Research Article

Bipartite Consensus of Heterogeneous Multiagent Systems Based on Distributed Event-Triggered Control

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In this paper, the bipartite consensus problem of heterogeneous multiagent systems composed of first-order and second-order agents is considered by utilizing the event-triggered control scheme. Under structurally balanced directed topology, event-triggered bipartite consensus protocol is put forward, and event-triggering functions consisting of measurement error and threshold are designed. To exclude Zeno behavior, an exponential function is introduced in the threshold. Bipartite consensus problem is transformed into the corresponding stability problem by means of gauge transformation and model transformation. By virtue of Lyapunov method, sufficient conditions for systems without input delay are obtained to guarantee bipartite consensus. Furthermore, for the case with input delay, sufficient conditions which include an admissible upper bound of the delay are obtained to guarantee bipartite consensus. Finally, numerical simulations are provided to illustrate the effectiveness of the obtained theoretical results.

1. Introduction

Due to the wide applications on multirobot collaboration, cooperative control of unmanned aerial vehicles, and attitude alignment of satellites, many researchers have devoted to the study of coordination control of multiagent systems [1–4]. As one of the fundamental problems for distributed coordination in multiagent systems, the consensus problem has attracted much attention from multidisciplinary researchers due to its theoretical and practical significance. Consensus means states of all agents can reach a common value under an appropriate control protocol. During the past decade, a vast amount of theoretical achievements have been made on consensus problems [5–15]. Among them, when the single final consensus state cannot satisfy the control requirement with the increase of system size and complexity, group consensus, which means states of different groups of agents may approach to different values, has been put forward [13–15]. In [13], Yu and Wang studied the group consensus problem with both switching topologies and transmission delays. In [14], Altafini first proposed a sufficient and necessary condition for first-order multiagent systems to achieve bipartite consensus. In [15], Jiang et al. extended the results in [14] to general linear multiagent systems with signs.

Due to the complexity of real systems, it is possible for agents to have different dynamics. Hence, it is meaningful to study heterogeneous multiagent systems composed of first-order and second-order agents [16–20]. In [16], based on the properties of nonnegative matrices, sufficient consensus criterion was obtained for the agents with bounded communication delays under fixed topology and switching topologies, respectively. In [17], the consensus problem for continuous-time heterogeneous multiagent systems under directed graphs was investigated. In [18], by designing novel consensus protocols for continuous and discrete heterogeneous multiagent systems under directed topology, Liu et al. proved that the corresponding system asymptotically achieves consensus if and only if the fixed directed topology contains a directed spanning tree. In [19], sufficient and necessary condition for consensus of heterogeneous multiagent systems was given under certain assumptions on
control parameters. In [20], some sufficient group consensus conditions which depend on input delays and control parameters were obtained for heterogeneous multiagent systems by using the frequency-domain analysis method and matrix theory.

However, all the control schemes in the aforementioned works depend on continuous or periodic communication among agents. In practical systems, as the communication bandwidth and system energy are usually limited, it is a waste of energy to communicate with others when it is unnecessary. In order to reduce the frequency of communication controller updates, the event-triggered scheme has been developed [21–33]. The basic idea of the event-triggered control is to replace the paradigm of periodic sampling (or continuous control) by aperiodic sampling. In [21], Dimarogonas et al. proposed distributed event-triggered consensus strategies for first-order multiagent systems, whose updates depended on the ratio of a certain measurement error with respect to the norm of a function of the states. Different from [21], event-triggering conditions based on sampled data were designed in [22–25], where energy can be saved by checking the event condition only at periodic sampling instants. In [23], Fan et al. proposed a self-triggered consensus protocol to further reduce the amount of state sampling. In [25], Liu et al. investigated the periodic event-triggered consensus problem of multiagent systems under directed topology which contains a spanning tree. In [26], Xie et al. put forward a novel event-triggered control strategy for second-order multiagent systems. In [27, 28], Tan et al. addressed the mean square consensus problem of leader-following stochastic multiagent systems with input delay. In [31], the bipartite consensus problem for first-order multiagent systems via the event-triggered control was investigated. In [32], Yin et al. considered the event-triggered consensus of heterogeneous multiagent systems and designed event-triggering functions by using the positions and velocities of agents separately under a directed topology. After that, Yin et al. investigated event-triggered discrete-time heterogeneous multiagent systems with random communication delays represented by a Markov chain in [33].

Motivated by the aforementioned works, the distributed event-triggered scheme is applied to solve the bipartite consensus problem of heterogeneous multiagent systems in this paper. Different from [32], instead of designing the event-triggering function for the second-order agents by using positions and velocities of agents separately, we introduce a variable \( y_i = x_i + (1/\alpha)v_i \) in the event-triggering function, which has special use in analysis. Moreover, variable \( y_i \) is also used in designing control protocol, which can weaken the requirement of control gain. Compared with the threshold designed in [32, 33], where each agent still needs to continuously monitor its neighbors’ states, the threshold in this paper only depends on the latest event-triggered states of agents. To exclude Zeno behavior, we introduce an exponential function in the threshold. As the input delay between the controller and the actuator is ubiquitous in practical systems, the consensus problem of multiagent systems with input delay has been investigated in [10, 20, 30]. To the best of our knowledge, there are no results concerning bipartite consensus of heterogeneous multiagent systems with input delay under the event-triggered scheme. Therefore, we also consider the case with input delay in this study.

The paper is organized as follows. The graph theory and the problem formulation are given in Section 2. Some important definitions and lemmas are also presented in Section 2. The main results of this paper are given in Section 3. Both cases without input delay and with input delay are investigated under topology which contains a directed spanning tree. Numerical examples are given in Section 4 to validate the effectiveness of the obtained results. Finally, conclusions are given in Section 5.

Notations: \( \mathbb{R} \) and \( \mathbb{N} \) represent the set of real and natural numbers, respectively. \( \mathbb{R}^n \) and \( \mathbb{R}^{m \times n} \) denote \( n \)-dimensional real vector space and the \( n \times m \) real matrix space, respectively, \( \mathbf{1}_n(\mathbf{0}_n) \) denotes a column vector with all 1(0) elements. \( \mathbf{0} \) indicates a zero matrix with a proper order. \( \mathcal{F} = \{1, \ldots, n\} \) indicates an index set.

Notation \( \text{diag}(b_1, \ldots, b_n) \) denotes a diagonal matrix. For a symmetric matrix \( P, P > 0 \) means \( P \) is positive definite. \( \lambda(B) \) represents the eigenvalue of matrix \( B \). \( \Re(\lambda(B)) \) represents the real part of \( \lambda(B) \). \( \lambda_{\max}(B) \) and \( \lambda_{\min}(B) \) represent the maximum eigenvalue and the minimum eigenvalue of matrix \( B \), respectively. \( |\cdot| \) and \( \|\cdot\| \) denote \( l_1 \)-norm and Euclidean norm, respectively, both for vectors and matrices.

2. Preliminaries

In this section, we first give a brief review of graph theory and introduce some lemmas which will be used. Then, the model is formulated.

2.1. Graph Theory. A weighted directed graph (digraph) \( G = (\mathcal{V}, \mathcal{E}, A) \) consists of a node set \( \mathcal{V} = \{v_1, \ldots, v_n\} \), an edge set \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \), and a weighted adjacency matrix \( A = [a_{ij}] \in \mathbb{R}^{m \times n} \) satisfying \( a_{ij} \neq 0 \) iff \( (v_i, v_j) \in \mathcal{E} \). An edge \( e_{ij} = (v_i, v_j) \in \mathcal{E} \) means that node \( v_i \) can receive information from node \( v_j \). We assume that \( (v_i, v_j) \notin \mathcal{E} \); hence, \( a_{ij} = 0 \) for all \( i \in \mathcal{I} \). Define node \( v_j \) as a neighbor of node \( v_i \) if \( a_{ij} \neq 0 \). The set of neighbors of node \( v_i \) is denoted by \( \mathcal{N}_i = \{v_j \in \mathcal{E}: e_{ij} \in \mathcal{E}\} \). The in-degree of node \( v_i \) is defined as \( \Delta_{ii} = \sum_{j=1}^{n} a_{ij}, i \in \mathcal{I} \). The Laplacian matrix \( L \) of a weighted digraph \( G \) is defined as \( L = \Lambda - A \), where \( \Lambda = \text{diag}(\Lambda_1, \ldots, \Lambda_n) \). A directed path from \( v_j \) to \( v_i \) is a finite ordered sequence of distinct edges of \( G \) with the form \( (v_j, v_{k_1}), (v_{k_1}, v_{k_2}), \ldots, (v_{k_l}, v_i) \). A digraph contains a directed spanning tree if there exists a node called the root node such that there exists a directed path from it to every other node.
Definition 1. (structural balance, see [14]). A digraph $G$ is said to be structurally balanced if all the nodes of $G$ can be partitioned into two nonempty subsets $V_1$ and $V_2$ such that $V_1 \cup V_2 = V$, $V_1 \cap V_2 = \emptyset$, and the following two conditions hold:

1. $a_{ij} \geq 0, \forall v_i, v_j \in V_q$, where $q \in \{1, 2\}$
2. $a_{ij} \leq 0, \forall v_i \in V_q, v_j \in V'$, where $q \neq r$ and $q, r \in \{1, 2\}$

Definition 2 (gauge transformation, see [14]). Gauge transformation is a change of orhant order in $\mathbb{R}^n$ performed by an orthogonal matrix $D$ which is defined as $D \in \mathbb{R} \triangleq \{\text{diag}[a_1, \ldots, a_n], a_i \in \{1, -1\}\}$. Obviously, $D = D^{-1}$. When the digraph is structurally balanced, the entries of $DAD$ can be guaranteed to be nonnegative by selecting appropriate $D$. Consequently, the nondiagonal entries of $DLD$ are nonpositive, and the sum of each row is zero.

The following are some lemmas which will be used in Section 3.

Lemma 1 (see [5]). Given a matrix $B = [b_{ij}] \in \mathbb{R}^{m \times n}$, where $b_{ij} \geq 0, b_{ij} \leq 0$ for $\forall i \neq j$, and $\sum_{i=1}^{m} b_{ij} = 0$ for $\forall i$. $B$ has at least one zero eigenvalue, and all of the nonzero eigenvalues are in the open right half plane. Furthermore, $B$ has exactly one simple zero eigenvalue if and only if the digraph associated with $B$ contains a directed spanning tree.

Lemma 2 (Young’s inequality, see [34]). Given $x, y \in \mathbb{R}$, for $\forall e \in \mathbb{R}_{0+}, |x| \leq e^{x/2} + y^2/2e$.

Lemma 3 (comparison principle, see [35]). Consider a differential equation $du/dt = f(t, u), t \geq t_0$, where $f(t, u)$ is continuous and satisfies local Lipschitz condition in $u$. Let $[t_0, T]$ be the maximum existence interval of the solution $u(t)$, where $T$ can be infinite. If, for $\forall t \in [t_0, T)$, $v = v(t)$ satisfies $dv/dt \leq f(t, v), t \geq t_0$, and $v(t_0) \leq u(t_0)$, then $v(t) \leq u(t), t \in [t_0, T]$.

Lemma 4 (see [36]). Assuming that all the eigenvalues of matrix $P_i$ are in the open left half plane, then for all $t \geq 0$, it holds that

$$\|e^{Pt}\| \leq \beta e^{-\gamma t},$$

where $\beta \geq 1$ and $\gamma$ is any positive constant which is smaller than $\text{Re} \left(\lambda_{min}(-P_i)\right)$.

2.2. Problem Formulation. Suppose that the considered heterogeneous multiagent system has a communication topology represented by a digraph $G$, which means each agent is regarded as a node of the interaction digraph. Assume that the system is composed of $m (1 \leq m < n)$ agents with second-order dynamics and $n - m$ agents with first-order dynamics. Without loss of generality, we assume that agents $v_i$ to $v_m$ are second-order agents, whose index set is denoted as $\mathcal{I}_1 = \{1, 2, \ldots, m\}$, and agents $v_{m+1}$ to $v_n$ are first-order agents, whose index set is denoted as $\mathcal{I}_2 = \{m + 1, m + 2, \ldots, n\}$.

Assumption 1. The communication topology is structurally balanced with the following groups: the second-order agents constitute $\mathcal{V}_1$ and the first-order agents constitute $\mathcal{V}_2$.

The dynamics of the system are described as

$$\begin{cases}
\dot{x}_i(t) = v_i(t), \\
\dot{v}_i(t) = u_i(t),
\end{cases} i \in \mathcal{I}_1,$n

$$\begin{cases}
\dot{x}_i(t) = u_i(t), \\
\dot{v}_i(t) = u_i(t),
\end{cases} i \in \mathcal{I}_2, \ t \geq 0,$n

where $x_i, v_i$, and $u_i \in \mathbb{R}$ are the position, velocity, and control input of the $i$th agent, respectively. Based on the event-triggered control scheme, each agent broadcasts its state information at the event-triggering time instants. The sequence of event-triggering time instants of agent $v_i$ is denoted as $\{t_{\text{on}}\}, m \in \mathbb{N}$, with $t_{\text{on}} = 0$.

The definition of bipartite consensus of (2) is given as follows.

Definition 3. Bipartite consensus of system (2) can be achieved for any initial conditions if it holds that

$$\begin{cases}
\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0, \\
\lim_{t \to \infty} v_i(t) = v_j(t) = 0, \\
\lim_{t \to \infty} |x_i(t) - x_j(t)| = 0, \ \forall i, j \in \mathcal{I}_1, \\
\lim_{t \to \infty} |x_i(t) + x_j(t)| = 0, \ \forall i \in \mathcal{I}_1, j \in \mathcal{I}_2.
\end{cases}$$

3. Consensus Analysis

For system (2), we propose the following bipartite consensus protocol:

$$u_i(t) = -av_i(t_{\text{on}} - \tau) + \alpha^2 \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(t_{\text{on}} - \tau) - y_j(t_{\text{on}} - \tau)) - \sum_{j \in \mathcal{N}_i} a_{ij}(y_j(t_{\text{on}} - \tau) + y_j(t_{\text{on}} - \tau)), \ i \in \mathcal{I}_1,$n

$$u_i(t) = \alpha \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_{\text{on}} - \tau) - x_i(t_{\text{on}} - \tau)) - \sum_{j \in \mathcal{N}_i} a_{ij}(y_j(t_{\text{on}} - \tau) + y_j(t_{\text{on}} - \tau)), \ i \in \mathcal{I}_2, t\geq \tau,$n

where $y_i(t - \tau) = x_i(t - \tau) + (1/\alpha)v_i(t - \tau), t \in [t_{\text{on}}, t_{\text{on}} + 1)$, $i \in \mathcal{I}_1$, $\tau \geq 0$ represents the input delay, $t_{\text{on}} = \text{argmin}_{m \in \mathbb{N}}$
respectively. Note that when $\tau > 0$, the control input $u_i(t)$ is chosen as zero for $t \in [0, \tau)$.

Remark 1. Compared with [17] where the condition $k_i > 2\sqrt{\max_{e \in \mathcal{F}} \| \ell_i \|}$ was required to guarantee consensus of heterogeneous multiagent systems, the control gain $\alpha$ is only required to be positive, even though the event-triggered scheme is applied here and it was absent in [17]. More specifically, from the following analysis, we can see that heterogeneous system (2) under (4) can be transformed into an equivalent homogeneous system containing $n + m$ agents by virtue of variable $y_i$, which makes the analysis easier.

For $t \in [t_m^1, t_{m+1})$, denote measurement errors as $e_{ix}(t) = x_i(t_{m+1} - t) - x_i(t - t), i \in \mathcal{F}$, $e_{yx}(t) = y_i(t_{m+1} - t) - y_i(t - t), i \in \mathcal{F}_1$, and $e_{iy}(t) = y_i(t_{m+1} - t) - y_i(t - t), i \in \mathcal{F}_2$. Event-triggering function for each agent is designed as

$$f_{i1}(t) = \left\| \begin{bmatrix} e_{ix}(t) \\ e_{iy}(t) \end{bmatrix} \right\|^2 - \sigma |p_i(t)|^2 - \zeta(t), \quad i \in \mathcal{F}_1,$$

$$f_{i2}(t) = |e_{ix}(t)|^2 - \sigma |q_i(t)|^2 - \zeta(t), \quad i \in \mathcal{F}_2,$$

where

$$u_i(t) = -\alpha(v_i(t) + e_{ix}(t)) + \alpha^2 \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t) - y_i(t) + e_{yx}(t) - e_{iy}(t)) - \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) + x_j(t) + (e_{ix}(t) + e_{ix}(t))), \quad i \in \mathcal{F}_1,$$

$$u_i(t) = \alpha \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t) - x_i(t) + e_{yx}(t) - e_{iy}(t)) - \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(t) + x_j(t) + e_{ix}(t) + e_{ix}(t)), \quad i \in \mathcal{F}_2.$$

Substituting (6) into system (2), we can obtain

$$\dot{x}_i(t) = \alpha (y_i(t) - x_i(t)), \quad i \in \mathcal{F}_1,$$

$$\dot{y}_i(t) = -\alpha(e_{iy}(t) - e_{ix}(t)) + \alpha \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t) - y_i(t) + e_{yx}(t) - e_{iy}(t)) - \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) + x_j(t) + (e_{ix}(t) + e_{ix}(t))), \quad i \in \mathcal{F}_1,$$

$$\dot{y}_i(t) = \alpha \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t) - x_i(t) + e_{yx}(t) - e_{iy}(t)) - \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(t) + x_j(t) + e_{ix}(t) + e_{ix}(t)), \quad i \in \mathcal{F}_2.$$

Based on the grouping of multiagent systems, the adjacency matrix $A$ of the digraph $\mathcal{G}$ can be described as

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix},$$

where $A_1 \in \mathbb{R}^{mxm}$, $A_2 \in \mathbb{R}^{mx(n-m)}$, $A_3 \in \mathbb{R}^{(n-m)xm}$, and $A_4 \in \mathbb{R}^{(n-m)x(n-m)}$. Set $\mathbf{x}_m(t) = [x_1(t), \ldots, x_m(t)]^T$, $\mathbf{y}_m(t) = [y_1(t), \ldots, y_m(t)]^T$, $\mathbf{x}_{m+1}(t) = [x_{m+1}(t), \ldots, x_n(t)]^T$, $\mathbf{e}_{mx}(t) = [e_{mx}(t), \ldots, e_{mx}(t)]^T$, $\mathbf{e}_{my}(t) = [e_{my}(t), \ldots, e_{my}(t)]^T$, $\mathbf{e}_{(m+1)x}(t) = [e_{(m+1)x}(t), \ldots, e_{(m+1)x}(t)]^T$, and $\mathbf{e}(t) = [\mathbf{e}_m(t), \mathbf{e}_{mx}(t), \mathbf{e}_{my}(t)]^T$. Thus, (7) can be written in a compact form as

$$\dot{x}(t) = -\Xi \dot{x}(t) - \Psi \mathbf{e}(t),$$

where

$$\Xi = \begin{bmatrix} \alpha I_m & -\alpha I_m & 0 \\ -\alpha \tilde{A}_1 & \alpha (\Lambda_1 + \Lambda_2) & \alpha \tilde{A}_2 \\ \alpha \tilde{A}_3 & 0 & -\alpha \tilde{A}_4 + \alpha (\Lambda_3 + \Lambda_4) \end{bmatrix},$$

$$\Psi = \begin{bmatrix} 0 & 0 & 0 \\ -\alpha I_m + \alpha (\Lambda_1 + \Lambda_2) & \alpha \tilde{A}_2 \\ \alpha \tilde{A}_3 & 0 & -\alpha \tilde{A}_4 + \alpha (\Lambda_3 + \Lambda_4) \end{bmatrix},$$

in which $\tilde{A}_i = A_i$ for $i = 1, 4$, $\tilde{A}_i = -A_i$ for $i = 2, 3$. 

3.1. Without Input Delay. First, we investigate bipartite consensus of (2) without input delay, i.e., $\tau = 0$. According to the definitions of measurement errors, bipartite consensus protocol (4) can be rewritten as

$$p_i(t) = -v_i(t_{m+1} - t) + \alpha \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_{m+1} - t) - y_i(t_{m+1} - t)) - \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t_{m+1} - t) + x_j(t_{m+1} - t)), \quad i \in \mathcal{F}_1; \quad q_i(t) = \alpha \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_{m+1} - t) - x_i(t_{m+1} - t)) - \sum_{j \in \mathcal{N}_i} a_{ij}(x_i(t_{m+1} - t) + x_j(t_{m+1} - t)), \quad i \in \mathcal{F}_2; \quad \zeta(t) = c_1 e^{-\sigma t}$ is an exponential function with $c_1 > 0$, $c_2 > 0$; $\sigma > 0$ is the event-triggering parameter which will be further defined later. When $f_{i1}(t) \geq 0$ or $f_{i2}(t) \geq 0$, the event of corresponding agent $v_i$ is triggered immediately.
\[ \Lambda_1 = \text{diag} \left\{ \sum_{j=1}^{m} a_{1j}, \ldots, \sum_{j=1}^{m} a_{mj} \right\}, \]
\[ \Lambda_2 = -\text{diag} \left\{ \sum_{j=m+1}^{n} a_{1j}, \ldots, \sum_{j=m+1}^{n} a_{mj} \right\}, \]
\[ \Lambda_3 = -\text{diag} \left\{ \sum_{j=1}^{n} a_{(m+1)j}, \ldots, \sum_{j=1}^{n} a_{nj} \right\}, \]
\[ \Lambda_4 = \text{diag} \left\{ \sum_{j=m+1}^{n} a_{(m+1)j}, \ldots, \sum_{j=m+1}^{n} a_{nj} \right\}. \]

Obviously, all entries of \( \tilde{A}_i \) (\( i = 1, 2, 3, 4 \)) are nonnegative. Based on Assumption 1, (9) can be written as
\[ \hat{\xi}(t) = -\hat{D}\bar{\Phi}(t) - \bar{\Psi}\hat{e}(t), \]
where \( \bar{\Phi}(t) = DK(t), \bar{e}(t) = De(t), \Xi = DED, \bar{\Psi} = D\Psi D \) with \( D = \text{diag}[I_{n+m}, I_{n+m}] \). Otherwise, the sum of entries in each row of \( \Xi \) is zero. Meanwhile, the diagonal entries are all nonnegative. Therefore, \( \Xi \) can be regarded as the Laplacian matrix corresponding to a digraph \( \mathcal{G}' \) with \( n+m \) nodes.

**Lemma 5** (see [18]). The digraph \( \mathcal{G}' \) corresponding to the matrix \( \hat{\Phi} \) contains a directed spanning tree if and only if digraph \( \mathcal{G} \) contains a directed spanning tree.

It is easy to see that the bipartite consensus problem of system (2) is equivalent to the consensus problem of system (12). Denote \( \psi(t) = E\bar{\xi}(t) \) and \( \sigma(t) = E\bar{e}(t) \), where \( E = [-I_{n+m-1} \ I_{n+m-1}] \). Correspondingly, \( \tilde{\xi}(t) = F\tilde{\xi}(t)1_{n+m1}(t) \) and \( \tilde{\sigma}(t) = F\tilde{\sigma}(t)1_{n+m1}(t) \), where \( F = [0_{n+m-1} \ I_{n+m-1}] \). Then, (12) can be rewritten as
\[ \tilde{\xi}(t) = -E\bar{D}F\tilde{\xi}(t) - E\bar{F}\bar{\Psi}\tilde{e}(t). \]

Then, from [7], we have the consensus problem of system (12) which can be solved asymptotically if and only if system (13) achieves stability asymptotically. Moreover, when the digraph \( \mathcal{G}' \) contains a directed spanning tree, all the eigenvalues of the matrix \( -E\Xi F \) have negative real parts. So, there exists a positive definite matrix \( P \) satisfying
\[
\begin{align*}
& P(-E\bar{D}F) + (-E\bar{F}\bar{\Psi})^TP = -I_{n+m-1}. \tag{14}
\end{align*}
\]
For further analysis, we select a positive number \( \epsilon > 0 \) such that
\[
\gamma = 1 - \epsilon\|P(-E\bar{F}\bar{\Psi})\|^2 > 0. \tag{15}
\]

**Theorem 1.** Suppose that Assumption 1 holds and the digraph \( \mathcal{G} \) contains a directed spanning tree. The bipartite consensus problem of heterogeneous system (2) can be solved under event-triggered control protocol (4) and event-triggering functions (5) when event-triggering parameter \( \sigma \) satisfies
\[
\sigma < \frac{\gamma e}{2\gamma e\|\bar{\Psi}\|^2 + 2\lambda_{\text{max}}(E^T E)\|\bar{\Psi}\|^2}. \tag{16}
\]

**Proof.** Construct the following Lyapunov function:
\[
V(t) = \tilde{\xi}^T(t)P\tilde{\xi}(t), \tag{17}
\]
where \( P \) is the positive definite matrix satisfying (14). The time derivative of \( V(t) \) is as follows:
\[
\dot{V}(t) = \tilde{\xi}^T(t)P\tilde{\xi}(t) + \tilde{\xi}^T(t)P\tilde{\xi}(t)
= \tilde{\xi}^T(t)[P(-E\bar{D}F) + (-E\bar{F}\bar{\Psi})^TP]\tilde{\xi}(t) + 2\tilde{\xi}^T(t)P(-E\bar{F}\bar{\Psi})\tilde{e}(t)
\leq -\tilde{\xi}^T(t)\tilde{\xi}(t) + \tilde{\xi}^T(t)P(-E\bar{F}\bar{\Psi})(-E\bar{F}\bar{\Psi})^TP\tilde{\xi}(t) + \tilde{\xi}^T(t)\tilde{e}(t)
\leq -\tilde{\xi}^T(t)\tilde{\xi}(t) + c\|P(-E\bar{F}\bar{\Psi})\|^2\|\tilde{\xi}(t)\|^2 + \frac{1}{\epsilon}\max(\lambda_{\text{max}}E^T E)\|\tilde{\xi}(t)\|^2. \tag{18}
\]

Based on event-triggering functions (5), we can get that
\[
\|\tilde{\xi}(t)\|^2 \leq \|\Psi(\tilde{\xi}(t) + \tilde{e}(t))\|^2 + n_c\epsilon e^{-\zeta t/f} \leq 2\sigma(\|\Psi\tilde{\xi}(t)\|^2 + \|\tilde{e}(t)\|^2) + n_c\epsilon e^{-\zeta t/f}. \tag{19}
\]
According to the definitions of \( \tilde{\xi}(t), \tilde{e}(t), \) and \( \Psi \) in (12), we have
\[
\|\tilde{e}(t)\|^2 \leq 2\sigma(\|\tilde{\xi}(t)\|^2 + \|\tilde{e}(t)\|^2) + n_c\epsilon e^{-\zeta t/f}. \tag{20}
\]
Combining with (16), it is easy to get that \( \sigma < 1/2\|\Psi\|^2 \).
Thus, \( \|\tilde{\xi}(t)\|^2 \leq 2\sigma(\|\Psi\tilde{\xi}(t)\|^2 + n_c\epsilon e^{-\zeta t/f}) < 2\|\tilde{\xi}(t)\|^2 \).
Therefore, (18) can be further extended to the following inequality:
\[
\dot{V}(t) \leq -\left( \gamma - \frac{\lambda_{\text{max}}(E^T E)(2\sigma\|\Psi\|^2)}{\epsilon(1 - 2\sigma\|\Psi\|^2)} \right)|\tilde{\xi}|^2
+ \frac{\lambda_{\text{max}}(E^T E)}{\epsilon(1 - 2\sigma\|\Psi\|^2)}n_c\epsilon e^{-\zeta t/f}. \tag{21}
\]
From (16), we have \( \gamma - \lambda_{\text{max}}(E^T E)(2\sigma\|\Psi\|^2)/\epsilon(1 - 2\sigma\|\Psi\|^2) > 0 \). Then, combining (21) with (17), we can get that
\[
\dot{V}(t) \leq -\varrho V + \varrho \tilde{\xi}^T(t)P\tilde{\xi}(t) - \left( \gamma - \frac{\lambda_{\text{max}}(E^T E)(2\sigma\|\Psi\|^2)}{\epsilon(1 - 2\sigma\|\Psi\|^2)} \right)|\tilde{\xi}|^2
+ \frac{\lambda_{\text{max}}(E^T E)}{\epsilon(1 - 2\sigma\|\Psi\|^2)}n_c\epsilon e^{-\zeta t/f} \leq -\varrho V + \varrho \lambda_{\text{max}}(P)|\tilde{\xi}|^2
+ \frac{\lambda_{\text{max}}(E^T E)}{\epsilon(1 - 2\sigma\|\Psi\|^2)}n_c\epsilon e^{-\zeta t/f} \tag{22}
\]
\[ \varphi(t) = \begin{cases} e^{-\varphi_0} + \frac{\lambda_{\text{max}}(E^T) m c}{1 - 2\sigma \| -\Psi \|^2} e^{-\| -\Psi \|^2 t}, & \text{if } q = c_2, \\ e^{-\varphi_0} + \frac{\lambda_{\text{max}}(E^T) m c}{1 - 2\sigma \| -\Psi \|^2} (e^{-c_2 t} - e^{-\varphi_0}), & \text{if } q \neq c_2. \end{cases} \] (23)

Obviously, \( \lim_{t \to \infty} \varphi(t) = 0 \). Hence, \( \lim_{t \to \infty} \xi(t) = 0 \), i.e., system (2) achieves bipartite consensus asymptotically.

The following theorem illustrates that Zeno behavior does not exist, i.e., there is no trajectory with infinite event-triggered time instants in a finite time interval.

**Theorem 2.** System (2) does not exhibit Zeno behavior under conditions of Theorem 1.

**Proof.** First, we consider agents in \( \mathcal{F}_1 \). Assume that \( t_m^i \) and \( t_{m+1}^i \) are two adjacent event-triggering time instants of agent \( \nu_i \), i.e., \( e_{ix}(t_m^i) = e_{ix}(t_{m+1}^i) = 0 \) and \( e_{iy}(t_m^i) = e_{iy}(t_{m+1}^i) = 0 \). Thus, \( e_{ix}(t_m^i) = e_{ix}(t_{m+1}^i) = 0 \).

Considering the derivative of \( \| [e_{ix}(t_i), e_{iy}(t_i)] \| \) over the interval \( [t_m^i, t_{m+1}^i] \), we have

\[ \frac{d}{dt} \| [e_{ix}(t_i), e_{iy}(t_i)] \| = \frac{d}{dt} \sqrt{e_{ix}^2(t_i) + e_{iy}^2(t_i)} \leq |y_i(t_i) - x_i(t_i)| + |y_i(t_i) - x_i(t_i) + p_i(t_i)| + |y_i(t_i) - x_i(t_i) - (x_i(t_i) - x_i(t_i))| + |y_i(t_i) - x_i(t_i) - (x_i(t_i) - x_i(t_i)) + p_i(t_i)|. \] (24)

From the proof of Theorem 1, we know that \( \lim_{t \to \infty} \xi(t) = 0 \). Thus, \( \xi(t) \) is bounded. According to the definition of \( \xi(t) \) and gauge transformation, for all \( t \in [t_m^i, t_{m+1}^i] \), \( y_i(t) - x_i(t) \), \( y_i(t_i) - x_i(t_i) \), and \( x_i(t_i) - x_i(t_i) \), \( i \in \mathcal{F}_2 \), and \( -x_i(t_i) - x_i(t_i) \), \( i \in \mathcal{F}_1 \), are all bounded. Thus, \( p_i(t) = -a(y_i(t) - x_i(t)) + a(y_i(t_i) - x_i(t_i)) + a \sum_{j \in N_i} a_j (y_j(t) - x_j(t)) - \sum_{j \in N_i} a_j (y_j(t) - x_j(t_i)) \) is bounded for \( t \in [t_m^i, t_{m+1}^i] \). Therefore, \( \frac{d}{dt} \| e_{ix}(t_i), e_{iy}(t_i) \| \) is bounded, i.e., there exists a constant \( M > 0 \) such that \( \frac{d}{dt} \| e_{ix}(t_i), e_{iy}(t_i) \| \leq M \). Then,

\[
\begin{bmatrix} e_{ix}(t) \\ e_{iy}(t) \end{bmatrix} \leq M(t - t_m^i), \quad t \in [t_m^i, t_{m+1}^i], \quad i \in \mathcal{F}_1. \] (25)

For \( i \in \mathcal{F}_1 \), since the event will not be triggered before \( \| [e_{ix}(t), e_{iy}(t)] \|^2 = c_1 e^{-c_2 t} \), by (25), a lower bound on \( [t_m^i, t_{m+1}^i] \), which is denoted as \( t_{m+1}^i \), satisfies \( c_1 e^{-c_2 t_{m+1}^i} = (M t_{m+1}^i)^2 e^{-c_2 t_{m+1}^i} \). That is, \( t_{m+1}^i = \sqrt{c_1 e^{-c_2 t_{m+1}^i}} /2M \). According to the definition of sequence limit, for \( \epsilon_0 > 0 \), there exists a positive integer \( N_0 \) such that \( t^* - \epsilon_0 < t^* \leq t_{m+1}^i \) for \( m \geq N_0 \). Therefore, when \( m \geq N_0 \), \( t_{m+1}^i \geq t_{m+1}^i + t_{m+1}^i = t_{m+1}^i + 2 \epsilon_0 \geq t^* + \epsilon_0 \). This contradicts with the fact that \( t_{m+1}^i \leq t^* \) for \( m \geq N_0 \). Hence, Zeno behavior is strictly excluded.

Next, we consider agents in \( \mathcal{F}_2 \). Similarly, assume that \( t_m^i \) and \( t_{m+1}^i \) are two adjacent event-triggering time instants of agent \( \nu_i \), i.e., \( e_{ix}(t_m^i) = e_{ix}(t_{m+1}^i) = 0 \). Since \( \dot{e}_{ix}(t) = -\dot{x}_i(t) = -u_i(t), i \in \mathcal{F}_2 \), it holds that, for all \( t \in [t_m^i, t_{m+1}^i] \), \( \| e_{ix}(t) \| \leq \int_{t_m^i}^{t} |u_i(s)|ds \). Similar to the above analysis, it is easy to get that \( u_i(t) \) is bounded for \( i \in \mathcal{F}_2 \), i.e., there exists a constant \( M' > 0 \) such that \( |u_i(t)| \leq M' \). Then,

\[
\| e_{ix}(t) \| \leq M' (t - t_m^i), \quad i \in \mathcal{F}_2, \quad (26)
\]

The event will not be triggered before \( \| e_{ix}(t) \|^2 = c_1 e^{-c_2 t} \) for \( i \in \mathcal{F}_2 \). Combining with (26), the Zeno behavior can be excluded. Since the proof is just as same as that for agents in \( \mathcal{F}_1 \), we omit it here.

In conclusion, all the agents of system (2) do not exhibit Zeno behavior under conditions of Theorem 1.

**Remark 2.** From the proof of Theorem 2, we can see that, for second-order agents, to prove that \( \frac{d}{dt} \| [e_{ix}(t), e_{iy}(t)] \| \) is bounded by virtue of variable \( y_i \), which plays a key role to exclude Zeno behavior. This further shows the importance of introducing \( y_i \).

### 3.2. With Input Delay

In this section, we investigate bipartite consensus of (2) with input delay, i.e., \( \tau > 0 \).

Substituting (4) into system (2) and according to the definitions of measurement errors, we can get that
\[
\begin{aligned}
\dot{x}_i(t) &= \alpha (y_i(t) - x_i(t)), \quad i \in \mathcal{I}_1, \\
\dot{y}_i(t) &= -\alpha (e_{iy}(t) - e_{ix}(t)) + \alpha \left[ \sum_{j \in \mathcal{N}_i} (x_j(t) - y_i(t) + e_{ix}(t) - e_{iy}(t)) - \sum_{j \in \mathcal{N}_i} a_{ij} y_j(t) + x_j(t) + e_{iy}(t) + e_{ix}(t) \right], \\
\dot{x}_i(t) &= \alpha \left[ \sum_{j \in \mathcal{N}_i} a_{ij} (x_i(t) - x_j(t) + e_{ix}(t) - e_{ix}(t)) - \sum_{j \in \mathcal{N}_i} a_{ij} (x_i(t) + x_j(t) + e_{ix}(t) + e_{ix}(t)) \right], \quad i \in \mathcal{I}_2.
\end{aligned}
\]

Then, (27) can be rewritten in a compact form as
\[
\dot{\xi}(t) = -H\xi(t) - \Xi(t) - \Psi \xi(t),
\]
where
\[
H = \begin{bmatrix}
\alpha I_m & -\alpha I_m & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix},
\]
\[
\Xi = \begin{bmatrix}
-\alpha \tilde{A}_1 & \alpha (\Lambda_1 + \Lambda_2) & \alpha \tilde{A}_2 \\
0 & -\alpha \tilde{A}_4 & \alpha (\Lambda_3 + \Lambda_4)
\end{bmatrix},
\]
and \( \tilde{A}_i, \Lambda_i, i = [1, 2, 3, 4] \) and \( \Psi \) have the same definitions as in (9). Analogically, according to Assumption 1, system (28) can be written as
\[
\dot{\tilde{\xi}}(t) = -\tilde{H}\tilde{\xi}(t) - \tilde{\Xi}(t) - \tilde{\Psi} \tilde{\xi}(t),
\]
where \( \tilde{H} = DHD, \tilde{D} = DE_D, \) and \( \tilde{\xi}(t), \tilde{\xi}(t), \tilde{\Psi}, \tilde{D} \) have the same definitions as in (12). By Newton–Leibniz formula, (30) can be rewritten as
\[
\dot{\tilde{\xi}}(t) = -H(t + \tilde{\xi}) \tilde{\xi}(t) - \tilde{\Psi} \xi(t) + \tilde{\Xi} \int_{t-\tau}^{t} \tilde{\xi}(s) ds.
\]

**Remark 3.** Based on Lemma 5, we can get the following result similarly. The digraph \( \mathcal{G} \) corresponding to the matrix \( \tilde{H} + \tilde{\Xi} \) has a directed spanning tree if and only if graph \( \mathcal{G} \) has a directed spanning tree. And the bipartite consensus problem of system (2) under control protocol (4) can be solved asymptotically if and only if system (31) achieves consensus asymptotically.

By using vectors \( \tilde{\xi}(t) \) and \( \tilde{\pi}(t) \) which have been denoted in (13), system (31) can be written as
\[
\dot{\tilde{\xi}}(t) = -E(\tilde{H} + \tilde{\Xi}) F \tilde{\xi}(t) - E\tilde{\Psi} F \tilde{\pi}(t) + E\tilde{E} \Xi \int_{t-\tau}^{t} \tilde{\xi}(s) ds.
\]

Then, system (30) achieves consensus asymptotically if and only if system (32) achieves stability asymptotically.

The following notations will be used in the next analysis. Set
\[
\begin{aligned}
a_1 &= \| -EHF \|, \\
a_2 &= \| -E\tilde{E} \Xi \|, \\
a_3 &= \| -E\tilde{\Psi} F \|,
\end{aligned}
\]
\[
\begin{aligned}
b_1 &= \sqrt{\lambda_{\text{max}}(E^T E) 2\sigma^2 - \tilde{\Psi} \tilde{\Xi} \tilde{\Xi}^T / (1 - 2\sigma - \tilde{\Psi} \tilde{\Xi} \tilde{\Xi}^T)}, \\
b_2 &= \sqrt{\lambda_{\text{max}}(E^T E) \sigma / (1 - 2\sigma - \tilde{\Psi} \tilde{\Xi} \tilde{\Xi}^T)}.
\end{aligned}
\]

**Theorem 3.** Suppose that Assumption 1 holds and the digraph \( \mathcal{G} \) contains a directed spanning tree. The bipartite consensus problem of system (2) can be solved under event-triggered control protocol (4) and event-triggering functions (5) when \( \sigma, \tau, \) and \( c_2 \) satisfy \( 0 < \sigma < \sigma_0, 0 < \tau < \tau_0, \) and \( 0 < c_2 < 2r, \) where
\[
\begin{aligned}
\sigma_0 &= \frac{r^2}{2\beta^2 a_3 \lambda_{\text{max}}(E^T E) - \tilde{\Psi} \tilde{\Xi} \tilde{\Xi}^T + 2r^2 - \tilde{\Psi} \tilde{\Xi}^T / 2}, \\
t_0 &= \frac{r - \beta a_3 b_1}{\beta (a_1 a_2 + a_2 (a_2 + a_3 b_1))},
\end{aligned}
\]
and \( \beta \) and \( r \) are the numbers obtained from Lemma 4 with \( P_1 = -E(\tilde{H} + \tilde{\Xi}) F. \) Moreover, system (2) does not exhibit Zeno behavior.

**Proof.** By integrating (32), we have
\[
\begin{aligned}
\tilde{\xi}(t) &= e^{-E(\tilde{H} + \tilde{\Xi}) F \int_{t-\tau}^{t} \tilde{\xi}(s) ds} \left[ \int_0^t e^{-E(\tilde{H} + \tilde{\Xi}) F (s-\theta)} \right] \\
&\quad \times \left\{ -E\tilde{\Psi} F \tilde{\pi}(\theta) + E\tilde{E} F \int_{\theta-\tau}^{\theta} \left[ -E\tilde{H} \tilde{\xi}(s) E\tilde{E} F \tilde{\xi}(s) - E\tilde{\Psi} F \tilde{\varsigma}(s) \right] ds d\theta \right\}.
\end{aligned}
\]

According to [7], we know that all the eigenvalues of the matrix \( -E(\tilde{H} + \tilde{\Xi}) F \) have negative real parts. Therefore, based on Lemma 4, it follows that
\[
\begin{aligned}
\| \tilde{\xi}(t) \| &\leq \beta e^{-\tau t} \| \tilde{\xi}(0) \| + \beta \int_0^t e^{-r(t-\theta)} \times \left\{ \| -E\tilde{\Psi} F \| \| \pi(\theta) \| \\
&+ \| E\tilde{E} F \| \times \int_{\theta-\tau}^{\theta} \| -E\tilde{H} \| \| \tilde{\xi}(s) \| ds d\theta \right\}.
\end{aligned}
\]

Based on event-triggering functions (5) and the requirement of \( \sigma \) in Theorem 3, we can get that
\[
\| \tilde{\pi}(t) \| \leq b_1 \| \tilde{\xi}(t - \tau) \| + b_2 e^{-(c_2/2r)}.
\]

By replacing (36) into (35), we can derive that
\[ \begin{align*}
\|\bar{x}(t)\| & \leq e^{-\tau t}\|\bar{x}(0)\| + \int_0^t e^{-(\tau-\theta)}\left[a_i b_i \|\bar{x}(\theta-t)\| + a_i b_i e^{-(c_i/2)\theta}\right] d\theta \\
& \quad + a_2 \int_{\theta-1}^{\theta} \left[a_i b_i \|\bar{x}(\theta-t)\| + a_i b_i \|\bar{x}(s-t)\| + a_i b_i \|\bar{x}(s-t)\|\right] ds \, d\theta \\
& \quad + a_i b_i e^{-(c_i/2)\theta}\|\bar{x}(\theta-t)\| ds \, d\theta.
\end{align*} \]  
\[(37)\]

Next, we prove that
\[ \|\bar{x}(t)\| < w(t) \triangleq \beta\|\bar{x}(0)\|e^{-\lambda t} + \chi e^{-(c_i/2)t}, \quad t \geq 0, \]  
\[(38)\]

where \( \lambda \in (0, r) \) satisfies
\[ \beta \lambda a_i b_i e^{\lambda t} + \beta \left[ a_i a_1 + a_2 (a_2 + a_1 b_1) e^{\lambda t}\right] (e^{\lambda t} - 1) < 1. \]

First, we prove the existence of \( \lambda \). Define a function \( h(\lambda) = \beta \lambda a_i b_i e^{\lambda t} + \beta \left[ a_i a_1 + a_2 (a_2 + a_1 b_1) e^{\lambda t}\right] (e^{\lambda t} - 1) - \lambda \). It is easy to get that \( h(0) = 0 \) and \( h(0) = \beta \left[ a_i a_1 + a_2 (a_2 + a_1 b_1)\right] \tau + \beta a_i b_i. \) Since \( \sigma < a_0 \), we can get that \( \beta a_i b_i < r. \) Meanwhile, \( h(0) < 0 \) since \( r > r_0. \) Therefore, there must exist a positive constant \( \lambda \in (0, r) \) such that \( h(\lambda) < 0. \) Consequently, (39) holds. Similarly, we can prove that the denominator of \( \chi \) is positive when \( 0 < c_i < 2r, \) which implies that \( \chi > 0. \) Next, we prove that (38) holds. Note that \( \|\bar{x}(t)\| \leq \beta\|\bar{x}(0)\|e^{-\lambda t} - \chi e^{-(c_i/2)t} < w(t) \) for \( t = 0. \) If (38) does not hold, there must exist \( t^* > 0 \) such that
\[ \|\bar{x}(t^*)\| = w(t^*) = \beta\|\bar{x}(0)\|e^{-\lambda t^*} + \chi e^{-(c_i/2)t^*}. \]

Replacing (38) into (37), we have
\[ \begin{align*}
\|\bar{x}(t^*)\| & < \beta\|\bar{x}(0)\| e^{-\lambda t^*} + \chi e^{-(c_i/2)t^*}
\end{align*} \]  
\[(40)\]

By some direct calculations, it follows that
\[ \|\bar{x}(t^*)\| < \beta\|\bar{x}(0)\| \left\{ e^{-\lambda t^*} + \left( e^{-\lambda t^*} - e^{-\lambda r} \right) \right\} \times \frac{\beta \lambda a_i b_i e^{\lambda t^*} + \beta \left[a_i a_1 + a_2 (a_2 + a_1 b_1) e^{\lambda t^*}\right] (e^{\lambda t^*} - 1)}{\lambda (r - \lambda)} \\
+ \left( e^{-\lambda t^*} - e^{-\lambda r} \right) \times \frac{\beta \left[(c_i/2) a_i b_i + a_2 b_i e^{-(c_i/2)r}\right]}{(c_i/2) (r - (c_i/2))} - \beta \left[a_i a_1 + a_2 (a_2 + a_1 b_1) e^{-(c_i/2)r}\right] (e^{-(c_i/2)r} - 1) + (c_i/2) a_i b_i e^{(c_i/2)r} \\
< \beta\|\bar{x}(0)\| e^{-\lambda t^*} + \chi e^{-(c_i/2)t^*} = w(t^*). \]  
\[(42)\]

This contradicts with (40). Hence, (38) holds, and bipartite consensus of system (2) under control protocol (4) and event-triggering function (5) can be solved asymptotically.
Figure 1: Directed topology which contains a directed spanning tree.

Figure 2: The position trajectory of each agent when $\tau = 0$.

Figure 3: The velocity trajectory of each agent when $\tau = 0$. 
The proof of exclusion of Zeno behavior is just as same as that for Theorem 2. The details are omitted here for brevity.

4. Simulation

In this section, we give some numerical simulations to demonstrate the effectiveness of the obtained theoretical results. Consider a heterogeneous multiagent system composed of four agents and under a direct topology shown in Figure 1. Obviously, $G$ is structurally balanced with $V_1 = \{v_1, v_2\}$, $V_2 = \{v_3, v_4\}$, where $V_1$ is composed of second-order agents and $V_2$ is composed of first-order agents. For simplicity, let $a_{ij} = 1$ if $a_{ij} \neq 0$. Let the initial states of system (2) be $x(0) = [2, -2, -4, -1]^T$, $v(0) = [-2, 3]^T$ and choose $\alpha = 0.18$. According to the topology in Figure 1, we have

$$
\Phi = \begin{bmatrix}
0.18 & 0 & -0.18 & 0 & 0 & 0 \\
0 & 0.18 & 0 & -0.18 & 0 & 0 \\
0 & 0 & 0.36 & 0 & -0.18 & -0.18 \\
-0.18 & 0 & 0.18 & 0 & 0 & 0 \\
0 & -0.18 & 0 & 0 & 0.36 & -0.18 \\
0 & 0 & 0 & 0 & 0 & 0.18
\end{bmatrix},
$$

$$
\Psi = \begin{bmatrix}
0.18 & 0 & 0.54 & 0 & -0.18 & -0.18 \\
0 & 0 & 0 & 0 & 0 & 0.18 \\
0 & 0 & 0 & 0 & 0 & 0.18 \\
-0.18 & 0 & 0.36 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.36 & -0.18 \\
0 & 0 & 0 & 0 & 0 & 0.18
\end{bmatrix}.
$$

Example 1. This example is for the case without input delay, i.e., $\tau = 0$. By solving the Lyapunov equation in (14), we can get that $P = \begin{bmatrix} 3.742 & -1.576 & 1.357 & 0.091 & 0.874 \\
-1.576 & 2.034 & -1.510 & 0.113 & -0.351 \\
1.357 & -1.510 & 4.135 & -0.473 & -0.555 \\
0.091 & 0.113 & -0.473 & 1.446 & 0.403 \\
0.874 & -0.351 & -0.555 & 0.403 & 2.829 \end{bmatrix}$.

By selecting $c = 0.1$, we can get $\gamma = 0.39$ from (15). By substituting parameters $c$ and $\gamma$ into (16), we can get that $0 < \sigma < 0.0088$. Therefore, set $\sigma = 0.007$ and $\zeta(t) = e^{-0.1 t}$. Figures 2 and 3 show that the heterogeneous multiagent
Figure 6: Event-triggering time instants of each agent when $\tau = 0$.

Figure 7: Evolution of $\|\xi(t)\|$ when $\tau = 0$.

Figure 8: The position trajectory of each agent when $\tau = 0.01$. 
system can achieve bipartite consensus under control protocol (4) and event-triggering functions (5). Figure 4 shows that \( \lim_{t \to \infty} y_i(t) = \lim_{t \to \infty} x_j(t) \), \( \forall i \in I_1, j \in I_2 \), which means the introduction of \( y_i \) does not change bipartite consensus of the heterogeneous multiagent system. Figure 5 exhibits that all states can achieve consensus after being applied with gauge transformation. Figure 6 shows the event-triggering time instants of all agents. Figure 7 shows the evolution of \( \|\xi(t)\| \).

Example 2. This example is for the case with input delay. Since \( \text{Re}(\lambda_{\min}(E(H + \Xi)F)) = 0.161 > 0 \), we can choose \( r = 0.08 \), \( \beta = 1 \) and \( c_2 = 0.1 \) according to Lemma 4 and conditions of Theorem 3. Therefore, set \( \zeta(t) = e^{-0.1t} \). By substituting the above parameters into (33), we can get that \( \sigma_0 = 0.0077 \) and \( \tau_0 = 0.0314 \). By choosing \( \sigma = 0.007 \) and \( \tau = 0.01 \), the following numerical simulations are carried out. Figures 8 and 9 show that the heterogeneous multiagent system can achieve bipartite consensus under control protocol (4) and event-triggering functions (5). By comparing Figures 2 and 8 or Figures 3 and 9, we can see that the case with input delay takes longer time to achieve bipartite consensus than the case without input delay. Figure 10 shows the event-triggering time instants of all agents under event-triggered control protocol (4). Figure 11 shows the evolution of \( \|\xi(t)\| \).

5. Conclusions and Future Work

In this paper, we have investigated event-triggered bipartite consensus for heterogeneous multiagent systems composed of first-order agents and second-order agents under a structurally balanced topology which contains a directed spanning tree. A special variable \( y_i = x_i + 1/\alpha v_i \) is used in designing event-triggered control protocol. By using gauge transformation, we transform a bipartite consensus problem under a structurally balanced signed network into a standard consensus problem under a nonnegative network. Sufficient conditions are obtained to guarantee bipartite consensus for both cases without input delay and with input delay. For the case with input delay, an upper bound of the delay is given to ensure bipartite consensus. It has been shown that no Zeno behavior occurs for heterogeneous multiagent systems under the presented event-triggered control protocol. Investigating bipartite consensus for heterogeneous multiagent systems under adaptive event-triggered control will be our future work.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.
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