1. Introduction

Transporting non-fixed heavy cargoes by vehicles (road trains, railroad freight trains, container trucks, trailers, plate trucks, etc.) may be accompanied by the cargoes hitting their restrictive devices (body’s boards, wagon walls, etc.). An other issue relates to the transportation of fragile, explosive, chemically, bacteriologically dangerous articles. Additional loads on cargoes are manifested when they are transported by multi-link vehicles where impact loads on their coupling mechanisms are additionally added (Fig. 1).

However, the impact phenomena that occur during the transportation of specifically non-impact-resistant cargoes, under the extreme modes of movement (sharp acceleration and braking) have not been studied in detail. This is especially evident in the case of cargo transportation by multi-link vehicles.

Fig. 1. Examples of multi-link vehicles: a — a three-link road train; b — a two-link road train; c — a multi-link train; d — a three-link bus
There are such structures of damping and vibration devices that reduce dynamic loads on cargoes. However, their use is predominantly random, not always justified, and, therefore, often ineffective.

When constructing impact-resistant and vibration protection systems for cargoes transported by multi-link vehicles, it is necessary to take into consideration both the characteristics of a cargo, of coupling devices, and transportation conditions. Accordingly, it is a relevant task to devise procedures for the analysis and synthesis of impact-resistant protective systems for cargo transportation.

2. Literature review and problem statement

Works [1, 2] describe the process of car movement during acceleration and define the dynamic forces that act on a cargo taking into consideration the viscous friction of the damper and the sliding friction coefficient between the cargo and the car body. Paper [3] reports the results of studying the condition of spring resonance exposed to dynamic loads without taking into consideration the friction that would cause the onset of exiting that state.

Article [4] considers the elastic impact of a non-fixed cargo during its transportation and notes that the phenomenon of an elastic impact causes additional loads on the cargo.

Study [5] into the rectilinear movement during vehicle braking while ascending proves the possibility of the emergence of corresponding impact phenomena that occur between the cargo and the vehicle, especially at the beginning of such braking.

Paper [6] emphasizes the need to take into consideration, in the designs of cranes, vibration impacts for the case of seismic instability in operating conditions, which can cause a series of peak loads. Ensuring the strength of the crane structure is provided for only by an additional margin of safety.

Works [1–6] relate to studies into impact phenomena; however, their authors did not obtain explicitly the value of the dynamic load on a cargo during the uneven movement of vehicles.

Structural solutions reported in [7–9] could be an option for overcoming vibration shock phenomena but the elastic elements of dampers do not provide for a steady restorative force, which could cause a double impact.

The elastic elements of a coupling device [9] smooth only possible jerks when moving or braking abruptly. Vibration shock phenomena are partially damped in the presence of friction in the guide elements, which is insufficient.

It is possible to reduce impact loads during the transportation of non-fixed or partially fixed oversized cargoes by using special elastic and dissipating elements. Longitudinal elastic and dissipative elements in a truck should be placed between the body (a cargo platform) and the cargo, or between the coupling links in a multi-link vehicle [8, 9].

Our analysis [1–9] reveals that the vibration protection of transported cargoes requires further research, in particular determining the critical stages in the transitional motion modes, as well as patterns of impact processes, and ways to resolve this issue.

3. The aim and objectives of the study

The purpose of this work is to investigate the possible impact of heavy non-fixed or partially fixed cargoes against restrictive devices during their transportation under transition modes of movement by multi-link vehicles, in order to determine the amount of dynamic loads on these cargoes and on the vehicles themselves. That would make it possible to devise a procedure for synthesizing vibration protection systems to be employed in protective devices on vehicles.

To accomplish the aim, the following tasks have been set:
- to consider the movement of a vehicle under transition modes and the impact of such modes on reducing the dynamic loads on a cargo;
- to investigate those impact phenomena that occur during the transportation of non-fixed or partially fixed oversized cargoes in order to reduce impact loads on cargoes during their transportation under extreme modes of movement;
- to offer a way to reduce impact loads on cargoes during their transportation under extreme modes of movement.

4. Methods for studying the impact phenomena during the transportation of oversized cargoes

To study the actual impact phenomena during the transportation of non-fixed or partially fixed cargoes, the simplest flat two- and three-mass elastic and viscoelastic estimation models were used. To investigate the actual impact phenomena during the transportation of non-fixed or partially fixed cargoes, an elementary theory of elastic impact was applied, which has allowed us to determine the maximum dynamic loads on cargoes in a closed form. When exploring such impact phenomena, the elastic-damping characteristics of the suspensions of vehicles and the influence of the actual micro profile of the road were not taken into consideration, since this could complicate the research process and would not make it possible to find in a closed form the maximum dynamic loads on cargoes and vehicles during their transportation under extreme modes of movement.

To solve the above-built system of ordinary differential equations, a fitting method was used, which has allowed us to determine in a closed form the values of the dynamic loads on cargoes and vehicles during their acceleration and braking.

To analyze the dynamic processes that arise during the transportation of non-fixed heavy cargoes, the most simplified estimation schemes were used (Fig. 1, a, b), which have made it possible to build such a system of differential equations that adequately describe the actual impact phenomena that occur during the transportation of non-fixed or partially fixed cargoes under extreme modes of motion, and have allowed us to analytically solve this system of differential equations and determine, in a closed form, the values of the maximum dynamic loads on heavy cargoes and vehicles.

These diagrams show the translational acceleration of a two-link vehicle (tractor) with a fixed driving force $P(t)$ hosting an elastically fixed cargo (Fig. 2, a) and a partially viscoelastic fastening of the cargo on the body of the tractor (Fig. 2, b), $m_3$ denotes the total weight of the trailer or semitrailer (Fig. 2, a), $m_2$ – the weight of the cargo. In Fig. 2, a, the following generalized coordinates are selected: $S_1$ and $S_2$ – absolute movements of the tractor and trailer, $S_1$ – relative movement of a cargo. First, consider the process of accelerating a two-link vehicle weighing $m_1$, which is elastically connected to a semitrailer weighing $m_2$ during the transportation of a load weighing $m_3$ elastically attached to the body of the tractor.
The system of differential equations that express the motion of a two-link vehicle with an elastic coupling between a trailer and a tractor and a load weighing $m_3$ elastically attached to the body of the tractor during its acceleration (the initial stage of movement) at a steady driving force $P(t)$ takes the form similar to that reported in [11–13]:

$$
\begin{align*}
&m_1 \frac{d^2 S_1}{dt^2} + a_1 \frac{dS_1}{dt} + C_{12} (S_1 - S_2) = b_1; \\
&m_2 \frac{d^2 S_2}{dt^2} - C_{12} (S_1 - S_2) + C_{23} S_2 = -F_{f2};
\end{align*}
$$

where $C_{12}$ and $C_{23}$ are the coefficients of rigidity of the coupling device between the tractor and cargo, $F_{f2}$ is the friction of the cargo against the body of the trailer.

The initial conditions for the acceleration process of a two-link vehicle with an elastic coupling between a trailer and a tractor are:

$$
\begin{align*}
&S_1 = \frac{F_{f2}}{C_{12}}; \\
&\text{At } t = t_* = 0; \\
&V_1 = V_{\text{max=rs}}; \\
&S_2 = 0; \\
&V_2 = 0.
\end{align*}
$$

This stage of accelerating a two-link vehicle with an elastic coupling between the trailer and the tractor ends provided:

$$
C_{23} S_2 = F_{f1}.
$$

Hence, we determine the time, starting and final speed, of moving the tractor relative to the trailer until the cargo shifts on the trailer:

$$
T_{\text{link}} \Rightarrow V_{\text{max=rs}} \Rightarrow S_{\text{max=rs}} \Rightarrow S_{\text{max=rs}} \Rightarrow V_{\text{max=rs}}.
$$

Thus, the movement of the cargo begins (the final stage) due to the acceleration of a two-link vehicle with an elastic coupling between the trailer and the tractor and the manifestation of the inertia force. At the same time, the cargo begins to move on the trailer, overcoming the friction forces between the cargo and the body of the trailer. The system of differential equations with a strong driving force $P(t)$ is similar to that given in [7, 11, 12]:

$$
\begin{align*}
&m_1 \frac{d^2 S_1}{dt^2} + a_1 \frac{dS_1}{dt} + C_{12} (S_1 - S_2) = b_1; \\
&m_2 \frac{d^2 S_2}{dt^2} - C_{12} (S_1 - S_2) + C_{23} S_2 = -F_{f2}; \\
&m_3 \frac{d^2 S_3}{dt^2} - C_{23} (S_2 - S_3) = -F_{f3},
\end{align*}
$$

Those integration constants that appear when solving the systems of differential equations (4) are determined from the final conditions (2), (3) for the initial stage of the movement of this mechanical system.

5. Results of studying impact phenomena during the transportation of oversized cargoes

Under the transition modes of vehicle movement, impact phenomena are possible, which, at some approximation, can be considered using an estimation scheme of vibration protection during the transportation of a non-fixed cargo shown in Fig. 2, a.

We believe that during acceleration or braking the resistance force to the rolling of a vehicle and the cargo movement force $P_{\text{fr}}$ and $P_{\text{br}}$, by analogy with [1, 2, 8–11], equal, accordingly, $(m_1 + m_2)g/f_s$ and $m_2 g f_s$, $f_s$ is the coefficient of total resistance to rolling the vehicle; $f$ is the coefficient of resistance to cargo movement; $g$ is the acceleration of gravity.

In order to simplify the study of impact phenomena, we believe that the driving or braking forces caused by the acceleration or braking of a vehicle are applied instantly and are of constant magnitude. At the same time, the coefficient of the reduced overall rigidity $c$ of the restrictive device and cargo at the place of their collision is considered unchanged.

We neglect the scattering of energy in the process when a cargo hits the viscoelastic restrictive devices on the vehicle body, inelastic tyre pliability, and fluctuations in the suspension of the vehicle, as well as the road micro profile.

The impact process, similarly to [4], is divided into four stages:

1. From the beginning of cargo movement to the moment the cargo travels the distance $l$, Fig. 2, a.
2. Loading the viscoelastic element of the restrictive device to a value equal to the load $F_{\text{max}}$.
3. Weakening of the viscoelastic element of the restrictive device.
4. Departure of the cargo from the viscoelastic element of the restrictive device.

The differential equations describing the movement of this system can be represented as:

$$
\begin{align*}
&m_3 \ddot{s}_i = P - P_{\text{fr}} - P_{\text{br}} \text{sign}(\dot{s}_i - \dot{s}_j) - \\
&- c(\dot{s}_i - \dot{s}_j) \sigma_0 (\dot{s}_j - \dot{s}_j) - v(\dot{s}_j - \dot{s}_j) \sigma_0 (\dot{s}_j - \dot{s}_j); \quad (5)
\end{align*}
$$

$$
\begin{align*}
&S_1 = \frac{F_{f2}}{C_{12}}; \\
&\text{At } t = t_* = 0; \\
&V_1 = V_{\text{max=rs}}; \\
&S_2 = 0; \\
&V_2 = 0.
\end{align*}
$$
\[ m\ddot{s}_2 = P_\sigma \text{sign}(s_1 - s_2) - 
- c(s_1 - s_2)\sigma_2 (s_1 - s_2) - v(\dot{s}_1 - \dot{s}_2)\sigma_2 (s_1 - s_2), \quad (6) \]

where \( \sigma_0 \) is the Heaviside unit function.

To solve the system of equations (5), (6), we use a conjugation method [12].

Solving the system of equations (5), (6) under the initial conditions \( t=0; S_1=0; S_2=0; S_2=t; S_2=0 \) and, considering that at the end of the initial stage the cargo would hit a viscoelastic restrictive device of the vehicle, we derive the time

\[ t = \sqrt{\frac{2m_2}{(P-P_\sigma)m_2-(m_1+m_2)P_\sigma}}. \quad (7) \]

The speed of collision between the cargo and a viscoelastic restrictive device is equal to:

\[ V = \dot{s}_1 = \frac{a}{m_1m_2}\sqrt{\frac{2m_1m_2}{a}}, \quad (8) \]

where

\[ a = (P-P_\sigma)m_1 - (m_1+m_2)P_\sigma. \quad (9) \]

Solving the system of equations (5), (6) according to the procedure given in [1, 4, 12], under the initial conditions \( t=0; s_1=s_2=0 \), we determine the maximum deformation of the elastic element of the restrictive device:

\[ s_{\text{max}} = S_1 = \sqrt{\frac{a + 2cl(m_1 + m_2)}{c(m_1 + m_2)}} + a. \quad (10) \]

The maximum load on the elastic element of the restrictive device at impact is:

\[ F_{\text{max}} = s_{\text{max}}c = \sqrt{\frac{a + 2cl(m_1 + m_2)}{m_1 + m_2}} + a. \quad (11) \]

Solving the system of equations (5), (6) under initial conditions \( t=0; S_1 - S_2 - S\neq -S1; S_1 - S_2 - S\neq 0 \), we find a decrease in the deformation of the elastic element of a two-link vehicle with an elastic coupling with a tractor after impact:

\[ S = S_{\text{III}} = \frac{1}{c(m_1 + m_2)} \]

\[ b - \left[ 2(P-P_\sigma)m_2 + d \right] \cos \sqrt{\frac{c(m_1 + m_2)}{m_1m_2}} t, \quad (12) \]

where

\[ b = (P-P_\sigma)m_2 + (m_1 + m_2)P_\sigma; \]

\[ d = \sqrt{a + 2cl(m_1 + m_2)}; \]

\[ a = (P-P_\sigma)m_1 - (m_1 + m_2)P_\sigma. \]

We determine the speed of movement of the cargo at the time of its detachment from the restrictive device of a two-link vehicle with an elastic coupling to the tractor:

\[ V_{\text{crc}} = \dot{S}_{\text{III}}(t_3) = \sqrt{\frac{2(P-P_\sigma)m_2 + d}{cm_1(m_1 + m_2)}}. \quad (13) \]

Equation (12) makes it possible to determine the conditions under which the cargo may leave the restrictive device of a two-link vehicle with an elastic coupling to the tractor after impact:

\[ -t_3 < t < t_f - a cargo departs from the restrictive device; \]

\[ t_3 \geq t_f - a cargo does not break away from the elastic element. \]

The time \( t_3 \) of the cargo departure from the restrictive device of a two-link vehicle with an elastic coupling to the tractor is determined from equation (8) under the condition \( S_{\text{III}} \):

\[ t_3 = \sqrt{\frac{m_1m_2}{c(m_1 + m_2)}} \arccos \frac{b}{2(P-P_\sigma)m_2 + d}. \quad (14) \]

Solving the system of equations (5), (6) with new initial conditions \( t=0; S_1 - S_2 - S=0; S_1 - S_2 = S_{\text{III}} - V_{\text{crc}} \), we find the distance at which the cargo can move away from the restrictive device of a two-link vehicle with an elastic coupling to the tractor after impact:

\[ l = S(t_3) = \sqrt{\frac{2(P-P_\sigma)m_2 + d}{cm_1(m_1 + m_2)}} \]

\[ = \sqrt{\frac{\pi}{c(m_1 + m_2)}} \left[ 2(P-P_\sigma)m_2 + d \right] - b^2; \quad (15) \]

where \( b \) and \( d \) are the parameters previously given in equation (12).

Fig. 3 shows the graphical dependences of maximum dynamic loads \( F_{\text{max}} \) on the restrictive device of a two-link vehicle with an elastic coupling to the tractor; Fig. 4, \( b \) – the nature of the change in distance \( l \) after impact between the cargo and the restrictive device of a two-link vehicle with an elastic coupling to the tractor, depending on the stiffness coefficient of an elastic element \( c \), on the ratio \( \frac{P}{P_\sigma} \) of the driving force to the resistance force to the rolling of a two-link vehicle with an elastic coupling to the tractor and the distance \( l \) between the cargo and the restrictive device at the initial moment of time at different ratios of masses \( \frac{m_1}{m_2} \) and ratios \( \frac{P}{P_\sigma} \).

Our calculations show that the value of the impact load on the restrictive device of a two-link vehicle with an elastic coupling to the tractor significantly depends on the ratios of masses \( \frac{m_1}{m_2} \) and the ratios \( \frac{P}{P_\sigma} \) of the tractor’s driving force to the resistance forces to vehicle movement. With a decrease in the ratio of masses \( \frac{m_1}{m_2} \) and an increase in \( \frac{P}{P_\sigma} \), the impact force increases significantly. Our analysis of equations (8) and (11) demonstrates that for a given mechanical model (Fig. 4, \( a \) ) an impact is possible for \( P = P_\sigma \geq 1 + \frac{m_1}{m_2} \).
Of particular practical interest are the graphical dependences shown in Fig. 5 as they demonstrate the direct exclusion of impact against a restrictive device at certain values of the coefficient of resistance to the movement of cargoes $f_k$ (a coefficient of friction between the cargo and the body of the vehicle) at the appropriate ratios.

With an increase in $f_k$, the impact avoidance is possible with a larger $f$. If the acceleration of the vehicle continues for a long time, then the repeated impacts of the cargo against limiting devices are possible, and the value of the impact load pulse would increase.

Fig. 4, b shows that if $\frac{P}{P_{j_k}} \geq 10$ under certain parameters of system (5), (6), the distance $L^*$ at which the cargo moves from the restrictive device after impact would be greater than the distance $l$ before the impact. A second impact would occur after a period equal to the sum of the time of stages when a two-link vehicle with an elastic coupling starts its movement:

$$T = t + t_m + t_i.$$

In particular, the duration of time $t_m$ is found by solving a system of equations (5) and (6) for the final stage equal to:

$$t_m = \sqrt{\frac{m_1 m_2}{c(m_1 + m_2)} \left( \frac{\pi}{2} \arctg \sqrt{\frac{2c}{(m_1 + m_2)^2} \frac{m_1}{m_2}} \right)}$$

The time $t_i'$ is determined from solving a system of equations (5) and (6) for the final stage:
The differential equations describing the movement of the mechanical system (Fig. 4, b) can be represented in the following form:

\[
\begin{align*}
    m\ddot{x}_1 &= P(t) - P_n - P_{\sigma} \text{sign}(\dot{x}_1 - \dot{x}_2) - c(x_1 - x_2)\sigma(x_1 - x_2) - v(\dot{x}_1 - \dot{x}_2)\sigma_0(x_1 - x_2); \\
    m\ddot{x}_2 &= P_n \text{sign}(\dot{x}_1 - \dot{x}_2) + c(x_1 - x_2)\sigma_0(x_1 - x_2) - v(\dot{x}_1 - \dot{x}_2)\sigma_0(x_1 - x_2).
\end{align*}
\]

(18)

where is the Heaviside unit function; \(P_{\sigma} = (m_1 + m_2)g/s\); \(P_n\) is the coefficient of resistance to vehicle’s rolling; \(g\) is the acceleration of gravity; \(f\) is the coefficient of friction of the slip of the cargo on the body of the vehicle.

The impact phenomena during the periods of acceleration and braking of vehicles, according to Fig. 4, b, are considered as two stages:

1. From the beginning of moving a system (18) to the moment the cargo travels the distance \(l\).
2. Loading a viscoelastic restrictive device to a value equal to the dynamic effort \(R\), the expression for which, in a closed form, was obtained after solving the system of equations (18).

To solve the system of equations (18), a conjugation method from [13–15] was used.

Solving the system of equations (18) under the initial conditions \(t=0; x=x_1=x_2=0\), we find the time over which the cargo would hit a restrictive device:

\[
t_1 = \frac{\sqrt{2m_1m_2l}}{a}.
\]

(19)

where

\[
a = (P - P_n)\frac{m_2}{m_1 + m_2} - (m_1 + m_2)P_n = \frac{m_2}{2}(P - g(m_1 + m_2)(f_k + f)).
\]

Determine the speed of collision between a cargo and a restrictive device, which is equal to:

\[
V = \dot{x}(t_1) = \frac{a}{m_1m_2} \sqrt{2m_1m_2l}.
\]

(20)

Solving the system of equations (18) according to the procedure from [1, 4, 11–13] under initial conditions \(t=0; x=x_1=x_2=0\), we determine the maximum deformation of a viscoelastic element of the restrictive device of the vehicle:

\[
x_{max} = Ae^{-\alpha t} \sin(k_0 t + \phi) + \frac{a}{c(m_1 + m_2)}.
\]

(21)

where

\[
\alpha = \frac{a}{m_1 + m_2} \times \sqrt{\frac{a(m_1 + m_2)(2cm_1m_2 - \sqrt{2cm_1m_2})}{cm_1m_2 + \sqrt{4cm_1m_2 - \sqrt{2cm_1m_2}}}.
\]

(22)

A damping coefficient of the viscoelastic element of the restrictive device \(n = \frac{m_1 + m_2}{m_1m_2}\).

The duration of the second stage is found from equation (21), which is equal to:

\[
t_2 = \frac{1}{k_0} \times \arctg \sqrt{\frac{2cm_1m_2(2cm_1m_2 - \sqrt{2cm_1m_2})}{cm_1m_2 + \sqrt{4cm_1m_2 - \sqrt{2cm_1m_2}}}}.
\]

(23)

We find the dynamic force \(R\), which is transmitted to the restrictive device of the vehicle during its acceleration or braking; it takes the form:

\[
R = c\dot{x}_{max} + v\ddot{x}_{max} = \frac{a}{(m_1 + m_2)} + Ae^{-\alpha t}(c - v)\sin(k_0 t + \phi) + uk\cos(k_0 t + \phi).
\]

(24)

In the absence of a viscous friction damper in the estimation scheme (Fig. 2, b), then the time is

\[
t_1 = \sqrt{\frac{2m_1m_2l}{a}} (\pi - \arctg \sqrt{\frac{2\pi l(m_1 + m_2)}{a}}).
\]

(24)

where

\[
k_0 = \frac{c(m_1 + m_2)}{m_1m_2}.
\]
\[
\phi = \arctg \left( \frac{a}{2c(m_1 + m_2)} \right) (k_t + \phi) = \frac{\pi}{2}
\]

Then the dynamic effort that is transferred to the restrictive device of the vehicle during its acceleration or braking takes the form:

\[
R = \frac{a + 2c(m_1 + m_2)}{m_1} \left( a + 2c(m_1 + m_2) \right)
\]

(25)

Fig. 5 shows the charts of dynamic efforts, which are perceived by the restrictive device of the vehicle during its acceleration or braking, depending on a change in the distance \( l \) and the coefficient of friction at different ratios of the masses of the vehicle to the masses of cargoes that are transported.

The graphical dependences in Fig. 3–5 are constructed at the following system parameters: \( m_1 = 100 \text{ kg}; m_2 = 500 \text{ kg}; c = 50 \text{ kN/m}; \nu = 100 \text{ Nf/m} \); \( l = 0.1 \text{ m} \); \( P = 1500 \text{ N} \); \( f_1 = 0.02 \); \( f = 0.2 \).

5.3. A way to reduce impact loads on a cargo

Our calculations show that the maximum dynamic efforts from the displacement of cargoes perceived by restrictive devices increase with an increase in the driving or braking forces \( P(0) \), distance \( l \) (Fig. 4–6).

These efforts reduce at the ratio of masses \( \mu = \frac{m_1}{m_2} \) and an increase in the coefficient of viscous friction of the damper \( r \), the friction coefficient \( f \) between a cargo and the body of a vehicle (Fig. 5, 6).

The charts demonstrate that the dynamic loads on the restrictive devices of vehicle bodies during acceleration or braking in the transportation of non-fixed cargoes would not occur always but are determined by the conditions that follow from equation (19):

\[
P > g(m_1 + m_2)(f_1 + f); \quad c > \nu \frac{m_1 + m_2}{4m_2}
\]

(26)

To reduce impact loads, the elastic-dissipative elements of the damping system are used, which provide for the restorative force shown in Fig. 7, a. The disadvantage of such structures is the instability of the restorative force during the movement of the damper. The dashed area indicates the presence of a restorative force when there is no disturbing force (the lower curve). It is necessary to synthesize structures with the appropriate desired law with constant restorative force, which is a compensation for the perturbing force, rather than an additional negative influence (Fig. 7, b). Such systems can consist not only of springs with linear characteristics but also of pneumatic, rubber elements of various cross-sections [7, 8, 16, 17].

After an impact, the mass of a non-fixed cargo \( m_2 \) acquires speed \( V_0 \). During the transition of the kinetic energy \( \frac{m_2V_0^2}{2} \) of the cargo \( m_2 \) to potential energy, this cargo would move to the extreme lower position with the coordinate \( x_1 \) (Fig. 7, a). Then, the following condition should be met:

\[
\frac{m_2V_0^2}{2} = \int_{x_1}^{x} F(x)dx.
\]

(27)

In terms of the protection and preservation of the integrity of a vehicle body, the value of force \( F_2(x_2) \) should not differ significantly from \( F_1(x_1) \).

If the \( x_2 \) coordinate on the upward approach remains greater than zero (enters the dashed region in Fig. 7, a), then the cargo weighing \( m_2 \) would not return to the zero (original) position.

The movement of the cargo weighing \( m_2 \) under the influence of forces \( F_1(x) \) and \( F_2(x) \) (Fig. 7, b) is described by the following differential equations:

\[
m \cdot x''_1 = -F_1(x),
\]

(28)

\[
m \cdot x''_2 = -F_2(x).
\]

(29)

The boundary conditions for these equations are as follows:

1) \( t = 0 \): \( x_1 = 0; x'_1 = V_0; \)

2) \( t = \tau \): \( x_1(\tau) = x_1(\tau); \)

3) \( t = T \): \( x_1(T) = 0, \)

(30)

where \( \tau \) is the time of movement to stop the cargo at the bottom; \( T \) is the period of one fluctuation of the cargo \( m_2 \) under the action of viscoelastic impact due to the extreme mode of movement of the vehicle.

Fig. 8, a shows the dependence of the coordinate of the cargo weighing \( m_2 \) on the time of collision with the
restrictive device. After the impact, the cargo weighing \( m_2 \) would move from a neutral position to the extreme lower one (Fig. 8, a). The speed of the cargo weighing \( m_2 \) at this moment is zero.

The movement of the cargo weighing \( m_2 \) upwards is described by equations (28) and (29). Taking into consideration conditions (30), the dependences of the coordinate position and the speed of movement of the cargo \( m_2 \) on the duration of time of the impact against the restrictive device due to the extreme mode of movement of the vehicle are shown in Fig. 8.

In the process of impact, the characteristic of the restorative force shown in Fig. 7, a is not the best. The characteristic shown in Fig. 7, b provides greater stability of the damping process. There is no such zone at all when at shifting a cargo downwards the restorative force is also directed down (when the cargo stops in this region, the cargo would not return to its zero position).

6. Discussion of results of studying the effect of impact loads on the transportation of non-fixed or partially fixed cargoes under extreme modes of movement

The main achievement and advantage of our devised procedure for studying those impact phenomena that occur during the transportation under extreme modes of movement of non-fixed or partially fixed cargoes is that, in contrast to all other methods for investigating similar impact phenomena, ours makes it possible to determine in a closed form the maximum dynamic loads on these cargoes (equations (11), (23), (25), Fig. 3, 5–7) and the possibility of selecting rational design parameters for the corresponding restrictive devices (equation (26), Fig. 3, a) in order to prevent or reduce the impact of these cargoes against restrictive devices.

The closest to the current paper is works [7, 18–22] whose authors studied the vibration protection of rolling stock and a crew with the use of a damping mechanical device. However, that works did not define the dynamic loads on cargoes during their transportation under the extreme modes of movement of a vehicle.

The disadvantage of this research is that we considered the simplified estimation schemes, which do not take into consideration the springing of vehicles and do not take into account, while moving, the disturbances due to the actual road profile. In addition, the estimation schemes assume that a vehicle moves due to a steady driving force with a steady acceleration under extreme modes of movement. However, during emergency braking, the acceleration is variable. Should the proposed estimation schemes take into consideration that the vehicle, during emergency braking, moves at variable acceleration, the resulting systems of differential equations (1), (4) to (6), and (18) would include variable coefficients so that solving these systems of differential equations with variable coefficients could prove much more complicated. That requires a method that would allow these systems of equations to be solved analytically, despite cumbersome solutions. In case of consideration of the vehicle springing and actual road profile, differential equation systems would be much more complex, nonlinear, and only special approximate numerical procedures could solve them, which requires the development of appropriate applied software. Such software could make it possible to investigate only certain aspects of the dynamic processes that occur during the transportation of cargoes under different operating conditions. However, such research is advisable and necessary to conduct.

7. Conclusions

1. When a vehicle moves under transition modes, non-fixed or partially fixed cargoes on the trailer begin to move
due to the forces of inertia, overcoming the friction forces between the cargo and the body of the trailer. In this case, the patterns of movement of the non-fixed or partially fixed cargoes are determined by the adhesion properties of the support surfaces between the body and the cargo and the manifestation of the corresponding forces of inertia due to the uneven movement of the vehicle. The value of the dynamic effort on a cargo decreases with an increase in the friction coefficient of the slip of the cargo on the trailer body and with a decrease in the ratio between the driving force and the resistance force to a two-link vehicle rolling (slow acceleration). If the friction coefficient of the slip of the cargo on the trailer body exceeds 0.4 (the cargo almost does not shift), then the dynamic effort on the cargo is minimal in the case of a small driving force and increases significantly with a significant increase in the driving force of the vehicle.

2. In the case of long acceleration and at certain values of the coefficient of resistance to the movement of cargoes (a friction coefficient between the cargo and the body of the vehicle), repeated impacts of the cargo against the restrictive devices are possible. It has been established that the value of the impact load pulse would increase, in this case, according to a nonlinear dependence. In particular, the value of the dynamic effort increases according to the parabolic law, with an increase in the rigidity of the elastic element between the tractor and the trailer, and with a decrease in the ratio of the tractor mass to the mass of the trailer $k$, and, if $k=2$, we observe the maximum efforts that act on the trailer with the cargo that would be minimal at $k=1,000$.

3. It has been proposed to use dampers with a characteristic that does not include a region where, when a cargo shifts downwards, the restorative force is also directed downwards (when the cargo stops in this region, the cargo would not return to zero position). The research results reported here allow us to estimate values of the mechanical parameters of restrictive devices: a distance between a cargo and the damper, the restorative force, the rigidity of viscoelastic restrictive devices, the coefficient of friction of the slip, the ratio of masses between the tractor, trailer, and cargo, etc. Based on the specified parameters, it becomes possible to select the required rational design characteristics for viscoelastic restrictive devices of a certain shape, length, and appropriate mass.

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