Can There be Quark Matter Core in a Strongly Magnetized Neutron Star?

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The effect of strong quantizing magnetic field on the nucleation of quark matter droplets and on the chemical evolution of nascent quark phase at the core of a neutron star are investigated. The surface energy of quark phase diverges logarithmically. As a consequence there can not be a first order transition to quark phase. However, a metal-insulator type of second order transition is possible unless the field strength exceeds $10^{20}$G. The study of chemical evolution of newborn quark phase shows that in $\beta$-equilibrium the system becomes energetically unstable.

The theoretical investigation of the effect of ultra-strong magnetic field on stellar matter in nuclear astrophysics has got a new dimension after the discovery of a few magnetars. The observed soft gamma repeaters discovered by BATSE\textsuperscript{1} and KONUS\textsuperscript{2} (see also\textsuperscript{3}) experiments and X-ray sources observed by ASCA, RXTE and BappoSAX\textsuperscript{4} show the presence of strong surface magnetic field up to $10^{15}$G. The discovery of these objects pose a great challenge to the existing models of magnetic field evolution, since they require a very rapid field decay in isolated neutron stars\textsuperscript{5}. To investigate the global properties of these strange objects, it also requires a detail investigation of stability of dense stellar matter in presence of ultra-strong magnetic field and know the exact equation of state of such strongly magnetized matter.

The dynamo mechanism, recently proposed by Thompson and Duncan\textsuperscript{6} suggests that the dipole magnetic field of a young neutron star can reach up to $10^{15}$G. It is generally expected that the internal magnetic field is a few orders of magnitude stronger than surface field strength. Since the strength of internal magnetic field of a neutron star strongly depends on the nature of dense stellar matter present at the core region, it may not necessarily be reflected in its surface magnetic field. However, there is an upper limit of internal magnetic field strength constrained by the scalar virial theorem, which gives $B_{\text{max}} \sim 2 \times 10^{8} (M/M_{\odot})(R/R_{\odot})^{-2} \text{G}$\textsuperscript{7,8}. For a typical neutron star of radius $R = 10$km and mass $M = M_{\odot}$, this upper limit is $\sim 10^{18}$G. Beyond this limit, the ultra-magnetized neutron stars become unstable.

Now there is also a strong belief that a transition to quark phase occurs at the core of a neutron star if the density exceeds a few times normal nuclear density. The transition could be a first order, with the nucleation of stable quark matter droplets in metastable hadronic matter by fluctuation at the core region. The transition could also be a second order type. Since hadrons (neucleons and hyperons) do not carry color quantum number, we may call the hadronic matter a color insulator. Whereas, in this regard, the quark phase is a color conductor. Therefore, such a second order structural phase transition at high density is analogous to the metal insulator transition in condensed matter physics, which takes place under high pressure.

In this letter our aim is to show that a first order transition to quark phase is absolutely forbidden at the core of a young pulsar if the magnetic field strength exceeds $10^{15}$G for which the Landau levels of the quarks are populated. On the other hand a second order transition is possible unless the magnetic field strength exceeds $10^{20}$G. Now the transition to quark phase in a young strongly magnetized neutron star is a strong interaction process (time scale $\sim 10^{-23}$sec.). The constituents (up ($u$), down ($d$) and strange ($s$) quarks and electrons) of the quark phase are however not in $\beta$-equilibrium immediately after their formation. The chemical equilibrium time scale (weak interaction time scale) is much longer than the droplet nucleation time scale. In this context we shall further like to show that if in the quark phase the magnetic field strength exceeds the typical value $4.4 \times 10^{13}$G, for which the Landau levels of

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electrons are populated in the chemical or β equilibrium condition quark phase becomes energetically unstable with respect to the corresponding hadronic phase.

Let us consider the nucleation of stable quark matter droplets due to fluctuation in a metastable hadronic matter. The rate of nucleation is given by

\[ I = I_0 \exp \left(-\frac{\sigma^3}{C}\right) \]  

(1)

where \( I_0 \) and \( C \) are finite constants, \( \sigma \) is the surface tension of the quark phase formed in metastable hadronic matter. The detail analytical structure of these constants are not important in our study. Then considering the MIT bag model of color confinement, we have the expression for surface tension of the quark phase in presence of a strong quantizing magnetic field of strength \( B \)

\[ \sigma_i = \frac{3TB}{8\pi} q_i \sum_{\nu=0}^{\infty} \int_0^\infty \frac{dk_z}{(k_z^2 + k_\perp^2)^{1/2}} \ln \left[ 1 + \exp \left( -\frac{\epsilon_{i}(\nu) - \mu_i}{T} \right) \right] G \]  

(2)

where \( i \) indicates the species (up, down or strange quarks), \( q_i \) is the flavor charge, \( k_\perp = (2\nu q_i B)^{1/2} \) is the transverse momentum, \( \nu \) is the Landau quantum number, \( \epsilon_{i}(\nu) = (k_z^2 + k_\perp^2 + m_i^2)^{1/2} \) is the modified form of single particle energy, \( m_i \) and \( \mu_i \) are the mass and chemical potential respectively and \( G = 1 - 2 \tan^{-1}(k/m_i)/\pi \).

Whereas in the non-magnetic or non-quantizing magnetic field case [14,15] (see also [16]),

\[ \sigma_i = \frac{3T}{32\pi^2} \int \frac{d^3k}{k} \ln \left[ 1 + \exp \left( \frac{\epsilon_i - \mu_i}{T} \right) \right] G \]  

(3)

Now it is just a matter of simple integration by parts to show from eqn.(2) that the surface tension of the quark phase diverges logarithmically for \( \nu = 0 \) (zeroth Landau level). The surface tension goes as \( \sim -\ln(\nu) \) as \( \nu \to 0 \) in presence of a quantizing magnetic field, whereas the surface tension as given in eqn.(3) for the non-magnetic case is a finite quantity. Therefore the rate of nucleation of quark matter droplets as given in eqn.(1) becomes identically zero. Which concludes that if the magnetic field strength at the core of a neutron star is strong enough to populate Landau levels of the quarks the nucleation of quark droplets become impossible, which means that under such strange condition a first order transition to quark phase is absolute forbidden.

However, a second order phase transition to quark matter can not be ruled out. This is a continuous transition. The surface tension of the new phase has no role in the process. The chemical potential, density and pressure change continuously at the phase boundary. Using these conditions at the phase boundary, we have obtained numerically the critical density for the phase transition as a function of magnetic field strength. In fig.1 we have plotted this variation. This figure shows that the critical density (solid curve) diverges at \( B \approx 10^{18} \text{G} \), which is of course too high to achieve at the core of a newborn neutron star. However, for relatively lower values of magnetic field strength, the critical densities are finite and well within the limit of central density of a stable neutron star. Therefore, a metal-insulator kind second order transition is possible even in presence of a strong magnetic field of astrophysical interest. The bag parameter also becomes unphysical beyond \( B \approx 10^{18} \text{G} \) (shown by dashed curve).

As we have already mentioned that such a transition takes place in the strong interaction time scale. Therefore, the produced quark phase will not be in β-equilibrium. It takes comparatively longer time (~ weak interaction time scale) to achieve chemical equilibrium in the system.

To study the chemical evolution of the system, we consider the following weak processes in quark matter phase: \( d \to u + e^- + \bar{\nu}_e \) (1), \( u + e^- \to d + \nu_e \) (2), \( s \to u + e^- + \bar{\nu}_e \) (3), \( u + e^- \to s + \nu_e \) (4), \( u + d \to u + s \) (5). We have further assumed that the neutrinos are non-degenerate (they leave the system immediately after their formation). We have noticed that the trapping of neutrinos (if the phase transition occurs at the core of a proto-neutron star) in the quark phase do not change our conclusions. The approach to chemical equilibrium of the system is therefore governed by the following sets of kinetic equations

\[ \frac{dY_u}{dt} = \frac{1}{n_B} [\Gamma_1 - \Gamma_2 + \Gamma_3 - \Gamma_4] \]  

(4)

\[ \frac{dY_d}{dt} = \frac{1}{n_B} [\Gamma_1 + \Gamma_2 - \Gamma_5^{(d)} + \Gamma_5^{(r)}] \]  

(5)

where \( n_B \) is the baryon number density, \( Y_i = n_i/n_B \) is the fractional abundance of the species \( i \) and \( \Gamma_j \)'s are the rates of the processes \( j = 1, 2, 3, 4, 5 \). The indices \( d \) and \( r \) are respectively for the direct and reverse processes for \( j = 5 \). The baryon number conservation and charge neutrality conditions give \( Y_e = 3 - Y_u - Y_d \) and \( Y_e = Y_u - 1 \).
respectively. For a neutron star of mass $\approx 1.4M_{\odot}$, the baryon number density at the centre is $3 - 4$ times normal nuclear density, temperature $\sim 10^9$K and proton fraction is about $4\%$. Then the initial conditions are $Y_u(t = 0) = 1.04$, $Y_d(t = 0) = 1.96$. As a consequence of baryon number conservation and charge neutrality, we have $Y_u(t = 0) = 0$ and $Y_d(t = 0) = 0.04$. Solving numerically the kinetic equations along with the conditions of baryon number conservation and charge neutrality and using the initial conditions as given above, we have obtained the time variation of fractional abundances for various species for a given baryon number density. In fig.2 we have plotted the time dependences of fractional abundances for various species in the zero magnetic field case. In this situation the fractional abundances for up, down and strange quarks saturate to their $\beta$-equilibrium values, whereas the electron fraction becomes extremely small. In fig.3 we have plotted the same variations when only electrons are affected by quantizing magnetic field $(B = 10^{14}$G). In this case the strangeness fraction first increases but ultimately both the down and strange quark abundances go to zero and the system effectively behaves like a up-quark core in the $\beta$-equilibrium condition. For the sake of charge neutrality the electron fraction also becomes very high in this particular physical situation in the equilibrium condition. In fig.4 we have shown the time evolution of the species when the magnetic field strength is strong enough $(B = 5 \times 10^{15}$G) to affect electrons and also up and down quarks. In this case the strange quarks are never produced. In the $\beta$-equilibrium condition, therefore, the main constituents are up and down quarks with a small fraction of electrons.

Now the phase transition to quark matter may occur at the core of a neutron star from dense hyperonic matter (with non-zero initial strangeness fraction) instead of pure nucleonic matter. In this case the initial strange quark abundance is finite. We have noticed that the qualitative nature of the curves do not change even if the initial hyperon fraction is $30\%$. In this particular scenario, in the field free case the fractional abundances of all the three quarks saturate to $\beta$-equilibrium values (as shown in fig.2). If only electrons are affected, the variation of strangeness fraction and down quark abundance are exactly identical with fig.3. Although, the initial strangeness fraction is non-zero, it ultimately vanishes in the $\beta$-equilibrium condition. Same is the case when the magnetic field is strong enough to affect electrons as well as up and down quarks. The equilibrium values for the fractional abundances changes only $\approx 5\%$ with respect to the corresponding values when the initial strangeness fraction is zero.

To study the stability of the quark matter system under various physical situations discussed above, we have calculated energy per baryon in the $\beta$-equilibrium condition for different baryon number densities. Under zero pressure condition, the energy per baryon is given by

$$\varepsilon = \frac{1}{n_B} \sum_{u,d,s,e} \mu_i n_i$$

We have obtained energy per baryon as a function of baryon number density for all the cases discussed in the text, including the hypothetical situation-an almost flavor symmetric quark matter in an external magnetic field (according to our calculation such a situation can not be achieved in the real world). In fig.5 we have shown the variation of energy per baryon with baryon number density for various physical scenarios. For the sake of comparison we have plotted the stability curve of neutron matter with Bethe-Johnson equation of state (dashed curve) [8]. From the nature of the curves we can conclude that the hypothetical quark matter scenario is the most stable configuration. The field free quark matter with and without degenerate neutrinos are also energetically stable up to 2-2.5 times normal nuclear density. This is consistent with the speculation of Witten [17]. Whereas, in all other physical situations, in presence of a strong magnetic field the quark matter is energetically unstable.

Therefore the final conclusions are (i) if the magnetic field strength exceeds $10^{15}$G, a first order transition to quark phase is absolutely forbidden at the core of a neutron star. (ii) A metal-insulator kind of second order transition is however possible unless the field strength is $> 10^{20}$G, which is of course too high to have at the core of a neutron star. (iii) Even if the transition is of second order in nature, because of two kinds of completely different time scales, the chemical evolution of the system in presence of a strong magnetic field $(\geq 4.4 \times 10^{13}$G) leads to energetically unstable configuration. Therefore quark matter core is absolutely impossible in an young pulsar with strong magnetic field. However, in very old neutron stars with much weaker magnetic field strength such a phase transition is possible by matter accretion which may increase the density of the core.

If the situation is such that a pulsar is very young and strongly magnetized and at the same time compact enough, then probably pion or / and kaon condensation will play the major role to make the matter energetically stable [8].

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FIG. 1. Critical density (solid curve) and MIT bag parameter (dashed curve) for metal-insulator type second order transition as a function of magnetic field strength. The magnetic field strength is expressed in terms of the critical field for electron
FIG. 2. Fractional abundances for various species in the zero magnetic field case.

FIG. 3. Same as fig. 2 when only electrons are affected by quantizing magnetic field \((B = 10^{14} \text{G})\).
FIG. 4. Same as figs. 2 and 3 but the magnetic field is strong enough to affect electrons as well as up and down quarks ($B = 5 \times 10^{16} \text{G}$).

FIG. 5. Variation of energy per baryon for different physical conditions (see text). Curves (a) is for neutron matter, (b) is for hypothetical quark matter placed in an external magnetic field, (c) is for quark matter in the field free case, (d) is for quark matter in presence of a strong magnetic field $B_m = 10^{14} \text{G}$, (e) is same as (d) for $B_m = 5 \times 10^{16} \text{G}$, (f) is for neutrino trapped quark matter in the field free case and (g) is same as (f) for $B_m = 5 \times 10^{16} \text{G}$.