On the Vacuum Propagation of Gravitational Waves

Xiao Liu

Perimeter Institute for Theoretical Physics, Waterloo, Ontario, Canada

(Dated: May 31, 2007)

We show that, for any local, causal quantum field theory which couples covariantly to gravity, and which admits Minkowski spacetime vacuum(a) invariant under the inhomogeneous proper orthochronous Lorentz group, plane gravitational waves propagating in such Minkowski vacuum(a) do not dissipate energy or momentum via quantum field theoretic effects.

1. INTRODUCTION

Gravitational waves propagate in empty space in general relativity. One basic class of such vacuum solutions is the plane waves in Minkowski spacetime. They describe, for example, the propagation of a gravitational wave, emitted by a bounded source, in a region far from its source\(^1\). The metric is given by the following exact solution to the vacuum Einstein equation

\[
\text{d}s^2 = \text{d}u^2 + h_{ij}(u)\text{d}x^i\text{d}x^j - \text{d}x^iv^i, \tag{1}
\]

where \(h_{ij}(u)\) is a symmetric, traceless, and otherwise arbitrary \(d-2\) by \(d-2\) matrix valued smooth function of \(u\). This describes a plane gravitational wave propagating along the light-like direction \(v\), with \(h_{ij}(u)\) specifying the space-dependent profile for the \((d-2)(d-1)/2-1\) polarizations of the wave in \(d\)-dimensional spacetime\(^2\). \(h_{ij}(u)\)'s in-dependence on the coordinate \(v\) shows explicitly that, among other things, no dissipation occurs. For \(h_{ij}(u)\) with finite support, the metric describes a plane gravitational wave in Minkowski spacetime with a finite duration.

As classical solutions to the equations of motion, these plane wave spacetimes are robust under a large class of deformations to the gravitational dynamics. Any local correction to the Hilbert action by adding higher powers of the Riemann tensor and covariant derivatives leaves the solutions intact. This follows from the fact that any scalar or second-rank tensor field constructed locally from the Riemann tensor and the covariant derivative necessarily vanish on these backgrounds\(^3\). Hence any higher curvature and higher derivative correction to the Hilbert action and to the Einstein equation does not materialize.

In quantum theory, the vacuum is no longer empty. As ripples of spacetime curvature travel through the vacuum, the zero-point fluctuations of the quantum field in the background are generically amplified to higher energy excitations. To calculate these effects, one needs to diagonalize the full time-dependent Hamiltonian for the quantum field at every instant of time, and to expand the in-vacuum state in the basis of the out-states. This calculation is in general difficult to carry out, except when the field theory is free. In the latter case, diagonalizing the full Hamiltonian boils down to solving free field equations in the time-dependent background, and the occupation number of the in-vacuum in each out-Fock space state is determined by the Bogolubov coefficients (see e.g.\(^2\)).

The metric \(\text{(1)}\) has a covariantly constant global null Killing vector field, which generates the translation along the plane wave \((v\text{-direction})\). Had it generated an evolution across the wave (along \(u\) for example), one would have been able to immediately conclude that no particle would be produced by the gravitational wave in any field theory, because one can choose to work with a (light cone)time-independent Hamiltonian by slicing the geometry properly. Nevertheless, in the case of free quantum fields\(^4\), the presence of this along-the-wave Killing field is enough to forbid mixing between positive and negative frequency modes under the evolution along the \(u\)-direction across the wave (but still allows mixing between positive/negative frequency modes with themselves respectively). This establishes that plane gravitational waves do not dissipate energy or momentum by exciting the vacuum of a free quantum field.

This calculation does not apply to interacting theories. In particular, it does not apply to the real world, since the relevant low energy physics is governed by the nonlinear interacting theory of photons, in which the leading interaction from integrating out the electron is \(\sim \alpha^2(\langle F^2 \rangle_2^2 + 7 \langle FF F \rangle_3^2)\). One may wonder whether, taking into account this interacting nature of the QED (and the Standard Model) vacuum, the gravitational wave could dissipate energy and momentum by producing extremely soft photons as it propagates\(^4\). Were this possible to happen, attenuation of extremely high frequency gravitational waves might accumulate over a cosmic distance

\(^1\) Electronic address: xliu@perimeterinstitute.ca
\(^2\) For results of recent attempts of experimental detection, see \(^1\)
\(^3\) For another coordinate system that makes explicit the symmetry of the plane wave front, see e.g.\(^2\)
\(^4\) Such photons, if any, are presumably mostly collinear to the gravitational wave on kinematic grounds.
2. GRAVITATIONAL PLANE WAVE SPACETIME

The spacetime defined by the metric (1) is geodesically complete, and contains no closed time-like or light-like curves. It admits $2d - 3$ Killing vector fields for generic choice of $h_{ij}(u)$, although only one of them

$$Z = \frac{\partial}{\partial v}.$$  \hspace{1cm} (2)

is manifest in the Brinkman coordinates (1). The $2(d - 2)$ non-manifest Killing vector fields are all in the form:

$$X = 2b_i'(u)x^i\partial_v + b_i(u)\partial_t,$$  \hspace{1cm} (3)

where $(b_1(u), \ldots , b_{d-2}(u))$ is a solution to:

$$b''_i(u) + h_{ij}(u)b_j(u) = 0.$$  \hspace{1cm} (4)

The ODE has $2(d - 2)$ independent solutions, which give rise to the same number of additional independent Killing vector fields. Killing fields associated to two solutions $b_i(u)$ and $\tilde{b}_i(u)$ satisfy

$$[X_b, X_{\tilde{b}}] = 2W[b, \tilde{b}]Z,$$  \hspace{1cm} (5)

where the Wronsky $W[b, \tilde{b}] = \sum_i (b_i(u)\tilde{b}_i(u) - b'_i(u)\tilde{b}'_i(u))$ is independent of $u$. In a suitable basis, they generate the Heisenberg algebra

$$[X_{(k)}, \bar{X}_{(l)}] = \delta_{kl}Z \quad \{k , l = 1, \ldots , d - 2\},$$  \hspace{1cm} (6)

with central element $Z$.

The Killing vector fields in (6) preserve each $u = \text{const}$ hypersurface, and generate on each such hypersurface the $d - 1$ translations and the $d - 2$ $x$-linearly-dependent translations along $v$. For any given Killing vector field, the actions on the constant $u$ hypersurfaces are $u$-dependent. To help characterize the algebraic aspect of this dependence, we introduce yet another vector field

$$H = \frac{\partial}{\partial u}.$$  \hspace{1cm} (7)

This is not a Killing field. It generates evolution along $u$ and would be upgraded into the light-cone Hamiltonian if we quantize field theories in the plane wave background. In the basis of (6)

$$[H, X_{(k)}] = -h_{kl}(u)\bar{X}_{(l)},$$  \hspace{1cm} \hspace{1cm} (8)

and $Z$ remains central. Observe that the algebra does not close, unless $h_{ij}(u)$’s are constant. This is what one expects, since $H$ does not generate isometries unless the metric does not depend on $u$.

We are interested in plane gravitational waves of finite duration, that propagate in otherwise flat spacetime. So we demand $h_{ij}(u)$ to vanish outside $[-T, T]$. In regions $|u| > T$ where (1) reduces to flat space, the Killing fields generate a subgroup of the subgroup of Poincare group that preserves the null hyperplanes $\{u = u_0\}$. The latter is the same subgroup that preserves the vector field $\partial/\partial v$, and is generated by the translation along $v$, the translations and rotations of the $x^i$’s among themselves, and the $d - 2$ additional vector fields:

$$2x^i\partial_v + u\partial_t.$$  \hspace{1cm} (9)

The last $d - 2$ fields are linear combinations of boosts and rotations. All the translations and the $d - 2$ boost-rotations extend to the whole plane wave spacetime, while the rotations among the $x^i$ themselves do not for generic $h_{ij}(u)$. The translations and boost-rotations account for the total $2d - 3$ global Killing vector fields.
3. PLANE WAVES AS ROBUST CLASSICAL SOLUTIONS IN EFFECTIVE FIELD THEORIES

The Einstein-Hilbert action is an effective action for gravity. Various higher dimensional operators may be added and are presumably indeed present, their effects small until spacetime curvature approaches the mass scale that suppresses these higher dimensional operators, at which point the applicability of the effective theory itself starts to break down. Two natural questions come to mind: (1) what modifications these corrections may bring to the gravitational wave solutions? (2) how strong the gravitational wave needs to be to invalidate the application of the effective theory itself? In normal scenarios, the cut-off scale for the gravity effective action is assumed to be not too low below the Planck scale or the string scale.

The answer to the first question is well-known (see, e.g. [6] [7]): any higher curvature and higher derivative corrections to the Einstein-Hilbert action, involving only the quantities derived from the metric itself, does not modify the geometry .

The reason for this, as already mentioned in the introduction, is that any scalar and non-trivial second-rank tensor field that can be constructed based on the metric, the curvature, and the covariant derivative, necessarily vanish in the background . Hence both the corrections to the action, and the corrections to the Einstein equation, vanish for the plane wave spacetime.

To see how the geometric property comes about, we compute the Riemann tensor and its covariant derivatives in the Brinkman coordinate basis. The only nonvanishing components of the Riemann tensor are

\[ R^{\alpha\beta\gamma\delta}_{\mu
u} = -h^{\alpha\beta}_{\mu\nu}, \]  

and those related to this by symmetry. Further inspection reveals that \( \nabla_{\alpha}\nabla_{\beta}\nabla_{\gamma}\nabla_{\delta}R^{\mu\nu\rho\sigma} \) vanishes unless every index is either \( u \) or one of the \( d-2 \) \( i \)'s, and the total number of \( i \)-index must be less than or equal to 2. In fact, by inspecting the basic operations in the construction of these higher rank tensor fields, a simple “sum rule” can be shown to hold: the total number of \( i \)-index for any nonvanishing component plus its degree as a homogeneous polynomial of the variables \( \{ x^i, i = 1, ..., d-2 \} \) always equals 2. Technically aside, the upshot for now is that any component of the above tensors with a \( v \)-index vanishes. Note also that \( g^{\mu\nu} \neq 0 \) if and only if \( \nu \) is \( v \), and that \( h^{ij}_{\mu\nu}(u) \) is traceless. It then follows by inspection that any nonvanishing scalar or second-rank tensor fields (except the metric itself) can be constructed because there are not too many lower \( u \) indices and no lower \( v \) index that there is no way to contract all of them.

Since the contribution of all higher dimensional operators are parametrically smaller (indeed, in this case, they vanish) than the leading contribution from the Einstein equation, the application of the effective theory is, by definition, valid regardless of how strong the gravitational wave is. This might seem a little confusing at the first glance, because the solutions allow \( h^{ij}_{\mu\nu}(u) \), the tidal force, arbitrarily large and arbitrarily fast varying.

There is no paradox here. The point is that \( h^{ij}_{\mu\nu}(u) \) and its variations are frame-dependent; and, around every point in the plane wave geometry, one can always boost to a free-falling frame in which all components of the tidal force and its gradients are smaller than 1 in any specified mass unit. So they can all be made parametrically small compared to the mass scales suppressing the corrections in the action. This follows from the non-compactness of the Lorentz group and the fact above that there is no non-vanishing Lorentzian scalar constructed locally by the Riemann tensor and its derivatives. To gain intuitions about it, we now see it directly from the metric .

Along the locus \( \{ x^i = 0, i = 1, ..., d-2 \} \), the basis \( \{ \frac{1}{2}(\partial_u + \partial_v), \frac{1}{2}(\partial_u - \partial_v), \partial_i, i = 1, ..., d-2 \} \) is already a Lorentz frame at each point, namely the metric tensor in this basis is \( diag(+1, -1, ..., -1) \). A boost \( \partial_u \mapsto \lambda \partial_u, \partial_v \mapsto \lambda^{-1}\partial_v, \partial_i \mapsto \partial_i \) (regarded as a linear transformation in the tangent space at a given point) leaves the metric invariant, but enforces \( \nabla_{\alpha_1}...\nabla_{\alpha_k}R^{\mu\nu\rho\sigma} \mapsto \lambda^{k+2}\nabla_{\alpha_1}...\nabla_{\alpha_k}R^{\mu\nu\rho\sigma} \) for every nonvanishing component, by the previously mentioned sum rule. Hence given any finite number of such tensors, we can always choose \( \lambda \) properly to make all the components of these tensors arbitrarily small. This finds the proper Lorentz frames point-wise along \( \{ x^i = 0, i = 1, ..., d-2 \} \).

Remember that the spacetime has a large isometry, which acts on each hypersurface \( \{ u = u_0 \} \) transitively, some of which, specifically, act as translations. So for any point \( P \) in the spacetime, there always is an isometry that brings that point to a point \( Q \) in \( \{ x^i = 0, i = 1, ..., d-2 \} \). The pullback to \( P \) from \( Q \) of the appropriate Lorentz frame at \( Q \) gives rise to the sought-after frame at \( P \) that makes all the components of the tidal force and its gradients small.

The boost we did to scale down the tidal force corresponds to speeding up in the direction that the wave propagates. This elongates the duration of the plane wave and lowers its frequency of variation, both of which are frame-dependent scales. Furthermore, as shown above, no frame-independent scale exists at all that observer can define in the spacetime. This implies that the solutions are valid for arbitrarily strong waves. On the other hand, as well-known, the field theoretic description does break down, but only as we start asking questions about the local physics on length scales approaching the cut-off scale.

---

8 To jump ahead a little bit, one may try to infer the absence of particle production based on an adiabatic reasoning. The author does not know how, along this line of reasoning, to rule out extensive production effects in cases without a mass gap. What we will do instead is to determine the effects by exploiting its property under boosts and other transformations.
To recapitulate, as long as we restrict ourselves to length scales above the cut-off, not only that the geometry \( \Pi \) is always valid as classical solutions to the effective action, but also that the application of the effective action itself is always valid in solving for these classical solutions.

4. \( \langle \text{vac}, \text{in}|T_{\mu\nu}^{\text{ren}}(x)|\text{vac}, \text{in} \rangle \) IN GENERALLY COVARIANT QUANTUM FIELD THEORIES

We ignored the presence of other fields in the last section by setting them to their values in a classical vacuum, which we assume to be a configuration of Minkowski space with a Poincare invariant profile for all the fields present. This is consistent at the classical level.  

Quantum mechanically, local observables in the vacuum are no longer sharply peaked at any particular values. The consequences are several-fold. First, zero-point motions give rise to cut-off dependent contributions to the effective Lagrangian density. These include, generally, a constant piece, acting effectively like a cosmological constant, and various other fields and curvature dependent terms. Unless one works in a UV finite theory like string theory, one can only determine the coefficients of these interactions by measurements. We will make no statements about these coefficients, except that we restrict ourselves to theories that admit (proper orthonochronous) Poincare invariant \( T_{\mu\nu}^{\text{ren}}(x) \) needs to satisfy. Remember that in the Minkowski space portion of the spacetime \( \Pi \), the isometries are part of the Poincare group which, by assumption, leaves \( |\text{vac}, \text{in} \rangle \) invariant. Combined with the boundary condition we imposed, this implies that \( \langle \text{vac}, \text{in}|T_{\mu\nu}^{\text{ren}}(x)|\text{vac}, \text{in} \rangle \) (for convenience, we will denote this quantity by \( \langle T_{\mu\nu} \rangle \)) is an invariant tensor under the isometry group. That is, for any transformation \( x \mapsto y = f(x) \) that satisfies \( (f^*g)_{\mu\nu} = g_{\mu\nu} \) we have

\[
\frac{\partial y^\rho}{\partial x^\mu} \frac{\partial y^\sigma}{\partial x^\nu} \langle T_{\rho\sigma}(y) \rangle = \langle T_{\mu\nu}(x) \rangle .
\] (11)

That this does not only hold in the before-wave region but also hold everywhere requires some explanation. Let \( \hat{U}[f] \) be the operator that realizes the isometry transformation \( f \) in the quantum field theory. This operator is \( u \)-dependent, and its action on each constant \( u \) hypersurface, which the isometry \( f \) preserves, is determined by the generating Killing vector field \( V_j \), which, in turn, is determined by \( \mathfrak{g} \) or equivalently by \( \mathfrak{s} \). We have

\[
\frac{\partial y^\rho}{\partial x^\mu} \frac{\partial y^\sigma}{\partial x^\nu} \hat{T}_{\rho\sigma}^{\text{ren}}(y) = \hat{U}[f]^\dagger(u) \cdot \hat{T}_{\mu\nu}^{\text{ren}}(x) \cdot \hat{U}[f](u) \quad (12)
\]

by the fact that \( \hat{T}_{\mu\nu}^{\text{ren}}(x) \) is an operator that transforms as a tensor; the arguments of \( \hat{T} \) in this equation (points \( x \) and \( y \)) share the same value for the \( u \)-coordinate.

We claim that, for all values of \( u \), \( \hat{U}[f](u) \) leaves the in-vacuum invariant. This is clear if \( |u| \geq T \) in which case it represents an element of the Poincare group that preserves the null hyperplanes \( u = \text{constant} \). It might seem less clear if \( |u| < T \), but it is also true. The point is, on each constant \( u \) hypersurface, the Killing vector field \( V_j \) that generates \( f \) can be expanded in the basis of vector fields \( \{\partial_u, \partial_i, x^j \partial_u, \partial_j, i = 1, \ldots, d - 2\} \) restricted to the same hypersurface. The corresponding operator \( \hat{O}[V_j](u) \) can thus be expanded in terms of the \( u \)-independent operators \( \{\hat{O}[\partial_u], \hat{O}[^i_\partial_u][\partial_i], \hat{O}[x^j \partial_u][\partial_i], i = 1, \ldots, d - 2\} \) with \( u \)-dependent coefficients. Since we know from the flat before-wave region that all the latter annihilate the in-vacuum, \( \hat{O}[V_j](u) \) must also do. Hence, \( \hat{U}[f](u) \) leaves it invariant:

\[
\hat{U}[f](u) |\text{vac}, \text{in} \rangle = |\text{vac}, \text{in} \rangle \quad (13)
\]

for all values of \( u \). Now we sandwich (12) between \( |\text{vac}, \text{in} \rangle \) and \( |\text{vac}, \text{in} \rangle \), simplify the right hand side via (13), and produce (11).

It follows from (11) that

\[
\mathcal{L}_V \langle T_{\mu\nu} \rangle = 0 ,
\] (14)
for any $V$ in the algebra (11). Writing out this equation explicitly we find that for $V = Z$
\[ \partial_v \langle T_{\mu \nu} \rangle = 0 \] (15)
and that for $V$ one of the $X$’s
\begin{align*}
0 &= b_i(u) (\partial_i \langle T_{\mu \nu} \rangle - 2 h_{ij}(u) x^j (\delta_\mu^i \langle T_{\nu \nu} \rangle + \delta_\nu^i \langle T_{\mu \nu} \rangle) \\
&+ b'_i(u) \langle T_{\mu \nu} \rangle (2 \delta_\mu^i \delta_\nu^j + \delta_\nu^i \delta_\mu^j) \\
&+ b'_i(u) \langle T_{\mu \nu} \rangle (2 \delta_\mu^i \delta_\nu^j + \delta_\nu^i \delta_\mu^j) \\
&+ b'_i(u) \langle T_{\mu \nu} \rangle (2 \delta_\mu^i \delta_\nu^j + \delta_\nu^i \delta_\mu^j) \tag{16}
\end{align*}
where, as before, $i, j = 1, \ldots, d-2$ and repeated indices are summed over regardless of their vertical positions. Since the $2d-4$ $(b_i(u), \ldots, b_d-2(u))$’s that define the $X$-type isometries constitute a complete basis of the solutions to (14), the functions multiplying $b_i(u)$ and $b'_i(u)$ in (16) must vanish separately. Working out their consequences we find
\begin{align*}
\langle T_{\mu \nu} \rangle &= \langle T_{\nu \mu} \rangle = \langle T_{\nu i} \rangle = \langle T_{\mu i} \rangle = 0, \\
\langle T_{\mu u} \rangle (u, x^i) &= 2 h_{ij}(u) x^j \langle T_{\mu u} \rangle (u), \\
\langle T_{ij} \rangle (u) &= -2 \langle T_{\mu \nu} \rangle (u) \delta_{ij}.
\end{align*}
(17)
In the above equations, we explicitly write out the coordinate(s) each component of $\langle T_{\mu \nu} \rangle$ is allowed to depend on; for example, $\langle T_{\mu \nu} \rangle$ can only depend on $u$. These all follow from solving the isometry constraints (14).

It is now clear that, that $\langle \text{vac, in}|\hat{T}_{\nu \mu}^{\text{ren}}(x)|\text{vac, in} \rangle$ is invariant under the full isometry group is very constraining: the expectation values of a $d$-dimensional second-rank symmetric tensor, that is $d(d+1)/2$ functions of $d$ variables each, reduce to a single unknown function $\langle T_{\mu \nu} \rangle$ of a single variable $u$! This is certainly only possible because we started in the in-vacuum and imposed proper boundary conditions, any excitations in the initial state or in the in-coming wave from $v \to -\infty$ will spoil the property.

To proceed further, we will need some dynamical equation, which is generally hard to write down. There is a simple one, the covariant conservation of the energy-momentum-stress tensor:
\[ \nabla^\mu \langle T_{\mu \nu} \rangle = 0. \] (18)
This condition is necessary for general covariance to be preserved at the quantum mechanical level (14). Now applying the results in (17), it simplifies to
\[ 2 \partial_u \langle T_{\mu \nu} \rangle + \partial_i \langle T_{\mu i} \rangle = 0. \] (19)
Further application of (17) immediately shows that (19) only gives nontrivial constraint when $\nu$ is $u$:
\[ \frac{d}{du} \langle T_{\mu \nu} \rangle (u) = 0. \] (20)
Hence $\langle T_{\mu \nu} \rangle$ is a constant.

What we have shown is that, up to an overall constant, there is precisely one covariantly constant symmetric second rank tensor field in the background (11) that is invariant under the full isometry group (14). Of course, the metric tensor itself satisfies these conditions, hence $\langle T_{\mu \nu} \rangle = \text{constant} \times g_{\mu \nu}$. Since we started from a Minkowski vacuum in which $\langle T_{\mu \nu} \rangle \equiv 0$ for $u < -T$, this constant must vanish $\dagger$.

We showed that $\langle \text{vac, in}|\hat{T}_{\nu \mu}^{\text{ren}}(x)|\text{vac, in} \rangle \equiv 0$ in the gravitational plane wave background. What does it mean? Had the in-vacuum evolved into an (locally discernable) excited out-state in the future flat region, this quantity would have been non-vanishing. Hence, for any local observer after the wave, the field appears to remain in a vacuum state. Put another way, in any finite (however large) region of space, the energy and momentum dissipated into the quantum field in that region by the gravitational wave vanish exactly.

On the other hand, the gravitational aspect of $\langle \hat{T}_{\nu \mu}^{\text{ren}} \rangle$’s significance is not at all clear at the conceptual level. Plausible statements had been made in the literature that suggest to feed it back to the Einstein equation to further correct the background metric in some sort of semiclassical approximation, but none had been made precise. Time-dependent backgrounds in string theory would hopefully be understood well enough to clarify its physical significance in future. Nevertheless, we note, given that the field theoretic aspects of the computation of $\langle \hat{T}_{\nu \mu}^{\text{ren}} \rangle$ is well-defined, the result $\langle \hat{T}_{\nu \mu}^{\text{ren}} \rangle = 0$ should be taken seriously. It may also be reassuring to note that, incidentally, this result nullifies further concerns of back-reaction on the metric at the semi-classical level.

5. REMARKS

When solving the free field wave equation in the plane gravitational wave spacetime (3), one finds that a monochromatic positive frequency solution in the before-wave region evolves into a superposition of positive frequency solutions after the wave passes by $\ddagger$. That is, the

\begin{itemize}
\item \textbf{\dagger} We thank K.Krasnov for pointing out the reference (3) which showed that the cosmological constant, if zero, is not renormalized by pure graviton loops up to two loops. If such result fails to hold at higher loops and/or after coupling with matter, $\langle T_{\mu \nu} \rangle = \text{constant} \times g_{\mu \nu}$ by itself means that no dissipation of energy and momentum into the matter sector occurs.

\item \textbf{\ddagger} To see this, one needs to transform the equations (3.1)-(3.3) of (4), which are given in the Rosen coordinate associated to the before-wave region, into the global Brinkman coordinate. One should also note that the singularities of the mode solutions do not represent a fundamental obstruction to quantizing the field theory. They disappear when wave packets are considered that have finite supports in directions transverse to the propagation of the wave. On the other hand, it does indicate formations of singularity when two infinite plane waves are collided (10).
\end{itemize}
creation operators do not mix with annihilation operators, but they do mix with themselves. In a free field theory, for which the physical ground state is the same as the Fock space ground state, one concludes that the field stays in the vacuum undisturbed. After interaction is turned on, one expects that the physical ground state spreads out in the Fock space. So it may appear that mixing between the positive frequency solutions themselves would generically lead to volume-extensive particle productions. We find the contrary.

The point has to do with the vacuum structure in light cone quantization. Remember that we sliced the geometry by constant $u$ hypersurfaces, which are light-like. The rules of light cone quantization for general field theories are not entirely clear, but it is generically expected that the physical vacuum, modulo the zero-modes problem, is the same as the Fock space vacuum \[\text{[11]},\] as a result of the positivity of the longitudinal light cone momentum. This vacuum furthermore is not affected when the light cone Hamiltonian becomes (light cone) time-dependent, again modulo the problem associated to the zero-modes. In a simple case like the $\lambda\phi^4$ theory, the light cone quantization in the plane wave spacetime can be carried out and explicitly shows that no particle production effects arise. On the other hand, our argument in the previous section holds for general field theories. It does not depend on any specifics of light cone quantization, but is consistent with expectations derived from it.

The message to take away is that, the propagation of gravitational waves in stable (proper orthochronous) Poincare invariant Minkowski spacetime vacua is robustly characterized by the classical solutions to general relativity. Vacuum fluctuations of field theoretic origin, regardless of their property, do not modify this behavior.

Exceptions, however, may arise if Lorentz invariance is spontaneously broken in a Minkowski vacuum. One such class of examples was constructed, at low energies, in \[\text{[12].}\] After coupled to gravity, the goldstone field $\pi(x)$ of these theories violates the equivalence principle and allows sources to anti-gravitate; $\pi(x)$ also develops a Jeans-like instability at large distances around the flat background. It may be interesting to study the propagation of gravitational waves in this class of Lorentz violating vacua, both at the classical and at the quantum level.

For practical purposes, it is perhaps important to study the propagation of a gravitational wave in a gas of particles (see e.g. \[\text{[13] \[14\] \[15\] for some earlier results}).

**Acknowledgments**

The author would like to thank Robert Brandenberger, Jaume Gomis, Gary Horowitz, Shamit Kachru, Justin Khoury, Slava Mukhanov, and Constantinos Skordis for helpful comments on the draft. He would also like to thank Freddy Cachazo and Amihay Hanany for suggestions of references on light cone quantization. Research at the Perimeter Institute for Theoretical Physics is supported in part by the Government of Canada through NSERC and by the Province of Ontario through MRI.

---

[1] B. Abbott et al., “Search for Gravitational-Wave Bursts in LIGO Data from the Fourth Science Run,” [arXiv: 0704.0943v1].  
[2] M. Blau, “Plane Waves and Penrose Limits,” Lecture Notes for the ICTP School on Mathematics in String and field Theory (June 2-13 2003).  
[3] N. D. Birrell and P. C. W. Davies, Quantum Fields in Curved Space, (Cambridge University Press, 1982).  
[4] G. W. Gibbons, “Quantized Fields Propagating in Plane-Wave Spacetimes,” Commun. Math. Phys. 45, 191 (1975).  
[5] J. Schwinger, “On Gauge Invariance and Vacuum Polarization,” Phys. Rev. 82, 664 (1951).  
[6] S. Deser, “Plane Waves Do Not Polarize the Vacuum,” J. Phys. A 8, 1972 (1975).  
[7] G. T. Horowitz and A. R. Steif, “Spacetime Singularities in String Theory,” Phys. Rev. Lett. 64, 260 (1990).  
[8] L. Alvarez-Gaume and E. Witten, “Gravitational Anomalies,” Nucl. Phys. B 234, 269 (1983).  
[9] M. H. Goroff and A. Sagnotti, “Ultraviolet Behavior of Einstein Gravity” Nucl. Phys. B 266, 709 (1986).  
[10] U. Yurtsever, “Instability of Killing-Cauchy Horizons in Plane-Symmetric Spacetimes,” Phys. Rev. D 36, 1662 (1987).  
[11] M. Burkardt “Light Front Quantization,” Adv. Nucl. Phys. 23, 1 (1996) [arXiv:hep-ph/9505259].  
[12] N. Arkani-Hamed, H. C. Cheng, M. A. Luty, and S. Mukohyama, “Ghost Condensation and a Consistent Infrared Modification of Gravity,” JHEP 0405, 074 (2004) [arXiv:hep-th/0312099].  
[13] H. Bondi, F. A. E. Pirani, and I. Robinson, “Gravitational Waves in General Relativity III. Exact Plane Waves,” Proc. Roy. Soc. London. Ser. A 251, 519 (1959).  
[14] H. Bondi and F. A. E. Pirani, “Energy Conversion by
Gravitational Waves,” Nature 332, 212 (1988).

[15] H. Bondi and F. A. E. Pirani, “Gravitational Waves in General Relativity XIII. Caustic Property of Plane Waves,” Proc. Roy. Soc. London. Ser. A 421, 395 (1989).