Ultimate State Evaluating Criteria of RC Structural Systems at Loss of Stability of Bearing Element

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Abstract. The paper deals with the deformation of reinforced concrete structural systems of multi-story buildings and structures under accidental impacts. One of these impacts is instantaneous removal of a bearing element of a system. Before this moment researches of survivability and resistance against progressive collapse of structural systems under such impacts were based on the strength criteria. At the same time instantaneous removal of structural element may be caused by loss of stability at evolutionary accumulation of corrosion damages or other physical impacts. That is why the deformation criteria of loss of stability of a structural system under accidental impacts are proposed in this paper. The energy approach and the mixed method of structural mechanics are applied to evaluate survivability and resistance against progressive collapse of a structural system at loss of stability of a bearing element. Two-level computational models are used to solve this problem. An example of calculation of 14-storey building under such impact in accordance to deformation criteria is presented. It is shown, that critical force, that lead the structural system to progressive collapse at evolutionary accumulation of damages is less then ultimate compressive force, which were calculated in accordance to strength criteria.

1. Introduction

Structural systems made of prefabricated concrete flat frames and bracing beams or space frames are widely used in mass civil building in Russia, EU, Great Britain, China and etc. At this moment it is developed and using significant numbers of various modifications of such framing structures, such as 1.020.1/87 series in Russia; prefabricated reinforced concrete structural system "REKON" [1, 2]; prefabricated system of large panel housing developed by Central Research Institute for Experimental Design of Housing [3, 4]; "RADIUS" system [5]; panel-frame constructive system, proposed by OASTC of RAACS [6, 7], PPB SARET system; system with prefabricated U-shape beams [8]; hybrid system with rigid reinforcement [9] and etc. For presented variants of framing structural systems prefabricated beams are connected with columns and overlap slabs by monolithing that gives rigid nodes and whole rigid overlap disk.

Gradual decreasing of additives price makes using of high-strength concrete of C60, C70 classes and stronger economically effective in mass building [12 - 14]. It is obvious, in this case cross sections sizes of bearing elements decrease, that leads to self-weight decreasing. It gives good effect for strength resistance of structural system at seismic actions and special accident impacts. At the same time, comparison of physical and mechanical parameters of concretes of C40 and C70 classes shows
that 117.6% increasing of ultimate compressive strength for concrete C70 corresponds only 35.7% increasing of it ultimate tensile strength and 26.2% increasing of deformation modulus. Hence, eccentrically compressed element cross section sizes determined by strength criterion are equal or more than the sizes that determined by stability criterion. Another factor, which can be critical for strength resistance of concrete constructive elements and leads to loss of stability, is accumulation of corrosion damages. At the same time, it should be noted that prevailing number of scientific papers devoted to progressive collapse and survivability of reinforced concrete framing structural systems deals with strength criterion and observes removal of the most loaded rod at mechanical impacts [15-18], for example vehicle impact, corrosion damage [19-22] or high temperature influence [23].

Therefore, the purpose of this paper is modeling of reinforced concrete prefabricated framing structural systems strength resistance to destruction at loss of stability one of the compressed bearing elements caused by accumulation of corrosion damages. For comparison it is carried out calculation of deforming and destructuring of such structural system by strength criterion.

2. Models and methods
Let us write equation for critical force of arbitrary compressed rod element of constructive system:

$$ N_{cr} = \frac{k_{cr}^2 B_{red}}{l^2}, $$

where the following designations are applied: $B_{red}$ is reduced bending stiffness of reinforced concrete element, that determined taking in account accumulation of different types of damages; $l$ is geometrical length of rod element; $k_{cr}$ is parameter of critical force which depends on the topology of structural system, loading and boundary conditions for rod working as element of constructive system:

$$ k_{cr} = \frac{\pi}{\nu}, $$

$\nu$ is effective length factor of rod element.

Despite there are values of effective length factor for single rod elements with ideal boundary conditions in technical literature and building standards, it is necessary to calculate $k_{cr}$ using space computational models of structure-foundation to account influence from nearby rods and increase the accuracy of calculation.

For calculation of centrally compressed reinforced concrete element the formula of ultimate internal force $N_{ult}$, determined by strength criterion, can be written in the form:

$$ N_{ult} = R_b \cdot A + R_{sc} A_s = A \cdot R_b + \mu R_{sc}, $$

where $R_b$ is ultimate compressive strength of concrete, $A$ is area of reinforced concrete element cross section, $R_{sc}$ is ultimate compressive strength of reinforcement, $\mu$ is percent of reinforcement in concrete.

Taking in account relationship for reduced area of cross section $A_{red} = A_b \left(1 + \mu \frac{E_s}{E_b}\right)$ we obtain:

$$ N_{ult} = R_b \cdot A_{red} \left(1 + \mu \frac{R_{sc}}{R_b}\right) \left(1 + \mu \frac{E_s}{E_b} \right). $$

If condition $N_{cr} < N_{ult}$ is satisfied, then it should calculate structural element by stability criterion. Substituting in the right and left parts of this inequality corresponding equations for $N_{cr}$ and $N_{ult}$ we obtain:

$$ \frac{k_{cr}^2 B_{red}}{l^2} < R_b \cdot A_{red} \left(1 + \mu \frac{R_{sc}}{R_b}\right) \left(1 + \mu \frac{E_s}{E_b} \right). $$
Saving in the left part of inequality the parameters characterizing system topology and transferring to right part all physical and mechanical parameters, we obtain

\[
\frac{k_{cr}^2 I_{red}}{l^2 A_{red}} < \frac{R_b}{E_{b,\tau}} \left( 1 + \mu \frac{R_s}{R_b} \right) \left( 1 + \mu \frac{E_s}{E_{b,\tau}} \right).
\]  

(5)

In previous inequality, \(\frac{I_{red}}{A_{red}}\) is square of reduced inertia radius of cross section \(i_{red}^2\). Taking it in account we obtain:

\[
\frac{k_{cr} i_{red}^2}{l^2} < \frac{R_b}{E_{b,\tau}} \left( 1 + \mu \frac{R_s}{R_b} \right) \left( 1 + \mu \frac{E_s}{E_{b,\tau}} \right),
\]

or taking square root from previous equation:

\[
\frac{k_{cr} i_{red}}{l} < \left[ \frac{R_b}{E_{b,\tau}} \left( 1 + \mu \frac{E_s}{E_{b,\tau}} \right) \right]^{1/2}.
\]  

(6)

However, it is more convenient to use inverse inequality, where the left part is reduced flexibility divided on \(\pi\):

\[
\frac{l}{k_{cr} i_{red}} > \left[ \frac{R_b}{E_{b,\tau}} \left( 1 + \mu \frac{E_s}{E_{b,\tau}} \right) \right]^{1/2}.
\]  

(7)

Account of long-term corrosion process for constructive elements of reinforced concrete framing structural systems is carried out on the basis of phenomenological model by V.M. Bondarenko variant [19, 20]. In accordance with this model the corrosion depth of reinforced concrete structure \(\delta \ t, t_0\) at colmatation (fading in time) can be written in the following form, the graphical interpretation of which is presented in figure1(a):

\[
\delta \ t, t_0 = \delta_{cr} \left( 1 - \left[ \alpha \ m -1 \ t - t_0 + \left( 1 - \frac{t_0 - t}{\delta_{cr}} \right)^{-m} \right] \right)^{1-m},
\]  

(8)

where \(\delta_{cr}\) is ultimate corrosion damage depth at fading in time process; \(\alpha\) and \(m\) are empirical parameters of corrosion velocity and loading level respectively; \(t\) and \(t_0\) are current time and initial moment of corrosion damage observation respectively. It is should be noted that \(m < 0\) corresponds to zone of avalanche accumulation of corrosion damage, \(m > 0\) is area of colmatation (fading in time) corrosion process, \(m = 0\) is boundary line which corresponds to filtration character of corrosion front moving. Parameters \(\delta_{cr}\), \(\alpha\) and \(m\) can be determined empirically and approximated by polynomial equations [19] as it is shown in figure1 (b):

\[
\delta_{cr} = \sum_{i=0}^{3} q_{\delta} \cdot \sigma / R_b^i, \quad \alpha = \sum_{i=0}^{3} q_{\alpha} \cdot \sigma / R_b^i, \quad m = \sum_{i=0}^{3} q_{m} \cdot \sigma / R_b^i,
\]

where \(q_{\delta}, q_{\alpha}, q_{m}\) are empirical parameters determined for aggressive environment condition and initial physical and mechanical characteristics of concrete.

Reduced bending stiffness of compressed structural element, which is operated at combination of strength resistance and aggressive environment resistance, can be written in the following form for stability analysis:
where $I_{\text{red},n}$, $I_{\text{red},n\delta}$, $I_{\text{red},s}$ are inertia moment of concrete layers without destruction, inertia moment of damaged concrete layers and inertia moment of reinforcement respectively. These inertia moments are taking about center of mass of reduced cross section of reinforced concrete element with corrosion damages as it is shown in figure 1 (c).

\begin{align}
B_{\text{red}} &= E_{b,z} I_{\text{red}} = E_{b,z} I_{\text{red},n} + I_{\text{red},n\delta} + I_{\text{red},s},\tag{9}
\end{align}

In these equations the following designations are taken: $a$ is distance between external surface of reinforced concrete element and reinforcement center of mass, $b$ and $h$ are height and width of cross section respectively, the center of mass of which placed from the low surface of concrete element at the distance $y_c$ as it is shown in figure 1:

\begin{align}
y_c &= S_{\text{red}} / A_{\text{red}},
\end{align}

where $S_{\text{red}}$ and $A_{\text{red}}$ are reduced statical moment of cross section and it area respectively.
Depth of cross section $\delta t$ damaged by corrosion but partially saving its mechanical characteristics takes the form:

$$\delta t = \delta t_0 - z^* t,$$

in which $\delta t, t_0$ is determined by the formula (8), and $z^* t = t \frac{dz}{dt}$ is simple relationship for determination of depth of entirely destructured concrete.

Account of reinforcement corrosion is carried out on the basis of simple relationships:

$$\begin{align*}
\omega_s &= 1 \quad \text{for } \delta t, t_0 < a - d/2; \\
\omega_s &= \frac{\pi}{4A_s} \left[ d - t - t_s \frac{dz}{dt} \right]^2 \quad \text{for } \delta t, t_0 > a - d/2; \\
\omega_s &= 0 \quad \text{for } z^* t \geq a - d/2,
\end{align*}$$

where $t_s$ is moment, when corrosion front reaches external surface of reinforcement, i.e. $\delta t, t_0 > a - d/2$, $d$ is diameter of longitudinal reinforcement.

3. Results and discussion

Comparison of calculation against progressive collapse by strength and stability criteria is carried out for framing structural system of 14-storey civil building, the initial computational model of which is presented in figure 2 (a). Prefabricated columns and beams are made of high-strength concrete of C100 class. Element lengths and its cross section sizes are applied on the basis of prefabricated structural system "REKON". Reinforcement of B500 class is determined by calculation and equal 1% of concrete cross section. Based on the analysis of the local destruction zones of the framing structure at sudden loss of stability of one of the outermost columns for analyzing the stress-strain state and determining the critical force as the second-level design scheme it is chosen a fragment in the form of a two-story frame. Reduced constructive elements of framing structure are modeling as rigid or flexible supports and forces acting in it as it is shown in figure 3. Possible zone of local destruction includes area between axes 1-5 along axis "C". It is supposed that left column of this fragment is subjected action of medium aggressive environment, which leads to destruction about 0.4-1.5 mm of concrete in the year. Taking it into account the following parameters of corrosion process from equation (8) are accepted on the basis of paper [20]: $\delta_{cr} = 20$ mm, $\alpha = 0.035$, $m = 2.2$. Calculation results are presented in the table 1.

![Figure 2. Initial finite-element computational model of prefabricated structural system with flat frames and bracing beams of 14-story house (a); mosaic of longitudinal forces $N$ (kN) in elements of structure at hazard loadings combination (b).](image-url)
Figure 3. Simplified second level computational model for reinforced concrete structural system to calculate against progressive collapse

Table 1. Computational results for second level model

| h, cm | b, cm | t, year | \( y_c \), cm | \( I_{red} \), cm\(^3\) | L, cm | \( EI_{red} \), kN*m\(^2\) | k | \( \lambda_{cr} \) | \( \lambda \) | \( N_{ult} \), kN | \( N_{cr} \), kN |
|-------|-------|---------|--------------|-----------------|-------|-----------------|-----|-------------|------|-------------|-------------|
| 20    | 45    | 0       | 10           | 33798           | 420   | 6055.5          | 5.99| 61.6        | 38.0 | 4666        | 12308        |
| 20    | 45    | 45      | 5.7          | 6342            | 420   | 1136.4          | 6.17| 61.6        | 64.3 | 2868        | 2455         |

Analysis of obtained results shows that at evolutionary accumulation of corrosion damages in outermost column of the fragment of structural system it emergency destruction occurs by stability criterion instead strength criterion i.e. \( N_{cr} = 2455 \text{ kN} < N_{ult} = 2868 \text{ kN} \). Special attention at modeling of progressive collapse of such structural system it is required evaluation force dynamical increasing in remaining reinforced concrete elements of structural system at sudden loss of stability of the compressed element.

4. Conclusions

It is obtained computational model to evaluate stability of compressed element of reinforced concrete framing structural system at evolutionary accumulation of corrosion damages.

Account of long-term colmatational process of corrosion damage accumulation in reinforced concrete is carried out on the basis of the variant of phenomenological model proposed by V.M. Bondarenko.

Comparison of ultimate strength and values of critical force for compressed elements shows that the reason of progressive collapse for loaded and damaged by corrosion reinforced concrete framing structural systems can be not only sudden removal of a bearing element by strength criterion but it loss of stability.

References

[1] Shembakov V A 2013 Zhilischnoe stroitelstvo vol. 3 pp. 26-29.
[2] Shembakov V A 2017 Zhilischnoe stroitelstvo vol. 3 pp. 9-15.
[3] Nikolaev S V, Shrejber A K, Etenko V P 2015 Zhilischnoe stroitelstvo vol. 2 pp. 3-7.
[4] Nikolaev S V 2016 Zhilischnoe stroitel'stvo vol. 3 pp. 15-25.
[5] Senchenkov A S, Demidov A R, Sokolov B S 2008 Beton i zhelezobeton vol. 5 pp. 2-4.
[6] Bukhtiyarova A S, Kolchunov V I, Rypakov D A, Filatova S A 2014 Stroitels'tvo i rekonstruktsiya Vol 56 (6) pp. 18-24.
[7] Emelianov S G, Fedorova N V, Kolchunov V I 2017 Stroiteln'ye materialy vol. 3 pp. 23-27.
[8] Tae-Sung Eom, Hong-Gun Park, Hyeon-Jong Hwang, Su-Min Kang J. of Structural Eng., Vol. 142 (2). doi: 10.1061/(ASCE)ST.1943-541X.0001378.
[9] Haider Hamad Ghayeb, Hashim Abdul Razak, N H Ramli Sulong 2017 Construction and Building Materials, Vol. 151, 258-278. doi: 10.1016/j.conbuildmat.2017.06.073.
[10] Yufan Huang, Enrico Mazzarolo, Bruno Briseghella, Tobia Zordan, Airon Chen 2018 Eng. Structures, Vol. 159 pp. 227-244. doi: 10.1016/j.engstruct.2018.01.012.
[11] Zhenmin Yuan, Chengshuang Sun, Yaowu Wang 2018 Automation in Construction, Vol. 88, 13-22. doi: 10.1016/j.autcon.2017.12.021.
[12] GuoY B, Gao G F, Jing L, V P W Shim 2017 International J. of Impact Eng. Vol. 108 pp. 114-135.
[13] Zvezdov A I, Falikman V R 2008 Zhilischnoe stroitel'stvo vol. 7 pp. 2-6.
[14] Kaprielov S S, Travush V I, Karpenko N I, Sheinfeld A V, Kardumjan G S, Kiselev Y U, Prigogenko O V 2008 Building materials vol. 3 pp. 9-13.
[15] Kodysh E N, Trekin N N, Chesnokov D A 2016 Promishlennoe I grazhdanskoe stroitel'stvo vol. 6 pp. 8-13.
[16] Setareh Amiri, Hamed Saffari, Javad Mashhadi 2018 Eng. Failure Analysis Vol. 84, 300-310. doi:10.1016/j.engfailanal.2017.11.011.
[17] Kolchunov V I, Savin S Y 2017 J. Appl. Eng. Sc. Vol 15 (3) pp. 325-331. doi:10.5937/jaes15-14602
[18] Fedorova N V, Koren'kov P A 2016 Stroitels'tvo i rekonstruktsiya Vol. 6 pp. 90-100.
[19] Bondarenko V M 2016 Silovoye deformirovaniye, korrozionnye povrezhdeniya i energosoprotivleniye zhelezobetona (Kursk: Publ. of the South-West State Univ.)
[20] Bondarenko V M, Kolchunov V I 2007 Izvestiya vuzov. Stroitels'tvo vol. 5 pp. 4-8.
[21] Selyayev V P, Selyayev P V, Alimov M F, Sorokin Ye V 2017 Stroitels'tvo i rekonstruktsiya vol. 6 pp. 49-58.
[22] Bondarenko V M, Klyuyeva N V 2008 Izvestiya vuzov. Stroitels'tvo Vol. 1 pp. 4-12.
[23] Fedorov V S, Levitskiy V E, Soloviev I A 2015 Stroitels'tvo i rekonstruktsiya Vol. 61(5) pp. 47-55.