Symmetric and asymmetric bifurcations of natural convection inside circular cavity in the route to chaos

Jilai Chen¹ and Jishuo Zhang², *

¹Faculty of Science and Engineering, University of Nottingham Ningbo China, Ningbo, China
²School of Geography, University of Nottingham United Kingdom, Nottingham, United Kingdom

*Corresponding author e-mail: hakan.zhang@outlook.com

Abstract. Natural convection occurs when pipe floating at the water surface during night, which can be regarded as a circular cross-section enclosure heated from below and cooled from the top. Numerical investigation is carried out with focuses on the flow configurations in the route to chaos. When $Ra$ is lower than $10^3$, convection is neglectable and conduction dominate the heat transfer. The flow becomes chaos and unsteady when $Ra$ increases to $10^8$. In the route to chaos, the number of cell increase with the decrease of the cell size. It is found that a symmetric to asymmetric bifurcation of flow configuration followed by a asymmetric to symmetric exist for $Ra$ between $10^3$ to $10^6$.

1. Introduction

Natural convection, also known as free convection, is extensively studied because natural convection is widely occurs in nature and engineering applications, such as solar collectors, electronic equipment cooling, heat exchanger and crystal growth processes [1, 2]. In general, the heat and mass transfer is influenced by the Rayleigh number ($Ra$), Prandtl number ($Pr$) and different types of thermal boundary conditions. With the increase of $Ra$, the heat transfer will be dominated by conduction then by convection. And the flow patterns will change with $Ra$ as will resulting in the variation of heat transfer rate. $Pr$, the ratio of diffusion of momentum and heat speed, can influence relative thickness of momentum and thermal boundary layer. As for the types of thermal boundary conditions, the isothermal and isoflux conditions are the most concerned ones. Natural convection can be caused by horizontal or vertical temperature gradients. The former one is relevant to the famous Rayleigh-Bénard convection.

The geometry of the model can also have significant impacts on the heat and mass transfer, natural convection in a few typical geometry has been investigated [3]. Natural convection in rectangular cavities are probably the most classic one. Bachelor [4] is one of the earliest researchers to investigate the natural convection in a two-dimensional cavity. The features of the main cell in the cavity are reported. Patterson et al. [5] examined the transient natural convection in a 2D rectangular, the scales for heat and mass transfer are proposed such as the thickness of laminar thermal boundary layer and velocity of boundary layer flow. In recent years, the turbulent flow and heat transfer are examined [6].

On the other hand, natural convection in the rectangular domain is also investigated in recent years, which are closely related to the heat and mass transfer in the attic space of building or that in a valleys
or rivers with V-shaped cross-section. Ridouane et al. [7] firstly reported a Pitchfork bifurcation exist in when Ra increases to pass $2 \times 10^5$, the flow configuration will turn to asymmetric from asymmetric. Bhowmick et al. [8] further analyzed natural convection in a V-shaped triangular cavity to modeling the flow in valleys. In their investigation, the Rayleigh number from $10^6$ to $10^8$ is considered. Three typical bifurcations are reported with the transition regimes of $Ra$ presented, i.e., a Pitchfork bifurcation at $Ra$ between $7.5 \times 10^3$ and $7.6 \times 10^3$, a Hopf bifurcation at $Ra$ between $1.5 \times 10^7$ and $1.6 \times 10^7$, and another bifurcation to a chaotic state at between $Ra$ between $5 \times 10^7$ and $6 \times 10^7$. On the other hand, Saha et al. [9] studied the transient natural convection in an isosceles triangular cavity. The development of convection is consisted of three stages: early, transitional and quasi-steady stages.

Besides the natural convection in rectangular and triangular cavities, that in circular cross-section cavities heated below and cooled at top is also of foundational and application importance. Taking the fluid inside a pipe or a tank floating on the water surface as an example. During night, the temperature of the air is usually higher than the water because of the high thermal capacity of the water. The natural convection of the fluid inside can be observed in this top cooling and bottom heating cavity. Inspired by the transition in the rectangular and triangular cavities, complex transition of fluid could exist in the circular cross-section cavity as well. Although the many investigations on the natural convection in a circular cross-section cavity have carried out [10], the transition route is still unclear. In this study, the transition of fluid in a circular cross-section cavity which is heated underneath and cooled above is examined. The emphasize is on the impact of $Ra$ on the transition route.

2. Physical model and numerical procedures

The issue we considered is the flow in a 2D circular cross-section cavity as sketched in Fig. 1. The cavity is heated underneath and cooled above, which is filled with air of Prandtl number of 0.71. The fluid is motionless initially, and natural convection will be generated once the thermal boundary condition is exerted on. The governing equations for the heat and mass transfer are

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0, \quad (1)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial r} + V \frac{\partial U}{\partial \theta} = \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} \right), \quad (2)$$

![Figure 1. Air filled floating tank with circular cross-section heated underneath and cooled above during night.](image-url)
\[ \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial r} + V \frac{\partial V}{\partial y} = - \frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} \right) + g \beta \Delta T, \]  

(3)

\[ \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \kappa \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \]  

(4)

Where, \( x \) and \( y \) are the coordinates, \( t \) is time. \( U \) and \( V \) are the velocity components. \( p \) and \( T \) are the pressure and temperature. \( \nu, \kappa, g \) and \( \beta \) are viscosity, heat conductivity, gravitational acceleration and coefficient of thermal expansion.

The system is governed by the Rayleigh number and Prandtl number, which written as

\[ Ra = \frac{g \beta \Delta T R^3}{\kappa \nu} \]  

(5)

\[ Pr = \frac{\nu}{\kappa} \]  

(6)

The equations (1)–(4) are solved using the finite volume method with the SIMPLE algorithm. The QUICK scheme is used to discretize the advection term, the second-order central difference scheme for the diffusion term, and second-order backward implicit time-marching scheme for the transient term.

Based on the sensitivity assessment of the size of element, 41228 elements are used to discrete the computational domain. As shown in Fig.2, finer grids are adopted for the near wall regions since velocity and temperature varies significantly.

**Figure 2.** Schematic of the grid, in which the radius of the circular cross-section is 0.05 m.

3. **Results and discussion**

In this work, \( Ra \) ranges from \( 1 \times 10^3 \) to \( 1 \times 10^8 \) are investigated. The isotherms and streamlines of the air inside are presented as shown in Figs. 3 to 8.

**Figure 3.** Isotherms of natural convection when \( Ra = 1 \times 10^3 \).
Fig. 3 indicates that the isotherms is perfect symmetric when the Rayleigh number is low, i.e., $Ra < 1 \times 10^3$. Heat transfer inside is dominated by conduction and the convection is insignificant. Convection becomes distinct with the increase of Rayleigh number. When the Rayleigh increase to $1 \times 10^4$, the symmetric is broken. The flow is dominated by a clockwise cell at the centre of cavity which with the length scale comparable to the diameter. Besides, two small cells located at the left-top corner and right-bottom corner can be observed. In fact, the main cell could be both clockwise and anticlockwise, which is dependent on the perturbations. That implies a pitchfork bifurcation occurs between $1 \times 10^3 < Ra < 1 \times 10^4$.

**Figure 4.** Isotherms (left) and streamlines (right) of natural convection when $Ra = 1 \times 10^4$.

The isotherms and streamlines for $Ra = 1 \times 10^5$ are presented in Fig. 5. Comparing Figs. 4 and 5, it suggests that the flow maintains the same configuration for $1 \times 10^4 < Ra < 1 \times 10^5$, i.e., a main cell accompanied with two small cells. That means no further bifurcation occurs for this range.

**Figure 5.** Isotherms (left) and streamlines (right) of natural convection when $Ra = 1 \times 10^5$.

Increasing the Rayleigh number to $1 \times 10^6$ further, it can be observed that the cell number increases from three to four as shown in Fig. 6. And the asymmetric flow configuration turn to symmetric again. The four cells have almost the same size and located at the four quadrants.

**Figure 6.** Isotherms (left) and streamlines (right) of natural convection when $Ra = 1 \times 10^6$.

The flow keeps the symmetric configuration featured by four cells up to $Ra = 1 \times 10^7$, see Fig. 7. However, the shape of cells are significant different from that low Rayleigh number. More
specifically, the cell is subround for $Ra = 1 \times 10^6$, whereas they are compressed in the vertical direction and become slender for $Ra = 1 \times 10^7$.

![Circular cavity with isotherms and streamlines](image)

**Figure 7.** Isotherms (left) and streamlines (right) of natural convection when $Ra = 1 \times 10^7$.

When the Rayleigh number increases to $1 \times 10^8$, the flow in the circular cavity goes to the chaos state as shown in Fig. 8. The large cells break into many smaller cells, and the flow becomes temporal unsteady. It can be foreseen that more bifurcations occurs for $1 \times 10^4 < Ra < 1 \times 10^5$. At the same time the intensity of convection has been significantly enhanced.

![Circular cavity with isotherms and streamlines](image)

**Figure 8.** Isotherms (left) and streamlines (right) of natural convection when $Ra = 1 \times 10^8$.

4. **Conclusion**

The natural convection inside circular cavity undergoes a complex bifurcation route from conduction to chaos convection with the increase of Raleigh number. Numerical investigation are carried out for $Ra$ from $10^3$ to $10^8$. The typical bifurcations are recognized via the isotherms and streamlines. A bifurcation from symmetric to asymmetric flow configuration occurs at Rayleigh number between $1 \times 10^4$ and $1 \times 10^5$. The symmetric-asymmetric bifurcation is followed by a asymmetric to symmetric bifurcation, which result in four cells at the four quadrants of the cavity. Increasing $Ra$ to $10^8$, the flow becomes chaos after more bifurcation.

**References**

[1] M.R. Ravi, R. Henkes, C.J. Hoogendoorn, On the high-Rayleigh-number structure of steady laminar natural-convection flow in a square enclosure. Journal of Fluid Mechanics, 262 (2016) 325-351.

[2] M. Corcione, Effects of the thermal boundary conditions at the sidewalls upon natural convection in rectangular enclosures heated from below and cooled from above. International Journal of Heat and Fluid Flow, 42 (2002) 199-208.

[3] D. Das, M. Roy, T. Basak, Studies on natural convection within enclosures of various (non-square) shapes – A review. International Journal of Heat and Mass Transfer, 106 (2017) 356-406.

[4] G.K., Batchelor, Heat transfer by free convection across a closed cavity between vertical boundaries at different temperature, Quarterly of Applied Mathematics, 12 (1954) 209-233.

[5] J. Patterson, J. Imberger, Unsteady natural convection in a rectangular cavity. Journal of Fluid Mechanics, 100(1980) 65-86.
[6] Y.H. He, K.Q. Xia, Temperature fluctuation profiles in turbulent thermal convection: a logarithmic dependence versus a power-law dependence, Physical Review Letters, 122(2019) 014503.

[7] E.H. Ridouane, A. Campo, Formation of a pitchfork bifurcation in thermal convection flow inside an isosceles triangular cavity. Physics of Fluids, 18 (2006) 251.

[8] S. Bhowmick, S.C. Saha, M. Qiao, F. Xu, Transition to a chaotic flow in a V-shaped triangular cavity heated from below. International Journal of Heat and Mass Transfer, 128 (2019) 76-86.

[9] S.C. Saha, Y.T. Gu, Natural convection in a triangular enclosure heated from below and non-uniformly cooled from top. International Journal of Heat and Mass Transfer, 80 (2015) 529-538.

[10] S. M. Mirabedin, F. Farhadi, Natural convection in circular enclosures heated from below for various central angles. Case Studies in Thermal Engineering, 8 (2016) 322-329.