A conceptual roadmap for setting up a system of units in the “New SI” context

Luca Mari
Università Cattaneo – LIUC, Castellanza (VA), Italy
lmari@liuc.it

Franco Pavese
formerly Consiglio Nazionale delle Ricerche, Istituto di Metrologia, Torino, Italy

Abstract. The definition of a system of units is analysed here as a structural problem, for which the physical theory provides the necessary conditions of realization but remains as backgrounder. The structurally more sophisticated option is provided by the so-called Global Constant Definitions, discussed in this paper through a conceptual roadmap intended to separate the components of the definitions based on theoretical foundation from those related to conventional features and, at the same time, to ensure the structural stability of the system and its partial modularity and therefore flexibility. The critical case of the so-called “New SI” is used as an illustration example.

1. Introduction

The quantity units used in measurement of ratio quantities provide an empirical reference to numerical values in mathematical equations that represent physical laws. Units have then the fundamental role of relating theory and experiment, and a system of units is a critical component of what metrology has to offer to empirical sciences and to their relation to societal needs. This role is crucial and cannot be under-evaluated, or taken for granted.

The so-called “New SI” [1], still in draft stage at the moment, is based on a structure, which we call here Global Constant Definitions (GCD), that is significantly different from the one (either Explicit Unit Definitions (EUD) or Explicit Constant Definitions (ECD) [2]) that has been adopted in defining systems of units so far. This proposes an interesting problem: how to fulfil the societal and metrological need of stability in time of the system without hindering the fundamental requisite of science to maintain the principled possibility of evolving in time [3]. A requisite of metrology is indeed the continuity of the units with time, to avoid error-prone conversion factors or massive recalibration processes, and thus user disconcert. A manifest breach of this principle would be, for example, a length unit that, though maintaining its name, would change its definition so that the new metre were double of the previous one. Of course, nobody can guarantee that the presently available knowledge on the system of quantities and constant quantities on which the New SI is based will

---

* One of the authors is a member of the Joint Committee on Guides in Metrology (JCGM) Working Group 2 (VIM). The opinion expressed in this paper does not necessarily represent the view of this Working Group.

1 In the BIPM documents the GCD is instead called “Explicit Constant Definition”.

---
remain unchanged in future. On the contrary, the history of science and technology has taught that new knowledge will plausibly be produced that will modify our understanding. The strategies of unit definition have evolved from (1) practical engineering requirements, related to the adoption of units defined as quantities of material standards, thus objects to be kept stable in time (e.g., the prototype of the kilogram), to (2) experimental physics or chemistry, where given kinds of objects have experimentally proved to have properties stable enough in time (e.g., the frequency of a given radiation of the caesium 133), to (3) stability in time to be proved by basic principles or laws of fundamental physics, which is the aim of the present evolution of the system of units. How to conciliate continuity and possibility of change is a structural challenge for all such strategies, but the third one, i.e., GCD, is specifically in a critical position on this matter, given that adapting unit definitions to new knowledge either might be formally impossible or might require changing the background theory. To this aim we propose here a modified version of GCD that makes it flexible to the embedding of new knowledge while maintaining all its structural merits.

2. A conceptual roadmap for unit stability under knowledge evolution

A critical issue of GCD is how possible future changes in physics could be taken into account in the system of units of which the system itself is based. It might happen that new basic physical equations between quantities will be established that produce a new structure of the system of quantities (as it happened with the introduction of relativity into the Newtonian mechanics) or that some quantities are found that are not to be time and space invariant as instead previously supposed. Of course such changes would produce a scientific revolution in Kuhn’s sense [3], requiring a brand new approach to accommodate the changes in the SI system of units. However, less dramatic changes might more likely occur, which a sound structure of the system of units should be able to accommodate without any revolution provided that its conceptual structure — while (i) based on the principles and tools of and contents of fundamental physics, and thus in particular on the currently accepted system of quantities and the set of fundamental constants and (ii) linked to the current SI so that the principle of continuity is fulfilled — has some flexibility.

The paper intends to describe a conceptual roadmap that satisfies both these requirements, by construing a system of units according to a two-stage structure that explicitly includes a fundamental system and a conventional one:

- both a system of quantities and a set of constants corresponding to the dimensions of the base of the system are assumed; this represents the fundamental system where the numerical value of each constant is 1;
- a conventional system is then considered linked to the above fundamental system, where it is admitted that the numerical values of the constants can have values different from 1, assigned according to the best available present knowledge so that in the change to the new system the units maintain their values as expected according to the principle of continuity.

We start from considering, as the consistent and systematic the strategy used in the “New SI”, called here “Global Explicit Constant Definition” (GCD). In such a system, the units are defined by taking as referent a set \( \{C_i\} \) of quantities modelled as constants, and taking a given numerical value \( k_i \) for each \( C_i \). Each unit \( [Q] \) is then defined as:

\[
[Q] = \prod_i (C_i / k_i)^n
\]

In other words, a system GCD is characterised as a system of units where the constants \( C_i \) have numerical values \( k_i \). If \( C_i \) are constants, fundamental or not, but considered universally valid and invariant in time and space according to the current physical or chemical current theory, the system of units has its most general definition relative to the current state of knowledge. However, this definition depends critically from the dependence on the set \( \{k_i\} \).

2 Actually the constants used in the “New SI” are, as indicated in [1], of four kinds: (i) fundamental constants of nature, (ii) specified atomic parameters, (iii) conversion factors, and (iv) technical constants.
Why it is critical? In a definition constrained by the choice of the set \( \{ k_i \} \), the quantities \( C_i \) modelled as constants have, in fact, a double role: (i) they connect quantities, (ii) they are parameters in physical/chemical laws, being themselves quantities of kind of objects. For example, the quantity linking energy and mass in the equation \( E = mc^2 \) is the speed of light in vacuum. The role (ii) makes in principle the quantities \( C_i \) measurable, via an empirical interaction with their kind of objects, the light in vacuum in the case of \( c \). Actually, the numerical value \( k_i \) of \( C_i \) has been obtained by means of measurements, making use of a system of units established on standards based on the magnitude of objects (strategy EUD) or of kind of objects (strategy ECD). The numerical value of these standards that will be based on GCD is mandated to remain consistent with the former according to the continuity principle indicated hereinbefore.

Being the quantities \( C_i \) possible objects of empirical interaction, thus allowing measurements referred to standards of units pre-defined, it is, in principle and in practice, possible that the measurement results are affected by systematic errors. Therefore, one must admit, for example, as possible the fact of new experimental evidence that the numerical value of speed of light is different from the currently stipulated one. If the units are defined independently from the relevant quantity (\( c \) for length), the situation is not problematic: one only has to correct the consensus value, expressed in the relevant unit, associated to the quantity, from \( k_i \) to \( k_i^* \).

On the contrary, what to do when the required change occurs after a system of units of the type GCD has been established also based on \( C_i \) is a delicate issue. One could

- keep the system unchanged, admitting that in fact it is not based on constants assumed as invariant in time and space;

or

- change the whole definition, where in the new system the numerical value associated to \( C_i \) is \( k_i^* \).

However, adjusting the strategy GCD could produce the required flexibility to a minor cost.

The problem is clearly structural, not involving physics or chemistry: thus, in the following we will refer to an hypothetical simplified system, only including units of time (duration), length and mass. In such a system, the “New SI” scheme would bring to the following type of definition (using data from the CCU 2016 Draft of SI Brochure):

**The SI is the system of units in which the unperturbed ground state hyperfine splitting frequency of the caesium 133 atom \( \Delta \nu_{Cs} \) is **9 192 631 770 Hz**, the speed of light in vacuum \( c \) is **299 792 458 m s\(^{-1}\)** and the Planck constant \( h \) is **6.6260693 \times 10^{-34} \) J s.**

We can abstain from making use of any structural feature by writing instead:

**The SI is the system of units in which the frequency \( C_1 \) is **9 192 631 770 Hz**, the speed \( C_2 \) is **299 792 458 m s\(^{-1}\)** and the action \( C_3 \) is **6.6260693 \times 10^{-34} \) J s.**

There is still the problem that this definition makes reference to units (hertz, metre, second, joule) which are the object of the definition itself, known only by admitting that the user already knows the previous definition. The simple solution is to rewrite as follows:

**The SI is the system of units in which the frequency \( C_1 \) is **9 192 631 770 units of frequency**, the speed \( C_2 \) is **299 792 458 units of speed** and the action \( C_3 \) is **6.6260693 \times 10^{-34} \) units of action.**

One interesting issue of this type of definition is that the quantities referred to, as the “fundamental bases” of the system, are frequency, speed and action, not time (duration), length and mass. The latter are anyway retrievable from the former by means of the quantity equations of physics, which do not depend on any specific unit.
The roadmap of deconstruction of the definition can now be completed, by understanding that the numerical values \( k_i \) do not have a structural role, thus by rewriting the definition in the most ‘fundamental’ form:

The SI-fundamental is the coherent system of units in which \( C_1 \) is the fundamental-unit of frequency, \( C_2 \) is the fundamental-unit of speed and \( C_3 \) is the fundamental-unit of action.

In this way, according to this definition, for example the Planck constant has a value 1 fundamental-unit of action.

Now it is possible to rebuild the system, linking the SI-fundamental with a SI-conventional (the present one), the latter with the aim to implement the continuity principle:

The SI is the coherent system of units in which
(A) the unit of time (duration) is the second, symbol s, the unit of length is the metre, symbol m, and the unit of mass is the kilogram, symbol kg;
(B) the frequency \( \Delta \nu_{Cs} \) is \( k_1 \) s\(^{-1}\), the speed \( c \) is \( k_2 \) m s\(^{-1}\), the action \( h \) is \( k_3 \) kg m\(^2\) s\(^{-2}\);
(C) the numerical values are \( k_1 = 9.192631770 \times 10^{-10}, k_2 = 2.99792458 \times 10^{-9}, k_3 = 6.6260693 \times 10^{-34} \) (or whatever values CODATA will propose when the system is decided; clause (C) specifies stipulated values, i.e., exact, derived from experimental uncertain values).

3. Conclusions
In summary, a system of units of this modified GCD type is defined according to the following logical scheme:
- a system of equations between quantities and a set of constant quantities \( \{C_i\} \) are assumed, their number corresponding to the dimension of the base of the system;
- from these components a fundamental system is derived first, where each \( C_i \) has a numerical value 1;
- a conventional system is linked to it, where one assume that each \( C_i \) can assume a value different from 1, in order to make possible the continuity with the previous system;
- in agreement with the best knowledge available at the time of the change, numerical values are assigned to the \( k_i \) such that the continuity is ensured. They are in fact those accepted in the previous system, under the hypothesis that the best knowledge available is the one immediately preceding the formulation of the new system.

The benefits of this conceptual structure are twofold:
- a structural stability of the system, which needs to be modified only in the event of a radical novelty, e.g., the re-formulation of fundamental equations between quantities, or the gained evidence that a quantity assumed to be invariant is not;
- a partial modularity of the system, thanks to the possibility of re-parametrisation of the structure, so that the correction of the value of a constant would only require to adjourn the stipulated value.

This conceptual structure of the definition clearly distinguishes between the components of the definition that arise from theory and the components that instead are conventional. Such a distinction also makes the whole system easier to understand in its fundamental structure, an issue extremely important due to the importance of the SI in our societies.

Acknowledgements
The authors thank Prof. Ingvar Johansson, for some clarifications he provided to a draft of this paper, and appreciate his manuscript [4] and Prof. Alessandro Giordani for the fruitful discussions on this subject.
References
[1] BIPM, CCU: 9th Brochure of the SI, draft of 11 December 2015, published on January 4, 2016 (2016 Draft), http://www.bipm.org/en/measurement-units/new-si (consulted on February 12, 2016).
[2] I.M. Mills, P.J. Mohr, T.J. Quinn, B.N. Taylor, E.R. Williams, Redefinition of the kilogram, ampere, kelvin and mole: a proposed approach to implementing CIPM recommendation (CI-2005), Metrologia 43, 227–246, 2006.
[3] T. Kuhn, The structure of scientific revolutions, University of Chicago Press, 1962
[4] I. Johansson, Constancy and Circularity in the SI, (2014) pp. 25, in http://metrologybytes.net/PapersUnpub/OpEds/Johansson_2014.pdf.