Analytical calculation of Coulomb corrections to $e^+e^-$ pair production at intermediate photon energies

R.N. Lee, A. I. Milstein, and V.M. Strakhovenko

Budker Institute of Nuclear Physics, 630090 Novosibirsk, Russia

Abstract

First correction to the high-energy asymptotics of the total $e^+e^-$ photoproduction cross section in the electric field of a heavy atom is obtained with the exact account of this field. The consideration is based on the use of the quasiclassical electron Green function in an external electric field. The influence of screening on the Coulomb corrections is examined in the leading approximation. It turns out that the high-energy asymptotics of the corresponding correction is independent of the photon energy. The detailed comparison of our results with experimental data is performed. This comparison has justified the analytical result and allowed us to elaborate a simple ansatz for the next-to-leading correction. Using this ansatz, good agreement with the experimental data is obtained for photon energies above a few MeV. In the region where both produced particles are relativistic, the corrections to the high-energy asymptotics of the electron (positron) spectrum are obtained. In addition, analogous corrections to the bremsstrahlung spectrum are derived starting from the corresponding results for pair production.

Key words: $e^+e^-$ photoproduction, bremsstrahlung, Coulomb corrections, screening

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I Introduction

Knowledge of the photoabsorption cross sections is very important in various applications, see, e.g., [Hubbell, 2000]. The relevant processes are the atomic

Email addresses: R.N.Lee@inp.nsk.su (R.N. Lee), A.I.Milstein@inp.nsk.su (A. I. Milstein), V.M.Strakhovenko@inp.nsk.su (V.M. Strakhovenko).
photoeffect, nuclear photoabsorption, incoherent and coherent photon scattering and $e^+e^-$ pair production. In the coherent processes, by definition, there is no excitation or ionization of an atom. The high-accuracy estimation of the corresponding cross sections is required. They have different dependence on the photon energy $\omega$. At $\omega \gtrsim 10\text{MeV}$, the cross section of $e^+e^-$ pair production becomes dominant (Hubbell et al., 1980). The coherent contribution $\sigma_{\text{coh}}$ to the pair production cross section is roughly $Z$ times larger than the incoherent one ($Z$ is the atomic number), thereby being the most important for heavy atoms. Just the coherent pair production is considered below.

The theoretical and experimental investigation of the coherent pair production has a long history, see (Hubbell et al., 1980). In the Born approximation, the cross section $\sigma_B$ is known for arbitrary photon energy (Bethe and Heitler, 1934; Racah, 1934). The account of the effect of screening is straightforward in this approximation and can be easily performed if the atomic form factor is known (Jost et al., 1950). For heavy atoms it is necessary to take into account the Coulomb corrections $\sigma_C$,

$$\sigma_{\text{coh}} = \sigma_B + \sigma_C.$$  \hspace{1cm} (1)

These corrections are higher order terms of the perturbation theory with respect to the atomic field. The magnitude of $\sigma_C$ depends on $\omega$ and the parameter $Z\alpha$ ($\alpha = 1/137$ is the fine-structure constant). The formal expression for $\sigma_C$, exact in $Z\alpha$ and $\omega$, was derived by Øverbø et al. (1968). This expression has a very complicated form causing severe difficulties in computations. The difficulties grow as $\omega$ increases, so that numerical results in (Øverbø et al., 1968) were obtained only for $\omega < 5\text{MeV}$.

For the high-energy region $\omega \gg m$ ($m$ is the electron mass), the consideration is greatly simplified. As a result, a rather simple form was obtained in (Bethe and Maximon, 1954; Davies et al., 1954) for the Coulomb corrections in the leading approximation with respect to $m/\omega$. However, the theoretical description of the Coulomb corrections at intermediate photon energies ($5 \div 100\text{MeV}$) has not been completed. At present, all estimates of $\sigma_C$ in this region are based on the ”bridging” expression derived by Øverbø (1977). This expression is actually an extrapolation of the results obtained for $\omega < 5\text{MeV}$. It is based on some assumptions on the form of the asymptotic expansion of $\sigma_C$ at high photon energy. It is commonly believed that the ”bridging” expression has an accuracy providing the maximum error in $\sigma_{\text{coh}}$ of the order of a few tens of percent.

Here we develop a description of $e^+e^-$-pair production at intermediate photon energies by deriving the next-to-leading term of the high-energy expansion of
σ_C. First we consider a pure Coulomb field and represent σ_C in the form

\[ \sigma_C = \sigma_C^{(0)} + \sigma_C^{(1)} + \sigma_C^{(2)} + \ldots \]  

(2)

The term \( \sigma_C^{(n)} \) has the form \( (m/\omega)^n S^{(n)}(\ln \omega/m) \), where \( S^{(n)}(x) \) is some polynomial. The \( \omega \)-independent term \( \sigma_C^{(0)} \) corresponds to the result of Davies et al. (1954). In the present paper we derive the term \( \sigma_C^{(1)} \). It turns out that \( S^{(1)} \) is \( \omega \)-independent in contrast to a second-degree polynomial suggested by Øverbø (1977). We propose a new ansatz for \( \sigma_C^{(2)} \), which provides a good agreement with available experimental data for \( \omega > 5 \text{MeV} \).

The high-energy expansion of the Coulomb corrections to the spectrum has the same form as (2). In the region \( \varepsilon_\pm \gg m \), we derive the term \( d\sigma_C^{(1)}/dx \), where \( \varepsilon_- \) and \( \varepsilon_+ \) are the electron and positron energy, respectively, \( x = \varepsilon_-/\omega \). The term \( d\sigma_C^{(1)}/dx \) may turn important, e.g., for description of the development of electromagnetic showers in a medium. The correction found is antisymmetric with respect to the permutation \( \varepsilon_+ \leftrightarrow \varepsilon_- \) and does not contribute to the total cross section. In fact, \( \sigma_C^{(1)} \) originates from two energy regions \( \varepsilon_+ \sim m \) and \( \varepsilon_- \sim m \), where the spectrum is not known. However, we emphasize that it differs drastically from the result obtained by Davies et al. (1954) for \( \varepsilon_\pm \gg m \), if the latter is formally applied at \( \varepsilon_- \sim m \) or \( \varepsilon_+ \sim m \).

For the first time, we estimate the effect of screening on \( \sigma_C \) at \( \omega \gg m \). In the leading approximation, we find the corresponding correction \( \sigma_C^{(\text{scr})} \), which is \( \omega \)-independent similar to \( \sigma_C^{(0)} \). So, for the atomic field, \( \sigma_C^{(\text{scr})} \) should be added to the right-hand side of Eq. (2). The screening correction to the spectrum is also obtained.

Tedious and cumbersome calculations have been performed to derive our formulas. All technical details are omitted here and will be presented elsewhere.

II General discussion

The cross section of \( e^+e^- \) pair production by a photon in an external field reads

\[ d\sigma_{\text{coh}} = \frac{\alpha}{(2\pi)^4 \omega} dp \, dq \, \delta(\omega - \varepsilon_+ - \varepsilon_-) |M|^2 \]  

(3)
where $\varepsilon = \varepsilon_p = \sqrt{p^2 + m^2}$, $\varepsilon = \varepsilon_q$, and $p$, $q$ are the electron and positron momenta, respectively. The matrix element $M$ has the form

$$M = \int d\mathbf{r} \bar{\psi}_p^+(\mathbf{r}) \dot{\varepsilon} \psi_q^-(\mathbf{r}) \exp(i\mathbf{k}\mathbf{r}) .$$

(4)

Here $\psi_p^+$ and $\psi_q^-$ are positive-energy and negative-energy solutions of the Dirac equation in the external field, $e_\mu$ is the photon polarization 4-vector, $k$ is the photon momentum. It is convenient to study various processes in external fields using the Green function $G(\mathbf{r}_2, \mathbf{r}_1|\varepsilon)$ of the Dirac equation in this field. This Green function can be represented in the form

$$G(\mathbf{r}_2, \mathbf{r}_1|\varepsilon) = \sum_n \frac{\psi_n^+(\mathbf{r}_2)\bar{\psi}_n^+(\mathbf{r}_1)}{\varepsilon - \varepsilon_n + i0} + \int \frac{d\mathbf{p}}{(2\pi)^3} \left[ \frac{\psi_p^+(\mathbf{r}_2)\bar{\psi}_p^+(\mathbf{r}_1)}{\varepsilon - \varepsilon_p + i0} + \frac{\psi_p^-(\mathbf{r}_2)\bar{\psi}_p^-(\mathbf{r}_1)}{\varepsilon + \varepsilon_p - i0} \right] ,$$

(5)

where $\psi_n^+$ is the discrete-spectrum wave function, $\varepsilon_n$ is the corresponding binding energy. The regularization of denominators in (5) corresponds to the Feynman rule. From (5),

$$\int d\Omega_q \psi_q^-(\mathbf{r}_2)\bar{\psi}_q^-(\mathbf{r}_1) = -i\frac{(2\pi)^2}{q\varepsilon_q} \delta G(\mathbf{r}_2, \mathbf{r}_1|\varepsilon-q),$$

$$\int d\Omega_p \psi_p^+(\mathbf{r}_1)\bar{\psi}_p^+(\mathbf{r}_2) = i\frac{(2\pi)^2}{p\varepsilon_p} \delta G(\mathbf{r}_1, \mathbf{r}_2|\varepsilon_p) ,$$

(6)

where $\Omega_p$ is the solid angle of $\mathbf{p}$, and $\delta G = G - \tilde{G}$. The function $\tilde{G}$ is obtained from (5) by the replacement $i0 \leftrightarrow -i0$.

Taking the integrals over $\Omega_p$ and $\Omega_q$ in (3), we obtain the electron spectrum, which is the cross section differential with respect to the electron energy $\varepsilon_-$ . Using the relations (6), we express this spectrum via the Green functions:

$$\frac{d\sigma_{\text{coh}}}{d\varepsilon_-} = \frac{\alpha}{\omega} \int d\mathbf{r}_1 d\mathbf{r}_2 e^{i\mathbf{k}\mathbf{r}} Sp \left\{ \delta G(\mathbf{r}_1, \mathbf{r}_2|\varepsilon_-) \dot{\varepsilon} \delta G(\mathbf{r}_2, \mathbf{r}_1|\varepsilon_+) \dot{\varepsilon}^* \right\} ,$$

(7)

where $\varepsilon_+ = \omega - \varepsilon_-$ and $\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_1$.

Due to the optical theorem, the process of pair production is related to the process of Delbrück scattering (coherent scattering of a photon in the electric field of an atom via virtual electron-positron pairs). At zero scattering angle, the amplitude $M_D$ of Delbrück scattering reads

$$\int d\mathbf{r} \bar{\psi}_p^+(\mathbf{r}) \dot{\varepsilon} \psi_q^-(\mathbf{r}) \exp(i\mathbf{k}\mathbf{r}) .$$

(4)
\[ M_D = 2i\alpha \int d\varepsilon \int \int dr_1 \, dr_2 \, e^{ikr} \text{Sp} \{ G(r_1, r_2|\varepsilon) \dot{\varepsilon} G(r_2, r_1|\varepsilon - \omega) \dot{\varepsilon}^* \} . \] (8)

It follows from Eqs. (7),(8) and the analytical properties of the Green function that

\[ \frac{1}{\omega} \text{Im} \, M_D = \sigma_{coh} + \sigma_{bf} . \] (9)

Here

\[ \sigma_{bf} = -\frac{2i\pi\alpha}{\omega} \int \int dr_1 \, dr_2 \, e^{ikr} \sum_n \text{Sp} \{ \rho_n(r_1, r_2) \dot{\varepsilon} G(r_2, r_1|\varepsilon_n - \omega) \dot{\varepsilon}^* \} , \]

\[ \rho_n(r_1, r_2) = \lim_{\varepsilon \rightarrow \varepsilon_n} (\varepsilon - \varepsilon_n) G(r_1, r_2|\varepsilon) . \] (10)

The quantity \( \sigma_{bf} \) coincides with the total cross section of the so-called bound-free pair production when an electron is produced in a bound state. In fact, due to the Pauli principle, there is no bound-free pair production on neutral atoms. Nevertheless, the term \( \sigma_{bf} \) should be kept in the r.h.s. of (9). In a Coulomb field, the total cross section \( \sigma_{bf} \) was obtained in (Milstein and Strakhovenko, 1993) for \( \omega \gg m \). In this limit, \( \sigma_{bf} \approx 1/m\omega \) and should be taken into account when using the relation (9) for the calculation of the corrections to \( \sigma_{coh} \) from. The main contribution to \( \sigma_{bf} \) comes from the low-lying bound states (Milstein and Strakhovenko, 1993) when screening can be neglected. So, in (9) we can use \( \sigma_{bf} \) obtained in (Milstein and Strakhovenko, 1993).

### III Coulomb corrections to the spectrum

In this section we consider the Coulomb corrections to the spectrum, \( d\sigma_C/dx \), for \( \varepsilon_\pm \gg m \) taking into account terms of the order \( m/\varepsilon_\pm \). Within this accuracy, the spectrum is determined by the region of small angles between vectors \( p, q, \) and \( k \) in (3). In this case the angular momenta of both particles are large, providing the applicability of the quasiclassical approximation. Thus we can use the quasiclassical Green function in (7). For a pure Coulomb field, this function was found in (Milstein and Strakhovenko, 1983); for arbitrary localized field, it was obtained in (Lee et al., 2000). In the latter paper, the first correction to the quasiclassical Green function was derived as well.

According to Davies et al. (1954), the higher order terms of the perturbation theory with respect to the external field (Coulomb corrections) are not seriously modified by screening. However, this question has not been studied.
quantitatively so far. The influence of screening on Coulomb corrections is investigated in detail in Section V. In the present Section we calculate \( d\sigma_C/d\varepsilon \) in a pure Coulomb field.

Using the results of Lee et al. (2000), we obtain from (7)

\[
\frac{d\sigma_C^{(0)}}{dx} + \frac{d\sigma_C^{(1)}}{dx} = -4\sigma_0 \left[ \left( 1 - \frac{4}{3}x(1-x) \right) f(Z\alpha) \right. \\
- \frac{\pi^3(1-2x)m}{8x(1-x)\omega} \left. \left( 1 - \frac{3}{2}x(1-x) \right) \text{Re} g(Z\alpha) \right],
\]

\[
f(Z\alpha) = \text{Re}\psi(1+iZ\alpha) + C, \quad g(Z\alpha) = Z\alpha \frac{\Gamma(1-iZ\alpha)\Gamma(1/2+iZ\alpha)}{\Gamma(1+iZ\alpha)\Gamma(1/2-iZ\alpha)},
\]

\[
x = \frac{\varepsilon_-}{\omega}, \quad \sigma_0 = \alpha(Z\alpha)^2/m^2,
\]

where \( \psi(t) = d\ln \Gamma(t)/dt \), \( C = 0.577... \) is the Euler constant. In (11), the term \( \propto f(Z\alpha) \) corresponds to the leading approximation \( d\sigma_C^{(0)}/dx \) (Davies et al., 1954), the term \( \propto \text{Re} g(Z\alpha) \) is the first correction \( d\sigma_C^{(1)}/dx \). In contrast to the leading term, this correction is antisymmetric with respect to the permutation \( \varepsilon_+ \leftrightarrow \varepsilon_- \) (or \( x \leftrightarrow 1-x \)) and, therefore, does not contribute to the total cross section. Besides, the correction is an odd function of \( Z\alpha \) due to the charge-parity conservation and the antisymmetry mentioned above. The antisymmetric contribution enhances the production of electrons at \( x < 1/2 \) and suppresses it at \( x > 1/2 \). Evidently, the opposite situation occurs for positrons. Qualitatively, such a behavior of the spectrum takes place for any \( \omega \) being the most pronounced at low photon energy (Øverbø et al., 1968). At intermediate photon energies, the spectrum (11) essentially differs from that given by the leading approximation. We illustrate this statement in Fig. 1, where \( \sigma_0^{-1}d\sigma_C/dx \) with correction (solid line) and without correction (dashed line) are plotted for \( Z = 82 \) and \( \omega = 50 \text{ MeV} \).

Due to the antisymmetry of \( d\sigma_C^{(1)}/dx \) at \( \varepsilon_+ \gg m \), the term \( \sigma_C^{(1)} \) in the total cross section may originate only from the energy regions \( \varepsilon_- \sim m \) and \( \varepsilon_+ = \omega - \varepsilon_- \sim m \). The quasiclassical approximation can not be used directly in these regions, and another approach is needed to calculate the spectrum. We are going to do this elsewhere. However, for the total cross section, it is possible to overcome this difficulty by means of dispersion relations (see Section IV).

As known (see, e.g., Berestetski et al., 1982), the spectrum of bremsstrahlung can be obtained from the spectrum of pair production. This can be performed by means of the substitution \( \varepsilon_+ \rightarrow -\varepsilon', \omega \rightarrow -\omega', \) and \( dx \rightarrow ydy \), where \( y = \omega'/\varepsilon, \omega' \) is the energy of an emitted photon, \( \varepsilon \) is the initial electron energy. Using (11), we obtain for the Coulomb corrections to the bremsstrahlung spectrum.
Fig. 1. The dependence of $\sigma_0^{-1}d\sigma_C/dx$ on $x$, see (11), for $Z = 82$, $\omega = 50MeV$. Dashed curve: leading approximation; solid curve: first correction is taken into account.

$$y \frac{d\sigma_C^\gamma}{dy} = -4\sigma_0 \left[ \left( y^2 + \frac{4}{3}(1-y) \right) f(Z\alpha) \right.$$  
$$ - \frac{\pi^3(2-y)m}{8(1-y)\varepsilon} \left( y^2 + \frac{3}{2}(1-y) \right) \text{Re} g(Z\alpha) \left. \right].$$  

(12)

This formula describes bremsstrahlung from electrons. For positrons, it is necessary to change the sign of $Z\alpha$ in (12). Our result (12) coincides with that obtained in (Baier and Katkov, 1976) if the obvious mistake in the latter is corrected by changing

$$\frac{1}{\gamma} \rightarrow \frac{1}{2} \left( \frac{m}{\varepsilon} + \frac{m}{\varepsilon - \omega'} \right) = \frac{(2-y)m}{2(1-y)\varepsilon}$$

in Eq.(22) of (Baier and Katkov, 1976).

### IV Coulomb corrections to the total cross section

For $\omega \gg m$, we derive $\sigma_C^{(1)}$ using the relation (9). In the leading approximation, the Coulomb corrections to the cross section of pair production were obtained in (Davies et al., 1954). Using this result and dispersion relations, the corresponding correction, $M_D^{(0)}$, to the forward Delbrück scattering amplitude $M_D$ were obtained in (Rohrlich, 1957). They read:
\[ \sigma_C^{(0)} = -\frac{28}{9} \sigma_0 f(Z\alpha), \quad M_{DC}^{(0)} = -i\frac{28}{9} \omega \sigma_0 f(Z\alpha), \quad (13) \]

where \( \sigma_0 \) and \( f(Z\alpha) \) are defined in (11).

Using the results of Lee et al. (2000) for the quasiclassical Green functions, we find within logarithmic accuracy

\[ \text{Re} M_{DC}^{(1)} = \frac{\alpha(Z\alpha)^2 \pi^3 \text{Im} g(Z\alpha)}{m} \ln \frac{\omega}{m}, \quad (14) \]

the function \( g(Z\alpha) \) is defined in (11). The logarithm appears as a result of integration in (8) over \( \varepsilon \) in the region where \( \varepsilon \gg m \) and \( \omega - \varepsilon \gg m \), thereby the quasiclassical approximation is applicable. Unlike \( \text{Re} M_{DC}^{(1)} \), the quantity \( \text{Im} M_{DC}^{(1)} \) is determined by the regions of integration over \( \varepsilon \), where \( \varepsilon \sim m \) and \( \omega - \varepsilon \sim m \), and, therefore, the quasiclassical approximation is invalid. Nevertheless, this quantity, which is related to \( \sigma_C^{(1)} \) (9), can be obtained from the dispersion relation for \( M_D \) (Rohrlich, 1957)

\[ \text{Re} M_D(\omega) = \frac{2}{\pi} \omega^2 P \int_0^\infty \frac{\text{Im} M_D(\omega') d\omega'}{\omega'(\omega'^2 - \omega^2)}. \quad (15) \]

Using this relation, it can be easily checked that the high-energy asymptotics (14) unambiguously corresponds to the \( \omega \)-independent high-energy asymptotics

\[ \text{Im} M_{DC}^{(1)} = -\frac{\alpha(Z\alpha)^2 \pi^4 \text{Im} g(Z\alpha)}{2m}, \quad (16) \]

Substituting (16) into (9) and using \( \sigma_{bf} \) from Milstein and Strakhovenko, 1993 in the form

\[ \sigma_{bf} = 4\pi \sigma_0 (Z\alpha)^3 f_1(Z\alpha) \frac{m}{\omega}, \quad (17) \]

we have for \( \sigma_C^{(1)} \)

\[ \sigma_C^{(1)} = -\sigma_0 \left[ \frac{\pi^4}{2} \text{Im} g(Z\alpha) + 4\pi (Z\alpha)^3 f_1(Z\alpha) \right] \frac{m}{\omega}. \quad (18) \]

The function \( f_1(Z\alpha) \) is plotted in Fig. 2.
The quantity $(\omega/m)\sigma_C^{(1)}/\sigma_C^{(0)}$ is shown in Fig. 3 (solid curve). It is seen that this ratio is numerically large for any $Z$. Therefore, the term $\sigma_C^{(1)}$ gives a significant contribution to $\sigma_C$ for intermediate photon energies. Dashed curve in Fig. 3 gives the same ratio when $\sigma_{bf}$ in (18) is omitted. It is seen that the relative contribution of the term $\propto f_1(Z\alpha)$ in (18) is numerically small.

Fig. 3. The quantity $(\omega/m)\sigma_C^{(1)}/\sigma_C^{(0)}$ as a function of $Z$ (solid curve). Dashed curve corresponds to the same quantity without the contribution of the bound-free pair production.
V Screening corrections

In two previous Sections the cross section of $e^+e^-$ pair production has been considered for a pure Coulomb field. The difference, $\delta V(r)$, between an atomic potential and a Coulomb potential of a nucleus leads to the modification of this cross section known as the effect of screening. In the Born approximation, this effect was studied long ago (see, e.g., Jost et al. 1950). Let us consider now $\sigma_C^{(scr)}$ characterizing the influence of screening on the Coulomb corrections. Recollect that the Coulomb corrections denote the higher-order terms of the perturbation theory with respect to the atomic field. So far it was only known that the correction $\sigma_C^{(scr)}$ is not large (Davies et al. 1954). Here we consider this issue quantitatively. The Coulomb corrections are determined by distances $r \sim 1/m$ where the difference $\delta V(r)$ is small. In our calculation of $\sigma_C^{(scr)}$, we retain the linear term of expansion in powers of $\delta V(r)$. We represent $\delta V(r)$ as

$$
\delta V(r) = \int \frac{dQ}{(2\pi)^3} e^{iQr} F(Q) \frac{4\pi Z\alpha}{Q^2},
$$

(19)

where $F(Q)$ is the atomic electron form factor. Using the results obtained in (Lee and Milstein, 1993; Katkov and Strakhovenko, 2001; Lee et al., 2000) for arbitrary atomic potential, we have for the correction to the spectrum

$$
\frac{d\sigma_C^{(scr)}}{dx} = \frac{32}{3} \sigma_0 m^2 \int_0^\infty \frac{dQ}{Q^3} F(Q) \int_0^\infty \frac{d\tau}{\sinh \tau} \left[ \frac{\sin(2Z\alpha\tau)}{2Z\alpha} - \tau \right]
$$
\[ R(\mu, a) = \frac{(\mu - 1)}{4\mu^2} \left\{ \frac{1}{2\sqrt{\mu}} \left[ 18 - 6\mu + a(\mu^2 + 2\mu - 3) \right] \ln \left[ \frac{\sqrt{\mu} + 1}{\sqrt{\mu} - 1} \right] \right. \]

\[ -18 - a(\mu - 3) \right\} \]

\[ \mu_{\pm} = 1 + \frac{8m^2 e^{\pm\tau} \sinh^2 \tau}{Q^2 (\cosh \tau + \cos \varphi)}, \quad a = 6x(1 - x). \]  

(21)

Integrating over \( x \), we obtain

\[ \sigma_C^{(\text{scr})} = \frac{32}{3} \sigma_0 m^2 \int_0^\infty \frac{dQ}{Q^3} F(Q) \int_0^\infty \frac{d\tau}{\sinh \tau} \left[ \sin(2Z\alpha \tau) \right] \]

\[ \times \left[ e^\tau R(\mu_{+}, 1) - e^{-\tau} R(\mu_{-}, 1) \right] . \]

(22)

Similar to \( \sigma_C^{(0)} \), this correction is \( \omega \)-independent. Shown in Fig. 4 is the \( Z \)-dependence of the ratio \( \sigma_C^{(\text{scr})}/\sigma_C^{(0)} \) calculated with the use of the form factors taken from Hubbell and Overbe, 1979. As seen from Fig. 4, this ratio is approximately described by the linear function \(-5.4 \cdot 10^{-4} \cdot Z\).

The corresponding correction to the bremsstrahlung spectrum is obtained from (20) by means of the same substitutions as in Section III. So that the quantity \( y^{-1}d\sigma_C^{(\text{scr})}/dy \) is given by the right-hand side of (20) if we set \( a = 6(y - 1)/y^2 \).

VI Comparison with experimental data and estimation of \( \sigma_C^{(2)} \)

The most detailed and accurate experimental data have been obtained just in the region of intermediate photon energies. In this region, the first correction \( \sigma_C^{(1)} \), obtained above, becomes large, see Fig. 3, and the next term \( \sigma_C^{(2)} \) in the expansion (2) may be significant. Since it has not been calculated, we use for \( \sigma_C^{(2)} \) the following ansatz:

\[ \sigma_C^{(2)} = \sigma_0 \left[ b \ln(\omega/2m) + c \right] \left( \frac{m}{\omega} \right)^2 , \]

(23)

where \( b \) and \( c \) are some functions of \( Z\alpha \). This form follows from the arguments similar to those presented by Davies et al., 1954.
Shown in Fig. 5 is the quantity

$$\Sigma = \frac{\omega}{m} \sigma_0^{-1} (\sigma_{coh} - \sigma_B - \sigma^{(0)}_C - \sigma^{(scr)}_C),$$  \hspace{1cm} (24)$$

where the experimental cross section $\sigma_{coh}$ for $Bi$ is taken from [Sherman et al., 1980], $\sigma_B$ is the Born cross section calculated with screening taken into account, $\sigma^{(0)}_C$ and $\sigma^{(scr)}_C$ are given by (13) and (22), respectively. This quantity is fitted by the formula $a + (m/\omega) [b \ln(\omega/2m) + c]$ (dashed curve) which corresponds to the sum $(\omega/m) \sigma_0^{-1} (\sigma^{(1)}_C + \sigma^{(2)}_C)$. The fitting parameters obtained by the linear regression method are $a = 25.79$, $b = -98.92$, $c = 2.43$. The quantity $a$ determines the asymptotics of the fit at $\omega \to \infty$ and should be compared with our result $a_{th} = (\omega/m) \sigma_0^{-1} \sigma^{(1)}_C = 25.66$ (solid line). So the value of $a_{th}$ is in a perfect agreement with that extracted from the experimental data. From Fig. 5, we conclude that the term $\sigma^{(2)}_C$ gives a noticeable contribution to $\Sigma$.

![Fig. 5. The values of $\Sigma$ (24) extracted from the experimental data for $Bi$ together with the fit $a + (m/\omega) [b \ln(\omega/2m) + c]$ (dashed curve) and the theoretical asymptotics $a_{th} = (\omega/m) \sigma_0^{-1} \sigma^{(1)}_C$ (solid line). Coefficients $a,b,$ and $c$ are given in the text.](image)

Note that the coefficient $c$ is small as compared to $b$. The same situation ($a \approx a_{th}, |b| \gg |c|$) takes place at fitting of experimental data for $Ta$ [Sherman et al., 1980]. So, we set $c = 0$ in Eq. (23) and use $b$ as the only fitting parameter. Fitting the data for $Bi$ with such ansatz, we obtain $b = -96.63$. A new curve practically coincides with that shown in Fig. 5.

The values of $\Sigma$ extracted from the experimental data for $Ta$ [Sherman et al., 1980] and $Pb$ [Rosenblum et al., 1952; Gimm and Hubbell, 1978] are shown...
in Fig. 6 and Fig. 7, respectively. Dashed curves represent the results of the fit, solid lines correspond to the asymptotics $\Sigma = a_{th}$.

Fig. 6. The values of $\Sigma$ (24) extracted from the experimental data for Ta together with the fit $a_{th} + (m/\omega)b \ln(\omega/2m)$ (dashed curve), $a_{th} = 21.88$, $b = -82.82$. The solid line represents the asymptotics $\Sigma = a_{th}$.

Fig. 7. The values of $\Sigma$ (24) extracted from the experimental data for Pb obtained by Rosenblum et al. (squares) and Sherman et al. (stars). The dashed curve corresponds to the fit $a_{th} + (m/\omega)b \ln(\omega/2m)$, $a_{th} = 25.29$, $b = -95.95$. The solid line represents the asymptotics $\Sigma = a_{th}$.

Within the accuracy of the fitting procedure, we obtain $b = -3.78 \cdot a_{th}$ for all three elements considered. This fact hints at the same $Z$-dependence of the
coefficients $a_{th}$ and $b$, at least for heavy atoms.

In the paper [Davies et al., 1954] it was mentioned that all available experimental data on $Pb$ above 5$MeV$ are well represented by the formula $\sigma_{coh} = \sigma_B + \sigma_C + 11.8 \cdot (m/\omega)\sigma_0$. Poor knowledge of the nuclear photoabsorption cross section at that time can not explain the large difference between the coefficient 11.8 and our result $a_{th} = 25.29$. The main source of this difference is the neglecting of the term $\sigma_C^{(2)}$. It is evident from Fig. 7 that an attempt to fit $\Sigma$ in the region below 18 $MeV$ by the constant only, leads immediately to the appreciably smaller coefficient $a$ than our value $a_{th}$.

It is interesting to compare our predictions for the Coulomb corrections to the total cross section with the results of Øverbø (1977). Shown in Figs. 8,9 is the ratio $S = (\sigma_{coh} - \sigma_B)/\sigma_C^{(0)}$, which is the Coulomb corrections in units of $\sigma_C^{(0)}$, (13). Our results are represented by solid curves, those of Øverbø are shown as dashed curves. The values of $S$ extracted from the experimental data are also shown. The results for $Bi$ are plotted in Fig. 8 with the experimental data taken from (Sherman et al., 1980). The results for $Pb$ are plotted in Fig. 9 with the experimental data taken from (Rosenblum et al., 1952; Gimm and Hubbell, 1978). It is seen that the difference between our results and those of Øverbø is small at relatively low energies and becomes noticeable as $\omega$ increases. According to our results, this difference tends to a constant $\sigma_C^{(scr)}/\sigma_C^{(0)}$ at $\omega \rightarrow \infty$. As a whole, the experimental data are in a better agreement with our results than with those of Øverbø.

![Graph](image-url)  
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Fig. 8. The $\omega$-dependence of $S = (\sigma_{coh} - \sigma_B)/\sigma_C^{(0)}$ for $Bi$. Solid curve: our result; dashed curve: the result of Øverbø (1977); experimental data from (Sherman et al., 1980).
For the $e^+e^-$ photoproduction, we have calculated the leading correction (11) to the electron spectrum in the region $\varepsilon_\pm \gg m$. This contribution noticeably modifies the spectrum at intermediate photon energy. It turns out that the correction is antisymmetric with respect to the permutation $\varepsilon_+ \leftrightarrow \varepsilon_- \text{ and hence does not contribute to the total cross section.}$ The leading correction to the total cross section, $\sigma^{(1)}_C$, originates from two regions, $\varepsilon_+ \sim m$ and $\varepsilon_- \sim m$. We have obtained $\sigma^{(1)}_C$ (18) using dispersion relations. In contrast to the form of the fit suggested by Overbo (1977), the quantity $\sigma^{(1)}_C$ does not contain any powers of $\ln(\omega/m)$. We have also performed the quantitative investigation of the influence of screening on the Coulomb corrections (20),(22). It is important that $\sigma^{(scr)}_C$ does not vanish in the high-energy limit. We have suggested a form for the next-to-leading correction, $\sigma^{(2)}_C$, to the total cross section. Altogether, the corrections found allow one to represent well the available experimental data.

Starting with the results obtained for the $e^+e^-$ photoproduction spectrum, we have obtained the corresponding correction to the bremsstrahlung spectrum as well.
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