New microscopic and macroscopic variables in dusty plasmas

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Abstract. The kinetic theory of dusty plasmas, in which the absorption of electrons and ions by dust particles is the dominating interaction process, is derived from first principles. The charging process is shown to imply a considerable modification of properties in comparison with usual plasmas. Not only the electric charge, but also the mass, the angular momentum and the inner energy of the dust particles are new dynamic variables. Their influence on the kinetic behaviour of dusty plasmas is also considered.

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1. Introduction

For the understanding of dusty plasma experiments [1]–[4] and astrophysical phenomena [5] it is essential to derive a kinetic theory in which new dynamic variables of the dust particles play a dominant role. In [6]–[8] the charging process was fully taken into account in the derivation of kinetic equations from non-equilibrium statistical mechanics. The mass was supposed to be constant. This situation may be realistic when plasma particles are absorbed by dust particles and also neutral gas regeneration occurs due to recombination of ions with electrons on the surface of the grains. In order to describe this neutral gas regeneration correctly it seems necessary to take into account the grain momentum change due to evaporation of neutrals from the grain and to modify the kinetic equations accordingly. If the distribution of the regenerated neutrals at the instant of evaporation is isotropic, however, then this effect can be expected to leave the kinetic equations approximately unaltered.

In section 2 this situation is considered and the derivation of the kinetic equations is sketched for the case that absorption of plasma particles is the dominant process. Approximate stationary solutions are obtained which show the existence of different effective temperatures for the velocity and charge distributions.

In section 3 Coulomb collisions and the collisions of grains with neutral atoms or molecules are taken into account especially with regard to the influence on the effective temperatures. It is shown that the experimental observation of dusty crystals melting as a consequence of decreasing gas pressure [1, 2] is in qualitative agreement with the theory developed.

In section 4 we consider the mass of the grains as the essential new dynamic variable. In order to simplify the theory and to concentrate on the effect of changing mass we restrict ourselves to the situation where neutral gas particles collide with neutral grains and are absorbed by them. The problem of mass transfer is typical for the conditions of experiments on grain synthesis [4] and the formation of new materials [3]. The (non-stationary) distribution function of the grains is determined. One of the main findings is that the effective temperature of the grains is lower than the temperature of the gas.

In section 5 the inner energy of grains is introduced as a new dynamic variable. Also the angular momentum should be treated as a new dynamic variable; no attention, however, is paid to the angular momentum, since it is the main topic of [9]. The evolution of the average inner energy is connected with the evolution of the average mass. Section 6 contains some conclusions.

2. Statistical and kinetic theory of dusty plasmas including the charging process

Considering a plasma consisting of electrons, ions and dust particles, and restricting ourselves to charging collisions in which plasma particles are absorbed by grains, we formulate a generalized Klimontovich formalism and derive the corresponding BBGKY hierarchy and the kinetic equation for the grains [6, 7]. Assuming that the charge of the plasma particles is small compared with the charge of the dust particles, and also that their velocity changes in collisions are small, we arrive at a Fokker–Planck-type kinetic equation. We also take the limit \(m_s/m_g \ll 1\), where \(m_s\) is the mass of electrons (\(s = e\)) or ions (\(s = i\)) and \(m_g\) is the mass of a grain. Assuming the distribution functions \(f_s\) to be Maxwellian we arrive at a solution for \(f_g\). For typical values of plasma parameters with weak plasma coupling guaranteeing small charge fluctuations,
\( \frac{e^2}{(aT_e)} \ll 1 \), \( a \) being the grain radius, this solution can be written as

\[
f_g(v, q) = \frac{n_{0g}}{\sqrt{2\pi a T_{\text{eff}}^*}} e^{-\frac{(q-q_0)^2}{2a T_{\text{eff}}^*}} \left( \frac{m_g}{2\pi T_{\text{eff}}^*} \right)^{3/2} e^{-m_e v^2 / 2T_{\text{eff}}^*}. \quad (1)
\]

where

\[
T_{\text{eff}}^* = \frac{(1+Z_i)T_e}{2} \frac{t+z}{1 + t + z} \quad (2)
\]

\[
T_{\text{eff}} \simeq 2T_i \frac{t+z}{t-z} \quad (3)
\]

Here \( n_{0g} \) is the averaged number density of grains, \( z = e^2 Z_g/(a T_e) \) (\( Z_g \) is the charge number of a grain and \( T_e \) the electron temperature), \( t = T_i/(Z_i T_e) \), \( S_e^2 = T_e / m_e \), \( Z_g = q_0 / e \), \( Z_i = |e_i / e_e| \) and \( q_0 \) is the equilibrium grain charge satisfying

\[
I(q_0, 0) = 2\sqrt{2\pi a^2 e^2 n_i S_i} \left[ 1 + \frac{z}{t} - \left( \frac{m_i}{m_e} \right)^{1/2} \left( \frac{T_e}{T_i} \right)^{1/2} \frac{n_e}{Z_i n_i} e^{-z} \right] = 0. \quad (4)
\]

Distribution (1) is a Maxwellian velocity and a Gaussian grain charge distribution with the temperatures \( T_{\text{eff}} \) and \( T_{\text{eff}}^* \) respectively. For \( t+z \gg 1 \) and \( Z_i = 1 \) we have \( T_{\text{eff}}^* = T_e \). In general for \( Z_i = 1 \) \( T_{\text{eff}}^* \) is smaller than the electron temperature. The Gaussian charge distribution in (1) with the effective temperature \( T_{\text{eff}}^* \), given by (2), exactly coincides (for \( Z_i = 1 \)) with the charge distribution obtained in [10] on basis of a simplified, but well physically justified, model for the process of grain charging in ionized gases. We point out that there is no region of the plasma parameters, for the conditions under consideration, where the distribution on charges is non-Gaussian, as was found in [11]. This is one of the important differences between the kinetic theory proposed in [6]–[8] and the one proposed by the authors of [11, 12].

The behaviour of the temperature \( T_{\text{eff}} \) is even more surprising. In the case of neutral grains \( (z=0) \) it is equal to \( 2T_i \). The factor 2 is due to the inelastic nature of the charging collisions: a part of the kinetic energy of the ions is transformed into additional kinetic energy of the grains. This is different from the situation of conventional Brownian motion where the temperatures of the Brownian and the light bombarding particles are equal.

Equation (3) implies the possibility of very high effective temperatures when \( z \) approaches \( t \) or even negative temperatures for \( z > t \). Physically these phenomena are connected with the decrease of the friction coefficient with increasing grain charge. The reason for this is the fact that the charge-dependent part of the ionic charging cross section is larger for ions moving with smaller relative velocities (motion parallel to the grain motion), so that the difference between the anti-parallel and parallel ion fluxes decreases with increasing charge. At \( z = t \) the friction coefficient is zero.

It is clear that, in this situation, where the effective temperature approaches infinity or becomes negative, the approximation of dominant charging collisions is no longer valid in the suggested form, i.e. Coulomb collisions and collisions with neutrals should be taken into account. Furthermore, as was recently shown [8], the friction and diffusion coefficients for charging collisions have to be calculated as functions of the grain’s velocity, which provides regularization of the effective temperature. In that case the effective temperature is a positive definite function of the dusty plasma’s parameters for all values \( z \) and arbitrary atom density. At the same time the conclusion about anomalously high temperature of grains and the mechanism of heating in the case of dominant absorption collisions remains valid.

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3. Elastic collisions

We disregard the grain–grain and the electron–grain Coulomb collisions because of the assumed smallness of the grain density and the negligible momentum transfer respectively. The ion–
grain Coulomb collisions are described by a Landau collision integral with a modified Coulomb logarithm. The modification is necessary because of the competition between the radius $a$ of a grain and the usual minimum impact parameter $r_{Li} = \pi |Z_{ieq}|/(8T_i)$. For $r_{Li} \leq a$ the Coulomb logarithm would include the contribution of collisions with particles reaching the grain surface, i.e. charging collisions. On the other hand it seems natural not to use the Debye length, $r_D = [T_i/(4\pi Z^2_{ieq}e^2n_i)]^{1/2}$, as the maximum impact parameter, but instead $r_D + a$, as in [7], because the screened potential of a finite-size grain is the DLVO potential [13]. In [14] a modification has been suggested to take into account the ion scattering at large angles in dusty plasmas. At the same time both the approximation in [7] and the one in [14] cannot provide a value of the logarithm, which is positive for all values of the dusty plasma's parameters. The new form has been obtained in [15] and can be written as

$$\ln \Lambda_i = \ln [(r_D + a + r_{Li})/(a + r_{Li})].$$  

(5)

In the limit $r_D \gg r_{Li} \gg a$ equation (5) reduces to the usual Coulomb logarithm $\ln(r_D/r_{Li})$; in the limit $r_{Li} + a \gg r_D$, typical for dusty plasmas, the Coulomb logarithm is not already large. In that case, as was found in [14] on the basis of physically justified renormalization of the maximal cut-off parameter for the electron and ion scattering by grains, the Coulomb logarithm has to be essentially modified. Inclusion of the finite grain size via the DLVO potential permits us (with use also of the approximation suggested in [14]) to find the completely positive form of the logarithm (5). It is necessary to mention that in many problems a more exact form of the logarithm is required, where instead of the impact parameter $r_{Li}$ the velocity-dependent length $r_{min} = Z_{ieq}^2/m_i v^2$ appears and the averaging of that logarithm on the velocity ion distribution is performed. We will not discuss this question here in detail.

At the same time consideration of this problem is not finished. The ideology of the cutting parameters for regularization of the Coulomb’s divergence initially was connected with a weak Coulomb interaction. For the case of strong interaction, when the logarithm is small, the region of distances larger then the maximal cut-off parameter can be essential. Nevertheless the approximation suggested in [14], as well as the generalization (5), are applicable in a wide domain of the dusty plasma parameters, where the usual approximation is not valid. The second interesting problem has been considered briefly in [15] and related to influence of the grain–grain correlations. This task has not yet been considered in detail and also has to be developed.

The collisions between neutral plasma particles and grains are described by a Fokker–Planck-type collision integral which can be derived from the Boltzmann equation. The ion–
and neutral–grain elastic collisions result in the following modification of equation (2):

$$T_{eff} = 2T_i \left[ \frac{1 + \frac{2}{r_{Li}} \ln \Lambda_i + \frac{n_e}{n_i} \left( \frac{n_e}{n_i} \right)^{1/2} \left( \frac{T_e}{T_i} \right)^{3/2}}{1 - \frac{2}{r_{Li}} \ln \Lambda_i + 2 \frac{n_e}{n_i} \left( \frac{n_e}{n_i} \right)^{1/2} \left( \frac{T_e}{T_i} \right)^{1/2}} \right].$$  

(6)

where the subscript $n$ refers to the neutral plasma particles.

If there are no neutral plasma particles ($n_n = 0$), the Coulomb collisions alone can produce a saturation of the grain temperature, but in the case of dominant charging collisions $T_{eff}$ can still
be anomalously large. This fact can be useful for qualitative explanation of the experimental observation of grain temperatures much higher than the ion temperature \[1, 2\].

Equation (6) shows that the effective temperature increases with decreasing neutral density. The influence of the neutral density is especially important at \(1 - z/t + 2(z/t)^2 \ln \Lambda_i \simeq 0\). Then a decrease of the neutral gas pressure can produce an anomalous growth of the effective temperature. That is in qualitative agreement with the experimental observation of the melting of dusty crystals by reduction of the gas pressure \[1, 2\]. We would like to stress that the plasma state in the experiments \[1, 2\] is essentially different from the homogeneous conditions considered in the theory, which we discuss here. Therefore we can only point out some general mechanism for grain heating. A more complete qualitative and especially quantitative analysis can be done by using the new results of \[8\] applied for the specific conditions of the experiments, which is a separate and interesting problem.

4. Changing grain mass

In order to obtain insight into the influence of changing mass of dust particles we now consider a system consisting only of neutral gas atoms (or molecules) and heavy neutral grains. We assume that all gas atoms hitting a grain are absorbed by the grain. In the statistical theory of the grain system, mass now plays the role of a new dynamic variable. It should be noted that the methods, applied to consideration of complete absorption, can be useful under the conditions of the experiments aimed at plasma synthesis of fine grains \[3, 4\]. However, the specific conditions, considered below, are different from those in the case of plasmas synthesis.

The appropriate kinetic equations for the grains and for the neutral gas were given in \[16, 17\]. In the equation for the latter not only neutral–grain collisions but also neutral–neutral collisions and re-supply from external sources were taken into account. Conservation laws can be derived; the total energy, however, is no longer a conserved quantity. The reason is obvious: a part of the kinetic energy of a colliding atom changes the kinetic energy of a grain, while the remainder is spent in the change of the inner energy of the grain.

The collision term in the kinetic equation for the grains is greatly simplified by expansion in powers of the small mass ratio \(m/M\) (atom mass divided by grain mass). The solution of the kinetic equation for the atoms is approximately a Maxwellian, if we assume a dominating role of interatomic collisions. Under these conditions the distribution function of the grains may be assumed to be isotropic and the kinetic equation for the grains becomes:

\[
\frac{df_g(P, M, t)}{dt} = n_n \left( \frac{8T_n m}{\pi} \right)^{1/2} \left\{ g(P, M) \frac{P}{M} + \frac{P}{3M} \frac{\partial g(P, M)}{\partial P} - \frac{\partial g(P, M)}{\partial M} \right\},
\]

where \(g(P, M) = \sigma(P, M) f_g(P, M)\), \(\sigma(P, M)\) is the cross section for atom–grain collisions, and \(n_n\) and \(T_n\) are the number density and the temperature of the atoms respectively.

If the last term of equation (7) is omitted, i.e. if the mass growth is neglected, then an exact stationary solution is a Maxwellian with temperature \(T_g = 2T_n\) in agreement with the results of section 2 for the case \(z = 0\).

It is possible to investigate the temperature evolution on basis of the general solution of equation (7). For simplicity we neglect, however, the mass dispersion and write:

\[
f_g(P, M, t) = F(P, t) \delta(M - \mu(t)),
\]
where $\mu(t)$ is the unknown mass of the grain, which is an increasing function of time, due to the process of absorption of the neutrals by grains. Substitution of equation (8) into equation (7) leads to the equation for determination of the function $\mu(t)$:

$$\frac{d\mu(t)}{dt} = n_n \left( \frac{8T_n m}{\pi} \right)^{1/2} \sigma[\mu(t)] \tag{9}$$

and to the equation for $F(P, t)$ which is solved by the Maxwellian

$$F(P, t) = n_g \left\{ \frac{2\pi}{\Delta_1(t)} \right\}^{-3/2} \exp\left[ -\frac{P^2}{\Delta_1(t)} \right] \tag{10}$$

with the time-dependent temperature $\Delta(t) = T_{\text{eff}}(t)\mu(t)$. It was shown for a particular dependence $\mu(P, M)$ on $M$ in [13] and proved for arbitrary dependence of $\mu(P, M)$ on $M$ in [14] that

$$T_{\text{eff}}(t) = \frac{4}{5} T_n + C \mu(t)^{-5/3}, \tag{11}$$

where $C$ is a constant of integration.

Asymptotically $\mu(t)$ approaches infinity and $T_{\text{eff}}$ approaches $(4/5)T_n$. The mass growth results in cooling of the dust component below the gas temperature, while without mass growth eventually $T_{\text{eff}}$ is twice $T_n$.

5. Inner energy of grains

Again we consider a system consisting of neutral gas atoms and neutral grains. We add the inner energy as a new dynamic variable. We do not consider the angular momentum, cf [9, 18], because it is considered in some detail in [9].

The Fokker–Planck equation for the grain distribution function $f_g(P, M, \varepsilon, t)$, where $\varepsilon$ represents the inner energy, takes the form [19]:

$$\frac{df_g}{dt} = \sigma(M) \frac{\partial}{\partial P_a} \left\{ -\beta_a f_g + \lambda_{\alpha\beta} P_{\beta} f_g + \frac{\partial}{\partial P_{\beta}} (\kappa_{\alpha\beta} f_g) \right\} - \frac{\partial (J \sigma f_g)}{\partial M} - \frac{\partial}{\partial \varepsilon} \left( \frac{3 J \sigma \xi f_g}{m} \right), \tag{12}$$

where $\sigma(M)$ is the cross section for absorption and the coefficients $J, \beta_a, \lambda_{\alpha\beta}, \kappa_{\alpha\beta}$ and $\xi$ are determined as different moments of the momentum calculated through integrals with the distribution function of the light particles [18].

We assume that the distribution function of the light particles is Maxwellian with temperature $T_n$. Then as a result we find for the evolution of the average inner energy per particle of the grains:

$$\langle \varepsilon(t) \rangle = \frac{\mu(0)}{\mu(t)} \langle \varepsilon(0) \rangle + 2T_n \left\{ 1 - \frac{\mu(0)}{\mu(t)} \right\}. \tag{13}$$

Clearly the grains are heated by the absorbing collisions with light particles if $\langle \varepsilon(0) \rangle < 2T_n$.

6. Conclusions

Situations may exist where the electric charge of the dust particles changes, but not the mass and the inner energy (at the same time the inner grain temperature, in general, can be different from the kinetic temperatures of the other plasma components). This is the case when the absorption of ions is combined with recombination and the emission of neutral atoms. The results
show that then the stationary distributions of the grain velocities and charges are described by
effective temperatures other than those of the plasma subsystem. These effective temperatures are
determined by the competition between charging collisions and elastic collisions. Grain–neutral
and Coulomb collisions tend to equalize the grain temperature to the temperature of neutrals or
ions respectively, while charging collisions can produce anomalous temperature growth. That
might be the main mechanism of the experimentally observed grain heating.

When the mass is not kept constant by recombination and emission of atoms, it should
also be introduced as a new dynamic variable. The mass of the grains will grow indefinitely
under such circumstances. The consequences are interesting. In the present paper the case of
neutral atoms and neutral grains was considered. The grains are eventually cooled to 80% of the
temperature of the ambient gas, while in the absence of mass growth the effective temperature
is about twice the temperature of the atoms.

It is relatively simple to generalize the obtained Fokker–Planck equations to include also
terms representing the angular momentum and the inner energy of the grains as new dynamic
variables. The evolution of the average inner energy in our simple model appears to be directly
coupled to the evolution of the mass. Asymptotically the inner energy expressed as a temperature
becomes twice the temperature of the ambient gas.

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