ON THE LOCALIZATION PRINCIPLE FOR THE AUTOMORPHISMS OF PSEUDOELLIPSOIDS

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Abstract. We show that Alexander’s extendibility theorem for a local automorphism of the unit ball is valid also for a local automorphism $f$ of a pseudoellipsoid $E^{n}_{(p_{1},\ldots,p_{k})}$, when

$$E^{n}_{(p_{1},\ldots,p_{k})} = \{ z \in \mathbb{C}^{n} : \sum_{j=1}^{n-k} |z_{j}|^{2} + |z_{n-k+1}|^{2p_{1}} + \cdots + |z_{n}|^{2p_{k}} < 1 \}.$$ 

provided that $f$ is defined on a region $U \subset E^{n}_{(p)}$ such that: i) $\partial U \cap \partial E^{n}_{(p)}$ contains an open set of strongly pseudoconvex points; ii) $U \cap \{ z_{i} = 0 \} \neq \emptyset$ for any $n-k+1 \leq i \leq n$. By the counterexamples we exhibit, such hypotheses can be considered as optimal.

1. Introduction

For a given $k$-tuple of integers $p = (p_{1},\ldots,p_{k})$, with each $p_{i} \geq 2$, let us denote by $E^{n}_{(p_{1},\ldots,p_{k})}$ (or, more simply, $E^{n}_{(p)}$) the pseudoellipsoid in $\mathbb{C}^{n}$ defined by

$$E^{n}_{(p_{1},\ldots,p_{k})} = \{ z \in \mathbb{C}^{n} : \sum_{j=1}^{n-k} |z_{j}|^{2} + |z_{n-k+1}|^{2p_{1}} + \cdots + |z_{n}|^{2p_{k}} < 1 \}.$$ 

When $k = 0$, we assume $E^{n}_{(p)}$ to be the unit ball $B^{n} = \{ z \in \mathbb{C}^{n} : |z| < 1 \}$. Now, let us consider the following definition.

Definition 1.1. We define a local automorphism of $E^{n}_{(p)}$ to be any biholomorphic map $f : U_{1} \subset E^{n}_{(p)} \rightarrow U_{2} \subset E^{n}_{(p)}$ between two connected open subsets of $E^{n}_{(p)}$ such that:

a) each of the intersections $\partial U_{i} \cap \partial E^{n}_{(p)}$, $i = 1,2$, contains a boundary open set $\Gamma_{i} \subset \partial E^{n}_{(p)}$;

b) there exists at least one sequence $\{ x_{k} \} \subset U_{1}$ which converges to a point $x_{o} \in \Gamma_{1}$, which is not a limit point of $\partial U_{1} \cap E^{n}_{(p)}$, and so that $\{ f(x_{k}) \}$ converges to a point $\hat{x}_{o} \in \Gamma_{2}$, which is not a limit point of $\partial U_{2} \cap E^{n}_{(p)}$.

We say that a local automorphism $f : U_{1} \subset E^{n}_{(p)} \rightarrow U_{2} \subset E^{n}_{(p)}$ extends to a global automorphism of $E^{n}_{(p)}$ if there exists some $F \in \text{Aut}(E^{n}_{(p)})$ such that $F|_{U_{1} \cap E^{n}_{(p)}} = f|_{U_{1} \cap E^{n}_{(p)}}$.

By a celebrated theorem of Alexander and its generalization obtained by Rudin ([Al, Ru]), when $E^{n}_{(p)} = B^{n}$, any local automorphism extends to a global one.
This crucial extendibility result is often referred to as the localization principle for the automorphisms of \(B^n\), and it has been extended or established under different but similar hypotheses for a wide class of domains besides the unit balls (see e.g. [DS, Pi, Pi1]). On the other hand, even if it is known that the pseudoellipsoids \(E_n(p)\) share many useful properties with \(B^n\) for what concerns the global automorphisms and the proper holomorphic maps (see for instance [We, La, LS, DS]), some simple examples show that Alexander’s theorem cannot be true in full generality for a pseudoellipsoid \(E_n(p)\) different from \(B^n\) (see e.g. Example 3.4 below).

Nonetheless, for each \(E_n(p)\), it is possible to determine, precisely and in an efficient way, the class of local automorphisms that can be extended to global ones. In this short note we give a characterization of such local automorphisms by means of the following generalization of Alexander’s theorem.

**Theorem 1.2.** Let \(f : U_1 \subset E_n(p) \to U_2 \subset E_n(p)\) be a local automorphism of a pseudoellipsoid \(E_n(p)\), with \(p = (p_1, \ldots, p_k)\), and satisfying the following two conditions:

i) there exists a sequence \(\{x_i\}\) as in (b) of Definition 1.1 whose limit point \(x_o \in \partial E_n(p)\) is Levi non-degenerate;

ii) for any \(n - k + 1 \leq i \leq n\), the intersection \(U_1 \cap \{z_i = 0\}\) is not empty.

Then \(f\) extends to a global automorphism \(f \in \text{Aut}(E_n(p))\).

We point out that the set \(\partial E_n(p) \cap \bigcup_{i=n-k+1}^{n} \{z_i = 0\}\) coincides with the set of points of Levi degeneracy of \(\partial E_n(p)\). So, Theorem 1.2 can be roughly stated by saying that \(f\) is globally extendible as soon as it admits a holomorphic extension to some open subset \(U \subset E_n(p)\), which intersects each of the hyperplanes containing the Levi degeneracy set of \(\partial E_n(p)\) and, at the same time, the boundary \(\partial U\) contains an open set of strongly pseudoconvex points of \(\partial E_n(p)\).

From Example 3.4 it will be clear that such hypotheses can be considered as optimal.

The properties of the pseudoellipsoid used in the proof are basically just two: (1) It admits a finite ramified covering over the unit ball; (2) Its automorphisms are “lifts” of the automorphisms of the unit ball that preserve the singular values of the covering. Since (2) is a consequence of (1), it is reasonable to expect that a similar result should be true for any arbitrary ramified covering of the unit ball.

About this more general problem, we refer to [KLS, KS] for what concerns the classification of the domains in \(\mathbb{C}^2\) that admit a ramified holomorphic covering over \(B^2\).

2. **On the Automorphisms of the Unit Ball**

First of all, we need to recall some basic facts on the automorphisms of the unit ball. Let us denote by \(i : \mathbb{C}^n \to \mathbb{C}P^n\) the canonical embedding

\[
i : \mathbb{C}^n \to \mathbb{C}P^n, \quad i(z) = \begin{bmatrix} z_1 \\
... \\
\vdots \\
z_n \\
1 \end{bmatrix}
\]

and let \(\tilde{\mathbb{C}}^n = i(\mathbb{C}^n) = \mathbb{C}P^n \setminus \{[w] : w_{n+1} = 0\}\). We recall that, via the embedding, \(B^n\) corresponds to the projective open set \(\tilde{B}^n = \{ [w] \in \mathbb{C}P^n : \langle w, w \rangle < 0 \}\).
where we denote by $\langle \cdot, \cdot \rangle$ the pseudo-Hermitian inner product on $\mathbb{C}^{n+1}$ defined by

\begin{equation}
\langle w, z \rangle = \bar{w}^t \cdot I_{n,1} \cdot z , \quad \text{where} \quad I_{n,1} \overset{\text{def}}{=} \begin{pmatrix} I_n & 0 \\ 0 & -1 \end{pmatrix} .
\end{equation}

It is also known that a holomorphic map $F : B^n \to B^n$ is an automorphism of $B^n$ if and only if the corresponding map $\hat{F} = i \circ F \circ i^{-1} : \hat{B}^n \to \hat{B}^n$ is a projective linear transformation which preserves the quadric $\partial \hat{B}^n = \{ [w] : \langle w, w \rangle = 0 \}$ (see e.g. [Ve]). This means that $\hat{F}$ is of the form

\begin{equation}
\hat{F}([z]) = [A \cdot z],
\end{equation}

where $A$ is a matrix in $\text{SU}_{n,1}$, i.e. such that $\hat{\mathcal{A}}^t I_{n,1} \hat{\mathcal{A}} = I_{n,1}$ and with $\det \hat{\mathcal{A}} = 1$.

The correspondence $F \mapsto \hat{F} = i \circ F \circ i^{-1}$ gives an isomorphism between $\text{Aut}(B_n)$ and $\text{SU}_{n,1}/K$, where $K = \{ e^{\frac{2\pi i}{n+1}} I_{n+1} , \ 0 \leq k \leq n \}$.

The identification of the elements of $\text{Aut}(B^n)$ with the corresponding projective linear transformations is often quite useful, for instance in order to establish the following fact (see also [We], §6).

**Lemma 2.1.** Let $F = (F_1, \ldots, F_n) \in \text{Aut}(B^n)$ be an automorphism such that

\begin{equation}
F(B^n \cap \{ z_i = 0 \}) \subset \{ z_i = 0 \}
\end{equation}

for all $n - k + 1 \leq i \leq n$. Then the components $F_i$ are of the following form:

\begin{equation}
F_j(z) = \frac{\sum_{\ell=1}^{n-k} A_{j\ell} z_\ell + b_j}{\sum_{\ell=1}^{n-k} c^\ell z_\ell + d}, \quad \text{for} \ 1 \leq j \leq n-k ,
\end{equation}

\begin{equation}
F_j(z) = \frac{1}{\sum_{\ell=1}^{n-k} c^\ell z_\ell + d}, \quad \text{for} \ n-k+1 \leq j \leq n ,
\end{equation}

for some $\theta_j \in \mathbb{R}$ and where $A = (A_{ij})$, $b = (b_j)$, $c = (c^\ell)$ and $d$ are such that

\[
\begin{pmatrix} A & b \\ c & d \end{pmatrix} \in \text{SU}_{n-k,1}.
\]

In particular, the maps $F_j$, $1 \leq j \leq n-k$, coincide with the components of an element of $\text{Aut}(B^{n-k})$, while $\sum_{j=1}^{n-k} c^\ell z_j + d \neq 0$ for any $z \in B^n$.

**Proof.** By hypothesis, the corresponding automorphism $\hat{F} = i \circ F \circ i^{-1} \in \text{Aut}(\hat{B}^n)$ maps all hyperplanes $H_i = \{ [w] \in \mathbb{C}P^n : w_i = 0 \}$ into themselves and hence fixes their poles relative to the quadric $\partial \hat{B}^n$, i.e. fixes all the points

\[
[e_i] = [0 : \ldots : 0 : 1 \ i-th \ place : 0 : \ldots : 0] , \quad n-k+1 \leq i \leq n .
\]

This implies that the matrix $\hat{A}$ which determines the projective transformation $\hat{F}$ is of the form

\[
\hat{A} = \begin{pmatrix} A & 0 & \cdots & 0 & b \\ 0 & e^{i\theta_{n-k+1}} & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & e^{i\theta_n} & 0 \\ c & 0 & \cdots & 0 & d \end{pmatrix},
\]

where $A$, $b$, $c$ and $d$ are such that $\hat{\mathcal{A}}' \overset{\text{def}}{=} \begin{pmatrix} A & b \\ c & d \end{pmatrix}$ belongs to $\text{SU}_{n-k,1}$. From this, (2.4) and (2.5) follow immediately. The last claim follows from the fact that the value $\sum_{\ell=1}^{n-k} c^\ell z_\ell + d$ is the last homogeneous coordinate of the element
Given a domain $B^n$ by Theorem 3.1, for any sufficiently small ball $E$ do ellipsoid

**Proof.** In all the following we will use the symbols $\Gamma$ for any condition.

Let $\hat{x} \in \partial D$ be a Levi non-degenerate point $\hat{x}$ for some $\Gamma$ and consider a sequence $\{\hat{x}_j\} \subset \partial D'$ at which $\partial D'$ is $C^2$ and strictly pseudoconvex, then $h$ extends continuously to all points of a neighborhood $V$ of $\hat{x}$ in $\overline{D}$.

We may now prove the following lemma.

**Lemma 3.2.** Let $f : U_1 \subset \mathcal{E}_p \rightarrow U_2 \subset \mathcal{E}_p$ be a local automorphism of a pseudoellipsoid $\mathcal{E}_p$ with $p = (p_1, \ldots, p_k)$ and assume that

i) there exists a sequence $\{x_i\}$ as in (b) of Definition 1.1 whose limit point $x_o \in \partial \mathcal{E}_p$ is Levi non-degenerate;

ii) for any $n-k+1 \leq i \leq n$, the intersection $U_1 \cap \{z_i = 0\}$ is not empty.

Then, up to composition with a coordinate permutation,

$$(z_1, \ldots, z_n) \mapsto (z_{\sigma(1)}, \ldots, z_{\sigma(n)}),$$

the map $f$ sends the points of the hyperplane $\{z_i = 0\}$ into the same hyperplane for $n-k+1 \leq i \leq n$.

**Proof.** In all the following we will use the symbols $\Gamma, x_o$ and $\hat{x}_o$ with the same meaning as in Definition 1.1.

First of all, notice that $\hat{x}_o \in \Gamma_1 \subset \partial \mathcal{U}_2$ satisfies the condition (P) and hence, by Theorem 3.1 for any sufficiently small ball $B_{\varepsilon}(\hat{x}_o)$, centered at $\hat{x}_o$ and of radius $\varepsilon$, the holomorphic map $f^{-1} : U_2 \rightarrow U_1$ extends continuously to all points of $B_{\varepsilon}(\hat{x}_o) \cap \Gamma_2$. In particular, we may assume that $f^{-1}(B_{\varepsilon}(\hat{x}_o) \cap \Gamma_2)$ is contained in a neighborhood of $x_o = f^{-1}(\hat{x}_o)$ in $\Gamma_1$ in which there are no Levi degenerate points.

Pick a Levi non-degenerate point $x'_o \in B_{\varepsilon}(\hat{x}_o) \cap \Gamma_2$ and consider a sequence $\{x'_o\} \subset B_{\varepsilon}(\hat{x}_o) \cap \Gamma_2$ which converges to $\hat{x}_o$. By construction, the sequence $\{x'_o = f^{-1}(\hat{x}'_o)\} \subset U_1$ converges to the Levi non-degenerate point $x'_o = f^{-1}(\hat{x}'_o) \in \Gamma_1$. It follows that, replacing $x_o$ by $x'_o$ and $\hat{x}_o$ by $\hat{x}'_o$ and by Theorem 3.1 applied to
f and $f^{-1}$, there is no loss of generality if we assume that $x_o$ and $\hat{x}_o$ are both Levi non-degenerate and that, for any sufficiently small $\varepsilon_1 > 0$, the map $f$ extends continuously to a map

$$f : U_1 \cup \left( B_{\varepsilon_1}(x_o) \cap \Gamma_1 \right) \to U_2 \cup (B_{\varepsilon}(\hat{x}_o) \cap \Gamma_2),$$

which is a homeomorphism onto its image.

Since the complex Jacobian matrices $J\pi^{(p)}|_{x_o}$ and $J\pi^{(p)}|_{\hat{x}_o}$ are of maximal rank (recall that $x_o$ and $\hat{x}_o \in \partial E^p$ are both Levi non-degenerate), from the fact that $x_o$ is not a limit point of $\partial U_1 \cap E^p$ and by the continuity of $f$ and $f^{-1}$ around $x_o$ and $\hat{x}_o$, respectively, we may choose $\varepsilon_1$ and $\varepsilon_2$ so that:

a) $\pi^{(p)}|_{B_{\varepsilon_1}(x_o)}$ and $\pi^{(p)}|_{B_{\varepsilon_2}(\hat{x}_o)}$ are both biholomorphisms onto their images;

b) $f(B_{\varepsilon_1}(x_o) \cap U_1) \subset B_{\varepsilon_2}(\hat{x}_o)$ and $f|_{B_{\varepsilon_1}(x_o) \cap U_1}$ extends to a homeomorphism between $B_{\varepsilon_1}(x_o) \cap U_1$ and $f(B_{\varepsilon_1}(x_o) \cap U_1)$ which induces a homeomorphism between $B_{\varepsilon_1}(x_o) \cap \Gamma_1$ and $f(B_{\varepsilon_1}(x_o) \cap \Gamma_1) \subset \Gamma_2$.

Notice that, by definition, $x_o$ is not a limit point of $\partial (B_{\varepsilon_1}(x_o) \cap U_1) \cap E^p$ and, by (b), $\hat{x}_o$ is not a limit point of $\partial f(B_{\varepsilon_1}(x_o) \cap U_1) \cap E^p$. So, if we set

$$U_1 \overset{\text{def}}{=} B_{\varepsilon_1}(x_o) \cap U_1, \quad U_2 \overset{\text{def}}{=} f(U_1) \subset B_{\varepsilon_2}(\hat{x}_o), \quad V_i \overset{\text{def}}{=} \pi^{(p)}(U_i), \quad i = 1, 2,$$

then the maps

$$f|_{U_1} : U_1 \to U_2$$

and

$$\tilde{f} = \pi^{(p)} \circ f \circ \pi^{(p)}|_{V_1} : V_1 \subset B^n \to V_2 \subset B^n$$

are local automorphisms of $E^p$ and of the unit ball, respectively.

By Rudin’s generalization of Alexander’s theorem ([Rud]), this implies that $\tilde{f}$ extends to a global automorphism of $B^n$, which we denote by $\tilde{f}$ as well. By construction, for any $z \in U_1 = \pi^{(p)}|_{V_1}$, we have

$$(3.2) \quad \tilde{f} \circ \pi^{(p)}(z) = \pi^{(p)} \circ f(z),$$

but since both sides have a holomorphic extension on $U_1$, we get that (3.2) must be true also for any $z$ in such a larger set.

In particular,

$$(3.3) \quad J(\tilde{f})|_{\pi^{(p)}(z)} \cdot J(\pi^{(p)}|_{z}) = J(\pi^{(p)}|_{f(z)}) \cdot J(f)|_{z}, \quad \text{for any } z \in U_1.$$ 

Since for any $z \in U_1$, $\det J(f)|_z \neq 0$ and

$$(3.4) \quad \{ J(\pi^{(p)}|_{z}) = 0 \} = \bigcup_{i=n-k+1}^n \{ z_i = 0 \},$$

equality (3.3) implies that, for any $n-k+1 \leq i \leq n$ and $z \in U_1 \cap \{ z_i = 0 \}$, the value of $J(\pi^{(p)}|_{f(z)})$ is 0. By (3.4), this means that $f(U_1 \cap \{ z_i = 0 \})$ is contained in the union $\bigcup_{i=n-k+1}^n \{ z_i = 0 \}$. Indeed, it is contained in exactly one of the hyperplanes $\{z_i = 0\}$, because $f$ is a biholomorphism and consequently $f(U_1 \cap \{ z_i = 0 \})$ is an irreducible analytic variety. From this the conclusion follows. \qed

We proceed by defining a rule that associates an automorphism of $B^n$ with any local automorphism of a pseudoellipsoid (see also [We], §6). Given a local automorphism $f : U \to \mathbb{C}^n$ of $E^p$, pick a point $x_o \in U \cap \partial E^p$ for which (b) of
Definition 1.1 holds and determine a small ball \( B_\varepsilon(x_0) \) centered in \( x_0 \) as in the proof of the previous lemma. Then, we denote by \( \tilde{f} \in \text{Aut}(B^n) \) the global automorphism of the unit ball that extends \( \tilde{f} \text{ def } \pi(p) \circ f \circ \pi(p)^{-1} \mid_{\pi(p)(V)} \), with \( V \text{ def } B_\varepsilon(x_0) \cap E^n_{(p)} \). By the identity principle of the holomorphic maps, such an automorphism \( \tilde{f} \) depends only on \( f \) and will be called the (global) automorphism of \( B^n \) associated with \( f \).

With the help of such a correspondence, we may state the following criterion for extendibility of local automorphisms.

**Proposition 3.3.** A local automorphism \( f : U_1 \subset E^n_{(p)} \to U_2 \subset E^n_{(p)} \) of a pseudoellipsoid \( E^n_{(p)} \), \( p = (p_1, \ldots, p_k) \), extends to a global automorphism \( \tilde{f} \in \text{Aut}(E^n_{(p)}) \) if and only if its associated automorphism \( \tilde{f} \in \text{Aut}(B^n) \) satisfies (2.3) at all points where \( f \) is defined (in this case, at all points of \( E^n_{(p)} \)). Then, by Lemma 3.2 and the fact that \( \pi(p) \left( E^n_{(p)} \cap \{ z_i = 0 \} \right) = B^n \cap \{ z_i = 0 \} \), the equality (3.3) implies that, up to a suitable permutation of coordinates, \( f \) satisfies (2.3) for any \( n - k + 1 \leq i \leq n \).

Conversely, assume that \( f = (f_1, \ldots, f_n) : U_1 \subset E^n_{(p)} \to U_2 \subset E^n_{(p)} \) is a local automorphism of \( E^n_{(p)} \) such that (up to a suitable permutation of coordinates) the associated automorphism \( \tilde{f} = (\tilde{f}_1, \ldots, \tilde{f}_n) \in \text{Aut}(B^n) \) satisfies (2.3) for any \( n - k + 1 \leq i \leq n \). From (2.4), (2.5) and (3.2), it follows that the components \( f_j \) of \( f \) are of the form

\[
\begin{align*}
(3.5) & \quad f_j(z) = \frac{\sum_{\ell=1}^{n-k} A_{j\ell} z_\ell + b_j}{\sum_{\ell=1}^{n-k} c_{\ell} z_\ell + d} , \quad \text{for } 1 \leq j \leq n-k, \\
(3.6) & \quad f_{n-k+j}(z) = e^{\alpha_j} z_j \left( \frac{1}{\sum_{\ell=1}^{n-k} c_{\ell} z_\ell + d} \right)^{p_j} , \quad \text{for } 1 \leq j \leq k,
\end{align*}
\]

for some fixed definitions of the \( p_j \)-th roots \( w \mapsto w^{1/j} \).

From (3.5) and (3.6) it follows immediately that \( f \) coincides with a globally defined automorphism of \( E^n_{(p)} \) (for the general expressions of the elements in \( \text{Aut}(E^n_{(p)}) \), see [WG, La]).

Now, Theorem 1.2 follows almost immediately. In fact, if \( f : U_1 \subset E^n_{(p)} \to U_2 \subset E^n_{(p)} \) is a local automorphism satisfying the hypotheses of the theorem, by Lemma 3.2 and (3.2), the associated automorphism \( \tilde{f} \in \text{Aut}(B^n) \) satisfies the hypotheses of Proposition 3.3 and the claim follows.

We conclude with the following simple construction of non-extendible local automorphisms of pseudoellipsoids.

**Example 3.4.** Let \( \tilde{f} \in \text{Aut}(B^n) \) be an automorphism which does not satisfy (2.3) for some \( n - k + 1 \leq j \leq n \). Pick a point \( w_o \in \partial B \cap \{ \prod_{j=n-k+1}^n z_j \neq 0 \} \) so that also its image \( f(w_o) \) is in \( \partial B \cap \{ \prod_{j=n-k+1}^n z_j \neq 0 \} \). Then, let \( z_0 \in \partial E^n_{(p)} \) so that \( \pi(p)(z_0) = w_o \) and consider a connected neighborhood \( U \) of \( z_0 \) with the
following two properties: a) \( \pi^{(p)}|_{\mathcal{U}} \) is a biholomorphism between \( \mathcal{U} \) and its image \( \pi^{(p)}(\mathcal{U}) \); b) \( \tilde{f}(\pi^{(p)}(\mathcal{U})) \) does not intersect \( \{ \prod_{j=n-k+1}^{n} z_j = 0 \} \) (a sufficiently small neighborhood \( \mathcal{U} \) surely satisfies both requirements). Then, we may consider the map

\[
f : \mathcal{U}_1 = \mathcal{U} \cap \mathcal{E}_n^{(p)} \to \mathcal{U}_2 = \mathcal{f}(\mathcal{U}) \cap \mathcal{E}_n^{(p)},
\]

By construction, \( f \) is a local automorphism of \( \mathcal{E}_n^{(p)} \) and its associated automorphism of \( \text{Aut}(B^n) \) is \( \tilde{f} \). By the hypotheses on \( \tilde{f} \) and by Proposition 3.3, \( f \) cannot extend to a global automorphism of \( \mathcal{E}_n^{(p)} \).

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