Hybrid Baryons in Large-$N_c$ QCD

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(October 14, 2018)

Abstract

We study the properties and couplings of hybrid baryons in the large-$N_c$ expansion. These are color-neutral baryon states which contain in addition to $N_c$ quarks also one constituent gluon. Hybrid baryons with both symmetric and mixed symmetric orbital wave functions are considered. We introduce a Hartree description for these states, similar to the one used by Witten for ordinary baryons. It is shown that the Hartree equations for $N_c (N_c - 1)$ quarks for symmetric (mixed symmetric) states in these states coincide with those in ordinary baryons in the large-$N_c$ limit. The energy due to the gluon field is of order $\Lambda_{QCD}$. Under the assumption of color confinement, our results prove the existence of hybrid baryons made up of heavy quarks in the large $N_c$ limit and provides a justification for the constituent gluon picture of these states. The couplings of the hybrid baryons to mesons of arbitrary spin are computed in the quark model. Using constraints from the large $N_c$ scaling laws for the meson-baryon scattering amplitudes, we write down consistency conditions for the meson couplings of the hybrid baryons. These consistency conditions are solved explicitly with results in agreement with those in the quark model for the respective couplings.
I. INTRODUCTION

It has been suggested for some time (for a review see [1]) that a new class of hadron states could exist beyond the framework of the quark model. These are the so-called hybrid or hermaphrodite hadrons and contain, in addition to constituent quarks, also constituent gluons. From an experimental point of view it is easiest to distinguish a hybrid meson from an usual one. This is made possible by the fact that the former can have exotic quantum numbers, which are forbidden for states containing only constituent quarks (e.g. $J^{PC} = 1^{-+}, 0^{--}$).

Perhaps for this reason, most of the theoretical effort has been devoted to the study of the hybrid mesons. Their properties have been investigated extensively using QCD-inspired methods such as the bag model [2,3], the flux tube model [4,5] and lattice QCD. Recently, some evidence has been reported by the E852 Collaboration for the existence of a $J^{PC} = 1^{-+}$ resonance [6].

In addition to hybrid mesons, hybrid baryons are expected to exist too. These are color-neutral states made up of $N_c$ quarks, each transforming under the fundamental representation of $SU(N_c)_{color}$ plus one constituent gluon transforming in the adjoint representation. The nontrivial color correlations between quarks and gluons constrain, via the Pauli principle, the spin-flavor structure of these states to be different from that of pure $(q^{N_c})_{singlet}$ states.

The lowest-lying hybrid baryons are expected to form a positive parity 70 multiplet of SU(6) lying around 1.5 GeV [4,5] and have the constituent gluon field in a TE mode with $J^P = 1^+$. It is tempting to identify the Roper resonance $N(1440)$ with such a hybrid baryon. Although this identification is not yet completely ruled out, (see [1] for a discussion of the various possibilities) it appears to conflict with existing photoproduction data. An analog of the Moorhouse selection rule derived in [2] forbids the photoproduction of a hybrid $J^P = 1/2^+$ state off protons, whereas experimentally the $N(1440)$ is photoproduced copiously. A similar reasoning can be used to argue that the nearby $I,J^{PC} = 1/2^+, 3/2^+$ state $N(1720)$ cannot be predominantly hybrid. This leaves as the only hybrid candidate the $I,J^{PC} = 1/2, 1/2^+$ state $N(1710)$.

The apparent absence of the hybrid baryons in this region is somewhat puzzling and raises the question of the very existence and stability of these states. It is therefore of some interest to prove these properties starting, as far as possible, from first principles. It is the purpose of this paper to provide such a proof, in the framework of large-$N_c$ QCD. More precisely, we prove that in the large-$N_c$ limit there exist stable hybrid baryons made up of heavy quarks. Our proof is based on the assumption of color confinement and on the validity of a Hartree description for these states.

In Sec. II we discuss the spin-flavor structure of the ground state hybrid baryons containing heavy quarks in large-$N_c$ QCD and show that it is closely related to that of orbitally excited baryons. Two types of hybrid baryons are considered: the first with symmetric orbital wave functions and mixed symmetry spin-flavor content, and the second with a mixed symmetric orbital wave functions and symmetric spin-flavor content. The close connection between hybrid baryons with symmetric orbital wave functions and ordinary baryons is made explicit in Sec. III, where it is proven that the two types of baryons lead to the same Hartree equations in the large-$N_c$ limit. Intuitively speaking, this is due to the fact that
the interquark forces in the two cases tend to the same value as \( N_c \) is taken to be large. The couplings of these states to mesons and hybrid mesons are discussed in Sec. IV and the corresponding \( N_c \) counting rules are established. These are used in Sec. V to derive consistency conditions for the spin-isospin structure of the couplings to mesons, which are then solved explicitly.

II. HYBRID BARYONS WITH HEAVY QUARKS

We will restrict ourselves in this paper to the tractable case of hybrid baryons made up only of heavy quarks, for which the constituent quark picture becomes exactly valid. The wavefunction of the hybrid can be written as

\[
\Psi_{B_h} = \frac{(-)^{\delta}}{\sqrt{N_c(N_c^2 - 1)}} \sum_{n=1}^{N_c^2 - 1} \chi^n(J_g, m_g) \otimes [\psi_{a_1 a_2 \ldots a_{N_c}}^n]_i \otimes \Psi_i(\vec{x}_1, \ldots, \vec{x}_{N_c}; S, I; m_S, \alpha)CG ,
\]

where \( \chi^n \) is the wavefunction of the gluon field which carries angular momentum \( J_g \). \( \psi^n \) is the color part (\( a_i \) are color indices for the quarks), \( \Psi \) is the orbital part of the quarks' wavefunction and \( CG \) is the Clebsch-Gordan coefficient \( \langle Jm | SJ_g; m_Sm_g \rangle \).

Since the hybrid is a color singlet, the quark part of the wavefunction \( \psi^n \) must transform in the adjoint representation of the color \( SU(N_c) \) group. The normalized color part of the wavefunction reads

\[
[\psi_{a_1 a_2 \ldots a_{N_c}}^n]_i = \sqrt{\frac{2}{(N_c - 1)!}} \epsilon_{a_1 a_2 \ldots a_{N_c} \ldots a_{N_c}^n} (t^n)_{a_i b_i} .
\]

The \( i^{th} \) quark will be called the external quark, and the remaining \( N_c - 1 \) quarks will be called core quarks. The Gell-Mann matrices \( t^n \) are normalized in the standard way by \( \text{Tr}(t^n t^m) = 1/2\delta_{nm} \). These color wavefunctions are normalized as

\[
\langle \psi_i^n | \psi_j^m \rangle = \delta_{nm} \quad (i = j) , \quad -\frac{1}{N_c - 1} \delta_{nm} \quad (i \neq j) .
\]

These relations and many others below can be obtained by making use of the identity

\[
\sum_{x=1}^{N_c^2 - 1} t^x_{ij} t^x_{kl} = \frac{1}{2} \delta_{kj} \delta_{il} - \frac{1}{2N_c} \delta_{ij} \delta_{kl} .
\]

The Pauli principle requires the hybrid wavefunction to be antisymmetric under any permutation of the quarks. This constrains the spin-flavor-orbital wavefunction to be described by a Young diagram which is the transpose of that corresponding to the color wavefunction (see Eq. (3)). Therefore the quarks in a hybrid can be effectively considered as identical particles subjected to a special statistics: their wavefunction must have permutation symmetry described by the second Young diagram in Eq. (3).
The quark color wavefunction (2) is only one of many other possible ways of constructing an adjoint from $N_c$ fundamental fields. The most general construction involves first arranging $N_c-j$ quarks into an antisymmetric representation. The remaining $j$ quarks are then coupled either into an antisymmetric representation or a representation whose Young diagram has the same form as the first diagram in Eq. (5). Each of these possibilities could in principle generate one distinct set of degenerate states. This ambiguity in constructing a hybrid baryon state is in contrast to the situation for ordinary baryons, where each subset of $j$ quarks is in a color antisymmetric state. However, all these states different from (2) disappear in the physical limit $N_c = 3$. Therefore we will focus in the following only on the hybrid baryon states (2).

It is plausible to assume that the ground state hybrids will have all the quarks in a state with zero orbital angular momentum $L = 0$; thus their orbital wavefunction is completely symmetric. This implies that the spin-flavor part of the wavefunction must have mixed symmetry. This is exactly the same as the spin-flavor wavefunction of an orbitally excited baryon with one quark in an excited state. Therefore the spin-flavor structure of the ground state hybrids can be read off immediately from [7] (for two light flavors) and from [8] (for three light flavors). Its wavefunction can be written in Hartree form as

$$\Psi_i = \Phi(\vec{x}_1)\Phi(\vec{x}_2)\cdots\Phi(\vec{x}_{N_c}) \otimes |S, I; m, \alpha\rangle_i,$$

with $\Phi$ one-particle wavefunctions and $|S, I; m, \alpha\rangle_i$ is the spin-flavor state with mixed permutation symmetry constructed in [7].

For example, assuming that the constituent gluon field carries quantum numbers $J^P = 1^+$, the resulting states are classified into three infinite towers of states with $|J - I| \leq \Delta$, $\Delta = 0, 1, 2$. Each tower is identified by a value of $\Delta$ and its states are degenerate in the large-$N_c$ limit. One obtains in this way the sequence of states

$$(I, J^P) = \left(\frac{1}{2}, \frac{1}{2}^+\right), \left(\frac{3}{2}, \frac{3}{2}^+\right), \left(\frac{5}{2}, \frac{5}{2}^+\right), \cdots \quad (\Delta = 0)$$

$$(\frac{1}{2}, \frac{1}{2}^+), (\frac{1}{2}, \frac{3}{2}^+), (\frac{1}{2}, \frac{1}{2}^+), \cdots \quad (\Delta = 1)$$

$$(\frac{3}{2}, \frac{1}{2}^+), (\frac{3}{2}, \frac{5}{2}^+), (\frac{3}{2}, \frac{1}{2}^+), \cdots \quad (\Delta = 2)$$.

These states are linear combinations of the states (11) with well-defined spin $S$. The corresponding relations are given below in (12).

In addition to the states (11) we will consider also a different type of hybrid baryons for which the quark wavefunction is given by

$$\Psi'_i = \Phi(\vec{x}_1)\Phi(\vec{x}_2)\cdots\Psi(\vec{x}_i)\cdots\Phi(\vec{x}_{N_c}) \otimes |I, m, \alpha\rangle.$$

The spin-flavor part $|I, m, \alpha\rangle$ is completely symmetric and the orbital part has mixed symmetry. Here $\Psi(\vec{x})$ is a one-body Hartree wavefunction which will be assumed to be also $s$-wave. $\Psi(\vec{x})$ can be taken to be orthogonal to $\Phi$ without any loss of generality. Indeed, the component of $\Psi$ along the direction of $\Phi$ will give a vanishing result when inserted into (10) due to the identity.
\[
\sum_{i=1}^{N_c} \varepsilon_{a_1 a_2 \ldots a_{N_c}} (t^n)_{a_i b_i} = 0.
\] (11)

This can be proved by taking the square and summing over \( a_i \) with the help of (3) which gives a vanishing result. These states are expected to lie above the ones described by (6). Assuming the same quantum numbers for the color octet gluon field, they form in the large \( N_c \) limit a \( \Delta = 1 \) tower with the spin-flavor content (8).

### III. EXISTENCE

The one-body Hartree wavefunction \( \Phi(\vec{x}) \) in (3) can be determined from the variational principle

\[
\delta \frac{\langle \Psi_{B_h} | \mathcal{H} | \Psi_{B_h} \rangle}{\langle \Psi_{B_h} | \Psi_{B_h} \rangle} = 0
\] (12)

where the Coulomb gauge Hamiltonian \( \mathcal{H} \) is given by

\[
\mathcal{H} = \frac{1}{2} \int \, d\vec{x} (\vec{E}_\perp^2 + \vec{B}^2) + \sum_n \frac{1}{2m_Q} \left( -i \nabla_n - g A^a(x_n) t_n^a \right)^2 + \frac{g^2}{4\pi} \sum_{m<n} \frac{t_m^a t_n^a}{|x_m - x_n|}
\] (13)

\[
- \frac{g^2}{4\pi} f_{abc} \sum_n \int \, d\vec{x} \frac{\vec{A}^b(\vec{x}) \cdot \vec{E}^c(\vec{x}) t_n^a}{|\vec{x} - \vec{x}_n|} + \frac{g^2}{8\pi} f_{abc} f_{ade} \int \, d\vec{x} \, d\vec{y} \, \frac{\left( \vec{A}^b \cdot \vec{E}^c(\vec{x}) \right) \left( \vec{A}^d \cdot \vec{E}^e(\vec{y}) \right)}{|\vec{x} - \vec{y}|}.
\]

The chromoelectric field is given by \( \vec{E}^a = -\partial_0 \vec{A}^a - \nabla A^{0a} + g f_{abc} A^{0b} \vec{A}^c \) and includes, in addition to the transverse part \( \vec{E}_\perp^a \), also a longitudinal component. In the expression for the Hamiltonian (13) it is the full chromoelectric field which appears, except in the first term.

We assume, as everywhere else, that the quarks are sufficiently heavy so that their interactions are purely Coulombic. We neglected in (13) terms of higher order in the inverse heavy quark mass. The expectation value of the color factor \( t^a_m t^a_n \) in the state (1), (6) can be easily computed with the result

\[
\mathcal{C} = \frac{\langle \Psi_{B_h} | \sum_n t^a_m t^a_n | \Psi_{B_h} \rangle}{\langle \Psi_{B_h} | \Psi_{B_h} \rangle} = -\frac{N_c^2 + 2N_c + 1}{2N_c(N_c - 1)}
\] (14)

for any \( m \neq n \). We made use here of the methods developed in [7] for computing matrix elements on the states with mixed symmetry |\( SI \rangle_i \). Keeping only the leading terms in \( N_c \) we obtain in this way the following expression for the expectation value of the Hamiltonian (13)

\[
\delta \left( \sum_{n=1}^{N_c} \int \, d\vec{x}_n \, \Phi^\dagger(\vec{x}_n) H_0(\vec{x}_n) \Phi(\vec{x}_n) + \frac{g^2}{4\pi} \mathcal{C} \sum_{m<n} \int \, d\vec{x}_n \, d\vec{x}_m \frac{|\Phi(\vec{x}_n)|^2 |\Phi(\vec{x}_m)|^2}{|x_m - x_n|} \right) = 0
\] (15)

with

\[
H_0 = -\frac{1}{2m_Q} \nabla^2 + \frac{ig}{m_Q} \vec{A}(x) \cdot \nabla + \frac{g^2}{2m_Q} \vec{A}(x) \cdot \vec{A}(x).
\] (16)
The terms proportional to $g$ and $g^2$ in (14) describe the interaction of the quarks with the transverse gluon field. The $O(g)$ term appears to contribute a term of order $N_c g = 1/\sqrt{N_c}$ to the hybrid mass. However, its matrix elements taken on the $s$-wave Hartree wavefunctions considered here vanish:

$$\int d\vec{x} \Phi(\vec{x}) \vec{A} \cdot \nabla \Phi(\vec{x}) = \frac{1}{2} \int d\vec{x} \partial_i (\Phi(\vec{x}) A^i \Phi(\vec{x})) - \frac{1}{2} \int d\vec{x} \Phi(\vec{x}) (\nabla \cdot \vec{A}) \Phi(\vec{x}) = 0.$$  \tag{17}

Therefore the leading correction to the hybrid baryon mass comes from the “seagull” term in (16) (quadratic in $A^a$) and is of order $g^2 N_c = N_c^0$. Also, the instantaneous Coulomb quark-gluon interactions in (13) contribute only to order $g^2 N_c = N_c^0$ to (16) and is unimportant for large $N_c$.

One can see thus that the large $N_c$ limit of the hybrid baryon Hartree equation (15) contains only the kinetic quark terms and the quark-quark Coulomb interactions, just as for ordinary baryons. Furthermore, for large $N_c$ this equation is exactly identical to the corresponding equation for an ordinary baryon $\Phi$ (for which the color factor $C$ takes the value $-/(N_c + 1)/(2N_c)$).

We conclude therefore that the one-particle Hartree wavefunctions $\Phi(\vec{x})$ in (13) are the same as in ordinary baryons. Hence the mass of a hybrid baryon grows linearly with $N_c$ in the same way as for an ordinary baryon, as expected on intuitive grounds. The different expectation values of the gluon field energy and gluon-quark interactions in (13) produce a finite mass difference among these types of states of order $N_c^0$. This mass difference can be interpreted phenomenologically as a constituent gluon mass.

Turning to the applicability of these results to the physical world with $N_c = 3$, one can see that the Coulomb quark-quark interaction in a hybrid is attractive for any $N_c \geq 3$. This suggests the existence of these states also in the physical case $N_c = 3$. However, for finite values of $N_c$ the quark-gluon interactions must be taken into account too. Calculations in the bag model [2,3] show that these additional interaction terms are likely to be small and can be treated as a perturbation.

The Hartree equation for the hybrid with orbital wavefunction with mixed symmetry (10) has the form

$$\delta \left\{ (N_c - 1) \int d\vec{x} \Phi^\dagger(\vec{x}) H_0 \Phi(\vec{x}) + \int d\vec{x} \Psi^\dagger(\vec{x}) H_0 \Psi(\vec{x}) + g^2 \frac{N_c (N_c - 1)}{2} \left( -\frac{(N_c + 1)(N_c - 2)}{2N_c^2} \int d\vec{x}_m d\vec{x}_n \frac{\Phi(\vec{x}_m) \Phi(\vec{x}_n)^2}{4\pi |\vec{x}_m - \vec{x}_n|} \right. \\
+ \frac{1}{N_c^2 (N_c - 1)} \int d\vec{x}_m d\vec{x}_n \frac{\Psi(\vec{x}_m) \Psi(\vec{x}_n)^2}{4\pi |\vec{x}_m - \vec{x}_n|} \\
- \frac{N_c - N_c - 1}{N_c^2 (N_c - 1)} \int d\vec{x}_m d\vec{x}_n \frac{\Phi^\dagger(\vec{x}_m) \Psi(\vec{x}_n) \Psi^\dagger(\vec{x}_n) \Phi(\vec{x}_n)}{4\pi |\vec{x}_m - \vec{x}_n|} \right) \\
- \frac{1}{2(N_c^2 - 1)} \langle \chi^y | (if_{yax} - df_{yax}) \int d\vec{x} d\vec{x}_n \frac{g^2 f_{abc}(A^b \cdot \vec{E}^c(\vec{x}) \Phi(\vec{x}_n)^2}{4\pi |\vec{x} - \vec{x}_n|} |\chi^x \rangle \\
- \frac{1}{2(N_c^2 - 1)} \langle \chi^y | (if_{yax} + df_{yax}) \int d\vec{x} d\vec{x}_n \frac{g^2 f_{abc}(A^b \cdot \vec{E}^c(\vec{x}) \Psi(\vec{x}_n)^2}{4\pi |\vec{x} - \vec{x}_n|} |\chi^x \rangle \right\} = 0.$$  \tag{18}

We included here the quark-quark and quark-gluon Coulomb interaction terms. From this equation it is easy to read off the couplings of the quarks with each other and with the gluon.
field. The $N_c - 1$ quarks in the core interact with each other by Coulomb interaction to leading order and with the gluon field at subleading order in $1/N_c$. Hence the latter term can be neglected in the equation for $\Phi$, which becomes thus identical to (13). Its solution is identical to the Hartree wavefunction in an ordinary baryon.

The dynamics of the external quark is more complex: it feels a repulsive force from the $N_c - 1$ quarks in the core of the order $1/N_c^2$ (the term in the third line of (18)) and an exchange interaction of order 1 (the fourth line of (18)) which can be either attractive or repulsive. However, the hybrid state can still be bound because the external quark interacts also with the gluon field to order 1 (the last term in (18)). We can estimate the nature of this interaction by assuming that the gluon field is classical (this is essentially the assumption made in bag model calculations [2,3]). Then the matrix elements of the gluon fields can be written in terms of these functions

$$\langle \chi^y(m_g) | \vec{A}^a(\vec{x}) | \chi^x(m_g) \rangle = i \omega |\chi(\vec{x})|^2 (\delta_{y \alpha} \delta_{\alpha \epsilon} - \delta_{y \epsilon} \delta_{\alpha \epsilon}) .$$

Introducing this expression into (18) one obtains the following expectation value of the Coulomb quark-gluon interaction

$$\langle \Psi_B' | H_{q-g} | \Psi_B' \rangle = -N_c g^2 \omega \left( \int d\vec{x} d\vec{x}_n \frac{|\chi(\vec{x})|^2 |\Phi(\vec{x}_n)|^2}{4\pi |\vec{x} - \vec{x}_n|} + \int d\vec{x} d\vec{x}_n \frac{|\chi(\vec{x})|^2 |\Psi(\vec{x}_n)|^2}{4\pi |\vec{x} - \vec{x}_n|} \right) ,$$

which is seen to be attractive.

The properties of these states following from their Hartree description are very similar to those of the hybrids with symmetric orbital wavefunction (6). Their mass grows linearly with $N_c$ in the same way as for ordinary baryons. The interaction energy with the gluon field and the energy of the external quark are responsible for a mass splitting among these states of order $N_c^0$.

It is interesting to compare the hybrid baryons to the hybrid mesons, recently discussed in the large $N_c$ limit in [10]. Let us consider, for illustration, a hybrid meson made up of a heavy quark-antiquark pair. Its wavefunction can be written analogously to (1)

$$\Psi_{Mh} = \frac{1}{\sqrt{N_c^2 - 1}} \sum_{n=1}^{N_c^2 - 1} \chi^n \otimes \psi_{a_1 a_2}^n(\vec{x}_1, \vec{x}_2) .$$

The color part of the quarks’ wavefunction is $\psi_{a_1 a_2}^n = \sqrt{2} (t^n)_{a_1 a_2}$, which gives the following color-factor for the quark-antiquark potential in such a bound state

\[ \text{The quark-gluon Coulomb interaction in the state (13) has exactly the same form as the first term in (22).} \]
\[ \langle \psi^n | t_1^x t_2^x | \psi^m \rangle = \frac{1}{2N_c} \delta_{nm} \quad (\vec{p} = -(t^x)^T). \]  

(24)

This produces a repulsive quark-antiquark force which vanishes in the large \( N_c \) limit. For comparison, the value of the corresponding color factor in a color-singlet meson state \( \psi = \frac{1}{\sqrt{N_c}} \delta_{a_1 a_2} \) is

\[ \langle \psi | t_1^x t_2^x | \psi \rangle = -\frac{N_c^2 - 1}{2N_c} \]  

(25)

which gives the usual attractive potential between a quark and antiquark in a meson.

These results show that if hybrid mesons are to exist in the large \( N_c \) limit (as shown in [10]) then the color-octet gluon field must supply the attractive force needed to bind the system. Furthermore, in the physical case \( N_c = 3 \) this quark-gluon attractive interaction must be strong enough to overcome the finite Coulomb quark-antiquark repulsion. This is in contrast to the case of the hybrid baryons (6), where the dynamical effect of the color-octet gluon field on the quarks can be neglected for large \( N_c \) and the quark-quark interaction is attractive for any \( N_c \) larger than 2. The hybrid mesons are somewhat similar to the hybrid baryons (10), in which the external quark is repelled by the core with a strength \( 1/N_c^2 \).

However, there is no analog for the exchange interaction term in hybrid baryons for the hybrid meson case.

We have not included here the contribution to the total energy (15) and (18) from the gluon field. In the same approximation as in (19) and (20), one simply replaces the field operators in (13) by the corresponding classical functions. The attractive quark-gluon interactions should localize the gluon field within a region of size \( 1/\Lambda_{QCD} \) surrounding the quarks. Suffice it to note that these contributions are all of order \( N_c \Lambda_{QCD} \).

To complete our proof of existence of the hybrid baryons, we still have to show that these states are relatively narrow. It is known that meson coupling to baryons can grow with \( N_c \). This mechanism could potentially cause the widths of these states to grow with \( N_c \) too, which would effectively render them unstable. We will show in the following section by means of computations in the quark model that some of these states are indeed narrow in the large-\( N_c \) limit.

IV. QUARK MODEL PREDICTIONS FOR HYBRID BARYONS

The Hartree picture of a hybrid baryon developed in Sect. II can be used in the same way as for ordinary baryons [11] to derive scaling laws for its couplings to mesons. The properties of the two types of hybrid baryons (3) and (10) will be seen to be different. We will consider them in the following in turn.

A. Hybrid baryons with symmetric orbital wavefunction

The coupling of a pion to a hybrid baryon (3) is parametrized in terms of the matrix element

\[ \langle \psi^n | t_1^x t_2^x | \psi^m \rangle = \frac{1}{2N_c} \delta_{nm} \quad (\vec{p} = -(t^x)^T). \]  

(24)

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\[ \langle S'I'J_g; m'_S\alpha'm'_y \rangle \sum_{n=1}^{N_c} \sigma_n \tau_n^{\alpha_a} |SIJ_g; m_S\alpha m_g \rangle = \]

\[ \frac{1}{N_c(N_c^2 - 1)} (-)^{\psi(S'I') + \psi(S'I)} \delta_{m_g m'_y}^{\alpha_a} \sum_{y=1}^{N_c^2-1} \sum_{k,k'=1}^{N_c} \nu' \langle S'I'; m'_S\alpha' |SI; m_S\alpha \rangle_k (\tilde{\psi}_k' \psi_k^\nu) = \]

\[ \times \left\{ k \langle S'I'; m'_S\alpha' |SI; m_S\alpha \rangle_k - \nu' \langle S'I'; m'_S\alpha' |SI; m_S\alpha \rangle_k \right\}. \quad (k \neq k') \]

The color part of the matrix element has been computed with the help of the relations \(8\). The matrix elements appearing in the last line have been computed already (see Eqs. (4.81) and (4.82) in \[7\]). The “physical” value of the matrix element is obtained, just as in ordinary baryons, after dividing with the square roots of the norms of the initial and final states

\[ \langle SIJ_g |SIJ_g \rangle = 3(2i + 1) \left\{ \frac{S}{2} \frac{I}{2} \frac{I'}{2} \right\}^2. \]

We obtain finally the following result for the matrix element \(26\)

\[ \langle S'I'J_g; m'_S\alpha'm'_y \rangle \sum_{n=1}^{N_c} \sigma_n \tau_n^{\alpha_a} |SIJ_g; m_S\alpha m_g \rangle = \]

\[ N_c (-)^{S+I'} \sqrt{(2S + 1)(2I + 1)} \left\{ \frac{S'}{I} \frac{S}{I'} \frac{1}{1} \right\} \langle S'm'_S |SI; m_S\alpha \rangle \langle I'\alpha' |I1; \alpha \rangle \delta_{m_g m'_y}. \]

We used here the same phase as in \[7\] for the mixed symmetry states \(\psi(SIi) = i + I + \frac{1}{2}\).

After dividing with the meson decay constant \(f_M \propto N_c^{\frac{1}{2}}\) this gives that mesons couple to hybrid baryons with a coupling of order \(N_c^{\frac{1}{2}}\), just as to ordinary baryons. It is interesting to note that the quark model result \(28\) coincides exactly with the one obtained in \[7\] for pion couplings to orbitally excited baryons.

It is easy to see that the decay mode \(B_h \rightarrow M + B\) is forbidden for hybrid baryons made up of heavy quarks. In this approximation the transition matrix element is proportional to the overlap of the color-singlet and color-octet wavefunctions respectively \(\langle \psi |\psi_n^{\nu} \rangle = 0\). On the other hand, the lowest lying hybrid baryons can decay to a pair \((M_h, B)\) in a \(p\)-wave, with \(M_h\) a hybrid meson and \(B\) an ordinary \(s\)-wave baryon.

The decays to an ordinary meson and baryon are induced through mixing \(M_h \leftrightarrow M\). This mixing is present already at order 1 in the \(1/N_c\) expansion, unless the respective state \(M_h\) has exotic quantum numbers. Therefore the scaling laws for the hybrid meson couplings are the same as those for ordinary mesons. As mentioned above, the mixing \(M_h \leftrightarrow M\) vanishes in the heavy quark limit as \(N_c^0/m_Q\). This has been taken into account in recent refinements of quarkonium physics, where a small octet \(QQg\) admixture has been included in addition to the \(\bar{Q}Q\) component \[11\].

In the following we will compute the decay amplitudes of a hybrid baryon in the quark model. The detailed form of the respective coupling depends on the quantum numbers of the final hybrid meson and is shown below for a few cases of physical interest.
The quantum numbers $J^{PC}$ of the respective hybrid meson state are shown in the brackets. We will be interested in the following only in decays $B_h \to B + M_h$ where both $B_h$ and $B$ have positive parity. Then an odd parity hybrid meson $M_h$ is emitted in an orbital $p$-wave. The decay amplitudes for these transitions can be extracted from (29) and are given, in the nonrelativistic limit, by

$$\mathcal{M}_a = \langle B | \sum_{n=1}^{N_c} (\vec{p} \cdot g \vec{B}^x(n)) t_n^x t_n^a | B_h \rangle$$ \hspace{1cm} (30) \hspace{1cm} (J^{PC} = 0^{-+})

$$\mathcal{M}_b = \langle B | \bar{p} \cdot \sum_{n=1}^{N_c} (\vec{\sigma} \times g \vec{B}^x(n)) t_n^x t_n^a | B_h \rangle$$ \hspace{1cm} (31) \hspace{1cm} (J^{PC} = 0^{--})

$$\mathcal{M}_c = \langle B | (\bar{p} \cdot \vec{\varepsilon}) \cdot \sum_{n=1}^{N_c} g \vec{B}^x(n) t_n^x t_n^a | B_h \rangle$$ \hspace{1cm} (32) \hspace{1cm} (J^{PC} = 1^{-+})

$$\mathcal{M}_d = \langle \delta_{ij} (\bar{p} \cdot \vec{\varepsilon}) - \varepsilon_i p_j \rangle \langle B | \sum_{n=1}^{N_c} g B^{i,x}(n) \sigma_n^i t_n^x \tau_n^a | B_h \rangle$$ \hspace{1cm} (33) \hspace{1cm} (J^{PC} = 1^{--})

$$\mathcal{M}_e = 2 \epsilon_{mqi} \varepsilon_{jq} p_m \langle B | \sum_{n=1}^{N_c} g B^{i,x}(n) \sigma_n^i t_n^x \tau_n^a | B_h \rangle$$ \hspace{1cm} (34) \hspace{1cm} (J^{PC} = 2^{-+}).

The matrix elements $\mathcal{M}_a$ and $\mathcal{M}_c$ are related in a simple way. They can be computed with the methods of [7] with the result

$$\langle I', m' \alpha' | \sum_{n=1}^{N_c} g B^{i,x}(n) \tau_n^a | (SJ_y)JI, m \alpha \rangle =$$

$$\frac{1}{\sqrt{6}} N_c (-)^{S-J+1+2I} \sqrt{(2I+1)(2J+1)} \frac{1}{2I' + 1} \langle 0 | g B \| \chi \rangle \delta_{S I'} \langle I' m' | J 1; m_i \rangle \langle I' \alpha' | I 1; \alpha a \rangle .$$

The reduced matrix element of the chromomagnetic field $\vec{B}^a$ is defined by

$$\int d\vec{r} \Phi^i(\vec{r}) \Phi(\vec{r}) \langle 0 | g B^a(\vec{r}) | \chi^n(J_y, m_y) \rangle = \langle 0 | g B \| \chi \rangle \langle 0 | J_y, 1; m_y, i \rangle \delta_{aa}$$

(36)

The remaining matrix elements (31), (33) and (34) can be expressed in terms of the following basic quantity.

$$\mathcal{M}_I^T = \langle T \ell | 11; ji | I', m' \alpha' | \sum_{n=1}^{N_c} g B^{i,x}(n) \sigma_n^j t_n^x \tau_n^a | (SJ_y)JI, m \alpha \rangle =$$

$$N_c (-)^{2J-I+S+T+1} \sqrt{(2I+1)(2J+1)(2S+1)(2T+1)} \left\{ \begin{array}{ccc} 1 & 1 & T \\ J & I' & S \\ 1 & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} 1 & 1 & 1 \\ S & I & I' \end{array} \right\} \times \langle 0 | g B \| \chi \rangle \langle I' m' | JT; m \ell \rangle \langle I' \alpha' | I 1; \alpha a \rangle .$$
The amplitude $\mathcal{M}_b$ is given by

$$\mathcal{M}_b = -i\sqrt{2}\sum_{\ell}(-)^\ell\mathcal{M}_t^{T=1}p^{-\ell}. \quad (38)$$

The matrix element (33) can be written most conveniently in terms of a tensor $t(T, \ell)$ defined by

$$t(T, \ell) = \langle T\ell|11; ij \rangle p^i\epsilon^j, \quad p^i\epsilon^j = \sum_{T,\ell} t(T, \ell) \langle T\ell|11; ij \rangle. \quad (39)$$

Then one has

$$\mathcal{M}_d = 2\mathcal{M}_t^{T=0}t(0) + \sum_{\ell}(-)^\ell\mathcal{M}_t^{T=1}t(1, -\ell) - \sum_{\ell}(-)^\ell\mathcal{M}_t^{T=2}t(2, -\ell). \quad (40)$$

The advantage of writing the decay amplitude in this form is that the angular distribution of the emitted vector mesons can be expressed in a simple form. For example, for a general amplitude

$$\mathcal{M}(\vec{p}, \vec{\epsilon}) = \sum_{T=0,1,2} c_T \sum_{\ell}(-)^\ell t(T, -\ell) \langle J'm'|JT; m\ell \rangle \quad (41)$$

one obtains the following spin-averaged angular distribution

$$\Gamma \approx \frac{1}{2J+1} \sum_{m, m'} |\mathcal{M}(\vec{p}, \vec{\epsilon})|^2 = \frac{2J' + 1}{2J+1} \sum_{T} \frac{|c_T|^2}{2T+1} \sum_{\ell} |t(T, \ell)|^2 \quad (42)$$

with

$$\sum_{\ell} |t(T, \ell)|^2 = \begin{cases} \frac{1}{3}|\vec{p} \cdot \vec{\epsilon}|^2 & (T = 0) \\
\frac{1}{2}|\vec{p} \cdot \vec{\epsilon}|^2 & (T = 1) \\
\frac{2}{3}|\vec{p} \cdot \vec{\epsilon}|^2 + \frac{1}{2}|\vec{p} \times \vec{\epsilon}|^2 & (T = 2) \end{cases} \quad (43, 44, 45)$$

It should be noted that the 3-dimensional scalar product $\vec{p} \cdot \vec{\epsilon}$ does not vanish for a longitudinally polarized vector meson.

Finally, the matrix element appearing in the amplitude (34) can be expressed in terms of (37) as

$$\langle B|\sum_{n=1}^{N_n} gB^{i,x}(\vec{r}_n)\sigma_n^j t^x_n \pi_n^a|B_h \rangle = \sum_{T,\ell} \mathcal{M}_t^{T} \langle T\ell|11; ij \rangle. \quad (46)$$

The amplitudes for decays into isoscalar hybrid mesons can be computed analogously with the help of the basic matrix elements.
\[ \langle I', m' \alpha' | \sum_{n=1}^{N_c} gB^i(\vec{r}_n) | (SJ_g)JI, m \alpha \rangle = 0 \]  \hspace{1cm} (47)

\[ \mathcal{M}_{\ell}^T = \langle T\ell|11; j i \rangle \langle I', m' \alpha' | \sum_{n=1}^{N_c} gB^{i,x}(\vec{r}_n) \sigma^j_n t^x_n | (SJ_g)JI, m \alpha \rangle = \]  \hspace{1cm} (48)

\[ N_c \frac{1}{\sqrt{6}} (-)^{T+1} \sqrt{\frac{(2J+1)(2S+1)(2T+1)}{2I'+1}} \left\{ \begin{array}{ccc} 1 & 1 & T \\ J & I' & S \end{array} \right\} \langle 0 \| gB \| \chi \rangle \times \delta_{JJ'} \delta_{\ell_1} \delta_{\alpha \alpha'} . \]  \hspace{1cm} (49)

After dividing by the decay constant \( f_{M_h(M)} \propto N_c^2 \), the quark model calculation shows that the width of these states due to the decay mode \( B_h \to B + M_h(M) \) is of order \( N_c^0 \) in the large-\( N_c \) limit. This would be true if the mass difference \( M_{B_h} - M_B \) was a constant in the same limit. Unfortunately, this turns out not to be the case due to a large mixing between the ordinary baryons and the hybrid baryons. This mixing appears first at order \( 1/m_Q \) and is induced by the chromomagnetic term in the nonrelativistic expansion.

The matrix element of this operator is given by

\[ \langle I'm'\alpha' | \frac{g}{2m_Q} \sum_{n=1}^{N_c} \vec{\sigma}_n \cdot \vec{B}(\vec{r}_n) | (SJ_g)JI, m \alpha \rangle = \]  \hspace{1cm} (49)

\[ \frac{1}{2m_Q} N_c \frac{1}{\sqrt{6}} (-)^{1+J+S} \sqrt{\frac{2S+1}{2J+1}} \delta_{JJ'} \delta_{\ell_1} \langle 0 \| gB \| \chi \rangle \delta_{mm'} \delta_{\alpha \alpha'} . \]  \hspace{1cm} (50)

We made use here of the result (48) (with \( T = 0 \)) for the matrix element of an isoscalar operator. The mixing amplitude grows with \( N_c \) like \( \sqrt{N_c}/m_Q \) which is to be contrasted with the corresponding meson-hybrid meson mixing which is of order \( N_c^0/m_Q \). Assuming the existence of only two such baryon states, it would appear that the physical states are given in the large \( N_c \) limit by the “ideal-mixing” combinations

\[ |B_{1,2} \rangle = \frac{1}{\sqrt{2}} (|B \rangle \pm |B_h \rangle) . \]  \hspace{1cm} (50)

An infinite mixing would also repel the states by an infinite amount, such that the mass separation \( M_{B_h} - M_B \) grows like \( N_c^2 \). A \( p \)-wave decay width will be of order \( N_c^2 \). We conclude thus that some of the ground state hybrids (6) are unstable in the large-\( N_c \) limit against \( 1/m_Q \) corrections (see the discussion following Eq. (75) in Sect. V). Whether this type of states exists for \( N_c = 3 \) is an open question which we cannot answer.

**B. Hybrid baryons with mixed symmetry orbital wavefunction**

The mixing of the states (10) with the ordinary baryons \( Q^3 \) induced at order \( 1/m_Q \) is parametrized by a matrix element analogous to (49)

\[ \langle I'm'\alpha' | \frac{g}{2m_Q} \sum_{n=1}^{N_c} \vec{\sigma}_n \cdot \vec{B}(\vec{r}_n) | (SJ_g)JI, m \alpha \rangle = \]  \hspace{1cm} (51)

\[ \frac{1}{2m_Q} \sqrt{\frac{2}{3}} \sqrt{I(I+1)} \delta_{JJ'} \delta_{\ell_1} \langle 0 \| gB \| \chi \rangle . \]  \hspace{1cm} (51)
One can see that the mixing amplitude of these states with the ordinary baryons vanishes in the large $N_c$ limit. Thus these states can be expected to be stable with respect to mixing in this limit, in contrast to the ground state hybrids (6).

The calculation of the couplings of the hybrid states (10) proceeds in an analogous way. Mesons couple to them with a strength of order $N_c^2$, just as to the states (5). The corresponding matrix element is

$$\langle I'J_g; m'_S\alpha'm'_g | JI_g; m_S\alpha m_g \rangle = \frac{1}{N_c(N_c^2-1)} \delta_{m_Sm_g} \sum_{n=1}^{N_c^2-1} \sum_{y=1}^{N_c} \langle I'm'_S\alpha' | \sum_{n=1}^{N_c} \sigma_n^i \tau_n^a | Im_Sm_g \rangle \langle \bar{\psi}_k^x \psi_k^y \delta_{kk'} \rangle =$$

$$\langle I'm'_S\alpha' | \sum_{n=1}^{N_c} \sigma_n^i \tau_n^a | Im_Sm_g \rangle = N_c \sqrt{\frac{2I+1}{2I'}+1} \langle I'm'_S|I1; m_Si\rangle \langle I'\alpha'|I1; \alpha a \rangle \delta_{m_Sm_g}.$$  \hspace{1cm} (52)

The decay amplitude of these hybrids into ordinary baryons plus (isovector) mesons are given by the following two basic matrix elements

$$\langle I', m'\alpha' | \sum_{n=1}^{N_c} gB^i(\bar{r}_n)\tau_n^a | JJ, m\alpha \rangle = \sqrt{\frac{2}{3}} \langle \bar{\psi}(\bar{r}, \tau) \psi \rangle \langle 0|gB|\chi \rangle \delta_{II'} \langle I'm'|J1; m\ell \rangle \langle I'\alpha'|I1; \alpha a \rangle \hspace{1cm} (53)$$

and

$$\mathcal{M}^e_I = \langle T|1i; ji \rangle \langle I', m'\alpha' | \sum_{n=1}^{N_c} gB^i(\bar{r}_n)\sigma_n^i \tau_n^a | JJ, m\alpha \rangle = \sqrt{\frac{1}{6}} N_c (-)^{2I-I'+T+1} \sqrt{\frac{(2I+1)(2J+1)(2T+1)}{2I'+1}} \left\{ \begin{array}{ccc} 1 & 1 & T \\ J & I' & I \end{array} \right\} \times \langle 0|gB|\chi \rangle \langle I'm'|JJ; m\ell \rangle \langle I'\alpha'|I1; \alpha a \rangle \hspace{1cm} (54)$$

The reduced matrix element of the chromomagnetic field $\vec{B}^a$ is defined for this case by

$$\int d\vec{r} \Psi^\dagger(\vec{r}) \Phi(\vec{r}) \langle 0|gB^a(\vec{r})|\chi^n(J_g,m_g) \rangle = \langle 0|gB|\chi \rangle \langle 0|J_g,1; m_g, i \rangle \delta_{an} \hspace{1cm} (55)$$

These results show that the decay widths of a hybrid baryon $\Psi_{B_h}^J$ into an ordinary baryon and an isovector meson with quantum numbers $J^{PC} = 0^{--},1^{--}$ are of order $1/N_c^2$ (see (52), (50) and (52)). On the other hand, the decays into mesons with quantum numbers $J^{PC} = 0^{--},1^{--},2^{--}$ are enhanced and have widths of order $N_c^0$ (see (54)). These couplings have the same $N_c$ scaling as the couplings of the orbitally excited baryons with mixed symmetry discussed in [7]. In the following section we will abstract these results of the quark model by keeping only their large $N_c$ scaling law to derive constraints on the spin-flavor structure of the couplings.
V. CONSISTENCY CONDITIONS FOR HYBRID BARYON COUPLINGS

In this section we derive a set of model-independent constraints on the hybrid baryons’ coupling to mesons using the method of consistency conditions (see [12] and references therein). These constraints arise from a mismatch in the large $N_c$ scaling laws for scattering amplitudes, computed in quark diagram and hadron diagram language respectively. For the latter one uses the scaling laws for vertices derived in the quark model, with an implicit assumption that they still hold if the quarks become light. We will study in the following the consistency conditions obtained from the scattering amplitude $B_h + M \rightarrow B + M_h$.

There are two different classes of diagrams contributing to this scattering amplitude to first order in $\alpha_s$. Typical diagrams are shown in Fig. 1. Using Eq. (1) and the wavefunction of a hybrid meson [23], the dependence on $N_c$ of these two contributions can be made explicit

\[
I_1 = \frac{\alpha_s}{N_c(N^2_c - 1)} \sum_{x,y=1}^{N^2_c-1} \sum_{i,j=1}^{N_c} \text{Tr} \left[ t^a \sqrt{2t^a} \right] \left[ \bar{\psi}_j t_i^a \psi_i^y \right] \left\langle \chi^x(M_h) \right| \chi^y(B_h) \right\rangle \mathcal{M}_1^1(p) \tag{56}
\]

\[
I_2 = \frac{\alpha_s}{N_c(N^2_c - 1)} \sum_{x,y=1}^{N^2_c-1} \sum_{i,j=1}^{N_c} \left[ \bar{\psi}_j t_i^a t_k^a \sqrt{2t^a} \psi_i^y \right] \left\langle \chi^x(M_h) \right| \chi^y(B_h) \right\rangle \mathcal{M}_2^2(p) \tag{57}
\]

\[
+ \sum_{i \neq k,j=1}^{N_c} \left[ \bar{\psi}_j t_i^a t_k^a \sqrt{2t^a} \psi_i^y \right] \left\langle \chi^x(M_h) \right| \chi^y(B_h) \right\rangle \mathcal{M}_3^3(p) \quad (j \neq k)
\]

where $\mathcal{M}^{1,2,3}_i(p)$ depend only on the momenta of the involved particles but not on $N_c$. In (57) one must sum only over terms with $j \neq k$ because only these diagrams can transfer momentum from the hybrid baryon to the meson. Furthermore, we took into account the fact that the momentum-dependent part of the diagram is different when the exchanged quark is identical or not with the external quark in the hybrid. For the hybrids (6) this difference appears because of the spin-flavor structure of the $|SI\rangle_i$ state and for the states (10) it is due to the different orbital wavefunction of the external $i^{th}$ quark.

The overlap of the gluonic wavefunctions is given by \[ \langle \chi^x(M_h) | \chi^y(B_h) \rangle = \mathcal{I}(p) \delta_{xy} \] with $\mathcal{I}(p)$ another function of the hybrid momenta. The expressions (56), (57) can be easily computed with the help of the relations (here $i$ is kept fixed)

\[
\sum_{j=1}^{N_c} \bar{\psi}_j t_i^a \psi_i^y = 0 \tag{58}
\]

\[
\sum_{j,k=1}^{N_c} \bar{\psi}_j t_i^a t_k^a \psi_i^y = \sqrt{\frac{2}{N_c}} \left( \frac{i}{4} f_{axy} - \frac{1}{4} d_{axy} \right) \quad (k = i, j \neq k) \tag{59}
\]

\[
= \sqrt{\frac{2}{N_c}} \left( -\frac{i}{4} f_{axy} + \frac{1}{4} d_{axy} \right) \quad (k \neq i, j \neq k)
\]

One finds

\[
I_1 = 0, \quad I_2 = -\frac{1}{2} \alpha_s \frac{N^2_c - 2}{N_c^2} \mathcal{I}(p) \sum_{i=1}^{N_c} \left( \mathcal{M}^2_i(p) - \mathcal{M}^3_i(p) \right) \simeq N_c^{-\frac{1}{2}}, \tag{60}
\]
where we took into account the scaling law for the strong coupling $\alpha_s N_c = \text{const.}$ and that each of the $N_c$ terms in the sum over $i$ is of order 1.

The same scattering amplitude can be expressed as a sum of hadronic diagrams with $B$ and $B_h$ appearing as intermediate states in the $s$-channel. The vertices in these graphs can be parametrized in terms of the following operators. In addition to $X^{ia}, Z^{ia}$ parametrizing pion couplings to the ground state [12] and hybrid baryons respectively, we introduce four new couplings defined as follows.

- $W^{ia}$ describes nondiagonal hybrid meson coupling between ordinary and hybrid baryons defined such that the vertex is given by a sum of terms of the form
  \[ -\frac{i}{M} t(T, \ell) N_c^\kappa (B|W^{ia}|B_h). \] (61)

- $X'^{ia}$ describes hybrid meson coupling to ordinary baryons.

- $Z'^{ia}$ describes hybrid meson coupling to hybrid baryons.

- $W'^{ia}$ describes nondiagonal ordinary meson coupling between ordinary and hybrid baryons, defined analogously to (61).

For generality, we assumed that the meson has an arbitrary spin $J$. Then the meson coupling in a $p$-wave can be written in terms of a tensor $t(T, \ell)$ with $T = J, J \pm 1$ constructed from the meson polarization vector and its momentum as in (52) for $J = 1$. The leading $N_c$ dependence of the vertex has been factored out, such that the $1/N_c$ expansion of $W^{ia}$ starts with a term of order 1. The value of $\kappa$ for different mesons and hybrid baryons can be extracted from the quark model calculations of Sect. IV.

In terms of these operators, the total scattering amplitude for $B_h + \pi(p) \rightarrow B + M_h(p')$ is written as

\[
\mathcal{T} = \frac{p \cdot t(T, j)}{f_\pi f_M} N_c^{1+\kappa} \left\{ \frac{1}{E(p)} \left( W^{jbt} Z^{ia} - X^{ia} W^{jbt} \right) + \frac{1}{E(p')} \left( X'^{jbt} W'^{ia} - W'^{ia} Z'^{jbt} \right) \right\}. \] (62)

For all cases considered in this paper (except for the coupling (53)), one has $\kappa \geq 0$. Imposing consistence with the quark diagram scaling law (50) requires that the operator relations in (62) vanish independently to leading order in $1/N_c$. We will be interested in the following only in the first of these relations, which will be used to constrain the coupling $W^{jbt}$

\[
W^{jbt}_0 Z^{ia}_0 - X^{ia}_0 W^{jbt}_0 = 0 \quad (63)
\]

\[
X'^{jbt}_0 W'^{ia}_0 - W'^{ia}_0 Z'^{jbt}_0 = 0. \quad (64)
\]

The solutions for $X^{ia}_0$ and $Z^{ia}_0$ are well known [12]. Their matrix elements on tower states are given by

\[
\langle J' I' \Delta'; m' \alpha'|X^{ia}_0 |JI \Delta; m\alpha\rangle =
\]

\[
g(X) \sqrt{(2I+1)(2J+1)} (-1)^{2I-J+I'-\Delta+1} \left\{ \begin{array}{ccc}
I' & 1 & I \\
J & \Delta & J'
\end{array} \right\} \delta_{\Delta\Delta'} \langle J'|m'|JI; mi\rangle \langle I'\alpha'|I1; \alpha a\rangle
\]

\[= \]
and analogous for $Z_{0}^{ia}$.

In the following we will solve the consistency conditions (63) for the matrix elements of $W^{ia}$. First, we parametrize the matrix elements of this operator on tower states in terms of reduced matrix elements $W, \bar{W}$ as

$$
\langle J' I' \Delta' (B); m' \alpha' | W_{0}^{ia} | J I \Delta (B_{h}); m \alpha \rangle = \langle J' I' \Delta' (B_{h}); m' \alpha' | W_{0}^{ia} | J I \Delta (B); m \alpha \rangle =
$$

$$
g(W) \sqrt{(2I + 1)(2J + 1)(-)^{J'+I'+J} W(J' I', JJ) \langle J' m' | JT; m \alpha \rangle \langle I' \alpha' | I1; \alpha a \rangle = (66)
$$

$$
g(W) \sqrt{(2I + 1)(2J + 1)(-)^{J'+I'+J} W(J' I', JJ) \langle J' m' | JT; m \alpha \rangle \langle I' \alpha' | I1; \alpha a \rangle .
$$

With this choice for the normalization factors, the reduced matrix elements satisfy the symmetry relation

$$
W(J' I', JJ) = \bar{W}(JJ, J'I') .
$$

It is easy now to transform the consistency conditions (63) into algebraic equations for the reduced matrix elements, by projecting them onto channels with well-defined spin $H$ and isospin $K$. We obtain

$$
\sum_{J'I'_{1}} (2I_{1} + 1)(2J_{1} + 1)(-)^{-J'+J_{1}} \left\{ \begin{array}{ccc}
I' & 1 & I_{1} \\
J_{1} & \Delta' & J'\end{array} \right\} \left\{ \begin{array}{ccc}
J & 1 & H \\
J' & T & J_{1}\end{array} \right\} \left\{ \begin{array}{ccc}
I & 1 & K \\
I' & 1 & I_{1}\end{array} \right\} W(J_{1} I_{1}, JJ) = (69)
$$

$$
= (-)^{I_{1}-2J+K+\Delta'+\Delta+T-1} \left\{ \begin{array}{ccc}
K & 1 & I \\
J & \Delta & H\end{array} \right\} W(J'I', HK).
$$

The solution of this equation can be written directly in analogy to Eq. (3.59) in [7] and is given by a sum of three $9j$ symbols with undetermined coefficients.\footnote{There is an additional arbitrariness of this solution, manifested in the possible presence of a phase factor $(-)^{2Jn_{1}+2Jn_{2}}$, with $n_{1}, n_{2}$ integers.}

$$
W(J'I', JJ) = \sum_{y=T,T\pm1} c_{y}(\Delta', \Delta) \left\{ \begin{array}{ccc}
\Delta' & I' & J' \\
\Delta & I & J \\
y & 1 & T\end{array} \right\} .
$$

It is important to note that, while phrased in the language of hybrid baryons couplings, the consistency conditions (63) and their solution (70) have a wider validity. They give at the same time the general solution for couplings of mesons of arbitrary spin to excited and ground state baryons (assuming only that the large $N_{c}$ scaling of the vertices is such that the consistency conditions (63) still hold). Therefore the results of this paper extend the results of [7] to include the couplings of mesons with any spin.

In the following we quote the results for the coefficients $c_{y}(\Delta', \Delta)$ in the quark model, using the results of Sec. IV. This will illustrate the solution (70) on a few particular cases.

For final ground state baryons containing only $u, d$ quarks ($\Delta' = 0$), the solution (70) reduces to the following simpler expression containing only 6$j$ symbols.
\[ W(I', JI\Delta) = c_{\Delta} \frac{(-1)^{1+J+I'+\Delta}}{\sqrt{(2I'+1)(2\Delta+1)}} \left\{ \begin{array}{ccc} T & 1 & \Delta \\ I & J & I' \end{array} \right\}, \quad (\Delta = T, T \pm 1). \tag{71} \]

To compare this against the results of Sec. IV.A, the matrix elements of the latter must be transformed from the basis \( |(IP)S, J_g; J\rangle \) to the basis \( |(PJ_g)\Delta, I; J\rangle \). The connection between the two is a simple recoupling relation (Eq. (3.23) in [7])

\[ |(PJ_g)\Delta, I; J\rangle = (-)^{-I-J} \sum_S \sqrt{(2S+1)(2\Delta+1)} \left\{ \begin{array}{ccc} I & 1 & S \\ 1 & J & \Delta \end{array} \right\} |(IP)S, J_g; J\rangle. \tag{72} \]

The quark model matrix element (35) for hybrid baryon decays into mesons with \( J^{PC} = 0^{-+}, 1^{--} \) gives, when expressed in the basis (72),

\[ W(I', JI\Delta) = \frac{N_c}{\sqrt{6}}(-)^{J-I'+\Delta} \sqrt{\frac{2\Delta+1}{2I'+1}} \left\{ \begin{array}{ccc} I & 1 & I' \\ 1 & J & \Delta \end{array} \right\} \langle 0|gB|\chi \rangle \tag{73} \]

which can be seen to have the form (71) with \( T = 1 \). In a similar way, the matrix elements (37) can be transformed to the \( |(PJ_g)\Delta, I; J\rangle \) basis with the result

\[ W(I', JI\Delta) = \frac{N_c}{\sqrt{2}}(-)^{J-I'+\Delta+1} \sqrt{\frac{(2\Delta+1)(2T+1)}{2I'+1}} \left\{ \begin{array}{ccc} \Delta & T & 1 \\ 1 & 1 & 1 \end{array} \right\} \left\{ \begin{array}{ccc} I & 1 & I' \\ T & J & \Delta \end{array} \right\} \langle 0|gB|\chi \rangle. \tag{74} \]

This takes also the form (71).

The structure of the mixing \( B - B_h \) induced by the chromomagnetic term (49) takes on a more transparent form in the basis of the states \( |(PJ_g)\Delta, I; J\rangle \). One obtains for the mixing amplitude the result

\[ \langle I'm'\alpha'|g_{2mQ} \sum_{n=1}^{N_c} \vec{\sigma}_n \cdot \vec{B}(\vec{r}_n) |(PJ_g)\Delta, I; Jm\alpha \rangle = \frac{1}{\sqrt{2}} \frac{1}{2mQ} \langle 0|gB|\chi \rangle \delta_{\Delta 0} \delta_{II'} \delta_{m'm} \delta_{\alpha'\alpha}, \tag{75} \]

which shows that only the members of the \( \Delta = 0 \) tower mix with the ground state ordinary baryons. Therefore only the hybrid baryons with symmetric orbital wavefunction (6) with \( \Delta = 0 \) can be considered as being ill-behaved in the large-\( N_c \) limit. The remaining towers (8) and (9) with \( \Delta = 1, 2 \) are not affected by this anomaly.

The matrix element (53) for the hybrid baryons with mixed symmetry orbital wavefunction cannot be expressed as (71). This is easily explained by noting that the \( N_c \) scaling of this particular coupling is not strong enough so that it should satisfy a consistency condition (63). Therefore the result (53) does not receive a model-independent justification in the large-\( N_c \) limit and must be regarded as a mere consequence of the quark model. The same is true for the matrix element of the chromomagnetic moment responsible for the mixing \( B - B_h \) for the states \( \Psi_{B_h} \) (51). However, the coupling (54) does grow fast enough with \( N_c \) so that the consistency conditions constrain it. We obtain for this matrix element

\[ W(I', JI\Delta) = \frac{1}{\sqrt{6}} \frac{1}{\sqrt{2}} \frac{1}{2mQ} \langle 0|gB|\chi \rangle \frac{(-)^{1+J+I'+\Delta}}{\sqrt{(2I'+1)(2\Delta+1)}} \left\{ \begin{array}{ccc} T & 1 & \Delta \\ I & J & I' \end{array} \right\} \tag{76} \]

which has precisely the form (71) with \( \Delta = 1 \).
VI. CONCLUSIONS

We have studied in this paper the existence and properties of hybrid baryons from the point of view of the large-$N_c$ expansion. Our arguments are limited to the case of baryons made up of heavy quarks, for which the constituent picture is exactly valid. For this case a nonrelativistic Hartree description for quarks of these states can be introduced, similar to the one commonly used for ordinary baryons in the large $N_c$ limit \cite{9}. Two types of hybrid baryons are discussed, the ground state hybrids \cite{6} and hybrids with mixed symmetric orbital wavefunction \cite{10}. Their spin-flavor structure is similar to that of ordinary orbitally excited baryons and ground state baryons respectively. The variational equation for the Hartree wavefunction turns out to coincide, in the large $N_c$ limit, with the one corresponding to the ordinary baryons. This proves the existence of hybrid baryons with heavy quarks in the large $N_c$ limit.

Using the Hartree picture one can study static properties of the hybrid baryons such as their mass and mixing with ordinary baryons. A surprising result concerns the large mixing amplitude of the ground state hybrid baryons with the ordinary baryons. A similar observation has been made in \cite{3} in the framework of the bag model. These authors find that for $N_c = 3$ the hybrid baryons mix strongly with the ground state baryons. In contrast, the mixing of the hybrids with mixed symmetry orbital wave functions \cite{10} is suppressed in the large $N_c$ limit. While the applicability of these large $N_c$ arguments to the real world is probably rather limited, this result suggests that the states of the $\Delta = 0$ tower of hybrids might be very broad (as discussed in the main text) and thus difficult to observe.

The scalings laws of the hybrid baryons’ couplings can be easily determined with the help of their Hartree wavefunctions. These are used in turn to write down consistency conditions for the couplings, similar to those introduced for determining the pion couplings of baryons by Dashen, Jenkins and Manohar \cite{12}. These consistency conditions are solved explicitly and their solutions are shown to coincide with the quark model result for these couplings.

In principle these results could be used to distinguish between positive parity hybrid baryons and radial excitations of the ordinary baryons (Roper resonances) by comparing their strong decay amplitudes into ordinary baryons and mesons. The dependence of these two amplitudes on the quantum numbers of the respective states is different: Eqs. (71) for the former and (53) for the latter respectively. Unfortunately, at present the practical power of this method is likely to be rather limited. Indeed, in order to use our predictions, each of these states has to be unambiguously assigned to one large $N_c$ tower (as the solutions (71), (53) depend on the tower quantum number $\Delta$). This, in turn, would be possible only if all, or at least most of the expected hybrid states have been identified. For the negative parity baryons this assignment is relatively easy and has been presented in \cite{8}. At the present moment this is far from being accomplished for the positive parity states. One can only hope that with improved experimental data, the results of this paper will be put to test in a not too distant future.

ACKNOWLEDGMENTS

The research of D.P. has been supported by the Ministry of Science and the Arts of Israel. The work of C-K.C. and T.M.Y. was supported in part by the National Science Foundation.
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FIG. 1. Typical diagrams showing color flows for scattering amplitudes \( B_h + M(p) \rightarrow B + M_h(p') \) to order \( \alpha_s \).