DETERMINATION OF THE QUANTUM PART OF THE TRULY NONPERTURBATIVE YANG-MILLS VACUUM ENERGY DENSITY IN THE COVARIANT GAUGE QCD

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Abstract

Using the effective potential approach for composite operators, we have formulated a general method of calculation of the truly nonperturbative Yang-Mills vacuum energy density in the covariant gauge QCD ground state quantum models. It is defined as an integration of the truly nonperturbative part of the full gluon propagator over the deep infrared region (soft momentum region). A nontrivial minimization procedure makes it possible to determine the value of the soft cutoff in terms of the corresponding nonperturbative scale parameter, which is inevitably present in any nonperturbative model for the full gluon propagator. We have shown for specific models of the full gluon propagator explicitly that the use of the infrared enhanced and finite gluon propagators lead to the vacuum energy density which is finite, always negative and it has no imaginary part (stable vacuum), while the infrared vanishing propagators lead to unstable vacuum and therefore they are physically unacceptable.

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I. INTRODUCTION

The nonperturbative QCD vacuum is a very complicated medium and its dynamical and topological complexity [1-3] means that its structure can be organized at various levels (classical, quantum). It can contain many different components and ingredients which contribute to the truly nonperturbative vacuum energy density (VED), one of the main characteristics of the QCD ground state. Many models of the QCD vacuum involve some extra classical color field configurations such as randomly oriented domains of constant color magnetic fields, background gauge fields, averaged over spin and color, stochastic colored background fields, etc. (see Refs. [1,4,5] and references therein). The most elaborated classical models are random and interacting instanton liquid models (RILM and IILM, respectively) of the QCD vacuum [6]. These models are based on the existence of the topologically nontrivial instanton-type fluctuations of gluon fields, which are nonperturbative, weak coupling solutions to the classical equations of motion in Euclidean space (see Ref. [6] and references therein).

Here we are going to discuss the quantum part of VED which is determined by the effective potential approach for composite operators [7-9]. It allows us to investigate the nonperturbative QCD vacuum, in particular Yang-Mills (YM) one, by substituting some physically well-justified Ansatz for the full gluon propagator since the exact solutions are not known. In the absence of external sources the effective potential is nothing but VED which is given in the form of the loop expansion where the number of the vacuum loops (consisting in general of the confining quarks and nonperturbative gluons properly regularized with the help of ghosts ) is equal to the power of the Plank constant, $\hbar$.

The full dynamical information of any quantum gauge field theory such as QCD is contained in the corresponding quantum equations of motion, the so-called Schwinger-Dyson (SD) equations for lower (propagators) and higher (vertices and kernels) Green’s functions. It is a highly nonlinear, strongly coupled system of four-dimensional integral equations for the above-mentioned quantities. The kernels of these integral equations are determined by the infinite series of the corresponding skeleton diagrams [10-12]. It is a general feature of nonlinear systems that the number of exact solutions (if any) can not be fixed a priori. Thus formally it may have several exact solutions. These equations should be also complemented by the corresponding Slavnov-Taylor (ST) identities [10-12] which in general relate the above mentioned lower and higher Green’s functions to each other. These identities are consequences of the exact gauge invariance and therefore "are exact constraints on any solution to QCD" [10]. Precisely this system of equations can serve as an adequate and effective tool for the nonperturbative approach to QCD [13,14].

Among the above-mentioned Green’s functions, the two-point Green’s function describing the full gluon propagator

$$iD_{\mu\nu}(q) = \left\{ T_{\mu\nu}(q) d(-q^2, \xi) + \xi L_{\mu\nu}(q) \right\} \frac{1}{q^2}, \quad (1.1)$$

has a central place [10-15]. Here $\xi$ is a gauge fixing parameter ($\xi = 0$, Landau gauge) and $T_{\mu\nu}(q) = g_{\mu\nu} - q_{\mu}q_{\nu}/q^2 = g_{\mu\nu} - L_{\mu\nu}(q)$. Evidently, its free perturbative (tree level) counterpart is obtained by simply setting the full gluon form factor $d(-q^2, \xi) = 1$ in Eq. (1.1). In particular, the solutions of the above-mentioned SD equation for the full gluon
propagator (1.1), are supposed to reflect the complexity of the quantum structure of the QCD ground state. As emphasized above, it is a highly nonlinear system of four-dimensional integrals containing many different, unknown in general, propagators, vertices and kernels [10-12]. Because of truncation schemes this system becomes the equation for the full gluon propagator only, but it remains nonlinear, nevertheless. Different truncations could lead to qualitatively different solutions, and the number of these solutions may be increased only. Moreover, to clearly distinguish between the exact or approximate solutions (if any), we do not know even the complete set of boundary conditions to attempt to uniquely fix solution of the truncated equation. We certainly know the boundary condition in the ultraviolet (UV) limit because of asymptotic freedom and certainly we do not know the corresponding boundary condition in the infrared (IR) because precisely of confinement (at this stage it is not even clear whether the two boundary conditions (in the UV and in the IR (if it can be established)) will be sufficient to completely fix the theory or not). Because of the above-discussed highly complicated mathematical structure of the SD equation for the full gluon propagator, there is no hope for exact solution(s). However, in any case the solutions of this equation can be distinguished by their behaviour in the deep IR limit (the UV limit is uniquely determined by asymptotic freedom), describing thus many different types of quantum excitations and fluctuations of gluon field configurations in the QCD vacuum. Evidently, not all of them reflect the real structure of the QCD vacuum.

The deep IR asymptotics of the full gluon propagator can be generally classified into the three different types: 1) the IR enhanced (IRE) or IR singular (IRS), 2) the IR finite (IRF) and 3) the IR vanishing (IRV) ones (for references see the corresponding sections below). Let us emphasize that any deviation in the behaviour of the full gluon propagator in the deep IR domain from the free perturbative one automatically assumes its dependence on a scale parameter (at least one) in general different from QCD asymptotic scale parameter, $\Lambda_{QCD}$. It can be considered as responsible for the nonperturbative dynamics (in the IR region) in the QCD vacuum models under consideration. If QCD itself is a confining theory, then such characteristic scale is very likely to exist. In what follows, let us denote it, say $\Lambda_{NP}$. This is very similar to asymptotic freedom which requires the above-mentioned asymptotic scale parameter associated with nontrivial perturbative dynamics in the UV region (scale violation). However, for calculation of the truly nonperturbative VED we need not exactly the deep IR asymptotics of the full gluon propagator, but rather its truly nonperturbative part, which vanishes when the above-mentioned nonperturbative scale parameter goes formally to zero, i.e., when only the perturbative phase survives. So we define the truly nonperturbative part of the full gluon form factor in Eq. (1.1) as follows:

$$d^{NP}(-q^2, \Lambda_{NP}) = d(-q^2, \Lambda_{NP}) - d(-q^2, \Lambda_{NP} = 0),$$

which, on one hand, uniquely determines the truly nonperturbative part of the full gluon propagator. On the other hand, the definition (1.2) explains the difference between the truly nonperturbative part $d^{NP}(-q^2)$ and the full gluon propagator $d(-q^2)$ which is nonperturbative itself. Let us note in advance, that in the realistic models for the full gluon propagator, the limit $\Lambda_{NP} \to 0$ is usually equivalent to the limit $-q^2 \to \infty$. In some cases, the model gluon propagator does not depend explicitly on the nonperturbative scale parameter (the dependence is hidden) then its behaviour at infinity should be subtracted. In the realistic models of the full gluon propagator its truly nonperturbative part usually coincides with its
deep IR asymptotics underlying thus the strong intrinsic influence of the IR properties of the theory on its nonperturbative dynamics.

It is well known, however, that VED in general is badly divergent in quantum field theory, in particularly QCD [16]. Thus the main problem is how to extract the truly nonperturbative VED which is relevant for the QCD vacuum quantum model under consideration. It should be finite, negative and it should have no imaginary part (stable vacuum). Why is it so important to calculate it from first principles? i.e., on the basis of some realistic Ansatz for the full gluon propagator only. As was emphasized above, this quantity is important in its own right as being nothing else but the bag constant (the so-called bag pressure) apart from the sign, by definition [16]. Through the trace anomaly relation [17] it helps in the correct estimation of such an important phenomenological nonperturbative parameter as the gluon condensate introduced in the QCD sum rules approach to resonance physics [18]. Furthermore, YM VED assists in the resolution of the $U(1)$ problem [19] via the Witten-Veneziano (WV) formula for the mass of the $\eta'$ meson [20]. The problem is that the topological susceptibility [19-23] needed for this purpose is determined by a two point correlation function from which perturbative contribution is already subtracted, by definition [20,23-25]. The same is valid for the above-mentioned bag constant which is much more general quantity than the string tension because it is relevant for light quarks as well. Thus to calculate correctly the truly nonperturbative VED means to understand correctly the structure of the QCD vacuum in different models.

We have already formulated a general method of calculation of the truly nonperturbative YM VED in the axial gauge QCD in Ref. [26], where the Abelian Higgs model [27] of the dual QCD [28] ground state was investigated. Moreover, we have calculated the truly nonperturbative VED (using particular method) in the covariant gauge QCD quantum vacuum model as well [29,30]. The main purpose of this paper (section II) is to formulate precisely a general method of calculation of the truly nonperturbative quantum part of YM VED in the covariant gauge QCD. In sections III, IV and V it is illustrated by by considering different covariant gauge QCD quantum models of its ground state by choosing three different types of the deep IR asymptotics of the full gluon propagator, IRE, IRF and IRV, respectively. The conclusions are presented in section VI.

II. THE TRULY NONPERTURBATIVE VACUUM ENERGY DENSITY

In this section we formulate a general method of numerical calculation of the quantum part of the truly nonperturbative YM VED in the covariant gauge QCD. Let us start from the gluon part of VED which to-leading order (log-loop level $\sim \hbar^{4\over 3}$) is given by the effective potential for composite operators [7] as follows:

$$V(D) = \frac{i}{2} \int\frac{d^nq}{(2\pi)^n} Tr\{\ln(D_0^{-1}D) - (D_0^{-1}D) + 1\}, \quad (2.1)$$

$^1$Next-to-leading and higher contributions (two and more vacuum loops) are numerically suppressed by one order of magnitude in powers of $\hbar$ at least and are left for consideration elsewhere.
where $D(q)$ is the full gluon propagator (1.1) and $D_0(q)$ is its free perturbative (tree level) counterpart. Here and below the traces over space-time and color group indices are understood. The effective potential is normalized as $V(D_0) = 0$, i.e., the free perturbative vacuum is normalized to zero. In order to evaluate the effective potential (2.1) we use the well-known expression

$$\text{Tr} \ln(D^{-1}D) = 8 \times \ln \det(D^{-1}D) = 8 \times 4 \ln \left[ \frac{3}{4} d(-q^2) + \frac{1}{4} \right].$$  \hspace{1cm} (2.2)$$

It becomes zero (in accordance with the above mentioned normalization condition) when the full gluon form factor is replaced by its free perturbative counterpart. This composition does not depend explicitly on a gauge choice. Going over to four-dimensional ($n = 4$) Euclidean space in Eq. (2.1), on account of (2.2), and evaluating some numerical factors, one obtains ($\epsilon_g = V(D)$)

$$\epsilon_g = -\frac{1}{\pi^2} \int dq^2 \ q^2 \left[ \ln[1 + 3d(q^2)] - \frac{3}{4} d(q^2) + a \right], \hspace{1cm} (2.3)$$

where constant $a = (3/4) - 2\ln 2 = -0.6363$ and the integration from zero to infinity is assumed. Substituting the definition (1.2) into the Eq. (2.3) and doing some trivial rearrangement, one obtains

$$\epsilon_g = -\frac{1}{\pi^2} \int dq^2 \ q^2 \left[ \ln[1 + 3d_{NP}(q^2, \Lambda_{NP})] - \frac{3}{4} d_{NP}(q^2, \Lambda_{NP}) \right] - \frac{1}{\pi^2} I_{PT}, \hspace{1cm} (2.4)$$

where we introduce the following notation

$$I_{PT} = \int dq^2 \ q^2 \left[ \ln[1 + 3d_{NP}(q^2, \Lambda_{NP})] - \frac{3}{4} d_{NP}(q^2, \Lambda_{NP}) \right]. \hspace{1cm} (2.5)$$

It contains the contribution which is mainly determined by the perturbative part of the full gluon propagator, $d(q^2, \Lambda_{NP} = 0)$. The constant $a$ also should be included since it comes from the normalization of the free perturbative vacuum to zero. If we separate the deep IR region from the perturbative one (which consists of the intermediate (IM) and UV regions since the IM region remains terra incognita in QCD) by introducing the so-called soft cutoff explicitly then we get

$$\epsilon_g = \epsilon_g - \frac{1}{\pi^2} \int_{q_0^2}^{\Lambda_{NP}} dq^2 \ q^2 \left[ \ln[1 + 3d_{NP}(q^2, \Lambda_{NP})] - \frac{3}{4} d_{NP}(q^2, \Lambda_{NP}) \right] - \frac{1}{\pi^2} (\tilde{I}_{PT} + I_{PT}), \hspace{1cm} (2.6)$$

where evidently

$$\tilde{I}_{PT} = \int_{q_0^2}^{\infty} dq^2 \ q^2 \left[ \ln[1 + 3d_{NP}(q^2, \Lambda_{NP})] - \frac{3}{4} d_{NP}(q^2, \Lambda_{NP}) \right]. \hspace{1cm} (2.7)$$

Thus the first integral represents contribution to YM VED which is determined by the truly nonperturbative piece of the full gluon propagator integrated over the deep IR region. In other words, just this term is the truly nonperturbative contribution to YM VED. This means that the two remaining terms in Eq. (2.6) should be subtracted by introducing corresponding counter terms into the effective potential. Thus in general the integral (2.5)
determining the contribution from the perturbative part of the full gluon propagator and the integral (2.7) determining the contribution from the perturbative region (IM plus UV) are of no importance for our present consideration. The above-mentioned necessary subtractions can be done in more sophisticated way by means of ghost degrees of freedom (see below).

The effective potential at the log-loop level for the ghost degrees of freedom is

\[ V(G) = -i \int \frac{d^np}{(2\pi)^n} Tr\{\ln(G_0^{-1}G) - (G_0^{-1}G) + 1\}, \]  

(2.8)

where \( G(p) \) is the full ghost propagator and \( G_0(p) \) is its free perturbative (tree level) counterpart. The effective potential \( V(G) \) is normalized as \( V(G_0) = 0 \). Evaluating formally the ghost term \( \epsilon_{gh} = V(G) \) in Eq. (2.8), we obtain \( \epsilon_{gh} = \pi^{-2}I_{gh} \). The integral \( I_{gh} \) depends on the ghost propagator, which remains arbitrary (unknown) within our approach. In principle, we have to sum up all contributions to obtain the total VED (the confining quark part of the vacuum energy density is not considered here). However, upon the substitution of definition (1.2) into the integral over the whole momentum range from zero to infinity (2.3), some terms appear there which may have unphysical singularities below the scale \( \Lambda_{QCD} \) (integral (2.5)). Thus the initial VED (2.3) is formal one, it suffers from unphysical singularities briefly mentioned above and it is badly divergent as well. In order to get physically meaningful expression, one have to subtract two integrals (2.5) and (2.7) from Eq. (2.3). We have done this subtraction with the help of ghost term by imposing the following condition \( \Delta = \tilde{I}_{PT} + I_{PT} - I_{gh} = 0 \). The nonperturbative gluon contribution to VED is determined by subtracting unwanted terms by means of the ghost contribution, i.e., defining \( \epsilon_g + \epsilon_{gh} = \epsilon_{YM} \) at \( \Delta = 0 \). Thus the truly nonperturbative YM VED becomes

\[ \epsilon_{YM} = \frac{1}{\pi^2} \int_0^{q_0^2} dq^2 q^2 \left[ \frac{3}{4} d^{NP}(q^2, \Lambda_{NP}) - \ln[1 + 3d^{NP}(q^2, \Lambda_{NP})] \right]. \]  

(2.9)

In many cases this subtraction is sufficient to obtain the expression for the truly nonperturbative YM VED. However, in some other cases the truly nonperturbative part of the full gluon propagator which enters Eq. (2.9) continues to suffer from unphysical singularities below the scale \( \Lambda_{QCD} \) (see discussion at the end of section V). As it was noticed, some additional terms should be included in our subtraction scheme in this case indicating that the chosen Ansatz for the full gluon propagator itself was not realistic.

A few general remarks are in order. In QCD nothing should explicitly depend on ghosts. By contributing into the closed loops only, the main purpose of their introduction is to cancel unphysical degrees of freedom of gauge bosons (maintaining thus the unitarity of \( S \)-matrix), for example to exclude the longitudinal components, the above mentioned unphysical singularities below the QCD scale, etc. This is the main reason why they are to be considered together with gluons always. In nonperturbative QCD in general and in our approach in particular the ghost propagator (or equivalently the ghost self-energy) still remains unknown (in this sense arbitrary) since the exact ghost-gluon vertex (which enters the corresponding SD equation) is not exactly known (in Refs. [31,32] some very specific truncation scheme is used in order to derive particular expression for this vertex). We know however, that the ghost propagator contribution to VED, regular or singular, should be combined with gluon contribution in order to cancel exactly the above-mentioned unphysical singularities of the gauge bosons which are inevitably present in any Ansatz for the full gluon propagator. In other words, if one knows the ghost propagator exactly then the above-mentioned
cancellation should proceed automatically (as usual in perturbative calculus if, of course, all calculations were correct). But if it is not known exactly (as usual in nonperturbative calculus) then one has to impose condition of cancellation as it was done in our case, \( \Delta = 0 \). Obviously, the above-mentioned condition of cancellation was imposed in the most general form. Instead of the introduction of some counter terms into the initial effective potential to cancel the most dangerous UV divergences presented in the integral (2.5), we have used the ghost term for this purpose as well. Thus our subtraction scheme is in agreement with general physical interpretation of ghosts to cancel all unphysical degrees of freedom of the gauge bosons [10,33].

The expression (2.9) is our definition of the truly nonperturbative YM VED as integrated out the truly nonperturbative part of the full gluon propagator over the deep IR region (soft momentum region, \( 0 \leq q^2 \leq q_0^2 \)). The soft cutoff \( q_0^2 \) (as a function of the nonperturbative scale) can be determined by the corresponding minimization procedure (see below).

A.

From this point it is convenient to factorize scale dependence of the truly nonperturbative YM VED (2.9). As was already emphasized above, \( d^{NP}(q^2) \) always contains at least one scale parameter (\( \Lambda_{NP} \)) responsible for the nonperturbative dynamics in the model under consideration. It is considered as free one within our general method, i.e., ”running” (when it formally goes to zero then the perturbative phase only survives in the model). Its numerical value (if any) will be used at final stage only to evaluate numerically the corresponding truly nonperturbative YM VED (if any). We can introduce dimensionless variables and parameters by using completely extra scale (which is always fixed in comparison with \( \Lambda_{NP} \)), for example flavorless QCD asymptotic scale parameter \( \Lambda_{YM} \) as follows:

\[
z = \frac{q^2}{\Lambda_{YM}^2}, \quad z_0 = \frac{q_0^2}{\Lambda_{YM}^2}, \quad b = \frac{\Lambda_{NP}^2}{\Lambda_{YM}^2}.
\] (2.10)

Here \( z_0 \) is a corresponding dimensionless soft cutoff while the parameter \( b \) has a very clear physical meaning. It measures the ratio between nonperturbative dynamics, symbolized by \( \Lambda_{NP}^2 \) and nontrivial perturbative dynamics (violation of scale, asymptotic freedom) symbolized by \( \Lambda_{YM}^2 \). When it is zero only perturbative phase remains in the model. In this case, the gluon form factor obviously becomes a function of \( z \) and \( b \), i.e., \( d^{NP}(q^2) = d^{NP}(z, b) \) and the truly nonperturbative VED (2.9) is \( (\epsilon_{YM} \equiv \epsilon_{YM}(z_0, b)) \)

\[
\Omega_g(z_0, b) = \frac{1}{\Lambda_{YM}^4} \epsilon_{YM}(z_0, b),
\] (2.11)

where the gluon effective potential at a fixed scale, \( \Lambda_{YM} \), [26,29,34] is introduced

\[
\Omega_g \equiv \Omega_g(z_0, b) = \frac{1}{\pi^2} \int_0^{z_0} dz \ z \left[ \frac{3}{4} d^{NP}(z, b) - \ln[1 + 3 d^{NP}(z, b)] \right].
\] (2.12)

This expression precisely allows us to investigate the dynamical structure of the YM vacuum. It is free of scale dependence since it has been already factorized in Eq. (2.11). It depends only on \( z_0 \) and \( b \) and a minimization procedure with respect to \( b \), \( \partial \Omega_g(z_0, b)/\partial b = 0 \), (usually
after integrated out in Eq. (2.12)) can provide a self-consistent relation between \( z_0 \) and \( b \), that is we get \( q_0 \) as a function of \( \Lambda_{NP} \). Let us note in advance that the final numerical results will depend on \( \Lambda_{NP} \) only as it should be for the nonperturbative part of YM VED (see sections III and IV below). Obviously, the minimization with respect to \( z_0 \) leads to trivial zero. In principle, through the relation \( \Lambda_{YM}^4 = q_0^4 z_0^{-2} \), it is possible to fix soft cutoff \( q_0 \) itself, but this is not the case indeed since then \( z_0 \) can not be varied.

\[ \text{B.} \]

On the other hand, the scale dependence can be factorized as follows:

\[ z = \frac{q^2}{\Lambda_{NP}^2}, \quad z_0 = \frac{q_0^2}{\Lambda_{NP}^2}, \quad (2.13) \]

i.e., \( b = 1 \). For simplicity (but not loosing generality) we use the same notations for the dimensionless set of variables and parameters as in Eq. (2.10). In this case, the gluon form factor obviously becomes a function of \( z \) only, \( d^{NP}(q^2) = d^{NP}(z) \) and the truly nonperturbative YM VED (2.9) becomes

\[ \epsilon_{YM}(z_0) = \frac{1}{\pi^2} q_0^3 z_0^{-2} \int_0^{z_0} dz z \left[ \frac{3}{4} d^{NP}(z) - \ln[1 + 3d^{NP}(z)] \right]. \quad (2.14) \]

Evidently, to fix the scale is possible in the two different ways. In principle, we can fix \( \Lambda_{NP} \) itself, i.e., introducing

\[ \bar{\Omega}_g(z_0) = \frac{1}{\Lambda_{NP}^4} \epsilon_{YM}(z_0) = \frac{1}{\pi^2} \int_0^{z_0} dz z \left[ \frac{3}{4} d^{NP}(z) - \ln[1 + 3d^{NP}(z)] \right]. \quad (2.15) \]

However, the minimization procedure again leads to the trivial zero, which shows that this scale can not be fixed.

In contrast to the previous case, let us fix the soft cutoff itself, i.e., setting [26,29,30]

\[ \bar{\Omega}_g(z_0) = \frac{1}{q_0^2} \epsilon_{YM}(z_0) = \frac{1}{\pi^2} z_0^{-2} \int_0^{z_0} dz z \left[ \frac{3}{4} d^{NP}(z) - \ln[1 + 3d^{NP}(z)] \right]. \quad (2.16) \]

In this case the perturbative phase is recovered in the \( z_0 \to \infty (\Lambda_{NP} \to 0) \) limit. Now the minimization procedure with respect to \( z_0 \) is nontrivial. Indeed, \( \partial \bar{\Omega}_g(z_0) / \partial z_0 = 0 \), yields the following "stationary" condition

\[ \int_0^{z_0} dz z \left[ \frac{3}{4} d^{NP}(z) - \ln[1 + 3d^{NP}(z)] \right] = \frac{1}{2} \frac{3}{4} \int_0^{d^{NP}(z_0)} d^{NP}(z_0) - \ln[1 + 3d^{NP}(z_0)] \right], \quad (2.17) \]

the solutions of which (if any) allow one to find \( q_0 \) as a function of \( \Lambda_{NP} \). On account of this "stationary" condition, the effective potential (2.16) itself becomes simpler for numerical calculations, namely

\[ \bar{\Omega}_g(z_0^{st}) = \frac{1}{2\pi^2} \left[ \frac{3}{4} d^{NP}(z_0^{st}) - \ln[1 + 3d^{NP}(z_0^{st})] \right]. \quad (2.18) \]

where \( z_0^{st} \) is a solution (if any) of the "stationary" condition (2.17) and corresponds to the minimum(s) (if any) of the effective potential (2.16). In the next sections, we illustrate how this method works by considering some quantum models of the covariant gauge QCD ground state explicitly.

8
III. THE IRE GLUON PROPAGATOR. ZME QUANTUM MODEL

Today there are no doubts left that the dynamical mechanisms of the important nonperturbative quantum phenomena such as quark confinement and dynamical (or equivalently spontaneous) chiral symmetry breaking (DCSB) are closely related to the complicated topologically nontrivial structure of the QCD vacuum [1-4,10]. On the other hand, it also becomes clear that the nonperturbative IR dynamical singularities, closely related to the nontrivial vacuum structure, play an important role in the large distance behaviour of QCD [35,36]. For this reason, any correct nonperturbative model of quark confinement and DCSB necessarily turns out to be a model of the true QCD vacuum and the other way around.

Our model of the true QCD ground state is based on the existence and importance of such kind of the nonperturbative, quantum excitations of the gluon field configurations (due to self-interaction of massless gluons only, i.e., without explicit involving some extra degrees of freedom) which can be effectively correctly described by the $q^{-4}$ behaviour of the full gluon propagator in the deep IR domain (at small $q^2$) [29,30]. These excitations are topologically nontrivial also since they lead to the nontrivial YM VED (see below). Thus our main definition (1.2) becomes

\[ d^{NP}(-q^2, \Lambda_{NP}) = d(-q^2, \Lambda_{NP}) - d(-q^2, \Lambda_{NP} = 0) = \frac{\Lambda^2_{NP}}{(-q^2)}. \] (3.1)

In the above-mentioned papers [29,30] the nonperturbative scale was denoted as $\bar{\mu}$, i.e., $\bar{\mu} \equiv \Lambda_{NP}$. In this way we obtain the generally accepted form of the deep IR singular asymptotics for the full gluon propagator (for some references see below)

\[ D_{\mu\nu}(q) \sim (q^2)^{-2}, \quad q^2 \to 0, \] (3.2)

which may be referred equivalently to as the strong coupling regime [10]. It describes the zero momentum modes enhancement (ZMME) dynamical effect in QCD at large distances. We prefer to use simply ZME (zero modes enhancement) since we work always in momentum space. This is our primary dynamical assumption in this section. The main problem due to this strong singularity is its correct treatment by the dimensional regularization method [37] within the distribution theory [38], which was one of highlights of our previous publications [29,30] (see also Ref. [39]). There exist many arguments in favor of this behaviour:

a). Such singular behaviour of the full gluon propagator in the IR domain leads to the area law for static quarks (indicative of confinement) within the Wilson loop approach [40].

b). The cluster property of the Wightman functions in QCD fails and this allows such singular behaviour like (3.2) for the full gluon propagator in the deep IR domain [41].

c). After the pioneering papers of Mandelstam in the covariant (Landau) gauge [42] and Baker, Ball and Zachariasen in the axial gauge [43], the consistency of the singular asymptotics (3.2) with direct solution of the SD equation for the full gluon propagator in the IR domain was repeatedly confirmed (see for example Refs. [13,14,44,45] and references therein).

d). Moreover, let us underline that without this component in the decomposition of the full gluon propagator in continuum theory it is impossible to "see" linearly rising potential between heavy quarks by lattice QCD simulations [46] not involving some extra (besides gluons and quarks) degrees of freedom. This should be considered as a strong lattice evidence...
(though not direct) of the existence and importance of $q^{-4}$-type excitations of gluon field configurations in the QCD vacuum. There exists also direct lattice evidence that the zero modes are enhanced in the full gluon propagator indeed [47].

e). Within the distribution theory [38] the structure of the nonperturbative IR singularities in four-dimensional Euclidean QCD is the same as in two-dimensional QCD, which confines quarks at least in the large $N_c$ limit [48]. In this connection, let us note that $q^{-4}$ IR singularity is the simplest nonperturbative power singularity in four-dimensional QCD as well as $q^{-2}$ IR singularity is the simplest nonperturbative power singularity in two-dimensional QCD. The QCD vacuum is much more complicated medium than its two-dimensional model, nevertheless, the above-mentioned analogy is promising even in the case of the nonperturbative dynamics of light quarks.

f). Some classical models of the QCD vacuum also invoke $q^{-4}$ behaviour of the gluon fields in the IR domain. For example, it appears in the QCD vacuum as a condensation of the color-magnetic monopoles (QCD vacuum is a chromomagnetic superconductor) proposed by Nambu, Mandelstam and ’t Hooft and developed by Nair and Rosenzweig (see Ref. [49] and references therein. For recent developments in this model see Di Giacomo’s contribution in Ref. [1]) as well as in the classical mechanism of the confining medium [50] and in effective theory for the QCD vacuum proposed in Ref. [51].

g). It is also required to derive the heavy quark potential within the recently proposed exact renormalization group flow equations approach [52].

h). It has been shown in our papers that the singular behaviour (3.2) is related directly to light quarks confinement and DCSB [29,30]. Moreover, a very good agreement has been obtained with the phenomenological values of the topological susceptibility, the mass of the $\eta'$ meson and the gluon condensate [21,22].

Thus we consider our main Ansatz (3.1), (3.2) as physically well-motivated. Let us emphasize that $d^{NP}(-q^2, \xi) = \Lambda_{NP}^2 / (-q^2)$ is the truly nonperturbative part of the full gluon propagator since it vanishes in the perturbative limit ($\Lambda_{NP}^2 \to 0$, when the perturbative phase survives only) and simultaneously it correctly reproduces the deep IR asymptotics of the full gluon propagator, i.e., $d^{NP}(-q^2)$ coincides with $d^{IR}(-q^2)$.

A.

The truly nonperturbative YM VED is given now by Eq. (2.9) with $d^{NP}(q^2) = \Lambda_{NP}^2 / q^2$ in Euclidean space. Let us first introduce the A-type set of dimensionless variables (2.10). Then $d^{NP}(q^2)$ becomes $d^{NP}(z, b) = b / z$. Performing almost trivial integration in the effective potential at a fixed scale (2.12), one obtains

$$\Omega_g(z_0, b) = \frac{1}{2\pi^2} \left[ 9b^2 \ln \left( 1 + \frac{z_0}{3b} \right) - \frac{3z_0}{2} b - z_0^2 \ln \left( 1 + \frac{3b}{z_0} \right) \right].$$

(3.3)

It is easy to show that as a function of $b$, the effective potential (3.3) linearly approaches zero from below and it diverges also linearly at infinity while as a function of $z_0$ itself it approaches zero from above and also diverges as $\sim -z_0$ at infinity. Thus as a function of $b$ it has a local minimum (relating $b$ to $z_0$) at which the truly nonperturbative YM VED will be always finite and negative. The minimization procedure with respect to $b$, $\partial \Omega_g(z_0; b) / \partial b = 0$,
yields the following "stationary" condition, \( \nu = 4 \ln(1 + (\nu/3)) \), where \( \nu = z_0/b \). Its solution is \( \nu^{\min} = 2.2 \). Using this "stationary" condition, the effective potential (3.3) can be written down as follows:

\[
\Omega_g(\nu^{\min}, b) = \frac{b^2 \nu^{\min}}{2\pi^2} \left[ \frac{3}{4} - \frac{3}{\nu^{\min}} \ln \left( 1 + \frac{3}{\nu^{\min}} \right) \right] = -0.1273b^2,
\]

so the truly nonperturbative YM VED (2.11) becomes

\[
\epsilon_{YM} = -0.1273 \times \Lambda_{NP}^4,
\]

where the relation \( \Lambda_{NP}^4 = b^2 \Lambda_{YM}^4 \) has been already used. Determined in this way, it is always finite (since characteristic scale of our model \( \Lambda_{NP} \) is finite, evidently it can not be arbitrary large), automatically negative (as it should be for the truly nonperturbative energy) and it has no imaginary part (stable vacuum). Obviously the characteristic scale of our model \( \Lambda_{NP} \) can not be determined within the YM theory alone. Its numerical value should be taken from good physical observable in full QCD by implementing the physically well-motivated scale-setting scheme. Precisely this has been done in our papers [29,30] where the nonperturbative VED was numerically evaluated from first principles. Moreover, in recent publications [21,22] it is shown that our numerical results are of the necessary order of magnitude in order to nicely saturate the large mass of \( \eta' \) meson in the chiral limit as well as the phenomenological value of the topological susceptibility. Thus the existence of the nontrivial VED in ZME quantum model, which agrees well with QCD topology, is one more serious argument in its favor. It is worthwhile to present numerical value for the soft cutoff in terms of \( \Lambda_{NP} \), namely \( q_0 = 1.48324 \Lambda_{NP} \). It follows from the solution of the "stationary" condition, of course.

\[\text{B.}\]

It is instructive to calculate the truly nonperturbative YM VED by choosing the B-type set of dimensionless variables (2.13). Then \( d^{NP}(q^2) = \Lambda_{NP}^2/q^2 \) becomes \( d^{NP}(z) = 1/z \). Performing almost trivial integration in the effective potential at a fixed scale (2.16) in this case, one obtains

\[
\bar{\Omega}_g(z_0) = \frac{1}{2\pi^2} z_0^{-2} \left[ 9 \ln \left( 1 + \frac{z_0}{3} \right) - \frac{3}{2} z_0 - z_0^2 \ln \left( 1 + \frac{3}{z_0} \right) \right].
\]

(3.6)

It is easy to show now that as a function of \( z_0 \), the effective potential (3.7) diverges as \( \sim z_0^{-1} \) at small \( z_0 \) and converges as \( \sim -z_0^{-1} \) at infinity (perturbative limit), see Fig. 1. Thus as a function of \( z_0 \) it has a local minimum at \( z_0 = 4 \ln(1 + (z_0/3)) \), the so-called "stationary" condition in this case. Its solution again is \( z_0^{\min} = 2.2 \). At "stationary" state the effective potential (3.6) can be written down as follows:

\[
\bar{\Omega}_g(z_0^{\min}) = \frac{1}{2\pi^2} \left[ \frac{3}{4} (z_0^{\min})^{-1} - \ln \left( 1 + \frac{3}{z_0^{\min}} \right) \right] = -0.0263,
\]

(3.7)

so the truly nonperturbative YM VED (2.16) becomes
\[ \epsilon_{YM} = -0.0263 q_0^4 = -0.1273 \times \Lambda_{NP}^4, \]  
(3.8)

where the relation \( q_0^4 = (z_0^{\text{min}})^2 \Lambda_{NP}^4 \) has been already used. Thus we have explicitly demonstrated that truly nonperturbative YM VED does not indeed depend on how one introduces dimensionless variables into the effective potential, i.e., \( \epsilon_{YM} = \Lambda_{NP}^4 \Omega_g(\nu^{\text{min}}, b) = q_0^4 \Omega_g(z_0^{\text{min}}) = -0.1273 \Lambda_{NP}^4 \). In some cases, the B-type calculation is preferable. For example, to calculate confining quark contribution into the total VED is much easier using precisely this set of the dimensionless variables (see our papers [29,30] and next section as well).
IV. THE IRF GLUON PROPAGATOR

Let us consider now a possible IRF behaviour of the full gluon propagator (in the Landau gauge) in the deep IR domain, which was suggested by recent lattice calculations in Ref. [53]. The main definition (1.2) in this case becomes

\[ d^{NP}(-q^2, M) = d(-q^2, M) - d(-q^2, M = 0) = ZAM^{2\alpha}(-q^2) \left(\frac{-q^2 + M^2}{M^2}\right)^{1+\alpha}. \] (4.1)

Here \( M \) is the mass scale parameter responsible for the nonperturbative dynamics in this model, i.e., \( M = \Lambda_{NP} \) in our notation. When the parameter \( M \) formally goes to zero, the perturbative phase only remains in this model. Again as in previous case, the truly nonperturbative part vanishes in the perturbative limit (\( M \to 0 \)) and it reproduces the IR asymptotics of the full gluon propagator correctly as well. The best estimates for the parameters \( M \) and \( A \) are \( M = (1020 \pm 100 \pm 25) \text{ MeV} \) and \( A = (9.8 + 0.1 - 0.9) \). As it was emphasized above, the numerical value of the parameter \( M \) will be used only at final stage in order to estimate numerically the truly nonperturbative YM VED in this model. The exponent in general is \( \alpha = 2 + \delta \), where \( \delta > 0 \) and small, while \( Z \approx 1.2 \) is the renormalization constant.
In this case, it is convenient to choose the B-type set of variables and parameters (2.13). Then $d^{NP}(q^2)$ in Euclidean space becomes
\[ d^{NP}(z) = \frac{a_1 z}{(1 + z)^{1+\alpha}}, \quad (4.2) \]
where the parameter $a_1 = ZA = 11.76$ is fixed. Substituting this into the effective potential (2.16), one obtains
\[ \Omega_g(z_0; a_1) = \frac{1}{q_0} \epsilon_{YM} = -\frac{1}{\pi^2} \times z_0^{-2} \left\{ I_1(z_0; a_1) - I_2(z_0; a_1) \right\}, \quad (4.3) \]
where integrals are given as follows:
\[
I_1(z_0; a_1) = \int_0^{z_0} dz z \ln \left( 1 + \frac{3a_1 z}{(1 + z)^{1+\alpha}} \right),
\]
\[
I_2(z_0; a_1) = \frac{3a_1}{4} \int_0^{z_0} dz z \frac{z}{(1 + z)^{1+\alpha}}.
\]
(4.4)

The asymptotic behaviour of the effective potential (4.3) depends on the asymptotic properties of the integral $I_1(z_0; a_1)$ since the integral $I_2(z_0; a_1)$ in Eq. (4.4) can be taken explicitly, namely (in what follows in this section, $\alpha = 2$)
\[
I_2(z_0; a_1) = \frac{3a_1}{4} \left( \ln(1 + z_0) + 2[(1 + z_0)^{-1} - 1] - \frac{1}{2}[(1 + z_0)^{-2} - 1] \right). \quad (4.5)
\]

From these expressions it is almost obvious that asymptotics of the effective potential (4.3) at $z_0 \to 0$, $\infty$ to-leading order can be easily evaluated analytically. The effective potential (4.3) as a function of the soft cutoff $z_0$ has two local minimums (see below). The corresponding "stationary" condition can be evaluated as follows:
\[
[I_1(z_0; a_1) - I_2(z_0; a_1)] = \frac{1}{2} z_0^2 \left\{ \ln \left( 1 + \frac{3a_1 z_0}{(1 + z_0)^3} \right) - \frac{3a_1 z_0}{4(1 + z_0)^3} \right\}.
\]
(4.6)

Using this "stationary" condition, the effective potential (4.3) at "stationary" state becomes
\[
\Omega_g(z_0^{st}; a_1) = -\frac{1}{2\pi^2} \left\{ \ln \left( 1 + \frac{3a_1 z_0^{st}}{(1 + z_0^{st})^3} \right) - \frac{3a_1 z_0^{st}}{4(1 + z_0^{st})^3} \right\},
\]
(4.7)

where $z_0^{st}$ is a solution(s) to the "stationary" condition (4.6). The two solutions of the "stationary" condition (4.6) corresponding to the two local minimums are $z_0^{st} = 0.19$ and $z_0^{st} = 2.37$ with almost equal numerical values for the corresponding effective potentials at "stationary" states, namely $\Omega_g(0.19; a_1) = -0.0309$ and $\Omega_g(2.37; a_1) = -0.0310$, respectively. However, the numerical values of the nonperturbative YM VED (4.3) are drastically different,
\[
\epsilon_{YM}(0.19) = -0.0309 q_0^4(0.19) = -0.00123 \times M^4 \quad (4.8)
\]
and
\[
\epsilon_{YM}(2.37) = -0.0310 q_0^4(2.37) = -0.174 \times M^4, \quad (4.9)
\]
where the relation $q_0^4 = (z_0^{min})^2 M^4$ and the corresponding values of $z_0^{min}(\equiv z_0^{st})$ were applied. How to distinguish between the two solutions for the truly nonperturbative YM VED (4.8) and (4.9)? This question is discussed in the following.
A. Discussion

In the first case, on account of the numerical value of the nonperturbative scale \( M \approx 1 \, \text{GeV} \), Eq. (4.8) numerically becomes

\[
\epsilon_{YM}(0.19) = -0.00123 \, \text{GeV}^4. \tag{4.10}
\]

It is the same order of magnitude as VED due to instantons [22]. Thus summing up this and instantons with ZME values, one obtains a fair agreement with chiral QCD topology [20]. Also the soft cutoff in this case is \( q_0 \approx 0.463M \approx 463 \, \text{MeV} \). This is quite reasonable value for the deep IR region (in continuum theory) where the smooth-type behaviour of the full gluon propagator effectively takes place.

In the second case, on account of the numerical value of the nonperturbative scale \( M \approx 1 \, \text{GeV} \), Eq. (4.9) numerically becomes

\[
\epsilon_{YM}(2.37) = -0.174 \, \text{GeV}^4. \tag{4.11}
\]

In Refs. [21,22] an analytical formalism has been developed which allows one to calculate the topological susceptibility as a function of the truly nonperturbative YM VED. The corresponding expression is

\[
\chi_t = -\left( \frac{4\xi}{3} \right)^2 \epsilon_{YM}, \tag{4.12}
\]

where parameter \( \xi \) has two different values, namely \( \xi_{NSVZ} = 2/11 \) and \( \xi_{HZ} = 4/33 \) (see Ref. [22]). Evaluating (4.12) numerically, on account of (4.11), one obtains, \( \chi_t^{NSVZ} = (550.8 \, \text{MeV})^4 \) and \( \chi_t^{HZ} = (259.6 \, \text{MeV})^4 \), while its phenomenological value is, \( \chi_t^{phen} = (180.36 \, \text{MeV})^4 \). Thus, Eq. (4.11) substantially overestimates the phenomenological value of the topological susceptibility (in both modes) and consequently the mass of \( \eta' \) meson in the chiral limit, indeed. The soft cutoff in this case is \( q_0 \approx 1.54M \approx 1.54 \, \text{GeV} \). It is also hard to imagine that the deep IR region (in continuum theory) can be effectively extended up to \( \approx 1.54 \, \text{GeV} \) especially for the smooth-type behaviour of the full gluon propagator there. The continuum limit of the scale parameter \( M \) is not known, so its realistic numerical value still remains to be well-established, and so does the selection from solutions, Eqs. (4.8) and (4.9). Let us note, that in accordance with the general scheme of our method we distinguish the nonperturbative scale of this model from the perturbative one but for simplicity we retain the same notation. Evidently, one will obtain the same numerical results for the truly nonperturbative YM VED by choosing the set of variables of A type.

V. THE IRV GLUON PROPAGATOR

The IRV full gluon propagator is represented by the so-called Zwanziger-Stingle (ZS) formula [54,55]

\[
d(-q^2) = \frac{(-q^2)^2}{(-q^2)^2 + \mu^4}, \tag{5.1}
\]
in the whole range, where $\mu^4$ is again the mass scale parameter responsible for the non-perturbative dynamics in this model, i.e., $\mu \equiv \Lambda_{NP}$, in our notation. When it is zero then the ZS gluon propagator (5.1) becomes free perturbative on $e$, indeed. Though the full gluon propagator (5.1) is nonperturbative itself, however its truly nonperturbative part is determined by the subtraction (1.2), i.e,

$$d^{NP}(q^2) = d(q^2, \mu^4) - d(q^2, \mu^4 = 0) = -\frac{\mu^4}{(-q^2)^2 + \mu^4}. \quad (5.2)$$

Since this expression is rather simple, it will be instructive to perform calculations in both schemes, A (2.10) and B (2.13). So let us start from A scheme.

A.

Within the A-type set of variables (2.10), $d^{NP}(q^2)$ from Eq. (5.2) becomes $d^{NP}(z, b) = -(b^2/b^2 + z^2)$ (Euclidean space). After the integration over four-dimensional Euclidean space in Eq. (2.12), one obtains

$$\Omega_g(z_0, b) = \frac{1}{8\pi^2} \left\{ -8b^2 \ln 2b^2 + 8b^2 \ln(2b^2 - z_0^2) + (b^2 + 4z_0^2) \ln(b^2 + z_0^2) - 4z_0^2 \ln(z_0^2 - 2b^2) - b^2 \ln b^2 \right\}. \quad (5.3)$$

From this expression it follows obviously that the effective potential (5.3) at any finite relation between the soft cutoff $z_0$ and parameter $b$ will always contain the imaginary part, which is a direct manifestation of the vacuum instability [56] in this model. Its asymptotics at $b \to 0, \infty$ to-leading order can be easily evaluated analytically. Omitting all intermediate calculations, one finally obtains $\Omega_g(z_0, b) \sim_{b \to 0} -(9/8\pi^2)b^2 \ln b^2$ and $\Omega_g(z_0, b) \sim_{b \to \infty} -(1/8\pi^2)[3 + 4 \ln(-2)]z_0^2$, confirming the vacuum instability. Let us also consider the corresponding formal "stationary" condition, $\partial\Omega_g(z_0, b)/\partial b = 0$, which yields

$$3t_0^2 + (1 + t_0^2) \ln(1 + t_0^2) + 8(1 + t_0^2) \ln(1 - \frac{t_0^2}{2}) = 0, \quad (5.4)$$

where $t_0^2 = (z_0^2/b^2)$. It has only trivial solution $t_0 = z_0 = 0$.

Thus the vacuum of this model is unstable, indeed, so it has no relation to quark confinement and DCSB. Our conclusion is in full agreement with conclusion driven in Ref. [57]. The particular-type expressions for the dressed-quark-gluon vertex free from ghost contributions were used in their investigation. Our result, however, is a general one since it does not require the particular choice of the dressed-quark-gluon vertex.

B.

Within the B-type set of variables (2.13), $d^{NP}(q^2)$ from Eq. (5.2) becomes $d^{NP}(z) = -(1/1 + z^2)$ (Euclidean space). After almost trivial integration over four-dimensional Euclidean space in Eq. (2.16), one obtains
\[ \bar{\Omega}_g(z_0) = \frac{1}{8\pi^2} z_0^{-2} \left\{ -8 \ln 2 + 8 \ln(2 - z_0^2) + (1 + 4z_0^2) \ln(1 + z_0^2) - 4z_0^2 \ln(z_0^2 - 2) \right\}. \]  

(5.5)

From this expression it obviously follows that the effective potential at any finite value of the soft cutoff \( z_0 \) will always contain the imaginary part, which is a direct manifestation of the vacuum instability [56] as it was indicated above. Its asymptotics at \( z_0 \to 0, \infty \) to-leading order can be easily evaluated analytically. Omitting all intermediate calculations, one finally obtains, \( \bar{\Omega}_g(z_0) \sim z_0^{-2} \) and \( \bar{\Omega}_g(z_0) \sim 3 + 4 \ln(-2) \), so the vacuum of this model is unstable, indeed. In order to confirm this, let us consider the corresponding formal "stationary" condition which is

\[ 3z_0^2 + (1 + z_0^2) \ln(1 + z_0^2) + 8(1 + z_0^2) \ln(1 - \frac{z_0^2}{2}) = 0. \]  

(5.6)

It has only trivial solution \( z_0 = 0 \).

In Ref. [57] it was proposed the modification of the ZS propagator (5.1) which took into the consideration the renormgroup improvements to-leading order for the running coupling constant in the UV region, namely

\[ d(-q^2) = \frac{(-q^2)^2}{(-q^2)^2 + \mu^4} \times \frac{const}{\ln\left(\tau + \frac{q^2}{\Lambda_{QCD}^2}\right)}. \]  

(5.7)

Here \( const \) obviously depends on the first coefficient of the \( \beta \)-function and an unphysical parameter \( \tau \) is introduced in order to regulate unphysical singularity – Landau pole – at \( q^2 = \Lambda_{QCD}^2 \) (Euclidean space). The truly nonperturbative part now is

\[ d^{NP}(q^2) = d(q^2, \mu^4) - d(q^2, \mu^4 = 0) = -\frac{\mu^4}{(-q^2)^2 + \mu^4} \times \frac{const}{\ln\left(\tau + \frac{q^2}{\Lambda_{QCD}^2}\right)}. \]  

(5.8)

However, it is possible to show that YM VED continues to contain imaginary part in this case as well. It is worth noting, that in derivation of the corresponding expression for YM VED (2.9) all terms depending in general on some unphysical parameters (in this case \( \tau \)) should be additionally subtracted by means of ghosts (as it was mentioned above in Section II just after Eq. (2.9)). Concluding, let us note that neither (5.2) nor (5.8) coincides with deep IR asymptotics of the corresponding full gluon propagators (5.1) and (5.7).

VI. CONCLUSIONS

In summary, we have formulated a general method how to numerically calculate the quantum part of the truly nonperturbative YM VED (the bag constant, apart from the sign, by definition) in the covariant gauge QCD quantum models of its ground state using the effective potential approach for composite operators. It is defined as integrated out the truly nonperturbative part of the full gluon propagator over the deep IR region (soft momentum region), Eq. (2.9). The nontrivial minimization procedure makes it possible to determine the value of the soft cutoff as a function of the corresponding nonperturbative scale parameter which is inevitably present in any nonperturbative full gluon propagator model.
If the chosen Ansatz for the full gluon propagator is realistic one, then our general method gives the truly nonperturbative YM VED which is always finite, automatically negative and it has no imaginary part (stable vacuum) (sections III and IV). Its numerical value does not, of course, depend on how one introduces the scale dependence by choosing different scale parameters as it was described above in subsections A and B of section II, i.e., both set of variables lead to the same numerical value of the truly nonperturbative YM VED.

From comparison of Eqs. (2.3) and (2.9), a prescription can be derived how one can obtain the relevant expression for the truly nonperturbative YM VED. For this purpose the full gluon propagator in Eq. (2.3) should be replaced by its truly nonperturbative part in accordance with Eq. (1.2). The constant $a$ should be omitted (it was already explained why) and the soft cutoff $q_0^2$ on the upper limit should be introduced. Now it looks like the UV cutoff. Nevertheless let us underline once more that it separates the deep IR region from the perturbative one, which includes the IM region as well. It has a clear physical meaning as determining the range where the deep IR asymptotics of the full gluon propagator is valid. By definition it can not be arbitrary large as the UV cutoff is. As far as one chooses Ansatz for the full gluon propagator, the separation ”NP versus PT” is exact because of the definition (1.2). The separation ”soft versus hard” momenta is also exact because of the above-mentioned minimization procedure. Thus the proposed determination of the truly nonperturbative YM VED is uniquely defined. The nontrivial minimization procedure can be done only by two ways. First, to minimize the effective potential at a fixed scale (2.11), (2.12) with respect to the physically meaningful parameter. When it is zero, the perturbative phase only survives in all models of the QCD ground state. Equivalently, we can minimize the auxiliary effective potential (2.16) as a function of the soft cutoff $z_0$ itself. When it goes to infinity then again the perturbative phase survives only. On the other hand, both effective potentials (2.12) and (2.16) should go to zero in the perturbative limit since the perturbative contributions have been already subtracted from the very beginning (see section 2). As was underlined above, both methods lead to the same numerical value for the truly nonperturbative YM VED.

We have shown explicitly that the IRE gluon propagator (3.2) as well as IRF (4.1) correspond to nontrivial VED which is always finite, negative and it has no imaginary part (stable vacuum). In this way they reflect some physical types of excitations of gluon field configurations in the QCD vacuum. At the same time, the IRV gluon propagators (5.1) and (5.2) lead to unstable vacuum and therefore are physically impossible. However, these results are by no means general. For example, to come to the same conclusion for the IRV gluon propagator obtained and investigated in Refs. [31,32] it is necessary to proceed along with lines of our method. Thus the proposed method is precisely general one and each particular model for the full gluon propagator should be separately analyzed within its framework. However, it seems to us that the unstable vacuum is a fundamental defect of all vacuum models based on the IRV-type behaviour of the full gluon propagator. It is worthwhile also noting that, in contrast to IRE gluon propagator, the smooth behaviour of the full gluon propagator in the IR domain is hard to relate to quark confinement and DCSB.

Thus our method can serve as a test of any different QCD vacuum models (quantum or classical) since it provides an exact criterion for the separation “stable versus unstable vacuum”. Vacuum stability in classical models is important as well. For example, we have already shown [26] that the vacuum of the Abelian Higgs model without string contributions
is unstable against quantum corrections.

There is no general method of calculation of the confining quark contribution to the total VED. In quantum theory it heavily depends on the particular solutions of the corresponding quark SD equation, on account of the chosen Ansatz for the full gluon propagator. If it is correctly calculated then it is of opposite sign to the nonperturbative gluon part and it is one order of magnitude less (see, for example our papers [21,22,29,30]). Our method is not a solution for the fundamental badly divergent problem of VED in QCD. Moreover, it is even not necessary to deal with this problem. What is necessary indeed, is to be able to extract the finite part of the truly nonperturbative VED in a self-consistent way. Just this is provided by our method which thus can be applied to any nontrivial QCD vacuum quantum/classical models.

In conclusion, let us make some remarks. In some cases together with the nonperturbative scale some other parameter(s) should be considered as ”running” in accordance with the general scheme of our method. For example, such situation will arise in the IRF model gluon propagator suggested by lattice calculations in Ref. [58] (see also Ref. [59]). In this case the general procedure of calculation of the truly nonperturbative YM VED (if any) remains, of course, unchanged. However, due to some technical details (for example, the corresponding ”stationary” condition (2.17) will be more complicated) this case requires a separate consideration. A brief recent reviews on both continuum and lattice gluon propagators can be found in Refs. [15,53]. An attempt of VED calculation by introduction of rather controversial gluon mass was made in a recent paper [60].

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