Very recently, the BESIII collaboration observed an excess over the known contributions of the $\sigma$ spectrum with the significance of 5.3 $\sigma$. The predicted mass $M_\sigma = 3.99 \pm 0.09$ GeV is in excellent agreement with the experimental data $3982.5^{+1.8}_{-2.6} \pm 2.1$ MeV from the BESIII collaboration, and supports assigning the $Z_{cs}(3985)$ to be the cousin of the $Z_c(3900)$ with the quantum numbers $J^{PC} = 1^{+-}$. We take into account the light flavor SU(3) mass-breaking effect, and estimate the mass spectrum of the diquark-antidiquark type hidden-charm tetraquark states with the strangeness based on our previous work.

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Key words: Tetraquark state, QCD sum rules

1 Introduction

Very recently, the BESIII collaboration observed an excess over the known contributions of the conventional charmed mesons near the $D_s^- D^{*0}$ and $D_s^- D^0$ mass thresholds in the $K^+$ recoil-mass spectrum with the significance of 5.3 $\sigma$ in the processes of the $e^+ e^- \rightarrow K^+ (D_s^- D^{*0} + D_s^- D^0)$ [1]. It is the first candidate of the charged hidden-charm tetraquark state with the strangeness. The Breit-Wigner mass and width of the new structure, $Z_{cs}(3985)$, were determined to be $3982.5^{+1.8}_{-2.6} \pm 2.1$ MeV and $12.8^{+5.3}_{-4.2} \pm 3.0$ MeV, respectively. According to the production mode, it is nature to draw the conclusion that the $Z_{cs}(3985)$ should be a cousin of the well-known $Z_c(3885/3900)$ with the strangeness, the $Z_c(3885)$ was observed in the process $e^+e^- \rightarrow (D\bar{D})^+\pi^+$, which means that the $Z_c(3885)$ and $Z_{cs}(3985)$ are governed by a similar production mechanism and have a similar quark structure $c\bar{c}q\bar{q}$ with the $q = d$ or $s$. After the discovery of the $Z_{cs}(3985)$, several possible explanations for its nature were proposed, such as the tetraquark state [3], molecular state [5], $D\bar{D} - D\bar{D}$ hadronic molecular state or dynamically generated resonance with the coupled-channel effects [4], re-scattering effect [4], and we can study its nature with the photo-production [6].

In Ref.[7], we study the axialvector hidden-charm tetraquark states without strangeness via the QCD sum rules, and explore the energy scale dependence of the QCD sum rules for the tetraquark states in details for the first time. The calculations support assigning the $X(3872)$ and $Z_c(3900)$ to be the diquark-antidiquark type tetraquark states with the quantum numbers $J^{PC} = 1^{++}$ and $1^{+-}$, respectively. In Ref.[5], we take the diquark and antidiquark operators as the basic building blocks to construct the scalar, axialvector and tensor currents to study the mass spectrum of the ground state hidden-charm tetraquark states without strangeness via the QCD sum rules in a comprehensive way, and revisit the possible assignments of the $X(3860)$, $X(3872)$, $X(3915)$, $X(3940)$, $X(4160)$, $Z_{c}(3900)$, $Z_c(4020)$, $Z_c(4050)$, $Z_c(4055)$, $Z_c(4100)$, $Z_c(4200)$, $Z_c(4250)$, $Z_c(4430)$, $Z_c(4600)$, etc.

In Refs.[7,8], we choose the axialvector current $\eta_\mu(x)$,

$$\eta_\mu(x) = \frac{\varepsilon^{ijk}\epsilon^{imn}}{\sqrt{2}} \left\{ u_j^T(x) C\gamma_5 c_k(x)\bar{d}_m(x)\gamma_\mu C\bar{c}_n(x) - u_j^T(x) C\gamma_\mu c_k(x)\bar{d}_m(x)\gamma_5 C\bar{c}_n(x) \right\},$$ (1)

to interpolate the $Z_c^+(3900)$, where the $i$, $j$, $k$, $m$, $n$ are color indices. The current $\eta_\mu(x)$ has the quantum numbers $J^{PC} = 1^{+-}$, in fact, the $Z_c^+(3900)$ have non-zero electric charge, and have no definite charge conjugation, only the electric neutral state $Z_c^0(3900)$ has definite charge.
conjugation. If the $Z^c_{c}(3985)$ is really a tetraquark state, irrespective of the diquark-antidiquark type or meson-meson type, its valence quarks are $c\bar c s\bar u$, and also has no definite electric conjugation, we suppose that it has definite conjugation, just like its cousins $c\bar c q\bar q$ and $c\bar c s\bar s$. In the present work, we tentatively assign the $Z^c_{c}(3985)$ to be the diquark-antidiquark type axialvector tetraquark state with the valence quarks $c\bar c s\bar u$ and study its mass with the QCD sum rules in details. Then we take into account the light flavor $SU(3)$ mass-breaking effect and explore the mass spectrum of the diquark-antidiquark type tetraquark states with the strangeness based on our previous work.

The article is arranged as follows: in Sect.2, we derive the QCD sum rules for the masses and pole residues of the axialvector tetraquark states in Sect.2; in Sect.3, we present the numerical results and discussions; and Sect.4 is reserved for our conclusion.

## 2 The QCD sum rules for the axialvector tetraquark states with strangeness

Firstly, let us write down the two-point correlation functions $\Pi_{\mu\nu}(p)$,

$$\Pi_{\mu\nu}(p) \equiv i \int d^4xe^{ip\cdot x}\langle 0|T\{J_{\mu}(x)J_{\nu}^\dagger(0)\}|0\rangle,$$

(2)

where the interpolating currents $J_{\mu}(x) = J^N_{\mu}(x)$ and $J^P_{\mu}(x)$,

$$J^N_{\mu}(x) = \frac{\epsilon_{ijk}\epsilon^{lmn}}{\sqrt{2}}\left\{u^T_j(x)C\gamma_5c_k(x)\bar{s}_m(x)\gamma_\mu C\bar{c}^T_n(x) - u^T_j(x)C\gamma_\mu c_k(x)\bar{s}_m(x)\gamma_5C\bar{c}^T_n(x)\right\},$$

$$J^P_{\mu}(x) = \frac{\epsilon_{ijk}\epsilon^{lmn}}{\sqrt{2}}\left\{u^T_j(x)C\gamma_5c_k(x)\bar{s}_m(x)\gamma_\mu C\bar{c}^T_n(x) + u^T_j(x)C\gamma_\mu c_k(x)\bar{s}_m(x)\gamma_5C\bar{c}^T_n(x)\right\},$$

(3)

the $i, j, k, m, n$ are color indices. We suppose that the interpolating currents $J^N_{\mu}(x)$ and $J^P_{\mu}(x)$ have the negative and positive charge conjugation respectively in the sense of the limit $J_{\mu}(x)|_{s\rightarrow u}$ or $J_{\mu}(x)|_{u\rightarrow s}$. In Refs.[3, 5], we observe that the axialvector tetraquark states $c\bar c u d$ with the quantum numbers $J^{PC} = 1^{+}\pm$ and $1^{++}$ have almost degenerated masses. Now we expect that the currents $J^N_{\mu}(x)$ and $J^P_{\mu}(x)$ couple potentially to the axialvector tetraquark states with almost the same masses, there are two axialvector tetraquark candidates $Z_{NJ/P}$ for the $Z_{c}(3985)$ if we neglect the charge conjugation, as it is not a very good quantum number.

At the hadron side, we insert a complete set of intermediate hidden-charm tetraquark states with the same quantum numbers as the current operators $J_{\mu}(x)$ into the correlation functions $\Pi_{\mu\nu}(p)$ to obtain the hadronic representation $\Pi_{\mu\nu}(p)$, and isolate the lowest axialvector hidden-charm tetraquark states $Z_{NJ/P}$, and obtain the results:

$$\Pi_{\mu\nu}(p) = \frac{\epsilon_{\mu\nu}}{M_Z^2 - p^2}\left(-g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2}\right) + \cdots,$$

$$= \Pi(p^2)\left(-g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{p^2}\right) + \cdots,$$

(4)

where the pole residues $\lambda_Z$ are defined by $\langle 0|J_{\mu}(0)|Z_{NJ/P}(p)\rangle = \lambda_Z\epsilon_{\mu}$, the $\epsilon_{\mu}$ are the polarization vectors of the axialvector tetraquark states $Z_{NJ/P}$. On the other hand, there are two-particle scattering state contributions from the $KJ/\psi$, $\eta_cK^*$, $D_sD^*$, $D^*D_s$, $\cdots$, as the quantum field theory does not forbid non-vanishing couplings to the two-particle scattering states if they have the same quantum numbers. In Ref.[11], we study the $Z_{c}(3900)$ as an axialvector tetraquark state with the quantum numbers $J^{PC} = 1^{+}-$ via the QCD sum rules in details by considering the two-particle scattering state contributions and nonlocal effects between the diquark and antidiquark constituents, and obtain the conclusion that the contribution of the $Z_{c}(3900)$ as a pole term plays
an un-substitutable role, we can saturate the QCD sum rules with or without the two-particle scattering state contributions. The net effects of the two-particle scattering states can be taken into account effectively by adding a finite width to the pole term. In the present case, the width $12.8^{+5.3}_{-4.4} \pm 3.0$ MeV of the $Z_{cs}(3985)$ is small enough to be neglected safely.

In the QCD side, we carry out the operator product expansion up to the vacuum condensates of dimension 10 in consistent way in the deep Euclidean region $P^2 = -p^2 \to \infty$ or $\gg \Lambda_{QCD}^2$ [7,8]. We take into account the vacuum condensates $\langle \bar{q}q \rangle$, $\langle \bar{s}s \rangle$, $\langle \bar{q}q, \sigma Gq \rangle$, $\langle \bar{s}g, \sigma Gs \rangle$, $\langle \frac{\alpha_s \bar{G}G}{\pi} \rangle$, $\langle \bar{q}q \bar{s}s \rangle$, $\langle \bar{q}q \rangle \langle \alpha_s \bar{G}G \rangle$, $\langle \bar{s}s \rangle \langle \alpha_s \bar{G}G \rangle$, $\langle \bar{q}q \bar{s}s \rangle$, $\langle \frac{\alpha_s \bar{G}G}{2} \rangle$ and $\langle \bar{q}g, \sigma Gq \rangle \langle \bar{s}g, \sigma Gs \rangle$, which are the vacuum expectations of the quark-gluon operators of the orders $O(\alpha_s^k)$ with $k \leq 1$. In calculations, we have assumed vacuum saturation for the higher dimensional vacuum condensates, which works well in the large $N_c$ limit.

Once we get the analytical expressions of the correlation functions $\Pi(p^2)$ at the quark-gluon degrees of freedom, then we obtain the QCD spectral densities through dispersion relation directly, and match the hadron side with the QCD side of the correlation functions $\Pi(p)$, take the quark-hadron duality below the continuum thresholds $s_0$, perform the Borel transform in regard to the variable $P^2 = -p^2$ and obtain the QCD sum rules:

$$\lambda^2 \exp \left( -\frac{M^2}{T^2} \right) = \int_{4m_s^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{T^2} \right),$$

(5)

where the explicit expressions of the QCD spectral densities $\rho_{QCD}(s)$ are neglected for simplicity.

We differentiate Eq.(5) with respect to $\frac{1}{T^2}$, then eliminate the pole residues $\lambda^2$ and obtain the QCD sum rules for the masses of the hidden-charm axialvector tetraquark states with the strangeness,

$$M^2_{Z} = \frac{-\int_{4m_s^2}^{s_0} ds \frac{d}{ds} \rho_{QCD}(s) \exp \left( -\frac{s}{T^2} \right)}{\int_{4m_s^2}^{s_0} ds \rho_{QCD}(s) \exp \left( -\frac{s}{T^2} \right)},$$

(6)

3 Numerical results and discussions

We take the standard values or conventional values of the vacuum condensates $\langle \bar{q}q \rangle = -(0.24 \pm 0.01$ GeV$)^3$, $\langle \bar{s}s \rangle = (0.8 \pm 0.1)(\bar{q}q)$, $\langle \bar{g}g, \sigma Gq \rangle = m_0^2(\bar{q}q)$, $\langle \bar{s}g, \sigma Gs \rangle = m_0^2(\bar{s}s)$, $m_0^2 = (0.8 \pm 0.1)$ GeV$^2$, $\langle \frac{\alpha_s \bar{G}G}{2 \pi} \rangle = 0.012 \pm 0.004$ GeV$^4$ at the energy scale $\mu = 1$ GeV [9,10,12], and take the $MS$ masses $m_c(m_c) = (1.275 \pm 0.025)$ GeV and $m_s(\mu = 2$ GeV$) = (0.095 \pm 0.005)$ GeV from the Particle Data Group [13], just like in our previous works [7,8]. Moreover, we consider the energy-scale dependence of the input parameters, such as the quark condensates, mixed quark condensates and
\( \overline{\text{MS}} \) masses from the renormalization group equation \[14\],

\[
\langle \bar{q}q \rangle (\mu) = \langle \bar{q}q \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{\mu^2}{4\pi^2} t},
\]

\[
\langle \bar{s}s \rangle (\mu) = \langle \bar{s}s \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{\mu^2}{4\pi^2} t},
\]

\[
\langle \bar{q}g_s\sigma G q \rangle (\mu) = \langle \bar{q}g_s\sigma G q \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{\mu^2}{4\pi^2} t},
\]

\[
\langle \bar{s}g_s\sigma G s \rangle (\mu) = \langle \bar{s}g_s\sigma G s \rangle (1\text{GeV}) \left[ \frac{\alpha_s(1\text{GeV})}{\alpha_s(\mu)} \right]^{\frac{\mu^2}{4\pi^2} t},
\]

\[
m_c(\mu) = m_c(m_c) \left[ \frac{\alpha_s(\mu)}{\alpha_s(m_c)} \right]^{\frac{\mu^2}{4\pi^2} t},
\]

\[
m_s(\mu) = m_s(2\text{GeV}) \left[ \frac{\alpha_s(\mu)}{\alpha_s(2\text{GeV})} \right]^{\frac{\mu^2}{4\pi^2} t},
\]

\[
\alpha_s(\mu) = \frac{1}{b_0} \left[ 1 - b_1 \log \frac{t}{b_0^2} + b_2^2 (\log^2 t - \log t - 1) + b_3 b_4 \right],
\]

where \( t = \log \frac{\mu^2}{\Lambda^2} \), \( b_0 = \frac{33-2n_f}{12\pi} \), \( b_1 = \frac{153-19n_f}{24\pi^2} \), \( b_2 = \frac{2857-803n_f+225n_f^2}{128\pi^4} \), \( \Lambda = 213\text{ MeV} \), 296 MeV and 339 MeV for the quark flavors \( n_f = 5, 4 \) and 3, respectively \[13\]. In the present work, we study the hidden-charm tetraquark states \( cc\bar{u}u \), it is better to choose the quark flavor numbers \( n_f = 4 \), and evolve all the input parameters to a typical energy scale \( \mu \), which satisfies the energy scale formula \( \mu = \sqrt{M_Z} - (2M_c)^2 \) with the effective c-quark mass \( M_c = 1.82\text{ GeV} \) \[8, 13, 16, 17\]. If we take the \( Z_{cs}(3985) \) as the ground state axialvector tetraquark candidate for the \( Z_{N/P} \), the optimal energy scales of the QCD spectral densities are \( \mu = 1.6\text{ GeV} \).

The energy gaps between the ground state and the first radial excited states of the hidden-charm tetraquark states are about 0.6 GeV \[8, 13, 19, 20\], we tentatively choose the continuum threshold parameters as \( \sqrt{s}_0 = M_Z + 0.55 \pm 0.10\text{ GeV} = 4.55 \pm 0.10\text{ GeV} \) and vary the Borel parameters \( T^2 \) to satisfy the pole dominance at the hadron side and convergence of the operator product expansion at the QCD side via trial and error. In doing so, we should bear in mind that if our numerical results do no support assigning the \( Z_{cs}(3985) \) to be the axialvector tetraquark state, we should refit the continuum threshold parameters and the energy scales of the QCD spectral densities to obtain the real ground state masses.

Finally, we obtain the Borel windows and pole contributions in the QCD sum rules for the currents \( J_{\mu}^N(x) \) and \( J_{\mu}^P(x) \), and show them explicitly in Table 1. From the table, we can see that the pole contributions are about (42–66)% of the central values exceed 50%, the pole dominance criterion at the hadron side is well satisfied. In Fig 1 we plot the absolute values of the contributions of the vacuum condensates with the central values of all the input parameters. From the figure, we can see that the largest contributions come from the quark condensates \( \langle \bar{q}q \rangle \) and \( \langle \bar{s}s \rangle \), the vacuum condensates of the dimensions 7, 8 and 10 play a tiny role, the operator product expansion converges very good. Now we can draw the conclusion that it is reliable to extract the tetraquark masses.

We take into account all the uncertainties of the input parameters, and obtain the values of the masses and pole residues of the hidden-charm axialvector tetraquark states with the strangeness \( S = 1 \), which are shown explicitly in Table 1 and Fig 2. From Table 1 we can see that the energy scale \( \mu = 1.6\text{ GeV} \) is consistent with the mass 3.99 GeV according to the energy scale formula \( \mu = \sqrt{M_Z} - (2M_c)^2 \) or the relation between the tetraquark masses and the energy scales of the QCD spectral densities \( M_F^2 = \mu^2 + 4M_c^2 \). In Fig 2 we plot the predicted masses with variations of the Borel parameters at much larger regions than the Borel windows, furthermore, we also plot the experimental value of the mass of the \( Z_{cs}(3985) \) from the BESIII collaboration \[1\]. From the figure,
we can see that there appear very flat platforms in the Borel windows. In the Borel windows, the mass of the $Z_{cs}(3985)$ overlaps with the central values of the masses of the axialvector tetraquark states with the strangeness. In fact, the central values of the masses of the axialvector tetraquark states with the negative and positive charge conjugation have degenerated masses. From the Table 1, we can see that the predicted masses $M = 3.99 \pm 0.09 \text{GeV}$ are in excellent agreement with the experimental value $3982.5^{+1.8}_{-2.5} \pm 2.1 \text{MeV}$ from the BESIII collaboration. and support assigning the $Z_{cs}(3985)$ to be the hidden-charm axialvector tetraquark state with the $J^{PC} = 1^{+-}$. We prefer the quantum numbers $J^{PC} = 1^{+-}$ to the quantum numbers $J^{PC} = 1^{++}$ as the $Z_{cs}(3985)$ was observed in the $D_{s}^{*} D^{0}$ and $D_{s}^{*} D^{0}$ mass spectrum, just like the $Z_{c}(3900)$, which was observed in the $D D^{*}$ mass spectrum.

In Ref. [8], we introduce the four-vector $t_{\mu} = (1, 0)$ to project the axialvector and vector component of the tensor diquark operators, and take the diquark operators $\varepsilon^{ijk} q_{k}^{T} C \gamma_{5} q_{l}^{a} (S)$, $\varepsilon^{ijk} q_{k}^{T} C q_{l}^{a} (P)$, $\varepsilon^{ijk} q_{k}^{T} C \gamma_{\mu} q_{l}^{a} (A)$, $\varepsilon^{ijk} q_{k}^{T} C \gamma_{\mu} \gamma_{5} q_{l}^{a} (V)$ and $\varepsilon^{ijk} q_{k}^{T} C \gamma_{\mu} \gamma_{5} q_{l}^{a} (V)$ in the color antitriplet as the basic building blocks to construct the tetraquark currents to study the mass spectrum of the hidden-charm tetraquark states without strangeness in a comprehensive way, where the $S$, $P$, $A/\bar{A}$ and $V/\bar{V}$ denote the scalar, pseudoscalar, axialvector and vector diquark operators, respectively, $\sigma_{\mu\nu}^{t} = \frac{1}{2} \left[ \gamma_{\mu}^{t}, \gamma_{\nu}^{t} \right]$, $\sigma_{\mu\nu}^{v} = \frac{1}{2} \left[ \gamma_{\mu}^{v}, \gamma_{\nu}^{v} \right]$, $\gamma_{\mu}^{t} = \gamma \cdot t_{\mu}$, $\gamma_{\mu}^{v} = \gamma_{\mu} \gamma_{5}$, $\gamma_{\mu} = \gamma_{\mu} - \gamma \cdot t_{\mu}$. In Table 2, we can see that there appear very flat platforms in the Borel windows. In the Borel windows, the central values of the masses of the axialvector tetraquark states with the strangeness. In fact, the central values of the masses of the axialvector tetraquark states with the negative and positive charge conjugation have degenerated masses.

Table 1: The Borel windows, continuum threshold parameters, ideal energy scales, pole contributions, masses and pole residues for the axialvector tetraquark states with the strangeness.

| $J^{PC}$ | $T^{2}(\text{GeV}^2)$ | $\sqrt{s_0}(\text{GeV})$ | $\mu(\text{GeV})$ | pole | $M(\text{GeV})$ | $\lambda(\text{GeV}^2)$ |
|----------|-----------------|-----------------|-----------------|-------|-----------------|-----------------|
| $1^{+-}$ | 3.0 - 3.4        | 4.55 ± 0.10     | 1.6             | (42 - 63)% | 3.99 ± 0.09    | (2.85 ± 0.45) × 10^{-2} |
| $1^{++}$ | 3.0 - 3.4        | 4.55 ± 0.10     | 1.6             | (41 - 62)% | 3.99 ± 0.09    | (2.85 ± 0.45) × 10^{-2} |

Figure 1: The absolute values of the contributions of the vacuum condensates with the central values of the input parameters, where the $N$ and $P$ represent the negative and positive charge conjugation, respectively.
Figure 2: The masses of the axialvector tetraquark states with variations of the Borel parameter $T^2$, where the $N$ and $P$ represent the negative and positive charge conjugation, respectively, the regions between the two vertical lines are the Borel windows, the Expt represents the experimental value of the mass of the $Z_{cs}(3985)$.

we present the mass-spectrum of the hidden-charm tetraquark states $c\bar{c}u\bar{d}$ obtained via the QCD sum rules in Ref.[8] with the possible assignments.

If we assign the $Z_{cs}(3985)$ to be the cousin of the $Z_c(3900)$ with the strangeness, the mass gap or the light flavor $SU(3)$ mass-breaking effect $M_{Z_{cs}(3985)} - M_{Z_c(3900)} = 94$ MeV, then we take the light flavor $SU(3)$ mass-breaking effect $m_s = 0.09$ GeV, and estimate the mass spectrum of the hidden-charm tetraquark states with the strangeness based on our previous work [8], which are shown explicitly in Table 3. We should bear in mind that the charge conjugation is not a very good quantum number in the present case. We cannot exclude that the $Z_{cs}(3985)$ can be assigned to be the axialvector hidden-charm tetraquark state with the quantum numbers $J^{PC} = 1^{++}$.

4 Conclusion

In this article, we choose the scalar and axialvector diquark operators and antidiquark operators as the basic building blocks to construct the tetraquark currents and study the diquark-antidiquark type axialvector tetraquark states $c\bar{c}u\bar{s}$ with the QCD sum rules by carrying out the operator product expansion up to the vacuum condensates of dimension 10 in a consistent way, and use the energy scale formula $\mu = \sqrt{M_Z - (2M_c)^2}$ with the effective $c$-quark mass $M_c = 1.82$ GeV to determine the ideal energy scales of the QCD spectral densities. The predicted tetraquark mass $M_Z = 3.99 \pm 0.09$ GeV is in excellent agreement with the experimental data $3982.5^{+1.8}_{-2.0} \pm 2.1$ MeV from the BESIII collaboration, and supports assigning the $Z_{cs}(3985)$ to be the cousin of the $Z_c(3900)$ with the quantum numbers $J^{PC} = 1^{-+}$. Furthermore, we obtain the mass of the corresponding tetraquark state $c\bar{c}u\bar{s}$ with the quantum numbers $J^{PC} = 1^{++}$, which can be confronted to the experimental data in the future. We take into account the light flavor $SU(3)$ mass-breaking effect about 90 MeV, and estimate the mass spectrum of the diquark-antidiquark type tetraquark states with the strangeness.

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Table 2: The possible assignments of the ground state hidden-charm tetraquark states, where the strangeness is implied.

| $Z_c(X_c)$ | $J^{PC}$ | $M_Z$(GeV) | Assignments |
|------------|----------|------------|-------------|
| $[uc]_S[dc]_S$ | $0^{++}$ | $3.88 \pm 0.09$ | ? X(3860) |
| $[uc]_A[dc]_A$ | $0^{++}$ | $3.95 \pm 0.09$ | ? X(3915) |
| $[uc]A[dc]_A$ | $0^{++}$ | $3.98 \pm 0.08$ | |
| $[uc]V[dc]_V$ | $0^{++}$ | $4.65 \pm 0.09$ | |
| $[uc]_P[dc]_P$ | $0^{++}$ | $5.35 \pm 0.09$ | |
| $[uc]_P[dc]_P$ | $0^{++}$ | $5.49 \pm 0.09$ | |
| $[uc]S[dc]_A - [uc]_A[dc]_S$ | $1^{+-}$ | $3.90 \pm 0.08$ | ? $Z_c(3900)$ |
| $[uc]A[dc]_A - [uc]_A[dc]_A$ | $1^{+-}$ | $4.02 \pm 0.09$ | ? $Z_c(4020, 4055)$ |
| $[uc]V[dc]_V + [uc]_V[dc]_V$ | $1^{+-}$ | $4.02 \pm 0.09$ | ? $Z_c(4020, 4055)$ |
| $[uc]_P[dc]_V + [uc]_V[dc]_P$ | $1^{+-}$ | $4.02 \pm 0.09$ | ? $Z_c(4020, 4055)$ |
| $[uc]_A[dc]_A + [uc]_A[dc]_A$ | $1^{++}$ | $4.08 \pm 0.09$ | ? $Z_c(4020, 4055)$ |
| $[uc]V[dc]_V - [uc]_V[dc]_V$ | $1^{++}$ | $5.46 \pm 0.09$ | ? $Z_c(4020, 4055)$ |

Table 3: The possible assignments of the ground state hidden-charm tetraquark states with the strangeness.

| $Z_c(X_c)$ | $J^{PC}$ | $M_Z$(GeV) | Assignments |
|------------|----------|------------|-------------|
| $[uc]_S[sc]_S$ | $0^{++}$ | $3.97 \pm 0.09$ | ? $Z_{cs}(3985)$ |
| $[uc]_A[sc]_A$ | $0^{++}$ | $4.04 \pm 0.09$ | |
| $[uc]A[sc]_A$ | $0^{++}$ | $4.07 \pm 0.08$ | |
| $[uc]V[sc]_V$ | $0^{++}$ | $4.74 \pm 0.09$ | |
| $[uc]_P[sc]_P$ | $0^{++}$ | $5.44 \pm 0.09$ | |
| $[uc]_P[sc]_P$ | $0^{++}$ | $5.58 \pm 0.09$ | |
| $[uc]S[sc]_A - [uc]_A[sc]_S$ | $1^{+-}$ | $3.99 \pm 0.09$ | ? $Z_{cs}(3985)$ |
| $[uc]A[sc]_A$ | $1^{+-}$ | $4.11 \pm 0.09$ | |
| $[uc]A[sc]_A$ | $1^{+-}$ | $4.11 \pm 0.09$ | |
| $[uc]V[sc]_V + [uc]_V[sc]_V$ | $1^{+-}$ | $5.75 \pm 0.10$ | |
| $[uc]V[sc]_V + [uc]_V[sc]_V$ | $1^{+-}$ | $5.55 \pm 0.09$ | |
| $[uc]_A[dc]_A$ | $2^{++}$ | $4.17 \pm 0.09$ | ? $Z_{cs}(3985)$ |
| $[uc]V[dc]_V$ | $2^{++}$ | $5.49 \pm 0.09$ | |
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