Nanosized superconducting constrictions.

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(August 19, 2019)

Nanowires of lead between macroscopic electrodes are pro-
duced by means of an STM. Magnetic fields may destroy the
superconductivity in the electrodes, while the wire remains
in the superconducting state. The properties of the resulting
microscopic Josephson junctions are investigated.

PACS number(s): 74.50.+r, 74.80.-g, 74.80.Fp

It is well known that the magnetic field supresses su-
perconductivity in type I materials only if the dimensions
of the sample are sufficiently large, compared to the co-
herence length [1,2]. Superconductivity survives in small
systems. A number of experiments showed the enhance-
ment of the critical field in thin films [3]. Advances in
nanotechnology have made possible to study this effect in
small metallic particles [4,5]. A different realization can
be achieved by applying a magnetic field above the bulk
critical value to microscopic constrictions [6,7]. Then,
it can happen that the constriction itself remains super-
conducting, while the electrodes become normal [8,9].
This novel device allows us to probe superconductivity at
small scales through transport measurements. Properties
like the critical current, and its dependence on field and
thickness, can be studied in detail. If the superconduct-
ing properties are not homogeneous along the constric-
tion, it can behave as a Josephson junction of nanoscopic
dimensions.

Narrow constrictions are generated by pressing an
STM tip made of a metal which remains normal at low
temperatures (Pt-Ir or Au) into a Pb substrate. The sub-
strate has an area of approximately 1 cm², and a thick-
ness of 0.5 mm. When the tip is first pressed into the
substrate, it gets covered by lead atoms. Upon succes-
sive raising and lowering of the tip, a lead bridge is formed
between the tip and the substrate. The aspect ratio of
this bridge can be varied by changing the position of
the tip in a controlled way and its size can be estimated
from the evolution of the conductance during this pro-
cess as detailed in ref. [11]. The advantage of working
with normal tips is that the magnetic field needs only
to be applied to the substrate and the constriction. A
magnetic field of 2.6 kG (approximately five times the
zero temperature critical field of lead) at the surface of
the sample was produced by a small permanent magnet
placed under the sample.

FIG. 1. Top panel: I-V characteristics of Pb constrictions
with and without an applied field. The inset shows a sketch
of the expected situation at the constriction. The applied
field is five times the bulk critical field of lead. Middle panel:
conductances for the same constrictions as at the top part.
Bottom panel: estimated dimensions of a typical constriction
following the procedure of ref. [11].
narrower than the penetration depth, the cylinder. This situation describes well cylinders much
 broader than the penetration depth, which also supports a conductance with and without a magnetic field. In the absence of an applied field, both quantities are propor-
tional to the current carried by the condensate is proportional to the current flowing along the cylinder. The critical
transition to the normal state is discontinuous, as in
at zero voltage, the Josephson effect shunts the con-
striction, leading to the observed peak in the conduc-
tivity. The residual resistivity is due to scattering in the
normal parts, outside the constriction. At finite voltages,
the number of channels, in turn, goes as the cross section over an area of atomic dimensions. Hence, the high voltage conductance gives a measurement of the cross section of the constriction.

At zero voltage, the Josephson effect shunts the con-
striction, leading to the observed peak in the conduc-
tivity. The residual resistivity is due to scattering in the
normal parts, outside the constriction. At finite voltages,
but below the gap of the superconducting region, reso-
ance processes due to Andreev scattering are observed.

Figure 2 shows the critical current versus normal state con-
ductance with and without a magnetic field. In the
absence of an applied field, both quantities are propor-
tional. This can be understood because the critical cur-
rent of a constriction measures the number of conducting channels within it.

A magnetic field above the bulk critical value reduces the
the critical current. The effect is more pronounced in the
wide constrictions. In order to analyze this effect, we calculate the free energy of a cylindrical supercon-
ductor in the presence of a field, which also supports a
current. Let us assume that the field is constant within
the cylinder. This situation describes well cylinders much
 narrower than the penetration depth, \( r \ll \lambda(T) \). The
current carried by the condensate is proportional to the
gradient of the phase of the order parameter, which we
write as \(|\psi| e^{i\phi}\). Then, the free energy, per unit length is:

\[
g = \pi r^2 \left[ \left( \alpha + \frac{\hbar^2}{2m} |\nabla \phi|^2 \right) |\psi|^2 + \frac{\beta}{2} |\psi|^4 + \frac{e^2 r^2}{8 mc^2} H^2 |\psi|^2 \right]
\]

(1)

where \( \alpha, \beta \) and \( m \) follow the standard notation \( \alpha, \beta \).

Minimizing with respect to \(|\psi|^2\), we obtain:

\[
|\psi|^2 = |\psi_0|^2 \left( 1 - \frac{\hbar^2 |\nabla \phi|^2}{2m|\alpha|} - \frac{e^2 h^2}{16} \right)
\]

(2)

where \( \psi_0 = \frac{\alpha}{\beta} \). In addition, \( \alpha = \frac{2e^2}{mc^2} H^2(T) \lambda^2(T) \).

Hence,

\[
|\psi|^2 = |\psi_0|^2 \left( 1 - \frac{\hbar^2 |\nabla \phi|^2}{2m|\alpha|} - \frac{e^2 h^2}{16} \right)
\]

(3)

where \( \epsilon = \frac{\alpha}{\beta} \) and \( h = \frac{H}{H_c(T)} \). Finally, we relate \( \nabla \phi \) to the current flowing along the cylinder. The critical
current, in the absence of a field, of a cylinder of radius

\[
I_c = \frac{B_c(T) r^2}{4 \pi \epsilon_0 \lambda_0(T)}.
\]

In terms of \( f = \frac{\psi}{\psi_0} \), we can write:

\[
f^2 = 1 - \frac{\hbar^2 c^2}{16} - \frac{4 i^2}{27 f^2} \]

(4)

where \( i = \frac{e}{mc} \).

Equation (4) allows us to determine the critical field of a cylinder with no current flowing:

\[
H_{cyl}(T) = 4H_c(T) \lambda(T).\]

This formula is valid for \( r < \lambda(T) \). For sufficiently large values of \( r \), solutions with vortices threading the
cylinder are also possible [12,13].

In the presence of a current, we obtain the critical field by first extracting \( h(f, i) \) from (4) and then calculating the
total current flowing along the cylinder. The critical
current, in the absence of a field, of a cylinder of radius

\[
I_c(T) = \frac{H_c(T) r^2}{4 \pi \epsilon_0 \lambda_0(T)}.
\]

The transition to the normal state is discontinuous, as in
a thin film [1]. When the current reaches its critical value, the
superconducting order parameter in the constriction
jumps to zero.

As a function of the radius of the constriction, eq. (4) predicts that the current density in a cylindrical superconductor in the presence of a field, which also supports a
current. Let us assume that the field is constant within
the cylinder. This situation describes well cylinders much
 narrower than the penetration depth, \( r \ll \lambda(T) \). The
current carried by the condensate is proportional to the
gradient of the phase of the order parameter, which we
write as \(|\psi| e^{i\phi}\). Then, the free energy, per unit length is:
can coexist. As the symmetry of the order parameter is different in each region, a phase boundary should be generated, and the critical current will be suppressed.

The general trend of $I_c$ as function of $r$ is consistent with the results shown in fig. 2a. In the presence of a field, we find that $I_c \propto r^2$ for narrow constrictions, $r \ll \lambda(T)$. For wider constrictions, $I_c$ is strongly supressed by a field. Finally, we have studied the dependence of $I_c$ on temperature. The theory presented here predicts that the critical temperature, in the presence of an applied field, in particular, the dependence of critical currents on width.

In conclusion, we present new devices in which to study superconductivity at small scales. In contrast to previous work, our superconducting regions are strongly coupled to its environment, which greatly facilitates the measurement of transport properties. The results presented here are in agreement with the expected behavior for a narrow superconducting cylinder in an applied field, in particular, the dependence of critical currents on width. This work was done with support from CICYT (Spain) through grant PB96-0875.

\[
I_c = \frac{H_c(0)r^2e}{3\sqrt{6}\lambda(0)} \left[ 1 - \left( \frac{r}{r_0} \right)^2 \right]^{1/2} \times \left[ 1 - \frac{H^2r^2e^2}{16H_c^2(0)\lambda^2(0)} \left( \frac{r}{r_0} \right)^2 \left( 1 - \left( \frac{r}{r_0} \right)^2 \right) \right]^{1/2}
\]

This expression predicts that $I_c$ has a linear dependence on $r^2$ at low temperatures, and curves downward as the temperature is increased, in agreement with fig. 3b.

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