Hot Stellar Populations in Globular Clusters:
a Photometrist’s View

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**Abstract.** We briefly review the recent results on hot horizontal-branch stars in globular clusters. Since the first Ivanfest, in 1992, there have been a number of new observational lines of evidence which have allowed significant progress in our understanding of blue-tail stars, though new, even more intriguing questions arise. Despite this progress, we still do not know the answer to the main question: why are there blue-HB-tail stars?

The new photometric data bases collected in the last few years, and the forthcoming multi-fiber observational campaigns on 10m-class telescopes, might be the key to solve this puzzle. We will show an example of how these data bases can disclose important properties of blue HB stars.

1. Introduction

Horizontal branches (HB) are probably the most challenging structures in the color magnitude diagrams (CMD) of old stellar populations. Despite the fact that all the HB stars have almost the same absolute luminosity and are at the same evolutionary stage, a simple look at a sample of observed globular-cluster (GC) CMDs (e.g., the HST snapshot survey in Piotto et al. 2002) shows a large variety of morphologies, suggesting immediately a complex scenario. As originally realized about 40 years ago by Sandage and Wallerstein (1960), Faulkner (1996), and van den Bergh (1967), the location of a star on the HB depends on a large number of parameters. Indeed, we now know that the temperature of a star on the zero age HB (ZAHB) depends on all the parameters involved in the stellar models. On the one hand, this fact makes the interpretation of the HB morphologies rather difficult. On the other hand, as the HBs behave as a sort of amplifiers of the stellar population properties, they are an ideal laboratory in which to study the initial conditions and the evolution of the single stars as well as of the parent cluster. Understanding the HB morphology has a large astrophysical impact. The HBs allow us to understand the evolution of Population II stars, and of specific systems (clusters or nearby galaxies). Moreover, as HB stars are bright (particularly at short wavelengths), they are key ingredients in any stellar-population synthesis models; and therefore the knowledge of their properties has a significant impact on understanding the evolution of distant, unresolved galaxies. If we note that HB stars (in particular the RR Lyrae) play a fundamental role in the calibration of the cosmic distance scale, we see that the study of the HBs is relevant also for cosmology.
Despite the frustration one might sometimes feel when confronted with the zoo of HB properties, the large effort devoted to their study is undoubtedly worth its while.

2. The Open Problems

A number of observational properties of HBs are not well understood, despite the fact that they have been known for more than 30 years. Among them we mention:

(1) The Oosterhoff-Sandage period shift, concerning the pulsation properties of the RR Lyrae variables (Oosterhoff 1939, Arp 1955, Sandage 1981);
(2) The dependence of the HB morphology on metallicity;
(3) Horizontal-branch bimodality (Harris 1974);
(4) The non-monotonic correlation between the HB extension and the metallicity, i.e., the evidence that there must be a “second parameter” (Sandage and Wildey 1967), or a combination of parameters (Buonanno et al. 1997) that determine the observed HB morphology;
(5) The “tilted HB” (Raimondo et al. 2002), i.e., a dependence of the HB luminosity on the temperature (cf. also Raimondo et al. in this book);
(6) The presence of blue tails (Fusi Pecci et al. 1993).

In the last ten years, since the first Ivanfest, most of the investigation of globular cluster HBs has been devoted to the blue tails (BTs), and we will concentrate on them in the remainder of this paper. The BT is an extension of the HB that runs almost vertically in the classical $V$ vs. $B - V$ (or $V$ vs. $V - I$) CMD, because of the insensitivity of the $B - V$ (or $V - I$) color to the temperature, for $T_e \geq 10,000$ K. The BT has a different morphology in different bandpasses. In the literature, there is some confusion concerning the name of the different branches of the hot HB; in the following, we will refer as BT to the part of the HB beyond $T_e = 10,000$ K, which corresponds observationally to the region where the HB becomes almost vertical in the $V$ vs. $B - V$ plane. We will refer to the piece of the HB between the blue edge of the RR Lyrae gap and this region as the blue HB (BHB). Finally, we will refer to the part of the BT populated by stars with $T_e \geq 23,000$ K as extreme horizontal branch (EHB). Observationally, the beginning of the EHB region corresponds to the recently identified “second jump” (see below).

The blue tails probably represent the most extreme of the mixed bag of anomalies that are sometime lumped into the term “second-parameter phenomena”. As an example, we can consider the HBs of NGC 5466, M92, and M15 (Fig. 1). Even if these three GCs have in practice the same metallicity (within 0.1 dex, Harris 1996), they have completely different HBs (Buonanno et al. 1985). NGC 5466 shows only a BHB. The M92 HB extends beyond $T_e = 10,000$ K, with a BT $\sim 1$ magnitude long in the $V$ vs. $B - V$ or $V - I$ CMD, and no discontinuity in the stellar distribution. M15 has a long BT, which extends well below the TO (as it can be clearly seen in the more populated $V$ vs. $B - V$ CMD of the cluster core, shown on the right side of Fig. 1), i.e., for more than 4 magnitudes, reaching the EHB. Both in the $V$ vs. $V - I$ and in the $V$ vs. $B - V$ CMD, there is a gap along the HB of M15, at $T_e \sim 10,000$ K, i.e., exactly where the HB of NGC 5466 seems to end.
In the last few years, a number of additional, unexpected, and mostly not clearly understood observational facts about the horizontal branch hot stars have come out, to further complicate the evolutionary scenario.

(1) It has been realized that HBs can extend well beyond $T_e = 30,000$ K (Sosin et al. 1997). These stars are the GC counterparts of the field blue subdwarfs (Newell 1973). The hottest HB stars lost most of their envelope during the red giant branch (RGB) phase, and underwent a late helium core flash (D’Cruz et al. 1996) while descending the white dwarf cooling sequence, with a deep mixing (Brown et al. 2001) which might explain why these objects are less luminous than predicted by canonical HB models. The high He content and the large enhancement of carbon found in a sample of these objects in $\omega$ Cen by Moehler et al. (2002) seems to support the flash-mixing hypothesis of Brown et al. (2000).
Thanks to the HST snapshot survey (Piotto et al. 2002), it has been realized that BTs are present at all metallicities, including the metal-rich clusters NGC 6388 and NGC 6441 (Rich et al. 1997), for which canonical models would predict only a red clump.

The BTs are present all over the cluster (Bedin et al. 2000), even in its outskirts. This is a particularly intriguing result, as the presence of BTs has been often attributed to an high density environment, in which close stellar encounters should favor tidal stripping, with a resulting shallower envelope, and therefore higher temperatures on the HB.

Peterson et al. (2002) have found a large fraction of binaries among the EHB stars.

The BTs have a number of discontinuities, such as:

(i) The gaps, i.e., regions underpopulated with stars (Sosin et al. 1997, see also Fig. 1). All the HBs with a BT in the snapshot sample (Piotto et al. 2002) show a gap. In some cases, these gaps might be simple statistical fluctuations in the distribution of the stars along the HB (Catelan et al. 1998). However, some gaps seem to appear at the same temperature (Ferraro et al. 1998), and/or at the same mass (Piotto et al. 1999) in all the BT clusters, pointing to some, common physical origin.

(ii) The first jump, at $T \sim 11,500$ K. Discovered for the first time by Grundahl, VandenBerg, and Andersen (1998) in M13, this discontinuity in the luminosity of the HBs in the Strömgren $u, u - y$ CMD seems to be a feature present in all the BTs which extends beyond $T_e = 11,500$ K (Grundahl et al. 1999). It is also clearly seen in the $U, U - V$ CMD (Fig. 3).

(iii) The discontinuity in the surface gravities (Moehler et al. 2000, and references therein). In the interval $11,500 \leq T_e \leq 21,500$ K spectroscopic observations suggest surface gravities smaller than predicted by canonical models.

(iv) The discontinuity in the surface abundance ratios. Confirming earlier results by Glaspey et al. (1989), Behr et al. (1999, 2000) have shown that all the stars with $T_e > 11,500$ K show an overabundance of heavy elements (Fe, Ti, N, P, etc.) with respect to the cluster metallicity, and an under-abundance of helium.

(v) The discontinuity in the stellar rotation rates (Behr et al. 2000, Recio-Blanco et al. 2002). As shown in Fig. 2, the projected rotation rates of BT stars show a sharp discontinuity at $T_e \sim 11,500$ K (Recio-Blanco et al. 2002), with the stars hotter than this temperature being all slow rotators, while some star cooler than $T_e = 11,500$ K rotating faster than 40 km/s (see also Recio-Blanco et al. in this volume for more details). Recio-Blanco et al. (2002) have suggested that the jump in stellar rotation happens at the same position of the photometric $u$-jump.

(vi) The “second jump”, at $T_e \sim 23,500$ K. This feature (Fig. 3) has been very recently discovered by Momany et al. (2002) in the $U$ vs. $U - B$ CMD of NGC 6752. The “second jump” seems to be an ubiquitous feature (Momany et al. 2003, in prep.). It is probably due to the combination of post-zero-age HB evolution and diffusion effects.

It is clear that some of these discontinuities appear at approximately the same temperature, and therefore might be the manifestation of the same physical phenomenon. The abundance anomalies discovered by Behr et al. (1999) are
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Figure 2. This figure shows all the projected rotation velocities of HB stars measured so far in GCs. The open circles are the new data from Recio-Blanco et al. (2002, 2003a, in prep., see also Recio-Blanco et al. in this volume). There is a clear discontinuity at the level of the \( u \)-jump, at \( T_e \sim 11,500 \) K.

interpreted as observational evidence of the onset of radiative levitation in stars hotter than \( T_e = 11,500 \) K. Following Greenstein, Truran, & Cameron (1967), gravitational settling of helium and radiative levitation of metals can occur in the stable, non-convective atmospheres of the hot, high-gravity HB stars (Michaud, Vauclair, & Vauclair 1983). Grundahl et al. (1999) have suggested that their \( u \)-jump can be the consequence of the metal enhancement caused by radiative levitation. Higher abundances can also (at least partially) account for the low-gravity problem affecting all the stars with \( T_e \geq 11,500 \) (Moehler et al. 2000). The fact that the change in the rotational velocity distribution of HB stars can be associated with the \( u \)-jump (Fig. 2), makes the entire scenario observationally consistent. The enhanced surface abundance of metals can boost mass (and angular-momentum) loss via radiation pressure on such elements, as suggested by Recio-Blanco et al. (2002). Recent models by Vink and Cassisi (2002) seem to confirm this scenario.

Despite these successes, we still do not understand the origin of the blue tails, or of the gaps along them, and we do not know why fast rotators among HB stars exist at all. However, the large photometric data bases that have been collected in recent years (e.g., Rosenberg et al. 2000a,b, and Piotto et al. 2002), and the even larger spectroscopic data bases that will arrive in the near future thanks to the new multi-fiber facilities on the 10m-class telescopes will undoubtedly help to solve these long-standing problems. An example of the kind of improvements we can expect is reported in the next Section.
Figure 3. Upper panel: The two jumps in the HB of NGC 6752, located at \( T_e \sim 11,500 \) K (the \( u \)-jump by Grundahl et al. 1999) and at \( T_e \sim 23,500 \) K (the “second jump” recently discovered by Momany et al. (2002), respectively. The open circles and squares are post-HB stars. Lower panel: As in the upper panel, but with the \( Z=0.0006 \) HB models superimposed. The thick line is the ZAHB, while the dotted lines represent the off-ZAHB evolutionary tracks. At the level of the “second jump” the evolutionary tracks become almost vertical, while the evolution time becomes slower and slower. Still, off-ZAHB evolution alone cannot explain this “second jump” (cf. Momany et al. 2002 for more details).

3. The Origin of BTs: New Observational Results

One of the main targets of the HST snapshot project (Piotto et al. 2002) was the investigation of the dependence of the GC HB morphology on cluster parameters. The photometric quality, the large stellar population sampled by the WFPC2 images in each cluster, and the large number (74) of observed GCs provide a unique data base for this investigation.

As shown by Djorgovski and Meylan (1994, see also Djorgovski in this volume), the GCs represent a complex, multi-parameter family. The correct statistical approach for the analysis of the relations among the many parameters which define the GC family is a multivariate analysis. This has been performed by Recio-Blanco (2003), and will be fully described in a forthcoming paper by Recio-Blanco et al. (2003b, in prep.). Here I want to anticipate a few new results,
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already evident in the simpler univariate analysis, to show the potentialities of this database.

One of the problems we had to solve for this analysis was the definition of a parameter able to describe the extension of the HB. There have been many suggestions in the literature (cf. Piotto et al. 1999 and references therein). Here we decided to use the effective temperature of the hottest stars on the HB. This implies fitting the HB with some theoretical models, but avoids all the problems that other empirical parameters have (Catelan et al. 1998, Ferraro et al. 1998, Piotto et al. 1999).

To fit a model to the HB we need a distance modulus and a reddening. In order to maintain internal consistency within the snapshot database, we have remeasured all the distance moduli and reddenings. The method will be fully described in Recio-Blanco et al. (2003c, in prep.). Here, suffice it to say that we adopted the same approach as in Zoccali et al. (2000), though in this case we worked directly in the HST F439W and F555W bands.

In the following, we will consider only the clusters with a BHB. Once the distance moduli and the reddenings had been determined, we fitted ZAHB models specifically developed by Cassisi (2003, prov. comm.) to the observed HBs. This procedure allowed us to evaluate the highest effective temperature \(T_e\) reached by the globular cluster HB and therefore its temperature extent. The errors in this temperature determination are difficult to estimate, as they depend not only on the errors in the distance modulus and reddening, but also on the number of stars in the HB, and on the temperature range we have to deal with. As a consequence, the largest errors occur for the smallest, low-central-concentration clusters, and for the most extended HBs, where the large bolometric correction in these photometric bands precludes an accurate estimate of \(T_e\). However, although the errors can be important, the general trend of the HB morphology with cluster parameters is not dramatically affected.

Though we performed a full, multivariate analysis, we started with the simplest univariate analyses.

The first correlation we identified is the HB morphology-metallicity dependence. However, the large dispersion of the data clearly indicates that there must be some other parameter which regulates the HB extent.

The most interesting results of this simple approach are summarized in Figs. 4 and 5. We isolated a subsample of 24 GCs with intermediate metallicity in order to (i) remove as much as possible the metallicity effect on the HB extent, and (ii) study the HB morphology in the metallicity interval where the second parameters are expected to be most effective (Fusi Pecci et al. 1993).

There is a significant correlation between the HB extension and the total cluster luminosity \((M_V)\) and the collisional parameter \((\Gamma_{\text{col}})\). The collisional parameter \(\Gamma_{\text{col}}\) is defined as the number of collisions per unit time in the cluster core, and it has been derived for a King-model cluster by King (2002):

\[
\Gamma_{\text{col}} = \log[5 \cdot 10^{-15} \sqrt{\sigma^3 \cdot \tau_c}]
\]

where \(\sigma\) is the central surface brightness in units of \(L_{\odot}/pc^2\):

\[
\sigma = 10^{-0.4(M_V - 26.41)}
\]
Figure 4. The effective temperature of the hottest HB star is plotted against the total absolute magnitude $M_V$ for a sample of 24 GCs with intermediate metallicity ($-1.90 \leq [\text{Fe/H}] \leq -1.35$). The open circles represent post-core-collapse clusters.

Figure 4 shows that the most luminous clusters tend to have the most extended HBs. Noteworthy, at a given total luminosity, the post-core-collapse (PCC) (open circles) clusters have the most extended HBs: the PCC define the upper boundary of the relation between log $T_e$ and $M_V$. Luminosity is the most fundamental observed quantity characterizing a stellar system, and for a set of old stellar clusters it is a good relative measure of its baryonic mass. Therefore, the observed trend suggests that more massive clusters tend to have more extended HBs.

Figure 5 shows the correlation between the HB extension and the collision rate. Clusters with higher probability of collisions among stars in the core (the snapshot CMDs refer to the stars in the cluster cores!), have, on average, more extended HBs. PCC clusters in Fig. 5 seem to behave as normal King-model clusters, but this can be due to the inadequacy of $\Gamma_{\text{col}}$ in representing the collision probability in a PCC cluster (indeed, $\Gamma_{\text{col}}$ was derived for a King-model cluster).

Formally, the collisional parameter is independent of $M_V$, as it has been calculated using the $\mu_V$ and $r_c$ parameters. However, we must expect some correlation between $\Gamma_{\text{col}}$ and $M_V$, because of the dependence of $M_V$ on $\mu_V$ (Djorgovski and Meylan 1994). In any case, multivariate analysis (Recio-Blanco et al. 2003c) shows that the correlations in Fig. 4 and 5 are both significant.

It is tempting to interpret the correlation between the HB extent and the collisional parameter as an indication that close stellar encounters (or collisions) can cause some modification in the evolution of the stellar population in GCs. It is also worth to note that the correlation between the HB extent and $\Gamma_{\text{col}}$ is more significant than the correlation between the HB extent and the central
cluster density (Fusi Pecci et al. 1993), indicating that $\Gamma_{\text{col}}$ is a more appropriate parameter to describe the collision effects in GC cores.

The dependence of the HB morphology on the total luminosity and therefore the cluster total mass is also noteworthy. It cannot be a simple consequence of the correlation of the HB extension with $\Gamma_{\text{col}}$. One possible interpretation (Recio-Blanco 2003) of the influence of M$_V$ and therefore, of cluster total mass, on the HB morphology can be derived from some recent results by D’Antona et al. (2002). These authors analyze the consequences, on the HB morphology, of helium variation due to self-pollution within GC stars. Self-pollution had already been proposed as an explanation for the chemical inhomogeneities (spread in the abundances of CNO, O–Na and Mg anti-correlation) observed in GC members from the main sequence to the RGB (e.g., Gratton et al. 2001). The ejecta of massive asymptotic giant branch stars, which would be the origin of the self-pollution, would not only be CNO processed, but also helium enriched. The models of D’Antona et al. (2002) take into account this possible helium enhancement. They find that a spread in the helium content does not affect the morphology of the main sequence, turn-off, and RGB in an easily observable way, but that it can play a role in the formation of blue tails. Indeed, structures with larger helium abundance are able to populate much bluer HB regions. Self-pollution, and therefore helium enrichment, are expected to be greater in more massive clusters, as they would be able to retain the material from the ejecta better than less massive ones. Therefore, we should expect more extended HBs in more massive clusters, as we observe. It would be extremely interesting to investigate whether the clusters with the most extended HB also show a Na and Mg anti-correlation, which would confirm this scenario (D’Antona et al. 2002).
In any case, helium variation due to self-pollution is not enough to explain the observed HB morphology. For example, higher mass loss is needed in metal-rich clusters than in metal-poor ones, to produce bluer HB stars. Close stellar encounters, in clusters with higher collision rates, contribute to explaining the complex observational scenario, as suggested by Fig. 5.

Another noteworthy example of the effect of the environment on the evolution of the stellar population of GCs is discussed by De Angeli and Piotto, in this volume. Using the same snapshot data base that has been used here, we find a dependence of the relative frequency of blue stragglers on both the total cluster luminosity and the collision rate. In particular, we find that clusters with higher collision rates show a smaller relative number of blue stragglers. We also find that the most luminous clusters have the smallest population of BSS.

4. Conclusions

There was a large debate in 1992, at the first Ivanfest, on the interplay between dynamical evolution and stellar evolution (Djorgovski and Piotto 1993, and other articles in the same volume). At that time the observational evidence was scarce and mostly contradictory. After 10 years, it seems we have collected enough observational information, mostly from photometric data, to confirm this effect. We hope that in ten years from now we can meet again, feasting Ivan’s 85th birthday, after having digested the huge amount of photometric data that we have collected with the recently developed wide field imagers, and the even larger data set that we expect to collect with the forthcoming multi-fiber spectroscopic facilities. It is easy to foresee that by that time, thanks to these observational inputs, and with the help of parallel improvements of the models, we will have a much deeper knowledge of the evolution processes at the basis of the formation of the complex and still not completely understood horizontal branches.

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