Simulation of the flow between rotating disks with Discontinuous Galerkin method

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Abstract. In this work Discontinuous Galerkin (DG) method is applied to the simulation of the incompressible, viscous flow between rotating disks. The code is based on the projection method and accepts unstructured meshes with hexahedral elements. It is explicit in time and allows for arbitrary partition of the computational domain between processors in parallel computing (communication patterns are automatically assigned). Two cases (configurations) are considered: flat disks (structured mesh) and disk with a step placed at the center of the cavity (unstructured mesh). In both configurations the upper disk is rotating, the other one and the remaining walls are stationary. The results for the flat disks are compared to DNS data and very good agreement is obtained. The second case shows capability of the new approach with handling complex geometries.

1. Introduction

The Discontinuous Galerkin method (Reed & Hill, 1973; Li, 2006; Cockburn & Shu, 2001; Karniadakis & Sherwin, 2005) has gained some popularity in recent years in the field of modeling of complex flows. As a part of the family of Finite Element methods (FEM) it is very flexible with respect to geometry of computational domain. Comparing to classical Continuous Galerkin (CG) methods DG is formulated locally – approximation basis is defined for a given element with no assumed connection to bases in the adjacent elements. This freedom allows for straightforward implementation of p-adaptation techniques and non-conforming elements but requires proper strategy for coupling between the elements, which comes by means of numerical flux – concept taken from Finite Volume (FV) methods.

The code – DioGenes – developed at Institute of Thermal Machinery is implementation of DGM for 3D incompressible, viscous flow. It accepts arbitrary unstructured FEM meshes with hexahedral elements (the only requirement is that the element faces must be flat). Beside better quality of such meshes, comparing to those with tetrahedral elements, there is possibility of direct construction of basis functions as tensor products of one-dimensional functions (Li, 2006; Karniadakis & Sherwin, 2005) and straightforward application of Gauss method for integration. The disadvantage is much larger storage required for the DG matrices which must be constructed for each of the elements seperately (because of non-constant Jacobians), although their size is related only to number of local degrees of freedom (not global, like in the case of classical CG FEM). The code is explicit in time and offers TVD Runge-Kutta methods of order from 1 to 3. Because of explicit and local formulation of the method, in single time step each element
communicates only with its neighbours (regardless of order of approximation in space) and parallelisation of the code could be implemented efficiently. In the present version, arbitrary partition of the mesh between the processors is allowed.

The flow analyzed in this work, both in simple rotating cavity with flat disks and disks with a step, is of large interest not only from the scientific point of view – it appears in many practical applications (e.g. turbines or other turbomachinery). The presence of three-dimensional boundary layers allows for analysis of mean flow structures on turbulence and transition effects. Usually this type of flow is characterized by regions in which it is laminar, transitional or strongly turbulent – this makes an accurate simulation of the flow quite a challenging task for a numerical method (Tuliszka & Zielinski, 2008).

2. Cavity with flat disks

The cavity is enclosed by two discs, one rotating with angular velocity \( \Omega \), and two stationary cylinders (Fig. 1). This configuration has been studied in the literature for long but usually employing high-order spectral methods with their well-known limitations to simple geometries (Tuliszka & Zielinski, 2008; Serre et al, 2004). Such case can be used to validate the code and to compare mean velocity profiles on the vertical line connecting the disks (through the center of the cavity).

Reynolds number \( Re \) for the considered case is defined as:

\[
Re = \frac{\Omega R_1^2}{\nu},
\]

where \( R_1 \) – outer radius of the disks and \( \nu \) – kinematic viscosity. In the presented calculations value \( Re = 7 \times 10^4 \) has been employed.

![Figure 1. The configuration of the disks.](image)

![Figure 2. Computational mesh for flat disks – 20 × 160 × 10 elements.](image)

In Fig. 3 instantenous flow velocity distributions in the cross-section of the cavity for two different times are shown. As the time unit \( \tau_n \) a natural time scale related to the angular velocity \( \Omega \) has been selected:

\[
\tau_n = \frac{1}{\Omega}.
\]

Time \( t = 2\pi \tau_n \) corresponds to one full rotation of the upper disk.

The comparison of the results with available DNS data required steady state of the flow, thus to speed up the calculations the simulation started with low order approximations in elements.
(\(P = 1\) – linear) and later the order was increased (\(P = 2\) – parabolic). The agreement with DNS data and Fluent results (with WALE subgrid scale model) is quite satisfactory, especially for the axial component (Fig. 4).

![Image](image1)

**Figure 3.** Flat disks – instantaneous flow velocity distributions in the cross-section of the cavity for two different times: radial component (a), axial (b) and azimuthal (c); vector plot (d)

![Image](image2)

**Figure 4.** Comparison of the DGM results with DNS data and Fluent results: mean azimuthal component (a) and axial component (b)

### 3. Disk with a step

To show the capability of the new approach, instead of using flat disks a step has been added to the stationary disk. This case requires unstructured mesh (Fig. 5), refined in the thinner region between the step and the upper disk. The instantaneous velocity distribution in the cross-section of the cavity is shown in Fig. 6.
In the flow between rotating disks, if Reynolds number is high enough, characteristic instability patterns may appear manifesting themselves as ring or spiral structures (Serre et al, 2004). They can be seen in velocity module distribution in various cross-sections of the cavity (Fig. 7) or isosurfaces of the axial component of the flow velocity (Fig. 8).
4. Summary

The developed code allows for simulation of the flow between rotating disks for simple and more complex geometries. It is validated for the case of flat disks against DNS data. The main advantage of the code is its capability of handling complex geometries with unstructured meshes.
and high-order approximations.

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