On trilinear terms in the scalar potential of 3-3-1 gauge models

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Abstract

The trilinear terms of the form $\sqrt{2} f \epsilon^{ijk} \rho_i \chi_j \phi_k$ in the scalar potential of a 3-3-1 gauge model are considered. When looking for the eigenbasis of the massive physical Higgs bosons that survive the spontaneous symmetry breakdown of the model - in light of the observed SM-like Higgs boson with mass $m_h \simeq 125$ GeV reported in 2012 at the LHC - one gets a strong constraint to the cubic term. It has to be $f \ll w$, in flagrant contradiction with the large one $f \simeq w$ which is propagated in the literature to date.

Introduction

In this letter we argue that the coupling of the trilinear terms of the form $\sqrt{2} f \epsilon^{ijk} \rho_i \chi_j \phi_k$ included in the scalar potential of 3-3-1 gauge models cannot range $f \simeq w$ (as it has long been considered in the literature, to our best knowledge). Here, by $w$ we mean the highest VEV in the model - the one responsible for the first step of the symmetry breaking in the electroweak sector $SU(3)_L \otimes U(1)_X \rightarrow SU(2)_L \otimes U(1)_Y$. The other two VEVs, namely $v$ and $u$ respectively, achieve the last step of the symmetry breaking to the universal electromagnetic group $SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{em}$. Obviously, $v, u \ll w$. Moreover, the $f$ coupling must be much smaller ($f \ll w$) in order for the model to supply plausible phenomenological consequences, at least regarding the low-energy regime with a special focus on the SM-like Higgs boson $H$. In our discussion here, a non-canonical approach [2] to gauge models with high symmetries is worked out in the particular case of 3-3-1 models. This approach - utterly equivalent with the main one exploited in the literature, as shown in Ref. [3] - leads to an appealing outcome. The couplings (charges) for the electric and neutral currents of all the fermion fields are exactly computed [4] and a one-parameter mass scale can be inferred for the boson mass spectrum [3, 4] of the model. Once these results obtained, a rich phenomenology can be systematically investigated and thus certain restrictions on the parameters can be inferred. However, the statement regarding the trilinear term in the potential remains valid regardless the approach involved in treating the gauge model, as we will conclude in the subsequent sections.
Why 3-3-1 gauge models?

In spite of its great success with respect to the particle physics, the theory of the Standard Model (SM) lacks in explanations for several important issues. In this connection, one can list some embarrassing questions still awaiting their appropriate answers: (i) why there are precisely three fermion families in nature? (ii) how does it come that the masses of leptons are ranging apparently so widely? (iii) what provides us with the observed pattern of the quark mass spectrum and mixings? (iv) what is the mechanism responsible for generating the tiny neutrino masses? (v) are these neutral particles of Dirac or Majorana nature? (vi) why the neutrino mixing pattern differs so sharply from the quark mixing pattern? (vii) what about the strong-CP issue? (viii) what are the best candidates for the so called “dark matter”? (ix) what about the flavor changing neutral currents (FCNC) and the restrictions their suppression imposes? (x) why lepton flavor mixing is allowed in the neutrino sector only?

Therefore, some extensions of the SM have emerged in the last decades, by simply enlarging the gauge group. Such models as $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ (in short 3-3-1, see Refs. [5] - [11]) or $SU(3)_C \otimes SU(4)_L \otimes U(1)_X$ (in short 3-4-1, see Refs. [12]) are intensively studied in the literature. These kind of models have many advantages and promising features. Among them we can single out some:

- These SM-extensions can explain the number of fermion families [5, 8], since a smart interplay among the families (which are not identical replicas to one another) takes place in order to make the model anomaly-free. The anomaly cancellation procedure requires the number of families be multiple of the number of QCD colors, but if one assumes furthermore the QCD asymptotic freedom condition, this number yields precisely $3!$

- These 3-3-1 gauge models do exhibit a natural Peccei-Quinn chiral symmetry [6] able to solve the strong-CP puzzle without need to artificially impose supplementary conditions.

- The charge quantization (somehow enforced in the SM) is achieved in the most natural manner within the framework of these models [7].

- They, also unfold a rich Higgs sector [9] to be investigated in detail in order to single out the SSB agents with the whole gamut of their properties.

- The neutrino phenomenology gets its proper framework [10], as these models supply the necessary ingredients for various seesaw or radiative mechanisms to be employed to generate the appropriate tiny masses for those eluding neutral particles.

- In addition, when it comes to the peculiar issue of the so called “dark matter” in the universe, one can find out some plausible candidates [11] among the rich spectrum of particles this class of models generously exhibits.
We will confine our list of 3-3-1 and 3-4-1 models advantages to the above presented arguments.

Furthermore, let’s briefly survey the procedure employed here for “solving” such models. It is a general method (conceived by Cotăescu\cite{2} two decades ago) that lead to a renormalizable model with high symmetry. The model’s dynamics comes - via Euler-Lagrange equations - from a Lagrangian density \( L = L_S + L_G + L_H + L_Y \) (with \( S \) - for spinor sector, \( G \) - for gauge sector, \( H \) - for Higgs sector, \( Y \) - for Yukawa sector) by imposing a certain gauge symmetry. Its novelty consists in a particular Higgs mechanism by means of which a general gauge group \( SU(3)_C \otimes SU(n) \otimes U(1) \) undergoes a spontaneous symmetry breaking (SSB), in a single step, up to \( SU(3)_C \otimes U(1)_{em} \) in a kind of geometrical manner. This approach assumes the existence of a single scalar variable \( \phi \sim (1, 1, 0) \) that acts as a norm for the \( n \)-dimensional vector space of the scalar multiplets in the model, once the orthogonality condition among scalar multiplets \( \phi_i^\dagger \phi_j = \phi^2 \delta_{ij} \) is postulated. The method also presumes a set of different parameters \( \eta_0, \eta_1, \ldots, \eta_n \) to be introduced from the very beginning in the scalar sector in order to finally supply a non-degenerate mass spectrum for all the vector bosons. However, this particular approach is utterly compatible with the canonical approach in the literature where \( n \) Higgs fields supply \( n \) vacuum expectation values (VEV), once a proper redefinition of the scalar fields is performed in the Cotăescu method. This was shown by the author in detail for the 3-3-1 models in Refs.\cite{3, 4} and for 3-4-1 models in Refs.\cite{12}. Hence, in the case of the 3-3-1 models the prescriptions for the boson masses provides us with a single remaining parameter (say \( a \)), out of the three initially considered \( \eta_1, \eta_2, \eta_3 \) that obey the trace condition \( Tr \eta^2 = 1 - \eta_0^2 \). The single remaining parameter has to be tuned according to the available data. At the same time, the electric and the weak charges are straightforwardly computed\cite{4}, since the new “would be hypercharges” \( X \) of the model are established and the set of versors \( \nu_i \) for the general Weinberg transformation (gWt)\cite{2} needed to separate the massive neutral bosons are properly chosen. The versors fulfill the natural relation \( \nu_i \nu^i = 1 \). Their particular choice is a matter of discriminating among the resulting 3-3-1 models. The general method allows for only two plausible 3-3-1 models: (i) \( \nu_0 = 1, \nu_1 = 0, \nu_2 = 0 \) which leads precisely to the “minimal model” (Pisano-Pleitez-Frampton)\cite{3,4} and (ii) \( \nu_0 = 0, \nu_1 = 0, \nu_2 = 1 \) which leads to the so called “no-exotic electric charge” or “right-handed neutrino model” (Hoang Ngoc Long)\cite{8-11}. One could conceive one more model for \( \nu_0 = 1, \nu_1 = 0, \nu_2 = 0 \) , but it proves itself meaningless since all the fermions in the triplet representation are then restricted to carry the same electric charge (which is of no physical use). As the phenomenological details of the general method\cite{2} were worked out in extenso elsewhere\cite{3, 4}, we do not enter in details again here. The fermion content as well as the gauge boson content can be found in Refs.\cite{8, 10, 11}. We briefly list here, in a self-explanatory notation:

**Leptons**
\[ f_{\alpha L} = \left( \begin{array}{c} N^c_{\alpha} \\ \nu_{\alpha} \\ e_{\alpha} \end{array} \right)_L \sim (1, 3, -1/3) \quad \quad (e_{\alpha L})^c \sim (1, 1, 1) \]

with \( \alpha = 1, 2, 3 \).

Quarks

\[ Q_{iL} = \left( \begin{array}{c} D_i \\ -d_i \\ u_i \end{array} \right)_L \sim (3, 3^*, 0) \quad , \quad Q_{3L} = \left( \begin{array}{c} T_3 \\ t \\ b \end{array} \right)_L \sim (3, 3, -1/3) \]

\[(b_{L})^c, (d_{iL})^c \sim (3, 1, 1/3) \quad \quad (t_{L})^c, (u_{iL})^c \sim (3, 1, -2/3) \]

\[(T_{3L})^c \sim (3, 1, -2/3) \quad \quad (D_{iL})^c \sim (3, 1, 1/3) \]

with \( i = 1, 2 \). The capital letters denote exotic leptons and quarks, even though their electric charges are not different from those of the fermions coming from the SM. In the parentheses we put the representations with respect to the gauge group of the model.

The gauge fields sector corresponding to \( SU(3)_L \) can be expressed as the adjoint representation of the group:

\[ A_\mu = \frac{1}{2} \left( \begin{array}{ccc} A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2} U^*_\mu & \sqrt{2} V^*_\mu \\ \sqrt{2} U^\dagger_\mu & -A^3_\mu + A^8_\mu / \sqrt{3} & \sqrt{2} W^*_\mu \\ \sqrt{2} V^\dagger_\mu & \sqrt{2} W^\dagger_\mu & -2 A^8_\mu / \sqrt{3} \end{array} \right) \]

One can easily identify the SM charged boson \( W^\pm \). However, there are two more off-diagonal exotic bosons: one charged \( (V^\pm) \) and one neutral \( (U = U^\dagger) \). The pure neutral bosons (Hermitian ones) \( A^{em} \) (massless), \( Z^0 \) (90.1GeV) and the new \( Z' \) (very massive) are obtained via \( gWt \) applied to the diagonal entries in the above matrix after one added the \( A_0 = XI \) corresponding to \( U(1)_X \). The masses of these bosons can be inferred from the general prescriptions of the Cotăescu method and that becomes a matter of tuning a single free parameter, let’s call it \( a \) running in \([0, 1]\) (see Ref.\[3, 4\]).

Assuming that the phenomenology favors \( a \to 0 \) (small values for the free parameter, rather than \( a \to 1 \)) - see for details Ref. \[3\] where the compatibility with the canonical approach with split VEVs is presented - one gets roughly the mass spectrum:

\[ m(U) \simeq m(V) = \frac{m(W)}{\sqrt{a}} \quad (1) \]

For \( Z' \), when considering \( \sin^2 \theta_W \simeq 0.223 \) \([13]\), one obtains \[3\]:

\[ m(Z') \simeq \frac{m(W)}{\sqrt{a}} \left( \frac{2 \cos \theta_W}{\sqrt{3 - 4 \sin^2 \theta_W}} \right) = 1.2 \frac{m(W)}{\sqrt{a}} \quad (2) \]

The approach allows now the tuning of the single free parameter \( a \) in order to get a realistic mass spectrum for the bosons involved in the class of 3-3-1.
method, becomes

\begin{align}
\langle \varphi \rangle = 1 TeV & \quad \langle \varphi \rangle = 5 TeV \\
a = 0.06 & \quad a = 0.0024
\end{align}

models analyzed above, in dependence of the breaking scale \( \sqrt{a} \langle \varphi \rangle = \langle \varphi \rangle_{SM} \) (formula (32) in Ref. [3]). The masses can be summarized in the Table 1 (where we took \( m(W) \approx 80.4 \text{GeV} \) as supplied by Ref. [13]).

Scalar potential

The scalar triplets for the 3-3-1 model under consideration here [3] stand in the following representations:

\[
\begin{pmatrix}
\rho^0 \\
\rho^0 \\
\rho^-
\end{pmatrix}, \quad
\begin{pmatrix}
\chi^0 \\
\chi^0 \\
\chi^-
\end{pmatrix} \sim (1, 3, -\frac{1}{3}), \quad
\begin{pmatrix}
\phi^+ \\
\phi^+ \\
\phi^0
\end{pmatrix} \sim (1, 3, \frac{2}{3})
\]

(3)

The most general potential allowed by the gauge invariance of the model can be put in the following form:

\[
V(\rho, \chi, \phi) = -\mu_1^2 \rho^+ \rho - \mu_2^2 \chi^+ \chi - \mu_3^2 \phi^+ \phi + \lambda_1 (\rho^+ \rho)^2 + \lambda_2 (\chi^+ \chi)^2 + \lambda_3 (\phi^+ \phi)^2 + \lambda_4 (\rho^+ \rho) (\chi^+ \chi) + \lambda_5 (\rho^+ \rho) (\phi^+ \phi) + \lambda_6 (\chi^+ \chi) (\phi^+ \phi) + \lambda_7 (\rho^+ \chi) (\rho^+ \phi) + \lambda_8 (\rho^+ \phi) (\phi^+ \rho) + \lambda_9 (\chi^+ \phi) (\phi^+ \chi) - (\sqrt{2} f \epsilon^{ijk} \rho_i \chi_j \phi_k + h.c.)
\]

(4)

which, under the orthogonality restriction \( \phi^+_i \phi_j = \varphi^2 \delta_{ij} \), required by the general method, becomes

\[
V(\rho, \chi, \phi) = -\mu_1^2 \rho^+ \rho - \mu_2^2 \chi^+ \chi - \mu_3^2 \phi^+ \phi + \lambda_1 (\rho^+ \rho)^2 + \lambda_2 (\chi^+ \chi)^2 + \lambda_3 (\phi^+ \phi)^2 + \lambda_4 (\rho^+ \rho) (\chi^+ \chi) + \lambda_5 (\rho^+ \rho) (\phi^+ \phi) + \lambda_6 (\chi^+ \chi) (\phi^+ \phi) - (\sqrt{2} f \epsilon^{ijk} \rho_i \chi_j \phi_k + h.c.)
\]

(5)

The orthogonality restriction in the general method is simply intended to avoid the unwanted Goldstone bosons that could survive the SSB. For our case
at hand, the initial set of scalar triplets \((\phi_i, \text{ with } i = 1, 2, 3, \text{ in Cotăescu’s notation \[2\]}\), actually consists of 18 real field variables. By imposing \textit{ab initio} the 9 orthogonal relations to the scalar potential, one restricts the number of the real field variables to only 9, of which 5 “are eaten” by the gauge bosons to become massive and 1 can be removed by the gauge fixing. The 3 remaining ones will supply precisely the three neutral Higgses we are left with.

The coupling of the trilinear term in the potential above bears obviously a mass dimension and it is assumed in the literature [\[9\]}, without any justification, to be \(f \simeq w\) (the highest VEV).

The minimum conditions

\[
\frac{\partial V}{\partial \rho} = 0 \big|_{\langle \rho \rangle = w}, \quad \frac{\partial V}{\partial \chi} = 0 \big|_{\langle \chi \rangle = v}, \quad \frac{\partial V}{\partial \phi} = 0 \big|_{\langle \phi \rangle = u} \quad (6)
\]

applied to the shifted potential restricted to the surviving Higgses

\[
V(H_\rho, H_\chi, H_\phi) = -\frac{1}{2} \mu_1^2 \left( H_\rho + \langle \rho \rangle \right)^2 + \frac{1}{2} \lambda_1 \left( H_\rho + \langle \rho \rangle \right)^4 - \frac{1}{2} \mu_2^2 \left( H_\chi + \langle \chi \rangle \right)^2 + \frac{1}{4} \lambda_2 \left( H_\chi + \langle \chi \rangle \right)^4
\]

\[
- \frac{1}{2} \mu_3^2 \left( H_\phi + \langle \phi \rangle \right)^2 + \frac{1}{4} \lambda_3 \left( H_\phi + \langle \phi \rangle \right)^4 + \frac{1}{4} \lambda_4 \left( H_\rho + \langle \rho \rangle \right)^2 \left( H_\chi + \langle \chi \rangle \right)^2
\]

\[
+ \frac{1}{4} \lambda_5 \left( H_\rho + \langle \rho \rangle \right)^2 \left( H_\phi + \langle \phi \rangle \right)^2 + \frac{1}{4} \lambda_6 \left( H_\chi + \langle \chi \rangle \right)^2 \left( H_\phi + \langle \phi \rangle \right)^2
\]

\[- 2 \sqrt{2} f \left( \frac{H_\rho + \langle \rho \rangle}{\sqrt{2}} \right) \left( \frac{H_\chi + \langle \chi \rangle}{\sqrt{2}} \right) \left( \frac{H_\phi + \langle \phi \rangle}{\sqrt{2}} \right) + h.c. \quad (7)
\]

supply - via the linear terms cancellation - the following relations:

\[-\mu_1^2 + \lambda_1 w^2 + \frac{\lambda_4}{2} v^2 + \frac{\lambda_5}{2} u^2 - f \frac{vu}{w} = 0\]

\[-\mu_2^2 + \lambda_2 v^2 + \frac{\lambda_4}{2} w^2 + \frac{\lambda_6}{2} u^2 - f \frac{uv}{v} = 0\]

\[-\mu_3^2 + \lambda_3 u^2 + \frac{\lambda_5}{2} w^2 + \frac{\lambda_6}{2} v^2 - f \frac{uw}{u} = 0\]

(8)

They lead straightforwardly to the following Higgs mass matrix

\[
M^2 = \begin{pmatrix}
2\lambda_1 w^2 + f \frac{vu}{w} & \lambda_4 uv - f u & \lambda_5 uw - f v \\
\lambda_4 uv - f u & 2\lambda_2 v^2 + f \frac{uw}{v} & \lambda_6 uv - f w \\
\lambda_5 uw - f v & \lambda_6 uv - f w & 2\lambda_3 u^2 + f \frac{uw}{u}
\end{pmatrix} \quad (9)
\]

Now, if we take into consideration the hypothesis propagated in the literature that \(f \simeq w\) one has to deal with the following matrix
\[ M^2 = w^2 \begin{pmatrix} 2\lambda_1 + \frac{uv}{w} & \lambda_4 \frac{(v-u)}{w} & \lambda_5 \frac{(u-v)}{w} \\ \lambda_4 \frac{(v-u)}{w} & 2\lambda_2 \frac{v^2}{w^2} + \frac{u}{v} & \lambda_6 \frac{uv}{w^2} - 1 \\ \lambda_5 \frac{(u-v)}{w} & \lambda_6 \frac{uv}{w^2} - 1 & 2\lambda_3 \frac{u^2}{w^2} + \frac{v}{u} \end{pmatrix} \] (10)

that becomes:

\[ M^2 \simeq w^2 \begin{pmatrix} 2\lambda_1 & 0 & 0 \\ 0 & \frac{u}{v} & -1 \\ 0 & -1 & \frac{v}{u} \end{pmatrix} \] (11)

by simply erasing the negligible ratios, under the usual assumption \( w \gg v, u \).

One can now notice that, somehow naturally, the lighter degrees of freedom are decoupled from the heavier one without imposing any supplementary restriction which in turn provides us with a safe behavior regarding the SM phenomenology which does not interfere with the new physics of the model. Notwithstanding, this leads to \( m^2(H) = 2\lambda_1 w^2 \) for the heavier Higgs and \( m^2(h_1) \simeq w^2, m^2(h_2) = 0 \) for the SM-like Higgs sector. In view of LHC results [1], the above obtained pair of SM-like Higgses are unacceptable physical solutions, as \( w \) ranges in the TeV region. This outcome simply must be ruled out.

We now follow a different strategy: enforce the decoupling hypothesis (as in Ref. [3]), but considering a more realistic \( f \simeq kw \). Under this assumption we will look for restrictions (if any) to be imposed on the coefficient \( k \). Decoupling the heaviest Higgs - entry 11 in the matrix (9) -, one has to deal with the following restrictions

\[ \lambda_4 \simeq f \frac{u}{w}, \quad \lambda_5 \simeq f \frac{v}{w} \] (12)

which provides us with the masses

\[ m^2(H) = 2\lambda_1 w^2 \] (13)

for the heaviest Higgs boson, and the matrix

\[ m^2 = \begin{pmatrix} 2\lambda_2 v^2 + \lambda_4 w^2 & \lambda_6 uv - \lambda_5 \frac{uv}{w} \frac{w^2}{v} \\ \lambda_6 uv - \lambda_4 \frac{uv}{w} \frac{w^2}{v} & 2\lambda_3 u^2 + \lambda_5 w^2 \end{pmatrix} \] (14)

for the SM-like pair of Higgs bosons.

By diagonalizing (14) one is led to the following masses

\[ m^2(h_1) \simeq f \left( \frac{u^2 + v^2}{uv} \right) w, \quad m^2(h_2) \simeq 0 \] (15)
This result could still seem troublesome in view of the LHC exclusion bounds for such an almost massless CP-even scalar \((h_2)\). But this scalar is actually sterile, since its couplings to all the bosons in the model vanish, as the eq.(35) in Ref. [3] explicitly shows. Formally, due to the parameter interplay in the Cotăescu’s method, the physical state corresponding to the third scalar field \((h_2)\) simply erases itself from the spectrum since \(H_\chi = \eta_2 \varphi\) and \(H_\phi = \eta_3 \varphi\) and the physical states \([3]\) corresponding to the massive SM-like Higgses now read

\[
 h_1 = \frac{\eta_2 H_\chi + \eta_3 H_\phi}{\sqrt{\eta_2^2 + \eta_3^2}} \quad h_2 = \frac{-\eta_3 H_\chi + \eta_2 H_\phi}{\sqrt{\eta_2^2 + \eta_3^2}} \quad (16)
\]

which lead to the precise identification: \(h_1 = \sqrt{\eta_2^2 + \eta_3^2} \varphi\) and \(h_2 = 0\), where \(\varphi\) is the Higgs field acting as an orthogonal norm in the vector space of the scalar fields. It obviously develops an overall VEV \(\langle \varphi \rangle\), while \(\eta_1\), \(\eta_2\) and \(\eta_3\) are the parameters previously introduced to split the latter into the three VEVs in the model (for more details see Ref. [2]).

In our particular 3-3-1 model following the prescriptions of the general method there are required three parameters [4], such as

\[
 \eta_1 = \sqrt{1 - a} \quad \eta_2 = \sqrt{\frac{a(1 - \tan^2 \theta_W)}{2}} \quad \eta_3 = \frac{\sqrt{a}}{\sqrt{2} \cos \theta_W} \quad (17)
\]

with the trace condition realized in the manner \(1 = \eta_1^2 + \eta_2^2 + \eta_3^2\).

Hence, the VEVs splitting is finally realized by the unique parameter \(a\) as follows

\[
 w = \sqrt{1 - a} \langle \varphi \rangle \quad v = \sqrt{\frac{a(1 - \tan^2 \theta_W)}{2}} \langle \varphi \rangle \quad u = \frac{\sqrt{a}}{\sqrt{2} \cos \theta_W} \langle \varphi \rangle \quad (18)
\]

So, according to eqs. (13), (10) and (11), one gets the physical CP-even Higgs spectrum in the following form:

\[
 m^2(H) = 2\lambda_1 (1 - a) \langle \varphi \rangle^2 \quad m^2(h) \approx k \left( \frac{2 \cos \theta_W}{\sqrt{1 - \tan^2 \theta_W}} \right) (1 - a) \langle \varphi \rangle^2 \quad (19)
\]

and nothing else!

Now one can establish a straightforward relation between the two surviving Higgses:

\[
 m(H) = \frac{\lambda_1}{k} \left( \frac{\sqrt{1 - \tan^2 \theta_W}}{\cos \theta_W} \right) m(h) \quad (20)
\]

Numerically, under the usual assumption that \(\lambda_1 \simeq 1\), this becomes
\[ m(H) \simeq \frac{1}{\sqrt{k}} 0.122 \text{ TeV} \] (21)

which ranges dependently on the trilinear coupling \( k \).

**Phenomenological scenario**

With \( \lambda_1 \simeq 1 \), the coefficient \( k \) cannot be \( k \sim 1 \), but much lesser than \( 1 \) - say of order \( \sim 10^{-3} \), for keeping the heavier Higgs somewhere around the TeV threshold. For a more accurate estimate one can equate (13) and (21)

\[ \sqrt{2\lambda_1(1-a)} \langle \varphi \rangle = \frac{1}{\sqrt{k}} 0.122 \text{ TeV} \] (22)

then yielding

\[ k = 0.007442 \left( \frac{\text{TeV}}{\langle \varphi \rangle} \right)^2 \] (23)

Under these circumstances, the conclusion is definitely \( f \ll w \)! For a reasonable \( \langle \varphi \rangle \sim 1 \text{ TeV} \), \( k \) must be of order \( \sim 10^{-3} \). In conclusion, the higher the overall breaking scale \( \langle \varphi \rangle \), the more suppressed the trilinear coupling.

We must mention that recently the same restriction assumed by the authors in Ref. [14] led to a plausible phenomenology of the neutrino sector where a type-II seesaw mechanism was employed to get tiny neutrino masses. At the same time in Ref. [15] the authors conclude that the stability of a plausible scalar Dark Matter candidate imposes a strong suppression for \( f \), while in [16] the same coupling discriminates among charged Higgs bosons of the model. During the review process of our manuscript our attention was drawn to the most recent work [17] dealing with 3-3-1 model’s scalar sector, where quite similar restrictions on \( f \) yielded from numerical analysis of the loop-induced Higgs decays \( (H \rightarrow Z\gamma, \gamma\gamma) \). In conclusion, our simple and rough result - based strictly on the hypothesis of decoupling the heavier Higgs neutral particle from the low-energy scale of the 3-3-1 model - opens up a promising phenomenological outcome to be further investigated.

**Conclusions**

In this letter we presented a rough analysis of the scalar potential of a 3-3-1 gauge model based on the parametrization supplied by the Cotăescu method. This approach recovers all the features supplied by the canonical approach, but its main result is that the trilinear coupling must be much lesser than \( w \) in order to infer a plausible mass for the Higgs bosons spectrum. Consequently, we obtain the mass of the heavier Higgs of the model as a magnitude depending only on the trilinear coupling, once the mass of the SM-like Higgs is firmly established 125 GeV.
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