Entanglement of Solid Vortex Matter: A Boomerang Shaped Reduction Forced by Disorder in Interlayer Phase Coherence in Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$

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We present evidence for entangled solid vortex matter in a glassy state in a layered superconductor Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$ containing randomly splayed linear defects. The interlayer phase coherence—probed by the Josephson plasma resonance—is enhanced at high temperatures, reflecting the recoupling of vortex liquid by the defects. At low temperatures in the vortex solid state, the interlayer coherence follows a boomerang-shaped reentrant temperature path with an unusual low field decrease in coherence, indicative of meandering vortices. We uncover a distinct temperature scaling between in-plane and out-of-plane critical currents with opposing dependencies on field and time, consistent with the theoretically proposed "splayed-glass" state.

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We found that the interlayer coherence is strongly affected by the presence of randomly splayed linear defects, which are expected to promote topological disorder effects, and the ability to cut and reconnect—all ultimately informing the state of coherence and entanglement of lines.

In high transition temperature ($T_c$) superconductors, the competing influences lead to new states of vortex matter. Bragg glass [6] with quasi-long-range translational order can arise in the presence of weak point defects, while a conceptually different Bose glass state [7] is promoted by topologically correlated columnar defects (CDs), where the vortices become unidirectionally localized. Another distinct “splayed glass” state has been predicted in the presence of directionally distributed CDs; this state differs dynamically from other glassy states, with forced entanglement of vortex lines by the splayed CDs as its chief characteristic.

The differentiating factor and a key to the nature of these states reside in the longitudinal vortex correlations. In layered superconductors, the quasi 2-D pancake vortices in the layers are connected by Josephson strings to form the lines. There, vortex correlations are closely related to the interlayer phase coherence (IPC) $\langle \cos \phi_{n,n+1} \rangle$, with $\phi_{n,n+1}$ being the gauge-invariant phase difference between the neighboring layers $n$ and $n+1$. IPC can be directly probed by Josephson plasma resonance (JPR) [9, 10, 11, 12], with resonance frequency $\omega_p$ given by

$$\omega_p^2(H,T) = \omega_p^2(0,T)\langle \cos \phi_{n,n+1} \rangle = \frac{8\pi^2 cs}{\epsilon_0 \Phi_0} J_c^c,$$

where $s$ is the interlayer spacing, $\epsilon_0$ the dielectric constant, $\Phi_0$ the flux quantum, and $J_c^c$ the $c$-axis critical current density.

In pristine Bi$_2$Sr$_2$CaCu$_2$O$_{8+y}$ (BSCCO), it has been demonstrated [13, 14] that the IPC suddenly decreases with field at the phase transition between the low-field Bragg glass and the high-field liquid phases, and that within the liquid phase vortices in each layer are well decoupled [15]. In BSCCO with the unidirectional CDs, known for their strong vortex pinning [16, 17], an enhanced IPC has been observed in the liquid state near $B_0/3$ ($B_0$ is the matching field where the vortex density coincides with the defect density) [18, 20, 21, 22]. Thus, the decoupled vortex liquid transforms to a recombined liquid state at higher fields, where the vortices tend to be confined and forced to align along the CDs. Forced topological entanglement has been deduced from transport in the liquid state of YBa$_2$Cu$_3$O$_7$ [23], where strong interlayer coupling prevents experimental access to IPC. In more anisotropic BSCCO and Hg-based systems [14], partial evidence for entanglement has been claimed from large critical current enhancements in polycrystalline samples with splayed CDs. Thus far, however, there is no direct demonstration of entangled vortex structure via IPC. Here we report on the IPC in BSCCO single crystals containing directionally distributed CDs. We demonstrate that the interlayer coherence in BSCCO with uniformly splayed CDs shows an anomalous ‘boomerang’-like reduction at low temperatures and low fields. The forced entanglement is evidenced by the enhanced in-plane critical current density $J_{c}^{ab}$ with the concurrent drop of $J_{c}^{c}$—the latter being a measure of vortex meandering. The found anticorrelation of the field and time dependence of $J_{c}^{c}$’s in the boomerang region of the phase space is in correspondence with the proposed framework in the splayed glass state [8].
Optimally-doped BSCCO single crystals were prepared by the floating zone method. Crystals with thicknesses (along the c axis) of ~20 μm were exposed to 0.8 GeV proton beam in Los Alamos Neutron Science Center (LANSCE). Bi nuclei within the crystals are fissioned by collisions with swift protons and the fission fragments create extended randomly oriented damage tracks with diameter of ~7 nm and length of ~6 μm [24]. In this study, we used two crystals with matching fields \( B_p \approx 0.5 \) and 1 T, as estimated from the relation between areal defect density and the proton fluence [24]. JPR experiments were performed by a cavity perturbation technique at a frequency \( \omega/2\pi \approx 28 \) GHz [11, 13, 18], from which we evaluate \( J_c^p \) by Eq. (1). Standard magnetization measurements were used to determine the creep rate \( S = -d \ln (\mu_0 H_c) / d \ln (t) \) as well as \( J_c^p \) as a function of field and time [19].

*Josephson Plasma Resonance.*— Figure 1 shows the field dependence of the inverse quality factor \( 1/Q \) (× microwave dissipation) and the frequency shift for the \( B_p \approx 1 \) T sample. Magnetic field \( H \) is increased after the zero-field-cooling condition. Clear JPR is observed as a peak in \( 1/Q \) and a step-like change in the frequency shift. At high temperatures, we always observe a single resonance line, in contrast to the case of BSCCO with parallel CDs along the c axis, where multiple resonances have been observed near \( B_p/3 \) [13, 20, 21].

At low temperatures, in addition to the main resonance line, we find another resonance (indicated by the arrows) at a lower field. In this second resonance, the frequency shift shows a step-like anomaly having an opposite direction from that of the main resonance. This goes to prove that the second resonance does not originate from some secondary low-\( T_c \) phase or inhomogeneity, but is an intrinsic feature of this system, as will be discussed later.

*High-Temperature Liquid State.*— Let us first discuss the single resonance in the liquid state at high temperatures. Figure 2(a) shows the temperature dependence of the resonance field \( H_p \). To compare this with previous reports [18], we normalize the temperature \( T \) by \( T_c \) and \( H_p \) by \( \omega^2 \) [9, 13]. Near \( T_c \), our data roughly follow the data for pristine BSCCO as well as for the one with parallel CDs [22]. However, at low \( T \) and high fields a distinct deviation is observed, indicating that the IPC in the system with randomly splayed CDs is enhanced relative to the pristine one. A quantitative comparison with a system containing the parallel CDs with the same \( B_p \) shows that the IPC enhancement in our case is considerably smaller. Moreover, we do not observe multiple resonances in the liquid state, which are only expected for largely enhanced IPC with non-monotonic field dependence of \( \langle \cos \phi_{n,n+1} \rangle / \langle H \rangle \) [23].

The angular field dependence is particularly telling. In BSCCO with CDs parallel to the c axis, it has been shown that in the low-field decoupled liquid state, the resonance field is determined mostly by the c-axis component over a wide range of field angles [21]. By contrast, in the high-field recoupled liquid state \( H_p \) is nearly angle-independent; indeed, this is a key signature of recoupling. Our low-field data at 70 K in Fig. 2(b) reproduces the previous results on the low-field decoupled liquid state; its c-axis component \( H_p \cos \theta_H \) is almost constant over a wide angle range. On the other hand, the high-field data

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The anisotropic system is described by \[ 25 \]. In our splayed system, the directions of the CDs are determined by the pinning strength of the defects \( \gamma \). Here \( \theta \) is the polar angle \( \tan \theta = \tan \theta \). The angle dependence of the trapping probability of vortices in the scaled isotropic system is determined by the pinning strength of the defects \( 26 \).

In our splayed system, the directions of the CDs are random and the angular dependence of the probability in the anisotropic system is described by

\[
P(\theta_H) = \int_0^{2\pi} \int_0^{\pi/2} \frac{\sin \theta_{\text{CD}} \sin \phi_{\text{CD}}}{1 + (\theta_H - \theta_{\text{CD}}/A)^2} d\phi_{\text{CD}} d\theta_{\text{CD}}.
\] (2)

Here \( \theta_{\text{CD}} \) is the polar angle between a CD and the \( c \) axis, and \( \phi_{\text{CD}} \) is its angle of longitude with respect to the field rotation plane. Relative angle between the field and the defect in the scaled isotropic system is given by \( \theta_{H-\text{CD}} = \cos^{-1}(\sin \theta_H \sin \theta_{\text{CD}} \cos \phi_{\text{CD}} + \cos \theta_H \cos \theta_{\text{CD}}) \) with \( \theta_{H-\text{CD}} = \tan^{-1}(\tan \theta_H / \gamma) \) and \( \phi_{\text{CD}} = \phi_{\text{CD}} \). As shown in Fig. 2(c), the \( c \)-axis component of the resonance field at 49 K can be fitted to Eq. 2 with parameters physically sensible \( 26 \). Thus, in the high-field liquid state vortices are largely trapped inside the CDs. We conclude then that here a recoupling crossover takes place in a field range much smaller than the matching field \( B_0 \), much akin to the recoupling induced by parallel CDs.

**Low-Temperature Glass State.**— In Fig. 3(b) we plot the temperature dependence of \( H_p \) normalized by \( B_0 \) for both samples. At high temperatures the two curves coincide, indicating that the IPC in this system is mostly determined by the defects. At low temperatures below \( \sim 30 \) K, \( H_p(T) \) undergoes a kink and then turns to decrease. This kink is consistent with previous reports in BSCCO \( 10 \), where \( H_p(T) \) in the vortex solid state drops almost linearly with decreasing \( T \) in the zero-field-cooling condition. Remarkably, \( H_p(T) \) in our system has an anomalous reentrant behavior—it is this boomerang-like trajectory that gives double resonance lines in single field sweep runs at temperatures below \( \sim 30 \) K\( 26 \).

Since we use a fixed microwave frequency and \( \omega_{\mu}(0, T) \) in Eq. (1) is only weakly temperature dependent at low temperatures \( 13 \), the resonance field curve \( H_p(T) \) provides a rough contour of IPC at low temperatures. Thus, the observation of the reentrant behavior at low temperatures in this system demonstrates that the field dependence of \( \langle \cos \phi_{n+1} \rangle \) at a fixed temperature is nonmonotonic [see also \( J_s(H) \) in Fig. 4(b)]: at low fields it increases with field and at high fields it decreases. The observed opposite steps at the two resonances in the frequency shift in Fig. 1 are in accord with this result.

More importantly, \( H_p(T) \) at low fields follows a contour of in-plane \( J_{c^0}^b \approx 3-4 \times 10^9 \) A/cm\(^2\) [Figs. 3(b) and (c)] as well as of the creep rate \( S \) [Fig. 3(a)]. This newly found scaling can be naturally understood when the meandering vortex structure (witnessed by the low \( H_p \) fa-
cilitates its pinning and reduces the vortex dynamics. In the proposed “splayed glass” phase [8], vortices entangle by confining their finite segments to different columnar paths [see sketch in Fig. 3(b)]. The dynamics is determined by the nucleation and the motion of kinks, which are impeded from sliding along the CDs at the crossing sites when the CDs are roughly localization length apart [8, 28]. In the uniformly splayed CD landscape, random exchanges of vortex lines with numerous kinks will show up as a reduction of IPC, which is proportional to \( J_{c}^{s} \) [see Eq. (1)].

In Figs. 4(a) and (b), we compare \( J_{c}^{a+b}(H) \) and \( J_{c}^{s}(H) \) with those in the pristine samples. From the boomerang \( \langle T \rangle \) in Fig. 3, at a fixed temperature we expect a non-monotonic \( J_{c}^{s}(H) \). Indeed, in sharp contrast with the pristine samples, at low fields \( J_{c}^{s} \) decreases with decreasing \( H \) [dashed lines in Fig. 3(b)]. Concurrently, the in-plane \( J_{c}^{a+b} \) rapidly increases, fast exceeding that of pristine one. We remark that this \( J_{c} \) anti-correlation is counter-intuitive in a coupled pancake system and unique to the splayed glass phase [29]. In pristine BSCCO, the reduction of longitudinal correlation due to decoupling is a consequence of in-plane correlations becoming dominant at higher fields. We also note that in the Bose glass state discussed in the system with parallel CDs such an anti-correlation between \( J_{c}^{s} \) and \( J_{c}^{a+b} \) is not expected. Thus the low-field \( J_{c}^{s} \) reduction discovered in our system is anomalous.

Also striking is that the two resonance lines split apart [Fig. 3(c)]—a slower field-sweep rate gives rise to a larger \( J_{c}^{s} \) [Fig. 3(b)]. We emphasize that the time relaxation of \( J_{c}^{a+b} \) shows an opposite trend [Fig. 4(a)]. All these findings are characteristics of splayed glass.

Thus, through our experiments we demonstrate unusual ‘line’ behaviors in a system with randomly oriented linear defects. The non-monotonic field dependence of \( \langle \cos \phi_{n,n+1} \rangle \), the reentrant “boomerang” path of the JPR field, the scaling of this path in the \( H-T \) phase space with the contours of iso-\( J_{c}^{a+b} \), all support a strong low-field suppression of interlayer phase coherence. While a full theory of \( \langle \cos \phi_{n,n+1} \rangle \) in a system of random distributions of CDs that includes point disorder is yet to be formulated, our findings in vortex structure and dynamics support the presence of a distinct “splayed glass” entangled state of matter.

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[1] T. Halpin-Healy and Y.-C. Zhang, Phys. Rep. 254, 215 (1995).
[2] M. Kardar and Y.-C. Zhang, Phys. Rev. Lett. 58, 2087 (1987).
[3] L. Krusin-Elbaum, T. Shibauchi, B. Argyle, L. Gignac, and D. Weller, Nature 410, 444 (2001); T. Shibauchi et al., Phys. Rev. Lett. 87, 267201 (2001).
[4] J. Maunukseka et al., Phys. Rev. Lett. 79, 1515 (1997).
[5] G. Blatter, M. V. Feigel’man, V. B. Geshkenbein, A. I. Larkin, and V. M. Vinokur, Rev. Mod. Phys. 66, 1125 (1994).
[6] T. Nattermann and S. Scheidt, Adv. Phys. 49, 607 (2000); T. Giamarchi and P. Le Doussal, Phys. Rev. B 52, 1242 (1995).
[7] D. R. Nelson and V. M. Vinokur, Phys. Rev. B 48, 13060 (1993).
[8] T. Hwa, P. Le Doussal, D. R. Nelson, and V. M. Vinokur, Phys. Rev. Lett. 71, 3545 (1993).
[9] Y. Matsuda, M. B. Gaifullin, K. Kumagai, K. Kadowaki, and T. Muchik, Phys. Rev. Lett. 75, 4512 (1995).
[10] Y. Matsuda, M. B. Gaifullin, K. I. Kumagai, M. Kosugi, and K. Hirata, Phys. Rev. Lett. 78, 1972 (1997).
[11] T. Shibauchi et al., Phys. Rev. B 55, R11977 (1997); T. Shibauchi, M. Sato, S. Ooi, and T. Tamegai, Phys. Rev. B 57, R5622 (1998).
[12] L. N. Bulaevskii, M. P. Maley, and M. Tachiki, Phys. Rev. Lett. 74, 801 (1995).
[13] T. Shibauchi et al., Phys. Rev. Lett. 83, 1010 (1999).
[14] M. B. Gaifullin, Y. Matsuda, N. Chikumoto, J. Shimoyama, and K. Kishio, Phys. Rev. Lett. 84, 2945 (2000).
[15] A. E. Koshelev, Phys. Rev. Lett. 77, 3901 (1996).
[16] L. Civale et al., Phys. Rev. Lett. 67, 648 (1991).
[17] M. Konczykowski et al., Phys. Rev. B 44, 7167 (1991).
[18] M. Sato, T. Shibauchi, S. Ooi, T. Tamegai, and M. Konczykowski, Phys. Rev. Lett. 79, 3759 (1997).
[19] L. Krusin-Elbaum et al., Phys. Rev. Lett. 81, 3948 (1998).
[20] M. Kosugi et al., Phys. Rev. Lett. 79, 3763 (1997); M. Kosugi et al., Phys. Rev. B 59, 8970 (1999).
[21] T. Hanaguri, Y. Tsuchiya, S. Sakamoto, A. Maeda, and D. G. Steel, Phys. Rev. Lett. 78, 3177 (1997); Y. Tsuchiya et al., Phys. Rev. B 59, 11568 (1999).
[22] S. Colson et al., Phys. Rev. B 69, 180510(R) (2004).
[23] D. López et al., Phys. Rev. Lett. 79, 4258 (1997).
[24] L. Krusin-Elbaum et al., Appl. Phys. Lett. 64, 3331 (1994).
[25] N. Kameda et al., Phys. Rev. B 72, 064501 (2005).
[26] G. Blatter, V. B. Geshkenbein, and A. I. Larkin, Phys. Rev. Lett. 68, 875 (1992).
[27] At zero field, the plasma frequency is expected to be large \( (\Delta_{c}^{s} > 7 \AA/\text{cm}^{2}) \), but we do not observe the third resonance line at low fields. This is likely due to inhomogeneous field distribution arising from the large \( J_{c}^{ab} \).
[28] T. Schuster et al., Phys. Rev. B 53, 2257 (1996).
[29] We note the important role of points defects (most effective at low T) in reducing the motion of kinks as well as in reducing the interlayer coherence.