Modeling of multicomponent radiatively driven stellar winds using a Newton-Raphson method

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Abstract. We present a simple method for the solution of one-component and multicomponent hydrodynamic equations based on the Newton-Raphson method. We show that this method can be used for the solution of stationary hydrodynamic equations. This method has been used for the calculation of the low density stellar wind models for which the multicomponent nature of the wind influences the overall wind structure.

1. Introduction

Radiatively driven stellar winds are accelerated by the absorption of radiation mainly in the resonance lines of such elements like C, N, O or Fe (see Castor, Abbott, & Klein 1975, hereafter CAK; Abbott 1982). These wind components have much lower density than the stellar wind itself, which is composed mainly by hydrogen and helium. The momentum obtained by the low-density absorbing wind component is transferred to the high-density nonabsorbing wind component via collisions between electrically charged particles (Springmann & Pauldrach 1992; Babel 1995). Clearly, multicomponent radiatively driven winds of hot stars have a multicomponent nature. This multicomponent nature is important mainly for the low-density stellar winds, which shall be described by the multicomponent hydrodynamic equations.

Krtička & Kubát (2000; 2001a; 2001b) calculated models of such multicomponent stellar winds using Newton-Raphson method. In this paper we describe in detail method of solution of these multicomponent hydrodynamic equations.

2. Model equations

Each wind component is described by a density $\rho_a$, radial velocity $v_{ra}$, temperature $T_a$ and electrical charge $q_a$. Typically, we take into account only three wind components (ie. absorbing ions, nonabsorbing ions and electrons).

Stationary spherically symmetric multicomponent radiatively driven stellar winds are described by the continuity equation for each component $a$ of the wind

$$\frac{d}{dr}(r^2 \rho_a v_{ra}) = 0,$$

(note, that in the case of continuity equation of electrons a right-hand term occurs due to the change of the wind ionization in the flow, see Krčtěka &
Kubát, 2001b), momentum and energy equations for each component $a$

\[ v_r \frac{dv_r}{dr} = g_a^{\text{rad}} - g - \frac{1}{\rho_a} \frac{d}{dr} \left( a_a^2 \rho_a \right) + \frac{q_a}{m_a} E + \sum_{b \neq a} g_{ab}^{\text{fric}}, \]  

(2)

\[ \frac{3}{2} v_r a_a^2 \frac{d \rho_a}{dr} + \frac{a_a^2 \rho_a}{r^2} \frac{d}{dr} \left( r^2 v_r a_a \right) = Q_a^{\text{rad}} + \sum_{b \neq a} Q_{ab}^{\text{ex}} + \sum_{b \neq a} Q_{ab}^{\text{fric}}, \]  

(3)

where $a_a$ is isothermal sound speed, $E$ is charge separation electric field, $g$ is gravitational acceleration, radiative acceleration $g_a^{\text{rad}}$ acts on free electrons and absorbing ions. Radiative acceleration acting on absorbing ions is taken in the CAK approximation with the finite disk correction factor (Pauldrach, Puls, & Kudritzki 1986) and with force multipliers after Abbott (1982), slightly modified for the multicomponent case (Krtička & Kubát 2000). Frictional acceleration $g_{ab}^{\text{fric}}$, frictional heating $Q_{ab}^{\text{fric}}$ and exchange of the heat $Q_{ab}^{\text{ex}}$ between wind components $a$ and $b$ are given by Burgers (1969; see also Krtička & Kubát 2001b).

The only transitions which contribute to the radiative heating/cooling term $Q_a^{\text{rad}}$ in the static atmosphere are bound-free and free-free transitions (Kubát, Puls, & Pauldrach 1999; see also Kubát, this volume). Because these transitions deposit energy directly to electrons, this radiative heating/cooling term is considered in electron energy equation. Mean intensity $J_\nu$ at the base of the wind is taken as an emergent radiation from a spherically symmetric static hydrogen model atmosphere (Kubát 2001). In the case of moving media additional heating/cooling term occurs, so-called Gayley-Owocki heating, which is caused by the dependence of a radiative force on a velocity via Doppler effect (Gayley & Owocki 1994; Krtička & Kubát 2001b).

The electrical charge $q_a$ is obtained using the approximate ionization equilibrium described by Mihalas (1978, eq. [5-46]).

The system of hydrodynamic equations (1, 2, 3) is closed using an equation for polarisation electric field in the form of

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 E \right) = 4\pi \rho = 4\pi \sum_a \frac{q_a \rho_a}{m_a}. \]  

(4)

Finally, the system of equations solved is supplemented by appropriate boundary conditions. We start to calculate our models at the critical point of nonabsorbing component. We also use zero current condition, the condition of quasi-neutrality and assume that the flow at the inner boundary is in radiative equilibrium and that the boundary temperature of all components is the same. Boundary density of absorbing ions is determined using abundance ratios.

3. Method of solution

Velocities, densities and other unknown variables are calculated at grid points. We selected logarithmically spaced grid in radii with accumulation of grid points near the stellar photosphere,

\[ r_i = qr_{i-1}, \quad q = \exp \left( \frac{\ln \frac{r_{i+1}}{r_1}}{NR - 1} \right), \]  

(5)
where \( i \) is a number of a given grid point and \( NR \) is number of grid points. We use typically 50–300 grid points.

For the approximation of the derivative of expression \( X \) we selected

\[
\frac{dX}{dr} \bigg|_{r=r_i} \approx \begin{cases} 
 y_i \frac{X_{i+1} - X_i}{\Delta r_i}, & i < NR, \\
 \frac{X_i - X_{i-1}}{\Delta r_i}, & i = NR,
\end{cases}
\]

(6)

where

\[
\Delta r_i = r_i - r_{i-1}, \quad \bar{r}_i = \frac{1}{2} (r_i + r_{i-1}), \quad y_i = \frac{r_i - \bar{r}_i}{\bar{r}_{i+1} - \bar{r}_i}.
\]

(7)

An appropriate approximation of the derivative is important, numerical tests showed that approximation (6) gives the best convergence of the models.

An approximation of hydrodynamical equations (1, 2, 3) together with the equation for a polarisation electric field (4), equation of ionization equilibrium, and boundary conditions can be formally written as

\[
P \psi = 0,
\]

(8)

where the vector describing the solution has the form of

\[
\psi = (\psi_1, \psi_2, \ldots, \psi_{NR})^T,
\]

(9a)

\[
\psi_i = \left( \sum_a \rho_{a,i}, \sum_a v_{ra,i}, \sum_a T_{a,i}, \sum_a z_{a,i}, E_i, \Delta v_{r,i} \right)
\]

(9b)

and \( P \) consists from all equations mentioned above.

For the solution of the nonlinear system of model equations (8) we selected Newton-Raphson method. The solution of Eqs.(8) can be obtained using iterative procedure in the form of

\[
J^n \delta \psi^{n+1} = -P^n \psi^n,
\]

(10)

where \( \psi^n \) denotes solution in the \( n \)-th iteration, \( \delta \psi^{n+1} \) is a correction of the solution and the Jacobian is

\[
J^n_{kl} = \frac{\partial P_k}{\partial \psi_l}.
\]

(11)

The Newton-Raphson method is similar to the well-known method of complete linearization (Auer & Mihalas 1969). The analytical form of Jacobi matrix \( J^n_{kl} \) can be easily obtained from Eqs.(8) and for the case of three-component radiatively driven stellar wind was calculated by Krtička (2001). However, the value of corrections \( \delta \psi^{n+1} \) in each iterative step is limited by \( 0.7 \psi^n \) during the calculation of a model below the critical point and by \( 0.1 \psi^n \) during the calculation of a model above the critical point. The velocity differences between wind components are limited similarly.

Another problem for the calculation of wind models is the inclusion of critical point condition. Critical point is a point where the wind velocity is equal to the sound speed. Generally, there are two possibilities of inclusion of the critical
point condition. Nobili & Turola (1988) showed that critical point condition can be directly included into the set of linearised equations. However, due to some numerical problems, we selected the so called "shooting method". We change the density at the base of the wind in order to fulfil the critical point condition. When the base density and model upstream the critical point are obtained, we calculate model downstream the critical point. The initial model for the Newton-Raphson iterations is selected in such a way that it converges to the correct branch of CAK solution appropriate for the flow downstream the critical point (see CAK).

For the solution of the system of linearised equations (10) we use numerical package LAPACK. We also applied the Gaussian elimination, however we found that the package LAPACK is faster and even more accurate.

Newton-Raphson method represents quite powerful tool for the solution of stationary hydrodynamic equations. The iterations converge very fast (typical relative change is of order $10^{-10}$ after 20 iterations), the number of equations solved can be easily extended and it is possible to calculate multidimensional models. On the other hand the model convergence depends on initial estimate and we should also check the dynamical stability of obtained solution.

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