An overview of branes in the plane wave background

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Abstract.
We give an overview of D-branes in the maximally supersymmetric plane wave background of IIB supergravity. We start by reviewing the results of the probe analysis. We then present the open string analysis and show how certain spacetime symmetries are restored using worldsheet symmetries. We discuss the construction of these branes as boundary states and summarize what is known about the dual gauge theory description.

We present in this contribution an overview of D-branes in the maximally supersymmetric plane wave background of IIB supergravity (henceforth called “the plane wave”),

\[
\begin{align*}
(ds^2) = 2dx^+dx^- + \sum_{I=1}^{8}(dx^I dx^I - \mu^2(x^I)^2(dx^+)^2), \\
F_{+1234} = F_{+5678} = 4\mu.
\end{align*}
\]

The flux breaks the symmetry of the directions transverse to the lightcone from $SO(8)$ to $SO(4) \times SO(4)$; we denote this splitting of the transverse coordinates as $(4, 4)$. It is useful to adopt the notation $(m, n)$ to indicate the transverse directions wrapped by a given brane.

The motivation for studying D-branes in this background is twofold. Firstly, the plane wave background is related to $AdS_5 \times S^5$ via a Penrose limit. This implies a correspondence between string theory on the plane wave background and gauge theory. Secondly, this background is one of the very few RR backgrounds on which string theory is exactly solvable. Given the importance of string theory on such backgrounds, it is worthwhile to obtain as thorough an understanding as possible in this case where the analysis is tractable.

D-branes can be analyzed by a variety of methods. In string perturbation theory they are surfaces on which strings can end and thus one can investigate possible D-branes by finding consistent boundary conditions for open strings on the background. In the Green-Schwarz (GS) formulation the spacetime symmetries (the symmetries of the D-brane) manifest themselves as global symmetries of the worldsheet action.
An overview of branes in the plane wave background

Table 1. D-branes visible in string perturbation theory and their supersymmetries. 
\((m,n)\) denotes the number of worldvolume coordinates wrapping the two sets of directions transverse to the lightcone. The branes in the open string channel also wrap the lightcone, but the corresponding boundary states do not. \(q^-\) denotes dynamical supersymmetries and \(q^+\) kinematical supersymmetries. The hatted quantities are symmetries that were restored using worldsheet symmetries. \(x_0\) denotes the constant position of the brane in transverse space.

| Branes          | Probe analysis | Open string analysis | Boundary States |
|-----------------|----------------|----------------------|-----------------|
| \(D_-\)        |                |                      |                 |
| \((m,m \pm 2)\) at \(x_0 = 0\) | 8 \(q^-\) + 8 \(q^+\) | 8 \(q^-\) + 8 \(q^+\) | 8 \(q^-\) + 8 \(q^+\) |
| \((m,m \pm 2)\) at \(x_0 \neq 0\) | 8 \(q^+\) | 8 \(\hat{q}^-\) + 8 \(q^+\) | 8 \(q^-\) + 8 \(q^+\) |
| \(D_+\)        |                |                      |                 |
| \((0,0)\) at \(x_0\) | 8 \(q^-\) | 8 \(q^-\) + 8 \(\hat{q}^+\) | 8 \(q^-\) + 8 \(q^+\) |
| \((0,4)\) with flux | 8 \(q^-\) | 8 \(q^-\) + 8 \(\hat{q}^+\) | 8 \(q^-\) + 8 \(q^+\) |
| \((m,n)\) at \(x_0\) | 0 | 8 \(\hat{q}^+\) | 8 \(q^+\) |

Table 2. Branes that wrap only one of the lightcone directions. These branes are not visible in the string perturbation theory in the lightcone gauge.

| Branes                  | SUSY            |
|-------------------------|-----------------|
| \((+,0,S^3)\) and \((+,S^3,0)\) | 16 \(q^-\) |
| \((+,0,1)\) and \((+,1,0)\) | 8 \(q^-\) |

Conformal invariance of the open string requires that the coordinates that specify the position of the brane, the worldvolume gauge field and the associated fermions satisfy the Dirac-Born-Infeld (DBI) field equations. One may thus obtain possible D-branes by solving the DBI field equations. The target space symmetries of the GS open string should appear as worldvolume symmetries of the DBI action.

D-branes can also be described using boundary states. The boundary state is a closed string state that imposes appropriate boundary conditions. Open-closed duality requires that D-branes have such a description and one may thus investigate possible D-branes by constructing boundary states. The symmetries that the boundary state preserves are the combinations of the closed string symmetries that leave the boundary state invariant.

In the next three sections we will discuss these three complimentary approaches; the results are summarised in the tables 1 and 2. Plane wave/gauge theory duality implies that the D-branes should have a gauge theory description and this will be discussed in section 4, followed by conclusions in section 5. The results described here were obtained in [4, 5, 6], with related work in [7, 8, 9, 10]. The emphasis in this contribution will be on the results and on the methods used rather on the technical details.
An overview of branes in the plane wave background

1. Probe analysis

As discussed, one may investigate possible D-branes by looking for solutions of the DBI field equations. The gauge invariant DBI field equations for bosonic fields were derived in generality in [4]. These equations can be effectively used to look for possible D-brane embeddings. In [4] we systematically analyzed embeddings with constant transverse scalars and zero worldvolume flux and furthermore discussed some specific cases with non-zero fluxes.

The DBI action is invariant under local kappa symmetry and target space supersymmetries. The combined transformations of the (target space) fermions are

$$\delta \theta = (1 + \Gamma) \kappa + \epsilon$$

where $\Gamma$ is the $\kappa$ symmetry projector and $\epsilon$ is the target space Killing spinor evaluated on the worldvolume. The worldvolume supersymmetry arises after gauge fixing the kappa symmetry. A convenient gauge fixing is the one that uses the kappa symmetry projector evaluated on the embedding, $(1 - \Gamma)\theta = 0$ [11]. Combining this projection with (2) we get the condition for unbroken supersymmetry

$$\Gamma \epsilon = \epsilon$$

where both the kappa symmetry projector $\Gamma$ and the Killing spinor $\epsilon$ are evaluated on the brane.

In the generic case, the solutions of (3) give the number of supersymmetries linearly realised on the brane. The open string analysis discussed in the next section, however, suggests that the DBI action on the plane wave background has additional supersymmetries. It is an interesting problem to analyze whether there are special class of backgrounds on which the DBI action exhibits more symmetries than those implied by the generic analysis.

The presence of a brane breaks some of the target space isometries and one can obtain new embeddings by acting with the broken generators. Since all properties of the new brane follow from those of the original brane, one may group the branes in equivalence classes: two branes are considered equivalent if they are related by the action of a broken isometry of the background. We call such branes “symmetry-related branes”. Since the translational Killing vectors in the plane wave are $x^+$ dependent, the same equivalence class may contain both static and time-dependent branes. In particular, static branes localized at the origin and certain time-dependent branes localized away from origin belong to same equivalence class. Notice also that from the point of view of open strings a brane localized at the origin and one located at a constant (non-zero) position are not symmetry-related.

The supersymmetric embeddings found are summarized in tables 1 and 2. The kappa symmetry analysis shows that the preserved supersymmetry of the brane depends both on the splitting $(m, n)$ and on the transverse positions. For instance $(m, m \pm 2)$ branes, which we denote $D_-$ branes, preserve 16 supercharges when located at the origin.
An overview of branes in the plane wave background

In the transverse space, but appear to preserve only the 8 kinematical supercharges when located away from the origin. Switching on constant worldvolume fluxes may in some cases allow the brane to be at specific non-zero position whilst preserving 16 supercharges.

In the plane wave one can also have branes wrapping only one lightcone direction, \( x^+ \). We should emphasise that these have non-degenerate induced metrics, cf corresponding null branes in flat space. Only specific embeddings seem to preserve any supersymmetry and these are listed in Table 2: there are D-strings and D3-branes which wrap arbitrary radius three spheres. The explicitly broken translational invariance in the \( x^- \) direction means that the lightcone momentum \( p^+ \) is not preserved, and thus neither are kinematical supercharges which anticommute to \( p^+ \).

Finally, one has instantonic branes which do not wrap the lightcone. These are described by closed string boundary states and are related to the branes wrapping the lightcone under open/closed duality. We will return to this subject later.

2. Open string analysis

In string perturbation theory D-branes are surfaces on which strings can end. To classify D-branes one may thus investigate consistent open string boundary conditions. At tree-level this amounts to requiring that the variational problem is well defined, i.e. requiring that \( \delta S = 0 \) implies the field equations. For a theory defined on a manifold (worldsheet) with a boundary this requires that boundary conditions are chosen such that all boundary terms arising in the variation are zero. Further conditions may arise at the quantum level.

Such an analysis of open strings in the plane wave background was presented in \[5\]. One finds that one can impose standard static D-brane boundary conditions on the bosonic coordinates, but also that more general time-dependent boundary conditions are possible. For instance, the ends of the string may trace a trajectory of an ellipse with angular velocity equal to the mass of the string. This is an example of a symmetry-related brane though not all time dependent branes we find are symmetry-related.

Appropriate boundary conditions for the fermions \( (\theta^1, \theta^2) \) amount to setting to zero on the worldsheet boundary a specific linear combination of them, \( (\theta^1 - \Omega \theta^2) \), where \( \Omega \) is a 16 \( \times \) 16 dimensional matrix. We find that D-branes have different properties depending on whether \( (\Omega \Pi)^2 = -1 \), where \( \Pi = \gamma^1 \gamma^2 \gamma^3 \gamma^4 \), or \( (\Omega \Pi)^2 = 1 \). We call the former branes \( D_- \) branes and the former \( D_+ \) branes; these two categories reflect the splitting \( (m, n) \) as indicated in table 1.

After the D-brane has been defined, we can investigate its symmetries by looking for symmetries of the worldsheet action with these boundary conditions. Generically the symmetries of the open string are the closed string symmetries that respect the

\[\|
\text{We call kinematical the supercharges that anticommute to the lightcone momentum and dynamical the ones that anticommute to the lightcone Hamiltonian, possibly along with other generators.}
\]
An overview of branes in the plane wave background

boundary conditions. The supersymmetries obtained in this fashion agree exactly with
the ones obtained in the probe analysis using (3).

In some circumstances, however, the worldsheet theory may admit additional
symmetries. This is the case for the open string on the plane wave. The (gauge-
fixed) worldsheet action in this case is that of free massive bosons and fermions. The
fact that the Lagrangian is quadratic in the fields implies that it is invariant up to a
total derivative under a transformation that is a shift of the fields by a parameter that
satisfies the free field equations. Let us illustrate this point with a free massive boson;
an exactly analogous discussion holds for free massive fermions. Consider the action

$$S = \int d\tau \int_0^\pi d\sigma \frac{1}{2} (\partial^\mu X \partial_\mu X + m^2 X^2).$$

Under the transformation

$$\delta X = \epsilon(\sigma, \tau)$$

we obtain

$$\delta S = \int d\tau d\sigma X \left( \nabla^2 - m^2 \right) \epsilon + \int d\tau \left[ (\partial_\sigma \epsilon) X \right]^{\sigma=\pi}_{\sigma=0}.$$

It follows that if $\epsilon(\sigma, \tau)$ satisfies the free field equation the variation results in only a
boundary term.

If in addition $\epsilon(\sigma, \tau)$ satisfies appropriate boundary conditions so that the boundary
term vanishes we obtain a new symmetry. Expanding $\epsilon(\sigma, \tau)$ in a basis we obtain in
this way a countably infinite number of symmetries. The corresponding Noether charges
evaluated on-shell are linear in the oscillators. These symmetries reflect the fact that the
mode expansion for the quantum field can be solved explicitly. They are a generalization
of the symmetries generated by the chiral currents, $J_n = z^n \partial X$, in the case of CFTs
associated with free massless bosons.

Since the open string worldsheet Lagrangian is the same as the closed string
Lagrangian, the variation of the open string action under any symmetry transformation
of the closed string either leaves the action invariant or results in a boundary term. In
the latter case one would conclude that this symmetry is broken. We have just
seen, however, that the open string action also varies into a boundary term under a
transformation of the form (4). It follows that if we can find appropriate $\epsilon(\sigma, \tau)$ such
that the boundary term in (5) cancels the boundary term in the variation of the action
under the closed string transformation, then the combined worldsheet and closed string
transformation is a good symmetry of the open string. We will denote the generators of
these new symmetries by the same symbol as the generator of the corresponding closed
string symmetry, but with a hat; this notation is used in table 1.

We have shown in [5, 6] that one can use this mechanism in order to restore
some apparently broken symmetries. In all such cases the violation of the closed string
symmetry depends solely on quantities that are determined by the boundary conditions,
and the violating terms can be adjusted to zero by changing the boundary condition.
For example, $D_-$ branes located at a constant transverse position $x_0$ appear to break all
dynamical supersymmetries, and the violating terms vanish when the transverse position
is set to zero, $x_0 = 0$. We find that in these cases one can combine a closed string
transformation with a transformation of the form (1) (and a corresponding fermionic transformation) to obtain a good symmetry of the open string.

In all these cases the brane located at the origin is symmetric without the need of extra worldsheet symmetries. The Noether charge for this symmetry is exactly equal on-shell to the Noether charge that generates the new symmetries for the brane located away from the origin. Taking into account the extra symmetries one finds that all on-shell conserved charges are exactly the same for static branes located at and away from the origin. The only exception is the lightcone Hamiltonian. The lightcone Hamiltonian for a brane located at \( x_0 \) contains an extra c-number contribution \( \Delta H \) that is equal to the energy that a classical open string with ends at \( x_0 \) has due to the harmonic oscillator potential. \( \Delta H \) itself is the on-shell value of a worldsheet charge. These considerations imply that the superalgebras for the brane at and away from the origin are almost identical. The only differences result from the difference in lightcone Hamiltonians.

Similar considerations lead to new kinematical symmetries in the cases where the closed string kinematical supersymmetries are not compatible with the boundary conditions. The fact that the worldsheet theory contains free fermions always implies that there is a corresponding kinematical supercharge: it is given on-shell by the zero mode of the free fermion.

The spectrum of the theory reflects the existence of all the symmetries discussed here. In particular, the states of the theory organize themselves into supermultiplets of the extra supersymmetries. For the \( D_- \) branes the theory has a unique vacuum state. The dynamical supersymmetries commute with the lightcone Hamiltonian, and thus its eigenstates form multiplets under the dynamical supersymmetry. The kinematical supercharges are spectrum generating. For the \( D_+ \) branes both the dynamical and the kinematical supercharges commute with the Hamiltonian and the ground state is degenerate.

3. Boundary states

Boundary states in the plane wave background were discussed in [7,6]. The boundary state is a closed string state that represents the addition of a boundary to the tree level worldsheet. It is constructed by imposing the boundary conditions on the worldsheet fields as operator relations. The appropriate boundary is now spacelike: at time \( \tau = \tau_0 \) a closed string is created (or annihilated) from the vacuum. The gluing conditions can be solved to obtain the boundary state as a coherent state of closed strings.

Having obtained the boundary state one may then check what symmetries it preserves. This can be done using the explicit expressions for the closed string generators in terms of modes. Acting on the boundary state and using the gluing conditions one can uniquely determine which of the closed string symmetries are preserved by the boundary state.

In all cases we find [4] that the symmetries of the corresponding boundary state match the symmetries of the brane in the open string channel. However, one does not
use any additional worldsheet transformations. In particular, in all cases the boundary states are annihilated by a combination of the closed string kinematical supercharge generators. Furthermore, the boundary state representing a $D_-$ brane located away from the origin preserves 8 closed string kinematical supercharges, and 8 linear combinations of the the closed string dynamical and kinematical supercharges.

4. Dual gauge theory

The branes whose dual description is best understood are the $D_-$ branes \cite{9,4}. The idea behind the construction is to add a $Dp$ brane in the sequence:

$$D3 \xrightarrow{\text{near-horizon}} AdS_5 \times S^5 \xrightarrow{\text{Penrose}} \text{plane wave}$$

We have shown in \cite{4} that the $D_-$ branes localized at the origin originate from half supersymmetric $AdS$ embeddings in $AdS_5 \times S^5$ in the Penrose limit. The latter in turn arise from supersymmetric intersections of the $Dp$ branes with $D3$ branes in the near horizon $D3$-brane limit. This sequence of relations allows one to identify the dual description of $D_-$ branes as being represented by a defect CFT. The latter captures the physics of the $3-p$ strings in the original $D3$-$Dp$ system.

In the BMN proposal \cite{2} the closed string ground state is constructed in the gauge theory by the operator $\text{Tr} Z^J$, where $Z = X^1 + iX^2$ and $X^i, i = 1, ..6$ are the six scalars of $\mathcal{N} = 4$ SYM. The $Z$’s represent the string bits and by taking the trace one obtains a closed string. To construct an open string we should not trace over the $Z$’s but instead put “quarks” at the end of the strings. These “quarks” are supplied by the defect theory: they are some of the massless modes in the spectrum of the $3-p$ strings. The open string vacuum is then represented by the defect operator $\bar{q}Z^Jq$, where the $q$’s are the scalars of the hypermultiplet in the case of ND=4 intersections, but are instead fermions for ND=8 intersections. In all cases one finds that this state indeed has the correct lightcone energy to represent the lightcone vacuum. We refer to \cite{4} for more details.

In table 3 we summarize what is known about the relations between branes in the plane wave, branes in $AdS_5 \times S^5$ and brane intersections in asymptotically flat spacetimes. These relations are instrumental in obtaining the dual gauge description of branes in the plane wave/gauge theory correspondence. The first two entries were discussed above. The $(+, -, 0, 0)$ D-string is related to the D-instanton via open-closed duality and as such may be related to instantons in the gauge theory. The $(+, -, 0, 4)$ branes most likely represent the Penrose limit of a baryon vertex (but they are not the Penrose limit of the known baryon vertex solution \cite{12}). Giant gravitons have also been discussed in \cite{10}, and the $(+, 0, 1)$ branes are expected to be related to monopoles.

5. Discussion

We presented an overview of branes in the plane wave background. Perhaps the most surprising result is the restoration of certain symmetries using worldsheet symmetries.
Table 3. Supersymmetric intersections in asymptotically flat spacetimes, supersymmetric embedding in $AdS_5 \times S^5$, branes in the plane wave and the dual description. $(r|D_p \perp D_3)$ denotes an orthogonal intersection of a $D_p$ brane with a $D_3$ brane over an $r$-brane. $(+, -, m, n)_{x_0}$ denotes a brane localized at $x_0$.

| Intersection | Embedding | brane | dual description |
|--------------|-----------|-------|------------------|
| $(n|D(2n+1)\perp D3)$ | $AdS_{n+2} \times S^n$ | $(+, -, n+1, n-1)_{x_0=0}$ | $(n+1) d$ dCFT |
| $(n|D(2n+5)\perp D3)$ | $AdS_{n+2} \times S^{n+4}$ | $(+, -, n+1, n+3)_{x_0=0}$ | $(n+1) d$ dCFT |
| $\ - \ - \ R \times S^3$ | $(+, -, 0, 0)$ | instantons? |
| $\ - \ - \ R \times S^3$ | $(+, -, 0, 2)$ | giant graviton? |
| $\ - \ - \ ?$ | $(+, -, 0, 4)$ with flux | baryon vertex? |
| $\ - \ - \$ giant graviton | $(+, 0, S^3)$ and $(+, S^3, 0)$ | giant graviton? |
| $\ - \ - \$ rotating $(A)dS_2$ string | $(+, 1, 0)$ and $(+, 0, 1)$ | monopoles? |

These results were obtained at tree level and an important question is whether string interactions will respect them. Notice that the symmetries of the open string match the symmetries of the boundary states only after the new symmetries are included. Furthermore, the symmetries of the boundary states are less subtle as they are always a proper subset of the closed string symmetries. These considerations suggest that the new symmetries are compatible with string interactions. If this is so, one should able to present arguments for the restoration of the symmetries that are valid for any worldsheet and also one should be able to find the extra worldvolume supersymmetries of the DBI action in the plane wave background.

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