Nonextensive triplet in a geological faults system

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Abstract – The San Andreas fault (SAF) in the USA is one of the most investigated self-organizing systems in Nature. In this paper, we studied some geophysical properties of the SAF system in order to analyze the behavior of earthquakes in the context of Tsallis’s q-Triplet. To this end, we considered 134573 earthquake events in the magnitude interval $2 \leq m < 8$, taken from the Southern Earthquake Data Center (SCEDC, 1932–2012). The values obtained (“q-Triplet” ≡ $\{q_{\text{stat}}, q_{\text{sen}}, q_{\text{rel}}\}$) reveal that the $q_{\text{stat}}$-Gaussian behavior of the aforementioned data exhibit long-range temporal correlations. Moreover, $q_{\text{sen}}$ exhibits quasi-monofractal behavior with a Hurst exponent of 0.87.

Introduction. – Earthquakes are among the most complex spatiotemporal phenomena investigated in the context of self-organized criticality (SOC), introduced in ref. [1]. In this regard, let us consider the so-called fault systems, a complex phenomenon related to the deformation and sudden rupture of some parts of the Earth’s crust driven by convective motion in the mantle. One of the first examples of self-organizing systems in Nature [2] is the San Andreas fault (SAF) in California. The SAF, one of the world’s longest and most active geological faults, is $\sim 1200$ km long, $\sim 15$ km deep, and about 20 million years old. It forms the boundary between the North American and Pacific plates and is classified as a right lateral strike-slip fault, although its movement also involves comparable amounts of reverse slip [3]. From the geophysical standpoint, a considerable number of investigations have been conducted in order to better understand the complexity of this system (see, e.g., [4] and references therein). In contrast to the complexity of earthquakes, empirical laws are extremely simple, e.g., the Gutenberg-Richter law, which gives the number of earthquakes with a magnitude $M > m$ [5], and the Omori law for temporal distribution of aftershocks [6].

Several studies have demonstrated that seismicity exhibits an out-of-equilibrium behavior that is being investigated by different authors, e.g., studies based on wavelet-based multifractal analysis [7] and nonextensive statistical mechanics [8–10], among others. In the present study, we consider a nonextensive formalism, which is a generalization of Boltzmann-Gibbs statistical mechanics (B-G statistics) for out-of-equilibrium systems and is described by the entropic parameter $q$. The celebrated Boltzmann-Gibbs (B-G) statistics is recovered at $q = 1$ [11–13]. This parameter measures the degree of nonextensivity in the stochastic process.

Tsallis statistics is based on the $q$-exponential and $q$-logarithm, two central functions defined by

$$
\exp_q(f) = [1 + (1 - q)f]^{1/1-q},
$$

and

$$
\ln_q(f) = \frac{f^{1-q} - 1}{1-q},
$$

which produces entropy $S_q$ [13], associated with $q$-statistics,

$$
S_q = k \frac{1 - \int [ PDF(x) ]^q dx}{q - 1} \quad (q \in \mathbb{R}),
$$

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where the Boltzmann-Gibbs entropy, the usual exponential and logarithm are recovered if $q = 1$.

This theory has been successfully applied to many complex physical systems such as geological faults [10] and astrophysical systems [14–16]. In 2004, Tsallis [17] proposed the existence of a three-parameter set ($q_{\text{stat}}, q_{\text{sen}}, q_{\text{rel}}$), also known as $q$-Triplet, characterized by metastable states in nonequilibrium, where $q_{\text{stat}} > 1$, $q_{\text{sen}} < 1$ and $q_{\text{rel}} > 1$. When $(q_{\text{stat}}, q_{\text{sen}}, q_{\text{rel}}) = (1, 1, 1)$, the set denotes the B-G thermal equilibrium state. Burlaga and Viñas [18] used this triplet to describe the behavior of two sets of daily magnetic-field strength performed by Voyager 1 in the solar wind in 1989 and 2002. In 2009, de Freitas and De Medeiros [16] presented a physical corroboration of the $q$-Triplet, based on analyses of the behavior of three sets of daily magnetic-field strength observed by different solar indices. More recently, Ferri, Savio and Plastino [19] showed a physical implication of this triplet for the ozone layer in Buenos Aires, Argentina.

The main aim of this study is to analyze the behavior of physical parameters directly reflecting seismic activity in the context of Tsallis $q$-Triplet’s formalism, and to compare the properties of this $q$-Triplet with those expected for a metastable or quasi-stationary dynamical system described by nonextensive statistics. In this context, we focus our attention on the magnitude values for SAF data $M(t)$ and their hourly variability $dM_h(t)$. Following the ideas presented in ref. [20], we focus our investigation on the “return” or fluctuation $dM_h(t) = M(t + \tau) - M(t)$, which denotes the differences between “avalanche” sizes obtained at time $t + \tau$ and at time $t$. With respect to seismic activity, this analysis also checks the validity of the $q$-Central Limit Theorem, the so-called $q$-CLT, recently conjectured by Umarov, Tsallis and Gell-Mann [21].

The remainder of this paper is organized as follows: in the second section, we present our seismic sample; the main results and discussions are presented in the third section; and, finally, conclusions are put forth in the last section.

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**Fig. 1:** (Colour on-line) Time series for the magnitude of earthquakes along the SAF. The peaks denote the maximum magnitudes.

**Fig. 2:** Map of the seismicity in the SAF system, showing epicenters of earthquakes considered in this study (source SCEDC).

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**The seismic data.** Figure 1 shows the time series for magnitude $M$ of earthquakes along the SAF, in the interval $2 \leq M < 8$, with 134573 events. These were taken from the Southern California Earthquake Data Center (SCEDC) from 1932 to 2012. This range was chosen because for small magnitudes it has the limitation of seismic monitoring in the area, since many such events are unregistered. Figure 2 illustrates the distribution of events considering the SAF map.

Figure 2 shows the data and the San Andreas fault system. This system is more than 800 miles long and extends to depths of at least 10 miles. The fault is a complex zone of crushed and broken rock ranging from a few hundred feet to a mile wide. Many smaller faults branch from and join the San Andreas fault zone. Almost any road cut in the zone shows a myriad of small fractures, fault gouge (pulverized rock), and a few solid pieces of rock [4]. The movement that occurs along the fault is a right-lateral strike-slip forming the tectonic boundary between the Pacific Plate and the North American Plate.

**Results and discussions.** In this section, we show the results after the estimation of the “$q$-Triplet” $\equiv \{q_{\text{stat}}, q_{\text{sen}}, q_{\text{rel}}\}$ based on SAF data from 1932 to 2012 (see fig. 1). These results are presented in three subsections, each associated to the properties of one of the $q$’s.

**On the behavior of the $q$-stationary parameter.** For the time series $M(t)$, increment fluctuations due to its
variability over the time scale $\tau$ are given as $dM_r(t) = M(t+\tau) - M(t)$. The values of $q_{stat}$ are derived from probability distribution functions (PDFs). These PDFs are obtained from the variational problem using the continuous version for the nonextensive entropy given by eq. (3),

$$PDF = A_q \left[ 1 + (q-1)B_q dM_r(t)^2 \right]^{\frac{1}{q-1}}, \quad (4)$$

the entropic parameter $q$ is related to the size of the tail in the distributions [15] and coefficients $A_q$ and $B_q$ for $q > 1$ are given by

$$A_q = \frac{\Gamma \left[ \frac{1}{q-1} \right]}{\Gamma \left[ \frac{3-q}{2q-2} \right]} \sqrt{\frac{q-1}{\pi} B_q} \quad (5)$$

and

$$B_q = \frac{1}{(3-q)\sigma_q^2} \quad (6)$$

for further details see ref. [22].

Following the same procedure described by [19], we varied the index $q$ between 1.0 and 2.0, making a linear adjustment in each computational iteration and evaluating the specific correlation coefficient $R^2$. The best linear fit is obtained for $q_{stat} = 1.364 \pm 0.04$ with $R^2 = 0.992$ as shown in fig. 3. It should be emphasized that this $q_{stat}$ value is fully consistent with the bounds obtained from several independent studies involving the nonextensive Tsallis framework (see, e.g., [23]). The PDF for the return $d_r M(t)$ on scale $\tau = 1$ is shown in fig. 4. On this scale we can conduct a closer investigation of a possible correlation between events. Our study used the Levenberg-Marquardt method [24,25] to compute PDFs with symmetric Tsallis distribution from eq. (4). In this adjustment, we found $B_q = 0.858 \pm 0.16$. These results are consistent with the value expected for nonlinear systems, where the random variable is the sum of strongly correlated contributions [15,18,26]. In this respect, we showed that PDFs for the return $dM_r(t)$ have fat tails with a $q$-Gaussian shape.

**On the behavior of the $q$-sensibility parameter.** Values of the $q_{sen}$-index are directly related to the system instability and the entropy growth. These values can be obtained from multifractal (or singularity) spectrum $f(\alpha)$, where $\alpha$ is the singularity strength or H"older exponent. Spectrum $f(\alpha)$ is derived via a modified Legendre transform, through the application of the MF DFA method [27]. This method consists of a multifractal characterization of a nonstationary time series, based on a generalization of the detrended fluctuation analysis (DFA). MF DFA performs best when the signal is a noise-like time series. However, there is also difficulty in visualizing the difference between walk- and noise-like time series. As suggested by [28], before application, it is necessary to run a DFA and verify if the value of the Hurst exponent is less than 1.2. For SAF data we obtain a Hurst exponent of 0.87, indicating that the MF DFA method can be employed directly without transformation of the time series.

The $q_{sen}$-index denotes sensitivity at initial conditions. For the present purposes, we used the expression defined by Lyra and Tsallis [29] for the relation between $q_{sen}$ and multifractality in dissipative systems, as follows:

$$\frac{1}{1 - q_{sen}} = \frac{1}{\alpha_{min}} - \frac{1}{\alpha_{max}}, \quad (7)$$

where $\alpha_{min}$ and $\alpha_{max}$ denote the roots of the best-fit.

The multifractal characterization of these data is shown in fig. 5. These spectra $f(\alpha)$, calculated for SFA data, show a narrow H"older exponent interval with $\alpha_{min} = 0.924 \pm 0.04$ and $\alpha_{max} = 1.051 \pm 0.11$. For multifractal
of a monofractal-like time series. The behavior of our sample is similar to that of a multifractal-like time series. The symbols represent the best adjustment using a cubic fit to the data.

As mentioned by [22], this behavior is related to local equilibrium, and then a much slower decay for larger $\tau$. In agreement with these authors, this constitutes a necessary condition for the application of the superstatistical model, as described in ref. [31].

See [32] for further details and an extensive discussion about the estimation of the Tsallis $q$-Triplet.

**Conclusions.** – We used a new approach to nonextensive formalism for hourly measurements of earthquakes along the SAF from 1932 to 2012. From these data we were able to estimate the values of the nonextensive three-index. We found that $q_{stat} = 1.364 \pm 0.04$, $q_{sen} = -6.647 \pm 0.35$ and $q_{rel} = 2.69 \pm 0.13$. It is important to underscore that the result of the $q_{stat}$ is consistent with the upper limit $q < 2$ obtained from several independent investigations [23]. In addition, the values of this triplet confirm the general scheme proposed by Tsallis [17]. These results reveal that this system is consistent with a nonequilibrium state, strongly suggesting that long-range correlations exist among the random variables involved in the physical process that controls seismic activity.

Finally, it is worth mentioning that the nonextensive three-index can be recalculated by considering a spatiotemporal analysis for earthquakes along the SAF. This issue will be addressed in a forthcoming communication.

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