Parametric modeling of sports prostheses based on the flat spring design formulas

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Abstract
Sports prosthesis for lower-extremity amputees has a mechanical structure similar to flat springs, and its elastic energy is expected to improve sports performance. However, it is quite challenging to represent the mechanical phenomena during the takeoff action with sports prosthesis because the contact point to the ground moves based on the direction and deformation of the prosthesis. The purpose of this study is to propose a parametric model of sports prosthesis based on the flat spring design formulas to represent the deformation and rolling contact with the ground with a reasonable computational cost. The shape of the prosthesis is modeled as serial elements, and it can easily be changed by using design parameters, such as the curvature and length of each element. The curvature of each element of the prosthesis is modified by the deflection angle of the flat spring model, and the contact point to the ground is calculated by considering the deformation and rolling contact. The spring properties obtained from the proposed model well agreed with the result of a finite element analysis. Moreover, simulation results revealed that the deformed shape of the prosthesis and the takeoff action in the long jump qualitatively agreed with the actual phenomena. As future research, the proposed model, coupled with the human body model, will be applied to a computer simulation system to optimize the shape of the prosthesis in order to improve sports performance.

Keywords: Human dynamics, Simulation, Rolling contact, Ground reaction force, Long jump

1. Introduction
Sports prosthesis is made of carbon fiber reinforced plastic (CFRP). Based on the elastic energy associated with the deformation of sports prosthesis, the performance of a person with prosthesis has been shown to be comparable with that of a healthy person in disabled sports; moreover, the possibility of exceeding the performance of a healthy person has been demonstrated in a recent study (Dyer et al., 2010; Hobara et al., 2015). However, the construction of a mechanical model for the elastic and deformation properties of sports prosthesis is crucial for an adequate understanding of the properties and development of more efficient prosthetic legs.

In studies on the mechanical properties of sports prosthesis, finite element methods are often used; besides, stress distribution and elastic deformation characteristics of prostheses when an external force is given have been investigated (Omasta et al., 2012; Saad et al., 2017). Since some researchers have considered the evaluation of viscoelastic properties of prosthetic legs by numerical analysis (Geil, 2002), it is thought that the modeling and analysis of the characteristics will not be a difficult task, considering only the prosthesis. However, in the use of actual sports prosthesis, it is necessary to consider the interaction with the human body movement and contact with the ground.

When contact is made with the ground, the shape of the prosthesis is deformed by the load, and the position of the contact point to the ground is changed dynamically as the prosthesis rolls around the contact point. Hence, the mechanical properties of the prosthesis cannot be sufficiently expressed as simple relations between static load and deflection, but it is necessary to consider the interaction with the human body movement and contact with the ground.
while considering the deformation. Furthermore, putting into consideration the optimal design of the shape of the sports prosthesis, evaluating the mechanical properties becomes even more complicated. In general, when solving structural optimization problems, such as mechanical structures, it is crucial to repeat the performance evaluation of the object several times on the computer, which, of course, increases the computational cost of using prosthetic models based on finite element methods to solve the optimization problem. Therefore, it is necessary to model the deformation characteristics and elastic properties of the prosthesis at a relatively low computational cost.

On the other hand, in the design and development of springs used in industrial products, a methodology to determine the relationship between spring shape and elastic properties at a reasonable computational cost has been established (Oguchi, 1976; JIS B2713, 2009). This methodology can systematically evaluate the elastic properties of shape if the shape can easily be handled like a flat plate. By considering a sports prosthesis as a type of flat spring, this methodology can be applied to the sports prosthesis design. This study aims to provide a computational model that evaluates the mechanical properties of prostheses at low computational cost based on flat spring design formulas. In other words, we propose a method that parametrically defines and modifies a prosthetic shape and its elastic deformation characteristics. Further, we solve the contact problem with the ground associated with prostheses and coupled the proposed model with the human body model. Moreover, the effectiveness and efficiency of the proposed model are demonstrated via comparison with the finite element analysis model of a simple shape. Finally, we discuss usefulness of the proposed model, such as the possibility of applying it to obtain an optimal design.

2. Method

2.1 Modeling assumptions

Figure 1 shows an example of a prosthetic leg (Ossur, Cheetah Xtreme), referenced as an initial model to create a prosthetic model. As shown in the figure, the prosthesis shape is represented by a complex curve but can be expressed as a combination of simple arcs or straight lines if divided into short elements. Although the thickness and width of the prosthesis vary depending on the location, the change is smooth, and the cross-sectional shape can be expressed as a simple rectangle. In addition, the CFRP, which is a prosthetic material, basically has uneven material properties, such as anisotropy and a laminated structure. However, we assumed that the bending property, which is critical for modeling of the mechanical properties of the prosthesis, can be expressed by the constant material properties (Young’s modulus) and the constant second moment of the cross-sectional shape. Further, the basic spring characteristics of the prosthesis can be expressed by two-dimensional motion in the sagittal plane, although the actual prosthesis contains three-dimensional deformation characteristics, such as twisting. Therefore, we expressed the mechanical properties of the prosthesis by a two-dimensional model with uniform material properties. In this model, dynamic properties, such as mass and moment of inertia, were not taken into account; thus, we only considered elastic properties, i.e., the relationship between load and deformation. The modeling of these dynamical properties is described in the latter part of this paper.

![Fig. 1](image) An example of sports prosthesis (Ossur, Cheetah Xtreme).

2.2 Formulation by flat spring theory

In this subsection, we describe the two-dimensional relationship between the load and deflection angle of a prosthesis represented by a flat spring. We expressed the prosthesis shape as a cantilever-shaped flat spring with one fixed terminal, as shown in Fig. 2. Further, we assumed the point of application of the load to be the origin of the
coordinate system. When the $X$-direction load in the $XY$ coordinate system is $P_x$, the $Y$-direction load is $P_y$, the deflection angle at the load point is $i$, the coordinate along the curved axis of the spring is $s$, and the spring length is $S$; the relationship between the deflection angle, $i$, and the load of a flat spring, $(P_x, P_y)$, is as follows:

$$i = \int_0^S \frac{M}{EI} ds = \frac{1}{EI} \left( P_y \int_0^s x ds - P_x \int_0^s y ds \right).$$

(1)

Here, $M$ is the bending moment acting on the minute element, $ds$, and $E$ is Young’s modulus and is set at 125 GPa, assuming CFRP in the fiber direction as a material. The value of $E$ is based on the tensile test results of CFRP for the prosthesis. Furthermore, $I$ is the moment of inertia of the area of the shape. When the cross-sectional shape is a rectangle plate of width, $b$, and thickness, $h$, the moment of inertia of area, $I$, is given by $I = bh^3/12$. Young’s modulus, $E$, and the moment of inertia of area, $I$, are, however, formulated as constants regardless of the location in this formulation. Here we defined $\Gamma_x$ and $\Gamma_y$ as follows:

$$\Gamma_x = \int_0^s y ds;$$

$$\Gamma_y = \int_0^s x ds.$$

(2)

Thus, the deflection angle, $i$, is given as,

$$i = \int_0^S \frac{M}{EI} ds = \frac{1}{EI} \left( P_y \Gamma_y - P_x \Gamma_x \right).$$

(3)

Here, $\Gamma_x$ and $\Gamma_y$ are, respectively, the shape coefficients in the static moment of the spring axis for the $X$- and $Y$-direction axes determined by the shape and position of the spring element.

Next, we consider a method for calculating specific shape coefficients. Although a prosthesis can be represented by a combination of multiple arcs and linear elements as described above, a straight element can be represented as an arc element with zero curvature; thus, the shape of a prosthetic leg can be expressed as a multi-linked shape with arc elements of length $S$ and curvature $k$ in series. Consequently, the shape of a prosthesis can be defined and modified by using the arc length and curvature as the design parameters of the prosthesis.

Here, we focus on the shape coefficient, $\alpha \Gamma$, of one arc element. Since the position and direction of the prosthesis change during human body movement, we defined the $xy$ coordinate system as shown in Fig. 3, in which the position acting as the load was set to be the origin. In this case, the shape coefficients, $\alpha \Gamma_x$ and $\alpha \Gamma_y$, of the arc element of length $S$ and curvature $k$, which is located at the position $(x_0, y_0)$ from the load action point, and inclined at an angle $\alpha$, are given as follows:

![Fig. 2 Formulation of the two-dimensional elastic property in a sports prosthesis by using a flat spring model.](image-url)
Fig. 3 Formulation of the shape coefficient in one arc element of a flat spring model.

\[ a^e \Gamma_y = \int_0^S xds = \int_0^S \left\{ x_0 + \frac{1}{k} \sin ks \cdot \cos \alpha - \frac{1}{k} (1 - \cos ks) \sin \alpha \right\} ds; \]

\[ a^e \Gamma_x = \int_0^S yds = \int_0^S \left\{ y_0 + \frac{1}{k} \sin ks \cdot \sin \alpha - \frac{1}{k} (1 - \cos ks) \cos \alpha \right\} ds. \]

(4)

Here, a small \( s \) is the coordinates along the arc element, and when integrating about these points, we obtained the following equations:

\[ a^e \Gamma_y = \frac{1}{k^2} \left\{ (k^2 x_0 - k \sin \alpha) S - \cos Sk \cdot \cos \alpha + \sin Sk \cdot \sin \alpha + \cos \alpha \right\}; \]

\[ a^e \Gamma_x = \frac{1}{k^2} \left\{ (k^2 y_0 + k \cos \alpha) S - \sin Sk \cdot \cos \alpha - \cos Sk \cdot \sin \alpha + \sin \alpha \right\}. \]

(5)

The curvature \( k \) of the shape parameter becomes zero if the arc element is a straight line. Since Eq. (5) is expressed in the form of division by the curvature \( k \), it becomes singular when the curvature is zero. To avoid the problem of division-by-zero curvature, we expressed Eq. (5) as a Taylor series of the arc length \( S \) up to the sixth order and obtained the following equations:

\[ a^e \Gamma_y = \frac{1}{720} S^6 k^4 \cos \alpha + \frac{1}{120} S^5 k^3 \sin \alpha - \frac{1}{24} S^4 k^2 \cos \alpha - \frac{1}{6} S^3 k \sin \alpha + \frac{1}{2} S^2 k \cos \alpha + \frac{1}{2} S \sin \alpha + x_0; \]

\[ a^e \Gamma_x = \frac{1}{720} S^6 k^4 \sin \alpha - \frac{1}{120} S^5 k^3 \cos \alpha + \frac{1}{24} S^4 k^2 \sin \alpha + \frac{1}{6} S^3 k \cos \alpha + \frac{1}{2} S^2 \sin \alpha + y_0. \]

(6)

The sixth order, i.e., the relatively high-order Taylor series, was employed in order to express the curvature of the arc element up to 90° at the center angle.

Further, we considered the modeling of the overall shape in the prosthesis. The prosthesis consists of multiple arcs or straight elements connected in series; additionally, the connection between the elements was assumed to be continuous. Given the origin position \( (x_e, y_e) \), inclination angle \( \alpha_e \), curvature \( k_e \), and element length \( S_e \) of the \( e \)th element, as shown in Fig. 4, the origin position \( (x_{e+1}, y_{e+1}) \) and the inclination angle \( \alpha_{e+1} \) of the next \( e+1 \)th element is determined as follows:

\[ x_{e+1} = \frac{1}{k_e} \sin k_e S_e \cdot \cos \alpha_e - \frac{1}{k_e} (1 - \cos k_e S_e) \cdot \sin \alpha_e + x_e; \]

\[ y_{e+1} = \frac{1}{k_e} \sin k_e S_e \cdot \sin \alpha_e - \frac{1}{k_e} (1 - \cos k_e S_e) \cdot \cos \alpha_e + y_e; \]
If this process is repeated recursively, the shape and posture of the entire prosthesis can be determined. The position coordinates of any point in the middle of the prosthetic element, such as the contact point with the ground, can also be obtained similarly if the element length $S_e$ is replaced with the distance to any middle point along the element shape. Furthermore, to avoid division-by-zero problems due to the curvature $k_e$, we approximated Eq. (7) using Taylor series.

$$\alpha_{e+1} = k_e S_e + \alpha_e.$$  \hspace{1cm} (7)

By repeating this process recursively as in Eq. (7), one can represent the overall shape of the prosthesis, including the deformation associated with the applied load.

2.3 Motion generation including contact with the ground

So far, we have derived the angle of deflection and the involved posture and deformation of the entire prosthesis, assuming the load, $P$, acting on the prosthesis is known. In this subsection, we attached prosthesis model which is combined flat spring model to human body model, and simulation of motion considering the contact with ground was performed. In the simulation of motion generation, including the human body model, we utilized motion displacements, such as the joint angle in the human body, as state variables. If the current force and moment, such as the ground reaction force and the joint driving moment, are determined from the current motion displacement and velocity in the state variables, the acceleration can be calculated using the equation of motion, which is explicitly solved in the forward direction. In other words, the steps for the calculation of the human motion simulation, including the prosthesis model, are as follows. Firstly, the relative positional relationship between the prosthesis and ground is determined by the simulation of the entire body movement. Secondly, the prosthetic leg is deformed (i.e., the deflection is given) not to get into the ground. Thirdly, the load acting on the prosthesis, i.e., the ground reaction force, is derived so as to realize the deflection amount. Lastly, the required ground reaction force is applied to the human body model, including the prosthesis, and the human body movement is generated. To realize the procedure of such human motion simulation, it is inappropriate to apply the derivation procedure described above to obtain the deformation by applying a load in the prosthesis model; rather, the opposite procedure should be used, that is, assuming the deformation condition of the prosthesis is known, the load required for the deformation should be derived. The interference with the ground should also be considered at the same time. Based on the above discussion, we utilized the following calculation procedure based on an optimization method to determine the required load value from the motion displacements.
1) First, the right horizontal direction is denoted as $x$, the vertically upward direction is denoted as $y$, and the ground height is assumed to be $y = 0$, as the spatial coordinates. In addition, the ground side is assumed to be deformed elastically, and the linear spring constant is defined as $k_g$, as explained in what follows.

2) The lowest point in the prosthetic leg is determined at the current posture, and contact with the ground is judged whether the height of this point falls below $y = 0$. At the time of the first contact with the ground, the $x$ coordinate of the contact point is defined as $x_0$, the contact point on the prosthetic side is set to $p_0$, and these points are recorded as the starting points of the rolling contact.

3) The load values, $\overline{P}_x$ and $\overline{P}_y$, are defined as unknowns (search parameters) in the optimization calculation. The optimization calculation is carried out for each discrete time in the simulation. The initial values in the optimization are set to zero at the start of the simulation and then to the load values before one discrete time.

4) The deflection angle, $i_p$, in each element of the prosthesis is determined based on this assumed load, and the contact point, $p_t$, on the prosthetic side and the position coordinates, $x$ and $y$, are obtained by Eqs. (7) and (8).

5) The following cost function for the optimization, $C$, is determined, and the load values, $\overline{P}_x$ and $\overline{P}_y$, which minimize it, are computed by the optimization. Thus, we have

$$C = (x - x_g - x_{RollingContact})^2 + (y - y_g)^2 + \xi \sum_i i_i^2;$$

$$x_g = \frac{\overline{P}_x}{k_g}, \quad y_g = \frac{\overline{P}_y}{k_g}.$$  \hspace{1cm} (9)

Here, $x_g$ and $y_g$ are the amounts of deformation of the ground when considering the elastic deformation of the ground, and $\xi$ is the weighting coefficient. Further, $x_{RollingContact}$ is an $x$-directional contact position, which is determined from the geometric relationship between the ground and prosthetic shape, starting at the position where the prosthesis first made contact with the ground, assuming that the prosthesis is in rolling contact without sliding against the ground. Precisely, $x_{RollingContact}$ is expressed as follows:

$$x_{RollingContact} = x_0 + ArcLength(p_0, p_t).$$  \hspace{1cm} (11)

Here, $ArcLength(p_1, p_2)$ is a function that obtains the arc length between the two points $p_1$ and $p_2$ on the prosthesis, and $x_0$, $p_0$, and $p_t$ are as defined earlier. As shown in Eq. (10), the linearly elastic property was also given with respect to the ground side. In the proposed model, the amount of deflection in the tangential (axial) direction of the prosthetic shape is zero even if any load is applied in the tangential direction. Therefore, there is a problem that the load in the tangential direction cannot be obtained backward. On the other hand, if the ground side has elastic properties, it is possible to determine the tangential load of the prosthesis from the relation between the displacement and load in the ground side. Of course, the ground side actually has some elasticity characteristic, and it is necessary to model it. From the above observation, we obtained the load so as to assume rolling contact, in order to satisfy the geometric relationship with the ground and minimize the sum of the deflection angles in the prosthetic elements. This load value becomes the ground reaction force acting on the prosthesis from the ground.

In this model, we did not take into account dynamic characteristics, such as mass and moment of inertia; instead, we only considered elastic properties, i.e., the relation between load and deformation. In general, the change of the center of gravity and the moment of inertia associated with deformation of the prosthesis is minimal, compared with the magnitude of inertia of the entire coupled system, including the human body model. The deformation of the prosthesis increases, especially when it is in contact with the ground. However, since the acceleration and angular velocity acting on the prosthetic leg are relatively small during the ground contact phase, the change of the inertia of the prosthesis with the deformation is considered to be negligible. Therefore, the position of the center of gravity and the moment of inertia are obtained assuming the state of the original shape that the load does not act, and these values were used as constant values regardless of the deformation state of the prosthesis.

Although the load acting on the prosthesis or the ground reaction force can be obtained by the method described above, we also imposed the following conditions. First, not only the elastic force was due to the spring characteristics of the prosthetic leg, but also the viscosity characteristic, which is proportional to the velocity at the contact point, was
also considered in the calculation of the ground reaction force. Second, the coefficient of friction at the contact point between the ground and prosthesis was determined; besides, the prosthesis is given a slide when the horizontal component of the ground reaction force exceeds the upper limit of the frictional force. The sliding phenomenon was achieved by forcibly moving the position coordinates \( x_{\text{RollingContact}} \) in Eqs. (9) and (11).

We implemented all the models by using the C-language programs we developed.

### 2.4 Simulation conditions

We performed the following analyses using the proposed model.

#### 2.4.1 Modeling of prosthesis shape

The length \( S_e \) and the curvature \( k_e \) of each element are the design parameters of the prosthesis. If these are given, any prosthesis shape can be generated. First, we simulated the prosthesis shown in Fig. 1 as a reference and constructed a model of its shape. Then, we changed the curvature of one element of the simulated shape and confirmed whether the shape is deformed parametrically as expected.

#### 2.4.2 Comparison with finite element analysis

In this subsection, we performed a comparison with finite element analysis model to verify the validity of the load–deflection relation of the proposed prosthesis model based on the flat spring design theory. Since it is our purpose to verify the validity of the computations, we used simple shape models, such as a straight line and an arc for the analysis. Figure 5 shows the outline of the analysis. We utilized a special software (ANSYS17.0, ANSYS, Inc.) for the finite element analysis and employed large deformation analysis method for the analysis. The numbers of elements were 17,184 in the linear shape model and 35,280 in the arc shape model. The upper end was fixed, and the load (external force) in the direction of 45° from the \( X \)-axis was applied at the lower end as constraints of the finite element analysis model as shown in the figure. Then, we determined the deflection amounts of the \( X \)-axis and \( Y \)-axis directions at the lower end. Further, we constructed flat spring models with the same shapes as the finite element analysis models and compared the amounts of deflection obtained from the flat spring models with those from the finite element analysis when the same load is applied to them. To verify the computational cost, we measured and compared the calculation times of both models using the same computer (CPU: Intel Xeon E5-2630v4 @ 2.20 GHz (10 cores)\( \times \)2 CPUs, Memory: 64 GB, OS: Microsoft Windows Server 2016 Standard).

![Finite element models](image)

Fig. 5  Finite element models. (a) The straight shape element has a thickness of 0.01 m, a width of 0.06 m, and a length of 0.459 m. (b) The arc shape element has a thickness of 0.01 m, a width of 0.06 m, and a curvature of 4.357 rad/m. The external force from 0 to 4000 N was applied to the models in the direction, as shown in the figure.

#### 2.4.3 Ground reaction force during takeoff action in the long jump

To verify the validity of the calculated ground reaction force, we coupled the prosthetic model with the human body model, and a takeoff action in the long jump was generated by computer simulations and compared with the measured results. The appendix contains the details of this human body model. In this study, we used the data measured
in the experiments previously conducted at Osaka University of Health and Sport Sciences, Japan. In this experiment, the amputee, whose left thigh was cut, performed the long jump. Two types of measurement data were collected: the ground reaction force measured by the force plate embedded in the ground near the takeoff board and the image data of the takeoff motion taken with a high-speed camera installed on the side of takeoff position. On the other hand, we performed a simulation of the takeoff action using a model that simulates the prosthesis mentioned earlier. Furthermore, we compared the kinematic motion data visually with the high-speed camera image. We also compared the simulated ground reaction force with that measured by the force plate.

3. Results
3.1 Modeling of prosthesis shape
Figure 6 shows a schematic diagram of the prosthesis shape constructed by using the proposed model. The model could express the referenced prosthesis shape (Cheetah Xtreme, Ossur) by the parameters shown in Table 1.

Figure 7 shows the results of changing the curvature of the tip part of the original/referenced prosthetic model. This model could express concavo-convex and straight shapes freely by merely rewriting the curvature data described in the input file from positive to negative.

3.2 Comparison with finite element analysis
Figure 8 shows the comparison of the relation between load and deflection in the finite element analysis and the flat spring models. As shown in the figure, the results of the proposed flat spring model are generally consistent with the finite element analysis. When comparing the computation time, the finite element analysis took about 20 min to perform the computation, whereas the proposed flat spring model could finish the computation in about 10 s.

![Model of the existing prosthesis foot. This model imitated the Cheetah Xtreme made by Ossur.](image)

Table 1 Design parameters of the model of the referenced prosthesis.

| Element No. | Length (m) | Curvature (rad/m) | Thickness (m) | Width (m) |
|-------------|------------|-------------------|---------------|-----------|
| 1           | 0.1445     | 0.00              | 0.01445       | 0.056     |
| 2           | 0.1100     | 14.30             | 0.01182       | 0.056     |
| 3           | 0.0079     | 0.00              | 0.01100       | 0.059     |
| 4           | 0.2070     | 3.03              | 0.00999       | 0.063     |
| 5           | 0.0759     | 0.00              | 0.00930       | 0.068     |
| 6           | 0.1110     | 5.13              | 0.00824       | 0.070     |
Fig. 7 Results of changing the curvature of Element No. 6 parametrically. The curvature of the original model is 5.13 rad/m; however, it was changed to -5.13 rad/m and 0 rad/m.

Fig. 8 The comparison of the force-deflection relation in the flat spring and FEM models: (a-1) indicates the deflection in X-direction for the straight shape model; (a-2) indicates the deflection in Y-direction for the straight shape model; (b-1) indicates the deflection in X-direction for the arc shape model, and (b-2) indicates the deflection in Y-direction for the arc shape model.
3.3 Ground reaction force during takeoff action in the long jump

Figure 9 shows the visual comparison between the image data taken with the high-speed camera and simulation results corresponding to these images in the takeoff action of the long jump. In the actual takeoff action, the result of the image data revealed that the prosthesis is deformed by contact with the ground and that the contact position with the ground moves back and forth to roll. On the other hand, the deformation phenomena of the prosthesis obtained by the simulation well represent the characteristics of the measurement results, as shown in the figure.

Figure 10 shows the results of the comparison between the measured and simulated values in the vertical and horizontal component waveforms of the ground reaction force. The change tendency of the ground reaction force patterns is generally consistent, although the ground contact period in the simulation was slightly shortened compared with that of the measurement; additionally, the horizontal reaction force of the simulation was smaller in the first half of the ground contact period.

Fig. 9 States of the deformation of prosthesis foot at the time of takeoff action; (a) shows the actual phenomena. These images were taken at a sampling frequency of 200 Hz and flipped left to right; (b) shows the simulation results. Red lines indicate the ground reaction force vectors. Numerical values in the figure indicate points of time [s] after contact of the prosthesis with the ground.
Fig. 10 The comparison between the measured and simulated ground reaction force during the takeoff action of the long jump: (a) shows the vertical component of the ground reaction force, and (b) shows the horizontal component. For the time axis (horizontal axis) of each graph, the moment when the prosthetic foot is grounded is the zero time.

4. Discussion

4.1 Modeling of prosthesis shape

In the proposed model, the shape can be set parametrically, as shown in Fig. 6, and the value can be changed easily, as shown in Fig. 7. Such setting and change of the model are performed just by running the calculation program after rewriting the input data for the modeling conditions. The proposed model is considered to be a useful tool for the design and analysis of the considered sports prosthesis because it can easily be constructed and changed compared with the finite element analysis method.

4.2 Comparison with finite element analysis

As observed from the comparison with the finite element analysis results in Fig. 8, the model proposed in this study can analyze almost the same elastic properties as the finite element model. The deformation characteristics of the material are assumed to be linear, the material properties are uniform, and the anisotropy and the lamination state of the carbon fiber are not taken into account in this model. However, since the nonlinear characteristics of the shape are fully considered, the nonlinear spring characteristics, including the interference between the $X$- and $Y$-directions, can be realized as a whole.

We analyzed only the elastic properties in this study, but it is also crucial to analyze the strength of the prosthesis, i.e., whether or not it is destroyed by the action of the load. In that regard, it was not possible to perform advanced strength analysis in this proposed model. However, this model can easily determine the bending stress from the bending moment acting on the prosthesis and the sectional modulus. Since the main reason the prosthesis is destroyed is that the bending stress reaches the stress limit, it is expected that some degree of strength analysis can be performed even with this proposed model.

Regarding the computational cost, the proposed model could calculate at an overwhelming speed compared with the finite element analysis. At first, we were worried about the computational cost in the proposed model because an optimization method was used when solving the inverse problem shown in Eq. (9), but the optimization problem had only two unknowns, and solving it did not lead to an enormous computational cost. A model with such low computational cost is considered to be very useful in the construction of a coupled model with the human body dynamic system; moreover, repetition of the calculation could yield an optimal design solution.

4.3 Ground reaction force during takeoff action in the long jump

The proposed model could realize the mechanical boundary conditions of the rolling contact while the prosthesis was deformed at a low computational cost. Although it is possible to achieve such results by using the finite element model, the computational cost is generally high, and the analysis condition setting when solving the contact problem also varies and is not always easy (Pfeiffer and Glocker, 2000). In addition, the human body dynamics model is generally formulated by the multi-body dynamics method, and the proposed prosthesis model can easily be integrated.
into the multi-body framework. Therefore, its ease of coupling with the human body model is also an advantage as a computational model.

Although the simulation results and the actual measurement results were generally consistent, some differences were observed in the ground contact period and horizontal reaction force. These inconsistencies are not due to the modeling problems of the mechanical properties of the prosthesis, but due to issues in the human body dynamics model and the motion generation simulation. In general, owing to experimental errors and modeling assumptions, measured ground reaction forces are often dynamically inconsistent with the model kinematics (Delp et al., 2007). In the future, we would like to approach this problem focusing on the simulation of the human body movement, as opposed to prosthesis modeling.

4.4 Limitations and ripple effects of this study

One limitation of this study is that the model is two-dimensional, and the exact three-dimensional shape is not taken into account. Additionally, the material properties are limited to linear and uniform properties. To be more precise, it is possible to change the thickness and width of each element, but it must be done uniformly within each element; besides, they cannot be changed smoothly and continuously. Therefore, it is difficult to create a blueprint for producing a prosthetic leg directly from the design parameters of the proposed model. Despite these limitations, the proposed model has some remarkable usefulness, as described above.

As a ripple effect of this model, as future research, we shall optimize the shape of the prosthesis to improve performance in sports competitions and enhance its compatibility with the human body model. The proposed model has been specialized in sports prosthesis, but in recent years, flat spring-shaped CFRP material has been used in a prosthetic foot for normal walking, and not for sports. We shall apply the proposed model to the design and evaluation of such prosthetic foot.

5. Conclusion

In this study, a computational model was constructed based on the flat spring design theory to efficiently analyze the elastic deformation characteristics associated with a prosthesis shape. The contact with the ground and the interaction with the human body model were also considered in the model. The results of the elastic characterization were comparable with those of finite element analyses; besides, the computational cost and convenience of parameter changes were overwhelmingly better compared with those of the finite element analysis. Therefore, the ripple effect, such as the application of the proposed model to optimization design, is expected.

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Appendix

Figure A shows the outline of the human body model in conjunction with the prosthetic model. As shown in the figure, the human body system was represented by a two-dimensional rigid-link model of a total of 10 links, including the right thigh, right calf, right foot, left thigh, pelvis, thorax (including head), both upper arms, and both forearms in accordance with a modeled athlete having the left thigh amputated. The body segment parameters, such as the mass and moment of inertia, were obtained by using the estimation equation of the body segment parameters for Japanese athletes (Ae et al., 1992) based on the body dimensions and weight of the subject. Because the left leg of the object is amputated in the thigh, the left knee joint is replaced with a passive prosthetic knee joint. The actual prosthesis is connected to the amputated femoral segment via a prosthetic socket. However, as there is no movement between these parts, the amputated femoral segment, prosthetic socket, and upper part of the prosthetic knee joint were represented together as one femoral segment in the model. Inertial parameters of the femoral segment, such as mass and position of the center of gravity, were determined by combining the mechanical properties of each of its elements. The proposed prosthesis model was connected below the prosthetic knee joint via an adapter.
The joints, other than the left knee joint, which is prosthetic and passive, were controlled by active driving joint moments. The joint moments were determined using a proportional-derivative (PD) control method as follows:

$$
\tau_i(t) = K_i \left( \overset{\text{ref}}{q}_i(t) - q_i(t) \right) + D_i \left( \overset{\text{ref}}{\dot{q}}_i(t) - \dot{q}_i(t) \right)
$$

(A1)

where $\tau_i$ is the joint moment of the $i$th joint, $K_i, D_i$ are the gains, $\overset{\text{ref}}{q}_i$ is the joint angle referred to, and $q_i$ is the joint angle of the simulation model. The referred joint angle $\overset{\text{ref}}{q}_i$ is parametrically defined by a spline function as follows:

$$
\overset{\text{ref}}{q}_i(t) = \text{spline}(n_{i1},\cdots,n_{i9},t)
$$

(A2)

where $\text{spline}(n_{i1},\cdots,n_{i9},t)$ indicates the spline function defined by the 9 knots $n_{i1},\cdots,n_{i9}$. In general, if the measured joint angles are applied as the referred joint angles and higher gains are used, the generated motion will be close to the measured motion; however, the joint moment patterns in this case will not be close to those of an actual human. Therefore, we employed a method to change the referred joint angles for the generated motion for a closer match to the motion measured with lower gains. The parameters $n_{i1},\cdots,n_{i9}$ in Eq. (A2) were determined by an optimization method to minimize the residuals between the measured and generated motion data. However, the gains $K_i, D_i$ were determined by trial and error. By using this model, the human body motion for 0.3 s before and after the takeoff action of the long jump was reproduced by a forward dynamics simulation. The initial displacement and velocity at the start of the simulation were determined by the measured data.

Fig. A An overview of the human body model with a prosthesis.

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