ON THE INTERPOLATION PROBLEM FOR THE POISSON EQUATION ON COLLOCATED MESHES

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Finite-volume collocated discretizations on unstructured meshes is the solution adopted for most of the general-purpose CFD codes such as ANSYS-FLUENT, OpenFOAM, etc. Despite the intrinsic errors due to the improper pressure gradient formulation, this approach is usually preferred over a staggered one due to its simple form. In this context, a fully-conservative discretization method for general unstructured grids was proposed in [1]: it exactly preserves the symmetries of the underlying differential operators on a collocated mesh. Likewise other collocated codes, to suppress the well-known checkerboard problem the Poisson equation is solved using a compact stencil. Using the same notation than in [1], this reads

\[ L p_c = M u^p_c \]  

where \( L = -\Omega^{-1} M^T \) is the Laplacian operator, \( p_c \) is the cell-centered pressure field, \( u^p_c \) is a face-normal velocity and \( \Omega_s \) is a diagonal matrix that contains the staggered control volumes. For staggered velocity fields, the projection onto a divergence-free space is a well-posed problem. This is not the case for collocated velocity fields. Namely, cell-centered velocity field, \( u^p_c \), needs to be interpolated to the faces, \( u^p_s = \Gamma_{c \to s} u^p_c \) using a cell-to-face interpolation, \( \Gamma_{s \to c} \). Then, the staggered gradient, \( G p_c = -\Omega^{-1} M^T \), of the pressure field obtained by solving Eq.(1) must be interpolated back to the cells. Namely, the overall procedure can be compactly written as follows

\[ u^{n+1}_c = (1 + \Omega^{-1} \Gamma^T c \to s M^T L^{-1} M \Gamma_{c \to s}) u^n_c. \]  

The new cell-centered velocity field will not be exactly incompressible, \( M \Gamma_{c \to s} u^{n+1}_c \approx 0_c \), and the overall procedure will inevitable introduce some artificial dissipation. Apart from this well-known drawbacks of using collocated formulations, instability issues may also appear for highly distorted meshes. Namely, let us consider that we recursively apply the pseudo-projection given in Eq.(2). Then, we obtain

\[ L p^{n+1}_c = M u^p_c + (L - L_c) p^0_c, \]  

where \( L_c \equiv -M \Gamma_{c \to s} \Omega_c^{-1} \Gamma^T c \to s M^T \) is the non-compact Laplacian operator. This can be viewed like a stationary iterative solver. The stability of this process will depend on the eigenvalues of \( L - L_c \), which subsequently depend on the interpolation operators. This will be carefully analysed and results for general unstructured grids will be presented.

REFERENCES

[1] F. X. Trias, O. Lehmkuhl, A. Oliva, C.D. Pérez-Segarra, and R.W.C.P. Verstappen. Symmetry-preserving discretization of Navier-Stokes equations on collocated unstructured meshes. Journal of Computational Physics, 258:246–267, 2014.