Millimetre-Range Forces in Superstring Theories with Weak-Scale Compactification

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Abstract

We show that theories in which supersymmetry is broken via Scherk-Schwarz compactification at the weak scale, possess at least one scalar particle with Compton wavelength in the millimetre range, which mediates a force with strength $1/3$ of gravity. Such forces are going to be explored in upcoming experiments using microelectromechanical systems or cantilever technology. We also present a simple way of understanding some decoupling aspects of these theories by analogy with finite-temperature field theory.

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1. Light scalars and theories of supersymmetry breaking

One of the most outstanding challenges of supersymmetric theories is the problem of the breaking of supersymmetry. It is directly connected with the cosmological constant problem for which no solution is in sight. In spite of this big hole in our theoretical understanding, the idea of softly broken supersymmetric theories [1] has allowed us to bypass these questions and to study phenomenological consequences of supersymmetric theories in the past 16 years. In the mean time, three distinct classes of theories have emerged for the breaking of supersymmetry:

- Gravity-mediated theories.
- Gauge-mediated theories.
- Theories with weak-scale compactification.

A main difference between these approaches is the scale of supersymmetry breaking. In gravity-mediated theories it is $10^{13}$ GeV. In gauge-mediated ones it is anywhere from over 10 TeV to $10^{12}$ GeV. In theories of weak-scale compactification the supersymmetry breaking scale is around the weak scale, $\sim 1$ TeV [2]. The different supersymmetry breaking scales imply important differences in the pattern of sparticle masses; these differences will be tested in the next decade at the LHC. The main purpose of this paper is to point out a possible difference between these theories that may emerge at very low energies, in the near future, at “tabletop” experiments costing orders of magnitude less than the LHC. These involve macroscopic gravitational strength forces and arise as follows.

In supersymmetric or superstring theories one often encounters scalar particles, called moduli, which parametrize the size and the shape of the extra compact dimensions. They couple with gravitational strength and are massless to all orders in perturbation theory. The moduli may get masses of the order of the Planck mass from non-perturbative effects; in this case they are not relevant to our considerations. It is, however, quite possible that they do not get masses until supersymmetry is broken. In this case their mass, because all
their couplings are gravitationally suppressed, will be of order

\[ m_{\text{moduli}} \sim \frac{F}{M_p}, \]  

(1.1)

where \( M_p = 2.4 \times 10^{18} \text{ GeV} \) is the reduced Planck mass, and \( F \) is the scale of supersymmetry breaking with dimensions of mass-squared. In gravity-mediated theories these moduli have weak scale masses and, since they are gravitationally coupled, they are not relevant to phenomenology. In gauge-mediated theories, if the scale of supersymmetry breaking is 10 TeV, the moduli can be quite light and have Compton wavelengths of a few microns \[ \text{[3]} \]. Finally, in theories with weak-scale compactification, the moduli can have Compton wavelengths of the order of a millimetre. In fact, taking the scale of supersymmetry breaking from \( F = (1 \text{ TeV})^2 \) to \( (10 \text{ TeV})^2 \) we find a range of \( \sim 1 \text{ mm to } 10 \mu\text{m} \) for the range of the Compton wavelengths of the moduli \[ \text{[4]} \]. Since the moduli are gravitationally coupled, they would lead to apparent deviations in Newton’s universal law of gravitation at these distances.

Of course, measuring gravitational strength forces at such short distances is quite challenging. Nevertheless, a number of recent heroic experimental proposals aim at looking for precisely such forces in this range \[ \text{[5]} \text{, } \text{[6]} \]. The most important background is the Van der Walls force. The Van der Walls and gravitational forces between two atoms are equal to each other when the atoms are about 100 \( \mu\text{m} \) apart. Since the Van der Walls force falls off as the 7th power of the distance, it rapidly becomes negligible compared to gravity at distances exceeding 100 \( \mu\text{m} \). Therefore, theories of weak-scale compactification offer us the distinct possibility of measuring gravitational-strength forces at distances in the millimetre range. In fact, as we shall argue later, these theories are very likely to contain at least one modulus of such large Compton wavelength. This modulus is the one associated with the size of the “large” compact dimension of the order of the weak scale. It is essentially the logarithm of the radius of the large extra dimension. As we will show later this modulus couples with a strength that is 1/3 that of gravity. In addition it is quite possible that
these theories possess other moduli which couple with strength up to $\sim 2000$ times gravity

The outline of this paper is as follows. We first sketch, in Section 2, the theories of weak-scale compactification and subsequently show, in Section 3, how the ultraviolet behaviour of these theories is tame, by using the analogy with finite-temperature field theory. As a result, although the soft supersymmetry breaking masses are “hard”, the vacuum energy density has no power law sensitivity to the cutoff $M_p$. Next and most important, in Section 4, we compute the mass range and the coupling of the large radius modulus to matter. Finally, in Section 5, we end with a summary of the special properties of weak-scale compactification theories.

Those results which are relevant for experiment are summarized in the figure. It depicts the strength of the modulus force relative to gravity versus distance. There we show all the presently allowed/excluded regions and the claimed capabilities of upcoming experiments. Particularly noteworthy are the horizontal lines corresponding to the “large radius modulus” and the dilaton. These theories can have several light moduli; their couplings are expected to lie between those of the dilaton and the large radius modulus.

2. Supersymmetry breaking with weak-scale compactification

A large compact dimension at the TeV range is one of the few general low-energy predictions of perturbative superstring theory, which relates its size $R$ with the scale of supersymmetry breaking. In fact, superstring theories do not possess any small dimensionful parameters associated with supersymmetry breaking $\mathbb{B}$. As a result, there are two ways to get a weak scale supersymmetry breaking mass in string theory. One is dimensional transmutation, which requires non-perturbative phenomena. The second is large radius compactification with $R \sim \text{TeV}^{-1}$. 
The main problem of such a large dimension is that all couplings become strong very rapidly above the decompactification scale, because of the contribution of the infinite tower of Kaluza-Klein (KK) excitations, which change the logarithmic evolution with the energy into a power, as expected in an effective higher-dimensional non-renormalizable field theory. Fortunately, this problem, which exists even before supersymmetry breaking is turned on, can be avoided in a particular class of string models that include the simple case of orbifold compactifications [2].

The spectrum of these theories consists of two sectors. The untwisted sector contains the states of the toroidal compactification, which are invariant under the action of the orbifold group. These are for instance all gauge multiplets. At energies above the decompactification scale, but much below the string scale, they are characterized by an integer wave number \( n \) corresponding to the quantized momentum along the extra large dimension. Since this sector is effectively higher-dimensional, the massive levels, \( n \neq 0 \), form multiplets of extended \( N \geq 2 \) supersymmetry (spontaneously broken to \( N = 0 \)). On the other hand, the massless modes form multiplets of \( N = 1 \) supersymmetry only, due to the orbifold projection. Finally, consistency of the theory implies in general the existence of a twisted sector corresponding to strings with centre of mass stacked at the orbifold fixed points. It follows that the resulting states, which typically are chiral, live on four-dimensional boundaries of the higher-dimensional theory. Thus, they are not accompanied by KK excitations and form multiplets of \( N = 1 \) (unbroken) supersymmetry.

It is now clear that the requirement for avoiding the large coupling problem, already before supersymmetry breaking, is to impose the vanishing of the contributions to the beta-functions of the massive untwisted tower of states, level by level. This is sufficient to be done at the one-loop level, since the effective \( N \geq 2 \) supersymmetry guarantees the absence of higher-loop corrections [2]. The simplest solution is when KK modes come into multiplets of \( N = 4 \) supersymmetry [3, 10] (possibly spontaneously broken to \( N \leq 2 \)).
Since $N = 4$ multiplets contain spin-1 fields, quark and lepton multiplets should come from the twisted sector due to phenomenological constraints, such as to avoid fast proton decay. The Higgs multiplet may originate from either sector, depending on model building \cite{12}. When it is untwisted, the Standard Model gauge group is enlarged in the higher-dimensional theory \cite{13}, while, when it is twisted, the KK excitations have just the quantum numbers of the $SU(3) \times SU(2) \times U(1)$ gauge group.

Since the presence of the infinite massive tower of $N = 4$ KK multiplets does not contribute to the beta-functions, the gauge couplings continue the logarithmic evolution, as determined by the usual $N = 1$ beta-functions. Inclusion of supersymmetry breaking gives negligible threshold effects, exponentially suppressed in the large radius limit \cite{2}. As a result, the value of the unification scale inferred from low energy data does not change. A possible worry may arise from string non-perturbative dynamics. In fact, although in these heterotic string models all four-dimensional couplings remain perturbative well above the decompactification scale, the ten-dimensional string coupling is strong, because of the large volume of the internal compactification space, which might be a source of stringy non-perturbative effects. These effects can be studied using duality and give rise to energy thresholds of the weakly coupled dual theory, which is type I string theory or M-theory, above which the effective field theory description breaks down \cite{14}. In the case of one large dimension, the thresholds of the dual theories turn out to be of the order of the unification scale, $\sim 10^{16}$ GeV, and are thus harmless for low energy physics and gauge coupling unification \cite{15}.

The simplest way of breaking supersymmetry spontaneously by compactification is the Scherk-Schwarz mechanism \cite{16, 17}. Higher-dimensional fields, instead of remaining periodic under a $2\pi$-rotation around the compact dimension, are allowed to pick up a (discrete) phase that can be absorbed by an R-symmetry transformation. This change of the bound-

\footnote{In general, additional constraints may have to be imposed to avoid potential growing of the Yukawa couplings.}
any condition amounts to shifting the momentum KK number $n$ by the (discrete) R-charge, which splits the supersymmetry multiplets. Note that only the untwisted sector of the theory feels this breaking, while twisted states do not carry internal momentum and remain untouched to lowest order. Communication of supersymmetry breaking arises through gauge interactions and can be studied in the context of the effective field theory \cite{12}. Since all mass splittings are of the order of the compactification scale, the scale of supersymmetry breaking is set by the same scale $R^{-1}$, which is at the TeV range.

This situation is very similar to that of spontaneously broken global supersymmetry. It is different from the situation where supersymmetry breaking originates from a new strongly interactive sector; the breaking scale is thus set by the corresponding QCD-like dynamical scale $\Lambda_s$. In the case of gauge-mediated models, due to the gauge coupling suppression, $\Lambda_s$ is at least of the order of 10–100 TeV \cite{18}, while for gravity-mediated models $\Lambda_s$ is at an intermediate scale of order $10^{13}$ GeV. We will see below that this difference has very important consequences.

A natural question arising in models with weak-scale compactification is to know which mechanism fixes the size of the extra dimension at the $\text{TeV}^{-1}$ scale. One possibility would be some strong supersymmetric dynamics at a higher scale. In fact, as mentioned in the introduction, the value of $R$ corresponds to a classical flat direction, which parametrizes the vacuum expectation value (VEV) of a modulus scalar field originating from the diagonal component of the metric along the extra dimension. The perturbative flatness could be lifted by non-perturbative effects. Such effects can either have string origin, or can be due to field theoretical strong gauge dynamics. However, the latter seems impossible since all low energy couplings are independent from $R$ before supersymmetry breaking, by construction. Furthermore, the former can be studied by going to the weakly coupled dual theory, as we mentioned before. It follows that as long as the radius modulus remains flat in the dual theory, its VEV cannot be fixed by supersymmetric non-perturbative dynamics.
Alternatively, $R$ is fixed only after taking into account the supersymmetry-breaking effects, which can drive electroweak symmetry breaking radiatively \cite{19}. The compactification scale could then be set by a new dynamical scale defined as the energy where the mass-squared of the Higgs becomes negative and leads to the breaking of the electroweak gauge symmetry. This mechanism, proposed in no-scale models \cite{20}, could be realized provided there are no quadratic divergences in the vacuum energy. As we will see below, this condition is fulfilled in our case and a precise study of this possibility deserves further investigation.

The main prediction of weak scale compactification models for high energy experiments is the existence of massive towers of KK modes with the quantum numbers of the Standard Model gauge bosons. These modes are unstable as they can decay into quarks and leptons within a short lifetime of the order of $10^{-26}$ s \cite{13}. Their clear experimental signature is the direct production, through for instance Drell-Yan processes in $pp$ and $p\bar{p}$ collisions \cite{21, 22}. At present energies, they lead only to small indirect effects, such as effective dimension-six four-fermion operators. The resulting limits come from bounds on compositeness and allow a compactification scale of one dimension to be as low as a few hundred GeV \cite{23, 13}.

3. Analogy with finite temperature

In order to be more explicit, without loss of generality for our conclusions, let us consider the simplest case of Scherk-Schwarz compactification of one large dimension using a $Z_2$ R-parity. It can be shown that such a generic and model-independent symmetry is the ordinary fermion number restricted to the untwisted sector of the theory \cite{2}. In this case all bosonic excitations are unaffected, while KK fermions split, since they become antiperiodic and obtain half-integer frequencies. It follows that the mass spectrum of the tower of
excitations which accompanies the massless states becomes:

vector multiplets : \[ M_B^2 = \frac{n^2}{R^2}; \quad M_F^2 = \frac{(n + \frac{1}{2})^2}{R^2} \]
twisted chiral multiplets : \[ M_T^2 = 0. \]  

(3.1)

Note that the above breaking is identical to the one obtained at finite temperature by replacing time with a space coordinate and identifying the temperature \( T \equiv R^{-1} \) in the five-dimensional theory. Using this analogy, we can understand some extremely soft ultraviolet properties of supersymmetry breaking in these models, such as the absence of quadratic divergences, or equivalently the vanishing of the supertrace \( \text{Str} M^2 \) in the effective potential. This is consistent with the fact that all soft breaking masses are in the TeV region. In fact, the vacuum energy density \( E \) in these models behaves as [24, 3]:

\[ E \sim (n_F - n_B) \frac{1}{R^4} + \mathcal{O}(e^{-R^2}), \]  

(3.2)

where \( n_F \) and \( n_B \) denote, respectively, the number of massless fermions and bosons after supersymmetry breaking. This result is independent of the requirement of \( N = 4 \) supersymmetry among the massive KK excitations, and can be understood as the analogue of the \( T^4 \) behaviour of the free energy at finite temperature.

To illustrate this phenomenon, let us consider the contribution of a single (five-dimensional) multiplet to the vacuum energy, in the field theory limit [24]:

\[
E = \frac{1}{2} \text{Str} \int \frac{d^4k}{(2\pi)^4} \ln(k^2 + M^2) = -\frac{1}{2} \text{Str} \int \frac{d^4k}{(2\pi)^4} \int_0^\infty ds \frac{e^{-s(k^2 + M^2)}}{s} \\
= -\frac{1}{32\pi^2} \int_0^\infty ds \frac{e^{-s(nF - nB)^2/R^2} - e^{-s(nF + nB)^2/R^2}}{s^3} \\
= -\frac{1}{32\pi^2} \int_0^\infty ds \frac{\pi R^2}{s} \left( \frac{1}{s} \right)^{1/2} \sum_m [1 - (-)^m] \frac{1}{s^2} \\
= -\frac{93 \zeta(5)}{2^{10} \pi^6} \frac{1}{R^4}, \]

(3.3)

where \( \zeta(5) \approx 1.037 \). From the second line of eq. (3.3), it follows that every single multiplet in the sum gives a quadratically divergent contribution which, in the proper time representation...
tation, behaves as $ds/s^2$ in the ultraviolet limit $s = 0$. The quartic divergence $ds/s^3$ cancels among bosons and fermions of the same multiplet, as expected. However, after summing over all modes the quadratic divergence disappears as well, which is clear from the convergent integral in the third line of eq. (3.3), where we performed a Poisson resummation. In other words, the contribution to the $S/M^2$ of the four-dimensional effective field theory describing the $n = 0$ mode is cancelled non-trivially by the infinite sum over the massive KK excitations with $n \neq 0$.

By analogy with finite temperature, the ultraviolet softness of these theories can also be understood by using an alternative description of Feynman propagators that makes manifest the Boltzman factors suppression at energies higher than the temperature. After summing over all KK excitations, the bosonic and the (square of) fermionic propagators yield:

$$
\Delta_B = \sum_n \frac{1}{k^2 + n^2/R^2} = \frac{\pi R}{k} \text{cth}(\pi R k) = \frac{\pi R}{k} \left\{ 1 + \mathcal{O}(e^{-2\pi R k}) \right\},
$$

$$
\Delta_F = \sum_n \frac{1}{k^2 + (n + \frac{1}{2})^2/R^2} = \frac{\pi R}{k} \text{th}(\pi R k) = \frac{\pi R}{k} \left\{ 1 + \mathcal{O}(e^{-2\pi R k}) \right\},
$$

where the leading behaviour in the ultraviolet limit is dictated by the five-dimensional supersymmetric field theory and the subleading corrections are exponentially suppressed. It follows that the short-distance limit of the difference of the two propagators is finite,

$$
\int \frac{d^3k}{(2\pi)^3} (\Delta_B - \Delta_F) < \infty,
$$

which shows that no new divergences are introduced after supersymmetry breaking. In particular, the vacuum energy behaves as $1/R^4 \equiv T^4$, which is determined by naive dimensional analysis.

At higher-loop level, an additional complication arises from the propagation of matter twisted fields that have no KK modes, but acquire soft scalar masses through radiative corrections. Then, the analogy with finite temperature holds only for graphs involving fields from the untwisted sector of the theory. Of course, graphs with only twisted fields
give vanishing contribution by supersymmetry. There remain the potentially dangerous diagrams with mixed propagation, which need more careful analysis. Consider for instance the two-loop supergraph containing a loop of a (massless) twisted state exchanging an untwisted mode from the tower of KK excitations. This diagram can be computed in two steps.

As a first step, we consider the one-loop correction to the two-point function of the untwisted mode, due to the propagation of the twisted fields. Obviously, this is a supersymmetric correction to the wave function renormalization of the KK mode, which makes, in particular, the coupling of all excited gauge bosons to run with the energy \[21\]. As a second step, we compute the contribution to the vacuum energy of the untwisted state—with one-loop corrected wave function—and perform the sum over all KK modes (3.1):

\[
E^{(2)}_{\text{mixed}} = \frac{1}{2} \gamma_T g^2 V_M^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2 \ln k^2}{k^2 + M^2},
\]

where \(\gamma_T\) is a numerical coefficient coming from the one-loop integration over the twisted states, \(g\) is the four-dimensional string coupling, and \(gV_M\) is the three-point vertex of the untwisted field with two twisted states. For generic orbifold, \(|V_M| = \delta M^2\), with \(\delta \leq 1\) a model-dependent constant related to the order of the orbifold twist [13]. In the large radius limit \(M^2 \to 0\) and for the leading contribution one can substitute \(V_M = 1\), since the sum over KK momenta \(n\) converges in eq. (3.6). Using now the mass formula (3.1) and introducing the proper time representation (3.3), eq. (3.6) becomes in the large \(R\) limit:

\[
E^{(2)}_{\text{mixed}} \sim \partial_\varepsilon |_{\varepsilon = 0} \text{Str} \int \frac{d^4k}{(2\pi)^4} (k^2)^{1+\varepsilon} \int_0^\infty ds e^{-s(k^2 + M^2)}
\]

\[
\sim \partial_\varepsilon |_{\varepsilon = 0} \int_0^\infty \frac{ds}{s^{3-\varepsilon}} \sum_n \left[ e^{-sn^2/R^2} - e^{-s(n+\frac{1}{2})^2/R^2} \right]
\]

\[
\sim \partial_\varepsilon |_{\varepsilon = 0} \int_0^\infty \frac{ds}{s^{3-\varepsilon}} \left( \frac{\pi R^2}{s} \right)^{1/2} \sum_m [1 - (-)^m] e^{-\pi^2 R^2/s}
\]

\[
\sim \frac{1}{R^4} \ln R.
\]

We will now show that the \(\sim 1/R^4\) behaviour of the vacuum energy (up to logarithms)
can in fact be understood as a result of the (global) supersymmetry algebra. In the globally supersymmetric limit, supersymmetry is spontaneously broken through the boundary conditions, and the goldstino is identified with the component of the five-dimensional gravitino along the extra dimension. In the \( N = 1 \) theory, there is only one zero mode that survives the orbifold projection, and the non-linearity in its transformation, which measures the supersymmetry breaking, is \([16]\):

\[
\delta \psi \sim \frac{1}{\kappa_5 R} \epsilon \sim R^{-3/2} \epsilon, \tag{3.8}
\]

where \( \kappa_5 = R^{1/2} M_p^{-1} \) is the five-dimensional gravitational coupling, and \( \epsilon \) is the supersymmetry transformation parameter. From the supersymmetry algebra then follows that the vacuum energy density of the theory must behave with the 3rd power of the gravitino mass, at least, which implies \( E \sim R^{-4} \) due to the evenness of the fermion number.

It should be stressed that the above behaviour of the vacuum energy holds in spite of the fact that sparticle masses are hard. In models with gauge-mediated supersymmetry breaking soft masses vanish with a power low above the messengers mass. On the contrary, in our case the mass shifts are hard, since they arise from a change of boundary conditions, and their behaviour is determined in the supersymmetric theory through the wave-function renormalizations. In the particular class of models we consider, all couplings and sparticle masses run logarithmically all the way up to the unification scale. For instance, gaugino masses run through the evolution of gauge couplings, which is unaffected by the presence of the infinite tower of massive excitations due to the effective \( N = 4 \) supersymmetry.

4. Moduli masses and millimetre range forces

We now study the question of moduli masses and their couplings to matter in theories with weak-scale compactification. As we already mentioned, all these theories contain a universal scalar modulus whose VEV determines the large radius \( R \). Furthermore, as we
argued in Section 2, its flatness is lifted only after supersymmetry breaking through a potential generated by radiative corrections. To lowest order, its mass can be read off from eq. (3.3), after taking into account that the field that has canonical kinetic terms is \( \phi = \ln R \), in units of the reduced Planck mass \( M_p \), as follows by direct dimensional reduction of the five-dimensional Einstein action:

\[
m_\phi = \left( \frac{93}{32\pi^6} N \right)^{1/2} \frac{R^{-2}}{M_p}.
\] (4.1)

Here, \( N \) is the number of massless untwisted multiplets (before supersymmetry breaking). Counting the Standard Model gauginos together with the higgsinos and the gravitino, one finds \( N \gtrsim 16 \), which yields:

\[
m_\phi \gtrsim 0.22 \frac{R^{-2}}{M_p}.
\] (4.2)

It is a general property of moduli that they are gravitationally coupled with their interactions to matter arising through the dependence of the low energy couplings. Since in the absence of supersymmetry breaking all dimensionless couplings are independent of the radius modulus, its coupling to matter is extremely suppressed. In the presence of supersymmetry breaking, it acquires direct couplings through the soft masses, which again lead to suppressed interactions. It turns out that the dominant coupling comes through the logarithmic running of gauge couplings after supersymmetry breaking. This dependence arises as follows.

Consider the nucleon mass term in the low energy effective Lagrangian:

\[
\mathcal{L}_N = m_N(\phi) \bar{N} N,
\] (4.3)

where the nucleon mass \( m_N(\phi) \), which depends on the canonically normalized radius modulus \( \phi = \ln R \), is proportional to the QCD scale \( \Lambda_{\text{QCD}} \). Since the sparticle masses are all proportional to \( 1/R \), the sparticle threshold \( \Lambda_{\text{SUSY}} \) is proportional to \( 1/R \). The coupling of the radius modulus to nucleons is therefore given by the derivative of the nucleon mass
relative to the modulus $\phi$:

$$\frac{\partial m_N}{\partial \phi} = \frac{\partial \Lambda_{\text{QCD}}}{\partial \ln \Lambda_{\text{SUSY}}}.$$  (4.4)

Since the graviton coupling to the nucleon is $m_N/M_p$, the coupling $\alpha$ of the modulus relative to gravity is

$$\alpha\phi = \frac{\partial \ln \Lambda_{\text{QCD}}}{\partial \ln \Lambda_{\text{SUSY}}} = 1 - \frac{b_{SS}}{b_{NS}} = \frac{4}{7},$$  (4.5)

where $b_{SS}$ is the supersymmetric beta-function, which includes all the sparticles, and $b_{NS}$ is the non-supersymmetric beta-function, which includes just the ordinary particles. In this formula we assumed for simplicity that all sparticles have the same mass $\sim \Lambda_{\text{SUSY}}$. Thus we see that the force between two pieces of matter mediated by the radius modulus is $(4/7)^2 \approx 1/3$ times the force of gravity.

In principle there can be other light moduli which couple with larger strengths. For example the dilaton, whose VEV determines the (logarithm of the) string coupling constant, if it does not acquire large mass from some dynamical supersymmetric mechanism, leads to the strongest effect. Its coupling is

$$\alpha_{\text{dilaton}} = \ln \frac{M_p}{\Lambda_{\text{QCD}}} \sim 44,$$  (4.6)

which corresponds to a strength $\sim 2000$ times bigger than gravity. Therefore, a generic prediction of these theories is the existence of moduli with Compton wavelengths in the millimetre range and with couplings ranging from $\sim$ gravitational to several times larger. The important point we want to emphasize here is that at least one of these, namely the radius modulus, appears to be a necessary consequence of the weak-scale compactification theories.

In the figure we depict our theoretical predictions together with information from present and upcoming experiments. The vertical axis is the strength, $\alpha^2$, of the force relative to gravity; the horizontal axis is the Compton wavelength of the exchanged particle; the upper scale shows the corresponding value of the large radius in TeV. The solid
Strength of the modulus force relative to gravity ($\alpha^2$) versus the Compton wavelength ($\lambda$) of the modulus. The upper scale shows the corresponding value of the large radius in TeV.
lines indicate the present limits from the experiments indicated [26]. The excluded regions lie above these solid lines. Our theoretical prediction for the radius modulus is the thin horizontal line at 1/3 times gravity. The dilaton-mediated force is the thin dashed horizontal line at $\sim 2000$ times gravity. The proposed experiments are sensitive to dilatons of Compton wavelengths as small as $10 \mu m$. Other moduli may mediate forces between the above two extremes.

The most important part of the figure is the dashed thick line; it is the expected sensitivity of two heroic proposed experiments by Price et al. and by Kapitulnik et al. [5, 6]. They use cantilever technology or micro-electromechanical systems utilizing microsensors based on silicon micromachining. On the time-scale of a year they will improve the present limits by almost 6 orders of magnitude and –at the very least– they will, for the first time, measure gravity to a precision of 1% at distances of $\sim 100 \mu m$.

As we see from the figure, if the ideas presented in this paper are correct, these experiments have a good chance of finding the force mediated by the radius modulus. In fact, if the scale of supersymmetry breaking is at its most likely range of a TeV, the force is just below the sensitivity of the Mitrofanov et al. experiment. Actually, scales smaller than a TeV are already excluded from this experiment. If the radius modulus exists, then it becomes inaccessible to the proposed experiments only if the scale of supersymmetry breaking is higher than 5 TeV. This scale is uncomfortably large for the hierarchy problem: it requires fine tunings in excess of one part in $\sim 100$. It is also too large for the LHC to have a shot at finding supersymmetric particles. It is therefore fair to conclude that if the proposed experiments do not find the force mediated by the radius modulus, they will shed strong doubts on the idea of weak-scale compactification.
5. What is special about weak-scale compactification theories?

There are key differences between weak-scale compactification and gauge- or gravity-mediated theories. One is that weak-scale compactification leads to direct supersymmetry breaking without the need for an intervening sector that feels primordial supersymmetry breaking. This is only possible because the underlying theory is higher-dimensional and therefore sidesteps the four-dimensional supertrace theorems which make direct supersymmetry breaking phenomenologically impossible in four dimensions. It has the crucial advantage that the largest supersymmetry-breaking splittings in the full theory are at the weak scale. Consequently the Compton wavelength of the moduli, as given by eq. (1.1), is large.

A related unique theoretical feature of these theories is the simultaneous occurrence of hard gluino and squark masses $\Lambda_{\text{SUSY}} \sim \text{TeV}$ together with vacuum energy that is $\Lambda_{\text{SUSY}}^4 \sim (\text{TeV})^4$, i.e. the vacuum energy is at most logarithmically sensitive to the ultraviolet cutoff. This opens the possibility that the VEVs of some moduli may be predominantly determined by low energy (weak scale) physics. This suggests that the weak-scale compactification theories provide a natural home for the no-scale models. In fact the no-scale mechanism for fixing the scale of supersymmetry breaking, as the place where the Higgs mass-squared turns negative, seems to be the most economical way of dynamically determining the compactification radius. The guaranteed absence of strong ultraviolet sensitivity makes this possible.

Another application of this is the mechanism of dynamical alignment. This is the idea that the orientation of the soft terms in flavour space depends on moduli whose VEVs are not determined by ultraviolet physics. This implies that their VEVs, for energetic reasons, will line-up with the fermion masses for the same reason spins align with a magnetic field. In fact, such a mechanism may be needed in these theories since the sfermion masses are hard and, consequently, can be distorted by Planckian flavour physics.
Another feature of weak-scale compactification is the existence of the large radius; this makes it extremely likely that the modulus corresponding to the large radius is massless until supersymmetry breaking takes over at the weak scale. Therefore, the existence of at least one light modulus seems unavoidable. In other words, if a force of $\sim 1/3$ times gravity in the millimetre range is not measured, it will cast serious doubt on the idea of weak-scale compactification.

Our most important conclusions are well summarized in the figure. There we see the future possibilities for the detection of sub-centimetre gravitational strength forces. These searches are important for two reasons. First they are, for the first time, going to detect gravity in a totally new range of distances. This is a sure thing, but very important nevertheless. Secondly, they may discover a new force of Nature which, according to the theories discussed here, will give us a first glimpse into extra dimensions of spacetime.

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