Dark matter and dark energy proposals: maintaining cosmology as a true science? *

George F R Ellis †

November 21, 2008

Abstract

I consider the relation of explanations for the observed data to testability in the following contexts: observational and experimental detection of dark matter; observational and experimental detection of dark energy or a cosmological constant $\Lambda$; observational or experimental testing of the multiverse proposal to explain a small non-zero value of $\Lambda$; and observational testing of the possibility of large scale spatial inhomogeneity with zero $\Lambda$.

1 Dark matter and testability

As discussed at this meeting, there is a great deal of astronomical evidence for dark matter: galaxy rotation curves and dynamical studies; the Baryon Acoustic Oscillations (BAO) and peaks in the Cosmic Blackbody Radiation (CBR) power spectrum; and other studies of Large Scale Structure (LSS). An important feature of this story is that dark matter was (somewhat reluctantly) observationally discovered, it was not predicted! However its nature is unknown, except that it is not baryonic, so there is much theoretical speculation about what it is.

As to the astronomical evidence, there is more coming, with many studies under way and detectors planned. Laboratory and accelerator tests are also planned, attempting to link astrophysics to detected particle properties. This will be a major coup if successful - it will identify dark matter physically. Accompanying this experimental work is a continuing development of different theories about the nature of dark matter, as evidenced at this meeting.

All this work is very much in the proper scientific spirit: make theories and test observationally and experimentally. Dark matter searches and tests are thriving, and I will not comment more on them here.

---

*Paper for CRAL-IPNL conference "Dark Energy and Dark Matter", Lyon 2008.
†Mathematics Department, University of Cape Town.
2 The Acceleration of the universe

The case of dark energy is quite different. The explanation of dark energy is a central preoccupation of present day cosmology. Its presence is indicated by the recent speeding up of the expansion of the universe as shown by the supernova observations discussed at this meeting, and is confirmed by other observations such as those of the cosmic background radiation anisotropies and LSS/BAO studies. Like dark matter, its existence was discovered, not predicted.

As discussed at this meeting, astronomical observations are being refined in many sophisticated ways and used to confirm the acceleration and test equations of state of hypothetical dark energy. However the interpretation of these observations is ambiguous, as discussed in section 4. It is therefore crucial to pursue the possibility of any other tests on the one hand, and theoretical explanations on the other. So how can we confirm its existence and nature?

2.1 Lab tests of Dark energy?

It is striking that at this meeting there are many proposals for laboratory or accelerator detection of dark matter, but none for dark energy. Indeed such tests in a lab or even the solar system are not feasible, in the case of the usual conception of DE as cosmological constant or quintessence; it simply has no significant effect at the relevant scales. The exception would be in the case of unified approaches to Dark Energy and Dark Matter, as sometimes discussed.

Such approaches need to be explored: they may be facets of the same problem, and then evidence for dark matter is also evidence for dark energy. But then we would have a force that would change (with scale) from attraction to repulsion: we would have to explain why and how, and propose how to test that change. The lab tests for dark matter would not explore the scales where it would become effective dark energy. It seems unlikely one would attain the required evidence for the dark energy effect.

2.2 Theoretical explanations?

Without lab tests, we have to rely on theoretical explanations for its nature. However that nature (whether constant, or varying) is a major problem for theoretical physics. It is not uniquely related to any known field or particles.

If the dark energy is in fact constant, the attempt to explain it from fundamental physics is a disaster — theoretical proposals for a cosmological constant from quantum field theory give an answer $10^{120}$ factors too large! The only way out seems to be the multiverse proposal, which is gaining ground, but is rather problematic as a scientific explanation, as discussed in section 3.

If it is varying, a quintessence field, we need to know its nature; but no compelling identification has been made. Perhaps it is due to modified gravitational theories: higher curvature terms or effects of higher dimensions. Perhaps it is due to some physical effect such as Bose-Einstein condensation; such options need to be explored. We can just deal with it at a phenomenological level in
terms of an arbitrary equation of state; but not all such equations of state are physically acceptable, as discussed in Section 5.4, and in any case that approach is not in the end satisfactory: ultimately it needs an underlying physical basis.

In all cases, the issue is how do we test these theoretical proposals? Many seem very arbitrary. Just writing down a Lagrangian does not prove such matter exists! If the explanation only explains one thing (the observed acceleration) and has no other testable outcome, it is an ad hoc explanation for that one thing rather than a unifying scientific proposal. It needs some other independent experimental or observational test but we don’t have another viable context for applying such tests.

So how do we justify our proposed theoretical explanations? Why this form of quintessence? Why a cosmological constant with the observed value? We need to see if there are any alternatives; and there are, as discussed below.

3 Explaining the cosmological constant via a multiverse

The idea of a multiverse – an ensemble of universes or of universe domains has received increasing attention in cosmology (see the articles in Carr [6] for an up to date survey), with suggestions including that it can occur

- in separate places, as particularly justified by chaotic inflation (Linde [21, 20], Guth [15, 16], Vilenkin [30])
- through the Everett quantum theory interpretation: other branches of the wavefunction of the universe (Deutsch [11])
- because of the landscape of string theory, imbedded in a chaotic cosmology (Susskind [26]).

A particular theoretical driver of these proposals is the "anthropic" issue: the realization that the universe is fine-tuned for life as regards both the laws of physics and as regards the boundary conditions of the universe (Barrow and Tipler [2], Rees [24, 25]). A multiverse with varied local physical properties is one possible scientific explanation: an infinite set of universe domains allows all possibilities to occur, so somewhere things work out OK: conditions for life will be fulfilled somewhere in the multiverse (NB: it must be an actually existing multiverse rather than a hypothetical one - this is essential for any such anthropic argument).

The application of this proposal is to explaining fundamental constants, and particularly explaining the small value of the cosmological constant (Weinberg [31, 32], Susskind [26]). Too large a value for \( \Lambda \) results in no structure and hence no life; so anthropic considerations in a multiverse mean that the value of \( \Lambda \) observed by any intelligent being will be small (in fundamental units), thus justifying an actual value extremely different from the ‘natural’ one predicted by physics: a difference of 120 orders of magnitude. This makes clear the true multiverse project: making the extremely improbable appear probable.
However the very nature of the scientific enterprise is at stake in the multiverse debate: the multiverse proponents are proposing weakening the nature of scientific proof in order to claim that multiverses provide a scientific explanation. This is a dangerous tactic (note that we are concerned with really existing multiverses, not potential or hypothetical). Two central scientific virtues are testability and explanatory power. In the cosmological context, these are often in conflict with each other (Ellis [12]). The extreme case is multiverse proposals, where no direct observational tests of the hypothesis are possible, as the supposed other universes cannot be seen by any observations whatever, and the assumed underlying physics is also untested and indeed probably untestable.

In this context one must re-evaluate what the core of science is: can one maintain one has a genuine scientific theory when direct and indeed indirect tests of the theory are impossible? If one claims this, one is altering what one means by science. One should be very careful before so doing. The key observational point is that the domains considered are beyond the particle horizon and are therefore unobservable. The assumption is we that can extrapolate to 100 Hubble radii, \(10^{100}\) Hubble radii, or much much more (infinity is often mentioned); but we have no data whatever about these domains. Given this extremely poor observational context, are there other reasons to believe the multiverse proposal?

Is it implied by known physics, that leads to chaotic inflation?
The key physics (Coleman-de Luccia tunneling, the string theory landscape) is extrapolated from known and tested physics to new contexts; the extrapolation is unverified and indeed is unverifiable; it may or may not be true. The physics is hypothetical rather than tested. Is the situation:

\[
\text{Known Physics} \Rightarrow \text{Multiverse ??}
\]

NO! The real situation is

\[
\text{Known Physics} \Rightarrow ? \quad \text{Hypothetical Physics} \Rightarrow \text{Multiverse}
\]

It is a great extrapolation from known physics. This extrapolation is untested, and may be untestable: it may or may not be correct.

Is it implied by inflation, which is justified by CBR anisotropy observations?
It is implied by some forms of inflation but not others; inflation is not yet a well defined theory, it is a family of theories. Not all forms of inflation lead to chaotic inflation, for example inflation can occur in small closed universes.

Is it implied by probability arguments?
It is claimed it is implied by a probability argument: the universe is no more special than need be to create life. Hence the observed value of the Cosmological constant is confirmation (Weinberg [31, 32]; Rees [24, 25]). But the statistical argument only applies if a multiverse exists; it is simply inapplicable if there
is no multiverse. In that case we only have one object we can observe; we can do many observations of that one object, but it is still only one object (one universe), and you can’t do statistical tests if there is only one existent entity (Ellis [12]). Furthermore, we don’t know the measure to use; but the result depends critically on this choice. Overall, this is a weak consistency test on multiverses, that is indicative but not conclusive, firstly because a probability argument cannot in fact be falsified, all it can do is confirm that some result is improbable; and secondly, because while consistency tests must be satisfied, they are not confirmation unless no other explanation is possible.

Is it testable through predicting closed spatial sections?
The claim is made (Susskind [26]) that only negatively curved RW models can emerge in a chaotic inflation multiverse, because Coleman-de Luccia tunneling only gives such models; so one can disprove chaotic inflation if one observationally determines that $k = +1$. But that claim about inflation is already disputed, as there are papers suggesting $k = +1$ tunneling is possible. In any case this model it depends on a very specific speculative mechanism, which has not been verified to actually work, and indeed such verification is probably impossible. Alternatively one can claim the idea is disproved if we determine the spatial sections are positively curved in the observed region, for then if they extend unchanged far enough this implies a closed universe and hence no chaotic inflation. But we could live in high density lump imbedded in a low density universe: the extrapolation of $k = +1$ geometry beyond the visual horizon may not be valid. Neither argument is conclusive!

However, chaotic inflation can be disproved if we observationally prove we live in a small universe: that is, we have already seen round the universe because it has small closed spatial sections. To test for this possibility we can search for identical circles in the CBR sky, plus a low CBR anisotropy power at large angular scales (which is what is observed) – see Frank Steiner’s contribution. This is an important test as it would indeed disprove the chaotic inflation variety of multiverse; but not seeing them would not prove a multiverse exists. Their non-existence is a necessary but not sufficient condition for a multiverse.

Is it theoretically preferable to other explanations of the way things are?
This is what many are claiming. Indeed the real argument for the proposal is that it is the only purely physical explanation for fine tuning of parameters that lead to our existence, in particular the value of the cosmological constant; but this is theoretical explanation, not supported by astronomical observation. So which is more important in cosmology: theory (explanation) or observations (tests against reality)? That is the core issue to be faced.

Is it an infinity of entities necessarily implied?
Often it is claimed there are physically existing infinities both of universes in a multiverse, and of spatial sections in each of the universes in the multiverse context, see e.g. Vilenkin [30]. But infinity is an unattainable state rather than a number, and is plausibly never attained in physical reality. Indeed David
Hilbert states "the infinite is nowhere to be found in reality, no matter what experiences, observations, and knowledge are appealed to" (Hilbert [17]). Furthermore, this claim is completely untestable: if we could see them, which we can’t, we could not count them in a finite time! The claimed existence of physically existing infinities is highly dubious, and is not a scientific statement, if science involves testability by either observation or experiment. This claim in the multiverse context emphasizes how tenuously scientific that idea is. It is not remotely testable.

Implication of all the above:
The multiverse idea is not provable either by observation, or as an implication of well established physics (cf. Gardner [13]). It may be true, but cannot be shown to be true by observation or experiment. However it does have great explanatory power: it does provide an empirically based rationalization for fine tuning, developing from known physical principles. Here one must distinguish between explanation and prediction. Successful scientific theories make predictions, which can then be tested. The multiverse theory can’t make any predictions because it can explain anything at all. Any theory that is so flexible is not testable because almost any observation can be accommodated. I conclude that multiverse proposals are good empirically-based philosophical proposals for the nature of what exists, but are not strictly within the domain of science because they are not testable. I emphasize that there is nothing wrong with empirically-based philosophical explanation, indeed it is of great value, provided it is labeled for what it is. I suggest that cosmologists should be very careful not to make methodological proposals that erode the essential nature of science in their enthusiasm to support such theories as being scientific (cf. Tegmark [27, 28]), for if they do so, there will very likely be unintended consequences in other areas where the boundaries of science are in dispute. It is dangerous to weaken the grounds of scientific proof in order to include multiverses under the mantle of ‘tested science’ for there are many other theories standing in the wings that would also like to claim that mantle.

4 Inhomogeneity and the Acceleration of the universe

The deduction of the existence of dark energy is based on the assumption that the universe has a Robertson-Walker (RW) geometry - spatially homogeneous and isotropic on a large scale. The observations can at least in principle be accounted for without the presence of any dark energy, if we consider the possibility of inhomogeneity. This can happen in two ways: locally via backreaction and observational effects, and via large scale inhomogeneity.
4.1 Small scale inhomogeneity: backreaction and observational effects

Acceleration due to backreaction from small scale inhomogeneities is discussed by Wiltshire at this meeting (see also Wiltshire \[33\], Buchert \[4\]). Wiltshire proposes that gravitational energy can provide a source of effective dark energy, leading to the possibility in principle of concordance cosmology without $\Lambda$. It is important to notice there are two effects here: firstly the backreaction from small scale inhomogeneity to the large scale geometry can generate a dynamic effect in the effective Friedmann equation for the cosmology, and secondly small scale inhomogeneity has significant effects on the propagation of photons in a lumpy universe, with potentially important effects on observations.

Whether these effects are sufficient to account for the apparent supernova observations is an important ongoing debate involving interesting modeling and general relativity issues, and particularly how one models a universe with large scale voids and the nature of the Newtonian limit in cosmology. In my view the jury is still out on this one, with many skeptical there is any significant effect and others suggesting it may be at least large enough to affect the cosmic relation between energy densities and expansion that leads us to deduce the spatial curvature is almost flat. Conceptual clarity on the modeling issues involved is required.

4.2 Large scale inhomogeneity: inhomogeneous geometry

Perhaps there is a large scale inhomogeneity of the observable universe such as that described by the Lemaître-Tolman-Bondi (LTB) pressure-free spherically symmetric models, and we are near the centre of a void. The idea that such models can explain the supernova observations without any dark energy is discussed by Céleriér at this meeting (and see also Céleriér \[7\]).

4.3 Can we fit the observations?

The LTB models have comoving coordinates

$$ds^2 = -dt^2 + B^2(r, t) + A^2(r, t)(d\theta^2 + \sin^2 \theta d\phi^2)$$

where

$$B^2(r, t) = A'(r, t)^2(1 - k(r))^{-1}$$

and the evolution equation is

$$(\dot{A}/A)^2 = F(r)/A^3 + 8\rho_M/3 - k(r)/A^2$$

with the energy density given by $F'(A'A^2)^{-1} = 8\rho_M$.

There are two arbitrary functions of the spatial coordinate $r$: namely $k(r)$ (curvature) and $F(r)$ (matter). This freedom enables us to fit the supernova observations with no dark energy or other exotic physics (this is a theorem, see
Mustapha et al. [23]). One can also fit the CBR observations because they refer to much larger values of $r$ (see e.g. Alexander et al. [1]). One should note here that at least some of the observed CBR dipole can then arise because we are a bit off-centre in the void, so one can re-evaluate the great attractor analysis in this context and the alignment of the dipole and quadrupole. Nucleosynthesis data can also be fitted; what is a bit more problematic is the BAO. The key comment to make is that different scales are probed by different observations and can in principle all be fitted by adjusting the free spatial functions at different distances.

A typical observationally viable model is one in which we live roughly centrally (within 10% of the central position) in a large void: a compensated underdense region stretching to $z \approx 0.08$ with $\delta \rho/\rho \approx -0.4$ and size $160/h$ Mpc to $250/h$ Mpc, a jump in the Hubble constant of about 1.20 at that distance, and no dark energy or quintessence field (Biswas et al. [3], Ishak et al. [18], Yoo et al. [34]).

4.4 Large scale inhomogeneity: dynamic evolution?

Given we can fit the observations by such a model, can we find dynamics (inflation followed by a HBB era) that can lead to such a model? It has the same basic dynamics as the standard model (evolution along individual world lines governed by the Friedmann equation) but with distant dependent parameters. Will inflation prevent it? This depends on the initial data, the amount of inflation, and the details of the unknown inflaton. If we are allowed the usual tricks of fiddling the inflationary potential and initial data, and adding in multiple fields as desired, then there is sufficient flexibility that it should certainly be possible.

4.5 Improbability

Many dismiss these models on probability grounds: It is improbable we are near the centre of such a model. But there is always improbability in cosmology. We can can shift it around, but it is always there. It might be in the nature of a Robertson-Walker geometry (the old view), in the inflationary potential and initial conditions (the current mainstream position), which specific universe domain we are in within a multiverse, or the spatial position in an inhomogeneous universe (the present proposal). Note that we are competing with a probability of $10^{-120}$ for $\Lambda$ in a RW universe; we do not have to get very high probabilities to outdo that improbability, which is what the multiverse proposal aims to handle.

Three comments are in order. First, a key feature of cosmology is that there is only one universe; and the very concept of probability does not apply to a single object, even though we can make many measurements of that single object to determine its detailed nature. Probability applies to the multiple measures we can make of the single universe, but not to issues doing with the existence of the universe itself (Ellis [12]). There is no physically realised ensemble to apply
that probability to, unless a multiverse exists in physical reality which is not proven, as discussed above: it’s a philosophical assumption. In essence, there simply is no proof the universe is probable; that is a philosophical assumption, which may not be true. The universe may be improbable!! Secondly, there is no well-justified measure for any such probability proposal even if we ignore the first problem. This is still an issue of debate.

And thirdly, a study by Linde et al \[22\] shows that (given a particular choice of measure) this kind of inhomogeneity actually is a probable outcome of inflationary theory, with ourselves being located near the centre! One cannot dismiss such models out of hand for probability reasons.

5 Observational tests of spatial homogeneity

Given the above context, direct observational tests of the Copernican(spatial homogeneity) assumption are of considerable importance. Given that we can both find inhomogeneous models to reproduce the observations without any exotic energy, as well as homogeneous models with some form of dark energy that explain the same observations, can we distinguish between the two? Ideally we need a model-independent test: is a RW geometry the correct metric for the observed universe region? Four kinds of tests are possible, as discussed below. Whatever position we may have on the issue of probability, in the end our philosophy on this question will have to give way to any such possible observational tests.

5.1 CBR based tests

Some tests use scattered CBR photons to check spatial homogeneity (Goodman \[14\]; Caldwell and Stebbins \[5\]). If the CBR radiation is anisotropic around distant observers (as will be true in inhomogeneous models), Sunyaev-Zeldovich scattered photons have a distorted spectrum that reflects the spatial inhomogeneity. However this test is somewhat model dependent - it is good for void models but misses, e.g., conformally stationary spacetimes. It also has to take into account other possible causes of spectral distortion.

5.2 Direct observational tests: behaviour near origin

The universe must not have a geometric cusp at the origin, as this implies a singularity there. Thus there are centrality conditions that must be fulfilled in the inhomogeneous models (Vanderveld et al \[29\]). The distance modulus behaves as $\Delta \delta m(z) = -(5/2)q_0z$ in standard $\Lambda$CDM models, but if this were true in a LTB void model without $\Lambda$ this implies a singularity (Clifton et al \[9\]). Observational tests of this requirement will be available from intermediate redshift supernovae in the future.
5.3 Direct observational tests: constancy of curvature

There are two geometric effects on distance measurements: curvature $\Omega_k$ bends null geodesics, expansion $H(z)$ changes radial distances. These are coupled in RW models, as expressed in the relation

$$d_L(z) = \frac{(1+z)}{H_0 \sqrt{-\Omega_k}} \sin \left( \sqrt{-\Omega_k} \int_0^z dz' \frac{H_0}{H(z')} \right).$$

While these effects are strictly coupled in RW geometries, they are decoupled in LTB geometries.

In RW geometries, we can combine the Hubble rate and distance data to find the curvature today:

$$\Omega_k = \frac{\left[H(z)D'(z)\right]^2 - 1}{H_0 D(z)^2}.$$

This relation is independent of all other cosmological parameters, including dark energy model and theory of gravity. It can be used at single redshift to determine $\Omega_k$. The exciting result of Clarkson et al (8) is that since $\Omega_k$ is independent of $z$, we can differentiate to get the consistency relation

$$C(z) := 1 + H^2 DD'' - D'^2 + HH' DD' = 0,$$

which depends only on a RW geometry: it is independent of curvature, dark energy, nature of matter, and theory of gravity. Thus it gives the desired consistency test for spatial homogeneity. In realistic models we should expect $C(z) \simeq 10^{-5}$, reflecting perturbations about the RW model related to structure formation. Errors may be estimated from a series expansion

$$C(z) = \left[ q_0^{(D)} - q_0^{(H)} \right] z + O(z^2)$$

where $q_0^{(D)}$ is measured from distance data and $q_0^{(H)}$ from the Hubble parameter. It is simplest to measure $H(z)$ from BAO data. It is only as difficult carrying out this test as carrying out dark energy measurements of $w(z)$ from Hubble data, which requires $H'(z)$ from distance measurements or the second derivative $D''(z)$.

This is the simplest direct test of spatial homogeneity, and its implementation should be regarded as a high priority: for if it confirms spatial homogeneity, that reinforces the evidence for the standard view in a satisfying way; but if it does not, it has the possibility of undermining the entire project of searching for a physical form of dark energy.

5.4 Indirect Observational tests

If the standard inverse analysis of the supernova data to determine the required equation of state, as discussed at this meeting, shows there is any redshift range where $w := p/\rho < -1$, this may well be a strong indication that one of these
geometric explanations is preferable to the Copernican (Robertson-Walker) assumption, for otherwise the matter model indicated by these observations is non-physical (it has a negative kinetic energy).

There is already data suggesting this may be the case, see e.g. Lima et al [19]. There are some attempts to generate matter models that will give this kind of behaviour without negative kinetic energies, but they are very speculative physically, supposing multiple unknown forms of matter or energy with arbitrarily proposed interactions between them. It all seems rather reminiscent of the Ptolemaic epicycles for describing the solar system. The physically most conservative approach is to assume no unusual dark energy or exotic interacting fields, but rather that an inhomogeneous geometry might be responsible for the observed apparent acceleration; this should be seriously considered as an alternative.

6 Conclusion

The issue of what is testable and what is not testable in cosmology is a key issue. Some dark energy proposals, specifically from multiverse advocates, propose weakening the link to observational tests, because they believe we have such a good theory that it must be right. But if a proposal is not testable, we certainly need to consider observationally testable alternatives.

The acceleration indicated by supernova data could possibly be due to small scale inhomogeneity that definitely exists, but may not be sufficiently significant to do the job. It could be due to large scale inhomogeneity that can probably do the job, but may not exist. Observational tests of the latter possibility are as important as pursuing the dark energy (exotic physics) option in a homogeneous universe. Theoretical prejudices as to the universe’s geometry, and our place in it, must bow to such observational tests.

We should stand firm and insist that genuine science is based on observational testing of plausible hypotheses. There is nothing wrong with physically motivated philosophical explanation: but it must be labeled for what it is. Overall: theory must be subject to experimental and/or observational test; this is the central feature of science. There is good progress in this respect as regards both dark matter and dark energy.

References

[1] Alexander, S., Biswas, T., Notari, A., and Vaid, D. 2007. ”Local void vs dark energy: confrontation with WMAP and Type IA supernovae” arXiv:0712.0370.

[2] Barrow, J. D. and Tipler, F. J. 1986. The Anthropic Cosmological Principle. Oxford: Oxford University Press.
[3] Biswas, T., Monsouri, R., and Notari, A. 2007. "Nonlinear Structure Formation and ‘Apparent’ Acceleration: an Investigation” JCAP 12, 017 [astro-ph/0606703].

[4] Buchert, T. 2008. "Dark Energy from structure: a status report”. Gen. Rel. Grav. 40, 467-527 [arXiv:0707.2153].

[5] Caldwell, R., and Stebbins, A. 2008. "A Test of the Copernican Principle", Phys Rev Lett. 100, 191302 [arXiv:0711.3459].

[6] Carr, B. 2007. *Universe or Multiverse?* Cambridge: Cambridge University Press.

[7] Célerier, M-N. 2007. "The Accelerated Expansion of the Universe Challenged by an Effect of the Inhomogeneities. A Review”. New Advances in Physics 1, 29 [astro-ph/0702416].

[8] Clarkson, C., Bassett, B., and Lu, T. H-C. 2008. "A general test of the Copernican Principle”. Phys Rev Lett, 101, 011301 [arXiv:0712.3457].

[9] Clifton, T., Ferreira, P. G., and Land, K. 2008. "Living in a Void: Testing the Copernican Principle with Distant Supernovae”, Phys Rev Lett, 101, 131302 [arXiv:0807.1443].

[10] Davies, P. C. W. 2004. "Multiverse cosmological models”, Mod. Phys. Lett. A19, 727.

[11] Deutsch, D. 1997. *The Fabric of reality: The science of parallel universes.* London: Allen Lane.

[12] Ellis, G. F. R. 2006. "Issues in the Philosophy of cosmology”. In *Handbook in Philosophy of Physics*, Ed J Butterfield and J Earman. Dordrecht: Elsevier, 1183 [astro-ph/0602280].

[13] Gardner, M. 2003. *Are Universes Thicker than Blackberries?.* New York: Norton.

[14] Goodman, J. 1995. "Geocentrism reexamined”, Phys Rev D 52, 1821 [astro-ph/9506068].

[15] Guth, A. H. 2001. "Eternal Inflation” Annals NY Acad Sci 950: 66 [astro-ph/0101507].

[16] Guth, A. H. 2007. "Eternal inflation and its implications”. J. Phys. A 40, 6811 [hep-th/0702178].

[17] Hilbert, D. 1964. "On the Infinite”. In *Philosophy of Mathematics*, ed P Benacerraf and H Putnam, Englewood Cliff, N. J.: Prentice Hall, 134.

[18] Ishak, M., Richardson, J., Whittington, D., and Garred, D. 2007. "Dark Energy or Apparent Acceleration Due to a Relativistic Cosmological Model More Complex than FLRW?” [arXiv:0708.2943].
[19] Lima, M.P., Vitenti, S., and Reboucas, M.J. 2008. "Energy conditions bounds and their confrontation with supernovae data." Phys Rev, D 77, 083518 [arXiv:0802.0706].

[20] Linde, A. D. 2003. "Inflation, Quantum Cosmology and the Anthropic Principle". In Science and Ultimate Reality: From Quantum to Cosmos, ed J D Barrow et al. Cambridge: Cambridge University Press, 426.

[21] Linde, A. D., Linde, D. A., and Mezhlumian, A. 1994. "From the Big Bang Theory to the Theory of a Stationary Universe", Phys. Rev. D 49, 1783 [gr-qc/9306035].

[22] Linde, A., Linde, D., and Mezhlumian, A. 1995. "Do We Live in the Center of the World?" Phys Lett B, 345, 203 [hep-th/9411111].

[23] Mustapha, N., Hellaby, C., and Ellis, G.F.R. 1999. "Large Scale Inhomogeneity vs Source Evolution: Can we Distinguish Them?" Mon Not Roy Ast Soc 292, 817 [gr-qc/9808079].

[24] Rees, M. J. 1999. Just Six Numbers: The Deep Forces that Shape the Universe. London: Weidenfeld and Nicholson.

[25] Rees, M. J. 2001. Our Cosmic habitat. Princeton: Princeton University Press.

[26] Susskind, L. 2005. The Cosmic Landscape: String Theory and the Illusion of Intelligent Design. New York: Little Brown.

[27] Tegmark, M. 2003. "Parallel Universes". Scientific American (May, 2003), 41.

[28] Tegmark, M. 2004. "Parallel Universes" in Science and Ultimate Reality: From Quantum to Cosmos, ed J D Barrow et al (Cambridge: Cambridge University Press), 459.

[29] Vanderveld, R.A., Flanagan, E.A. and Wasserman, I. 2006. "Mimicking Dark Energy with Lemaitre-Tolman-Bondi Models: Weak Central Singularities and Critical Points" Phys. Rev. D 74, 023506 [astro-ph/0602476].

[30] Vilenkin, A. 2006. Many Worlds in One: The Search for Other Universes, New York: Hill and Wang.

[31] Weinberg, S. W. 2000. “A priori distribution of the cosmological constant”. Phys. Rev. D 61, 103505 [astro-ph/0002387].

[32] Weinberg, S. W. 2007. “The cosmological constant problems” In Universe or Multiverse?, ed. B. Carr. Cambridge: Cambridge University Press [hep-th/0511037].
[33] Wiltshire, D. 2008. ”Dark energy without dark energy”. In Dark Matter in Astroparticle and Particle Physics: Proceedings of the 6th International Heidelberg Conference, eds H.V. Klapdor-Kleingrothaus and G.F. Lewis, World Scientific, Singapore, 565 [arXiv:0712.3954].

[34] Yoo, C-M., Kai, T., Nakao, K-I, 2008. ”Solving the Inverse Problem with Inhomogeneous Universes” [arXiv:0807.0952].