Sensitivity limits on heavy-light mixing $|U_{\mu N}|^2$ from lepton number violating $B$ meson decays

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We consider the lepton number violating decays $B \to \mu^{\pm} \mu^{\pm} \pi^{\mp}$ and $B \to D^{(*)} \mu^{\pm} \mu^{\pm} \pi^{\mp}$ which may be detected at LHCb and Belle-II experiments; and $B \to \mu^{\pm} e^{\mp} \nu$ and $B \to D^{(*)} \mu^{\pm} e^{\mp} \nu$ decays which may be detected at Belle-II experiment. The projected total number of produced $B$ mesons is $4.8 \times 10^{12}$ at LHCb upgrade and $5 \times 10^{10}$ at Belle-II. For the case that the above decays are not detected, we deduce the new upper bounds (sensitivity limits) for the mixing parameter $|U_{\mu N}|^2$ of heavy sterile neutrino with sub-eV light neutrino, as a function of the sterile neutrino mass in the interval $1.75 \text{ GeV} < M_N < 5.0 \text{ GeV}$. We take into account the probability of decay of the sterile neutrino $N$ within the detector, taking as the effective detector length $L = 2.3 \text{ m}$ at LHCb upgrade and $L = 1 \text{ m}$ at Belle-II. In the interval $1.75 \text{ GeV} < M_N < 3 \text{ GeV}$, the most stringent bounds can be obtained with the decays $B \to \mu^{\pm} \mu^{\pm} \pi^{\mp}$ at LHCb upgrade. The sensitivity limits are expected to be in general more stringent at LHCb upgrade than at Belle-II, principally because the number of produced $B$ mesons in LHCb upgrade is expected to be by about two orders of magnitude larger than at Belle-II. We conclude that the LHCb upgrade and Belle-II experiments have the potential to either find a new heavy Majorana neutrino $N$, or to improve significantly the sensitivity limits (upper bounds) on the heavy-light mixing parameter $|U_{\mu N}|^2$, particularly in the mass range $1.75 \text{ GeV} < M_N < 3 \text{ GeV}$. This work is a continuation and refinement of our previous work [1] on the subject.

Keywords: rare meson decays; sterile neutrino; mixing parameters of sterile neutrino and sub-eV neutrino

I. INTRODUCTION

The existence of sterile neutrinos has not been proven yet. However, their existence is suggested by various scenarios which can explain the detected differences of masses of the three known light neutrinos. Furthermore, most of such scenarios suggest that the neutrinos are Majorana fermions. Since Majorana fermions, unlike the Dirac fermions, are their own antiparticles, they can participate not just in the lepton number conserving (LNC) processes, but also in the lepton number violating (LNV) processes. LNV processes are appreciable if the Majorana neutrinos are sufficiently massive. Various scenarios suggest that mixing of sterile neutrinos with the known Standard Model (SM) flavor neutrinos leads to neutrinos which are significantly heavier than the known light neutrinos. The main questions facing the neutrino physics beyond the SM are: (1) Are the neutrinos Majorana or Dirac? (2) How heavy are the new mass eigenstates $N$? (3) What are the values of the heavy-light mixing parameters $U_{\ell N}$, i.e., the mixing parameters of a massive $N$ neutrino with the SM flavor neutrinos $\nu_{\ell}$ ($\ell = e, \mu, \tau$)?

Whether the neutrinos are Majorana particles can be determined in neutrino experiments with various LNV processes. Among the most known such experiments are those with the neutrinoless double beta decay ($0\nu\beta\beta$) [2], rare LNV decays of mesons [3, 11] and of $\tau$ lepton [12, 13], and specific scattering processes [14, 17].

Observation of neutrino oscillations [18] can determine (small) mass differences between neutrinos, and thus prove that the neutrinos have mass. The neutrino oscillations of the SM flavor neutrinos have been observed [19, 21]. If sterile neutrinos exist and if their mixing with the SM flavor neutrinos leads to almost degenerate heavy neutrinos, also such neutrinos can oscillate among themselves [22, 23].

The neutrino sector can also have CP violation [24], which plays an important role in the leptogenesis [25]. Resonant CP violation of neutrinos appears when we have two heavy almost degenerate neutrinos. It can appear in scattering processes [26], in semileptonic rare meson decays [9, 27, 28], and in purely leptonic rare meson decays [8, 9]. Among the models with almost degenerate heavy neutrinos are the neutrino minimal standard model (νMSM) [29, 30] and low-scale seesaw models [31].

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As mentioned, extended sectors of Majorana neutrinos appear in models which explain the very small masses of the three light neutrinos. Such models are the original seesaw models [32] (the heavy neutrinos there have masses $M_N \gg 1$ TeV), and seesaw models with heavy neutrinos with lower masses $M_N \sim 1$ TeV [33], and $M_N \sim 1$ GeV [15, 29, 34–37]. In such models, the heavy-light mixing parameters are in general less suppressed than in the original seesaw models.

In this work, we will work in a generic framework where we have one massive neutrino $N$ which mixes with the SM flavor neutrinos $\nu_\ell$ ($\ell = e, \mu, \tau$). We will evaluate the rates of some rare decays of $B$ mesons at the future LHCb upgrade and Belle-II experiments, namely, the LNV decays with one on-shell Majorana massive neutrino $N$: $B \to (D^{(*)})\ell^\pm N \to (D^{(*)})\mu^\pm \mu^\pm X^\mp$, where $X^\mp$ is either a pion $\pi^\mp$, or a lepton-neutrino pair $\ell \nu_\ell$ (this latter option only at Belle-II). This work is based on our previous work [1], but now the obtained results are more specific and directly applicable to the calculation of the sensitivity limits on the $|U_{\mu N}|^2$ mixing parameter, as a function of mass $M_N$, achievable at LHCb upgrade and at Belle-II, where the projected total number of produced $B$ mesons is $4.8 \times 10^{12}$ [39] and $5 \times 10^{10}$ [38], respectively. Unlike in Ref. [1], here we do not make any assumptions on the size of the probability $P_N$ of the produced neutrino $N$ to decay within the detector (in [1] we assumed that either $P_N \approx 1$ or $P_N \ll 1$). Detailed explanation on this issue is given in Sec. III and in Appendix A.

Similar analyses for the upper bounds on $|U_{\mu N}|^2$ from the absence of the rare $B$-meson decays were made for the Belle-I measurements in Ref. [40], and for LHCb (run I) measurements in Refs. [41] and reconsideration thereof in Ref. [42].

In Sec. II we summarize the framework in which we work, and the decay widths which are relevant for the decay rates that we want to obtain. The summarized formulas for these decay widths are presented in subsections of Sec. II and Appendix A. In Sec. III we present the probability $P_N$ of the produced on-shell neutrino $N$ to decay within the detector, and the integration formulas which account for the effect of this probability on the effective rate for the mentioned LNV decays. In Appendix B we present detailed formulas for the Lorentz factors and the probabilities $P_N$ for the various considered decays. In Sec. IV we present the results of the numerical evaluations, in the form of the obtained sensitivity limits on $|U_{\mu N}|^2$, as a function of $M_N$, that can be achieved by LHCb upgrade and Belle-II experiments. In Sec. V we discuss the obtained results and make conclusions.

II. DECAY WIDTHS FOR $B \to (D^{(*)})\ell^1 N \to (D^{(*)})\ell_1 \ell_2 X$

Here we briefly summarize the results of Ref. [1] for the decay widths of the rare decays of $B$ mesons via on-shell sterile neutrino $N$. The on-shellness of $N$ implies the factorization

$$\Gamma \left( B \to (D^{(*)})\ell^1 N \to (D^{(*)})\ell_1 \ell_2 X \right) = \Gamma \left( B \to (D^{(*)})\ell^1 N \right) \frac{\Gamma(N \to \ell_2 X)}{\Gamma_N} . \quad (1)$$

Here, $\ell_j$ ($j = 1, 2$) are generical names for charged leptons; later we will use $\ell_1 = \ell_2 = \mu^\pm$. The second factor on the right-hand side of Eq. (1) represents the effect of the subsequent decay of the produced heavy on-shell neutrino $N$ into $\ell_2 + X$, where $X$ will be either a charged pion $\pi$, or a leptonic pair $\ell_3 \nu_3$.

The first factor in Eq. (1), $\Gamma \left( B \to (D^{(*)})\ell^1 N \right)$, is well known when no $D^{(*)}$ meson is produced; when $D^{(*)}$ is produced, this factor was obtained and evaluated in Ref. [1]. The formulas for this factor are summarized in subsections A-C, as well as some (here relevant) differential decay widths for these decays $B \to (D^{(*)})\ell^1 N$. The second factor in Eq. (1) includes the exclusive decay width $\Gamma(N \to \ell_2 X)$ which is well known, either for $X = \pi$ or $X = \ell_3 \nu_3$. For both cases, the expressions for these decay widths are summarized in subsections D-E. The denominator of the second factor in Eq. (1), namely the total decay width $\Gamma_N$ of neutrino $N$, was evaluated numerically in [27] for the case of Majorana $N$ (cf. also [30] for the case of $N$ Majorana or Dirac); the expression for $\Gamma_N$ and its evaluation is presented in Appendix A.

All the mentioned decay widths involve the (suppressed) heavy-light mixing parameters $U_{\ell N}$ ($\ell = e, \mu, \tau$) appearing in the coupling of the heavy $N$ neutrino with the $W$ boson and $\ell$ lepton. These parameters are part of the (extended) Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, i.e., the light flavor neutrino states $\nu_\ell$ (with flavor $\ell = e, \mu, \tau$) are the following combination of the three light mass eigenstates $\nu_k$ and of the heavy mass eigenstate $N$: \[ \nu_\ell = \sum_{k=1}^{3} U_{\ell k} \nu_k + U_{\ell N} N . \quad (2) \]
A. Decay width $\Gamma(B \to \ell_1 N)$

The decay width for the process $B \to \ell_1 N$, where $\ell_1$ is a charged lepton ($\ell_1 = e, \mu, \tau$) and $N$ is a (massive) neutrino, is

$$\Gamma(B^\pm \to \ell_1^\pm N) = |U_{\ell_1 N}|^2 T(B^\pm \to \ell_1^\pm N)$$

(3)

where the canonical decay width $\Gamma$, i.e., the part without the heavy-light mixing factor, is

$$\Gamma(B^\pm \to \ell_1^\pm N) = \frac{G_F^2 f_B^2}{8\pi} |V_{ub}|^2 M_B^2 \lambda^{1/2}(1, y_N, y_1) [(1 - y_N) y_N + y_1 (1 + 2y_N - y_1)]$$

(4)

Here, $G_F$ is the Fermi coupling constant ($G_F = 1.1664 \times 10^{-5}$ GeV$^{-2}$), $f_B$ is the decay constant of the $B$-meson, $V_{ub}$ its CKM matrix element, and in the mass dependent parts the following notations are used:

$$y_N = \frac{M_N^2}{M_B^2}, \quad y_1 = \frac{M_1^2}{M_B^2}$$

(5a)

$$\lambda^{1/2}(x, y, z) = [x^2 + y^2 + z^2 - 2xy - 2yz - 2zx]^{1/2}$$

(5b)

We denote the mass of $\ell_1$ as $M_1$ throughout this paper. We use the values $|V_{ub}| = 0.00409$ and $f_B = 0.1871$ GeV [56] (cf. also [57]).

B. Decay width $\Gamma(B \to D\ell_1 N)$

We now consider the decay $B \to D\ell_1 N$, cf. Fig. 1. For the general case of a massive neutrino $N$ (and a massive charged lepton $\ell_1$), the general expression for the decay width of the process $B \to D\ell_1 N$ was obtained in Ref. [1].

There, the differential decay width $d\Gamma(B^- \to D^0 \ell_1^- \bar{N})/dq^2$ was presented.

Here we present the “more differential” cross section $d\Gamma(B^- \to D^0 \ell_1^- \bar{N})/dq^2 d\Omega_d d\Omega_{\bar{N}}$, which is needed for calculation of the effective (true) branching ratio $Br_{\text{eff}}(B \to D\ell_1 N \to D\ell_1 \ell_2 X)$ of Eq. (36). The differential of the decay width is

$$d\Gamma(B^- \to D^0 \ell_1^- \bar{N}) = \frac{1}{2M_B (2\pi)^2} \frac{1}{d_3} |T|^2$$

(6)

where $d_3$ is the differential for the three-particle final phase space

$$d_3 = \frac{d^3 \bar{p}_D}{2E_D(p_D)} \frac{d^3 \bar{p}_1}{2E_{\ell_1}(\bar{p}_1)} \frac{d^3 \bar{p}_N}{2E_N(p_N)} \delta^{(4)}(p_B - p_D - p_1 - p_N)$$

$$= d_2 (B^- \to D^0(p_D) W^*(q)) dq^2 d_2(W^*(q) \to \ell_1(p_1) \bar{N}(p_N))$$

(7)
and the two-particle final phase space differentials are

\[
d_2(B^- \to D^0(p_D) W^*(q)) = \frac{1}{8} \lambda^{1/2} \left(1, \frac{M_D}{M_B}, \frac{q^2}{M_B^2} \right) d\Omega', \tag{8a}
\]

\[
d_2(W^*(q) \to \ell^-_1 (p_1) N(p_N)) = \frac{1}{8} \lambda^{1/2} \left(1, \frac{M_N^2}{M_N^2}, \frac{M_N^2}{q^2} \right) d\Omega_p, \tag{8b}
\]

The decay amplitude \( \mathcal{T} \) appearing in Eq. (6) is

\[
\mathcal{T} = U_{\ell_1 N} V_{cb} \frac{G_F}{\sqrt{2}} \left[ \bar{u}(\ell_1) (p_1) \gamma_\mu (1 - \gamma_5) u(N) (p_N) \right] \left\{ \left( 2p_D + q \right)^\mu - \frac{(M_B^2 - M_D^2)}{q^2} q^\mu \right\} F_1(q^2) + \frac{(M_B^2 - M_D^2)}{q^2} q^\mu F_0(q^2),
\]

where \( F_1(q^2) \) and \( F_0(q^2) \) are the form factors of the \( B \to D \) transition, and we consider them to be real.

In terms of the reduced canonical decay amplitude \( \tilde{\mathcal{T}} \) defined via the relation

\[
|\tilde{\mathcal{T}}|^2 = |U_{\ell_1 N}|^2 |V_{cb}|^2 G_F^2 |\mathcal{T}|^2,
\]

we can then express the differential decay width \( \Gamma_B \) in a somewhat more explicit form

\[
\frac{d\Gamma(B^- \to D^0 \ell^-_1 N)}{dq^2d\Omega_q^2d\phi_q} = \frac{|U_{\ell_1 N}|^2 |V_{cb}|^2 G_F^2}{4 M_B (4\pi)^3} |\tilde{\mathcal{T}}|^2 \lambda^{1/2} \left(1, \frac{M_B^2}{M_D^2}, \frac{q^2}{M_B^2} \right) \lambda^{1/2} \left(1, \frac{M_B^2}{q^2}, \frac{M_B^2}{q^2} \right),
\]

where \( \hat{p}_1 \) is the direction of \( \ell^-_1 \) in the \( W^* \)-rest frame (\( \Sigma \)), and \( \hat{q}' \) is the direction of \( W^* \to (\ell^-_1 N) \) pair in the \( B \)-rest frame (\( \Sigma' \)). We use the expression (10) for the decay amplitude, and calculate the square of its absolute magnitude, \( |\tilde{\mathcal{T}}|^2 \), summing over the helicities of the final particles. We then obtain for the square of the reduced canonical amplitude, \( |\tilde{\mathcal{T}}|^2 \), introduced via Eq. (10), the following expression:

\[
|\tilde{\mathcal{T}}|^2 = \frac{1}{q^2} F_1(q^2) (F_0(q^2) - F_1(q^2)) \left( M_B^2 - M_D^2 \right) \left[ M_B^2 \left( -4 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right) + 2 M_B^2 - 2 M_D^2 + 2 M_N^2 - q^2 \right]
+ M_N^2 \left( 4 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right) - M_N^2 + q^2 - M_B^2
+ \frac{1}{2} F_1(q^2)^2 \left( M_B^2 \left( 8 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right) - 4 M_B^2 - 2 M_D^2 + 3 q^2 \right) - 8 M_B^2 \left( 8 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right) - 8 M_B^2 \left( 8 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right) - 8 q^2 \left( 8 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right)
+ 16 \cos \theta_1 |p_D||\tilde{p}_N| + p_D^0 p_1^0 \right)^2 + M_N^4 + M_N^2 - M_N^2 q^2 \right]
+ \frac{1}{2(q^2)^2} (F_0(q^2) - F_1(q^2))^2 \left( M_B^2 - M_D^2 \right)^2 \left[ - M_N^4 + \left( 2 M_B^2 + q^2 \right) - M_N^4 + M_N^2 q^2 \right].
\]

Here, we denoted as \( p_1 \) the 4-momentum of \( \ell_1 \) (in \( W^* \)-rest frame \( \Sigma \)), and \( \theta_1 \) is the angle between \( \tilde{p}_1 \) and \( \hat{q} = \hat{q}' \). We also used in Eq. (12) the following quantities:

\[
|\tilde{p}_N| = |\tilde{p}_1| = \frac{1}{2} \sqrt{q^2} \lambda^{1/2} \left(1, \frac{M_B^2}{q^2}, \frac{M_N^2}{q^2} \right),
\]

\[
|\tilde{p}_D| = \frac{M_B^2}{2\sqrt{q^2}} \lambda^{1/2} \left(1, \frac{M_D^2}{M_B^2}, \frac{q^2}{M_B^2} \right) = \frac{M_B|q|}{\sqrt{q^2}},
\]

\[
p_1^0 = \frac{1}{2\sqrt{q^2}} (q^2 - M_N^2 + M_D^2),
\]

\[
p_0^D = \frac{1}{2\sqrt{q^2}} (M_B^2 - M_D^2 - q^2).
\]

They are all in the \( W^* \)-rest frame (\( \Sigma \)). We can see from these expressions that the absolute square of the reduced canonical amplitude, \( |\tilde{\mathcal{T}}|^2 \), and thus the differential decay width \( \Gamma_B \), depend only on the variables \( q^2 \) (square of the invariant mass of \( W^* \)) and on \( \cos \theta_1 \) [note: \( d\Omega = d\phi_1 d(\cos \theta_1) \)]. They are thus independent of the direction \( \hat{q}' \), i.e., of the direction of \( W^* \) in the \( B \)-rest frame.
The expressions \([12]\) and \([11]\) contain two form factors, \(F_1\) and \(F_0\). The form factor \(F_1(q^2)\) is well known \([15]\) and can be expressed in terms of a variable \(w(q^2)\)

\[
w = \frac{(M_B^2 + M_D^2 - q^2)}{2M_B M_D},
\]

\[
z(w) = \frac{\sqrt{w + 1} - \sqrt{2}}{\sqrt{w + 1} + \sqrt{2}}.
\]

According to Ref. \([45]\), \(F_1(q^2)\) has the following power expansion in \(z(w(q^2)):\)

\[
F_1(q^2) = F_1(w = 1) (1 - 8\rho^2 z(w) + (51\rho^2 - 10) z(w)^2 - (252\rho^2 - 84) z(w)^3).
\]

The free parameters \(\rho^2\) and \(F_1(w = 1)\) in this expansion have been determined by the Belle Collaboration, Ref. \([46]\)

\[
\rho^2 = 1.09 \pm 0.05,
\]

\[
|V_{cb}|F_1(w = 1) = 48.14 \pm 1.56 \times 10^{-3}.
\]

In our numerical evaluations we use the above central values, and \(|V_{cb}| = 40.12 \times 10^{-3}\) \([46]\).

The form factor \(F_0(q^2)\) is not well known at present, principally because it contributes only when the masses of \(N\) and \(\ell_1\) are not very small as can be deduced from Eq. \([12]\). In our case \(F_0(q^2)\) is important, and it was presented in Ref. \([11]\) by using the truncated expansion for \(F_0\) in powers of \(w(q^2) - 1\) of Ref. \([47]\)

\[
F_0(q^2) = \frac{(M_B + M_D)}{2\sqrt{M_B M_D}} \left[ 1 - \frac{q^2}{(M_B + M_D)^2} \right] f_0(w(q^2)),
\]

\[
f_0(w) \approx f_0(w = 1) \left[ 1 + \rho_0^2 (w - 1) + (0.72\rho_0^2 - 0.09)(w - 1)^2 \right].
\]

Here, we use the value \(f_0(w = 1) \approx 1.02\) \([47]\) \([48]\) which is obtained from the heavy quark limit. The other free parameter \(\rho_0\) in Eq. \([17b]\) is then fixed by requiring the absence of spurious poles at \(q^2 = 0\): \(F_0(0) = F_1(0) \approx 0.690\). This yields the value \(\rho_0^2 \approx 1.102\) and \((0.72\rho_0^2 - 0.09) \approx 0.704\).

For the curves of these form factors \(F_1(q^2)\) and \(F_0(Q^2)\), as a function of positive \(q^2\), we refer to Ref. \([11]\) (Fig. 2 there).

C. Decay width \(\Gamma(B \to D^*\ell_1N)\)

We now consider the decay \(B \to D^*\ell_1N\), i.e., the same type of decay as in the previous Sec. \([113]\) but now instead of the (pseudoscalar) \(D\) meson we have vector meson \(D^*\). The expressions for the (differential) decay widths are now more complicated, because \(D^*\) is a vector particle. For the case of massive neutrino \(N\) (and massive lepton \(\ell_1\)), these expressions were obtained in Ref. \([11]\), using the approach of Ref. \([49]\). The needed differential decay width, after summation over helicities and over the three polarizations of \(D^*\), turns out to be \([11]\)

\[
d\Gamma = \frac{1}{8\pi^3} \frac{|U_{\ell_1N}|^2 |V_{cb}| G_F^2}{M_B^2} \chi^{1/2} \sqrt{|q^2|} \left[ 2 \left( 1 - \frac{(M_N^2 + M_1^2)}{q^2} \right) - \chi \sin^2 \theta_1 \right] \Gamma_H + \Gamma_H^\prime
\]

\[
- \eta 2 \chi^{1/2} \cos \theta_1 \left( \Gamma_H + \Gamma_H^\prime \right) + 2 \left[ \left( 1 - \frac{(M_N^2 + M_1^2)}{q^2} \right) - \chi \cos^2 \theta_1 \right] \Gamma_H^\prime
\]

\[
+ 4 \left( \frac{M_N^2 - M_1^2}{q^2} \right) \chi^{1/2} \cos \theta_1 \Gamma_H^\prime + 2 \left[ \frac{(M_N^2 - M_1^2)}{q^2} + \frac{(M_N^2 + M_1^2)}{q^2} \right] \Gamma_H^\prime.
\]

Here, the factor \(\eta = \pm 1\) appears at one term proportional to \(\cos \theta_1\); \(\eta = +1\) if \(\ell_1^-\) is produced, and \(\eta = -1\) if \(\ell_1^+\) is produced. \(^2\) Further, the following notations are used:

\[
|\vec{q}| = \frac{1}{2} M_B \chi^{1/2} \left( 1, \frac{M_{D^*}^2}{M_B^2}, \frac{q^2}{M_B} \right),
\]

\[
\chi \equiv \lambda \left( 1, \frac{M_1^2}{q^2}, \frac{M_N^2}{q^2} \right).
\]

\(^1\) It can be checked that the difference \(\|\vec{T}^2 - \vec{T}^2\|/F_0 \to 0\) is zero when \(M_1 = M_N = 0\).

\(^2\) The quantity \([48]\) is written in Ref. \([11]\) in Eq. (C19) for the case \(\eta = -1\); the quantity \(d\Gamma/dq^2\) used there is independent of \(\eta\).
and $\bar{H}_{\pm 1}$, $\bar{H}^0$ and $\bar{H}^3$ are expressions containing the form factors $V$ and $A_j$ ($j = 0, 1, 2, 3$) appearing in the $B$-$D^*$ matrix elements

$$\bar{H}_{\pm 1} = (M_B + M_{D^*})A_1(q^2) \mp V(q^2) \frac{|q^2|2M_B}{(M_B + M_{D^*})}, \quad (20a)$$

$$\bar{H}^3 = \frac{M_B^2}{2M_{D^*} \sqrt{q^2}} \left[(M_B + M_{D^*})A_1(q^2) \left(1 - \frac{(q^2 + M_{D^*}^2)}{M_B^2}\right) - 4A_2(q^2) \frac{|q^2|^2}{(M_B + M_{D^*})}\right], \quad (20b)$$

$$\bar{H}^0 = \frac{M_B|q^2|}{M_{D^*} \sqrt{q^2}} \left[(M_B + M_{D^*})A_1(q^2) - (M_B - M_{D^*})A_2(q^2) + 2M_{D^*} (A_0(q^2) - A_3(q^2))\right]. \quad (20c)$$

The $A_3$ form factor is not independent, it is a linear combination of $A_1$ and $A_2$

$$A_3(q^2) = \frac{(M_B + M_{D^*})}{2M_{D^*}} A_1(q^2) - \frac{(M_B - M_{D^*})}{2M_{D^*}} A_2(q^2). \quad (21)$$

Among the other four form factors, three ($V, A_1$ and $A_2$) are well known, they were recently determined to a high precision \[50\] in terms of the parametrization of Ref. \[45\]

$$A_1(q^2) = \frac{1}{2} R_* (w + 1) F_+(1) \left[1 - 8\rho_*^2 z(w) + 53\rho_*^2 - 15\right] z(w) - 231\rho_*^2 - 91\right) z(w)^3], \quad (22a)$$

$$V(q^2) = \frac{2}{R^2(w + 1)} \left[R_1(1) - 0.12(w - 1) + 0.05(w - 1)^2\right], \quad (22b)$$

$$A_2(q^2) = \frac{2}{R^2(w + 1)} \left[R_2(1) + 0.11(w - 1) - 0.06(w - 1)^2\right]. \quad (22c)$$

The notation $R_* = 2\sqrt{M_B M_D}/(M_B + M_{D^*})$ is used here, and $w = w(q^2)$ and $z = z(w(q^2))$ are given in Eqs. \[14\] (with $M_D \rightarrow M_{D^*}$). The values of the three parameters in Eqs. \[22\] were determined in Ref. \[20\]

$$\rho_*^2 = 1.214(\pm 0.035), \quad 10^3 F_+(1) |V_{cb}| = 34.6(\pm 1.0), \quad (23a)$$

$$R_1(1) = 1.401(\pm 0.038), \quad R_2(1) = 0.864(\pm 0.025). \quad (23b)$$

We use the central values in the present work.

The form factor $A_0$, on the other hand, is not well known. It is relevant only if the masses of $N$ or $\ell_1$ are nonnegligible, which is the case here. Employing the heavy quark limit relations between $A_1$ and $A_2$, the relation \[21\] gives a relation between $A_2$ and $A_3$. Using this relation in the heavy quark limit relation $A_0 \approx A_2$, we then obtain the following approximation for the form factor $A_0$ in terms of $A_3$:

$$A_0(q^2) \approx A_3(q^2) \frac{- q^2}{2M_{D^*}(M_B + M_{D^*})} = \frac{(M_B + M_{D^*})^2}{2M_{D^*}(M_B + M_{D^*}) - q^2} \left(1 - \frac{(M_B - M_{D^*})}{M_B + M_{D^*}} A_2(q^2)\right) A_1(q^2), \quad (24)$$

This relation satisfies the relation $A_0(0) = A_3(0)$ which is obligatory since it reflects the absence of the pole at $q^2 = 0$ in the $B$-$D^*$ matrix elements. We refer for any further details on these points to Ref. \[1\].

**D. Decayment width for $N \rightarrow \ell^\pm \pi^\mp$**

The decay width $\Gamma(N \rightarrow \ell^\pm \pi^\mp)$ is proportional to the heavy-light mixing factor $|U_{\ell N}|^2$

$$\Gamma(N \rightarrow \ell^\pm \pi^\mp) = |U_{\ell N}|^2 \Gamma(N \rightarrow \ell^\pm \pi^\mp). \quad (25)$$

Here, the canonical decay width $\Gamma$ is (e.g., cf. Refs. \[6\] \[9\] \[23\] \[27\])

$$\Gamma(N \rightarrow \ell^\pm \pi^\mp) = \frac{1}{16\pi} |V_{u\ell}|^2 G_F^2 f^2 \left(\frac{M_N^2}{2} x_N^d \right) \left[1 - x_N - 2x_\ell - x_N (x_\pi - x_\ell)\right], \quad (26)$$

where $f_\pi (\approx 0.1304$ GeV) is the decay constant of pion, and we use the notations

$$x_N = \frac{M_\pi^2}{M_N^2}, \quad x_\ell = \frac{M_\ell^2}{M_N^2}. \quad (27)$$
E. Decay width for $N \rightarrow \ell_2\ell_3\nu$

If the heavy neutrino $N$ is produced by the decay $B \rightarrow (D^{(*)})\ell_i^+N$, the neutrino can decay into various leptonic channels $\ell_2\ell_3\nu$. We can have the leptonic decays of $N$ of the lepton number conserving (LNC) type $N \rightarrow \ell_2^\pm \ell_3^\mp \nu_\ell$, and of the lepton number violating (LNV) type $N \rightarrow \ell_2^\pm \ell_3^\mp \nu_\ell$

$$\Gamma^{(LNC)}(N \rightarrow \ell_2^\pm \ell_3^\mp \nu_\ell) = |U_{\ell_2N}|^2 \Gamma(N \rightarrow \ell_2\ell_3\nu),$$

$$\Gamma^{(LNV)}(N \rightarrow \ell_2^\pm \ell_3^\mp \nu_\ell) = |U_{\ell_2N}|^2 \Gamma(N \rightarrow \ell_2\ell_3\nu).$$

Here, the charged leptons can be $\mu, e$ or $\tau$. The canonical decay widths $\Gamma(N \rightarrow \ell_2\ell_3\nu)$ have in the general case (with masses of leptons) the following form [6, 8, 9]:

$$\Gamma(N \rightarrow \ell_2\ell_3\nu) = \frac{G_F^2 M_N^2}{192\pi^3} \mathcal{F}(x_2, x_3),$$

where we denoted $x_j = M_j^2/M_N^2$ ($M_j$ is the mass of $\ell_j$), and the function $\mathcal{F}$ is [8]

$$\mathcal{F}(x_2, x_3) = \left\{ \begin{array}{c}
\lambda^{1/2}(1, x_2, x_3) [(1 + x_2)(1 - 8x_2 + x_2^2) - x_3(7 - 12x_2 + 7x_2^2)] \\
-7x_2^2(1 + x_2) + x_3^2 - 24(1 - x_2^2)x_2^2 \ln 2 \\
+ 12 \left[ -x_2^2(1 - x_3^2) \ln x_2 + (2x_2^2 - x_2^3(1 + x_2^2)) \ln(1 + x_2 + \lambda^{1/2}(1, x_2, x_3) - x_3) \\
+ x_2^2(1 - x_2^2) \ln \left( \frac{(1 + x_2)^2 + (1 - x_2)\lambda^{1/2}(1, x_2, x_3) - x_3(1 + x_2)}{x_3} \right) \right] \right\}. \quad (30)
$$

The function $\mathcal{F}$ is symmetric under the exchange of the two arguments. When one lepton is massless (or almost massless, i.e., lepton $e$), this expression reduces to the well-known result

$$\mathcal{F}(x, 0) = \mathcal{F}(0, x) = f(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x. \quad (31)$$

III. DECAY PROBABILITY OF HEAVY NEUTRINO IN THE DETECTOR; EFFECTIVE BRANCHING RATIO

If all the neutrinos $N$ decay within the detector with probability one, then the decay width Eq. (1) is also the effective (true) decay width, and the effective branching ratio is obtained by dividing it by the decay width of the $B$ meson $\Gamma_B$. However, since the neutrino $N$ is weakly coupled to SM particles, it often does not decay within the detector and, consequently, the mentioned decays $B \rightarrow (D^{(*)})\ell_i^+N$ are not observed although $N$ may be produced in the $B$-decays. The effect of the decay of $N$ can be accounted for by multiplying the above decay width Eq. (1) by the decay (nonsurvival) probability $P_N$ of $N$ within the detector

$$P_N = 1 - \exp \left[ -\frac{L}{\tau_N \gamma_N \beta_N} \right] = 1 - \exp \left[ -\frac{L\Gamma_N}{\gamma_N \beta_N} \right], \quad (32)$$

where $L$ is the maximum possible flight length of $N$ within the detector, $\beta_N$ is the velocity of $N$ in the lab frame, $\tau_N = 1/\Gamma_N$ is the lifetime of $N$ in its rest frame, and $\gamma_N = (1 - \beta_N^2)^{-1/2}$ is the Lorentz time dilation factor [7, 8, 10, 27, 51, 53].

For Belle-II, the $B$ meson pairs will be produced in SuperKEKB in central collisions of $e^-(p_1)$ and $e^+(p_2)$, which will produce a moving $\Upsilon(4S)$, the latter decaying into a $B$ meson pair (either $B^+B^-$ or $B^0\bar{B}^0$). In the lab frame, the $e^\pm$ have the momenta

$$p_j = (E_j, 0, 0, (-1)^{j+1}E_j) \quad (j = 1, 2), \quad (33)$$

with the values $E_1 = 7.007$ GeV and $E_2 = 3.993$ GeV. This then produces the invariant mass $(p_1 + p_2)^2 = M_{\Upsilon(4S)}^2$, where $M_{\Upsilon(4S)} = 10.579$ GeV [43]. The kinetic energy of the produced $\Upsilon(4S)$ is $K_\Upsilon = E_1 + E_2 - M_{\Upsilon(4S)} = 0.421$ GeV, which is semirelativistic, leading to the Lorentz factor in the lab frame

$$\gamma_\Upsilon = \frac{E_1 + E_2}{M_{\Upsilon(4S)}} = 1.0398 \Rightarrow \beta_\Upsilon = (1 - 1/\gamma_\Upsilon^2)^{1/2} = 0.274. \quad (34)$$
When Υ(4S) produces $B$ meson pair, the kinetic energy of the produced $B$ mesons is about 0.010 GeV in the Υ(4S)-rest frame, which is negligible. Therefore, we consider the velocity of the produced $B$ mesons in the lab frame to be the same as the velocity of Υ(4S)

$$\beta_B = \beta_\Upsilon = 0.274, \quad \gamma_B = \gamma_\Upsilon = 1.0398, \quad (p_B)_{\text{lab}} = M_B\beta_B\gamma_B = 1.504 \text{ GeV}. \quad (35)$$

In the decays $B \to D^{(*)}\ell_1N$, we will denote the rest frame of the off-shell $W^*$ (i.e., of $\ell_1N$ pair) as $\Sigma'$; the $B$-rest frame as $\Sigma''$; the laboratory frame as $\Sigma'''$. With these notations, the effective (true) branching ratio is calculated

$$\text{Br}_{\text{eff}}(B \to D^{(*)}\ell_1N \to D^{(*)}\ell_1\ell_2X) = \int dq^2 \int d\Omega_{q'} \int d\Omega_{p_1} \frac{d\Gamma(B \to D^{(*)}\ell_1N) \Gamma(N \to \ell_2X)}{dq^2d\Omega_{q'}d\Omega_{p_1} \Gamma_{N}\Gamma_B} \times \left\{ 1 - \exp \left[ \frac{-L\Gamma_N}{\sqrt{(E_N''(q^2; q', p_1)/M_N)^2 - 1}} \right] \right\}, \quad (36)$$

where in the denominator inside the exponent we have the Lorentz factor

$$\beta_N''\gamma_N'' = \sqrt{(E_N''(q^2; q', p_1)/M_N)^2 - 1}, \quad (37)$$

in the laboratory frame, which is a function of $W^*$ ($= \ell_1N$) momentum $q'$ (in the $B$-rest frame) and of the direction $p_1$ of the momentum $p_1$ of the produced charged lepton $\ell_1$ (in the $W^*$-rest frame). The expression (37) as an explicit function of $q'^2$, $q'$ and $p_1$ is derived in Appendix B. It depends on the angle $\theta_q$ between the direction of $\hat{\beta}_B$ (in the $W^*$-rest frame) and $\hat{q}'$ of $W^*$ (in the $B$-rest frame $\Sigma'$), as well as on the spherical angles $\theta_1$ and $\phi_1$ of the vector $\hat{p}_1$ of $\ell_1$ in the $W^*$-rest ($\Sigma'$) frame, in a specific 3-dimensional system of coordinates in the frame $\Sigma'$ (cf. Eq. [6][B] in Appendix B). On the other hand, the differential decay width $d\Gamma(B \to D^{(*)}\ell_1N)/(dq^2d\Omega_{q'}d\Omega_{p_1})$ depends only on $q'^2$ and $\theta_1$, as shown in subsections II B IC. Due to the mentioned dependence in the decay (nonsurvival) factor $P_N$, integration over these momenta is needed, as indicated in Eq. (36). The differential decay widths $d\Gamma(B \to D^{(*)}\ell_1N)/(dq^2d\Omega_{q'}d\Omega_{p_1})$ are given in subsections II B IC. All this implies that the integration Eq. (36) has the following form:

$$\int_{(M_B-M_{D^{(*)}})^2}^{(M_B-M_N)^2} dq^2 2\pi \int_{-1}^{+1} d(cos\theta_q) \int_{-1}^{+1} d(cos\theta_1) \int_{0}^{2\pi} d\phi_1 f(q'^2, \theta_q, \theta_1, \phi_1). \quad (38)$$

If no mesons $D^{(*)}$ are produced in the decays, then the differential decay width is even simpler, as it depends only on the direction $\hat{p}_N'$ of the on-shell $N$ in the $B$-rest frame, and the expression (36) simplifies

$$\text{Br}_{\text{eff}}(B \to \ell_1N \to \ell_1\ell_2X) = \int d\Omega_{p_1} \frac{d\Gamma(B \to \ell_1N) \Gamma(N \to \ell_2X)}{\Gamma_{N}\Gamma_B} \times \left\{ 1 - \exp \left[ \frac{-L\Gamma_N}{\sqrt{(E_N''(p_{1}/M_N)^2 - 1}} \right] \right\}. \quad (39)$$

The differential decay width is $d\Gamma(B \to \ell_1N)/d\Omega_{p_{1}'} = \Gamma(B \to \ell_1N)/(4\pi)$ since $B$ is a pseudoscalar, and the expression of $\Gamma(B \to \ell_1N)$ is given in subsection II A. The nonsurvival probability $P_N$ is in the case of Eq. (39) also simpler, because it (and the energy of $N$ in the lab frame, $E_N''(p_{1}')$) depends only on the direction $\hat{p}_N'$ of $N$ in the $B$-rest frame. The expression $E_N''(p_{1}')$ is given in Appendix B.

On the other hand, in the LHCb experiment, the entire procedure described in this Section, designed for a given momentum $(p_B)_{\text{lab}} \equiv p_B'$ of $B$ in the laboratory frame (cf. Eq. [35][B] for Belle-II where $p_B = 1.504 \text{ GeV}$), has to be repeated for various values of momenta $p_B''$. The obtained effective branching ratios then have to be averaged over these momenta $p_B''$. We took into account that the lab momentum $p_B'$ of the produced $B$ mesons in LHCb is distributed over a large interval, cf. the shaded curve in Fig. [2][B]. We separated this distribution in ten bins of equal weight (equal number of events), cf. Fig. [2][B], and calculated the results of Figs. [3][a]-(d) by averaging over these ten bins. For each bin, we took in our evaluations the value of the $B$ meson momentum to be such that, within the bin interval, the number of events to the left and to the right of it [according to the shaded curve of Fig. [2][a]] are equal; e.g., in the last bin, 223 GeV $< p_N < 403$ GeV, the average momentum value taken is $p = 273$ GeV.

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3 Note that $q'^2 = q^2$ is frame independent.

4 We thank Sheldon L. Stone (LHCb Collaboration) for providing us with the distribution, from Ref. [35][B], appearing here as Fig. 2[a].
IV. NUMERICAL RESULTS FOR SENSITIVITY LIMITS ON $|U_{\mu N}|^2$ AT LHCb UPGRADE AND BELLE-II

We assume that in the considered decays, the produced on-shell neutrino $N$ has the available length of $L = 1$ m for flight within the detector, at Belle-II and $L = 2.3$ m at LHCb upgrade.\footnote{This length $L$ is considered here to be independent of the position of the vertex where $N$ is produced and independent of the direction in which the produced $N$ travels. It can be called here the effective detector length for the neutrino $N$. In the case of LHCb, the length of the Vertex Locator (VELO) is about 1 m \cite{53}; the effective detector length could be extended beyond that locator, to $L = 2.3$ m \cite{39} \cite{56}.} We consider that at Belle-II, the total number of $5 \times 10^{10}$ $B$-mesons will be produced \cite{38}, and at LHCb upgrade this number will be about $4.8 \times 10^{12}$ \cite{39}. We assume that there are no background events for the considered lepton number violating (LNV) decays $B \to D^{(*)}\mu^+\mu^-N \to D^{(*)}\mu^+_0\mu^-0\pi^\mp$; and $B^{\pm} \to \mu^\mp N \to \mu^\mp\mu^\pm X^\mp$. Here, $X^\pm$ stands either for $\pi^\pm$ (LHCb and Belle-II), or the lepton pair $e^\pm\nu_e$ (Belle-II), and $B$ stands for $B^0$, $B^0$ or $B^\pm$. In these events, we have no QED background because no $\mu^+\mu^-$ pairs appear in the final states.

The effective branching ratios of the mentioned decay modes depend crucially on the heavy-light mixing parameter $|U_{\mu N}|^2$. The sensitivity limit on $|U_{\mu N}|^2$ at 95 % confidence limit is obtained for $N_{\text{events}} = 3.09 \times10^7$. Therefore, the sensitivity limits on $|U_{\mu N}|^2$ are obtained by requiring $\langle \text{Br}_{\text{eff}} \rangle = 3.09/(4.8 \times 10^{12})$ at LHCb upgrade, and $\langle \text{Br}_{\text{eff}} \rangle = 3.09/(5 \times 10^{10})$ at Belle-II, where we recall that the projected total number of produced $B$ mesons at LHCb upgrade and at Belle-II is $4.8 \times 10^{12}$ and $5 \times 10^{10}$, respectively.

The values of $\langle \text{Br}_{\text{eff}}(B \to D\mu X) \rangle (X = \pi$ or $e\nu_e)$ are obtained by taking the arithmetic average of the values of $\text{Br}_{\text{eff}}$ for the four LNV decay modes: $B^- \to D^{0}\mu^-\mu^- X^+$, $B^0 \to D^{*+}\mu^-\mu^+ X^+$ and their charge conjugates. Analogously, $\langle \text{Br}_{\text{eff}}(B \to D\mu X) \rangle$ is the arithmetic average over the four analogous LNV decays as mentioned before, having now $D$ instead of $D^*$. We note that the total decay widths of $B^0$ and $B^{\pm}$ differ somewhat, $\Gamma_{B^0}/\Gamma_{B^+} = 1.078 \cite{43}$, and we took this into account. In our calculations we neglected, however, the small difference between the masses of $D^+$ and $D^0$ (about 5 MeV), and between the masses of $D^{*+}$ and $D^{*0}$ (about 3 MeV); we used $m_{D} \approx 1.865$ GeV and $m_{D^*} \approx 2.010$ GeV.

Further, for the LNV decays of $B$ at $D^{(*)}$, $B \to \mu\mu X$, we do not have four, but only two modes, due to the electric charge restriction: $B^{\pm} \to \mu^\pm \mu^\mp X^\mp$. For such decays, the average $\langle \text{Br}_{\text{eff}}(B \to \mu\mu X) \rangle$ is taken only over these two LNV modes. In these latter cases, we have to take into account that the total number of produced charged $B$ mesons is only half of the total number of produced $B$ mesons. Hence, the sensitivity limits on $|U_{\mu N}|^2$ are obtained in these cases by requiring $\langle \text{Br}_{\text{eff}}(B \to \mu\mu X) \rangle = 3.09/(2.4 \times 10^{12})$ at LHCb upgrade, and $\langle \text{Br}_{\text{eff}} \rangle = 3.09/(2.5 \times 10^{10})$ at Belle-II.

We note that the charge-conjugated versions of the decays, i.e., the decays of $B^0$ vs $\bar{B}^0$, and of $B^+ \text{ vs } B^-$, give in general the same results. The only exception are the decays in which $D^*$ vector meson is produced. This is so because of the factor $\eta = \pm 1$ in the expression \cite{18}, in one term there proportional to $\cos \theta_1$, which changes sign. The effect of this sign change does not entirely cancels out in the integration \cite{36} for the effective branching ratio, because the

FIG. 2: (a) (left-hand figure) The lab momentum ($p_B'$) distribution of the produced $B^0$ mesons in LHCb \cite{53}. We take the shaded figure as the representative case; (b) (right-hand figure) the distribution of the left-hand shaded curve in ten bins of equal weight (equal number of events).
expression $E'_{\nu}(q^2;q',\hat{p}_1)$ in the neutrino $N$ decay probability also has dependence on $\cos \theta_1$.

We assume in our formulas that only the mixings $|U_{\mu N}|^2$ are nonzero; if other mixings ($|U_{\tau N}|^2$, $|U_{\tau N}|^2$) are nonzero, the obtained upper bounds for $|U_{\mu N}|^2$ are in general less restrictive (higher).\(^6\)

The results for the decays with $\pi\pm$ in the final state, for LHCb upgrade, are given in Figs. 3(a)-(d). In Figs. 3(a)-(c), the present direct experimental bounds are included for comparison, along with our results - the obtained prospective sensitivity limits for LHCb upgrade. Fig. 3(d) shows the LHCb sensitivity limits for the three considered decays, for mutual comparisons. Further, we note that the decay modes $B \to (D^{(*)})\mu^{\pm}\mu^{\mp}e^{\mp}\nu_e$ cannot be detected at LHCb.

The results for the considered decays at Belle-II, either with $\pi\pm$ or with $e^{\pm}\nu_e$ in the final state, are given in Figs. 4(a)-(d). In Figs. 4(a)-(c), the present experimental bounds are included for comparison. In Fig. 4(d), the prospective Belle-II sensitivity limits for all the six considered decays are presented, for mutual comparisons.

V. DISCUSSIONS AND CONCLUSIONS

From Figures 3 and 4, we can see that the decays where $D^*$ and $D$ are produced give quite strong new sensitivity limits on $|U_{\mu N}|^2$ in the mass interval 1.75 GeV < $M_N$ < 3 GeV. This is a reflection of the fact that the presence of $D^{(*)}$ mesons leads to a significantly weaker CKM suppression in the decay rates, because $|V_{cb}|^2 \approx 10^2|V_{ub}|^2$. However,\(^6\)

\(^6\) If $\bar{N}$ (and $N$) were Dirac, it would produce, e.g., a pair $\mu^+\mu^-$ or a pair $e^+e^-$, which have a strong QED background, and would thus not be useful. Or it could produce a pair $\mu^{\pm}e^{\mp}$; this could give important contribution, but only in the scenario where both $U_{\mu N}$ and $U_{\tau N}$ are nonnegligible, i.e., the scenario not considered here.
when \( M_N > 3 \text{ GeV} \), such decays are kinematically suppressed, and then only the (CKM-suppressed) decays \( B \rightarrow \mu \mu X \) give useful sensitivity limits, as seen in Figs. 3(c), (d) and Figs. 4(c), (d). Further, we see in Figs. 3 that in general the sensitivity limits are more restrictive (lower) when \( X = e\nu \) than when \( X = \pi \).

Comparing Figs. 3 with Figs. 4 we can see that the decays \( B \rightarrow (D^{(*)})\mu^\pm \mu^\mp \pi^\mp \), which can be measured at both LHCb and Belle-II experiments, give more stringent (lower) sensitivity limits on \( |U_{\mu N}|^2 \) at LHCb upgrade experiment. This is so primarily because the expected number of produced \( B \) mesons at LHCb upgrade (4.8 \times 10^{12}) is by two orders of magnitude larger than the number at Belle-II (5 \times 10^{10}). Yet another factor contributing to the more stringent bounds is the effective detector length, which is assumed to be larger at LHCb upgrade (\( L = 2.3 \text{ m} \) vs \( L = 1 \text{ m} \) at Belle-II). The difference between the two sets of the sensitivity limits is somewhat reduced by the fact that the lab energy of the produced \( B \) mesons in LHCb is significantly higher than in Belle-II; as a consequence, the produced on-shell \( N \) neutrinos move in the LHCb case faster and are thus less likely to decay within the detector. If, on the other hand, the acceptance factors decrease the effective number of produced \( B \) mesons, or if the effective detector length \( L \) turns out to be smaller, the sensitivity limits for \( |U_{\mu N}|^2 \) go up, in general as approximately proportional to \( 1/\sqrt{N_L} \) for not very heavy neutrinos (\( M_N < 3 \text{ GeV} \)).

This approximate proportionality comes from the following behavior. For the values of \( |U_{\mu N}|^2 \) which are of the order of magnitude of the presented upper bounds, we have at \( M_N \lesssim 2.5 \text{ GeV} \) small \( N \)-decay probabilities, \( P_N \ll 1 \), and therefore our expressions imply in such a case the approximate proportionality \( Br_{\text{eff}} \propto |U_{\mu N}|^4 L \). However, for \( M_N \gtrsim 4.5 \text{ GeV} \) we have \( P_N \approx 1 \) and thus the approximate proportionality \( Br_{\text{eff}} \propto |U_{\mu N}|^2 \) (and \( L \)-independent). We verified these approximate proportionality also numerically with our expressions. Approximate \( L \)-independence of \( Br_{\text{eff}} \) occurs already at \( M_N \gtrsim 3 \text{ GeV} \).

In Ref. [53], a similar analysis was made for the decay \( B^+ \rightarrow \mu^+ N \rightarrow \mu^+ \mu^- \pi^- \) at Belle-II, where the same total number of \( B \) meson pairs was assumed as here, \( 5 \times 10^{10} \). They obtained lower, i.e., more restrictive sensitivity limits on \( |U_{\mu N}|^2 \) than we do for this decay for Belle-II. The reason for the difference cannot be the fact that they did not
take into account the movement of $B$-mesons in the lab frame (this effect changes the sensitivity limits only weakly).

The reason for the difference lies possibly in the evaluated values of the total decay width $\Gamma_N$ as a function of $M_N$. We evaluated this decay width according to the formulas and Figures in Appendix A, based on Refs. [4, 5], and we applied those evaluations in Refs. [9, 27].

The experimental bounds on $|U_{\mu N}|^2$ presented in Figs. 3(a)-(c) and Figs. 4(a)-(c) are from various experiments: DELPHI [59], BEBC [60], NuTeV [61], NA3 [62], CHARM II [63], and Belle [40].

We evaluated this decay width according to the formulas and Figures in Appendix A, based on Refs. [4, 5], and we applied those evaluations in Refs. [9, 27].

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Appendix A: Total decay width of neutrino $N$

We summarize here the formulas needed for evaluation of the total decay width of a massive sterile neutrino $N$, $\Gamma(N \to \ell \ell' \nu' \nu)$.

The formulas for the widths for leptonic decays and semileptonic decay are given in Ref. [5] (Appendix C there), for $M_N \lesssim 1$ GeV. For higher masses $M_N$, the calculation of the semileptonic decay widths cannot be performed in this way because not all the resonances are known. For such higher masses, the decay widths for semileptonic decays were calculated in Refs. [4, 12] by the inclusive approach based on duality. In this approach, the various (pseudoscalar and vector) meson channels were calculated by quark-antiquark channels. This was applied for $M_N \geq M_{\eta'} \approx 0.958$ GeV. Below we write the expressions given in Ref. [4] for the decay width channels. In some of these formulas, twice the decay width appears $[2\Gamma(N \to \ldots)]$, where the factor two is applied if $N$ is a Majorana neutrino, and factor one if it is Dirac neutrino. This is so because when Majorana neutrino decays to charged particles, the decay in charge conjugate channel is equally possible; this is not possible if $N$ is Dirac particle.

The leptonic decays are

$$2\Gamma(N \to \ell^- \ell'^+ \nu' \nu) = |U_{\ell N}|^2 \frac{G_F}{96\pi^3} M_N^5 I_1(y, 0, y', 0) (1 - \delta_{\ell \ell'}) , \quad \text{(A1a)}$$

$$\Gamma(N \to \nu \ell \ell'^- + ) = |U_{\ell N}|^2 \frac{G_F}{96\pi^3} M_N^5 \left[ (g_L^{(\text{lept})} g_R^{(\text{lept})}) + \delta_{\ell \ell'} (g_L^{(\text{lept})} g_R^{(\text{lept})}) I_2(0, 0, y, y') + \left( (g_L^{(\text{lept})})^2 + (g_R^{(\text{lept})})^2 + \delta_{\ell \ell'} (1 + 2g_L^{(\text{lept})}) I_1(0, 0, y, y') \right) \right] \quad \text{(A1b)}$$

$$\sum_{\nu} \sum_{\nu'} \Gamma(N \to \nu \nu' \nu' \nu') = \sum_{\ell} |U_{\ell N}|^2 \frac{G_F}{96\pi^3} M_N^5 \quad \text{(A1c)}$$

The factor 2 is included in Eq. (A1a) when $N$ is Majorana, because in such a case both decays, $N \to \ell^- \ell'^+ \nu' \nu$ and $N \to \ell'^- \ell' \nu' \nu'$ are contributing ($\ell \neq \ell'$).

Further, the following semileptonic decays contribute when $M_N < M_{\eta'} \approx 0.968$ GeV, involving pseudoscalar ($P$)
and vector (V) mesons:

\[ 2\Gamma(N \to \ell^- P^+) = |U_{\ell N}|^2 \frac{G_F^2}{8\pi} M_N^2 f_P^2 |V_P|^2 F_P(y_{\ell}, y_P) \]  
(A2a)

\[ \Gamma(N \to \nu_{\ell} P^0) = |U_{\ell N}|^2 \frac{G_F^2}{64\pi} M_N^3 f_P^2 (1 - y_{\ell}^2)^2 \]  
(A2b)

\[ 2\Gamma(N \to \ell^- V^+) = |U_{\ell N}|^2 \frac{G_F^2}{8\pi} M_N^2 f_V^2 |V_V|^2 F_V(y_{\ell}, y_V) \]  
(A2c)

\[ \Gamma(N \to \nu_{\ell} V^0) = |U_{\ell N}|^2 \frac{G_F^2}{2\pi} M_N^2 f_V^2 \kappa_V (1 - y_V^2)^2 (1 + 2y_V^2). \]  
(A2d)

Again, the factor 2 appears in the charged meson channels if \( N \) is Majorana. The factors \( V_P \) and \( V_V \) appearing in the above expressions stand for the CKM matrix elements of the valence quarks of the mesons. The constants \( f_P \) and \( f_V \) are the corresponding decay constants of these mesons. Their values are given in Table 1 of Ref. [4].

The contributing pseudoscalar mesons here are: \( P^\pm = \pi^\pm, K^{\ast\pm} \); \( P^0 = \pi^0, K^0, K^{\ast0}, \eta \). The contributing vector mesons here are: \( V^\pm = \rho^\pm, K^{\ast\pm} \); \( V^0 = \rho^0, \omega, K^{\ast0}, K^{\ast\ast0} \).

On the other hand, for higher mass \( M_N \geq M_{\eta'} (=0.9578 \text{ GeV}) \), the quark-hadron duality is used and the sum of the widths of the semileptonic decay modes are represented by the following widths into quark-antiquark decay modes [4]:

\[ 2\Gamma(N \to \ell^- U D) = |U_{\ell N}|^2 \frac{G_F^2}{32\pi} M_N^2 |V_{UD}|^2 I_Y(y_{\ell}, y_U, y_D) \]  
(A3a)

\[ \Gamma(N \to \nu_{\ell} \bar{q}q) = |U_{\ell N}|^2 \frac{G_F^2}{32\pi} M_N \left[ g_L^{(\text{lep})} g_R^{(U)} I_2(0, y_q, y_{\bar{q}}) + \left( g_L^{(U)} + g_R^{(U)} \right)^2 \right] I_1(0, y_q, y_{\bar{q}}) \]  
(A3b)

In all the formulas (A1)–(A3), the notations

\[ y_Y \equiv M_Y/M_N \quad (Y = \ell, \nu_{\ell}, P, V, q) \]  
(A4)

are used. We denoted in Eq. (A3): \( U = u, c \); \( D = d, s, b \); \( q = u, d, c, s, b \). The used values of the quark masses in our evaluations are: \( M_u = M_d = 3.5 \text{ MeV} \); \( M_s = 105 \text{ MeV} \); \( M_c = 1.27 \text{ GeV} \); \( M_b = 4.2 \text{ GeV} \).

As mentioned earlier, in the evaluation of the total decay width \( \Gamma_N \), if \( N \) is Majorana we add the expressions (A3a) and (A3b); if \( N \) is Dirac, the expressions should be added, but with the expressions (A3a) multiplied by 1/2. The same is valid in the case when we sum the expressions (A1) and (A2).

In Eqs. (A1b) and (A3b), the following SM neutral current couplings appear:

\[ g_L^{(\text{lep})} = \frac{1}{2} + \sin^2 \theta_W, \quad g_R^{(\text{lep})} = \sin^2 \theta_W \]  
(A5a)

\[ g_L^{(U)} = \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W, \quad g_R^{(U)} = -\frac{2}{3} \sin^2 \theta_W \]  
(A5b)

\[ g_L^{(D)} = \frac{1}{2} + \frac{1}{3} \sin^2 \theta_W, \quad g_R^{(U)} = \frac{1}{3} \sin^2 \theta_W \]  
(A5c)

In Eq. (A2d), the neutral current couplings \( \kappa_V \) (for the neutral vector mesons) are

\[ \kappa_V = \frac{1}{3} \sin^2 \theta_W \quad (V = \rho^0, \omega) \]  
(A6a)

\[ \kappa_V = -\frac{1}{4} + \frac{1}{3} \sin^2 \theta_W \quad (V = K^{\ast0}, K^{\ast\ast0}) \]  
(A6b)

Further, in the above expressions, the following expressions \( I_1, I_2, F_P \) and \( F_V \) were used:

\[ I_1(x, y, z) = 12 \int_{x+y}^{1-z^2} \frac{ds}{s} (s - x^2 - y^2)(1 + z^2 - s) \lambda^{1/2}(s, x^2, y^2) \lambda^{1/2}(1, s, z^2) \]  
(A7a)

\[ I_2(x, y, z) = 24yz \int_{x+y}^{1-z^2} \frac{ds}{s} (1 + x^2 - s) \lambda^{1/2}(s, y^2, z^2) \lambda^{1/2}(1, s, x^2) \]  
(A7b)

\[ F_P(x, y) = \lambda^{1/2}(1, x^2, y^2) \left[ (1 + x^2)(1 + x^2 - y^2) - 4x^2 \right] \]  
(A7c)

\[ F_V(x, y) = \lambda^{1/2}(1, x^2, y^2) \left[ (1 - x^2)^2 + (1 + x^2)y^2 - 2y^4 \right]. \]  
(A7d)
Here, the $\lambda^{1/2}$ function is given in Eq. (5b).

All these formulas then give the total decay width $\Gamma(N \to \text{all})$ as a function of $M_N$. This total decay width can be written in the following form:

$$\Gamma_N = \tilde{K} \tilde{\Gamma}_N(M_N).$$  \hspace{1cm} (A8)

The corresponding canonical (i.e., without the heavy-light mixing factors) decay width expression is

$$\tilde{\Gamma}_N(M_N) \equiv \frac{G_F^2 M_N^5}{96\pi^3}.$$  \hspace{1cm} (A9)

The factor $\tilde{K}$ in Eq. (GNwidth) contains the dependence on the heavy-light mixing factors, and it has the form

$$\tilde{K}(M_N) \equiv \tilde{K} = N_{\ell N} |U_{e N}|^2 + N_{\mu N} |U_{\mu N}|^2 + N_{\tau N} |U_{\tau N}|^2.$$  \hspace{1cm} (A10)

The dimensionless coefficients $N_{\ell N}(M_N)$ here are numbers $\sim 1-10$ which are functions of the mass $M_N$, and they are determined by the above formulas given in this Appendix. We present in Figs. 5 the resulting coefficients $N_{\ell N}(M_N)$ as a function of neutrino mass $M_N$, for the case of Dirac and Majorana neutrino $N$. The figures are from Ref. [27] for Majorana $N$, and [9] for Dirac $N$.

It is interesting to notice a small kink in the curves of Figs. 5 at $M_N = M_\eta' (=0.9578 \text{ GeV})$. The kink is there because at $M_N \geq M_\eta'$ the use of quark-hadron duality is made, i.e., we replace the semileptonic decay channel contributions by those of the quark-antiquark channel. As a consequence, we can conclude that the quark-hadron duality works well at $M_N \geq M_\eta'$. A partial exception is the case $\ell = \tau$ because $\tau$ lepton has a large mass.

Appendix B: Lorentz factors of on-shell $N$ in laboratory frame

In this Appendix we calculate the energy $E''_N$ of the produced heavy neutrino $N$ in the laboratory frame $\Sigma''$ [the rest frame of $\Upsilon(4S)$] in the reaction $B \to D^{(*)}\ell_1 N$, cf. Sec. III. We recall that our notations are: $\Sigma$ is the rest frame of the virtual $W^*$ (i.e., of the $\ell_1$-$N$ pair); $\Sigma'$ is the rest frame of the $B$ meson; and $\Sigma''$ is the laboratory frame.

As explained in Sec. III, the velocity of the produced mesons $B$ in the laboratory ($\Sigma''$) frame, $\vec{\beta}_B$, is (practically) the same as the velocity of $\Upsilon(4S)$ there, Eqs. (34)-(35). The momentum $p_N$ transforms between the $\Sigma''$ (lab) frame and the $\Sigma'$ (B-rest) frame in the following way:

$$E''_N = \gamma_B \left( E'_N + \beta_B (\vec{p}'_N \cdot \vec{\beta}_B) \right),$$ \hspace{1cm} (B1a)

$$(\vec{p}'_N \cdot \vec{\beta}_B) = \gamma_B \left( (\vec{p}'_N \cdot \vec{\beta}_B) + \beta_B E'_N \right),$$ \hspace{1cm} (B1b)

$$(\vec{p}'_N)_{\perp} = (\vec{p}'_N)_{\perp}.$$ \hspace{1cm} (B1c)

![FIG. 5: The coefficients $N_{\ell N}(M_N)$ ($\ell = e, \mu, \tau$) appearing in Eqs. (A8)-(A10), as a function of the mass of the heavy sterile neutrino $N$. When $N$ is Dirac, the left-hand figure applies; when it is Majorana, the right-hand figure applies.](image-url)
where in the last line (…)⊥ denotes the component of the vector perpendicular to \( \hat{\beta}_B \equiv \hat{z} \), i.e., perpendicular to the direction of movement of \( B \) [+ of \( \Upsilon(4S) \)] in the lab frame \( \Sigma'' \).\(^{7}\)

The momentum \( p_N \) transforms between the \( B \) rest frame \( \Sigma' \) and the \( W^* \) rest frame \( \Sigma \) in the following way:

\[
E_N' = \gamma_W(q^2) (E_N(q^2) - \beta_W(q^2)|\vec{p}_N(q^2)|\cos \theta_1),
\]

\[
(\vec{p}_N' \cdot \vec{q}') = \gamma_W(q^2) ( -|\vec{p}_N(q^2)|\cos \theta_1 + \beta_W(q^2)E_N(q^2)).
\]

Here, \( \theta_1 \) is the angle between \( \vec{q}' \equiv \hat{z} \) and \( \vec{p}_1 \) of \( \ell_1 \) in the \( \Sigma' \) frame of \( \ell_1-N \). The corresponding quantities in the \( \Sigma \) frame, as a function of the squared invariant mass of \( W^*, Q^2 \), are

\[
E_N = \frac{1}{2\sqrt{q^2}}(q^2 + M_B^2 - M_1^2),
\]

\[
|\vec{p}_N| = |\vec{p}_1| = \frac{1}{2\sqrt{q^2}}\lambda^{1/2} \left(1, \frac{M_1^2}{q^2}, \frac{M_B^2}{q^2}\right),
\]

the Lorentz factors for the transition between \( \Sigma' \) and \( \Sigma \) are

\[
\gamma_W(q^2) = \left(1 + \frac{q^2}{q^2}\right)^{1/2}, \quad \beta_W(q^2) = \left(\frac{q^2}{|q|^2} + 1\right)^{-1/2},
\]

where the magnitude \( |\vec{q}'| \) of the 3-momentum of \( W^* \) in \( \Sigma' \) (\( B\)-rest frame) is

\[
|\vec{q}'| = \frac{1}{2}M_B\lambda^{1/2} \left(1, \frac{M_1^2}{M_B^2}, \frac{q^2}{M_B^2}\right).
\]

In order to combine all these relations Eqs. \( B1 \)−\( B5 \), to obtain \( E_N' \) as a function of \( q^2, \vec{q}' \) and \( \vec{p}_1 \), we must express the \( B \)-meson velocity direction \( \hat{\beta}_B \) in a 3-dimensional coordinate system in \( \Sigma \). We introduce such a system in the following way: \( \hat{z} \) is defined as \( \hat{z} = \hat{z}' = \hat{q} \), i.e., the direction of \( W^* \) in the \( B\)-rest frame \( \Sigma' \). Then the vectors \( \hat{q}' \) and \( \hat{\beta}_B \) define a plane, the angle between \( \hat{q}'(= \hat{z}) \) and \( \hat{\beta}_B \) is \( \theta_q \) \((0 \leq \theta_q \leq \pi)\), and the axis \( \hat{x} \) in this plane is such that \( (\hat{\beta}_B)_x = \sin \theta_q \) \((>0)\). We recall that \( \hat{\beta}_B \) is the direction vector of \( B \) in \( \Sigma'' \) (lab) frame. The axis \( \hat{y} \) is then obtained in the usual way, \( \hat{y} = \hat{z} \times \hat{x} \), cf. Fig. 6. As a result, we have

\footnote{Strictly speaking, we should use the notation \( \vec{\beta}_B' \) for the velocity of \( B \) meson in the lab, but we prefer the simplified notation \( \vec{\beta}_B \) for this vector.}
\[ \hat{\beta}_B = \sin \theta_q \hat{x} + \cos \theta_q \hat{q}' \]  
\[ \Rightarrow (\hat{p}_N^\prime \cdot \hat{\beta}_B) = (\hat{p}_N^\prime \cdot \hat{q}') \cos \theta_q + (\hat{p}_N^\prime \cdot \hat{x}) \sin \theta_q, \]  
(B6a)

We can now take into account that \( \hat{p}_N^\prime \cdot \hat{x} = \hat{p}_N^\prime \cdot \hat{x} \), because these are components perpendicular to the boost direction \( \hat{q}' = \hat{z} \) between \( \Sigma' \) and \( \Sigma \). Since in \( \Sigma \) we have \( \hat{p}_1 = -\hat{p}_N^\prime \), we thus have

\[ \hat{p}_N^\prime \cdot \hat{x} = -\hat{p}_N^\prime \cdot \hat{x} = -[\hat{p}_1] \sin \theta_1 \cos \phi_1 = -[\hat{p}_N^\prime] \sin \theta_1 \cos \phi_1, \]  
(B7)

where \( \theta_1 \) and \( \phi_1 \) are the spherical coordinates of \( \hat{p}_1 \) in \( \Sigma (0 \leq \theta_1 \leq \pi; 0 \leq \phi_1 < 2\pi) \), cf. Fig. 6. Substitution of Eq. (B7) into Eq. (B6b), and taking into account the relation (B2b), then gives

\[ (\hat{p}_N^\prime \cdot \hat{\beta}_B) = [\gamma_W(q^2) (-[\hat{p}_N^\prime(q^2))] \cos \theta_1 + \beta_W(q^2)E_N(q^2)] \cos \theta_q - [\hat{p}_N^\prime(q^2)] \sin \theta_1 \cos \phi_1 \sin \theta_q]. \]  
(B8)

Using this expression, and the expression for \( E''_N \) of Eq. (B2a), in the Lorentz transformation (B1a), we finally obtain the energy \( E''_N \) of the \( N \) neutrino in the lab frame in terms of \( q^2, q' \) (i.e., \( \theta_q \)) and \( \hat{p}_1 \) (i.e., \( \theta_1 \) and \( \phi_1 \))

\[ E''_N(q^2; \theta_q, \theta_1, \phi_1) = \gamma_B \left\{ \gamma_W(q^2) \left( E_N(q^2) - \beta_W(q^2)\hat{p}_N(q^2) \right) \cos \theta_1 \right\} \]

\[ + \beta_B \left[ \gamma_W(q^2) \left( -[\hat{p}_N^\prime(q^2)] \cos \theta_1 + \beta_W(q^2)E_N(q^2) \right) \cos \theta_q - [\hat{p}_N^\prime(q^2)] \sin \theta_1 \cos \phi_1 \sin \theta_q \right\}. \]  
(B9)

Here, the expressions \( \gamma_W(q^2) \) and \( \beta_W(q^2) \) are given in Eq. (B4), and the expressions for \( E_N(q^2), [\hat{p}_N(q^2)] \) and \( [\hat{q}] \) are given in Eqs. (B3) and (B5).

For the decay \( B \rightarrow DN \) the same expressions apply, with the only difference that instead of \( M_D \), we have \( M_D \).

However, when the decay is without \( D^* \), namely \( B \rightarrow \ell_i N \), the expression for \( E''_N \) gets simplified significantly, and has only dependence on the direction \( \hat{p}_N^\prime \) of the \( N \) neutrino in the \( B \)-rest frame (\( \Sigma' \))

\[ E''_N = \gamma_B(E'_N + \cos \theta_N \beta_B |\hat{p}_N^\prime|), \]  
(B10)

where \( \theta_N \) is the angle between \( \hat{\beta}_B \) and \( \hat{p}_N^\prime \) in the \( B \)-rest frame (\( \Sigma' \)), and we have

\[ E'_N = \frac{M_B^2 + M_N^2 - M_1^2}{2M_B}, \]  
(B11a)

\[ |\hat{p}_N^\prime| = \frac{1}{2} M_B \lambda^{1/2} \left( 1, \frac{M_2^2}{M_B^2}, \frac{M_3^2}{M_B^2} \right). \]  
(B11b)

From Eqs. (B10) and (B11) we see that in this case \( E''_N \) depends only on \( \theta_N \), the angle between \( \hat{\beta}_B \) and \( \hat{p}_N^\prime \). The integration differential in Eq. (39) thus reduces simply to \( d\Omega_{\hat{p}_N^\prime} \rightarrow 2\pi d(\cos \theta_N) \).

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