Non-linear electrodynamics and the variation of the fine structure constant

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ABSTRACT

It has been claimed that during the late-time history of our Universe, the fine structure constant of electromagnetism, $\alpha$, has been increasing. The conclusion is achieved after looking at the separation between lines of ions like CIV, MgII, SiII, FeII, among others in the absorption spectra of very distant quasars, and comparing them with their counterparts obtained in the laboratory.

However, in the meantime, other teams have claimed either a null result or a decreasing $\alpha$ with respect to the cosmic time. Also, the current precision of laboratory tests does not allow one to either confirm or reject any of these astronomical observations. Here, we suggest that as photons are the sidereal messengers, a non-linear electrodynamics (NLED) description of the interaction of photons with the weak local background magnetic fields of a gas cloud absorber around the emitting quasar can reconcile the Chand et al. and Levshakov et al. findings with the negative variation found by Murphy et al. and Webb et al., and also to find a bridge with the positive variation argued more recently by Levshakov et al. We also show that NLED photon propagation in a vacuum permeated by a background magnetic field presents a full agreement with constraints from Oklo natural reactor data. Finally, we show that NLED may render a null result only in a narrow range of the local background magnetic field which should be the case of both the claims by Chand et al. and by Srianand et al.

Key words: atomic data – magnetic fields – methods: analytical – techniques: spectroscopic – galaxies: magnetic fields – quasars: absorption lines.

1 INTRODUCTION

The gauge coupling constants $\alpha_G$, $\alpha$, $\alpha_W$ and so on, of theories of fundamental interactions as gravity, electroweak, strong and Grand Unified Theories are by definition constant; that is, they are not supposed to depend on time nor on the spatial location. The cosmological variation of fundamental physical constants was first addressed by Dirac in his theory on the Large Number Hypothesis (Dirac 1937). The impact of this study on fundamental research has extended from multidimensional to the modern Kaluza–Klein and brane-world theories. However, if one conceives, for instance, the variation in time of a fundamental parameter as the gravitational constant, then one should be prepared to admit a modification of the standard model of electroweak and strong interactions. Pathways into those directions have been pursued and their predictions have been demanding more precision tests of such novel theories. In the perspective of such potential changes, physicists and astronomers have devised skills over more than six decades, methods and techniques in an attempt to detect and measure any putative variation of these fundamental constants.

Each formulation of any of the basic physical theories quoted earlier has its own way to make it evident the possible space–time or simply cosmological evolution of those gauge couplings. Usual tests of the validity of the ‘constant’ hypothesis use either experimental setups in laboratories as well as local geophysical data or deep-space astronomical observations, all of them having in mind to at least put tight bounds on their eventual time evolution. For instance, a laboratory setup may allow to compare the rate between atomic clocks built on different composition ($A$, $Z$). Meanwhile, the analysis of absorption spectra from high-redshift quasars is at the kernel of most astrophysical techniques to ask about any cosmological evolution of $\alpha$. In all these cases, the main physical reason for this search is that the splitting ratio of absorption lines depends on $\alpha$ following the Bethe–Weizäcker relation;

$$\frac{\Delta \lambda}{\lambda} \approx (Z\alpha)^2 + O(\alpha)^4.$$  (1)
where $\Delta \lambda = \lambda_1 - \lambda_2$. This is the difference of two spectral lines of wavelength $\lambda_1$ and $\lambda_2$. This splitting ratio implies that the variation of $\alpha$ at redshift $z$ compared to its local (laboratory) value in first approximation reads

$$\frac{\Delta \alpha}{\alpha} = \frac{1}{2} \frac{\Delta (\lambda_1/\alpha) - 1}{\Delta \lambda/\alpha_0}.$$ (2)

Although most of the above-quoted experimental searches produced null results (Bahcall, Steinhardt & Schlegel 2004; Quast, Reimers & Levshakov 2004; Srianand et al. 2004; Tzanavaris et al. 2005), evidence for the time variation of the fine structure constant reported from high-redshift quasar absorption systems (Webb et al. 1999, 2001; Murphy et al. 2001a, b; Murphy, Webb & Flambaum 2003; Levshakov et al. 2006a, 2007) has recently appeared. In similar lines, geophysical methods that use both the natural nuclear reactor that was active $1.8 \times 10^9$ yr ago in Oklo Uranium Mine in Gabon (West Africa),\footnote{This phenomenon was discovered by the French atomic energy commission (Commissariat à l’Energie Atomique) in 1972, and consists in an abnormally low relative concentration of the isotopes of Samarium – the isotopic ratio of the $^{149}$Sm and $^{150}$Sm isotopes. The standard isotopic ratio is $0.9$, whereas the actual measurement gave $0.02$. It is as if$^{149}$Sm were depleted due to neutron capture of thermal neutrons when the natural reactor was active. To understand this anomaly in the isotopic ratio one can invoke a different value of the capture resonance energy at the time of the reaction. This threshold energy depends on $\alpha$, as already shown by the Bethe–Weizäcker formula, quoted above in equation (1).} and the analysis of natural long-lived $\beta$ decayers in geological minerals and meteorites showed no variation (Olive et al. 2004). Let us emphasize that the latter data as well as the Oklo one are out of the cosmological context.

Bounds on the variation of $\alpha$ in the early universe can be obtained using data from the cosmic microwave background (CMB) radiation and from the abundances of light elements. Although these bounds are not as stringent as the mentioned above, they are important because they refer to a different cosmological era (see Mosquera et al. 2008) for an account of bounds on $\alpha$ from the early universe based on abundances of $D$, $^4$He, $^3$He and $^7$Li, and also see the recent review by García-Berro, Isern & Kubinsky 2007).

Just very recently, a very innovative experimental proposal in which the effects of variation in the value of the fine structure constant at extremely high redshifts, $z_{\text{rec}} \lesssim z \lesssim 30$, on the absorption of the CMB radiation at the 21 cm of the hyperfine transition of neutral atomic hydrogen have been investigated by Khatri and Wandelt (2007). It is shown there that the 21 cm signal is very sensitive to variations of $\alpha$, so that a change in the value of $\alpha$ by 1 per cent changes the mean brightness temperature decrement of the CMB due to 21 cm absorption by $\gtrsim 5$ per cent over the redshift range $z < 50$. That paper also demonstrates that the signal from the 21 cm absorption by neutral hydrogen between redshifts of 30 and 200 is extremely sensitive to the value of $\alpha$ during that epoch. By assuming that the technological hurdles for detecting this absorption signal in the long wavelength radio band can be overcome, the change of $\alpha$ from the era of redshift $\sim 100$ can be constrained to 1 part in $10^6$ (Wandelt, private communication).

Our objective in this paper is to call to the attention of workers in the field that NLED can help to understand the current controversial observations indicating a putative variation of alpha. In a future communication, we plan to address each individual source that has been observed by all of the research groups cited in this paper to confront each of them with the predictions of the NLED theory brought into the discussion here.

\footnote{This phenomenon was discovered by the French atomic energy commission (Commissariat à l’Energie Atomique) in 1972, and consists in an abnormally low relative concentration of the isotopes of Samarium – the isotopic ratio of the $^{149}$Sm and $^{150}$Sm isotopes. The standard isotopic ratio is $0.9$, whereas the actual measurement gave $0.02$. It is as if$^{149}$Sm were depleted due to neutron capture of thermal neutrons when the natural reactor was active. To understand this anomaly in the isotopic ratio one can invoke a different value of the capture resonance energy at the time of the reaction. This threshold energy depends on $\alpha$, as already shown by the Bethe–Weizäcker formula, quoted above in equation (1).}

2 QUASAR-ABSORPTION SPECTRA AND THE CONTROVERSY ON VARIATION OF $\alpha$

Since long ago astronomers realized that ideal laboratories to search for a potential signature of cosmic evolution of the fine structure constant are high-redshift quasars whose spectra present absorption resonance lines of several alkaline ions like $\text{CIV}$, $\text{MgII}$, $\text{FeII}$, including the $\text{SiIV}$ doublet system that is obtained from a source at redshift between $2.5 \leq z \leq 3.33$ (see Mosquera et al. 2008, and references therein). Also $\text{OIII}$ emission lines have been observed (Bahcall et al. 2004). Other authors used transitions between species with far different atomic masses from which they obtained a single data consistent with time varying of $\alpha$ at redshift $0.5 \lesssim z \lesssim 3.5$ (Webb et al. 1999; Murphy et al. 2001a, b, 2003). Notwithstanding, no variation was found from other recent independent analysis of similar observations (Quast et al. 2004; Srianand et al. 2004; Grupe, Pradhan & Frank 2005). Meanwhile, Levshakov et al. (2005) introduced a different procedure in which a high-resolution spectrograph is used to observe pairs of $\text{FeII}$ lines during individual exposures. No variation of $\alpha$ for redshifts $z \sim 1.15$ and $z \sim 1.839$ was found. Very recently, however, a reanalysis of the spectrum of the quasar Q 1101-264 exhibits signatures of variability with a confidence level of $1\sigma$ (Levshakov et al. 2007).

From another side, techniques that compare molecular and radio lines provided more stringent constraints (Murphy et al. 2003). Also bounds on cosmological evolution of $\alpha$ at redshift $z = 0.2467$ obtained from satellite observations at wavelength $\lambda = 18$ cm for the oxyhydril (OH) conjugate lines have been reported (Darling 2004). Moreover, Kanekar et al. (2005) compared the redshifts of the $\text{H}i$ and OH main absorption lines of the different components in an absorber source at $z = 0.765$, and also of the lens toward the object B0218+357 at $z = 0.685$, to place stringent constraints on changes in the parameter $F$ defined as $F = g_0(\alpha^2/\mu)^{1/3}$. Finally, a full analysis of the bounds on cosmological variation of $\alpha$ obtained by comparing the optical and radio redshifts is presented by Mosquera et al. (2008) based on data from Wolfe et al. (1976), Spinrad & Mackee (1979), Cowie & Songaila (1995) and Tzanavaris et al. (2007).

In summary, the controversy is still lively, pushed ahead since the moment in which Webb et al. (2001), Murphy et al. (2001a, b, c, d) and Dzuba et al. (2001) claimed a negative variation of the fine structure constant as a function of the redshift. In the meantime, Levshakov et al. (2006a) have found a positive variation of $\alpha$ between the two redshifts quoted above. The null results of Chand et al. (2004) and Srianand et al. (2004) could appear as an intermediate case making the transition between the former two but it has been challenged recently by Murphy, Webb & Flambaum (2007), who have pointed out a large number of errors in the statistical analysis performed by Chand et al. (2004). Indeed, Murphy et al. (2007) show that when these are corrected for one finds $\Delta \alpha/\alpha = -(0.64 \pm 0.36) \times 10^{-5}$, which is a six-fold larger uncertainty than that quoted by Chand et al. (2004). In passing, such a value of $\Delta \alpha/\alpha$ is consistent with the value found by Webb et al. (2001).

In view of those discrepant data one may wonder whether some basic systematic effects, not yet accounted for, may be involved in the aforementioned observations. As a possible pathway to unveiling these effects let us recall that our knowledge in astrophysics and cosmology comes mostly from the information gathered from...
electromagnetic (EM) waves (as far as gravitational waves start to be detected in the near future). In this perspective, it has been proved that electrodynamics in a vacuum is subject to non-linear effects (Burke et al. 1997). Hence, it is legitimate to address the question of the possible variation of the fine structure constant, α, within the framework of NLED. It is interesting to notice that based on Maxwell EM theory, Murphy et al. (2001c) considered large magnetic fields as a potential cause of systematic errors in their measurements of Δalp (see Murphy et al. 2001c, Section 2.6). They concluded that the intracluster magnetic field strengths are nine orders of magnitude below the strength required to cause substantial effects. As we show below, the latter conclusion can be reversed by considering the NLED Lagrangian density proposed by Novello, Pérez Bergliaffa & Salim (2004), which was introduced as a realization of dark energy to provide an explanation of the Type Ia supernovae observation-inspired interpretation of a late-time acceleration of the expansion of the universe.

3 NLED: PHOTON VACUUM NON-LINEAR INTERACTION AND THE VARIATION OF THE FINE STRUCTURE CONSTANT

A non-linear Lagrangian is one for which the second or even higher derivatives of the Lagrangian with respect to the Maxwell invariant \( F = F^\mu_\nu F^\nu_\mu \) is not null! That is, \( L_{NF} \neq 0 \), where \( L_F = \partial L / \partial F \) and \( L_{NF} = \partial L / \partial F^2 \). This is clearly not the case for Maxwell’s theory. Therefore, one way to guide ourselves to build a NLED Lagrangian able to account for the unavoidable non-linear interaction of photons from distant quasars with the intergalactic background EM field is to realize that those background fields are extremely weak, so that in order to have any practical influence on the photon dynamics the Lagrangian should depend on the \( F \) field in a non-trivial fashion. This is our approach in what follows. We first start with the dynamics of the photon non-linear propagation in a vacuum. Then we will focus on the structure of that specific Lagrangian, and its dynamical consequences.

3.1 Photon dynamics in NLED

In this Section, we investigate the effects of non-linearities in the evolution of EM waves in a vacuum, where the waves are described onwards as the surface of discontinuity of the EM field. Extremizing the Lagrangian \( L(F) \), with \( F(A_\mu) \), with respect to the potentials \( A_\mu \) yields the following field equation (Plebanski 1970)

\[
\nabla_\nu (L_F F^\nu_\mu) = 0, 
\]

where \( \nabla_\nu \) defines the covariant derivative. Besides this, we have the cyclic identity:

\[
\nabla_\nu F^{\mu\nu} = 0 \Leftrightarrow F^\nu_{\mu\nu} + F^{\mu\nu}_\nu + F^{\nu}_\nu = 0. 
\]

Taking the discontinuities of the field equation we get

\[
L_F f_{\alpha\beta} k^\gamma + 2L_F F^\mu_\nu f_{\alpha\beta} F^{\nu\mu} k_\nu = 0, 
\]

which together with the discontinuity of the Bianchi identity yields:

\[
f_{\alpha\beta} k_\nu + f_{\gamma \alpha} k_\beta + f_{\beta \gamma} k_\alpha = 0. 
\]

In order to obtain a scalar relation, we contract this equation with \( k^\nu F^\mu_\nu \), resulting

\[
(F^\mu_\nu f_{\alpha\beta} \delta^{\nu\rho} + 2 F^{\mu\alpha}_\nu f_{\beta\gamma} k_\nu) k_\mu = 0. 
\]

It is straightforward to see that here we find two distinct solutions: (i) when \( F^\mu_\nu f_{\alpha\beta} = 0 \), case in which such mode propagates along standard null geodesics, and (ii) when \( F^\mu_\nu f_{\alpha\beta} = \chi \). In the case in which \( \chi \) does not vanish we obtain from equations (5) and (7), the propagation equation for the field discontinuities being given by (Novello et al. 2004)

\[
\left( g^{\mu\nu} - \frac{4 L_{FF}}{L_F} F^\mu_\nu F^\nu_\mu \right) k_\mu k_\nu = 0. 
\]

This equation proves that photons propagate following a geodesic that is not that one of the background space–time described by \( g^{\mu\nu} \). Rather, they follow the effective metric given by equation (8).

If one now takes the \( x^0 \) derivative of this expression, we can easily obtain (Mosquera Cuesta, de Freitas Pucheco & Salim 2006; Mosquera Cuesta & Salim 2004a,b)

\[
k_{\mu\nu} k^\nu = \frac{4 L_{FF}}{L_F} F^\mu_\nu F^\nu_\mu k_\mu k_\nu. 
\]

This expression shows that the non-linear Lagrangian introduces a term acting as a force that accelerates the photon along its path. It is therefore essential to investigate what are the effects of this peculiar prediction. The occurrence of this phenomenon over cosmological distance scales may have a non-negligible effect on the physical properties that are ascribed to a given source from its astronomical observables. One example of this is the cosmological redshift (Mosquera Cuesta, Salim & Novello 2007). Since the photon ought to travel very long distances from cosmic sources until be detected on Earth, then its interaction with the background intergalactic EM fields should modify the putative (nominal) value of the redshift, or equivalently, the actual luminosity distance, associated to its emitting source, compared to the proper distance computed in a standard fashion in the context of a particular cosmology. In similar lines, we show next that in the case of EM radiation coming from far distance radio-galaxies and quasars the interaction of the photon with local intergalactic background EM fields may significantly modify the actual position of a particular absorption line, from which a potential variation of \( \alpha \) can be estimated. In this way, an observer on Earth is prone to say that effectively \( \alpha \) has changed for this particular observation. But, how exactly does it change?

3.2 NLED and cosmological variation of \( \alpha \)

In order to investigate whether the photon non-linear interaction with background fields over large distances does affect the position of absorption lines, next we present the Lagrangian formulation of this NLED theory.

3.2.1 A motivation to look for NLED effects in cosmology

In Novello et al. (2004) several general properties of NLED in cosmology were reviewed by assuming that the action for the EM field is that of Maxwell with an extra term, namely:\n
\[\text{Notice that this Lagrangian is gauge invariant, and that hence charge conservation is guaranteed in this theory.}\]
\[
S = \int \sqrt{-g} \left( -\frac{F}{4} + \frac{\gamma}{F} \right) d^4x,
\]

(10)

where \( F \equiv F_{\mu\nu} F^{\mu\nu} \).

Physical motivations for bringing in this theory have been provided in Novello et al. (2004). Besides, an equally unavoidable motivation comes from the introduction of both the Heisenberg–Euler and Born–Infeld NLED, which are valid in the regime of extremely high-magnetic field strengths. Those studies constituted the first demonstration that a Quantum Electrodynamics (QED) description of the atomic world would be necessary, at least; at the one-loop level to overcome the ultraviolet catastrophe. Both theories have been extensively investigated in the literature (see for instance Mosquera Cuesta & Salim 2004a,b; Mosquera Cuesta et al. 2006), and the long list of references therein). Since in nature not only such very strong magnetic fields exist, then it appears to be promising to investigate also those super weak field frontiers that should outcome over intergalactic distances, that is over the space encircling the absorption line emitting galaxy.

Regarding this phenomenological Lagrangian in equation (10), at first, one notices that for high values of the field \( F \), the dynamics resembles Maxwell’s one except for small corrections associate to the parameter \( \gamma \), while at low \( B\)-field strengths, that is \( F \to 0 \), it is the \( 1/F \) term that dominates. Clearly, this last term should drastically affect the photon–\( B \) field interaction over the intergalactic space. In this respect, the consistency of this theory with observations, including the recovery of the well-established Coulomb law, was shown by Novello et al. (2004) using the cosmic microwave radiation bound, and also after discussing the anomaly in the dynamics of Pioneer 10 spacecraft by Mbelek et al. (2007). Both analysis provide small enough values for the coupling constant \( \gamma \).

Therefore, the EM field described by equation (10) can be taken as source in Einstein equations, to obtain a toy model for the evolution of the universe which displays accelerate expansion caused when the non-linear EM term takes over the term describing other matter fields. This NLED theory yields ordinary radiation plus a dark energy component with \( w < -1 \) (phantom-like dynamics). Introducing the notation, the EM field can act as a source for the FRW model if \( \langle E_i \rangle|_\nu = 0 \), \( \langle B_i \rangle|_\nu = 0 \), \( \langle E_iB_j \rangle|_\nu = 0 \), \( \langle E_iE_j \rangle|_\nu = -\frac{1}{2}E^2g_{ij} \), and \( \langle B_iB_j \rangle|_\nu = -\frac{1}{2}B^2g_{ij} \). When these conditions are fulfilled, a general non-linear Lagrangian \( L(F) \) yields the energy-momentum tensor \( (L = dL/dF, \ L_{FF} = d^2L/dF^2)^9 \)

\[
\langle T_{\mu\nu} \rangle|_\nu = (\rho + p)v_\mu v_\nu - p g_{\mu\nu},
\]

\[
\rho = -L - 4E^2L_F, \quad p = L + 4\left( E^2 - 2B^2 \right)L_F.
\]

Hence, when there is only a magnetic field, the fluid can be thought of as composed of ordinary radiation with \( p_1 = \frac{1}{3} \rho_1 \) and also of another fluid with EOS \( p_2 = -\frac{4}{3} \rho_2 \). It is precisely this component with negative pressure that may drive accelerate expansion throughout the Friedmann equations.

It is important to notice that this theory has been successfully applied to both the studies of the late-time cosmic acceleration inferred from Type Ia supernovae observations (Novello et al. 2004), and also to explain the non-trivial phenomenon known as the ‘Anomaly of the Pioneer Spacecraft’ (Mbelek et al. 2006), a problem that has been around over nearly two decades. In a separate communication (Mosquera Cuesta et al. 2007) we show how the cosmological redshift is modified in virtue of this unaccounted effect from non-linear propagation of EM radiation from cosmic sources. The consequences of this effect appear to be dramatic for current astro-nomical investigations. Finally, we also advance that the Compton Effect itself is radically modified by this unaccounted NLED effect Mbelek & Mosquera Cuesta (in preparation).

Turning back to the general NLED Lagrangian \( L = L(F) \), one notices that in the presence of sources the field equation (3) becomes [see Novello et al. 2004, equation (18)]

\[
\nabla_i(\epsilon_0 L_F F^{\mu\nu} - \frac{\epsilon}{c^2} F^{\mu\nu}) = J^{\mu}/c^2,
\]

(12)

where \( F = F_{\mu\nu}F^{\mu\nu} = 2(B^2c^2 - E^2) \). Hence, the definition of the NLED effective permittivity in a vacuum reads

\[
\epsilon_{0\text{NLED}} = -4 \epsilon_0 L_F,
\]

(13)

and consequently the definition of the NLED effective fine structure constant

\[
\alpha_{\text{NLED}} = \alpha(-4 L_F)^{-1},
\]

(14)

where \( \alpha = e^2/(4\pi \epsilon_0 h c) \) with \( e \) the electron charge, \( h \) the Planck constant and \( c \) the speed of light. Once again, from this expression it becomes clear that Maxwell’s theory cannot explain any variation of alpha.

If one introduces the particular Lagrangian density\(^8\)

\[
L = -1/4 F + \gamma/F, \quad \text{where} \quad \gamma = -(Bic)^4
\]

(15)

**Footnotes:**

5 Also called the Rayleigh–Jeans catastrophe, it was a prediction of early 20th century classical physics that an ideal blackbody at thermal equilibrium will emit radiation with infinite power. The term ultraviolet catastrophe was first used in 1911 by Paul Ehrenfest, although the concept goes back to 1905 study of blackbody by Planck; the word ultraviolet refers to the fact that the problem appears in the short wavelength region of the EM spectrum. Since the first appearance of the term, it has also been used for other predictions of a similar nature, as in quantum electrodynamics and such cases as ultraviolet divergence in Quantum Field Theory (QFT).

6 That is why we did not address the description of the standard physical picture called for when one considers the EM interaction inside a hydrogen atom, for instance. At such a distance scale the electric fields produced by both the electron shell and the nucleus overrun even the Maxwell limit, so that one is forced to invoke Heisenberg–Euler or Born–Infeld NLED to more accurately figure out what is electrically going on at that level, rather than invoking this \( F + F^{-1} \) theory.

7 Due to the isotropy of the spatial sections of the Friedman–Robertson–Walker (FRW) model, an average procedure is needed if EM fields are to act as a source of gravity (Tolman and Ehrenfest 1930). Thus a volumetric spatial average of a quantity \( X \) at the time \( t \) by \( \langle X \rangle|_\nu \equiv \lim_{\nu \to 0} V \int X \sqrt{-g} \ D^4x \), where \( V = \int \sqrt{-g} \ D^4x \), and \( V_0 \) is a sufficiently large time-dependent volume. [Here the metric sign convention (+−−−) applies.]

8 Let us remark that since we are assuming that \( \langle B_i \rangle|_\nu = 0 \), the background magnetic fields induce no directional effect in the sky, in accordance with the symmetries of the standard cosmological model.

9 Under the same assumptions, the EM field associate to Maxwell Lagrangian generates the stress-energy tensor defined by equation (11) but now \( \rho = 3p = \frac{1}{2}(E^2 + B^2) \).

10 In order to prevent the Lagrangian under consideration from being singular for \( F = 0 \) (case of pure radiation) and for some value in the case of the electric field solely, one may consider a more general expression like \( L = -\frac{1}{4} F + \sqrt{F^{\mu\nu}B_{\mu\nu}} \), which is the sum of the Maxwell Lagrangian and the inverse of the Born-Infeld Lagrangian, and the Latter is known to be non-singular and stable. Thus, one recovers the Lagrangian \( L = -\frac{1}{4} F + \frac{1}{2} F^{\mu\nu}B_{\mu\nu} \) for \( |F| \gg \beta > 0 \). Notice that the limiting case \( F \to 0(|F|\ll \beta) \) is equivalent to Maxwell theory plus a cosmological constant term \( \Lambda = \gamma/2\beta \); see note (27) in Mbelek et al. (2007).

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(being γ a universal constant, and B1 a critical non-zero limiting value for the magnetic field strength) equation (15) can be rewritten as [see Novello et al. 2004, equations (12)]

\[ \left[ 1 + (4 \gamma F^2/F^{\mu_s}) \right] = 0. \]  

(16)

As in previous works (Novello et al. 2004; Mbelek et al. 2007), we consider hereafter the case with an average magnetic field and with a null mean electric field. Therefore, equation (14) yields

\[ \alpha_{\text{NLED}} = \alpha \left[ 1 - (B_1/B)^4 \right]^{-1}. \]  

(17)

Once again, were the NLED effect null, the B1 field would be zero. Clearly, since \( L_F < 0 \) (positive energy condition) and hence \( B > B_1 \), equation (17) leads to a positive departure of \( \alpha_{\text{NLED}} \) from the unperturbed fine structure constant \( \alpha \).

### 4 ENCOMPASSING BOTH LOCAL NLED AND COSMOLOGICAL EFFECTS ON \( \alpha \) MEASUREMENTS

Unification theories which are generalization of general relativity (GGR) like superstring theory, scalar-tensor or multidimensional gravitational theories predict the variation of the effective fine structure constant \( \alpha_{\text{GGR}} \) with respect to the redshift \( z \) (or the cosmic time) in the cosmological context (García-Berro et al. 2007; Mbelek & Lachieze-Rey 2003; Gardner 2003; Sandvik, Barrow & Magueijo 2002). Hence, the observed fine structure constant, \( \alpha_{\text{obs}} \), might be understood as resulting from a combination of both local NLED and cosmological GGR effects, so that

\[ \alpha_{\text{obs}}(z) = \alpha_{\text{GGR}}(z) \left[ 1 - (B_1/B)^4 \right]^{-1}. \]  

(18)

We show below that this approach can help to reconcile the negative variation claimed by Webb et al. (2001) and Murphy et al. (2001a,b,c), with the recent positive variation found by Levshakov et al. (2007) which amounts

\[ \langle \Delta \alpha_{\text{obs}}/\alpha \rangle = (0.543 \pm 0.252) \times 10^{-5}, \]  

(19)

between the redshifts \( z_1 = 1.15 \) and \( z_2 = 1.84 \) (Levshakov et al. 2006a). Indeed, on Earth, at present \( (z = 0) \), in the best laboratory conditions the residual magnetic fields (including the geomagnetic field) are such that

\[ B_{\text{lab}} \gg B_1, \text{ say, } B_{\text{lab}} \gg 1 \mu G \]  

(20)

so that

\[ \alpha_{\text{lab}} = \alpha. \]  

(21)

Thus, by setting

\[ \Delta \alpha_{\text{GGR}}(z) = \alpha_{\text{GGR}}(z) - \alpha, \text{ and } \Delta \alpha_{\text{obs}}(z) = \alpha_{\text{obs}}(z) - \alpha, \]  

(22)

one obtains

\[ \frac{\Delta \alpha_{\text{obs}}(z)}{\alpha} = \frac{(\Delta \alpha_{\text{GGR}}(z)/\alpha) + (B_1/B)^4}{1 - (B_1/B)^4}. \]  

(23)

Averaging over the redshift range of a given sample of absorbers, equation (23) above yields

\[ \langle \Delta \alpha_{\text{obs}}/\alpha \rangle = \frac{(\Delta \alpha_{\text{GGR}}(z)/\alpha) + (B_1/B)^4}{1 - (B_1/B)^4}. \]  

(24)

We will use the relation \( B = 6 \mu \text{H}_1 \text{cm}^{-3} \times 2.2 \mu G \) proposed by J. P. Vallée for the magnetic field strength within intergalactic gas clouds (Vallée 2004). This way one finds

\[ 0.4 \mu G \leq B < 1.5 \mu G \]  

(25)

for the sample of Murphy et al. (2001c).11 This result implies that \( B \gg B_1 \), and therefore, it also means that the NLED contribution is quite negligible for this sample given the present estimate \( B_1 = (0.008 \pm 0.002) \mu G \) (Mbelek et al. 2006). Thus, we conclude from the measurements of Webb et al. (2001) and Murphy et al. (2001a,b,c,d) that

\[ \frac{\Delta \alpha_{\text{GGR}}}{\alpha} \leq (-0.543 \pm 0.116) \times 10^{-5} < 0 \]  

(26)

over the redshift range \( 0.2 < z < 3.7 \) (Murphy et al. 2003). Therefore, equation (24) can be rewritten as

\[ \langle \Delta \alpha_{\text{obs}}/\alpha \rangle = \frac{(B_1/B)^4 - 1}{(B/B_1)^4 - 1}, \]  

(27)

where we have set \( B_1 = B_1 (\Delta \alpha_{\text{GGR}}/\alpha)^{-1} = (0.165 \pm 0.033) \mu G \).

Thus, comparing the magnitude \( B \) of the mean magnetic field within an intergalactic gas cloud absorber to \( B_1 \), one of the three following conclusions may be reached:

(i) no variation of \( \alpha_{\text{obs}} \) should be observed from any sample of absorbing intergalactic gas cloud such that \( B \leq B_1 \). This could be the case of Chand et al. (2004), Srianand et al. (2004) samples, but see Murphy et al. (2007) quoted above and the answer of Srianand et al. (2007),

(ii) a negative variation of \( \alpha_{\text{obs}} \) should be observed from any sample of absorbing intergalactic gas cloud such that \( B > B_1 \) [case of Murphy et al. (2001a,b,c,d), Webb et al. (2001) samples],

(iii) a positive variation of \( \alpha_{\text{obs}} \) should be observed from any sample of absorbing intergalactic gas cloud such that \( B < B_1 \) [case of Levshakov et al. (2007) sample].

Meanwhile, notice that the H\(_1\) column densities are provided neither by Srianand et al. (2004, 2007), Chand et al. (2004) nor by Levshakov et al. (2007) for their studies on the cosmological variation of the fine structure constant. However, Srianand et al. (2004) pointed out that they have avoided sub-Damped Lyman Alpha systems, that is, \( N(\text{H}^+) \geq 10^{19} \text{ cm}^{-2} \). Nevertheless, Boksenberg & Snijders (1981) derived limits to the neutral hydrogen column density of

\[ 5 \times 10^{17} \text{ cm}^2 < N(\text{H}) \leq 210^{19} \text{ cm}^2, \]  

(28)

from their observations of the \( z_{\text{obs}} = 1.8387 \) richest absorption system identified in the optical region towards Q 1101–264.

Hence, on account that \( N(\text{H}) \sim N(\text{H})/1000, \)

\[ \frac{1}{2} \times 10^{15} \text{ cm}^2 < N(\text{H}) \leq \frac{1}{5} \times 10^{17} \text{ cm}^2, \]  

(29)

which implies

\[ 0.069 < \frac{B}{\mu G} < 0.144 \]  

(30)

in accordance with the magnitude of the magnetic field

\[ 0.075 < \frac{B}{\mu G} < 0.227 \text{ and } 1.110^{15} < \frac{N(\text{H})}{\text{cm}^{-2}} < 2.610^{17} \]  

(31)

derived from equation (27) and the new measurement of \( \langle \Delta \alpha_{\text{obs}}/\alpha \rangle \) by Levshakov et al. (2007). We have estimated the distances of the

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11 see Murphy et al. (2001c) relations

\[ N(\text{H}) \sim \frac{N(\text{H})}{1000} \text{ and } -3 < \log_{10} \left( \frac{N(\text{H})}{\text{cm}^{-2}} \right) < 0. \]

in Section 4.1.2 and Table-1 in Section 4.1.3.
gas cloud absorbers from the Hubble law by using $H_0 = 68 \text{ km s}^{-1} \text{ Mpc}^{-1}$. We emphasize that the estimates of $B$ in equations (25) and (30) are consistent with the available astrophysical data on the magnetic field strength within intergalactic gas clouds.

5 CONCLUSION

By using a non-linear electrodynamics theory in which the effective Lagrangian is the first order approximation (after Maxwell’s term) of a polynomial series of inverse powers of the EM invariant quantity $F$, we have presented a consistent explanation of the controversial results regarding a hypothetical variation of the fine structure constant $\alpha$ since the recombination era.

In these lines, one can state that the large set of observations of quasar absorption systems are mapping the structure of the intergalactic magnetic field in several directions between the Earth and absorbers in the sky, in addition to the expansion of the universe as a whole. In other words, the fact that the ballpark of the observations seem to be controversial could be interpreted as an indication that the strength of the local magnetic field in each of the observed systems is most likely different from each other. Hence, any conclusive statement on the actual cosmological evolution of the fine structure constant $\alpha$ rests on a better understanding of the intergalactic magnetic field strength and structure over the whole sky, and also on more accurate measurements of the relative magnitude of the fine splitting between resonance absorption lines from far away quasars.

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