GLUON CONDENSATES AT FINITE BARYON DENSITIES AND TEMPERATURE

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We derive here the equation of state for quark matter with a nontrivial vacuum structure in QCD at finite temperature and baryon density. Using thermofield dynamics, the parameters of thermal vacuum and the gluon condensate function are determined through minimisation of the thermodynamic potential, along with a self-consistent determination of the effective gluon and quark masses. The scale parameter for the gluon condensates is related to the SVZ parameter in the context of QCD sum rules at zero temperature. With inclusion of quarks in the thermal vacuum the critical temperature at which the gluon condensate vanishes decreases as compared to that containing only gluons. At zero temperature, we similarly obtain the critical baryon density for the same to be about 0.36 $\text{fm}^{-3}$.
I. INTRODUCTION

Hot dense hadronic matter in relativistic heavy ion collisions or in early universe is likely to behave as quark gluon plasma. The study of this in the context of quantum chromodynamics (QCD) is nontrivial and nonperturbative [1,2]. For gauge fields only we had seen earlier [3] that such a vacuum structure with gluon condensates can emerge dynamically. We examine here the same problem including quarks at zero or finite temperature, and at finite baryon density, but again with only gluon condensates [2,3]. We expect that the effect of gluons will dominate vacuum structure due to the colour factor.

The method used here is variational and nonperturbative, since only equal time algebra is taken as input. The ansatz functions for variation are determined with the minimisation of energy density at zero temperature, and, of the thermodynamic potential at finite temperatures and baryon densities. This is a continuation of the earlier programme [3] using thermofield dynamics method (TFD) [4] to study the problem at finite temperature and leads to a simple generalisation of Bogoliubov transformations as a part of the technology at zero temperature.

We organise the paper as follows. In section II, we consider quark matter at finite temperature using as stated the method of thermofield dynamics [4]. In section III, we calculate the thermodynamic potential for the system of quark matter at finite temperature and baryon density and then minimise the same to derive the equation of state. In section IV we discuss the results. Similar techniques with explicit ground state construction have also been applied to study hot nuclear matter [5] where scalar isoscalar two pion condensates replace the $\sigma$-field and for chiral symmetry breaking [6] with quark condensates.

Nonperturbative nature of QCD has been studied for a long time in the context of condensates [1,2,7] or scale symmetry breaking [8] as well as effective field theories [3,4] or lattice gauge theories [10]. The present approach is complementary to the above.
II. QUARK MATTER AT FINITE TEMPERATURE

We shall very briefly recapitulate the notations of Ref. [3], now with quark fields so that finite baryon density effects can be included. Let us consider the QCD Lagrangian given as

\[ \mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{matter}} + \mathcal{L}_{\text{int}} \]  

where

\[ \mathcal{L}_{\text{gauge}} = -\frac{1}{2} G^{a\mu\nu} \left( \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g f^{abc} W^b_\mu W^c_\nu \right) + \frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} \]  

\[ \mathcal{L}_{\text{matter}} = \bar{\psi}(i\gamma^\mu \partial_\mu - m_0)\psi \]  

and

\[ \mathcal{L}_{\text{int}} = g \bar{\psi} \gamma^\mu \frac{\lambda^a}{2} W^a_\mu \psi \]  

where \( W^a_\mu \) are the SU(3) colour gauge fields. We shall quantise in Coulomb gauge [11] and write the electric field \( G^a_{0i} \) in terms of the transverse and longitudinal parts as

\[ G^a_{0i} = T G^a_{0i} + \partial_i f^a, \]

where \( f^a \) is to be determined. We take at time \( t=0 \) [3]

\[ W^a_i(\vec{x}) = (2\pi)^{-3/2} \int \frac{d\vec{k}}{\sqrt{2\omega(\vec{k})}} (a^a_i(\vec{k}) + a^a_i(-\vec{k})^\dagger) \exp(i\vec{k}.\vec{x}) \]  

and

\[ T G^a_{0i}(\vec{x}) = (2\pi)^{-3/2} i \int d\vec{k} \sqrt{\frac{\omega(\vec{k})}{2}} \left( -a^a_i(\vec{k}) + a^a_i(-\vec{k})^\dagger \right) \exp(i\vec{k}.\vec{x}), \]

where, \( \omega(k) \) is arbitrary [11] and for equal time algebra we have

\[ \left[ a^a_i(\vec{k}), a^b_j(\vec{k}')^\dagger \right] = \delta^{ab} \Delta_{ij}(\vec{k}) \delta(\vec{k} - \vec{k}') \]

with

\[ \Delta_{ij}(\vec{k}) \]
\[ \Delta_{ij}(\vec{k}) = \delta_{ij} - \frac{k_i k_j}{k^2}. \]  
(6)

The equal time quantization condition for the fermionic sector is given as

\[ [\psi^i_\alpha(x, t), \psi^j_\beta(y, t)^\dagger]_+ = \delta^{ij} \delta_{\alpha\beta} \delta(\vec{x} - \vec{y}). \]  
(7)

We now also have the field expansion for fermion field \( \psi \) at time \( t=0 \) given as

\[ \psi_i(\vec{x}) = \frac{1}{(2\pi)^{3/2}} \int \left[ U_i(\vec{k})c^i_{Ir}(\vec{k}) + V_s(-\vec{k})\tilde{c}^i_{Is}(-\vec{k}) \right] e^{i\vec{k}\cdot\vec{x}}d\vec{k}, \]  
(8)

where \( U \) and \( V \) are given by

\[ U_i(\vec{k}) = \begin{pmatrix} \cos \frac{\chi(k)}{2} \\ \vec{\sigma} \cdot \hat{k} \sin \frac{\chi(k)}{2} \end{pmatrix} u_{Ir}; \quad V_s(-\vec{k}) = \begin{pmatrix} \vec{\sigma} \cdot \hat{k} \sin \frac{\chi(k)}{2} \\ \cos \frac{\chi(k)}{2} \end{pmatrix} v_{Is}, \]  
(9)

where the function \( \chi(k) \) could be arbitrary. Here we approximate the same as \( \cos \chi(k) = m_Q/\epsilon(k) \) and \( \sin \chi(k) = k/\epsilon(k), \) with \( \epsilon(k) = (k^2 + m_Q^2)^{1/2}. \) For free fields e.g., \( m_Q = m_0. \) However, for interacting fields, we shall determine \( m_Q \) in a self consistent manner as will be discussed later. The above are consistent with the equal time anticommutation conditions provided

\[ [c^i_{Ir}(\vec{k}), \tilde{c}^j_{Is}(\vec{k}')]_+ = \delta_{rs} \delta^{ij} \delta(\vec{k} - \vec{k}') = [\tilde{c}^i_{Ir}(\vec{k}), c^j_{Is}(\vec{k}')]_+, \]  
(10)

where \( i \) and \( j \) refer to the colour and flavour indices.

In Coulomb gauge, the expression for the Hamiltonian density, \( \mathbf{T}^{00} \) from equation (1) is given as

\[ \mathbf{T}^{00} = : \frac{1}{2} T G^a_{0i} T G^a_{0i} + \frac{1}{2} W^a_i(-\vec{\nabla}^2)W^a_i + g f^{abc} W^a_i W^b_j \partial_i W^c_j \\
+ \frac{g^2}{4} f^{aef} f^{bce} W^b_j W^c_i W^e_j W^f_i + \frac{1}{2} (\partial_i f^a)(\partial_i f^a) \\
+ \bar{\psi}(-i\gamma^i \partial_i + m_0)\psi - g\bar{\psi}\gamma^\mu \chi^a W^a_\mu \psi : \]  
(11)

where : : denotes the normal ordering with respect to the perturbative vacuum, say \( |\text{vac} \rangle \), defined through \( a^a_i(\vec{k}) |\text{vac} \rangle > = 0, c^i_{Ir}(\vec{k}) |\text{vac} \rangle > = 0 \) and \( \tilde{c}^i_{Ir}(\vec{k})^\dagger |\text{vac} \rangle > = 0. \) In order to solve for the operator \( f^a \), we first note that
\[ f^a = -W^a_0 - g f^{abc} (\nabla^2)^{-1} (W^b_i \partial_i W^c_0). \]  

(12)

Proceeding as earlier \[3\] with a mean field type of approximation we obtain,

\[
\vec{\nabla}^2 W^a_0(\vec{x}) + g^2 f^{abc} f^{cde} <vac', \beta | W^b_i(\vec{x}) \partial_i (\vec{\nabla}^2)^{-1} (W^d_j(\vec{x}) | vac', \beta > \partial_j W^e_0(\vec{x})) = J^a_0(\vec{x}),
\]

(13)

where,

\[ J^a_0 = g f^{abc} W^b_i \partial_i G^c_0 - g \bar{\psi} \gamma^0 \frac{\lambda^a}{2} \psi. \]

(14)

We note that at zero temperature, \( | vac'; \beta = \infty > = | vac' > \) was the nonpertubative ground state as discussed in Ref. \[3\]. The extra fermionic contributions in equations (11) and (14) may be noted, and thus the expressions of Ref. \[3\] will get modified at finite temperatures and densities. We define \( | vac' > \) through a unitary transformation, in a similar manner to Gross-Neveu model considered earlier \[13\], given as

\[ | vac' >= U | vac >, \]

(15)

where

\[ U = \exp(B^\dagger - B), \]

(16)

In Ref. \[3\], it was shown that at zero temperature, we may have

\[ B^\dagger = \frac{1}{2} \int f(\vec{k}) a^{a, i}(\vec{k}) \dagger a^{a, i}(-\vec{k}) \dagger d\vec{k}, \]

(17)

where \( f(\vec{k}) \) describes gluon condensates. For the consideration of vacuum destabilisation we should also take quark condensates \[4\]. However, we shall not do the same here because of the following reason. Our method here shall consist of minimisation of the thermodynamic potential. We had earlier determined the temperature dependent mass like term for the gluon in a self consistent manner \[3\]. We shall here also need to determine the same for the quark fields. The numerical computation with a variation for the vacuum structure of
gluon and quark condensates becomes forbidding. Besides this, we believe that the gluons being in the adjoint representation shall contribute more strongly to QCD interactions, and therefore dominate the vacuum structure. However, we do include the effect of quark sector for vacuum structure at finite temperatures through thermofield dynamics \[4\].

In the following, we shall consider the effect of temperature as well as finite baryon density on the behaviour of the gluon condensates for vacuum structure. We use the method of thermofield dynamics to consider the above which is convenient for our purpose while dealing with operators and expectation values. The thermal average of an operator is replaced by expectation value in an extended Hilbert space associated with thermal doubling \[4\]. For the present case of including temperature and finite baryon density effects, the thermal vacuum is given as

\[ |\text{vac}', \beta > = U_G(\beta)U_Q(\beta)|\text{vac} > \]  

where \( U_G \) and \( U_Q \) are unitary operators involving thermal excitations of gluons and quarks respectively. For the gluon sector, we have the old expression \[3\]

\[ U_G(\beta) = \exp (B_G(\beta)^\dagger - B_G(\beta)), \]  

with

\[ B_G(\beta)^\dagger = \int_{\theta(\vec{k}, \beta)} \theta^i_a(\vec{k})^\dagger \theta^i_a(\vec{k})^\dagger d\vec{k}. \]  

In addition, for quark sector we have,

\[ U_Q(\beta) = \exp (B_Q(\beta)^\dagger - B_Q(\beta)), \]  

with

\[ B_Q(\beta)^\dagger = \int \left[ \theta_-(\vec{k}, \beta)c^i_{lr}(\vec{k})^\dagger \bar{c}^i_{lr}(\vec{k}) + \theta_+(\vec{k}, \beta)\bar{c}^i_{lr}(\vec{k})^\dagger c^i_{lr}(\vec{k})^\dagger \right] d\vec{k}. \]  

In the above the underlined operators \[3\] correspond to the extra Hilbert space in TFD. Further, \( \theta(\vec{k}, \beta), \theta_\pm(\vec{k}, \beta) \) are arbitrary functions to be determined from the minimisation of the thermodynamic potential. For example, for free fields these functions are given by

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\[
sinh^2 \theta(\vec{k}, \beta) = \frac{1}{\exp(\beta \omega(\vec{k}, \beta)) - 1} \tag{23a}
\]

and
\[
sin^2 \theta_\pm(\vec{k}, \beta) = \frac{1}{\exp(\beta(\epsilon(\vec{k}, \beta) \pm \mu)) + 1} \tag{23b}
\]

where \(\omega(k, \beta) = \sqrt{k^2 + m_G^2}, \epsilon(k, \beta) = \sqrt{k^2 + m_Q^2}\) and \(\mu\) is the quark chemical potential.

However, for interacting fields these will be different and we shall approximately determine them in a self consistent manner. We have seen that such a structure as in equation (18) introduces a thermal Bogoliubov transformation for gluon fields [3]. We now have, in addition, the parallel transformation in quark sector given as

\[
\begin{pmatrix}
c_i^I r(\vec{k}) \\
\xi_i^I r(-\vec{k})^\dagger \\
c_i^I r(-\vec{k}) \\
\xi_i^I r(\vec{k})^\dagger
\end{pmatrix} =
\begin{pmatrix}
\cos \theta_- & \sin \theta_- & 0 & 0 \\
-\sin \theta_- & \cos \theta_- & 0 & 0 \\
0 & 0 & \cos \theta_+ & \sin \theta_+ \\
0 & 0 & -\sin \theta_+ & \cos \theta_+
\end{pmatrix}
\begin{pmatrix}
c_i^I r(\vec{k}, \beta) \\
\xi_i^I r(-\vec{k}, \beta)^\dagger \\
c_i^I r(-\vec{k}, \beta) \\
\xi_i^I r(\vec{k}, \beta)^\dagger
\end{pmatrix}. \tag{24}
\]

Our job now is to evaluate the expectation value of \(T^{00}\) with respect to \(|\text{vac}'; \beta >\). For this purpose, we have the earlier equations [3]

\[
<\text{vac}'; \beta | : W^a_i(\vec{x})W^b_j(\vec{y}) : |\text{vac}'; \beta > = \delta^{ab} \times (2\pi)^{-3} \int d\vec{k} e^{i\vec{k}.(\vec{x} - \vec{y})} \frac{F_+(\vec{k}, \beta)}{\omega(k, \beta)} \Delta_{ij}(\vec{k}), \tag{25}
\]

\[
<\text{vac}'; \beta | : T G^a_{0i}(\vec{x}) T G^b_{0j}(\vec{y}) | \text{vac}'; \beta > = \delta^{ab} \times (2\pi)^{-3} \int d\vec{k} e^{i\vec{k}.(\vec{x} - \vec{y})} \Delta_{ij}(\vec{k}) \omega(k, \beta) F_-(k, \beta). \tag{26}
\]

In the above the temperature dependant \(F_\pm(k, \beta)\) are given as

\[
F_\pm(\vec{k}, \beta) = \cosh 2\theta \left(\sinh^2 f(k) \pm \frac{\sinh 2f(k)}{2}\right) + \sinh^2 \theta(k, \beta) \tag{27}
\]

where \(\sinh^2 \theta(\vec{k}, \beta)\) is given by equation (23a). For the quark fields we have the parallel equations given as

\[
<\psi^i_{\alpha}(\vec{x})^\dagger \psi^j_{\beta}(\vec{y}) >_{\text{vac}'; \beta} = (2\pi)^{-3} \delta^{ij} \int \left(\Lambda_-(\vec{k}, \beta)\right)^{\beta\alpha}_{\alpha\beta} e^{-\vec{k}.(\vec{x} - \vec{y})} d\vec{k}, \tag{28a}
\]
\[<\psi_\alpha(x)\psi_\beta(y)^\dagger>_{\text{vac}',\beta} = (2\pi)^{-3}\delta^{ij} \int (\Lambda_+(k,\beta))_{\alpha\beta} e^{k\cdot(x-y)}dk, \tag{28b}\]

where
\[\Lambda_\pm(k,\beta) = \mp\frac{1}{2}[(\sin^2 \theta_- - \sin^2 \theta_+) + (\gamma^0 \cos \chi + \vec{\alpha}\cdot\vec{k}\sin \chi)(\sin^2 \theta_- + \sin^2 \theta_+)]. \tag{29}\]

Using equations (11), (25), (26) and (28), we then obtain the expectation value of \(T^{00}\) with respect to \(|\text{vac}';\beta\rangle\) as

\[\epsilon_0(\beta) \equiv <\text{vac}';\beta|T^{00}|\text{vac}';\beta> = C_F(\beta) + C_1(\beta) + C_2(\beta) + C_3(\beta)^2 + C_4(\beta), \tag{30}\]

where
\[C_F(\beta) = <\psi(-i\gamma^i\partial_i + m_0)\psi >_{\text{vac}',\beta}\]
\[= \frac{6}{\pi^2} \int k^2 dk \epsilon(k)(\sin^2 \theta_-(k,\beta) + \sin^2 \theta_+(k,\beta))(m_Q m_0 + k^2), \tag{31a}\]

\[C_1(\beta) = <\frac{1}{2}\Gamma G^a_{0i} G^a_{0i} >_{\text{vac}',\beta}\]
\[= \frac{4}{\pi^2} \int \omega(k) k^2 F_-(k,\beta) dk, \tag{31b}\]

\[C_2(\beta) = <\frac{1}{2}W^a_i(-\nabla^2)W^a_i >_{\text{vac}',\beta}\]
\[= \frac{4}{\pi^2} \int \frac{k^4}{\omega(k)} F_+(k,\beta) dk \tag{31c}\]

\[C_3(\beta)^2 = <\frac{1}{4}g^2 f^{aef} f^{abc} W^b_i W^c_j W^e_i W^f_j >_{\text{vac}',\beta}\]
\[= \left(\frac{2g}{\pi^2} \int \frac{k^2}{\omega(k,\beta)} F_+(k,\beta) dk\right)^2, \tag{31d}\]

and
\[C_4(\beta) = <\frac{1}{2}(\partial_i f^a)(\partial_i f^a) >_{\text{vac}',\beta},\]
\[= 4 \times (2\pi)^{-6} \int dk G_1(k,\beta) + G_2(k,\beta) \frac{k^2 + \phi(k,\beta)}{k^2 + \phi(k,\beta)}. \tag{31e}\]
In the above,

\[
G_1(\vec{k}, \beta) = 3g^2 \int dq F_+ (|\vec{q}|, \beta) F_- (|\vec{k} + \vec{q}|, \beta) \frac{\omega(|\vec{k} + \vec{q}|, \beta)}{\omega(|\vec{q}|, \beta)} \times \left(1 + \frac{(q^2 + \vec{k}.\vec{q})^2}{q^2(k + q)^2}\right),
\]

(32a)

\[
G_2(\vec{k}, \beta) = -g^2 \int dq \left[ \left(1 + \frac{m_{Q}(\beta)^2}{\epsilon(\vec{q}, \beta)\epsilon(\vec{k} - \vec{q} - \vec{k}, \beta)} + \frac{\vec{q}.(\vec{k} - \vec{q})}{\epsilon(\vec{q}, \beta)\epsilon(\vec{k} - \vec{q}, \beta)} \right) \times \left(\sin^2 \theta_-(\vec{q}, \beta) \sin^2 \theta_- (\vec{k} - \vec{q}, \beta) + \sin^2 \theta_+(\vec{q}, \beta) \sin^2 \theta_+(\vec{k} - \vec{q}, \beta) \right) \\
- \left(1 - \frac{m_{Q}(\beta)^2}{\epsilon(\vec{q}, \beta)\epsilon(\vec{k} - \vec{q}, \beta)} - \frac{\vec{q}.(\vec{q} - \vec{k})}{\epsilon(\vec{q} - \vec{k}, \beta)\epsilon(\vec{q} - \vec{k})} \right) \times \left(\sin^2 \theta_-(\vec{q}, \beta) \sin^2 \theta_-(\vec{k} - \vec{q}, \beta) + \sin^2 \theta_+(\vec{q}, \beta) \sin^2 \theta_+(\vec{k} - \vec{q}, \beta) \right) \right]
\]

(32b)

and

\[
\phi(k, \beta) = \frac{3g^2}{8\pi^2} \int \frac{dk'}{\omega(k', \beta)} F_+(k, \beta) \left(k^2 + k'^2 - \frac{(k^2 - k'^2)^2}{2kk'} \log \left|\frac{k + k'}{k - k'}\right| \right).
\]

(32c)

The expressions above are the same as earlier, except for the additional contribution of \(C_F(\beta)\) from the fermionic terms, as well as the fermionic contribution through \(G_2(\vec{k}, \beta)\) in the expansion for \(C_4(\beta)\) arising from the auxiliary fields.

Since for interacting fields, the form of the functions \(\omega(\vec{k}, \beta)\) and \(\epsilon(\vec{k}, \beta)\) are not known, we parametrise them in the free field form with temperature dependent effective mass parameters for the gluon and quark fields given as

\[
\omega(\vec{k}, \beta) = \sqrt{k^2 + m_G(\beta)^2}; \quad \epsilon(\vec{k}, \beta) = \sqrt{k^2 + m_Q(\beta)^2},
\]

(33)

with \(m_G(\beta)\) and \(m_Q(\beta)\) are to be calculated self consistently as below.

We identify the gluon mass \(m_G(\beta)\) as earlier through the selfconsistency requirement that

\[
m_G(\beta)^2 = \frac{2g^2}{\pi^2} \int \frac{k^2}{\omega(k, \beta)} F_+(k, \beta) dk.
\]

(34)

This is derived through single contraction contribution from the quartic terms.
We similarly identify the effective quark mass, $m_Q$ from the sum of the single contractions of the term quartic in the field $\psi$ of $\mathcal{T}^{00}$ given by equation (11) as well as from the mass term $m_0 \bar{\psi} \psi$. Writing the sum of the single contraction terms of $\mathcal{T}^{00}_{\text{int}}$ as

$$\bar{\psi}^i_\alpha(x) \mathcal{M}_{\alpha\beta}(\nabla x) \psi^j_\beta(x), \quad (35)$$

we identify the effective quark mass, $m_Q(\beta)$ as $m_Q(\beta) = m_0 + m'_Q(\beta)$, with $m'_Q$ given through the relation

$$M_{\alpha\beta}(\nabla x)\bigg|_{|\nabla x| \to 0} = m'_Q(\beta) \delta_{\alpha\beta}. \quad (36)$$

Writing $\psi^i(x)$ in the momentum space as

$$\psi^i(x) = (2\pi)^{-3/2} \int \bar{\psi}^i(k) \exp(i \vec{k} \cdot \vec{x}) d\vec{k}, \quad (37)$$

we thus have

$$\frac{4}{3} \times \frac{g^2}{(2\pi)^6} \int \frac{d\tilde{k}_1 d\tilde{k}_2 d\tilde{k}}{(\tilde{k}_1 - \tilde{k}_2)^2 + \phi(\tilde{k}_1 - \tilde{k}_2, \beta)} \bar{\psi}^i_\alpha(\tilde{k}) \left( \gamma^0 \Lambda_+(\tilde{k}_1, \beta) \right)_{\alpha\beta} \tilde{\psi}^j_\beta(\tilde{k})$$

$$= (2\pi)^{-3} \int \bar{\psi}^i_\alpha(\tilde{k}) M_{\alpha\beta}(\tilde{k}) \psi^j_\beta(\tilde{k}) d\tilde{k} d\tilde{k}' \quad (38).$$

Hence, with the identification $M_{\alpha\beta}(\tilde{k})\big|_{|\tilde{k}| \to 0} = m'_Q(\beta) \delta_{\alpha\beta}$, the effective quark mass, $m_Q(\beta)$ as given by

$$m_Q(\beta) = m_0 - \frac{g^2}{3\pi^2} \int \frac{k^2}{k^2 + \phi(k, \beta)} x \frac{m_Q(\beta)}{\epsilon(k, \beta)} \left( \sin^2 \theta_-(\tilde{k}, \beta) + \sin^2 \theta_+(\tilde{k}, \beta) \right). \quad (39)$$

This is the self consistency requirement for the quark mass, and is solved iteratively where the input $m_Q(\beta)$ of the right hand side through $\epsilon(k, \beta)$ becomes equal to the output $m_Q(\beta)$ of the left hand side.

We shall extremise over the thermodynamic potential containing $\epsilon_0(\beta)$. For this purpose, as earlier we shall take

$$\sinhf(\tilde{k}) = Ae^{-Bk^2/2}, \quad (40)$$

which corresponds to taking a gaussian distribution for perturbative gluons in nonperturbative vacuum. The energy density, $\epsilon_0(\beta)$ in terms of the dimensionless quantities $x = \sqrt{B}k, \mu_G = \sqrt{B}m_G(\beta), \mu_Q = \sqrt{B}m_Q(\beta)$ and $y = \frac{\beta}{\sqrt{B}}$ then gets parametrised as
\[ \epsilon_0(A, \beta) = \frac{1}{B^2}(I_F(y) + I_1(A, y) + I_2(A, y) + I_3(A, y)^2 + I_4(A, y)) \]
\[ \equiv \frac{1}{B^2}F(A, y), \quad (41) \]

where
\[ I_F(y) = \frac{6}{\pi^2} \int \frac{x^2dx}{\epsilon(x, y)}(\sin^2 \theta_-(\vec{x}, y) + \sin^2 \theta_+(\vec{x}, y))(\mu_Q(y)\mu_0 + x^2), \quad (42a) \]

and
\[ I_4(A, y) = 4 \times (2\pi)^{-6} \int d\vec{x}'G_1(\vec{x}, y) + G_2(\vec{x}, y) \]
\[ = \frac{4\times(2\pi)^{-6} \int d\vec{x}'G_1(\vec{x}, y) + G_2(\vec{x}, y)}{x^2 + \phi(x, y)}. \quad (42b) \]

In the above, \( G_1(\vec{x}, y) = G(\vec{x}, y) \) of \[3\], and
\[ G_2(\vec{x}, y) = -g^2 \int d\vec{x}' \left[ \left( 1 + \frac{\mu_Q(y)^2}{\epsilon(\vec{x}', y)\epsilon(\vec{x}' - \vec{x}, y)} \right) + \frac{\vec{x}'(\vec{x}' - \vec{x})}{\epsilon(\vec{x}', y)\epsilon(\vec{x}' - \vec{x}, y)} \right] \]
\[ \times \left( \sin^2 \theta_-(\vec{x}', y) \sin^2 \theta_-(\vec{x}' - \vec{x}, y) + \sin^2 \theta_+(\vec{x}', y) \sin^2 \theta_+(\vec{x}' - \vec{x}, y) \right) \]
\[ - \left( 1 - \frac{\mu_Q(y)^2}{\epsilon(\vec{x}', y)\epsilon(\vec{x}' - \vec{x}, y)} \right) \]
\[ \times \left( \sin^2 \theta_-(\vec{x}', y) \sin^2 \theta_+(\vec{x}' - \vec{x}, y) + \sin^2 \theta_+(\vec{x}', y) \sin^2 \theta_-(\vec{x}' - \vec{x}, y) \right) \]. \quad (43)

The other expressions \( I_1(A, y), I_2(A, y), I_3(A, y) \) and \( \phi(x, y) \) are the same as in \[3\]. The above integrals contain \( \mu_G(y) \) and \( \mu_Q(y) \) which are determined from the self consistency requirements \[3\]
\[ \mu_G(y)^2 = \frac{2g^2}{\pi^2} \int \frac{x^2dx}{\omega(x, y)} \left[ \left( A^2e^{-x^2} + Ae^{-x^2/2}(1 + A^2e^{-x^2})^{1/2} \right) \left( 1 + \frac{2}{\exp(y\omega(x, y)) - 1} \right) \right. \]
\[ \left. + \frac{1}{\exp(y\omega(x, y)) - 1} \right], \quad (44) \]

and
\[ \mu_Q(y) = \mu_0 - \frac{g^2}{3\pi^2} \int dx \frac{x^2}{x^2 + \phi(x, y)} \times \frac{\mu_Q(y)}{\epsilon(x, y)} \left( \sin^2 \theta_-(x, y) + \sin^2 \theta_+(x, y) \right). \quad (45) \]

\( \mu_G(y) \) and \( \mu_Q(y) \) are solved through an iterative procedure \[3\]. We shall now calculate the thermodynamic potential and minimise the same.
III. EXTREMISATION OF THERMODYNAMIC POTENTIAL AND RESULTS

At zero temperature, we had considered extremisation of energy density to obtain the vacuum structure. At finite temperatures and baryon densities, the relevant quantity for extremisation is the thermodynamic potential, which at temperature $T = 1/\beta$ is given as

$$F(A, \beta) = \epsilon_0(A, \beta) - \frac{1}{\beta} (S_G + S_F) - \mu_B \rho_B.$$  \hspace{1cm} (46)

Here $\mu_B = 3\mu$ is the baryon chemical potential corresponding to the baryon number density $\rho_B$ given as, with two quark flavours,

$$\rho_B = \frac{1}{3} \times 2 \times 3 \times 2 \times \frac{1}{(2\pi)^3} \int \left( \sin^2 \theta_- - \sin^2 \theta_+ \right) d\vec{k},$$  \hspace{1cm} (47)

and $\mu$ as in equation (23b) is the quark chemical potential. Further, in the above, $\epsilon_0(A, \beta)$ is as given in equation (30), and the entropy densities $S_G$ and $S_F$ for the gluon and quark fields are given as

$$S_G = -2 \times 8 \times (2\pi)^{-3} \int d\vec{k} \left( \sinh^2 \theta \log(\sinh^2 \theta) - \cosh^2 \theta \log(\cosh^2 \theta) \right)$$ \hspace{1cm} (48)

and

$$S_F = -3 \times 2 \times 2 \times (2\pi)^{-3} \int d\vec{k} \left( \sin^2 \theta_- (\vec{k}, \beta) \log(\sin^2 \theta_- (\vec{k}, \beta)) \right. + \left( \cos^2 \theta_- (\vec{k}, \beta) \right) \log(\cos^2 \theta_- (\vec{k}, \beta)) + \sin^2 \theta_+(\vec{k}, \beta) \log(\sin^2 \theta_+(\vec{k}, \beta)) + \left( \cos^2 \theta_+(\vec{k}, \beta) \right) \log(\cos^2 \theta_+(\vec{k}, \beta)).$$  \hspace{1cm} (49)

Clearly, the factor $2 \times 8$ in (48) above comes from the transverse and colour degrees of freedom for the gluon fields and the factor $3 \times 2 \times 2$ in (49) comes from the colour, flavour and spin degrees of freedom for the quarks as well as for the antiquarks. For the minimisation of thermodynamic potential we may scale out the dimensional parameter $1/B^2$ and write

$$F(A, \beta) \equiv \frac{1}{B^2} F_1(A, y) = \frac{1}{B^2} \left[ F(A, y) - \frac{1}{y} (S_G(A, y) + S_F(A, y)) - \mu'_B \rho'_B \right].$$ \hspace{1cm} (50)
where \( F(A, y) = B^2 \epsilon_0(A, \beta) \), \( \mu_B' = \sqrt{B} \mu_B, \rho_B' = B^{3/2} \rho_B \) and the entropy densities \( S_G(A, y) \) and \( S_F(A, y) \) in dimensionless units are given as

\[
S_G(A, y) = -\frac{8}{\pi^2} \int x^2 dx \left\{ \left( \frac{1}{\exp(y\omega(x, y))} \right) \log \left( \frac{1}{\exp(y\omega(x, y))} \right) - \left( 1 + \frac{1}{\exp(y\omega(x, y))} \right) \log \left( 1 + \frac{1}{\exp(y\omega(x, y))} \right) \right\} \tag{51}
\]

and

\[
S_F(A, y) = -\frac{6}{\pi^2} \int x^2 dx \left\{ \left( \frac{1}{\exp(y(\epsilon(x, y) - \mu))} \right) \log \left( \frac{1}{\exp(y(\epsilon(x, y) - \mu))} \right) + \left( 1 - \frac{1}{\exp(y(\epsilon(x, y) - \mu))} \right) \log \left( 1 - \frac{1}{\exp(y(\epsilon(x, y) - \mu))} \right) \right. \\
+ \left( \frac{1}{\exp(y(\epsilon(x, y) + \mu))} \right) \log \left( \frac{1}{\exp(y(\epsilon(x, y) + \mu))} \right) + \left( 1 - \frac{1}{\exp(y(\epsilon(x, y) + \mu))} \right) \log \left( 1 - \frac{1}{\exp(y(\epsilon(x, y) + \mu))} \right) \right\} . \tag{52}
\]

We note that for each \( A \), the gluon and quark masses are determined self consistently through equations (44) and (45) for the evaluation of the right hand side of equation (50). We extremise \( F_1(A, y) \) of equation (50) with respect to the parameter \( A \) and obtain the optimum value of \( A \) as \( A_{\text{min}} \) at a given temperature \( T \) and baryon density \( \rho_B \). In fig.1 we plot \( A_{\text{min}} \) versus \( T \) at zero baryon density for \( g^2/4\pi = 0.8 \) against the earlier curve which was without quark excitations in the thermal vacuum. We find that the critical temperature is about 205 MeV in contrast to 275 MeV of Ref. \[3\] where quark excitations were not included. We have taken here the coupling constant \( g^2/4\pi = 0.8 \) and the Lagrangian quark mass \( m_0=300 \text{ MeV} \) as a typical value for the constituent quark mass. Such a result is similar to that of lattice QCD where inclusion of two flavours of quarks decreases the critical temperature \[15\] to about 150 MeV \[16\] from about 235 MeV of quenched approximation \[17\]. For quark mass \( m_0 \) less than 300 MeV, \( T_C \) decreases below 205 MeV, but for very small quark masses, the numerical calculations tend to be unreliable. \( A_{\text{min}} \) is plotted in fig.2 as a function of baryon density \( \rho_B \) for temperatures 0 and 100 MeV. We find that \( A_{\text{min}} \) decreases with increase in baryon density \( \rho_B \), and vanishes at and above a critical value, \( (\rho_B)_{\text{crit}} \) of \( \rho_B \). The values of \( (\rho_B)_{\text{crit}} \) for temperatures 0 and 100 MeV are 0.36 fm\(^{-3} \) and
0.88 fm$^{-3}$ respectively. Clearly in fig. 2 $A_{\min}$ decreases with $\rho_B$ and hence in fig. 1 for finite $\rho_B$ the $A_{\min}$ versus $T$ curve will always lie below the solid curve, yielding a smaller critical temperature. In fig.3, we plot the effective gluon mass as a function of $\rho_B$ for the above temperatures. We see that it starts decreasing with increase in the baryon density, $\rho_B$, and for zero temperature, it becomes zero at and above $\langle \rho_B \rangle_{\text{crit}}=0.36$ fm$^{-3}$ and for $T=100$ MeV, it decreases and remains almost a constant above $\langle \rho_B \rangle_{\text{crit}}=0.88$ fm$^{-3}$. This is because, when the condensate function vanishes, the effective gluon mass depends only upon the thermal contributions as in equation (44) and is independent of the baryon density. We then plot in fig.4, the effective quark mass, $m_Q$ as function of $\rho_B$, which starts decreasing with increase in $\rho_B$ and there is a discontinuity at $\langle \rho_B \rangle_{\text{crit}}$ above which the quark mass decreases slowly. This discontinuity in the mass is associated with abrupt vanishing of condensate function with $A_{\min}$ approaching zero at the critical baryon density. Similar behaviour was observed for the effective nucleon mass by Ellis et al in Ref. 9, where however the critical baryon density at which the gluon condensates vanishes was much higher, being of the order of 2.37 fm$^{-3}$. This discontinuity might indicate a possible link between vanishing of gluon condensates and chiral symmetry restoration.

We next estimate the SVZ parameter using the value of $A_{\min}$. This at any temperature is given as

$$\frac{g^2}{4\pi^2} <: G_{\mu\nu}^a G^{a\mu\nu} :\rangle_{\text{vac}} = \frac{1}{B^2} \frac{g^2}{\pi^2} \left[ - I_1(A, y) + I_2(A, y) + I_3(A, y)^2 - I_4(A, y) \right]_{A=A_{\min}}, \quad (53)$$

with $I_1(A, y), I_2(A, y), I_3(A, y)$ and $I_4(A, y)$ as in equation (41) 3. In fig.5, we plot the SVZ parameter given above as a function of $\rho_B$. It starts decreasing with increase in $\rho_B$ and for $T=0$, becomes minimum at $\langle \rho_B \rangle_{\text{crit}}$ and then increases with increase in $\rho_B$. It may be noted that although the condensate function vanishes at $\langle \rho_B \rangle_{\text{crit}}$, the SVZ parameter is nonzero and increases with density. This is because the contribution from the fermionic sector to $I_4$ is negative and increases in magnitude with density. Similar behaviour is also seen for $T=100$ MeV except that the magnitude here is smaller as we have extra positive thermal contributions to $I_1$ and $I_4$. 

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We also calculate the pressure $P$ given as \[18\]

$$P(\beta) = -\mathcal{F}(A, \beta)igg|_{A=A_{\text{min}}}. \quad (54)$$

In fig. 6, we plot the pressure as function of the baryon number density, $\rho_B$ for the temperatures 0 and 100 MeV. We see that the pressure increases with increase in $\rho_B$. We may compare the present nonperturbative results with the perturbative estimation of the pressure \[19\] given as

$$P_{\text{pert}} = \frac{1}{2\pi^2} \left[ \mu_f k_f (\mu_f^2 - 2.5m^2_f) + 1.5m^2_f ln \left( \frac{\mu_f + k_f}{m_f} \right) \right]$$

$$- \frac{\alpha_s}{\pi^3} \left[ 1.5 \times \left( \mu_f k_f - m^2_f ln \left( \frac{\mu_f + k_f}{m_f} \right) \right)^2 - k^4_f \right], \quad (55)$$

where $k^2_f = \mu^2_f - m^2_f$ and $\mu_f = \mu_B/3$ is the baryon chemical potential. In the above we have taken $\alpha_s = 0.8$ and considered the case for two flavours of quarks with masses 300 MeV each. This perturbative equation of state for zero temperature is plotted as the dashed curve in Fig. 6. As may be seen, the present equation of state is stiffer than the perturbative equation of state.

**IV. DISCUSSIONS**

In the present paper, we have extended the study of nontrivial ground state structure of QCD with gluon condensates at zero and finite temperatures \[3\] to the case of nonzero baryon densities. The calculations as earlier are done in Coulomb gauge. We would have also liked to include the quark condensates for the vacuum structures in these calculations. It would have been nice to demonstrate that this effect is not large, as we believe to be the case. However, simultaneous consideration of gluon condensates and quark condensates with appropriate self consistency requirements becomes computationally prohibitive and has not been attempted here.

The modified gluon condensate function and the quark distribution functions are obtained here through minimisation of the thermodynamic potential with, as stated, a *self*
consistent determination for the quark and gluon mass parameters. At different temperatures, the gluon condensates disappear at critical values of the baryon density. Inclusion of thermal excitations in quark sector as here decreases the critical temperature from 275 MeV to 205 MeV even for zero baryon density, and shall be lower at finite baryon densities. At zero temperature, the above critical baryon density is about 0.36/fm$^3$. The results are similar to that of lattice QCD [15–17].

We note that the present approach is based on QCD Lagrangian of quarks and gluons without the introduction of effective fields [3], but with a nonperturbative variational approach and an explicit vacuum structure [3]. This may be relevant for quark gluon plasma where baryon structures are likely to dissolve at high densities or temperatures [20–22].

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Figure Captions

**Fig.1:** We plot $A_{\text{min}}$ as function of $T$ in MeV. The dashed curve corresponds to the case of not including thermal excitations in the quark sector. Lowering of critical temperature with inclusion of quarks in the thermal vacuum may be noted.

**Fig.2:** We plot $A_{\text{min}}$ as function of $\rho_B$ in fm$^{-3}$ for temperatures 0 and 100 MeV.

**Fig.3:** We plot the gluon mass, $m_G$ in MeV as function of $\rho_B$ in fm$^{-3}$ for temperatures 0 and 100 MeV.

**Fig.4:** We plot the fermion mass, $m_Q$ in MeV as function of $\rho_B$ in fm$^{-3}$ for temperatures 0 and 100 MeV. The discontinuity in the same at critical baryon density may be noted.

**Fig.5:** We plot the SVZ parameter in units of $10^{-2} \text{GeV}^4$ as function of $\rho_B$ in fm$^{-3}$ for temperatures 0 and 100 MeV.

**Fig.6:** We plot the pressure, $P$ in units of MeV/fm$^3$ as function of $\rho_B$ in fm$^{-3}$ for temperatures 0 and 100 MeV. The perturbative equation of state as given by equation (55) for temperature, $T=0$ is given as the dashed curve of the same figure.