Sharing of Unlicensed Spectrum
by Strategic Operators

Fei Teng, Dongning Guo, and Michael L. Honig
Department of Electrical Engineering and Computer Science, Northwestern University
Evanston, IL 60208, USA

Abstract

Facing the challenge of meeting ever-increasing demand for wireless data, the industry is striving to exploit large swaths of spectrum which anyone can use for free without having to obtain a license. Major standards bodies are currently considering a proposal to retool and deploy Long Term Evolution (LTE) technologies in unlicensed bands below 6 GHz. This paper studies the fundamental questions of whether and how the unlicensed spectrum can be shared by intrinsically strategic operators without suffering from the tragedy of the commons. A class of general utility functions is considered. It is first shown that a simple static sharing scheme allows a given set of operators to reach a subgame perfect Nash equilibrium for mutually beneficial sharing. The question of how many operators will choose to enter the market is also addressed by studying an entry game. A sharing scheme which allows dynamic spectrum borrowing and lending between operators is then proposed to address time-varying traffic and proved to achieve subgame perfect Nash equilibria with significantly higher revenue than static sharing. Implications of the results to practical LTE in unlicensed bands (LTE-U) are also discussed. The goal of this paper is to provide a theoretical foundation for LTE-U standardization and deployment.

Index Terms

Dynamic sharing, entry game, LTE-U, repeated game, unlicensed spectrum.

I. INTRODUCTION

Hundreds of megahertz of unlicensed spectrum under 10 GHz is currently available and more will likely be allocated in the near future. Unlike licensed frequency bands, an unlicensed band is free to use by anyone as long as basic constraints on the transmit power spectral density (PSD)
are satisfied, except in a few regions and bands, where an additional simple protocol (such as listen-before-talk) also needs to be followed. A fundamental question is how multiple wireless operators can effectively share the “free” unlicensed spectrum while mitigating the effects of the tragedy of the commons [1], namely, the spectral efficiency becomes severely degraded due to excessive interference from overuse.

Recent work on using unlicensed spectrum has studied on WiFi offloading from a cellular network [2]–[4]. In comparison with WiFi, Long Term Evolution (LTE) technology has benefits of high efficiency and robust mobility [5], [6]. Most operators and vendors believe that LTE in unlicensed spectrum (LTE-U) will seamlessly extend cellular networks and require no separate management as WiFi offloading would.

Challenging coexistence issues arise with multiple LTE-U and WiFi operators. It is a consensus within the 3rd Generation Partnership Project (3GPP) that LTE-U should not disrupt concurrent WiFi services [7], [8]. More importantly, since every LTE-U operator is incentivized to make the maximum use of the free spectrum, without an effective scheme for cooperation, many operators are likely to suffer from severe interference, leading to the tragedy of the commons. Two fundamental questions are addressed in this work: 1) Can intrinsically selfish and strategic operators cooperate for their mutual benefit? and 2) if so, how should strategic operators with dynamic traffic cooperate?

Given the non-cooperative and strategic feature of wireless operators, it is natural to cast the spectrum sharing problem in the framework of game theory and mechanism design. There have been some game-theoretic studies of spectrum sharing among non-cooperative parties (e.g., [9]–[19]). In particular, [12] laid the groundwork for answering the preceding fundamental questions in a limited scenario where the number of operators is fixed, the utility function is the Shannon rate, and each operator is subject to a total transmit power constraint. The schemes in [12] and [13] also need each operator to measure the exact power spectral profile of every other operator, which is hard to implement in practice. The authors of [19] devised a pricing mechanism for power control in unlicensed spectrum, where LTE-U operators can negotiate for the revenue obtained from the unlicensed spectrum.

In this paper, we study coexistence of multiple non-cooperative operators with time-varying
traffic. Such traffic variations are likely to be quite pronounced in densely deployed small cells [20]. A class of general utility functions is considered, with Shannon capacity being a special case. We start with sharing schemes for a simple scenario where a given fixed set of operators are colocated and orthogonal spectrum sharing is preferred due to higher overall spectral efficiency. We first establish the effectiveness of a simple static sharing scheme where each operator spectrum use does not vary with traffic levels. Then, assuming operators arrive sequentially, we show that the number of operators willing to invest a fixed cost in order to share the spectrum is limited and depends on the investment cost and externalities. The study on static sharing is a straightforward extension of our prior work [21]. With the total network revenue in mind, we then introduce a dynamic sharing scheme that adapts to the operators’ traffic conditions. The sharing problem is formulated as a repeated game with private information and communication. We devise a dynamic sharing profile, where the operators share information about their traffic intensities, and in any time slot, operators with low traffic intensities loan spectrum to those operators with high traffic intensities (with anticipation that borrowers will reciprocate). The proposed profile is shown to be subgame perfect Nash equilibrium with truthful reporting of traffic intensities. The practical implication is that operators are likely to enter such sharing agreements with dynamic spectrum trade for mutual benefits.

This study suggests simple spectrum sharing schemes for the deployment of LTE in unlicensed bands. Device with no intelligence are generally not strategic, thus the choice of the actual equilibrium allocation is likely to be determined by standards bodies such as the 3GPP and WiFi Alliance. A credible punishment scheme is necessary (and can be easily implemented in devices) to deter deviation, but the punishment state is never visited if all operators consistently comply with the proposed schemes. All operators must agree to the selected utility vector in advance and follow the profiles according to standards.

The rest of the paper is organized as follows. Section II presents the basic system model. Section III proposes a static sharing scheme. Section IV studies dynamic sharing schemes that allow operators to trade spectrum. Numerical results are shown in Section V. Concluding remarks are given in Section VI.

1The standards could also allow devices to negotiate the utility vector.
II. SYSTEM MODEL

Assume $n$ operators share a certain band or bands of unlicensed spectrum denoted as a real-number set $S \subset \mathbb{R}$, which is in general a union of some finite intervals. The total amount of spectrum is $W = \int_S 1 df \text{ Hz}$. On the timescale of interest, it is conveniently assumed that the spectrum in $S$ is homogeneous. It is also assumed that the operators can monitor the power spectral density of each other in every slot\(^2\).

We focus on a discrete-time formulation, where time is slotted and all operators are fully synchronized at the slot level. The traffic intensity of operator $i$ at time slot $t$ is defined as the traffic level during the time interval of slot $t$, and the intensity is revealed only to operator $i$ at the beginning of time slot $t$. Each operators’s traffic conditions are private information. We assume that the traffic intensity of operator $i$ is an exogenous random process denoted by $\{\Lambda_i^t, t \geq 0\}$ independent of other operators.

Each operator determines its transmit PSD at the beginning of each slot based on its prior information, and maintains the same PSD over the entire slot\(^3\). For ease of notation, denote the transmit PSD of operator $i$ at time slot $t$, normalized by the flat noise PSD, as $p_i^t(f)$. Throughout this paper, it is assumed that the only exogenous constraint on the PSD is regulatory, where the transmit PSD must be upper bounded by $P$, i.e., $p_i^t(f) \leq P$ for all $i$, $t$, and $f$. In general, the utility of operator $i$ is a function of the PSDs chosen by all operators, as well as its own traffic intensity $\lambda_i^t$, denoted as $u(p_i^t, p_{-i}^t, \lambda_i^t)$, where $p_{-i}^t$ denotes collectively the PSDs of operators other than $i$. The total revenue of operator $i$ over the infinite time horizon is defined as

$$V^i = (1 - \delta) \sum_{t=0}^{\infty} \delta^t u(p_i^t, p_{-i}^t, \lambda_i^t)$$

(1)

where $\delta \in [0, 1)$ denotes the discount of future utility, and the factor $(1 - \delta)$ makes it convenient to compare the total revenue with a one-slot utility. In practice, an operator is concerned with the utilities over the course of many slots (at least days), hence the discount factor is typically very close to 1.

It is common in practice that operators deploy their transmitters on the same tower or the

\(^2\)Alternatively, one operator may infer about other operators’ usage based on its own quality of service, without active monitoring. This is not considered here.

\(^3\)In a more nuanced setting, the selected PSD is a mask that constrains the actual PSD which can vary over a slot.
towers close to each other. For simplicity, we assume for most of our discussion that the cells of operators completely overlap, all transmitters are colocated, and all receivers are one unit distance away from the transmitters. The key principles here apply to more general scenarios with partially overlapped cells. For operator $i$, the signal-to-interference-and-noise ratio (SINR) at time $t$ and frequency $f$ is expressed as

$$\gamma_i^t(f) = \frac{p_i^t(f)}{1 + \sum_{j \neq i} p_j^t(f)}.$$  

(2)

In general, let the “usefulness” per Hz at the vicinity of a certain frequency $f$ be $r(\gamma_i^t(f))$, where $r(\cdot)$ is strictly increasing (hence increasing the SINR makes the frequency more useful). In this paper, we also assume that the system operates in the interference-limited regime. In particular, $r(\cdot)$ satisfies:

$$r(P) > \sup_{n \geq 2} \left[ n \cdot r\left( \frac{P}{(n-1)P+1} \right) \right].$$  

(3)

Condition (3) implies that the sum utility that multiple operators extract by simultaneously sharing the same piece of spectrum with maximum transmit power is less than that can be extracted from exclusive use of the spectrum. A practically relevant example of $r(\cdot)$ is $r(\gamma) = \log(1 + \gamma)$ where $\int_S r(\gamma_i^t(f))df$ is the Shannon capacity of the additive white Gaussian noise channel. In that case, it is easy to show that if $P > 1.62$, then (3) is always satisfied.

For concreteness, we introduce one specific form of the utility function. The utility of an operator depends on the accumulated spectrum usefulness and the traffic intensity. Let the utility of operator $i$ at time $t$ take the following form:

$$u(p_i^t, p_i^{-t}, \lambda_i^t) = \pi \left( \frac{1}{r(P)} \int_S r(\gamma_i^t(f))df, \lambda_i^t \right).$$  

(4)

Without loss of generality, let $u(\cdot, \cdot, \cdot)$ be bounded and non-negative. Under orthogonal sharing, operator $i$ maximizes its utility by transmitting at peak power over its spectrum, yielding utility $\pi(w_i^t, \lambda_i^t)$, where $w_i^t$ is the bandwidth occupied by operator $i$ at time slot $t$. Indeed, the first argument on the right side of (4) can be viewed as the effective exclusive bandwidth occupied by operator $i$ that yields the same utility.

In this paper, we consider a class of functions $\pi$ satisfying some additional conditions:
a) $\pi(x, \lambda)$ is continuous, strictly increasing and strictly concave in $x$ for every $\lambda$. As a consequence, the incremental utility of adding spectrum usefulness decreases as the initial amount of spectrum usefulness increases. Precisely, for every $x, \lambda$, and $\Delta > 0$,

$$\pi(x + \Delta, \lambda) - \pi(x, \lambda) < \pi(x, \lambda) - \pi(x - \Delta, \lambda).$$

(5)

b) $\pi(x, \lambda)$ is finite and strictly supermodular, that is, adding an incremental amount of spectrum usefulness yields higher improvement in the utility as the traffic increases. Precisely, for every $x, \xi > \lambda$, and $\Delta > 0$,

$$\pi(x + \Delta, \lambda) - \pi(x, \lambda) < \pi(x + \Delta, \xi) - \pi(x, \xi).$$

(6)

Without coordination, each operator would choose to transmit over the full spectrum using the maximum power, referred to as the full-spectrum strategy. Clearly, if all operators employ the full-spectrum sharing, they all suffer the maximum interference, which results in poor spectral efficiency in the assumed interference-limited regime. Under full-spectrum sharing, the utility of operator $i$ at time $t$ is

$$\pi_f(\lambda^i_t) = \pi \left( \frac{W}{r(P)} \left( \frac{P}{P(n - 1) + 1} \right), \lambda^i_t \right),$$

(7)

and the expected utility of operator $i$ is

$$u^i_f = E \left[ \pi_f(\Lambda^i_t) \right].$$

(8)

Alternatively, if the operators avoid interfering with each other by using different parts of the spectrum, the spectral efficiency and the sum utility become much higher. This suggests that it may be beneficial for strategic operators to cooperate.

This paper addresses the situation where operators’ traffic is dynamic in general. A smaller slot length leads to better tracking of traffic intensities, which results in higher revenues. However, due to the limitations of techniques in spectrum monitoring and information exchange among operators, the slot length cannot be arbitrarily small. The slot length should be chosen to balance the benefit of spectrum agility with the cost, robustness, and other practical issues. Throughout
this paper, a fixed slot length is assumed, which is conceived to be between a few seconds to several minutes.

III. STATIC SHARING SCHEMES

In this section, we develop relatively simple sharing schemes, where the PSD of an operator does not depend on the traffic dynamics at equilibrium. We begin with the simplest case of two operators in Section III-A, then generalize to the case of arbitrary number of operators in Section III-B. In Section III-C, we study the situation where the number of operators is not determined a priori, and each newly arrived operator needs to choose whether to invest in order to use the spectrum.

Lemma 1. If (3) holds, orthogonal sharing with a uniform partition of spectrum achieves higher utility than full-spectrum sharing for all operators.

Proof: Under orthogonal sharing with a uniform partition of spectrum, the expected utility of operator $i$ in one slot is $u_i^o = \mathbb{E} \left[ \pi \left( \frac{W}{n}, \Lambda_i \right) \right]$. By assumption (3),

$$W_r \left( \frac{P}{1 + (n-1)P} \right) < \frac{W}{n} r(P).$$

(9)

Since $\pi(x, \lambda)$ is increasing in $x$ for every $\lambda$, we have $\pi \left( \frac{W}{n}, \lambda \right) > \pi \left( \frac{W}{r(P)}, r \left( \frac{P}{P(n-1)+1} \right), \lambda \right)$ for every $\lambda$. Hence, $u_i^o$ is larger than $u_j^f$ given in (8).

A. The Two-Operator Case

First, consider a system with two operators whose PSDs are chosen initially and remain fixed through the course of their transmissions. This is a one-shot game. The strategy space $\mathcal{P}$ is the set of feasible power spectral densities. A strategy profile $(p^1, p^2) \in \mathcal{P} \times \mathcal{P}$ is a strict Nash equilibrium if an operator becomes worse off by unilaterally deviating from it, i.e., $u(p^1, p^2, \lambda^1) > u(q, p^2, \lambda^1)$ and $u(p^2, p^1, \lambda^2) > u(q, p^1, \lambda^2)$ for every $\lambda^1, \lambda^2$, and $q \in \mathcal{P}$. The minimax utility of an operator is defined as the smallest utility that the other operators can force the operator to receive, without knowing his actions.

Lemma 2. In a one-shot game, all operators using the full-spectrum strategy is the unique (strict) Nash equilibrium. In particular, $\pi_f(\lambda^i)$ is the minimax utility of operator $i$ with traffic $\lambda^i$. 7
The proof is trivial due to assumption \((3)\) as each operator maximizes its utility by using the highest PSD regardless of the other operators’ choice.

In practice, an operator can vary its PSD over time. The operators’ sharing problem can be modeled as a repeated game over an infinite sequence of equal epochs, where an operator’s PSD remains fixed during each epoch but may be chosen arbitrarily depending on the history of the game. The players are the \(n\) operators; the action space \(\mathcal{P}\) is the set of feasible power spectral densities; the strategy space of an operator is the set of complete plans of actions that defines what PSD the operator will use in every possible event where the operator needs to act; and the payoff is the expected total revenue over the infinite time horizon in \((1)\). As was observed in \([12]\), the repeated game allows a richer set of Nash equilibria. In this work, we consider profiles which are subgame perfect Nash equilibria. The subgame perfect Nash equilibrium is a refined and stronger notion than the Nash equilibrium \([23]\), in the sense that such equilibria are not merely the consequence of non-credible threats, and thus are rationale and likely outcomes in practice.

**Lemma 3.** Let \((u^1, u^2)\) be a feasible utility pair, i.e., there exists a pair of PSDs the two operators can adopt to attain \((u^1, u^2)\) as their utilities. If \((u^1, u^2) > (u^1_f, u^2_f)\), then there is a subgame perfect Nash equilibrium where the corresponding expected total revenue pair is \((u^1, u^2)\) for some future discount factor \(\delta\) sufficiently close to 1.

**Proof:** We construct such a subgame perfect Nash equilibrium. Let the utility pair \((u^1, u^2)\) be attained by the PSD pair \((p^1, p^2)\), i.e., \(E[u(p^1, p^2, \Lambda^1_t)] = u^1\) and \(E[u(p^2, p^1, \Lambda^2_t)] = u^2\).

**Profile 1.** The strategy of each operator \(i \in \{1, 2\}\) is: Use PSD \(p^i\) in slot 0 and continue to use \(p^i\) in each subsequent slot, as long as the other operator \(i'\) \((i' \neq i)\) uses PSD \(p^{i'}\) in the previous slot; otherwise, transmit maximum power over the full spectrum hereafter.

It is convenient to refer to the slots in which the operators use \((p^1, p^2)\) as the cooperation state, and the remaining slots (if any) as the punishment state. We can use the one-shot deviation principle \([23, \text{Theorem 4.2}]\) to verify that Profile \([1]\) is a subgame perfect Nash equilibrium. In particular, no operator has any incentive to ever deviate from \((p^1, p^2)\). Indeed, once triggered, the punishment is everlasting to deter deviation from cooperation. Hence Lemma \([3]\) holds. ■
The cooperation defined in Profile 1 is not robust in practice. In case of any perceived deviation, even if due to false detection of the other operator’s spectrum usage, the operators will be trapped in the punishment state to yield poor utilities. We next consider a profile that provides incentives for deviating operators to return to the cooperation state. The idea is to return to the cooperation state after spending sufficiently many slots in the punishment state. Denote

$$U(\lambda) = \sup_{p^1, p^2} u(p^1, p^2, \lambda).$$

Choose the length of punishment $T$ such that for every $\lambda$ and $i = 1, 2$,

$$U(\lambda) - \pi \left( \frac{W}{2}, \lambda \right) < T(u^i - u^i_f),$$

which implies that one’s loss due to punishment is greater than the one-shot gain by deviating if there is no future discount.

**Profile 2.** The system starts from the cooperation state. Operator $i$ uses $p^i$ in slot 0. In each subsequent slot, the system state evolution and the strategy of operator $i \in \{1, 2\}$ depends on the state as follows:

I. (Cooperation state) If the PSDs were $(p^1, p^2)$ in the previous slot, then use $p^i$ in this slot; otherwise, transit to the punishment state.

II. (Punishment state) Transmit maximum power over full spectrum for $T$ slots before returning to the cooperation state.

**Lemma 4.** For any feasible vector $(u^1, u^2) > (u^i_f, u^j_f)$, Profile 2 is a subgame perfect Nash equilibrium that achieves the expected total revenue pair $(u^1, u^2)$, as long as the punishment duration $T$ satisfies (11) and the future discount factor $\delta$ is sufficiently close to 1.

The proof is similar to Theorem 2 in [24] for general repeated games. According to Lemmas 3 and 4 in order to achieve the total revenue vector $(u^1, u^2) > (u^i_f, u^j_f)$, the operators announce their strategies and agree to the selected utility vector in advance.

Under orthogonal sharing with a uniform partition of spectrum, the expected utility for operator $i$ is denoted as $u^i_o$. When $n = 2$, $(u^1_o, u^2_o)$ dominates $(u^i_f, u^j_f)$. From Lemma 4, the expected total revenue pair $(u^1_o, u^2_o)$ can be achieved by a Nash equilibrium in the repeated game, as depicted...
Fig. 1: Static spectrum sharing with equal and orthogonal partitions of spectrum in the cooperation state.

in Fig. 1. According to the corresponding strategy, each operator transmits in an exclusive half of the unlicensed spectrum and is unwilling to deviate.

Lemma 1 has important practical implications. In particular, it is much easier to verify orthogonal sharing by monitoring the PSD support than the exact PSD of another operator. Moreover, orthogonal sharing allows an operator to use its share of the spectrum as if it were licensed spectrum.

B. The $n$-Operator Case

The preceding sharing schemes can be easily extended to the case of $n$ operators. Denote a set of PSDs corresponding to equal and orthogonal sharing as $(p^i_o)^n_{j=1}$ such that $\mathbb{E}[u(p^i_o, p^{-i}_o, \Lambda_i)] = u^i_o$. According to Lemma 1, we have $u^i_o > u^j_i$. Define the maximum utility of operator $i$ with traffic intensity $\lambda$ in one time slot as $\bar{U}(\lambda) = \sup_p u(p, p^{-i}, \lambda)$. We denote the equal-sharing bandwidth as

$$w = \frac{W}{n}. \quad (12)$$

Then choose $T$ such that for every $i$ and $\lambda$,

$$\bar{U}(\lambda) - \pi(w, \lambda) < T(u^i_o - u^j_i) \quad (13)$$

Consider the following straightforward generalization of Profile 2.

Profile 3. The system starts from the cooperation state. Operator $i$ uses $p^i$ in slot 0. In each

$^4$It is easy to introduce a protocol to determine which values the operators choose to use.
subsequent slot, the system state evolution and the strategy of operator \( i \in \{1, \ldots, n\} \) depends on the state as follows:

I. (Cooperation state) If the PSDs were \((p^1, \ldots, p^n)\) in the previous slot, then use \(p^i\) in this slot; otherwise, transit to the punishment state.

II. (Punishment state) Transmit maximum power over full spectrum for \(T\) slots before returning to the cooperation state.

**Lemma 5.** Profile 3 is a subgame perfect Nash equilibrium for \(n\) operators as long as (3) and (13) hold and the future discount factor \(\delta\) is sufficiently close to 1.

The proof is a simple generalization of the 2-operator case (Lemma 4) and is omitted.

The static sharing scheme discussed so far is that all operators contending in a given area follow orthogonal sharing with equal partition of spectrum. Lemma 5 states that the operators have no incentive to deviate from such a strategy profile. It is not difficult to show that if the operators agree on a nonuniform split, equilibria can also be achieved under Profile 3. It is conceivable that the spectrum is divided in proportions to the relative amount of traffic the operators have. However, such a method is problematic as operators may inflate their traffic levels. We address the truthfulness issue in Section IV.

**C. Entry Problem**

The preceding discussions are based on the assumption that a given fixed number of operators share the spectrum in a given area. We now study the situation where an arbitrary number of strategic operators may arrive in a sequential manner. In order for an operator to use any part of the spectrum, it must make an investment (e.g., on network infrastructure) in advance. For simplicity, we assume all operators have identically distributed traffic intensities, power constraints, and investment costs (assumed to be \(c\)). If \(n\) operators have invested to share the spectrum, the expected utility of each operator in one slot is

\[
u_f(n) = \mathbb{E} \left[ \pi \left( \frac{W}{r(P)}r(P), \Lambda_i^1 \right) \right]
\]  

(14)
under full-spectrum sharing or is

\[ u_o(n) = E \left[ \pi \left( wr(P), \Lambda_i^t \right) \right] \]  

(15)

under equal and orthogonal sharing. The difference here is that an incumbent operator may change its action upon investment by a new operator. It is natural for each incumbent operator to choose between two actions, ‘to punish’ and ‘to cooperate’, when a new operator starts to share any spectrum. If all operators use full spectrum to punish the new operator, then everyone achieves the utility of \( u_f(n + 1) \); if all cooperate with the new operator by using \( \frac{1}{n+1} \) fraction of the spectrum, then each achieves the utility of \( u_o(n + 1) \).

According to Lemma 1, we have \( u_o(n) - u_f(n) \geq 0 \) for every \( n \). It is easy to verify that both \( u_f(n) \) and \( u_o(n) \) vanish as \( n \to \infty \). Therefore, if \( u_f(1) \geq c \), there exists \( n^* \) such that \( u_f(n^* + 1) < c \) and \( u_f(n^*) \geq c \). If \( u_f(1) < c \), we let \( n^* = 0 \). Evidently, larger \( c \) results in smaller \( n^* \). Denote \( U(\lambda) \) as the maximum utility of an operator with traffic \( \lambda \), which is achieved if the operator uses the full-spectrum strategy while other operators make no transmissions. Let \( T(n) \) be such that for every \( \lambda \),

\[ U(\lambda) - \pi (w, \lambda) < T(n) (u_o(n) - u_f(n)) \]  

(16)

**Profile 4.** The \( i \)-th operator to arrive does not invest if \( i > n^* \). If \( i \leq n^* \), the operator invests and performs the following in each slot thereafter:

- If there are \( n \leq n^* \) active operators, use Profile 3 for \( n \) operators.
- If there are more than \( n^* \) operators, always use the full-spectrum strategy.

**Theorem 1.** Profile 4 is a subgame perfect Nash equilibrium, as long as (3) and (16) hold and the future discount factor \( \delta \) is sufficiently close to 1.

**Proof:** We use the one-shot deviation principle in [23, Theorem 4.2] to prove the desired result. It suffices to show that if any operator deviates from Profile 4 in a single slot and then returns to conform to Profile 4, the operator suffers a net loss. For incumbent operators, first consider the one-shot deviation in the cooperation state for \( n \leq n^* \). Assuming an operator deviates at slot \( t \) in the cooperation state and conforms afterwards, the punishment starts at slot
If there is no new entrant during the punishment slots, then the utilities are identical to the case with a fixed number of operators. Hence the deviating operator’s utility decreases according to Lemma \[5\]. On the other hand, if a new operator starts transmission before the punishment ends, the new operator transmits maximum power over the full spectrum, further reducing the payoff of the deviator. The same arguments apply to the case where multiple new operators arrive before returning to cooperation. Moreover, the preceding analysis applies to the situation where the new operator \(i \leq n^*\) starts transmission but deviates by using a different PSD than required by the current state.

Since the minimax utility for incumbent operator with traffic \(\lambda\) is \(\pi_f(\lambda)\), deviation in the punishment state only lengthens the punishment and postpones the larger utility. For new entrant \(i\), if \(i \leq n^*\) (respectively, \(i > n^*\)), the total revenue is larger (smaller) than the investment cost. Hence, (one stage) deviation from Profile 4’s investment decision is not profitable. Therefore, Profile 4 is a subgame perfect Nash equilibrium.

According to Theorem \[1\], under the proposed profile, there will be at most \(n^*\) active operators in the market and all of operators obtain larger revenue than full-spectrum sharing. The proposed profile provides a way to share the spectrum efficiently when there are an indefinite number of strategic operators, thereby mitigating the effects of the tragedy of the commons.

IV. DYNAMIC SHARING SCHEMES

The static sharing strategies discussed in Section III are in general not the most efficient under dynamic traffic conditions. At any given time, some operators may have light traffic and hence excess spectrum, while others may have heavy traffic and hence experience a spectrum shortage. In this section, we allow the operators to adapt their spectrum usage to the traffic conditions in each time slot.

A sharing scheme can be either direct or indirect. With a direct scheme, all operators report their traffic intensities, and each operator uses a designated strategy according to all reported traffic intensities. With an indirect scheme, each operator may report other signals (i.e., desired actions) instead of traffic intensities, and then determines the spectrum to utilize according to ones own traffic intensity and other operators reports. By the Revelation Principle of Bayesian games \[25, Theorem 2\], for every Nash equilibrium of an indirect scheme, there exists a direct
scheme that is payoff-equivalent and in which truthful revelation is a Nash equilibrium. Thus, we focus on direct schemes in this paper.

Spectrum trade can take place either with or without monetary payment. Spectrum trade with payment has several drawbacks. It may encourage spectrum “trolls”—operators who serve few or no customer, yet demand payment so as to not cause interference. Besides, pricing, metering, and billing for spectrum usage require efforts beyond the physical layer and can be costly. This study is hence restricted to spectrum trade without monetary payment. The basic idea is that an operator in need of extra spectrum may borrow from another operator but needs to return the spectrum in the future.

The spectrum sharing with dynamic traffic is formulated as a repeated game with private information and communication. In each time slot, operators report their traffic intensities and adjust their spectrum usage based on the reported traffic intensities. Since the operators are assumed to be strategic, they do not have to report their actual traffic intensities. To achieve efficient spectrum sharing, we seek a direct scheme such that it is in the best interest of every operator to truthfully report their traffic.

Denote the net spectrum balance of operator $i$ at time $t$ as $b_{it}$. The assumption $b_{it} > 0$ (respectively, $b_{it} < 0$) means that up to time $t$ operator $i$ has lent more (respectively, less) spectrum than borrowed. For simplicity, we set a balance constraint as $|b_{it}| \leq \bar{b}$ for all $i$ and $t$ to preclude infinite borrowing (e.g., in the manner of a Ponzi scheme).

Throughout this section, we assume that in each slot, the traffic intensity of operator $i$ is either high ($\Lambda_{it} = 1$) or low ($\Lambda_{it} = 0$). It is further assumed that $(\Lambda_{it})$ are independent and identically distributed for every $i$. The two-level traffic assumption is meaningful in practice where the sharing protocol needs to be made simple. The main principles developed here can be extended to more general traffic assumptions.

A. The Two-Operator Case

We begin with the two-operator case. Let $\Delta \in (0, w]$ be the amount of spectrum that the operator with high traffic intensity borrows from the operator with low traffic intensity in one slot. Consider the following strategy profile.
Profile 5. The strategies of the two operators mirror each other. Let the system start from the cooperation state with beginning balances $b_{10} = b_{20} = 0$. Operator 1’s strategy at time $t$ depends on the state as follows:

I. Cooperation state:

a. Reveal its own traffic intensity $\lambda_1^t$, and learn $\lambda_2^t$ from operator 2.

b. If $\lambda_1^t > \lambda_2^t$ and $b_i^t - \Delta > -\bar{b}$, then use $w + \Delta$ Hz for transmission and reduce the balance to $b_{i+1}^t = b_i^t - \Delta$; if $\lambda_1^t < \lambda_2^t$ and $b_i^t + \Delta \leq \bar{b}$, then use $w - \Delta$ Hz for transmission and increase the balance to $b_{i+1}^t = b_i^t + \Delta$; otherwise, let $b_{i+1}^t = b_i^t$ and use $w$ Hz for transmission.

II. Punishment state: If any operator is detected to deviate from this profile in the previous slot, then use maximum power over the full spectrum for $T$ slots and then return to the cooperation state. The balances remain unchanged during the punishment state.

Because spectrum trade occurs only if there is sufficient balance, the balance $b_i^t$ remains within $[-\bar{b}, \bar{b}]$ at all times. Without loss of generality, let $\bar{b} = k\Delta$ where $k$ is a positive integer. There are two types of deviations: undetectable deviation, i.e., lying about one’s own traffic, and detectable deviation, i.e., using a different amount of spectrum than dictated by the profile.

Theorem 2. There exist $\Delta > 0$ and $T$ such that Profile 5 is a subgame perfect Nash equilibrium, as long as (3), (5) and (6) hold, $P(\Lambda_1^t = 0, \Lambda_2^t = 1) > 0$, $P(\Lambda_1^t = 1, \Lambda_2^t = 0) > 0$, and the future discount factor $\delta$ is sufficiently close to 1.

We use the one-shot deviation principle [23, Theorem 4.2] to prove this theorem. Undetectable and detectable deviations are addressed respectively. We relegate the detailed proof of Theorem 2 to Appendix A.

B. The n-Operator Case

The sharing scheme introduced in Section IV-A can be easily extended to the case of multiple operators. A trading policy needs to be set up, so that the borrowers know who the respective lenders are. Assume operators have perfect recall of trading history. This implies that each operator knows the current balances of all operators.
We will discuss a scheme for $n$ operators sharing $W$ Hz unlicensed spectrum. Denote the set of operators who report a high traffic intensity and have balance no less than $-\overline{b} + \Delta$ as $A_1$, and the set of operators who report a low traffic intensity and have balance no greater than $\overline{b} - \Delta$ as $A_0$. The trading policy $Q$ is that the operator with the $i$-th largest balance in $A_1$ borrows $\Delta$ Hz from the operator with the $i$-th smallest balance in $A_0$, for any $i \leq \min\{|A_1|, |A_0|\}$. This pairing and subsequent trade is one possible scheme. In general, we can map the current balances and reported traffic conditions to a desired spectrum allocation, where the key findings remain the same.

**Profile 6.** Let the system start from the cooperation state with $b_i^0 = 0$, for all $i$. Operator $i$’s strategy at time $t$ depends on the state as follows:

I. Cooperation state:
   a. Reveal the traffic intensity $\lambda_i^t$ and learn traffic intensities from other operators;
   b. If chosen to trade by the trading policy $Q$ and $\lambda_i^t = 1$, then use $w + \Delta$ Hz for transmission and let $b_i^{t+1} = b_i^t - \Delta$; if chosen to trade and $\lambda_i^t = 0$, then use $w - \Delta$ Hz for transmission and let $b_i^{t+1} = b_i^t + \Delta$; if not chosen, let $b_i^{t+1} = b_i^t$ and use $w$ Hz for transmission.

II. Punishment state: If any operator is detected to deviate from this profile in the previous slot, then use maximum power over the full spectrum for $T$ slots and then return to the cooperation state. The balances remain unchanged during the punishment state.

**Theorem 3.** There exist $\Delta > 0$ and $T$ such that Profile 6 is a subgame perfect Nash equilibrium, as long as (3), (5) and (6) hold, the future discount factor $\delta$ is sufficiently close to 1 and for every $i$, there exist $j$ such that $\Pr(\Lambda_i^t > \Lambda_j^t) > 0$.

Theorem 3 is proved in Appendix B.

In this section, we have shown the existence of subgame-perfect Nash equilibria with dynamic spectrum sharing. Profiles 5 and 6 are subgame-perfect Nash equilibria with truthful reporting of traffic intensities when the future discount is sufficiently close to 1. Dynamic sharing schemes can achieve substantial improvement in spectral efficiency in comparison with static sharing schemes.
V. NUMERICAL RESULTS

In this section, some numerical results are presented for the proposed schemes. Let $r(\gamma) = \log_2(1 + \gamma)$, $\delta = 0.99$, and $W = 100$ MHz.

First we show some numerical results for the entry game. Denote $n$ as the number of active operators. Assume $\pi(x, \lambda) = \lambda r(P)x$, $P = 100$ (i.e., 20 dB), and $E\Lambda_t^i = \frac{1}{2}$ for every $t$ and $i$. Here $n^*$ satisfies

$$50 \log_2 \left(1 + \frac{100}{100(n^* - 1) + 1}\right) \geq c \geq 50 \log_2 \left(1 + \frac{100}{100n^* + 1}\right).$$

Fig. 2 shows the relationship between the maximum number of active operators ($n^*$) and the investment cost ($c$). The trend in Fig. 2 appears to be approximately exponential. More operators are willing to invest as the cost goes down. Evidently, without the entry cost, there will be infinitely many operators, who all receive zero revenue.

Next, we present numerical results for the proposed static and dynamic schemes. We consider two operators and assume that the traffic intensity of an operator is either high ($\Lambda_t^i = 1$) or low ($\Lambda_t^i = 0$). Let $\pi(x, \lambda) = (24\lambda + 1)^{0.5}(r(P)x)^{0.9}$, where $\pi(\cdot, \cdot)$ is a simple example of the Cobb-Douglas production function, which is widely used in economics to represent the technological relationship between the amount of output and the amounts of multiple inputs.
Fig. 3: Total revenue for different sharing schemes: $\bar{b} = 50$ MHz.

Fig. 4: Improvement in total revenue by the proposed dynamic sharing for different balance limits: $\log_2(1 + P) = 8$.

(traffic and spectrum here) [27]. Assume that the traffic conditions of the two operators are independent and the probabilities of low traffic intensity for the two operators are 0.75 and 0.5, respectively. The expected total revenues from full-spectrum sharing for the operators are $2 \left( w \log_2 \left( 1 + \frac{P}{P+1} \right) \right)^{0.9}$ and $3 \left( w \log_2 \left( 1 + \frac{P}{P+1} \right) \right)^{0.9}$; The expected total revenues in the proposed static scheme (Profile 2) for the two operators are $2 \left( w \log_2 \left( 1 + P \right) \right)^{0.9}$ and $3 \left( w \log_2 \left( 1 + P \right) \right)^{0.9}$. In the proposed dynamic scheme (Profile 5), $\Delta$ is chosen to maximize the sum revenue of both
operators and let Profile 5 be a subgame perfect Nash equilibrium.

We compare the proposed static and dynamic schemes with full-spectrum sharing in Fig. 3. The proposed sharing schemes outperform full-spectrum sharing dramatically. As $\gamma$ goes up, the gain of the proposed sharing schemes increases. The proposed static scheme is better than full-spectrum sharing when $P > 2.1$ dB, and the proposed dynamic scheme offers additional gain over the static sharing. When $P = 30$ dB, the proposed dynamic scheme has 400% improvement over full-spectrum sharing and 16% improvement over the proposed static scheme. Fig. 4 shows the improvement from the proposed dynamic scheme over full-spread sharing under different balance limits. As $\bar{b}$ goes up, the gain approaches 500% when $\log_2(1 + P) = 8$. Even if $\bar{b}$ is small, the proposed dynamic scheme still provides a significant improvement.

VI. CONCLUSION

In this paper, we have used game theory techniques to study unlicensed spectrum sharing by multiple strategic operators. A static sharing scheme was first proposed for operators to share the spectrum in a given area and was shown to reach a subgame perfect Nash equilibrium. It has also been shown that the number of strategic operators willing to invest is limited due to entry barriers and externalities. A dynamic scheme for trading bandwidth has also been proposed, where operators with low traffic loads lend spectrum to those with high traffic loads, subject to a cumulative balance constraint on loaned bandwidth, which induces truthful reporting. Numerical results show that the proposed schemes can provide a substantial increase in spectral efficiency relative to full (uncoordinated) spectrum sharing. It is worth noting that a credible punishment scheme is in general necessary to deter deviation from the cooperation state.

APPENDIX A

PROOF OF THEOREM 2

We use the one-shot deviation principle \[23, \text{Theorem 4.2}\]. The proof consists of two parts, which address undetectable and detectable deviations, respectively.

In the first part of this proof, we discuss the one-shot deviation of reporting a false traffic intensity, which is assumed to be undetectable. From one-shot deviation principle, after the deviation, both operators conform to Profile 5. The subsequent actions of both operators are
functions of the balance and the random traffic conditions. The key to the proof is to recognize
that, because it is the most beneficial for an operator to borrow (respectively, lend) when spectrum
is most (respectively, least) needed, an operator cannot gain by unilaterally lying about its own
traffic.

Without loss of generality, suppose operator 1 lies about his traffic intensity, whereas operator 2
reveals its true traffic intensity. The traffic loads \((\lambda_1^t, \lambda_2^t)\) have four possible realizations: \((0,0)\),
\((0,1)\), \((1,0)\), and \((1,1)\). We discuss the case of \(\lambda_1^t = \lambda_2^t = 0\) in detail. The other three cases are
similar and omitted. If \(b_1^t \geq -\overline{b} + \Delta\) and operator 1 lies to report high traffic, then operator 1
borrows spectrum to use bandwidth \(w + \Delta\); otherwise, operator 1 uses bandwidth \(w\). Hence,
when operator 1 lies to report high traffic at time \(t\), operator 1 may benefit by using \(\Delta\) more
bandwidth. The one-slot gain of operator 1 by lying is

\[
G = \begin{cases} 
\pi(w + \Delta, 0) - \pi(w, 0), & \text{if } b_1^t \geq -\overline{b} + \Delta \\
0, & \text{otherwise.}
\end{cases}
\]  

(18)

We develop a detailed proof with full justification (especially of the change of limits). If
operator 1 does not use more spectrum from lying, i.e., \(G = 0\), the deviating case is identical
to the truth-telling case, so operator 1 does not gain by lying in this case. Hence we assume
otherwise subsequently. If operator 1 uses more spectrum by lying, then due to the balance
constraint, there will be one slot in the future, denoted as \(t^*\), when operator 1 either borrows
\(\Delta\) less (because it first hits the minimum balance) or lends \(\Delta\) more (because it first hits the
maximum balance) in comparison with the truth-telling case. The actions after slot \(t^*\) are the
same as in the truth-telling case.

Let \(p_\tau\) (respectively, \(q_\tau\)) denote the probability that the first time operator 1 borrows less
(respectively, lends more) than the truth-telling case is \(\tau\) slots after deviation. Clearly, \(\sum_{\tau=1}^{\infty} (p_\tau + q_\tau) = 1\). Note that \(p_\tau\) and \(q_\tau\) implicitly depend on the balance at the time of deviation. Since
\(P(\Lambda_1^t = 0, \Lambda_2^t = 1) > 0\) and \(P(\Lambda_1^t = 1, \Lambda_2^t = 0) > 0\), we have \(\sum_{\tau=1}^{\infty} p_\tau > 0\) and \(\sum_{\tau=1}^{\infty} q_\tau > 0\).
Define the expected loss at the slot with \(\tau\) slots after deviation as

\[
m_\tau = p_\tau (\pi(w + \Delta, 1) - \pi(w, 1)) + q_\tau (\pi(w, 0) - \pi(w - \Delta, 0)).
\]  

(19)
By assumption, $\pi(\cdot, \cdot)$ is increasing in the first argument, so $m_\tau > 0$. The total expected loss in the future is

$$L = \sum_{\tau=1}^{\infty} \delta^\tau m_\tau. \tag{20}$$

We show that $L > G$ if $\delta$ is sufficiently close to 1, i.e., the loss exceeds the one-slot gain. By (6) and the continuity of $\pi$,

$$\pi(w, 0) - \pi(w - \Delta, 0) < \pi(w, 1) - \pi(w - \Delta, 1) \tag{21}$$

$$= \pi(w + \Delta, 1) - \pi(w, 1) + o(\Delta). \tag{22}$$

Hence there must exist $\Delta > 0$ which satisfies

$$\pi(w, 0) - \pi(w - \Delta, 0) < \pi(w + \Delta, 1) - \pi(w, 1). \tag{23}$$

Then, $\sum_{\tau=1}^{\infty} m_\tau$ is upper bounded, since from (19) and (23)

$$\sum_{\tau=1}^{\infty} m_\tau < \sum_{\tau=1}^{\infty} (p_\tau + q_\tau)(\pi(w + \Delta, 1) - \pi(w, 1)) \tag{24}$$

$$= \pi(w + \Delta, 1) - \pi(w, 1). \tag{25}$$

Since $m_\tau \geq 0$, $\sum_{\tau=1}^{l} m_\tau$ is nondecreasing in $l$. Due to monotonicity and boundedness, $\sum_{\tau=1}^{l} m_\tau$ converges as $l$ goes to infinity. Also, $|\delta^\tau m_\tau| \leq m_\tau$ since $\delta \in [0, 1)$. According to [26, Theorem 7.10], $\sum_{\tau=1}^{l} \delta^\tau m_\tau$ converges uniformly. Also, by [26, Theorem 7.11], we have

$$\lim_{\delta \to 1} \lim_{l \to \infty} \sum_{\tau=1}^{l} \delta^\tau m_\tau = \lim_{l \to \infty} \lim_{\delta \to 1} \sum_{\tau=1}^{l} \delta^\tau m_\tau \tag{26}$$

$$= \lim_{l \to \infty} \sum_{\tau=1}^{l} m_\tau. \tag{27}$$
Thus, by (19), (20), (23) and (27),

\[
\lim_{\delta \to 1} L = \sum_{\tau=1}^{\infty} m_{\tau} \quad (28)
\]
\[
> \sum_{\tau=1}^{\infty} (p_\tau + q_\tau) (\pi(w,0) - \pi(w - \Delta,0)) \quad (29)
\]
\[
= \pi(w,0) - \pi(w - \Delta,0). \quad (30)
\]

Because \( L \) increases with \( \delta \), there exists \( \delta_0 \), such that for every \( \delta > \delta_0 \),

\[
L > \pi(w,0) - \pi(w - \Delta,0). \quad (31)
\]

Due to (5), (18) and (31), we have \( L \geq G \). Therefore, if both operators have low traffic intensities, then operator 1 suffers net loss in the revenue by reporting high traffic intensity. Using similar arguments, one can show that an operator always suffers a net loss by unilaterally lying in the traffic under all traffic conditions.

In the second part of this proof, we consider detectable one-shot deviation, which will trigger the punishment state. We shall show that when the length of punishment is sufficiently long, it wipes out more than the gain from one-shot deviation.

Here we consider the case where operator 1 deviates, so the superscript will be dropped when it is for operator 1. Denote the maximum utility of operator 1 with traffic \( \lambda \) in one time slot as \( \overline{U}(\lambda) \) given by (10). Let the expected total revenue of operator 1 starting from balance \( b \) without deviation be denoted as \( V(b) \). Because it is a repeated game where the only memory in the system is the balance, \( V(b) \) does not depend on time.

Consider a detectable one-shot deviation, where operator 1 deviates in the cooperation state at time \( t \) and then conforms thereafter. Operator 1 receives at most \( \overline{U}(\lambda_t) \) in the slot when operator 1 deviates, experiences the punishment for \( T \) slots and achieves \( V(b_{t+1}) \) thereafter. The total revenue of operator 1 from deviation is upper-bounded by

\[
V_{dev}(b_t) = \overline{U}(\lambda_t) + \sum_{\tau=t+1}^{t+T} \delta^{\tau-t} \pi_f(\lambda_\tau) + \delta^{T+1} V(b_{t+1}). \quad (32)
\]
If each operator conforms to the profile, operator 1 obtains

\[ V(b_t) = \pi(w_t, \lambda_t) + \sum_{\tau=t+1}^{t+T} \delta^{\tau-t} \pi(w_\tau, \lambda_\tau) + \delta^{T+1} V(b_{t+T+1}) \]  

(33)

where \( w_t \) is the bandwidth of operator 1 in slot \( t \) when both operators conform. The gain by the deviation is less than

\[ V_{dev}(b_t) - V(b_t) = \bar{U}(\lambda_t) - \pi(w_t, \lambda_t) + \delta^{T+1} (V(b_{t+1}) - V(b_{t+T+1})) \]

(34)

- \[ t \to t+T \sum_{\tau=t+1}^{t+T} \delta^{\tau-t} (\pi(w_\tau, \lambda_\tau) - \pi_f(\lambda_\tau)). \]

We first show that \( \bar{U}(\lambda_t) - \pi(w_t, \lambda_t) \) and \( \delta^{T+1} (V(b_{t+1}) - V(b_{t+T+1})) \) in (34) are upper bounded. Since the utility function is finite, \( \bar{U}(\lambda_t) - \pi(w_t, \lambda_t) \) is upper bounded, assuming the upper-bound is \( z_1 \). If both operators conform, operator 1’s utilities starting from balance \( b \) and \( b + \Delta \) are only different at one slot in the future, denoted as \( t^* \). Thus,

\[ V(b + \Delta) - V(b) \]

\[ = \begin{cases} 
\delta^{t^*} (\pi(w + \Delta, 1) - \pi(w, 1)), & \text{if it first hits the minimum balance} \\
\delta^{t^*} (\pi(w, 0) - \pi(w - \Delta, 0)), & \text{if it first hits the maximum balance}.
\end{cases} \]  

(35)

From (23), (35) and the fact that \( \delta < 1 \),

\[ V(b + \Delta) - V(b) \leq \pi(w + \Delta, 1) - \pi(w, 1). \]  

(36)

From (36) and the fact that \( k = \frac{\delta}{\Delta} \) and \( \delta < 1 \),

\[ \delta^{T+1} (V(b_{t+1}) - V(b_{t+T+1})) \leq V(b_{t+1}) - V(b_{t+T+1}) \]

\[ \leq V(\bar{b}) - V(-\bar{b}) \]  

(37)

\[ \leq \sum_{m=1}^{2k} (V(-\bar{b} + m \cdot \Delta) - V(-\bar{b} + (m - 1) \cdot \Delta)) \]  

(38)

\[ \leq 2k(\pi(w + \Delta, 1) - \pi(w, 1)). \]  

(39)

Thus, \( \delta^{T+1} (V(b_{t+1}) - V(b_{t+T+1})) \) is upper bounded by \( z_2 = 2k(\pi(w + \Delta, 1) - \pi(w, 1)) \).

In the following, we show that \( \sum_{\tau=t+1}^{t+T} \delta^{\tau-t} (\pi(w_\tau, \lambda_\tau) - \pi_f(\lambda_\tau)) \) in (34) can be arbitrarily
large for sufficiently large $T$ and $\delta$. By the assumption that $\pi(x, \lambda)$ is supermodular, for any $\xi > \lambda$,

$$\pi(w, \xi) - \pi_f(\xi) > \pi(w, \lambda) - \pi_f(\lambda).$$

(41)

According to (5) and (41), we have

$$\pi(w + \Delta, 1) - \pi_f(1) > \pi(w, 1) - \pi_f(1)$$

(42)

$$> \pi(w, 0) - \pi_f(0)$$

(43)

$$> \pi(w - \Delta, 0) - \pi_f(0).$$

(44)

From Lemma 1, there exist $\Delta > 0$ and $z_3 > 0$ such that the right side of (44) is positive. Since $\pi(w_\tau, \lambda_\tau)$ takes value from $\{\pi(w + \Delta, 1), \pi(w, 1), \pi(w, 0), \pi(w - \Delta, 0)\}$ when both operators conform,

$$\pi(w_\tau, \lambda_\tau) - \pi_f(\lambda_\tau) \geq z_3.$$  

(45)

There exists $T$ such that $z_1 + z_2 - z_3 T < 0$, i.e., the loss by punishment is larger than the gain by one-shot deviation.

Therefore, if $T$ is large enough and $\delta$ is sufficiently close to 1, the detectable one-shot deviation in the cooperation state results in a lower revenue.

If operator 1 deviates in the punishment state at time $t$, operator 1 obtains at most $\pi_f(\lambda_t)$ in the slot when operator 1 deviates, since $\pi_f(\lambda_t)$ is the min-max utility. Then the deviation only lengthens the punishment and postpones the larger utility in coordination state. Therefore, any detectable one-shot deviation is undesired.

To summarize, if $\delta$ is sufficiently close to 1, any one-shot deviation by operator 1 is non-profitable. The same conclusion applies to operator 2 by symmetry. Therefore, there exists $\Delta > 0$ and $T$, such that if $\delta$ is sufficiently close to 1, the strategy profile is a subgame perfect Nash equilibrium.
Appendix B

Proof of Theorem 3

Similar as in Theorem 2, both undetectable and detectable deviations need to be addressed. The proof for the case of detectable deviation is essentially the same as that in Theorem 2. Hence, here we only discuss the one-shot deviation of reporting a false traffic intensity, which is undetectable. The proof is based on the one-shot deviation principle.

For the 2-operator case, an operator can access at most $\Delta$ more spectrum in one slot by lying in comparison with the truthful reporting. However, for the $n$-operator case, it is possible for an operator to be chosen for trade no matter what the operator reports. If an operator is chosen to borrow (lend), the operator uses $\Delta$ Hz more (less) spectrum. Thus, an operator can access up to $2\Delta$ Hz more spectrum in one slot by lying.

Consider the event where the traffic intensity of operator $i$ is low. Denote $F$ as the event that operator 1 would be chosen as a borrower if it lies to report a high traffic intensity. Denote $E$ as the event that operator $i$ would be chosen as a lender if it tells the truth. When operator $i$ lies to report a high traffic intensity, operator $i$ may benefit by using more bandwidth in comparison with the truth-telling case. The gain by lying to report high traffic intensity is expressed as follows depending on the sub-events:

$$G = \begin{cases} 
\pi(w + \Delta, 0) - \pi(w, 0), & \text{if } F \cap \overline{E} \\
\pi(w, 0) - \pi(w - \Delta, 0), & \text{if } \overline{F} \cap E \\
\pi(w + \Delta, 0) - \pi(w - \Delta, 0), & \text{if } F \cap E \\
0, & \text{if } \overline{F} \cap \overline{E}.
\end{cases}$$

(46)

All operators conform to Profile 6 after the one-shot deviation. Similar as the proof in Theorem 2, due to the balance constraint, if operator $i$ uses more bandwidth by lying, operator $i$ either borrows less (because it first hits the minimum balance) or lends more (because it first hits the maximum balance) in the future in comparison with the truth-telling case.

It suffices to show that an operator cannot gain by lying about its own traffic for different cases in (46). The proof of the cases of $\overline{F} \cap E$ and $F \cap \overline{E}$ is similar to Theorem 2. In the case of $\overline{F} \cap \overline{E}$, there is no change in spectrum use and revenue by lying. If $F \cap E$ occurs, operator $i$ uses...
$2\Delta$ Hz more spectrum when the deviation occurs. According to Profile $6$, operator $i$ will use $\Delta$ Hz less spectrum in two slots in the future. Using the analogous argument as (31) in Theorem $2$, we can show that in each of the two slots when the loss occurs, the expected one-shot loss is greater than $\pi(w, 0) - \pi(w - \Delta, 0)$. Thus, the expected total loss of the one-shot deviation is greater than

$$2(\pi(w, 0) - \pi(w - \Delta, 0)).$$

From (5),

$$2(\pi(w, 0) - \pi(w - \Delta, 0)) > \pi(w + \Delta, 0) - \pi(w, 0) + \pi(w, 0) - \pi(w - \Delta, 0) \quad (48)$$

$$> \pi(w + \Delta, 0) - \pi(w - \Delta, 0). \quad (49)$$

That is, the expected total loss is greater than the one-shot gain when $F \cap E$ happens. Thus, if operator $i$ has a low traffic intensity, to report a high traffic intensity brings no gain.

The case where the traffic intensity of operator $i$ is high can be treated similarly, and hence is omitted. Thus, if $\delta$ is sufficiently close to 1, any one-shot undetectable deviation by operator $i$ is non-profitable.

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