Research on the roundness error separation method for precision spindle based on harmonic wavelet de-noise and mathematical statistics method

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Abstract. Based on the online measurement and evaluation technologies of roundness error for precision machine tool spindle, not only the processing quality of spindle component can be evaluated, but also form error and surface roughness of machining workpiece can be forecasted, and then the suitable compensation control methods can be implemented to improve the processing accuracy of workpiece. According to the geometric characteristics of the roundness error, the combinatorial method of mathematical statistics and harmonic wavelet de-noise is proposed to separate the roundness error of the spindle. First, the collected signal by the sensor is filtered by harmonic wavelet de-noise method, and then the roundness error signal is separated by the mathematical statistics method. The simulation analysis shows that the high separating accuracy of roundness error can be obtained even in the case of small sample by the proposed method in the paper.

1. Introduction
Roundness error is an important quality index of the precision machine tool spindle, and it has a certain impact on the form-position error and surface quality of the machining workpiece. Based on the measurement and calculation of roundness error for precision spindle, the influence of roundness error on the machining accuracy of workpiece can be analysed, and the compensation control method for roundness error can be implemented so as to improve the machining accuracy of workpiece [1,2]. Therefore, roundness error is a very important monitoring parameter in the production and application of precision machine tool spindle.

With the development of detection technology, the sub-micron or even nano-scale measurement has been achieved for displacement signals. However, a large number of noise signals will be unavoidably mixed in the measurement signal along with the high-precision measurement. Therefore, how to accurately extract and evaluate the roundness error signal is the focus of related research. Based on the analysis of the four evaluation methods of roundness error, a least square method and the minimum area method was put forward by Liu Qingmin et al. [3]. Based on the dynamic characteristics of local least squares circle parameters, a highly robust evaluation method of workpiece roundness was designed by Cao Zhimin et al. [4]. Based on the geometric characteristics of roundness error, the roundness evaluation method of adaptive region search algorithm was proposed by Gong Yuling et al. [5]. Based on the evaluation results of roundness error, the expression guide of measurement uncertainty and Monte Carlo method were used to evaluate the uncertainty of roundness error by Wang Dongxia et al. [6].
The above separation and evaluation methods for roundness error mostly involve complex theoretical calculation problems. The traditional separation method for roundness error based on mathematical statistics is relatively simple and easy to popularize, but there are also some problems such as low separating accuracy, large statistical samples and so on. To solve these problems, a roundness error separation method is proposed based on harmonic wavelet de-noise and mathematical statistics, and its advantages of proposed method are verified by simulation in the paper.

2. The separating method of roundness error

2.1. The geometric characteristics of roundness error

The radial and periodic properties are two main geometric characteristics of roundness error. The radial property means that the roundness error is reflected in the radius direction of the axis circumference. The periodic property means that the roundness error has the same rotating frequency with the spindle.

Based on the above characteristics, the periodic property of roundness error can be expressed by Fourier series, which is shown in equation (1):

\[
 r(\omega t) = r_0 + \sum_{i=1}^{\infty} a_i \cos i\omega t + \sum_{i=1}^{\infty} b_i \sin i\omega t = r_0 + \sum_{i=1}^{\infty} c_i \sin(i\omega t + \alpha_i)
\]

And, \( c_i = \sqrt{a_i^2 + b_i^2} \), \( \alpha_i = \arctan \frac{a_i}{b_i} \)

In equation (1), \( r(\omega t) \) is the radius at the phase of \( \omega t \), \( r_0 \) is the theoretical radius of spindle, \( a_i \) and \( b_i \) are the Fourier coefficient, \( c_i \) and \( \alpha_i \) are the amplitude and phase of the i-th harmonic component respectively.

When \( i=1 \), it means that the base circle has a certain eccentricity, which is shown as an eccentric circle with fixed radius in the polar coordinates. When \( i=2 \), it is shown as an ellipse in the polar coordinates. When \( i=3 \), it is shown as a triangular circle in the polar coordinates. The n-th harmonic wave is shown as an n-edged circle in the polar coordinates by analogy. According to the definition of roundness error, the influences of eccentricity, surface roughness (higher harmonic component) and surface waviness on roundness error should be eliminated, that is, the roundness error function can be expressed as equation (2).

\[
 \Delta r(\omega t) = \sum_{i=2}^{\infty} a_i \cos i\omega t + \sum_{i=2}^{\infty} b_i \sin i\omega t = \sum_{i=2}^{\infty} c_i \sin(i\omega t + \alpha_i)
\]

In equation (2), \( n \) denotes the highest order of the harmonics, and the signal, which the order is higher than \( n \), can be regarded as the noise and surface roughness signals.

2.2. The roundness error separation method based on mathematical statistics method

In practical application, the radial vibration displacement signals of the spindle can be directly measured by the sensors, and the roundness error signal of the spindle can be separated by mathematical statistics method. The principle of mathematical statistics method is expressed as follows.

The integer period sampling method is applied to collect the vibration displacement signal on monitoring points, and the sampling frequency is assumed as \( f_s \), the rotating frequency of spindle is assumed as \( f \), the sampling points number per cycle can be calculated as \( N = f_s/f \). If the period number of sampling is assumed as \( M \), the \( N*\)M displacement values can be recorded.

Taking the rotation centre of the spindle as the coordinate, the Cartesian coordinate system is established firstly. If the distance between the intersection point of the spindle outline and the positive direction of the X axis and the coordinate origin is assumed as \( d \), the equation (3) can be gained for any sampling point \( i \) under the rotating condition of spindle.
In equation (3), $d_i$ is the distance between the intersection point of the spindle outline and the positive direction of the X axis and the coordinate origin at the i-th sampling point position. $x_i$ and $r_i$ are the rotation error and shape error of spindle at the i-th sampling point position respectively.

Taking into account the periodicity of spindle rotation, the mean value of multiple times sampling at the same position can be obtained and expressed as equation (4).

$$\sum_{j=1}^{M} d_{(i+N\times j)} = \sum_{j=1}^{M} x_{(i+N\times j)} + \sum_{j=1}^{M} r_{(i+N\times j)} \quad (i = 1, 2, \ldots, N)$$

$$D_i = \frac{\sum_{j=1}^{M} d_{(i+N\times j)}}{M}, \quad X_i = \frac{\sum_{j=1}^{M} x_{(i+N\times j)}}{M}, \quad R_i = \frac{\sum_{j=1}^{M} r_{(i+N\times j)}}{M}$$

In theory, if the sample length is large enough, the rotation error tends to be constant (expressed as $X_i \to I$), and the roundness error at the i-th sampling point can be obtained and expressed as equation (5).

$$R_i = D_i - I \quad (i = 1, 2, \ldots, N)$$

3. The simulation analysis for rotation error of spindle

3.1. The parameters settings for simulation signal

According to the form of equation (1), the 8-th harmonic (the roundness error is 8 edges circle) signal is taken as an example to simulate and analyse the roundness error in the paper. The diameter of the spindle is set as 20 mm, the speed of spindle is set as 600 rpm, and the sampling frequency is set as 6000. The other simulation parameters are shown in Table 1.

| Order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|-------|---|---|---|---|---|---|---|---|
| $c_i$ (mm) | 0.1 | 0.08 | 0.05 | 0.04 | 0.02 | 0.01 | 0.005 | 0.002 |
| $\alpha_i$ | random | random | random | random | random | random | random | random |

Based on the above parameters, the uniform white noise signal (the amplitude is 0.05mm) is added to the simulation signal to simulate the surface roughness error of spindle. The four-cycle simulation signal is shown in Figure 1 by the integer period sampling method.

![Figure 1. The simulated signal of spindle shape and position error](image-url)
Based on the method of mathematical statistics, the outlines of the spindle before and after the rotation error is removed are shown in Figure 2.

![Diagram](image)

(a) The outlines with rotation error of the spindle  
(b) The outlines without rotation error of the spindle

Figure 2. The roundness error signal

According to the definition of roundness error, the roundness errors in Figure 2 (a) and Figure 2 (b) are calculated respectively. The values of roundness errors in Figure 2 (a) and (b) are 0.58145mm and 0.43427mm respectively. It can be seen that the most rotation errors of the spindle are eliminated by comparing the values of roundness errors in Figure 2 (a) with (b), and the rest message can be regarded as roundness errors.

3.2. Improvement of roundness error separation accuracy

According to the principle of the mathematical statistics method, the more the number of cycles of whole-period sampling is, the higher the separating accuracy of roundness error is. To illustrate the problem, the roundness errors values of simulation signals with different sampling cycles are calculated respectively, and the calculating results are shown in Table 2. From Table 2, it can be seen that the separation accuracy of roundness error can be improved by enlarging the sample length of monitoring signal, but the improving effect of the accuracy is not significant. In practical applications, increasing the number of sampling cycles makes the signal acquisition time longer, and the sample length increases, the amount of data calculation is larger, which is not conducive to analyse and calculate the sampling signal.

According to the principle of mathematical statistics method, the roundness error separating accuracy can be seriously affected by the surface roughness of spindle and the high frequency noise signal in signal measurement. In order to improve the separating accuracy of roundness error, the appropriate filtering and re-noise method is chosen to reduce the interference signal firstly, and then mathematical statistics method is used to separate the roundness error in the paper.

By analysing the common de-noising methods, the wavelet de-noise has good time-frequency characteristics and it is widely used to processed the stationary and non-stationary signal. Based on the wavelet theory, the harmonic wavelet analysis can analyse the non-stationary and strong noise signals, and the harmonic wavelet has good phase preservation function for each harmonic component [7]. This feature is very advantageous to separate the roundness errors with periodic characteristics. Please refer to the relevant references for the theory of harmonic wavelet. In this paper, the method of harmonic wavelet de-noise is used to de-noise the vibration signal of the spindle firstly, and then the roundness error is fitted by mathematical statistics. The simulation signals after noise reduction are shown in Figure 3(a). The roundness errors with the radius of spindle are shown in Figure 3(b). The roundness error value after noise reduction is shown in Table 2.
Figure 3. The time domain signal and roundness errors after noise reduction

Table 2. The fitting data of roundness errors after noise reduction

| The period number of sampling | The fitting data of roundness errors before noise reduction | The fitting data of roundness errors after noise reduction |
|-------------------------------|-------------------------------------------------------------|----------------------------------------------------------|
|                               | The value of roundness errors | The relative error of roundness errors | The value of roundness errors | The relative error of roundness errors |
| 4                             | 0.43427mm | 4.3427% | 0.27773mm | 2.7773% |
| 12                            | 0.39419mm | 3.9419% | 0.27629mm | 2.7629% |
| 36                            | 0.37659mm | 3.7659% | 0.27344mm | 2.7344% |
| 108                           | 0.37196mm | 3.7196% | 0.2731mm | 2.731% |
| 324                           | 0.35729mm | 3.5729% | 0.27263mm | 2.7263% |

Notes: The relative error = The value of roundness errors / Radius of spindle x 100%, and the minimum inclusion zone method is used to evaluate the roundness errors.

Comparing the values of roundness errors before and after noise reduction in Table 2, it can be seen that the separation accuracy of roundness error can be significantly improved by using harmonic wavelet de-noising method to eliminate the interference of surface roughness and other high frequency signals. At the same time, the separation accuracy of roundness error is not improved obviously by increasing sampling cycles after the sampled signal has been filtered. Therefore, the higher roundness error separation accuracy can be achieved by the combination method of harmonic wavelet de-noise and mathematical statistics method in the case of small samples.

4. Conclusions
Based on the radial and periodic characteristics of roundness errors, the roundness errors of the spindle can be directly separated by the mathematical statistics method. The separation accuracy of roundness errors can be improved by increasing the number of sampling cycles, but the improvement of the accuracy is not very obvious. In order to achieve a certain separation accuracy, then sample data must be huge, which is not conducive to signal analysis and calculation.

In order to achieve high separation accuracy for roundness errors, the harmonic wavelet de-noise method is used to eliminate the interference of surface roughness and other high frequency signals firstly, and then the mathematical statistics method is used to separate the roundness error of spindle. The simulation analysis indicates that the separating accuracy of roundness error can be significantly improved based on the combination method of harmonic wavelet de-noise and mathematical statistics method in the case of small samples, and the separating calculation accuracy is stable.

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References
[1] Zhang, G.Q., Yu, H.C., Zhao, Z.X., Wang, W.B., Wang, R.Z. (2018) Research on the influence of the roundness error for aerostatic spindle rotating accuracy. Manufacturing Technology & Machine Tool, 3:55-59
[2] Brinkmann, S., Bodschwinna, H., Lemke, H. W. (2001) Accessing roughness in three-dimensions using Gaussian regression filtering. International Journal of Machine Tools and Manufacture, 41(13): 2153-2161.
[3] Liu, Q.M., Zhang, L., Wu, L.Q., Ji J.B. (2016) Roundness error evaluation of non-uniformly distributed data points based on machine vision. ACTA Metrologica Sinica, 37(6):567-570
[4] Cao, Z.M., Lv, X.L., Han J., Wu, Y., Song, H.M., Zhao, L.H. (2017) A robust roundness error evaluation method based on dynamic feature analysis of least square circles. Control and Instruments in Chemical Industry, 44(6):563-566
[5] Gong, Y.L., Xu, X.D., Su, Z.N., Gong, H. (2017) Roundness error evaluation based on the algorithm of self-adaptive region searching. Manufacturing Automation, 39(3):56-59
[6] Wang, D.X., Wen, X.L., Qiao, F.F. (2018) Estimation of uncertainty in measuring the workpiece circularity error. Optics and Precision Engineering, 26(10):2438-2445
[7] Li, J.M., Wei, H.J., Wei, L.D., Yang, Z.Y., Liu, C., Liu, H. (2017) Frictional vibration signals based on harmonic wavelet and detrended fluctuation analysis. Journal of Vibration and Shock, 36(15):235-239