SPH code for dynamical and chemical evolution of disk galaxies.

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1 Introduction

• Galaxy formation in the Universe – collapse the baryons within DMH potential wells (White & Rees (78)). Observational support: COBE detection (Bennett et al. (93)).

• Formation of self – gravitating inhomogeneities of protogalactic size (Dar (95)). Origin of initial angular momentum (Steinmetz & Bartelmann (95)).

• Smoothed Particle Hydrodynamics (SPH) (Monaghan (92)). TREE – SPH code (Hernquist & Katz (89)). GRAPE – SPH code (Steinmetz & Muller (94, 95)).

• Extension of our N – body/SPH method (Berczik & Kolesnik (96), Berczik & Kravchuk (96)). New ”energetic” criteria for SF and more realistic account of returned chemical enriched gas fraction via SNII, SNIa and PN events.

2 The CD–SPH code

• The SPH code. Continuous hydrodynamic fields in SPH are described by the interpolation functions constructed from the known values of these functions at randomly positioned particles (Monaghan (92)).

\[
\rho(r_i) = \sum_{j=1}^{N} m_j \cdot \frac{1}{2} \cdot [W(r_{ij}; h_i) + W(r_{ij}; h_j)].
\]

The equations of motion for particle \(i\) are:

\[
\frac{d}{dt} r_i = v_i.
\]
\[
\frac{d\mathbf{v}_i}{dt} = -\nabla_i \frac{P_i}{\rho_i} + \mathbf{a}_{vis} - \nabla_i \Phi_i - \nabla_i \Phi_i^{ext}.
\]

The energy equation in the particle representation has the form:

\[
\frac{d u_i}{dt} = \frac{P_i}{\rho_i} \cdot \nabla_i \mathbf{v}_i + \frac{\Gamma_i(\rho_i, T_i) - \Lambda_i(\rho_i, T_i)}{\rho_i}.
\]

The system of equations is closed by adding the equation of state:

\[
P_i = \rho_i \cdot (\gamma - 1) \cdot u_i.
\]

- Time integration. To solve the system of equations we use the standard algorithm of leapfrog integrator (Hernquist & Katz (89)). The integrator has a second order accuracy in the time step \( \Delta t \). To define \( \Delta t \) we use the relation (Hiotelis & Voglis (91)):

\[
\Delta t = \min_i \{ C_n \cdot \min \left[ \sqrt{\frac{h_i}{|\mathbf{a}_i|}}, \frac{h_i}{|\mathbf{v}_i|}, \frac{h_i}{c_i} \right] \}.
\]

- The star formation algorithm. We modify the standard SPH star formation algorithm (Katz (92)), taking into account the presence of chaotic motions in the gaseous environment and the time lag between initial development of suitable conditions for star formation and star formation itself (Berczik & Kravchuk (96)). It states that in the separate "gas" particle the SF can start if the absolute value of the "gas" particle gravitational energy exceeds the sum of its thermal energy and energy of chaotic motions:

\[
|E_i^{gr}| > E_i^{th} + E_i^{ch}.
\]

It seems reasonable that the chosen "gas" particle produce stars only if the above condition holds over the time interval exceeding its free fall time:

\[
t_{ff} = \sqrt{\frac{3 \cdot \pi}{32 \cdot G \cdot \rho}}.
\]

We check the number of SF acts in selected "gas" particle \( i \). If the number of SF acts becomes greater than \( N_{max}^{SF} = 25 \) we stop any SF activity in these particles.

We also define which "gas" particles remain cool, i.e. \( t_{cool} < t_{ff} \). These conditions we rewrite in the manner presented in the paper by Navarro & White (93): \( \rho_i > \rho_{crit} \). Here we use the value of \( \rho_{crit} = 0.03 \text{ cm}^{-3} \).

When the collapsing particle \( i \) has been defined we create the new "star" particle with mass \( m^{star} \) and updated the "gas" particles \( m_i \) using these simple equations:

\[
\begin{align*}
    m^{star} &= \epsilon \cdot m_i, \\
    m_i &= (1 - \epsilon) \cdot m_i.
\end{align*}
\]
In our Galaxy on the scale of giant molecular clouds the typical values for SF efficiency are in the range $\epsilon \approx 0.01 \div 0.4$ (Duerr et al. (82), Wilking & Lada (83)). We define $\epsilon$ as:

$$\epsilon = 1 - \frac{E_{th}^i + E_{ch}^i}{|E_{gr}^i|}.$$ 

In the code, we set the absolute maximum value of the mass of such "star" particles $m_{max}^{star} = 10^7 M_\odot$.

At the moment of the birth, the positions and velocities of new "star" particles are set equal to those of parent "gas" particles. Subsequently, the "star" particle interact with the rest of "gas" and "star" particles or "dark – matter" only by gravitation. The gravitational smoothing length for these (Plummer – like) particles is set equal to $h_{star}$.

- The thermal SNII feed-back. For the thermal budget of the ISM, SNIIs play main role. Following to Katz (92) and Friedli & Benz (95), we assume that the explosion energy is converted totally to the thermal energy. The total energy released by SNII explosions ($10^{44}$ J per SNII) within "star" particles is calculated at each time step and distributed uniformly between the surrounding (i.e. $r_{ij} < h_{star}$) "gas" particles (Raiteri et al. (96)).

- The chemical enrichment of gas. In our SF scheme, every new "star" particle represents a separate, gravitationally closed, star formation macro region (like a globular clusters). The "star" particle has its own time of birth $t_{begSF}$ which is set equal to the moment of the particle formation. After formation of these particles due to SNII, SNIa and PN events, return the chemically enriched gas to surrounding "gas" particles. For description of this process we use the approximation proposed by Raiteri et al. (96). We concentrate our treatment only on the production of $^{16}$O and $^{56}$Fe, but try to describe the full galactic time evolution of these elements, from the beginning up to present time (i.e. $t_{evol} \approx 13.0$ Gyr).

For example, if the mass of new "star" particle (with metallicity $Z = 10^{-4}$) is equal to $10^4 M_\odot$, it produces next numbers of events: $\Delta N_{SNII} \approx 52.5$, $\Delta N_{PN} \approx 1770$, $\Delta N_{SNIa} \approx 8.48$ during total time of evolution $t_{evol}$.

The total masses (H, He, $^{56}$Fe, $^{16}$O) returned to the surrounding "gas" particles, due to these processes the are (in solar masses): $\Delta m^H \approx 2644$, $\Delta m^{He} \approx 881$, $\Delta m^{Fe} \approx 8.8$, $\Delta m^{O} \approx 120$.

- The cold dark matter halo. In the literature we have found some, sometimes contraversial, profiles of Cold Dark Matter Haloes (CDMH) in the galaxies (Burkert (95), Navarro (98)). For resolved structures of CDMH: $\rho_{halo}(r) \sim r^{-1.4}$ (Moore et al. (97)). The structure of CDMH, as shown in high-resolution N – body simulations, can be described by: $\rho_{halo}(r) \sim r^{-1}$ (Navarro et al. (96), Navarro et. al (97)). Finally, in paper by Kravtsov et al. (97) we find that the cores of DM dominated galaxies may have a central profiles: $\rho_{halo}(r) \sim r^{-0.2}$.

In our calculations, as a first order aproximation, it is assumed that the model galaxy halo contains the CDMH component with Plummer – type density profiles (Douphole & Colin (95)). Therefore for the external force which exercises onto the "gas" and "star" particles by CDMH we can write as:
Figure 1: The distribution of "star" and "gas" particles in the final step.

\[-\nabla_i \Phi^\text{ext}_i = -G \cdot \frac{M_{\text{halo}}}{(r_i^2 + b_{\text{halo}}^2)^{3/2}} \cdot r_i.\]

3 Results and discussion

• Initial conditions. The SPH calculations were carried out for \(N_{\text{gas}} = 2109\) "gas" particles. According to Navarro & White (93) and Raiteri et al. (96), such number seems to be quite enough to provide qualitatively correct description of the system behaviour. Even such small number of "gas" particles produces a \(N_{\text{star}} = 31631\) "star" particles at the end of calculation.

The value of the smoothing length \(h_i\) was chosen requiring that each "gas" particle had \(N_B = 21\) neighbours within \(2 \cdot h_i\). Minimal \(h_{\text{min}}\) was set equal to 1 kpc. For "star" particles we use the fixed gravitational smoothing length \(h_{\text{star}} = 1\) kpc.

As initial model (relevant for CDM – scenario) we took constant – density homogeneous triaxial configuration of gas \((M_{\text{gas}} = 10^{11} M_\odot)\) within the dark matter halo \((M_{\text{halo}} = 10^{12} M_\odot)\). We set \(A = 100\) kpc, \(B = 75\) kpc and \(C = 50\) kpc for semiaxes of system. We
set the smoothing parameter of CDMH: $b_{halo} = 25$ kpc. The gas component was assumed to be initially cold, $T_0 = 10^4$ K.

The gas was assumed to be involved into the Hubble flow ($H_0 = 65$ km/s/Mpc, $\Omega_0 = 1$) and into the solid – body rotation around $z$ – axis. We added the small random components of velocities ($\Delta | \mathbf{v} | = 10$ km/s) to account for the chaotic motions of fragments.

The spin parameter in our simulation is $\lambda \approx 0.08$. This parameter is defined in Peebles (69) as:

$$\lambda = \frac{|\mathbf{L}_0| \cdot \sqrt{|E^{gr}_0|}}{G \cdot (M_{\text{gas}} + M_{\text{dm}})^{5/2}}.$$

If the angular momentum is acquired through the tidal torque of the surrounding matter, the standard spin parameter does not exceed $\lambda \approx 0.11$ (Steinmetz & Bartelmann (95), $\lambda \approx 0.07^{+0.04}_{-0.05}$).

• Conclusion. This simple model provides good, self – consistent picture of the process of galaxy formation. The dynamical and chemical evolution of modelled disk – like galaxy is coincident with the results of observations for our own Galaxy. Some basic distributions of gas and star parameters are given in figures:

- Fig. 1. The distribution of ”star” and ”gas” particles in the final step.
- Fig. 2. $V_{\text{rot}}(r)$. The rotational velocity distribution of gas in the final step.
- Fig. 3. $\sigma_*(r)$, $\sigma_{\text{gas}}(r)$. The column density distribution in the disks of gas and stars in the final step.
- Fig. 4. $T(r)$. The temperature distribution of gas in the final step.
- Fig. 5. $SFR(t)$. The time evolution of the SFR in galaxy.
- Fig. 6. $[\text{Fe/H}](t)$. The age metallicity relation of the ”star” particles in the ”solar” cylinder ($8 < r < 10$ kpc).
- Fig. 7. $N_*(\text{[Fe/H]})$. The metallicity distribution of the ”star” particles in the ”solar” cylinder ($8 < r < 10$ kpc).
- Fig. 8. $[\text{O/Fe}](\text{[Fe/H]})$. The $[\text{O/Fe}]$ vs. $[\text{Fe/H}]$ distribution of the ”star” particles in the ”solar” cylinder ($8 < r < 10$ kpc).
- Fig. 9. $[\text{O/H}](r)$. The $[\text{O/H}]$ radial distribution.

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Figure 2: The rotational velocity distribution of gas in the final step.

Figure 3: The column density distribution in the disks of gas and stars in the final step.
Figure 4: The temperature distribution of gas in the final step.

Figure 5: The time evolution of the SFR in galaxy.
Figure 6: The age metallicity relation of the "star" particles in the "solar" cylinder (8 kpc $< r < 10$ kpc).

Figure 7: The metallicity distribution of the "star" particles in the "solar" cylinder (8 kpc $< r < 10$ kpc).
Figure 8: The \([O/Fe]\) vs. \([Fe/H]\) distribution of the "star" particles in the "solar" cylinder (8 kpc < \(r\) < 10 kpc).

Figure 9: The \([O/H]\) radial distribution.