Numerical analysis of the effect of isotropic and kinematic hardening of anisotropic targets in impact loading

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Abstract. The paper presents numerical simulation of the dynamic loading of a metal tansotropic target with steel projectiles of various shapes. Equations of the continuum mechanics are used. The target loading velocities are 300m/sec and 600m/sec. The change of the target material’s yield stress in various directions in the process of elastoplastic straining was analysed using the isotropic hardening model. The influence of the effect of an isotropic kinematic hardening on the target’s deflected mode when the target is under the impact loading was studied. The behaviour and features of the material’s limits of plasticity zones changes distribution when it is tensioned and compressed and when the target is subject to impact loading are shown.

1. Introduction
The paper discussed the yield stress zones formation in the initially anisotropic material of the target when the target is under impact loading. The framework of isotropic and kinematic hardening models is employed. Various material hardening models are used to clarify the defining relations describing the behaviour of anisotropic mediums. The hypotheses of isotropic, kinematic and combined (isotropic-kinematic) hardening have proven to be the most useful.

In the isotropic hardening law formulated by Hill and Hodge, one assumes that when subject to plastic flow, the yield surface is expanding, keeping its shape and its position with respect to the origin of the stress space unchanged. Isotropic hardening law does not account for the Bauschinger effect that was observed in the materials studied.

A hardening law is called kinematic if under this law the initial yield surface undergoes transfer, keeping its sizes and shape constant. Experimental studies [1, 2] showed that initially anisotropic aluminium alloys satisfy kinematic hardening based on Ziegler law.

The goal of our work is to study, using Ziegler hardening law, the influence of the effects of isotropic and kinematic hardening on the behaviour and features of the distribution of changes in the target of the plasticity limits changes zones on tension and compression in the target when the target is under impact loading.

2. The basic equations of the model
The axes of Cartesian coordinate system in which the stress and strain tensors are described are chosen to coincide with the main axes of the material anisotropy. The system of equations describing non-stationary adiabatic motions of compressible medium includes the continuity equation, continuous medium motion equations and energy equation [3] both for isotropic and anisotropic mediums:

-continuity equation

$$\frac{\partial \rho}{\partial t} + \text{div} \rho \mathbf{u} = 0, \quad (1)$$

-equation of motion

$$\rho \frac{d\mathbf{u}^k}{dt} = \frac{\partial \sigma^{k\ell}}{\partial x_{\ell}} + F^k, \quad (2)$$

-energy equation
Here \( \rho \) is the medium density, \( \overline{\mathbf{v}} \) is the velocity vector, \( F^k \) are components of the vector of mass force, \( \sigma_{ij} \) are contravariant components of the symmetric stress tensor, \( E \) is the specific integral energy.

\[
\frac{dE}{dt} = \frac{1}{\rho} \sigma^{ij} e_{ij}. \tag{3}
\]

\( e_{ij} \) are the components of the symmetric tensor of strain rates, \( \nu_i \) are the components of the velocity vector, \( i, j = 1, 3 \).

In addition it is necessary to write down the equations that characterize physical properties of examined medium. We shall assume that the total strain can be represented as the sum of elastic and plastic strains, the elastic strain is given by generalized Hooke’s law:

\[
\frac{d\sigma_{ij}}{dt} = C_{ijkl} e_{kl}, \tag{5}
\]

\( C_{ijkl} \) - are the elastic constants.

In order to construct the defining relationship of the plasticity for anisotropic medium, it is necessary to determine the initial yield condition, hardening law, describing the change of the loading surface, and flow rule, that describes the position of the plastic strain incremental vector. As for the loading surface, the gradient law is valid, i.e. vector increments of the plastic strain are normal at the regular points on the yield surface:

\[
d\varepsilon_{ij}^p = d\lambda \left( \frac{\partial F}{\partial \sigma_{ij}} \right),
\]

\( d\lambda \) - positive increment of scalar function which depends on \( d\sigma_{ij}, \sigma_{ij}, \varepsilon_{ij}, F \) - plasticity function, \( i, j = 1, 6 \).

We shall assume that the following conditions are satisfied in the course of elastoplastic strain of anisotropic materials modelling [4]: plastic material flow does not depend on hydrostatic stress (for materials that have a low anisotropic degree), elastic behaviour does not depend on the plastic strain, and the material is incompressible in the plastic domain. In this case when stress and strain tensors are decomposed into spherical and deviatoric parts, there are 5 independent components of deviator stress tensor and 5 independent components of deviator strain tensor. Therefore, the loading process can be considered in the 5-dimensional space of stress and strain deviators, taking the hydrostatic stress into account. Since the loading velocity does not exceed 600m/sec., barotropic medium model is used in the course of hydrostatic stress calculation.

The loading and strain processes of orthotropic material are presented in the 5-dimensional vector spaces of the stress \( S_i \) and strain \( \varepsilon_i \) in agreement with the theory of A.A. Ilyushin. Instead of 6 interdependent functions \( S_{ij} \) A.A. Ilyushin introduces 5 independent functions \( S_i \) so that the transformations are one-to-one and linear [5]. The transformations of the components of stress deviator from 6-dimensional space to 5-dimension space can be written in following way:

\[
S_1 = \frac{3}{2} S_{11}, \quad S_2 = \sqrt{2} \left( \frac{1}{2} S_{11} + S_{22} \right),
\]

\[
S_3 = \sqrt{2} S_{12}, \quad S_4 = \sqrt{2} S_{23}, \quad S_5 = \sqrt{2} S_{31}. \tag{8}
\]

In equation (8) \( S_i \) and \( S_j \) are the components of stress deviators in 5- dimensional and 6- dimensional Euclidean real spaces respectively. Components of strain deviators, written in 5- dimensional and 6- dimensional spaces, are transformed according to the same rule.
Since the considered aluminium alloys demonstrate properties of both isotropic and kinematic hardening [1] in the process of plastic strain we can write the law of the flow surface variation that takes into account isotropic and kinematic hardening in the 5-dimensional space of stress deviators.

Kovalchuk-Kosarchuk plasticity condition [2], taking into account isotropic and kinematic material hardening, when parameter $\eta$ varies, is transformed into Mises-Hill plasticity condition (when $\eta = 1$), or into Tresca plasticity condition, modified for anisotropic materials (when $\eta = 0$):

\[
F(S_i, \alpha_i, R) = [\eta \sqrt{\left(\frac{(S_i - \alpha_i)^2}{r_i^2} + \frac{(S_j - \alpha_j)^2}{r_j^2}\right)} + (1 - \eta)(C_1(S_i - \alpha_i) + C_2(S_j - \alpha_j))]^2 + \\
\frac{(S_i - \alpha_i)^2}{r_i^4} + \frac{(S_j - \alpha_j)^2}{r_j^4} + \frac{(S_k - \alpha_k)^2}{r_k^4} - R^2 = 0,
\]

\[R(\psi) = 1 + \xi \psi,\]

where $\psi = \int (d\mathcal{E}^p d\mathcal{E}^p)_{\frac{1}{2}}^{1}$, $\psi$ - is the accumulated plastic strain, $R$- is the function of isotropic hardening, $\xi$ – is the constant of isotropic material hardening, $i = \mathbf{1, 3}$.  

In this paper $\eta = 1$ and this corresponds to the plasticity condition by Mises-Hill.

Hooke’s law for anisotropic mediums in the 5-dimentional space is:

\[dS_j = D_{kj} (d\mathcal{E}_k - d\mathcal{E}^p_k) = D_{kj} (d\mathcal{E}_k - d\lambda \frac{\partial F}{\partial S_k}),\]

where $j, k = \mathbf{1, 3}; D_{kj}$ are the components of the elastic constants matrix for 5-dimentional space; $\mathcal{E}, \mathcal{E}^p$ - total and plasticity strain respectively.

For materials whose properties do not depend on the hydrostatic stress value, the Ziegler law of kinematic hardening can be written [6]:

\[d\alpha_i = d\zeta (S_i - \alpha_i),\]

where $d\zeta \geq 0$ is the functional of the loading history, $\alpha_i$ are the components of displacement vector of the loading surface centre in the space of stress in the direction of $i$.

\[\alpha = \sqrt{\alpha_i \alpha_i}\] is the magnitude of the displacement of the loading surface centre in the stress space. If $d\mathcal{E}^p_k = 0$, then $d\zeta = 0$.

Using the experimental data obtained for aluminium alloys the papers presents:

\[\alpha_i = A_0 \frac{S_r(k_v)}{S_{r_0}} \psi^{-p},\]

$S_r(k_v)$ is the yield stress of the initial material when loaded along the ray trajectory, which coincides with the direction of instantaneous stress vector; $k_v$ are the components of the unit vector, which coincides with the direction of instantaneous stress vector; $S_{r_0}$ is the yield stress when it is tensioned in the direction of axis $S_i$; $A_0$ and $p$ which are the constants of the material.

$dS = 0$, if $d\psi = 0$.

The functional of the loading depends on the accumulated plastic strain, accumulated plastic strain increment, stress deviator components, the components of displacement vector of the loading surface centre and material constants:

\[
d\zeta = pA_0 \frac{S_r(k_v)}{S_{r_0}} \psi^{-p+1} d\psi \frac{1}{\sqrt{(S_i - \alpha_i)(S_i - \alpha_j)}}.
\]
Thus, the displacement law of the loading surface of stress deviators in the 5-dimensional space can be written:

\[ d\alpha_j = pA_0 \frac{S_j(k_\nu)}{S_{ki}} \psi^{p-1} d\psi(S_j - \alpha_j) \frac{1}{\sqrt{(S_j - \alpha_j)(S_i - \alpha_i)}}. \]

Stresses, identified in the element, rigidly rotated in space, are recalculated using the Jaumann derivative and are written in coordinates:

\[ \frac{D\sigma_{ij}}{Dt} = \frac{d\sigma_{ij}}{dt} - \sigma_{ik} \omega_{kj} - \sigma_{jk} \omega_{ki}, \]

where \( \omega_{ij} = \frac{1}{2} (\nabla_i u_j - \nabla_j u_i) \).

3. Problem statement

The numerical modelling of impact was carried out in a three-dimensional formulation by the finite element method [7]. The projectile material is isotropic steel. The contact surface between the projectile and the target has the frictionless sliding.

The cylindrical target thickness is 30mm. The material of the target is 2024 transotropic aluminium alloy with the following elasticity and plasticity specifications:

- \( \rho = 2700 \text{ kg/m}^3 \), \( E_1 = 92.1 \text{ GPa} \), \( E_2 = E_3 = 86.7 \text{ GPa} \), \( \nu_{12} = 0.34 \), \( \nu_{31} = 0.32 \), \( \nu_{23} = 0.33 \),
- \( G_{12} = G_{13} = 31 \text{ GPa} \), \( G_{23} = 33 \text{ GPa} \), \( \sigma_{1y} = 350 \text{ MPa} \), \( \sigma_{2y} = \sigma_{3y} = 290 \text{ MPa} \), \( \tau_{13} = \tau_{13y} = 300 \text{ MPa} \), \( \tau_{23} = 240 \text{ MPa} \), \( \eta = 1 \).

Here \( E_i \) - Young modulus, \( G_{ij} \) - shear modulus, \( \nu_{ij} \) - Poisson ratios.

The target loading is modelled by spherical and cylindrical shape projectiles. In the course of study of the zones of isotropic hardening into the target material the diameters of cylindrical projectile are 7mm and 4.94mm, the length of cylinders are 7mm and 14mm respectively and the spherical projectile diameter is 8mm. The cylindrical shape projectiles are used to study the kinematic hardening zones formation. In the first case, the target diameter and length are 15mm, in the second case, the diameter is 30mm if the masses are identical. The elastoplastic flow of the projectile material is described by Prandtl-Reuss model. The steel yield stress is 1GPa. As a numerical method the method of final elements modified by G.R. Johnson for impact problems is used [8].

4. Formation of isotropic hardening zones

The formation of isotropic hardening zones inside the target material that occurs as a result of the impact loading, depends on physical-mechanical properties of the target material, as well as kinematic and geometric parameters of the target loading. When projectiles of various configurations, but of identical mass and velocity are used, the hardening zones inside the target material differ in the degree of the hardening and in 3-dimensional configuration of these zones. The paper does not consider the destruction of the target material in order to exclude the influence of the used anisotropic destruction criterion type on the results. As a consequence, elastoplastic material deformation is also considered in the domain where the material can be destroyed.

For the cases of the target loading with projectiles of various configurations at a 300m/sec., projectile decay curves are demonstrated in fig. 2a. The differences in projectile shapes result in significant differences in the projectile rebound times and in the values of the accumulated plastic strain in the target and projectile. At the beginning of the process, out of three cases presented, the minimal contact area between the projectile and target occurs in the case of spherical projectile and it causes a noticeable deformation of the projectile and the lowest projectile deceleration at the first 5 µs of the process. The maximal hardening of the target material occurs when it is loaded with spherical projectile. Fig. 3 presents the hardening zones of the material, where the maximal accumulated plastic strain (in the target zone, adjacent to the contact surface) is 0.2. The target material hardening in the
case of the loading with spherical projectiles is very deep. It is explained by the small contact surface and by the extension of the zone of the hardening deep into the target at the beginning of the process.

Compact projectile has the maximal contact surface at the beginning of the process. Therefore, the projectile deceleration is faster – over 8 μs. At 20 μs of the process there are the rebounds of the projectiles of all forms from the target. Fig. 3a, 3b, 3c demonstrate 3-dimensional configurations of spherical, compact and extended projectiles and the distribution of the zones of hardening inside the target sections. For the cases of target loading with cylindrical projectile, the values of the maximal accumulated plastic strain under the projectile inside the target coincide – 0.1, while the zone of hardened target material when loaded with compact projectile increases 3-4 times in depth.

Fig. 2 Projectiles velocity decay curves in the interaction with the target.

![Fig. 2 Projectiles velocity decay curves in the interaction with the target.](image)

Fig. 3 Zones of the hardened material inside the target at V0=300m/sec.

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Braking deceleration curves for the cases of loading of the target by the identical configuration projectiles at V=300m/s are shown in fig. 2b. In this case the large strain of projectiles of all shapes is observed and the first occurred rebound from the target was of the spherical projectile. The contact area of both spherical and compact projectiles and target eventually become approximate, and if the projectiles masses are identical then projectile velocity decay curves also become close.

At time 2μs the maximal accumulated strain inside the target is observed when it is loaded by spherical projectile and is equal to 0.22 (fig. 4a). The volume of the hardened material zone of the target for the cases of loading by three projectile shapes at time 2μs is maximal for compact projectile (fig. 5a). At 5μs into the process, the maximal accumulated plastic strain inside the target is observed when it is loaded by spherical projectile and is equal to 0.27. Eventually the hardened material zone inside the target extends greater when it is loaded by spherical projectile (fig. 4b). It is explained by greater deformation of the spherical projectile at the beginning of the process and therefore by extending of the contact area of the projectile and target. Within the framework of the model of isotropic hardening of a material, the change of the yield stress occurs when tensioned and compressed. Because of the decrease of the plastic strain values inside the target and as the distance from the contact area increases the degree of the target material hardening decreases, therefore in this part of the target the yield stress of the material remains almost constant. For extended shape projectiles the rebound from a target occurs at time 18μs, thus the volume of the hardened material of the target is minimal. In this case in contrast to spherical projectile the contact area of extended
projectile and target increases slower (fig. 6a). Thus the maximal accumulated plastic strain at time 5µs of the process is equal 0.2.

Fig. 4. Zones of the hardened material inside the target at V₀=600m/sec.

Fig. 5. Zones of the hardened material inside the target at V₀=600m/sec.

Fig. 6. Zones of the hardened material inside the target at V₀=600m/sec.

5. The formation of zones of kinematic hardening

The formation of zones of kinematic hardening inside the material of the target is investigated assuming the absence of the isotropic hardenings. Fig. 7 demonstrates the solid line of the projectiles velocity decay curve (d=30mm and d=15mm) during the interaction with the target for the case when kinematic hardening of the material of the target is not taken into account, i.e. α=0. The dashed line is for projectiles velocity decay curve when kinematic hardening of the material of the target is taken into account, i.e. α>0.

Figures 7a and 7b shows that the accounting of kinematic hardening does not influence integral characteristics of the process of the impact loading of the target.

The distribution of the fields of total strains at 10µs and 20µs of the process of the loading by disk-shaped projectile is shown in fig. 8a and 8b. In contrast to the fields of stresses which distribution occurs in the wave-like fashion, the change of the fields of strains occurs with increasing of their
values during the entire process in the directions which form a zone under projectile and in a direction of conical surface.

Fig. 7 Change of projectile velocity in interaction with a target: a) disk-shaped projectile (d=30mm), b) compact projectile (d=15mm).

Fig. 8. Distribution of $\varepsilon_x$ isolines in cross section ZX.

Fig. 9. Distribution of $\alpha_1$ isolines in cross section ZX

Fig. 9 presents the distributions of additional stresses $\alpha_1$ at 10µs and 20µs after the impact. The distributions of additional stresses, as well as total strains are not related to the wave-like distribution of the stresses inside the target. The distribution $\alpha_2$ in the directions which are perpendicular load axis at 10µs and 20µs of the process are shown in fig. 10. Distribution $\alpha_2$ has the character which is identical to $\alpha_1$, but has an opposite sign. It is explained by the method of the impact loading of the target.

During the entire process of the impact loading under the projectile the zone of compression strain in the direction of the axis of the impact loading and the zone of tension strain - in the perpendicular direction are formed.

Figures 9a, 9b and 10a and 10b demonstrate the hardening when compressed in the direction of one axis and the hardening when it is tensioned in the perpendicular direction in the identical points of the target. Therefore the accounting of kinematic hardening irrespective of the level of additional stresses does not influence integral characteristics of the process of the loading, i.e. the projectile deceleration (fig. 7).

The zone of the hardening of the material when it is tensioned is formed and extended in the direction of the back surface of the target from the moment of the plastic strain onset. Eventually the
zone of the hardening of the material when it is compressed is formed inside the target at the angle of 45°. Thus, there are additional stresses of the opposite signs inside the target in the orthogonally related directions (fig. 9 and 10). It causes the change of the values of the yield stress of the material. Such character of the change of the target material’s yield stress is explained by the method of loading.

Fig. 10. Distribution of additional stresses inside the target in the direction of the axis OY at 10µs and 20µs respectively.

In this case the material of the target has the plastic compression strain in the impact direction, and it has the plastic tension strain in the perpendicular directions.

6. Conclusion
1. In the considered loading velocity range (300m/sec – 600m/sec) the increase of loading velocity leads to the extending of the contact area of projectile and target and therefore to the increase of the volume of the hardened material inside the target.
2. The numerical simulation of the impact loading of the target with regard to kinematic hardening of the transtropic material by the example of 2024 aluminum alloy shows, that:
   a) effects of kinematic hardening of the transtropic material of the target does not influence integral characteristics of the process of the loading because at points of the target in the orthogonally related directions there are additional stresses of the opposite signs;
   b) the values of the tensor components of additional stresses ($\alpha_{ij}$) are not related to the wave-like pattern of the stressed state of the target, but they are related only to the distribution of the values of the components of the total strain tensors in the corresponding direction.

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