Optical signatures of a fully dark exciton condensate

Monique Combescot\textsuperscript{1}, Roland Combescot\textsuperscript{2,3}, Mathieu Alloing\textsuperscript{4} and François Dubin\textsuperscript{1,4}

\textsuperscript{1} Institut des Nanosciences de Paris, Université Pierre et Marie Curie, CNRS - 2 pl. Jussieu, 75005 Paris, France
\textsuperscript{2} Laboratoire de Physique Statistique, Ecole Normale Supérieure, UPMC Paris 06, Université Paris Diderot, CNRS 24 rue Lhomond, 75005 Paris, France
\textsuperscript{3} Institut Universitaire de France - 103 boulevard Saint-Michel, 75005 Paris, France
\textsuperscript{4} ICFO-The Institute of Photonic Sciences - 3 Av. Carl Friedrich Gauss, 08860 Castelldefels (Barcelona), Spain

received 25 November 2013; accepted in final form 17 February 2014
published online 6 March 2014

PACS 73.63.Hs – Quantum wells
PACS 78.47.jd – Time resolved luminescence
PACS 03.75.Hh – Static properties of condensates; thermodynamical, statistical, and structural properties

Abstract – We propose optical means to reveal the presence of a dark exciton condensate that does not yield any photoluminescence at all. We show that i) the dark exciton density can be obtained from the blueshift of the excitonic absorption line induced by dark excitons; ii) the polarization of the dark condensate can be obtained from the blueshift dependence on the probe photon polarization as well as from the Faraday effect. All these effects result from carrier exchanges between dark and bright states.

Copyright © EPLA, 2014

Introduction. – The experimental observation of exciton Bose-Einstein condensation faced major difficulty for decades [1]. Experiments mostly looked for signatures through intense photoluminescence emitted by bright excitons with total spin (±1), thus overlooking that optically inactive states, i.e., dark states with total spin (±2), have a lower energy [2]. This fact was known already [3–6] but its consequence on exciton condensation was put forward very recently only. We showed [7] that, in the dilute regime, the exciton condensate must be purely dark. So, it cannot be detected through photoluminescence. Yet, above a density threshold, the condensate gets a bright coherent component [8] through carrier exchanges which couple dark and bright excitons: the condensate becomes “gray” and can be detected through the weak photoluminescence emitted by its bright part.

In very recent experiments [9], we have revealed such a “gray” condensate in the vicinity of a fragmented ring-shaped exciton gas [9–12]. We have shown that i) a far denser dark exciton gas coexists with bright excitons, ii) the bright component exhibits a coherence length 10 times larger than the thermal de Broglie wavelength and a linear polarization expected for degenerate (±1) states, iii) these features disappear when the bath temperature gets larger than a few kelvins. Revealing a fully dark exciton condensate at density smaller than the threshold for the appearance of a bright component remains a real challenge. This would be highly valuable because we would then have a direct evidence for exciton condensation in a dark state. Such a condensation can already be guessed [2] in a trap confining both, bright and dark excitons, as electrostatic traps [13–16], traps induced by stress [17] or even optical traps [18]. Indeed, under a decrease of the bath temperature, exciton condensation must lead to a darkening of the trap center where the potential energy is minimum. In this context, we can mention recent experiments pointing out a decrease of the luminescence in a wide electrostatic trap for temperature below a few degrees kelvin [19].

In this letter, we propose an all-optical approach to reach both, the density and the spin polarization of fully dark exciton condensates [8].

i) The exciton density can be quantitatively deduced from the absorption spectrum of a probe beam: carrier exchanges between an exciton gas, either bright or dark, and the bright exciton created by a probe photon yield a blueshift of the absorption line which increases with exciton density. If this blueshift occurs in a fully dark region of the sample, it evidences the presence of a dense gas of dark excitons, these excitons being necessarily condensed. Indeed, as the sample temperature differs from zero, some thermally excited dark excitons also exist which follow a
Bose-Einstein distribution. Since for usual experimental temperatures, the thermal energy is large or comparable to the energy splitting between dark and bright exciton states, a number of excited bright excitons similar to the number of dark ones should also exist. So, if the number of excited dark excitons were not very small, bright excitons would be experimentally seen through their photoluminescence and the exciton gas would no longer be dark—which is in contradiction with having a fully dark sample.

ii) The spin polarization of the dark exciton condensate can be reached through polarization effects with probe photons tuned on excited exciton level. It can also be obtained from Faraday effect: indeed, for linearly polarized dark excitons, the polarization plane of a linearly polarized unabsorbed beam stays unaffected while for any other dark exciton polarization, the polarization plane of the light would rotate. These features are in contrast with polarizations involving bright excitons: in the exciton optical Stark effect, the blueshift of a probe beam strongly depends on pump and probe polarizations even for probe photons tuned on the ground state, while a bright exciton gas with linear polarization induces a “Faraday oscillation”, the polarization changing from linear to elliptical and back to linear again as photons propagate through the sample [20].

Absorption line shift induced by dark excitons.- Hulin and coworkers [21] have discovered that the absorption line of a probe beam blueshifts when the semiconductor is irradiated by an unabsorbed pump beam. This shift results from carrier exchanges between the real (bright) exciton created by a probe photon and virtual (bright) excitons coupled to the unabsorbed pump beam. We here consider a similar shift, virtual bright excitons being replaced by the (real) dark excitons present in the sample as a result of exciton condensation. To get this shift, we follow the procedure detailed in ref. [22]. Before proceeding, let us note that, after completion of this work, Natilov et al. [23] have also pointed out the interest of pump-probe techniques for studying optically indirect excitons in double quantum wells heterostructures.

Without dark excitons, the initial state before photon absorption is the zero electron-hole pair state \( |0\rangle \) with energy 0. The final state after the absorption of a probe photon with momentum \( Q_p \) tuned on the exciton level \( \nu_p \), is the excitonic state \( B_p^\dagger |0\rangle \) with

\[
B_p^\dagger = g_{+1}B_{\nu_p}^\dagger Q_{\nu_p}^+ + g_{-1}B_{\nu_p}^\dagger Q_{\nu_p}^-,
\]

its energy being \( E_p = \nu_p + Q_p^2/2(m_e + m_h) \), where \( m_e \) and \( m_h \) are the electron and hole masses, and \( \nu_p \) the exciton relative motion energy, while \( g_{+1} \) are the \( \sigma_+ \) amplitudes of the probe photon. In narrow quantum wells, the creation operator \( B_{\nu_p}^\dagger \) of a bright exciton with total spin \( S_i = \pm 1 \) in state \( i \equiv (\nu, Q) \) reads, in terms of the free electron and free hole creation operators, \( a_{p;\pm 1/2}^\dagger \) and \( b_{p;\pm 3/2}^\dagger \) as

\[
B_{\nu_p}^\dagger = \sum_p a_{p;\pm 1/2}^\dagger b_{\nu_p;\pm 3/2}^\dagger (p|\nu_p),
\]

where \( \gamma_e = 1 - \gamma_h = m_e/(m_e + m_h) \). For probe photons having an elliptical polarization \( \phi \) with main axes \( (X, Y) \) tilted by \( \theta \) from the \( (x, y) \) well axes, the prefactors in eq. (1) are \( g_{+1} = e^{-i\theta} \cos(\phi - \pi/4) \) and \( g_{-1} = e^{i\theta} \sin(\phi - \pi/4) \). Photons linearly polarized along \( x \) correspond to \( (\theta = 0, \phi = 0) \) while \( \sigma_+ \) photons correspond to \( (\phi = \pi/4) \).

Let \( \langle \Psi_N \rangle \) be the initial state in the presence of a dark exciton condensate, and \( E_N \) its energy. For a dilute exciton gas, \( \langle \Psi_N \rangle \) at lowest order in the interactions reads as

\[
\langle \Psi_N \rangle \simeq (D^\dagger)^N |0\rangle \equiv (g_{+1}B_{00,0;+2}^\dagger + g_{-1}B_{00,0;-2}^\dagger)^N |0\rangle
\]

which is an adequate approximation for the validity of the blueshift we derive below. Creation operators for dark excitons \( S_i = \pm 2 \) are related to electron-hole creation operators through

\[
B_{\nu_p}^\dagger = \sum_p a_{p;\pm 1/2}^\dagger b_{\nu_p;\pm 3/2}^\dagger (p|\nu_p),
\]

the dark-bright exciton splitting having negligible effects on the exciton relative motion wave function \( \langle \Psi_N \rangle \)

The final state \( |\Phi_{N+1}\rangle \) after photon absorption is \( B_p^\dagger |\Psi_N\rangle \). Let \( |X_{N,p}\rangle \) be its normalized form. To get \( |\Phi_{N+1}\rangle \), we introduce the projector \( P_L \) over the subspace orthogonal to \( |X_{N,p}\rangle \) defined through

\[
I = |X_{N,p}\rangle \langle X_{N,p}| + P_L.
\]

Inserting this identity in front of \( |\Phi_{N+1}\rangle \) in \( (H_{sc} - E_{N+1})|\Phi_{N+1}\rangle = 0 \), where \( H_{sc} \) is the semiconductor Hamiltonian and multiplying the resulting equation by \( P_L \), yields

\[
P_L |\Phi_{N+1}\rangle = \left[P_L (E_{N+1} - H_{sc})P_L \right]^{-1} P_L H_{sc} |X_{N,p}\rangle \langle X_{N,p}|\Phi_{N+1}\rangle.
\]

When used into the Schrödinger equation projected onto \( |X_{N,p}\rangle \), we get

\[
E_{N+1} = (X_{N,p}|H_{sc}|X_{N,p}) + (X_{N,p}|H_{sc}P_L \left[P_L (E_{N+1} - H_{sc})P_L \right]^{-1} P_L H_{sc}|X_{N,p}\rangle.
\]

The shift of the probe photon absorption line induced by the dark exciton condensate is given by

\[
\Delta = (E_{N+1} - E_N) - E_p.
\]

A simple way to get it from eq. (6) is to introduce the “creation potential” \( V_p^\dagger \) of the \( B_p^\dagger \) exciton defined as

\[
[H_{sc}, B_p^\dagger] = E_p B_p^\dagger + V_p^\dagger.
\]
Since \((H_{\text{ex}} - \mathcal{E}_N)|\Psi_N\rangle = 0\), we then find
\[
H_{\text{ex}}B_p^\dagger|\Psi_N\rangle = (E_p + \mathcal{E}_N)B_p^\dagger|\Psi_N\rangle + V_p^\dagger|\Psi_N\rangle.
\] (9)
So, for \(H_{\text{ex}}\) acting either on the right or on the left in the first term of eq. (6), we end with
\[
\Delta = \frac{1}{2} \frac{\langle \Psi_N|B_pV_p^\dagger + V_pB_p^\dagger|\Psi_N\rangle}{\langle \Psi_N|B_pB_p^\dagger|\Psi_N\rangle}
\]
\[
\frac{\langle \Psi_N|V_p^\dagger \left( \Delta + E_p + \mathcal{E}_N - H_{\text{ex}} \right) \Gamma_p^\dagger \right|^{-1} |\Psi_N\rangle}{\langle \Psi_N|B_pB_p^\dagger|\Psi_N\rangle}. \tag{10}
\]

The first term comes from one Coulomb interaction between the probe exciton and the condensate while the second term corresponds to correlations.

We can calculate these terms by using the many-body formalism for composite bosons we have recently developed [24]. Its extension to composite bosons with spin degrees of freedom can be found in ref. [25]. Yet, a physically more enlightening way to perform many-body calculations is to use Shiva diagrams.

The denominator in eq. (10) is represented by the diagram of fig. 1(a) without the wavy Coulomb line. Its dominant term, shown in fig. 1(b), is given by
\[
\langle \Psi_N|\Psi_N\rangle \simeq \langle 0|D_N(D_N^\dagger)^N|0\rangle = N!F_N,
\] (11)
where \(F_N\) comes from carrier exchanges. For large \(N\), it is exponentially small, but \(F_{N-1}/F_N = 1 + \mathcal{O}(\eta)\) where \(\eta = N(a_N/L)^d\) is the dimensionless gas parameter ruling many-body effects between \(N\) excitons in \(d\) dimension.

The next-order term, shown in fig. 1(c), contains a factor \(N^2\) from the \(N\) ways on each side to choose the dark exciton involved in carrier exchange. Contributions of each exchange process shown in figs. 2(b), (c) add up, leading to a \(\sum_{\epsilon} [\lambda(0,0)_{p}\lambda(0,0)_{p}]|a_{\epsilon}|^2 + \lambda(0,0)_{p}|a_{\epsilon}|^2|a_{\epsilon}|^2\) factor. So, the next-order term of \(\langle \Psi_N|\Psi_N\rangle\) reads
\[
- N^2 \left[ \lambda(0,0)_{p} \left( |a_1a_2|^2 + |a_{-1}a_{-2}|^2 \right) + \lambda(0,0)_{p} \left( |a_1a_2|^2 + |a_{-1}a_{-2}|^2 \right) \left( N - 1 \right) F_{N-1} \right]. \tag{12}
\]

As \(\lambda(0,0)_{p} = \lambda_2(a_N/L)^d\) with \(\lambda_2 = 33\pi/2\) and \(\lambda_2 = 4\pi/5\), this term indeed is \(\eta\) smaller than the main term given in eq. (11). For \(\nu_p = \nu_0\), the polarization part reduces to \(|a_1|^2 + |a_{-1}|^2|a_{-2}|^2 + |a_{-2}|^2|a_1|^2|a_{-1}|^2|a_{-2}|^2 = 1\) regardless of the bright and dark exciton polarizations. If \(\eta'\) instead of \(\eta\), dark excitons are involved in the exchange processes, the corresponding term, proportional to \(\eta'N^2\), would have a \(|a_1|^2|a_{-2}|^2 + |a_{-2}|^2|a_1|^2\) factor which depends on the dark exciton polarization but still not on the probe photon polarization. It is yet possible to get a dependence on probe polarization and possibly test the linear polarization of the dark condensate by using probe photons not tuned on the exciton ground state, \(\nu_p \neq \nu_0\).

We now turn to \(\langle \Psi_N|B_pV_p^\dagger|\Psi_N\rangle\) linear in Coulomb interaction, shown in fig. 1(a). Its dominant terms in \(\eta\) are shown in fig. 3. Again, a factor \(N^2\) arises from the \(N\) ways on each side to choose the dark exciton interacting with the bright exciton created by the probe beam. The direct process, shown in Fig. 3(a), cancels because the exciton states remain unchanged [24]. We are left with the exchange Coulomb processes between one bright and one dark exciton shown in fig. 3(b). For \(\nu_p = \nu_0\), they lead to
\[
\langle \Psi_N|B_pV_p^\dagger|\Psi_N\rangle \simeq - N^2 \xi_{\text{exch}}(0,0) \left[ |a_1a_2|^2 + |a_{-1}a_{-2}|^2 \right]
\]
\[
+ |a_1a_{-2}|^2 + |a_{-1}a_{-2}|^2 \left( N - 1 \right) F_{N-1}. \tag{13}
\]

Again, the bracket reduces to 1 regardless of the bright and dark polarizations. As for \(\langle \Psi_N|B_pB_p^\dagger|\Psi_N\rangle\), exchange Coulomb processes involving \(N'\) dark excitons instead of
This correlation term changes the numerical prefactor to the shift when one among fig. 1(c) as the probe exciton interacts with a single dark shift calculated in eq. (14) does not depend on dark excitons. Hence, the fact that the blueshift of the excitonic line reads at lowest order in exchange, while no exchange exists for the \( \sigma_- \) part. So, the \( n_{\pm} \) indices are different and the polarization plane of the linearly polarized light rotates. The situation is far more complex when the bright excitons present in the sample have a linear polarization, let say along \( X \). We may think that the \( \sigma_+ \) and \( \sigma_- \) parts of the linearly polarized light have equal exchanges with the exciton gas; so, \( n_{+} = n_- \) and nothing should happen. Yet, unabsorbed photons with polarization along \( X \) or \( Y \) should react differently to excitons with \( X \) polarization. Since the linear polarization of the exciton gas does not differentiate \( \sigma_+ \) from \( \sigma_- \) rotation, the polarization plane of the unabsorbed beam cannot rotate. It actually oscillates [20], with a polarization changing from linear to elliptical (or circular) and back to linear again.

We here reconsider this effect when the excitons present in the sample are dark. The major change comes from the fact that the two parts of a linearly polarized light see \( S = 2 \) excitons by carrier exchange, hole for \( \sigma_+ \) and electron for \( \sigma_- \); so, dark excitons should not affect the light, regardless of their polarization. To tackle this effect, we follow ref. [20] which leads to both, Faraday rotation and Faraday oscillation in the case of bright excitons.

Let \( \alpha_{\pm} \) create an unabsorbed probe photon with energy \( \omega_p \) and circular polarization \( \sigma_\pm \). In the absence of photonic-semiconductor coupling, the two states

\[
|\Psi_N^{(1)}\rangle = \alpha_{+} |0\rangle \otimes |\Psi_N\rangle
\]

are degenerate in energy, \( (H_{sc} + H_{ph} - E_N - \omega_p)|\Psi_N^{(1)}\rangle = 0 \), with \( H_{ph} \) being the photon Hamiltonian.

In the presence of coupling, the system Hamiltonian reads \( H' = H_{sc} + H_{ph} + W \) with \( W = U + U^\dagger \) and \( U^\dagger = \sum \Omega_j B_{j}^\dagger S \). To get the \( H' \) eigenstates, we proceed as we did for eq. (6) except that the \( |\Psi_N^{(1)}\rangle \) subspace is now degenerate. We introduce the projector \( P_L \) over the subspace orthogonal to \( |\Psi_N^{(1)}\rangle \) defined as

\[
I = \frac{|\Psi_N^{(1)}\rangle \langle \Psi_N^{(1)}|}{\langle \Psi_N^{(1)}|\Psi_N^{(1)}\rangle} + \frac{|\Psi_N^{-1}\rangle \langle \Psi_N^{-1}|}{\langle \Psi_N^{-1}|\Psi_N^{-1}\rangle} + P_L.
\]

When inserted into the Schrödinger equation \( (H' - E_{N+1})|\Psi_{N+1}\rangle = 0 \) multiplied by \( P_L \), we obtain an expression for \( P_L|\Psi_{N+1}\rangle \) similar to eq.(5). Using it into the Schrödinger equation projected onto \( |\Psi_N^{(1)}\rangle \), we get 2 equations for \( |\Psi_{N+1}'\rangle \) which have a non-zero solution for

\[
E_N + \omega_p + \Delta_{+1,0} - E_{N+1}' = 0, \quad E_N + \omega_p + \Delta_{-1,0} - E_{N+1}' = 0.
\]

The coupling between dark excitons and \( \sigma_\pm \) unabsorbed photons reads at lowest order in \( U \) as

\[
\Delta_{S',S} = \frac{\langle \Psi_N^{(S')}|U|E_N + \omega_p - H_{sc} - H_{ph}|\Psi_N\rangle}{\langle \Psi_N|\Psi_N\rangle}.
\]

---

Monique Combescot et al.

Fig. 3: Dominant terms of \( \langle \Psi_N|B_p V_p^\dagger|\Psi_N\rangle \) in dark exciton density.

1 depend on the dark exciton polarization but not on the probe polarization for photons tuned on the ground state, \( \nu_p = \nu_0 \). Again, we need \( \nu_p \neq \nu_0 \) to get a polarization dependence and through it, possibly test the dark exciton polarization. As \( \xi_{\text{exc}} = -\xi_d(aX/L)^d R_X \), where \( R_X = c^2/2aX \) is the 3D exciton Rydberg while \( \xi = 26 \pi/3 \) and \( \xi = 8 \pi - 315 \pi^2/512 \approx 6.057 \), first-order exchange Coulomb processes with the dark condensate thus yield a blueshift of the excitonic absorption line

\[
\Delta \approx \eta \xi_d R_X + O(\eta^2)
\]

which increases with dark exciton density but is independent of the probe polarization for photons tuned on the exciton ground state.

The second term of eq. (10) also brings an \( \eta \) contribution to the shift when one among \( N \) dark excitons is involved. This correlation term changes the numerical prefactor \( \xi_d \) in the probe shift but not the structure of the result.

We wish to note that a small number of thermally excited dark excitons would not affect our result since the shift calculated in eq. (14) does not depend on dark excitons being condensed or not. This is clearly seen from fig. 1(c) as the probe exciton interacts with a single dark exciton; correlations between dark excitons specific of the condensate do not play any role. These correlations would only enter at higher order in the dark exciton density. Hence, the fact that the blueshift of the excitonic line reveals the existence of a condensate does not come from the expression eq. (14) of this shift, but from our above analysis where we show that a large dark exciton density together with no bright excitons as in a fully dark situation, can only be explained by the presence of a dark condensate. Specifically, the absence of observed luminescence puts an upper limit on the bright exciton density which, as explained, puts itself an upper limit on the density of thermally excited dark excitons. If this last number is incompatible with the density of dark excitons resulting from the observed shift, there necessarily is a dark exciton condensate.

**Dark excitons interacting with linearly polarized unabsorbed photons.** Another effect driven by carrier exchanges is the interaction between excitons and an unabsorbed photon beam having a linear polarization. When the excitons are bright, the problem is rather simple when the unabsorbed photon beam is circularly polarized [25], let say \( \sigma_+ \). Through the virtual excitons to which the \( \sigma_+ \) part of a linearly polarized light is coupled, this \( \sigma_+ \) part sees the exciton gas by electron exchange and by hole exchange, while no exchange exists for the \( \sigma_- \) part. So, the \( n_{\pm} \) indices are different and the polarization plane of the linearly polarized light rotates. The situation is far more complex when the bright excitons present in the sample have a linear polarization, let say along \( X \). We may think that the \( \sigma_+ \) and \( \sigma_- \) parts of the linearly polarized light have equal exchanges with the exciton gas; so, \( n_{+} = n_- \) and nothing should happen. Yet, unabsorbed photons with polarization along \( X \) or \( Y \) should react differently to excitons with \( X \) polarization. Since the linear polarization of the exciton gas does not differentiate \( \sigma_+ \) from \( \sigma_- \) rotation, the polarization plane of the unabsorbed beam cannot rotate. It actually oscillates [20], with a polarization changing from linear to elliptical (or circular) and back to linear again.

We here reconsider this effect when the excitons present in the sample are dark. The major change comes from the fact that the two parts of a linearly polarized light see \( S = 2 \) excitons by carrier exchange, hole for \( \sigma_+ \) and electron for \( \sigma_- \); so, dark excitons should not affect the light, regardless of their polarization. To tackle this effect, we follow ref. [20] which leads to both, Faraday rotation and Faraday oscillation in the case of bright excitons.

Let \( \alpha_{\pm} \) create an unabsorbed probe photon with energy \( \omega_p \) and circular polarization \( \sigma_\pm \). In the absence of photonic-semiconductor coupling, the two states

\[
|\Psi_N^{(1)}\rangle = \alpha_{+} |0\rangle \otimes |\Psi_N\rangle
\]

are degenerate in energy, \( (H_{sc} + H_{ph} - E_N - \omega_p)|\Psi_N^{(1)}\rangle = 0 \), with \( H_{ph} \) being the photon Hamiltonian.

In the presence of coupling, the system Hamiltonian reads \( H' = H_{sc} + H_{ph} + W \) with \( W = U + U^\dagger \) and \( U^\dagger = \sum \Omega_j B_{j}^\dagger S \). To get the \( H' \) eigenstates, we proceed as we did for eq. (6) except that the \( |\Psi_N^{(1)}\rangle \) subspace is now degenerate. We introduce the projector \( P_L \) over the subspace orthogonal to \( |\Psi_N^{(1)}\rangle \) defined as

\[
I = \frac{|\Psi_N^{(1)}\rangle \langle \Psi_N^{(1)}|}{\langle \Psi_N^{(1)}|\Psi_N^{(1)}\rangle} + \frac{|\Psi_N^{-1}\rangle \langle \Psi_N^{-1}|}{\langle \Psi_N^{-1}|\Psi_N^{-1}\rangle} + P_L.
\]

When inserted into the Schrödinger equation \( (H' - E_{N+1})|\Psi_{N+1}\rangle = 0 \) multiplied by \( P_L \), we obtain an expression for \( P_L|\Psi_{N+1}\rangle \) similar to eq.(5). Using it into the Schrödinger equation projected onto \( |\Psi_N^{(1)}\rangle \), we get 2 equations for \( |\Psi_{N+1}'\rangle \) which have a non-zero solution for

\[
E_N + \omega_p + \Delta_{+1,0} - E_{N+1}' = 0, \quad E_N + \omega_p + \Delta_{-1,0} - E_{N+1}' = 0.
\]

The coupling between dark excitons and \( \sigma_\pm \) unabsorbed photons reads at lowest order in \( U \) as

\[
\Delta_{S',S} = \frac{\langle \Psi_N^{(S')}|U|E_N + \omega_p - H_{sc} - H_{ph}|\Psi_N\rangle}{\langle \Psi_N|\Psi_N\rangle}.
\]
The state $U^\dagger\Psi_N^{(S)}$ contains $N$ dark excitons plus one virtual exciton $j$ with spin $S$. Contribution to $\Delta_{S,S'}$ linear in dark exciton density comes from interaction of this $(j,S)$ exciton with one among $N$ dark excitons.

i) These interactions can be pure carrier exchanges as in figs. 4(a), (b). They bring a contribution in $\eta$ multiplied by a polarization factor given by

$$
\delta_{S,S'}\left\{ \delta_{S,1} \left[ |a_2|^2 \lambda^0_{0',j} + |a_{-2}|^2 \lambda_{0',0} \right] + \delta_{S,-1} \left[ |a_{-2}|^2 \lambda^0_{0',j} + |a_2|^2 \lambda_{0',0} \right] \right\}.
$$

This term is diagonal and identical for $S = \pm 1$ if $|a_2|^2 = |a_{-2}|^2$, i.e., when dark excitons are linearly polarized.

ii) The $(j,S)$ exciton can also have direct Coulomb processes with dark excitons, as in figs. 4(c), (d). Diagram 4(c) leads to a contribution in $\delta_{S',S} \left[ |a_2|^2 + |a_{-2}|^2 \right]$ again diagonal and independent of the dark polarization while the diagram of fig. 4(d), responsible for $\Delta_{1,-1} \neq 0$ when bright excitons are present in the sample, reduces to zero when these excitons are dark.

So, there is no interaction between dark and bright excitons possibly leading to a Faraday oscillation through $\Delta_{1,-1} \neq 0$. In addition, there also is no energy splitting between circularly polarized photons, $\Delta_{1,1} \neq -\Delta_{-1,-1}$, possibly leading to a Faraday rotation when the dark condensate is linearly polarized. As a result, the absence of Faraday effect is an optical signature that excitons condense into a linearly polarized dark state.

**Conclusion.** – We have shown that the presence of a fully dark exciton condensate can be optically revealed in a completely dark region of the sample through a blueshift of the excitonic absorption line. This shift increases with the dark exciton density but depends on the probe photon polarization for photons not tuned on the ground-state exciton only. We also show that, unlike for bright excitons, an unabsorbed photon beam with linear polarization is unaffected by the presence of a condensate of dark excitons having a linear polarization.

---

**REFERENCES**

[1] Moskalenko S. A. and Snoke D. W., *Bose-Einstein Condensation of Excitons and Biexcitons* (Cambridge University Press) 2000.

[2] Combescot M. and Leuenberger M., *Solid State Commun.*, 149 (2009) 567.

[3] Eckardt W., Losch K. and Bimberg D., *Phys. Rev. B*, 20 (1979) 3303.

[4] Blackwood E. et al., *Phys. Rev. B*, 50 (1994) 12246.

[5] Amand T. et al., *Phys. Rev. Lett.*, 78 (1997) 1355.

[6] Vina L., *J. Phys.: Condens. Matter*, 11 (1999) 5929.

[7] Combescot M., Betbeder-Matibet O. and Combescot R., *Phys. Rev. Lett.*, 99 (2007) 176403.

[8] Combescot R. and Combescot M., *Phys. Rev. Lett.*, 109 (2012) 026401.

[9] Alloing M., Beian M., Fuster D., Gonzalez Y., Gonzalez L., Combescot R., Combescot M. and Dubin F., arXiv:1304.4101 (2013).

[10] Alloing M., Beian M., Fuster D., Gonzalez Y., Gonzalez L., Combescot R., Combescot M. and Dubin F., arXiv:1210.3176 (2012).

[11] Butov L. V., Gossard A. C. and Chemla D. S., *Nature*, 418 (2002) 751.

[12] High A. A. et al., *Nature*, 483 (2012) 584.

[13] Rapaport R. et al., *Phys. Rev. B*, 70 (2005) 075428.

[14] High A. A. et al., *Phys. Rev. Lett.*, 103 (2009) 087403.

[15] Gartner A. et al., *Phys. Rev. B*, 76 (2007) 085304.

[16] Alloing M., Lemaître A., Galopin E. and Dubin F., *Sci. Rep.*, 3 (2013) 1578.

[17] Sinclair N. W. et al., *Phys. Rev. B*, 83 (2011) 245304.

[18] Combescot M., Moore M. G. and Piermarocchi C., *Phys. Rev. Lett.*, 106 (2011) 206404.

[19] Shilo Y. et al., *Nat. Commun.*, 4 (2013) 2335.

[20] Combescot M. and Betbeder-Matibet O., *Solid State Commun.*, 149 (2009) 835.

[21] Mysyrowicz A. et al., *Phys. Rev. Lett.*, 56 (1986) 2748.

[22] Combescot M., *Phys. Rep.*, 221 (1992) 167.

[23] Nalitov A. V., Vladimirova M., Kavokin A. V., Butov L. V. and Gippius N. A., arXiv:1311.0154v2. Here, the authors consider two coupled quantum wells and investigate how pump-probe techniques relying on direct excitons could allow the study of indirect excitons which are weakly active optically.

[24] Combescot M., Betbeder-Matibet O. and Dubin F., *Phys. Rep.*, 463 (2008) 215.

[25] Combescot M. and Betbeder-Matibet O., *Phys. Rev. B*, 74 (2006) 125316.