STUDYING THE HIGGS POTENTIAL VIA $e^+e^- \rightarrow Zhh$.

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The physics prospect at future linear $e^+e^-$ colliders for the study of the Higgs triple self-coupling via the process of $e^+e^- \rightarrow Zhh$ is investigated. The measurement of this cross section leads us to the first non-trivial information on the Higgs potential. We found that the Standard Model and the model without the Higgs self-coupling can be distinguished at the level of one standard deviation for a rather light Higgs mass with $100 \text{ fb}^{-1}$ integrated luminosity. In the MSSM, the cross section is enhanced if the production of at least one of heavy Higgs bosons ($H$ or $A$) and its subsequent decay ($H \rightarrow hh$, $A \rightarrow Zh$) are kinematically allowed. When such processes are not allowed, the cross section in the region of small $m_A$ is significantly suppressed relative to the SM cross section.

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1. Introduction

In the Standard Model (SM) of particle physics, there are three types of interactions of fundamental particles, gauge interactions, Yukawa interactions and the Higgs boson self-interaction. It has already been confirmed by experiments that the interactions between fermions and vector bosons are described by gauge theories. The gluon triple coupling has also been established. At LEP II, LHC and future linear colliders we will be able to examine the triple couplings of weak bosons and photons, the Higgs couplings to $W^\pm$ and $Z$ bosons and the Yukawa couplings of heavy fermions.

To measure the Higgs self-coupling, the most promising way is to observe the ‘Higgs double-production’ processes, the processes with two Higgs bosons in the final state. The cross sections of these processes in the SM have been evaluated at $e^+e^-$ colliders$^1$–$^4$, at hadron colliders$^5$–$^{11}$ and at $\gamma\gamma$ colliders$^3$,$^{12}$. The cross sections at linear colliders have been examined both in the SM and the model with an anomalous Higgs triple coupling$^1$–$^{12}$. Since the cross section is relatively large and all the final states can be identified without large missing momentum, the process $e^+e^- \rightarrow Zhh$ is the best among the various Higgs double-production processes to look for the Higgs self-coupling during the first stage of a future linear collider.

In this paper we investigate the physics prospect of future linear $e^+e^-$ colliders for the study of the Higgs-boson triple self-coupling via the process $e^+e^- \rightarrow Zhh$, which leads us to the first non-trivial information on the Higgs potential. In Sec. 2 we examine the SM and a model without a Higgs self-coupling, and we evaluate their distinguishability. Also, we survey the minimal supersymmetric standard model in Sec. 3. Conclusions will be given in Sec. 4.

2. The Standard Model

In the unitary gauge of the SM, at the tree level, there are four Feynman diagrams relevant to the process $e^+e^- \rightarrow Zhh$ (Fig. 1). Three of them come purely from the gauge interactions, while the other one has a Higgs-boson self-coupling vertex. The Higgs self-coupling constant carries information about the Higgs potential, and in the SM it is expressed by the Higgs mass.

\[ g_{hhh} = 3\lambda v = 3m_h^2/v. \]

Examination of this relation will be the first non-trivial test for the shape of the Higgs potential.

Unfortunately, it is known that the cross section of this process is not large$^1$–$^3$, and we cannot expect a sufficiently precise determination of the cross section to study various models with anomalous Higgs self-couplings. Therefore we focus only on the problem of whether the SM can be distinguished from a model without the triple Higgs-boson self-coupling (NT)$^4$–$^6$. 

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The dependence of the cross sections in these models on the Higgs-boson mass is plotted in Fig. 2 for $\sqrt{s} = 500$ GeV and 1 TeV. Here we assume no beam polarization. The cross sections are much smaller than 1 fb, and they quickly drop as they approach the kinematic limit. We found that the dominant Feynman amplitude in the unitary gauge is the one with the $ZZhh$ contact coupling (Fig. 1(c)) in the most of the parameter space. All of the diagrams interfere with each other, but the amplitude of the diagrams Fig. 1(c) and (d) have always the same argument. The difference between the two models is not so large; however, it is notable that the cross section in the SM is always greater than that in the NT. The ratio of the cross sections of the two models depends on the Higgs-boson mass and the collider energy; it varies from about 0.3 to 0.9.

Assuming 100 fb$^{-1}$ of integrated luminosity, 100% detection efficiency and no beam polarizations, we can estimate the statistical significance of the difference between the two models as presented in Fig. 3. The expected number of the signal events in a model (e.g. SM or NT) is $N_{\text{model}} = \sigma_{\text{model}} \times 100 \text{ fb}^{-1}$, where $\sigma_{\text{model}}$ is the cross section of $e^+e^- \rightarrow Zhh$ process in the model. Such a model will be rejected at the 1 $\sigma$ level (stat. only) if the statistical significance,

$$S = |N_{\text{model}} - N_{\text{obs}}| / \sqrt{N_{\text{obs}}} ,$$

exceeds unity, where $N_{\text{obs}}$ is the observed number of events, and the denominator in R.H.S. represents the statistical fluctuation of $N_{\text{obs}}$. If the SM is correct, we can expect $N_{\text{obs}} = N_{SM}$ in average, and from Fig. 3, we found that the NT will be rejected for a Higgs-boson mass of up to 170 GeV with $\sqrt{s} = 500$ GeV, and up to 320 GeV at a 1 TeV collider.
Figure 2: The Higgs-mass dependence of the $e^+e^- \rightarrow Zhh$ cross section in the SM (solid) and in the NT (dashed), for $\sqrt{s} = 500$ GeV and 1000 GeV.

Figure 3: The statistical significance $S$ defined in Eq.(2) with $N_{model} = N_{NT}$ and $N_{obs} = N_{SM}$ as functions of the Higgs mass for $\sqrt{s} = 500$ GeV (solid) and 1000 GeV (dashed). We assumed perfect detection efficiency, 100 fb$^{-1}$ of the integrated luminosity and no beam polarization.
Figure 4: The additional Feynman diagrams for $e^+e^- \rightarrow Zhh$ in the unitary gauge in the MSSM. All the relevant diagrams are in this figure and in Fig. 1.

The dependence of the cross section on beam polarization is simply factored out, since the electron is coupled only to the $Z$ boson, as is shown in Fig. 1. The ratio of the cross section with both beams polarized to that with no polarization is expressed as follows:

$$\frac{\sigma(P_{e^-}, P_{e^+})}{\sigma(0, 0)} = (1 - P_{e^-} \cdot P_{e^+}) - \frac{1 - 4\sin^2\theta_W}{1 - 4\sin^2\theta_W + 8\sin^4\theta_W}(P_{e^-} - P_{e^+})$$

$$\simeq (1 - P_{e^-} \cdot P_{e^+}) - 0.144(P_{e^-} - P_{e^+}), \quad (3)$$

where $P_{e^-}$ and $P_{e^+}$ are the polarizations of the electron beam and the positron beam, respectively, and $\theta_W$ is the Weinberg angle. Highly polarized beams with right-handed electrons ($P_{e^-} \sim +1$) and left-handed positrons ($P_{e^+} \sim -1$) are preferred to suppress background processes such as $e^+e^- \rightarrow W^+W^-Z$. If the both of the beams can be perfectly polarized, the factor of Eq.3 is as large as 1.7.

3. The MSSM

In the Minimal Supersymmetric extension of the Standard Model (MSSM), there are three additional diagrams (Fig. 4) in addition to Fig. 1 in the SM, which are relevant to the process $e^+e^- \rightarrow Zhh$, one with the exchange of a heavy Higgs boson ($H$) (Fig. 4(a)) and two with the exchange of a pseudo-scalar ($A$) (Fig. 4(b,c)). (In the MSSM, we denote the light Higgs boson as “$h$”.) In some regions of the SUSY parameters and the collider energy, the real production of the $H$ and the $A$ and their subsequent decays through $H \rightarrow hh$ and $A \rightarrow Zh$ are possible, and in such cases a large cross section is obtained.
Figure 5: The $m_A$ dependence of the $e^+e^-\to Zhh$ cross-section in the MSSM (solid) for $\tan\beta = 2$ and 10 at $\sqrt{s} = 500$ GeV. We take $m_t = 170$ GeV and $m_{\text{stop}} = 1$ TeV. In this figure the Higgs mass is given as a function of $m_A$. For comparison, the SM (dashed) and the NT (dotted) cross-sections with the same Higgs mass are also shown.

The dependence on the beam polarization of the cross section is precisely the same as in the SM.

In the MSSM, once $m_A$, $\tan\beta$, the top-quark mass and the top-squark mass ($m_{\text{stop}}$) have been specified, the light Higgs mass, $m_h$, is predicted. We choose $m_t = 170$ GeV and $m_{\text{stop}} = 1$ TeV and plot the MSSM $e^+e^-\to Zhh$ cross-section versus $m_A$ in Fig. 5. In this figure, the light Higgs mass $m_h$ is given as a function of $m_A$. For comparison we also plot the SM and NT cross-sections with the same $m_h$ as in the MSSM. Large cross section can arise in the MSSM in the region where the poles of the heavy Higgs and/or the pseudo-scalar are hit. This is especially remarkable for the heavy Higgs pole. However, the cross section rapidly approaches to the SM value as $m_A$ goes large. In this limit the lightest Higgs boson behaves just like the SM Higgs boson.

The contour plots of the cross section in the plane of $m_A$ and $\tan\beta$ in the MSSM with $\sqrt{s} = 300$ GeV can be found in Fig. 6. The lightest Higgs mass $m_h$ varies from 48 GeV to 124 GeV in this figure. The process $e^+e^-\to Zhh$ is kinematically
Figure 6: The contour plot of cross section of $e^+e^- \rightarrow Zhh$ in the MSSM at $\sqrt{s} = 300$ GeV. The top and the stop masses are the same as in Fig. 5.
Figure 7: The ratio of the $e^+e^- \rightarrow Zhh$ cross-section in the MSSM to that in the SM. The collider energy is 300 GeV, and the Higgs mass in the SM is adopted to be the same as $m_h$ in the MSSM at each point in the figure. The top and the stop masses are the same as in Fig. 5.
forbidden in the upper right region. In the left-end region of the figure, the cross section is as remarkably large as 100 fb, and there is also a horizontal area at the bottom of the figure where the cross section exceeds to 10 fb. Both areas of large cross section correspond to the pole of the heavy Higgs.

Fig. 7. shows the ratio of the cross sections of the MSSM and the SM. Again, the resonant regions correspond to a large ratio. It is quite interesting that there is an area with a cross-section ratio sizably smaller than unity at around $m_A = 70 – 130$ GeV and $\tan \beta > 2.5$. In this region, no internal Higgs boson is on-shell, and the $ZZh$ coupling is suppressed.

4. Conclusions

The process $e^+e^- \rightarrow Zhh$ is important to examine the Higgs potential.

We found that the SM and the NT can be distinguished at the level of one standard deviation for a rather light Higgs mass with 100 fb$^{-1}$ integrated luminosity.

In the MSSM case, the cross section is enhanced if the $H$ or $A$ pole is accessible. In the case where no pole is accessible, the cross section is significantly suppressed relative to the SM at around $m_A = 70 – 130$ GeV and $\tan \beta > 2.5$. In this region the MSSM can be distinguished from the SM, if enough luminosity is available.

To perform these studies at a future linear collider, a high luminosity, a high efficiency and high beam polarizations are required.

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