MAGNETIC FIELD LIMITATIONS ON ADVECTION-DOMINATED FLOWS

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ABSTRACT

Recent papers discussing advection-dominated accretion flows (ADAF) as a solution for astrophysical accretion problems should be treated with some caution because of their uncertain physical basis. The suggestions underlying ADAF involve ignoring the magnetic field reconnection in heating of the plasma flow, assuming electron heating due only to binary Coulomb collisions with ions. Here we analyze the physical processes in optically thin accretion flows at low accretion rates including the influence of an equipartition random magnetic field and heating of electrons due to magnetic field reconnection. The important role of the magnetic field pointed out by Shvartsman comes about because the magnetic energy density, $E_m$, increases more rapidly with decreasing distance than the kinetic energy density, $E_k$ (or thermal energy density). Once $E_m$ grows to a value of order $E_k$, further accretion to smaller distances is possible only if magnetic flux is destroyed by reconnection. For the smaller distances it is likely that there is approximate equipartition, $E_m \approx E_k$. Dissipation of magnetic energy is associated with the destruction of magnetic flux. We discuss reasons for believing that the field annihilation leads to appreciable electron heating. Such heating significantly restricts the applicability of ADAF solutions, and it leads to a radiative efficiency of the flows of $\sim 25\%$ of the standard accretion disk value.

Subject headings: accretion, accretion disks — galaxies: active — MHD — plasmas — stars: magnetic fields — X-rays: stars

1. INTRODUCTION

Recent papers (see Narayan, Barret, & McClintock 1997; Menou, Quataert, & Narayan 1997; Narayan, Mahadevan, & Quataert 1999 and references therein; Ichimaru 1977) considering advection-dominated accretion flows (ADAF) as a solution for many astrophysical problems should be treated with some caution because of its uncertain physical basis. The suggestions underlying ADAF include the neglect of the heating of the accretion flow due to magnetic field annihilation and the assumption of electron heating due only to binary collisions with protons (ions). These issues were first pointed out by Bisnovatyi-Kogan & Lovelace (1997, hereafter BKL) and have subsequently been discussed further by Quataert (1998), Blackman (1998), and others.

The work by Quataert (1998) envisions an almost uniform magnetic field in the accretion flow, which makes the theory of Alfvénic turbulence of Goldreich & Sridhar (1995) applicable. The linear damping of short-wavelength Alfvén modes is then calculated to determine the relative importance of heating of electrons and ions. We believe that this picture is inconsistent because the magnetic field is strongly nonuniform, in fact, turbulent with $\delta B/B \sim 1$, and magnetic flux is necessarily destroyed in the flow. We have a fundamentally different picture of magnetohydrodynamic (MHD) turbulence in accretion flows (BKL) (see Figs. 1 and 2), which shows that there are many neutral layers of the magnetic field with associated current sheets where field reconnection and annihilation occurs. At these neutral layers, there is sporadic ohmic dissipation of magnetic energy. This picture is qualitatively similar to that for the chaotic magnetic field of the corona of the Sun that we discuss later. The reconnection in ADAF flows is driven (or forced) reconnection due to the compression of the flow, and it unavoidably involves flux annihilation. Thus, the model for free reconnection of Lazarian & Vishniac (1999), which has negligible flux annihilation, is not applicable to ADAF flows.

Competition between rapid accumulation of observational data, mainly from Hubble Space Telescope, X-ray satellites, and the rapid development of theoretical models, creates a situation where a model is sometimes disproved during the time of its publication. Some ADAF models give examples of this. One is connected with an explanation of the unusual spectrum of the galaxy NGC 4258, which has a nonmonotonic dependence. It was claimed that such spectrum may be explained by an ADAF model (Lasota et al. 1996). However, recent observations (Herrnstein et al. 1998) give new data in the X-ray from the Rossi X-Ray Timing Explorer (RXTE) (Cannizzo et al. 1998) that show features not explained by ADAF. High-resolution radio continuum observations of this object (Herrnstein et al. 1998) put severe constraints on the ADAF model, indicating that it does not apply inside $\sim 100$ Schwarzschild radii.

Another example is connected with an attempt to prove the existence of the event horizons of the black holes in ADAF models. Figure 7 of the work by Menou, Quataert, & Narayan (1997), (or Fig. 2 from Narayan, Garcia, & McClintock 1997) was presented as a proof of the existence of the event horizon of black holes and at the same time as a triumph of the ADAF model. Unfortunately, the data for this figure appear to be incomplete, and a full set of observational data mars this picture (Chen et al. 1997).

Radio observations of low-luminosity cores of elliptical galaxies, where ADAF models could explain the X-ray luminosity, put severe constraints on the models (Di Matteo...
et al. 1998). The observed radio flux is much smaller than predicted by the ADAF model with energy equipartition magnetic fields, and require magnetic energy density much below equipartition, which is implausible.

It is, of course, difficult to justify a physical model by astronomical observations without a firm physical basis. Here, we analyze the different processes in an optically thin accretion flow at low accretion rates. Of particular importance is inclusion of the influence of a chaotic magnetic field that is likely embedded in the accreting plasma. The accretion flow necessarily destroys magnetic flux by reconnection. We give reasons for believing that the reconnection gives appreciable heating of the electrons. This electron heating invalidates the usual ADAF solutions as well as advection-dominated inflow/outflow solutions (Blandford & Begelman 1999). This casts doubt on the attempts to connect ADAF models with the existence of the event horizons of black holes. Also, it makes it unlikely that ADAF models explain the low luminosity of massive black holes in nearby galactic nuclei.

In § 2 we discuss the basic equations and give a solution for the time-averaged magnetic field in a quasispherical accretion flow. In § 3 we analyze the energy dissipation in accretion flows where there is equipartition between magnetic energy and flow energy. In § 4 we point out the relevance of observations of magnetic energy dissipation in the solar corona to that in accretion flows. Conclusions of this work are given in § 5.

2. MAGNETIC FIELD ENHANCEMENT IN QUASISPHERICAL ACCRETION

Matter flowing into a black hole from a companion star or from the interstellar medium is likely to be magnetized. Due to the more rapid increase of the magnetic energy density in comparison with kinetic energy densities, the dynamical influence of the magnetic field becomes more and more important as the matter flows inward. Shvartsman (1971) argued that beginning even at large radii there should be approximate equipartition of magnetic and kinetic energies. Of course, the main energy release in an accretion flow occurs in the region of small radii. This equipartition is usually accepted in recent ADAF models of accretion (for example, Narayan & Yi 1995).

In a quasispherical accretion flow of a perfectly conducting plasma, \( d(B \cdot dS)/dt = 0 \), where \( dS \) is a surface area element and \( d/dt \) is the derivative following the flow \( (v = v_r \hat{r} + v_\phi \hat{\phi}) \) in spherical coordinates). With \( dS = r^2 d\Omega \), \( v_r \approx v_r(r) \), and \( d\Omega \) a fixed solid angle increment, one finds that \( B_r \propto 1/r^2 \) independent of the toroidal velocity \( v_\phi \). Writing \( B_r = B_0(r_0/r)^2 \), the radius at which \( pv^2/2 = B^2/(8\pi) \) is

\[
 r_{\text{equiv}} = \left( \frac{gB_0^2r_0^4}{GM} \right)^{2/3},
\]

where \( v_K \equiv (GM/r)^{1/2} \) and the accretion speed is assumed \( v_r = -g v_K \), with \( g = \text{const} \lesssim 1 \). For example, for a black hole mass \( M = 10^6 M_\odot \), \( M = 0.1 M_\odot \) yr\(^{-1} \), \( B_0 = 10^{-3} \) G, \( r_0 = 1 \) pc, and \( g = 0.1 \), we find \( r_{\text{equiv}} \approx 6 \times 10^{14} \) cm, which is much larger than the Schwarzschild radius, \( r_S \approx 3 \times 10^{11} \) cm. Thus the magnetic field resulting from flux freezing is dynamically important in quasispherical accretion flows (Shvartsman 1971), and this is independent of dynamo processes or MHD instabilities. Further accretion for \( r < r_{\text{equiv}} \) is possible only if magnetic flux is destroyed by reconnection and the magnetic energy is dissipated. Figure 1 shows a sketch of the instantaneous poloidal magnetic field configuration.

For example, for a flow with time-averaged velocity \( v = v_r \hat{r} + v_\phi \hat{\phi} \), an approximate solution for the time-averaged magnetic field for infinite conductivity can readily be obtained in the vicinity of the equatorial plane. In this region it is reasonable to assume \( \mathbf{B} = B_r \hat{r} + B_\phi \hat{\phi} \); that is,
$B_\phi$ is neglected. Thus,
\[ \nabla \cdot B = 0 = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 B_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (B_\phi). \]

Near the equatorial plane, $\sin \theta \approx 1$. This equation is then satisfied by taking
\[ B_r = \frac{\text{const}}{r} \exp \left( i \int dr_k + i m \phi \right), \quad (2) \]
\[ B_\phi = f(r) \exp \left( i \int dr_k + i m \phi \right), \quad (3) \]
with the actual fields given by the real parts and with
\[ k \cdot B = k_r B_r + k_\phi B_\phi = 0, \quad (4) \]
where $k \equiv \hat{r} k_r + \hat{\phi} k_\phi$, $k_\phi \equiv m/r$ is the azimuthal wavenumber, and $m = \pm 1, \pm 2, \ldots$, etc. The stationary magnetic field satisfies $\nabla \times (v \times B) = 0$, and this requires
\[ v \times B = \hat{\theta} (v_\phi B_r - v_r B_\phi) = 0, \quad \text{or} \quad B_\phi = (v_\phi/v_r) B_r. \]
That is, $v \propto B$ and $k \cdot \mathbf{v} = 0$. Due to equation (3),
\[ k_r = -(v_\phi/v_r) k_\phi. \quad (5) \]
In ADAF accretion models, $v_\phi/v_r = \text{constant}$, so that $k_r \propto 1/r$. Thus, $|B_r| = |v_\phi/v_r||B_\phi| \propto 1/r^2$, and $f(r) \propto 1/r^2$.

The nature of the field is most easily obtained from the flux function $r A_\phi$ (with $A$ the vector potential), where
\[ B_r = -\frac{1}{r^2} \frac{\partial}{\partial \phi} (r A_\phi), \quad B_\phi = \frac{1}{r} \frac{\partial}{\partial r} (r A_\phi), \]
for $\sin \theta \approx 1$. Note that $(\mathbf{B} \cdot \nabla) (r A_\phi) = 0$. Thus the frozen-in field is described by
\[ r A_\phi = \text{const} \exp \left[ \text{im} \left( \frac{v_\phi}{v_r} \ln r + \phi \right) \right], \quad (6) \]
where it is assumed that $v_\phi > 0, v_r < 0$, and $v_\phi/v_r = \text{const}$. The fact that $|r A_\phi| = \text{const}$ corresponds to the conservation of the ingoing or outgoing flux in a given tube. A given field line satisfies
\[ r = \text{const} \exp \left( - \phi \left| \frac{v_\phi}{v_r} \right| \right). \quad (7) \]

Figure 2 shows the nature of the equatorial field for $m = 2$.

Consider now the problem of magnetic field amplification in an accretion flow that is not infinitely conducting. The flow is described by the MHD equations,
\[ \rho \frac{dB}{dt} = -\nabla p + \rho g + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \eta \nabla^2 \mathbf{v}, \]
\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J}, \quad \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \]
\[ \mathbf{J} = \sigma \left( \mathbf{E} + \mathbf{v} \times \frac{\mathbf{B}}{c} \right), \quad (8) \]
where $\mathbf{v}$ is the flow velocity, $\mathbf{B}$ the magnetic field, $p$ the plasma pressure, $\sigma$ the plasma conductivity, $\eta$ the dynamic viscosity (with $v = \eta/\rho$ the kinematic viscosity), $\mathbf{J}$ the current density, and $E$ the electric field. These equations can

be combined to give the induction equation,
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta_m \nabla \times \mathbf{B}), \]
\[ \approx \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta_m^2 \mathbf{B}, \quad (9) \]
where $\eta_m \equiv c^2/(4\pi \sigma)$ is the magnetic diffusivity, and the approximation involves neglecting $\nabla \mathbf{v}_m$.

In equations (8) and (9) both the viscosity, $\nu$, and the magnetic diffusivity, $\eta_m$, have the same units and both are assumed to be due to turbulence in the accretion flow. Thus, it is reasonable to express both transport coefficients using the “alpha” prescription of Shakura (1972) and Shakura & Sunyaev (1973),
\[ \nu = \alpha c_\ell \ell, \quad \eta_m = \alpha_m c_\ell \ell, \quad (10) \]
where $\ell$ is the outer scale of the turbulence, and $c_\ell = \sqrt{\rho/\mu}$ is the isothermal sound speed. Bisnovatyi-Kogan & Ruzmaikin (1976) introduced $\alpha_m$ and proposed that $\alpha_m \sim \alpha$. Note that $\alpha$ and $\alpha_m$ characterize a turbulent MHD flow in which there is a Kolmogorov cascade of energy from large scales ($\ell_o$) to much smaller scales where the actual (microscopic) dissipation of energy occurs.

We obtain stationary $B$ field solutions for accretion flows with time-averaged flow, $v = \bar{v}_r + \hat{\theta} \bar{v}_\phi$, and with finite diffusivity, $\eta_m$, in the form of equations (2) and (3), but with $k_r$ complex, $k_r = k_r^* + ik_\phi^*$. Equation (3) still applies, but equation (5) does not. In its place we have from equation (9),
\[ 0 \approx ik \times (v \times B) - \eta_m k^2 B, \quad (11) \]
where the approximation involves neglecting terms of order $1/(|k| r) \ll 1$ or smaller. Solution of (11) gives
\[ k_r^* v_r + k_\phi^* v_\phi \approx 0, \quad \text{and} \quad k_r^* \approx -\frac{\eta_m k^2}{|v_r|}, \quad (12) \]
where in this equation $k^2 = (k_r^*)^2 + k_\phi^2$. Consistent with the first part of equation (12), we take $k_r^* = n_r/r$, where $n_r$ is const. For the ADAF flows, we write the second part of equation (10) as $\eta_m = \alpha_m c_\ell r$. The second part of (12) then gives
\[ k_r^* = -\left( \frac{\alpha_m c_\ell}{|v_r|} \right) \frac{n_r^2 + m^2}{r}, \quad (13) \]
where the quantity $\alpha_m c_\ell/|v_r|$ is const.

The imaginary part of $k_r$, $k_\phi$, gives an additional radial dependence of $B_r$ and $B_\phi$ (and $r A_\phi$), multiplying the earlier expressions (eqs. [2], [3], and [6]) by a factor $r^\delta$, where $\delta = (\alpha_m c_\ell/|v_r|)(n_r^2 + m^2)$. For radial distances $r < r_{\text{equi}}$, we have equipartition with magnetic energy density $E_m \propto r^{-5/2}$ so that $|B_r| \propto r^{-3/4}$. This requires $\delta = 3/4$ so that $r A_\phi \propto r^{3/4}$, which corresponds to the flux within a given tube decreasing as $r$ decreases. The accretion speed is
\[ |v_r| = \frac{3}{4} (n_r^2 + m^2) \alpha_m c_\ell. \quad (14) \]
Note that for the present solution the accretion depends on the magnetic diffusivity $\eta_m$ but not the viscosity, $\nu$; more general solutions have $v_r$ dependent on both viscosity and diffusivity. Validity of this solution requires $|k_r^*|/|k| = (3/4)(n_r^2 + m^2)^{1/2} \ll 1$. The field line pattern is similar to that in Figure 2, but the number of field lines in a tube $\propto r^{3/4}$. 
The magnitude of the magnetic field follows from the requirement that the inflow of angular momentum carried by the matter, \( \dot{L}_{\text{mech}} = 4\pi r^2 \dot{\sigma}_m (\sin \theta \dot{\rho}_v \nu_v, \dot{\nu}_v, \dot{\nu}_v / \nu_v < 0) \), equal the outflow carried by the field, \( \dot{L}_{\text{mag}} = -r^3 B, B_0 > 0 \), which gives \( \rho \nu_v^2 \approx B^2 / 4\pi \). More generally, turbulent viscosity carries away part of the angular momentum, in which case the magnetic field strength is reduced. For \( r \leq r_{\text{equi}} \), we have \( B_r r_{\text{equi}} (r_{\text{equi}} / r)^{3/4} \), and zero angular momentum flux gives

\[
B_0 = \left[ \frac{4(n_2^2 + m^2)\alpha_m (\nu_v / \nu_v) \sqrt{G M M \dot{\rho} M \dot{r}}}{3\rho / 2} \right]^{1/2}.
\]

This gives \( B_0 \approx 55 \, G \) for \( r_{\text{equi}} = 10^{15} \, M \), \( M = 10^6 M_\odot \), \( M = 0.1 \, M_\odot \, \text{yr}^{-1}, \alpha_m = 0.1, \, c_v / \nu_v = 0.1, \) and \( n_2^2 + m^2 = 10^2 \).

3. STATIONARY MAGNETIC FIELD DISTRIBUTION IN QUASISPERICAL ACCRETION

In optically thin accretion disks at low accretion rates, the density of the matter is low and energy exchange between electrons and ions due to binary collisions is slow. In this situation, due to different mechanisms of heating and cooling for electrons and ions, the electrons and ions may have different temperatures. This was first discussed by Shapiro, Lightman, & Eardley (1976), but advection was not included. Narayan & Yi (1995) point out that advection in this case is extremely important. It may carry the main energy flux into a black hole, giving a low radiative efficiency of the accretion of \( 10^{-3} \) to \( 10^{-4} \) (ADAF). This conclusion is valid only when the effects, connected with magnetic field annihilation and heating of matter because of it, are neglected.

The ADAF solutions assume an alpha prescription for the viscosity. Thus, as the ion temperature increases, the viscosity and the viscous dissipation also increases. If the energy losses by ions are small, then there can be a kind of “thermo-viscous” instability where heating increases the viscosity and the increase in viscosity increases the heating. Development of this instability leads to formation of ADAF. In the ADAF solution the ion temperature is of the order of the virial temperature \( \sim G M \rho / r \). This means that even for high initial angular momentum, the disk is very thick, forming a quasi-spherical accretion flow.

The presence of a finite magnetic field considerably changes the picture of ADAF. It was shown by Shvartsman (1971) that the radial component of the magnetic field increases so rapidly in a quasispherical inflow that equipartition between magnetic and kinetic energy is reached in the flow far from the black hole horizon. In the region of the flow where the main energy production takes place there is equipartition. In the presence of an equipartition magnetic field the efficiency of radiation during accretion of matter to a black hole increases enormously, from \( \sim 10^{-8} \) up to \( \sim 0.1 \) (Shvartsman 1971) due to the efficiency of a magneto-bremstrahlung radiation. Thus, the possibility of an ADAF regime for a spherical accretion was noticed long ago. To support the condition of equipartition, continuous magnetic field reconnection is necessary, leading to annihilation of the magnetic flux, and corresponding energy dissipation due to ohmic heating of the matter. Bisnovatyi-Kogan & Ruzmaikin (1974) showed that the radiative efficiency of quasispherical accretion to a black hole could be as high as \( \sim 30\% \) owing to ohmic heating.

As mentioned, the poloidal magnetic field energy density tends to grow locally as \( E_B \propto r^{-7} \) with decreasing distance \( r \). However, as a result of equipartition \( E_m \propto r^{-5/2} \). The difference in the radial gradients of \( E_m \) and \( E_m \) must therefore go into ohmic heating of the matter. This leads to the important result

\[
T \frac{dS}{dr}_{\text{ohm}} = \frac{3}{2} \frac{B^2}{8\pi r},
\]

with \( S \) the entropy per unit mass, \( T \) measured in energy units, and \( B = \langle B^2 \rangle^{1/2} \), where the angle brackets indicate a time average over the turbulence (Bisnovatyi-Kogan & Ruzmaikin 1974).

In quasispherical accretion flows, equipartition between magnetic and kinetic energy of the flow was proposed by Shvartsman (1971),

\[
\frac{B^2}{8\pi} \sim \frac{1}{2} \rho \nu_v^2 = \frac{\rho GM}{r},
\]

which corresponds to an Alfven speed \( v_\text{A} \equiv B / \sqrt{4\pi \rho} \sim \sqrt{2v_\text{k}}, \) with \( v_\text{k} = \sqrt{GM/r} \) the Keplerian speed. For disk accretion, where there is more time for a field dissipation, Shakura (1972) proposed equipartition between magnetic and turbulent energy. For an alpha prescription for the viscosity, where the turbulent velocity is \( v_t = v_a c_s \), this leads to the relation

\[
\frac{B^2}{8\pi} \sim \frac{1}{2} \rho \nu_v^2 = \frac{1}{2} \nu_a^2 c_s^2.
\]

In ADAF solutions where the ion temperature is of the order of the virial temperature, equations (17) and (18) give comparable estimates for \( B^2 / 8\pi \) if \( \nu_a \sim 1 \). Note, however, that in the following the exact value of \( v_\text{A} \) is not assumed.

For quasispherical accretion, the local electric field strength can be estimated as \( \mathcal{E} \sim v_t B/c \). The heating of the matter by ohmic dissipation is then given by

\[
T \frac{dS}{dr}_{\text{ohm}} = -\frac{\sigma e^2}{\rho v_t} \sim -\frac{\sigma e^2 B^2}{\rho v_t c_s^2}.
\]

Using equations (10) and \( v_t = v_a c_s \), we find that equation (19) is compatible with (16) if \( c_s^2 / \alpha_m \sim (3/4) \langle \epsilon / \rho \rangle v_t / c_s \).

Equations for the radial variations of the ion and electron temperatures can be written as

\[
\frac{dE_i}{dt} - \frac{p_i}{\rho^2} \frac{dp_i}{dt} = \mathcal{H}_i + (1 - g) \mathcal{H}_B - Q_{\text{i}},
\]

\[
\frac{dE_e}{dt} - \frac{p_e}{\rho^2} \frac{dp_e}{dt} = \mathcal{H}_e + g \mathcal{H}_B + Q_{\text{e}} - \mathcal{C}_{\text{brems}} - \mathcal{C}_{\text{cy}}
\]

which differ from the corresponding equations of BKL by separating out the viscous and ohmic heating contributions. Here, \( d/dt = \partial / \partial t + v_i \partial / \partial r \), which is equal to \( v_i \partial / \partial r \) for stationary conditions, \( \mathcal{C}_{\text{brems}} \) represents the bremsstrahlung cooling, and \( \mathcal{C}_{\text{cy}} \) the cyclotron cooling including the self-absorption (Trubnikov 1958, 1973), which is important (see expressions given in BKL). The rate of the energy exchange between ions and electrons due to the binary collisions was obtained by Landau (1937) and Spitzer (1940) as

\[
Q_{\text{i}} = \frac{4\sqrt{2 \pi n e^2}}{m_i m_e} \frac{T_i^3}{T_e^3}^{-3/2} \ln(\Lambda),
\]

where

\[
T_a = \left( \frac{T_e + T_i}{2} \right)^{3/2} \ln(\Lambda).
\]
with $\ln(\Lambda) = \mathcal{O}(20)$ the Coulomb logarithm. We may express parameters of the accretion flow as

$$|v_r| = \frac{\alpha c_s^2}{v_k \mathcal{J}}, \quad \rho = \frac{M}{4\sqrt{2\pi \alpha}} \frac{v_k^2}{r^2 c_s^3},$$

(23)

where $\mathcal{J} \equiv 1 - (r_{in}/r)^{1/2}$.

The rates of viscous heating of ions $\mathcal{H}_n$ and of electrons $\mathcal{H}_e$ can be written as

$$\mathcal{H}_n = \frac{3}{2} \frac{\alpha v_k c_s^2}{r}, \quad \mathcal{H}_e = K_v \mathcal{H}_n.$$  

(24)

For a single temperature plasma, the microscopic viscosity coefficient of electrons gives $K_v = \sqrt{m_e/m_i}$ (Chapman & Cowling 1953; Braginskii 1965). As discussed by BKL, the microscopic viscosities are much too small to be important in the collisionless regime of accretion flows. In the collisionless regime, $K_v$ could be significantly smaller than $\sqrt{m_e/m_i}$.

In equations (20) and (21) the rate of ohmic heating of ions is written as $(1 - g)\mathcal{H}_n$ and that of ions as $g\mathcal{H}_B$, with $0 \leq g \leq 1$. Using equations (16) and (24), we find

$$\frac{\mathcal{H}_B}{\mathcal{H}_n} = \frac{|v_r| B^2/(8\pi \rho)}{\alpha v_k c_s^2} = \frac{1}{2} \frac{v_k^2}{\alpha c_s^2} \frac{|v_r|}{v_k}.$$  

(25)

In view of equation (21), this ratio is equal to $v_k^2/(2v_k^2 \mathcal{J})$. For equipartition (17), this ratio is simply $1/\mathcal{J}$, which is unity except close to the inner radius of the flow.

The partition of the ohmic heating between electrons and ions, measured by the quantity $g$, has a critical influence on the model, assuming that $Q_{ie}$ is due only to binary collisions (eq. [22]) and is not altered by plasma turbulence. Observations of the solar corona outlined in § 4 point to the importance of magnetic field reconnection events and indicate that a significant part of the magnetic field energy is dissipated by accelerating electrons.

It follows from the physical picture of the magnetic field reconnection that transformation of magnetic energy into heat is connected with the time rate of change of the magnetic flux and the associated electric field in the neutral layer which accelerates particles. The electric force acting on the electrons and the protons is the same, but the accelerations are larger by a factor $m_i/m_e$ for electrons. During a short time the electrons gain much larger energy than the ions. Additional particle acceleration and heating is expected at shock fronts, appearing around turbulent cells where reconnection occurs. In BKL, equations of the form of equations (20) and (21) were solved in the approximation of nonrelativistic electrons with the viscous and ohmic heating terms combined. Using equation (25), the total (viscous + ohmic) heating of electrons and of ions is

$$\mathcal{H}_i = \left(1 + \frac{2v_k^2}{v_A^2} \mathcal{J} - g\right) \mathcal{H}_B, \quad \mathcal{H}_e = g \mathcal{H}_B.$$  

(26)

In the expression for cyclotron emission, self-absorption was taken into account using the analysis of Trubnikov (1958, 1973). The results of calculations for $g \sim 0.5$–1 show that almost all of the energy of the electrons is radiated. Thus the radiative efficiency of the two-temperature, optically thin spherical accretion flow is not less than $\sim 25\%$. Note that the account of the effect of plasma turbulence on the energy exchange rate $Q_{ie}$ could lead to a radiative efficiency close to the value it has for optically thick, geometrically thin disks.

4. RECONNECTION AND HEATING OF ELECTRONS

Studies of the solar corona have led to the general conclusion that the energy buildup in the chaotic coronal magnetic field by slow photospheric driving is released by magnetic reconnection events or flares on a wide range of scales and energies (Benz 1997). The observations support the idea of “fast reconnection,” not limited to a rate proportional to an inverse power of the magnetic Reynolds number (Parker 1979, 1990). The data clearly show the rapid acceleration of electrons (e.g., hard X-ray flares, Tsuneta 1996) and the acceleration of ions (e.g., gamma-ray line events and energetic ions; Reames et al. 1997), but the detailed mechanisms of particle acceleration are not established.

An essential aspect of the coronal heating is the continual input of energy to the chaotic coronal magnetic field due to motion of the photospheric plasma. As emphasized here, there is also a continual input of energy to the chaotic magnetic field in a quasispherical accretion flow due to compression of the flow which gives $|B| \propto 1/r^2$ for a perfectly conducting plasma. The fact that the ratio of the plasma to magnetic pressures is small in the corona but of order unity in the accretion flow may, of course, affect the details of the plasma processes. (The configuration of magnetic field in the geomagnetic tail of the Earth is distinctly different from the chaotic coronal field, and this makes the geotail reconnection—where the magnetic energy is released mainly as bulk kinetic energy—less similar to the field in accretion flows.)

Figure 3 shows a sketch of driven (or forced) magnetic field reconnection. Plasma flows into the neutral layer from above and below with a speed of the order of the turbulent velocity $v_t$. Magnetic flux is destroyed in this layer as $\mathcal{J}$ above and below with a speed of the order of the turbulent velocity $v_t$. Magnetic flux is destroyed in this layer as $\mathcal{J}$.
there is an electric field in the $z$-direction $\mathbf{E}_z = -(1/c)\partial A_z/\partial t = \ell (v B_0/c)$, where $B_0$ is the field well outside of the neutral layer. This is the same as the estimate of the electric field given by BKL, and we discuss it further as an illustration of a mechanism of electron acceleration. (Other processes that may also be important for electron acceleration in reconnection regions are MHD turbulence and shock acceleration in the downstream region. See LaRosa et al. 1996 and Blackman 1998.) In the vicinity of the neutral layer (|$y| \ll \ell$) this electric field is typically much larger than the Dreicer electric field for electron runaway $E_0 = 4\pi e^4 (n_e/k T_e) \ln \Lambda$ (Parail & Pogutse 1965), and this leads to a streaming motion of electrons in the $z$-direction. Note that it is much more difficult to have ion runaway because of the large gyro radii of ions (BKL; Lesch 1991; Romanova & Lovelace 1992). Of course, in the almost uniform magnetic field assumed by Quataert (1998) runaway of particles are not expected. A study of particle acceleration in collisionless reconnection by Moses, Finn, & Ling (1993) indicates that the heating of electrons is comparable to that of ions. The theory of electron acceleration in reconnection events has been developed by LaRosa et al. (1996) and Tsuneta & Naito (1998).

The streaming of the electrons can give rise to a number of different plasma instabilities, the growth of which gives an anomalous resistivity. Galeev & Sagdeev (1983) discuss the quasilinear theory for such conditions and derive expressions for the rate of heating of electrons,

$$\frac{dT_e}{dt} = \frac{1}{n} \int d^3k \gamma_k W_k \frac{k \cdot u_e}{\omega_k}, \quad (27)$$

and ions,

$$\frac{dT_i}{dt} = \frac{1}{n} \int d^3k \gamma_k W_k, \quad (28)$$

where $W_k$ is the wavenumber energy spectrum of the turbulence, $\gamma_k$ is the linear growth rate of the mode with real frequency $\omega_k$, $n$ is the number density, and $u_e$ is the electron drift velocity. If the quantity $k \cdot u_e/\omega_k$ is assumed constant and taken out of the integral sign in equation (27), then the ratio of heating of electrons to that of ions is

$$\frac{dT_e}{dT_i} = \frac{u_e}{\omega_k/k}. \quad (29)$$

This result is independent of the instability type. Galeev & Sagdeev (1983) point out that for most instabilities, equation (29) predicts faster heating of electrons than ions. Earlier, Lesch (1991), Field & Rogers (1993), and Di Matteo (1998) emphasized the important role of reconnection in accelerating electrons to high, nonthermal energies in accretion flows of AGNs.

5. CONCLUSIONS

Observational evidence for a black hole in the center of our Galaxy and in the nuclei of other galaxies (Cherepashchuk 1996; Haswell 1999) make it necessary to revise or generalize theoretical models of accretion flows. Improvements of the models include account of advective terms and account of the influence of equipartition magnetic fields. Conclusions based on ADAF solutions for optically thin accretion flows at low mass accretion rates are at present open to question. This is because the ADAF solutions neglect the unavoidable magnetic field annihilation, which may give significant electron heating. In contrast with the ADAF solutions, we argue that the radiative efficiency is $\gtrsim 25\%$ of the standard value (for an optically thick, geometrically thin disk). For equipartition between magnetic and kinetic energies, we argue that half of the dissipated energy of the accretion flow results from the destruction of the magnetic field. Further, we give reasons that suggest that at least half of this energy goes into accelerating electrons in reconnection events analogous to those in the corona of the Sun. It is possible that a full treatment of the ion-electron energy exchange due to the plasma turbulence further increases the radiative efficiency to its standard value (see also Fabian & Rees 1995).

Some observational data that were interpreted as evidence for the existence of the ADAF regime have disappeared after additional accumulation of data. The most interesting example of this sort is connected with the claim of “proof” of the existence of event horizons of black holes due to manifestation of the ADAF regime of accretion (Narayan, Garcia, & McClintock 1997). Analysis of a more complete set of observational data (Chen et al. 1997) shows that the statistical effect claimed as an evidence for ADAF disappears. This example shows the danger of “proving” a theoretical model with preliminary observational data. It is even more dangerous when the model is physically not fully consistent, because then even a reliable set of the observational data cannot serve as a proof of the model.

Thus, there are fundamental reasons for questioning the application ADAF models to underluminous AGNs, where the observed energy flux is much smaller than expected for standard accretion disk models. Two possible explanations may be suggested. One is based on more accurate estimations of the accretion mass flow into the black hole, which could be overestimated. Another possibility is based on existence of jets and/or outflows that expel most of the matter supplied at large distances $M_{\text{acc}}(\infty)$ Bisnovatyi-Kogan 1999. The accretion to the black hole may be much smaller than $M_{\text{acc}}(\infty)$ with the result that the accretion luminosity is much smaller without the radiative efficiency being small. (Model of Blandford & Begelman 1999 takes into account outflows, but requires the same assumption as earlier ADAF models that there is no heating of electrons. BKL and the present work argue against this assumption.) Many compact astrophysical objects are observed to have jets or outflows, including active galaxies and quasars, old compact stars in binaries, and young stellar objects. The formation of jets and outflows is very probable under conditions where an ordered magnetic field threads a thin disk (see reviews by Bisnovatyi-Kogan 1993; Lovelace, Ustyugova, & Koldoba 1999). To extend this interpretation, we suggest that underluminous AGNs may lose the main part of their energy to the formation of jets or outflows. This suggests a search for a correlation between existence of jets or outflows and underluminous galactic nuclei.

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