The main frequencies of solar core natural oscillations

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Abstract

The oscillations of a spherical body consisting of hot electron-nuclear plasma are considered. It is shown that there are two basic modes of oscillations. The estimation of the main frequencies of the solar core oscillation gives a satisfactory fit of the calculated spectrum and the measurement data.

1 Introduction. The main parameters of star cores

The substance of a star interior exists as hot dense electron-nuclear plasma. In the general case, the equilibrium of a hot plasma in a gravitational field can be written as

$$\nabla P + \gamma g + \rho E = 0$$  \hspace{1cm} (1)

where $\nabla P$ is the pressure gradient, $\gamma$ is the mass density and $\rho$ is the electric density induced by the gravity in plasma, satisfying

$$4\pi \rho = \text{div } E,$$  \hspace{1cm} (2)

$$- 4\pi G\gamma = \text{div } g.$$  \hspace{1cm} (3)

It is conventionally accepted to think that the equilibrium exists at $E = 0$ and

$$\nabla P + \gamma g = 0.$$  \hspace{1cm} (4)

At that the gradient pressure induces the increasing of the density and the temperature depthward a star. According to the virial theorem the full energy of star is equal to one half of its gravitational (potential) energy. It means the order of value of the full energy

$$E_{(E=0)} \approx -\frac{GM_{\text{star}}^2}{R_{\text{star}}},$$  \hspace{1cm} (5)
where $M_{\text{star}}$ and $R_{\text{star}}$ are the mass and the radius of a star. However, it is possible to see that Eq. (1) can be reduced to another equilibrium condition

$$\gamma g + \rho E = 0$$

(6)

at $\nabla P = 0$. Because of a high plasma density into a star core [1], the significant part of a star substance is concentrated in its core, and the star energy in the order of value is

$$E_{(\nabla P=0)} \approx -\frac{GM_{\text{star}}^2}{R_{\text{core}}}.$$ 

(7)

As a core radius $R_{\text{core}} < R_{\text{star}}$, the state with $\nabla P = 0$ is energetically preferable for the central core of a plasma body. The problem of equilibrium plasma in self-gravitating field is considered in detail in [1].

The absence of the pressure gradient results in constant plasma density. It is not difficult to see, that the plasma state with constant density corresponds to the energy minimum. Indeed, taking into account of inter-particle interaction, the free energy of the hot plasma is [2],

$$F_{\text{plasma}} = F_{\text{id}} + \frac{\pi^{3/2}}{4} \left( \frac{r_B e^2}{kT} \right)^{3/2} N_e n_e - \frac{2\pi^{1/2}}{3} \left( \frac{\langle Z \rangle + 1}{kT} \right)^{3/2} e^3 N_e n_e^{1/2},$$

(8)

where $F_{\text{id}}$ is the free energy of the ideal gas, $N_e$ is the full number of electrons in a system, $n_e$ is the electron density. In this equation the second item describes the correction for quantum properties of electron gas and third item describes the electron-ion interaction in plasma. The full energy of ideal gas doesn’t depend on particle density by definition and the derivative from $F_{\text{id}}$ is negligible small. At a constant full number of particles in the system and at a constant temperature, the equilibrium state exists at the minimum of energy

$$\left( \frac{\partial F_{\text{plasma}}}{\partial n_e} \right)_{N,T} = 0,$$

(9)

which allows one to obtain the steady-state value of the density of hot non-relativistic plasma

$$\Theta = \frac{16^3}{9\pi^2} \left( \frac{\langle Z \rangle + 1}{3} \right)^3 cn^{-3},$$

(10)

where $r_B = \frac{\hbar^2}{me}$ is the Bohr radius, $\langle Z \rangle$ is the averaged charge of nuclei plasma is composed from.

The equilibrium conditions give possibility to calculate all main parameters of the star core [1]:

the equilibrium temperature:

$$T = \left( \frac{10^7}{\pi^4} \right)^{1/3} \left( \frac{\langle Z \rangle + 1}{3} \right)^{1/3} \frac{hc}{kT_B} \approx 2 \cdot 10^7 (\langle Z \rangle + 1) K;$$

(11)
the equilibrium mass of core:

\[ M = 1.56 \left( \frac{10}{\pi^3} \right)^{1/2} \left( \frac{\hbar c}{G m_p^2} \right)^{3/2} \left( \frac{Z}{A} \right)^2 m_p \approx 6.47 M_{\text{Ch}} \left( \frac{Z}{A} \right)^2, \]  

(12)

where \( M_{\text{Ch}} = \left( \frac{\hbar c}{G m_p^2} \right)^{3/2} m_p = 3.42 \cdot 10^{33} \text{g} \) is the Chandrasechar mass, \( \langle A \rangle \) is averaged mass number of nuclei plasma consists from, \( m_p \) is the proton mass;

the equilibrium radius of the star core:

\[ R = \left( \frac{3}{2} \right)^{1/6} \left( \frac{10}{\pi} \right)^{1/6} \left( \frac{\hbar c}{G m_p^2} \right)^{1/2} \frac{r_B}{\langle Z \rangle + 1} \langle A/Z \rangle. \]  

(13)

and the pressure into a core:

\[ P = \frac{GM^2}{8\pi R^4}. \]  

(14)

It is important to underline that the steady-state parameters of a core are depending only on chemical composition of a star, that is expressed through two variable parameters \( \langle Z \rangle \) and \( \langle A/Z \rangle \). These parameters are unknown apriori.

## 2 The sound speed in a hot plasma

The pressure of a high temperature plasma is a sum of the plasma pressure (ideal gas pressure) and the pressure of black radiation:

\[ P = n_e kT + \frac{a}{4} (kT)^4, \]  

(15)

and its entropy is

\[ s = \frac{1}{m'} \ln \left( \frac{kT}{n_e} \right)^{3/2} + \frac{a(kT)^3}{n_e}, \]  

(16)

where \( m' = \frac{A}{Z} m_p \) is the mass related to one electron and \( a = \frac{4\pi^2}{45\hbar^3 c^3} \).

The sound speed \( c_s \) can be expressed by a Jacobian [3]:

\[ c_s^2 = \frac{D(p,s)}{D(p,s)} = \left( \frac{D(p,s)}{D(n_e,T)} \right) \]  

(17)

or

\[ c_s = \left\{ \frac{5 kT}{3 \langle A/Z \rangle m_p} \left[ 1 + \frac{2a^2(kT)^6}{5n_e[n_e + 2a(kT)^3]} \right] \right\}^{1/2}. \]  

(18)

For \( T = T \) and \( n_e = \Theta \) we have:

\[ \frac{a(kT)^3}{n_e} = \frac{a(kT)^3}{\Theta} = \frac{1}{2}. \]  

(19)
Finally we obtain:

\[ c_s = \left( \frac{5}{3} \left( \frac{10}{\pi^3} \right)^{1/3} \frac{1/0.05}{(A/Z)m_p r_B} \right)^{1/2} \approx 5.42 \times 10^7 \left( \frac{\langle Z \rangle + 1}{\langle A/Z \rangle} \right)^{1/2} \text{ cm/s}. \]  

(20)

3 The basic elastic oscillation of a spherical core

Star cores consist of a dense high temperature plasma which is a compressible matter. The basic mode of elastic vibrations of a spherical core is related with its radius oscillation. For the description of this type of oscillation, the potential \( \phi \) of displacement velocities \( v_r = \frac{\partial \psi}{\partial r} \) can be introduced and the motion equation can be reduced to the wave equation expressed through \( \phi \) [3]:

\[ c_s^2 \Delta \phi = \ddot{\phi}, \]  

(21)

and a spherical derivative for periodical in time oscillations (\( \sim e^{-i\Omega t} \)) is:

\[ \Delta \phi = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial \phi}{\partial r} \right) = -\frac{\Omega_s^2}{c_s^2} \phi. \]  

(22)

It has the finite solution for the full core volume including its center

\[ \phi = \frac{A}{r} \sin \frac{\Omega_s r}{c_s}, \]  

(23)

where \( A \) is a constant. For small oscillations, when displacements on the surface \( u_R \) are small (\( u_R / R = v_R / \Omega_s R \to 0 \)) we obtain the equation:

\[ t g \frac{\Omega_s R}{c_s} = \frac{\Omega_s R}{c_s}, \]  

(24)

which has the solution:

\[ \frac{\Omega_s R}{c_s} \approx 4.49. \]  

(25)

Taking into account Eq. (20), the main frequency of the core radial elastic oscillation is

\[ \Omega_s = 4.49 \left\{ \frac{10.5}{(3/2)^{7/3} \pi} \left[ \frac{G m_p}{r_B^3} \right] \left( \frac{A}{Z} \right) \left( \frac{\langle Z \rangle + 1}{\langle A/Z \rangle} \right)^3 \right\}^{1/2}. \]  

(26)

It can be seen that this frequency depends on \( \langle Z \rangle \) and \( \langle A/Z \rangle \) only.

Some values of frequencies of radial sound oscillations \( F = \Omega_s / 2\pi \) calculated from this equation for selected \( A/Z \) at \( Z = 1 \) and \( Z = 2 \) are shown in third column of Table 1.

Table 1.
The measured frequencies of surface vibrations for some of stars [4] are shown in right part of this table. The data for ν Indus and ξ Hydrae also exists [4], but characteristic frequencies of these stars are below 0.3 mHz and they have some another mechanism of excitation, probably. One can conclude from the data of Table 1 that the core of the Sun is basically composed by helium-10. It is not a confusing conclusion, because according to Eq.(14), the pressure which exists inside the solar core amounts to $10^{19}$ dyne/sm$^2$ and it is capable to induce the neutronization process in plasma and to stabilize neutron-excess nuclei. Using these parameters ($\langle Z \rangle = 2, \langle A/Z \rangle = 5$), we obtain that the solar core radius $R \approx 9.4 \cdot 10^9$ cm ($R/R_\odot \approx 0.13$) and its temperature $T \approx 6.1 \cdot 10^7$ K. It is interesting that the solar core mass amounts to $9.6 \cdot 10^{32}$ g, i.e. almost exactly one half of full mass of the Sun is concentrated in its core.

4 The low frequency oscillation of the density of a neutral plasma

According to Eq.(10) a hot plasma has the density $\Theta$ at its equilibrium state. The local deviations from this state induce processes of a density oscillation since the plasma tends to return to its steady-state density. If we consider small periodic oscillations of core radius

$$R = R + u_R \sin \omega t,$$

where a radial displacement of plasma particles is small ($u_R \ll R$), the oscillation process of plasma density can be described by the equation

$$\frac{dE_{\text{plasma}}}{dR} = M\ddot{R}.$$
From this
\[
\omega^2_b = \frac{3\pi^{3/2}}{2} kT \left( \frac{e^2}{r_B kT} \right)^{3/2} \frac{r_B^3 \Theta}{R^2 \langle A/Z \rangle m_p}
\]
(29)
or
\[
\omega_\Theta = \left\{ \frac{\sqrt{\pi} 2^4}{\sqrt{10}} (3/2)^{3/2} \left[ \frac{Gm_p}{r_B} \langle A \rangle \langle Z \rangle^{4.5/2} \left[ (Z) + 1 \right] \right] \right\}^{1/2},
\]
(30)

where \( \alpha = \frac{e^2}{\hbar c} \) is the fine structure constant. These low frequency oscillations of neutral plasma density are like to phonons in solid bodies. At that oscillations with multiple frequencies \( k\omega_\Theta \) can exist. Their power is proportional to \( 1/k \), as the occupancy these levels in energy spectrum must be reversely proportional to their energy \( k\hbar \omega_\Theta \). As result, low frequency oscillations of a plasma density constitute set of vibrations
\[
\sum_{k=1}^{\infty} \frac{1}{k} \sin(k\omega_\Theta t).
\]
(31)

5 The main frequencies of the solar core oscillation

The set of the low frequency oscillations with \( \omega_\Theta \) can be induced by sound oscillations with \( \Omega_s \). At that displacements obtain the spectrum:
\[
u_R \sim \sin \Omega_s t \cdot \sum_{k=0}^{\infty} \frac{1}{k} \sin k\omega_\Theta t \sim \xi \sin \Omega_s t + \sum_{k=1}^{\infty} \frac{1}{k} \sin (\Omega_s \pm k\omega_\Theta) t,
\]
(32)

where \( \xi \) is a coefficient \( \approx 1 \).

This spectrum is shown in Fig.1b.

The central frequency of experimentally measured distribution of the solar oscillation is approximately equal to [Fig.1a]
\[F_\odot \approx 3.23 \text{ mHz} \]
(33)

and the experimentally measured frequency splitting in this spectrum is approximately equal to
\[f_\odot \approx 67.5 \text{ \mu Hz} \]
(34)

(see Fig.2b). At \( \langle Z \rangle = 2 \) and \( \langle A/Z \rangle = 5 \) the calculated frequencies of basic modes of oscillations (from Eq.26 and Eq.30) are
\[F = \frac{\Omega_s}{2\pi} = 3.19 \text{ mHz}; \quad f_\Theta = \frac{\omega_\Theta}{2\pi} = 66.0 \text{ \mu Hz}.\]
(35)

It is important to note that there are two ways for the core chemical composition determination. Thus chemical parameters \( \langle Z \rangle \) and \( \langle A/Z \rangle \) can be obtained by fitting at the known frequency of basic oscillation \( F \) as it was done above (according to Table 1). Another way - to express these parameters through \( F \)
and $f$. The betweenness relation of two this frequencies gives a possibility for a
direct determination of averaged parameters of nuclei which the core composed
from:

$$\langle Z \rangle = \frac{4.49^{4/3} \times 10^{1/3} \times 10.5^{2/3}}{2^{8/3} \pi \alpha} \left( \frac{f}{F} \right)^{4/3} - 1 \quad (36)$$

and

$$\langle A/Z \rangle = \frac{3^7 \alpha^3 r_B^3}{5 \cdot 10.5^3 G m_p} \left( \frac{\pi^2 F^3}{4.49^3 f^2} \right)^2 \quad (37)$$

Using experimentally obtained frequencies (Eq. (33) and Eq. (34)), we have
for the solar core

$$\langle Z \rangle_\odot = 2.03 \quad (38)$$

and

$$\langle A/Z \rangle_\odot = 4.99. \quad (39)$$

Thus both these ways of determination of chemical parameters give prac-
tically identical results that demonstrates the adequacy of our consideration.

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Figure 1: (a) The measured power spectrum of solar oscillation. The data were obtained from the SOHO/GOLF measurement [5]. (b) The calculated spectrum described by Eq. (32) at $Z = 2$ and $A/Z = 5$. 

\[ F \]

\[ P \text{ (mHz)}^2 / \text{Hz} \]

\[ \nu \text{ (mHz)} \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad \text{mHz} \]

Arbitrary units

\[ 10^0 \quad 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad \text{mHz} \]
Figure 2: (a) The power spectrum of solar oscillation obtained by means of Doppler velocity measurement in light integrated over the solar disk. The data were obtained from the BISON network [6]. (b) An expanded view of a part of the frequency range.
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