Quantization of AdS$_3$ Black Holes in External Fields

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Abstract

2+1-dimensional Anti-deSitter gravity is quantized in the presence of an external scalar field. We find that the coupling between the scalar field and gravity is equivalently described by a perturbed conformal field theory at the boundary of AdS$_3$. We derive the explicit form of this coupling, which allows us to perform a microscopic computation of the transition rates between black hole states due to absorption and induced emission of the scalar field. Detailed thermodynamic balance then yields Hawking radiation as spontaneous emission, and we find agreement with the semi-classical result, including greybody factors. This result also has application to four and five dimensional black holes in supergravity. However, since we only deal with gravitational degrees of freedom, the approach is not based on string theory, and does not depend, either, on the validity of Maldacena’s AdS/CFT conjecture.

June 1998

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1 Introduction

Suggestions that black hole radiation should have its origin in transitions between discrete states of a thermally excited system have a long history [1]. The development of a picture of this sort, however, has always been hampered by the lack of a proper quantum description of the black hole. Ideally, one would like to quantize a system whose classical dynamics is described by an action of the form

$$I[g, \Psi] = I_{\text{grav}}[g] + I_m[\Psi; g],$$

where $I_{\text{grav}}[g]$ is typically the Einstein-Hilbert action (possibly including a cosmological constant) for the gravitational degrees of freedom $g$, and $I_m[\Psi; g]$ is the action for some matter field(s) $\Psi$ coupled to the geometry. In view of the difficulty to quantize this system in a complete way, Hawking proposed to treat $g$ as a classical, fixed background, in the presence of which the field $\Psi$ is quantized [2]. The drawback in this approach is that the black hole is unaffected by the emission of radiation. No reference to black hole microstates is made, and accounting for back reaction has proven to be a notoriously difficult problem.

In this paper we suggest a different route, in which the microstates of the black hole play an explicit role. This approach is more akin to the old fashioned treatment of radiation from, say, an atom. The latter, in fact, provides a useful analogy: take a (quantized) atom in a classical, external electric field. As is well known, this external field couples to the electric dipole moment operator of the atom, which in this way induces a coupling between otherwise stable energy eigenstates of the atom. The atom can be excited by absorbing energy from the external radiation field, and it can also decay via induced emission of radiation, by giving away energy to the field. Under the assumption of thermodynamic equilibrium, a classical argument of Einstein shows that spontaneous emission must occur, with rate given in terms of the coefficients for absorption and induced emission. It is a variation of that approach that we aim to develop here. This is, we treat the gravitational field $g$ as quantum degrees of freedom, whereas the matter field $\Psi$ will remain classical.

In view of the lack of a consistent quantum theory of four-dimensional gravity we will work in the framework of Anti-deSitter (AdS) gravity $2 + 1$ dimensions. $(2 + 1)$-dimensional gravity with a negative cosmological constant is known to have black hole solutions [3] which have proved to be a useful laboratory for the study of the microscopical properties of black holes. At the same time $(2 + 1)$-dimensional gravity is almost trivial. More precisely, it is topological, at least in the absence of matter fields. As a consequence the dynamics of the gravitational degrees of freedom is described by a conformal field
theory (CFT) at the boundary i.e. the asymptotic region at infinity in $AdS_3$.

The coupling of matter fields to $(2 + 1)$-gravity is not topological, however. But since we treat $\Psi$ classically, matter will be on shell in the bulk of the black hole geometry. As we shall see, this reduces the coupling to gravity to a perturbation of the boundary CFT. This coupling to the boundary degrees of freedom is the analogue of the coupling of an electric field to the dipole moment of the atom. Since all matter fields couple to the gravitational field (through their energy-momentum tensor), we will choose to work with the simplest example: a scalar field with minimal coupling to gravity. The approach, however, can be readily extended to other fields.

Our results have also bearings for certain higher dimensional black holes, namely those for which the near horizon geometry reduces to an $AdS_3$ black hole. These are precisely the generalized four- and five-dimensional black holes for which a microscopic description of the low energy dynamics in terms of string theory has been found recently [4, 5, 6, 7, 8]. One may therefore speculate that the important structure present in these higher dimensional black holes is the near horizon $AdS_3$ gravity, which has a natural conformal field theory associated with it. String theory may be but one way to describe it.

The picture of black hole radiation that emerges from this approach is ‘holographic’, in that all the interactions take place at the asymptotic boundary of $AdS_3$. It is closely related to (and in fact, inspired by) the extremely successful description of black hole radiation in string theory [3, 8, 10], in which the microscopic theory is fully quantum. The latter, however, relies essentially on a conjectured correspondence between $AdS$ gravity and the CFT on its boundary [11]. In contrast, in the present approach this correspondence is an automatic consequence of the topological nature of $(2+1)$-gravity. This enables us to present what, to our knowledge, is the first explicit derivation of the coupling of the external field to the CFT on the boundary of $AdS$.

Our approach is also conceptually somewhat similar to the derivation of black hole radiance in the Ashtekar program [12]. The degrees of freedom involved in both cases have a clear, purely gravitational origin. The technical implementation is, however, rather different. Most significantly, in [12] the microstates are localized in the black hole horizon (like in Carlip’s approach [13]), whereas in the present approach they appear at the asymptotic boundary [14, 15]. One might hope that the results reported here could help to relate these apparently different formulations.
2 \textit{AdS}_3 \textit{gravity in the presence of external fields}

In this paper we take gravitational action in (1) to be the standard three-dimensional Einstein-Hilbert action with a negative cosmological constant,

\[ I_{EH} = -\frac{1}{16\pi G} \int d^3x \sqrt{-g} \left( R + \frac{2}{\ell^2} \right), \]

where \( \Lambda = -1/\ell^2 \) is the cosmological constant. The identification of this theory with a boundary conformal field theory has been described by several authors \([16, 17, 18]\) (see also \([19]\)), and our description of it will accordingly be rather cursory. One starts by mapping 3-dimensional gravity to a Chern-Simons (CS) theory \([20, 21]\). Using the 3-bein \( e^a_\mu \) and spin connection \( \omega^a = \varepsilon^a_{\ bc} \omega^{bc} \) to define two \( SL(2, \mathbb{R}) \) Chern-Simons gauge potentials \( A \) and \( \tilde{A} \)

\[ A^a_\mu = \omega^a_\mu + \frac{e^a_\mu}{\ell}, \quad \tilde{A}^a_\mu = \omega^a_\mu - \frac{e^a_\mu}{\ell}, \]

the Einstein-Hilbert action (2) can be expressed as the difference of two Chern-Simons (CS) actions, \( I_{EH} = I[A] - I[\tilde{A}] \), where\footnote{Traces are taken in the gauge group, where we choose the basis \( T_+ = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \ T_- = \frac{1}{2} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}, \ T_3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \)}

\[ I[A] = \frac{k}{4\pi} \int_M \text{Tr} \left( A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right), \]

with \( k = -\frac{\ell}{4G} \). Gauge transformations in this theory correspond to diffeomorphisms in (3), and can be used to gauge away all the degrees of freedom in the bulk. However, if the manifold has a boundary, only gauge transformations that vanish at the boundary leave the CS-action invariant. The dynamics of the residual degrees of freedom is, in turn, described by a CFT. We follow the analyses in \([17, 18]\), and work within the canonical formalism.

In what follows we choose, as our radial coordinate, the proper radius \( \rho \), rescaled by \( \ell \) to make it dimensionless. The boundary, which is at very large \( \rho \), is parametrized by \( t, \varphi \), or alternatively by the lightcone coordinates

\[ u = \frac{t}{\ell} + \varphi, \quad v = \frac{t}{\ell} - \varphi. \]

Furthermore we choose the boundary conditions \( A_v = \tilde{A}_u = 0 \) for the CS-potentials. As we will see below, these boundary conditions are compatible with the existence of black hole solutions, but may still leave too much freedom. In order to have a variational principle
diffeomorphisms \( \xi, \eta \) the form

\[
\text{preserve these boundary conditions and gauge choices have infinitesimal parameters of }
\]

Hence \( a \) sectors with opposite chiralities combine to give a single non-chiral WZ W theory \([16]\). It boundary \([22]\). Furthermore, it has been argued that in three-dimensional gravity the two boundary \([22]\). As shown in \([18]\), in a canonical analysis, where \( A_t \) is a Lagrange multiplier, we need \( \xi^i \neq \bar{\xi}^i \) in order to generate time-like diffeomorphisms.

\[
\int_{\partial M} \text{Tr} \left( A^2_\varphi \right). \tag{6}
\]

and similarly for \( I[\bar{A}] \). Now choose a gauge where

\[
A_\varphi = b(\rho)^{-1} \partial_\varphi b(\rho), \quad \bar{A}_\varphi = b(\rho) \partial_\rho b(\rho)^{-1}, \tag{7}
\]

with \( b(\rho) = \exp(\rho T_3) \). Solving the Gauss’s constraint \( F_{\rho \varphi} = 0 \) we express

\[
A_\varphi = b(\rho)^{-1} a(u) b(\rho) = \begin{pmatrix} a^3(u) & e^{-\rho} a^+(u) \\ e^\rho a^-(u) & -a^3(u) \end{pmatrix},
\]

\[
\bar{A}_\varphi = -b(\rho) \bar{a}(v) b(\rho)^{-1} = -\begin{pmatrix} \bar{a}^3(v) & e^\rho \bar{a}^+(v) \\ e^{-\rho} \bar{a}^-(v) & -\bar{a}^3(v) \end{pmatrix}. \tag{8}
\]

(Upper indices in \( a^a, \bar{a}^a \), correspond to group indices). The gauge transformations that preserve these boundary conditions and gauge choices have infinitesimal parameters of the form \( \eta = b^{-1} \lambda(u) b, \; \tilde{\eta} = b\bar{\lambda}(v) b^{-1} \). These, in turn, can be expressed in terms of diffeomorphisms \( \xi^i(u), \tilde{\xi}^i(v) \) (i = \( \rho, \varphi \)) by means of the relations \( \eta = \xi^i A_i, \; \tilde{\eta} = \tilde{\xi}^i \bar{A}_i \) \footnote{As shown in \([18]\), in a canonical analysis, where \( A_t \) is a Lagrange multiplier, we need \( \xi^i \neq \bar{\xi}^i \) in order to generate time-like diffeomorphisms.}

Hence

\[
\delta A_\varphi = \begin{pmatrix} \frac{1}{2} \partial_\varphi \xi^\rho + \partial_\varphi (\xi^\varphi a^3) & e^{-\rho} [\partial_\varphi (\xi^\varphi a^+) - \xi^\rho a^+] \\ e^\rho [\partial_\varphi (\xi^\varphi a^-) + \xi^\rho a^-] & -\frac{1}{2} \partial_\varphi \xi^\rho - \partial_\varphi (\xi^\varphi a^3) \end{pmatrix},
\]

\[
\delta \bar{A}_\varphi = -\begin{pmatrix} \frac{1}{2} \partial_\varphi \tilde{\xi}^\rho - \partial_\varphi (\tilde{\xi}^\varphi \bar{a}^3) & -e^\rho [\partial_\varphi (\tilde{\xi}^\varphi \bar{a}^+) + \tilde{\xi}^\rho \bar{a}^+] \\ e^{-\rho} [\partial_\varphi (\tilde{\xi}^\varphi \bar{a}^-) - \tilde{\xi}^\rho \bar{a}^-] & -\frac{1}{2} \partial_\varphi \tilde{\xi}^\rho + \partial_\varphi (\tilde{\xi}^\varphi \bar{a}^3) \end{pmatrix}. \tag{9}
\]

It is often helpful to think of the diffeomorphisms along the boundary as infinitesimal conformal transformations \( u \rightarrow u + \xi^\varphi(u), \; v \rightarrow v - \tilde{\xi}^\varphi(v) \). Under these transformations the fields \( a^a(u), \bar{a}^a(v) \) transform as conformal primary fields with weights \((1, 0)\) and \((0, 1)\), respectively. This is not unexpected. Chern-Simons theory, upon imposing boundary conditions as above, reduces to a chiral Wess-Zumino-Witten (WZW) theory at the boundary \([22]\). Furthermore, it has been argued that in three-dimensional gravity the two sectors with opposite chiralities combine to give a single non-chiral WZW theory \([16]\). It is then easy to see that the fields \( a^a(u), \bar{a}^a(v) \) are precisely the components of the level \( k \), left/right Kac-Moody currents in this WZW model.

For later use, we now give the asymptotic form of the metrics that are described by the connections \([3], [8]\). After solving for the 3-beins in \([8]\), and taking into account that,
from the boundary conditions, \( A_u = A_\varphi \) and \( \tilde{A}_v = -\tilde{A}_\varphi \), one gets

\[
d s^2 = \ell^2 d \rho^2 - \ell^2 e^{2\rho} a^{-}(u) \, \tilde{a}^{+}(v) \, d u \, d v + \ldots
\]

(10)

where for the sake of brevity we omit terms that are sub-leading at large \( \rho \).

While the system presented so far could be taken as a starting point for quantization, it appears that it has to be further reduced in order to isolate the black hole degrees of freedom. In particular, the boundary WZW theory does not account properly for the Bekenstein-Hawking entropy \([18, 23]\). A possible condition is to further impose that the induced metric on the boundary remains fixed under the allowed diffeomorphisms (9). It was shown in \([19]\) that with these extra boundary conditions the algebra of asymptotic symmetry generators contains a classical central charge. This was used in \([14, 15]\) to argue that, using Cardy’s partition function formula, the geometrical black hole entropy is indeed the same as that of the boundary CFT (subtleties in the application of this formula to the present situation are discussed in \([23]\)).

In our coordinates, fixing the geometry induced at the boundary is tantamount to keeping \( g_{\varphi \varphi} \propto e^{2\rho} a^{-}(u) \, \tilde{a}^{+}(v) \) fixed under residual diffeomorphisms. Geometrically this can be interpreted as keeping the worldsheet volume of the asymptotic conformal field theory invariant. On the other hand, using (9) it is easy to see that this constraint relates the diffeomorphisms along the boundary \( \xi^\varphi, \tilde{\xi}^\varphi \) to the radial displacement

\[
\rho \rightarrow \rho + \xi^\rho(u) + \tilde{\xi}^\rho(v)
\]

(11)

by

\[
\xi^\rho = -\partial^\rho \xi^\varphi, \quad \tilde{\xi}^\rho = -\partial^\rho \tilde{\xi}^\varphi.
\]

(12)

From the point of view of the WZW theory the relation (12) is implemented by the ‘improved’ Virasoro generator \([24]\)

\[
L = L_{\text{sug}} + k \partial_\varphi a^3,
\]

(13)

with classical central charge \( c = 6k \). Here \( L_{\text{sug}} \) is the Sugawara stress-energy tensor associated to the Kac-Moody algebra of \( a^{\pm}, a^3 \). We note in passing that the form \( c = 6k \) had previously been taken as an indication that the underlying algebra is that of a super conformal field theory \([28]\). This interpretation leads to the puzzle why the black hole should know about supersymmetry. The present result suggests that there may be an alternative interpretation of this.

It is well known that the constraints described above are precisely those imposed in the WZW to Liouville reduction \([24, 16]\). More details on this will be given elsewhere. At present we just note that the constraint (12) implies that under conformal transformations
the proper distance \( \rho \) and the Liouville field \( \phi \) transform in the same way. It is therefore natural to identify \( \rho \rightarrow -\phi(u,v) \). Having reduced the gravitational action to a conformal field theory at the boundary, one can quantize the latter using CFT techniques.

Next we consider the external field \( \Psi \), in the form of a minimally coupled scalar field. Matter fields perturb the dynamics of the metric by acting as sources of energy and momentum. The field \( \Psi \) is treated classically, i.e. taken to satisfy the classical wave equation in the bulk of \( AdS_3 \). One may think of this as the curved space equivalent of taking a homogeneous external field in the case of the atom in a radiation field. In this approximation one does not resolve the detailed structure of the bulk. The matter action then reduces to a boundary term

\[
I_m = -\frac{1}{16\pi G} \int \sqrt{-g} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \rightarrow -\frac{1}{16\pi G} \mathcal{B}^\rho(\infty) \quad \text{where}
\]

\[
\mathcal{B} = \frac{1}{2} \int_{\partial M} \sqrt{-g} g^{\mu\nu} (\psi^\dagger \partial_\mu \psi + \psi \partial_\mu \psi^\dagger)
\]

(14)

denotes the boundary term.

Requiring \( \Psi \) to satisfy the classical wave equation in a background that is asymptotically of the form (11) fixes its asymptotic form to

\[
\Psi(\rho, \varphi, t) = \left( 1 - ie^{-2\rho} \right) \psi_+(t, \varphi) + \left( 1 + ie^{-2\rho} \right) \psi_-(t, \varphi).
\]

(15)

We have decomposed the wave into components +, − with positive (ingoing) and negative (outgoing) flux respectively [25]. Substitution of this and the asymptotic metric (11) into (14) then leads to

\[
\mathcal{B} = \frac{\ell}{\mathcal{I}} \int du dv \mathcal{O}(u, v) (\psi_+ \psi_+^\dagger - \psi_- \psi_-^\dagger),
\]

(16)

where

\[
\mathcal{O}(u, v) = a^-(u) \tilde{a}^+(v).
\]

(17)

For definiteness, we take the dependence in \( t \) and \( \phi \) to be of the form

\[
\psi_\pm(t, \varphi) = e^{i(\omega_\pm t - m_\pm \varphi)}.
\]

(18)

Then we find

\[
\mathcal{B} = 2\ell \int du dv \mathcal{O}(u, v) \sin(\omega t - m \varphi),
\]

(19)

where \( \omega = \omega_+ - \omega_- \), \( m = m_+ - m_- \). This is our main result: the external field introduces a perturbation of the CFT at the boundary at infinity by a primary operator (17) with conformal weight (1, 1).

\footnote{Here we neglect a log-term which is of higher order in the frequency [25].}
Note that upon reduction to the Liouville theory one keeps $e^{2\phi} a^- \tilde{a}^+ \pm$ fixed. According to our remarks above one is then led to identify
\[ \mathcal{O}(u, v) = e^{2\phi}. \] (20)

In this case we can think geometrically of the conformal field theory as a ‘string at infinity’ which adjusts its proper radial position such as to keep its worldsheet volume constant. The scalar field couples to the position of the string. This is described in the conformal field theory language by the coupling (20), which is the gravitational analog of the coupling of an external electric field to the dipole moment operator of an atom. This approximation should be limited to transitions between neighbouring black hole states, that is, with small energy differences, as the effect of the change in the geometry in the bulk on the scalar field is neglected.

3 Black hole radiation

We now apply the results of the previous section to the specific case of interest, the BTZ black hole [3, 20]. In lightcone coordinates $u, v$ and proper radius $\rho$ the black hole has metric
\[ ds^2 = -\ell^2 \sinh^2 \rho \left( z_+ du + z_- dv \right)^2 + \ell^2 d\rho^2 + \frac{\ell^2}{4} \cosh^2 \rho \left( z_+ du - z_- dv \right)^2. \] (21)

This coordinate patch covers the region outside the (outer) horizon of a non-extremal black hole. Here,
\[ z_{\pm} = \sqrt{8G(M \pm J\ell)}, \] (22)
parametrize the family of non-extremal black hole solutions. For the black hole, the conformal operators $a, \tilde{a}$ of the previous section take the expectation values $\langle a^\pm \rangle = z_+/2$, $\langle \tilde{a}^\pm \rangle = z_-/2$, $\langle a^3 \rangle = \langle \tilde{a}^3 \rangle = 0$.

Note that an arbitrary non-extremal black hole can be obtained from (21) by a constant rescaling $(u, v) \to (\lambda u, \tilde{\lambda} v)$ [3]. In the quantum theory $z_{\pm}$ are replaced by operators $a, \tilde{a}$ and conformal transformations change the eigenvalues of the mass and angular momentum operators in the usual manner. It is interesting to notice that extremal black holes correspond to the limit where the horizon is at infinite proper distance from the asymptotic region, which in view of the above identification may be related to the (non-normalizable) ground state of Liouville theory ($\phi \to -\infty$).

The black hole corresponds to a thermal state of the left and right moving sectors of the CFT [27]. The effective temperature of each sector \footnote{Properly, they are combinations of the (Hawking) temperature and chemical potential associated to angular momentum.} can be found from the energy
and entropy formulas,

$$\varepsilon_R = V^{-1}L_0 = \frac{z^2}{16G}, \quad s_R = 2\pi \sqrt{\frac{cN_R}{6}} = \frac{\pi \ell z_+}{4G},$$

$$\varepsilon_L = V^{-1}\tilde{L}_0 = \frac{z^2}{16G}, \quad s_L = 2\pi \sqrt{\frac{cN_L}{6}} = \frac{\pi \ell z_-}{4G},$$

(23)

where $V$ is the volume of the boundary CFT and $N_R, N_L$ are the eigenvalues of $L_0, \tilde{L}_0$, resp. The corresponding left- and right moving temperatures are therefore

$$T_{R,L}^{-1} = \frac{\partial s_{R,L}}{\partial \varepsilon_{R,L}} = \frac{2\pi \ell}{z_{\pm}}.$$  

(24)

These are related to the Hawking temperature as $2T_H^{-1} = T_R^{-1} + T_L^{-1}$. After properly rotating to Euclidean time these effective temperatures correspond to the inverse periods of the lightcone variables [27]. Note that (24) is rather insensitive to the details of the concrete realisation of the underlying boundary CFT. Indeed only the relation between energy and entropy enters.

Having the coupling (19), we can now compute transition amplitudes occurring in the presence of a matter field. As explained above this interaction vertex should correctly describe the transition between black hole states with small energy difference. Note that it is not required that the initial state itself has low energy. In particular it should describe correctly the low frequency decay rates of highly excited black holes.

The calculation will be similar to that in [28]. From (14), the transition amplitude between an initial and a final state in the presence of an external flux with frequency and angular momentum $\omega, m$ is then given by

$$M = \ell \int dudv \langle f|O(u,v)|i\rangle e^{-i(\omega\ell - m)\frac{u}{2}} e^{-i(\omega\ell + m)\frac{v}{2}},$$

(25)

where $i, f$, denote the initial and final black hole state respectively. If this term corresponds to emission, then the term in (19) with the opposite frequency will give absorption, but at this moment this is still a matter of convention. The important point is that calculation of transition amplitudes is reduced to the computation of correlation functions of $(1,1)$ primary fields. In particular it does not rely on the identification (20), which, to some, may seem a little far fetched.

We proceed to compute the decay rate. For simplicity we set $m = 0$. Squaring the amplitude (25) and summing over final states leads to

$$\sum_{f} |M|^2 = \ell^2 \int dudu'dv dv' \langle i|O(u,v)O(u',v')|i\rangle e^{-i\omega\ell \frac{u-u'}{2}} e^{-i\omega\ell \frac{v-v'}{2}},$$

(26)

$$^5$$In the string theory description of four and five dimensional black holes, this regime is mapped to the low energy dynamics of near extremal black holes.
Since the black hole corresponds to a thermal state, we must average over initial states weighed by the Boltzmann factor. This means that we must take finite temperature two point functions, which for fields of conformal weight one are given by

\[
\langle \mathcal{O}(0,0) \mathcal{O}(u,v) \rangle_{T_R,T_L} = \left[ \frac{\pi T_R}{\sinh(\pi T_R u)} \right]^2 \left[ \frac{\pi T_L}{\sinh(\pi T_L v)} \right]^2,
\]

provided \( T >> V^{-1} \). These have the right periodicity properties in the Euclidean section. The remaining integrals can be performed by contour techniques of common use in thermal field theory. Whether we deal with emission or absorption depends on how the poles at \( u = 0, v = 0 \) are dealt with. The choice for emission leads to integrals of the type

\[
\int du \frac{e^{-\frac{\pi}{x}(u-i\epsilon)}}{\sinh^2(xu)} = \frac{\pi \omega}{x^2} \sum_{n=1}^{\infty} e^{-\frac{\pi \omega n}{x}} = \frac{\pi \omega}{x^2} \left( e^{\frac{\pi \omega}{x}} - 1 \right)^{-1}.
\]

The resulting emission rate is then given by

\[
\Gamma = \frac{\omega \pi^2 \ell^2}{(e^{\frac{\pi \omega}{xL}} - 1)(e^{\frac{\pi \omega}{xR}} - 1)},
\]

where we have included a factor \( \omega^{-1} \) for the normalization of the outgoing scalar. Eq. (29) reproduces correctly the semiclassical result [29, 25], therefore providing a microscopical derivation of the decay of \( \text{AdS}_3 \) black holes relying exclusively on the gravitational degrees of freedom.

**Acknowledgements**

We are grateful to Peter Bowcock, Steven Carlip, Ian Kogan and Miguel Ortiz for enlightening discussions. RE is supported by EPSRC through grant GR/L38158 (UK), and by UPV through grant 063.310-EB225/95 (Spain).

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