Gravitino Warm Dark Matter Motivated by the CDF $ee\gamma\gamma$ Event

M. Kawasaki

Institute for Cosmic Ray Research, The University of Tokyo, Tanashi, Tokyo 188, Japan

Naoshi Sugiyama

Department of Physics, Kyoto University, Kyoto 606-01, Japan

T. Yanagida

Department of Physics, School of Science, The University of Tokyo, Tokyo 113, Japan

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Abstract

The $ee\gamma\gamma + E_T$ event observed by the CDF at Fermilab is naturally explained by dynamically supersymmetry breaking models and suggests the presence of the light gravitino which can be a warm dark matter. We consider large scale structure of the universe in the worm dark matter model and find that the warm dark matter plays almost the same role in the formation of the large scale structure as a cold dark matter if its mass is about 0.5keV. We also study the Ly $\alpha$ absorption systems which are presumed to be galaxies at high redshifts and show that the baryon density in the damped Ly $\alpha$ absorption systems predicted by the warm dark matter model is quite consistent with the present observation.

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Low-energy supersymmetry (SUSY) is a very attractive candidate beyond the standard model, since it provides a natural solution to the gauge hierarchy problem [1,2]. If there exists the SUSY it must be spontaneously broken. The hidden sector model in $N = 1$ supergravity [3] is widely used for a realization of the SUSY breaking. Although this model has many attractive features, it suffers from a serious cosmological problem, i.e. the Polonyi problem [4]. There is, however, no such a problem in an alternative model [5] where the SUSY is broken dynamically by some new strong gauge interactions. In this class of models the SUSY breaking is mediated to the ordinary sector by the ordinary gauge interactions and another problem in the SUSY standard model, i.e. the flavor changing neutral current problem, is also solved automatically.

This dynamical model predicts the SUSY breaking scale $F$ to be low as $(100 - 1000)$ TeV and the gravitino mass $m_{3/2}$ in the range of $10eV - 1keV$. Therefore, the usual lightest SUSY particle (LSP) decays into the gravitino and cannot be a stable cold dark matter (CDM) in the universe. Instead of it the gravitino is a true LSP and can form a dark matter. Because its mass is so small, the gravitino has larger a velocity dispersion than the CDM. Such a type of dark matter is called a warm dark matter (WDM) [6].

It has been, recently, pointed out [7] that the dynamical SUSY-breaking model naturally explains the $ee\gamma\gamma + E_T$ event observed by the CDF experiment [8]. The event is explained by sequent decays [7]: $\tilde{e}^- (\tilde{e}^+) \rightarrow e^- (e^+) + \tilde{B}$ and $\tilde{B} \rightarrow \gamma + \tilde{G}$ where $\tilde{e}$, $\tilde{B}$ and $\tilde{G}$ are selectron, bino and gravitino, respectively. The decay length of the bino into a photon is given by

$$c\tau_{\tilde{B}} \simeq 5 \left( \frac{M_{\tilde{B}}}{100GeV} \right)^{-5} \left( \frac{m_{3/2}}{0.5keV} \right)^2 m, \quad (1)$$

where $M_{\tilde{B}}$ is the bino mass which should be $(38 - 100)$ GeV [7]. For $m_{3/2} \lesssim 0.5keV$ and $M_{\tilde{B}} = 100GeV$, the decay length is less than about 5m, which is consistent with the bino

\[\text{The longitudinal component of the gravitino (Goldstino) couples to matter with strength proportional to } F^{-1} \sim 1/\sqrt{m_{3/2}M_p} (M_p: \text{ Planck mass}). \text{ Thus the decay is very fast for the light gravitino.}\]
decay inside the CDF detector. Thus, the light gravitino is well motivated.

In this letter, we show that the warm (gravitino) dark matter whose mass is about 0.5keV \(^2\) plays almost the same role in the large-scale structure formation as CDM. We also study damped Ly \(\alpha\) absorption systems which are presumed to be the progenitors of present-day spiral galaxies in the WDM model and find that the observed mass of neutral hydrogens in Ly \(\alpha\) systems is consistent with the prediction by the WDM model. Colombi et al. \([10]\) has recently studied the large scale structure formations by WDM. However they have considered a very light WDM (\(\sim 100eV\)). Such a light WDM may cause a serious problem against the damped Ly \(\alpha\) systems.

Before the universe becomes colder than the gravitino mass (\(T > \sim 10^7(m_{3/2}/1keV)K\)), the gravitino behaves as a relativistic particle. Therefore the free streaming of the gravitino smears out small-scale density fluctuations and leads to a sharp cutoff in the power spectrum of the density fluctuations. The cutoff scale (\(=\) free streaming scale) is given by

\[
R_{fs} = 0.2 \left(\frac{g}{100}\right)^{-4/3} (\Omega h^2)^{-1} \text{Mpc}, \tag{2}
\]

where \(g\) is the effective number of particle degrees of freedom when the gravitino decoupled (\(g \simeq 200\) for the particle content of the minimal SUSY standard model and hereafter we take \(g = 200\)), \(h\) the Hubble constant in units of 100km/s/Mpc and \(\Omega\) the present density parameter of the gravitino which is related to the gravitino mass \(m_{3/2}\) by \(\Omega h^2 = (g/100)^{-1}(m_{3/2}/\text{keV})\). Since we only consider a gravitino dominated universe, \(\Omega \simeq \Omega_0\) where \(\Omega_0\) is the total density parameter at present. Assuming a scale invariant Harrison-Zeldovich spectrum, we can write the power spectrum \(P(k)\) of WDM as \([11]\)

\[
P(k) = A k |T(k)|^2, \tag{3}
\]

\(^2\)If we take, e.g. \(m_{3/2} \simeq 0.3\text{keV}\), we get \(c\tau_B \simeq 2\text{m}\). In this case the density parameter of the bino is \(\simeq 0.5\) and hence we need an additional contribution to \(\Omega\) to get the flat universe. However the results in the text are unchanged if the additional contribution comes from CDM \([9]\).
\[ T(k) = \exp \left[ -\frac{kR_{fs}}{2} - \frac{(kR_{fs})^2}{2} \right] T_0(k), \quad (4) \]
\[ T_0 = \frac{\ln(1 + 2.34q)}{2.34q} \left[ 1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4 \right]^{-1/4}, \quad (5) \]

where \( q \) is defined as \( q \equiv k/\Omega_0 h^2 / \exp(-\Omega_B - \sqrt{h/0.5\Omega_B/\Omega_0}) \text{Mpc}^{-1} \) taking into account the dependence on the baryon density \( \Omega_B \) \cite{12}, \( T(k) \) is the transfer functions for WDM, and \( A \) the normalization constant which is determined by COBE DMR 4 year data \cite{13}. Notice that for a CDM-dominated universe the power spectrum \( P_{\text{CDM}} \) is given by \( P_{\text{CDM}}(k) = Ak|T_0(k)|^2 \).

The WDM power spectrum for \( \Omega_0 = 1, h = 0.5 \) are shown in Fig. 1 together with the CDM power spectrum with the same cosmological parameters. Here, we have taken \( m_{3/2} = 0.5 \text{keV} \) corresponding to the \( \Omega_0 = 1 \) universe. Since the cutoff scale is relatively small (\( \sim 0.3 \text{Mpc}^{-1} \)) the power spectrum relevant for the large-scale structure (\( k \lesssim 1 \text{hMpc}^{-1} \)) is almost the same as the CDM one. (This contrasts the hot dark matter spectrum which has a cutoff of the order of 0.1Mpc^{-1}.) Therefore, the WDM model with \( \Omega_0 = 1 \) has the same problem as CDM one, i.e. the shape and the magnitude do not fit the observational data from the galaxy surveys \cite{14} which are also shown in Fig. 1. In particular, the amplitude of the power spectrum normalized by COBE is too large for \( k = (0.03 - 0.3) \text{hMpc}^{-1} \). The amplitude contradicts not only the only surveys but also the recent analysis of velocity fields \cite{13} (shaded region) which could directly reflect the mass distribution. If we take smaller value for \( A \), the power spectrum \( P(k) \) better fits to the data. Since the tensor mode \cite{16} or isocurvature mode \cite{17} may significantly contribute to \( \delta T/T \) in COBE scales, it is possible that the actual normalization of \( A \) is smaller. For example, we show the power spectrum normalized by \( \sigma_8 = 0.8 \) with the same cosmological parameters in Fig. 1. Here \( \sigma_8 \) is the mass overdensity within spheres of radius \( 8h^{-1}\text{Mpc} \). Notice that the COBE normalization gives \( \sigma_8 = 1.2 \). As is seen in Fig. 1 \( P(k) \) with \( \sigma_8 = 0.8 \) is in a good agreement with the velocity field data.

The difference between WDM and CDM is more significant for galaxies or smaller sys-
tems. The existence of the cutoff in the WDM spectrum delays the galaxy formation compared with in the CDM case. Damped Ly α absorption systems observed in QSO spectra are important since they give us information about galactic systems in the early universe. It is presumed that the damped Ly α absorptions observed in QSO spectra are due to neutral hydrogens contained in galactic systems at high redshifts ($z \sim 1 - 4$). Therefore, the observed damped Ly α absorptions can give us interesting information about baryons contained in the galactic systems \cite{18} and can set a constraint on galaxy formation models. In fact, this constraint is very stringent for the mixed dark matter (MDM = hot + cold dark matter) model \cite{19,20} since few galaxies are formed at high redshifts in the MDM model.

Here we study the damped Ly α constraint on the WDM model. Following ref. \cite{19}, we use the Press-Schechter theory \cite{21} to estimate the comoving number density $N(z, M)$ of the dark matter halos with mass between $M$ and $M + dM$ at redshift $z$:

$$N(z, M)dM = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M} \frac{\delta_c}{D_1(z)} \left[ -\frac{1}{\sigma^2(M)} \frac{\partial \sigma(M)}{\partial M} \right] \exp \left[ -\frac{\delta_c^2}{2\sigma^2(M)D_1^2(z)} \right],$$

where $\rho_0$ is the mean comoving mass density, $\delta_c$ is the overdensity threshold for the collapse ($= 1.68$, corresponding to the prediction of the spherical collapse model), $D_1(z)$ is the function for the growth of the perturbations ($D_1(z) = (1 + z)^{-1}$ for $\Omega_0 = 1$) and $\sigma^2(M)$ is the rms mass fluctuation in a top-hat window with radius $r_M = [M/(4\pi\rho_0/3)]^{1/3}$, given by

$$\sigma^2(M) = \frac{1}{2\pi^2} \int P(k)W^2(kr_M)k^2 \, dk,$$

$$W(x) = \frac{3}{x^3}(\sin x - x \cos x).$$

Since we assume that the damped Ly α absorptions are due to the neutral hydrogens in galactic systems, we need identify the halos with a certain mass range as galaxies. For this purpose, it is convenient to use circular velocity $v_c$ which is related to $M$ by

$$M = 2.45 \times 10^{11} M_\odot (1 + z)^{-3/2}(\Omega_0 h^2)^{-1/2} \times (v_c/100 \text{ km/s})^{3\Omega_0^{0.3}}.$$

(9)
For the spiral galaxies the circular velocity $v_c$ is in the range of $100 - 250\text{km/s}$. Smaller objects may also contribute to the absorption. However the lower limit may not be less than $50\text{km/s}$ since baryons in such small halo cannot be cooled enough to form a gaseous disk \cite{22}. The upper limit is also uncertain since gaseous disks might survive when halo merging occurs \cite{19}. In Fig. 2(a) we show the density parameter of baryons in galactic systems ($\Omega_D$) predicted in the WDM model for $\Omega_0 = 1, h = 0.5$, and $\Omega_B = 0.05$ together with observational data \cite{15}. Notice that the adopted $\Omega_B$ is consistent with the prediction by big bang nucleosynthesis ($\Omega_B h^2 = 0.0125 \pm 0.0025$ \cite{23}). In this case, the predicted $\Omega_D$ with $v_c = 100 - 250\text{km/s}$ is above the data. Since some fraction of baryons become stars and do not contribute to the absorption, the data should be taken to be lower limit to $\Omega_D$. Therefore the prediction by the WDM model is quite consistent with the observation. For comparison, we also show the predictions by CDM and MDM in Fig. 2(a). The MDM model ($\Omega_{\text{hot}} \simeq 0.3$ and $\Omega_{\text{cold}} \simeq 0.7$) can explain the data of the large scale structure in the universe better than the CDM model \cite{24}. However, as already mentioned, since the galaxy formation in the MDM model seems too late, it is difficult to explain the damped Ly $\alpha$ absorption \cite{19,20}. One can also consider the (warm + hot) dark matter model \cite{25} which may explain the large scale structure of the universe. However, this model has the same difficulty in explaining the damped Ly $\alpha$ absorption systems as the MDM model. In the case of CDM, the predicted $\Omega_D$ is larger than that predicted by WDM at higher redshifts. Thus, the survey of the damped Ly $\alpha$ systems at $z \gtrsim 5$ may possibly distinguish two models.

As is seen before, the WDM power spectrum with $\Omega_0 = 1$ and COBE normalization gives poor fit to the data at large scales, which leads us to consider the WDM power spectrum with smaller normalization constant. In Fig. 2(b) the predicted $\Omega_D$ is shown for $\Omega_0 = 1, h = 0.5, \Omega_B = 0.05, \sigma_8 = 0.8$. In the figure, it is seen that the WDM model is still quite consistent with the observation even if we take the smaller normalization for the power spectrum.

If the light fermion forms dark matter in a galactic halo, its phase space density in the galactic core might be larger than that allowed by the Fermi statistics \cite{26}. This phase space
constraint puts a stringent constraint on the mass of the dark matter fermion. From the study on the stellar motions in dwarf galaxies, the mass should be larger than about 500eV. Since the analysis of dwarf galaxies may contain systematic errors, this constraint may not be taken seriously. In our case, the dark matter is gravitino and its mass is about 500eV for $\Omega_0 = 1$ and $h = 0.5$. Therefore the phase space constraint is satisfied even if we take the constraint from the dwarf galaxies.

In summary, the $ee\gamma\gamma + E_T$ event observed by CDF is naturally explained by dynamical SUSY-breaking models and suggests the light gravitino. The light gravitino is a LSP and can be a WDM. We study the formation of the large scale structure of the universe and the adamped Ly $\alpha$ absorption systems in the WDM model. It is found that the WDM plays almost the same role in the formation of the large scale structure as the CDM if its mass is about 0.5keV. The difference between CDM and WDM becomes more significant when one considers the damped Ly $\alpha$ absorption systems which are presumed to be galaxies at high redshifts. The baryon density in the damped Ly $\alpha$ absorption systems predicted by the WDM model is quite consistent with the observational data. The future observation of damped Ly $\alpha$ systems at higher redshifts may distinguish the WDM model from the CDM one. Measurements of cosmic microwave background anisotropies may provide another possible way to distinguish WDM from CDM [27] although the difference is very small and only appears on fine angular scales. We need to wait for new generation satellite experiments(e.g., MAP and COBRAS/SAMBA).
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FIGURES

FIG. 1. The matter power spectra $P(k)$ in WDM models for the $\Omega_0 = 1, h = 0.5$ and $\Omega_B = 0.05$ with $n = 1$ adiabatic fluctuations. The power spectra normalized by the COBE 4 year data (solid) and by $\sigma_8 = 0.8$ (dotted) are plotted. An adiabatic CDM model with COBE 4 year normalization is also plotted (dashed). The observational data of galaxy surveys are taken from Peacock and Dodds [14]. Shaded regions are the best fitted value of Mark III catalog of peculiar velocities of galaxies by Zaroubi et al. [15] with 30% errors.

FIG. 2. (a) Evolution of the Density parameter $\Omega_D$ of the baryons contained in galactic systems whose circular velocity is between 100 and 250 km/s using Press-Schechter theory and the power spectrum normalized by COBE. Solid, dashed and dotted-dashed curves denote the predictions by WDM, CDM and MDM, respectively. Symbols represent for the observational data by Wolfe et al. [18]. (b) Same as (a) with the power spectrum normalized by $\sigma_8 = 0.8$. 

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