A SIMULATION OF CONVECTIVE DYNAMO IN THE SOLAR CONVECTIVE ENVELOPE: MAINTENANCE OF THE SOLAR-LIKE DIFFERENTIAL ROTATION AND EMERGING FLUX

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ABSTRACT

We report the results of a magnetohydrodynamic (MHD) simulation of a convective dynamo in a model solar convective envelope driven by the solar radiative diffusive heat flux. The convective dynamo produces a large-scale mean magnetic field that exhibits irregular cyclic behavior with oscillation time scales ranging from about 5 to 15 yr and undergoes irregular polarity reversals. The mean axisymmetric toroidal magnetic field is of opposite signs in the two hemispheres and is concentrated at the bottom of the convection zone. The presence of the magnetic fields is found to play an important role in the self-consistent maintenance of a solar-like differential rotation in the convective dynamo model. Without the magnetic fields, the convective flows drive a differential rotation with a faster rotating polar region. In the midst of magneto-convection, we found the emergence of strong super-equipartition flux bundles at the surface, exhibiting properties that are similar to emerging solar active regions.

Key words: dynamo – magnetohydrodynamics (MHD) – Sun: interior

Online-only material: animation, color figures

1. INTRODUCTION

Despite the turbulent nature of solar convection, the Sun’s large-scale magnetic field exhibits remarkable order and organization such as the 11 yr sunspot cycle (e.g., Maunder 1922) and the Hale’s polarity rule of the bipolar active regions (Hale et al. 1919; Hale & Nicholson 1925). In recent years, global fully dynamic three-dimensional (3D) convective dynamo simulations have been making headway in producing the solar-like cyclic behavior of the large-scale magnetic field (e.g., Ghizaru et al. 2010; Racine et al. 2011; Kapyla et al. 2012; Augustson et al. 2013) and the self-consistent formation of buoyant, active region-like emerging tubes from dynamo generated strong toroidal fields (Nelson et al. 2011, 2013, 2014). Most of these simulations have differential rotation with cylindrical iso-rotation contours throughout the convection zone (CZ; see review by Charbonneau 2013), and some are considering rotation rate three times the solar rate. In this paper, we present a convective dynamo simulation driven by the solar radiative diffusive heat flux and maintains a differential rotation profile that resembles more closely to the solar differential rotation in the CZ in terms of the pole-equator contrast and the more conical iso-contours of rotation in the mid-latitude region. The convective dynamo produces a large-scale mean magnetic field with irregular cyclic behavior and polarity reversals very similar to a convective dynamo presented in Miesch et al. (2011). We demonstrate in this paper the important role the magnetic fields play in the maintenance of the solar-like differential rotation. We also show the emergence of strong super-equipartition flux tubes near the surface that exhibit some properties similar to emerging solar active regions.

2. THE NUMERICAL MODEL

We solve the following anelastic MHD equations using a finite-difference spherical anelastic MHD code (Fan 2008; Fan et al. 2013):

\[
\nabla \cdot (\rho_0 \mathbf{v}) = 0, \tag{1}
\]

\[
\rho_0 \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = \nabla \cdot \mathbf{F} = 2 \rho_0 \mathbf{v} \times \mathbf{\Omega} - \nabla p_1 + \rho_1 \mathbf{g} + \frac{1}{4\pi} (\nabla \times \mathbf{B}) \times \mathbf{B} + \nabla \cdot \mathbf{D} \tag{2}
\]

\[
\rho_0 T_0 \left[ \frac{\partial s_1}{\partial t} + (\mathbf{v} \cdot \nabla) (s_0 + s_1) \right] = \nabla \cdot (K \rho_0 T_0 \nabla s_1) - (\mathbf{D} \cdot \nabla) \mathbf{v} + \frac{1}{4\pi} \eta (\nabla \times \mathbf{B})^2 - \nabla \cdot \mathbf{F}_{rad} \tag{3}
\]

\[
\nabla \cdot \mathbf{B} = 0 \tag{4}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) - \nabla \times (\eta \nabla \times \mathbf{B}), \tag{5}
\]

\[
\frac{\rho_1}{\rho_0} = \frac{\rho_1}{T_1} - \frac{T_1}{T_0}, \tag{6}
\]

\[
s_1 = \frac{T_1}{T_0} - \frac{\eta - 1}{\gamma - 1}, \tag{7}
\]

In the above, \(s_0(r), \rho_0(r), \rho_1(r), T_0(r), \) and \(\mathbf{g} = -g_0(r) \hat{\mathbf{r}}\) denote the profiles of entropy, pressure, density, temperature, and the gravitational acceleration of a time-independent reference state of hydrostatic equilibrium and nearly adiabatic stratification, \(c_p\) is the specific heat capacity at constant pressure, \(\gamma\) is the ratio of specific heats, and \(\mathbf{v}, \mathbf{B}, s_1, \rho_1, \rho_0, T_1,\) and \(T_0\) are the velocity, magnetic field, entropy, pressure, density, and temperature to be solved that describe the changes from the reference state. \(\mathbf{\Omega}\) denotes the solid body rotation rate of the Sun and is the rotation rate of the frame of reference, where \(\Omega = 2.7 \times 10^{-6} \text{ rad s}^{-1}\). \(\mathbf{D}\) is the viscous stress tensor: \(D_{ij} = \rho_0 \nu [S_{ij} - (2/3)(\nabla \cdot \mathbf{v}) \delta_{ij}]\), where \(\nu\) is the kinematic viscosity, \(\delta_{ij}\) is the unit tensor, and \(S_{ij}\) is the strain rate tensor given by the following in spherical polar coordinates:

\[
S_{ij} = \frac{1}{2} \left( \frac{\partial v_i}{\partial r} + \frac{\partial v_j}{\partial r} - \frac{\partial v_i}{\partial \theta} \sin \theta \right).
\]
coordinates:

\[ S_{rr} = 2 \frac{\partial v_r}{\partial r} \]  

(8)

\[ S_{\theta \theta} = \frac{1}{r} \frac{\partial v_\theta}{\partial \theta} + \frac{1}{r} \frac{\partial v_r}{\partial \theta} + \frac{1}{\rho_0} \frac{\partial \rho_0}{\partial r} \]  

(9)

\[ S_{\phi \phi} = \frac{2}{r \sin \theta} \frac{\partial v_\phi}{\partial \phi} + \frac{2 v_r}{r} \cos \theta \]  

(10)

\[ S_{r \theta} = S_{\theta r} = \frac{1}{r} \frac{\partial v_r}{\partial \theta} + r \frac{\partial (v_\theta r)}{\partial r} \]  

(11)

\[ S_{\phi \theta} = S_{\theta \phi} = \frac{1}{r \sin \theta} \frac{\partial v_\phi}{\partial \theta} + \frac{\sin \theta}{r} \frac{\partial (v_\theta r)}{\partial \theta} \]  

(12)

\[ S_{r \phi} = S_{\phi r} = \frac{1}{r} \frac{\partial v_r}{\partial \phi} + r \frac{\partial (v_\phi)}{\partial r} \]  

(13)

\[ \hat{F}_{\text{rad}} = \frac{-16 \sigma_T T_0^4 d T_0}{3 \kappa \rho_0} \]  

(14)

is the radiative diffusive heat flux, where \( \sigma_T \) is the Stephan–Boltzmann constant, \( \kappa \) is the Rosseland mean opacity.

The simulation domain is a partial spherical shell with \( r \in [r_i, r_o] \), spanning from \( r_i = 0.722 R_S \) at the base of the CZ to \( r_o = 0.971 R_S \) at about 20 Mm below the photosphere, where \( R_S \) is the solar radius, \( \theta \in [\pi / 2 - \Delta \theta, \pi / 2 + \Delta \theta] \) with \( \Delta \theta = \pi / 3 \), and \( \phi \in [0, 2 \pi r] \). The domain is resolved by a grid with 96 grid points in \( r \), 512 grid points in \( \theta \), and 768 grid points in \( \phi \). J. Christensen-Dalsgaard’s (JCD) solar model (Christensen-Dalsgaard et al. 1996) is used for the reference profiles of \( T_0, \rho_0, \rho_0 \) and \( g_0 \) in the simulation domain. We assumed that \( s_0 = 0 \) for the reference state. The heating (the last term in Equation (3)) due to the solar radiative diffusive heat flux drives a radial gradient of \( s_1 \) that drives the convection. We set the thermal diffusivity \( K = 3 \times 10^{13} \text{cm}^2 \text{s}^{-1} \), the viscosity \( \nu = 10^{12} \text{cm}^2 \text{s}^{-1} \), and the magnetic diffusivity \( \eta = 10^{12} \text{cm}^2 \text{s}^{-1} \) at the top of the domain, and they all decrease with depth following a 1/ \( r \) profile. The stratification of the domain includes approximately four density scale heights between the top and the bottom, and thus the above diffusivities decrease to \( K = 4.02 \times 10^{12} \text{cm}^2 \text{s}^{-1} \), \( \nu = 1.34 \times 10^{11} \text{cm}^2 \text{s}^{-1} \), \( \eta = 1.34 \times 10^{11} \text{cm}^2 \text{s}^{-1} \) at the bottom of the CZ domain. The rationale for our choice of such depth-dependent diffusivities for the numerical experiments here is that, if the dominant spatial scales of convection decreases with height, it may be expected that more heat is transported by the unresolved scales as one moves toward the top of the simulation domain, and hence the greater diffusivities there. Furthermore, with a low magnetic diffusivity and viscosity in the deep CZ, should buoyant magnetic structures develop, they would be able to better preserve their magnetic buoyancy and rise. Given the above diffusivities, the various diffusive timescales for the simulation are estimated as follows. The viscous and magnetic diffusive timescales \((\Delta r)^2 / \nu \) and \((\Delta r)^2 / \eta \) range from about 71 yr near the bottom of CZ to about 10 yr near the top, and the thermal diffusive timescale \((\Delta r)^2 / K \) ranges from about 2.4 yr near the bottom to about 0.3 yr near the top, where we have used the depth of the CZ domain \( \Delta r \) for the estimate.

**3. RESULTS**

### 3.1. Overview of the Convective Dynamo

Figure 1 shows the magnetic and kinetic energies in the statistically steady convective flows in the simulation domain over a time span of about 74 yr. The total magnetic energy \( E_m \) maintained by the dynamo is about 10% of the total kinetic energy \( E_k \) of the convective envelope. The energy of the azimuthally averaged (mean) magnetic field \( E_{m, \text{mean}} \) only constitutes a small fraction of \( E_m \), oscillating from about 1% to 10% of \( E_m \).
Figure 2. Depth variation of the mean entropy gradient established in the statistical steady state of the dynamo simulation.

Figure 3. Horizontally integrated energy fluxes due to respectively, radiative diffusion $L_{\text{rad}}$ (red curve), convection $L_{\text{conv}}$ (black curve), thermal conduction $L_{\text{cond}}$ (yellow curve), kinetic energy flux $L_{\text{kin}}$ (blue curve), viscous flux $L_{\text{vis}}$ (dashed line), Poynting flux $L_{\text{poyn}}$ (green curve), resistive flux $L_{\text{res}}$ (cyan curve), and the sum of all $L_{\text{tot}}$ (dash–dotted black curve), as a function of depth (see the text for the expressions of $L_{\text{rad}}, L_{\text{conv}}, L_{\text{cond}}, L_{\text{kin}}, L_{\text{vis}}, L_{\text{poyn}},$ and $L_{\text{res}}$).

Figure 4. Peak downflow speed, the rms convective speed, and their corresponding equipartition magnetic field strengths as a function of depth.

The enthalpy flux of the return are put into the thermal energy in the entropy equation as a loss of the total energy and hence a loss of about 13% of the total energy flux exiting the domain at the top compared to the total energy flux $(L_s)$ entering the domain from the bottom. We note that the explicit resistive dissipation of the magnetic field (due to $\eta$), and both the explicit viscous dissipation (due to $v$) and the numerical diffusion of momentum are put into the thermal energy in the entropy equation as resistive and viscous heating to maintain energy conservation. From Figure 3, it can be seen that the enthalpy flux of the resolved convection transports about 66% of the solar luminosity in the middle of the CZ, and due to the high thermal diffusivity $K$, thermal conduction also transports a substantial fraction of the solar luminosity (about 36% at the middle of the CZ). The kinetic energy flux of the convective flows is downward and peaks at about 16% of the solar luminosity. The energy fluxes due to the Poynting flux (mostly downward), resistive, and viscous transport are all much smaller. Figure 4 shows the depth variation of the peak downflow (solid black curve), and the rms speed $v_{rms}$ (dash–dotted black curve), of the statistical steady convective flows in the domain. Note that in computing $v_{rms}$, we take out the azimutally
Reidentification is moderately more turbulent as characterized by the strength $B_{eq}$ (dash–dotted red curve). It can be seen that the equipartition field the peak downflow speed (solid red curve) and the rms speed the corresponding magnetic field strength in equipartition with fluctuating parts of the velocity components. Also shown are averaged velocity components and only sum up the azimuthally fluctuating parts of the velocity components. Also shown are the corresponding magnetic field strength in equipartition with the peak downflow speed (solid red curve) and the rms speed (dash–dotted red curve). It can be seen that the equipartition field strength $B_{eq}$ corresponding to the peak downflow speed reaches $\approx 63$ kG, while $B_{eq}$ corresponding to the rms speed is $\sim 10$ kG for the deep and mid-convection zone, and decrease to about 5000 G near the top boundary at about $r = 0.971 R_s$. Following Käpylä et al. (2012), we compute the following non-dimensional numbers characterizing the convective flows. The Reynolds number $R_e = u_{rms}/k_f$ ranges from about 130 at the bottom to about 50 at the top, and with a mid-convection zone value of about 128, where $k_f = 2\pi/(r_c-r_t)$ and $u_{rms} = ((3/2)(v_r^2+v_\theta^2))$ is the rms velocity averaged over each depth, omitting the contribution from the azimuthal velocity. The Coriolis number $C_O = (2\Omega/u_{rms,all}k_f) = 1.3$, where $u_{rms,all} = ((3/2)(v_r^2+v_\theta^2))$ with the averaging () done for the entire domain. We can compare the values of these non-dimensional numbers with the corresponding ones in Käpylä et al. (2012): $R_e = 36$ and $C_O = 7.6$. It appears the convective flow in our dynamo simulation is moderately more turbulent as characterized by the larger $R_e$, especially in the deeper layers of the CZ. Our Coriolis number $C_O$ is significantly lower, indicating that our convective dynamo is operating in a significantly less rotationally dominant regime. If we were to scale their typical rms velocity to be similar to ours $u_{rms,all} \approx 100$ m $s^{-1}$, then their $C_O$ would imply a significantly more rapidly rotating stellar envelope (with the solar CZ depth) at about five times the solar rotation rate.

Figure 5(a) shows the latitude-time variation of the mean (azimuthally averaged) toroidal magnetic field at a depth near the bottom of the CZ. The mean toroidal magnetic field tends to be of opposite signs for the two hemispheres, and exhibits an irregular cyclic behavior with oscillations of the field strength on timescales ranging from about 5 to about 15 yr and undergoes irregular sign/polarity reversals. The strongest mean toroidal field is concentrated near the bottom of the CZ (see Figure 5(b)), peaking at about 7 kG. Figure 5(c) shows a shell-slice of $B_\phi$ at a depth near the bottom of the CZ, at a cycle maximum phase indicated by the green line in Figure 5(a). It shows that strong toroidal fields $B_\phi$ of a preferred sign (opposite for the two hemispheres) are concentrated in individual channels or filaments in each hemisphere, reaching a peak field strength of about 30 kG, which exceeds the field strength in equipartition with the local rms convective speed ($B_{eq} \approx 13$ kG), but is below the equipartition field strength corresponding to the peak down flow speed ($B_{eq} \approx 63$ kG). Thus, these strong field filaments are not passively advected by convective flows, but would be pinned down by the strong down flows if in their paths.

3.2. Maintenance of the Solar-like Differential Rotation

Figure 6(a) shows the time and azimuthally averaged rotation rate in the convective envelope self-consistently maintained in the convective dynamo simulation. It shows a solar-like differential rotation profile (e.g., Thompson et al. 2003) with a faster rotation rate at the equator than at the polar region by about 30% of the mean rotation rate, and more conical shaped iso-rotation contours in the mid-latitude zone. The
Figure 6. Top row panels from left to right show the time and azimuthally averaged mean meridional profile of angular velocity (a), meridional flow mass flux (b), angular momentum flux density in the $r_{\perp}$ direction due to the Reynolds stress (c), the viscous stress (d), and the Maxwell stress (e) resulting from the dynamo simulation. Middle row panels from left to right show the same as those of the top row except for the results from the corresponding HD simulation and there is not a panel for the Maxwell stress. Bottom row panels from left to right show the same as the middle row, except for the results from the HVHD simulation. See the text for the expressions for the various angular momentum flux density $RS$, $VS$, and $MS$.

(A color version of this figure is available in the online journal.)

time and azimuthally averaged mean meridional flow pattern is shown in Figure 6(b) in terms of the mass flux function $f$ where $\rho_0(\mathbf{v}) = \mathbf{V} \times [(f/r \sin \theta) \hat{\phi}]$. The meridional circulation has a complex multi-cell structure with a counter-clockwise (clockwise) cell pattern in the low latitude region of the northern (southern) hemisphere, i.e., a poleward near-surface flow in the low latitude region.

Interestingly, we find that the presence of the magnetic field is necessary for the self-consistent maintenance of the solar-like differential rotation profile in the current parameter regime. We have carried out a hydrodynamic simulation (hereafter referred to as the HD case) which is identical to the present convective dynamo simulation except that the magnetic field is set to zero, and found that a very different differential rotation profile (Figure 6(f)) is established in the statistical steady state of the HD simulation. It shows a significantly larger differential rotation with a faster rotation rate in the polar region than at mid-latitudes and the equator. The iso-rotation contours are also
more cylindrical. The meridional flow (Figure 6(g)) shows a more prominent counter-clockwise (clockwise) cell pattern in the northern (southern) hemisphere in the middle depths in the CZ, with much weaker reversed cells in the near surface layer.

To compare the transport of angular momentum in the dynamo and the HD cases, we show in Figures 6(c)–(e) the meridional profile of the angular momentum flux density in the \( r_\perp \) direction (perpendicular to and away from the rotational axis) due, respectively, to the Reynolds stress of the rotationally influenced convection (panel (c)):

\[
RS = \rho_0 r_\perp \langle v_{r_\perp} v_{\phi} \rangle,
\]

the viscous stress (panel (d)):

\[
VS = \rho_0 v_{r_\perp} \langle \langle S_{\phi r} \rangle \sin \theta + (S_{\phi \theta}) \cos \theta \rangle,
\]

and the Maxwell stress (panel (e)):

\[
MS = -r_\perp \frac{1}{4\pi} \langle B_{\phi} B_{r_\perp} \rangle,
\]

where \( \langle \rangle \) denotes time and azimuthal averages and \( \prime \) denotes the azimuthally varying component. The meridional profiles of the angular momentum flux density RS and VS for the corresponding HD case are shown in Figures 6(h) and (i). We find that there is a significant difference in the angular momentum flux density by the Reynolds stress between the dynamo and HD cases. The RS for the dynamo case shows an overall more outward transport in its meridional distribution compared to that for the corresponding HD case. Near the lower boundary in the mid-latitude range, the presence of the concentrated magnetic field (which helps to damp the convective downflows) results in a more enhanced outward RS flux at the lower boundary layer. The angular momentum flux density MS due to the Maxwell stress in the dynamo case (Figure 6(e)) is found to oppose RS, and its strength is greatest at the lower boundary layer where the magnetic field concentrates. The angular momentum flux density VS simply acts to reduce the differential rotation as expected for both the dynamo and the HD cases. The difference between the dynamo and the HD cases is more clearly seen by evaluating the net angular momentum fluxes in the \( r_\perp \) direction integrated over individual concentric cylinders of radii \( r_\perp \) centered on the rotation axis, as shown in Figure 7(a) for the dynamo case and Figure 7(b) for the corresponding HD case. It can be seen that the net angular momentum flux due to RS is outward throughout (except near the top boundary) for the dynamo simulation (black curve in Figure 7(a)), which drives a faster rotation in the outer equatorial region. The net angular momentum flux due to VS is mainly counteracted by the net flux due to MS by the Maxwell stress, with the remaining difference balanced by the significantly smaller net fluxes due to VS and the meridional flow. In contrast, the HD simulation shows a significant inward angular momentum transport due to the Reynolds stress (black curve in Figure 7(b)) across the inner cylinders in the high to mid-latitude region. This drives a faster rotation in the polar region. Thus, it appears that the presence of the magnetic field alters the convective flows such that the resulting Reynolds stress from the convective motions produces a more outward (away from the rotation axis) net transport of the angular momentum needed to drive a solar-like differential rotation. The Rossby number \( R_\alpha = \overset{\text{rms, all}}{v}/(\Omega H_p) \) is about 0.74 for the dynamo case and 0.96 for the HD case, where \( \overset{\text{rms, all}}{v} = 125 \text{ m s}^{-1} \) for the dynamo case and 157 m s\(^{-1}\) for the HD case, is the rms velocity (with the azimuthally averaged mean flow velocity taken out) averaged over the entire volume, and \( H_p \) is the pressure scale height at the bottom of the CZ. The Rossby number measures the importance of the Coriolis force in the force balance. The lower Rossby number in the dynamo simulation shows that the magnetic fields suppress the convective motions so that they are more rotationally constrained.

A recent systematic study by Gastine et al. (2014) of rotating stellar convection considering a wide range of models shows that the differential rotation profile transitions from being solar-like, with a faster rotating equator, to being anti-solar, with a faster polar rotation rate, at a value of about 1 for the Rossby number. This result is found to be quite general, independent of the detailed model setup (presence of a magnetic field, thickness...
of the convective layer, density stratification). Our HD case with \( R_D = 0.96 \) appears to be very close to the transition, and a reduction of \( \sim 23\% \) of the overall rms velocity and \( R_D \) by the presence of the magnetic field in the dynamo case is able to significantly alter the angular momentum transport by the rotationally constrained convection, leading to a transition into the solar-like differential rotation. We note that even though the HD case is quite close to the transition, its anti-solar differential rotation appears to be a stable solution not dependent on the history, i.e., not one of two bistable states (Gastine et al. 2014). We have arrived at the statistically steady HD solution with the anti-solar differential rotation by either starting from the initial setup with a seed velocity field as described in Section 2, or by starting from the statistically steady state dynamo solution (for which the differential rotation is solar-like) and zero out the magnetic field. The solar-like differential rotation obtained in the convective dynamo simulation also appears to be a stable solution that is not dependent on the history. The differential rotation at the pole and equator remain statistically steady without exhibiting any systematic drift for the \( \gtrsim 74 \) yr period (comparable to the maximum viscous timescale near the bottom of CZ) we have run after the dynamo solution has reached a statistical steady state. Also, as we start the dynamo simulation from the initial setup described in Section 2, we find that the convective dynamo goes through an earlier phase of anti-solar differential rotation (for \( \sim 6 \) yr) before it evolves toward the solar-like differential rotation profile as the mean entropy gradient and the convective energy flux settle down to their statistical steady state. Thus, it appears that the solar-like differential rotation is the preferred stable solution in the dynamo case.

The transition to a solar-like differential rotation can alternatively be achieved in the non-magnetic hydro simulations by simply increasing the viscosity to reduce \( R_D \). We have run another hydrodynamic simulation (hereafter referred to as the HVHD case, meaning “high viscosity hydro”) where we increase the viscosity \( \nu \) by 5 times (with the same \( \rho \nu^{-1/2} \) depth dependence) compared to the HD case (or the dynamo case). The resulting rms velocity of the statistical steady convection reached is \( \nu_{\text{rms, all}} = 120 \) m s\(^{-1}\) and the Rossby number \( R_D = 0.71 \), much closer to those of the dynamo case. The bottom row panels of Figure 6 show, respectively, the resulting differential rotation profile (Figure 6(j)), meridional circulation (Figure 6(k)), the angular momentum flux density in the \( r_\perp \) direction, RS (Figure 6(l)) due to the Reynolds stress, and VS (Figure 6(m)) due to the viscous stress. The integrated net (outward) angular momentum fluxes across concentric cylinders of radius \( r_\perp \) centered on the rotation axis is shown in Figure 7(c). It is found that a solar-like differential rotation profile (Figure 6(j)) with faster rotating equator and with a more conical iso-rotation contours in mid-latitude zones is established, although the contrast of rotation rate between the equator and the polar region is larger, about \( 44\% \) of the mean rotation rate (compared to about \( 32\% \) in the dynamo case). The mean meridional circulation (Figure 6(k)) shows a counter-clockwise (clockwise) cell pattern in the low-latitude region of the northern (southern) hemisphere, i.e., a poleward near-surface flow in the low latitude region, similar to the dynamo case (Figure 6(b)). We find that the angular momentum transport in the \( r_\perp \) direction due to the Reynolds stress for the HVHD case is very similar to that for the dynamo case, both in the meridional profile of the flux density (Figure 6(i)) compared to Figure 6(c)) as well as in the integrated net flux across the constant \( r_\perp \) concentric cylinders (Figure 7(c) compared to Figure 7(a)). However, this similar outward net angular momentum flux by the Reynolds stress is now balanced almost entirely by the transport due to the viscous stress, with the absence of the Maxwell stress which is the major component that balances the angular momentum flux by the Reynolds stress in the dynamo case. The comparison between the dynamo and the HVHD cases suggests an effective role of enhanced viscosity played by the magnetic fields, which (1) suppresses the large-scale convective motions such that they are more rotationally constrained (lower \( R_D \)) to produce an outward transport of the angular momentum by the Reynolds stress, necessary to drive a solar-like differential rotation, and (2) takes up the main role to balance the Reynolds stress transport with the Maxwell stress instead of the viscous stress.

Further in Figure 8, we show the various horizontally integrated energy fluxes through the domain for the HD case (upper panel) and the HVHD case (lower panel), in comparison with the energy fluxes shown in Figure 3 for the dynamo case. It can be seen that the dynamo case and the HVHD case show a similar convective energy flux \( L_{\text{conv}} \) (reaching about \( 65\% \) \( L_s \) in the dynamo case and about \( 60\% \) \( L_s \) in the HVHD case). In both the dynamo and the HVHD cases, the downward kinetic energy flux \( L_{\text{kin}} \) (reaching about \( 16\% \) \( L_s \) in the dynamo and \( 10\% \) \( L_s \) in the HVHD case) is significantly reduced compared to the HD case (reaching about \( 45\% \) \( L_s \)). In fact, in the HD case (upper panel).
the CZ as a measure of the transport speed. Thus, the advective transport acts more quickly.

In the midst of magneto-convection, we find occasional active regions like flux emergence events in the top layer of the simulation domain. Such an example is shown in Figure 10, where panels (a), (b), (c), and (d) show, respectively, snapshots of $B_\psi$, $B_\theta$, $v_r$, and $v_\phi$ at a constant $r$ slice at the depth of 30 Mm below the photosphere. The location of the emerging bipolar region is indicated by an arrow in the panels. It is characterized by a diverging bipolar pattern in $B_\psi$ (panel (a)) and the emergence of a strong toroidal field patch reaching a peak field strength of 9800 G (panel (b)) (also see the animation in the online journal). The emerging region corresponds to an up flow region in $v_r$ (panel (c)), but the upward velocity is not significantly different from that of other up flow convective cells. The zonal velocity $v_\phi$ of the emerging region shows a diverging pattern, and when averaged over the emerging region, is $\sim 100$ m s$^{-1}$ faster than the mean zonal velocity of that latitude. Figure 10(e) shows the subsurface 3D magnetic field configuration in the convective envelope by showing field lines traced from randomly seeded points throughout the volume. The field lines are colored based on their azimuthal field $B_\phi$ as indicated by the color table. It can be seen that relatively more coherent bundles of strong toroidal flux are embedded in the turbulent magnetic fields. In Figure 10(f), regions of strong field strength where the Alfvén speed ($v_A$) exceeds the rms convective velocity ($v_{rms}$) for the corresponding depth is outlined with the equipartition iso-surfaces (with $v_A/v_{rms} = 1$), which are again colored based on the $B_\phi$ value on the iso-surfaces. There is a systematic preference for these strong flux regions to be green or of negative $B_\phi$ (red or of positive $B_\phi$) in the northern (southern) hemisphere. The arrows in Figures 10(e) and (f) mark the toroidal flux bundle with super-equipartition field strength that gives rise to the emerging region.

Figure 11 shows a more zoomed in view of the thermodynamic properties of the emerging region at the same depth as that shown in the upper four panels of Figure 10. We see that there is a systematic reduction of density (i.e., buoyant; see Figure 11(b)) and pressure (Figure 11(d)) in the emerging region compared to the surrounding, although the reduction magnitude is rather moderate compared to the fluctuations seen in strong downflow lanes and in strong vertical flux tubes in the downflow lanes. The temperature change in the emerging region compared to the surrounding is smaller, partly due to the large thermal conduction. Averaged over the emerging region (area enclosed in the yellow contour in Figure 11(a), $\langle \rho_1/\rho_0 \rangle \approx -1.6 \times 10^{-5}$, $\langle p_1/p_0 \rangle \approx -1.1 \times 10^{-5}$, and $\langle T_1/T_0 \rangle \approx 0.5 \times 10^{-5}$). It can be seen that the temperature change is relatively small compared to the density and pressure change in the emerging region. This suggests that the buoyancy or reduction in density is mainly due to the reduction in gas pressure provided largely by the presence of the magnetic pressure, instead of mainly due to an increase in temperature. In other words, the buoyancy contribution is more from the magnetic buoyancy than the thermal buoyancy.

As can be seen from Figure 10(e), the emerging flux region is the apex of a roughly east-west oriented (toroidal) super-equipartition flux bundle which remain relatively coherent for some distance, before the two ends connect in complex ways to other flux systems. The following end of the coherent flux bundle extends into the middle of the CZ. The fact that the emerging flux has a prograde zonal speed of $\sim 100$ m s$^{-1}$ relative to the mean zonal speed of the latitude indicates that it is not a toroidal flux tube rising in isolation from the bottom of the CZ. If it

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3.3. Emerging Flux

In the convective dynamo, the large-scale mean toroidal field as shown in Figure 5(b) is produced by the latitudinal differential rotation shearing a dipolar poloidal mean field. The reason that the mean toroidal field is concentrated towards the bottom of the CZ is mainly due to a downward advective transport of the magnetic energy in the bulk of the CZ, as represented by $\langle v_r(B_\theta^2 + B_\phi^2)/8\pi \rangle$ shown in Figure 9(a). This causes the distribution of the magnetic energy (for both the mean field and the small-scale field) to be strongly concentrated toward the bottom (see Figure 9(b)). The decrease with depth of the magnetic diffusivity $\eta$ would also promote stronger fields toward the bottom but is less important here because of the small magnitude of $\eta$ and the long diffusive timescale: $(\Delta r)^2/\eta \sim 10$ yr, using the peak $\eta$ value near the surface and the depth of the CZ domain $\Delta r$. The advective timescale for the downward magnetic energy transport across the CZ is $\Delta r/v_m \sim 0.5$ yr is significantly shorter, where we have used $u_m = \langle v_r (B_\theta^2 + B_\phi^2)/(B_\theta^2 + B_\phi^2) \rangle$ evaluated at the middle of the CZ as a measure of the transport speed. Thus, the advective transport acts more quickly.

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- **Figure 9.** (a) Advective flux of the horizontal magnetic field energy, and (b) the distribution of horizontal magnetic field energy as a function of depth.
Figure 10. Panels (a), (b), (c), and (d) show, respectively, snapshots of $B_r$, $B_\phi$, $v_r$, and $v_\phi$ at a shell slice at the depth of 30 Mm below the photosphere, displayed on the full sphere in Mollweide projection. An animation showing the evolution of $B_r$, $B_\phi$, $v_r$ at the 30 Mm depth, and also $B_\phi$ at a depth near the bottom of the CZ, over a period of about 13 days centered around the time instant shown in this figure is also available in the online journal. Panels (e) and (f) show, respectively, 3D views of the magnetic field lines and the equipartition field iso-surfaces of $v_a/v_{rms} = 1$ with $v_a$ being the Alfvén speed and $v_{rms}$ being the rms convective velocity for the corresponding depth.

(An animation and a color version of this figure are available in the online journal.)

were, it would have a retrograde flow due to angular momentum conservation as is found in many previous studies of isolated rising flux tubes in the rotating solar CZ (e.g., Caligari et al. 1995; Fan 2008, 2009). The emerging flux bundle must have well mixed with the local plasma through reconnections, and is continually sheared and amplified by the differential rotation and the local flows against resistive dissipation. The sequence of images in Figure 12 show the sub-surface development of the super-equipartition emerging flux bundle (marked by the arrow) over a nine-day period prior to the time of the flux emergence event shown in Figure 10. It shows that local shear in the upper CZ contributes significantly to the development of the emerging flux bundle. The left column images show iso-volumes of super-equipartition fields with the surface of the volume colored by $B_\phi$. The right column images show representative field lines traced from the iso-volume corresponding to the emerging flux bundle. It can be seen that a segment of a super-equipartition flux bundle in the middle of the CZ is sheared and stretched in the prograde direction into a hairpin turn with the upper side of the hairpin forming the emerging flux bundle reaching the top boundary.

We have done a statistical study of the super-equipartition emerging fields. For a time period of about one year centered at the cycle maximum phase (green line in Figure 5(a)) and at an interval of 12 hr, we find in the shell slice at 30 Mm depth all the area where the emerging horizontal field exceeds $\sqrt{2}$ times the field strength in equipartition with the rms convective velocity of that depth. For each pixel (or grid point) of the selected emerging field area, we compute a tilt angle of the horizontal field vector based on the local $B_\phi$ and $B_\theta$. The resulting tilt angle distribution of all the pixels is shown in Figure 13. The quadrant of the tilt angle is such that, if the sign of the azimuthal field $B_\phi$ is consistent with Hale’s polarity rule of the cycle, i.e., negative (positive) in the northern (southern) hemisphere, then the tilt angle falls in quadrants I and IV. If the horizontal field vector is tilted clockwise (anti-clockwise) from the cycle preferred azimuthal field direction in the northern (southern) hemisphere by an acute angle, consistent with the mean tilt of solar active regions, then the tilt angle falls in quadrant I. We find from Figure 13 that there is a preference for Hale’s polarity rule for the emerging azimuthal field by a ratio of 2.4 to 1 in the area. For those pixels satisfying Hale’s rule, the mean tilt angle is 7.5 with an estimated uncertainty of 1.6. Thus, the super-equipartition emerging fields have a statistically significant mean tilt similar to the active region mean tilt (e.g., Stenflo & Kosovichev 2012).

4. DISCUSSIONS

We have presented a 3D convective dynamo driven by the solar radiative diffusive heat flux in the solar CZ, and
with a latitudinal gradient of entropy imposed at the bottom, representing the tachocline-induced thermal variation that can break the Taylor–Proudman constraint in the CZ (Rempel 2005). The convective dynamo produces a large-scale mean field that undergoes irregular cycles and polarity reversals, and self-consistently maintains a solar-like differential rotation with faster rotation at the equator than at the polar region by about 30% and more conical iso-rotation contours in mid-latitudes (e.g., Thompson et al. 2003).

The irregular cyclic behavior of the mean field in our model differs from those in the literature (e.g., Ghizaru et al. 2010; Racine et al. 2011; Käpylä et al. 2012). By comparing the Reynolds number $R_e$ and the Coriolis number $C_D$ (as defined in Käpylä et al. 2012) achieved in our dynamo run with those of the cyclic dynamo simulation presented in Käpylä et al. (2012) (see Section 3.1), we find that their convective dynamo is operating in a significantly more rotationally constrained regime with their $C_D$ being about five times ours. Our convective flows appear to be only moderately more turbulent compared to theirs as reflected in the similar order of magnitude for the $R_e$ values. If we consider our convective rms velocity to be similar to theirs, then their dynamo would be effectively operating in a stellar envelope that is rotating at about five times the solar rotation rate. This is probably the main reason we obtain a very different mean field dynamo behavior compared to that of Käpylä et al. (2012).

Our irregularly cycling convective dynamo model also differs significantly in many ways from the cyclic convective dynamo model described in Ghizaru et al. (2010); Racine et al. (2011). Their dynamo model used an implicit large eddy code with no explicit viscosity, magnetic diffusion, and thermal diffusion. However, our convective dynamo appears to be operating in a more turbulent regime if we compare the convective downflow speed obtained: about 25 m s$^{-1}$ at $r = 0.954 R$, in (Ghizaru et al. 2010) versus our $\sim 300$ m s$^{-1}$ at the same depth. Given that both models use the solar rotation rate for the convective envelope, this suggests that their dynamo model is also operating in a significantly more rotationally constrained regime with a significantly lower Rossby number compared to ours. The Newtonian cooling treatment of the entropy equation used in Ghizaru et al. (2010); Racine et al. (2011) is very different from our treatment of the energy transport which forces the solar luminosity through the convective domain. Furthermore, their model includes a sub-adiabatically stratified overshoot layer at the bottom of the CZ which our model does not have. All these

Figure 11. Zoomed in view centered on the emerging region on the horizontal surface at the same depth (30 Mm) as shown in Figure 10 with (a) showing $B_\phi$ and a yellow contour marking the emerging region where $B_\phi < -5000$ G and the ratio of the Alfvén speed over the rms convective speed $v_a/v_{rms} > 1$, (b) showing density change $\rho_1$ relative to the reference state $\rho_0$ at that height, (c) showing temperature change $T_1$ relative to the reference state $T_0$, and (d) showing pressure change $p_1$ relative to the reference state $p_0$.

(A color version of this figure is available in the online journal.)
contribute to the significant differences in the resulting dynamo behavior.

In both Käpylä et al. (2012) and Racine et al. (2011), because of the significantly higher $C_O$ or lower $R_o$, the convective flows in their corresponding hydro cases in the absence of the magnetic fields are already driving a solar-like differential rotation, and the addition of the magnetic fields in their dynamo cases appear to mainly reduce the differential rotation (Charbonneau 2013). On the other hand, for our convective dynamo in a significantly less rotationally constrained regime with $R_o$ closer to 1, the presence of the magnetic field is found to play an important role for the maintenance of the solar-like differential rotation, without which a faster rotating polar region results, as is shown with the corresponding HD simulation. A solar-like differential rotation profile can alternatively be achieved in the hydro case by increasing the viscosity as shown in the HVHD simulation. A solar-like differential rotation profile can alternatively be achieved in the hydro case by increasing the viscosity as shown in the HVHD simulation. Comparing between the dynamo case and the HVHD case indicates that in several aspects the magnetic field plays an effective role of enhanced viscosity to (1) suppress the large scale convective motions such that they become more rotationally constrained to produce an outward transport of angular momentum by the Reynolds stress needed to drive a solar-like differential rotation, (2) take up the main role of balancing the Reynolds stress transport with the Maxwell stress transport instead of the viscous stress under low viscosity conditions, and (3) reduce the downward kinetic energy flux. Our resulting differential rotation self-consistently maintained in the convective dynamo simulation is in fairly good agreement with the observed solar differential rotation in both the pole-equator contrast and also the more conical shaped iso-rotation contours in the mid-latitude zone. The more conical iso-rotation contours are achieved by the latitudinal gradient of entropy imposed at the lower boundary as has been described in Rempel (2005); Miesch et al. (2006). The latitudinal gradient in the CZ provides the necessary balance in the $\phi$ component of the vorticity equation to allow for a non-cylindrical differential rotation to be established (Rempel 2005). Thus, the role of the imposed latitudinal entropy gradient is to change the shape of the iso-rotation contours from cylindrical to more conical. It is not the reason for the change of differential rotation from anti-solar (fast rotating pole) to solar-like (faster equator) between the HD and the magnetic cases, both of which have the same latitudinal entropy gradient imposed. That change is brought about by the change in the direction of Reynolds stress transport of the angular momentum.

Figure 12. Left column images show iso-volumes of super-equipartition fields (where $v_a/v_{rms} > 1$) for a period of 9 days prior to the time of the flux emergence event shown in Figure 10, with the arrow marking the evolution of the emerging flux bundle that produces the flux emergence event shown in Figure 10. Right column images show representative field lines traced from points in the iso-volume corresponding to the emerging flux bundle.

(A color version of this figure is available in the online journal.)
We also note here the possible effect of the small deviation from total energy conservation in our dynamo simulation as seen in Figure 3, where the total energy flux $L_{\text{tot}}$ exiting the domain at the top is about 13% less than the input solar luminosity. As pointed out in Section 3.1, this is due to the loss of the magnetic energy dissipated by the numerical diffusion, which is not put back into the thermal energy and hence does not have to be carried out by thermal conduction at the top. This would mean a slightly weakened driving of convection where a reduced solar luminosity (reduced by up to about 13% near the top) is forced through the domain. The heat fluxes by thermal conduction and convection would have been slightly higher if the energy conservation were strictly adhered to. This, however, does not affect our finding that the magnetic field takes up the role of an enhanced viscosity, and if it damps the large-scale convective motions to the level similar to that of the HVHD case (with convection carrying roughly 66% of the solar luminosity) a solar-like differential rotation can be achieved by the resulting Reynolds stress. The larger question that remains is how to maintain the solar differential rotation with a convection that can transport nearly 100% of the solar luminosity without resorting to an ad-hoc thermal diffusion (conduction) that carries a substantial fraction (about 36% at the middle of the CZ) of the solar luminosity.

In our dynamo simulation the large-scale mean toroidal field, antisymmetric with respect to the equator, is concentrated at the bottom of the CZ, unlike many of the recent convective dynamo simulations with cylindrical iso-rotation contours and significantly faster (than solar) rotation rates (e.g., Nelson et al. 2013; Augustson et al. 2013), where strong wreath-like toroidal field structures are present in the equatorially confined region of the bulk of the CZ. In the 3D magnetic field of the present simulation (Figure 10), occasional more coherent toroidal flux bundles of super-equipartition field strength are embedded in the small scale fields, as discussed in Section 3.3. Some of these super-equipartition flux bundles rise to the surface to produce active region like flux emergence events. Although these emerging flux bundles show significant magnetic buoyancy, they are not flux bundles rising in isolation from the bottom of the CZ, but are a product of continued reconnection and shear amplification by local flows in the CZ. There is a preference for the azimuthal field in the strong emerging field regions to conform to Hale’s polarity rule by a ratio of 2.4 to 1 in area, and a statistical significant mean tilt angle of $7.5 \pm 1.6^\circ$ for the emerging horizontal fields, consistent with the active region mean tilt. However, the violation from Hale’s rule is far greater than that of solar active regions. This is because of the very weak mean field component (~5% of the total magnetic energy) in the current convective dynamo. It is very likely that our dynamo model is still significantly over-estimating the giant-cell convective speed (Hanasoge et al. 2012) and the Sun’s dynamo is operating in a significantly more rotationally constrained regime. This is indicated by the more rotationally constrained convective dynamo models of Käpylä et al. (2012); Augustson et al. (2013), which are able to achieve a more regular, solar-cycle-like cyclic behavior and stronger mean field by effectively increasing the rotation rate. With a significantly lower magnetic diffusion (than have been achieved by current global convective dynamo models), the solar dynamo may be operating in a significantly stronger field regime with a much more suppressed giant-cell convection and lower Rossby number. This may lead to a stronger Reynolds stress transport of angular momentum that needs to be balanced by a stronger Maxwell’s stress and hence the generation of a stronger mean field. However, the question remains how such suppressed convective flows transport the solar luminosity through the CZ, although the convective energy transport in the more magnetic-buoyancy-dominated regime may be quite different. Furthermore, the inclusion of an overshoot layer below the base of the CZ may be important for the operation of the solar cycle dynamo. The convective dynamo model of Ghizaru et al. (2010); Racine et al. (2011) has achieved a much stronger large-scale mean field component, with the mean field magnetic energy comparable to that of the small-scale magnetic energy during cycle maxima (compared to the 10% in our model), by allowing penetration and storage of the strong mean field in the stable overshoot layer.

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REFERENCES

Augustson, K., Brun, A. S., Miesch, M. S., & Toomre, J. 2013, ApJL, submitted (ArXiv:1310.8417)
Caligari, P., Moreno-Insertis, F., & Schüssler, M. 1995, ApJ, 441, 886
Charbonneau, P. 2013, JPhCS, 440, 012014
Christensen-Dalsgaard, J., Dappen, W., Aukov, S. V., et al. 1996, Science, 272, 1286
Fan, Y. 2008, ApJ, 676, 680
Fan, Y. 2009, LSRP, 6, 4
Fan, Y., Featherstone, N., & Fang, F. 2013, ApJ, submitted (ArXiv:1305.6370)
Gastine, T., Yadav, R. K., Morin, J., Reiners, A., & Wicht, J. 2014, MNRAS, 438, L76
Ghizaru, M., Charbonneau, P., & Smolarkiewicz, P. K. 2010, ApJL, 715, L133
Hale, G. E., Ellermann, F., Nicholson, S. B., & Joy, A. H. 1919, ApJL, 49, 153
Hale, G. E., & Nicholson, S. B. 1925, ApJ, 62, 270
Hanasoge, S. M., Duvall, T. L., & Sreenivasan, K. R. 2012, PNAS, 109, 11928
Käpylä, P. J., Mantere, M. J., & Brandenburg, A. 2012, ApJL, 755, L22
Maunder, E. 1922, MNRAS, 82, 534
Miesch, M. S., Brown, B. P., Browning, M. K., Brun, A. S., & Toomre, J. 2011, in IAU Symp. 271, Astrophysical Dynamics: From Stars to Galaxies, ed. N. H. Brummell, A. S. Brun, M. S. Miesch, & Y. Ponty (Cambridge: Cambridge Univ. Press), 261
Miesch, M. S., Brun, A. S., & Toomre, J. 2006, ApJ, 641, 618
Nelson, N. J., Brown, B. P., Brun, A. S., Miesch, M. S., & Toomre, J. 2011, ApJL, 739, L38
Nelson, Y., Brown, B. P., Brun, A. S., Miesch, M. S., & Toomre, J. 2013, ApJ, 762, 73
Nelson, N. J., Brown, B. P., Sacha Brun, A., Miesch, M. S., & Toomre, J. 2014, SoPh, 289, 441

Racine, É., Charbonneau, P., Ghizaru, M., Bouchat, A., & Smolarkiewicz, P. K. 2011, ApJ, 735, 46
Rempel, M. 2005, ApJ, 622, 1320
Stenflo, J. O., & Kosovichev, A. G. 2012, ApJ, 745, 129
Stone, J. M., & Norman, M. L. 1992, ApJS, 80, 791
Thompson, M. J., Christensen-Dalsgaard, J., Miesch, M. S., & Toomre, J. 2003, ARA&A, 41, 599