WHAT DO WE REALLY KNOW ABOUT COSMIC ACCELERATION?

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ABSTRACT

Essentially all of our knowledge of the acceleration history of the universe (even the discovery of the acceleration itself) is predicated on the validity of general relativity through the Friedmann equation. Assuming only a flat spacetime that is homogeneous and isotropic on large scales and described by a metric theory of gravity, we use SNe Ia to analyze the acceleration history. We infer robust conclusions about cosmic acceleration that are independent of a particular parameterization of the acceleration history. Specifically, we find (1) very strong (5 \sigma) evidence for a period of acceleration, (2) strong evidence that the acceleration has not been constant, (3) evidence for an earlier period of deceleration, and (4) only weak evidence that the universe has been accelerating since \( z \approx 0.3 \).

Subject headings: cosmology: theory — gravitation

1. INTRODUCTION

A still puzzling feature of our universe is its accelerated expansion. Measurements of Type Ia supernovae (SNe Ia) provide direct evidence (Riess et al. 1998; Perlmutter et al. 1999; Astier et al. 2006) and CMB anisotropy measurements provide important indirect evidence (Tegmark et al. 2004). Within the context of general relativity, the acceleration can be explained if the present mix of matter and energy in the universe includes a dark energy component that contributes about 2/3 of the critical density. Dark energy is defined by its large negative pressure, \( p_x < -\rho_x \), and nearly spatially uniform distribution (Turner & White 1997; Turner 1998).

In the dark energy picture, the acceleration history of a flat universe is simply fixed by the dark energy content \( \Omega_x \) and its equation-of-state parameter, \( w \equiv p_x/\rho_x \). The universe begins with a period of decelerated expansion and then, for \( w = constant < -1/3 \), crosses over to accelerated expansion at redshift

\[
z_c = \left[ \frac{\Omega_M}{(3w+1)(\Omega_M - 1)} \right]^{1/w} - 1 \sim 0.5;
\]

see Figure 1. A \( \Lambda \)CDM model with \( w = -1 \) and \( \Omega_M = 0.30 \pm 0.05 \) is a reasonable fit to the SNe and a wealth of other cosmological data, as is a more general \( \omega \)CDM model with \( w = -1.05 \pm 0.14 \) and \( \Omega_M = 0.29 \pm 0.03 \) (Tegmark et al. 2004). This in fact is the basic evidence for a period of accelerated expansion. The existence of an early period of decelerated expansion is also supported by the success of the gravitational instability theory of structure formation and of big bang nucleosynthesis (BBN). In particular, unless the equations governing the growth of small-density perturbations are radically different, it is not possible to grow structure from small perturbations in an accelerating universe (Turner & White 1997). In addition, it is difficult for a universe that accelerates during BBN to reproduce the light-element abundances unless one artificially adjusts the cosmic neutrino asymmetry (Carroll & Kaplinghat 2002).

While the energy of the quantum vacuum, mathematically equivalent to a cosmological constant, is the simplest explanation for the dark energy and cause of cosmic acceleration, all attempts to compute its numerical value have been unsuccessful, leading either to divergent results or numbers that are orders of magnitude too large; this is known as the cosmological constant problem (Weinberg 1988; Carroll 2001). Because it is possible that the energy of the quantum vacuum is zero or too small to explain cosmic acceleration, theoretical physicists have explored a plethora of possible explanations for the dark energy, from a very light scalar field (Ratra & Peebles 1988; Wetterich 1988; Frieman et al. 1995; Coble et al. 1997; Caldwell et al. 1998) to the influence of extra dimensions (Deffayet et al. 2002), and have even considered the possibility that there is no need for dark energy and that it arises from herefore neglected “ordinary” physics within general relativity (Kolb et al. 2005). A fair summary of the present state of affairs is that there is little understanding of why the universe is accelerating.

Given that state of affairs, it is not unreasonable to think more broadly, even entertaining the possibility that the explanation may well have to do with gravity theory itself (Carroll et al. 2004; Capozziello et al. 2003; Freese & Lewis 2002; Arkani-Hamed et al. 2002; Dvali & Turner 2003). After all, we are confident that Einstein’s theory must be modified to make it compatible with quantum mechanics, and perhaps cosmic acceleration has something to do with that modification. However, relaxing the assumption of general relativity (i.e., Einstein’s equation) means that we must abandon the Friedmann equation, which relates the expansion rate to the matter and energy densities. We might then wonder what we really know about cosmic acceleration, or even if the universe is really accelerating, since almost everything we have inferred about it depends on the assumption of the existence of dark energy.

Even when the assumption of general relativity is dropped by ignoring the Friedmann equation, SNe Ia still provide a direct kinematic probe of the expansion. One needs only to retain the weaker assumption of an isotropic and homogeneous space-time (which is supported by ample observational data) described by a metric theory of gravity (because there are few if any viable nonmetric theory alternatives).

Beginning with Turner & Riess (2002), a number of authors have studied the expansion history via this cosmographic or “Friedmannless” approach, each of them drawing from the myriad of possible kinematic parameters. The choices (defined in § 2.1)
include the cosmic scale factor $R(t)$ (John 2004, 2005, 2006), the expansion rate $H(z)$ (Lazkoz et al. 2005; Wang & Tegmark 2005), and the deceleration parameter $q(z)=[d \ln H/d \ln (1+z)]-1$ (Turner & Riess 2002; Elgaroy & Multamaki 2006). It has even been suggested (Blandford et al. 2004) that “cosmic jerk,” $j \equiv -[(d^3R/dt^3)/H^3]$, might be a good kinematic choice. One can also use an effective dark energy density defined by the Friedmann equation (Wang & Gamvich 2001; Tegmark 2002).

Operationally, these parameters are all equivalent since each can be expressed using integrals or derivatives of the others, but we feel that the deceleration parameter $q(z)$ is both convenient and undervalued. In general relativity, $q$ is related to the cosmic equation of state:

$$q = \frac{1}{2} (\Omega + 3 w_{\text{tot}}),$$

where $\Omega$ is the total density in units of the critical density and $w_{\text{tot}} \equiv p/\rho$ is the combined equation-of-state parameter for all components. For example, in a flat $\Lambda$CDM universe, $q(z)$ progresses from 1 to 0.5 to $-1$, crisply defining the radiation, matter, and dark energy dominated phases. Such information is obtained from $H(z)$ only through its derivative, and we find no strong motivation to go beyond $q$ to higher derivatives. Although the case $j = 1$ conveniently corresponds to a model that goes from matter-dominated behavior to cosmological constant–dominated behavior, $j = 1$ does not account for the early radiation-dominated epoch, and constant $j$ models do not span the space of interesting acceleration histories. Moreover, $q < 0$ immediately indicates accelerated expansion (and something new!), and it was, after all, the measurement of negative $q$ that surprised cosmologists at the turn of the century.

We would like to know what the SNe Ia themselves are telling us about the cosmic acceleration history. With the Friedmannless approach, there is always the concern that certain conclusions can be model dependent, i.e., that they depend on the parameterization of $q(z)$. A safe strategy is to allow ample freedom in the model, but this course of reducing our ability to get interesting constraints on any one model parameter. Wang & Tegmark (2005) skirt the modeling issue by directly reconstructing $H(z)$ from SNe Ia data, but it is harder to extract model-independent information about acceleration or dark energy in this way. Likewise, Daly & Djorgovski (2003, 2004, 2005) have developed a method to determine global trends in a suite of parameters including $q(z)$. Our goal is not to reconstruct $q(z)$, nor are we looking for a $q(z)$ model that nicely fits the SNe Ia data. Rather, we wish to discover features that any viable model of the acceleration history must have.

In this paper, we assume a flat Robertson-Walker metric, and, without using the Friedmann equation, we show that the following can be inferred from the SNe Ia data: (1) very strong (5 $\sigma$) evidence for a period of accelerated expansion, (2) strong evidence that the acceleration has not been constant, (3) evidence for an earlier period of deceleration, and (4) only weak evidence that the universe has not been decelerating since $z \sim 0.3$.

2. FRIEDMANNLESS COSMOLOGY

2.1. Kinematics

With only the assumption that spacetime can be described by a metric theory that is on the large isotropic and homogeneous, the Robertson-Walker metric still pertains:

$$ds^2 = -dt^2 + R(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega \right],$$

where $R(t)$ is the usual cosmic scale factor and we assume spatial flatness ($k = 0$) for simplicity (we will relax this assumption later). Likewise, the cosmic scale factor at a given epoch is related in the usual way to the redshift that a free-streaming photon has suffered since that epoch: $R(t)/R(0) = 1/(1+z)$. The epoch-dependent expansion and deceleration rates are defined as usual and are related to one another by

$$H(z) \equiv \frac{\dot{R}}{R},$$

$$q(z) \equiv -\frac{\ddot{R}}{RH^2} = \frac{d}{dt} \ln H^{-1} - 1 = \frac{d \ln H}{d \ln (1+z)} - 1,$$

$$H(z) = H_0 \exp \left[ \int_0^z [1 + q(u)] d \ln (1 + u) \right].$$

The relationship between detected energy flux $F$ of a source at redshift $z$ and its intrinsic luminosity $L$ is also unaffected,

$$F \equiv \frac{L}{4\pi dl^2},$$

$$dl = (1+z) \int_0^z \frac{du}{H(u)},$$

where $dl$ is the luminosity distance. Defining the distance modulus $\mu$ for a standard candle (or standardizable candle, e.g., SNe Ia) in the usual way,

$$\mu(z) \equiv [m_B(z) - M_B] = 5 \log (dl/Mpc) + 25,$$

the observable $\mu(z)$ is related to the acceleration history, $q(z)$, by

$$\mu(z) = 25 + 5 \log \left[ \frac{1+z}{H_0(1\text{ Mpc})} \times \int_0^z du \exp \left( -\int_0^u [1 + q(v)] d \ln v \right) \right].$$

Here $M_B$ and $m_B(z)$ are respectively the absolute and apparent magnitudes of the source. Equation (8) provides the
fundamental relationship between the deceleration history and SN Ia measurements.

2.2. Simple Kinematic Models

Our goal is to determine what we can robustly learn about the acceleration history. To begin, we use two simple parameterized models, \( q(z) = q_0 + zdq/dz \) and piecewise constant \( q(z) \), in part to illustrate how the conclusions one draws can depend on the model itself. In §2.3, we turn to a principal component analysis, which is much less susceptible to this malady. For all these analyses we use the gold set of SNe Ia culled by Riess et al. (2004), which contains 157 well studied SNe Ia between \( z = 0.01 \) and \( z = 1.76 \). Throughout, our parameter fits include a nuisance parameter, \( M \), and our constraints always include marginalization over \( M \).

Riess et al. (2004) analyzed their own gold set using the linear model, which has an 11% goodness of fit (\( \chi^2/\text{dof} = 173/153 \)), on par with the best flat \( \Lambda \)CDM with \( \Omega_m = 0.31 \), \( \chi^2/\text{dof} = 177/155 \). The fact that this model finds \( q_0 < 0 \) gives us confidence that the universe has accelerated recently. Furthermore, the positive slope is certainly an indication that \( q(z) \) was higher in the past. But can we trust that there was ever a transition? Unfortunately, this linear model always implies a transition redshift when its two parameters have opposite signs. Hence, it is unclear whether the transition is real or just an artifact. To illustrate, suppose that Riess et al. had found \( z_t = 3 \); since their farthest SNe Ia has \( z = 1.76 \), it would have been clear that \( z_t \) is implied not by the data but by the model itself. Since \( z_t \) actually does lie in the observed redshift range, evidence for a transition exists but remains unconvincing.

An alternate parameterization is a piecewise constant acceleration with two distinct epochs (Turner & Riess 2002):

\[
q(z) = \begin{cases} q_0 & \text{for } z \leq z_t, \\ q_1 & \text{for } z > z_t, \end{cases}
\]

(9)

where \( q_0 \) and \( q_1 \) correspond to the average values of \( q \) in their respective epochs. Figure 2 shows our confidence contours in the \( q_0 - q_1 \) plane for various values of \( z_t \). The best-fit model has \( z_t = 0.08, q_0 = -2.0, q_1 = 0.11, \chi^2/\text{dof} = 173/153 \), and a 13% goodness of fit; however, the \( \chi^2 \) minimum in \( z_t \) is broad and the constraints on \( q_0 \) and \( q_1 \) vary significantly with \( z_t \). Moreover, in the limit of \( z_t < 0.1 \), the model becomes essentially a one-zone model, as there are no SNe to constrain \( q_0 \) and \( q_1 \) and the error on \( q_0 \) (\( q_1 \)) blows up. Marginalizing over \( z_t \) with a prior that takes this into account, \( z_t \in [0.1, 1] \), one obtains the final constraint contour in Figure 2. Again there is strong evidence for recent acceleration (negative \( q_0 \) contains over 99.9% of the probability) and weaker evidence for past deceleration (positive \( q_1 \) contains 91% of the probability). The marginalized distribution for \( z_t \) is very broad (see Fig. 3) because \( q_0 \) and \( q_1 \) are poorly constrained for low and high \( z_t \), and meaningful constraints to \( z_t \) cannot be obtained.

As shown in Figure 1, the best fitting kinematic models appear different from \( \Lambda \)CDM; in particular, they both find a lower redshift for the transition to acceleration. To address the significance of this, we generated 1000 mock SNe Ia data sets, assuming \( \Lambda \)CDM with \( \Omega_m = 0.31 \), each with 157 mag, and errors taken from the gold set. We fit both kinematic models to the simulated data sets and tabulated the best-fit parameters. Figures 3 and 4
show that the kinematic parameters we expect to measure in a \( \Lambda \)CDM universe are consistent with the constraints given by the gold set. If there is a difference between the actual acceleration history of the universe and \( \Lambda \)CDM, these kinematic models and the present data do not have the statistical power to demonstrate it. Both the two-epoch and linear \( q(z) \) models provide direct evidence for a recent period of acceleration and weaker evidence for a past period of deceleration. However, as discussed, they each have their shortcomings. To further illustrate, we ask the provocative question: could the universe be decelerating today? First, as a practical matter, the Hubble flow only begins to dominate local peculiar motions for \( z > 0.01 \); this implies that any knowledge of the expansion dates back to at least \( 0.01H_0^{-1} \approx 0.1 \) Gyr. So as far as we know, the universe could have stopped expanding 100 Myr ago! Next, consider the standard Taylor expansion of the luminosity distance,

\[
d_L = H_0^{-1}z + \frac{1}{2}(1 - q_0)H_0^{-1}z^2 + \ldots
\]

Because the acceleration term is second order in \( z \), one is further limited in detecting its effect by the accuracy of measuring \( d_L \). 10% uncertainty in \( H_0 \) implies that objects at redshift greater than \( z \sim 0.1 \) are needed, even to measure \( q_0 \) with an error of order unity. Thus, direct evidence of acceleration/deceleration more recent than 1 Gyr ago will be difficult to obtain; this can be seen quantitatively in Figure 2, where for \( z_t < 0.1 \) the error ellipse on \( q_0 \) blows up.

The reason for asking whether or not the universe is accelerating today is not just academic; there exist dark energy models that predict a history of alternating acceleration and deceleration (Dodelson et al. 2000; German & de la Macorra 2004). To address this issue we consider a three-epoch model,

\[
q(z) = \begin{cases} 
q_0 & \text{for } z \leq z_t, \\
q_1 & \text{for } z_t < z < z_e, \\
0.5 & \text{for } z \geq z_e,
\end{cases}
\]

motivated by the evidence for a recent epoch of acceleration and an earlier epoch of deceleration. The parameters \( q_0 \) and \( z_t \) allow us to address the question of whether or not the universe has been accelerating recently. For the borderline case, we set \( q_0 = 0 \); Figure 5 shows our goodness of fit as a function of \( z_t \) after minimizing \( \chi^2 \) with respect to \( M \) and \( q_1 \). With an early transition at \( z_t = 0.3 \), a long epoch of recent deceleration is consistent with the data at the 10% level. Larger values of \( q_0 \) will decrease the goodness of fit, but note that there may exist better fitting parameterizations that raise it. In summary, the present SNe Ia data cannot rule out the possibility that the universe has actually been decelerating for the past 3 Gyr (i.e., since \( z = 0.3 \)).

2.3. Principal Component Analysis

Here, we follow Huterer & Starkman (2003) and let the data itself guide our parameterization. Through the use of a Fisher matrix (Dodelson 2003; Bond et al. 1998), we will be able to discover accurately measured features of the cosmic acceleration history. We start by dividing the redshift range of the SNe Ia into \( N \) bins of width \( \Delta z \) and write

\[
q(z) = \sum_{i=1}^{N} q_i c_i(z),
\]
where the \( c_i(z) \) equal unity inside the \( i \)th bin and zero outside (note that as \( N \to \infty \), arbitrary \( q(z) \) can be specified). Using only the gold set error bars, equations (7), and (12), it is straightforward to numerically compute the \((N+1) \times (N+1)\) Fisher matrix \( F \) for the \( q_i \) and the nuisance parameter \( M \). Its inverse, \( F^{-1} \), approximates the full covariance matrix that one would calculate from the entire gold set. We have found from experience that \( F \) is insensitive to the choice of fiducial parameters; therefore, for simplicity, we compute \( F \) using \( q_i = 0 \) for all \( i \). After marginalizing \( F \) over \( M \), we find a new basis of orthonormal functions \( e_i(z) \) which diagonalizes \( F \). The deceleration rate can be expanded in terms of these modes or “principal components”:

\[
q(z) = \sum_{i=1}^{N} \alpha_i e_i(z).
\]

The eigenvalues of \( F^{-1} \) estimate the uncertainties \( \sigma (\alpha_i) \) in the coefficients \( \alpha_i \). The advantage of the principle component basis is that the \( \alpha_i \) are uncorrelated, i.e., any pair of coefficients has a nondegenerate error ellipse. With \( \Delta z = 0.05 \), the six modes of \( q(z) \) with the smallest errors are shown in Figure 6.

We can use the principal components to reconstruct \( q(z) \), i.e., using equation (13) as our model, we can fit the \( \alpha_i \) to SNe Ia data. Since \( F \) is insensitive to fiducial parameters, we can keep the \( \sigma (\alpha_i) \) as error estimates. Note that if \( F \) were sensitive to fiducial parameters, we would have to ensure that the best-fit model was the same as the fiducial model before calculating the \( \sigma (\alpha_i) \); such a model can be found iteratively. Table 1 shows the results of parameter fits which use progressively more modes, starting with the most well constrained. The 1st and 2nd mode coefficients are respectively about 5 and 2 \( \sigma \) negative while higher modes are consistent with zero due to increasingly large variances. Increasing the number of modes slightly alters the fit; however, since the data are insensitive to higher modes and since the coefficients are uncorrelated, we find it unlikely that keeping more modes will change the coefficients in Table 1 by 1 \( \sigma \).

It is sensible to reconstruct \( q(z) \) using only the well-measured modes (Huterer & Starkman 2003). Doing so gives us a coarse yet accurate picture of the deceleration parameter in its “mode space,” but it also introduces a bias: the reconstruction will lack features of the true \( q(z) \) which are poorly probed by SNe Ia. Consider for instance the two-mode reconstruction illustrated in Figure 7; it approaches zero at high redshift, but it is clear that we cannot trust this feature. It is also clear that the reconstruction will look different if we include more modes. So which features of the reconstruction can we trust?

First, we can trust that the universe has at some time accelerated. To see this, we note that the 1st mode is completely positive. Since the modes are orthonormal, the coefficients \( \alpha_i \) are given by

\[
\alpha_i = \int dz q(z) e_i(z).
\]

**TABLE 1**

RECONSTRUCTION OF \( q(z) \)

| Number of Modes in Fit | \( \chi^2/\text{dof} \) | \( \alpha_1 \) | \( \alpha_2 \) | \( \alpha_3 \) | \( \alpha_4 \) |
|------------------------|-------------------|-------------|-------------|-------------|-------------|
| 1                      | 177.7/155 = 1.15  | -1.10       | ...         | ...         | ...         |
| 2                      | 173.2/154 = 1.12  | -1.08       | -1.61       | -1.10       | ...         |
| 3                      | 172.6/153 = 1.13  | -1.08       | -1.71       | -1.10       | ...         |
| 4                      | 172.6/152 = 1.14  | -1.08       | -1.56       | -1.30       | 0.61        |
| \( \sigma (\alpha_i) \) |                   | 0.22        | 0.72        | 1.52        | 2.54        |

**Notes:** Reconstruction of \( q(z) \) by eq. (13) using well-constrained principal components. The table shows the results of parameter fits which use progressively more modes. The gray band shows the error corresponding to 1 \( \sigma \) uncertainties in both \( \alpha_1 \) and \( \alpha_2 \). The solid line is \( q(z) \) for LCDM with \( \Omega_M = 0.3 \). As in Fig. 1, the kinematic model suggests a later transition to acceleration than \( \Lambda \) CDM does.

**Fig. 5.—**Maximum goodness of fit for three-epoch \( q(z) \) models that are not accelerating today. Setting \( q(z < z_c) = 0 \) and \( q(z > z_c) = 0.5 \), the solid, long-dashed and short-dashed lines show \( z_c = 0.5, 0.3, \) and 0.15 respectively.

**Fig. 6.—**Six most well-constrained principal components of \( q(z) \). They are normalized so that \( e_i(0) > 0 \) and \( \int e_i(z)^2 dz = 1 \). The errors on the mode amplitudes, \( \sigma (\alpha_i) \), are estimated from the Fisher matrix.

**Fig. 7.—**Reconstruction of \( q(z) \) using its two most well-constrained principle components (eq. [13]). The short-dashed line is the best two-mode fit to SNe Ia data; it has a 14\% goodness of fit. The gray band shows the error corresponding to 1 \( \sigma \) uncertainties in both \( \alpha_1 \) and \( \alpha_2 \). The solid line is \( q(z) \) for LCDM with \( \Omega_M = 0.3 \). As in Fig. 1, the kinematic model suggests a later transition to acceleration than \( \Lambda \) CDM does.
It follows that $\alpha_1$ is negative only if $q(z) < 0$ for some $z$. Our estimate that $\alpha_1$ is 5 $\sigma$ negative therefore implies a 5 $\sigma$ detection of cosmic acceleration at some redshift. The shape of $e_i(z)$ implies that this redshift is likely to be low. This conclusion is not model dependent since we are free to decompose $q(z)$ using any complete basis we like. One could make a similar argument using e.g., a polynomial or Fourier expansion, but the principle component expansion is advantaged since the uncertainty in $\alpha_1$ is small and nondegenerate with other parameters.

Next, we can also trust that $q(z)$ is not constant. Fitting a constant $q$ model to the gold set yields $q = -0.29$ with a 6% goodness of fit, but fitting only to modes that have been convincingly measured shows that constant $q$ really fails. Using the reconstructions in Table 1, we fit a constant $q$ model directly to the mode coefficients. The best fit to these reconstructions is also $q = -0.29$; it is essentially determined by the first two modes, which have the smallest errors. We obtain the following goodness of fit:

$$P(\chi^2 > \chi^2_{\text{min}}) = \begin{cases} 
1.0\% & \text{first 2 modes}, \\
0.4\% & \text{first 3 modes}, \\
1\% & \text{first 4 modes}.
\end{cases}$$

The main reason for the poor fit is easily seen in Figure 8: if $q$ were constant, the first two coefficients should have roughly opposite values, but they do not. Higher modes will tend to improve the fit by adding degrees of freedom without significantly increasing $\chi^2$.

Throughout, we have assumed a flat universe; here, we relax that assumption to illustrate how our conclusions depend on it. The generalized equation for luminosity distance is then

$$d_L = (1 + z)f \left[ \int_0^z \frac{du}{H(u)} \right],$$

$$f(x) = \begin{cases} 
\sin(xK^{1/2})/(K)^{1/2} & K > 0, \\
x & K = 0, \\
\sinh(x[-K]^{1/2})/(-K)^{1/2} & K < 0,
\end{cases}$$

where $K \equiv k/R(0)^2$. Positive curvature brightens SNe [decreases $d_L(z)$] and can therefore be used to mimic deceleration at high redshifts. As noted by Elgarøy & Multamäki (2006), a positively curved universe with constant negative acceleration fits the gold set surprisingly well, and allowing $q$ to vary does not significantly improve the fit. Similarly, we find that the errors on the $\alpha_i$ increase when curvature is allowed to vary, and a constant $q$ model cannot be confidently ruled out even when fit to well-measured modes as above (e.g., two modes gives a 2% goodness of fit). We notice a similar trend among previous cosmographic studies. For instance, assuming a flat universe, Daly & Djorgovski (2004) find evidence in the gold set for deceleration before $z \approx 0.4$, as in Riess et al. (2004); however, John (2005) does not assume flatness and finds no such evidence. In addition to increasing the errors on the $\alpha_i$, including a curvature parameter changes the shape of the principle components so that the first mode is no longer completely positive; thus, our previous argument for a 5 $\sigma$ detection of accelerated expansion is no longer valid.

3. CONCLUSIONS

Understanding dark energy may ultimately require us to abandon or change the familiar equations of general relativity. Hence it is crucial that we keep track of which observations and conclusions depend on the assumption of general relativity and which are more robust. Without the Friedmann equation, it is still possible to describe the universe with a Robertson-Walker metric and infer the expansion history via the dimming and cosmological redshifting of SNe Ia. We used this fact to learn about cosmic acceleration kinematically.

Focusing on the deceleration parameter, $q(z)$, we aimed to determine what can be learned independently of particular parameterizations of the expansion history. We have shown, under the assumption of a flat Robertson-Walker metric, that the gold set of SNe Ia from Riess et al. (2004) contains the following information:

1. **Very strong evidence that the universe once accelerated.**—Various models imply this, and strong independent evidence comes from measuring the best constrained principal component of $q(z)$. The acceleration is likely to have been relatively recent in cosmic history ($z \lesssim 0.5$).

2. **Strong evidence that $q$ was higher in the past.**—The constant $q$ model has a poor goodness of fit, especially when fit in mode space using only well-measured modes. In addition, the linear $q(z)$ model indicates that the average $dq/dz$ is positive.

3. **Weak evidence that the universe once decelerated.**—Both the two-epoch and two-mode models of $q(z)$ favor past deceleration with transitions to acceleration after $z = 0.5$. The linear model also predicts a transition. However, it’s plausible that this is a model-dependent feature.

4. **Little or no evidence that the universe is currently accelerating.**—It is manifestly difficult to constrain $q(z < 0.1)$ with SNe Ia. Using a three-epoch model, we further showed that a long period (e.g., $z = 0 \rightarrow 0.3$) of recent deceleration is consistent with data.

5. **$\Lambda$CDM and wCDM are acceptable fits to SNe Ia data.**—While there are tantalizing indications that the actual acceleration history may be different, none of the kinematic models studied here have revealed robust features about cosmic acceleration that differ from $\Lambda$CDM or wCDM. Moreover, none of the kinematic models fit the data significantly better.

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