The influence of small-scale magnetic field on the evolution of inclination angle and precession damping in the framework of 3-component model of neutron star

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Abstract. The evolution of inclination angle and precession damping of radio pulsars is considered. It is assumed that the neutron star consists of 3 "freely" rotating components: the crust and two core components, one of which contains pinned superfluid vortices. We suppose that each component rotates as a rigid body. Also the influence of the small-scale magnetic field on the star’s braking process is examined. Within the framework of this model the star simultaneously can have glitch-like events combined with long-period precession (with periods 10⁻¹⁰⁴ years). It is shown that the case of the small quantity of pinned superfluid vortices seems to be more consistent with observations.

1. Introduction
Radio pulsars can be considered as exceptionally stable clocks. But sometimes besides smooth braking their rotation suffers some irregularities called glitches [1, 2]. Usually it is related to superfluid pinning. Pinned vortices do not allow superfluid to participate in pulsar braking. It causes the sudden unpinning of vortices at some moments and the transfer of angular momentum from superfluid to the crust [2]. That is observed as a glitch. There are some evidences that isolated neutron stars may precess with large periods $T_p$ of precession. Pulsar B1821-11 precesses with period $T_p \sim 500$ days [3]. Some other pulsars show periodic variations which may be explained by precession with $T_p \sim 100 - 500$ days [4]. The changing of pulse profile of Crab pulsar also may be related to precession with $T_p \sim 10^5$ years [5]. The increase of radio luminosity of Geminga pulsar [6] may be related to precession with $T_p > 10$ years as well. The non-regular pulse variations known as "red noise" [1, 2] with timescales $T \sim 1$ month $- 10^4$ years also can be related to precession [7]. The problem is that the pinning of vortices leads to precession with periods $T_p \sim P \cdot (I_{tot}/L_g) \sim (10^2 - 10^6) P$ [8], where $P$ is pulsar period, $I_{tot}$ is moment of inertia of the star, $L_g$ is the angular momentum of pinned superfluid. Such precession seems to damped very quickly and it is incompatible with existence of the long-period precession [8]. In this paper we consider a model of rotating neutron star, proposed in [9]. It allows the coexistence of long-term precession and quasi-glitch events. We also take into account the influence of the small-scale magnetic field on pulsar braking.
2. Basic equations
We assume that neutron star consists of 3 components which we will call the crust (c-component), g-component and r-component.

The crust (c-component). We suppose that it rotates as a rigid body with angular velocity \( \dot{\Omega}_c \). It is the outer component so the angular velocity \( \dot{\Omega}_c \) is the observed pulsar angular velocity \( \dot{\Omega}, \Omega = 2\pi/P \). We suppose that

\[
\dot{M}_c = I_c \dot{\Omega}_c \quad \text{and} \quad \dot{M}_c = \dot{\Omega}_c + \dot{N}_{gc} + \dot{N}_{rc},
\]

where \( \dot{M}_c \) is angular momentum of the crust, \( I_c \) is its moment of inertia, \( \dot{\Omega}_c \) is the external torque of magnetospheric origin acting on the crust, \( \dot{N}_{gc} \) and \( \dot{N}_{rc} \) are the torques acting on the crust due to its interaction with \( g \) and \( r \) components correspondingly.

g-component. It is the one of two inner components. We assume that it consists of normal matter rotating as a rigid body with angular velocity \( \dot{\Omega}_g \) and superfluid matter firmly pinned to the normal matter so that superfluid vortices rotate together with the normal matter with angular velocity \( \dot{\Omega}_g \):

\[
\dot{M}_g = I_g \dot{\Omega}_g + \dot{\mathbf{L}}_g, \quad \dot{M}_g = \dot{\mathbf{N}}_{cg} + \dot{\mathbf{N}}_{rg} \quad \text{and} \quad \dot{\mathbf{L}}_g = \left[ \dot{\mathbf{N}}_g \times \dot{\mathbf{L}}_g \right], \tag{2}
\]

where \( \dot{M}_g \) is the total angular momentum of g-component, \( I_g \) is the moment of inertia of its normal matter, \( \dot{\mathbf{L}}_g \) is angular momentum of pinned superfluid, \( \dot{\mathbf{N}}_{cg} \) and \( \dot{\mathbf{N}}_{rg} \) are the torques acting on g-component due to its interaction with the crust and r-component correspondingly.

r-component. It is the second inner component. We assume that it rotates as a rigid body with angular velocity \( \dot{\Omega}_r \):

\[
\dot{M}_r = I_r \dot{\Omega}_r \quad \text{and} \quad \dot{M}_r = \dot{\mathbf{N}}_{cr} + \dot{\mathbf{N}}_{gr}, \tag{3}
\]

where \( \dot{M}_r \) is the angular momentum of r-component, \( I_r \) is its moment of inertia, \( \dot{\mathbf{N}}_{cr} \) and \( \dot{\mathbf{N}}_{gr} \) are the torques acting on the r-component due to its interaction with the crust and g-component correspondingly. In the crust frame of reference the equations of rotation (1)-(3) can be rewritten as

\[
\dot{\mathbf{N}}_{ij} = \Omega_j \Omega_i, \quad \dot{\mathbf{N}}_{ij} = -\dot{\mathbf{N}}_{ji}, \quad \dot{\mathbf{N}}_{ij} = \dot{\mathbf{N}}_{ij}/I_j, \quad i,j = c,g,r, \quad \dot{\mathbf{S}}_{ext} = \dot{\mathbf{K}}_{ext}/I_c \quad \text{and} \quad \dot{\mathbf{g}}_g = \dot{\mathbf{L}}_g/I_g.
\]

In the sake of simplicity we suppose that

\[
\dot{\mathbf{N}}_{ij} = -I_j \left( \alpha_{ij} \dot{\mathbf{e}}_\Omega || \dot{\mathbf{e}}_\Omega + \beta_{ij} \dot{\mathbf{e}}_\Omega ^+ + \gamma_{ij} [\dot{\mathbf{e}}_\Omega \times \dot{\mathbf{e}}_\Omega ^+] \right), \tag{8}
\]

where \( \alpha_{ij}, \beta_{ij}, \gamma_{ij} \) are some constants, \( \dot{\mathbf{e}}_\Omega = \dot{\Omega}/\Omega \) and we have introduced parallel component \( \dot{\mathbf{A}}^\parallel = \dot{\mathbf{A}} \cdot \dot{\mathbf{e}}_\Omega \) and perpendicular component \( \dot{\mathbf{A}}^\perp = \dot{\mathbf{A}} - \dot{\mathbf{A}}^\parallel \dot{\mathbf{e}}_\Omega \) for any vector \( \dot{\mathbf{A}} \).

First let us consider the equilibrium state for zeroth external torque \( \dot{\mathbf{K}}_{ext} = 0 \). In this case, the whole star rotates as a rigid body (\( \dot{\mathbf{m}}_{ij} = 0 \)) and, hence, \( \dot{\mathbf{R}}_{ij} = 0 \). Equations (4)-(7) may be written as

\[
\dot{\mathbf{\Omega}} = 0, \quad \dot{\mathbf{g}}_g = 0 \quad \text{and} \quad \dot{\mathbf{g}}_g = \omega_g \dot{\mathbf{e}}_\Omega. \tag{9}
\]
Let us further consider a small perturbation to the equilibrium state. We will treat values \( \tilde{\mu}_{ij} \), \( \tilde{\omega}_{c} \) and \( S_{ext} \) as small perturbations and neglect any term quadratic in these values. Hence, Equations (10)-(17) may be written as

\[
\begin{align*}
\dot{\tilde{\mu}} &= R_{gc}^\parallel + R_{rc}^\parallel + S_{ext}^\parallel, \\
\dot{\tilde{\mu}}_{cg} &= R_{cg}^\parallel + R_{rg}^\parallel - R_{gc}^\parallel - R_{rc}^\parallel - S_{ext}^\parallel, \\
\dot{\tilde{\mu}}_{cr} &= R_{cr}^\parallel - R_{gg}^\parallel - R_{gr}^\parallel - S_{ext}^\parallel, \\
\tilde{\mathbf{\omega}} &= 0, \\
\Omega \dot{\tilde{\mathbf{\omega}}} &= R_{gc}^\parallel + R_{rc}^\parallel + S_{ext}^\parallel, \\
\dot{\tilde{\omega}}_{c} &= (\tilde{\mathbf{\omega}}_{g} - \Omega) [\tilde{\mathbf{e}}_{\Omega} \times \tilde{\mu}_{c}^\parallel] + [\tilde{\mathbf{\Omega}} \times \tilde{\omega}_{c}^\parallel] = \tilde{R}_{rg}^\parallel - \tilde{R}_{gc}^\parallel - \tilde{R}_{rc}^\parallel - S_{ext}^\parallel, \\
\dot{\tilde{\omega}}_{cr} &= [\tilde{\mathbf{\Omega}} \times \tilde{\mu}_{cr}] = \tilde{R}_{cr}^\parallel + \tilde{R}_{gr}^\parallel - \tilde{R}_{rc}^\parallel - \tilde{R}_{gc}^\parallel - S_{ext}^\parallel, \\
\dot{\tilde{\omega}}_{g} &= -\frac{\tilde{\omega}_{g}}{\tilde{\Omega}} \left( \tilde{R}_{gc}^\parallel + \tilde{R}_{rc}^\parallel + S_{ext}^\parallel + [\tilde{\Omega} \times \tilde{\mu}_{cg}] \right). 
\end{align*}
\]

In order to calculate the magnetospheric torque \( \tilde{K}_{ext} \) acting on the crust we use the model proposed in [13]. It is assumed that neutron star is braking simultaneously by both magnitodipolar and current losses. Hence,

\[
\tilde{K}_{ext} = -\frac{I_{tot}}{\tau_{0}} \left( \tilde{\mathbf{e}}_{\Omega} - (1 - \alpha) \cos \chi \tilde{e}_{m} - R_{eff} [\tilde{\mathbf{e}}_{\Omega} \times \tilde{e}_{m}] \right),
\]

where \( \tilde{\mathbf{m}} = m \tilde{e}_{m} \) is the dipolar magnetic moment of neutron star, \( \chi \) is the inclination angle (the angle between \( \tilde{\mathbf{e}}_{\Omega} \) and \( \tilde{e}_{m} \), see fig.3), \( \tau_{0} = \frac{3}{2m^{3}c^{2}I_{tot}} \), \( I_{tot} = I_{c} + I_{g} + I_{r} \), the coefficient \( R_{eff} \) is related to the magnetic field inertia [14, 15]. In the paper we assume that \( R_{eff} = \frac{9}{16} \frac{r_{ns}}{r_{c}} \approx 5 \cdot 10^{5} \left( \frac{P_{r}}{r} \right) [16] \), where \( r_{ns} \) is neutron star radius. The coefficient \( \alpha \) is related to the value of the current flowing through the pulsar tubes. In this paper we assume that there are only ”inner gaps” with free electron emission from neutron star surface in pulsar tube. Hence, the magnitude of the current depends on the structure of surface small-scale magnetic field (see fig.1). The value of \( \alpha \) averaged over precession angle \( \phi_{\Omega} \) (see fig.3)

\[
< \alpha > (\chi) = \frac{1}{2\pi} \int_{0}^{2\pi} \alpha(\chi, \phi_{\Omega}) d\phi_{\Omega}
\]

is shown in fig.2. Here, \( \nu = B_{sc}/B_{dip} \), \( B_{sc} \) is the induction of small-scale magnetic field, \( B_{dip} = 2m/r_{ns}^{3} \) is the induction of dipolar field on the magnetic pole of neutron star.

3. Quasistatic approximation

Firstly let us take into account that \( \tau_{rel} \ll \tau_{0} \), where \( \tau_{rel} \sim \max (1/\alpha_{ij}, 1/\beta_{ij}, 1/\gamma_{ij}) \sim (1 - 10^{7})s \) is relaxation time [2]. Hence, we can consider the rotation of neutron star under the small slowly varying torque \( \tilde{K}_{ext} \). In this case, it is possible to neglect terms \( \tilde{\mu}_{ij} \) and \( \tilde{\omega}_{c} \) in (10)-(17). Using (10)-(12) we can immediately obtain that

\[
\dot{\tilde{\Omega}} = \frac{K_{ext}}{I_{tot}}
\]
so the star brakes as a rigid body with moment of inertia equal to $\tilde{I}_{tot}$. Equations (11) and (12) give us

\begin{align}
\mu_{cg}^\parallel &= \frac{S_{ext}^{||} I_c}{\tilde{I}_{tot}}, \quad \frac{\alpha_{cr} + \alpha_{rg} + \alpha_{gr}}{\alpha_{cg} \alpha_{cr} + \alpha_{rg} \alpha_{cr} + \alpha_{cg} \alpha_{gr}}, \\
\mu_{cr}^\parallel &= \frac{S_{ext}^{||} I_c}{\tilde{I}_{tot}}, \quad \frac{\alpha_{cg} + \alpha_{rg} + \alpha_{gr}}{\alpha_{cg} \alpha_{cr} + \alpha_{rg} \alpha_{cr} + \alpha_{cg} \alpha_{gr}}.
\end{align}

Let us introduce a complex number $A^\perp$ associated to arbitrary perpendicular vector $\tilde{A}^\perp$ in the following way: if $\tilde{e}_z = \tilde{e}_\Omega$ then $\tilde{A}^\perp = A_x \tilde{e}_x + A_y \tilde{e}_y$ and $A^\perp = A_x + i A_y$. Hence, equations (15)-(17) give us

\begin{align}
\mu_{cg}^\perp &= \frac{S_{ext}^{\perp}}{\Delta^\perp} \cdot (i \Omega + \xi_{cr} + \xi_{gr}), \\
\mu_{cr}^\perp &= \frac{S_{ext}^{\perp}}{\Delta^\perp} \cdot (i \Omega + \xi_{gr}).
\end{align}
\[ \omega_g^\perp = \frac{S_{\text{ext}}^\perp}{\Omega \Delta} \cdot \left( \omega^\parallel (i\Omega + \xi_{cr} + \xi_{gr}) - \Omega \xi_{cg} + \xi_{cg} \xi_{cr} + \xi_{cg} \xi_{gr} + \xi_{rg} \xi_{cr} \right), \] (25)

where \( \xi_{pq} = \beta_{pq} + i\gamma_{pq} \) and

\[ \Delta = \Omega^2 - i\Omega (\xi_{cr} + \xi_{rc} + \xi_{gr} + \xi_{gc}) - (\xi_{gc} \xi_{cr} + \xi_{gc} \xi_{gr} + \xi_{rc} \xi_{gr}). \] (26)

In the case of weak viscosity \( |\xi_{pq}| \ll \Omega \) we have

\[ \bar{\mu}_c^\perp \approx \frac{1}{\Omega} [\vec{e}_\Omega \times \vec{S}_\text{ext}^\perp], \quad \bar{\mu}_r^\perp \approx \frac{1}{\Omega} [\vec{e}_\Omega \times \vec{S}_\text{ext}^\perp] \quad \text{and} \quad \bar{\omega}_g^\perp \approx \frac{\omega^\parallel}{\Omega^2} [\vec{e}_\Omega \times \vec{S}_\text{ext}^\perp]. \] (27)

In the last equation we also assume that \( |\xi_{pq}| \ll \omega^\parallel |. \)

Taking into account equations (21) - (24) and (18) one can rewrite equations (10) and (14) as

\[ \dot{\Omega} = -\frac{\Omega}{\tau_0} \left( \sin^2 \chi + \alpha \cos^2 \chi \right), \] (28)

\[ \dot{\chi} = -\frac{\tilde{I}_\text{tot}}{I_c} \frac{1}{\tau_0} \sin \chi \cos \chi \cdot (B (1 - \alpha) - \Gamma R_{\text{eff}}), \] (29)

\[ \dot{\phi}_\Omega = -\frac{\tilde{I}_\text{tot}}{I_c} \frac{1}{\tau_0} \cos \chi \cdot (B R_{\text{eff}} + \Gamma (1 - \alpha)), \] (30)

where coefficients \( B \) and \( \Gamma \) are defined as

\[ B + i\Gamma = \frac{\Omega}{\Delta} \cdot (\Omega - i(\xi_{cr} + \xi_{gr})). \] (31)

In the case of weak viscosity \( |\xi_{pq}| \ll \Omega \) we have

\[ B \approx 1 - \frac{\gamma_{gc} + \gamma_{rc}}{\Omega} \quad \text{and} \quad \Gamma \approx \frac{\beta_{ge} + \beta_{rc}}{\Omega}. \] (32)

And consequently equation (30) gives that as long as inclination angle \( \chi \) differs from 0° and 90° the star will precess with period

\[ T_p = \frac{\tau_0}{R_{\text{eff}}} \cdot \frac{I_c}{\tilde{I}_\text{tot}} \cdot \frac{2\pi}{\cos \chi} \sim (10^{-5} - 10^{-3}) \tau_0 \sim 10 - 10^3 \text{ years}. \] (33)

Also it is worth to note that as long as quasistatic approximation is valid the pinned superfluid \( \bar{L}_g \) does not influence on star rotation.

We suppose that the period of precession \( T_p \ll \left( I_c/\tilde{I}_\text{tot} \right) \cdot \tau_0 \ll \tau_0 \). Hence, we can average equations (28) and (29) over precession obtaining equation

\[ \frac{d\chi}{dP} = -\frac{1}{P} \cdot \frac{\tilde{I}_\text{tot}}{I_c} \cdot \sin \chi \cos \chi \cdot \frac{B (1 - < \alpha >) - \Gamma R_{\text{eff}}}{\sin^2 \chi + < \alpha > \cos^2 \chi}. \] (34)

4. Quasi-glitch events

According to equation (24) the direction of \( \bar{L}_g \) follows the vector \( \vec{e}_\Omega \) so the pinned superfluid vortices are directed almost along the crust rotation axis. Consequently, the pinned vortices precess together with \( \vec{e}_\Omega \). However, due to ideal pinning (see the last equation in (2)) the magnitude of vector \( \bar{L}_g \) does not change at all. Hence, due to pulsar braking the difference
between velocities of superfluid and normal matter will grow. So the glitch must occur in $g$-component at some moment. We suppose that the glitch occurs and relaxes at time-scale much smaller than the period of precession $T_p$. It allows us to consider $\tilde{\Omega}$ as a constant vector in equations (11), (12), (13)-(17) and, for simplicity, neglect the acting of external torque $\tilde{S}_{\text{ext}}$.

Let us assume that before glitch the neutron star rotates as a rigid body. At some moment ($t = 0$) a small amount of angular momentum $\Delta L_x = \Delta L_g \tilde{\Omega}$ is is transferred from superfluid part to normal part of $g$-component. Then the solution of equation (34) for different initial inclination angles $\chi$ and consequently growing of $R$ We can see that in most cases the star forgets initial value of inclination angle $\sigma$ in the case of $r\text{-component like normal matter with superfluid and there are normal viscosity between the crust and } g\text{-component interact with } r\text{-component like normal matter with superfluid and there are normal viscosity between the crust and } g\text{-component interact}$

\begin{equation}
\Omega(t) = \Delta \Omega \left(1 - e^{-p_+ t} - Q(1 - e^{-p_- t})\right),
\end{equation}

where $\Delta \Omega = \frac{\Delta \Omega_\infty}{I}$, $\Delta \Omega_\infty = \frac{\Delta \Omega_e}{I_{\text{tot}}}$, the coefficients $p_+$ and $p_-$ ($p_+ > p_-$) are the roots of equation

\begin{equation}
p^2 - (\alpha_{cg} + \alpha_{rg} + \alpha_{cr} + \alpha_{gr} + \alpha_{gc} + \alpha_{rc}) p + \\
+ (\alpha_{gc} + \alpha_{rg} + \alpha_{cg}) (\alpha_{cr} + \alpha_{gr} + \alpha_{rc}) + (\alpha_{rc} - \alpha_{rg}) (\alpha_{gr} - \alpha_{gc}) = 0
\end{equation}

and

\begin{equation}
Q = \frac{I_{\text{tot}} \alpha_{cg} - I_{c} p_+}{I_{\text{tot}} \alpha_{cg} - I_{c} p_-}
\end{equation}

In the case of $\alpha_{cg} \gg \left(1 + \frac{I_{c}}{I_{r}}\right) \alpha_{rc}, \left(1 + \frac{I_{c}}{I_{r}}\right) \alpha_{rg} \text{ we have}

\begin{equation}
p_+ \approx \left(1 + \frac{I_{g}}{I_{c}}\right) \alpha_{cg}, \quad p_- \approx \frac{I_{\text{tot}}}{I_{c} + I_{g}} (\alpha_{cr} + \alpha_{gr}) \quad \text{and} \quad 1 - Q \approx \frac{I_{c} + I_{g}}{I_{\text{tot}}}
\end{equation}

If one want to relate these quasi-glitch event to observed glitches then $1/p_+$ and $1/p_-$ should be interpreted as the glitch growth and relaxation times respectively. We obtain that $1/p_+ \leq 1$ min $10$ $2$ and $1/p_- \approx 1 - 10^{-2}$ days $1$. Unfortunately, $1 - Q \approx 10^{-2} - 10^{-1}$ in our model. That may be not so bad for glitches in some pulsars like Crab ($Q \geq 0.8$ $2$) or J0205+6446 ($Q \approx 0.77$ $11$), but obviously contradicts glitches in most pulsars for which $Q \ll 1$ $11$ $12$. In particular, our model does not describe glitches in Vela pulsar $Q \leq 0.2$ $2$.

5. Results

In present paper we will use the following component interaction model. The crust and $g\text{-component interact with } r\text{-component like normal matter with superfluid and there are normal viscosity between the crust and } g\text{-component interact}$

\begin{equation}
\beta_{cr} = \beta_{gr} = \Omega \frac{\sigma}{1 + \sigma^2}, \quad \alpha_{cr} = \alpha_{gr} = 2 \beta_{gc}, \quad \gamma_{cr} = \gamma_{gr} = -\sigma \beta_{gc}, \quad \beta_{cg} = \alpha_{cg} \quad \text{and} \quad \gamma_{cg} = 0.
\end{equation}

The solution of equation (34) for different initial inclination angles $\chi$ and initial period $P = 10$ ms in the case of $\sigma = 10^{-10}, \alpha_{cg} = 10^{-1} s^{-1}, I_c/I_{\text{tot}} = 10^{-2}, I_g/I_{\text{tot}} = 10^{-3}$ is shown in fig. 5 and 6. We can see that in most cases the star forgets initial value of inclination angle $\chi$ very rapidly and evolves to equilibrium inclination angle $\chi_{eq}$, at which

\begin{equation}
\sin \chi \cos \chi \cdot (B (1 - < \alpha >) - \Gamma R_{eff}) \approx 0.
\end{equation}

The subsequent evolution of angle $\chi$ is caused by the slow changing of equilibrium angle $\chi_{eq}$ due to pulsar braking and consequently growing of $R_{eff}$. The solution (34) for initial inclination angle $\chi = 45^\circ$, initial period $P = 10$ ms and different values of $\nu$, in case of $\alpha_{cg} = 10^{-1} s^{-1}$, $I_c/I_{\text{tot}} = 10^{-2}, I_g/I_{\text{tot}} = 10^{-3}$ is shown in fig. 7. In this case $1 - Q \approx 10^{-2}$. The same but for $I_c/I_{\text{tot}} = 10^{-1}, I_g/I_{\text{tot}} = 10^{-5}$ is shown in fig. 8. In this case $Q \approx 0.9$. The increase of $I_c$ leads
to slower evolution to equilibrium angle $\chi_{eq}$. The equilibrium inclination angle evolves slowly due to both the decrease of $I_g$ and, hence, lesser dissipation, and the increase of $I_c$ and, hence, large precession period $T_p$. And, consequently, evolutionary tracks may pass through the most pulsars. The same but for $I_g/I_{tot} = 10^{-3}$, $I_r/I_{tot} = 10^{-3}$ is shown in fig. 11-13. In this case, $I_c \approx I_{tot}$ and $Q \approx 10^{-3}$. The star rotates almost as a rigid body. Consequently, the inclination angle changes very slowly and the pulsars usually do not reach the equilibrium angle during their life.

![Figure 5](image1)

Figure 5. The evolution of inclination angle $\chi$ for $\nu = 0.0$ (left panel) and $\nu = 0.5$ (right panel), $I_c/I_{tot} = 10^{-2}$, $I_g/I_{tot} = 10^{-3}$, $\alpha_{cg} = 10^{-1}$ s$^{-1}$, $\sigma = 10^{-10}$. Observed inclination angles $\beta_2$ at 10 cm is taken from [17], red stars correspond to $C > 0$, blue dots corresponds to $C < 0$.

![Figure 6](image2)

Figure 6. The same as in fig. 5 but for $\nu = 0.8$ (left panel) and $\nu = 1.0$ (right panel).

6. Discussion
We consider a model proposed in [9] that allows simultaneously the long-period precession and quasi-glitch events with taking into account the influence of the small-scale magnetic field on pulsar braking. For simplicity we consider only the case of axial symmetric precession and
The evolution of inclination angle $\chi$ for different $\nu$, $\sigma = 10^{-10}$ (left panel), $\sigma = 10^{-6}$ (right panel), $\alpha_g = 10^{-1}$ s$^{-1}$, $I_c/I_{\text{tot}} = 10^{-2}$, $I_g/I_{\text{tot}} = 10^{-3}$. Initial inclination angle $\chi = 45^\circ$, initial period $P = 10$ ms. Observed inclination angles $\beta_2$ at 10 cm is taken from [17], red stars correspond to $C > 0$, blue dots to $C < 0$.

Figure 7. The same as in fig. 5 but for $I_c/I_{\text{tot}} = 10^{-1}$, $I_g/I_{\text{tot}} = 10^{-5}$.

do not take into account that the presence of the small-scale magnetic field makes precession triaxial [15]. The main problem of proposed model is the exact nature of components, especially, of $g$-component. Strictly speaking, we only postulate the existence of components with some properties.

Let us speculate a little about possible nature of these components. The $g$-component may consist of tangles of closed fluxoids, normal matter inside the tangles and pinned superfluid (see fig. 14). These tangles "freely" flow inside the star core. The collapse of closed fluxoids is prevented by repulsion of pinned vortices. Similar configurations are considered in [20], where vortices are pinned to fluxoids forming the regular toroidal magnetic field inside the neutron star core, and [21], where the "freely" flowing of magnetic field tangles is discussed. We expect that $L_g \sim 0.1 I_g \Omega$. Hence, if we suppose that $L_g \sim 10^{-2} I_{\text{tot}} \Omega$ [2], then $I_g \sim 10^{-3} I_{\text{tot}}$. In this case, $c$-component is exactly the crust, so $I_c \sim 10^{-2} I_{\text{tot}}$ [22] and $r$-component consists of normal and
superfluid matter located outside the tangles, so \( I_r \approx I_{tot} \). We suppose that strong interaction between the crust and the tangles may be related to small number of fluxoids got out the tangles. It is worth to note that a tangle located deep inside the core weakly interacts with the crust and may produce something like "slow glitch" \[23\]. The main problem of such configuration is the stability of tangles and its partial destruction during glitches.

The \( g \)-component also may be created by small rigid core which can exist in central region of the star \[19\] (see fig. \[15\]) with superfluid vortices pinned to it. However, in this case, we must assume that the vortices are extremely rigid, so a vortex pinned by its central part to the rigid core is fixed outside the rigid core as well. Consequently, if we assume that the radius of rigid core \( r_g \sim (0.1 - 0.2) r_{ns} \) then the moment of inertia of "normal" matter inside the rigid core \( I_g \sim 0.1(r_g/r_{ns})^3 \sim (10^{-6} - 10^{-5}) I_{tot} \). However, it controls the vortices movement and, hence, the superfluid flow in volume \( \sim r_g \cdot r_{ns} \), see fig. \[19\]. Consequently, we can estimate the angular momentum of pinned superfluid: \( L_g \sim (r_g/r_{ns})^2 I_{tot} \Omega \sim 10^{-2} I_{tot} \Omega \). In this case, \( c \)-component

**Figure 9.** The same as in fig. \[6\] but \( I_c/I_{tot} = 10^{-1}, I_g/I_{tot} = 10^{-5} \). Left panel corresponds to \( \nu = 0.8 \), right panel to \( \nu = 1.0 \).

**Figure 10.** The same as in fig. \[7\] but \( I_c/I_{tot} = 10^{-1}, I_g/I_{tot} = 10^{-5} \). Left panel corresponds to \( \sigma = 10^{-10} \), right panel to \( \sigma = 10^{-6} \).
Figure 11. The same as in fig. 5 but for \( I_g/I_{\text{tot}} = 10^{-3} \), \( I_r/I_{\text{tot}} = 10^{-3} \). Left panel corresponds to \( \nu = 0 \), right panel to \( \nu = 0.5 \).

Figure 12. The same as in fig. 6 but for \( I_g/I_{\text{tot}} = 10^{-3} \), \( I_r/I_{\text{tot}} = 10^{-3} \). Left panel corresponds to \( \nu = 0.8 \), right panel to \( \nu = 1.0 \).

consists of the crust and normal matter of the core: \( I_c \sim 10^{-1} I_{\text{tot}} \) \(^{22} \), and \( r \)-component is superfluid which is not pinned to rigid core \( I_r \approx I_{\text{tot}} \). We suppose that the strong interaction between \( g \)-component and the crust requires that the magnetic field penetrates from crust up to boundaries of rigid core. Consequently, core superconductivity must be absent (at least outside the rigid core).

In the paper we assume that the interaction between \( c \)-component and \( g \)-component is strong in order to provide rapid angular momentum transfer from \( g \)-component to the crust. Hence, if \( I_g \sim I_{\text{tot}} \), precession is damped very rapidly and the inclination angle quickly evolves to \( \chi \approx 0^\circ \) or \( \chi \approx 90^\circ \). So in order to save precession during pulsar life time it is necessary to suppose that \( I_g \ll I_{\text{tot}} \). We can reduce this restriction if we suppose that the friction between the crust and \( g \)-component increases during glitch. For example, we can assume that the crust and \( g \)-component interact due to weak viscous friction between two glitches but during the the glitch the angular momentum is rapidly transferred by Kelvin waves \(^{23} \), Alfen or sound waves. We
The tangle of fluxoids with pinned vortices is show on the left. Pinned vortices are shown by wavy green lines. The tangles are shown on the right. Pinned vortices are shown by straight pink lines, free vortices are shown by straight pink lines, closed fluxoids are shown by blue lines. Two flowing in star core tangles are shown on the right. Pinned vortices are shown by straight pink lines, free vortices are shown by wavy green lines. The $g$-component is shown by dark gray circles and core superfluid outside $g$-component is shown by yellow area.

\textbf{Figure 13.} The same as in fig. but for $I_g/\bar{I}_{tot} = 10^{-3}$, $I_r/\bar{I}_{tot} = 10^{-3}$. Left panel corresponds to $\sigma = 10^{-10}$, right panel to $\sigma = 10^{-6}$.

can also reduce the restriction if we assume that $g$-component is slowly destroyed. For example, the tangles of fluxoids slightly destroyed during each glitch.

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\textbf{Figure 14.} The tangle of fluxoids with pinned vortices is show on the left. Pinned vortices are shown by straight pink lines, closed fluxoids are shown by blue lines. Two flowing in star core tangles are shown on the right. Pinned vortices are shown by straight pink lines, free vortices are shown by wavy green lines. The $g$-component is shown by dark gray circles and core superfluid outside $g$-component is shown by yellow area.
Figure 15. The star with small rigid core. Vortices pinned to rigid core are shown by straight pink lines, free vortices are shown by wavy green lines. The $g$-component is shown by dark gray circle and core superfluid outside the $g$-component is shown by yellow area.

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