitting Constraints

In order to develop the equations of state, several constraints were used to control the shape of the thermodynamic surface. In Table 5, a list of the main constraints for \(n\)-hexadecane is given. The table shows the properties for which a constraint was needed, the kind of constraint imposed, and the range over which it was applied. Because the thermodynamic behavior of \(n\)-hexadecane and \(n\)-docosane is, for the most part, the same when viewed on a reduced basis (such as with \(\tau\) and \(\delta\)), the
Table 5 Constraints for $n$-hexadecane

| Property                  | Constraint                                      | Range                        |
|---------------------------|-------------------------------------------------|------------------------------|
| Speed of sound            | Negative slope and 3rd derivative Positive curvature and 4th derivative | 1 MPa, (35–115) K            |
| Isochoric heat capacity   | Negative slope and 3rd derivative Positive curvature and 4th derivative | 3 MPa, (430–625) K           |
| Rectilinear diameter      | Zero curvature                                  | Liquid saturation line, from 650 K to $T_c$ |
| Phase identification parameter | Negative curvature                           | 1000 K, (1.08–1.85) mol·dm$^{-3}$ |
| Phase identification parameter | Zero curvature                               | 725 K, (0.80–0.95) mol·dm$^{-3}$ |
| Phase identification parameter | Positive derivatives                         | 1 MPa, (200–420) K           |
| Phase identification parameter | Positive derivatives                         | Liquid saturation line, (580–650) K |
| 4th virial coefficient    | Positive values                                | (1300–3000) K                |
| Ideal curve               | Zero curvature                                 | (1000–1200) K                |
| Joule inversion curve     | Positive curvature                             | (0.02–2.00) mol·dm$^{-3}$    |
| Joule–Thomson inv. curve  | Positive curvature                             | (0.05–2.50) mol·dm$^{-3}$    |

Fig. 1 Temperature as a function of density for $n$-hexadecane along several isobars: the straight line is the rectilinear diameter

Constraints used for $n$-hexadecane can be applied directly to the fitting of $n$-docosane after the temperature and density ranges of the constraints have been properly adjusted, and thus they are not reported here.

As an example, the rectilinear diameter (the average of the vapor and liquid saturated densities) was constrained to be linear by imposing zero curvature from 650 K
to the critical temperature; it results in the behavior shown in Fig. 1. In preliminary fits, the fourth virial coefficient was negative around 1500 K to 2500 K, thus it was forced to give positive values in the temperature range between 1300 K and 3000 K, as presented in Fig. 2.

The phase identification parameter (PIP) [91] required many constraints. An example is the constraint acting on the isobar at 1 MPa over the temperature range from 200 K to 420 K that forced the slope, curvature, and third and fourth derivatives to all be positive. A further constraint was used to obtain negative curvature

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**Fig. 2** Virial coefficients for n-hexadecane, with units of dm$^3$·mol$^{-1}$ for the second virial coefficient $B$, units of dm$^6$·mol$^{-2}$ for the third virial coefficient $C$, and units of dm$^9$·mol$^{-3}$ for the fourth virial coefficient $D$

**Fig. 3** Phase identification parameter (PIP) as a function of density for n-hexadecane along isotherms from 100 K to 1000 K
of the isotherm at 1000 K between 1.08 mol·dm$^{-3}$ and 1.85 mol·dm$^{-3}$ (see Fig. 3). More details about the phase identification parameter and the fourth virial coefficient are reported in the following section.

6 Comparison to Experimental Data

The experimental data used for this work are summarized in Table 6 for $n$-hexadecane and in Table 7 for $n$-docosane. The experimental data were converted to kelvins (ITS-90) for temperatures, megapascals for pressures, and moles per cubic decimeter for densities.

In order to estimate the uncertainties of the equations of state, all available experimental data are considered. The uncertainties are estimates of the combined expanded uncertainties with a coverage factor of two. The accuracy of the equations of state were determined by statistical comparisons between the properties calculated from the equations of state and the experimental values.

Tables 6 and 7 show the average absolute deviations (AAD) for any generic property $X$, as follows:

$$\text{AAD} = \frac{1}{n} \sum_{i=1}^{n} |\Delta X_i|,$$

where $n$ is the number of data and $\Delta X$ is defined as

$$\Delta X = 100 \cdot \left( \frac{X_{\text{data}} - X_{\text{calc}}}{X_{\text{data}}} \right).$$

6.1 Comparison to $n$-hexadecane Experimental Data

In Fig. 4, the deviations of experimental vapor pressures from the equation are shown. The measurements cover a range of temperatures between 298 K and 600 K. Most of the experimental data show scatter within 2 %. The data of Morgan and Kobayashi [49], Viton et al. [74], and Camin et al. [17] are best represented. The three data sets overlap between 460 K and 470 K, and the data show deviations on average within 0.1 %. Above 460 K, the Morgan and Kobayashi and Camin et al. data agree with the equation within 0.2 %. The data of Morgan and Kobayashi were carried out for temperatures up to 583 K, and the equation maintains deviations below 0.2 %. For temperatures lower than 460 K, the data of Morgan and Kobayashi show deviations from the equation below 0.6 %.

The deviations of saturated liquid densities for $n$-hexadecane are shown in Fig. 5. Most of the data have deviations lower than 0.1 % between 273 K and 373 K. In the temperature range between 303 K and 373 K, the data of Bolotnikov et al. [15], Prak et al. [63, 64] show deviations lower than 0.05 %. The measurements of Plebanski et al. [62] were performed over a wide range of temperature (from about 300 K to
### Table 6  Experimental data for \( n \)-hexadecane

| Authors                      | # Pnts | Temperature range (K) | Pressure range (MPa) | Density range (mol·l\(^{-1}\)) | AAD (%) |
|------------------------------|--------|-----------------------|----------------------|---------------------------------|---------|
| **Vapor pressure**           |        |                       |                      |                                 |         |
| Abedinzadegan Abdi and Meisen (1998) | [1]    | 8                     | 393–482              | 0–0.013                         | 7.36    |
| Camin et al. (1954)          | [17]   | 16                    | 463–560              | 0.007–0.101                     | 0.066   |
| Eggertsen et al. (1969)      | [28]   | 10                    | 299–413              | 0                               | 3.38    |
| Francis and Wood (1926)      | [33]   | 4                     | 410–435              | 0.001–0.003                     | 6.61    |
| Grenier-Loustalot et al. (1981) | [38]   | 4                     | 358–418              | 0–0.001                         | 4.85    |
| Krafft (1882)                | [40]   | 6                     | 424–561              | 0.001–0.101                     | 2.75    |
| Lee et al. (1992)            | [43]   | 5                     | 504–589              | 0.026–0.188                     | 1.76    |
| Mills and Fenton (1987)      | [46]   | 11                    | 389–560              | 0–0.102                         | 0.841   |
| Morgan and Kobayashi (1994)  | [49]   | 20                    | 393–583              | 0–0.164                         | 0.223   |
| Myers and Fenske (1955)      | [50]   | 26                    | 354–559              | 0–0.101                         | 3.25    |
| Parks and Moore (1949)       | [56]   | 10                    | 298–323              | 0                               | 10.1    |
| Siitsman et al. (2014)       | [69]   | 5                     | 486–561              | 0.015–0.102                     | 0.863   |
| Viton et al. (1996)          | [74]   | 24                    | 303–467              | 0–0.008                         | 2.92    |
| Zuiderweg (1952)             | [83]   | 31                    | 371–464              | 0–0.007                         | 5.8     |
| TDE                          | [84]   | 43                    | 282–561              | 0–0.102                         | 20      |
| **Saturated liquid density** |        |                       |                      |                                 |         |
| Aminabhavi et al. (1992)     | [2]    | 6                     | 298–323              | 3.33–3.41                       | 0.117   |
| Asfour et al. (1990)         | [5]    | 4                     | 293–313              | 3.36–3.42                       | 0.057   |
| Banos et al. (1992)          | [11]   | 7                     | 293–323              | 3.32–3.42                       | 0.023   |
| Boelhouwer (1960)            | [13]   | 4                     | 303–393              | 3.11–3.39                       | 0.018   |
| Bolotnikov et al. (2005)     | [15]   | 17                    | 293–373              | 3.17–3.42                       | 0.025   |
| Calingaert et al. (1941)     | [16]   | 4                     | 293–373              | 3.17–3.42                       | 0.043   |
| Diaz Pena and Tardajos (1978)| [20]   | 4                     | 298–333              | 3.29–3.4                        | 0.016   |
| Doolittle (1964)             | [21]   | 6                     | 323–573              | 2.49–3.35                       | 0.524   |
| Authors                          | # Pnts | Temperature range (K) | Pressure range (MPa) | Density range (mol·l⁻¹) | AAD (%) |
|---------------------------------|--------|-----------------------|----------------------|--------------------------|---------|
| Dymond et al. (1979)            | 4      | 298–373               |                      | 3.17–3.4                 | 0.065   |
| Dymond and Young (1980)         | 11     | 298–393               |                      | 3.11–3.4                 | 0.075   |
| Dymond et al. (1980)            | 7      | 298–358               |                      | 3.22–3.4                 | 0.056   |
| El-Banna and El-Batouti (1998)  | 7      | 283–313               |                      | 3.36–3.43                | 0.055   |
| Espeau and Ceolin (2006)        | 12     | 298–573               |                      | 2.44–3.36                | 1.61    |
| Findenegg (1970)                | 7      | 293–333               |                      | 3.29–3.42                | 0.015   |
| Graaf et al. (1992)             | 5      | 375–538               |                      | 2.65–3.17                | 0.311   |
| Krafft (1882)                   | 4      | 291–372               |                      | 3.18–3.42                | 0.104   |
| Lauer and King (1956)           | 7      | 298–368               |                      | 3.19–3.4                 | 0.04    |
| Paredes et al. (2011)           | 6      | 293–343               |                      | 3.26–3.42                | 0.03    |
| Plebanski et al. (1986)         | 11     | 300–490               |                      | 2.79–3.39                | 0.067   |
| Prak et al. (2013)              | 9      | 293–373               |                      | 3.17–3.42                | 0.017   |
| Prak et al. (2014)              | 9      | 293–373               |                      | 3.17–3.42                | 0.026   |
| Queimada et al. (2003)          | 6      | 293–343               |                      | 3.26–3.42                | 0.021   |
| Schiessler et al. (1946)        | 5      | 273–372               |                      | 3.18–3.48                | 0.03    |
| Vogel (1946)                    | 4      | 293–360               |                      | 3.22–3.42                | 0.174   |
| Wu et al. (1998)                | 4      | 293–313               |                      | 3.35–3.41                | 0.065   |
| TDE                             | 117    | 281–414               |                      | 3.05–4.02                | 0.115   |
| Density                         |        |                       |                      |                          |         |
| Amorim et al. (2007)            | 54     | 318–413               |                      | 6.89–62.1                | 3.07–3.5| 0.126 |
| Banipal et al. (1991)           | 72     | 318–373               |                      | 0.1–10                   | 3.17–3.37| 0.023 |
| Boelhouwer (1960)               | 44     | 303–393               |                      | 0.101–118                | 3.11–3.56| 0.156 |
| Chang et al. (1998)             | 21     | 333–413               |                      | 0.1–30                   | 3.05–3.39| 0.132 |
| Authors                          | # Pnts | Temperature range (K) | Pressure range (MPa) | Density range (mol·l⁻¹) | AAD (%) |
|---------------------------------|--------|-----------------------|----------------------|-------------------------|---------|
| Doolittle (1964)                | [21]   | 60                    | 323–573              | 5–500                   | 2.57–4  | 0.756   |
| Dymond et al. (1979)            | [26]   | 27                    | 298–373              | 0.1–450                 | 3.17–3.9| 0.061   |
| Dymond and Harris (1992)        | [24]   | 23                    | 298–348              | 0.1–279                 | 3.25–3.77| 0.06    |
| Glaser et al. (1985)            | [34]   | 63                    | 303–360              | 2.03–17.7               | 3.21–3.44| 0.221   |
| Gouel (1978)                    | [36]   | 75                    | 315–392              | 5.17–40.6               | 3.13–3.47| 0.068   |
| Matthews et al. (1987)          | [44]   | 10                    | 323–564              | 1.42–3.5                | 2.54–3.33| 0.313   |
| Outcalt et al. (2010)           | [54]   | 108                   | 291–470              | 0.083–50.7              | 2.86–3.46| 0.108   |
| Snyder and Winnick (1970)       | [70]   | 93                    | 298–358              | 0.101–290               | 3.22–3.75| 0.095   |
| Tanaka et al. (1991)            | [71]   | 16                    | 298–348              | 0.1–150                 | 3.25–3.64| 0.065   |
| Wu et al. (2011)                | [78]   | 31                    | 324–523              | 14.1–262                | 2.84–3.63| 0.15    |
| Wuerflinger et al. (2001)       | [79]   | 27                    | 298–313              | 0.1–90                  | 3.36–3.58| 0.078   |
| Wuerflinger and Sandmann (2000) | [80]   | 104                   | 303–348              | 0.1–280                 | 3.27–3.79| 0.683   |
| Zolghadr et al. (2013)          | [82]   | 165                   | 313–393              | 0.34–11.4               | 3.14–3.39| 0.352   |
| Enthalpy of vaporization        |        |                       |                      |                         |         |
| Morawetz (1968)                 | [47]   | 4                     | 298                  |                         | 2.1     |
| Morawetz (1972)                 | [48]   | 1                     | 298                  |                         | 0.762   |
| Heat capacity                   |        |                       |                      |                         |         |
| Baba et al. (1992)              | [8]    | 11                    | 323–423              | 5                       | 3.71    |
| Banipal et al. (1991)           | [10]   | 60                    | 318–373              | 0.1–10                  | 0.124   |
| Benson et al. (1971)            | [12]   | 4                     | 298–358              | 0.101                   | 0.315   |
| Gollis et al. (1962)            | [35]   | 3                     | 311–422              | 0.101                   | 3.64    |
| Lainez et al. (1989)            | [41]   | 1                     | 298                  |                         | 0.739   |
| Petit and Ter Minassian (1974)   | [58]   | 14                    | 298–454              | 0.101                   | 1.16    |
| Authors                        | # Pnts | Temperature range (K) | Pressure range (MPa) | Density range (mol·l⁻¹) | AAD (%) |
|-------------------------------|--------|-----------------------|----------------------|-------------------------|---------|
| Tardajos et al. (1986)        | [72]   | 1                     | 298                  |                         | 0.243   |
| Wilhelm et al. (1986)         | [76]   | 1                     | 298                  |                         | 1.03    |
| Saturation heat capacity      |        |                       |                      |                         |         |
| Finke et al. (1954)           | [32]   | 84                    | 11.9–320             |                         | 0.101   |
| Speed of sound                |        |                       |                      |                         |         |
| Aminabhavi and Gopalakrishna (1994) | [3]   | 3                     | 298–318              |                         | 0.693   |
| Awwad and Pethrick (1984)     | [7]    | 1                     | 298                  | 0.101                   | 0.144   |
| Ball and Trusler (2001)       | [9]    | 65                    | 298–373              | 0.1–101                 | 0.416   |
| Boelhouwer (1967)             | [14]   | 74                    | 293–473              | 10–140                  | 0.259   |
| Bolotnikov et al. (2005)      | [15]   | 17                    | 293–373              | 0.1                     | 0.125   |
| Khasanshin and Shchemelev (2001) | [39] | 29                    | 303–433              | 0.1–49.1                | 0.106   |
| Nascimento et al. (2015)      | [51]   | 18                    | 313–333              | 0.1–25                  | 0.116   |
| Neruchev et al. (2005)        | [52]   | 43                    | 293–713              | 0.1                     | 3.21    |
| Outcalt et al. (2010)         | [54]   | 16                    | 291–343              | 0.083                   | 0.029   |
| Paredes et al. (2011)         | [55]   | 6                     | 293–343              | 0.101                   | 0.034   |
| Plantier et al. (2000)        | [61]   | 9                     | 303–383              |                         | 0.638   |
| Prak et al. (2014)            | [64]   | 5                     | 293–333              | 0.1                     | 0.025   |
| Tardajos et al. (1986)        | [73]   | 1                     | 298                  |                         | 0.407   |
490 K), with a maximum deviation of 0.07 % from the equation and better than 0.02 % between 340 K and 420 K.

All of the density data available are represented by the equation with deviations lower than about 1.5 % and most are within 0.3 %, as shown in Fig. 6. The data of Banipal et al. [10] are represented by the equation within 0.1 % for temperatures between 318 K and 373 K, and at pressures up to 10 MPa. At ambient pressure, the deviation is reduced to 0.05 %. The deviations of the data of Snyder and Winnick [70] are below 0.1 % at 298 K. Between 0.6 MPa and 28 MPa, the data are within 0.1 %. The measurements performed by Outcalt et al. [54] agree with the equation of state within 0.2 %, but the deviations are consistently negative.

The deviations in speed of sound data from the equation are shown in Fig. 7. The only available measurements at temperatures higher than 473 K are those in the work of Neruchev et al. [52]. The data show good agreement with the equation and are less than 0.5 % for temperatures up to 693 K. At pressures between 10 MPa and

| Authors                        | # Pnts | Temperature range (K) | Pressure range (MPa) | Density range (mol·l⁻¹) | AAD (%) |
|-------------------------------|--------|-----------------------|----------------------|-------------------------|---------|
| Vapor pressure                |        |                       |                      |                         |         |
| Chickos and Hanshaw (2004)    | [19]   | 11 298–575            | 0–0.021              |                         | 7.18    |
| Francis and Wood (1926)       | [33]   | 8 462–509             | 0–0.004              |                         | 21.3    |
| Grenier-Loustalot et al. (1981)| [38]  | 4 379–434             |                      |                         | 3.16    |
| Morgan and Kobayashi (1994)   | [49]   | 12 453–573            | 0–0.021              |                         | 0.459   |
| Piacente et al. (1991)        | [60]   | 115 341–489           |                      |                         | 28.4    |
| Piacente et al. (1994)        | [59]   | 23 372–410            |                      |                         | 26.4    |
| Sasse et al. (1988)           | [67]   | 16 353–462            |                      |                         | 3       |
| Young (1928)                  | [81]   | 3 494–519             | 0.001–0.004          |                         | 5.44    |
| Saturated liquid density      |        |                       |                      |                         |         |
| Melaugh et al. (1976)         | [45]   | 1 323                 |                      | 2.5                     | 0.088   |
| Queimada et al. (2005)        | [66]   | 3 323–343             |                      | 2.45–2.49               | 0.057   |
| Dutour et al. (2001)          | [23]   | 8 323–393             |                      | 2.34–2.49               | 0.053   |
| Neruchev et al. (1967)        | [53]   | 15 333–473            |                      | 2.17–2.48               | 0.059   |
| Density                       |        |                       |                      |                         |         |
| Peters et al. (1988)          | [57]   | 48 323–368            | 2.05–16.1            | 2.38–2.51               | 0.945   |
| Heat capacity                 |        |                       |                      |                         |         |
| Durupt et al. (1996)          | [22]   | 6 373–473             | 0.101                |                         | 5.23    |
| Atkinson et al. (1969)        | [6]    | 8 320–450             | 0.101                |                         | 0.255   |
| Speed of sound                |        |                       |                      |                         |         |
| Dutour et al. (2001)          | [23]   | 126 323–393           | 0.101–150            |                         | 0.559   |
| Neruchev et al. (1967)        | [53]   | 15 333–473            | 0.1                  |                         | 0.064   |
Fig. 4 Comparisons of \( \text{C}_{16}\text{H}_{34} \) vapor pressures \( p_\sigma \) calculated with the equation of state and experimental data.

Fig. 5 Comparisons of \( \text{C}_{16}\text{H}_{34} \) saturated liquid densities \( \rho' \) calculated with the equation of state and experimental data.
Fig. 6  Comparisons of $C_{16}H_{34}$ densities $\rho$ calculated with the equation of state and experimental data.
50 MPa, the data of Ball and Trusler [9], Nascimento et al. [51], Khasanshin and Shchemelev [39], and Boelhouwer [14] are fitted to within 0.4 %. Above 50 MPa, the measurements of Boelhouwer, which were carried out up to 120 MPa, spread no more than 0.3 %. For the other available data above 50 MPa (Ball and Trusler), the deviations increase at higher pressures. The deviations become increasingly negative down to −3.2 % at 100 MPa. The speed of sound data at ambient pressure between 290 K and 373 K are represented by the equation within 0.2 %.
For the isobaric heat capacity, there are fewer experimental data than those of the properties previously discussed. The measurements, shown in Fig. 8, are within 6% from the equation. The work with the largest number of measurements is that of Banipal et al. [10]. They are well represented by the equation of state; the deviations are not higher than 0.3% in the temperature range from 318 K to 373 K. At 298 K, the measurements of Benson et al. [12] deviate from the equation by less than 0.02%. The deviations of their data increase to 0.5% for higher temperatures (up to 358 K), and all of them are negative.

### 6.2 Comparison to \( n \)-docosane Experimental Data

For \( n \)-docosane, there are significantly fewer data points than for \( n \)-hexadecane. The vapor pressure is the most studied property for this fluid, covering a wide range of temperature (see Fig. 9). The data set of Morgan and Kobayashi [49] is best represented by the equation at high temperatures, between 453 K and 573 K, with an average deviation of 0.5%. The data given in Sasse et al. [67] show different behaviors depending on the temperature. Between 393 K and 423 K, the data spread about the equation within 2%. Below 393 K, they all show deviations between −4% and −8%. Above 423 K the data differ with the equation consistently around 2%. The data of Chickos and Hanshaw [19] show deviations consistently around −2% at temperatures higher than 450 K, while the data and equation differ more as the temperature decreases.

All the experimental saturated liquid densities deviate from the equation of state within ±0.2%, as shown in Fig. 10. The measurements of Dutour et al. [23] deviate

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**Fig. 8** Comparisons of \( \text{C}_{16}\text{H}_{34} \) heat capacities \( c_p \) calculated with the equation of state and experimental data
more from the equation as the temperature increases, starting from $-0.001\%$ at $323.15\,\text{K}$ to $-0.12\%$ at $393.15\,\text{K}$. Queimada et al. \[66\] measured three saturated liquid density points, which deviate from the calculated values by around $-0.06\%$. The measurements of Neruchev et al. \[53\] cover a wider range of temperature, from $333\,\text{K}$ to $473\,\text{K}$; all of them agree with the equation of state within $\pm0.15\%$. 

**Fig. 9** Comparisons of $\text{C}_{22}\text{H}_{46}$ vapor pressures $p_v$ calculated with the equation of state and experimental data

**Fig. 10** Comparisons of $\text{C}_{22}\text{H}_{46}$ saturated liquid densities $\rho'$ calculated with the equation of state and experimental data
Peters et al. [57] published the only $p\nu T$ data reported for $n$-docosane, with temperatures from 323 K to 368 K and pressures up to 16 MPa. As shown in Fig. 11, the data show a systematic average deviation of $-1\%$ from the equation of state. The data at pressures near ambient differ from the saturated liquid density data by this amount. The data show a consistent offset from the equation at all pressures.

The measurements of Neruchev et al. [53] and Dutour et al. [23] are the only available data for $n$-docosane for the speed of sound as shown in Fig. 12. Between 353 K and 373 K the two sets overlap, and the deviations from the equation and
both sets are lower than 0.2 % at ambient pressure. Over their whole temperature range (from 333 K to 473 K) the Neruchev data are accurately represented by the equation of state, with a maximum deviation of 0.2 %. The data presented by Dutour et al. deviate from the equation of state within 1 %, but for the measurements performed at ambient pressure, the maximum deviation is less than 0.3 %.

For the isobaric heat capacity, just two sets of data are available: Atkinson et al. [6] and Durupt et al. [22]. The two data sets show substantial differences at the higher temperatures as shown in Fig. 13. The deviations of the Atkinson et al. data are within 0.05 % and −0.5 %, while the deviations of the Durupt et al. data increase with increasing temperatures, from 2.2 % at 273 K and up to 8 % at 473 K.

### 6.3 Virial Coefficients

One of the most important validation tests in the development of equations of state comes from the analysis of its virial coefficients.

\[
Z(T, \rho) = 1 + B(T)\rho + C(T)\rho^2 + D(T)\rho^3 + \ldots \tag{15}
\]

In Fig. 2, a plot of the second, third, and fourth virial coefficients is given as a function of temperature. The third virial coefficient \(C\) for \(n\)-hexadecane has a maximum value of 1.849 dm\(^6\)·mol\(^{-2}\) at 486.3 K, and is always positive above a temperature of 367.14 K. The maximum value of the fourth virial coefficient \(D\) is 1.120 dm\(^9\)·mol\(^{-3}\) at 489.3 K, has a minimum positive value at 1304 K, and a second maximum around 7536 K (for any temperature between 1304 K and above).

![Fig. 13 Comparisons of C\(_{22}\)H\(_{46}\) heat capacities \(c_p\) calculated with the equation of state and experimental data](image-url)
These values and the shape of the function were achieved through the use of the constraints reported in Table 5 and explained in the section “Fitting constraints.”
7 Extrapolation Behavior

The REFPROP software [92] was used to generate diagrams for inspection of the extrapolation behavior. One of the most important parameters used to verify the correct behavior of the equations is the phase identification parameter (PIP) defined in the work of Venkatarathnam and Oellrich [90]; this is an extremely sensitive property from which small inconsistencies can be seen that are not visible with other properties.

In Fig. 14, the PIP for n-hexadecane as a function of temperature is shown, along isobars and at saturation states. The plot shows positive curvature in the PIP over most of the liquid region, except for the region between 150 K and about 200 K. At very low temperatures (below 20 K), the curvature is still negative. This behavior of the PIP is also present in the plots for R-1234ze(E) reported in the work of Thol and Lemmon [93] and for R-245ca reported in Zhou and Lemmon [94].

Similar behavior is observed in the plot of the PIP versus temperature for n-docosane, shown in Fig. 15. The curvature is almost always positive over the liquid region, although it becomes slightly negative between 175 K and 205 K. The PIP for n-docosane, even at temperatures lower than 15 K, has positive curvature.

The validation of the equations can also be done by checking the behavior of other properties such as density, speed of sound, heat capacity, and the ideal curves. All of these properties show the expected trends as explained in other publications on equations of state, such as the work for the equation of state of propane [88] or R-125 [95].

The ideal curves are curves along which one property of a real fluid is equal to the hypothetical ideal gas. This definition can refer to any property, but usually the ideal curves of the compressibility factor, $Z(T, \rho)$, and its derivatives are only considered, as follows:

Ideal curve:

$$ Z = 1 $$

Boyle curve:

$$ \left( \frac{\partial Z}{\partial \rho} \right)_T = 0 $$

Joule–Thomson curve:

$$ \left( \frac{\partial Z}{\partial T} \right)_\rho = 0 $$

Joule inversion curve:

$$ \left( \frac{\partial Z}{\partial T} \right)_\rho = 0 $$
The ideal curves are used to judge the behavior of the equation of state. Even if the curves do not provide numerical information, reasonable shapes of the curves, such as the plots for propane [88], indicate correct extrapolation behavior of the...
equation of state extending to high pressures and temperatures far in excess of the likely thermal stability of the fluid. Consequently, the behavior of the ideal curves should always be analyzed and checked during the equation development process. Figures 16 and 17 show the plots of the ideal curves obtained for \( n \)-hexadecane and \( n \)-docosane, which have the expected shape, giving confidence in the proper extrapolation behavior at high temperatures and pressures.

8 Conclusions

Fundamental equations of state for \( n \)-hexadecane and \( n \)-docosane were developed and presented here. The equations can be used to calculate all the thermodynamic properties of these alkanes over the entire fluid region.

For \( n \)-hexadecane, the uncertainty in vapor pressure is 0.5 %. For saturated liquid density, the uncertainty is 0.05 % for temperatures up to 400 K and increases to 0.2 % at higher temperatures. The estimated uncertainty in densities is 0.1 % from the triple point to 450 K for pressures below 50 MPa. Outside this range, the uncertainty is 0.5 %. The speed of sound has an uncertainty of 0.25 %. The uncertainty in isobaric heat capacity is estimated to be 0.25 %.

For \( n \)-docosane, the uncertainty in vapor pressure is about 5 % and the uncertainty in saturated liquid density is 0.1 %. At pressures up to 20 MPa, the uncertainty in density is about 1 %; no estimation can be provided at higher pressures. For speed of sound, the uncertainty is less than 0.5 % at ambient pressure and increases to 1 % at higher pressures. The uncertainty in heat capacity is 3 %.

9 Supplementary Information

The files containing the parameters of the equations of state for \( n \)-hexadecane and \( n \)-docosane are available for the use in REFPROP [92], TREND [96], and CoolProp [97].

Supplementary Information The online version contains supplementary material available at https://doi.org/10.1007/s10765-022-03059-0.

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Declarations

Conflict of interest The authors have no competing interests to declare that are relevant to the content of this article.

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