Type III Neutrino Seesaw, Freeze-In Long-Lived Dark Matter, and the $W$ Mass Shift

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Abstract

In the framework of seesaw neutrino masses from heavy fermion triplets ($\Sigma^+, \Sigma^0, \Sigma^-$), the addition of a light fermion singlet $N$ and a heavy scalar triplet $(\rho^+, \rho^0, \rho^-)$ has some important consequences. The new particles are assumed to be odd under a new $Z_2$ symmetry which is only broken softly, both explicitly and spontaneously. With $N - \Sigma^0$ mixing, freeze-in long-lived dark matter through Higgs decay becomes possible. At the same time, the $W$ mass is shifted slightly upward, as suggested by a recent precision measurement.
Introduction: The well-known seesaw mechanism for tiny Majorana neutrino masses has three simple tree-level realizations \([1]\), depending on the heavy intermediary particles involved. Whereas Type I [heavy fermion singlet \(N\)] and Type II [heavy scalar triplet \((\xi^+, \xi^0, \xi^0)\)] are routinely considered in numerous papers, Type III [heavy fermion triplet \((\Sigma^+, \Sigma^0, \Sigma^-)\)] is rather less studied \([2,3]\). In this paper, in addition to three copies of heavy \(\Sigma\) for obtaining three light Majorana neutrinos, a new \(Z_2\) symmetry is assumed under which one new light singlet fermion \(N\) and one new heavy scalar triplet \((\rho^+, \rho^0, \rho^-)\) are odd, and all other fields are even. Whereas all dimension-four terms of the resulting Lagrangian must respect \(Z_2\) [which forbids \(N\) from coupling to lepton doublets through the usual Higgs doublet \(\Phi\) of the standard model of quarks and leptons (SM)], this symmetry is broken explicitly by the dimension-three soft term \(\Phi^\dagger \rho \Phi\), resulting thus in a small nonzero vacuum expectation value \(v_\rho\) for \(\rho^0\).

Three important consequences follow. (A) The \(W\) mass gets a slight upward shift \([4]\). (B) The \(\phi^0 - \rho^0\) and \(\Sigma^0 - N\) mixings allow the SM Higgs boson \(h\) to decay to \(NN\). (C) \(N\) decays through its mixing with the heavy \(\Sigma^0\) which couples to \(\nu \phi^0\), converting thereby to \(\nu \bar{f} f\), where \(f\) is the heaviest fermion kinematically allowed. Hence \(N\) is possibly a long-lived dark-matter candidate, produced in a freeze-in scenario \([5]\) through rare \(h\) decay \([6]\).

Higgs Potential: The Higgs potential \(V\) of this proposal consists of the SM Higgs doublet \(\Phi\) and the new real scalar triplet \(\rho\) which is odd under the assumed \(Z_2\), i.e.

\[
V = -\mu_0^2(\Phi^\dagger \Phi) + \frac{1}{2} m_1^2 (\vec{\rho} \cdot \vec{\rho}) + \frac{1}{2} \lambda_1 (\Phi^\dagger \Phi)^2 + \frac{1}{8} \lambda_2 (\vec{\rho} \cdot \vec{\rho})^2 + \frac{1}{2} \lambda_3 (\Phi^\dagger \Phi)(\vec{\rho} \cdot \vec{\rho}) + \sqrt{2} \mu_1 (\Phi^\dagger (\vec{\sigma} \cdot \vec{\rho}) \Phi, \tag{1}\]

where the \(\mu_1\) trilinear term breaks \(Z_2\) softly. Let

\[
\phi^0 = \frac{1}{\sqrt{2}} (v_0 + h), \quad \rho^0 = v_1 + s, \tag{2}\]
Then $v_{0,1}$ are determined by

$$0 = -\mu_0^2 + \frac{1}{2} \lambda_1 v_0^2 + \frac{1}{2} \lambda_3 v_1^2 - \frac{\mu_1 v_1}{\sqrt{2}},$$

(3)

$$0 = v_1 \left( m_1^2 + \frac{1}{2} \lambda_2 v_1^2 + \frac{1}{2} \lambda_3 v_0^2 \right) - \frac{\mu_1 v_0^2}{2 \sqrt{2}}.$$  

(4)

For large and positive $m_1^2$, the scalar seesaw solution [7] is

$$v_0^2 \simeq 2 \frac{\mu_0^2}{\lambda_1}, \quad v_1 \simeq \frac{\mu_1 v_0^2}{2 \sqrt{2} m_1^2}.$$  

(5)

The $2 \times 2$ mass-squared matrix spanning $h$ and $s$ is then

$$\mathcal{M}_{hs}^2 \simeq \begin{pmatrix} \frac{\lambda_1 v_0^2}{\sqrt{2}} & -\frac{\mu_1 v_0}{\sqrt{2}} \\ -\frac{\mu_1 v_0}{\sqrt{2}} & m_1^2 \end{pmatrix},$$

(6)

with $h - s$ mixing given by

$$\theta_{hs} \simeq \frac{\mu_1 v_0}{\sqrt{2} m_1^2} \simeq \frac{2 v_1}{v_0}.$$  

(7)

To explain the new precision measurement of the $W$ mass [4], i.e.

$$M_W = 80.4335 \pm 0.0094 \text{ GeV},$$  

(8)

which is several standard deviations above the prediction of the SM ($v_1 = 0$), a central value of $v_1 \simeq 3.68$ GeV may be extracted from the analysis of Ref. [8].

Singlet-Triplet Fermion Mixing: Neutrinos obtain seesaw masses through the heavy fermion triplets from the Yukawa couplings

$$\mathcal{L}_Y = \sqrt{2} f_{\nu} (\bar{\nu}, \bar{l})_L (\bar{\sigma} \cdot \bar{\Sigma})_R \left( \begin{array}{c} \phi^0 \\ -\phi^- \end{array} \right),$$

(9)

resulting in the Dirac mass $m_{\nu \Sigma} = f_{\nu} v_0 / 2$, and then the usual seesaw Majorana neutrino mass $m_\nu = f_{\nu}^2 v_0^2 / 4 m_\Sigma$. The $\nu - S$ mixing is $\sqrt{m_\nu / m_\Sigma}$, and the coupling of $\Sigma$ to $\nu h$ is $\sqrt{m_\nu m_\Sigma} / v_0$.

Since $N$ is odd under $Z_2$, it does not couple to $\nu$ through $\phi^0$. However, it does couple to $\Sigma$ through $\rho$, i.e.

$$\mathcal{L}'_Y = f_N \bar{N}_L (\bar{\rho} \cdot \bar{\Sigma})_R.$$  

(10)
The $N - \Sigma^0$ mixing is then $f_N v_1 / m_\Sigma$ and the $s$ coupling to $N N$ is $f_N^2 v_1 / m_\Sigma$.

**Higgs Decay to $NN$** : Since $N$ is a singlet fermion, the Higgs boson $h$ does not couple to $NN$ directly. It does so first through $h - s$ mixing, then through $N - \Sigma^0$ mixing, as shown in Fig. 1.

![Figure 1: Decay of $h$ to $NN$.](image)

The effective coupling is then

$$f_h \approx \left( \frac{2v_1}{v_0} \right) \left( \frac{f_N^2 v_1}{m_\Sigma} \right) = \frac{2f_N^2 v_1^2}{v_0 m_\Sigma}. \quad (11)$$

The decay rate of $h \to NN + \bar{N}\bar{N}$ is [9]

$$\Gamma_h = \frac{f_h^2 m_h}{8\pi} \sqrt{1 - 4r^2(1 - 2r^2)}, \quad (12)$$

where $r = m_N / m_h$. Now $N$ is assumed light and a candidate for long-lived dark matter. The correct relic abundance is obtained [10] if $f_h \sim 10^{-12} r^{-1/2}$, provided that the reheat temperature of the Universe is above $m_h$ but well below $m_\rho$ and $m_\Sigma$.

**Long-Lived Dark Matter** : The singlet fermion $N$ is assumed light and decays only through its mixing with $\Sigma^0$ which couples to $\nu h$. Through the virtual Higgs, its coupling to $\nu\bar{f}f$ is then given by

$$G_N = \frac{f_N v_1}{m_\Sigma} \left( \frac{\sqrt{m_\nu m_\Sigma}}{v_0} \right) \frac{1}{m_h^2} \left( \frac{m_f}{v_0} \right), \quad (13)$$

where $f$ is a fermion allowed kinematically in the decay, as shown in Fig. 2. The decay rate is analogous to that of muon decay, i.e.

$$\Gamma_N = \frac{G_N^2 m_N^5}{48(4\pi)^3}. \quad (14)$$
Consider for example $m_N = 0.1$ GeV, then $f_h \sim 10^{-12}(0.1/125)^{-1/2}$ from freeze-in Higgs decay implies

$$m_\Sigma/f_N^2 \sim 3.1 \times 10^9 \text{ GeV}$$

(15)

for $v_1 = 3.68$ GeV. In $N \rightarrow \nu \bar{f} f$ decay, only an electron-positron pair is possible, hence $m_f = m_e$ and

$$G_N \sim 3.5 \times 10^{-22} \text{ GeV}^{-2}$$

(16)

for $m_\nu = 0.1$ eV. The $N$ lifetime is then

$$\tau_N \sim 5.1 \times 10^{28} \text{ s},$$

(17)

many orders of magnitude greater than the age of the Universe and satisfies bounds from all cosmological considerations [11].

**Conclusion**: In this paper, a first example of long-lived freeze-in dark matter is presented in the context of Type III seesaw neutrino masses using heavy fermion triplets $\Sigma$. The key is the addition of a light fermion singlet $N$ and a real scalar triplet $\rho$, both odd under a softly broken $Z_2$ symmetry. The rare decay of the SM Higgs to $NN$ accounts for the dark matter relic abundance of the Universe, with $N$ having a lifetime many orders of magnitude greater than the age of the Universe. This is accomplished with $m_N = 0.1$ GeV, $m_\Sigma \sim 10^9$ GeV, and $\langle \rho^0 \rangle = 3.68$ GeV, which also explains the shift in the $W$ boson mass, observed recently.

**Acknowledgement**: This work was supported in part by the U. S. Department of Energy Grant No. DE-SC0008541.
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