A Novel View of Exotic Hadrons and the Covariant $\tilde{U}(12)$-Classification Scheme

Kenji Yamada

Department of Engineering Science, Junior College Funabashi Campus, Nihon University, Funabashi, Chiba 274-8501, Japan

Abstract. I classify exotic hadrons into two types, “genuine” and “hidden” exotics, and propose that the “hidden” exotics would be interpreted as “chiralons” in the $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme of hadrons. Based upon this conjecture I investigate the mass spectrum of 1S, 1P, and 2S states for the $c\bar{c}$ system by use of a phenomenological mass formula with the spin-dependent interactions and present possible assignments for exotic neutral charmonium-like states which were recently discovered. It is also mentioned that some $J^{PC}$-exotic states predicted in this scheme might correspond to those found in a recent lattice QCD calculation of the charmonium spectrum.

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INTRODUCTION

In recent years there have been discovered numerous exotic states which have unexpected and puzzling nature, such as charmonium-like states, XYZ series, and $\Theta(1540)$, and also there seem to be too many observed mesons, especially in the light $(u,d)$-quark sector, to be classified as the conventional quark-model $q\bar{q}$ states [1]. These experimental situations have revived multiquark hadrons and their theoretical studies which were declining before them. A serious complication of most models suggesting multiquark states is that they would predict too many states to exist, that is, the known problem of “state inflation”, and further, though many theoretical proposals for interpreting exotic charmonium-like states have put forward, no single framework of models can accommodate most of those states.

THE $\tilde{U}(12)_{SF} \times O(3,1)_L$-CLASSIFICATION SCHEME OF HADRONS

The $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme [2] is a covariant system of classification for hadrons which gives covariant quark representations for composite hadrons with definite Lorentz and chiral transformation properties. In the rest frame of hadrons the SU(2)$_{\gamma}$ intrinsic spin symmetry is extended to $U(4)_S$ by adding a new degree of freedom on the $\rho$-spin, SU(2)$_{\rho}$, being connected with decomposition of Dirac matrices, $\gamma = \rho \times \sigma$. Then an extended $U(12)_{SF}$ spin-flavor symmetry, including the flavor SU(3)$_F$, has as its subgroups both the nonrelativistic spin-flavor and chiral symmetry, as SU(6)$_{SF} \times SU(2)_\rho$ and U(3)$_L \times U(3)_R \times SU(2)_\sigma$. The $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme is obtained through the covariant generalization from $U(12)_{SF} \times O(3)_L$ by boosts, separating the spin and space degrees of freedom. Thus the $U(12)_{SF}$ symmetry in the hadron rest frame is embedded in the covariant $\tilde{U}(12)$-representation space, which includes subgroups as $\tilde{U}(4)_D \times SU(3)_F$, $\tilde{U}(4)_D$ being the pseudounitary homogeneous Lorentz group for Dirac spinors.

An essential ingredient of the $\tilde{U}(12)_{SF} \times O(3,1)_L$-classification scheme is that quarks have the $\rho$-spin degree of freedom, which is discriminated by the eigenvalues $r = \pm$ of $\rho_{3}$ in the hadron rest frame. This implies that not only conventional quarks with $r = +$ but also new type quarks with $r = -$ are building blocks of hadrons. Thus hadron states are characterized, aside from flavors, by the quantum numbers, the net constituent $\rho$-spin $S^{(\rho)}$, its third component $S^{(\rho)}_3$, the ordinary net constituent $\sigma$-spin $S$, the net orbital angular momentum $L$, and the total spin $J$.

For $q\bar{q}$ meson states the meson spin $J$, parity $P$, and charge-conjugation parity $C$ are given by

$$J = L + S, \quad P = (-1)^{L+|S^{(\rho)}_3|}, \quad C = (-1)^{L+S+S^{(\rho)}+1}. \quad (1)$$
The ground \((L = 0)\) states of \(q\bar{q}\) mesons are composed of states with the \(J^{PC}\) quantum numbers, two pseudoscalars \(P^{(0^{-+})}, P^{(2^+)}\), two scalars \(S_A^{(1)}(0^{++}), S_B^{(1)}(0^{++})\), two vectors \(V^{(1^{--})}, V^{(2)}(1^{--})\), and two axial-vectors \(A^{(2)}(1^{+-}), B^{(3)}(1^{+-})\), where the symbol \(\chi\) represents at least one of the constituents having the negative eigenvalue \(\varepsilon = -\) of \(\rho_l\), referred to as “chiralons”. There are also orbital and radial excitations for each of these eight types of meson. The \(\rho\)-spin quantum numbers of ground states and their excitations are given by

\[ (S^{(\rho)}, S_3^{(\rho)}) = \begin{cases} 
(1, 1) & \text{for } P \text{ and } V \text{ sectors} \\
(1, -1) & \text{for } P^{(\chi)} \text{ and } V^{(\chi)} \text{ sectors} \\
(1,!0) & \text{for } S_A^{(\chi)} \text{ and } B^{(\chi)} \text{ sectors} \\
(0, 0) & \text{for } S_B^{(\chi)} \text{ and } A^{(\chi)} \text{ sectors} 
\end{cases} \quad (2) \]

where the \((1, 1)\) sector corresponds to the conventional \(q\bar{q}\) states. Here I simply assume that all ground and excited states are physically realized, though it is a highly dynamical problem.

**A NOVEL CLASSIFICATION OF EXOTIC HADRONS AND EXOTIC CHARMONIUM-LIKE STATES**

Exotic hadrons are defined, in the conventional quark model, as mesonic and baryonic states with quantum numbers forbidden for \(q\bar{q}\) and \(qqq\) states or anomalous features which cannot be explained as those states. Here I classify exotic hadrons into the following two types:

**“Genuine” exotics** mesonic and baryonic states with flavor quantum numbers forbidden for the conventional quark-model \(q\bar{q}\) and \(qqq\) states.

**“Hidden” exotics** mesonic and baryonic states which are constructed by adding extra flavor-singlet light-\(q\bar{q}\) or gluon components to the conventional \(q\bar{q}\) and \(qqq\) states, and pure gluonic ones.

Then I conjecture that the hidden exotics would be interpreted as chiralons in the \(\tilde{U}(12)_{SF} \times O(3, 1)_L\)-classification scheme. Mesonic \(q\bar{q}\) chiralons are considered to be asymptotically four quark states \((q\bar{q} + \text{flavor-singlet light } q\bar{q}), q\bar{q}-\text{gluon states } (q\bar{q} + \text{gluon(s)}), \text{or pure gluonic states } (gg, ggg)\), while baryonic \(qqq\) chiralons are considered to be asymptotically five quark states \((qqq + \text{flavor-singlet light } q\bar{q})\) or \(qqq-\text{gluon states } (qqq + \text{gluon(s)})\).

**The charmonium spectrum in the \(\tilde{U}(12)_{SF} \times O(3, 1)_L\)-classification scheme**

Based upon the above conjecture I undertake explaining numerous charmonium-like states, which have unexpected and puzzling nature, in the \(\tilde{U}(12)_{SF} \times O(3, 1)_L\)-classification scheme. Here I will restrict myself to discussing the neutral charmonium-like \(XY\) states which have large hadronic transition rates to lower conventional charmonia. The corresponding observed states, together with measured properties, are collected in Table 1.

To assign these nine charmonium-like states to \(J^{PC}\) multiplets in this scheme, I make predictions on the mass spectrum of \(1S, 1P,\) and \(2S\) states for the \(c\bar{c}\) system, assuming a phenomenological mass formula in which spin-dependent interactions are taken into account as

\[ M(n^{2S+1}L_J) = \mathcal{M}(nL; S^{(\rho)}, S_3^{(\rho)}) + c_{LS}(nL)\langle L \cdot S \rangle + c_T(nL)\langle S_T \rangle + c_{SS}(nL)\langle S_Q \cdot S_Q \rangle, \quad (3) \]

where \(\mathcal{M}(nL; S^{(\rho)}, S_3^{(\rho)})\) are the spin-averaged masses of \(1S, 1P, 2S\) states for the \((S^{(\rho)}, S_3^{(\rho)}) = (1, 1), (1, -1), (1, 0), (0, 0)\) sectors and \(c_{LS}(nL), c_T(nL), c_{SS}(nL)\) are the constant parameters which represent the contributions of spin-orbit, tensor, and spin-spin interactions, respectively. I take the excitation energies of \(1P\) and \(2S\) states to be equal for all the \((1, 1), (1, -1), (1, 0), (0, 0)\) sectors, so there are six independent parameters concerning the spin-averaged masses, \(\mathcal{M}(1S), \mathcal{M}(1P), \mathcal{M}(2S)\) for the \((1, 1)\) sector and \(\mathcal{M}(1S)\)’s for each of the \((1, -1), (1, 0), (0, 0)\) sectors. I also assume the following mass relations for the scalar and axial-vector states with \((1, 0)\) and \((0, 0)\):

\[ M(0^{++}; S^{(\rho)} = 1) < M(0^{+-}; S^{(\rho)} = 0), \]

\[ M(1^{+-}; S^{(\rho)} = 1) < M(1^{++}; S^{(\rho)} = 0), \quad (4) \]
and the respective mass differences between the $S^p(0) = 0$ and 1 states are identical. The $c_{SS}(1S)$, $c_{SS}(2S)$, $c_{LS}(1P)$, $c_T(1P)$, and $c_{SS}(1P)$ are taken to be common values for all the $(1,1), (1,-1), (1,0), (0,0)$ sectors, except for the sign of $c_{SS}$ for the $(1,0)$ and $(0,0)$ sectors, and thus there are five independent parameters, $c_{SS}(1S)$, $c_{SS}(2S)$, $c_{LS}(1P)$, $c_T(1P)$, and $c_{SS}(1P)$.

To fix values of the model parameters I use measured masses of the $\eta_c(1S), J/\psi(1S), h_c(1P)$, $\chi_{c0,1,2}(1P), \eta_c(2S), \psi(2S)$ [1], $X(3872)$ [5], $X(3915)$ [6], $X(4350)$ [8], assuming that the last two states have $J^{PC} = 0^{++}$, and extracted values are as follows:

$$M(1S; 1, 1) = 3067.8 \text{ MeV}, \quad M(1P; 1, 1) = 3525.3 \text{ MeV}, \quad M(2S; 1, 1) = 3673.8 \text{ MeV},$$

$$M(1S; 1, -1) = 4003.7 \text{ MeV}, \quad M(1S; 1, 0) = 3826.6 \text{ MeV}, \quad M(1S; 0, 0) = 3901.5 \text{ MeV},$$

and

$$c_{SS}(1S) = 116.6 \text{ MeV}, \quad c_{SS}(2S) = 49.0 \text{ MeV},$$

$$c_{LS}(1P) = 35.0 \text{ MeV}, \quad c_T(1P) = 40.6 \text{ MeV}, \quad c_{SS}(1P) \approx 0 \text{ MeV}.$$  \hfill (5)

Using these values the charmonium spectrum of $1S$, $1P$, $2S$ states are calculated, and then comparing the calculated masses of the predicted $J^{PC}$ states with the measured masses and $J^{PC}$ of the relevant $XY$ states, I assign those $XY$ states to appropriate places. The results are given in Table 2, where the “$Y(4280)$” is a hypothetical state mentioned by the CDF collaboration as follows [7]: \textbf{There is a small cluster of events approximately one pion mass higher than the first structure. However, the statistical significance of this cluster is less than 3}\sigma. In Table 3 the suggested assignments for the relevant $XY$ states are summarized in comparison with experiment. From this table it is seen that the theoretical masses for the relevant $XY$ states are in excellent agreement with experiment.

As is seen from Table 2, there exist some $J^{PC}$-exotic states, $0^{+-}(h_{c0})$ in the $1S$ and $2S$ levels and $0^{--}(\psi_{0})$, $1^{+-}(\eta_{c1})$ in the $1P$ level\(^\text{2}\), which is a remarkable feature of the $\bar{U}(12)_{SP} \times O(3,1)_L$-classification scheme. Of particular interest is that a recent lattice QCD calculation of the charmonium spectrum showed the existence of exotic $1^{+-}(\eta_{c1})$ and $0^{+-}(h_{c0})$ states with masses of 4300(50) MeV and 4465(65) MeV, respectively [9]. These masses might be compared with the corresponding predicted ones of 4240 (or 4359) MeV and 4544 MeV for the $1^{+-}(1^{P(2B)})$ or $1^{P(2B)}$ and $0^{+-}(2^{1} S_{0}^{(2B)})$ states. A subsequent lattice QCD analysis on the radiative decay of the exotic $\eta_{c1}$ state at 4300(50) MeV,

\(^{1}\text{The } \psi(2S) \text{ is assumed to be a pure } 2^{3}S_{1} \text{ state, neglecting a small admixture of the } 1^{3}D_{1} \text{ state.}\)

\(^{2}\text{In the light } (u,d) \text{-quark sector there are presently experimental observations of the two exotic mesons with } J^{P}(J^{PC}) = 1^{+}(1^{--}), \pi_{c}(1400) \text{ and } \pi_{c}(1600) \text{ [1], which can be assigned to the } 1^{1} P_{1}^{(2B)} \text{ and } 1^{3} P_{1}^{(2B)} \text{ states.}\)
TABLE 2. Theoretical mass spectrum (in MeV) of conventional and chiralonic charmonia. The fitted mass values are underlined. The n and L are radial and orbital angular-momentum quantum numbers.

| P     | V    | P(x) | V(x) | S_A(x) | B(x) | S_B(x) | A(x) |
|-------|------|------|------|--------|------|--------|------|
| S_1   | 1    | 1    | 1    | 1      | 1    | 1      | 0    |
| S_1   | 1    | 1    | -1   | -1     | 0    | 0      | 0    |
|       |      |      |      |        |      |        |      |
| n = 1 | 0^+  | 1^-  | 0^-  | 1^-    | 0^+  | 1^-    | 0^+  |
| L = 0 | 2980 | 3097 | 3916 | 4033   | 3914 | 3797   | 3989 | 3872 |
| \(\eta_c(1S)\) | J/\(\psi(1S)\) | Y(3940) | Y(4008) | X(3915) | X(3872) |
|       |      |      |      |        |      |        |      |
|       | 1^3P_0 | 1^3P(x) | 1^3P(x) | 1^3P(x) | 1^3P(x) | 1^3P(x) | 1^3P(x) |
|       | 0^+   | 0^-   | 0^-   | 0^-    | 0^-   | 0^-    |
|       | 3415  | 4351  | 4144  | 4248   |
| Z_0(1P) | | X(4350) | Y(4140) |
| n = 1 | 1^-   | 1^+   | 1^-   | 1^-    | 1^-   | 1^-    |
| L = 1 | 3526 | 3511 | 4461 | 4447   | 4255 | 4240   | 4359 | 4344 |
| \(h_c(1P)\) | \(\chi_c(1P)\) | Y(4260) | Y(4360) |
|       | 1^3P_2 | 1^3P(x) | 1^3P(x) | 1^3P(x) | 1^3P(x) | 1^3P(x) | 1^3P(x) |
|       | 2^+   | 2^+   | 2^-   | 2^-    |
|       | 3556  | 4492  | 4286  | 4390   |
| Z_2(1P) | “Y(4280)” |
| n = 2 | 0^+   | 1^-   | 0^-   | 1^-    | 0^+   | 1^-    | 0^+   |
| L = 0 | 3637 | 3686 | 4573 | 4622   | 4469 | 4420   | 4544 | 4495 |
| \(\eta_c(2S)\) | \(\psi(2S)\) | Y(4660) |

^* \(J^P\)-exotic states.

\(\eta_c \rightarrow J/\psi\gamma\), suggests that a \(q\bar{q}\) pair in the \(\eta_c\) is in a spin triplet [10] and thus it might be favorable to being assigned to \(1^3P_1(x^B)\). Then the three states \(Y(4140)\), “\(\eta_c(\sim 4300)\)” , “\(Y(4280)\)” would form the \(1^3P_1(x^B)(0^+, 1^-, 2^+)\) multiplet.

In the present study another interesting prediction is obtained that there exists a spin-singlet partner, \(\eta_c(2^1S_0(x^E))\), of the \(Y(4660)\), assigned above to \(\psi(2^3S_1(x^F))\), where the mass splitting between them is the same as that of the conventional charmonia, \(\psi(2S)\) and \(\eta_c(2S)\), being \(\approx 49\) MeV.\(^3\)

\(^3\) The same prediction was given in a completely different approach of the hadronic molecule model with heavy-quark spin symmetry [11].
I have investigated the mass spectrum of 1S, 1P, and 2S states for the c\bar{c} system in the $\bar{U}(12)_{SF} \times O(3,1)_L$-classification scheme and have seen that the observed spectrum of the neutral charmonium-like $XY$ states is described well. The resultant assignments suggest that the $J^{PC}$ quantum numbers of the $X(3915)$ and $X(4350)$ are $0^{++}$, while both the $Y(3940)$ and $Y(4140)$ have $0^{-+}$, and there exists a charmonium-like state with a mass of $\approx 4280$ MeV and $J^{PC} = 2^{-+}$ which seems to correspond to the hypothetical state “$Y(4280)^{+}$”. It is also expected that there exist some $J^{PC}$-exotic states, such as $1^{-+}(\eta_{c1})$ and $0^{++}(h_{c0})$ states around 4.3 GeV and 4.5 GeV, respectively. These results will be tested in the coming experiment.

In a future work, to put the present interpretation for the neutral charmonium-like $XY$ states on a firm basis, it is necessary to explain the decay properties of these $XY$ states that they do not decay dominantly into $D^{(*)} \bar{D}^{(*)}$, but have large hadronic transition rates to lower conventional charmonia.

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