New Optical Soliton Solutions for Coupled Resonant Davey-Stewartson System with Conformable Operator

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New optical soliton solutions for coupled resonant Davey-Stewartson system with conformable operator

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Abstract. This paper investigates the novel soliton solutions of the coupled fractional system of the resonant Davey-Stewartson equations. The fractional derivatives are considered in terms of conformable sense. Accordingly, we utilize a complex traveling wave transformation to reduce the proposed system to an integer-order system of ordinary differential equations. The phase portrait and the equilibria of the obtained integer-order ordinary differential system will be studied. Using suitable mathematical assumptions, the new types of bright, singular, and dark soliton solutions are derived and established in view of the hyperbolic, trigonometric, and rational functions of the governing system. To achieve this, illustrative examples of the fractional Davey-Stewartson system are provided to demonstrate the feasibility and reliability of the procedure used in this study. The trajectory solutions of the traveling waves are shown explicitly and graphically. The effect of conformable derivatives on behavior of acquired solutions for different fractional orders is also discussed. By comparing the proposed method with the other existing methods, the results show that the execute of this method is concise, simple, and straightforward. The results are useful for obtaining and explaining some new soliton phenomena.

Keywords: Soliton phenomena, Fractional-order resonant system, Davey-Stewartson equation, Conformable operator, hyperbolic and trigonometric

1. Introduction

Since the advent of the concept of fractional calculus, fractional differential equations (FDEs) have attracted the interest of many researchers due to their importance in accurately demonstrating the dynamics of abundant real-world systems in various fields of sciences such as physics, diffusion, biology, chaos theory, chemistry, engineering, economics, commerce, and many others [1-7]. In particular, more and much attention has been paid to constructing exact and approximate solutions to Des in fractional sense. Anyhow, in contemporary literature, various analytical and numerical methods have been developed and implemented to obtain approximate, exact, traveling wave, and soliton solutions to FDEs. For instance, the variational iteration method, residual power series method, reproducing kernel Hilbert space method, multistep generalized differential transform method, homotopy perturbation method, homotopy analysis method, extended trial equation method, \((G'/G)\)-expansion method, undetermined of coefficients method, Riccati-Bernoulli Sub-ODE method, and among others [8-16]. Moreover, some of the important aspects of FDEs have been studied in many studies including the Cauchy problems and solution stability [17, 18]. On the other hand, not like the differential derivatives of integer order, there are many different fractional operators that are used in dealing with biological, physical, and engineering systems. Riesz-Caputo,
Caputo-Hadamard, Grünwald-Letnikov, Erdélyi-Kober, Sonin-Letnikov, Caputo-Fabrizio, Marchaud, Weyl, and Atangana-Baleanu, are some of these fractional operators [19-23].

Davey and Stewartson introduced the Davey-Stewartson (DS) equation in 1974, that is a notable and significant model in dynamics of fluids to characterize 3-dimensional wave packet evolution of finite depth on water within weak nonlinearity. Furthermore, the DS equation is a model for short and long-waves resonances, and other types of propagating waves. The DS system originates from multi-scale test respecting propagate of surface waves along horizontally sea surface. A coupled pair of nonlinear DS equations in two dependent variables can be reduced to the (1+1)-dimensional nonlinear Schrodinger (NLS) equation via a suitable dimensional reduction [24]. There are several studies which have been introduced to investigate the exact and approximate solutions for DS system. In particular, we have the inverse scattering transform [25], Exp-function method [26], Jacobi elliptic function method [27], direct algebraic method [28], homotopy perturbation method [29], G0/G)-expansion method [30], spectral transform [31], Darboux transformation [32], and among others.

This paper investigates new optical soliton solutions for a conformable coupled resonant Davey-Stewartson (CCRDS) system. We seek to explore two types of bright soliton solutions using two different formulas in terms of hyperbolic and rational functions. In addition, we establish singular and dark soliton solutions in terms of trigonometric and hyperbolic, respectively. We consider the CCRDS system in the form [33]:

\[
i T^\alpha_t \phi + \gamma^2 T^2_x \phi + T^2_y \phi - 2 \gamma^2 T^2_x |\phi| \phi - 2 \gamma^4 T^2_y |\phi| \phi - \phi \psi + \lambda |\phi|^2 = 0,
\]

\[
T^2_x \psi - \gamma^2 T^2_x \psi - 2 \lambda T^2_x |\phi|^2 = 0,
\]

where \(\phi \equiv \phi(x, y, t)\) represents amplitude of a surface wave packet and real quantity \(\psi \equiv \psi(x, y, t)\) may be regarded the velocity potential of the mean flow interacting with the surface wave, while the independents variables \(x, y\) and \(t\) is the dimensionless variable, propagation coordinate and time, respectively. Also, \(\gamma^2 = -1\) dictates the hyperbolic nature, and \(\gamma^2 = 1\) dictates the elliptic nature of the system (1), while \(\lambda\) is a constant coefficient. It is worthy that the system (2) at integer derivative orders, \(\alpha = 1\), reduces to the following resonant NLS equation [34]:

\[
i \phi_t + \gamma^2 \phi_{xx} - 2 \gamma^2 |\phi|_{xx} \phi - \lambda |\phi|^2 = 0.
\]

The rest of the paper is organized as Section 2 presents the definition of the conformable derivative and their properties. The analysis of the governing system via a complex transformation is formulated in Section 3. In the same section, the dynamical system and the classification of its equilibria are studied. In Section 4, the bright, singular, and dark soliton solutions are established for the CCRDS system. Some conclusions that be gained throughout the paper have been presented in Section 5.

2. Conformable fractional derivative overview

The conformable fractional derivative is a new simple definition of a derivative of fractional order analogous to the definition of the ordinary derivative introduced by Khalil et al. [35]. Given a function \(f(t): (0, +\infty) \rightarrow \mathbb{R}\). Then the conformable fractional derivative of order \(\alpha\) of \(f\) is given as

\[
T^\alpha_t f(t) = \lim_{\epsilon \to 0} \frac{f(t + \epsilon t^{1-\alpha}) - f(t)}{\epsilon},
\]
for all \( t > 0, \alpha \in (0,1] \). The function \( f \) is \( \alpha \)-conformable differentiable at a point \( t \) if the limit in (3) exists. The conformable derivative accomplishes many renowned required properties [36].

**Theorem 2.1.** [36] If the functions \( f_1 = p_1(t) \) and \( f_2 = p_2(t) \) are \( \alpha \)-conformable differentiable at any point \( t > 0, \alpha \in (0,1] \), then:

i) \( T_t^\alpha (c_1 f_1 + c_2 f_2) = a T_t^\alpha f_1 + b T_t^\alpha f_2, \forall c_1, c_2 \in \mathbb{R} \).

ii) \( T_t^\alpha (t^n) = nt^{n-\alpha}, \forall n \in \mathbb{R} \).

iii) \( T_t^\alpha (c) = 0, \) where \( c \) is any constants.

iv) \( T_t^\alpha (f_1 f_2) = f_1 T_t^\alpha f_2 + f_2 T_t^\alpha f_1 \).

v) \( T_t^\alpha \left( \frac{f_1}{f_2} \right) = \frac{T_t^\alpha f_1 - f_1 T_t^\alpha f_2}{f_2^2} \).

vi) If \( f_1 \) is differentiable, then we have \( T_t^\alpha (f_1(t)) = t^{1-\alpha} \frac{df_1}{dt} \).

The conformable differential operator obeys some important crucial properties like the chain rule.

**Theorem 2.2.** [36] Suppose \( f_1(t) \) is \( \alpha \)-conformable differentiable function and \( f_2(t) \) is differentiable and well-defined in the range of \( f_1(t) \). Then, we have

\[
T_t^\alpha \left( f_1(t) \frac{df_2}{dt}(t) \right) = t^{1-\alpha} \frac{d}{dt} \left( f_1(t) \frac{df_2}{dt}(t) \right).
\]

(4)

### 3. Complex traveling wave transformation and equilibria classifications

In this section we seek to utilize a suitable complex traveling wave transformation for the CCRDS system (2) to translate it into integer-order ordinary differential equation (ODE) system. Furthermore, we study the planner dynamical system that corresponds to the integer-order ODE, then determine its equilibrium points and classification. The governing system (2) can be reduced into integer-order differential system via the following complex traveling wave transformation:

\[
\phi(x, y, t) = \Phi(\xi(x, y, t)) e^{i \Sigma(x, y, t)}, \psi(x, y, t) = \Psi(\xi(x, y, t)),
\]

where \( \Phi(\xi) \) and \( \Psi(\xi) \) gives the structure of the wave profile with

\[
\xi(x, y, t) = i \theta \left( \frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} - \mu \frac{t^\alpha}{\alpha} \right), \Sigma(x, t) = \rho \frac{x^\alpha}{\alpha} + \delta \frac{y^\alpha}{\alpha} + \sigma \frac{t^\alpha}{\alpha},
\]

(6)

where \( \mu \) refers to the velocity of soliton, \( \rho \) and \( \delta \) are the soliton frequency, and \( \sigma \) represents the wave number of the soliton. Substitute the transformation (5) with (6) into the CCRDS system (1) together using the above properties of the conformable derivative, then simplify the result to obtain

\[
\theta^2 (y^2 + 2y^4 - 1) \frac{d^2 \Phi}{d\xi^2} + \theta(-\mu - 2\rho y^2 - 2\delta) \frac{d\Phi}{d\xi} + (\sigma - \rho^2 y^2 - \delta^2) \Phi - \Phi \Psi + \lambda \Phi^3 = 0,
\]

\[
-\theta^2 (1 - y^2) \frac{d^2 \Psi}{d\xi^2} + 4\lambda \theta^2 \frac{d^2 \Phi}{d\xi^2} \Phi + 4\lambda \theta^2 \left( \frac{d\Phi}{d\xi} \right)^2 = 0.
\]

(7)

By integrating twice the second equation in (7) with respect to \( \xi \) and considering the integration constants to be zero, we get the following relation

\[
\Psi = \frac{2\lambda}{1 - y^2} \Phi^2.
\]

(8)
Using the relation (8) in the first equation of (7), we reach to the following integer-order ODE
\[
\theta^2(y^2 + 2y^4 - 1) \frac{d^2\Phi}{d\xi^2} + \theta(\mu - 2\rho y^2 - 2\delta) \frac{d\Phi}{d\xi} - (\sigma - \rho^2y^2 - \delta^2)\Phi - \lambda \frac{1 + y^2}{1 - y^2} \Phi^3 = 0. \tag{9}
\]

In the context of the classification of equilibria, we introduce the assumption \( \Phi = \frac{d\Phi}{d\xi} \) to get the planner dynamical system that corresponds to the integer-order ODE (9) in the form
\[
\frac{d\Phi}{d\xi} = \Phi, \quad \frac{d\Phi}{d\xi} = \frac{-\theta(\mu - 2\rho y^2 - 2\delta)\Phi + (\sigma - \rho^2y^2 - \delta^2)\Phi + \lambda \frac{1 + y^2}{1 - y^2} \Phi^3}{\theta^2(y^2 + 2y^4 - 1)} \tag{10}
\]

The system (10) can be written as
\[
\frac{-\theta^2(y^2 + 2y^4 - 1)d\Phi}{\theta(\mu - 2\rho y^2 - 2\delta)\Phi - (\sigma - \rho^2y^2 - \delta^2)\Phi - \lambda \frac{1 + y^2}{1 - y^2} \Phi^3} = \frac{d\Phi}{\Phi}. \tag{11}
\]

Cross the multiplication of (11) and integrate both sides. The Hamiltonian function of the system (10) becomes
\[
\mathcal{H}(\Phi, \Phi) = 2\theta^2(y^2 + 2y^4 - 1)\Phi^2 + 4\theta(\mu - 2\rho y^2 - 2\delta)\Phi\Phi - 2(\sigma - \rho^2y^2 - \delta^2)\Phi^2 - \lambda \frac{1 + y^2}{1 - y^2} \Phi^4 = h \in \mathbb{R}. \tag{12}
\]

The investigation of the equilibrium points, \((\Phi, \Phi)\), for the dynamical system (10) can be listed as follows:

**Case I:** There are unique equilibrium point, \(E_1(0,0)\), if one of the conditions is satisfied: \(|y| = 1\), or \(|y| > 1\) and \(\sigma \geq \rho^2y^2 + \delta^2\), or \(|y| < 1\) and \(\sigma \leq \rho^2y^2 + \delta^2\).

**Case II:** There are three equilibrium points \(E_1(0,0)\) and \(E_{2,3} \left( \pm \sqrt{1 - y^2}(\delta^2 + y^2^2\rho^2 - \sigma)/(1 + y^2)\mu, 0 \right)\), if one of the conditions is satisfied: \(|y| > 1\) and \(\sigma < \rho^2y^2 + \delta^2\), or \(|y| < 1\) and \(\sigma > \rho^2y^2 + \delta^2\).

The linearized system of the dynamical system (10) has the following coefficients matrix
\[
\mathcal{C}(\Phi, \Phi) = \begin{bmatrix}
0 & 1 \\
-\delta^2 - y^2\rho^2 + \sigma + \frac{3(1 + y^2)\lambda}{-1 + y^2} \Phi^2 & -2\delta + \mu - 2y^2\rho \\
(1 + y^2 + 2y^4)\theta^2 & (1 + y^2 + 2y^4)\theta
\end{bmatrix} \tag{13}
\]

The matrix \(\mathcal{C}\) in (12) have the determinant, \(\mathcal{J}(\Phi, \Phi)\), and trace, \(\mathcal{T}(\Phi, \Phi)\), as the follows:
\[
\mathcal{J}(\Phi, \Phi) = \frac{\delta^2 + y^2\rho^2 - \sigma - \frac{3(1 + y^2)\lambda}{-1 + y^2} \Phi^2}{(1 + y^2 + 2y^4)\theta^2}, \quad \mathcal{T}(\Phi, \Phi) = -\frac{-2\delta + \mu - 2y^2\rho}{(1 + y^2 + 2y^4)\theta}. \tag{14}
\]

Accordingly, we can obtain the values of \(\mathcal{J}(\Phi, \Phi)\) at the observed equilibrium points to have
\[
\mathcal{J}(E_1) = \frac{\delta^2 + y^2\rho^2 - \sigma}{(1 + y^2 + 2y^4)\theta^2}, \quad \mathcal{J}(E_{2,3}) = -\frac{2(\delta^2 + y^2\rho^2 - \sigma)}{(1 + y^2 + 2y^4)\theta^2}. \tag{15}
\]
Using the planar dynamical system theory of a planar integrable system, the equilibrium points can be classified as: if $J < 0$, then it’s a saddle, if $\mathcal{T}^2 - 4J > 0$, $\mathcal{T} > 0$ and $J > 0$, then it’s unstable node, if $\mathcal{T}^2 - 4J > 0$, $\mathcal{T} < 0$ and $J > 0$, then it’s stable node, if $\mathcal{T}^2 - 4J < 0$, $\mathcal{T} > 0$, then it’s unstable spiral, if $\mathcal{T}^2 - 4J < 0$, $\mathcal{T} < 0$, then it’s stable spiral, if $\mathcal{T} = 0$ and $J > 0$, then it’s a center, if $J = 0$ with zero Poincar`e index, then the equilibrium point is cusp.

Figure 1 shows the phase portrait for the dynamical system (10) at selected parameters values. The color points in Figure 1 represent the equilibrium points at the selected parameters. We found that the classification of the equilibria can listed as: (a) $E_1(0,0), J[E_1] \approx 1.6 > 0, \mathcal{T}[E_1] \approx 3.04 > 0, \mathcal{T}[E_1]^2 - 4J[E_1] \approx 2.8 > 0$; unstable node, and $E_{2,3}(\pm 1.37833, 0), J[E_{2,3}] \approx -3.2 < 0$; saddle, (b) $E_1(0,0), J[E_1] \approx -0.9 < 0$; saddle, and $E_{23}(\pm 3.158, 0), J[E_{23}] \approx 1.9 > 0, \mathcal{T}[E_{2,3}] \approx -1.8 < 0, \mathcal{T}[E_{2,3}]^2 - 4J[E_{2,3}] \approx -4.7 < 0$; stable spiral, (c) $E_1(0,0), J[E_1] \approx -1.02 < 0$; saddle, and $E_{2,3}(\pm 3.146, 0), J[E_{2,3}] \approx 2.04 > 0, \mathcal{T}[E_{2,3}] = 0$; center, (d) $E_1(0,0), J[E_1] \approx 1.6 > 0, \mathcal{T}[E_1] = 0$; center, and $E_{2,3}(\pm 1.378, 0), J[E_{2,3}] \approx -3.2 < 0$; saddle points. Consequently, that clearly appears in the Figure 1.
Figure 1. The phase portraits of the system (10) at: (a) $\gamma = 0.9, \lambda = 0.1, \theta = \rho = \delta = 1, \sigma = 0.0002$ and $\mu = 0.2$; (b) $\gamma = 0.009, \lambda = 0.1, \theta = \rho = \delta = 1, \sigma = 0.002$ and $\mu = 0.2$; (c) $\gamma = 0.1, \lambda = 0.1, \theta = \rho = \delta = 1, \sigma = 0.0001$ and $\mu = 2.02$; (d) $\gamma = 0.9, \lambda = 0.1, \theta = \rho = \delta = 1, \sigma = 0.0002$ and $\mu = 3.62$.

4. Exact traveling wave solutions for CCRDS system

In this section, we look to deducing exact traveling wave solutions for the governing system (2) using direct method via appropriateness assumptions to solutions for the integer-order ODE (9). We will construct two types of bright soliton solutions with aid two various formulas in terms of hyperbolic and rational functions. Furthermore, singular, and dark soliton have will obtain using suitable formulas.

4.1 Bright-I soliton solutions

Assume that the integer-order ODE (9) has a solution in terms of hyperbolic functions as

$$
\Phi(\xi) = \frac{\Pi_1 \text{sech}(\omega \xi)}{\sqrt{1 + \Pi_2 \text{sech}^2(\omega \xi)}},
$$

where $\Pi_1, \Pi_2$ and $\omega$ are constants to be determined. Substitute the formal solution (16) into the integer-order ODE (9) and simplify the result, then group the coefficients of the similar terms in the numerator and set it are to zero, we get an algebraic system. Solve the obtained algebraic system to get:

**Case 1.**

$$
\mu = 2(\rho \gamma^2 + \delta), \omega^\pm = \pm \sqrt{-\frac{-\delta^2-\gamma^2\rho^2+2\sigma}{(-1+\gamma^2+2\gamma^4)\theta^2}}, \Pi_2^\pm = \pm \sqrt{-\frac{-2(\delta^2+\gamma^2\rho^2-\sigma)(1-\gamma^2)}{\lambda(1+\gamma^2)}}, \Pi_2 = -1.
$$

**Case 2.**

$$
\mu = 2(\rho \gamma^2 + \delta), \omega^\pm = \pm \sqrt{-\frac{-\delta^2-\gamma^2\rho^2+2\sigma}{(-1+\gamma^2+2\gamma^4)\theta^2}}, \Pi_2^\pm = \pm \sqrt{-\frac{2(\delta^2+\gamma^2\rho^2-\sigma)(1-\gamma^2)}{\lambda(1+\gamma^2)}}, \Pi_2 = 0.
$$

Using the given values in the solutions sets (17) and (18) in the formal solution (16) along the relation (9), then inserting the results into (5) with aid of (6), we get the following bright-I soliton solutions for the governing system (2) that correspond to case 1.

$$
\phi_{1,2,3,4}(x,y,t) = \pm \sqrt{-2(\delta^2+\gamma^2\rho^2-\sigma)(1-\delta^2)} \text{sech} \left( \sqrt{-\frac{-\delta^2-\gamma^2\rho^2+2\sigma}{(-1+\gamma^2+2\gamma^4)\theta^2}} \left( i \theta \left( \frac{\gamma^\alpha + \rho^\alpha - \mu^\alpha}{\alpha} \right) \right) \right) \times \exp \left( i \left( \frac{\gamma^\alpha}{\alpha} + \delta \frac{\rho^\alpha}{\alpha} + \sigma \frac{\mu^\alpha}{\alpha} \right) \right),
$$

where $\alpha$ is a constant.

$$
(19)
$$
\[ \psi_{1,2,3,4}(x,y,t) = \frac{2\lambda}{1-\gamma^2} \left( \pm \sqrt{\frac{2(\delta^2+y^2\rho^2-\sigma^2)(1-\gamma^2)}{\lambda(1+\gamma^2)}} \right) \left( \pm \sqrt{\frac{-\delta^2-y^2\rho^2+\sigma}{(1-\gamma^2+2\gamma^4)\theta^2}} \right) \left( i\theta \left( \frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} - \mu^\alpha \frac{t^\alpha}{\alpha} \right) \right)^2 \].

Consequently, the bright-I soliton solutions for the governing system (2) that correspond to case 2 are given as:

\[ \phi_{5,6,7,8}(x,y,t) = \pm \sqrt{\frac{2(\delta^2+y^2\rho^2-\sigma^2)(1-\gamma^2)}{\lambda(1+\gamma^2)}} \left( \pm \sqrt{\frac{-\delta^2-y^2\rho^2+\sigma}{(1-\gamma^2+2\gamma^4)\theta^2}} \right) \left( i\theta \left( \frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} - \mu^\alpha \frac{t^\alpha}{\alpha} \right) \right) \times \text{Exp} \left( i \left( \rho \frac{x^\alpha}{\alpha} + \delta \frac{y^\alpha}{\alpha} + \sigma \frac{t^\alpha}{\alpha} \right) \right). \]  

\[ \psi_{5,6,7,8}(x,y,t) = \frac{2\lambda}{1-\gamma^2} \left( \pm \sqrt{\frac{2(\delta^2+y^2\rho^2-\sigma^2)(1-\gamma^2)}{\lambda(1+\gamma^2)}} \right) \left( \pm \sqrt{\frac{-\delta^2-y^2\rho^2+\sigma}{(1-\gamma^2+2\gamma^4)\theta^2}} \right) \left( i\theta \left( \frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} - \mu^\alpha \frac{t^\alpha}{\alpha} \right) \right)^2. \]  

To illustrate the physical naturality for the deduced solutions in (19) and (20), we depicted the profile of surfaces for the gotten solutions. Figures 2 and 3 presents the 3D and 2D of \(|\phi_5(x,y,t)|\) and \(|\psi_5(x,y,t)|\), respectively, at \(\omega^-\) and \(\Pi^-\), where the derivatives are considered at integer and fractional orders.
Figure 2. The bright-I soliton solution at: $\gamma = 0.03, \lambda = 0.03, \theta = 0.1, \rho = 0.1, \delta = 0.02$ and $\sigma = 1$ where $x \in [-10,10]$ and $t \in [0,1]$ such that: (a) 3D plot of $|\phi_5(x,0,t)|$ at $\alpha = 1$; (b) 2D plot of $|\phi_5(x,0,0)|$ at $\alpha = 1$; (c) 3D plot of $|\phi_5(x,0,t)|$ at $\alpha = 0.7$; (d) 2D plot of $|\phi_5(x,0,0)|$ at $\alpha = 0.7$.

Figure 3. The bright-I soliton solution at: $\gamma = 0.03, \lambda = 0.03, \theta = 0.1, \rho = 0.1, \delta = 0.02$ and $\sigma = 1$ where $x \in [-10,10]$ and $t \in [0,1]$ such that: (a) 3D plot of $|\psi_5(x,0,t)|$ at $\alpha = 1$; (b) 2D plot of $|\psi_5(x,0,0)|$ at $\alpha = 1$; (c) 3D plot of $|\psi_5(x,0,t)|$ at $\alpha = 0.7$; (d) 2D plot of $|\psi_5(x,0,0)|$ at $\alpha = 0.7$. 
To more illustrate, Figure 4 shows the 2D plots of $|\phi_8(x,0,0)|$ and $|\psi_8(x,0,0)|$ at $\omega^+$ and $\Pi^+$, where the derivatives orders considered in various fractional sense, that give more understand of their effect on the behavior of bright-I solitons.

![Figure 4](image)

**Figure 4.** Effect of fractional derivatives on the behavior of bright-I soliton at: $\gamma = 0.03, \lambda = 0.02, \theta = 0.1, \rho = 0.3, \delta = 0.02$ and $\sigma = 1$ where $x \in [-20,10]$, red; $\alpha = 0.8$, blue; $\alpha = 0.6$, orange; $\alpha = 0.55$ and green; $\alpha = 0.4$ such that: (a) 2D plot of $|\phi_8(x,0,0)|$, (b) 2D plot of $|\psi_8(x,0,0)|$.

### 4.2 Bright-II soliton solutions

The bright-II soliton solution for the governing system (2) can be obtained through the following rational term assumption of the solution for the integer-order ODE (9):

$$\Phi(\zeta) = \frac{1}{\sqrt{\Pi_3 + \Pi_4 \zeta^2}}$$

(21)

where the constants $\Pi_3$ and $\Pi_4$ can be determined by inserting the assumption (21) into the integer-order ODE (9), simplifying the result, then collecting the coefficients of similar terms in the numerator and setting are to be zero. We are falling into the following cases

**Case 1.**

$$\mu = 2(\rho \gamma^2 + \delta), \Pi_3 = -\frac{1}{2}, \Pi_4 = -\frac{(-1 + \gamma^2)(\delta^2 + \gamma^2 \rho^2 - \sigma) + 2(1 + \gamma^2)\lambda}{10(-1 + \gamma^2)(-1 + \gamma^2 + 2\gamma^4)\theta^2}.$$  

(22)

The bright-II soliton solution that corresponds to these values (22) is given by

$$\phi_1(x,y,t) = \frac{1}{\sqrt{\frac{1}{2} \left( 1 - \frac{1}{10(-1 + \gamma^2)(-1 + \gamma^2 + 2\gamma^4)\theta^2} \right)}} \times \text{Exp} \left( i \left( \frac{\rho \gamma^2}{\alpha} + \delta \frac{\gamma^2}{\alpha} + \frac{\gamma^2}{\alpha} \mu \frac{\alpha}{\alpha} \right) \right).$$

(23)

$$\psi_1(x,y,t) = \frac{2\lambda}{1 - 2\gamma} \left( \frac{1}{\sqrt{\frac{1}{2} \left( 1 - \frac{1}{10(-1 + \gamma^2)(-1 + \gamma^2 + 2\gamma^4)\theta^2} \right)}} \right)^2.$$  

**Case 2.**

$$\mu = 2(\rho \gamma^2 + \delta), \sigma = \rho \gamma^2 + \delta^2, \Pi_3 \neq 2, \Pi_4 = -\frac{(1 + \Pi_3)(1 + \gamma^2)\lambda}{(-2 + \Pi_3)(1 - \gamma^2)(-1 + \gamma^2 + 2\gamma^4)\theta^2}.$$  

(24)
Accordingly, we have the following bright-II soliton solution

\[
\phi_2(x, y, t) = \frac{1}{\sqrt{\Pi}} \left( \frac{(1+\Pi)(1+y^2)\lambda}{(-2+\Pi)(1-y^2)(1+y^2+2y^\alpha)} \right)^{\alpha/2} \times \exp \left( i \left( \frac{x^\alpha}{\alpha} + \frac{\rho^\alpha}{\alpha} + \frac{\delta^\alpha}{\alpha} + \frac{\sigma^\alpha}{\alpha} \right) \right),
\]

\[
\psi_2(x, y, t) = \frac{2\lambda}{1-\gamma^2} \left( \frac{1}{\Pi} \left( \frac{(1+\Pi)(1+y^2)\lambda}{(-2+\Pi)(1-y^2)(1+y^2+2y^\alpha)} \right)^{\alpha/2} \right)^2.
\] (25)

Figure 5 and 6 present the graphical representation of the bright-II soliton solutions \( \phi_1(x, y, t) \) and \( \psi_1(x, y, t) \) in (23), respectively, at selected parameters in 3D and 2D plots where the derivatives are considered in integer and fractional orders. Moreover, the profile of the obtained bright-II soliton solutions in (25) for the CCRDS system (2) are plotted in 3D and 2D in Figures 7 and 8, respectively, at various fractional derivatives orders.

**Figure 5.** The bright-I soliton solution at: \( \gamma = 0.01, \lambda = 0.001, \theta = 0.1, \rho = 1, \delta = 0.3 \) and \( \sigma = 0.0002 \) where \( x \in [-50,50] \) and \( t \in [0,5] \) such that: (a) 3D plot of \( |\phi_5(x,0,t)| \) at \( \alpha = 1 \); (b) 2D plot of \( |\phi_5(x,0,t)| \) at \( \alpha = 1 \) and \( t \in \{0,5,10,15,20,25\} \); (c) 3D plot of \( |\phi_5(x,0,t)| \) at \( \alpha = 0.99 \); (d) 2D plot of \( |\phi_5(x,0,t)| \) at \( \alpha = 0.99 \) and \( t \in \{0,5,10,15,20,25\} \).
Figure 6. The bright-II soliton solution at: $\gamma = 0.01, \lambda = 0.001, \theta = 0.1, \rho = 1, \delta = 0.3$ and $\sigma = 0.0002$ where $x \in [-50,50]$ and $t \in [0,5]$ such that: (a) 3D plot of $|\psi_5(x,0,t)|$ at $\alpha = 1$; (b) 2D plot of $|\psi_5(x,0,t)|$ at $\alpha = 1$ and $t \in \{0,5,10,15,20,25\}$; (c) 3D plot of $|\psi_5(x,0,t)|$ at $\alpha = 0.85$; (d) 2D plot of $|\psi_5(x,0,t)|$ at $\alpha = 0.85$ and $t \in \{0,5,10,15,20,25\}$.

Figure 7. Effect of fractional derivatives on the behavior of bright-II soliton at: $\gamma = 0.1, \lambda = 0.005, \theta = 0.2, \rho = 1, \delta = 0.7$ and $\Pi_3 = 0.8$ where $x \in [-100,500]$ and $t \in [0,1]$, red; $\alpha = 0.99$, blue; $\alpha = 0.9$, green; $\alpha = 0.8$ and yellow; $\alpha = 0.62$ such that: (a) 3D plot of $|\phi_2(x,0,t)|$, (b) 2D plot of $|\phi_2(x,0,0)|$. 
Figure 8. Effect of fractional derivatives on the behavior of bright-II soliton at: \( \gamma = 0.1, \lambda = 0.005, \theta = 0.2, \rho = 1, \delta = 0.7 \) and \( \Pi_3 = 0.8 \) where \( x \in [-100,100] \) and \( t \in [0,1] \), red; \( \alpha = 0.99 \), blue; \( \alpha = 0.86 \), green; \( \alpha = 0.73 \) and yellow; \( \alpha = 0.63 \) such that: (a) 3D plot of \( |\psi_2(x,0,t)| \), (b) 2D plot of \( |\psi_2(x,0,0)| \).

4.3 Singular soliton solutions

The singular soliton solution for the CCRDS system (2) deduced upon the integer-order ODE (9) has a solution involving hyperbolic function in the form:

\[
\Phi(\xi) = \frac{\Pi_5}{\sqrt{1 + \Pi_6 \sinh(\omega \xi)}}
\]

where \( \omega, \Pi_5 \) and \( \Pi_6 \) are constants to be determined. Applying the same above approach leads to the following solution set

\[
\mu = 2(\rho \gamma^2 + \delta), \omega^\pm = \pm 2 \sqrt{-\frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)}{(-1 + \gamma^2 + 2 \gamma^4)}}, \Pi_5^\pm = \pm 2 \sqrt{-\frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(\gamma^2 - 1)}{(-1 + \gamma^2 + 2 \gamma^4)}}, \Pi_6^\pm = \pm i.
\]

Therefore, we gained the singular soliton solutions for the governing system (2) as

\[
\begin{align*}
\phi_{1,\ldots,8}(x,y,t) &= \pm 2 \sqrt{-\frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(\gamma^2 - 1)}{(-1 + \gamma^2 + 2 \gamma^4)}}, \\
&\times \text{Exp} \left( i \left( \rho^\alpha \frac{x^\alpha}{\alpha} + \delta \frac{y^\alpha}{\alpha} + \sigma \frac{t^\alpha}{\alpha} \right) \right), \\
\psi_{1,\ldots,8}(x,y,t) &= \frac{2 \lambda}{1 - \gamma^2} \left( \pm 2 \sqrt{-\frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(\gamma^2 - 1)}{(-1 + \gamma^2 + 2 \gamma^4)}}, \right) \times \text{Exp} \left( i \left( \rho^\alpha \frac{x^\alpha}{\alpha} + \delta \frac{y^\alpha}{\alpha} + \sigma \frac{t^\alpha}{\alpha} \right) \right)^2.
\end{align*}
\]

We exhibited the constructed singular soliton solutions (28), graphically, to show their physical characteristics at integer and fractional derivatives orders by utilizing suitable parameters in 3D and 2D plots. For example, the singular soliton solution \( \phi_6(x,y,t) \) and \( \psi_6(x,y,t) \), that obtained by considered \( \omega^+, \Pi_5^+ \) and \( \Pi_6^+ \), depicted in Figure 9.
Figure 9. The singular soliton solution at: $\gamma = 0.9, \lambda = 1, \theta = 1, \rho = 0.1, \delta = 0.1$ and $\sigma = 0.1$ where $x \in [2,5]$ and $t \in [0,5]$ such that: (a) 3D plot of $|\phi_6(x, 0, t)|$ at $\alpha = 1$; (b) 3D plot of $|\phi_6(x, 0, t)|$ at $\alpha = 0.75$; (c) 3D plot of $|\psi_6(x, 0, t)|$ at $\alpha = 1$; (d) 3D plot of $|\psi_6(x, 0, t)|$ at $\alpha = 0.75$.

In the following, Figure 10 presents the profile, in 2D, of the singular soliton solution $\phi_8(x, y, t)$ and $\psi_8(x, y, t)$, that obtained by considered $\omega^-, \Pi_5^-$ and $\Pi_6^-$, at fractional orders of derivatives.

Figure 10. Effect of fractional derivatives on the behavior of singular soliton at: $\gamma = 0.8, \lambda = 0.1, \theta = 2, \rho = 0.4, \delta = 0.2$ and $\sigma = 0.2$ where $x \in [0,3.5]$; red; $\alpha = 1$, blue; $\alpha = 0.8$, green; $\alpha = 0.75$ and orange; $\alpha = 0.65$ such that: (a) 2D plot of $|\phi_8(x, 0, 0)|$, (b) 2D plot of $|\psi_8(x, 0, 0)|$.

4.4 Dark soliton solutions

This soliton kind observed using the following hyperbolic function hypothesis of the solution of integer-order ODE (9):
\[ \Phi(\xi) = \Pi_7 \tanh^x(\omega \xi), \]  
(29)

where \( \omega, \chi \) and \( \Pi_7 \) are constants to be determined. Using the assumption (29) into the integer-order ODE (9) gives

\[
\Pi_7 \theta^2 \chi \omega^2 (-1 + \gamma^2 + 2\gamma^4)(\chi - 1) \tanh(\xi \omega)^{-2+\chi} 
+ \Pi_7 \left( (\delta^2 + \gamma^2 \rho^2 - \sigma) - 2(-1 + \gamma^2 + 2\gamma^4)\theta^2 \chi^2 \omega^2 \tanh(\xi \omega)^x \right) 
+ \Pi_7 \theta^2 \chi \omega^2 (-1 + \gamma^2 + 2\gamma^4)(\chi + 1) \tanh(\xi \omega)^2 + \chi 
+ \Pi_7 \theta \chi \omega (-2\delta + \mu - 2\gamma^2 \rho) \tanh(\xi \omega)^{-1+\chi} 
- \Pi_7 \theta \chi \omega (-2\delta + \mu - 2\gamma^2 \rho) \tanh(\xi \omega)^1 + \chi 
- \frac{(1 + \gamma^2)\lambda}{1 - \gamma^2} \Pi_7^2 \tanh(\xi \omega)^3 + \chi 
= 0. 
\]  
(30)

By applying the balancing principle on the expression (30), we get the exponent \( \chi \) that falls to be 1. Substitute \( \chi = 1 \) in (30), then group the coefficients of the same linearly independent terms and set it to be zero, we have the following solutions for \( \omega \) and \( \Pi_7 \)

\[
\mu = 2(\rho \gamma^2 + \delta), \omega^{\pm} = \pm \sqrt{ \frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(1 - \gamma^2)}{2(-1 + \gamma^2 + 2\gamma^4)\theta^2} }, \Pi_7^{\pm} = \pm \sqrt{ \frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(1 - \gamma^2)}{2(-1 + \gamma^2 + 2\gamma^4)\theta^2} }. 
\]  
(31)

Insert these gained values into (29) along with (5) to get the dark soliton solutions for the CCRDS system in the form

\[
\phi_{1,2,3,4}(x, y, t) = \pm \sqrt{ \frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(1 - \gamma^2)}{2(-1 + \gamma^2 + 2\gamma^4)\theta^2} } \tanh \left( \pm \sqrt{ \frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(1 - \gamma^2)}{2(-1 + \gamma^2 + 2\gamma^4)\theta^2} } \left( i \theta \left( \frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} \right) - \mu t^\frac{\alpha}{\alpha} \right) \right) \times \text{Exp} \left( i \left( \rho \frac{x^\alpha}{\alpha} + \delta \frac{y^\alpha}{\alpha} + \sigma \frac{t^\alpha}{\alpha} \right) \right), 
\]

\[
\psi_{1,2,3,4}(x, y, t) = \frac{2\lambda}{1 - \gamma^2} \left( \pm \sqrt{ \frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(1 - \gamma^2)}{2(-1 + \gamma^2 + 2\gamma^4)\theta^2} } \tanh \left( \pm \sqrt{ \frac{(\delta^2 + \gamma^2 \rho^2 - \sigma)(1 - \gamma^2)}{2(-1 + \gamma^2 + 2\gamma^4)\theta^2} } \left( i \theta \left( \frac{x^\alpha}{\alpha} + \frac{y^\alpha}{\alpha} \right) - \mu t^\frac{\alpha}{\alpha} \right) \right) \right)^2. 
\]

(32)

Interpretation of physical behavior for the obtained dark soliton solutions are done via graphical representation for \( \phi_1(x, y, t) \) and \( \psi_1(x, y, t) \) at \( \omega^+ \) and \( \Pi_7^+ \). Figure 11 shows the profile of \( |\phi_1(x, 0, t)|^2 \) and \( |\psi_1(x, 0, t)|^2 \) at suitable parameters where the orders of derivatives are considered in integer and fractional orders.
Figure 11. The dark soliton solution at: $\gamma = 0.0009, \lambda = 1, \theta = 1, \rho = 0.001, \delta = 0.7$ and $\sigma = 0.1$ where $x \in [-10,10]$ and $t \in [0,1]$ such that: (a) 3D plot of $|\phi_1(x,0,t)|^2$ at $\alpha = 1$; (b) 3D plot of $|\phi_1(x,0,t)|^2$ at $\alpha = 0.75$; (c) 3D plot of $|\psi_1(x,0,t)|^2$ at $\alpha = 1$; (d) 3D plot of $|\psi_1(x,0,t)|^2$ at $\alpha = 0.75$.

Furthermore, we plotted the surface of $|\phi_2(x,0,t)|^2$ and $|\psi_2(x,0,t)|^2$ in 3D and 2D at different fractional derivatives orders and the reason due to understand the impress of the fractional derivatives on the physical naturality of the obtained dark soliton solution.

Figure 12. Effect of fractional derivatives on the behavior of dark soliton at: $\gamma = 0.001, \lambda = 0.2, \theta = 0.2, \rho = 0.001, \delta = 0.5$ and $\omega = 0.01$ where $x \in [-20,10]$ and $t \in [0,1]$, red; $\alpha = 0.99$, blue; $\alpha = 0.8$, green; $\alpha = 0.73$ and yellow; $\alpha = 0.7$ such that: (a) 3D plot of $|\phi_2(x,0,t)|$, (b) 2D plot of $|\phi_2(x,0)|$. (c) 3D plot of $|\psi_2(x,0,t)|$, (d) 2D plot of $|\psi_2(x,0)|$. 

5. Conclusion
This article investigated the soliton solutions of the coupled fractional resonant Davey-Stewartson system. The fractional derivative has been described by means of conformable derivative concept. The governing system is translated to integer-order ODE using complex transformation. The phase portrait and equilibria has been profitably studied for the fractional RDS system. Three types of soliton solutions have constructed, which are bright, singular, and dark solitons in terms of hyperbolic, trigonometric, and rational functions. The dynamic features of different types of traveling waves are analyzed in detail through numerical simulation. Meanwhile, the profiles of the surface for the deduced solutions have been depicted in 2D and 3D for several fractional derivatives orders. From the acquired results, it can be concluded that the procedures followed in this analysis can be implemented in a simple and straightforward manner to create new exact solutions of many other partial fractional differential equations in terms of conformable operator.

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