Universal Seesaw Mass Matrix Model
and Neutrino Phenomenology

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Abstract

Stimulated by the recent development of the “universal seesaw mass matrix model”, an application of the model to the neutrino mass matrix is investigated: For the charged lepton and down-quark sectors, the model explains the smallness of their masses $m_f$ by the conventional seesaw mechanism $M_f \simeq m_L M_F^{-1} m_R$ ($M_F$ is a mass matrix of hypothetical heavy fermions $F$). On the other hand, the observed fact $m_t \sim \Lambda_L = O(m_L)$ (electroweak scale $\Lambda_L = 174$ GeV) seems to reject the applying of the seesaw mechanism to the up-quark sector. However, recently, it has been found that, by taking $\det M_F = 0$ for the up-quark sector $F = U$, we can understand the question of why only top quark has a mass of the order of $\Lambda_L$ without the seesaw-suppression factor $O(m_R)/O(M_F)$. For neutrino sector, the mass matrix $M_\nu$ is given by $M_\nu \simeq m_L M_F^{-1} m_T^L$ ($F = N$), so that the masses $m_\nu$ are suppressed by a factor $O(m_L)/O(m_R)$ compared with the conventional quark and charged lepton masses. The model can naturally lead to a large mixing $\sin^2 2\theta \simeq 1$. Also another model is investigated within the framework of the universal seesaw model: the model leads to three sets of the almost degenerate two Majorana neutrinos which are large mixing states between the left-handed neutrinos $\nu_{Li}$ and $SU(2)_L \times SU(2)_R$ singlet neutrinos $N_{1i}$ ($i = e, \mu, \tau$), so that the model can give a simultaneous explanation of the atmospheric and solar neutrino data.

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1. Introduction

Recent progress of the non-accelerator and accelerator experiments has gloriously accumulated knowledge of the masses and mixings of neutrinos. One of the most challenging problems in the particle physics is to give a unified understanding of quark and lepton masses and mixings. Especially, the study of the neutrino mass matrix will give us a valuable clue to the unified understanding of the quarks and leptons. Now, the study of the unified mass matrix models is just timely.

As one of such the unified mass matrix models, the so-called “universal seesaw” mass matrix model [1] has recently revived. The seesaw mechanism was first proposed [2] in order to answer the question of why neutrino masses are so invisibly small. Then, in order to understand that the observed quark and lepton masses are considerably smaller than the electroweak scale, the mechanism was applied to the quarks [1]: A would-be seesaw mass matrix for $(f,F)$ is expressed as

$$M = \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} = m_0 \begin{pmatrix} 0 & Z_L \\ \kappa Z_R & \lambda Y_f \end{pmatrix},$$  \hspace{1cm} (1.1)

where $f = u, d, \nu, e$ are the conventional quarks and leptons, $F = U, D, N, E$ are hypothetical heavy fermions, and they belong to $f_L = (2,1)$, $f_R = (1,2)$, $F_L = (1,1)$ and $F_R = (1,1)$ of SU(2)$_L \times$SU(2)$_R$. The matrices $Z_L$, $Z_R$ and $Y_f$ are of the order one. The matrices $m_L$ and $m_R$ take universal structures for quarks and leptons. Only the heavy fermion matrix $M_F$ takes a structure dependent on $f = u, d, \nu, e$. For the case $\lambda \gg \kappa \gg 1$, the mass matrix (1.1) leads to the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R.$$  \hspace{1cm} (1.2)

However, the observation of the top quark of 1994 [3] aroused a question: Can the observed fact $m_t \simeq 180 \text{ GeV} \sim \Lambda_L = O(m_L)$ be accommodated to the universal seesaw mass matrix scenario? Because $m_t \sim O(m_L)$ means $M_F^{-1} m_R \sim O(1)$. For this question, a recent study gives the answer “Yes”: Yes, we can do [4,5] by putting an additional constraint

$$\text{det} M_F = 0.$$  \hspace{1cm} (1.3)

on the up-quark sector ($F = U$). Then, we will easily be able to understand why only top quark $t$ acquires the mass $m_t \sim O(m_L)$. In the next section, we will review the mass generation scenario on the basis of the universal seesaw mass matrix model with the constraint (1.3). Also, in the next section, a rough sketch of the neutrino mass generation scenarios within the framework of the universal seesaw mass matrix model is given. One (Model A) is a straightforward extension of (1.2), so that the neutrino
mass matrix $M_\nu$ is given by $M_\nu \simeq -m_L M_N^{-1} m_R$. The smallness of the neutrino masses $m_\nu \ll m_f (f = e, u, d)$ is explained by assuming $\lambda_\nu \gg \lambda$ ($\lambda \equiv \lambda_e = \lambda_u = \lambda_d$), although the assumption is somewhat artificial. Another one (Model B) is a model without a new scale parameter such as $\lambda_\nu$: the light neutrino mass matrix $M_\nu$ is given by

$$M_\nu \simeq -m_L M_N^{-1} m_L^T,$$

so that the light neutrino masses $m_\nu$ are given with the order of

$$m_\nu \sim O \left( \frac{1}{\lambda} m_0 \right) \sim O \left( \frac{1}{\kappa} m_{\text{charged lepton}} \right).$$

Therefore, the model B can explain the smallness of the neutrino masses without assuming an additional scale parameter such as $\lambda_\nu$ in the model A. The third scenario (Model C) is very attractive to the neutrino phenomenology, because the model can lead to three sets of almost degenerate two Majorana neutrinos (the pseudo-Dirac neutrinos [6]). Every model of these can naturally give a large mixing $\sin^2 2\theta \simeq 1$.

In Sec. 3, we give a more explicit model of the universal seesaw mass matrix with some special structures of $m_L$, $m_R$ and $M_F$. In Secs. 4, 5, and 6, the models A, B, and C are discussed, respectively. Finally, Sec. 7 is devoted to the summary and concluding remarks.

2. Energy scales and fermion masses

For convenience, we take the diagonal basis of the matrix $M_F$. Then, the condition (1.3) means that the heavy fermion mass matrix $M_F$ in the up-quark sector is given by

$$M_U = \lambda m_0 \begin{pmatrix} O(1) & 0 & 0 \\ 0 & O(1) & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

although the other heavy fermion mass matrices $M_F$ ($F \neq U$) are given by

$$M_F = \lambda m_0 \begin{pmatrix} O(1) & 0 & 0 \\ 0 & O(1) & 0 \\ 0 & 0 & O(1) \end{pmatrix}, \quad (F \neq U).$$

Note that for the third up-quark the seesaw mechanism does not work (see Fig. 1).
The mass generation at each energy scale is as follows. First, at the energy scale \( \mu = \Lambda_S \), the heavy fermions \( F \), except for \( U_3 \), acquire the masses of the order of \( \Lambda_S \). Second, at the energy scale \( \mu = \Lambda_R \), the SU(2)_R symmetry is broken, and the fermion \( u_{R3} \) generates a mass term of the order of \( \Lambda_R \) by pairing with \( U_{L3} \). Finally, at \( \mu = \Lambda_L \), the SU(2)_L symmetry is broken, and the fermion \( u_{L3} \) generates a mass term of the order \( \Lambda_L \) by pairing with \( U_{R3} \). The other fermions \( f \) acquire the well-known seesaw masses (1.2). The scenario is summarized in Table 1.

Table 1. Fermion mass generation scenario

| Energy scale | \( d- \& e \)-sectors | \( u \)-sector (\( i = 1, 2 \)) | \( (i=3) \) |
|--------------|------------------------|-------------------------------|------------|
| At \( \mu = \Lambda_S \sim \lambda m_0 \) | \( m(F_L, F_R) \sim \Lambda_S \) | \( m(U_{Li}, U_{Ri}) \sim \Lambda_S \) | \( m(U_{L3}, U_{R3}) = 0 \) |
| At \( \mu = \Lambda_R \sim \kappa m_0 \) | \( m(f_R, F_L) \sim \Lambda_R \) | \( m(u_{Li}, U_{Li}) \sim \Lambda_R \) | \( m(u_{R3}, U_{L3}) \sim \Lambda_R \) |
| At \( \mu = \Lambda_L \sim m_0 \) | \( m(f_L, F_R) \sim \Lambda_L \) | \( m(u_{Li}, U_{Ri}) \sim \Lambda_L \) | \( m(u_{L3}, U_{R3}) \sim \Lambda_L \) |
| \( \downarrow \) | \( m(f_L, f_R) \sim \frac{\Lambda_L \Lambda_R}{\Lambda_S} \) | \( \downarrow \) | \( m(u_{Li}, u_{Ri}) \sim \frac{\Lambda_L \Lambda_R}{\Lambda_S} \) |

Thus, we can understand why only top quark \( t \) acquires the mass \( m_t \sim O(m_L) \). The other quarks and charged leptons acquire masses suppressed by a factor \( \kappa/\lambda = \Lambda_R/\Lambda_S \). A suitable choice of the mass matrix parameters will be able to give reasonable quark masses and Cabibbo-Kobayashi-Maskawa [7] (CKM) matrix parameters (for example, see the next section).

Next, we discuss the neutrino mass generation. Within the framework of the universal seesaw mass matrix model, we will discuss the following three scenarios.

One (Scenario A) is a trivial extension of the present model: we introduce a further
large energy scale $\Lambda_S'$ in addition to $\Lambda_S$, and we assume that $M_F \sim \Lambda_S$ ($F = U, D, E$), while $M_N \sim \Lambda_S'$ ($\Lambda_S' \gg \Lambda_S$). The mass matrix $M_\nu$ for the conventional light neutrinos (Dirac neutrinos) is given by

$$M_\nu \simeq -m_L M_N^{-1} m_R.$$  \hspace{1cm} (2.3)

The Dirac neutrino masses $m_\nu^i$ are suppressed by a factor $\Lambda_S'/\Lambda_S$ compared with the charged lepton masses $m_e^i$.

Another one (Scenario B) is more attractive because we do not introduce an additional energy scale. The neutral heavy leptons are singlets of $\text{SU}(2)_L \times \text{SU}(2)_R$ and they do not have $\text{U}(1)$-charge. Therefore, it is likely that they acquire Majorana masses $M_M$ together with the Dirac masses $M_D \equiv M_N$ at $\mu = \Lambda_S$. Then, the mass matrix $M_\nu$ for the conventional light neutrinos (Majorana neutrinos) is given by

$$M_\nu \simeq -m_L M_N^{-1} m_L^T,$$  \hspace{1cm} (2.4)

so that the masses $m_\nu^i$ are given with the order of

$$m_\nu \sim \frac{\Lambda_S^2}{\Lambda_S} = \frac{1}{\kappa} \frac{\Lambda_L \Lambda_R}{\Lambda_S} \sim O \left( \frac{1}{\kappa} m_e^i \right),$$  \hspace{1cm} (2.5)

where $\kappa = \Lambda_R/\Lambda_L$. In order to explain the smallness of $m_\nu$, the model [8,9] requires that the scale $\Lambda_R$ must be extremely larger than $\Lambda_L$. The details are discussed in Sec. 5.

On the other hand, the scenario A allows a case with a lower value of $\Lambda_R$. If we consider $\kappa \sim 10$, then we can expect abundant new physics effects as discussed in Ref.[10]. Therefore, although the model B is attractive from the theoretical point of view, the model A is also attractive from the phenomenological point of view. The model B is discussed in Sec. 4.

There is a model which is phenomenologically more attractive. In the present model, there is no distinction between $N_L$ and $N_R$, because both fields are $\text{SU}(2)_L \times \text{SU}(2)_R$ singlets and do not have $\text{U}(1)$ charges. Therefore, if $\nu_L$ ($\nu_R$) acquire masses $m_L$ ($m_R$) together with the partners $N_R$ ($N_L$), they may also acquire masses $m'_L$ ($m'_R$) together with the partners $N'_L$ ($N'_R$), where $\psi^c$ denotes charge conjugate state $C \psi^T$. In the limit of $m'_L = m_L$ and $m'_R = m_R$, we obtain six massless Majorana neutrino states: three are states which consist of almost left-handed neutrino $\nu_L$, and the other are states which consist of $N_1 \equiv (N_L - N'_R)/\sqrt{2}$. A suitable choice of the differences between $m'_L$ and $m'_R$ and between $m'_R$ and $m_R$ can lead to three sets of almost degenerate two Majorana neutrinos which are large mixing states between $\nu_L$ and $N_1$. The almost degenerate states with a large mixing between $\nu_{L\mu}$ and $N_{1\mu}$ and those with a large mixing between $\nu_{Le}$ and $N_{1e}$ are favorable to the explanation of the observed data of the atmospheric [11] and solar [12] neutrinos, respectively. However, in the model C, we must introduce a new parameter
which characterizes the differences $m'_L - m_L$ and $m'_R - m_R$ by hand, differently from the model B. The details of the model C are discussed in Sec. 6.

These neutrino mass generation scenarios are roughly summarized in Table 2.

Table 2. Neutrino mass generation scenarios

| Energy scale | Scenario A | Scenario B | Scenario C |
|--------------|------------|------------|------------|
| At $\mu = \Lambda_S^\nu$ | $m(N_L, N_R) \sim \Lambda_S^\nu$ | $m(N_L, N_R) \sim \Lambda_S$ | $m(N_L, N_R) \sim \Lambda_S$ |
| At $\mu = \Lambda_S$ | $m(N_L, N_R) \sim \Lambda_S$ | $m(N_L, N_R) \sim \Lambda_S$ | $m(N_L, N_R) \sim \Lambda_S$ |
| At $\mu = \Lambda_R$ | $m(\nu_R, N_L) \sim \Lambda_R$ | $m(\nu_R, N_L) \sim \Lambda_R$ | $m(\nu_R, N_L) \sim \Lambda_R$ |
| At $\mu = \Lambda_L$ | $m(\nu_L, N_R) \sim \Lambda_L$ | $m(\nu_L, N_R) \sim \Lambda_L$ | $m(\nu_L, N_R) \sim \Lambda_L$ |
| | $\downarrow$ | $\downarrow$ | $\downarrow$ |
| | $m(\nu_L, \nu_R) \sim \frac{\Lambda_L A_R}{\Lambda_S^\nu}$ | $m(\nu_L, \nu_R) \sim \frac{\Lambda_L^2}{\Lambda_S}$ | $m(\nu_L, \nu_R) \sim 0$ |

3. Democratic seesaw mass matrix model

So far, we have not assumed explicit structures of the matrices $Z_L, Z_R$ and $Y_f$. Here, in order to give a realistic numerical example, we put the following working hypotheses [4]:

(i) The matrices $Z_L$ and $Z_R$, which are universal for quarks and leptons, have the same structure:

$$Z_L = Z_R \equiv Z = \text{diag}(z_1, z_2, z_3),$$  \hfill (3.1)

with $z_1^2 + z_2^2 + z_3^2 = 1$, where, for convenience, we have taken a basis on which the matrix $Z$ is diagonal.

(ii) The matrices $Y_f$, which have structures dependent on the fermion sector $f = u, d, \nu, e$, take a simple form [(unit matrix)+(a rank one matrix)]:

$$Y_f = 1 + 3 b_f X.$$  \hfill (3.2)

(iii) The rank one matrix is given by a democratic form

$$X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix},$$  \hfill (3.3)
on the family-basis where the matrix $Z$ is diagonal.

(iv) In order to fix the parameters $z_i$, we tentatively take $b_e = 0$ for the charged lepton sector, so that the parameters $z_i$ are given by

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}}, \quad (3.4)$$

from $M_e \simeq m_L M^{-1}_E m_R = (\kappa/\lambda)m_0 Z \cdot 1 \cdot Z$.

The mass spectra are essentially characterized by the parameter $b_f$. The fermion masses $m_f$ versus $b_f$ are illustrated in Fig. 2. At $b_f = 0$, the charged lepton masses have been used as input values for the parameters $z_i$. Note that at $b_f = -1/3$, the third fermion mass takes a maximal value, which is independent of $\kappa/\lambda$. Also note that at $b_f = -1/2$ and $b_f = -1$, two fermion masses degenerate.

![Fig. 2. Masses $m_i$ ($i = 1, 2, \cdots, 6$) versus $b_f$ for the case $\kappa = 10$ and $\kappa/\lambda = 0.02$. The solid and broken lines represent the cases $\text{arg} b_f = 0$ and $\text{arg} b_f = 18^\circ$, respectively. The figure was quoted from Ref. [13].](image)

We take $b_u = -1/3$ for up-quark sector, because, at $b_u = -1/3$, we can obtain the maximal top-quark mass enhancement (see Fig. 2)

$$m_t \simeq \frac{1}{\sqrt{3}} m_0, \quad (3.5)$$

and a successful relation

$$\frac{m_u}{m_c} \simeq \frac{3}{4} \frac{m_e}{m_\mu}, \quad (3.6)$$
independently of the value of $\kappa/\lambda$. The value of $\kappa/\lambda$ is determined from the observed ratio $m_c/m_t$ as $\kappa/\lambda = 0.0198$. Considering the successful relation

$$\frac{m_dm_s}{m_b^2} \simeq 4 \frac{m_em_\mu}{m_t^2} ,$$

(3.7)

for $b_d \simeq -1$, we seek for the best fit point of $b_d = -e^{i\beta_d} (\beta_d^2 \ll 1)$. The observed ratio $m_d/m_s$ fixes the value $\beta_d$ as $\beta_d = 18^\circ$. Then we can obtain the reasonable quark mass ratios [4], not only $m_t^d/m_j^d$, but also $m_t^u/m_j^d$:

$$m_u = 0.000234 \text{ GeV}, \quad m_c = 0.610 \text{ GeV}, \quad m_t = 0.181 \text{ GeV},$$

$$m_d = 0.000475 \text{ GeV}, \quad m_s = 0.0923 \text{ GeV}, \quad m_b = 3.01 \text{ GeV}.$$

(3.8)

Here, we have taken $(m_0\kappa/\lambda)_q/(m_0\kappa/\lambda)_e = 3.02$ in order to fit the observed quark mass values at $\mu = m_Z$ [14]

$$m_u = 0.000233 \text{ GeV}, \quad m_c = 0.677 \text{ GeV}, \quad m_t = 0.181 \text{ GeV},$$

$$m_d = 0.000469 \text{ GeV}, \quad m_s = 0.0934 \text{ GeV}, \quad m_b = 3.00 \text{ GeV}.$$

(3.9)

We also obtain the reasonable values of the CKM matrix parameters:

$$|V_{us}| = 0.220 , \quad |V_{cb}| = 0.0598 ,$$

$$|V_{ub}| = 0.00330 , \quad |V_{td}| = 0.0155 .$$

(3.10)

(The value of $|V_{cb}|$ is somewhat larger than the observed value. For the improvement of the numerical value, see Ref. [13].)

4. Model A: a straightforward extension to the neutrinos

The most straightforward extension of the model to the neutrinos is to consider the mass matrix of the neutrino sector is also given by (1.1), so that the mass matrix $M_\nu$ for the conventional light neutrinos is given by (2.3), i.e.,

$$M_\nu \simeq -m_L M_N^{-1} m_R = -\frac{\kappa}{\lambda_\nu} m_0 Z Y_\nu^{-1} Z .$$

(4.1)

The smallness of the neutrino masses $m_\nu$ is given by assuming $\lambda_\nu \gg \lambda$ ($\lambda \equiv \lambda_e = \lambda_u = \lambda_d$).
Our interest is in a large mixing solution. As anticipated from Fig. 2, the large mixing solutions are given at $b_f \simeq -1/2$ and $b_f \simeq -1$, at which the mass degenerates $m_2^f \simeq m_3^f$ and $m_1^f \simeq m_2^f$ occur, respectively:

[Case A] $b_\nu \simeq -1/2$:

$$m_1^\nu \simeq 2 \frac{m_\mu}{m_\tau} \frac{\kappa}{\lambda_\nu} m_0, \quad m_2^\nu \simeq m_3^\nu \simeq \sqrt{\frac{m_\mu}{m_\tau} \frac{\kappa}{\lambda_\nu}} m_0,$$

$$U \simeq \left( \begin{array}{ccc}
1 & \frac{m_e}{2m_\mu} & \frac{m_e}{2m_\mu} \\
\frac{m_e}{m_\mu} & \frac{1}{\sqrt{2}} & \pm \frac{1}{\sqrt{2}} \\
\frac{m_e}{m_\tau} & \pm \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array} \right). \quad (4.2)$$

[Case B] $b_\nu \simeq -1$:

$$m_1^\nu \simeq m_2^\nu \simeq \sqrt{\frac{m_e m_\mu}{m_\tau^2} \frac{\kappa}{\lambda_\nu}} m_0, \quad m_3^\nu \simeq \frac{1}{2} \frac{\kappa}{\lambda_\nu} m_0,$$

$$U \simeq \left( \begin{array}{ccc}
\frac{1}{\sqrt{2}} & \pm \frac{1}{\sqrt{2}} & -\frac{m_e}{m_\tau} \\
\pm \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{m_\mu}{m_\tau} \\
\frac{m_\mu}{m_\tau} & \frac{m_\mu}{m_\tau} & 1
\end{array} \right). \quad (4.3)$$

These cases are favorable to the large mixing picture suggested by the atmospheric neutrino data [11]. However, it is hard to give the simultaneous explanation of the atmospheric and solar neutrino [12] data, because the case $b_\nu \simeq -1/2$ ($b_\nu \simeq -1$) can give $\sin^2 2\theta_{23} \simeq 1$ ($\sin^2 2\theta_{12} \simeq 1$), while the case leads to $\Delta m_{atm}^2 \equiv \Delta m_{32}^2 \ll \Delta m_{solar}^2 \equiv \Delta m_{21}^2$ ($\Delta m_{atm}^2 \equiv \Delta m_{21}^2 \ll \Delta m_{solar}^2 \equiv \Delta m_{32}^2$), where $\Delta m_{ij}^2 = (m_i^\nu)^2 - (m_j^\nu)^2$. We must seek for another explanation for the solar neutrino data.

### 5. Model B: a model without any new scale parameters

The neutral lepton mass matrix which is sandwiched between $(\nu_L, \nu_R, \overline{N}_L, \overline{N}_R)$ and
\((\nu_L^c, \nu_R, N_L^c, N_R)\)^T, where \(\nu_L^c \equiv (\nu_L)^c \equiv C\nu_L^T\) and so on, is given by

\[
M = \begin{pmatrix}
0 & 0 & 0 & m_L \\
0 & 0 & m_T^R & 0 \\
0 & m_R & M_M & M_D \\
m_L^T & 0 & M_D^T & M_M
\end{pmatrix},
\]

(5.1)

where \(M_D (\equiv M_N)\) and \(M_M\) are Dirac and Majorana mass matrices of the neutral heavy fermions \(N_i\). The heavy fermions \(F_i\) belong to \((1, 1)\) of \(SU(2)_L \times U(2)_R\). Besides, the neutral heavy leptons \(N_i\) do not have the \(U(1)\)-charge. Therefore, it is likely that when the Dirac masses \((M_D)_{ij}\) are generated between \(N_{Li}\) and \(N_{Rj}\), the Majorana masses \((M_M)_{ij}\) are also generated between \(N_{Li}\) and \(N_{cLj}\) (\(N_{cRi}\) and \(N_{Rj}\)) with the same structure at the same energy scale \(\mu = \lambda m_0\). Hereafter, we assume that

\[
M_M = M_D \equiv M_N \equiv \lambda m_0 Y_\nu.
\]

Then, we obtain the following twelve Majorana neutrinos [9]: (i) three heavy Majorana neutrinos with masses of the order of \(\lambda m_0\), whose mass matrix is approximately given by

\[
M_{\text{heavy}} \simeq 2M_N = 2\lambda m_0 Y_\nu,
\]

(5.3)

(ii) three sets of almost degenerate two Majorana neutrinos (the pseudo-Dirac neutrino [6]) with masses of the order of \(\kappa m_0\), whose mass matrix is approximately given by

\[
M_{PS-D} \simeq \begin{pmatrix}
-\frac{1}{4}m_R M_N^{-1} & m_R & 1 & \frac{1}{\sqrt{2}} m_T^R \\
1 & 0 & \frac{1}{\sqrt{2}} & m_R \\
\frac{1}{\sqrt{2}} & m_R & 0 & 0
\end{pmatrix} = \kappa m_0 \begin{pmatrix}
-\frac{\kappa}{4\lambda} Z & 1 & \frac{1}{\sqrt{2}} Z \\
\frac{1}{\sqrt{2}} Z & 0 & \end{pmatrix} \left(\begin{array}{c}
\nu \\
\nu
\end{array}\right),
\]

(5.4)

and (iii) three light Majorana neutrinos with masses of the order of \((1/\lambda)m_0\), whose mass matrix is approximately given by

\[
M_{\nu} \simeq -m_L M_N^{-1} M_L^T = -\frac{m_0}{\lambda} Z Y_\nu^{-1} Z.
\]

(5.5)

The neutrinos which are described by the mass matrix (5.5) consist of almost left-handed neutrinos \(\nu_{Li}\). Therefore, as far as the conventional light neutrinos are concerned, the model B is identical with the model A by substituting \(1/\lambda\) for \(\kappa/\lambda_\nu\) in the model A. The numerical results have been given in Ref. [9] in detail. For example, for the case \(b_\nu = -(1/2)e^{i\beta_\nu} (\beta_\nu = 0.12^\circ)\), we obtain

\[
m_1' = 0.00164 \text{ eV}, \quad m_2' = 0.695 \text{ eV}, \quad m_3' = 0.707 \text{ eV},
\]

(5.6)
\[ \Delta m_{32}^2 = 0.016 \text{ eV}^2, \quad \Delta m_{21}^2 = 0.483 \text{ eV}^2, \]  
(5.7)

\[ \sin^2 2\theta_{\text{atm}} \equiv -4\text{Re}(U_{\mu 2}U_{\tau 2}^* U_{\mu 3}^* U_{\tau 3}) = 0.995, \]  
(5.8)

\[ \sin^2 2\theta_{\text{LSND}} \equiv 4|U_{e1}|^2|U_{\mu 1}|^2 = 0.0191, \]  
(5.9)

\[ \langle m_\nu \rangle \equiv |m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2| = 0.00267 \text{ eV}. \]  
(5.10)

\[ \Lambda_L = m_0 = 3.12 \times 10^2 \text{ GeV}, \]  
\[ \Lambda_R = \kappa m_0 = 1.90 \times 10^{11} \text{ GeV}, \]  
\[ \Lambda_S = \lambda m_0 = 3.20 \times 10^{13} \text{ GeV}. \]  
(5.11)

Here, the numerical result (5.11) has been obtained from the input value \( \Delta m_{32}^2 \equiv \Delta m_{32}^2 = 0.016 \text{ eV}^2 \) and the relations

\[ m_t \simeq \frac{1}{\sqrt{3}} m_0 \simeq 180 \text{ GeV} \quad \text{(at } \mu = m_Z) , \]  
(5.12)

and

\[ (\kappa/\lambda) m_0 = m_{\tau} + m_{\mu} + m_e = 1.850 \text{ GeV} , \quad \text{(at } \mu = m_Z) . \]  
(5.13)

The results (5.7) - (5.9) can successfully explain the atmospheric neutrino data [11] and the neutrino oscillation (\( \nu_\mu \to \nu_e \)) experiment by the liquid scintillator neutrino detector (LSND) [15] at Los Alamos. However, the model B fails to explain the solar neutrino data straightforwardly, as well as the model A.

### 6. Model C: a model with light pseudo-Dirac neutrinos

In the present model, there is no distinction between \( N_L \) and \( N_R \), because both fields are \( SU(2)_L \times SU(2)_R \) singlets and do not have \( U(1) \) charges. Therefore, if \( \nu_L \) (\( \nu_R \)) acquire masses \( m_L \) (\( m_R \)) together with the partners \( N_R \) (\( N_L \)), they may also acquire masses \( m'_L \) (\( m'_R \)) together with the partners \( N'_L \) (\( N'_R \)). Then, the mass matrix for the neutrino sector is given by

\[
M = \begin{pmatrix} 0 & 0 & m'_L & m_L \\ 0 & 0 & m'_R & m'_L \\ m'_L^T & m_R & M_M & M_D \\ m'_L & m'_R & M_D^T & M_M \end{pmatrix}.
\]  
(6.1)
By rotating the fields \((\nu_L^c, \nu_R, N_L^c, N_R)\) by

\[
R_{34} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}, \tag{6.2}
\]

the 12 \times 12 mass matrix (6.1) becomes

\[
M' = R_{34} M R_{34}^T = \frac{1}{\sqrt{2}} \begin{pmatrix}
0 & 0 & m'_L - m_L & m'_L + m_L \\
0 & 0 & (m_R - m'_R)^T & (m_R + m'_R)^T \\
(m'_L - m_L)^T & m_R - m'_R & \sqrt{2}(M_M - M_D) & 0 \\
(m'_L + m_L)^T & m_R + m'_R & 0 & \sqrt{2}(M_M + M_D)
\end{pmatrix}, \tag{6.3}
\]

where we have used \(M_D^T = M_D\).

In the \(N_L \leftrightarrow N_R^c\) symmetric limit, i.e., in the limit of \(m'_L = m_L, m'_R = m_R\) and \(M_M = M_D \equiv M_N\), the mass matrices for the neutrinos \(N_1, N_2\) and \(\nu_R\) are given by

\[
M(N_1) = 0, \tag{6.4}
\]

\[
M(N_2) \simeq 2M_N, \tag{6.5}
\]

\[
M(\nu_R) \simeq -m_R^T M_N^{-1} m_R, \tag{6.6}
\]

where \(N_1\) and \(N_2\) are defined by

\[
N_{1i} = \frac{1}{\sqrt{2}}(\nu_{Li} - N_{Ri}^c), \quad N_{2i} = \frac{1}{\sqrt{2}}(\nu_{Li} + N_{Ri}^c). \tag{6.7}
\]

Furthermore, for the model with the relation \(m_R \propto m_L\) such as a model given in Sec. 3, we obtain

\[
M(\nu_L) = 0. \tag{6.8}
\]

Only when we assume a sizable difference between \(m'_L\) and \(m_L\) (and also between \(m'_R\) and \(m_R\)), we can obtain visible neutrino masses \(m(\nu_L) \neq 0\) and \(m(N_1) \neq 0\). A suitable choice of the differences \(m'_L - m_L\) and \(m'_R - m_R\) can lead to three sets of the pseudo-Dirac neutrinos \((\nu_{\nu i+}^{ps}, \nu_{\nu i-}^{ps})\) \((i = e, \mu, \tau)\) which are large mixing states between \(\nu_{Li}\) and \(N_{1i}\), i.e.,

\[
\nu_{\nu i \pm}^{ps} \simeq \frac{1}{\sqrt{2}}(\nu_{Li} \pm N_{1i}), \tag{6.9}
\]
and whose masses are almost degenerate. The ps-Dirac neutrinos $\nu_{\mu\pm}^{ps}$ are favorable to the explanation of the large mixing suggested from the atmospheric neutrino date [11] and the ps-Dirac neutrinos $\nu_{e\pm}^{ps}$ are favorable to that of the large-angle solution of the Mikheyev-Smirnov-Wolfenstein (MSW) solutions [16] for the solar neutrino data [12]. (A suitable choice can also give that the lightest two neutrino states are favorable to the small-angle solution of the MSW solutions.) The numerical results from the model C will be given elsewhere in the collaboration with Fusaoka [17].

If the scenario C is true, we will not be able to observe a large-angle mixing in appearance experiments such as $\nu_\mu \rightarrow \nu_\tau$ oscillation experiments by CHORUS [18] and NOMAD [19]. The large-angle mixing will be observed only in the disappearance experiments, because the mixing partners of $\nu_\mu$ and $\nu_e$ are SU(2)$_L \times$SU(2)$_R$ singlet neutrinos $N_{1\mu}$ and $N_{1e}$, respectively.

However, for example, when we take $m'_L = (1 - \varepsilon_L)m_L$ and $m'_R = (1 - \varepsilon_R)m_R$ [$\varepsilon = \text{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3)$], the these $\varepsilon$ are must be of the order of $10^{-11}$. Where such the scale of $\varepsilon \sim 10^{-11}$ comes from is open question in the scenario C.

7. Concluding remarks

In conclusion, we have discussed possible three neutrino-mass-generation scenarios within the framework of the universal seesaw mechanism, especially, on the basis of the “democratic seesaw mass matrix model”, which can answer the question why only top quark $t$ acquires the mass of the order of the electroweak scale $\Lambda_L = O(m_L)$, and can give reasonable quark masses and mixings in terms of the charged lepton masses.

The scenario A is a trivial extension of the model for quarks, and it is not so attractive, because the smallness of the neutrino masses is explained by introducing an additional energy scale parameter $\Lambda^\nu_S$ ($\Lambda^\nu_S \gg \Lambda_S \equiv \Lambda^e_S = \Lambda^\mu_S = \Lambda^\tau_S$), by hand. In contrast with the scenario A, the scenario B is a model without a new energy scale such as $\Lambda^\nu_S$. The smallness of the neutrino masses is explained by the suppression factor $1/\kappa \equiv \Lambda_R/\Lambda_S$. The choice of the parameter value $b_\nu \simeq -1/2$ can give reasonable values of $\Delta m^2_{atm} = \Delta m^2_{32} \sim 10^{-2}$ eV$^2$ and $\sin^2 2\theta_{23} \simeq 1$, but it fails to explain the solar neutrino data.

The scenario C is the most attractive model from the phenomenological point of view. The model leads to three sets of the pseudo-Dirac neutrinos $(\nu_{\mu\pm}^{ps}, \nu_{e\pm}^{ps})$ ($i = e, \mu, \tau$), which are large mixing states between $\nu_{Li}$ and $N_{1i} \equiv (N_{Li} - N_{Ri})/\sqrt{2}$. If the scenario C is true, we will not be able to observe a large-angle mixing in the appearance oscillation experiments such as $\nu_\mu \rightarrow \nu_\tau$ oscillation experiments by CHORUS and NOMAD. The large-angle mixing will be observed only in the disappearance experiments, because the mixing partners of $\nu_\mu$ and $\nu_e$ are SU(2)$_L \times$SU(2)$_R$ singlet neutrinos $N_{1\mu}$ and $N_{1e}$, respectively. However, in the scenario C, the smallness of the neutrino masses must be explained.
by a small breaking of the $N_L \leftrightarrow N_R$ symmetry, by hand. This is our future task.

At present, it seems to be hard to explain all of the neutrino data within the framework of the conventional light three family neutrinos. Since the scenario C leads to three sets of the light ps-Dirac neutrinos (i.e., six almost massless neutrinos), the model can give us abundant phenomenological predictions. It will be worth while studying the scenario C furthermore from the phenomenological point of view as well as from the theoretical point of view.

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